Predicting the dynamics of the maximum and optimal profits of innovative enterprises

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Abstract. This article proposes new economic and mathematical models of the dynamics of the development of industrial enterprises that introduce innovative technologies. The level of attraction of innovative technologies by the enterprise is set by dimensionless coefficients of innovations, which depend on time and affect the increase in output and decrease in costs. The influence of not only production costs, but also transaction costs on the size of the enterprise’s profit was investigated, the sources of which are the forced costs of searching and processing economic information, financing negotiation procedures, developing and concluding contracts with partners, organizing the protection of property rights, paying for the opportunistic behavior of employees and enterprise managers, etc. It is found, that transaction costs force to maximize not only the profit function of the enterprise, but also the target transaction utility function, which takes into account the redistribution of profits in the interests of the enterprise management and for the implementation of socially oriented programs. The size of the part of the profit spent in the interests of the enterprise management and for the implementation of socially oriented programs is set by dimensionless redistribution coefficients. It is shown, that in the presence of transaction costs, the enterprise, instead of the maximum value of profit, achieves a lower optimal value of profit.

1. Introduction
The activity of a manufacturing enterprise is carried out in the sphere of material production and in a certain social environment. The result of the impact of the enterprise on its own production is certain transformations, which are accompanied by output, production costs and profits. The interaction of an enterprise with the social sphere is carried out in the form of a certain set of transactions that generate transaction costs and redistribution of profits. The production activity of the enterprise forms its production transformation costs, and the interaction of the enterprise with the social environment generates non-production transaction costs [1, 2].

Transaction costs can include the cost of finding economic information, the cost of measuring the parameters of various goods, the cost of negotiating and negotiating contracts, the cost of creating specifications and protecting property rights, the cost of managing opportunistic behavior, etc. In some cases, transaction costs are the result of strengthening measures taken by management to improve the quality of products with new enhanced consumer properties. A significant part of the transaction costs can be directed by the management to social programs for personnel, to programs for continuing education of employees, to the environment, to scientific and charitable projects, etc. [3–12].

The implementation of such programs contributes to the growth of quality products and
sales volumes, develops the innovative component of the enterprise, and attracts new volumes of investment. At the same time, the implementation of these programs generates significant transaction costs and reduces the company’s profit. The presence of transaction costs forces the management of the enterprise to maximize not the profit function, but the transaction utility function, which takes into account the opportunistic interests of the management and the outflow of part of the enterprise’s profit for non-production needs. These circumstances hinder the achievement of the maximum possible profit of the enterprise, instead of which it is necessary to limit itself to its optimal value [13–19].

If an enterprise undergoes a certain re-equipment or modernization through the introduction of innovative technologies, then the parameters of the production function, the function of total costs and profits change over time. As a result of such innovative processes, instead of the point values of the maximum profit and the corresponding volumes of production factors, a whole spectrum of such values is obtained, which is a function of time. By appropriately controlling the parameters of the introduction of innovative technologies, it becomes possible to predict the maximum profit of an enterprise at the right time [20, 21].

Thus, the task of creating mathematical models for calculating the economic indicators of the enterprise, taking into account the level of transaction costs, seems to be very relevant.

2. Statement of the problem

In the general case, the set of resources with which the enterprise provides the production and release of its products can be represented in the form of a \( n \)-dimensional vector of the space \( \mathbb{R}^{m+n} \) of volumes of production factors

\[
Q = (Q_1, Q_2, \ldots, Q_m, S_1, S_2, \ldots, S_n),
\]

where the components of the vector \( Q_i \) are the main and labor resources involved in production, and the components of the vector \( S_i \) are the resources that provide the non-production transactional activity of the enterprise. It should be noted that factors of production \( Q_i \) are sources of only production costs, and resources \( S_i \) are sources of both production and transaction costs.

An arbitrary multifactorial production function \( TR \) that provides the volume of output of a manufacturing enterprise can be written as

\[
TR = F(Q_1, Q_2, \ldots, Q_m, S_1, S_2, \ldots, S_n).
\]

Within the framework of the economic and mathematical model considered here, we restrict ourselves to the Cobb–Douglas multiplicative production function

\[
TR = P \cdot \prod_{i=1}^{m} Q_i^{a_i} \cdot \prod_{j=1}^{n} S_j^{c_j},
\]

where \( a_i, c_j \) are the elasticities of output with respect to the corresponding resources \((0 < a_i < 1), \ (0 < c_j < 1)\), \( P \) is the cost of products produced per unit volume of resources.

