Mathematical processing of the signal in the track circuit for determining the location of the rolling stock

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Abstract. The article considers the issues of determining the location of rolling stock on the section of the railway track. Application of mathematical apparatus of conformal mappings is proposed and justified. Principle of conformal transformations of track circuit signal into the coordinate of place of superposition of the shunt is disclosed. Visualization of conformal transformations is presented using the example of a 420 Hz track circuit. The rules for determining the coordinate of the place of superposition of the shunt is well-phrased. Factors affecting the accuracy of determining the location of rolling stock on the section of the railway track have been determined.

1. Introduction

Increasing the speed of passenger and freight trains is a priority for the development of the railways of the Russian Federation. It is impossible to increase train speeds without providing all the necessary traffic safety conditions. One of these conditions is the track clear along the route [1]. Track circuits act as the most common track clear detection devices on railroads in the Russian Federation. The use of track circuits ensures the necessary level of traffic safety, but limits the capacity of railways due to the low accuracy of determining the coordinates of rolling stock [2]. Axle counting sensors used in many countries of the world also cannot provide the necessary accuracy of determining the coordinates of rolling stock, and also do not provide control of rail integrity. Thus, it is necessary to pay attention to improving the positioning systems of coordinates of rolling stock, while not forgetting about traffic safety.

Currently, the most accurate determination of the location of rolling stock is required on the gravity humps, because the influence of any negative factors on it is maximum [3, 4].

2. Methods

The authors propose a technique for determining the coordinates of rolling stock, based on the use of a mathematical apparatus of conformal mappings used to analyze the change in parameters of rail-wires line.

This mathematical apparatus allows real-time determination of the areas of the input resistance of the rail-wires line, which makes it possible to determine the coordinate of rolling stock on the track with high accuracy by changing the input resistance [2].

For clarity, consider the example of track circuit (Figure 1).
Assume that the carrier frequency of the signal current is 420 Hz, the length of the rail-wires line is 625 meters, rail-wires line insulation resistance range is 0.5 to 2.5 Ohms/km. According to [5], the right half-plane in conformal mapping will be changed to a circle centered at a point whose coordinates are calculated by formula (1) and a radius calculated by formula (2). Construction of areas of normal states of the quadripole of rail-wires line for different values of insulation resistance is shown in figure 2. The area of the normal state of the rail-wires line will be called the area of the input resistances calculated for the free and serviceable rail-wires line.

\[
W = \frac{2\alpha AC + AD\lambda + B\bar{C}\lambda}{2\alpha|C|^2 + 2\text{Re}(CD\lambda)},
\]

(1)

\[
R = \frac{(AD - BC)\lambda}{2\alpha|C|^2 + 2\text{Re}(CD\lambda)}.
\]

(2)

where \(A, B, C, D\) are the coefficients of the quadripole of rail-wires line; 
\(\overline{B}, \overline{C}, \overline{D}, \overline{\lambda}\) are the conjugated complex values; 
\(\lambda, \alpha\) are the coefficients.

As the load of the quadripole of rail-wires line, an area of all kinds of values with a non-negative real part of complex resistances, representing the right half-plane of the complex plane, is considered.

The conformal transformation of the right half-plane results in a plurality of input resistance values having the shape of a circle lying in the right half-plane. The size and location of the area depends entirely on the parameters of the quadripole.

We will analyzing the areas of the input resistance of the rail-wires line when superimposing a single shunt \(R_{sh}\) at its various points. We will shunting it by transverse resistance every 12.5 m.

