Multidisciplinary design of the wing for aircraft preliminary design purposes

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Abstract. In this paper, an aerodynamic and wing structure is investigated by low-fidelity methods. Bell-shaped lift distribution was rediscovered in the last decade as a perspective alternative to traditional wing design. This leads to lower aerodynamic drag than elliptical lift distribution for a given lift force and root bending moment. Root bending moment is used as a surrogate model of wing structure weight. It is relatively raw simplification introduced by Prandtl to estimate the weight of the spar as a main part of the wing structure. For a more accurate wing weight estimation, the main parts of the wing are dimensioned under CS-23 regulation in this work. The design procedure starts with defining the elementary parameters of the wing shape (chord/twist distribution, wingspan). After geometry generating a non-linear lifting line is used to calculate aerodynamic characteristics for all regime, determined from the flight envelope. The dimensions of a spar, ribs and skin are calculated in the next step of the procedure for given bending moment, load and torque moment distribution. The structure of the wing is assumed as a two-spar, manufactured by aluminum. A target of design is to find out the shape of the wing for given weight. The solution is verified by CFD calculation.

1. Introduction
From the beginning of aviation, an optimal shape of the wing is investigated. Most of the first aviation pioneers tried to find out inspiration in nature such as airfoil shape as fish or wing of the bird. Ludwig Prandtl developed an analytical method to estimate aerodynamic forces and moments acting on the wing with a finite span wing [1]. He also determined an optimal aerodynamic solution for a given wingspan, required lift, and maximum bending moment – elliptical lift distribution. In the next paper [2] he wrote the previous solution is optimal only for a wing with a given wingspan (fig. 1).

If we consider only required lift and bending moment, we could create wing with lower aerodynamic drag. Unfortunately, he developed this knowledge no longer and thus remained unused. Similar results Reimar Horten uses on its gliders. He also investigated a distribution of induced drag. Next researchers were studied effect of lift distribution on aerodynamic drag and they got similar results [3] (fig. 2), [4] and [5]. In this century Albion Bowers published his paper [6], in which a bell shaped lift distribution is completely described. All these papers use a bending moment as a surrogate model to estimate the weight of the wing.

In this paper, a detailed wing structure is designed to better estimate the influence of different wing shapes and its span load on the mass of the wing. Elementary parts of the wing such as rig, spar, and skin are calculated under plate stability consideration.
2. Computational procedure

We assume three wings with generalized shape and two more wings designed to reach a bell shaped lift distribution. For the purpose of optimization procedure, a lifting line theory is used, and the validation of optimal wing is done using CFD.

2.1. Lifting line theory

The lifting line theory was developed by Ludwig Prandtl [1] and uses a vortex scheme based on the Biot-Savart law that determines the value of induced variables. Basic characteristic in LLT is downwash velocity $w$:

$$w(y_0) = \frac{1}{4\pi} \int_{-b/2}^{b/2} \frac{d\Gamma}{y-y_0}$$

(1)

where $y_0$ is side position, $\Gamma$ is circulation and $b$ is wing span. This equation is in explicit form because a circulation depends on lift and lift is depending on the downwash velocity and angle of attack respectively. Moreover, the calculation process is adjusted to use nonlinear airfoil aerodynamic characteristics. For that reason, two-dimensional airfoil characteristics were calculated in CFD for different Reynolds numbers to cover the whole airfoil regime on the wing. After that, these characteristics are used in lifting line theory. Thus, an induced and viscous drag should be calculated. This concept was verified in previous work [7].

2.2. Optimization of wings geometry

As an optimization procedure a response surface methodology is used [8],[9]. This method uses a fact, that cost function could be defined as a linear combination of design vector. Then a least squared method can be used for solving the problem. This approach enables to create local smooth surface in which we can find an optimal solution. In this paper a total aerodynamic drag is minimized with consideration of maximum root bending moment. Total aerodynamic lift is prescribed and is interpolated from polar for desired angle of attack.
Design parameters used in optimization for different wings are defined in the following table:

| Wing shape       | Design parameters                                      |
|------------------|--------------------------------------------------------|
| Rectangle        | Chord, wing span                                       |
| Elliptic         | Root chord, wing span                                   |
| Trapezoid        | Root and tip chord, wing span                           |
| Twisted trapezoid| Root and tip chord, wing span, twist distribution        |
| BSLD             | Root chord, wing span, chord distribution               |

### 2.3 Calculation using CFD

For CFD computations five variants of different wing platforms were generated. By reason that structural analysis will be done consequently, the quarter point of the root profile of all wing platforms were set to the same null coordinate. Wings were placed into sufficiently large domain of cylinder shape. These geometrical operations were done in CATIA program. In the next step the meshing procedure was realized in the Pointwise program. Structured rectangular grid was largely set on the surface of the wing. In case of a complex geometry shape triangular elements were used. Volume is formed by tetrahedral elements.

