Reflective scattering from unitarity saturation

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Abstract

Proceeding from optical analogy we propose a new physical interpretation of unitarity saturation leading to antishadowing as a reflective scattering. This interpretation of antishadowing is related to the non-perturbative aspects of strong interactions and follows from the specific property of the unitarity saturation when elastic $S$-matrix $S(s, b)_{b=0} \to -1$ at $s \to \infty$. The analogy with Berry phase and experimental consequences of the proposed interpretation as reflective scattering at the LHC and in the cosmic rays studies are discussed.
Introduction

The fundamental problems of the nonperturbative QCD are related to confinement and spontaneous chiral symmetry breaking phenomena. Those phenomena deal with collective, coherent interactions of quarks and gluons, and results in formation of the asymptotic states, which are the colorless, experimentally observable particles. Among the problems of strong interactions, the total cross-section growth with energy constitutes one of the important questions. The nature of this energy dependence is not completely understood since underlying microscopic mechanism is due to the nonperturbative QCD. An essential role here belongs to elastic scattering where all hadron constituents interact coherently and therefore elastic scattering can also serve as an important tool to the confinement studies. Owing to the experimental efforts during recent decades it has became evident that the coherent processes survive at high energies.

General principles play a guiding role in the soft hadron interaction studies, and, in particular, unitarity which regulates the relative strength of elastic and inelastic processes is the most significant one. It is important to note here that unitarity is formulated for the asymptotic colorless hadron on-mass shell states and is not directly connected to the fundamental fields of QCD — quarks and gluons. The same is valid for the analyticity which is relevant for the scattering amplitudes of the observable particles only. As it was noted in [1], it is not clear what these fundamental principles imply for the confined objects.

Even the extension of $S$–matrix and $s$–channel unitarity to the off–mass shell particle scattering leads to the significant changes in the predictions for the behavior of the observables: it does not rule out now the power–like asymptotical behavior of the total cross–sections, i.e. unitarity in this case does not lead to the Froissart-Martin bound in the small $x$–region [2].

Our goal here is to provide a new physical interpretation for the scattering mode based on the saturation of the unitarity relation for the on–mass shell particles, when elastic scattering not only survives, but prevails at super high energies in hadron collisions at small impact parameter values. We discuss in this note this rather unexpected behavior on the base of optical analogy, and list some of the respective experimental signatures in the studies of hadronic interactions at the LHC and in cosmic rays. It is important to note that some of these predictions are due to this new interpretation of unitarity saturation as a reason for the appearance of the reflective scattering at superhigh energies.
1 Saturation of unitarity and energy evolution of the geometric scattering picture

Unitarity or conservation of probability, which can be written in terms of the scattering matrix as

\[ SS^+ = 1, \]

implies an existence at high energies of the two scattering modes - shadowing and antishadowing. Existence of antishadowing has been known long ago [3, 4], in the context of the rising total cross-sections and transition beyond the black disk limit it was discussed in the framework of the rational unitarization scheme in [7]. The most important feature of antishadowing is self-damping [3] of the inelastic channels contribution. Corresponding geometric picture and physical effects will be discussed further. As it was noted in Introduction, the operator \( S \) is defined in the Hilbert space spanned over vectors corresponding to the observable physical particles and the unitarity is formulated in terms of the physical particles.

The attempts to construct \( S \)-matrix in perturbative QCD [9] have difficulties related to the problem of confinement [8, 10] and the problem of the undefined scale dependence entering \( S \)-matrix through \( \alpha_s \) [11]. Moreover, as it was shown in [12], the color fields can be represented by harmonic oscillators (gluons) only in the limit when the strong interaction coupling constant tends to zero, i.e. at \( Q^2 \to \infty \).

After these necessary remarks, one can proceed further with unitarity condition, apply the standard procedure and derive from Eq. (1) unitarity relation for the partial wave amplitudes \( f_l(s) \) :

\[ \text{Im} f_l(s) = |f_l(s)|^2 + \eta_l(s), \]

where elastic \( S \)-matrix is related to the amplitude as

\[ S_l(s) = 1 + 2if_l(s) \]

and \( \eta_l(s) \) stands for the contribution of the intermediate inelastic channels to the elastic scattering with the orbital angular momentum \( l \). The relation (2) turns out to be a quadratic equation in the case of the pure imaginary scattering amplitude. Therefore, the elastic amplitude appears to be not a single-valued function of \( \eta_l \). But, only one of the two solutions of the equation corresponding to the relation (2) is considered almost everywhere

