Quantitative credit risk monitoring using purchase order information

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Abstract

This paper proposes advanced credit risk assessment using purchase order information from borrower firms. It first introduces a structural credit risk model based on purchase orders and demonstrates the applicability of the model to practical credit risk monitoring with a case study. The estimated default probabilities reflect trends in purchase order volumes and customers’ default risk. The proposed model realizes more frequent credit risk monitoring than typical monitoring based on financial statements. Financial institutions can monitor the actual business conditions of borrower firms using the model.

Keywords Credit risk, default probability, purchase order, structural model

\textbf{Research Activity Group} Mathematical Finance

1. Introduction

Financial institutions provide capital for firms by lending, and they must assess and monitor their borrowers’ credit risk, which is the risk associated with financial losses caused by defaults. In financial practice, firm monitoring is typically occurs regularly through financial statement analysis. However, there are several problems with this traditional method. First, financial statements, which should describe actual business conditions, may not describe them accurately, particularly in the case of non-listed small or medium-size firms. Second, because financial statements represent static information that only describes the firm at one point in time, financial institutions cannot see possible changes in a firm’s condition using only financial statements. Thus, these traditional monitoring methods do not provide a real-time snapshot of the changes in business conditions. In addition, traditional monitoring methods rely heavily on costly human resources and time.

We propose a quantitative credit risk model using purchase order (PO) information and demonstrate real-time firm monitoring using the model. PO information includes attributions of customers, date of PO receipts and PO volumes. Monitoring with PO information enables financial institutions to capture precise business conditions on a real-time basis, which is not the case when using only financial statements. Moreover, a valuation system combined with automated systems for obtaining PO data and credit risk models using PO information can enhance the effectiveness of monitoring. This advanced firm monitoring can be considered a FinTech application.

In our model, we obtain the asset value of the borrower firm from PO volume; the default occurs when the asset value falls below the debt value. Because the structure of default occurrences is explicitly modelled with the level of asset values, our model is a type of structural credit risk model. Previous works on structural models directly model the stochastic process of corporate values (e.g. [1]) or obtain asset values by aggregating future earnings (e.g. [2, 3]). On the other hand, we model PO volume transition first, and then generate the earnings and asset values according to PO volume in our modeling framework. One of the advantages of our model is that we reflect the borrowing firm’s business conditions, such as trends in PO volumes and credit quality of customers in the credit risk assessment.

2. Modelling framework

This section provides a structural framework of credit risk modelling based on PO information.

We model uncertainty in the economy in a filtered complete probability space $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\}_{t \in T})$, where $\{\mathcal{F}_t\}_{t \in T}$ is a complete filtration with discrete time space $T = \{0, 1, 2, \ldots, \infty\}$, and $\mathbb{P}$ is the physical probability measure. The target firm for credit risk assessment is the borrower-side of a PO. We denote the set of corresponding customers by $I = \{1, 2, 3, \ldots, I\}$. We denote PO volumes ordered by customer $i \in I$ by $\{O^i_t\}_{t \in T}$, which are $\mathcal{F}_t$-adopted stochastic process. Then, the proceeds of sales at time $t$ are given by

$$S_t = \sum_{i=1}^{I} \left( O^i_{t-h} 1_{\{t < T_i\}} + (1 - LGD^i) O^i_{t-h} 1_{\{t \geq T_i\}} \right),$$

(1)

where $\mathcal{F}_t$-stopping time $T_i$ indicates the default time of customer $i$, $h \geq 0$ is the time-lag between PO arrivals and collection of the sales proceeds. The constant $LGD^i$ is the rate of loss given defaults.

We obtain production cost by PO volumes and cost
function
\[ C_t = f(\{O_{t-s}^i\}_{i \in I}). \] (2)

Here, function \( f : \mathbb{R}^I \to \mathbb{R} \) is a cost function and constant \( g(\leq h) \) is the time-lag between PO arrival and the corresponding cost defrayment.

Then, the cumulative ordinary profits and losses are
\[ P_t = \sum_{s=0}^{t} (S_s - C_s). \]

In addition to ordinary profit and losses, we consider accumulated impairment losses with
\[ I_t = \min(V_t - \tilde{V}_0, 0). \]

Here, \( \tilde{V}_t \) is the present value of operating earnings.

We obtain earnings before tax (EBT) and net earnings with ordinary profits and impairment losses. That is, the cumulative EBT and net earnings are given by
\[
\begin{align*}
EBT_t &= P_t + I_t, \quad (3) \\
E_t &= EBT_t 1_{\{EBT_t \leq 0\}} + (1 - G) EBT_t 1_{\{EBT_t > 0\}}, \quad (4)
\end{align*}
\]
where \( G \) is the corporate tax rate.

Finally, the corporate value is
\[ V_t = V_0 + E_t. \]

The debtor defaults when the net capital becomes negative. Then, the default time is
\[ \tau = \min\{t \in T \setminus \{0\}|H_t < 0\}. \]

The net capital is \( H_t \) and obtained using \( H_t = V_t - D_t \) with debt value \( D_t \).

