Abstract—We consider the problem of resource allocation in a wireless cellular network, in which nodes have both open and private information to be transmitted to the base station over block fading uplink channels. We develop a cross-layer solution, based on hybrid ARQ transmission with incremental redundancy. We provide a scheme that combines power control, flow control, and scheduling in order to maximize a global utility function, subject to the stability of the data queues, an average power constraint, and a constraint on the privacy outage probability. Our scheme is based on the assumption that each node has an estimate of its uplink channel gain at each block, while only the distribution of the cross channel gains is available. We prove that our scheme achieves a utility, arbitrarily close to the maximum achievable utility given the available channel state information.

I. INTRODUCTION

Recently, information theoretic security has gained significant attention, provisioning an ultimate goal of guaranteed security against adversaries with unlimited computational resources. The foundations of physical layer secrecy have been initially developed in [1], [2] and different variants of the problem -mainly for the wireless channel- have been revisited vastly. For example, in [3] channel fading has been exploited for secrecy, and more recently, multiple antennas [4] and cooperative relays [5] have been utilized to increase the achievable secrecy rates. Despite the significant volume of work in information theoretic secrecy, most of work has focused on physical layer techniques and on a single link. The area of wireless theoretic secrecy remains in its infancy, especially as it relates to the design of wireless networks and its impact on network control and protocol development.

To that end, we investigated [6] the cross-layer resource allocation problem with information theoretic security. There, we considered a system in which nodes collect both open and private information, store them in separate queues and transmit them to the base station over block fading uplink channels. We first introduced the concept of private opportunistic scheduling and showed that it achieves the maximum sum private rate achievable in uplink wireless systems. We subsequently developed a joint flow control and scheduling scheme and showed that it achieves a performance, close to the optimal. In [6], we assumed a constant power transmission and that information is encoded over individual packets. In this paper, we extend our results to the scenario with hybrid ARQ (HARQ) transmission based on incremental redundancy (INR), which basically relies on mutual information accumulation at each retransmission. Furthermore, we include the possibility that nodes transmit at varying power levels, subject to an average power constraint. We assume that the transmitter has an estimate of its uplink channel and only the distribution of the cross channels to the every other node. We develop a dynamic cross-layer control scheme which maximizes aggregate utility subject to power and privacy constraints. We prove the optimality of our scheme by Lyapunov optimization theory. Finally, we numerically characterize the performance of the dynamic control algorithm with respect to several network parameters.

The HARQ transmission scheme we use is similar to the one employed in [10], which considers a block fading wire-tap channel with a single source-destination pair and an (external) eavesdropper and develops sequences of Wyner codes to be transmitted in subsequent transmissions of a block of information. The main challenge of incorporating information encoding across many blocks into our solution was that, it is not possible to dynamically update the resource allocation, based on the amount of information leakage to the other nodes at each retransmission, since the amount of leakage is unknown to the transmitting node. Furthermore, the privacy outage probability of subsequent retransmissions of a given block cannot be decoupled from each other, eliminating the possibility of using standard Lyapunov techniques. We resolve that issue by utilizing the Markov inequality so that the desired decoupling occurs at the expense of some loss in performance. We believe our new technique contributes to the field of network control [7], [8], since it enables the use of Lyapunov techniques in the analysis of the schemes such as HARQ, which is based on encoding information over many blocks.

The rest of the paper is organized as follows. Section II describes the system model and we provide a brief summary of INR HARQ. Section III gives the problem formulation. In Section IV we give our joint flow and scheduling algorithm. Lastly, Section V contains the numerical results of the effect of system parameters on the performance of the algorithm. Section VI concludes this work by summarizing the contributions.

II. SYSTEM MODEL

Network Model: We consider a multiuser uplink network as illustrated in Fig. 1. The system consists of $n$ nodes and a base station. The traffic injected by each of these nodes, consists of both open and private packets. Nodes wish to transmit those packets to the base station via the uplink channel, which we will refer to as the main channel. All private messages of each node need to be transmitted to the base station, privately from the other nodes. They overhear transmissions from the
transmitting node over the cross channels. Hence, nodes will treat each other as “internal eavesdroppers” when transmitting private information.

