Heavy-Flavour Production at HERA

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Abstract. We review the theoretical and experimental status of heavy-flavour production at HERA. The results presented include some outstanding issues in charm and beauty photoproduction, charm production in DIS and quarkonium production.

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1. Introduction

The production of heavy flavours (charm and beauty) in high energy ep collisions at HERA provides new opportunities to study the dynamics of perturbative QCD and to extract information on the proton and photon structure.

The heavy-flavour cross section at HERA is dominated by photoproduction events where the electron is scattered by a small angle, producing photons of almost zero virtuality. The dynamics of charm and beauty photoproduction can be probed at HERA in a wide kinematical region, the available centre-of-mass energy being more than one order of magnitude higher than in fixed-target experiments. Resolved-photon interactions, where the photon fluctuates into parton constituents, which undergo hard scattering with the partons from the proton, contribute significantly at HERA energies and allow one to study the photon structure. Because of the high cross sections in photoproduction, some sort of tagging must be done to allow the data to be recorded to tape in the experiment. A charm meson may be tagged and a jet finder may also be used, with a certain number of high transverse energy jets demanded. In the first case information on the charm meson is available. In the second case the observables of the meson, the jet and the correlation between the two may be examined. This sheer wealth of variables and currently available statistics is what makes photoproduction most interesting. All the previous studies from normal photoproduction may be repeated with a charm tag, but thought should be given to original analyses.

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As a complement, heavy-flavour production at HERA can be studied in deep inelastic scattering (DIS). The finding of a scattered positron and the distribution of these events in bins of the parton momentum fraction $x$ and the photon virtuality $Q^2$ is well documented in the literature. With the additional requirement of a tagged charm meson (typically $D^*$) a way is found to measure the charm tagged content of the proton structure function $F_2$ ($F_2^c$). Since $F_2^c$ is evolved from the gluon content, it provides a measurement of the gluon density in the proton. Large statistics in the normal $F_2$ measurement and the added measurement of $F_2^c$ constrain the theory greatly. Inclusive $F_2$ is predicted to contain between 10% and 25% charm and it must therefore be properly taken into account in global analyses of structure function data. Differential cross sections for $D^*$ production in DIS have also become available and allow a more detailed study of the charm production mechanisms than is possible with the inclusive $F_2^c$ measurement alone.

The production of heavy-quark bound states has been the subject of intense study during the past few years. Exciting phenomenological developments followed from the application of non-relativistic QCD (NRQCD), an effective theory that disentangles physics on the scale of the heavy-quark mass, relevant to the production of a heavy-quark pair, from physics on the scale of the bound state’s binding energy, relevant to the formation of the quarkonium. The NRQCD approach implies that so-called colour-octet processes, in which the heavy-quark–antiquark pair is produced at short distances in a colour-octet state and subsequently evolves non-perturbatively into a physical quarkonium, must contribute to the cross section. Although gluon fragmentation into colour-octet quark pairs appears as the most plausible explanation of the large direct $\psi$ production cross section observed at the Tevatron, NRQCD factorization is still not definitely established on a quantitative level. The analyses of inclusive charmonium production at HERA offer unique possibilities to test general features of the quarkonium production mechanisms and to establish the phenomenological significance of colour-octet processes.

In the following we will discuss various aspects of charm and beauty production at HERA, organized into the three above-mentioned areas: photoproduction, deep inelastic scattering and quarkonium production. Additional information may be found in [1–3].

2. Photoproduction

The photon structure is being probed in new kinematic regimes at HERA. Tagging of charmed mesons or semileptonic decays in photoproduction events is providing a wealth of information on charm in the photon and the processes involved in its production. Thanks to the overall higher cross section for photoproduction events, there exists a large statistics sample from the 1996 and 1997 data sets alone ($37 \text{ pb}^{-1}$). Since photoproduction of charm and beauty is much less affected by non-perturbative and higher-order effects than hadroproduction cross sections, the HERA measurements can provide important tests of the heavy-flavour production dynamics.
The photon–proton cross section may be written as a sum of a direct and a resolved-photon contribution:

\[
\sigma_{\gamma P}(Q^2) = \sum_i \int dP f_i^P(x_P, \mu_P) \sigma_{i\gamma}(x_P, \alpha_s(\mu_R), \mu_P, \mu_R, \mu_\gamma, Q^2)
\]

\[
+ \sum_{ij} \int dP d\gamma f_i^P(x_P, \mu'_P) f_j^\gamma(x_\gamma, \mu_\gamma) \sigma_{ij}(x_P, x_\gamma, \alpha_s(\mu'_R), \mu'_P, \mu'_R, \mu_\gamma, Q^2),
\]

(1)

where \( f_i^P(x_P, \mu_P) \) and \( f_j^\gamma(x_\gamma, \mu_\gamma) \) denote the density functions for parton \( i \) and \( j \) in the proton and in the photon, respectively. The renormalization scale is labelled \( \mu_R \), and \( Q^2 \) is the hard scale of the interaction. The factorization scales \( \mu_P \) and \( \mu_\gamma \) are related to the subtraction of divergences associated with the collinear emission of partons from the incoming parton in the proton and in the incoming photon, respectively.

