Unshared Secret Key Cryptography: Finite Constellation Inputs and Ideal Secrecy Outage

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Abstract—The Unshared Secret Key Cryptography (USK), recently proposed by the authors, guarantees Shannon’s ideal secrecy and perfect secrecy for MIMO wiretap channels, without requiring secret key exchange. However, the requirement of infinite constellation inputs limits its applicability to practical systems. In this paper, we propose a practical USK scheme using finite constellation inputs. The new scheme is based on a cooperative jamming technique, and is valid for the case where the eavesdropper has more antennas than the transmitter. We show that Shannon’s ideal secrecy can be achieved with an arbitrarily small outage probability.

I. INTRODUCTION

Symmetric-key cryptography (e.g. AES [1]) has traditionally been the major technology for providing a secure gateway for communication and data exchanges at the network layer. One weakness of this approach is that the transmitter (Alice) and the legitimate receiver (Bob) must trust some secure communications channel to transmit the secret key, and there may be a chance that others (Eve) can discover the secret key during transmission. Physical layer security (PLS) is an alternative way of providing non-key based security solutions [2]. The PLS approaches leverage state-of-the-art channel coding (e.g. polar codes [3]) to enhance security at the physical layer. The general problem of PLS is the requirement of an infinite-length wiretap code to approach the secrecy capacity [4]. This limits the applicability of these schemes to practical communication systems.

Our previous work [5]–[7] has shown that it is possible to protect the confidential message without requiring either secret key exchange or wiretap codes. In particular, we proposed the Unshared Secret Key Cryptography (USK) to comply with two security goals: (i) the secret key is not needed by Bob to decipher, (ii) the secret key is fully affecting Eve’s ability to decipher the ciphertext. Although those two goals seem to contradict each other, this can be reconciled by aligning a one-time pad (OTP) secret key within the null space of a MIMO channel between Alice and Bob. In this way, the OTP nulls out at Bob, but adds a certain degree of uncertainty to the received signal at Eve. The USK is rooted in the artificial noise (AN) technique [9], and has been shown to achieve Shannon’s ideal secrecy and perfect secrecy [7]. An interesting case is ideal secrecy: an encryption algorithm is ideally secure if no matter how much of ciphertext is intercepted by Eve, there is no unique solution of the plaintext but many solutions of comparable probability [9]. An ideal cryptosystem has information-theoretic security (i.e., cryptanalytically unbreakable), but not Shannon’s perfect secrecy (i.e., plaintext and ciphertext are mutually independent) [9].

The original USK scheme [7] may be regarded as being of theoretical interest only, since it bases on two assumptions: (i) infinite lattice constellation input, (ii) Alice has more antennas than Eve. The first assumption is used to prove the ideal secrecy, and the second assumption ensures that Eve cannot run zero-forcing (ZF) attack to remove the OTP [10]. In this work, we remove these assumptions and show how the idea of USK can be applied to practical systems. We put forward a new security model and measure:

1) Finite constellation inputs: we use finite input alphabets based on QAM signalling.
2) Cooperative jammers: the OTP is generated from the third-party jammers. This renders the USK viable for the cases where Eve has more antennas than Alice.
3) Ideal secrecy outage: we show that Shannon’s ideal secrecy can be achieved for finite constellation input with an arbitrarily small outage probability.

Section II presents the system model. Section III describes the USK cryptosystem with finite constellation input. Section IV analyzes the security of the USK cryptosystem. Section V sets out the theoretical and practical conclusions. The Appendix contains the proofs of the theorems.

Notation: Matrices and column vectors are denoted by upper and lowercase boldface letters, and the Hermitian transpose, inverse, pseudo-inverse of a matrix B by $B^H$, $B^{-1}$, and $B^†$, respectively. We use the standard asymptotic notation $f(x) = O(g(x))$ when $\limsup_{x \to \infty} |f(x)/g(x)| < \infty$. The real, complex, integer, and complex integer numbers are denoted by $\mathbb{R}$, $\mathbb{C}$, $\mathbb{Z}$, and $\mathbb{Z}[i]$, respectively. $H(\cdot)$, $H(\cdot|\cdot)$, and $I(\cdot;\cdot)$ represent entropy, conditional entropy, and mutual information, respectively.

II. SYSTEM MODEL

A. Cooperative Jamming

The security model is based on our recently proposed MIMO cooperative jamming scheme using artificial noise [11]. This model is quite different from the conventional cooperative...
jamming scheme (multiuser jammers over AWGN channels) in [12]. The detailed differences can be referred to [11].

