THE GUNN PETERSON EFFECT:
A TEST FOR A BLACK HOLES INDUCED
PHOTOIONIZATION OF THE INTERGALACTIC MEDIUM

Marina Gibilisco
Queen Mary and Westfield College
Astronomy Unit, School of Mathematical Sciences
Mile End Road, London E1, 4NS;
and Dipartimento di Fisica - Università di Milano
Sezione di Astrofisica,
Via Celoria, 16, 20133 Milano, Italy.

Abstract:

Many experimental evidences indicate the presence of a ionizing background radiation flux at large redshifts, whose nature is doubtful. A lot of informations about the characteristics of such a background can be obtained both from the study of the Gunn Peterson effect and from the so-called “proximity effect”.

In some previous works I suggested the possibility that this ionizing flux comes from the quantum evaporation of primordial black holes (PBHs): here, I discuss the constraints that the experimental measurements put upon the free parameters of this reionization model and I try to verify its reliability. In particular, the radiation intensity of the background at the hydrogen Lyman edge, as inferred from the proximity effect, enables me to determine an upper limit to the PBHs average relics mass; due to our poor knowledge of the ultimate fate of the evaporating black holes, this limit represents an important theoretical information. In the second part of this paper I study the absorption of the ionizing background due to \( \text{Ly}_\alpha \) clouds: in particular, I discuss this phenomenon in presence of different absorption levels and I calculate the HI Gunn Peterson optical depth \( \tau_{\text{GP}}(z) \); from a comparison with the experimental data of Giallongo et al. \( \tau_{\text{GP}, \text{HI}} < 0.02 \pm 0.03 \) I obtain a constraint on the intergalactic medium density parameter, namely \( \Omega_{\text{IGM}} < 0.020 \).

A study of the characteristics of the absorbers is also performed: I determine the hydrogen gas density \( n_{H,e} \) and the column density \( N_{\text{HI}} \) for \( \text{Ly}_\alpha \) clouds; a satisfactory agreement with the available experimental data is obtained in the case of expanding, adiabatically cooled clouds. Finally, the same kind of analysis is performed for He II: in this case, the theoretical optical depth I obtain is smaller than the preliminary experimental lower limit of Jakobsen et al. \( \tau_{\text{GP}} > 1.7 \).
1. INTRODUCTION

The Gunn Peterson effect [1] is known since 1965 as a test of our knowledge of the ionization history of the Universe: the presence of an uniform distribution of neutral hydrogen at a redshift \( z \) should indeed be proved by the presence of an absorption trough in the blueside of the \( \lambda = 1216 \) resonance absorption line of the observed quasars; in fact, the \( \lambda = 1216 \) resonance absorption line, is shifted by a factor \( 1 + z \) into the visible, blue part of the spectrum. However, no trough is experimentally observed, in spite of the presumable sharpness of this effect; this fact probably reveals our unsatisfactory knowledge of the status of the Universe during the post-recombination era.

On the basis of the pioneering Schmidt’s observations [2] of the quasar 3C9 at \( z = 2.016 \), Gunn and Peterson [1] obtained an integrated \( \lambda = 1216 \) optical depth \( \tau_{GP} \leq 0.5 \), corresponding to an intergalactic neutral hydrogen number density

\[
n_{HI}(z) = \tau_{GP}(z) (1 + z) (1 + 2q_0 z)^{1/2} \left( \frac{m_p \nu_a H_0}{\pi e^2 f} \right) = \frac{\tau_{GP}(z) (1 + z) (1 + 2q_0 z)^{1/2}}{4.14 \times 10^{10} h^{-1}}, \tag{1.1}
\]

In eq. (1.1), \( H_0 \) is the present value of the Hubble constant, equal to 100 \( h \) \( km \ s^{-1} \) \( Mpc^{-1} \), \( 0.4 \leq h \leq 1 \), \( q_0 = 1/2 \) is the deceleration parameter, \( \nu_a = 2.46 \times 10^{15} \text{sec}^{-1} \) is the HI \( \lambda = 1216 \) frequency, \( m_p \) the proton mass and \( f = 0.416 \) is the oscillator strength corresponding to the \( \lambda = 1216 \) transition [1]. From eq. (1.1), \( n_{HI} \) at \( z = 2 \) is equal to \( 6 \times 10^{-11} h \) \( cm^{-3} \); subsequent works [3] still lowered this limit, that now approximately is 15 times smaller than the Gunn Peterson original estimate; for instance, Steidel and Sargent found [4] \( n_{HI}(z = 0.0) < 9.0 \times 10^{-14} h \) \( cm^{-3} \).

These values are smaller that the cosmic abundances: the inobservance of the \( \lambda = 1216 \) absorption trough is probably attributable to a high ionization level of the Universe rather than to an effective paucity of neutral hydrogen.

Many causes of such a ionization have been postulated: for instance, shock heating phenomena [5], [6], [7], high mass stars in primordial galaxies and unseen quasars, hidden due to an effect of dust obscuration by intervening galaxies [8].

All these hypotheses are indeed constrained by the recent observations of high redshift quasars \( z > 4 \); in fact, the effectiveness of the Gunn Peterson test increases with the redshift, the optical depth \( \tau_{GP} \) being proportional to \( (1 + z)^4.5 \).

Apparently, galaxies and unseen quasars cannot be the only photoionizing sources: in fact, the average ionizing intensity per unit of frequency and steradian they produce is smaller that the value suggested by the so-called proximity effect; this effect is seen [9], [10] as a decrement in the counted number of absorbing \( \lambda = 1216 \) clouds in the immediate proximity of the known quasars: the \( \lambda = 1216 \) forest pattern is reduced as a consequence of the higher average ionization characterizing these clouds.

The lower limit for the average intensity \( J_{-21} \) [9], [10] at the hydrogen Lyman limit (\( \lambda = 912 \) \( \text{Å} \)) is \( J_{-21} \sim 1 \) and it is independent on the redshift in the range \( 1.8 < z < 3.5 \); however, this lower limit for \( J_{-21} \) is yet higher than the maximum flux coming from the known, counted quasars [10].

Thus, probably we need some additional radiation sources: in the following, I would like to discuss a model in which the evaporation of primordial black holes produces the high energy photons flux which ionizes the IGM at small redshifts.

This model has been investigated in other papers: in particular, I discussed the effect of an exponential, late and fast reionization of the Universe on the polarization of the Cosmic Microwave Background in [11] while I discussed the details of the reionization processes induced by quantum evaporation of PBHs in [12], [13]. Here I will test the reliability of this model in the light of the Gunn Peterson effect, performing a comparison with the available experimental data.

This paper is structured as follows: in Sect. 2 I discuss the main characteristics of the evaporating primordial black holes, particularly their mass evolution in presence of quarks and gluons jets emission; I
also recall the main equations giving the photon emission spectrum: more details about the time evolution of the ionization degree \( x \) and of the plasma temperature \( T_e \) can be found in [13].

In Sect. 3, I discuss the average ionization intensity \( J_{-21} \) coming from the photons emitted by PBHs during their evaporation: the lower limit \( J_{-21} \sim 1 \) suggested by the proximity effect is employed to estimate the average PBH relics mass, \( \overline{M}_{rel} \), left after the evaporation of these objects. The agreement between theory and experiment is only possible for a PBH average relics mass \( \overline{M}_{rel} \sim 10^{-18} \text{ g} \); that means we need a complete evaporation of these primordial objects, the most effective reionization being produced at the end of the PBHs life. A deeper investigation about this point is under study: in fact, the quantum gravity effects might change the BH evolution when its mass lowers under the Planck mass.

In Sect. 3 I also present the main results of this work: I calculate the HI Gunn Peterson optical depth \( \tau_{GP} \), both in the case of a homogeneous IGM and in the more realistic case of an IGM presenting some inhomogeneities; as in ref. [8], a clumping factor \( f \) takes into account the presence of moderate overdensities in the IGM. The cases of low (LA), medium (MA) and high (HA) photoelectric absorption by \( Ly_\alpha \) clouds are all examined for different values of the density parameter \( \Omega_{IGM} \), namely \( \Omega_{IGM} = 0.010, 0.015, 0.020 \); larger values produce a too high \( \tau_{GP} \) that disagrees with the experimental data concerning high redshift \((z \sim 4)\) quasars.

In Sect. 4, I discuss in some detail the clumped structures present in the IGM in the form of \( Ly_\alpha \) clouds, I determine the efficiency of ionization \( G_H \) and I present my results for the hydrogen number density \( n_{H,c} \), the related column density \( N_{H1} \) and the density parameter \( \Omega_{La} \).

Two important quantities are involved in this calculation: the Doppler shift parameter \( b \) and the cloud temperature \( T_c \); some evidence of a positive correlation \( b - T_c \) and \( b - N_{H1} \) was suggested in ref. [14] and it has been related to the presence of expanding, adiabatically cooled clouds in [15]: my analysis seems to confirm such a possibility.

