Prediction for the decay width of a charged state near the $D_s\bar{D}^*/D_s^*\bar{D}$ threshold

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Very recently it was predicted the existence of a charged state near the $D_s\bar{D}^*/D_s^*\bar{D}$ threshold. This state, that we call $Z^+_c$, would be the strange partner of the recently observed $Z^+_c(3900)$. Using standard techniques of QCD sum rules, we evaluate the three-point function for the vertices $Z^+_c J/\psi K^+$, $Z^+_c\eta K^+$ and $Z^+_c D_s^* \bar{D}^0$ and we make predictions for the corresponding decay widths in this channel.

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In a pioneering work, using the initial single pion emission mechanism (ISPE), the authors of ref. \textsuperscript{1} have predicted the existence of a charged state, close to the $D^*\bar{D}$ threshold, in the hidden-charm dipion decay of the charmonium-like structure $Y(4260)$. This state, called $Z^+_c(3900)$, was soon after observed by the BESIII and BELLE collaborations in $e^+e^-\rightarrow J/\psi\pi^+\pi^-$ at $\sqrt{s} = 4260$ MeV \textsuperscript{2,3}. This observation was also confirmed by the authors of ref. \textsuperscript{4} using CLEO-c data. Stimulated by this discovery, the authors of ref. \textsuperscript{5} have extended the ISPE mechanism to include the kaon, the chiral partner of the pion. They call it the initial single chiral particle emission (ISChE) mechanism. Under the ISChE mechanism it is possible to study the hidden-charm dikaon decay of a charmonium-like states. In particular, studying the hidden-charm dikaon decay of the charmonium-like structure $Y(4660)$, the authors of ref. \textsuperscript{6} find a sharp peak structure close to the $D_s\bar{D}^*/D_s^*\bar{D}$ threshold. Therefore, a charged charmonium-like structure with hidden-charm and open-strange channels with mass close to the $D_s\bar{D}^*/D_s^*\bar{D}$ threshold, which we call $Z^+_c$, should be seen in the $Y(4660) \rightarrow J/\psi K^+ K^-$ decay.

The mass of a $J^{P} = 1^+ D_s\bar{D}^*$ molecular state was first predicted, using the QCD sum rules (QCDSR) method \textsuperscript{6,8} in ref. \textsuperscript{7}. They found $m_{Z^+_c} = (3.97 \pm 0.08)$ GeV, which is very close to the $D_s^+ \bar{D}^0$ threshold at 3.976 GeV. In this work we use the method of QCDSR to study some hadronic decays of $Z^+_c$, considering the $Z_c$ as a tetraquark state, similar to what was done for the $Z_c^+(3900)$ state in ref. \textsuperscript{10}. Therefore, the interpolating field for $Z^+_c$ is given by:

$$j_\alpha = \frac{i\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}}[(u^T_a C\gamma_5 c_b)(\bar{s}_d\gamma_\alpha C\bar{c}_c^T) - (u^T_a C\gamma_\alpha c_b)(\bar{s}_d\gamma_5 C\bar{c}_c^T)] ,$$

(1)

where $a, b, c, ...$ are color indices, and $C$ is the charge conjugation matrix. The mass obtained in QCDSR for the $Z_c$ state described by the current Eq. (1) is the same as the one obtained in \textsuperscript{7}, as expected from the results presented in ref. \textsuperscript{11}. Therefore, here we evaluate only the decay width. For a comprehensive review of the use of different currents to describe four-quark states we refer the reader to \textsuperscript{12}.

We will consider four decay channels: $Z^+_c \rightarrow J/\psi K^+$, $Z^+_c \rightarrow \eta K^{*+}$, $Z^+_c \rightarrow \bar{D}^0 \bar{D}_s^+$ and $Z^+_c \rightarrow D^0 \bar{D}_s^*$. Besides these four discussed decay channels, $Z^+_c \rightarrow \chi_{c0} K^+$ via P-wave is allowed, where the sum of the masses of $\chi_{c0}$ and Kaon is about 3912 MeV less than the central value of the mass of $Z^+_c$ \textsuperscript{8}. However, in this work we will not include this channel in our discussion since this P-wave decay and small phase space can suppress the decay width of $Z^+_c$ compared with these two S-wave hidden-charm decay channels $Z^+_c \rightarrow J/\psi K^+$ and $Z^+_c \rightarrow \eta_c K^{*+}$.

