The nonminimal scalar multiplet: duality, sigma-model, beta-function

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ABSTRACT

We compute in superspace the one-loop beta-function for the nonlinear sigma-model defined in terms of the nonminimal scalar multiplet. The recently proposed quantization of this complex linear superfield, viewed as the field strength of an unconstrained gauge spinor superfield, allows to handle efficiently the infinite tower of ghosts via the Batalin-Vilkovisky formalism. We find that the classical duality of the nonminimal scalar and chiral multiplets is maintained at the quantum one-loop level.

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1 Introduction

It is well known that an alternative description of the scalar multiplet is provided by the complex linear superfield \([1]\). The equivalence of the two formulations can be exhibited by means of a duality transformation which relates the two multiplets to each other. Starting with the scalar multiplet \(\Phi\), with \(\bar{D}^\alpha \Phi = 0\), one can write a first order action

\[
S = -\int d^4 x \, d^4 \theta \left[ \Sigma \Sigma + \Sigma \Phi + \bar{\Sigma} \bar{\Phi} \right] \tag{1.1}
\]

where \(\Sigma\) is an auxiliary superfield. Using the equations of motion to eliminate the superfields \(\Sigma, \bar{\Sigma}\), one obtains the usual chiral superfield action. Eliminating instead the superfields \(\Phi, \bar{\Phi}\), whose equations of motion impose the linearity constraint \(\bar{D}^2 \Sigma = D^2 \bar{\Sigma} = 0\), leads to the linear superfield action. In the same manner one can start with the linear multiplet \(\Sigma\) satisfying the constraint \(\bar{D}^2 \Sigma = 0\), and write the first order action

\[
S = \int d^4 x \, d^4 \theta \left[ \Phi \Phi + \Sigma \Phi + \bar{\Sigma} \bar{\Phi} \right] \tag{1.2}
\]

where now \(\Phi\) has to be thought of as auxiliary and unconstrained. We note that the duality transformation exchanges the field equations with the constraints, as standard electric-magnetic duality leads to the exchange of the Maxwell equations with the Bianchi identities.

More generally, in terms of these multiplets, it is possible to formulate supersymmetric \(\sigma\)-models. We consider superfields \(\Phi^\mu, \bar{\Phi}^{\bar{\mu}}, \Sigma^\mu, \bar{\Sigma}^{\bar{\mu}}\) with \(\mu, \bar{\mu} = 1, \ldots, n\) and write the first order action

\[
S = \int d^4 x \, d^4 \theta \left[ K(\Phi, \bar{\Phi}) + \Sigma \Phi + \bar{\Sigma} \bar{\Phi} \right] \tag{1.3}
\]

where \(\Sigma\) are linear superfields, \(\bar{D}^2 \Sigma = 0\), and \(\Phi\) initially unconstrained (we do not indicate the indices \(\mu, \bar{\mu}\) on the superfields in order to simplify the notation). Varying with respect to \(\Sigma\) one obtains the chirality constraint on \(\Phi\), so that the quadratic terms, being total derivatives, can be dropped and one is left with the standard \(\sigma\)-model action for chiral superfields. On the other hand the dual model is obtained once the variation with respect to \(\Phi\) is considered: the equations

\[
\Sigma = -\frac{\partial K}{\partial \Phi}, \quad \bar{\Sigma} = -\frac{\partial K}{\partial \bar{\Phi}} \tag{1.4}
\]

have to be solved, \(\Phi = \Phi(\Sigma, \bar{\Sigma}), \bar{\Phi} = \bar{\Phi}(\Sigma, \bar{\Sigma})\), so that the dual action becomes

\[
S = \int d^4 x \, d^4 \theta \, \tilde{K}(\Sigma, \bar{\Sigma}) \tag{1.5}
\]
where $\tilde{K}$ is the Legendre transform of $K$

$$\tilde{K}(\Sigma, \bar{\Sigma}) = [K(\Phi, \bar{\Phi}) + \Sigma \Phi + \bar{\Sigma} \bar{\Phi}]|_{\Phi = \Phi(\Sigma, \bar{\Sigma}), \bar{\Phi} = \bar{\Phi}(\Sigma, \bar{\Sigma})}$$  \hspace{1cm} (1.6)

Thus the duality transformations are implemented exchanging the potentials

$$K(\Phi, \bar{\Phi}) \rightarrow \tilde{K}(\Sigma, \bar{\Sigma})$$  \hspace{1cm} (1.7)

At the level of the matrix given by the second derivatives of the potential, i.e. defining

$$G = \begin{pmatrix} \frac{\partial^2 K}{\partial \Phi \partial \bar{\Phi}} & \frac{\partial^2 K}{\partial \Phi \partial \bar{\Phi}} \\ \frac{\partial^2 K}{\partial \bar{\Phi} \partial \Phi} & \frac{\partial^2 K}{\partial \bar{\Phi} \partial \Phi} \end{pmatrix}$$  \hspace{1cm} (1.8)

and similarly for $\tilde{G}(\Sigma, \bar{\Sigma})$, it is easy to show that

$$G(\Phi, \bar{\Phi}) \rightarrow \tilde{G}(\Sigma, \bar{\Sigma}) = -G(\Phi, \bar{\Phi})^{-1}$$  \hspace{1cm} (1.9)

(These transformations generalize the ones valid for bosonic $\sigma$-models with an isometry [2].)

The duality properties of these theories are well understood in a superfield formulation. On the other hand the situation is not so clear in terms of component fields. The two multiplets have the same content of physical degrees of freedom, but they differ in their auxiliary field structure and it appears that the elimination of these auxiliary fields via their equations of motion might lead to $\sigma$-models quite different in the two cases [3]. Therefore in attempting to understand duality issues at the quantum level it seems safer to stick to a superspace approach. While for the chiral superfield the quantization is not an issue, a direct quantum formulation of the complex linear superfield is not known. Indeed we are able to perform functional integration and differentiation in chiral superspace [1], but we do not have a corresponding setup in the linear case. One way out is to solve the linearity constraint $\bar{D}^2 \Sigma = 0$ in terms of unconstrained gauge superfields $\sigma^\alpha, \bar{\sigma}^{\dot{\alpha}}$ whose quantization however leads to an infinite tower of ghosts. In a recent paper [4] a complete solution of the problem has been obtained by using the Batalin-Vilkovisky [5] approach to gauge fix the infinite sequence of invariances. Although conceptually straightforward the method looks complicated and any practical application difficult to envisage. One aim of the present paper is to prove that some of these difficulties are not real obstacles: we adopt the recently proposed quantization procedure and investigate the quantum duality properties of the complex linear $\sigma$-model. Since these theories are
not renormalizable in four dimensions, in the following we restrict our attention to the
two-dimensional situation.

