Coexistence of vector chiral order and Tomonaga-Luttinger liquid in the frustrated three-leg spin tube in a magnetic field

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The frustrated three-leg antiferromagnetic spin-$\frac{1}{2}$ tube with a weak interchain coupling in a magnetic field is investigated by means of Abelian bosonization techniques. It is clearly shown that a vector chiral long-range order and a one-component Tomonaga-Luttinger liquid coexist in a wide magnetic-field region from a state with a small magnetization to a nearly saturated one. The chiral order is predicted to still survive in the intermediate plateau state. We further predict that (even) when the strength of one bond in the three rung couplings is decreased (increased), an Ising-type quantum phase transition takes place and the chirality vanishes (no singular phenomena occur and the chiral order is maintained). Even without magnetic fields, the chiral order would also be present if the spin tube possesses easy-plane anisotropy.

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I. INTRODUCTION

Quantum spin tubes—namely, systems of coupled spin chains with a periodic boundary condition (PBC) for the interchain (rung) direction—have been studied since late 1990s. In particular, odd-leg spin tubes have attracted much interest, because the antiferromagnetic (AF) rung coupling yields frustration. It has been recognized now that coupled spin chains including spin tubes offer far richer physics compared with single spin chains. Recently, a few new spin-tube compounds such as a three-leg tube $[(\text{CuCl}_2 \text{tachH})_3 \text{Cl}] \text{Cl}_2$ and a nine-leg one $\text{Na}_2 \text{V}_3 \text{O}_7$ have been fabricated. The presence of these materials has further promoted the study of spin tubes.

For nonfrustrated AF spin-S ladders—namely, coupled AF chains with an open boundary condition for the rung—the following universal feature (generalization of Haldane’s prediction) has been elucidated: if both $2S$ and the number of chains, $N$, are odd, the low-energy physics is described by a gapless, one-component Tomonaga-Luttinger liquid (TLL) and no symmetry breakings take place, whereas for all the other cases ($2S \times N = \text{even}$), the system has a gapped spectrum with conserving all symmetries. This even-odd property, however, does not always hold in frustrated coupled spin chains. In the three-leg AF spin-$\frac{1}{2}$ tube where the rung triangle induces geometrical frustration, it is known that the one-site translational symmetry along the chain direction is spontaneously broken and a finite gap between a ground state and the lowest excitation exists.

In this paper, we point out another striking difference between the three-leg nonfrustrated ladder and the frustrated tube: namely, utilizing bosonization techniques, we show in a clear way that a vector chiral long-range order resides in the magnetic-field-induced TLL phase of the three-leg AF spin-$\frac{1}{2}$ tube at least in the weak-rung-coupling regime. In the chiral phase, the parity symmetry for the rung direction is spontaneously broken. On the contrary, the chiral order is absent in the corresponding ladder. For one-dimensional gapped $U(1)$-symmetric coupled spin chains, (i) the field-induced magnon-condensed gapless state is usually described by a standard TLL and conserves all symmetries and (ii) many physicists have focused only on whether nontrivial magnetization plateau states exist or not. Therefore, the possibility of chirality in spin tubes has not been considered well so far. We also show that the chiral order continues in the intermediate plateau state of the spin tube. These predictions mean that the three-leg spin tube possesses both gapless and gapped chiral-ordered states. We further discuss a rung-coupling modification in the three-leg spin tube: the strength of one bond of three rung couplings is changed. If the strength of the changed bond is extremely increased (decreased), the system approaches a two-leg ladder plus single chain (a three-leg ladder). Both limits possess no geometrical frustration. Interestingly, it is found that in the case of increasing the bond strength, the chiral order always remains unbroken, but an Ising-type quantum phase transition takes place and the chirality disappears at the Ising critical point in the other case.

The organization of the paper is as follows. In Sec. II we define a spin tube model and briefly explain the bosonization method. The next two sections are the main part of the paper. We show the mechanism of the vector chiral order in Sec. III. Section IV is devoted to investigating effects of the rung deformation. Finally, conclusions and brief discussions about our results are presented in Sec. V.

