Two-Dimensional Electron-Hole Systems in a Strong Magnetic Field: Composite Fermion Picture for Multi-Component Plasmas

Arkadiusz Wójs\textsuperscript{1,2}, Izabela Szlufarska\textsuperscript{1,2}, Kyung-Soo Yi\textsuperscript{1,3}, and John J. Quinn\textsuperscript{1}
\textsuperscript{1}Department of Theoretical Physics, University of Tennessee, Knoxville, Tennessee 37996, USA
\textsuperscript{2}Institute of Physics, Wroclaw University of Technology, Wroclaw 50-370, Poland
\textsuperscript{3}Physics Department, Pusan National University, Pusan 609-735, Korea

Electron-hole systems on a Haldane sphere are studied by exact numerical diagonalization. Low lying states contain one or more types of bound charged excitonic complexes $X_k^\pm$, interacting through appropriate pseudopotentials. Incompressible ground states of such multi-component plasmas are found. A generalized multi-component Laughlin wavefunction and composite Fermion picture are shown to predict the low lying states of an electron-hole gas at any value of the magnetic field.

71.10.Pm, 73.20.Dx, 73.40.Hm, 71.35.Ji

**Introduction.** Recently there has been considerable interest in two dimensional systems containing both electrons and holes in the presence of a strong magnetic field.\cite{1} In such systems, neutral ($X^0$) and charged excitons ($X^-$) and larger exciton complexes ($X_k^\pm$), k neutral $X^0$’s bound to an electron) can occur. The excitonic ions $X_k^\pm$ are long-lived Fermions whose energy spectra contain Landau level structure.\cite{2} In this paper we investigate by exact numerical diagonalization small systems containing $N_e$ electrons and $N_h$ holes ($N_e \geq N_h$), confined to the surface of a Haldane sphere. For $N_h = 1$ these systems serve as simple guides to understanding photoluminescence.\cite{3} For larger values of $N_h$ it is possible to form a multi-component plasma containing electrons and $X_k^\pm$ complexes.\cite{1} We propose a model\cite{4,5} for determining the incompressible quantum fluid states of such plasmas, and confirm the validity of the model by numerical calculations. In addition, we introduce a new generalized composite Fermion (CF) picture\cite{6,7,8} for the multi-component plasma and use it to predict the low lying bands of angular momentum multiplets for any value of the magnetic field.

**Bound States.** In a sufficiently strong magnetic field, the only bound electron-hole complexes are the neutral exciton $X^0$ and the spin-polarized charged exciton $X_k^\pm$ (electron $e^- \equiv X_0^+$, charged exciton $X^- \equiv X_1^-$, charged biexciton $X_{2}^-$, etc.).\cite{9,10,11} All other complexes found at weaker magnetic fields (e.g. spin-singlet charged excitons or spin-singlet biexciton) unbind.\cite{12} The angular momenta of complexes $X^0$ and $X_k^\pm$ on a Haldane sphere with monopole strength $2S$ are $l_{X^0} = 0$ and $l_{X_k^\pm} = |S| - k$.\cite{13} The binding energies of an exciton, $\varepsilon_0 = -E_{X^0}$, and of excitonic ions, $\varepsilon_k = E_{X_{k-1}^+} + E_{X_k^-} - E_{X_k^0}$ ($E_A$ is the energy of complex $A$) are listed in Tab. 1 for several different values of $2S$.\cite{14} It is apparent that $\varepsilon_0 > \varepsilon_1 > \varepsilon_2 > \varepsilon_3$. Depending on the ratio $N_e : N_h$, we expect to find different combinations of complexes that have the largest total binding energy. When $N_e = N_h$ we expect $N_h$ neutral excitons $X^0$ to form. When $N_e \geq 2N_h$ the low lying states will contain $N_h$ charged excitons $X^-_k$ and $N_e - 2N_h$ free electrons $e^-$. For $N_h < N_e < 2N_h$ we expect to find larger charged exciton complexes.

**Pseudopotentials.** Whether the states with largest binding energy form the lowest energy band of the electron-hole system depends on the interaction between charged complexes $X_k^\pm$. The interaction of a pair of charged particles $A$ and $B$ of angular momentum $l_A$ and $l_B$ can be described by a pseudopotential $V_{AB}(L)$ where $L = l_A + l_B$ is the total pair angular momentum.\cite{15} It is convenient to plot pseudopotentials as a function of the relative angular momentum $R = l_A + l_B - L$.\cite{16,17} Fig. 1 shows $V_{AB}(R)$ for the pairs $e^- e^-$, $e^- X^-$, $X^- X^-$, and $e^- X_{2}^-$, at the monopole strength $2S = 17$. Roughly, the pseudopotential parameters $V_{AB}(R)$ calculated for different pairs $AB$ and for a given $2S$ lie on the same curve. Small differences between energies $V_{AB}(R)$ calculated for different pairs at the same $R$ are due to different values of $l_A$ and $l_B$ and to the finite size and polarization of composite particles. Only the latter effect, important at small $R$, persists for $2S \to \infty$, i.e. in the planar geometry.

