Two-Analytical Comparison on Force Measurements and The Wave Pattern

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Abstract. This work presents a comparison of two analytical methods on the components of resistance, transverse and divergent waves, and the profile and contour of the waves. Michell's thin linearization theory is applied to estimate the wave resistance and analysis of the method compared to the solution of the wave resistance coefficient based on the Rankine source method. The hull of series 60 is utilized as the initial hull, and varying the shape of the bow to the mid-ship with the hull offset variable. Computation of the wave resistance coefficient at low Froude number (Fr<0.3) shows the results of the wave resistance from the thin ship theory are higher, but it gives a graphical agreement with the Rankine source method. The wave profile provides a very similar profile line for both of them. However, the thin ship theory does not establish a significant difference in the wave profile from the initial hull of the improved hull variation. Comparison of contour displays is presented in different versions of the wave contour. Still, it affords a good contour match on the initial hull and the results of hull variations from both methods. At high Froude number (Fr>0.7), the initial hull and variations in the hull have minimal effects with small differences in wave resistance, profile and contour.

Keywords: analytical methods, Michell's thin theory, Rankine source method, series 60.

1. Introduction
Numerical algorithms in evaluating hydrodynamic performance in the preliminary design stage can produce a substantial reduction in the cost of the ship. Technical analysis with Computational Fluid Dynamics (CFD) is commonly utilized in engineering design problems. This method can make excellent and fast calculations in the process of improving the shape of the hull. However, the excessive preprocessing of computation takes a really long time requires an enormous RAM capacity. Sometimes even have to use several processors working at once to get the results of hydrodynamic performance. Nevertheless, the model test method is the best approach in specifying the best form of the hull even though it requires a great deal of cost, time and effort.

This work represents the method of resistance calculating the hull form based on thin ship theory approach. Several studies from [1] - [5] have widely discussed this theory in several hull shapes, both monohull and Multihull. In general, their answers indicated that the computation of wave resistance based on the thin ship theory from Michell is quite good both in monohull and Multihull with the test data on the towing tank. [6] Developed this theory into a simple CFD method. Moreover, [7] delivered a simple tool "Michlet and Flotilla" for hydrodynamic calculations. The second author of this paper used a computation of [7] in investigating the hydrodynamic characteristic of pentamaran and compared with the experimental, [8] - [10]. Those tools perform hydrodynamic calculations which are
entirely consistent with the results of experiments and able to display the contour of the waves behind the ship.

The aim of this research is to investigate the accuracy of Michell's thin ship theory when compared to the Rankine source method commonly used by many researchers. This method is also considered as a fast, efficient and high-precision numerical method of waves in potential flow theory. In the conception of the design phase, this method is useful for medium-speed, and high-speed ship in reducing wave resistance. [11] combined the Rankine source method and experiments to get the configuration of the ship with minimum resistance by modifying the hull of [12]. [13] used the B-Spline function parameter to optimize the hull of the series 60 based on the Rankine source method with the nonlinear programming (NLP). This study took the hull form provided by [13] and compared with the solutions of the wave resistance coefficient based on Michell's thin ship theory with a program of [10]. These investigations are also carried out on resistance components, transverse and divergent waves, wave profiles and contour wave in the near-field.

2. Theoretical approach
Wave resistance is according to thin ship theory [1] based on the ratio of length to width of the shape of the ship as thin. The centre of the formation of a wave profile as $z = \zeta(x, y)$ moves at various angles $\theta$ propagation relative to the negative $x$-axis to the motion of the ship. The wave propagation is expressed by

$$\zeta(x, y) = R_w \int_{-\pi/2}^{\pi/2} A(\theta) e^{-ik(\theta)\omega(x,y,\theta)} d\theta$$

where $A(\theta)$ is a complex amplitude function, concerned with the wave elevation, expressed as a free wave spectrum or the Konchin function as in [2]; $k(\theta) = k_0 \sec^2 \theta$ wave number; $R_w$ is the wave resistance and $\omega$ as a phase function $\omega = (x, y, \theta) = x \cos \theta + y \sin \theta$. Then the equation (1) be expressed as:

$$\zeta(x, y) = R_w \int_{-\pi/2}^{\pi/2} A(\theta) e^{-ik(\theta)\omega(x,y,\theta)} \left[ x \cos \theta + y \sin \theta \right] d\theta$$

(2)

where the wave amplitude function $A(\theta)$ is determined by the factor of hull form that expressed by:

$$A(\theta) = -\frac{2i}{\pi} k_0^2 \sec^4 \theta \iint Y(x,z) e^{ik_0 z \sec^2 \theta + ik_0 \sec \theta} dx dz$$

(3)

The wave system as the wave resistance, $R_w$ is:

$$R_w = \frac{2}{\pi} \rho V^2 k_0^4 \int_{-\pi/2}^{\pi/2} \left| A(\theta) \right|^2 \cos^3 \theta d\theta$$

