A time-space varying speed of light and the Hubble Law in static Universe

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We consider a hypothetical possibility of the variability of light velocity with time and position in space which is derived from two natural postulates. For the consistent consideration of such variability we generalize translational transformations of the Theory of Relativity. The formulae of transformations between two rest observers within one inertial system are obtained. It is shown that equality of velocities of two particles is as relative a statement as simultaneity of two events is. We obtain the expression for the redshift of radiation of a rest source which formally reproduces the Hubble Law. Possible experimental implications of the theory are discussed.

I. INTRODUCTION

Recently a number of papers have been published\[1\] - \[12\] in which the possibility of light speed variability with time has been investigated. It has been shown that models of Varying Light Speed might resolve some cosmological problems, such as: the flatness problem, the quasi-flatness problem\[4\], the horizon problem etc.

Investigations of the possibility of variability of fundamental constants with time have a long history and there are various approaches to the problem\[8\] - \[12\]. After it had become clear that such a fundamental constant as the curvature radius of our Universe varies with time, a doubt arose about the constancy of other physical constants. An excellent review of the research devoted to the variability of physical constants with time can be found in Ref.\[4\].

It is obvious that introduction of variability of light speed with time is not possible without considerable modification of the Theory of Relativity. Recently a generalization of the Lorentz transformation (so-called Projective Lorentz Transformation), has been obtained\[13\], \[14\]. Within this approach the variability of light speed with time and distance arises naturally from the analysis of transformations between two observers within different inertial reference systems. In the Projective Theory of Relativity, besides the fundamental speed $c$ there exists a new constant $\lambda$ that determines the magnitude of corrections to the Theory of Relativity. If $\lambda = 0$, we return to the Lorentz transformations and the Theory of Relativity.

In this paper we consider in detail a possibility of the variability of light speed with time, and show that to describe it consistently it is necessary to modify not only the Lorentz transformation but also the translational transformations between two rest observers. This eliminates some contradictions and makes the physical picture clearer.

In Sec. II, on the basis of two simple postulates we obtain the form of a functional dependence of light speed on time and distance. Agreement between the variability of light speed and the relativistic principles requires modification of the Theory of Relativity, which is considered in Sec. III for the case of rest observers within one inertial reference system. It is shown in Sec. IV that the variability of the light speed with time and distance results in Hubble’s redshift for rest sources. The formulae for aberration are obtained which can also be interpreted in terms of Hubble Law. Possible experimental implications of the theory are studied in Sec. V.

II. A TIME-SPACE VARYING SPEED OF LIGHT

First of all let us consider in general the possibility of the variability of light speed with time. Our purpose is to obtain most simple and natural mode of variability of speed with time. In particular, it is preferable that photons\[1\] still move without acceleration.

We require that the following postulates hold:

1. The Light Speed varies with time and distance: $C \rightarrow C(t, \vec{r})$

2. The speed of a particular photon is constant along its trajectory.

The first postulate seems obvious from the relativistic point of view. If a physical constant varies with time it must vary with distance as well. The second one in some respect introduces the variability of light speed with time minimally. This means that though in some point of space $r_0$ the light speed varies with time $C(t, r_0)$, if we observe the movement of the particular photon, we will find it travelling uniformly along the trajectory $\vec{r} = \vec{r}_0 + \vec{C}(t_0, \vec{r}_0)(t - t_0)$, at a constant speed $\vec{C}_0 = \vec{C}(t_0, \vec{r}_0) = \vec{C}(t, \vec{r})$, where $\vec{r}_0$, $t_0$ are some fixed point and moment of time.

* By "photon" we mean a light signal or wave packet that is much smaller than the distance it is travelling.
time. In other words, the function of light speed $\bar{C}(t, \vec{r})$ must satisfy the following functional equation:

$$\bar{C} \left( t, \vec{r}_0 + \bar{C}(t_0, \vec{r}_0)(t - t_0) \right) = \bar{C}(t_0, \vec{r}_0)$$ (1)

for any $t, t_0, \vec{r}_0$.

