Implications of electron acceleration for high-energy radiation from gamma-ray bursts

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ABSTRACT

In recent work we suggested that photons of energy $>100$ MeV detected from GRBs by the Fermi Satellite are produced via synchrotron emission in the external forward shock with a weak magnetic field – consistent with shock compressed upstream magnetic field of a few tens of micro-Gauss. Here we investigate whether electrons can be accelerated to energies such that they radiate synchrotron photons with energy up to about 10 GeV in this particular scenario. We do this using two methods: (i) we check if these electrons can be confined to the shock front; and (ii) we calculate radiative losses while they are being accelerated. We find that these electrons remain confined to the shock front, as long as the upstream magnetic field is $\gtrsim 10\mu$G, and don’t suffer substantial radiative losses, the only condition required is that the external reverse shock emission be not too bright: peak flux less than 1 Jy in order to produce photons of 100 MeV, and less than $\sim 100$ mJy for producing 1-GeV photons. We also find that the acceleration time for electrons radiating at 100 MeV is a few seconds (in observer frame), and the acceleration time is somewhat longer for electrons radiating at a few GeV. This could explain the lack of $>100$ MeV photons for the first few seconds after the trigger time for long GRBs reported by the Fermi Satellite, and also the slight lag between photons of GeV and 100 MeV energies. We model the onset of the external forward shock light curve in this scenario and find it consistent with the sharp rise observed in the 100-MeV light curve of GRB080916C and similar bursts.

Key words: radiation mechanisms: non-thermal - methods: analytical - gamma-rays: bursts, theory

1 INTRODUCTION

The Fermi Satellite has detected 18 GRBs (Gamma-ray Burst) at energies $>100$ MeV by LAT (Large Area Telescope). This emission can be described as follows. The first 100 MeV photons arrive $\sim 1$ s (in the host galaxy frame) after the trigger time, for long GRBs; the trigger time is the time when low energy photons ($\sim 1$ MeV) are first detected by the GBM (Gamma-ray Burst Monitor aboard Fermi). The 100 MeV light curve rises fast until it peaks and then it decays as a single power-law for a long duration of time (of order $10^3$ s) – much longer than the duration of the lower energy photons detected by GBM – until it falls below the detector’s sensitivity. Radiation above 100 MeV from GRBs has been suggested to be produced via the synchrotron mechanism in the external forward shock (Kumar & Barniol Duran 2009, 2010); the external forward shock scenario was first proposed by Rees & Mészáros (1992), Mészáros & Rees (1993), Paczynski & Rhoads (1993), and since then it has been used widely, see, e.g., Mészáros & Rees (1997), Sari, Piran, Narayan (1998), Dermer & Mitman (1999), for a comprehensive review see, e.g., Piran (2004) and references therein. After our initial suggestion, many groups have also considered and provided evidence for this origin of the $>100$ MeV radiation (Gao et al. 2009; Corsi, Guetta, Piro 2010; De Pasquale et al. 2010; Ghirlanda, Ghisellini, Nava 2010; Ghisellini, Ghirlanda, Nava 2010). The magnetic field required for this model is consistent with being produced via shock-compressed seed magnetic field in the CSM (circum-stellar medium) of strength of a few tens of micro-Gauss. The peak of the 100 MeV light curve can be attributed to the deceleration time which is the time it takes for the GRB-jet to transfer about half of its energy to the external medium.

We investigate in this work whether electrons in the external forward shock can be accelerated to sufficiently high Lorentz factors, even for a small CSM magnetic field of a few tens of $\mu$G, so that the synchrotron radiation can extend to $\sim 10$ GeV as seen by Fermi/LAT for a number of GRBs. We study the electrons acceleration in the context of diffusive shock acceleration (e.g., Krymski 1977, Axford, Leer & Skadron 1978, Bell 1978, Blandford & Ostriker 1978, Blandford & Eichler 1987), which was developed for non-relativistic shocks and has now been developed to consider relativistic shocks (semi-) analytically (e.g. Gallant & Achterberg 2010).
1999, Achterberg et al. 2001) and recently using 2D particle-in-cell simulations (e.g. Spitkovsky 2008a,b, Keshef et al. 2009). We assume that the electrons acceleration proceeds in the Bohm diffusion limit and that the magnetic field downstream is simply shock-compressed upstream magnetic field (other possibilities are considered in, e.g., Milosavljević & Nakar 2006, Sironi & Ostriker 2007, Goodman & MacFadyen 2008, Couch, Milosavljević, Nakar 2008).

