Jeans instability in a tidally disrupted halo satellite galaxy

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Accepted 2010 October 6. Received 2010 October 5; in original form 2010 May 25

ABSTRACT
We use a hybrid test particle/N-body simulation to integrate four million massless test particle trajectories within a fully self-consistent $10^5$ particle N-body simulation. The number of massless particles allows us to resolve fine structure in the spatial distribution and phase space of a dwarf galaxy as it is disrupted in the tidal field of a Milky Way type galaxy. The tidal tails exhibit nearly periodic clumping. By running simulations with different satellite particle mass, halo particle mass, number of massive and massless particles and with and without a galaxy disc, we have determined that the instabilities are not due to numerical noise, amplification of structure in the halo or shocking as the satellite passes through the disc of the galaxy. We measure Jeans wavelengths and growth time-scales in the tidal tail and show that the Jeans instability is a viable explanation for the clumps. We find that the instability causes velocity perturbations of order $10$ km s$^{-1}$. Clumps in tidal tails present in the Milky Way could be seen in stellar radial velocity surveys as well as number counts. We find that the unstable wavelength growth is sensitive to the simulated mass of dark matter halo particles. A simulation with a smoother halo exhibits colder and thinner tidal tails with more closely spaced clumps than a simulation with more massive dark matter halo particles. Heating by the halo particles increases the Jeans wavelength in the tidal tail affecting substructure development, suggesting an intricate connection between tidal tails and dark matter halo substructure.

Key words: instabilities – galaxies: dwarf – galaxies: halo.

1 INTRODUCTION
There is evidence for past and ongoing accretion of small objects by the Milky Way halo, the most dramatic object being the disrupted Sagittarius dwarf galaxy (Sgr; Ibata, Gilmore & Irwin 1994). Disrupting satellites leave behind tidal tails and, on longer time-scales, stellar streams (Bekki & Freeman 2003; Helmi et al. 2003; Meza et al. 2005; Penarrubia et al. 2005; Purcell, Bullock & Zentner 2007; Helmi 2008). Previous studies of stellar streams have used N-body simulations to study the disruption and evolution of merging galaxies (e.g. Helmi et al. 2006; Johnston et al. 2008; Gomez & Helmi 2010). However, most previous simulations have not placed many particles in the disrupting object itself. A large number of particles are required to resolve structure in the velocity distribution in a small local volume (e.g. Minchev et al. 2009; Gomez & Helmi 2010). After satellite disruption, satellite particles are distributed all over the galaxy.

Early simulations of dwarf galaxy disruption necessarily contained few particles. The focus of many of these simulations was constraining the orientation and shape of the halo from observations of the Sgr stream rather than searching for substructure in the tails themselves. The simulations by Johnston, Hernquist & Bolte (1996) and Johnston (1998) contained only $10^3$ particles in the dwarf galaxy and those by Helmi et al. (2006) only had 5000 particles. The simulations by Bullock & Johnston (2005) had $10^4$ massive particles but an additional $1.2 \times 10^5$ massless test particles in their dwarf satellites. The simulations discussed by Gomez & Helmi (2010) are described in more detail by Villalobos & Helmi (2008) and contained $3 \times 10^5$ particles in the satellite.

Clumping in tidal tails has been seen in simulations previously described by Barnes & Hernquist (1992) with a hybrid N-body/smoothed particle hydrodynamics (SPH) simulation and by Wetzstein, Naab & Burkert (2007) with both a N-body, and hybrid N-body/SPH code. Both found tidal dwarf galaxies forming in the tidally disrupted tails of galaxies involved in a major merger. Wetzstein et al. (2007) found that overdensities occur but do not form bound objects in pure N-body simulations with a sufficiently high resolution. However, the addition of gas in a high-resolution simulation again allows formation of the dwarf galaxies.

While substructure in the form of clumps has not been detected in the Sgr streams, it has been seen in tidal tails associated with smaller objects. Clumped structure observed in the tidal debris of the globular cluster Palomar 5 (Odenkirchen et al. 2003) is interpreted in terms of oscillations in the cluster (Gnedin, Lee & Ostriker 1999) caused by a previous passage through the Galactic
disc (Odenkirchen et al. 2003; Dehnen et al. 2004), epicyclic perturbations excited during tidal disruption (Kupper, Macleod & Heggie 2008) and Jeans instability (Quillen & Comeropeta 2010). Alternative possibilities accounting for structure in cluster tidal tails include the effect of dark matter subhaloes, as explored by Ibata et al. (2002), Mayer et al. (2002), Johnston, Spergel & Hayden (2002), Penarrubia et al. (2006), Siegal-Gaskins & Valluri (2008) and Carlb erg (2009).

Here, we strive to carry out N-body simulations with a larger number of particles placed in the disrupting dwarf galaxy. We use the simulations to look in detail at the structure of the tidal tails during and following disruption from the dwarf galaxy.

2 NUMERICAL INTEGRATIONS

We first describe the N-body integrator used. We then describe our modifications to the integrator that allow us to simultaneously integrate test (massless) particles. We then describe the initial conditions used in our simulations and list the different simulations carried out.

