Nonlinear analysis of constructions from different materials based on unified plastic constitutive relations

Mark Panasyuk, Aleksandr Petrakov and Natalia Petrakova

Donbas National Academy of Civil Engineering and Architecture, 2 Derzhavina st., Donetsk region, Makeevka 23, Ukraine, 286123

Abstract. In the paper the plastic constitutive relations based on the hypotheses of the plastic flow theory, unified for different materials, such as soil ground, concrete, metal etc. are proposed. These plastic constitutive relations are based on the hypotheses of the Mises-Schleicher-Botkin theory of failure. The strength characteristics of materials, determined by standard methods, are redefined for octahedral sites, they are invariants for. The problem is solved by the finite element method (FEM). The calculation model MFOE made up of singular finite elements (tetrahedral, triangular plates, rods). This fact increases the accuracy of the description of loading paths in finite elements and ensures a unique correspondence of the constitutive equations to elementary volumes of the construction. The use of singular finite elements is associated with a large amount of access memory for storing the stiffness matrix of the system. So, the range of practically solvable problems limits significantly. To eliminate this contradiction, the Newton's iteration method – SOR method is developed for equilibrium equations solving in the design of finite elements structures. An iterative algorithm is obtained. It does not require assembly of the system stiffness matrix for its implementation. The volume of operational information is proportional to the number of finite elements in the system. Due to the traditional approach, which requires the assembly of the system stiffness matrix, the amount of operational information is proportional to the square of the degree of kinematic indeterminacy of the system. While using the iterative algorithm, the size of the stiffness matrix and the time of solving are reduced. The results of a nonlinear analysis of soil massifs and concrete structures are represented.

1. Introduction
Methods for nonlinear analysis of structural units were formulated for specific materials. In such a case, plastic constitutive relations used for concrete, masonry, metal and foundation soil are substantially different. This way, for concrete, masonry and metal, constitutive equations are used. They are based on the hypotheses of the deformation plasticity theory [7,11,14,16]. While attractive this theory, significant difficulties in describing complex incl. variables loading paths arise [14].

A number of elaborations closely connected with usage of nonlinear elasticity is especially important. These are the so-called mixed issues of elasticity and plasticity theory [6,15]. The disadvantages of these equations include the inability to separate the fracture mode by shear and bulk strains. In this regard, the flow of material along shear strains is accompanied by the flow of material along bulk strains. This makes it impossible to analyze arbitrary loading paths.

In the methods of soil foundations calculating, constitutive equations, based on the hypotheses of the plastic flow theory are used [2,3,8,9,13]. These constitutive equations assume the separation of strains into shear and bulk, so make it possible to take into account arbitrary loading histories.

In this paper, the plastic constitutive relations unified for all types of materials are demonstrated. These constitutive equations are based on the hypotheses of the plastic flow theory and the octahedral
Mises-Schleicher-Botkin theory of failure. The signs for stresses and strains in mathematical manipulations are taken in accordance with the rules of structural theory.

2. Unified plastic constitutive relations

The initial tensors of stresses and strains can be represented by stress vectors $\mathbf{\sigma}$ and strain vectors $\mathbf{\epsilon}$ in six-dimensional Euclidean spaces. These spaces are transformed into six-dimensional vector spaces of shear stresses $\sigma_i$ and shear strains $\epsilon_i$, when the norm is the stress intensity $\sigma_i$ and the strain intensity $\epsilon_i$. Thus, the transformation formula is:

$$
\begin{align*}
\sigma_i &= \Sigma \sigma; \\
\epsilon_i &= \Sigma^T \sigma_i + \delta \cdot \sigma_0; \\
\epsilon_i &= E \epsilon; \\
\sigma_i &= E^T \epsilon_i + \delta \cdot \epsilon_0
\end{align*}
$$

(1)

$\sigma_0$ and $\epsilon_0$ are average stresses and strains.