II. MODEL AND METHOD

In this paper, we mainly consider the three-leg AF spin-$\frac{1}{2}$ tube with a homogeneous rung coupling. The
Hamiltonian is defined as

$$\mathcal{H} = \sum_{n=1}^{3} \left[ J \tilde{S}_{n,j} \cdot \tilde{S}_{n,j+1} + J_{\perp} \tilde{S}_{n,j} \cdot \tilde{S}_{n+1,j} - H S_{n,j}^z \right], \quad (1)$$

where $\tilde{S}_{n,j}$ is spin-$\frac{1}{2}$ operator on site $j$ of the $n$th chain ($n = 1\text{-}3$), $J > 0$ ($J_{\perp} > 0$) is the AF intrachain (rung) coupling, and the PBC $\tilde{S}_{n,j} = \tilde{S}_{n,j}$ is imposed. We begin with the independent three chains in a field $\mathcal{H}$. From Abelian bosonization, the $n$th chain in the low-energy limit is mapped to a Gaussian model $\mathcal{H}_n = \int dx \left\{ K (\partial_x \phi_n)^2 + K^{-1} (\partial_x \phi_n)^2 \right\}$, where $\phi_n$ is the scalar boson field and $\theta_n$ is the dual to $\phi_n$. The TLL parameter $K$ and the spin-wave velocity $v$ depend on $J$ and $\mathcal{H}$; when $\mathcal{H}$ is changed from 0 to the saturation field $2J$, $v(K)$ monotonically decreases (increases) from $\pi J a_0/2$ (1/2) to 0 (1) [$a_0$: lattice constant]. The spin operators are also bosonized as

$$S_{n,j}^0 \approx M + a_0 \partial_x \phi_n / \sqrt{\pi} + (\pi/2) A_1 \sin((\pi/4) \phi_n + 2\pi M j) + \cdots, \quad (2a)$$

$$S_{n,j}^+ \approx \varepsilon^{\sqrt{\pi}} \left\{ (\pi/2) B_0 + \sin((\pi/4) \phi_n + 2\pi M j) + \cdots \right\}, \quad (2b)$$

where $M(H) = \langle S_{n,j}^0 \rangle$, and $A_1$ and $B_1$ are nonuniversal constants of $O(1)$. This formula indicates that the period of $\phi_n (\theta_n) = \sqrt{\pi} (\sqrt{4\pi})$. Using $\mathcal{H}_n$ and formula (2), we can straightforwardly derive the following effective Hamiltonian of the model (1) in the weak-rung-coupling regime:

$$\mathcal{H}_{\text{eff}} = \int dx \sum_{n=1}^{3} \frac{v}{2} \left\{ K (\partial_x \phi_n)^2 + K^{-1} (\partial_x \phi_n)^2 \right\} + 2M J_1 \sum_{n} \partial_x \phi_n + \frac{J_{a_0}}{\pi} \sum_{n} \partial_x \phi_n \partial_x \phi_{n+1} + \frac{1}{2} J_{\perp} \sum_{n} \left[ B_0 \cos((\pi/4) \phi_n + 2\pi M j) + \cdots \right], \quad (3)$$

where $\phi_4 = \phi_1$, $\theta_4 = \theta_1$, and we have written only the important part among all the rung-coupling terms and neglected terms with oscillating factors $\varepsilon^{i\pi M j}$ or $(\pi/2)$, which are irrelevant. In the bosonization picture, symmetries of the spin tube (1) are represented as follows: a $U(1)$ rotation around the $S^z$ axis $S_{n,j}^0 \rightarrow e^{i\pi/2} S_{n,j}^0$, the one- site translation along the chain $S_{n,j}^0 \rightarrow S_{n,j+1}^0$, that along the rung $S_{n,j}^0 \rightarrow S_{n,j+1}^0$, the site-parity transformation for the chain $S_{n,j}^0 \rightarrow S_{n,j}^0$, and that for the rung $S_{n,j}^0 \leftrightarrow S_{n,j}^0$, respectively, correspond to $\theta_n \rightarrow \theta_n + \gamma/\sqrt{\pi}$, $(\phi_n, \theta_n) \rightarrow (\phi_n + \sqrt{\pi}(M + 1/2), \theta_n + \sqrt{\pi})$, $(\phi_n, \theta_n) \rightarrow (\phi_{n+1}, \theta_n + \sqrt{\pi}/2, \theta_n(-x)) \rightarrow (\phi_n(-x) + \sqrt{\pi}/2, \theta_n(-x))$ and $(\theta_1, \phi_1) \leftrightarrow (\phi_3, \theta_3)$. These symmetries strongly restrict possible terms in $\mathcal{H}_{\text{eff}}$. In other words, the symmetries reduce the number of coupling constants. As a result, for all vertex operators without oscillating factors and conformal spin, only $\sum_n \cos((\sqrt{2\pi} \phi_n - \phi_{n+1}))$ and $\sum_n \cos((\sqrt{2\pi} \theta_n - \theta_{n+1}))$ are allowed to exist in Eq. (3). The most relevant terms with $l = 1$ indeed appear in Eq. (3). Using the effective theory (4), we study the low-energy properties of the spin tube below. We stress here that almost all the discussions below are applicable to other three-leg spin tubes with symmetry-conserved perturbations (XXZ anisotropy, further-neighbor interactions, etc.).

