The SuperWorlds of SU(5) and SU(5) × U(1):
A Critical Assessment and Overview

JORGE L. LOPEZ\(^{(a),(b)}\), D. V. NANOPOLLOS\(^{(a),(b),(c)}\), and A. ZICHICHI\(^{(d)}\)

\(^{(a)}\)Center for Theoretical Physics, Department of Physics, Texas A&M University
College Station, TX 77843-4242, USA
\(^{(b)}\)Astroparticle Physics Group, Houston Advanced Research Center (HARC)
The Woodlands, TX 77381, USA
\(^{(c)}\)CERN, Theory Division, 1211 Geneva 23, Switzerland
\(^{(d)}\)CERN, 1211 Geneva 23, Switzerland

ABSTRACT

We present an overview of the simplest supergravity models which enforce radiative breaking of the electroweak symmetry, namely the minimal SU(5) supergravity model and the class of string-inspired/derived supergravity models based on the flipped SU(5) × U(1) structure supplemented by a minimal set of additional matter representations such that unification occurs at the string scale (∼ 10^{18} \text{ GeV}). These models can be fully parametrized in terms of the top-quark mass, the ratio \( \tan \beta = v_2/v_1 \), and three supersymmetry breaking parameters \( m_1/2, m_0, A \). The latter are chosen in the minimal SU(5) model such that the stringent constraints from proton decay and cosmology are satisfied. In the flipped SU(5) case we consider two string-inspired supersymmetry breaking scenarios: SU\((N,1)\) no-scale supergravity and a dilaton-induced supersymmetry breaking scenario. Both imply universal soft supersymmetry breaking parameters: \( m_0 = A = 0 \) and \( m_0 = \sqrt{3} m_{1/2}, A = -m_{1/2} \) respectively. We present a comparative study of the sparticle and Higgs spectra of both flipped SU(5) models and the minimal SU(5) model and conclude that all can be partially probed at the Tevatron and LEP II (and the flipped models at HERA too). In both flipped SU(5) cases there is a more constrained version which allows to determine \( \tan \beta \) in terms of \( m_t, m_\tilde{g} \) and which leads to much sharper and readily accessible experimental predictions. We also discuss the prospects for indirect experimental detection: a non-trivial fraction of the parameter space of the flipped SU(5) models is in conflict with the present experimental allowed range for the \( b \to s\gamma \) rare decay mode, and the one-loop electroweak radiative corrections imply the 90\% CL upper bound \( m_t < \sim 175 \text{ GeV} \).
1 Introduction

The Standard Model of electroweak and strong interactions is well established by now. In fact, the effects of the top quark in one-loop electroweak processes predict its mass (within \( \approx 20\% \)) centered around \( \approx 145 \text{ GeV} \) \(^1\). Therefore, its expected direct experimental detection at the Tevatron in the near future will complete the set of Standard Model predictions for the vector and fermion sectors. The scalar sector is another story. The simplest electroweak symmetry breaking scenario with a single Higgs boson is only mildly constrained experimentally, with a lower bound of \( m_H \gtrsim 60 \text{ GeV} \) \(^2\) and no firm indirect experimental upper bound, although this situation will change once the top quark mass is measured \(^3\). On the other hand, interesting upper bounds on \( m_H \) follow from various theoretical assumptions, such as perturbative unitarity at tree- \( (m_H \lesssim 700 \text{ GeV}) \) \(^4\) and one-loop \( (m_H \lesssim 400 \text{ GeV}) \) \(^5\) levels, and the stability of the Higgs potential \( (m_H \lesssim 500 \text{ GeV}) \) \(^6\). In practice, with the advent of the SSC and LHC, experimental information about the TeV scale is likely to clarify the composition of the Higgs sector. Nevertheless, despite all these efforts the structure of the Standard Model and its corresponding Higgs sector will remain basically unexplained.

It has therefore become customary to turn to the physics at very high energies to search for answers to these theoretical questions. The most promising theories of this kind contain two new ingredients: supersymmetry and unification. Together these can explain the origin of the weak scale (\( i.e. \), the gauge hierarchy problem) relative to the very high energy unification \( (M_U) \) or Planck \( (M_P) \) scales \(^7\). Furthermore, this class of theories predict a new set of relatively light \( (\lesssim O(1 \text{ TeV})) \) particles consisting of partners for the Standard Model particles but with spin offset by \( 1/2 \) unit. In fact, the new set of particles appears ever more likely to overlap little with the mass scales of the standard ones, thus their present unobserved status. Moreover, the Standard Model Higgs boson will then appear as one of the new particles but with mass close to \( M_Z \), thus avoiding naturally the theoretical problems mentioned above.

Unfortunately, the introduction of supersymmetry also increases significantly the number of unknown parameters in the theory, mainly because this symmetry must be softly broken at low energies. Indeed, to describe a generic low-energy supersymmetric model (the so-called minimal supersymmetric standard model (MSSM)) neglecting the first- and second-generation Yukawa couplings, the CKM angles, and possible CP violating phases, we need the following set of parameters (the values of \( \sin^2 \theta_w, \alpha_3, \alpha_e, M_Z \) are taken as measured parameters):

(a) The Yukawa \( (\lambda_t, \lambda_b, \lambda_\tau) \) and Higgs mixing \( (\mu) \) superpotential couplings. (We can trade the Yukawa couplings for \( m_t, \tan \beta; m_b, m_\tau \), with \( \tan \beta = v_2/v_1 \) the ratio of Higgs vacuum expectation values, and \( m_b, m_\tau \) given.)

(b) The soft-supersymmetry breaking trilinear \( (A_t, A_b, A_\tau) \) and bilinear \( (B) \) scalar couplings (corresponding to the superpotential couplings in \( (a) \)).
(c) The soft-supersymmetry breaking left-left and right-right entries in the squark and slepton mass matrices for the first and second \((m_{Q,U,D,\tilde{D}}, m_{L,E,\tilde{E}})\), and third \((m_{Q_3,U_3,D_3,\tilde{D}_3}, m_{L_3,E_3})\) generations.

(d) The soft-supersymmetry breaking gaugino masses \(m_{\tilde{g}}, m_{\tilde{W}}, m_{\tilde{B}}\).

(e) The Higgs sector parameter (at tree-level), \(e.g.,\) the pseudoscalar Higgs boson mass \(m_A\).

The above 21 unknown parameters make any thorough analysis of this class of models rather impractical, and have allowed in the past only limited explorations of this parameter space. If we now add the gauge unification constraint \((\alpha_i(M_U) = \alpha_U, i = 1, 2, 3)\), the assumption of universal soft-supersymmetry breaking at a scale \(\Lambda_{susy} = M_U\), and high-energy dynamics (in the form of renormalization group equations (RGEs) for all the parameters involved), the set of parameters in (b) reduces to \(A = A_t = A_b = A_{\tau} and B\), those in (c) to \(m_0 = m_{Q,U,D,\tilde{D}} = m_{Q_3,U_3,D_3,\tilde{D}_3} = m_{L,E,\tilde{E}} = m_{L_3,E_3}\), and those in (d) to \(m_{1/2} = m_{\tilde{g}} = m_{\tilde{W}} = m_{\tilde{B}}\); these relations are valid only at the scale \(\Lambda_{susy}\). The number of parameters has been dramatically reduced down to eight.

Let us now add low-energy dynamics by demanding radiative breaking of the electroweak symmetry. The tree-level Higgs potential is given by

\[
V_0 = (m_{H_1}^2 + \mu^2)|H_1|^2 + (m_{H_2}^2 + \mu^2)|H_2|^2 + B\mu(H_1H_2 + h.c.) + \frac{1}{8}g_2^2(H_2^\dagger \sigma H_2 + H_1^\dagger \sigma H_1)^2 + \frac{1}{8}g'^2\left(|H_2|^2 - |H_1|^2\right)^2, \tag{1}
\]

where \(H_1 \equiv \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}\) and \(H_2 \equiv \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}\) are the two complex Higgs doublet fields, \(g' = \sqrt{5/3}g_1\) and \(g_2\) are the \(U(1)_Y\) and \(SU(2)_L\) gauge couplings, and \(B\mu\) is taken to be real and negative. This potential has a minimum if \(\partial V_0/\partial \phi_i = 0\), with \(\phi_i\) denoting the eight real degrees of freedom of \(H_1\) and \(H_2\). In particular, for \(\phi_i = \text{Re} H_i^0\) one obtains two constraints which allow the determination of \(\mu\) and \(B\),

\[
\mu^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2}M_Z^2, \tag{2}
\]

\[
B\mu = -\frac{1}{2} \sin 2\beta (m_{H_1}^2 + m_{H_2}^2 + 2\mu^2) < 0, \tag{3}
\]

up to the sign of \(\mu\). In these expressions, \(m_{H_1}^2, m_{H_2}^2\) are soft-supersymmetry breaking masses equal to \(m_0^2\) at \(\Lambda_{susy}\). Since the whole set of Higgs masses and couplings (at tree-level) follows from \(m_A^2\) (and \(\tan \beta\)), and one can easily show that \(m_A^2 = -2B\mu/\sin 2\beta\), the parameter in (e) is also determined. (This result also holds at one-loop although the expression for \(m_A^2\) is more complicated in this case.)

\footnote{The parameters in \(V_0\) must satisfy further consistency constraints to insure that this is a true minimum of the tree-level Higgs potential. As discussed in Ref. [8], the one-loop effective potential satisfies most of these constraints automatically.}
The final parameter count in the class of models we consider is then just five: $m_t, \tan \beta, m_{1/2}, m_0, A$ (plus the sign of $\mu$). Note also that $\sin^2 \theta_w$ (as well as $M_U$ and $\alpha_U$) gets determined (from $\alpha_3$ and $\alpha_e$) by the gauge unification condition. What are the \textit{a priori} expected values of $m_{1/2}, m_0, A$? In principle choosing a suitable supergravity model (\textit{i.e.}, a suitable hidden sector) one could have arbitrary values for these parameters. In fact, such generic scenarios are worthwhile studying. However, further well motivated constraints on this sector of the theories turn out to yield quite predictive models. Below we consider two string-inspired supersymmetry breaking scenarios where $m_0$ and $A$ are fixed functions of $m_{1/2}$. We also consider a generic scenario which is constrained by the proton lifetime and which results in strict restrictions on the allowed choices for these three parameters.

In this paper we study and contrast two well motivated examples belonging to the class of unified supersymmetric models described above, namely the class of flipped $SU(5)$ supergravity models and the minimal $SU(5)$ supergravity model. Our purpose is to describe their features and compare their predictions. However, we emphasize throughout that the underlying motivation for these models is different: flipped $SU(5)$ is an archetypal string model, whereas minimal $SU(5)$ is an archetypal traditional grand unified model. The conceptual differences between these two frameworks are significant, \textit{e.g.}:

1. In string models the gauge couplings of the various gauge groups (which are typical in these constructions\cite{9}) all unify at a calculable scale near the Planck scale.\footnote{This result holds as long as all gauge groups are represented by level-one Kac-Moody algebras, otherwise various factors appear in the unification relations \cite{10}.} In fact, the decoupling effect of the infinite tower of string massive states can be incorporated exactly\cite{11,12,13}. In contrast, in traditional GUT models the unification scale is not predicted by the theory, although it can be easily determined by running the low-energy gauge couplings until they unify.

2. Grand unified gauge groups describing the observable sector of string models are rare\cite{14}, if not disfavored. One can easily obtain the Standard Model ($SU(3) \times SU(2)$) gauge groups\cite{13,14}, the Pati-Salam ($SU(4) \times SU(2) \times SU(2)$) gauge group\cite{17}, and the flipped $SU(5)$\cite{18,19} gauge group\cite{20}. Of these, only flipped $SU(5)$ actually unifies the non-abelian gauge couplings of the Standard Model. A proliferation of $U(1)$ factors is the norm. It is amusing to note that flipped $SU(5)$ can be argued to be the simplest unified gauge group\cite{13,21}, in that $SU(5)$ does not provide for neutrino masses, and $SO(10)$ includes flipped $SU(5)$.

3. The spectra of string models are correlated with the corresponding gauge groups\cite{22} and the fermion matter representations are automatically anomaly-free. In contrast, in traditional GUT models anomalies are actual constraints on the possible models and the representations one can choose from are only limited by gauge invariance. For instance, the missing partner mechanism\cite{23} to effect
the doublet-triplet splitting of the Higgs pentaplets in \( SU(5) \) requires the introduction of large representations which in a string model can only occur when complicated higher-level Kac–Moody algebra realizations of the gauge group are invoked \[22\].

4. All interactions in string models can be calculated once the model is specified. This full specification of the effective supergravity and the superpotential has no analog in GUT models, where all these are free parameters.

This paper is organized as follows. In Sec. 2 we discuss the typical characteristics of string models. In Sec. 3 we present in detail the flipped \( SU(5) \) models which we consider here. In Sec. 4 we discuss the string-inspired supersymmetry breaking scenario that we explore later. In Sec. 5 we consider the experimental predictions for all the sparticle and one-loop corrected Higgs boson masses in the flipped \( SU(5) \) models, and deduce several simple relations among the various sparticle masses. In Sec. 6 we repeat this analysis for special more constrained cases of the chosen supersymmetry breaking scenario. In Sec. 7 we discuss the minimal \( SU(5) \) supergravity model, including the constraints from proton decay and the neutralino relic density. In Sec. 8 we discuss the prospects for direct experimental detection of these particles at Fermilab, LEPI,II, and HERA, while in Sec. 9 we consider the corresponding indirect detection signatures. Finally, in Sec. 10 we summarize our conclusions.

## 2 Typical Characteristics of String Models

The ultimate unification of all particles and interactions has string theory as the best candidate. If this theory were completely understood, we would be able to show that string theory is either inconsistent with the low-energy world or supported by experimental data. Since our present knowledge of string theory is at best fragmented and certainly incomplete, it is important to consider models which incorporate as many stringy ingredients as possible. The number of such models is expected to be large, however, the basic ingredients that such “string models” should incorporate fall into few categories: (i) gauge group and matter representations which unify at a calculable model-dependent string unification scale; (ii) a hidden sector which becomes strongly interacting at an intermediate scale and triggers supersymmetry breaking with vanishing vacuum energy and hierarchically small soft supersymmetry breaking parameters; (iii) acceptable high-energy phenomenology, \( e.g. \), gauge symmetry breaking to the Standard Model (if needed), not-too-rapid proton decay, decoupling of intermediate-mass-scale unobserved matter states, etc.; (iv) radiative electroweak symmetry breaking; (v) acceptable low-energy phenomenology, \( e.g. \), reproduce the observed spectrum of quark and lepton masses and the quark mixing angles, sparticle and Higgs masses not in conflict with present experimental bounds, and not-too-large neutralino cosmological relic density.

All the above are to be understood as constraints on potentially realistic string models. Since some of the above constraints can be independently satisfied in specific
models, the real power of a string model rests in the successful satisfaction of all these constraints within a single model. In what follows, “string-derived” models refer to models which can be derived rigorously from string, even if not all their interactions have been determined explicitly; “string-inspired” models are those field-theoretical models which are believed to be in principle derivable from string, although most likely not exactly reproducible; finally “string” models refer generically to both kinds, although perhaps describe more accurately the conceptual framework these models are examples of.

String model-building is at a state of development where large numbers of models can be constructed using various techniques (so-called formulations) [9]. Such models provide a gauge group and associated set of matter representations, as well as all interactions in the superpotential, the Kähler potential, and the gauge kinetic function. The effective string supergravity can then be worked out and thus all the above constraints can in principle be enforced. In practice this approach has never been followed in its entirety: sophisticated model-building techniques exist which can produce models satisfying constraints (i), (iii), (iv) and part of (v); detailed studies of supersymmetry breaking triggered by gaugino condensation have been performed for generic hidden sectors; and extensive explorations of the soft-supersymmetry breaking parameter space satisfying constraints (iii), (iv), and (v) have been conducted.

In searching for good string model candidates, we are faced with two kinds of choices to be made: the choice of the gauge and matter content of the model, and the choice of the supersymmetry breaking mechanism. Fortunately, a string theory theorem provides significant enlightenment regarding the first choice: models whose gauge groups are constructed from level-one Kac-Moody algebras do not allow adjoint or higher representations in their spectra [22]. This implies that the traditional GUT groups (SU(5), SO(10), E6) are excluded since the GUT symmetry would remain unbroken. Exceptions to this theorem exist if one uses the technically complicated higher-level Kac-Moody algebras [14], but these models are beset with constraints [22]. If one imposes the aesthetic constraint of unification of the Standard Model non-abelian gauge couplings, then flipped SU(5) [18, 19, 20, 21] emerges as the prime candidate, as we shortly discuss. String models without non-abelian unification, such as the standard-like models of Refs. [15, 16] and the Pati-Salam-like model of Ref. [17] possess nonetheless gauge coupling unification at the string scale, even though no larger structure is revealed past this scale. However, the degree of phenomenological success which some of these models enjoy, usually rests on some fortuitous set of vanishing couplings which are best understood in terms of remnants of higher symmetries.