The factorization of collinear initial-state singularities has to be performed at NLO where Feynman diagrams like Figure 1 contribute. Although this diagram forms part of the \( \mathcal{O}(\alpha_s^2) \) cross section, if the virtuality \( k_T^2 \) of the propagator labelled in the figure is less than \( \mu_\gamma^2 \), then its contribution is already included in the LO-resolved process. Hence the separation of direct and resolved reactions is ambiguous beyond LO and only the sum is meaningful. This is explicitly shown in (1), where the factorization scale \( \mu_\gamma \) appears in both the direct and the resolved contributions. Total cross sections, single-inclusive distributions as well as heavy-quark correlations have been calculated to next-to-leading order accuracy [4].

In higher orders, potentially large terms \( \sim \alpha_s \ln(p_T^2/m_c^2) \) arise from the collinear branching of gluons or photons into heavy-quark pairs or from the collinear emission of gluons by a heavy quark at large transverse momentum. For \( p_T \gg m_c \) these terms may spoil the convergence of the perturbation series and should be resummed by absorbing the collinear \( \ln(p_T^2/m_c^2) \)-singularities into heavy-quark parton densities and fragmentation functions. Applications of this approach to charm and \( D^* \) photoproduction at \( p_T > m_c \) can be found in [5]. For a comparison of the different theoretical schemes, see [6].

Experimentally, the first topic to be studied, in general, is the production of charmed mesons, which may be identified by their decay products. Both H1 and ZEUS

![Figure 1. A NLO direct photoproduction process.](image)
have shown results on the $D^*$ meson in photoproduction \cite{7,8}. This channel is favoured because of its clear signal. A comparison of ZEUS data and different NLO theory predictions for the $D^*$ pseudorapidity distribution is shown in Figure 2. The data lie above the NLO theory when the $D^*$ meson travels in the forward direction, as is also seen in the DIS case. Although the discrepancy might not be considered very significant at present, some interesting suggestions were made for its possible causes. They are listed here and discussed below:

(i) ‘String effects’ between the proton remnant and the $c$ quark \cite{11}.
(ii) Low-$x_\gamma$ non-perturbative gluon in the photon \cite{12}. This would give rise to more charm and is currently poorly constrained by LEP data.
(iii) Effects attributed to the Peterson et al. fragmentation function used.
(iv) More beauty than predicted by the theory.

Item (i) can be examined by comparing the available LO Monte Carlo programs to the data. In the fragmentation process a string would be linked between the charm quark and the forward travelling proton remnant. The tension in this string could be responsible for pulling the quark (and hence the meson) forward. This effect is not implemented in the NLO calculations, so LO Monte Carlo would be more forward than NLO calculations. This comparison was carried out at the recent HERA Monte Carlo Workshop \cite{13} and a better agreement is seen between the shape of the LO curves and

Figure 2. The pseudorapidity distribution of the tagged $D^*$ for $p_T^{D^*} > 2, 3, 4, 6$ GeV. The points are ZEUS data for the 1996 and 1997 data sample \cite{7}. The inner error bars are the statistical errors and the outer error bars are the statistical and systematic errors added in quadrature. The curves are different theoretical NLO predictions \cite{9,5}. The variable $\epsilon$ is the non-perturbative parameter in the Peterson et al. fragmentation function \cite{10}. 
Figure 3. The pseudorapidity distribution of the tagged $D^*$ for $p_{\perp}^{D^{*}} > 2$ GeV and $p_{\perp}^{D^{*}} > 4$ GeV. The points are ZEUS preliminary data for the 1996 and 1997 data sample. The inner error bars are the statistical errors and the outer error bars are the statistical and systematic errors added in quadrature. The solid histogram is HERWIG [14] and the dashed histogram is PYTHIA [15], both having been area normalised to the data.

the data (see Figure 3). However, there are many other effects implemented in the Monte Carlo (including item (iii)). The process of separating them is still ongoing.

Previously ZEUS demonstrated [16] its sensitivity to the low-$x_{\gamma}$ regions, by the use of the observable

$$x_{\gamma}^{\text{OBS}} = \frac{\sum_{\text{jets}} E_{T}^{\text{jet}} e^{-\eta_{\text{jet}}}}{2yE_{e}},$$

where the sum is over the two jets with highest transverse energy. In LO, $x_{\gamma}^{\text{OBS}}$ can be interpreted as an observable related to the fraction of the photon’s momentum participating in the hard-scale interaction. At higher $x_{\gamma}^{\text{OBS}}$ (the usual cut is 0.75) the direct processes dominate, and at lower $x_{\gamma}^{\text{OBS}}$ the resolved processes dominate. If a charm meson or semileptonic decay is then tagged in these events, statements can be made about the charm component of the photon. This is of interest because charm in the photon may come from the low-$x_{\gamma}$ non-perturbative gluon, a parton density of the photon still badly constrained by LEP [17].

