Estimation of Spatial Panels
Estimation of Spatial Panels

Lung-fei Lee
The Ohio State University
Columbus, Ohio 43210
USA
lflee@econ.ohio-state.edu

Jihai Yu
Peking University
China
jihai.yu@gsm.pku.edu.cn

University of Kentucky
Lexington, KY 40506
USA
jihai.yu@uky.edu

Full text available at: http://dx.doi.org/10.1561/0800000015
Editorial Scope

Foundations and Trends® in Econometrics will publish survey and tutorial articles in the following topics:

- Identification
- Model Choice and Specification Analysis
- Non-linear Regression Models
- Simultaneous Equation Models
- Estimation Frameworks
- Biased Estimation
- Computational Problems
- Microeconometrics
- Treatment Modeling
- Discrete Choice Modeling
- Models for Count Data
- Duration Models
- Limited Dependent Variables
- Panel Data
- Dynamic Specification
- Inference and Causality
- Continuous Time Stochastic Models
- Modeling Non-linear Time Series
- Unit Roots
- Cointegration
- Latent Variable Models
- Qualitative Response Models
- Hypothesis Testing
- Interactions-based Models
- Duration Models
- Financial Econometrics
- Measurement Error in Survey Data
- Productivity Measurement and Analysis
- Semiparametric and Nonparametric Estimation
- Bootstrap Methods
- Nonstationary Time Series
- Robust Estimation

Information for Librarians
Foundations and Trends® in Econometrics, 2010, Volume 4, 4 issues. ISSN paper version 1551-3076. ISSN online version 1551-3084. Also available as a combined paper and online subscription.
Estimation of Spatial Panels

Lung-fei Lee\textsuperscript{1} and Jihai Yu\textsuperscript{2}

\textsuperscript{1} Department of Economics, The Ohio State University, Columbus, Ohio 43210, USA, lflee@econ.ohio-state.edu
\textsuperscript{2} Guanghua School of Management, Peking University, 100871, China, jihai.yu@gsm.pku.edu.cn; Department of Economics, University of Kentucky, Lexington, KY 40506, USA, jihai.yu@uky.edu

Abstract

Spatial panel models have panel data structures to capture spatial interactions across spatial units and over time. There are static as well as dynamic models. This text provides some recent developments on the specification and estimation of such models. The first part will consider estimation for static models. The second part is devoted to the estimation for spatial dynamic panels, where both stable and unstable dynamic models with fixed effects will be considered.

For the estimation of a spatial panel model with individual fixed effects, in order to avoid the incidental parameter problem due to the presence of many individual fixed effects, a conditional likelihood or partial likelihood approach is desirable. For the model with both fixed individual and time effects with a large and long panel, a conditional likelihood might not exist, but a partial likelihood can be constructed. The partial likelihood approach can be generalized to spatial panel

\textsuperscript{*}The authors are grateful to a referee for valuable comments to improve the presentation of this paper.
models with fixed effects and a space–time filter. If individual effects are independent of exogenous regressors, one may consider the random effects specification and its estimation. The likelihood function of a random effects model can be decomposed into the product of a partial likelihood function and that of a between equation. The underlying equation for the partial likelihood function can be regarded as a within equation. As a result, the random effects estimate is a pooling of the within and between estimates. A Hausman type specification test can be used for testing the random components specification vs. the fixed effects one. The between equation highlights distinctive specifications on random components in the literature.

For spatial dynamic panels, we focus on the estimation for models with fixed effects, when both the number of spatial units $n$ and the number of time periods $T$ are large. We consider both quasi-maximum likelihood (QML) and generalized method of moments (GMM) estimations. Asymptotic behavior of the estimators depends on the ratio of $T$ relative to $n$. For the stable case, when $n$ is asymptotically proportional to $T$, the QML estimator is $\sqrt{nT}$-consistent and asymptotically normal, but its limiting distribution is not properly centered. When $n$ is large relative to $T$, the QML estimator is $T$-consistent and has a degenerate limiting distribution. Bias correction for the estimator is possible. When $T$ grows faster than $n^{1/3}$, the bias corrected estimator yields a centered confidence interval. The $n$ and $T$ ratio requirement can be relaxed if individual effects are first eliminated by differencing and the resulting equation is then estimated by the GMM, where exogenous and predetermined variables can be used as instruments. We consider the use of linear and quadratic moment conditions, where the latter is specific for spatial dependence. A finite number of moment conditions with some optimum properties can be constructed. An alternative approach is to use separate moment conditions for each period, which gives rise to many moments estimation.

The remaining text considers estimation of spatial dynamic models with the presence of unit roots. The QML estimate of the dynamic coefficient is $\sqrt{nT^3}$-consistent and estimates of all other parameters are $\sqrt{nT}$-consistent, and all of them are asymptotically normal. There are cases that unit roots are generated by combined temporal and
spatial correlations, and outcomes of spatial units are cointegrated. The asymptotics of the QML estimator under this spatial cointegration case can be analyzed by reparameterization. In the last part, we propose a data transformation resulting in a unified estimation approach, which can be applied to models regardless of whether the model is stable or not. A bias correction procedure is also available.

The estimation methods are illustrated with two relevant empirical studies, one on regional growth and the other on market integration.
## Contents

1 Introduction 1

2 Static Spatial Panels — Fixed Effects Models 13

2.1 A Spatial Panel Model with Individual Effects 13

2.2 A Spatial Panel Model with Both Individual and Time Effects 20

2.3 A General Spatial Panel Model with Space–Time Filters 25

3 Static Spatial Panels — Random Effects Models 33

3.1 Random Individual Effects 35

3.2 The Hausman Specification Test 40

3.3 The Long Panel Case with Time Effects 42

3.4 Monte Carlo Results 45

4 Spatial Dynamic Panels — Stable Models with Fixed Effects 47

4.1 A Spatial Dynamic Panel Model with Individual Effects 48

4.2 A Spatial Dynamic Panel Model with Time Varying $W_{nt}$ 51

4.3 A Spatial Dynamic Panel Model with Both Individual and Time Effects 56

4.4 GMM Estimation 61
### Spatial Dynamic Panels — Unstable Models with Fixed Effects

5.1 Unit Roots in Spatial Panels
5.2 Spatial Cointegration
5.3 Estimation of Models with Explosive Roots

### Some Empirical Applications

6.1 Regional Growth
6.2 Market Integration

### Conclusions

### Some Basic Technical Theorems and Proofs

A.1 Basic Technical Theorems
A.2 Theorems for Some Statistics for a General Spatial Panel Model

### Notations

B.1 Notations for Static Spatial Panels — Fixed Effects Models
B.2 Notations for Static Spatial Panels — Random Effects Models
B.3 Notations for Spatial Dynamic Panels — Stable Models with Fixed Effects
B.4 Notations for Spatial Dynamic Panels — Unstable Models with Fixed Effects

### References
Introduction

The last decade has seen a growing literature on panel data models with cross sectional dependence. The current text presents some recent developments in the specification and estimation of panel data models with spatial interactions. Spatial econometrics consists of econometric techniques dealing with interactions of economic units in space, which can be of physical or economic characteristics. The spatial autoregressive (SAR) model by Cliff and Ord (1973) has received the most attention in economics. Early development in estimation and testing for cross sectional data in econometrics can be found in Anselin (1988, 1992), Cressie (1993), Kelejian and Robinson (1993), Anselin and Florax (1995), Anselin and Rey (1997) and Anselin and Bera (1998), among others. Under the panel data setting, spatial panel data models are of great interest, because they enable researchers to take into account dynamics and control for unobservable heterogeneity.