The coefficients of the quadripole of rail-wires line \(A_{sh}, B_{sh}, C_{sh}\) and \(D_{sh}\) in this mode are according to formula (3):

\[
\begin{pmatrix}
A_{sh} & B_{sh} \\
C_{sh} & D_{sh}
\end{pmatrix}
= \begin{pmatrix}
\frac{1}{Z_w} & \text{ch}(\gamma l_2) & Z_w \cdot \text{sh}(\gamma l_2) \\
\text{ch}(\gamma l_2) & 1 & \text{ch}(\gamma l_1)
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 1 & Z_w \cdot \text{sh}(\gamma l_1) & \text{ch}(\gamma l_1)
\end{pmatrix},
\]

(3)

where \(R_{sh}\) is the shunt resistance (transverse resistance); 
\(\gamma\) is the wave propagation factor in rail-wires line, 1/km 
\(Z_w\) is the wave resistance of rail-wires line, Ohm\cdot km
$l_1$, $l_2$ are the coordinates of the shunt from the end and beginning of the rail line, respectively.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Example of conformal transformation on a complex plane}
\end{figure}

3. Results
Figure 3 shows the construction of areas of normal states of the rail-wires line (dotted line), as well as the movement of areas depending on the location of the shunt with a nominal value of 0.015 Ohm (shunt resistance of four-axle carriage) and 0.5 Ohm (defect with point insulation resistance), with an interval of 12.5 meters, with an insulation resistance of the rail line of 2.5 Ohm/km. Centers of displayed areas are indicated by crosses.

Let's analyze the results:

- The lower the introduced resistance of the shunt, the higher the accuracy of determining the coordinate of place of superposition of the shunt due to the absence of intersections of the areas of input resistances.
- The further of the distance from the beginning of the rail-wires line to the place of superposition of the shunt, the greater the region of possible values of the input resistances of the rail-wires line.
- For areas having common intersections, at different shunt resistance ratings, the place of superposition of the shunt cannot be uniquely determined by the point belonging to several areas.

Thus, the accuracy of determining the coordinate of the shunt from the obtained measurements of the input complex resistance of the rail-wires line will depend on the size of the input resistance area, on the selected sampling step during its calculation, as well as the rating of the resistance of the introduced shunt. It is not possible to determine the coordinate of the place of superposition of the shunt by calculating the coordinates of the centers of these areas (see formula 1) and binding the measurement results to these points, since these points are just abstract values of the input resistances for specific unknown values of the load of the quadripole of rail-wires line. In order for the specific (actual) value of the input resistance of the rail-wires line to be uniquely compared with the place of superposition of the shunt, it is necessary to exclude the intersection of the input resistance areas.
Enter the rules for determining the coordinate of the place of superposition of the shunt:

- each input resistance value of the quadripole of rail-wires line uniquely belongs to only one area, which corresponds to a certain coordinate of the place of superposition of the shunt;
- the first calculation area is calculated for the 1 meter coordinate of the place of superposition of the shunt;
- discreteness of calculation and sampling of obtained results is taken equal to 1 meter;
- two calculated adjacent areas corresponding to certain coordinates have only one common point of intersection (do not overlap each other). If such a point does not exist, at this sampling of calculations, then the nearest adjacent area corresponding to the requirements is selected $|W_i - W_k| - |R_i + R_k| \rightarrow \min$ (ideally, the distance between the centers of adjacent areas is equal to the sum of their radii), where $W_k$, $W_i$ are the coordinates of the centers of the area $k$ and the area $i$, respectively, $R_k$, $R_i$ are the radii of the area $k$ and the area $i$, respectively;
- in case the input resistance value ($W_j$) does not belong to adjacent design areas (see Figure 4), belonging to the area of the corresponding specific coordinate of the place of superposition of the shunt is determined by the following rule: $W_j$ belongs to area $i$ if inequality is performed $|W_i - W_j| - R_i \leq |W_k - W_j| - R_k$, otherwise, the obtained value belongs to the region $k$.

**Figure 3.** Conformal mapping $f$ of the rail-wires line input resistance areas at shunting by transverse resistance with nominal value of 0.015 Ohm (a) and 0.5 Ohm (b).
In compliance with the above rules allows you to determine the coordinate of the place of superposition of the shunt, with a certain discreteness, depending on the sampling step and the nominal value of the introduced shunt. We determine the accuracy of calculation of the coordinate of the place of superposition of the shunt at values of shunt resistance from 0.015 to 0.5 Ohm (see Figure 5).