Demanding a jet also provides another, even harder, scale to the calculation. Typical experimental jet transverse energies are of the order of 6 or 7 GeV, well above the charm mass. When $Q^2 \gg m_c^2$, charm should behave as a light quark. At the scales probed with present data it is unclear to what extent this is true, and the problem arises of how to treat the intermediate $Q^2$ region. Similar theoretical issues arise for charm production in DIS and will be discussed in the next section.
Figure 4. The $x_{\text{OBS}}^\gamma$ distribution for tagged $D^*$ decays from charm, beauty and light quarks. The points are ZEUS preliminary data points for the 1996 and 1997 data sample. The inner error bars are the statistical errors and the outer error bars are the statistical and systematic errors added in quadrature. In the top plot the shaded histograms show the direct and resolved contributions as predicted by HERWIG 5.9. The solid line is the two contributions added together. In the bottom plot the same data is shown (rebinned into four bins) against a NLO calculation with two choices of parameters.

ZEUS have published a measurement of $x_{\text{OBS}}^\gamma$ with a tagged $D^*$ meson, shown in Figure 4. This shows a clear tail to low $x_{\text{OBS}}^\gamma$, which in terms of LO Monte Carlo programs can only be explained by a large (40%) component of LO-resolved processes. This is heavily dependent on the amount of charm in the parton density function used. A NLO calculation, performed in a factorization scheme where heavy quarks are exclusively generated at short distances from light-parton scattering, cannot describe this long tail and falls short of both the data and the LO Monte Carlo. This could be due to a large non-perturbative part of the gluon in the photon, or could be attributable to effects from the fragmentation function used.

Heavy-hadron fragmentation functions were discussed as an interesting area of study and not just as a possible explanation as item (iii) on the above list. The extraction of fragmentation functions from experimental data and the value of the corresponding non-perturbative parameters are closely related to the description of the perturbative part of the production cross section. Different theoretical schemes for calculating heavy-quark production require different heavy-hadron fragmentation functions. (See [13] for
recent NLO extractions of $D^*$ fragmentation functions from $e^+e^-$ data.) Suggestions have been made for defining and directly measuring fragmentation functions at HERA \[19\]. However, the invariant mass of the subprocess cannot be determined directly and has to be reconstructed from observables in the final state (e.g. jet variables). It therefore makes sense to define the fragmentation parameter in terms of the jets. One example would be:

$$z = \frac{P \cdot p(D^*)}{P \cdot p(jet)} = \frac{(E - p_z)_{D^*}}{(E - p_z)_{jet}},$$

where $P$ is the four momentum of the proton and $p(D^*)$ and $p(jet)$ are those of the meson and the jet, respectively. A modified fragmentation function may then be defined as

$$D_{jet \to D^*}(z) = \frac{1}{\sigma_{0 \to D^*}} \frac{d\sigma_{jet \to D^*}(z)}{dz}.\quad (4)$$

Only a convolution of the $c \to D^*$ and $c \to$ jet fragmentation functions may be measured, so direct comparison with the LEP data is impossible. With suitably large statistics, it may be possible to do a search for scaling violations with HERA data alone. Work is now proceeding on the measurement of charm fragmentation functions at HERA.

Finally, the subject of open beauty production was brought up. H1 already had released their preliminary measurement and comparison with the LO theory \[20\]. H1 quote, for the kinematic range $Q^2 < 1$ GeV$^2$, $0.1 < y < 0.8$, $p_T > 2.0$ GeV and $35^\circ < \theta^\mu < 130^\circ$, the preliminary visible cross section of $\sigma_{ep \to e^-+b\bar{b}+X}^{vis} = 0.93 \pm 0.08 \pm 0.017$ nb and also quote a predicted cross section from AROMA 2.2 \[21\] of $\sigma_{ep \to e^-+b\bar{b}+X}^{vis} = 0.191$ nb. This states that the preliminary measurement of the cross section of open beauty was up to a factor of 5 higher than the LO theory as predicted by the Monte Carlo AROMA. It should be noted that AROMA is designed to produce only a direct component. The magnitude of the resolved component is not taken into account in the comparison.

At the time of the workshop, no result from ZEUS was forthcoming. However, before going to press, a preliminary result was released of a beauty measurement by the study of semileptonic decays to electrons \[22\]. The measurement is an inclusive electron measurement

$$e^+p \to e^- + \text{dijets} + X$$

in the kinematic region:

- $Q^2 < 1$ GeV$^2$, $0.2 < y < 0.8$.
- Two jets with $E_T > 7.6$ GeV and $|\eta| < 2.4$.
- An electron in the final state with $p_T > 1.6$ GeV and $|\eta| < 1.1$.

