Transition from accelerated to decelerated regimes in JT and CGHS cosmologies

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In this work we discuss the possibility of positive-acceleration regimes, and their transition to decelerated regimes, in two-dimensional (2D) cosmological models. We use general relativity and the thermodynamics in a 2D space-time, where the gas is seen as the sources of the gravitational field. An early-Universe model is analyzed where the state equation of van der Waals is used, replacing the usual barotropic equation. We show that this substitution permits the simulation of a period of inflation, followed by a negative-acceleration era. The dynamical behavior of the system follows from the solution of the Jackiw-Teitelboim equations (JT equations) and the energy-momentum conservation laws. In a second stage we focus the Callan-Giddings-Harvey-Strominger model (CGHS model); here the transition from the inflationary period to the decelerated period is also present between the solutions, although this result depend strongly on the initial conditions used for the dilaton field. The temporal evolution of the cosmic scale function, its acceleration, the energy density and the hydrostatic pressure are the physical quantities obtained in through the analysis.

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I. INTRODUCTION

The proposal of theories of gravity in lower dimensions has been studied intensively \[1\]. These models offer interesting results that, if properly analyzed, can be used to solve problems in realistic theories. Cosmological solutions has been also under analysis in these formulations \[2,3,4\]. Taking the particular case of two-dimensional (2D) theories, the gravity model proposed by Teitelboim and Jackiw \[5,6\] (JT model) provided consistent results at the classical and quantum levels \[1\]. This model works as the “closest counterpart” of general relativity in 2D formulations, since the Einstein field equations furnish no dynamics in 2D \[1\]. In the JT model a scalar field (related to the dilaton \[7\]) is present. In the “Einsteinianness” interpretation the dilaton is working solely as an auxiliary variable to guarantee the correct equations of motion for the geometrical quantities \[1\]. Another analysis of the JT model, in a cosmological context \[2\], considered the dilaton as a dynamical field. The best cosmological results obtained using the JT model include the description of a Universe filled with ordinary matter or/and electromagnetic radiation, where its behavior is described in terms of the temporal evolution of the scale parameter, the energy density and the total pressure \[2,4,7\]. A remarkable point here is that positive accelerated regimes (such as an inflationary Universe) were possible only under very special conditions, such as a negative vacuum energy density for the inflation and violation of the weak energy condition for the matter/radiation constituents \[2\].

Inspired in string theory, Callan et al. \[8\] proposed the so-called CGHS model. In this theory the dilaton is invariably a dynamical field; in fact this model is formally related to the JT model, through a field mapping, when we consider the dilaton as part of the true degrees of freedom of the gravitational field \[5\]. Here the cosmological results include, again, the description of a matter/radiation filled Universe \[2\]. The possibility of description of inflation or dark energy regimes depend basically on the same special conditions present in the JT model \[2\].

In this work, as a sequence to our previous effort \[7\], we want to discuss the existence of positive-acceleration solutions in 2D cosmologies; these related to the description of an inflation period and its transition to a decelerated period. We show that a consistent formulation is possible when the usual barotropic equation of state is replaced by the van der Waals equation (VDW), as was proposed by Capozziello et al. \[4\] in 4D models and explored by one of these authors \[10\]. We show that an inflationary scenario is possible in the JT model, with a compatible behavior of the physical quantities. In fact, we obtain the temporal evolution of the scale factor, its acceleration, and the energy density of the VDW constituent. We also obtain an accelerated regime, with a transition to a decelerated era, in the CGHS model, although in this case the behavior of the acceleration field depends strongly on the initial conditions of the dilaton field.

The manuscript is structured as follows. In Section II we make a brief review of JT and CGHS models in a cosmological context. In Section III, accelerated regimes with their transition to deceleration are focused in the JT model when we take into account the VDW equation of state. Moreover, we describe positive accelerated regimes in the CGHS model, using again the VDW equation, together with a discussion of the results. Units have been chosen so that \( G = c = k = 1 \).
II. COSMOLOGY IN JT AND CGHS MODELS

In this section we make a brief review of the JT and CGHS models in a cosmological context, focusing on the results present in [2, 4, 6].

One fundamental point in 2D gravity is that the relation between the metric tensor and the sources is modified. This is because, as it is well-known, the Einstein field equations give no dynamics in the 2D case [1]; in fact this is a consequence of general coordinate transformation and conformal invariance [1].

Another important feature of 2D models is that they show considerable less mathematical complexity and at the same time they preserve the gauge principles that are used to construct their 4D counterparts. One impressive result is that in 2D models the quantization of the gravitational field is possible [1], opening the possibility, for the first time, between other perspectives, of full quantum cosmological models for the very early Universe [1, 2].

