Optimization of fuel-cell tram operation based on two dimension dynamic programming

Wenbin Zhang¹, Xuecheng Lu¹, Jingsong Zhao¹ and Jianqiu Li²,³

¹ Department of Scientific Research. Military Transportation University, Tianjin 300161, PR China;
² Department of Automotive Engineering, State Key Laboratory of Automotive Safety and Energy, Tsinghua University, Beijing 100084, PR China; Collaborative Innovation Center of Electric Vehicles in Beijing, Beijing 100081, PR China.
³ lijianqiu@tsinghua.edu.cn

Abstract. This paper proposes an optimal control strategy based on the two-dimension dynamic programming (2DDP) algorithm targeting at minimizing operation energy consumption for a fuel-cell tram. The energy consumption model with the tram dynamics is firstly deduced. Optimal control problem are analyzed and the 2DDP strategy is applied to solve the problem. The optimal tram speed profiles are obtained for each interstation which consist of three stages: accelerate to the set speed with the maximum traction power, dynamically adjust to maintain a uniform speed and decelerate to zero speed with the maximum braking power at a suitable timing. The optimal control curves of all the interstations are connected with the parking time to form the optimal control method of the whole line. The optimized speed profiles are also simplified for drivers to follow.

1. Introduction

Railway transportation shares huge parts of energy consumption in the national economy. It is of great significance in studying its energy saving problem. Under the premise of types, lines and operating schedules, improving the operation method is an economically viable route to achieve energy saving in trains.

The operation method refers the operation taken by the train crew during the start, stop and running periods, such as the moment of addition and subtraction of the handle, the level of the target speed and the length of time to maintain a constant speed. Trams are typical railway vehicles and they are popular in large and medium-sized cities for their merits of great passenger capacity, low energy consumption and long service life [1]. The trams’ interstation length is about 1 km, which is much shorter than railway vehicles’. From one station to the next one, the tram may only experience acceleration and deceleration once. The energy consumption is affected by many factors, such as line conditions, speed and weight.

The existing research follows the following sequence: an energy consumption model is given and transformed into a constrained multi-objective optimization problem. Optimization methods such as genetic algorithm (GA), maximum value optimization principle (MVOP) and neural network (NN) are applied to solve the problem. Scheepmaker et al. [2] developed an energy-efficient train operation model based on optimal control algorithm that determined the joint optimal cruising speed and coasting point for individual train trips. The model was applied in a case study of a regional train line.
in the Netherlands. Li et al. [3] put forward "ramp divided into three parts" joystick manipulating principle to describe the train energy-saving optimization problem and solved it with GA. Jin et al. [4] proposed an intelligent computation model to generate the optimal energy-saving train operation diagram whose local optimization was realized by a NN and global optimization was a GA. Wang et al. [5] divided the whole process of train operation into multiple running states like gentle slope, upgrade and downgrade and tried to seek an optimum speed curve as to achieve minimum energy consumption of trains from departure to destination under satisfying the multi-objective constraints including safety, punctuality and comfort. Huang et al. [6] demonstrated an energy-efficient approach considering both the trip time and driving strategy to reduce the traction energy by optimizing the train operation for multiple infestations and applied multi-population genetic algorithm (MPGA) to settle the problem. Zhang Yong et al. [7] calculated the actual operation data of HXD3 electric locomotive and obtained the additional resistance parameters and gave the model of indirectly calculating the energy consumption of electric locomotive. Compared with the actual energy consumption, the accuracy of model 90%. For one 59.2 km long line section, the researcher selected eight key points, encoded the speed and the handle position of each key point, and applied GA to obtain the optimal operation method, saving 30% energy compared to the actual energy consumption.

From the studies above, an energy-saving trajectory can be expressed an optimal speed curve and GA is the commonly used method. However, the GA can be implemented only for the test data are obtained in advance. It will not be available for scenarios without test data. The fuel-cell tram is still in the design stage and the line test data are unknown. We want to devise a set of strategies that will guide tram driver before running. The dynamic programming (DP) algorithm is a global optimization algorithm. It can find the optimal solution by traversing all feasible, even in the absence of test data. DP has been successfully applied on velocity profile optimization by Hellström et al. [8] and Ozatay et al. [9] to minimize the fuel consumption. The problem studied has two states generating a state plane. Therefore, we propose a new type of algorithm: two-dimensional dynamic programming (2DDP) algorithm to design the tram operation.

