Timing of gamma-ray pulsars: search in seven-parametric space

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Abstract

Timing of Geminga gamma-ray pulsar is done using data of COS B and EGRET. It is shown, that errors in angular coordinates of sources similar to Geminga strongly influence a determination of \( \dot{\nu} \), so that at angular precision less than \( 10^{-3} \) arc sec determination of the value of \( \dot{\nu} \) by means of criterion gives error more than 100%. Attempts have been done to improve coordinates of a gamma-ray pulsar using timing analysis. In addition to search of \( \nu \), \( \dot{\nu} \) and \( \ddot{\nu} \), a technique is first developed permitting search of two angular coordinates, absolute speed value and direction of a proper motion. In that way timing of gamma-ray pulsars gives amount of information compatible with radiopulsars, but using of real data gives much poorer precision. In gamma-ray sources with rare pulses the periodicity criteria are quite different from the ones in radio region. Data on the coordinates and proper motion of Geminga, obtained from timing studies, do not contradict inside the errors to its identification with \( G'' \) star and its proper motion. These errors are larger than ones in optical measurements, but are smaller than corresponding errors in X-ray and \( \gamma \)-ray data. Estimations of the gamma-ray pulsar coordinates and its proper motion could be obtained independently on its optical or radio component, and are available in their absence.

1 Introduction

The discovery of Geminga as a “true” pulsar, but without visible radioemission, gave additional evidence to the idea that hard gamma-ray emission (\( E \geq 30 \) MeV) is an inherent property of pulsar radiation. Before this only the Vela pulsar, the strongest hard gamma-ray source and the young Crab pulsar gave hints of this possibility. Observations on the sky, made by EGRET on CGRO [28] [30] have shown that only young pulsars with age not exceeding several tens thousand years and, possibly, millisecond pulsars [31] give observable flux in gamma radiation. It is not yet clear, what is the mechanism of this gamma radiation and whether it has a threshold character, or if there is a gradual decrease of hard gamma ray flux with an age.

The existence of the Geminga pulsar indicates, that there could be other gamma ray pulsars with no radioemission, which are exhibited in EGRET observations as ordinary point-like sources, see i.e. [28] [11].

Determination of pulsations in a hard gamma ray source is a very difficult problem, connected with rareness of arriving quanta \( \delta t \gg P \), and small total number of quanta. When the value of the period is known from other observations (radio or X-ray), timing analysis gives the possibility to reproduce this periodicity also in gamma region [1], [25]. When there is no information about the period, it could, in principle, be found from pure gamma data [4], but this could take enormous amount of computer time and has not been in full realized in practice.
A position on the sky of pulsars with no radioemission cannot be established precisely; the best position obtained from X-ray observations are between 3′′ (Einstein) and 5′′ (ROSAT) for 90% level \[1, 2\]. Optical identification of Geminga with very faint > 25′′.5 object have been done in \[4\] and later measurements of its proper motion \[6\] and parallax \[13\] can be considered as an evidence of reality of this identification.

Timing of Geminga in hard gamma region based on COS–B \[5\], \[20\] and EGRET \[25\] data gave anomalously high braking index \(n = \dot{\nu}/\ddot{\nu}^2 \sim 10 – 30\), corresponding to very high second derivative \(\ddot{\nu}\). While there is a possibility, that it is connected with poor precision of \(\ddot{\nu}\) determination, it is worth to investigate other explanations. It was suggested in \[7\], that high value of \(n(\ddot{\nu})\) results from errors in its coordinates, leading to incorrect barycenter reduction procedure, which spoils the timing procedure. This problem is well known for pulsar timing, where the error in coordinates give a one year periodical deviations from the smooth curve in the pulse arriving time, what permits to improve pulsar position to amazing precision of the order and even better than VLBI observations \[19, 27\].

Here we describe a method for investigation of timing of gamma pulsars, represented by periodical objects with rare pulses, which gives possibility to determine 7 parameters of a gamma pulsar: frequency \(\nu\), its two derivatives \(\dot{\nu}\) and \(\ddot{\nu}\), angular coordinates \(\alpha\) and \(\delta\) of the source, absolute value \(v\) and direction of a velocity of a proper motion, characterized by an angle \(\theta\). When registered pulses are rare, so that their time separation \(\delta t\) is much larger then the period \(P\), the method of investigation is quite different from the one, used for radiopulsars. On the artificial sample of data, which properties simulate Geminga, but in contrary, have very narrow light curve (\(\delta\)-function), no systematic errors and no false quanta, we have managed to determine coordinates and proper motion parameters with very high precision. This precision is decreasing when we go to pulses with a finite width, in presence of background and systematic errors.

Application of this method to real data sample of Geminga from COS–B and EGRET was not so successful, because of smooth light curve, “nonperfectness” of data, possible glitches. We present here results for most probable position and proper motion characteristics, determined by using of periodicity criteria, which are not in contradiction with more precise optical data, and has better coordinate precision then \(\gamma\) - ray or \(X\) - ray data. Correlation properties of timing criteria, used for sources with rare pulses are investigated, before applying them to a timing procedure.