**Figure 4.** Determination of point affiliation for non-intersecting areas

As can be seen from the figure, with a rail-wires line length of 625 m, in a wide range of changes in shunt resistance, the inaccuracy in determining the coordinate of the place of superposition of the shunt may be reach 100 (for a very bad shunt) meters. It should be borne in mind that in real
conditions the resulting resistance of the axles of the carriage, and even more so the train, is significantly less than the value of 0.06 Ohm (standard shunt). For a four-axle carriage (resistance 0.015 Ohm) the inaccuracy in determining the coordinate of the place of superposition of the shunt does not exceed 10 meters.

An increase in the accuracy of determination or deterioration of shunt sensitivity can lead to the occurrence of a case of crossing several areas in which the obtained value of the input resistance will belong to several areas corresponding to different coordinate of the place of superposition of the shunt at once, as shown in Figure 3, b. Thus, the disadvantages of this technique of determining the coordinate of the place of superposition of the shunt are the discreteness of determining the coordinate, wherein the inaccuracy can reach more than 10% of the length of the rail-wires line, which does not give a complete and continuous picture of the change in the state of the rail-wires line. In order to eliminate this drawback, it is necessary to consider characteristic points within the obtained areas (see Figure 5).

In addition to the point of the center of the input resistance region of the rail-wires line, during conformal mapping there are two more uniquely defined points characteristic of each region, by which it is possible to specify the place of superposition of the shunt on the rail-wires line (see Figure 2). These points are obtained by conforming two loads (intersecting the lines of the imaginary and real axes of the output resistances). As the load of the quadrupole, the range of values of complex resistances is considered, from the value $Z = 0$ – short-circuit to $Z = \infty$ open-circuit. Since the real part of the complex resistance is always non-negative, after the transformation according to the rules of conformal mappings, we get two intersecting circles, in the places whose intersections we get: zero ($W_0$) and infinity ($W_\infty$) points of imaginary and real axes, respectively [4, 5].

Consider the trajectory of these points to calculate the complex input resistance of the rail-wires line under the following conditions: carrier frequency of the signal current 420 Hz, length of the rail line 625 meters, shunt resistance 0.0015-0.5 Ohm, shunt overlap interval 12.5 meters, insulation resistance of the rail-wires line 2.5 Ohm/km. The figure 6 shows the change in the coordinates of points of zeros (a), and infinities (b) of the resulting calculations. For a more visual representation figure shows an enlarged fragment important area of all transformations.

Analysis of simulation results showed that:

- in case of short-circuit at the output of the quadruple of rail-wires line (zero point $W_0$ is considered), determination of the coordinate of the place of superposition of the shunt at the end of the rail-wires line is difficult (see Figure 6, a);
- in case of open-circuit at the output of the quadruple of rail-wires line (infinity points are considered $W_\infty$) determination of the coordinate of the place of superposition of the shunt uniquely throughout its length (see Figure 6, b).

4. Conclusions

1. Possibility of application of mathematical apparatus of conformal mappings confirmed by analytical calculations.

2. The graphical display of the state areas of the rail-wires line can be used as a visual and effective means of investigating the functional dependencies of its electrical characteristics on internal (conductivity of insulation, change in the resistance of rail connector and insulating rail joint) and external (interference from electric locomotives, power lines) parameters in a wide range of their change.

3. To eliminate the discreteness of the determination coordinate of the place of superposition of the shunt, it is advisable to consider the load corresponding to infinity points (the presence of insulating rail joints at the end of the rail-wires line).

4. The coordinate of the place of superposition of the shunt can be determined by comparing the calculated curves and the experimentally obtained values of the input resistance of the rail-wires line.
5. Analysis of the rail-wires line and determination of its parameters allows a graphical solution, the accuracy of which is sufficient. The use of automatic classification of rail-wires lines conditions extends the possibilities of their research in each of the main operating modes, which is especially relevant for difficult operating conditions (eg [6], [7], [8], [9]).

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