Using a Modified Turian–Yuan Model to Enhance Heterogeneous Resistance in Municipal Sludge Transportation Pipeline

Hai Lu, Lijun Chen, Jianhui Wang, Xiaoyu Zhang, Guang Li, Jing Wang, Wenluo Chen, and Bojiao Yan*

ABSTRACT: Based on the Turian–Yuan heterogeneous resistance model, the simulation results of three urban sludge pipelines with a volumetric concentration of 2.38, 3.94, and 5.39% were analyzed. The reasons for the large deviation of the simulation results under high Reynolds number conditions were also analyzed. The results showed that the deviation of the simulation was mainly caused by the difference between the sludge volumetric concentration \(C_V\), the settlement resistance coefficient \(\kappa_2\), and the values of the two parameters in the Turian–Yuan heterogeneous resistance model. Consequently, it was necessary to optimize the index \(m_1\) of \(C_V\) and the index \(m_2\) of \(\kappa_2\). Taking mean square deviation as the objective function, using Matlab programming, the abovementioned two indexes were optimized by the simulated annealing algorithm. The optimized index \(m_1\) of \(C_V\) was 0.887, and the index \(m_2\) of \(\kappa_2\) was \(-0.162\). Hence, a modified Turian–Yuan heterogeneous resistance model was obtained. The model verified that the minimum value of the regression coefficient, \(R^2\), of the simulated value reached 0.9701, proving that, the model can be used to simulate the heterogeneous resistance of urban sludge pipeline transportation.

1. INTRODUCTION

In modern industry, hydraulic conveying of pipes is often preferred when selecting the conveying means of pulverized coal and other granular mineral materials because of its merits of cleanliness, high efficiency, and easy automatic control. In transport, due to the intervention of settling solid particles, there are mutual contacts and momentum exchanges between particles and pipes, as well as between particles and particles. At the same time, the fine particle causes the change in the viscosity of water and other conveying media, which makes the hydraulic conveying process extremely complex and this is a typical two-phase flow problem.\(^{1-3}\) The research content of two-phase flow is very extensive, but in practical production, the most concerning issue is the calculation of flow resistance because the size of flow resistance determines the type of equipment and the size of the pipeline. It is also very important to reduce energy consumption in the process of transportation. Many scholars have carried out a large number of experiments on the two-phase flow resistance.\(^{4-13}\) According to their results, the experiential model or semiempirical model for calculating the flow resistance was obtained on experimentally. However, there are still some deficiencies in the generality of these models basically due to the variety of test materials they used, including the various mineral particles and the physical properties of these test materials which are quite different.\(^{12,14,15}\)

The two-phase flow resistance model that has been established so far is mainly based on diffusion theory and gravity theory.\(^{12}\) The basic idea of the previous concept is that solid particles and fluid particles participate in diffusion in the two-phase flow. Subsequently, the two-phase flow can be regarded as a pure liquid with a uniform density, while the latter is considered to be due to gravity consumption. The flow resistance of two-phase flow is regarded as the sum of clear water resistance and additional resistance. Meanwhile, the energy consumption of the two-phase flow is superior to the pure liquid. Since most solid particles have sedimentary properties, there must be a vertical concentration gradient in the pipeline to aid hydraulic transport. In other words, the actual two-phase flow is mostly heterogeneous. Therefore, it is inaccurate to treat two-phase flow as a pure liquid with uniform density according to the diffusion theory, especially when the solid concentration in two-phase flow is high. On the contrary, the two-phase flow resistance model based on gravity theory takes into account both the flow resistance of clear water and the additional resistance caused by the intervention of solid particles. Furthermore, from the perspective of the form and theoretical analysis of the model, this kind of model can reflect the influencing factors of resistance composition in two-phase flow. Simultaneously, the accuracy of...
the calculation of two-phase flow and they are obtained by applying regression methods. It may not be possible to match the test data well when applying it to the calculation of two-phase flow resistance under certain conditions. Therefore, when applied to the simulation of actual flow resistance, such resistance models must be optimized by the actual test data before more accurate simulation results can be obtained.

Consequently, for the sake of environmental protection, land conservation, and operation cost reduction, a large amount of municipal sludge produced in municipal wastewater treatment plants must be centralized, treated, and disposed of after pipeline hydraulic transportation. This system has been applied in developed countries such as Japan, the Netherlands, and the United States. The hydraulic transport process of these municipal sludge also belongs to the category of two-phase flow. However, owing to the permeability of sludge particles and the moisture content of sludge is often very high, in most cases approximately greater than 97%, the existing two-phase flow resistance model’s calculation has a low accuracy when used for the hydraulic transportation of sludge pipes. Therefore, this study is aimed to select the appropriate existing calculation model and improve it for the accurate calculation of flow resistance in the hydraulic transportation process of urban sludge pipeline. This phenomenon is significant according to the specific characteristics of urban sewage sludge to optimize the equipment and pipeline model and pipeline transportation parameters and to save the cost in the process of sludge pipeline transportation.

2. TEST MATERIALS AND METHODS

2.1. Test Materials. The municipal sludge used for the improvement and verification of the flow resistance model was prepared by the dewatered sludge of the sewage treatment plant in the western suburbs of Changchun City, which was diluted with effluent from the sewage plant. The moisture content of the original dewatered sludge was 75.2%. Volume ratios of 9.42:1, 5.29:1, and 3.60:1 of effluent and dewatered sludge of sewage plant were used to prepare the experimental sludge with different moisture contents 97.62, 96.06, and 94.61%, respectively. The volumetric concentrations were 2.38, 3.94, and 5.39% correspondingly, and the mixture densities were 105.26, 1009.05, and 1014.96 kg/m³, respectively. Besides, the particle size distribution and morphological characteristics of sludge were analyzed using a laser particle size analyzer and scanning electron microscopy. The results are shown in Figures 1 and 2.

