Propagation mechanism of fracture zones in single-hole rock mass under high in-situ stress

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Abstract. The blasting process is affected by in-situ stress in deep buried tunnels. The design of blasting parameters with consideration of in-situ stress is important in damage control for deep buried tunnel excavation. In this work, a two-dimensional theoretical model of single-hole blasting crack propagation is established, and the relationship between crack propagation and in-situ stress is studied from the perspective of fracture mechanics. Using the finite element dynamic analysis software LS-DYNA, the single-hole blasting failure process under high in-situ stress is numerically simulated, and the crack propagation mechanism of single-hole blasting under high in-situ stress is explored. Results show that the ultimate crack growth length depends on the interaction of detonation gas and in-situ stress. The internal pressure of explosive gas and in-situ stress ($\sigma_x, \sigma_y$) are linearly superimposed on the boundary to control the stress intensity factor at the crack tip. The range of the fracture zone decreases as in-situ stress increases. When the stress level is less than 20 MPa, the change in the crack growth length is obvious with the increase of in-situ stress. When the stress level is greater than 20 MPa, the change degree of crack growth length is relatively small as in-situ stress increases. This result is due to the nonlinear pressure of the gas escaping along the crack direction. The relationship between crack growth length and in-situ stress is nonlinear and thus makes the stress intensity factor at the crack tip reach the critical threshold.

1. Introduction
Blast design for shallow blasting has matured with the development of the engineering rock blasting industry. For activities involving high in-situ stress, such as the mining of deep mines and deep traffic tunnels, their effects on the distribution of damage zones have been widely studied in a large volume of research, and the guiding effect of in-situ stress has been identified in single-hole blasting and group-hole blasting[1-3]. However, the use of empirical methods in engineering work has limited the research into the effects of in-situ stress during blasting excavation. The study on the formation mechanism of blasting damage zones under high confining pressure conditions is of great significance for group-hole blasting control technology. For mine blasting in particular, fragile support makes the damage control of pillars especially important[4,6]. A number of studies have explored blasting under high in-situ stress by conducting field tests and numerical simulations. Yang[11] reported crack propagation in a PMMA blasting simulation test on the basis of a caustic curve using a high-speed camera. The crack propagation mechanism under high in-situ stress conditions was observed, and the guiding effect of in-situ stress on the direction of the large principal stress was discussed. In-situ stress changed the crack fracture mode from mode I to mixed modes I–II. Peng[10] studied the distribution characteristics of cracks under nonuniform confining pressure by simulating charge initiation and rock blasting. The phenomenon in which blasting cracks always follow the initial stress direction in the presence of initial stress was noted and found to be consistent with the report by Yang. FV Donzé[1] used a discrete element method and then pointed out...
that crack propagation speed affects the final degree of radial crack propagation. The influence of the lateral pressure coefficient on crack propagation was qualitatively studied using a simulation model. Many studies have investigated the impact of blasting failure on rock mass. The current understanding is that the impact of explosive blasting on surrounding rocks is caused by the simultaneous action of detonation waves and blasting gas. The tangential tension generated by the stress compression wave causes radial cracks. Released strain energy causes circumferential cracks. According to previous studies, the current work proposes a theoretical calculation method for crack propagation length under in-situ stress conditions. Then, a finite element numerical simulation method is built to verify the theory. Rock blasting under hydrostatic pressure and nonuniform confining pressure given different in-situ stress conditions is also studied.

2 Theory of crack propagation under in-situ stress
The relationship between the dynamic fracture toughness of rocks and the stress intensity factors of crack tips is a decisive factor for crack growth propagation. The present research on the propagation of rock blasting cracks under blasting loads has mainly focused on the problem of one-dimensional cracks. Several studies have investigated two-dimensional cracks in in-situ stress fields. Numerical simulation is the main method for studying the impact of in-situ stress fields on rock fragmentation under two-dimensional in-situ stress conditions, and it reveals the mechanism of crack propagation under high in-situ stress conditions. However, this method is limited in terms of the description of the mechanism principle. In the current work, the crack propagation mode under the inhomogeneous in-situ stress field is discussed in combination with previous research results.