This measurement is inclusive of all beauty, charm and light-quark decays, but with all detector effects removed (e.g. conversion electrons). The $x_\gamma^{\text{OBS}}$ distribution (Figure 4) shows, as with the $D^*$ study, a clear tail to low $x_\gamma^{\text{OBS}}$, which needs a significant fraction of resolved LO Monte Carlo to describe it. The $x_\gamma^{\text{OBS}}$ distribution also peaks at high values, which is consistent with the observation of direct processes. The preliminary fit
Figure 5. The $x_\gamma^{\text{OBS}}$ distribution for tagged semileptonic decays from charm, beauty and light quarks. The points are ZEUS preliminary 1996 and 1997 data points. The inner error bars are the statistical errors and the outer error bars are the statistical and systematic errors added in quadrature. The shaded histograms show the direct and resolved contributions as predicted by HERWIG 5.9. The solid line is the two contributions added together.

on the fraction of the LO resolved contribution yields $35 \pm 6\%$ and agrees well with the HERWIG 5.9 [14] prediction of $40\%$. The agreement in shape between the data and the LO Monte Carlo is good.

Next the $p_T^{\text{rel}}$ distribution was examined. In this study $p_T^{\text{rel}}$ is the momentum of the electron transverse to the jet axis of the closest jet (the jets were reconstructed using a clustering algorithm, KTCLUS [23]). The electrons from beauty decays are expected to dominate at high $p_T^{\text{rel}}$ because of the larger beauty mass. The obtained distribution is shown in Figure 6. A fit was done allowing the fraction of beauty to vary with respect to the charm and light quarks contribution (added in the ratios given by HERWIG). A preliminary beauty fraction of $20 \pm 6^{+12}_{-7}\%$ is needed to fit the data (using the 40\% resolved component), in agreement with the HERWIG prediction of 17\%.

ZEUS quotes a preliminary visible cross section for beauty production of

$$\sigma^{\text{vis}}_{b\bar{b}}(e^+p \rightarrow e^- + \text{dijet} + X) = 39 \pm 11^{+23}_{-16} \text{ pb}$$

in the kinematic region described above.

Finally, a preliminary overall normalization factor of 3.7 is necessary to fit the Monte Carlo to the data. This normalization factor is for the complete sample and not
Figure 6. The $p_T^{rel}$ distribution for tagged semileptonic decays from charm, beauty and light quarks. The points are ZEUS preliminary 1996 and 1997 data points. The inner error bars are the statistical errors and the outer error bars are the statistical and systematic errors added in quadrature. The shaded histograms show the beauty and charm added to other components as predicted by HERWIG 5.9. The solid line is the two contributions added together.

just for the beauty component. It should be noted that these results depend on the parton density functions used in the analysis (for the photon GRV-LO [24] and for the proton CTEQ4 [25]) and also on the charm and beauty masses used ($m_c = 1.55$ GeV and $m_b = 4.95$ GeV). It should also be noted that for normal dijet photoproduction a factor of 1.8 is generally needed to normalize the Monte Carlo to the data. In the $D^*$ with dijets study, a factor of 2.6 was necessary.

There is clear consistency between HERWIG 5.9 and the ZEUS data in the percentages and $p_T^{rel}$-distribution shapes. However, they disagree with the large normalization factor. Taking all the caveats in the previous paragraph into account and the different measured kinematic regimes, it is hard to make a statement about the agreement between the experiments at present. A comparison of the visible cross section with a NLO calculation as well as alternative experimental techniques using microvertex detection should allow a clarification of the issue in the future.
3. Charm Production in DIS

The production of charm quarks in DIS has become an important theoretical and phenomenological issue. The charm contribution to the total structure function $F_2$ at small $x$, at HERA, is sizeable, up to $\sim 25\%$; through the charm contribution to the scaling violations the treatment of charm also has a significant impact on the interpretation of the NMC data. Besides being an interesting theoretical problem due to the presence of two hard scales, $m_c$ and $Q$, a proper description of charm-quark production in DIS is thus required for a global analysis of structure function data and a precise extraction of the parton densities in the proton.