If the downstream magnetic field is simply the shock-compressed large-scale upstream field, then the field component perpendicular to the shock normal is amplified, while the parallel component is not. In this case, the downstream magnetic field will be mainly pointing to the direction perpendicular to the shock front normal, therefore particles trying to cross the shock front from downstream to upstream will find it difficult to catch up with the shock front, which moves with a speed of \( \sim c/3 \) with respect to the downstream medium (see, e.g., Achterberg et al. 2001, Lemoine, Pelletier & Revenu 2006, Pelletier, Lemoine & Marcus with Koj 2009). One way that the particles might return to the upstream is if there is efficient cross-field diffusion of particles, which might occur if turbulent magnetic field is produced downstream (Jokipii 1987, Achterberg & Ball 1994, Achterberg et al. 2001). In principle, the turbulent magnetic field could dominate the shock-compressed field throughout the downstream region. However, it seems that although some turbulence is present just downstream of the shock front it does not persist across the entire downstream region (see recent simulations by Sironi & Spitkovsky 2010 that show that magnetic field is amplified only right behind the shock front and returns to the shock-compressed value far downstream). In this case, much of the radiation is produced by particles swept downstream where the turbulence has died out and the magnetic field is consistent with the shock-compressed value. We also note that as long as the thickness of the turbulent magnetic field layer is smaller than the thickness of the shocked fluid divided by \((B_t/B_d)^2\) then the energy loss in the turbulent layer is small; \(B_t\) is the turbulent magnetic field strength and \(B_d\) is the shock-compressed magnetic field. Therefore, in this work we neglect energy loss in the turbulent magnetic field layer since it persists for a very short distance compared to the thickness of the shocked fluid (see, e.g., Keshef et al. 2009 and references therein).

This work is organized as follows. In Section 2 we address the question of high-energy electron confinement upstream and downstream of the shock front, and also radiative losses suffered by electrons in between acceleration. Also, in Section 2, we discuss the lag of the \(>100\) MeV light curves observed by Fermi LAT for several GRBs in light of our results on electron acceleration. In Section 3, we calculate the rise of the external forward shock light curve, taking into consideration the non-zero time to accelerate electrons to high enough energies so they can radiate at \(>100\) MeV. We present our conclusions in Section 4.

2 ELECTRON ACCELERATION FOR \(>100\) MEV EMISIION

2.1 Electron confinement

It is widely believed that electrons in non-relativistic shocks undergo diffusive shock acceleration. (e.g. Krymski 1977, Axford et al. 1978, Bell 1978, Blandford & Ostriker 1978, Blandford & Eichler 1987). In the context of relativistic shocks, it has been shown that electrons gain energy each time they cross the shock front by a factor of \(\sim 2\), except on the first crossing when they gain energy by a factor of the Lorentz Factor (LF) of the shock front (Achterberg et al. 2001).

In order for electrons to turn around while up/down stream and cross the shock front, their Larmor radius should be smaller than the size of the system, i.e. electrons should be confined to the system in order to be accelerated. In this subsection, we explore the confinement of electrons in the external forward shock model when the magnetic field in the unshocked medium, upstream of the shock front, is a few tens of \(\mu G\) in strength, and the magnetic field in the shocked medium, downstream of the shock front, is simply the shock-compressed upstream field.

The highest photon energy detected for Fermi GRBs is on the order of 10 GeV. We first calculate the random LF in the downstream co-moving frame, \(\gamma_e\), of electrons radiating 10 GeV photons via synchrotron radiation, because these electrons have the largest Larmor radius and thus give us stricter confinement requirements. The synchrotron frequency in observer frame is given by:

\[
\nu_{\text{syn}} = \frac{eB_d\gamma_e}{2\pi m_e c(1+z)}
\]

where \(\Gamma\) is the bulk LF of the shocked fluid measured in the upstream rest frame (lab frame), \(B_d\) is the magnetic field downstream (measured in the local rest frame), \(z\) is the redshift, \(m_e\) and \(e\) are the electron’s mass and charge, respectively, and \(c\) is the speed of light (Rybicki & Lightman 1979). We convert the synchrotron frequency to 10 GeV, i.e.,

\[
\nu_{\Gamma} = h\nu_{\text{syn}} / 1.6 \times 10^{-2} \text{ erg},
\]

where \(h\) is the Planck constant and 10 GeV corresponds to \(1.6 \times 10^{-2} \text{ erg}\). Using the convention \(Q_\nu = \nu^{1/2} / \mu \text{ cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}\) and solving the last expression for \(\gamma_e\) yields

\[
\gamma_e = 1.5 \times 10^5 \nu_{\Gamma}^{1/2} (1+z)^{1/2} \Gamma^{-1} B_u^{-1/2} R_{\text{syn}}^{-1/2},
\]

(1)

where \(B_u\) is the magnetic field upstream, which is the magnetic field in the CSM. To obtain (1) we have assumed that the magnetic field in the downstream region is \(B_d = 4\Gamma B_u\) (Gallant & Achterberg 1999, Achterberg et al. 2001); note that the shock front LF measured in the lab frame is \(\Gamma = \sqrt{2}\Gamma\), Blandford & McKee 1976), i.e. \(B_d\) is the shock-compressed magnetic field in the upstream (lab frame), which is what we have found for Fermi GRBs (Kumar & Barniol Duran 2009, 2010).

