Great attention has been devoted to an experimental and theoretical study of dc biased series arrays of Josephson junctions [1, 2]. Such a system displays diverse fascinating nonlinear classical and macroscopic quantum-mechanical phenomena. E.g. a resistive state of Josephson junction series arrays can show synchronized behavior [1, 2], and this effect has been used in Josephson voltage standard devices [3]. As we turn to a region of small dc bias currents and low temperatures, the macroscopic quantum-mechanical phenomena start to play a role. Thus, a quantum phase superconductor-insulator transition has been observed in artificially prepared series arrays of a small size Al/Al₂O₃/Al junctions [4]. All these effects strongly depend on the interaction between Josephson junctions.

This field of research, i.e. the macroscopic quantum phenomena in spatially extended superconducting systems, has been boosted even further by recent discovery of macroscopic quantum tunneling (MQT) in single crystals of layered high-$T_c$ superconductors [5, 6]. At low temperatures the MQT determines the escape rate of the switching from the superconducting state to a resistive one. Although the MQT of a single Josephson junction has been studied long time ago in low-$T_c$ superconductors [5, 6], MQT in layered high-$T_c$ superconductors has shown many novel features. A most unexpected result is that the MQT escape rate $\Gamma_{MQT}$ in high-$T_c$ superconductors is four order of magnitude larger than the MQT escape rate for a lumped Josephson junction having the same parameters [4]. Moreover, the crossover temperature $T^*$ from the thermal fluctuation regime to the MQT regime increases in respect to a lumped Josephson junction. In these experiments it was also found that the escape rate $\Gamma_T$ in the thermal fluctuation regime did not differ from the escape rate of a single Josephson junction.

Layered high-$T_c$ superconductors can be modelled as a stack (a series array) of intrinsic Josephson junctions [4]. A modern fabrication technique allows to prepare single crystals of layered high-$T_c$ superconductors with an extremely homogeneous distribution of critical currents of intrinsic Josephson junctions, and a low level of dissipation [5, 6]. Since in the model of independent Josephson junctions the escape rate $\Gamma$ is just proportional to $N$, an enhancement of the MQT observed in layered high-$T_c$ superconductors stems from an interaction between intrinsic Josephson junctions. All experimental observations receive a natural explanation in a simple model [10] of Josephson junctions series array with an intrinsic charge interaction between nearest-neighbor Josephson junctions [11, 12]. Moreover, such a model allows quantitative comparison with experimental results, and a good agreement has been found as the Debye screening length is of the order of superconducting layer thickness [12].

However, the authors of Ref. [6] proposed an other model in order to explain a giant increase of the MQT escape rate. In this model intrinsic Josephson junctions are globally coupled due to the presence of an external shunting impedance $Z$ (see schematic in Fig. 1). An influence of electromagnetic environment, and in particular, a shunting impedance $Z$ on the MQT in a lumped Josephson junction has been studied long time ago in Refs. [13, 14, 15]. It was shown that the presence of a small shunting impedance can lead to a strong suppression of the MQT in a lumped Josephson junction. Therefore, natural questions arise: what is a role of shunting impedance $Z$ in the macroscopic quantum dynamics of large Josephson junction series arrays ($N >> 1$) and what is a most appropriate model in order to explain a giant increase of the MQT escape rate in Josephson junction series arrays?

In order to answer these questions we carry out a quantitative analysis of the escape rate $\Gamma(J)$ in globally coupled Josephson junction series arrays. A global coupling is provided by an external shunting impedance $Z$ (see Fig.
and \( Q \) in series. In order to obtain the escape rate \( \Gamma \) we use
\[
\sum_{n} \frac{1}{2\omega_p} [\dot{\phi}_n(\tau)]^2 + \frac{1}{2\omega_R} q(\tau)^2 + \sum_{n} U_n + \frac{i}{\omega_R} \sum_{n} q(\tau) \dot{\phi}_n(\tau),
\]
and the Lagrangian of a series array with the shunting impedance is written as
\[
L = \sum_{n} \frac{1}{2\omega_p} [\dot{\phi}_n(\tau)]^2 + \frac{1}{2\omega_R} q(\tau)^2 + \frac{1}{2} q(\tau)^2 + \sum_{n} U_n + \frac{i}{\omega_R} \sum_{n} q(\tau) \dot{\phi}_n(\tau),
\]
where \( j = I/I_c \) is the normalized external dc current, and \( I_c \) is the nominal value of the critical current of a single junction. Here, \( \omega_p \) is the plasma frequency of a single Josephson junction in the absence of dc bias. The Lagrangian is expressed in units of \( E_J \), where \( E_J \) is the Josephson energy of a single junction. The \( q = Q/\sqrt{E_J C} \) is the normalized charge flowing through the impedance \( Z \). The shunting impedance is characterized by the resonance frequency \( \omega_R = 1/\sqrt{LC} \), where \( L \) and \( C \) are the impedance inductance and capacitance, accordingly. The coupling between the Josephson junction series array and the shunting impedance branch is described by parameter \( \alpha = \sqrt{L_J/I} \), and \( L_J = 2e/(hI_c) \) is the Josephson inductance.

