Two-flavor lattice QCD study of the axial charges of $N(1535)$ and $N(1650)$

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We show the first lattice QCD results on the axial charges $g_{\pi N^*}$ of $N^*(1535)$ and $N^*(1650)$. The measurements are performed with two flavors of dynamical quarks employing the renormalization-group improved gauge action at $\beta=1.95$ and the mean-field improved clover quark action with the hopping parameters, $\kappa=0.1375$, $0.1390$ and $0.1400$. We construct $2 \times 2$ correlation matrices and diagonalize them so that the signs of $N^*(1535)$ and $N^*(1650)$ are properly separated. Wraparound contributions in the correlator, which can be another source of signal contaminations, are eliminated by imposing the Dirichlet boundary condition in the temporal direction. The axial charge of $N^*(1535)$ is found to take small values as $g_{\pi N^*}$, whereas that of $N^*(1650)$ is approximately 0.5, which is almost independent of quark masses and consistent with the predictions by the naive nonrelativistic quark model.

1. Introduction

Chiral symmetry together with its spontaneous breaking is one of the key ingredients in the low-energy hadron or nuclear physics. Due to its spontaneous breaking, up and down quarks, whose current masses are of the order of a few MeV, acquire the large constituent masses of a few hundreds MeV, and are consequently responsible for about 99% of mass of the nucleon and hence that of our world. One would thus consider that chiral condensate $\langle \bar{\psi} \psi \rangle$, the order parameter of the chiral phase transition, plays an essential role in the hadron-mass genesis in the light quark sector. On the other hand, chiral symmetry gets restored in systems where hard external energy scales such as high-momentum transfer, temperature($T$), baryon density and so on exist, owing to the asymptotic freedom of QCD. Then, several questions may arise: Are all hadronic modes massless in such systems? Can hadrons be massive even without non-vanishing chiral condensate?

An interesting possibility was suggested some years ago by DeTar and Kunihiro\cite{1}, who showed that nucleons can be massive even without the help of chiral condensate due to the possible chirally invariant mass terms, which give degenerated finite masses to the members in the chiral multiplet (a nucleon and its parity partner) even when chiral condensate is set to zero: In order to demonstrate this possibility for a finite-$T$ case, they introduced a linear sigma model which has a nontrivial chiral structure in the baryon sector and a mass-generation mechanism essentially different from that by the spontaneous chiral symmetry breaking. Interestingly enough, their chiral doublet model has recently become a source of debate as a possible scenario of observed parity doubling in excited baryons\cite{2,3,4,5,6,7}, although their original work\cite{1} was intended for an application to finite-$T$ systems.

It is thus an intriguing problem to clarify the chiral structure of excited baryons in the light quark sector beyond model considerations. One of the key observables which are sensitive to the chiral structure of the baryon sector is axial charges\cite{1}. The axial charge of a nucleon $N$ is encoded in the three-point function $\langle N|A^a_\mu|N \rangle = \bar{u} \gamma_\mu \gamma_5 \tau^a \frac{g_{\pi N^*}}{2} (q^2 + q_\mu g_5 h A(q^2)) u$. (1)

Here, $A^a_\mu = \gamma_\mu \gamma_5 \tau^a Q$ is the isovector axial cur-
rent. The axial charge $g_A$ is defined by $g_A(q^2)$ with the vanishing transferred momentum $q^2 = 0$. It is a well-known fact that the axial charge $g_A^{NN}$ of $N(940)$ is 1.26. Though the axial charges in the chiral broken phase can be freely adjusted with higher-dimensional possible terms and cannot be the crucial clues for the chiral structure [34], they would reflect the internal structure of baryons and would play an important role in the clarification of the low-energy hadron dynamics.

In this paper, we show the first unquenched lattice QCD study [8] of the axial charges $g_A^{N^*N^*}$ of $N^*(1535)$ and $N^*(1650)$, two lowest nucleon resonances in the negative parity channel. We employ $16^3 \times 32$ lattice with two flavors of dynamical quarks, generated by CP-PACS collaboration [9] with the renormalization-group improved gauge action and the mean-field improved action [8].

We then have to construct an optimal operator which dominantly couples resonances in the negative parity channel. We employ 16 × 32 lattice with two flavors of dynamical quarks, generated by CP-PACS collaboration [8].

Our main concern is the axial charges of the negative-parity nucleon states in the sea and valence quarks. $c_{\text{sea}}$, $c_{\text{val}}$ = 0.1375, 0.1390 and 0.1400, which correspond to quark masses of $\sim 150$, 100, 65 MeV and the related $\pi$-$\rho$ mass ratios are $m_{\pi}/m_{\rho} = 0.804(1)$, 0.752(1) and 0.690(1), respectively. Statistical errors are estimated by the jackknife method with the bin size of 10 configurations.

2. Formulation

Our main concern is the axial charges of the negative-parity nucleon resonances $N^*(1535)$ and $N^*(1650)$ in channel. We then have to construct an optimal operator which dominantly couples to $N^*(1535)$ or $N^*(1650)$. We employ the following two independent nucleon fields,

$$N_1(x) \equiv \epsilon_{abc} u^a(x) (u^b(x) C\gamma_5 d^c(x))$$

(2)

and

$$N_2(x) \equiv \epsilon_{abc}\gamma_5 u^a(x) (u^b(x) C d^c(x))$$

(3)

in order to construct $2 \times 2$ correlation matrices and to separate signals of $N^*(1535)$ and $N^*(1650)$. Here, $u(x)$ and $d(x)$ are Dirac spinors for u- and d-quark, respectively, and $a, b, c$ denote the color indices. Even after the successful signal separations, there still remain several signal contaminations: Signal contaminations by scattering states and wraparound effects.

