Manipulation of multiphoton blockade in a two atoms cavity-QED system

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Abstract. We present a study of manipulating the multiphoton blockade phenomenon in a single mode cavity with two ladder-type three-level atoms. Combining the cavity QED with electromagnetically induced transparency technique, we show that it is possible to actively manipulate the photon blockade when two atoms are in phase radiations. As a result, the two-photon blockade can be changed to three-photon blockade by changing the control field Rabi frequency. In the case of out-phase radiations, we show that the three-photon blockade can be improved with enhanced mean photon number. In addition, we show that the nonclassical field with sub-Poissonian distribution can be changed to the classical field with super-Poissonian distribution by tuning the Rabi frequency of the control field. The results presented in this work open up the possibility for achieving a two-photon gateway operation, which could be used in network of atom-cavity systems to control the quantum property of photons leaking from the cavity.

Keywords: Three-photon blockade, Nonclassical field, Two-photon gateway

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1. Introduction

In an atom-cavity QED system driven by a coherent field, a single photon can suppress the transmission of other photons due to the strong coupling between the atom and cavity, which is well known as the two-photon blockade. This phenomenon which is a close analogy to the phenomenon of Coulomb blockade was first proposed by Imamoglu et al. \cite{1}. Later, Birnbaum et al. demonstrate the photon blockade phenomenon experimentally in a single atom-cavity QED system with strong coupling strength, where
the quantum statistic property of the incident photon stream was changed from the Poissonian distribution to sub-Poissonian distribution if the frequency of photons was tuned to one of the states of the lowest doublet dressed states [2].

Due to it’s potential applications in quantum communication and computation, the study of two-photon blockade has received extensive attention in past few decades. Many research groups have carried out experimental and theoretical works on photon blockade in various systems with strong coupling strength, including the circuit QED systems [3, 4, 5, 6], artificial atoms on a chip [7, 8], optomechanical systems [9, 10, 11, 12, 13, 14], and atom-cavity QED system [15, 16, 17, 18, 19, 20, 21, 22, 23]. Recently, a novel physical mechanism for achieving photon blockade was proposed by Liew and Savona [24] under the weak nonlinearity regime, which is known as the unconventional photon blockade and is based on the destructive quantum interference under special system parameters [25, 26, 27, 28]. The corresponding experimental works were demonstrated in quantum dot cavity QED system [29] and superconducting circuit QED system [30], respectively.

Although the two-photon blockade has been studied extensively, there are few researches on multiphoton blockade because it is challenging in experiments. A direct method to realize three-photon blockade is by increasing the incident field intensity so that two-photon excitations can be measured. However, the strong field intensity will result in the broadening of the dressed states, which prohibits the observation of the three-photon blockade behavior. These characteristics have been observed by Hamsen et al. [15]. Another method to realize three-photon blockade is based on the collective decay of two atoms trapped in a single mode cavity with different coupling strengths [19]. If two atoms have out-phase radiations, the three-photon blockade with two-photon bunching behavior can be achieved since the two-photon excitations is dominant and one-photon excitations are forbidden.

As we all known, the electromagnetically induced transparency (EIT) is a technique that eliminates the absorption of a electromagnetic field by the medium via quantum destructive interference [31, 32]. It is well known that the EIT technique results in many interesting features based on quantum interference effect, including the group velocity reduction [33, 34], light storage [35, 36], giant Kerr nonlinearity [37, 38] and so on. In this paper, we combine the EIT technique with the cavity QED system, and study the manipulation of multiphoton blockade in a two atoms cavity QED system, where two atoms interact with a pump field and a control field simultaneously, forming the EIT configuration. We show that the effects of EIT technique on the photon blockade strongly depend on the radiations of two atoms. When two atoms radiate in-phase, it is possible to active control the quantum property of the cavity field, and the photon blockade phenomenon can be changed by tuning the control field Rabi frequency. However, in the case of out-phase radiations, we show that the multiphoton blockade phenomenon can be improved by just changing the control field Rabi frequency. Based on these characteristics, we also demonstrate two kinds of gateway with different functions, which could be used in network of atom-cavity systems to control the quantum
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property of cavity photons.

2. Model

As depicted in Fig. 1, this two-atom cavity QED system consisting of a single-mode cavity and two identical ladder-type three-level atoms interacts with a pump field $\eta$ and a control field $\Omega_c$, respectively. The cavity mode and the pump field drive the $|g\rangle \leftrightarrow |m\rangle$ transition simultaneously, while the control field drives the $|m\rangle \leftrightarrow |e\rangle$ transition. Clearly, the system is the same as that proposed in Ref. [19] when the control field is turned off (i.e., setting $\Omega_c = 0$).

