On the Space-Time Uncertainty Principle and Holography

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Abstract

In this note further evidence is collected in support of the claim that the space-time uncertainty principle implies holography, both within the context of Matrix Theory and the framework of the proposed duality between certain conformal field theories and M-theory/string theory on AdS backgrounds.
1. Introduction

The holographic principle [1] asserts that quantum theory of gravity should be described via a boundary theory. In particular, the number of dynamical degrees of freedom should satisfy the Bekenstein-Hawking bound [2].

The background dependent proposals for a non-perturbative formulation of M-theory, such as Matrix theory [3] (M-theory in the infinite momentum frame), or proposals for a non-perturbative formulation of IIB string theory on $AdS_5 \times S^5$ (and similarly M-theory on $AdS_4 \times S^7$ and $AdS_7 \times S^4$) [3,4,5,6] satisfy the Bekenstein-Hawking bound on the number of degrees of freedom as required by the holographic principle [3, 6]. (The boundary conformal field theories (CFT) in the latter case live on the boundary of the corresponding $AdS$ space).

It is our aim in this note to collect further evidence in support of the claim, recently presented by Li and Yoneya [8], which states that the holographic principle is implied by the space-time uncertainty principle [7,8,9] both in the context of Matrix theory and CFT/AdS duality. As it turns out, in both cases, the space-time uncertainty principle insures that the ultraviolet behavior of the boundary Dp-brane [10] world-volume theory is directly related to the infrared behavior of the theory in the bulk [7,8,9], which in turn implies holography.

2. Space-time Uncertainty Principle in String Theory and M-theory

First we briefly review the space-time uncertainty principle following the original work of Yoneya [7]. For a more complete account the reader is referred to the recent beautiful survey of Li and Yoneya [8].

The origin of the space-time uncertainty principle in string theory is in the time-energy uncertainty relation of quantum mechanics [7,8] $\Delta T \Delta E \geq \hbar$. In string theory, the energy
is approximately proportional to the string length $X$, so that $\Delta E \sim \frac{\hbar}{\alpha'} \Delta X \sim \frac{\hbar}{l_s} \Delta X$. Thus the following space-time uncertainty relation is valid

$$\Delta T \Delta X \geq l_s^2. \quad (1)$$

The above formulation of the space-time uncertainty principle can be related to the opposite scaling of the longitudinal (including time) and transverse directions of a string (or a Dp-brane in general) [7, 8]. Short scales (UV) on the world-volume correspond to long scales (IR) in target space. In other words, if the space-time coordinates $X_i(\sigma_m)$ scale as

$$X_i(\sigma_m) \rightarrow \lambda X_i(\sigma_m), \quad (2)$$

then the world-volume coordinates $\sigma_m$ must scale inversely as

$$\sigma_m \rightarrow \lambda^{-1} \sigma_m. \quad (3)$$

In [7,8] it has been also pointed out that the space-time uncertainty principle in string theory is intimately related to the original Regge/resonance duality (s-t channel duality of the old dual resonance models). For example, the Regge regime corresponds to the limit $\Delta X \rightarrow \infty$. On the other hand, the resonance regime corresponds to the limit $\Delta T \rightarrow \infty$.

The space-time uncertainty principle of the theory of closed strings is also related to the existence of a massless spin 2 particle in the spectrum [7,8]. The argument goes as follows: high-energy scattering gedanken experiments probe the short time scales; hence, $\Delta T \rightarrow 0$. The amplitude of the scattering process is proportional to $\Delta X_{long} \sim l_s^2/\Delta T \sim E$, which in turn implies the existence of a massless spin 2 particle, the graviton. (The intercept of the leading Regge trajectory $\alpha(t)$ is 2, which follows from the dependence of the scattering amplitude on energy, $E^{\alpha(t)-1} = E$.)

On the other hand, the region $\Delta T \rightarrow \infty$, which according to the space-time uncertainty principle corresponds to $\Delta X \rightarrow 0$, is associated with short distance processes in
string theory, i.e. with the physics of D-branes [9,10,11]. It has been also pointed out by Yoneya that the space-time uncertainty principle is closely related to the fact that perturbative string theory is conformally invariant [7].

