Imprints of anisotropy on the power spectrum in matter dominated bouncing universe as background

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Abstract In this paper, we aim to investigate the effects of the anisotropy on the scale-invariant power spectrum considering the matter-dominated collapsing universe as background and look for the deviations from the scale invariance. Having set up this background, we consider a test massless scalar field and work out the correlations for the field, first by using the perturbative approach in which the anisotropic background is approximated with an effective isotropic metric represented by the metric of matter dominated collapsing universe, second by directly solving the field equation numerically, and then obtain the power spectrum for the range of modes which are of cosmological interest. Using both techniques, we get an upper bound on the deviation in the power spectrum from the scale invariance. We also work out the power spectrum for much smaller modes and look at whether it is possible to explain the observed anomalies in CMB via the matter bounce scenario.

1 Introduction

The beginning of the universe has been a long-standing question for cosmologists. We have always wondered whether the universe began with a singularity or not. The Big Bang paradigm that consists of an initial singularity seems to be the most natural thought, as imagining a point-sized universe with an infinite density and temperature with all the fundamental interactions unified by a yet unknown framework is the most accessible course of action. Nonetheless, no one can exclude the possibility of a cyclical cosmological evolution of the universe where its size never shrinks to zero. Even quantum cosmologies support this latter perspective [1]. The observations of cosmic microwave background (CMB) and the large scale structure [2,3] strongly support the argument that the primordial curvature perturbations which generate nearly scale-invariant power spectrum act as seeds for the structures of our universe. This spectrum is the result of correlations between the quantum vacuum fluctuations [4–6] during the cosmic inflation; an era of exponentially accelerated expansion of our universe right after the Big Bang [7,8]. Not just the inflation, a corresponding scale-invariant power spectrum is obtained in the case of a matter-dominated collapsing universe, as pointed out in [9,10]. This kind of model, which is succeeded by a non-singular bounce, is called the matter bounce scenario and can act as an alternative to the inflation generating the observed power spectrum of the primordial fluctuations [11,12]. To understand the transition between the expanding and contracting phases of the universe, new physics is required. This transition can be singular if the ekpyrotic scenario [13] is to be considered or non-singular. For the latter case various methods have been developed, for example, the modifications of gravitational action in torsion gravity [14,15], Horava–Lifshitz gravity [16–18] or by introducing the Galileon field [19,20] or a ghost condensate [21–23] consisting of matter that violates the positive energy condition. For a more detailed review, one can look at [24].

Now, one of the main issues with bouncing cosmologies is the anisotropic instabilities, known as Belinski–Khalatnikov–Lifshitz (BKL) instability [25]. It occurs because, during the contracting phase, the rate at which the energy densities of the dust and radiation matter field increase is much lesser than the rate of increase in the energy densities contributed by the back-reaction of the anisotropies. Hence, it becomes necessary that to have a bounce that is nearly isotropic, one has to fine-tune the initial conditions to such accuracies that the anisotropies never dominate. However, there is another way to resolve this problem, that is by introducing a scalar field accompanied by a steep negative poten-
tial that always dominates over the anisotropies during the contracting phase [26] justifying the argument that neglecting the anisotropies can be done in an ekpyrotic scalar field scenario. It has also been shown how one can combine ekpyrotic contraction era and non-singular bounce using a negative exponential potential and a scalar field with a Horndeski-type non-standard kinetic term [27]. This model, also discussed in [28], shows how during the entire cosmological evolution of the matter-ekpyrotic bounce, the anisotropies remain small, successfully avoiding the BKL instability. The model is further explored in [29] via loop quantum cosmology perspective and also in [30] where the authors have worked with two matter fields, one a scalar field that causes bounce and the other a matter field dominating at the beginning of the contracting phase.

As the model discussed in [27,28] is free from the above-mentioned instability and is a well-established matter-ekpyrotic bounce scenario, we consider it as our background model. The existence of duality in the scale invariance aspect of the power spectrum [9] between an exponentially expanding and collapsing universe motivated us to look for the imprints of anisotropy on the power spectrum if the background is matter-dominated. The imprints caused by the breaking of rotational invariance have been discussed in [31] but considering the matter-dominated collapsing era as a background was not explored yet. We aimed to search for these imprints in this work. After starting with a period of matter-dominated contraction, we considered a test massless scalar in this background and worked out the correlations for the fluctuations, we were also able to solve the field equation numerically and found the quantitative results for the scale invariance part of the power spectrum and the deviation caused by the presence of anisotropy.

