Pile–soil interactions in frozen soil foundations based on frost heave rate and soil stiffness variations

Yalong Zhou\textsuperscript{1}, Xu Wang\textsuperscript{1}\textsuperscript{,2}* , Deren Liu\textsuperscript{1}, Fei He\textsuperscript{1} and Fujun Niu\textsuperscript{3}

\textsuperscript{1}School of Civil Engineering, Lanzhou Jiaotong University, Lanzhou, Gansu 730070, China
\textsuperscript{2}National and Provincial Joint Engineering Laboratory of Road & Bridge Disaster Prevention and Control, Lanzhou 730070, China
\textsuperscript{3}State Key Laboratory of Frozen Soil Engineering, Northwest Institute of Eco-Environment and Resources, Chinese Academy of Sciences, Lanzhou 730000, China
*Corresponding author’s email: publicwang@163.com

Abstract. The frost heave rate and stiffness of soil in frozen soil foundations varies remarkably with the freezing depth under the influence of the freezing-end temperature, temperature gradient, and overburden stress. The stress level of frozen soil increases with increasing frost penetration, leading to greater stiffness and a smaller frost heave rate. In this work, the freezing-end temperature and temperature gradient are introduced to the traditional Takashi model via the frost heave layering method and the overburden stress is modified to establish an empirical prediction model of frost heave rate and stiffness. Analytical solutions of load transfer differential equation of single pile considering stiffness increase and frost heave rate decrease along the frost penetration are derived on the basis of the mechanism of pile–soil interactions. The calculation results are in good agreement with measured values, thus confirming the correctness of the analytical solution. Variations in frost heave rate and soil stiffness with increasing depth are then combined to form four models, and changes in axial force and tangential frost heave force as a function of frost penetration are calculated and analyzed on the basis of these models. Results show that calculation of the frost heave rate and stiffness of soil along the frost penetration depth according to normal values will significantly underestimate the axial force of on the pile.

1. Introduction
Implementation of The Belt and Road Initiative has led to increased engineering activities, such as construction, railway, highway, oil and gas pipeline, and electric power engineering, in permafrost regions. Foundation forms, such as prefabricated buried and bored pile foundations, are often applied in frozen soil engineering [1]. Frost jacking of foundations is a common engineering disaster in frozen soil engineering. Under the action of multiple freeze–thaw cycles, the pile foundation may experience frost jacking displacement and even pull-out failure. Serious frost jacking displacement of
transmission towers and other foundations may result in failure of these structures, thereby causing massive economic losses [2-4].

The active layer of permafrost foundations shows frost heave when it is frozen during the cold season. In the presence of a pile foundation, the frost heave deformation of the foundation soil is constrained and a frost heave force is produced on the foundation. Pile–soil interactions in frozen soil foundations are much more complex than those in general soil. Mindlin elastic theory and the load transfer method are commonly used to analyze general pile–soil interactions. However, these approaches may not be applicable to pile–soil interactions in frozen soil foundations. Zhou Y. [5] believed that the Linell and Kaplar formula could closely simulate field measurements and put forward a simplified formula for calculating frost heave force based on the field observation data of Yanjiagang Station in Northeast China. Lai Y. [6] used the superposition principle and Mindlin formula of elastic half space to propose an integral equation for calculating the three-dimensional frost heave force of pile foundations. The distribution of tangential and horizontal frost heave forces as a function of depth was obtained through simple discretization. He F. [7] combined the findings of Lai Yuanming and other scholars with classical viscoelastic mechanics theory and gave an expression of the three-dimensional viscoelastic problem of the frost heave force of pile foundations through theoretical derivation. Wu Y. [8] introduced the interfacial element of pile–soil interactions and established a calculation model of pile foundations in frozen soil considering the coupled effects of temperature, water, and stress fields. Lu [9] proposed a model to predict the frost heave effect on a single pile embedded in frozen soil. Abdulrahman [1] systematically studied the load transfer law of pile foundations in frozen soil.

