Field strength correlators in QCD: new fits to the lattice data

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Abstract

We discuss the results obtained by fitting the lattice data of the gauge–invariant field strength correlators in QCD with some particular functions which are commonly used in the literature in some phenomenological approaches to high–energy hadron–hadron scattering. A comparison is done with the results obtained in the original fits to the lattice data.

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1. Introduction

The gauge–invariant two–point correlators of the field strengths in the QCD vacuum are defined as
\[
D_{\mu\rho,\nu\sigma}(x) = g^2 \langle 0 \mid \text{Tr} \left\{ G_{\mu\rho}(0) S(0, x) G_{\nu\sigma}(x) S^\dagger(0, x) \right\} \mid 0 \rangle ,
\] (1.1)

where \( G_{\mu\rho} = T^a G^a_{\mu\rho} \) and \( T^a \) are the matrices of the algebra of the colour group SU\( (N_c) \) in the fundamental representation (For \( N_c = 3 \), \( T^a = \lambda^a/2 \), where \( \lambda^a \) are the Gell–Mann matrices). The trace in (1.1) is taken with respect to the colour indices. Moreover, in Eq. (1.1),
\[
S(0, x) \equiv \text{P exp} \left( ig \int_0^x dz^\mu A_\mu(z) \right) ,
\] (1.2)
with \( A_\mu = T^a A^a_\mu \), is the Schwinger phase operator needed to parallel–transport the tensor \( G_{\nu\sigma}(x) \) to the point 0. “P” stands for “path ordering”: for simplicity, we take \( S(0, x) \) along the straight–line path from 0 to \( x \).

These field–strength correlators play an important role in hadron physics. In the spectrum of heavy \( Q\bar{Q} \) bound states, they govern the effect of the gluon condensate on the level splittings [1, 2, 3]. They are the basic quantities in models of stochastic confinement [4, 5, 6] and in the description of high–energy hadron scattering [7, 8, 9, 10]. In some recent works [11, 12], these correlators have been semi–classically evaluated in the single–instanton approximation and in the instanton dilute–gas model, so providing useful information about the role of the semiclassical modes forming the QCD vacuum.

In the Euclidean theory, translational, \( O(4)– \) and parity invariance require the correlator (1.1) to be of the following form [1, 2, 3]:
\[
D_{\mu\rho,\nu\sigma}(x) = (\delta_{\mu\nu}\delta_{\rho\sigma} - \delta_{\mu\sigma}\delta_{\rho\nu}) \left[ D(x^2) + D_1(x^2) \right] + (x_\mu x_\nu \delta_{\rho\sigma} - x_\mu x_\sigma \delta_{\rho\nu} + x_\rho x_\sigma \delta_{\mu\nu} - x_\rho x_\nu \delta_{\mu\sigma}) \frac{\partial D_1(x^2)}{\partial x^2} ,
\] (1.3)
where \( D \) and \( D_1 \) are invariant functions of \( x^2 \).

These functions \( D(x^2) \) and \( D_1(x^2) \) have been directly determined (in the Euclidean theory) by numerical simulations on a lattice in the quenched (i.e., pure gauge) theory, with gauge–group SU\( (2) \) [13], in the quenched SU\( (3) \) theory in the range of physical distances between 0.1 and 1 fm [14, 15] and also in full QCD, i.e., including the effects of dynamical fermions [16]. In another approach [17], they have been extracted (in the
quenched SU(3) theory) from lattice calculations of the heavy–quark potential, under the assumptions of the model of the stochastic vacuum \cite{4, 5, 6}.

It is convenient to define a $\mathcal{D}_\parallel(x^2)$ and a $\mathcal{D}_\perp(x^2)$ as follows:

\[
\begin{align*}
\mathcal{D}_\parallel &\equiv \mathcal{D} + D_1 + x^2 \frac{\partial D_1}{\partial x^2}, \\
\mathcal{D}_\perp &\equiv \mathcal{D} + D_1. 
\end{align*}
\]  

(1.4)

In Figs. 1–2 we display the results for $\mathcal{D}_\parallel/\Lambda_L^4$ and $\mathcal{D}_\perp/\Lambda_L^4$ versus the physical distance in fermi units, obtained in the quenched (i.e., pure–gauge) theory, with gauge group SU(3): data are taken from Refs. \cite{14, 15}. $\Lambda_L$ is the fundamental constant of QCD in the lattice renormalization scheme, in the pure–gauge case: its value, extracted from the string tension \cite{18}, turns out to be about 4.92 MeV for the gauge group SU(3) and in the absence of quarks.

