Next-to-leading-order QCD corrections to the decay of Z boson into $\chi_c(\chi_b)$

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(Dated: September 10, 2018)

Abstract

Based on the framework of nonrelativistic Quantum Chromodynamics (NRQCD), we carry out next-to-leading order (NLO) QCD corrections to the decay of Z boson into $\chi_c$ and $\chi_b$, respectively. The branching ratio of $Z \to \chi_c(\chi_b) + X$ is about $10^{-5}$ ($10^{-6}$). It is found that, for $Z \to \chi_c(\chi_b) + X$, the single gluon fragmentation diagrams of $^3S_1^{[8]}$, which first appear at the NLO level, can provide significant contributions, leading to a great enhancement on the leading-order results. Consequently the contributions from the color octet (CO) channels will account for a large proportion of the total decay widths. Moreover, the introduction of the CO processes will thoroughly change the color singlet (CS) predictions on the ratios of $\Gamma_{\chi_{c1}}/\Gamma_{\chi_{c0}}$, $\Gamma_{\chi_{c2}}/\Gamma_{\chi_{c0}}$, $\Gamma_{\chi_{b1}}/\Gamma_{\chi_{b0}}$ and $\Gamma_{\chi_{b2}}/\Gamma_{\chi_{b0}}$, which can be regarded as an outstanding probe to distinguish the CO and CS mechanism. With regard to the CS ($^3P_{J}^{[1]}$) channels, the heavy quark pair associated processes serve as the leading role, however, in the case of $\chi_b$, $Z \to b\bar{b}[^3P_{J}^{[1]}] + g + g$ can also contribute significantly. Summing over all the feeddown contributions from $\chi_{cJ}$ and $\chi_{bJ}$, respectively, we find $\Gamma(Z \to J/\psi + X)|_{\chi_{c}-\text{feeddown}} = (0.28 - 2.4) \times 10^{-5}$ and $\Gamma(Z \to \Upsilon(1S) + X)|_{\chi_{b}-\text{feeddown}} = (0.15 - 0.49) \times 10^{-6}$.

PACS numbers: 12.38.Bx, 12.39.Jh, 13.38.Dg, 14.40.Pq

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I. INTRODUCTION

As one of the most successful theories describing the production of heavy quarkonium, nonrelativistic Quantum Chromodynamics (NRQCD) [1] has proved its validity in many processes [2–13], including the hadroproductions of $J/\psi$, $\eta_c$ and $\Upsilon$, and the photo-/electro-productions of $J/\psi$ etc. Despite these successes, NRQCD still faces many challenges, for example the NRQCD predictions significantly overshoot the measured total cross section of $e^+e^- \rightarrow J/\psi + X_{\text{non-}\bar{c}c}$ released from BABAR and Belle collaborations [14]; the $J/\psi$ polarization puzzle is still under debate [15–17]. One key factor responsible for these problems is that there are three LDMEs to be determined, bringing about difficulties in fitting the LDMEs and drawing a definite conclusion.

In comparison with $J/\psi$, $\chi_c$ is more “clean”, namely at leading-order (LO) accuracy in $v$, $^3S_1^{[8]}$ is the unique color octet (CO) state involved, which is beneficial to carry out further study. In addition, considering the branching ratios of $\chi_c \rightarrow J/\psi + \gamma$ are sizeable, the $\chi_c$ feeddown may have a significant effect on the yield and/or polarization of $J/\psi$, for example the inclusion of the $\chi_c$ feeddown will evidently make the polarization trend of the hadroproduced $J/\psi$ more transverse. On the experiment side, $\chi_c$ can be easily detected by hunting the ideal decay processes, $\chi_c \rightarrow J/\psi \rightarrow \mu^+\mu^-$. In conclusion, comparing to $J/\psi$, $\chi_c$ has its own advantages on further studying heavy quarkonium, deserving a separate investigation.

In the past few years, there have been a number of literatures concerning the studies of the $\chi_c$ and $\chi_b$ productions, mainly focusing on the hadroproduction processes [3–5, 8, 18–21]. Ma et al. [22] for the first time accomplished the next-to-leading order (NLO) QCD corrections to the $\chi_c$ hadroproductions. Later on Zhang et al. [23] carried out a global analysis of the copious experimental data on the $\chi_c$ hadroproduction, indicating that almost all the existing measurements can be reproduced by the NLO predictions based on NRQCD. To further check the validity and universality of the $\chi_c$ related LDMEs, it is indispensable to utilize them in other processes.