The proportional costs of an enterprise with such resources are given by the expression

\[
TC = \sum_{i=1}^{m} A_Q^i \cdot Q_i + \sum_{j=1}^{n} A_S^j \cdot S_j + TFC,
\]

where \( A_Q^i, A_S^j \) are the costs of costs for unit volumes of resources, respectively, \( TFC \) are the fixed costs of the enterprise.
The expression for the profit of the enterprise is written as

\[ PR = P \cdot \prod_{i=1}^{m} Q_i^{a_i} \cdot \prod_{j=1}^{n} S_j^{c_j} - \sum_{i=1}^{m} A_Q \cdot Q_i - \sum_{j=1}^{n} A_S \cdot S_j - TFC. \] (5)

The highest income of the considered enterprise corresponds to the maximum of the profit function (5). In practice, along with function (5), the enterprise has to maximize the target transaction utility function. This function takes into account the redistribution of profits in the interests of the company’s management and for the implementation of socially oriented programs. It depends on profit and resources and is taken here as linear

\[ U = U(PR, S_1, S_2, \ldots, S_n) = PR + \sum_{j=1}^{n} q_j \cdot S_j, \] (6)

where \( q_j \) are the coefficients of the utility function (5). It should be noted that all the coefficients of the utility function (5) are non-negative (\( \forall j : q_j \geq 0 \)).

If innovative technologies are constantly being introduced into the production of the enterprise under consideration in order to maintain stability, then the indicators of the production function and costs are not constant values, but change over time \( t \).

In this article, we will restrict ourselves to a variant of the enterprise model in which the output of its products is provided by one production factor \( Q(t) \) and one non-production resource \( S(t) \). Then formulas (3)–(6) take the form

\[ TR(t) = P(t) \cdot Q(t)^{a(t)} \cdot S(t)^{c(t)}, \] (7)
\[ TC(t) = A_Q(t) \cdot Q(t) + A_S(t) \cdot S(t) + TFC. \] (8)
\[ PR(t) = P(t) \cdot Q(t)^{a(t)} \cdot S(t)^{c(t)} - A_Q(t) \cdot Q(t) - A_S(t) \cdot S(t) - TFC. \] (9)
\[ U(t) = PR(t) + q(t) \cdot S(t). \] (10)

The process of introducing innovations into the production of an enterprise is assumed to be progressive, therefore, over time, output should increase, and costs should decrease \([10, 11]\).

The influence of the target transaction utility function (10) of the enterprise on the redistribution of profits in the interests of the enterprise management and for the implementation of socially oriented programs is completely determined by the parameter \( q(t) \).

This parameter satisfies the inequality

\[ q_0 \leq q(t) \leq q_r(t). \] (11)

The lower bound of inequality (11) \( q_0 \) corresponds to the redistribution of profits between the production needs of the enterprise and the non-production needs of the enterprise management for the development of socially oriented programs at the initial moment of time \( t = 0 \).

In a particular case, when \( q_0 = 0 \), the enterprise does not finance any non-production programs at all and the utility function coincides with the profit function.

The upper bound of inequality (11) \( q = q_r(t) \) corresponds to a situation in which at the current moment of time the enterprise spends all its profit on social programs.

The values of resources \( Q_F(t) \) and \( S_F(t) \) at which the profit of the enterprise at the current moment of time vanishes are found from the equation

\[ P(t) \cdot Q_F(t)^{a(t)} \cdot S_F(t)^{c(t)} - A_Q(t) \cdot Q_F(t) - A_S(t) \cdot S_F(t) - TFC = 0. \] (12)
The values of the upper bound of inequality (11) $q_F(t)$ at the current time are found from the equation

$$q_F(t) = \frac{\partial P R(t)}{\partial S F(t)} = A_S(t) - \frac{P(t) \cdot Q_F(t)^{\alpha(t)} \cdot c(t)}{S_F(t)^{1-\alpha(t)}}. \tag{13}$$

If the process of introducing innovative technologies at an enterprise enters a stationary final mode, then instead of inequality (11), one can use the inequality

$$q_0 \leq q(t) \leq q_\infty, \tag{14}$$

where $q_\infty = \lim_{t \to \infty} q_F(t)$.

