Dark Energy Constraints from the Cosmic Age and Supernova

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Using the low limit of cosmic ages from globular cluster and the white dwarfs: $t_0 > 12$ Gyr, together with recent new high redshift supernova observations from the HST/GOODS program and previous supernova data, we give a considerable estimation of the equation of state for dark energy, with uniform priors as weak as $0.2 < \Omega_m < 0.4$ or $0.1 < \Omega_m h^2 < 0.16$. We find cosmic age limit plays a significant role in lowering the upper bound on the variation amplitude of dark energy equation of state. We propose in this paper a new scenario of dark energy dubbed Quintom, which gives rise to the equation of state larger than $-1$ in the past and less than $-1$ today, satisfying current observations. In addition we’ve also considered the implications of recent X-ray gas mass fraction data on dark energy, which favors a negative running of the equation of state.

Age limits of our universe are among the earliest motivations for the existence of the mysterious dark energy. Namely, observations of the earliest galaxies could set a low limit on the age of the universe. In 1998, two groups [1, 2] independently showed the accelerating expansion of our universe basing on Type IA Supernova (SNe Ia) observations of the redshift–distance relations. The recently released first year WMAP data [3] support strongly the concordance model with dark energy taking part of ~ 2/3. The most recent discovery of 16 SNe Ia [4] with the Hubble Space Telescope during the GOODS ACS Treasury survey, together with former SNe Ia data alone could provide a strong hint for the existence of dark energy. Riess et al. [4] provided evidence at > 99% for the existence of a transition from deceleration to acceleration using supernova data alone.

Despite our current theoretical ambiguity for the nature of dark energy, the prosperous observational data (e.g. supernova, CMB and large scale structure data and so on) have opened a robust window for testing the recent and even early behavior of dark energy using some simple parameterization for its equation of state (e.g., Ref. [5]) or even reconstruction of its recent density [6, 7, 8]. Both recent WMAP fit and more recent fit by Riess et al. find the behavior of dark energy is to great extent in consistency with a cosmological constant. In particular when the equation of state is not restricted to be a constant, the fit to observational data improves dramatically [9, 10, 11, 12]. Huterer and Cooray [10] produced uncorrelated and nearly model-independent band power estimates (basing on the principal component analysis [12]) of the equation of state of dark energy and its density as a function of redshift, by fitting to the recent SNe Ia data they found marginal (2-σ) evidence for $W(z) < -1$ at $z < 0.2$, which is consistent with other results in the literature [6, 7, 10, 11, 12, 15-17].

The recent fit to first year WMAP and other CMB data, SDSS and 172 SNe Ia data [18] by Tegmark et al [19] provided the most complete and up-to-date fit. Although SNe Ia data accumulated more after that, Ref. [11] should still be a very profitable benchmark for current fit of the observables. However, when considering the behavior of dark energy alone, one has to do more since Ref. [10] only dealt with constant equation of state before the recent release of 16 more SNe Ia data by Ref. [4]. In fact a complete fit to full observational data still remains impossible provided one wants to reconstruct the full behavior of dark energy, despite the using of the most efficient Markov Chain Monte Carlo (MCMC) method. Under such circumstance Wang et al. and Riess et al. fitted dark energy to SNe Ia data, 24 or linear growth factor and a parameter related (up to a constant) to the angular size distance to the last scattering surface. In fact even the angular size distance is model dependent, as can be seen from Ref. [5] it differs for the six-parameter vanilla model and when an additional parameter $\alpha$ (running of the spectral index) is added. Regarding the constraint from the cosmic age, Krauss [21] used the WMAP fitted value for seven parameters: $t_0 = 13.7 \pm 0.2$ Gyr, together with H ST [22] bound and assuming some specific relation between $\Omega_m$ and $h$, he got a lower bound on constant equation of state: $W > -1.2$, which is in strikingly agreement with WMAP result. Generally speaking, age limit can give an upper limit rather than lower limit, as shown by Cepa [23].

The low limit to the cosmic age can be directly obtained from dating the oldest stellar populations. Globular clusters (GC) in the Milky Way are excellent laboratory for constraining cosmic ages. Carretta et al. [24] gave the best estimate for the age of GCs to be Age $= 12.9 \pm 2.9$ Gyr at 95% level. The limit for age of GCs is around 11-16 Gyr [3]. White dwarf dating provides a good approach to the main sequence turn-off. Richer et al. [25] and Hansen et al. [26] found an age of 12.7 $\pm 0.7$ Gyr at 2σ level using the white dwarf cooling sequence method. For a full review of cosmic age limit see Ref. [3]. The low limit to cosmic age serves as the ‘anti-smoking gun’ in excluding models which lead to shorter age. In this paper we use $t_0 > 12.0$ Gyr as the bound on cosmic age. Other constraints we use are only uniformly in range $0.2 < \Omega_m < 0.4$ or $0.1 < \Omega_m h^2 < 0.16$. As can be seen from Ref. [11], such constraint is much looser than the six parameters + $W$ set and comparable to the
Firstly we delineate the effect of age limit in ΛCDM cosmology. In the full paper we assume a flat space, i.e. \( \Omega_k = 0 \). In the left panel of Fig. 1 we vary \( \Omega_m \) from 0 to 1 and the Hubble parameter \( h \) from 0 to 1.4. The red area is excluded by \( t_0 > 12.0 \) Gyr, the area between the two black solid lines is given by the 1σ HST limit. If we conservatively assume in ΛCDM cosmology the cosmic age is no more than 20 Gyr, the area with blue color will be excluded. It can give a rough estimate for current fraction of matter in the universe, although much looser than SNe Ia constraint as shown in the right panel. The supernova data we use is the "gold" set of 157 SNe Ia published by Riess et al. in [4]. In the below we constrain the Hubble parameter to be uniformly in 3σ HST region: 0.51 < \( h < 0.93 \).