2.1 Stress intensity factor of crack tip
Suppose that a star-shaped crack is generated because of explosive detonation in a single hole. In all directions, the crack length is the same, and the total number of cracks is \(k\) \((k > 1\) and is an integer). To describe the nonuniform in-situ stress condition, this study subjects the supposed plane to the pressure of \(\sigma_x\) in the horizontal direction and \(\sigma_y\) in the vertical direction. In addition to in-situ stress effect, explosion gas pressure \(p\) perpendicularly acts on the crack surface. The theoretical model is shown in Figure 1.

\[
\begin{align*}
\sigma_x & \quad \sigma_y \\
\sigma_y & \quad \sigma_x
\end{align*}
\]

Figure 1. Schematic of theoretical model

Constructing a complex function is a conventional method for accurately calculating the stress intensity factors of crack tips. Chen obtained the stress intensity factor at the tip of a star crack under single confining pressure. In the current work, we extend the confining pressure condition from one dimension to two dimensions on the basis of Chen’s research. A conformal function of a plane figure is constructed to ensure the formation of a unit circle indicating the area on the plane excluding the crack. The conformal function is expressed as follows:

\[
\omega(\zeta) = a2^{\frac{3}{k}} \left[ (1 + \zeta^k)^{\frac{k}{2}} + \sqrt{(1 + \zeta^k)^{\frac{k}{2}}} \right]^{\frac{1}{k}}
\]

The relations for mode I cracks and mode II cracks are as follows:
\[ K_I^{\phi} - iK_{II}^{\phi} = 2\sqrt{\pi} \lim_{\zeta \to +\pi} \frac{\varphi(\zeta)}{n\omega(\zeta)} \]  \hspace{1cm} (2)

where \( K_I \) and \( K_{II} \) are the stress intensity factors for mode I cracks and mode II cracks, respectively. The stress intensity factors of the crack tips in different directions can be obtained by determining the complex potential on the \( z \)-plane. The two complex potentials on the plane can be written as follows:

\[ \varphi(\zeta) = \frac{1 + \mu}{8\pi} (X + iY) \ln \zeta + B(\zeta) + \varphi_0(\zeta) \]  \hspace{1cm} (3)

\[ \psi(\zeta) = \frac{1 - \mu}{8\pi} (X - iY) \ln \zeta + (B' + iC') \omega(\zeta) + \psi_0(\zeta) \]  \hspace{1cm} (4)

where \( \varphi(\zeta) \) and \( \psi(\zeta) \) are the functions related to the construction of a conformal function and \( X, Y, B, B', C' \) are the parameters related to the boundary conditions. The solution of the equation is determined by the boundary conditions, which reflect the information of external stress. Eq. (2) takes the following form\[^{19}\]:

\[ f = K_I^{\phi} - iK_{II}^{\phi} = \frac{Aa^2 \sin^2 \left( \frac{n\pi}{k} \right)}{\sqrt{\pi} \left[ 1 + (k - 1) e^{2\pi i} \right] \cos^2 \left( \frac{n\pi}{k} \right)} \]  \hspace{1cm} (5)

Where

\[ K_I^{\phi} = \text{Re}[f] = \frac{3^2 \frac{2}{a} A_I \sqrt{\pi a}}{\sqrt{k}} \]  \hspace{1cm} (6)

\[ K_{II}^{\phi} = \text{Im}[f] = \frac{3^2 \frac{2}{a} A_{II} \sqrt{\pi a}}{\sqrt{k}} \]  \hspace{1cm} (7)

where \( A, A_I \), and \( A_{II} \) are the parameters reflecting the stress boundary, with \( A_I \) and \( A_{II} \) being the boundary parameters of the stress intensity factors of modes I and II, respectively. These parameters can be obtained by the superposition of the stress intensity factors. The specific expressions are as follows:

\[ A_I = -\left[ p - \sigma_x \sin \left( \frac{2n\pi}{k} \right) \sigma_y \cos \left( \frac{2n\pi}{k} \right) \right] \]  \hspace{1cm} (8)

\[ A_{II} = \left[ \sigma_x + \sigma_y \cos \left( \frac{2n\pi}{k} \right) \sigma_y \sin \left( \frac{2n\pi}{k} \right) \right] \]  \hspace{1cm} (9)

The empirical formula method is adopted to calculate the explosive gas pressure \( p \) as follows:

\[ p = p_{\text{max}} \cdot \frac{ap_{\text{max}}}{v_{\text{f}}} \]  \hspace{1cm} (10)

where \( p \) is the explosion gas pressure, \( p_{\text{max}} \) is the maximum pressure of the explosion gas, \( \alpha \) is the crack length, \( v_{\text{f}} \) is the crack growth rate, and \( t_{\text{f}} \) is the decay time of the explosion gas.