The electrons’ LF in the rest frame of the upstream plasma is \(\gamma_{\text{e} \Gamma}\), therefore, the Larmor radius in the upstream is given by

\[
R_{\text{e} \Gamma} = \frac{m_e c^2 \gamma_{\text{e}} \Gamma}{eB_u} = (2.6 \times 10^{19} \text{ cm}) \nu_{\Gamma}^{1/2} (1+z)^{1/2} B_u^{3/2} R_{\text{syn}}^{-1/2},
\]

(2)

where we made use of (1) to eliminate \(\gamma_e\). Comparing the Larmor radius with the size of the system upstream, \(R\), which is given by the blast wave radius in the host galaxy rest frame — \(R = 2c\Gamma^2 t/(1+z) \sim 10^{17} \text{ cm}\) (where \(t \sim \text{ few seconds}\) and \(\Gamma \sim 10^3\) is the blast wave Lorentz factor, e.g. Abdo et al. 2009a) — we find that \(R_{\text{e} \Gamma} \gg R\). This might suggest that electrons of \(\gamma_e \sim 10^5\) are not confined to the system. However, an electron upstream of the shock front travels only a distance \(\sim R_{\text{e} \Gamma}/\Gamma\) before returning to the downstream, because by the time the angle between electrons’ velocity vector and the normal to the shock front exceeds \(\sim 1/\Gamma\), the shock front catches up with the electron and sweeps it back downstream (Achterberg et al. 2001). Therefore, for electron confinement upstream one should compare \(R_{\text{e} \Gamma}/\Gamma\) with \(R\):

\[
R_{\text{e} \Gamma}/\Gamma = 0.26 \nu_{\Gamma}^{1/2} (1+z)^{1/2} \frac{1}{\Gamma^3} \frac{1}{B_u^{3/2} R_{\text{syn}}^{1/2}} = 1.1 \nu_{\Gamma}^{1/2} B_u^{3/2} R_{\text{syn}}^{1/2} (E_{54}/n_0)^{3/8},
\]

(3)

where \(E\) is the energy in the blast wave, \(t\) is the time since the burst trigger in observer frame, and \(n_0\) is the number density of particles in the CSM; in deriving the second equality we made use of the time
dependence of $\Gamma$ and $R$ in the external forward shock scenario for a homogeneous CSM (Sari, Piran, Narayan 1998). For $B_{\mathrm{u},-5} \approx 4$ found for the Fermi bursts, $R_{L,u} / (\Gamma R) \lesssim 0.2$, and, thus, electrons radiating at 10 GeV cannot escape from the upstream side of the shock front; note that this conclusion holds for at least several hours in the observer frame.

One should also check for electron confinement downstream. Here, the Larmor radius is smaller than it is upstream, because the magnetic field is larger by at least a factor of 4T due to shock compression. Therefore, the requirement for the confinement of electrons downstream is automatically satisfied whenever it is satisfied upstream.

We conclude that there is no problem confining external forward shock electrons that radiate $\sim$10 GeV synchrotron photons by the CSM magnetic field of strength $\gtrsim 10 \mu G$.

### 2.2 Radiative losses during electron acceleration

Electrons suffer radiative losses while being accelerated that could prevent them from reaching LFs of $\sim 10^8$ that are needed for radiating photons of 10 GeV via the synchrotron process. In this section, we ascertain whether or not the radiative losses suffered by electrons – due to synchrotron and inverse-Compton processes – are small compared with the energy gain in each round of crossing the shock front. We do this by comparing the total radiative cooling time-scale, $t_{\text{cool}}$, which is the time-scale for electrons to lose half of their energy, with the acceleration time-scale.

For the case of ultra-relativistic shocks when the downstream magnetic field is simply the shock compressed upstream field, the upstream and downstream residency times for electrons are approximately equal, when particle diffusion is in the Bohm limit (Gallant & Achterberg 1999, Achterberg et al. 2001). Thus, the time it takes for electrons to make one complete cycle across the shock front is about twice the upstream residency time, and the upstream residency time is on the order of the gyro-time in the shock front co-moving frame (Baring 2004). In the lab frame, their upstream residency time is on the order of the time it takes them to travel a distance $\sim R_{L,u}/\Gamma$. Since the Larmor radius ($R_{L,u}$) increases with increasing electron energy, the last shock crossing dominates the total upstream residency time. Thus, the time, in the co-moving frame of the blast wave, that electrons spend during the last cycle of crossing the shock front (upstream $\rightarrow$ downstream $\rightarrow$ upstream) before getting accelerated to Lorentz factor $\gamma_c$ – given by (1) – is:

$$t'_s \sim \frac{2R_{L,u}}{c} = (1.7 \times 10^3 s) \nu_{\mathrm{10}}^{-1/2} (1 + z)^{1/2} \Gamma_{-3}^{-2} B_{\mu G}^{-3/2}. \quad (4)$$

Taking into account the energy loss that these electrons experience because of radiative cooling, the acceleration time-scale, in the blast wave co-moving frame, is given by:

$$t'_{\text{acc}}(\gamma_c) \approx t'_e(\gamma_c) + t'_s(\gamma_c), \quad (5)$$

where $t'_e(\gamma_c)$ is the elapsed time since the beginning of the explosion when $t'_e(\gamma_c) = t'_{\text{cool}}(\gamma_c)/2$ (shock front crossing time should be equal to at least half of the the radiative cooling time in order to reach a particular $\gamma_c$). At $t'_e$, the electron barely reaches $\gamma_c$, therefore, it needs an extra time on the order of $\sim t'_s$ to fully reach the desired $\gamma_c$. If $t'_e > t'_{\text{cool}}/2$, then the radiative cooling is too large and prevents the electron from reaching the desired $\gamma_c$. In the subsections below we discuss synchrotron and inverse-Compton losses and calculate the radiative cooling time.

