$N = 2$ Current Algebras for Non-semi-simple Groups

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Abstract

We examine the problem of constructing $N = 2$ superconformal algebras out of $N = 1$ non-semi-simple affine Lie algebras. These $N = 2$ superconformal theories share the property that the super Virasoro central charge depends only on the dimension of the Lie algebra. We find, in particular, a construction having a central charge $c = 9$. This provides a possible internal space for string compactification and where mirror symmetry might be explored.

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1 Introduction

The study of $N = 2$ superconformal algebras is of fundamental importance in string theory and topological field theory [1]. It was realised in string theories that $N = 2$ superconformal invariance on the world-sheet leads to $N = 1$ space-time supersymmetry, which is crucial for phenomenological applications of string theories. To be able to extract physics out of string theory, the ten-dimensional space-time $\mathcal{M}_{10}$ has to be compactified as $\mathcal{M}_4 \times \mathcal{K}_6$, where $\mathcal{M}_4$ is Minkowski space and $\mathcal{K}_6$ is an internal space which is, to lowest order in perturbation theory, Ricci flat, complex and Kähler; that is a Calabi-Yau space. However, the physics one reaches depends very much on which Calabi-Yau manifold one chooses. Therefore it is natural to look for some criteria which cut down on the number of the classically allowed vacua for string theory.

One way of finding this selection procedure among all possible string vacua is provided by $N = 2$ superconformal theories. Indeed, the internal space $\mathcal{K}_6$ can be replaced by any $N = 2$ superconformal theory having a central charge $c = 9$. The problem of string compactification reduces then to studying the space of all possible $N = 2$ superconformal field theories. In this space one moves from one conformal field theory to another (that is, from one string vacuum to another) by perturbing by the so-called truly marginal operators in the so-called chiral ring of the $N = 2$ superconformal theory. This chiral ring [2] is at the heart of many interesting features of $N = 2$ superconformal algebras and in particular mirror symmetry [3].

Mirror symmetry relates two Calabi-Yau manifolds $\mathcal{K}_6$ and $\tilde{\mathcal{K}}_6$ which are a priori topologically and geometrically different. If $(\mathcal{K}_6, \tilde{\mathcal{K}}_6)$ is a mirror pair then $h^{(1,1)}_{\mathcal{K}_6} = h^{(2,1)}_{\mathcal{K}_6}$ and $h^{(2,1)}_{\mathcal{K}_6} = h^{(1,1)}_{\tilde{\mathcal{K}}_6}$, where $h^{(p,q)}_\mathcal{K}$ is a $(p,q)$-form on $\mathcal{K}$. Moreover, the non-linear sigma models corresponding to $\mathcal{K}_6$ and $\tilde{\mathcal{K}}_6$ must also yield isomorphic $N = 2$ superconformal field theories. At the level of the $N = 2$ superconformal theory, $h^{(1,1)}_{\mathcal{K}_6}$ and $h^{(2,1)}_{\mathcal{K}_6}$ are identified with the so-called (antichiral,chiral) truly marginal operators while $h^{(2,1)}_{\mathcal{K}_6}$ and $h^{(1,1)}_{\tilde{\mathcal{K}}_6}$ are identified with the so-called (chiral,chiral) truly marginal operators [3]. Therefore, one obtains two different string vacua depending on whether one deforms the $N = 2$ superconformal theory by an (antichiral,chiral) or by a (chiral,chiral) truly marginal operators. It is crucial to mention that the (antichiral,chiral) and the (chiral,chiral) truly marginal operators differ only by the sign of their left $U(1)$ charge.