For this purpose, considering that copious $Z$ boson events can be produced at LHC, the axial vector part of the $Z$-vertex allows for a wider variety of processes and the relative large mass of $Z$ boson can make the perturbative calculations more reliable, we will, for the first time, perform a systematic study on the decay of the $Z$ boson into $\chi_c$ within the framework
of NRQCD. Note that, due to the larger mass of the $b\bar{b}$ mesons, the typical coupling constant and relative velocity of bottomonium are smaller than those of charmonium, subsequently leading to better convergent results over the expansion in $\alpha_s$ and $v^2$ than the charmonium cases. Thus, in this article, the $\chi_b$ productions via $Z$ boson decay will also be systematically investigated.

The rest paragraphs are organized as follows: In Sec. II we give a description on the calculation formalism. In Sec. III, the phenomenological results and discussions are presented. Sec. IV is reserved as a summary.

II. CALCULATION FORMALISM

Within the NRQCD framework, the decay width of $Z \rightarrow \chi_c(\chi_b) + X$ can be written as:

$$d\Gamma = \sum_n d\Gamma_n \langle \mathcal{O}^H(n) \rangle$$

(1)
where $d\hat{\Gamma}_n$ refers to the perturbative calculable short distance coefficients, representing the production of a configuration of the $Q\bar{Q}$ intermediate state with a quantum number $n$, and $\langle O^H(n)\rangle$ is the universal non-perturbative LDME. According to NRQCD, for $\chi_c$ and $\chi_b$ related processes, only two states should be taken into considera tions at LO accuracy in $v$, namely $^3S_1^{[8]}$ and $^3P^J_1$. Taking $\chi_c$ as an example, up to $\mathcal{O}(\alpha s^2)$, for $n = ^3S_1^{[8]}$ we have

\begin{align*}
\text{LO} : & \quad Z \rightarrow c\bar{c}[^3S_1^{[8]}] + g, \\
\text{NLO} : & \quad Z \rightarrow c\bar{c}[^3S_1^{[8]}] + g \text{ (virtual)}, \\
& \quad Z \rightarrow c\bar{c}[^3S_1^{[8]}] + g + g, \\
& \quad Z \rightarrow c\bar{c}[^3S_1^{[8]}] + u_g + \bar{u}_g \text{ (ghost)}, \\
& \quad Z \rightarrow c\bar{c}[^3S_1^{[8]}] + u + \bar{u}, \\
& \quad Z \rightarrow c\bar{c}[^3S_1^{[8]}] + d(s) + \bar{d}(\bar{s}), \\
\text{NLO}^* : & \quad Z \rightarrow c\bar{c}[^3S_1^{[8]}] + c + \bar{c}, \\
& \quad Z \rightarrow c\bar{c}[^3S_1^{[8]}] + b + \bar{b},
\end{align*}

(2)

and in the case of $n = ^3P^J_1$, there are two involved channels as listed below:

\begin{align*}
Z & \rightarrow c\bar{c}[^3P^J_1] + g + g, \\
Z & \rightarrow c\bar{c}[^3P^J_1] + c + \bar{c}.
\end{align*}

(3)

The typical Feynman diagrams corresponding to Eqs. (2) and (3) are presented in Figs. 1, 2 and 3, including 51 diagrams for $^3S_1^{[8]}$ (2 LO diagrams, 6 counter-terms, 15 one-loop, 18 diagrams for real corrections and 10 NLO* diagrams) and 10 diagrams for $^3P^J_1$. Note that, as shown in Eq. (2), the real correction processes $Z \rightarrow c\bar{c}[^3S_1^{[8]}] + g + \bar{q}$ have been classified into two categories, namely $q = u$ and $q = d(s)$, and in Fig.(1e), the diagrams involving fermion loops of $u, c$ and $d, s, b$ are also divided into two groups.
For purpose of isolating the ultraviolet (UV) and infrared (IR) divergences, we adopt the dimensional regularization with \( D = 4 - 2\epsilon \). The on-mass-shell (OS) scheme is employed to set the renormalization constants for the heavy quark mass, the heavy quark field and gluon filed, namely \( Z_m \), \( Z_2 \) and \( Z_3 \), respectively. The modified minimal-subtraction (\( \overline{MS} \)) scheme is adopted for the QCD gauge coupling, \( Z_g \), \((Q = c, b)\)

\[
\delta Z_m^{OS} = -3C_F\frac{\alpha_s N_c}{4\pi} \left[ \frac{1}{\epsilon_{UV}} - \gamma_E + \ln \frac{4\pi \mu^2}{m_Q^2} + \frac{4}{3} + \mathcal{O}(\epsilon) \right],
\]

\[
\delta Z_2^{OS} = -C_F\frac{\alpha_s N_c}{4\pi} \left[ \frac{1}{\epsilon_{UV}} + \frac{2}{\epsilon_{IR}} - 3\gamma_E + 3\ln \frac{4\pi \mu^2}{m_Q^2} + 4 + \mathcal{O}(\epsilon) \right],
\]

