EFFECTS OF KERR STRONG GRAVITY ON QUASAR X-RAY MICROLENSING

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ABSTRACT

Recent quasar microlensing observations have constrained the sizes of X-ray emission regions to be within about 10 gravitational radii of the central supermassive black hole. Therefore, the X-ray emission from lensed quasars is first strongly lensed by the black hole before it is lensed by the foreground galaxy and star fields. We present a scheme that combines the initial strong lensing of a Kerr black hole with standard linearized microlensing by intervening stars. We find that X-ray microlensed light curves incorporating Kerr strong gravity can differ significantly from standard curves. The amplitude of the fluctuations in the light curves can increase or decrease by ∼0.65–0.75 mag by including Kerr strong gravity. Larger inclination angles give larger amplitude fluctuations in the microlensing light curves. Consequently, current X-ray microlensing observations can under or overestimate the sizes of the X-ray emission regions. We estimate this bias using a simple metric based on the amplitude of magnitude fluctuations. The half-light radius of the X-ray emission region can be underestimated by up to ∼50% or overestimated by up to ∼20% depending on the spin of the black hole, the emission profile, and the inclination angle of the observer.

Key words: accretion, accretion disks – black hole physics – gravitational lensing: strong – quasars: general – X-rays: galaxies

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1. INTRODUCTION

It is well known that most active galactic nuclei (AGNs) are copious X-ray emitters. However, despite efforts for decades, the origin of this emission is still unclear. Standard AGN accretion disk theory predicts disk temperatures (∼10^9 K) too low to emit X-rays (∼10^9 K for hard X-rays). Instead, X-ray emission is generally believed to be generated by reprocessing of optical/UV disk photons by hot electrons in a corona above the accretion disk via unsaturated multiple inverse Compton scattering. The two-phase (cold disk plus hot corona) accretion disk model (Haardt & Maraschi 1991, 1993) predicts that the observed spectrum contains three components: a direct power-law component from the X-ray corona (Zdziarski et al. 1994), a reflection component from the disk (Guilbert & Rees 1988; Lightman & White 1988) with metal emission lines (in particular, the Fe Kα line; Fabian et al. 1989; Laor & Netzer 1989), and thermal radiation from the cold accretion disk (Shakura & Sunyaev 1973). Despite the general belief in the existence of the X-ray corona, fundamental questions remain unanswered: what is its size and geometrical structure, how are its hot electrons heated, and why does it radiate so much (∼10%) of the accretion luminosity?

A major difficulty in solving the AGN corona puzzle is that its small angular size makes it unresolvable by current telescopes. Many estimates of the corona size are based on variability arguments. The observed rapid X-ray variability (as short as a few hours) was important early evidence for small X-ray emission sizes. Recently, techniques based on a Bayesian Monte Carlo analysis method (Kochanek 2004; Poindexter & Kochanek 2010) together with accumulated high-quality data (e.g., Chen et al. 2012) are making quasar microlensing a very powerful tool in constraining corona geometry (Blackburne et al. 2006, 2011, 2013; Pooley et al. 2006; Morgan et al. 2008, 2012; Chartas et al. 2008a, 2008b, 2013; Moragne et al. 2008, 2010; Mosquera et al. 2013). These observations have conclusively constrained the quasar X-ray emission size to be of order ∼10r_g (r_g ≡ GM_{BH}/c^2, the black hole’s gravitational radius), much smaller than the optical emission size. Furthermore, Chen et al. (2011) detected energy-dependent X-ray microlensing in Q 2237+0305, and their results suggest that the hard X-ray emission might come from regions smaller than the soft X-rays. Based on Chandra monitoring data for six gravitationally lensed quasars, Chen et al. (2012) found that the rest-frame equivalent widths of the Fe Kα line are significantly higher than those measured in typical AGNs. This implies that the Fe Kα line emission is microlensed more strongly than the X-ray continuum, and therefore suggests that the iron line emitting region is more compact than the continuum emission region (Chen et al. 2012). If true, studying quasar X-ray emission (continuum or metal lines) probes the innermost regions of AGNs, the region where relativistic effects of the central black hole are important.

Quasar X-ray microlensing observations study the gravitational lensing of the X-ray emission by random foreground star fields in intervening galaxies. However, the traditional interpre-
tation of these observations assumes a flat spacetime for the source plane, and constrains the projected source area along the line of sight. When the X-ray source is within a few gravitational radii of the central supermassive black hole powering the AGN, both the flux profile and images of the X-ray emission are significantly altered (Bardeen et al. 1972; Cunningham 1975; Fabian et al. 1989; Chen & Halpern 1989; Laor & Netzer 1989; Laor 1991; Rauch & Blandford 1994; Bromley et al. 1997; Beckwith & Done 2004; Popović et al. 2006; Schnittman & Koliuk 2010; Abolmasov & Shakura 2012; Chen et al. 2013). In other words, a quasar's X-ray emission is gravitationally lensed by the central black hole at the very beginning of its trip to a distant observer, well before it is lensed by any foreground mass concentrations. We refer to the strong lensing by a Kerr black hole as “Kerr lensing” throughout this paper. Kerr lensing differs from standard linear lensing theory in a few important aspects (Chen et al. 2013), and we emphasize two of them here. First, the linear approximation is not valid since the X-ray sources are very close to the central black holes (a few \( r_g \)), and consequently, there is no simple lens equation which can be used to find the source position given the image position on the sky. Second, the redshifts produced by the gravity field of the central black hole and the relativistic motion of the X-ray source are very important for Kerr lensing in contrast to standard lensing theory in which the redshift comes solely from cosmology. Strong bending and redshift effects of Kerr lensing change the shape, size, and profile of the X-ray source, all of which are input parameters for the foreground microlensing. The constraints obtained from current quasar X-ray microlensing should consequently be expanded by including Kerr gravity. Because Kerr lensing depends on important parameters such as inclination angle, black hole spin, and the velocity flow of the X-ray sources, quasar microlensing might become an even more powerful tool in probing the innermost region of AGNs. We investigate these ideas in this paper.

In Section 2, we present a model combining Kerr lensing and microlensing. In Section 3, we investigate the effects of Kerr gravity on quasar X-ray microlensing via two concrete examples using our model. We conclude in Section 4. We assume a flat Friedman–Lemaître–Robertson–Walker cosmology with \( \Omega_m = 0.3 \) and \( \Omega_\Lambda = 0.7 \) throughout this paper.

### 2. A MODEL COMBINING KERR STRONG LENSING AND MICROLENSING

In order to investigate the effects of Kerr strong lensing on foreground microlensing, we need a model to combine Kerr lensing and standard microlensing. In this section, we first briefly discuss ray-tracing in Kerr spacetime and compare it with microlensing ray-tracing. Next we show that for normal quasar gravitational lensing systems at cosmological redshifts an “effectively flat” source plane between the background quasar and the foreground lens plane can be conveniently defined as the interface between Kerr ray-tracing and microlensing ray-tracing, i.e., as the ending point of the backward microlensing ray-tracing and the starting point of the Kerr ray-tracing. We then show that parallel light approximation is adequate for Kerr ray-tracing. This makes it possible to generate the image of Kerr lensing and the microlensing magnification pattern independently in this source plane, and significantly reduces the computing time. The key result of this section is Equation (2) which combines Kerr lensing and microlensing smoothly.