To solve this equation, let us consider the trajectory of the moving photon. Since $\bar{C}_0 = \bar{C}(t_0, \vec{r}_0)$, $\vec{r}_0$ is a function of $\bar{C}_0$ and $t_0$. Thus the trajectory of the photon

$$\vec{r} = \vec{r}_0 + \bar{C}_0 \cdot (t - t_0) = \vec{F}_1(\bar{C}_0, t_0) + \bar{C}_0 t$$ (2)

or, since $\bar{C}_0 = \bar{C}(t, \vec{r})$ we have

$$\vec{F}_1 \left( \bar{C}(t, \vec{r}), t_0 \right) = \vec{r} - \bar{C}(t, \vec{r})t.$$ (3)

The fixed moment of time $t_0$ can be chosen arbitrarily and does not depend on the current position $\vec{r}$ and time $t$, thus the function $\vec{F}_1$ does not depend on $t_0$. So, the most general solution of the equation (1) has the following form:

$$\bar{C}(t, \vec{r}) = \vec{F} \left( \vec{r} - \bar{C}(t, \vec{r})t \right)$$ (4)

where $\vec{F}(\xi)$ is an arbitrary function.

To make the function $\vec{F}(\xi)$ a more specific one we need to introduce additional postulates. It is however easy to see that resolving Eq. (1) in elementary functions is only possible if $\vec{F}$ is linear: $\vec{F}(\xi) = \vec{c} + \lambda \vec{c} \xi$, where $\vec{c}$ and $\lambda c^2$ are constants. Therefore, the simplest non-trivial dependence of the light speed on time and distance satisfying the above formulated axioms has the following form:

$$\bar{C}(t, \vec{r}) = \frac{\vec{c} + \lambda \vec{c} \vec{r}}{1 + \lambda \vec{c} \vec{r}}$$ (5)

The constant $\lambda$ is a new fundamental constant which determines the magnitude of effects caused by dependence of light speed on time and distance. In particular, if $\lambda = 0$, the light speed is constant and is equal to the constant $c$ ($\vec{c} = c \vec{n}$, where $\vec{n}$ is a unit vector). The initial moment of time $t = 0$ corresponds to the present moment when the fixation of units of measurement takes place. The unit of time is chosen so that the light velocity is equal to $C(0, 0) = c = 299792458$ m s$^{-1}$ at that moment $(t = 0)$.

If the parameter $\lambda$ is small, the effects connected with the light speed variability with time and distance will manifest themselves in long times $t$ and at big distances $r$ from an observer. That is, only at cosmological scale.

As it was mentioned in Sec.I, consistent introduction of light speed variability with time and distance requires a considerable generalization of the Theory of Relativity. In Refs. [14], [15] it was shown how such a generalization can be applied to the Lorentz transformation.

The functional dependence (5) requires a generalization of transformation between two rest observers within one inertial reference system. Indeed, let us consider the rest observer at the origin $x = 0$ which at the moment $t = 0$ emits a light signal in the direction of the second rest observer at the point $x = R$. The speed of this signal equals $C(0, 0) = c$, and propagating according to the second postulate at the constant speed $c = C(t, ct)$, it reaches the second observer at the moment of time $t = R/c$. However, the second observer cannot reflect this signal with the same speed because in that case it would return to $x = 0$ with the speed "$c$" which is greater than the speed of light for that moment:

$$c > C \left( \frac{2R}{c}, 0 \right) = \frac{c}{1 + 2\lambda c R}.$$ (6)

It is especially strange from the point of view of the observer at $x = 0$, because for him the light speed

$$\bar{C}(t, 0) = \frac{\vec{c}}{1 + \lambda \vec{c}^2 t}$$ (7)

is isotropic, and he can receive and emit signals with the same speed in any direction (for the given moment of time).