\[
\delta Z_3^{\overline{MS}} = \frac{\alpha_s N_c}{4\pi} \left[ \beta_0(n_{lf}) - 2C_A \right] \left[ \frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} \right] - \frac{4}{3}T_F \left( \frac{1}{\epsilon_{UV}} - \gamma_E + \ln \frac{4\pi \mu^2}{m_{\gamma}^2} \right) - \frac{4}{3}T_F \left( \frac{1}{\epsilon_{UV}} - \gamma_E + \ln \frac{4\pi \mu^2}{m_{\gamma}^2} \right) + \mathcal{O}(\epsilon),
\]

\[
\delta Z_g^{\overline{MS}} = -\frac{\beta_0(n_f)}{2} \frac{\alpha_s N_c}{4\pi} \left[ \frac{1}{\epsilon_{UV}} - \gamma_E + \ln(4\pi) + \mathcal{O}(\epsilon) \right], \tag{4}
\]

where \( \gamma_E \) is the Euler’s constant, \( \beta_0(n_f) = \frac{11}{3}C_A - \frac{4}{3}T_F n_f \) is the one-loop coefficient of the \( \beta \)-function, and \( \beta_0(n_{lf}) \) is identical to \( \frac{11}{3}C_A - \frac{4}{3}T_F n_{lf} \), with \( n_f(= 5) \) denoting the number of active quark flavors and light quark flavors respectively. \( N_c = \Gamma[1 - \epsilon]/(4\pi \mu^2/(4m^2))^{\epsilon}. \)

In SU(3)_c, the color factors are given by \( T_F = \frac{1}{2}, C_F = \frac{4}{3} \) and \( C_A = 3 \). To subtract the IR divergences in the real correction channels as listed in Eq. \([2]\) the two cutoff slicing strategy \([24]\) is utilized.

To calculate the D-dimension trace of the fermion loop involving \( \gamma_5 \), under the scheme described in \([25]\), we write down all the amplitudes from the same starting point (such as the \( Z \)-vertex) and abandon the cyclicity. As a crosscheck for the correctness of the treatments on \( \gamma_5 \), we have calculated the QCD NLO corrections to the similar process, \( \Gamma(Z \to c\bar{c}[3S_1^1] + \gamma) \), and obtain exactly the same \( K \) factor as in \([26]\).

To deal with the color singlet (CS) processes, \( \Gamma(Z \to c\bar{c}(b\bar{b})[3P_j^{1^1}] + g + g) \), which involve soft singularities, we first classify \( \Gamma(Z \to c\bar{c}[3P_j^{1^1}] + g + g) \) into two terms (the \( b\bar{b} \) cases can
be obtained in a similar way),

\[
d\Gamma(Z \to c\bar{c}[3P^1_J] + g + g) = d\hat{\Gamma}_{3P^1_J}^{(4)}(\mathcal{O}\chi_c(3P^1_J)) + d\hat{\Gamma}_{3S^1_1}^{(4)}(\mathcal{O}\chi_c(3S^1_1))^{NLO},
\]

then we have

\[
d\hat{\Gamma}_{3P^1_J}^{(4)}(\mathcal{O}\chi_c(3P^1_J)) = d\Gamma(Z \to c\bar{c}[3P^1_J] + g + g) - d\hat{\Gamma}_{3S^1_1}^{(4)}(\mathcal{O}\chi_c(3S^1_1))^{NLO}.
\]

Both \(d\Gamma(Z \to c\bar{c}[3P^1_J] + g + g)\) and \(\langle \mathcal{O}\chi_c(3S^1_1) \rangle^{NLO}\) have IR singularities, which can cancel each other. The soft part of \(d\Gamma(Z \to c\bar{c}[3P^1_J] + g + g)\) can be written as (“s” means soft)

\[
d\Gamma(Z \to c\bar{c}[3P^1_J] + g + g)|_s = -\frac{\alpha_s}{3\pi m_c} u^s_c N_c^2 - \frac{1}{N_c} d\hat{\Gamma}_{3S^1_1}^{(4)}(\mathcal{O}\chi_c(3P^1_J))
\]

with

\[
u^s_c = \frac{1}{\epsilon_{IR}} + \frac{E}{|p|} \ln \left( \frac{E + |p|}{E - |p|} \right) + \ln \left( \frac{4\pi^2}{s\delta_s^2} \right) - \gamma_E - \frac{1}{3},
\]

where \(N_c\) is identical to 3 for \(SU(3)\) gauge field. \(E\) and \(p\) denote the energy and 3-momentum of \(\chi_c\), respectively. \(\delta_s\) is the usual “soft cut” employed to impose an amputation on the energy of the emitted gluon.