![Figure 1.](image.png) (Color online) Schematic of the two atoms cavity QED system. The wavelength of this single model cavity is $\lambda_{cav}$, and the corresponding angular frequency is $\omega_{cav} = 2\pi/\lambda_{cav}$. Each atom has three energy levels labeled as $|g\rangle$, $|m\rangle$ and $|e\rangle$, respectively. A coherent pump field with Rabi frequency $\eta$ (angular frequency is $\omega_p$) drives the $|g\rangle \leftrightarrow |m\rangle$ transition, and a strong control field with Rabi frequency $\Omega_c$ (angular frequency is $\omega_c$) drives the $|m\rangle \leftrightarrow |e\rangle$ transition. The one-photon detunings are defined as $\Delta_m = \omega_m - \omega_g - \omega_p$ and $\Delta_c = \omega_e - \omega_m - \omega_c$. Here, the distance between two atoms is $\Delta z$. The spontaneous emission rate of state $|m\rangle$ ($|e\rangle$) is $\gamma_m$ ($\gamma_e$), and the cavity decay rate is $\kappa$.

In general, the dynamical behavior of this cavity QED system can be described by using the master equation, i.e.,

$$\frac{d\rho}{dt} = -i[H, \rho] + L_\kappa \rho + L_\gamma \rho,$$

where $\rho$ is the density-matrix operator of the atom-cavity QED system. Under rotating-wave and electric dipole approximations, the system Hamiltonian can be written as $H = H_0 + H_I + H_L$ with

$$H_0 = \hbar \sum_{i=1,2} (\Delta_c \sigma_{ee}^i + \Delta_m \sigma_{mm}^i + \Delta_{cav} a^\dagger a),$$

$$H_I = \sum_{i=1,2} g_i (a \sigma_{mg}^i + a^\dagger \sigma_{gm}^i),$$

$$H_L = \eta \sum_{i=1,2} (\sigma_{mg}^i + \sigma_{gm}^i) + \Omega_c \sum_{i=1,2} (\sigma_{me}^i + \sigma_{em}^i).$$
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where \( H_0 \) is the energy of atoms and the cavity field, \( H_I \) represents the interaction between atoms and the cavity field, and \( H_p \) is the coherent driving term involving the pump field and control field. The coupling strength between the \( i \)-th atom and cavity \( g_i = g \cos(2\pi z_i/\lambda_{cav}) \) is dependent to the position of the \( i \)-th atom \( z_i \), where \( \lambda_{cav} \) is the wavelength of the cavity mode (the corresponding angular frequency is \( \omega_{cav} = c/\lambda_{cav} \)). Here, \( a \) and \( a^\dagger \) are the annihilation and creation operators of the cavity mode, respectively. \( \sigma_{jk}^i = |j\rangle^i \langle k| (j, k = \{g, m, e\}) \) denotes the atomic operator of the \( i \)-th atom. The detunings are defined as \( \Delta_{cav} = \omega_{cav} - \omega_p, \Delta_m = \omega_m - \omega_g - \omega_p, \Delta_e = \omega_e - \omega_g - (\omega_p + \omega_e) = \Delta_m + \Delta_c \) with \( \Delta_c = \omega_e - \omega_m - \omega_c \) and \( \hbar \omega_j \) \((j = \{g, m, e\})\) being the energy of state \( |j\rangle \).

The last two terms in Eq. (1) are the Liouvillian operators representing the decay of the atom and cavity, respectively, which are given by

\[
\mathcal{L}_{cav} \rho = \kappa (2a a^\dagger - a^\dagger a - \rho a a^\dagger),
\]

\[
\mathcal{L}_{atom} \rho = \sum_{i=1,2} \left[ \gamma_e (2 \sigma_{me}^i \rho \sigma_{em}^i - \sigma_{em}^i \sigma_{me}^i \rho - 2 \rho \sigma_{em}^i \sigma_{me}^i) + \gamma_m (2 \sigma_{gm}^i \rho \sigma_{mg}^i - \sigma_{mg}^i \sigma_{gm}^i \rho - 2 \rho \sigma_{mg}^i \sigma_{gm}^i) \right],
\]

with \( \kappa \) being the cavity decay rate, and \( \gamma_m (\gamma_e) \) being the spontaneous emission rate of the state \(|m\rangle \) (|e\rangle).

