Revisit of Y-junctions for strings with currents: transonic elastic case

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We studied the formation of Y-junctions for transsonic elastic strings. Using the general solution for this type of strings, which is described by left- and right-moving modes, we obtained the dynamics of Y-junctions. Considering the linearized ansatz for straight strings, we constructed the region in “angle-velocity” space for which the formation of Y-junctions due to strings collisions is allowed. We argue that the obtained result is valid for all current carrying straight strings.

I. INTRODUCTION

Cosmic strings are hypothetical objects that were originally described by Tom Kibble [1]. They appear as a prediction of numerous models of early universe [2]. To highlight some of them it is worthwhile to mention brane inflation [3,8], supersymmetric grand unified theory [9-13] and theories of high energy particle physics [15-17].

Some types of cosmic strings allow the existence of bound states, named as Y-junctions. They might appear due to collisions of distinct strings that form trilinear vertices, see figure 1. Y-junctions are common for non-abelian strings [18], complex field configurations [19] and cosmic strings from brane inflation (cosmic superstrings) [5]. It was demonstrated that it is possible to obtain kinematic constraints for production of Y-junctions using approximation that cosmic strings are infinitely thin (are described by Nambu-Goto action) [20,21]. The result of this research states that for magnetic (space-like current) and electric (time-like current) superconducting strings the formation of Y-junction is impossible, unless the newly formed string is described by a more general equation of state.

This study is aimed to revisit the problem of Y-junctions formation for particular case of transonic elastic strings. We re-examine the exact solution for these strings [44,47], obtain left-/right-moving modes, and in line with [20] we derive kinematic conditions under which the production of Y-junction is possible. The result obtained in this paper is in conflict with conclusion of the work [48]. The explanation of this situation is given in the end of section VI.

II. SOLUTION IN MINKOWSKI SPACE FOR TRANSONIC ELASTIC STRINGS

In this section we revisit the exact solution for transonic elastic strings, originally obtained in [44,47], with the method developed in [49]. We start consideration from the action

\[ S = -\mu_0 \int f(\kappa) \sqrt{-\gamma} d\sigma d\tau, \]  

(1)

where \( \mu_0 \) is a constant defined by the symmetry breaking scale, \( \{\sigma, \tau\} \) are coordinates on the string worldsheet (Latin indexes “a-d” run over 0, 1) with induced metric

\[ \gamma_{ab} \equiv \epsilon_{ab} \varphi,_{a} \gamma,_{b}, \]  

(2a)

\[ \kappa \equiv \varphi,_{a} \varphi,_{b} \gamma^{ab}, \]  

(2b)

\[ \gamma \equiv \frac{1}{2} \epsilon_{abcd} \gamma_{ab} \gamma_{cd}, \]  

(2c)

\[ \{a, b, c, d\} \] run over 1, 2, 3, 4, \( \epsilon_{1234} = 1 \).
can be considered as an effective description of wiggly strings\cite{46,47} and some particular limits of superconducting strings (see sections 5.8, 5.9 in\cite{50}).

Using\cite{4}, the explicit form of\cite{5} can be written as
\begin{equation}
\begin{aligned}
c_E^2 &= \frac{T}{U^2},
\end{aligned}
\end{equation}
where $c_E = c_L = 1$. It is anticipated to have supersonic strings ($c_E > c_L$) for most of regimes of superconducting strings\cite{33,52}. Meanwhile, the transonic model
\begin{equation}
c_L = c_E \leq 1
\end{equation}
can be considered as an effective description of wiggly strings\cite{46,47} and some particular limits of superconducting strings (see sections 5.8, 5.9 in\cite{50}).

Using\cite{4}, the explicit form of\cite{5} can be written as
\begin{equation}
\begin{aligned}
c_E^2 &= \frac{f - 2\kappa f_\alpha^\prime \Theta[\kappa f_\alpha^\prime]}{f - 2\kappa f_\alpha^\prime \Theta[\kappa f_\alpha^\prime]},
\end{aligned}
\end{equation}
\begin{equation}
\begin{aligned}
c_L^2 &= \frac{f' - 2(f_\alpha + \kappa f_\alpha^\prime) \Theta[\kappa f_\alpha^\prime]}{f' - 2(f_\alpha + \kappa f_\alpha^\prime) \Theta[\kappa f_\alpha^\prime]}.
\end{aligned}
\end{equation}

Substituting\cite{7} into condition\cite{6} for transonic strings, one can obtain the equation for $f(\kappa)$
\begin{equation}
\begin{aligned}
(f'_e)^2 + 2f'_e = 0 \Rightarrow f = \sqrt{c_1\kappa + c_2},
\end{aligned}
\end{equation}
where $c_1$ and $c_2$ are constants of integration.