Such a seeming non-equality of two rest observers shows that it is necessary to consider in detail the relation not only between the measurements performed by observers in different inertial frames of reference but also between observers within the same inertial frame. These transformations along with the Projective Lorentz Transformations [14], [15] provide the necessary generalization of the Theory of Relativity.

III. GENERALIZATION OF TRANSLATIONAL TRANSFORMATIONS.

Let us consider two rest observers within one inertial system who are situated at the points $x = 0$ and $x = R$. We denote coordinates and times of events as measured by the first and second observers respectively by $x, y, z, t$ and $X, Y, Z, T$. The question is as follows: "What is the most natural way to generalize translational transformations?"

$$\begin{align*}
X &= x - R, \\
Y &= y \\
T &= t
\end{align*} \quad \Rightarrow \quad \begin{align*}
X &= X(x, y) \\
Y &= Y(x, y) \\
T &= T(x, y, t)
\end{align*}$$ (8)

(Below we will only consider two dimensions $(x, y)$, because all the formulae for $y$ and $z$ components are equivalent.)

To solve the stated problem we use the Principle of Parametrical Incompleteness [15] which consists in the following. The set of axioms of classical mechanics is complete and any statement formulated within the theory framework can be either proved or denied on the basis of these axioms. Reducing the number of axioms...
where the constant \( \sigma \) is connected from relativistic mechanics in the limit of small velocities. We conventionally obtain classical mechanics in this limit.\( \sigma \) has the property that any geodesic line is projected into a straight line. Beltrami's space touches the space at the point where the observer is situated, and it possesses the property that any geodesic line is projected on it as a straight line. In the simplest case of a two-dimensional sphere, Beltrami's map is a plane tangent to the sphere. The projection on the plane is made from the centre of the sphere. Physical and geometrical distances to some point are connected by the equation \( S_{\text{phys}} = \tan(S_{\text{geom}}) \), and for Lobachevsky space of negative curvature by \( S_{\text{phys}} = \tanh(S_{\text{geom}}) \). Analogous relations between geometrical and physical values also exist in the velocity space of the Theory of Relativity.

Now let us consider the transformation of time. Suppose that all events lying in some plane normal to the \( x \) axis occur simultaneously from the point of view of one observer. Then they will be simultaneous from the point of view of another observer as well. It means that

\[
X = \frac{x - R}{1 - \alpha R x}, \quad Y = \frac{\gamma(R) y}{1 - \sigma(R) x}, \quad T = \frac{a(R) x + b(R) t + c(R) + d(R) y}{1 - \sigma(R) x},
\]

where \( \sigma(R), \gamma(R) a(R), b(R), c(R), d(R) \) are some unknown functions. The linear fractional transformations \( \mathbb{H} \) are well-known as the most general geometrical transformations imaging a straight line into a straight line. This is the main point of the second axiom.

Axioms

1. The transformations of coordinates and time are continuous, differentiable and single valued-functions.
2. If from the point of view of one observer a free particle moves uniformly, it will move uniformly from the point of view of another observer.
3. The observers negotiate a units of length so that their relative distance is equal to \( R \).
4. All the observers are equal and the transformations compose a group.
5. Space is isotropic.

The first axiom is standard for the majority of physical constructions. The second one is actually a definition of inertial reference systems and time. We define the time so that the movement of a free particle is as simple as possible. The third one is a definition of units of length: two rest observers, assume, by mutual agreement, that the distance between them is equal to \( R \). These axioms are very strong and completely fix the functional form of transformations. We can show (see Appendix), that the most general transformations satisfying the first three axioms are:

\[
X = \frac{x - R}{1 - \sigma(R) x}, \quad Y = \frac{\gamma(R) y}{1 - \sigma(R) x}, \quad T = \frac{a(R) x + b(R) t + c(R) + d(R) y}{1 - \sigma(R) x},
\]

where \( \sigma(R), \gamma(R) a(R), b(R), c(R), d(R) \) are some unknown functions. The linear fractional transformations \( \mathbb{H} \) are well-known as the most general geometrical transformations imaging a straight line into a straight line. This is the main point of the second axiom.

