BILEPTON RESONANCE IN ELECTRON-ELECTRON SCATTERING

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ABSTRACT

Theoretical background for bileptonic gauge bosons is reviewed, both the SU(15) GUT model and the 3-3-1 model. Mass limits on bileptons are discussed coming from $e^+e^-$ scattering, polarized muon decay and muonium-antimuonium conversion. Discovery in $e^-e^-$ at a linear collider at low energy (100 GeV) and high luminosity ($10^{33}/cm^2/s$) is emphasised.

Introduction.

It is a stunning historical fact that $e^-e^-$ collisions have never been studied at a center of mass energy above 1.12 GeV as published in 1971 by Richter et al.1 There were plans to explore $e^-e^-$ at DESY but these were abandoned when money ran out.

The three large projects in HEP for the US (and internationally) for the foreseeable future are: NLC, MC and VLHC. Of these the NLC is for the first decade of the twenty-first century; the other two are for the second decade. The NLC is presently a multi-billion dollar project primarily aimed at $e^+e^-$. A topic of this workshop is: should it have also $e^-e^-$ capability?

Why has $e^-e^-$ been so neglected? Firstly $e^+e^-$ is where $Z'$ can be found - often cited as the most conservative extension of the Standard Model (SM). By contrast $e^-e^-$ is an exotic, empty channel because it has double electric charge and lepton number $L = 2$. Surely, $e^-e^-$ would allow only checks of higher-order quantum electrodynamics. But physics is an experimental science!

$e^-e^-$ Resonance.

Such a resonance must have $L = 2$ and $Q = 2$. It must be a boson. For spin zero a doubly-charged Higgs scalar, the coupling is a free parameter and is generically small. For a spin one gauge boson, the coupling is large and prescribed. Bilepton gauge bosons give a pronounced peak at $s = M^2$. But, as our main emphasis here, the resonance tail is detectable at much lower energy.

Bilepton gauge bosons were first suggested in the context of SU(15) grand unification.
First recall that in $SU(5)$ grand unification with families each in $5 + \bar{10}$ the reason for $B$ violation is that the second rank tensor $\bar{10}$ has indefinite $B$ and $L$ quantum numbers.

If $SU(5)$ had fermions only in the 5 then $B$ and $L$ would necessarily be conserved perturbatively.

The presence of the $\bar{10}$ is what causes the indeterminacy of $B$ and $L$ and allows mediation of proton decay in the gauge sector.

Since proton decay remains elusive the idea in $SU(15)$ is to prohibit it in the gauge sector. The 15 helicity states in each family are assigned to a 15 of $SU(15)$. Whereupon each gauge boson has definite $B$ and $L$ according to which pair of the fundamental fermions it couples.

The first family is assigned to:

$$15 = (u_R^u, u_L^G, u_L^B, d_R^B, d_L^d, \bar{u}_L^\bar{u}, \bar{u}_L^B, \bar{d}_L^\bar{d}, \bar{d}_L^B, \bar{e}_L^\bar{e}, \nu_{eL}, e^e_L)$$

and similarly for the second and third families.

It is clear that all of the 224 gauge bosons of $SU(15)$ have definite $B$ and $L$.

Anomaly cancellation is by mirror fermions - disfavored aesthetically but not phenomenologically.

The pattern of spontaneous symmetry breaking is:

$$SU(15) \xrightarrow{M_G} SU(12)_q \times SU(3)_l$$

$$\xrightarrow{M_B} SU(6)_L \times SU(6)_R \times U(1)_B \times SU(3)_l$$

$$\xrightarrow{M_A} SU(3)_C \times SU(2)_L \times U(1)_Y$$

In the breaking at $M_A$ color $SU(3)_C$ is embedded in $SU(6)_L \times SU(6)_R$ as $(3 + 3, 1) + (1, \bar{3} + \bar{3})$. 
SU(2)_L is embedded in SU(6)_L × SU(3)_l with 6_L = 3(2)_L and 3_L = 2_L + 1_L

U(1)_Y is contained in SU(6)_R × U(1)_B × SU(3)_l according to:

\[ Y = \sqrt{3} \Lambda + \sqrt{\frac{2}{3}} B + \sqrt{3} Y \]

with \( \Lambda, B \) and \( Y \) generators of SU(6)_R, U(1)_B and SU(3)_l, respectively, normalized as SU(15) matrices with

\[ Tr(\Lambda^a \Lambda^b) = 2 \delta^{ab}. \]