#### 2.1. Kerr Ray-tracing versus Microlensing Ray-tracing

The spacetime of a black hole with nonzero angular momentum is described by the Kerr metric (Kerr 1963) which depends on only two parameters: the black hole mass \( M_{\text{BH}} \) and spin \( a \). As stated earlier, there is no simple lens equation for Kerr strong lensing. We have developed a ray-tracing code using a fifth-order Runge–Kutta algorithm with adaptive step size control in Chen et al. (2013) to remedy this problem (see also Dexter & Agol 2009; Vincent et al. 2011). Ray-tracing has also become a standard technique in gravitational microlensing (e.g., Kayser et al. 1986; Schneider & Weiss 1987). In standard lensing theory photons travel along straight lines, except at the lens plane where bending happens instantaneously. The time consuming part in generating microlensing magnification patterns in the source plane is adding bending angles caused by all the random stars (from a few tens to tens of thousands, depending on the specific problem). To find the magnification in pixels near the caustics where the magnification diverges for point sources, thousands of (or even more) rays need to be traced. Algorithms based on a hierarchical tree code (Wambsganss 1999), Fourier transforms (Kochanek 2004), tessellation methods (Mediavilla et al. 2006), or high-performance hardwares such as graphics processing unit (Bate & Fluke 2012) have been developed to improve the speed of this process. In contrast to ray-tracing in microlensing where the bending angle needs to be computed only once for each ray, in Kerr lensing, it can take a few hundred steps to trace a ray from a distant point (e.g., \( r_{\text{abs}} = 10^9 r_g \)) to a point on the accretion disk near the black hole with desired relative accuracy, e.g., \( 10^{-9} \). Smaller and smaller step sizes are used in our code to obtain the desired accuracy when the photon is close to the black hole, and for each step, many computations involving the (complicated) Kerr metric tensor must be made.

#### 2.2. Combining Kerr Lensing and Microlensing

To combine standard microlensing and Kerr strong lensing accurately and efficiently, a few key issues have to be addressed. First, since the ambient spacetime of the quasar X-ray corona is strongly curved, the foreground microlensing ray-tracing has to end well before it experiences the curvature of the Kerr black hole. For this purpose, we define an effectively flat source plane slightly closer to the observer than the true source plane but far enough from the Kerr black hole not to be affected by its strong curvature (see Figure 1). Such a source plane, orthogonal to the optical axis, is easy to construct. Second, a (backward) ray starting from the observer and bent by the lensing star field arrives at the source plane along a direction slightly different from the optical axis (i.e., the line of sight determined by the inclination angle of the observer). For the purpose of Kerr ray-tracing, we can assume that all such rays are parallel to the line of sight. If not, then we could not split the Kerr ray-tracing part of the calculation from the microlensing part and we need the full set of seven-dimensional (7D) phase space coordinates for photons at the source plane as the input of the Kerr ray-tracing. This would increase the numerical computing time enormously, especially for an extended source moving in the source plane. Fortunately, the angular spread of such rays is so small that they can all be considered as traveling parallel to the optical axis. Consequently, we are able to construct the image of the Kerr source in the microlensing source plane and then convolve it with the microlensing amplification pattern.
To make this splitting work, we need to match the length units in Kerr and microlensing ray-tracing. The natural length units for Kerr and microlensing ray-tracing are respectively the gravitational radius $r_g$ of the black hole, and the Einstein ring $\theta_E$ of a mean stellar-mass lens. As an illustrative example of a supermassive black hole, we use $M_{\text{BH}} = 10^9 \, M_\odot$ (a loose mass range of supermassive black hole powering AGNs is between $10^6 \, M_\odot$ and $10^{12} \, M_\odot$; see, e.g., Bian & Zhao 2002; Wang et al. 2003; Morgan et al. 2010). To estimate the size of the Einstein ring in the source plane in terms of the gravitational radius $r_g$, we choose a typical quasar lensing system with lens and source redshift $z_d = 0.5$ and $z_s = 1.0$, respectively. We assume that the mean stellar lens mass in the foreground microlensing star field is $(m_{\text{stellar}} = 0.3 \, M_\odot)$, consistent with recent microlensing simulations (e.g., Mosquera & Kochanek 2011). We emphasize that the conclusions of this paper will not change if we choose moderately different parameters. This choice is not special, but made merely for convenience.

The angular diameter distance of a source at $z_s = 1$ is $D_s = 1653 \, \text{Mpc} = 3.45 \times 10^{13} \, r_g$ for the black hole mass and cosmology assumed. We choose the source plane in the foreground microlensing at the distance $r_{\text{obs}} = 10^6 \, r_g$ from the Kerr black hole toward the observer and orthogonal to the line of sight (Figure 1). Because the most significant non-vanishing components of the Riemann curvature tensor of the Kerr spacetime are of order $M_{\text{BH}}/r^3$, we are safe in ignoring the curvature of the Kerr black hole at this distance and are safe in assuming that this plane is flat for the purpose of foreground microlensing ray-tracing. Since $r_{\text{obs}}/D_s$ is of order $10^{-7}$, the microlensing magnification pattern in this plane is numerically the same as a fictitious flat plane at redshift $z_s$. In short, the backward ray-tracing for the foreground gravitational microlensing can be done with no modifications.

For the mean microlens mass assumed ($0.3 \, M_\odot$), the Einstein ring angle is $\theta_E \approx 4.4 \times 10^{-12} \, \text{rad}$ (or $D_s \theta_E = 152.6 \, r_g$). For a microlensing magnification pattern in the source plane of size $10 \theta_E \times 10 \theta_E$ with 1024 pixels in each dimension, the pixel size is $\sim 1.5 \, r_g$. If we start the backward ray-tracing for Kerr lensing from the source plane $(10^6 \, r_g$ from the black hole), using $\theta_E$ as the characteristic bending angle for foreground microlensing, the error induced by treating rays starting from the source plane as parallel light toward the black hole is of order $r_{\text{obs}} \theta_E = 10^{-6} \, r_g$, much smaller than the resolution needed in the foreground microlensing even if we increase the desired resolution by factor of 1000 to a pixel size of $\sim 0.001 \, r_g$.

If the mass of the central massive black hole is $10^6 \, M_\odot$ or $10^{12} \, M_\odot$ instead of $10^9 \, M_\odot$ as used in this paper, the parallel light approximation for the Kerr lensing will still be valid, and so will splitting Kerr ray-tracing and microlensing ray-tracing.