At the classical level the chiral and the linear $\sigma$-models defined on dual backgrounds
represent different parametrizations of the same theory. The manipulations that bring
one theory into the other are essentially based on functional integrations performed in a
different order. Consequently in order to address quantum issues, the obvious question
to ask is about quantum duality of the properly regularized theories. The simplest object
which directly relates to the renormalization properties of a given model, is the $\beta$-function.
We compute the one-loop $\beta$-function for the nonlinear $\sigma$-model in terms of the complex
linear superfields and compare it with the well known corresponding result [6, 7] for the
$\sigma$-model in terms of chiral superfields.

In the next section we give a concise description of the quantum-background field
approach, best suited for perturbative calculations. In section 3 we show how to treat
the infinite tower of fields that enter in the quantization of the linear superfield and how
to obtain an effective propagator to be used in section 4, where the one-loop $\beta$-function
is explicitly computed. We conclude with some comments and problems open to future
investigations. Few useful identities are collected in an appendix.

2 The background field method

First we briefly review the situation for the nonlinear $\sigma$-model defined in terms of chiral
superfields [7]. The action can be written as

$$S = \int d^2 x \ d^4 \theta \ K(\Phi, \bar{\Phi})$$

(2.1)

where $\Phi^\mu$, $\bar{\Phi}^\bar{\mu}$ are interpreted as complex coordinates of a Kähler manifold. In order
to perform perturbative calculations one shifts the superfields with a linear quantum-
background splitting

$$\Phi \rightarrow \Phi + \Phi_0 \ , \ \bar{\Phi} \rightarrow \bar{\Phi} + \bar{\Phi}_0$$

(2.2)

and expand the action around the background $\Phi_0$, $\bar{\Phi}_0$. One separates the free kinetic
action of the quantum fields from the interaction vertices

$$S = \int d^2 x \ d^4 \theta \ \left[ \Phi^\mu \bar{\Phi}^\bar{\rho} \delta_{\mu \bar{\rho}} + [K_{\mu \bar{\rho}}(\Phi_0, \bar{\Phi}_0) - \delta_{\mu \bar{\rho}}]\Phi^\mu \bar{\Phi}^\bar{\rho}
+ \frac{1}{2} K_{\mu \bar{\rho}}(\Phi_0, \bar{\Phi}_0) \Phi^\mu \Phi^\nu + \frac{1}{2} K_{\bar{\mu} \bar{\rho}}(\Phi_0, \bar{\Phi}_0) \bar{\Phi}^{\bar{\mu}} \bar{\Phi}^{\bar{\rho}} + \ldots \right]$$

(2.3)
where

\[ K_{\mu_1\ldots\mu_n\bar{\nu}_1\ldots\bar{\nu}_m} \equiv \frac{\partial}{\partial \Phi^{\mu_1}} \ldots \frac{\partial}{\partial \Phi^{\mu_n}} \ldots \frac{\partial}{\partial \bar{\Phi}^{\bar{\nu}_1}} \ldots \frac{\partial}{\partial \bar{\Phi}^{\bar{\nu}_m}} K(\Phi, \bar{\Phi}) \]  

(2.4)

Quantum calculations can be performed using superspace Feynman diagrams and conventional D-algebra techniques. The quadratic action in (2.3) is sufficient for one-loop computations. Using dimensional regularization in \(2 - 2\epsilon\) dimensions, the one-loop divergent contribution to the Kähler potential is given by

\[ K^{(1)} = \frac{1}{\epsilon} \text{tr} \log K_{\mu\bar{\nu}} \]  

(2.5)

The corresponding renormalization of the Kähler metric is obtained in terms of the one-loop \(\beta\)-function

\[ \beta^{(1)}_{\mu\bar{\nu}} = -\partial_\mu \partial_{\bar{\nu}} \text{tr} \log K_{\nu\bar{\nu}} = -R_{\mu\bar{\nu}} \]  

(2.6)

being \(R_{\mu\bar{\nu}}\) the Ricci tensor of the manifold.

In order to study the corresponding problem for the \(\sigma\)-model written in terms of complex linear superfields, we start with the general superspace action

\[ S = \int d^2x \ d^4\theta \ F(\Sigma, \bar{\Sigma}) \]  

(2.7)

where \(\bar{D}^2\Sigma = D^2\bar{\Sigma} = 0\). As we have mentioned above, if \(F = \hat{K}\) with \(\hat{K}\) given in (1.0) the two models described by the actions in (2.7) and (2.1) are classically dual to each other. Now we want to step up to the quantum level.

We follow the same approach described for the nonlinear \(\sigma\)-model in terms of chiral superfields and use the background field method in perturbation theory, shifting

\[ \Sigma \to \Sigma + \Sigma_0 \quad , \quad \bar{\Sigma} \to \bar{\Sigma} + \bar{\Sigma}_0 \]  

(2.8)

in (2.7). As in the previous case the quantum fields appear explicitly, while the background dependence is contained in the interaction vertices through derivatives of the \(F\) potential

\[ S = \int d^2x \ d^4\theta \left[ -\Sigma^\mu \bar{\Sigma}^\bar{\nu} \delta_{\mu\bar{\nu}} + [F_{\mu\bar{\nu}}(\Sigma_0, \bar{\Sigma}_0) + \delta_{\mu\bar{\nu}}] \Sigma^\mu \bar{\Sigma}^\bar{\nu} \right. \\
+ \frac{1}{2} F_{\mu\bar{\nu}}(\Sigma_0, \bar{\Sigma}_0) \Sigma^\mu \Sigma^\nu + \frac{1}{2} F_{\mu\bar{\nu}}(\Sigma_0, \bar{\Sigma}_0) \bar{\Sigma}^\mu \bar{\Sigma}^\bar{\nu} + \ldots \]  

(2.9)

with the definitions

\[ F_{\mu_1\ldots\mu_n\bar{\nu}_1\ldots\bar{\nu}_m} \equiv \frac{\partial}{\partial \Sigma^{\mu_1}} \ldots \frac{\partial}{\partial \Sigma^{\mu_n}} \ldots \frac{\partial}{\partial \bar{\Sigma}^{\bar{\nu}_1}} \ldots \frac{\partial}{\partial \bar{\Sigma}^{\bar{\nu}_m}} F(\Sigma, \bar{\Sigma}) \]  