Explicitly, these normalized SU(15) generators are

\[ \Lambda = \frac{1}{\sqrt{3}} \text{diag}(000000, -1 - 1 - 1111, 000) \]

\[ B = \sqrt{\frac{3}{2}} \text{diag}(\frac{1 1 1 1 1 1}{3 3 3 3 3 3}, -\frac{1}{3} - \frac{1}{3} - \frac{1}{3} - \frac{1}{3} - \frac{1}{3} - \frac{1}{3}, 000) \]

and \( Y = \frac{1}{\sqrt{3}} \text{diag}(000000, 000000, 2 - 1 - 1) \)

RENORMALIZATION GROUP

\[ \mu d \alpha_i(\mu)/d\mu = B_i \alpha_i^2(\mu) \]

with matching conditions, at \( M_A: \)
\[
\alpha_{3C}^{-1}(M_A) = \frac{1}{2} \alpha_{6L}^{-1}(M_A) + \frac{1}{2} \alpha_{6R}^{-1}(M_A)
\]

\[
\alpha_{2L}^{-1}(M_A) = \frac{3}{4} \alpha_{6L}^{-1}(M_A) + \frac{1}{4} \alpha_{3l}^{-1}(M_A)
\]

\[
\alpha_{1Y}^{-1}(M_A) = \frac{9}{20} \alpha_{6R}^{-1}(M_A) + \frac{1}{10} \alpha_B^{-1}(M_A) + \frac{9}{20} \alpha_{3l}^{-1}(M_A)
\]

at \(M_B\):

\[
\alpha_{6L}(M_B) = \alpha_{6R}(M_B) = \alpha_B(M_B) = \alpha_{12q}(M_B)
\]

and at \(M_G\):

\[
\alpha_{12q}(M_G) = \alpha_{3l}(M_G) = \alpha_{15}(M_G)
\]

The results can be tabulated, as shown in this Table of typical values for the three breaking scales of \(SU(15)\)

| \(M_A(\text{GeV})\) | \(M_B(\text{GeV})\) | \(M_G(\text{GeV})\) |
|-----------------|-----------------|-----------------|
| 250            | \(4.0 \times 10^6\) | \(6.0 \times 10^6\) |
| 500            | \(5.8 \times 10^6\) | \(8.9 \times 10^6\) |
| \(10^3\)       | \(8.3 \times 10^6\) | \(1.3 \times 10^7\) |
| \(2 \times 10^3\) | \(1.2 \times 10^7\) | \(1.9 \times 10^7\) |

There is one input parameter, say \(M_A\).

\(M_B\) and \(M_G\) are outputs.

At low energies \((M_A)\) the gauge bosons under \(SU(6)_L \times SU(6)_R \times U(1)_B \times SU(3)_l\) are, with respect to the standard model:

\[
35_L = (8, 3)_0 + (8, 1)_0 + (1, 3)_0
\]
\( 35_R = 2(8,1)_0 + (8,1)_{\pm 1} + (1,1)_0 + (1,1)_{\pm 1} \)

\[ 1_B = (1,1)_0 \]

\[ 8_I = (1,3)_0 + (1,1)_0 + (1,2)_{\pm 3/2} \]

All are interesting but the last-listed \((1,2)_{\pm 3/2}\) are the bileptonic gauge bosons which can show up in Moller scattering. (\textit{e.g.} \(e^-e^- \rightarrow \mu^-\mu^-\)).

Clearly such bileptons are a general feature of the embedding

\[ SU(2)_L \subset SU(3) \]

and have the electric charges

\((Y^{-+}, Y^-) \quad (L = +2)\)

with antiparticles

\((Y^{++}, Y^+) \quad (L = -2).\)

This feature of \(SU(15)\) grand unification re-emerges in the \(3-3-1\) model\(^3\) to which we now turn.

\(3-3-1\) is more economic, and anomaly cancellation is more elegant, compared to \(SU(15)\).

To introduce the 3-3-1 model, the following are motivating factors:

1. Consistency of a gauge theory requires cancellation of all chiral anomalies. Such cancellation occurs for a quark-fermion family and is enough (almost) to fix all charges.

2. This does not explain \(N_f > 1\) but is sufficiently impressive to suggest that \(N_f = 3\) may be explicable by anomaly cancellation in an extension. This requires extended families have non-zero anomaly and not all three families treated similarly.

3. The third family is exceptional because of the top quark mass, and suggests \(+1 +1 -2\) cancellation.
4. There is such a -2 in the SM as the ratio of quark charges.

5. Extension of $SU(2)_L$ to $SU(3)_L$ will have the same lepton couplings of the bileptons as in $SU(15)$.

For the 3-3-1 model the gauge group is:

$SU(3)_C \times SU(3)_L \times U(1)_X$

The first family quarks are assigned to

\[
\begin{pmatrix}
u \\
\bar{u} \\
\bar{d} \\
\bar{D}
\end{pmatrix}_L
\]

The triplet is a 3 of $SU(3)_L$.