Many researchers and scholars have carried out a considerable amount of work on the theoretical formulas of pile–soil interactions in frozen soil foundations and obtained valuable results. However, the available formulas basically assume that the frozen soil foundation is a uniform single soil layer and ignore changes in frost heave rate and stiffness as a function of frost penetration. In practical engineering, the soil is not composed of a single layer and the frost heave rate and stiffness of frozen soil vary markedly under different stress levels; in fact, the greater the stress level, the smaller the frost heave rate and the greater the soil stiffness [10-14]. This study uses the frost heave layering method [15] to introduce the cold-end temperature and temperature gradient to the traditional Takashi frost heave model and modifies the overlying stress to establish an empirical prediction model of frost heave rate and stiffness. A theoretical formula to solve the load transfer of a single pile in frozen soil foundations is then derived on the basis of the decrease in frost heave rate and increase in soil stiffness as a function of frost penetration according to the mechanism of pile–soil interactions. The calculation results are compared with actual measurements, and findings indicate that the proposed formula could provide a reliable theoretical basis for engineering design.

2. Simplified model of the interaction between frozen soil and pile foundations

2.1. Model of the layered frost heave of soil

The soil along the freezing depth affected by frost heave is divided into \( n \) layers, the cold-end temperature and temperature gradient are introduced to the traditional Takashi frost heave model, and the overlying stress is corrected. Then, the frost heave rate of layer \( i \) is [16]:

\[ h_i = \frac{1}{\kappa_i} \left( \frac{1}{2} \Delta T_i - \Delta T_0 \right) \]

where \( h_i \) is the frost heave rate of layer \( i \), \( \kappa_i \) is the thermal conductivity of layer \( i \), \( \Delta T_i \) is the temperature difference between the cold end and the ambient temperature, and \( \Delta T_0 \) is the temperature difference between the cold end and the initial temperature.


\[ \varepsilon_i = X_1 + \frac{X_3}{(\sigma_i + X_2)\sqrt{\text{grad}T_i}} + X_4\sqrt{\text{grad}T_i} \]  \hspace{1cm} (1)

\[ \sigma_i = \sigma_{i1} + \gamma_i(z - Z_{i1}) \]  \hspace{1cm} (2)

\[ \text{grad}T_i = \frac{(T_0 - T_C)}{h} \]  \hspace{1cm} (3)

where \( \varepsilon_i \) is the frost heave rate, \( \sigma_i \) is the effective stress in the freezing direction (kPa), \( X_1, X_2, X_3, \) and \( X_4 \) are the four soil test parameters, \( T_0 \) is the temperature at the freezing front of soil layer \( i \), \( T_C \) is the cold-end temperature of soil layer \( i \), and \( h \) is the thickness of soil layer \( i \).

We assume \( n \) layers of soil along the depth affected by frost heave. In this case, the lower limit depth of each layer of soil is \( Z_i \) (\( i = 1, 2, \ldots, n \)), \( Z_n \) is the depth of the frozen soil foundation, and the final frost heave amount of layer \( i \) is:

\[ h_i = \int_{Z_{i1}}^{Z_i} \varepsilon_i dZ \]  \hspace{1cm} (4)

Therefore, the total amount of the frost heave of the frozen soil foundation at depth \( z \) is obtained as follows:

\[ H_{\text{ult}}(z) = \sum_{j=i+1}^{n} h_j + h_i \quad (Z_{i+1} \leq z \leq Z_i) \]  \hspace{1cm} (5)

### 2.2. Pile–soil load transfer model

The theoretical formula of foundation pile interactions in frost heave soil is obtained under the following assumptions:

(1) The frozen soil foundation is divided into two layers. The upper layer is the active layer where frost heave occurs with a thickness of \( h_1 \). The lower layer is the permafrost layer with no frost heave and a thickness of \( h_2 \).

