The implementation of fuzzy trapezoidal critical path method in a housing project

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Abstract. Project scheduling is one of the most important elements in a project. Scheduling using deterministic methods often has constraints in estimating the completion time for a project. Using deterministic methods that there is only 1 duration to determine the completion of an activity where this will be risky if there is a delay, so in this study try to implement the scheduling method using a trapezoidal fuzzy critical path, where there are 4 durations is used to estimate the completion of a job. In this analysis, the project data was obtained from the VB housing project. Based on the available data, several research scenarios were obtained to find the effect of changes in the factors contained in the trapezoidal fuzzy method towards the changing of the final duration and critical path in the project. The results of this analysis were found that the changes in the critical path could be influenced by changes in alpha values and the duration of the activity range. The total project duration range is affected by the shape of the fuzzy diagram.

1. Introduction
Conventional scheduling methods that are deterministic such as precedence diagrams and arrow diagrams are commonly used techniques in planning the scheduling of a project [1], [2], [3]. In applying these two commonly used methods often have constraints in estimating the final duration, path and critical activities that occur in the project [4], [5]. To resolve this, alternative scheduling using fuzzy critical path method is used [6], [7], [8]. The fuzzy critical path is a method for identifying critical paths on a project network with activity duration using fuzzy numbers [9]. This method provides a way to find a true critical paths in a fuzzy project network. Therefore, in this study try to compare the use of fuzzy critical path method with the traditional critical path method for planning schedule and the duration of the construction project. In addition to the application of fuzzy critical path method in this study will find the effect of changes in factors contained in the fuzzy critical path method to the duration, path and critical activities.

2. Method
2.1. Data Acquisition
Scheduling data for this study was taken from the housing project in Bekasi, West Java. The project consists of one type of house with a land area of 66 m², a building area of 33 m² and only has one floor. Data collected in the form of activity list carried out for a house, the duration of each activity and the type of relationship between activities.

2.2. Trapezoidal Fuzzy Critical Path Method
The representation of trapezoidal curve is same with triangle curve, except that there are several points in trapezoidal curve have a membership value of one [10]. Representation of the trapezoidal curve shown in figure 1.
Here it is membership function for the representation of trapezoidal curve:

\[
\mu_T(x) = \begin{cases} 
\frac{x - t_1}{t_2 - t_1}, & t_1 \leq x \leq t_2 \\
1, & t_2 \leq x \leq t_3 \\
\frac{t_4 - x}{t_4 - t_3}, & t_3 \leq x \leq t_4 \\
0, & \text{otherwise}
\end{cases}
\]

\[
\tilde{T}_L(\alpha) = t_1 + (t_2 - t_1)\alpha \\
\tilde{T}_U(\alpha) = t_4 - (t_4 - t_3)\alpha \\
\alpha \in [0, 1]
\]

Explanation:
t1 : Fastest completion duration 
t2 : Average completion duration 
t3 : Average completion duration 
t4 : Latest completion duration 
\(\alpha\) : Degree of membership 
\(\tilde{T}_L\) : Fastest parametric form 
\(\tilde{T}_U\) : Latest parametric form

2.3. Vector Similarity
Fuzzy durations that have been obtained will be processed using a mathematical process that is vector similarity, this process is used for processing fuzzy duration data into a vector and then the results will be obtained a critical path ranking in the alpha (\(\alpha\)) range from 0 to 1 [10]. Figure 2 shows the process of calculation using vector similarity.
2.4. Method
Data obtained from the project will be processed by the trapezoidal fuzzy critical path method with the following steps:
1. Input the parameter of a project network.
2. Calculate each path of the fuzzy project networks.
3. Calculate the fuzzy total duration and determine the critical path (vector similarity process).
4. Calculate the fuzzy earliest time parameters of activities.
5. Calculate the fuzzy latest time parameters of activities.
6. Calculate the total floats and free floats of activities.

3. Results and discussion
3.1. Scheduling data
Table 1 shows the scheduling data uses as an example also with their dependency.

