Production of charged spin-two gauge bosons
in gluon-gluon scattering

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Abstract

We are considering the production of charged spin-two gauge bosons in the gluon-gluon scattering and calculating polarized cross sections for each set of helicity orientations of initial and final particles. The angular dependence of this cross section is being compared with the gluon-gluon scattering cross section in QCD.
1 Introduction

An infinite tower of massive particles of high spin naturally appears in the spectrum of different string field theories. It is generally expected that in the tensionless limit or, what is equivalent, at high energy and fixed angle scattering the string spectrum becomes effectively massless [1, 2, 3, 4, 5, 6]. In the open string theory with Chan-Paton charges these massless states can combine into the infinite tower of non-Abelian tensor gauge fields [7] and one could guess that the corresponding Lagrangian quantum field theory should be described by some kind of extension of the Yang-Mills theory.

A possible extension of Yang-Mills theory which includes non-Abelian tensor gauge fields was suggested recently in [10, 11, 12]. The non-Abelian gauge fields are defined as rank-(s+1) tensor gauge fields $A^a_{\mu_1...\mu s}$. The gauge invariant Lagrangian describing tensor gauge bosons of all ranks has the form [10, 11, 12]

$$ L = L_1 + L_2 + g_3 L_3 + ..., \tag{1} $$

where $L_1$ is the Yang-Mills Lagrangian and $L_s (s = 2, 3, ..)$ are Lagrangian forms invariant with respect to the extended gauge transformations [10, 11, 12]. The Lagrangian $L$ defines cubic and quartic self-interactions of charged gauge quanta carrying spin larger than one. For the lower-rank tensor gauge fields the Lagrangian has the following form [10, 11, 12]:

$$ L_1 = - \frac{1}{4} G^a_{\mu \nu} G^a_{\mu \nu} , $$

$$ L_2 = - \frac{1}{4} G^a_{\mu \nu \lambda} G^a_{\mu \nu \lambda} - \frac{1}{4} G^a_{\mu \nu \lambda} G^a_{\mu \nu \lambda} + \frac{1}{4} G^a_{\mu \nu \lambda} G^a_{\mu \nu \lambda} + \frac{1}{2} G^a_{\mu \nu} G^a_{\mu \nu}, \tag{2} $$

where the generalized field strength tensors are:

$$ G^a_{\mu \nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu, $$

$$ G^a_{\mu \nu \lambda} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} ( A^b_\mu A^c_\nu + A^b_\nu A^c_\mu ), $$

$$ G^a_{\mu \nu \lambda \rho} = \partial_\mu A^a_{\nu \lambda \rho} - \partial_\nu A^a_{\mu \lambda \rho} + g f^{abc} ( A^b_\mu A^c_{\nu \lambda \rho} + A^b_{\nu \lambda \rho} A^c_\mu + A^b_\mu A^c_{\nu \lambda} + A^b_\nu A^c_{\mu \lambda} ). \tag{3} $$

*Tensor gauge fields $A^a_{\mu_1...\mu s}(x)$, $s = 0, 1, 2, ...$, are totally symmetric with respect to the indices $\lambda_1...\lambda_s$. A priori the tensor fields have no symmetries with respect to the first index $\mu$. In particular, we have $A^a_{\mu \lambda} \neq A^a_{\lambda \mu}$ and $A^a_{\mu \lambda \rho} = A^a_{\mu \rho \lambda} \neq A^a_{\lambda \mu \rho}$. The adjoint group index $a = 1, ..., N^2 - 1$ in the case of $SU(N)$ gauge group.*
The definition of the Lagrangian forms $L_s$ for higher-rank fields can be found in the previous publications [10, 11, 12]. The above expressions define interacting gauge field theory with infinite many gauge fields. Not much is known about physical properties of such gauge field theories and in the present paper we shall focus our attention on the lower-rank tensor gauge field $A^\alpha_{\mu\lambda}$, which describes in this theory charged gauge bosons of spin two. We are interested in studying the first nontrivial interaction processes. In particular, we shall consider production of charged spin-two gauge bosons by a pair of vector gauge bosons.

Our intention in this article is to calculate the leading-order differential cross section of spin-two tensor gauge boson production by a pair of vector gauge bosons in the process $V + V \rightarrow T + T$ and to analyze the angular dependence of the polarized cross sections for each set of helicity orientations of initial and final particles. The process is illustrated in Fig.4 and receives contribution from four Feynman diagrams shown in Fig.5-Fig.8.

Below we shall present the Feynman diagrams for the given process, the expressions for the corresponding vertices and transition amplitudes. Then we shall calculate the polarized cross sections for each set of helicity orientations of the initial and final particles (see formulas (33), (34), (35) and (37)) and shall compare them with the corresponding cross section of the vector gauge bosons $V + V \rightarrow V + V$ in Yang-Mills theory (see formulas (46), (47), (48) and (38)). In Appendix A we are reviewing the well known result for the three-level scattering $V + V \rightarrow V + V$ [9] and in Appendix B we shall demonstrate the gauge invariance of the transition amplitude.

2 Summary of Feynman rules

The Feynman rules for the Lagrangian (1) can be derived from the functional integral over the gauge boson fields $A^a_\alpha$, $A^a_{\alpha\alpha'}$, ... [10, 11, 12]. The indices of the symmetry group $G$ are $a, b = 1, ..., d(G)$, where $d(G)$ is the number of generators of the group $G$. The standard vector propagator is given by the expression

$$D^{\alpha\beta}_{ab}(k) = \frac{i}{k^2} \eta^{\alpha\beta} \delta_{ab}. \quad (4)$$

The second-rank tensor gauge field $A_{a\alpha}$ with 16 components describes in this theory three physical transversal polarizations [10, 11, 12]. The kinetic operator of the tensor gauge bosons

$$H_{a\alpha\gamma\gamma}(k) = (-\eta_{\alpha\gamma} \eta_{\alpha\gamma} + \frac{1}{2} \eta_{\alpha\gamma} \eta_{\gamma\gamma} + \frac{1}{2} \eta_{\alpha\gamma} \eta_{\gamma\gamma}) k^2 + \eta_{\alpha\gamma} k_{\alpha} k_{\gamma} + \eta_{\alpha\gamma} k_{\alpha} k_{\gamma}.$$
where the residue can be represented as a sum of \( \lambda \)

\[
\lambda = \lambda_\mu^\nu(k)
\]