2.1 Hybrid N-body and test particle integration

The N-body integrator used is a direct-summation code called φGRAPE (Harfst et al. 2007) that employs a fourth-order Hermite integration scheme with hierarchical commensurate block time-steps (Makino & Aarseth 1992). Instead of using special purpose GRAPE hardware, we use the SAPPORO subroutine library (Gaburov, Harfst & Portegies Zwart 2009) that closely matches the GRAPE-6 subroutine library (Makino et al. 2003) but allows the force and jerk computations to be done on graphics processing units (GPUs). Force computation is done in double precision, though jerk computation is not.

We have modified the integrator so that massless particles can be integrated simultaneously along with the massive particles. This is done by adding an extra parameter to the φGRAPE code, Nmassive, that is the number of massive particles. The massless and massive particles are integrated together but only the massive particles are part of the data set used to calculate the forces and jerks (Makino et al. 2003). The GRAPE-6 subroutine library passes Nj massive particles to the GRAPE boards (or in our case to the GPU) with the subroutine call _6_SET_I_PARTICLE (see the GRAPE-6 User Guide).1 Instead of passing all particles, we pass the total number of massive particles, Nmassive, to this routine. The accelerations and jerks on the i particles are computed by summing terms from each of the Nj particles; this is done in the GRAPE-6 subroutines _6CALC_FIRSTHALF and _6CALC_LASTHALF that are called in the gravity computation step of φGRAPE. The number of j particles in these two subroutine calls is also changed to Nmassive. The remaining computation steps, e.g. the predictor step, the corrector step and the identification of active particles, remain unchanged and are done on all particles.

Computation of accelerations and jerks in an N-body simulation require O(N^2) computations for N particles. In our hybrid code O(Nmassive^2) computations are done on the massive particles and O(Nmassive x Nmassless) are done on the massless particles. For example a simulation of 10^7 massive and 10^8 massless particles would require O(10^15) computation steps. This is less than integrating 10^7 massive particles, which would require O(10^13) computation steps. Thus, the hybrid scheme is a way to integrate additional particles and better resolve structure in phase space while not compromising the speed of the simulation. The hybrid scheme adopted here made it possible for us to integrate four million dwarf galaxy particles during tidal disruption in the context of a self-consistent N-body simulation with a live halo on a desktop computer containing off the shelf graphics cards. While test particle simulations can simulate this number of trajectories without difficulty (e.g. Minchev et al. 2009; Quillen et al. 2009), test particle simulations alone (not combined with N-body) are not self-consistent.

The test-particle/N-body hybrid scheme used here is similar to the particle cloning technique often used in celestial mechanics integrations to improve understanding of statistical properties of orbits (e.g. Masaki & Kinoshita 2003; Kaib et al. 2009). Test particles have been used previously to better resolve structure in cosmological simulations. For example, those by Bullock & Johnston (2005) placed 1.2 x 10^5 test particles in their dwarf galaxies which contained only 10^5 massive particles.

2.2 Initial conditions

Initial particle distributions were created separately for the Milky Way type galaxy and the dwarf galaxy.

The initial conditions for the model Milky Way were made with a numerical phase distribution function using the method discussed by Widrow, Pym & Dubinski (2008) and their numerical routines which are described by Kuijken & Dubinski (1995), Widrow & Dubinski (2005) and Widrow et al. (2008). The code computes a gravitational potential for bulge, disc and halo components, then computes a distribution function for each component. N-body initial condition files are then computed for each component. The galactic bulge is consistent with a Sersic law for the projected density. The halo profile has cusp strength γ. The disc falls off exponentially with radius and as a sech^2 with height. Parameters are those for the standard Milky Way model, listed in table 2 by Widrow et al. (2008).

The number of massive particles in each of the bulge, disc, halo and dwarf components is shown in Table 1. The total mass of the disc is 5.31 x 10^10 M⊙, the bulge is 8.27 x 10^9 M⊙ and the halo is 4.42 x 10^11 M⊙. The halo is live.

The dwarf galaxy initial conditions were created using a King model. The model is described by two parameters, a velocity dispersion, σ, and a concentration, c. The concentration, c = r_t/r_c, is the ratio of the tidal radius, r_t, setting the outer boundary to the core radius, r_c. Of interest is the central density ρ_c = [9σ^2/(4πG)]^1/3. This central density sets the approximate location of complete tidal disruption in the background galaxy. In Table 2 we list the properties of the dwarf galaxy. The total mass of the dwarf galaxy is 2.70 x 10^6 M⊙.

