The Design and Test of Elliptical-Circular Bicycle Sprockets

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Abstract. Most people have different strength and frequency of the movements between the left and the right body side. In riding a bicycle with circular sprocket, the torques produced by the right leg and the left leg are generally different. Usually, the dominant leg exerts larger force than the non-dominant leg. Based on these conditions, in this paper, the application of elliptical-circular bicycle sprockets with the right size and position on the leg can provide more advantages compared to the ordinary circular sprocket in cycling. The crank torque comparisons reveal that the application of the elliptical-circular sprocket can reduce the torque produced by the right leg, and increase the torque produced by the left leg. This proves that the elliptical-circular sprocket is useful for balancing the force produced between the left leg and the right leg. Then, from the leg joint comparisons, the elliptical-circular sprocket can reduce the torque in the hips, knees, and the ankles. This means that the use of elliptical-circular sprocket can reduce the legs fatigue in riding a bicycle.

1. Introduction

Most people have different strength and frequency of the movements between the left and the right body side [1]. Some studies have supported that the motion of humans are usually the right dominant of activities requiring mobilization and left dominant of activities requiring stabilization and postural strength [2-6].

Velotta et al. [7] reported that leg dominance seems to be a function of the type of activity a subject required to perform. When the task is manipulative in nature, most subjects use the right leg (most people are right-side preference), but when the task involves stabilization, most of the subjects in the study use left leg to perform the task. Spry et al. [3] reported that right leg dominant people tended to use the right leg for manipulative skills and the left leg for supportive or weight-bearing skills. The reverse was true for the left leg dominant people. Peters [8] defined the dominant leg as ‘the leg used in order to manipulate an object or to lead out in movement’. Sadeghi et al. [9] reported during stabilization task, the opposite leg is used for dynamic counter-balance and since the right leg is better at manipulative activities, the left leg is used for stabilization. Duque et al. [10] investigated torque variation from pedal motion in cycling. Torques produced by the right foot and the left foot in pedalling are different. Different peak values indicate an asymmetry due to emphasizing an unequal pushing action during the downstroke. Faharani et al. [11] and Bini et al. [12] have similar result when investigating peak crank torque in pedalling as shown in Figure 1.
In this paper, the radius of the ordinary circular sprocket (OC sprocket) were manipulated by adjusting the number of teeth and the radius when the non-dominant leg are pedalling in order to increase the level of leg torque. The sprocket produced in this study is a combination of ellipse and circle, namely elliptical-circular sprocket (EC sprocket).

2. Methods
2.1. Instrumentation
The measurement tool applied in this paper is a bicycle test platform that transmits the obtained data signals to the computer through various sensors and presents the required data via the software. The front and rear rotary encoders shown in Figures 2a are used to measure the rotational speed of the crank and the rear wheel, respectively. The bottom bracket and the smart training platform shown in Figures 2b and 2c, are equipped with pulleys and belts to drive the encoder to measure the rotation speed of the crank and the rear wheel. The strain gauges are placed on the front and back sides of the left and right cranks shown in Figure 2d and 2e. The wireless receiving module transmits the strain gauge signal generated by the cranks of both sides during pedalling to another wireless receiving module in the storage box connected to the computer.

In this paper, the bicycle load is created by the smart training platform produced by Wahoo Fitness and use a Bluetooth to connect the mobile phone to adjust the resistance as shown in Figure 2c. The simulated weight is 175 pounds (79.38 kg). The electromagnetic load applicator provides a stable electromagnetic load for the overall riding process, while dynamometer shown in Figure 2a measures the force generated by the training platform. The CCD camera is used to catch the illuminator device on the leg and on the pedal crank for the calculation of the angle formed.

Figure 1. (a) The averaged crank torque on level terrain at preferred cadence [11], (b) Crank torque measured by dominant and non-dominant pedals [12].