2 Barycentric corrections: account for angular coordinates correction and proper motion

For timing analysis all data must be presented in the same coordinate system, which as a rule is connected with a barycenter of the Solar system. Consider first a situation, when angular coordinates of a source \(\alpha\) and \(\delta\) are known exactly. On Fig. 4 \(xyz\) is a coordinates system, that remains at rest with respect to distant stars. The point O is the barycenter of the Solar system. The space probe is in the point S with Cartesian coordinates \((x_0, y_0, z_0)\). A direction to a source is defined by a straight line with coordinate angles \(\alpha\) and \(\delta\), where \(\alpha\) angle is counted in \(xy\) plane counterclockwise from the positive direction of \(x\)-axis. An angle \(\delta\) is countered in a plain, perpendicular to \(xy\) plane. \(SB\) is a perpendicular from the space probe (point S) onto a line from the barycenter to a source. \(BC\) is a perpendicular from the
point $B$ to the $xy$ plane, and $CD$ is a line parallel to $OB$; points $C$ and $D$ belong to the $xy$ plane. The unit vector pointed from the barycenter to a source has Cartesian coordinates $(\cos \alpha \cos \delta, \sin \alpha \cos \delta, \sin \delta)$. The scalar product of this vector and the radius-vector of the space probe is the length of the segment $OB$.

$$OB = x_0 \cos \delta \cos \alpha + y_0 \cos \delta \sin \alpha + z_0 \sin \delta$$

A time interval during which light flies the length $OB$ is a barycenter correction, if a source is far enough and $SB$ is a part of a flat wave front. This time interval $\Delta T$, which must be added to the moment of each event in the point $S$ to obtain a corresponding barycenter moment, is determined as

$$\Delta T = \frac{1}{c} (x_0 \cos \delta \cos \alpha + y_0 \cos \delta \sin \alpha + z_0 \sin \delta)$$

In a common choice $\alpha$ and $\delta$ coincide with right ascension and declination, when $xy$ is an Earth’s equatorial plain for some fixed epoch and $x$-axis points to spring equinox at the same epoch. Procedure of calculation of $\Delta T$ with precise account of Earth and satellite motion is described in \cite{24}.

Suppose that coordinates $\alpha$ and $\delta$ are known not exactly with corresponding errors $d\alpha$ and $d\delta$. Then for small errors we may find from (1) corresponding barycenter corrections in linear approximation

$$\delta T = \frac{\partial \Delta T}{\partial \alpha} d\alpha + \frac{\partial \Delta T}{\partial \delta} d\delta =$$

$$= \frac{1}{c} \left( -x_0 \cos \delta \sin \alpha + y_0 \cos \delta \cos \alpha \right) d\alpha + \frac{1}{c} \left( -x_0 \sin \delta \cos \alpha - y_0 \sin \delta \sin \alpha + z_0 \cos \delta \right) d\delta$$

Consider for simplicity a case when a space probe orbit around the Sun is circular and, consequently, its angular velocity $\omega$ is a constant. Then $x_0 = R \cos \omega t$, $y_0 = R \sin \omega t$, $z_0 = 0$ and

$$\Delta T = \frac{R}{c} \left( \omega t \cos \delta \cos \alpha + \sin \omega t \cos \delta \sin \alpha \right) = \frac{R}{c} \cos \delta \cos (\omega t - \alpha)$$

where $R$ is a radius of its orbit. The correction (2) then is reduced to

$$\delta T = \frac{R}{c} \left[ \cos \delta \sin (\omega t - \alpha) d\alpha - \sin \delta \cos (\omega t - \alpha) d\delta \right].$$

Assume that only first and second derivatives of $\nu$ are essential, and a frequency of the signal may be represented by

$$\nu = \nu_0 + \dot{\nu}_0 t + \frac{\ddot{\nu}_0}{2} t^2,$$

where index ”0” is referred to the epoch $t = 0$. Define a current arrival time of photons from the source, measured on the satellite, as $\tilde{t}$. When the source coordinates are known exactly, barycenter arrival time $t$ is found as

$$t = \tilde{t} + \Delta T,$$
and having barycenter arrival times it is possible to find $\nu_0$, $\dot{\nu}_0$ and $\ddot{\nu}_0$ using the criteria from previous sections.

When source coordinates are not known exactly and their possible errors are $d\alpha$ and $d\delta$, the error in barycenter correction is determined by (4). In order to estimate an input of these errors on timing characteristics let us compare phases of the arriving signal calculated from measurements $\phi'$ (with errors) and in true barycenter time $\phi$, so that

$$\phi' = \int_0^{t'} \nu' dt', \quad \phi = \int_0^t \nu dt \tag{7}$$

Here $t'$ is the time calculated from (6), and $\nu'$ is a frequency found after barycenter corrections, containing errors. Times $t$ and $t'$ correspond to the same event, so we may rewrite $\phi'$ in true barycenter coordinates as

$$\phi' = \int_0^t \left[ 1 + \frac{d\delta T}{dt} \right] dt \tag{8}$$

Using (4) and (5) in (8) we obtain after integration and account of (4),(5)

$$\phi' = \text{const} + \int_0^t \nu dt + \nu \delta T - \int_0^t \delta T(\dot{\nu}_0 + \ddot{\nu}_0 t)dt$$

$$= \text{const} + \int_0^t \nu dt + \left( \nu - \frac{\dot{\nu}_0}{\omega^2} \right) \delta T - (\dot{\nu}_0 + \ddot{\nu}_0 t) \int_0^t \delta T dt. \tag{9}$$