The transformation operators in formulas (1) are defined by the expressions:

$$
\Sigma = \frac{1}{\sqrt{2}} \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}; \quad \Sigma^T = \frac{\sqrt{2}}{3} \begin{bmatrix} C & 0 \\ 0 & D \end{bmatrix}; \\
E = \frac{\sqrt{2}}{3} \begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix}; \quad E^T = \frac{1}{\sqrt{2}} \begin{bmatrix} C & 0 \\ 0 & B \end{bmatrix}; \\
\delta = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}; \\
A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}; \quad B = \sqrt{6} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \\
C = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}; \quad D = \frac{\sqrt{3}}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.
$$

(2)

To derive the plastic constitutive relation, we use the hypothesis of the plastic flow theory where the vector of total shear stress is collinear to the vector of plastic strain rate. Since a numerical algorithm is elaborated, the rates are replaced by increments (finite differences). The derivation is based on the procedure of shear strain separating into elastic (e) and plastic (p) components. The initial equations are:

$$
\epsilon_i = \epsilon_{i(0)} + d\epsilon_{ie} + d\epsilon_{ip}; \\
d\epsilon_{ie} = 1/3G \cdot d\sigma_i; \\
d\epsilon_{ip} = \lambda \cdot \sigma_i,
$$

(3)

$G$ is shear modulus.

The strength diagram (loading surface) represents the Mises-Schleicher-Botkin theory of failure equations. The parameters of these equations are calculated from standard test results. Coulomb–Mohr failure equation is traditionally used for derivation. The bilinear strength diagram (Fig.1,2) is assigned in this paper. The second leg of the diagram, as a rule, is used for rock and concrete materials. This diagram is transformed into the line graph parallel to the axis of average stresses for metallic materials.

**Figure 1.** The strength diagram in the compression-extension concrete test [5] for the determination of the strength parameters in the Coulomb–Mohr failure equation

**Figure 2.** The strength diagram of concrete for the determination of the strength parameters in the Mises-Schleicher-Botkin failure equation

The strength diagram (Fig.2) is represented by the following equations:
\[ \sigma_{i,\text{max}1} = \sigma_0 \cdot \tan(\varphi_{1,\text{oct}}) + C_{1,\text{oct}}; \quad \sigma_{i,\text{max}2} = \sigma_0 \cdot \tan(\varphi_{2,\text{oct}}) + C_{2,\text{oct}}; \]
\[ \sigma_{i,\text{max}} = \min(\sigma_{i,\text{max}1}, \sigma_{i,\text{max}2}). \]  

(4)

The concrete strength parameters Coulomb–Mohr failure equation are determined by the formulas from the Figure 1:

\[ \sin(\varphi_i) = \frac{f_{cd} + f_{ctd}}{f_{cd} - f_{ctd}}; \quad C_1 = \frac{f_{cd}}{2} \cdot \frac{1 + \sin(\varphi_i)}{\cos(\varphi_i)}; \quad \varphi_2 = 0; \quad C_2 = \frac{f_{cd}}{2}. \]  

(5)

The concrete strength parameters, which are referred to the octahedral plane, are calculated by the following formulas:

\[ c_{\text{oct}} = 2 \cdot c \cdot \cos(\varphi); \quad \varphi_{\text{oct}} = \arctan(3 \cdot \sin(\varphi)) \]  

(6)

The similar correspondences were obtained for soil testing. Direct shear apparatus:

\[ c_{\text{oct}} = \sqrt{3} \cdot \cos(\varphi) \cdot c; \quad \varphi_{\text{oct}} = \arctan(\sqrt{3} \cdot \sin(\varphi)) \]  

(7)

Stabilometr:

\[ c_{\text{oct}} = \frac{6 \cdot \cos(\varphi)}{3 - \sin(\varphi)} \cdot c; \quad \varphi_{\text{oct}} = \arctan\left(\frac{6 \cdot \cos(\varphi)}{3 - \sin(\varphi)} \cdot \tan(\varphi)\right) \]  