At scales $Q \lesssim m_c$, charm production in DIS is calculated in the so-called fixed flavour number scheme (FFNS) from hard processes initiated by light quarks ($u, d, s$) and gluons, where all effects of the charm quark are contained in the perturbative coefficient functions. The FFNS incorporates the correct threshold behaviour, but for large scales, $Q \gg m_c$, the coefficient functions in the FFNS at higher orders in $\alpha_s$ contain potentially large logarithms $\ln^i(Q^2/m_c^2)$, which may need to be resummed. Such a resummation can be achieved by including the heavy quark as an active parton in the proton. The simplest approach incorporating this idea is the so-called zero mass variable flavour number scheme (ZM-VFNS), where heavy quarks are treated as infinitely massive below some scale $Q \sim m_H$ and massless above this threshold. This scheme has been used in global fits for many years, but it has an error of $O(m_H^2/Q^2)$ and is not suited for quantitative analyses unless $Q \gg m_H$. Considerable effort has been made to devise a scheme for heavy-flavour production that interpolates between the FFNS close to threshold and the ZM-VFNS at large $Q$. Here, we will focus on two such schemes, the Aivaziz–Collins–Olness–Tung (ACOT) [26] and the Thorne–Roberts (TR) [27] scheme, which have been used in recent global analyses of parton distributions. These generalized VFNSs include the heavy quark as an active parton flavour and involve matching between the FFNS with three active flavours and a four-flavour scheme with non-zero heavy-quark mass. They employ the fact that the mass singularities associated with the heavy-quark mass can be resummed into the parton distributions without taking the limit $m_H \rightarrow 0$ in the short-distance coefficient functions, as done in the ZM-VFNS. To all orders (and neglecting intrinsic heavy-quark contributions [28]) the FFNS, the ACOT scheme, and the TR scheme are identical, but the way of ordering the perturbative expansion is not unique and the results differ at finite order in perturbation theory. A NLO comparison of the predictions for $F_2^c$ within the FFNS [29][30], the ZM-VFNS and the TR scheme is shown in Figure 7. While the ZM-VFNS does not provide an adequate theoretical description at low values of $Q$, the difference between the FFNS and the TR-scheme is moderate, typically $\lesssim 20\%$. A similar behaviour is observed for the ACOT-VFNS.

Experimentally, ZEUS have released their $F_2^c$ measurement using their full 1996 and 1997 data samples [31]. H1 have released their measurement using only the 1994 to 1996 data sample [31]. Figure 8 shows the $Q^2$ and $x$ dependence of $F_2^c$ compared to

† For theoretical details and the discussion of other schemes, see [3].
the NLO FFNS calculation [30]. The measurements are in good agreement with the theoretical prediction. Given the experimental error and the theoretical uncertainty due to the variation of the charm-quark mass, as shown in Figure 8, present data on $F_2^c$ cannot discriminate between the FFNS and the ACOT and TR schemes. Global structure function fits, however, seem to favour the VFNSs over the FFNS [3].

The resummation of $\ln^i(Q^2/m_c^2)$ terms at higher orders in the FFNS may not be necessary for the present analysis of neutral-current structure functions, but it is required for a stable theoretical prediction of the charged-current structure function $F_3^c$ [33]. As analysed in [33], the higher-order corrections to the lowest-order flavour excitation mechanism $W^\pm + s \rightarrow c/\bar{c}$ are large and dominated by $W$-boson–gluon fusion $W^\pm + g \rightarrow c/\bar{c} + \bar{s}/s$ at $x \lesssim 0.1$. The extraction of the strange quark density from the charged-current structure functions at small $x$ thus requires a good knowledge of the gluon density [‡].

Another source of potentially large higher-order corrections to charm production in DIS comes from the emission of soft gluons in the threshold region. This effect has been studied within the FFNS for charm production through photon–gluon fusion in [35]. The fact that heavy-quark production at HERA energies may be sensitive to threshold effects is due to the large gluon density at small $x$, which enhances the contribution of soft-gluon emission near the elastic limit. An all-order resummation of threshold logarithms has been performed in [35] at next-to-leading logarithmic accuracy; an approximate NNLO result has been derived for $F_2^c$ and the single-particle inclusive distribution $dF_2^c/dp_T$. Figure 9 shows the effect of the approximate NNLO corrections for the inclusive structure function. Plotted are the $K$-factors $F_2^c(\text{NLO})/F_2^c(\text{LO})$ (solid

\‡ For a discussion of charged-current charm electroproduction in the ACOT scheme, see [34].
Figure 8. The dependence of $F_2^c$ on $Q^2$ (left) and $x$ (right) compared to the NLO FFNS calculation [30] using the GRV94 parton densities [32].

line) and $F_2^c$ (NNLO)/$F_2^c$ (NLO) (dashed line). It can be concluded that the soft-gluon effects are well under control for $x \lesssim 0.01$, where the size of the NNLO corrections, evaluated at the central scale $\mu = m_c$, is negligible. It has also been shown in [33] that the inclusion of NNLO terms stabilizes the theoretical prediction for $F_2^c$ by reducing the scale dependence of the NLO result.