For static models, the following spatial panel model with both spatial lag and spatial disturbances is a typical one:

\[
Y_{nt} = \lambda_0 W_n Y_{nt} + X_{nt} \beta_0 + c_{n0} + U_{nt},
\]

\[
U_{nt} = \rho_0 M_n U_{nt} + V_{nt}, \quad t = 1, 2, \ldots, T,
\]

(1.1)
Introduction

where $\mathbf{Y}_{nt} = (y_{1t}, y_{2t}, \ldots, y_{nt})'$ and $\mathbf{V}_{nt} = (v_{1t}, v_{2t}, \ldots, v_{nt})'$ are $n \times 1$ vectors, and $v_{it}$ is i.i.d. across $i$ and $t$ with zero mean and variance $\sigma_0^2$. $\mathbf{W}_n$ is an $n \times n$ nonstochastic spatial weights matrix that generates the spatial dependence on $y_{it}$ among cross sectional units, which may or may not be row-normalized. $\mathbf{X}_{nt}$ is an $n \times k$ matrix of nonstochastic time varying regressors, $\mathbf{c}_{n0}$ is an $n \times 1$ vector of individual effects, $\mathbf{M}_n$ is an $n \times n$ spatial weights matrix for the disturbance process. In practice, $\mathbf{M}_n$ may or may not be $\mathbf{W}_n$.

For static panel data models with spatial interactions, we can have random effects or fixed effects specifications. For the random effects specification, Anselin (1988) provides a panel regression model with error components and SAR disturbances, and Baltagi et al. (2003) consider specification tests for spatial correlation in that spatial panel regression model. The Anselin and Baltagi et al. model is $\mathbf{Y}_{nt} = \mathbf{X}_{nt}\beta_0 + \mathbf{c}_{n0} + \mathbf{U}_{nt}, \mathbf{U}_{nt} = \lambda_0 \mathbf{W}_n \mathbf{U}_{nt} + \mathbf{V}_{nt}$, where $\mathbf{c}_{n0}$ is an $n \times 1$ vector of individual error components, and the spatial correlation is in $\mathbf{U}_{nt}$. Kapoor et al. (2007) propose a different specification with error components and a SAR process in the overall disturbance, and suggest a method of moments (MOM) estimation. The specification in Kapoor et al. (2007) is $\mathbf{Y}_{nt} = \mathbf{X}_{nt}\beta_0 + \mathbf{U}_{nt}^+ + \mathbf{U}_{nt} = \lambda_0 \mathbf{W}_n \mathbf{U}_{nt}^+ + \mathbf{d}_{n0} + \mathbf{V}_{nt}$, where $\mathbf{d}_{n0}$ is a vector of individual error components. Fingleton (2008) adopts a similar approach to estimate a spatial panel model with SAR dependent variables, random components and a spatial moving average (SMA) structure in the overall disturbance. By the transformation ($\mathbf{I}_n - \lambda_0 \mathbf{W}_n$), the data generating process (DGP) of Kapoor et al. (2007) becomes $\mathbf{Y}_{nt} = \mathbf{X}_{nt}\beta_0 + \mathbf{c}_{n0} + \mathbf{U}_{nt}$ where $\mathbf{c}_{n0} = (\mathbf{I}_n - \lambda_0 \mathbf{W}_n)^{-1}\mathbf{d}_{n0}$ and $\mathbf{U}_{nt} = \mathbf{U}_{nt}^+ - (\mathbf{I}_n - \lambda_0 \mathbf{W}_n)^{-1}\mathbf{d}_{n0}$. The $\mathbf{U}_{nt} = \lambda_0 \mathbf{W}_n \mathbf{U}_{nt} + \mathbf{V}_{nt}$ forms a SAR process. This model implies spatial correlations in both the individual and disturbance components, $\mathbf{c}_{n0}$ and $\mathbf{U}_{nt}$, having the same spatial effect parameter. Nesting the Anselin (1988) and Kapoor et al. (2007) models, Baltagi et al. (2007) suggest an extended model without restrictions on implied SAR structures in the error component and the remaining disturbance.

As an alternative to the random effects specification, Lee and Yu (2010b) investigate the quasi-maximum likelihood (QML) estimation of spatial panel models under the fixed effects specification. The fixed
effects model has the advantage of robustness in that fixed effects are allowed to depend on included regressors in the model. It also provides a unified model framework because different random effects models in Anselin (1988), Kapoor et al. (2007) and Baltagi et al. (2007) reduce to the same fixed effects model.

We have two approaches to estimate the spatial panel data models with individual fixed effects. The first is called the “direct approach”, where common parameters and the individual effects are jointly estimated. The second is called the “transformation approach”, where the individual effects are eliminated first before estimation. For the direct ML approach, it will yield consistent estimates for the spatial and regression coefficients, except for the variance parameter when $T$ is finite. Thus, the results are similar to Neyman and Scott (1948). The transformation approach is the method of conditional likelihood, which is applicable when sufficient statistics can be found for the fixed effects. For the linear regression and logit panel models, the time average of the dependent variables for each cross sectional unit provides a sufficient statistic (see Hsiao, 1986). For the normal panel regression model, the conditional likelihood can be constructed from some transformed data. We investigate the use of similar transformations to the spatial panel model. By using the deviation from the time mean transformation, individual effects can be eliminated. The transformed equation can then be estimated by the QML approach. This transformation approach can be justified as a conditional likelihood approach. For the model with both individual and time fixed effects, one may combine the transformations by deviations from time means and also deviations from cross section means to eliminate those effects. The transformed equation can be regarded as well-defined equation system when the spatial weights matrix is row-normalized. The resulting likelihood function can be interpreted as a partial likelihood (Cox 1975; Wong 1986).

The spatial panel data models have a wide range of applications such as agricultural economics (Druska and Horrace 2004), transportation research (Frazier and Kockelman 2005), public economics (Egger et al.)

\footnote{As is illustrated in Neyman and Scott (1948), for the linear panel regression model with fixed effects, the ML estimates of the regression coefficients are consistent, while the MLE of the variance parameter is inconsistent when $T$ is finite.}
and good demand (Baltagi and Li 2006), to name a few. The above panel models are static ones which do not incorporate time-lagged dependent variables in the regression equation.