To compute the flux at the observer from an extended source near the black hole which is lensed by both the supermassive black hole and the foreground microlensing stars, we need two quantities. The first is the magnification pattern in the source plane caused by foreground microlensing (Figure 1). This can be generated independently of the Kerr lensing, although the required resolution depends on the source size. The second quantity we need is the Kerr lensing image of the source on the source plane, including its intensity profile. This can be computed using our Kerr ray-tracing code with the parallel light approximation, independently of the foreground microlensing. For each pixel in the source plane, we need only trace one ray starting from the center of this pixel and traveling parallel to the optical axis toward the black hole. In short, an extended light source near the black hole is (inversely) mapped onto the nearby source plane which has been placed orthogonal to the line of sight and at rest in the background cosmology. This mapping includes the image, its redshift, and its intensity profile. Kerr-Micro lensing light curves are then generated by using the two-dimensional (2D) Kerr image as the source for the microlensing magnification pattern.

A short description of the intensity calculations follows: we let $\nu_o$ be the photon’s frequency measured by a cosmic observer

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Schematic light path for Kerr+Micro lensing ray-tracing. Here, $\theta$ is the disk inclination angle and $r_{\text{isco}}$ is the innermost stable circular orbit. A ray arriving at the observer in a direction $\hat{n}$ (with $\theta$ the angle between the light ray and the optical axis) is backward traced to $\hat{\xi}$ (lens plane), then to $\eta$ (source plane) by microlensing ray-tracing, and then to $(x^a, p^a)$ on the X-ray source near the Kerr black hole by Kerr ray-tracing. The photon frequencies $\nu_o, \nu_\eta, \nu_\xi$, and $\nu_\xi$ are measured at the observer, source plane, and the source, respectively. The Kerr image (in the source plane) is treated as the source for the foreground microlensing ray-tracing. $r_{\text{obs}}$ is greatly exaggerated with respect to the angular diameter distance $D_s$.}
\end{figure}
when the photon is received and \( v_o \) be the frequency measured by an observer at rest with respect to the corona at emission. We split the redshift into two parts, the cosmological redshift and the “Kerr redshift”:

\[
\frac{v_o}{v_c} = \frac{v_o}{v_r} \frac{v_r}{v_c} = \frac{1}{1 + \frac{v_r}{v_o}} = \frac{g}{1 + \frac{v_o}{v_r}}.
\]

(1)

where \( v_r \) is the photon frequency measured in the source plane defined above. The Kerr redshift factor \( g \equiv \frac{v_o}{v_r} \) depends on the inclination angle \( \theta \) of the accretion disk, the 2D impact vector \( \eta \) in the source plane with respect to the optical axis, and on the relativistic flow of the corona (Figure 1). Let \( I(x^a, p^a) \) be the specific intensity profile of the source (e.g., the hot corona emitting X-rays), where \((x^a, p^a)\) are the 7D phase space coordinates \((p^a, p_a = 0 \text{ for massless particles})\), the observed monochromatic flux is

\[
F_{v_o} = \int \int \int \int \int \int \int I_{v_o}(\hat{n}) \cos \theta d\Omega_o = \int \int \int \int \int \int I_{v_o}(\hat{n})d\Omega_o = \int \frac{L_{v_o}(\eta)}{D_L^2 (1 + z)^3} A(\eta) d^2 \eta = \frac{1}{1 + \frac{v_o}{v_r}} \int \frac{g^3(\eta) I[\chi^a(\eta), p^a(\eta)] A(\eta)}{D_L^2} d^2 \eta.
\]

(2)

where \( \theta \) is the angle between the optical axis and the incoming ray, \( D_L \) and \( D_S \) are 2D integral domains (i.e., the images of the X-ray source) in the foreground lens plane and the source plane, respectively. \( A \) is the microlensing magnification factor of the source plane, and the composite map \( \xi \rightarrow \eta \rightarrow (x^a, p^a) \) is realized by microlensing and Kerr ray-tracing, respectively (we have used \( \cos \theta \approx 1 \) in the second step). The effect of foreground microlensing is contained in the magnification factor \( A \). The effect of Kerr lensing is multifold: (1) the integral domain \( D_S \) for the case of Kerr lensing is enlarged and distorted compared with that of the case without Kerr lensing; (2) the intensity profile of the Kerr image is changed by the Kerr light bending, e.g., a pixel in the source plane will be mapped to a different place in the corona from that of straight line ray-tracing; and (3) the Kerr redshift combined with relativistic beaming also changes the profile significantly (e.g., the \( g^3 \) factor in the integral above).

3. EFFECTS OF KERR STRONG GRAVITY ON QUASAR X-RAY MICROLENSING

We investigate the effects of Kerr lensing on quasar X-ray microlensing observations in this section using the model developed in Section 2. We describe the example X-ray source and lens models in Section 3.1. The importance of including Kerr lensing for quasar X-ray microlensing is established in a few steps. First, we show that there is a strong-lensing effect by directly comparing standard microlensing light curves with Kerr+Macro lensing light curves. We then show that source parameters such as the inclination angle and black hole spin are encoded in Kerr+Macro lensing light curves. Next, we show that the half-light radius of a Kerr-lensed X-ray disk can be significantly different from that of a flat unlensed disk. Finally, we show that current microlensing size estimates can be biased due to the neglect of Kerr strong gravity.

3.1. Example Source and Lens Configurations

For the X-ray source model, we need to specify the corona geometry, its size, its intensity, and its relativistic velocity profile. We choose a simple geometry for the X-ray source: a thin X-ray disk immediately above the accretion disk moving with Keplerian flow. This is partially motivated by the “sandwich” corona model (Haardt & Maraschi 1991, 1993). We assume a simple double power law in radius and frequency model for the X-ray emission profile,

\[
I_o(v, r) \propto \frac{1}{r^n} \frac{1}{\nu^3 - 1},
\]

(3)

where \( r \) is the radial coordinate of the Kerr metric, \( n \) specifies the steepness of the radial profile, and \( \Gamma \) is the photon index. Using Equation (2), the observed monochromatic flux at frequency \( v_o \) is

\[
F_{v_o} = \frac{1}{(1 + z)^3} \frac{1}{D_L^2} \frac{1}{v_o} \int \int r^{\Gamma + 2}(\eta) A(\eta) d^2 \eta.
\]

(4)

where we have dropped the unimportant constant. We take the value \( \Gamma = 2.0 \) customarily used for quasar X-ray emission (Chen et al. 2012), and \( n = 3 \) or 0. The source profile is radially steeper, and more concentrated toward the center for the \( n = 3 \) case. In the following, we call this background X-ray source model a Kerr-lensed disk when including Kerr strong gravity, and a flat disk when ignoring Kerr lensing.

As for the spin of the black hole, we experimented with \( a = 0 \) (a Schwarzschild black hole) and \( a = 0.998M_{\odot} \) (an extreme Kerr black hole; Thorn 1974). We compare results for three inclination angles \( \theta = 15^\circ, 45^\circ, \) and \( 75^\circ, \) covering the range from nearly face-on to nearly edge-on. We limit the radial extent of the X-ray emission to \( r_{\text{ISCO}} < r < r_{\text{disk}}, \) where \( r_{\text{ISCO}} \) is the innermost stable circular orbit (Bardeen et al. 1972). The inner cutoff \( r_{\text{ISCO}} \) depends on the black hole spin and is \( 1.24 r_g \) and \( 6 r_g \) for \( a = 0.998M_{\odot} \) and 0, respectively. The maximum disk size we tested is \( r_{\text{disk}} = 50 r_g \). The minimum \( r_{\text{disk}} \) is \( 2.5 r_g \) and \( 10 r_g \) for \( a = 0.998M_{\odot} \) and \( a = 0 \), respectively. We focused on the \( r_{\text{disk}} = 20 r_g \) case in our analysis. An X-ray emission region of this size is consistent with existing quasar microlensing observations (see Section 1). Since most of the current constraints from quasar X-ray microlensing are upper bounds, a smaller size is possible (Morgan et al. 2012). The Kerr lensing effect will be more important for a smaller X-ray emitting region. A summary of the example source model is given in Table 1. Figure 2 shows intensity images of Kerr-lensed X-ray emitting disks for three inclination angles, two spins, and two radial profiles \( r_{\text{disk}} = 50 r_g \).