\[ f_l(s) = \frac{i}{2}(1 - \sqrt{1 - 4\eta_l(s)}), \quad \text{i.e.} \quad |f_l| \leq 1/2, \]

while another one

\[ f_l(s) = \frac{i}{2}(1 + \sqrt{1 - 4\eta_l(s)}), \quad \text{i.e.} \quad 1/2 \leq |f_l| \leq 1 \]
is ignored. Eq. (4) corresponds to the typical shadow picture when elastic amplitude $f_l(s)$ for each value of $l$ is determined by the contribution of the inelastic channels $\eta_l(s)$ and for small values of the function $\eta_l(s)$: $f_l(s) \simeq i\eta_l(s)$. Note, that the value $|f_l| = 1/2$ corresponds to the black disk limit where absorption is maximal, i.e. $\eta_l(s) = 1/4$. Unitarity limit for the partial wave amplitude is unity, i.e. twice as much as the black disk limit. The saturation of the unitarity limit is provided by Eq. (5): it leads at the small values of $\eta_l(s)$ to the relation $f_l(s) \simeq i(1 - \eta_l(s))$. It means that elastic amplitude is increasing when contribution of the inelastic channels is decreasing. Therefore, the term of antishadowing was used.

It is important to consider the arguments in favor of the solution (5) neglecting. They are simple: it is well known that analytical properties in the complex $t$-plane lead to decrease with $l$ at large values of $l > L(s)$ of both the amplitude $f_l(s)$ and the contribution of inelastic channels $\eta_l(s)$ at least exponentially, i.e. $\lim_{l \to \infty} f_l(s) = 0$ and $\lim_{l \to \infty} \eta_l(s) = 0$. It is evident that the solution (5) does not fulfill the requirement of simultaneous vanishing $f_l(s)$ and $\eta_l(s)$ at $l \to \infty$ and this is the reason for its neglecting. But the requirement of simultaneous vanishing $f_l(s)$ and $\eta_l(s)$ effective at large values of $l$ only and, respectively, the solution (5) should be neglected at large values of $l$. At small and moderate values of $l$ both solutions (4) and (5) have equal rights to exist.

But how both the above mentioned solutions can be reconciled and realized in the uniform way and what physics picture can underlie the solution (5), which anticipates saturation of the unitarity limit for the partial wave amplitude? In what follows we consider the case and its consequences. To provide a geometric meaning to the scattering picture we will use an impact parameter representation. The unitarity relation written for the elastic scattering amplitude $f(s, b)$ is similar to (2) in the high energy limit and has the following form

$$\text{Im} f(s, b) = |f(s, b)|^2 + \eta(s, b).$$  \hspace{1cm} (6)

There is no universally accepted method to implement unitarity in high energy scattering [4]. The recent approaches to solve the problem of high-energy limit in QCD based, e.g. on the AdS scattering, apriori suppose the eikonal function corresponding to the black disk [5]. The reduced unitarity limit for the amplitude based on Eq. (4) is used also in the models of gluon saturation (cf. [6] and references therein).

In principle, a choice of particular unitarization scheme is not completely ambiguous. The two above mentioned solutions of unitarity equation (4) and (5) can be naturally reconciled and uniformly reproduced by the rational ($U$-matrix)
form of unitarization. The arguments based on analytical properties of the scattering amplitude were put forward \[13\] in favor of this form. In the \(U\)-matrix approach the elastic scattering matrix in the impact parameter representation is the following linear fractional transform:

\[
S(s, b) = \frac{1 + iU(s, b)}{1 - iU(s, b)} \tag{7}
\]

\(U(s, b)\) is the generalized reaction matrix, which is considered to be an input dynamical quantity. The transform (7) is one-to-one and easily invertible. Inelastic overlap function \(\eta(s, b)\) can also be expressed through the function \(U(s, b)\) by the relation

\[
\eta(s, b) = \frac{\text{Im}U(s, b)}{|1 - iU(s, b)|^2}, \tag{8}
\]

and the only condition to obey unitarity in the form of Eq. (6) is \(|\text{Im}U(s, b)| \geq 0\).