(2) The pile is a uniform cross-sectional pile, the nonlinear compression characteristics of the pile concrete are not considered, the compression deformation of pile is elastic, and no crushing or pulling phenomenon occurs.

(3) The frost heave rate decreases with increasing depth of frost penetration, and the variation relationship is determined by Equations (1), (2), and (3). The relationship between the frost heave and frost penetration is determined by Equations (4) and (5).

(4) The shear modulus of the pile–soil transfer spring increases with the depth, where \( G_s(z) = G_{s0} + kZ \).

(5) The pile–soil interface does not slide. The displacement of the pile and soil is positive in the downward direction, and the axial force of pile is positive during compression.

A schematic of the pile–soil load transfer model is shown in Fig. 1. Here, the length of the pile is \( L \), the radius of the pile is \( r_0 \), the active layer is divided into \( n \) layers from top to bottom, and the permafrost layer is divided into \( m \) layers. Thus, the thickness of each micro segment is \( h = L/(n + m) \).
Figure. 1 Pile–soil load transfer model. (a) The pile–soil load transfer model considering linear elastic spring. (b) Variation of the frost heave amount as a function of frost penetration. (c) Variation of the shear modulus as a function of depth.

3. Load transfer law of the pile foundation under frost heave

Relevant research results show that the frost heave \( w_s(z, r) \) of the soil around a pile foundation is a function of the depth \( z \) and radial distance \( r \) from the pile axis. This relationship could be expressed by the following formula [17]:

\[
 w_s(z, r) = -H_{ult}(z) + f(z) \ln (r_m / r) \tag{6}
\]

where \( H_{ult}(z) \) is the extent of frost heave of soil beyond \( r_m \) radius at depth \( z \), \( f(z) \) is the vertical displacement function related to depth, and \( r_m \) is the maximum influence radius of the pile in the surrounding soil and generally calculated as \( r_m = 7d_0 \) [7].

From the displacement continuity condition \( w_s(z, r_0) = w_p(z) \) at the pile surface \( (r = r_0) \). Introduction of Equation (6) reveals that:

\[
 f(z) = -\frac{H_{ult}(z) + w_p(z)}{\ln (r_m / r_0)} \tag{7}
\]

where \( w_p(z) \) is the axial displacement of the pile.

Introducing Equation (7) to Equation (6) yields the following relationship:

\[
 w_s(z, r) = -H_{ult}(z) + \frac{H_{ult}(z) + w_p(z)}{\ln (r_m / r_0)} \ln (r_m / r) \tag{8}
\]

Combined with elasticity, the influence of radial displacement could be ignored, and the shear strain at any point in the soil is:

\[
 \frac{\tau_s(z, r)}{G_s} = -\frac{\partial w_s(z, r)}{\partial r} = \frac{H_{ult}(z) + w_p(z)}{r \ln (r_m / r_0)} \tag{9}
\]

where the relationship between the shear modulus \( G_s \) and the elastic modulus \( E \) of the soil is determined by the formula \( G_s = E/(1 + \nu) \) and \( \nu \) is Poisson's ratio.

The frictional resistance at the pile–soil interface under frost heave could be obtained by introducing \( r = r_0 \) to Equation (9) as follows:
\[ \tau(z, r_0) = \frac{G_s[H_{ult}(z) + w_p(z)]}{r_0 \ln(r_m / r_0)} \]  

\[ \frac{\partial P(z)}{\partial z} = -2\pi r_0 \tau(z, r_0) \]  

\[ \frac{\partial w_p(z)}{\partial z} = -\frac{P(z)}{E_p A_p} \]  

Equation (11) could be introduced to Equation (12), and the differential equation of the pile could be established according to Equation (10) as follows:

\[ \frac{d^2 w_p(z)}{dz^2} = \frac{2\pi G_s[H_{ult}(z) + w_p(z)]}{E_p A_p \ln(r_m / r_0)} \]  