Integrating \( \Theta \) over \( q(\tau) \) we obtain the effective action \( S_{\text{eff}} \) that depends on the variables \( \phi_n(\tau) \) only
\[
S_{\text{eff}}(\phi_n) = \sum_{n} \int_{0}^{\frac{\omega_B}{\omega}} d\tau \frac{1}{2\omega_p} [\dot{\phi}_n(\tau)]^2 + U_n
+ \frac{\alpha^2}{2} \int_{0}^{\frac{\omega_B}{\omega}} \int_{0}^{\frac{\omega_B}{\omega}} d\tau_1 d\tau_2 \cdot G_T(\tau_1 - \tau_2) \left[ \sum_{n} \phi_n(\tau_1) \right] \left[ \sum_{m} \phi_n(\tau_2) \right],
\]
where the kernel \( G_T(\tau) \) is determined as
\[
G_T(\tau) = \frac{k_B T}{\hbar} \sum_{m} \frac{e^{i\omega_m \tau}}{\omega_m^2 + \omega_R^2},
\]
\[\omega_m = m(2\pi k_B T)/\hbar, \ m = \pm 1, \pm 2, \ldots\]
Thus, the last term in Eq. (3) presents an effective global charge interaction, that is due to current fluctuations flowing through an external shunting impedance.

In the escape experiments \( E_J > \hbar \omega_p \), and the switching to a resistive state occurs as the dc current \( I \) is close to \( I_c \), and therefore \( (j-1) \ll 1 \). In this case the potential \( U_n(\varphi) \) is written as
\[
U_n(\varphi) = (1 - j) \phi_n(\tau) - \frac{\phi_n^3(\tau)}{6}. \tag{5}
\]
The escape rate is determined by the particular solution \( \phi_n(\tau) \) providing the extremum of effective action \( S_{\text{eff}} \). At high temperatures such a solution is determined by extremum points of the potential \( U_n \), and it is written as
\[
\phi_n^T = 2\sqrt{2(1 - j)\delta_n - \sqrt{2(1 - j)}},
\]
where \( \delta_n \) is a junction number where the fluctuation occurs. Since this solution does not depend on the time \( \tau \), we can immediately conclude that the last term in Eq. (3) does not give contribution to the escape rate exponent
\[
\Gamma \simeq \exp(-S(\phi_n^T)/\hbar). \tag{5}
\]
Note here that an absence of the dependence of the escape rate exponent in the thermal fluctuation regime on a number of junctions \( N \) is

![FIG. 1: Schematic of a dc biased layered high-Tc superconductor and a series array of Josephson junctions. A strongly localized instanton (dashed line) and a charge instanton with long tails (solid lines) are shown.](image)
where the eigenfunctions \( \phi_n \) are the solution of the non-local and inhomogeneous equation:

\[
\xi^2 \phi_n + 2\alpha^2 \omega_0^2 \sum_n \phi_n - 2\omega_0^2 \delta_{ln} \phi_n = (\lambda - \omega_0^2) \phi_n ,
\]

where \( \lambda \) are the eigenvalues of the Eq. (7), \( \omega_0 = \omega_0^0 [2(1 - j)]^{1/4} \) is the dc bias dependent frequency of oscillations on the bottom of potential well, \( U_n(\phi) \). The crossover temperature \( T^* \) is determined by the condition that there is the eigenvalue \( \lambda = 0 \) \[14, 18\]. In a global coupling case (Eq. 7) the crossover temperature is obtained as a solution of the particular transcendent equation:

\[
\xi^4 + \frac{2N\alpha^2 \omega_0^2}{\xi^2 + \omega_R^2} \left[ \xi^2 - (1 - 2/N)\omega_0^2 \right] = \omega_0^4,
\]

where \( \xi = \frac{2\pi k_B T^*}{\hbar} \). Thus, one can see that the crossover temperature \( T^* \) is strongly suppressed for short arrays \( N \approx 1 \) for both cases, namely, "inductive" \( (\omega_R \ll \omega_0, \text{and } \alpha \ll 1) \) or "capacitive" \( (\omega_R \gg \omega_0, \text{and } \alpha >> 1) \) types of an external impedance. However, \( T^* \) recovers to the value \( T^* = \hbar \omega_0 / (2\pi k_B) \) for long arrays \( N >> 1 \). Typical dependencies of \( T^*(N) \) are shown in Fig. 2.