Another way to warrant unitarity is to represent elastic \(S\)-matrix in the exponential form:

\[
S(s, b) = \exp[2i\delta(s, b)], \tag{9}
\]

where \(\delta(s, b)\) is a phase shift \([\delta(s, b) \equiv \delta_R(s, b) + i\delta_I(s, b)]\) and the inequality \(\delta_I(s, b) \geq 0\) is needed to satisfy unitarity. Both representations (7) and (9) provide \(|S(s, b)| \leq 1\) but contrary to (7), \(S \neq 0\) for any finite value of \(\delta(s, b)\) when \(S\) is written in the form (9). To trace a further difference between them, let us consider for simplicity the case of pure imaginary \(U\)-matrix and make the replacement \(U \rightarrow iU\). The \(S\)-matrix has the following form

\[
S(s, b) = \frac{1 - U(s, b)}{1 + U(s, b)}. \tag{10}
\]

At this point we should make an important remark on the models, which use rational or exponential forms of the amplitude unitarization. We note that most of the models provide increasing dependence of these functions with energy (e.g. power-like one) and their exponential decrease with impact parameter \(b\). Thus, any model of this kind for \(U\)-matrix is not compatible with any similar eikonal model, predicting crossing the black disk limit. The value of energy corresponding to this limit for central collisions \(S(s, b)|_{b=0} = 0\) will be denoted as \(s_0\) and it is determined by the equation \(|U(s, b)|_{b=0} = 1\). Thus, in the energy region \(s \leq s_0\) the scattering in the whole range of impact parameter variation has a shadow nature and correspond to solution (4), the \(S\) matrix varies in the range \(0 \leq S(s, b) < 1\). But when the energy is higher than this threshold value \(s_0\), the scattering picture at small values of impact parameter \(b \leq R(s)\) corresponds to the solution Eq. (5), where \(R(s)\) is the interaction radius. The \(S\)-matrix variation region is
\(-1 < S(s, b) \leq 0 \) at \( s \geq s_0 \) and \( b \leq R(s) \). We are going to discuss emerging physical picture of the scattering in this particular region of impact parameters and very high energies. The schematic energy evolution of \( S \)-matrix of the impact parameter dependence is depicted on Fig. 1. It is evident that at \( s > s_0 \) there is a region of impact parameters where \( S \)-matrix is negative, i.e. phases of incoming state and outgoing state differ by \( \pi \). Thus, there is a close analogy here with the light reflection off a dense medium, when the phase of the reflected light is changed by \( 180^0 \). Therefore, using the optical concepts [14], the above behavior of \( S(s, b) \) should be interpreted as an appearance of a reflecting ability of scatterer due to increase of its density beyond some critical value, corresponding to refraction index noticeably greater than unity. In another words, the scatterer has now not only absorption ability (due to presence of inelastic channels), but it starts to be reflective at very high energies and its central part \((b = 0)\) approaches to the completely reflecting limit \((S = -1)\) at \( s \to \infty \). It would be natural to expect that this reflection has a diffuse character. In another words we can describe emerging physical picture of very high energy scattering as scattering off the reflecting disk (approaching to complete reflection at the center) which is surrounded by a black ring. The reflection which is a result of antishadowing leads to \( S(s, b)|_{b=0} \to -1 \) asymptotically.

The inelastic overlap function \( \eta(s, b) \) gets a peripheral impact parameter dependence in the region \( s > s_0 \) (Fig.2). Such a dependence is manifestation of the self–damping of the inelastic channels at small impact parameters. The function

![Figure 1: Impact parameter dependence (schematic) of the function \( S(s, b) \) for three values of energy \( s_1 < s_0 < s_2 \).](image)

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Figure 2: Impact parameter dependence of the inelastic overlap function in the unitarization scheme with antishadowing. Arrows indicate directions of the energy evolution.