Due to the unquenched gauge configurations, the negative parity nucleon states could decay to $\pi$ and $N$, and their scattering states could inevitably get into the spectrum. The sum of the pion mass $M_\pi$ and the nucleon mass $M_N$ is however in our setups heavier than the masses of the lowest two states (would-be $N^*(1535)$ and $N^*(1650)$) in the negative parity channel. We then do not suffer from any scattering-state signals. The other possible contamination is wraparound effects [10]. Since we perform unquenched calculations, the excited nucleon $N^*$ can decay into $N$ and $\pi$, and even when we have no scattering state $|N^1\pi\rangle$, we could have another type of “scattering states”. The baryonic correlator $\langle N^*(t_{\text{src}})N^*(t_{\text{src}}) \rangle$ can still accommodate, for example, the following term.

$$\langle \pi|N^*(t_{\text{src}})|N\rangle \langle N|N^*(t_{\text{src}})|\pi\rangle \times e^{-E_N t_{\text{src}} - t_{\text{src}}}.$$  

(4)

Here, $N_t$ denotes the temporal extent of a lattice. Such a term is quite problematic and mimic a fake plateau at $E_N - E_\pi$ in the effective mass plot because it behaves as $\sim e^{-E_N t_{\text{src}} - t_{\text{src}}}$. In order to eliminate such contributions, we impose the Dirichlet condition on the temporal boundary for valence quarks, which prevents valence quarks from going over the boundary. (Wraparound effects can be found even in quenched calculations [10].)

We employ zero-momentum-projected point-type operators for the sinks, and employ wall-type operators in the Coulomb gauge for the sources. After we diagonalize the $2 \times 2$ correlation matrices, we can construct optimized operators for $N(1535)$ and $N(1650)$ states. Once we construct optimized operators, we can easily compute the (non-renormalized) vector and axial charges $g_{V,A}^{+,[\text{lat}]}$ for the positive- and negative-parity nucleons via three-point functions with the so-called sequential-source method [11]. In prac-
We finally reach the renormalized charges $g_A^{\pm} = \tilde{Z}_A g_A^{\pm[\text{lat}]}$ with the prefactors $\tilde{Z}_A \equiv 2\kappa u_0Z_A \left(1 + b_A \frac{m_q}{m_\pi} \right)$, which are estimated with the values listed in Ref. [9].

3. Results

We first take a stock of the axial charge $g_A^{0+}$ of the ground-state positive-parity nucleon, which is well known and can be the references. The axial charge $g_A^{0+}$ of the positive parity nucleon is shown in Fig. 2 as open squares. The axial charge of the positive parity nucleon shows little quark-mass dependence, and they lie around the experimental value 1.26.

We finally show the axial charges of the negative-parity nucleon resonances in Fig. 2. One finds at a glance that the axial charge $g_A^{-}$ of $N^*(1535)$ takes quite small value, as $g_A^{-} \sim 0.1$ and that even the sign is quark-mass dependent. While the wavy behavior might come from the sensitiveness of $g_A^{-}$ to quark masses, this behavior may indicate that $g_A^{-}$ is rather consistent with zero. These small values are not the consequence of the cancellation between u- and d-quark contributions. The u- and d-quark contributions to $g_A^{-}$ are in fact individually small [8]. On the other hand, the axial charge $g_A^{-}$ of $N^*(1650)$ is found to be about 0.55, which has almost no quark-mass dependence. The striking feature is that these axial charges, $g_A^{0-} \sim 0$ and $g_A^{1-} \sim 0.55$, are consistent with naive nonrelativistic quark model calculations [12][13], $g_A^{0-} = -\frac{4}{9}$ and $g_A^{1-} = \frac{5}{9}$. Such values are obtained if we assume that the wave functions of $N^*(1535)$ and $N^*(1650)$ are $|l = 1, S = \frac{1}{2} \rangle$ and $|l = 1, S = \frac{3}{2} \rangle$ neglecting the possible state mixing. (Here, $l$ denotes the orbital angular momentum and $S$ the total spin.)
In the chiral doublet model \cite{1,2}, the small $g_{A^0}^{N^*N^*}$ is realized when the system is decoupled from the chiral condensate $\langle \bar{\psi} \psi \rangle$. The small $g_{A^0}^{N^*}(1535)$ then does not contradict with the possible and attempting scenario, the chiral restoration scenario in excited hadrons \cite{2}. If this scenario is the case, the origin of mass of $N^*(1535)$ (or excited nucleons) is essentially different from that of the positive-parity ground-state nucleon $N(940)$, which mainly arises from the spontaneous chiral symmetry breaking. However, the non-vanishing axial charge of $N^*(1650)$ unfortunately gives rise to doubts about the scenario.

4. Conclusions

We have performed the first lattice QCD study of the axial charges $g_{A^0}^{N^*N^*}$ of $N^*(1535)$ and $N^*(1650)$, with two flavors of dynamical quarks employing the renormalization-group improved gauge action at $\beta=1.95$ and the mean-field improved clover quark action with the hopping parameters, $\kappa=0.1375, 0.1390$ and 0.1400. We have found the small axial charge $g_{A^0}^{0-}$ of $N^*(1535)$, whose absolute value seems less than 0.2 and which is almost independent of quark mass, whereas the axial charge $g_{A^0}^{1-}$ of $N^*(1650)$ is found to be about 0.55. These values are consistent with naive nonrelativistic quark model predictions, and could not be the favorable evidences for the chiral restoration scenario in (low-lying) excited hadrons. Further investigations on the axial charges of $N^*(1535)$ or other excited baryons will cast light on the chiral structure of the low-energy hadron dynamics and on where hadronic masses come from.

5. acknowledgments

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