#### 2.2.1 Synchrotron losses

The synchrotron cooling time-scale (in the blast wave co-moving frame) in the upstream of the shock front is $t'_{\text{syn},u} = 6\pi m_e c / \sigma_T B_{\mu G}^2 \gamma_c^2$, where $\sigma_T$ is the Thomson scattering cross-section. We find that the synchrotron cooling time for an external forward shock electron with LF given by (1) is:

$$t'_{\text{syn},u} = (5.2 \times 10^3 s) \nu_{\mathrm{10}}^{-1/2} (1 + z)^{-1/2} \Gamma_{-3}^{-1} B_{\mu G}^{-3/2}. \quad (6)$$

Since $t'_{\text{syn},u} > t'_s$ by a factor of 30, then synchrotron cooling in the upstream is unimportant for electrons radiating at 10 GeV.

Next, we calculate synchrotron losses in the downstream. Since $t'_{\text{syn}} \propto B^{-2}$, the synchrotron loss rate is larger downstream because of the larger magnetic field. For shock-compressed magnetic field downstream, $B$ is larger than upstream field by a factor 4 (in the blast wave co-moving frame), and therefore $t'_{\text{syn},d} = t'_{\text{syn},u}/16$. The effective synchrotron cooling time for electrons of LF given in (1) is $t'_{\text{syn}} \approx [1/2t'_{\text{syn},d} + 1/2t'_{\text{syn},u}]^{-1}$, which gives:

$$t'_{\text{syn}} = (6.1 \times 10^3 s) \nu_{\mathrm{10}}^{1/2} (1 + z)^{-1/2} \Gamma_{-3}^{-1} B_{\mu G}^{-3/2}. \quad (7)$$

We see from (4) that $t'_{\text{syn}} \sim 4t'_s$ for electrons that produce synchrotron photons of 10 GeV energy, and therefore the maximum synchrotron photon energy — obtained by setting $t'_e = t'_{\text{syn}}$ — is $\nu_{\text{max, syn}} \sim 40\nu_{\mathrm{10}} (1 + z)^{-1} \, \text{GeV}$ (see, e.g., Guilbert, Fabian, Rees 1983, de Jager et al. 1996, Cheng & Wei 1996).

#### 2.2.2 Inverse-Compton losses

In this subsection we calculate the inverse-Compton (IC) cooling time-scale for electrons. The inverse-Compton cooling time depends on the energy density of photons, and on the electron LF. Electrons in the external forward shock region are exposed to photons from three different sources of radiation: (a) prompt $\sim$MeV $\gamma$-ray radiation which carries most of the energy release in GRBs; (b) synchrotron radiation produced in the external forward shock heated CSM and (c) radiation produced in the external reverse shock heated GRB-jet. We will consider all of these sources in our estimate for the IC cooling time. All calculations will be carried out in the rest frame of the shocked CSM.

The IC cooling time is given by:

$$t'_I \approx \frac{3m_e c^2}{4 \sigma_T \nu_c} \nu F(\nu) \gamma_c, \quad (8)$$

where $F(\nu)$ is the energy flux in radiation per unit frequency in the co-moving frame of the shocked CSM, $\nu$ is photon frequency in observer frame, and $\sigma$ is the cross-section for interaction between electrons and photons; $\sigma \approx \sigma_T$ (Thomson cross-section) when $\nu < \Gamma m_e c^2 / [(1 + z) \hbar \nu_c] \equiv \nu_{\text{kn}}$, and for $\nu \gg \nu_{\text{kn}}$, $\sigma \approx \sigma_T (\nu / \nu_{\text{kn}})^{-1}$. Thus, an approximate equation for the IC cooling time is:

$$t'_I \approx \frac{3m_e c^2}{4 \sigma_T \nu_c} \left[ F(\nu < \nu_{\text{kn}}) + \frac{\nu_{\text{kn}}}{\nu_p} \nu_p \left( \nu > \nu_{\text{kn}} \right) \right]^{-1}, \quad (9)$$

where $F(\nu < \nu_{\text{kn}})$ is photon energy flux in the shock co-moving frame below the frequency $\nu_{\text{kn}}$, and $F(\nu > \nu_{\text{kn}})$ is the flux above $\nu_{\text{kn}}$. The frequency at the peak of the $\nu F(\nu)$ spectrum is $\nu_p$ (in observer frame, i.e., co-moving synchrotron peak frequency boosted by a factor of $\Gamma$ and redshift corrected) and $\nu_{\text{kn}}$, the Klein-Nishina frequency in the observer frame, for an electron of LF $\gamma_c$, is $\hbar \nu_{\text{kn}} \approx (5 \, \text{eV}) \Gamma_{-3}^{-1/2} (1 + z)^{-1}$. We note that for $\nu_{\text{kn}} < \nu_p$, only the first term in (9) should be kept.
The co-moving energy flux in radiation is related to the observed bolometric luminosity by:

\[ F'(\gtrsim \nu_p) \sim \frac{L_{obs}}{4\pi R^2 t^2}. \]  
(10)

