Glueball Wave Functions in $U(1)$ Lattice Gauge Theory

Mushtaq Loan
Department of Physics, Zhongshan (Sun Yat-Sen) University, Guangzhou 510275, China
School of Physics, The University of New South Wales, Sydney, NSW 2052, Australia

Yi Ying
Department of Physics, Chong Qing University, Chong Qing 400030, China

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Standard Monte Carlo simulations have been performed for 3-dimensional $U(1)$ lattice gauge model on improved lattices to measure the wave functions and the size of the scalar and tensor glueballs. Our results show the radii of $\sim 0.60$ and $\sim 1.12$ in the units of string tension, or $\sim 0.28$ and $\sim 0.52$ fm, for the scalar and tensor glueballs, respectively. At finite temperature we see clear evidence of the deconfined phase, and the transition appears to be similar to that of the two-dimensional XY model as expected from universality arguments. Preliminary results show no significant changes in the glueball wave functions and masses in the deconfined phase.

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I. INTRODUCTION

The prediction of glueball masses has long been attempted in lattice gauge theory calculations [1, 2, 3, 4, 5]. These calculations show that the lowest-lying scalar, tensor and axial vector glueballs lie in the mass region of 1-2.5 GeV. While there is a long history of glueball mass calculations in lattice QCD, little is known about the glueballs besides their masses. Accurate lattice calculations of their size, matrix elements and form factors would help considerably in their experimental identification.

Glueball wave functions and sizes have been studied in the past [6, 7, 8, 9], but much of the early work contains uncontrolled systematic errors, most notably from discretisation effects. The scalar glueball is particularly susceptible to such errors for the Wilson gauge action, due to the presence of a critical end point of a line of phase transitions (not corresponding to any physical transition found in QCD) in the fundamental-adjoint coupling plane. As this critical end-point (which defines the continuum limit of a $\phi^4$ scalar field theory) is neared, the coherence length in the scalar channel becomes large, which means that the mass gap in this channel becomes small; glueballs in other channels seem to be affected very little. Results in which the scalar glueball was found to be significantly smaller than the tensor were most likely due to contamination of the scalar glueball from this non-QCD critical point [7]. On the other hand, the calculations using operator overlaps obtained from variational optimization for improved lattice gauge action, which are gauge invariant. The choice of such loops eliminate the need for gauge fixing [10]. Although the calculations in Ref. [7] have produced some interesting results,

II. WAVE FUNCTIONS OF GLUEBALLS

In contrast with the techniques used in Ref. [7], we measure our lattice operators from spatially connected Wilson loops. Glueballs are colour singlet states and one should be able to construct them with closed-loop paths which are gauge invariant. The choice of such loops eliminate the need for gauge fixing [1]. Nevertheless, some careful analysis is needed, since Wilson loops are not gauge invariant even when closed. A straightforward procedure to address the controversy over glueball size is to measure the glueball wave function, much in the same way as the meson and baryon wave functions were measured [10].

As a first test of our method we study the low-lying scalar and tensor glueballs and their wave functions for the 3-dimensional $U(1)$ model with tadpole improved Symanzik gauge action. This theory is perhaps the best understood of all lattice gauge theories at zero temperature, which makes it testing ground for new analytic and numerical methods. The relevance of the model to QCD at finite temperature [11], including confinement, chiral symmetry breaking [12] and existence of a mass gap [13], suggest that a comparison of the respective mass spectra should be informative. Our techniques for calculating the glueball wave functions from Wilson loop operators are outlined in Sec. II. We present and discuss our results at zero temperature in Sec. III. We extend our method to examine the wave functions and masses at finite temperature in this section. Here we briefly discuss an interpretation of the deconfinement in terms of the power-law behaviour of the correlation function. Section IV is devoted to the summary and concluding remarks.
the approach suffers from a basic problem: the observables are calculated from a lattice version of the 2-glue operator, which risks a mixture of glueball states with flux states\textsuperscript{2}.