In the detailed discussions below we consider two type of parameterizations for the equation of the state of the dark energy,

\[ W(z) = W + W'z, \]  

(taken as Model A) so that

\[ X = a^{3(1+W-W')e^{3W'(a^{-1}-1)}}, \]  

where \( a \) is the cosmic scale factor. The other form was proposed by Refs. [5, 20] (taken as Model B):

\[ W(z) = W_1 + W_a z/(1 + z), \]  

which leads to

\[ X = a^{-3(1+W_1+W_a)}e^{3W_a(a-1)}. \]

Both models (as well as other models such as firstly proposed in [4]) make good approximations to probe the behavior of dark energy around the present epoch, while the former model leads to poor parameterization at very large redshift. But as argued by Riess et al. [4] this is acceptable for showing the late behavior of dark energy.

SNe Ia data alone proves to be a weak constraint on above models, as shown in the right panel of Fig. 2 and also in Ref. [5] where the authors used flux averaging method [11, 31]. In the left panel of Fig. 2, the region on up right corner is fully excluded by the age limit \( (t_0 > 12) \) Gyr. The role of age limit can be easily seen from Eqs. (1-6). In Model A larger \( W' \) leads to larger \( X \) in early epochs, hence corresponds to smaller ages which can be directly constrained. Similar case works for Model B. Age constraint still works when adding the prior on \( \Omega_m \) or \( \Omega_m h^2 \). In Figs. 3-4 we show the corresponding effects when adding different priors for SNe Ia on right panels and show the role of age correspondingly in the left panels. The dashed lines are the 1σ regions and the small dots denote the best fit parameters. Up right regions of the left panels are excluded by the cosmic age limit. We can see the age limit reduces significantly the upper regions and consequently changes the best fit values of the model parameters. One can also find from Fig. 2 and

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\(^1\) Our prior on \( \Omega_m \) is also consistent with the median statistics study on mass density by Chen and Ratra [27], for more investigations on the effects of priors see Refs. [5, 20, 22].
So, its equation of state $\rho = \frac{1}{2} Q^2 + V(Q)$, $p = \frac{1}{2} Q^2 - V(Q)$.

The equation of state $W = (-\frac{1}{2} Q^2 - V)/(1 - \frac{1}{2} Q^2 + V)$ is located in the range of $W \leq -1$. Neither the quintessence nor the phantom alone can fulfill the transition from $W > -1$ to $W < -1$ and vice versa. But at least a system containing two fields, one being the quintessence with the other being the phantom field, can do this job. The combined effects will provide a scenario where at early time the quintessence dominates with $W > -1$ and lately the phantom dominates with $W$ less than $-1$, satisfying current observations. As an example, we consider a model:

$$\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi_1 \partial^{\nu} \phi_1 - \frac{1}{2} \partial_{\mu} \phi_2 \partial^{\nu} \phi_2 - V_0 [\exp(-\frac{\lambda}{m_p} \phi_1) + \exp(-\frac{\lambda}{m_p} \phi_2)] ,$$

where $\phi_1$ and $\phi_2$ stand for the quintessence and phantom.

Fig. 2: Right panel: $2\sigma$ SNe Ia limit alone on Model A dark energy. Left panel: $2\sigma$ SNe Ia limit and age limit ($t_0 > 12$Gyr) on Model A dark energy. The dots inside the two panels show the best fit parameters.

As mentioned above, the present data seem to favor an evolving dark energy with the equation of state being below $-1$ around present epoch evolved from $W > -1$ in the past. This can also be seen from the best fit parameters of our Figs. 2-5. If this result is confirmed in the future, it has important implications for the theory of dark energy. Firstly, the cosmological constant as a candidate for dark energy will be excluded and dark energy must be dynamical. Secondly, the simple dynamical dark energy models considered vastly in the literature like the quintessence or the phantom can not be satisfied either.

In the quintessence model, the energy density and the pressure for the quintessence field are $\rho = \frac{1}{2} \dot{Q}^2 + V(Q)$, $p = \frac{1}{2} \dot{Q}^2 - V(Q)$.

Fig. 3: Age and SNe limits on Model A with differrent priors as noted inside. The dots inside the $1\sigma$ dashed lines denote the best fit parameters.

Although the k-essence like models can have $W < -1$, it has been proved later by Ref. 11 to be difficult to get $W$ across -1 during evolution.
In Fig. 4 we illustrate the evolution of the effective equation of state of such a system with $-\ln(1 + z)$.

