Dispersion of Inferred SNe Ia/SNe II Ratios for Current Models of Supernova Nucleosynthesis

Shigehiro Nagataki, Masa-aki Hashimoto, and Katsuhiko Sato

1 Department of Physics, School of Science, the University of Tokyo, 7-3-1 Hongo, Bunkyoku, Tokyo 113
E-mail(TY): Nagataki@utaphp1.phys.s.u-tokyo.ac.jp

2 Department of Physics, Faculty of Science, Kyusyu University, Ropponmatsu, Fukuoka 810

3 Research Center for the Early Universe, School of Science, the University of Tokyo, 7-3-1 Hongo, Bunkyoku, Tokyo 113

(Received 1997 August 27; accepted 1997 November 21)

Abstract

We have estimated the dispersion of the inferred relative frequencies of Type-Ia and Type-II supernovae \((N_{\text{Ia}}/N_{\text{II}})\) in our Galaxy by fitting the numerical results of supernova nucleosynthesis to the solar-system abundances. The ratio \(N_{\text{Ia}}/N_{\text{II}}\) is estimated to be 0.056–0.14, which is consistent with the observation, if the model of Woosley and Weaver (1995, WW95) is adopted for Type-II supernovae (SNe II). On the other hand, the upper limit of \(N_{\text{Ia}}/N_{\text{II}}\) becomes too large for the model of Hashimoto (1995, Ha95). However, Ha95 can fit the solar values better than WW95 as far as the abundant nuclei are concerned. These results mean that Ha95 can reproduce the main nuclei of the solar-system well and that WW95 can reproduce the solar values over a wide mass-number range. We also note that \(N_{\text{Ia}}/N_{\text{II}}\) tends to become smaller if the delayed detonation model is adopted for Type-Ia supernovae (SNe Ia). The dispersion of \(N_{\text{Ia}}/N_{\text{II}}\) obtained in the present investigation is larger than that concluded by Tsujimoto et al. (1995), which will have an influence on the estimate of the average life time of SNe Ia’s progenitors and/or the star-formation rate history in our galaxy.

Key words: Chemical evolution — Solar system — Sun: abundances — Supernovae
1. Introduction

Supernovae play an important role in ejecting heavy elements of $A \geq 12$ produced during stellar evolution and/or an explosion. There are two types of supernovae: Type-I and Type-II supernovae. Type-Ia supernovae (SNe Ia) are thought to be thermonuclear explosions of accreting white dwarfs (e.g., Nomoto et al. 1994). On the other hand, Type-II supernovae (SNe II) and Type-Ib/Ic supernovae (SNe Ib/Ic) are thought to be collapse-driven supernovae (e.g., Bethe 1990). Many calculations of them have been performed and their chemical compositions have been calculated.

As for SNe Ia, the deflagration model has been thought to be more favored than the detonation model, since $^{56}\text{Ni}$ is over-produced in the detonation model. However, concerning the amount of $^{56}\text{Ni}$, the deflagration model can explain the spectrum of SNe Ia fairly well (e.g., Nomoto et al. 1997a). In particular, a model W7 (Nomoto et al. 1984) has been considered to be 'standard', and the calculated chemical composition has been used widely to explain the solar-system abundances. As for SNe II, two groups have systematically calculated the nucleosynthesis (e.g., Woosley, Weaver 1995; Hashimoto 1995). Although their calculations assumed different physical process such as the criterion of convection, the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate, which is crucial for constructing stellar models, and initial shock-wave conditions, which determines the peak temperature during the explosion, they succeeded in explaining the chemical compositions of SNe II with a fair degree of precision. In fact, the solar-system abundances are well reproduced within a factor of 2–3 by the combination of their results for a wide range of mass numbers, except for some nuclei (Hashimoto 1995; Timmes et al. 1995.)

