Hydrodynamic description of Weyl fermions in condensed state of matter

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Abstract. Due to the many unique transport properties of Weyl semimetals, they are promising materials for modern electronics. We investigate the electrons in the strong coupling approximation near Weyl points based on their representation as massless Weyl fermions. We have constructed a new fluid model based on the many-particle quantum hydrodynamics method to describe the behavior of electrons gas with different chirality near Weyl points in the low-energy limit in the external electromagnetic fields, based on the many-particle Weyl equation and many-particle wave function. The derived system of equations forms a closed apparatus for describing the dynamics of the electron current, spin density and spin current density. Based on the proposed model, we considered small perturbations in the Weyl fermion system in an external uniform magnetic field and predicted the new type of eigenwaves in the systems of the electrons near the Weyl points.

1 Introduction

Weyl fermions are stable and chiral massless particles, both a particle and an antiparticle \cite{1}. Weyl fermions can be realized as emergent low-energy quasiparticle excitations in the condensed state system or Weyl semimetals \cite{2,3}. Semimetals, are the crystals, the conductivity of which is determined by the topological properties of the wave function in the bulk of the sample of semimetal \cite{4,5}. Recently, these materials were discovered experimentally \cite{6}. In the strong-coupling approximation, strongly interacting electrons can jump from one atom to another, and can be imaged by the weakly interacting electron quasiparticles \cite{7}. In semimetals, this corresponds to the formation of the spatial electronic band structure, where the Weyl-like points would appear in the band structure if two non-degenerate bands cross each other at a single node. Near this point the quasiparticles are massless and can be described by the Hamiltonian for the Weyl fermions \cite{8}. In the Weyl semimetal, it is precisely three-dimensional chiral charge carriers with a linear spectrum near the Weyl node, which are more stable than two-dimensional ones in graphene. Therefore, in the Weyl semimetal, the anomalous Hall effect, the presence of negative magnetoresistance, and a chiral magnetic effect are possible \cite{9,10}.

The study of Weyl fermions is an important and topical problem in the physics of the condensed state of matter. In addition to bulk states, charge particles in Weyl semimetals have the gapless surface states \cite{11}. The existence of the surface states such as Fermi arcs leads to the strong nonlocality of transport in thin-layer samples and to the anomalous quantum oscillations \cite{12}. The Weyl semimetals exhibit the important properties in an external magnetic and electric fields. In Ref. \cite{13} the Landau levels and quantum oscillations of the density of states in the Weyl semimetal in crossed magnetic and electric fields had been investigated and an expression for the energy spectrum of such system had been derived. The transport processes which are specific to an ideal gas of relativistic Weyl fermions had been studied in Ref. \cite{14}. The semiclassical kinetic theory of Weyl particles in the presence of external electromagnetic fields in an uniformly rotating coordinate frame, by keeping the usually ignored centrifugal force terms, had been derived \cite{15}. The phase-space dynamics of the Weyl and Dirac particles is directly linked. A hydrodynamic form of the Weyl equation for the neutrino wave function had been derived in Ref. \cite{16}. A hydrodynamic field theory of the three-dimensional fractional quantum Hall effect, which was recently proposed to exist in magnetic Weyl semimetals, when the Weyl nodes are gapped by strong repulsive interactions, had been developed in Ref. \cite{17}.

Weyl fermions have been actively studied in the context of superfluid \textsuperscript{3}He. The Weyl points can exist in the fermionic spectrum of superfluid \textsuperscript{3}He-A and in the core of the quantized vortices in \textsuperscript{3}He-B \cite{18}. The effect-
tive or synthetic “electric” and “magnetic” fields can be exist and act on the Weyl quasiparticles in the topological semiconductors, because the position of the Weyl points in momentum space depends on the coordinates of space. The orbital-motive force in superfluid \( ^3 \text{He-A} \) originates from chiral Weyl fermions had been derived in Ref. [19]. Tetrad formalism for the Weyl fermions and the chiral/axial anomaly in superfluid \( ^3 \text{He-A} \) had been discussed in Ref. [20].