The analysis of different quadrants complicates the problem because of the multivalency of complex powers. If \( k \) is an even number so that the analysis model is completely symmetrical, then the crack growth in the first quadrant is considered for the convenience of calculation.

### 2.2 Crack suspensive conditions

Maximum circumferential stress theory is applied to classify crack suspensive conditions for multiple crack modes in engineering. The theory compares the fracture toughness of rocks with superimposed results to determine the fracture angle. The basic assumptions are as follows:

1. The crack forms along the direction of the maximum circumferential stress;
(2) If the circumferential stress in this direction reaches a critical value, then the crack grows unstably. For I–II compound cracks, the stress at the crack tip is expressed as follows:

\[
\sigma_r = \frac{1}{2\sqrt{2\pi r}} [K_1 (3 - \cos \theta) \cos \frac{\theta}{2} + K_\| (3 \cos \theta - 1) \sin \frac{\theta}{2}]
\]

(11)

\[
\sigma_\theta = \frac{1}{\sqrt{2\pi r}} \cos \frac{\theta}{2} [K_1 \cos^2 \frac{\theta}{2} - \frac{3}{2} K_\| \sin \theta]
\]

(12)

\[
\tau_{r\theta} = -\frac{1}{2\sqrt{2\pi r}} \cos \frac{\theta}{2} [K_1 \sin \theta + K_\| (3 \cos \theta - 1)]
\]

(13)

where \( r \) is the length of the crack, \( \theta \) is the crack angle, \( \sigma_r \) is the radial stress at the crack tip, \( \sigma_\theta \) is the circumferential stress at the crack tip, and \( \tau_{r\theta} \) is the shear stress at the crack tip.

Within this model, we derive the following extreme conditions:

\[
\sin (3 \cos \theta) = 0
\]

(14)

According to the assumption (2), the fracture criterion is established as

\[
(\sigma_\theta)_{\text{max}} = (\sigma_\theta)_c
\]

(15)

\((\sigma_\theta)_c\) can be verified by testing the fracture toughness over mode I crack propagation.

The acceptable criterion for crack suspension is given as

\[
\cos \frac{\theta}{2} [K_1 \cos^2 \frac{\theta}{2} - \frac{3}{2} K_\| \sin \theta] \geq K_1 c
\]

(16)

3 Numerical simulation of blast cracks

3.1 Model failure criterion

A theoretical model can effectively illustrate the mechanism of crack growth. However, the theoretical description lacks essential intuitiveness. In the current work, numerical simulation is conducted using LS-DYNA to clearly describe the crack growth process. The formation of a crush zone and fracture zone around the blasthole can be observed during the explosion. The crush zone is mainly formed by compression and shear effect caused by the pressure of the blasting wave while the fracture zone is mainly formed by the tensile effect. When simulating the impact of blasting load on the rock mass, we combine the theoretical descriptions of the crush and fracture zones to determine the failure mode of the rocks. To simulate the failure elements, we delete the infinite elements that meet the failure criterion conditions by using the keyword *MAT_ADD_EROSION. The invalidation criteria are set as follows:

\[
\begin{cases}
\sigma > \sigma_{cd} \\
\sigma > \sigma_{id}
\end{cases}
\]

(17)

where \( \sigma_{cd} \) is the uniaxial dynamic compressive strength of the rock mass and \( \sigma_{id} \) is the uniaxial dynamic tensile strength of the rock mass.

3.2 Rock constitutive and explosive JWL equation

The instantaneous strain rate of the rock mass is extremely large during the blasting process. The rock mass model in this work is a plastic hardening model with a strain rate effect. The *MAT_PLASTIC_KINEMATIC model, which is a widely used plastic constitutive model that considers the change of strain rates, is also applied herein. The relevant rock material parameters are taken from Chentaigou Iron Mine, Anshan City, Liaoning Province. The specific parameters are shown in Table 1.