By using the results of W7 and Ha95, the relative frequencies of SNe Ia and SNe II in our galaxy was obtained to be $N_{\text{Ia}}/N_{\text{II}} = 0.15$ (Tsujimoto et al. 1995), which is consistent with the fact that the observed estimate of the SNe Ia frequency is about 10% of the total supernovae occurrence (van den Bergh, Tammann 1991). However, there is one question in their calculations. They concluded that the contribution of SNe Ia to the solar-system abundances is in the range $r = 0.09 \pm 0.01$ against possible uncertainties in their analysis (see subsection 2.2). However, they arbitrarily selected 31 species to be fitted, and neglected other nuclei when they derived the proper value of $r$. We think that it is worth examining whether the dispersion of the $N_{\text{Ia}}/N_{\text{II}}$ changes or not by the choice of the nuclei to be fitted. Moreover, we should examine whether the proper value of $r$ can be really predicted from numerical calculations without such an arbitrariness.
Another motivation for the present study is the model dependence of the prediction of $N_{\text{Ia}}/N_{\text{II}}$. Recently, based on the progress of numerical calculations for supernovae, the chemical compositions of their ejecta have been recalculated. In particular, the effect of the delayed detonation has been taken into consideration for SNe Ia. As for the W7 model, the flame speed is assumed to be relatively high, which is inconsistent with that of multi-dimensional calculations; the simple deflagration can not generate an energetic explosion, or can not lead to an explosion in the multi-dimensional calculations (Arnett, Livne 1994; Khokhlov 1995). To solve this problem, a mechanism of delayed detonation is proposed by Khokhlov (1991). Recently, nucleosynthesis in SNe Ia was recalculated based on the assumption of delayed detonation (Nomoto et al. 1997b), and has presented a more excellent explanation for the observation of SNe Ia. For example, the velocity profiles of intermediate nuclei, such as Si and S, become wider compared with that of the W7 model, which is consistent with the observations. We also note that the chemical compositions of SNe II are different between Ha95 and WW95 (Hashimoto 1995; Woosley, Weaver 1995).

To investigate the capability of reproducing the solar-system abundances with using the available results, we will explore the model dependence and derive the dispersion based on numerical calculations. It is important to estimate the dispersion, since the change in the relative frequency of SNe Ia and SNe II will have an influence on the estimate of the average lifetime of SNe Ia’s progenitors and/or the star-formation history in our galaxy.

We describe our calculation method in section 2. The results are presented in section 3. A summary and a discussion are given in section 4.

2. Models and Method of Calculations

2.1. Models of SNe Ia and SNe II

As for the composition of SNe Ia, we use the result of W7 for the simple deflagration model and WDD2 for the delayed detonation model, respectively (Nomoto et al. 1984; Nomoto et al. 1997b). We also use the results of Ha95 and WW95 for SNe II. More precisely, we use for Ha95 the results of helium-star models with masses of $M_\alpha = 3.3, 4, 5, 6, 8, 16$, and $32$, which approximately correspond to main-sequence stars of $M_{\text{ms}} = 13, 15, 18, 20, 25, 40$, and $70 M_\odot$, and for WW95 S13A, S15A, S18A, S20A, S25A, and S40A models. The averaged synthesized yields in $10-50 M_\odot$ stars are obtained with a weight of Salpeter’s initial mass function, as follows:

$$M_{i,\text{II}} = \frac{\int M_i(m)m^{-(1+x)}dm}{\int m^{-(1+x)}dm}.$$  \hspace{1cm} (1)
The differences in the calculated chemical compositions are shown in figure 1. The open circles denote W7/WDD2 and the dots represent WW95/Ha95. The abscissa represents the mass number. The plotted 46 nuclei, which are the same nuclei as those in table 1 of Tsujimoto et al. (1995), are summarized in table 1.

