Maneuvering Target Tracking Based on Multiple-model Multipath Bernoulli Filter

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Abstract. Aiming at the problem of maneuvering single target tracking in over the horizon radar system under the clutter environment, a novel multiple-model multipath Bernoulli filter (MM-MPBF) algorithm is proposed. First, the multipath target measurement model is established based on the finite set statistics, and then combining the multiple models method with the multipath Bernoulli filter to deal with the single maneuvering target tracking problem in over the horizon radar system. Finally, the simulation experiment is used to verify the performance of the MM-MPBF algorithm.

1. Introduction
Over the horizon radar (OTHR) transmits electromagnetic wave to achieve long range detection and tracking through the wave reflection of ionosphere. Due to the existence of multiple ionosphere, the major problem of target tracking is that a target can generate multiple measurements through multiple paths. If the tracking algorithm can use all the measurements, the target tracking performance in OTHR will be greatly improved. However the traditional target tracking algorithm only considers one measurement corresponds to each target at most, which can not directly track multipath targets.

Recently, many random finite set (RFS) algorithms have been applied to target tracking system [1-4]. Many improved algorithms based on RFS have been used to solve the problem of the multipath target tracking in OTHR system. To solve the multipath problem in OTHR tracking system, first author has proposed two algorithms [5,6] based on the cardinality balanced multitarget multi-Bernoulli filter and probability hypothesis density filter. Furthermore, for single target tracking in OTHR, multipath Bernoulli filter (MPBF) is proposed in [7] based on the traditional Bernoulli filter. However, most of the algorithms focus on non-maneuvering target, and maneuvering target tracking is a thorny problem due to the dynamic model uncertainty. This paper focuses on the problem of the single maneuvering multipath target tracking and proposes a novel multiple-model multipath Bernoulli filter (MM-MPBF). The MM-MPBF combines the multiple models (MM) algorithm [8] with the MPBF to solve the problems of maneuvering target tracking in OTHR system.

2. The Multipath Bernoulli Filter
2.1. Measurement Model of Multipath Target
Fig.1 shows the model of multipath target tracking model in OTHR system. At time k, multipath target state vector is $x_k = [\rho_k, h_k, \rho_{h_k}, \hat{h}_k]^T$. 

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As shown in Fig. 1, the transmitter transmits electromagnetic wave to detect and track targets through ionosphere reflection. Due to the existence of multiple ionosphere, there are multiple reflected waves in one target, which is called multipath target. Generally, it is assumed that there are only two ionospheres with height $h_E$ and $h_F$ respectively, it means that there are four possible propagation modes in OTHR [7]. The multipath target measurement model is $z_k = [Rg(k), Rr(k), Az(k)]'$, where $Rg=r_1 + r_2$, $Rr$ and $Az=\pi/2-\theta$ are the slant range, Doppler and azimuth.

$$z_k = \begin{cases} h_1(x_k) + \omega_{k,1} & \text{mode EE} \\ h_2(x_k) + \omega_{k,2} & \text{mode EF} \\ h_1(x_k) + \omega_{k,3} & \text{mode FF} \\ h_2(x_k) + \omega_{k,4} & \text{mode FE} \end{cases}$$  \hspace{1cm} (1)

In OTHR, the measurement from detection and clutter can be represented as an RFS. At time $k$, a RFS of measurements can denote by $Z^k = \{z_{k,1}, z_{k,2}, \ldots, z_{k,N_z}\}$, where $z_{k,1}, z_{k,2}, \ldots, z_{k,N_z}$ are the received measurements, and the measurements can be expressed as:

$$Z^k = Z_{k,1}(x_k) \cup Z_{k,2}(x_k) \cup Z_{k,3}(x_k) \cup Z_{k,4}(x_k) \cup C_k$$  \hspace{1cm} (2)

where $Z_{k,1}(x_k), l=1,2,3,4$ denotes the RFS of detection from the $l$th path and $C_k$ is the RFS of clutter.

### 2.2. The Multipath Bernoulli Filter in OTHR

The target state $X_k$ can be expressed as a Bernoulli RFS with

$$\pi_k(X_k) = \begin{cases} 1-r_k & X_k = \emptyset \\ r_k \cdot p_k(x_k) & X_k = \{x_k\} \end{cases}$$  \hspace{1cm} (3)

where $r_k$ is the target existence probability, $p_k(x_k)$ is the spatial probability density.

