Measurement of $CP$ Violation Parameters with a Dalitz Plot Analysis of $B^\pm \rightarrow D_{\pi^+\pi^-\pi^\mp}$

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We report the results of a CP violation analysis of the decay $B^\pm \to D_{s+\pi^-\pi^0}K^\pm$, where $D_{s+\pi^-\pi^0}$ indicates a neutral $D$ meson detected in the final state $\pi^+\pi^-\pi^0$, excluding $K^+\pi^-$. The analysis makes use of 324 million $e^+e^-\to B\bar{B}$ events recorded by the BABAR experiment at the PEP-II $e^+e^-$ storage ring. By analyzing the $\pi^+\pi^-\pi^0$ Dalitz plot distribution and the $B^\pm \to D_{s+\pi^-\pi^0}K^\pm$ branching fraction and decay rate asymmetry, we calculate parameters related to the phase $\gamma$ of the CKM unitarity triangle. We also measure the magnitudes and phases of the components of the $D^0 \to \pi^+\pi^-\pi^0$ decay amplitude.

PACS numbers: 13.25.Hw, 12.15.Hb, 11.30.Er

An important component of the program to study CP violation is the measurement of the angle $\gamma = \arg (-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$ of the unitarity triangle related to the Cabibbo-Kobayashi-Maskawa quark mixing matrix. The decays $B \to D^{(*)0}K^{(*)}$ can be used to measure $\gamma$ with essentially no hadronic uncertainties, exploiting interference between $b \to u\bar{s}s$ and $b \to c\bar{s}s$ decay amplitudes. In one of the measurement methods, $\gamma$ is extracted by analyzing the $D$-decay Dalitz plot distribution in $B^\pm \to D\bar{K}^\pm$ with multi-body $D$ decays. This method has only been used with the Cabibbo-favored decay $D \to K^0_s\pi^+\pi^-$ [2, 3], and Cabibbo-suppressed decays are expected to be similarly sensitive to $\gamma$ [2]. We present here the first CP-violation study of $B^\pm \to D\bar{K}^\pm$ with a multibody, Cabibbo-suppressed decay, $D \to \pi^+\pi^-\pi^0$.

The data used in this analysis were collected with the BABAR detector at the PEP-II $e^+e^-$ storage ring, running on the $\Upsilon(4S)$ resonance. Samples of simulated Monte Carlo (MC) events were analyzed with the same reconstruction and analysis procedures. These samples include an $e^+e^-\to B\bar{B}$ sample about five times larger than the data; a continuum $e^+e^-\to q\bar{q}$ sample, where $q$ is a $u$, $d$, $s$ or $c$ quark, with luminosity equivalent to the data; and a signal sample about 300 times larger than the data, with both phase space $D$ decays and decays generated according to the amplitudes measured by CLEO [3]. The BABAR detector and the methods used for particle reconstruction and identification are described in Ref. [3].

The reader is referred to Ref. [10] for details of the event selection criteria. Briefly, we use event-shape variables to suppress the continuum background, and identify kaon and pion candidates using specific ionization and Cherenkov radiation. The invariant mass of $D$ candidates must satisfy $1830 < M_{D^0} < 1895$ MeV/$c^2$. We require $5272 < m_{ES} < 5300$ MeV/$c^2$, where $m_{ES} = \sqrt{E_{CM}^2/4 - \mathbf{p}_B^2}$, $E_{CM}$ is the total $e^+e^-$ center-of-mass (CM) energy, and $\mathbf{p}_B$ is the $B$ candidate CM momentum. Events must satisfy $-70 < \Delta E < 60$ MeV, where $\Delta E = E_B - E_{CM}/2$ and $E_B$ is the $B$ candidate CM energy. We exclude the decay mode $D \to K^0_s\pi^0$, which is a previously studied $CP$ eigenstate not related to the method of Ref. [2], by rejecting candidates with $489 < M(\pi^+\pi^-) < 508$ MeV/$c^2$ or for which the distance between the $\pi^+\pi^-$ vertex and the $B^-$ candidate decay vertex is more than 1.5 cm. We reject $B^\pm \to D_{s+\pi^-\pi^0}K^\pm$ candidates in which the $K^\pm\pi^\mp$ invariant mass satisfies $1840 < M(K^\pm\pi^\mp) < 1890$ MeV/$c^2$, to suppress $B^\pm \to D_{K^-\pi^+\rho^0}$ decays. We require $d > 0.25$, where $d$ [10] is a neural net variable that separates signal candidates (which peak toward $d = 1$) from those with a misreconstructed $D$ (peaking toward $d = 0$). In events with multiple candidates, we keep the candidate whose $m_{ES}$ value is closest to the nominal $B^\pm$ mass [11].