### III. VECTOR CHIRAL ORDER

Let us introduce new boson fields $\Phi_0 = \sum_{n} \phi_n/\sqrt{3}$, $\Phi_1 = (\phi_1 - \phi_3)/\sqrt{2}$, $\Phi_2 = (\phi_1 + \phi_3 - 2\phi_2)/\sqrt{6}$, $\Theta_0 = \sum_n \theta_n/\sqrt{3}$, $\Theta_1 = (\theta_1 - \theta_3)/\sqrt{2}$, and $\Theta_2 = (\theta_1 + \theta_3 - 2\theta_2)/\sqrt{6}$. The relationship between old and new bosons is illustrated in Fig. 1. As one will see later, these fields help us to detect a vector chiral order. Using these new fields, we can diagonalize the boson bilinear part in Eq. (3). Consequently, the effective Hamiltonian is rewritten as

$$\mathcal{H}_{\text{eff}} = \int dx \sum_{q=0}^{2} \frac{v_q}{2} \left\{ K_q (\partial_x \Phi_q)^2 + K^{-1}_q (\partial_x \Phi_q)^2 \right\} + 2J_1 \sum_{q=0}^{2} \left( B_0^2 \cos((\pi/4) \phi_n + 2\pi M j) + \cdots \right) \left( \frac{1}{2} J_{a_0} \right), \quad (4)$$

where the potential $V[\alpha, \beta]$ is defined as

$$V = 2 \cos \left( \frac{\sqrt{\pi} \alpha}{2} \right) \cos \left( \sqrt{\frac{3\pi}{2}} \beta \right) + \cos \left( \sqrt{2\pi} \alpha \right). \quad (5)$$

The new TLL parameters $K_q$ and velocities $v_q$ are evaluated as

$$K_0 = K f_{a_0}^{-1}, \quad v_0 = v f_0, \quad K_{1,2} = K f_{g}^{-1} = K_g, \quad v_{1,2} = v_f = v_g, \quad (6)$$

where $f_0 = (1 + 2K J_{a_0})^{1/2}$ and $f_g = (1 - K J_{a_0})^{1/2}$. One finds that the Hamiltonian (1) does not contain
any vertex operator with $\Phi_0$ or $\Theta_0$. This is strongly supported by symmetries of the tube. The $\Phi_0$ sector hence provides a one-component TLL. The derivative term $\partial_\phi \Phi_0$ can be absorbed into the Gaussian part and yields only a small correction of $O(1)$. From the formula (2), the $z$-independent part can be absorbed into the Gaussian part and is sufficiently small. This chiral plateau state is reminiscent of the narrow chiral phase in the classical AF $XY$ model on a triangular lattice.\textsuperscript{20} It is noteworthy that by controlling the strength of the magnetic field, one can realize both gapless and gapped chiral states in single spin tube.

FIG. 2: (a) Potential $V[\Theta_1, \Theta_2]$. (b) Physically relevant zone (enclosed by the green line) in the projected $\Theta_1-\Theta_2$ plane. $K_n$ play the role of the primitive translation vectors. The rung-parity transformation causes $\Theta_1 \rightarrow -\Theta_1$, and the rung translation by one site does $K_n \rightarrow K_{n+1}$.