For a geometrical interpretation of mirror symmetry in the context of Calabi-Yau sigma models one requires that the $N = 2$ superconformal algebra has a central charge $\hat{c} \equiv 3c$
equal to an integer number. So far, the only rigorously established example of mirror symmetry relies on another model, namely the exactly solvable $N = 2$ minimal model \[4\], where the mirror manifold is obtained by an orbifold construction \[5\]. It is, therefore, crucial to search for other models where mirror symmetry might be better understood. It is the purpose of this paper to provide, as a first step in this direction, $N = 2$ superconformal algebras having integer conformal anomaly $\hat{c}$. These algebra are based on non-semi-simple affine current algebras. Wess-Zumino-Novikov-Witten (WZNW) models based on non-semi-simple Lie algebras have recently been extensively analysed \[6–14\].

The analogue of our construction for semi-simple affine algebras has already been performed in \[15\]. All these $N = 2$ constructions are in the spirit of Kazama-Suzuki models \[16\]. Attempts in understanding mirror symmetry and string compactification in Kazama-Suzuki models have also been made in \[17\].

The paper is organised as follows. In section two we review the $N = 1$ supersymmetric current algebra based on non-semi-simple Lie algebras. We also show how this can be cast into a tensor product of a bosonic current algebra and an algebra of free Majorana fermions. In section three we tackle the problem of constructing $N = 2$ algebras from $N = 1$ non-semi-simple affine current algebras. The conditions for the existence of this construction are then found. We apply our method to a wide set of Lie algebras possessing non-degenerate invariant bilinear forms. Explicit examples of our construction are also given.

2 The Supersymmetric Current Algebra

Let $G$ be a Lie algebra (semi-simple or non-semi-simple) whose commutators are

$$[T^I, T^J] = f^{IJ}_K T^K .$$  \tag{2.1}

The supercurrent algebra built on this Lie algebra is given by the operator product expansions of the supercurrents $J^I(Z)$ (left-handed part) \[18\]

$$J^I(Z_1)J^J(Z_2) = h^{IJ}Z_{12}^{-1} + f^{IJ}_K Z_{12}^{-1/2} J^K(Z_2) .$$  \tag{2.2}

In these expressions $Z = (z, \theta)$ denotes the holomorphic coordinate of two-dimensional superspace and the symbol $Z^M_{ij}, M \in \mathbb{Z}$, is defined by

$$Z^M_{ij} = \begin{cases} (z_i - z_j - \theta_i \theta_j)^M, & M \in \mathbb{Z} \\ (\theta_i - \theta_j)(z_i - z_j - \theta_i \theta_j)^{M-1/2}, & M \in \mathbb{Z} + \frac{1}{2} \end{cases} .$$  \tag{2.3}
The associativity of the above product expansion shows that $h^{IJ}$ is an invariant of the group obeying
\[ f^{IJ}_K h^{KL} + f^{IL}_K h^{KJ} = 0 \]  
(2.4)
At this stage we do not know if $h^{IJ}$ needs to be invertible.

The super energy-momentum tensor is assumed to take the general form
\[ T(Z) = \Omega_{IJ} : D J^I J^J : (Z) + M_{IJK} : J^I : J^J J^K :: (Z) , \]  
(2.5)
where $\Omega_{IJ}$ is symmetric and $M_{IJK}$ is totally antisymmetric. The super covariant derivatives is $D = \frac{\partial}{\partial \theta} + \theta \frac{\partial}{\partial z}$ obeying $D^2 = \partial$.

