On the dynamics of air craters observed on droplet surface during impact on immiscible liquid pool

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(Received xx; revised xx; accepted xx)

We study drop impacts on immiscible liquid pools and investigate the formation of air craters on droplet surface during penetration through the pool using experimental and theoretical analysis. We attribute the formation of air craters to the sudden deceleration of the droplet. Viscous force is the primary contributor to the rapid deceleration that leads to the formation of air craters on the droplet’s surface. The droplet response to the external impulsive decelerating force induces oscillatory modes on the droplet surface exposed to the air forming capillary waves that superimpose to form air craters of various shapes and sizes. A critical Weber number based on the diameter exists ($We_c = 10$), beyond which significant depth air craters during the penetration process are detectable. We introduce a non-dimensional parameter ($\Gamma$) that is the ratio of drag force to the capillary force acting on the droplet. We show that droplets forming air craters of significant depths have $\Gamma > 1$. Further, we also demonstrate that Legendre polynomials can locally approximate the central air crater jet profile. We also decipher that the air crater response time scale ($T$) varies as the square root of impact Weber number ($T\sim We^{1/2}$).

Key words: Authors should not enter keywords on the manuscript, as these must be chosen by the author during the online submission process and will then be added during the typesetting process (see http://journals.cambridge.org/data/relatedlink/jfm-keywords.pdf for the full list)

1. Introduction

The study of drop impact physics began with the seminal works of Worthington in the late nineteenth century [Worthington 1877, Worthington & Cole 1897] and continues till date due to applications in various industries like manufacturing, printing, food-processing, bio-medical, and pharmaceuticals [Bolleddula et al. 2010, Pasandideh-Fard et al. 2002, Roy et al. 2019]. The early pioneering work of Worthington led to the discovery of various spatio-temporal physics during drop impacts on solids and liquids. Drop impacts on liquids generates aesthetically beautiful patterns like crowns, corona splash, and jets, to name a few [Gekle & Gordillo 2010, Gordillo & Gekle 2010] and are used in various artistic contexts and platforms. The mechanism underlying the beauty is multi-scale and multi-physics in origin and has attracted many scientists and engineers to study drop impacts on liquids theoretically, numerically, and experimentally [Rioboo

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A droplet impacting on a liquid media can result in a wide variety of phenomena like splashing, bouncing, coalescence, and formation of jets, corollas, and crowns depending on the impact parameter space characterized by various non-dimensional groups like impact Weber number, Reynolds number, Ohnesorge number, Bond number, Froude number to name a few (Thoroddsen et al. 2002; Yarin et al. 2017). The various non-dimensional numbers represent the ratios of various competing effects governing the dynamics of drop impacts. Several phenomena like the formation of flower-like pattern in the liquid film, entrapment of air, generation and propagation of ejecta sheet widen the landscape of possibilities of drop impact physics on liquids (Thoroddsen 2002; Zhang et al. 2012; Marcotte et al. 2019). During the impinging process, just prior to impact, a thin lubricating air layer gets entrapped underneath the droplet (Thoroddsen et al. 2003; Bartolo et al. 2006). The entrapped air between the drop and the liquid must be displaced/drained out before proper contact between the drop and the pool. The pressure in the lubricating air layer is inversely proportional to the air layer thickness and hence increases monotonically till the entrapped air pressure reaches the capillary pressure of the droplet (Roy et al. 2022). The excess pressure above the capillary pressure causes a dimple to form just beneath the impacting droplet. The air layer ruptures at a point leading to the first local contact of the drop and the formation of entrapped air bubbles. The formation of air bubbles in drop impact systems can be detrimental to specific technologies such as printing, coating, and several cooling applications (Aziz & Chandra 2000; Yarin et al. 2017; Aksoy et al. 2020). However, the entrapment of air bubbles is a boon to aquatic life forms since these entrapped air bubbles are medium of gaseous exchange between the atmosphere and the water bodies that sustain marine life forms (Woolf et al. 2007). For impact on similar liquids, after the initial entrapped air layer rupture phase, the droplet forms an air crater in the liquid layer/pool, collapsing to form Worthington jets. Gekle and Gordillo studied the Worthington jet formed during the impact of a circular disc on water using detailed boundary-integral simulations and analytical modeling (Gekle & Gordillo 2010; Gordillo & Gekle 2010). They discovered that the flow structure inside the jet could be divided into three regions; the acceleration region, ballistic region, and tip region. Majority of the previous study on drop impact on liquids have focused on impacts on similar liquids (Shetabivash et al. 2014; Yarin et al. 2017; Castillo-Orozco et al. 2015; Castrejón-Pita et al. 2016; Hasegawa & Nara 2019). However, literature of drop impact on immiscible pools are relatively sparse (Yakhshi-Tafti et al. 2010; Dhuper et al. 2021; Che & Matar 2018; Minami & Hasegawa 2022). Drop impact on immiscible liquid pools is very important and is found in various industrial, engineering, and natural systems. Droplet interactions in immiscible systems are inherently different, as mentioned in some of the literature available (Che & Matar 2018). In general, drop impact on liquids produces Worthington jets (Worthington & Cole 1897) on the liquid pool formed due to the collapse of an air crater in the liquid pool. However, we have found that the air craters formed in immiscible impact systems are significantly different from those reported in the literature for miscible liquids. Primarily, the air crater on the surface of the impacting droplet resembles central air craters found in drop impact on superhydrophobic substrates (Yamamoto et al. 2018) at low impact Weber number. The air craters on the drop surface can be understood based on the response dynamics of the droplet subjected to external forces, as was demonstrated by...
Drop impact on immiscible liquid pool