In Fig. 3 we display the results for $\mathcal{D}_\parallel/\Lambda_F^4$ and $\mathcal{D}_\perp/\Lambda_F^4$ versus the physical distance in fermi units, obtained in full QCD, with gauge group SU(3) and $N_f = 4$ flavours of staggered fermions with a quark mass $m_q = 0.01$ in lattice units: data are taken from Ref. \cite{16}. $\Lambda_F$ is an effective $\Lambda$–parameter for QCD in the lattice renormalization scheme, for the gauge group SU(3) and $N_f = 4$ flavours of quarks. It was defined in Ref. \cite{16}, where the value $\Lambda_F \simeq 1.07$ MeV was derived and used.

In Refs. \cite{4, 5, 10} best fits to the lattice data with the functions

\[
\begin{align*}
\mathcal{D}(x^2) &= A_0 e^{-|x|/\lambda_A} + \frac{a_0}{|x|^4} e^{-|x|/\lambda_a}, \\
\mathcal{D}_1(x^2) &= A_1 e^{-|x|/\lambda_A} + \frac{a_1}{|x|^4} e^{-|x|/\lambda_a}, 
\end{align*}
\]  

(1.5)

were performed: the results obtained for all the various cases are reported in Table I. From these results one sees that, in order to obtain a fit with a good $\chi^2/N$, it is necessary to include a perturbative–like term behaving as $1/|x|^4$ (indeed, a term of this form is predicted by ordinary perturbation theory; see for example Ref. \cite{19} and references therein) in addition to the nonperturbative exponential term in the parametrization for both $\mathcal{D}$ and $\mathcal{D}_1$. In Table I we also report the results from the fits to the restricted set of data of the correlators obtained in Ref. \cite{4}, corresponding to physical distances $|x| \geq 0.4$ fm: in this case a fit with only the nonperturbative exponential terms for $\mathcal{D}$ and $\mathcal{D}_1$ turns out to be acceptable.
In this paper we report the results obtained by fitting the lattice data of the gauge-invariant field strength correlators in QCD, displayed in Figs. 1–4, with some particular functions which are commonly used in the literature in some phenomenological approaches to high-energy hadron–hadron scattering. Therefore, we think that these results may be useful to those people working with this alternative parametrization. In Sect. 2 we give a brief review on the new parametrization used for the best fits and the results so obtained are discussed. In Sect. 3 some quantities of physical interest are extracted from our best fits and compared with the corresponding results obtained in the original fits to the lattice data.

2. New fits to the lattice data

In this section we discuss the results obtained by fitting the lattice data reported in Figs. 1–4 with the following functions:

\[
D(x^2) = \frac{\pi^2}{3} \kappa G_2 \tilde{D}(x^2) + \frac{a_0}{|x|^4} e^{-|x|/\lambda a}, \\
\tilde{D}_1(x^2) = \frac{\pi^2}{6} (1 - \kappa) G_2 \tilde{D}_1(x^2) + \frac{a_1}{|x|^4} e^{-|x|/\lambda a},
\]

(2.1)

where \(\tilde{D}(x^2)\) and \(\tilde{D}_1(x^2)\) are so defined:

\[
\tilde{D}(x^2) = \left( \frac{3\pi |x|}{8a} \right) K_1 \left( \frac{3\pi |x|}{8a} \right) - \frac{1}{4} \left( \frac{3\pi |x|}{8a} \right)^2 K_0 \left( \frac{3\pi |x|}{8a} \right), \\
\tilde{D}_1(x^2) = \left( \frac{3\pi |x|}{8a} \right) K_1 \left( \frac{3\pi |x|}{8a} \right).
\]

(2.2)

\(K_0\) and \(K_1\) are the modified Bessel functions. These expressions for the “nonperturbative” parts of \(D\) and \(\tilde{D}_1\), i.e., the expressions reported in Eqs. (2.1) and (2.2), with the exclusion of the “perturbative–like” \(1/|x|^4\) pieces, are extensively used in the literature in many phenomenological approaches to high-energy hadron–hadron scattering (see for example Refs. [10, 20, 21] and references therein). They were proposed for the first time in Ref. [10], where also a preliminary fit to lattice data was performed. However, at that time only lattice data obtained in the quenched SU(3) theory, in the range of physical distances
between 0.4 and 1 fm, were available [14]. Therefore, we think that a re–analysis of this parametrization, fitting also the new lattice data now available in the quenched SU(3) theory in the range of physical distances between 0.1 and 1 fm [15] and in full QCD [16], may be useful for the practitioners in this field.

