Measurement of topological invariants in a 2D photonic system

Sunil Mittal1,2, Sriram Ganeshan1,3, Jingyun Fan1, Abolhassan Vaezi4 and Mohammad Hafezi1,2*

A hallmark feature of topological physics is the presence of one-way propagating chiral modes at the system boundary1,2. The chirality of edge modes is a consequence of the topological character of the bulk. For example, in a non-interacting quantum Hall model, edge modes manifest as mid-gap states between two topologically distinct bulk bands. The bulk-boundary correspondence dictates that the number of chiral edge modes, a topological invariant called the winding number, is completely determined by the bulk topological invariant, the Chern number3. Here, for the first time, we measure the winding number in a 2D photonic system. By inserting a unit flux quantum at the edge, we show that the edge spectrum resonances shift by the winding number. This experiment provides a new approach for unambiguous measurement of topological invariants, independent of the microscopic details, and could possibly be extended to probe strongly correlated topological orders.

Recently, there has been a surge of interest in investigating topological states with synthetic gauge fields. Synthetic gauge fields have been realized in various atomic16–19 and photonic systems8,9. In particular, topological photonic edge states have been imaged in two recent concurrent experiments10,11 and the robustness of their transport has been quantitatively confirmed both in the microwave12 and telecom domains13. Several other interesting studies have investigated topological states in 1D14–16, 2D17–21 and also 3D22 synthetic structures. Topological states are characterized by topologically invariant integers5. In fermionic systems, conductance measurements reveal these integer invariants. However, direct measurement of these integers is non-trivial in bosonic systems, mainly because the concept of conductance is not well defined23,24. Whereas these integers have been measured in 1D bosonic systems5,25,26, the 2D bosonic case has been limited to atomic lattices7.

Here, we experimentally demonstrate that selective manipulation of the edge can be exploited to measure topological invariants, that is, the winding number of the edge states. We implement an integer quantum Hall system using a fixed, uniform synthetic gauge field in the bulk and couple an extra, tunable gauge field only to the edge. The edge state energy spectrum flows as a function of this tunable flux. With the insertion of a unit quantum of flux, the edge state resonances move by ±1, which is the winding number of edge states in our system. This spectral flow can be directly observed in an experiment as the flow of transmission resonances, and thus provides a direct measurement of the winding number. For this demonstration, we employ the unique ability of our photonic system to selectively manipulate edge states—a feature that is challenging to achieve in current electronic and atomic systems.

To model the spectral flow of a quantum Hall edge, with winding number \( k = 1 \), we consider a linear edge dispersion \( E_p = \nu p \) where \( E_p \) is the energy, \( \nu \) is the group velocity, and \( p \) is the momentum along the edge. When a gauge flux (\( \theta \)) is coupled to the edge, the momentum is replaced by the covariant momentum:

\[
E_p = \nu \left( p - q \frac{\theta}{L} \right)
\]

where \( L \) is the length of the edge and \( q \) is the charge of the edge excitations. For non-interacting photons, the charge \( q = 1 \). Note that the corresponding vector potential is simply \( \theta/L \). For a finite system, the quantization of momentum on the edge results in

\[
E_n = \frac{2\pi\nu}{L} \left( n - \frac{\theta}{2\pi} \right)
\]

where \( n \) is an integer. Thus, the insertion of \( \theta = 2\pi \) flux shifts \( E_n \rightarrow E_{n-1} \) resulting in an anomalous spectral flow, as shown in Fig. 1a. This is in contrast to the case of a topologically trivial system where there is no net shift and, zero and \( 2\pi \) fluxes are equivalent to each other. However, in quantum Hall systems, these linear-dispersion mid-gap modes continuously interpolate between two topologically distinct bulk bands and the difference between the Chern numbers of these bulk bands dictates the shift, and hence the winding number of the edge states. For winding numbers larger than one \( (k > 1) \), there exist \( k \) edge modes in the bandgap and hence \( k \) copies of this relation, that is for a unit quantum of flux insertion, the edge state resonances shift by \( k \) units. Such a spectral flow is similar to Laughlin’s charge pump23,26 but with an important distinction: here, the gauge flux is coupled only to the edge of the system (see Supplementary Information). Moreover, the observable in our interferometric measurement is expected to be an integer, in contrast to a previous scheme27 that measured Hall drift, a continuous variable. More generally, our scheme provides a powerful universal probe (independent of microscopic details) of topological order that can be generalized to the case of strongly correlated topological systems.