The fluid–solid coupling method is used to directly simulate the explosion process of high-energy explosives. The explosive unit volume expands and generates pressure to the surrounding medium.
after the explosive is ignited. This method is widely used and exerts good effects. Its state equation is the \textit{JWL} state equation written as
\begin{equation}
P_{\text{eos}} = A \left(1 - \frac{\omega}{R_1 V}\right) e^{-\frac{R_1 V}{V}} + B \left(1 - \frac{\omega}{R_2 V}\right) e^{-\frac{R_2 V}{V}} + \frac{\omega E_0}{V}\end{equation}
where $P_{\text{eos}}$ is the pressure determined by the \textit{JWL} state equation; $V$ is the relative volume; $E_0$ is the initial specific internal energy per unit volume; and $A$, $B$, $R_1$, $R_2$, and $\omega$ are the material constants related to explosives. Rock emulsion explosive #2 is selected in this work. The specific explosive parameters are shown in Table 2.

### Table 1 Rock mechanics parameters of rock mass

| Density (kg/m³) | Young’s modulus (GPa) | Poisson’s ratio | Bulk modulus (GPa) | Shear modulus (GPa) | Compressive strength (MPa) | Tensile strength (MPa) | Dynamic compressive strength (MPa) | Dynamic tensile strength (MPa) |
|-----------------|------------------------|----------------|------------------|-----------------|---------------------------|----------------------|----------------------------------|-------------------------------|
| 3460            | 24                     | 0.21           | 13               | 10              | 67                        | 4.2                  | 80                               | 25.2                          |

### Table 2 Explosive parameters of \textit{JWL} equation

| $P$ (kg/m³) | $D$ (m/s) | $A$ (GPa) | $B$ (GPa) | $R_1$ | $R_2$ | $\omega$ | $E_v$ (J/m³) |
|-------------|-----------|-----------|-----------|-------|-------|---------|---------------|
| 1200        | 4000      | 214.4     | 0.182     | 4.2   | 0.9   | 0.15    | 4.192×10⁹    |

### 3.3 Numerical model establishment

The diameter of the blast hole is 50 mm in the numerical simulation. The charge diameter is also 50 mm. To study the detailed process of rock mass blasting and fragmentation, we take the cross section perpendicular to the grain as the research object. The cross section is 2 m long and 2 m wide. Normal constraints are imposed on the blast hole axis. In this way, the actual state of the force of the blast hole is simulated. The schematic of the model is shown in Figure 2.

![Figure 2. Calculation model](image)

The model is used to simulate the rock fragmentation under the absence of in-situ stress. The calculation result is shown in Figure 3. The calculated crack length is 0.837 m, which is in good agreement with existing research results\cite{8,9,11}.

### 4 Analysis and discussion

#### 4.1 Crack propagation under different in-situ stress conditions

To analyze the rock fragmentation mechanism under low, moderate, and high in-situ stress levels, we select 10 working conditions in which the in-situ stress is 0–50 MPa with an interval of 5 MPa. The crack propagation mode under each working condition is studied. As the explosive grid cells are not completely symmetric during the calculation, the pressure caused by the explosive in all directions during the calculation is not consistent. Moreover, the crack length and damage zone in all directions are not exactly the same. The average value of the radii of the fracture and crush zones in the four directions is counted as the final value under this working condition. Then, the theoretical calculation
results are compared with the numerical results. The range of the fracture zone changes with in-situ stress, as shown in Figure 4.

**Figure 4.** Range of fracture zone changes with in-situ stress level

The results of the numerical simulation and theoretical calculation show that the development of blasting cracks is affected by in-situ stress. The development of the cracks is restrained by the in-situ stress field. The crack length decreases with the increase of in-situ stress. The expansion of the cracks in the fracture zone is mainly controlled by the tensile stress failure criterion. The clamping effect of the in-situ stress reduces the tensile stress of the unit and restricts the further expansion of cracks.