The unusual properties of the low-energy quasiparticle excitations can be described by the methods ranging from the quantum field theory to the chiral kinetic and hydrodynamics theory. A steady-state non-local response in Weyl semimetals in the hydrodynamic regime had been studied by using the consistent hydrodynamic formalism [21]. In the Drude model, which describe the electrons transport, electron scattering on impurities and phonons dominates over the electron-electron scattering. But, if electron-electron scattering prevails over electron-impurity and electron-phonon scattering, charged particles form a fluid, the properties of which can be described in terms of hydrodynamic formalism. Also, in the hydrodynamic regime, the momentum and energy-current relaxations are independent processes. An additional condition for the usage of chiral hydrodynamics is the smallness of the mean free path in comparison with the wavelength \( k << 1/L_m f_p \). The charge current is determined not only by the local electromagnetic field and temperature gradient, but also by the hydrodynamic flow velocity, which is described by the momentum balance equation. A theory of thermodlectric transport in weakly disordered Weyl semimetals, where the electron-electron scattering time is faster than the electron-impurity scattering time, had been derived based on the hydrodynamical description of the relativistic fluids at each Weyl node [22]. The chiral kinetic theory [23,24] and chiral hydrodynamics were constructed within the theoretical description [25,26].

The dynamics of a system of electrons in the strong coupling approximation can be modeled by a viscous fluid flow, and the hydrodynamic approach can be applied [27]. The experimental signatures of hydrodynamic electron flow in the Weyl semimetal tungsten diphosphide (WP2) had been observed recently [27], using thermal and magneto-electric transport experiments. The transition from a conventional metallic state at higher temperatures to a hydrodynamic electron fluid below 20 K had been found. Thus, an important task is to describe the collective effects in Weyl semimetals based on the theory of hydrodynamics. Attempts have been made to use the model of classical hydrodynamics to describe the behavior of electrons in the strong coupling approximation [28,29].

On the other hand, as wave processes, processes of information transfer, diffusion and other transport processes occur in the three-dimensional physical space, a need arises to turn to a mathematical method of physically observable values which are determined in a 3D physical space. A quantum mechanics description for the systems of \( N \) interacting particles is based upon the many-particle Schrödinger equation (MPSE) that specifies a wave function in a 3N-dimensional configuration space. To do so we should derive equations those determine dynamics of functions of three variables, starting from MPSE. This problem has been solved with the creation of a method of many-particle quantum hydrodynamics (MPQHD). Therefore, the reason for development such method is analogous with the motivation of the density functional theory.

In our research we propose a further development of the MPQHD method for the hydrodynamic description of Weyl fermions in the condensed state of matter. We assume, that the electron-electron interactions dominate over the electron-impurity and electron-phonon interactions. We demonstrate the original derivation of the system of equations of hydrodynamic type. The developed model can be used in the future to study the transport of electrons in various types of Weyl semimetals.

2 Quantum hydrodynamics formulation of Weyl equation

The quasiparticle states in Weyl semimetals are described by a Weyl equation. The many-particle Weyl equation can be introduce for the massless spin-1/2 particles in the form

\[
i \hbar \frac{\partial \psi}{\partial t} = \sum_{j=1}^{N} \left( \lambda_j v_{f_{\alpha}} (\hat{\sigma}_x p_{j\alpha} + \hat{\sigma}_y q_j) \right) \psi, \tag{1}\]

where \( q_j \) is a charge of \( j \)-th particle, \( \lambda_j = \pm 1 \) is the chirality of \( j \)-th particle, \( v_{f_{\alpha}} \) is a Fermi velocity, which is the same for all electrons of the system (near the Weyl points). The Fermi velocity is not a vector, but a set of three parameters (two of them coincide each other). They are the plane Fermi velocity \( v_{f_\perp} = v_{f_{\alpha}} = v_{f_{\alpha}}' \), and the Fermi velocity \( v_{f_\parallel} = v_{f_{\alpha}}' \) parallel to the anisotropy axis. Hence, the first term on the right-hand side of Eq. (1) can be presented in more detailed form: \( \lambda_j v_{f_{\alpha}} (\hat{\sigma}_x p_{j\alpha} + \hat{\sigma}_y q_j) \). Hence, the value of parameter \( v_{f_{\alpha}} \) is bounded to the projection of the momentum. It is bound to other functions below appearing via the momentum contribution. The difference in the Fermi velocity for different directions of space reflects the presence of anisotropy near the Weyl points. The operator of covariant derivative is \( \hat{D}_{\alpha} \), where \( \hat{D}_{\alpha} \) is the momentum operator near the Weyl node, \( \varphi_j \) and \( A_{\alpha} \) are the scalar and vector potentials respectively, \( \psi(\hat{R}, t) = \psi(r_1, r_2, ..., r_N, t) \) is the many-particle wave function of the particles with right and left helicity in 3N configuration space, and \( a, b, d \) is the spin index \( = -1/2, 1/2 \). The wave function has the complex form of matrix. The Pauli matrices \( \sigma_{\alpha} \) in the Weyl Eq. (1) have the well known form.
\[
(\sigma^x)_{ab} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (\sigma^y)_{ab} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},
\]
\[
(\sigma^z)_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (\sigma^0)_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.
\] (2)