Spatial panel data can include both spatial and dynamic effects to investigate the state dependence and spatial correlations. To include time dynamic features in spatial panel data, an immediate approach is to use the time lag term as an explanatory variable. In a conventional dynamic panel data model with individual fixed effects, the MLE of the autoregressive coefficient is biased and inconsistent when \( n \) tends to infinity but \( T \) is fixed (Nickell 1981; Hsiao 1986). By taking time differences to eliminate the fixed effects in the dynamic equation and by the construction of instrumental variables (IVs), Anderson and Hsiao (1981) show that IV methods can provide consistent estimates. When \( T \) is finite, additional IVs can improve the efficiency of the estimation. However, if the number of IVs is too large, the problem of many IVs arises as the asymptotic bias would increase with the number of IVs.

For spatial dynamic models, Korniotis (2010) investigates a spatial time lag model with fixed effects, and considers a bias adjusted within estimator, which generalizes Hahn and Kuersteiner (2002). Elhorst (2005) estimated a dynamic model with spatial disturbances by unconditional maximum likelihood method, and Mutl (2006) investigates the model using a three-step GMM. Su and Yang (2007) derive the QMLEs of the above model under both fixed and random effects specifications. Yang et al. (2006) propose a generalized dynamic error component model that accounts for the effects of functional form and spatial dependence. For a general model with both time and space dynamics, we term it the spatial dynamic panel data (SDPD) model to better link the terminology to the dynamic panel data literature (see, e.g., Hsiao 1986; Alvarez and Arellano 2003). Yu et al. (2008) study the stable SDPD models where the individual time lag, spatial time lag and contemporaneous spatial lag are all included. For the estimation of SDPD models, we can use the QMLE when the number of periods \( T \) is large. When \( T \) is relatively small, we can rely on GMM where lagged values can be used as IVs. Elhorst (2010) uses Monte Carlo to investigate small sample performances of various ML and GMM estimators when \( T \) is finite.
When both $n$ and $T$ are large, the incidental parameter problem in the MLE becomes less severe as each individual fixed effect can be consistently estimated. However, the presence of asymptotic bias may still cause the distribution of estimates not centered properly. Similar issue on asymptotic bias occurs for estimates of SDPD models. As the presence of asymptotic bias is an undesirable feature of these estimates, a bias correction procedure is needed. Kiviet (1995), Hahn and Kuersteiner (2002), and Bun and Carree (2005) have constructed bias corrected estimators for the conventional dynamic panel data model by analytically modifying the within estimator. For the QMLE of the SDPD model, analytic bias correction is also possible (Yu et al., 2008).

A general SDPD model can be specified as, for $t = 1, 2, \ldots, T$,

$$Y_{nt} = \lambda_0 W_n Y_{nt} + \gamma_0 Y_{n,t-1} + \rho_0 W_n Y_{n,t-1} + X_{nt} \beta_0 + c_{n0} + \alpha_{t0} I_n + V_{nt}, \quad (1.2)$$

where $c_{n0}$ is an $n \times 1$ column vector of fixed effects and $\alpha_{t0}$’s are time effects. Comparing it to the static model, we have included the dynamic terms $Y_{n,t-1}$ and $W_n Y_{n,t-1}$ in (1.2). Here, $\gamma_0$ captures the pure dynamic effect and $\rho_0$ captures the spatial-time effect.

These SDPD models can be applied to various fields such as growth convergence of countries and regions (Ertur and Koch, 2007), regional markets (Keller and Shiue, 2007), labor economics (Foote, 2007), public economics (Revelli, 2001; Tao, 2005; Franzese, 2007).

To investigate the dynamics of this model, one may investigate eigenvalues of $A_n$ under the assumption that $W_n$ is diagonalizable. Let $\varpi_n = \text{diag}\{\varpi_{n1}, \varpi_{n2}, \ldots, \varpi_{nn}\}$ be the $n \times n$ diagonal eigenvalues matrix of $W_n$ such that $W_n = \Gamma_n \varpi_n \Gamma_n^{-1}$ where $\Gamma_n$ is the corresponding eigenvector matrix. As $A_n = S_n^{-1}(\gamma_0 I_n + \rho_0 W_n)$, the eigenvalues matrix of $A_n$

\[\text{as an example in practice with a row-normalized } W_n, |\lambda_0| < 1 \text{ will guarantee that } S_n \text{ is invertible.}\]
is $D_n = (I_n - \lambda_0 \omega_n)^{-1}(\gamma_0 I_n + \rho_0 \omega_n)$ such that $A_n = \Gamma_n D_n \Gamma_n^{-1}$. When $W_n$ is row-normalized, all the eigenvalues are less than or equal to 1 in absolute value, where it definitely has some eigenvalues being 1 (Ord, 1975). Let $m_n$ be the number of unit eigenvalues of $W_n$ and let the first $m_n$ eigenvalues of $W_n$ be the unity. Then, $D_n$ can be decomposed into two parts, one corresponding to the unit eigenvalues of $W_n$, and the other corresponding to the remaining eigenvalues of $W_n$ smaller than 1. Define $\mathcal{J}_n = \text{diag}\{l_{m_n}, 0, \ldots, 0\}$ with $l_{m_n}$ being an $m_n \times 1$ vector of ones and $\hat{D}_n = \text{diag}\{0, \ldots, 0, d_{n,m_n+1}, \ldots, d_{nn}\}$, where $|d_{ni}| < 1$, for $i = m_n + 1, \ldots, n$, are assumed. As $\mathcal{J}_n \cdot \hat{D}_n = 0$, we have $A_n = (\gamma_0 + \rho_0)\mathcal{J}_n \Gamma_n \Gamma_n^{-1} + B_n$ where $B_n = \Gamma_n \hat{D}_n \Gamma_n^{-1}$ for any $h = 1, 2, \ldots$.