In our setting, we consider a MIMO wiretap system model that includes a transmitter (Alice), a legitimate receiver (Bob), and a passive eavesdropper (Eve), with $N_A$, $N_B$, and $N_E$ antennas, respectively. Additionally, a set of $N$ friendly jammers $\{J_i\}_i^N$ are used to protect against eavesdropping, where each one has $N_{J_i}$ antennas, respectively. We assume that $N_B \geq N_A$ and $N_{J_i} > N_B$. Let

$$N_J = \sum_{i=1}^N N_{J_i},$$

be the total number of antennas among all the jammers.

Alice sends a information vector $u$, which is uniformly selected from a $M$-QAM constellation $\mathcal{Q}^N$, where $\mathbb{H}(\mathcal{Q}) = \mathbb{H}(\mathcal{Q}) = \{0, 1, ..., \sqrt{M} - 1\}$. For simplicity, we ignore the shifting and scaling operations at Alice to minimize the transmit power.

Let the matrices $\{\hat{H}_{J_i}\}_{i}^N \in \mathbb{C}^{N_B \times N_{J_i}}$ represent the channels from $J_i$ to Bob, for $1 \leq i \leq N$. Suppose that $J_i$ knows $\hat{H}_{J_i}$, using the AN scheme [8], the $i$th jammer transmits

$$x_{J_i} = Z_i v_i,$$

where $Z_i = \text{null}(\hat{H}_{J_i}) \in \mathbb{C}^{N_B \times (N_{J_i} - N_B)}$ represents an orthonormal basis of the null space of $\hat{H}_{J_i}$, i.e., $\hat{H}_{J_i} Z_i = 0_{N_B \times (N_{J_i} - N_B)}$. Each $J_i$ randomly and independently (without any predefined distribution) chooses a vector $v_i \in \mathbb{C}^{N_{J_i} - N_B}$.

We set a peak jamming power constraint $P_J$, i.e.,

$$P_J \geq \sum_{i=1}^N ||x_{J_i}||^2 = \sum_{i=1}^N ||v_i||^2.$$

The signals received by Bob and Eve are given by

$$z = H u + \sum_{i=1}^N \hat{H}_{J_i} x_{J_i} + n_B = H u + n_B,$$

$$y = G u + \sum_{i=1}^N \hat{H}_{J_i} E \hat{v} + n_E,$$

where $\hat{H}_{JE} = [\hat{H}_{JE,1}, \ldots, \hat{H}_{JE,N}]$, $\hat{v} = \text{diag}(\{v_i\}_{i}^N)$, and $\hat{v} = [\hat{v}_1^T, \ldots, \hat{v}_N^T]^T$.

The matrices $H \in \mathbb{C}^{N_B \times N_A}$, $G \in \mathbb{C}^{N_B \times N_A}$ represent the channel from Alice to Bob and Alice to Eve, respectively, while $\hat{H}_{JE,i} \in \mathbb{C}^{N_B \times N_{J_i}}$ is the channel from jammer $J_i$ to Eve. All channel matrices are assumed to be mutually independent (i.e., all terminals are not co-located) and have i.i.d. entries $\sim \mathcal{C}(0, 1)$. Except for Eve, no one knows $G$ and $\hat{H}_{JE}$. We assume that $n_B$ and $n_E$ are white Gaussian noise (AWGN) vectors at Bob and Eve, respectively, with i.i.d. entries $\sim \mathcal{C}(0, \sigma_B^2)$ and $\mathcal{C}(0, \sigma_E^2)$.

**Remark 1:** The vector $\hat{v}$ is independently generated by jammers, but not needed by Bob to decipher, while it is fully affecting Eve’s ability to decipher the original message $u$. This enables us to interpret $\hat{v}$ as an unshared OTP.

**C. Shannon’s Ideal Secrecy**

We consider a cryptosystem where a sequence of $K$ messages $\{m_i\}_i^K$ are enciphered into the cryptograms $\{y_i\}_i^K$ using a sequence of secret keys $\{k_i\}_i^K$. We assume that $\{m_i\}_i^K$ and $\{k_i\}_i^K$ are mutually independent. Let $L_i$ be the space size of $k_i$. Shannon introduced the concept of ideal secrecy in [9] as: “No matter how much material is intercepted, there is not a unique solution but many of comparable probability.” In this work, we give a formal definition of ideal secrecy.

