Prompt Charmonium Production in $Z$ Decays

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Abstract

The color-evaporation model quantitatively describes all data on photoproduction and hadroproduction of charmonium. Although the model is in part nonperturbative, the associated parameters can for instance be determined from the charmonium photoproduction data. At this point its predictions for the prompt production of $\psi$'s at the $Z$ pole are made with no free parameters. We show here that this approach successfully describes all data on the production of prompt $\psi$'s in $Z$ decays.

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I. INTRODUCTION

There has been a renewed interest in studying the mechanism by which charmonium is produced, triggered mostly by some puzzling data from the Fermilab Tevatron. The standard perturbative QCD calculations using the color-singlet model failed to explain the data, occasionally by orders of magnitude \([1]\). The data incited a complete review of the treatment of color in QCD and can, in fact, be explained by allowing perturbative color octet \(c\bar{c}\) states to evolve into the asymptotic colorless charmonium states. This prescription is present in both the color-evaporation \([2–5]\) and in the color-octet models \([6]\).

At the \(Z\) resonance the majority of \(\psi\)'s are produced via \(b\)-hadron decays, with a branching ratio \(B(\Z \rightarrow \psi + X)\) of approximately \(4.0 \times 10^{-3}\) \([7]\). Recently the OPAL collaboration also presented evidence for the prompt production of \(\psi\) in hadronic \(Z\) decays \([8]\).

We study here the prompt charmonium production in \(Z\) decays in the framework of the color-evaporation model. The predictive power of the model stems from the fact that its free, nonperturbative parameters can be fixed by charmonium photoproduction data, which leads to very well defined predictions for the inclusive charmonium branching ratios. Although part of the mechanism is essentially nonperturbative, it can be described by a minimal set of free parameters which are few in comparison with the color-octet model.

We show that the color-evaporation model predicts an inclusive branching ratio of \(Z\) into prompt \(\psi\) in agreement with the LEP results \([8,9]\). The dominant process is \(Z \rightarrow c\bar{c}q\bar{q}\). It predicts an energy spectrum which is very different from the one predicted by the color-singlet model as well as a substantially larger branching ratio of \(Z\) into charmonium.

II. THE MODEL AND ITS PARAMETERS

The color-evaporation approach, which actually predates the color-singlet approach, quantitatively describes all charmonium photo- and hadroproduction data \([10,11]\). The model simply states that charmonium production is described by the same dynamics as \(DD\)
production, \textit{i.e.}, by the formation of a colored $c\bar{c}$ pair. Rather than imposing that the $c\bar{c}$ pair is in a color-singlet state in the short-distance perturbative diagrams, it is argued that the appearance of color-singlet asymptotic states solely depends on the outcome of large-distance fluctuations of quarks and gluons. These large-distance fluctuations are probably complex enough for the occupation of different color states to approximately respect statistical counting. In other words, the formation of color-singlet states is a nonperturbative phenomenon. In fact, it does not seem logical to enforce the color-singlet property of the $c\bar{c}$ pair at short distances, given that there is an infinite time for soft gluons to readjust the color of the pair before it appears as an asymptotic $\psi$, $\chi_c$ or, alternatively, $D\bar{D}$ state. It is indeed hard to imagine that a color-singlet state formed at a range $m_{\psi}^{-1}$ automatically survives to form a $\psi$. This formalism was proposed almost twenty years ago \cite{3-5} and subsequently abandoned for no good reason.

In the color-evaporation model the sum of the cross sections of all onium and open charm states is described by

$$
\sigma_{\text{onium}} = \frac{1}{9} \int_{2m_c}^{2m_D} dm \frac{d\sigma_{c\bar{c}}}{dm},
$$

(1)

and

$$
\sigma_{\text{open}} = \frac{8}{9} \int_{2m_c}^{2m_D} dm \frac{d\sigma_{c\bar{c}}}{dm} + \int_{2m_D}^{2m_c} dm \frac{d\sigma_{c\bar{c}}}{dm},
$$

(2)

where the cross section for producing heavy quarks, $\sigma_{c\bar{c}}$, is computed perturbatively, irrespective of the color of the $c\bar{c}$ pair, and $m$ is the invariant mass of the $c\bar{c}$ pair. The coefficients $\frac{1}{9}$ and $\frac{8}{9}$ represent the statistical probabilities that the $3 \times \bar{3}$ charm pair is asymptotically in a singlet or octet state \cite{10}.