Charm production in DIS has not only been studied for inclusive structure functions but differential cross sections for $D^*$ production are available as well [8,31]. Differential distributions allow a study of the underlying production mechanism more detailed than what is possible with the total cross section alone. A comparison of recent HERA data for differential $D^*$ cross sections in DIS with the NLO FFNS calculation [30] shows good overall agreement, with a possible slight excess of data in the forward region for large $\eta(D^*)$ and in the central $x(D^*)$ distribution (see Figure 9) ($x(D^*)$ is the fractional momentum of the $D^*$ in the $\gamma^*p$ frame). We have verified that the excess is not driven by the small-$Q$ region of the data, so a possible resolved photon contribution can be ruled out. Hadronization effects may well be responsible for the deviation from the parton-level calculation, as discussed for $D^*$ photoproduction in the previous section.

Recently, new theoretical results for differential distributions in the ACOT scheme have become available [36]. An event generator Monte Carlo program has been constructed, which allows to produce arbitrary differential distributions and to impose experimental cuts. Figure 11 shows a comparison between the ACOT prediction for
the $p_T$ distribution in $D^*$ DIS and recent H1 data [8]. Given the uncertainty in the choice of the non-perturbative $D^*$ fragmentation function, which affects the $p_T$ slope of the theory, the agreement is good. In Figure 11 the Peterson et al. form [10] has been adopted, with $\epsilon = 0.078$. A smaller value for $\epsilon$, as suggested by recent analyses of $e^+e^-$ data [37], would increase the cross section at large $p_T$ and thereby improve the theoretical description of the data further. A more detailed comparison between HERA data and VFNS predictions will be carried out in the future.

Experimentally much effort is now being put into reducing the systematic and statistical errors, to gain as much as possible from the current data samples. First, detailed systematic studies are in progress for standard $F_2$, and these will eventually be applied to the measurement of $F_2^c$. Some of the systematic error on $F_2^c$ is because of the extrapolation, due to the forward excess seen by both collaborations [8,31]. Work is under way to implement ‘string effects’ described in the previous section into NLO calculations. These appear to correct the calculation so as to agree with the data in the forward direction.

The statistical errors may not be much improved by 1998 and 1999 data taking, so work is under way to examine different channels. Already ZEUS have shown a tagged semileptonic decay channel [38], and other channels are under investigation (e.g. $D^* \rightarrow K\pi\pi\pi\pi_s$). Since these are statistically independent samples, it may be possible to combine them to reduce the statistical errors. Finally it is hoped that the new HERA luminosity upgrade planned for next year and the inclusion of results from microvertex detectors into the analyses will provide vastly increased statistics as well.
Figure 10. The differential $D^*$ cross section measured by ZEUS \[31\] with respect to $Q^2$, $x$, the $\gamma^*p$ centre-of-mass energy $W$, the transverse momentum and pseudorapidity of the $D^*$ and $x(D^*)$. The shaded band shows the predictions of \[30\] including the effect of varying $m_c$ between 1.2 and 1.6 GeV.

4. Quarkonium Production

The production of quarkonium at high-energy colliders has been the subject of considerable interest during the past few years. New experimental results from $p\overline{p}$, $ep$ and $e^+e^-$ colliders have become available, some of which revealed dramatic shortcomings of earlier quarkonium models (see \[39\] for recent reviews). In theory, progress on factorization between perturbative and the quarkonium bound state dynamics has been made. The ‘colour-singlet model’ (CSM) has been superseded by a consistent and rigorous approach, based on non-relativistic QCD (NRQCD) \[40\], an effective field theory that includes the so-called colour-octet mechanisms. On the other hand, the ‘colour evaporation’ model (CEM) \[41\] of the early days of quarkonium physics has been revived \[42\]. Despite these developments, the range of applicability of these approaches to the practical case of charmonium is still subject to debate, as is the quantitative verification of factorization. The problematic aspect is that, because the charmonium mass is still not very large with respect to the QCD scale, non-factorizable corrections may not be suppressed enough, if the quarkonium is not part of an isolated jet, and the expansions in NRQCD may not converge very well. In this situation cross checks between various processes, and predictions of observables such as quarkonium polarization and differential cross sections, are crucial in order to assess the importance of different quarkonium production mechanisms, as well as the limitations of a particular theoretical approach.
According to the NRQCD formalism, inclusive quarkonium production can be viewed as a two-step process, where the production of a heavy-quark pair $Q\bar{Q}[n]$ in a certain angular momentum and colour state $n$ is described by a perturbatively calculable short-distance cross section $d\hat{\sigma}_n$ and the subsequent quarkonium formation is parametrized by a non-perturbative matrix element $\langle O^H \rangle_n$. The magnitude of the non-perturbative transition probabilities is determined by the intrinsic velocity $v$ of the bound state and the production cross section can be expressed as a double expansion in $\alpha_s$ and $v$.

In the CEM and the related ‘soft colour interaction’ (SCI) model \[43\], the cross section for a specific charmonium state is given as a fraction of the inclusive $c\bar{c}$ production cross section integrated up to the open charm threshold. The production fraction is assumed to be universal in the CEM, while in the SCI model it depends somewhat on the partonic state and the possible string configurations. In both models the colour and spin quantum numbers of the intermediate $c\bar{c}$ state are irrelevant, and gluon emission during hadronization is assumed to be unsuppressed.

