Cavity cooling of micro-mechanical oscillators has seen tremendous experimental progress in the past few years, also stimulated by theoretical analysis showing that the quantum ground state of such oscillators is achievable. In essence, through cavity mediated interaction the mechanical vibrations at kHz-MHz frequencies are converted to strongly damped sidebands of light modes at optical frequencies, that contain essentially no thermal excitations (thermal photons). Standard sideband cooling is effective only when the linewidth of the optical resonator $\kappa$ is smaller than the mirror vibration frequency $\omega_m$ and one has a sufficient optomechanical coupling. For a given mirror reflectance a small enough $\kappa$ can be reached using longer cavities, but strong optomechanical coupling then requires a very large intracavity field and input power.

Here we propose the alternative use of atomic, molecular or solid state electronic transitions as extremely localized narrow bandwidth oscillators, for the cooling process. These are coupled to the vibrational modes via a cavity light mode. Strong mirror-light and light-atom coupling can be achieved while still maintaining a narrow and tailorable resonance behavior. As the simplest generic example we will consider ensembles of two-level atoms with a narrow transition suitably detuned to the intrinsic decay rate of the mechanical oscillator $\gamma_m$ to yield $\bar{\gamma}_m = \Gamma + \gamma_m \gg \gamma_m$. For an oscillator of frequency $\omega_m$, this produces cooling to an effective temperature $T \simeq \hbar \omega_m / [k_B \ln(1 + 1/\bar{n})]$ ($k_B =$Boltzmann’s constant) where $\bar{n} = \gamma_m n/\bar{\gamma}_m + n_{res}$. Therefore cooling leads to a reduction of the initial average thermal vibration quantum number $n$ by a factor $\gamma_m/\bar{\gamma}_m$, but one has also an additional residual occupancy $n_{res} = A_s/\bar{\gamma}_m$ that is due to the residual Stokes scattering rate. Hence one best uses a high-finesse optical cavity (with $\kappa < \omega_m$, as illustrated in Fig.1) resonant with the blue anti-Stokes sideband leading to cooling to $n_{res} \simeq (\kappa/2\omega_m)^2$. 

Cavity assisted sideband cooling is based on unequal scattering of pump photons into higher/lower energy (anti-Stokes/Stokes) photons; the cavity modifies the density of optical modes around the laser frequency such that a higher density at the anti-Stokes frequency leads to a higher scattering rate into the more energetic sideband. Denoting the corresponding rates with $A_{as}$ and $A_s$, an effective extraction rate of vibrational quanta $\Gamma = A_{as} - A_s$ can be added to the intrinsic decay rate of the mechanical oscillator $\gamma_m$ to yield $\bar{\gamma}_m = \Gamma + \gamma_m \gg \gamma_m$. For an oscillator of frequency $\omega_m$, this produces cooling to an effective temperature $T \simeq \hbar \omega_m / [k_B \ln(1 + 1/\bar{n})]$ ($k_B =$Boltzmann’s constant) where $\bar{n} = \gamma_m n/\bar{\gamma}_m + n_{res}$. Therefore cooling leads to a reduction of the initial average thermal vibration quantum number $n$ by a factor $\gamma_m/\bar{\gamma}_m$, but one has also an additional residual occupancy $n_{res} = A_s/\bar{\gamma}_m$ that is due to the residual Stokes scattering rate. Hence one best uses a high-finesse optical cavity (with $\kappa < \omega_m$, as illustrated in Fig.1) resonant with the blue anti-Stokes sideband leading to cooling to $n_{res} \simeq (\kappa/2\omega_m)^2$. 