The color-evaporation model assumes a factorization of the production of the $c\bar{c}$ pair, which is perturbative and process dependent, and the materialization of this pair into a charmonium state by a mechanism that is nonperturbative and process independent. This assumption is reasonable given that the characteristic time scales of the two processes are very different: the time scale for the production of the pair is the inverse of the heavy-quark mass, while the formation of the bound state is associated to the time scale $1/\Lambda_{\text{QCD}}$. 
Comparison with the $\psi$ data requires knowledge of the fraction $\rho_{\psi}$ of produced onium states that materialize as $\psi$'s, \textit{i.e.},

$$\sigma_{\psi} = \rho_{\psi} \sigma_{\text{onium}},$$

(3)

where $\rho_{\psi}$ is assumed to be a constant. This assumption is in agreement with the low-energy data \[12,13\]. Notice that a single nonperturbative factor $\rho$ describes a given charmonium state, regardless of the spin and orbital angular momentum of the charm pair. Therefore the color-evaporation model is more economical in nonperturbative parameters than the color-octet mechanism.

In Ref. \[11\], we determined the factor $\rho_{\psi}$ from an analysis of photoproduction of charmonium. Using this value we were able to accommodate all data on the hadroproduction of charmonium without introducing any new parameters. This is a very remarkable result given that the subprocess responsible for the charmonium hadroproduction changes from $q\bar{q}$ fusion to $gg$ fusion as the center-of-mass energy is increased. The fragmentation factor $\rho_{\psi}$ turned out to be 0.50 (0.43) when we adopted the GRV 94 HO (MRS-A) parton distribution functions for the proton \[14\] (\[15\]).

III. PROMPT DECAY OF $Z$ INTO CHARMONIUM

In the color-evaporation model the width for inclusive $Z$ decay into prompt charmonium is:

$$\Gamma(Z \to \text{prompt charmonium}) = \frac{1}{9} \int_{2m_{c}}^{2m_{D}} \frac{d\hat{\Gamma}_{c\bar{c}}}{dm},$$

(4)

where $\hat{\Gamma}$ is the partonic width for producing a $c\bar{c}$ pair. In order to obtain the partial width into a specific charmonium state we multiply the above expression by the appropriate fragmentation factor $\rho$, which was determined from charmonium photoproduction data \[10,11\]. Notice that the predictions for the $Z$ decay into charmonium are parameter-free. Therefore the production of charmonium at the $Z$ pole is a clean test of the validity of the color-evaporation model.
We have evaluated all the tree-level partonic amplitudes using the package MADGRAPH [10]. The leading-order process in $\alpha_s$ is $Z \to c\bar{c}g$, which leads to the production of a charmonium state and a hard jet. The partial width $Z \to \psi g$ is only $6 \times 10^{-7}$ GeV, for $m_c = 1.45$ GeV, $\rho_\psi = 0.50$, and $\alpha_s(2m_c) = 0.235$. It is small because the virtual quark propagator suppresses the amplitude by a factor of the order $m_c/m_Z$. This situation persists even when taking into account the next-to-leading order corrections. Therefore, this decay mode is too small to be seen at LEP I.

Although formally of higher order in $\alpha_s$, the dominant process for the inclusive decay of the $Z$ into charmonium is $Z \to c\bar{c}q\bar{q}$, where $q = u, d, s, c,$ and $b$. We show in Fig. 1 the Feynman diagrams contributing to this process. Table I contains the color-evaporation-model predictions for the $Z$ partial width into the different partonic final states [17]. In order to access the uncertainties in the determination of the fragmentation factor $\rho_\psi$ from the photo-production of $\psi$ data, we considered the values $\rho_\psi = 0.50$ and 0.43 which were obtained from $\psi$ photoproduction using the GRV 94 HO structure functions with $m_c = 1.45$ GeV and MRS-A distribution functions with $m_c = 1.43$ GeV, respectively [11]. Notice that the branching fraction of $Z$ into prompt $\psi$ is $(1.7-1.8) \times 10^{-4}$. This is to be contrasted with the color-singlet model which predicts a branching fraction for direct $\psi$ in $Z$ decay of the order $3 \times 10^{-5}$ [18]. The color-evaporation model leads to a branching fraction larger by almost an order of magnitude.

Using vertex detectors the LEP collaborations have recently been able to separate the prompt production of $\psi$ from those originating from $b$-hadron decays [8,9]. The OPAL collaboration reports that

$$B(Z \to \text{prompt } \psi + X) = (1.9 \pm 0.7 \pm 0.5 \pm 0.5) \times 10^{-4}.$$ 

Therefore, the color-evaporation-model prediction is in excellent agreement with the LEP results, illustrating how this approach gives a complete picture of the charmonium production in hadron-hadron, $\gamma$-hadron, and $Z$ decays. At this point we should point out that the color-octet model predicts a branching ratio $Z \to \psi + X$ to be $2.9 \times 10^{-4}$ [8], which is also
in agreement with the experimental data.