Table 1. Numbers of particles in each galactic component.

| Simulation no. | Disc | Bulge | Halo | Dwarf_massive | Dwarf_massless |
|----------------|------|-------|------|---------------|---------------|
| Run 1          | 77000| 37000 | 8000 | 9072          | 4 x 10^6       |
| Run 2          | 77000| 37000 | 8000 | 60000         | 4 x 10^6       |
| Run 3          | 77000| 37000 | 8000 | 60000         | 3.94 x 10^6    |
| Run 4          | 0    | 37000 | 8000 | 60000         | 4 x 10^6       |

Notes. Particle numbers for the four simulations. Total mass in the bulge, disc and halo components is the same for all runs (except for Run 4 which does not include a disc). When there are more particles in a given component the mass of each particle is lower. All particles in a given component have the same mass.

1 http://www.artcompsci.org/makino/softwares/GRAPE6/
The initial conditions of each simulation are identical except for the following changes. Run 1 has a lower number of massive particles present in the dwarf galaxy than the other three runs, though the total mass in the dwarf is the same. Run 2 is our standard run. Runs 3 and 4 have 10 times more dark matter particles than the Runs 1 and 2, though the total halo mass is the same in all runs. Run 4 is lacking a disc. There is a small but insignificant change in the number of massless particles in the dwarf galaxy in Run 3.

### 2.3 Orbit

We choose a polar orbit for the dwarf satellite. The initial position of the satellite is 15 kpc from the centre of the Milky Way type galaxy along the direction of the plane of the disc, and 20 kpc perpendicular to the disc. The velocity is 200 km s\(^{-1}\) directed toward the disc. The orbital time is approximately 1 Gyr. The orbit of the dwarf can be seen in the snapshots as shown in Figs 1–3. Each figure corresponds to a separate run, with Fig. 1 corresponding to Run 2, Fig. 2 corresponding to Run 3 and Fig. 3 corresponding to Run 4. The parameters of the King model for the satellite were adjusted so that the satellite produced strong tidal tails but did not completely disrupt during its first pericentre passage.

### 2.4 Additional details about the simulations

The softening length for all particles in all simulations was 0.1 kpc and was chosen to be the average initial spacing between dwarf galaxy particles. Total energy was conserved between 99.996 and 99.999 per cent in all simulations. If the chosen softening length were larger we would have failed to see structure on kpc scales. If the softening length were chosen to be smaller we would have seen unrealistic acceleration of a small fraction of the particles to high velocities from close approaches.

All simulations were performed for a time-scale of 3 Gyr on a single desktop computer with two GTX 295 GPUs. Each run took between 5 and 10 d.

### 3 RESULTS

We first discuss the morphology of the tidal tails as seen in the different runs. We compare tidal tail structure as a function of the number of simulated massive particles in the dwarf and number of particles in the halo. We explore the possibility that the Jeans instability is the cause of the clumping apparent in the simulations.

#### 3.1 Morphology

Snapshots from simulation Runs 2–4 are shown in Figs 1–3, respectively. Run 1 is not shown as it is visibly indistinguishable from Run 2. The figures show projections into the x–z plane of number density (histograms) in Galactocentric coordinates. The galaxy disc lies in the z = 0 plane or horizontally on these plots. All dwarf galaxy particles are included and both massive and test particles are shown. Halo, disc and bulge particles are not shown. Evolution of the dwarf galaxy is shown from 0.33 Gyr to the end of the simulation at 3.0 Gyr in increments of one-third Gyr. We saw no differences between massive and massless dwarf galaxy particle distributions so they are not displayed separately.

In these snapshots, it is evident that the tidally disrupted tails of the dwarf galaxy exhibit clumps, or nearly periodically spaced density enhancements. When viewed sequentially as a movie this effect appears almost smoke like in behaviour. The density enhancements can be seen in most of the snapshots past 1.0 Gyr, and more clearly in the left-hand panels of Figs 6 and 7. The spacing between clumps is 2–5 kpc in Run 2 but shorter in Run 3 and of order 1–2 kpc. The density enhancements are in the form of nearly periodically spaced ridges that are oriented perpendicular to the orbit. These density oscillations do not exhibit any kinking in the tails nor compression in the radial direction of the tail. They appear visually only as compression in the direction of motion along the tail, which will be further described in Section 3.3. We note that clumps are particularly prominent after the tail particles have passed through pericentre. This is particularly evident looking at the bottom panels of Figs 1 and 2.

From comparisons of Run 1 to Run 2 we can test the possibility that the substructure is caused by numerical heating within the tail itself from the massive dwarf galaxy particles in the tail. For Run 2 we use a factor of 6.6 times more massive particles in the dwarf galaxy than in Run 1 so the mass of each dwarf particle is 6.6 times larger in Run 1 than Run 2. Yet the two simulations have similar morphologies. Noise caused by low numbers of massive particles in the dwarf is not likely responsible for the clumping. Likewise small groupings of particles in the dwarf do not appear to cause clumps in the tails. The study by Asphaug & Benz (1996) also explored the sensitivity of a tidally disrupted object to the number of massive particles in the object (in their case the break-up of a comet). They similarly found no sensitivity of the clump size and spacing to the number of simulated particles.