Figure 1. Schematic of the experimental setup used for high-speed imaging (not to scale). The components of the setup are labelled using upper-case alphabets as mentioned: (A) Syringe pump, (B) Syringe and Hypodermic Needle, (C) Stroboscopic high intensity light source, (D) Impacting water droplet, (E) Acrylic container, (F) Silicone oil pool, (G) Computer with image acquisition software (H) High speed camera.

Harper et al. and Simpkins et al. [Harper et al. 1972, Simpkins & Bales 1972]. Recent studies have shown various topology of air crater/singular jet formation in immiscible impacts [Yang et al. 2020]. However, the detailed mechanism of the jets produced on the surface of the droplets is not well understood and remains elusive.

In this work, we study the role of impact Weber number ($We = \frac{\rho_w V_0^2 2R_0}{\sigma_{aw}}$) on the formation of air craters on the surface of impacting water droplet impinging on an immiscible liquid pool of silicone oil with particular focus on the mechanism of air crater formation using experimental and theoretical analysis. The remaining text is organized as follows. In section 2, we provide the details of the experimental setup and data processing algorithms. Section 3 outlines the results and discussions with various subsections. Subsection 3.1 provides a global overview of the important results obtained. Subsection 3.2 discusses droplet penetration dynamics and the scales of the various forces acting on the droplet. Subsection 3.3 details ways of characterizing droplet deformation from the spherical shape. Subsection 3.4 provides a criterion for forming air craters of significant depth. Subsection 3.5 computes the center of mass velocity of the droplet. Section 3.6 outlines the details of droplet response / Air crater and jet characteristics. We conclude with the results and discussions in conclusion (section 4).
2. Materials and Methods

Fig. 1 shows a schematic representation of the experimental setup. The various components are labelled as (A) syringe pump (New Era Pump Systems, NE-1010), (B) syringe and a hypodermic needle, (C) stroboscopic light source, (D) water droplet, (E) acrylic container, (F) silicone-oil pool, (G) computer with image acquisition software and an (H) high-speed camera (Photron SA5) with Tokina and Navitar zoom lens. De-ionised water droplet of radius \( R_0 = 1.1 \text{mm} \) was used as the impacting liquid droplet. Silicone oil was used as the liquid pool, and the pool depth was maintained constant at \( H_p \sim 5 \text{mm} \). The kinematic viscosity of the pool was \( \nu_s = 350 \text{cSt} \) and density was \( \rho_s \sim 970 \text{kg/m}^3 \). All experiments were conducted at room temperature of \( T \sim 298 \text{K} \). The droplet impacts energy, and hence impact Weber number was varied by changing the free fall height of the droplet from the free surface of the liquid pool using a vertical linear stage. We explored the air crater dynamics in the impact Weber number regime of \( \text{We} \sim 2 - 145 \). The high-speed imaging was performed at 10000 frames per second and with a spatial resolution of 1.5 micrometers per pixel. The geometrical and kinematic quantities like droplet size and velocity were computed from the high-speed videos using Image processing techniques in ImageJ [Schneider et al. 2012] and Python [Van Rossum & Drake 2009]. The geometrical parameters from the images that were taken from an inclination were corrected using appropriate trigonometric transformation [Che & Matar 2018].