2. Tram energy consumption model

The fuel-cell tram is running along a rail line of 8.28 km. There are 11 stations, corresponding to 10 interstations. When it travels through one interstation, the energy consumption $E$ (kJ) is the integral of the wheel driving force $F_T$ (kN) to the driving distance $X$ (m). The driving force is the motor output power $P$ (kW) divided by the tram speed $V$ (km/h).

$$E = \int F_T \, dX = \int PV^{-1} \, dX$$

Figure 1. (a) Motor torque vs. rotation speed and (b) the driving force vs. tram speed.

$$E = \int F_T \, dX = \int PV^{-1} \, dX$$
The motor operating characteristics are expressed by the relationship between torque $T_q$ (N.m) and rotation speed $n$ (rad/min) as shown in Figure 1(a). Besides, the driving and braking acceleration are limited to ensure passenger comfort. The maximum acceleration speed $a_{\text{max}}$ (m.s$^{-2}$) and deceleration speed $a_{\text{min}}$ (m.s$^{-2}$) are pre-settled. The dotted lines in Figure 1(a) shows the motor characteristic under limitation of the acceleration and deceleration speed. The curve for the wheel driving force $T_F$ and the tram speed $V$ given in Figure 1(b) and has the similar shape as Figure 1(a). The equation can expressed as:

$$P = f_1(n, T_q) = f_2(V, F_T)$$

According to Newton's second motion law, the tram is accelerated by the driving force $F_T$ (kN) overcoming the load force $F_L$ (kN) including: the basic resistance $F_0$ (kN), the ramp resistance $F_i$ (kN) and the corner resistance $F_R$ (kN). The forces are described as:

$$\delta m \frac{dV}{dt} = F_T - F_L = F_0 + F_i + F_R$$

where $m$ (tons) is the tram mass and $\delta$ is the rotation mass transfer factor.

Eq. (4) gives the three resistances. The basic resistance $F_0$ is associated with the tram speed $V$. $p_0$ (N/kN) is the dimensionless basic resistance coefficient expressed as a quadratic equation of $V$, with a, b and c are the equation’s coefficients. According to the actual situation of the line, the tram will limit the maximum speed of each section. This paper limits the maximum speed $V_{\text{max}}$ (km/h) for the whole line. The ramp resistance $F_i$ and the corner resistance $F_R$ have nothing to do with speed but are related to the line characteristics. $p_i$ is the ramp resistance coefficient (N/kN) or the segmented slope (%), which is expressed as the function of the driving distance $X$. $p_R$ is the corner resistance coefficient (N/kN) related to the corner radius R(m). For the almost straight line, R is a constant value.

$$\begin{cases}
F_0 = mgp_0 / 1000, p_0 = a + bV + cV^2 \\
F_i = mgp_i / 1000, p_i = f_3(X), X = \int V dt \\
F_R = mgp_R / 1000, p_R = 650 / (R - 55)
\end{cases}$$

The basic resistance is the main source of energy consumption and the only controllable energy consumption factors, for the ramp resistance and the corner resistance do not change with speed and under any operation modes, they consume the same energy. The parameters of the tram energy consumption model are list in Table 1.

| Parameter/unit | Value | Parameter/unit | Value |
|----------------|-------|----------------|-------|
| m (tons)       | 72.4  | c(km$^{-2}$.h$^2$) | 0.00262 |
| a              | 5     | R(m)           | 10$^6$ |
| $b$(km$^{-1}$.h) | 0.06  | $\delta$       | 1.07  |
| $a_{\text{min}}$(m.s$^{-2}$) | -1.1 | $a_{\text{max}}$(m.s$^{-2}$) | 0.6 |

Table 1. Parameters and values of the model.
3. Optimal control problem analysis

The DP is quite effective to solve the global optimization problem[10]. For the fuel-cell tram in this paper, the optimal control sequence of motor output power \{P\} is to be calculated, so that the cost during one interstation time horizon can be minimized. Its mathematical description is as follows.

\[
J(V, X) = \min_P E
\]

subject to:
\[
0 \leq V \leq V_{\text{max}}
\]
\[
a_{\text{min}} \leq \frac{dV}{dt} \leq a_{\text{max}}
\]
\[
0 \leq X \leq X_{\text{end}}
\]
\[
P_{\text{min}} \leq P \leq P_{\text{max}}
\]

where \(J\) is the cost function, representing the minimum total value of energy consumption \(E\), the tram speed \(V\) and mileage \(X\) are two state variables. The motor output power \(P\) is the decision variable. Since there are two state variables, the DP is called 2DDP.