(2.10)
Up to this point we have repeated the same steps as in the chiral superfield example. In the present case however, we have to deal with the fact that a direct superspace quantization of the complex linear superfield is not available. As explained in ref. [4], one way to proceed is to solve the linearity constraint \( \Sigma = \bar{D}_{\dot{\alpha}} \sigma^\alpha, \bar{\Sigma} = D_\alpha \sigma^\alpha \) in terms of unconstrained spinor superfields \( \sigma^\alpha, \bar{\sigma}^\dot{\alpha} \). The gauge invariance introduced in this manner needs to be fixed and the quantization gives rise to an infinite tower of ghosts. The Batalin-Vilkovisky method provides a systematic procedure for obtaining the gauge-fixed action. Due to the appearance of an infinite number of ghost fields the final result looks rather intricate and difficult to use in applications. In fact we show here that choosing gauge-fixing functions flat (i.e. independent) with respect to the background external fields, all the ghosts essentially decouple. More precisely, the canonical transformations, which in the Batalin-Vilkovisky approach are necessary to go from the classical basis to the gauge-fixed basis, produce nondiagonal terms between the quantum gauge spinors \( \sigma^\alpha, \bar{\sigma}^\dot{\alpha} \) and some of the ghost fields. In the following we perform explicitly the diagonalization which leads to the relevant kinetic terms needed for the evaluation of the one-loop \( \beta \)-function. As a non trivial check of the consistency of our calculation we compute in a general gauge and prove that the resulting physics (e.g. the \( \beta \)-function) is independent of the gauge parameters.

### 3 Diagonalization and effective propagator

We express the quantum linear superfields, viewed as the field strength of some gauge spinors, in terms of \( \sigma^\alpha, \bar{\sigma}^\dot{\alpha} \) so that the classical kinetic term in (2.9) becomes

\[
S_{cl} = - \int d^2x \ d^4\theta \ \bar{\sigma}^\dot{\alpha} D_{\dot{\alpha}} D_\alpha \sigma^\alpha
\]

(3.1)

In ref. [4] the gauge invariances, i.e. the infinite chain of transformations with zero modes which are responsible for the appearance of a tower of ghosts, have been discussed in detail. We refer the reader to that paper for the complete derivation of the Batalin-Vilkovisky quantization of the model. Here we review only the few steps which are relevant for our successive calculations.

The quantization of the classical action in (3.1) starts by defining the minimal extended action

\[
S_{min} = S_{cl} + \Phi_A^* \delta \Phi^A
\]

(3.2)

where generically \( \Phi_A, \delta \Phi_A \) denote a field and its gauge variation, while \( \Phi_A^* \) denotes the corresponding antifield. Then one adds to (3.2) non-minimal fields and constructs the
extended action in classical basis. Gauge fixing is performed by canonical transformations through the introduction of a gauge fermion $\Psi(\Phi)$

$$\Phi^*_A \to \Phi^*_A + \frac{\delta \Psi}{\delta \Phi^*_A}$$

(3.3)

It is the iterative succession of canonical transformations which is responsible for the mixing of the various fields and ultimately requires a nontrivial diagonalization. Now we outline the procedure on our specific model.

As mentioned above, since the gauge fixing does not introduce any explicit coupling with the external background fields, we need only consider those contributions in the nonminimal action and in the gauge fermion which give rise to mixed, nondiagonal terms between the ghosts and the quantum fields $\sigma^\alpha$, $\bar{\sigma}^\dot{\alpha}$. This amounts saying that in the pyramid of fields produced by the Batalin-Vilkovisky construction, given schematically in table 1 for the first four levels ($A_i$ is an abbreviation for the symmetrized set of indices $(\alpha_1 \ldots \alpha_i)$), it suffices to concentrate on the fields and antifields of ghost number zero, which are those in the upper left diagonal. Therefore in the following we systematically ignore the remaining fields: while important for the complete Batalin-Vilkovisky construction, they do not play any role for the determination of the effective propagator which will enter in the $\beta$-function calculation.

In order to complete (3.1) to an invertible kinetic term one introduces gauge-fixing functions

$$F^\dot{\alpha}_\alpha = D_\alpha \bar{\sigma}^\dot{\alpha} , \quad F^\alpha_\dot{\alpha} = \bar{D}_\dot{\alpha} \sigma^\alpha$$

(3.4)

and corresponding (antighost) fields $b^\alpha_\dot{\alpha}$ and their complex conjugates $\bar{b}^\dot{\alpha}_\alpha$. Indicating the antifields of the antighosts respectively by $b^{*\alpha}_\dot{\alpha}$ and $\bar{b}^{*\dot{\alpha}}_\alpha$, we add to $S_{\text{min}}$ the non–minimal term (we do not indicate the integration symbol and an overall minus sign, $- \int d^2x d^4\theta$, in the definition of the action )

$$S_{\text{nm},1} = \bar{b}^{*\dot{\alpha}}_\alpha b^{*\alpha}_\dot{\alpha}$$

(3.5)

Then we perform a canonical transformation generated by the gauge fermion

$$\Psi_1 = k[b^{\dot{\alpha}}_\alpha \bar{D}_\alpha \sigma^\alpha + \bar{\sigma}^\dot{\alpha} D_\dot{\alpha} \bar{b}^{\dot{\alpha}}_\alpha]$$

(3.6)

$k$ being the gauge parameter\(^1\) (note that in ref. [4] $k$ was chosen equal to one). This implies the substitutions

$$b^{*\alpha}_\dot{\alpha} \to b^{*\alpha}_\dot{\alpha} + k \bar{D}_\dot{\alpha} \sigma^\alpha$$

$$\bar{b}^{*\dot{\alpha}}_\alpha \to \bar{b}^{*\dot{\alpha}}_\alpha + k \bar{\sigma}^\dot{\alpha} D_\alpha$$

(3.7)

\(^1\)For simplicity we choose $k$ and all further gauge parameters real.
Table 1: Fields up to fourth level. The first column gives the level, and the second one indicates whether the fields are bosonic or fermionic. The ghost numbers of the field and of the antifield are also indicated.