We assume the time to be slotted. Over each block (of time), the amount of open information, $A_i^o(k)$, and private information, $A_i^p(k)$ injected in the queues at node $j$ are both chosen by node $j$ at the beginning of each block. Private and open packets have a fixed size $R_j^o$ and $R_j^p$ respectively. Open and private information are stored in separate queues with sizes $Q_j^o(k)$ and $Q_j^p(k)$ respectively. At any given block, only one node transmits either open or private information (but not both) and a scheduler decides on which node will transmit.

In addition, the scheduler decides the transmission powers for open and private transmission, which are denoted by $P_j^o(k)$ and $P_j^p(k)$ for user $j$’s private and open transmissions, respectively. We use indicator variables, $\mathcal{I}_j^p(k)$ and $\mathcal{I}_j^o(k)$, which take on a value 0 or 1, depending on whether or not open or private information is transmitted by node $j$ over block $k$.

Channel Model: We assume the block length to be identical to $N$ channel uses. Both the main and the cross channels experience independent identically distributed (i.i.d) block fading, in which the channel gain is constant over a block and it is varying independently from block to block. We denote the instantaneous achievable rate for the main channel of node $j$ and the cross channel between nodes $j$ and $i$ by $R_j^o(k)$ and $R_{ji}^o(k)$ respectively. Even though our results are general for all channel state distributions, in numerical evaluations, we assume all channels to be Rayleigh fading. Let $h_j^o(k)$ and $h_{ji}^o(k)$ be power gains of the main channel for node $j$ and the cross channel between node $j$ and node $i$, respectively. We normalize the power gains such that the additive Gaussian noise has unit variance. Then, the instantaneous achievable rate are:

$$R_j^o(k) = \log (1 + P_j^o(k) h_j^o(k))$$

$$R_{ji}^o(k) = \log (1 + P_j^o(k) h_{ji}^o(k)).$$

Similarly, the instantaneous achievable rate for the uplink channel of node $j$ for open messages, $R_j^o(k)$ is:

$$R_j^o(k) = \log (1 + P_j^o(k) h_j^o(k)).$$

We assume that the transmitter has access to a noisy estimate of its main channel gain and merely the distribution of its cross-channel gains. After each transmission, the base station informs the transmitting node about the amount of mutual information accumulated over that block, i.e., $R_j^o(k)$ or $R_{ji}^o(k)$.

Coding: We assume that a fixed INR HARQ scheme is employed at each node. We first explain the details of the version of the scheme for private packets: Node $j$ collects each packet of $\hat{R}_j^o$ bit and encodes it into a codeword $x_j^M$ called the mother code, which is then divided into individual groups symbols, $[x_1^N, x_2^N, \ldots, x_M^N]$, of length $N$ channel uses. The mother code is encoded by using Wyner code of $C(\hat{R}_j^o/M, \hat{R}_j^o/M, MN)$. After the partitioning is realized, the first transmission of the packet forms a codeword of Wyner code $C(\hat{R}_j^o, \hat{R}_j^o, N)$. At the possible $m^\text{th}$ transmission, the combined codewords, $[x_1^N, x_2^N, \ldots, x_M^N]$ form a codeword of length $mN$ as $C(\hat{R}_j^o/m, \hat{R}_j^o/m, mN)$. The maximum number of retransmissions is $M$ and we assume that $M$ is sufficiently large to keep the probability of decoding failure due to exceeding the maximum number of retransmissions approximately identical to zero. At each retransmission, base station combines the codeword of length $N$ with the previously transmitted codewords of the same packet. For a packet with content $W_j$, let the vector of symbols received by node $i \neq j$ be $Y_i = [y_1^N, \ldots, y_m^N]$ at the end of $m^\text{th}$ retransmission of the packet by node $j$. To achieve perfect privacy, following constraint must satisfied by node $j$, for all $i \neq j$.

$$\frac{1}{mN} I(W_j; Y_i) \leq \epsilon,$$  \hspace{1cm} (4)

for all $\epsilon > 0$. Note that the amount, $\hat{R}_j^o$, of encoded private information and the amount, $\hat{R}_j^o$, of bits that encapsulate the private information is fixed and do not change from one packet to another. For INR, the mutual information accumulation for the private packet in the main channel and eavesdropper channels over block $n$ can be found (as detailed in [10]) as:

$$D_j^o(n) = \sum_{k=n_j^i+1}^{n} \log \left(1 + P_j^o(k) h_j^o(k) \mathcal{I}_j^p(k) \right),$$

$$D_{ji}^o(n) = \sum_{k=n_j^i+1}^{n} \log \left(1 + P_j^o(k) h_{ji}^o(k) \mathcal{I}_j^p(k) \right),$$

respectively, where $n_j^i$ is the block index at which $(l-1)^{\text{th}}$ private packet is successfully decoded by the base station. Note that, if $\hat{R}_j < D_j^o(n)$ at block $n$, we say that the successful decoding of the private packet took place.