Denote $W_n = \Gamma_n \mathcal{J}_n \Gamma_n^{-1}$. Then, for $t \geq 0$, $Y_{nt}$ can be decomposed into a sum of a possible stable part, a possible unstable or explosive part, and a time effect part:

$$Y_{nt} = Y_{nt}^s + Y_{nt}^u + Y_{nt}^\alpha,$$

(1.4)

where

$$Y_{nt}^s = \sum_{h=0}^{\infty} B_n^h S_n^{-1}(c_{n0} + X_{n,t-h}\beta_0 + V_{n,t-h}),$$

$$Y_{nt}^u = W_n \left\{ \left( \frac{\gamma_0 + \rho_0}{1 - \lambda_0} \right)^{t+1} Y_{n,-1} + \frac{1}{(1 - \lambda_0)} \sum_{h=0}^{t} \left( \frac{\gamma_0 + \rho_0}{1 - \lambda_0} \right)^h (c_{n0} + X_{n,t-h}\beta_0 + V_{n,t-h}) \right\},$$

$$Y_{nt}^\alpha = \frac{1}{(1 - \lambda_0)} l_n \sum_{h=0}^{\infty} \alpha_{t-h,0} \left( \frac{\gamma_0 + \rho_0}{1 - \lambda_0} \right)^h .$$

The $Y_{nt}^u$ can be an unstable component when $\frac{\gamma_0 + \rho_0}{1 - \lambda_0} \geq 1$. With $\lambda_0 < 1$, $\gamma_0 + \rho_0 + \lambda_0 > 1$ is equivalent to $\frac{\gamma_0 + \rho_0}{1 - \lambda_0} > 1$ and, in that case, $Y_{nt}^u$ can be explosive. The component $Y_{nt}^\alpha$ captures the time effect due to the time dummies. The $Y_{nt}^\alpha$ can be rather complicated as it depends on what

\footnote{We note that $d_{ni} = (\gamma_0 + \rho_0 \omega_n)/(1 - \lambda_0 \omega_n)$. Hence, if $|\gamma_0| + |\lambda_0| + |\rho_0| < 1$, we have $d_{ni} < 1$ as $|\omega_n| \leq 1.$}
the time dummies would represent. The $Y_{nt}$ can be explosive when $\alpha_0$ represents some explosive functions of $t$, even when $\gamma_0 + \rho_0 + \lambda_0$ were smaller than 1. Without a specific time structure for $\alpha_0$, it is desirable to eliminate this component for the estimation. If the absolute values of the elements in $D_n$ are less than 1, $Y_{nt}^s$ will be a stable component. The $Y_{nt}^s$ can be a stable component unless $\gamma_0 + \rho_0 + \lambda_0$ is much larger than 1. If the sum $\gamma_0 + \rho_0 + \lambda_0$ were too big, some of the eigenvalues $\lambda_i$ in $Y_{nt}^s$ might become larger than 1.

The spatial cointegration case is the situation where $\gamma_0 + \rho_0 + \lambda_0 = 1$ but $\gamma_0 \neq 1$. The unit eigenvalues of $A_n$ correspond exactly to those unit eigenvalues of $W_n$ via the relation $D_n = (I_n - \lambda_0 \bar{w}_n)^{-1}(\gamma_0 I_n + \rho_0 \bar{w}_n)$. $W_n$ has some unit eigenvalues, but not all of them are equal to 1 because $\text{tr}(W_n) = 0$, and hence the sum of eigenvalues of $W_n$ is zero. Hence, some eigenvalues of $A_n$, but not all, are equal to 1. If $c_n$ and/or the time mean of $X_n \beta_0$ are nonzero, the $\sum_{h=0}^t c_n \chi_{n,t-h} \beta_0$ will generate a time trend. The $\sum_{h=0}^t V_{n,t-h}$ will generate a stochastic trend. These imply the unstability of $Y_{nt}$.

The unit roots case has all eigenvalues of $A_n$ being 1. It occurs when $\gamma_0 + \rho_0 + \lambda_0 = 1$ and $\gamma_0 = 1$, because $A_n = (I_n - \lambda_0 W_n)^{-1}(\gamma_0 I_n + \rho_0 W_n) = (I_n - \lambda_0 W_n)^{-1}(I_n - \lambda_0 W_n) = I_n$. For this unit roots case, the unit eigenvalues of $A_n$ are not linked to the eigenvalues of $W_n$. Because $W_n^s$ is defined completely from $W_n$, the decomposition in (1.4) is not revealing for the unit roots case; instead, one has

$$Y_{nt} = Y_{nt-1} + S_n^{-1} (X_{nt} \beta_0 + c_n + \alpha_0 l_n + U_{nt}). \tag{1.5}$$

Some implications of spatial and dynamic effects in terms of the coefficients $\lambda_0$, $\gamma_0$ and $\rho_0$ can be revealed via marginal impacts of regressors. Suppose that we are interested in (an average) total (expected) impact resulting from changing a regressor by the same amount across all spatial units in some time periods, say, from the time period $t_1$ to $t$, where $t_1 \leq t$. For simplicity, we assume that $x_{nt}$ is a single regressor, and consider the situation that $W_n$ is row-normalized and does not depend on $x$. Thus, we have from the reduced form equation that $\frac{\partial E(Y_{nt})}{\partial x} = \beta_0 \sum_{h=0}^{t-t_1-1} A_n^h S_n^{-1} I_n$, where $I_n$ is an $n$-dimensional vector of ones. As $W_n$ is row-normalized such that $W_n l_n = l_n$, $\frac{\partial E(Y_{nt})}{\partial x} = \sum_{h=0}^{t-t_1-1} A_n^h S_n^{-1} I_n \beta_0 = l_n \sum_{h=0}^{t-t_1-1} \left( \frac{\gamma_0 + \rho_0}{1 - \lambda_0} \right)^h \cdot \frac{\beta_0}{1 - \lambda_0}$, where every unit will receive the same impact.
Introduction

There are several cases of interest:

(1) $t_1 = t$, i.e., the marginal change of $x$ occurs for all spatial units at the current period $t$. In that case, $\frac{\partial E(Y_{nt})}{\partial x} = \ln(\frac{\beta_0}{1-\lambda_0})$, which is the marginal impact due to spatial interactions. The $\beta_0$ is the marginal effect of $x$ and $\frac{1}{1-\lambda_0}$ represents the spatial multiplier effect.

(2) $t_1 < t$, i.e., the marginal change of $x$ occurs for all spatial units from the past period $t_1$ to the current period $t$. In this case, $\frac{\partial E(Y_{nt})}{\partial x} = \ln\left[1 + (\frac{\gamma_0 + \rho_0}{1-\lambda_0}) + \cdots + (\frac{\gamma_0 + \rho_0}{1-\lambda_0})^{t-t_1}\right] \frac{\beta_0}{1-\lambda_0}$. If $t_1 = t - 1$, the marginal impact for each spatial unit becomes $\ln\left[1 + (\frac{\gamma_0 + \rho_0}{1-\lambda_0})\right] \frac{\beta_0}{1-\lambda_0}$. This marginal impact is composed of the marginal impact $\frac{\beta_0}{1-\lambda_0}$ of changing $x$ in the current period $t$ to $E(Y_{nt})$ and also an impact due to changing $x$ in the last period $t - 1$. The change of $x$ at $t - 1$ has the marginal impact with spatial multiplier $\frac{\beta_0}{1-\lambda_0}$ on $Y_{n,t-1}$. This marginal change of $Y_{n,t-1}$ generates its marginal impact $\frac{\gamma_0 + \rho_0}{1-\lambda_0}$ on $E(Y_{nt})$ through both the time filter $(\gamma_0 I_n + \rho_0 W_n)$ and the space filter $S_{n-1}^{-1}$. Thus, the marginal impact on changing $x$ in the last period is the product $(\frac{\gamma_0 + \rho_0}{1-\lambda_0}) \frac{\beta_0}{1-\lambda_0}$. The marginal impact on changing $x$ from a past period can be deducted recursively, and the total impact accumulates effects of those changes. For both the unit roots case ($\gamma_0 = 1$ and $\lambda_0 + \rho_0 = 0$) and the spatial cointegration case ($\lambda_0 + \gamma_0 + \rho_0 = 1$ with $\gamma_0 \neq 1$), they imply $\frac{\gamma_0 + \rho_0}{1-\lambda_0} = 1$; hence, their total marginal impact is simply the product of the marginal impact $\frac{\beta_0}{1-\lambda_0}$ at each time period multiplied by the total number of time periods of changing $x$, i.e., $\frac{\partial E(Y_{nt})}{\partial x} = \ln(\frac{\beta_0}{1-\lambda_0})(t - t_1 + 1)$.