Figure 2. (a) Developed bicycle test platform, (b) The bottom bracket with belts to drive the front encoder, (c) the smart training platform produced by Wahoo Fitness, (d) strain gauge on the left crank, and (e) strain gauge on the right crank.
2.2. Novel sprocket design for averaging torque applied to the left and right
To overcome the unequal torque applied by the left and the right legs, the pitch curve of the sprocket driven by the left foot can be changed to be the ellipse, while the pitch curve for the right foot is not changed. The result will be an elliptical-circular sprocket (a combination of ellipse and circle). The sprocket then has two different shaped sections. The long axis is for the left foot forces section, thereby generating a large torque. The elliptical eccentricity needs to be limited, otherwise, the bicycle is likely caused a power drop in riding [13]. For the number of teeth, the number of teeth in the left and right section (ellipse and circle) must be even and the pitch must be equal. The combination of the sprocket is presented in table 1. In order to make sure the smooth in combination of two shapes, the radius of the short axis of the ellipse is equal to the radius of the circular section.

| Type           | Left foot section sprocket | Right foot section sprocket |
|----------------|----------------------------|----------------------------|
|                | Number of teeth | Eccentricity | Long axis (mm) | Short axis (mm) | Number of teeth | radius (mm) |
| EC 0.45-0 U    | 34            | 0.4502       | 72.5518        | 64.6806         | 32             | 64.6806     |
| EC 0.59-0 U    | 36            | 0.5913       | 80.3301        |                |                |             |

2.3. Sprocket generation
In this paper, the sprocket generation is based on the equations proposed by Chang [14] to make the sprocket. Firstly, the pitch curve must be calculated, and then the sprocket tooth profile shown in Figure 3 can be configured in the normal direction of ellipse. The circumference \( S \) of the pitch curve of ellipse is set to be an integral multiple of the chain pitch.

\[
S = \int_{\phi}^{\pi/2} \frac{2\pi a (1-\varepsilon^2) \sqrt{\varepsilon^2 + 2 \varepsilon \cos \phi + 1}}{(1+\varepsilon \cos \phi)^2} \, d\phi
\]

Further, the calculation can be solved by Simpson’s 1/3 rule integral formula. Since the circumference of the pitch curve needs to be multiple integrals of the pitch, no incomplete tooth shape will occur when all the teeth profiles are placed. In this study, the bi-section method is used to obtain the position angle of each tooth. After obtaining the each position of tooth, the coordinate of each tooth can be obtained. The normal direction of sprocket on the ellipse can be established with the right-hand rule by:

\[
n = \frac{\varepsilon + \cos \phi}{\sqrt{\varepsilon^2 + 2 \varepsilon \cos \phi + 1}} i + \frac{\sin \phi}{\sqrt{\varepsilon^2 + 2 \varepsilon \cos \phi + 1}} j
\]
then use the inverse trigonometric function to convert the normal direction to the angle:

$$\theta = \tan^{-1}\left(\frac{\sin \phi}{\varepsilon + \cos \phi}\right)$$  \hspace{1cm} (4)

Substituting the eccentricity and the number of teeth required in the foot segment of Table 1 into the equation, then the elliptical-circular sprocket can be generated by using AutoCAD software. JIS U type tooth profile [15] is considered to be the teeth shape. The resulting elliptical sprocket curve is cut in half, then using the annular array method, the tooth can then be placed on the circular sprocket. The combined graphics of the sprocket are shown in Figure 4.

![Combined Graphics of Sprocket](image)

**Figure 4(a).** Elliptical-circular sprocket eccentricity 0.45-0U, and (b) eccentricity 0.59-0U.

### 3. Results and discussion

#### 3.1. Crank speed

Figure 5 shows the comparison of the crank speed between the applications of the ordinary circular sprocket (OC sprocket), the elliptical-circular sprocket (EC sprocket) 0.45-0U, and 0.59-0U. From the figure, it reveals that the speed of the OC sprocket crank is stable at around 46-48 rpm. For EC 0.45-0U, the crank speed is greater than the OC sprocket. When the right leg is pedalling at 0°-180°, the crank speed is stable between 48-50 rpm. But when the left leg starts pedalling at 180°, the speed rises slightly then goes down until 270°, reaches around 44 rpm. This is because the stroke is in the largest sprocket radius position. After that, the speed increases again until 360°. For EC 0.59-0U, the crank speed is higher than the others when the crank position is at 0-180°. It shows the same phenomenon with the EC 0.45-0U, the speed rises slightly then goes down until 270°, reaches around 41 rpm. This speed is the lowest compared to the other sprocket because it has the largest major axis.