The main idea of the many-particle quantum hydrodynamics method [30–35], which is used in this paper, is to average the operators of observable physical quantities over quantum states. Below, averaging of the quantity \( \chi(R, t) \) over the quantum states of particles is given by an expression of the form
\[
\int dR \sum_{j=1}^N \delta(r - r_j) \chi(R, t) = \left\langle \chi(R, t) \right\rangle.
\] (3)

The state of the spinning massless particles is characterized by the total density in the neighborhood of \( r \) in a physical space. The total concentration of the particles \( n(r, t) \) is thus determined as the quantum average of the concentration operator \( \hat{n} = \sum_j \delta(r - r_j) \) in the coordinate representation
\[
n(r, t) = \left\langle \psi^\dagger(R, t) \psi(R, t) \right\rangle = n_+ + n_-.
\] (4)

where \( ^\dagger \) is the Hermitian conjugation. Lets introduce the additional scalar physical quantity \( \nu(r, t) \) depending on the chirality of the particles and representing the difference in the concentrations of massless particles with different chiralities
\[
\nu(r, t) = \left\langle \lambda_j \psi^\dagger(R, t) \psi(R, t) \right\rangle = n_+ - n_-.
\] (5)

Differentiating the expressions for the total concentration (4) with respect to time and using the Weyl Eq. (1), the continuity equation for particle density \( n(r, t) \) can be derived in the form
\[
\partial_t n(r, t) + v_f^{\alpha\nu} \partial_\alpha n(r, t) = 0,
\] (6)

where the analog of mass current in the continuity equation in the fluid hydrodynamics is derived in the form of the vector physical quantity, which represents the difference between spin densities of particles with different chiralities
\[
j^\alpha(r, t) = \left\langle \lambda_j \psi^\dagger(R, t) \sigma^\alpha_j \psi(R, t) \right\rangle = s^\alpha_+ - s^\alpha_-.
\] (7)

This result follows from the fact that the velocity of massless particles is connect with spin, which is codirectional or opposite to the direction of motion of the particle. Symbol \( v_f^{\alpha\nu} \) in the continuity Eq. (6) is not a vector. Subindex “\( \alpha \)” refers to the one of two values of this parameter, which are bound to the value of the vector index \( \alpha \) existing in the same term.

Continuity equation for the quantity \( \nu(r, t) \) can be derived in a similar way in the form
\[
\partial_t \nu(r, t) + v_f^{\alpha\nu} \partial_\alpha \nu(r, t) = 0,
\] (8)

where the divergence of the spin density vector appears during the derivation of the Eq. (8) in the microscopic form
\[
s^\alpha(r, t) = \left\langle \psi^\dagger(R, t) \sigma^\alpha_j \psi(R, t) \right\rangle = s^\alpha_+ + s^\alpha_-. \] (9)

The important property of the Weyl fermions is the existence of chiral anomaly, when for the fermions with different chiralities the currents of the left-handed and right-handed Weyl fermions cannot be preserved separately. But within the framework of the method of quantum hydrodynamics, the equation of dynamics for the quantity \( \nu(r, t) \) has the form of a conservation law.