Now we are to calculate the transition rate of \(3S^1_1\) into \(3P^1_J\). Under the dimensional regularization scheme, we have

\[
\langle \mathcal{O}\chi_c(3S^1_1) \rangle^{NLO} = -\frac{\alpha_s}{3\pi m_c} u^s_c N_c^2 - \frac{1}{N_c} \langle \mathcal{O}\chi_c(3P^1_J) \rangle,
\]

where, on the basis of \(\mu\)-cuttoff scheme, \(u^s_c\) has the form of

\[
u^s_c = \frac{1}{\epsilon_{IR}} - \gamma_E - \frac{1}{3} - \ln \left( \frac{4\pi^2}{\mu^2} \right).
\]

Substituting Eqs. (7), (8), (9) and (10) into Eq. (6), we finally obtain the finite short distance coefficient for \(3P^1_J\), namely \(d\hat{\Gamma}_{3P^1_J}\). Having eliminated all the singularities, we will move on to perform the numerical calculations.
III. NUMERICAL RESULTS AND DISCUSSIONS

Before presenting the phenomenological results, we first demonstrate the choices of the parameters in our calculations. To keep the gauge invariance, the masses of $\chi_c$ and $\chi_b$ are set to be $2m_c$ and $2m_b$ respectively, with $m_c = 1.5 \pm 0.1$ GeV and $m_b = 4.9 \pm 0.2$ GeV. $m_Z = 91.1876$ GeV. $\alpha = 1/137$. In the calculations for the NLO, the NLO* and two $3P_j^{[1]}$ processes, as listed in Eqs. (2) and (3), we employ the two-loop $\alpha_s$ running, and one-loop $\alpha_s$ running for LO. We take $m_c(m_b)$ as the value of $\mu_A$ for $\chi_c(\chi_b)$. The values of $\langle O^{\chi_c(\chi_b)}(3S_1^{[8]}) \rangle$ are taken as

$$\langle O^{\chi_c}(3S_1^{[8]}) \rangle = 2.15 \times 10^{-3} \text{ GeV}^3,$$

$$\langle O^{\chi_b}(3S_1^{[8]}) \rangle = 9.40 \times 10^{-3} \text{ GeV}^3,$$

(11)

from Refs. [8] and [23]. In the case of $3P_j^{[1]}$ channels, the relation $\langle O^{\chi_c(\chi_b)}(3P_j^{[1]}) \rangle = \frac{9}{2\pi}(2J + 1)|R'_p(0)|^2$ is adopted, where $|R'_p(0)|^2 = 0.075 \text{ GeV}^5$ for $\chi_c$ and $1.417 \text{ GeV}^5$ for $\chi_b$.

In our calculations, the mathematica packages Malt@FDC are employed to deal with the virtual corrections, and FDC packages serve as the agent to evaluate the contributions from the hard non-collinear parts. Both the cancellation of divergence and the independence on cutoff have been checked carefully.

A. Phenomenological results for $\chi_c$

The NRQCD predictions for $\Gamma(Z \to \chi_{cJ} + X)$ with $J = 0, 1, 2$ are demonstrated in Tables. I I I and III respectively. One can see that the branching ratios for $\chi_c$ are on the order

| $\mu_r$ [GeV] | $m_c(\text{GeV})$ | $3S_1^{[8]}|_{\text{LO}}$ | $3S_1^{[8]}|_{\text{NLO}}$ | $3S_1^{[8]}|_{\text{NLO}^*}$ | $3P_0^{[1]}|_{gg}$ | $3P_0^{[1]}|_{cc}$ | $\Gamma_{\text{total}}$ [KeV] | $\text{Br}(10^{-5})$ |
|--------------|-----------------|------------------|------------------|------------------|-----------------|-----------------|-----------------|-----------------|
| $m_c$        | $1.4$           | $1.20 \times 10^{-2}$ | $14.9$ | $8.26$ | $5.63 \times 10^{-2}$ | $27.0$ | $50.2$ | $2.02$ |
| $2m_c$       | $1.5$           | $1.09 \times 10^{-2}$ | $10.9$ | $6.05$ | $4.27 \times 10^{-2}$ | $18.1$ | $35.1$ | $1.41$ |
| $1.6$        | $9.99 \times 10^{-3}$ | $8.12$ | $4.53$ | $3.30 \times 10^{-2}$ | $12.5$ | $25.1$ | $1.01$ |
| $m_Z$        | $1.4$           | $5.30 \times 10^{-3}$ | $2.99$ | $1.66$ | $1.13 \times 10^{-2}$ | $5.43$ | $10.1$ | $0.41$ |
| $1.5$        | $4.95 \times 10^{-3}$ | $2.31$ | $1.28$ | $9.06 \times 10^{-3}$ | $3.84$ | $7.45$ | $0.30$ |
| $1.6$        | $4.64 \times 10^{-3}$ | $1.82$ | $1.01$ | $7.36 \times 10^{-3}$ | $2.78$ | $5.61$ | $0.23$ |
It is observed that the QCD NLO corrections can enhance the LO results significantly, by which signifies a detectable prospect of these decay processes at LHC or other platforms. 