Here and farther relations

$$\dot{\delta T} = -\omega^2 \delta T, \quad \int (\int \delta T dt) dt = \frac{\delta T}{\omega^2}, \quad \frac{d\delta T}{dt} = -\omega^2 \int \delta T dt \tag{10}$$

are used, and it follows from (4)

$$\int \delta T dt = -\frac{R}{c\omega} \left[ \cos \delta \cos(\omega t - \alpha) d\alpha + \sin \delta \sin(\omega t - \alpha) d\delta \right]. \tag{11}$$

Differentiating (9) we obtain an input of the angular coordinate errors into the values of frequency and its derivatives

$$\nu' = \frac{d\phi'}{dt} = \nu \left( 1 + \frac{d\delta T}{dt} \right),$$

$$\dot{\nu}' = \frac{d^2\phi'}{dt^2} = \dot{\nu} \left( 1 + \frac{d\delta T}{dt} \right) - \nu \omega^2 \delta T,$$  \quad \tag{12}

$$\ddot{\nu}' = \frac{d^3\phi'}{dt^3} = \ddot{\nu} + (\ddot{\nu} - \nu \omega^2) \frac{d\delta T}{dt} - 2\dot{\nu} \omega^2 \delta T,$$

where

$$\dot{\nu} = \dot{\nu}_0 + \ddot{\nu}_0 t, \quad \ddot{\nu} = \ddot{\nu}_0,$$  \quad \tag{13}

and current values $\nu'$, $\dot{\nu}'$ and $\ddot{\nu}'$ are connected with corresponding values at $t = 0$ as
\[\nu' = \nu_0' + \dot{\nu}_0't + \frac{\ddot{\nu}_0't^2}{2}\]
\[\nu_0' = \nu' - \dot{\nu}'t + \frac{\ddot{\nu}'t^2}{2}\]
\[\dot{\nu}_0' = \dot{\nu}' - \ddot{\nu}'t\]
\[\ddot{\nu}_0' = \ddot{\nu}'\]  
(14)

The detailed variant of previous calculations can also be found in [9].

In presence of a proper motion of the source the errors \(d\alpha\) and \(d\delta\) change linearly in first approximation as
\[d\alpha = d\alpha_0 + \alpha_0 t, \quad d\delta = d\delta_0 + \delta t.\]  
(15)

It leads to farther complication of the formula (9)-(12). Note that in simulations we deal not with these formulae, but directly with arrival times of photons \(t_i, t_i'\) and \(t_i\). In the problem of timing of radiopulsars the precision of observational data is very high, so appearance of the periodical 1 year component gives a direct indication to the errors in angular coordinates of the pulsar, possibility to improve them [27, 19], and to determine a proper motion. In periodic sources with rare pulses a quality of data is much worse and other methods, based on above mentioned criteria must be used.

3 Mathematical simulation

For checking a possibility to use criteria considered above for determination of corrections to the angular coordinates and proper motion, in addition to frequency and its two derivatives, artificial sample of data was produced. A pulse shape was taken as \(\delta\)-function with a frequency of the signal changing in time according to (5), what corresponds to a phase dependence
\[\phi = \phi_0 + \nu_0 t + \frac{\dot{\nu}_0 t^2}{2} + \frac{\ddot{\nu}_0 t^3}{6}.\]  
(16)

We need to find time moments, corresponding to phase values \(\phi_i = 2\pi i\). Two sets of input parameters were considered. The time \(t = 0\) is related to a point of the orbit, where \(\alpha = 0\).

(i) \(\phi_0 = 0, \nu_0 = 4 \text{s}^{-1}, \dot{\nu}_0 = -2 \cdot 10^{-8} \text{s}^{-2}, \ddot{\nu}_0 = 3 \cdot 10^{-16} \text{s}^{-3}\),
\[\omega = 6.060171 \cdot 10^{-6} \text{s}^{-1}\]  
(17)

(ii) \(\phi_0 = 0, \nu_0 = 4 \text{s}^{-1}, \dot{\nu}_0 = -2 \cdot 10^{-13} \text{s}^{-2}, \ddot{\nu}_0 = 3 \cdot 10^{-26} \text{s}^{-3}\),
\[\omega = 1.991063802 \cdot 10^{-7} \text{s}^{-1}\]

The source parameters are chosen to satisfy a relation \(\nu_0\dot{\nu}_0/\dot{\nu}_0^2 = 3\), supposed to be valid for ejecting pulsars [22]. Because of low reliability of \(\ddot{\nu}\) detecting in gamma observations, some authors [23] set it equal to zero. The modeling year duration \((2\pi/\omega)\) is equal to 12 days in the first case; and is a true value of 365.2422 days in the second, when the moment \(t = 0\) corresponds to 21 March. Note that in the second case the values of \(\nu_0\) and \(\dot{\nu}_0\) are chosen very close to that of Geminga [20].
One possible way to find \( t_i \) is to use Burmann-Lagrange expression, which links the Taylor coefficients of direct and inverse functions. We have used instead a procedure, valid for a general law of a phase dependence \( \phi(t) \), based on a Taylor expansion formula

\[
t = \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{d^n t}{d\phi^n} \right)_0 \phi^n = \sum_{n=0}^{\infty} \frac{A_n}{n!} \phi^n.
\]