There are four constraints that must be met: the speed does not exceed the maximum value \(V_{\text{max}}\), the acceleration is limited between the maximum acceleration \(a_{\text{max}}\) and the minimum acceleration \(a_{\text{min}}\), the mileage is within one interstation length \(X_{\text{end}}\), and the motor power is limited between its maximum power \(P_{\text{max}}\) and minimum power \(P_{\text{min}}\) calculated by the curves in Figure.1(a).

Set the sample time \(t_s\) as 1 second, \(k\) represents the length of the time axis, the experienced time from the start point to the end point is \(T_{\text{end}}\), which is divided into \(N_t\) segments and expressed as:

\[
T_k \in [0, T_{\text{end}}], k = 1...N_t, T_{\text{end}} = N_t t_s
\]

The maximum tram speed is divided \(N_v\) parts and the maximum mileage is divided \(N_x\) parts. The two states belong to the following range:

\[
V_i \in [0, V_{\text{max}}], i = 1...N_v,
\]
\[
X_j \in [0, X_{\text{end}}], j = 1...N_x
\]

The control variable of motor output power \(P\) is divided into \(N_p\) segments and expressed as:

\[
P_l \in [P_{\text{min}}, P_{\text{max}}], l = 1...N_p
\]

Combined with Eq. (6) ~ (8), the \(k\)-th motor power causes the \(k\) step state to migrate to \(k+1\) step state.

\[
(V_{ik}, X_{jk}) \rightarrow (V_{ik+1}, X_{jk+1})
\]

The energy consumption is discretized as:

\[
E = \sum_{k=1}^{N_t} P_{ik} \Delta X_k
\]

The optimization problem is rewritten as the following equation:

\[
J = \min_P \sum_{k=1}^{N_t} \sum_{i=1}^{N_v} \sum_{j=1}^{N_x} \sum_{l=1}^{N_p} (P_{ijkl} t_s + \frac{P_{ijkl} - F_{ijkl} V_{ik} t_s}{2V_{ik} t_s^2})
\]
Figure 2 gives the sketch of the 2DDP algorithm. In addition to the time axis, there are two axes of velocity and position representing the two states. The states at a certain moment correspond to one point of the plane composed by velocity and position. The state transition is a transition from one point in a plane at k-th step to another point in the adjacent plane at k+1-th step.

![2DDP algorithm sketch.](image)

As Figure 3 shows the 2DDP calculation flowchart which follows four steps: parameter settings, forward calculation traversing all the feasible solutions, reverse calculation searching for the optimal path and solution and forward calculation to obtain the optimal state sequence with the optimal decision value. The reverse calculation process is the key part of the proposed algorithm and outlined as follows [8]:

1. Let $J_{N_T} = 0$.
2. Let $k = N_T - 1$.
3. Let $J_k (X_j, V) = \min_{R_{max}, R_{min}} \left[ J_{k+1} (X_j, V_j) + J_{k+1 \rightarrow i} \right]$, $V \in [0, V_{max}]$, $X_j \in [0, X_{end}]$.
4. Repeat (3) for $k = N_T - 2, N_T - 3, \ldots, 0$.
5. The optimal cost is $J_0$ and the sought control is the optimal control set from the initial state.

4. Results and discussion

Calculating results are shown in Figure 4. Optimal speed profiles for the first interstation is given in Figure 4 (a). The tram accelerates to about 50 km/h with the maximum driving force, and then runs at this speed in near steady state. When the tram is near to the stop station, it brakes at the maximum braking force. The time when the speed is 0 is just the driving distance reaches the length of interstation. Figure 4 (b) gives the optimal motor output power at any time, which makes sure that the tram consumes the least amount of energy and can reach the destination accurately. Figure 4 (c) and Figure 4 (e) shows the optimal speed profiles for the second and third interstation, while Figure 4 (d) and Figure 4 (f) are the optimal motor output power correspondingly. There are ten interstations, the other seven scenarios are similar to the first three ones.