Furthermore, Li and Yoneya [9] showed that the description of Dp-branes in terms of $U(N)$ super Yang-Mills theories is also compatible with the space-time uncertainty principle. In this case one examines the behavior of the low energy theory of $N$ D0-branes, i.e. the action of the $\mathcal{N} = 16$ $U(N)$ supersymmetric Yang-Mills quantum mechanics [3, 12] (with $l_s = 1$)

\[
S = \int dt Tr \frac{1}{g_s} \left( \frac{1}{2} (D_t X_a)^2 + i \bar{\theta} D_t \theta + \frac{1}{4} [X_a, X_b]^2 - \bar{\theta} \Gamma^a [\theta, X_a] \right).
\] (4)

Here $D_t X_a = \partial_t + [A_0, X_a]$. As Li and Yoneya [9] noticed, this expression is invariant under (in $A_0 = 0$ gauge)

\[
X \to g_s^{1/3} X, \quad t \to g_s^{-1/3} t,
\] (5)

which is consistent with (1). The scale $g_s^{-1/3} l_s$ is nothing but the 11-d Planck length of Matrix theory [3]. There are many nice physical applications of (5) [9]. For example, the order of magnitude estimate of the distances probed by D0-branes is given by $\Delta X \sim \sqrt{v l_s}$ (this follows from $\Delta T \Delta X \sim \frac{(\Delta X)^2}{v} \sim l_s^2$). The spreading of a D0-brane wave packet is estimated from $\Delta X \sim \sqrt{v l_s}$ and $X \sim g_s^{1/3} l_s$ to be of the order of $g_s l_s / v$. All these results can be also obtained from the Born-Oppenheimer approximation for the coupling between the diagonal and off-diagonal $U(N)$ matrix elements as in [11].

The argument of Li and Yoneya [9] can be generalized to other Dp-branes. The longitudinal and transverse lengths scale oppositely with the powers of the string coupling constant $g_s^{-1/(3-p)}$ and $g_s^{1/(3-p)}$ (for $p = 3$ the super Yang-Mills theory is conformally invariant). As pointed out by Yoneya [7], these scalings are consequences of the fact
that the interactions between Dp-branes are mediated via open strings, whose respective longitudinal and transverse lengths satisfy the space-time uncertainty relation.

The mathematical structure behind the space-time uncertainty principle is at the moment unknown. The simplest Lorentz covariant formulation of the space-time uncertainty principle, as proposed by Yoneya [7], is to assume non-commutativity of all space-time coordinates

$$[X_\mu, X_\nu]^2 \sim I.$$  \hspace{1cm} (6)

This relation leads immediately to eq. (1). Unfortunately, at the moment, there is no satisfactory covariant formulation of M-theory which would incorporate this statement of the space-time uncertainty principle. (See [13] for some preliminary comments regarding this question.)

But the space-time uncertainty principle in string theory can be generalized to M-theory. As shown in [8], the space-time uncertainty relation of Matrix theory reads

$$\Delta X_- \Delta X_a \Delta X_+ \sim l_{11}^3.$$  \hspace{1cm} (7)

Here $X_-$ denotes the longitudinal direction and $X_+$ - the global time of Matrix theory. This follows from the fact that $\alpha' = l_{11}^3/R$, $l_{11}$ being the 11-d Planck length. In Matrix theory $\Delta X_- \sim R$, the extent of the eleventh dimension; hence, (7) is compatible with (1). Ideally, the M-theory space-time uncertainty relation (7) should be used as a guide towards a covariant formulation of Matrix theory.

Here we wish to point out that (7) is consistent with the covariant formulation of 11-d membrane [12], if the latter is suitably discretized (for example, along the lines of [13]) in order to take into account the fact that the membrane world-volume should be at least of order $l_{11}^3$.  

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First (7) implies the following scaling of space-time coordinates

$$X_\mu \rightarrow g^{1/3} X_\mu.$$ \hspace{1cm} (7a)

Notice that this relation is consistent with the classical membrane Nambu-Goto action

$$S_{\text{mem}} \sim \frac{1}{g} \int d^3 \xi \sqrt{-\text{det} g_{ij}},$$

where $g_{ij} = \partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu}$ is the induced metric on the 3-volume of the membrane (for simplicity we consider only the bosonic case as in [13]). In light cone-gauge [12], (7a) reduces to (5).