To experimentally observe and measure this spectral flow, we implement the integer quantum Hall model in a photonic system: a 2D square annulus of ring resonators with a uniform synthetic magnetic field in the bulk and a tunable gauge field coupled only to the lattice edge (Fig. 1b–e). The uniform magnetic field with flux \( \phi_e = 2\pi/4 \) radians per plaquette is synthesized using asymmetric placement of site and link resonators, as previously described in ref. 10. To couple a tunable gauge field to the edge, we fabricate metal heaters above the link ring waveguides on the lattice edge (Fig. 1c–e). These heaters use the thermo-optic effect to modify

---

1Joint Quantum Institute, NIST/University of Maryland, College Park, Maryland 20742, USA. 2Department of Electrical Engineering and Institute for Research in Electronics and Applied Physics, University of Maryland, College Park, Maryland 20742, USA. 3Condensed Matter Theory Center, University of Maryland, College Park, Maryland 20742, USA. 4Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14853, USA. *e-mail: hafezi@umd.edu
Figure 1 | Schematic of the set-up used to measure the winding number. a, Spectral flow of edge states with coupled gauge flux \( \theta \) for \( q = 1 \) and \( k = 1 \). When \( \theta = 2\pi \), edge states move by one position. b,c, Square annulus of coupled ring resonators that implements the integer quantum Hall effect, with an outer edge of 10 sites and an inner edge of 4 sites. The uniform synthetic magnetic field coupled only to the edge states is introduced by fabricating heaters on link resonators situated on the lattice edges. The total field coupled only to the edge states is introduced by fabricating heaters on link resonators situated on the lattice edges. The total flux \( \theta \) introduced by the heaters is the sum of the individual phases incurred in the link resonator arms. d, SEM image showing the heaters fabricated on top of the link resonators (green). e, Schematic of the waveguide cross-section showing the ring resonators, the metal heaters and the metal routing layer.

Figure 2 | Anomalous spectral flow of edge state resonances. a,b, Measured (a) and simulated (b) transmission (\( T \), in linear scale) as a function of \( \theta \) and \( \omega \) (frequency) for spin-up excitation. For a 2\( \pi \) increase in gauge flux, the edge state resonances move by one position, giving \( k = +1.0 \) (1) for CW edge band and \( k = -1.0 \) (2) for the ACW band. Insets: Zoom-in of the edge state bands. c,d, Measured (c) and simulated (d) transmission spectra (log scale) for \( \theta/2\pi = 0 \), 1. For this 2\( \pi \) increase in flux, the measured spectra approximately match in the edge state regions. The green and red shaded regions indicate the CW and ACW edge state bands, respectively. The transmission is normalized such that the maximum is unity. Here \( \omega_0 \) is the resonance frequency of the resonators and \( J \) is the coupling rate between site resonators.

the accumulated phase of light propagating through the waveguides and hence result in a gauge flux.

Our system supports two pseudo-spin components, up and down, which circulate in opposite directions in the ring resonators\(^{10}\). Each pseudo-spin component experiences a uniform magnetic field, and we can selectively excite each pseudo-spin by appropriately choosing the input port (Fig. 1c). Furthermore, with this particular choice of uniform magnetic field, the energy spectrum exhibits two bandgap regions\(^{3}\). For spin-up excitation, the low-energy bandgap corresponds to clockwise (CW) circulating edge states confined on the outer edge, and anticlockwise (ACW) circulating edge states on the inner edge, as shown with green arrows in Fig. 1b with their corresponding winding numbers. Similarly, the high-energy bandgap corresponds to edge states both on the outer and the inner edges, however, they circulate opposite to those in the low-energy bandgap, as shown in Fig. 1b by red arrows. A pseudo-spin flip swaps the position of CW and ACW edge bands in the transmission spectrum, and also flips the sign of gauge flux \( \theta \) (ref. 29).