In these four channels there is always a vector and a pseudoscalar mesons as final states. For the last three cases the pseudoscalar mesons are described by pseudoscalars currents:

$$j^V_{5\nu} = i\bar{c}_a\gamma_5 c_a, \quad j^D_{5\nu} = i\bar{c}_a\gamma_5 u_a, \quad \text{and} \quad j^{D*}_{5} = i\bar{s}_a\gamma_5 c_a.$$ 

(2)

However, it is well known that the kaon can not be well described, in QCDSR, by a pseudoscalar current \textsuperscript{13}. Therefore, in the case of the $Z^+_c \rightarrow J/\psi K^+$ decay, we use an axial current to describe the kaon.

$$j^K_{5\nu} = \bar{s}_a\gamma_5 \gamma_\nu u_a.$$ 

(3)
For the vector mesons we use the currents

\[ j^{\psi}_{\mu} = \bar{c}\gamma_{\mu}c, \quad j^{D^*}_{\mu} = \bar{c}\gamma_{\mu}u, \quad j^{D^*}_{\mu} = \bar{s}\gamma_{\mu}c, \quad \text{and} \quad j^{K^*}_{\mu} = \bar{s}\gamma_{\mu}u. \]  

(4)

The QCDSR calculation of these four vertices are based on the three-point function given by:

\[ \Pi_{\mu\nu\alpha}(p, p', q) = \int d^4x \, d^4y \, e^{ip' \cdot x} e^{iq \cdot y} \Pi_{\mu\nu\alpha}(x, y), \]  

(5)

with

\[
\begin{align*}
\Pi_{\mu\nu\alpha}(x, y) &= \langle 0 | T [ j^{\psi}_{\mu}(x) j^{K^*}_{\nu}(y) j^{K^*}_{\alpha}(0) ] | 0 \rangle, \\
\Pi_{\mu\alpha}(x, y) &= \langle 0 | T [ j^{\psi}_{\mu}(x) j^{K^*}_{\alpha}(y) j^{K^*}_{\alpha}(0) ] | 0 \rangle, \\
\Pi_{\mu\nu}(x, y) &= \langle 0 | T [ j^{D^*}_{\mu}(x) j^{D^*}_{\nu}(y) j^{D^*}_{\alpha}(0) ] | 0 \rangle, \\
\Pi_{\mu\alpha}(x, y) &= \langle 0 | T [ j^{D^*}_{\mu}(x) j^{D^*}_{\nu}(y) j^{D^*}_{\alpha}(0) ] | 0 \rangle,
\end{align*}
\]

(6)

for the four decays. In Eq. (4) \( p = p' + q \).

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**FIG. 1:** CC diagram which contributes to the OPE side of the sum rule.

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To assure that the non-trivial color structure of the current in Eq. (1) is maintained in the QCDSR calculation, in the OPE side we will consider only the diagrams with non-trivial color structure, as in ref. [10]. These diagrams are called color-connected (CC) diagrams. In the case of the \( Z_+ \rightarrow J/\psi K^+ \) decay, one of the CC diagrams that contribute to the OPE side is shown in Fig. 1. Possible permutations (not shown) of the diagram in Fig. 1 also contribute.