In this paper we take a more direct approach to the problem. We measure the observables in a three-step procedure. First, we calculate the lattice operator
\[ \Phi(\vec{r}, t) = \sum_\mathbf{x} [\phi(\vec{x}, t) + \phi(\vec{x} + \vec{r}, t)], \]
where \( \phi \) is the plaquette operator and \( \Phi \) measures the two plaquette or two-loop component of the glueball wave function. The \( r \) dependence will be reflected in the length of links required to close the loops. From a suitable linear combinations of rotation, parity inversions and real or imaginary parts of the operators involved in \( \Phi \), one can construct glueball operators with desired quantum numbers. Since we want to explore the nature of wave functions, we focus only on the low-lying “symmetric” and “antisymmetric” scalar channels (which are the cosine and sine, respectively of the Wilson loop in question) and tensor glueball states.

The wave function and mass are obtained from the correlation function:
\[ C(\vec{r}, t) = \langle \Phi^\dagger(\vec{r}, t)\Phi(0, 0) \rangle, \]
where one needs to subtract the vacuum contribution from the correlator for \( 0^{++} \) state. The source can be held fixed while the sink takes on the \( r \) dependence. This proves to be helpful in maintaining a good signal. The disentangling of the glueball and torelon is usually taken care of automatically by the choice of Wilson or Polyakov loops.

To increase the overlap with the lowest state and reduce the contamination from higher states, we exploit the APE link smearing techniques \[14\]. The procedure is implemented by an iterative replacement of the original spatial link variable by a smeared link. This results in correlations which reach their asymptotic behaviour at small time separations. In addition, the noise from ultraviolet fluctuations is reduced. The smearing parameter is fixed to 0.7 and ten iterations of the smearing process are used. To find the optimum smearing value, \( n \), we examine the ratio (at \( r = 0 \) and 1)
\[ C(r, t + 1)/C(r, t), \]
which should be maximum for good ground state dominance. Using \( 1 \times 1 \) loop as template, the best signal is obtained with four smearing steps, with \( 1 \times 1 \) and \( 2 \times 2 \) loops being almost indistinguishable. At \( \beta = 2.0 \), the signal in \( 1 \times 1 \) showed a slow convergence with \( n \), hence \( 2 \times 2 \) loops were preferred for optimum overlap. A typical value which proved to be sufficient for this case was \( n = 2 \).

A second pass was made to measure the optimized correlation matrices
\[ C_{ij}(t) = \langle \Phi(r_i, t)\Phi(r_j, 0) \rangle - \langle \Phi(r_i) \rangle \langle \Phi(r_j) \rangle. \]

Let \( \psi^{(k)} \) be the radial wave function of the \( k \)-th eigenstate of the transfer matrix, then
\[ C_{ij}(t) = \sum_k \langle \psi^{(k)}(r_i) \psi^{(k)}(r_j) e^{-m_k t} \rangle. \]
The glueball masses and the wave functions are extracted from the Monte Carlo average of \( C_{ij}(t) \) by diagonalizing the correlation matrices \( C(t) \) for successive times \( t \):
\[ C(t) = \tilde{R}(t)D(t)R(t), \]
where \( D \) is a diagonal matrix of the eigenvalues and \( R \) a rotation matrix whose columns are the eigenvectors of \( C \). Each eigenvector of \( C \) matches an eigenstate \( \psi^{(k)}(r) \) of the complete transfer matrix. As the wave function is largest at the origin, one would first determine the glueball mass with the optimal separation, and then fix that mass for all \( r \), and extract the wave function for less optimal separations. Similar to the case of mesons \[13\], the wave function is expected to decrease exponentially with the \( r \) at large separations and is therefore fitted with the simple form
\[ \psi(r) \equiv e^{-r/r_0} \]
to determine the effective radius \( r_0 \). The effective mass can be read off directly from the largest eigenvalue corresponding to the lowest energy
\[ m_{\text{eff}} = \log \left[ \frac{\lambda_0(r = 0, t = 1)}{\lambda_0(r = 0, t = 2)} \right] \]

III. SIMULATION RESULTS AND DISCUSSION

A. Results at zero temperature

Most of our Monte Carlo calculations are carried out on \( 16^2 \times 16 \) lattice with periodic boundary conditions (\( 16^2 \) is the space-like box and 16 is the extension in Euclidean time direction). The gauge configurations are generated using the Metropolis algorithm. After the equilibration, configurations are stored every 250 sweeps; 2000 stored configurations are used in the measurement of glueball masses. Measurements made on the stored configurations are binned into 10 blocks with each block containing an average of 200 measurements. The mean and the final errors are obtained using single-elimination jackknife method with each bin regarded as an independent

\textsuperscript{2} The link-link operator used in Ref. \[1\] sums up a large number of loops; some of these loops have a zero winding number and project on glueballs - others have non-zero winding number and project on flux states also called torelons.
data point. Three sets of measurements were taken at 
\( \beta = 2.0, 2.25 \) and 2.5. Some finite-size consistency checks 
are done at \( \beta = 2.25, 2.5 \) on a \( 20^2 \times 20 \) lattice.