It is of interest to obtain a closed apparatus of quantum hydrodynamics, consisting of a system of equations for the basic physical quantities. For this purpose we need to follow the dynamics of vectors \( s^\alpha(r, t) \) and \( j^\alpha(r, t) \) in time. By repeating the above procedure for spin density vector \( s^\alpha \) and vector \( j^\alpha \), the balance equations for these quantities can be derived in the way
\[
\partial_t s^\alpha(r, t) + v_f^{\alpha\nu} \partial_\alpha s^\alpha(r, t) = \frac{2}{\hbar} v_f^{\alpha\nu} \varepsilon^{\alpha\beta\gamma} \tau_{\gamma}^\beta(r, t),
\] (10)

and
\[
\partial_t j^\alpha(r, t) + v_f^{\alpha\nu} \partial_\alpha j^\alpha(r, t) = \frac{2}{\hbar} v_f^{\alpha\nu} \varepsilon^{\alpha\beta\gamma} \Lambda_{\beta}^\gamma(r, t).
\] (11)

As we can see the balance Eqs. (10) and (11) do not include the effect of the fields of external forces. But tensors of the second rank \( \tau_{\gamma}^\beta \), \( \Lambda_{\beta}^\gamma \) appear at the right-hand sides of the Eqs. (10) and (11), the physical meaning of which requires clarification. The terms at the right-hand sides of the equations characterize the new force fields act on the spin densities of the particles with different chiralities. In the process of derivation of the system of Eqs. (10) and (11) tensors \( \tau_{\gamma}^\beta \) and \( \Lambda_{\beta}^\gamma \) can be obtained in the microscopic form
\[
\tau_{\gamma}^\beta(r, t) = \left\langle \frac{\lambda_j}{2} \left( \psi^\dagger(R, t) \sigma^\alpha_j D^\beta_j \psi(R, t)
\right. \right. \\
\left. \left. + (D^\beta_j \psi)^\dagger(R, t) \sigma^\alpha_j \psi(R, t) \right) \right\rangle, \] (12)

and
\[
\Lambda_{\beta}^\gamma(r, t) = \left\langle \frac{1}{2} \left( \psi^\dagger(R, t) \sigma^\alpha_j D^\beta_j \psi(R, t)
\right. \right. \\
\left. \left. + (D^\beta_j \psi)^\dagger(R, t) \sigma^\alpha_j \psi(R, t) \right) \right\rangle. \] (13)

As we can see, the quantity \( \Lambda_{\beta}^\gamma \) represents the total spin current density of the fluid of massless particles.
Now it is important to derive the evolution equations for the tensors $\Lambda^{\alpha\beta}$ and $\tau^{\alpha\beta}$, which characterizes the difference of spin currents of the particles with different chiralities. Differentiating tensor quantities (12) and (13) with respect to time and using the Weyl equation (1), we arrive at the system of equations for the evolution of the spin current density

\[
\partial_t \tau^{\alpha\beta} + v f^{\alpha\beta\gamma} \partial_\beta J^{\gamma} + \frac{\hbar}{2} v f^{\alpha\beta\gamma} \varepsilon^{\alpha\beta\gamma} n B^{\gamma} + \frac{2}{\hbar} \varepsilon^{\alpha\beta\gamma} v f^{\alpha\beta\gamma} \delta^{\alpha\beta\gamma} = q s^{\alpha} E^{\beta},
\]

and the axial current density

\[
\partial_t \tau^{\alpha\beta} + v f^{\alpha\beta\gamma} \partial_\beta \tau^{\gamma} + \frac{\hbar}{2} v f^{\alpha\beta\gamma} \varepsilon^{\alpha\beta\gamma} \partial_\beta \partial_\delta J^{\gamma} = q s^{\alpha} E^{\beta},
\]

The second terms at the left-hand side of Eqs. (14) and (15) represent the influence of nonuniform spin density fields. The third terms at the right-hand side of equation represent the density of the force acting on a charge density and spin density in an external electromagnetic field. The second equation (19) is the equation for the evolution of the axial current density $\tau^{\alpha}$. The second term at the right-hand side of equation for the axial current density (19) represents the action of torque in an external magnetic field.

In external uniform magnetic and electric fields, the system of Eqs. (6), (8), (10), (11), (14), (15), (18) and (19) should lead to known effects, such as a linear spectrum in a magnetic field. In addition, the developed model should predict the appearance of new effects that are absent in the one-particle description.