2.2. Test Device. The sludge flow resistance test device used in the test is shown in Figure 3. The major parts were test pipes, sludge pump, sludge tank, control valves, pressure tapings, pressure sensors, reflux pipe, observation segment, and vortex flowmeter.

The effective volume of the sludge tank was 500 L (diameter = 100 cm, height = 65 cm, over height = 20 cm), wherein the sludge pump was installed (model QW40-15, flow 40 m³/h, head 15 m, power 2.2 kW). A plexiglass observation segment of 1 m in length was arranged on the reflux pipe (PP-R pipe, D = 51 mm) to observe the flow phenomenon in the pipe. A vortex flowmeter (LWY-40, Hong Kong Huixiang Automation System Co., Ltd) was set on the reflux pipe with a measuring range of 50 m³/h and an accuracy of 0.01 m³/h to detect the flow of sludge in the pipeline. The model of the pressure sensor was RS-YB07. There are 4 test pipe sections in the device, each with a valve control. The main parameters of pipe sections are as follows in Table 1. Measuring sections are placed in the straight pipes in such a way that the mixture flow structure in the sections is not affected by bends and other sources of flow disturbance. Generally, the effectiveness of the flow is assessed by the ratio of distance between the first upstream pressure transducer and upstream flow T-diversion (bend) to the inner diameter of the measuring section. In previous studies, Gong et al. (2013) set this ratio as 10 in their simulating research. In experimental studies, Sudo. et al. (1998), Kaushal et al. (2005), Matousek et al. (2002), and Delgado et al. (2005) set these values as

---

Figure 1. Frequency distribution of test sludge particle size.

Figure 2. Scanning electron microscopy image of test sludge.

---

Table 1. Measuring sections are placed in the straight pipes in such a way that the mixture flow structure in the sections is not affected by bends and other sources of flow disturbance. Generally, the effectiveness of the flow is assessed by the ratio of distance between the first upstream pressure transducer and upstream flow T-diversion (bend) to the inner diameter of the measuring section. In previous studies, Gong et al. (2013) set this ratio as 10 in their simulating research. In experimental studies, Sudo. et al. (1998), Kaushal et al. (2005), Matousek et al. (2002), and Delgado et al. (2005) set these values as...
Figure 3. Sludge flow resistance test device diagram. 1. No. 1 Pipe section, 2. No. 2 Pipe section, 3. No. 3 Pipe section, 4. No. 4 Pipe section, 5. Sludge pump, 6. Sludge tank, 7. Control valve, 8. Pressure tapings, 9. Pressure sensor, 10. Reflux pipe, 11. Observation segment, and 12. Vortex flowmeter.

Table 1. Main Parameters of Pipe Sections

| item | pipe length, L (mm) | inner diameter, D (mm) | ratio L to D | material |
|------|----------------------|------------------------|--------------|---------|
| no. 1 | 5000 | 61.2 | 82:1 | PPR |
| no. 2 | 5000 | 38.3 | 131:1 | PPR |
| no. 3 | 5000 | 45.2 | 111:1 | UPVC |
| no. 4 | 5000 | 35.0 | 143:1 | stainless steel |

40, 57.9, 44, and 16.5, respectively. Therefore, in this research, accordingly, the measuring sections start 1200 mm (i.e., in the 19.6, 31.3, 26.5, and 34.3 times inner diameter distance, respectively) behind the upstream bend and T-diversions.

2.3. Test Plan. The sludge pump was immersed below the liquid level of the sludge tank, and the sludge was sucked from the bottom and around the sludge pump to ensure that the sludge in the sludge tank was evenly mixed. The four test pipes were operated separately. In the experiment, under the premise of ensuring full flow, the opening degree of the valve was constantly adjusted to control the flow of sludge in the operating pipe section for the sludge of different volumetric concentrations. In addition, the reading of flowmeter and pressure sensor was recorded for the simulation analysis and improvement of the model, as well as the verification of the improved model. For each determined flow rate, 20 pressure sensor readings were recorded within 20 min at an interval of 1 min. Statistical analysis of the measured pressure results was expressed as mean ± error bars.

2.4. Selection of the Resistance Model. The model of sludge pipeline transportation resistance was based on the theoretical research and experimental results of solid—liquid two-phase flow resistance. It was deduced that the current research results of the resistance model of solid—liquid two-phase flow can be summarized into the following four categories.

The first category was the results of the experiments. Durand et al. obtained from flow tests using sand with a density, $\rho_s$, of 2.65 g/cm³ and a density of 1.60 and 3.95 g/cm³, respectively. Based on the results of additional tests of plastic sand and emery sand, the relationship among the average flow velocity of settling slurry, the volume concentration of slurry in the pipe, the diameter of the pipe, and the additional hydraulic gradient loss of the pipe were objectively revealed. In the form of friction loss coefficient and Freund’s number, a representative hydraulic gradient calculation of settling solid–liquid two-phase medium pipe was proposed. Calculation formula, Durand formula, as

$$\frac{J - J_w}{CJ_w} = K_b \left( \frac{\mu_m^2 \sqrt{C_D}}{gD(\delta - 1)} \right)^{-1.5}$$

(1)

where $J$ is the gradient of heterogeneous flow; $J_w$ is the hydraulic gradient of clear water; $C$ is the volume concentration; $K_b$ is the coefficient; $\mu_m$ is the average flow velocity of the solid—liquid mixture; $C_D$ is the resistance coefficient; $g$ is the acceleration of gravity; $D$ is the inner diameter of the pipeline; $\delta$ is the ratio of solid particle density, and $\rho_s$ is the clear water density.