Besides the very economic GUT symmetry breaking mechanism in flipped SU(5) [18, 19] – which allows it to be in principle derivable from superstring theory [20] – perhaps one of the more interesting motivations for considering such a unified gauge group is the natural avoidance of potentially dangerous dimension-five proton decay operators [21]. In Ref. [24] we constructed a supergravity model based on this gauge group, which has the additional property of unifying at a scale \( M_U = \mathcal{O}(10^{18}) \) GeV, as expected to occur in string-derived versions of this model [12]. As
such, this model constitutes a blueprint for string model builders. In fact, in Ref. [25] one such model was derived from string and served as inspiration for the field theory model in Ref. [24]. The string unification scale should be contrasted with the naive unification scale, $M_U = \mathcal{O}(10^{16}\text{GeV})$, obtained by running the Standard Model particles and their superpartners to very high energies. This apparent discrepancy of two orders of magnitude [26, 27] creates a gap which needs to be bridged somehow in string models. It has been shown [28] that the simplest solution to this problem is the introduction in the spectrum of heavy vector-like particles with Standard Model quantum numbers. The minimal such choice [29, 30], a quark doublet pair $Q, \bar{Q}$ and a $1/3$–charge quark singlet pair $D, \bar{D}$, fit snugly inside a $10, \bar{10}$ pair of flipped $SU(5)$ representations, beyond the usual $3 \cdot (10 + \bar{5} + 1)$ of matter and $10, \bar{10}$ of Higgs.

In this model, gauge symmetry breaking occurs due to vacuum expectation values (vevs) of the neutral components of the $10, \bar{10}$ Higgs representations, which develop along flat directions of the scalar potential. There are two known ways in which these vevs (and thus the symmetry breaking scale) could be determined:

(i) In the conventional way, radiative corrections to the scalar potential in the presence of soft supersymmetry breaking generate a global minimum of the potential for values of the vevs slightly below the scale where supersymmetry breaking effects are first felt in the observable sector [21]. If the latter scale is the Planck scale (in a suitable normalization) then $M_U \sim M_{Pl}/\sqrt{8\pi} \sim 10^{18}\text{GeV}$.

(ii) In string-derived models a pseudo $U(1)$ anomaly arises as a consequence of truncating the theory to just the massless degrees of freedom, and adds a contribution to its $D$-term, $D_A = \sum q_i^A |\langle \phi_i \rangle|^2 + \epsilon$, with $\epsilon = g^2 \text{Tr} U_A(1)/192\pi^2 \sim (10^{18}\text{GeV})^2$ [31].

To avoid a huge breaking of supersymmetry we need to demand $D_A = 0$ and therefore the fields charged under $U_A(1)$ need to get suitable vevs. Among these one generally finds the symmetry breaking Higgs fields, and thus $M_U \sim 10^{18}\text{GeV}$ follows.

In general, both these mechanisms could produce somewhat lower values of $M_U$. However, $M_U \gtrsim 10^{16}\text{GeV}$ is necessary to avoid too rapid proton decay due to dimension-six operators [32]. In these more general cases the $SU(5)$ and $U(1)$ gauge couplings would not unify at $M_U$ (only $\alpha_2$ and $\alpha_3$ would), although they would eventually “superunify” at the string scale $M_{SU} \sim 10^{18}\text{GeV}$. To simplify matters, below we consider the simplest possible case of $M_U = M_{SU} \sim 10^{18}\text{GeV}$. We also draw inspiration from string model-building and regard the Higgs mixing term $\mu h\bar{h}$ as a result of an effective higher-order coupling [33, 34, 35], instead of as a result of a light singlet field getting a small vev (i.e., $\lambda h\phi \rightarrow \lambda \langle \phi \rangle h\bar{h}$) as originally considered [19, 21]. An additional contribution to $\mu$ is also generically present in supergravity models [36, 33, 37].

The choice of supersymmetry breaking scenario is less clear. Below we show that the phenomenologically acceptable choices basically fall in two categories:

1. The no-scale ansatz [38], which ensures the vanishing of the (tree-level) cosmological constant even after supersymmetry breaking. This framework also arises in the low-energy limit of superstring theory [33]. In a theory which contains heavy fields, the minimal no-scale structure $SU(1, 1)$ [10] is generalized to
which implies that the scalar fields do not feel the supersymmetry breaking effects. In practice this means that the universal scalar mass ($m_0$) and the universal cubic scalar coupling ($A$) are set to zero. The sole source of supersymmetry breaking is the universal gaugino mass ($m_{1/2}$), i.e.,

$$m_0 = 0, \quad A = 0. \quad (4)$$

2. The dilaton $F$-term scenario, which also leads to universal soft supersymmetry breaking parameters

$$m_0 = \frac{1}{\sqrt{3}} m_{1/2}, \quad A = -m_{1/2}. \quad (5)$$

In either case, after enforcement of the above constraints, the low-energy theory can be described in terms of just three parameters: the top-quark mass ($m_t$), the ratio of Higgs vacuum expectation values ($\tan \beta$), and the gluino mass ($m_{\tilde{g}} \propto m_{1/2}$). Therefore, measurement of only two sparticle or Higgs masses would determine the remaining thirty. Moreover, if the hidden sector responsible for these patterns of soft supersymmetry breaking is specified, the gravitino mass ($m_0$) will also be determined and the supersymmetry breaking sector of the theory will be completely fixed.

In sum, we see basically two unified string supergravity models emerging as good candidates for phenomenologically acceptable string models, both of which include a flipped $SU(5)$ observable gauge group supplemented by matter representations in order to unify at the string scale $M_U \sim 10^{18}$ GeV \[28, 29, 30\], and supersymmetry breaking is parametrized by either of the scenarios in Eqs. (4,5).

We should remark that a real string model will include a hidden sector in addition to the observable sector discussed in what follows. The model presented here tacitly assumes that such hidden sector is present and that it has suitable properties. For example, the superpotential in Eq. (3) below, in a string model will receive contributions from cubic and higher-order terms, with the latter generating effective observable sector couplings once hidden sector matter condensates develop \[33\]. The hidden sector is also assumed to play a fundamental role in triggering supersymmetry breaking via e.g., gaugino condensation. This in turn would make possible the mechanism for gauge symmetry breaking discussed above. Probably the most important constraint on this sector of the theory is that it should yield one of the two supersymmetry breaking scenarios outlined above.

### 3 The SU(5) x U(1) Models

The model we consider is a generalization of that presented in Ref. \[19\], and contains the following flipped $SU(5)$ fields:

1. three generations of quark and lepton fields $F_i, \bar{f}_i, l^c_i, i = 1, 2, 3$;

2. two pairs of Higgs $10, \bar{10}$ representations $H_i, \bar{H}_i, i = 1, 2$;

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SU($N, 1$) \[41\] which implies that the scalar fields do not feel the supersymmetry breaking effects. In practice this means that the universal scalar mass ($m_0$) and the universal cubic scalar coupling ($A$) are set to zero. The sole source of supersymmetry breaking is the universal gaugino mass ($m_{1/2}$), i.e.,

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2. two pairs of Higgs $10, \bar{10}$ representations $H_i, \bar{H}_i, i = 1, 2$;
3. one pair of “electroweak” Higgs 5,5 representations \( h, \bar{h} \);
4. three singlet fields \( \phi_{1,2,3} \).

Under \( SU(3) \times SU(2) \) the various flipped \( SU(5) \) fields decompose as follows:

\[
\begin{align*}
F_i &= \{Q_i, d_i^c, u_i^c\}, \quad \bar{f}_i = \{L_i, u_i^c\}, \quad \bar{l}_i^c = e_i^c, \\
H_i &= \{Q_H, d_H^c, \nu_{H_i}\}, \quad \bar{H}_i = \{Q_{H_i}, d_{H_i}^c, \nu_{H_i}\}, \\
h &= \{H, D\}, \quad \bar{h} = \{\bar{H}, \bar{D}\}.
\end{align*}
\]

The most general effective superpotential consistent with \( SU(5) \times U(1) \) symmetry is given by

\[
W = \lambda_{ij} F_i F_j h + \lambda_{ij}^{\prime} F_i \bar{f}_j h + \lambda_{ij}^{\prime\prime} \bar{f}_i^c \bar{h} + \mu h \bar{h} + \lambda_{ij}^{\prime} H_i H_j h + \lambda_{ij}^{\prime\prime} \bar{H}_i \bar{H}_j \bar{h}
+ \lambda_{ij}^{\prime} H_i F_j h + \lambda_{ij}^{\prime\prime} \bar{H}_i \bar{f}_j h + \lambda_{ij}^{\prime\prime\prime} \bar{f}_i^c \bar{H}_j \phi_k + w_{ij}^{\prime} H_i \bar{H}_j + \mu_{ij}^{\prime} \phi_i \phi_j.
\]

Symmetry breaking is effected by non-zero vevs \( \langle \nu_{H_i} \rangle = V_i, \langle \nu_{H_i}^{\prime} \rangle = \bar{V}_i \), such that \( V_1^2 + V_2^2 = V_1^2 + \bar{V}_2^2 \).

### 3.1 Higgs doublet and triplet mass matrices

The Higgs doublet mass matrix receives contributions from \( \mu h \bar{h} \rightarrow \mu H \bar{H} \) and \( \lambda_{ij}^{\prime} H_i \bar{f}_j h \rightarrow \lambda_{ij}^{\prime} V_i L_j \bar{H} \). The resulting matrix is

\[
\mathcal{M}_2 = \begin{pmatrix}
H \\
\bar{H}
\end{pmatrix}
= \begin{pmatrix}
H_L & \lambda_{ij}^{\prime} V_i \\
\lambda_{ij}^{\prime\prime} V_i & \lambda_{ij}^{\prime\prime\prime} V_i
\end{pmatrix}.
\]

To avoid fine-tunings of the \( \lambda_{ij}^{\prime\prime\prime} \) couplings we must demand \( \lambda_{ij}^{\prime\prime\prime} \equiv 0 \), so that \( \bar{H} \) remains light.

The Higgs triplet matrix receives several contributions: \( \mu h \bar{h} \rightarrow \mu D \bar{D} \); \( \lambda_{ij}^{\prime} H_i F_j h \rightarrow \lambda_{ij}^{\prime} V_i d_j^c D \); \( \lambda_{ij}^{\prime} H_i H_j h \rightarrow \lambda_{ij}^{\prime} V_i d_j^c D \); \( \lambda_{ij}^{\prime} \bar{H}_i \bar{H}_j \bar{h} \rightarrow \lambda_{ij}^{\prime} \bar{V}_i d_j^c \bar{D} \); \( w_{ij}^{\prime} d_i^c d_j^c \). The resulting matrix is

\[
\mathcal{M}_3 = \begin{pmatrix}
D & d_{H_1}^c & d_{H_2}^c & d_{H_3}^c & d_1^c & d_2^c & d_3^c \\
d_{H_1}^c & \lambda_{ij}^{\prime} V_i & w_{11} & w_{12} & 0 & 0 & 0 \\
d_{H_2}^c & \lambda_{ij}^{\prime} V_i & w_{21} & w_{22} & 0 & 0 & 0 \\
d_{H_3}^c & \lambda_{ij}^{\prime} V_i & w_{31} & w_{32} & 0 & 0 & 0
\end{pmatrix}.
\]

\[4\]To be understood in the string context as arising from cubic and higher order terms [42, 33].
\[5\]The zero entries in \( \mathcal{M}_3 \) result from the assumption \( \langle \phi_k \rangle = 0 \) in \( \lambda_{ij}^{\prime\prime\prime} F_i \bar{H}_j \phi_k \).
Clearly three linear combinations of \( \{D, d_{H_{1,2}}, d_{c_{1,2,3}}\} \) will remain light. In fact, such a general situation will induce a mixing in the down-type Yukawa matrix \( \lambda^{ij}_1 F_i F_j h \rightarrow \lambda^{ij}_1 Q_i d^c_{c_{1,2,3}} \), since the \( d^c_{c_{1,2,3}} \) will need to be re-expressed in terms of these mixed light eigenstates. This low-energy quark-mixing mechanism is an explicit realization of the general extra-vector-abeyance (EVA) mechanism of Ref. [43]. As a first approximation though, in what follows we will set \( \lambda^{ij}_1 = 0 \), so that the light eigenstates are \( d^c_{c_{1,2,3}} \).

### 3.2 Neutrino see-saw matrix

The see-saw neutrino matrix receives contributions from:

\[
\lambda^{ij}_2 F_i \bar{f}_j \bar{h} \rightarrow m^{ij}_u \nu^c_i \nu_j; \quad \lambda^{ijk}_6 F_i \bar{H}_j \phi_k \rightarrow \lambda^{ijk}_6 \bar{V}_k \nu^c_i \phi_j; \quad \mu^{ij}_i \phi_i \phi_j.
\]

The resulting matrix is:

\[
\mathcal{M}_\nu = \begin{pmatrix}
\nu_i & \nu^c_i & \phi_j \\
m^{ij}_u & 0 & 0 \\
0 & \lambda^{kji} \bar{V}_k & \mu^{ij}
\end{pmatrix}.
\]  

(12)

### 3.3 Numerical scenario

To simplify the discussion we will assume, besides \( \lambda^{ij}_1' = \lambda^{ij}_2' = 0 \), that

\[
\lambda^{ij}_4 = \delta^{ij}_4 \lambda^{(i)}_4, \quad \lambda^{ij}_5 = \delta^{ij}_5 \lambda^{(i)}_5, \quad \lambda^{ijk}_6 = \delta^{ij}_6 \delta^{ik}_6 \lambda^{(i)}_6,
\]

and

\[
\mu^{ij}_i = \delta^{ij}_i \mu_i, \quad w^{ij} = \delta^{ij}_w.
\]

These choices are likely to be realized in string versions of this model and will not alter our conclusions below. In this case the Higgs triplet mass matrix reduces to

\[
\mathcal{M}_3 = \begin{pmatrix}
D & d^c_{H_1} & d^c_{H_2} \\
\mu & \lambda^{(1)}_4 V_1 & 0 \\
\lambda^{(2)}_5 \bar{V}_1 & w_1 & 0 \\
\lambda^{(2)}_5 \bar{V}_2 & 0 & w_2
\end{pmatrix}.
\]  

(15)

Regarding the \((3, 2)\) states, the scalars get either eaten by the \(X, Y\) \(SU(5)\) heavy gauge bosons or become heavy Higgs bosons, whereas the fermions interact with the

---

6Note that this mixing is on top of any structure that \( \lambda^{ij}_1 \) may have, and is the only source of mixing in the typical string model-building case of a diagonal \( \lambda_4 \) matrix.

7We neglect a possible higher-order contribution which could produce a non-vanishing \( \nu^c_i \nu^c_j \) entry [43].

8In Ref. [13] the discrete symmetry \( H_1 \rightarrow -H_1 \) was imposed so that these couplings automatically vanish when \( H_2, \bar{H}_2 \) are not present. This symmetry (generalized to \( H_i \rightarrow -H_i \)) is not needed here since it would imply \( w^{ij} = 0 \), which is shown below to be disastrous for gauge coupling unification.

---
$\tilde{X}, \tilde{Y}$ gauginos through the following mass matrix \[25\]

$$
\mathcal{M}_{(3,2)} = \begin{pmatrix}
    Q_{H_1} & Q_{H_2} & \tilde{Y} \\
    w_1 & 0 & g_5 V_1 \\
    0 & w_2 & g_5 V_2 \\
    g_5 V_1 & g_5 V_2 & 0
\end{pmatrix}.
$$

(16)

The lightest eigenvalues of these two matrices (denoted generally by $d_H$ and $Q_H$ respectively) constitute the new relatively light particles in the spectrum, which are hereafter referred to as the “gap” particles since with suitable masses they bridge the gap between unification masses at $10^{16}$ GeV and $10^{18}$ GeV.