By controlling the choice of function $q(t)$, it is possible to choose such a mode of operation of the enterprise, in which both the economic component of production and its social orientation will remain quite effective.

Let us introduce a dimensionless indicator $H(t)$ of the introduction of innovations at the enterprise and a dimensionless indicator of the redistribution of profits in the interests of the enterprise management and for the implementation of socially oriented programs $R(t)$. The functions $H(t)$ and $R(t)$ are bounded ($0 \leq H(t) \leq 1$, $0 \leq R(t) \leq 1$), continuous and continuously differentiable on the interval ($0 \leq t < \infty$).

The value of the function $H = 0$ corresponds to the beginning of the process of introducing innovations into production, the values of the function $H \to 1$ correspond to the completion of the process of introducing innovations into the production of the enterprise.

The function value $R = 0$ corresponds to the beginning of the profit redistribution process in the interests of the enterprise management and for the implementation of socially oriented programs, the function values $R \to 1$ correspond to the completion of the profit redistribution process in the interests of the enterprise management and for the implementation of socially oriented programs.

Suppose, that the increment of the innovation indicator $\Delta H$ for a certain short period of time $\Delta t$ will be proportional to the deviation of the function $H(t)$ from the maximum unit value, and the increment in the redistribution indicator $\Delta R$ for a certain short period of time $\Delta t$ will be proportional to the deviation of the function from the maximum unit value

$$\begin{align*}
\Delta H(t) & = \frac{\lambda}{T} \cdot (1 - H(t)) \cdot \Delta t, \\
\Delta R(t) & = \frac{\mu}{T} \cdot (1 - R(t)) \cdot \Delta t,
\end{align*} \tag{15}$$

where $\lambda$ is a parameter characterizing the rate of implementation of innovative technologies in production, $\mu$ is a parameter characterizing the rate of redistribution of profits, $T$ is the maximum time limit of the considered process of introducing innovations.

Passing to the limit under the condition $t \to 0$ leads to differential equations for the functions $H(t), R(t)$

$$\begin{align*}
\frac{dH(t)}{dt} & = \frac{\lambda}{T} \cdot (1 - H(t)), \\
\frac{dR(t)}{dt} & = \frac{\mu}{T} \cdot (1 - R(t)),
\end{align*} \tag{16}$$

whose solutions with initial conditions $H(0) = 0, R(0) = 0$ give

$$\begin{align*}
H(t) & = 1 - e^{-\frac{\lambda}{T} \cdot t}, \\
R(t) & = 1 - e^{-\frac{\mu}{T} \cdot t}.
\end{align*} \tag{17}$$
As a result of the innovative activity of the enterprise and management measures for the redistribution of profits, the function of the value of the product produced per unit volume of the resource \( P(t) \), the function of the elasticity of the output \( a(t) \), \( c(t) \), the cost coefficients \( A_Q(t) \), \( A_S(t) \) and the function \( q(t) \) will change over time in accordance with the formulas

\[
\begin{align*}
P(t) &= P_0 + (P_\infty - P_0) \cdot H(t), \\
a(t) &= a_0 + (a_\infty - a_0) \cdot H(t), \\
c(t) &= c_0 + (c_\infty - c_0) \cdot H(t), \\
A_Q(t) &= A_Q^0 + (A_Q^\infty - A_Q^0) \cdot H(t), \\
A_S(t) &= A_S^0 + (A_S^\infty - A_S^0) \cdot H(t), \\
q(t) &= h \cdot (q_0 + (q_\infty - q_0) \cdot R(t)),
\end{align*}
\]  
(18)

where \( P_0, P_\infty \) are the initial and final values of the quantity \( P(t) \), \( a_0, a_\infty \) are the initial and final values of the quantity \( a(t) \), \( c_0, c_\infty \) are the initial and final values of the quantity \( c(t) \), \( A_Q^0, A_Q^\infty \) are the initial and final values of the quantity \( A_Q(t) \), \( A_S^0, A_S^\infty \) are the initial and final values of the quantity \( A_S(t) \), \( q_0, q_\infty \) are the initial and final values of the quantity \( q(t) \), is the limiting coefficient of redistribution of the enterprise’s profit. When \( h = 0 \) all profits are invested in the development of production, while \( h = 1 \) all profits are gradually invested in the development of social programs and serving the opportunistic interests of the leadership.