**Definition 1:** A secrecy system is ideal when

$$\text{Pr}(m_i | \{y_i\}_i^K) = \text{Pr}(m_i | y_i) = 1/L_i, \text{ for all } i.$$  

**Remark 3:** In terms of Eve’s equivocation, using the entropy chain rule, ideal secrecy is achieved when

$$H(\{m_i\}_i^K | \{y_i\}_i^K) = \sum_{i=1}^K H(m_i | y_i) = \sum_{i=1}^K \log L_i,$$

where $L_i \geq 2$ for all $i$. This condition will be used as our design criterion for ideal secrecy.
D. Lattice Preliminaries

To describe our scheme, it is convenient to introduce some lattice preliminaries. An \( n \)-dimensional complex lattice \( \Lambda_C \) in a complex space \( \mathbb{C}^m \) \((n \leq m)\) is the discrete set defined by:

\[
\Lambda_C = \{ \mathbf{Bu} : \mathbf{u} \in \mathbb{Z}[i]^n \},
\]

where the basis matrix \( \mathbf{B} = [b_1 \cdots b_n] \) has linearly independent columns.

\( \Lambda_C \) can also be easily represented as \( 2^n \)-dimensional real lattice \( \Lambda_R \) \([13]\). In what follows, we introduce some lattice parameters of \( \Lambda_C \), which have a corresponding value for \( \Lambda_R \).

The Voronoi region of \( \Lambda_C \), defined by

\[
V_i(\Lambda_C) = \{ \mathbf{y} \in \mathbb{C}^m : \| \mathbf{y} - \mathbf{x}_i \| \leq \| \mathbf{y} - \mathbf{x}_j \|, \forall \mathbf{x}_i \neq \mathbf{x}_j \},
\]
gives the nearest neighbor decoding region of lattice point \( \mathbf{x}_i \). The volume of any \( V_i(\Lambda_C) \), defined as \( \text{vol}(\Lambda_C) \triangleq |\det(\mathbf{B}^H \mathbf{B})| \), is equivalent to the volume of the corresponding real lattice. The effective radius of \( \Lambda_C \), denoted by \( r_{\text{eff}}(\Lambda_C) \), is the radius of a sphere of volume \( \text{vol}(\Lambda_C) \) \([14]\). For large \( n \), it is approximately

\[
r_{\text{eff}}(\Lambda_C) \approx \sqrt{n/(4\pi)}\text{vol}(\Lambda_C)^{1/3}.
\]

III. Unshared Secret Key Cryptosystem With Finite Constellation Inputs

In this section, we show that the idea of USK can be applied to practical systems using finite constellation inputs. We define the concept of secrecy outage and a secrecy outage probability. We will later show how such probability can be made arbitrarily small by considering larger constellation size and jamming power.

A. Encryption

We consider a sequence of \( K \) mutually independent messages \( \{m_i\}_{i=1}^K \), where each one contains \( n \) mutually independent information bits. For each \( m_i \), Alice maps the corresponding \( n \) bits to \( N_A \) elements of \( \mathbf{u} \) for one channel use. Each element of \( \mathbf{u} \) is uniformly selected from a finite lattice constellation \( \mathcal{Q}^N_A \), where \( \mathcal{R}(\mathcal{Q}) = \mathcal{Q} = \{0, 1, \ldots, \sqrt{M} - 1\} \). Consequently, we have

\[
n = N_A \log_2 M.
\]

Let \( \mathbf{u}_i \) be the transmitted vector corresponding to message \( m_i \).

Across the \( K \) channel uses, we apply a sequence of secret keys \( \{\hat{v}_i\}_{i=1}^K \) to protect \( \{\mathbf{u}_i\}_{i=1}^K \). We consider secure transmissions in a fast fading MIMO wiretap channel, i.e., all the channels \( \mathbf{H}_i, \mathbf{G}_i, \mathbf{H}_J, \mathbf{H}_E, \mathbf{H}_E, \mathbf{H}_J, \mathbf{H}_E, \mathbf{H}_J \) are assumed to be mutually independent and change independently for every channel use. We assume that \( \{\hat{v}_i\}_{i=1}^K \) and \( \{\mathbf{u}_i\}_{i=1}^K \) are mutually independent. Using \([10]\), we only need to demonstrate the encryption process for one transmitted vector \( \mathbf{u}_i \), corresponding to a message \( m_i \). We first interpret the signal model \( \hat{s} \) as an encryption algorithm:

\[
y_i = \mathbf{G}_i \mathbf{u}_i + \mathbf{H}_E \mathbf{Z}_i \hat{v}_i.
\]

