Statistics of confined turbulent flows with and without rotation effects

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Abstract. A volume penalization method is implemented in a tri-periodic pseudo-spectral code. The numerical stability constraint of the method is studied and released with an original implicit formulation of the penalization term. The accuracy of the method in representing no-slip boundaries is tested by considering the impact of a three-dimensional vortex ring on a wall. This technique is then used to assess the influence of cylindrical boundaries on turbulent flow fields and on their structure at given radial positions. The Lagrangian study shows that the axial direction of the open-ends cylinder is privileged for the orientation of the turbulent structures. After these cases in which kinetic energy decays, we finally test rotating the outer cylindrical walls to force “naturally” the flow towards kinetic energy statistical equilibrium.

1. Introduction

The canonical character of homogeneous turbulence makes it a durable topic of interest for scientists as several questions are still pending about the phenomenology of turbulence. Kolmogorov theory (see Kolmogorov (1941); Batchelor (1953); Frisch (1995)) has dramatically improved our understanding of turbulence, but it only rigorously applies to homogeneous isotropic turbulence. In actual flows, e.g. in engineering, environmental, or geophysical contexts, the concept of homogeneity is never fully verified, as boundaries are always present in nature. Moreover, many kinds of distortions apply to the turbulent flow, among which mean deformations or shear, or body forces such as the buoyancy force in density stratified flows, the Coriolis force in rotating flows, the Lorentz force in electrically conducting fluids submitted to a magnetic field.

In this numerical work, we introduce two elements of additional complexity to the case of isotropic homogeneous turbulence: the presence of solid boundaries, and of an external body force, the Coriolis force. In so doing, we would like to overcome the mere context of isotropic turbulence, as well as consider the global modification of the structure of turbulence and its statistics, when it is confined within solid walls. Of course, when the boundaries are far enough from the fluid volume considered, turbulence may be considered homogeneous, but how far is enough?

To begin with, simple geometries can be considered, with sufficient symmetry properties to relieve the statistical analysis from a complete three-dimensional dependence. A first candidate is a cylindrical container with open ends, which also has the advantage of being compatible—
in terms of symmetries —with the application of solid body rotation with an axis aligned with that of the cylinder.

Solid body rotation applied to homogeneous turbulence has been extensively studied (Hopfinger et al. (1982); Bardina et al. (1985); Jacquin et al. (1990); Cambon et al. (1997)) and is still currently being investigated (e.g. experimentally by Moisy et al. (2011)). Rotating turbulence exhibits elongated flow structures and modified dynamics, and it is of interest to observe the effect of a finite enclosure onto the structures of rotating turbulent flows.

In the great majority of the numerical codes used to take into account the role of solid boundaries on fluid flows, a refinement of the numerical grid is achieved close to the walls or obstacles on which the conditions are imposed, so as to fairly represent the sharp gradients in the boundary layers. Unfortunately, it can be shown that the greatest accuracy— regarding the rapidity of its convergence —is obtained with spectral codes. The major drawback of these codes is that they can only be used in very few regular geometry (homogeneous flows, channel flows, pipe flows, etc).

Another method for introducing walls or obstacles in numerical codes is to use Immersed Boundary (IB) methods (see Mittal & Iaccarino (2005) for an extensive review), following the pioneering work of Peskin (1972, 1977). Rather than treat the presence of the walls with explicit boundary conditions, these methods consist in adding a term in the Navier–Stokes equations that forces the flow to rest in the volumic wall regions. The formulation of the method is such that the additional forcing is equivalent to the global pressure modification due to the action of the walls on the flow. One advantage of this method is the fact that the mesh does not need to conform to the shape of the solid body, and thus several geometries can be envisaged without additional effort. From Peskin to present time, lots of modifications and refinements of this method have been done.

In this article, the results are obtained with the volume penalization method which was first introduced by Arquis & Caltagirone (1984). This numerical technique, described in section 2, was proven to converge to the Navier–Stokes equations with a bound of the error of the order $\eta^{1/2}$, where $\eta$ is the penalization parameter (Angot et al. (1999), Carbou & Fabrie (2003)). Several works were performed in the case of two-dimensional turbulence, in e.g. Schneider (2005), Keetels et al. (2007b, 2009), Kadoch et al. (2008). Note that in most of the above studies, the numerical stiffness due to the presence of the penalization term introduces a stability constraint related to the penalization parameter $\eta$, the better the representation of the wall the stronger the constraint. We rather use here the implicit treatment of the penalization term proposed in Jause-Labert & Godeferd (2011b) and show in section 3 a validation of the accuracy of the method by computing a wall-impacting vortex, a classical test case in two dimensions, less common in three dimensions.