Since with the development of the process of introducing innovations at the enterprise, output increases and costs decrease, inequalities \( P_0 \leq P_\infty, a_0 \leq a_\infty, c_0 \leq c_\infty, q_0 \leq q_\infty \) and \( A_Q^0 \geq A_Q^\infty, A_S^0 \geq A_S^\infty \) take place.

3. Modeling the dynamics of economic indicators in the short-term period of the production enterprise

Firstly, we investigate the activities of the enterprise in the short term. In this case, changes in the main and labor resources can be neglected and it can be assumed that \( Q(t) = \text{const} \), \( A_Q(t) = \text{const} \) and \( a(t) = \text{const} \). Then formulas (7)–(10) take the form

\[
\begin{align*}
TR(t) &= P(t) \cdot Q^a \cdot S(t)^{c(t)}, \\
TC(t) &= A_Q \cdot Q + A_S(t) \cdot S(t) + TFC, \\
PR(t) &= P(t) \cdot Q^a \cdot S(t)^{c(t)} - A_Q \cdot Q - A_S(t) \cdot S(t) - TFC, \\
U(t) &= PR(t) + q(t) \cdot S(t).
\end{align*}
\]  
(19) (20) (21) (22)

Formula (21) for the profit of an enterprise shows that its maximum value \( PR_{\text{max}} \) and the corresponding value of the production factor \( S_{\text{max}} \) will depend on time.

The quantity \( S_{\text{max}} \) is found from the condition

\[
\frac{dPR(t)}{dS(t)} = c(t) \cdot \left( (P(t) \cdot Q^a \cdot S(t)^{c(t)} - 1) - \alpha_S(t) \right) = 0,
\]  
(23)

where \( \alpha_S(t) = \frac{A_S(t)}{c(t)} \).

Solving equation (23) for \( S(t) \), we find the function \( S_{\text{max}}(t) \)

\[
S_{\text{max}}(t) = \left( \frac{P(t) \cdot Q^a}{\alpha_S(t)} \right)^\frac{1}{1 - c(t)}.
\]  
(24)
Substituting expression (24) into the formula for profit (20), we obtain the maximum profit function

\[ PR_{\text{max}}(t) = P(t) \cdot Q^a \cdot S_{\text{max}}(t)^{c(t)} - A_Q \cdot Q - A_S(t) \cdot S_{\text{max}}(t) - TFC. \] (25)

Values (24) and (25) are bounded from below and from above by their limiting values

\[
\begin{align*}
    cS_{\text{max}}^0 & \leq S_{\text{max}}(t) \leq S_{\text{max}}^\infty, \\
    PR_{\text{max}}^0 & \leq PR_{\text{max}}(t) \leq PR_{\text{max}}^\infty,
\end{align*}
\] (26)

where

\[
\begin{align*}
    S_{\text{max}}^0 &= \left( \frac{P_0 \cdot Q^a}{\alpha_S^0} \right) \frac{1}{1 - c_0}, & \alpha_S^0 &= \frac{A_S^0}{c_0}, \\
    S_{\text{max}}^\infty &= \left( \frac{P_\infty \cdot Q^a}{\alpha_S^\infty} \right) \frac{1}{1 - c_\infty}, & \alpha_S^\infty &= \frac{A_S^\infty}{c_\infty}, \\
    PR_{\text{max}}^0 &= P_0 \cdot Q^a \cdot (S_{\text{max}}^0)^{c_0} - A_Q \cdot Q - A_S^0 \cdot S_{\text{max}}^0 - TFC, \\
    PR_{\text{max}}^\infty &= P_\infty \cdot Q^a \cdot (S_{\text{max}}^\infty)^{c_\infty} - A_Q \cdot Q - A_S^\infty \cdot S_{\text{max}}^\infty - TFC.
\end{align*}
\] (27)