We considered two foreground microlensing models: a simple Chang–Refsdal lens (Chang & Refsdal 1979) and a random star field (Schneider et al. 1992) with mean lens mass \( \langle M_{\text{lens}} \rangle = 0.3 M_{\odot} \) (Mosquera & Kochanek 2011); see Table 1. The simple, but nontrivial caustic structure of a Chang–Refsdal lens is ideal for studying the effect of Kerr lensing on foreground microlensing. The random star field model is more realistic for a quasar at cosmic redshift seen through a foreground lensing galaxy. We fix the lens and source redshifts at \( z_d = 0.5 \) and \( z_s = 1.0, \) respectively. The magnification patterns for the two models are shown in Figure 3. We also show the redshift images of lensed X-ray disks in the source plane for inclination angles of \( \theta = 15^\circ \) and \( 75^\circ, \) and black hole spins of \( a = 0.998M_{\odot} \) and 0.

3.2. Microlensing Light Curves versus Kerr+Macro Lensing Light Curves

To show that Kerr strong gravity does affect quasar X-ray microlensing observations, we need to compare the light curves
Figure 2. Intensity plots of Kerr-lensed X-ray emitting disks moving with Keplerian flow. The color bars are in logarithmic scale (normalization is arbitrary). \( r_{\text{disk}} = 50 r_g \). We show results for four source models (spin \( a = 0 \) or 0.998 \( M_{\text{BH}} \) and radial profile \( n = 3 \) or 0) in the four rows, and three inclination angles \( \theta = 15^\circ, 45^\circ, \) and 75\(^\circ\) in the first, second, and third columns. The star in each panel marks the peak surface brightness location. For nearly face-on cases with flat radial profile \( (n = 0) \), the peak brightness happens on the left boundary of the disk (the approaching side) because of gravitational and Doppler shifts. The black curve in each panel is the critical intensity contour line delimiting the half-light region (used to compute the half-light radius). The intensity profile of a Kerr-lensed disk can be significantly different from that of an unlensed disk. Consequently, the half-light radius of a Kerr-lensed disk is different from the unlensed case.

(A color version of this figure is available in the online journal.)

Table 1

| Parameter | Meaning                  | Values                  |
|-----------|--------------------------|-------------------------|
| \( \theta \) | Inclination             | 15\(^\circ\), 45\(^\circ\), 75\(^\circ\) |
| \( a \) | Black hole spin         | 0, 0.998 \( M_{\text{BH}} \) |
| \( n \) | Radial profile          | 0, 3                    |
| \( \Gamma \) | Photon index           | 2                       |
| \( r_{\text{disk}} \) | Disk radius            | 2.5 \( r_g \) \( \leq \) \( r_{\text{disk}} \) \( \leq \) 50 \( r_g \) |
| \( r_{\text{ISCO}} \) | Disk inner cutoff      | 1.24 \( r_g \), 6 \( r_g \)^0 |
| \( M \) | Black hole mass         | \( 5 \times 10^8 M_\odot \), \( 10^9 M_\odot \) |
| Chang & Refsdal | \( M_{\text{em}} \) | Lens mass               | 0.3 \( M_\odot \) |
| \( (\gamma_1, \gamma_2) \) | Shear                   | (0, 0.3)                |
| Random star field | \( M_{\text{em}} \) | Mean lens mass          | 0.3 \( M_\odot \)^b |
| \( (\gamma_1, \gamma_2) \) | Lens mass               | \( 0.01 M_\odot \) \( \leq \) \( M_{\text{em}} \) \( \leq \) \( 1.6 M_\odot \) |
| \( \kappa_s \) | Shear                   | (0.2, 0)                |
| \( \kappa_c \) | Continuous 2D mass density | 0.1^b                |
| \( \kappa_c \) | Discrete 2D mass density | 0.6^b                 |

Notes.

^a \( r_{\text{ISCO}} = 1.24 r_g \) and 6 \( r_g \) for black hole spin \( a = 0.998 M_{\text{BH}} \) and 0, respectively.

^b Refer to Mosquera & Kochanek (2011).
between the two cases. Were the Kerr+Micro lensing light curves different from standard microlensing light curves only by a constant, the results would not be very interesting, because this constant shift is degenerate with the source luminosity and the (macro) galactic lensing magnification. However, we expect Kerr+Micro lensing light curves to differ from standard curves in a nontrivial way because Kerr lensing changes the size, shape, and profile of the source which are inputs for the foreground microlensing; see Figure 2. In regions of the source plane without complicated caustic structures, the gradient of the magnification is small, and the Kerr+Micro lensing light curves differ from those of microlensing by roughly a constant, i.e., by the amount the flux at the source plane is reduced or increased by Kerr lensing (Chen et al. 2013). However, when the source is crossing a caustic or regions with clustered caustic structure (see Figure 3), the difference between the two lensing schemes should deviate from merely a constant. To quantify this nontrivial difference, we measure the amplitude of the magnitude fluctuation, i.e., the difference between the maximum and minimum magnitude, $m_{\text{max}} - m_{\text{min}} \equiv m_{\text{max}-\text{min}}$. This quantity can be used to estimate the source size because light curves of smaller sources have larger amplitudes of fluctuation. We will focus on this quantity when comparing Kerr+Micro lensing with standard microlensing variability.

Figures 4 and 5 plot the light curves for the two source trajectories in Figure 3 (red lines) and for source size $r_{\text{disk}} = 20 r_g$. We show the magnitude $m \equiv 2.5 \log_{10} \mu$, where $\mu \equiv f_{\text{lensed}}(\theta)/f_{\text{unlensed}}(\theta)$ is the lensing magnification. Using this ratio modulates out the source luminosity and the $\cos \theta$ part of the projection effect. We show the light curves for four source models ($a = 0.998 M_{\odot}$ at 0, $n = 3$ or 0), and three inclination angles in the first and third rows. We also show the inclination dependence of the Kerr+Micro lensing light curves by plotting $m(t; 45^\circ) - m(t; 15^\circ)$ and $m(t; 75^\circ) - m(t; 15^\circ)$ in the second and fourth rows (dashed and dotted curves, respectively) with and without the projection effect (i.e., the $\cos \theta$ factor, cyan and magenta, respectively). The spin dependence of the Kerr+Micro lensing is shown by $m(t; a = 0.998 M_{\odot}) - m(t; a = 0)$ in the fifth row. We tabulate the amplitude fluctuation $m_{\text{max}-\text{min}}$ for the light curves in Figures 4 and 5 in Table 2.