If the accumulated information at one of eavesdroppers exceeds $\hat{R}_j - \hat{R}_j^p$, perfect privacy constraint (4) is violated and we say that the privacy outage occurs. Then, the privacy outage probability over block $n$ for the private packet is calculated as:

$$P_j^p(n) = \mathbb{P} \left( \hat{R}_j - \hat{R}_j^p < \max_{i \neq j} D_j^o(n) \right).$$ \hspace{1cm} (7)

For the case of open transmission, at the transmitter, the information and CRC bits are encoded by a mother code [9]. In each transmission, only the systematic part of the codeword and a selected number of parity bits are transmitted. Decoding is attempted at the receiver side by combining all previously transmitted codes. This procedure is again called INR HARQ, and mutual information accumulated for the private packet in the main channel of user $j$ over block $n$ is:

$$D_j^{\alpha, o}(n) = \sum_{k=n_j^i+1}^{n} \log \left(1 + P_j^o(k) h_j^o(k) \mathcal{I}_j^{\alpha, o}(k) \right),$$

\hspace{1cm} (8)

\footnote{Note that, if $\mathcal{I}_j^{\alpha, o}(k) = 1$ and the size of the private queue is smaller than $\hat{R}_j^o$, then the transmitter uses dummy bits to complete it to $\hat{R}_j^o$.}
where $n_j^{l-1}$ corresponds to the block index, where $(l-1)^{th}$ open message is successfully decoded by the base station. Here, we assume that fixed length packets are encoded with a rate of $R_j^l$, arrive to the open queue at node $j$. If the accumulated information is larger than the fixed rate, i.e., $R_j^l < D_j^{o}(n)$, the decoding of the open message is successful.

### III. Problem Formulation

In this section, we formulate the problem as a network utility maximization (NUM) problem. Our objective is to choose the admission rate and transmission power in order to achieve a long term private and open rates close to the ergodic.

Let $U_j^p(x)$ and $U_j^o(x)$ be the utilities obtained by node $j$ from the transmission of $x$ private and open bits, respectively. We assume that $U_j^p(\cdot)$ and $U_j^o(\cdot)$ are non-decreasing concave functions and the utility of a private information is higher than the utility of open transmission at the same rate, i.e., $U_j^p(x) > U_j^o(x)$. In addition, it is assumed that the arrival processes are ergodic.

To state the problem clearly, we define the expected service and the expected arrival rates of the private and open queues at each node as follows. First, the amount of private information transmitted from node $j$ in block $k$ is $R_j^p(k) \triangleq \frac{\tilde{R}_j^p}{R_j^o} R_j(k)$, since $\frac{\tilde{R}_j^p}{R_j^o}$ is the fraction of the private information encapsulated within $R_j(k)$ bits of transmitted data. Let $\mu_j^p$ and $\mu_j^o$ denote the expected service rates of private and open traffic queues, respectively, i.e., $\mu_j^p = \mathbb{E} \left[ R_j^p(k) \frac{R_j^o}{R_j(k)} \right]$ and $\mu_j^o = \mathbb{E} \left[ R_j^o(k) R_j^o(k) \right]$. Note that, the “effective” expected rate of private information received at the base station without a privacy outage is $\mu_j^{p,e}$. Hence, node $j$ effectively obtains an utility of $U_j^p(\mu_j^{p,e})$ from its private transmissions. We assume that the utility gained by a packet suffering a privacy outage reduces from that of a private packet to that of an open packet. Thus, node $j$ obtains an utility of $U_j^o(x_j^p - \mu_j^{p,e} + x_j^o)$ from all transmitted open messages as well as the messages that have been encoded privately, but have undergone a privacy outage. Finally, let the expected arrival rate to the private queue of node $j$ be $x_j^p \triangleq \mathbb{E} \left[ A_j^p(k) \right]$ and the expected arrival rate to the open queue of node $j$ be $x_j^o \triangleq \mathbb{E} \left[ A_j^o(k) \right]$. We consider the following optimization problem:

$$\max \sum_{j=1}^n \left( U_j^p(\mu_j^{p,e}) + U_j^o(x_j^p - \mu_j^{p,e} + x_j^o) \right)$$

subject to

$$x_j^p \leq \mu_j^p, \forall j$$
$$x_j^o \leq \mu_j^o, \forall j$$
$$\frac{\mu_j^{p,e}}{\mu_j^o} \geq 1 - \gamma_j$$
$$\mathbb{E} \left[ P_j(k) + P_j^o(k) \right] \leq \alpha_j, \forall j.$$