Guided by the phenomenological requirement on the gap particle masses, i.e., $M_{Q_H} \gg M_{d_H}$ \[29\], we consider the following explicit numerical scenario

$$
\lambda_4^{(2)} = \lambda_5^{(2)} = 0, \quad V_1, \tilde{V}_1, V_2, \tilde{V}_2 \sim V \gg w_1 \gg w_2 \gg \mu,
$$

(17)

which would need to be reproduced in a viable string-derived model. From Eq. (14) we then get $M_{d_{H_2}} = M_{\tilde{d}_{H_2}} = w_2$, and all other mass eigenstates $\sim V$. Furthermore, $\mathcal{M}_{(3,2)}$ has a characteristic polynomial $\lambda^3 - \lambda^2 (w_1 + w_2) - \lambda (2V^2 - w_1 w_2) + (w_1 + w_2)V^2 = 0$, which has two roots of $O(V)$ and one root of $O(w_1)$. The latter corresponds to $\sim (Q_{H_1} - Q_{H_2})$ and $\sim (\tilde{Q}_{H_1} - \tilde{Q}_{H_2})$. In sum then, the gap particles have masses $M_{Q_H} \sim w_1$ and $M_{d_{H}} \sim w_2$, whereas all other heavy particles have masses $\sim V$.

The see-saw matrix reduces to

$$
\mathcal{M}_\nu = \nu^i \begin{pmatrix}
    \nu_i & \nu_i^c & \phi_i \\
    0 & m^i_u & 0 \\
    m^i_u & 0 & \lambda^{(i)} \tilde{V}_i \\
    0 & \lambda^{(i)} \tilde{V}_i & \mu^i
\end{pmatrix},
$$

(18)

for each generation. The physics of this see-saw matrix has been discussed in Ref. \[14\] and more generally in Ref. \[15\], where it was shown to lead to an interesting amount of hot dark matter ($\nu_\tau$) and an MSW-effect ($\nu_e, \nu_\mu$) compatible with all solar neutrino data. Moreover, the out-of-equilibrium decays of the $\nu^c$ “flipped neutrino” fields in the early Universe induce a lepton number asymmetry which is later processed into a baryon number asymmetry by non-perturbative electroweak processes \[16, 15\]. All these phenomena can occur in the same region of parameter space.

### 3.4 Proton decay

The dimension-six operators mediating proton decay in this model are highly suppressed due to the large mass of the $X, Y$ gauge bosons ($\sim M_U = 10^{18}$ GeV). Higgsino mediated dimension-five operators exist and are naturally suppressed in the minimal model of Ref. \[19\]. The reason for this is that the Higgs triplet mixing term $\mu h \tilde{h} \rightarrow \mu D\tilde{D}$ is small ($\mu \sim M_Z$), whereas the Higgs triplet mass eigenstates obtained
from Eq. (11) by just keeping the $2 \times 2$ submatrix in the upper left-hand corner, are always very heavy ($\sim V$). The dimension-five mediated operators are then proportional to $\mu/V^2$ and thus the rate is suppressed by a factor or $(\mu/V)^2 \ll 1$ relative to the unsuppressed case found in the standard $SU(5)$ model.

In the generalized model presented here, the Higgs triplet mixing term is still $\mu D \bar{D}$. However, the exchanged mass eigenstates are not necessarily all very heavy. In fact, above we have demanded the existence of a relatively light ($\sim w_1$) Higgs triplet state ($d_H^c$). In this case the operators are proportional to $\mu \alpha_i \bar{\alpha}_i / M_i^2$, where $M_i$ is the mass of the $i$-th exchanged eigenstate and $\alpha_i, \bar{\alpha}_i$ are its $D, \bar{D}$ admixtures. In the scenario described above, the relatively light eigenstates ($d_H^{c2}, d_H^{\bar{c}2}$) contain no $D, \bar{D}$ admixtures, and the operator will again be $\propto \mu/V^2$.

Note however that if conditions (17) (or some analogous suitability requirement) are not satisfied, then diagonalization of $M_3$ in Eq. (15) may re-introduce a sizeable dimension-five mediated proton decay rate, depending on the value of the $\alpha_i, \bar{\alpha}_i$ coefficients. To be safe one should demand $\mu \alpha_i \bar{\alpha}_i / M_i^2 < \frac{1}{10^{17} \text{GeV}}$. (19)

For the higher values of $M_{d_H^{c}}$ in Table 1 (see below), this constraint can be satisfied for not necessarily small values of $\alpha_i, \bar{\alpha}_i$.

### 3.5 Gauge coupling unification

Since we have chosen $V \sim M_U = M_{SU} = 10^{18}$ GeV, this means that the Standard Model gauge couplings should unify at the scale $M_U$. However, their running will be modified due to the presence of the gap particles. Note that the underlying flipped $SU(5)$ symmetry, even though not evident in this respect, is nevertheless essential in the above discussion. The masses $M_Q$ and $M_{d_H^{c}}$ can then be determined, as follows

$$
\ln \frac{M_{Q_H}}{m_Z} = \pi \left( \frac{1}{2\alpha_e} - \frac{1}{3\alpha_3} - \frac{\sin^2 \theta_w - 0.0029}{\alpha_e} \right) - 2 \ln \frac{M_U}{m_Z} - 0.63, \quad (20)
$$

$$
\ln \frac{M_{d_H^{c}}}{m_Z} = \pi \left( \frac{1}{2\alpha_e} - \frac{7}{3\alpha_3} + \frac{\sin^2 \theta_w - 0.0029}{\alpha_e} \right) - 6 \ln \frac{M_U}{m_Z} - 1.47, \quad (21)
$$

where $\alpha_e, \alpha_3$ and $\sin^2 \theta_w$ are all measured at $M_Z$. This is a one-loop determination (the constants account for the dominant two-loop corrections) which neglects all low- and high-energy threshold effects, but is quite adequate for our present purposes. As shown in Table 1 (and Eq. (21)) the $d_H^{c}$ mass depends most sensitively on $\alpha_3(M_Z) = 0.118 \pm 0.008$ [50], whereas the $Q_H$ mass and the unified coupling are rather insensitive.

---

9Here we assume a common supersymmetric threshold at $M_Z$. In fact, the supersymmetric threshold and the $d_H^{c}$ mass are anticorrelated. See Ref. [20] for a discussion.
Table 1: The value of the gap particle masses and the unified coupling for $\alpha_3(M_Z) = 0.118 \pm 0.008$. We have taken $M_U = 10^{18}$ GeV, $\sin^2 \theta_w = 0.233$, and $\alpha_e^{-1} = 127.9$.

| $\alpha_3(M_Z)$ | $M_{d_H}$ (GeV) | $M_{q_H}$ (GeV) | $\alpha(M_U)$ |
|-----------------|-----------------|-----------------|---------------|
| 0.110           | $4.9 \times 10^4$ GeV | $2.2 \times 10^{12}$ GeV | 0.0565         |
| 0.118           | $4.5 \times 10^6$ GeV | $4.1 \times 10^{12}$ GeV | 0.0555         |
| 0.126           | $2.3 \times 10^8$ GeV | $7.3 \times 10^{12}$ GeV | 0.0547         |

Figure 1: The running of the gauge couplings in the flipped $SU(5)$ model for $\alpha_3(M_Z) = 0.118$ (solid lines). The gap particle masses have been derived using the gauge coupling RGEs to achieve unification at $M_U = 10^{18}$ GeV. The case with no gap particles (dotted lines) is also shown; here $M_U \approx 10^{16}$ GeV.
4 String-inspired Supersymmetry Breaking Scenarios

Supersymmetry breaking in string models can generally be triggered in a phenomenologically acceptable way by non-zero $F$-terms for: (a) any of the moduli fields of the string model ($\langle F_M \rangle$) [31], (b) the dilaton field ($\langle F_D \rangle$) [37], or (c) the hidden matter fields ($\langle F_H \rangle$) [22]. It has been recently noted [37] that much model-independent information can be obtained about the structure of the soft supersymmetry breaking parameters in generic string supergravity models if one neglects the third possibility ($\langle F_H \rangle = 0$) and assumes that either: (i) $\langle F_M \rangle \gg \langle F_D \rangle$, or (ii) $\langle F_D \rangle \gg \langle F_M \rangle$.

In case (i) the scalar masses are generally not universal, i.e., $m_i = f_im_0$ where $m_0$ is the gravitino mass and $f_i$ are calculable constants, and therefore large flavor-changing-neutral-currents (FCNCs) [53] are potentially dangerous [54]. The gaugino masses arise from the one-loop contribution to the gauge kinetic function and are thus suppressed ($m_{1/2} \sim (\alpha/4\pi)m_0$) [31, 32, 54]. The experimental constraints on the gaugino masses then force the squark and slepton masses into the TeV range [54]. It is interesting to note that this supersymmetry breaking scenario is not unlike that required for the minimal $SU(5)$ supergravity model in order to have the dimension-five proton decay operators under control [17, 56, 49], which requires $m_{1/2}/m_0 \lesssim 3^{-1}$, as discussed in Sec. 7.2. This constraint entails potential cosmological troubles: the neutralino relic density is large and one needs to tune the parameters to have the neutralino mass be very near the Higgs and $Z$ resonances [57, 56, 49, 58, 59] (see Sec. 7.3). Clearly, such cosmological constraints are going to be exacerbated in the case (i) scenario ($m_{1/2}/m_0 \ll 1$) and will likely require real fine-tuning of the model parameters.

An important exception to case (i) occurs if $f_i \equiv 0$ and all scalar masses at the unification scale vanish ($\langle F_M \rangle_{m_0=0}$), as is the case in unified no-scale supergravity models [38], where the minimal no-scale structure $SU(1,1)$ [14] is generalized to $SU(N,1)$ [11] in the presence of the heavy GUT fields. This special case automatically restores the much needed universality of scalar masses, and in the context of no-scale models also entails $A = 0$, see Eq. 11. A special case of this scenario occurs when the bilinear soft-supersymmetry breaking mass parameter $B(M_U)$ is also required to vanish. With the additional ingredient of a flipped $SU(5)$ gauge group, all the above problems are naturally avoided [24], and interesting predictions for direct [64, 61, 62, 63] and indirect [64, 63, 60] experimental detection follow.

If supersymmetry breaking is triggered by $\langle F_D \rangle$ (case (ii)), one obtains universal soft-supersymmetry gaugino and scalar masses and trilinear interactions [37] and the soft-supersymmetry breaking parameters in Eq. (3) result. As well, there is a special more constrained case where $B(M_U) = 2m_0 = \frac{\sqrt{3}}{2}m_{1/2}$ is also required, if one demands that the $\mu$ parameter receive contributions solely from supergravity.
With the complement of a flipped $SU(5)$ structure, this model has also been seen to avoid all the difficulties of the generic $\langle F_M \rangle$ scenario \[37\]. This supersymmetry breaking scenario has been studied recently also in the context of the minimal supersymmetric Standard Model (MSSM) in Ref. \[38\].

More generally, presumably it should be possible to find suitable hidden sectors where an arbitrary choice of $m_0, m_{1/2}, A$ is realized, keeping the vacuum energy at zero. This is the attitude taken below when studying the minimal $SU(5)$ supergravity model, where the proton decay constraint imposes severe restrictions on the allowed choices of these three parameters. In the context of string-inspired models (i.e., the flipped $SU(5)$ case), in what follows we restrict ourselves to the two supersymmetry breaking scenarios in Eqs. (4,5) and their special cases ($B(M_U) = 0$ and $B(M_U) = 2m_0$, respectively).

5 Flipped Phenomenology: General Case

The procedure to extract the low-energy predictions of the models outlined above is rather standard by now (see e.g., Ref. \[39\]): (a) the bottom-quark and tau-lepton masses, together with the input values of $m_t$ and tan $\beta$ are used to determine the respective Yukawa couplings at the electroweak scale; (b) the gauge and Yukawa couplings are then run up to the unification scale $M_U = 10^{18}$ GeV taking into account the extra vector-like quark doublet ($\sim 10^{12}$ GeV) and singlet ($\sim 10^6$ GeV) introduced above \[29, 24\]; (c) at the unification scale the soft-supersymmetry breaking parameters are introduced (according to Eqs. (4,5)) and the scalar masses are then run down to the electroweak scale; (d) radiative electroweak symmetry breaking is enforced by minimizing the one-loop effective potential which depends on the whole mass spectrum, and the values of the Higgs mixing term $|\mu|$ and the bilinear soft-supersymmetry breaking parameter $B$ are determined from the minimization conditions; (e) all known phenomenological constraints on the sparticle and Higgs masses are applied (most importantly the LEP lower bounds on the chargino and Higgs masses), including the cosmological requirement of not-too-large neutralino relic density.

5.1 Mass ranges

We have scanned the parameter space for $m_t = 130, 150, 170$ GeV, tan $\beta = 2 \to 50$ and $m_{1/2} = 50 \to 500$ GeV. Imposing the constraint $m_{\tilde{g}, \tilde{q}} < 1$ TeV we find

$$\langle F_M \rangle_{m_0=0} : \quad m_{1/2} < 475 \text{ GeV}, \quad \text{tan} \beta \lesssim 32,$$

$$\langle F_D \rangle : \quad m_{1/2} < 465 \text{ GeV}, \quad \text{tan} \beta \lesssim 46.$$ (22) (23)

These restrictions on $m_{1/2}$ cut off the growth of most of the sparticle and Higgs masses at $\approx 1$ TeV. However, the sleptons, the lightest Higgs, the two lightest neutralinos,
Table 2: The value of the $c_i$ coefficients appearing in Eq. (28), the ratio $c_\tilde{g} = m_\tilde{g}/m_{1/2}$, and the average squark coefficient $\bar{c}_\tilde{q}$, for $\alpha_3(M_Z) = 0.118 \pm 0.008$. Also shown are the $a_i, b_i$ coefficients for the central value of $\alpha_3(M_Z)$ and both supersymmetry breaking scenarios ($M : \langle F_M \rangle_{m_0=0}, D : \langle F_D \rangle$). The results apply as well to the second-generation squark and slepton masses.

| $i$ | $c_i(0.110)$ | $c_i(0.118)$ | $c_i(0.126)$ | $i$ | $a_i(M)$ | $b_i(M)$ | $a_i(D)$ | $b_i(D)$ |
|-----|-------------|-------------|-------------|-----|----------|----------|----------|----------|
| $\nu, \tilde{e}_L$ | 0.406 | 0.409 | 0.413 | $\tilde{e}_L$ | 0.302 | +1.115 | 0.406 | +0.616 |
| $\tilde{e}_R$ | 0.153 | 0.153 | 0.153 | $\tilde{e}_R$ | 0.185 | +2.602 | 0.329 | +0.818 |
| $\tilde{u}_L, \tilde{d}_L$ | 3.98 | 4.41 | 4.97 | $\tilde{u}_L$ | 0.302 | -2.089 | 0.406 | -1.153 |
| $\tilde{u}_R$ | 3.68 | 4.11 | 4.66 | $\tilde{u}_R$ | 0.991 | -0.118 | 1.027 | -0.110 |
| $\tilde{d}_R$ | 3.63 | 4.06 | 4.61 | $\tilde{d}_R$ | 0.956 | -0.016 | 0.994 | -0.015 |
| $c_\tilde{q}$ | 1.95 | 2.12 | 2.30 | $\tilde{d}_L$ | 0.991 | +0.164 | 1.027 | +0.152 |
| $\bar{c}_\tilde{q}$ | 3.82 | 4.07 | 4.80 | $\tilde{d}_R$ | 0.950 | -0.033 | 0.989 | -0.030 |

and the lightest chargino are cut off at a much lower mass, as follows\(^{10}\)

$$\langle F_M \rangle_{m_0=0} : \begin{cases} m_{\tilde{e}_R} < 190 \text{ GeV}, & m_{\tilde{e}_L} < 305 \text{ GeV}, & m_{\tilde{\nu}} < 295 \text{ GeV} \\ m_{\tilde{\tau}_1} < 185 \text{ GeV}, & m_{\tilde{\tau}_2} < 315 \text{ GeV} \\ m_h < 125 \text{ GeV} \\ m_{\chi_1^0} < 145 \text{ GeV}, & m_{\chi_1^0} < 290 \text{ GeV}, & m_{\chi_1^+} < 290 \text{ GeV} \end{cases}$$ \hspace{1cm} (24)

$$\langle F_D \rangle : \begin{cases} m_{\tilde{e}_R} < 325 \text{ GeV}, & m_{\tilde{e}_L} < 400 \text{ GeV}, & m_{\tilde{\nu}} < 400 \text{ GeV} \\ m_{\tilde{\tau}_1} < 325 \text{ GeV}, & m_{\tilde{\tau}_2} < 400 \text{ GeV} \\ m_h < 125 \text{ GeV} \\ m_{\chi_1^0} < 145 \text{ GeV}, & m_{\chi_1^0} < 285 \text{ GeV}, & m_{\chi_1^+} < 285 \text{ GeV} \end{cases}$$ \hspace{1cm} (25)

It is interesting to note that due to the various constraints on the model, the gluino and (average) squark masses are bounded from below,

$$\langle F_M \rangle_{m_0=0} : \begin{cases} m_{\tilde{g}} \gtrsim 245 \text{ (260) GeV} \\ m_{\tilde{q}} \gtrsim 240 \text{ (250) GeV} \end{cases} \quad \langle F_D \rangle : \begin{cases} m_{\tilde{g}} \gtrsim 195 \text{ (235) GeV} \\ m_{\tilde{q}} \gtrsim 195 \text{ (235) GeV} \end{cases}$$ \hspace{1cm} (26)

for $\mu > 0 (\mu < 0)$. Relaxing the above conditions on $m_{1/2}$ simply allows all sparticle masses to grow further proportional to $m_{\tilde{g}}$.