2.2. Calculation Method for the Solar-System Abundances

In an analysis of the reproduction of the solar-system abundances, we adopt the same method with Tsujimoto et al. (1995). At first, as the sum of $M_{i,\text{Ia}}$ we define $M_{\text{Ia}}$ which are the total heavy element mass of SNe Ia. We also define $M_{\text{II}}$ in the same way.

Next, we define the abundance pattern $x_i$ as

$$x_i = rM_{i,\text{Ia}}/M_{\text{Ia}} + (1 - r)M_{i,\text{II}}/M_{\text{II}} \quad (0 \leq r \leq 1),$$

which is to be compared with the solar $x_i(\odot)$, defined as $Z_i/\sum_i Z_i$, where $Z_i$ is the observed abundance of the $i$-th element per unit mass (Anders, Grevesse 1989). The most probable value of $r = r_p$ is determined by minimizing the following function (Yanagida et al. 1990):

$$g(r) = \frac{1}{n} \sum_{i=1}^{n} (\log x_i - \log x_i(\odot))^2,$$

where $i$ runs over the heavy elements and their isotopes chosen in the minimization procedure. We summarize those elements in table 1. We note that $r$ has a different meaning from that of the relative frequency $N_{\text{Ia}}/N_{\text{II}}$. The relation between them is

$$\frac{N_{\text{Ia}}}{N_{\text{II}}} = \frac{\omega_{\text{II}} M_{\text{II}}}{\omega_{\text{Ia}} M_{\text{Ia}} (1 - r_p)},$$

where $\omega_{\text{Ia}}$ and $\omega_{\text{II}}$ represent the mass fraction of heavy elements ejected into the interstellar gas from SNe Ia and SNe II, respectively. These values were estimated to be 0.27 and 0.22 in the solar neighborhood from the numerical calculation (Tsujimoto et al. 1995). We calculated $g(r)$ in five cases. We used all 46 nuclei (Case 1). Next, we used the selected 31 nuclei (Case 2) and the most abundant 6 nuclei (Case 3). Then, 20 elements, each of which is a summation of the isotopic abundances, were examined (Case 4). Finally, the selected 14 elements were fitted (Case 5). Furthermore, we also performed the same analysis for LMC and SMC with the same elements with Tsujimoto
et al. (1995) to check our conclusion. The value of $M_{II}/M_{Ia}$ for each case is summarized in table 2. We note that Tsujimoto et al. (1995) concluded that $r_p = 0.09 \pm 0.01$ and $N_{Ia}/N_{II} = 0.15$ in their analysis by adopting Case 2 and Case 5 for the results of W7+Ha95.

3. Results

3.1. The Solar System Abundances Pattern

We show in figure 2 the form of $g(r)$ for four models: W7+Ha95, W7+WW95, WDD2+Ha95, and WDD2+WW95. The thick solid, long-dashed, short-long-dashed, short-dashed, and dot lines correspond to Case 1, 2, 3, 4, and 5, respectively.

Generally speaking, $g(r)$ has a minimum value at $r \sim 0.1$ in most cases. In particular, $g(r)$ has minimum values for all cases if WW95 is adopted. On the other hand, $g(r)$ is not sensitive to $r$ for the W7+Ha95 and WDD2+Ha95 models in Case 1 and 4. However, $g(r)$ has a deep minimum value in the case of 3 if Ha95 is adopted. These results mean that WW95 can reproduce the solar values well in many nuclei and that Ha95 can reproduce them well for selected abundant nuclei. The values of $r_p$ for all cases are summarized in table 3. We should note that the range of $r_p$ is wider than that of Tsujimoto et al. (1995).

We can see the effect of delayed detonation. The values of $r_p$ tend to become smaller if WDD2 is adopted. We also feel that the minimum values of $g(r_p)$ seem to become smaller in WDD2. This means that the solar-system abundances are fitted better by WDD2 than W7. We examine this tendency by adopting our analysis to SMC and LMC in the next subsection.

