Effects of surface drilling grouting on deformation properties of thin rock stratum above goaf

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Abstract. Strata movement of coal mining for surface drilling grouting of overlying loose layers is distinctly different from that of traditional caving mining. In order to calculate strata movement accurately after full-size grouting, based on the shear transfer mechanism along foundation horizontal, a mechanical model of strata movement was established using the theory of Pasternak double-parameter beam on elastic foundation, and the deflection differential equations of rock beam were derived. Then the influence mechanisms of varied grouting parameters were discussed and the results showed that, the maximum deformation change value of roof increase non-obviously with the increasing of elastic modulus ratio, while decrease non-linearly with the grouting thickness. Thus can provide theoretical reference for strata displacement control in the same or similar projects.

1. Introduction
A thin rock stratum above coal seam can be found widely scattered in some coalfields of China and other countries, and its sudden breakage has significant impact on mining activities, even catastrophic losses. Usually, controlling the roof deformation and roof stability have remained universally technical challenge facing scientific researchers of mining engineering. The key strata theory about strata movement and it’s control has been introduced and promoted at home and abroad, for example, pressure arch theory [1, 2], hinged rock block theory [3], voussoir beam theory [4-6], elastic foundation beam theory [7, 8] and plate theory [9]. For the thin rock stratum above coal seam, its related mechanisms have also been studied by researchers. Using analytical solutions based on the theory of an elastic thin plate, Li et al. [10] studied the relationship between the elastic foundation coefficient and the roof deformation. Huang [11] obtained the critical unbroken conditions of main roof using the principle of virtual work. Using numerical simulation, theoretical analysis and in-site measurement, Fang et al. [12] studied the overlying strata movement rules, the result indicates that bedrock thickness and mechanical properties and thickness of surface soil are the key influential elements for the stable structure of “stacked layer of blocks”. Du et al. [13] analysed the deformation and breakage law of semi-arch structural model of immediate roof in Sima Coal Mine of Lu’an in China. Li et al. [14] analysed the regularity of strata behaviour in mining face under different ratio of rock and loadings. The aforementioned investigations are mainly for untreated roof layers, as to the behaviour of reinforced roof, its correlative reinforcement effect should be thoroughly studied.

In this paper we take the grouting technology and site strata in Ordos Coal Field in China. The roof behaviour is outlined based on the mathematical model and analytic solving. A method of Pasternak elastic foundation beam is proposed for deformation assessment of roof stratum, and the transfer matrix method is used for the solving of beam equations. Finally, the influential effects of grouting parameters are presented.
2. Site description
The Stone GeTai Coal Mine is located in Ordos, Inner Mongolia of China. This study area (mining area 22303) lies in the fourth panel of this mine. According to the field drilling investigation in this mine and adjacent mining experience, the overlying strata with thin bedrock and alluvium have revealed. The main strata profile is nearly horizontal distribution with an angle of 1° or so. The cover depth of coal seam is about 60-100 m, including approximately 50 m of loose bed. The roof stratum is mainly composed of fine-grained sandstone with a thickness of 10-50 m. The schematic section view of 10 m thick rock stratum of this mine is illustrated in Figure 1. Under such geological conditions, the fracture in coal roof grows quickly during mining, so the coal mining length will be shortened covering the ultrathin bedrock and the mining efficiency will be much lowered down. Thus, additional reinforcement is needed to stabilize the overlying strata on the basis of the concept of establishing pressure arches by grouting in loose bed.

3. Mathematical model and solution
Previously, it was well believed that the roof layer cracked primarily by bending induced tension. To analyze the mechanisms of the roof, we assume that the roof acts as an Euler-Bernoulli beam which is clamped at each end, and the overlying reinforced soil layer can be simulated as Pasternak double-parameter elastic foundation considering the shear transfer mechanism along horizontal, then the solutions for the maximum beam deflection can also be obtained using the beam differential equation. This simplified analytical model is illustrated in Figure 2. The Cartesian coordinate axes $x$, $y$ and $z$ are defined in terms of the un-deformed beam. The beam is taken to be made of homogeneous material with thickness $h$, breadth $B$ and length $L$. The vertical stress at both ends of the beam which we denoted as $\rho g H * BL/2$ is viewed as equivalently transmitted from the overlying strata above goaf, $\rho$ is the average density of the strata and $g$ is the acceleration due to gravity, $H$ is the depth of overlying strata. $k$ and $G_p$ denote equivalent foundation reaction coefficient and shear transfer coefficient respectively.
Using the analysis relevant to beam element with the width $B$ and force equilibrium conditions, the vertical displacement along the Euler-Bernoulli beam is denoted by $w(x)$, then the bending moment $M(x)$ and shearing force $Q(x)$ can be expressed as the following

$$\frac{dQ(x)}{dx} = p(x)B^*$$
$$\frac{dM(x)}{dx} = Q(x)$$

where $p(x)$ is the vertical reaction of beam foundation and $B^*$ is the equivalent calculation width, given by

$$p(x) = kw(x) - G_\rho \frac{dw^2(x)}{dx^2}$$

$$B^* = B + \frac{G_\rho}{k}$$

Based on the theory of material mechanics, the bending moment about the origin has magnitude

$$M(x) = EI \frac{dw^2(x)}{dx^2}$$

Where $E$ represents the Young’s modulus of the roof rock and $I$ is the moment of area about the y-axis.

