Title: Why understanding multiplex social network structuring processes will help us better understand the evolution of human behavior

Running title: Multiplex networks and behavior

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Anthropologists have long appreciated that single-layer networks are insufficient descriptions of human interactions—individuals are embedded in complex networks with dependencies. One debate explicitly about this surrounds food sharing. Some argue that failing to find reciprocal food sharing means that some process other than reciprocity must be occurring, whereas others argue for models that allow reciprocity to span domains. The analysis of multi-dimensional social networks has recently garnered the attention of the mathematics and physics communities. Multilayer networks are ubiquitous and have consequences, so processes giving rise to them are important social phenomena. Recent models of these processes show how ignoring layer interdependencies can lead one to miss why a layer formed the way it did, and/or draw erroneous conclusions. Understanding the structuring processes that underlie multiplex networks will help understand increasingly rich datasets, which give better, richer, and more accurate pictures of social interactions.

**Multilayer and multiplex networks**

![Multiplex networks diagram](image)

Fig 1: Multiplex networks. The set of individuals is the same across all layers. Individuals are connected to themselves across different layers but can be connected to different sets of people on different layers.

Social networks are representations of relationships that allow us to use methods from graph theory. Networks consist of individuals connected to each other by ties. Multilayer networks is a broad category that includes all networks consisting of more than one set of nodes and edges. Multiplex networks is the subset of multilayer networks with two basic properties: (1) all layers share the same set of nodes (i.e., each node is replicated in each layer) and (2) all individuals are connected only to themselves across layers, see Fig 1. One example of a multiplex network is social networks with different domains of interactions, such as hunt, farm, and drink. In such a domain-specific multiplex network, all individuals could do all those things (the same set of nodes is shared across domains), but they may do different things with different sets of people (e.g.,).
Network structuring processes

We can consider a benchmark model with no constraints. Without costs or interdependencies, individuals would optimize each of their networks by rearranging their relationships. Individuals, however, may not be able to do this due to features of the existing network itself or other reasons, e.g. time constraints. The rules for how a network changes based on the current features of the networks and the individuals that compose them are called network structuring processes. These conditions affect the likelihood of a tie arising between two individuals in a given layer or change individuals' network-based outcomes due to their pattern of ties.

We briefly highlight a few multiplex structuring processes. Ties might arise in multiple domains between the same individuals because features of the individuals that make a tie likely in one domain are also operating in other domains. This may include things like personality or risk tolerance: individuals who are wary of being caught alone after dark may fish together in midday. Ties between individuals in one domain may be more likely if they are connected in other domains. Examples are incidental network membership (discussed in detail below), as well as benefits to bundling relationships: a person who is a great hunter but poor fisher might offer to be a partner in both domains with someone who is a poor hunter but great fisher. Individuals may struggle to reorganize their networks if the probability of removing a tie depends on other layers of the network. This includes such processes as constraining outside options: the excellent hunter might threaten to not hunt with the excellent fisher if the excellent fisher does not fish with them. Finally, outcomes may be the result of interactions between layers. This includes processes such as alignment of incentives: if a hungry hunter is a poor hunter, then that individual’s partners might share additional food with them so that hunting returns are higher for everyone.

Incidental network membership

We now discuss one important but specific example of such multiplex network generating processes, to illustrate some of our main points. The process of incidental network membership rests on a few key premises. First, relationships require time and effort. Second, organisms do not have infinite time and resources. Third, relationships in some domains have a higher net benefit. If these premises are true, then organisms will prioritize optimizing networks in the domains with the highest net benefit. Given finite time and resources, organisms may optimize their entire network by extending a relationship with a partner on one important domain into a less important domain—even if that individual is not an optimal partner in the other domain. This can result in non-optimal networks when considered at the single layer.

As an illustration, the Makushi of southern Guyana grow and process cassava into a product that is shelf-stable for years by parching the cassava with beef fat to remove the water to make what they call farine.5. Processing cassava to make farine involves many steps, which must occur concurrently. Because of this, it is the best use of time to have several people working together on different stages of the process, constantly adding more cassava to the farine pan. Indeed, women (who do most of this work) have preferred cassava parching partners and are
rather consistent in their use of those partners (CA observation). These women spend large amounts of time together, talking constantly. It is common to hear women seeking out advice on their personal lives or reproductive decisions. Since these women have already received such information as a by-product of their cassava processing, they may not be motivated to pay an additional cost to recruit better partners in their advice network for only marginally better information. By increasing the efficiency of one dimension of the multiplex network (cassava processing), inefficiencies have been introduced on another dimension (the reproductive advice network).

A model of a network structuring process

We now discuss a formal model of a network structuring process in more detail. This model examines how coupling between layers of a multiplex network impacts the optimality of the network.6 Specifically, this model looks at how a network of two coupled layers impacts Heider balance in the network. Heider balance is a type of network optimality where all the link weights (strength of association between nodes, which range from -1 to +1) forming a cycle of individuals multiply to a positive value (see Fig 2 for examples of Heider balanced and unbalanced networks).7 Colloquially, this is summarized by the aphorisms the friend of my friend is my friend, the enemy of my friend is my enemy, etc. We can also say that a node is in Heider balance when the component of the network that it is connected to by a distance of one edge, its close neighborhood, is in Heider balance. A multiplex is in Heider balance when all domain networks are in Heider balance (they end up being the same in the two-layer case).