For each $B^\pm \to D_{s+\pi^-\pi^0}K^\pm$ candidate, we compute the neural net variable $q$ [10]. The distribution of $B\bar{B}$ events peaks toward $q = 1$, while that of continuum peaks at $q = 0$. For $\nu \in \{q, d\}$, we define the variables $\nu' \equiv \tan^{-1}[(\nu - \frac{1}{2}(\nu_{\max} + \nu_{\min}))/\frac{1}{2}(\nu_{\max} - \nu_{\min})]$, where $\nu_{\max} = d_{\max} = 1$, $q_{\min} = 0.1$, and $d_{\min} = 0.25$ are the allowed ranges for $q$ and $d$. The $\nu'$ variables can be conveniently fit with Gaussians, as described later.

As in Ref. [10], we identify in the MC samples ten event types, one signal and nine different backgrounds. We list them here with the labels used to refer to them throughout the paper. $DK_{sig}$: $B^\pm \to D_{\pi^+\pi^-\pi^0}K^\pm$ events that are correctly reconstructed; these are the only events considered to be signal. $DK_{bgs}$: $B^\pm \to D_{\pi^+\pi^-\pi^0}K^\pm$ events that are misreconstructed; namely, some of the particles used to form the final state do not originate from the $B^\pm \to D_{\pi^+\pi^-\pi^0}K^\pm$ decay. $\Delta \pi_D$ ($\Delta \pi_B$): $B^- \to D^0_{\pi^-\pi^-\pi^0}$, $D^0 \to \pi^+\pi^-\pi^0$ decays, where the decay $D^0 \to \pi^+\pi^-\pi^0$ is correctly reconstructed (misreconstructed). $DK_X$: $B^\pm \to D_{K^\pm\pi^\mp}^{(*)0}$ events not containing the decay $D \to \pi^+\pi^-\pi^0$. $\Delta \pi_X$: $B \to D_{\rho^-}^{(*)\pi^-}$, $B \to D_{\rho^0}^{(*)\pi^-}$, $BBC_D$ ($BBC_B$): all other $B\bar{B}$ events with a correctly reconstructed (misreconstructed) $D$ candidate. $qq\pi$ ($qq\rho$): continuum $e^+e^-\to q\bar{q}\pi$ events with a correctly reconstructed (misreconstructed) $D$ candidate.

The measurement of the $CP$ parameters proceeds in three steps, each involving an unbinned maximum likelihood fit. In step 1, we measure the complex Dalitz plot amplitude $\alpha(s_+, s_-)$ for the decay $D^0 \to \pi^+\pi^-\pi^0$, where $s_\pm = m^2(\pi^\pm\pi^0)$ are the squared invariant masses of the $\pi^\pm\pi^0$ pairs. In step 2, we extract the numbers of $B^+$ and $B^-$ signal events and background yields. We obtain the $CP$ parameters in step 3.

We parameterize $\alpha(s_+, s_-)$ using the isobar model, $\alpha(s_+, s_-) = |\alpha_{NR}\alpha_{NR} + \sum_r \alpha_r \epsilon^{\alpha_{NR}r} A_r(s_+, s_-)|/N_s$, where the first term represents a nonresonant contri-
bution, the sum is over all intermediate two-body resonances \( r \), and \( N_\alpha \) is such that \( \int ds_+ds_-|\alpha (s_+, s_-)|^2 = 1 \).