The requirement of fulfillment of group properties (axiom 4) means that there are at least three equal observers for whom:

\[
x_2 = \frac{x_1 - R_1}{1 - \sigma_1 x_1}, \quad x_3 = \frac{x_2 - R_2}{1 - \sigma_2 x_2} = \frac{x_1 - R_3}{1 - \sigma_3 x_1},
\]

where \( \sigma_i = \sigma(R_i) \). These equations are satisfied only if

\[
\frac{\sigma(R_1)}{R_1} = \frac{\sigma(R_2)}{R_2} = \alpha = \text{const}
\]

and

\[
R_3 = \frac{R_1 + R_2}{1 + \alpha R_1 R_2}.
\]

Since relative distances \( R_1 \) and \( R_2 \) are arbitrary, \( \alpha \) is a fundamental constant which is the same for all the observers.

The reverse transformation corresponds to substitution \( R \rightarrow -R \), and since

\[
y = \frac{1 - \alpha R^2}{\gamma(R)} \frac{Y}{1 + \alpha R X},
\]

we have \( \gamma(R) \gamma(-R) = 1 - \alpha R^2 \). The isotropy of space (axiom 5) implies that transformations are invariant under inversion of the spatial axes \( y \rightarrow -y \), \( Y \rightarrow -Y \), \( R \rightarrow -R \) etc. This leads to the fact that the function \( \gamma(R) \) is even, and for the space transformations we obtain

\[
X = \frac{x - R}{1 - \alpha R x}, \quad Y = \frac{y \sqrt{1 - \alpha R^2}}{1 - \alpha R x}.
\]
In case of light moving along the same function expressed in coordinates of each observer have:

\[ b(R) = \sqrt{1 - aR^2}, \quad c(R) = \frac{a(R)}{aR} (\sqrt{1 - aR^2} - 1) \]  

(15)

The composition of transformations \( t_2 = f(t_1, x_1, R_1), \) \( t_3 = f(t_2, x_2, R_2) = f(t_1, x_1, R_3) \) is possible only if

\[ \frac{a(R_1)}{R_1} = \frac{a(R_2)}{R_2} = \lambda = \text{const.} \]  

(16)

So, we obtain:

\[ T = \frac{t\sqrt{1 - aR^2} + \lambda Rx + (\sqrt{1 - aR^2} - 1)\lambda/\alpha}{1 - \alpha Rx} . \]

(17)

We should note that the synchronization procedure is derived automatically: the event that happens between observers at equal distances from these observers, \( x = -X = (1 - \sqrt{1 - aR^2})/\alpha R, \) is simultaneous for them: \( T = t. \)

Using transformations (14) and (17) it is easy to obtain transformations for the speed of particle as measured by each of the observers \( U = d\vec{X}/dT, \) \( \vec{u} = d\vec{x}/dt: \)

\[ U_X = \frac{u_x \sqrt{1 - aR^2}}{1 + \lambda Ru_x - \alpha R(x - u_xt)} \quad (18) \]

\[ U_Y = \frac{u_y + \alpha R(yu_x - xu_y)}{1 + \lambda Ru_x - \alpha R(x - u_xt)} . \]  

(19)

If the particle moves uniformly \( \vec{r} = \vec{r}_0 + \vec{u}t, \) transformations of speed do not vary with time but vary with the “initial” position of the particle \( \vec{r}_0. \)

Here we should note that, if \( \alpha = (\lambda c)^2, \) the formula (3) for the light speed \( \vec{C}(t, \vec{r}) \) possesses the following properties:

1. \( \vec{C}(t, \vec{r}) \) is invariant for both observers. It means that, if \( \vec{C}(t, \vec{r}) \) is transformed as a speed (18), (19), the same function expressed in coordinates of each observer stands on the right and on the left of the transformations (13), (11). In case of light moving along the axis we have:

\[ C(T, X) = \frac{C(t, x) \sqrt{1 - (\lambda cR)^2}}{1 + \lambda RC(t, x) - (\lambda c)^2 R(x - C(t, x)t)} \]  

(20)

The movement in an arbitrary direction is considered in the next section.