The two tensors $\Omega_{IJ}$ and $M_{IJK}$ are then determined by requiring that the supercurrents $J^I(Z)$ are primary operators of dimension $\frac{1}{2}$ with respect to the super stress tensor and that $T(Z)$ satisfies the super Virasoro algebra. These requirements lead uniquely to [10]
\[ \Omega_{IK} h^{KJ} = \frac{1}{2} \delta^J_I \ , \ \Omega_{IK} f^{KL}_J + \Omega_{JK} f^{KI}_L = 0 \]
\[ M_{IJK} = \frac{2}{3} f_{IJK} \equiv \frac{2}{3} \Omega_{IP} \Omega_{JQ} f^{PQ}_K \ . \]  
(2.6)
Therefore $\Omega^{IJ}$ (and $h^{IJ}$) is an invertible invariant bilinear form of the group $G$. Using these expressions for $M_{IJK}$ and $h^{IJ}$, the super energy-momentum tensor does indeed satisfy a super Virasoro algebra with central charge given by [10]
\[ c = \frac{3}{2} \dim(G) - \gamma^{IJ} \Omega_{IJ} \ . \]  
(2.7)
In these expression $\gamma^{IJ}$ is the Killing-Cartan invariant bilinear form defined by
\[ \gamma^{IJ} = f^{KL}_I f^{LJ}_K \]  
(2.8)
and is degenerate for non-semi-simple Lie algebras. It was shown in [11] that for non-semi-simple Lie algebras, which cannot be decomposed into a product of semi-simple and non-semi-simple bits, we necessarily have $\gamma^{IJ} \Omega_{IJ} = 0$. Therefore, the super Virasoro central charge depends only on the dimension of the Lie algebra.

The supercurrent algebra can be made to look like a direct sum of a bosonic current algebra and an algebra of free Majorana fermions. This can be achieved by decomposing $J^I(Z)$ as
\[ J^I(Z) = \psi^I(z) + \theta J^I(z) \]  
(2.9)
and introducing a modified current $\tilde{J}^I(z)$ defined by
\[ \tilde{J}^I(z) = J^I(z) + R^I_{JK} : \psi^J \psi^K : (z) \ , \]  
(2.10)
where $R^I_{\ JK}$ is antisymmetric in the indices $J$ and $K$, and is determined by requiring that the fermions $\psi^I(z)$ and the bosonic currents $\hat{J}^I$ decouple. Indeed, the following operator product expansions

$$
\psi^I(z_1)\psi^J(z_2) = \frac{h^{IJ}}{(z_1 - z_2)} \\
\hat{J}^I(z_1)\psi^J(z_2) = 0 \\
\hat{J}^I(z_1)\hat{J}^J(z_2) = \frac{(h^{IJ} - 1/2 \gamma^{IJ})}{(z_1 - z_2)^2} + f^{IJ}_K \hat{J}^K(z_2) \\
$$

(2.11)

hold only if the tensor $R^I_{\ JK}$ satisfies [10]

$$
R^I_{\ JK} = \Omega_{JL} f^{LI}_{\ K} .
$$

(2.12)

The super stress tensor is also written in components as

$$
T(Z) = \frac{1}{2} G(z) + \theta T(z) .
$$

(2.13)

In terms of the modified currents we have

$$
T(z) = \Omega_{IJ} :\hat{J}^I \hat{J}^J : (z) - \Omega_{IJ} :\psi^I \partial \psi^J : (z) \\
G(z) = 2\Omega_{IJ} :\psi^I \hat{J}^J : (z) - \frac{2}{3} f_{IJK} :\psi^I :\psi^J \psi^K :: (z) .
$$

(2.14)

The components $T(z)$ and $G(z)$ satisfy the usual $N = 1$ superconformal algebra. The contribution to the central charge due to the stress tensor of the bosonic currents $\hat{J}^I(z)$ is $[\dim(G) - \gamma^{IJ} \Omega_{IJ}]$ while that of the fermions $\psi^I(z)$ is given by $1/2 \dim(G)$. These two contributions add up to give, as expected, the central charge for the $N = 1$ algebra calculated in (2.8).