The glueball correlation function for the \( 0^{++} \) channel against \( t \) at \( \beta = 2.25 \) is shown in Fig. 1. It can be 
seen that the expected behaviour of correlation function 
is attained virtually straight away. The absolute errors 
in the correlation functions are expected to be indepen-
dent of \( t \) for large \( t \). Our errors are consistent with this 
expectation.

Effective mass plot for \( \beta = 2.5 \) simulation is presented 
in Fig. 2. For \( 0^{++} \) and \( 0^{--} \) channels each it was possible to find a fit region \( t_{\text{min}} \) and \( t_{\text{max}} \) in which convincing 
plateaus were observed. The effective masses are found 
stable using different values of \( t \) in Eq. (7), which sug-
gests that the glueball ground state is correctly projected.

At \( \beta = 2.5 \), we noticed considerable fluctuations in the 
tensor mass at large \( t \). An acceptable fit was only possible for \( t = [2 - 5] \).

The Wave functions are extracted at time-separations 
\( t = 1 \) and 2. We found a little variation (less than two 
percent) in the eigenvectors of \( C(t) \) with \( t \) which suggests that there is no mixing with states of distinct masses. 

Typical plots of the wave functions, normalised to unity at the origin, for the symmetric and antisymmetric scalar 
glueballs, at \( \beta = 2.0, 2.25 \) and 2.5 are shown in Figs. 3 and 4 respectively. For guiding the eyes the Monte Carlo 
points of the same \( \beta \)-value are connected with straight 
lines. The scalar wave function shows the expected be-

FIG. 1: Correlation function for the \( 0^{++} \) channel against \( t \).

FIG. 2: Effective mass plot for scalar and tensor glueball 
states for \( \beta = 2.5 \). The dashed horizontal lines indicate the 
plateau values.

FIG. 3: Scalar \( 0^{++} \) glueball wave functions measured on a 
\( 16^3 \) lattice for various values of \( \beta \).

antisymmetric channel we notice the presence of negative 
contributions in the glueball wave function for \( r > 6 \) 
at \( \beta = 2.5 \). However these contributions do not persist 
when the lattice size is increased from \( L = 16 \) to 20. This 
would mean that these effects are unphysical and can be 
described as a finite volume artifact. For this reason we 
extract the effective radius of the antisymmetric state 
from the wave function obtained at larger volume. The 
symmetric scalar glueball wave function, on the other 
hand, barely changes sign.

Fig. 5 shows the wave function for the tensor glueball, 
at \( \beta = 2.0, 2.25 \) and 2.5. The tensor wave function re-
mains positive and shows the expected flatness. It can 
be seen that tensor glueball is much more extended than the 
scalar as one moves towards higher \( \beta \) values. This 
would imply that tensor is therefore more sensitive to 
the finite-size effects, which is very visible in the distor-
tion of the wave function for large \( r \) at \( \beta = 2.5 \). Naively 
we would expect that the spatial size at which we begin 
to encounter large finite size effects to be related to the 
size of the glueball.

The expected finite-size scaling behaviour of the mass 
gap near the continuum critical point in this model is 
not known; but Weigel and Janke [16] have performed 
a Monte Carlo simulation for an \( O(2) \) spin model in 
three dimensions which should lie in the same univers-
sality class, obtaining

\[ M \approx \frac{1.3218}{L} \] (8)
for the magnetic gap. In order to ascertain the finite-size effect on our measurements, we performed extra simulations on a $20^2 \times 20$ lattice at $\beta = 2.25$ and 2.5. The mass and size of $0^{++}$ channel are almost unchanged as the lattice size increases from 16 to 20. We also find that our estimates for the tensor state are consistent with no finite volume dependence at $\beta = 2.25$. However, the tensor mass was found to increase by about 4% and the effective radius by about 7% from 16 to 20 lattices at $\beta = 2.5$. We do not have enough data extrapolate mass and the radius to the infinite volume limit or to check whether the difference is due to statistical errors or whether there is an incomplete convergence. Given that no mass reductions of sufficient magnitudes were found as the lattice volume is changed, none of our states could be interpreted as a trelon pair.