In detail, for the \( i \)th channel use, the jammers randomly and independently (without any predefined distribution) choose a OTP \( \hat{v}_i \), from a ball of radius \( \sqrt{P_J} \):

\[
S \triangleq \{ \mathbf{v} \in \mathbb{C}^N - N \mathbf{N}_x : ||\mathbf{v}||^2 \leq P_J \},
\]

and encrypts \( \mathbf{u}_i \) to \( \mathbf{y}_i \) in \([13]\) using \( \hat{v}_i \), such that \( \mathbf{G}_i \mathbf{u}_i \) is the \( k_i \)th closest lattice point to \( \mathbf{y}_i \), within the finite lattice \( \Lambda_{F,i} \triangleq \{ \mathbf{G}_i \mathbf{u}_i, \mathbf{u} \in \mathcal{Q}^N_A \} \).

The value of \( k_i \) ranges from 1 to \( L_i \), where

\[
L_i = |\mathcal{S}_{R_{\max,i}} \cap \Lambda_{F,i}|,
\]

and \( \mathcal{S}_{R_{\max,i}} \) is a sphere centered at \( \mathbf{y}_i \) with radius:

\[
R_{\max,i}(P_J) \triangleq \max_{||\mathbf{v}||^2 \leq P_J} ||\mathbf{H}_E \mathbf{Z}_i \hat{v}_i|| = \sqrt{\lambda_{\max,i}} P_J.
\]

where \( \lambda_{\max,i} \) is the largest eigenvalue of \( (\mathbf{H}_E \mathbf{Z}_i)^H (\mathbf{H}_E \mathbf{Z}_i) \).

As shown in Fig. 1, the full and empty dots correspond to the infinite lattice

\[
\Lambda_{C,i} \triangleq \{ \mathbf{G}_i \mathbf{u}, \mathbf{u} \in \mathcal{Q}^N_A \}.
\]

The finite subset of \( \Lambda_{C,i} \), \( \Lambda_{F,i} \), is demonstrated by the full dots. Consequently, \( L_i \) represents the number of full dots within the sphere \( \mathcal{S}_{R_{\max,i}} \).

The security problem lies in how much Eve knows about \( k_i \). Since we assume that the realizations of \( \mathbf{G}_i, \mathbf{H}_E, \mathbf{Z}_i \) are known to Eve, \( k_i \) is a function of \( \hat{v}_i \). Since \( \hat{v}_i \) is randomly and independently selected by the jammers and is never shared with anyone, Eve can neither know its realization nor its distribution. Thus, given \( \mathbf{y}_i \), Eve is not able to estimate the distribution of the index \( k_i \), or more specifically, she only knows that \( \mathbf{G}_i \mathbf{u}_i \in \mathcal{S}_{R_{\max,i}} \cap \Lambda_{F,i} \).

**Remark 4:** The index \( k_i \) can be interpreted as the effective one-time pad secret key, whose randomness comes from \( \hat{v}_i \). The effective key space size is \( L_i \).

B. Analyzing Eve’s Equivocation

We then show that for each \( \mathbf{u}_i \), Eve cannot obtain a unique solution but \( L_i \) indistinguishable candidates. The posterior probability that Eve obtains \( \mathbf{u}_i \), or equivalently, finds \( k_i \), from the cryptogram \( \mathbf{y}_i \), is

\[
\text{Pr} \{ \mathbf{u}_i | \mathbf{y}_i \} = \text{Pr} \{ k_i | \mathbf{y}_i \} = \text{Pr} \{ \mathbf{u}_i | \mathbf{u}_i \in \mathcal{U}_i \},
\]

where

\[
\mathcal{U}_i = \{ \mathbf{u}' : \mathbf{G}_i \mathbf{u}' \in \mathcal{S}_{R_{\max,i}} \cap \Lambda_{F,i} \}.
\]
Due to the use of uniform constellation $Q^{N_A}$, according to Bayes’ theorem, we have
\[ P_r \{ u_i | u_j \in U_i \} = 1/L_i. \tag{22} \]

To recover the message $m_i$, Eve has to recover the vector $u_i$, or equivalently, find $k_i$. Therefore, Eve’s equivocation is given by
\[ H(m_i | y_i) = H(k_i | y_i) = H(u_i | y_i). \tag{23} \]
Moreover, since $u_i$ is independent of $u_j$ and $y_j$, we have
\[ H(\{u_{i1}^{K} \} | \{y_{i1}^{K} \}) = \sum_{i=1}^{K} H(u_i | y_i) = \sum_{i=1}^{K} \log L_i. \tag{24} \]

