Matrix Model on Z-Orbifold

Akiko MIYAKE and Akio SUGAMOTO

Department of Physics, Ochanomizu University,
2-1-1, Otsuka, Bunkyo-ku, Tokyo 112-8610, Japan

Abstract

Six dimensional compactification of the type IIA matrix model on the Z-orbifold is studied. Introducing a \( \mathbb{Z}_3 \) symmetry properly on the three mirror images of fields in the \( N \)-body system of the supersymmetric D0 particles, the action of the Matrix model compactified on the Z-orbifold is obtained. In this matrix model \( \mathcal{N} = 1 \) supersymmetry is explicitly demonstrated.
Introduction

Recent development of the string theory makes it possible to investigate the strong coupling regime of the theory as well as the inter-relationship between various string models (or phases) by the duality. To establish the duality (exchange of "electricity" and "magnetism") it is necessary to have a "magnetic" object of string, in addition to the original "electric" string.

Once it is recognized by Witten [1] that the dynamics of the extended objects, D-branes [2], playing an important role in the duality, can be described in terms of the dimensionally reduced Supersymmetric Yang-Mills Theory (SYM), the non-perturbative study of string becomes more familiar and more realistic. In particular, the dynamics of the simplest point-like D0 branes in 11 dimensions may play the fundamental role in such a study [3]. Since the coordinates of a gas, consisting of $N$ D0-branes, are represented by $N \times N$ hermitian matrices, its dynamics is given in terms of the SYM theory dimensionally reduced to one temporal dimension, that is the quantum mechanics of the SUSY Matrix model. Hereafter, we refer this model simply as the Matrix model.

If the Matrix model is quantized in the light-cone gauge, the model reproduces the discrete version of the super-membrane theory, since the discretized row and column indices $\sigma$ and $\rho$ of the Matrix, $X^\mu(t)_{\sigma\rho}$, naturally become the coordinates of a membrane, $X^\mu(t, \sigma, \rho)$ [4].

Compactification of the Matrix model is an interesting topic, in particular when the model is located on the orbifold space, \textit{i.e.}, the space of torus divided by its discrete symmetry.

First, since the 11d type IIA (yet-unknown) M-theory compactified on a simplest orbifold space, $S^1/\mathbb{Z}_2$, is shown to be connected with the 10d $E(8) \times E(8)$ heterotic string theory [5], from which we obtain the well-known E(6) grand unified theory, after compactification of 6d space by Calabi-Yau manifold [6]. The Matrix model describing this (yet-unknown) M-theory may be the type IIA Matrix model.

Then, the compactification of this type IIA Matrix model on the orbifold space $S^1/\mathbb{Z}_2$ is demonstrated explicitly by [7], [8], [9], [10], and others.

Second, when we intend to obtain "realistic" 4d string models, by compactifying the extra six dimensions in the 10d heterotic string models, the orbifold compactification is a good method of breaking the supersymmetries. Since the orbifold space does not differ so much from the torus compactification, we can explicitly estimate various physical quantities (amplitudes) in this orbifold com-
pactification, without much difficulty compared with the torus compactification. Among the orbifold compactifications, we have found various realistic models with three generations, having extra U(1) gauge groups and $\mathcal{N}=1$ supersymmetry [11], [12], [13].

If the M-theory is compactified on a simple torus, $T^6$, it is claimed that the extra degrees of freedom, representing the wrapped D6 branes, become massless and should be included, in order that the compactified M-theory reproduces the strong coupling region of the corresponding type IIA superstring theory [14]. Then, the theory becomes very complicated, but the D6 brane may play the role of an “electromagnetic dual” of the D0 brane. These extra degrees of freedom coming from the wrapped D6 branes may decouple in the other 6d compactification space, such as the Calabi-Yau manifold [15].

In this paper, we study the second problem mentioned above, that is the six dimensional compactification of the quantum mechanics of the type IIA Matrix model by the $\mathbb{Z}$-orbifold [16], the most fundamental space among the various orbifold spaces. Therefore, in our study, we include only the D0-brane degrees of freedom (“magnetic” or “electric” part), without incorporating the extra degrees of freedom (“electric” or “magnetic” part) existing in some cases of the M-theory.

If we consider our approach as the study of the type IIA Matrix model on the non-compact orbifold, $\mathbb{C}^3/\mathbb{Z}_3$, then our obtained Matrix model may become a candidate of M-theory on $\mathbb{C}^3/\mathbb{Z}_3$, since the wrapping of D6 branes is not possible in this case.

The Matrix model on the $\mathbb{Z}$-orbifold is obtained from the $3N \times 3N$ Matrix model, by imposing the $\mathbb{Z}_3$ symmetry of the $\mathbb{Z}$-orbifold, and $\mathcal{N}=1$ supersymmetry is explicitly checked in the obtained model.

By generalizing our analysis to the more elaborate orbifold spaces, we may find the more realistic analysis of string model on the basis of the standard model with three generations of quarks and leptons.