The second family of quarks is assigned similarly:

\[
\begin{pmatrix} c \\
\bar{c} \\
\bar{s} \\
\bar{S}
\end{pmatrix}_L
\]

The third family of quarks is assigned differently:

\[
\begin{pmatrix} T \\
\bar{t} \\
\bar{b} \\
\bar{b}
\end{pmatrix}_L
\]

The triplet in this case is a 3* of $SU(3)_L$.

The X quantum numbers of the triplets are equal to the electric charges of the central members. That is, for the three families of quarks, $X = -\frac{1}{3}, -\frac{1}{3}, +\frac{2}{3}$.

The leptons are assigned to 3*'s as follows:

\[
\begin{pmatrix}
\nu_e \\
e^- \\
\nu_e \\
\nu_e \\
\mu^- \\
\nu_\mu \\
\mu^- \\
\nu_\tau \\
\tau^- \\
\nu_\tau \\
\tau^-
\end{pmatrix}_L
\]
These three antitriplets have $X = 0$.

Let us see how anomalies cancel. Recall that anomaly cancellation is crucial in many situations of model-building beyond the standard model e.g. chiral color and in string theory.

The color anomaly $(3_L)^3$ cancels because QCD is vectorlike.

The anomaly $(3_L)^3$ is non-trivial. Taking $N_C$ colors and $N_l$ light neutrinos the anomaly cancels only if $N_C = N_l = 3$.

The remaining anomalies

$$(3_C)^2X, (3_L)^2X, X^3, X(T_{\mu\nu})^2$$

also all cancel.

In particular, each family has a non-zero anomaly for $X^3$, $(3_L)^2X$ and $(3_L)^3$; in each case the anomalies cancel proportionately to $+1 + 1 - 2$, as anticipated in the earlier discussion.

To break the symmetry requires several Higgs multiplets.

First an $X = +1$ triplet $\Phi$ with VEV $< \Phi > = (0, 0, U)$ breaks $331$ to $321$ and gives masses to the D, S and T quarks as well as the gauge bosons $Z'$ and $Y$. The scale $U$ sets the scale for the new physics.

Electroweak symmetry breaking requires two further triplets $\phi$ and $\phi'$ with $X = 0$ and $X = -1$ respectively. Their VEVs give mass to d, s, t and to u, c, b respectively. The first VEV also gives a family-antisymmetric contribution to the charged leptons. To obtain a general mass matrix for charged leptons necessitates adding a sextet with $X = 0$.

THE NEW PHYSICS SCALE U

There is a lower bound from precision electroweak data:

$$Z - Z'$$ mixing dictates $M(Z') > 300 GeV$.

FCNC limits give a similar bound. For FCNC it is crucial that the third family be the one treated asymmetrically. Otherwise the FCNC disagree with experiment.
UPPER BOUND ON U:

A bound on $U$ arises because the embedding of 321 in 331 requires $\sin^2\theta < 1/4$ because for $\sin^2\theta = 1/4$ the coupling $g_X$ diverges. This fixes $U < 3\text{TeV}$ using $\sin^2\theta(M_Z) = 0.231$. Hence $M(Y)$ cannot be higher than 1.5 TeV.

LEP data:

The highest precision high-energy data is from LEP. It gives $M(Y) > 120\text{GeV}$.

The best lower bounds come from low energy experiments:

(1) Polarized muon decay:

$M(Y^\pm) > 230 \text{GeV}$.

(2) Muonium-Antimuonium conversion:

$M(Y^{\pm\pm}) > 850\text{GeV}$.

Just to recapitulate some of the points made at the beginning:

$e^+e^-$ collisions have never been studied above c.o.m. energy 1.12 GeV. An NLC should have $e^+e^-$ capability.

**Accomplishment of $e^+e^-$ Collisions at NLC.**

In the post-SSC era it is desirable to avoid a third comma in the cost $C$, *i.e.* $C < $1B.

How can this be achieved?

The cost of an NLC is roughly linear in the energy.

A 500GeV NLC was costed last year at $7.9B$, although I have been told informally that that cost might be lowered below $5B$. Thus 100GeV could be below $1B$?
Therefore the first fundable step could focus on luminosity rather than energy and be a 100GeV machine with luminosity $\sim 10^{33}$. This is sufficiently above LEP to give a Giga-Z and allows an opportunity to do new machine physics.

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Note Added

In a recent work [P.H. Frampton and A. Rasin, UNC Report IFP-781-UNC (February 2000)] we have updated the cross-section estimates for $e^-e^- \rightarrow \mu^-\mu^-$ in 6 which used the $SU(15)$ model. In the simpler 331-model the cross-section is about one order of magnitude higher than the results in 6.