**q** loop triangle anomalous diagrams (Fig. 1(e)) and those (Fig. 1(j)) associated with a final gluon fragmentation (SGF) diagrams that emerge first at the NLO level, such as the one-

of 10^{-5}. To be specific, considering the uncertainties induced by the choices of the values of \( \mu_r \) and \( m_c \), we have

\[
\begin{align*}
\text{Br}(Z \to \chi_{c0} + X) &= (0.23 - 2.02) \times 10^{-5}, \\
\text{Br}(Z \to \chi_{c1} + X) &= (0.47 - 4.06) \times 10^{-5}, \\
\text{Br}(Z \to \chi_{c2} + X) &= (0.62 - 5.14) \times 10^{-5},
\end{align*}
\]

which signifies a detectable prospect of these decay processes at LHC or other platforms. It is observed that the QCD NLO corrections can enhance the LO results significantly, by 2 – 3 orders, which can be attributed to the kinematic enhancements via the \( 3S_1^{[8]} \) single gluon fragmentation (SGF) diagrams that emerge first at the NLO level, such as the one-loop triangle anomalous diagrams (Fig. 1(e)) and those (Fig. 1(j)) associated with a final \( q\bar{q} \) \(( q = u, d, s \) pair. By the same token, the NLO* channels can also provide considerable contributions, about one half of the NLO results. Consequently the CO channels will play
a vital role in the decay processes of $Z \to \chi_c + X$. To show the CO significance evidently, we introduce the following ratios

$$\frac{\Gamma_{\chi_c^0}}{\Gamma_{\chi_c^0}} \left/ \frac{\Gamma_{\chi_c^0}}{\Gamma_{\chi_c^0}} \right. = (46.1 - 50.3)\%,$$

$$\frac{\Gamma_{\chi_c^1}}{\Gamma_{\chi_c^1}} \left/ \frac{\Gamma_{\chi_c^1}}{\Gamma_{\chi_c^1}} \right. = (68.9 - 72.4)\%,$$

$$\frac{\Gamma_{\chi_c^2}}{\Gamma_{\chi_c^2}} \left/ \frac{\Gamma_{\chi_c^2}}{\Gamma_{\chi_c^2}} \right. = (90.1 - 91.4)\%, \tag{13}$$

where $\Gamma_{\chi_c^J} (J = 0, 1, 2)$ denotes the sum of the contributions from NLO and NLO*, and $\Gamma_{\chi_c^J}$ is the total results by adding together the CO and CS contributions. In addition to the crucial impacts on the total widths, the CO channels are also capable of significantly influencing the predictions on the ratios of $\chi_c^1/\chi_c^0$ and $\chi_c^2/\chi_c^0$, as shown below

\[
\begin{align*}
\text{CS} : & \quad \Gamma_{\chi_c^1}/\Gamma_{\chi_c^0} = 1.159 - 1.162, \\
\text{NRQCD} : & \quad \Gamma_{\chi_c^1}/\Gamma_{\chi_c^0} = 2.007 - 2.087, \\
\text{CS} : & \quad \Gamma_{\chi_c^2}/\Gamma_{\chi_c^0} = 0.471 - 0.480, \\
\text{NRQCD} : & \quad \Gamma_{\chi_c^2}/\Gamma_{\chi_c^0} = 2.558 - 2.756. \tag{14}
\end{align*}
\]

One can see that the CS results have been thoroughly changed by the inclusion of the CO states. These conspicuous differences can be regarded as an outstanding probe to distinguish the CO and CS mechanism.

On the aspect of CS, the $c$ quark fragmentation dominated channels, namely $Z \to \bar{c}c^{[3P^1]} + c + \bar{c}$, serve as the leading role, and $Z \to \bar{c}c^{[3P^1]} + g + g$ contribute moderately, about 0.24%, 5% and 10% of the total CS predictions, corresponding to $J = 0, 1, 2$ respectively.