In Sect. 5, I perform the same calculation in the case of ionized helium and I discuss the relevance of the Gunn-Peterson effect for \( He \, II \); the intergalactic helium should be present in the form of singly ionized \( He \, II \) rather than in the neutral form, proving again the high ionization status of the IGM at high redshift. Jakobsen et al. [16] recently claimed the observation of the \( He \, II \) 304 \AA Gunn-Peterson absorption from diffuse IGM at \( z \sim 3.2 \), with a total optical depth \( \tau_{GP}(He\, II) > 1.7 \); the theoretical value I calculated is smaller than this preliminary limit, namely equal to 0.76.

Finally, in Sect. 6 I summarize the results obtained by using this model of PBHs-induced photoionization of the IGM and I present my conclusions.

2. THE QUANTUM EVAPORATION OF PRIMORDIAL BLACK HOLES.

The primordial black holes are very interesting structures, from a theoretical point of view; however, the processes that caused their formation and, in general, their overall evolution are poorly known; that despite of the relevant effects their quantum evaporation may have on the present status of the Universe.

Following a widely accepted idea, the black holes formation should characterize the early instants of the Universe after the Big Bang; however, the nature of the phenomena acting to create such structures is not clear and many theories have been advanced [17].

The creation of a black hole is induced by the contraction of a mass to a size less than its gravitational radius; large mass stars at the end of their evolution may typically represent some good candidates to form BHs. Indeed, a gravitational contraction of such a size is quite unlikely for small stars and therefore the formation of black holes having a small mass \((M \leq 10^{17} \text{ g})\) is only possible in presence of a huge compression, i.e. at the early stages of the cosmological expansion.

These small mass, primordial black holes present an intense Hawking’s quantum evaporation: in fact, the blackbody temperature of the emission is inversely proportional to their mass [18]:

\[
kT = \frac{\hbar c^3}{8\pi GM} \sim 1.06 \left[ \frac{M}{10^{17} \text{ g}} \right]^{-1} \text{ GeV}.
\] \hspace{1cm} (2.1)

The nature of the emitted particles clearly depends on the blackbody temperature: the mass loss makes the BH hotter and hotter, thus enabling the emission of more and more massive particles. BHs having a mass
larger than $10^{17}$ g can only emit massless particles [19], [20], [21]; however, when the BH mass falls below $10^{14}$ g, hadrons emission becomes possible: for temperatures above the QCD confinement scale $\Lambda_{QCD}$, a relevant emission of quarks and gluons jets does start; thus, the resulting spectrum is no more a blackbody one [20], [21], [22].

As I discussed in [12], [13], the photons emitted by PBHs during their quantum evaporation may be one of the causes of the reionization of the Universe: indeed, they seem very efficient to produce a late and sudden (nearly exponential) rise of the ionization degree for a reionization redshift $z_R \leq 30$, while for $30 < z_R \leq 60$ they are still able to cause a partial reionization.

Note that the plasma heating due to the photons / electrons interactions is not strong enough to induce a relevant distortion of the CBR spectrum: that is in agreement with the recent FIRAS upper limit on the reionization time which corresponds to the evaporation time for an object having a mass near the critical one ($M_c \sim 4.4 \times 10^{14} h^{-0.3} g$ is the mass of a PBH that survives till the present epoch).

In the following, I will briefly recall the fundamental equations describing the quantum evaporation of a BH: more details can be found in [12], [13] and references therein.

The initial mass $M_i$ of a PBH is connected to its lifetime by the following general formula [22]:

$$t_{evap} \sim 1.19 \times 10^3 \frac{G^2 M_i^3}{\hbar c^3} f(M_i) \sim 6.24 \times 10^{-27} f(M_i)^{-1} M_i^3 \text{ sec};$$

(2.2)

the function $f(M)$ in eq. (2.2) contains the contributions of the different species of particles, it is normalized to the unit for very massive ($M \geq 10^{17}$ g) BHs and reads as follows:

$$f(M) = 1.569 + 0.569 \left[ \exp \left( \frac{-M}{4.53 \times 10^{14}} \right)_{\mu} + 6 \exp \left( \frac{-M}{1.60 \times 10^{14}} \right)_{\nu,d} + 
+ 3 \exp \left( \frac{-M}{9.60 \times 10^{13}} \right)_{s} + 3 \exp \left( \frac{-M}{2.56 \times 10^{13}} \right)_{e} + 6 \exp \left( \frac{-M}{2.68 \times 10^{13}} \right)_{\tau} + 
+ 3 \exp \left( \frac{-M}{9.07 \times 10^{12}} \right)_{b} + 3 \exp \left( \frac{-M}{0.48 \times 10^{12}} \right)_{\ell} + 
+ 0.963 \exp \left( \frac{-M}{1.10 \times 10^{14}} \right)_{\text{gluons}} \right].$$

(2.3)

In eq. (2.3) the first addendum in the right-hand side expresses the contribution of electrons, positrons, photons and neutrinos; heavier particles are considered in the remaining terms, following their relative importance and with a factor 3 taking into account the color charge for the quarks; the denominators in the exponential terms are defined as the product $\beta_{s_j} M_j$, where $M_j$ is the mass of a black hole whose temperature is equal to the rest mass $\mu_j$ of the $j$ specie and $\beta_{s_j}$ is a spin-dependent factor defined [22] in such a way the energy of a BH having $M = \beta_{s_j} M_j$ has a peak at $\mu_j$.

When the BH mass falls below $10^{14}$ g and the temperature $T$ becomes larger than the confinement scale $\Lambda_{QCD}$, it is no longer possible to neglect the quarks and gluons emission in the calculation of the BH mass evolution: following [20], [21], [22], one should write this evolution as

$$\frac{dM}{dt} = -\sum_j \frac{1}{2\pi\hbar} \int \Gamma_j \left[ \exp \left( \frac{8\pi G Q M}{\hbar c^3} \right) - (-1)^{2s_j} \right]^{-1} \times \frac{Q dQ}{c^2};$$

(2.4)

in eq. (2.4), $\Gamma_j$ is the absorption probability for the $j$ particle having a spin $s_j$ [24] and one sums on all the emitted species [22]; this equation means the emission of a parent particle $j$ with total energy $Q$ decreases the BH mass by $Q/c^2$. After the integration over the energy $Q$, eq. (2.4) can be rewritten as [22]:

$$\frac{dM}{dt} = -5.34 \times 10^{25} f(M) M^{-2} \text{ g sec}^{-1}.$$ 

(2.5)
Now, the Hawking emission rate of particles having an energy in the range \((E, E + dE)\) from a black hole having an angular velocity \(\omega\), an electric potential \(\phi\) and a surface gravity \(\kappa\) is [18]:

\[
\frac{dN}{dt} = \frac{\Gamma dE}{2\pi \hbar} \left( e^{\exp \left( \frac{E - n\hbar \omega - e\phi}{\hbar \kappa / 2\pi c} \right)} \pm 1 \right)^{-1},
\]

(2.6)

where the signs ± respectively refer to fermions and bosons and \(\Gamma\) is the absorption probability of the emitted species. In the case of photon emission, \(\Gamma\) reads [24]:

\[
\Gamma_{s=1} = \frac{4A}{9\pi} \left( \frac{M}{M_{PL}} \right)^2 \left( \frac{\omega}{\omega_{PL}} \right)^4;
\]

(2.7)

in eq. (2.7) \(A\) is the surface area of the BH and the Planck mass and energy assure we are working with dimensionless quantities, as in [24].

Here, I neglect the charge and the angular momentum of PBHs, a quite reasonable and simplifying assumption due to the fact their loss via the quantum evaporation happens on a time scale shorter than the one characterizing the mass evaporation [25].

As pointed out in [26] in the emission spectrum we need a fragmentation function in order to take into account the production of quarks and gluons jets:

\[
\frac{dN_x}{dt dE} = \sum_j \int_0^{+\infty} \frac{\Gamma_j(Q, T)}{2\pi \hbar} \left( \exp \frac{Q}{T} \pm 1 \right)^{-1} \frac{dg_{jx}(Q, E)}{dE} \ dQ;
\]

(2.8)

here \(x\) and \(j\) respectively label the final and the directly emitted particles while the last factor, containing the fragmentation function \(g_{jx}\), expresses the number of particles with energy in the range \((E, E + dE)\) coming from a jet having an energy equal to \(Q\) [26]:

\[
\frac{dg_{jx}(Q, E)}{dE} = \frac{1}{E} \left( 1 - \frac{E}{Q} \right)^{2m-1} \theta(E - km_h c^2);
\]

(2.9)

in eq. (2.9) \(m_h\) is the hadron mass, \(k\) is a constant \(\sim O(1)\) and \(m\) is an index equal to 1 for mesons and 2 for baryons.

After selecting the dominant contribution to the integral over \(Q\) and summing over the final states, eq. (2.8) becomes [26]:

\[
\frac{dN}{dt dE} \sim E^2 \exp \left( \frac{-E}{T} \right) \quad \text{for} \ E >> T \quad Q \sim E, \quad (2.10a)
\]

\[
\frac{dN}{dt dE} \sim E^{-1} \quad \text{for} \ T >> E >> m_h \quad Q \sim T, \quad (2.10b)
\]

\[
\frac{dN}{dt dE} \sim \frac{dg}{dE} \quad \text{for} \ E \sim m_h << T \quad Q \sim m_h; \quad (2.10c)
\]

eqs. (2.10a), (2.10b) and (2.10c) respectively hold for the dominant value of \(Q\) written on the right.