We predict ground state cooling of a micro-mechanical oscillator, i.e. a vibrating end-mirror of an optical cavity, by resonant coupling of mirror vibrations to a narrow internal optical transition of an ensemble of two level systems. The particles represented by a collective mesoscopic spin model implement, together with the cavity, an efficient, frequency tailorable zero temperature loss channel which can be turned to a gain channel of pump. As opposed to the case of resolved-sideband cavity cooling requiring a small cavity linewidth, one can work here with low finesses and very small cavity volumes to enhance the light mirror and light atom coupling. The tailored loss and gain channels provide for efficient removal of vibrational quanta and suppress reheating. In a simple physical picture of sideband cooling, the atoms shape the cavity profile to enhance/inhibit scattering into higher/lower energy sidebands. The method should be applicable to other cavity based cooling schemes for atomic and molecular gases as for molecular ensembles coupled to stripline cavities.

Micro-mechanical oscillator ground state cooling via intracavity optical atomic excitations

C. Genes and H. Ritsch
Institute for Theoretical Physics, University of Innsbruck, and Institute for Quantum Optics and Quantum Information, Austrian Academy of Sciences, Technikerstrasse 25, A-6020 Innsbruck, Austria

D. Vitali
Dipartimento di Fisica, Università di Camerino, I-62032 Camerino (MC), Italy
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We predict ground state cooling of a micro-mechanical oscillator, i.e. a vibrating end-mirror of an optical cavity, by resonant coupling of mirror vibrations to a narrow internal optical transition of an ensemble of two level systems. The particles represented by a collective mesoscopic spin model implement, together with the cavity, an efficient, frequency tailorable zero temperature loss channel which can be turned to a gain channel of pump. As opposed to the case of resolved-sideband cavity cooling requiring a small cavity linewidth, one can work here with low finesses and very small cavity volumes to enhance the light mirror and light atom coupling. The tailored loss and gain channels provide for efficient removal of vibrational quanta and suppress reheating. In a simple physical picture of sideband cooling, the atoms shape the cavity profile to enhance/inhibit scattering into higher/lower energy sidebands. The method should be applicable to other cavity based cooling schemes for atomic and molecular gases as for molecular ensembles coupled to stripline cavities.
The novel mechanism that we propose in this Letter can be well understood from the picture of cavity cooling shown in Fig.1 and is illustrated in Fig.1c. The added ensembles of two-level atoms, one prepared in the ground state (type 1), and the other one inverted (type 2), modify the cavity susceptibility, and consequently change the cavity induced scattering to the sidebands. Even in the limit of a cavity bandwidth much larger than the mirror vibration frequency the atoms still change the response function to inhibit Stokes scattering and enhance scattering into the anti-Stokes sideband. As we will see later, the first mechanism is much more efficient than the second in that, while it is characterized by slower cooling rates, it ensures a low non-zero temperature near the gain cooling. The difference comes from the fact that while the cavity filled with ground state atoms forms an effective zero-temperature bath, the noise inherent in any amplification process implies an effective non-zero temperature near the gain maximum of the inverted atoms. Cooling in the second case is then bound by the effective bath temperature.

Hamiltonian - A Fabry-Perot optical cavity is driven by a laser of frequency $\omega_1$ near one of the cavity resonances $\omega_c$ with detuning $\Delta_0 = \omega_c - \omega_i$. One of the mirror cavities is allowed to vibrate at the mechanical frequency $\omega_m$ and with an effective vibrating mass $m$. Two clouds of $N_{1,2}$ two-level atoms (TLA) with level splittings $\omega_{1,2}$ (detuned from the laser by $\Delta_{1,2} = \omega_{1,2} - \omega_i$) are positioned inside the cavity mode. The Hamiltonian of the system is $(h=1)$

$$H = H_0 + H_1 + H_{dis},$$

$$(1a)$$

$$H_0 = \omega_i a^\dagger a + \frac{\omega_{in}}{2}(p^2 + q^2) + \omega_1 S_{1z} + \omega_2 S_{2z},$$

$$(1b)$$

$$H_1 = g_1 \left( S_{1z} a + S_{1z} a^\dagger \right) + g_2 \left( S_{2z} a + S_{2z} a^\dagger \right),$$

$$(1c)$$

$$-G_0 a^\dagger a q + i\epsilon (a^\dagger e^{-i\omega_1 t} - ae^{i\omega_1 t}),$$