The MPBF is an extension of traditional Bernoulli filter based on the Bayesian framework, which uses the prediction and update to propagates the posterior density $\pi_k$.

The posterior density $\pi_{k-1}$ is a Bernoulli RFS and can be expressed as $\pi_{k-1} = \{r_{k-1}, p_{k-1}(x_{k-1})\}$, the recursion of the MPBF can be described as follows:

**Prediction:**

$$r_{k|k-1} = p_{B,k} (1-r_{k-1}) + r_{k-1} \left( p_{S,k} \cdot p_{k-1} \right)$$  \hspace{1cm} (4)

$$p_{k|k-1}(x_k) = \frac{p_{B,k} (1-r_{k-1})}{r_{k|k-1}} f_{B,k}(x_k) + \frac{r_{k-1}}{r_{k|k-1}} \left( f_{k|k-1}(x_k | \cdot) \cdot p_{S,k} \cdot p_{k-1} \right)$$  \hspace{1cm} (5)

**Update:**

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**Figure 1.** Model of multipath target tracking in OTHR.
\[ r_k = \frac{L_k(Z_k \mid \cdot), p_k(x_k) \cdot p_k(x_k)}{e^{-\frac{1}{2} r_k}}, + L_k(Z_k \mid \cdot), p_k(x_k)} \]  

\[ p_k(x_k) = \frac{L_k(Z_k \mid x_k), p_k(x_k)}{L_k(Z_k \mid \cdot), p_k(x_k)} \]  

Where \( L_k(Z_k \mid x_k) \) is multipath likelihood function (further details, see [7]).

3. The Multiple-model MPBF

Due to the dynamic model uncertainty, maneuvering target tracking is a difficult problem in OTHR system. Normally the multiple models (MM) algorithm can effectively solve the problem of dynamic model uncertainty. In this section, a MM-MPBF algorithm is proposed to track single maneuvering multipath target.

We assume that \( x_k \) represents the target state and \( m_k \in \{1, K, M\} \) represents the label of the dynamic model. The models transition probability is

\[ f_{k|k-1} (m_k = b \mid m_{k-1} = a) = h_{ab} \]  

Let \( y_k = (x_k, m_k) \) denote the new target state vector, which can be represented as the Bernoulli RFS parameter set \( \{r_k, p_k, m_k\} \). Note that the proposed MM-MPBF is similar to the MM method in mixing stage, and a group of single model MPBF (SM-MPBF) also apply. Then, the MM-MPBF algorithm is:

1) Mixing:

If the model-dependent posterior density denote \( \pi_{k-1} = \bigcup_{a=1}^{M} \pi_{k-1} (m_{k-1} = a) \), where \( \pi_{k-1} (m_{k-1} = a) = (r_{k-1}^{(a)}, p_{k-1}^{(a)}(x_{k-1}), m_{k-1} = a) \), then the mixed initial posterior density is

\[ \tilde{\pi}_{k|k-1} = \bigcup_{b=1}^{M} \tilde{\pi}_{k|k-1} (m_k = b) \]  

where

\[ \tilde{\pi}_{k|k-1} (m_k = b) = r_{k-1}^{(b)}, \tilde{p}_{k|k-1}^{(b)}(x_k), m_k = b \]  

\[ \tilde{p}_{k|k-1}^{(b)} = \sum_{a=1}^{M} r_{k-1}^{(a)} h_{ab} \]  

\[ \tilde{p}_{k|k-1}^{(b)}(x_k) = \sum_{a=1}^{M} p_{k-1}^{(a)} h_{ab} \]  

2) Prediction:

If the initial posterior density for the MM-MPBF is matched to the model \( b \), then the model-dependent predicted posterior density is

\[ \pi_{k|k-1} (m_k = b) = (r_{k|k-1}^{(b)}, p_{k|k-1}^{(b)}(x_k), m_k = b) \]  

where

\[ r_{k|k-1}^{(b)} = p_{R,k} (1 - \tilde{p}_{k|k-1}^{(b)}) + \tilde{p}_{k|k-1}^{(b)} \left( p_{S,k}, \tilde{p}_{k|k-1}^{(b)}(x_k) \right) \]  

\[ p_{k|k-1}^{(b)}(x_k) = p_{R,k} (1 - \tilde{p}_{k|k-1}^{(b)}) \tilde{F}_{k|k-1} \left( r_{k|k-1}^{(b)}(x_k), p_{k|k-1}^{(b)}(x_k), m_k \right) + \tilde{p}_{k|k-1}^{(b)} \left( f_{k|k-1}(x_k \mid \cdot), p_{S,k} \cdot \bar{\tilde{p}}_{k|k-1}^{(b)}(x_k) \right) \]  

