Neutrinos Interacting with Polarizable Media*

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Abstract

We study Cherenkov and transition radiation of neutral spin 1/2 particles which carry magnetic moments or electric dipole moments. In particular, we estimate the radiation caused by the solar neutrino flux in dielectric media.

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1 Introduction

There is some hope for discovering physics beyond the Standard Model through neutrino properties like masses, mixings and magnetic moments (MM) or electric dipole moments (EDM). The electromagnetic interactions generated by these moments can be probed by laboratory $\nu - e^-$ scattering yielding the bounds

$$\mu_{\nu e}^2 + 2.1 \mu_{\nu e}^2 < 1.16 \cdot 10^{-18} \mu_B^2$$

and

$$\sqrt{\mu_{\nu e}^2 + d_{\nu e}^2} < 5.4 \cdot 10^{-7} \mu_B$$

(1)

where $\mu_B$ is the Bohr magneton and $\mu$, $d$ denote MM and EDM, respectively. Note that with the neglect of neutrino masses MMs and EDMs cannot be distinguished and thus $\mu^2 + d^2$ is the relevant parameter. Bounds for the $\tau$ neutrino from $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ are an order of magnitude weaker than the above limit.

In this report we want to discuss the possibility of getting information on neutrino MMs and EDMs by using their interaction with polarizable media. It is well known that charged particles have such interactions but also neutral ones though much weaker as long as they have electromagnetic properties. Given a dielectric medium one can have two opposite situations:

i) Cherenkov radiation (CR)

The medium is homogeneous and infinitely extended. In this case radiation is only possible if the velocity of the particle is larger than the velocity of light in the medium, i.e. $v > 1/n$ where $n$ is the refractive index.

ii) Transition radiation (TR)

There are two different homogeneous media each of which fills a half-space. In this case radiation is emitted when the particle crosses the boundary. There is no lower limit on $v$.

Quantum theoretical treatments of TR of charged particles can be found in [7] and [10]. The latter reference is remarkable because there CR and TR for charged and neutral particles is considered in some approximation.

In the following the quantization of the electromagnetic field in the presence of a polarizable medium will be discussed in section 2. Then we will enter into CR in section 3 and consider TR in section 4. In both cases our discussion will be generally applicable to neutral particles with MMs and EDMs in the ultra-relativistic limit by which we mean the limit of vanishing masses. For finite mass effects we refer the reader to refs. [11] and [12] for CR and TR, respectively. In the numerical examples concerning neutrinos we will consider the solar neutrino flux as source and normalize to a MM of $10^{-10} \mu_B$ (1). Section 5 will contain the conclusions.

2 Quantization of the Electromagnetic Field

We envisage a situation where a dielectric medium is distributed in space with its properties given by the dielectric “constant” $\varepsilon(\vec{x})$ varying in space but being time-independent. We assume that the permeability of the medium is 1. Then in the absence of free charges and currents
\((p_f = 0, \vec{J}_f = 0)\) and with time dependence of \(\vec{E}, \vec{B}\) given by \(e^{-i\omega t}\) Maxwell’s equations reduce to

\[
\text{curl curl} \vec{E} - \varepsilon \omega^2 \vec{E} = 0, \quad \text{div} (\varepsilon \vec{E}) = 0, \quad \vec{B} = \frac{1}{i\omega} \text{curl} \vec{E}.
\]

(2)

This allows to dispense with the discussion of \(\vec{B}\) most of the time.

With \(\varepsilon(\vec{x}) = \bar{n}(\vec{x})^2\) we get a refractive index varying in space. If there is some region where \(\varepsilon(\vec{x}) = \varepsilon = n^2\) constant then a plane wave in that region is characterized by

\[
e^{-i(\omega t - \vec{k} \cdot \vec{x})} \quad \text{with} \quad \vec{k}^2 = n^2 \omega^2.
\]

(3)

Now we assume that the medium is distributed in such a way that one can split up the fields in

\[
\vec{E}_j(\vec{k}, x) \sim \bar{e}_j e^{-i(\omega t - \vec{k} \cdot \vec{x})} + \text{“scattered” part}
\]

with \(j = I, II\) indicating the polarizations. This allows to label the solutions of (2) by the momentum \(\vec{k}\) of the incident plane wave. If in addition the fields are normalized such that

\[
\frac{1}{2} \int d^3x \left( \bar{e}(\vec{x}) \vec{E}_j(\vec{k}, x) \cdot \vec{E}_j(\vec{k}', x) + \vec{B}_j(\vec{k}, x) \cdot \vec{B}_j(\vec{k}', x) \right) = (2\pi)^3 \delta_{jj'} \delta(\vec{k} - \vec{k}')
\]

\[
\int d^3x \left( \bar{e}(\vec{x}) \vec{E}_j(\vec{k}, x) \cdot \vec{E}_j(\vec{k}', x) + \vec{B}_j(\vec{k}, x) \cdot \vec{B}_j(\vec{k}', x) \right) = 0
\]