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**Matrix Model on Z-Orbifold**

We start with the type IIA Matrix model, *i.e.* the 10 dimensional Super Yang-Mills theory (SYM) dimensionally reduced to the single temporal dimension. The
action of this model is written by

$$S_M = \text{Tr} \int d\tau \left( \frac{1}{2R} (D_\tau X^I)^2 - \frac{R}{4} [X^I, X^J]^2 + \Theta^T D_\tau \Theta + i R \Theta^T \Gamma_I [X^I, \Theta] \right).$$  \hspace{1cm} (1)

Here, $I, J = 0, \ldots, 9$ label the Minkowski space-time indices, $R$ is the radius of the compactified 11-th dimension, $\Gamma_I$ is the 10d gamma matrix, and $X^I$ and $\Theta$ are $3N \times 3N$ hermitian matrices, representing the gauge fields and the gaugino fields of the original SYM, respectively. The gaugino fields $\Theta$ are given by the Majorana-Weyl spinors, having eight degrees of freedom on the mass shell. In the terminology of the $3N$-body D0 brane system, $X^I$ are the bosonic coordinates of the D0 branes. The $D_\tau$ is the covariant derivative defined by

$$D_\tau \equiv \partial_\tau - i [A_\tau, \ ].$$  \hspace{1cm} (2)

Starting with this action, we study the action of the Matrix model compactified on the $\mathbf{Z}_3$-orbifold of $\mathbf{T}^6/\mathbf{Z}_3$. For this purpose, it is convenient to use the complex notations for the six spacial dimensions which we are going to compactify. By introducing complex coordinates $Z_i, (i = 1, 2, 3)$,

$$Z_i \equiv X^{2i+2} + iX^{2i+3},$$  \hspace{1cm} (3)

the torus lattice $\mathbf{T}^6$ of the $\mathbf{Z}_3$-orbifold is defined by

$$Z_i \simeq Z_i + r \simeq Z_i + r e^{\pi i/3},$$  \hspace{1cm} (4)

where $r$ is the compactification size of the six spacial dimensions. This lattice has a $\mathbf{Z}_3$ symmetry, under

$$Z_i \simeq e^{2\pi i/3} Z_i,$$  \hspace{1cm} (5)

for $i = 1, \ldots, 3$, so that we can divide the torus lattice by this $\mathbf{Z}_3$ symmetry, and obtain the $\mathbf{Z}_3$-orbifold.

Denoting uncompactified and compactified coordinates by $X^\mu_i$ and $Z_i$, respectively ($\mu = 0, \ldots, 3$ and $i = 1, \ldots, 3$), where the complex coordinates $Z_i$ are assumed to satisfy the condition of the torus lattice (4).

We have to impose furthermore the $\mathbf{Z}_3$ symmetry (5) on the bosonic as well as the fermionic coordinates. Under the $\mathbf{Z}_3$ symmetry, three points (or three mirror images) on the torus are identified. Corresponding to this identification we have to prepare three copies (or three mirror images) of the fields, and identify them by
the $\mathbb{Z}_3$ symmetry up to the complex phases. Therefore, we start from the $3N \times 3N$ matrices for the $N$-body system of the D0 branes.

Now the $\mathbb{Z}_3$ invariance we impose on the bosonic and fermionic fields are given as follows:

\begin{align}
X_{\mu}^\prime &= MX_{\mu}^\prime M^\dagger, \\
Z_i &= \alpha_i MZ_i M^\dagger, \\
\Theta &= \hat{\alpha} M\Theta M^\dagger,
\end{align}

where the matrix $M$ is the generator of the $\mathbb{Z}_3$ transformation on the fields, and it satisfies $M^3 = 1_{3N \times 3N}$. The complex phases $\alpha_j$ and $\hat{\alpha}$ for the bosonic and fermionic fields respectively appearing under the $\mathbb{Z}_3$ transformation also satisfy $M^3 = 1$ and $(\alpha_j)^3 = (\hat{\alpha})^3 = 1$. Then, $M$, $\alpha_j$ and $\hat{\alpha}$ can be written as follows:

\begin{align}
M &= \begin{pmatrix} 0 & e^{i\phi_3} & 0 \\
0 & 0 & e^{i\phi_1} \\
e^{i\phi_2} & 0 & 0 \end{pmatrix}, \\
\alpha_j &= \exp(2\pi i \frac{n_j}{3}), \\
\hat{\alpha} &= \exp(2\pi i \sum_{j=2}^4 n_j b_j^\dagger b_j),
\end{align}

where $n_j$ is an integer and a relation, $\phi_1 + \phi_2 + \phi_3 = 2\pi n$ ($n =$ integer), is assumed to hold. In Eq. (11), there appear the raising and lowering operators of the five “spins” constituting the 10d spinor. These are defined, as usual, by

\begin{align}
\begin{cases}
b_0 = \frac{1}{2}(\Gamma^1 - \Gamma^0) \\
b_j = \frac{1}{2}(\Gamma^{2j} - i\Gamma^{2j+1}) , \quad j = 1, \ldots, 4
\end{cases}
\end{align}

and $\Gamma^\mu$ satisfy

\begin{align}
\{\Gamma^\mu, \Gamma^\nu\} &= 2\eta^{\mu\nu} \\
(\Gamma^0)^2 &= -1 , \quad (\Gamma^i)^2 = 1.
\end{align}

Then, we can understand in Eqs.(7) and (8) that the $\mathbb{Z}_3$ rotation of the complex coordinates induces exactly that of the spinors by $\hat{\alpha}$, and the matrix $M$ interchanges cyclically the 3 “mirror” images of the bosonic and fermionic fields.

The 10d spinor on the mass shell, can be expressed as a set of $\{\Theta_{\text{\tiny \#}}\}$, where $+$ means the “spin” at the position $j = 0, \ldots, 4$ is occupied, while -
means the “spin” is unoccupied. Due to the Majorana condition $B \Theta = \Theta^*$, with $B = \Gamma^3 \Gamma^5 \Gamma^7 \Gamma^9$, we have the following relation:

$$
\Theta^*_{(-,s_2,s_3,s_4,s_5)} = (s_2 s_3 s_4 s_5) \Theta_{(-,-s_2,-s_3,-s_4,-s_5)},
$$

(14)

where $s_j = \pm$. Then, we can decompose the Majorana fermion $\Theta$ into 3 components, $\Theta_1$, $\Theta_\omega$, and $\Theta_{\omega^2}$, depending on the eigen-value $\alpha = 1, \omega, \omega^2$ for the operation $\hat{\alpha}$, respectively, where $\omega = \exp(2\pi i/3)$.