where $H_0$ is the free part, $H_1$ is the interaction part which will not be explicitly shown here but included in the Langevin equations. The free part includes the cavity mode, mirror and atom free energy where $a$ is the annihilation field operator with $[a, a^\dagger] = 1$, and $p$ and $q$ are the mirror quadratures with $[p, q] = i$ and the atoms are described by a set of collective spin operators satisfying $[S_{1,2z}, S_{1,2z}] = S_{1,2z}$ and $[S_{1,2z}, S_{1,2\pm}] = \pm S_{1,2\pm}$. The interaction contains the atom-field Tavis-Cummings term and the radiation pressure field-mirror coupling where $g_{1,2}$ is the single atom-field coupling strength while $G_0 = (\omega_c/L)^2 \sqrt{\hbar/m\omega_m}$ ($L$ is the cavity length) is the single photon-mirror optomechanical coupling. The laser driving shows up in $H_1$ as a displacement term of amplitude $\epsilon$ (directly connected to the input laser power). Not included in the Hamiltonian is a driving term that leads to an effective population inversion in type 2 atoms.

Linearization - For simplicity we perform a transformation from the spin algebra to a harmonic oscillator algebra for the atomic system via $c_1 = S_{1z} - \sqrt{N_1}$ and $c_2 = S_{2z} - \sqrt{N_2}$, such that: $\omega_1 S_{1z} \simeq \omega_1 (-N_1/2 + c_1^\dagger c_1)$ and $\omega_2 S_{2z} \simeq \omega_2 (N_2/2 - c_2^\dagger c_2)$ and we have $[c_i, c_j^\dagger] = \delta_{ij}$. The effect of $H_{dis}$ in Eq. (1) can now be added in a Langevin equation approach via the quantum noises $\dot{a}_{in}$, $\dot{p}_{in}$, $c_{1,in}$ and $c_{2,in}$ with the following nontrivial correlations: $\langle a_{in}(t)a_{in}^\dagger(t') \rangle = 2\kappa_0 \delta(t - t')$, $\langle p_{in}(t), p_{in}(t') \rangle = \gamma_m (2n + 1) \delta(t - t')$, $\langle c_{1,in}(t)c_{1,in}^\dagger(t') \rangle = 2\gamma_0 \delta(t - t')$ and $\langle c_{2,in}(t)c_{2,in}^\dagger(t') \rangle = 2\gamma_0 \delta(t - t')$. The cavity and atom reservoirs are at zero-temperature and are fully characterized by the decay rates $\kappa_1$ and $\gamma_2$ while the thermal mirror reservoir is described by the damping $\gamma_m$ and thermal occupancy $n$. Notice that $\gamma_1$ is the actual decay rate of atoms of type 1 to their real ground state; $\gamma_2$ is instead the effective optical pumping rate to the excited state and can be considered as an effective “decay” rate. The other levels involved in the optical pumping process are irrelevant for cooling and are neglected in the present two-level description.

As in [12], one can linearize the equations of motion for the atom-field-mirror system around steady state values to derive a set of quantum linearized Langevin equations (QLLEs)

$$\dot{q} = \omega_m p,$$

$$(2a)$$

$$\dot{p} = -\omega_m q - \gamma_m p + G \left( a + a^\dagger \right) + p_{in},$$

$$(2b)$$

$$\dot{a} = \left( -\kappa + i\Delta_f \right) a + iGq - iG_1 c_1 - iG_2 c_2 + a_{in},$$

$$(2c)$$

$$\dot{c}_1 = \left( -\gamma_1 + \Delta_1 \right) c_1 - iG_1 a + c_{1,in},$$

$$(2d)$$

$$\dot{c}_2 = \left( -\gamma_2 - \Delta_2 \right) c_2 - iG_2 a^\dagger + c_{2,in},$$

$$(2e)$$

where all the couplings are collectively enhanced, $G_{1,2} = g_1,2 \sqrt{N_{1,2}}$ and $G = G_0 \alpha$, with $\alpha$ the steady state intracavity field amplitude (which can always be assumed real). In Eqs. (2) $a$, $q$, $p$ and $c_{1,2}$ operators denote steady state fluctuations, while $\Delta_f = \Delta_0 - G_2/\omega_m$ is the effective cavity detuning.