The amplitude for the decay chain \( D^+ \to rC, r \to AB \) is 
\[
A_r(s_+, s_-) = F_r F_s (m_r^2 - M_{AB}^2 - i m_r \Gamma_r (M_{AB}^2))^{-1},
\]
where \( m_r \) is the peak mass of the resonance and \( M_{AB}^2 = m_A^2 - m_B^2 \) is the squared invariant mass of the \( AB \) pair, \( F_r \) is a spin-dependent form factor, and \( \Gamma_r (M_{AB}^2) \) is the mass-dependent width for the resonance \( r \).

In step 1, we determine the parameters \( \sigma_{NR}, \alpha, \phi_{NR}, \) and \( \phi_{C} \) by fitting a large sample of \( D^0 \) and \( \bar{D}^0 \) mesons, flavor-tagged through their production in the decay \( D^{\pm} \to C^0 \pi^\pm \).

Of the \( D^0 \) candidates in the signal region \( 1848 < M_D < 1880 \) MeV/c\(^2\), we obtain from the fit \( N_S = 44780 \pm 250 \) signal and \( N_B = 830 \pm 70 \) background events. To obtain the parameters \( \alpha (s_+, s_-) \), we fit these candidates with the probability distribution function (PDF) \( N_S |\alpha (s_+, s_-)|^2 \varepsilon (s_+, s_-) + N_B |f_B (s_+, s_-)|^2 \), where the background PDF \( f_B (s_+, s_-) \) is a binning distribution obtained from events in the sideband \( 1930 < M_D < 1990 \) MeV/c\(^2\), and \( \varepsilon (s_+, s_-) \) is an efficiency function, parameterized as a two-dimensional third-order polynomial determined from MC. To within the MC-signal statistical uncertainty, \( \varepsilon (s_+, s_-) = \varepsilon (s_+, s_-) \).

The fit for step \( i = 2, 3 \) uses the PDF
\[
P_{\xi, i}^C = \sum_{i} \frac{N_i}{2 \eta} (1 - CA_i) P_{\xi, i}^{C((i))} (\xi_i) \int P_{\xi, i}^{C((i))} (\xi'_i) d\xi_i d\xi'_i,
\]
where \( \xi_i \) is the set of \( n_i \) event variables \( \xi_i = \{ \Delta E, q', d' \} \), \( \xi_2 = \{ \Delta E', q', s_+, s_- \} \), and \( t \) corresponds to one of the ten event types listed above. \( N_i = \sum_{t} N_{t_+}^i + N_{t_-}^i \) is the number of events of type \( t \), \( A_t = (N_{t_-}^i - N_{t_+}^i)/N_{t_0} \) is their charge asymmetry, \( C = \pm 1 \) is the electric charge of the \( B \) candidate, and \( \eta = \sum N_i \). Using MC, we verify that the \( \xi_i \) and \( \xi_j \) \( (i \neq j) \) distributions are uncorrelated for each event type. Therefore, the PDFs \( P_{\xi, i}^{C((i))} \) are the products
\[
P_{\xi, i}^{2((i))} (\Delta E, q', d') = E_i (\Delta E) Q_i (q', C_i (d'))
\]
\[
P_{\xi, i}^{3((i))} (\Delta E, q', s_+, s_-) = E_i (\Delta E) Q_i (q', C_i (s_+, s_-)).
\]

The signal Dalitz plot PDF \( D_{\xi_i}^{\xi} (s_+, s_-) \) are obtained from the data as described below. Those of all other functions in Eq. (2) are obtained from the MC samples. The functions \( E_i (\Delta E) \) are parameterized as the sum of a Gaussian and a second-order polynomial. The PDFs \( Q_i (q') \) and \( C_i (d') \) are the sum of a Gaussian and an asymmetric Gaussian. The PDF parameters are different for each event type. Assuming no CP violation in the background, we take \( D_{\xi, t}^C (s_+, s_-) = D_{\xi, t}^C (s_+, s_-) + A_t = 0 \) for \( t \neq D_{K_{s}} \).