3.2. The SMC and LMC Abundance Patterns

In subsection 3.1, we found that both $r_p$ and $g(r_p)$ become smaller if WDD2 is adopted. We will check this tendency by adopting our method of analysis to other galaxies: SMC and LMC. As was done by Tsujimoto et al. (1995), 12 and 10 elements were taken into consideration for SMC and LMC, respectively (see table 1). The results are given in table 4. We found the same tendency of $r_p$ and $g(r_p)$, which supports our conclusion. We also note the $r_p$s obtained in this analysis tend to be larger than those of the solar-system abundances, which is consistent with Tsujimoto et al. (1995), even if WDD2 is adopted.
3.3. Relative frequency of SNe Ia/SNe II

The ratio \( N_{\text{Ia}}/N_{\text{II}} \) depends on the three values of \( \omega_{\text{II}}/\omega_{\text{Ia}} \), \( M_{\text{II}}/M_{\text{Ia}} \), and \( r_p \). In our analysis, \( \omega_{\text{II}}/\omega_{\text{Ia}} \) is constant (subsection 2.2). WW95 tend to make \( M_{\text{II}}/M_{\text{Ia}} \) smaller than Ha95. WDD2 also make it lower, though its degree is weak. The range of \( N_{\text{Ia}}/N_{\text{II}} \) in table 3 is estimated from \( r_p \) and \( M_{\text{II}}/M_{\text{Ia}} \) for Case 1–5. The values in the parenthesis in table 3 are not so accurate, since \( g(r) \) is insensitive to \( r \). We note that \( N_{\text{Ia}}/N_{\text{II}} \) is estimated to be 0.056–0.14, which would be consistent with the observation if the model of WW95 is adopted. On the other hand, the upper limit of \( N_{\text{Ia}}/N_{\text{II}} \) becomes too large for Ha95. We also note that \( N_{\text{Ia}}/N_{\text{II}} \) tends to become smaller if WDD2 is adopted. Generally speaking, the range of \( N_{\text{Ia}}/N_{\text{II}} \) is fairly wider compared with the conclusion of Tsujimoto et al. (1995).

3.4. Other Analysis

The method described in subsection 2.2 is not the only one that can be used to determine \( r_p \). In this subsection we examine another method to obtain \( r_p \): \( \chi^2 \) fitting. We use two elements, O and Fe, for the \( \chi^2 \) fitting. The \( r_p \) is determined to be \( 7.5 \times 10^{-2} – 1.0 \times 10^{-1} \), \( (6.8–9.1) \times 10^{-2} \), \( (1.1–1.3) \times 10^{-1} \), and \( 9.8 \times 10^{-2} – 1.2 \times 10^{-1} \) (99% C.L.) for W7+Ha95, WDD2+Ha95, W7+WW95, and WDD2+WW95, respectively. However, we must say that all models are excluded unless proper nuclei are chosen for the fitting, since the uncertainty of the observation is much smaller than that of the theoretical prediction. That is why these ranges mentioned above are less persuasive than those discussed in subsection 3.1. In other words, we must wait for progress concerning theoretical calculations to use this method effectively.

3.5. Present Situation of Numerical Simulations

In figure 3, we show the solar abundance pattern \( x_i/x_i(\odot) \) (see, equation (2)) predicted by numerical calculations using \( r_p \) of Case 2. The open circles represent WW95 and the dots denote Ha95. There are some nuclei which are less produced in the range of mass numbers 35–50. For example, Ha95 has a less-production problem of some elements, such as Ar, K, Ti. WW95 also has a problem of less production of \(^{41}\text{K}, ^{44}\text{Ca}, \text{and Ti}\). Cu and Zn are also less produced by Ha95. On the other hand, some nuclei in the range 50–65 are over produced. These problems remain to be solved to predict \( r_p \) more precisely.
4. Summary and Discussion