Run 2 and Run 3 are the same except that Run 3 contains more halo particles. The mass of each halo particle in Run 2 is 5.5 × 10\(^7\) M\(_\odot\) whereas each halo mass in Run 3 is a tenth of this. Here we do see a difference between the tidal tail morphologies. Run 3 (with lower mass halo particles) has denser and narrower tails. Previous simulations have seen thickening of tidal tails caused by heating from subhaloes (Ibata et al. 2002; Johnston et al. 2002; Mayer et al. 2002; Penarrubia et al. 2006). The wider tails seen in Run 2 (Fig. 1) compared to those Run 3 (Fig. 2) are consistent with heating caused by the larger masses of the halo particles in Run 2. The mass of our halo particles in Run 2 is similar to the lowest mass subhaloes simulated by Ibata et al. (2002). Their simulations were carried out in a smooth static background potential but included softened subhaloes.

Previous simulations have found that dark matter subhaloes can also cause structure in a tidal tail (Siegal-Gaskins & Valluri 2008; Carlberg 2009). Siegals-Gaskins & Valluri (2008) also used a smooth static background potential and included subhaloes with masses in the range of 6 × 10\(^6\)–6 × 10\(^8\) M\(_\odot\) as well as a smaller number of halo particles with masses up to 4.6 × 10\(^9\). Clumps in the tails can be excited by nearby dark matter subhaloes (see fig. 3 by Siegals-Gaskins & Valluri 2008). Here, however, we see more prominent but also more closely spaced clumps in tails when the dark matter particles are of lower mass (Run 3; see Fig. 2). The

### Table 2. King satellite model.

| Parameter | Value |
|-----------|-------|
| c         | 0.672 |
| r_0       | 0.5 kpc |
| ρ_0       | 0.4 M_⊙ kpc^{-3} |
| σ         | 25 km s^{-1} |

Notes. Parameters for the satellite are described by a King model with parameters above.
larger number of clumps in Run 3 compared to Run 2 suggests that the dark matter particles are not the cause of the substructure. Siegal-Gaskins & Valluri (2008) saw clumps in the tails that were simulated in a smooth static potential and lacked subhaloes (see their fig. 3 top left-hand panel). This suggests that if we were to carry out simulations in a smooth potential (or with more and even lower mass halo particles) that we would continue to see clumps.

Run 4 lacks a stellar disc. In this simulation (see Fig. 3) we still see clumping in the tails. However, as the clumping is still present in Run 4 we conclude that shocking from passage through the disc is not the cause. Note that the difference in the orbit shown in Fig. 3 from Figs 1 and 2 is to be expected as the simulation is missing the potential from the galactic disc.

In summary, our simulations show periodic clumping in the tidal tails of a disrupting dwarf satellite. By comparing two simulations with different numbers of massive particles in the dwarf we rule out amplification of numerical noise from massive particles in the tails as an explanation for clumps. The simulation lacking a galactic disc also exhibits clumps, thus disc shocking cannot account for the tail structure. The mass of the dark matter particles does affect the tail morphology, but in a way opposite to that expected. When the halo particles are more massive the clumps are smaller and less closely spaced rather than larger, suggesting that heating by the halo has reduced the extent of the clumping instability.

We note that the clumps we see in the tidal tails would be difficult to see in a simulation containing fewer dwarf galaxy particles. Most previous simulations may have lacked the substructure we see here because they had fewer particles in the dwarf galaxy. It is possible that the simulations by Siegal-Gaskins & Valluri (2008) do display Jeans instabilities (see their figs 8 and 13 which may show periodic clumps), however, it is not easy to tell as they plotted each particle individually rather than made histograms as we have done here.
3.2 Jeans instability

Self-gravity could be responsible for the substructure seen in the simulated tidal tails. To test this hypothesis we measure the Jeans wavelength at different regions in the tails and times during the simulations.

The Jeans wavelength is

$$\lambda_J \equiv \sqrt{\frac{\pi \sigma^2}{G \rho_0}}$$

Planar perturbations on wavelengths longer than the Jeans wavelength are unstable and grow on a time-scale

$$t_{\text{growth}} \sim (4 \pi G \rho_0)^{-1/2},$$

whereas those wavelengths shorter than the Jeans wavelength are damped via a process similar to Landau damping (e.g. Chap. 5 in Binney & Tremaine 1987).

If the Jeans instability is responsible for the clumping then prior to clump formation we should find that the Jeans wavelength is comparable to the distance between the clumps.

Formally, the growth time-scale is infinite at the Jeans wavelength. However, perturbations at all wavelengths larger than the Jeans wavelength are unstable. The wavelength of maximum growth rate should be larger than but of order of the Jeans wavelength (e.g. Fridman & Polyachenko 1984; Binney & Tremaine 1987; Quillen & Comparetta 2010). The exact growth rate, however, is non-trivial to predict for the case where the wavenumber of the perturbation times the width of the tail is on order or greater than 1, as seen in Quillen & Comparetta (2010) for the regime investigated in these simulations. We expect that unstable perturbations with wavelength just longer than the Jeans wavelength are those with the fastest growth.