### 3 Hydrodynamic of Weyl fermions with single chirality

Let us simplify the problem and consider the one-helical state. Dual structure of the hydrodynamic equations obtained above related to the interplay of two values of chirality. If we consider the evolution of the system of particle with positive chirality we find simplified set of four hydrodynamic equations. The continuity equation has no changes in compare with its original form (6)

\[
\partial_t n + v f^{\alpha\nu} \partial_\alpha j^\nu = 0.
\]

The current of particles $j^\alpha$ leads to the equation of kinematic nature

\[
\partial_t j^\alpha + v f^{\alpha\nu} \partial_\alpha n = \frac{2}{\hbar} v f^{\alpha\nu} \varepsilon^{\alpha\beta\gamma} \Lambda^{\beta\gamma},
\]

No interaction contributes in equation (21), but additional function $\Lambda^{\gamma\beta}$ enters it. It shows the way to extend the set of hydrodynamic equations to include the action of the electromagnetic field on the collection of the Weyl fermions. The evolution of tensor $\Lambda^{\alpha\beta}$ connected to the dynamic of the momentum density $j^\alpha$. These equations have the following form which is simplified in compare with the similar equations obtained above in the general regime

\[
\partial_t J^{\alpha} + v f^{\alpha\beta\gamma} \partial_\beta J^{\gamma} = q n E^{\alpha} + \frac{q}{\hbar} v f^{\alpha\beta\gamma} \varepsilon^{\alpha\beta\gamma} j^{\beta} B^{\gamma},
\]

and

\[
\partial_t \tau^{\alpha} + v f^{\alpha\beta\gamma} \partial_\beta \tau^{\gamma} = q n E^{\alpha} + \frac{q}{\hbar} v f^{\alpha\beta\gamma} \varepsilon^{\alpha\beta\gamma} s^{\beta} B^{\gamma}.
\]

We have a closed system of equations that determine the dynamics of massless charged Weyl particles against the background of external electromagnetic fields. The
Eq. (22) is the momentum balance equation, according to which the dynamics of the flow velocity is affected by the electric field acting on the particle number density and the magnetic field acting on the spin density. The Eq. (23) represents the spin current density evolution equation. As we can see from the right-hand side of equation, the evolution of the spin current is associated with the action of an external electric field on the spin density and with the action of an external magnetic field on the concentration of the number of particles. The effect may have both homogeneous and not-uniform electromagnetic fields.

4 Wave propagation in system of Weyl fermions with positive chirality

One of the primary goals of this article is to derive dispersion characteristics of eigenwaves in the systems of the electrons near the Weyl points in the external uniform magnetic field. In this section we consider the systems of charged Weyl particles with spin aligned to the velocity $\lambda = +1$, $q = -1$. The external magnetic field aligned with $z$ direction $B = \{0, 0, B_0\}$. The wave propagates in the parallel direction $k = \{0, 0, k_\parallel\}$. For the Fermi velocity $\nu_f$ to do that let’s analyze small perturbations of physical variables from the stationary state

$$n = n_0 + \delta n, \quad j^\alpha = 0 + \delta j^\alpha, \quad J^\alpha = 0 + \delta J^\alpha, \quad \Lambda^{\alpha\beta} = 0 + \delta \Lambda^{\alpha\beta}. \quad (24)$$

Considering small deviations from the equilibrium position in the equations for the particle number density, spin density, and spin current density, we arrive at the two dispersion equations for the longitudinal wave

$$-\omega^2 + k_\parallel^2 \nu_f^2 = -\frac{4}{\hbar} \nu_f^2 \frac{eB_0}{c} k_\parallel \frac{\nu_f}{\omega} \quad (25)$$

and transversal wave

$$\omega^2 - k_\perp^2 \nu_f^2 = \frac{2}{\hbar} \nu_f^2 \frac{eB_0}{c} k_\perp \nu_f \omega. \quad (26)$$

In the limit of low magnetic fields, the frequency spectrum (25) is characterized by a dispersion relation for a stable wave

$$\omega = \sqrt{k_\parallel^2 \nu_f^2 + \frac{4}{\hbar} \nu_f^2 \frac{eB_0}{c}}, \quad (27)$$

as can be seen from the dispersion law, the frequency for the Weyl fermion system in an external magnetic field $B_0$ is shifted relative to the frequency of eigenwaves

$$\omega = \nu_f k_\parallel \quad (28)$$

in the absence of external field, when the wave propagates at the Fermi velocity $\nu_f$. The linear spectrum is derived for the positive chirality. As can be seen from the expression (27), the dispersion law for the system in the low magnetic fields coincides with the spectrum of Landau levels for the case of $n = 2$.