The major and critical difference between four plotted pseudopotentials lies in the allowed values of $R$. If all $A$ and $B$ were point charges, the allowed pair angular momenta for two identical Fermions ($A = B$) would be $L = 2l_A - j$, where $j$ is an odd integer, i.e. $R = 1, 3, \ldots$ and $R \leq 2l_A$. For two distinguishable particles ($A \neq B$), the values of $L$ would satisfy $|l_A - l_B| \leq L \leq l_A + l_B$, i.e. $R = 0, 1, 2, \ldots$ and $R \leq 2\min(l_A, l_B)$. However, if $A$ or $B$ is a composite particle, one or more pair states with largest $L$ (smallest $R$) are forbidden, and the corresponding pseudopotential parameters are effectively infinite ($AB$ repulsion has a hard core). For $A = X_{kA}^+$ and $B = X_{kB}^-$, the smallest allowed $R$ can be

| $2S$ | $\varepsilon_0$ | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_3$ |
|------|----------------|----------------|----------------|----------------|
| 10   | 1.3295043      | 0.0728337      | 0.0411069      | 0.0252268      |
| 15   | 1.3045679      | 0.0671086      | 0.0395282      | 0.0262927      |
| 20   | 1.2919313      | 0.0647886      | 0.0381324      | 0.0260328      |
deduced from the mappings between the electron-hole and two-spin systems,

$$R_{\text{min}}^{AB} = 2 \min(k_A, k_B) + 1. \quad (1)$$

Thus, in Fig. 1, \(R_{e-X^-} \geq 1\), \(R_{X^-X^-} \geq 3\), etc.

Low lying states of a system of \(N_e\) electrons and \(N_h\) holes can contain a number of charged complexes \(X_k^+\) (\(X^-\) and possibly larger ones) interacting with one another and with electrons through appropriate pseudopotentials. It has been shown that the Laughlin \(\nu = 1/m\) state occurs in the gas of (identical) Fermions if the pseudopotential increases faster than linearly as a function of \(L(L+1)\) in the vicinity of \(R = m\). As seen in the inset in Fig. 1, this is true for both \(V_{e-e^-}\) and \(V_{X^-X^-}\), and also (at even values of \(R\)) for \(V_{e-X^-}\) and \(V_{e-X^-}\). In Ref. 3 we found Laughlin states of one-component \(X^-\) gas formed at \(N_e = 2N_h\). In the present note we concentrate on a more general situation, where more than one kind of charged particles occur in an electron-hole system, and find incompressible fluid states of such multi-component plasma.

**Numerical Results.** As an illustration, we present first the results of exact diagonalization performed for the system with \(N_e = 8\) and \(N_h = 2\). We expect low lying bands of states containing the following combinations of complexes: (i) \(4e^- + 2X^-\), (ii) \(5e^- + X^-X^-\), (iii) \(5e^- + X^- + X^0\), and (iv) \(6e^- + 2X^0\). All groupings (i)–(iv) contain an equal number \(N = N_e - N_h\) of singly charged complexes, however, both the angular momenta of involved complexes and the relevant hard cores are different. The total binding energies are: \(\varepsilon_i = 2\varepsilon_0 + 2\varepsilon_1\), \(\varepsilon_{ii} = 2\varepsilon_0 + \varepsilon_1 + \varepsilon_2\), \(\varepsilon_{iii} = 2\varepsilon_0 + \varepsilon_1\), and \(\varepsilon_{iv} = 2\varepsilon_0\). Clearly, \(\varepsilon_i > \varepsilon_{ii} > \varepsilon_{iii} > \varepsilon_{iv}\).

However, which of the groupings contains the (possibly incompressible) ground state depends upon not only the total binding energy, but the interactions between all the charged particles which depends on \(2S\).

In Fig. 2, we show the low energy spectra of the \(8e + 2h\) system at \(2S = 9\) (a), \(2S = 13\) (c), and \(2S = 14\) (e). Filled circles mark the non-multiplicative states, and the open circles and squares mark the multiplicative states with one and two decoupled excitons, respectively. In frames (b), (d) and (f) we plot the low energy spectra of different charge complexes interacting through appropriate pseudopotentials (see Fig. 1), corresponding to four possible groupings (i)–(iv). By comparing left and right frames, we can identify low lying states of type (i)–(iv) in the electron-hole spectra.