The second category was the research results which took certain theories as the guide and then auxiliary to determine the coefficient in the theoretical formula by the experiment. The scholars represented by D. M. Newitt studied the basic problem of analyzing the energy needed to transport solid particles, and according to the fact that the power consumed by the flow to bear the suspension of solid particles is equal to the energy obtained by the suspended solid particles in the flow, in that;

$$E_b = K_\delta (\rho - \rho_s) gV_1$$

(2)

where $K_\delta$ is a constant; $g$ is the concentration in the pipe; and $V_1$ is the final settling velocity of a single particle, so a formula for calculating the hydraulic gradient of settling slurry similar to the gravity theory in sediment research with certain theoretical basis is proposed.

The third category was the theoretical results using the dimensional analysis method as the research means. Turian—Yuan was based on the regression analysis of a large number of test data by the dimensional analysis method; a formula for calculating the hydraulic gradient of pipes under various flow states of settling slurry was proposed. To calculate the additional hydraulic gradient caused by particle intervention, formula 3 was introduced. At the same time using dimensional analysis, the result of the friction coefficient and the following parameters are found with a function of formula 4.
Table 2. Summarized List of the Research Results of Existing Representative Two-phase Flow Resistance Models

| author(s) | resistance models | parameters | applicable object |
|-----------|-------------------|------------|-------------------|
| Durand (1950) | \( i_m = i_w + 82C_1 \left( \frac{v_m^2}{gD(\delta - 1)} \right)^{1.5} \) | | based on gravity theory and a large number of experimental results; suitable for coarse particles and low solid concentration according to different conditions, the coefficient should be changed; suitable for coarse particles and low solid concentration based on experiments; suitable for coarse particles with diameter larger than 0.2 mm based on theoretical deduction, and the composition of flow resistance is discussed; suitable for slurries, but with poor practicability based on theoretical deduction and data verification; suitable for slurries, but with poor practicability based on theoretical deduction and data verification; suitable for coarse particles Pipe diameter 12.6–699 mm, solid particle size 0.05–38 mm, and solid particle density 1.16–11.3 g/cm³, the average flow rate of the solid–liquid mixture was 0.009–6.7 m/s |
| D. M. Newitt (1961) | \( i_m = i_w + 1100C_1(\delta - 1) \frac{\rho gD}{\varepsilon_m\varepsilon_w} \) | | |
| Jian Dai (1985) | \( \Delta_p = \frac{n_i - i_m}{i_w} \) where \( i_w = \frac{\Delta p_m - \Delta p}{\delta_{ad}} \) | | |
| Guangwen Chen (1994) | \( i_m = \frac{2v_m^2}{gD} + C(\delta - 1) \frac{\omega}{\varepsilon_m} + D_c \frac{C_{iv}}{D(1 - C/C_m)} \) | | |
| Sundqvist A (1996) | \( i_m = \zeta_i + i_w \) | | |
| Jianxin Xia (2002) | \( i_m = 2\mu_s(\delta - 1)C, \) where \( i_m = i_w + C(\delta - 1) - \frac{v^2}{gD} \) | | |
| Turian–Yuan (1977) | \( i_m = i_w + 0.551C, 0.869C_{iv} = 0.168 + (\frac{v^2}{gD}) (\mu_s - \rho/\rho) \) | | |
\[ J_s = \lambda_s \frac{v^2_{\text{in}}}{2gD} \quad (3) \]

\[ \lambda_s = f(C, \lambda_w, C_D, Fr) = K_s C^{m_1} \lambda_w^{m_2} C_D^{m_3} Fr^{m_4} \quad (4) \]

where \( K_s \) is the coefficient and \( m_1, m_2, m_3, \) and \( m_4 \) are all indexes.

The fourth category was the results of using pure theoretical analysis as research means. Carstens et al. made a theoretical study on the relationship between the factors that determine the flow mechanism and characteristics of heterogeneous flow. Aykanswa started from the study on the relationship between the velocity distribution and the concentration distribution and investigated the theoretical relationship between the hydraulic gradient and the concentration and velocity. Therefore, Roco and Shook gave their numerical solution in a study. Table 2 summarizes a list of the existing representative research results of two-phase flow resistance models.

Among the abovementioned research results, the first type of model was the empirical formula obtained through experiments, which failed to reveal the interaction mechanism between liquid and solid particles in the solid–liquid two-phase flow and the generation mechanism of slip motion between them. The specific application conditions included the solid particle size: 0.2–25 mm, and the solid particle density range: 1.5–3.95 g/cm³, pipe diameter range was 40–580 mm. The second type of model was a semiempirical formula proposed by theoretical analysis, which is quite controversial because the flow directly bears the energy consumption of the moving part, and its turbulence can promote the work of suspended particles, but this part of the capacity is not directly driven by the flow energy. Besides, because only 25 mm pipe diameter was used for the test, the reliability of the coefficient in the hydraulic gradient formula needed to be demonstrated. The third type of model was the resistance model obtained by regression analysis based on the research done by the scholars themselves, and the data range included: pipe diameter 12.6–699 mm, solid particle size 0.03–38 mm, and solid particle density 1.16–11.3 g/cm³, the average flow rate of the solid–liquid mixture was 0.009–6.7 m/s. The model can be applied in most cases, but it could not explain the interaction between the liquid and solid and the flow mechanism of heterogeneous flow. For the fourth type of model, because of the limitations of the theoretical understanding of the complex...
two-phase flow of the solid and liquid, the research results only stayed at the theoretical level and could not be applied to the actual situation.\textsuperscript{1,2,4}

It should be noted that the abovementioned calculation models of pipeline friction loss were empirical or semiempirical models based on a large number of measured data. Not all formulas applied to the calculation of sludge in sewage plant examined in this research. Future studies can consider the analysis and selection of suitable models for calculation according to the slurry conditions to be studied.