Now let us determine the optimal values of profit, which take into account the influence of the target transaction utility function on the operation of the enterprise. Substitution of profit function (21) into transaction utility function (22) gives

\[ U(t) = P(t) \cdot Q^a \cdot S(t)^{c(t)} - A_Q \cdot Q - A_S(t) \cdot S(t) - TFC + q(t) \cdot S(t). \] (28)

To calculate the optimal value of the resource, taking into account the influence of the target transaction utility function, we equate to zero its derivative of function (28)

\[ \frac{dU(t)}{dS(t)} = c(t) \cdot \left( P(t) \cdot Q^a \cdot S(t)^{c(t) - 1} - \alpha_S(t) \right) + q(t). \] (29)

The solution to equation (29) gives the optimal resource value \( S_{\text{opt}}(t) \)

\[ S_{\text{opt}}(t) = \left( \frac{P(t) \cdot Q^a}{\eta_S(t)} \right)^{1 - c(t)}. \] (30)

where \( \eta_S(t) = \alpha_S(t) - \frac{q(t)}{c(t)}. \)

It follows from the nonnegativity of the coefficients \( c(t), q(t) \) that the inequality holds \( \eta_S(t) < \alpha_S(t) \), comparing with the help of which the values of quantities (24) and (30) we obtain

\[ S_{\text{opt}}(t) \geq S_{\text{max}}(t). \] (31)

The optimal profit function has the form

\[ PR_{\text{opt}}(t) = P(t) \cdot Q^a \cdot S_{\text{opt}}(t)^{c(t)} - A_Q \cdot Q - A_S(t) \cdot S_{\text{opt}}(t) - TFC. \] (32)

Thus, it follows from relations (21), (27), and (28) that

\[ PR(S_{\text{opt}}(t)) \leq PR(S_{\text{max}}(t)). \] (33)
Equations (12) and (13) take the form

\[ P(t) \cdot Q^a \cdot S_F(t)^{c(t)} - A_Q \cdot Q - A_S(t) \cdot S_F(t)^{c(t)} \]
\[ S_F(t) - TFC = 0, q_F(t) = A_S(t) - \frac{P(t) \cdot Q(t)^a \cdot c(t)}{S_F(t)^{1-c(t)}}. \]

Figure 1 shows a graph of the surface of the profit function (21) and the lines of its contact with the surfaces of indifference of the target transaction utility function \( U(PR, S, t) = U(PR_{opt}, S_{opt}) \) for different values of the parameter \( h \). Solid spatial lines correspond to solutions of equations (30), (32) and show changes in time of the maximum and optimal profit of the enterprise. The solid line on the coordinate plane \( PR = 0 \) corresponds to the solution of equation (34) with respect to the function \( S_F \).

The lines in figure 1, corresponding to the parameters \( h = 0 \) and \( h = 1 \), represent the upper and lower boundaries of all possible options for redistributing enterprise profits between production and non-production costs. One of these options is built for parameter values \( h = 0.75 \).

Figure 2 shows the projections onto the coordinate plane \( S = 0 \) of the spatial lines of tangency of the graph of the surface of the profit function (21) with the surfaces of indifference of the target transaction utility function \( U(PR, S, t) = U(PR_{opt}, S_{opt}) \) for different values of the parameter \( h \).

4. Modeling the dynamics of economic indicators in the long-term period of the production enterprise
Consider now the long-term period of the enterprise. In this case, the production factor \( Q = Q(t) \) and elasticity \( a = a(t) \) are variable, and the functions of output, costs and profits are described by formulas (7)–(10)

The maximum possible values of the profit function (36) are found from the conditions

\[ \begin{align*}
\frac{\partial PR(t)}{\partial Q(t)} &= a(t) \cdot (P(t) \cdot Q(t)^{a(t)} \cdot S(t)^{c(t)} - \alpha_Q(t)) = 0, \\
\frac{\partial PR(t)}{\partial S(t)} &= c(t) \cdot (P(t) \cdot Q(t)^{a(t)} \cdot S(t)^{c(t)} - \alpha_S(t)) = 0,
\end{align*} \]

where \( \alpha_Q(t) = \frac{A_Q(t)}{a(t)} \).
determined by the formulas

Thus, the values of resources at which the profit of the enterprise takes the maximum value are

