Brane Fluctuation and anomalous muon magnetic moment

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Abstract

We study the effects of extra dimensions on the muon anomalous magnetic moment with brane fluctuation. Since the coupling is naturally suppressed if brane fluctuation is considered by exponential softening factor for heavier states, the contribution from the whole Kaluza-Klein graviton tower is shown to be finite. The recent BNL E821 result is accommodated with \( D = 4 + \delta \) dimensional gravitational scale, \( M_D \), in the range of \( M_D \simeq 1.0 - 5.1 \) TeV (\( \delta = 2 \)), and \( M_D \simeq 1.0 - 8.0 \) TeV (\( \delta = 6 \)) with the brane tension parameter \( f = (4\pi^2\tau)^{1/4} \), in the range \( f = 1 - 10 \) TeV.

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The report of measurement of the anomalous $g$ value for the positive muon from Brookhaven AGS experiment 821, based on data collected in 1999, has attracted great interests [1]. Combining recent theoretical and experimental uncertainties, the new world average shows $2.6\sigma$ deviation from the standard model (SM) prediction. It may be the first evidence that the standard model must be extended by new physics at TeV scale. The reported result is given as

$$\delta a_\mu \equiv a_\mu(\text{exp}) - a_\mu(\text{SM}) = 43(16) \times 10^{-10},$$

where $a_\mu(\text{exp})$ is world averaged value. Since four times larger data is already recorded in 2000 and measurements with negative muon are undertaking now, it is very interesting situation to consider the implications of the speculative advocates of the new physics on the anomaly. Most popular approaches include weak scale supersymmetry, large or warped extra dimensional scenario, extension of the gauge structure and many other alternatives (see e.g., [3]). In this paper, we consider the extra dimensional scenario suggested by Arkani-Hamed et.al. with brane fluctuation.

If all the matter fields are confined on the brane-world, 1-loop contribution of the Kaluza-Klein excitations of the graviton is known to explain the recent BNL anomaly [4], [5], [6]. (The case in which SM fields propagate in the bulk is also studied [7] and see [8] for general arguments for graviton loops and brane observables.) We note that in both cases-with large or warped extra dimensions- physical cutoff at weak scale should be imposed to get finite results from the non-renormalizable theory. Even these truncation procedures are not un-natural in treating effective theory [9], it seems not generally true that the exactly same expressions for the muon anomalous moment remains valid until the cutoff scale. In this letter, we study the another possibility to get rid of divergence by considering brane fluctuation [10], [11], [12], [13].

We take the case that all the gauge and fermion fields of SM are confined on 3-brane and only the gravitational field can propagate through the bulk. In that case, $D = 4 + \delta$ dimensional gravitational scale, $M_D$, is related to our four dimensional Planck scale $M_P$.
by \( M_P^2 = M_D^{2+\delta} R^\delta \). Where \( R \) is the size of the extra dimension. From this relation, we can understand weakness of gravity or largeness of Planck energy scale: if the size of the extra dimension is large enough, the Planck scale also is very big. It is phenomenologically interesting case, if \( M_D \) is as small as TeV scale.

The gravitational interaction with SM fields which are confined on the fluctuating brane can be described by the action:

\[
S \supset -\frac{1}{M_P} \int d^D x T_{\mu\nu}(x) g^{\mu\nu}(x, \vec{y}) \delta^D(\vec{y} - \vec{\phi}(x)) \tag{2}
\]

where the Nambu-Goldstone boson \( \vec{\phi}(x) \), which came from the spontaneous translational symmetry breaking, represent the brane fluctuating in the \( \vec{y} \) direction at point \( x \) in our 3-brane. The Nambu-Goto action with tension \( \tau = f^4/4\pi^2 \) describe the dynamics of \( \vec{\phi}(x) \):

\[
\int d^4x (-\frac{\tau}{2} \partial_\mu \vec{\phi}(x) \cdot \partial^\mu \vec{\phi}(x) + \cdots) \tag{3}
\]
on the flat background. After expanding the gravitational field around the compact extra dimension as