### 3 THE IONIZING PHOTON FLUX

The possible origin of the ionizing flux has been studied in many papers [8], [27], [28], [29], [30], [31]. Generally, quasars are considered the best candidate sources for the photoionization of the Universe; however, a background generated by quasars only is not consistent with the predictions of the proximity effect and with the data concerning the evolution of \(Ly_\alpha\) clouds. For instance, in ref. [31] the Authors claimed that these quasars should ionize the IGM too late and produce a too large Gunn Peterson optical depth (note however that their conclusions have often been questioned).
In the following, I will discuss the mean specific intensity of the radiation field coming from the evaporation of PBHs as a function of the final mass of their relics; then, I will calculate the Gunn Peterson optical depth associated with the resonant HI $L_{\alpha}$ absorption.

### 3.1 The Radiation Intensity

**At the Hydrogen Lyman Edge**

At the hydrogen Lyman edge, $\nu_L = c/912 \AA = 3.29 \times 10^{15} \text{ sec}^{-1}$, and for an observer at a redshift $z_{\text{obs}}$, the mean intensity of the photons flux in erg $\text{cm}^{-3} \text{ sec}^{-1} \text{ Hz}^{-1} \text{ sr}^{-1}$ is [32]:

$$J_{912}(z_{\text{obs}}) = \frac{c}{4\pi H_0} \int_{z_{\text{obs}}}^{z_{\text{max}}} \frac{(1+z_{\text{obs}})^3}{(1+z)^3} \frac{\epsilon_T(\nu, z) \exp\left[-\tau_{\text{eff}}(912, z_{\text{obs}}, z)\right]}{(1+z)^2 (1+2q_0z)^{0.5}} \, dz; \quad (3.1.1)$$

in eq. (3.1.1) $\epsilon_T(\nu, z)$ is the total emissivity of a primordial population of BHs, calculated at a frequency $\nu = \nu_L (1+z) (1+z_{\text{obs}})^{-1}$ and at a redshift $z$, expressed in erg $\text{cm}^{-3} \text{ sec}^{-1} \text{ Hz}^{-1}$; $\tau_{\text{eff}}(912, z_{\text{obs}}, z)$ is an effective optical depth that, as I will discuss below, takes into account the probability that these photons are absorbed.

The emissivity $\epsilon$ can be obtained by calculating the total photon number density from eq. (2.10c). In our case, the condition $E \ll T$ holds: we are effectively interested in the last stages of the evaporation, when the BHs temperature is very high; moreover, the processes that are relevant for this study (the ionization and the recombination) dominate [20], [21] for an energy $E \leq 14 \text{ KeV}$.

For a jet fragmentation function given by eq. (2.9), the photon density in the unit time and energy can be written as follows:

$$\frac{\partial n_{\gamma}}{\partial \omega \partial t}|_{\text{Tot}} = \frac{\partial n_{\gamma}}{\partial \omega \partial t}|_{\text{Mes}} + \frac{\partial n_{\gamma}}{\partial \omega \partial t}|_{\text{Bar}}, \quad (3.1.2)$$

where the mesons and baryons contributions have been singled out.

For $Q = m_{\text{hadr}} \sim 300 \text{ MeV}$, eq. (3.1.2) reads

$$\frac{\partial n_{\gamma}}{\partial \omega \partial t}|_{\text{Tot}} = \frac{1}{\omega} \left(1 - \frac{\omega}{Q}\right) + \frac{1}{\omega} \left(1 - \frac{\omega}{Q}\right)^3, \quad (3.1.3)$$

the dominating contribution being the mesonic one; after reducing to the proper dimensions, I obtain the volume emissivity of a single evaporating PBH:

$$\frac{\partial n_{\gamma}}{\partial \omega \partial t} = \epsilon(\omega) = \frac{3.20 \times 10^{-15}}{\omega (\text{GeV})} \text{ erg cm}^{-3} \text{ sec}^{-1} \text{ Hz}^{-1}. \quad (3.1.4)$$

After considering the frequency shift $\nu = \nu_L (1+z) (1+z_{\text{obs}})^{-1}$, eq. (3.1.4) becomes:

$$\epsilon(\nu, z) = \frac{3.36 \times 10^{-40}}{\nu_L} \frac{(1+z_{\text{obs}})}{(1+z)} \text{ erg cm}^{-3} \text{ sec}^{-1} \text{ Hz}^{-1}. \quad (3.1.5)$$

Finally, the total emissivity is:

$$\epsilon_T(\nu, z) = N_{\text{PBH}}(z) \epsilon(\nu, z), \quad (3.1.6)$$

where the parameter $N_{\text{PBH}}(z)$ represents the population of PBHs at a redshift $z$.

Its value can roughly be estimated as follows: I write the PBHs density as

$$\rho(z) \sim \frac{M_{\text{rel}}(z) N_{\text{PBH}}(z)}{R^3(z)}, \quad (3.1.7)$$

where $M_{\text{rel}}(z)$ is the average mass of the PBHs relics at a redshift $z$. Then, the scale factor evolves as in the radiation dominated epoch:

$$R(t) \sim R_0 (t/t_0)^{1/2}; \quad (3.1.8)$$
\( R_0 = 1.25 \times 10^{28} \text{ cm} = 1.4 \times 10^{10} \text{ lyr} \) in a standard cosmological model, see ref. [33].

Now, I assume the PBHs density evolution is approximately described by a power law with index 2/3; the formation time coincides with the Big Bang and the initial density is \( \rho_i \sim 4.28 \times 10^{24} \text{ g cm}^{-3} \), [13] as one infers on the basis of the present experimental limits on \( \Omega_{PBH} \) \( (\Omega_{PBH} < (7.6 \pm 2.6) \times 10^{-9} \ h^{-1.95 \pm 0.15}) \) [20], [21]. Then, one finally gets [13]:

\[
N_{PBH}(z) = R_0 \rho_i \left( \frac{t_i}{t_0} \right)^{2/3} (1 + z)^{1/4} \frac{1}{M_{rel}(z)},
\]

i.e.:

\[
N_{PBH}(z) = 9.67 \times 10^{-10} \frac{(1 + z)^{1/4}}{M_{rel}(z)} \text{ cm}^{-2}.
\]  

The most difficult parameter to estimate in eq. (3.1.10) is the average relics mass \( M_{rel} \) at the reionization epoch \((z \sim 20 \div 30)\);

This parameter can be evaluated by searching a basic agreement with the data coming from the proximity effect and concerning the average intensity of the ionizing background; an agreement is only possible for \( M_{rel} \sim 10^{-18} \text{ g} \), i.e. for black holes totally evaporated or may be for relics well lighter than the Planck mass. However, for these objects the quantum gravity effects might be very relevant, thus a deeper theoretical analysis about this point is under study.

Turning back to the effective optical depth, it can be written as [34]:

\[
\tau_{eff}^A(912, z_{obs}, z) = \int_{z_{obs}}^{z} \int_{0}^{\infty} \frac{\partial^2 N}{\partial N_{HI} \partial z'} \times [1 - \exp(-N_{HI} \sigma'_{z})] \ d N_{HI} \ dz';
\]

in eq. (3.1.11) \( \sigma'_{z} \) is the hydrogen photoionization cross section, \( N_{HI} \) is the hydrogen column density of the absorber and the derivative \( \frac{d N}{dz} \) represents the column density distribution, depending on the assumed model of attenuation [32]; in fact, we can have various level of absorption and the approximate integration of eq. (3.1.11), respectively in the cases of low, medium and high absorption, gives [32]:

\[
\tau_{eff}^L(912, z_{obs}, z) \simeq \left[ 0.0118 x_{obs}^3 (x^{0.4} - x_{obs}^{0.4}) + 2.35 x_{obs}^{1.5} \ln \left( \frac{x}{x_{obs}} \right) - 0.78 x_{obs}^{3} (x_{obs}^{-1.5} - x^{-1.5}) - 0.003 (x^{1.5} - x_{obs}^{1.5}) \right],
\]

\[
\tau_{eff}^M(912, z_{obs}, z) \simeq \left[ 0.244 x_{obs}^3 (x^{0.4} - x_{obs}^{0.4}) + 2.35 x_{obs}^{1.5} \ln \left( \frac{x}{x_{obs}} \right) - 0.78 x_{obs}^{3} (x_{obs}^{-1.5} - x^{-1.5}) - 0.003 (x^{1.5} - x_{obs}^{1.5}) \right],
\]

\[
\tau_{eff}^H(912, z_{obs}, z) \simeq \left[ 0.097 x_{obs}^{1.56} (x^{1.84} - x_{obs}^{1.84}) - 0.0068 x_{obs}^{3} (x^{0.4} - x_{obs}^{0.4}) - 8.06 \times 10^{-5} (x^{3.4} - x_{obs}^{3.4}) \right],
\]

where \( x_{obs} = 1 + z_{obs} \) and \( x = 1 + z \).