In order to get some quantitative information on the effective radius, the glueball wave functions are fitted in the range $3 \leq r \leq 8$ by the form (6). This form fits the data rather well for the scalar glueball with the best-fit estimates obtained with a $\chi^2/\mathrm{NDF}$ of 0.92 - 0.67. Due to distortion$^3$ in the tensor wave function at small $r$ at $\beta = 2.5$, a meaningful fit was possible only in the range $6 \leq r \leq 8$. The effective radius obtained was confirmed by examining the plateau in the ratio $\log[\psi(r)/\psi(r+1)]$. Note, that our logarithmically plotted wave functions (Fig. 6) are merely illustrations.

To summarize: in the weak coupling region a spectrum of massive $0^{++}$, $0^{--}$ and $2^{++}$ glueballs is indicated with $m(0^{--}) < m(0^{++}) < m(2^{++})$.

Since there is a good signal for wave function persisting long enough to demonstrate convergence to the asymptotic value, it seems to be reasonable to estimate mass ratios with our present method. The estimates of masses and $r_0$, in lattice units, at various $\beta$ values are shown in Tables I, II and III.

### TABLE I: Masses of scalar glueballs in lattice units for two spatial extensions, $L = 16$ and 20.

| $\beta/L$ | 16  | 20  |
|-----------|-----|-----|
| $0^{++}$  | 0.803(6) | 0.441(4) |
| $0^{--}$  | 0.523(3) | 0.527(3) | 0.266(3) | 0.261(4) |

| $\beta/L$ | 16  | 20  |
|-----------|-----|-----|
| $0^{++}$  | 0.364(3) | 0.369(2) |
| $0^{--}$  | 0.182(2) |

Our results for lattice masses and mass ratios are generally, within statistical errors, in agreement with the existing Euclidean estimates$^3$, if perhaps a little

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$^3$ Because of the distortion and impossible complete elimination of all the excited-states, especially near $r \sim 0$, it follows that Eq. 6 holds only in the limited interval, which does not include the vicinity of $r \sim 0$.  

TABLE II: Sizes of scalar glueballs in lattice units for two spatial extensions, \( \beta = 16 \) and 20.

| \( \beta/L \) | \( \theta^{++} \) | \( \theta^{-} \) |
|--------------|---------|---------|
| 2.0          | 1.39(16) | 1.01(12) |
| 2.25         | 2.75(43) | 2.79(34) |
| 2.5          | 5.13(1.07) | 5.20(94) |

TABLE III: Mass and size of tensor glueballs in lattice units for two spatial extensions, \( \beta = 16 \) and 20.

| \( \beta/L \) | \( m \) | \( \sigma \) |
|--------------|-------|-----|
| 2.0          | 1.18(1) | 5.02(68) |
| 2.25         | 0.82(2) | 0.814(6) |
| 2.5          | 0.544(2) | 0.583(1) |

TABLE IV: Glueball sizes in the units of string tension.

| \( \beta \) | \( K(=\langle a^2 \rangle) \) | \( a_{eff} \) | \( r_{0^{++}}\sqrt{\sigma} \) | \( r_{0^{-}}\sqrt{\sigma} \) | \( r_{2^{++}}\sqrt{\sigma} \) |
|-------------|-----------------|-------------|-----------------|-----------------|-----------------|
| 2.0         | 0.0508(5)       | 0.0856      | 0.31(14)        | 0.24(9)         | 1.13(18)        |
| 2.25        | 0.0221(3)       | 0.0481      | 0.40(17)        | 0.32(16)        | 1.14(22)        |
| 2.5         | 0.0119          | 0.0272      | 0.56(21)        | 0.50(19)        | 1.11(25)        |

