Correlation and multifractality in climatological time series

I T Pedron
Paraná Western State University
Campus de Marechal Cândido Rondon, Paraná, 85960-000, Brazil
E-mail: itpedron@yahoo.com.br

Abstract. Climate can be described by statistical analysis of mean values of atmospheric variables over a period. It is possible to detect correlations in climatological time series and to classify its behavior. In this work the Hurst exponent, which can characterize correlation and persistence in time series, is obtained by using the Detrended Fluctuation Analysis (DFA) method. Data series of temperature, precipitation, humidity, solar radiation, wind speed, maximum squall, atmospheric pressure and random series are studied. Furthermore, the multifractality of such series is analyzed applying the Multifractal Detrended Fluctuation Analysis (MF-DFA) method. The results indicate presence of correlation (persistent character) in all climatological series and multifractality as well. A larger set of data, and longer, could provide better results indicating the universality of the exponents.

1. Introduction
The behavior of the climatological variables affect directly crucial aspects of the people’s daily lives. Typically measures of these variables are a sequence of values that constitute a time series and time series analysis tools can contribute effectively to the study of climatological variables. This work is focused on the investigation of correlations and memory effects in such series and to capture underlying multifractality. It is applied the Detrended Fluctuation Analysis method (DFA), a well-established method for the detection of long-range correlations. Usually trends may mask the effect of correlations. DFA can systematically eliminate polynomial of different order. This method was proposed in [1] and has successfully been applied to many different fields, and particularly in the study of variables associated with the weather and climate. The DFA method gives the Hurst exponent (H) and estimating such exponent from the given data is an effective way to determine the nature of correlation in it. Values of H in the range (0, 0.5) characterize anti-persistence, whereas those in the range (0.5,1) characterize persistent long-range correlations. The value $H=0.5$ is associated with uncorrelated noise. However, when the series points to the existence of more than one exponent for its characterization we are dealing with multifractal behavior. Multifractal signals are far more complex than monofractal signals and require more exponents (theoretically infinite) to characterize their scaling properties. In this work multifractality in time series data of climatological variables are studied using the Multifractal Detrended Fluctuation Analysis (MF-DFA) proposed in [2]. It is a modified version of DFA to detect multifractal properties of time series and provides a systematic means to identify and quantify the multiple scaling exponents in the data.

2. Data set and method
The data series were obtained in Cascavel meteorological station (24º53’ S, 53º33’ W, 719 m) located in the South of Brazil. The following series of (daily) data were studied: average temperature,
precipitation, relative humidity of air, solar radiation, wind speed, maximum squall, atmospheric pressure, and random series. All data set is within the period from Sep-1998 to Feb-2009.

The MF-DFA method is a generalization of the standard DFA, being based on identification of the scaling of the $m$th-order moments of the time series which may be non-stationary. The modified MF-DFA procedure consists of a sequence of steps and detailed information about computation can be found in [2]. The first steps are essentially identical to the conventional DFA procedure. First we construct the profile $X(i)$ as $X(i) = \sum k x'_k$. The index $i$ counts the data points in the record, $i=1,2,..., N$. For eliminating the periodic seasonal trends was obtained daily differences $x'_i = x_i - \bar{x}_i$ where $\bar{x}_i$ represents the average value for each calendar date $i$. The profile is then divided into $N_s = int (N/ s)$ non-overlapping segments of length $s$. To accommodate the fact that some of data points may be left out, the procedure is repeated from other end of the data set [2]. The local trend is determined by using the least-squared fit to each segment $\nu$ and we obtain the detrended time series $X'_i (i) = X(i) - p_i(i)$ where $p_i(i)$ is the polynomial fit to the $i$th segment. In this work it is used linear fit (DFA1). The variance of the detrended time series is calculated for each segment as

$$F^2(\nu, s) = \frac{1}{s} \left\{ \sum_{i=1}^{s} X^2_s \left[ (\nu - 1)s + i \right] \right\}.$$  

Averaging over all segments the $m$th order fluctuation can be obtained and

$$F_m (s) = \left\{ \frac{1}{2N_s} \sum_{\nu=1}^{2N_s} \left[ F^2(\nu, s) \right]^{m/2} \right\}^{1/m}.$$  

For $m=2$ the standard DFA procedure is retrieved. If the original series are long-range power-law correlated the fluctuation function will vary as $F_m (s) \propto s^{h(m)}$ (for large values of $s$). Note that $h(m)$ is the generalized Hurst exponent and $h(2)$ is the usual exponent $H$ previously mentioned. A multifractal description can also be obtained from considering partitions functions

$$Z_m(s) = \sum_{\nu=1}^{N_s} |X_{\nu s} - x_{(\nu-1)s}|^m \propto s^{\tau(m)}$$

where $\tau(m)$ is the Renyi exponent [3]. A linear scaling of $\tau(m)$ with $m$ is characteristic of a monofractal data set, whereas a nonlinear scaling is indicative of multifractal behavior. The exponent $h(m)$ is related to the Renyi exponent $\tau(m)$ by

$$\tau(m) = mh(m) - 1.$$  

It is also possible to verify the multifractality degree defining the ratio $H_</H_>$ where $H_<$ is the slope of the function $\tau(m)$ versus $m$ for $m < 0$ and $H_>$ is the equivalent for $m > 0$. By definition such relation is equal to unity in monofractal signals and deviation from this value indicate multifractal properties.