The model proposed by Teitelboim and Jackiw [4] focused on the fact that in 2D the full geometrical information for the 2D space-time is encapsulated in the curvature scalar \( R \). The action for this model is given by

\[
S = \int d^2x \sqrt{-g} \left\{ N(x) \left[ R(x) - \Lambda + 8\pi T^\mu_\mu(x) \right] \right\},
\]  

where \( T^\mu_\mu(x) \) is the trace of the energy-momentum tensor of the sources and \( \Lambda \) is a cosmological constant. Using the variational principle for the scalar field \( N(x) \) (which is related to the dilaton [2]) the equation of motion that follow is

\[
R(x) = -8\pi T^\mu_\mu(x) + \Lambda.
\]  

When we apply the variational principle to the components of the metric we obtain a relation that defines the scalar field \( N(x) \) in terms of the other fields. In this strict “Einsteinian” interpretation the scalar field is taken as an auxiliary field; that means, this formulation is the closest general relativity counterpart in 2D. On the other hand, in a “Brans-Dicke” interpretation, the scalar field \( N(x) \) is dynamical [4]. We will consider the dilaton as a dynamical field in the CGHS model (see end of this and next section).

In this hypothetical 2D Universe the assumptions of spatial homogeneity and isotropy are also invoked. The Robertson-Walker metric has the following form in a 2D Riemannian space

\[
ds^2 = (dt)^2 - a(t)^2(dx)^2,
\]  

where \( a(t) \) is the cosmic scale factor, that in this case encloses the complete information about the evolution of the gravitational field created by the sources. In fact the usual geometrical quantities (Ricci tensor, curvature scalar) turn out to be

\[
R_{00} = \frac{\ddot{a}}{a}, \quad R_{11} = -\ddot{a}a, \quad R = 2\frac{\ddot{a}}{a}.
\]  

The cosmological solutions of the JT model include a Universe filled with radiation or matter [2, 7]; the equations of motion (together with the conservation law \( T^\mu_\nu = 0 \) for a perfect fluid where \( T^\mu_\mu = \text{diag}(\rho, -p) \) can be expressed as

\[
\frac{\ddot{a}}{a} = -4\pi(p - \rho) + \frac{\Lambda}{2},
\]  

\[
\dot{\rho} + \frac{\dot{a}}{a}(\rho + p).
\]  

The system of differential equations above can be solved for \( a(t) \) and \( \rho(t) \) for given initial conditions, once the equation of state \( p = p(\rho) \) is specified.

Inflationary scenarios appear only when unusual hypothesis are taken into account. Negative energy density and violation of the weak energy condition [2] are the most representative; on the other hand, new features appear in the cosmological solutions when the auxiliary field is taken as a dynamical variable [4]. In fact, this approach is formally linked to the analysis of the so-called CGHS model [8].

The CGHS model was proposed initially for the physical investigation of 2D black-holes [8]. The action, inspired in string theories [8], includes the dilaton field in a similar way to the 4D Jordan-Brans-Dicke theories (although a complete analogy cannot be done [1]). As was mentioned before, the dilaton is seen as a dynamical field in this case. The equations of motion are

\[
e^{-2\phi} [R_{\mu\nu} - \Lambda g_{\mu\nu} - 2\nabla_\mu \nabla_\nu \phi] = -8\pi T_{\mu\nu},
\]  

\[
R - \Lambda - 4(\nabla \phi)^2 + 4\nabla^2 \phi = 0.
\]  

where \( \phi \) is the dilaton and \( T_{\mu\nu} \) is the energy-momentum tensor of the sources. For a perfect fluid constituent these gravitational field equations read

\[
\frac{\ddot{a}}{a} = -4\pi e^{2\phi}(p - \rho) + \frac{\dot{a}^2}{a} + \dot{\phi} + \frac{\Lambda}{2},
\]  

\[
\dot{\phi} = 4\pi e^{2\phi}(\rho + p) + \frac{\ddot{a}}{a}.
\]  

If we know the equation of state \( p = p(\rho) \) and prescribe initial conditions, it is possible to obtain from the system of differential equations [2, 5] and [10] the evolution equations for the fields \( \rho(t), a(t) \) and \( \phi(t) \).

The regimes coming out include dust and radiation filled Universes with the possible inclusion of a cosmological constant [2, 4]. The existence of positive accelerated solutions depend, as in the JT model, on the imposition of unusual conditions as negative energy density for the scalar field that represents the inflaton.

In the following section we present a solution to this problem considering the van der Waals state equation (VDW) [3] to model the behavior of an early Universe.