Second, note that the M-theory space-time uncertainty relation (7) is consistent with the energy-time uncertainty relation, once we take into account that the energy of the membrane is proportional to its area. Of course, strictly speaking this argument is not correct, because the membrane is an effective object in M-theory. On the other hand this should be true for the case of a discretized covariant membrane [13], whose volume is bounded from below by $l_{11}^3$.

Finally, note that (7) implies a relativistic bound on the uncertainty of the transverse coordinate $\Delta X \sim \sqrt{c/\Delta X^+} \quad l_{11}^{3/2}$, (c is the velocity of light), once we take into account that $c\Delta X^+ \sim \Delta X$. One expects, in analogy with quantum field theory where the uncertainty for the coordinate of a relativistic particle corresponds to a momentum uncertainty proportional to the threshold energy for particle-antiparticle production, that processes which lead to a bound on $\Delta X$ should involve brane-anti-brane interactions, and should be described in any covariant version of Matrix theory consistent with (7).

3. Space-time Uncertainty Principle, Holography and Matrix Theory

The point of this section is to discuss evidence in favor of the claim that the space-time uncertainty principle implies holographic behavior in Matrix theory [8].

We start by reviewing the work of Jevicki and Yoneya [14]. First, these authors show that there is a hidden $SU(1,1)$ symmetry of the Matrix theory action (4). The symmetry
is generated by scale transformations (D) (implied by the space-time uncertainty relation),
time translations (H), and special conformal transformations (K) (under which the string
constant, treated as a part of background field variables, also changes [14]).

The explicit actions of D, H and K are given by [14]

\[ \delta_D X_a = X_a, \quad \delta_D A_0 = A_0, \quad \delta_D t = -t, \quad \delta_D g_s = 3g_s \]

\[ \delta_H X_a = 0, \quad \delta_H A_0 = 0, \quad \delta_H t = 1, \quad \delta_H g_s = 0 \]

\[ \delta_K X_a = 2tX_a, \quad \delta_K A_0 = 2tA_0, \quad \delta_K t = -t^2, \quad \delta_K g_s = 6tg_s. \]

These transformations form an \( SU(1,1) \) algebra

\[ [\delta_D, \delta_H] = \delta_H, \quad [\delta_D, \delta_K] = -\delta_K, \quad [\delta_H, \delta_K] = 2\delta_D. \] (8b)

Furthermore, Jevicki and Yoneya [14] demonstrate that the above conformal symmetry
(which stems from the space-time uncertainty principle) taken together with the SUSY non-
renormalization theorem [15] implies the lagrangian of Becker-Becker-Polchinski-Tseytlin
(BBPT) [16], which is an effective lagrangian of a D0-brane probe in the background of
heavy D0-brane sources. (The BBPT lagrangian [16] is computed in the framework of
a discrete light-cone formulation of Matrix theory [17], where the number of partons \( N \)
is kept finite.) This lagrangian is valid in the limit of large distances and small relative
velocities [16] (with \( l_s = 1 \))

\[ S_{eff} = -\int dt \frac{1}{g_s h(r)}(\sqrt{1 - h(r)v^2} - 1), \] (9a)

where the static D0-brane background is described by \( h(r) \equiv h_{-\pm}(r) = \frac{15N}{2g_s^2 r^7} \).

Now, in the limit of mean field theory, the BBPT action implies holography. This
particular fact has been already demonstrated in the study of neutral black hole - like
bound states of D0-branes in the infinite momentum frame [18]. The characteristic size
of such a bound state is essentially the Schwarzschild radius $R_h$. The characteristic velocity of D0-branes in the infinite momentum frame is determined by the inverse of the boosting parameter, or basically from the Heisenberg uncertainty relation $v R_h \sim 1$. The Bekenstein-Hawking scaling relation for the black-hole entropy $S \sim R_h^{D-2}/G_D$, in D-spacetime dimensions, follows after the virial theorem is applied to (9a) (at the transition point $S \sim N$ [18,19]). Thus we conclude, though indirectly, that the space-time uncertainty principle together with the SUSY non-renormalization theorem implies holography.