As can be seen from Figures 4, 5, and Table 2, Kerr lensing does have effects on X-ray microlensing light curves. In particular, the amplitude of the magnitude fluctuation, $m_{\text{max}-\text{min}}$ can differ by $\sim 0.5$–0.6 mag between a flat X-ray disk and a Kerr-lensed disk for both foreground lens models considered in this paper. An important result is that the Kerr+Micro lensing light curve depends significantly on the inclination angle, while the standard microlensing light curve does not. Beyond the $\cos \theta$ projection effect, which we have factored out, the standard microlensing light curves for three inclination angles, shown in blue curves, are almost indistinguishable from each other; see Figures 4, 5, and Table 2. The fact that Kerr+Micro lensing light curves depend on inclination angles is not really surprising because it is well known among relativists that the influence of Kerr strong gravity on AGN X-ray emission, continuum, or iron line emission is inclination dependent (e.g., Cunningham 1975; Chen et al. 2013). In short, if there is an effect, then it should be found to be inclination dependent. Specifically, we find that a larger inclination angle results in a larger amplitude of magnitude fluctuation; see Table 2 ($m_{\text{max}-\text{min}}$ increases with $\theta$ for Kerr+Micro lensing). This is not difficult to understand. For a corona with a steep radial intensity profile ($n = 3$) observed nearly face-on, the severe gravitational redshift suffered by the central region of the corona makes

Figure 3. Magnification pattern (in the source plane) of a simple Chang–Refsdal lens ($2 \theta_E \times 2 \theta_E$) and a random star field ($9 \theta_E \times 9 \theta_E$). The resolutions are $6000 \times 6000$ (left) and $7200 \times 7200$ (right). The insets are the Kerr lensing images of an X-ray disk of size $r_{\text{disk}} = 20 r_g$ by a supermassive black hole ($M_{\text{BH}} = 5 \times 10^8 M_{\odot}$ left, $10^9 M_{\odot}$ right) with spin $a = 0.998 M_{\odot}$ (left panel) and $a = 0$ (right panel) observed at inclination angle $\theta = 15^\circ$ and $75^\circ$ (upper/lower insets), respectively. We scaled the disk image by a factor of two (left panel) and five (right panel) with respect to the source window for clarity. The thin red lines are trajectories of the X-ray source in the source plane. The color bars show the Kerr redshift factor $g = \nu_\ell/\nu_e$ for $\theta = 15^\circ$ (top scale) and $\theta = 75^\circ$ (bottom scale). A disk with higher inclination angle, or larger spin (smaller $r_{\text{ISCO}}$) spans a larger redshift interval.

(A color version of this figure is available in the online journal.)
Figure 4. X-ray microlensing light curves for the Chang–Refsdal lens model, showing the magnification in magnitudes ($m \equiv 2.5 \log_{10} \mu$, and $\mu$ is the lensing magnification) as a function of time in pixel units. We show results for four source models, spin $a = 0$ or $0.998 M_{\odot}$, and radial profile $n = 3$ (steep, the first column) or 0 (flat, the second column) as indicated in the panels. The solid, dashed, and dotted curves are for inclination angles $\theta = 15^\circ$, $45^\circ$, and $75^\circ$, respectively. The red and blue curves are, respectively, for Kerr+Micro lensing or microlensing only. The dependence of standard microlensing curves on the inclination angle is so weak that it is hard to distinguish these curves by eye. In the second and the fourth rows, we show the inclination dependence of the Kerr+Micro lensing curves by plotting $m(t; 45^\circ) - m(t; 15^\circ)$ and $m(t; 75^\circ) - m(t; 15^\circ)$ (magenta dashed and dotted curves, respectively). The cyan curves plot the same things but including the geometrical projection effect (the $\cos \theta$ factor). In the fifth row, we show the spin dependence of the Kerr+Micro lensing light curves by plotting $m(t; a = 0.998) - m(t; a = 0)$ for each inclination angle.

(A color version of this figure is available in the online journal.)

the source emission less concentrated toward the center, and consequently, the Kerr+Micro lensing light curves show smaller fluctuation amplitudes than standard microlensing light curves. For an observer at high inclination angle, the X-ray emission is more concentrated in the Doppler blueshifted regions (caused by relativistic beaming) which produces the larger fluctuations we see in the Kerr+Micro lensing light curves.

In the previous example, we have fixed $r_{\text{disk}} = 20 r_g$. In Figure 6, we show $m_{\text{max}} - m_{\text{min}}$ as a function of emission size $r_{\text{disk}}$ ($2.5 r_g < r_{\text{disk}} < 50 r_g$) for the same four background source models and three inclination angles restricted to the random star field model. The results are consistent with the previous example. The inclination angle dependence of $m_{\text{max}} - m_{\text{min}}$ is much more significant for Kerr-lensed disks than for flat disks. The results for flat disks, shown as dashed curves, are almost indistinguishable from each other. Larger inclination angles give larger amplitude fluctuation in the microlensing light curves. The amplitude of magnitude fluctuation, $m_{\text{max}} - m_{\text{min}}$, can increase by ~0.65 mag or decrease by ~0.75 mag including Kerr strong gravity. If a very large amplitude of magnitude
fluctuation is observed, e.g., \( m_{\text{max}} - m_{\text{min}} \gtrsim 3.2 \), then a Kerr-lensed disk is strongly favored (none of the four flat models can produce \( m_{\text{max}} - m_{\text{min}} \) of this size). Since the amplitude of magnitude fluctuation, \( m_{\text{max}} - m_{\text{min}} \), of a Kerr-lensed disk can be either larger or smaller than that of a flat disk, current quasar X-ray microlensing observations might have over or underestimated the X-ray emission sizes by ignoring Kerr strong gravity. We will discuss this point in detail in Section 3.4.

The spin dependence of microlensing light curves, with or without Kerr strong lensing, is shown in the third row of Figure 6. We show the difference in \( m_{\text{max}} - m_{\text{min}} \), i.e., \( \Delta m_{\text{max}} - m_{\text{min}} \equiv m(a = 0.998)_{\text{max}} - m(a = 0)_{\text{max}} - m(a = 0)_{\text{min}} \) as a function of \( r_{\text{disk}} \) for two radial profiles \( (n = 0 \text{ or } 3) \) and for three inclination angles. For the case of a steep radial profile, the spin dependence of \( m_{\text{max}} - m_{\text{min}} \) of a flat X-ray disk is in fact more significant than a Kerr-lensed disk, mainly because an \( a = 0.998 M_{\odot} \) disk has a smaller \( r_{\text{ISCO}} (1.24 r_g) \) than an \( a = 0 \) disk \( (r_{\text{ISCO}} = 6 r_g) \) and therefore has a more concentrated emission profile (or smaller half-light radius, see Section 3.4). Consequently, the \( a = 0.998 M_{\odot} \) case shows a much larger \( m_{\text{max}} - m_{\text{min}} \) than the \( a = 0 \) case.5 For Kerr-lensed disks, the concentrated emission in the region \( 1.24 r_g < r < 6 r_g \) was washed out by gravitational redshifts, particularly for the nearly face-on case \( (\theta = 15^\circ) \); see the (3, 1) panel of Figure 6. For a flat radial profile, the unlensed source emission is uniformly distributed over the X-ray disk, therefore, the extra emission between \( 1.24 r_g < r < 6 r_g \)