We can recognize that there are three cases for $\hat{\alpha}$, namely,

$$(n_2, n_3, n_4) = (1, 1, 1), (1, 1, 2), (1, 2, 2).$$

(15)

In the case of $(n_2, n_3, n_4) = (1, 1, 1)$, $\Theta_1$ is one (Majorana) 4d spinor whose charge conjugated field is included within itself, $\Theta_\omega$ are the 3 component 4d spinors, and $\Theta_{\omega^2}$ are the charge conjugation of $\Theta_\omega$. These $\Theta_1$, $\Theta_\omega$, and $\Theta_{\omega^2}$ transform $\{1\}$, $\{3\}$, and $\{3^*\}$ under the flavor SU(3) symmetry which corresponds to the rotational symmetry in the 3d complex spaces with axes $Z_i(i = 1, \ldots, 3)$. In the $(1, 1, 2)$ case $\Theta_\omega$ are the charge conjugation of $\Theta_1$, and $\Theta_{\omega^2}$ are invariant under the charge conjugation, while in the $(1, 2, 2)$ case, $\Theta_1$ are the charge conjugation of $\Theta_{\omega^2}$, and $\Theta_\omega$ are invariant under the charge conjugation.

In the following, we adopt the case of $(1, 1, 1)$, for simplicity. Then, one finds that $3 \times 3$ block matrix structure of the uncompactifited transverse coordiantes, the compactified coordiantes, and the fermionic variables reads

$$
X^\mu_{/\gamma} = \begin{pmatrix}
H & A e^{i\phi_3} & A^\dagger e^{-i\phi_2} \\
A^\dagger e^{-i\phi_3} & H & A e^{i\phi_1} \\
A e^{i\phi_2} & A^\dagger e^{-i\phi_1} & H
\end{pmatrix}^{\mu}_{3N \times 3N}
$$

(16)

$$
Z_i = \begin{pmatrix}
B_{1i} & e^{-i\phi_3} B_{2i} & e^{-i\phi_2} B_{3i} \\
e^{-i\phi_3} B_{3i} & \alpha_{1}^{-1} B_{1i} & \alpha_{2} e^{-i\phi_2} B_{2i} \\
e^{i\phi_2} B_{2i} & \alpha_{1}^{-1} e^{-i\phi_1} B_{3i} & \alpha_{2}^{-1} B_{1i}
\end{pmatrix}_{3N \times 3N}
$$

(17)

$$
\Theta = \sum_a \Theta^a T^a
$$

(18)

$$
T^a = \begin{pmatrix}
\hat{H} & \hat{A} e^{i\phi_3} & \hat{A}^\dagger e^{-i\phi_2} \\
\hat{A}^\dagger e^{-i\phi_3} & \hat{H} & \hat{A} e^{i\phi_1} \\
\hat{A} e^{i\phi_2} & \hat{A}^\dagger e^{-i\phi_1} & \hat{H}
\end{pmatrix}^a_{3N \times 3N}
$$

(19)

for $\Theta$ with $\alpha = 1$,

$$
T^a = \begin{pmatrix}
\hat{B}_1 & \alpha^{-1} e^{i\phi_3} \hat{B}_2 & \alpha^{-2} e^{-i\phi_2} \hat{B}_3 \\
\hat{e}^{-i\phi_3} \hat{B}_3 & \alpha^{-1} \hat{B}_1 & \alpha^{-2} e^{i\phi_1} \\
\hat{e}^{i\phi_2} \hat{B}_2 & \alpha^{-1} e^{-i\phi_1} \hat{B}_3 & \alpha^{-2} \hat{B}_1
\end{pmatrix}^a_{3N \times 3N}
$$

for $\Theta$ with $\alpha = \omega, \omega^2$. 

6
where
\[
\Omega_j \equiv \begin{pmatrix} 1 & \alpha_j^{-1} & \alpha_j^{-2} \end{pmatrix}.
\] (20)

In the above expressions, $H$ and $\hat{H}$ stand for arbitrary hermitian $N \times N$ matrices, and $A$, $B_{1i}$, $B_{2i}$, $B_{3i}$, $\hat{A}$, $\hat{B}_{1}$, $\hat{B}_{2}$ and $\hat{B}_{3}$ stand for arbitrary complex $N \times N$ matrices.

The original action of the type IIA Matrix model compactified on the $\mathbb{Z}$-orbifold is now deformed to
\[
S = \text{Tr} \int d\tau \left\{ \frac{1}{2R} (D_{\tau} X_{\mu}^i)^2 + \frac{1}{2R} (D_{\tau} Z_i) (D_{\tau} Z_i^\dagger) - \frac{R}{4} \left( [X_{\mu}^i, X_{\nu}^i]^2 + 2 [X_{\mu}^i, Z_j] [X_{\nu}^i, Z_j] + \frac{1}{2} [Z_i, Z_j] [Z_i^\dagger, Z_j^\dagger] + \frac{1}{2} [Z_i, Z_j^\dagger] [Z_i^\dagger, Z_j] \right) \right. \\
\left. + \Theta_i D_{\tau} \Theta + R (\Theta \Gamma_i [X_i^i, \Theta] + \Theta b_{i+1} [Z_i, \Theta] + \Theta b_{i+1}^\dagger [Z_i^\dagger, \Theta]) \right\}. (21)
\]