In the following it will be useful to define $\epsilon_m(\omega) = \omega_m/\left( \omega_m^2 - \omega^2 - i\gamma_m \omega \right)$, $\epsilon_f(\omega) = 1/\left[ \kappa + i \left( \Delta_f - \omega \right) \right]$, $\epsilon_1(\omega) = 1/\left[ \gamma_1 + i \left( \Delta_1 - \omega \right) \right]$ and $\epsilon_2(\omega) = 1/\left[ \gamma_2 - i \left( \Delta_2 + \omega \right) \right]$ as the mirror, field, and atom bare susceptibilities. Fourier transforming Eqs. (2) one can derive the following set of coupled equations

$$\left[ \epsilon_m(\omega) \right]^{-1} q = G \left( a + a^\dagger \right) + p_{in},$$

$$(3a)$$

$$\left[ \epsilon_f(\omega) \right]^{-1} a = iGq + a_{in},$$

$$(3b)$$

where the effective field input noise is $a_{in} = a_{in} - i[G_1 \epsilon_1(\omega)c_{1,in} + G_2 \epsilon_2(-\omega)c_{2,in}].$ The field response function is modified by the atoms according to

$$\left[ \epsilon_f(\omega) \right]^{-1} = \left[ \epsilon_f(\omega) \right]^{-1} + G_1^2 \epsilon_1(\omega) - G_2^2 \epsilon_2(-\omega).$$

Physical picture. Cavity response - In the absence of atoms (setting $G_{1,2} = 0$) Eqs. (2) describe a typical optomechanical setup, which in the good-cavity limit $\kappa \ll \omega_m$ (resolved-sideband limit) and $\Delta_f = \omega_m$ leads to optimal cooling of the mirror state. This effect can be simply understood in terms of preferential scattering of sidebands at frequencies $\omega_{as,s} = \omega \pm \omega_m$ off the mirror, as illustrated in Fig.1, where the cavity response (i.e., $\left[ \epsilon_f(\omega) \right]$) is plotted around $\omega_m$ to the mechanical sidebands is depicted. The choice of $\omega_c = \omega_{as}$ allows the enhancement of the cooling sideband. When atoms are placed inside the cavity, their effect on the cavity response function leads to a modification of the cavity mode structure
around the resonance $\omega_c$ quantified by Eq. 4. This is illustrated in Fig. 1b for the bad-cavity limit where $\kappa \gg \omega_m$, and resonant cavity driving $\omega_l = \omega_c$. The type 1 atoms, placed at $\Delta_1 = -\omega_m$ induce a dip in the cavity profile at $\omega_m$ while the gain medium increases the cavity response at $\omega_{as}$. What results is an inhibition of scattering processes into the Stokes sideband and enhancement of scattering into the cooling sideband.

Based on the mode structure tailoring picture, we can now make some quantitative remarks on the efficiency of cooling that is expected from such a scheme. With the usual definition of the atom cooperativity $C_{1,2} = G_{1,2}^2/(\kappa_1,2)$, one can see that the dip value at $\omega_m$ scales down as $(1 + C_1)^{-1}$ while the peak at $\omega_{as}$ has a maximum that scales up $(1 - C_2)^{-1}$.

The widths of the dip/peak are $\gamma_1 = \gamma_1 (1 + C_1)$ and $\gamma_2 = \gamma_2 (1 - C_2)$, respectively representing the enhanced/reduced light–induced atomic linewidths. In the limit where the scattered sidebands fit inside the dip/peak, i.e. the resulting cooling rate is smaller than $\gamma_1, \gamma_2$, one then expects an inhibition of the Stokes peak (enhancement of the anti-Stokes peak) by a factor of the order of $(1 + C_1)^{-1} (1 - C_2)^{-1}$, respectively.