The diagram in Fig. 1 contributes to many structures. However, as we can see below, only the structures \( q_\nu g_{\mu\alpha} \) and \( q_\nu p'_\mu p'_\alpha \) also appear in the phenomenological side. Following [10] we choose to work with the \( q_\nu p'_\mu p'_\alpha \) structure. Therefore in the OPE side and in the \( q_\nu p'_\mu p'_\alpha \) structure we obtain:

\[
\Pi^{(\text{OPE})} = \frac{\langle \bar{q}g_\sigma Gq \rangle + \langle \bar{s}g_\sigma Gs \rangle}{24\sqrt{2\pi^2}} \frac{1}{q^2} \int_0^1 \, d\alpha \, \frac{\alpha(1-\alpha)}{m_{cs}^2 - \alpha(1-\alpha)p^2}. 
\]

(7)

The phenomenological side of the sum rule can be evaluated by inserting intermediate states for \( Z_{cs}, J/\psi \) and \( K \) into Eq. (5). We get:

\[
\Pi^{(\text{phen})}_{\mu\nu\alpha}(p, p', q) = \frac{\lambda_{Z_{cs}} m_{ \psi K} f_{K} g_{Z_{cs} \psi K}(q^2)}{(p^2 - m_{cs}^2)(p'^2 - m_{cs}^2)(q'^2 - m_{K}^2)} \left( -g_{\mu\lambda} + \frac{p'_\mu p'_\lambda}{m_{cs}^2} \right) \left( -g_{\nu\lambda} + \frac{p_\nu p_\lambda}{m_{cs}^2} \right) + \cdots.
\]

(8)

The contribution of the excited states are included by the dots. These include pole-continuum and continuum contributions. The form factor, \( g_{Z_{cs} \psi K}(q^2) \), appearing in Eq. (7), is defined as the generalization for a off-shell kaon, of the on-mass-shell coupling constant \( g_{Z_{cs} \psi K} \). The coupling constant can be extracted from the effective lagrangian

\[
\mathcal{L} = g_{Z_{cs} \psi K} Z_{cs}^{\mu} \psi K + \text{cc}.
\]

(9)
From the lagrangian in Eq. (9) we get:

\[ \langle J/\psi(p')K(q)\rangle = g_{Z\psi\psi}(q^2)\varepsilon_{\psi}^*(p')\varepsilon(q), \]

where \( \varepsilon_{\psi}(p) \) and \( \varepsilon_{\mu}(p') \) are the polarization vectors of the \( Z_{cs} \) and \( J/\psi \) mesons respectively.

The coupling \( \lambda_{Z\psi} \) and the meson decay constants \( f_\psi \) and \( F_K \) appearing in Eq. (8) are defined through the current-state couplings:

\[
\begin{align*}
\langle 0|J_{\mu}|J/\psi(p')\rangle &= m_\psi f_\psi \varepsilon_{\mu}(p'), \\
\langle 0|J_{\mu}^K|K(q)\rangle &= i q_\mu F_K, \\
\langle Z_{cs}(p)|J_\alpha|0\rangle &= \lambda_{Z_{cs}} \varepsilon_{\alpha}(p).
\end{align*}
\]

If one neglects the kaon mass in the right hand side of Eq. (8) we can extract directly the coupling constant, \( g_{Z\psi\psi} \), instead of the form factor, like in (10, 14). Therefore, isolating the \( q_\mu p'_\alpha \) structure in Eq. (8) and making a single Borel transformation to both state couplings:

\[
\langle 0|j_{\mu}^K|J/\psi(p')\rangle = m_\psi f_\psi \varepsilon_{\mu}(p'),
\]

\[
\langle 0|J_{\mu}^K|K(q)\rangle = i q_\mu F_K,
\]

\[
\langle Z_{cs}(p)|j_\alpha|0\rangle = \lambda_{Z_{cs}} \varepsilon_{\alpha}(p).
\]

For the two-point sum rule (9):

As commented above, the dots in Eq. (8) include pole-continuum and continuum contributions. The parameter \( B \) in Eq. (12) is introduced to take into account the contributions associated with pole-continuum transitions, which are not suppressed when only a single Borel transformation is done in a three-point function sum rule, as shown in (15, 18).

The numerical values for quark masses and QCD condensates used in this calculation are listed in Table I.

| Parameters | Values |
|------------|--------|
| \( m_c \) | (1.18 – 1.28) GeV |
| \( \langle \bar{q}q \rangle \) | \(- (0.23 \pm 0.03)^3 \) GeV³ |
| \( m_\psi^2 \equiv \langle \bar{q}qGq \rangle / \langle \bar{q}q \rangle \) | (0.8 \pm 0.1) GeV² |
| \( \beta/\langle \bar{q}q \rangle \) | 0.8 |

The numerical values of the meson masses and decay constants used in all calculations are given in Table II.