Here we impose the gauge fixing condition under the background field of $X_{d0}$, that is
\[
\partial_{\tau} X^0 - i [X^i_{d}, X^i] = 0, (i = 1, \ldots, 9). (22)
\]

By decomposing the $3N \times 3N$ matrices into the sum of the tensor products $(N \times N$ matrices) $\otimes (3 \times 3$ matrices), we have
\[
X_{\mu}^i = H^\mu \otimes 1_3 + A^\mu \otimes M + A^{\mu \dagger} \otimes M^\dagger, (23)
\]
\[
Z_i = (B_{1i} \otimes 1_3 + B_{2i} \otimes M + B_{3i} \otimes M^\dagger) \Omega_i, (24)
\]
\[
\Theta = \Theta_i (H_{i}^\mu \otimes 1_3 + A_{i}^\mu \otimes M + A_{i}^{\mu \dagger} \otimes M^\dagger) \\
+ \Theta_{i}(B_{1i}^a \otimes 1_3 + B_{2i}^a \otimes M + B_{3i}^a \otimes M^\dagger) \Omega_i \\
+ \Theta_{i}(B_{1i}^{a \omega} \otimes 1_3 + B_{2i}^{a \omega} \otimes M + B_{3i}^{a \omega} \otimes M^\dagger) \Omega_i^\dagger, (25)
\]
\[
C = C_{gh} \otimes 1_3 + A_{gh} \otimes M + A_{gh}^\dagger \otimes M^\dagger, (26)
\]
\[
\bar{C} = C_{gh}^\dagger \otimes 1_3 + A_{gh}^\dagger \otimes M^\dagger + A_{gh} \otimes M,
\]

where $C$ and $\bar{C}$ are $3N \times 3N$ ghost and anti-ghost fields, $C_{gh}$ is the anti-commuting $N \times N$ hermitian matrix and $A_{gh}$ is the anti-commuting $N \times N$ arbitrary complex matrix.

Performing the trace with respect to $3 \times 3$ matrices, the bosonic action, the fermionic action and the ghost action of our model are obtained. The resulting actions are given in Eqs. (32), (33) and (34) in the appendix, where $\Omega_i = \Omega$. 

7
This is the action of the type IIA Matrix model in which the six spacial dimensions are compactified on the $\mathbb{Z}$-orbifold. The effective action of this model in the presence of the classical configuration of $N$ D0-branes, and the analysis of the Matrix model being compactified on the spaces $S^1/\mathbb{Z}_2 \times T^6/\mathbb{Z}_3$, will be given in our forthcoming paper [17].

Finally, we study the structure of supersymmetries. More precisely, for our choice of $(n_2, n_3, n_4) = (1, 1, 1)$, the isotopic structure of the bosonic variable, $X^\mu$, and the fermionic variable, $\Theta_1$, is the same, both expressed in terms of $H$ and $A$, so that a naive counting gives $3N^2$ degrees of freedom to both of them. After a usual gauge fixing, two degrees of freedom among the vector fields, $X^\mu$, remain to be physical, and the fermionic field, $\Theta_1$, on the mass shell, contains two $\{1\}$’s under the flavor SU(3) symmetry. Hence, the bosonic and fermionic degrees of freedom for the uncompactified dimensions are the same number, $6N^2$. As for the pair of bosonic fields, $Z^{i=1,...,3}$, and the fermionic fields, $\Theta_\omega$, the isotopic structure of both of them are expressed by $B_{1i}$, $B_{2i}$, $B_{3i}$, $\hat{B}_1$, $\hat{B}_2$ and $\hat{B}_3$, giving $2N^2$ degrees of freedom. The degrees of freedom coming from the Lorentz structure gives the factor three to $Z^{i=1,...,3}$, while the same factor three exists in $\Theta_\omega$ on the mass shell, being attributable to $\{3\}$ under the flavor SU(3) symmetry. The same counting holds for the other pair of $Z^{i=1,...,3}$ and $\Theta_{\omega^2}$. Hence, the bosonic and fermionic degrees of freedom in the compactified dimensions are both $18N^2$.

Therefore, the uncompactified dimensions give $6N^2$ vector multiplets of (gauge bosons, gauginos), while the compactified dimensions give $18N^2$ chiral multiplets of (higgs, higgsinos).

Our starting action is that of SYM, which is invariant under the following supersymmetry, generated by the generator $Q$ and its spinorial transformation parameter $\epsilon$:

$$\bar{\epsilon}Q = -\text{Tr} \left\{ \left( \bar{\epsilon} \Gamma^I \Theta \right) \left( D_r X_I \right) + \left( 2\Sigma^{0I} D_r X_I - R \Sigma^{IJ} [X_I, X_J] \right) \epsilon_\alpha \Phi^\dagger_\alpha \right\}, \quad (27)$$

where $\Sigma^{IJ} = 1/4[\Gamma^I, \Gamma^J]$.

After decomposing the bosonic and fermionic fields as well as the spinorial parameter $\epsilon$ into the various components, following the analysis of the $\mathbb{Z}$-orbifold compactification given above, we find the following expression for the terms of $\bar{\epsilon}Q$, having time-derivative of the bosonic fields, or the terms with the conjugate momenta of the bosonic fields:

$$\bar{\epsilon}_\alpha \text{Tr} \left( \Gamma^I_{\alpha\beta} \dot{X}_I \Theta_\beta \right) = 3\epsilon_s^{[234]} \text{Tr} \left( \dot{H}^{2+i3} \dot{H} + \dot{A}^{2+i3} \dot{A}^\dagger + (\dot{A}^{2-i3})^\dagger \dot{A} \right) \tilde{\psi}_s^{[1234]}$$
the steps given in [8] by demonstrating furthermore 

where \( \hat{H}^{2+i3} = \hat{H}^2 + i\hat{H}^3 \), and so on.