Note that this picture can be, at least formally, generalized to the charged black holes discussed in [20] (I thank S. Chaudhuri and M. Li for many discussions regarding this issue): For a large boost the energy $E$ and momentum $P$ of a charged black hole are according to [20] $E \sim \mu A \exp \alpha$, $P \sim \mu A \exp \alpha$, following the notation of [20]. Also, the black hole mass scales as $M \sim \mu A \sim R_h^{D-3} C^{-1/2}$ where $A(\beta_i) \equiv (\lambda + \sum_{i=1}^n \cosh 2 \beta_i)$ and $C(\beta_i) \sim A(\beta_i)^{-2} B(\beta_i)^{2(D-3)/D-2}$ and $B(\beta_i) \equiv \prod_{i=1}^n \cosh \beta_i$, again following the notation of [20]. Thus, the black hole longitudinal momentum is determined as $P \sim R_h^{D-3} C^{-1/2} \exp \alpha$. Let us concentrate on the special point $P = N/R$. The boosting parameter $\exp \alpha$ is given by $\exp \alpha \sim C^{1/2} R_h/R$. This parameter determines by how much the box of size $R$ has to expand to accommodate a charged black hole [20] of the horizon radius $R_h$. Suppose we postulate that the radius of the bound state $r_b$ is related to the horizon radius $R_h$ via $R_h \sim r_b C^{1/2}$. The physical meaning of this relation is simple: the line elements of [20] roughly have the Reissner-Nordström form, which implies that there should exist two characteristic scales describing the physics of charged black holes. The presence of charges is hidden in the complicated function $C$, which we take as a phenomenological input. The two scales naturally coalesce into one, $R_h$, if we consider a neutral black hole. The characteristic partonic velocity is determined by the inverse of the boosting parameter $\exp \alpha$, that is,
\[ v \sim \frac{R}{R_h} C^{-1/2}. \] Then the application of the virial theorem to the BBPT effective lagrangian gives the Bekenstein-Hawking formula at the special point \( N \sim S \), i.e. \( S \sim N \sim \frac{R_h^{D-2}}{G_D} \). The virial theorem and the first law of thermodynamics imply the equation of state (after we utilize the Bekenstein-Hawking scaling) \( S \sim (NT/R)^{D-2} C^{D-2} \). This relation tells us that \( R_h \sim (NTG_D/R)^{D-4} C^{D-2} \), which is exactly the relation between the size of the bound state \( r_b \) and the horizon radius \( R_h \), if the size of the bound state is defined to scale as \( r_b \sim (NTG_D/R)^{D-4} \). Also, the infinite momentum dispersion relation \( E_{lc} \sim M^2 R/N \) leads to \( \mu \sim (NT/R)^{(D-3)/(D-4)} (C^{D-2}/A) \), which agrees with [20]. When \( C \to 1 \) we recover the familiar expression for the Schwzschild black hole, as expected.

Another way to see holography of (9a) is to realize that the BBPT effective action is nothing but the Einstein-Hilbert action \( S_{EH} = -\frac{1}{16\pi G} \int dx^{11} \sqrt{g} R \), written in the infinite momentum frame [21], in the limit of large distances and small relative velocities. Then by formally applying the Gibbons-Hawking argument [22, 23] to the euclidean partition function defined by the above action one finds that the entropy satisfies the Bekenstein-Hawking scaling. One might expect that this formal counting is valid in this case because one is only looking at the low-energy behavior of Matrix theory. But a word of caution is needed here. Strictly speaking, we cannot use the Gibbons-Hawking argument given the current dictionary between Matrix theory and 11-d supergravity [24]. In the linearized 11-d supergravity one cannot talk about black hole bound states, so the above scaling arguments do not apply. On the other hand there exists a very beautiful argument due to Jacobson [25], which essentially states that a holographic theory of gravity which satisfies the usual axioms of quantum field theory in curved space-time, implies the full non-linear structure of general relativity. In our case this would translate into a statement that holography in Matrix theory (as implied by the space-time uncertainty principle) together with locality
should lead to the full non-linear structure of 11-d supergravity. But to be able to directly apply Jacobson’s argument [25] we also need a fully covariant version of Matrix theory.

Fortunately, we can do better than to observe holography in Matrix theory in the background of static D0-brane sources. Using the recent powerful results of Okawa and Yoneya [26] on the full structure of 3-body interactions in Matrix theory at finite $N$, we can argue that holography is valid even when many-body interactions of D0-branes are taken into account.