For other event types, \( D_{\xi, t}^C (s_+, s_-) = \varepsilon (s_+, s_-) D_{\xi, t}^C (s_+, s_-) \), where the efficiency function \( \varepsilon (s_+, s_-) \) has different parameters for well-reconstructed and misreconstructed \( D \) candidates.

The signal Dalitz PDF accounts for interference between the \( b \to u \bar{c} s \) and \( b \to c \bar{t} \bar{s} \) amplitudes, using an \( I = 0 \)-dominated final state.

TABLE I: Result of the fit to the \( D^{\pm} \to D^0 \pi^\pm \) sample, showing the amplitudes ratios \( R_{C} \equiv \alpha_{C}/\alpha_{C} \), phase differences \( \Delta \phi_{r} \equiv \phi_{r} - \phi_{r} \), and fit fractions \( f_{C} \equiv \int |A_{r}(s_+, s_-)|^2 ds_+ds_- \). The first (second) errors are statistical (systematic).

| State          | \( R_{C} (%) \) | \( \Delta \phi_{r} (\%) \) | \( f_{C} (%) \) |
|---------------|-----------------|--------------------------|----------------|
| \( \rho^{+} \) | 100             | 0                        | 67.8 \pm 0.06   |
| \( \rho^{0} \) | 58.8 \pm 0.6    | 16.2 \pm 0.6             | 26.0 \pm 0.5    |
| \( \rho^{-} \) | 71.4 \pm 0.8    | -2.0 \pm 0.6             | 34.6 \pm 0.8    |
| \( \rho^{+} \) | 25.4 \pm 1.6    | -146.2 \pm 18.2          | 0.11 \pm 0.07   |
| \( \rho^{0} \) | 33.6 \pm 6.4    | 10.8 \pm 1.3             | 0.30 \pm 0.11   |
| \( \rho^{-} \) | 82.5 \pm 5.4    | 16.4 \pm 3.3             | 1.79 \pm 0.22   |
| \( \rho^{+} \) | 225 \pm 18.4    | -17.2 \pm 2.3            | 4.1 \pm 0.7     |
| \( \rho^{0} \) | 251 \pm 15.3    | -17.2 \pm 2.3            | 5.0 \pm 0.6     |
| \( \rho^{-} \) | 200 \pm 11.7    | 5.0 \pm 3.3              | 3.2 \pm 0.4     |
| \( f_{0}(980) \) | 1.50 \pm 0.12    | -59.5 \pm 4.4            | 0.25 \pm 0.04   |
| \( f_{1}(1370) \) | 6.3 \pm 0.9     | 156.4 \pm 9.6            | 0.37 \pm 0.11   |
| \( f_{1}(1500) \) | 5.8 \pm 0.6     | 124 \pm 9.4              | 0.39 \pm 0.08   |
| \( f_{1}(1710) \) | 11.2 \pm 1.4    | 51.8 \pm 8.7             | 0.31 \pm 0.07   |
| \( f_{2}(1270) \) | 104 \pm 32.1    | -171 \pm 3.4             | 1.32 \pm 0.08   |
| \( \sigma(400) \) | 6.9 \pm 0.6     | 8 \pm 4.2               | 0.82 \pm 0.10   |