![Fig 2: Networks which are a) balanced and b) unbalanced in the sense of Heider. Dashed and solid lines represent links of negative (-1) and positive (+1) weights, respectively. In a) all four individuals are in Heider balance, as is the network. In b), the network is unbalanced and only the individuals with an outline in black have a close neighborhood (i.e., all individuals whom they are directly connected to) in Heider balance. This shows that balance is a property of both individuals and networks.](image)

Change in link weights in each domain can be captured with a system of differential equations. The link weight between two individuals in a domain at each time step is determined by their current link weight in the focal domain, their current link weight in the other domain, and the sum of the product of link weights including one other person. How much the link weight between two individuals on the other domain impacts their link weight in the focal domain is called coupling
and can vary in strength. The coupling between layers in this model can be asymmetrical such that a link weight in layer A changes more in response to the link weight in layer B than the reverse. An example of this would be that people already processing cassava together can give reproductive advice, but those already giving reproductive advice may not have cassava around to process together. The analysis finds that if one layer is much more strongly driven by the other layer than the reverse, Heider balance is achieved because the dominant layer will drag the other layer to its state. Likewise, if layers are completely disconnected from each other, Heider balance is achieved independently on each of the networks. But for the parameter space between those extremes, Heider balance may not always be achieved. Furthermore, the parameter space in the coupling strength for which Heider balance is achieved decreases as network size increases (Fig 3).

![Graph](image)

**Fig 3:** Parameter space leading to systems always reaching Heider balance is decreasing with network size. Figure shows the probability of networks of size $N$ with randomly generated initial link weights and two layers reaching Heider balance for the multiplex ($P_{HB}$). $\beta_1$ and $\beta_2$ are coupling coefficients. For instance, $\beta_1$ represents the influence of the link weights in layer 2 on the link weights in layer 1, and $\beta_2$ the reverse. Panels are divided on the diagonal to show the results for four values of network size. The color at each pixel shows $P_{HB}$, for that combination of $\beta_1$ and $\beta_2$, ranging from black ($P_{HB}=0$) to white ($P_{HB}=1$).

This model leads us to a central formal finding in the nascent study of multiplex network structuring processes: looking at domain-specific networks without appreciating the multiplex structure can lead us to the wrong conclusions (e.g., assuming each layer of the network rather than the entire network is being optimized). This applies to papers that concentrate solely on network structure and/or formation, as well as papers studying the effect of networks on outcomes. If we examine a single domain to find optimality, we are unlikely to find optimality simply because we have not examined the whole network: agents will be optimizing across their entire multiplex. Further, using network measures in an analysis of outcomes tacitly assumes optimality in that network, and predicting outcomes based on these single-layer networks can lead to incorrect conclusions.

**Analyzing incidental network membership**
The existence of these multiplex network structuring processes leads us to conclude that we have to incorporate the structure of an individual’s ties across domains or else we risk being substantially incorrect. In order to do this appropriately, we need to develop models and techniques for analyzing multiplexed settings. There has been relatively little work on multilayer and multiplex networks to date, but one of the areas to first receive attention is the concept of interdependence. Researchers have proposed different measures of interdependence, but we still have an incomplete understanding of any of them.

Multiplex structuring processes complicate traditional network analysis. The multiplex structuring process may be a common cause of measures on each layer (e.g., centrality). Without a method to parse the centrality unique to each network from the centrality in both networks due to the structuring process, we are unlikely to recover the true effect of each network on the outcome of interest, possibly leading to incorrect conclusions. Given that we know processes such as incidental network membership lead to the coupling of networks, and that measures on coupled networks are not independent, we expect the creation of tools incorporating the structure of multiplex networks to be an active and productive area of research.

Conclusion
All humans are embedded in multiplex social networks: we have different partners in different domains of interaction. Multiplex structuring processes result from the nature of the multiplex structure itself and from the natures of the individuals involved. Four categories of multiplex structuring processes that we have discussed here are based on different ways in which interdependencies between layers arise: similarity of individual characteristics, inter-domain dependency, cross-domain complementarity, and spillover of interactions across domains.

We illustrated these processes with discussion of two specific examples. First, in incidental network membership a tie is formed between two individuals in a certain domain not because they are optimal partners for each other, but by virtue of them being connected (perhaps optimally) in another, more important layer. This illustrates the potential inefficiencies that may arise when one domain drives the formation of another. The second example we discussed, was a recent model based on coupling between layers of a multiplex. An example of this sort of coupling across domains showed that it is possible to have large areas of the parameter space where network optimality may not be reached. These two examples show that multiplex structuring processes can lead to non-optimal networks, and that we should incorporate multiplex networks and their structuring processes into our analysis of the evolution of human behavior. While the development of techniques to incorporate these into our analysis is just beginning, there are already some promising directions and we expect that many more will be generated. Appreciating the multiplex and linked nature of the domains of interaction humans are involved in will not only add richness to our understanding but will prevent us from being incorrect in our analyses.

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