Indeed, the effective lagrangian that describes 3-body interactions of D0-branes according to Okawa and Yoneya has two pieces (eqs. 2.53 and 2.55 of [26]):

\[
L_V = - \sum_{a,b,c} \frac{(15)^2 N_a N_b N_c}{64 R^5 M^{18}} v_{ab}^2 v_{ca}^2 (v_{ab} \cdot v_{ca}) r_{ab}^{-7} r_{ca}^{-7}
\]

and

\[
L_Y = - \sum_{a,b,c} \frac{(15)^3 N_a N_b N_c}{96 (2\pi)^4 R^5 M^{18}} [v_{bc}^2 v_{ca}^2 (v_{cb} \cdot \nabla_c) (v_{ca} \cdot \nabla_c) \\
+ \frac{1}{2} v_{ca}^4 (v_{cb} \cdot \nabla_c)^2 + \frac{1}{2} v_{bc}^4 (v_{ca} \cdot \nabla_c)^2 - \frac{1}{2} v_{ba}^2 v_{ac}^2 (v_{cb} \cdot \nabla_c) (v_{bc} \cdot \nabla_b) \\
+ \frac{1}{4} v_{bc}^4 (v_{ba} \cdot \nabla_b) (v_{ca} \cdot \nabla_c)] \Delta(a, b, c)
\]

where $\Delta(a, b, c) \equiv \int d^9 y |x_a - y|^{-7} |x_b - y|^{-7} |x_c - y|^{-7}$ and $a, b, c$ are particle labels.

It is now easy to see that both $L_V$ and $L_Y$ lead to the Bekenstein-Hawking scaling if the relative velocities of particles saturate the Heisenberg uncertainty bound $v r \sim 1$ at the transition point $S \sim N$ in the large $N$ limit. The $L_Y$ term is much more subtle than $L_V$ in this respect, but the scaling argument still works. The effective lagrangians for 2- and 3-body interactions follow from the general structure of (4), which is compatible with the space-time uncertainty principle, thus supporting the claim that the space-time uncertainty principle underlies holography in Matrix theory.

We end this section with a simple observation about the hidden $SU(1, 1) \sim SO(2, 1)$ symmetry in Matrix theory [14]: In [27] it was argued that in the large $N$ limit there exists
an $SO(1, 2) \times SO(16)$ symmetry in Matrix theory. This should be compared to the well-known fact that the 11-d supergravity can be formulated in such a way so that the same symmetry is made manifest [28]. Notice that the $SO(16)$ symmetry is not an isometry of any compactification of the 11-d supergravity. Rather this $SO(16)$ naturally appears as a local symmetry of the 11-d supergravity dimensionally reduced to 3-d [29].

4. Space-time Uncertainty Principle, Holography and AdS/CFT Duality

In this section we comment on the claim that the space-time uncertainty principle implies holography [8] in the context of CFT/AdS duality [30].

The argument of Susskind and Witten [30] relies on the connection between the UV behavior of the boundary conformal field theory and the IR behavior of the bulk AdS theory of gravity. In fact, this UV-IR relation of Susskind and Witten is nothing but the statement of the space-time uncertainty principle, as we have seen in section 1. (eqs. (2) and (3)).

The argument for the holographic bound goes as follows [30]: The number of degrees of freedom $N_{dof}$ per unit volume in the boundary CFT theory ($\mathcal{N} = 4$ super $SU(N)$ Yang-Mills in 4-d) scales as

$$N_{dof} \sim N^2/\delta^3,$$

where $\delta$ is an UV cut-off of the world-volume $CFT_4$ theory which lives on the boundary of $AdS_5$. The spatial volume of the boundary theory is $V_3 = R_b^3/\delta^3$ which implies

$$N_{dof} \sim V_3 N^2/R_b^3.$$

But CFT/AdS duality, following the original work of Maldacena [4], relates $N$ and the radius of $AdS_5$

$$R_b = l_s(g_s N)^{1/4},$$

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which in combination with eq. (10) gives the Bekenstein-Hawking bound.