3. Results and Discussions

3.1. Global Overview

We discover that the mechanism behind air crater formation is intricately related to the penetration dynamics of the impinging droplet through the liquid pool and depends explicitly on the impact Weber number (Fig. 2). Fig. 2 shows the high-speed snapshots of a water droplet impacting a silicone liquid pool at various impact Weber numbers (\( \text{We} = 4, 16, 65, 145 \)). The timestamps are in milliseconds, and the scale bars represent 500 \( \mu \text{m} \). The interaction between the silicone pool and water droplet occurs through the interplay of viscous, buoyancy, droplet weight, air-water, and water-oil surface tension forces. Beyond a critical impact Weber number \( \text{We}_c \sim 10 \), air craters of significant depths (crater depth comparable to droplet size) are formed. No significant air craters are observed during the penetration phase for impact Weber number smaller than the critical Weber number \( \text{We} < \text{We}_c \). The penetration time scale is largely increased due to the lubrication effect of the entrapped air layer formed under the droplet. The air layer rupture occurs at a time scale of the order of \( t_r \sim 2 \times 10^{-1} \text{s} \). The transient dynamics of the droplet (surface oscillation) occur early above the pool for \( t < t_r \). For \( t > t_r \), the droplet penetrates smoothly without any surface oscillation. The deceleration of the droplet is delayed due to the lubrication effect of the entrapped air layer formed under the droplet. The deceleration force during the penetration phase is of the order of \( F \sim \mathcal{O}(4 \times 10^{-4}) \text{N} \) for \( \text{We} = 4 \). The air craters formed for \( \text{We} > \text{We}_c \) occur due to the rapid deceleration experienced by the impacting droplet over a short period (impulsive deceleration). For impact Weber number larger than the critical Weber number \( \text{We} > \text{We}_c \), the air layer rupture time scale is of the order of \( t_r \sim \mathcal{O}(5 \times 10^{-4}) \text{s} \), about two orders of magnitude faster than for low Weber number \( \text{We} \sim 1 \). We have shown using scaling analysis that the force responsible for the sudden deceleration is the viscous drag. The viscous drag force is of the order of \( F_r \sim \mathcal{O}(1.5 \times 10^{-3}) \text{N} \) for \( \text{We} = 16 \) which is comparatively larger than the buoyancy/weight of the droplet (\( \mathcal{O}(5 \times 10^{-5}) \text{N} \)). Further, we deduce a criterion for significant air crater depths. We show that the ratio of stokes viscous drag pressure to capillary pressure
(\Gamma = \Delta p_s/\Delta p_c) is the parameter characterizing the formation and the size of air craters. \(\Delta p_s/\Delta p_c \sim 1\) corresponds to low Weber number states (\(We=4\)). The ratio \(\Gamma\) increases as we increase the impact Weber number. The shape of the air crater central jet depends on the response of the droplet subjected to impulsive retardation. We show that the shape of the central air jet can be approximated with a fourth-order Legendre polynomial. We observe that the air crater formation/response time scales for all Weber number greater than the critical lies within the capillary time scale of \(t_c \sim \sqrt{m/\sigma} \sim O(9 \times 10^{-3}\,s)\). However, a complete air crater retraction time scale \((T)\) is a monotonic function of impact Weber number \((T \sim We^{1/2})\). The crater retraction time scale increases with an increase in impact Weber number. When the air crater dynamics subsides, the droplet penetrates slowly under the influence of weight and buoyancy and attains an almost steady-state submerged configuration.