3 The $N = 2$ Construction

In order to build the $N = 2$ superconformal current algebra, we need another $N = 1$ supercurrent. This we write as

$$
\tilde{G}(z) = 2D_{IJ} :\psi^I \hat{J}^J : (z) - \frac{2}{3} S_{IJK} :\psi^I :\psi^J \psi^K :: (z) ,
$$

(3.1)

where $S_{IJK}$ is totally antisymmetric. The quantities $D_{IJ}$ and $S_{IJK}$ are determined by the closure of the $N = 2$ superconformal algebra. Let us also define

$$
G^+ = \frac{1}{\sqrt{2}} (G + i\tilde{G}) , \quad G^- = \frac{1}{\sqrt{2}} (G - i\tilde{G}) .
$$

(3.2)
We would like now to demand the closure of the $N = 2$ superconformal algebra. By demanding that
\[ G^\pm(z)G^\pm(w) = 0 \tag{3.3} \]
we find a set of equations (spelled out in details in ref. [19]) which are solved by
\[
\begin{align*}
D_{IJ} &= -D_{JI}, \quad D_{IK}\Omega^{KL}D_{LJ} = -\Omega_{IJ} \tag{3.4} \\
f_{IJK} &= D_{IP}D_{JQ}f^{PQ}_K + D_{KP}D_{IQ}f^{PQ}_J + D_{JP}D_{KQ}f^{PQ}_I \tag{3.5} \\
S_{IJK} &= D_{IP}D_{JQ}D_{KL}f^{PQL} \tag{3.6}
\end{align*}
\]

The conditions (3.4) means that $D_{IJ}$ is an almost complex structure on $G$, while equation (3.5) is a constraint on the structure constants $f^{IJ}_K$. The last equation defines, however, the tensor $S_{IJK}$. The group indices are raised and lowered using the invariant bilinear form $\Omega^{IJ}$ and its inverse $\Omega_{IJ}$.

The $U(1)$ current, $H(z)$, necessary for the existence of the $N = 2$ superconformal algebra is read from the following operator product expansion
\[ G^+(z)G^-(w) = \frac{2c}{3(z-w)^3} + \frac{2H(w)}{(z-w)^2} + \frac{2T(w) + \partial H(w)}{(z-w)} \tag{3.7} \]
and is found to be
\[ H(z) = i \left[ \left( D_{IJ} + D_{PQ}f^{PQ}_L f^{L}_{IJ} \right) \psi^I \psi^J : (z) - D_{PQ}f^{PQ}_I \bar{\psi}^I(z) \right] . \tag{3.8} \]

The central charge $c$ and the energy-momentum tensor $T(z)$ are those of the previous section. By virtue of the above conditions, we also have
\[ H(z)G^\pm(w) = \pm \frac{G^\pm(w)}{(z-w)} . \tag{3.9} \]

The operator product expansions in (3.3), (3.7) and (3.9) are the minimal data necessary to guarantee that we have a $N = 2$ superconformal algebra [19,20].

4 $N = 2$ Construction for Lie Algebras with an Invariant Metric

Let us begin this section by recalling how one constructs Lie algebras with invariant and nondegenerate metrics. This construction unifies all the non-semi-simple Lie algebras known so far [21,11]. Let $h$ be any Lie algebra and let $h^*$ denotes its dual. If we choose a basis $\{H^a\}$ for $h$ then the dual basis for $h^*$ is $\{H_a\}$ such that $< H^a, H_b > = \delta^a_b$. We also assume that $h$ possesses an invariant bilinear form (possibly degenerate) which we denote
A Lie algebra structure is defined on the vector space $h \oplus h^*$ in the following manner. On $h$ one has the Lie bracket $[H^a, H^b] = f^{ab}_c H^c$, whereas $h^*$ is made abelian $[H_a, H_b] = 0$. The mixed brackets are given by $[H^a, H_b] = - f^{ac}_b H_c$.