3.2.1 Measuring Jeans wavelength and growth rates

We calculate the Jeans wavelength using both massive and massless particles in the tidal tail. Including only the massive particles results in an insufficient particle sample to compute the velocity dispersion in a bin small enough to resolve the substructure. Since
we include both massive and massless particle in computing the density $\rho_0$ in equation (1), we normalize the density by the ratio of massless plus massive to massive particles. We include only those particles within a half kpc of the plane containing the dwarf galaxy orbit (the $y = 0$ plane). In bins of $0.25 \times 0.25 \text{kpc}^2$ in the $x, z$ directions and 1 kpc wide in the $y$ direction we computed sums involving both the massive and massless particles. To estimate the density in the bin, we count the number of particles, multiply by the mass of the massive particles and divide by the ratio of the number of massless plus massive to massive particles. The velocity dispersion in the bin was computed from all particles in the bin and using all velocity components. The Jeans wavelength was then computed from the mass density and velocity dispersion using equation (1).

Growth rates are computed from the mass density using equation (2). The growth rate is calculated for the same regions that were used in the computation of the Jeans lengths. Here only the massive particles were included in the computation of the growth rate as we only require the density and we did not need to estimate the local velocity dispersion.

Jean wavelengths computed in the tails at different time-steps are shown in Figs 4 and 5. At the same time-steps and locations in the tail we also show the growth time-scales and projected density.

In Fig. 4 the top left-hand panel shows the Jeans wavelength for the tidally disrupted dwarf in Run 2 at a time-step of 0.46 Gyr. For the upper right portion of the tail, there are sections of the tidal tail that have Jeans lengths of 2–3 kpc. This is similar to the spacing between the clumps seen later in the simulation and is shown in the projected density in Fig. 4 in the bottom right-hand panel.

In Fig. 4 the top right-hand panel shows the growth rate in Gyr for the same sections of the tidal tail used in the computation of the Jeans length in the top left-hand panel. For the same sections of the tidal tail that had Jeans lengths of 2–3 kpc, the growth rate is about 0.2–0.3 Gyr.

If the observed clumping is due to the Jeans instability, we should see clumping become evident in the simulation after about a growth time-scale. The two bottom panels in Fig. 4 show projected number densities of all particles in the tidally disrupted dwarf. The bottom left-hand panel is a snapshot at the same time as the upper panels, at 0.46 Gyr, while the bottom right-hand panel is at 0.82 Gyr.
Clumping has formed within 0.36 Gyr as expected from the growth rate estimate, and the clumps are about 3.5–4 kpc apart as would be expected from the Jeans wavelengths exhibited by the tail earlier in the simulation.

The same progression is illustrated for Run 3 in Fig. 5 for the simulation with lower mass halo particles. Here the upper right portion of the tail has a Jeans length of around 2 kpc in the upper left-hand panel, and a growth time-scale for that region of 0.3–0.4 Gyr is seen in the upper right-hand panel. Although no clumping is seen in the projected density in the lower left-hand panel at 0.47 Gyr, clumps have formed in the tail a growth time-scale later. These are shown in the bottom right-hand panel at 0.87 Gyr and have a shorter spacing (compared to Fig. 4) as expected from the shorter Jeans wavelength exhibited earlier.

Within the context of first-order linear perturbation theory (e.g. Binney & Tremaine 1987) the growth time-scale for the Jeans instability is the inverse of an exponential growth rate. If perturbations present in the tail were small, then they would require many exponential growth time-scales before they cause detectable density contrasts. However, we see substructure in a time that is only on the order of a single growth time-scale. Gravitational collapse can be self-similar (Shu 1977). As the growth is expected to be non-linear the density contrast may become high on a single collapse time-scale. The original perturbations that grow may not be small, and examination of the density at early times in the simulation suggests that this may be the case. We can ask what is the source of the initial perturbations? In Section 3.1, a comparison of Run 1 with Run 2 showed that the clumping size-scale was not dependent on the
numbers of satellite particles. This suggests that numerical noise associated with low numbers of satellite particles is not the source of the initial perturbations. One possibility is that the halo particles are the source of the perturbations. This situation differs from that ordinarily encountered in simulations of galactic discs. A comparison of Run 2 with Run 3 showed that the halo with larger mass particles heated the tails and so had clumps with larger spacings. Consequently the halo may not only be seeding the perturbations, but since it heats the tails, the halo also affects the fastest growing wavelength.

To summarize, we find that the spacings between the clumps are consistent with Jeans wavelengths measured earlier during the simulation. The delay time-scale is similar to the growth timescales needed to develop the instability. This suggests that the Jeans instability is a viable explanation for the periodic substructure we see in our simulations.

### 3.3 Features in velocity space

The periodically spaced overdensities are visible in space coordinates and so could be visible in stellar number counts on the sky. However, removal of background number counts introduces noise in a measurement of density in a tidal tail (e.g. Yanny et al. 2009). As background number counts may be high it may be difficult to detect low-amplitude density perturbations in a tidal tail from number counts alone. Here we consider the possibility that the clumps also cause structure in the radial velocity field.