### 5.2 Mass relations

The neutralino and chargino masses show a correlation observed before in this class of models \([0, 24]\), namely

$$m_{\chi_1^0} \approx \frac{1}{2} m_{\chi_2^0}, \quad m_{\chi_1^\pm} \approx m_{\chi_1^\pm} \approx M_2 = (\alpha_2/\alpha_3)m_{\tilde{g}} \approx 0.28 m_{\tilde{g}}.$$ \hspace{1cm} (27)

\(^{10}\)In this class of supergravity models the three neutrinos ($\tilde{\nu}$) are degenerate in mass. Also, $m_{\tilde{\mu}_L} = m_{\tilde{\tau}_L}$ and $m_{\tilde{\mu}_R} = m_{\tilde{\tau}_R}$. 

15
This is because throughout the parameter space $|\mu|$ is generally much larger than $M_W$ and $|\mu| > M_Z$. In practice we find $m_{\chi_2^0} \approx m_{\tilde{\chi}_1^\pm}$ to be satisfied quite accurately, whereas $m_{\chi_3^0} \approx \frac{1}{2} m_{\chi_2^0}$ is only qualitatively satisfied, although the agreement is better in the $\langle F_D \rangle$ case. In fact, these two mass relations are much more reliable than the one that links them to $m_{\tilde{g}}$. The heavier neutralino ($\chi_{3,4}^0$) and chargino ($\chi_2^\pm$) masses are determined by the value of $|\mu|$; they all approach this limit for large enough $|\mu|$. More precisely, $m_{\chi_4^0}$ approaches $|\mu|$ sooner than $m_{\chi_3^0}$ does. On the other hand, $m_{\chi_3^0}$ approaches $m_{\chi_2^\pm}$ rather quickly.

The first- and second-generation squark and slepton masses can be determined analytically

$$
\bar{m}_i = \left[ m_{1/2}^2 (c_i + \xi_0^2) - d_i \frac{\tan^2 \beta - 1}{\tan^2 \beta + 1} M_W^2 \right]^{1/2} = a_i m_{\tilde{g}} \left[ 1 + b_i \left( \frac{150}{m_{\tilde{g}}} \right)^2 \frac{\tan^2 \beta - 1}{\tan^2 \beta + 1} \right]^{1/2},
$$

where $d_i = (T_{3i} - Q) \tan^2 \theta_w + T_{3i}$ (e.g., $d_{\tilde{u}_L} = \frac{1}{2} - \frac{1}{6} \tan^2 \theta_w$, $d_{\tilde{e}_R} = -\tan^2 \theta_w$), and $\xi_0 = m_0/m_{1/2} = 0.33$. The coefficients $c_i$ can be calculated numerically in terms of the low-energy gauge couplings, and are given in Table 2 for $\alpha_3(M_Z) = 0.118 \pm 0.008$. In the table we also give $c_{\tilde{g}} = m_{\tilde{g}}/m_{1/2}$. Note that these values are smaller than what is obtained in the minimal $SU(5)$ supergravity model (where $c_{\tilde{g}} = 2.90$ for $\alpha_3(M_Z) = 0.118$) and therefore the numerical relations between the gluino mass and the neutralino masses are different in that model. In the table we also show the resulting values for $a_i, b_i$ for the central value of $\alpha_3(M_Z)$. Note that the apparently larger $\tan \beta$ dependence in the $\langle F_M \rangle_{m_0=0}$ case (i.e., $|b_i(M)| > |b_i(D)|$) is actually compensated by a larger minimum value of $m_{\tilde{g}}$ in this case (see Eq. (28)).

The “average” squark mass, $m_{\tilde{q}} \equiv \frac{1}{8} (m_{\tilde{u}_L} + m_{\tilde{u}_R} + m_{\tilde{d}_L} + m_{\tilde{d}_R} + m_{\tilde{\ell}_L} + m_{\tilde{\ell}_R} + m_{\tilde{\tau}_L} + m_{\tilde{\tau}_R}) = (m_{\tilde{g}}/c_{\tilde{g}}) \sqrt{\bar{c}_i + \xi_0^4}$, with $\bar{c}_i$ given in Table 2, is determined to be

$$
m_{\tilde{q}} = \begin{cases} (1.00, 0.95, 0.95) m_{\tilde{g}}, & \langle F_M \rangle_{m_0=0} \\ (1.05, 0.99, 0.98) m_{\tilde{g}}, & \langle F_D \rangle \end{cases}
$$

for $\alpha_3(M_Z) = 0.110, 0.118, 0.126$ (the dependence on $\tan \beta$ is small). The squark splitting around the average is $\approx 2\%$.

These masses are plotted in Fig. 2. The thickness and straightness of the lines shows the small $\tan \beta$ dependence, except for $\tilde{\nu}$. The results do not depend on the sign of $\mu$, except to the extent that some points in parameter space are not allowed for both signs of $\mu$: the $\mu < 0$ lines start-off at larger mass values. Note that

$$
\langle F_M \rangle_{m_0=0} : \begin{cases} m_{\tilde{e}_R} \approx 0.18 m_{\tilde{g}} \\ m_{\tilde{e}_L} \approx 0.30 m_{\tilde{g}} \\ m_{\tilde{e}_R}/m_{\tilde{e}_L} \approx 0.61 \end{cases} \quad \langle F_D \rangle : \begin{cases} m_{\tilde{e}_R} \approx 0.33 m_{\tilde{g}} \\ m_{\tilde{e}_L} \approx 0.41 m_{\tilde{g}} \\ m_{\tilde{e}_R}/m_{\tilde{e}_L} \approx 0.81 \end{cases}
$$

These are renormalized at the scale $M_Z$. In a more accurate treatment, the $c_i$ would be renormalized at the physical sparticle mass scale, leading to second order shifts on the sparticle masses.
Figure 2: The first-generation squark and slepton masses as a function of the gluino mass, for both signs of $\mu$, $m_t = 150\text{ GeV}$, and both supersymmetry breaking scenarios under consideration. The same values apply to the second generation. The thickness of the lines and their deviation from linearity are because of the small $\tan \beta$ dependence.
The third generation squark and slepton masses cannot be determined analytically. In Fig. 3 we show $\tilde{\tau}_{1,2}, \tilde{b}_{1,2}, \tilde{t}_{1,2}$ for the choice $m_t = 150$ GeV. The variability on the $\tilde{\tau}_{1,2}$ and $\tilde{b}_{1,2}$ masses is due to the $\tan \beta$-dependence in the off-diagonal element of the corresponding $2 \times 2$ mass matrices ($\propto m_{\tau,b}(A_{\tau,b} + \mu \tan \beta)$). The off-diagonal element in the stop-squark mass matrix ($\propto m_t(A_t + \mu/\tan \beta)$) is rather insensitive to $\tan \beta$ but still effects a large $\tilde{t}_1 - \tilde{t}_2$ mass splitting because of the significant $A_t$ contribution. Note that both these effects are more pronounced for the $\langle F_D \rangle$ case since there $|A_{t,b,\tau}|$ are larger than in the $\langle F_M \rangle$ case with $m_0 = 0$. The lowest values of the $\tilde{\tau}_{1,2}$ mass go up with $m_t$ and can be as low as $m_{\tilde{\tau}_1} \sim 160, 170, 190$ GeV; $m_{\tilde{\tau}_2} \sim 155, 150, 170$ GeV; $m_{\tilde{\tau}_3} \sim 88, 112, 150$ GeV; $m_{\tilde{\tau}_4} \sim 92, 106, 150$ GeV; $m_{\tilde{\tau}_5} \sim 107, 117, 125$ GeV for $m_t = 130, 150, 170$ GeV and $\mu < 0$ ($\mu > 0$).

The one-loop corrected lightest CP-even ($h$) and CP-odd ($A$) Higgs boson masses are shown in Fig. 4 as functions of $m_{\tilde{g}}$ for $m_t = 150$ GeV. Following the methods of Ref. [61] we have determined that the LEP lower bound on $m_h$ becomes $m_h > 60$ GeV, as the figure shows. The largest value of $m_h$ depends on $m_t$; we find

$$m_h < \begin{cases} 106, 115, 125 \text{ GeV} & \langle F_M \rangle_{m_0=0} \\ 107, 117, 125 \text{ GeV} & \langle F_D \rangle \end{cases}$$

for $m_t = 130, 150, 170$ GeV. It is interesting to note that the one-loop corrected values of $m_h$ for $\tan \beta = 2$ are quite dependent on the sign of $\mu$. This phenomenon can be traced back to the $\tilde{t}_1 - \tilde{t}_2$ mass splitting which enhances the dominant $\tilde{t}$ one-loop corrections to $m_h$ [71], an effect which is usually neglected in phenomenological analyses. The $\tilde{t}_{1,2}$ masses for $\tan \beta = 2$ are drawn closer together than the rest. The opposite effect occurs for $\mu < 0$ and therefore the one-loop correction is larger in this case. The sign-of-$\mu$ dependence appears in the off-diagonal entries in the $\tilde{t}$ mass matrix $\propto m_t(A_t + \mu/\tan \beta)$, with $A_t < 0$ in this case. Clearly only small $\tan \beta$ matters, and $\mu < 0$ enhances the splitting. The $A$-mass grows fairly linearly with $m_{\tilde{g}}$ with a $\tan \beta$-dependent slope which decreases for increasing $\tan \beta$, as shown in Fig. 4. Note that even though $m_A$ can be fairly light, we always get $m_A > m_h$, in agreement with a general theorem to this effect in supergravity theories [72]. This result also implies that the channel $e^+e^- \rightarrow hA$ at LEPI is not kinematically allowed in this model.

5.3 Neutralino relic density

The computation of the neutralino relic density (following the methods of Refs. [73, 74]) shows that $\Omega_{\chi h^2} \lesssim 0.25 (0.90)$ in the no-scale (dilaton) model. This implies that in these models the cosmologically interesting values $\Omega_{\chi h^2} \lesssim 1$ occur quite naturally. These results are in good agreement with the observational upper bound on $\Omega_{\chi h^2}$ [73]. Moreover, fits to the COBE data and the small and large scale structure of the
Figure 3: The $\tilde{\tau}_{1,2}$, $\tilde{b}_{1,2}$, and $\tilde{t}_{1,2}$ masses versus the gluino mass for both signs of $\mu$, $m_t = 150$ GeV, and both supersymmetry breaking scenarios. The variability in the $\tilde{\tau}_{1,2}$, $\tilde{b}_{1,2}$, and $\tilde{t}_{1,2}$ masses is because of the off-diagonal elements of the corresponding mass matrices.
Figure 4: The one-loop corrected $h$ and $A$ Higgs masses versus the gluino mass for both signs of $\mu$, $m_t = 150$ GeV, and the two supersymmetry breaking scenarios. Representative values of $\tan \beta$ are indicated.
Universe suggest a mixture of ≈ 70% cold dark matter and ≈ 30% hot dark matter together with \( h_0 \approx 0.5 \). The hot dark matter component in the form of massive tau neutrinos has already been shown to be compatible with the flipped \( SU(5) \) model we consider here, whereas the cold dark matter component implies \( \Omega_\chi h_0^2 \approx 0.17 \) which is reachable in these models.

6 Flipped Phenomenology: Special Cases

6.1 The strict no-scale case

We now impose the additional constraint \( B(M_U) = 0 \) to be added to Eq. (4), and obtain the so-called strict no-scale case. Since \( B(M_Z) \) is determined by the radiative electroweak symmetry breaking conditions, this added constraint needs to be imposed in a rather indirect way. That is, for given \( m_\tilde{g} \) and \( m_t \) values, we scan the possible values of \( \tan \beta \) looking for cases where \( B(M_U) = 0 \). The most striking result is that solutions exist only for \( m_t < \sim 135 \text{ GeV if } \mu > 0 \) and for \( m_t \gtrsim 140 \text{ GeV if } \mu < 0 \). That is, the value of \( m_t \) determines the sign of \( \mu \). Furthermore, for \( \mu < 0 \) the value of \( \tan \beta \) is determined uniquely as a function of \( m_t \) and \( m_\tilde{g} \), whereas for \( \mu > 0 \), \( \tan \beta \) can be double-valued for some \( m_t \) range which includes \( m_t = 130 \text{ GeV} \) (but does not include \( m_t = 100 \text{ GeV} \)). In Fig. 5 (top row) we plot the solutions found in this manner for the indicated \( m_t \) values.

All the mass relationships deduced in the previous section apply here as well. The \( \tan \beta \)-spread that some of them have will be much reduced though. The most noticeable changes occur for the quantities which depend most sensitively on \( \tan \beta \). In Fig. 5 (bottom row) we plot the one-loop corrected lightest Higgs boson mass versus \( m_\tilde{g} \). The result is that \( m_\tilde{g} \) is basically determined by \( m_t \); only a weak dependence on \( m_\tilde{g} \) exists. Moreover, for \( m_t \lesssim 135 \text{ GeV} \Leftrightarrow \mu > 0 \), \( m_h \lesssim 105 \text{ GeV} \); whereas for \( m_t \gtrsim 140 \text{ GeV} \Leftrightarrow \mu < 0 \), \( m_h \gtrsim 100 \text{ GeV} \). Therefore, in the strict no-scale case, once the top-quark mass is measured, we will know the sign of \( \mu \) and whether \( m_h \) is above or below 100 GeV.

For \( \mu > 0 \), we just showed that the strict no-scale constraint requires \( m_t \lesssim 135 \text{ GeV} \). This implies that \( \mu \) cannot grow as large as it did previously in the general case. In fact, for \( \mu > 0 \), \( \mu_{\text{max}} \approx 745 \text{ GeV} \) before and \( \mu_{\text{max}} \approx 440 \text{ GeV} \) now. This smaller value of \( \mu_{\text{max}} \) has the effect of cutting off the growth of the \( \chi_{3,4}^0, \chi_2^\pm \) masses at \( \approx \mu_{\text{max}} \approx 440 \text{ GeV} \) (c.f. \( \approx 750 \text{ GeV} \)) and of the heavy Higgs masses at \( \approx 530 \text{ GeV} \) (c.f. \( \approx 940 \text{ GeV} \)).

6.2 The special dilaton scenario case

In our analysis above, the radiative electroweak breaking conditions were used to determine the magnitude of the Higgs mixing term \( \mu \) at the electroweak scale. This quantity is ensured to remain light as long as the supersymmetry breaking parameters remain light. In a fundamental theory this parameter should be calculable and its
Figure 5: The value of $\tan \beta$ versus $m_{\tilde{g}}$ in the strict no-scale case (where $B(M_U) = 0$) for the indicated values of $m_t$. Note that the sign of $\mu$ is determined by $m_t$ and that $\tan \beta$ can be double-valued for $\mu > 0$. Also shown is the one-loop corrected lightest Higgs boson mass. Note that if $\mu > 0$ (for $m_t < 135$ GeV) then $m_h < 105$ GeV; whereas if $\mu < 0$ (for $m_t > 140$ GeV) then $m_h > 100$ GeV.

value used to determine the $Z$-boson mass. From this point of view it is not clear that the natural value of $\mu$ should be light. In specific models on can obtain such values by invoking non-renormalizable interactions \cite{33,34}. Another contribution to this quantity is generically present in string supergravity models \cite{32,33,37}. The general case with contributions from both sources has been effectively dealt with in the previous section. If one assumes that only supergravity-induced contributions to $\mu$ exist, then it can be shown that the $B$-parameter at the unification scale is also determined \cite{37},

\[ B(M_U) = 2m_0 = \frac{2}{\sqrt{3}}m_{1/2}, \tag{33} \]

which is to be added to the set of relations in Eq. (3). This new constraint effectively determines $\tan \beta$ for given $m_t$ and $m_{\tilde{g}}$ values and makes this restricted version of the model highly predictive.