(5)

then the quantization is straightforwardly performed by

\[
\hat{E}(x) = \sum_{j=I,II} \int \frac{d^3k}{(2\pi)^3/2} \sqrt{\frac{\omega}{2}} \left( a_j(\vec{k}) \vec{E}_j(\vec{k}, x) + a_j^\dagger(\vec{k}) \vec{E}_j(\vec{k}, x)^* \right)
\]

(6)

and

\[
[a_j(\vec{k}), a_j^\dagger(\vec{k}')] = \delta_{j,j'} \delta(\vec{k} - \vec{k}'), \quad [a_j(\vec{k}), a_{j'}(\vec{k}')] = 0.
\]

(7)

For \(\vec{B}\) a relation analogous to (6) holds. Then the Hamilton operator is given by

\[
H_\gamma = \frac{1}{2} \int d^3x : (\bar{e}(\vec{x}) \vec{E}^2(x) + \vec{B}^2(x)) : = \sum_{j=I,II} \int d^3k \omega a_j^\dagger(\vec{k})a_j(\vec{k}).
\]

(8)

The correctness of the quantization procedure is checked by the time evolution

\[
i[H_\gamma, \hat{E}(x)] = \dot{\hat{E}}(x), \quad i[H_\gamma, \hat{B}(x)] = \dot{\hat{B}}(x).
\]

(9)

Given equis. (3) this check is nearly trivial. Depending on the physical problem it might not be necessary to find a complete system of solutions (2,4). Only those electromagnetic fields appearing in the problem are required.

In the following we want to treat two special cases of the reaction \(\mathcal{P} \rightarrow \mathcal{P}' + \gamma\) in the presence of the medium where \(\mathcal{P}, \mathcal{P}'\) are arbitrary particles with the same electric charge which are assumed to have tree level electromagnetic interactions at least in some effective theory. We are thus considering the tree graph of the above reaction but with the photon being represented by (4) instead of vacuum plane waves. With \(p_i, p_f, k\) being the 4-momenta of \(\mathcal{P}, \mathcal{P}', \gamma\), respectively, and thus \(p_i^2 = m_i^2, p_f^2 = m_f^2\), the process \(\mathcal{P} \rightarrow \mathcal{P}' + \gamma\) is only possible in vacuum if \(m_i > m_f\). The presence of the dielectric medium, however, in general allows this reaction even for \(m_f \geq m_i\) in
a certain range. Since we have assumed the medium to be static the energy is still conserved but in general momentum is not. We can summarize the situation in the following way:

\[ q \equiv p_i - p_f \Rightarrow \begin{cases} 
\text{vacuum:} & q = k \\
\text{medium:} & q^0 = \omega \text{ but } \vec{q} \neq \vec{k} \text{ in general.}
\end{cases} \]

As mentioned in the introduction we are interested in the reaction

\[ \nu(p_i, s_i) \rightarrow \nu'(p_f, s_f) + \gamma_j(k) \tag{10} \]

where \( \nu, \nu' \) are neutral spin 1/2 particles with polarizations \( s_i, s_f \), respectively, and the tree level electromagnetic interaction is given by (transition) MMs and/or EDMs, i.e.

\[ \langle p_f, s_f | J_{\text{em}}^\mu(x) | p_i, s_i \rangle = -\frac{i}{(2\pi)^3} \left( \frac{m_f m_i}{E_f E_i} \right)^{1/2} e^{-iq \cdot \vec{x}} u_f(\mu + i\sigma_5) \sigma^\mu q_{\nu} u_i \tag{11} \]

with \( u_i \equiv u(p_i, s_i) \) etc.