2. \( \vec{C}(t, \vec{r}) \) is the maximal possible speed for the given point of space \( \vec{r} \) and given moment of time \( t. \)

On the basis of these two properties we call \( \vec{C}(t, \vec{r}) \) the speed of light.

Therefore, for the consistent introduction of the varying with time and distance light velocity (3) into the theory, it is necessary to generalize the translational transformations for rest observers:

\[ X = \frac{x - R}{1 - (\lambda c)^2 R_x}, \quad Y = \frac{y\sqrt{1 - (\lambda c)^2 R_x^2}}{1 - (\lambda c)^2 R_x} . \]  

(21)

\[ 1 + \lambda c^2 T = \frac{\sqrt{1 - (\lambda c)^2 R_x^2}}{1 - (\lambda c)^2 R_x} (1 + \lambda c^2 t) . \]  

(22)

If two observers move at a relative speed \( v, \) the generalized Lorentz transformations have the following form [4, 17]:

\[ x' = \frac{\gamma(x - vt)}{1 + \lambda v\gamma x - \lambda c^2(\gamma - 1)t}, \]

\[ y' = \frac{y}{1 + \lambda v\gamma x - \lambda c^2(\gamma - 1)t}, \]

\[ t' = \frac{\gamma(t - uv/c^2)}{1 + \lambda v\gamma x - \lambda c^2(\gamma - 1)t} . \]

(23)

(24)

(25)

where \( \gamma = 1/\sqrt{1 - v^2/c^2} \) is the Lorentz factor. The formulae (23)-(25) form the basis of kinematics of the Projective Theory of Relativity, within which the speed of light varies with time but at the same time is an invariant of the theory.

The contradiction considered in the Sec. II is easy to resolve now. From the point of view of the first observer the signal reaches the second observer \( x = R \) at the moment of time \( t = R/c \) with the speed \( u = c. \) From the point of view of the second observer the speed of the signal (18) and the moment of time (22) are equal to:

\[ U = c\sqrt{1 - (\lambda c)^2 R^2} , \quad \lambda c^2 T = \sqrt{1 + \lambda c^2 R^2} - 1. \]  

(26)

The observer reflects the signal with the same speed \( U \rightarrow -U. \) However, due to the transformations of speed its speed relative to the first observer (18) equals:

\[ u = -c\frac{1 - \lambda cR}{1 + \lambda cR} . \]  

(27)

In a time \( t = R/c + R/|u| \) the signal returns to the first observer to the point \( x = 0, \) and has the same speed as any other light signal at that moment of time:

\[ C \left( \frac{R}{c} + \frac{R}{|u|}, 0 \right) = c\frac{1 - \lambda cR}{1 + \lambda cR} = |u| \]  

(28)

Therefore, if two particles have the same speed from the point of view of one observer, they will have different speeds for another observer. The equality of speeds is as relative a notion as the simultaneity of events is. This
happens because the Projective Transformations do not conserve parallelism of straight lines.

If we add to the initial system of axioms the requirements of absolutivity of equality of two speeds and absolutivity of time, we obtain the complete axiom system in which incompleteness connected with undefinable constants \( \lambda = 0, \alpha = 0 \) disappears. If we exclude these axioms, we obtain the more general parametrically incomplete theory with new fundamental physical constants. This is the Principle of Parametrical Incompleteness.