(8)

The compression diagram (Fig.3) is described by the piecewise equation:

\[ \text{if } \varepsilon_0 \leq \varepsilon_{0,\text{lim}} \quad \sigma_0 = 3K \cdot \varepsilon_0; \quad \text{if } \varepsilon_0 \geq 2\varepsilon_{0,\text{lim}} \quad \sigma_0 = 0; \]
\[ \quad \text{else } \sigma_0 = \sigma_{0,\text{lim}} \cdot \left(1 - \frac{\varepsilon_0 - \varepsilon_{0,\text{lim}}}{\varepsilon_{0,\text{lim}}}\right) \]  

(9)

\[ \sigma_{0,\text{lim}} = C_{1,\text{oct}} \cdot \tan(\varphi_{1,\text{oct}}) \cdot \varepsilon_{0,\text{lim}}; \quad \varepsilon_{0,\text{lim}} = \sigma_{0,\text{lim}} / (3K), \]

K is the modulus of the material bulk strain.

**Figure 3.** The diagram of concrete compression (volumetric deformation)

The compression diagram (9) is associated with the strength diagram (4). The shear diagram is described by the vector equation under condition of arbitrary loading:

\[ \sigma_i = \sigma_{i(0)} + 3 \cdot G \cdot d\varepsilon_i; \quad |\sigma_i| \leq \sigma_{i,\text{max}} \]  

(10)
The clay loam soil with following -
- is the initial vector of the shear stress; \( \mathbf{d} \) is the vector of shear strain increment; \( \sigma_{\text{max}} \) is the strength of the material according to the strength diagram (4).

The shear diagram (10) is associated with the strength diagram according to equations (4).

The problem solving. It is given: the stress and strain vectors \( \sigma_{(0)} \) and \( \varepsilon_{(0)} \), which characterize the reached level of the strain-stress state, the strain increment vector \( \mathbf{d} \). The stress vector \( \sigma \) is required to be determined.

The initial vectors are transformed into the vectors of shear stresses and strains \( \sigma_{(0)}, \varepsilon_{(0)} \) in accordance with the formulas (1). The initial equations (3), it is possible to form a set of equations in reference to the vectors of elastic and plastic strain:

\[
\frac{1}{\lambda} \cdot d \varepsilon_{ip} - 3G \cdot d \varepsilon_{ip} = \sigma_{i(0)};
\]
\[
d \varepsilon_{ip} + d \varepsilon_{ie} = d \varepsilon.
\]

Eliminating the vector of elastic strain increment \( \mathbf{d} \) from the set of equations, the equation is:

\[
d \varepsilon_{ip} = 3G \cdot \lambda / (1 + 3G \cdot \lambda) \cdot (1 / 3G \cdot \sigma_{i(0)} + d \varepsilon_{i}) = \lambda \cdot \sigma_{i}
\]

The coefficient \( \lambda \) of the similarity for the shear stress vectors \( \sigma_{i} \) and the plastic strain rate \( \mathbf{d} \) is determined by the solution of the vector equation (12):

\[
\lambda = \frac{\sqrt{a}}{\sigma_{i}} - \frac{1}{3G}; \quad a = \sum_{j=1}^{6} \left( \frac{1}{3G} \sigma_{i(0), j} + d \varepsilon_{i,j} \right)^{2}.
\]

\( \sigma_{i} \) is the intensity of the total shear stress vectors (modulus of the vector \( \sigma_{i} \)) in accordance to the shear diagram (10).

The sequential application of formulas (5) and (3) gives the opportunity to determine the components of the vectors \( \mathbf{d} \) and \( \sigma_{i} \). Ultimately, the components of the stress tensor are determined by the components of the shear stress deviator and average stress due to formulas (1). In this case, the correspondence between the average stresses and the average strains is taken from the compression diagram (9).