One can write down the equation of state for transonic strings using the expressions\cite{4} together with\cite{5}
\begin{equation}
\begin{aligned}
UT = f(f - 2\kappa f_\alpha^\prime) = c_2 = m^2,
\end{aligned}
\end{equation}
where $m$ is a mass dimensional constant.

We can define $c_1 = \pm m^2$ and absorb $m^2$ into the definition of $\mu_0$. These manipulations allow us to establish the function $f(\kappa)$ for transonic elastic strings in the following form, as also presented in\cite{44,47},
\begin{equation}
\begin{aligned}
f(\kappa) = \sqrt{1 - \kappa}, \quad \kappa \in (-\infty, 1], \quad UT = 1.
\end{aligned}
\end{equation}

It is known that the transonic model has the general wave-like solution\cite{44}. Let’s use the method from\cite{49} to demonstrate that there are only two types of strings, whose equations of motion can be reduced to the wave equation: chiral (see\cite{44,53}) and transonic elastic strings. We start consideration by writing down the equations of motion for the action\cite{1} in Minkowski space\cite{54}
\begin{equation}
\begin{aligned}
\partial_a \left[ T^{ab} x_b^\prime \right] = 0, \quad (11a)
\partial_a \left[ \sqrt{-\gamma} \gamma x_b^\prime \beta_c, b \right] = 0, \quad (11b)
\end{aligned}
\end{equation}
where
\begin{equation}
\begin{aligned}
T^{ab} = \sqrt{-\gamma} \left( \gamma^{ab} f - 2f'_e \gamma^{ac} \gamma^{bd} \varphi_c \varphi_d \right) = \sqrt{-\gamma} (\gamma^{ab} f + \theta^{ab})
\end{aligned}
\end{equation}
(notice the change of the sign in\cite{12} due to misprint in equation (6) of\cite{54}).

Parametrization invariance of the string worldsheet allows us to make the transformation
\begin{equation}
\begin{aligned}
T^{ab} \rightarrow \eta^{ab},
\end{aligned}
\end{equation}
if their determinants are equal\cite{49}
\begin{equation}
\begin{aligned}
\det T^{ab} = \det \eta^{ab} = -1.
\end{aligned}
\end{equation}
Let’s expand the determinant of $T^{ab}$
\begin{equation}
\begin{aligned}
\det T^{a}_e = -f^2 - f \text{ Tr} \theta^c_e - \det \theta^{ae}_e.
\end{aligned}
\end{equation}
It is easy to check that \( \det \theta^a_i = 0 \), hence, we are left only with
\[
\det T^a_i = - f^2 + 2 f f'_\kappa \Tr \left[ \gamma^{ac} \gamma^{bd} \varphi_c \varphi_d \right] = 0,
\]
for right- and left-moving modes.

Using relations (22), one can write down the current (23) as
\[
\varphi = \frac{1}{2} \left( F(\sigma_+) + G(\sigma_-) \right).
\]

We still have freedom to chose the normalization for \([a']\) and \([b']\). For simplicity and in order to keep consistency with the standard description, to have \(|a'| = 1\) and \(|b'| = 1\) when correspondent current components vanish, we chose the following normalization
\[
(a')^2(\sigma_+) = 1 - F'^2(\sigma_+), \quad (b')^2(\sigma_-) = 1 - G'^2(\sigma_-)
\]
for right- and left-moving modes.

Using relations (22), one can write down the current (23) as
\[
\kappa = \frac{2 F' G'}{1 + F' G'} - a' \cdot b'.
\]
\(X^\mu(\tau)\) and \(\Phi(\tau)\) define values for \(x^\mu_i\) and \(\varphi_i\) at the point where strings are connected, the index \(i = 1, 2, 3\) denotes each of the three strings (the summation over index \(i\) is carried out only when it is written explicitly).

Varying the action \((25)\) with respect to \(x^\mu_i\) and \(\varphi_i\), we obtain the equations of motion \((11)\) and \((11)\) for each type of strings. Using \((16)\) and \((20)\) the boundary terms from equations of motion, which are proportional to \(\delta(s_i(t) - \sigma_i)\), can be expressed as

\[
\begin{align*}
\mu_i \eta^\mu\varpi^\alpha \lambda_{\alpha i} &= f^i, \\
2\mu_i f_i f^\mu \eta^\alpha \varphi_\alpha \lambda_{\beta i} &= g_i,
\end{align*}
\]

where \(\lambda_{\alpha i} = \{\dot{s}_i, -1\}\).