We have explored the dispersion of $N_{\text{Ia}}/N_{\text{II}}$ using the four models Ha95, WW95, W7, and WDD2. As for $r_p$, $g(r)$ clearly has minimum values if WW95 is adopted. On the other hand, $g(r)$ is insensitive to $r$ for the W7+Ha95 and WDD2+Ha95 models in Case 1 and 4. However, $g(r)$ has a deep minimum value in the case of 3 if Ha95 is adopted. These results mean that WW95 reproduces the solar values well in many nuclei and that Ha95 reproduces them well especially with regard to abundant nuclei. Among the most abundant 6 nuclei, the abundance of $^{56}\text{Fe}$ is different most between WW95 and Ha95. This would be because only WW95 determines the mass cut by a hydrodynamical calculation. This suggests the difficulty of the determining the mass cut from the hydrodynamics, even if the spherical explosion is assumed. $M_{\text{II}}/M_{\text{Ia}}$ tends to become lower for WW95 than for Ha95. Although WDD2 also make it lower, its degree is weak.

As a result, the range of $N_{\text{Ia}}/N_{\text{II}}$ becomes 0.056–0.14 for WW95+SNe Ia, which is consistent with the observation, and 0.077–(0.61) for Ha95+SNe Ia. We also note that $N_{\text{Ia}}/N_{\text{II}}$ tends to become smaller if the delayed detonation model is adopted for Type-Ia supernovae. Since the delayed-detonation model may be a more realistic model than the simple defraglation model, this tendency should be stressed. Generally speaking, the dispersion of $N_{\text{Ia}}/N_{\text{II}}$ obtained in our study is larger than that concluded to be $\sim 10\%$ by Tsujimoto et al. (1995), as can be seen in table 3. We think that this is the present situation of the predictability of numerical calculations, and that this dispersion must always be taken into account. For example, our results will bring the dispersion of both the average lifetime of SNe Ia's progenitors and history of the star-formation rate in our galaxy. Moreover, the critical mass, which represents the upper-mass limit of SNe II progenitor, may also be changed.

A realistic s-process calculation will increase the amount of some less-produced elements in Ha95, as is shown by Prantzos et al. (1990). It would hold true for WW95. This suggests that the neutron-capture process during stellar evolution is also important for some elements. Since WW95 used a larger network for the 'post processing', they obtained neutron-rich elements. On the other hand, Ha95 used a small network during stellar evolution. As a result, neutron-rich nucleosynthesis seems to be insufficient. We also note that there is a possibility that the chemical composition of SNe Ia could be changed if the mass-accretion rate is changed (Nomoto et al. 1997b). An asymmetric explosion in SNe II may also help to solve these problems, since it would produce more $^{44}\text{Ca}$ and Ti than
the spherical calculations (Nagataki et al. 1997a). The initial conditions of the shock wave should also be considered (Nagataki et al. 1997b) because they affect the production of the iron-peak elements.

Finally, let us to say that there remains still some physical uncertainties to be studied in order to clarify the uncertainty of supernova nucleosynthesis, which could be greater than the dispersion obtained in the present study. Therefore, we must trace the physical uncertainty in order to find the reason for the over/under production problems. Such efforts will cause the dispersion of $N_{\text{Ia}}/N_{\text{II}}$ to be smaller, and we should then be able to discuss the chemical evolution in the universe more precisely.

We are grateful to an anonymous referee for useful comments. We also thank for Ken’ichi Nomoto for useful discussions. This research has been supported in part by a Grant-in-Aid for the Center-of-Excellence (COE) Research (07CE2002) and for the Scientific Research Fund (05243103, 07640386, 3730) of the Ministry of Education, Science, Sports and Culture in Japan and by Japan Society for the Promotion of Science Postdoctoral Fellowships for Research Abroad.
References