Note that $x_j^{o,e}$ in (15) can be interpreted as the long term average arrival rate for packets which do not incur privacy outage. Thus, the portion of packets, kept private from eavesdroppers should be greater than $1 - \gamma_j$. Also note that since objective function is an increasing function of $x_j^{o,e}$ (15) is satisfied with equality at the optimal point.

### IV. Dynamic Control

In this section, we present an opportunistic scheduling algorithm maximizing the total expected utility of network while satisfying the constraints (16)-(17). In the following, we assume that there is an infinite backlog of data at the transport layer of each node providing both private and open messages. The private messages are encoded by Wyner code at a fixed rate $(\hat{R}_j, \hat{R}_j^o)$. However, the challenge here is that the privacy outage probability in (9) depends on the past transmissions and the scheduling decision may affect future transmissions, i.e., the events that successful decoding occurs by an eavesdropper over subsequent retransmissions are non-iid. Then, utilizing standard Lyapunov optimization techniques [12] to solve our problem is not possible. To address this issue, we need to quantify the privacy outage probability over each block independently. For that purpose, we make use of Markov’s inequality:

$$\mathbb{P} \left( \hat{R}_j - \hat{R}_j^o < \max_{j \neq j} D_j^o(n) \right) \leq \frac{1}{\hat{R}_j - \hat{R}_j^o} \left( 1 - \prod_{j \neq j} (1 - \mathbb{E} \left[ D_j^o(n) \right]) \right)$$

$$= \frac{1}{\hat{R}_j - \hat{R}_j^o} \sum_{j \neq j} \mathbb{E} \left[ D_j^o(n) \right] \prod_{j \neq j} (1 - \mathbb{E} \left[ D_j^o(n) \right])$$

$$\leq \frac{1}{\hat{R}_j - \hat{R}_j^o} \sum_{j \neq j} \mathbb{E} \left[ D_j^o(n) \right] = \frac{1}{\hat{R}_j - \hat{R}_j^o} \sum_{j \neq j} \sum_{k \neq j} \mathbb{E} \left[ \mathcal{E}(k) R_j(k) \right].$$

where (18) follows from Markov inequality, and (19) is due to the fact that we choose the pair $(\hat{R}_j, \hat{R}_j^o)$ such that $\mathbb{E} \left[ D_j^o(n) \right] < \hat{R}_j - \hat{R}_j^o$ for all $j$. Recall that $D_j^o(n)$ is the accumulated information at the eavesdropper $i$. According to the Markov inequality, the fraction of private packets that suffer a privacy outage is thus upper bounded by $\frac{1}{\hat{R}_j - \hat{R}_j^o} \sum_{j \neq j} \mathbb{E} \left[ R_j(k) \right]$. 

for any block $k$. Hence, Markov inequality enables us to quantify the amount of information leakage to the other nodes independently over each block $k$, being $\frac{R_p}{R_q}\sum_{j,\in}\mathbb{E}[R_{ji}(k)]$. However, since the Markov inequality is merely a bound, the constraint set over which we solve the problem shrinks. Hence some performance is sacrificed. In the simulations, we numerically analyze the amount of shrinkage in the constraint set due to the use of the Markov inequality and show that it is not significant under most scenarios.

The dynamics of private and open traffic queues, $Q_j^p(k)$ and $Q_j^o(k)$, respectively, are given as follows:

$$Q_j^p(k+1) = [Q_j^p(k) - \mathcal{F}_j^{p}(k)] + A_j^p(k),$$

and

$$Q_j^o(k+1) = [Q_j^o(k) - \mathcal{F}_j^{o}(k)] + A_j^o(k),$$

where $x^+ = \max(0,x)$.