Replacing equation (3) for $M(x)$ in equation (1), we obtain

$$\frac{d^4w}{dx^4} - \frac{G_\rho B^*}{EI} \frac{d^3w}{dx^3} + \frac{kB^*}{EI} w = 0$$

The above deduced equation (4) is the required differential equation for $w(x)$, we may solve it to obtain the general solution

$$w(x) = C_1 \sinh(\alpha x) \cos(\beta x) + C_2 \cosh(\alpha x) \cos(\beta x) + C_3 \sinh(\alpha x) \sin(\beta x) + C_4 \cosh(\alpha x) \sin(\beta x)$$

where,

$$\alpha = \sqrt{\frac{kB^*}{4EI} + \frac{G_\rho B^*}{4EI}}$$

$$\beta = \sqrt{\frac{kB^*}{4EI} - \frac{G_\rho B^*}{4EI}}$$

Using equations (5) and (6), the other variables $M(x)$ and $Q(x)$ are found as

$$\begin{bmatrix} M(x) \\ Q(x) \end{bmatrix} = \begin{bmatrix} A_1 & A_2 & A_3 & A_4 \\ D_1 & D_2 & D_3 & D_4 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix}$$

in which,

$$A_1 = \frac{(\alpha^2 - \beta^2) F_1 - 2\alpha\beta F_4}{\alpha^2 - \beta^2}$$
$$A_2 = -\frac{\alpha^2 - \beta^2}{\alpha^2 - \beta^2} F_2 - 2\alpha\beta F_3$$
$$A_3 = \frac{\alpha^3 - 3\alpha^2 \beta}{\alpha^2 - \beta^2} F_1 + 2\alpha\beta F_2$$
$$A_4 = \frac{\alpha^3 - 3\alpha^2 \beta}{\alpha^2 - \beta^2} F_1 + 2\alpha\beta F_2$$

$$D_1 = \frac{\beta^3 - 3\alpha^2 \beta}{\alpha^2 - \beta^2} F_3 + (\alpha^3 - 3\alpha^2 \beta^2) F_2$$
$$D_2 = -\frac{\beta^3 - 3\alpha^2 \beta}{\alpha^2 - \beta^2} F_3 + (\alpha^3 - 3\alpha^2 \beta^2) F_2$$
$$D_3 = \frac{(3\alpha^2 \beta - \beta^3) F_3 + (\alpha^3 - 3\alpha^2 \beta^2) F_2}{\alpha^2 - \beta^2} + G_\rho B^* F_2$$
$$D_4 = \frac{(3\alpha^2 \beta - \beta^3) F_3 + (\alpha^3 - 3\alpha^2 \beta^2) F_2}{\alpha^2 - \beta^2} + G_\rho B^* F_2$$
Using the internal force equilibrium and deformation coordination conditions and the transfer matrix method, the roof rock is divided into \( n \) beam sections according to the grouting influence ranges. On the interval of \( x_i \) and \( x_{i+1} \), the resulting algebra can easily be exploited

\[
\begin{align*}
& F_1 = \sinh (\alpha x) \cos (\beta x) \\
& F_2 = \cosh (\alpha x) \cos (\beta x) \\
& F_3 = \sinh (\alpha x) \sin (\beta x) \\
& F_4 = \cosh (\alpha x) \sin (\beta x) \\
\end{align*}
\]

(10)

Then, let \( \lambda = \frac{G_p B^*}{EI} \) and the transfer matrix \( X_i \) is

\[
X_i = \begin{bmatrix}
F_1 & F_1' & -E I F_1^* & -E I F_1'^* + G_p B^* F_1^* \\
F_2 & F_2' & -E I F_2^* & -E I F_2'^* + G_p B^* F_2^* \\
F_3 & F_3' & -E I F_3^* & -E I F_3'^* + G_p B^* F_3^* \\
F_4 & F_4' & -E I F_4^* & -E I F_4'^* + G_p B^* F_4^*
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 3\alpha^2 - \beta^2 - \lambda & 0 & -1 \\
0 & 2\alpha(\alpha^2 + \beta^2) & 0 & 0 \\
0 & 0 & 2\beta \alpha & 0 \\
2\beta \alpha & 0 & 0 & 2E I \alpha \beta
\end{bmatrix}
\]

(12)

Then, let \( S(x) = \begin{bmatrix} w(x) \theta(x) M(x) Q(x) \end{bmatrix}^T \), we get the solution along the beam extension

\[
S(x=L) = \prod_{i=1}^{n} X_i S(x=0)
\]

(13)