3. Case study

This section provides a case study on credit risk monitoring using PO information.

3.1 Data

Our sample data is the historical PO records of Kojima Industries Co. Kojima manufactures interior and exterior automobile components. Its main customers (buyers) are auto manufacturers such as Toyota Motor Co., Denso Co., and so on. The samples are monthly POs data from January, 2011 to December, 2014. Fig. 1 shows the monthly PO volumes. We recognize the seasonality of PO volumes; for example, the relative decrease every August and December.

3.2 Empirical model

Kojima’s customers, such as Toyota Motor Company, belong to the automotive sector. In this example, when customers belong to the same industrial sector, the PO volumes of each customer tend to co-move with changes in business conditions. Thus, we suppose that the model can capture these co-movements in PO volumes. In addition, there are default correlations among customers; therefore, we consider them in our model.

Our sample data are monthly observations; thus, the unit of time \( T \) is one month. Monthly PO volume data often have seasonal effects, which we address by modelling the one-year difference in the log-PO volumes
\[ R_t^j = \log(O_t^j) - \log(O_{t-12}^j). \]

We employ the time-series model \( R_t^j \), described as follows:
\[ R_t^j = \alpha_i + \beta_i R_{t-1}^j + \sigma_i \left( \rho_i W_t + \sqrt{1 - \rho_i^2} \epsilon_{i,t} \right). \]

Here, \( W_t \sim N(0,1) \), \( \epsilon_{i,t} \sim N(0,1) \) and these random variables are independent of time, where \( N(m, v) \) is a normal distribution with mean \( m \) and variance \( v \).

This model captures the correlation of PO volumes by common factor \( W_t \) and specifies the strength of the correlation by the factor loading \( \{\rho_i\} \). Idiosyncratic risks are captured by \( \epsilon_{i,t} \).

We employ the following model estimation procedures. First, we estimate parameters \( \{\alpha_i, \beta_i, \sigma_i\} \) by the AR (Auto-Regressive) model
\[ R_t^j = \alpha_i + \beta_i R_{t-1}^j + \epsilon_{i,t}, \quad \epsilon_{i,t} \sim N(0, \sigma_i^2) \]
with the observed one-year difference in the log-PO volumes. Then, we estimate \( \rho_i \) with obtained residuals \( \epsilon_{i,t} \). Here, we calculate the sample residual correlation matrix \( \{\hat{\rho}_{ij}\} \) and obtain the estimated parameters by minimizing the sum of the square difference
\[ \sum_{i=1}^{I} \sum_{j=1}^{I} (\hat{\rho}_{ij} - \rho_{ij})^2. \]

We model customers’ POs ranked in the top nine PO volume \( (i = 1, 2, \ldots, 9) \). The sum of the top nine PO volumes accounts for approximately 96% of all PO volumes. In addition, we model the aggregated remainder \( (i = 10) \). Thus, the number of customers in the case study is \( I = 10 \). The default risk of most customers is relatively low because the range of their credit ratings is from AA+ to A-.

Table 1 describes the estimated parameters of the PO model. Because most of the estimated values of the auto-regression coefficients \( \{\beta_i\} \) are positive, we recognize the existence of PO trends in our sample data. The p-values of the Ljung-Box test show no significant autocorrelation in the residuals, and the model is not rejected. Here, we conduct the Kolmogorov-Smirnov (K-S) test, which tests goodness of fit of a realized series of residuals and the associated distribution \( N(0, 1) \). From the K-S test, we confirm that the customers’ PO volumes that are not rejected at the 1% significant level is 100% for the in-sample and 83.4% for the out-of-sample tests.

Referring to the estimated values of factor loading \( \rho_i \) in Table 1, the estimates of the correlation between any
two customers are over 0.4, based on the PO volume correlation between customer \(i\) and \(j\) with \(\rho_{ij}\). This implies that there are pairs of customers who are highly correlated in POs in our samples.

We obtain PO volumes \(\{O_{it}\}_{t \in T}\) from \(\{R_{it}\}\) as

\[
O_{it} = \{O_{it-12} \times \exp(R_{it})\}_{1 \leq t \leq T_i}.
\]

Here, \(T_i\) is the default time of customer \(i\).

Sales proceeds are calculated with equation (1). We set the value of \(LGD^g\) conservatively as \(LGD^g = 1\). The time lag between the order and collection is set to two months (\(h = 2\)).

We model customer defaults with a Merton-type one-factor Gaussian-copula model. The probability of default (PD) until time \(t+1\), under the condition that customer \(i\) survives at least until time \(t\), is the probability that the credit quality \(X^t\) goes below the default barrier \(Q^t\). That is, the probability of customer \(i\) defaulting is \(P(X^t < Q^t)\). The credit quality \(X^t\) is given by

\[
X^t = \tilde{p}_i \tilde{W} + \sqrt{1 - \tilde{p}_i^2} \tilde{\epsilon}_i,
\]

\(\tilde{W} \sim N(0, 1), \quad \tilde{\epsilon}_i \sim N(0, 1)\)

and these random variables are independent of time. Then, the PD of customer \(i\) is

\[
P(X^t < Q^t) = \Phi(Q^t), \tag{5}
\]

where \(\Phi(\cdot)\) is the cumulative normal distribution. The default correlation among customers is captured by common factor \(\tilde{W}\), and the strength of the default correlation is specified by \(\tilde{p}\). We estimate the default barriers \(Q^t\) according to (5) with the historical probabilities of defaults of the associated credit ratings of customer \(i\). For unrated customers, we assume a credit rating of BBB.