Anders E., Grevesse N. 1989, Geochim. Cosmochim. Acta 53, 197

Arnett W.D., Livne E. 1994, ApJ 427, 315

Bethe H.A. 1990, Rev. Mod. Phys. 62, 801

Hashimoto M. 1995, Prog. Theor. Phys. 94, 663

Khokhlov A.M. 1991, A&A 246, 383

Khokhlov A.M. 1995, ApJ 449, 695

Nagataki S., Hashimoto M., Sato K., Yamada S. 1997a, ApJ 486, 1026

Nagataki S., Hashimoto M., Yamada S. 1997b, PASJ 49,

Nomoto K., Iwamoto K., Kishimoto N. 1997a, Science 276, 1378

Nomoto K., Thielemenn F.-K., Yokoi K. 1984, ApJ 286, 644

Nomoto K., Yamaoka H., Shigeyama T., Kumagai S., Tsujimoto T. 1994, in Supernovae (Les Houches, Session LIV),

   ed J. Audouze, S. Bludman, R. Mochkovitch, J. Zinn-Justin (Elsevies Sci. Publ., Amsterdam) p199

Nomoto, K. et al. 1997b, Nucl. Phys. A, in press.

Prantzos N., Hashimoto M., Nomoto K. 1990, A&A 234, 211

Timmes F.X., Woosley S.E., Weaver T.A. 1995, ApJS 98, 617

Tsujimoto T., Nomoto K., Yoshii Y., Hashimoto M., Yanagida S., Thielemenn F.-K. 1995, MNRAS 277, 945

van den Bergh S., Tammann G. 1991, ARA&A 29, 363

Woosley S.E., Weaver T.A. 1995, ApJS 101, 181

Yanagida S., Nomoto K., Hayakawa S. 1990, in Proc. 21st International Cosmic Ray Conference 4, 44
### Table 1. Nuclei used in the analysis.

