Searching for top quark pair production cross-section at LHeC and FCC-eh

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Abstract – The deep inelastic scattering mode of $t\bar{t}$ pair production at the proposed LHeC and FCC-eh is considered. We present a method to extract the top reduced cross-section related to the transversal structure function $F_{2}(x, Q^{2})$ parameterization. Numerical calculations with known kinematics of the LHeC and FCC-eh colliders are presented. The results obtained for charm and beauty pair production are comparable with the experimental data. We show that for a wide range of the momentum transfer into the top quark pair, the reduced cross-section is well described by center-of-mass energies.

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Introduction. – The highest center-of-mass energy in deep inelastic scattering of electrons on protons at HERA was reached for $\sqrt{s}$ ≃ 320 GeV [1, 2]. In recent years, the particle physics landscape has greatly evolved due to the appearance of the project of the Large Hadron electron Collider (LHeC) with the electron-proton center-of-mass energy at $\sqrt{s}$ ≃ 1.3 TeV. This collider can be the high-energy $ep/eA$ collider based at CERN [3, 4]. The LHeC energy is about 30 times the center-of-mass energy range of $ep$ collisions at HERA. The high-luminosity LHC program would be uniquely complemented by the LHeC, which was designed in an extended Conceptual Design Report (CDR) in [3]. The LHeC leads into the region of high parton densities at low $x$ values where center-of-mass energy is approximately 1 TeV. The kinematic range in the $(x, Q^{2})$ plane of the LHeC for electron and positron neutral current (NC) in the perturbative region is well below $x \approx 10^{-6}$ and extends up to $Q \approx 1$ TeV. Also this behavior will be checked at the Future Circular Collider (FCC) programme which runs at beyond a TeV in center-of-mass energy [4]. In this collider the FCC-eh with 50 TeV proton beams colliding with 60 GeV electrons. In this $ep$ collision the center-of-mass energy reaches ≃3.5 TeV. The LHeC and FCC-eh collisions lead into the region of high parton densities at small Bjorken $x$. Deep inelastic scattering measurements at FCC-eh and LHeC will allow the determination of the parton distribution functions at very small $x$. These measurements are pertinent in investigations of lepton-hadron processes at ultra-high energy (UHE) neutrino astroparticle physics [4, 5]. By electron-proton ($ep$) colliders, the top quark can be produced in pair in the deep inelastic scattering (DIS) through neutral current (NC) production. The top quark distribution in leptonproduction is dominated by the photon-gluon fusion (BGF) where an incident virtual photon interacts with a gluon from the target nucleon (see fig. 1). The total cross-section for top-pair quark production at the FCC-eh is 663 fb. At the LHeC, the $t\bar{t}$ production cross-sections, associated with the electron energies $E_{e} = 60$, 140 and 300 GeV, are 0.023, 0.120 and 0.380 pb, respectively. These values are smaller than the $\gamma p$ collision where the top pair cross-section is as large as 0.700 pb at $E_{\gamma} = 60$ GeV [6, 7]. These colliders have a broad top physical potential which can be consulted through refs. [8–10]. The $t\bar{t}$ production in DIS at the LHeC can be used to measure the $t\bar{t}\gamma$ vertex where the cross-section depends on it. In contrast at the LHC the vertex is probed through $t\bar{t}\gamma$ production. Indeed, pair production in DIS is sensitive to the gluon distribution in proton. The cross-sections, in the LHeC and FCC-eh colliders, will permit a complete unfolding of the heavier quark distributions in a hugely extended kinematic range of $Q^{2}$. By using DIS pair heavier production in an un-accessed range of $Q^{2}$ and $x$, one can study top component of the structure function. Also the nonlinear dynamics must be observed at very low $x$ values ($x < 10^{-6}$).