\( \eta(s, b) \) reaches its maximum value at \( b = R(s) \), while the elastic scattering (due to reflection) occurs effectively at smaller values of the impact parameters, i.e. \( \langle b^2 \rangle_{el} < \langle b^2 \rangle_{inel} \). Note that

\[
\langle b^2 \rangle_i = \frac{1}{\sigma_i} \int b^2 \frac{d\sigma_i}{db^2} db^2,
\]

where \( i = tot, el, inel \) and

\[
\text{Im} f(s, b) \equiv \frac{1}{4\pi} \frac{d\sigma_{tot}}{db^2}; \quad |f(s, b)|^2 \equiv \frac{1}{4\pi} \frac{d\sigma_{el}}{db^2}; \quad \eta(s, b) \equiv \frac{1}{4\pi} \frac{d\sigma_{inel}}{db^2}.
\]

The quantity \( \langle b^2 \rangle \) is a measure of the particular reaction peripherality and the following sum rule takes place for \( \langle b^2 \rangle_{inel} \):

\[
\langle b^2 \rangle_{inel}\sigma_{inel}(s) = \sum_{n \geq 3} \langle b^2 \rangle_n \sigma_n(s).
\]

It is useful to return to the exponential representation for the \( S \)-matrix (9) at this point. It is evident that this form with pure imaginary phase shift (eikonal) would never give the negative values of the function \( S(s, b) \), it will always vary in the range \( 0 < S(s, b) < 1 \). However it is not the case when \( \delta_R(s, b) \) is not zero. If \( \delta_R(s, b) = \pi/2 \), the antishadowing can be reproduced in the exponential form of the unitarization, i.e. the function \( S(s, b) \) will vary in the range \( -1 < S(s, b) < 0 \).

In order to combine shadowing at large values of \( b \) with antishadowing in central collisions the real part of the phase shift should have the dependence

\[
\delta_R(s, b) = \frac{\pi}{2} \theta(R(s) - b).
\]

Such a behavior of \( \delta_R(s, b) \) just takes place in the \( U \)-matrix form of unitarization. Indeed, the phase shift \( \delta(s, b) \) can be expressed in terms of the function
\[ U(s, b) \text{ as following} \]

\[ \delta(s, b) = \frac{1}{2i} \ln \frac{1 - U(s, b)}{1 + U(s, b)} \]  \hspace{1cm} (11) \]

It is clear that in the region \( s > s_0 \) the function \( \delta(s, b) \) has a real part \( \pi/2 \) in the region \( 0 < b \leq R(s) \), while \( \delta_I(s, b) \) goes to infinity at \( b = R(s) \) (Fig. 3). The

\[ \text{Figure 3: Impact parameter dependence of the real (left panel) and imaginary part (right panel) of the phase shift at } s > s_0. \]

picture leads to discontinuity in the phase shift\(^2\) and it resembles the scattering on the cut-off optical potentials \[16]. \]

It also leads to another interesting similarity, namely it allows one to consider \( \delta_R \) as an analog of the geometric Berry phase in quantum mechanics which appears as a result of a cyclic time evolution of the Hamiltonian parameters \[17\]. This interesting phenomenon can be observed in many physical systems \[18, 19\] and is, in fact, a feature of a system that depends only on the path it evolves along of. In the case of pure imaginary elastic scattering amplitude the contribution of the inelastic channels \( \eta \) can be considered as a parameter which determines due to unitarity (but not in a unique way) the elastic \( S \)-matrix. We can vary variable \( s \) (and/or \( b \)) in a way that the parameter \( \eta \) (which has a peripheral \( b \)-dependence, cf. Fig. 2) evolves cyclically from \( \eta_i < 1/4 \) to \( \eta_{\text{max}} = 1/4 \) and again to the value \( \eta_f \), where \( \eta_f = \eta_i \) (loop variation). As a result the non-zero phase appears \( (\delta_R = \pi/2 \text{ at } b \leq R(s)) \) and this phase is independent of the details of the energy evolution.

Thus, we can summarize that the physical scattering picture beyond the black disk limit evolves with energy by simultaneous increase of the reflective ability (i.e. \( |S(s, b)| \) increases with energy and \( \delta_R = \pi/2 \)), and decrease of the absorptive ability \( 1 - |S(s, b)|^2 \) at the impact parameters \( b < R(s) \).

\(^2\)The discontinuity of \( \delta_I(s, b) \) was noticed first by V.A. Petrov \[15\].
We address now the elastic scattering amplitude $F(s, t)$, which is a Fourier-Bessel transform of the $f(s, \beta)$. In the $U$-matrix unitarization method it is determined by the singularities in the impact parameter complex $\beta = b^2$ plane. Those singularities include the poles which positions are determined by the solutions of the equation $1 + U(s, \beta) = 0$ and the branching point at $\beta = 0$ which follows from the spectral representation for the function $U(s, \beta)$

$$U(s, \beta) = \frac{\pi^2}{s} \int_{t_0}^{\infty} \rho(s, t') K_0(\sqrt{t'\beta})dt'.$$

(12)

With account of the analytical properties dictated by Eq. (12) we can use parameterization in the form [20, 22]

$$U(s, \beta) = g(s) \exp(-M\sqrt{\beta^2/\xi}),$$

(13)

where $M$ is the total mass of $N$ constituent quarks in the colliding hadrons and $\xi$ is a parameter. Contribution of the poles determines the elastic scattering amplitude in the region of small and moderate values of $-t$. The amplitude dependence in this region provides diffraction peak and shows up dip-bump structure of the differential cross-section. It reproduces also Orear behavior at larger values of $-t$.