In the present work, we perform pseudo-spectral direct numerical simulations (DNS) of turbulence, introducing a cylindrical geometry with an axis along that of rotation; thus we extend the analysis proposed by Kadoch et al. (2008) to the case of three-dimensional turbulence, in which vortex stretching completely modifies the behaviour with respect to two-dimensional turbulence. In addition, we consider the anisotropic effect of rotation, and compare the Eulerian structure and the dispersion properties of decaying turbulence to that of decaying homogeneous turbulence, aiming at isolating the most important phenomena linked to the applied external rotation and to confinement (presented in section 4). We shall especially consider the inhomogeneous regions in the flow and the influence of the distance to the wall on turbulence statistics. Conditional probability density functions of velocity and conditional length scales will be presented. Lagrangian results will also be discussed in relation with the structuration of turbulence. In section 5, we consider the rotation of the cylindrical boundaries around the axis of symmetry, the aim being the limitation of the decay of kinetic energy. Conclusions are drawn in section 6.
2. Description of the volume-penalization method

We consider the flow of an incompressible Newtonian fluid of density $\rho$ and of kinematic viscosity $\nu$ in a domain $D$, surrounded by a solid domain $S$. The penalization technique consists in solving the Navier–Stokes equations with an extra penalization term as shown by equation (1) in the whole domain $D \cup S$, with an implicit summation over the repeated subscript $j$:

$$\partial_t u_i + u_j \partial_j u_i = -\rho^{-1} \partial_i p + \nu \partial_j \partial_j u_i - \eta^{-1} \chi (u_i - u_{s,i})$$  \tag{1}

where $u_i$ is the velocity component, $p$ is a modified pressure term and $u_{s,i}$ is the velocity component of the solid wall (input parameter). $\eta$ is called the penalization parameter and is related to the porosity of the solid media, the smaller the more impermeable. $\chi$ is the mask function defining the geometry of the fluid domain:

$$\chi (x, t) = \begin{cases} 1 & \text{if } x \in S \\ 0 & \text{if } x \in D. \end{cases}$$  \tag{2}

We must add to equation (1) the equation of conservation of mass which is simply, for an incompressible field: $\partial_t u_j = 0$.

The spectral counterpart of equation (1) in the Fourier space is:

$$\partial_t \hat{u}_i + \nu k^2 \hat{u}_i = P_{ij} \left[ \varepsilon_{jmn} \mathcal{F} \{ u_m \omega_n \} - \eta^{-1} \mathcal{F} \{ \chi (u_j - u_{s,j}) \} \right]$$  \tag{3}

where $\varepsilon$ is the Kronecker symbol. $\mathcal{F}$ denotes the Fourier coefficient of a spatial function and $\mathcal{F}$ is the symbol used for a Fourier Transform. $k_i$ is the wave vector component in the $i$ direction. The elimination of the pressure in this equation is due to the mass conservation equation, that is, in the spectral space, $k_j \hat{u}_j = 0$: for an incompressible field, $\hat{u}(k)$ is orthogonal to $k$, explaining the presence of a projection operator $P_{ij} = \delta_{ij} - k_i k_j k^{-2}$ in equation (3) which yields a pressure-free equation.

In order to implement the penalization technique in our existing tri-periodic pseudo-spectral code, we take advantage of the round-trip in the physical space to add the penalization term to the nonlinear one. This method is easy to implement although it introduces the following numerical stability condition (see Angot et al. (1999)):

$$\eta \geq \Delta t / 2$$  \tag{4}

If one desires to compute a flow with no-slip wall boundary conditions, one wishes $\eta$ to be as close to zero as possible, which is clearly incompatible with a time step also vanishing. Even though this issue can easily be solved in two-dimensional turbulence (e.g. Kadoch et al. (2008); Schneider (2005); Keetels et al. (2007a)) by increasing the number of time steps in affordable computational times, this stability condition makes this method not suitable for the study of long-term three-dimensional turbulence, where the computational effort required for each time step is much greater. This problem was overcome in Jause-Labert & Godeferd (2011b) where an implicit formulation for the penalization term is explained and validated. The following numerical scheme is illustrated here in a first-order Euler scheme for clarity (the following computations use a 3rd order Adams-Bashforth scheme):