IV. THE HUBBLE LAW. EXPANSION OF THE STATIC UNIVERSE.

An interesting consequence of the results of previous sections arises when the Doppler effect is analyzed within one inertial system.

1. Hubble Law

Let us consider a remote rest source with coordinates \( \vec{R} \) emitting light in the direction of observer which is situated at the origin \( x = 0 \). According to observer’s clock the light pulse emitted at the moment of time \( t_1 \) reaches it at the moment \( t_2 \). Since the speed of this signal is constant \( C(R, t_1) = C(0, t_2) \) and it moves in the direction towards the observer \( \vec{c} = -c\vec{R}/R \), we have the following relation between \( R, t_1, t_2 \):

\[
(t_2 - t_1)c = R + \lambda c^2 R t_2.
\]

Let us assume that light pulses are emitted with the period \( \Delta T \), and are received with the period \( \tau \). Since the source’s time \( T \) and the observer’s time \( t \) are related by Eq. (22), the interval \( \Delta T \) equals \( \Delta T = \sqrt{1 - (\lambda cR)^2} \), for \( \vec{x} = \vec{R} \). Thus the period of emission is \( \tau = \Delta t_1/\sqrt{1 - (\lambda cR)^2} \), and introducing the parameter of redshift \( z \) we finally obtain:

\[
1 + z = \frac{\tau}{\tau_0} = \sqrt{1 + \lambda cR \over 1 - \lambda cR}.
\]

Interpreting the redshift according to Doppler’s formula, we obtain the Hubble law: \( \vec{V} = \lambda c^2 \vec{R} \), but such an interpretation would not be correct in this case.

2. Distance Measurement

We can obtain the same result by the following speculations. Suppose, the observer at \( x = 0 \) makes a radiocating experiment measuring the distance to the rest object at \( x = R \). At the moment of time \( t_1 \) this observer emits a light signal at the speed of \( C(0, t_1) \) receiving it at time \( t_2 \) at the speed of \( C(0, t_2) \). If the observer (despite different speeds of the emitted and reflected signals) assumed the distance to the object to be equal to \( l = (t_2 - t_1)c/2 \), he would, probably, conclude that the object moves away from him at Hubble’s speed:

\[
l = c \left( \frac{R}{C(t_1)} + \frac{R}{C(t_2)} \right) = R + \lambda c^2 R \frac{t}{t + V t},
\]

where \( t = (t_2 + t_1)/2 \). Such an interpretation would not, of course, be correct. If the observer emitted signals at speed \( u < C(0, t_2) \), he could (with appropriate conditions of reflection) receive them at the same speed, and the distance \( l = (t_2 - t_1)u/2 \) would be unchanging and equal to \( R \).

3. Aberration of Light

Let us obtain another useful equation which also can be interpreted in terms of the Hubble speed. An expression similar to that for aberration in the Theory of Relativity arises for the rest light source and receiver. Suppose, the light travels in some direction \( (\cos \omega, \sin \omega) \) relative to the observer at \( x = 0 \), and in direction \( (\cos \Omega, \sin \Omega) \) relative to the observer at \( x = R \). Then the components of light velocity will be equal to

\[
C_x = c \frac{\cos \omega + \lambda c x}{1 + \lambda c^2 t}, \quad C_y = c \frac{\sin \omega + \lambda c y}{1 + \lambda c^2 t}.
\]

If we put \( \frac{t}{t_0} = \frac{1}{1 - (\lambda cR)^2} \) and similar equations for the second observer in \( [18], [19] \), where \( \alpha = (\lambda c)^2 \), we would obtain the identity for any \( x, y, t \), only if:

\[
\sin \Omega = \frac{\sqrt{1 - (\lambda cR)^2} \sin \omega}{1 + \lambda cR \cos \omega}, \quad \cos \Omega = \frac{\cos \omega + \lambda cR \cos \omega}{1 + \lambda cR \cos \omega}.
\]

These formulae formally coincide with those for aberration in the Theory of Relativity if we set \( \lambda cR = V/c \). So, we again come to the Hubble formula.