In Figs 6 and 7, for Runs 2 and 3, respectively, we have plotted radial velocity versus radius (right-hand panels, in galactocentric coordinates) for several sections of the tidal tails exhibiting clumping (shown in number density plots in the left-hand panels). In these figures, we show on the left the projected density for small regions of the tidal tail. For each panel on the left there is a corresponding
Figure 6. Left-hand panels: projected number densities at three different locations and times in Run 2 (0.86, 1.70 and 2.08 Gyr from top to bottom). We have chosen regions with prominent clumping in the tidal tails. Right-hand panels: for particles shown on each left-hand panel the radial velocity (y-axis) is plotted against radius (x-axis). Radial velocity perturbations are seen in the density clumps. These perturbations are of order 10 km s$^{-1}$ and might be detectable in a real tidal tail with high-resolution spectroscopy. Perturbative motions are along the direction of orbital motion in the tidal tail. They are not due to wiggling or bending of the tail. In between the clumps the mean velocities are divergent, and in the clumps they are convergent, consistent with compressive motions due to Jeans instability. There are higher velocity dispersions inside the clumps themselves implying that the development of the instability has heated the tail.
Figure 7. Same as Fig. 6 but for Run 3 and at times (0.77, 1.60 and 1.85 Gyr from top to bottom). Clumps are more closely spaced here than in Run 2. In this simulation the tails are colder and denser because of reduced heating by the halo. As a result the Jeans wavelength is smaller and shorter wavelength perturbations have grown in the tail. Velocity perturbations are smaller than shown in Fig. 6 for Run 2 with larger halo particles.

Panel on the right showing the distribution of radial velocity $v_r$ versus galactocentric radius $r$. The radial velocity component would be consistent with a radial velocity measurement by an observer near the Galactic centre. We chose these components to roughly illustrate phenomena that would be seen for a distant tidal stream as observed from the Sun. However we did not project components from a specific location outside the galactic centre which would have required us to specify an arbitrary location within the context of this simulation.

In Figs 6 and 7, the same regions that display substructure (clumps) in spatial coordinates also exhibit substructure in the velocity plots. It is important to note that though we have plotted the
velocity in the radial direction of a spherical galactocentric coordinate, we find that the velocity gradient points along the path of the orbit and is consistent with longitudinal compression along the tidal tail and along the orbit. These plots should not be misinterpreted in terms of bending of the tail.

In Figs 6 and 7, the densest regions in the left-hand panels show the density corresponds to bright vertical regions on the $v_{\parallel}$ versus $r$ plots shown on the right. These regions have larger ranges in the radial velocity component, so larger velocity dispersions. The clumps have larger velocity dispersions than interclump regions.

Steps in the $v_{\parallel}$ versus $r$ plots correspond to changes in the mean velocity. Steps are particularly visible in the bottom-right-hand panel of Fig. 6. The smooth drop in velocity with increasing radius in the bottom-right-hand panel is caused by the orbit. If one was to subtract a smooth mean curve from this panel a sinuoidal-like oscillation would remain. This corresponds to positive and negative velocity perturbations about the mean orbital velocity. After subtraction the zeros of the sinusoidal oscillation lie in the interclump region. These correspond to velocities diverging from the mean orbital velocity. The maxima of the sinusoidal oscillation correspond to clumps where the velocities are converging. Thus, the motions are diverging in the interclump regions and converging in the clumps. This is the motion expected for longitudinal compressive motions oriented along the direction of the orbit and along the tail.

From the mass continuity equation, we can check whether the velocity in the tail is consistent with the growth time-scales for clumping. The mass continuity equation is

$$\frac{\Delta \rho}{\Delta t} + \nabla \cdot (\rho v) = 0. \quad (3)$$

To order of magnitude this gives, $(\Delta \rho/\Delta t) \approx \rho (\Delta v/\Delta x)$, where $\Delta x$ is the spacing between clumps, $\Delta \rho$ the difference between clump and interclump density, $\Delta v$ the size of the velocity perturbations and $\Delta t$ the time-scale of the instability. Solving for $\Delta v$:

$$\Delta v \sim \frac{\Delta x \Delta \rho}{\Delta t \rho}. \quad (4)$$

For spacing between the clumps, $\Delta x$, of 3 kpc, growth time-scales, $\Delta t$, of 0.3 Gyr and a density contrast, $\Delta \rho/\rho$, of 2, we estimate a $\Delta v$ of 20 km s$^{-1}$. This is about the size of the velocity jumps between the clumps in the panels shown in Figs 6 and 7, and implies that the velocity jumps we see in the simulation are consistent with longitudinally compressive motions on the growth time-scale of the instability.