Due to the sedimentation properties of the urban sludge, the sludge exhibited a heterogeneous characteristic when transported at a flow velocity within a practical range, that is to say, the sludge had a vertical concentration gradient in the pipe flow. Therefore, the heterogeneous flow resistance model based on gravity theory was used to simulate sludge flow resistance. The Turian–Yuan heterogeneous resistance model is one of the more widely used models in gravity theory.\textsuperscript{5,12,18} Its model form is shown in formula 5

\[
i_n = i_0 + K_Y C_v C_D \left[ \frac{v^2}{gD} \left( \frac{\rho_s - \rho}{\rho} \right) \right]^{n_1} i_0^{n_2}
\]  

(5)

where \(i_n\) — hydraulic gradient of the two-phase mixture (%); \(i_0\) — clear water hydraulic gradient (%); \(C_v\) — volumetric concentration of the two-phase mixture (%); \(C_D\) — particle settling resistance coefficient, when \(1000 < R_e < 2 \times 10^5\), it is generally 0.4–0.43, of which \(R_e\) is particle Reynolds number. \(v\) — conveying velocity (m/s); \(g\) — acceleration of gravity (m/s\(^2\)); \(D\) — pipe diameter (m); \(\rho_s\) — solid density (kg/m\(^3\)); \(\rho\) — water density (kg/m\(^3\)); \(K_Y\) — additional hydraulic gradient coefficient; and \(m_1\), \(m_2\), \(m_3\), and \(m_4\) — additional hydraulic gradient index.

The values of the typical additional hydraulic gradient coefficient and index were as follows: \(K_Y = 0.551\), \(m_1 = 0.869\); \(m_2 = 0.168\); \(m_3 = -0.694\), and \(m_4 = 1.2\), so the Turian–Yuan heterogeneous resistance model was expressed as

\[
i_n = i_0 + 0.551 C_v^{0.869} C_D^{-0.168} \left[ \frac{v^2}{gD} \left( \frac{\rho_s - \rho}{\rho} \right) \right]^{-0.694} i_0^{1.2}
\]  

(6)

From eq 6, the first term represents the hydraulic gradient of clear water and the second term represents the additional hydraulic gradient due to the intervention of solid particles. Therefore, by making the expression of the total hydraulic gradient in Turian–Yuan heterogeneous resistance model very reasonable. In addition, the influencing factors of two-phase flow resistance such as two-phase flow volumetric concentration, \(C_v\), particle settling resistance coefficient, \(C_D\), tube diameter, \(D\), solid density, \(\rho_s\), were considered in the model. The model is an explicit expression with less calculation, is easy to solve, and has high precision. At the same time, through comparison, it was found that the conditions of the sludge sample and pipeline system used in the test met the applicable scope of the Turian–Yuan model. In a nutshell, it is feasible to simulate sludge flow resistance with the Turian–Yuan heterogeneous resistance model. Origin used data fitting to analyze the accuracy and simulation of the model.

3. RESULTS AND DISCUSSION

3.1. Analysis of Simulation Results of the Turian–Yuan Heterogeneous Resistance Model. The Turian–Yuan heterogeneous resistance model was used to simulate and analyze the flow resistance of the three types of sludge in the pipeline with volumetric concentration, \(C_v\), of 2.38, 3.94, and 5.39%, respectively. The data fitting to analyze the accuracy of the model and the simulation results were compared with the measured results shown in Figure 4.

As shown in Figure 4, the simulated values of the three sludge hydraulic gradients with a volumetric concentration of 2.38, 3.94, and 5.39% using the Turian–Yuan heterogeneous resistance model are slightly higher than the measured values at lower flow velocities (\(R_e\) was less than 8500–9000). However, significantly higher than the measured values at higher flow velocities (\(R_e\) was greater than 8500–9000). Hence, the Turian–Yuan heterogeneous resistance model had a higher accuracy when \(R_e\) was not very large, but when \(R_e\) was large, the Turian–Yuan heterogeneous resistance model had a large analogue deviation to the sludge flow resistance. Therefore, before applying the Turian–Yuan heterogeneous resistance model in simulation municipal sludge flow resistance, it is necessary to modify the relevant parameters or indexes in the model to improve its accuracy.

3.2. Improvement of the Turian–Yuan Heterogeneous Resistance Model. It can be deduced from formula 5 that when the Turian–Yuan heterogeneous resistance model was used to simulate the hydraulic gradient, the hydraulic gradient in the model was composed of the hydraulic gradient and an additional hydraulic gradient. Hence, the structure of the model was reasonable. Also, the model was derived from a large amount of test data by regression analysis, so the coefficients \(K_Y\) and \(m_1\) in the model did not need to be verified. In unison, the density of physical property parameters of solid particles in sludge was a definite quantity, which conformed to the application conditions of the Turian–Yuan heterogeneous resistance model. Although the velocity, \(v\), and diameter, \(D\), of the transport parameters, had a greater impact on the overall hydraulic gradient, this effect was related to sludge \(C_v\) and \(C_D\), so the index \(m_1\) in the model did not need to be checked. Thus, the model accuracy can be significantly influenced by optimizing other indexes.

From the abovementioned analysis, it can be seen that \(C_v\) and \(C_D\) in the Turian–Yuan heterogeneous resistance model depended on the physical properties of the sludge on the one hand and at the same time was associated with the transport conditions such as the density of solid particles, \(\rho_s\), the flow velocity, \(v\), and pipe diameter, \(D\). For that reason, the two parameters that had a great influence on the accuracy of the model were optimized.