Cooling - For a proper derivation of cooling rates and residual occupancy we start with Eqs. 3. Notice the similarity with typical cavity-assisted cooling of a mirror, which can be obtained as a limiting case when $G_{1,2} = 0$ for which $\tilde{a}_{in} = \tilde{a}_{in}$ and $\dot{\epsilon}_f (\omega) = \epsilon_f (\omega) \bar{F} \left( \frac{\kappa}{\kappa^2} \right)$. The mechanical oscillator of Eq. 3(b) is driven by a thermal zero-average Langevin force $p_{in}$, and by a field–atom induced Langevin force $F = G \left[ \tilde{a}_{in} + \tilde{a}_{in}^\dagger \right]$. Following the approach of [10], one can find the cooling rate induced by the field–atom system and the mirror occupancy directly from the spectrum of the force $F, C_{1}(\omega) F (\omega') = S_F (\omega) \delta (\omega + \omega')$. From Eq. 3(b) one derives

$$S_F (\omega) = 2G \left[ |\epsilon_f (\omega)|^2 \left( \kappa + \gamma_1 G_1^2 |\epsilon_1 (\omega)|^2 \right) + |\epsilon_f (-\omega)|^2 \gamma_2 G_2^2 |\epsilon_2 (\omega)|^2 \right]. \quad (5)$$

The cooling and heating rates (anti-Stokes and Stokes, respectively) are computed as $A_{r,s} = S_F (\pm \omega_m) / 2$. We shall consider from now $\Delta_F = 0$, i.e., the cavity resonant with the laser, so that there is no cooling in the absence of atoms.

Cooling with ground state atoms - We restrict our discussion in the following to a cavity filled with atoms of type 1 (setting $G_2 = 0$) and derive the effective cooling rate and residual occupancy under resonance condition $\Delta_1 = -\omega_m$. First we stress that the effective temperature of the cavity is zero: $(\tilde{a}_{in}^\dagger (\omega) \tilde{a}_{in} (\omega')) = 0$. We identify the optimal regime as the bad-cavity regime $\kappa \gg \omega_m$, where the effect of the atoms on the cavity profile is to efficiently inhibit Stokes scattering for a bandwidth $\gamma_1$ around $\omega_m$. We ask that the bandwidth $\gamma_1$ be larger than the scattered sideband halfwidth, which in this limit and with the requirement $\gamma_1 \ll \omega_m$, one can approximate $|\epsilon_1 (\omega_m)| \simeq 1/(2\omega_m), |\epsilon_1 (-\omega_m)| \simeq 1/\gamma_1, |\epsilon_f (\omega_m)| \simeq 1/\kappa$ and $|\epsilon_f (-\omega_m)| \simeq 1/\kappa (1 + C_1)$. The scattering rates can be derived from Eq. 4 as $A_{as} \simeq G^2 / \kappa$ and $A_s \simeq G^2 / \kappa (1 + C_1)$ and consequently the cooling rate

$$\Gamma_1 \simeq \frac{G^2}{\kappa} \frac{C_1}{1 + C_1}. \quad (6)$$

The residual occupancy can be computed as $n_{r,es} = A_s/\gamma_m \simeq \frac{A_s}{\Gamma_1} \rightarrow C_1^{-1}$ (in the limit of large cooperativity). This result replaces the resolved-sideband limit final occupancy $n_{r,es} = (\kappa/2\omega_m^2)$ of the purely optomechanical case with no atoms. The advantage of adding ground state atoms within the cavity is obvious from the scaling of $n_{r,es}$ with the controllable and in principle unbounded parameter $N_1$. In fact, in the strong resolved-sideband limit $\kappa \ll \omega_m$, the cavity is far-off-resonance, and one has to use large input power for optimal cooling, while with atoms one can make $n_{r,es}$ smaller and smaller by increasing $N_1$. Illustrations of the dependence of $\gamma_m/\gamma_m$ and $n_{r,es}$ on the normalized cooling strengths $G_1/\omega_m$ when $G_2 = 0$ is shown in Fig. 2, where red lines correspond to the exact numerical solution of Eqs. 3 in steady state to derive the 4-mode covariance matrix, while black lines show the corresponding previously derived analytical expressions. The set of parameters we have made use of is $\omega_m/2\pi = 10$ MHz, $\gamma_m = 10^{-5} \times \omega_m, \kappa = 100 \times \omega_m, \gamma_1 = \omega_m/100, \gamma_2 = \omega_m$ and $n = 100$.