The small $x$ range is also relevant to the interactions of cosmic ultra-high energy neutrinos (e.g., the scattering of cosmic neutrinos from hadrons) which is related to charm and very low $x$ PDFs in comparison with emerging
data from the IceCube Collaboration [11]. The top quark contribution with mass \(172 \pm 0.5 \text{GeV} \) measured by ATLAS [12] and CMS [13] is special among all quarks.

At low \(Q^2\) where heavy quarks are not considered as active, the most standard pQCD scheme for heavy flavors is the fixed flavor number scheme (FFNS). For \(Q^2 > m_h^2\) (where \(m_h\) is the heavy quark mass), the variable flavor number schemes (VFNS) have been introduced. For realistic kinematics it has to be extended to the case of a general-mass VFNS (GM-VFNS) [14]. In GM-VFNS one should take into account quark mass, as one of the ingredients used in this scheme is the replacement of \(x\) by the rescaled variable \(\chi\) because

\[
\chi = x \left(1 + \frac{4m_h^2}{Q^2}\right).
\]

Within the GM-VFNS, heavy quark densities arise via the \(g \to Q\overline{Q}\) evolution. It would be interesting to confront with the top distribution at small \(x\) in the LHeC and FCC-eh projects.

The layout of the present paper is as follows. After reviewing the essential features of the heavy quark pair production in the next section, we calculate the production top quark cross-section of the subprocess \(\gamma^* g \to t\overline{t}\) at the LHeC and FCC-eh kinematics. To determine our numerical results, we consider heavy quark cross-sections predicted by the proton structure function parameterization in the third section. Finally, we give our summary and conclusions in the last section.

Theory. – In the small-\(x\) range, where only the gluon contribution is dominant, the heavy quark contributions \(F_k^{Q\overline{Q}}(x, Q^2)\) are given by these forms (for \(k = 2, L\)):

\[
\begin{align*}
F_2^{Q\overline{Q}}(x, Q^2) &= C_2^{Qg}(x, \xi) \otimes G(x, \mu^2), \\
F_L^{Q\overline{Q}}(x, Q^2) &= C_L^{Qg}(x, \xi) \otimes G(x, \mu^2),
\end{align*}
\]

where \(F_2^{Q\overline{Q}}\) and \(F_L^{Q\overline{Q}}\) refer to the heavy-quarks transverse and longitudinal structure functions, respectively. The \(G(x, Q^2)\) and \(g(x, Q^2)\) represent the gluon momentum distribution and gluon density, respectively,

\(G(x, Q^2) = xg(x, Q^2)\). Here \(C_{(g,k)}\) are the coefficient functions at LO and NLO approximation and \(\mu\) is the mass factorization scale. They are presented in ref. [15] in the following form:

\[
\begin{align*}
C_{k,g}(z, \zeta) &\to C_{k,g}^0(z, \zeta) + \frac{\alpha_s(\mu^2)}{4\pi} \left[ C_{k,g}^1(z, \zeta) \\
&+ C_{k,g}^1(z, \zeta) \ln \frac{\mu^2}{m_h^2} \right].
\end{align*}
\]

(2)

The symbol \(\otimes\) denotes convolution according to the usual form, \(f(x) \otimes g(x) = \int_x^1 dy f(y)g(x/y)\) while for the heavy-quark production the lower limit should be replaced by \(ax\) where \(a = 1 + 4\xi\) and \(\xi = \frac{m_h^2}{Q^2}\). The deep inelastic heavy-quarks structure functions related to the reduced cross-section are given by

\[
\sigma_{\gamma^*}\overline{Q}(x, Q^2) = F_2^{Q\overline{Q}}(x, Q^2) - \frac{y^2}{y_+} F_L^{Q\overline{Q}}(x, Q^2),
\]

(3)

where \(y = Q^2/sx\) is the inelasticity with \(s\) the ep center-of-mass energy squared and \(Y_+ = 1 + (1-y)^2\). The small \(x\) asymptotic behavior of the gluon density can be exploited by the following form:

\[
g(x, Q^2)\bigg|_{x \to 0} \sim \frac{1}{x^{1+\lambda_y}}.
\]