Assuming \( \beta^2 = \frac{2\pi G_s}{E_p A_p \ln(r_m / r_0)} \):

\[ \frac{d^2 w_p(z)}{dz^2} - \beta^2 w_p(z) = \beta^2 H_{ult}(z) \]  

The differential equation of the pile satisfies the following equations over the whole range of pile lengths studied:

\[ \begin{cases} 
\frac{d^2 w_p(z)}{dz^2} - \beta^2 w_p(z) = \beta^2 H_{ult}(z), (0 \leq z \leq Z_n) \\
\frac{d^2 w_p(z)}{dz^2} - \beta^2 w_p(z) = 0, (Z_n \leq z \leq L) 
\end{cases} \]

This derivation is based on on-line elastic theory and yields the shear modulus. The geotechnical material is a type of nonlinear material. The frost heave rate and shear modulus of soil do not remain constant with increasing freezing depth; therefore, the above differential equations have no analytical solution and the finite difference method must be used to solve these equations. The pile and frozen soil foundation are divided into \( n \) units in the frozen depth range and \( m \) units below the freezing depth.
and the length of each unit is \( h \). The first and second derivatives of \( w_p(z) \) are as follows:

\[
\begin{align*}
&w_{p,1} = (w_{p,1} - w_{p,0}) / h \\
&w_{p,1} = (w_{p,1} + 2w_{p,2} + w_{p,3}) / h^2
\end{align*}
\] (16)

Equation (15) can be discretized into the following differential equation:

\[
\begin{align*}
w_{p,i+2} - 2w_{p,i+1} + (1 + \beta^2 h^2)w_{p,i} &= \beta^2 h^2 H_{ult,i}, & (0 \leq z \leq Z_n) \\
w_{p,i+2} - 2w_{p,i+1} + (1 + \beta^2 h^2)w_{p,i} &= 0, & (Z_n \leq z \leq L)
\end{align*}
\] (17)

The differential form of the boundary conditions is as follows:

\[
\begin{align*}
w_{p,2} - w_{p,1} &= 0 \\
w_{p,n+m+1} - w_{p,n+m} &= -\frac{K_b}{E_p} h w_{p,n+m+1}
\end{align*}
\] (18)

where \( K_b \) is the resistance coefficient of the foundation soil at the pile end.

Writing Equation (17) in the form of a matrix yields the following formula:

\[
\begin{bmatrix}
-1 & 1 & 1 \\
1 + \beta^2 h^2 & -2 & 1 \\
1 + \beta^2 h^2 & -2 & 1 \\
\vdots & \vdots & \vdots \\
1 + \beta^2 h^2 & -2 & 1 \\
1 + \beta^2 h^2 & -2 & 1 \\
1 + \beta^2 h^2 & -2 & 1 \\
-1 & \frac{K_b}{E_p} h & 1 + \beta^2 h^2
\end{bmatrix}
\begin{bmatrix}
w_{p,1} \\
w_{p,2} \\
w_{p,3} \\
\vdots \\
w_{p,n} \\
w_{p,n+1} \\
w_{p,n+2} \\
\vdots \\
w_{p,n+m} \\
w_{p,n+m+1}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
\vdots \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\] (19)

The \( \{w_{p,i}\} \) sequence is obtained from the above formula, and the axial force can be calculated as follows:

\[
\{p_i\} = \left\{ -E_p A_p \left( w_{p,i+1} - w_{p,i} \right) / h \right\}
\] (20)

4. Example verification

In this work, a cast-in-place concrete pile with a diameter of 0.3 m is considered. The length and elastic modulus of this pile are 2.5 m and 20 GPa, respectively. The foundation soil is sandy clay, the frost penetration is 2 m, the frost jacking displacement of the pile is 0.01 m, and the free frost heave amount \( H_{ult}(z = 0) \) is 0.192 m. The elastic modulus and Poisson’s ratio of the frozen soil are 243.24 MPa and 0.3, respectively. We assume that no relative sliding occurs at the pile–soil interface, that is, the frost jacking displacement of pile is equal to the frost heave of the soil on the pile surface. The whole pile is pulled under the action of frost jacking, and the resistance coefficient of the foundation soil at the pile end is taken as 0.

