The universal behavior of one-dimensional, multi-species branching and annihilating random walks with exclusion

Géza Ódor
Research Institute for Technical Physics and Materials Science,
H-1525 Budapest, P.O. Box 49, Hungary

A directed percolation process with two symmetric particle species exhibiting exclusion in one dimension is investigated numerically. It is shown that if the species are coupled by branching \((A \rightarrow AB, B \rightarrow BA)\) a continuous phase transition will appear at zero branching rate limit belonging to the same universality class as that of the dynamical two-offspring (2-BARW2) model. This class persists even if the branching is biased towards one of the species. If the two systems are not coupled by branching but hard-core interaction is allowed only the transition will occur at finite branching rate belonging to the usual 1+1 dimensional directed percolation class.

The study of phase transitions in low dimensions is an interesting and widely investigated topic \([12]\) (since the mean-field solution is not valid). The research of non-equilibrium phase transitions occurring in one dimensional coupled systems has drawn interest nowadays \([8,14]\). Several models have been found with transitions that do not belong the robust directed percolation (DP) class \([13,17]\) or to the parity conserving (PC) class \([3,18,19]\) which are the most prominent ones among one component systems. Particle blocking which is common in one dimension has not been taken into account in field theoretical description of these models yet \([20,21]\). It has been known for some time already \([23]\) that the pair contact process \([24]\) can be regarded as a coupled system that exhibits DP class static exponents while the spreading ones depend on initial densities \([25]\). The field theoretical investigation of Janssen \([21]\) predicts that in coupled DP systems the symmetry between species is unstable and generally a phase transition belongs to the class of unidirectionally coupled DP where coupling between pairs of species is relevant in one direction only. Such systems have been shown to describe also certain surface roughening processes \([10,13]\).

Recently we have shown \([12]\) that in the two-component annihilating random walk \((AB \rightarrow \emptyset, BB \rightarrow \emptyset)\) owing to the hard-core interaction of particles dynamical exponents are non-universal. Some consequences of hard core effects for random walks in one dimension have been known for some time already \([22]\).

Very recently simulations \([6,7]\) gave numerical evidence that in the two-component branching and annihilating random walk (2-BARW2) the lack of particle exchange between different species results in new universality classes in contrast to widespread beliefs that bosonic field theory can well describe these systems. The critical exponents obtained numerically suggest that the location of offspring particles at branching is the relevant factor that determines the critical behavior. In particular if the parent separates the offsprings:

1) \(A \rightarrow BAB\) the steady state density will be higher than in the case when they are created on the same site:
2) \(A \rightarrow ABB\) for a given branching rate because in the former case they are unable to annihilate with each other.

This results in different order parameter exponents for the symmetric (2-BARW2s) and the asymmetric (2-BARW2a) cases \((\beta_s = 1/2 vs \beta_a = 2\) for 1) and 2\), respectively.

Hard-core effects are conjectured to cause a series of new universality classes in one dimension \([1]\). In this paper I point out that probably only a few universality classes emerge as the consequence of particle exclusion, because other symmetries and conservation laws (like that of the PC class) will become irrelevant.

In the present study first I show that in case of the two-component single off-spring BARW model (2-BARW1) defined as

\[
\begin{align*}
A^{\sigma_A/2} & AB, & A^{\sigma_A/2} & BA \\
B^{\sigma_B/2} & BA, & B^{\sigma_B/2} & AB \\
AA & \xrightarrow{\lambda} \emptyset, & BB & \xrightarrow{\lambda} \emptyset \\
AO & \xrightarrow{d} OA, & BO & \xrightarrow{d} OB \\
AB & \xrightarrow{d} BA 
\end{align*}
\]

a continuous phase transition will occur at zero branching rate limit \((\sigma = 0)\) like in the 2-BARW2 model where they are equivalent and therefore the exponents on the critical point must be the same as those determined in \([3,7,12]\). Furthermore I show that the order parameter exponent describing the singular behavior of the steady state density near the critical point coincides with that of the 2-BARW2s model.