Before discussing the results of the fits, we shall remind the reader of some technical details about the parametrization (2.1). The functions $\tilde{D}(x^2)$ and $\tilde{D}_1(x^2)$ are normalized to 1 in $x = 0$:

$$\tilde{D}(0) = \tilde{D}_1(0) = 1.$$  

(2.3)

As we shall discuss in the next section, this implies that the parameter $G_2$ in Eq. (2.1) should be identified with the gluon condensate. The parameter $\kappa$ measures the non–Abelian character of the correlator: in fact, $\kappa = 0$ in an Abelian theory, if there are no magnetic monopoles, while there is no reason for the $D$–term to vanish in a non–Abelian theory. The expression for $\tilde{D}(x^2)$ in Eq. (2.2) comes from the following ansatz:

$$\tilde{D}(x^2) = \frac{27}{64} a^{-2} \int d^4 k e^{i k x} \frac{k^2}{\left[k^2 + \left(\frac{3\pi}{8\kappa}\right)^2\right]^2},$$  

(2.4)

where the length–scale $a$ enters into this parametrization as a “correlation length”, being defined as

$$a \equiv \int_0^{+\infty} d|x|\tilde{D}(x^2).$$  

(2.5)

The correlation function $\tilde{D}(x^2)$ is negative at large distances, with the following asymptotic behaviour:

$$\tilde{D}(x^2) \sim -\frac{1}{4} \sqrt{\frac{\pi}{2}} \left(\frac{3\pi|x|}{8a}\right)^\frac{3}{2} \exp\left(-\frac{3\pi|x|}{8a}\right).$$  

(2.6)

The function $\tilde{D}_1(x^2)$ is chosen such that

$$\left(4 + x_\mu \frac{\partial}{\partial x_\mu}\right) \tilde{D}_1(x^2) = 4\tilde{D}(x^2),$$  

(2.7)

which leads to

$$\tilde{D}_1(x^2) = \frac{1}{|x|^4} \int_0^{x^2} dz^2 \tilde{D}(z^2),$$  

(2.8)

and then to the expression in Eq. (2.2).
The results of the fits to the lattice data using the parametrization (2.1)–(2.2) for $D$ and $D_1$ are reported in Table II. The continuum lines in Figs. 1–4 have been obtained using the parameters of these best fits, in the cases where all the parameters were free. The dashed lines in Figs. 1–2 correspond to the nonperturbative parts only in Eq. (2.1) and are derived using the same parameters used for the corresponding continuum lines. They are drawn in this particular case (quenched SU(3) theory) in order to illustrate the role of the perturbative–like terms.

From the results in Table II, one sees that, in order to obtain fits with a good $\chi^2/N$, it is necessary to include the perturbative–like terms $\sim 1/|x|^4$ in the parametrization for $D$ and $D_1$. In fact one also sees directly from Figs. 1–2 that these terms are necessary to well describe the behaviour of the correlators at small distances (down to 0.1 fm), while they are less important in the range of distances between 0.4 and 1 fm. The coefficients $a_0$ and $a_1$ turns out to be comparable with (even if slightly smaller than) the coefficients derived in the corresponding cases in Table I, obtained using the parametrization (1.5). Even restricting the set of quenched data to those obtained in Ref. [14], corresponding to physical distances $|x| \geq 0.4$ fm, one finds that a fit with only the nonperturbative terms in Eq. (2.1), i.e., fixing $a_0 = a_1 = 0$, is more acceptable (when compared to the same fit applied to the entire set of data between 0.1 and 1 fm), but the $\chi^2/N$ is still too high ($\sim 3.8$). When all the parameters in the fit are free, the $\chi^2/N$ turns out to be acceptable in all the various cases examined, even if it is systematically a bit larger than the $\chi^2/N$ obtained in the corresponding cases examined in Table I, using the original parametrization (1.5).