Figure 2a,b show the measured and simulated transmission spectra as a function of \( \theta \) for spin-up excitation. The edge states of the outer edge (bright regions) and the bulk states (dark region
in the middle) are easily identifiable. As $\theta$ increases, the energy of the CW edge states decreases, whereas the energy of the ACW edge states increases. For a $2\pi$ increase in flux, the edge state resonances move by one resonance to replace the position once held by their neighbours. This flow indicates that the measured winding number is $k = +1.0 \pm 1$ for the CW circulating edge states, and $k = -1.0 \pm 2$ for the ACW band. The transmission spectra shown in a,c are from two different devices.

Figure 3 | Local nature of coupled gauge flux and spectral flow for spin-flip excitation. a,b, Measured (a) and simulated (b) transmission spectra (linear scale, normalized to unity) as a function of $\theta$ when only the inner edge is heated. The transmission resonances of the edge states associated with the outer edge do not change. c,d, Measured (c) and simulated (d) transmission spectra for spin-down excitation. The CW and ACW circulating states have now exchanged positions and move in opposite directions (compared with Fig. 2a,b), giving $k = 1.2 \pm 2$ for the CW edge band and $k = -1.2 \pm 2$ for the ACW band.

Figure 4 | Absence of spectral flow in a topologically trivial ring geometry. a, Schematic of a ring geometry used to investigate gauge coupling to topologically trivial states. The ring supports CW and ACW circulating modes. External flux can be coupled to these states by using heaters on the link resonators. b,c, Measured (b) and simulated (c) transmission as a function of $\theta$ and $\omega$. The CW and ACW modes move in opposite directions but for a unit increase in flux they do not change position. d,e, Measured (d) and simulated (e) transmission spectra at $\theta = 0$, $2\pi$ overlap well.

in the middle) are easily identifiable. As $\theta$ increases, the energy of the CW edge states decreases, whereas the energy of the ACW edge states increases. For a $2\pi$ increase in flux, the edge state resonances move by one resonance to replace the position once held by their neighbours. This flow indicates that the measured winding number is $k = +1.0 \pm 1$ for the CW circulating edge states, and $k = -1.0 \pm 2$ for the ACW circulating edge states. We note that only the outer edge contributes to the transmission spectra, because the edge states are tightly confined to the lattice boundary. Moreover, such sharpness of edge states allows us to locally create the extra gauge flux $\theta$ by placing the heaters only on the outermost rings at the boundary. Also note that the flow of resonances observed here is very distinct in appearance and physical origin from what is observed in a Fabry-Perot cavity, where increasing the length of the cavity always reduces the resonant frequencies. However, in our observations, CW and ACW edge state resonances shift in opposite directions, and resonance peaks shift exactly by their winding number.

Figure 2c,d show the overlap of the observed and simulated transmission spectra at $\theta/2\pi = 0$ and 1, showing an excellent qualitative agreement between them. We attribute the small discrepancy in the observed and simulated results to lattice disorder introduced by the non-local response of the heaters (see Supplementary Information). The discrepancy is more pronounced for the ACW state because of the finite size effects in travelling a shorter path from the input to the output port. Also note that compared with measured spectra, the simulated spectra show high transmission through the two bulk bands. This is because the bulk bands are
not topologically protected and therefore disorder in the experimen-
tal system suppresses transmission through these bands.

To verify the local character of the gauge coupling, we selectively
couple a non-zero flux $\theta$ to the inner edge by heating the link rings
on the inner edge. We observe that heating only the inner edge does
not shift the edge states (Fig. 3a,b), which indicates the strong con-
finement of edge states at the lattice boundary. Furthermore, we
excited the system with a flipped pseudo-spin (compared with Fig. 2).
Figure 3c,d shows the measured and simulated transmission spec-
tra as a function of $\theta$. As the spin flip reverses both the sign of
the coupled flux $\theta$ and the position of the edge bands in the trans-
mittance spectrum, the resulting spectrum is similar to Fig. 2.