Combining (9) and (10) we find

\[ t_{IC}^i \sim \frac{3\pi R^2 T_{26}^2 m_e c^2}{\sigma_T L_{obs} \gamma_\epsilon} \left( \frac{\nu_{kn}}{\nu_p} \right)^{-1} \left[ 1 + \left( \frac{\nu_{kn}}{\nu_p} \right)^{\alpha} \right]^{-1}, \]
(11)

where \( \alpha \) is the spectral index, i.e. \( F'(\nu) \propto \nu^\alpha \) for \( \nu_{kn} < \nu < \nu_p \); for \( \alpha > 0 \) the term in the square bracket is of order unity. The above equation is valid only when \( \nu_{kn} < \nu_p \).

As mentioned before, there are three different sources of photons that interact with electrons in the the external forward shock. We analyze these cases separately.

**Case (a):** The prompt \( \gamma \)-ray emission in GRBs – the origin of which is still uncertain – often has a low spectral index \( \alpha \sim 0 \), and the spectrum peaks at \( \nu_p, \epsilon \sim 1 \). The luminosity of this component is the highest of the three cases considered; \( L_{obs,52} \sim 10^5 \) for Fermi GRBs that have \( >10^5 \) MeV emission. The cooling time, obtained from (11), for this case is

\[ t_{IC,a}^i = \left( 2.2 \times 10^3 \right) \frac{R_{d,17}^5 \Gamma_{p,6.6} (1 + z)}{L_{obs,53}} \left[ 1 + \left( \frac{\nu_{kn}}{\nu_p} \right)^{\alpha} \right]^{-1}. \]  
(12)

**Case (b):** The external forward shock synchrotron spectrum peaks at \( \sim 100 \) keV (before the deceleration time), and the spectral index between \( \nu_{kn} \) and \( \nu_p \) is \( \alpha \sim 1/3 \). The luminosity from the external forward shock is \( L_{obs,52} \sim 0.1 \) at the deceleration radius \( (R_d) \), and at smaller radius it decreases as \( \sim R^{-1} \). Therefore, we find from (11) that, for \( R \leq R_d \),

\[ t_{IC,b}^i = \left( 2.2 \times 10^3 \right) R_{d,17}^5 \Gamma_{p,6.6} (1 + z) L_{obs,53}^{-1} \left[ 1 + \left( \frac{\nu_{kn}}{\nu_p} \right)^{\alpha} \right]^{-1}. \]  
(13)

**Case (c):** If the GRB-jet is composed of protons and electrons, then the interaction of the jet with the CSM will heat up these particles by the reverse shock propagating into the cold jet, and the synchrotron radiation produced would be very effective at cooling electrons in the forward shock region. This is because the peak of the reverse shock emission at the deceleration time is at a few eV (Sari & Piran 1999a), which is of order \( \nu_{kn} \), for electrons of \( \gamma_\epsilon \sim 10^6 \). Since \( \nu_p \sim \nu_{kn} \), then we can keep only the first term in (9), and use (10) for flux in the calculation of the cooling time. The observed luminosity (at the deceleration time) is given by

\[ L_{obs,52} \approx \frac{4\pi d_L^2 \nu_p,5 \Gamma_\epsilon L_{53}^d \Gamma_{d,17}^{-1}}{\Gamma_{p,6.6} L_{53}^d \Gamma_\epsilon^{-1}}. \]

The total IC cooling time-scale for electrons in the external forward shock region, due to the radiation produced in the reverse shock heated GRB-jet, is

\[ t_{IC,c}^i \approx \left( 400 \right) R_{d,17}^5 \Gamma_{p,6.6} \left( \Gamma_{d,17}^{-1} L_{53}^d \Gamma_{p,6.6} \Gamma_{d,17}^{-1} L_{53}^d \right)^{\alpha}. \]

(14)

where \( \Gamma_d \) is the LF of the GRB-jet at the deceleration time, \( F_{p,d} \) is in Jy and \( \nu_p,d \) are the observed external reverse shock peak flux and peak energy at the deceleration time, respectively. Thus, the IC cooling time-scale for electrons in the external forward shock region, due to the radiation produced in the reverse shock heated GRB-jet, is

\[ t_{IC,c}^i = \left( 400 \right) R_{d,17}^5 \Gamma_{p,6.6} \left( \Gamma_{d,17}^{-1} L_{53}^d \Gamma_{p,6.6} \Gamma_{d,17}^{-1} L_{53}^d \right)^{\alpha}. \]

and, finally, the total radiative cooling time, \( t_{cool}^i \), is given by

\[ t_{cool}^i = \left[ t_{IC,a}^i + t_{IC,b}^i + t_{IC,c}^i \right]^{-1}. \]

(15)