The effective nature of the absorbers is still unclear: probably, they are large clouds (up to 1 Mpc) containing a significant baryonic fraction [35]; many recent analyses identify such absorbers with evolved densities fluctuations in the intergalactic medium, produced by large scale flows and inhomogeneities of the dark matter component of the Universe [36]; anyway, these absorbers both severely attenuate the flux coming from the ionization sources and generally delay the growth of HIII regions [32].

By using eqs. (3.1.5), (3.1.6) and (3.1.10) in eq. (3.1.1), I obtain:

\[
J(z_{obs}) = \frac{c}{4 \pi H_0} \int_{z_{obs}}^{z_{max}} \frac{(1 + z_{obs})^4}{(1 + z)^{25/4}} \exp \left[ -\tau_{eff}(912, z_{obs}, z) \right] dz;
\]

\[
\int_{z_{obs}}^{z_{max}} \frac{(1 + z_{obs})^4}{(1 + z)^{25/4}} \exp \left[ -\tau_{eff}(912, z_{obs}, z) \right] dz;
\]
\[ \frac{9.67 \times 10^{-10}}{M_{rel}} \frac{3.36 \times 10^{-40}}{\nu_L} dz. \quad (3.1.15) \]

After recalling the definition of the reduced intensity
\[ J_{-21}(z_{\text{obs}}) = J_{912}(z_{\text{obs}})/10^{-21}, \quad (3.1.16) \]
I plot in fig. 1 the behaviour of \( J_{-21}(z_{\text{obs}}) \) vs \( z_{\text{obs}} \) obtained from eq. (3.1.15), in the cases of low (LA), medium (MA) and high (HA) absorption; I take \( M_{rel} = M_{\text{rel}}(z_{\text{reion}}) \sim 10^{-18} \) g.

The Gunn-Peterson optical depth associated with the resonant \( Ly_\alpha \) absorption can be written in the following form [1]:
\[ \tau_{GP}(z_{\text{obs}}) = \left( \frac{\pi e^2 f_\alpha}{m_e c \nu_\alpha H_0} \right) \frac{n_{HI,d}(z)}{(1 + z)(1 + 2q_0 z)^{1/2}}, \quad (3.1.17) \]
where \( n_{HI,d} \) is the HI density of the IGM diffuse component and \( f_\alpha \) is the oscillation strength of the \( Ly_\alpha \) transition.

Rewriting eq. (3.1.17) as a function of the intensity \( J_{-21}(z_{\text{obs}}) \) one obtains [32]:
\[ \tau_{GP}(z_{\text{obs}}) = 3 T_4^{-0.75} (\Omega_{IGM} h^2)^2 (3 + \alpha) (1 + z)^{4.5} f(z) (J_{-21}(z_{\text{obs}}))^{-1}; \quad (3.1.18) \]
eq \[ \text{eq. (3.1.18) is evaluated by inserting the temperature } T_4 = T/10^4 \text{ K}^\circ \text{ of the ionized intercloud medium, as it results from the PBHs induced photoionization processes [12], [13]. A plot of } T_4 \text{ (in K}^\circ \text{) as a function of the redshift } z \text{ is shown in fig. 2; in eq. (3.1.18) } \alpha \text{ is the power spectrum index of the metagalactic flux at high } z (J \sim \nu^{-\alpha}); \text{ in our case} \]
\[ J_{-21}(z_{\text{obs}}, \nu) = \left( \frac{\nu}{\nu_\alpha} \right) J_{-21}(z_{\text{obs}}), \quad (3.1.19) \]
thus \( \alpha = 1. \)

If compared to the original equation of Gunn and Peterson, Eq. (3.1.18) contains an additional factor, namely the clumping factor \( f(z) \): the presence of inhomogeneities in the IGM may influence in a relevant way the processes of absorption [5], [6], [7], [8], [37] and, indeed, the observation of \( Ly_\alpha \) clouds is a clear proof of the existence of remarkable overdensities.

The function \( f \) describes moderate overdensities \( (1 < \rho/\bar{\rho} \leq 10) \) and it is defined as follows [8]:
\[ \frac{< \rho^2 >}{< \rho >^2} = f, \quad (3.1.20) \]
while larger inhomogeneities are directly identified with \( Ly_\alpha \) clouds.

The values of \( f \) at some different redshifts are listed in tab. 1; these values have been obtained by Carlberg and Couchman [38] through a numerical simulation.

Another important parameter entering in eq. (3.1.18) is the density parameter for the intergalactic medium, \( \Omega_{IGM} \): here I tested 3 values, namely \( \Omega_{IGM} = 0.010, 0.015, 0.020 \), for all the cases of low, medium and high absorption and both for an idealized, perfectly homogeneous IGM \( (f(z) = 1) \) and in presence of moderate inhomogeneities \( (f(z) \text{ as in tab. 1}) \). Figs. 3a, 3b, 4a, 4b, 5a, 5b show the behaviour of \( \tau_{GP} \) vs \( z_{\text{obs}} \) for all these cases; in tab. 2a, 2b, 3a, 3b, 4a, 4b I listed the values of \( \tau_{GP} \) calculated for two reference redshifts, \( z_{\text{obs}} = 3, z_{\text{obs}} = 4.3 \), for which some experimental data are known; tab. 5 finally gives a picture of the present experimental \( \tau_{GP} \) measurements, that also includes the recent data of Galli et al. [39].

Finally, in figs. 6a, 6b and 6c I resume the behaviour of \( \tau_{GP}(z) \) for the different values of the density parameter \( \Omega_{IGM} \) and for various levels of absorption.

### 3.2 Discussion of the Results

As I told, the average photon intensity \( J_{-21} \) determined from the proximity effect is employed to constrain the final average mass of the PBHs relics, that should be nearly totally evaporated at the time of the reionization.
The calculation of the Gunn Peterson optical depth is strongly model-dependent: in fact, eq. (3.1.18) contains both the temperature of the ionized IGM and the power spectrum index $\alpha$ of the metagalactic flux at high $z$, i.e. two peculiar predictions of the model.

Looking firstly at fig. 1, one can remark that the radiation intensity evolution depends on the behaviour of the effective optical depth, eq. (3.1.11); the same behaviour is obtained for $J_{-21}$ in the cases of low and medium absorption, only rescaled by a factor weighting the first cubic term in eqs. (3.1.12) and (3.1.13).

On the contrary, in the case of a high absorption level, eq. (3.1.14) for $\tau_{eff}$ produces a steeper curve.

The behaviour of $J_{-21}$ vs. $z$ differs in a sensitive way from the one predicted, for instance, in [8], where the sources of the ionization one assumes are quasars and high-mass stars in primordial galaxies. At early epochs, the intergalactic gas is neutral and the Universe is opaque to the radiation, while the volume emissivity of photons, $\varepsilon(\nu, z)$, is not so high to produce a relevant ionization; thus, the intensity $J_{-21}$ in fig. 2 goes to zero at high redshifts.

In ref. [8] the rise of the intensity $J_{-21}$ begins at $z \sim 5$ and stops at $z \sim 2.5$, due to fact that, at this redshift, the quasars start to decline; on the contrary, the behaviour of the intensity shown in fig. 1 is nearly exponential, due to the peculiar emission process I considered.

Looking at tabs. 2a, 2b, 3a, 3b, 4a and 4b, together with the experimental available data for the Gunn Peterson optical depth shown in tab. 5, we can observe that:

a) the experimental data generally refer to different values of the redshift $z$ and therefore, in order to allow a comparison, I considered two reference redshifts $z = 3$ and $z = 4.3$ for which many measurements have been performed.

b) The clumping factor $f$ enters in a linear way in eq. (3.1.18); thus, the presence of some inhomogeneities ($f > 1$) in the intergalactic medium increases the GP optical depth; a clumped configuration of the IGM makes it more opaque to the radiation.

c) I tested three values of the IGM density parameter, i.e. $\Omega_{IGM} = 0.010, 0.015, 0.020$; values larger than 0.020 produce a too high optical depth $\tau_{GP}$, disagreeing with the results of Giallongo et al. [39] (that however are affected by significant uncertainties). The disagreement is particularly serious in the inhomogeneous case ($f > 1$) with a medium / high absorption.

I can obtain a satisfactory agreement with the data of Giallongo et al. [39] for $\Omega_{IGM} = 0.010$ and a low absorption level; smaller values of $\Omega_{IGM}$ are in principle acceptable but, as I will show, they do not allow to obtain a consistent value of the diffuse medium density parameter, $\Omega_D = \Omega_{IGM} - \Omega_{Ly\alpha}$.