To extrapolate our effective radii to the continuum limit, we take the dimensionless products of sizes so that the scale, \( a \), in which they are expressed cancels. We choose to take products of the effective radii, \( r_{0.2}/a \), to \( a\sqrt{\sigma} \) since the string tension is our most accurately calculated quantity. As in the \((3+1)D\) confining theories, we expect that dimensionless product of physical quantities, such as \( r_{0.2}\sqrt{\sigma} \), will approach their continuum limit with correction of \( O(a_{eff}^2) \), where \( a_{eff} \) is the effective lattice spacing in “physical units” when the mass gap has been renormalized to a constant [17]. The string tension, \( K(=\langle a^2 \rangle) \), is obtained by using the Wilson loop averages and fitting the on-axis data with \( V(r) \). In Fig. 7, we show the product \( r_{0.2}\sqrt{\sigma} \) plotted against \( a_{eff} \). Since the products are plotted against \( a_{eff} \), the continuum extrapolations are simple straight lines. We notice that the product \( r_{0.2}\sqrt{\sigma} \) varies only slightly over the fitting range. The non-zero lattice spacing values of the product are within 0.04 - 0.29 and 0.01 - 0.02 standard deviations of the extrapolated zero lattice spacing results for the scalar and tensor glueballs respectively. The striking feature of this plot is the little variation of the product with \( a_{eff} \). This will make for very accurate and reliable continuum extrapolations. Linear extrapolations to the continuum limit yield values of \( 0.60 \pm 0.05 \) and \( 1.12 \pm 0.03 \), in the units of string tension, for the scalar and tensor states, respectively. In contrast to the tensor, the scalar glueball size shows significant finite-spacing errors. By setting the string tension to 420 MeV, we obtain the physical radii of 0.28(7) and 0.52(5) fm, for the scalar and tensor glueballs, respectively. Our results show the size of the tensor glueball roughly two times as large as the scalar glueball. The extension of the method to the \( SU(3) \) lattice gauge theory [19] yielded very similar results for the glueball size in the limit \( a \rightarrow 0 \). These estimates agree with the rough estimates of glueball sizes obtained at various temperatures in Ref. [9]. This is an improvement over the estimates obtained in Ref. 7 where the predicted radius for the tensor glueball (~ 0.8 fm) was found four times larger than scalar glueball radius.

FIG. 7: Glueball radii in the units of string tension as a function of the effective spacing, \( a_{eff} \). Extrapolations to the continuum limit are shown as dashed lines.

**B. Finite temperature results**

To check the consistency of our method, we performed a study on an asymmetric lattice: \( 16^2 \times 4 \) at \( \beta = 2.25 \). The procedure itself is a straightforward extension of the procedure adopted in the previous subsection. We do not plan to study the high temperature aspects of this model here but focus on the behaviour of the glueball mass and wave function in the deconfined region.

The physical temperature \( T = 1/(aN_t) \), is given via the lattice parameters as follows:

\[
T/\sqrt{\sigma} = \frac{1}{N_t \sqrt{K}}.
\]

For completeness, we give a temperature estimate of 1.125 in the units of string tension. By setting the string tension to 420 MeV, we estimate a physical temperature of \( T \sim 1.25T_c \), where the \( T_c \sim 360 \) MeV at pseudo-critical coupling \( \beta_c = 1.87(2) \) [20]. One expects [21]...
that the high temperature phase has a massless photon and the linear potential is replaced by the two dimensional logarithmic Coulomb potential. This logarithmic behaviour is equivalent to a power-law dependence of the Wilson loop correlation function,

$$C(r) = \langle P^+(r)P(0) \rangle \sim |r|^{-\eta(T)},$$

with an exponent which decreases as $T$ increases. Furthermore, since the high-temperature phase of the gauge theory corresponds to the ordered phase of the spin system, the predicted power-law behaviour of the correlation function is just like that of a two-dimensional U(1)-invariant spin system - a 2-D XY model.

Fig. 8 shows a plot of correlation functions versus separation. The straight line indicates the fit to the form \(10\). The finite temperature phase transition is visible in the change of the correlation function from exponential to power-law behaviour.

