The Interesting Dynamics of the 55 Cancri System

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Abstract. Recent studies of the 55 Cancri system suggest the existence of three planets with periods of \( \sim 15 \) days, \( \sim 45 \) days, and \( \sim 5500 \) days (Marcy et al. 2002). The inner two planets are near the 3:1 mean motion commensurability and it is likely that these two planets became trapped in the resonance while farther from the star and migrated together. As the innermost planet begins to dissipate energy through tides the planets break out of the resonance. The final state of the system gives important information about its past history, such as the migration timescale that led to capture.

1. Introduction

Tidal evolution leading to capture into resonance has been invoked to explain the many commensurabilities in our solar system (e.g. Goldreich 1965), but the problem of breaking resonance due to tidal effects is less well-studied. The presence of even a small amount of dissipation can alter the dynamical evolution of the system. In this case, the dissipation causes the resonance to accelerate the circularization of the inner planet compared to what would happen in isolation.

The resonant interaction between the planets in the 55 Cancri system imply that fully self-consistent fits are necessary to obtain the orbital elements of the system. The 3:1 resonance covers a narrow range of semi-major axes, so determining the location of the system within the resonance will require accurate fits, which requires additional data. Direct integrations of the orbital elements given by one fully self-consistent fit (Laughlin 2002) indicate that the resonant angles are circulating and the system is not in resonance.

To study the long term dynamics of the system, we use the classical disturbing function together with Lagrange’s planetary equations to lowest order in eccentricity (e.g. Murray & Dermott 1999). We numerically integrate these equations using a variable time step Bulirsch-Stoer integrator.

We consider tidal evolution in the regime where energy is dissipated and angular momentum is conserved. The eccentricity damping rate is given by \( \dot{e} = -e/\tau_e \) and \( \tau_e = GMm e^2 Q/an E_0 \) where \( G \) is the gravitational constant, \( M \) is the mass of the star, \( m \) is the mass of the inner planet, \( Q \) parameterized tidal energy dissipation, \( a \) is the planet’s semi-major axis, \( n \) is the planet’s mean motion, and \( E_0 \) is the maximum amount of energy stored in the tidal deformation of the planet. The planet’s eccentricity e-folding time in isolation is given by \( \tau_e \).
2. Restricted three body problem

In order to develop an understanding of the problem, we study a simplified case: the circular restricted three body problem where the inner planet is massless and the outer planet is \(1 M_J\). The resonant libration timescale is \(\sim 100\) orbits of the outer planet, the secular evolution time is \(\sim 3500\) orbits. We assume that tidal dissipation occurs only in the inner planet. The tidal evolution timescale is typically much longer than the dynamical and secular timescales, so we choose a small value for \(Q\). This does not change the character of the dynamics as long as the tidal timescale remains long compared to other timescales.

Figure 1. The time evolution of the inner planet under the influence of tidal dissipation in the circular restricted three body problem. On the left, the system starts deep in the 3:1 resonance, and on the right, the system is initially far from the exact resonance. The outer planet supplies energy and angular momentum to the inner planet, resulting in an accelerated circularization of the inner planet compared to what would happen if the inner planet were isolated.

As tides remove energy the outer planet supplies the inner planet with enough energy to maintain the commensurability, but an adiabatic invariant demands that the resonant angle’s libration amplitude increase as the eccentricity decreases, eventually breaking the resonance. The energy flow is arranged by forcing the resonant angle librate to about a mean value slightly displaced from its equilibrium value of \(\pi\). The flow of angular momentum between the planets is determined by the same resonant angle and this results in faster circularization of the inner planet’s orbit compared to the situation where the inner planet is isolated. In the case of the 3:1 resonance, the inner planet circularizes five times faster in the presence of an outer companion.

3. Breaking the resonance

The resonant angle’s libration amplitude can be written as
\[ \Delta a_i = 4a_i \sqrt{Cm_o a_i / 3M a_o e_i^{j/2}} \sin(\Delta \phi/4) \]

where the subscripts \( i, o \) refer to the inner and outer planets, \( m \) is the mass of a planet, \( M \) is the mass of the star, \( j \) is the order of the resonance, \( \phi \) is the resonant angle, \( C \) is a constant determined by the resonance under consideration, and \( \Delta a \), \( \Delta \phi \) refer to the full width of the libration. As energy is removed, the only one of these factors to change is the inner planets eccentricity which determines that the resonant angle will begin to circulate when:

\[ e_{\text{break}} = e_{\text{init}} (\sin(\Delta \phi_{\text{init}}/4))^{2/j} \]

where the subscript \( \text{init} \) refers to the values when tidal dissipation became important. Once the resonance is broken the inner planet evolves to a circular orbit only under the influence of tides, where the final semi-major axis is \( a_{\text{final}} = a_{\text{res}} (1 - e_{\text{break}}^2) \) and \( a_{\text{res}} \) is that corresponding to the exact commensurability. The final period ratio of the two planets gives a constraint on how deep the two planets were in resonance when tidal effects became important.

Knowing the eccentricity decay timescale as augmented by the dynamics of the resonance allows us to write the time required to break the resonance after tidal effects become important as:

\[ \tau_{\text{break}} = -2(1 + 2j) \ln(\sin(\Delta \phi/4))/j \]

Figure 2. Semi-major axis and eccentricity as the system evolves out of resonance. The lines indicate the maximum semi-major axis libration while in resonance. In both plots, the planet’s eccentricity is initially high and it evolves toward zero. The plot on the left is a system that starts near the exact commensurability. The inner planet has a very low eccentricity when the resonance is finally broken. The system plotted on the left starts far from the exact resonance and the breaking occurs while the inner planet still has substantial \( e_i \). The asymptotic period ratio \( (P_i/P_o) \) is substantially smaller than the previous case..
Figure 3. The time required to break the resonance according to numerical integrations of the first order secular theory compared to the value predicted by equation 3. This expression takes into account the accelerated circularization caused by the change in the dynamics induced by the slight dissipation.

4. Conclusions

We used the classical disturbing function together with Lagrange’s planetary equations accurate to lowest order in eccentricity to study resonance breaking through tidal dissipation. It is possible to derive analytic expressions for the inner planet’s eccentricity when the resonance is broken and the time required to break the resonance (equations 2, 3). These expressions are in good agreement with the secular theory (figure 3).

This is applicable to the 55 Cancri system because the inner planet is near the regime where tidal effects are starting to play a role and current fits to the radial velocity curve give orbital elements where the planets are not in the resonance. We argue that the two planets became trapped in resonance while farther from the star and then broke the resonance when tidal dissipation began to play a role in the inner planet’s evolution.

References

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