In (9) it was shown that the Borel window where the two-point function for \( Z_{cs} \) shows good OPE convergence and pole dominance is in the range \( 2.0 \leq M^2 \leq 3.0 \) GeV². Therefore, we use here this same Borel window. In Fig. 2 we show, through the circles, the right-hand side (RHS) of Eq. (12), i.e., the OPE side of the sum rule, as a function of the Borel mass. We can fit the OPE results with the analytical expression in the left-hand side (LHS) of Eq. (12). We get: \( A = 1.28 \times 10^{-4} \) GeV⁻⁴ and \( B = -1.03 \times 10^{-3} \) GeV⁵, using \( \sqrt{s_0} = 4.5 \) GeV. Using the value obtained for \( A \) through the fit and the expression in Eq. (13) we get for coupling constant: \( g_{Z\psi\psi} = 2.57 \) GeV. Considering the uncertainties given in the parameters in Tables I and II, we obtain:

\[ g_{Z\psi\psi} = (2.58 \pm 0.30) \text{ GeV.} \]

With the value of \( g_{Z\psi\psi} \) we can estimate the decay width using the expression (10):

\[
\Gamma(Z_{cs}^+ \rightarrow J/\psi K^+) = \frac{p^+(m_{Z_{cs}}, m_\psi, m_K)}{8\pi m^2_{Z_{cs}}} \frac{1}{3} \lambda^2_{Z_{cs}} \psi K \left( 3 + \frac{p^+(m_{Z_{cs}}, m_\psi, m_K)}{m^2_\psi} \right)^2,
\]

where \( \langle J/\psi(p')K(q)\rangle = g_{Z\psi\psi}(q^2)\varepsilon_{\psi}^*(p')\varepsilon(q) \), and \( \langle 0|j_{\mu}^K|J/\psi(p')\rangle = m_\psi f_\psi \varepsilon_{\mu}(p'), \langle 0|J_{\mu}^K|K(q)\rangle = i q_\mu F_K, \langle Z_{cs}(p)|j_\alpha|0\rangle = \lambda_{Z_{cs}} \varepsilon_{\alpha}(p) \).
### TABLE II: Meson masses and decay constants.

| Quantity   | Value     | Ref. |
|------------|-----------|------|
| $m_{\psi}$ | 3.1 GeV   | [20] |
| $m_{\eta_c}$ | 2.98 GeV | [20] |
| $m_{D_s^*}$ | 2.01 GeV | [20] |
| $m_{D_s}$  | 2.11 GeV  | [20] |
| $m_D$      | 1.97 GeV  | [20] |
| $m_{K_s}$  | 1.87 GeV  | [20] |
| $m_{K}$    | 0.892 GeV | [20] |
| $f_{\psi}$ | 0.405 GeV | [20] |
| $f_{\eta_c}$ | 0.35 GeV | [21] |
| $f_{D_s^*}$ | 0.33 GeV | [22] |
| $f_{D_s}$  | (0.24 ± 0.08) GeV | [23] |
| $f_{D^*}$  | (0.24 ± 0.02) GeV | [14] |
| $f_D$      | (0.18 ± 0.02) GeV | [14] |
| $f_K$      | (0.16 ± 0.02) GeV | [20] |
| $f_{K_s}$  | (0.22 ± 0.01) GeV | [20] |

![Diagram](image-url)  
**FIG. 2:** Dots: the RHS of Eq.(12), as a function of the Borel mass for $\sqrt{s_0} = 4.5$ GeV. The solid line gives the fit of the QCDSR results through the LHS of Eq.(12).

where

$$p^*(a, b, c) = \frac{\sqrt{a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 - 2b^2c^2}}{2a}.$$  

(16)

Here, the mass of $Z_{cs}^+$ is taken as (3.97 ± 0.08) GeV, which is from the QSR calculation [9]. We obtain:

$$\Gamma(Z_{cs}^+ \rightarrow J/\psi K^+) = (11.2 \pm 3.5) \text{ MeV}. \quad (17)$$