From the outset we note that only solutions with $\mu < 0$ exist. This is not a completely obvious result, but it can be partially understood as follows. In tree-level approximation, $m_A^2 > 0 \Rightarrow \mu B < 0$ at the electroweak scale. Since $B(M_U)$ is required to be positive and not small, $B(M_Z)$ will likely be positive also, thus forcing
Table 3: The range of allowed sparticle and Higgs masses in the restricted dilaton scenario. The top-quark mass is restricted to be \( m_t < 155 \text{ GeV} \). All masses in GeV.

| \( m_t \) | 130  | 150  | 155  |
|----------|------|------|------|
| \(  \tilde{g} \) | 335 – 1000 | 260 – 1000 | 640 – 1000 |
| \( \chi_1^0 \) | 38 – 140 | 24 – 140 | 90 – 140 |
| \( \chi_2^0, \chi_1^\pm \) | 75 – 270 | 50 – 270 | 170 – 270 |
| \( \tan \beta \) | 1.57 – 1.63 | 1.37 – 1.45 | 1.38 – 1.40 |
| \( h \) | 61 – 74 | 64 – 87 | 84 – 91 |
| \( \tilde{l} \) | 110 – 400 | 90 – 400 | 210 – 400 |
| \( \tilde{q} \) | 335 – 1000 | 260 – 1000 | 640 – 1000 |
| \( A, H, H^+ \) | > 400 | > 400 | > 970 |

\( \mu \) to be negative. A sufficiently small value of \( B(M_U) \) and/or one-loop corrections to \( m_A^2 \) could alter this result, although in practice this does not happen. A numerical iterative procedure allows us to determine the value of \( \tan \beta \) which satisfies Eq. (33), from the calculated value of \( B(M_Z) \). We find that

\[
\tan \beta \approx 1.57 – 1.63, 1.37 – 1.45, 1.38 – 1.40 \quad \text{for} \quad m_t = 130, 150, 155 \text{ GeV} \tag{34}
\]

is required. Since \( \tan \beta \) is so small (\( m_h^{\text{tree}} \approx 28 – 41 \text{ GeV} \)), a significant one-loop correction to \( m_h \) is required to increase it above its experimental lower bound of \( \approx 60 \text{ GeV} \) \([61]\). This requires the largest possible top-quark masses (and a not-too-small squark mass). However, perturbative unification imposes an upper bound on \( m_t \) for a given \( \tan \beta \) \([78]\), which in this case implies \([69]\)

\[
m_t \lesssim 155 \text{ GeV}, \tag{35}
\]

which limits the magnitude of \( m_h \)

\[
m_h \lesssim 74, 87, 91 \text{ GeV} \quad \text{for} \quad m_t = 130, 150, 155 \text{ GeV}. \tag{36}
\]

Lower values of \( m_t \) are experimentally disfavored.

In Table 3 we give the range of sparticle and Higgs masses that are allowed in this case. Clearly, continuing top-quark searches at the Tevatron and Higgs searches at LEPI,II should probe this restricted scenario completely.

7 The Minimal SU(5) Supergravity Model

The minimal SU(5) supergravity model \([79]\) needs to be specified clearly in order to avoid the common misconception that it is simply the so-called MSSM with the low-energy gauge couplings meeting at very high energies. Two of its elements are
particularly important: (i) it is a supergravity model \[80\] and as such the soft supersymmetry breaking masses which allow unification are in principle calculable and are assumed to be parametrized in terms of \(m_{1/2}, m_0, A\); and (ii) there exist dimension-five proton decay operators \[81\], which are much larger than the usual dimension-six operators, and require either a tuning of the supersymmetry breaking parameters or a large Higgs triplet mass scale, to obtain a sufficiently long proton lifetime \[82, 83, 84, 85, 47, 48, 56, 49\].

The \(SU(5)\) symmetry is broken down to \(SU(3) \times SU(2) \times U(1)\) via a vev of the neutral component of the adjoint 24 of Higgs. The low-energy pair of Higgs doublets are contained in the \(5,5\) Higgs representations. Of the various proposals to split the proton-decay-mediating Higgs triplets from the light Higgs doublets, perhaps the most appealing one is the so-called “missing partner mechanism” \[23\], whereby a 75 of Higgs breaks the gauge symmetry and the \(5,5\) pentaplets are coupled to \(50,\bar{50}\) representations \((50 \cdot 75 \cdot 5, \bar{50} \cdot 75 \cdot \bar{5})\). The doublets remain massless, while the triplets acquire \(\sim M_U\) masses. We should remark that this non-minimal symmetry breaking mechanism is not the one that is usually considered in studies of high-energy threshold effects in gauge coupling unification, where one usually assumes that it is the 24 which effects the breaking.

### 7.1 Gauge and Yukawa coupling unification

This problem can be tackled at several levels of sophistication, which entail an increasing number of additional assumptions. The most elementary approach consists of running the one-loop supersymmetric gauge coupling RGEs starting with the precisely measured values of \(\alpha_e, \alpha_3, \sin^2 \theta_w\) at the scale \(M_Z\) and discovering that the three gauge couplings meet at the scale \(M_U \sim 10^{16}\) GeV \[86\]. More interesting from the theoretical standpoint is to assume that unification must occur, as is the case in the minimal \(SU(5)\) supergravity model, and use this constraint to predict the low-energy value of \(\sin^2 \theta_w\) in terms of \(\alpha_e\) and \(\alpha_3\). The next level of sophistication consists of increasing the accuracy of the RGEs to two-loop level and parametrize the supersymmetric threshold by a single mass parameter between \(\sim M_Z\) and \(\sim \text{few TeV}\) \[26, 87, 88, 89\]. More realistically, one specifies the whole light supersymmetric spectrum in detail \[32, 90, 91, 92, 48, 93, 94, 27, 95, 96\], as well as some subtle effects such as the evolution of the gaugino masses (EGM) effect \[93, 11\], and the effect of the Yukawa couplings on the two-loop gauge coupling RGEs \[17\]. A final step of sophistication attempts to model the transition from the \(SU(3) \times SU(2) \times U(1)\) theory onto the \(SU(5)\) theory by means of high-energy threshold effects which depend on the masses of the various GUT fields \[90, 11, 93, 48, 27, 93\] as well as on coefficients of possible non-renormalizable operators \[93, 99\]. This last step does away completely with the concept of a single “unification mass”. In fact, until this last step is actually accounted for somehow, one is not dealing with a true unified theory since otherwise the gauge couplings would diverge again past the unification scale, \(i.e.,\) “physics is not euclidean geometry”.

It is interesting to note that the original hope that precise knowledge of the low
energy gauge couplings would constrain the scale of the low-energy supersymmetric particles, has not bear fruit [98, 23, 91], mainly because of the largely unknown GUT threshold effects. More precisely, the supersymmetric particle masses can lie anywhere up to \( \sim \) few TeV provided the parameters of the GUT theory are adjusted accordingly.

Another consequence of the \( SU(5) \) symmetry is the relation \( \lambda_b(M_U) = \lambda_r(M_U) \) which when renormalized down to low energies gives a ratio \( m_b/m_r \) in fairly good agreement with experiment [100]. This problem can also be tackled with improving degree of sophistication [24, 88, 91, 102, 103, 97, 104, 105] and even postulating some high-energy threshold effects [97, 105]. In practice, the \( \lambda_b(M_U) = \lambda_r(M_U) \) constraint entails a relationship between \( m_t \) and \( \tan \beta \), i.e., \( \tan \beta = \tan \beta(m_t, m_b, \alpha_3) \), as follows: (i) the values of \( m_b \) and \( m_r \), together with \( \tan \beta \) determine the low-energy values of \( \lambda_b \) and \( \lambda_r \); (ii) the input value of \( m_t \) determines the low-energy value of \( \lambda_t \); (iii) running these three Yukawa couplings up to the unification scale one discovers the above relation between \( \tan \beta \) and \( m_t \) if the Yukawa unification constraint is satisfied.

In actuality, the dependence on \( m_b \) and \( \alpha_3 \) is quite important. We note that for arbitrary choices of \( m_t \) and \( \tan \beta \), one obtains values of \( m_b \) typically close to or above 5 GeV, whereas popular belief would like to see values below 4.5 GeV. Strict adherence to this prediction for \( m_b \) requires that one be in a rather constrained region of the \((m_t, \tan \beta)\) plane, where \( \tan \beta \sim 1 \) or \( \gtrsim 40 \) [24, 102, 97, 103], or that \( m_t \) be large (above 180 GeV). We do not impose this stringent constraint on the parameter space, hoping that further contributions to the quark masses (as required in \( SU(5) \) GUTs to fit the lighter generations also [106]) will relax it somehow.

As noted above, the parameter space of this model can be described in terms of five parameters: \( m_t, \tan \beta, m_{1/2}, m_0, A \). In Ref. [56] we performed an exploration of the following hypercube of the parameter space: \( \mu > 0, \mu < 0, \tan \beta = 2 - 10 \) (2), \( m_t = 100 - 160 \) (5), \( \xi_0 = 0 - 10 \) (1), \( \xi_A = -\xi_0, 0, +\xi_0 \), and \( m_{1/2} = 50 - 300 \) (6), where the numbers in parenthesis represent the size of the step taken in that particular direction. (Points outside these ranges have little (a posteriori) likelihood of being acceptable.) Of these 92,235 \( \times 2 = 184,470 \) points, \( \approx 25\% \) passed all the standard constraints, i.e., radiative electroweak symmetry breaking and all low-energy phenomenology as described in Ref. [98]. The most important constraint on this parameter space is proton decay, as discussed below. First we discuss some aspects of the gauge coupling unification calculation.

As a first step we used one-loop gauge coupling RGEs and a common supersymmetric threshold at \( M_Z \), to determine \( M_U, \alpha_U \), and \( \sin^2 \theta_w \), once \( \alpha_3(M_Z) = 0.113, 0.120 \) and \( \alpha_3^{-1}(M_Z) = 127.9 \) were given. In Ref. [49] we refined our study including several important features: (i) recalculation of \( M_U \) using two-loop gauge coupling RGEs including light supersymmetric thresholds, (ii) exploration of values of \( \alpha_3 \) throughout its \( \pm 1 \sigma \) allowed range, and (iii) exploration of low values of \( \tan \beta \) (\( < 2 \)) (which maximize the proton lifetime). We used the analytical approximations to the solution of the two-loop gauge coupling RGEs in Ref. [73] to obtain \( M_U, \alpha_U, \sin^2 \theta_w \). The supersymmetric threshold was treated in great detail [73] with all the sparticle masses obtained from our procedure [56]. Since, the sparticle masses vary as
Table 4: The value of the one-loop unification mass $M_U^{(0)}$, the two-loop and supersymmetric threshold corrected unification mass range $M_U^{(1)}$, the ratio of the two, and the range of the calculated $\sin^2 \theta_w$ for the indicated values of $\alpha_3$ (the superscript $+ (-)$ denotes $\mu > 0 (< 0)$) and $\alpha_w^{-1} = 127.9$. The $\sin^2 \theta_w$ values should be compared with the current experimental $\pm 1 \sigma$ range $\sin^2 \theta_w = 0.2324 \pm 0.0006$ [95]. Lower values of $\alpha_3$ drive $\sin^2 \theta_w$ to values even higher than for $\alpha_3 = 0.118$. All masses in units of $10^{16}$ GeV.

|      | $\alpha_3 = 0.126^+$ | $\alpha_3 = 0.126^-$ | $\alpha_3 = 0.118^+$ | $\alpha_3 = 0.118^-$ |
|------|----------------------|----------------------|----------------------|----------------------|
| $M_U^{(0)}$ | 3.33                 | 3.33                 | 2.12                 | 2.12                 |
| $M_U^{(1)}$ | 1.60 – 2.13          | 1.60 – 2.05          | 1.02 – 1.35          | 1.02 – 1.30          |
| $M_U^{(1)}/M_U^{(0)}$ | 0.48 – 0.64          | 0.48 – 0.61          | 0.48 – 0.64          | 0.48 – 0.61          |
| $\sin^2 \theta_w$ | 0.2315 – 0.2332     | 0.2313 – 0.2326    | 0.2335 – 0.2351    | 0.2332 – 0.2345    |

one explores the parameter space, one obtains ranges for the calculated values. In Table 4 we show the one-loop value for $M_U$ ($M_U^{(0)}$), the two-loop plus supersymmetric threshold corrected unification mass range ($M_U^{(1)}$) [as expected $M_U$ is reduced by both effects], the ratio of the two, and the calculated range of $\sin^2 \theta_w$. Note that for $\alpha_3 = 0.118$ (and lower), $\sin^2 \theta_w$ is outside the experimental $\pm 1 \sigma$ range ($\sin^2 \theta_w = 0.2324 \pm 0.0006$), whereas $\alpha_3 = 0.126$ gives quite acceptable values.

We do not specify the details of the GUT thresholds and in practice take two of the GUT mass parameters (the masses of the $X, Y$ gauge bosons $M_V$, and the mass of the adjoint Higgs multiplet $M_\Sigma$) to be degenerate with $M_U$. Since below we allow $M_H < 3M_U$, Table 4 indicates that in our calculations $M_H < 6.4 \times 10^{16}$ GeV. In Ref. [48] it is argued that a more proper upper bound is $M_H < 2M_V$, but $M_V$ cannot be calculated directly, only $(M_V^2M_\Sigma)^{1/3} < 3.3 \times 10^{16}$ GeV is known from low-energy data [48]. If we take $M_\Sigma = M_V$, this would give $M_H < 2M_V < 6.6 \times 10^{16}$ GeV, which agrees with our present requirement. Below we comment on the case $M_\Sigma < M_V$.

7.2 Proton decay

In the minimal $SU(5)$ supergravity model only the dimension-five–mediated proton decay operators are constraining. In calculating the proton lifetime we consider the typically dominant decay modes $p \to \bar{u}_{\mu, r}K^+$ and neglect all other possible modes.

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12 We should note that these ranges are obtained after all constraints discussed below have been satisfied, the proton decay being the most important one.
Schematically the lifetime is given by\[^{13}\]

\[
\tau_p \equiv \tau(p \to \bar{\nu}_\mu \tau K^+) \sim \left| M_H \sin 2\beta \frac{1}{f} \frac{1}{1 + y^{tK}} \right|^2.
\]

(37)

Here \(M_H\) is the mass of the exchanged GUT Higgs triplet which on perturbative grounds is assumed to be bounded above by \(M_H < 3M_U\); \[^{3, 17, 18}\] thus \(\sin 2\beta = 2\tan\beta/(1 + \tan^2\beta)\), thus \(\tau_p\) ‘likes’ small \(\tan\beta\) (we find that only \(\tan\beta \lesssim 6\) is allowed); \(y^{tK}\) represents the calculable ratio of the third- to the second-generation contributions to the dressing one-loop diagrams. An unknown phase appears in this ratio (which has generally \(|y^{tK}| \ll 1\) and we always consider the weakest possible case of destructive interference. Finally \(f\) represents the sparticle-mass–dependent dressing one-loop function which decreases asymptotically with large sparticle masses.

In Fig. 3 (top row) we show a scatter plot of \((\tau_p, m_\tilde{g})\). The various ‘branches’ correspond to fixed values of \(\xi_0\). Note that for \(\xi_0 < 3\), \(\tau_p < \tau_p^{\text{exp}} = 1 \times 10^{32} y\) (at 90\% C.L. \[^{107}\]). Also, for a given value of \(\xi_0\), there is a corresponding allowed interval in \(m_\tilde{g}\). The lower end of this interval is determined by the fact that \(\tau_p \propto 1/f^2\), and \(f \approx m_{\tilde{g}1}/m_{\tilde{g}2} \propto 1/m_\tilde{g}(c + \xi_0^2)\), in the proton-decay–favored limit of \(\mu > M_W\); thus \(m_\tilde{g}(c + \xi_0^2) > \text{constant}\). The upper end of the interval follows from our requirement \(m_\tilde{g}(\sqrt{c + \xi_0^2}) < 1\text{ TeV}\). Statistically speaking, the proton decay cut is quite severe, allowing only about \(\sim 1/10\) of the points which passed all the standard constraints, independently of the sign of \(\mu\).

Note that if we take \(M_H = M_U\) (instead of \(M_H = 3M_U\)), then \(\tau_p \to \frac{1}{3}\tau_p\) and all points in Fig. 3 would become excluded. To obtain a rigorous lower bound on \(M_H\), we would need to explore the lowest possible allowed values of \(\tan\beta\) (in Fig. 3, \(\tan\beta \geq 2\)). Roughly, since the dominant \(\tan\beta\) dependence of \(\tau_p\) is through the explicit \(\sin 2\beta\) factor, the upper bound \(\tau_p \lesssim 8 \times 10^{32} y\) for \(\tan\beta = 2\), would become \(\tau_p \lesssim 1 \times 10^{33} y\) for \(\tan\beta = 1\). Therefore, the current experimental lower bound on \(\tau_p\) would imply \(M_H \gtrsim M_U\). Note also that SuperKamiokande \((\tau_p^{\text{exp}} \approx 2 \times 10^{33} y)\) should be able to probe the whole allowed range of \(\tau_p\) values.