In general to realize the transition of $W$ around $-1$, one needs to consider models of dark energy with more complicated dynamics and interactions with gravity and matter. This class model of dark energy, which we dub “Quintom”, is different from the quintessence or phantom in the determination of the evolution and fate of the universe. Generically speaking, the phantom model has to be more fine tuned in the early epochs to serve as dark energy today, since its energy density increases with expansion of the universe. Meanwhile the Quintom model as illustrated in Fig.5 can preserve the tracking behavior of quintessence, where less fine tuning is needed. We will leave the detailed investigation of the Quintom models in a separated publication, however will mention briefly two of the possibilities below in addition to the one in Eq. (9). One will be the scalar field models with non-minimal coupling to the gravity where the effective equation of the state can be arranged to change from above -1 to below -1 and vice versa. For a single scalar field coupled with gravity minimally, one may consider a model with a non-canonical kinetic term with the following effective Lagrangian:

$$
\mathcal{L} = \frac{1}{2} f(T)\partial_{\mu}Q\partial^{\mu}Q - V(Q),
$$

where $f(T)$ in the front of the kinetic term is a dimensionless function of the temperature or some other scalar fields. During the evolution of the universe when $f(T)$ changes sign from positive to negative it gives rise to an realization of the interchanges between the quintessence and the phantom scenarios.

Recently Allen et al. [46] have provided new observational data basing on Chandra measurements of the X-ray gas mass fraction in 26 X-ray luminous galaxy clusters. Under the assumption that the X-ray gas mass fraction measured within $r_{2500}$ is constant with redshift the $f_{\text{gas}}$ data in the range $0.07 < z < 0.9$ can be used directly to constrain cosmological models. We use their data and fit to Model A, we set the same gaussian prior on $\Omega_m$ as Ref. [44] meanwhile varying $h$ uniformly in range $0.51 \sim 0.93$ and $0 < \Omega_m < 1$. The $\chi^2$ value is defined as

$$
\chi^2 = \left( \sum_{i=1}^{26} \frac{[f_{\text{SCDM}}(z_i) - f_{\text{gas}, i}]^2}{\sigma_{f_{\text{gas}, i}}^2} \right).
$$

FIG. 4: The same as Fig.3 for Model B.

FIG. 5: The evolution of the effective equation of state of the double scalar fields given in Eq. (9). The parameters are chosen as: $V_0 = 8.38 \times 10^{-126} m_p^4$, $\lambda = 20$. We set the initial conditions as: $\phi_1 = -1.7 m_p$, $\phi_2 = -0.2292 m_p$, which lead to $\Omega_m = 0.30$, $w_{\text{eff}} = -2.44$. 

Ref. 46, with SNe Ia constraints also shown for comparison. We delineate the resulting 2\( \times \) 1\( \times \) dimensional constraints on \( W' \) in light of the data \( f_{\text{gas}} \) from Chandra observations from Ref. 46 with SNe Ia constraints also shown for comparison.

\[
\frac{\Omega_b h^2 - 0.0214}{0.0020} + \left( \frac{b - 0.824}{0.089} \right)^2.
\]

(11)

For a detailed description of the data and fitting see Ref. 46. We delineate the resulting 2\( \times \) dimensional and 1\( \times \) dimensional plots in Fig.6, together with previous SNe Ia results for comparison (here we do not include age constraint for simplicity). Very interestingly we get \( W' < 0 \) with the center value \((W, W') = (-0.2, -4.875)\) when varying fine grids, which indicates the universe may not be accelerating today but in the very near past. However the smallest redshift of \( f_{\text{gas}} \) data gives \( z = 0.077900 \), this shows also some poor parameterization of model A. As shown in our paper and also vastly in the literature the results of fitting depends somewhat in the parameterizations. Moreover, in any case if nonzero \( W' \) gets more favored with the accumulation of observational data, it gives strong implications for dark energy \( "\text{metamorphosis}" \). We also find although the two data sets give consistent results to model A, there seems to be some discrepancy. In Fig.6, the contours do overlap in 1\( \sigma \) regions, but only in a small area and the right panel is more distinctive: \( f_{\text{gas}} \) favors a negative \( W' \) at more than 1.3\( \sigma \) while a positive \( W' \) is favored around 1.1\( \sigma \) with the prior on \( \Omega_m \) for SNe Ia. Basically the X-ray data probe the late behavior of the angular-diameter distance and SNe Ia probes the luminosity distance. Therefore, there seems to be some discrepancy between them and this may possibly be due to some new physics.

In summary in this paper we consider the effect of cosmic age and supernova limits on the variation of \( W \). Our results show that age limit plays a significant role in lowering the variation of amplitude on the equation of state. Current SNe Ia observation seems to favor a variation of \( W \) from \( > -1 \) in the recent past and \( < -1 \) today. If such a result holds on with the accumulation of observational data, this would be a great challenge to current cosmology. We give a simplest example of Quintom which can satisfy the current implications on the equation of state on dark energy, and discuss briefly the possibility of building Quintom models.

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