The argument of [30] can be readily generalized for the case of membranes and five branes [8]. The relevant number of degrees of freedom in the former case should scale as $N^{3/2}$ and in the latter as $N^3$. The latter scaling might be naturally interpreted if we had a non-abelian formulation of theories involving two-forms, which we don’t.

But the relationship between $N^{3/2}$ and $N^3$ could be understood from the following picture: Imagine a covariant formulation of Matrix theory in which the world-volume of the M-theory membrane is suitably discretized, as described in section 2. The covariant formulation should also naturally include the covariant M-theory five-brane. Given the fact that in Matrix theory [31] the longitudinal five-brane wrapped around the longitudinal direction can be seen by stacking two orthogonal transverse membranes, we can envision stacking two orthogonal discretized membrane world-volumes in covariant Matrix theory to get a covariant M-theory five-brane. Then in order to match the energy densities of two objects the number of degrees of freedom of the theory that describes the discretized membrane should be square root of the number of degrees of freedom that describe the covariant M-theory five-brane. Hence $N^3$ degrees of freedom of the five-brane theory imply $N^{3/2}$ degrees of freedom of the membrane theory, as it should be.

Actually, we can extend the argument Susskind and Witten, to include conformal field theories in other number of dimensions. (This has been noticed in conversations with M. Li.) For the cases considered above the number of degrees of freedom in d-dimensions goes as $N^{d/2}$. Then, there should exist a superconformal quantum mechanics with $N^{1/2}$ degrees of freedom, and a five dimensional superconformal theory with $N^{5/2}$ degrees of freedom. The two dimensional superconformal field theory with $N$ abelian degrees of freedom also fits this pattern. The argument of Susskind and Witten which gives Bekenstein-Hawking
scaling in the end, works nicely in each of these cases.

5. **Background Independent Holography via 2-Hilbert spaces?**

The above discussion of the space-time uncertainty relations and holography is obviously background dependent. (One possible exception to this statement is the theory proposed by Hořava [32].) In conclusion to this note, we wish to point out that background independent holography might be understood via the concept of 2-Hilbert spaces, in analogy with a kinematical set-up proposed by Crane [33] in relation to 3 + 1 dimensional quantum general relativity (see the article by Smolin [33] for a nice review).

The main idea of Crane [33] is that a background independent holographic theory, such as quantum gravity, should not be described in terms of a single Hilbert space, but in terms of a linear structure which is spanned by basis elements which are also taken to be Hilbert spaces (such structures are called 2-Hilbert spaces; see the review of Baez [33] for the precise mathematical set-up within the framework of category theory.)

Very roughly, according to Baez [33], the basic objects of 2-Hilbert spaces are finite dimensional Hilbert spaces (replacing vectors as the basic elements of Hilbert spaces). The zero object (the analog of the zero vector) is a zero-dimensional Hilbert space; the analog of adding two vectors is forming the direct sum; the analog of multiplying a vector by a complex number is tensoring an object by a Hilbert space, and the analog of the inner product of two vectors is a bifunctor taking each pair of objects $a, b$ of a 2-Hilbert space to the set of morphisms from $a$ to $b$.

Thus, following Crane, the background independent 2-Hilbert space of the bulk quantum theory of gravity should be constructed in terms of component Hilbert spaces of the appropriate boundary theories. We want to illustrate this idea within the framework of Matrix theory: A Hilbert space in Matrix theory is represented by block diagonal matrices
[3]. Denote such a Hilbert space by $H^i_{\text{bound}}$. In order to realize the idea of Crane [33] we suppose that each of these Hilbert spaces can be taken as a component of a 2-Hilbert space ($Hilb$). In other words, an element $H_2$ of $Hilb$ can be written as a linear combination of $H^i_{\text{bound}}$

$$H_2 = \sum_i c_i H^i_{\text{bound}}. \quad (13)$$

Now, let each $H_{\text{bound}}$ satisfy the holographic bound (i.e. let the dimension of a particular $H_{\text{bound}}$ be determined by the appropriate boundary theory). This implies the holographic bound on the dimension of the bulk 2-Hilbert space, and therefore - background independent holography. Coming back to our membrane analogy from section 2., the corresponding 2-Hilbert space can be concretely envisioned as a collection of transverse membranes, each representing the Hilbert space of a particular background.

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