$$\hat{u}_i^{n+1} = P_{ij} \left[ \mathcal{F}^{-1} \left\{ \frac{(1 - \nu k^2 \Delta t) \hat{u}_j^n - \Delta t P_{jl} \left[ \mathcal{F} \{ \varepsilon_{lmq} u_m^n \omega_q^n \} \right]}{1 + \eta^{-1} \Delta t \chi^{n+1} u_{s,j}^{n+1}} \right\} \right]$$  \tag{5}

where $n$ denotes the time-step $t^n$ at which a variable is evaluated. Implementing the penalization term in this way allows to suppress any penalization-related numerical stability constraint. The
price to pay is an additional round-trip in physical space, but overall, the net computational gain is clear with respect to using an explicit time-scheme.

Finally, one must also note that a sharp transition from 1 to 0 in the mask function definition introduces numerical oscillations. To avoid such unphysical phenomenon, we choose to smooth the mask function as done by Kolomenskiy & Schneider (2009) by convoluting it with a Gaussian filter. Then, we will denote the smoothed mask function with $\chi$, and the original sharp one with $\chi_0$.

3. Numerical validation of the 3D penalization method

In Jause-Labert & Godeferd (2011b), several benchmarks were performed to validate the accuracy of the method. It is shown that the reflection of inertial or internal-gravity waves on penalized walls is well reproduced when compared to what was obtained in Godeferd & Lollini (1999) with a channel flow code—periodic in two directions and confined in the third with a discretization based on Chebychev polynomials. It has also been found that the spatial convergence is relatively low ($\in [N^{-0.4}, N^{-1}]$).

In this paper, we focus on the convergence with $\eta$ of the method to the Navier–Stokes equations with no-slip boundary conditions. Angot et al. (1999) found that the bound of the error on the velocity field evolves as $\eta^{1/4}$. This analytical estimation was refined by Carbou & Fabrie (2003) with $\eta^{1/2}$. We propose here to confirm such results by considering the impact of a vortex ring on a penalized wall and compare with the experiment of Walker et al. (1987) and to the simulations by Orlandi & Verzicco (1993).

In the following simulations, we set the resolution to $N_x = N_y = N_z = 512$ and the domain size to $L_x = L_y = L_z = 6\pi$. The mask function is:

$$\chi_0(x) = \begin{cases} 1 & \text{if } x \in [0, L_x/6] \cup [5L_x/6, L_x] \\ 0 & \text{if } x \in [L_x/6, 5L_x/6] \end{cases}$$

and the initial condition is a vortex ring of radius $r_0$, of thickness $a$ and of circulation $\Gamma$. It is defined from its vorticity distribution in cylindrical coordinates with unit vectors $(e_r, e_\theta, e_x)$:

$$\omega(r, \theta, x) = \omega_\theta e_\theta = \left(\Gamma / \pi a^2\right) \exp\left(-\left((x - x_0)^2 + (r - r_0)^2\right) / a^2\right) e_\theta$$

This defines a vortex ring with self-induced velocity $V_0$ in the direction of negative $x$, which is linked with $\Gamma$ and the ratio $a/r_0$ by Lamb’s approximation. A Reynolds number can then be defined by $Re_v = 2V_0r_0/\nu$. So as to confront our results to existing experimental data, we set $Re_v = 564$ as in Walker et al. (1987).

Figure 1 shows the evolution of the vortex ring, before, during and after the impact. We can clearly see the formation of secondary and tertiary vortices and finally the transformation into a three-dimensional flow, as described by Walker et al. (1987). This computation was achieved with a penalization parameter $\eta = 10^{-8}$. To demonstrate the convergence, we carry out the exact same computation with parameters $\eta \in [10^{-2}, 10^{-7}]$ and we compute the averaged difference on the enstrophy field after 3000 timesteps so that the comparison is done just after the impact. Figure 2 shows, in logarithmic scales, the evolution of the root-mean-square difference versus $\eta$. It exhibits that the convergence is of order 1 for the enstrophy, meaning an order 0.5 for the velocity, as expected by Carbou & Fabrie (2003).