The pole and cut contributions are decoupled dynamically when $g(s) \to \infty$ at $s \to \infty$ [20, 22]. At large angles the contribution from the branching point is a dominating one. The large angle or small impact parameter scattering being a result of the reflection has a power-like angular distribution dependence:

$$\frac{d\sigma}{dt} \propto \left(\frac{1}{s}\right)^{N+3} \omega(\theta).$$

(14)

The power-like dependence of the differential cross sections in large angle scattering closely interrelates with the rise of the total cross sections at high energies since both are determined by the dependence of $g(s) \to \infty$ at $s \to \infty$ in the unitarity saturating scheme.

For the symmetric case of $pp$-interaction the scattering is the same in the forward and backward hemispheres. The more interesting case is the one where interacting particles are not identical, for example, $\pi N$-scattering. In this case the behavior of the differential cross-section in the forward hemisphere is completely analogous to the above case of $pp$-scattering where overall behavior of the differential cross-section incorporates diffraction cone, Orear type and power-like dependencies. But in the backward hemisphere the poles contributions are suppressed compared to the cut contribution in the whole region of the variation of the variable $u$, i.e. the power-like dependence will take place at all values of $u$ and there would be no diffraction cone and Orear type dependence [20] in the backward hemisphere. It is not surprising, indeed, if we will recollect reflecting nature of the scattering at small impact parameters.
2 Observable effects at the LHC energies

In the $U$–matrix unitarization scheme asymptotical behavior of the cross–sections and the ratio of elastic to total cross-section are different from the asymptotic equipartition of elastic and inelastic cross–sections based in the black disk limit when

$$\frac{\sigma_{el}(s)}{\sigma_{tot}(s)} \to \frac{1}{2}.$$ 

Under the $U$-matrix unitarization the rise of the elastic cross–section is predicted to be steeper than the rise of the inelastic cross–section beyond some threshold energy. Asymptotically

$$\sigma_{tot}(s) \sim \sigma_{el}(s) \sim \ln^2 s, \quad \sigma_{inel}(s) \sim \ln s, \quad (15)$$

This is due to saturation of the upper unitarity limit for the partial–wave amplitude in the $U$–matrix approach $|f_l| \leq 1$, while the restriction $|f_l| \leq 1/2$ dictated by the black disk limit leads to the above mentioned equipartition of the cross-sections. Note, that despite the asymptotics for $\sigma_{el}$ and $\sigma_{inel}$ are different, the quantities $\langle b^2 \rangle_{el}$ and $\langle b^2 \rangle_{inel}$ have the same energy dependence, proportional to $\ln^2 s$ at $s \to \infty$.

The above asymptotic dependencies take place for various forms of the function $U(s, b)$. Explicit forms for the function $U(s, b)$ can be obtained, e.g. using geometrical, Regge or chiral quark models for the $U$–matrix. Of course, it is useful to have numerical estimates for the cross-sections at the LHC energies. These estimates are model dependent ones and can vary rather strongly depending on the choice of the particular model parameterizations for $U$–matrix. However, both the Donnachie-Landshoff and dipole Pomeron parameterizations of $U$-matrix, used in [21] and the model [22] are in agreement that the black disk limit will be passed at $\sqrt{s} = 2$ TeV. The latter model provides the following values at the LHC energy $\sqrt{s} = 14$ TeV: $\sigma_{tot} \simeq 230$ mb and $\sigma_{el}/\sigma_{tot} \simeq 0.67$ [23]. Thus, there is an interesting possibility that the reflective scattering mode could be discovered at the LHC by measuring $\sigma_{el}/\sigma_{tot}$ ratio which would be greater than the black disk value $1/2$. However, the asymptotical regime (15) is expected in the model at $\sqrt{s} > 100$ TeV only.