| Level | Field | Antifield | Ghost Numbers |
|-------|-------|-----------|---------------|
| 0     | F     | $\sigma^{(0,-1)}$ |               |
| 1     | F     | $b^{(0,-1)}$ | $\sigma^{(1,-2)}$ |
| B     | $d^{(0,-1)}_a$ | $\nu$ | $\mu$ |
|       |       | $\nu^{(1,-2)}$ | $\mu^{(2,-3)}$ |
| 2     | F     | $d^{(1,-2)}_a$ | $b^{(-2,1)}$ | $\sigma^{(2,-3)}$ |
| B     | $\rho^{(-1,0)}$ | $\nu^{(1,-2)}$ | $\mu^{(2,-3)}$ |
|       |       | $\rho^{(1,-2)}$ | $\mu^{(2,-3)}$ |
| 3     | F     | $e^{(1,-2)}_A$ | $d^{(2,-3)}_A$ | $b^{(-3,2)}$ | $\sigma^{(3,-4)}$ | $\varsigma^{(3,-4)}$ |
| B     | $\rho^{(-1,0)}$ | $\nu^{(1,-2)}$ | $\mu^{(2,-3)}$ | $\lambda^{(4,-5)}$ |
|       |       | $\rho^{(1,-2)}$ | $\mu^{(2,-3)}$ | $\lambda^{(4,-5)}$ |
| 4     | F     | $e^{(2,-3)}_A$ | $d^{(3,-4)}_A$ | $b^{(-4,3)}$ | $\sigma^{(4,-5)}$ | $\varsigma^{(4,-5)}$ |
|       |       | $\rho^{(-1,0)}$ | $\nu^{(1,-2)}$ | $\mu^{(2,-3)}$ | $\lambda^{(4,-5)}$ |
|       |       | $\rho^{(1,-2)}$ | $\mu^{(2,-3)}$ | $\lambda^{(4,-5)}$ |

We then obtain from the non-minimal action

$$S_1 \rightarrow k \bar{b}_\alpha \bar{D}_\alpha \sigma^\alpha + k \bar{\sigma}^{\dot{\alpha}} D_\alpha b^{\dot{\alpha}} + k^2 \bar{\sigma}^{\dot{\alpha}} D_\alpha \bar{D}_\dot{\alpha} \sigma^\alpha$$  (3.8)

The last term combines with $S_{nl}$ and leads to a quadratic gauge-fixed action

$$S_{Q,1} = \bar{\sigma}^{\dot{\alpha}} [\bar{D}_\dot{\alpha} D_\alpha + k^2 D_\alpha \bar{D}_\dot{\alpha}] \sigma^\alpha$$  (3.9)

The rest gives

$$S_{*,1} = \bar{b}^{\dot{\alpha}} b_\alpha + k \bar{\sigma}^{\dot{\alpha}} D_\alpha \sigma^\alpha + k \bar{\sigma}^{\dot{\alpha}} D_\alpha b^{\dot{\alpha}}$$  (3.10)

Continuing, after the first level gauge-fermion in (3.9) we introduce

$$\Psi_2 = b^{\dot{\alpha}} \left( D_{\dot{\beta}} d^{(\dot{\beta} \alpha)} + i \bar{\nu} \partial^{\dot{\alpha}} \nu \right) + \left( \bar{d}^{(\dot{\alpha} \dot{\beta})} \bar{D}_{\dot{\beta}} - \bar{\nu} \bar{\nu} \partial^{\dot{\alpha}} \right) b^{\dot{\alpha}} + \ldots$$  (3.11)

where we have considered only the part which is relevant for our present discussion. Note that the gauge fermion at odd levels connects fields of that level with fields of the previous
level, already present in the action constructed that far: in this case the gauge parameter cannot be rescaled. At even levels instead one connects fields of that level with fields below, not yet present in the action: therefore appropriate field redefinitions of the new fields allow to rescale some of the gauge parameters to one. In (3.11) and in the following, below, not yet present in the action: therefore appropriate field redefinitions of the new fields cannot be rescaled. At even levels instead one connects fields of that level with fields of lower levels.

The canonical transformations induced by (3.11) are given by

\[
\begin{align*}
    b_{\dot{a}}^\alpha & \rightarrow b_{\dot{a}}^\alpha + D_\beta d_{\dot{a}}^{(\beta\alpha)} + i\partial_{\dot{a}}^\alpha \nu \\
    \bar{b}_{\dot{a}}^{\dot{\alpha}} & \rightarrow \bar{b}_{\dot{a}}^{\dot{\alpha}} + \bar{d}_{\dot{a}}^{(\dot{\alpha}\dot{\beta})} \bar{D}_\beta - \bar{\nu} \partial_{\dot{a}}^{\dot{\alpha}}
\end{align*}
\]

(3.12)

The action receives accordingly the following contributions

\[
S_2 \rightarrow k\tilde{\sigma}^\alpha D_\alpha i\partial^\alpha \nu - k\bar{\nu} i\partial_{\dot{a}}^{\dot{\alpha}} \bar{D}_\dot{a} \sigma^\alpha + \left[\bar{d}_{\dot{a}}^{(\dot{\alpha}\dot{\beta})} \bar{D}_\beta - \bar{\nu} i\partial_{\dot{a}}^{\dot{\alpha}}\right] \left[D_\beta d_{\dot{a}}^{(\beta\alpha)} + i\partial_{\dot{a}}^\alpha \nu\right]
\]

(3.13)

Clearly, as we had anticipated, we have produced mixed (non diagonal) terms between the various fields, $\sigma^\alpha$, $\nu$ and $d_{\dot{a}}^{(\dot{\alpha}\dot{\beta})}$. Note however that mixed terms between the $\sigma^\alpha$s and the ghosts $d_{\dot{a}}^{(\dot{\alpha}\dot{\beta})}$ do not arise for symmetry reasons (e.g. $\bar{\sigma}^\alpha D_\alpha D_\beta d_{\dot{\alpha}}^{(\beta \dot{\alpha})} = 0$). This pattern repeats itself at every step.

We proceed introducing the gauge fermion

\[
\Psi_3 = k_1 e_{(\alpha\beta)}^{(\dot{\alpha}\dot{\beta})} \bar{D}_\dot{a}^{(\dot{\alpha}\dot{\beta})} + k_1 \bar{d}_{\dot{a}}^{(\dot{\alpha}\dot{\beta})} \bar{D}_\beta e_{(\dot{\alpha}\dot{\beta})}^{(\alpha\beta)} + k_1' \rho^{\dot{\alpha}} \bar{D}_\dot{a} \nu - k_1' \bar{\nu} D_\alpha \rho^\alpha + \ldots
\]

(3.14)

with canonical transformations

\[
\begin{align*}
    \rho^*_\dot{a} & \rightarrow \rho^*_\dot{a} + k_1' \bar{D}_\dot{a} \nu \\
    \bar{\rho}^*_\dot{a} & \rightarrow \bar{\rho}^*_\dot{a} - k_1' \bar{\nu} D_\alpha \\
    e^{*}_{(\alpha\beta)} & \rightarrow e^{*}_{(\alpha\beta)} + k_1 \bar{D}_\beta d_{\dot{a}}^{(\alpha\beta)} \\
    \bar{e}^{*}_{(\dot{\alpha}\dot{\beta})} & \rightarrow \bar{e}^{*}_{(\dot{\alpha}\dot{\beta})} + k_1 \bar{d}_{\dot{a}}^{(\alpha\beta)} \bar{D}_\beta
\end{align*}
\]