In Eqs. (6)–(9), the in-situ stress and pressure of the explosive gas are $p-\sigma_x$ and $p-\sigma_y$, respectively, for the stress boundary parameter. This relationship is linear to gas pressure. In the explosion process, the escape of explosive gas promotes the expansion of the crack process, and in-situ stress inhibits this process. High in-situ stress makes this phenomenon increasingly obvious. However, as in-situ stress increases, the extent of the reduction in the fracture range gradually decreases. The numerical simulation results show that when the local stress level increases from 0 MPa to 20 MPa, the range of the fracture area decreases by 48.5%. When the local stress increases from 0 MPa to 40 MPa, the range of the fracture zone is reduced by 62.9%. The theoretical results are consistent with the simulation results. For this nonlinear phenomenon, an explanation is given in Eqs. (8) and (9).

Although the relationship between gas pressure and in-situ stress is linear, the gas pressure is nonlinear with time and the displacement of the crack direction, thereby leading to such nonlinear results.

### 4.2 Influence of lateral pressure coefficient on the range of fracture zone

The fracture zone shows an irregular shape in the horizontal and vertical directions when the lateral pressure coefficient ($\eta$) is not equal to 1. The clamping effect is the strongest in the side with the highest stress and thus limits the further expansion of the fracture zone. A relatively long and short crack length side pattern is formed. The crack growth pattern given an in-situ stress level of 20 MPa and $\eta$ of 0.1 is shown in Figure 5.

To analyze the influence of $\eta$ on crack propagation under different in-situ conditions, we calculate the crack propagation when $\eta$ is 0.1–1.0 under the condition of 20 MPa. $\eta$ is controlled to be less than 1 to ensure the same in-situ stress. The calculation rules of the crushing and fracture zones are the same as those described previously. The calculation results are shown in Figure 6.
Figure 5. Single-hole crack propagation (when $p = 20$ MPa and $k = 0.1$)

Figure 6. Ratio of the long axis to the short axis of the crack zone under different in-situ stresses and different lateral pressure coefficients

As shown in Figure 6, when the initial in-situ stress level is the same, the ratio of the long axis to the short axis decreases and eventually approaches 1 as $\eta$ increases. As $\eta$ increases, the extent of ratio change is relatively reduced. Eqs. (6)–(9) show that $\eta$ reflects the changes in the magnitude of $\sigma_x$ and $\sigma_y$. For example, when $\eta$ is equal to 0.5, $\sigma_x = 0.5\sigma_y$; when $n = 0$, $A_1 = -p(\sigma_y)$, and $A_{||} = 0$; when $n = 4/k$, $A_1 = -p(\sigma_y)$, and $A_{||} = 0$. For the two different cracks which are perpendicular and parallel to the maximum principal stress, no shear stress is observed, the stress intensity factor of mode II is 0, and the magnitude of the stress intensity factor of mode I depends on $p$, $\sigma_x$, and $\sigma_y$. The greater the principal stress is, the stronger the suppression effect is. This relation is consistent with the law reflected in Figure 6. Another interesting pattern is that this crack stress guiding phenomenon is particularly obvious under high in-situ stress.

5 Conclusion

In this work, a two-dimensional crack propagation model is established. Through finite element numerical simulation, the blasting crack propagation of single-hole blasting under different in-situ stress levels is studied. Our analysis indicates that single-hole blasting cracks are sensitive to initial in-situ stress conditions. As the initial in-situ stress strengthens the restriction on the expansion of blasting, the range of the fracture zone decreases with the increase in in-situ stress. When the in-situ stress reaches 20 MPa, the range of the fracture zone is reduced by 48.5% relative to the condition without in-situ stress. When the in-situ stress reaches 40 MPa, the range of the fracture zone is reduced by 62.9% relative to the condition without in-situ stress. The reason is that the pressure of the gas escaping along the crack direction is nonlinear. The relationship between the crack growth length and in-situ stress is nonlinear such that the stress intensity factor at the crack tip reaches the critical threshold. In the inhomogeneous in-situ stress field, the development of explosion cracks shows obvious stress orientation, and the clamping effect of stress inhibits the development of explosion cracks. The cracks develop in the direction parallel to the large principal stress and are perpendicular to the large principal stress. This crack stress guiding phenomenon is increasingly obvious under high in-situ stress.

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