The quantity \(1+\lambda_y\) is equal to the intercept of the so-called BFKL pomeron. Then eq. (1) can be rewritten as

\[
F_k^{Q\overline{Q}}(x, Q^2) = G(x, \mu^2) \left[ \int_x^1 \frac{dy}{y} C_{k,g}(y, \xi) y^{\lambda_y} \right].
\]

To summarize and simplify the equations, we make the following statement:

\[
f(x) \otimes g(x) = \int_x^1 \frac{dy}{y} f(y) g(y).
\]

Thus, the above equation can be rewritten in the form convenient for further discussion,

\[
F_k^{Q\overline{Q}}(x, Q^2) = G(x, \mu^2) \left[ C_{k,g}(x, \xi) \otimes x^{\lambda_y} \right].
\]

(4)

The reduced cross-section for heavy quarks is expressed in terms of the gluon distribution as we have it:

\[
\sigma_{\gamma^*}\overline{Q} = G(x, \mu^2) \left[ C_{2,g}(x, \xi) \otimes x^{\lambda_y} - \frac{y^2}{Y_+} C_{L,g}(x, \xi) \otimes x^{\lambda_y} \right] .
\]

(5)

At small \(x\) the gluon determination comes from the extension of range and precision in the measurement of \(F_2\) and \(\partial F_2/\partial \ln Q^2\). Several methods of relating the \(F_2\) scaling violations to the gluon density at small \(x\) have been suggested previously [16]. These relations estimate the logarithmic slopes \(F_2\) with respect to the gluon distribution. Recently a relation between the gluon distribution...
and $F_2$ and $\partial F_2/\partial \ln Q^2$ has been presented in [17], with the result

$$G(x, Q^2) = \frac{1}{\Theta_{qq}(x, Q^2)} \left[ \frac{\partial F_2(x, Q^2)}{\partial \ln Q^2} - \Phi_{qq}(x, Q^2) F_2(x, Q^2) \right].$$

Indeed the measurement of $F_2(x, Q^2)$ and $\partial F_2(x, Q^2)/\partial \ln Q^2$ determine $G(x, Q^2)$ at low $x$ kinematic region. The parameterization $F_2(x, Q^2)$ was suggested in ref. [18] by BDH (i.e., Block, Durand and Ha) and also suggested for the longitudinal structure function by KKCZ (i.e., Kaptari, Kotikov, Chernikova and Zhang). In eq. (6), kernels for the quark and gluon sectors (denoted by $\Phi$ and $\Theta$) are presented by the following forms:

$$\Theta_{qq}(x, Q^2) = P_{qq}(x, \alpha_s) \ln^2 x,$n

$$\Phi_{qq}(x, Q^2) = P_{qq}(x, \alpha_s) \ln^2 x,$$

where the splitting functions up to NNLO were found in ref. [19] by the following form:

$$P_{ij}(x, \alpha_s(Q^2)) = P_{ij}^{\text{LO}}(x) + \frac{\alpha_s(Q^2)}{2\pi} P_{ij}^{\text{NLO}}(x) + \left( \frac{\alpha_s(Q^2)}{2\pi} \right)^2 P_{ij}^{\text{NNLO}}(x).$$

The exponents $\lambda_x$ and $\lambda_y$ are defined by the derivatives of the distribution functions in the form