The variation of the action \((25)\) with respect to \(X^\mu_i\) and \(\Phi\) gives us

\[
\begin{align*}
\sum_i f^\mu_i &= 0, \\
\sum_i g_i &= 0,
\end{align*}
\]

which can be rewritten using solutions \((17)\) and \((21)\) together with expressions \((20)\) in the following way

\[
\begin{align*}
\sum_i \mu_i [a'_i(1 + \dot{s}_i) - b'_i(1 - \dot{s}_i)] &= 0, \\
\sum_i \mu_i [F'_i(1 + \dot{s}_i) - G'_i(1 - \dot{s}_i)] &= 0.
\end{align*}
\]

Finally, variation of the action \((25)\) with respect to \(f^\mu_i\) and \(g_i\) provides us the following relations

\[
\begin{align*}
x^\mu_i(s_i(\tau), \tau) &= X^\mu(\tau), \\
\varphi_i(s_i(\tau), \tau) &= \Phi(\tau).
\end{align*}
\]

Differentiating \((29)\), using the exact solutions \((17)\) and \((21)\) we obtain

\[
\begin{align*}
(1 + \dot{s}_i)a'_i + (1 - \dot{s}_i)b'_i &= 2X(t), \\
F'_i(1 + \dot{s}_i) + G'_i(1 - \dot{s}_i) &= 2\Phi(t).
\end{align*}
\]

Manipulating vectors \(a'_i, b'_i\) and using \((28)\) with \((30)\), it is possible to obtain the following equations

\[
\begin{align*}
a'_k(1 + s_k) &= \frac{2}{\mu} \sum_i (1 - \dot{s}_i) \mu_i b'_i - (1 - \dot{s}_k) b'_k, \\
F'_k(1 + s_k) &= \frac{2}{\mu} \sum_i (1 - \dot{s}_i) \mu_i G'_i - (1 - \dot{s}_k) G'_k,
\end{align*}
\]

and

\[
X = \frac{1}{\mu} \sum_i (1 - \dot{s}_i) \mu_i b'_i.
\]

We parametrize the string worldsheets in such way that modes \(a'_i(\sigma_+), F'_i(\sigma_+)\) move outwards the string connection, while \(b'_i(\sigma_-)\) and \(G'_i(\sigma_-)\) move towards the string connection. Such choice means that \(b'_i(\sigma_-)\) and \(G'_i(\sigma_-)\) are initial values that define \(a'_i(\sigma_+)\) and \(F'_i(\sigma_+)\) by equations \((31)\). The first three equations for vectors \(a'_i(\sigma_+)\) in \((31)\) can be squared and using the normalization conditions \((22)\) we eliminate \(a'_i(\sigma_+)\). Hence, we have the system of six independent algebraic equations \((31)\) and six variables that can be found: three variables \(s_i\) and three variables \(F'_i(\sigma_+)\).

It is illustrative to compare values of \(s_i\) for strings with currents and without. For this purpose we fix angles between \(b'_i(\sigma_-)\), define string constants \(\mu_i\) and evaluate the system of equations \((31)\) for different values of \(G'_i(\sigma_-)\). An example of such dependence is shown in figure 3.

![Figure 3](image)

IV. COLLISIONS OF TRANSONIC ELASTIC STRINGS

It is always possible to chose small region, where collided strings can be considered straight. We are going to study kinematic conditions for straight strings to produce a Y-junction.

We decompose the straight string solution as a linear combination of “bare” and current carrying parts \((33)\)

\[
x_i = y_i + z_i,
\]

where the “bare” part is given by

\[
\begin{align*}
y_{1,2} &= \{-\gamma^{-1}_c \cos \alpha; \mp \gamma^{-1}_e \cos \alpha; \pm \nu \tau\}, \\
y_3 &= \{\gamma^{-1}_c \cos \theta; \gamma^{-1}_u \sin \theta; \nu \tau\},
\end{align*}
\]
while the current carrying part is described by
\[ z_i = -g_i \sigma (\dot{y}_i - y_i') - f_i \sigma (\dot{y}_i + y_i'), \]
with \( \gamma_i^{-1} = \sqrt{1 - v_i^2} \).

Constants \( f_i \) and \( g_i \) in (35) represent the current contribution for left- and right-moving modes.