The actual value of \(\alpha_3(M_Z)\) used in the calculations (\(\alpha_3 = 0.120\) in Fig. 3) has a non-negligible effect of some of the final results, mostly due to its effect on the value of \(M_H = 3M_U\): larger values of \(\alpha_3\) increase \(M_U\) and therefore \(\tau_p\), and thus open up the parameter space, and vice versa. For example, for \(\alpha_3 = 0.113 (0.120)\) we get \(m_{\tilde{g}} \lesssim 550 (800) \text{ GeV}\), \(\xi_0 \geq 5 (3)\), and \(\tau_p \lesssim 4 (8) \times 10^{32} y\).

The refinement on the calculation of the unification mass described above, to include two-loop effects and light supersymmetric thresholds, has a significant effect on the calculated value of the proton lifetime \[^{19}\], since we take \(M_H = 3M_U\). With the more accurate value of \(M_U\) we simply rescale our previously calculated \(\tau_p\) values which satisfied \(\tau_p^{(0)} > \tau_p^{\text{exp}}\), and find that \(\tau_p^{(1)} = \tau_p^{(0)}[M_U^{(1)}/M_U^{(0)}]^2 > \tau_p^{\text{exp}}\) for only\(^{13}\)

\[^{13}\]Throughout our calculations we have used the explicit proton decay formulas in Ref. \[^{3}\].

\[^{14}\]This relation assumes implicitly that all the components of the 24 superfield are nearly degenerate in mass \[^{18}\].
Figure 6: Scatter plot of the proton lifetime $\tau_p \equiv \tau(p \to \bar{\nu}_\mu \tau K^+)$ versus the gluino mass for the hypercube of the parameter space explored. The unification mass is calculated in one-loop approximation assuming a common supersymmetric threshold at $M_Z$, and $M_H = 3 M_U$ is assumed. The current experimental lower bound is $\tau_p^{exp} = 1 \times 10^{32}$ y. The various ‘branches’ correspond to fixed values of $\xi_0$ as indicated (the labelling applies to all four windows). The bottom row includes the cosmological constraint. The upper bound on $m_{\tilde{q}}$ follows from the requirement $m_{\tilde{q}} < 1$ TeV.
Figure 7: The calculated values of the proton lifetime into $p \rightarrow \bar{\nu}K^+$ versus the lightest chargino (or second-to-lightest neutralino) mass for both signs of $\mu$, using the more accurate value of the unification mass (which includes two-loop and low-energy supersymmetric threshold effects). Note that we have taken $\alpha_3 + 1\sigma$ in order to maximize $\tau_p$. Note also that future proton decay experiments should be sensitive up to $\tau_p \approx 20 \times 10^{32}$ y.

$\lesssim 25\%$ of the previously allowed points. The value of $\alpha_3$ has a significant influence on the results since (see Table 3) larger (smaller) values of $\alpha_3$ increase (decrease) $M_U$, although the effect is more pronounced for low values of $\alpha_3$. To quote the most conservative values of the observables, in what follows we take $\alpha_3$ at its $+1\sigma$ value ($\alpha_3 = 0.126$). As discussed above, this choice of $\alpha_3$ also gives $\sin^2 \theta_w$ values consistent with the $\pm 1\sigma$ experimental range. Finally, in the search of the parameter space above, we considered only $\tan \beta = 2, 4, 6, 8, 10$ and found that $\tan \beta \lesssim 6$ was required. Our present analysis indicates that this upper bound is reduced down to $\tan \beta \lesssim 3.5$. Here we consider also $\tan \beta = 1.5, 1.75$ since low $\tan \beta$ maximizes $\tau_p \propto \sin^2 2\beta$. These add new allowed points (i.e., $\tau_p^{(0)} > \tau_p^{exp}$) to our previous set, although most of them ($\gtrsim 75\%$) do not survive the stricter proton decay constraint ($\tau_p^{(1)} > \tau_p^{exp}$) imposed here.

In Fig. 7 we show the re-scaled values of $\tau_p$ versus the lightest chargino mass $m_{\tilde{\chi}_1^\pm}$. All points satisfy $\xi_0 \equiv m_0/m_{1/2} \gtrsim 6$ and $m_{\tilde{\chi}_1^\pm} \lesssim 150$ GeV, which are to be contrasted with $\xi_0 \gtrsim 3$ and $m_{\tilde{\chi}_1^\pm} \lesssim 225$ GeV derived using the weaker proton decay constraint [47, 56]. The upper bound on $m_{\tilde{\chi}_1^\pm}$ derives from its near proportionality to $m_{\tilde{\chi}_2}$, $m_{\tilde{\chi}_1^\pm} \approx 0.3m_{\tilde{\chi}_2}$ [17, 56], and the result $m_{\tilde{\chi}_2} \lesssim 500$ GeV. The latter follows from the proton decay constraint $\xi_0 \gtrsim 6$ and the naturalness requirement $m_{\tilde{\chi}_2} \approx \sqrt{m_0^2 + 6m^2_{1/2}} \approx \frac{1}{3} m_{\tilde{\chi}_2} \sqrt{6 + \xi_0^2} < 1$ TeV. Within our naturalness and $H_3$ mass
assumptions, we then obtain\(^\text{15}\)

\[
\tau_p < 3.1 \times 10^{32} \text{y} \quad \text{for} \quad \mu > 0 (\mu < 0).
\]

(38)

The \(p \to \bar{\nu}K^+\) mode should then be readily observable at SuperKamiokande and Gran Sasso since these experiments should be sensitive up to \(\tau_p \approx 2 \times 10^{33} \text{y}\). Note that if \(M_H = 2.3 \times 10^{17} \text{GeV}\) \cite{18}, then in Eq. (38) \(\tau_p \to \tau_p < 4.0 (4.8) \times 10^{33} \text{y}\), and only part of the parameter space of the model would be experimentally accessible. However, to make this choice of \(M_H\) consistent with high-energy physics (\(i.e., M_H < 2M_V\)) one must have \(M_V/M_\Sigma > 42\).

### 7.3 Neutralino relic density

The study of the relic density of neutralinos requires the knowledge of the total annihilation amplitude \(\chi \chi \to \text{all}\). The latter depends on the model parameters to determine all masses and couplings. Previously \cite{73,74} we have advocated the study of this problem in the context of supergravity models with radiative electroweak symmetry breaking, since then only a few parameters (five or less) are needed to specify the model completely. In particular, one can explore the whole parameter space and draw conclusions about a complete class of models. The ensuing relationships among the various masses and couplings have been found to yield results which depart from the conventional minimal supersymmetric standard model (MSSM) lore, where no such relations exist. In the minimal \(SU(5)\) supergravity model we have just shown that its five-dimensional parameter space is strongly constrained by the proton lifetime. It was first noticed in Ref. \cite{57} that the neutralino relic density for the proton-decay allowed points in parameter space is large, \(i.e., \Omega_\chi h^2_0 \gg 1\), and therefore in conflict with current cosmological expectations: requiring that the Universe be older than the oldest known stars implies \(\Omega_0 h^2_0 \leq 1\) \cite{75}.

In Refs. \cite{57,56,49} the neutralino relic density has been computed following the methods of Refs. \cite{73,74}. In Fig. 6 (bottom row) we show the effect of the cosmological constraint on the parameter space allowed by proton decay. Only \(\sim 1/6\) of the points satisfy \(\Omega_\chi h^2_0 \leq 1\). This result is not unexpected since proton decay is suppressed by heavy sparticle masses, whereas \(\Omega_\chi h^2_0\) is enhanced. Therefore, a delicate balance needs to be attained to satisfy both constraints simultaneously. Note that the subset of cosmologically allowed points does not change the range of possible \(\tau_p\) values, although it depletes the constant-\(\xi_0\) ‘branches’.

The effect of the cosmological constraint is perhaps more manifest when one considers the correlation between \(m_h\) and \(m_{\chi^\pm_1}\) after imposing the (\(\text{e.g.}, \text{weaker}\)) proton decay constraint, but with and without imposing the cosmological constraint. This contrast is shown in Fig. 8.

\(^{15}\)Note that in general, \(\tau_p \propto M^2_{H_1} [m^2_{\tilde{q}}/m_{\chi^\pm_1}]^2 \propto M^2_{H_1} [m_{\tilde{g}}(6 + \xi_0^2)]^2\) and thus \(\tau_p\) can be made as large as desired by increasing sufficiently either the supersymmetric spectrum or \(M_H\).
The allowed region in parameter space which satisfies the *weaker* proton decay constraint, before and after the imposition of the cosmological constraint. Note that when the cosmological constraint is imposed (bottom row), an interesting correlation between the two particle masses arises.

The main conclusion is that the relic density can be small only near the $h$- and $Z$-pole resonances, *i.e.*, for $m_\chi \approx \frac{1}{2} m_{h,Z}$ [58, 58], since in this case the annihilation cross section is enhanced. It is important to note that in this type of calculations the thermal average of the annihilation cross section is usually computed using an expansion around threshold (*i.e.*, $\sqrt{s} = 2m_\chi$) [108]. In Ref. [109] however, it has been pointed out that the resulting thermal average can be quite inaccurate near poles and thresholds of the annihilation cross section, which is precisely the case for the points of interest in the minimal $SU(5)$ model. In Ref. [58, 58] the relic density calculation has been redone following the more accurate methods of Ref. [109]. The result is that the poles are broader and shallower, and thus the cosmological constraint is weakened with respect to the standard (using the expansion) procedure of performing the thermal average. However, qualitatively the cosmologically allowed region of parameter space is not changed. This result is shown in Fig. 8, where the points in parameter space allowed by the stricter proton decay constraint and cosmology are
shown in the \((m_{\chi^\pm}, m_h)\) plane.

### 7.4 Mass ranges and relations

Since the proton decay constraint generally requires \(|\mu| \gg M_W\) (and to a somewhat lesser extent also \(|\mu| \gg M_2\)), the lightest chargino will have mass \(m_{\chi_1^+} \approx M_2 \approx 0.3m_\tilde{g}\), whereas the two lightest neutralinos will have masses \(m_\chi \approx M_1 \approx \frac{1}{2}M_2\) and \(m_\tilde{\chi}_0^2 \approx M_2\) \([17, 70]\). Thus, within some approximation we expect the relation in Eq. (27) to be satisfied in this model also. Inclusion of the cosmological constraint does not affect significantly the range of sparticle masses in Sec. 7.2. The value of \(\alpha_3\) does not affect these mass relations either, although the particle mass ranges do change

\[
m_\chi < 85 \ (115) \text{ GeV}, \quad m_{\tilde{\chi}_0^2, \chi_1^+} < 165 \ (225) \text{ GeV}, \quad \text{for } \alpha_3 = 0.113 \ (0.120). \tag{39}
\]

The reason is simple: higher values of \(\alpha_3\) increase \(M_U\) and therefore \(M_H (= 3M_U)\), which in turn weakens the proton decay constraint. We also find that the one-loop corrected lightest Higgs boson mass \((m_h)\) is bounded above by

\[
m_h \lesssim 110 \ (100) \text{ GeV}, \tag{40}
\]

independently of the sign of \(\mu\), the value of \(\alpha_3\), or the cosmological constraint; the stronger bound holds when the stricter proton decay constraint is enforced. In Fig. 9 we have shown \(m_h\) versus \(m_{\chi_1^+}\) for \(\tan \beta = 1.5, 1.75, 2\); for the maximum allowed \(\tan \beta\) value (\(\approx 3.5\)), \(m_h \lesssim 100\) GeV. Note that for \(\mu > 0\), \(m_h \approx 50\) GeV, and \(m_{\chi^\pm} \gtrsim 100\) GeV, there is a sparsely populated area with highly fine-tuned points in parameter space \((m_t \approx 100\) GeV, \(\tan \beta \approx 1.5, \xi_A \equiv A/m_{1/2} \approx \xi_0 \approx 6\)\). This figure shows an experimentally interesting correlation when the cosmological constraints are imposed,

\[
m_h \gtrsim 72 \text{ GeV} \Rightarrow m_{\chi_1^+} \lesssim 100 \text{ GeV}. \tag{41}
\]

The bands of points towards low values of \(m_h\) represent the discrete choices of \(\tan \beta = 1.5, 1.75\). The voids between these bands are to be understood as filled by points with \(1.5 \lesssim \tan \beta \lesssim 1.75\). For \(m_{\chi_1^+} > 106 \ (92)\) GeV (for \(\mu > 0\) \((\mu < 0)\)), we obtain \(m_h \lesssim 50 \ (56)\) GeV and Higgs detection at LEP should be immediate. The correlations among the lightest chargino and neutralino masses imply analogous results for \((m_h, m_{\tilde{\chi}_0^2})\) and \((m_h, m_{\chi})\),

\[
m_h \gtrsim 80 \text{ GeV} \Rightarrow m_{\tilde{\chi}_0^2} \lesssim 90 \ (110) \text{ GeV}, \quad m_\chi \lesssim 48 \ (60) \text{ GeV}, \tag{42}
\]

for \(\alpha_3 = 0.113 \ (0.120)\). These correlations can be understood in the following way: since we find that \(m_A \gg M_Z\), then \(m_h \approx |\cos 2\beta| M_Z + \text{(rad. corr.)}\). In the situation we consider here, we have determined that all of the allowed points for \(m_\tilde{g} > 400\) GeV correspond to \(\tan \beta = 2\). This implies that the tree-level contribution to \(m_h\) is \(\approx 55\) GeV. We also find that the cosmology cut restricts \(m_t < 130\ (140)\) GeV for
Figure 9: The points in parameter space of the minimal $SU(5)$ supergravity model which satisfy the stricter proton decay constraint and the cosmological constraint with the relic density computed in approximate and accurate way. Note the little qualitative difference between the two sets of plots.
\( \mu > 0 (\mu < 0) \) in this range of \( m_{\tilde{g}} \). Therefore, the radiative correction contribution to \( m_h^2 (\propto m_{\tilde{q}}^2) \) will be modest in this range of \( m_{\tilde{g}} \). This explains the depletion of points for \( m_h \gtrsim 80 \text{ GeV} \) in Fig. 1 and leads to the mass relationships in Eqs. (11,12).

In this model the only light particles are the lightest Higgs boson \( (m_h \lesssim 100 \text{ GeV}) \), the two lightest neutralinos \( (m_{\chi_1^0} \approx \frac{1}{2} m_{\chi_2^0} \lesssim 75 \text{ GeV}) \), and the lightest chargino \( (m_{\chi_1^\pm} \approx m_{\chi_2^0} \lesssim 150 \text{ GeV}) \). The gluino and the lightest stop can be light \( (m_{\tilde{g}} \approx 160 - 460 \text{ GeV}, m_{\tilde{t}_1} \approx 170 - 825 \text{ GeV}) \), but for most of the parameter space are not within the reach of Fermilab.

In Ref. [61] it has been shown that the actual LEP lower bound on the lightest Higgs boson mass is improved in the class of supergravity models with radiative electroweak symmetry breaking which we consider here, one gets \( m_h \gtrsim 60 \text{ GeV} \). In Sec. 8.2 below we discuss the details of this procedure. For now it suffices to note that the improved bound \( m_h \gtrsim 60 \text{ GeV} \) mostly restricts low values of \( \tan \beta \) and therefore the minimal \( SU(5) \) supergravity model where \( \tan \beta \lesssim 3.5 \) [19]. Above we obtained upper bounds on the light particle masses in this model \((\tilde{g}, h, \chi_1^0, \chi_2^0)\) for \( m_h > 43 \text{ GeV} \). In particular, it was found that \( m_{\chi_1^\pm} \gtrsim 100 \text{ GeV} \) was only possible for \( m_h < \sim 50 \text{ GeV} \). The improved bound on \( m_h \) immediately implies the following considerably stronger upper bounds

\[
\begin{align*}
m_{\chi_1^0} &\lesssim 52(50) \text{ GeV}, \quad (43) \\
m_{\chi_2^0} &\lesssim 103(94) \text{ GeV}, \quad (44) \\
m_{\chi_1^\pm} &\lesssim 104(92) \text{ GeV}, \quad (45) \\
m_{\tilde{g}} &\lesssim 320 (405) \text{ GeV}, \quad (46)
\end{align*}
\]

for \( \mu > 0 (\mu < 0) \).