Figure 1: The vector - \( D_{\mu\nu}^\alpha(k) \) and tensor - \( \Delta_{\alpha'\beta'\gamma'\lambda'\lambda}(k) \) gauge boson propagators are conventionally drawn as thin and thick wave lines.

\[
- \frac{1}{2}(\eta_{\alpha'\gamma'}\kappa_\alpha\kappa_\gamma + \eta_{\alpha\gamma'}\kappa_\alpha\kappa_\gamma + \eta_{\alpha\lambda'}\kappa_\alpha\kappa_\lambda + \eta_{\gamma'\lambda'}\kappa_\gamma\kappa_\lambda)
\]  

(5)

is a gauge invariant operator \( k_\alpha H_{\alpha\gamma'\gamma} = 0, \ k_\lambda H_{\alpha\gamma'\gamma} = 0 \). It describes the propagation of massless particles with helicities two and zero because the equation \( H_{\alpha\gamma'\gamma}(k) f^{\gamma'\gamma}(k) = 0 \) has three independent solutions of the helicity two and zero. The propagator \( \Delta_{\alpha'\beta'\gamma'}^\alpha(k) \) is defined through the equation \( H_{\alpha'\beta'\gamma'}^f(k) \Delta_{\alpha'\beta'\gamma'}^\gamma(k) = i\eta_{\alpha'\beta'}\eta_{\lambda'\lambda} \) and has the following form:

\[
\Delta_{\alpha'\beta'\gamma'}^\alpha(k) = -\frac{i}{k^2} \Pi_{\alpha'\beta'\gamma'} \delta^{\alpha\gamma'}
\]

(6)

where the residue can be represented as a sum of \( \lambda = \pm 2 \) and \( \lambda = 0 \) helicity states:

\[
\Pi_{\alpha'\beta'\gamma'} = (\eta_{\alpha'\beta'}\eta_{\alpha'\gamma'} + \eta_{\alpha'\gamma'}\eta_{\alpha'\beta'} - \eta_{\alpha'\alpha'}\eta_{\beta'\gamma'}) + \frac{1}{3}(\eta_{\alpha'\beta'}\eta_{\alpha'\beta'} - \eta_{\alpha'\beta'}\eta_{\alpha'\beta'}). 
\]

(7)

The standard Yang-Mills three-vector boson interaction vertex \( VVV \) in the momentum representation has the form

\[
\gamma_{\alpha'\beta'\gamma'}^{abc}(k,p,q) = -gf_{abc} F_{\alpha'\beta'\gamma'}(k,p,q) = -gf_{abc} [\eta_{\alpha\beta}(p - k)_\gamma + \eta_{\alpha\gamma}(k - q)_\beta + \eta_{\beta\gamma}(q - p)_\alpha]. 
\]

(8)

The interaction vertex of vector gauge boson \( V \) with two tensor gauge bosons \( T \) - the VTT vertex - has the form\(^1\)

\[
\gamma_{\alpha\beta\gamma'\gamma'}^{abc}(k,p,q) = -gf_{abc} F_{\alpha\beta\gamma'\gamma'}, 
\]

(9)

where

\[
F_{\alpha\beta\gamma'\gamma'}(k,p,q) = [\eta_{\alpha\beta}(p - k)_\gamma + \eta_{\alpha\gamma}(k - q)_\beta + \eta_{\beta\gamma}(q - p)_\alpha] \eta_{\gamma'\gamma'} - \frac{1}{2} [ (p - k)_\gamma(\eta_{\alpha'\beta'}\eta_{\alpha\delta} + \eta_{\alpha\delta}\eta_{\beta'\gamma'}) + (k - q)_\beta(\eta_{\alpha'\gamma'}\eta_{\alpha\delta} + \eta_{\alpha\delta}\eta_{\gamma'\gamma'}) \\
+ (q - p)_\alpha(\eta_{\alpha'\beta'}\eta_{\beta'\gamma'} + \eta_{\beta'\beta'}\eta_{\gamma'\gamma'}) + (p - k)_\alpha\eta_{\alpha'\beta'}\eta_{\gamma'\gamma'} + (p - k)_\gamma\eta_{\beta'\beta'}\eta_{\gamma'\gamma'} + (q - p)_\gamma\eta_{\alpha'\beta'}\eta_{\gamma'\gamma'}. 
\]

(10)

\(^1\)See formulas (62), (65) and (66) in \([12]\).
Figure 2: The interaction vertex for vector gauge boson V and two tensor gauge bosons T the VTT vertex - $\mathcal{V}_{abc\gamma\delta}(k, p, q)$ in non-Abelian tensor gauge field theory [12]. Vector gauge bosons are conventionally drawn as thin wave lines, tensor gauge bosons are thick wave lines. The Lorentz indices $\alpha\gamma$ and momentum $k$ belong to the first tensor gauge boson, the $\gamma\delta$ and momentum $q$ belong to the second tensor gauge boson, and Lorentz index $\beta$ and momentum $p$ belong to the vector gauge boson.

The Lorentz indices $\alpha\gamma$ and momentum $k$ belong to the first tensor gauge boson, the $\gamma\delta$ and momentum $q$ belong to the second tensor gauge boson, and Lorentz index $\beta$ and momentum $p$ belong to the vector gauge boson. The vertex is shown in Fig.2. Vector gauge bosons are conventionally drawn as thin wave lines, tensor gauge bosons are thick wave lines.