Table 1. The main quantities used in our analysis for three Fermi GRBs. \( \alpha \) is the approximate spectral energy index, during the prompt emission phase, below the peak of the spectrum \( (\nu_p \propto \nu^\alpha \) for \( \nu < \nu_p \), \( \nu_p \) is the observed peak of the spectrum; \( B_{u,-5} \) is the average upstream magnetic field, in units of \( 10^\mu \) Gauss, obtained by modeling of the data for these bursts (Kumar & Barniol Duran 2010); \( L_{prompt,53}^{p,d} \) and \( L_{obs,51}^{ES} \) are the approximate observed isotropic equivalent luminosities of the prompt \( \gamma \)-rays and external reverse shock peak flux at the deceleration time, respectively. Data are taken from Abdo et al. (2009a,b,c).

| GRB     | \( \alpha \) | \( \nu_p \) [MeV] | \( B_{u,-5} \) | \( L_{prompt,53}^{p,d} \) | \( L_{obs,51}^{ES} \) |
|---------|-------------|-----------------|--------------|-----------------|-----------------|
| 080916C | 0.6         | 0.5             | 4            | 1.5             | 30              |
| 090510  | 0.4         | 2.8             | 2            | 3.6             | 1.6             |
| 090920B | 0.4         | 0.7             | 2            | 1.2             | 1.7             |

2.3 Application to Fermi GRBs

In this section, we analyze 3 GRBs detected by Fermi: GRB080916C, 090510 and 090920B (Abdo et al. 2009a,b,c). The relevant data for each burst are tabulated in Table 1. We apply the above general results to these three GRBs, and determine the time it would take for electrons in the external forward shock for these bursts to be accelerated – via shock acceleration – to LFs capable of producing synchrotron photons of energies 100 MeV and 1 GeV (Table 2).
Table 2. $t_{acc}/t_d$ is the ratio of the time for electron acceleration to a specific energy (corresponding to synchrotron frequency given in column 2) and the deceleration time; it is a measure of the delay, with respect of the trigger time, for photons of a given energy to arrive at the observer when the external reverse shock emission is smaller than given in the last column of the Table. The observed time delay of photons in column 2 with respect to the trigger time is given by $t_{max} - F_{p,d}^max$ (in Jy) is the maximum possible observed external reverse shock peak flux, so that electrons can be accelerated to produce photons of energy given in column 2 at $t_d$.

| GRB    | $\nu$ | $t_{acc}/t_d$ | Expected $t_{acc}/t_d$ | Observed $t_{max}$ | $F_{p,d}^max$ |
|--------|-------|---------------|-------------------------|-------------------|---------------|
| 080916C | 100 MeV | 0.3           | 0.6                     | 0.30              |               |
|        | 1 GeV  | 0.6           | 1.4                     | 0.02              |               |
| 090510  | 100 MeV | 0.3           | 0.5                     | 9.90              |               |
|        | 1 GeV  | 0.6           | 1                       | 0.90              |               |
| 090902B | 100 MeV | 0.3           | 0.3                     | 1.20              |               |
|        | 1 GeV  | 0.7           | 1                       | 0.10              |               |

GRB090910: For GRB090910 (Abdo et al. 2009b) there was a short delay in the detection of $>100$ MeV photons by $\sim 0.1$ s (we take the trigger time to be $\sim 0.5$ s after the GBM trigger, because of the presence of a precursor). The 100 MeV light curve peaked at $\sim 0.2$ s (which we associate with the deceleration time), and the arrival of the first $>100$ MeV photons was at $t/t_d \sim 0.5$. Higher energy photons arrived later: $>1$ GeV photons started arriving at $t_d$, and $\sim 10$ GeV photons arrived slightly after $t_d$. As shown in Table 2 these results are roughly consistent with our estimates within a factor of 2.

GRB090902B: The 100 MeV light curve for this burst peaked at $\sim 10$ s, which we identify as $t_d$, and the first $>100$ MeV photons were detected at $\sim 3$ s after the trigger time (Abdo et al. 2009c), i.e. $t/t_d \sim 0.3$. Most of the GeV photons arrived at $\sim t_d$. The first 10 GeV photon is detected at $\sim 12$ s. The highest energy photon detected was $\sim 30$ GeV at 80 s, i.e. at $\sim 8t_d$. As shown in Table 2 the non-zero acceleration time of the emitting electrons is taken into account. We plot the specific flux (normalized to the flux at $t_d$, $F_{p,d}$) versus time (normalized to the deceleration time, $t_d$). We show two cases: (1) $R_0 < R_d < R_f$ (red solid line), and (2) $R_d < R_0 < R_f$ (blue dashed line). The light curve temporal slope, $\beta = d\ln(f/f_0)/d\ln(t/t_d)$. The horizontal black dotted line shows the asymptotic value of the temporal decay index if we take electrons to accelerate instantaneously; for $t < t_d$ the light curve would rise as $t^{3\beta}$. However, we find that the external forward shock light curve rises faster than $t^{3\beta}$ due to the finite time it takes electrons to accelerate.

3 Steep Rise of the High-Energy Photon Light Curve

In this section, we calculate the onset of light curves of high-energy photons ($>100$ MeV). According to the standard external forward shock model, and assuming instantaneous acceleration of electrons, the observed flux rises as $t^3$ when the CSM-density is homogeneous. We show here that the light curve rises much more steeply – similar to what is seen by Fermi/LAT data – when finite time for electron acceleration is taken into consideration, as was suggested by Kumar & Barniol Duran (2009).