| No | Activities            | Successor | Dependency | T1 | T2 | T3 | T4 |
|----|-----------------------|-----------|------------|----|----|----|----|
| 1  | Preparation           | 2         | Finish to Start | 2  | 3  | 3  | 4  |
| 2  | Earthwork             | 3         | Finish to Start | 2  | 3  | 3  | 4  |
| 3  | Foundation            | 4         | Finish to Start | 3  | 5  | 6  | 7  |
| 4  | Sloof 1               | 5,6       | Finish to Start | 2  | 3  | 3  | 4  |
| 5  | Column                | 7,8       | Finish to Start | 5  | 7  | 7  | 9  |
| 6  | Sloof 2               | 9         | Finish to Start | 2  | 3  | 3  | 4  |
| 7  | Wall                  | 10,11     | Finish to Start | 7  | 10 | 11 | 12 |
| 8  | Door and window frame | 12        | Finish to Start | 1  | 2  | 2  | 3  |
| 9  | Plumbing              | 13        | Finish to Start | 4  | 5  | 5  | 6  |
| 10 | Upper tie beam        | 14        | Finish to Start | 4  | 6  | 6  | 7  |
| 11 | Wall plaster          | 16        | Finish to Start | 5  | 7  | 7  | 9  |
| 12 | Floor covering        | 15        | Finish to Start | 7  | 9  | 9  | 10 |
| 13 | Sanitation            | 15        | Finish to Start | 4  | 5  | 6  | 7  |
| 14 | Roof                  | 16        | Finish to Start | 7  | 9  | 10 | 11 |
| 15 | Door and window       | 19        | Finish to Start | 1  | 2  | 2  | 3  |
| 16 | Mechanical electrical | 17,18     | Finish to Start | 2  | 3  | 3  | 4  |
| 17 | Ceiling               | 19        | Finish to Start | 5  | 7  | 7  | 9  |
| 18 | Mechanical electrical | 19        | Finish to Start | 1  | 2  | 2  | 3  |
| 19 | Paint                 | -         | -           | 7  | 10 | 10 | 12 |
3.2. Scheduling result with fuzzy critical path method

Based on the data obtained and using the calculation methods mentioned above, Table 2 shows the result of scheduling using the trapezoidal fuzzy critical path method. Figure 3 shows the result from the application of the trapezoidal fuzzy critical path method.

| α   | J1  | J2  | J3  | J4  | J5  | J6  | Critical path | Critical activity | Total project duration |
|-----|-----|-----|-----|-----|-----|-----|---------------|--------------------|------------------------|
| 0   | 0.918 | 0.884 | 0.846 | 0.769 | 0.637 | 0.577 | J1           | J1                 | 1-2-3-4-5-7-10-14-16-17-19 | 46 | 83 |
| 0.1 | 0.926 | 0.885 | 0.846 | 0.769 | 0.637 | 0.576 | J1           | J1                 | 1-2-3-4-5-7-10-14-16-17-19 | 48 | 81.7 |
| 0.2 | 0.935 | 0.887 | 0.847 | 0.770 | 0.638 | 0.575 | J1           | J1                 | 1-2-3-4-5-7-10-14-16-17-19 | 50 | 80.4 |
| 0.3 | 0.943 | 0.889 | 0.848 | 0.771 | 0.639 | 0.575 | J1           | J1                 | 1-2-3-4-5-7-10-14-16-17-19 | 52 | 79.1 |
| 0.4 | 0.950 | 0.891 | 0.850 | 0.773 | 0.640 | 0.576 | J1           | J1                 | 1-2-3-4-5-7-10-14-16-17-19 | 54 | 77.8 |
| 0.5 | 0.958 | 0.894 | 0.852 | 0.775 | 0.642 | 0.576 | J1           | J1                 | 1-2-3-4-5-7-10-14-16-17-19 | 56 | 76.5 |
| 0.6 | 0.965 | 0.897 | 0.855 | 0.778 | 0.644 | 0.577 | J1           | J1                 | 1-2-3-4-5-7-10-14-16-17-19 | 58 | 75.2 |
| 0.7 | 0.971 | 0.900 | 0.858 | 0.780 | 0.646 | 0.578 | J1           | J1                 | 1-2-3-4-5-7-10-14-16-17-19 | 60 | 73.9 |
| 0.8 | 0.976 | 0.903 | 0.860 | 0.783 | 0.648 | 0.579 | J1           | J1                 | 1-2-3-4-5-7-10-14-16-17-19 | 62 | 72.6 |
| 0.9 | 0.981 | 0.906 | 0.862 | 0.785 | 0.650 | 0.580 | J1           | J1                 | 1-2-3-4-5-7-10-14-16-17-19 | 64 | 71.3 |
| 1   | 0.986 | 0.910 | 0.865 | 0.789 | 0.653 | 0.582 | J1           | J1                 | 1-2-3-4-5-7-10-14-16-17-19 | 66 | 70 |

Figure 3 shows that using the fuzzy critical path method, 2 durations are generated in the range of alpha = 0 to alpha = 1. The actual duration of the completion of this house is 70 days. Figure 3 shows that the actual duration of the project is between 2 duration ranges by this method. This shows that this method can accommodate scheduling for this project. Figure 3 also shows the influence of alpha (α) makes the estimated range of final duration of the project smaller when going to alpha (α) = 1. The application of this method also shows that there is no change in the critical path in the range of alpha = 0 to alpha = 1 for this project.