One can notice that the coupling in this case is smaller than $g_{Z_{cs}\psi\pi}$, obtained in [10]. One of the possible reasons for that is the fact that the OPE side, in the $Z_{cs}$ case, is smaller than the corresponding one for $Z_c^+(3900)$, due to the presence of the strange-quark condensate. Also, the current-coupling parameter $\lambda_{Z_{cs}}$ is bigger than $\lambda_{Z_c}$. In addition, the phase space of $Z_{cs}^+$ decay into $J/\psi K^+$ is smaller than that of $Z_c^+(3900) \rightarrow J/\psi\pi$, which is a reason why $\Gamma(Z_{cs}^+ \rightarrow J/\psi K^+)$ is less than half of the $\Gamma(Z_c^+(3900) \rightarrow J/\psi\pi)$.

Let us consider now the $Z_{cs}^+ \rightarrow \eta_c K^{*+}$ decay. Considering only CC diagrams, like the one in Fig.1, we get for the
where $Q$ is isolating the $q$ states. The decay constants for vector ($V$) and pseudoscalar ($P$) states are defined through the coupling of the current with the states:

$$
\langle 0| j^{V}_{\mu}(q) = m_{V} f_{V} \varepsilon_{\mu}(q),
$$

$$
\langle 0| j^{P}_{\mu}(q) = \frac{f_{P} m_{P}^{2}}{m_{q_{1}} + m_{q_{2}}},
$$

where $m_{q_{1}}$ and $m_{q_{2}}$ are the masses of the constituents quarks of the pseudoscalar meson $P$.

We get for the phenomenological side

$$
\Pi^{(\text{phen})}_{\mu \alpha}(p, p', q) = \frac{-i \lambda Z_{cs} m_{K'} f_{K} f_{\eta_{c}} m_{\eta_{c}}^{2}}{2 m_{c}(p^{2} - m_{Z_{cs}}^{2})(p'^{2} - m_{\eta_{c}}^{2})(q^{2} - m_{K'}^{2})} \left( -g_{\mu \lambda} + \frac{g_{\mu \lambda}}{m_{p}^{2}} \right) \left( -g_{\alpha} + \frac{p_{\alpha} p_{\lambda}}{m_{Z_{cs}}^{2}} \right) + \cdots.
$$

Isolating the $q_{\alpha} p'_{\mu}$ structure in Eq. (20) and making a single Borel transformation on both $P^{2} = P'^{2}$, we get:

$$
C \left( e^{-m_{\eta_{c}}^{2}/M^{2}} - e^{-m_{Z_{cs}}^{2}/M^{2}} \right) + D e^{-s_{0}/M^{2}} = \frac{Q^{2} + m_{c}^{2}}{Q^{2}} \frac{m_{c}(\langle \bar{q} g \sigma G q \rangle + \langle \bar{q} g \sigma G s \rangle)}{96 \sqrt{2} \pi^{2}} \int_{0}^{1} \frac{d\alpha}{m_{c}(1 - \alpha)}.
$$

(21)

where $Q^{2} = -q^{2}$ and the parameter $C$ is given in terms of the form factor:

$$
C = \frac{g_{Z_{cs} \eta_{c} K^{*}}(Q^{2}) \lambda Z_{cs} m_{K'} f_{K} f_{\eta_{c}} m_{\eta_{c}}^{2}}{2 m_{c} m_{Z_{cs}}^{2} (m_{Z_{cs}}^{2} - m_{\eta_{c}}^{2})}.
$$

(22)

FIG. 3: QCDSR results for the form factor $g_{Z_{cs} \eta_{c} K^{*}}(Q^{2})$ as a function of $Q^{2}$ and $M^{2}$ for $\sqrt{s_{0}} = 4.5$ GeV.