The analysis of charmonium production in high energy $ep$ collisions at HERA appears to be a powerful tool to test the NRQCD approach and to discriminate NRQCD from the CEM or SCI model. The production of charmonium at HERA is dominated by photon–gluon fusion, where the photon interacts directly with a gluon from the proton. Resolved processes are expected to contribute significantly to the lower end-point of the $J/\psi$ energy spectrum and might be probed at HERA for the first time in the near future. Higher-twist phenomena such as diffractive charmonium production \[44\] are not included in the leading-twist calculations of NRQCD and the CEM: it can be eliminated by either a cut in the $J/\psi$ transverse momentum $p_T \gtrsim 1$–2 GeV, or by a cut in the $J/\psi$
energy variable \( z \equiv p_p \cdot k_\psi / p_p \cdot k_\gamma \lesssim 0.9 \) (in the proton rest frame, \( z \) is the ratio of the \( J/\psi \) to \( \gamma \) energy, \( z = E_\psi / E_\gamma \)).

Within NRQCD and at leading order in the intrinsic velocity \( v \) of the bound state, the inelastic photoproduction of \( J/\psi \) comes from an intermediate \( \bar{c}c \) pair in a colour-singlet \( ^3S_1 \) state and coincides with the colour-singlet model result. (The notation for the angular momentum configuration is \( ^{2S+1}L_J \) with \( S \), \( L \) and \( J \) denoting spin, orbital and total angular momentum, respectively.) Cross sections \([45]\), polar \([46]\), and polar and azimuthal \([47]\) decay angular distributions have been calculated for the direct-photon contribution. The angular integrated cross section is known to next-to-leading order in \( \alpha_s \) \([48]\). Relativistic corrections to the colour-singlet channel at \( \mathcal{O}(v^2) \) were calculated in \([49]\) and were found to be small in the inelastic region \( z \lesssim 0.9, p_T \gtrsim 1 \text{ GeV} \). The colour-singlet contribution, including next-to-leading corrections in \( \alpha_s \), is known to reproduce the current \( J/\psi \) photoproduction data \([51]\) adequately. But there is still a considerable amount of uncertainty in the normalization of the theoretical prediction, which arises from the value of the charm quark mass and the wave function at the origin, as well as the choice of parton distribution functions and renormalization/factorization scale.

At order \( v^4 \sim 0.05-0.1 \) relative to the colour-singlet contribution, the \( J/\psi \) can also be produced through the intermediate colour-octet configurations \( ^3S_1, ^1S_0 \) and \( ^3P_J \). In the inelastic region, they have been considered in \([52]\) for the direct photon contribution and in \([53]\) for resolved photons, in which case the photon participates in the hard scattering through its parton content. Colour-octet contributions to the total photoproduction cross section (integrated over all \( z \) and \( p_T \)) are known to next-to-leading order \([54]\). The polarization of inelastically produced \( J/\psi \) due to these additional production mechanisms has been calculated recently \([55]\).

The colour-octet production channels are kinematically different from the colour-singlet one, because the \( ^1S_0 \) and \( ^3P_J \) configurations can be produced through \( t \)-channel exchange of a gluon already at lowest order in \( \alpha_s \). (For the \( ^3S_1 \) octet this is true for the resolved process.) This leads to a significantly enhanced amplitude, in particular in the large-\( z \) region. The colour-octet contributions to \( J/\psi \) photoproduction are indeed strongly peaked at large \( z \) \([52]\). Such a shape is not supported by the data, which at first sight could lead to a rather stringent constraint on the octet matrix elements \( \langle O_8[n] \rangle, n \in \{ ^1S_0, ^3P_0 \} \), and to an inconsistency with the values obtained for these matrix elements from other processes. However, it is premature to interpret the discrepancy between the theoretical predictions for colour-octet contributions and the HERA data at large \( z \) as a failure of the NRQCD approach itself, as long as not all theoretical uncertainties that reside in the calculation of the short-distance cross sections have been assessed carefully. Potentially large QCD effects include higher-twist contributions \([56]\), higher-order corrections and intrinsic transverse momentum of the partons in the proton \([57]\). Moreover, the peaked shape of the \( z \)-distribution is derived neglecting the energy

\[ § \] This result has been called into question by a recent analysis \([53]\), which finds a sizeable \( \mathcal{O}(v^2) \) correction in the large-\( z \) region even after a cut in the transverse momentum, \( p_T \gtrsim 1 \text{ GeV} \), has been applied.
transfer in the non-perturbative transition $c\bar{c}[n] \rightarrow J/\psi + X$. In reality the peak may be considerably smeared as a consequence of resumming kinematically enhanced higher-order corrections in $\nu^2$ and no constraint or inconsistency can be derived from the end-point behaviour of the $z$-distribution at present. As a consequence, the role of octet contributions to the direct process remains unclear. The resolved photon contribution, on the other hand, could be entirely colour-octet-dominated. The $z$-distribution should then begin to rise again at small $z$, if the colour-octet matrix elements are as large as suggested by NRQCD velocity scaling rules and fits to hadroproduction data.