In order to get the constants of \( C_1-C_4 \), the boundary conditions at both ends of the beam (\( x=0 \) and \( x=L \)), are

\[
\begin{align*}
\frac{dw}{dx} & \bigg|_{x=0} = 0 \\
\frac{dw}{dx} & \bigg|_{x=L} = 0
\end{align*}
\]

(14)

4. Discussion of the results

According to normal test, the mechanical properties obtained from samples of the roof rock stratum follows: uni-axial compressive strength \( \sigma_c = 79.9 \) MPa, uni-axial tensile strength \( \sigma_t = 2.63 \) MPa, elastic modulus \( E = 6829 \) MPa, Poisson's ratio \( \mu = 0.22 \). Based on the grouting technology, the grout diffusion radius \( r = 1.0 \) m, height \( h^* = 20 \) m, the foundation parameters are: \( k = 2.4 \times 10^6 \) N/m\(^3\), \( G_p = 7.69 \times 10^8 \) N/m for unreinforced area in loose bed; and \( k = 3.75 \times 10^6 \) N/m\(^3\), \( G_p = 5.0 \times 10^8 \) N/m for reinforced area.

We chose the isolated coal pillar spacing \( L = 60 \) m, roof width \( B = 1 \) m, thickness \( h = 10 \) m, the displacement and curvature magnitude along beam section are as shown in Figure 3-Figure 4, by Matlab programming.
In figure 3, the minimum displacement occurs at \( x=0 \) and \( x=L \) and the maximum displacement at \( x=L/2 \). In figure 4, the curvature has two turning points which are that \( w'(x = 0) > 0 \) and \( w'(x = L) > 0 \) while \( w'(x = L/2) < 0 \). In the meanwhile, under the given established conditions in the article, the calculation results of Winkler and Pasternak, in a certain degree, have obvious differences. Therefore, as to the reinforced foundation, the influence of shear effect on the deformation of roof may be very obvious.

5. Parameter analyses

To analyze the influence mechanisms of drilling grouting on roof deformation, using the Pasternak model proposed above, we start by considering non-dimensional variables \( \tilde{E} \), \( \tilde{H} \) and \( \tilde{w} \) where

\[
\tilde{E} = \frac{E_2}{E_1}, \quad \tilde{H} = \frac{h^*}{h}, \quad \tilde{w} = \frac{W_2 - W_1}{W_1} \times 100\%
\]

(15)

(16)

where, \( E_1 \) and \( E_2 \) denote the elastic modulus of no grouting and grouting region, accordingly \( W_1 \) and \( W_2 \) denote the maximum relative displacement, respectively; \( h^* \) is the grouting reinforced height.

According to [16], the foundation coefficients \( k \) and \( G_0 \) can be expressed as the relationship of the elastic modulus and thickness of strata. Then in the next paragraph, we will study the influential mechanism of grouting material and geometric factors.

5.1. Elastic modulus ratio \( \tilde{E} \)

The elastic modulus ratio \( \tilde{E} \) reflects the change of reinforced strata by drilling grouting. Figure 5 shows us the non-dimensional displacement \( \tilde{w} \) for values of \( \tilde{E} \) in the range 1-25 with a step size of 1.

In figure 5, we see that the non-dimensional displacement \( \tilde{w} \) increases nonlinearly from 0 to 0.7% as \( \tilde{E} \) increases from 1 to 25, but the amplification is not obvious on the whole. It increase quickly within the scope of 1-5, and then slows down with the increase of the modulus.

5.2. Grouting thickness ratio \( \tilde{H} \)

Figure 6 shows us the non-dimensional displacement \( \tilde{w} \) for values of \( \tilde{H} \) in the range 0.5-5 with a step size of 0.5.

In figure 6, we see that the non-dimensional displacement \( \tilde{w} \) decreases nonlinearly with the increase of the thickness of grouting strata, attenuation amplitude is gradually increasing with the increase of grouting thickness ratio \( \tilde{H} \), at \( \tilde{H} >3 \), the attenuation rate speedsup sharply, even to infinite. On the other hand, for different values of elastic modulus ratio \( \tilde{E} \), the larger the modulus ratio is the more obvious attenuation we obtain. Thus, appropriate optimization of grouting parameters is crucial for decreasing the roof deformation and improving the coal extraction efficiently.
6. Conclusions
A simplified mathematical model of the deformation behaviour of thin rock roof during the extraction of coal seam has been proposed that accounts for the compressive stresses created by the overlying strata. Based on Pasternak foundation beam and transfer matrix theory, the roof deformation is investigated and the influence parameters of drilling grouting have been discussed: (1) Pasternak double parameter foundation model is more reasonable to simulate the shear transfer behaviour of reinforced stratum; (2) Different boundary condition assumption has important influence on the deformation characteristics of different calculation length of rock beam; (3) The key factors affecting the characteristics and development regularity of overlying rock beam deformation depends mainly on the elastic modulus, grout diffusion radius and the thickness of grouting.

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