Calculated values: \( P_0 = 20; P_\infty = 25; Q = 1.5; a = 0.52; c_0 = 0.33; c_\infty = 0.35; A_Q = 0.4; A_Q^S = 1.7; A_S^S = 1.5; TFC = 20; T = 20; \lambda = 5; \mu = 6; \) \( q_\infty = 0.8865; S_{max}^0 = 10.2484; S_{max}^\infty = 20.5962; PR_{max}^0 = 147726; PR_{max}^\infty = 36.7752; S_{opt}^0 = 10.2484; S_{opt}^\infty = 50.7558; PR_{opt}^0 = 147726; PR_{opt}^\infty = 24.2998.\)

The system of equations (39) can be written in the form

\[
\begin{align*}
P(t) \cdot Q(t)^{a(t)} \cdot S(t)^{c(t)} &= \alpha_Q(t) \cdot Q(t), \\
P(t) \cdot Q(t)^{a(t)} \cdot S(t)^{c(t)} &= \alpha_S(t) \cdot S(t).
\end{align*}
\]

Equations (37) show that the quantities \( S_{max}(t) \) and \( Q_{max}(t) \) are related by the relation

\[ S_{max}(t) = \frac{\alpha_Q(t)}{\alpha_S(t)} \cdot Q_{max}(t). \]

Substituting formula (38) into the first equation of system (37), we find

\[ P(t) \cdot Q_{max}(t)^{a(t) + c(t) - 1} \cdot \left( \frac{\alpha_Q(t)}{\alpha_S(t)} \right)^{c(t)} = \alpha_Q(t). \]

Thus, the values of resources at which the profit of the enterprise takes the maximum value are determined by the formulas

\[
\begin{align*}
Q_{max}(t) &= \left( \frac{P(t)}{\alpha_Q(t)^{1-c(t)} \cdot \alpha_S(t)^{c(t)}} \right) \frac{1}{1 - a(t) - c(t)}, \\
S_{max}(t) &= \left( \frac{P(t)}{\alpha_Q(t)^{a(t)} \cdot \alpha_S(t)^{1-a(t)}} \right) \frac{1}{1 - a(t) - c(t)}.
\end{align*}
\]

The maximum profit value is calculated by the formula

\[ PR_{max}(t) = P(t) \cdot Q_{max}(t)^{a(t)} \cdot S_{max}(t)^{c(t)} \cdot \alpha_Q(t) \cdot Q_{max}(t) \cdot A_Q(t) \cdot S(t) \cdot S_{max}(t) - TFC. \]

Now let us calculate the optimal profit of the enterprise, taking into account the target transaction utility function (10). For this, it is necessary to jointly maximize the profit function

Figure 2. Projections of spatial lines of tangency of the graph of the surface of the profit function (21) with the surfaces of indifference of the target transaction utility function \( U(PR, S, t) = U(PR_{opt}, S_{opt}) \) for different values of the parameter \( h. \)
and the target transaction utility function (10). It is obvious that the optimal values of resources, profit function and transaction utility function are found from the conditions

\[
\begin{align*}
\frac{\partial U(t)}{\partial Q(t)} &= a(t) \cdot \left( P(t) \cdot Q(t)^{a(t)-1} \cdot S(t)^{c(t)} - \alpha Q(t) \right) = 0, \\
\frac{\partial U(t)}{\partial S(t)} &= c(t) \cdot \left( P(t) \cdot Q(t)^{a(t)} \cdot S(t)^{c(t)-1} - \alpha S(t) + q(t) \right) = 0.
\end{align*}
\]  

(42)

The solution to equation (42) has the following form

\[
\begin{align*}
Q_{\text{opt}}(t) &= \left( \frac{P(t)}{\alpha Q(t) \cdot \eta S(t)^{c(t)}} \right) \frac{1}{1 - a(t) - c(t)}, \\
S_{\text{opt}}(t) &= \left( \frac{P(t)}{\alpha Q(t) \cdot \eta S(t)^{1-a(t)}} \right) \frac{1}{1 - a(t) - c(t)}.
\end{align*}
\]  