3. Results and discussion

The $H$ exponent for the studied data series are presented in Table 1. Temperature and precipitation values agree with values presented in some works [4, 5] despite some references claim that the scaling exponent is not universal for the temperature data [6, 7]. Contrary to results presented in the studies cited, the precipitation showed here a more pronounced multifractality compared with temperature.
Remembering that $H > 0.5$ is related with correlation and persistence the high value for it in the relative humidity shows a stronger persistence in the relative humidity fluctuations, results also obtained in [8, 9]. This feature can be associated with the water behavior and probably has a local characterization. For wind speed the H exponent is less than that presented in [10,11] but is closest to those found in [12,13]. One can ensure the universality of the correlations but its exponents are related to local circulation patterns. These conclusions can also be extended to the multifractal behaviour. The other variable present relatively weak correlation, a not surprising result. In contrast with our result, solar radiation present high degree of anti-persistence in [14]. All these variables are strongly correlated but apparently the results for the scaling exponents of their series is not related in a simple way. In Table 1 it is also possible to observe the ratio $H_{\downarrow}/H_{\uparrow}$ which indicates the deviation of the monofractal behavior. Monofractal signals are characterized by the unity in this relation. In this sense, Figure 1 shows the behavior of the exponent $\tau(m)$ with different values of $m$. Non-linearity in these curves indicates multifractal properties as shown in Table 1. It is necessary to apply the procedure in a larger number of stations to obtain more conclusive results and to characterize a particular site through their exponents. Forthcoming works will be addressed in this task.

**Table 1.** Hurst exponent H for the climatological series from Cascavel – PR meteorological station and the ratio $H_{\downarrow}/H_{\uparrow}$ indicating the multifractality degree (value 1 means monofractality in the data series).

| variable           | H    | $H_{\downarrow}/H_{\uparrow}$ |
|--------------------|------|-------------------------------|
| humidity           | 0.722| 1.099                         |
| wind speed         | 0.628| 1.373                         |
| temperature        | 0.625| 1.157                         |
| solar radiation    | 0.593| 1.036                         |
| maximum squall     | 0.560| 0.962                         |
| precipitation      | 0.557| 1.651                         |
| atmospheric pressure| 0.543| 1.417                         |
| randomic series    | 0.495| 1.018                         |

**4. Conclusion**

In this work the DFA method was applied to detect long range correlation in several climatological series of one station. Results indicate that climatological series present correlation in some way. Long memory or persistent effects imply the possibility to predict conditions in the future based on historical records. Furthermore, to analyze their scaling properties it was applied the MF-DFA method to the data set. Results indicate that the climatological series are multifractal, but in different degrees. These characteristics mean that the processes may be governed by more than one scaling exponent to capture the complex dynamics inherent in the data and show the non-stationary character of these series. It should be noted here that it is worthy to consider the problem taking into account a larger set of stations and longer series of data to generalize the new results and recover others in the literature. Both methods are powerful to analyze the statistical behavior of the series and to obtain important information on the climate from its historical data.
Figure 1: The multifractal exponent $\tau(m)$ versus several moments $m$. Different slopes indicate multifractal characterization for the data series.
5. References

[1] Peng C-K, Buldyrev S V, Havlin S, Simons M, Stanley H E, Goldberger A L 1994 Phys. Rev. 49 1685

[2] Kantelhardt J W, Zschiegner S A, Koscielny-Bunde E, Bunde A, Havlin S, Stanley H E 2002 Physica A 316 87

[3] Barabasi A and Vicsek T 1991 Phys. Rev. A 44 2730

[4] Koscielny-Bunde E, Bunde A, Havlin S, Roman H E, Goldreich Y, Schellnhuber HJ 1998 Phys. Rev. Lett. 81 729

[5] Bunde A, Havlin 2002 Physica A 314 15

[6] Király A, Jánosi I M 2005 Meteorol. Atmos. Phys. 88 119

[7] Rybski D, Bunde A, von Storch H 2008 J. Geophys. Res. 113 D02106

[8] Chen X, Guangxing L, Zuntao F 2007 Geophys. Res. Lett. 34 L07804

[9] Guangxing L, Chen X, Zuntao F 2007 Physica A 383 585

[10] Govindan RB, Kantz H 2004 Europhys. Lett. 68 184

[11] Koçak K 2009 Energy 34 1980

[12] Kavasseri R G, Nagarajan R 2005 Chaos, Solitons & Fractals 24 165

[13] Feng T, Fu ZT, Deng X, Mao J 2009 Physics Letters A 373 4134

[14] Harrouni S, Guessoum A 2009 Chaos, Solitons & Fractals 41 1520