As a control experiment, to investigate the difference in gauge
coupling to chiral and non-chiral states, we fabricate a ring of reso-
nators (Fig. 4a) that corresponds to 1D tight-binding model. This
system supports topologically trivial Bloch modes, circulating in the
CW and ACW directions, which are degenerate in the absence of
a magnetic flux. Figure 4bc show the experimentally observed and
the simulated transmission spectra as a function of $\theta$. The
flux lifts the degeneracy of the CW and ACW states and with an
increase in the flux, these states move in opposite directions.
Therefore, for $\theta = 2\pi n$, there is no net shift of the resonances, with
measured $k = 0.03(8)$. The absence of spectral flow proves that this
system is topologically trivial. As shown in Fig. 4d, we observe the
transmission spectra at $\theta/2\pi n = 0$, 1, exhibit an excellent overlap.
However, compared with the simulation, we realize that not all
the states can be resolved in the experimental data. Such discrepancy
is due to the absence of topological protection of non-chiral states
against disorder, which leads to coupling between the CW and
ACW modes.

Here, we used a simple model to describe spectral flow of a chiral
device coupled to a gauge field, for non-interacting bosons. However,
the emergence of anomalous spectral flow is more general1–3, and
can be modelled using the elegant framework of conformal field
theory of chiral bosons, where this behaviour is known as a chiral anomalous4,5,33. This framework allows the investigation of systems
with strong photon–photon interaction, such as circuit–QED2,2 and
coupled cavity systems, and also ultracold atoms with topological
features. In particular, the presence of strong interactions could
lead to the emergence of various fractional quantum Hall
states in the steady-state regime. Spectral flow could be an interest-
ning way to explore the fascinating features of many-body topological
states in systems with synthetic gauge fields.

Methods

Methods and any associated references are available in the online
version of the paper.

Received 24 August 2015; accepted 13 January 2016; published online 22 February 2016

References

1. Wen, X.-G. Quantum Field Theory of Many-Body Systems (Oxford Univ. Press, 2004).

2. Bernevig, B. & Hughes, T. Topological Insulators and Topological Superconductors (Princeton Univ. Press, 2013).

3. Hatsugai, Y. Chern number and edge states in the integer quantum Hall effect. Phys. Rev. Lett. 71, 3697–3700 (1993).

4. Miyake, H. et al. Realizing the Harper Hamiltonian with laser-assisted tunneling in optical lattices. Phys. Rev. Lett. 111, 185302 (2013).

5. Jotzu, G. et al. Experimental realization of the topological Haldane model with ultracold fermions. Nature 515, 237–240 (2014).

6. Spielman, I. B. Detection of topological matter with quantum gases. Ann. Phys. 525, 597–807 (2013).

7. Aidelsburger, M. et al. Measuring the Chern number of Hofstadter bands with ultracold bosonic atoms. Nature Phys. 11, 162–166 (2015).

8. Hafezi, M. & Taylor, J. M. Topological physics with light. Phys. Today 67, 68–69 (2014).

9. Lu, L. et al. Topological photonics. Nature Photon. 8, 821 (2014).

10. Hafezi, M. et al. Imaging topological edge states in silicon photonics. Nature Photon. 7, 1001–1005 (2013).

11. Rechtsman, M. C. et al. Photonic Floquet topological insulators. Nature 496, 196–200 (2013).

12. Wang, Z. et al. Observation of unidirectional backscattering-immune topological electromagnetic states. Nature 461, 772–775 (2009).

13. Mittal, S. et al. Topologically robust transport of photons in a synthetic gauge field. Phys. Rev. Lett. 113, 087403 (2014).

14. Kraus, Y. et al. Topological states and adiabatic pumping in quasicrystals. Phys. Rev. Lett. 109, 106402 (2012).

15. Hu, W. et al. Measurement of a topological edge invariant in a microwave network. Phys. Rev. X 5, 011012 (2015).

16. Zeuner, J. M. et al. Observation of a topological transition in the bulk of a non-hermitian system. Phys. Rev. Lett. 115, 040402 (2015).

17. Umucalilar, R. & Carusotto, I. Artificial gauge field for photons in coupled cavity arrays. Phys. Rev. A 84, 043804 (2011).