**Remark 5:** From \((11)\) and \((24)\), ideal secrecy is achieved if
\[ L_i \geq 2, \text{ for all } i. \tag{25} \]

**C. Ideal Secrecy Outage**

We then study how to satisfy the condition in \((25)\). Note that the values in \(\{L_i\}_1^{K}\) are known to Eve, but not Alice. From Alice’s perspective, according to \((17)\) and \((21)\), $L_i$ is a function of $G_i, H_{JE,i}, Z_i, \hat{v}_i$, thus a random variable. Although Alice cannot know the exact values in \(\{L_i\}_1^{K}\), she may be able to evaluate the joint probability $Pr \{ L_1 \geq d, ..., L_K \geq d \}$, for any $2 \leq d \leq M^{N_A}$.

We refer to the event
\[ L_i < d, \tag{26} \]
as the **ideal secrecy outage**. We refer to the probability
\[ P_{out}(d, K) = 1 - Pr \{ L_1 \geq d, ..., L_K \geq d \}, \tag{27} \]
as the secrecy outage probability. From \((25)\) and \((27)\), if $P_{out}(d, K) \to 0$, then
\[ L_i \geq d, \text{ for all } i. \tag{28} \]

In the next section, we will show that Alice can ensure $P_{out}(d, K) \to 0$ by increasing the jamming power $P_J$ and constellation size $M$.

**IV. THE SECURITY OF USK**

In this section, we show that the USK with the finite constellation $Q^{N_A}$ provides Shannon’s ideal secrecy with an arbitrarily small outage.

**A. A Useful Lemma**

We define
\[ \Theta(P_J) = \frac{2R_{max}(P_J)}{\sqrt{M}r_{eff}(\Lambda_C)}, \tag{29} \]
where $\Lambda_C$ is given in \((19)\), $r_{eff}(\Lambda_C)$ is given in \((12)\), and $R_{max}(P_J)$ is given in \((13)\).

From Alice’s perspective, $\Theta(P_J)$ is a function of $G_i, \hat{H}_{JE,i}, \hat{Z}_i$, thus is a random variable. To prove the main theorem, we first introduce the following lemma.

**Lemma 1:**
\[ Pr \{ \Theta(P_J) < x \} \leq \prod_{j=1}^{N_A} B \frac{N_E N_j g(x, j)}{N_E N_j g(x, j) + N_E - j + 1} \tag{30} \]
where
\[ g(x, j) = \frac{x^2 MN_A (N_E - j + 1)}{4 \pi e P_J N_E N_1 (N_E - N \cdot N_B)}, \tag{31} \]
and $B_{a,b}(x)$ is the regularized incomplete beta function \([15]\
\[ B_{a,b}(x) \triangleq \sum_{j=0}^{a+b-1} \binom{a+b-1}{j} x^j (1-x)^{a+b-1-j}. \tag{32} \]

**Proof:** See Appendix A.

**B. Ideal Secrecy Outage Probability**

An upper bound on $P_{out}(d, K)$ in \((27)\) can be derived using Lemma \[1\]

**Theorem 1:** Given $\varepsilon < 1$, $d \geq 2$, $M \geq e^{-3-2/N_{min}\kappa(d)^2}$, and $P_J = e^{2/N_{min}\kappa(d)^2}/\Phi^{2N_A/N_E}$, then
\[ P_{out}(d, K) < O(\varepsilon), \tag{33} \]
where
\[ N_{min} \triangleq \min \{ N_E - N_A + 1, N_A \}, \tag{34} \]
\[ \kappa(d) \triangleq d^{1/(2N_E)} / \sqrt{\pi}, \tag{35} \]
\[ \Phi \triangleq \left( \frac{N_E - N_A}{N_E} \right)^{N_A} \tag{36} \]
i.e., ideal secrecy is achieved with probability $1 - O(\varepsilon)$.

**Proof:** See Appendix B.

Theorem \[1\] shows that for finite constellation $Q^{N_A}$, the ideal secrecy outage can be made arbitrarily small. Given a target pair \(\{\varepsilon, d\}\), we can easily compute the required values of $P_J$ and $M$ to realize the USK cryptosystem.

**Example 1:** Fig. 2 examines the value of $P_{out}(2, 1)$ as a function of $\varepsilon$. We choose $P_J$ and $M$ according to
\[ P_J = e^{-2/N_{min}\kappa(d)^2}/\Phi^{2N_A/N_E} \text{ and } M \geq e^{-3-2/N_{min}\kappa(d)^2}. \]
We observe that $P_{out}(2, 1) = 1.6 \times 10^{-4}$ when $P_J = 3.5926$ and $M = 256$. This simulation confirms that the secrecy outage for the finite constellation $Q^{N_A}$ can be made arbitrarily small by increasing $P_J$ and $M$.