Therefore, we can conclude that the expressions (2.1)–(2.2) are a good parametrization of the correlators $D$ and $D_1$, in the range of physical distances where the lattice data are available (i.e., 0.1–1 fm for quenched QCD and 0.3–1 fm for full QCD). However, the parametrization (1.5) appears to be slightly preferable. In the next section we shall discuss some quantities of physical interest which can be extracted from the results of the best fits in Tables I and II.

3. Discussion

Three quantities of physical interest can be extracted from our fits to the lattice data:
The correlation length $l_G$ of the gluon field strengths, defined as (“np” stands for “nonperturbative”, in the sense explained in the previous sections)

$$l_G \equiv \frac{1}{\mathcal{D}^{(np)}(0)} \int_0^{+\infty} d|x| \mathcal{D}^{(np)}(x^2) . \quad (3.1)$$

The so-called “gluon condensate”, defined as

$$G_2 \equiv \langle \frac{\alpha_s}{\pi} : G_{\mu\nu}^a G_{\mu\nu}^a : \rangle \quad (\alpha_s = \frac{g^2}{4\pi}) . \quad (3.2)$$

The parameter $\kappa$, defined as

$$\kappa = \frac{\mathcal{D}^{(np)}(0)}{\mathcal{D}^{(np)}(0) + \mathcal{D}_1^{(np)}(0)} . \quad (3.3)$$

The results obtained are summarized in Table III.

The quantities $l_G$ and $G_2$ play an important role in phenomenology. The correlation length is relevant for the description of vacuum models [4, 5, 6]. The relevance of the gluon condensate was first pointed out by Shifman, Vainshtein and Zakharov (SVZ) [22]. It is a fundamental quantity in QCD, in the context of the SVZ sum rules.

The physical meaning of the $\kappa$ parameter has been already discussed in the previous section: it measures the non–Abelian character of the correlator, since one expects that $\kappa = 0$ in an Abelian theory with no magnetic monopoles present. This parameter appears explicitly in the parametrization (2.1), by virtue of Eq. (2.3). Instead, using the parametrization (1.5) one finds the following expression for $\kappa$:

$$\kappa = \frac{A_0}{A_0 + A_1} . \quad (3.4)$$

From the results reported in Table III, one sees that, in both parametrizations (1.5) and (2.1)–(2.2), the parameter $\kappa$ appears to decrease when increasing the quark mass, tending towards the pure–gauge value (obviously, when evaluating the field strength correlators (1.1), the quenched, i.e., pure–gauge, limit coincides with the large quark–mass limit, $m_q \to \infty$). In other words, the non–Abelian character of the correlator $\mathcal{D}$ appears to
increase when approaching the chiral limit \( m_q \to 0 \). The values of \( \kappa \) obtained using the parametrization (2.1)–(2.2) in all the cases examined are a bit larger than the corresponding values of \( \kappa \) obtained using the parametrization (1.5).

Now let us discuss the results for the correlation length \( l_G \). Using the parametrization (1.5) one easily finds \( l_G = \lambda_A \), while in the parametrization (2.1)–(2.2) one has \( l_G = a \), according to Eqs. (2.5) and (2.3). From the results in Table III, one sees that both \( \lambda_A \) and \( a \) decrease by increasing the quark mass, when going from chiral to quenched QCD. Obviously, the difference between \( \lambda_A \) and \( a \) is due to the different parametrization used for the correlators.

Now we come to the gluon condensate. As pointed out in Ref. [16], the lattice provides us with a regularized determination of the correlators. We shall briefly repeat here the argumentation originally reported in Ref. [16], for the benefit of the reader. At small distances \( x \) a Wilson operator–product–expansion (OPE) [23] is expected to hold. The regularized correlators will then mix to the identity operator \( \mathbf{1} \), to the renormalized local operators of dimension four, \( \frac{\alpha_s}{\pi} : G_{\mu\nu}^a G_{\mu\nu}^a : \) and \( m_f : \bar{q}_f q_f : \) (\( f = 1, \ldots, N_f \), \( N_f \) being the number of quark flavours), and to operators of higher dimension:

\[
\frac{1}{2\pi^2} D_{\mu\nu,\mu'\nu'}(x) \sim C_1(x)\langle \mathbf{1} \rangle + C_g(x)G_2 + \sum_{f=1}^{N_f} C_f(x)m_f\langle : \bar{q}_f q_f : \rangle + \ldots \ . \tag{3.5}
\]