Variations in axial force and tangential stress with increasing depth could be obtained through the calculation method described earlier in this paper, as shown in Figs. 3 and 4, respectively.
The average unit tangential frost heave force obtained through field measurements is 84.28 kN/m² [6], and the calculation result determined via the method presented in [6] is 91.18 kN/m². The error between these values is 8.2%. By comparison, the result obtained via the calculation method presented in the present paper is 89.06 kN/m². This value reflects an error of only 5.7% relative to the actual measurements. The error calculated from the method described in this paper is 2.5% less than that in [6]. These results show that the pile–soil interaction model based on variations in frost heave rate and stiffness with increasing frost penetration can well simulate the actual situation in the field and that the method described in this paper is correct and reasonable.

5. Parameter analysis
The frost heave rate shows two distribution patterns, i.e., it decreases with increasing depth or remains unchanged. The shear modulus also shows two distribution patterns: it increases with increasing depth or remains unchanged. We combine the distribution patterns of frost heave rate and shear modulus to form four models. In model 1, the frost heave rate decreases with increasing depth and the shear modulus remains unchanged. In model 2, the frost heave rate remains unchanged with increasing depth and the shear modulus increases. In model 3, the frost heave rate remains unchanged with increasing depth and the shear modulus remains unchanged. Finally, in model 4, the frost heave rate decreases with increasing depth and the shear modulus increases. The load transfer characteristics of
the pile and soil are calculated using the four models, and the variations in axial force and tangential stress with increasing depth determined by these models are obtained as shown in Figs. 5 and 6, respectively.

![Figure 5](image-url) Variation of the pile axial force as a function of depth in different models.

![Figure 6](image-url) Variations in tangential stress with increasing depth in different models.

The calculation results of models 1, 2, and 3 are compared with those of model 4. Figures 5 and 6 reveal that model 1 overestimates the tangential frost heave force and axial force of the pile at the shallow part of the frost heave soil layer when the average shear modulus is used. Moreover, the model underestimates the anchoring effect of the permafrost soil layer on the pile; thus, the maximum axial force of the pile increases relative to that in model 4. Model 2 underestimates the tangential frost heave force and axial force of pile at the shallow part of the frost heave soil layer when a constant frost heave rate is used; however, the maximum axial force of pile body increases by approximately 24% compared with that obtained from model 4. Model 3 adopts the average shear modulus and constant frost heave rate; in this case, the maximum axial force obtained is approximately 22% lower compared with that obtained from model 4, and the tangential frost heave force remains constant despite increases in depth.

6. Conclusion
(1) The cold-end temperature and temperature gradient are introduced to the traditional Takashi frost heave model by using the frost heave layering method, the overlying stress is modified, and an empirical prediction model of frost heave rate and stiffness is established. Based on the mechanism of pile–soil interaction, a theoretical formula of load transfer in a single pile in a frozen soil foundation is derived on the basis of the decrease in frost heave rate and increase in stiffness along the frost penetration. The calculation results are verified by field test results.

(2) When the average shear modulus is used, the tangential frost heave force and axial force of pile at the shallow part of the frost heave soil layer are overestimated, the anchoring effect of the permafrost soil layer on the pile is relatively underestimated, and the maximum axial force of the pile increases. The tangential frost heave force and axial force of pile at the shallow part of the frost heave soil layer are underestimated when a constant frost heave rate is used, but the maximum axial force of the pile body is approximately 24% higher. The maximum axial force is approximately 22% lower and the tangential frost heave force remains constant despite increases in depth when the average shear modulus and constant frost heave rate are adopted.