Here, \( \epsilon \) and \( \Theta \) are 10d Majorana-Weyl fermions, so that their components are defined as follows:

\[
\bar{\epsilon} = <0|b_1\epsilon_s^{[1]} + <0|b_4b_3b_2\epsilon_s^{[234]} + \sum_{i=2}^{4} <0|b_i\epsilon_s^{[i]} + \frac{1}{2}\sum_{i,j=2}^{4} <0|b_jb_ib_1\epsilon_s^{[1ij]} \] (29)

\[
\Theta = \psi_s|0 > + \psi_s^{[1234]}b_1b_2b_3b_4|0 > + \sum_{i=2}^{4} \psi_s^{[i]}b_i^\dagger b_i^\dagger|0 > + \frac{1}{2}\sum_{i,j=2}^{4} \psi_s^{[ij]}b_i^\dagger b_j^\dagger|0 >, \quad \text{ (30)}
\]

with the Majorana conditions of \( \epsilon_s^{[1]} = \epsilon_s^{[234]} \), \( \epsilon_s^{[i]} = -\frac{1}{2}\epsilon_{1ijk}\epsilon_s^{[1jk]} \), and \( \psi_s^* = \psi_s^{[1234]} \), \( \psi_s^{*[ij]} = -\frac{1}{2}\epsilon_{ijkl}\psi_s^{[kl]} \).

Now, we find that the bosonic field \( Z_i^1 \) is transformed to the fermionic field \( (\Theta_1)_s \), by the SUSY transformation with the parameter \( \epsilon_s^{[234]} \), while the bosonic field \( B_i \) is transformed to the fermionic field \( (\Theta_{\omega^2})^{[ij]} \) by the SUSY transformation with the parameter \( \epsilon_s^{[ij]} \). When \( \Omega = \Omega^{-1} \), \( B_i \) is transformed to \( (\Theta_{\omega})^{[1i]} \). In other words, the SUSY transformation obtained here for the whole fields in our model, is defined by restricting the transformation parameter \( \epsilon \) as

\[
\epsilon_s^{[234]} = \epsilon_s^{[i]} = \epsilon_s^{*[1]} = -\frac{1}{2}\epsilon_{1ijk}\epsilon_s^{*[1jk]} \equiv \eta, \quad \text{ (31)}
\]

where \( \eta \) is a common SUSY transformation parameter, expressed in terms of a 4d chiral fermion on the mass shell. Hence, the \( \mathcal{N} = 1 \) SUSY is explicitly demonstrated.

In this model, \( (X^\mu, \Theta_1) \) can be considered as a vector multiplet, while \( (Z_i^1, \Theta_{\omega^2}) \) can be three chiral multiplets. If these three chiral multiplets correspond to three generations of quarks and leptons, this \( Z_3 \) orbifold model becomes more interesting. In order to clarify the existing gauge group of our model, we have to follow the steps given in [8] by demonstrating furthermore \( Z_2 \) orbifoldong and S-duality transformation.

The number of supersymmetry depends on the compactified spaces. Different choices of the 6d orbifold spaces may lead to a different analysis of the 4d Matrix models.
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Appendix

$$S \text{ (bosonic action)} = 3\text{Tr} \int \frac{dt}{2R} \left\{ \left( \dot{H}^i - i[H_0, H^i] + [A_0, A^i] + [A_0^t, A^i]\right)^2 + 2 \left( \dot{A}^i - i([H_0, A^i] + [A_0, H^i] + [A_0^t, A^i]) \right) \right. \right.$$  

$$\left. \left( \dot{A}^t - i([H_0, A^t] + [A_0, A^t] + [A_0^t, H^t]) \right) \right.$$  

$$\left\{ \Delta_{1i} - i([H_0, B_{3i}] + (A_0 B_{3i} - \alpha B_{3i} A_0) + (A_0^t B_{3i} - \alpha^{-1} B_{3i} A_0)) \right.$$  

$$\left\{ \Delta_{2i} - i([H_0, B_{2i}] + (\alpha A_0 B_{2i} - B_{2i} A_0) + (\alpha^{-1} A_0^t B_{2i} - B_{2i} A_0)) \right.$$  

$$\left\{ \Delta_{3i} - i([H_0, B_{1i}] + (\alpha A_0^t B_{1i} - B_{1i} A_0) + (\alpha^{-1} A_0^t B_{2i} - B_{2i} A_0)) \right.$$  

$$\frac{R}{4} \left\{ ([H_0, H^i] + [A^i, A^i] + [A^t, A^i])^2 + 2 ([H_0, B_{3i}] + (A_0 B_{3i} - \alpha B_{3i} A_0) + (A_0^t B_{3i} - B_{3i} A_0)) \right.$$  

$$\left\{ ([H_0, B_{2i}] + (\alpha A_0 B_{2i} - B_{2i} A_0) + (\alpha^{-1} A_0^t B_{2i} - B_{2i} A_0)) \right.$$  

$$\left\{ ([H_0, B_{1i}] + (\alpha A_0^t B_{1i} - B_{1i} A_0) + (\alpha^{-1} A_0^t B_{2i} - B_{2i} A_0)) \right.$$  

$$\left\{ ([H_0, B_{3j}] + (\alpha A_0 B_{2j} - \alpha B_{3j} A^i) + (A_0^t B_{3j} - \alpha^{-1} B_{2j} A^i)) \right.$$  