Numerical tests of soil in a triaxial compression machine. The clay loam soil with following strength and deformation characteristics, determined in a shear machine, odometer and calculated by the formulas (7) was chosen for observations:

\[
\varphi = 22^\circ; \varphi_{\text{oct}} = 33^\circ; \ c = 30kPa; \ \ c_{\text{oct}} = 48,2kPa; \ \ E = 10MPa; \ \ \nu = 0,35;
\]

\[
G = \frac{E}{2(1+\nu)} = 3,7MPa; \ \ K = \frac{E}{3(1-2\nu)} = 11,1MPa.
\]

The soil sample undergoes varying loading with regard to the average stress \( \sigma_{0} \) and strain intensity \( \varepsilon_{i} \). In the loading process, the ground flow is reached by shear stresses at various levels of average stress. When the shear strain is completely unloaded, the shear stresses decrease to a certain level and increase to a limit value corresponding to the ground flow. The diagram of soil shear in the numerical test (Fig.4) is algorithm-based (1-13). Table 1 represents the coordinates of the shear diagram flex points.

| \( \sigma_{0}, \text{kPa} \) | 300 | 300 | 300 | 150 | 150 | 150 | 150 | 150 |
|---------------------------|-----|-----|-----|-----|-----|-----|-----|-----|
| \( \varepsilon_{i}, 10^{-3} \) | 0   | 21,8| 28,9| 28,9| 46,2| 34,6| 17,3| 0   |
| \( \sigma_{i}, \text{kPa} \)  | 0   | 242,6| 242,6| 145,4| 145,4| 17,2| 145,4| 145,4|
The analysis of the shear diagram (Fig.4) indicates the approximation of plastic constitutive relations (1-13), incl. the description of the soil strain-stress state analysis under complex loading. Considering the fact that the shear stresses do not reach zero indicates at full unloading, so there is the misalignment of the total shear stresses vectors and strain vectors during the complex loading process.

3. Newton’s iteration method – SOR method for solving FOE problems

For solving the non-linear problems, the calculation model MFOE [19] made up of singular finite elements (tetrahedras, triangular plates, rods) are used in this paper. This method allows to set up the single-valued correspondence between the plastic constitutive relations and the elementary volumes of the structure. The resulting calculation problems associated with the economical usage of the random access memory are resolved by using the iteration methods for solving FOE problems.

Newton’s iteration method [12] in the mathematical sense consists of two procedures: the first is the linearization of composed function by Newton’s iteration method; the second is the solving of linear algebra system of equations by the SOR method (the successive over-relaxation method).

The iteration procedure of m-step Newton’s iteration–SOR method [12] is:

\[
\begin{align*}
\begin{bmatrix}
\frac{\partial f_i(z^k)}{\partial z_j} & \sum_{j=1}^{n} \frac{\partial f_j(z^k)}{\partial z_j} & (z^k_{j+1} - z^k_j) & \sum_{j=1}^{n} \frac{\partial f_j(z^k)}{\partial z_j} (z^k_{j+1} - z^k_j) + f_i(z^k)
\end{bmatrix} \cdot z_i^{k+1} &= z_i^{k+1} - \omega \cdot \left( \frac{\partial f_i(z^k)}{\partial z_i} \right)^{-1}.
\end{align*}
\]

(14)

\( z_i \) is the displacement of the \( i \)-th node of the calculation model; \( i = 1, \ldots, n \); \( n \) is the degree of kinematic uncertainty of the system; \( f_i(z) \) is a nonlinear trim equation; \( \frac{\partial f_i(z)}{\partial z_j} \) is the element of the system stiffness matrix; \( p \) is the number of the SOR iteration within \( k \)-th Newton's iteration; \( \omega \) is the relaxation coefficient.