To determine $g_{Z_{cs} \eta_{c} K^{*}}(Q^{2})$ we use Eq. (21) and its derivative with respect to $M^{2}$ to eliminate $D$ from Eq. (21). The form factor $g_{Z_{cs} \eta_{c} K^{*}}(Q^{2})$ is shown in Fig. 3 as a function of both $M^{2}$ and $Q^{2}$. To extract $g_{Z_{cs} \eta_{c} K^{*}}(Q^{2})$ we need first to establish the Borel window where the sum rule is as much independent of the Borel mass as possible. From Fig. 3 we notice that this happens in the region $4.0 \leq M^{2} \leq 10.0$ GeV$^{2}$. 
masses and condensates. As always the phenomenological side is obtained by considering the contribution of the form factor to a region of $Q^2$ where the QCDSR is not valid. To do that we parametrize the QCDSR results for $g_{Z_{cs}, n, K^*}(Q^2)$ using a monopole form:

$$g_{Z_{cs}, n, K^*}(Q^2) = \frac{g_1}{g_2 + Q^2}. \quad \text{(23)}$$

The fit gives $g_1 = 78.35$ GeV$^{-2}$ and $g_2 = 24.3$ GeV. In Fig. 4 we also show, through the line, the fit of the QCDSR results, using Eq. (23). The coupling constant is obtained by using Eq. (23) and $Q^2 = -m_{K^*}^2$:

$$g_{Z_{cs}, n, K^*} = g_{Z_{cs}, n, K^*}(-m_{K^*}^2) = (3.4 \pm 0.3) \text{ GeV}. \quad \text{(24)}$$

The uncertainty in Eq. (24) comes from variations in $s_0$, $\lambda_{Z_{cs}}$, and $m_c$ in the ranges given in Tables I and II. Using this in Eq. (15), and varying $m_{Z_{cs}}$ in the range $m_{Z_{cs}} = (3.97 \pm 0.08)$ GeV we get

$$\Gamma(Z_{cs}^+ \to \eta_c K^{*+}) = (10.8 \pm 6.2) \text{ MeV}. \quad \text{(25)}$$

Next we consider the decays $Z_{cs}^+ \to D_s^+ \bar{D}^{*0}$ and $Z_{cs}^+ \to D_s^{*+} \bar{D}^0$. Here we give only the expressions for $Z_{cs}^+ \to D_s^+ \bar{D}^{*0}$. The expression for $Z_{cs}^+ \to D_s^{*+} \bar{D}^0$, can be easily obtained from the prior by exchanging the corresponding mesons masses and condensates. As always the phenomenological side is obtained by considering the contribution of the $Z_{cs}$, $D_s$, and $D^{*}$ mesons to the correlation function in Eq. (8):

$$\Pi_{\mu \alpha}(p, q) = \frac{-i\lambda_{Z_{cs}} m_{D^*} f_D f_{D_s} m_{D_s}^2 g_{Z_{cs}, D^*} (q^2)}{(m_c + m_s)(p^2 - m_{Z_{cs}}^2)(p^2 - m_{D_s}^2)} \left(-g_{\mu \alpha} + \frac{p'_\mu p'_\lambda}{m_{D_s}^2}\right) \left(-g_{\mu \alpha} + \frac{p_\mu p_\lambda}{m_{Z_{cs}}^2}\right) + \cdots. \quad \text{(26)}$$

Following the OPE side we consider only the CC diagrams and we work with the $p'_\mu p'_\alpha$ structure. We get:

$$\Pi_{\mu \alpha}(p, q) = \frac{-i m_{Z_{cs}}}{48\sqrt{2} \pi} \left[\frac{\langle \bar{q} g_\sigma G_q \rangle}{m_c^2 - q^2} \int_0^1 \frac{da}{m_c^2 - (1 - \alpha)p^2} - \frac{\langle q g_\sigma G_q \rangle}{m_c^2 - p^2} \int_0^1 \frac{da}{m_c^2 - (1 - \alpha)q^2}\right]. \quad \text{(27)}$$