To understand the physical mechanism clearly, we rewrite the Hamiltonian of the system in dressed state picture by using \(|GG, n\rangle, |MG\pm, n - 1\rangle, |EG\pm, n - 1\rangle, |MM, n - 2\rangle, |EM\pm, n - 2\rangle \) and \(|EE, n - 2\rangle\) as basis in \( n \)-photon space (the definition of these basis are given in the appendix). Considering the case of \( \Delta_c = 0 \) and assuming \( \omega_{cav} = \omega_m - \omega_g \) for mathematical simplicity, we have \( \Delta_m = \Delta_e = \Delta_{cav} \equiv \Delta_p \). Under the weak pump field approximation, the effects of the pump field can be treated as a perturbation to the system, the Hamiltonian in one-photon space is expressed as

\[
H_{1ph} = \begin{pmatrix}
0 & g_+ / \sqrt{2} & g_- / \sqrt{2} & 0 & 0 \\
g_+ / \sqrt{2} & 0 & 0 & \Omega_c & 0 \\
g_- / \sqrt{2} & 0 & 0 & 0 & \Omega_c \\
0 & \Omega_c & 0 & 0 & 0 \\
0 & 0 & \Omega_c & 0 & 0
\end{pmatrix},
\]

(7)

and, in two-photon space, the matrix of the Hamiltonian is given by

\[
H_{2ph} = \begin{pmatrix}
0 & g_+ & g_- & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
g_+ & 0 & 0 & \Omega_c & 0 & 0 & 0 & 0 & 0 & 0 \\
g_- & 0 & 0 & 0 & \Omega_c & 0 & \frac{g_+}{2} & -\frac{g_-}{2} & 0 & 0 \\
0 & \Omega_c & 0 & 0 & 0 & \frac{g_+}{2} & -\frac{g_-}{2} & 0 & 0 & 0 \\
0 & 0 & \Omega_c & 0 & 0 & -\frac{g_+}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \Omega_c & 0 & 0 & -\frac{g_+}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & g_+ / \sqrt{2} & -g_- / \sqrt{2} & 0 & 0 & \sqrt{2} \Omega_c & \sqrt{2} \Omega_c & 0 \\
0 & 0 & 0 & -g_+ / \sqrt{2} & g_- / \sqrt{2} & 0 & 0 & 0 & \sqrt{2} \Omega_c & \sqrt{2} \Omega_c \\
0 & g_+ / \sqrt{2} & -g_- / \sqrt{2} & 0 & 0 & \sqrt{2} \Omega_c & 0 & 0 & 0 & 0 \\
0 & 0 & g_+ / \sqrt{2} & -g_- / \sqrt{2} & 0 & 0 & \sqrt{2} \Omega_c & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sqrt{2} \Omega_c & 0 & 0 & 0 & 0
\end{pmatrix},
\]

(8)
where $g_{\pm} = g_1 \pm g_2 = g(\pm \cos \phi_z)$ with $\phi_z = 2\pi \Delta z/\lambda_{\text{cav}}$. Diagonalizing Eqs. (7) and (8), we can obtain the corresponding eigenvalues and eigenstates forming the dressed states in one- and two-photon space (see Appendix). Furthermore, we can also obtain the transition strength by calculating the operator $\eta \sum_{i=1,2}(\sigma_{m}^{i} + \sigma_{mg}^{i})$. These dressed states along with some of the important transitions are shown in Fig. 2.

3. THE CASE OF $\phi_z = 0$

In this section, we consider the case that the two atoms have the same coupling strengths, i.e., $g_1 = g_2 = g$. Numerically solving Eq. (1), we can obtain the cavity excitation spectrum and photon-photon correlation function, which show the main difference between this three-level cavity QED system and the standard two-level cavity QED system. As shown in Fig. 3 we plot the mean photon number $\langle a^\dagger a \rangle$ [panel (a)] and the equal-time photon-photon correlation function $g^{(2)}(0) = \langle a^\dagger a^\dagger aa \rangle/\langle a^\dagger a \rangle^2$ in logarithmic unit [panel (b)] versus the normalized detuning $\Delta p/\kappa$. Here we choose the control field

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{(Color online) The dressed-state energy structures for $\phi_z = 0$ [panel (a)] and $\phi_z = \pi$ [panel (b)], respectively. The red arrows represent the one-photon transitions, but the blue ones represent the two-photon transitions. Here, we only show several main pathways of two photon transitions. The black arrows with red cross denote that the transitions are forbidden.}
\end{figure}
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Figure 3. (Color online) Panels (a) and (b) display the mean photon number $\langle a^\dagger a \rangle$ and the equal-time photon-photon correlation function $\log_{10}[g^{(2)}(0)]$ as a function of the normalized detuning $\Delta_p/\kappa$, respectively. The Rabi frequency of the control field is chosen as $\Omega_c/\kappa = 0$ (blue dashed curves) and 20 (red solid curves), respectively. Other system parameters are given by $\eta/\kappa = 0.2$, $g/\kappa = 20$, $\gamma_m/\kappa = 1$ and $\gamma_e/\kappa = 0.01$. The black dash-dotted line indicates $\log_{10}[g^{(2)}(0)] = 0$ for a coherent field.