(3) $t_1 = -\infty$, i.e., the marginal change of $x$ occurs from infinite past to the current period. For the stable SDPD process, one has a convergent series. The total marginal impact would be $\frac{\partial E(Y_{nt})}{\partial x} = \ln(\frac{\beta_0}{1-(\lambda_0 + \gamma_0 + \rho_0)})$. The spatial and dynamic effects are combined into the multiplier effect $\frac{1}{1-(\lambda_0 + \gamma_0 + \rho_0)}$. 

Full text available at: http://dx.doi.org/10.1561/0800000015
For estimation of those models with QML approaches, the QMLEs may have different rates of convergence. For the stable case, the rates of convergence of QMLEs are $\sqrt{nT}$, as shown in Yu et al. (2008) and reported in a subsequent section. For the spatial cointegration case, Yu et al. (2007) show that the QMLEs for such a model are $\sqrt{nT}$ consistent and asymptotically normal, but, the presence of the unstable components will make the estimators’ asymptotic variance matrix singular. Consequently, a linear combination of the spatial and dynamic effects estimates can converge at a higher rate. For the unit roots case, the QMLEs of $\gamma_0$ is $\sqrt{nT^3}$ consistent and other estimates are $\sqrt{nT}$ consistent; however, the estimate of sum of $\rho_0 + \lambda_0$ is $\sqrt{nT^3}$ consistent. For the explosive case, we will rely on a data transformation in order to estimate the model.

The rest of the text is organized as follows. Static Spatial Panels — Fixed Effects Models uses either the conditional likelihood or the partial likelihood approaches to estimate the spatial panel model with individual fixed effects. Static Spatial Panels — Random Effects Models investigates the spatial panel model with a general space–time filter under random effects specification. It is shown that the estimates under the random effects specification is a pooling of the within and between estimates, and a Hausman type specification test can be used for testing the random components specification vs. the fixed effects one. Spatial Dynamic Panels — Stable Models with Fixed Effects study both QML and GMM estimation of stable SDPD models with fixed effects. The QML approach is applicable when $T$ is large, and a bias correction procedure can eliminate the dominant bias of the QMLE. The $n$ and $T$ ratio requirement can be relaxed if individual effects are first eliminated by differencing and the resulting equation is then estimated by the GMM, where exogenous and predetermined variables can be used as instruments. Spatial Dynamic Panels — Unstable Models with Fixed Effects cover QML estimation of SDPD models in the presence of unit roots. A data transformation is proposed to estimate various SDPD models, and can provide regular $\sqrt{nT}$-consistent and asymptotic normal estimates as long as the stable component is present. Finally, two empirical applications are presented to illustrate the proposed estimation methods. Some technical theorems and notations are provided in Appendices.
Introduction

Even though we have different model specifications, there are some basic common features for all of them. The following common assumptions will be used throughout the text for both static and dynamic models.\footnote{Matlab codes for those estimation methods are available upon request for readers who are interested.} In addition to these, specific assumptions for different models will be listed when needed.

**Assumption 1.** All the spatial weights matrices (i.e., $W_n$) are non-stochastic with zero diagonals and are uniformly bounded in both row and column sums in absolute value (for short, UB).\footnote{We say a (sequence of $n \times n$) matrix $P_n$ is uniformly bounded in row and column sums in absolute value if $\sup_{n \geq 1} \|P_n\|_\infty < \infty$ and $\sup_{n \geq 1} \|P_n\|_1 < \infty$, where $\|P_n\|_\infty = \sup_{1 \leq i \leq n} \sum_{j=1}^n |p_{ij,n}|$ is the row sum norm and $\|P_n\|_1 = \sup_{1 \leq j \leq n} \sum_{i=1}^n |p_{ij,n}|$ is the column sum norm.}

**Assumption 2.** The relevant disturbances (i.e., $v_{it}$ or $e_{it}$) are i.i.d. across $i$ and $t$ with zero mean and finite variance, and their higher than fourth moments exist.

**Assumption 3.** The true spatial effect coefficients (i.e., $\lambda_0$) are in the interior of their parameter spaces. The spatial transformation matrices (i.e., $I_n - \lambda W_n$) are invertible on the compact parameter spaces of spatial effects, and their inverses are UB uniformly in the parameter spaces.

**Assumption 4.** The elements of $X_{nt}$ are nonstochastic and bounded, uniformly in $n$ and $t$. When we have $n \times k_z$ time invariant regressor $z_n$, it is also nonstochastic and bounded uniformly in $n$.

For some cases, we focus on row-normalized spatial weights matrices that are popular in empirical applications. Under these situations, we make that explicit and extend Assumption 1 to

**Assumption 1'.** All the spatial weights matrices are row-normalized and satisfy Assumption 1.

**Assumption 5.** $n$ goes to infinity, where $T$ can be finite or an increasing function of $n$.

**Assumption 5'.** $T$ goes to infinity, where $n$ is an increasing function of $T$. 
Assumption 5”. \( T \) goes to infinity, where \( n \) can be finite or an increasing function of \( T \).