As shown in [12], each of the constraints $[13]-[17]$ can be represented by a virtual queue, and when these virtual queues are stabilized the constraints are also satisfied.

$$Q_j^{p,e}(k+1) = [Q_j^{p,e}(k) - \mathcal{F}_j^{p,e}(k)] + A_j^{p,e}(k),$$

$$Z_j(k+1) = \left( Z_j(k) - A_j^{p,e}(k) + A_j^o(k)(1 - \gamma_j) \right)^+,\quad (24)$$

$$Y_j(k+1) = \left[ Y_j(k) + \mathcal{F}_j^{p}(k)P_j(k) + \mathcal{F}_j^{o}(k)P_j^o(k) - \alpha_j \right]^+,\quad (25)$$

where virtual queues in (24) represent the constraints in [13]-[17] respectively. In addition, $R_j^{p,e}(k)$ denotes the private information sent to the base station without privacy outage over block $k$. By using the result of Markov inequality in (20), we obtain $R_j^{p,e}(k)$ as $R_j^{o}(k) - \frac{R_p}{R_q}\sum_{j,\in}\mathbb{E}[R_{ji}(k)]$. The first term corresponds to the amount of information received by the base station and the second term to the amount of information captured by the eavesdroppers.

**Control Algorithm:** The algorithm executes the following steps in each block $k$:

1. **Flow Control:** For some $V > 0$, each node $j$ injects $A_j^p(k)$, and $A_j^o(k)$ bits to respective queues and update the virtual queue with $\alpha_j^{p,e}$. Note that $\alpha_j^{p,e}$ can be interpreted as private bits for which perfect secrecy constraint is intended to be satisfied.

$$\left( A_j^p(k), \alpha_j^{p,e}(k), A_j^o(k) \right) = \arg \max \left\{ V \left[ U_j^p(k) + U_j^o(k) - \alpha_j^{p,e}(k) \right] + A_j^p(k) \right\}$$

$$\left( Q_j^p(k)A_j^p(k) + Q_j^o(k)A_j^o(k) + Z_j(k)(A_j^o(k)(1 - \gamma_j) - \alpha_j^{p,e}(k)) \right)$$

2. **Scheduling:** At any given block, scheduler chooses which node will transmit and the amount of power used during transmission of private messages. In other words, schedule node $j$ and transmit privately encoded ($\mathcal{F}_j = 1$), or open bits ($\mathcal{F}_j = 1$), with transmit power $P_j$ and $P_j^o$.

$$\left( \mathcal{F}_j^{p}(k), \mathcal{F}_j^{o}(k), P_j(k), P_j^o(k) \right) = \arg \max \left\{ \mathcal{F}_j^{p}(k)Q_j^{p,e}(k) + \mathcal{F}_j^{o}(k)Q_j^o(k) \right\}$$

$$\left( R_j^{o}(k) - Y_j(k)(\mathcal{F}_j^{p}(k)P_j(k) + \mathcal{F}_j^{o}(k)P_j^o(k)) \right),$$

where expectation is over the distribution of channel estimation error over block $k$.

1. **Optimality of Control Algorithm:** The optimality of the algorithm can be shown using the Lyapunov optimization theorem. Let $Q_j^p(k) = (Q_j^{p,1}(k), \ldots, Q_j^{p,n}(k))$, $Q_j^o(k) = (Q_j^{o,1}(k), \ldots, Q_j^{o,n}(k))$, $\mathcal{F}_j^{p}(k) = (\mathcal{F}_j^{p,1}(k), \ldots, \mathcal{F}_j^{p,n}(k))$, $Z_j(k) = (Z_1(k), \ldots, Z_n(k))$, $Y_j(k) = (Y_1(k), \ldots, Y_n(k))$ be the vectors of real and virtual queues. We consider a quadratic Lyapunov function of the form:

$$L(k) = \frac{1}{2} \sum_k \left[ (Q_j^p(k))^2 + (Q_j^o(k))^2 + (\mathcal{F}_j^{p}(k))^2 + (Z_j(k))^2 + (Y_j(k))^2 \right].$$

One-step expected Lyapunov drift, $\Delta(k)$, is the difference between the value of Lyapunov function at the $(k+1)$th block and $(k)$th block. The following lemma provides an upper bound on $\Delta(k)$.