Rabi frequency as $\Omega_c/\kappa = 0$ (i.e., two-level system, blue dashed curves) and $\Omega_c/\kappa = 20$ (red solid curves), respectively. Other system parameters are chosen as $\eta/\kappa = 0.2$, $g/\kappa = 20$, $\gamma_m/\kappa = 1$ and $\gamma_e/\kappa = 0.01$. When the control field is turned off, one can observe two peaks in the cavity excitation spectrum at $\Delta_p = \pm \sqrt{2} g$, corresponding to the frequencies of one-photon excitations (i.e., $\Psi(0) \rightarrow \Psi^{(1)}_{\pm 2}$ transitions, see the blue dashed curve in panel (a)). The quantum features of the cavity field can be characterized by the photon-photon correlation function $g^{(2)}(0)$. As shown in panel (b), the value of $g^{(2)}(0)$ at one-photon excitation is smaller than unity (i.e., $\log_{10}[g^{(2)}(0)] < 0$), which implies that the two-photon blockade behavior and the nonclassical cavity field with sub-Poissonian distribution can be achieved.

When the control field is turned on, the energies of the states $\Psi^{(1)}_{\pm 2}$ shift as the control field intensity increases (see Fig. 2(a) and the appendix). As a result, the width between two peaks in the cavity excitation spectrum becomes larger than that in the absence of the control field (see the red solid curve in panel (a)), which is given by $\Gamma_w = 2\sqrt{2g^2 + \Omega_c^2}$. Compared with the case of $\Omega_c = 0$, an attractive feature of this three-level cavity QED system is the improvement of the two-photon blockade phenomenon. It is clear to see that the values of $g^{(2)}(0)$ at one-photon excitations decrease to the order of $10^{-2}$ when the control field is turned on (see red curve in panel (b)). According to our calculations, the equal-time photon-photon correlation $g^{(2)}(0) \approx 8 \times 10^{-3}$, which is an evidence of perfect two-photon blockade. Since the frequency to realize two-photon blockade (nonclassical field generation) can be adjusted by the control field, it is possible to realize single photon gateway (from classical field to single photon), which has already been studied in some literature [39, 40, 41, 42].

Increasing the pump field Rabi frequency (for example, $\eta/\kappa = 1.5$), one can observe not only the one-photon excitations but also the two-photon excitations as shown in
Fig. 4(a) and (b), no matter the control field is turned on or off. In the absence of the control field ($\Omega_c = 0$), there are four peaks in the cavity excitation spectrum, corresponding to the frequencies $\Delta = \pm \sqrt{2}g$ (one-photon excitations) and $\Delta = \pm \sqrt{6}g/2$ (two-photon excitations), respectively [19]. At frequencies of one-photon excitations (see panel (a)), one can obtain $g^{(2)}(0) \approx 0.2$, i.e., the two-photon blockade. The system parameters are the same as those used in Fig. 3. However, it is difficult to observe the three-photon blockade (i.e., $g^{(2)}(0) > 1$ and $g^{(3)}(0) < 1$). As shown in panel (a), the regime for realizing the three-photon blockade is extremely narrow (see the inserted plot) so that the frequency of the pump field must be controlled precisely, which is very difficult in experiments and technique noise becomes a fatal problem. When the control field is turned on, the energy shifts cause a significant improvement of the photon...
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blockade phenomenon (see panel (b) with $\Omega_c/\kappa = 35$). At the frequencies of one-photon excitations, it is found that $g^{(2)}(0) \approx 0.136$, which is near 2 times smaller than that in the absence of the control field. Moreover, at the frequencies of two-photon excitations, the frequency regime for realizing the three-photon blockade regime is much broader than the case of $\Omega_c = 0$. For example, one can obtain $g^{(2)}(0) \approx 1.13$ and $g^{(3)}(0) \approx 0.025$ with mean photon number $n = 0.11$ at the frequency of two-photon excitation, which implies the three-photon blockade behavior.