3.3. Scheduling result from various scheduling scenarios

Various scheduling scenarios are made by changing factors in the critical path fuzzy method. In this scenario, the factors that are changed in the critical path fuzzy method are the dependency of the activity. The data used will be presented in Table 3 and Table 4. Figure 4 shows the change process between the start to start dependency to finish to start dependency.
Table 3. Scheduling data for start to start dependency

| No | Activities | Successor | Dependency | Fuzzy duration |
|----|------------|-----------|-------------|---------------|
|    |            |           |             | T1 | T2 | T3 | T4 |
| 1  | A          | B         | Finish to Start +0 | 2  | 3  | 3  | 4  |
| 2  | B          | C         | Start to Start +2  | 3  | 6  | 6  | 9  |
|    |            | D         | Finish to Start +0 | 3  | 6  | 6  | 9  |
| 3  | C          | E         | Finish to Start +0 | 6  | 7  | 7  | 8  |
| 4  | D          | E         | Finish to Start +0 | 1  | 3  | 3  | 5  |
| 5  | E          |           | Finish to Start +0 | 2  | 4  | 4  | 6  |

Table 4. Scheduling data for finish to start dependency

| No | Activities | Successor | Dependency | Fuzzy duration |
|----|------------|-----------|-------------|---------------|
|    |            |           |             | T1 | T2 | T3 | T4 |
| 1  | A          | B1        | Finish to Start +0 | 2  | 3  | 3  | 4  |
| 2  | B1         | B2        | Finish to Start +0 | 6  | 7  | 7  | 8  |
|    |            | C         | Finish to Start +0 | 6  | 7  | 7  | 8  |
| 3  | B2         | D         | Finish to Start +0 | 1  | 3  | 3  | 5  |
| 4  | C          | E         | Finish to Start +0 | 2  | 4  | 4  | 6  |
| 5  | D          | E         | Finish to Start +0 | 2  | 4  | 4  | 6  |
| 6  | E          |           | Finish to Start +0 | 2  | 4  | 4  | 6  |

Figure 4. Changes in dependency between activities

The effects of the difference between the dependency of start to start and finish to start is on the duration of activities that were previously separated into 2 and 4 days for activity B on the finish to start dependency then on the start to start dependency which was combined into a single unit with a duration of 6 days. By using the same calculation process, figure 5 shows the result from the application of trapezoidal fuzzy critical path method for this scenario.
Figure 5 shows that there is a change in the line where it shows the change in the critical path. From figure 5 it also shows that the critical path has changed when \( \alpha = 0.2 \) in the finish to start dependency, but in the start to start dependency, changes in the critical path occur at \( \alpha = 0.4 \). In this case, it can be concluded two things. First, the change in the critical path caused by the influence of \( \alpha \). Second, change in the critical path in start to start dependency occurs at a higher \( \alpha \) condition due to the shorter path of activity.

4. Conclusion

Based on the results of the implementation that has been done on the VB housing project, it is proved that the fuzzy trapezoidal method can accommodate the scheduling of the project by the actual duration of the project which is between in 2 durations that produced by the fuzzy trapezoidal method. Based on the results of the scenario, it was found that the change in the critical path could occur due to the influence of \( \alpha \) and the dependency of activity also affected the change in the critical path where in this case the dependency between finish to start dependency had a change in the critical path in the smaller \( \alpha \) than the start to start dependency.

5. References

[1] Soeharto I 1999 Manajemen Proyek Dari Konseptual Sampai Operasional (Jakarta: Erlangga)
[2] Hafnidar A 2016 Manajemen Proyek Konstruksi (Yogyakarta: Deep Publish)
[3] Husen A 2010 Manajemen Proyek (Perencanaan, Penjadwalan & Pengendalian Proyek) edisi revisi (Yogyakarta: Andi)
[4] Chen S P 2007 Analysis of critical path in a project network with fuzzy activity times European J. of Oper. Research 183(1) 442–59
[5] Chu C C, P E Zheng and D D Wang Analysis and modelling of fuzzy network planning considering complicated activity relationships J. Tianjin Univ, 39(5) 631-636
[6] Oladeinde M H and Itsisor D O 2013 Application of fuzzy theory to project scheduling with critical path method J. of Appl. Sci. and Envr. Manage. 17(1) 161–66 JASEM ISSN 11198362.
[7] Dubois D and Prade H 1980 Fuzzy Sets and System, Theory and application (New York: Academic press)
[8] Klim G J and Yuan B 1995 Fuzzy Sets and Fuzzy Logic, Theory and Applications (Upper Saddle River: Prentice Hall Inc.)
[9] Timothy J Ross 1995 Fuzzy Logic with Engineering Application (New York: McGraw-Hill Inc.)
[10] Hu C M and Liu D J 2018 Improved critical path method with trapezoidal fuzzy activity durations J. Constr. Eng. Manage. 144(9) 04018090