(4)

where $Y(x,z)$ is a data set of ships with $x$ from the bow to the stern, $y$ to the right, and $z$ up from the free surface; $w$ is the centre of the ship, $V$ is the speed of the ship, and $\rho$ is water density. The integral in the bar of the equation (4) is a complex amplitude function $A(\theta)$, then wave resistance $R_w$ can be written:

$$R_w = \frac{\pi}{2} \rho V^2 \int_{-\pi/2}^{\pi/2} \left| A(\theta) \right|^2 \cos^3 \theta d\theta$$

(5)

$R_w$ is a combination of a divergent and transverse wave. The propagation angle which sets the maximum limit of the numerical integration of the wave resistance and the wave amplitude integral function replaces the maximum limit of integration $\pi/2$. Transverse waves to be those propagating
between $\theta = 0$ and $|\theta| = \arcsin(1/\sqrt{3}) = 35.3^\circ$. The minimum $\theta$ value for divergent waves occurs at the wave angle and is $35^\circ 16'$. Then in determining total resistance $R_T$ that the sum of wave resistance $R_W$ and viscous resistance $R_V$.

$$R_T = R_W + R_V$$

(6)

$$R_T = R_W + R_V (k+1)$$

(7)

Form factor $(k+1)$ is determined following [14], $k = 0.0097(\theta_{entry} + \theta_{exit})$, where $\theta_{entry}$ and $\theta_{exit}$ are the degrees of half-angles the bow and stern at the waterplane. Thus, the total resistance coefficient $C_T$ based on the thin ship is calculated by:

$$C_T = \frac{R_T}{0.5 \rho U^2 S}$$

(8)

where $S$ is the wetted area of the ship. And wave resistance coefficient $C_w$:

$$C_w = \frac{R_W}{0.5 \rho U^2 S}$$

(9)

While in Rankine source method, the factor $k$ calculated by:

$$k = \frac{\left(\frac{1}{3}\gamma + 0.031 \rho \gamma \right)}{(1.3(1 - C_b) - 0.031 l_{cb})}$$

(10)

where $\gamma = \left(\frac{b}{L}\right)$, $V$ is volume displacement, $b$ is hull breadth, $C_b$ is the block coefficient, $\gamma$ is the degree of stern, $L$ is ship length between perpendiculars, and $l_{cb}$ is the longitudinal position of the centre of buoyancy. $R_W$ is determined based on [15]:

$$R_W = 1/2 \rho U^2 L^2 C_w$$

(11)

The formulation of friction resistance coefficient $C_f$ of both, Michell integral method and Rankine source method with a standard of the 1957 ITTC that is:

$$C_f = 0.0755/ \left(\log_{10} \text{Re} - 2\right)^2$$

(12)

Moreover, based on equation (9) to determine the transversal and divergent wave resistance, with the same divider ($\rho, V$, and $S$) in the formation of $C_w$, the transversal wave resistance coefficient ($C_w$-trans) and the divergent resistance coefficient ($C_w$-div) are defined:

$$C_w\text{-trans} = \frac{R_{w\text{-trans}}}{0.5 \rho V^2 S}$$

(13)

$$C_w\text{-div} = \frac{R_{w\text{-div}}}{0.5 \rho V^2 S}$$

(14)

3. Modelling hull shape

This study is using the tool of [7] in investigating the components of resistance and describe the profile and contour of the wave. The hull $y = \pm Y(x, z)$ is defined by inputting the offset setting as the grid along with the hull station and waterline as a data. Conforming to the objectives of this study, the hull used four models of series 60 adopted the hull form provided by [13], with main dimension as in Table 1. Each hull model with offset setting as the grid into 21 stations and 21 water lines with red dot identification, as presented in figure 1. The body plans of four series 60 are shown in figure 2,
identified as the IniI hull (black line), Sch-1 hull (blue line), Sch-2 hull (red line) and Sch-3 hull (green line).

| Table 1. The dimension of hull |
|-------------------------------|
| Dimension                     | Symbol | Value (m) |
| Length overall                | Loa    | 2         |
| Breadth                       | B      | 0.270     |
| Depth                         | D      | 0.137     |
| Draught                       | T      | 0.107     |
| Coefficient block             | Cb     | 0.6       |

Figure 1. Offset set of hull station and waterline

Figure 2. Four of series 60: black line as IniI, blue as Sch-1, red as Sch-2 and green as Sch-2

4. Comparison of variance models
The results of the computation of the component of resistance coefficients based on thin ship theory are presented in figure 3, and figures 5 – 8. While the wave profiles in figures 9 – 14 and the wave contour are captured in figures 15–18.