In Fig. 4(c) and (d), we show the manipulation of photon blockade by plotting the mean photon number, equal-time second-order and third-order field correlation functions as a function of the normalized control field Rabi frequency $\Omega_c/\kappa$. Here, we choose $\Delta_p/\kappa = 40$, and the coupling strength is taken as $g/\kappa = 20$ in panel (c) and $g/\kappa = 25$ in panel (d), respectively. It is clear to see that the cavity field behavior can be changed from two-photon blockade to three-photon blockade by increasing the control field intensity. Therefore, a controllable nonclassical field gateway, allowing one or two photons to leak from the cavity, can be achieved. In addition, the switch interval, i.e., the intensity difference of the control field to realize one- or two-photon blockade, can be enhanced if the coupling strength $g$ is increased. The physical mechanics of this gateway can be understood easily. When the control field is very weak, the pump field is far off-resonant to all states because of the large detuning $\Delta_p$. As a result, the mean photon number is close to zero. If one increase the control field intensity, the energy of the one-photon excitation will be shifted to be resonant to the pump field, yielding two-photon blockade phenomenon (see the left peaks in blue dashed curves). Further increasing the control field intensity, the pump field excites the two-photon excitation so that three-photon blockade occurs (see the right peaks in blue dashed curves). We must point out that, in our proposal, the mean photon number at two peaks is detectable when this gateway works.

4. THE CASE OF $\phi_z = \pi$

Now, we consider the case that two atoms have different coupling strengths (i.e., $g_1 = -g_2 = g$). In this case, the two-photon excitation becomes dominant because $\Psi_0 \leftrightarrow \Psi^{(1)}_{\pm}$ transitions are not allowed [19]. As a result, two side peaks (see Fig. 3(a)), corresponding to the two-photon excitations, can be observed in the cavity excitation spectrum when the control field is absent (i.e., $\Omega_c = 0$). Correspondingly, the three-photon blockade behavior can be observed at two-photon excitation frequencies (i.e., $\Delta/\kappa = \pm \sqrt{6}g/2$) with $g^{(2)}(0) > 1$ and $g^{(3)}(0) < 1$. The central peak in the cavity excitation spectrum arises from the multiphoton excitation process [19], which results in a classical field generation (i.e., $g^{(2)}(0) > 1$ and $g^{(3)}(0) > 1$). In the presence of the control field, for example, $\Omega_c/\kappa = 20$, there are four peaks corresponding to two-photon excitations (see Fig. 2(b)) in the cavity excitation spectrum as shown in Fig. 5(b). Compared with the case of $\Omega_c/\kappa = 0$, it is clear to see that the mean photon number is enhanced and the three-photon blockade phenomenon can be significantly
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Figure 5. (Color online) Panels (a) and (b): Plots of the mean photon number (blue dashed curves), equal-time second-order (green solid curves) and third-order (red dashed curve) field correlation functions for $\Omega_c/\kappa = 0$ (a) and $\Omega_c/\kappa = 20$ (b). Here, we choose $\eta/\kappa = 2$, $\phi_z = \pi$ and other system parameters are the same as those used in Fig. 3. The black dash-dotted line indicates $g^{(2)}(0) = g^{(3)}(0) = 1$. Panels (c) and (d): The demonstration of two-photon gateway with detuning $\Delta/\kappa = \sqrt{6}g/2$. The coupling strengths are chosen as $g/\kappa = 20$ in panel (c) and $g/\kappa = 25$ in panel (d), respectively.

improved. At the detuning $\Delta/\kappa = \pm\sqrt{\alpha + \sqrt{\beta}/(2\sqrt{2})}$, one can obtain $g^{(2)}(0) \approx 2.37$ and $g^{(3)}(0) \approx 0.015$ with mean photon number $n = 0.1$.

Based on the above characteristics of the cavity field offered by the control field, we show in Fig. 5(c) and (d) that this scheme can realize the two-photon gateway, which allows the photons leak from the cavity with anti-bunching behavior (two photons together) or bunching behavior. Here, we take the detuning $\Delta/\kappa = \sqrt{6}g/2$, the coupling strengths $g/\kappa = 20$ in panel (c) and $g/\kappa = 25$ in panel (d), respectively. Other system parameters are the same as those in Fig. 3. When the control field is weak, the pump field is only resonant to the two photon excitation so that the three-photon blockade can be observed with $g^{(2)}(0) > 1$ and $g^{(3)}(0) < 1$ (see panels (c) and (d) left peaks). Increasing the control field intensity, the energies of all states shift, and the pump field becomes resonant/near resonant to many states, which results in the multiphoton excitations. Therefore, the photons leaking from the cavity become classic and exhibit
bunching behavior. We also show that the switch interval for achieving the change from anti-bunching to bunching can be enhanced by increasing the coupling strength.