5 This spin dependence of microlensing light curves for flat X-ray disks is physically simple, but does not seem to have been pointed out previously.
Figure 6. $m_{\text{max-min}}$ as a function of emission size $r_{\text{disk}}$. The microlensing magnification pattern and the source trajectory are shown in the right panel of Figure 3. We show results for four source models ($a = 0$ or 0.998 $M_{\odot}$ and $n = 3$ or 0) and three inclination angles $\theta = 15^\circ$, $45^\circ$, and $75^\circ$ (red, cyan, and blue curves) assuming flat or Kerr-lensed disks (dashed and solid curves). The inclination angle dependence of $m_{\text{max-min}}$ is much more significant for Kerr-lensed disks than flat disks. The spin-dependence of $m_{\text{max-min}}$, i.e., $m_{\text{max-min}}(a = 0.998) - m_{\text{max-min}}(a = 0)$, is shown in the third row. For the $n = 3$ case, the spin dependence of $m_{\text{max-min}}$ is more significant for flat X-ray disks. For the $n = 0$ case, the spin dependence of $m_{\text{max-min}}$ for both flat and Kerr disks is insignificant. The horizontal lines mark the values of $m_{\text{max-min}}$ which we used to estimate the bias of size constraints of current microlensing observations (see Section 3.4).

(A color version of this figure is available in the online journal.)

Table 2

| Lens | Source | Microlensing | Kerr+Micro Lensing$^b$ |
|------|--------|-------------|------------------------|
|      | $a = 0.998$, $n = 3$ | $\theta = 15^\circ$ | $\theta = 45^\circ$ | $\theta = 75^\circ$ | $\theta = 15^\circ$ | $\theta = 45^\circ$ | $\theta = 75^\circ$ |
| I    | $a = 0.998$, $n = 0$ | 1.70        | 1.70                   | 1.71                   | 1.96 (+0.05) | 1.85 (+0.14) | 1.86 (+0.16) |
|      | $a = 0$, $n = 3$    | 1.88        | 1.88                   | 1.88                   | 1.76 (+0.03) | 1.86 (+0.15) | 1.90 (+0.18) |
|      | $a = 0$, $n = 0$    | 1.71        | 1.71                   | 1.71                   | 2.14 (+0.04) | 2.00 (+0.12) | 2.19 (+0.31) |
| II   | $a = 0.998$, $n = 3$ | 2.78        | 2.78                   | 2.79                   | 2.14 (+0.04) | 2.00 (+0.12) | 2.19 (+0.31) |
|      | $a = 0.998$, $n = 0$ | 1.58        | 1.59                   | 1.60                   | 1.76 (+0.07) | 1.89 (+0.30) | 2.05 (+0.46) |
|      | $a = 0$, $n = 3$    | 1.87        | 1.88                   | 1.88                   | 1.63 (+0.04) | 1.84 (+0.25) | 1.95 (+0.36) |
|      | $a = 0$, $n = 0$    | 1.58        | 1.59                   | 1.59                   | 1.91 (+0.04) | 2.24 (+0.37) | 2.44 (+0.55) |

Notes. Source size $r_{\text{disk}} = 20 r_g$.

$^a$ I: Chang–Refsdal lens model; II: random star field.

$^b$ The number in the parenthesis is the differential magnitude fluctuation between Kerr+Micro lensing and microlensing for the same inclination angle $\theta$, i.e., $m_{\text{max-min}}(\theta) - m_{\text{max-min}}(\theta)$. 
Figure 7. Half-light radii of Kerr-lensed X-ray disks. We plot $r_{\text{half}}$ as a function of the emission size $r_{\text{disk}}$. We show results for four source models and three inclination angles ($\theta = 15^\circ$, $45^\circ$, and $75^\circ$, as red, cyan, and blue curves). The data were given in Table 3. The solid and dashed curves show lensed and unlensed $r_{\text{half}}$, respectively. The relative correction $(r_{\text{Kerr}} - r_{\text{flat}})/r_{\text{flat}}$ is shown in the second and fourth rows. Relativistic effects change $r_{\text{half}}$ significantly. For a flat radial profile ($n = 0$, the second column) the lensed disk has a smaller $r_{\text{half}}$ than the unlensed disk. The reduction in $r_{\text{half}}$ is about 5%–40% depending on the spin and inclination angle. For a steep radial profile $n = 3$ with $a = 0.998 M_{\odot}$, the dominating gravitational redshift effect makes the intensity profile much less concentrated toward the center, particularly for the nearly face-on case. Consequently, $r_{\text{Kerr}}$ can be $\sim$3 times larger than $r_{\text{flat}}$. This number is reduced to $\sim$20% when the disk is observed near edge-on ($\theta = 75^\circ$).

(A color version of this figure is available in the online journal.)

Figure 7. Half-light radii of Kerr-lensed X-ray disks. We plot $r_{\text{half}}$ as a function of the emission size $r_{\text{disk}}$. We show results for four source models and three inclination angles ($\theta = 15^\circ$, $45^\circ$, and $75^\circ$, as red, cyan, and blue curves). The data were given in Table 3. The solid and dashed curves show lensed and unlensed $r_{\text{half}}$, respectively. The relative correction $(r_{\text{Kerr}} - r_{\text{flat}})/r_{\text{flat}}$ is shown in the second and fourth rows. Relativistic effects change $r_{\text{half}}$ significantly. For a flat radial profile ($n = 0$, the second column) the lensed disk has a smaller $r_{\text{half}}$ than the unlensed disk. The reduction in $r_{\text{half}}$ is about 5%–40% depending on the spin and inclination angle. For a steep radial profile $n = 3$ with $a = 0.998 M_{\odot}$, the dominating gravitational redshift effect makes the intensity profile much less concentrated toward the center, particularly for the nearly face-on case. Consequently, $r_{\text{Kerr}}$ can be $\sim$3 times larger than $r_{\text{flat}}$. This number is reduced to $\sim$20% when the disk is observed near edge-on ($\theta = 75^\circ$).

(A color version of this figure is available in the online journal.)

does not contribute significantly to the total flux. Consequently, the spin dependence of both flat and Kerr-lensed disks is insignificant.

### 3.3. Kerr Lensing Correction to Half-light Radii of X-Ray Coronae

What microlensing observations really constrain is the half-light radius $r_{\text{half}}$ of the X-ray source, with weak dependence on the emission profiles (Mortonson et al. 2005). Before we estimate the bias of current X-ray microlensing observations caused by ignoring Kerr gravity, we need to understand how Kerr gravity changes the half-light radius of the X-ray corona.\(^6\) The content in this subsection is solely about Kerr gravity, and is independent of microlensing. Nonetheless, the results will be needed in the next subsection. Readers not interested in details are invited to skip the last paragraph of this subsection.