3.2. Droplet penetration dynamics (Scales of various forces acting on the droplet)

Fig. 3(a) depicts a schematic representation of an impacting droplet on a silicone oil pool. The oil pool is shown in light yellow. The schematic is drawn at a time instant \(t (t > t_r)\) greater than the air rupture time scale, and the droplet is at a partially submerged state. CM represents the centre of mass of the impinging droplet. The contact line of three different fluids (air, water, and oil) is marked as a white dotted line. The submerged portion/volume of the droplet immersed below the oil surface is quantitatively characterized and measured by the penetration depth \(h(t)\) and penetration width \(w(t)\) (Fig. 3(a)). We assume that the droplet deforms into an ellipsoid during the penetration process. The ellipsoid is characterized by the semi-major axis \(a(t)\) and the semi-minor axis \(b(t)\) respectively (Fig. 3(a)). The forces acting on the droplet are the weight \(F_{mg}\) downwards, buoyancy force \(F_b\) upwards, and the viscous drag force \(F_v\) upwards. The coordinate axis \(z\) is chosen to be positive downwards. From mass conservation of the impacting droplet we have

\[
\frac{4}{3} \pi R_0^3 = \frac{4}{3} \pi a^2(t)b(t) = m
\]
where $\rho_w$ is the density of the impacting droplet (water here), $R_0$ is the initial radius of the droplet and $m$ is the mass of the impacting droplet. For droplet radius of $R_0 = 1.1 mm$, the mass of the impinging water droplet is $m \sim O(5.58 \times 10^{-6} kg)$. Simplifying equation (3.1) we have

$$a^2(t)b(t) = R_0^3$$

The dynamical equation (Newton’s second law of motion) for the impinging liquid droplet is given by

$$F_{mg} - F_b(t) - F_v(t) = m \frac{dV_{CM}(t)}{dt} = ma_{CM}$$

with initial condition for $V_{CM}$ as

$$V_{CM}(t = 0) = V_0$$

where $V_{CM}$ is the centre of mass velocity, $V_0$ is the impact velocity. The time instant just before the penetration of the droplet is defined as $t = 0$. $F_{mg} = mg$ is the weight of the impacting droplet where $g = 9.81 m/s^2$ is the acceleration due to gravity, $F_b$ is the buoyant force on the droplet and $F_v$ is the viscous drag force on the droplet due to the interaction with the silicone oil pool. For $R_0=1.1 mm$, the weight of the droplet is of the order of $F_{mg} \sim O(5.47 \times 10^{-5} N)$. The submerged droplet volume in the silicone oil pool can be approximated by an ellipsoidal cap approximation (refer to Fig. 3(a))

$$v(t) = \frac{\pi a^2(t)h^2(t)}{3b^2(t)}(3b(t) - h(t))$$

where $h(t)$ is the depth of penetration of the droplet. The buoyancy force is the weight of the displaced silicone oil, and therefore $F_b(t)$ becomes

$$F_b(t) = v(t)\rho_s g = \frac{\pi a^2h^2\rho_s g}{3b^2}(3b - h)$$

Using equation (3.2) in (3.6) the buoyancy force becomes

$$F_b(t) = \frac{\rho_s g \pi R_0^3 h^2}{3b^2}(3b - h)$$

At $h = b$ (i.e. when the droplet (ellipsoid) is half submerged) the buoyancy force becomes

$$F_b = \frac{2}{3} \rho_s \pi R_0^3 g \sim O(2.65 \times 10^{-5} N)$$

Notice that the order of magnitude of buoyancy force is the same as the weight of the droplet, and the buoyancy force becomes important at long time scales to determine the maximum steady-state submerged depth of the droplet. During the early stage of impact ($t \sim 2 ms$) the semi-major axis is approximately equal to the semi-minor axis ($a \approx b$) and hence the order of magnitudes of $a$ and $b$ are $O(R_0)$. Equation (3.6) can therefore be approximated for computing the buoyancy force at early impact time as

$$F_b(t) = v(t)\rho_s g \frac{\pi h^2 \rho_s g}{3}(3R_0 - h)$$

The viscous force on the droplet $F_v$ can be approximated by the Stoke’s drag law [Moghisi & Squire 1981] as given by

$$F_v(t) \sim 3\pi \mu_s w(t) \frac{dh}{dt}$$

where $\mu_s$ is the viscosity of the silicone oil pool. Fig. 3(b) represents the scales of viscous force, buoyancy forces compared to the weight of the droplet plotted in a logarithmic...
3.3. Characterizing droplet deformation from spherical shape

In general as the droplet penetrates the liquid pool, the droplet undergoes deformation and evolves as an ellipsoid (Fig. 3(a)). We have measured and characterized the amount of deviation from a spherical shape by calculating the function $f(t)$ defined as

$$f(t) = h^2(t) + \frac{w^2(t)}{4} - 2R_0h(t)$$

(3.11)