The scenario leading to the TLL here is already known well\textsuperscript{7,11} However, as one will see below, characteristics of the frustrated spin tube\textsuperscript{11} are hidden in the form of the potential $V$. Figure 2(a) and Eq. (5) show that six minimum points ($\Theta_1, \Theta_2) = (\pm \sqrt{2\pi}/3, \pm \sqrt{2\pi}/3)$ and ($\pm 2\sqrt{2\pi}/3, 0$) lie around the origin $(0,0)$. [Inversely, $(0,0)$ is the unique minimum of the potential $J_x V$ for $J_x < 0$.] Not all, however, are physically meaningful, because phase fields $\theta_n$ have a period. Projecting the physically relevant cubic space $\theta_n \in [-\sqrt{2\pi}, \sqrt{2\pi}]$ onto the $\Theta_1-\Theta_2$ plane (see Fig. 1), we obtain a diamond zone as in Fig. 2(b). Therefore, it is enough to consider only two minimum points $(\Theta_1^\dagger, \Theta_2) = (\pm \sqrt{2\pi}/3, \sqrt{2\pi}/3)$ if $\Theta_1^\dagger = -\Theta_1$ does not hold. Since the rung-parity transformation $(\phi_1, \phi_1) \leftrightarrow (\phi_2, \phi_3)$ causes $\Theta_1 \rightarrow -\Theta_1$, pinning $\Theta_1$ to these two minima implies the spontaneous breakdown of the rung-parity symmetry. As a candidate of the rung-parity order parameter, we can propose a vector chirality $\kappa_{n,j} = S_{n,j} \times S_{n+1,j} \times S_{n+2,j}$ which changes sign via the rung-parity operation. From the formula (2), the $z$ component of a chirality is evaluated as

$$\langle \kappa_{3,j}^z \rangle \approx -B^2 \left( \sin \left( \sqrt{2\pi} \Theta_1 \right) \right) + \cdots = \mp \text{finite},$$

for $\Theta_1^\pm$. Remarkably, the leading term of the chirality does not contain the massless fields ($\Phi_0, \Theta_0$). Similarly, $\langle \kappa_{3,j}^z \rangle$ are shown to be equivalent to $\langle \kappa_{3,j}^z \rangle$. We thus conclude that the vector chiral long-range order exists in the field-induced TLL phase. The chirality correlation function exhibits an exponential decay:

$$\langle \kappa_{3,j}^z \rangle^2 = C_0 e^{-|j|/\sqrt{|J|^2} + \cdots}.$$ Since the scaling dimensions of $\Theta_1$ and $\Theta_2$ are, respectively, evaluated as 2 and 1, the first $\Theta_1$ sector to obtain a gapped gapless and gapped chiral states in single spin tube.

IV. RUNG DEFORMATION

In this section, we discuss the rung deformation. Let us modify the coupling between the first and third chains as

$$J_{L} \vec{S}_{1,j} \cdot \vec{S}_{3,j} \rightarrow J_{L}(1 + \delta) \vec{S}_{1,j} \cdot \vec{S}_{3,j}. \quad (8)$$

This explicitly violates the rung translational symmetry, but conserves the parity symmetry between the first and third chains. Therefore, $\langle \kappa_{3,j}^z \rangle$ is still valid as a rung-parity order parameter. The cases of $\delta = -1$ and $\delta \rightarrow +\infty$, respectively, correspond to a three-leg ladder and a system of a two-leg ladder plus single chain. Note here that if we set $J_{L} = \text{const} \ll J$, the present weak-rung-coupling approach is available even for the case of a large $|\delta| \gg 1$. A finite $\delta$ brings new bosonic terms $\delta J_{L} M \partial_\phi (\phi_1 + \phi_3)/\sqrt{\pi}$, $\delta J_{L} a_0 \partial_\phi \phi_1 \phi_3/\pi$, $\delta J_{L} B^2 a_0 \cos(\sqrt{2\pi} \Theta_1)$, etc. The first term can be absorbed into the Gaussian part via the shift of $\Phi_q$, which does not affect dual fields $\Theta_q$ and yields a small deviation from the uniform magnetization, $\langle S_{3,j}^z \rangle \neq \langle S_{1,j}^z \rangle$. It is not surprising because (as mentioned above) $\delta$ breaks the rung translational symmetry. The second boson bilinear term is expected not to qualitatively influence the low-energy physics. Actually, introducing a new basis different from $(\Phi_q, \Theta_q)$, we can diagonalize all the bilinear terms.
terms. The main effect of the rung modification originates from the third cosine term which varies the form of the potential $V_\delta[\Theta_1, \Theta_2]$ as follows:

$$V[\Theta_1, \Theta_2] \rightarrow V_\delta[\Theta_1, \Theta_2] = 2\cos\left(\sqrt{\frac{\pi}{2}}\Theta_1\right)\cos\left(\sqrt{\frac{3\pi}{2}}\Theta_2\right) + (1+\delta)\cos\left(\sqrt{2\pi}\Theta_1\right).$$

The deformed potential $V_\delta$ is illustrated in Fig. 3. From $V_\delta$, one finds that when $\delta$ increases [decreases and reaches $\delta_{cl} = -0.5$, the two minimum points still survive with keeping $\Theta_1 \neq 0$] [approach with each other and meet at $(\Theta_1, \Theta_2) = (0, \sqrt{2\pi}/3)$]. The chirality $\langle \kappa^2_{3,j} \rangle$ is hence shown to be always finite when an arbitrary positive $\delta$ is applied. In other words, once an infinitesimal exchange is added between a two-leg AF ladder and single chain, a vector chiral order immediately emerges. On the other hand, for the case of $\delta < 0$, the vector chiral order is predicted to vanish at a certain point $\delta = \delta_c$ and the low-energy physics is described by the TLL of the $\Phi_0$ sector for $\delta < \delta_c$. The true value of $\delta_c$ must be renormalized from the classical value $\delta_{cl}$ by the effect of the Gaussian part and must depend on $J_{\perp}/J$ and $H/J$.

In order to further understand the physics near the phase transition at $\delta = \delta_c$, it is necessary to go beyond the semiclassical analysis of $V_\delta$. If $\cos(\sqrt{3\pi}/2\Theta_2)$ in $V_\delta$ is allowed to be replaced with its expectation value, the effective Hamiltonian for the $\Phi_1$ sector is written as the following double sine-Gordon (dSG) model:

$$\mathcal{H}[\Phi_1, \Theta_1] \approx \int dx \left[ K_\parallel (\partial_x \Theta_1)^2 + K_\perp (\partial_x \Phi_1)^2 \right] + \frac{J_1}{a_0} \beta_0^2 \left[ C_1 \cos\left(\sqrt{\frac{\pi}{2}}\Theta_1\right) - (1+\delta)\cos\left(\sqrt{2\pi}\Theta_1\right) \right],$$

where $C_1 = \langle \cos(\sqrt{3\pi}/2\Theta_2) \rangle$. It is believed that for $K_\parallel > 1/4$, the dSG model exhibits a second-order Ising transition by tuning the coupling constants. Indeed, as $\delta$ passes through a critical value $\delta_c$, the potential of the dSG model changes from a double-well type to a single-bottom one. We thus conclude that the transition at $\delta = \delta_c$ is in the Ising universality class. The semiclassical analysis of $V_\delta$ becomes more reliable with $V_\delta$ being more relevant. It is hence expected that the value $\delta_c$ monotonically decreases and approaches $\delta_{cl}$ when $K_\parallel \approx K$ increases—i.e., when $H$ becomes close to the upper critical field. Equation (3.11) and Fig. 12 in Ref. 22 also support this prediction.