Non-Res 57 \pm 7 \pm 8 \pm 11 \pm 4 \pm 2 \pm 0.21 \pm 0.12
Only the complex parameters \( z_\pm \) are free in the step-3 fit. This fit minimizes the function
\[
\mathcal{L} = -\sum_{e=1}^{N_{ev}} \log \mathcal{P}_3 \left( \xi_e^* \right) + \frac{1}{2} \chi^2 ,
\]
where \( N_{ev} \) is the number of events in the data sample. The term \( \chi^2 = \sum_{n=1}^{2} X_n \nu_{uv}^{-1} X_n \) increases the sensitivity of the fit by using the results of the step-2 fit via
\[
X_1 = N_{DK} - (n_- + n_+),
\]
\[
X_2 = A_{DK} - (n_+ - n_-)/(n_- + n_),
\]
where
\[
n_{\pm} = N^0 \int \mathcal{D}D_{DK} (s_+, s_-) ds_+ ds_-
\]
are the expected numbers of \( B^\pm \) signal events. In Eq. (6), \( N^0 \) is the product of the number \( N_{B^\pm B^-} \) of charged \( B^+ B^- \) pairs in the dataset, the branching fractions \( B(B^- \to D^0 K^-) \) [11] and \( B(D^0 \to \pi^+ \pi^- \pi^0) \) [13], and the total reconstruction efficiency \( \epsilon = 1.4\% \). The error matrix \( V_{uv} \) is the sum of two components: the step-2 fit error matrix \( V_{\text{stat}} \), which is almost diagonal (the correlation coefficient is $-2.8\%$), and the \( N^0 \) systematic error matrix \( V_{\text{syst}} \). Here \( V_{12} = V_{22} = 0 \), and \( V_{11} = \sum_{c=1}^{4} (N^0 \sigma_c^2) \), where \( \sigma_c^2 \) are the relative errors on the four components \( N_{B^\pm B^-} \) \((1.1\%), \epsilon \ (3.3\%) \), \( B(D^0 \to \pi^+ \pi^- \pi^0) \) \((3.8\%) \) [13], and \( B(B^- \to D^0 K^-) \) \((5.9\%) \) [11].

We parameterize \( z_\pm \) with the polar coordinates
\[
\rho_\pm \equiv |z_\pm - x_0|, \quad \theta_\pm \equiv \tan^{-1} \left( \frac{3[z_\pm]}{\mathcal{R}[z_\pm] - x_0} \right),
\]
where \( x_0 \) is a coordinate transformation parameter,
\[
x_0 \equiv -\int \mathcal{R} [\alpha(s_+, s_-) \alpha^*(s_-, s_+)] ds_+ ds_- = 0.850.
\]
This parameterization is optimal due to the polar symmetry of \( n_\pm = N^0 (1 + \rho_\pm^2 - x_0^2) \). Other parameterizations, such as \((|A_u/A_c|, \gamma, \delta)\) or \((|R[z_\pm]|, \mathcal{R}[z_\pm])\), result in significant nonlinear correlations between the fit variables, which cannot be parameterized with an error matrix, and bias the fit result. The polar coordinates enable a significant improvement in sensitivity due to the \( \chi^2 \) term in Eq. (4), and are determined from parameterized simulation to be unbiased. The step-3 fit yields
\[
\rho_- = 0.72 \pm 0.11 \pm 0.04, \quad \theta_- = (173 \pm 42 \pm 2)^\circ, \\rho_+ = 0.75 \pm 0.11 \pm 0.04, \quad \theta_+ = (147 \pm 23 \pm 1)^\circ,
\]
where the first errors are statistical and the second are systematic, due only to \( V_{11}^\text{syst} \). The largest correlation coefficient is \( c_{\rho-\rho+} = 14\% \), originating from \( V_{11}^\text{syst} \). All others are 1\% or less. Contours of constant \( \mathcal{L} \) values are shown in Fig. 1(d). Projections of the data and the PDF onto \( s_+ \) and \( s_- \) are shown in Fig. 1(e-f).

Additional systematic errors due to the analysis procedure are evaluated for the signal branching fraction, charge asymmetry, \( \rho_\pm \), and \( \theta_\pm \). The uncertainty in the model used for \( \alpha(s_+, s_-) \) is the largest source of error on the \( CP \) parameters: \( \sigma_{\rho_\pm} = 0.03, \sigma_{\theta_\pm} = 14\% \), \( \sigma_{v_{\rho_\pm}} = 11\% \). This error is evaluated by removing all but the \( \rho(770), \rho(1450), f_0(980) \), and nonresonant terms in \( \alpha(s_+, s_-) \); adding an \( f_2(1525) \), an \( \omega \), and a nonresonant P-wave contribution; varying the meson “radius” parameter in \( F \) [12]; and propagating the errors from Table 1 Uncertainties due to the masses and widths of the \( \rho(1700) \) and \( \omega \) resonances are small by comparison. Other errors are due to uncertainties on background yields that are
fixed in the fits, finite MC sample size, a possible reconstruction efficiency charge asymmetry, and uncertainties in the background PDF shapes, evaluated by comparing MC and data in signal-free sidebands of the variables $M_D$, $\Delta E$, and $m_{ES}$. We also evaluate errors due to possible charge asymmetries in $DKX$ and $DK_{bgd}$, uncertainties in particle identification and the efficiency functions, the finite $s_{\pm}$ measurement resolution, the background PDF $f_B$ in the $D^*$ sample, $D$-flavor mistagging in the $D^*$ sample, and correlations between the $D$ flavor and the kaon charge in $q_{\ell D}$ events. These errors add in quadrature to $\sigma^{\text{syst}}_{\rho_\pm} = 0.05, \sigma^{\text{syst}}_{\theta_\pm} = 19^\circ, \sigma^{\text{syst}}_{\rho_\pm} = 13^\circ$, and are combined with the systematic errors of Eqs. (9).