**Remark 6:** Using the proof of Theorem \[1\] we can show
\[ Pr \{ L_1 = 1, ..., L_K = 1 \} < O(\varepsilon^K). \tag{37} \]

In order to enhance security, Alice can scramble the $nK$ message bits in \(\{m_i^{(1)}\}_1^{K}\) by using an invertible binary $nK \times nK$ matrix $S$ to produce the sequence
\[ \{m_i\}_1^{K} = \{m_i^{(1)}\}_1^{K} S. \tag{38} \]

Only with a probability of the order of $\varepsilon^K$ given in \((37)\) Eve would be able to uniquely recover \(\{m_i^{(1)}\}_1^{K}\) by inverting $S$. In all other cases, occurring with probability $\varepsilon^K$, where Eve recovers only $K' < K$ sub-blocks $m_i'$, the system in \((38)\) will have $2^{n(K-K')}$ different solutions. This enhances the overall ambiguity of Eve’s when guessing the entire message $\{m_i^{(1)}\}_1^{K}$. 

Moreover, since $G, \mathbf{H}_{ij}, \mathbf{Z}_i$ are mutually independent, $R_{\text{max}}(P_3)$ and $r_{\text{eff}}(\Lambda_C)$ are independent.

Then, we have

$$
\Pr\left\{ \frac{2R_{\text{max}}(P_3)}{\sqrt{Mr_{\text{eff}}(\Lambda_C)} < x} \right\} 
\geq \Pr\left\{ P_3 \left\| \mathbf{H}_{ij} \right\|_F^2 < \frac{x^2 M}{4(Nj - N \cdot NB)} \right\}
$$

$$
\geq \Pr\left\{ \frac{\sum_{j=1}^{N_A} \chi^2(N_E - j + 1)}{N_A} < \frac{x^2 M N_A}{4\pi e P_3(Nj - N \cdot NB)} \right\}
$$

$$
= \prod_{j=1}^{N_A} \Pr\left\{ \frac{\sum_{j=1}^{N_A} \chi^2(N_E - j + 1)}{N_A} < \frac{x^2 M N_A}{4\pi e P_3(Nj - N \cdot NB)} \right\}
$$

where $g(x, j)$ is given in (31), and $F(k_1, k_2)$ represents an $F$-distributed random variable with $k_1$ and $k_2$ degrees of freedom. (a) holds due to the inequality of geometric and harmonic means. (b) holds by induction on the fact that if the non-negative random variables $A_i$, $1 \leq i \leq N$, are mutually independent, given a constant $C > 0$,

$$
\Pr\left\{ \sum_{i=1}^{N} A_i < C \right\} > \Pr\left\{ A_1 \leq C/N; \sum_{i=2}^{N} A_i \leq C(N - 1)/N \right\}
$$

$$
= \Pr\left\{ A_1 \leq C/N \right\} \Pr\left\{ \sum_{i=2}^{N} A_i \leq C(N - 1)/N \right\} .
$$

Since the cdf of $F(k_1, k_2)$ can be expressed using the regularized incomplete beta function $\text{B}_i$, the final expression of (44) is given in (30).

\[ \text{B. Proof of Theorem 1} \]

From Alice’s perspective, $L_i$ is a function of $G_{ij}, \mathbf{H}_{ij}, \mathbf{Z}_i$, and $\varphi_i$. Since $\{G_{ij}\}_1^K, \{\mathbf{H}_{ij}\}_1^K, \{\mathbf{Z}_i\}_1^K, \{\varphi_i\}_1^K$ are mutually independent, $\{L_i\}_1^K$ are mutually independent. From (27),

$$
\Pr\{ \sum_{i=1}^{N} L_i < d \} = 1 - \prod_{i=1}^{K} \Pr\{ L_i \geq d \} .
$$

We then evaluate $\Pr\{ L_i < d \}$. For simplicity, we remove the index $i$. We define

$$
D = |S_{R_{\text{max}}} \cap \Lambda_C| .
$$

According to [7, Th. 2], with $P_3 = e^{-2/N_{\text{min}}(k)a^2}/\sqrt{2N_A/N_B}$, the jammers can ensure

$$
\Pr(D < d) < O(\varepsilon) ,
$$

\[ \text{V. CONCLUSIONS} \]