In the Lagrangian (2) we have the standard four vector boson interaction vertex VVVV

$$\mathcal{V}_{\alpha\beta\gamma\delta}(k, p, q, r) = -ig^2 f^{lac} f^{lbd}(\eta_{\alpha\beta}\eta_{\gamma\delta} - \eta_{\alpha\delta}\eta_{\beta\gamma})$$

$$-ig^2 f^{lad} f^{lbe}(\eta_{\alpha\beta}\eta_{\gamma\delta} - \eta_{\alpha\gamma}\eta_{\beta\delta})$$

$$-ig^2 f^{lab} f^{lce}(\eta_{\alpha\gamma}\eta_{\beta\delta} - \eta_{\alpha\delta}\eta_{\beta\gamma})$$

and a new interaction of two vector and two tensor gauge bosons - the VVTT vertex,

$$\mathcal{V}_{\alpha\beta\gamma\delta\epsilon}(k, p, q, r) = -ig^2 f^{lac} f^{lbd}(\eta_{\alpha\beta}\eta_{\gamma\delta} - \eta_{\alpha\delta}\eta_{\beta\gamma})$$

$$-ig^2 f^{lad} f^{lbe}(\eta_{\alpha\beta}\eta_{\gamma\delta} - \eta_{\alpha\gamma}\eta_{\beta\delta})$$

$$-ig^2 f^{lab} f^{lce}(\eta_{\alpha\gamma}\eta_{\beta\delta} - \eta_{\alpha\delta}\eta_{\beta\gamma})$$

$$+ig^2 f^{lac} f^{lbd}[\eta_{\alpha\beta}(\eta_{\gamma\delta}\eta_{\gamma\delta} + \eta_{\gamma\gamma}\eta_{\delta\delta})$$

$$-\eta_{\beta\gamma}(\eta_{\alpha\delta}\eta_{\gamma\delta} + \eta_{\alpha\delta}\eta_{\beta\delta})$$

$$-\eta_{\alpha\delta}(\eta_{\beta\gamma}\eta_{\gamma\delta} + \eta_{\beta\delta}\eta_{\gamma\gamma})$$

$$+\eta_{\gamma\delta}(\eta_{\alpha\beta}\eta_{\gamma\delta} + \eta_{\alpha\beta}\eta_{\delta\delta})]$$

4
Figure 3: The quartic vertex with two vector gauge bosons and two tensor gauge bosons - the VVTT vertex \( \gamma_{\alpha\beta\gamma\delta}^{\mu\nu}(k, p, q, r) \) in non-Abelian tensor gauge field theory [12]. Vector gauge bosons are conventionally drawn as thin wave lines, tensor gauge bosons are thick wave lines. The Lorentz indices \( \gamma \) and momentum \( q \) belong to the first tensor gauge boson, \( \delta \) and momentum \( r \) belong to the second tensor gauge boson, the index \( \alpha \) and momentum \( k \) belong to the first vector gauge boson and Lorentz index \( \beta \) and momentum \( p \) belong to the second vector gauge boson.

\[
+\frac{i}{2} g^2 f^{\mu\nu\rho\sigma} f^{\hi\j\k\l} [ \eta_{\alpha\beta}(\eta_{\gamma\delta}\eta_{\gamma\delta} + \eta_{\gamma\delta}\eta_{\gamma\delta}) \\
- \eta_{\alpha\gamma}(\eta_{\beta\delta}\eta_{\gamma\delta} + \eta_{\beta\delta}\eta_{\gamma\delta}) \\
- \eta_{\beta\delta}(\eta_{\alpha\gamma}\eta_{\gamma\delta} + \eta_{\alpha\gamma}\eta_{\gamma\delta}) \\
+ \eta_{\gamma\delta}(\eta_{\alpha\gamma}\eta_{\beta\delta} + \eta_{\alpha\gamma}\eta_{\beta\delta}) ] \\
+\frac{i}{2} g^2 f^{\mu\nu\rho\sigma} f^{\hi\j\k\l} [ \eta_{\alpha\gamma}(\eta_{\beta\delta}\eta_{\gamma\delta} + \eta_{\beta\delta}\eta_{\gamma\delta}) \\
- \eta_{\beta\gamma}(\eta_{\alpha\delta}\eta_{\gamma\delta} + \eta_{\alpha\delta}\eta_{\gamma\delta}) \\
- \eta_{\alpha\delta}(\eta_{\beta\gamma}\eta_{\gamma\delta} + \eta_{\beta\gamma}\eta_{\gamma\delta}) \\
+ \eta_{\gamma\delta}(\eta_{\alpha\delta}\eta_{\beta\gamma} + \eta_{\alpha\delta}\eta_{\beta\gamma}) ], \tag{12}
\]

In summary, we have the Yang-Mills vertex VVV (8), the new vertex VTT (9) together with the Yang-Mills vertex VVVV (11) and the new vertex VVTT (12) (see Fig. 2-3).

3 Cross section and Matrix Elements

The scattering process is illustrated in Fig. 1. Working in the center-of-mass frame, we make the following assignments: \( k_1^\mu = E(1, 0, 0, 1) \), \( k_2^\mu = E(1, 0, 0, -1) \), and \( q_1^\mu = E(1, \sin\theta, 0, \cos\theta) \), \( q_2^\mu = E(1, -\sin\theta, 0, -\cos\theta) \), where \( k_{1,2} \) are momenta of the vector bosons \( V + V \) and \( q_{1,2} \) momenta of the tensor gauge bosons \( T + T \). All particles are massless \( k_1^2 = k_2^2 = q_1^2 = q_2^2 = 0 \). In the center-of-mass frame the momenta satisfy the relations...
The scattering $V + V \rightarrow T + T$, shown in the center-of-mass frame. The $k_1, k_2$ are momenta of the vector gauge bosons $V$ and $q_1, q_2$ are momenta of the tensor gauge bosons $T$.

$\vec{k}_1 = -\vec{k}_2, \quad \vec{q}_2 = -\vec{q}_1$. The invariant variables of the process are:

$$s = 2(k_1 \cdot k_2), \quad t = -\frac{s}{2}(1 - \cos \theta), \quad u = -\frac{s}{2}(1 + \cos \theta),$$

where $s = (2E)^2$ and $\theta$ is the scattering angle. It is convenient to write the differential cross section in the center-of-mass frame with tensor boson produced into the solid angle $d\Omega$ as

$$d\sigma = \frac{1}{2s} |M|^2 \frac{1}{32\pi^2} d\Omega,$$

where the final-state density is $d\Phi = \frac{1}{32\pi^2} d\Omega$.