The particle system was simulated on a lattice with size \(L = 4 \times 10^4\) and periodic boundary conditions for different \(\sigma\)-s (with \(\lambda = d = 1 - \sigma\) condition). The initial condition was uniformly random distribution of \(A\)-s and \(B\)-s with a total concentration 0.5. The evolution of the density was followed until steady state has been
reached plus $t \sim 10^4$ Monte Carlo sweeps (throughout the whole paper $t$ is measured in units of Monte Carlo sweeps (MCS) of the lattices). As Figure 1 shows a phase transition occurs at $\sigma_A = \sigma_B = 0$ indeed.

The order parameter exponent has been determined with the local slope analysis of the data

$$\beta_{eff}(\sigma) = \frac{\ln \rho_i - \ln \rho_{i-1}}{\ln \sigma_i - \ln \sigma_{i-1}},$$

providing an estimate for the true asymptotic behavior of the order parameter

$$\beta = \lim_{\sigma \to 0} \beta_{eff}(\sigma).$$

As one see on Figure 2 $\beta_{eff}$ extrapolates to $\beta = 0.50(1)$ with a strong correction to scaling like in case of the 2-BARW2s model [6]. The coincidence of this off-critical exponent in addition to the equivalence of processes at the critical point assures that they belong to the same universality class.

If we destroy the symmetry between species by the branching rates: $\sigma_A = \sigma_B/2$ we still get the same order parameter exponents ($\beta = 0.50(1)$) for both species (Fig.2). Therefore this universality class is stable with respect to coupling strengths unlike the coloured and flavoured directed percolation [21].

It is also insensitive whether or not the parity of particles is conserved meaning that the $A \rightarrow BAB$ process can be decomposed to a sequence of $A \rightarrow AB, AB \rightarrow BAB$ processes. This may seem to be quite obvious when particle exchange is not allowed, and if locality is assumed. By the choice of parameters $d = 1 - \sigma$ in the neighbourhood of the critical point the the diffusion is strong and the locality condition is not met. Still the two process share the same critical behavior.

If we decouple the two systems and allow hard-core exclusion only:

\begin{align*}
A \rightarrow AA & \quad (8) \\
B \rightarrow BB & \quad (9) \\
AA \rightarrow O & \quad , \quad BB \rightarrow O & \quad (10) \\
AO \leftrightarrow OA & \quad , \quad BO \leftrightarrow OB & \quad (11) \\
AB \leftrightarrow BA & \quad (12) \\
\end{align*}

the critical point will be shifted to $\sigma = 0.81107(1)$ and DP like density decay can be observed on the local slopes defined as

$$\alpha_{eff}(t) = -\frac{\ln[\rho(t)/\rho(t/m)]}{\ln(m)}$$

(where we use $m = 8$ usually) (see Figure 3).
One can not observe any relevant correction to scaling here, the most straight curve corresponding to the critical one ($\sigma = 0.81107$) extrapolates to $\alpha = 0.158(2)$ which agrees very well with the $\beta/\nu_1 = 0.159464(6)$ value of the 1 + 1 DP class value that can be found in the literature [20]. This is different from the case of coupled annihilating random walk, where the blocking causes marginal perturbation to the standard decay process [22].

One can generalize the results by taking into account that neighbouring AA and BB offsprings decay very quickly and therefore irrelevant for the leading scaling behavior. 

**Conjecture:** In coupled, one dimensional N-component BARW systems with particle exclusion and branching processes like: $A \rightarrow BABB$, $A \rightarrow BAAA$, $A \rightarrow BAC$ ... leaving behind non-reacting neighbouring particles which block each other the universality class of a phase transition will be the same as that of 1-BARW2s. If the branching creates only pairs that can annihilate immediately (like: $A \rightarrow BAAB$ ... etc.) the class of transition will be the same as that of the 2-BARW2a model. We can also conclude that in case of reaction-diffusion processes where spontaneous decay is allowed: $2A \rightarrow A$, $A \rightarrow 0$ the blocking effect between dissimilar species is irrelevant.