In this work, we showed how to construct a practical unshared secret key (USK) cryptosystem using finite constellation inputs and some helpers. The new USK scheme is specially designed for the scenario $N_E \geq N_A$, where the original USK scheme is not valid. We have shown that Shannon’s ideal secrecy can be obtained with an arbitrarily small outage probability, by simply increasing the constellation size and jamming power. Our results provide new ideas for the innovations and combinations of cryptography and physical layer security. Future work will generalize USK to relaying networks.

\[ \text{APPENDIX} \]

\[ \text{A. Proof of Lemma 1} \]

Recalling that

$$
R_{\text{max}}(P_3) = \max_{||\Phi|| \leq P_3} \left\| \mathbf{H}_{ij} \mathbf{Z} \right\|_F ,
$$

$$
r_{\text{eff}}(\Lambda_C) = 2\sqrt{N_A/(\pi e)} \det(G^H G)^{1/2} .
$$

From (18), applying Cauchy–Schwarz inequality,

$$
R_{\text{max}}^2(P_3) \leq P_3 \left\| \mathbf{H}_{ij} \mathbf{Z} \right\|_F^2 \leq P_3 \left\| \mathbf{H}_{ij} \right\|_F^2 \left\| \mathbf{Z} \right\|_F^2
$$

$$
= P_3 (Nj - N \cdot NB) \left\| \mathbf{H}_{ij} \right\|_F^2 .
$$

From Alice perspective, $\mathbf{H}_{ij}$ is a complex Gaussian random matrix with i.i.d. components. Thus, $\left\| \mathbf{H}_{ij} \right\|_F^2$ can be expressed in terms of a Chi-squared random variable:

$$
\left\| \mathbf{H}_{ij} \right\|_F^2 = \frac{1}{2} \chi^2(2N_E N_j) .
$$

According to (16), $r_{\text{eff}}(\Lambda_C)$ can be expressed in terms of $N_A$ independent Chi-squared variables:

$$
r_{\text{eff}}(\Lambda_C) = \sqrt{N_A/(\pi e)} \left( \prod_{j=1}^{N_A} \frac{1}{2} \chi^2(2(N_E - j + 1)) \right)^{1/2} .
$$

Fig. 2. $P_{\text{out}}(2, 1)$ vs. $\varepsilon$ with $N_A = N_B = 2$, $N_{11} = N_{12} = 3$, and $N_E = 4$. 
where \( N_{\min} \) is given in (24), \( \kappa(d) \) is given in (35), and \( \Phi \) is given in (56). We can upper bound \( \Pr \{ L < d \} \) by

\[
\Pr \{ L < d \} = \Pr \{ L < d \mid D \geq d \} \Pr \{ D \geq d \} + \Pr \{ L < d \mid D < d \} \Pr \{ D < d \} \\
\leq \Pr \{ L < d \ \mid D \geq d \} + \Pr \{ D \geq d \} + O(\varepsilon) \\
\leq \Pr \{ L < d \} + O(\varepsilon). \tag{49}
\]

We then evaluate \( \Pr \{ L < d \} \).

\[
\Pr \{ L < d \} = \Pr \{ L < d \mid \Theta(P) \} < \varepsilon \Pr \{ \Theta(P) \} < \varepsilon \\
+ \Pr \{ L < d \mid \Theta(P) \} \geq \varepsilon \Pr \{ \Theta(P) \} \geq \varepsilon \\
\leq \Pr \{ L < d \mid \Theta(P) \} < \varepsilon + \Pr \{ \Theta(P) \} \geq \varepsilon, \tag{50}
\]

where \( \Theta(P) \) is given in (29).

We then evaluate the two terms in (50), separately.

1) \( \Pr \{ L < d \mid \Theta(P) \} < \varepsilon \).

Recalling that \( y = Gu + H_{\text{FE}} \hat{Z} v \) and \( A_{\text{F}} = \{ Gu, u \in \mathbb{Q}^{N_{\text{A}}} \} \), (51) Since \( L = |S_{R_{\max}} \cap A_{\text{F}}| \), we begin by checking the boundary of \( A_{\text{F}} \). Let \( O \) be the center point of \( A_{\text{F}} \). According to (17), for the Gaussian random lattice basis \( G \), the boundary of \( A_{\text{F}} \) can be approximated by a sphere \( S_{\text{FE}} \) centered at \( O \) with radius \( \sqrt{M_{\text{eff}}(A_{\text{F}})} \), where \( M_{\text{eff}}(A_{\text{F}}) \) is given in (12).

