Settlement calculation method for single pile in negative friction case considering consolidation of surrounding soils

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ABSTRACT

Surcharge loading, phreatic level decline and other factors cause negative skin friction on the peripheral surface of piles, which increases engineering insecurity. Negative skin friction has been researched deeply by filed, scale and numerical test. However, few studies have considered the influences of drainage boundary conditions on pile settlement considering consolidation of surrounding soils. Based on 1D perfect elasto-plastic model, this paper demonstrates that pile-soil elastic relative displacement at the position of neutral point is zero, and derives a simplified settlement calculation method. Calculation results show that change of neutral point position makes the pile-soil relative displacement not zero. Finally, the proposed method is verified by FEM method and used to analyze the influence of different drainage cases on the pile settlement.

Keywords: negative skin friction; single pile; neutral point; pile settlement; consolidation

1 INTRODUCTION

Negative skin friction (NSF) often happens as a result of the downward movement of the soil relative to the pile constructed in consolidating ground. Reasons for NSF include surcharge loading, under-consolidation soil and phreatic level (Zhou 2007). NSF reduces pile bearing capacity and increases axial force and settlement, leading to serviceability problems (Li 2014). Based on filed experiments, Fellenius (2006) observed that NSF can produce additional pile axial force for more than 3000kN. Walker and Darvall (2013) reported that surcharge loading could lead to 35mm ground settlement and NSF developed to the depth of 18m in single pile foundation.

Neutral point, at where the pile axial force is the largest and NSF transfers to positive skin friction, is the key issue to research NSF (Fellenius 1984). Based on full-scale pile test within 1960s~1990s, Fellenius (2006) reported that pile-soil skin friction increased significantly, besides, the neutral point came up with the dissipation of pore water pressure. The NSF development characters for friction pile are studied by centrifuge tests (Lam et al. 2009; Ng et al. 2008). Tong and Zhu (2006) and Liu (2012) used FEM method to acknowledge this problem and got same conclusions.

Position of neutral point is typically determined by two lines for summing axial loads from the top down and from the bottom up, and the intersection is the position of neutral point, as shown in Fig.1. Because small pile-soil relative displacements are required to reach capacity, the shaft friction is assumed to be mobilized along the full length. Based on view that skin friction is zero at the neutral point, Fellenius (1984) postulated that soil settles equally with pile at position of neutral point. In addition, Technical Code for Building Pile Foundations (China Academy of Building Research 2008) and Foundation Engineering (Zhou 2007) also support this argument and use it to calculate downdrag settlement in NSF case.

Different drainage cases have different change rules for position of neutral point: position of neutral point goes down in top-drainage case, goes up in bottom-drainage case, and changes little in double-drainage case. However, positions of neutral point in different drainage cases are same at the end of consolidation. If pile-soil relative displacement is zero at the position of neutral point, the final pile settlements will be equivalent because positions of neutral point and soil settlements are same in different drainage case.

However, pile settles differently in different drainage case, Wang et al. (2013) found that final pile settlement in the top-drainage case is 14% higher than in the bottom-drainage case. And phenomenon that pile settlement in the top-drainage case is highest also gets support of our FEM calculation results. In top-drainage case, pile settles more than in the bottom-drainage, and...
different reaches to a non-negligible value of 14%. So, the argument that pile-soil relative displacement is zero at the neutral point has its limitation when consider the consolidation process.

Based on analysis of 1-D prefect elasto-plastic model, this paper discusses the correctness of view that pile-soil relative displacement is zero at the position of neutral point. Then, a simplified method for settlement calculation of single pile in NSF case is derived and used to analysed inner reason of different final settlements in different drainage cases. Finally, this paper compares proposed method results with FEM method to verify the applicability.

2 ELASTIC RELATIVE DISPLACEMENT

For 1-D perfect elasto-plastic model, relationship between fore and displacement is as follow.

\[
\begin{align*}
F &= k\Delta_s \\
\Delta &= \Delta_s + \Delta_p
\end{align*}
\]

Elastic deformation \(\Delta_s\) is zero when \(F\) is zero. If \(\Delta_s\) happens as fig. 2, the final \(\Delta_s\) will equal to \(\Delta_p\), not zero. And final \(\Delta_p\) equals maximum deformation minus elastic deformation capacity.

Let \(z(t)\) is the depth of neutral point at time \(t\). Because the position of neutral point \(z(t)\) is changing with process of consolidation, the shaft friction at depth of \(z(t)\) is not zero before time \(t\), and the relative displacement is not zero. If the relative displacement is large enough to produce plastic displacement, the relative displacement will not be zero at the position of neutral point. In conclusion, shaft friction at the position of neutral point is zero, indicating that plastic relative displacement, not relative displacement, is zero.

Depth of neutral point at the end of consolidation is \(z(\infty)\). In top-drainage case, the position of neutral point goes down all the time, so \(z(\infty)\) is the lower than \(z(t)\). At intermediate degrees of consolidation, \(z(\infty)\) is in the positive skin friction area, and pile settles more than soil. If the relative displacement is large enough to bring plastic relative displacement, pile will slip downward relative to soil, which makes pile settle more than soil at depth of \(z(\infty)\).