We note that the clumps have larger velocity dispersion compared to the interclump velocity dispersions. This implies that the Jeans wavelength measured after the clumps have grown is larger than that present prior to the growth of the instability. When we measure the Jeans wavelength in a region exhibiting clumps we find that it is much larger than the spacing between the clumps. This is not necessarily a contradiction as the current clumps grew when the tail was colder. Not surprisingly, the growth of the instability itself heats the tail as gravitational energy is converted to kinetic energy. However, this does present a problem for calculating a Jeans wavelength from an observed tail that already exhibits the instability in the form of clumping. When we calculate the Jeans wavelength from the regions shown after clump formation in Figs 4 and 5 we find that it exceeds the distance between the clumps. As there is an increase in the velocity dispersion caused by the instability, it is likely that tidal tails can be thickened by the growth of Jeans instabilities. This is possibly an issue in the interpretation of heating of tails from dark matter substructure alone (Ibata et al. 2002; Johnston et al. 2002; Siegal-Gaskins & Valluri 2008; Carlberg 2009).

In Section 3.1 we noted (see Figs 1 and 2) that the clumping was particularly prominent following galactic pericentre passage. The density of the tail increases as the tail passes through pericentre allowing shorter wavelengths to become unstable.

Previous studies have found structure in phase space in tidal tails. For example, phase wrapping has caused clumps to appear in the velocity field (Helmi & deZeeuw 2000; Minchev et al. 2009). However, this process requires many orbital times to develop and appears in the velocity distribution after the tail has wrapped multiple times around the galaxy. Consequently this type of phase space structure is unlikely to be confused with the periodic features seen in the radial velocity plots shown in Figs 6 and 7.

In summary, we observe correlations between velocity dispersion, mean velocity and density in the tidal tails consistent with a compressive instability. Such correlations might in the future be used to identify clumping via the Jeans instability and heating from the Jeans instability to differentiate it from clumping due to halo substructure.

4 DISCUSSION

Here we discuss differentiating between the Jeans instability and other mechanisms of structure formation in tidal tails. We contrast our results with alternate explanations for clumping in tidal tails. Furthermore, we compare our results with simulations shown in Siegal-Gaskins & Valluri (2008), which explore correlations between subhaloes in the dark matter halo and substructure in tidal tails. Lastly, we consider the possibility that external tidal forces may suppress the formation of the clumps.

4.1 An alternate mechanism

An alternate explanation for the periodic overdensities in tidal tails was proposed by Kupper et al. (2008, 2010), where they attribute the cause of the clumping to epicyclic motions of the escaped stars in the tails. The clumps correspond to places where the stars in the tail slow down in their epicyclic motion.

The analytic models explored by Kupper et al. require the distance to the first overdensity to be located some multiple of the tidal radius away from the core of the disrupting object. The time it takes the first clump to form is related to the inverse of the epicyclic frequency, and is the time it takes stripped stars to reach the distance to the first clump. After twice that duration, the stars have progressed twice as far and form a second overdensity. Thus, in the case of a circular orbit and constant tidal field (Kupper et al. 2008), after every multiple of this time-scale another clump forms.

There are differences between the morphology we see in our simulations than that expected for epicyclic overdensities. Though the clumps in our simulations form at intervals similar to the tidal radius of the initial dwarf galaxy, the distance to the first clump is much larger than the distance between the clumps. The periodic spacing between the clumps is not the same as the distance to the first clump as the epicyclic explanation would predict. Whereas the strength of the clumps decreases as a function of distance from the parent body, here we see strong clumps forming large distances from the parent body. We find that the timing of the creation of the clumps in our simulations differ from that expected from an epicyclic explanation. Clumps in a region grow simultaneously, not in order of distance from the dwarf galaxy core. While epicyclic overdensities are a possible explanation for clumps...
seen in Palomar 5’s tail they are unlikely to be the explanation for the clumps seen in the simulations presented here.

4.2 Halo substructure

Clumping in tidal debris has previously been investigated by Siegal-Gaskins & Valluri (2008), who found that subhaloes were responsible for increased substructure in tidal tails. This can be seen in their sky projections in fig. 3 comparing star particles in simulations with and without the presence of subhaloes. A smooth halo (top left-hand panel of their fig. 3) produces less clumpy debris than a halo with substructure (top right-hand panel). Similar to our simulations, they also find the substructure is observable in velocity space, as demonstrated in their fig. 6 showing radial velocity versus radius of their star particles, again in simulations with and without subhaloes.

While most previous works (e.g. Ibata et al. 2002; Johnston et al. 2002; Mayer et al. 2002; Penarrubia et al. 2006; Carlb erg 2009) found that halo substructure heats tidal tails, Siegal-Gaskins & Valluri (2008) found in some cases that the tails in smooth haloes were colder and denser than in haloes with more substructure. It is possible that Jeans instability at shorter wavelengths was responsible for additional heating in these simulations. If so the connection between tidal tail and halo substructure may be more complex than previously considered.

4.3 Suppression due to external tides

An interesting question to consider is if it is possible for external tides to suppress the growth of the instability. If the tidal tails happen to be oriented in a radial direction, the tidal shear could potentially counteract the gravitational force between the particles forming the clumps.