We shall calculate the polarized cross sections for the reaction $V + V \rightarrow T + T$, to the lowest order in $\alpha = g^2/4\pi$. The lowest-order Feynman diagrams contributing to the annihilation process of a pair of vector bosons into a pair of tensor gauge bosons are shown in Fig.5 Fig.8. In order $g^2$, there are five diagrams. Vector gauge bosons $V$ are conventionally drawn as thin wave lines and tensor gauge bosons $T$ as a thick wave lines.

The probability amplitude of the process can be written as a sum of four terms corresponding to each diagram. For the first diagram Fig.5 we shall get

$$iM_{I}^{ab,cd} =$$

$$\nu^{\rho\sigma\rho'}_{\lambda\lambda'}(-q_2, k_1, -p) \Delta_{\sigma\rho'}^{\mu\nu}(p, k_2, -q_2) c_{k_1}^{\mu}, c_{k_2}^{\nu}, e_{q_1}^{s\lambda\lambda'}, e_{q_2}^{s\rho\rho'},$$

for the second diagram Fig.6 the amplitude is

$$iM_{II}^{ab,cd} =$$

$$\nu^{\rho\sigma\rho'}_{\lambda\lambda'}(-q_2, k_1, -p') \Delta_{\sigma\rho'}^{\mu\nu}(p', k_2, -q_1) c_{k_1}^{\mu}, c_{k_2}^{\nu}, e_{q_1}^{s\lambda\lambda'}, e_{q_2}^{s\rho\rho'},$$
Figure 5: The t-channel diagram corresponding to the creation of tensor gauge bosons by vector bosons $V + V \rightarrow T + T$.

For the third diagram Fig. 7 it is

\[ iM_{III}^{ab,cd} = \gamma_{\mu\nu\rho\rho'}(k_1, k_2, -p'') D_{\sigma\ell}(p'') \mathcal{V}_{\lambda\lambda'\rho\rho'}(-q_1, p'', -q_2) e^\mu_{k_1} e^\nu_{k_2} e^{*\lambda\lambda'}_{q_1} e^{*\rho\rho'}_{q_2}, \]  

and finally for the fourth diagram Fig. 8 we have

\[ iM_{IV}^{ab,cd} = \gamma_{\mu
u\rho\rho'}(k_1, k_2, -q_2, -q_1) e^\mu_{k_1} e^\nu_{k_2} e^{*\lambda\lambda'}_{q_1} e^{*\rho\rho'}_{q_2}, \]  

where $e^\mu_{k_1}$ is the wave function of the first vector boson and $e^\nu_{k_2}$ - of the second. The final tensor gauge bosons wave functions are $e^{*\lambda\lambda'}_{q_1}$ and $e^{*\rho\rho'}_{q_2}$.

The total amplitude is a sum of four terms:

\[ iM = iM_I + iM_{II} + iM_{III} + iM_{IV}. \]  

Here we have been considering only the first nontrivial diagrams Fig. 5 to Fig. 8 but because the Lagrangian (1) contains also high-rank tensor gauge fields of increasing order one should include their contribution into the total amplitude as well. In the actual calculations we shall use the tensor propagator (6) with

\[ \Pi_{\alpha\alpha'\beta\beta'} = 2(\eta_{\alpha\beta}\eta_{\alpha'\beta'} + \eta_{\alpha\beta'}\eta_{\alpha'\beta} - \eta_{\alpha\alpha'}\eta_{\beta\beta'}) - \frac{2}{9}(\eta_{\alpha\beta}\eta_{\alpha'\beta'} - \eta_{\alpha\beta'}\eta_{\alpha'\beta}), \]

which, as it appears, sums the diagrams with high-rank tensor fields in the intermediate states. The total amplitude is gauge invariant, that is, if we take longitudinal wave

\[ \text{The details will be given elsewhere.} \]
function for gauge bosons, then the transition amplitude vanishes (see Appendix B for details).

Our intention is to calculate the physical matrix elements in the helicity basis for initial vector and final tensor gauge bosons. This calculation of polarized cross sections is very similar to the gluon-gluon scattering in QCD [8, 9]. The right- and left-handed vector wave functions are:

\[
e_R(k_1)^\mu = \frac{1}{\sqrt{2}} (0, 1, i, 0), \quad e_L(k_1)^\mu = \frac{1}{\sqrt{2}} (0, 1, -i, 0)
\]

\[
e_R(k_2)^\mu = e_L(k_1)^\mu, \quad e_L(k_2)^\mu = e_R(k_1)^\mu,
\] (20)

where \(k_1^\mu = (E, 0, 0, 1)\), \(k_2^\mu = (E, 0, 0, -1)\) and the tensor gauge boson wave functions for circular polarizations along the \(\vec{q}_1\) direction are

\[
e_R^{\mu\alpha}(q_1) = \frac{1}{2} (0, \cos \theta, i, -\sin \theta) \otimes (0, \cos \theta, i, \sin \theta),
\]

\[
e_L^{\mu\alpha}(q_1) = \frac{1}{2} (0, -\cos \theta, i, \sin \theta) \otimes (0, -\cos \theta, i, \sin \theta)
\] (21)

It is easy to check that the wave functions (21) are orthonormal \(e_R^{*\mu\alpha}(q_1)e_L(q_1)_{\alpha\nu} = 0\), \(e_R^{*\mu\alpha}(q_1)e_R(q_1)_{\mu\alpha} = 1\), \(e_L^{*\mu\alpha}(q_1)e_L(q_1)_{\mu\alpha} = 1\) and fulfil the equations

\[
q_1^\mu e_{\mu\alpha}(q_1) = q_1^\alpha e_{\mu\alpha}(q_1) = 0, \quad q_2^\mu e_{\mu\alpha}(q_2) = q_2^\alpha e_{\mu\alpha}(q_2) = 0,
\] (22)

The helicity states for the second tensor gauge boson are \(e_R^{\mu\nu}(q_2) = e_L^{\mu\nu}(q_1)\), \(e_L^{\mu\nu}(q_2) = e_R^{\mu\nu}(q_1)\), where \(q_1^\mu = (E, E \sin \theta, 0, E \cos \theta)\) and \(q_2^\mu = (E, -E \sin \theta, 0, -E \cos \theta)\).
Figure 7: The s-channel diagram for the process $V + V \rightarrow T + T$.