(3.15)

These shifts affect the nonminimal part of the action at the third level

\[
S_{nm,3} = \bar{e}^{*}_{(\dot{\alpha}\dot{\beta})} e_{(\dot{\alpha}\dot{\beta})}^{*} - \bar{\rho}^*_\dot{a} i\partial^\alpha \rho^*_\dot{a}
\]

(3.16)

in the following way

\[
S_3 = k_1^2 \bar{\nu} D_\alpha i\partial^\alpha \bar{D}_\dot{a} \nu - \bar{\rho}^*_\dot{a} i\partial^\alpha \rho^*_\dot{a} - k_1' \rho^*_\dot{a} i\partial^\alpha \bar{D}_\dot{a} \nu + k_1' \bar{\nu} D_\alpha i\partial^\alpha \rho^*_\dot{a} + k_1^2 \bar{d}_{\dot{a}}^{(\dot{\alpha}\dot{\beta})} \bar{D}_\beta d_{\dot{\alpha}}^{(\beta\dot{\alpha})} + \bar{e}^{*}_{(\dot{\alpha}\dot{\beta})} e_{(\dot{\alpha}\dot{\beta})}^{*} + k_1 \bar{d}_{\dot{a}}^{(\alpha\beta)} D_\beta e_{(\dot{\alpha}\dot{\beta})}^{(\alpha\beta)} + k_1 \bar{e}^{*}_{(\dot{\alpha}\dot{\beta})} D_\beta d_{\dot{\alpha}}^{(\beta\dot{\alpha})}
\]

(3.17)
At the fourth level, in complete analogy with the choice made at the second level in (3.11), we introduce the gauge fermion

\[ \Psi_4 = e_{(a \beta)}^{(\dot{a} \dot{\beta})} \left( D_\gamma f^{(a \beta \gamma)}_{(a \beta)} + i \partial^a \tau^a_{\dot{\beta}} \right) + \left( \bar{f}^{(a \beta \dot{\gamma})}_{(a \beta)} D_\gamma - \bar{\tau}^\beta_{\dot{\beta}} i \partial^a \right) e^{(a \beta)}_{(a \beta)} \]

+ \kappa \rho^a_{\dot{\alpha}} D_\alpha \tau^a_{\dot{\alpha}} - \kappa_1 \bar{\tau}^\alpha_{\dot{\alpha}} D_\alpha \rho^a + \ldots \] (3.18)

(Note that we have rescaled the fields \( f^{(a \beta \gamma)}_{(a \beta)} \) and \( \tau^\beta_{\dot{\beta}} \) to set equal to one two of the three gauge parameters that should appear in (3.18).) Now we have to perform the substitutions

\[ \r^a_{\dot{\alpha}} \rightarrow \r^a_{\dot{\alpha}} + k_1' \bar{D}_\alpha \tau^a_{\dot{\alpha}} \]
\[ \bar{\r}^a_{\dot{\alpha}} \rightarrow \bar{\r}^a_{\dot{\alpha}} - k_1' \bar{D}_\alpha \]
\[ \bar{e}^{(a \beta)}_{(a \beta)} \rightarrow \bar{e}^{(a \beta)}_{(a \beta)} + D_\gamma f^{(a \beta \gamma)} + i \partial^a \tau^a_{\dot{\beta}} \]
\[ e^{(a \beta)}_{(a \beta)} \rightarrow e^{(a \beta)}_{(a \beta)} + f^{(a \beta \gamma)} D_\gamma - \bar{\tau}^\beta_{\dot{\beta}} i \partial^a \] (3.19)

When inserted in (3.17) the shifts in (3.19) give rise to

\[ S_4 \rightarrow k_1' k_2' \bar{\tau}^a_{\dot{\alpha}} i \partial^a \bar{D}_\beta \nu - k_1' k_2' \bar{\nu} i \partial^a \bar{D}_\beta \nu^a_{\dot{\beta}} + k_1'' \bar{\tau}^\beta_{\dot{\beta}} \bar{D}_\beta i \partial^a D_\beta \tau^a_{\dot{\beta}} \]

+ \left( \bar{f}^{(a \beta \dot{\gamma})}_{(a \beta)} D_\gamma - \bar{\tau}^\beta_{\dot{\beta}} i \partial^a \right) \left( D_\gamma f^{(a \beta \gamma)}_{(a \beta)} + i \partial^a \tau^a_{\dot{\beta}} \right) \] (3.20)

Here again we see new non diagonal terms between the ghosts \( \nu \) and \( \tau^a_{\dot{\alpha}} \), but no crossed terms between the \( d^a_{\dot{\alpha}} \)'s and the \( f^{(a \beta \gamma)}_{(a \beta)} \)'s. This is a general feature which can be easily implemented at higher levels given the structure of the symmetrized indices on the various fields. In fact at this point it is simple to understand how the story continues and one can easily write all at once the infinite sum of terms that enter the quadratic part of the gauge-fixed action. To this end it is convenient to introduce a compact notation and rename appropriately the fields in the upper left diagonal of table 1. At even levels we call the fermionic fields \( d^A_{\dot{A}} \) and the bosonic fields \( \nu^{\dot{A}}_{\dot{A}} \), where \( n = 0, 1, \ldots \) (but \( \nu \) does not exist for \( n = 0 \)), so that, for example, at zero level we have set \( d^A_{\dot{A}} \equiv \sigma^\alpha_{\dot{\alpha}} \), \( \bar{d}^\alpha \equiv \bar{\sigma}^\alpha_{\dot{\alpha}} \) and so on. At odd levels in the same way, with obvious identifications with respect to the fields in table 1, we call the fermionic fields \( e^{\dot{A}}_A \), the bosonic fields \( \rho^{\dot{A}}_{\dot{A}} \), \( n = 1, 2, \ldots \) (the field \( \rho \) does not exist for \( n = 1 \)). Thus we can write the classical action and the non-minimal terms relevant for our purpose as

\[ S_{cl} + S_{nm} = \bar{\sigma}^\alpha D_\alpha D_\nu \sigma^\alpha + \sum_{n=0}^{\infty} e^{*A_{n+1}}_{A_{n+1}} e^{*A_{n+1}}_{A_{n+1}} - \sum_{n=1}^{\infty} \rho^{*A_{n-1}}_{\dot{A}_{n-1}} i \partial^\alpha \rho^{*A_{n-1}}_{\dot{A}_{n-1}} \] (3.21)
Using the same notation we obtain the total gauge fermion
\[ \Psi = \sum_{n=0}^{\infty} k_n e^{(\hat{\alpha} \hat{A}_n)} D_\alpha D_{\hat{A}_n} + \sum_{n=1}^{\infty} k_n \rho^{(\hat{\alpha} \hat{A}_{n-1})} \bar{D}_\alpha \nu_{\hat{A}_{n-1}} + \sum_{n=0}^{\infty} k_n e^{(\hat{\beta} \hat{A}_n)} D_\beta D_{\hat{A}_n} + \sum_{n=0}^{\infty} e^{(\hat{\alpha} \hat{A}_n)} i \bar{\partial}_\alpha \nu_{\hat{A}_n} + \sum_{n=1}^{\infty} k_n \rho^{\hat{A}_n} \bar{D}_\alpha \nu_{\hat{A}_{n-1}} + \sum_{n=0}^{\infty} k_n \bar{D}_\alpha \nu_{\hat{A}_{n-1}} \] (3.22)