4 CHARACTERISTICS OF THE Ly$\alpha$ CLOUDS

4.1 THE THEORY

Here I want to discuss the characteristics of the IGM regions presenting very large overdensities, $\rho/\overline{\rho} \geq 10$, i.e. the configurations known as Ly$\alpha$ clouds; many informations can be obtained by studying the Ly$\alpha$ absorption phenomena within these regions. A very interesting idea is the possibility that these clouds are expanding and adiabatically cooled: the observations of Pettini et al. [14] suggested that the Ly$\alpha$ forest lines with a low column density $N_{H1}$ ($N_{H1} \leq 10^{14} \text{ cm}^{-2}$) have also very small velocity widths $b$. They found a positive correlation between $b$ and $N_{H1}$; typical values for these parameters are [14], [15]:

$$< b > \sim 11 \pm 3 \text{ km sec}^{-1} \quad \text{for} \quad \log N_{H1} = 13;$$

$N_{H1}$ is expressed in $\text{cm}^{-2}$ and $b$ (also called "Doppler parameter") is expressed as

$$b^2 = V_{\text{bulk}}^2 + \frac{2kT}{m},$$

i.e. it is connected to the cloud characteristics, as the temperature and the bulk velocity.

Generally, these clouds are considered as small, dense and very filamentous structures, rather than large and highly ionized objects; however, a different interpretation has been proposed in [15], [40] where
one assumes that these low density clouds are indeed expanding structures, which progressively cool in an adiabatic way; the expansion cooling may bring highly ionized clouds out of the thermal ionization equilibrium, thus enabling lower values of temperature \( T << 10^4 \text{ K} \) and of the velocity widths but not necessarily a low ionization level.

In the following, I will investigate the characteristics of \( \text{Ly}\alpha \) clouds by studying their absorption of the PBH photons; in particular, I will calculate the ionization efficiency, the HI column density and the baryonic content of the \( \text{Ly}\alpha \) clouds, expressed by the density parameter \( \Omega_{\text{Ly}\alpha} \). We will see that, in such a model of PBH-induced reionization, senseful results for \( \Omega_{\text{Ly}\alpha} \) can be obtained only in presence of expanding and adiabatically cooled clouds; thus, my results seem to support the hypotheses of ref. [15].

4.2 IONIZATION EFFICIENCY, HI DENSITY AND DENSITY PARAMETER \( \Omega_{\text{Ly}\alpha} \)

Here I will recall the main equations that are useful to study the ionization problem. In the case of photoionization equilibrium between the cloud gas and the ionizing radiation and for a ionizing flux given by eq. (3.1.15), the ionization efficiency is [32]:

\[
G_H = \int_{\nu_L}^{\infty} \frac{4\pi J_\nu \sigma_\nu}{h_p} \frac{d\nu}{\nu} \frac{1}{J_{912}}, \tag{4.2.1}
\]

where \( h_p \) is the Planck constant, \( J_\nu/J_{912} \) is given by eq. (3.1.19) and the hydrogen photoionization cross section is [41]

\[
\sigma_\nu = A_0 \left( \frac{\nu_L}{\nu} \right)^4 \exp \left[ 4 - \left( 4 \arctan \varepsilon / \varepsilon \right) \right] \frac{1}{1 - \exp \left( -2\pi / \varepsilon \right)}, \tag{4.2.2}
\]

holding for \( \nu = \nu_L \) and with

\[
A_0 = \frac{2^8 \pi}{3 e^4} \left( \frac{1}{137} \right) \pi r_0^2 = 6.30 \times 10^{-18} \text{ cm}^2, \tag{4.2.3}
\]

\[
\varepsilon = \sqrt{\frac{\nu}{\nu_L} - 1}. \tag{4.2.4}
\]

Eq. (4.2.1) can be recast in the simpler form

\[
G_H \sim 2.11 \times 10^6 \nu_L \int_{\nu_L}^{\nu_{\text{max}}} \frac{\sigma_\nu}{\nu^2} d\nu, \tag{4.2.5}
\]

where a suitable upper cut in the integration (\( \nu_{\text{max}} = 1. \times 10^{17} \text{ sec}^{-1} \)) is inserted in order to numerically handle the integration, after testing that this choice does not seriously influence the final result. I obtain:

\[
G_H = 3.52 \times 10^{-12} \text{ erg}^{-1} \text{ sec}. \tag{4.2.6}
\]

The total hydrogen gas density \( n_{H,c} \) of a spherical cloud of average column density \( N_{HI} = 10^{14} \text{ cm}^{-2} \) is related to its diameter \( D \) in the following way [32]:

\[
n_{H,c}(z) = \left[ \frac{3 G_H J_{912}(z)(10^{14} \text{ cm}^{-2})}{2 (1 + 2\chi) \alpha_A(T_c) D} \right]^{1/2}, \tag{4.2.7}
\]

where \( \alpha_A = 4.2 \times 10^{-13} T_{4,c}^{-0.75} \text{ cm}^3 \text{ sec}^{-1} \) is the coefficient of recombination to all the levels of hydrogen [41], \( T_{4,c} \) is the cloud temperature (K) divided by 10^4 and \( \chi \) is the ratio He/H, equal to 1/12. The column density \( N_{HI} \), averaged over all the lines of sight, is connected to the diameter \( D \) of the cloud through the following formula [32]:

\[
<N_{HI}> = \frac{2}{3} n_{H,c} D, \tag{4.2.8}
\]
and the Lyα clouds density parameter, Ω_{Lyα} is

\[ \Omega_{Lyα} = f_c \frac{n_{H,c}}{n_{H, crit}}; \]  \tag{4.2.9}

here the closure hydrogen density is

\[ n_{H, crit}(0) = \frac{3H_0^2}{8\pi Gm_H(1 + 4\chi)} = 2.11 \times 10^{-6} h^2 \text{ cm}^{-3}, \]  \tag{4.2.10}

and the volume filling factor \( f_c \) of the Lyα forest is [32]:

\[ f_c(z) \sim \frac{2H_0}{c} D(z) (1 + z)^{4.4} (1 + 2q_0z)^{0.5}. \]  \tag{4.2.11}

Finally, the size of the Lyα clouds is connected to the Doppler parameter \( b \) by the approximate formula [32]:

\[ D(z) \sim < b > [H_0(1 + z) (1 + 2q_0z)^{0.5}]^{-1}. \]  \tag{4.2.12}

Choosing the standard values \( T_c = 2 \times 10^4 \text{ K} \) and \( < b > = 35 \text{ km sec}^{-1} \) [15], I obtained for the cloud hydrogen gas density \( n_{H,c} \) and for the column density \( N_{HI} \) the values listed in tab. 6a for two reference redshifts, \( z = 2.5 \) and \( z = 0.0 \).

The baryonic density parameter at the present time for the Lyα clouds I calculated from eq. (4.2.9) is

\[ \Omega_{Lyα} = 0.016; \]  \tag{4.2.13}

This high value means that a large fraction of baryons resides in the Lyα clouds, while the diffuse component of the IGM (\( \Omega_D = \Omega_{IGM} - \Omega_{Lyα} \)) should have a density parameter totally negligible. A more reliable result can be obtained if we consider the clouds as dynamically expanding structures, a possibility that also could explain the (probably) observed correlation between the Doppler parameter \( b \) and the column density \( N_{HI} \).

In this case, the temperature \( T_c \) lowers to 200 K° and the size grows up to 1 Mpc; the results I obtain are listed in tab. 6b. For this choice of \( T_c \) and \( D \), the present density parameter is

\[ \Omega_{Lyα} = 0.010, \]  \tag{4.2.14}

in all the cases of low, medium and high absorption (see tab. 7). Consequently, the diffuse medium should have a present density parameter

\[ 0 \leq \Omega_D \leq 0.010, \]  \tag{4.2.15}

depending on the effective value chosen for \( \Omega_{IGM} \).

Above, I calculated the hydrogen density for the clumped component of the IGM; now, the Gunn Peterson optical depth \( \tau_{GP}(z) \) theoretically obtained in the previous section, can be used in eq. (3.1.17) in order to predict the density \( n_{HI, d} \) of its diffuse component.

Eq. (3.1.17) can be rewritten as follows:

\[ \tau_{GP} = \frac{8.28 \times 10^{10} n_{HI, d}(z)}{(1 + z)^{3/2}}, \]  \tag{4.2.16}

and therefore:

\[ n_{HI, d}(z) = 1.21 \times 10^{-11} \tau_{GP}(z) (1 + z)^{3/2}. \]  \tag{4.2.17}

The density \( n_{HI, d} \) calculated for three reference redshifts, \( z = 0 \), \( z = 2 \) and \( z = 2.64 \) are listed in tabs. 8a, 8b and 8c for \( \Omega_{IGM} = 0.010, 0.015, 0.020 \) and for the cases of low, medium and high absorption.
5. THE HE II GUNN PETERSON TEST

An interesting extension of the Gunn Peterson test is the study of the HeII 304 Å resonance line of the singly ionized helium; both the theoretical and the experimental efforts are converging on this problem since the appearance of the results of Jakobsen et al. [16], concerning the probable detection of the HeII 304 Å Gunn Peterson absorption from the IGM at \( z \sim 3.2 \).

The primordial helium in the intergalactic medium should be strongly ionized at high redshifts and it should be mainly seen in the form of singly ionized HeII, rather than neutral HeI [16]; this fact has been confirmed by the failure in detecting the Gunn Peterson absorption in the HeI 584 Å line of the neutral helium [42].