$f(t) = 0$ represents a spherical undeformed droplet while the penetration occurs. Larger non zero values of $f(t)$ depicts the deviation from spherical symmetry. $f(t)$ has a dimension of square length and therefore can be represented in a dimensionless way.
Figure 4. (a) Snapshots depicting the penetration depth \( h(t) \) and width \( w(t) \) respectively for impact Weber number of 4, 16 and 145. (b) \( \Gamma(t) = \frac{\Delta p_s}{\Delta p_c} \) as a function of time \( (t_r < t < t_c) \) for impact Weber number of 4, 16 and 145 respectively.

by normalizing with respect to \( R_0^2 \). Equation (3.11) therefore becomes

\[
\frac{f(t)}{R_0^2} = \frac{h^2(t)}{R_0^2} + \frac{w^2(t)}{4R_0^2} - 2\frac{h(t)}{R_0^2} - 2h(t)R_0
\]

(3.12)

Fig. 3(c) and 3(d) shows the evolution of \( f(t)/R_0^2 \) plotted as a function of time using equation (3.12) for various impact Weber number \( (We = 4, 16, 145) \). It can be inferred from the variation of \( f(t) \) that for low Weber numbers \( (We \sim 4) \), the droplet shape while penetration is very close to spherical. However, increasing the impact Weber number \( (We \sim 16, 145) \) the droplet deformation is substantial as can be observed by the non-zero values of \( f(t)/R_0^2 > 1 \). Regardless of the impact Weber number, beyond the capillary time \( t_c \), the droplet penetrates, maintaining a spherical shape. It could be inferred that the deviation from the spherical shape due to penetration is a short time scale phenomenon \( (t < t_c) \). The dominant force experienced by the droplet on penetration is the viscous force (refer to Fig. 3(b)). We observe the formation of significant air craters due to sudden viscous drag acting on the droplet beyond critical impact Weber number of 10. Fig. 4(a) depicts the geometrical characteristics of the deforming droplet/air craters formed during penetration for three different impact Weber numbers equal to 4, 16, and 145. We observe that significant air craters does not form for low Weber number \( (We \sim 4 \text{ shown here}) \) due to delayed air layer rupture \( (t_r \sim O(2 \times 10^{-1} \text{s})) \). The impulsive nature of the drag force does not manifest during the penetration process due to delayed air layer rupture, and the viscous force becomes gradual for \( We < We_c \). Air craters of significant shape and size are observed for larger Weber numbers \( (We \sim 16 \text{ and } We \sim 145 \text{ shown here}) \).

### 3.4. Air crater formation criterion

The air craters observed for \( We > We_c \) (refer to Fig. 4(a)) occur due to impulsive viscous drag force acting on the droplet across the immersed surface/volume in the
silicone oil bounded by the contact line (refer to the schematic shown in Fig. 3(a)). We introduce a non-dimensional parameter $\Gamma(t)$ as the ratio of the stokes drag pressure to capillary pressure across the air-water interface in the air crater formation time scale ($t_r < t < t_c$). The parameter can be represented as

$$\Gamma(t) = \frac{\Delta p_s}{\Delta p_c} \sim \frac{\mu_s w(t)V_0 R_0}{\sigma_{aw} w^2(t)}$$

(3.13)

where $\Delta p_s \sim \mu_s w(t)V_0 / w^2(t)$ is the pressure force due to the stokes drag and $\Delta p_c \sim \sigma_{aw} / R_0$ is the capillary pressure across the air-water interface. $\Gamma(t) \sim 1$ signifies that the impulsive drag is almost balanced out by the capillary force acting across the air-water interface of the droplet.