As was pointed out in the introduction to this paper, the $\chi_c$ feeddown may have a substantial impact on the production of $J/\psi$. Therefore we employ the branching ratios of $\chi_c$ to $J/\psi$ as listed in [27], specifically \(\text{Br}(\chi_c^0 \to J/\psi + \gamma) = 1.4\%\), \(\text{Br}(\chi_c^1 \to J/\psi + \gamma) = 34.3\%\) and \(\text{Br}(\chi_c^2 \to J/\psi + \gamma) = 19.0\%\), to evaluate $\Gamma(Z \to J/\psi + X)$ via the $\chi_c$ feeddown. Summing over all the contributions from $\chi_c^0$, $\chi_c^1$ and $\chi_c^2$, we finally obtain

$$\Gamma(Z \to J/\psi + X)_{\chi_c\text{-feeddown}} = (0.28 \sim 2.4) \times 10^{-5}, \tag{15}$$

which is about one order of magnitude smaller than the experimental data released from the L3 Collaboration at LEP [28].
B. Phenomenological results for $\chi_b$

| $\mu_r$ | $m_b$(GeV) | $3S_1^{[8]}|_{\text{LO}}$ | $3S_1^{[8]}|_{\text{NLO}}$ | $3S_1^{[8]}|_{\text{NLO}^*}$ | $3P_0^{[4]}|_{gg}$ | $3P_0^{[4]}|_{b\bar{b}}$ | $\Gamma_{\text{total}}$ | Br$(10^{-7})$ |
|--------|-----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 4.7    | 9.76 $\times 10^{-3}$ | 0.272 | 0.148 | 8.84 $\times 10^{-3}$ | 0.677 | 1.11 | 4.46 |
| $2m_b$ | 4.9       | 9.26 $\times 10^{-3}$ | 0.225 | 0.121 | 7.46 $\times 10^{-3}$ | 0.535 | 0.888 | 3.57 |
| 5.1    | 8.82 $\times 10^{-3}$ | 0.187 | 9.95 $\times 10^{-2}$ | 6.34 $\times 10^{-3}$ | 0.426 | 0.719 | 2.89 |

| $\mu_r$ | $m_b$(GeV) | $3S_1^{[8]}|_{\text{LO}}$ | $3S_1^{[8]}|_{\text{NLO}}$ | $3S_1^{[8]}|_{\text{NLO}^*}$ | $3P_0^{[4]}|_{gg}$ | $3P_0^{[4]}|_{b\bar{b}}$ | $\Gamma_{\text{total}}$ | Br$(10^{-7})$ |
|--------|-----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 4.7    | 2.92 $\times 10^{-2}$ | 0.814 | 0.445 | 0.153 | 0.653 | 2.06 | 8.28 |
| $2m_b$ | 4.9       | 2.78 $\times 10^{-2}$ | 0.674 | 0.362 | 0.128 | 0.512 | 1.68 | 6.74 |
| 5.1    | 2.64 $\times 10^{-2}$ | 0.562 | 0.299 | 0.109 | 0.405 | 1.37 | 5.50 |

| $\mu_r$ | $m_b$(GeV) | $3S_1^{[8]}|_{\text{LO}}$ | $3S_1^{[8]}|_{\text{NLO}}$ | $3S_1^{[8]}|_{\text{NLO}^*}$ | $3P_0^{[4]}|_{gg}$ | $3P_0^{[4]}|_{b\bar{b}}$ | $\Gamma_{\text{total}}$ | Br$(10^{-7})$ |
|--------|-----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 4.7    | 1.90 $\times 10^{-2}$ | 0.357 | 0.188 | 6.47 $\times 10^{-2}$ | 0.276 | 0.886 | 3.56 |
| $m_Z$  | 4.9       | 1.83 $\times 10^{-2}$ | 0.303 | 0.157 | 5.54 $\times 10^{-2}$ | 0.221 | 0.736 | 2.96 |
| 5.1    | 1.75 $\times 10^{-2}$ | 0.258 | 0.131 | 4.77 $\times 10^{-2}$ | 0.178 | 0.616 | 2.47 |

Based on NRQCD, the predicted decay widths via $Z \rightarrow \chi_{bJ} + X$ are presented in Tables IV, V, and VI corresponding to $J = 0, 1, 2$, respectively. From the data in the three Tables, it is apparent that the branching ratio for $Z \rightarrow \chi_{bJ} + X$ is about one or two orders of magnitudes smaller than that of $\chi_c$, around $10^{-7} - 10^{-6}$. Taking into account the uncertainties induced by $\mu_r$ and the mass of $b$ quark, we have

$$\text{Br}(Z \rightarrow \chi_{b0} + X) = (1.29 - 4.46) \times 10^{-7},$$

$$\text{Br}(Z \rightarrow \chi_{b1} + X) = (2.47 - 8.28) \times 10^{-7},$$

$$\text{Br}(Z \rightarrow \chi_{b2} + X) = (0.31 - 1.02) \times 10^{-6}. \quad (16)$$