Now we turn to the MQT regime, where the extremum point of the action \( S_{eff}\{\varphi_n\} \) is the "tau-dependent" instanton (bounce) solution. At zero temperature and in the presence of an external impedance \( Z \), a spatial-temporal instanton solution satisfies the equation:

\[
\frac{1}{\omega_p^2} \ddot{\varphi}_n(\tau) + \alpha^2 \sum_m \int_0^\infty d\tau G_0(\tau - \tau_m) \dot{\varphi}_m - \frac{dU_n}{d\varphi} = 0 ,
\]

where \( G_0(\tau) = \frac{1}{2\pi R} \exp(-\omega_R|\tau|) \). The solution of Eq. (3) has a following form: a large bounce solution localized on a particular junction \( l, \varphi_l(\tau) = f(\tau) \), and a small spatial-temporal tail solution distributed over a whole array (see schematic in Fig. 1, dashed line). The Fourier transform of the instanton tail is obtained as

\[
\sum_{n \neq l} \varphi_n(\omega) = -\frac{\alpha^2 (N - 1) \omega_0^2 \omega^2 g_0(\omega)}{\omega^2 + \omega_0^2 + \alpha^2 (N - 1) \omega_0^2 \omega^2 g_0(\omega)} f(\omega) ,
\]

where \( g_0(\omega) \) and \( f(\omega) \) are the Fourier-transform of \( G_0(\tau) \) and \( f(\tau) \), respectively. The bounce solution \( f(\tau) \) localized on the junction \( l \) is determined self-consistently from the equation:

\[
\frac{1}{\omega_p^2} \ddot{f} + \alpha^2 \int_0^\infty d\tau G_1(\tau - \tau) \dot{f}(\tau) - (1 - j) + \frac{f^2}{2} = 0 ,
\]

\[
G_1(\tau) = \int_0^\infty d\omega \frac{(\omega^2 + \omega_0^2) e^{i\omega \tau}}{2\pi \omega^2 + \omega_0^2 + \alpha^2 (N - 1) \omega_0^2 \omega^2 g_0(\omega)} .
\]

In the absence of a global coupling the instanton solution is strongly localized on a particular junction (see schematic in Fig. 1, dashed line), i.e \[1]\)

\[
\varphi_n(\tau) = f_0(\tau) \delta_{nl} = \frac{3\sqrt{2(1 - j)}}{\cosh^2(\omega_0 \tau/2)} \delta_{nl} .
\]

Substituting (10) in the expression (3) for the effective action \( S_{eff} \), and using a perturbative approach (similarly to Refs. \[13, \[15\]), i.e. \( f(\tau) \simeq f_0(\tau) \), we obtain the MQT escape rate (in physical units) as

\[
\Gamma_{MQT} \simeq \Gamma_0 \exp \left[ \frac{-72\epsilon f}{15h \omega_p} 2^{1/4}(1 - j)^{5/4}(1 + \chi) \right] ,
\]

where

\[
\chi = \frac{30\pi^2 \omega_0^2 g_p^2}{\omega_0^2} \int_0^\infty dx \frac{x^4(x^2 + 1) g_0(x) \sinh^{-2}(\pi x)}{x^2 + 1 + \frac{\omega_0^2}{\omega_0^2} (N - 1)x^2 g_0(x)} ,
\]

where

\[
g_0(x) = 1/[x^2 + (\omega_R/\omega_0)^2] .
\]

Here, the parameter \( \Gamma_0 \) is just proportional to \( N \). A parameter \( \chi \) having a positive value, characterizes a suppression of the MQT due to the presence of the charge.
interaction between Josephson junctions of the array and an external shunting impedance. For short Josephson junction array ($N \approx 1$), such a MQT suppression can be rather large for moderate values of $\alpha$. However, as we turn to large Josephson junction arrays ($N \gg 1$) a standard MQT behaviour is recovered. Quantitatively an enhancement of MQT depends strongly on parameters $\alpha$ and $\omega_R$. The expression (14) can be simplified in two limits: $\omega_R \gg \omega_0$ ("capacitative impedance") and $\omega_R \ll \omega_0$ ("inductive impedance") as

$$\chi = 5\alpha^2 \omega_p^2 \frac{1}{\omega_R^2 + \alpha^2 \omega_0^2} \omega_R \gg \omega_0;$$

$$\chi = 5\alpha^2 \omega_p^2 \frac{1}{\omega_0^2} \omega_R \ll \omega_0.$$ (16)

Typical dependencies of the MQT escape rate $\Gamma_{MQT}$ on the dc bias current $I$ for various values of $N$ are presented in Fig. 3. One can see a giant enhancement of the MQT escape rate as we turn from short to long Josephson junction arrays. This enhancement results from a decrease of the slope of the bias current dependence escape rate. Comparing our theoretical predictions with the experimental curves published in Ref. [6] (see Fig. 5 in Ref. [4]) we find a good agreement for both the crossover temperature $T^*$ and the dependence of $\Gamma_{MQT}(I)$ for the inductive type of a shunting impedance. Therefore, in order to choose between two models, i.e. a nearest-neighbor intrinsic charge interaction or external global charge coupling, one needs additional independent measurements of Debye screening length [10] or to tune the MQT by variation of $Z$.

In conclusion we have shown that the dissipative (decoherence) effects can be strongly suppressed in long ($N \gg 1$) Josephson junction series arrays with a global charge interaction. The both dissipation and global charge interaction can be introduced through an external shunting impedance. This effect manifests itself as a giant enhancement of the MQT escape rate for the switching from the superconductive state to a resistive one (see Fig. 3). A giant MQT enhancement is explained through an excitation of spatial-temporal charge instanton distributed over a whole array.

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