One can easily check that the terms in (3.6) and (3.11) correspond to the \( n = 0 \) contributions in (3.22), while the gauge fermions at the third and fourth level in (3.14) and (3.18) are reproduced by the \( n = 1 \) coefficients of the various sums in the above expression. The gauge fermion in (3.22) implies canonical transformations that, once inserted in (3.21), reconstruct the quadratic gauge-fixed action
\[ S_{gf} = \sum_{n=0}^{\infty} \left[ \bar{d}^{(\hat{\beta} \hat{A}_n)} \left( \bar{D}_\beta D_\beta + k_n^2 D_\beta \bar{D}_\beta \right) d^{(\hat{\beta} \hat{A}_n)} + k_n \bar{D}^{(\hat{\beta} \hat{A}_n)} D_{(\hat{\alpha} \hat{A}_{n+1})} i \bar{\partial}_{\hat{A}_{n+1}} \right] + k_n \bar{D}^{(\hat{\beta} \hat{A}_n)} + \sum_{n=1}^{\infty} \frac{1}{n} k_n^2 \bar{D}_\alpha \nu_{\hat{A}_{n-1}}^2 D^2 \nu_{\hat{A}_{n-1}} - \sum_{n=1}^{\infty} k_n \nu_{\hat{A}_{n-1}} \bar{D}_\alpha \nu_{\hat{A}_{n-1}} \] (3.23)

As anticipated above, the transformations induced by the gauge fermions keep producing nondiagonal terms, in which the physical fields \( \sigma^\alpha \) mix with the ghosts, the ghosts mix with other ghosts, and so on, in an infinite sequence. So, at first sight, in order to obtain the \( < \sigma^\alpha \sigma^\alpha > \) propagator it seems compulsory to perform a complete diagonalization of the quadratic, kinetic terms in the quantum action. We want to show now that this infinite series of operations can actually be avoided and that with few steps one can decouple the \( \sigma^\alpha \)'s from the ghosts. Since only the \( \sigma^\alpha \), \( \bar{\sigma}^\alpha \) fields interact with the external background, this is all we need for the calculation of the one-loop \( \beta \)-function of the nonlinear \( \sigma \)-model. In order to explain the procedure it is simpler to visualize the kinetic terms as the infinite
Thus we cancel the $d\nu$ terms with a shift $d \rightarrow d + \nu$, thus producing new $\nu\nu$ and $\nu\tau$ terms. Then we simply obtain the final kinetic term for $\bar{\nu}_\tau \sigma$ terms with a shift $\nu \rightarrow \nu + \sigma$ and taking advantage of the fact that, contrary to the expectation, this move does not produce $\sigma\tau$ terms. At this point the ghosts, while still mixed with each others, do not have any crossing with the physical $\sigma^\alpha$'s. More specifically, from (3.23) we consider the part of the action explicitly indicated in (3.24)

$$S_{gf} = \bar{\sigma}^\alpha (\bar{D}_\alpha D_\alpha + k^2 D_\alpha \bar{D}_\alpha)\sigma^\alpha + (k\bar{\sigma}^\alpha D_\alpha i\partial^\alpha \nu + h.c.)$$

$$+ \bar{\nu}(-2\Box + k_1^2 D_\alpha i\partial^\alpha \bar{D}_\alpha)\nu + (\bar{d}^\alpha \bar{D}_\beta \partial^\alpha \nu + h.c.) + \bar{d}^\alpha (\bar{D}_\beta D_\beta + k_2 D_\beta \bar{D}_\beta) d^\alpha$$

$$- (k_1 \bar{\tau}_\beta i\partial^\alpha \bar{D}_\alpha d^\beta + h.c.) + (k_1' k_2' \bar{\bar{\tau}}_\alpha i\partial^\alpha D^2 \nu + h.c.) + \ldots$$

(3.25)

Thus we cancel the $d\nu$ crossed terms with the appropriate shift of the $d$ field

$$d^\alpha \rightarrow d^\alpha + \frac{4(k_1^2 - 1)}{3 - 2k_1^2} \bar{\bar{\tau}}_\alpha \partial^\alpha \bar{D}^\alpha D^\beta + \frac{2}{3 - 2k_1^2} \partial^\alpha \bar{D}^\alpha \nu$$

(3.26)

Correspondingly the additional contributions to the kinetic action of $\nu$ are

$$\delta S_{\nu\nu} = 3\bar{\nu} \left[ D_\alpha \bar{D}^2 D^\alpha + \frac{2}{3 - 2k_1^2} D^2 \bar{D}^2 \right] \nu$$

(3.27)

while the new $\tau\nu$ crossed terms are

$$\delta S_{\tau\nu} = \frac{3k_1}{3 - 2k_1^2} \left( \bar{\bar{\nu}} \partial^\alpha \bar{\bar{\tau}}_\alpha - \bar{\bar{\bar{\tau}}}_\alpha \partial^\alpha \bar{D}^2 \nu \right)$$

(3.28)
At this stage, adding the terms in (3.27) to the original quadratic $\bar{\nu}\nu$ terms in (3.13) and (3.17) we obtain

$$S_{\bar{\nu}\nu} = \bar{\nu} \left[ -2\Box + \left( \frac{6}{3 - 2k'_1} - 2k''_1 \right) D^2 \bar{D}^2 + (3 - k''_1) D_\alpha \bar{D}^2 D^\alpha \right] \nu \equiv \bar{\nu} \tilde{W}_\nu \nu \quad (3.29)$$

The final step requires the diagonalization of the $\sigma \nu$ terms (see eq. (3.13))

$$\bar{\nu} \tilde{W}_\nu \nu - k \bar{\nu} \dot{i} \partial_\alpha \dot{\bar{D}}_\alpha \sigma^\alpha + k \bar{\sigma}^\dot{\alpha} D_\alpha \partial^\alpha \nu \quad (3.30)$$