This equation (14) does not have significant advantages since it requires the assembly of the system stiffness matrix for its implementation, even if in the form of a band matrix. The problem of further equations transformations (14) is in featuring of derivatives only from the displacement \( z_i \) calculated at a given step of iteration. In the latter case, to implement the iterative process, it is sufficient to have stiffness matrix for individual finite elements. In this case, the procedure for assembling the system’s stiffness matrix from the calculation process is eliminated.

After a number of mathematical transformations, using the ideas presented in the paper [17], a modified procedure of the Newton’s iteration–SOR method is obtained in the notation adopted in the FOE [19]:
\[ U_{i,q}^{k,p+1} = U_{i,q}^{k,p} - \omega \left( \sum_{s \in S_i} \frac{\partial r_{is,q}(U^k)}{\partial U_{i,q}} \right) \left( \sum_{j=1}^{n} \frac{\partial r_{js,i}(U^k)}{\partial U_{j,i}} + \frac{\partial r_{ms,j}(U^k)}{\partial U_{m,j}} - U_{m,j}^k \right) \]

\[ + \frac{\partial r_{ps,l}(U^k)}{\partial U_{l,i}} \left( U_{l,i}^{k,p+\delta} - U_{l,i}^k \right) + r_{l,i,q}(U^k) \]

\[ + P_{l,i,q} \]

where \( U_{i,q} \) is the displacement of the \( i \)-th node in the \( q \)-th direction; \( i = 1, \ldots, n \); \( n \) is the number of nodes in the system; direction \( q \), \( l = 1(X), 2(Y), 3(Z) \); \( S_i \) is the list of finite elements numbers, converging in \( i \)-th node; \( s \), \( j \), \( m \), \( p \) are the numbers of the standard (accepted in the FOE theory) \( i \), \( j \), \( m \), \( p \) are nodes of the \( s \)-th finite element from the list \( S_i \); \( r_{is,j} \), \( r_{js,i} \), \( r_{ms,j} \), \( r_{ps,l} \) are reactions in \( i \), \( j \), \( m \), \( p \) nodes of the \( s \)-th finite element from the list \( S_i \) in the direction \( l \); \( \partial r(U)/\partial U \) is the element of the stiffness matrix of the finite element; \( P_{l,i,q} \) is the load in the \( i \)-th node of the \( q \)-th direction; \( p \) is the number of the SOR iteration within \( k \)-th Newton's iteration; \( \omega \) is the relaxation coefficient taken in the interval \((0,1]\), according to the data of the paper [18], it allows to expand the local convergence set of iterative process; as a default it is assumed to be equal to unity.

The iteration procedure (15) is implemented by using the permutation indexes in the stiffness matrices of finite elements and in the matrices of system nodes displacement. These procedures are formal and do not cause particular difficulties, but provide a compact form of operational information storing.

The iterative method according to the formula (15) is tested with the help of the research software complex (RSC) Panama. RSC Panama is the author’s development in the Visual C ++. Comparative calculations of the FOE models using the Libra PC [18] are performed.

The results of comparative calculations are represented in the Table 2 and Fig. 5.

**Table 2.** The results of calculation processes studied in the Libra PC and RSC Panama

| Model number | Characteristics of models | PC Lira | RSC Panama |
|--------------|--------------------------|---------|------------|
|              | number of equations | number of nodes | number of finite elements | finite element type | stiffness matrix | Density stif. mat. | Time working min. | control displacement mm | stiffness matrix | Time working min. | control displacement mm |
| 1            | 6144 | 2304 | 9000 | T | 25.9 | 6 | 0.58 | 32.7 | 10.4 | 0.45 | 32.6 |
| 2            | 13230 | 4851 | 20000 | T | 113.0 | 6 | 2.28 | 25.0 | 23.0 | 1.0 | 24.9 |
| 3            | 46128 | 16337 | 72000 | T | 1696 | 7 | 85.6 | 36.3 | 82.9 | 8.0 | 35.9 |
| 4            | 100860 | 35301 | 160000 | T | 9043 | 8 | 1227.6 | NR | 184.3 | 15.0 | 27.0 |
| 5            | 533286 | 88882 | 131299 | P | 1988 | 1 | 283.0 | - | - | - | - |