18. Fang, K., Yu, Z. & Fan, S. Realizing effective magnetic field for photons by controlling the phase of dynamic modulation. Nature Photon. 6, 782–787 (2012).

19. Khanikaev, A. B. et al. Photonic topological insulators. Nature Mater. 12, 233–239 (2013).

20. Tzeng, L. D. et al. Non-reciprocal phase shift induced by an effective magnetic flux for light. Nature Photon. 8, 701–705 (2014).

21. Ma, T., Khanikaev, A. B., Mousavi, S. H. & Shvets, G. Guiding electromagnetic waves around sharp corners: topologically protected photonic transport in metawaveguides. Phys. Rev. Lett. 114, 124501 (2015).

22. Lu, L. et al. Experimental observation of Weyl points. Science 349, 622–624 (2015).

23. Ozawa, T. & Carusotto, I. Anomalous and quantum Hall effects in lossy photonic lattices. Phys. Rev. Lett. 112, 133902 (2014).

24. Hafezi, M. Measuring topological invariants in photonic systems. Phys. Rev. Lett. 112, 210405 (2014).

25. Atala, M. et al. Direct measurement of the Zak phase in topological Bloch bands. Nature Phys. 9, 795–800 (2014).

26. Bouca, L. et al. An Aharonov–Bohm interferometer for determining Bloch band topology. Science 347, 288–292 (2015).

27. Laughlin, R. Quantized Hall conductivity in two dimensions. Phys. Rev. B 23, 5632–5633 (1981).

28. Halperin, B. I. Quantized Hall conductance, current carrying edge states and extended states in 2D disordered potential. Phys. Rev. B 25, 2185–2190 (1982).

29. Hafezi, M. et al. Robust optical delay lines with topological protection. Nature Phys. 7, 907–912 (2011).

30. Levkivskyi, I. P. et al. Mach–Zehnder interferometry of fractional quantum Hall edge states. Phys. Rev. B 80, 045319 (2009).

31. Ganzhorn, S. Quantum Effects in Condensed Matter Systems in Three, Two and One Dimensions (PhD thesis, Stony Brook Univ. (2012).

32. Houck, A. A. et al. On-chip quantum simulation with superconducting circuits. Nature Phys. 8, 292–299 (2012).

33. Carusotto, I. & Ciuti, C. Quantum fluids of light. Rev. Mod. Phys. 85, 299–366 (2013).

34. Kapit, E. et al. Induced self-stabilization in fractional quantum Hall states of light. Phys. Rev. X 4, 031039 (2014).

Acknowledgements

This research was supported by the Air Force Office of Scientific Research grant no. FA9550-14-1-0267, Army Research Office, Office of Naval Research, Bethe postdoctoral fellowship, National Science Foundation Career grant, Laboratory for Physical and Condensed Matter Theory Center, Microsoft and the Physics Frontier Center at the Joint Quantum Institute. We thank M. Levin, A. G. Abanov, A. Lobos, A. Migdall and J. Taylor for fruitful discussions and E. Barnes, M. Davanco and E. Goldschmidt for useful comments on the manuscript.

Author contributions

S.M. and M.H. conceived and designed the experiment. S.M. and J.F. performed the experiment. All authors contributed significantly in analysing the data and editing the manuscript.

Competing financial interests

The authors declare no competing financial interests.
Methods

Heaters. The heaters used in these devices are 110 nm thick metal (Ti) pads, with typical resistance of 117 Ω, such that a current flowing through them generates heat and modifies the refractive index and the accumulated phase of light propagating through the waveguide. A typical heater requires ≈76.5 mW to induce a 2π phase. Instead of employing a single heater that may introduce a significant disorder in the lattice, we distribute the heaters along the edges while maintaining an accumulated flux of 2π (see Supplementary Information).

Measurement of winding number. To estimate the value of $k$, we measure the frequency shift of an edge state resonance for 2π flux insertion; and its separation from the neighbouring resonance at $\theta = 0$. The integer $k$, in a given band, is then the mean of the ratio of shift of resonances to their separation. The standard deviation of this ratio is the error on $k$. 