First we easily compute the inverse of the operator in (3.29)

$$\tilde{W}_\nu^{-1} = -\frac{1}{2\Box} + \frac{3 - 3k''_1 + 2(k_1 k'_1)^2}{4k'_1 - 6k''_1 + 4(k_1 k'_1)^2} \frac{D^2 \bar{D}^2}{\Box^2} \frac{3 - k''_1}{2(1-k''_1)} \frac{D_\alpha \bar{D}^2 D^\alpha}{\Box^2} \quad (3.31)$$

Then we shift the $\nu$ field in such a way to cancel the $\sigma \nu$ terms

$$\nu \rightarrow \nu + k \bar{W}^{-1}_\nu \nu \equiv k \bar{W}^{-1}_\nu \nu \dot{i} \partial_\alpha \dot{\bar{D}}^\alpha_\sigma^\alpha \quad (3.32)$$

The relevant fact is that (3.32) does not give rise to $\tau \sigma$ contributions: in fact substituting (3.32) in (3.29) and (3.28) one obtains

$$\delta S_{\tau \sigma} = k \left( k'_1 k''_1 - \frac{3k_1}{3 - 2k''_1} \right) \bar{\sigma}^\dot{\alpha} \dot{i} \partial_\alpha \dot{\bar{D}}^2 \tilde{W}^{-1}_\nu \nu \dot{i} \partial_\beta \dot{\bar{D}}^\beta \sigma^\beta + \text{ h.c.} \quad (3.33)$$

From the expression of $\tilde{W}^{-1}_\nu$ in (3.31) it is immediate to check that all the terms in (3.33) vanish trivially. Finally no more crossed couplings of the $\sigma$-fields are present.

Thus we are left with new quadratic terms $\bar{\sigma}^\dot{\alpha} \sigma^\alpha$ which combine with the original $\bar{D}_\alpha D_\alpha + k^2 D_\alpha D_\dot{\alpha}$ and give as total kinetic operator

$$\tilde{W}_{\alpha \dot{\alpha}} = \bar{D}_\dot{\alpha} D_\alpha + \frac{k^2}{2} D_\alpha \bar{D}_\dot{\alpha} - \frac{k^2 + (k k'_1)^2}{2 - 2k''_1} \frac{i \partial_{\alpha \dot{\alpha}} D^2 \bar{D}^2}{\Box} + \frac{k^2}{2} \frac{i \partial_{\alpha \dot{\alpha}} D_\beta \bar{D}^2 D^\beta}{\Box} \quad (3.34)$$

The $\sigma^\alpha \bar{\sigma}^\dot{\alpha}$ propagator is given by

$$< \sigma^\alpha \bar{\sigma}^\dot{\alpha} > = (\tilde{W}^{-1})^{\alpha \dot{\alpha}} = -\frac{i \partial^{\alpha \dot{\alpha}}}{\Box} + \frac{3(k k'_1)^2 + 4 - 2k''_1}{4(k k'_1)^2} \frac{i \partial^{\alpha \dot{\alpha}} D^2 \bar{D}^2}{\Box^2} + \frac{3k^2}{2} \frac{i \partial^{\alpha \dot{\alpha}} D_\beta \bar{D}^2 D^\beta}{\Box^2} + \frac{2 - k''_1}{4k^2} \frac{i \partial^{\alpha \dot{\alpha}} i \partial_\beta \bar{\sigma}^\beta D_\beta \bar{D}^\beta}{\Box^2} \quad (3.35)$$

Now we have collected all the ingredients which are necessary for the computation of the one-loop $\beta$-function for the linear multiplet $\sigma$-model. We follow the same procedure as in the corresponding $N = 2$ chiral multiplet calculation [4].

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4 One-loop beta-function

Going back to (2.9) it appears that the quantum fields $\sigma^\alpha$, $\bar{\sigma}^{\dot{\alpha}}$ are always coupled to the external background through their field-strengths $\Sigma$, $\bar{\Sigma}$. This implies that in the perturbative calculations only the $<\Sigma\Sigma>$ propagator does enter. Thus it suffices to consider

$$<\Sigma\Sigma> = D_\alpha <\sigma^\alpha\bar{\sigma}^{\dot{\alpha}}> \bar{D}_{\dot{\alpha}} = \frac{D^2\bar{D}^2}{\Box} + \frac{D_\alpha D^2 D^\alpha}{\Box} \equiv \Pi \tag{4.1}$$

The above result shows that the effective propagator is automatically independent of the gauge parameters introduced in the gauge-fixing procedure. This provides a very good check of the methods used in the quantization of the linear multiplet.

Finally we are ready to compute the one-loop divergence: the effective Feynman rules can be obtained directly from the expansion of

$$\exp(S_{eff}) = \exp(-\Sigma\Pi^{-1}\Sigma + \Sigma V\bar{\Sigma} + \frac{1}{2}\Sigma U\Sigma + \frac{1}{2}\bar{\Sigma} \bar{U}\bar{\Sigma}) \tag{4.2}$$

where $\Pi$ is given in (1.1) and (cfr. (2.9))

$$V \equiv (F_{\mu\bar{\mu}} + \delta_{\mu\bar{\mu}}), \quad U \equiv F_{\mu\nu} \tag{4.3}$$

The one-loop divergent contributions are computed using standard $D$-algebra techniques very similar to the ones used in [7]. We recall that in order to obtain local divergent structures, it is sufficient to consider only the contributions with no derivatives acting on the external background, so that the covariant spinor derivatives $D_\alpha$, $\bar{D}_{\dot{\alpha}}$ are freely integrated by parts on the internal quantum lines of the diagrams and the algebra is easily completed.