Given \( \Theta(P) < \varepsilon \) and \( \varepsilon < 1 \), we have \( \sqrt{M_{\text{eff}}(A_{\text{F}})} > 2R_{\max}(P) \). We define a concentric sphere \( S_{\text{FE}} \) with radius \( \sqrt{M_{\text{eff}}(A_{\text{F}})} - 2R_{\max}(P) \), where \( R_{\max}(P) \) is given in (18). We then check when \( L = d \), \( \Theta(P) \) falls outside of \( S_{\text{FE}} \), using triangle inequality, we have

\[
\| y - O \| \leq \| Gu - O \| + \| H_{\text{FE}} \hat{Z} v \| \leq \sqrt{M_{\text{eff}}(A_{\text{F}})} - R_{\max}(P). \tag{52}
\]

We then check the locations of the \( D \) elements in \( S_{R_{\max}} \cap A_{\text{C}} \) (47), denoted by \( \{ Gu_t \} \), \( 1 \leq t \leq D \). Note that

\[
\| Gu_t - y \| \leq R_{\max}(P). \tag{53}
\]

From (52) and (53), using triangle inequality, for all \( t \),

\[
\| Gu_t - O \| \leq \| y - O \| + \| Gu_t - y \| \leq \sqrt{M_{\text{eff}}(A_{\text{C}})}. \tag{54}
\]

Therefore, \( S_{R_{\max}} \cap A_{\text{C}} \subset A_{\text{F}} \), i.e., \( L = D \).

If \( GV \notin S_{\text{FE}} \), there is a probability that \( L < D \). Therefore, we have

\[
\Pr \{ L < D \mid \Theta(P) \} < \varepsilon \Pr \{ Gu \notin S_{\text{FE}} \}. \tag{55}
\]

Since \( Gu \) is uniformly distributed over \( S_{\text{FE}} \), we have

\[
\Pr \{ Gu \in S_{\text{FE}} \} = \frac{\text{vol}(S_{\text{FE}})}{\text{vol}(S_{\text{FE}})} = 1 - \Theta(P)^2 N_{\text{A}} > (1 - \varepsilon)^2 N_{\text{A}} \tag{56}
\]

Based on (55) and (56), we have

\[
\Pr \{ L < D \mid \Theta(P) \} < \varepsilon < 1 - (1 - \varepsilon)^2 N_{\text{A}} = O(\varepsilon). \tag{57}
\]

2) \( \Pr \{ \Theta(P) \geq \varepsilon \} \) Using Lemma 1 with \( M \geq \varepsilon^{-3 - 2/N_{\min} \kappa(d)^2} \), we have

\[
\Pr \{ \Theta(P) \leq \varepsilon \} \geq \prod_{j=1}^{N_{\text{A}}} B_{a,b(j)}(1 - \frac{b(j)}{ag(\varepsilon,j) + b(j)}) \geq \prod_{j=1}^{N_{\text{A}}} \left(1 - O(\varepsilon^{b(j)})\right) \tag{58}
\]

where \( \hat{N} = N_{\text{E}} - N_{\text{A}} + 1 \) and

\[
\alpha = N_{\text{E}} N_{\text{j}} \text{ and } b(j) = N_{\text{E}} - j + 1. \tag{59}
\]

(a) and (b) hold due to the facts that

\[
B_{a,b(j)}(x) = 1 - \frac{b(j)}{ag(\varepsilon,j) + b(j)}(1 - x), \tag{60}
\]

\[
B_{a,b(j)}(x) = O(x^{b(j)}), \text{ for } x \to 0. \tag{61}
\]

Consequently, we have

\[
\Pr \{ \Theta(P) \geq \varepsilon \} < 1 - \left(1 - O(\varepsilon^{\hat{N}})\right)^{N_{\text{A}}} = O(\varepsilon^{\hat{N}}). \tag{62}
\]

By substituting (50), (57) and (62) to (49), we have

\[
\Pr \{ L < d \mid \Theta(P) \} < O(\varepsilon). \tag{63}
\]

From (46) and (55), if \( M \geq \varepsilon^{-3 - 2/N_{\min} \kappa(d)^2} \) and \( P_{\text{E}} = \varepsilon^{-2/N_{\min} \kappa(d)^2} / \Phi^2 N_{\text{A}} / N_{\text{E}} \), we have

\[
P_{\text{PAS}}(d, K) < 1 - (1 - O(\varepsilon))^K = O(\varepsilon). \tag{64}
\]

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