The zero diagonal assumption for the \( W_n \) matrix helps the interpretation of the spatial effect, as self-influence shall be excluded in practice. As a result, the trace of a spatial weights matrix is zero and hence the sum of all its eigenvalues is zero. In many empirical applications with a non-negative spatial weights matrix, each of the rows of that spatial weights matrix sums to 1, which ensures that all the weights are between 0 and 1. Row-normalized spatial weights matrix provides simple interpretation of the spatial interaction effect as an average neighborhood effect. For such a spatial weights matrix, because its spectral radius (the largest eigenvalue in absolute value) is 1, dynamic features of the SDPD model is easier to understand. Assumption 2 provides \( i.i.d. \) regularity assumptions for the disturbances. If there is unknown heteroskedasticity, the MLE (QMLE) will not be consistent. Methods such as the GMM in Lin and Lee (2010) and the G2SLS in Kelejian and Prucha (2010) may be designed for that situation. The UB condition in Assumption 3 is originated by Kelejian and Prucha (1998, 2001) and also used in Lee (2004, 2007), which limits the spatial correlation to a manageable degree. Invertibility of spatial transformation matrices in Assumption 3 guarantees that the reduced form of the spatial process is valid and the true parameter lies in the interior of the parameter space, which rules out spatial (near) unit roots problem in a cross section setting. As usual, compactness is a condition for theoretical analysis on nonlinear functions. When \( W_n \) is row-normalized, a compact subset of \((-1, 1)\) has often been taken as the parameter space for \( \lambda \) in theory. When exogenous variables \( X_{nt} \) are included in the model, it is convenient to assume that they are uniformly bounded as in Assumption 4. If elements of \( X_{nt} \) are allowed to be stochastic and unbounded, appropriate moment conditions can be imposed instead. Assumption 5 specifies that we have a large number of spatial units, while the time period \( T \) could be either large or small. For some direct estimation approaches, we need both \( n \) and \( T \) large, as in Assumption 5’. When we have a dynamic feature in the panel data model, we need a large \( T \) condition as in Assumption 5”, unless we specify a separate process for the initial value observation.
Alvarez, J. and M. Arellano (2003), ‘The time series and cross-subsection asymptotics of dynamic panel data estimators’. *Econometrica* **71**, 1121–1159.

Amemiya, T. (1971), ‘The estimation of the variances in a variance-components model’. *International Economic Review* **12**, 1–13.

Amemiya, T. (1985), *Advanced Econometrics*. Cambridge, MA: Harvard University Press.

Anderson, T. W. (1959), ‘On asymptotic distributions of estimates of parameters of stochastic difference equations’. *Annals of Mathematical Statistics* **30**, 676–687.

Anderson, T. W. and C. Hsiao (1981), ‘Estimation of dynamic models with error components’. *Journal of the American Statistical Association* **76**, 598–606.

Anderson, T. W. and C. Hsiao (1982), ‘Formulation and estimation of dynamic models using panel data’. *Journal of Econometrics* **18**, 47–82.

Anselin, L. (1988), *Spatial Econometrics: Methods and Models*. The Netherlands: Kluwer Academic.

Anselin, L. (1992), ‘Space and applied econometrics’. In: Anselin (ed.): Special issue, *Regional Science and Urban Economics*, vol. **22**.
Anselin, L. and A. K. Bera (1998), ‘Spatial dependence in linear regression models with an introduction to spatial econometrics’. In: A. Ullah and D. E. A. Giles (eds.): Handbook of Applied Economics Statistics. New York: Marcel Dekker.

Anselin, L. and R. Florax (1995), New Directions in Spatial Econometrics. Berlin: Springer-Verlag.

Anselin, L., J. Le Gallo, and H. Jayet (2008), ‘Spatial panel econometrics’. In: The Econometrics of Panel Data: Fundamentals and Recent Developments in Theory and Practice. Berlin, Heidelberg: Springer.

Anselin, L. and S. Rey (1997), ‘Spatial econometrics’. In: L. Anselin and S. Rey (eds.): Special Issue International Regional Science Review, vol. 20.

Arellano, M. (1993), ‘On the testing of correlated effects with panel data’. Journal of Econometrics 59, 87–97.

Arellano, M. and O. Bond (1991), ‘Some tests of specification for panel data: Monte carlo evidence and an application to employment equations’. Review of Economic Studies 58, 277–297.

Arellano, M. and O. Bover (1995), ‘Another look at the instrumental-variable estimation of error-components models’. Journal of Econometrics 68, 29–51.

Arellano, M. and J. Hahn (2005), ‘Understanding bias in nonlinear panel models: Some recent developments’. In: R. Blundell, W. K. Newey, and T. Persson (eds.): Advances in Econometric and Economics: Theory and Applications, Ninth World Congress, vol. II. Econometric Society Monographs.

Baicker, K. (2005), ‘The spillover effects of state spending’. Journal of Public Economics 89, 529–544.

Baltagi, B. (1995), Econometric Analysis of Panel Data. New York: John Wiley and Sons.

Baltagi, B., G. Bresson, and A. Pirotte (2009), ‘Forecasting with spatial panel data’. Working Paper, Syracuse University.

Baltagi, B., P. Egger, and M. Pfaffermayr (2007), ‘A generalized spatial panel data model with random effects’. Working Paper, Syracuse University.

Baltagi, B. and D. Li (2006), ‘Prediction in the panel data model with spatial correlation: The case of liquor’. Spatial Economic Analysis 1, 175–185.
References

Baltagi, B., S. H. Song, and W. Koh (2003), ‘Testing panel data regression models with spatial error correlation’. *Journal of Econometrics* **117**, 123–150.

Bekker, P. A. (1994), ‘Alternative approximations to the distributions of instrumental variable estimators’. *Econometrica* **62**, 657–681.

Bhargava, A., L. Franzini, and W. Narendranathan (1982), ‘Serial correlation and fixed effects model’. *Review of Economic Studies* **49**, 533–549.

Blundell, R. and S. Bond (1998), ‘Initial conditions and moment restrictions in dynamic panel data models’. *Journal of Econometrics* **87**, 115–143.

Brueckner, J. K. (1998), ‘Testing for strategic interaction among local governments: the case of growth controls’. *Journal of Urban Economics* **44**, 438–467.

Brueckner, J. K. and L. A. Saavedra (2001), ‘Do local governments engage in strategic property tax competition?’. *National Tax Journal* **54**, 203–229.

Bun, M. and M. Carree (2005), ‘Bias-corrected estimation in dynamic panel data models’. *Journal of Business & Economic Statistics* **3**, 200–211.

Bun, M. and J. F. Kiviet (2006), ‘The effects of dynamic feedbacks and LS and MM estimator accuracy in panel data models’. *Journal of Urban Economics* **132**, 409–444.

Case, A., J. R. Hines, and H. S. Rosen (1993), ‘Budget spillovers and fiscal policy interdependence: Evidence from the states’. *Journal of Public Economics* **52**, 285–307.

Chamberlain, G. (1982), ‘Multivariate regression models for panel data’. *Journal of Econometrics* **18**, 5–46.

Chao, J. C. and N. R. Swanson (2005), ‘Consistent estimation with a large number of weak instruments’. *Econometrica* **73**, 1673–1692.

Cliff, A. D. and J. K. Ord (1973), *Spatial Autocorrelation*. London: Pion Ltd.

Cox, D. R. (1975), ‘Partial likelihood’. *Biometrika* **62**, 269–276.

Cox, D. R. and N. Reid (1987), ‘Parameter orthogonality and approximate conditional inference’. *Journal of the Royal Statistical Society. Series B (Methodological)* **49**, 1–39.