4 Helicity Amplitudes

Now we can calculate all sixteen matrix elements between states of definite helicities. The scattering amplitude (18) for any particular choice of helicities contains four terms. By plugging explicit expressions for propagators (4), (6), (19), vertices (8), (11), (9), (12) and helicity wave functions (20), (21) into the matrix elements (14), (15), (16) and (17) we can find their explicit form. For the t-channel amplitude (14), corresponding to the diagram Fig.5, we shall get the following sequence of sixteen polarization amplitudes

$$i \mathcal{M}_I(LL \rightarrow LL) = i \mathcal{M}_I(RR \rightarrow RR) = -g^2 \frac{4}{3} f_{ace} f_{bde} (1 + \cos \theta)(2 - \cos \theta)$$

$$i \mathcal{M}_I(LL \rightarrow LR) = i \mathcal{M}_I(RR \rightarrow LR) = 0$$

$$i \mathcal{M}_I(LL \rightarrow RL) = i \mathcal{M}_I(RR \rightarrow RL) = 0$$

$$i \mathcal{M}_I(LL \rightarrow RR) = i \mathcal{M}_I(RR \rightarrow LL) = -g^2 \frac{4}{3} f_{ace} f_{bde} (7 - 2 \cos \theta + \cos 2\theta)$$

For the u-channel diagram Fig.6 the amplitude (15) gives

$$i \mathcal{M}_{II}(LL \rightarrow LL) = i \mathcal{M}_{II}(RR \rightarrow RR) = -g^2 \frac{4}{3} f_{ace} f_{bde} (1 - \cos \theta)(2 + \cos \theta)$$

$$i \mathcal{M}_{II}(LL \rightarrow LR) = i \mathcal{M}_{II}(RR \rightarrow LR) = 0$$

$$i \mathcal{M}_{II}(LL \rightarrow RL) = i \mathcal{M}_{II}(RR \rightarrow RL) = 0$$

$$i \mathcal{M}_{II}(LL \rightarrow RR) = i \mathcal{M}_{II}(RR \rightarrow LL) = -g^2 \frac{4}{3} f_{ade} f_{bce} (7 + 2 \cos \theta - \cos 2\theta)$$

$$i \mathcal{M}_{II}(LR \rightarrow LL) = i \mathcal{M}_{II}(LR \rightarrow RR) = -g^2 \frac{4}{3} f_{ade} f_{bce} (1 - \cos \theta)(3 + 2 \cos \theta)$$

$$i \mathcal{M}_{II}(LR \rightarrow LR) = 0, \quad i \mathcal{M}_{II}(RL \rightarrow RL) = 0.$$

(23)
Figure 8: The contact diagram for the process $V + V \rightarrow T + T$.

\[ i \mathcal{M}_{II}(LR \rightarrow LR) = 0, \quad i \mathcal{M}_{II}(LR \rightarrow RL) = 0 \]
\[ i \mathcal{M}_{II}(RL \rightarrow LL) = i \mathcal{M}_{II}(RL \rightarrow RR) = -g^2 \frac{f_{ade} f_{bce}}{2} (1 - \cos \theta)(3 + 2 \cos \theta) \]
\[ i \mathcal{M}_{II}(RL \rightarrow LR) = 0, \quad i \mathcal{M}_{II}(RL \rightarrow RL) = 0. \quad (24) \]

For the s-channel diagram Fig.7 the polarization amplitudes (16) are

\[ i \mathcal{M}_{III}(LL \rightarrow LL) = i \mathcal{M}_{III}(RR \rightarrow RR) = -g^2 \frac{i}{2} (f_{ace} f_{bde} - f_{ade} f_{bce}) \cos \theta \]
\[ i \mathcal{M}_{III}(LL \rightarrow LR) = i \mathcal{M}_{III}(RR \rightarrow LR) = 0 \]
\[ i \mathcal{M}_{III}(LL \rightarrow RL) = i \mathcal{M}_{III}(RR \rightarrow RL) = 0 \]
\[ i \mathcal{M}_{III}(LL \rightarrow RR) = i \mathcal{M}_{III}(RR \rightarrow LL) = -g^2 \frac{i}{2} (f_{ace} f_{bde} - f_{ade} f_{bce}) \cos \theta \]
\[ i \mathcal{M}_{III}(LR \rightarrow LL) = i \mathcal{M}_{III}(LR \rightarrow RR) = 0 \]
\[ i \mathcal{M}_{III}(LR \rightarrow LR) = 0, \quad i \mathcal{M}_{III}(LR \rightarrow RL) = 0 \]
\[ i \mathcal{M}_{III}(RL \rightarrow LL) = i \mathcal{M}_{III}(RL \rightarrow RR) = 0 \]
\[ i \mathcal{M}_{III}(RL \rightarrow LR) = 0, \quad i \mathcal{M}_{III}(RL \rightarrow RL) = 0. \quad (25) \]