The CEM and the SCI model also predict a strong colour-octet enhancement at small and large values of $z$, again due to $t$-channel gluon-exchange diagrams. As in the case of the NRQCD analysis, the impact of QCD effects (higher-twist contributions, higher-order corrections and intrinsic transverse momentum) has to be analysed before a final conclusion can be drawn. Since the qualitative behaviour of the $J/\psi$ energy distribution is similar in all existing approaches that include colour-octet contributions, it is not well suited to discriminate between NRQCD, CEM and SCI.

The most powerful way to test NRQCD against the CEM/SCI is the analysis of charmonium polarization. The polarization analysis in NRQCD is based on the symmetries of the NRQCD Lagrangian, of which spin and rotational symmetry are crucial, and the charmonium polarization can be calculated from the spin orientation of the intermediate $c\bar{c}$ state. In contrast to the NRQCD approach, the CEM/SCI predict charmonium to be produced always unpolarized. The models assume unsuppressed gluon emission from the $c\bar{c}$ pair during hadronization, which randomizes spin and colour. This assumption is wrong in the heavy-quark limit where spin symmetry is at work and soft gluon emission does not flip the heavy-quark spin. Nonetheless, since the charm-quark mass is not very large with respect to the QCD scale, the applicability of heavy-quark spin symmetry to charmonium physics has to be tested by confronting the NRQCD predictions to experimental data. Two instructive examples for $J/\psi$ photoproduction at HERA are shown in Figure 12, namely the polar-angle parameter $\lambda$ and the azimuthal-angle parameter $\nu$ in the leptonic $J/\psi$ decay, as a function of the energy fraction $z$ and the transverse momentum $p_T$, respectively. The polarization signatures predicted from colour-octet processes within NRQCD are distinctive and can also be used to extract information on the relative weight of colour-singlet and colour-octet contributions. Polarization signatures have also been studied for $\Upsilon$ production in $pN$ collisions at HERA-B. NRQCD predicts the $\Upsilon$ mesons to be produced transversely polarized, again in contrast to the CEM/SCI, which imply unpolarized quarkonium.

Another powerful way of discriminating NRQCD against the CEM is the measurement of $\chi_c$ photoproduction. In NRQCD the $\sigma(\chi_c)/\sigma(J/\psi)$ ratio is process-dependent and strongly suppressed in photoproduction with respect to hadroproduction. On the other hand, the assumption of a single universal long-distance factor in the CEM implies a universal $\sigma(\chi_c)/\sigma(J/\psi)$ ratio, which is not, however, supported by
the comparison of charmonium production in fixed-target hadron collisions and early photoproduction experiments. A search for $\chi_c$ production at HERA, resulting in a cross section measurement or upper cross section limit would be crucial to settle this issue.

Recently, first experimental results on inclusive $J/\psi$ electroproduction have been presented and compared to a leading-order NRQCD calculation including colour-singlet as well as colour-octet contributions. At large photon virtualities, the analysis of $J/\psi$ leptoproduction is under better theoretical control than photoproduction, since higher-order corrections and higher-twist contributions should become less important as $Q^2$ increases. Also, at sufficiently large $Q^2$, diffractive contributions are expected to be suppressed. The experimental data for differential cross sections in the kinematic region $Q^2 > 3.7$ GeV$^2$ are shown in Figure 13, together with the NRQCD predictions. The colour-singlet channel underestimates the data by a factor $\sim 2$–3, but there still is a considerable amount of uncertainty in the normalization from the value of the charm-quark mass, the wave-function at the origin, higher-order corrections as well as the choice of parton distribution functions and renormalization/factorization scale. The normalization of the theoretical prediction is significantly improved when adding colour-octet contributions; the differential distributions, however, seem to favour the shape predicted by the colour-singlet cross section. More theoretical work and more experimental information extending to larger $Q^2$ values are needed before firm conclusions can be drawn.

Various other observables and final states can be accessed with higher-statistics data, like photoproduction of $\psi'$ and $\Upsilon$ particles, associated $J/\psi + \gamma$ production as well as fragmentation contributions at large $p_T$. The future analyses at HERA will offer unique possibilities to test the NRQCD approach and to study the mechanism of quarkonium production in general.
Figure 13. Differential cross sections for $ep \rightarrow eJ/\psi X$ with $E_{\text{fwd}} > 5$ GeV, $Q^2 > 3.7$ GeV$^2$ and $p_T^2 > 3.4$ GeV$^2$ compared to theoretical predictions within NRQCD [64].

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