4. Decaying confined turbulence in a steady cylinder

Once the method validated, we use it to study the global effects of confinement on turbulence. To that extent, we perform, as exhaustively explained in Jause-Labert & Godeferd (2011a),
**Figure 1.** Enstrophy field at different instants of the impact of the vortex ring ($\eta = 10^{-8}$). Top: Volume rendering. Bottom: Cut on a plane of symmetry. (a) and (d): $t = 0.002s$; (b) and (e): $t = 0.016s$; (c) and (f): $t = 0.024s$.

**Figure 2.** Convergence of the penalization method with the parameter $\eta$, from the rms error based on the enstrophy field $Z = \omega_j \omega_j / 2$. The straight solid line shows the power-law dependence.
penalized simulations of decaying turbulent flows in an open-ends cylinder of diameter $D = 0.9 \times L_x$. In addition to the effect of confinement, we choose to take into account the effects of background rotation, meaning that the intensity of the Coriolis force $F_c = 2\Omega \times u$—or equivalently the Rossby number $Ro_\lambda = U/2\Omega \lambda$ based on Taylor’s microscale $\lambda$—is another parameter of our simulations. Figure 3 shows the enstrophy field at the beginning of the decay with a Reynolds number based on Taylor’s Microscale $Re_\lambda = 75$ and $Ro_\lambda = 0.5$, the axis of rotation being parallel with the axis of the cylinder. With this choice of $Ro_\lambda$, the effects of the rotation are moderate and are of the same order of the inertial effects, which explains the fact that the structures are not clearly elongated along the axis of rotation, as in some previous works of low $Ro_\lambda$ rotating homogeneous turbulence. The discretization of the 3D periodic box is $N_x = N_y = N_z = 288$, its size is $L_x = L_y = L_z = 2\pi$ and this computation is denoted CYL2. During the whole decay process, we also follow 20000 trajectories, which allows us to extract Lagrangian statistics. The picture 3(b) shows the time evolution of a few trajectories for the computation CYL1 without rotation ($Ro_\lambda \to \infty$). In order to evaluate the effect of the confinement, we carry out homogeneous simulations with the same Reynolds and Rossby numbers. They are denoted HIT for the non-rotating case and HAT for the rotating one.

Figure 4 shows some of the extracted statistics (see the article in preparation, Jause-Labert & Godeferd (2011a), for an extended review of the results obtained). Figure 4(a) highlights one of the main effects of cylindrical confinement in the non-rotating case: for $r \geq 0.7$, the radial root-mean-square ($\text{rms}$) velocity starts to decrease, while the azimuthal $\text{rms}$ velocity remains constant until $r \approx 0.9$, and the vertical $\text{rms}$ velocity increases in this range ($0.7 \leq r \leq 0.9$). This shows that the decay of the energy in the radial direction due to the presence of the wall is compensated by a gain of vertical energy in this region. It suggests that there is a preferential direction for the energy when approaching the solid boundaries. In fact, the vertical direction also seems to be privileged over the azimuthal one in other statistics.

Figure 4(b) shows the time evolution of directional length scales which definition is given by
Figure 4. Statistics obtained from the penalized simulations. (a) Time-averaged radial evolution of the normalized \( \text{rms} \) velocities in cylindrical coordinates for the simulation CYL1. \( r = 0 \) is the axis of the cylinder and \( r = 1 \) denotes the penalized boundary. (b) Time evolution of the radial length scale \( L_{rr} \) for the confined simulations CYL1 and CYL2. (c) Lagrangian statistics: one-particle mean horizontal dispersion in the 4 simulations. The results of the non-rotating cases are purposely shifted upward for clarity.

the following equation:

\[
L_{rr}(t) = \frac{1}{2} \int_{0}^{R/3} \frac{\langle u_r(r + le_r)u_r(r) \rangle_\odot}{\sqrt{\langle u_r(r + le_r)^2 \rangle_\odot \langle u_r(r)^2 \rangle_\odot}} dl. \tag{8}
\]

\( \langle \rangle_\odot \) denotes the averaging over one of three regions: the inner region \( \odot \) where \( r \leq 1/3 \), the intermediate one \( \odot \) where \( 1/3 \leq r \leq 2/3 \) and the wall region \( \odot \) where \( r \geq 2/3 \). We can see that radial turbulent length scales are shorter when approaching the wall, in both the rotating and the non-rotating simulations. We also note that the curves for the rotating case are recognizable from the oscillations which are at the same frequency as the rotation rate.