Cressie, N. (1993), *Statistics for Spatial Data*. New York: Wiley.
Dhrymes, P. J. (1978), *Introductory Econometrics*. New York: Springer Verlag.

Donald, C. G. and W. K. Newey (2001), ‘Choosing the number of instruments’. *Econometrica* 69, 1161–1191.

Druska, V. and W. C. Horrace (2004), ‘Generalized moments estimation for spatial panel data: Indonesian rice farming’. *American Journal of Agricultural Economics* 86, 185–198.

Egger, P., M. Pfaffermayr, and H. Winner (2005), ‘An unbalanced spatial panel data approach to US state tax competition’. *Economics Letters* 88, 329–335.

Elhorst, J. P. (2005), ‘Unconditional maximum likelihood estimation of linear and log-linear dynamic models for spatial panels’. *Geographical Analysis* 37, 85–106.

Elhorst, J. P. (2010), ‘Dynamic panels with endogenous interaction effects when \(T\) is small’. *Regional Science and Urban Economics* 40, 272–282.

Ertur, C. and W. Koch (2007), ‘Growth, technological interdependence and spatial externalities: Theory and evidence’. *Journal of Applied Econometrics* 22, 1033–1062.

Fingleton, B. (2008), ‘A generalized method of moments estimators for a spatial panel model with an endogenous spatial lag and spatial moving average errors’. *Spatial Economic Analysis* 3, 27–44.

Foote, C. L. (2007), ‘Space and time in macroeconomic panel data: Young workers and state-level unemployment revisited’. Working Paper No. 07–10, Federal Reserve Bank of Boston.

Franzese, R. (2007), ‘Spatial econometric models of cross-subsectional interdependence in political science panel and time-series-cross-subsection data’. *Political Analysis* 15, 140–164.

Frazier, C. and K. M. Kockelman (2005), ‘Spatial econometric models for panel data: Incorporating spatial and temporal data’. *Transportation Research Record: Journal of the Transportation Research Board* 1902, 80–90.

Greene, W. (2004), ‘The behavior of the fixed effects estimator in nonlinear models’. *The Econometrics Journal* 7, 98–119.

Griffith, D. (2003), *Spatial Autocorrelation and Spatial Filtering*. Berlin: Springer Verlag.
Hahn, J. and G. Kuersteiner (2002), ‘Asymptotically unbiased inference for a dynamic panel model with fixed effects when both \( n \) and \( T \) are Large’. *Econometrica* 70, 1639–1657.

Hahn, J. and H. R. Moon (2006), ‘Reducing bias of MLE in a dynamic panel model’. *Econometric Theory* 22, 499–512.

Han, C. and P. C. B. Phillips (2006), ‘GMM with many moment conditions’. *Econometrica* 74, 147–192.

Hausman, J. A. (1978), ‘Specification tests in econometrics’. *Econometrica* 46, 1251–1271.

Horn, R. and C. Johnson (1985), *Matrix Algebra*. Cambridge University Press.

Hsiao, C. (1986), *Analysis of Panel Data*. Cambridge University Press.

Hsiao, C., M. H. Pesaran, and A. K. Tahmiscioglu (2002), ‘Maximum likelihood estimation of fixed effects dynamic panel data models covering short time periods’. *Journal of Econometrics* 109, 107–150.

Im, K. S., M. H. Pesaran, and S. Shin (2003), ‘Testing for unit roots in heterogeneous panels’. *Journal of Econometrics* 115, 53–74.

Islam, N. (1995), ‘Growth empirics: A panel data approach’. *The Quarterly Journal of Economics* 110, 1127–1170.

Johansen, S. (1991), ‘Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models’. *Econometrica* 59, 1551–1580.

Kapoor, N. M., H. H. Kelejian, and I. R. Prucha (2007), ‘Panel data models with spatially correlated error components’. *Journal of Econometrics* 140, 97–130.

Kelejian, H. H. and I. R. Prucha (1998), ‘A generalized spatial two-stage least squares procedure for estimating a spatial autoregressive model with autoregressive disturbance’. *Journal of Real Estate Finance and Economics* 17, 99–121.

Kelejian, H. H. and I. R. Prucha (2001), ‘On the asymptotic distribution of the Moran I test statistic with applications’. *Journal of Econometrics* 104, 219–257.

Kelejian, H. H. and I. R. Prucha (2010), ‘Specification and estimation of spatial autoregressive models with autoregressive and heteroskedastic disturbances’. *Journal of Econometrics* 157, 53–67.
Kelejian, H. H. and D. Robinson (1993), ‘A suggested method of estimation for spatial interdependent models with autocorrelated errors, and an application to a county expenditure model’. *Papers in Regional Science* **72**, 297–312.

Keller, W. and C. H. Shin (2007), ‘The origin of spatial interaction’. *Journal of Econometrics* **140**, 304–332.

Kiefer, N. M. (1980), ‘Estimation of fixed effects models for time series of cross subsections with arbitrary intertemporal covariance’. *Journal of Econometrics* **14**, 195–202.

Kiviet, J. (1995), ‘On bias, inconsistency, and efficiency of various estimators in dynamic panel data models’. *Journal of Econometrics* **68**, 81–126.

Korniotis, G. M. (2010), ‘Estimating panel models with internal and external habit formation’. *Journal of Business and Economic Statistics* **28**, 145–158.

Lancaster, T. (2000), ‘The incidental parameter problem since 1948’. *Journal of Econometrics* **95**, 391–413.

Lee, L.-F. (2004), ‘Asymptotic distributions of quasi-maximum likelihood estimators for spatial econometric models’. *Econometrica* **72**, 1899–1925.

Lee, L.-F. (2007), ‘GMM and 2SLS estimation of mixed regressive, spatial autoregressive models’. *Journal of Econometrics* **137**, 489–514.

Lee, L.-F. and X. Liu (2010), ‘Efficient GMM estimation of high order spatial autoregressive models with autoregressive disturbances’. *Econometric Theory* **26**, 187–230.

Lee, L.-F. and J. Yu (2010a), ‘Efficient GMM estimation of spatial dynamic panel data models’. Manuscript, Ohio State University.

Lee, L.-F. and J. Yu (2010b), ‘Estimation of spatial autoregressive panel data models with fixed effects’. *Journal of Econometrics* **154**, 165–185.

Lee, L.-F. and J. Yu (2010c), ‘QML estimation of spatial dynamic panel data models with time varying spatial weights matrices’. Manuscript, Ohio State University.

Lee, L.-F. and J. Yu (2010d), ‘Some recent developments in spatial panel data models’. *Regional Science and Urban Economics* **40**, 255–271.
Lee, L.-F. and J. Yu (2010e), ‘A spatial dynamic panel data model with both time and individual fixed effects’. *Econometric Theory* **26**, 564–597.