A half-light radius $r_{\text{half}}$ was originally defined for sources with spherical symmetry to be the radius of the circular disk centered at the peak surface brightness and containing half of the total flux. For non-spherically symmetric objects, a half-light radius

\(^6\) Although strong lensing of AGN X-ray emission by Kerr black hole has been discussed extensively before, the current paper is the first detailed discussion of half-light radius of a Kerr-lensed X-ray disk.
can be similarly defined to be the radius of a disk whose area is the same as the half-light region (from the peak brightness point down to a fixed surface brightness). We compare the half-light radii of flat X-ray disks, $r_{\text{disk}}^{\text{flat}}$, with those of Kerr-lensed disks, $r_{\text{half}}^{\text{Kerr}}$. The results are shown in Figures 2 and 7, and Table 3. Figure 2 shows the intensity profile, peak brightness location, and the half-light region of images of X-ray disks ($r_{\text{disk}} = 50 r_g$) lensed by Kerr strong gravity. The $r_{\text{half}}$ for $2.5 r_g < r_{\text{disk}} < 50 r_g$ are given in Table 3 and shown in Figure 7 for different source models and inclination angles (solid and dashed curves are for Kerr and flat disks, respectively; red, cyan, and blue curves are for inclination angle $\theta = 15^\circ, 45^\circ$ and $75^\circ$).

Strong gravity and relativistic flows change $r_{\text{half}}$ in a few respects. First, the gravitational light bending increases the image area and tends to increase the half-light radius. This area distortion is more significant for high inclination angles (Chen et al. 2013). Second, the differential gravitational redshifts between different regions of the source emission change the intensity profiles of lensed disks. This also changes $r_{\text{half}}$. For example, the surface brightness distribution of an X-ray disk with a steep radial profile ($n = 3$) will be much less concentrated toward the center after the gravitational redshift effects have been taken into account. Consequently, the gravitational redshift tends to increase $r_{\text{half}}$ for this case. On the other hand, for a uniform intensity profile ($n = 0$), differential gravitational redshifts focus the emission to regions less severely redshifted, this tends to reduce $r_{\text{half}}$ (i.e., $r_{\text{Kerr}}^{\text{half}} < \sqrt{A/2\pi} = r_{\text{flat}}^{\text{half}}$, where $A$ is the disk area). Third, the Doppler shift and relativistic beaming tend to focus the intensity to a small region where the emission is strongly blueshifted. If the unlensed emission is already concentrated in this region, the lensed profile will then be even more concentrated, resulting in a smaller $r_{\text{half}}$ value. If not so concentrated, this lensing produced (local) peak brightness location might compete with other regions of intrinsically strong emission, and can give a larger $r_{\text{half}}$. Doppler shifts are more significant for high inclination angles (nearly edge-on). In reality, $r_{\text{half}}$ is influenced by these effects simultaneously. For example, for $n = 3$, $a = 0$, $\theta = 15^\circ$, the gravitational redshift effect dominates. Consequently, $r_{\text{Kerr}}^{\text{half}}$ can be $\sim 3$ times larger than $r_{\text{flat}}^{\text{half}}$ for $r_{\text{disk}} \gtrsim 20 r_g$; see the (4, 1) panel of Figure 7. For $n = 0$, $a = 0$ or $0.998 M_{\text{BH}}$, $r_{\text{Kerr}}^{\text{half}}$ is smaller than $r_{\text{flat}}^{\text{half}}$ by $10\%$–$40\%$ depending on the spin and inclination angle (see the second column of Figure 7). For the $n = 3$, $a = 0$ case, $r_{\text{Kerr}}^{\text{half}}$ can be either smaller or larger than $r_{\text{flat}}^{\text{half}}$ (from $-20\%$ to $+40\%$) depending on the emission size and the inclination angle (see the (1, 1) and (2, 1) panel of Figure 7).

### 3.4. Bias of Current Microlensing Size Estimates

Kerr lensing changes the size, the shape, and the intensity profile of quasar X-ray emission regions. Consequently, the microlensing light curves (e.g., the amplitude of magnitude fluctuation, $m_{\text{max}}-m_{\text{min}}$) are different for flat and Kerr-lensed disks. Current quasar microlensing observations might have slightly over or underestimated the X-ray emission sizes. We now tentatively estimate this bias using the $m_{\text{max}}-m_{\text{min}}$ metric.
Assuming that some amplitude of fluctuation, $m_{\text{max-min}}$, is observed, we compute the emission size $r_{\text{disk}}$ and half-light radius $r_{\text{half}}$ needed to produce the same $m_{\text{max-min}}$ for both flat and Kerr-lensed disk models. The difference between these two cases measures the bias of current microlensing measurements. We restrict ourselves to the random star field lens model and consider the same four source models and three inclination angles as before (see Figures 6). For each panel in Figure 6, we test a few $m_{\text{max-min}}$ values (marked by thin black lines). First, we find the corresponding $r_{\text{disk}}$ values using Figure 6, i.e., inverting the $r_{\text{disk}} \rightarrow m_{\text{max-min}}$ mapping for each curve. We then read off the value of the half-light radius $r_{\text{half}}$ corresponding to this $r_{\text{disk}}$ value using Figure 7 and Table 3. Since there is no analytical form for any of the curves in Figures 6 and 7, the composite map $m_{\text{max-min}} \rightarrow r_{\text{disk}} \rightarrow r_{\text{half}}$ is realized numerically by interpolation. The results for the two steps above are shown in Figures 8 and 9, respectively. Since microlensing observations constrain the half-light radius $r_{\text{half}}$, instead of $r_{\text{disk}}$, we will focus on the size estimates of $r_{\text{half}}$ in the following.

For most cases we considered, we found an underestimate of the half-light radius (up to $\sim 50\%$); see Figure 9. The $r_{\text{half}}$ for a Kerr-lensed disk, marked in red, is above the $r_{\text{half}}$ for a flat disk, marked in blue, for all cases we have tested except for the two cases marked by arrows; see the bottom right panel. For example, consider the $n = 0, a = 0$ model with $m_{\text{max-min}} = 2.0$ (see the top right panel of Figures 6). Assuming that flat X-ray disks results in an underestimate of $r_{\text{disk}}$ by $20\%, 40\%$, and $49\%$ (see the top right panel of Figure 8) and an underestimate of $r_{\text{half}}$ by $18\%, 33\%$, and $45\%$ (see the top right panel of Figure 9) for inclination angle $\theta = 15^\circ, 45^\circ$, and $75^\circ$, respectively. A larger emission size is needed for a Kerr-lensed disk because for a given $r_{\text{disk}}$, a Kerr disk has a smaller $r_{\text{half}}$ (see the (2, 2) panel of Figure 7) and this gives a larger $m_{\text{max-min}}$. In order to produce the same $m_{\text{max-min}}$ as a flat disk, we need to increase the source size for Kerr-lensed disks. We found an overestimate of $r_{\text{half}}$ only for the $n = 0, a = 0.998 M_{\odot}$, and $\theta = 15^\circ$ case, i.e., a disk with a flat radial profile and a large spin, observed nearly face-on (see the bottom panel of Figure 9). For this model, an overestimate of $r_{\text{half}}$ by $\sim 20\%$ is possible when the disk size is small, $r_{\text{disk}} \lesssim 6 r_s$. For the case $a = 0.998 M_{\odot}$, $n = 3$, and $\theta = 15^\circ$, if we observe $m_{\text{max-min}} = 2.8$, then an emission size $r_{\text{disk}}^{\text{Kerr}} = 14.8 r_s$ is needed assuming a flat disk, $\sim 3$ times larger than $r_{\text{disk}}^{\text{Kerr}} = 4.0 r_s$ assuming a Kerr-lensed disk (see the bottom left panel of Figure 8). This is because the severe gravitational redshift suffered by the central regions of a Kerr disk makes the lensed intensity profile much less concentrated than the intrinsically steep radial profile and therefore, a smaller $r_{\text{disk}}^{\text{Kerr}}$ (but much less concentrated) can produce a similar amplitude of magnitude fluctuation $m_{\text{max-min}}$ as a larger but more concentrated flat disk.