Since $\langle \kappa^2_{3,j} \rangle \approx (\sin(\sqrt{2\pi}\Theta_1)) \approx \sqrt{2\pi}\Theta_1$ holds near the transition, the chirality would play the role of the Ising order parameter. Comparing our dSG model and results [Eqs. (29) and (58)] in Ref. 21, we can find

$$\langle \kappa^2_{3,j} \rangle \sim (\delta - \delta_c)^{1/8},$$

near $\delta \rightarrow \delta_c + 0$. [The mean-field analysis of $V_\delta$ leads to $\langle \kappa^2_{3,j} \rangle \sim (\delta - \delta_c)^{1/2}$]. The chirality correlation function behaves as $\langle \kappa^2_{3,j} \kappa^2_{3,0} \rangle \sim C_2/|j|^{1/4}$ just at the critical point. Moreover, near the point, it follows the results in Ref. 22.

Sakai et al. have recently predicted that when $\delta$ is added to the spin tube in zero field, the spin gap rapidly disappears. Combining this prediction and ours, we construct the phase diagram on the $\delta$-$H$ plane as in Fig. 4.

V. CONCLUSIONS AND DISCUSSIONS

We have studied the three-leg AF spin-1/2 tube with a weak rung coupling under a magnetic field making use of Abelian bosonization techniques. In Sec. III we have definitely elucidated without the help of any artificial approximation that the vector chiral long-range order occurs in the magnetic-field-driven one-component TLL phase. Due to the chiral order, the discrete rung-parity symmetry is spontaneously broken, but the $U(1)$ spin-rotational symmetry remains unbroken. We have also been predicted that the chiral order survives even in the intermediate plateau state with $M = 1/6$. It is remarkable that one can obtain both gapless and gapped chiral
states by changing the strength of the magnetic field. In Sec. IV, we have investigated effects of the rung deformation in Eq. (3). It has been shown that when a positive $\delta$ is introduced with fixing $J_\perp \delta$ the chiral order and the TLL always continue, whereas when $\delta$ is decreased the chiral order vanishes at a certain Ising critical point $\delta = \delta_c < 0$ and then a standard TLL phase appears (see Fig. 4). This implies that vector chirality correlations are favored in two-leg spin ladders as well as in three-leg tubes.

Quite recently, based on the spin-wave picture, we have predicted that the same chiral order still survives a certain regime in the vicinity of saturation of the spin tube with $\delta = 0$. Therefore, it is inferred that the tube possesses the chiral order in a quite wide range of the magnetic field. One immediately finds that the classical three-leg spin tube always has a vector chiral order from the zero field to the upper critical one. We thus may say that the classical nature is strong in the weak-rung-coupling area of the quantum spin tube. Actually, as well known, the AF spin-$1/2$ chain, which is the starting point of our analysis, has a large instability toward a classical Néel ordering.

The discussion in this paper tells us that even without magnetic fields, a vector chiral order could also occur if a spin tube consists of three spin chains with a large $K(> 1/2)$: for instance, the condition $K > 1/2$ is satisfied in a chain with easy-plane $XXZ$ anisotropy. It is predicted that chiral orders exist in a certain parameter regime of easy-plane zigzag spin chains. Our results also suggest that a field-induced vector chiral order can appear in other frustrated odd-leg spin tubes—i.e., five-, seven-, nine-leg tubes, etc., tubes.

Finally, we briefly mention the possibility of experimentally detecting the vector chirality. The chiral order predicted in this paper must be destroyed by an effect of thermal fluctuation. However, chirality correlations are expected to still be strong if the temperature is sufficiently low. Moreover, weak three-dimensional interactions among tubes can stabilize the long-range chiral order: the gapless chiral order in the tube is expected to change into a conventional umbrella spin structure due to the interactions. Four-point spin correlation functions include features of the above chiral order or strong chiral correlation. Such functions, in principle, are measured in polarized neutron scattering, electromagnetic-wave resonance, etc.