For the boundary layer simulation sixty prismatic layers were generated. For the capture of the vortex structure over the wing, density regions with huge number of cells were created in the vicinity of these wings. Averaged number of elements of computed wing platform was about forty million. CFD analysis were processed in EDGE program. This solver is acceptable for both inviscid and viscous compressible flow. For the wing analysis were chosen Navier Stokes averaged equations with W&J EARSM + Hellsten k-omega Turbulence model and Central scheme discretization. For the next structural analysis overall lift, drag and pitch moment and their distributions over the wing were obtained from these computations. The fluid fields were analyzed in TECPLOT program to see pressure distribution and vortex structure that can be compared with the numerical output.
Figure 4. Surface grid on the wing and the symmetry plane. It can be seen prismatic layers and the density region on the symmetry plane. Transition between prismatic layers and outer polyhedral grid is gradually modelled. Surface of the wing is formed by structured mesh except the wing tip profile, where is the unstructured mesh with some triangular elements.

In following pictures the result for elliptical wing is depicted. Comparison between computational methods shows agreement both local aerodynamic characteristics (lift distribution) and integrated characteristics (polar).

Figure 5 cross-section vorticity and Iso-surface of Q-criterion (upper left); aerodynamic polar (upper right); skin friction coefficient (left down); lift distribution at desired lift force (right down)

The result of optimization is shown in the following picture. All wings have same lift force and root bending moment. There are aerodynamic drag force and aspect ratio for individual wing geometry. Percentage values of drag are related to elliptical wing.
3. Wing stress analysis

As mentioned above, previous work considers the weight of the wing as a function of root bending moment. This simplification could be used for similar wing geometry. In this work, a more detailed weight analysis was performed. In the first steps of the strength design, we first define the flight envelope. This is determined by speeds and flight load factor under CS-23 regulation. The flight envelope consists of gust and maneuvers envelope. The combination of these envelopes creates the final envelope and the result is depicted in the following picture.

![Flight envelope](image)

**Figure 7. Flight envelope**

Aerodynamic loads and pitching moment distributions were taken from CFD. Shear force, bending and torque moment distribution were calculated for main points from the flight envelope (A, C, D, E, F, G). A safety factor equals to 1.5 is used for the purpose of the structural design. As a common material for structural parts, an aluminum alloy AW-2024 is used. In the first step of the design procedure, a rib distribution was chosen. Then, flanges dimension was calculated in order to transmit bending moment. Further thicknesses of the web and skin are designed for a double-spar airfoil section with two cells, in this case. The shear flow of cell analysis is solved. As an input, we used local shear load and torque moment. Every element of flanges, web, and skin panels across the wingspan were dimensioned under consideration of column buckling. For that reason, empirical characteristics to determine critical stress are used [10].
After the design process, the total weight of the structural part was calculated. Mass distribution for the rectangular wing is depicted in figure 9. A comparison of different wing geometry shows the following graph. The weight of wings is relative to the elliptical wing. The heaviest structure that transmits aerodynamic forces has a rectangular wing. Other wings have quite similar weight.

**Conclusion**

In this paper, a comparison of the different shapes of the wing was presented. Aerodynamic efficiency and structural weight were investigated for the given requirement of lift force and root bending moment. The shape of the wing was optimized by the lifting line method and validated by CFD calculation. This design shows the wings with bell-shaped lift distribution have lower total aerodynamic drag than three generalized wings. After that, the design of the main structural part was done to estimate the weight of the wing. This comparison shows a similar weight for all wing except the rectangular wing, which has a 15 % higher weight.

**Acknowledgement**

This result was achieved within the institutional support of the Ministry of industry and trade for the development of the research organization.
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