### 3 Cherenkov Radiation

Since here the medium is assumed to be infinitely extended and homogeneous it is the only case where one has \( q = k \) and

\[ \vec{E}_j(k, x) = \frac{1}{n} \vec{e}_j e^{-i(\omega t - \vec{k} \cdot \vec{x})} \tag{12} \]

The factor \( 1/n \) comes from the normalization condition (5). The angle \( \theta = \zeta(p_i, k) \) under which the photon is emitted is given by

\[ \cos \theta = \frac{1}{vn} \left[ 1 + (n^2 - 1) \frac{\omega}{2E} + \frac{m_f^2 - m_i^2}{2E\omega} \right] \tag{13} \]

where \( v \) is the velocity of \( \nu \) and \( E \equiv E_i \). With

\[ \frac{\omega}{E} \ll 1, \quad \frac{|m_f^2 - m_i^2|}{E\omega} \ll 1 \tag{14} \]

it reduces to the well-known Cherenkov angle

\[ \cos \theta = \frac{1}{vn}. \tag{15} \]

As discussed in the previous chapter there are two cases:

\[ m_f \geq m_i \Rightarrow n > 1, \quad v > 1/n \quad \text{CR} \]
\[ m_f < m_i \Rightarrow \text{decay } \nu_i \rightarrow \nu_f + \gamma \text{ in the medium.} \]

For an upper limit on \( m_f \) in the first case see [11]. Only in the first case one has genuine CR but in the limit (14) both cases become indistinguishable. Since we are interested in \( m_i, m_f \to 0 \) this feature is inherent in our discussion. Therefore, for simplicity, we will only talk about CR
radiation in the following. For TR the situation is analogous and the distinction between the two cases will not be mentioned there anymore.

In the limit of vanishing masses the transition rate for CR (summed over $s_f$ and the photon polarizations) is given by the simple formula

$$dR = \frac{\alpha}{4m_e^2} \frac{\mu^2 + d^2}{\mu_B} \left(n - \frac{1}{n}\right)^2 \omega^2 d\omega$$  \hspace{1cm} (16)$$

where $m_e$ is the electron mass and $\alpha$ the fine structure constant. Note that $dR$ is independent of $E$ and the initial polarization $s_i$.

Let us now apply (16) to the case of solar neutrinos. If $p_a$ is the probability to find $\nu_a$ ($a = e, \mu, \tau$) in the solar neutrino flux then actually

$$\mu_{\text{eff}}^2 = \sum_{a,b} (|\mu_{ba}|^2 + |d_{ba}|^2)p_a$$  \hspace{1cm} (17)$$
is probed in CR where $\mu_{ba}, d_{ba}$ are transition moments from neutrino flavour $a$ to $b$. (Also in TR $\mu_{\text{eff}}$ is the relevant quantity.) Since we need a large volume of a medium with $n > 1$ it is obvious to take water with $n \simeq 1.335$ in the range of visible light $\bar{\omega}_1 = 1.7 \text{ eV} < \bar{\omega} < \bar{\omega}_2 = 3 \text{ eV}$ (water is practically opaque in the ultraviolet region) and with the Cherenkov angle $41,5^0$ for $v = c$. Then the number of photons emitted by the neutrinos in the solar neutrino flux $I$ during a time interval $T$ and in an observation volume $V$ is given by

$$N_\gamma = \frac{TIV}{c} \int_{\omega_1}^{\omega_2} dR = \frac{T}{1 \text{ day}} \frac{I}{6 \cdot 10^{10} \text{cm}^{-2}\text{s}^{-1}} \frac{V}{1 \text{km}^3} \left(\frac{\mu_{\text{eff}}}{10^{-10} \mu_B}\right)^2 \cdot 46.4.$$  \hspace{1cm} (18)$$

In equ. (18) the number of photons has been normalized to the solar neutrino flux as obtained by the solar standard model[13].

Finally we want to estimate CR of ultrarelativistic neutrons with $\mu_n \simeq -10^{-3} \mu_B$. If one has a beam with $i_n$ neutrons per second then

$$N_\gamma \sim 10^{-15} i_n T \frac{a}{1 \text{ cm}}$$  \hspace{1cm} (19)$$

where $a$ is the thickness of the medium.

Both results (18,19) show that it is exceedingly difficult to measure CR of neutral particles.