From (33) one can find that
\[ a_i' = (1 - 2f_i)(\dot{y}_i + y_i'), \quad |a_i'|^2 = (1 - 2f_i)^2, \]
\[ b_i' = (1 - 2g_i)(\dot{y}_i - y_i'), \quad |b_i'|^2 = (1 - 2g_i)^2. \]

Comparing constants \( f_i \) and \( g_i \) in (36) with (22), we establish the relations
\[ f_i = 1 - \sqrt{1 - F_i^2}, \quad g_i = 1 - \sqrt{1 - G_i^2}. \]

In order to find out for which velocities \( v \) and angles \( \alpha \) the third string can be produced (which means that \( \dot{s}_3 > 0 \)), we need to derive the orientation (angle \( \theta \)) of newly created string and its velocity \( u \). To obtain these variables we follow the procedure of [21], i.e., we write \( \sigma \rightarrow s_3(\tau) \) in expressions (34) and (35):
\[ \dot{X} = \{ T_1(\tau) \gamma_1^{-1} \cos \theta; \ T_2(\tau) \gamma_1^{-1} \sin \theta; \ T_2(\tau) u \}, \]
where \( T_1(\tau) = \dot{s}_3(\tau) + g_3(1 - \dot{s}_3(\tau)) - f_3(1 + \dot{s}_3(\tau)) \) and \( T_2(\tau) = 1 - f_3(1 + \dot{s}_3(\tau)) + g_3(1 - \dot{s}_3(\tau)) \).

Combining (35) with (32) one can obtain the vector equation, from which \( \theta \) and \( u \) are determined via \( b'_i \).

To summarize, we have nine equations: six equations from (28) and three equations from (32). Therefore, we can derive eight variables \( F'_i, s_i, u, \theta \) defining another eight variables \( \mu, G'_i, v, \alpha \). The vector equality (32) does not provide three independent equations, but only two, similarly as in [21]. Having all this information, we can numerically solve this system of algebraic equations. As a result, we obtain the region of velocities \( v \) and angles \( \alpha \) for which colliding strings give rise to Y-junctions (which means \( \dot{s}_3 > 0 \)), see figure 4.

It is important to highlight that the ansatz (33) for linearized straight strings satisfies equations of motions (11a), (11b) for arbitrary function \( f(\kappa) \). It happens because all components, such as \( \gamma_{ab}, \psi_{\alpha}, \kappa, \dot{f}(\kappa) \) and \( f'_{\kappa}(\kappa) \) are constants. Since any string near the collision point can be treated as a straight one, we anticipate that calculations presented above are applicable to all current carrying strings. The absence of an analytic solution for the general case does not allow us to establish connections for \( f_i \) and \( g_i \) as it was done for transonic elastic strings in [67], however, we anticipate that qualitative picture for all superconducting strings will be the same.

V. CONCLUSIONS

We revisited the exact solution for elastic transonic strings in Minkowski space [41, 67] with the method developed in [49]. The exact solution allowed us to consider left- and right-moving modes, which made it possible to treat the production of Y-junctions in a similar manner as it was done in [21].

The system of equations (31) allowed us to obtain \( \dot{s}_i \), the rate of string lengths change, requiring the definition of incoming components of the current \( G'_i \). The values of incoming current components \( G'_i \) should be determined by strings properties. Thus, in the case of cosmic superstrings the values of \( G'_i \) might be defined similarly to tensions of connected strings by saturated BPS state (see [4, 27, 28] for details)

\[ \mu_{p,q} = \mu_F \sqrt{(p - qC_0)^2 + q^2/g_0^2}. \]

For superconducting and wiggly cosmic strings with Y-junctions, the definitions of \( G'_i \) should arise from the values of tensions and mass per unit lengths [4]. The exact definition of \( G'_i \) for particular type of strings needs further investigation and goes beyond the scope of this paper, hence, we treated \( G'_i \) as free parameters.

In section IV we found kinematic constraints that should be satisfied to give rise to a Y-junction. In particular, we obtained a range of velocities \( v \) and angles \( \alpha \) of collided strings when \( \dot{s}_3 > 0 \), as shown in figure 4.

While the full analyses of kinematic constraints was carried out for transonic elastic strings, we anticipate the same qualitative result for all current carrying strings. This conjecture comes from the fact that near the collision point we always chose small region where strings can be described by linearized ansatz (33). The straight string form (45) satisfies equations of motion (11a), (11b) for arbitrary function \( f(\kappa) \) and leads to conclusion that kinematic constraints should be valid for all current carrying strings.
In the present study we do not face the problem of overdetermined system of equations, as it was reported in [HS]. We expect that the problem in [HS] appears due to the choice of conformal gauge
\[ \dot{x}^\mu \dot{x}_\mu = 0, \quad \dot{x}^\mu \dot{x}_\mu = -x^\mu x_\mu, \]

together with \( \tau \) coinciding with physical time \( t \). The implicit equality \( \tau = t \) comes up in the definition of the action for connected strings given in section III of [HS]. It can be seen from the fact that \( \tau \) is the same for all string worldsheets and that the vertex, which connects three strings, has only \( \tau \) dependence. We assume that relaxing one of these three conditions (two conditions from the conformal gauge and the condition \( t = \tau \)) one can resolve the problem of overdetermined equations.

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