Similar to the case of $\chi_c$, the NLO QCD corrections can also enlarge the LO results significantly, by about 10-20 times, and the impact of NLO$^*$ channels are as always sizeable. The
TABLE VI: The decay widths (unit: KeV) of $\Gamma(Z \rightarrow \chi_{b2} + X)$. $\mu_A = m_b$.

| $\mu_r$ / $m_b$ (GeV) | $3S_1^8$|LO | $3S_1^8$|NLO | $3S_1^8$|NLO* | $3P_2^{[1]}|_{gg}$ | $3P_2^{[1]}|_{bb}$ | $\Gamma_{\text{total}}$ | Br($10^{-7}$) |
|-----------------------|----------------|----------------|----------------|---------------|---------------|----------------|----------------|----------------|----------------|----------------|
| 4.7                   | $4.88 \times 10^{-2}$ | 1.360 | 0.743 | 0.160 | 0.270 | 2.53 | 10.2 |
| 2$m_b$                | $4.63 \times 10^{-2}$ | 1.130 | 0.605 | 0.136 | 0.212 | 2.08 | 8.35 |
| 5.1                   | $4.41 \times 10^{-2}$ | 0.937 | 0.497 | 0.116 | 0.168 | 1.72 | 6.91 |
| 4.7                   | $3.18 \times 10^{-2}$ | 0.595 | 0.312 | $6.78 \times 10^{-2}$ | 0.114 | 1.09 | 4.38 |
| $m_Z$                 | $3.04 \times 10^{-2}$ | 0.504 | 0.261 | $5.87 \times 10^{-2}$ | 0.0915 | 3.69 |
| 5.1                   | $2.92 \times 10^{-2}$ | 0.430 | 0.219 | $5.12 \times 10^{-2}$ | 0.774 | 3.11 |

Ratios of the CO contributions to the total widths are slightly smaller than the $\chi_c$ cases, to be specific

$$\Gamma_{\chi_{b0}}^{\text{CO}} / \Gamma_{\chi_{b0}}^{\text{total}} = (37.8 - 40.6)\%,$$

$$\Gamma_{\chi_{b1}}^{\text{CO}} / \Gamma_{\chi_{b1}}^{\text{total}} = (51.5 - 63.3)\%,$$

$$\Gamma_{\chi_{b2}}^{\text{CO}} / \Gamma_{\chi_{b2}}^{\text{total}} = (83.0 - 83.9)\%,$$  \hspace{1cm} (17)

Where $\Gamma_{\chi_{bJ}}^{\text{CO}}$, with $J = 0, 1, 2$, represents the sum of the NLO and NLO* contributions, and $\Gamma_{\chi_{bJ}}^{\text{total}}$ denotes the total widths, including both the CO and CS effects. The relative smallness of these ratios can be partly ascribed to that the larger mass of $m_b$ will weaken the significance of the enhancements via SGF. Regarding the ratios of $\chi_{b1}/\chi_{b0}$ and $\chi_{b2}/\chi_{b0}$, the NRQCD predictions are still far different from those built on the CS mechanism, namely

$$\text{CS} : \quad \Gamma_{\chi_{b1}} / \Gamma_{\chi_{b0}} = 1.175 - 1.188,$$

$$\text{NRQCD} : \quad \Gamma_{\chi_{b1}} / \Gamma_{\chi_{b0}} = 1.868 - 1.923,$$

$$\text{CS} : \quad \Gamma_{\chi_{b2}} / \Gamma_{\chi_{b0}} = 0.626 - 0.657,$$

$$\text{NRQCD} : \quad \Gamma_{\chi_{b2}} / \Gamma_{\chi_{b0}} = 2.286 - 2.420.$$  \hspace{1cm} (18)

Analogous to $\chi_c$, the dissimilarities between these ratios are also very beneficial to check the validity of the CO mechanism.

In contrast to the previously stated “moderation” of the effects via $Z \rightarrow c\bar{c}[3P_0^{[1]}] + g + g$, in the case of $\chi_b$, the channel $Z \rightarrow b\bar{b}[3P_0^{[1]}] + g + g$ can provide remarkable contributions,
namely

\[
\frac{\Gamma_{^3P^0_0}}{\Gamma_{^3P^1_0}} \sim 1.5\%,
\]
\[
\frac{\Gamma_{^3P^1_1}}{\Gamma_{^3P^1_1}} \sim 20\%,
\]
\[
\frac{\Gamma_{^3P^2_2}}{\Gamma_{^3P^1_1}} \sim 40\%,
\]

where, corresponding to \(J = 0, 1, 2\) respectively, the label \(\Gamma_{^3P^i_J}^{\text{gg}}\) refers to \(\Gamma(Z \to b\bar{b}[^3P^i_J] + g + g)\), and \(\Gamma_{^3P^i_J}^{\text{CS}}\) is for the sum of \(\Gamma(Z \to b\bar{b}[^3P^i_J] + g + g)\) and \(\Gamma(Z \to b\bar{b}[^3P^i_J] + b + \bar{b})\).