Detectable air craters during the penetration process can only be observed if the viscous force becomes comparable and overcomes the capillary force across the droplet interface, which corresponds to $\Gamma(t) > 1$. We also show that for $\Gamma(t) \leq 1$ for $t < t_c$, no significant air craters are observed, as is the case for low impact Weber number. Increasing Weber number increases $\Gamma(t)$ and hence the ratio of Stokes viscous force to capillary force across the air-water interface of the droplet. Equation (3.13) can be rewritten in terms of the capillary number $Ca = \mu_s V_0 / \sigma_{aw}$ as

$$\Gamma(t) = \frac{\Delta p_s}{\Delta p_c} \sim Ca \frac{R_0}{w(t)}$$

(3.14)

Figure 4(b) represents $\Gamma(t) = \Delta p_s / \Delta p_c$ as a function of time for $t_r < t < t_c$ for Weber numbers 4, 16 and 145. It can be inferred from Fig. 4(b) that significant air craters are
formed for $\Gamma > 1$ where the impulsive viscous forces become larger than the capillary force across the air-water interface. For small Weber number, $We < We_c = 10$, $\Gamma \lesssim 1$ signifying that the viscous forces are smaller than the capillary force across the air-water interface due to surface tension.

3.5. Evaluating the centre of mass velocity ($V_{CM}$)

We observe that the order of magnitude of air crater jet velocity is comparable to the centre of mass velocity of the droplet ($V_{CM}$). Using equation (3.9) and (3.10) for the buoyancy and viscous terms respectively in equation (3.3), the centre of mass acceleration of the droplet can be expressed as

$$\frac{dV_{CM}(t)}{dt} = \frac{1}{m} \left( mg - \frac{\pi h^2 \rho_s g}{3} (3R_0 - h) - 3\pi \mu_s w(t) \frac{dh}{dt} \right)$$ (3.15)

Integrating equation (3.15) we have

$$\int_{V_0}^{V_{CM}(t)} \frac{dV_{CM}(t)}{dt} dt = \int_0^t \frac{1}{m} \left( mg - \frac{\pi h^2 \rho_s g}{3} (3R_0 - h) - 3\pi \mu_s w(t) \frac{dh}{dt} \right) dt$$ (3.16)

Simplifying equation (3.16) we have

$$V_{CM}(t) = V_0 + gt - \frac{\pi \rho_s g}{3} \int_0^t h^2 (3R_0 - h) dt - 3\pi \mu_s \int_0^t w(t) \frac{dh}{dt} dt$$ (3.17)
Figure 7. (a) High speed snapshots depicting the evolution of air layer rupture forming a bubble during penetration through silicone oil for impact Weber number \( We = 145 \). The timestamps are in milliseconds and the scale bar represents 500\( \mu \)m. (b) Temporal evolution of kinematic parameters like penetration width and depth. (c) Temporal evolution of the various forces acting on the impacting droplet. The resultant of the various forces is also shown in red. Refer to supplementary Movie 3 for \( We = 145 \) here.

Notice that the acceleration due to gravity term \( gt \) and the buoyancy term (third term on the right hand side) is negligible compared to the viscous term (last term on the right hand side) for \( t_r < t < t_c \). The last term representing the viscous forces is the most dominant for \( t_r < t < t_c \).

Fig. 5, Fig. 6, and Fig. 7 show various kinematic and dynamic quantities for impact Weber numbers of 4, 16, and 145, respectively. Fig. 5(a) depicts the high-speed snapshots during the penetration process for an impact Weber number of 4. The penetration dynamics for low Weber numbers are drastically different as the droplet, and air layer interaction is significant. The air layer rupture and formation of a subsequent bubble can be observed in Fig. 5(a). Further, as the droplet penetrates through the silicone oil pool, the droplet shape is close to spherical (refer to Fig. 3(c) and Fig. 3(d)). Fig. 5(b) shows the temporal evolution of submerged droplet normalized width \( (w(t)/R_0) \) and normalized depth \( (h(t)/2R_0) \) respectively. Using the experimental value of \( w(t) \) and \( h(t) \), we have calculated the resultant force acting on the droplet. All the individual forces (\( F_{mg}, F_b \) and \( F_v \)) along with the resultant force (in red \( F_{mg} + F_b + F_v \)) is plotted in Fig. 5(c). Notice that during the early time of penetration for \( t < t_c \), the dominant force is only the viscous drag. In general, we observe that the resultant force is almost entirely due to viscous drag with buoyancy and weight almost balancing each other as time increases. Using the resultant external forces acting on the droplet (Fig. 5(c)), the centre of mass velocity \( (V_{CM}) \) can be evaluated using equation (3.17). We evaluate centre
Figure 8. (a) Comparison of the resultant force $\Sigma F(t)$ in $mN$ acting on the droplet during the penetration process as a function of time $t$ in $ms$ for impact Weber number ($We$) of 4, 16 and 145 evaluated using the right hand side of equation (3.15). (b) Comparison of the centre of mass velocity of the droplet ($V_{CM}$) in $m/s$ plotted as a function of time $t$ in $ms$ for impact Weber number ($We$) of 4, 16 and 145. (c) A high speed snapshot depicting the crater depth $h_c$. (d) Crater depth ($h_c$) evolution as a function of time in $ms$.