The HeII Ly\( \alpha \) resonance line is redshifted at \( \lambda_{\text{obs}} \sim 304 (1 + z) \) Å and therefore it is observable in the extreme UV; thus, the He II Gunn Peterson test could potentially give a lot of information on the spectrum of the UV background at very short wavelengths. Distant quasars at redshifts \( z > 3 \) are the most useful objects to study, in order to detect this effect in an experimentally suitable range of wavelengths, \( \lambda \geq 1200 \) Å, where the absorption by hydrogen in our galaxy does not produce noise.

From a theoretical point of view, the same kind of analysis used for the hydrogen can be performed for HeII, with only few differences; firstly, the HeII ionization edge is located at \( \lambda = 228 \) Å, corresponding to a frequency \( \nu_{L, \text{He}} = c/228 \) Å; from eqs. (3.1.17) and (3.1.18) it is possible to estimate [32] the ratios of the intensities of the diffuse radiation field at the hydrogen Lyman edge (\( \nu_{L, H} = c/912 \) Å) and at the helium edge (\( \nu_{L, \text{He}} = c/228 \) Å); for a low attenuation model, Meiksin and Madau found [32]:

\[
J_{912} / J_{228} \sim 8, \quad (5.1)
\]

while in a medium attenuation model [28] one approximately has

\[
J_{912} / J_{228} \sim 25. \quad (5.2)
\]

From eqs. (5.1) and (5.2) I simply obtained the intensity \( J_{228} \), that I plotted in fig. 7.

Using these intensities in eq. (3.1.18) I obtain the results shown in tabs. 9a, 9b, 10a, 10b and 11a, 11b for the Gunn Peterson optical depth at two reference redshifts, \( z = 3 \) and \( z = 4.3 \) and for \( \Omega_{\text{IGM}} = 0.010, 0.015, 0.020 \). A plot of \( \tau_{\text{GP}}(z) \) for the values of \( \Omega_{\text{IGM}} \) and both in the homogeneous and inhomogeneous cases is shown in figs. 8a, 8b, 9a, 9b, 10a and 10b; finally, in figs. 11a and 11b I resumed the behaviour of \( \tau_{\text{GP}}(z) \) for all the values of \( \Omega_{\text{IGM}} \) and for a fixed attenuation level.

The analysis of §4.2 can be repeated, simply observing that the Ly\( \alpha \) clouds absorption process is just modified as follows: the He absorption cross section is connected to the H cross section through the relation [8]:

\[
\sigma_{\text{HeII}}(\nu) = \frac{1}{4} \sigma_{\text{H}} \left( \frac{\nu}{4} \right); \quad (5.3)
\]

The ionization efficiency \( G_{\text{He}} \) can be simply calculated by using eq. (5.3) in eq. (4.2.1) and changing the intensities; thus, I calculate

\[
G_{\text{He}} = 2.25 \times 10^{-10} \text{ erg}^{-1} \text{ sec}, \quad (5.4)
\]

Finally, the helium density \( n_{\text{He}, c} \) and the column density \( N_{\text{HeII}} \) are given by some equations analogous to eqs. (4.2.7), (4.2.8), with a ionization efficiency given by eq. (5.4), an intensity \( J_{228} \) coming from eqs. (5.1) (LA case) and (5.2) (MA case) and a coefficient of recombination for He given by [41]:

\[
\alpha_{\text{A}}(T) = 2.607 \times 10^{-13} T_{4}^{-0.75}. \quad (5.5)
\]

The results are listed in tab. 12a for \( T_{c} = 2 \times 10^{4} \) K° and in tab. 12b for adiabatically expanding, cooled clouds at \( T_{c} = 200 \) K°.

A comparison of the Gunn Peterson optical depth here obtained with the experimental value of Jakobsen et al. [16] (\( \tau_{\text{HeII}} > 1.7 \) at \( z \sim 3 \)) is also possible but this lower limit should be considered with some prudence.
indeed, it is very conservative and probably affected by some uncertainties, due to the fact that the Authors
cannot distinguish between HeII absorption coming from the line blanketing in the discrete lines of the Ly$_\alpha$
forest and the real Gunn Peterson trough.

Looking at tabs. 9a, 9b, 10a, 10b and 11a, 11b we can remark that in such a PBH-induced model of
reionization, the Gunn Peterson optical depths obtained for HeII are quite similar to the ones for HI: in the
case of inhomogeneous clouds with a medium absorption level and for $\Omega_{IGM} = 0.020$, the maximum value
for $\tau_{GP \ HeII}$ at $z = 3$ is 0.76. The disagreement with the lower limit of ref. [16] should be tested in the light
of future, more precise measurements.

6. CONCLUSIONS

In this paper I assumed that the quantum evaporation of Primordial Black Holes having a mass $M \sim
10^{14}$ g provides the radiation that photoionizes the intergalactic medium at some early epochs in the past.

This particular model has been tested in the light of the Gunn Peterson test for both HI and HeII: I
calculated the optical depths associated with the resonant Ly$_\alpha$ absorption by clouds and I compared my
theoretical results with the last experimental data. The agreement is satisfactory, particularly if we assume
an inhomogeneous IGM with a density parameter $\Omega = 0.010$ and a low attenuation by the clouds. In any
case, a constraint on $\Omega_{IGM}$ can be inferred from this analysis: the agreement theory / experiment is possible
only if $\Omega_{IGM} < 0.020$.

I also studied the characteristics of the Ly$_\alpha$ clouds and I found that my model confirms the possibility
they are expanding and adiabatically cooled.

A comparison theory / experiment for the HeII Gunn Peterson effect is certainly premature, due to the
paucity of the experimental results regarding HeII: the HeII optical depth I calculated is smaller than the
lower limit claimed by Jakobsen et al. [16] ($\tau_{GP} > 1.7$) and near to the values I found for HI. Finally, I can
conclude that the reionization model here proposed seems to work quite well; anyway, a better knowledge of
the ultimate fate of the PBHs relics, particularly of their final masses, surely represents a definitive test for
it.

ACKNOWLEDGEMENTS

This work has been financially supported by the University of Milano that I want to thank; in particular,
a sincere and affectionate thank goes to all the people in the Astrophysics Section of the University of Milano
for their friendly support and sympathy. I thank also the Queen Mary and Westfield College for its hospitality.

REFERENCES

[1] Gunn, J.E., Peterson, B.A., Astroph. J., 1965, 142, 1633.
[2] Schmidt, M., Astroph. Journ., 1965, 141, 1295.
[3] Davidson, A., Hartig, G.F., Fastie, W.G., Nature, 1977, 269, 203.
[4] Steidel, C.C., Sargent, W.L.W., Astroph. Journ., 1987, 318, L11.
[5] Ostriker, J.P., Ikeuchi, S., Astroph. Journ., 1983, 268, L63.
[6] Ikeuchi, S., Ostriker, J.P., Astroph. Journ., 1986, 301, 522.
[7] Chiang, W., Ryu, D., Vishniac, E.T., Astroph. Journ., 1989, 339, 603.
[8] Miralda-Escudé, J., Ostriker, J.P., Astroph. Journ., 1990, 350, 1.
[9] Carswell, R.F., Morton, D.C., Smith, M.G., Stockton, A. N., Turnshek, D.A., Weymann, R.J., Astroph.
Journ., 1984, 278, 486;
Tytler, D., Astroph. Journ., 1987, 321, 69;
Murdoch, H.S., Hunstead, R.W., Pettini, M., Blades, J.C., Astroph. Journ., 1986, 309, 19.
[10] Bajtlik, S., Duncan, R.C., Ostriker, J.P., Astroph. Journ., 1988, 327, 570.
[11] Gibilisco, M., Intern. Journ. Of Mod. Phys. 1995, 10A, 3605.
[12] Gibilisco, M., "Reionization of the Universe induced by Primordial Black Holes ", Accept. for Pub. in Intern. Journ. of Mod. Phys A, May 1996.
[13] Gibilisco, M., "The influence of quarks and gluons jets coming from Primordial Black Holes on the Reionization of the Universe", submitted to Annals of Physics, 1996b.
[14] Pettini, M., Hunstead, R. W., Smith L.J., Mar, D.P., MNRAS, 1990, 246, 545.
[15] Duncan, R.C., Vishniac, E.T., Ostriker, J.P., Astroph. Journ., 1991, 368, L1.
[16] Jakobsen, P., Boksenberg, A., Deharveng, J.M., Greenfield, P., Jedrzejewski, R., Paresce, F., Nature, 1994, 370, 35.
[17] Novikov, I., "Black Holes and the Universe", 1990, Cambridge Univ. Press.
[18] Hawking, S.W., Commun. Math. Phys., 1975, 43, 199.
[19] Carr, B.J., Astroph. J., 1976, 206, 8.
[20] Mac Gibbon, J.H., Carr, B.J., Astroph. J., 1991, 371, 447.
[21] Mac Gibbon, J.H., Webber, B.R., Phys. Rev., 1990, D41, 3052.
[22] Mac Gibbon, J. H., Phys. Rev., 1991, D44, 376.
[23] Mather, J.C. et al., Astroph. J., 1994, 420, 439.
[24] Page, D. N., Phys. Rev., 1976, D13, 198.
[25] Page, D. N., Phys. Rev., 1977, D16, 2402.
[26] Carr, B. J., Astronomical and Astroph. Transactions, 1994, Vol. 5, 43.
[27] Bechtold, J., Weymann R. J., Liu, Z., Malkan, M., Astroph. J., 1987, 315, 180.
[28] Madau, P., Astroph. J., 1992, 389, L1.
[29] Songaila, A., Cowie, L. L., Lilly, S. J., Astroph. J., 1990, 348, 371.
[30] Miralda-Escudé, J., Ostriker, J. P., Astroph. J., 1992, 392, 15.
[31] Shapiro, P.R., Giroux, M.L., Astroph. J., 1987, 321, L107.
[32] Meiksin, A., Madau, P., Astroph. J., 1993, 412, 34.
[33] Misner, C. W., Thorne, K.S., Wheeler, J. A.: "Gravitation", 1973, p. 738, W. H. Freeman and Co., San Francisco.
[34] Paresce, F., McKee, C., Bowyers, S., Astroph. J., 1980, 240, 387.
[35] Haebl, M.G., Preprint Babbage [astro-ph/9512024], December 1995.
[36] Cen, R., Miralda-Escudé, J., Ostriker, J. P., Rauch, M., Astroph. J., 1994, 437, L9.
[37] Foltz, C.B., Weymann, R.J., Röser, H.J., Chaffee, F.H., Astroph. J., 1984, 281, L1.
[38] Carlberg, R.G., Couchman, H.M.P., Astroph. J., 1989, 340, 47.
[39] Giallongo, E. et al., Astroph. J., 1994, 425, L1.
[40] Bond, J.R., Szalay, A.S., Silk, J., Astroph. J., 1988, 324, 627.
[41] Osterbrock, D.E., "Astrophysics of Gaseous Nebulae and AGN",1989, Oxford Univ. Press.
[42] Beaver, E.A., Astroph. J., 1991, 337, L1; Tripp, T.M., Green, R.F., Bechtold, J., Astroph. J., 1990, 364, L29; Reimers, D. et al., Nature, 1992, 360, 561.
Tab. 1: Clumping function $f$ coming from a numerical simulation of Carlberg and Couchman (Carlberg & Couchman 1989)