**Lemma 1:**

$$\text{Lemma 1:}$$

$$\Delta(k) \leq B - \sum_j \mathbb{E} \left[ Q_j^p(k)(\mathcal{F}_j^{p}(k)R_j^o(k) - A_j^o(k))| Q_j^p(k) \right]$$

$$- \sum_j \mathbb{E} \left[ Q_j^o(k)(\mathcal{F}_j^{o}(k)R_j^o(k) - A_j^o(k))| Q_j^o(k) \right]$$

$$- \sum_j \mathbb{E} \left[ Z_j(k)(\mathcal{F}_j^{p}(k) - (1 - \gamma_j)A_j^p(k)) | Z_j(k) \right]$$

$$- \sum_j \mathbb{E} \left[ Y_j(k)(\alpha_j - \mathcal{F}_j^{o}(k)P_j(k) - \mathcal{F}_j^{p}(k)P_j^o(k)) | Y_j(k) \right]$$

where $B > 0$ is a constant.

**Proof:** In an interference-limited practical wireless system both the the transmission power and the transmission rate are bounded. Assume that the arrival rates are also bounded by $A_j^{p,max}, \alpha_j^{p,e,max}, A_j^{o,max}$. By simple algebraic manipulation one can obtain a bound for the difference $(Q_j^o(k+1))^2 - (Q_j^o(k))^2$ and also for other queues to obtain the result (27).

We now present our main result showing that our proposed dynamic control algorithm can achieve a performance arbitrarily close to the solution of the problem with the outage constraint tightened via Markov’s inequality.

**Theorem 1:** If $R_j^o(k) < \infty$ and $R_j^p(k) < \infty$ for all $j, k$, then dynamic control algorithm satisfies:

$$\liminf_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N-1} \sum_{j=1}^{n} \mathbb{E} \left[ U_j^p(k) + U_j^o(k) \right] \geq U^* - \frac{B}{V}$$

$$\limsup_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N-1} \sum_{j=1}^{n} \mathbb{E} \left[ Q_j^o(k) \right] \leq B + \frac{V(U-U^*)}{\varepsilon_1}$$

$$\limsup_{N \to \infty} \frac{1}{N} \sum_{k=0}^{N-1} \sum_{j=1}^{n} \mathbb{E} \left[ Q_j^p(k) \right] \leq B + \frac{V(U-U^*)}{\varepsilon_2}$$

where $B, \varepsilon_1, \varepsilon_2 > 0$ are constants, $U^*$ is the optimal aggregate utility and $\hat{U}$ is the maximum possible aggregate utility.

**Proof:** The proof of Theorem 1 is given in Appendix.

**V. Numerical Results**

In our numerical experiments, we consider a network consisting of four nodes and a single base station. The main channel between the node and the base station, and the cross-channels between nodes are modeled as iid Rayleigh fading Gaussian channels. The power gains of the main and cross-channels are exponentially distributed with means uniformly
chosen in the intervals [25, 50], [0.5, 1.5], respectively. The main channel power gain is estimated by an unbiased estimator based on the a priori channel measurements. As discussed in [11], the estimation error of such an estimator, \( e_j(k) \), can be modeled with a zero mean finite variance Gaussian random variable, i.e., \( e_{ji}(k) \sim N(0, \sigma^2) \) for all \( k \). We take \( \sigma = 1 \). In addition, we assume only the knowledge of distribution for \( \sigma \).

We consider logarithmic private and open utility functions where the private utility is \( \kappa \) times more than open utility at the same rate. More specifically, for a scheduled node \( j \), \( U^p_j(k) = \kappa \log(1 + R^p_j(k)) \), and \( U^o_j(k) = \log(1 + R^o_j(k)) \). We take \( \kappa = 5 \) in all experiments. We perform the simulation over five realizations of \( R_j, R^o_j, R^p_j \), and \( R^o_j, R^p_j \) are uniformly chosen in the interval [15, 25] and \( R^o_j \) in the interval [5, 10].