III. INFLATION IN JT AND CGHS MODELS

In this section we consider the Jackiw-Teitelboim model (JT model), to describe an inflationary scenario in a 2D Universe. The scalar field $N(x)$, mentioned in the precedent section, is taken as an auxiliary field. This Universe is seen as a fluid with its thermodynamical state ruled by a van der Waals (VDW) equation. This state equation was considered in a cosmological context in the works \[9, 10\]. In classical thermodynamics, the VDW equation is a refinement of the ideal gas equation of state, when the volume of particles and the long range interaction between the particles is taken into account for a dense fluid. In a 2D cosmological context we shall neglect the term related to the long range interaction and write the VDW equation of state as

$$p = \frac{w \rho}{1 - \alpha \rho},$$  \hspace{1cm} (11)

where $w$ and $\alpha$ are constants. When we introduce the above expression in equations (5) and (6) we get

$$\frac{\dot{a}}{a} = -4\pi \left( \rho - \frac{w \rho}{1 - \alpha \rho} \right) + \frac{\Lambda}{2}, \quad \dot{\rho} + \frac{\dot{a}}{a} \left( \rho + \frac{w \rho}{1 - \alpha \rho} \right) = 0.$$  \hspace{1cm} (12)

The system of differential equations (12) is solved numerically by prescribing the following initial conditions: $a(0) = 1$, $\dot{a}(0) = 1$ and $\rho(0) = 1$. Moreover, in order to plot the figure 1, we have chosen $\alpha = 0.5$, $\Lambda = 0.002$ and two values for $w$, namely, 0.9 and 0.8. The time evolution of the physical quantities obtained show (see figure 1) an expansion, with a transition from a positive accelerated regime, that would correspond to the inflationary period, followed by a decelerated period where the VDW equation approaches a barotropic equation of state, since the energy density is decreasing. At later times the Universe returns to an accelerated epoch, owing to the presence of the cosmological constant. Whereas the behavior of the acceleration field in 2D is analogous to the one obtained in four dimensions \[10\], the physical interpretations are quite different. In 4D the transition from an early accelerated regime to a decelerated epoch is due to the fact that the pressure of the VDW fluid changes from a negative value, where it behaves like an inflaton, to a positive value, where it behaves like a matter dominated fluid. In the 2D case, the pressure of the VDW fluid in the earliest times is positive and larger than the energy density whereas at later times it follows a barotropic equation of state. The transition from a decelerated regime to an accelerated epoch is due to a presence of a cosmological constant which has also different interpretations; while in the 4D case the cosmological constant plays the role of a dark energy this does not occur in the 2D case, since the sign of the energy density in the acceleration equation \[5\] is negative, and a negative cosmological constant would contribute more for the deceleration of the 2D Universe. If one considers a vanishing cosmological constant in the 2D case the deceleration of the Universe leads to a "Big Crunch". This last result was also obtained in the work \[7\] where a barotropic equation of state was used to model the cosmological fluid. For a fixed value of the parameter $w$ that permit the transition from an accelerated regime to a decelerated one (for $\alpha = 0.5$ the critical value is $w = 0.98$) the results show that the smaller the value of $\alpha$ the more drastic is the transition. Furthermore, for a fixed $\alpha$ the smaller the value of $w$ the transition to the decelerated regime is more pronounced (see figures 1 and 2). In all cases, after the transition occurs, the accelerated regime returns only when a cosmological constant term is present. On the other hand, for big values of the time variable the scale factor shows a collapsing Universe, dominated by a constituent with a barotropic equation of state when the cosmological constant term is absent.

We have also investigated the accelerated regimes that follow from the CGHS model by considering again the
VDW equation of state. We have solved numerically the system of differential equations (6), (9) and (10) for the energy density, the cosmic scale factor and the dilaton, respectively. The initial conditions chosen were: \( \rho(0) = 1, a(0) = 1, \dot{a}(0) = 1, \phi(0) = 0, \dot{\phi}(0) = d \) were \( d \) is a constant that determines the transition from the different regimes obtained, as we discuss below. The behavior of the acceleration depends strongly on the initial condition for the time derivative of the dilaton \( d \). In fact, the possibility of describing a transition from an accelerated period and a decelerated one starts to make sense when we take values of \( d < -0.62 \). The time evolution of the cosmic scale factor and of its acceleration field show a dramatically different behavior when compared to the ones obtained in the JT model. The acceleration vanishes for large values of \( t \) even with a non-vanishing cosmological constant. Furthermore, a vanishing cosmological constant implies that the cosmic scale factor tends to a constant value, i.e., the 2D Universe tends to a static Universe. For a non-vanishing cosmological constant the solution shows a 2D Universe in permanent expansion with the energy density decreasing accordingly. Again, as the Universe is expanding the VDW equation approaches a barotropic equation of state. For a fixed value of the parameter \( w \) the results show that the smaller the value of \( \alpha \) the more drastic is the transition. On the other hand, for a fixed \( \alpha \) the smaller the value of \( w \) the transition to the decelerated regime gets more drastic. In all cases, after the transition occurs, the accelerated regime never returns.

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