Lee, L.-F. and J. Yu (2010f), ‘A unified transformation approach for the estimation of spatial dynamic panel data models: Stability, spatial cointegration and explosive roots’. In: A. Ullah and D. E. A. Giles (eds.): *Handbook of Empirical Economics and Finance*. Chapman and Hall/CRC.

Levin, A., C.-F. Lin, and C.-S. J. Chu (2002), ‘Unit root tests in panel data: Asymptotic and finite sample properties’. *Journal of Econometrics* **108**, 1–24.

Lin, X. and L.-F. Lee (2010), ‘GMM estimation of spatial autoregressive models with unknown heteroskedasticity’. *Journal of Econometrics* **157**, 34–52.

Liu, X., L. Lee, and C. Bollinger (2010), ‘An efficient GMM estimator of spatial autoregressive models’. *Journal of Econometrics* **159**, 303–319.

Maddala, G. S. (1971), ‘The use of variance components models in pooling cross subsection and time series data’. *Econometrica* **39**, 341–358.

Maddala, G. S. and S. Wu (1999), ‘A comparative study of unit root tests with panel data and a new simple test’. *Oxford Bulletin of Economics and Statistics* **61**, 631–652.

Magnus, J. R. (1982), ‘Multivariate error components analysis of linear and nonlinear regression models by maximum likelihood’. *Journal of Econometrics* **19**, 239–285.

Moon, H. R. and B. Perron (2004), ‘Testing for a unit root in panels with dynamic factors’. *Journal of Econometrics* **122**, 81–126.

Mundlak, Y. (1978), ‘On pooling time series and cross subsection data’. *Econometrica* **46**, 69–85.

Mutl, J. (2006), ‘Dynamic panel data models with spatially correlated disturbances’. PhD thesis, University of Maryland, College Park.

Mutl, J. and M. Pfaffermayr (2011), ‘The Hausman test in a Cliff and Ord panel model’. *Econometrics Journal* **14**, 48–76.

Nerlove, M. (1971), ‘A note on error components models’. *Econometrica* **39**, 383–396.
Neyman, J. and E. L. Scott (1948), ‘Consistent estimates based on partially consistent observations’. *Econometrica* **16**, 1–32.

Nickell, S. (1981), ‘Biases in dynamic models with fixed effects’. *Econometrica* **49**, 1417–1426.

Nielsen, B. (2001), ‘The asymptotic distribution of unit root tests of unstable autoregressive processes’. *Econometrica* **69**, 211–219.

Nielsen, B. (2005), ‘Strong consistency results for least squares estimators in general vector autoregressions with deterministic terms’. *Econometric Theory* **21**, 534–561.

Okui, R. (2009), ‘The optimal choice of moments in dynamic panel data models’. *Journal of Econometrics* **151**, 1–16.

Ord, J. K. (1975), ‘Estimation methods for models of spatial interaction’. *Journal of the American Statistical Association* **70**, 120–297.

Parent, O. and J. P. LeSage (2008), ‘A space-time filter for panel data models containing random effects’. Manuscript, University of Cincinnati.

Pesaran, M. H. (2007), ‘A simple panel unit root test in the presence of cross subsection dependence’. *Journal of Applied Econometrics* **22**, 265–312.

Pesaran, M. H. and E. Tosetti (2010), ‘Large panels with common factors and spatial correlations’. Working paper, Cambridge University.

Phillips, P. C. B. (1995), ‘Fully modified least squares and vector autoregression’. *Econometrica* **63**, 1023–1078.

Phillips, P. C. B. and T. Magdalinos (2007), ‘Limit theory for moderate deviations from a unit root’. *Journal of Econometrics* **136**, 115–130.

Phillips, P. C. B. and D. Sul (2003), ‘Dynamic panel estimation and homogeneity testing under cross subsection dependence’. *Econometrics Journal* **6**, 217–259.

Revelli, F. (2001), ‘Spatial patterns in local taxation: Tax mimicking or error mimicking?’’. *Applied Economics* **33**, 1101–1107.

Rey, S. J. and B. D. Montouri (1999), ‘US regional income convergence: A Spatial econometric perspective’. *The Journal of the Regional Studies Association* **33**, 143–156.

Rincke, J. (2010), ‘A commuting-based refinement of the contiguity matrix for spatial models, and an application to local police expenditures’. *Regional Science and Urban Economics* **40**, 324–330.
Ruud, P. A. (2000), *An Introduction to Econometric Theory*. New York: Oxford University Press.

Shiue, C. H. (2002), ‘Transport costs and the geography of arbitrage in eighteen-century China’. *American Economic Review* 92, 1406–1419.

Sims, C. A., J. H. Stock, and M. W. Watson (1990), ‘Inference in linear time series models with some unit roots’. *Econometrica* 58, 113–144.

Solow, R. M. (1956), ‘A contribution to the theory of economic growth’. *Quarterly Journal of Economics* 70, 65–94.

Su, L. and Z. Yang (2007), ‘QML estimation of dynamic panel data models with spatial errors’. Working Paper, Beijing University and Singapore Management University.

Tao, J. (2005), ‘Spatial econometrics: Models, methods and applications’. PhD thesis, Ohio State University.

Theil, H. (1971), *Principles of Econometrics*. New York: John Wiley and Sons.

Tiefelsdorf, M. and D. A. Griffith (2007), ‘Semiparametric filtering of spatial autocorrelation: The eigenvector approach’. *Environment and Planning A* 39, 1193–1221.

Wallace, T. D. and A. Hussain (1969), ‘The use of error components models in combining cross-subsection and time-series data’. *Econometrica* 37, 55–72.

White, J. S. (1958), ‘The limiting distribution of the serial correlation coefficient in the explosive case I’. *Annals of Mathematical Statistics* 29, 1188–1197.

White, J. S. (1959), ‘The limiting distribution of the serial correlation coefficient in the explosive case II’. *Annals of Mathematical Statistics* 30, 831–834.

Wong, W. H. (1986), ‘Theory of partial likelihood’. *The Annals of Statistics* 14, 88–123.

Yang, Z., C. Li, and Y. K. Tse (2006), ‘Functional form and spatial dependence in dynamic panels’. *Economic Letters* 91, 138–145.

Yu, J., R. de Jong, and L.-F. Lee (2007), ‘Quasi-maximum likelihood estimators for spatial dynamic panel data with fixed effects when both n and T are large: A nonstationary case’. Manuscript, Ohio State University.
References

Yu, J., R. de Jong, and L.-F. Lee (2008), ‘Quasi-maximum likelihood estimators for spatial dynamic panel data with fixed effects when both $n$ and $T$ are large’. *Journal of Econometrics* 146, 118–134.

Yu, J. and L.-F. Lee (2010), ‘Estimation of unit root spatial dynamic panel data models’. *Econometric Theory* 26, 1332–1362.