Therefore, the sum rule in the $p'_\mu p'_\alpha$ structure is:
where the parameter $E$ is defined in terms of the form factor $g_{Z_{cs}, D^*} (Q^2)$:

$$E = g_{Z_{cs}, D^*} (Q^2) \lambda_{Z_{cs}, f_{D^*} f_{D_0}} m_{D_0}^2 (m_{D_0} + m_s) (m_{Z_{cs}} - m_{D_0}^2).$$

where $m_{D_0}$ is defined in terms of the form factor. Again, to extract the coupling constant we have to extrapolate the QCDSR results to $Q^2 = -m_{D_{cs}}^2$. To do that we use an exponential form

$$g_{Z_{cs}, D^*} (Q^2) = g_1 e^{-g_2 Q^2},$$

(30)

to fit the QCDSR results. We have used an exponential form is this case since it was not possible to fit the QCDSR results with the monopole form in Eq. (23). However, as shown in [14], both forms are acceptable to describe hadronic form factors. We get $g_1 = 0.94$ GeV and $g_2 = 0.00$ GeV$^{-2}$. The line in Fig. 6 shows the fit of the QCDSR results for $\sqrt{s_0} = 4.5$ GeV, using Eq. (30). We get for the coupling constant:

$$g_{Z_{cs}, D^*} = g_{Z_{cs}, D^*} (-m_{D_0}^2) = (1.4 \pm 0.3) \text{ GeV}.$$

(31)

With this coupling and using the bigger value predicted for the $m_{Z_{cs}}$ mass in [9] (since for values of the mass bellow the threshold the decay is not possible) we get for the decay width in this channel:

$$\Gamma (Z_{cs}^+ \rightarrow D_{s0}^+ D_s^0) = (1.5 \pm 1.5) \text{ MeV}.$$

(32)

For the $Z_{cs}^+ \rightarrow D_{s0}^+ D_s^0$, doing a similar analysis we arrive at:

$$g_{Z_{cs}, D^*} = g_{Z_{cs}, D^*} (-m_{D_s}^2) = (1.4 \pm 0.4) \text{ GeV},$$

(33)

that leads to a similar result:

$$\Gamma (Z_{cs}^+ \rightarrow D_{s0}^+ D_s^0) = (1.4 \pm 1.4) \text{ MeV}.$$

(34)
FIG. 6: QCDSR results for $g_{s_{z_{cs}}D_{s}D_{s}^{*}}(Q^2)$, as a function of $Q^2$, for $\sqrt{s_0} = 4.5$ GeV (squares). The solid line gives the parametrization of the QCDSR results through Eq. (30).

I. CONCLUSIONS

In this work we have estimated, using the QCDSR approach, the decay widths of the charmonium-like structure with hidden-charm and open-strange, that we call $Z_{cs}^+$. This state was predicted in [5] under the ISChE mechanism, and should be seen in the hidden-charm dikaon decay of a charmonium-like state $Y(4660)$. We have studied four decay channels and have considered only color connected diagrams. This is justified by the fact that we expect the $Z_{cs}$ state to be a genuine tetraquark state, with a non-trivial color configuration. The obtained couplings, with the respective decay widths, are given in Table III.

Table III: Coupling constants and decay widths in different channels.

| Vertex          | coupling constant (GeV) | decay width (MeV) |
|-----------------|-------------------------|-------------------|
| $Z_{cs}^+J/\psi K^+$ | $2.58 \pm 0.30$         | $11.2 \pm 3.5$    |
| $Z_{cs}^+\eta \ K^{*+}$ | $3.4 \pm 0.3$          | $10.8 \pm 6.2$    |
| $Z_{cs}^+D_{s}^+D_{s}^{*0}$ | $1.4 \pm 0.3$         | $1.5 \pm 1.5$    |
| $Z_{cs}^+D_{s}^{0}D_{s}^{*+}$ | $1.4 \pm 0.4$       | $1.4 \pm 1.4$    |

Considering these four decay channels we get a total width $\Gamma = (24.9 \pm 12.6)$ GeV for $Z_{cs}$ which is smaller than the total decay width of its non-strange partner the $Z_{c}^+(3900)$: $\Gamma = (46 \pm 22)$ MeV from BESIII [2], and $\Gamma = (63 \pm 35)$ MeV from BELLE [3].

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