A related consequence is that the mass relation \( m_{\chi_2^0} > m_{\chi_1^0} + m_h \) is not satisfied for any of the remaining points in parameter space and therefore the \( \chi_1^0 \to \chi_2^0 h \) decay mode is not kinematically allowed. Points where such mode was previously allowed led to a vanishing trilepton signal in the reaction \( p\bar{p} \to \chi_1^\pm \chi_2^0 \) at Fermilab (thus the name ‘spoiler mode’) [60]. The improved situation now implies at least one event per \( 100 \text{ pb}^{-1} \) for all remaining points in parameter space (see Sec. 8.1).

8 Prospects for Direct Experimental Detection

The sparticle and Higgs spectrum discussed in Secs. 5, 6, 7 can be directly explored partially at present and near future collider facilities, as we now discuss for each model considered above.

8.1 Tevatron

(a) The search and eventual discovery of the top quark will narrow down the parameter space of these models considerably. Moreover, in the two special flipped
SU(5) cases discussed in Sec. 4 this measurement will be very important: (i) in the strict no-scale case (Sec. 6.1) it will determine the sign of $\mu$ ($\mu > 0$ if $m_t \lesssim 135$ GeV; $\mu < 0$ if $m_t \gtrsim 140$ GeV) and whether the Higgs mass is above or below $\approx 100$ GeV, and (ii) it may rule out the restricted dilaton scenario (Sec. 6.2) if $m_t > 150$ GeV.

(b) The trilepton signal in $p\bar{p} \rightarrow \chi_2^0 \chi_1^\pm X$, where $\chi_2^0$ and $\chi_1^\pm$ both decay leptonically, is a clean test of supersymmetry [110] and in particular of this class of models [60]. The trilepton rates in the no-scale flipped SU(5) and in the minimal SU(5) models have been given in Ref. [60]; in Fig. 10 we show these (in the no-scale case $m_t = 130$ GeV has been chosen). One can show that with $\mathcal{L} = 100$ pb$^{-1}$ of integrated luminosity basically all of the parameter space of the minimal SU(5) model should be explorable. Also, chargino masses as high as $\approx 175$ GeV in the no-scale model could be explored, although some regions of parameter space for lighter chargino masses would remain unexplored. We expect that somewhat weaker results will hold for the dilaton model, since the sparticle masses are heavier in that model, especially the sleptons which enhance the leptonic branching ratios when they are light enough [60].

(c) The relation $m_{\tilde{q}} \approx m_{\tilde{g}}$ for the $\tilde{u}_{L,R}, \tilde{d}_{L,R}$ squark masses in the flipped SU(5) models should allow to probe the low end of the squark and gluino allowed mass ranges, although the outlook is more promising for the dilaton model since the allowed range starts off at lower values of $m_{\tilde{g}, \tilde{q}}$ (see Eq. (26)). An important point distinguishing the two models is that the average squark mass is slightly below (above) the gluino mass in the no-scale (dilaton) model, which should have an important bearing on the experimental signatures and rates [111]. In the dilaton case the $t_1$ mass can be below 100 GeV for sufficiently low $m_t$, and thus may be detectable. As the lower bound on $m_t$ rises, this signal becomes less accessible. The actual reach of the Tevatron for the above processes depends on its ultimate integrated luminosity. The squark masses in the minimal SU(5) model ($m_{\tilde{q}} \gtrsim 500$ GeV) are beyond the reach of the Tevatron.

8.2 LEPI

The current LEPI lower bound on the Standard Model (SM) Higgs boson mass ($m_H > 61.6$ GeV [112]) is obtained by studying the process $e^+e^- \rightarrow Z^*H$ with subsequent Higgs decay into two jets. The MSSM analog of this production process leads to a cross section differing just by a factor of $\sin^2(\alpha - \beta)$, where $\alpha$ is the SUSY Higgs mixing angle and $\tan \beta = v_2/v_1$ is the ratio of the Higgs vacuum expectation values [113]. The published LEPI lower bound on the lightest SUSY Higgs boson mass ($m_h > 43$ GeV) is the result of allowing $\sin^2(\alpha - \beta)$ to vary throughout the MSSM parameter space and by considering the $e^+e^- \rightarrow Z^*h, hA$ cross sections. It is therefore possible that in specific models (which embed the MSSM), where $\sin^2(\alpha - \beta)$ is naturally restricted
Figure 10: The number of trilepton events at the Tevatron per 100 pb$^{-1}$ in the minimal $SU(5)$ model and the no-scale flipped $SU(5)$ model (for $m_t = 130$ GeV). Note that with 200 pb$^{-1}$ and 60% detection efficiency it should be possible to probe basically all of the parameter space of the minimal $SU(5)$ model, and probe chargino masses as high as 175 GeV in the no-scale model.
to be near unity, the lower bound on $m_h$ could rise, and even reach the SM lower bound if $\text{BR}(h \to 2\text{jets})$ is SM-like as well. This has been shown to be the case for the supergravity models we discuss here, and more generally for supergravity models which enforce radiative electroweak symmetry breaking [61].

Non-observation of a SM Higgs signal puts the following upper bound in the number of expected 2-jet events.

$$#\text{events}_{\text{SM}} = \sigma(e^+e^- \to Z^*H)_{\text{SM}} \times \text{BR}(H \to 2\text{jets})_{\text{SM}} \times \int L\,dt < 3. \quad (47)$$

The SM value for $\text{BR}(H \to 2\text{jets})_{\text{SM}} \approx \text{BR}(H \to b\bar{b} + \ell\bar{\ell} + gg)_{\text{SM}} \approx 0.92$ [113] corresponds to an upper bound on $\sigma(e^+e^- \to Z^*H)_{\text{SM}}$. Since this is a monotonically decreasing function of $m_H$, a lower bound on $m_H$ follows, i.e., $m_H > 61.6$ GeV as noted above. We denote by $\sigma_{\text{SM}}(61.6)$ the corresponding value for $\sigma(e^+e^- \to Z^*H)_{\text{SM}}$. For the MSSM the following relations hold

$$\sigma(e^+e^- \to Z^*h)_{\text{SU(5)}} = \sin^2(\alpha - \beta)\sigma(e^+e^- \to Z^*H)_{\text{SM}}, \quad (48)$$

$$\text{BR}(h \to 2\text{jets})_{\text{SU(5)}} = f \cdot \text{BR}(H \to 2\text{jets})_{\text{SM}}. \quad (49)$$

From Eq. (47) one can deduce the integrated luminosity achieved,

$$\int L\,dt = 3/(\sigma_{\text{SM}}(61.6)\text{BR}_{\text{SM}}). \quad (50)$$

This immediately implies the following condition for allowed points in parameter space [61] [114]

$$f \cdot \sin^2(\alpha - \beta) < P(61.6/M_Z)/P(m_h/M_Z), \quad (51)$$

where we have used the fact that the cross sections differ simply by the coupling factor $\sin^2(\alpha - \beta)$ and the Higgs mass dependence which enters through a function $P$ [113]

$$P(y) = \frac{3y(3y^4 - 8y^2 + 20)}{\sqrt{4 - y^4}} \cos^{-1}\left(\frac{y(3 - y^2)}{2}\right) - 3(y^4 - 6y^2 + 4) \ln y - \frac{1}{2}(1 - y^2)(2y^4 - 13y^2 + 47). \quad (52)$$

The cross section $\sigma_{\text{SU(5)}}(m_h)$ for the minimal $SU(5)$ model also corresponds to the SM result since one can verify that $\sin^2(\alpha - \beta) > 0.9999$ in this case. For the flipped model there is a small deviation ($\sin^2(\alpha - \beta) > 0.95$) relative to the SM result for some points [61]. In the calculation of $\text{BR}(h \to 2\text{jets})_{\text{SU(5)}}$ which enters in the ratio $f$, we have included all contributing modes, in particular the invisible $h \to \chi_0^0\chi_0^0$ decays. The conclusion is that these models differ little from the SM and in fact the proper lower bound on $m_h$ is very near 60 GeV, although it varies from point to point in the parameter space.

In Ref. [61] it was also shown that this phenomenon is due to a decoupling effect of the Higgs sector as the supersymmetry scale rises, and it is communicated to
the Higgs sector through the radiative electroweak symmetry breaking mechanism. The point to be stressed is that if the supersymmetric Higgs sector is found to be SM-like, this could be taken as indirect evidence for an underlying radiative electroweak breaking mechanism, since no insight could be garnered from the MSSM itself.

Note that since the lower bound on the SM Higgs boson mass could still be pushed up several GeV at LEPI, the strict dilaton scenario in Sec. 6.2 (which requires $m_h \approx 61 - 91 \text{GeV}$) could be further constrained at LEPI.

## 8.3 LEPII

(a) At LEPII the SM Higgs mass could be explored up to roughly the beam energy minus 100 GeV \[118\]. This will allow exploration of almost all of the Higgs parameter space in the minimal $SU(5)$ model \[52\]. In the flipped $SU(5)$ models, only low $\tan\beta$ values could be explored, although the strict no-scale case will probably be out of reach (see Figs. 44). The $e^+e^- \rightarrow hA$ channel will be open in the flipped $SU(5)$ models for large $\tan\beta$ and low $m_{\tilde{g}}$. This channel is always closed in the minimal $SU(5)$ case (since $m_A > \sim 1 \text{TeV}$). It is important to point out that the preferred $h \rightarrow b\bar{b}, c\bar{c}, gg$ detection modes may be suppressed because of invisible Higgs decays ($h \rightarrow \chi_1^0 \chi_1^0$) for $m_h \approx 80 \text{GeV}$ ($m_h > 80 \text{GeV}$) by as much as 30%/15% (40%/40%) in the minimal/flipped $SU(5)$ model \[62\].

(b) Chargino masses below the kinematical limit ($m_{\chi_1^\pm} < \sim 100 \text{GeV}$) should not be a problem to detect through the “mixed” mode with one chargino decaying leptonically and the other one hadronically \[52\], i.e., $e^+e^- \rightarrow \chi_1^+ \chi_1^-, \chi_1^+ \rightarrow \chi_1^0 q\bar{q}$, $\chi_1^- \rightarrow \chi_1^0 l^- \bar{\nu}_l$. In Fig. 41 and Fig. 12 (top row) we show the corresponding event rates in the minimal $SU(5)$ and no-scale flipped $SU(5)$ models. Recall that $m_{\chi_1^\pm}$ can be as high as $\approx 290 \text{GeV}$ in the flipped models, whereas $m_{\chi_1^\pm} < 100 \text{GeV}$ in the minimal $SU(5)$ model. Interestingly enough, the number of mixed events do not overlap (they are much higher in the minimal $SU(5)$ model) and therefore, if $m_{\chi_1^\pm} < 100 \text{GeV}$ then LEPII should be able to exclude at least of the models.

(c) Selectron, smuon, and stau pair production is partially accessible for both the no-scale and dilaton models (although more so in the no-scale case), and completely inaccessible in the minimal $SU(5)$ case. In Fig. 12 (bottom row) we show the rates for the most promising (dilepton) mode in $e^+e^- \rightarrow \tilde{e}_R^+ \tilde{e}_R^-$ production in the no-scale model.

## 8.4 HERA

The elastic and deep-inelastic contributions to $e^-p \rightarrow \tilde{e}_R^-\chi_1^0$ and $e^-p \rightarrow \tilde{\nu}_\chi_1^-\tilde{\nu}$ at HERA in the no-scale flipped $SU(5)$ model should push the LEPI lower bounds on the lightest selectron, the lightest neutralino, and the sneutrino masses by $\approx 25 \text{GeV}$ with $\mathcal{L} = 100 \text{pb}^{-1}$ \[63\]. In Fig. 13 we show the elastic plus deep-inelastic contributions to
Figure 11: The number of “mixed” events (1-lepton+2jets+$\bar{p}$) events per $\mathcal{L} = 100\, \text{pb}^{-1}$ at LEP II versus the chargino mass in the minimal $SU(5)$ model.

Figure 12: The number of “mixed” events (1-lepton+2jets+$\bar{p}$) events per $\mathcal{L} = 100\, \text{pb}^{-1}$ at LEP II versus the chargino mass in the no-scale model (top row). Also shown (bottom row) are the number of di-electron events per $\mathcal{L} = 100\, \text{pb}^{-1}$ from selectron pair production versus the lightest selectron mass.
Figure 13: The elastic plus deep-inelastic total supersymmetric cross section at HERA ($ep \rightarrow \text{susy} \rightarrow eX + \not{p}$) versus the lightest selectron mass ($m_{\tilde{e}_R}$) and the sneutrino mass ($m_{\tilde{\nu}}$). The short- and long-term limits of sensitivity are expected to be $10^{-2}$ pb and $10^{-3}$ pb respectively.

9 Prospects for Indirect Experimental Detection
9.1 $b \to s\gamma$

There has recently been a renewed surge of interest on the flavor-changing-neutral-current (FCNC) $b \to s\gamma$ decay, prompted by the CLEO 95% CL allowed range\cite{116}

$$BR(b \to s\gamma) = (0.6 - 5.4) \times 10^{-4}.$$ (53)

Since the Standard Model (SM) prediction looms around $(2 - 5) \times 10^{-4}$ depending on the top-quark mass ($m_t$), a reappraisal of beyond the SM contributions has become topical \cite{117,118,64,119}. We use the following expression for the branching ratio $b \to s\gamma$\cite{118}

$$BR(b \to s\gamma) = \frac{BR(b \to c\ell\nu)}{6\alpha \pi I(m_c/m_b) \left[1 - \frac{2}{3\pi} \alpha_s(m_b) f(m_c/m_b)\right]^2},$$ (54)

where $\eta = \alpha_s(M_Z)/\alpha_s(m_b)$, $I$ is the phase-space factor $I(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x$, and $f(m_c/m_b) = 2.41$ the QCD correction factor for the semileptonic decay. The $A_s$, $A_g$ are the coefficients of the effective $bs\gamma$ and $bsg$ penguin operators evaluated at the scale $M_Z$. Their simplified expressions are given in the Appendix of Ref. \cite{118}, where the gluino and neutralino contributions have been justifiably neglected \cite{120} and the squarks are considered degenerate in mass, except for the $\tilde{t}_1, \tilde{t}_2$ which are significantly split by $m_t$.

For the minimal $SU(5)$ supergravity model we find\cite{64}

$$2.3 (2.6) \times 10^{-4} < BR(b \to s\gamma)_{\text{minimal}} < 3.6 (3.3) \times 10^{-4},$$ (55)

for $\mu > 0 (\mu < 0)$, which are all within the CLEO limits. One can show that $BR(b \to s\gamma)$ would need to be measured with better than 20% accuracy to start disentangling the minimal $SU(5)$ supergravity model from the SM. In Fig. 14 we present the analogous results for the no-scale (top row) and strict no-scale (bottom row) flipped $SU(5)$ supergravity models. The results are strikingly different than in the prior case. One observes that part of the parameter space is actually excluded by the new CLEO bound, for a range of sparticle masses. Perhaps the most surprising feature of the results is the strong suppression of $BR(b \to s\gamma)$ which occurs for a good portion of the parameter space for $\mu > 0$. It has proven to be non-trivial to find a simple explanation for the observed cancellation. See Ref. \cite{119} for the implications of this indirect constraint on the prospects for direct experimental detection of these models.

9.2 $\epsilon_{1,2,3}$

A complete study of one-loop electroweak radiative corrections in the supergravity models we consider here has been made in Ref. \cite{65}. These calculations include some recently discovered important $q^2$-dependent effects, which occur when light charginos
(m_{\chi^\pm} \lesssim 60 - 70 \text{ GeV}) are present \cite{121}, and lead to strong correlations between the chargino and the top-quark mass. Specifically, one finds that at present the 90\% CL upper limit on the top-quark mass is m_t \lesssim 175 \text{ GeV} in the no-scale flipped \textit{SU(5)} supergravity model. These bounds can be strengthened for increasing chargino masses in the $50 - 100 \text{ GeV}$ interval. For example, in the flipped model for $m_{\chi^\pm} \gtrsim 60 (70) \text{ GeV}$, one finds $m_t \lesssim 165 (160) \text{ GeV}$. As expected, the heavy sector of both models decouples quite rapidly with increasing sparticle masses, and at present, only $\epsilon_1$ leads to constraints on the parameter spaces of these models. For future reference, it is important to note that global SM fits to all of the low-energy and electroweak data constrain $m_t < 194, 178, 165 \text{ GeV}$ for $m_{H_{SM}} = 1000, 250, 50 \text{ GeV}$ at the 90\% CL respectively \cite{107}.