In strong magnetic fields, the dispersion dependence changes significantly, differing from the spectrum of the Landau zones. In strong magnetic fields, an unstable wave solution arises, which is very predictable, since the magnetic field, directed perpendicular to the direction of wave propagation, tends to rotate the Weyl fermions in the plane $yz$. In the regime of long wave length the dispersion relation (25) takes the form

$$\omega = \frac{3}{\hbar} \nu_f^2 \frac{eB_0}{c} k_\parallel \nu_f \omega. \quad (29)$$

Equation (29) predicts an unexpected result for Weyl fermions in strong magnetic fields.

The law of dispersion for the low magnetic fields (27) is nonlinear, which is very specific. The spectrum of one-electron states in a magnetic field represents the Landau levels. The zero Landau level has linear dispersion, the sign of which depends on the state of chirality. In the absence of an external magnetic field, the dispersion law (28) coincides with the linear spectrum of the zero Landau level, as expected. But, magnetic field strongly changes the spectrum making it nonlinear. The nonlinear behavior of the dispersion laws may be related to the fact that we considered the collective dynamics of a system of a large number of Weyl fermions, while the Hamiltonian of the one-particle model gives Landau zones.

5 Conclusion

Weyl fermions have long been a hypothetical particle, but with the discovery of Weyl semimetals, Weyl fermions were found in the form of quasiparticles. Weyl fermions are highly mobile and can overcome obstacles that slow down ordinary electrons. They are not scattered by crystal defects, their movement is stable and not supported by the influence of noise. Due to their unique properties, fermions can be used to create fast electronic devices.

Therefore, an important task is the development of theoretical models for describing the behavior of electrons in the state of Weyl fermions. In this article, based on the method of many-particle quantum hydrodynamics, we consider the hydrodynamical model of a fluid of massless particles with charge in the external electromagnetic fields. It is assumed that the electron-electron scattering prevails over the electron-impurity and electron-phonon scattering, and charged particles form a fluid, the properties of which can be described in terms of hydrodynamic formalism. Based on the many-particle Weyl equation (1) and many-particle wave function $\psi(R, t)$, a closed system of equations of hydrodynamic type was derived, which consists of eight equa-
tions: the equations of continuity (6) and (8), spin density evolution equations (10), (11), spin currents density evolution equations (14) and (15), momentum balance equations for the total current and axial current densities (18), (19). The equations for the balance of the spin current and the balance of the momentum contain force fields representing the action of the external electromagnetic fields. The constructed hydrodynamic model for describing chiral charge carriers in external fields differs from the previously developed models of classical hydrodynamics for describing Weyl fermions [22,25,26].

We used a new mathematical model to study the wave dispersion in a system of the Weyl particles and predicted the new type of eigenwaves in the systems of the electrons near the Weyl nodes in the external uniform magnetic field, which is directed along the “z” axis. For the case of the longitudinal and transversal waves (25) and (26) in the absence of an external magnetic field, the dispersion law characterizes the linear dependence of the wave frequency on the wave number when the wave propagates at the Fermi velocity. The magnetic field strongly changes the dispersion laws.

Thus, in this article, at the first stage, a new theoretical model for the description of the Weyl fermion gas based on the fluid model of many-particle quantum hydrodynamics is obtained. This new model has significant differences from the classical hydrodynamic models. Completely new equations are obtained and the effect of an external uniform magnetic field on the dynamics of eigenwaves in a system with positive chirality, which is essentially nonlinear, is predicted. A further task for the development of this method is to study the effect of an electric field on the Landau zones, on the chiral properties of Weyl semimetals, especially the effect of chiral anomaly in the parallel electric and magnetic fields, and also on surface states in the form of Fermi arcs. The next step is also the development of this model in the direction of describing various types of Weyl semimetals. Moreover, further development of the model requires taking into account the interactions between Weyl fermions.

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Author contributions

All authors contributed equally to the paper.

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