![FIG. 8: The logarithmic plot of the correlation function at $\beta = 2.25$. The straight line indicates power-law behaviour.](image)

in the change of the correlation function from exponential to power-law behaviour. Thus it becomes evident that $T > T_c$ in our simulation. It can be seen that form \(10\) fits the data rather well. Nonetheless, our Monte Carlo simulations were unable to confirm that the exponent is moving towards the value of 0.25 (that of the 2-D XY model \([22]\)) predicted for the continuum theory. Our estimated value for the exponent is four times larger than the predicted value. This indicates that our $\beta$ value of 2.25 is not large enough to give us reason to hope that we are approaching continuum physics. An interesting feature to explore in this context is whether the coupling to the matter fields in the leading order ($\beta \to \infty$) calculations will move the critical exponent towards the predicted value.

Fig. 9 shows the scalar and tensor wave functions obtained through the same analysis as in Figs. 3 and 5. Our results indicate that no significant changes occur in the scalar and tensor wave functions. Glueball masses appear almost unaffected. By comparing the results at $T = 0$ and $T = 360$, we observe an effective mass reduction, $(am_G(T \sim 0) - am_G(T \sim 360))$, of about 4%, with statistical uncertainties typically on less than a percent level, for $0^{++}$ and $2^{++}$ glueball modes. This appear to be a very small change since we expect a rather continuous mass reduction of glueballs in the deconfined phase. This might be due to the fact that for zero momentum the power-law behaviour of the correlation function leads at short distances to the spin-wave results, which prevents us from seeing the massless excitations. The non-vanishing effective masses would suggest the presence of glueball modes above $T_c$. Other work on finite temperature SU(3) \([1, 22]\) has also confirmed the survival of correlations above $T_c$ in the scalar and tensor colour-singlet modes. However, these studies have shown that thermal mass changes rather continuously across the critical temperature. The existence of the effective mass gives rise to the possibility that some of the nonperturbative effects survive in the deconfined phase, and the colour-singlet modes exist as metastable states above $T_c$. The metastable states in the ordered phase (large $\beta$) appear to be caused by the unusually large separation of a vortex pair, which may take many sweeps to recombine. Near the transition the number of vortices increase, and some of them begin to unbind. This eventually drives the system into a disordered phase as one moves to the region $T < T_c$. Whether bound or metastable modes, the glueballs can decay into two or more gluons thus acquiring finite width which is expected to become less negligible in the deconfined phase. Thus it becomes important to take into account the effect of width in best-fit analysis.

This might also explain a very modest reduction of our masses at $T > T_c$. However, from this study, it is not possible to determine whether such colour singlet modes really survive above $T_c$ as metastable states. An extensive systematic analysis, of unquenched improved lattice QCD at finite temperature, along these lines is under way \([24]\).

![FIG. 9: Scalar and tensor glueball wave functions measured on $16^2 \times 4$ lattice at $\beta = 2.25$.](image)
IV. SUMMARY AND CONCLUSION

We have studied wave functions and sizes of scalar and tensor glueballs using improved 3-dimensional U(1) lattice model. In this preliminary study we take a more direct approach to the problem; instead of fixing a gauge or a path for the gluons, we measure the correlation functions from our lattice operators from spatially connected Wilson loops which, being the expectation values of closed-loop paths, are gauge invariant. This approach has the advantage that the disentangling of the glueball and torelon is usually taken care of automatically by the choice of Wilson or Polyakov loops. We observed that the size of tensor glueball is roughly two times larger than the size of the scalar glueball. We have successfully extended our method to extract the results of glueball sizes for SU(3) lattice gauge model and obtained some encouraging results [19]. We believe that our estimates are more reliable than the results obtained in Ref. [7], where the size of the tensor glueball was found to be \( \sim 0.8 \text{ fm} \), four times as large as its scalar counterpart. The predicted zero lattice spacing results are not actually found by extrapolation to zero lattice spacing, but are obtained instead from calculations at \( \beta \) of 2.2 of glueball size, with no accurate representation of the effect of the absence of extrapolation. Also the results were of limited interest because of their manifest dependence on the gauge chosen and the problem of disentangling of the glueballs and torelons.

Finally, for completeness, we extended our method to measure the wave function and mass for a finite temperature deconfinement phase. For the lowest 0\(^{++}\) and 2\(^{++}\) glueballs, no significant mass reduction was observed in deconfined phase, while the wave functions remain almost unchanged. The existence of the effective mass indicates that colour-singlet modes may survive in the deconfined phase as metastable states. In such a case glueball decay and decay width, as spectral component, in the deconfinement phase are the most feasible candidates for a more reliable analysis for the future studies.

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