We group the various graphs into two sets: the graphs which contain only $\Sigma V \bar{\Sigma}$ vertices and all the others. The first set gives rise to a sum of terms, which before completion of the $D$-algebra, we write schematically in the form

$$-\text{tr} \log(1 - \Pi V) = \text{tr} \sum \frac{1}{n} (\Pi V)^n \tag{4.4}$$

We obtain the relevant, non vanishing contributions if, integrating by parts all the covariant derivatives inside the loop, we end up with exactly two $D$’s and two $\bar{D}$’s. Making use of the relation

$$\left(\frac{D^2\bar{D}^2 + D_\alpha \bar{D}^2 D^\alpha}{\Box}\right)^n = \frac{D^2\bar{D}^2 + D_\alpha \bar{D}^2 D^\alpha}{\Box} \to -\frac{1}{\Box} D^2 \bar{D}^2 \tag{4.5}$$
it is fairly straightforward to obtain the one-loop divergence from this first group of diagrams

\[
F^{(1)}_1 \rightarrow \frac{1}{\epsilon} \text{tr} \sum \frac{1}{n} (\mathcal{V})^n = -\frac{1}{\epsilon} \text{tr} \log(1 - \mathcal{V})
\] (4.6)

Then we consider diagrams which contain a dependence on the background of the type \( \mathcal{U} \) and \( \bar{\mathcal{U}} \). These contributions can be efficiently accounted for, defining first an effective \( < \ddot{\Sigma} \ddot{\Sigma} > \) propagator with the \( \mathcal{V} \)-type vertices resummed. This can be accomplished most easily keeping in mind that, as mentioned above, we can drop all the terms where covariant spinor derivatives act on the background fields. Performing explicitly the sum of all the \( \mathcal{V} \) vertices we obtain

\[
< < \ddot{\Sigma} \ddot{\Sigma} > > = \frac{1}{\square} (D^2 \bar{D}^2 + D_\alpha \bar{D}^2 \bar{D}^\alpha) \frac{1}{1 - \mathcal{V}} \equiv \ddot{\Pi}
\] (4.7)

In terms of \( \ddot{\Pi} \) the second class of one-loop diagrams can be written, before completion of the \( D \)-algebra, as

\[
\text{tr} \left[ \frac{1}{2} \mathcal{U} \bar{\mathcal{U}} \ddot{\Pi} + \frac{1}{4} (\mathcal{U} \ddot{\Pi} \bar{\mathcal{U}})^2 + \frac{1}{6} (\mathcal{U} \ddot{\Pi} \bar{\mathcal{U}})^3 + \ldots \right]
\]

\[
= \frac{1}{2} \text{tr} \sum_{n=1}^{\infty} (\mathcal{U} \ddot{\Pi} \bar{\mathcal{U}})^n
\] (4.8)

In this case the \( D \)-algebra is performed using

\[
\left[ \frac{(D^2 \bar{D}^2 + D_\alpha \bar{D}^2 \bar{D}^\alpha)}{\square} \left( \bar{D}^2 \bar{D}^2 + \bar{D}_\beta \bar{D}^2 \bar{D}^\beta \right) \right]^n = \frac{D_\alpha \bar{D}^2 \bar{D}^\alpha}{\square} \rightarrow -2 \bar{D}^2 \bar{D}^2
\] (4.9)

so that the one-loop divergence from the second set of diagrams is given by

\[
F^{(1)}_2 \rightarrow -\frac{1}{\epsilon} \text{tr} \log(1 - \mathcal{U} \frac{1}{1 - \mathcal{V}^2} \bar{\mathcal{U}})
\] (4.10)

Adding the results in (4.6) and (4.10) we obtain the total one-loop divergent contribution

\[
F^{(1)} \rightarrow -\frac{1}{\epsilon} \left[ \text{tr} \log(1 - \mathcal{V}) + \text{tr} \log(1 - \mathcal{U} \frac{1}{1 - \mathcal{V}^2} \bar{\mathcal{U}}) \right]
\] (4.11)

Using the definitions in (4.3) we rewrite the result

\[
F^{(1)} \rightarrow -\frac{1}{\epsilon} \text{tr} \log[-(F_{\mu \bar{\nu}} - F_{\mu \nu} F^{-1}_{\nu \bar{\nu}})]
\] (4.12)
Now we compare the above expression with what expected from duality correspondence with the nonlinear $\sigma$-model in terms of chiral superfields. The result in (1.12) has its counterpart in (2.3), where the one-loop divergent contribution to the Kähler potential of the $N = 2$ theory is exhibited. On the other hand in section 1 we have seen that at the level of the matrices given by the second derivatives of the potentials the duality transformations are given in (1.9). This is exactly the correspondence established by the results in (2.3) and (1.12).

5 Conclusions

We have computed the one-loop $\beta$-function of the nonlinear $\sigma$-model for nonminimal multiplets and checked that the result maintains the classical duality properties of the model with respect to the $N = 2$ chiral one.

Our calculation presents a highly nontrivial and concrete application of the quantization of the complex linear superfield via the Batalin-Vilkovisky procedure [4]. It is remarkable that in this application the relevant sector of the tower of ghosts are the so-called ‘extra ghosts’ rather than the ghosts or antighosts. In spite of the presence of the infinite tower of gauge invariances and of the corresponding infinite number of ghost fields our result is obtained with no need of formal manipulations and in a rather cute and straightforward manner.

Several issues deserve now further investigations.

It would be interesting to extend the analysis to higher loops and see whether the classical duality transformations remain unchanged or else, as for bosonic $\sigma$-models [8], perturbative corrections are induced. Since the $N = 2$ chiral $\sigma$-model has a vanishing $\beta$-function at two and three loops one could study the corresponding situation for the model of nonminimal multiplets.

The geometry associated to the chiral $N = 2$ theory has been thoroughly understood and well defined geometrical objects are constructed in terms of the Kähler potential. A complete geometrical interpretation is still lacking for the dual theory (however see ref. [9]).

Finally one could consider mixed models constructed in terms of both chiral and complex linear superfields, e.g. of the type used for the supersymmetric description of the low-energy QCD action [3,10], and introduce, in addition, couplings to the gauge Yang-Mills superfields.
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A Conventions

We list here some of the relations involving spinor covariant derivatives that we have repeatedly used in sections 3 and 4.

\[ \{D_\alpha, \bar{D}_\dot{\alpha}\} = i\partial_{\alpha\dot{\alpha}} \quad \text{(A.1)} \]

\[ [D_\alpha, \bar{D}^2] = -i\partial_{\alpha\dot{\alpha}} \bar{D}^{\dot{\alpha}} \quad \text{(A.2)} \]

\[ D^2 \bar{D}^2 D^2 = \Box D^2 \quad \text{(A.3)} \]

\[ D_\alpha \bar{D}^2 D^\alpha = \bar{D}_\dot{\alpha} D^2 \bar{D}^{\dot{\alpha}} \quad \text{(A.4)} \]

\[ D_\alpha i\partial^{\alpha\dot{\alpha}} D_{\dot{\alpha}} = -2D^2 D^2 - D_\alpha D^2 D^\alpha \quad \text{(A.5)} \]

\[ \delta^{(4)}(\theta - \theta') D^2 \bar{D}^2 \delta^{(4)}(\theta - \theta') = \frac{1}{2} \delta^{(4)}(\theta - \theta') D^\alpha D^2 D_\alpha \delta^{(4)}(\theta - \theta') = \delta^{(4)}(\theta - \theta') \quad \text{(A.6)} \]
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