### 4 Transition Radiation

We consider a slab of dielectric medium situated in space between the planes $z = -a/2$ and $z = a/2$. Thus we have two surfaces. The wave vectors of plane wave solutions of Maxwell’s equations can be parametrized by

$$\vec{k} = \omega \begin{pmatrix} \sin \alpha \cos \phi \\ \sin \alpha \sin \phi \\ \cos \alpha \end{pmatrix}, \quad \vec{k}_m = n \omega \begin{pmatrix} \sin \beta \cos \phi \\ \sin \beta \sin \phi \\ \cos \beta \end{pmatrix}$$  \hspace{1cm} (20)$$
corresponding to vacuum and medium, respectively. In addition there are the wave vectors $\vec{k}_r, \vec{k}_{mr}$ corresponding to reflected waves outside and inside the dielectric slab, respectively. By Snell’s law we obtain

$$\sin \alpha = n \sin \beta$$  \hspace{1cm} (21)$$
functions express momentum conservation. For the full expression of
One can show that this leads to the following consequences:

\[ \vec{E}_{II}(\vec{k}, x) = e^{-i\omega t} \cdot \begin{cases} \vec{e}_{II} e^{i\vec{k} \cdot \vec{x}} + \vec{e}_{II} a_{II}^I e^{i\vec{k}_{II} \cdot \vec{x}}, & z < -a/2 \\
\vec{e}_{II} a_{II}^I e^{i\vec{k}_{II} \cdot \vec{x}}, & z > a/2. \end{cases} \] (22)

For \( \vec{E}_{II}(\vec{k}, x) \) and the exact definitions of the polarization vectors and the coefficients in \( \vec{E}_{II}(\vec{k}, x) \) obtained by continuity conditions see ref. [12]. For simplicity we have only discussed \( k_z > 0 \).

The computation of the probability \( W \) for the process (10) is extremely simplified by the use of the relation

\[ i(E_i - E_f) \int d^4x A_\mu(x) \langle p_f, s_f | J_{em}^\mu(x) | p_i, s_i \rangle = \int d^4x \vec{E}(x) \cdot \langle p_f, s_f | J_{em}^\mu(x) | p_i, s_i \rangle. \] (23)

With eqn. (11) the expressions

\[ \mathcal{E}_f(\vec{k}, q_z) = \int dz e^{-iq_z z} \left. \vec{E}_f(\vec{k}, x) \right|_{x^0 = x^1 = x^2 = 0} \] (24)

appear in the probability amplitudes by \( z \)-integration whereas in \( x \) and \( y \)-directions the \( \delta \)-functions express momentum conservation. For the full expression of \( d^6W \), eqn. (24) and further computational details we refer the reader again to ref. [12]. We now concentrate on the limit \( m_i, m_f \to 0 \) and on the case where the incident particle flux is orthogonal to the dielectric layer. One can show that this leads to the following consequences:

- The TR does not depend on the initial polarization \( s_i \) of the particle.
- Only photon polarization \( II \) contributes.
- The situation is invariant under rotation around the \( z \)-axis and therefore the differential probability of (10) is independent of the angle \( \phi \).

With \( \theta \) being the angle between \( \vec{k} \) and the \( z \)-axis one obtains the rather simple result [12]

\[ d^2W = \sum_{\eta=\pm 1} \frac{\sin^3 \theta d\theta \omega^3 d\omega}{2\pi^2} \frac{E}{P (\mu^2 + d^2)} \left( n - \frac{1}{n} \right)^2 \cdot \frac{1}{q_z^2 - k_z^2} \left| S_{II}^\eta (q_z - \omega - k_{mz}) + S_{II}^\eta (q_z - \omega + k_{mz}) \right|^2 \] (25)

with

\[ q_z = E - \eta P, \quad P = (E^2 - 2E\omega + \omega^2 \cos^2 \theta)^{1/2}, \]

\[ S_{II}^\eta = a_{II}^m \frac{\sin \frac{\eta}{2} (k_{mz} - q_z)}{k_{mz} - q_z}, \quad S_{II}^\eta = a_{II}^m \frac{\sin \frac{\eta}{2} (k_{mz} + q_z)}{k_{mz} + q_z} \] (26)

\( \eta = \pm 1 \) corresponds to forward and backward scattering, respectively, of the fermion.

Dielectric media usually become transparent for photons in the X-ray range. There one has a simple expression for the dielectric constant, namely \( \varepsilon = n^2 = 1 - \omega_p^2/\omega^2 \) with \( \omega_p \) being
the plasma frequency. In the following we will take \( \omega_p = 20 \) eV of polypropylene as a typical example. Furthermore, the limit \( \omega \gg \omega_p \) and a realistic thickness \( a \) of the dielectric layer leads to \( \omega \gg 1 \) (e.g. \( a \omega \approx 10^6 \) for \( a = 0.01 \) mm and \( \omega = 20 \) keV). It is easy to check that now only small angles \( \theta \) contribute and that the following approximations can be made:

\[
|e_m^I| \to 1, \quad a_{mr}^I \to 0,
\]

\[
q_z \simeq \omega + \frac{\omega^2 \theta^2}{2(E - \omega)}, \quad k_{mz} - q_z \simeq -\frac{\omega}{2} \left( \frac{\omega_p^2}{\omega} + \frac{E}{E - \omega} \theta^2 \right)
\]