One can show that an expansion of the vacuum polarization tensors to order $q^2$, results in three independent physical parameters. In the first scheme introduced to study these effects \cite{122}, namely the $(S, T, U)$ scheme, a SM reference value for $m_t, m_{H_{SM}}$ is used, and the deviation from this reference is calculated and is considered
to be “new” physics. This scheme is only valid to lowest order in $q^2$, and is therefore not applicable to a theory with new, light ($\sim M_Z$) particles. In the supergravity models we consider here, each point in parameter space is actually a distinct model, and a SM reference point is not meaningful. For these reasons, in Ref. [65] the scheme of Refs. [123, 121] was chosen, where the contributions are absolute and valid to higher order in $q^2$. This is the so-called $\epsilon_{1,2,3}$ scheme. Regardless of the scheme used, all of the global fits to the three physical parameters are entirely consistent with the SM at 90% CL.

With the assumption that the dominant “new” contributions arise from the process-independent (i.e., “oblique”) vacuum polarization amplitudes, one can combine several observables in suitable ways such that they are most sensitive to new effects. It is important to note that not all observables can be included in the experimental fits which determine the $\epsilon_{1,2,3}$ parameters, if only the oblique contributions are kept [65].

It is well known in the MSSM that the largest contributions to $\epsilon_1$ (i.e., $\delta \rho$ if $q^2$-dependent effects are neglected) are expected to arise from the $\tilde{t}-\tilde{b}$ sector, and in the limiting case of a very light stop, the contribution is comparable to that of the $t$-$b$ sector [124]. The remaining squark, slepton, chargino, neutralino, and Higgs sectors all typically contribute considerably less. For increasing sparticle masses, the heavy sector of the theory decouples, and only SM effects with a light ($\sim 100$ GeV) Higgs mass survive. However, for very light chargino, a $Z$-wavefunction renormalization threshold effect can introduce a substantial $q^2$-dependence in the calculation, thus modifying significantly the standard $\delta \rho$ results. For completeness, in Ref. [65] the complete vacuum polarization contributions from the Higgs sector, the supersymmetric chargino-neutralino and sfermion sectors, and also the corresponding contributions in the SM were included.

In Fig. 15 we show the calculated values of $\epsilon_1$ versus the lightest chargino mass ($m_{\chi^+_1}$) for the sampled points in the minimal (no-scale flipped) $SU(5)$ supergravity model. In the no-scale flipped $SU(5)$ case three representative values of $m_t$ were used, $m_t = 100, 130, 160$ GeV, whereas in the minimal $SU(5)$ case several other values for $m_t$ in the range $90 \text{ GeV} \leq m_t \leq 160 \text{ GeV}$ were sampled. In both models, but most clearly in the no-scale model one can see how quickly the sparticle spectrum decouples as $m_{\chi^+_1}$ increases, and the value of $\epsilon_1$ asymptotes to the SM value appropriate to each value of $m_t$ and for a light ($\sim 100$ GeV) Higgs mass. The threshold effect of $\chi^+_1$ is manifest as $m_{\chi^+_1} \rightarrow \frac{1}{2} M_Z$ and is especially visible for $\mu < 0$ in both models. This effect is not expected to be very accurate as $m_{\chi^+_1} \rightarrow \frac{1}{2} M_Z$. However, according to Ref. [121], for $m_{\chi^+_1} > 50 \text{ GeV}$, this correction agrees to better than 10% with the one obtained in a more accurate way.

Recent values for $\epsilon_{1,2,3}$ obtained from a global fit to the LEP (i.e., $\Gamma_l, A_{FB}^{tb}, A_{pol}$) and $M_W/M_Z$ measurements are [125],

$$\epsilon_1 = (-0.9 \pm 3.7) \times 10^{-3}, \quad \epsilon_2 = (9.9 \pm 8.0) \times 10^{-3}, \quad \epsilon_3 = (-0.9 \pm 4.1) \times 10^{-3}. \quad (56)$$

For $\epsilon_1$ it is clear that virtually all the sampled points in the minimal $SU(5)$ supergravity model are within the $\pm 1.64 \sigma$ (90% CL) bounds (denoted by the two horizontal
Figure 15: The total contribution to $\epsilon_1$ as a function of the lightest chargino mass $m_{\chi^\pm_1}$ for the minimal $SU(5)$ model (upper row) and the no-scale flipped $SU(5)$ model (bottom row). Points between the two horizontal solid lines are allowed at 90% CL. The three distinct curves (from lowest to highest) in the no-scale case correspond to $m_t = 100, 130, 160$ GeV. Since several values for $90 \leq m_t \leq 160$ GeV were sampled, the trends for fixed $m_t$ are not very clear from the figure. Nonetheless, the points just outside the 1.64σ line correspond to $m_t = 160$ GeV, which are therefore excluded at the 90% CL. In the no-scale model, the upper bound on $m_t$ depends sensitively on the chargino mass. For example, for $m_t = 160$ GeV, only light chargino masses would be acceptable at 90% CL. In fact, in Ref. [65] the region $130$ GeV $\leq m_t \leq 190$ GeV was scanned in increments of 5 GeV and obtained the maximum values for $m_{\chi^\pm_1}$ allowed by the experimental value for $\epsilon_1$ at 90% CL. These are given in the Table I. One can immediately see the strong correlation between $m_t$ and $m_{\chi^\pm_1}$: as $m_t$ rises, the upper limit to $m_{\chi^\pm_1}$ falls, and vice versa. In particular, for $m_t \leq 150$ GeV all values of $m_{\chi^\pm_1}$ are allowed, while one could have $m_t$ as large as 160 (175) GeV for $\mu > 0$ ($\mu < 0$) if the chargino mass were light enough.
Table 5: Maximum allowed chargino mass ($m_{\chi^\pm}$) for different $m_t$ (in GeV) at 90%CL in the no-scale flipped $SU(5)$ model. In the entries Y(N) means all points are within (outside) the LEP bounds at 90%CL.

| $m_t$ | $\mu > 0$ | $\mu < 0$ |
|-------|-----------|-----------|
| 145   | Y         | Y         |
| 150   | Y         | Y         |
| 155   | 68        | 95        |
| 160   | 66        | 72        |
| 165   | N         | 63        |
| 170   | N         | 58        |
| 175   | N         | 53        |
| 180   | N         | N         |

10 Conclusions

The recent surge of interest in supersymmetric models, spurred by the precise LEP measurements of the gauge couplings and their unification at very high energies [26], has made it clear that some sort of organizing principle is needed to tame the zoo of supersymmetric particles at low energies, as encompassed by the minimal supersymmetric standard model (MSSM). Even though it is usually not acknowledged that this model needs at least twenty-one parameters for its full description, the fact remains that this is the case. One reason for disregarding this fact is that many phenomenological calculations of interest have been performed and numerical results obtained by making ad-hoc assumptions about the values of these parameters, giving the erroneous impression that the results so-obtained are largely insensitive to changes in these assumptions. This mindset has been explicitly exposed by now on a variety of calculations. These have been performed in the context of the class of unified supergravity models which we consider here, and have been found to yield quite different results than previously expected. A few examples include: the calculation of the cosmological relic density of neutralinos in the MSSM [126] and in supergravity models without [127, 73, 77] and with [128, 74] radiative electroweak symmetry breaking; the enhancement of the leptonic branching ratios of the chargino in the presence of light sleptons [60]; the large sparticle mass dependence of the $b \to s\gamma$ branching ratio [64] which can completely wash out the charged Higgs contribution [117]; etc. It then becomes apparent that one must make further well motivated theoretical assumptions to make further headway into this problem. As discussed above, supergravity models with radiative electroweak symmetry breaking accomplish this goal very economically, needing just three supersymmetry breaking parameters, the ratio $\tan \beta$, and the soon-to-be-measured top-quark mass to be fully described. We have discussed two classes of such models, the class of string-inspired flipped $SU(5)$ models and the
traditional minimal $SU(5)$ supergravity model.

The simplest, string-derivable, supergravity model has as gauge group flipped $SU(5)$ with supplementary matter representations to ensure unification at the string scale ($\sim 10^{18} \text{ GeV}$). This basic structure is complemented by two possible string supersymmetry breaking scenarios: $SU(N,1)$ no-scale supergravity and dilaton-induced supersymmetry breaking. These two variants should be considered to be idealizations of what their string-derived incarnation should be. The specification of the hidden sector is crucial to the determination of the supersymmetry breaking scenario at work. A thorough exploration of the parameter spaces of the two models yields interesting results for direct and indirect experimental detection at present or near future colliders. In this regard, the no-scale model is more within reach than the dilaton model, because of its generally lighter spectrum. In both supersymmetry breaking scenarios considered, there is a more constrained special case which allows $\tan \beta$ to be determined in terms of $m_t$ and $m_\tilde{g}$. In the strict no-scale case we find a striking result: if $\mu > 0$, $m_t \lesssim 135 \text{ GeV}$, whereas if $\mu < 0$, $m_t \gtrsim 140 \text{ GeV}$. Therefore the value of $m_t$ determines the sign of $\mu$. Furthermore, we found that the value of $m_t$ also determines whether the lightest Higgs boson is above or below 100 GeV. In the restricted dilaton case there is an upper bound on the top-quark mass ($m_t \lesssim 155 \text{ GeV}$) and the lightest Higgs boson mass ($m_h \lesssim 91 \text{ GeV}$). Thus, continuing Tevatron top-quark searches and LEPI,II Higgs searches could probe this restricted scenario completely. In Table 6 we give a summary of the general properties of these models and a comparison of their spectra.

The minimal $SU(5)$ supergravity model is strongly constrained by the proton lifetime and the cosmological neutralino relic density. The former constraint implies light chargino masses and heavy scalar masses (beyond the reach of present or near future colliders), while the second constraint nearly enforces the mass relations $m_\chi \approx \frac{1}{2} m_{h,Z}$. The prospects for detection of the light particles in this model ($\chi_1^0, \chi_1^\pm, h$) is very promising in the near future. It is interesting to remark that the resulting allowed choices for the supersymmetry breaking parameters bear close resemblance to those predicted in moduli-induced string-inspired supersymmetry breaking scenarios. In Table 7 we give a summary of the general properties of this models and its spectrum.

We conclude that these well motivated supergravity models (especially the strict versions of the string-inspired/derived models) could soon be probed experimentally. The various ingredients making up the flipped $SU(5)$ models are likely to be present in actual fully string-derived models which yield the set of supersymmetry breaking parameters in Eqs. (4,5). The search for such models is imperative, although it may not be an easy task since in traditional gaugino condensation scenarios Eqs. (4,5) are usually not reproduced (see however Refs. [129, 130]). Moreover, the requirement of vanishing vacuum energy may be difficult to fulfill, as a model with these properties and all the other ones outlined in Sec. 4 is yet to be found. This should not be taken as a discouragement since the harder it is to find the correct model, the more likely it is to be in some sense unique.
Table 6: Major features of the $SU(5) \times U(1)$ string-inspired/derived model and a comparison of the two supersymmetry breaking scenarios considered. (All masses in GeV).

| SU(5) × U(1)                                                                                           | \(\langle F_M \rangle_{m_0=0}\) (no-scale)                                                                 | \(\langle F_D \rangle\) (dilaton)                                                                 |
|----------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------|
| • Easily string-derivable, several known examples                                                      | • Parameters 3: \(m_{1/2}, \tan \beta, m_t\)                                                            | • Parameters 3: \(m_{1/2}, \tan \beta, m_t\)                                                            |
| • Symmetry breaking to Standard Model due to vevs of \(10, \overline{10}\) and tied to onset of supersymmetry breaking | • Universal soft-supersymmetry breaking automatic                                                         | • Universal soft-supersymmetry breaking automatic                                                         |
| • Natural doublet-triplet splitting mechanism                                                           | • Dark matter: \(\Omega h_0^2 < 0.25\)                                                                     | • Dark matter: \(\Omega h_0^2 < 0.90\)                                                                     |
| • Proton decay: \(d = 5\) operators very small                                                          | • \(m_{1/2} < 475\) GeV, \(\tan \beta < 32\)                                                            | • \(m_{1/2} < 465\) GeV, \(\tan \beta < 46\)                                                            |
| • Baryon asymmetry through lepton number asymmetry (induced by the decay of flipped neutrinos) as processed by non-perturbative electroweak interactions | • \(m_{\tilde{g}} > 245\) GeV, \(m_{\tilde{q}} > 240\) GeV                                               | • \(m_{\tilde{g}} > 195\) GeV, \(m_{\tilde{q}} > 195\) GeV                                               |
| • \(m_0 = 0, A = 0\)                                                                                    | • \(m_{\tilde{q}} \approx 0.97 m_{\tilde{g}}\)                                                           | • \(m_{\tilde{q}} \approx 1.01 m_{\tilde{g}}\)                                                           |
| • Dark matter: \(\Omega h_0^2 < 0.25\)                                                                     | • \(m_{\tilde{R}} \approx 0.18 m_{\tilde{g}}, m_{\tilde{E}_L} \approx 0.30 m_{\tilde{g}}\)              | • \(m_{\tilde{R}} \approx 0.33 m_{\tilde{g}}, m_{\tilde{E}_L} \approx 0.41 m_{\tilde{g}}\)              |
| • \(m_{\tilde{R}} / m_{\tilde{E}_L} \approx 0.61\)                                                     | • \(m_{\tilde{R}} / m_{\tilde{E}_L} \approx 0.81\)                                                      | • \(m_{\tilde{R}} / m_{\tilde{E}_L} \approx 0.81\)                                                      |
| • \(60\) GeV \(< m_h < 125\) GeV                                                                      | • \(2 m_{\chi_1^0} \approx m_{\chi_2^0} \approx m_{\chi_1^\pm} \approx 0.28 m_{\tilde{g}} \lesssim 290\)  | • \(2 m_{\chi_1^0} \approx m_{\chi_2^0} \approx m_{\chi_1^\pm} \approx 0.28 m_{\tilde{g}} \lesssim 285\)  |
| • \(m_{\chi_3^0} \approx m_{\chi_4^0} \approx m_{\chi_2^\pm} \approx |\mu|\)                            | • Spectrum easily accessible soon                                                                         | • \(m_{\chi_3^0} \approx m_{\chi_4^0} \approx m_{\chi_2^\pm} \approx |\mu|\)                            |
| • Spectrum accessible soon                                                                              | • Strict no-scale: \(B(M_U) = 0\)                                                                        | • Special dilaton: \(B(M_U) = 2m_0\)                                                                     |
| • \(\tan \beta = \tan \beta(m_t, m_{\tilde{g}})\)                                                      | • \(m_t \lesssim 135\) GeV \(\Rightarrow \mu > 0, m_h \lesssim 100\) GeV                                 | • \(\tan \beta = \tan \beta(m_t, m_{\tilde{g}})\)                                                      |
| \(m_t \gtrsim 140\) GeV \(\Rightarrow \mu < 0, m_h \gtrsim 100\) GeV                                    | • \(m_t \lesssim 135\) GeV \(\Rightarrow \mu > 0, m_h \lesssim 100\) GeV                                 | • \(\tan \beta \approx 1.4 - 1.6, m_t < 155\) GeV                                                      |
| • \(m_t \gtrsim 140\) GeV \(\Rightarrow \mu < 0, m_h \gtrsim 100\) GeV                                    | • \(m_h \approx 61 - 91\) GeV                                                                           | • \(m_h \approx 61 - 91\) GeV                                                                           |

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Table 7: Major features of the minimal SU(5) supergravity model and its spectrum (All masses in GeV).

| SU(5) |
|-------|
| • Not easily string-derivable, no known examples |
| • Symmetry breaking to Standard Model due to vevs of $24$ and independent of supersymmetry breaking |
| • No simple doublet-triplet splitting mechanism |
| • Proton decay: $d = 5$ operators large, strong constraints needed |
| • Baryon asymmetry? |

| Spectrum |
|---------|
| • Parameters 5: $m_{1/2}, m_0, A, \tan \beta, m_t$ |
| • Universal soft-supersymmetry breaking automatic |
| • $m_0/m_{1/2} > 3$, $\tan \beta \lesssim 3.5$ |
| • Dark matter: $\Omega_{\chi^0} h_0^2 \gg 1$, $1/6$ of points excluded |
| • $m_{\tilde{g}} < 400 \text{ GeV}$, $m_{\tilde{q}} > m_{\tilde{t}} > 2m_{\tilde{g}} \gtrsim 500 \text{ GeV}$ |
| • $m_{\tilde{t}_1} > 45 \text{ GeV}$ |
| • $60 \text{ GeV} < m_h < 100 \text{ GeV}$ |
| • $2m_{\chi^0_1} \approx m_{\chi^0_2} \approx m_{\chi^\pm_1} \approx 0.28m_{\tilde{g}} \lesssim 100$ |
| • $m_{\chi^0_3} \sim m_{\chi^0_4} \sim m_{\chi^\pm_2} \sim |\mu|$ |
| • Chargino and Higgs easily accessible soon |

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