The analysis procedure is validated in several ways. Conducting the analysis on the MC sample yields results consistent with the generated values. We carry out the step-3 fit on a sample of $1800 \pm 70$ $B^\to D^0_{\pi^+\pi^-\pi^0} \pi^-$ events, obtaining the background Dalitz plot distribution from the $\Delta E$ sideband. The fit yields $\rho_- = 0.815 \pm 0.034, \theta_- = (186 \pm 7)^\circ, \rho_+ = 0.854 \pm 0.035, \theta_+ = (192 \pm 7)^\circ$, consistent with $\rho_\pm = x_0, \theta_\pm = 180^\circ$, which corresponds to $x_\pm = 0$. We verify the signal efficiency by measuring the branching fraction $B(B^\to D^0 \pi^-)$ with $D^0 \to K^-\pi^+\pi^0$ and $D^0 \to \pi^+\pi^-\pi^0$. We compare the fit variable distributions of data and MC events in signal-free sidebands. Good agreement is found in all cases.

In summary, using a sample of $(324.0 \pm 3.6) \times 10^6 e^+e^- \to B\bar{B}$ events, we observe $170 \pm 29 B^\pm \to D^0_{\pi^+\pi^-\pi^0}\pi^\pm$ events. We calculate the branching fraction and decay rate asymmetry

$$B(B^\pm \to D^0_{\pi^+\pi^-\pi^0}\pi^\pm) = (4.6 \pm 0.8 \pm 0.7) \times 10^{-6},$$

$$A(B^\pm \to D^0_{\pi^+\pi^-\pi^0}\pi^\pm) = -0.02 \pm 0.15 \pm 0.03,$$

and the CP-violation parameters

$$\rho_- = 0.72 \pm 0.11 \pm 0.06, \quad \theta_- = (173 \pm 42 \pm 19)^\circ,$$

$$\rho_+ = 0.75 \pm 0.11 \pm 0.06, \quad \theta_+ = (147 \pm 23 \pm 13)^\circ,$$

where the first errors are statistical and the second are systematic. The parameters $\rho_\pm, \theta_\pm$ are defined in Eq. (10). While the errors on $\theta_\pm$ are too large for a meaningful determination of $\gamma$ with these results alone, our errors on $\rho_\pm$ are small enough to make a non-negligible contribution to the overall precision of $\gamma$ in a combination of all measurements related to $\gamma$. In addition, we measure the magnitudes and phases of the components of the amplitude of the decay $D^0 \to \pi^+\pi^-\pi^0$ in the isobar model.

We are grateful for the excellent luminosity and machine conditions provided by our PEP-II colleagues, and for the substantial dedicated effort from the computing organizations that support BaBar. The collaborating institutions wish to thank SLAC for its support and kind hospitality. This work is supported by DOE and NSF (USA), NSERC (Canada), IHEP (China), CEA and CNRS-IN2P3 (France), BMBF and DFG (Germany), INFN (Italy), FOM (The Netherlands), NFR (Norway), MIST (Russia), MEC (Spain), and PPARC (United Kingdom). Individuals have received support from the Marie Curie EIF (European Union) and the A. P. Sloan Foundation.

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