It is very likely that the transition of a very recently introduced ladder model [3] also belongs to this class. This model is composed of two one dimensional subsystems following BARW at the critical point and coupled by ladder links. In the supercritical region by updating an active site one can create an offspiring on the other subsystem or increase the inactivity level of that site. For small coupling strength ($s = 1$) the very few blocking events can not introduce relevant blocking on the other subsystem and the scaling exponents agree with those of the coupled BARWe model without exclusion [20]. For stronger coupling strength ($s = 2$) there are more blocking possibility resulting in 1-BARW2s scaling exponents.

In conclusion I have shown that the one dimensional two species coupled BARW with exclusion and one offspiring has the same critical transition point as that of the 2-BARW2s model investigated earlier. The hard-core interaction itself is not sufficient to cause deviation in scaling behavior from that of DP. A conjecture is given with regard the universality classes in coupled BARW systems exhibiting particle exclusion.

**Acknowledgements:**

The author would like to thank T. Antal for the stimulating discussions and N. Menyhárd for critically reading the manuscript. Support from Hungarian research fund OTKA (Nos. T-25286 and T-23552) and from Bólyai (No. BO/00142/99) is acknowledged.

[1] J. Marro and R. Dickman, *Nonequilibrium phase transitions in lattice models*, Cambridge University Press, Cambridge, 1999.
[2] H. Hinrichsen, preprint, cond-mat/0001070.
[3] M. J. Howard and U. C. Täuber, J. Phys. A 30, 7721 (1997).
[4] G. Ódor, Phys. Rev. E 62, R3027 (2000).
[5] H. Hinrichsen, eprint cond-mat/0004343.
[6] S. Kwon, J. Lee and H. Park, Phys. Rev. Lett. 85, 1682 (2000).
[7] G. Ódor, cond-mat/0008383, to be published in Phys. Rev. E.
[8] A. Lipowski, cond-mat/0007411.
[9] U.C. Täuber, M.J. Howard, and H. Hinrichsen, Phys. Rev. Lett. 80, 2165 (1998); Y.Y. Goldschmidt, Phys. Rev. Lett. 81, 2178 (1998); Y.Y. Goldschmidt, H. Hinrichsen, M.J. Howard, and U.C. Täuber, Phys. Rev. E 59, 6381 (1999); H.K. Janssen, cond-mat/9901188.
[10] H. Hinrichsen and G. Ódor, Phys. Rev. Lett. 82, 1205 (1999).
[11] H. Hinrichsen and G. Ódor, Phys. Rev. E 60, 3842 (1999).
[12] J. E. de Freitas, L. S. Lucena, L. S. da Silva and H. Hilhorst, Phys. Rev. E 61, 6330 (2000).
[13] F. van Wijland, K. Oerding and H. Hilhorst, Physica A 251, 179 (1998).
[14] S. Trimper, U.C. Täuber and G.M. Schütz, cond-mat/0001387.
[15] H. K. Janssen, Z. Phys. B 42, 151 (1981).
[16] P. Grassberger, Z. Phys. B 47, 365 (1982).
[17] W. Kinzel, in *Percolation Structures and Processes*, ed. G. Deutscher, R. Zallen, and J. Adler, Ann. Isr. Phys. Soc. 5.
[18] P. Grassberger, F. Krause and T. von der Twer, J. Phys. A:Math.Gen., L105 17 (1984).
[19] For further references see : N. Menyhárd N. and G. ´Odor, cond-mat/0001104; Brazilian J. of Physics 30, 113 (2000).
[20] J. L. Cardy and U. C. Täuber, J. Stat. Phys. 90, 1 (1998).
[21] H. K. Janssen, preprint, cond-mat/0006129.
[22] G. Ódor and N. Menyhárd, Phys. Rev. E. 61, 6404 (2000).
[23] G. Mennon, M. Barma and D. Dhar, J. Stat. Phys. 86, 1237 (1997).
[24] I. Jensen and R. Dickman, Phys. Rev. E 48, 1710 (1993).
[25] M. A. Muñoz, G. Grinstein, R. Dickman and R. Livi, Phys. Rev. Lett. 76, 451 (1996) and Physica D 103, 485 (1997).
[26] I. Jensen, J. Phys. A 32, 5233 (1999).