(3) In this paper, the simplified model of the interaction between frozen soil and a pile takes into account changes in frost heave rate and stiffness with increasing depth, which resembles actual engineering situations, and the calculation results agree well with the field measurements. The research results could provide a reliable theoretical basis for pile foundation engineering designs in permafrost regions.

Acknowledgments
This work was supported by the National Natural Science Foundation of China (Grant No. 41902272) and the Research Fund of the State Key Laboratory of Frozen Soil Engineering (Grant No. SKLFSE201607).

References
[1] Abdulrahman, A. A. H. (2019) Load Transfer and Creep Behaviour of Pile Foundations in Frozen Ground. PhD Thesis, Carleton University, Ottawa, Canada.
[2] Yu, J. (2007) Design of low temperature triaxial testing machine and experimental study on cyclic freeze-thaw on the mechanical properties of silty caly. PhD Thesis, Graduate School of the Chinese Academy of Sciences, Beijing.
[3] Wang, S. J., Jin, L., Mu, K., Zhu, D., Dong, Y. (2017) Distresses and countermeasures of highway subgrade in plateau permafrost regions. Chinese Engineering Science, 19(6):140-146.
[4] Xiong, Z. (2005) Study on deformation mechanism and treatment measures of pier and abutment in permafrost area along Qinghai-Tibet Railway. PhD Thesis, China Academy of Railway Science, Beijing.
[5] Zhou, Y. (1985) According to the frost heave within the scope of foundation constraints to calculate the frost heave force. Journal of Glaciology and Geocryology, 7(4): 335-346.
[6] Lai, Y., Zhu, Y., Wu, Z. (1998) A simple integral equation method for three-dimensional frost heaving force problem of piles. Journal of the China Railway Society, 20(6): 93-97.
[7] He, F., Wang, X., Jiang, D., et al. (2015) Research on three-dimensional viscoelastic frost heaving force problem of pile foundation. Rock and Soil Mechanics, 36(9): 2510-2516.
[8] Wu, Y., Zhu, Y., Guo, C., et al. (2005) Multi-field coupling analysis model of pile foundation in cold region and its application. Chinese Science (Series D: Earth Science), 35(4): 378-385.

[9] Lu, J. F., Yin, J., Shuai, J. (2018) A model for predicting the frost-heave effect of a pile embedded in the frozen soil. Cold Regions Science and Technology, 146: 214-222.

[10] Zheng, H., Kanie, S., Niu, F., & Li, A. (2015) Three-dimensional frost heave evaluation based on practical Takashi's equation. Cold Regions Science and Technology, 118, 30-37.

[11] Wang, Y. T., Wang, D.Y., Ma, W., et al. (2016) Experimental Study of development of cryostructure and frost heave of Qinghai-Tibet silty clay under one-dimensions freezing. Rock and Soil Mechanics, 37(5):1333-1342.

[12] Bronfenbrener L., Bronfenbrener R. (2010) Modeling frost heave in freezing soils. Cold Regions Science and Technology, 61(1): 43-64.

[13] Cheng, P. F., Yin, C. J. (2014) Analysis of frost heaving characteristics of silty clay in seasonal frozen region. Journal of Highway and Transportation Research and Development, 31(1), 44-49.

[14] Zhang, D. F., Yang, J., Li, L. Y. (2016) Analytical solutions of pile-soil interaction in expansive soil foundation with expansion rate and stiffness variation along the depth. Engineering Mechanics, 33(12), 86-93.

[15] Chen, J., Cheng, G. D., Zhang, X. F. (2012) Study on the Observation of Delamination Frost-Heave Amount. Journal of Glaciology and Geocryology, 26(4):466-473.

[16] Geng, L. (2016) Mechanical model and numerical prediction on deformation of soil under the action of frost heave. PhD Thesis, Harbin University of Technology, Harbin, China.

[17] Xiao, H. B., Zhang, C. S., Wang, Y. H., et al. (2011) Pile-soil interaction in expansive soil foundation: Analytical solution and numerical simulation. International journal of geomechanics, 11(3), 159-166.