Accepted designations: T – tetrahedron; P – plate; NR – the calculation is not realized due to unreal time of the account.
Figure 5. The diagram of the account time in the Libra PC and RSC Panama

It is learned that in the RSC Panama the size of the stiffness matrix linearly depends on the number of finite elements of the system. In the Libra PC the size of the stiffness matrix is proportional to the square of the kinematic uncertainty of the system and remains substantially larger than in the RSC Panama. Thus, with the number of equations of 50 000, the size of the stiffness matrix in the Libra PC is about 20 times larger than in the RSC Panama.

4. The results of concrete samples numerical studies
All numerical tests are performed with prisms 100x100x300 mm and cubes 100x100x100 mm. The destructive loads depending on the sample shape are analyzed. The bounding action of the prisms’ flat ends is analyzed. The material under analysis is concrete (strength class C20/25 (B25). The Poisson ratio is 0,25. For the short-term tests E = E_0, for the long-term tests E = 0,4 E_0, E_0 is the initial modulus of concrete deformation. The loads are carried out in the displacement of the sample flat ends.

The initial data for concrete (strength class C20/25 (B25) [1,5] and the derived characteristics (6) are given in the Table 3.

| Deformation | Strength characteristics (MPa, deg.) |
|-------------|--------------------------------------|
| E, GPa      | 23 (9,2)                             |
| f_{cd}      | 14,5                                 |
| \varphi_1   | 59,9                                 |
| c_1         | 1,99                                 |
| c_1,oct     | 1,996                                |
| G, GPa      | 9,2 (3,7)                            |
| \sigma_{0,\text{lim}} | 0,769                        |
| \varphi_2   | 0                                    |
| c_2         | 7,25                                 |
| c_2,oct     | 14,5                                 |

Note: The deformation characteristics of concrete under long-term loading are indicated in parentheses.

The results of numerical studies are represented by the graphs in Figures 6-9.

Figure 6. Comparative diagrams of a prism and cube deformation
Figure 7. Varying of the transverse strain ratio when loading concrete samples
When the prism is destroyed (Fig. 6) the average stress in the concrete is equal to the design resistance of the concrete to the compression (fcd) given in the initial data (Table 3.). Thus, the design model adequately describes the work of the concrete structure, the prism is. When the cube is destroyed (Fig. 6) the average stress in the concrete exceeds the calculated concrete resistance of compression by 17%. According to the reference [1], this excess should be 25 … 33.3%. The explanation is underestimated strength of concrete under one-axial tension (fctd), given in the initial data (Table 3.). The axial strain, corresponding to the maximum stresses in concrete, is on average 1.7‰. This is consistent with the data of the norms for the design of reinforced concrete structures [5]. Further loading of the samples is accompanied by a decrease in the compressive stress in the concrete. In the technical publications this phenomenon is called “the work of concrete on a descending branch”. When the limiting compressibility of concrete is reached, the axial strain corresponds to 3.44‰ [5], the reduction of compressive stresses is 8.2% for prisms and 12.7% for cubes. In reference documents this value is not standardized. According to the research works [10], the decrease in the design resistance to compression of concrete along the descending branch can vary from 7% to 47%.

**Figure 8.** Load trajectories of characteristic FE prisms

**Figure 9.** Load trajectories of characteristic FE cubes

The transverse strain ratio (Figure 7.) changes while loading. To a loading level of 0.7-0.8 it is equal to the Poisson’s ratio given in the initial data. When reaching the maximum stresses in concrete, it rises to 0.5, which corresponds to the plastic flow of the material. In the limiting state, this ratio reaches values of 0.6-0.64. It follows from the above analysis, that the concrete model allows one to take into account, in the framework of the theory of plastic flow, dilatancy phenomena associated with bulk expansion (decompaction) of the material.