In bottom-drainage case, because position of neutral point goes up all the time, the opposite happened. As a result, pile will slip upward relative to soil, and pile settles less than soil at depth of \(z(\infty)\).

In different drainage cases, depths of \(z(\infty)\) and soil settlements are equivalent. So, final pile settlement in the top-drainage case is larger than in the bottom-drainage case, which consists with the test results.

3 SETTLEMENT CALCULATION METHOD

Lager soil settlement makes the pile-soil friction reverse and causes NSF. Because stiffness of pile is far greater than of soil, compression of pile is far less than soil settlement in NSF case. Therefore, taking pile as a rigid body and ignoring pile compression will make little effect on analysis of pile settlement.

Based on pile-soil elasticity relative displacement being zero at the depth of neutral point, pile settlement calculation methods are put forward as follow. Let \(z(t)\) is the depth of neutral point at time \(t\). Pile settlement at time \(t\) equals soil settlement at neutral point \(z(t)\) plus pile-soil plastic relative displacement at neutral point \(z(t)\). Pile-soil plastic relative displacement at time \(t\) equals maximum pile-soil relative displacement at \(z(t)\) depth before time \(t\) minus limit elastic relative displacement.

\[
\begin{align*}
\Delta_p(t) &= s_{soil}(z(t), t) + \Delta_p(z(t), t) \\
\Delta_p(z(t), t) &= \max_{t' \in [0, t]} \Delta(z(t), t') - \Delta_p \quad \text{(top-drainage)} \\
\Delta_p(z(t), t) &= \min_{t' \in [0, t]} \Delta(z(t), t') + \Delta_p \quad \text{(bottom-drainage)} \\
\Delta(z(t), t') &= s_{pil}(t') - s_{soil}(z(t), t')
\end{align*}
\]

Where \(s_{pil}(t)\) = pile settlement at time \(t\); \(s_{soil}(z(t), t)\) = soil settlement of depth \(z(t)\) at time \(t\); \(\Delta_p(z(t), t)\) = pile-soil plastic relative displacement of depth \(z(t)\) at
time \( t \); \( \Delta_e = \) pile–soil limit elastic relative displacement; \( \Delta(z(t), t') = \) pile–soil relative displacement of depth \( z(t) \) at time \( t' \); \( s_{soil}(z(t), t') = \) soil settlement of depth \( z(t) \) at time \( t' \), which can be obtained by seepage consolidation theory.

In formula (3), if \( \max[\Delta(z(t), t')] - \Delta_e = 0 \), there is no plastic relative displacement at neutral point and take \( \Delta_p(z(t), t') = 0 \) as zero.

If position of neutral point does not move \( (z(t) \) is constant), \( \Delta(z(t), t') \) in formula (4) equals zero constantly and the plastic relative displacement does not occur; If position of neutral point changed little, the absolute value of \( \Delta(z(t), t') \) will be small and the plastic will not happen.

In top-drainage case, position of neutral point goes down and \( \Delta(z(t), t') \) is positive, so the plastic relative deformation is positive and pile settles more than soil at depth of neutral point. In bottom-drainage case, position of neutral point goes up and \( \Delta(z(t), t') \) is negative, as a result, the plastic relative deformation is negative and pile settles less than soil at neutral point. For double-drainage, position of neutral point changes little and plastic relative displacement is small. However, if position of neutral point changes greatly, consider double-drainage as top-drainage when neutral point rises, and as bottom-drainage when neutral point goes down.

Choose discretizing time \( t_j, j = 1, 2, 3 \cdots, k \), and make the problem more convenient.

\[
s_{soil}(z(t_j), t_j) = s_{soil}(z(t_j), t_j) + \Delta_p(z(t_j), t_j)
\]

\[
\Delta_p(z(t_j), t_j) = \begin{cases} 
\max \Delta(z(t_j), t_j) - \Delta_e & \text{top-drainage} \\
\min \Delta(z(t_j), t_j) + \Delta_e & \text{bottom-drainage} 
\end{cases}
\]

\[
\Delta(z(t_j), t_j) = s_{pile}(t_j) - s_{soil}(z(t_j), t_j)
\]

The steps of the modified neutral point solution are summarized as follow.

1. Calculate pore pressure using consolidation theory and get depth of neutral point \( z(t) \) using the traditional method proposed by Fellenius (1984).
2. Calculate soil settlement by integrating vertical strains, and get \( s_{soil}(z(t_j), t_j) \).
3. Take \( \Delta_p(z(t_j), t_j) = 0 \) and calculate pile settlement at initial moment \( t_1 \).
4. Calculate \( \Delta(z(t_j), t_j) \) according to pile settlement at time \( t_j (j < k) \) refer to formula (7); Calculate \( \Delta_p(z(t_j), t_j) \) refer to formula (6); Calculate \( s_{pile}(t_j) \) refer to formula (5);
5. Repeat step 4 and get pile settlements at discretizing time.