And finally for the contact diagram Fig.8 the polarization amplitudes (17) are

\[ i \mathcal{M}_{IV}(LL \rightarrow LL) = i \mathcal{M}_{IV}(RR \rightarrow RR) = g^2 \frac{i}{4} (f_{ace} f_{bde} + f_{ade} f_{bce}) \sin^2 \theta \]
\[ i \mathcal{M}_{IV}(LL \rightarrow LR) = i \mathcal{M}_{IV}(RR \rightarrow LR) = 0 \]
\[ i \mathcal{M}_{IV}(LL \rightarrow RL) = i \mathcal{M}_{IV}(RR \rightarrow RL) = 0 \]
\[ i \mathcal{M}_{IV}(LL \rightarrow RR) = i \mathcal{M}_{IV}(RR \rightarrow LL) = g^2 \frac{i}{4} (f_{ace} f_{bde} + f_{ade} f_{bce}) \sin^2 \theta \]
\[ i \mathcal{M}_{IV}(LR \rightarrow LL) = i \mathcal{M}_{IV}(LR \rightarrow RR) = g^2 \frac{i}{4} (f_{ace} f_{bde} + f_{ade} f_{bce}) \sin^2 \theta \]
\[ i \mathcal{M}_{IV}(LR \rightarrow LR) = 0, \quad i \mathcal{M}_{IV}(LR \rightarrow RL) = 0 \]
\[ i \mathcal{M}_{IV}(RL \rightarrow LL) = i \mathcal{M}_{IV}(RL \rightarrow RR) = g^2 \frac{i}{4} (f_{ace} f_{bde} + f_{ade} f_{bce}) \sin^2 \theta \]
\[ i \mathcal{M}_{IV}(RL \rightarrow LR) = 0, \quad i \mathcal{M}_{IV}(RL \rightarrow RL) = 0. \quad (26) \]
Thus only eight amplitudes out of sixteen are nonzero: \( V_L V_L \rightarrow T_L T_L, \ V_R V_R \rightarrow T_R T_R, \ V_L V_L \rightarrow T_R T_R, \ V_R V_R \rightarrow T_L T_L, \ V_L V_R \rightarrow T_R T_R, \ V_R V_L \rightarrow T_L T_L, \ V_R V_L \rightarrow T_R T_R. \) We can get total helicity amplitudes (18) summing corresponding terms from each diagram:

\[
iM_{LL-LL} = -\frac{ig^2}{4} \left[ f^{ace} f^{bde} \left( 4 + 5 \cos \theta + \cos 2\theta \right) + f^{ade} f^{bce} \left( 4 - 5 \cos \theta + \cos 2\theta \right) \right], \tag{27}
\]

\[
iM_{LL-RR} = -\frac{ig^2}{4} \left[ f^{ace} f^{bde} \left( 3 + \cos \theta \right) + f^{ade} f^{bce} \left( 3 - \cos \theta \right) \right], \tag{28}
\]

\[
iM_{LR-LL} = -\frac{ig^2}{4} \left[ +(1 + \cos \theta)(2 - \cos \theta)f^{ace} f^{bde} + +(1 - \cos \theta)(2 + \cos \theta)f^{ade} f^{bce} \right]. \tag{29}
\]

To compute the cross section, we must square matrix elements (27), (28), (29) and then average over the symmetries of the initial bosons and sum over the symmetries of the final tensor gauge bosons. This gives

\[
\frac{1}{d(G)^2} \sum_{col} |M_{LL-LL}|^2 = \frac{\alpha^4}{32} \frac{C_2^2(G)}{d(G)} (124 - 23 \cos 2\theta + 3 \cos 4\theta), \tag{30}
\]

\[
\frac{1}{d(G)^2} \sum_{col} |M_{LL-RR}|^2 = \frac{\alpha^4}{32} \frac{C_2^2(G)}{d(G)} (55 + \cos 2\theta), \tag{31}
\]

\[
\frac{1}{d(G)^2} \sum_{col} |M_{LR-LL}|^2 = \frac{\alpha^4}{128} \frac{C_2^2(G)}{d(G)} (61 - 32 \cos 2\theta + 3 \cos 4\theta), \tag{32}
\]

where the invariant operator \( C_2 \) is defined by the equation \( t^a t^a = C_2 \). We can calculate now the leading-order polarized cross sections for the tensor gauge boson production \( V + V \rightarrow T + T \).

**Helicity cross-sections.** Plugging matrix elements (30), (31), (32), into our general cross-section formula in the center-of-mass frame (13) yields:

\[
d\sigma_{LL-LL} = \frac{\alpha^2}{s} \frac{C_2^2(G)}{128 d(G)} (124 - 23 \cos 2\theta + 3 \cos 4\theta) \ d\Omega, \tag{33}
\]

\[
d\sigma_{LL-RR} = \frac{\alpha^2}{s} \frac{C_2^2(G)}{128 d(G)} (55 + \cos 2\theta) \ d\Omega, \tag{34}
\]
\[ d\sigma_{LR\rightarrow LL} = \frac{\alpha^2}{s} \frac{C_G^2}{512d(G)} (61 - 32\cos 2\theta + 3\cos 4\theta) \, d\Omega, \] (35)

where \( \alpha = \frac{g^2}{4\pi} \).

Unpolarized cross section. Adding up all sixteen helicity amplitudes and dividing the result by four in order to average over the initial boson spins we can get unpolarized cross section. Thus summing over helicities

\[ \frac{1}{4d(G)} \sum_{\text{col, het}} |M|^2 = \frac{1}{4d(G)} \sum_{\text{col}} 2|\mathcal{M}_{LL\rightarrow LL}|^2 + 2|\mathcal{M}_{LL\rightarrow RR}|^2 + 4|\mathcal{M}_{LR\rightarrow RR}|^2 = \]

\[ = g^2 \frac{g^4}{128} \frac{C_G^2}{d(G)} \left( 419 - 76\cos 2\theta + 9\cos 4\theta \right) \] (36)

for unpolarized cross section we shall get

\[ d\sigma = \frac{\alpha^2}{s} \frac{C_G^2}{d(G)} \frac{419 - 76\cos 2\theta + 9\cos 4\theta}{512} \, d\Omega, \] (37)

where for the \( SU(N) \) group we have \( \frac{C_G^2}{d(G)} = \frac{N^2}{(N^2 - 1)} \). The production cross section of tensor gauge bosons (37) has characteristic behavior with its maximum at \( \theta = \pi/2 \) and decrease for small angles.