$$\lambda_x = \partial \ln f_i(x, Q^2)/\partial \ln(1/x),$$

where $i = s, g$ and $f^{(g)}$ are the singlet structure and gluon distribution functions, respectively. The original behavior for eq. (8) was the expected expectation at sufficiently small values of $x$. In ref. [20] an exponent for the gluon distribution function at $Q^2 = 1 \text{GeV}^2$ for MSTW08 NLO computed which the fitted value with its uncertainties is obtained to be $-0.428_{-0.065}^{+0.067}$ at low values of $x$. In addition, the effective exponent values for the gluon structure function at $Q^2 = 10 \text{GeV}^2$ and $x = 10^{-4}$ were evaluated by means of different parameterizations (NNPDF3.0 [21], MMHT14 [22], CT14 [23], ABM12 [24] and CJ15 [25]). The results are as follows: $-0.20, -0.15, -0.29, -0.15$ and $-0.14$, respectively. For the gluon distribution, the intercept value by the fixed coupling leading log(1/x) BFKL solution is defined by $\lambda_x \approx -0.5$ (which is the so-called hard-pomeron exponent).

At low values of $x$, the transition from a low-$Q^2$ to a high-$Q^2$ domain is predicted with respect to the $Q^2$ dependence of the effective exponent. The asymptotic form of these exponents have been predicted [20] by the following forms:

$$\lambda_s \rightarrow -\frac{\gamma}{\rho} + \frac{3}{4\sigma \rho}, \quad \lambda_g \rightarrow -\frac{\gamma}{\rho} + \frac{1}{4\sigma \rho},$$

where $\gamma = \frac{12}{\pi^2}$ and $\beta_0 = 11 - \frac{2n_f}{3}$ ($n_f$ is the active flavor number). The variables $\sigma$ and $\rho$ are defined as

$$\sigma \equiv \left[ \ln \frac{x_0}{x} \ln \frac{Q^2}{\Lambda^2} \right]^{1/2}$$

and

$$\rho \equiv \left[ \ln \frac{Q^2}{\Lambda^2} / \ln \left( \frac{Q^2}{\Lambda^2} \right) \right]^{1/2}.$$
The heavy-quarks masses are set to $m_c = 1.5 \pm 0.15$ GeV and $m_b = 4.5 \pm 0.25$ GeV. The charm and beauty structure functions are obtained from the measured cross-sections in [29] and studied in [30,31] phenomenologically successfully in recent years. The $b$-quark density is important in Higgs production at the LHC. Also the $t$-quark density will be important to study the Higgs boson at the LHeC and FCC-eh in UHE and nonlinear $gg$ interaction effects [2] at very low values of $x$.

In fig. 2, phenomenological predictions of the charm and beauty reduced cross-sections are compared to the combined HERA data [29]. The renormalisation and factorisation scale for the heavy quarks is set to $\sqrt{s} = 318$ GeV) for charm and beauty quark production in the combined HERA data. In this figure the charm and beauty reduced cross-sections are determined using a strong coupling constant $\alpha_s^{\text{charm}}(M_Z) = 0.105 \pm 0.002$ which correspond to $\alpha_s^{\text{beauty}}(M_Z) = 0.116 \pm 0.002$. The uncertainty of the reduced charm and beauty cross-sections are due to the $F_2$ parameterization, singlet exponent and mass uncertainties. Consistency between the determined results with respect to the experimental data can be observed. These results are also comparable with results in refs. [30,31]. We observe that the uncertainties of $\sigma^{\text{charm}}_F$ are lower than the experimental uncertainties. Here there are two reasons for this process. Usually, the data obtained for $b\bar{b}$ pair-production in DIS have larger uncertainties than $c\bar{c}$ pair-production. Also, the $F_2$ parameterization is based on $n_f = 4$ as suggested by BDH in ref. [18].