It is worth mentioning that, to satisfy the conservation of \(C\)-parity, at \(B\) factories, the processes \(e^+e^- \rightarrow \gamma^* \rightarrow c\bar{c}(b\bar{b})[^3P^i_J] + g + g\) are forbidden, leaving alone the heavy quark pair associated channels as the unique CS processes. Moreover the center-of-mass energy at \(B\) factories, namely 10.6 GeV, is too small to allow for \(e^+e^- \rightarrow \gamma^* \rightarrow (b\bar{b})[^3P^i_J] + b\bar{b}\). From these points of view, the decay of \(Z\) boson seems to be more suitable for studying \(\chi_c\) and \(\chi_b\).

At last, as a further step towards providing a sound estimate on the \(\Upsilon(1S)\) production via \(Z\) decay, we make use of the branching ratios of \(\chi_b\) to \(\Upsilon(1S)\), namely \(\text{Br}(\chi_{b0} \to \Upsilon(1S) + \gamma) = 1.94\%\), \(\text{Br}(\chi_{b1} \to \Upsilon(1S) + \gamma) = 35.0\%\) and \(\text{Br}(\chi_{b2} \to \Upsilon(1S) + \gamma) = 18.8\%\), to calculate the impact of the \(\chi_b\) feeddown on \(\Gamma(Z \to \Upsilon(1S) + X)\). Taking into account all the contributions from \(\chi_{b0}\), \(\chi_{b1}\) and \(\chi_{b2}\), we finally obtain

\[
\Gamma(Z \to \Upsilon(1S) + X)\big|_{\chi_b-\text{feeddown}} = (0.15 \sim 0.49) \times 10^{-6}.
\]

IV. SUMMARY

In this paper, we have systematically investigated the decay of \(Z\) boson into \(\chi_c\) and \(\chi_b\), respectively. It is found that the branching ratio for \(Z \to \chi_c + X\) is on the order of \(10^{-5}\), and \(10^{-6}\) for the \(\chi_b\) case, which implies that these decay processes are able to be detected. It is observed that, the \(^3S^1_{1}\)[8] SGF diagrams that first emerge at the NLO level will significantly enhance the LO results by about 2-3 orders for \(c\bar{c}\), and 10-20 times for \(b\bar{b}\). For the same reasons, the NLO* processes can also contribute considerably, about 50% of the NLO results. Consequently, the CO contributions will play a vital, even dominant, role in the decay processes, \(Z \to \chi_c(\chi_b) + X\). Aside from the significance in the total widths, the \(^3S^1_{1}\)[8] state also has a remarkable effect on the predictions on the ratios of \(\Gamma(\chi_{c2})/\Gamma(\chi_{c0})\),
\[ \Gamma(\chi_{c1})/\Gamma(\chi_{c0}), \Gamma(\chi_{b1})/\Gamma(\chi_{b0}), \Gamma(\chi_{b2})/\Gamma(\chi_{b0}) \] thoroughly change the results based on the CS mechanism. On the aspect of CS, the heavy quark pair associated channels, namely 
\[ Z \rightarrow Q\bar{Q}[^{3}\not P_{J}^{[1]}] + Q\bar{Q}, \] play a leading role, however, the processes 
\[ Z \rightarrow Q\bar{Q}[^{3}\not P_{J}^{[1]}] + gg \] can also provide significant contributions, especially for \( \chi_{b} \). Taking into considerations the \( \chi_{cJ} \) and \( \chi_{bJ} \) feeddown contributions respectively, we find 
\[ \Gamma(Z \rightarrow J/\psi + X)|_{\chi_{cJ} \text{-feeddown}} = (0.28 - 2.4) \times 10^{-5} \] and 
\[ \Gamma(Z \rightarrow \Upsilon(1S) + X)|_{\chi_{bJ} \text{-feeddown}} = (0.15 - 0.49) \times 10^{-6}. \] In summary, the decay of \( Z \) boson into \( \chi_{c}(\chi_{b}) \) can be regarded as an ideal laboratory to further identify the significance of color octet mechanism.

V. ACKNOWLEDGMENTS

Acknowledgments: We would like to thank Wen-Long Sang for helpful discussions on the treatments on \( \gamma_{5} \). This work is supported in part by the Natural Science Foundation of China under the Grant No.11705034., by the Project for Young Talents Growth of Guizhou Provincial Department of Education under Grant No.KY[2017]135, and the Key Project for Innovation Research Groups of Guizhou Provincial Department of Education under Grant No.KY[2016]028.

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