This cross section should be compared with the analogous cross section in QCD. Indeed, let us compare this result with the gluon-gluon scattering [9]. The \( V + V \rightarrow V + V \) cross section is

\[ d\sigma = \frac{\alpha^2}{s} \frac{C_G^2}{d(G)} \frac{(4 - \sin^2\theta)^3}{32\sin^4\theta} \, d\Omega. \] (38)

This cross section increases at small angles \( \theta \sim 0, \pi \) and therefore the scattering is mostly going into forward and backward directions and has its minimum in transverse direction at \( \theta = \pi/2 \). The production cross section of spin-two gauge bosons (37) shows dramatically different behavior with its maximum in the transverse direction at \( \theta = \pi/2 \) and decrease in forward and backward directions. One can only speculate that at high enough energies, may be at LHC energies, we may observe the standard spin-one gauge boson together with its new partner, spin-two gauge boson.

5 Appendix A

Here we shall review the well known result for the three-level gluon scattering \( V + V \rightarrow V + V \) [9] in order to compare it with the tensor scattering considered above: \( V + V \rightarrow \)}
$T + T$. Because of the parity and crossing symmetry from 16 possible amplitudes only 5 are different. We have the following equalities:

\[
i \mathcal{M}(LL \to LL) = i \mathcal{M}(RR \to RR), \quad i \mathcal{M}(LL \to RR) = i \mathcal{M}(RR \to LL), \\
i \mathcal{M}(LL \to LR) = i \mathcal{M}(RR \to LR) = i \mathcal{M}(LR \to LL) = i \mathcal{M}(LR \to RR) = \\
i \mathcal{M}(LR \to LR) = i \mathcal{M}(RL \to RR), \quad i \mathcal{M}(LR \to RL) = i \mathcal{M}(RL \to LR).
\]

Four Feynman diagram contribute into this scattering. The contribution of the t-channel diagram can be expressed in the form:

\[
i \mathcal{M}_I(LL \to LL) = i g^2 f^{\text{ace}} f^{bde} (39 - 24 \cos \theta + \cos 2\theta) \cot^2 \frac{\theta}{2} \\
i \mathcal{M}_I(LL \to LR) = i g^2 f^{\text{ace}} f^{bde} \left( \frac{3 + \cos \theta}{\sin^2 \frac{\theta}{2}} \right) \sin^4 \frac{\theta}{2} \\
i \mathcal{M}_I(LL \to RR) = i g^2 f^{\text{ace}} f^{bde} \left( \frac{3 - \cos \theta}{\cos^2 \frac{\theta}{2}} \right) \cos^4 \frac{\theta}{2} \\
i \mathcal{M}_I(LR \to LR) = i g^2 f^{\text{ace}} f^{bde} \left( \frac{3 + \cos \theta}{\sin^2 \frac{\theta}{2}} \right) \cos^4 \frac{\theta}{2} \\
i \mathcal{M}_I(LR \to RL) = i g^2 f^{\text{ace}} f^{bde} \left( \frac{3 - \cos \theta}{\cos^2 \frac{\theta}{2}} \right) \sin^4 \frac{\theta}{2}, \tag{39}
\]

of the u-channel diagram as

\[
i \mathcal{M}_{II}(LL \to LL) = i g^2 f^{\text{abe}} f^{\text{cde}} (39 + 24 \cos \theta + \cos 2\theta) \tan^2 \frac{\theta}{2} \\
i \mathcal{M}_{II}(LL \to LR) = i g^2 f^{\text{abe}} f^{\text{cde}} \left( \frac{3 - \cos \theta}{\cos^2 \frac{\theta}{2}} \right) \cos^4 \frac{\theta}{2} \\
i \mathcal{M}_{II}(LL \to RR) = i g^2 f^{\text{abe}} f^{\text{cde}} \left( \frac{3 + \cos \theta}{\sin^2 \frac{\theta}{2}} \right) \sin^4 \frac{\theta}{2} \\
i \mathcal{M}_{II}(LR \to LR) = i g^2 f^{\text{abe}} f^{\text{cde}} \left( \frac{3 + \cos \theta}{\sin^2 \frac{\theta}{2}} \right) \cos^4 \frac{\theta}{2} \\
i \mathcal{M}_{II}(LR \to RL) = i g^2 f^{\text{abe}} f^{\text{cde}} \left( \frac{3 - \cos \theta}{\cos^2 \frac{\theta}{2}} \right) \sin^4 \frac{\theta}{2}, \tag{40}
\]

of the s-channel diagram as

\[
i \mathcal{M}_{III}(LL \to LL) = -i g^2 f^{\text{abe}} f^{\text{cde}} \cos \theta \\
i \mathcal{M}_{III}(LL \to LR) = 0 \\
i \mathcal{M}_{III}(LL \to RR) = -i g^2 f^{\text{abe}} f^{\text{cde}} \cos \theta \\
i \mathcal{M}_{III}(LR \to LR) = 0 \\
i \mathcal{M}_{III}(LR \to RL) = 0 \tag{41}
\]
and of the contact diagram as

\[ iM_{IV}(LL \rightarrow LL) = -ig^2 \left[ f^{abc} f^{cde} \cos \theta + f^{ace} f^{bde} (1 - \sin^4 \frac{\theta}{2}) + f^{ade} f^{bce} (1 - \cos^4 \frac{\theta}{2}) \right] \]

\[ iM_{IV}(LL \rightarrow LR) = -ig^2 \left( f^{ace} f^{bde} + f^{ade} f^{bce} \right) \sin \theta \]

\[ iM_{IV}(LL \rightarrow RR) = -ig^2 \left[ f^{abc} f^{cde} \cos \theta + f^{ace} f^{bde} (1 - \cos^4 \frac{\theta}{2}) + f^{ade} f^{bce} (1 - \sin^4 \frac{\theta}{2}) \right] \]

\[ iM_{IV}(LR \rightarrow LR) = ig^2 (f^{ace} f^{bde} + f^{ade} f^{bce}) \cos \theta \]

\[ iM_{IV}(LR \rightarrow RL) = ig^2 (f^{ace} f^{bde} + f^{ade} f^{bce}) \sin^2 \theta. \] (42)