The impact of \(C_v\) and \(C_D\) on the accuracy of the Turian–Yuan heterogeneous resistance model can be analyzed as follows.\textsuperscript{23–29} First, although the Turian–Yuan heterogeneous resistance model can be used to simulate the flow resistance of solid–liquid two-phase flow with volumetric concentration \(C_v\) = 0.6–42%, the experimental data under high concentration \((C_v = 10–40\%)\) were mainly used in the parameter and exponential regression of the model. Therefore, the accuracy was not ideal when applied to the hydraulic calculation of the relatively low concentration of two-phase flow.\textsuperscript{26} As the \(C_v\) of urban sludge was relatively low, usually about 0.80–4.00%, the strength of the flocculation network structure formed between solid particles was relatively weak. Therefore, the friction resistance between particles and the relative movement between particles and clear water decreased, which was replicated in the reduction of hydraulic gradient on the macrolevel. Also, the solid particles in urban sludge were mainly activated sludge, which was
characterized by a large specific surface area. Therefore, the settling resistance coefficient \( C_\Omega \) was much larger than that of sediment, clay, water, and coal slurry, which are widely involved in the Turian–Yuan heterogeneous resistance model. At the same time, coupled with the up-floating effect of the oil contained in the sludge on the sludge particles, the settling resistance coefficient \( C_\Omega \) of the sludge particles further increased. The flow state evolved to a homogeneous flow with a higher degree of homogenization, which further reduced the flow resistance of sludge.

In summary, the actual flow resistance of municipal sludge was lower than the simulation value of the Turian–Yuan heterogeneous resistance model due to the difference of physical properties between municipal sludge and ordinary solid materials. Therefore, to make the Turian–Yuan heterogeneous resistance model application in the simulation of sludge flow resistance more accurate, it is necessary to optimize the index \( m_1 \) of sludge volumetric concentration \( (C_v) \) and the index \( m_2 \) of sludge particle settling resistance coefficient \( (C_\Omega) \).

### 3.2.1 Demonstration on Optimization of the Index \( m_1 \) of the Volumetric Concentration, \( C_v \)

The volumetric concentration had a great influence on the physical properties of the sludge, especially the conveying properties in the pipeline. It can be attributed to the influence of sludge flow state and flow resistance. The transportation of sludge in the pipeline belongs to the two-phase flow of lower concentration because of the relatively small amount of solid particles contained, the microform was weak in the structure of the floc net between solid particles, and the macrolevel was characterized by a decrease in viscosity. Because of this, its flow state and resistance characteristics were quite different from that of high-concentration solid–liquid phase media. When the sludge was in different flow states, that is, homogeneous flow or heterogeneous flow, the flow resistance varied greatly due to the different interaction mechanisms between sludge particles and between sludge particles, pipe walls and flow.

To study the distribution law of sludge solid particles in pipelines to determine their homogeneity (according to the introduction of relevant literature), there are five theories, namely, diffusion theory, mixed theory, energy theory, similar theory, and random theory. Although their starting points are different, the conclusions are the same or similar to those derived from turbulent diffusion theory. According to the classical turbulence diffusion theory, owing to turbulence movement, suspended solid particles diffused from the region of high concentration to that of low concentration and the distribution of turbulent flow velocity changed. Furthermore, compared with the turbulent velocity of clear water under the same boundary conditions, the distribution of turbulent velocity was uneven.

The upward exchange rate of suspended particles due to turbulence is given as

\[
\dot{E}_s = \frac{dC_v}{dy}
\]

Because in the turbulent fluid, the shear stress \( \tau \) at any point is given as

\[
\tau = \rho E_m \frac{du}{dy}
\]

and

\[
\frac{du}{dy} = \frac{1}{k y} \sqrt{\frac{\tau}{\rho}}
\]

where \( \frac{du}{dy} \)—velocity gradient at any point \( (s^{-1}) \); \( \rho \)—density of the liquid \( (kg/m^3) \); \( \tau \)—pipe wall shear stress \( (Pa) \); \( k \)—Karman constant \( (k=0.4) \); and \( v^* \)—friction flow velocity \( (m/s) \).

Open channel flow is given as

\[
\tau = \tau_y \left( 1 - \frac{y}{y_m} \right)
\]

where \( y_m \)—the height from the canal bottom to the free surface \( (m) \).

From formulae 10 and 11, eq 13 is derived as

\[
\frac{du}{dy} = \frac{v^*}{ky}
\]

By integrating eqs 13, 14 is given as

\[
\frac{u - u_{max}}{v^*} = \frac{1}{k} \ln \left( \frac{y}{y_m} \right)
\]

where \( u_{max} \)—velocity (m/s) at the free surface.

By differentiating eq 14, \( E_m \) can be written as

\[
E_m = kv^* \left( 1 - \frac{y}{y_m} \right)
\]

considering eq 8

\[
E_s = \beta k v^* y \left( 1 - \frac{y}{y_m} \right)
\]

Substitute \( E_s \) in eq 7 and integrate is written as

\[
\ln \frac{C}{C_h} = \alpha \psi \ln \left( \frac{y}{y_m} \right)
\]

where \( a \)—the height \( (m) \) of a known datum plane and \( C_h \)—volumetric concentration of the datum plane \( (\%) \).

Although eq 17 is mostly applied to open channel flow, it can be applied to pipeline transportation of solid–liquid two-phase flow after deformation and introduction of limited conditions. The form is as follows

\[
C = \left( \frac{h}{h_m} \right)^Z
\]
In two-phase flow pipeline transport, the Karman constant $k$ of the two-phase flow is less than 0.4, the higher the concentration, the smaller the $k$. For two-phase fluids with a $\beta$ value greater than 1.0, when particle size is 0.1 mm, $\beta = 1.3$, and when particle size is 0.16 mm, $\beta = 1.5$. However, to make the design safer, generally $\beta = 1.0$ is preferable.

The log of the concentration ratio is given by:

$$\log \frac{C}{C_A} = -\left(\frac{1.8\omega_0}{\beta kP^*}\right)$$

where $C$—the volumetric concentration (%) from the top of the tube at 0.08D; $C_A$—the volumetric concentration at the center of the tube (%); $\omega_0$—settling velocity (m/s) of solid particles.