$$\left\{ ([H_0, B_{2j}] + (\alpha A_0 B_{3j} - B_{3j} A^i) + (\alpha^{-1} A_0^t B_{1j} - B_{1j} A^i)) \right.$$  

$$\left\{ ([H_0, B_{1j}] + (\alpha A_0^t B_{1j} - B_{1j} A^i) + (\alpha^{-1} A_0^t B_{2j} - B_{2j} A^i)) \right.$$  

$$\left\{ ([B_{1i}, B_{1j}] + (\alpha B_{2i} B_{3j} - \alpha B_{3i} B_{2j} + (\alpha B_{3i} B_{2j} - \alpha^{-1} B_{2i} B_{3j})) \right.$$  

$$\left\{ ([B_{1i}, B_{1j}] + (\alpha^{-1} B_{2i} B_{3j} - \alpha B_{3i} B_{2j} + (\alpha B_{3i} B_{2j} - \alpha^{-1} B_{2i} B_{3j}) \right.$$  

$$\left\{ ([B_{2i}, B_{2j}] + (\alpha B_{3i} B_{1j} - \alpha^{-1} B_{1j} B_{3i}) + (\alpha^{-1} B_{1i} B_{3j} - B_{3j} B_{1i})) \right.$$  

$$\left\{ (\alpha^{-1} B_{2i} B_{2j} + (\alpha B_{3i} B_{1j} - B_{1j} B_{3i}) + (B_{1i} B_{3j} - \alpha B_{3j} B_{1i})) \right\}$$
\[
\frac{1}{2} \left[ (\alpha^{-1}[B_{3i}, B_{3j}] + (\alpha B_{1i}B_{2j} - B_{2j}B_{1i}) + (B_{3i}B_{1j} - \alpha B_{1i}B_{2j})) \\
\left(\alpha[B_{3i}, B_{3j}] + (B_{3i}B_{2j} - \alpha^{-1}B_{2j}B_{1i}) + (\alpha^{-1}B_{2j}B_{1i} - B_{1j}B_{2i}) \right) \right]
\]
\[(A^t \Theta_{\omega, 2} B_{2, \omega} - \alpha (\Theta_{\omega, 2} B_{2, \omega}) A^t)]
\[+ (\Theta_{\omega, 2} B_{2, \omega} \Gamma_{\theta} [H^t, (\Theta_{\omega, 2} B_{2, \omega})] + (A^t (\Theta_{\omega, 3} B_{1, \omega}) - \alpha^{-1} (\Theta_{\omega, 2} B_{1, \omega}) A^t)]
\[+ (A^t (\Theta_{\omega, 2} B_{3, \omega}) - \alpha (\Theta_{\omega, 2} B_{3, \omega}) A^t)]
\[+ (\Theta_{\omega, 2} B_{3, \omega} \Gamma_{\theta} [H^t, (\Theta_{\omega, 2} B_{3, \omega})] + (A^t (\Theta_{\omega, 2} B_{2, \omega}) - \alpha^{-1} (\Theta_{\omega, 2} B_{2, \omega}) A^t)]
\[+ (A^t (\Theta_{\omega, 2} B_{1, \omega}) - \alpha (\Theta_{\omega, 2} B_{1, \omega}) A^t)]
\[+ (\Theta_{1} \hat{H} b_{i+1}^t [B_{1, i}, (\Theta_{\omega, 2} B_{1, \omega})] + \alpha^{-1} [B_{2, i}, (\Theta_{\omega, 2} B_{3, \omega})] + \alpha [B_{3, i}, (\Theta_{\omega, 2} B_{3, \omega})])
\[+ (\Theta_{1} \hat{A} b_{i+1}^t ((\alpha B_{1, i} (\Theta_{\omega, 2} B_{2, \omega}) - (\Theta_{\omega, 2} B_{2, \omega}) B_{1, i}) + (B_{2, i} (\Theta_{\omega, 2} B_{1, \omega}) - \alpha^{-1} (\Theta_{\omega, 2} B_{1, \omega}) B_{2, i})
\[+ (\alpha^{-1} B_{3, i} (\Theta_{\omega, 2} B_{3, \omega}) - \alpha (\Theta_{\omega, 2} B_{3, \omega}) B_{3, i})
\[+ (\Theta_{1} \hat{A}^t b_{i+1}^t ((-\alpha^{-1} B_{1, i} (\Theta_{\omega, 2} B_{3, \omega}) - (\Theta_{\omega, 2} B_{3, \omega}) B_{1, i}) + (\alpha B_{2, i} (\Theta_{\omega, 2} B_{2, \omega}) - \alpha^{-1} (\Theta_{\omega, 2} B_{2, \omega}) B_{2, i})
\[+ (\alpha B_{3, i} (\Theta_{\omega, 2} B_{3, \omega}) - \alpha^{-1} (\Theta_{\omega, 2} B_{3, \omega}) B_{3, i})
\[+ (\Theta_{1} \hat{B}_{i+1}^t [B_{1, i}, (\Theta_{\omega, 2} B_{1, \omega})] + \alpha^{-1} [B_{2, i}, (\Theta_{\omega, 2} B_{3, \omega})] + \alpha [B_{3, i}, (\Theta_{\omega, 2} B_{3, \omega})])
\[+ (\Theta_{1} A^t b_{i+1}^t ((\alpha^{-1} B_{1, i} (\Theta_{\omega, 2} B_{2, \omega}) - (\Theta_{\omega, 2} B_{2, \omega}) B_{1, i}) + (\alpha B_{2, i} (\Theta_{\omega, 2} B_{1, \omega}) - \alpha^{-1} (\Theta_{\omega, 2} B_{1, \omega}) B_{2, i})