In general, energies calculated from pseudopotentials \(V_{AB}\) in Fig. 2 underestimate energies of the corresponding electron-hole system if \(N\) and \(2S\) are large. This can be partially understood in terms of polarization effects.
in the two-particle pseudopotentials. For a particular grouping and value of $2S$, it is possible to calculate pseudopotentials that give a very good fit to the electron-hole spectrum. The “correct” pseudopotentials for the $8e+2h$ system are close to those of a pair of point charges with appropriate angular momenta $l_A$ and $l_B$, except for the hard cores.

It is unlikely that a system containing a large number of different species (e.g., $e^-$, $X^-$, $X_2^-$, etc.) will form the absolute ground state of the electron-hole system. However, different charge configurations can form low lying excited bands. An interesting example is the $12e+6h$ system at $2S^\prime = 17$. The 6$X^-$ grouping (v) has the maximum total binding energy $\varepsilon_v = 6\varepsilon_0 + 6\varepsilon_1$. Other expected low lying bands correspond to the following groupings: (vi) $e^- + 5X^- + X^0$ with $\varepsilon_v = 6\varepsilon_0 + 5\varepsilon_1$ and (vii) $e^- + 4X^- + X_2^-$ with $\varepsilon_v = 6\varepsilon_0 + 5\varepsilon_1 + \varepsilon_2$.

Although we are unable to perform an exact diagonalization for the $12e+6h$ system in terms individual electrons and holes, we can use appropriate pseudopotentials and binding energies of groupings (v)–(vii) to obtain the low lying states in the spectrum. The results are presented in Fig. 3. There is only one 6$X^-$ state (the $L = 0$ Laughlin state of the electron-hole system) and two bands of states in each of groupings (vi) and (vii). A gap of 0.0626 $e^2/\lambda$ separates the $L = 0$ ground state from the lowest excited state.

**Generalized Laughlin Wavefunction.** It is known that if the pseudopotential $V(R)$ decreases quickly with increasing $R$, the low lying multipoles avoid (strongly repulsive) pair states with one or more of the smallest values of $R$. For the (one-component) electron gas on a plane, avoiding pair states with $R < m$ is achieved with the factor $\prod_{i<j} (x_i - x_j)^m$ in the Laughlin $\nu = 1/m$ wavefunction. For a system containing a number of distinguishable types of Fermions interacting through Coulomb-like pseudopotentials, the appropriate generalization of the Laughlin wavefunction will contain a factor $\prod_{i<j} (x_i^{(a)} - x_j^{(b)})^{m_{ab}}$, where $x_i^{(a)}$ is the complex coordinate for the position of $i$th particle of type $a$, and the product is taken over all pairs. For each type of particle one power of $(x_i^{(a)} - x_j^{(b)})$ results from the antisymmetrization required for indistinguishable Fermions and the other factors describe Jastrow type correlations between the interacting particles. Such a wavefunction guarantees that $R_{ab} \geq m_{ab}$, for all pairings of various types of particles, thereby avoiding large pair repulsion Fermi statistics of particles of each type requires that all $m_{aa}$ are odd, and the hard cores defined by Eq. (3) require that $m_{ab} \geq R_{ab}^{\min}$ for all pairs.

**Generalized Composite Fermion Picture.** In order to understand the numerical results obtained in the spherical geometry (Figs. 2 and 3), it is useful to introduce a generalized CF picture by attaching to each particle fictitious flux tubes carrying an integer number of flux quanta $\phi_0$. In the multi-component system, each $a$-particle carries flux $(m_{aa} - 1)\phi_0$ that couples only to charges on all other $a$-particles and fluxes $m_{ab}\phi_0$ that couple only to charges on all $b$-particles, where $a$ and $b$ are any of the types of Fermions. The effective monopole strength seen by a CF of type $a$ (CF-$a$) is

$$2S^*_a = 2S - \sum_b (m_{ab} - \delta_{ab})(N_b - \delta_{ab})$$  \hspace{1cm} (2)

For different multi-component systems we expect generalized Laughlin incompressible states (for two components denoted as $[m_{AA}, m_{BB}, m_{AB}]$) when all the hard core pseudopotentials are avoided and CF’s of each kind fill completely an integer number of their CF shells (e.g. $N_a = 2l_a^* + 1$ for the lowest shell). In other cases, the low lying multipoles are expected to contain different kinds of quasiparticles (QP-$A$, QP-$B$, . . . ) or quasihole (QH-$A$, QH-$B$, . . . ) in the neighboring incompressible state.