Figure 5. Vertical distribution of sludge concentration in the test pipe sections. (a) no. 1 pipe section, $C_w = 2.38\%$; (b) no. 1 pipe section, $C_w = 3.94\%$; (c) no. 1 pipe section, $C_w = 5.39\%$; (d) no. 2 pipe section, $C_w = 2.38\%$; (e) no. 2 pipe section, $C_w = 3.94\%$; (f) no. 2 pipe section, $C_w = 5.39\%$; (g) no. 3 pipe section, $C_w = 2.38\%$; (h) no. 3 pipe section, $C_w = 3.94\%$; (i) no. 3 pipe section, $C_w = 5.39\%$; (j) no. 4 pipe section, $C_w = 2.38\%$; (k) no. 4 pipe section, $C_w = 3.94\%$; and (l) no. 4 pipe section, $C_w = 5.39\%$. 

ACS Omega 2021, 6, 7199−7211
Formula 18 is of high accuracy and has been widely used in engineering practice. In addition, for

\[ \nu^b = \left( \frac{\tau_w}{\rho_m} \right)^{1/8} \]  

(19)

where, \( \overline{\nu} \) —the average flow velocity in the pipe (m/s); \( \lambda_m \) —the Darcy resistance coefficient of the pipe; and \( \rho_m \) —the density of the two-phase fluid (kg/m^3).

The vertical distribution of sludge concentration in the pipeline can be calculated using formula 18. Figure 5 shows the vertical distribution of sludge concentration in four test pipes calculated based on formula 18 when sludge volumetric concentrations were 2.38, 3.94, and 5.93%, respectively. It can be seen from Figure 5 that the sedimentation property of the sludge solid particles under the action of gravity was greatly affected by the volumetric concentration \( C_s \) of the sludge and the average flow velocity. When the \( C_s \) was high, the viscosity of the sludge increased significantly. Meanwhile, the buoyancy force of the sludge particles was larger when it sunk, so it was easier to achieve homogeneity. On the other hand, when the sludge \( C_s \) was low, the viscosity of the sludge was smaller due to the lack of enough smaller grain size solid particles in the sludge. Sludge particles are less resistant to sinking and therefore appear to be easier to sink.30—32 As the flow velocity increased, the sedimentation characteristics of solid particles were disturbed by the intensification of turbulence, and the particles were vortexed to enter the mainstream flow region, thereby allowing the particles to be fully mixed throughout the pipeline. The distribution of vertical concentrations was more uniform, as shown in the figure, when the flow velocity, \( \overline{\nu} \), was 4.0 and 5.0 m/s.

Based on the abovementioned analysis, the homogenization degree of sludge was higher when the average flow velocity increased, while the Turian—Yuan heterogeneous resistance model is mainly applicable to heterogeneous fluids. Therefore, the index \( m_l \) of sludge volumetric concentration \( C_s \) must be optimized to ensure the simulation accuracy of the Turian—Yuan heterogeneous resistance model at a higher flow velocity.

### 3.2.2. Demonstration on Optimization of the Index \( m_l \) of the Settlement Resistance Coefficient, \( C_D \)

When particles settle in the liquid, the resistance, \( F \), of the liquid affects them, which is given as

\[ F = C_D \frac{\pi d^2 \rho_0 \nu^2}{4} \]  

(20)

where \( d \) —the equivalent diameter of the particles (m); \( \rho \) —the density of the liquid (kg/m^3); and \( C_D \) —resistance coefficient.

Particles begin to settle at an accelerated rate, once the balance between gravity, buoyancy, and resistance is reached, the settlement is of the equal speed. The time from acceleration to the equal speed is very short, so the whole process can be seen as an equal speed-settling problem.

The effective gravity after removing buoyancy, \( W \), which is calculated using eq 21

\[ W = \frac{\pi}{6} d^3 (\rho_0 - \rho) \]  

(21)

where \( \rho_0 \) —the density of solid particles (kg/m^3)

From \( F = W \), the general equation for calculating \( \nu^b \) can be obtained, shown as 22.

\[ \omega_0 = \frac{4}{3} \pi g \rho_0 \frac{1}{1 - \frac{\rho_0}{\rho}} \]  

(22)

Then

\[ \Delta = \frac{\rho_0}{\rho} \]  

(23)

In eq 23, \( \Delta \) is defined as the density ratio of solid particles to liquid after removing buoyancy. Equation 22 deforms as follows

\[ \omega_0 = \frac{4}{3} \pi g \rho \Delta \]  

(24)

The resistance coefficient, \( C_D \), in eq 24 is related to Reynolds number, \( R_{eD} \), of particles as in eq 25

\[ R_{eD} = \frac{d \omega_0 \rho}{\mu} = \frac{d \omega_0}{\nu} \]  

(25)

where \( \mu \) —the dynamic viscosity of the liquid (Pa·s) and \( \nu \) —the kinematic viscosity of the liquid (m²/s).

Then

\[ \nu = \frac{\mu}{\rho} \]  

(26)

For the laminar flow region of low \( R_{eD} \), surface resistance plays a major role, as shown in eq 27

\[ C_D = \frac{24}{R_{eD}} \]  

(27)

For the turbulent region of high \( R_{eD} \), the inertia resistance plays a major role, and the resistance coefficient is almost independent of the Reynolds number of particles. Generally, \( C_D = 0.4—0.43 \) is chosen. For the transition region, between the laminar flow region and turbulent flow region, there is no specific expression of \( C_D = f \left( R_{eD} \right) \), which is determined experimentally. Figure 6 shows the \( C_D-R_{eD} \) relationship of spherical particles.
3.2.3. Optimization Solution of Indexes \( m_1 \) and \( m_2 \).