The condition $\Gamma_1 + \gamma_m < \gamma_2$ puts a lower bound on $G_1$ which is given by $G$. Moreover, with the increase of $G_1$, the field response at $\omega_m \left( |\epsilon_f (\omega_m)|^2 \right)$ diminishes as $1 + (G_1^2/2\omega_m^2)$ from the value of $1/\kappa^2$ assumed above, which in consequence amounts to the limitation that $G_1 < \sqrt{\omega_m}$.

The cooling rate is independent of $\Gamma_1$ as long as $G_1 > G$, however the residual occupancy is limited by $n_{r,es} \simeq C_1^{-1} > \gamma_1/\omega_m$, which is the justification for working with small $\gamma_1$.

Cooling with inverted atoms - We shift our discussion now to the opposite case of a cavity filled with atoms of type 2, at $\Delta_2 = \omega_m$ and set $G_1 = 0$. The correlations of the input noises are now $\langle \tilde{a}_{in}^\dagger (\omega) \tilde{a}_{in} (\omega') \rangle = 2C_2 \omega_m^2 |\epsilon_2 (\omega)|^2 \delta (\omega + \omega')$ and $\langle \tilde{a}_{in} (\omega) \tilde{a}_{in}^\dagger (\omega') \rangle = 2\kappa \delta (\omega + \omega')$ which show that the

\[ \text{FIG. 2: Relative cooling rate } \gamma_m/\gamma_m \text{ and residual occupancy } n_{r,es} \text{ (in the insets) as a function of the normalized parameters } G_1/\omega_m \text{ and } G_2/\omega_m \text{ for } G = \omega_m. \text{ The red line corresponds to analytical expressions while the black line shows the exact results (see text). a) Cooling with ground state atoms and } G_2 = 0. \text{ b) Cooling with inverted atoms and } G_1 = 0. \text{ The other parameters are } \omega_m/2\pi = 10 \text{ MHz, } \gamma_m = 10^{-5} \times \omega_m, \kappa = 100 \times \omega_m, \gamma_1 = \omega_m/100, \gamma_2 = \omega_m \text{ and } n = 100. \]
cavity is at a nonzero effective temperature. This results in a frequency-dependent thermal cavity occupancy \( n(\omega) = \gamma_2 G_2^2 \varepsilon_2(\omega)^2 \kappa^{-1} (1 - \gamma_2 G_2^2 \varepsilon_2(\omega)^2 \kappa^{-1}) \) which evaluated at \( \omega_m \) leads to \( n(\omega_m) = C_2 / (1 - C_2) \). This will set a lower bound for the achievable mechanical occupancy because the oscillator cannot be cooled to less than the occupancy of the effective cooling reservoir. In the bad cavity limit the atoms produce a peak of bandwidth \( \gamma_2 \) around \( \omega_{as} \). We ask again for the condition that the anti-Stokes sideband fits inside the peak, i.e. \( \Gamma_2 + \gamma_m < \gamma_2 \). In this limit and with the requirement \( \gamma_2 \ll \omega_m \), one can approximate \( |\varepsilon_2(\omega_m)| \approx 1 / (2 \omega_m) \), \( |\varepsilon_f(\omega_m)| = 1 / \gamma_2 \), \( |\varepsilon_f(-\omega_m)| \approx 1 / (1 - C_2) \kappa \) and \( |\varepsilon_f(-\omega_m)| \approx 1 / \kappa \). The scattering rates are \( A_{as} \approx G^2 / \kappa (1 - C_2)^2 \) and \( A_s \approx G^2 / \kappa (1 + C_2 / (1 - C_2)^2) \) and the cooling rate is
\[
\Gamma_a \approx \frac{G^2 C_2}{\kappa (1 - C_2)}. \tag{7}
\]