In addition to $h$ and $h^*$, we introduce another Lie algebra $g$ with a nondegenerate invariant metric. This invariant metric is denoted $\omega^{ij}$ relative to a basis $\{X^i\}$. The Lie bracket on $g$ is $[X^i, X^j] = f^{ij}_k X^k$. The action of $h$ on $g$ is given by antisymmetric derivations [21,11]. Explicitly, this means the existence of mixed structure constants, $f^a_{ij}$, satisfying [21,11]

\[
\begin{align*}
  f^{ai}_k \omega^{kj} &= - f^{aj}_k \omega^{ki} \\
  f^{ij}_k f^{ak}_l &= f^{ai}_k f^{kj}_l + f^{ik}_l f^{aj}_k .
\end{align*}
\] (4.1)

These last relations define on the vector space $g \oplus h \oplus h^*$ the following non-vanishing brackets [21,11]

\[
\begin{align*}
  [X^i, X^j] &= f^{ij}_k X^k + f^{ai}_k \omega^{kj} H_a \\
  [H^a, H^b] &= f^{ab}_c H^c \\
  [H^a, X^i] &= f^{ai}_k X^k \\
  [H^a, H_b] &= - f^{ac}_b H_c .
\end{align*}
\] (4.2)

This algebra admits an invariant metric given by

\[
\Omega^{IJ} = \begin{pmatrix} X^j & H^b & H_b \\
\omega^{ij} & 0 & 0 \\
H^a & 0 & \omega^{ab} \delta^a_b \end{pmatrix} .
\] (4.3)

This bilinear form is nondegenerate and its inverse is given by

\[
\Omega_{IJ} = \begin{pmatrix} \omega_{ij} & 0 & 0 \\
0 & 0 & \delta^a_b \delta^b_a \\
0 & \delta^a_b & -\omega^{ab} \end{pmatrix} ,
\] (4.4)

where $\omega_{ij}$ is the inverse of $\omega^{ij}$. Notice that we have a family of invariant metrics, $\Omega^{IJ}$, parametrised by $\omega^{ab}$, the invariant bilinear forms of $h$.

In order to apply the formulae of the $N = 2$ construction to the above Lie algebra, we need to find the corresponding complex structure $D_{IJ}$. For this we suppose that we have a complex structure $d_{ij}$ on $g$ obeying

\[
d_{ij} = - d_{ji} , \quad d_{ik} \omega^{kl} d_{lj} = - \omega_{ij} .
\] (4.5)
and that the structure constants of $g$ satisfy

$$f_{ijn} = d_{ik}d_{jl}f^{kl}_{ni} + d_{nk}d_{il}f^{kl}_{ji} + d_{jk}d_{nl}f^{kl}_{li}.$$  \hspace{1cm} (4.6)

These two requirements mean that we have a $N = 2$ superconformal conformal algebra if we restrict ourselves to the Lie algebra $g$.

We found that the complex structure corresponding to the invariant metric $\Omega^{IJ}$ is given by

$$D_{IJ} = \begin{pmatrix} d_{ij} & 0 & 0 \\ 0 & 0 & i\delta^a_b \\ 0 & -i\delta^b_a & 0 \end{pmatrix}.$$  \hspace{1cm} (4.7)

Using the structure constants $f^{IJ}_K$ of the Lie algebra (4.2) and the expressions for $D_{IJ}$ and $\Omega^{IJ}$, we find that equation (3.5) is satisfied only if

$$f_{a^b}^{c} = 0$$

$$d_{ij}f^{aj}_k = d_{kj}f^{aj}_i.$$  \hspace{1cm} (4.8)

Let us turn our attention to some more concrete examples.

5 Examples

As a first example of our construction, let us consider the $n$-dimensional Heisenberg group whose non-vanishing brackets are given by

$$[\alpha_i, \alpha_j^\dagger] = \delta_{ij}I$$

$$[N, \alpha_i] = -\alpha_i$$

$$[N, \alpha_i^\dagger] = \alpha_i^\dagger,$$  \hspace{1cm} (5.1)

where $\{i = 1, \ldots, r = \frac{n-2}{2}\}$ and $N = \delta^{ij}\alpha_i\alpha_j^\dagger$. The group $g$ in this case is abelian and is generated by $\{\alpha_1^i,\alpha_1^{i\dagger},\ldots,\alpha_r,\alpha_r^\dagger\}$ while the group $h$ is one-dimensional and is generated by $\{H^a = N\}$ and its dual $h^\ast$ is generated by $\{H_a = I\}$. The invertible invariant bilinear form of this Lie algebra is [12]