The mixing to the identity operator \( C_1(x) \) shows up as a \( c/|x|^4 \) behaviour at small \( x \). The mixings to the operators of dimension four \( C_g(x) \) and \( C_f(x) \) are expected to behave as constants for \( x \to 0 \), while the other Wilson coefficients in the OPE (3.5) are expected to vanish when \( x \to 0 \) (for dimensional reasons). The coefficients of the Wilson expansion are usually determined in perturbation theory and are known to be plagued by the so–called “infrared renormalons” (see for example Ref. [24] and references therein). In the same spirit of Ref. [16], we shall assume that the renormalon ambiguity can be safely neglected in the extrapolation for \( x \to 0 \) of our correlators. With the normalization of Eq. (3.5), this gives \( C_g(0) \simeq 1 \). On the same line, the contribution from the quark operators in (3.5) can be neglected, for the reasons explained in Ref. [16]. Within these approximations, one immediately recognizes that the parameter \( G_2 \) in the parametrization (2.1)–(2.2), with the normalization condition (2.3), coincides with the gluon condensate as defined in (3.2). Moreover, when using the parametrization (1.5) for the correlators, one obtains the
following expression for the gluon condensate:

\[ G_2 \simeq \frac{6}{\pi^2} (A_0 + A_1) \, . \]  

(3.6)

For both parametrizations (1.5) and (2.1)–(2.2), the gluon condensate \( G_2 \) appears to increase with the quark mass, as expected [25], tending towards the asymptotic (pure–gauge) value. However, the values of \( G_2 \) extracted from the fits using the parametrization (2.1)–(2.2) are more than a factor two smaller than the corresponding values of \( G_2 \) extracted from the fits using the parametrization (1.5), in all the cases examined.

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Table Captions

Tab. I. Results obtained from a best fit to the data of the gauge–invariant field strength correlators with the functions (1.5), in the various cases that we have examined: “q” stands for “quenched” data in the range of physical distance between 0.1 and 1 fm [13]; “q*” stands for “quenched” data in the range of physical distance between 0.4 and 1 fm [14]; “f (I)” and “f (II)” stand for “full–QCD” data with quark mass (in lattice units) 0.01 and 0.02 respectively [10]. An asterisk (*) near the value of some parameter means that the parameter was fixed to that value.

Tab. II. Results obtained from a best fit to the data of the gauge–invariant field strength correlators with the functions (2.1)–(2.2), in the various cases that we have examined: the notation used is the same as in Table I.

Tab. III. The values of some quantities of physical interest extracted from the best–fit results in Tables I (“exp”) and II (“bessel”) in the cases where all the parameters were free. The notation used is the same as in Tables I and II. Reported errors refer only to our determination and do not include the uncertainty on the physical scale.
| theory | $A_0/\Lambda^4 \times 10^{-8}$ | $A_1/\Lambda^4 \times 10^{-8}$ | $1/(\lambda A \Lambda)$ | $a_0$ | $a_1$ | $1/(\lambda a \Lambda)$ | $\chi^2/N$ |
|--------|-----------------|-----------------|-----------------|-----|-----|-----------------|---------|
| q      | 3.34(20)        | 0.70(10)        | 182(3)          | 0.69(6) | 0.46(3) | 94(15)         | 1.7     |
| q      | 8.37(22)        | 2.94(9)         | 233(2)          | 0(*)  | 0(*)  | 0(*)           | 33      |
| q*     | 3.11(61)        | 0.83(23)        | 183(8)          | 0.22(12) | 0.12(7) | 0(343)         | 1.4     |
| q*     | 3.62(19)        | 1.23(7)         | 183(8)          | 0(*)  | 0(*)  | 0(*)           | 1.3     |
| f (I)  | 174(24)         | 20(10)          | 544(27)         | 0.71(3) | 0.45(3) | 42(11)         | 0.5     |
| f (I)  | 438(17)         | 303(9)          | 642(8)          | 0(*)  | 0(*)  | 0(*)           | 51      |
| f (II) | 348(42)         | 46(21)          | 631(23)         | 0.66(3) | 0.39(3) | 61(20)         | 0.7     |
| f (II) | 734(21)         | 354(10)         | 713(7)          | 0(*)  | 0(*)  | 0(*)           | 27      |