To find the coefficients \( A_n = \left( \frac{d^n t}{d\phi^n} \right)_0 \), \( n \geq 1 \), use an evident equality

\[
\phi \frac{dt}{d\phi} = 1.
\]

That gives

\[
A_1 = \frac{dt}{d\phi_0} = \frac{1}{\phi_0}.
\]

Differentiating (19) \((n-1)\) times over \( t \) we obtain a relation, linear to \( A_n \), what permits to express \( A_n \) as a function of \( A_m, m \leq n-1 \), and \( \dot{\phi}, \ddot{\phi} \). As an example, after 5 differentiation we get

\[
A_1 \phi_0 + 6A_2 \phi_0 \dot{\phi}_0 + 15A_2 \phi_0 \ddot{\phi}_0 + 10A_2 \phi_0 \dddot{\phi}_0 + 15A_3 \phi_0 \ddot{\phi}_0 + 60A_3 \phi_0 \dddot{\phi}_0 \phi_0 \\
+ 15A_3 \phi_0 ^3 + 20A_4 \phi_0 ^3 \phi_0 + 45A_4 \phi_0 ^3 \ddot{\phi}_0 + 15A_5 \phi_0 ^4 \phi_0 + A_6 \phi_0 ^6 = 0,
\]

where index \"0\" indicates time \( t = 0 \). For (16) with \( \phi_0 = 0 \) at \( m \geq 4 \) we have \( \dot{\phi}_0 = \nu_0 \), \( \ddot{\phi}_0 = \nu_0 \), \( \dddot{\phi}_0 = \nu_0 \) and get from (21) an equation for \( A_6 \)

\[
10A_2 \nu_0 ^2 + 60A_3 \nu_0 \nu_0 \nu_0 + 15A_3 \nu_0 ^3 + 20A_4 \nu_0 ^3 \nu_0 + 45A_4 \nu_0 ^3 \nu_0 ^2 + 15A_5 \nu_0 ^4 \nu_0 + A_6 \nu_0 ^6 = 0
\]

The first criterion \( K_1 \) of periodicity \([1, 16, 17, 18]\) was used to investigate periodicity properties of series of pulses. For a purpose of testing short intervals of \"observation\" were taken in different parts of the year. The error in coordinates was taken equal to 5 arc seconds in absolute value \( \sqrt{d\alpha^2 + d\delta^2} \), but the deviations from the initial point were taken in eight different directions, separated by 45°. The values of \( \nu_0', \nu_0'', \nu_0''' \) that have been detected by \( K_1 \) criterion coincide in both cases with a very high precision with theoretical ones from (12)-(14), see Table 1. It may be seen from Table 1 a strong influence of the errors on the determination of \( \nu_0'' \) by using a criterion. While the error in \( \nu_0'' \) is almost linearly proportional to the error in \( \sqrt{d\alpha^2 + d\delta^2} \), it is evident that at an angular error larger than \( 10^{-3} \) arc sec direct determination of \( \nu_0'' \) by criterion becomes impossible. This may be a reason for a high breaking index of Geminga \([20, 7]\).

Let us now formulate a problem of timing of a gamma pulsar, which gives a possibility for a search of its timing properties together with angular coordinates and a proper motion. Assume that a gamma pulsar simulated by computer is emitting signals with a frequency changing according to (5), satisfying condition \( \nu_0 \nu_0 / \nu_0^2 = 3 \).

Let the signal registered on the probe is reduced to the barycenter time, using the source coordinates \( \alpha_0 \) and \( \delta_0 \) (base point), which contain errors \( d\alpha_0 \) and \( d\delta_0 \) respectively. We
suspect also a proper motion of the source defined by following parameters: at the moment $t_0 = 0$ the source has coordinates $\alpha_0 + d\alpha_0$ and $\delta_0 + d\delta_0$ and a velocity modulus $\dot{\theta} = \text{const.}$ A velocity direction is defined by an angle $\theta$, which is counted clockwise from the positive $\alpha$-axis direction. The current source coordinates are consequently

$$\alpha = \alpha_0 + d\alpha_0 + v_< t \cos \theta, \quad \delta = \delta_0 + d\delta_0 + v_< t \sin \theta, \quad (23)$$

Thus, we have seven parameters that are needed to be found self-consistently:

- $\nu_0$ – frequency of the source signal
- $\dot{\nu}_0$ – first derivative of the frequency
- $\ddot{\nu}_0$ – second derivative of the frequency
- $d\alpha_0$ – shift in right ascension from the base point $(\alpha_0, \delta_0)$ at $t_0$ epoch
- $d\delta_0$ – shift in declination from the base point $(\alpha_0, \delta_0)$ at $t_0$ epoch
- $v_<$ – proper angular velocity of the source in celestial coordinates
- $\theta$ – direction of velocity $v<$, counted clockwise from the positive $\alpha$-axis direction

with additional restriction: $\nu_0 \dot{\nu}_0 / \nu_0^{\ddot{\nu}} = 3$ following from the model of the pulsar radiation [22].