Our multi-component CF picture can be applied to the system of excitonic ions, where the CF angular momenta are given by $l_{X^-} = |S_{X^-}| - k$. As an example, let us first analyze the low lying $8e+2h$ states in Fig. 2. At $2S = 9$, for $m_{e-e^-} = m_{e^-X^-} = 3$ and $m_{e^-X^-} = 1$ we predict the following low lying multipoles in each grouping: (i) $2S^*_e = 1$ and $2S^*_X^- = 3$ gives $l_{X^-}^* = l_{X^-}^* = 1/2$. Two CF-$X^-$’s fill their lowest shell ($L_{X^-} = 0$) and we have two QP-$e^-$’s in their first excited shell, each with angular momentum $l_{e^-}^* + 1 = 3/2$ ($L_{e^-} = 0$ and 2). Addition of $L_{e^-}$ and $L_{X^-}$ gives total angular momenta $L = 0$ and 2. We interpret these states as those of two QP-$e^-$’s in the incompressible [331] state. Similarly, for other groupings we obtain: (ii) $L = 2$; (iii) $L = 1, 2,$ and 3; and (iv) $L = 0$ ($\nu = 2/3$ state of six electrons).

At $2S = 13$ and 14 we set $m_{e^-e^-} = m_{e^-X^-} = 3$ and $m_{e^-X^-} = 2$ and obtain the following predictions. First, at $2S = 13$: (i) The ground state is the incompressible [332] state at $L = 0$; the first excited band should therefore contain states with one QP-QH pair of either kind. For the $e^-$ excitations, the QP-$e^-$ and QH-$e^-$ angular momenta are $l_{e^-}^* = 3/2$ and $l_{e^-}^* + 1 = 5/2$, respectively, and the allowed pair states have $L_{e^-} = 1, 2, 3,$ and 4. However, the $L = 1$ state has to be discarded, as it is known to have high energy in the one-component (four}
electron) spectrum. For the $X^-$ excitations, we have $t_{X^-}^* = 1/2$ and pair states can have $L_{X^-} = 1$ or 2. The first excited band is therefore expected to contain multiplets at $L = 1$, $2^2$, 3, and 4. The low lying multiplets for other groupings are expected at: (ii) $L = 2$ and 3; (iii) $2S_{X^-}^* = 3$ gives no bound $X_2^-$ state; setting $m_{e^-X^-} = 1$ we obtain $L = 2$; and (iv) $L = 0$, 2, and 4. Finally, at $2S = 14$ we obtain: (i) $L = 1$, 2, and 3; (ii) incompressible $3^+$ state at $L = 0$ ($m_{e^-X^-}$ is irrelevant for one $X^-$) and the first excited band at $L = 1$, 2, 3, 4, and 5; (iii) $L = 1$; and (iv) $L = 3$.

For the 12e + 6h spectrum in Fig. 3 the following CF predictions are obtained: (v) For $m_{X^-X^-} = 3$ we obtain the Laughlin $\nu = 1/3$ state with $L = 0$. Because of the hard core of $V_{X^-X^-}$, this is the only state of this grouping. (vi) We set $m_{e^-X^-} = 3$ and $e^-X^- = 1, 2, 3$. For $m_{e^-X^-} = 1$ we obtain $L = 1, 2, 3^2, 4^2, 5^3, 6^3, 7^3, 8^2, 9^2, 10, 11$. For $m_{e^-X^-} = 2$ we obtain $L = 1, 2, 3, 4, 5$, and 6. For $m_{e^-X^-} = 3$ we obtain $L = 1$. (vii) We set $m_{X^-X^-} = 3$, $m_{e^-X^-} = 1$, $m_{X^-X^-} = 3$, and $m_{e^-X^-} = 1, 2, 3$. For $m_{e^-X^-} = 1$ we obtain $L = 2, 3, 4^2, 5^2, 6^3, 7^2, 8^2, 9, 10$. For $m_{e^-X^-} = 2$ we obtain $L = 2, 3, 4, 5$, and 6. For $m_{e^-X^-} = 3$ we obtain $L = 2$. In groupings (vi) and (vii), the sets of multiplets obtained for higher values of $m_{e^-X^-}$ are subsets of the sets obtained for lower values, and we would expect them to form lower energy bands since they avoid additional small values of $R_{e^-X^-}$. However, note that the (vi) and (vii) states predicted for $m_{e^-X^-} = 3$ (at $L = 1$ and 2, respectively) do not form separate bands in Fig. 3. This is because the $V_{X^-X^-}$ pseudopotential increases more slowly than linearly as a function of $L(L + 1)$ in the vicinity of $R_{e^-X^-} = 3$ (see Fig. 3). In such case the CF picture fails.