We estimate default correlations among customers with their stock price data. For non-listed customers, we employ TOPIX sector indices of the corresponding sector and consider these as the customer’s stock data. We estimate parameters by minimizing the sum of the square difference between the historical correlation matrix of stock prices and the correlation matrix obtained through factor loadings. Table 2 summarizes the estimates of the factor loadings.

For simplicity, we assume that the function of ordinary cash flow, excluding sales proceeds, is given by the linear function

\[
\tilde{I}_t = \min(\tilde{V}_t - \tilde{V}_0, 0),
\]

\[
\tilde{V}_t = \sum_{s=t}^{\infty} \frac{P^s[\hat{P}_s|F_t]}{(1 + r)^{s-t}}.
\]

Here, \(\hat{P}_t\) is operating earnings. The constant \(r\) is the firm’s weighted average cost of capital (WACC). Operating earnings are given by

\[
\hat{P}_t = S_t - \hat{C}_t,
\]

where operating costs \(\hat{C}_t = \hat{f}(\{O_{it-12}\})\) with the following linear operating cost function:

\[
\hat{f}(\{O_{it-12}\}) = a \sum_{i=1}^{T_i} (O_{it-12}^t 1_{t \leq T_i}) + b.
\]

The process to estimate the operating cost function is similar to that of ordinary cash flow functions. To estimate the parameters of the operating costs function, we use historical annual PO volumes and the operating costs in the P/L. We use a linear regression of realized operating costs by employing annual sales as an explanatory variable. The coefficients from the linear regression are annual values, which we transform into monthly values. We obtained \(a = 0.905\) and \(b = 7.83 \times 10^8\) as coefficients of the ordinary cash flow function.

When calculating \(\hat{V}_t\), we set the end of the forecast period for operating earnings as the \(M\)-th time point from an evaluation time. Then, we obtain the net present value of operating earnings after the end of the forecast period with the terminal value. Thus, the net present value of operating earnings is the sum of the present value of operating earnings during the forecast period.
and the terminal value:

\[ \tilde{V}_t = \sum_{s=t}^{M-1} \mathbb{E}^{\mathcal{F}_t}[\tilde{P}_s | \mathcal{F}_t] + \mathbb{E}^{\mathcal{F}_t}[\tilde{P}_M | \mathcal{F}_t] \cdot \frac{(1 + r)^{M-t}}{r(1 + r)^{M-t}}. \]

We calculate the corporate value of profits by setting the end of the forecast period to five years \((M = 60)\). Then, we obtain the adjusted impairment losses using \(I_t = \tilde{V}_t \times \frac{V_0}{V_0}\), where \(V_0\) is the book value of operating profit and losses. We obtain the WACC for Kojima using the capital asset pricing model (CAPM). The range WACC is 3.52% \(\sim\) 3.84%. Finally, \(EB T_t = P_t + I_t\) gives the cumulative EBT.

We set the corporate tax at \(G = 0.4\) in (4) to calculate net earnings \(E_t\).

Using the settings above, we simulate the future PO volumes, calculate corporate values, and obtain Kojima’s PD. We run the simulation using a monthly timeframe with 100,000 trials, count the number of default trials in which the net capital becomes negative, and obtain the PD as the ratio of the number of default trials to the number of all trials.

3.3 Results

We calculate one-year forward PD for Kojima for every month in 2014. In addition to calculating PD under the realized PO scenario of PO volumes and customers’ credit quality, we calculated PD under stressed PO scenarios in which Kojima’s customers’ credit ratings declines. Fig. 2 shows the estimated PDs under the realized PO scenario. The estimated PDs increase, reflecting the decrease in PO volumes. Fig. 3 shows the estimated PD under the stressed PO scenario in which customers’ credit ratings decline in May 2014. Fig. 3 shows that Kojima’s PD increases, reflecting the lower credit ratings of its customers.

These results show that credit risk modelling based on PO information enables financial institutions to monitor the credit risk affected by changes in borrower firms’ business conditions, such as PO volumes and their customers’ credit quality on a real-time basis. This real-time monitoring represents advanced default prediction in lending operations and reduces the costs associated with traditional monitoring methods, which require substantial human resources and time.

4. Concluding remarks

This study proposed a quantitative credit risk model based on PO information and illustrated its use with a case study using real-time monitoring of a firm’s business conditions. The results of the case study reveal the effectiveness of using PO information to monitor the credit risks of borrower firms. The method of obtaining PDs proposed herein can reduce the cost of monitoring borrower firms. If borrower firms supply PO information to financial institutions regularly, financial institutions can offer borrower firms appropriate, timely support if necessary.

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