\[
g_{\mu\nu}(x, \vec{y}) = \sum_{\vec{n}} g_{\mu\nu}^{\vec{n}}(x) e^{i\vec{n} \cdot \vec{y}/R}, \tag{4}
\]
and taking normal ordering the exponential in perturbation framework, the interaction action includes an exponential ‘softening factor’:

\[
S \supset -\frac{1}{M_P} \sum_{\vec{n}} \int d^4x e^{-\frac{1}{2} m_{\vec{n}}^2 \Delta} g_{\mu\nu}^{\vec{n}}(x) T_{\mu\nu}(x) \tag{5}
\]

where \( m_{\vec{n}}^2 = \vec{n} \cdot \vec{n}/R^2 \) are the mass of the KK modes and \( \Delta \) is the free propagator of \( \vec{\phi}(x) \). If brane tension \( \tau \) is given, \( \Delta \) is understood as

\[
\Delta \equiv \langle \phi(x)\phi(y) \rangle \big|_{(x-y)^2 \to M_D^{-2}} \simeq \frac{M_D^2}{f^4} \tag{6}
\]
since the present effective theory is valid only at scales lower than \( M_D \). Note that the mass dimension of \( \Delta \) is \(-2\) since \([\tau] = -4\). Eq. (\ref{mass dim}) clearly shows that if the effect of brane fluctuations is correctly included, the coupling of the higher KK gravitons are exponentially suppressed.
Now let us consider the effects of the KK graviton from the fluctuating brane on \(a_\mu\). The anomalous magnetic moment of the muon is the coefficient of the operator 
\[ (e/4M_\mu)\bar{\mu}\sigma^{\alpha\beta}\mu F_{\alpha\beta}, \]
where \(\sigma^{\alpha\beta} = (i/2)[\gamma^\alpha, \gamma^\beta]\) is the Lorentz generator for spin-1/2 spinors. By considering loop induced connection to \(\mu\mu\gamma\) vertex, we can calculate this term. At 1-loop order, a few Kaluza-Klein graviton mediated diagrams can contribute to \((g-2)\) of the muon. Usually, radion contribution to anomalous magnetic moment is less than one order smaller than that of KK graviton, we just ignore that effect in this study.

It is convenient to express the contribution of the KK gravitons as
\[
\delta a^{KK} = \frac{1}{16\pi^2}\left(\frac{M_\mu}{M_P}\right)^2 A,
\]
where \(A\) is essentially effective degrees of freedom of contributing KK gravitons and it could be obtained by summing over all relevant contributions of all the relevant KK states in the Feynman diagrams (see Fig.1). The factor \((1/16\pi^2)\) is the usual loop factor and the suppression factor \((M_\mu/M_P)^2\) came from the gravitational coupling strength.

In the case of rigid brane, \(A\) can be approximated as
\[
A(\text{Rigid}) \approx 5 \int n^{\delta-1} \Omega_\delta dn
\]
at the limit of our interests: \((M_\mu/M_{KK})^2 \to 0\). Here 5 comes from the non-decoupling contribution from the each KK graviton mode and \(\Omega_\delta = 2\pi^{\delta/2}/\Gamma(\delta/2)\) is solid angle in \(\delta\)-dimension space. \(N_\Lambda\) denotes the maximum quantum number of KK state at the cut-off scale \(\Lambda \sim M_D\) such that \(N_\Lambda \sim M_D R\). After integration, the final form of the \(\delta a^{KK}\) from the rigid brane is
\[
\delta a^{KK}(\text{Rigid}) \approx \frac{1}{16\pi^2} \frac{10\pi^{\delta/2}}{\Gamma(\delta/2)} \left(\frac{M_\mu}{M_D}\right)^2
\]
and this result is same with the result of [4].