To investigate this we present an order of magnitude exploration of the accelerations involved. We can compare a tidal acceleration at a position one wavelength from a clump centre to the acceleration of the clumping particles. The tidal acceleration along the radial direction is \( GM(<D)/\Delta x/D^2 \), where \( M(<D) \) is the mass of the galaxy within galactocentric radius \( D \), and \( \Delta x \) is the instability wavelength. This can be simplified to approximately \( \Omega^2 \Delta x \), where \( \Omega \) is the galactic rotation rate at \( D \). The acceleration of the instability is \( \Delta x/\Delta t^2 \), where \( t \) is the growth time-scale. Thus we need only compare \( \Omega \) to \( \Delta t^{-1} \). We can compute these two quantities for our setting. At 30 kpc the orbital period is about 1 Gyr, the angular rotation rate is the inverse of that times 2\( \pi \). The growth time-scale for clumping in Run 2 is 0.3 Gyr so the strength of the tidal acceleration is less than that of the acceleration of the clumps and growth is not suppressed.

The condition is the same as comparing a galactic mean density to 4\( \pi \) times the tail density (the 4\( \pi \) comes from the growth time-scale) – this is quite similar to the density ratio criterion for tidal disruption. We know the dwarf galaxy disrupted tidally, otherwise there would be no tidal tails, so the density ratio itself would be less than 1. The factor of 4\( \pi \) seems to allow the instability to overcome tidal forces in the case of our simulation. However, this might not be the case if the tail density is lower by an order of magnitude than we find here.

5 SUMMARY AND CONCLUSION

We have used hybrid test particle/N-body integrations to increase the number of particles integrated within an N-body integrator so that we can more accurately resolve substructure. We have used our code to study the tidal disruption of a dwarf galaxy in a polar orbit of a Milky Way type galaxy. We have placed additional test particles in the dwarf galaxy so that we can resolve fine structure in the tidal tails.

In our simulations, we have found that stellar tidal tails can exhibit periodic ridges oriented perpendicular to the orbit. Such structure had not previously been noticed in similar N-body simulations, possibly because the tidal tails did not contain sufficient numbers of particles. However, similar structure has been seen in the tidal tails of major mergers in both low-resolution N-body simulations and high-resolution N-body/SPH simulations (Wetzstein et al. 2007). In this setting the clumps are interpreted in terms of a gravitational instability.

By comparing simulations with different numbers of massive dwarf and halo particles and with and without a disc, we have considered several explanations for the formation of clumps. We have ruled out the following methods for the formation of periodically spaced overdensities in our simulation: shocking by the galactic disc, numerical noise associated with underpopulating the dwarf galaxy, amplification of structure in the halo and epicyclic motions in the tidal tails.

We have measured the Jeans wavelength prior to the growth of the substructure and found that the tails are unstable to Jeans instability. The wavelengths of subsequently formed clumps are approximately consistent with the Jeans wavelength measured in the tail prior to formation. The time-scale for growth is approximately consistent with the estimated growth time-scale. These estimates suggest that Jeans instability is a viable interpretation of the clumps exhibited by our simulations. We find that the spacing between clumps is sensitive to the mass of our simulated dark matter halo particles. This is likely because heating by dark matter particles can increase the Jeans wavelength of the tidal tail. Dark matter particles or subhaloes could also be responsible for seeding the instability.

We find that the clumps are also visible in radial velocity projections suggesting that Jeans instabilities may be observable in tidal tails in our galaxy not only using number counts of stars but also in phase space using comparisons of radial velocity versus distance or position on the sky.

In the future, we expect increasingly rich data samples expanding the number of stars in the Milky Way with measured properties. These surveys may make it possible to probe for or rule out substructure in tidal tails such as exhibited by our simulations. If Jeans instabilities occur in tidal tails then associated heating caused by them should not be interpreted in terms of heating by halo substructure alone. Furthermore, the fastest growing unstable wavelength may be sensitive to heating from the dark matter substructure implying that there may be a complex connection between halo and tidal tail substructure. We note that clumps are particularly prominent following pericentre passage. Future observational surveys would be most likely to find evidence of clumping in tidal tails at post pericentre orbital locations.

The simulations carried out here were made using a modest direct N-body code on a desktop computer. Future studies could test the results presented here with a more sophisticated N-body code (such as a tree code), and integrate more particles by doing the simulations on a supercomputer. Further studies can be carried out in different mass and stripping regimes for the disrupting object, such as globular clusters. We would also be interested in carrying out simulations that would more fully probe the possible connection between dark matter substructure and tidal tail
morphology by simulating a more detailed halo than considered here.

ACKNOWLEDGMENTS

We thank Larry Widrow for giving us and helping us with his code GALACTICICS. We thank Jeff Bailin, Chris Purcell and Heidi Newberg for helpful communications. The King model was generated using code made available\(^2\) by Sergey Mashchenko. We thank Evgenii Gaburov and Stefan Harfst for making \(\phi\)GRAPE and SAPPORO available. Support for this work was provided by NSF through award AST-0907841.

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\(^2\)http://www.physics.mcmaster.ca/syam/software.html

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