So for the total amplitudes we have

\[ iM_{LL\rightarrow LL} = 4ig^2 \left[ \left( \frac{1}{1 - \cos \theta} \right) f^{ace} f^{bde} + \left( \frac{1}{1 + \cos \theta} \right) f^{ade} f^{bce} \right] \] (43)

\[ iM_{LR\rightarrow LR} = 2ig^2 \cos^2 \frac{\theta}{2} \cot \frac{\theta}{2} \left( \cot \frac{\theta}{2} f^{ace} f^{bde} + \tan \frac{\theta}{2} f^{ade} f^{bce} \right) \] (44)

\[ iM_{LR\rightarrow RL} = ig^2 (1 - \cos \theta)^2 \left[ \left( \frac{1}{1 - \cos \theta} \right) f^{ace} f^{bde} + \left( \frac{1}{1 + \cos \theta} \right) f^{ade} f^{bce} \right] \] (45)

together with \( iM_{LL\rightarrow LR} = 0, iM_{LL\rightarrow RR} = 0 \). Thus only three of 16 helicity amplitudes are nonzero. Squaring the matrix elements one can get

\[ |M_{LL\rightarrow LL}|^2 = \frac{16g^4}{d(G)} C_2^2(G) \left( \frac{3 + \cos^2 \theta}{\sin^4 \theta} \right), \] (46)

\[ |M_{LR\rightarrow LR}|^2 = \frac{g^4}{d(G)} C_2^2(G) \left( \frac{3 + \cos^2 \theta}{\sin^4 \theta} \right) (1 + \cos \theta)^4, \] (47)

\[ |M_{LR\rightarrow RL}|^2 = \frac{g^4}{d(G)} C_2^2(G) \left( \frac{3 + \cos^2 \theta}{\sin^4 \theta} \right) (1 - \cos \theta)^4. \] (48)

Using these formulas and (13) one can easily get the cross section (38). It is also instructive to compare the above helicity amplitudes (39)-(42) with the corresponding helicity amplitudes for the tensor gauge bosons (23)-(26). The characteristic feature of the squared amplitudes (46) - (48) is that they increase at small angles \( \theta \sim 0, \pi \) and therefore the scattering is mostly going into forward and backward directions. In contrast to that behavior tensor gauge boson amplitudes (30)-(32) are decreasing at \( \theta \sim 0, \pi \) and increasing in the transverse direction \( \theta \sim \pi/2 \).
6 Appendix B

To check on mass-shell gauge invariance of the amplitude (18) let one of the tensor gauge boson wave function be longitudinal:

$$e_{q_2}^{\rho \rho'} = q_2^\rho \xi_{\rho'} + q_2^{\rho'} \xi_{\rho}. $$

On mass-shell gauge transformations should fulfill the following conditions: $q_2^2 = 0$, $q_2 \cdot e_{q_2} = 0$, $te_{q_2} = 0$. These equations are satisfied if $q_2 \cdot \xi = 0$ and therefore

$$\xi_0 = \xi_1 \sin \theta + \xi_3 \cos \theta,$$

where $q_2^\mu = E(1, -\sin \theta, 0, -\cos \theta)$. To see explicitly how the cancelation between diagrams takes place let us take the rest of the vector bosons left polarized and one of the tensor bosons right polarized. In that case we shall get the following amplitudes:

$$iM_{I}^{long} = i g^2 f^{face} f^{bde} E \cos^3 \frac{\theta}{2} \left[ (\xi_0 - \xi_3) \cos \frac{\theta}{2} - (\xi_1 + i \xi_2) \sin \frac{\theta}{2} \right] \equiv f^{face} f^{bde} M_1,$$

$$iM_{II}^{long} = i g^2 f^{ade} f^{bce} E \sin^3 \frac{\theta}{2} \left[ (\xi_0 + \xi_3) \sin \frac{\theta}{2} - (\xi_1 - i \xi_2) \cos \frac{\theta}{2} \right] \equiv f^{ade} f^{bce} M_2,$$

$$iM_{III}^{long} = -\frac{ig^2}{4} (f^{face} f^{bde} - f^{ade} f^{bce}) E \sin \theta (\xi_1 \cos \theta - i \xi_2 - \xi_3 \sin \theta) \equiv (f^{face} f^{bde} - f^{ade} f^{bce}) M_3,$$

$$iM_{IV}^{long} = \frac{ig^2}{4} (f^{face} f^{bde} + f^{ade} f^{bce}) E \sin \theta \cos \theta (i \xi_2 - \xi_1 \cos \theta + \xi_3 \sin \theta) \equiv (f^{face} f^{bde} + f^{ade} f^{bce}) M_4,$$

where

$$M_1 = ig^2 E \cos^3 \frac{\theta}{2} \left[ (\xi_0 - \xi_3) \cos \frac{\theta}{2} - (\xi_1 + i \xi_2) \sin \frac{\theta}{2} \right],$$

$$M_2 = ig^2 E \sin^3 \frac{\theta}{2} \left[ (\xi_0 + \xi_3) \sin \frac{\theta}{2} - (\xi_1 - i \xi_2) \cos \frac{\theta}{2} \right],$$

$$M_3 = -\frac{ig^2}{4} E \sin \theta (\xi_1 \cos \theta - i \xi_2 - \xi_3 \sin \theta),$$

$$M_4 = \frac{ig^2}{4} E \sin \theta \cos \theta (i \xi_2 - \xi_1 \cos \theta + \xi_3 \sin \theta).$$

(49)
For the total amplitude we shall get

$$i\mathcal{M}^{long} = f^{ace} f^{bde} \left[ M_1 + M_3 + M_4 \right] + f^{ade} f^{bce} \left[ M_2 - M_3 + M_4 \right],$$  \hspace{1cm} (50)$$

and it nullifies if

$$M_1 + M_3 + M_4 = 0, \quad M_2 - M_3 + M_4 = 0$$  \hspace{1cm} (51)$$
or, equivalently: \quad M_1 + M_2 + 2M_4 = 0, \quad M_1 - M_2 + 2M_3 = 0.$$

Using explicit expressions (49) one can check that these equations are identically satisfied for arbitrary functions $\xi_1, \xi_2, \xi_3$.

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