At present, the algorithms used for global optimization mainly includes an artificial neural network and the simulated annealing algorithm. Among them, the simulated annealing algorithm is a point-to-point search algorithm, whose overall performance is better than the artificial neural network. Therefore, the simulated annealing algorithm was adopted to optimize the index \( m_1 \) of CV and the index \( m_2 \) of CD. The simulated annealing algorithm uses the principle of solid annealing to heat the solid to a high enough temperature and then slowly cool it down\(^{33,34}\). With the increase in temperature (heating), the internal particles of the solid turn out to be disordered, the internal energy become larger and gradually tend in an orderly state when they cool down. The equilibrium state is reached at each temperature point and finally reaches the ground state at normal temperature and the internal energy is minimized. The simulated annealing algorithm can be grouped into three parts: solution space, target function, and initial solution. The basic idea of the simulated annealing algorithm includes\(^{33,35,36}\):

1. Initialization: initial temperature, \( T_i \) (sufficiently large), initial solution, \( S_i \) (the initial value of algorithm iteration), iteration of each \( T \) value \( n \) times;
2. For \( j = 1, \ldots, n \) was performed using the following steps (3) to (6);
3. Generating \( S' \) as a new solution;
4. Calculate the increment of each step \( \Delta t' = C(S') - C(S) \), where \( C(S) \) is the objective evaluation function;
5. When \( \Delta t' < 0 \), \( S' \) is accepted as a new solution, otherwise \( S' \) is accepted as a new solution by the probability of \( \exp(-\Delta t'/T) \);
6. When the termination condition was satisfied, the output solution can be regarded as the optimal solution and the program can be terminated at the same time;
7. \( T \) gradually decreases, and when \( T \geq 0 \), turn to step 2.

In the fitting process of the hydraulic gradient of the sludge pipeline transport, the mean square deviation was used as the target function, and the target function is shown in eq 28.

Figure 7. Comparison between simulated values of the modified Turian–Yuan heterogeneous resistance model and measured values of the hydraulic gradient. (a) no. 1 pipe section, (b) no. 2 pipe section, (c) no. 3 pipe section, and (d) no. 4 pipe section.
where $F(x)$—objective function; $i_c$—the $i$th observation value; and $i_m$—calculated value.

Matlab was used for programming processing and according to the previous analysis, the range of values of $C_v$ was 0.008 to 0.04 and the range of values of $C_D$ was 0.428 to 0.446. Finally, the index $m_1$ of the optimized sludge volumetric concentration $C_v$ was 0.887 determined by the simulated annealing method, and the index $m_2$ of the optimized sludge particle settlement resistance coefficient $C_D$ was $-0.162$.

In summary, the optimized Turian−Yuan heterogeneous resistance model, that is, the improved Turian−Yuan heterogeneous resistance model is given as

$$i_m = i_0 + 0.551C_v^{0.887}C_D^{-0.162} \left( \frac{v^2}{gD} \left( \frac{\rho_s - \rho}{\rho} \right) \right)^{-0.694} i_0^{1.2}$$  \hspace{1cm} (29)  

where $i_m$—sludge hydraulic gradient (%); $i_c$—clear water hydraulic gradient (%); $C_v$—sludge volumetric concentration (%), and the range of values is 0.008–0.04, $C_D$—particle settling resistance coefficient, when $1000 < R_{ed} < 2 \times 10^5$, it is generally 0.4–0.43, of which $R_{ed}$ is particle Reynolds number. $v$—conveying velocity (m/s); $g$—acceleration of gravity (m/s²); $D$—pipe diameter (m); $\rho_s$—solid density (kg/m³); and $\rho$—water density (kg/m³).

The simulation results of the measured hydraulic gradient of municipal sludge using the modified Turian−Yuan heterogeneous resistance model on the no. 1 to no. 4 pipes is shown in Figure 7.

### 3.3. Verification of the Modified Turian−Yuan Heterogeneous Resistance Model.

In order to confirm the effectiveness of the improved Turian−Yuan heterogeneous resistance model to simulate sludge flow resistance, the additional measured hydraulic gradient data of pipeline no.1 to no. 4 were used to verify the results, as shown in Figure 8. The comparison of the simulation accuracy of the Turian−Yuan heterogeneous resistance model before and after improvement is shown in Table 3. From Table 3, the improved Turian−Yuan
heterogeneous resistance model had a high accuracy on simulating hydraulic gradient during sludge pipeline transportation. In the case of 2.38, 3.94, and 5.39% concentrations in no. 1 to no. 4 pipelines, the minimum value of the regression coefficient $R^2$ of simulated values reached 0.9701, which is acceptable for the model. The applicable conditions for improved the Turian–Yuan heterogeneous resistance model includes (1) the sludge of the municipal sewage plant; (2) sludge volume concentration range was 2.38–5.39%, sludge density range: 1005.26–1014.96 kg/m³; and (3) transportation flow rate range was 0.5–4.47 m/s.

4. CONCLUSIONS

Based on the Turian–Yuan heterogeneous resistance model, the index, $m_1$, of sludge volume concentration, $C_w$, and the index, $m_2$, of sludge particle settling resistance coefficient $C_D$ were corrected. Hence, a modified Turian–Yuan heterogeneous model of sludge pipeline transportation resistance is obtained. The accuracy of the improved Turian–Yuan heterogeneous resistance model was verified using the measured data of a hydraulic gradient. The results showed that the simulation value were in good agreement with the measured value, which confirmed that the improved Turian–Yuan heterogeneous resistance model met the accuracy requirements of the sludge pipeline transportation flow resistance simulation.

In future, research should consider the effect of temperature on sludge fluidity to improve the existing resistance calculation model.