4.1. Resistance components
The comparison of the wave resistance coefficient at Froude number $Fr$ 0.24 – 0.32 based on Michell's thin ship calculation is shown in figure 3. This Froude number is specified according to the results of calculations based on the Rankine source method from [13] in figure 4.

Comparison of the wave resistance coefficient based on thin ship theory (figure 3) and Rankine method (figure 4) shows a slight agreement on Sch-1 or Scheme 1 has the most significant wave resistance than others. The results of wave resistance from thin ship theory higher than Rankine method. It can be predicted that the thin ship theory is less precise at low speed, less than Fr 0.4. Computed by the Rankine method gave Initial hull lower than other hulls at the specified design speed, Fr 0.285 see ref. [13] where no analysis carried out at high speeds ($Fr > 0.32$).

The estimate of Michlet tool based on Michell's thin ship in figures 5 – 6 for total resistance coefficient and the wave resistance coefficient at variations $Fr$ 0.1 – 1.0. Whereas figures 7 – 8 shows the transverse wave resistance coefficient and the divergent wave resistance coefficient. High Fr was
chosen to investigate the series 60 hull while running at a higher speed than Zhang’s research. It is has conformed with the near-thin hulls that generally for the high-speed ship.

**Figure 3.** Wave resistance coeff. ($C_W$) based on Michell’s thin ship at low speed

**Figure 4.** Wave resistance coeff. ($C_W$) based on Rankine method by Zhang (2009)

**Figure 5.** Total resistance coefficient ($C_T$) based on Michell’s thin ship

**Figure 6.** Wave resistance coefficient ($C_W$) based on Michell’s thin ship
4.2. Wave profile the near-field

Figure 9 – 14 gives a wave profile of the series 60 hull variation. Figures 9, 11 and 13 show the wave profiles by Rankine source method of Zhang’s research. Whereas figures 10, 12 and 14 are the near-field profiles of the wave from Flotilla tool of [10] based on Michell’s thin ship. The profile is taken by the wave-cut method at position $y/B$ 0.5 from the centerline at higher wave resistance in low speed, $Fr > 0.3$.

Looking at figures 9, 11 and 13, the near-field profiles of wave profile by Rankine source method show a reasonably clear difference between Initial and three improved hulls. In contrast to the results of thin ship theory in figures 10, 12 and 14, which does not indicate a significant difference in the wave profile even though the calculation gives considerable enough different value. However, cutting the wave profile of the two methods gives a profile line that is quite similar.
4.3. Wave contour in the near-field
Some researchers indicate that wave contour describes the wave resistance generated by the ship. The wave contour of the series 60 hull of Flotilla is presented in figures 15 – 18. Capturing of a contour wave is taken at position x / L: -0.5 to 1.5 from the hull. Minimal changes in the Sch-1 to Sch-3 hulls from the initial hull of series 60 as parent hull, resulting in the wave contour is enough substantial differences from Michell's thin ship. While a little differences shape of hull brings a wave contour of four hulls was obtained very slight change from capturing of Zhang's research.

Figures 15 shows the contour at the initial hull of two methods gives a very similar picture. In figure 16, the contours of the two methods look quite different. From capturing of Michell's thin ship method, the Sch-1 hull produces a broader spread of waves. When viewed from figure 17 (Sch-2) and figure 18 (Sch-3), the capturing of the wave contour of the Sch-1 hull (figure 16) appears to be longer and broader.
It is sufficient to support the results of calculations on the wave resistance in Figure 3, where the Sch-1 hull produces the highest value than the others. Comparison of contour viewing from the initial and three improved hulls showed the Sch-1 hull produces waves bigger than others hull. Although presented in a different version of the wave contour, there appears to be a good contour match on the initial hull and three improved hulls from both methods.

5. Conclusion
Comparisons between the two numerical methods have shown that Michell’s thin ship theory is less precise at low Froude numbers, Fr <0.4. The formula approach in defining the form factor $k$ from the two methods, forms the basis of significant deviations of resistance where the viscous factor is still very influential at low speeds. Nevertheless, this method can provide graphical results that are entirely consistent with the Rankine method of total resistance, wave resistance, profile and wave contour.

The wave resistance at low speeds based on the theory of thin ships in the initial hull and the three hulls improved gives a little compatibility in the graphics with the Rankine method, although with a substantial difference in value. Whereas at high speeds, the waves produced by the initial hull and three enhanced hulls carry small differences in profile and contour. This is possible because small changes in shape have minimal effects at high speeds.

The use of Michell’s thin ship theory is only confined to practical matters for the initial stages of ship design in calm water conditions. The concept of ship design also requires a more comprehensive analysis related to seakeeping, ship manoeuvrability, as well as topics relating to water limitations. Furthermore, it is necessary to compare with other methods related to the three-dimensional flow around the hull and evaluate the results with experiments at higher speeds.

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