(27)

and

\[
dW \simeq \frac{(\mu^2 + d^2)}{2\pi^2} \left( \frac{\omega_p}{\omega} \right)^4 \omega^3 d\omega \int_0^\infty d\theta \theta \left\{ \frac{\sin \frac{a\omega}{\omega} \left[ \left( \frac{\omega_p}{\omega} \right)^2 + \frac{E}{E - \omega} \theta^2 \right]}{\frac{\sin y}{y}} \right\}^2 = \frac{(\mu^2 + d^2) a}{4\pi^2} d\omega \omega^2 E - \omega \left( \frac{\omega_p}{\omega} \right)^4 \int_{y_0}^\infty \left( \frac{\sin y}{y} \right)^2
\]

with \( y_0 = \frac{1}{4} a\omega \left( \frac{\omega_p}{\omega} \right)^2 \).

To apply (28) we consider the numerical example \( a = 0.01 \) mm and \( \hbar \omega_1 = 20 \) keV \( \leq \hbar \omega \leq \hbar \omega_2 \). Then with \( \omega_2 \gg \omega_1 \) and \( y_0 \simeq 0.25 \) we estimate the probability \( W \) for the emission of a photon in the energy range \([\omega_1, \omega_2]\) when one fermion is crossing the slab by

\[
W = \int_{\omega_1}^{\omega_2} dW \sim \frac{1}{8\pi} (\mu^2 + d^2) \frac{\omega_p^4 a}{\omega_1} \sim 10^{-12} \frac{\mu^2 + d^2}{\mu_B^2}.
\]

(29)

This very small probability in conjunction with the upper limit on the neutrino MM and EDMs cannot be overcome by the large solar neutrino flux. Taking \( \mu_{\text{eff}}/\mu_B = 10^{-10} \) and \( 10^5 \) foils with \( 10 \) m\(^2\), results in an order of magnitude of \( 10^{-4} \) photons per year. A larger photon yield can only be expected for a sizeable fraction of \( \tau \) neutrinos in the solar neutrino flux with \( \mu_{\tau\tau} \) close to the experimental upper limit [4].

In the case of ultrarelativistic neutrons with an optimistic flux of \( 10^{15} \) particles per second and \( 10^3 \) foils one expects around one photon per second.

5 Conclusions

Finally we want to present a short summary of our results and stress some physical aspects of the considered radiation mechanism.

- In this report we have considered CR and TR of ultrarelativistic neutral spin 1/2 particles with MM and EDM. Because of the nature of the considered electromagnetic interaction, a spin flip of the fermion must occur. We want to stress therefore that only the quantum theoretical calculation gives a correct result whereas the classical calculation gives zero in the limit of vanishing particle masses [14].

- We have discussed two ranges of photon energy, visible light and frequencies much larger than the plasma frequency. Between the two ranges, media are usually non-transparent. Thus we can summarize the situation in the following way.

\[
- \gamma \text{ in keV range: } \varepsilon = 1 - \frac{\omega_p^2}{\omega^2} < 1 \Rightarrow \text{only TR possible.}
\]
– γ in optical range: \( \varepsilon = n^2 > 1 \) and finite dielectric slabs \( \Rightarrow \) The above described computation of TR gives the exact result for radiation but for \( a \to \infty \) (i.e. \( a \omega \gg 1 \)) formula (24) develops the form \( \text{ad} R \) of CR with the angle of emission \( \theta' \) given by \( \cos \theta' = \sqrt{2 - n^2} \). This angle is obtained from the Cherenkov angle \( \cos \theta = 1/n \) by taking into account refraction at the surface of the layer. Thus we effectively have CR radiation in the optical range and we confirm that for \( n > 1 \) equ. (25) is the general expression for radiation.

• CR and TR are independent of the initial polarisation \( s_i \). Therefore, also spin flipped neutrinos would be counted in the solar neutrino flux.

• Unfortunately, for neutrinos the photon yield is very low because of the upper bounds on their MMs and EDMs.

• There might be some hope to see such effects for neutrons.

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\footnote{We have checked this only semiquantitatively.}
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