4 CASE STUDY

Refer to paper (Wang et al. 2013), the calculation numerical model is established as shown in Fig. 4. Pile-soil interface friction angle was set as 21°, and at-rest earth pressure coefficient \( K_0 \) was set as 0.5. At the tip of the pile, a constant upward load of 114kN was imposed to simulate full development of the tip resistance during downdrag. The pile head load is set to be 349kN. Xiao et al. (2011) reported the calculation formula for limit elastic relative displacement. \( \Delta_e \) calculated by Xiao’s formula is 12mm.

The settlements calculation results in different drainage cases are shown in Table 1 to 3. At the depth of neutral position, plastic relative displacement of pile relative to soil is upward direction in top-drainage case, downward direction in bottom-drainage case, and zero in double-drainage case.

Table 1. Settlement calculation (Top-drainage).

| Degree of consolidation | \( z(t) \) / m | \( S_{soil}(z(t_j)) \) /mm | \( \Delta(z(t_j), t_j) \) Max/mm | \( \Delta_{p}(z(t_j), t_j) \) Max/mm | \( S_{pile}(t_j) \) /mm |
|-------------------------|----------------|--------------------------|--------------------------|--------------------------|--------------------------|
| 25%                     | 7.0            | 27.7                     | 0                        | 0                        | 27.7                     |
| 50%                     | 8.0            | 130.8                    | 18.6                     | 6.6                      | 137.4                    |
| 75%                     | 9.0            | 239.7                    | 23.8                     | 11.8                     | 254.5                    |
| 100%                    | 9.7            | 343.0                    | 34.5                     | 22.5                     | 365.5                    |

Table 2. Settlement calculation (Double-drainage).

| Degree of consolidation | \( z(t) \) / m | \( S_{soil}(z(t_j)) \) /mm | \( \Delta(z(t_j), t_j) \) Max/mm | \( \Delta_{p}(z(t_j), t_j) \) Max/mm | \( S_{pile}(t_j) \) /mm |
|-------------------------|----------------|--------------------------|--------------------------|--------------------------|--------------------------|
| 25%                     | 9.6            | 83.3                     | 0                        | 0                        | 83.3                     |
| 50%                     | 9.6            | 168.6                    | 0                        | 0                        | 168.6                    |
| 75%                     | 9.7            | 253.8                    | 0.8                      | 0                        | 253.8                    |
| 100%                    | 9.7            | 343.0                    | 0.8                      | 0                        | 343.0                    |

Table 3. Settlement calculation (Bottom-drainage).

| Degree of consolidation | \( z(t) \) / m | \( S_{soil}(z(t_j)) \) /mm | \( \Delta(z(t_j), t_j) \) Max/mm | \( \Delta_{p}(z(t_j), t_j) \) Max/mm | \( S_{pile}(t_j) \) /mm |
|-------------------------|----------------|--------------------------|--------------------------|--------------------------|--------------------------|
| 25%                     | 11.9           | 147.2                    | 0                        | 0                        | 147.2                    |
| 50%                     | 11.3           | 213.9                    | 3.5                      | 0                        | 209.9                    |
| 75%                     | 10.4           | 274.6                    | 14.3                     | 2.3                      | 272.3                    |
| 100%                    | 9.7            | 343.0                    | 19.8                     | 7.8                      | 335.2                    |
Change curves for positions of neutral point in three different drainage cases are shown in Figure 4. Position of the neutral point goes down obviously in top-drainage case, goes up in bottom-drainage case and changes little in double-drainage. In different drainage cases, the positions of neutral point are different at intermediate degrees of consolidation, but same at the end.

![Figure 4](image)

**Fig. 4.** Position change for neutral point in three cases

Change curves for pile settlement in three different drainage cases are shown in Figure 5. In top-drainage case, speed of pile settlement development is slower at the beginning, faster at the end. In bottom-drainage case, the speed is faster at the beginning, slower at the end. And in double-drainage case, the speed is constant. Comparisons of final pile settlement are as follows: top-drainage (365.5mm) > double-drainage (343.0mm) > bottom-drainage (335.2mm).

FEM can consider various boundary conditions, and ensure good accuracy, making it be widely used in civil engineering. This paper uses finite element software Plaxis to analyze the model shown in Fig. 3. Calculation results by FEM are as shown in Fig. 5. They are consistent with results calculated by the method in this paper, verifying the correctness of the method.

5 CONCLUSIONS

Based on the argument that pile-soil elastic relative displacement is zero, simplified settlement calculation method, which can consider the influence of neutral point change on pile settlement, is proposed. Compared with FEM, the proposed method has distinguished advantages on calculation amount and load transfer path. It gets high accuracy with good engineering practice. Main conclusions are summarized as follow.

1. Change of position of neutral point will lead pile-soil relative displacement not to be zero at neutral point, but plastic relative displacement to be zero.

2. A simplified calculation method for pile settlement in NSF case are presented and used to analysis effect of different drainage case on pile settlement. Comparisons of pile settlement at same degrees of consolidation are as follows: top-drainage < double-drainage < bottom-drainage at the beginning, top-drainage > double-drainage > bottom-drainage at the end.

3. Finite element model is established to verify the validity of the proposed method. Results calculated by proposed method and FEM are exactly identical.

![Figure 5](image)

**Fig. 5.** Pile settlement develop in three cases

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