In data simulation we fix parameters $\nu_0, \dot{\nu}_0, \ddot{\nu}_0$ as (ii) in (17), find true $\alpha$ and $\delta$ from (23) with $d\alpha_0 = 2''$, $d\delta_0 = 3''$, $v_\,< 0^\circ/2$ per year, $\theta = 60^\circ$, and create the simulated data, i.e. the sequence of time moments of pulses. Time moments (true barycenter) found for a source from (19)-(22) are then recalculated for a probe using correct coordinates and (23). Now to a set of time moments "registered" by a probe from the source with subscribed coordinates $\alpha_0$ and $\delta_0$, containing errors, we apply the algorithm for searching the periodic signal to extract all the seven parameters from the simulated data set. Namely, we consider a number of different sets of 7 mentioned parameters. For each set we evaluate supposed errors, introduced in the data due to errors in a position and in a proper motion of the object. After that we subtract these supposed errors from the data. Then, assuming the data free of errors the periodicity criterion value was calculated. Remind, that if we assume the data free of errors, it means that the phases of the pulses must obey the simple relation (16). The criterion reaches its absolute maximum only for an exact set of parameters. There appear a number of local (or false) maxima. Fig. 5 demonstrates a typical structure of the criterion depending on two parameters: $\nu$ and $\dot{\nu}$, when other five ones are fixed (see also [23]). Because the height of a “false” maxima is close to “true” one, it is very difficult to separate the absolute maximum among a series of local ones.

We are looking for an absolute maximum, using a grid in 4-dimensional space $(d\alpha_0, d\delta_0, v_<, \theta)$, that was defined by the following way: $d\alpha_0$ varies from $-5''$ to $5''$, $d\delta_0$ varies from $-5''$ to $5''$, $v_<$ varies from $0''$/year to $0'05$/year and $\theta$ varies from $0^\circ$ to $350^\circ$. Three other parameters $(\nu_0, \dot{\nu}_0, \ddot{\nu}_0)$ were detected jointly for each point of above grid, using the grid $6 \times 6 \times 6$ with 12 times consecutively diminishing steps for all three axes. The maximum, detected at the previous step was placed to the center of the grid and all the scale multiplied by the factor of 0.4. This procedure was repeated for 12 times, so that the last grid steps are $0.4^{12} \approx 1.7 \cdot 10^{-5}$ times as small as the first ones. All seven parameters chosen for modeling were found with a precision, limited only by computational grid connected with a power of the computer, using criterion $K_1$. So, for a clean set of data the proposed procedure of searching is working effectively. Situation is becoming much more controversial when we apply it to real data existing to the moment.
4 Application to Geminga

4.1 Analysis of COS–B data

COS–B mission had been operated since August, 1975 till April 1982, and had observed Geminga in five shifts. Their numbers are 00, 14, 39, 54 and 64. Some measurements of Geminga position, reduced to the epoch 1950.0, are summarized in Table 2 and are plotted in Fig. 6.

It was obtained in [20] from COS–B data $\nu = 4.217 \text{ Hz}, \dot{\nu} = -1.952 \cdot 10^{-13} \text{ Hz}\cdot\text{s}^{-1}$, and a large value of $\ddot{\nu} = (28 \pm 16) \cdot 10^{-26} \text{ Hz}\cdot\text{s}^{-2}$, corresponding to a braking index $n = 31 \pm 18$. Barycenter corrections have been done for standing Geminga with coordinates No. 3,8 in Table 2. Quanta selection used in our analysis has been done by two different ways:

1) all the quanta in the circle of $r = 5^\circ$ around Geminga position; the interval 54 was excluded because of low reliability; it is total of 1505 quanta.

2) all the quanta of the energy $E > 50$ MeV laying in the circle $r = 12.5 \cdot E^{-0.16}$, where $E$ is measured in MeV and $r$ in degrees [12]; total of 1883 quanta.

The second selection is close to that used in [4]. A problem of a quanta selection criterion is a very delicate one because it is practically impossible for a single quantum to decide, was it really radiated by Geminga or belongs to a background. Both mentioned selections are noisy, but the first one is worse.

Geminga was considered as a moving object. The base coordinates, that are used for initial barycenter reduction are the position measured by Einstein’s satellite in 1981 (see Table 2). Geminga motion was defined by its velocity, direction and initial position at the epoch 1979, March, 14.0 [20]. The obtained barycenter time moments for each quantum was additionally reduced to the barycenter, using expression (2). A motion of the probe defined by $x_0(t), y_0(t), z_0(t)$ was taken from databases of COS B [24] or EGRET. The object coordinates were calculated by this procedure separately for each quantum according to a supposed object motion.