Now we focus our attention on the phenomenological prediction for the top quark production in the LHeC and FCC-eh collisions with center-of-mass energy 1.3 TeV and 3.5 TeV, respectively. The $t\bar{t}$ production at the Tevatron collider and LHeC were discussed in refs. [14] and [28] at NNLO. The total cross-section for top quark production at $t\bar{t}$ photoproduction is 1.14 pb as reported in refs. [2–4]. Determination of $\alpha_s$ at the LHeC is according to the H1 result at NNLO ($\alpha_s(M_Z^2) = 0.1157 \pm 0.0020\text{(exp.)} \pm 0.0029\text{(thy.)}$) with 0.2% uncertainty from the LHeC and 1% when combined with HERA [2]. Here we note that the LHeC uncertainties are simulated [2–4]. The singlet and gluon exponents are determined in accordance with data in refs. [20] and [32]. The average value of the parameter $y$ was chosen equal to $\langle y \rangle = 0.5$, since the minimum and maximum values of the top reduced cross-section are determined by the inelasticity $y = 1$ and $y = 0$, respectively. Figure 3 shows the theoretical prediction for $\sigma^{\text{charm}}_F$ as a function of $x$ using the parameterization of $F_2(x,Q^2)$. The solid curves are correspondent to the scale choice $Q^2 = \frac{1}{4} m_t^2$, $Q^2 = m_t^2$ and $Q^2 = 4m_t^2$. Model uncertainties arise from the variations of the $F_2$ parameterization, top quark mass and the singlet exponent behavior. Our numerical results accompanied with the statistical errors are summarized in this figure (i.e., fig. 3). Here the top reduced cross-sections are plotted as a function of $x$ at values of $\frac{1}{4} m_t^2 \leq Q^2 \leq m_t^2$ (GeV)$^2$. We observe that the reduced cross-sections, $\sigma^{\text{charm}}_F$, are in the range of 0.01–0.4 as $x$ decreases.

In fig. 4, we compared the results of top reduced cross-section for center-of-mass energies $\sqrt{s} = 1.3$ and 3.5 TeV separately. In this figure $\sigma_t$ is plotted for $100 < Q^2 < 4m_t^2$ (GeV)$^2$) and it is assumed that inelasticity is constant in this process, $y = 0.5$. At fixed center-of-mass energy, $\sqrt{s}$, the variables are related by the following rewritten form based on the rescaled variable $\chi$: $Q^2 = s \chi y$. The effects of the $y$ constant for the reduced cross-sections have been shown in this figure. In $Q^2$ range a enhancement is

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig2.png}
\caption{The charm and beauty components of the reduced cross-section given by $\sigma^{\text{charm}}_F$ and $\sigma^{\text{beauty}}_F$ as a function of $x$ at $Q^2 = 120$ GeV$^2$ accompanied with statistical errors. Experimental data are from the H1 and ZEUS Collaborations, ref. [29].}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig3.png}
\caption{Theoretical predictions for $\sigma^{\text{charm}}_F$ at $y = 0.5$ as a function of $x$ using the $F_2$ parameterization. The curves are calculated using $Q^2 = \frac{1}{4} m_t^2$, $m_t^2$ and $4m_t^2$ (GeV)$^2$ accompanied with statistical errors.}
\end{figure}
obtain explicit expression for observable until $Q^2 \simeq m_t^2$. Then a depletion is observable, because for $Q^2 > m_t^2$ we do not expect the inelasticity to be 0.5. The validity of these results to the top reduced cross-section could be checked in the future at the proposed LHeC and FCC-eh colliders.

Summary and conclusion. – We have studied the production of top-pair quarks in new electron proton collisions (i.e., LHeC and FCC-eh). The subprocess $\gamma^* g \rightarrow t\bar{t}$ will be one kind of important production channels at LHeC and FCC-eh. The production of charm and beauty quarks was studied in the basic processes of $c\bar{c}$ and $b\bar{b}$ production at HERA. The method relies on the DGLAP evolution equations and the proton structure function parameterization. We focus on the kinematic region of low-$x$ and high-$Q^2$ values which are proposed at new colliders. The obtained explicit expression for $\sigma_{\gamma^* g \rightarrow t\bar{t}}$ is entirely determined by the $F_{2D}^{DDH}$ parameterization which extended to values of high $Q^2$. The results of numerical calculations for charm and beauty as well as comparisons with available experimental data are presented. We considered the top reduced cross-section behavior at low values of $x$ in a wide range of $Q^2$ values.

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