We calculate the rise of the light curve using a simple model. The external shock emission at some frequency $\nu$ is zero until the blast wave reaches a radius $R_0$, which is set by the time-scale for electrons to be accelerated to a LF so that they start radiating at $\nu$; this time is calculated in Section 2. Electron distribution function in the neighborhood of the desired LF is assumed to grow with radius as $\propto R^\beta$, and the distribution attains its asymptotic powerlaw shape at some radius $R_f \sim 2R_0$. The rise of the light curve depends on $R_0$, $R_f$, $x$ and the deceleration radius; the rise also depends weakly on the type of CSM and the energy spectral index.

We show in Fig. 1 light curves for two different regimes $R_0 < R_d < R_f$ (case 1) and $R_d < R_0 < R_f$ (case 2), for $R_0 < R_f < R_d$ the light curve is similar to case 1 except that between $t_d$ (the observer time corresponding to radius $R_f$) and $t_d$ the light curve rises as $t^{3\beta}$. Guided by the estimates provided in $^2$ we take $R_0 \sim R_d/3$ for 100 MeV photons, whereas for 10 GeV $R_0 \sim R_d$.

We show the results for these choices of parameters in Fig. 1; note the very steep rise of light curves which appear similar to the fast rise of the observed 100 MeV light curve for GRB080916C as reported in Abdo et al. (2009a).

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1 At this time, the LF has dropped by a factor of $8^{1/8} \sim 2$, and at $z = 1.8$, $v_{\text{max, syn}} \sim 10$ GeV, a factor of $\sim 4$ smaller than the observed value. It can be shown that inhomogeneous magnetic fields lead to an increase of $v_{\text{max, syn}}$ by about an order of magnitude.
4 CONCLUSIONS

In this paper we have investigated the acceleration of electrons via diffusion shock acceleration in the external forward shock of GRBs, and its implications for the high-energy photon detection by the Fermi Satellite. The external shock model, with a weak magnetic field, has been proposed as the origin of the observed >100 MeV emission detected by the Fermi Satellite from a number of GRBs (Kumar & Barniol Duran 2009, 2010). We find that high-energy electrons of Lorentz factor $\sim 10^7$, required for producing $\sim$10 GeV photons via the synchrotron process, can indeed be accelerated in an external shock that is moving through a CSM with a magnetic field of strength a few tens of $\mu$G; they remain confined to the shock front as long as the upstream magnetic field is $\gtrsim 10\mu$G.

We have also calculated the time it takes for electrons to be accelerated to a Lorentz factor $\sim 10^7$ so that they can radiate synchrotron photons at $\sim$100 MeV. We find this acceleration time to be a few seconds in the observer frame; this calculation took into account radiation losses suffered during the acceleration process. This result offers a straightforward explanation as to why, for most Fermi GRBs, 100 MeV photons are not observed right at the trigger time but a little later. This also explains why 100 MeV photons are observed before GeV radiation: it takes electrons radiating at GeV energies even longer time to accelerate. Taking this acceleration time into consideration while calculating high-energy light curves, we find that the light curve rises very rapidly – much faster than it does for the external forward shock model with instantaneous electron acceleration for which the flux rises as $t^3$ when the CSM has uniform density (the $t^3$ rise reflects the increasing number of swept-up electrons before the blast wave decelerates).

The detection of the first 100 MeV photons at some fraction of the deceleration time, the longer delays in the detection of higher energy photons and the fast rise of the 100 MeV light curve, follow the expectation of the external forward shock model when the finite time for electron acceleration is taken into account. Detection of synchrotron photons of different energies provides an upper limit for the radiation flux produced in the reverse shock heated GRB-jet. For instance, the peak flux for the external reverse shock emission — if the peak of the spectrum is at a few eV — couldn’t have been larger than about 300 mJy close to the deceleration time, for GRB080916C, otherwise it would prevent electrons from accelerating to a Lorentz factor of $\sim 10^7$ so that they can produce synchrotron photons of 100 MeV energy at early times (see Table 2). Similarly, the reverse-shock flux should be $\gtrsim 20$ mJy for GRB 080916C in order that electrons in the forward shock are accelerated to a LF so that they produce 1 GeV photons.

We speculate that the lack of $>$100MeV emission during the prompt phase of GRBs might be due to the presence of a bright optical source with observed flux larger than about 100 mJy, which would prevent electrons from reaching high Lorentz factors. This, coupled with the fact that GRBs with the largest LFs, which have small deceleration time, are the most likely bursts to be detected by Fermi (Kumar & Barniol Duran 2009) might explain the detection/non-detection of $>$100MeV radiation from GRBs.

We note that the shock-compressed magnetic field scenario requires some cross-field diffusion of particles - presumably generated by turbulence - to allow them to travel back to the upstream (e.g. Achterberg et al. 2001, Lemoine et al. 2006). This turbulent layer probably occupies a small fraction of the downstream region as suggested by recent simulations by Sironi & Spitkovsky (2010). Therefore, the picture that seems to emerge from numerical simulations and Fermi observations, is that there might be a small region of turbulence behind the shock front that aids in the acceleration of particles across the shock, but that the radiation is mainly produced by particles that are swept downstream where the value of the downstream field is consistent with simple shock-compression of upstream field.