The results, with using the second selection from the mentioned above, are not very certain. The criterion appear to exhibit a gently sloping maximum at the following model parameters: velocity $\dot{v}_< = 0''2 - 0''3$ per year, direction $\theta = 40^\circ - 60^\circ$ and the initial coordinate offsets $d\alpha_0$ and $d\delta_0$ at the mentioned epoch are $-2''$ for both $\alpha$- and $\delta$-axes, but the uncertainty here is high and may reach 2'' for both coordinates. As to the periodicity parameters, they are in a good agreement with [20], except the second derivative $\ddot{v}_0$, which is a bit smaller but lays within the error box of standing Geminga. The motion of Geminga, obtained in our investigation does not contradict to the motion of G'' star [6].

Unfortunately, this solution is not a unique one, and there are a number of other maxima of approximately the same height. When we search for a global maximum in 7-dimensional space it is extra difficult to detect “the main maximum” among a series of other local maxima. For example, there is an accessory maximum at $v_\phi = 0''1 - 0''2$ per year, $\theta = 340^\circ - 360^\circ$ and very badly detected initial offsets (it can only be said that they both are negative). The periodicity parameters here are approximately the same as above, but $\ddot{v}_0$ appears to be negative.
4.2 Analysis of EGRET data

EGRET experiment is operating since April 1988. There are 9 periods of observations where Geminga was not far from the center of a view field (less than 30°, the standard requirement). The following sessions was used for data investigations: 2, 3, 4, 5, 10, 21, 2130, 2210, 3100. The standard procedure for the barycenter correction was used [32, 33], but we have used coordinates of the object from [3], line 8 in Table 2, different from those, indicated in EGRET data base. We have used the same Geminga coordinates for both satellites, COS-B and EGRET. They are the best fit Einstein position, but in the last case we were to reduce them to the epoch 2000.0, because it is used in an appropriate barycenter reduction routines. Reduced coordinates are the following:

\[
\alpha_{2000} = 98^\circ 28'30''90 = 98^\circ 47525 \quad \delta_{2000} = +17^\circ 46'11''6 = +17^\circ 76989 \quad (24)
\]

And in EGRET data base the coordinates are:

\[
\alpha_{2000} = 98^\circ 48 = 98^\circ 28'48'' \quad \delta_{2000} = +17^\circ 77 = +17^\circ 46'12''
\]

Other authors [14, 23] use the coordinates close to (24), they differ less than 1'' from the center of Einstein error box (lines 3, 8 in the Table 2).

We have used a number of techniques for quanta selection and have compared the results. The selections used are the following:

1) all the quanta of the energy \( E > 70 \text{ MeV} \) laying in the circle \( r = 5.85 \cdot (E/100)^{-0.534} \), where \( E \) is measured in MeV and \( r \) in degrees [29]; it is so-called the standard selection; total of 6751 quanta.

2) all the quanta of the energy \( E > 1500 \text{ MeV} \) in the circle \( r = 2^\circ \); total of 365 quanta.

3) all the quanta of the energy \( E > 2000 \text{ MeV} \) in the circle \( r = 2^\circ \); total of 223 quanta.

4) all the quanta of the energy \( E > 2000 \text{ MeV} \) in the circle \( r = 0^\circ 5 \) round the Geminga position; total of 100 quanta.

5) all the quanta of the energy \( E > 3000 \text{ MeV} \) in the circle \( r = 0^\circ 5 \) round the Geminga position; total of only 51 quanta.

The criterion value for unmoving Geminga, resting in Einstein’s HRI position is \( K_1 = 0.0572 \) and the parameters of periodicity are \( \nu_0 = 4.21775012925 \text{ Hz}, \nu'_0 = -1.95312 \cdot 10^{-13} \text{ Hz.s}^{-1}, \nu''_0 = (20 \pm 12) \cdot 10^{-26} \text{ Hz.s}^{-2} \). An appropriate light curve is shown in Fig. 7. The results of search in 7-dimensional space for different selections are:

1) There is a gently sloping maximum in criterion value at the following parameters: \( \nu_0 = 4.2177501295 \text{ Hz}, \nu'_0 = -1.9532 \cdot 10^{-13} \text{ Hz.s}^{-1}, \nu''_0 = (22 \pm 13) \cdot 10^{-26} \text{ Hz.s}^{-2}, v_\varphi = 0^\circ 3 - 0^\circ 4 \) per year, \( \theta = 40^\circ - 60^\circ \), \( d\alpha_0 \) and \( d\delta_0 \) are defined with very low precision and both lay in the interval \(-2''\) to \(0''\). The criterion value \( K_1 = 0.05732 \). The second derivative \( \ddot{\nu} \) here is of rather large value, so that the braking index is approximately equal to 25.
2) There is a gentle maximum in criterion value at the following parameters: \( \nu_0 = 4.2177501273 \text{ Hz}, \dot{\nu}_0 = -1.952711 \cdot 10^{-13}\text{ Hz s}^{-1}, \ddot{\nu}_0 \approx (0 \pm 10) \cdot 10^{-26}\text{ Hz s}^{-2}, \nu_\prec = 0"3 \) per year or more, \( \theta = 40^\circ - 60^\circ, \alpha_0 = 0" - (-2") \), \( \delta_0 = 0" - (-1") \) and criterion value \( K_1 = 0.22357 \). The second derivative here is very close to zero and the braking index is, respectively, also low and could be close to its theoretical value.