Proceeding from an increasing weight at the LHC energies of the reflective scattering compared to the shadow scattering, we can suppose that the dip-bump structure in the $d\sigma/dt$ in $pp$ scattering might be less prominent at large values of $-t$ and hadronic glory effect (enhancement of backscattering probability) would be observed. Unfortunately, it is difficult to give a more definite predictions for

\[^3\]Of course, quantitative predictions are different for the different models while qualitative features coincide.
differential cross-sections, since other effects such as real part of the amplitude and/or contributions from helicity-flip amplitudes should be taken into account.

The listed above values for the global characteristics of $pp$ – interactions at the LHC are different from the other model predictions. First, the total cross-section is predicted to be twice as much the common predictions in the range 95-120 mb \[24, 25\] due to strong increase of the $\sigma_{el}(s)$. The prediction for the inelastic cross-section is $\sigma_{inel}(s) = 75$ mb and coincides with the predictions of the other models \[25\]. Thus, reflective scattering would not result in worsening the background situation at the LHC, since the elastic scattering provides only two extra particles in the final state. However, the prediction for the total cross-section overshoots even the existing cosmic ray data. But extracting the total proton–proton cross sections from cosmic ray experiments is far from being straightforward (cf. e.g. \[24, 25\] and references therein). Indeed, those experiments are sensitive to the model dependent parameter called inelasticity and they do not the elastic channel. If we will use for the ratio $\sigma_{el}/\sigma_{tot}$ the value 0.67 at the LHC energy $\sqrt{s} = 14$ TeV and recalculate the total cross-section obtained from the cosmic ray data we will get the value about 227 mb in good agreement with the predicted value. This agreement, however, is merely an indirect confirmation of the model prediction for the total cross-section, since as it was noted there is an ambiguity in the elastic cross-section determination. The behavior of the ratio $\sigma_{el}/\sigma_{tot}$ at $s \to \infty$ does not imply decreasing energy dependence of $\sigma_{inel}$. The inelastic cross-section $\sigma_{inel}$ increases monotonically and grows as $\ln s$ at $s \to \infty$. Such a dependence of $\sigma_{inel}$ is in good agreement with the experimental data and, in particular, with the observed falling slope of the depth of shower maximum distribution \[26\]. We will discuss some of the cosmic ray related issues in the next section.

3 Reflective scattering in cosmic rays

As it was already noted, the important role in the studies of cosmic rays belongs to the inelasticity parameter $K$, which is defined as ratio of the energy going to inelastic processes to the total energy. The energy dependence of $K$ is not evident and cannot be directly measured. The number of models predict its decreasing energy dependence while other models insist on the increasing energy behavior at high energies \[27\]. Adopting simple ansatz of geometric models where parameter of inelasticity is related to inelastic overlap function we can use the following equation \[28\]

$$\langle K \rangle = 4 \frac{\sigma_{el}}{\sigma_{tot}} \left(1 - \frac{\sigma_{el}}{\sigma_{tot}}\right)$$

to get a qualitative knowledge on the inelasticity energy dependence.
The estimation based on the particular model with antishadowing [23] leads to increasing dependence with energy till $E \approx (3 - 4) \cdot 10^7$ GeV. In this region inelasticity reaches maximum value $\langle K \rangle = 1$, since $\sigma_{el}/\sigma_{tot} = 1/2$ and then starts to decrease at the energies where this ratio goes beyond the black disk limit $1/2$. Such qualitative non-monotonous energy dependence of inelasticity is the result of transition to the reflective scattering regime.

Reflective scattering results in relative suppression of particle production at small impact parameters:

$$
\bar{n}(s) = \frac{1}{\sigma_{inel}(s)} \int_0^{\infty} \bar{n}(s, b) \frac{d\sigma_{inel}}{db^2} db^2
$$

(16)

due to the peripherality of $d\sigma_{inel}/db^2$. Thus, the main contribution to the integral multiplicity $\bar{n}(s)$ comes from the region of $b \sim R(s)$ and the distinctive feature of this mechanism is the ring-like shape of particle production which will lead to correlations in the transverse momentum of the secondary particles. It means that the enhancement of particle production at fixed impact distances $b \sim R(s)$ would lead to higher probability of the circular events. Such events would reflect the production geometry with complete absorption at the impact distances equal to the effective interaction radius $R(s)$. The enhancement of the peripheral particle production would destroy the balance between orbital angular momentum in the initial and final states; most of the particles in the final state would carry out orbital angular momentum. To compensate this orbital momentum the spins of the secondary particles should become lined up, i.e. the spins of the particles in the ring-like events should demonstrate significant correlations. Of course the observation of such effects is difficult due to the multiple interaction of the secondary particles in the atmosphere. However, the circular event has been observed experimentally [29].