4. "Expansion" of the static Universe

If we admit the possibility light speed of variability with time, we will necessarily come to the following cosmological model. The Universe is a stationary space of constant curvature (the Lobachevsky space). The curvature is not connected with the presence of matter and is an intrinsic property of the empty space. The course of time in the Universe is defined so that it would look as simple as possible. This leads to the flat pseudo-Euclidean space-time.

The evolution of the Universe is connected with the decreasing of the speed of light with time and 14(?) billion years ago the speed of light was equal to infinity. We now take this moment as the origin of time, i.e. make the shift \( t \rightarrow t - 1/\lambda c^2 \) in all the formulae . Because of the infinite speed of interactions, the early Universe was homogeneous and hot. However, there was no singularity of matter. All the clocks in the Universe were synchronized \( (C = \infty) \) and pointed at the zero time mark:

\[
T = \sqrt{1 - (\lambda cR)^2} \frac{t}{1 - (\lambda c)^2 R^2}.
\]
With the course of time the speed of light was decreasing, the Universe was cooling, and the clocks located at the distance \( r = R \) from us started to advance compared to our clock:

\[
T = \frac{t}{\sqrt{1 - (\lambda c R)^2}} > t
\]  

(36)

Nevertheless, we observe the Universe in its past state

\[
T_v = \sqrt{\frac{1 - \lambda c R t}{1 + \lambda c R}} = \frac{t}{1 + z} < t,
\]  

(37)

because the speed of light is finite \( C(t, 0) = (\lambda c t)^{-1} \) (here \( z \) is the parameter of redshift).

The frequency of the light we receive from remote rest sources is shifted to the red. The farther the source is situated from us the more the frequency of the light is shifted to the red, in agreement with the Hubble Law.

The distance \( R_m = 1/\lambda c \) is the maximal possible distance an observer can measure, and at the same time is the radius of curvature of the Lobachevsky space. At any moment of time according to our clocks \( t \) we see areas situated at the distance \( R_m \) from us at the moment of time \( T = 0 \) according to the local clock. The infinite value of the red shift parameter \( z \) corresponds to these areas.

Although the Hubble Law is realized automatically in this cosmological model, it is obvious that including matter and gravitation into consideration can change the properties of our Space in some way, for instance, to make it expand. In this case the Hubble effect will consist of two components - the usual Doppler redshift and the shift connected with the new fundamental constant \( \lambda \). As a result, the actual age of our Universe could be much greater than the value derived from the Hubble Law.

V. CONCLUSION: VARYING SPEED OF LIGHT AND EXPERIMENT

Let us discuss applicability of the proposed theory to the real World. Since Hubble’s effect is naturally described within the Projective Theory of Relativity, it would be interesting to associate Hubble’s constant \( H = 65 \text{ km/sec/Mpc} = 6.7 \times 10^{-11} \text{ year}^{-1} \) with the constant \( \lambda c^2 \). In this case the change of the light velocity with time would be as follows \( (r = 0, t = 0) \):

\[
\frac{\Delta C}{C} = -\lambda c^2 \Delta t = -6.7 \times 10^{-11} \frac{\Delta t}{\text{year}}.
\]  

(38)

Obviously, the dimensional value \( C(t, 0) \) can be expressed in terms of some units of length and time, e.g. the atomic units \( h^2/mc^2 \) and \( h^3/mc^4 \). In particular, the dimensionless combination \( \alpha(t) = e^2/hC(t) \) should change. The laboratory value of \( \alpha \) is known at present (1997) with accuracy of \( 4 \times 10^{-9} \): \( \alpha^{-1} = 137.03599993(52) \), which is close enough to change \( \alpha \).