In Fig. 2 we show the energy distribution \((d\Gamma/dz\text{ with } z = 2E\psi/M_Z)\) of prompt \(\psi\) for the dominant process \(Z \rightarrow c\bar{c}q\bar{q}\), where we summed over all quark flavors. The band between the dashed curves is the distribution predicted by the color-evaporation approach, when we consider the different values for \(\rho_\psi\) given above in order to access the theoretical uncertainties. For comparison, we show the same distribution for the dominant process \((Z \rightarrow \psi q\bar{q})\) computed in the context of the color-octet (singlet) model represented by the upper (lower) solid curve [19].

As we can see from Fig. 2, the color-evaporation distribution is softer than the one predicted by the color-singlet model; it dominates however over the entire allowed range for \(z\). Moreover, the color-evaporation mechanism and the color-octet model predict very similar distributions. Taking into account the theoretical uncertainties in the determination of the nonperturbative parameters describing these models, it is impossible to distinguish between them on the basis of this distribution. Therefore the measurement of the energy spectrum of the prompt \(\psi\) is not a distinctive signature of the color-octet model, contrary to what is claimed in Ref. [19]. In the OPAL analysis of the production of prompt \(\psi\)'s [8], it is shown that the color-octet model describes the momentum distribution of prompt \(\psi\). Since this model and the color-evaporation mechanism have similar predictions for this distribution, we can conclude that this last model also describes the available data.

**IV. CONCLUSIONS**

The predictions of the color-evaporation model for the production of prompt \(\psi\) on \(Z\) decays have no free parameters once we use the photoproduction of charmonium to constrain the nonperturbative parameters of the model. We showed that this approach gives a good description of the available data on the production of prompt \(\psi\) at LEP I. Taking into account the success of this model to describe the photo- and hadroproduction of charmonium, we can conclude that this model gives a robust and simple parametrization of all
charmonium physics.

We would like to stress that the color-evaporation model has the same degree of success as the color-octet mechanism; however, the number of free parameters in the color-evaporation approach is smaller. The $\psi$'s produced through the color-evaporation mechanism are basically unpolarized since the polarization information is lost because of the multiple soft-gluon exchanges $^{[20]}$. On the other hand, the (non)polarization of $\psi$ is hard to explain in the framework of the color-octet model $^{[21][22]}$. Therefore, the measurement of the polarization of the produced charmonium may very well be a tool to discriminate between these competing descriptions.

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TABLE I. Partial decay widths for $Z \rightarrow \psi + X$. The results were obtained using $\rho_\psi = 0.50 [0.43]$, $m_c = 1.45 [1.43] \text{ GeV}$, and $\alpha_s(2m_c) = 0.235 [0.236]$.

| $X$         | $\Gamma(Z \rightarrow \psi + X) \text{ (GeV)}$ | $\text{Br}((Z \rightarrow \psi + X)$ |
|-------------|-----------------------------------------------|----------------------------------|
| $g$         | $6.0 \times 10^{-7} [5.6 \times 10^{-7}]$     | $2.4 \times 10^{-7} [2.3 \times 10^{-7}]$ |
| $u\bar{u}, d\bar{d}, s\bar{s}$ | $2.8 \times 10^{-4} [2.6 \times 10^{-4}]$     | $1.1 \times 10^{-4} [1.0 \times 10^{-4}]$ |
| $c\bar{c}$  | $7.8 \times 10^{-5} [7.3 \times 10^{-5}]$     | $3.1 \times 10^{-5} [2.9 \times 10^{-5}]$ |
| $b\bar{b}$  | $8.4 \times 10^{-5} [7.9 \times 10^{-5}]$     | $3.4 \times 10^{-5} [3.2 \times 10^{-5}]$ |
| $q\bar{q} \text{ (all } q)$ | $4.4 \times 10^{-4} [4.1 \times 10^{-4}]$     | $1.8 \times 10^{-4} [1.7 \times 10^{-4}]$ |
FIG. 1. Feynman diagrams leading to $Z \to c\bar{c}q\bar{q}$. In the case $q = c$ we must also add the crossed diagrams.
FIG. 2. Energy spectrum $d\Gamma/dz$ of the $\psi$ from the dominant partonic process $Z \to c\bar{c}q\bar{q}$. The upper (lower) solid curve stands for the prediction of the color octet (singlet) model according to Ref. [19]. The upper (lower) dashed curves is the distribution predicted by the color evaporation approach when we take $\rho_\psi = 0.50 \ (0.43)$, $m_c = 1.45 \ (1.43)$ GeV, and $\alpha_s(2m_c) = 0.235 \ (0.236)$. 