5. Conclusion

To conclude, we have studied the optical control of the quantum properties of the cavity field in the atom-cavity QED system with two ladder-type three-level atoms. By combining the cavity QED system with the electromagnetically induced transparency technique, we show that the quantum fluctuation of the cavity field can be controlled by tuning the Rabi frequency of the control field. When two atoms are in-phase radiations, for example, we show that the frequency to realize the two-photon blockade can be actively controlled by the EIT technique since the dressed states are shifted by the control field. We also show that, increasing the pump field Rabi frequency, both two-photon and three-photon blockades can be observed in this system. Manipulating the dressed state from one-photon excitation to two-photon excitation, it is possible to realize a nonclassical field gateway which allows one photon or two photons leaking from the cavity. In the case of out-phase radiations, we show that the three-photon blockade phenomenon can be significantly improved with enhanced photon numbers. Furthermore, tuning the control field Rabi frequency, the nonclassical cavity field with sub-Poissonian distribution can be changed to a classical field with super-Poissonian distribution, which provides a possibility to realize two-photon gateway. Based on this two atoms cavity QED system with EIT technique, many new features can be explored and may result in possible applications in quantum communication and computation.

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Appendix A. The eigenvalues and eigenstates of Eqs. (4) and (5)

The Hamiltonian of the system in dressed state picture can be rewritten by using $|GG, n\rangle$, $|MG\pm, n-1\rangle$, $|EG\pm, n-1\rangle$, $|MM, n-2\rangle$, $|EM\pm, n-2\rangle$ and $|EE, n-2\rangle$ as basis in $n-$photon space, which are defined as

$$|GG, n\rangle = |gg, n\rangle$$

$$|MG\pm, n-1\rangle = \frac{1}{\sqrt{2}}(|mg, n-1\rangle \pm |gm, n-1\rangle)$$

$$|EG\pm, n-1\rangle = \frac{1}{\sqrt{2}}(|eg, n-1\rangle \pm |ge, n-1\rangle)$$

$$|MM, n-2\rangle = |mm, n-2\rangle$$

$$|EM\pm, n-2\rangle = \frac{1}{\sqrt{2}}(|eg, n-2\rangle \pm |ge, n-2\rangle)$$

$$|EE, n-2\rangle = |ee, n-2\rangle$$
In two-photon space, the eigenvalues can be obtained by solving Eq. (5), yielding
\[ \lambda_0^{(2)} = \lambda_0^{(2)} = \lambda_1^{(2)} = \pm \sqrt{g^2 + \Omega_e^2}, \quad \lambda_2^{(2)} = \pm \sqrt{\alpha - \sqrt{\beta}/\sqrt{2}} \] with \( \alpha = 7g^2 + 5\Omega_e^2 \) and \( \beta = 25g^4 + 6g^2\Omega_e^2 + 9\Omega_e^4 \). Correspondingly, the eigenstates are given by

\[ |\Psi_0^{(2)}\rangle = \frac{\Omega_e^2}{\sqrt{2g^2}} |GG, 2\rangle - \frac{\sqrt{2}\Omega_e}{g} |EG+, 1\rangle + |EE, 0\rangle \] (A.10)

\[ |\Psi_0^{(2)}\rangle = \frac{\Omega_e^2 - g^2}{\sqrt{2g^2}} |GG, 2\rangle - \frac{\sqrt{2}\Omega_e}{g} |EG+, 1\rangle + |MM, 0\rangle \] (A.11)

\[ |\Psi_0^{(2)}\rangle = \frac{-g}{\Omega_e^2} |MG-, 1\rangle + |EM-, 0\rangle \] (A.12)

\[ |\Psi_1^{(2)}\rangle = \frac{\Omega_e}{g} |MG-, 1\rangle \pm \frac{\sqrt{g^2 + \Omega_e^2}}{g} |EG-, 1\rangle + |EM-, 0\rangle \] (A.13)

\[ |\Psi_2^{(2)}\rangle = \frac{\pm \gamma + \sqrt{\beta}}{3\sqrt{2\Omega_e^2}} |GG, 2\rangle \mp \frac{\sqrt{\alpha - \sqrt{\beta}(\gamma + \sqrt{\beta})}}{12g\Omega_e^2} |MG+, 1\rangle \]
\[ - \frac{(\gamma - 6g^2) + \sqrt{\beta}}{6\sqrt{2g}\Omega_e^2} |EG+, 1\rangle \pm \frac{\sqrt{\alpha - \sqrt{\beta}}}{2\Omega_e^2} |EM+, 0\rangle \]
\[ + \frac{-\gamma + 6\Omega_e^2 - \sqrt{\beta}}{6\Omega_e^2} |MM, 0\rangle + |EE, 0\rangle \] (A.14)