| Species | Case 1 | Case 2 | Case 3 | Case 4 | Case 5 | LMC | SMC | W7  | WDD2 | Ha95 | WW95 |
|---------|--------|--------|--------|--------|--------|-----|-----|-----|------|------|------|
| 16O     |        |        |        |        |        |     |     | 1.43E-01 | 6.93E-02 | 1.80E-00 | 1.14E-00 |
| 18O     |        |        |        |        |        |     |     | 8.25E-10 | 4.62E-07 | 4.61E-03 | 4.04E-03 |
| 20Ne    |        |        |        |        |        |     |     | 2.02E-03 | 9.13E-04 | 2.12E-01 | 1.65E-01 |
| 21Ne    |        |        |        |        |        |     |     | 8.46E-06 | 1.47E-06 | 1.08E-03 | 6.07E-04 |
| 22Ne    |        |        |        |        |        |     |     | 2.49E-03 | 1.96E-06 | 1.83E-02 | 1.88E-02 |
| 23Na    |        |        |        |        |        |     |     | 6.32E-05 | 1.30E-05 | 6.51E-03 | 5.13E-03 |
| 24Mg    |        |        |        |        |        |     |     | 8.50E-03 | 4.76E-03 | 8.83E-02 | 3.83E-02 |
| 25Mg    |        |        |        |        |        |     |     | 4.05E-05 | 2.39E-05 | 1.44E-02 | 9.15E-03 |
| 26Mg    |        |        |        |        |        |     |     | 3.18E-05 | 3.57E-05 | 2.01E-02 | 1.18E-02 |
| 27Al    |        |        |        |        |        |     |     | 9.86E-04 | 2.74E-04 | 1.48E-02 | 7.96E-03 |
| 28Si    |        |        |        |        |        |     |     | 1.50E-01 | 2.71E-01 | 1.05E-01 | 1.14E-01 |
| 29Si    |        |        |        |        |        |     |     | 8.61E-04 | 3.87E-04 | 8.99E-03 | 3.61E-03 |
| 30Si    |        |        |        |        |        |     |     | 1.74E-03 | 6.35E-04 | 8.05E-03 | 4.02E-03 |
| 31P     |        |        |        |        |        |     |     | 4.18E-04 | 1.80E-04 | 1.21E-03 | 1.14E-03 |
| 32S     |        |        |        |        |        |     |     | 8.41E-02 | 1.65E-01 | 3.84E-02 | 5.34E-02 |
| 33S     |        |        |        |        |        |     |     | 4.50E-04 | 2.49E-04 | 1.78E-04 | 3.18E-04 |
| 34S     |        |        |        |        |        |     |     | 1.90E-03 | 2.50E-03 | 2.62E-03 | 4.33E-03 |
| 35Cl    |        |        |        |        |        |     |     | 1.34E-04 | 9.83E-05 | 1.01E-04 | 3.80E-04 |
| 37CL    |        |        |        |        |        |     |     | 3.98E-05 | 3.36E-05 | 1.88E-05 | 7.59E-05 |
| 36Ar    |        |        |        |        |        |     |     | 1.49E-02 | 3.35E-02 | 6.62E-03 | 9.05E-03 |
| 38Ar    |        |        |        |        |        |     |     | 1.06E-03 | 1.45E-03 | 1.37E-03 | 2.22E-03 |
| 39K     |        |        |        |        |        |     |     | 8.52E-05 | 9.00E-05 | 6.23E-05 | 2.38E-04 |
| 41K     |        |        |        |        |        |     |     | 7.44E-06 | 7.12E-06 | 5.07E-06 | 5.31E-06 |
| 40Ca    |        |        |        |        |        |     |     | 1.23E-02 | 3.45E-02 | 5.77E-03 | 6.53E-03 |
| 44Ca    |        |        |        |        |        |     |     | 8.86E-06 | 2.07E-05 | 5.53E-05 | 2.49E-05 |
| 46Ti    |        |        |        |        |        |     |     | 1.71E-05 | 1.76E-05 | 7.48E-06 | 2.60E-05 |
| 47Ti    |        |        |        |        |        |     |     | 6.04E-07 | 1.24E-06 | 2.11E-06 | 4.94E-06 |
| 48Ti    |        |        |        |        |        |     |     | 2.03E-04 | 8.53E-04 | 1.16E-04 | 3.17E-05 |
| 49Ti    |        |        |        |        |        |     |     | 1.69E-05 | 6.71E-05 | 5.98E-06 | 4.10E-06 |
| 50Ti    |        |        |        |        |        |     |     | 1.26E-05 | 4.60E-04 | 3.81E-10 | 6.39E-06 |
| 50Cr    |        |        |        |        |        |     |     | 2.71E-04 | 5.24E-04 | 4.64E-05 | 8.16E-05 |
| 52Cr    |        |        |        |        |        |     |     | 5.15E-03 | 2.01E-02 | 1.15E-03 | 2.32E-04 |
| 53Cr    |        |        |        |        |        |     |     | 7.85E-04 | 2.26E-03 | 1.19E-04 | 2.52E-05 |
| 54Cr    |        |        |        |        |        |     |     | 1.90E-04 | 2.03E-03 | 2.33E-08 | 1.54E-05 |
| 55Mn    |        |        |        |        |        |     |     | 8.23E-03 | 1.88E-02 | 3.86E-04 | 1.94E-04 |
| 54Fe    |        |        |        |        |        |     |     | 1.04E-01 | 7.08E-02 | 3.62E-03 | 6.52E-03 |
| 56Fe    |        |        |        |        |        |     |     | 6.13E-01 | 6.15E-01 | 8.44E-02 | 1.66E-02 |
| 57Fe    |        |        |        |        |        |     |     | 2.55E-02 | 1.39E-02 | 2.72E-03 | 5.90E-04 |
| 58Co    |        |        |        |        |        |     |     | 1.02E-03 | 8.06E-04 | 7.27E-05 | 1.43E-04 |
| 58Ni    |        |        |        |        |        |     |     | 1.28E-01 | 3.34E-02 | 3.63E-03 | 7.93E-03 |
| 60Ni    |        |        |        |        |        |     |     | 1.05E-02 | 4.15E-03 | 1.75E-03 | 9.18E-04 |
| 62Ni    |        |        |        |        |        |     |     | 2.66E-03 | 1.36E-03 | 5.09E-04 | 2.94E-04 |
| 63Cu    |        |        |        |        |        |     |     | 1.79E-06 | 4.25E-05 | 8.37E-07 | 2.61E-05 |
| 65Cu    |        |        |        |        |        |     |     | 6.83E-07 | 1.59E-05 | 4.07E-07 | 7.47E-05 |
| 64Zn    |        |        |        |        |        |     |     | 1.22E-05 | 6.97E-07 | 1.03E-05 | 3.93E-05 |
| 66Zn    |        |        |        |        |        |     |     | 2.12E-05 | 3.18E-05 | 8.61E-06 | 1.23E-04 |