of mass velocity ($V_{CM}$) for early penetration time ($t < t_c$) as the approximation under which equation (3.17) is derived becomes increasingly accurate for small time duration. The effect of the impact Weber number can be understood by analyzing the scale of $F_v$ as a function of the Weber number. It can be inferred from Fig. 3(b) that on increasing the impact Weber number, the viscous force $F_v$ changes significantly. The resultant force on the droplet is approximately equal to that of the viscous force. Therefore on increasing the impact Weber number, the ratio of stokes viscous pressure to capillary pressure of the droplet $\Gamma$ increases resulting in the formation of air crater/reversed jets of various shapes and sizes (refer to Fig. 4(a) and Fig. 4(b)). Fig. 6(a) shows a sequence of high-speed snapshots showing the evolution of the air crater for $We = 16$. Fig. 6(b) plots the evolution of normalized penetration depth and width of the impacting droplet for impact Weber number of 16. The individual and resultant forces acting on the droplet are shown in Fig. 6(c). Fig. 7(a) shows the air crater formation and evolution for impact Weber number of 145. The crater formed is significantly different compared to the low Weber number impact. For Weber number of 16, a single air jet is formed due to the capillary waves interfering. Whereas for Weber number of 145, a cascade of capillary waves is formed, forming a bigger intricate air crater. The air crater normalized depth ($h(t)/2R_0$) and normalized width ($w(t)/R_0$) is plotted as a function of time in Fig.
Figure 9. (a) Image depicting the profile of the reversed jet formed during air crater formation/collapse at impact Weber number of 16. (b) Comparison of the experimental profile of the central jet with 4-th degree Legendre polynomial. (c) Image depicting the profile of the reversed jet formed during air crater formation/collapse at impact Weber number of 145. (d) Comparison of the experimental profile of the central jet with 4-th degree Legendre polynomial. (e) Experimental and theoretical comparison of air crater time scale \( T \) as a function of impact Weber number \( (We) \) based on equation (3.22).

Using the experimental air crater geometrical parameters, the individual and the resultant force acting on the impacting droplet were calculated and are shown in Fig. 7(c). The resultant force \( \Sigma F(t) \) acting on the impinging droplet during the penetration process is plotted as a function of time for various impact Weber number in Fig. 8(a) for comparison purposes \( (We = 4, 16, 145) \). The centre of mass velocity \( (V_{CM}) \), the solution for equation (3.17) is plotted in Fig. 8(b) for various impact Weber numbers. The centre
of mass velocity decreases as the droplet experiences retarding forces as it penetrates the liquid silicone oil pool. We can further observe that the centre of mass velocity shows a damped oscillatory response for \( t \sim t_c \). Fig. 8(c) shows a typical air crater geometry and its characteristics scale (air crater depth \( h_c \)) formed on the surface of an impacting droplet. The evolution of air crater depth \( h_c \) as a function of time \( t \) is plotted in Fig. 8(d) for impact Weber numbers of 16, 65 and 145. Note that the initial slope of \( h_c \) vs. \( t \) plot is ordered according to the centre of mass velocity for various Weber numbers, respectively. \( We = 145 \) has the highest initial slope followed by \( We = 65 \) and \( We = 16 \).

3.6. Transient Droplet Response / Air Crater and jet characteristics

The air crater/jets are formed due to the transient response of the droplet to impulsive forces acting on the droplet. The sudden deceleration of the droplet induces droplet oscillation that causes air craters of various shapes and sizes to form. Further, relative to the droplet the external force acting could also be formulated based on the relative effect of the surrounding air field. The transient response of the droplet surface based on sudden retardation can be characterized by surface displacement \( \eta = r(\mu, t)/R_0; \) where \( r(\mu, t) \) is the radial location of the droplet interface at any angular coordinate \( \theta \) at time \( t \) (refer to Fig. 3(a)) and \( R_0 \) is the initial droplet radius. We closely follow this part of the analysis based on the work of Harper et al. and Simpkins et al. \cite{Harper1972, Simpkins1972}. The perturbation result for the surface displacement based on a linear theory proposed by Harper et al. \cite{Harper1972} is given as