Figure 8. Bias of emission size $r_{\text{disk}}$ induced by ignoring Kerr strong lensing. We plot $r_{\text{disk}}$ as a function of the measured amplitude of magnitude variation $m_{\text{max-min}}$. We show results for source models ($a = 0$ or $0.998 M_{\odot}$ and $n = 3$ or $0$) and three inclination angles ($\theta = 15^\circ, 45^\circ$, and $75^\circ$, as stars, squares, and diamonds). We slightly offset the x coordinate of the $\theta = 15^\circ$ and $75^\circ$ case to the left (right) with respect to the $\theta = 45^\circ$ case for clarity. We found an underestimation of $r_{\text{disk}}$ for most cases, except for $a = 0.998 M_{\odot}, n = 3$ (marked by arrows in the bottom left panel). This is because the gravitational redshift effect makes the intensity profile of a Kerr-lensed disk much less concentrated toward the center, particularly for the nearly face-on case.
In terms of half-light radius (which is what microlensing really constrains), we still found an underestimate (∼19%; see the bottom left panel of Figure 9). This underestimate in \( r_{\text{half}} \) does not contradict the overestimate in \( r_{\text{disk}} \). For a fixed \( r_{\text{disk}} \), an unlensed disk has much smaller half-light radius than a Kerr-lensed disk (see the dashed and solid red curves in the (3, 1) panel of Figure 7). Consequently, the half-light radius of a flat disk with \( r_{\text{disk}} = 14.8 \, r_g \) is smaller than that of a lensed disk with \( r_{\text{disk}} = 4.0 \, r_g \).

We have focused on the amplitude of magnitude fluctuation \( (m_{\text{max}} - m_{\text{min}}) \) when comparing the microlensing light curves of Kerr-lensed disks and flat disks. Some other features of Kerr-lensed light curves are worth mentioning. For example, when the source is crossing a caustic, the shape of the spike in the light curve, e.g., its width (the duration of the caustic crossing) can be different for a Kerr-lensed disk and a flat disk; see Figures 4 and 5. We point out that Kerr lensing can change the caustic crossing time significantly. This can be easily seen for the simple Chang–Refsdal lens model (Figure 4). For example, for the case with inclination angle \( \theta = 75^\circ \), \( a = 0.998 \, M_{\text{BH}} \), and steep profile (bottom left panel of Figure 4), the two caustic crossing events of Kerr+Micro lensing light curves are later than those of standard microlensing light curves by ∼3.8 and 7.0 months, respectively (assuming that the relative crossing velocity is \( v_s = 300 \, \text{km s}^{-1} \) in the source plane; see Mosquera & Kochanek 2011), presumably caused by the focusing of emission to the approaching side of the disk (the left side, Figure 3). A simultaneous monitoring of gravitational lensed quasars in both X-ray and optical bands with densely sampled X-ray light curves might reveal this feature because the effect of strong gravity is much less in the optical band (see Chen et al. 2013).

4. CONCLUSION

We have combined Kerr lensing and standard microlensing to produce what we call Kerr+Micro lensing light curves. We have studied the effect of Kerr lensing on microlensing observations using a simple X-ray source geometry and two simple models for the foreground microlensing. The strong lensing of the X-ray emission by the central supermassive black hole changes the size, shape, and profile of the original X-ray emission. Kerr strong gravity changes the observed quasar X-ray microlensing light curves in a nontrivial way. Kerr+Micro lensing light curves have a larger amplitude of magnitude fluctuation for larger inclination angle. In particular, Kerr lensing can reduce or increase the amplitude of the magnitude fluctuation of microlensing light curves by ∼0.65–0.75 mag depending on the inclination angle of the observer, the spin of the black hole, and the intensity profile of the corona (see Figure 6).
Consequently, current quasar microlensing observations might have over or underestimated the X-ray emission sizes. We estimate this bias using a simple metric based on the amplitude of magnitude fluctuation (refer to Section 3.4). For most of the cases we considered, we found an underestimate of the half-light radius (up to ~50%). An overestimate of half-light radius (up to ~20%) was only found for the $a = 0.998$, $n = 0$, $\theta = 15^\circ$ case (Figure 9). We conclude that current microlensing size measurements generally underestimate the true size of the X-ray emission regions, and that more accurate constraints should be obtainable from microlensing observations by including Kerr lensing. Furthermore, it should be possible to measure the inclination angle and the black hole spin by analyzing Kerr+Micro lensing light curves. This new lensing model might help in breaking the parameter degeneracy of other methods measuring AGN black hole spin and/or inclination angle, e.g., by broad Fe K\alpha line shape (Reynolds & Fabian 2008; de La Calle Pérez et al. 2010), continuum fitting (Davis et al. 2006; Shafee et al. 2006; Czerny et al. 2011), polarization of the continuum emission (Dovciak et al. 2008; Li et al. 2009; Schnittman & Krollik 2009), and high-frequency quasi-periodic oscillations (Török et al. 2005; Middleton et al. 2011; Das & Czerny 2011).

Applications to other X-ray models such as an X-ray ball or an X-ray disk above the black hole (Chen et al. 2013) are straightforward and we expect the results to be similar, provided that the X-ray source is of a similar size to those assumed here, and that they are close to the central black hole. We have chosen a cutoff of the X-ray emission at the innermost stable circular orbit. In principle, it is possible to extend the X-ray emission region to within $r_{\text{ISCO}}$, where the gas follows so-called plunging trajectories along the geodesics (Agol & Krollik 2000). We expect that the effect of Kerr lensing will be more important for that case. As a simple demonstration, we have only probed Kerr lensing effects by using two typical magnification patterns each with a single crossing path. To fully characterize the effects of Kerr lensing for microlensing applications, detailed modeling of a known microlensed quasar with well-sampled light curves in the X-ray band, such as Q 2237+0305 (Chen et al. 2012) and RXJ 1131−1231 (Chartas et al. 2012), combining the double lensing technique introduced in this paper and Bayesian Monte Carlo analysis (Kochanek 2004) will be pursued in future work.

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