○ Nuclei used in the analysis of $g(r)$. 
Table 2. $M_{II}/M_{Ia}$ for each model.

| Case        | W7+Ha95 | W7+WW95 | WDD2+Ha95 | WDD2+WW95 |
|-------------|---------|---------|-----------|-----------|
| 46 nuclei   | 1.85    | 1.23    | 1.79      | 1.19      |
| 31 nuclei   | 1.86    | 1.24    | 1.80      | 1.20      |
| 6 nuclei    | 2.33    | 1.53    | 2.07      | 1.36      |
| 20 elements | 1.85    | 1.23    | 1.79      | 1.19      |
| 14 elements | 1.86    | 1.24    | 1.80      | 1.20      |

Table 3. The most proper value of $r_p$.

| Case        | W7+Ha95       | W7+WW95       | WDD2+Ha95     | WDD2+WW95     |
|-------------|---------------|---------------|---------------|---------------|
| 46 nuclei   | (2.8E-01)     | 1.1E-01       | (1.4E-01)     | 6.1E-02       |
| 31 nuclei   | 9.2E-02       | 9.2E-02       | 5.0E-02       | 5.4E-02       |
| 6 nuclei    | 8.5E-02       | 1.0E-01       | 7.6E-02       | 9.2E-02       |
| 20 elements | (2.9E-01)     | 1.1E-01       | (1.8E-01)     | 7.7E-02       |
| 14 elements | 8.1E-02       | 9.6E-02       | 6.4E-02       | 6.3E-02       |

$N_{Ia}/N_{II}$: 1.3E-01–(6.1E-01)  1.0E-01–1.4E-01  7.7E-02–(3.3E-01)  5.6E-02–1.1E-01

Parenthesis represents that $g(r)$ is insensitive to $r$. In that case, $r_p$ is hard to be determined.
Table 4. The most proper value of $r_p/g(r_p)$ for SMC and LMC.

| Object | W7+Ha95       | W7+WW95     | WDD2+Ha95 | WDD2+WW95 |
|--------|---------------|-------------|-----------|-----------|
| LMC    | 1.6E-01/5.2E-02 | 1.6E-01/6.5E-02 | 1.1E-01/3.6E-02 | 9.6E-02/4.7E-02 |
| SMC    | 1.9E-01/4.7E-02 | 1.9E-01/4.1E-02 | 1.5E-01/3.7E-02 | 1.2E-01/2.9E-02 |
Figure Captions

Fig. 1. Comparison of chemical compositions. The open circles represent WDD2/W7 and the dots denote WW95/Ha95.

Fig. 2. $g(r)$ for each model. The thick-solid, long-dashed, short-long-dashed, short-dashed, and dot lines correspond to Case 1, 2, 3, 4, and 5, respectively.

Fig. 3. Comparison with solar values. The open circles represent Ha95+SNe Ia and the dots denote WW95+SNe Ia. Upper: W7 is used for SNe Ia. Lower: WDD2 is used.
Fig. 2.

Fig. 3.