There exists also the possibility that the CSM seed field is actually a few $\mu$G and some instability produced ahead of the shock amplifies it to the value of a few tens of $\mu$G we infer by our modeling of Fermi GRBs (Kumar & Barniol Duran 2009, 2010). These instabilities have been studied by, e.g., Milosavljević & Nakar 2006, Sironi & Goodman 2007, Goodman & MacFadyen 2008, Couch et al. 2008. However, this possible amplification of a factor of $\sim 10$ is much smaller than the amplification customarily invoked to explain afterglow observations.

We received a preprint from Piran & Nakar (2010) soon after this paper was completed. They have also considered the acceleration of electrons in the external shock.

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REFERENCES

Abdo A.A. et al., 2009a, Science, 323, 1688
Abdo A.A. et al., 2009b, Nature, 462, 331
Abdo A.A. et al., 2009c, ApJ, 706, L138
Achterberg A., Ball L., 1994, A&A, 284, 687
Achterberg A., Gallant Y. A., Kirk J. G., Guthmann A. W., 2001, MNRAS, 328, 393
Axford W. I., Leer E., Skadron G., 1978, Proc. 15 Int. Cosmic Ray Conf. (Plovdiv) 11, B’lgarska Akademiena na Naukite, Sofia, Bulgaria, p. 132
Baring M.G., 2004, Nucl. Phys. B, 136, 198
Bell A.R., 1978, MNRAS, 182, 147
Blandford R. D., McKee C. F., 1976, Phys. Fluids, 19, 1130
Blandford R.D., Ostriker J.P., 1978, ApJL, 221, 29
Blandford R.D., Eichler D., 1987, Phys. Rep., 154, 1
Cheng K.S., Wei D.M., 1996, MNRAS, 283, L133
Corsi A., Guetta D., Piro L., 2010, ApJ, 720, 1008
Couch S.M., Milosavljević M., Nakar E., 2008, ApJ, 688, 462
de Jager O.C., Harding A.K., Michelson P.F., Niel H.I., Nolan P.L., Sreekumar P., Thompson D.J., 1996, ApJ, 457, 253
De Pasquale M. et al. 2010, ApJL, 709, 146
Dermier C.D., Mirman K.E., 1999, ApJ, 513, L5
Gallant Y. A., Achterberg A., 1999, MNRAS, 305, L6
Gao W.-H., Mao J., Xu D., Fan Y.Z., 2009, ApJL, 706, 33
Ghirlanda G., Ghisellini G., Nava L., 2010, A&A, 510, 7
Ghisellini G., Ghirlanda G., Nava L., 2010, MNRAS, 403, 926
Goodman J., MacFadyen A., 2008, J. Fluid Mech., 604, 325
Guilbert P.W., Fabian A.C., 1983, MNRAS, 205, 593
Jokipii J.R., 1987, ApJ, 511, 28
Keshet U., Katz B., Spitkovsky A., Waxman E., 2009, ApJ, 693, L127
Krymskii G. F., 1977, Sov. Phys. Dokl., 22, 327
Kumar P., Barniol Duran R., 2009, MNRAS, 400, L75

\( \text{Note that this possible trend in the data goes in the opposite direction than in the prompt } \sim 1 \text{ MeV emission, where higher energy photons arrive earlier than lower energy photons in long GRBs and there is no lag detected for short GRBs (Norris et al. 1986, Norris & Bonnell 2006).} \)
Kumar P., Barniol Duran R., 2010, MNRAS, 409, 226
Lemoine M., Pelletier G., Revenu B., 2006, ApJ, 645, L129
Mészáros P., Rees M.J., 1993, ApJ, 405, 278
Mészáros P., Rees M.J., 1997, ApJ, 476, 232
Milosavljević M., Nakar E., 2006, ApJ, 651, 979
Norris J.P., Share G.H., Messina D.C., Dennis B.R., Desai U.D., Cline T.L., Matz S.M., Chupp E.L., 1986, ApJ, 301, 213
Norris J.P., Bonnell J.T., 2006, ApJ, 643, 266
Paczynski B., Rhoads J.E., 1993, ApJ, 418, L5
Pelletier G., Lemoine M., Marcowith A., 2009, MNRAS, 393, 587
Piran T., 2004, RvMP, 76, 1143
Piran T., Nakar E., 2010, ApJ, 718, L63
Rees M.J., Mészáros P., 1992, MNRAS, 258, 41P
Rybicki G. B., Lightman A. P., 1979, Radiative Processes in Astrophysics. Wiley-Interscience Press, New York
Sari R., Piran T., Narayan R., 1998, ApJ, 497, L17
Sari R., Piran T., 1999a, ApJ, 520, 641
Sari R., Piran T., 1999b, ApJ, 517, L109
Sironi L., Goodman J., 2007, ApJ, 671, 1858
Sironi L., Spitkovsky A., 2010, ApJ, submitted (arXiv:1009.0024)
Spitkovsky A., 2008a, ApJ, 673, L39
Spitkovsky A., 2008b, ApJ, 682, L5