Here let us make clear one point about testing the dependence of the light velocity on time. There are two entities in our theory: \( C(t, \vec{r}) \) and \( c \). The first one is the light velocity and the maximal possible speed of material objects, the second one is the fundamental speed arising from the parametrical incompleteness of the axioms of the theory. Only after generalization of Quantum Electrodynamics for the case of the Projective Theory of Relativity would it be possible to say which of the constants would enter into \( \alpha \). The fine-structure constant may, thus, depend on \( c: \alpha = e^2/hc \), and do not change with time.

Recently, a new direct limit on \( |\dot{\alpha}/\alpha| < 10^{-14} \text{year}^{-1} \), has been derived from spectral properties of distant \( (z = 1 \pm 3.5) \) quasars [16], [17]. However, this does not mean that \( \alpha \) is falsified by experiment. Indeed, we observe an object which is situated at the distance \( R \) from us in its past state at the moment \( t = -R/c \) according to the local clock. That time corresponds to the local time of an object \( T = -z/(1 + z)\lambda c^2 \) and, therefore, the light velocity measured by the observer, which is situated near the object, equals \( C(0, T) = c(1 + z) \). From his point of view, the dimensionless combination \( \alpha(T) = e^2/hC(0, T) \), is \( 1 + z \) times less than our measurement shows at the present moment of time \( t = 0 \). According to the rule of transformation for speeds measured by distant observers, the light emitted by the object at the speed of \( c(1 + z) \) is equal to \( c \) from our point of view and, therefore, \( \alpha = e^2/hc \). That is why the measurement of \( \alpha \) based on the spectra of quasars does not allow us to test the change of light velocity in time.

It is likely that only direct laboratory measurement of the light velocity in terms of the atomic units of length and time would provide a direct test for \( \alpha \).

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APPENDIX

Let us consider arbitrary independent differentiable transformations of the coordinate \( x \) and time \( t \):
\[ X = f(x), \quad T = g(x,t). \quad (39) \]

We require the system of coordinates \((x, t)\) and \((X, T)\) to satisfy the definition of inertial reference systems:

\[ \frac{du}{dt} = 0 \implies \frac{dU}{dT} = 0, \quad (40) \]

i.e. a free particle moves uniformly from the point of view of all observers.

By definition, the speeds are \(u = dx/dt\) and \(U = dX/dT\), thus:

\[ U = \frac{uf_x}{g_x u + g_t}, \quad (41) \]

where \(g_x = \partial g(x,t)/\partial x\), etc. Differentiating \((41)\) on \(T\) \((dT = (g_x u + g_t)dt)\) and taking into account that the coefficients of the obtained polynomial in \(u\) must be equal to zero (since \(u\) is arbitrary) we obtain the system of differential equations:

\[ \begin{align*}
    f_{xx} g_x &= g_{xx} f_x, \\
    f_{xx} g_t &= 2g_{xt} f_x, \\
    g_{tt} f_x &= 0.
\end{align*} \quad (42-44) \]

Solving this system, we obtain:

\[ f(x) = \frac{ax + b}{1 + \alpha x}, \]
\[ g(x,t) = \frac{\gamma t + a' x + b'}{1 + \alpha x}. \]

In a more general case of two dimensions, linear fractional transformations have the following form \([18]\):

\[ \begin{align*}
    X &= \frac{ax + by + c}{1 + \alpha x + \beta y}, \\
    Y &= \frac{\bar{a}x + \bar{b} y + \bar{c}}{1 + \alpha x + \beta y}, \\
    T &= \frac{\gamma t + a' x + b' y + c'}{1 + \alpha x + \beta y}.
\end{align*} \quad (45) \]

It is assumed that the third axiom is equivalent to the following equations:

\[ \begin{cases}
    X(0,y) = -R, & X(R, y) = 0, \\
    Y(R,0) = 0, & Y(0,0) = 0.
\end{cases} \quad (46) \]

and we obtain the Eq.(4).