$$\Omega^{IJ} = \begin{pmatrix} A_1 & \ldots & A_r \\ \vdots & \ddots & \vdots \\ A_r & \ldots & A_1 \\ b & -a & 0 \\ -a & 0 & \end{pmatrix}, \quad A_1 = A_2 = \ldots = A_r = \begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix}.$$  \hspace{1cm} (5.2)
The corresponding complex structure is found to be

\[
D_{IJ} = \begin{pmatrix}
B_1 & & & \\
& \ddots & & \\
& & B_r & \\
0 & & \frac{i}{a} & 0 \\
-\frac{1}{a} & & 0 & \\
\end{pmatrix}, \quad B_1 = B_2 = \ldots = B_r = \begin{pmatrix} 0 & \frac{i}{a} \\ -\frac{1}{a} & 0 \end{pmatrix}.
\] (5.3)

Since \( f^{ij}_{\ k} = 0 \), equation (4.6) is trivially satisfied. As required by equation (4.8) we also have \( f^{ab}_{\ c} = 0 \) and it is straightforward to verify that \( d_{ik}f_{jk}^{ak} = d_{jk}f_{ik}^{ak} \).

The second example we present here concerns the centrally extended two-dimensional Euclidean group generated by \( \{P_1, P_2, J, T\} \) with the commutation relations [6]

\[
[P_i, P_j] = \epsilon_{ij}T \\
[J, P_i] = \epsilon_{ij}P_j,
\] (5.4)

where we identify \( \{X^i = P_1, P_2\}, \{H^a = J\} \) and \( \{H_a = T\} \). The invertible invariant bilinear form \( \Omega_{IJ} \) and the complex structure \( D_{IJ} \) are given by

\[
\Omega_{IJ} = \begin{pmatrix}
a & 0 & 0 & 0 \\
0 & a & 0 & 0 \\
0 & 0 & b & a \\
0 & 0 & a & 0 \\
\end{pmatrix}, \quad D_{IJ} = \begin{pmatrix} 0 & \frac{1}{a} & 0 & 0 \\ -\frac{1}{a} & 0 & 0 & 0 \\
0 & 0 & \frac{i}{a} & 0 \\
0 & 0 & 0 & -\frac{1}{a} \end{pmatrix}.
\] (5.5)

Here also \( f^{ij}_{\ k} = f^{ab}_{\ c} = 0 \) and \( d_{ik}f_{jk}^{ak} = d_{jk}f_{ik}^{ak} \).

To conclude, we have explored the possibility of constructing \( N = 2 \) superconformal algebras out of \( N = 1 \) non-semi-simple affine Lie algebras. The conditions under which the \( N = 2 \) superconformal algebra exists are spelled out. We present two non-trivial examples which explicitly solve these conditions.

The crucial feature of these \( N = 2 \) superconformal algebras is that they all have integer conformal anomaly \( \hat{c} \). This would allow for a geometrical interpretation of the truly marginal perturbations of these \( N = 2 \) superconformal theories. In particular, when \( c = 9 \) (like in the case of the 6-dimensional Heisenberg group), the super WZNW model would provide the internal space for string compactification. The string backgrounds corresponding to these non-semi-simple groups furnish exact solutions to the beta functions of the non-linear sigma models to all orders in perturbation theory [14].

Another important issue regarding this internal space for string compactification is to find its mirror. For this we need to determine the chiral ring for these \( N = 2 \) superconformal algebras. A possible way to proceed in determining this chiral ring would be to use a free field representation of these \( N = 2 \) superconformal theories. This is a natural thing to do.
since the central charge is just an integer number. Finally, it would be also interesting to
explore the relation between our construction and the constructions of Getzler [20] and of
Spindel et al. [22].

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