The rates depicted in the graphs are per node arrival and service rates averaged over all realizations of \( R_j \) and \( R^o_j \), i.e., the unit of the plotted rates are bits/channel use/node. All nodes have the same privacy outage probability \( \gamma \). In Fig. 2(a), we investigate the effect of the tolerable privacy outage probability. It is interesting to note that private service rate increases with increasing tolerable outage probability, \( \gamma \). This is due to the fact that for low \( \gamma \) values, the privacy outage condition is very tight, and this condition is satisfied by transmitting infrequently only when the channel is at its best condition and with low transmit power. The highest private service rate is realized when \( \gamma = 0.3 \), which suggests that 30% of the private packets undergo privacy outage. In Fig. 2(b), the effect of average power constraint, \( \alpha \) is investigated. As expected, for a tight power constraint, all rates are lower, since selected powers are smaller. The highest rates are obtained when \( \alpha = 1 \), and after \( \alpha = 1 \), the power constraint becomes inactive. In addition, the bound on privacy outage probability obtained by Markov inequality is 0.22, which is obtained by averaging the resulting values of the bound over all simulations, whereas the privacy outage probability calculated as in (7) is approximately 0.18. In most of the scenarios, this difference is not significant as long as privacy outage constraint is satisfied.

VI. Conclusion

We consider the problem of resource allocation in a wireless cellular network, in which nodes have both open and private information to be transmitted to the base station over block fading uplink channels. We have developed a cross-layer dynamic control algorithm in the presence of imperfect knowledge based on hybrid ARQ transmission with incremental redundancy. We explicitly took into account the privacy and power constraints and prove the optimality of our scheme by Lyapunov optimization theory. The main challenge that we faced is that, due to encoding of information across many blocks, the privacy outage probability of subsequent retransmissions of a given block cannot be decoupled from each other. We overcame this challenge by introducing a novel technique based on the Markov inequality.

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Appendix

Lyapunov Optimization Theorem suggests that a good control strategy is the one that minimizes the following:
\[
\Delta^U(k) = \Delta(k) - V \sum_j \left( U_j^p(k) + U_j^o(k) \right) - \left( Q_j^p(k), Q_j^o(k) \right)
\] (28)

where \( U_j^p(k) \) and \( U_j^o(k) \) are private and open utility obtained in block \( k \).

By using (27), we may obtain an upper bound for (28), as follows:

\[
\Delta^U(k) < B - \sum_j \left[ Q_j^p(k) \right] - \sum_j \left[ Q_j^o(k) \right] - \sum_j \left[ R_j^p(k) - A_j^o(k) \right] - \sum_j \left[ R_j^o(k) - A_j^p(k) \right] - \sum_j \left[ Y_j(k) \right] - \sum_j \left[ Z_j(k) \right]
\] (29)

Thus, by rearranging the terms in (29), it is easy to observe that our proposed dynamic network control algorithm minimizes the right hand side of (29) with the available channel information.

Assume that there exists a stationary scheduling and rate control policy that chooses the users and their transmission powers independent of queue backlogs and only with respect to the channel statistics. Let \( U^* \) be optimal value of the objective function of the problem (13) by the stationary policy. Also let \( x^{p,e}_j \) and \( x^{o,e}_j \) be optimal effective private, private and open traffic arrivals. In addition, let \( P_j^* \) be optimal transmission power for user \( j \). Note that, the expectations of right hand side (RHS) of (29) can be written separately due to independence of backlogs with scheduling and rate control policy. Since the rates and transmission power are strictly interior of the feasible region, the stationary policy should satisfy the following:

\[
\mathbb{E} \left[ R_j^p(k) R_j^o(k) \right] \geq x^{p,e}_j + \epsilon_1 \quad \mathbb{E} \left[ R_j^p(k) R_j^o(k) \right] \geq x^{o,e}_j + \epsilon_2
\]

\[
\mathbb{E} \left[ R_j^p(k) R_j^o(k) \right] \geq x^{p,e}_j + \epsilon_3 \quad \mathbb{E} \left[ R_j^p(k) R_j^o(k) \right] \geq x^{o,e}_j + \epsilon_4
\]

Recall that our proposed policy minimizes RHS of (29), hence, any other stationary policy has a higher RHS value. By using optimal stationary policy, we can obtain an upper bound for the RHS of our proposed policy. Inserting (30) into (29) and using the independence of queue backlogs with scheduling and rate policy, we obtain the following bound:

\[
RHS < B - \sum_j \epsilon_1 \mathbb{E} \left[ Q_j^p(k) \right] - \sum_j \epsilon_2 \mathbb{E} \left[ Q_j^o(k) \right] - \sum_j \epsilon_3 \mathbb{E} \left[ Q_j^{p,e}(k) \right] - \sum_j \epsilon_4 \mathbb{E} \left[ Q_j^{o,e}(k) \right]
\]

Now, we can obtain bounds on performance of the proposed policy and the sizes of queue backlogs as given in Theorem 1.