\[ |\Psi_3^{(2)}\rangle = \frac{-\gamma + \sqrt{\beta}}{3\sqrt{2\Omega_e^2}} |GG, 2\rangle \pm \frac{\sqrt{\alpha + \sqrt{\beta}(-\gamma + \sqrt{\beta})}}{12g\Omega_e^2} |MG+, 1\rangle \]
\[ + \frac{(\gamma - 6g^2) + \sqrt{\beta}}{6\sqrt{2g}\Omega_e^2} |EG+, 1\rangle \pm \frac{\sqrt{\alpha + \sqrt{\beta}}}{2\Omega_e^2} |EM+, 0\rangle \]
\[ + \frac{-\gamma + 6\Omega_e^2 + \sqrt{\beta}}{6\Omega_e^2} |MM, 0\rangle + |EE, 0\rangle \] (A.15)

with \( \gamma = -5g^2 + 3\Omega_e^2 \).

Likewise, one can obtain the dressed states in the case of \( \phi_z = \pi \), where \( g_1 = -g_2 = g \). In one-photon space, the eigenvalues are given by \( \lambda_0^{(1)} = 0, \lambda_1^{(1)} = \pm \Omega_e \) and
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\[ \lambda_{2\pm}^{(1)} = \pm \sqrt{2g^2 + \Omega_c^2} \] with the corresponding eigenstates

\[ |\Psi_0^{(1)}\rangle = -\frac{\Omega_c}{\sqrt{2g}}|GG, 1\rangle + |EG-, 0\rangle \] (A.16)

\[ |\Psi_{1\pm}^{(1)}\rangle = \pm |MG+, 0\rangle + |EG+, 0\rangle \] (A.17)

\[ |\Psi_{2\pm}^{(1)}\rangle = \frac{\sqrt{2g}}{\Omega_c}|GG, 1\rangle \pm \frac{\sqrt{2g^2 + \Omega_c^2}}{\Omega_c}|MG-, 0\rangle + |EG-, 0\rangle \] (A.18)

In two-photon space, we can find nine eigenvalues, which reads \( \lambda_0^{(2)} = \lambda_{0\pm}^{(2)} = 0 \), \( \lambda_{1\pm}^{(2)} = \pm \sqrt{g^2 + \Omega_c^2} \), \( \lambda_{2\pm}^{(2)} = \pm \sqrt{\alpha - \beta} \) and \( \lambda_{3\pm}^{(2)} = \pm \sqrt{\alpha + \beta} \). The corresponding eigenstates are given by

\[ |\Psi_{0\pm}^{(2)}\rangle = -\frac{\Omega_c^2}{\sqrt{2g}}|GG, 2\rangle + \frac{\sqrt{2g}}{\Omega_c}|EG-, 1\rangle + |EE, 0\rangle \] (A.19)

\[ |\Psi_{0\pm}^{(2)}\rangle = \frac{g^2 - \Omega_c^2}{\sqrt{2g}}|GG, 2\rangle + \frac{\sqrt{2g\Omega_c}}{g}|EG-, 1\rangle + |MM, 0\rangle \] (A.20)

\[ |\Psi_{0\pm}^{(2)}\rangle = \frac{g}{\Omega_c}|MG+, 1\rangle + |EM-, 0\rangle \] (A.21)

\[ |\Psi_{1\pm}^{(2)}\rangle = -\frac{\Omega_c}{g}|MG+, 1\rangle \pm \frac{g}{\Omega_c}|EG+, 1\rangle + |EM-, 0\rangle \] (A.22)

\[ |\Psi_{2\pm}^{(2)}\rangle = \frac{\gamma + \sqrt{\alpha}}{3\sqrt{2\Omega_c}}|GG, 2\rangle \pm \frac{\sqrt{\alpha - \beta}(\gamma + \sqrt{\beta})}{12g\Omega_c^2}|MG-, 1\rangle \]

\[ + \frac{\gamma - 6g^2 + \sqrt{\beta}}{6\sqrt{2g}\Omega_c}|EG-, 1\rangle \pm \frac{\sqrt{\alpha - \beta}}{2\Omega_c}|EM+, 0\rangle \]

\[ + \frac{-\gamma + 6\Omega_c^2 - \sqrt{\beta}}{6\Omega_c^2}|MM, 0\rangle + |EE, 0\rangle \] (A.23)