Finally, we show on figure 4(c) one Lagrangian statistical quantity which is the one-particle dispersion in the horizontal plane (normal to the axis of rotation and symmetry of the cylinder) averaged over more than 99.7% of the 20000 trajectories. Less than 0.3% of the tracers marginally enter the solid region during the whole decay and those trajectories are then not taken into account for the computation of the Lagrangian statistics. This figure shows that the short-term behaviour of the dispersion is the same in confined or in homogeneous simulations and we recover the classical ballistic region where the mean-square displacement is \( \Delta X^2 \sim t^2 \). After this stage, the horizontal dispersion exhibits a clear effect of confinement: unlike homogeneous simulations, where a Brownian-like behaviour is expected, with an evolution \( \sim t \) — that returns to a \( \sim t^2 \) evolution for the rotating simulation —, the dispersion is reduced by the presence
5. Forced turbulence with rotating boundaries?

In the previous simulations, energy decays significantly due to bulk viscosity and boundary-layer-like dissipation, as in unforced homogeneous turbulence simulations. Long-term evolution studies are therefore not possible, unless increasing dramatically the Reynolds number, at a cost. Therefore, we expect to take advantage of the penalization method and its capability of moving the boundaries— or in that case simply giving them an azimuthal velocity —so as to inject energy in the flow, conveyed by the wall-close velocity gradients thus generated. The following question is raised: does the rotation of the boundary feeds the turbulence with enough kinetic energy, so that one can obtain a statistically stationary turbulent flow with an adequate rotation rate, or does it lead at large time, when the turbulent energy is completely dissipated, to a solid-body rotation of the fluid?

We carry out 3 simulations for 3 different rotation rates of the solid region: \( \Omega_s = \pi \) for R1, \( \Omega_s = 2\pi \) for R2 and \( \Omega_s = 10\pi \) for R3. The mask function is then, with \( L = L_x/2 = L_y/2 \):

\[
\chi_0(x, y) = \begin{cases} 
1 & \forall (x, y), 
\left( (x - L)^2 + (y - L)^2 \right)^{1/2} \in [0.85L, 0.95L], \\
0 & \text{elsewhere}.
\end{cases}
\]

(9)

Because of the long-time behaviour we want to observe, we need to compute an important number of time-steps, which involves a moderate resolution. We choose \( N_x = N_y = N_z = 128 \), and \( \eta = 10^{-8} \). Figure 5 shows the evolution of the azimuthally-averaged radial, azimuthal and vertical velocities for the three simulations. We can clearly see that the flow tends to turn into a solid-body rotation as the only meaningful component is the azimuthal one after a certain time. Moreover, the profile of this component with the radial position tends to be linear which is the definition of the solid body rotation. Figure 6 shows, in logarithmic scales, the temporal evolution of each component for the three simulations. It helps us understand that the turbulent kinetic energy decays in the radial and vertical directions. We can see that in the three simulations, the radial and vertical rms velocities seems to decay in the same way, the
Figure 6. Time evolution of the radial, azimuthal and vertical $\text{rms}$ velocities for the three rotating enclosure simulations.

curve of the radial component always being under the vertical one as it is the one normal to the solid wall. Let us also note that the solid-body rotation involves those components to vanish. However, they approach this state slowly, which means that the rotation of the boundaries tends to create a solid-body rotation for the long-term behaviour of the flow but nonetheless slows down the decay of the turbulence initially introduced.

6. Conclusions and perspectives

We use a volume penalization method in a tri-periodic pseudo-spectral code in order to study the decay of a turbulent flow field, with and without an additional Coriolis force. We characterize the turbulent state of the confined flow, and compare it to homogeneous turbulence. Both the Eulerian structure, from e.g. vortex orientation and inter-component energy redistribution, and the Lagrangian dispersion statistics are affected by the presence of the cylindrical container. We then took advantage of one of the possibilities offered by the method by rotating the cylindrical boundaries so as to modify the dynamics of the flow and slow down the turbulence decay. At the considered values of Reynolds number and rotation velocity, the flow tends to the laminar rigid rotation solution. A different strategy, with time-dependent boundary velocity, will be tested in future simulations, to achieve the goal of turbulent statistically steady state.

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