3) Update:
If the predicted posterior density is
\[ \pi_{k|k-1}(m_k = b) = (r_k^{(b)}, p_{k|k-1}^{(b)}(x_k), m_k = b), \]
then the model-dependent updated posterior density can be approximated as:
\[ \pi_k(m_k = b) = (r_k^{(b)}, p_k^{(b)}(x_k), m_k = b) \] (16)
where
\[ r_k^{(b)} = \frac{\left\langle L_k^{(b)}(Z_k | \cdot), p_k^{(b)}(\cdot) \right\rangle}{\left\langle L_k^{(b)}(Z_k | \cdot), p_{k|k-1}^{(b)}(\cdot) \right\rangle} \] (17)
\[ p_k^{(b)}(x_k) = \frac{L_k^{(b)}(Z_k | x_k), p_k^{(b)}(x_k)}{\left\langle L_k^{(b)}(Z_k | \cdot), p_{k|k-1}^{(b)}(\cdot) \right\rangle} \] (18)
where \( n_k^{(b)}(Z_k | \cdot) \) is the multipath target likelihood function corresponding to the model \( b \).
Note that we use particle filter to solve the problem of nonlinear measurement models.

4. Numerical Simulations
The performance of MM-MPBF will be demonstrated through a manoeuvring target tracking example in OTHR system. We verify the performance of the MM-MPBF algorithm with the SM-MPBF and MM Bernoulli filter (MM-BF) through different numerical simulation scenarios.
In this tracking scenario, we assume that the target dynamic model set consists of two potential dynamic models (further details, see [8]), and the model transition probability matrix can be given by
\[ \begin{bmatrix} h_{k|k} \end{bmatrix} = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix} \] (19)
In this tracking simulation scheme, we assumed that one maneuvering target is moving, which appears at scan \( k = 1-80 \) with initial state \( x_0 = (110km, 0.15km/s, 1000km, 0.15km/s) \). The target moves with constant velocity between scan \( k=1-20, 40-45 \) and \( 60-80 \), and other time target performs maneuvering turning motion executes a coordinated turn with \( 10^\circ/\)s. The maneuvering target true trajectory is shown in Fig. 2. We assume that \( p_{D,k} = 0.7 \) and \( p_{S,k} = 0.95 \) (target detection probability and survive probability). We model the clutter as a Poisson RFS and assume that there are 50 clutters in each scan. Other experiment parameters are same as [7].
The tracking results of one simulation experiment comparing the MM-MPBF with the MM-BF and SM-MPBF are shown in Fig. 3. Both of the MM-MPBF and MM-BF can track the maneuvering multipath target in OTHR system. However, the SM-MPBF algorithm can not track the maneuvering multipath target effectively because it only considers just one dynamic model. If the target dynamic model changes, the algorithm will fail to tracking.

Figure 2. Target trajectory.

Figure 3. The estimation trajectory.
The optimal subpattern assignment (OSPA) distance [7] with different algorithms can be seen in Fig. 4-5. The experimental results demonstrate that both MM-MPBF and MM-BF algorithms have relatively small OSPA distance, it means that MM-MPBF and MM-BF algorithms can effectively track maneuvering multipath target. As is shown in Fig. 5, the OSPA of the MM-MPBF is smaller than the MM-BF, the performance of the MM-MPBF is better. This is because the MM-MPBF uses all the measurements from multiple paths to track the target. As is shown in Fig. 5, the SM-MPBF only tracks the target effectively in the first 20 scans, which is due to the target's dynamic model matches with the SM-MPBF in the first 20 scans. When the dynamic model does not match, the SM-MPBF can not track the maneuvering multipath target effectively.

5. Conclusion

The MM-MPBF has been proposed to solve the problem of single maneuvering multipath target tracking. The MM-MPBF is based on the RFS, which combines the interacting multiple models algorithm with the MPBF algorithm to solve the problems of maneuvering target tracking in OTHR. The performance of the MM-MPBF is verified by simulation experiment.

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