| $f$  | $z$  |
|------|------|
| 7.42 | 0.00 |
| 7.98 | 0.81 |
| 6.93 | 1.36 |
| 2.48 | 2.80 |
| 1.74 | 4.63 |
Tab. 2a: Gunn Peterson optical depth: homogeneous case \((f = 1)\) with a density parameter \(\Omega_{IGM} = 0.010\)

| ABSORPTION MODEL | \(z = 3\) | \(z = 4.3\) |
|-------------------|------------|-------------|
| Low Abs.          | 0.002      | 0.014       |
| Medium Abs.       | 0.003      | 0.030       |
| High Abs.         | 0.003      | 0.036       |

Tab. 2b: Gunn Peterson optical depth: inhomogeneous case \((f > 1)\) with a density parameter \(\Omega_{IGM} = 0.010\)

| ABSORPTION MODEL | \(z = 3\) | \(z = 4.3\) |
|-------------------|------------|-------------|
| Low Abs.          | 0.005      | 0.026       |
| Medium Abs.       | 0.008      | 0.057       |
| High Abs.         | 0.008      | 0.068       |
Tab. 3a: Gunn Peterson optical depth: homogeneous case $(f = 1)$ with a density parameter $\Omega_{IGM} = 0.015$

| ABSORPTION MODEL | $z = 3$ | $z = 4.3$ |
|------------------|---------|-----------|
| Low Abs.         | 0.004   | 0.031     |
| Medium Abs.      | 0.007   | 0.068     |
| High Abs.        | 0.007   | 0.081     |

Tab. 3b: Gunn Peterson optical depth: inhomogeneous case $(f > 1)$ with a density parameter $\Omega_{IGM} = 0.015$

| ABSORPTION MODEL | $z = 3$ | $z = 4.3$ |
|------------------|---------|-----------|
| Low Abs.         | 0.010   | 0.057     |
| Medium Abs.      | 0.017   | 0.128     |
| High Abs.        | 0.017   | 0.152     |
Tab. 4a: Gunn Peterson optical depth: homogeneous case 
\((f = 1)\) with a density parameter \(\Omega_{IGM} = 0.020\)

| ABSORPTION MODEL | \(z = 3\) | \(z = 4.3\) |
|------------------|-----------|-------------|
| Low Abs.         | 0.008     | 0.055       |
| Medium Abs.      | 0.012     | 0.121       |
| High Abs.        | 0.013     | 0.144       |

Tab. 4b: Gunn Peterson optical depth: inhomogeneous case \((f > 1)\) with a density parameter \(\Omega_{IGM} = 0.020\)

| ABSORPTION MODEL | \(z = 3\) | \(z = 4.3\) |
|------------------|-----------|-------------|
| Low Abs.         | 0.018     | 0.102       |
| Medium Abs.      | 0.030     | 0.230       |
| High Abs.        | 0.030     | 0.270       |
Tab. 5: Gunn Peterson optical depth: experimental data

| $\tau_{GP}$ | $z$   | year | Work                      |
|------------|------|------|---------------------------|
| 0.01       | 3.0  | 1992 | Giallongo et al. $^a$     |
| 0.01 ± 0.03| 3.0  | 1994 | Giallongo et al. $^b$     |
| 0.02 ± 0.03| 4.3  | 1994 | Giallongo et al. $^b$     |
| < 0.05     | 3.8  | 1992 | Webb et al. $^c$          |
| 0.04       | 4.0  | 1992 | Webb et al. $^c$          |
| 0.04 ± 0.01| 4.1  | 1992 | Webb et al. $^c$          |
| < 0.02 ± 0.03| 2.64| 1994 | Steidel and Sargent $^d$  |

a) Giallongo et al., APJ 398, L12, 1992.
b) Giallongo et al., APJ 425, L1, 1994.
c) Webb et al., MNRAS 255, 319, 1992.
d) Steidel and Sargent, APJ, 318, L11, 1987.
Tab. 6a: $L_{\text{Ly}\alpha}$ clouds characteristics: hydrogen density $n_{Hc}$ and column density $N_{HI}$ at a reference redshift $z = 2.5$ and for $T_C = 2 \times 10^4$ K°.

| ABS. MODEL  | $n_{Hc}$ cm$^{-3}$ | $N_{HI}$ cm$^{-2}$ |
|-------------|--------------------|--------------------|
| Low Abs.    | $1.04 \times 10^{-4}$ | $2.28 \times 10^{19}$ |
| Medium Abs. | $8.56 \times 10^{-5}$ | $1.88 \times 10^{19}$ |
| High Abs.   | $8.91 \times 10^{-5}$ | $1.96 \times 10^{19}$ |

Tab. 6b: $L_{\text{Ly}\alpha}$ clouds characteristics: hydrogen density $n_{Hc}$ and column density $N_{HI}$ at a reference redshift $z = 2.5$ and for $T_C = 200$ K°.

| ABS. MODEL  | $n_{Hc}$ cm$^{-3}$ | $N_{HI}$ cm$^{-2}$ |
|-------------|--------------------|--------------------|
| Low Abs.    | $1.84 \times 10^{-5}$ | $4.05 \times 10^{18}$ |
| Medium Abs. | $1.52 \times 10^{-5}$ | $3.35 \times 10^{18}$ |
| High Abs.   | $1.58 \times 10^{-5}$ | $3.48 \times 10^{18}$ |
Tab. 7: Density parameter $\Omega_{\text{Ly}\alpha}$ for $\text{Ly}\alpha$ clusters for a cluster temperature $T_c = 200\, \text{K}$

| ABS. MODEL   | $z = 2.5$ | $z = 0.0$ |
|--------------|-----------|-----------|
| Low Abs.     | 0.013     | 0.010     |
| Medium Abs.  | 0.011     | 0.010     |
| High Abs.    | 0.012     | 0.010     |
Tab. 8a: Hydrogen density $n_{H,d}$ at redshift $z = 0$, $z = 2.00$ and $z = 2.64$ characterizing the diffuse component of the IGM; $\Omega_{IGM} = 0.010$ and $f > 1$

| ABS. MODEL     | $n_{H,d}(z = 0) \text{ cm}^{-3}$ | $n_{H,d}(z = 2.00) \text{ cm}^{-3}$ | $n_{H,d}(z = 2.64) \text{ cm}^{-3}$ |
|----------------|-----------------------------------|-----------------------------------|-----------------------------------|
| Low Abs.       | $4.91 \times 10^{-17}$            | $8.99 \times 10^{-14}$            | $2.47 \times 10^{-13}$            |
| Medium Abs.    | $5.01 \times 10^{-17}$            | $1.19 \times 10^{-13}$            | $3.83 \times 10^{-13}$            |
| High Abs.      | $4.02 \times 10^{-17}$            | $1.01 \times 10^{-13}$            | $3.62 \times 10^{-13}$            |
Tab. 8b: Hydrogen density $n_{H,d}$ at redshift $z = 0$, $z = 2.00$ and $z = 2.64$ characterizing the diffuse component of the IGM; $\Omega_{IGM} = 0.015$ and $f > 1$