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1. K. Kawano and M. Takahashi, J. Phys. Soc. Jpn. 66, 4001 (1997).
2. H. J. Schulz, cond-mat/9605075.
3. D. C. Cabra, A. Honecker, and P. Pujol, Phys. Rev. Lett. 79, 5126 (1997); Phys. Rev. B 58, 6241 (1998).
4. R. Citro, E. Orignac, N. Andrei, C. Itoi, and S. Qin, J. Phys.: Condens. Matter 12, 3041 (2000).
5. P. Millet, J. Y. Henry, F. Mila and J. Gal, J. Solid State Chem. 147, 676 (1999).
6. J. Schnack, H. Nojiri, P. Kögerler, G. J. T. Cooper and L. Cronin, Phys. Rev. B 70, 174420 (2004).
7. A. Lüscher, R. M. Noack, G. Misguich, V. N. Kotov and F. Mila, Phys. Rev. B 70, 060405(R) (2004).
8. J.-B. Fouet, A. Läuchli, S. Pilgram, R. M. Noack and F. Mila, Phys. Rev. B 73, 014409 (2006).
9. K. Okunishi, S. Yoshikawa, T. Sakai, and S. Miyashita, Prog. Theor. Phys. Suppl. 159, 297 (2005).
10. M. Sato, Phys. Rev. B 72, 104438 (2005).
11. M. Sato and M. Oshikawa, Phys. Rev. B 75, 014404 (2007).
12. For example see A. O. Gogolin, A. A. Nersesyan and A. M. Tsvelik, Bosonization and Strongly Correlated Systems (Cambridge University Press, Cambridge, England, 1998); T. Giamarchi, Quantum Physics in One Dimension (Oxford University Press, New York, 2004).
13. G. Sierra, J. Phys. A 29, 5299 (1996); Lect. Notes Phys. 478, eds. G. Sierra and M. A.Martín-Delgado (Springer-Verlag, 1997); see also cond-mat/9610057.
14. A. G. Rojo, Phys. Rev. B 53, 9172 (1996).
15. S. Dell’Arlinga, E. Ercolessi, G. Morandi, P. Pieri, and M. Roncaglia, Phys. Rev. Lett. 78, 2457 (1997).
16. In this paper, we discuss a vector chiral order with spontaneously breaking the rung-parity symmetry in a spin tube. Note, however, that there is another mechanism of a vector chiral order induced by a rung-parity-breaking three-spin chiral term which can emerge via a virtual process on an electron tube system under a magnetic field. Of course, a spin tube Hamiltonian with such chiral terms does not possess the rung-parity symmetry, and the resulting finite expectation value of vector chirality does not mean any spontaneous symmetry breaking. For more detailed discussions, see D. Sen and R. Chitra, Phys. Rev. B 51, 1922 (1995); R. Chitra and R. Citro, Phys. Rev. B 63, 054441 (2001).
17. S. Lukyanov and A. Zamaldelchikov, Nucl. Phys. B 493, 571 (1997).
18. T. Hikihara and A. Furusaki, Phys. Rev. B 69, 064427 (2004).
19. M. Oshikawa, M. Yamanaka, and I. Affleck, Phys. Rev. Lett. 78, 1984 (1997).
20. For example see S. Miyashita and H. Shiba, J. Phys. Soc. Jpn. 53, 1145 (1984); S. Lee and K. C. Lee, Phys. Rev. B 57, 8472 (1998); L. Capriotti, R. Vaia, A. Cuocci and V. Tognetti, Phys. Rev. B 58, 273 (1998).
21. M. Fabrizio, A. O. Gogolin and A. A. Nersesyan, Nucl. Phys. B 580, 647 (2000).
22. Z. Bajnok, L. Palla, A. Cuccoli and V. Tognetti, Phys. Rev. B 58, 273 (1998).
23. T. T. Wu, B. M. McCoy, C. A. Tracy, and E. Barouch,
Phys. Rev. B 13, 316 (1976).

24 T. Sakai, M. Matsumoto, K. Okunishi, K. Okamoto, and M. Sato, Physica E 29, 633 (2005).

25 M. Sato, cond-mat/0703396 (unpublished).

26 M. Sato and T. Sakai, Phys. Rev. B 75, 014411 (2007).

27 A. A. Nersesyan, A. O. Gogolin and F. H. L. Eßler, Phys. Rev. Lett. 81 910 (1998); M. Kaburagi, H. Kawamura and T. Hikihara, J. Phys. Soc. Jpn. 68, 3185 (1999); T. Hikihara, M. Kaburagi, and H. Kawamura, Phys. Rev. B 63, 174430 (2001); A. K. Kolezhuk, Phys. Rev. B 62, R6057 (2000); P. Lecheminant, T. Jolicoeur, and P. Azaria, Phys. Rev. B 63, 174426 (2001).