3) There is a gentle maximum in criterion value at approximately the following parameters: \( \nu_0 = 4.2177501261 \text{ Hz}, \dot{\nu}_0 = -1.952611 \cdot 10^{-13}\text{ Hz s}^{-1}, \ddot{\nu}_0 \approx (-2.4 \pm 10) \cdot 10^{-26}\text{ Hz s}^{-2}, \nu_\prec = 0"3 - 0"4 \) per year, \( \theta = 40^\circ - 80^\circ, \alpha_0 = 0" - (-2") \), \( \delta_0 = -1" - (-3") \) and the criterion value \( K_1 = 0.25535 \). It was impossible to determine the parameters with higher precision.

4) There is a gentle maximum in criterion value at the following parameters: \( \nu_0 = 4.2177501344 \text{ Hz}, \dot{\nu}_0 = -1.954477 \cdot 10^{-13}\text{ Hz s}^{-1}, \ddot{\nu}_0 = (68 \pm 40) \cdot 10^{-26}\text{ Hz s}^{-2}, \nu_\prec = 0"3 - 0"4 \) per year or more, \( \theta = 220^\circ - 260^\circ, \alpha_0 \) and \( \delta_0 \) are negative and the criterion value \( K_1 = 0.3072 \). Because of a very poor statistics this selection as well as the following one can be considered as a test only. The line of motion here is the same as in the previous cases, but the direction is opposite.

5) There is a relatively good maximum despite of a very poor statistics at the following parameters: \( \nu_0 = 4.2177501220 \text{ Hz}, \dot{\nu}_0 = -1.95282 \cdot 10^{-13}\text{ Hz s}^{-1}, \ddot{\nu}_0 = (12 \pm 10) \cdot 10^{-26}\text{ Hz s}^{-2}, \nu_\prec = 0"2 - 0"3 \) per year, \( \theta = 20^\circ - 60^\circ, \alpha_0 = 0" - (-2") \), \( \delta_0 \approx 0" \) and the criterion value \( K_1 = 0.44441 \). The reliability of this result is not high, but large criterion value indicates that we may deal with real motion of the object.

4.3 Analysis of combined COS-B and EGRET data

We have combined COS-B and EGRET data with the following selection criteria:

**COS-B:** \( E > 50 \text{ MeV} \) and the standard conditions for \( r \): \( r = 12.5 \cdot E^{-0.16} \)

**EGRET:** \( E > 70 \text{ MeV} \) and the standard conditions for \( r \): \( r = 5.85 \cdot (E/100)^{-0.534} \)

There are total of 1883 + 6751 = 8634 quanta.

The combined series is not self-contradicting. There is a gentle maximum in criterion value at the following parameters: \( \nu_0 = 4.21775012323 \pm 0.000000000025 \text{ Hz}, \dot{\nu}_0 = (-1.952554 \pm 0.000025) \cdot 10^{-13}\text{ Hz s}^{-1}, \ddot{\nu}_0 = (-2.5 \pm 10) \cdot 10^{-26}\text{ Hz s}^{-2}, \nu_\prec = 0"5 - 0"6 \) per year, \( \theta = 55^\circ - 65^\circ, \alpha_0 = -2" - (-4") \), \( \delta_0 = 1" - 2" \) and the criterion value \( K_1 = 0.04937 \).

Note that 1σ errors above were obtained by approximate estimations.

5 Discussion

According to our investigations the coordinates and motion of Geminga obtained from timing of gamma pulsar is in a satisfactory agreement with the motion of C" star when separately COS-B or EGRET data are used. Parameters following from the combined data set are in much worse agreement. There are two possible reasons of it. First there could exist a systematic error between the data of two probes; and second, period could behave
nonmonotonously between 1982 and 1988 years, and the period jump (pulsar glitch) of the order of $\frac{\Delta P}{P} \geq 10^{-10}$ could already spoil the parameters obtained by criteria.

The value of the second derivative $\ddot{\nu}$ does not coincide with the theoretical one, however it is lower than in the previous investigations, but there are weighty reasons to explain this phenomenon. The second derivative is a very sensitive variable, and even $0\,\prime\prime\,001$ error in angular coordinates changes $\ddot{\nu}$ by the value, comparable with the result (see Table 1). If there was a jump in pulsar period, it may also cause the incorrect value of a variable. Possibility to improve angular resolution by timing is strongly limited by small number of quanta and existence of considerable background.

As a result of application to Geminga of the developed method of timing of gamma pulsars we have obtained that determination of true value of $\ddot{\nu}$ is possible only at very high precision (better then $0\,\prime\prime\,001$) of angular localization. At good statistics of gamma pulsars corresponding improvements would become possible from timing analysis. New types of gamma ray telescopes based on very wide aperture ($\geq 2.5\,\pi$ steradian) and higher threshold of a few hundred MeV [8, 21] would permit to get higher angular resolution ($\sim 1$ arc min), reducing influence of a background, get $\sim 100$ better statistics due to continuous monitoring of larger part of the sky in this region.

For the existing data of Geminga from COS B and EGRET it was obtained, using only gamma-ray data, that criterion value reaches its maximum at nonzero value of a proper motion. The coordinates of the source were confirmed with precision $\sim 2\,\prime$ what is better then follows from X ray data, but, of course, is worse then the precision obtained in the optical observations.