Loading trajectories of characteristic finite elements located at the end and in the middle of concrete samples are studied. When plastic deformation occurs at the end of the prism (Figure 8.), unloading takes place, both in shear and bulk stresses. At the end of loading, a volumetric extension occurs in the FE. At the end of the cube, the initial plastic flow goes into unloading, firstly, only by volumetric stresses and then by shear stresses. Volume expansion in FE was not achieved.

The difference in the load-bearing (capacity) of the prism and the cube is explained by the fact that in the cube the ends are in the state of compression, while the ends of the prism are destroyed as a result of volume expansion.

**5. The soil massif limiting states research findings**

The dimension of the soil massif model (Table 2., model No.3) are 30x30x16 m. The boundary conditions are given in the form of fixed settlements under the base of the massif and horizontal fastenings along the normal to the lateral surfaces of the massif. The dimensions of the basement are 4x4 m. The depth of foundation laying from the surface of the massif is 1.0 m. The options for loading the soil massif through an absolutely flexible and absolutely rigid stamp are considered. The characteristics of the foundation soil are given in Section 1. For the indicated soil and foundation
parameters, the control parameters calculated according to the standards for the design of the bases [4] are: the calculated soil resistance \( R \) is 224.8 kPa; the ultimate pressure \( P_{\text{lim}} \) is 778.5 kPa.

The settlements of the foundations are represented in the form of graphs as a function of the average pressures under foundation base (Figures 10, 11).

![Figure 10](image1.png)  **Figure 10.** Ground settling with simple loading

![Figure 11](image2.png)  **Figure 11.** Ground settling at variable loading

A comparison is made of the settlements of the absolutely rigid (RF) and medium settlements of the absolutely flexible (FF) foundations. To test the results, the relevant plots of the elastic settlements obtained in the Libra PC are listed.

Up to pressures below the sole equal to the calculated resistance of the soil \( R \), the settlements of the flexible and rigid foundations, calculated by the elastic and elastic-plastic model coincide. The average settlements of the flexible foundations with settlements of the hard foundations are also coincide. At pressures below the foot of the foundation, exceeding \( R \), settlements of the flexible foundations develop in the elastic-plastic stage and significantly exceed the settlements of the hard foundations. When the \( P_{\text{lim}} \) reaches the base of the foundation, the flexible foundations lose its resistance. In this case, the settlements of the hard foundations continue to develop in the elastic-plastic stage. The rigid foundations lose its resistance at an average pressure of 1.5 \( P_{\text{lim}} \). Settlements of the flexible foundations during unloading and repeated loading (Figure 11) are developing according to the laws described in the technical publications.

As the results of calculations, the illustrations of the most characteristic stages of the plastic flow zones development in a soil massif with different types of foundations are given. The figures show a mosaic of loading levels in red intensities.

![Figure 12](image3.png)  **Figure 12.** Loading levels
Based on the results of the studies, the adequacy of the proposed calculation models for nonlinear analysis of soil massifs is noted.

6. Conclusion
The unified plastic constitutive relations of structures from different materials based on the hypotheses of the plastic flow theory are proposed. The adequacy of the proposed equations is confirmed by the results of a nonlinear analysis of concrete structures and soil massifs.

The calculation models for nonlinear analysis are recommended to be constructed on the basis of singular finite elements. In this case, the simple correspondence is set up between the constitutive equations and the elementary volumes of the constructions. The calculation difficulties arise in such a case are effectively eliminated by using iterative methods for solving FOE problems.