\[
\eta = 1 + \epsilon \sum_{n=0}^{\infty} \frac{n(2n+1)}{4\beta_n^2} C_n P_n(\mu)[\cos(\beta_n t) - 1] \tag{3.18}
\]

where \( \epsilon = \rho_a/\rho_w \) is the density ratio of air to water characterizing the amplitude of the perturbation, \( n \) is a positive integer and \( C_n \) are the weighting coefficients given by,

\[
C_n = \int P_e^{(0)} P_n(\chi) d\chi \tag{3.19}
\]

where \( \chi = \cos \psi \). \( P_n(\mu) \) represents the \( n \)-th order legendre polynomials where \( \mu = \cos \theta \) and

\[
\beta_n = \left( \frac{(n-1)n(n+2)}{4 We R_0} \right)^{1/2} \tag{3.20}
\]

where \( \beta_n \) denotes the frequency of \( n \)-th Legendre mode and \( We R_0 \) denotes the Weber number defined with respect to the initial radius \( R_0 \) (\( We = 2 We R_0 \)). \( P_e^{(0)} \) represents the leading term of the external pressure \( P_e \) in the perturbation expansion with \( \epsilon \) as the perturbation parameter acting on the droplet during the penetration process.

\[
\beta \sim We^{-1/2} \tag{3.21}
\]

Refer to Fig. 3(a) for the definition of various angular coordinates like \( \theta \) and \( \psi \). Note that near the droplet top surface that corresponds to \( \psi = 0 \) and \( \theta = \pi \), the droplet shape approximates that of a Legendre polynomial locally. Fig. 9(a) and 9(b) show the comparison of the air jet profile at the centre with the fourth-order Legendre polynomial for \( We = 16 \). Similar comparison is shown in Fig. 9(c) and 9(d) for \( We = 145 \). In principle, the combined air crater deformation can be thought of as the superposition of various Legendre modes. The period of half oscillation of the air jets scales as

\[
T \sim \beta^{-1} \sim We^{1/2} \tag{3.22}
\]

Therefore, from equation (3.22), we observe that the period of air crater dynamics
increases monotonically with the impact Weber number. As inferred from Fig. 9(e), we observe the corresponding monotonic behavior experimentally.

4. Conclusion

In conclusion, we unearthed the role of the Weber number on the formation of air craters detected on the outer surface of the droplet, impinging an immiscible liquid pool using high-speed imaging and scaling analysis. We found that below a critical Weber number $We_c = 10$, the penetration process was smooth without forming air craters/jets. The smooth penetration was due to the delayed air layer rupture time scale ($t_r \sim O(2 \times 10^{-1} s)$). For impact Weber number larger than the critical Weber number ($We > We_c = 10$), the air layer rupture time scale is reduced by three orders of magnitude ($t_r \sim O(5 \times 10^{-4} s)$). The droplet experiences an impulsive force that results in the formation of capillary waves, causing air craters to form on the air-water interface. The major cause of the air crater formation is the sudden deceleration of the impinging droplet. The retardation of the droplet is primarily due to viscous forces and is a monotonically increasing function of impact Weber number. We discovered that significant depth air craters during penetration are formed when the ratio of viscous force and capillary force exceeds unity ($\Gamma = \Delta p_s/\Delta p_c > 1$). Further, the air crater time scale is also a monotonically increasing function of Weber number ($T \sim We^{1/2}$). The air jet profiles, in general, are the superposition of various Legendre polynomials.

Supplementary movie captions

Movie 1: Supplementary movie for impact Weber number ($We = 4$). Recorded at 10000 FPS, playback speed 100FPS.

Movie 2: Supplementary movie for impact Weber number ($We = 16$). Recorded at 10000 FPS, playback speed 10FPS.

Movie 3: Supplementary movie for impact Weber number ($We = 145$). Recorded at 10000 FPS, playback speed 10FPS.

Acknowledgement

The authors are thankful for the funding received from the Defence Research and Development Organization (DRDO) Chair Professorship.

Declaration of Interests

The authors declare no conflict of interest.
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