| ABS. MODEL  | $n_{H,d}(z = 0)$ cm$^{-3}$ | $n_{H,d}(z = 2.00)$ cm$^{-3}$ | $n_{H,d}(z = 2.64)$ cm$^{-3}$ |
|-------------|-----------------------------|-------------------------------|-------------------------------|
| Low Abs.    | $1.10 \times 10^{-16}$      | $2.02 \times 10^{-13}$       | $5.71 \times 10^{-13}$       |
| Medium Abs. | $1.12 \times 10^{-16}$      | $2.67 \times 10^{-13}$       | $8.62 \times 10^{-13}$       |
| High Abs.   | $9.04 \times 10^{-17}$      | $2.28 \times 10^{-13}$       | $8.15 \times 10^{-13}$       |
Tab. 8c: Hydrogen density $n_{H, d}$ at redshift $z = 0$, $z = 2.00$ and $z = 2.64$ characterizing the diffuse component of the IGM; $\Omega_{IGM} = 0.020$ and $f > 1$

| ABS. MODEL   | $n_{H,d}(z = 0)$ cm$^{-3}$ | $n_{H,d}(z = 2.00)$ cm$^{-3}$ | $n_{H,d}(z = 2.64)$ cm$^{-3}$ |
|--------------|-----------------------------|------------------------------|------------------------------|
| Low Abs.     | $1.96 \times 10^{-16}$      | $3.60 \times 10^{-13}$       | $1.01 \times 10^{-12}$       |
| Medium Abs.  | $1.61 \times 10^{-16}$      | $4.06 \times 10^{-13}$       | $1.45 \times 10^{-12}$       |
| High Abs.    | $2.03 \times 10^{-16}$      | $4.75 \times 10^{-13}$       | $1.53 \times 10^{-12}$       |
Tab. 9a: HeII Gunn Peterson optical depth: homogeneous case \((f = 1)\) with a density parameter \(\Omega_{IGM} = 0.010\)

| ABSORPTION MODEL | \(z = 3\) | \(z = 4.3\) |
|------------------|----------|----------|
| Low Abs.         | 0.037    | 0.204    |
| Medium Abs.      | 0.190    | 1.420    |

Tab. 9b: HeII Gunn Peterson optical depth: inhomogeneous case \((f > 1)\) with a density parameter \(\Omega_{IGM} = 0.010\)

| ABSORPTION MODEL | \(z = 3\) | \(z = 4.3\) |
|------------------|----------|----------|
| Low Abs.         | 0.015    | 0.109    |
| Medium Abs.      | 0.079    | 0.758    |
Tab. 10a: HeII Gunn Peterson optical depth: homogeneous case \((f = 1)\) with a density parameter \(\Omega_{IGM} = 0.015\)

| ABSORPTION MODEL | \(z = 3\) | \(z = 4.3\) |
|------------------|-----------|-------------|
| Low Abs.         | 0.083     | 0.460       |
| Medium Abs.      | 0.428     | 3.195       |

Tab. 10b: HeII Gunn Peterson optical depth: inhomogeneous case \((f > 1)\) with a density parameter \(\Omega_{IGM} = 0.015\)

| ABSORPTION MODEL | \(z = 3\) | \(z = 4.3\) |
|------------------|-----------|-------------|
| Low Abs.         | 0.035     | 0.246       |
| Medium Abs.      | 0.178     | 1.706       |
Tab. 11a: HeII Gunn Peterson optical depth: homogeneous case ($f = 1$) with a density parameter $\Omega_{IGM} = 0.020$

| ABSORPTION MODEL | $z = 3$ | $z = 4.3$ |
|------------------|---------|----------|
| Low Abs.         | 0.148   | 0.818    |
| Medium Abs.      | 0.760   | 5.679    |

Tab. 11b: HeII Gunn Peterson optical depth: inhomogeneous case ($f > 1$) with a density parameter $\Omega_{IGM} = 0.020$

| ABSORPTION MODEL | $z = 3$ | $z = 4.3$ |
|------------------|---------|----------|
| Low Abs.         | 0.062   | 0.437    |
| Medium Abs.      | 0.317   | 3.032    |
Tab. 12a: $Ly_\alpha$ clouds characteristics: helium density $n_{He}c$ and column density $N_{He II}$ at a reference redshift $z = 2.5$ and for $T_C = 2 \times 10^4$ K°

| ABSORPTION MODEL | $n_{He} cm^{-3}$ | $N_{HI} cm^{-2}$ |
|------------------|-----------------|-----------------|
| Low Abs.         | $4.11 \times 10^{-5}$ | $9.06 \times 10^{18}$ |
| Medium Abs.      | $1.92 \times 10^{-5}$ | $4.23 \times 10^{18}$ |

Tab. 12b: $Ly_\alpha$ clouds characteristics: helium density $n_{He}c$ and column density $N_{He II}$ at a reference redshift $z = 2.5$ and for $T_C = 200$ K°

| ABSORPTION MODEL | $n_{He} cm^{-3}$ | $N_{HI} cm^{-2}$ |
|------------------|-----------------|-----------------|
| Low Abs.         | $2.32 \times 10^{-4}$ | $5.09 \times 10^{19}$ |
| Medium Abs.      | $1.08 \times 10^{-4}$ | $2.38 \times 10^{19}$ |
FIGURE CAPTIONS:

Fig. 1: Average ionizing intensity per unit of frequency and steradian $J_{921}(z)$ at the hydrogen Lyman edge $\nu_L = c/912 \, \text{Å} \, (\text{erg cm}^{-3} \, \text{sec}^{-1} \, \text{Hz}^{-1} \, \text{sr}^{-1})$; full line: low attenuation model; dashed line: medium attenuation model; dotted line: high attenuation model.

Fig. 2: Evolution of the temperature of the intergalactic medium as a function of the redshift $z \, (\text{eV})$.

Fig. 3a: Gunn Peterson optical depth $\tau_{GP}(z)$ associated with the resonant HI $L_y$ absorption for an inhomogeneous IGM with $\Omega_{IGM} = 0.010$ and with a clumping factor $f < 1$; full line: low attenuation model; dashed line: medium attenuation model; dotted line: high attenuation model.

Fig. 3b: Gunn Peterson optical depth $\tau_{GP}(z)$ associated with the resonant HI $L_y$ absorption for a homogeneous IGM with $\Omega_{IGM} = 0.010$ and with a clumping factor $f = 1$.

Figs. 4a-4b: The same plots of figs. 3a and 3b but for $\Omega_{IGM} = 0.015$.

Figs. 5a-5b: The same plots of figs. 3a and 3b but for $\Omega_{IGM} = 0.020$.

Fig. 6a: Gunn Peterson optical depth $\tau_{GP}(z)$ associated with the resonant HI $L_y$ absorption for the three values of $\Omega_{IGM}$ in the case of an inhomogeneous IGM ($f > 1$) with low absorption; full line: $\Omega_{IGM} = 0.010$, dashed line: $\Omega_{IGM} = 0.015$, dotted line: $\Omega_{IGM} = 0.020$.

Fig. 6b: The same plot of fig. 6a but in the case of medium attenuation.

Fig. 6c: The same plot of fig. 6a but in the case of high attenuation.

Fig. 7: Average ionizing intensity per unit of frequency and steradian $J_{228}(z)$ at the Helium Lyman edge $\nu_L = c/228 \, \text{Å} \, (\text{erg cm}^{-3} \, \text{sec}^{-1} \, \text{Hz}^{-1} \, \text{sr}^{-1})$; full line: low attenuation model; dashed line: medium attenuation model; dotted line: high attenuation model.

Fig. 8a: Gunn Peterson optical depth $\tau_{GP}(z)$ associated with the resonant HeII $L_y$ absorption for an inhomogeneous IGM with $\Omega_{IGM} = 0.010$ and with a clumping factor $f < 1$; full line: low attenuation model; dashed line: medium attenuation model.

Fig. 8b: Gunn Peterson optical depth $\tau_{GP}(z)$ associated with the resonant HeII $L_y$ absorption for a homogeneous IGM with $\Omega_{IGM} = 0.010$ and with a clumping factor $f = 1$.

Figs. 9a-9b: The same plots of figs. 8a and 8b but for $\Omega_{IGM} = 0.015$.

Figs. 10a-10b: The same plots of figs. 8a and 8b but for $\Omega_{IGM} = 0.020$.

Fig. 11a: Gunn Peterson optical depth associated with the resonant HeII $L_y$ absorption for the three chosen values of $\Omega_{IGM}$ in the case of an inhomogeneous IGM ($f > 1$) with low absorption; full line: $\Omega_{IGM} = 0.010$, dashed line: $\Omega_{IGM} = 0.015$, dotted line: $\Omega_{IGM} = 0.020$.

Fig. 11b: The same plot of fig. 6a but in the case of medium attenuation.