\[+ (\alpha B_{3, i} (\Theta_{\omega, 2} B_{3, \omega}) - \alpha^{-1} (\Theta_{\omega, 2} B_{3, \omega}) B_{3, i})
\[+ (\Theta_{1} \hat{B}_{i+1}^t [B_{1, i}, (\Theta_{\omega, 2} B_{1, \omega})] + \alpha^{-1} [B_{2, i}, (\Theta_{\omega, 2} B_{3, \omega})] + \alpha [B_{3, i}, (\Theta_{\omega, 2} B_{3, \omega})])
\[+ (\Theta_{1} \hat{A}^t b_{i+1}^t ((\alpha^{-1} B_{1, i} (\Theta_{\omega, 2} B_{3, \omega}) - (\Theta_{\omega, 2} B_{3, \omega}) B_{1, i}) + (\alpha B_{2, i} (\Theta_{\omega, 2} B_{2, \omega}) - \alpha^{-1} (\Theta_{\omega, 2} B_{2, \omega}) B_{2, i})
\[+ (\alpha B_{3, i} (\Theta_{\omega, 2} B_{3, \omega}) - \alpha^{-1} (\Theta_{\omega, 2} B_{3, \omega}) B_{3, i})
\[+ (\Theta_{1} A^t b_{i+1}^t ((\alpha B_{1, i} (\Theta_{\omega, 2} B_{2, \omega}) - (\Theta_{\omega, 2} B_{2, \omega}) B_{1, i}) + (\alpha B_{2, i} (\Theta_{\omega, 2} B_{1, \omega}) - \alpha^{-1} (\Theta_{\omega, 2} B_{1, \omega}) B_{2, i})
\[+ (\alpha B_{3, i} (\Theta_{\omega, 2} B_{3, \omega}) - \alpha^{-1} (\Theta_{\omega, 2} B_{3, \omega}) B_{3, i})
\[+ (\Theta_{1} \hat{B}_{i+1}^t [B_{1, i}, (\Theta_{\omega, 2} B_{1, \omega})] + \alpha^{-1} [B_{2, i}, (\Theta_{\omega, 2} B_{3, \omega})] + \alpha [B_{3, i}, (\Theta_{\omega, 2} B_{3, \omega})])
\[+ (\Theta_{1} \hat{A}^t b_{i+1}^t ((\alpha^{-1} B_{1, i} (\Theta_{\omega, 2} B_{3, \omega}) - (\Theta_{\omega, 2} B_{3, \omega}) B_{1, i}) + (\alpha B_{2, i} (\Theta_{\omega, 2} B_{2, \omega}) - \alpha^{-1} (\Theta_{\omega, 2} B_{2, \omega}) B_{2, i})
\[+ (\alpha B_{3, i} (\Theta_{\omega, 2} B_{3, \omega}) - \alpha^{-1} (\Theta_{\omega, 2} B_{3, \omega}) B_{3, i})
\[+ (\Theta_{1} A^t b_{i+1}^t ((\alpha B_{1, i} (\Theta_{\omega, 2} B_{2, \omega}) - (\Theta_{\omega, 2} B_{2, \omega}) B_{1, i}) + (\alpha B_{2, i} (\Theta_{\omega, 2} B_{1, \omega}) - \alpha^{-1} (\Theta_{\omega, 2} B_{1, \omega}) B_{2, i})
\[+ (\alpha B_{3, i} (\Theta_{\omega, 2} B_{3, \omega}) - \alpha^{-1} (\Theta_{\omega, 2} B_{3, \omega}) B_{3, i})
\[+ (\Theta_{1} \hat{B}_{i+1}^t [B_{1, i}, (\Theta_{\omega, 2} B_{1, \omega})] + \alpha^{-1} [B_{2, i}, (\Theta_{\omega, 2} B_{3, \omega})] + \alpha [B_{3, i}, (\Theta_{\omega, 2} B_{3, \omega})])\]
\[
+\alpha[B_{3l}^i, (\Theta_{\omega^2}B_{2l}^i)]]
+ (\Theta_{\omega^2}B_{1l}^i) b_{l+1} \left( [B_{1l}^i, (\Theta_{\omega}H)] + (B_{2l}^i(\Theta_{\omega}A) - \alpha(\Theta_{\omega}A)B_{2l}^i) + (B_{3l}^i(\Theta_{\omega}A^\dagger) - \alpha^{-1}(\Theta_{\omega}A^\dagger)B_{3l}^i) \right)
+ (\Theta_{\omega^2}B_{2l}^i) b_{l+1} \left( (\alpha^{-1}B_{1l}^i(\Theta_{\omega}A) - (\Theta_{\omega}A)B_{1l}^i) + (\alpha^{-1}B_{2l}^i(\Theta_{\omega}A^\dagger) - (\Theta_{\omega}A^\dagger)B_{2l}^i) + \alpha^{-1}[B_{3l}^i, (\Theta_{\omega}H)] \right)
+ (\Theta_{\omega^2}B_{3l}^i) b_{l+1} \left( (\alpha B_{1l}^i(\Theta_{\omega}A^\dagger) - (\Theta_{\omega}A^\dagger)B_{1l}^i) + \alpha[B_{2l}^i, (\Theta_{\omega}H)] + (\alpha B_{3l}^i(\Theta_{\omega}A) - \alpha^{-1}(\Theta_{\omega}A)B_{3l}^i) \right) \right\} \tag{33}
\]