■ AUTHOR INFORMATION

Corresponding Author
Bojiao Yan — College of Civil Engineering and Architecture, Changchun Sci-Tech University, Changchun City 130600, Jilin Province, The People’s Republic of China; orcid.org/0000-0002-2788-6177; Email: yanbojiao1985@126.com

Authors
Hai Lu — Key Laboratory of Songliao Aquatic Environment, Ministry of Education, Jilin Jianzhu University, Changchun City 130118, Jilin Province, The People’s Republic of China; orcid.org/0000-0003-2877-0755
Lijun Chen — China Northeast Municipal Engineering Design and Research Institute Co. Limited, Changchun City 130000, Jilin Province, The People’s Republic of China
Jianhui Wang — Key Laboratory of Songliao Aquatic Environment, Ministry of Education, Jilin Jianzhu University, Changchun City 130118, Jilin Province, The People’s Republic of China
Xiaoyu Zhang — Key Laboratory of Songliao Aquatic Environment, Ministry of Education, Jilin Jianzhu University, Changchun City 130118, Jilin Province, The People’s Republic of China
Guang Li — Key Laboratory of Songliao Aquatic Environment, Ministry of Education, Jilin Jianzhu University, Changchun City 130118, Jilin Province, The People’s Republic of China
Jing Wang — Key Laboratory of Songliao Aquatic Environment, Ministry of Education, Jilin Jianzhu University, Changchun City 130118, Jilin Province, The People’s Republic of China
Wenlou Chen — Key Laboratory of Songliao Aquatic Environment, Ministry of Education, Jilin Jianzhu University, Changchun City 130118, Jilin Province, The People’s Republic of China

Complete contact information is available at: https://pubs.acs.org/10.1021/acsomega.1c00503

Notes
The authors declare no competing financial interest.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (no.52070087 and no.51808254) and Jilin Province Development and Reform Commission (2019C055-6).

REFERENCES

(1) Luk, J.; Mohamadabadi, H. S.; Kumar, A. Pipeline transport of biomass: Experimental development of wheat straw slurry pressure loss gradients. Biomass Bioenergy 2014, 64, 329–336.
(2) Kania, J. J. Economics of coal transport by slurry pipeline versus unit train: A case study. Energy Econ. 1984, 6, 131–138.
(3) Senapati, P. K.; Mishra, B. K.; Parida, A. Analysis of friction mechanism and homogeneity of suspended load for high concentration fly ash & bottom ash mixture slurry using rheological and pipeline experimental data. Powder Technol. 2013, 250, 154–163.
(4) Ma, X.; Duan, Y.; Li, H. Wall slip and rheological behavior of petroleum-coke sludge slurries flowing in pipelines. Powder Technol. 2012, 230, 127–133.
(5) Turian, R. M.; Ma, T. W.; Hsu, F. L. G.; Sung, D. J. Characterization, settling, and rheology of concentrated fine particulate mineral slurries. Powder Technol. 1997, 93, 219–233.
(6) Wilson, K. C.; Clift, R.; Sellgren, A. Operating points for pipelines carrying concentrated heterogeneous slurries. Powder Technol. 2002, 123, 19–24.
(7) Durand, R. The hydraulic transportation of coal and other materials in pipes; Colloq National Coal Board: London, 1952; pp 72–84.
(8) Yarar, B.; Dogan, Z. M. Mineral Processing Design; Springer: Dordrecht, 1987; pp 116–142.
(9) Ghanta, K. C.; Purohit, N. K. Pressure drop prediction in hydraulic transport of bi-dispersed particles of coal and copper ore in pipeline. Can. J. Chem. Eng. 1999, 77, 127–131.
(10) Sundqvist, Å.; Sellgren, A.; Addie, G. Pipeline friction losses of coarse sand slurries: Comparison with a design model. Powder Technol. 1996, 89, 9–18.
(11) Matoušek, V. Pressure drops and flow patterns in sand-mixture pipes. Exp. Therm. Fluid Sci. 2002, 26, 693–702.
(12) Matoušek, V. Research developments in pipeline transport of settling slurries. Powder Technol. 2005, 156, 43–51.
(13) Matoušek, V. Predictive model for frictional pressure drop in settling-slurry pipe with stationary deposit. Powder Technol. 2009, 192, 367–374.
(14) Kaushal, D. R.; Sato, K.; Toyota, T.; Funatsu, K.; Tomita, Y. Effect of particle size distribution on pressure drop and concentration
profile in pipeline flow of highly concentrated slurry. *Int. J. Multiphase Flow* 2005, 31, 809−823.

(15) Kaushal, D. R.; Tomita, Y. Solids concentration profiles and pressure drop in pipeline flow of multisized particulate slurries. *Int. J. Multiphase Flow* 2002, 28, 1697−1717.

(16) Kaushal, D. R.; Thinglas, T.; Tomita, Y.; Kuchii, S.; Tsukamoto, H. CFD modeling for pipeline flow of fine particles at high concentration. *Int. J. Multiphase Flow* 2012, 43, 85−100.

(17) Roco, M. C.; Shook, C. A. Computational method for coal slurry pipelines with heterogeneous size distribution. *Powder Technol.* 1984, 39, 159−176.

(18) Turian, R. M.; Yuan, T.-F. Flow of Slurries in Pipelines. *AIChE J.* 1977, 23, 232−243.

(19) Jiang, Y. W. Pipeline Transportation of Sludge from Municipal Sewage Treatment Plants. *Met. Mine Des. Construct.* 2000, 32, 32−56.

(20) Jacobs, B. E. A. Design of Slurry Transport Systems; Elsevier: London and New York, 1991; pp 201−224.

(21) Liao, M. A.; Franco, J. M.; Partal, P.; Gallegos, C. Experimental study of grease flow in pipelines wall slip and air entrainment effects. *Chem. Eng. Process.* 2005, 44, 805−817.

(22) Jacobs, B. E. A. Design of Slurry Transport Systems; Elsevier: London and New York, 1991; pp 201−224.

(23) Delgado, M. A.; Franco, J. M.; Partal, P.; Gallegos, C. Experimental study of grease flow in pipelines wall slip and air entrainment effects. *Chem. Eng. Process.* 2005, 44, 805−817.

(24) Jacobs, B. E. A. Design of Slurry Transport Systems; Elsevier: London and New York, 1991; pp 201−224.