| theory | $G_2/\Lambda^4 \times 10^{-8}$ | $\kappa$ | $1/(a \Lambda)$ | $a_0$ | $a_1$ | $1/(\lambda a \Lambda)$ | $\chi^2/N$ |
|--------|-----------------|---------|-----------------|-----|-----|-----------------|---------|
| q      | 0.95(5)         | 0.89(2) | 122(2)          | 0.56(4) | 0.38(3) | 48(12)         | 2       |
| q      | 2.36(5)         | 0.82(1) | 150(1)          | 0(*)  | 0(*)  | 0(*)           | 54      |
| q*     | 0.62(8)         | 0.85(4) | 116(3)          | 0.50+0.25−0.07 | 0.22+0.10−0.04 | 0(33)   | 1.5    |
| q*     | 1.14(4)         | 0.80(1) | 122(1)          | 0(*)  | 0(*)  | 0(*)           | 3.8     |
| f (I)  | 53(7)           | 0.95(5) | 387(16)         | 0.66(2) | 0.41(2) | 0.0(1)        | 0.5     |
| f (I)  | 185(4)          | 0.60(1) | 457(4)          | 0(*)  | 0(*)  | 0(*)           | 84      |
| f (II) | 97(10)          | 0.94(3) | 436(13)         | 0.62(3) | 0.36(2) | 0.0(5)        | 1       |
| f (II) | 284(5)          | 0.73(1) | 502(3)          | 0(*)  | 0(*)  | 0(*)           | 53      |

| theory | fit | $G_2$ (GeV$^4$) | $\kappa$ | $\lambda A$, $a$ (fm) |
|--------|-----|----------------|---------|-----------------------|
| f (I)  | exp | 0.015(3)       | 0.90(6) | 0.34(2)               |
| f (I)  | bessel | 0.007(1)       | 0.95(5) | 0.48(2)               |
| f (II) | exp | 0.031(5)       | 0.88(6) | 0.29(1)               |
| f (II) | bessel | 0.0127(13)     | 0.94(3) | 0.42(1)               |
| q      | exp | 0.144(11)      | 0.83(3) | 0.220(4)              |
| q      | bessel | 0.056(3)       | 0.89(2) | 0.328(5)              |
FIGURE CAPTIONS

Fig. 1. The function $D_\parallel/\Lambda_F^4$ versus the physical distance in fermi units, obtained in the quenched (i.e., pure–gauge) theory, with gauge group SU(3) ($\Lambda_L \simeq 4.92$ MeV). Data are taken from Refs. [14, 15], while the curves are obtained from our best fit with the functions (2.1)–(2.2) [fit no. 1 in Table II]: the continuum line corresponds to the entire correlator, while the dashed line corresponds to its nonperturbative part only.

Fig. 2. The same as in Fig. 1 for the function $D_\perp/\Lambda_F^4$.

Fig. 3. The functions $D_\perp/\Lambda_F^4$ (upper curve) and $D_\parallel/\Lambda_F^4$ (lower curve) versus the physical distance in fermi units, obtained in full QCD, with gauge group SU(3) and $N_f = 4$ flavours of staggered fermions with a quark mass $m_q = 0.01$ in lattice units ($\Lambda_F \simeq 1.07$ MeV). Data are taken from Ref. [16], while the curves are obtained from our best fit with the functions (2.1)–(2.2) [fit no. 5 in Table II].

Fig. 4. The same as in Fig. 3 for a quark mass $m_q = 0.02$ in lattice units ($\Lambda_F \simeq 1.07$ MeV). Data are taken from Ref. [16], while the curves are obtained from our best fit with the functions (2.1)–(2.2) [fit no. 7 in Table II].
quenched theory

corr_para

physical distance (fm)
quenched theory

corr_perp

physical distance (fm)
full QCD

(m = 0.01)
full QCD

\((m = 0.02)\)