\[S \quad \text{(ghost action)}\]
\[
= - \int dt \text{Tr} \left( \hat{C} \partial^\theta D_0 C - \hat{C}[X_{cl}^i, [X^i, C]] \right)
\]
\[
= -3 \text{Tr} \int dt \left[ C_{gh}^i(\partial_0)^2 C_{gh} + A_{gh}^i(\partial_0)^2 A_{gh} + A_{gh}(\partial_0)^2 A_{gh}^i \right.
- \iota \left( C_{gh}^i[H_0, C_{gh}] + C_{gh}^i[A_0, A_{gh}] \right) + C_{gh}^i[A_0, A_{gh}^i]
+ A_{gh}^i[H_0, A_{gh}] + A_{gh}^i[A_0, A_{gh}] + A_{gh}[A_0, C_{gh}] \\
+ A_{gh}[H_0, A_{gh}^i] + A_{gh}[A_0, C_{gh}] + A_{gh}[A_0, A_{gh}^i] \right)
- \left( C_{gh}^{i\dagger}[H_{cl}^i, [H^i, C_{gh}]] + C_{gh}^i[H_{cl}^i, [A^i, A_{gh}]] + C_{gh}^i[H_{cl}^i, [A^i, A_{gh}^i]] \right)
+ A_{gh}^i[H_{cl}^i, [H^i, A_{gh}]] + A_{gh}^i[H_{cl}^i, [A^i, A_{gh}]] + A_{gh}^i[H_{cl}^i, [A^i, A_{gh}^i]]
+ A_{gh}[H_{cl}^i, [H^i, A_{gh}]] + A_{gh}[H_{cl}^i, [A^i, A_{gh}]] + A_{gh}[H_{cl}^i, [A^i, A_{gh}^i]] \right)
+ \frac{1}{2} \left\{ C_{gh}^{i\dagger} \left( [B_{1cl}^i, [B_{1l}^i, C_{gh}]] \right.
+ B_{icl}^i[B_{2l}^i, A_{gh} - \alpha A_{gh} B_{2l}^i] - (B_{2l}^i A_{gh} - \alpha A_{gh} B_{2l}^i) B_{1cl}^i \\
+ B_{icl}^i[B_{3l}^i, A_{gh} - \alpha^{-1} A_{gh} B_{3l}^i] - (B_{3l}^i A_{gh} - \alpha^{-1} A_{gh} B_{3l}^i) B_{1cl}^i \\
+ [B_{1cl}^i, [B_{2l}^i, C_{gh}]] + B_{icl}^i(\alpha^{-1} B_{2l}^i A_{gh} - A_{gh}^i B_{2l}^i) - (\alpha^{-1} B_{2l}^i A_{gh} - A_{gh}^i B_{2l}^i) B_{1cl}^i \\
+ B_{icl}^i(\alpha B_{3l} A_{gh} - A_{gh} B_{3l}^i) - (\alpha B_{3l} A_{gh} - A_{gh} B_{3l}^i) B_{1cl}^i \right) \right. \\
+ A_{gh}^i \left( B_{icl}^i B_{2l}^i A_{gh} - \alpha A_{gh} B_{2l}^i \right) - (\alpha^{-1} B_{2l}^i A_{gh} - A_{gh} B_{2l}^i) B_{1cl}^i \\
+ B_{icl}^i(B_{2l}^i A_{gh} - \alpha^{-1} A_{gh} B_{2l}^i) - (\alpha^{-1} B_{2l}^i A_{gh} - A_{gh} B_{2l}^i) B_{1cl}^i \\
+ B_{icl}^i[B_{3l}^i, C_{gh}] - B_{icl}^i[B_{2l}^i, C_{gh}] B_{1cl}^i \right. \\
+ B_{icl}^i[B_{1l} A_{gh} - \alpha^{-1} A_{gh} B_{1l}^i] - (\alpha B_{1l} A_{gh} - A_{gh} B_{1l}^i) B_{1cl}^i \\
+ \alpha^{-1} B_{1cl}^i[B_{2l}, C_{gh}] - [B_{2l}, C_{gh}] B_{1cl}^i \\
+ B_{icl}^i(\alpha B_{3l} A_{gh} - \alpha^{-1} A_{gh} B_{3l}^i) - (\alpha^{-1} B_{3l} A_{gh} - A_{gh} B_{3l}^i) B_{1cl}^i \right) \right.
+ A_{gh} \left( B_{1cl}^i B_{2l}^i A_{gh} - \alpha^{-1} A_{gh} B_{2l}^i \right) - (\alpha B_{2l} A_{gh} - A_{gh} B_{2l}^i) B_{1cl}^i \\
+ B_{icl}^i[B_{2l}^i, C_{gh}] - (\alpha B_{2l}^i, C_{gh}) B_{1cl}^i \\
+ B_{icl}^i[B_{3l} A_{gh} - \alpha A_{gh} B_{3l}^i] - (\alpha B_{3l} A_{gh} - A_{gh} B_{3l}^i) B_{1cl}^i \\
+ B_{icl}^i(\alpha^{-1} B_{2l} A_{gh} - \alpha A_{gh} B_{2l}^i) - (\alpha B_{2l} A_{gh} - A_{gh} B_{2l}^i) B_{1cl}^i \\
+ B_{icl}^i(\alpha B_{3l} A_{gh} - \alpha^{-1} A_{gh} B_{3l}^i) - (\alpha^{-1} B_{3l} A_{gh} - A_{gh} B_{3l}^i) B_{1cl}^i \right\} \right. \\
+ \alpha B_{1cl}^i[B_{3l}, C_{gh}] - [B_{3l}, C_{gh}] B_{1cl}^i \right\} \tag{34} \]
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