![Crank Speed Comparison](image)

**Figure 5.** Crank speed comparison.

#### 3.2. Crank torque

Figure 6 shows the comparison of the crank torque, which is similar to the results proposed by Bini and Hume [10]. In 0° – 180°, the force is applied by the right leg, while in 180° – 360° the force is applied by the left leg. The resulting torque difference between the right leg and the left leg is about 40%. The main reason is the strength of the left leg is less than the right leg. From the result of EC 0.59-0U, the right leg is pedalling at 0°-180°, the crank torque is smaller than the OC crank torque. When the left leg is pedalling at 180°-360°, EC 0.59-0U can produce more torque. From the comparisons, it reveals that the use of EC 0.59-0U and EC 0.45-0U is able to increase torque on the left leg and reduce the torque difference produced by the right leg and the left leg. This torque difference is around 27.7% using EC 0.59-0U, and around 31% using EC 0.45-0U.
3.3. Leg joint torque
The leg-bicycle system can be modelled as a closed loop of five-bar linkage with the frame as the fixed link. There are consist of three joint human legs: hip joint, knee joint, and ankle joint, as shown in Figure 7.

Figure 8 shows the hip joint torque compared with the application of OC sprocket, EC 0.45-0U sprocket, and EC 0.59-0U sprocket. The right hip joint generates peak torque when the crank angle position is 115°-135°. And the left hip joint generates peak torque when the crank angle position is 270°-300°. Even though in that position, the torque produced in the crank is not at the peak value. When using elliptical-circular sprocket (EC), especially EC 0.45-0U the peak torque produced by the hip joint can be the minimum.

Figure 9 shows that the right knee joint generates peak torque compared with the application of OC sprocket at 90°-130°. And the left knee joint generates peak torque when the crank angle position at around 270°. Using EC 0.59-0U sprocket, the peak of the right knee joint can be reduced, but the torque value increases on the left knee when the crank position is at 270°, larger than the torque produced by the OC sprocket. The peak torque on the right knee joint can be reduced significantly by EC 0.45-0U.

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**Figure 6.** Crank torque comparison.

**Figure 7.** 5-bar linkage model of the leg-bicycle system [16].

**Figure 8.** Hip joint torque comparison.

**Figure 9.** Knee joint torque comparison.
The results shown on Figure 10 are similar to the knee joint torque comparison shown in Fig. 10. It appears that the peak value of torque is produced by the right ankle joint when the crank is in the position at around 90° using OC sprocket. This torque can be reduced by using EC sprocket. Using EC 0.45-0U, the torque on the right ankle joint can be reduce more than the application of EC 0.59-0U.

4. Conclusions

This paper shows how the elliptical-circular sprocket is designed to balance the torque produced by the dominant leg and the non-dominant leg in cycling, and compared with the ordinary circular sprocket. From the crank torque comparisons, it reveals that the right leg and the left leg produce different torques. The use of the elliptical-circular sprocket can reduce the torque produced by the right leg, and increase the torque produced by the left leg. This proves that the elliptical-circular sprocket is useful for balancing the force produced between the left leg and the right leg. In this case, the effect of elliptical-circular sprocket 0.59-0U is better than 0.45-0U.

From the leg joint comparisons, it can be seen that the use of the elliptical-circular sprocket can also reduce the torque in the hips, knees, and the ankles. The elliptical-circular sprocket 0.45-0U is better than the other sprocket in reducing torque in the leg joint. This means that the use of elliptical-circular sprocket can reduce the legs fatigue in riding a bicycle.

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