\[ |\Psi_{3\pm}^{(2)}\rangle = -\frac{\gamma + \sqrt{\beta}}{3\sqrt{2\Omega_c}}|GG, 2\rangle + \frac{\sqrt{\alpha + \beta}(-\gamma + \sqrt{\beta})}{12g\Omega_c^2}|MG-, 1\rangle \]

\[ - \frac{-\gamma + 6g^2 + \sqrt{\beta}}{6\sqrt{2g}\Omega_c}|EG-, 1\rangle \pm \frac{\sqrt{\alpha + \beta}}{2\Omega_c}|EM+, 0\rangle \]

\[ + \frac{-\gamma + 6\Omega_c^2 + \sqrt{\beta}}{6\Omega_c^2}|MM, 0\rangle + |EE, 0\rangle \] (A.24)

References

[1] Imamoğlu A, Schmidt H, Woods G and Deutsch M 1997 Phys. Rev. Lett. 79(8) 1467
[2] Birnbaum K M, Boca A, Miller R, Boozer A D, Northup T E and Kimble H J 2005 Nature 436 87
[3] Hoffman A J, Srinivasan S J, Schmidt S, SpieL L, Aumentado J, Türeci H E and Houck A A 2011 Phys. Rev. Lett. 107(5) 053602
[4] Liu Y x, Xu X W, Miranowicz A and Nori F 2014 Phys. Rev. A 89 043818
[5] Wang X, Miranowicz A, Li H R and Nori F 2016 Phys. Rev. A 93 063861
[6] Felicetti S, Rossatto D, Rico E, Solano E and Forn-Díaz P 2018 Phys. Rev. A 97 013851
Faraon A, Fushman I, Englund D, Stoltz N, Petroff P and Vučković J 2008 Nat. Phys. 4 859
Reinhard A, Volz T, Winger M, Badolato A, Hennessy K J, Hu E L and Imamoglu A 2012 Nat. Photon. 6 93
Rabl P 2011 Phys. Rev. Lett. 107(6) 063601
Ludwig M, Safavi-Naeini A H, Painter O and Marquardt F 2012 Phys. Rev. Lett. 109(6) 063601
Liao J Q and Nori F 2011 Phys. Rev. Lett. 107(6) 063601
Ludwig M, Safavi-Naeini A H, Painter O and Marquardt F 2012 Phys. Rev. Lett. 109(6) 063601
Liu J C H and Savona V 2010 Phys. Rev. Lett. 104(18) 183601
Majumdar A, Bajcsy M, Rundquist A and Vučković J 2012 Phys. Rev. Lett. 108(18) 183601
Bamba M, Imamoglu A, Carusotto I and Ciuti C 2011 Phys. Rev. A 83(2) 021802
Gerace D and Savona V 2014 Phys. Rev. A 89(3) 031803
Flayac H and Savona V 2017 Phys. Rev. A 96(5) 053810
Vaneph C, Morvan A, Aiello G, Féchant M, Aprili M, Gabelli J and Estève J 2018 Phys. Rev. Lett. 121(4) 043602
Snijders H J, Frey J A, Norman J, Flayac H, Savona V, Gossard A C, Bowers J E, van Exter M P, Bouwmeester D and Löffler W 2018 Phys. Rev. Lett. 121(4) 043601
Fleischhauer M, Imamoglu A and Marangos J P 2005 Rev. Mod. Phys. 77 633
Mücke M, Figueroa E, Bochmann J, Hahn C, Murr K, Ritter S, Villas-Boas C J and Rempe G 2010 Nature 465 755
Zhang J, Hernandez G and Zhu Y 2008 Opt. Lett. 33 46–48
Nikoghosyan G and Fleischhauer M 2010 Phys. Rev. Lett. 105(1) 013601
Heinze G, Hubrich C and Halfmann T 2013 Phys. Rev. Lett. 111(3) 033601
Katz O and Firstenberg O 2018 Nat. Commun. 9 2074
Schmidt H and Imamoglu A 1996 Opt. Lett. 21 1936–1938
Rebić S, Twamley J and Milburn G J 2009 Phys. Rev. Lett. 103(15) 150503
Michler P, Imamoglu A, Mason M, Carson P, Strouse G and Buratto S 2000 Nature 406 968
Chang W H, Chen W Y, Chang H S, Hsieh T P, Chyi J I and Hsu T M 2006 Phys. Rev. Lett. 96(11) 117401
Lin X, Dai X, Pu C, Deng Y, Niu Y, Tong L, Fang W, Jin Y and Peng X 2017 Nat. Commun. 8 1132
Senellart P, Solomon G and White A 2017 Nat. Nanotechnol. 12 1026