The residual occupancy in the limit of \( C_2 \to 1 \) diverges as \( n_{res} \approx C_2 / (1 - C_2) \), which is the occupancy of the effective reservoir at the Stokes sideband. However, \( C_2 \) cannot be increased arbitrarily close to 1 since there is a constraint \( \Gamma_2 + \gamma_m < \gamma_2 \) which leads to \( G^2 / (\gamma_2 \kappa) < (1 - C_2)^2 \). Graphical illustration of the increase of the normalized cooling rate \( \gamma_m / \gamma_2 \) as a function of \( G_2 / \omega_m \) at fixed \( G_1 = 0 \) is provided in Fig. 2b. The analytical result is valid up to values of \( G_2 / \kappa \) where \( \gamma_m \) is of the order of \( \gamma_2 \). The inset shows the agreement between the analytical and numerical results for \( n_{res} \) when \( G_2 \) is large such that \( C_2 \to 1 \).

**Discussion** - The use of ground state atoms for cooling presents the advantage of a low residual occupancy proportional to \( C_1^{-1} \). The downside is that cooling is slow at a rate that saturates at \( G^2 / \kappa \) in the limit \( C_1 \to 1 \). The use of inverted atoms considerably speeds up the cooling process that takes place at a rate \( \Gamma_2 \gg G^2 / \kappa \) for \( C_2 \) close to unity, at the price, however, of a high \( n_{res} \). One would ideally combine the two methods for optimal cooling. However, such a mixed scenario is not particularly advantageous. In fact, in the limit of \( \gamma_{1,2} \ll \omega_m \), one obtains a combined cooling rate that is practically the one induced by the anti-Stokes enhancement \( \Gamma_2 \simeq \Gamma_2 \), but the residual occupancy \( n_{res} \approx n_{res}^1 (1 - C_2) + n_{res}^2 \approx n_{res} \) is still large, reflecting the negative effect of the high effective temperature induced by the inverted atoms.

A few remarks on the system stability have to be made at this point. The analysis can be made by applying the Routh-Hurwitz criterion [15] on Eqs. (2). A cavity with a moving mirror driven at resonance is always stable and its stability is not perturbed by the addition of ground state atoms at \( \Delta_1 = -\omega_m \). The inverted atoms, however, do impose a limitation on the system’s stability, which can be simply cast as \( C_2 < 1 \) and trivially fulfilled by the proper choice of \( N_2 \).

The question of the validity of bosonisation also has to be discussed. For the ensemble of two level atoms to resemble an harmonic oscillator one has to ask that the single atom excitation induced by the cavity field is small. This means \( g_{1,2}^2 \left( \omega_m^2 + \gamma_{1,2}^2 \right) \ll \alpha^{-2} < 1 \), which, as detailed in [14], can be fulfilled for a dipole forbidden transition or for atomic vapor cell much larger than the cavity mode.

**Conclusions** - We have shown a new mechanism for mirror ground state cooling where atoms are used to dissipate the thermal energy of the mechanical oscillator. In a simple picture this can be seen as cavity-assisted cooling with a cavity profile tailored via use of atoms at the Stokes and anti-Stokes sidebands. The effect is to inhibit scattering processes that lead to heating while enhancing the cooling process. We notice that there are alternative ways of cooling through cavity spectrum tailoring. A recent example is given in [16] where tailoring is obtained when the micro-mechanical resonator directly modulates the cavity bandwidth. More generally speaking, appropriate spectrum tailoring can be always be obtained through quantum interference [17], as for example proposed and demonstrated for cooling trapped ions [18].

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