References
[1] Bondarenko V.M., Suvorkin D.G. Zhelezobetonnyie i kamennyie konstruktsii. – M.: Vyissh.shk., 1987. – 384 s.
[2] Boyko I. P., Saharov O. S., Saharov V. O. Vzaemodiya konstruktsiy bagatopoverhovih budivel z urahuvannyam y`yazkoplastichyi roboti gruntovogo masivu pri seysmichnih navantazhennyah / Svit geotehniki. – 2014. – № 1. – s. 17-21
[3] Brinkgreve R.B.J, Vermeer P.A. Plaxis. Finite Elements Code for Soil and Rock Analysis. Materials Models Manual. Part 3. – A.A. Balkema, Roterdam, Brookfield, 1998.
[4] DBN V.2.1-10-2009. Osnovu ta fundamentu sporud. Kiiv: Ministerstvo regionalnogo rozvitku ta budvinzvtnia Ukrainy. – 2009. – 102 s.
[5] DBN V.2.6-98:2009. Betonni ta zalizobetonni konstrukzii. – Kiiv: Ministerstvo regionalnogo rozvitku ta budvinzvtnia Ukrainy. – 2011. – 71 s.
[6] Fadeev A.B., Preger A.L. Reshenie smeshannoy oseissimntrichnoy zadachi teorii uprugosti i plastichnosti metodom konechnyh elementov // Osnovaniya, fundamentyi i mehanika gruntov. – 1984. – №4. – s. 25-27.
[7] Geniev G. A., Kisyuk V.N., Tyupin V.A., Teoriya plastichnosti betona i zhelezobetona. – M.: Stroytzdat, 1974. – 216 s.
[8] Klovanič S. F. Metod konechnyh elementov v neleneynih zadachah inzhenernoy mehaniki. – Zaporozhe: Izd. Zhurnala “Svit geotehniki”. 2009. – 400 s.
[9] Klovanič S. F. Model techeniya svyaznyih gruntov // Svit geotehniki. – 2012. – № 1. – s. 16-20
[10] Metodicheskie rekomendatsii po utochnennomu raschetu zhelezobetonnyih elementov s uchetom polnoy diagrammy szhatiya betona. – K.: NIISK, 1987. – 25s.
[11] Moskvitin V. V. Plastichnost pri peremennych nagruzheniyah. – M.: MGU, 1965. – 263 s.
[12] Ortega D. Zh., Reybold V. Iteratsionnyie metodi resheniya nelineynyih sistem uravneniy so mnogimi neizvestnymi. – M.: Mir, 1975. – 558 s.
[13] Pershina S. V., Slobodyanik A. V. PLAXIS – programmyiy kompleks dlya rascheta deformatsiy i ustoichivosti geotehnicheskikh sooruzheniy metodom konechnyh elementov // Zbirnik naukoviyh prats (galuzeeve mashinobuduvannya, budivnictstvo), vip. 12. – Półtava: PolNTY, 2003. – s. 158-163.
[14] Petakov A. A. Issledovanie nekotoryih gipotez slozhnogo nagruzheniya. – Stroitelnaya mehanika i raschet sooruzheniy, 1992, №3. – s. 30-36.
[15] Petakov A. A. Prakticheskie metody resheniya uprugoplasticheskih zadach pri slozhnom nagruzhenii betonnyih konstruktsiy // Sovremennyie problemy stroitelstva. – Donetskiy PromstroynIIproekt, OOO Lebed, 1999. – s. 65-69
[16] Petakov A. A. Primenenie deformatsionnyy teorii plastichnosti dlya analiza slozhnyih nagruzheniy. – Stroitelnaya mehanika i raschet sooruzheniy, 1984, № 2. – s. 13-18.
[17] Petakov A. A. Reshenie nelineynyih zadach metodom raspredeleniya usiliy i peremescheniy // Stroitelnaya mehanika i raschet sooruzheniy, 1982, №4. – s. 16-20.
[18] Программный комплекс для расчета и проектирования конструкций LIRA. Руководство пользователя. Книга 1. – Киев: NIIFSS, 2002. – 147 с.
[19] Zienkiewicz O. C. The Finite Element Method. Fifth Edition. V.2, Solid Mechanics. – Butterworth-Heinemann, 2000. – 459 p