The agreement of our CF predictions with the data in Figs. 2 and 3 (marked with lines) is really quite remarkable and strongly indicates that our multi-component CF picture is correct. We were indeed able to confirm predicted Jastrow type correlations in the low lying states by calculating their coefficients of fractional parentage. We have also verified the CF predictions for other systems that we were able to treat numerically. If exponents $m_{ab}$ are chosen correctly, the CF picture works well in all cases.

Summary. Charged excitons and excitonic complexes play an important role in determining the low energy spectra of electron-hole systems in a strong magnetic field. We have introduced general Laughlin type correlations into the wavefunctions, and proposed a generalized CF picture to elucidate the angular momentum multiplets forming the lowest energy bands for different charge configurations occurring in the electron-hole system. We have found Laughlin incompressible fluid states of multi-component plasmas at particular values of the magnetic field, and the lowest bands of multiplets for various charge configurations at any value of the magnetic field. It is noteworthy that the fictitious Chern-Simons fluxes and charges of different types or colors are needed in the generalized CF model. This strongly suggests that the effective magnetic field seen by the CF’s does not physically exist and that the CF picture should be regarded as a mathematical convenience rather than physical reality. Our model also suggests an explanation of some perplexing observations found in photoluminescence, but this topic will be addressed in a separate publication.

We thank P. Hawrylak and M. Potenski for helpful discussions. AW and JJQ acknowledge partial support from the Materials Research Program of Basic Energy Sciences, US Department of Energy. KSY acknowledges support from the Korea Research Foundation (Project No. 1998-001-D00305).

1. K. Kheng, R. T. Cox, Y. Merle d’Aubigne, F. Bassani, K. Saminadayar, and S. Tatarenko, Phys. Rev. Lett. 71, 1752 (1993); H. Buhmann, L. Mansouri, J. Wang, P. H. Beton, N. Mori, M. Heinzi, and M. Potenski, Phys. Rev. B 51, 7969 (1995).
2. A. J. Shields, M. Pepper, M. Y. Simmons, and D. A. Ritchie, Phys. Rev. B 52, 7841 (1995); G. Finkelstein, H. Shtrikman, and I. Bar-Joseph, Phys. Rev. B 53 1709 (1996).
3. X. M. Chen and J. J. Quinn, Phys. Rev. B 50, 2354 (1994); ibid. 51, 5578 (1995).
4. A. Wójcik and P. Hawrylak, Phys. Rev. B 51 10 880, (1995).
5. E. I. Rashba and M. E. Portnoi, Phys. Rev. Lett. 70, 3315 (1993); V. M. Apalkov, F. G. Pikus, and E. I. Rashba, Phys. Rev. B 52, 6111 (1995).
6. J. J. Palacios, D. Yoshioka, and A. H. MacDonald, Phys. Rev. B 54, 2296 (1996).
7. A. Wójcik, P. Hawrylak, and J. J. Quinn, Physica B 258–258A, 490 (1998); Phys. Rev. Lett. (submitted, cond-mat/9810082).
8. I. V. Lerner and Yu. E. Lozovik, Sov. Phys. JETP 53, 763 (1981); A. H. MacDonald and E. H. Rezayi, Phys. Rev. B 42, 3224 (1990).
9. F. D. M. Haldane, Phys. Rev. Lett. 51, 605 (1983); T. T. Wu and C. N. Yang, Nucl. Phys. B 107, 365 (1976).
10. B. I. Halperin, Helv. Phys. Acta 56, 75 (1983).
11. R. Laughlin, Phys. Rev. Lett. 50, 1395 (1983).
12. J. Jain, Phys. Rev. Lett. 63, 199 (1989).
13. F. D. M. Haldane and E. H. Rezayi, Phys. Rev. Lett. 60, 956 (1988).
14. A. Wójcik and J. J. Quinn, Solid State Commun. 108, 493 (1998); ibid. 110, 45 (1999); Phys. Rev. B (submitted, cond-mat/9903143).
15. A. Wójcik and J. J. Quinn, Physica E 3, 181 (1998).
16. P. Sikto, S. N. Yi, K.-S. Yi, and J. J. Quinn, Phys. Rev. Lett. 76, 3396 (1996).
17. A. de Shalit and I. Talmi, Nuclear Shell Theory, Academic Press, New York and London 1963.