We now consider the case that brane fluctuation is included. With brane fluctuation, exponential softening factor naturally suppress the effects from all the higher KK modes. The \(A\) can be casted as
\[
A(\text{Fluctuating}) \approx 5 \int n^{\delta-1} \Omega_\delta e^{-n^2\Delta/R^2} dn.
\]
Note that there are two vertex points for gravitational coupling in each diagrams and so doubly suppressed factor appears in the above equation. By defining dimensionless variable \( \mathcal{L}^2 \equiv R^2/\Delta \), the integration gives simply gamma function as

\[
\mathcal{A} \text{(Fluctuating)} \approx 5 \mathcal{L}^\delta \Omega_\delta \int x^{\delta-1} e^{-x^2} dx = \frac{5}{2} \mathcal{L}^\delta \Omega_\delta \Gamma(\delta/2).
\]  

(11)

Note that in this case we did not introduce the cutoff at the integration region. But we still need to introduce the cutoff when we consider the free propagator for the fluctuating field. Finally, the total contribution to anomalous magnetic moment of muon from the KK graviton modes from the fluctuating brane can be approximated as

\[
\delta a_{\mu}^{KK} \text{(Fluctuating)} \approx \frac{5}{16\pi^2} n^{\delta/2} \left( \frac{f \delta M_\mu}{M_D^{\delta+1}} \right)^2.
\]  

(12)

The result is sensitive to the number of extra dimensions (\( \delta \)).

As examples we explicitly show the expressions for the case \( \delta = 2 \) and \( \delta = 6 \) respectively.

\[
\delta a_{\mu}^{KK} \approx 11.1 \times 10^{-10} \left( \frac{f}{\text{TeV}} \right)^4 \left( \frac{\text{TeV}}{M_D} \right)^6 \quad (\delta = 2)
\]  

(13)

\[
\delta a_{\mu}^{KK} \approx 11.0 \times 10^{-9} \left( \frac{f}{\text{TeV}} \right)^{12} \left( \frac{\text{TeV}}{M_D} \right)^{14} \quad (\delta = 6).
\]  

(14)

If we ascribe the recent BNL anomaly to the KK graviton contribution, for \( f/\text{TeV} \) in the range 1-10, then \( M_D \simeq 1.0 - 5.1 \) TeV if \( D = 6 \) and \( M_D \simeq 1.0 - 8.0 \) TeV if \( D = 10 \). The Fig.2 describes the allowed parameter space \( (f, M_D) \) for the muon anomaly. The lower side curves show the allowed region for \( D = 6 \) and upperside curves show the region for \( D = 10 \).

In summary, we have studied the effects of Kaluza-Klein tower of graviton on the \((g-2)\) factor of muon with brane fluctuation regularization at 1-loop level. After normal ordering the fluctuation \( \sim e^{i\vec{n} \cdot \vec{\phi}(x)/R} \), we obtain the exponentially suppressed factor \( \sim \exp(-m_{KK}^2 R/2) \) for heavier states. By this suppression, we get the finite result for muon anomalous magnetic moment without simple neglecting procedure beyond the cutoff scale. The recently reported deviation in anomalous magnetic moment is possibly accomodated in the corresponding parameter space in the range of
\[ 1.0 \leq M_D/\text{TeV} \leq 5.1 \quad (D = 6) \quad (15) \]
\[ 1.0 \leq M_D/\text{TeV} \leq 8.0 \quad (D = 10) \quad (16) \]

with the tension parameter is chosen to be \( f = 1 - 10 \text{ TeV}. \)

In the Randall-Sundrum model, our brane-world is set to have negative tension [14] and such regularization does not work. But there is still a possibility with brane thickness effect.

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FIG. 1. The Kaluza-Klein mediated 1-loop diagrams for \((g-2)\) of the muon. The solid and wavy line denotes muon and photon on-shell. The spring-shape lines denote spin-2 Kaluza-Klein states.

FIG. 2. The allowed parameter space \((f, M_D)\) to compensating recent BNL E821 result of the muon anomalous magnetic moment. The (black)solid line and the (red) dotted line denotes the cases for \(\delta = 2\) (or \(D = 6\)) and \(\delta = 6\) (or \(D = 10\)) respectively.