Observations of radio pulsars have shown, that their optical and X-ray luminosity is decreasing with time much more rapid, that radio and hard gamma radiation. So at increasing sensitivity we expect a discovery of tens of new gamma-ray pulsars similar to Geminga, may be without X-ray and optical counterparts. For such objects method of timing of gamma-ray pulsars developed above would be a main and may be a single means of investigation of such sources by data processing.

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Table 1.
Properties of a source with angular coordinates and timing characteristics close to Geminga from (17), case (ii), calculated using criterion (1). Barycenter corrections have been done using angular coordinates with indicated errors.

| Source coordinates: $\alpha = \text{6}^\circ\text{30}''\text{00}^s$, $\delta = \text{+17}^\circ\text{30}''\text{00}''$ |
|---------------------------------------------------------------|
| **Month, day** | **Errors** | **Theoretical values** |
| $d\alpha,$ $''$ | $d\delta,$ $''$ | $\bar{\nu}_0,$ $s^{-1}$ | $\dot{\nu}_0,$ $10^{-13}$ $s^{-2}$ | $\ddot{\nu}_0,$ $10^{-26}$ $s^{-3}$ |
| May, 20 | 5.00 | 0.00 | 3.99999999748 | $-1.9738$ | $-28600$ |
| May, 20 | 2.24 | 2.24 | 3.99999999744 | $-1.9879$ | $-9620$ |
| May, 20 | 0.00 | 5.00 | 3.99999999681 | $-1.9992$ | 7140 |
| May, 20 | $-2.24$ | 2.24 | 3.99999999970 | $-2.0114$ | 16000 |
| May, 20 | $-5.00$ | 0.00 | 4.000000000252 | $-2.0262$ | 28600 |
| May, 20 | $-2.24$ | $-2.24$ | 4.00000000256 | $-2.0121$ | 9620 |
| May, 20 | 0.00 | $-5.00$ | 4.00000000319 | $-2.0008$ | $-7140$ |
| May, 20 | 2.24 | $-2.24$ | 4.00000000030 | $-1.9989$ | $-16000$ |
| July, 10 | 5.00 | 0.00 | 3.99999999621 | $-1.9695$ | $-35700$ |
| Aug., 20 | 5.00 | 0.00 | 4.00000000544 | $-1.9852$ | $-22300$ |
| Aug., 20 | 0.00 | $-5.00$ | 4.00000001020 | $-2.0155$ | 9110 |
| Nov., 20 | 5.00 | 0.00 | 4.00000008067 | $-2.0725$ | 29150 |
| Nov., 20 | $-2.24$ | 2.24 | 3.99999996211 | $-1.9631$ | $-16200$ |
| No. | $\alpha$      | $\delta$     | Error | Comments                      |
|-----|---------------|--------------|-------|-------------------------------|
| 1   | 97°44′43″7    | +17°48′27″5  | 12″   | ROSAT PSPC, 1991, Sep. 19-21, [2] |
| 2   | 97°44′51″7    | +17°48′36″0  | $\approx$ 5′   | ROSAT HRI, 1991, Mar. 19, [2] |
| 3   | 97°44′47″2    | +17°48′33″0  | 3′2   | Einstein, 1981, Mar. 18, [2]  |
| 4   | 97°44′45″9    | +17°48′32″7  | 0″46  | G″ star, 1984, [3]            |
| 5   | 97°44′45″9    | +17°48′32″6  | 0″5   | G″ star, 1986, Feb. 3, [3]    |
| 6   | 97°44′46″5    | +17°48′33″0  | 0″68  | G″ star, 1987, [3]            |
| 7   | 97°44′47″2    | +17°48′33″6  | 0″16  | G″ star, 1992, [3]            |
| 8   | 97°44′47″2    | +17°48′33″0  | 3′0   | Einstein, 1981, Mar. 18 [3]   |
Figure 4: Geminga 40-bin light curve from ERGET data, for standing Geminga with coordinates No. 8 from Table 2 and standard quanta selection.

Epoch 1979, March, 14.0
\[ \nu_0 = 4.21775012925465 \text{ Hz} \]
\[ \dot{\nu}_0 = -1.95316166 \times 10^{-13} \text{ Hz s}^{-1} \]
\[ \ddot{\nu}_0 = 19.66 \times 10^{-26} \text{ Hz s}^{-2} \]
\[ n = \frac{\nu_0 \ddot{\nu}_0}{\dot{\nu}_0^2} = 21.73 \]
Figure captions

Figure 1: On the barycentric correction.

Figure 2: On the absolute maximum structure.

Figure 3: On the Geminga position.

Figure 4: Geminga 40-bin light curve from ERGET data, for standing Geminga with coordinates No. 8 from Table 2 and standard quanta selection.
Figure 1: On the barycentric correction
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This figure "FIG1.GIF" is available in "GIF" format from:

http://arxiv.org/ps/astro-ph/9707190v1
This figure "FIG2.GIF" is available in "GIF" format from:

http://arxiv.org/ps/astro-ph/9707190v1