Observing the speed profiles in Figure 4, it is found that each profile has three stages: a maximum torque drive to a certain speed, followed by cruising at a constant speed and a maximum brake torque to zero speed. Besides, in the acceleration and braking process, the maximum drive/brake torque are applied and the slope of the speed curves seem straight, which guide us to use “even acceleration 14s + uniform speed + uniform deceleration 8s” curve to approximate the speed curve shown as near-optimal speed profile in Figure 5. The speed deviations of the acceleration and braking sections are controlled within 2%, and the cruising section adopts a constant value to remove the fluctuation.
Moreover, the 30 seconds’ stop at each station is considered to form the speed curve of the whole line. Compared to the optimal profile, the near-optimal one is much easier for drivers to follow.

Figure 4. Speed, mileage and optimal motor power profiles for the first three interstations.
5. Conclusions
Tram traction process consists of starting, cruising or idling, and braking. From the energy consumption optimization point of view, an optimal control strategy is to find the optimal speed curve under certain constraints, making the tram arrive on time and consume the least energy. Constraints include tram-related and line-related parts such as speed, line segment length, curve radius, grade and so on.

Due to the tram line interstation is very small, the tram only need to go through an acceleration and deceleration to finish an interstation. During each interstation, the tram can follow three stages. In the first stage, the tram starts to accelerate to the set speed as soon as possible with the maximum traction power. The second stage is the driving process when the traction power is dynamically adjusted to maintain a uniform speed as far as possible. The third stage is targeted at to select a suitable timing to decelerate to 0 with the maximum braking power.

In the design stage of fuel cell tram without real vehicle operating data, the two-dimensional dynamic programming strategy can effectively solve the optimal operation of the fuel-cell trams. The optimized control method is a little complicated, a near-optimal one is proposed for the driver to follow. Limited by the absence of test data, further research may include the comparison of the results in this paper with the actual operating data.

Acknowledgment
This work is supported by National Natural Science Foundation of China (Grant No. 51576113 and U1564209), Ministry of Science and Technology of China (Grant No. 2015BAG06B01), and Tsinghua University (the independent research plan Z02-1 Grant No. 20151080411) and CRRC Qingdao Sifang Co., Ltd (Grant No. SF/JS-LiangZi-2014-542).

References
[1] Zhang W, Li J, Xu L, Ouyang M, Liu Y, Han Q, et al. 2016 Comparison study on life-cycle costs of different trams powered by fuel cell systems and others Int J Hydrogen Energy
[2] Scheepmaker G M, Goverde R M P 2015 The interplay between energy-efficient train control and scheduled running time supplements. Journal of Rail Transport Planning & Management 5(4) 225-239
[3] Li Y S, Hou Z S. 2007 Study on energy-saving control for train based on genetic algorithm[J]
Journal of System Simulation 19(2) 384-387

[4] Jin W, Li C, Hu F, et al. 2000 A study on intelligent computation of methods of optimization operation for train// International Workshop on Autonomous Decentralized Systems, 2000. Proceedings. IEEE Xplore 97-102.

[5] Wang L, Wang P, Qi-Ya H U, et al. 2016 Study on Optimization of Locomotive Driving Strategy based on Multi-objective Constraints Railway Transport & Economy

[6] Huang Y, Ma X, Su S, et al. 2015 Optimization of Train Operation in Multiple Interstations with Multi-Population Genetic Algorithm Energies 8(12) 14311-14329

[7] Yong Z, Tan N, Hui D 2013 Optimization control method of electric locomotive based on genetic algorithm[J] Journal of Beijing Jiaotong University 37(2) 108-113

[8] Hellström E, Ivarsson M, Åslund J, et al. 2009 Look-ahead control for heavy trucks to minimize trip time and fuel consumption Control Engineering Practice 17(2) 245-254

[9] Ozatay E, Onori S, Wollaeger J, et al. 2014 Cloud-Based Velocity Profile Optimization for Everyday Driving: A Dynamic-Programming-Based Solution[J]. IEEE Transactions on Intelligent Transportation Systems 15(6) 2491-2505

[10] Xu L, Ouyang M, Li J, et al. 2013 Application of Pontryagin's Minimal Principle to the energy management strategy of plugin fuel cell electric vehicles Int J Hydrogen Energy 38(24) 10104-10115