It might be useful to note that the rescattering processes are affected by the reflective scattering. At the energies and impact parameters where reflective scattering exists there will be no rescatterings, i.e. the head-on collisions will experience a reflective elastic scattering at the energies $s > s_0$ and due to this fact less number of secondary particles will be detected in EAS at the ground level. This should provide a faster decrease of the energy spectrum reconstructed from EAS, i.e. it will result in the appearance of the knee. Thus, the hadron interaction and mechanism of particle generation will be changing in the region of $\sqrt{s} = 3 - 6$ TeV. Indeed, the energy spectrum which follows a simple power-like law $\sim E^{-\gamma}$ changes its slope in this energy region, i.e. index $\gamma$ increases from 2.7 to 3.1. The interpretation of the cosmic-ray data is complicated since the primary energies of cosmic particles are far beyond of the energies of modern accelerators with fixed targets and existing simulation programs merely extrapolate the present knowledge on the hadron interaction dynamics in the unexplored energy region. This
might be an oversimplification and here we would like to interpret the existence of the knee in the cosmic ray energy spectrum as the effect of changing hadron interaction mechanism related to the appearance of the reflective scattering at small impact parameters.

**Conclusion**

In this note we considered saturation of the unitarity limit in head-on (and small impact parameter) hadron collisions, i.e. when $S(s, b)|_{b=0} \rightarrow -1$ at $s \rightarrow \infty$. Approach to the full absorption in head-on collisions, in another words, the limit $S(s, b)|_{b=0} \rightarrow 0$ at $s \rightarrow \infty$ does not follow from unitarity, it is merely a result of the assumed saturation of the black disk limit. This limit is a direct consequence of the exponential unitarization with an extra assumption on the pure imaginary nature of the phase shift. On the other hand, the reflective scattering is a natural interpretation of the unitarity saturation based on the optical concepts in high energy hadron scattering. Such reflective scattering can be interpreted as a result of the continuous increasing density of the scatterer with energy, i.e. when density goes beyond some critical value relevant for the black disk limit saturation, the scatterer starts to acquire a reflective ability. Having in mind the quark-gluon hadron structure it would be tempting to find the particular microscopic mechanisms related to the collective quark-gluon dynamics in head-on collisions which can be envisaged as the origin of the reflection phenomenon. One can try to speculate at this point and relate the appearance of the reflective scattering to the Color-Glass Condensate in QCD (cf. e.g. [30] and references therein) merely ascribing the reflective ability to Glazma. However, as it was mentioned in the Introduction, the evident obstacle which preclude a direct link to the microscopic mechanisms is the problem of confinement. The unitarity and consequently its saturation and reflective scattering are the concepts relevant for the hadronic degrees of freedom. The concept of reflective scattering itself is general, and results from the $S$-matrix unitarity saturation related to the necessity to provide an indefinite total cross section growth at $s \rightarrow \infty$.

Thus, at very high energies ($s > s_0$) two separate regions of impact parameter distances can be anticipated, namely the outer region of peripheral collisions where the scattering has a typical shadow origin, i.e. $S(s, b)|_{b>R(s)} > 0$ and the inner region of central collisions where the scattering has a combined reflective and absorptive origin, $S(s, b)|_{b<R(s)} < 0$. The transition to the negative values of $S$ leads to the appearance of the real part of the phase shift, i.e. $\delta_R(s, b)|_{b<R(s)} = \pi/2$.

The generic geometric picture at fixed energy beyond the black disc limit can be described as a scattering off the partially reflective and partially absorptive disk
surrounded by the black ring which becomes grey at larger values of the impact parameter. The evolution with energy is characterized by increasing albedo due to the interrelated increase of reflection and decrease of absorption at small impact parameters. This picture implies that the scattering amplitude at the LHC energies is beyond the black disk limit at small impact parameters (elastic $S$-matrix is negative) and it provides explanation for the regularities observed in cosmic rays studies.

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