(43)

Comparison of formulas (40) and (43) shows that the inequalities

\[
Q_{\text{opt}}(t) \geq Q_{\text{max}}(t), \quad S_{\text{opt}}(t) \geq S_{\text{max}}(t).
\]  

(44)

The optimal value of the profit of the enterprise is expressed by the ratio

\[
PR_{\text{opt}}(t) = P(t) \cdot Q_{\text{opt}}(t)^{a(t)} \cdot S_{\text{opt}}(t)^{c(t)} - A_Q(t) \cdot Q_{\text{opt}}(t) - A_S(t) \cdot S_{\text{opt}}(t) - TFC.
\]  

(45)

From relations (41), (44) and (45) it follows that

\[
PR(Q_{\text{opt}}(t), S_{\text{opt}}(t)) \leq PR(Q_{\text{max}}(t), S_{\text{max}}(t)).
\]  

(46)

Now let us apply the obtained formulas for calculating the maximum possible values of the profit function, and the optimal values of the profit function.

In the case under consideration of the long-term period of the enterprise’s operation, the surface of the profit function \( PR = PR(Q, S, t) \), the surface of indifference of the target transaction utility function \( U(PR, Q, S, t) = U_{\text{opt}} \) and the spatial lines of their contact are four-dimensional objects and cannot be depicted graphically.

Therefore, figure 3 shows projections onto the coordinate plane \( S = 0 \) of the spatial lines of tangency of the graph of the surface of the profit function (9) with the surfaces of indifference

**Figure 3.** Projections onto the coordinate plane \( S = 0 \) of spatial lines of tangency of the graph of the surface of the profit function (9) with the surfaces of indifference of the target transaction utility function \( U(\text{PR}, Q, S, t) = U_{\text{opt}} \) for different values of the parameter \( h \). Calculated values: \( P_0 = 100; P_\infty = 105; a_0 = 0.25; a_\infty = 0.251; c_0 = 0.263; c_\infty = 0.264; A_Q^0 = 20; A_Q^\infty = 18; A_S^0 = 25; A_S^\infty = 23; TFC = 50; T = 20; \lambda = 5; \mu = 6; q_\infty = 10.7465. \)
of the target transaction utility function \( U(PR, Q, S, t) = U_{\text{opt}} \) for different values of the parameter \( h \).

For the parameter value \( h = 0 \), the initial and final values of resources and profit are equal:
\[
Q_0^\text{max} = 1.4406; \quad Q^\infty_{\text{max}} = 1.9743; \quad S_0^\text{max} = 1.2124; \quad S^\infty_{\text{max}} = 1.6251; \quad PR_0^\text{max} = 6.1264; \quad PR^\infty_{\text{max}} = 18.6861.
\]

For the parameter value \( h = 1 \), the initial and final values of resources and profit are equal:
\[
Q_0^\text{opt} = 1.4406; \quad Q^\infty_{\text{opt}} = 2.7814; \quad S_0^\text{opt} = 1.2124; \quad S^\infty_{\text{opt}} = 4.2975; \quad PR_0^\text{opt} = 6.1264; \quad PR^\infty_{\text{opt}} = 0.5583.
\]

For the parameter value \( h = 0.85 \), the initial and final values of resources and profit are equal:
\[
Q_0^\text{opt} = 1.4406; \quad Q^\infty_{\text{opt}} = 2.6005; \quad S_0^\text{opt} = 1.2124; \quad S^\infty_{\text{opt}} = 3.5508; \quad PR_0^\text{opt} = 6.1264; \quad PR^\infty_{\text{opt}} = 8.0125.
\]

5. Conclusion

New economic and mathematical models have been developed for the dynamics of the development of industrial enterprises that introduce innovative technologies.

The influence of production and transaction costs on the size of the maximum and optimal profit in the short and long term periods of the enterprise has been investigated.

Numerical analysis of the obtained models of innovative enterprises shows that instead of the maximum level of profit, the enterprise can only achieve its lower optimal level, corresponding to the optimal value of the transaction utility function.

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