The outline of this paper is as follows: in the next section, we discuss the model proposed in [27,28] for a non-singular matter bounce and establish our background as a matter-dominated collapsing universe. Moving to the Sect. 3, we work out the power spectrum and the deviations from the scale-invariant part by means of two techniques, one a perturbative approach while the other a direct approach which is discussed in Sects. 3.1 and 3.2 in detail. Then in Sect. 3.3 we do the numerical estimates for the power spectra on the background universe. Then we conclude our work in Sect. 4 and discuss potential future directions.

2 The model

In this section, we give an overview of the work done in [27,28], where the authors have developed an effective model that invokes a smooth bounce through the dynamics of a single scalar field and a flat homogeneous geometry. The metric considered here is a flat, homogeneous but anisotropic, namely the Bianchi-I type, which lacks the rotational invariance and is written as:

$$dx^2 = -dt^2 + a^2(t) \left[ e^{2\theta_1(t)} dx^2 + e^{2\theta_2(t)} dy^2 + e^{2\theta_3(t)} dz^2 \right],$$

where $e^{\theta_i(t)}$ represents the anisotropy in scale factor. The FRW universe in an Einstein gravity could yield a successful homogeneous, isotropic, non-singular bounce by violating the Null Energy condition [28].

The Lagrangian for such a universe filled with a scalar field $\phi$ and a matter fluid component is

$$L[\phi(x)] = M_p^2 \left( 1 - g(\phi) \right) X + \beta X^2 - V(\phi) + \gamma X \Box \phi,$$

(2.1)

here, $M_p^2$ is the Planck mass square defined to be $1/8\pi G$. The parameter $\beta$ is positive and bounds the kinetic term from below during high-energy processes, thereby preventing a ghost condition during contraction. The Galileon type operator $\gamma X$ takes care of the gradient instabilities that could creep in due to ghost condensation. The second order derivative in time would be able to establish the ghost condition for a bounce. Nonetheless, the equation of motion obtained from such a Lagrangian is second order. The Null Energy Condition (NEC) is achieved from a negative kinetic part of the equation of motion for $\phi$. The dimensionless function $g(\phi)$ is given by

$$g(\phi) = \frac{-2g_0}{e^{-\sqrt{\gamma \phi}} + e^{b_y \sqrt{\gamma \phi}}},$$

(2.2)

and serves to violate the NEC when approaching bounce ($\phi \to 0$), for the non-singular case. Here, $g(\phi)$ has to dominate in the quadratic kinetic term to satisfy for a phase of ghost condensation, which leads to this violation of NEC just before the bounce. The function $g$ starts with very small values for large $\phi$ and attains unity approaching the bounce ($\phi \to 0$). The particular potential

$$V(\phi) = -\frac{2V_0}{e^{-\sqrt{\gamma \phi}} + e^{b_y \sqrt{\gamma \phi}}},$$

(2.3)

is for an Ekpyrotic contraction to follow. The exponential potential $V(\phi)$ is always negative for positive $V_0$. It allows for an attractor behaviour in a contracting cosmology by allowing trajectories of the space-time evolve into one region. The initial condition, $\phi_0$ could be taken from an arbitrary point far in the past, to be an asymptotically large negative value. The potential thus forces the scalar field towards the bounce with $\dot{\phi} > 0$. Also the equation of state is tuned as such to allow an attractor solution namely, $\dot{\rho}_m > \dot{\rho}_m, \dot{\rho}_0$. 

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2.1 Background evolution

The canonical kinetic term in the Lagrangian is
\[ X = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi = \frac{1}{2} \dot{\phi}^2. \]

And a second kinetic term,
\[ \square \phi \equiv \frac{1}{\sqrt{-g}} \partial_{\mu} (\sqrt{-g} g^{\mu \nu} \partial_{\nu} \phi) = \ddot{\phi} + 3H \dot{\phi}, \]

for a homogeneous background. For a minimally coupled action,
\[ S = \int d^4 x \sqrt{-g} \left[ R - K(\phi, X) + G(\phi, X) \square \phi \right], \quad (2.4) \]

varying with respect to the metric gives the energy momentum tensor,
\[ T_{\mu \nu} = \frac{2}{\sqrt{-g}} \delta S / \delta g^{\mu \nu}, \quad (2.5) \]
\[ \Rightarrow T_{\mu \nu} = g_{\mu \nu} \left[ V(\phi) - \frac{1}{2} (1 - g(\phi)) M_P^2 \dot{\phi}^2 - \frac{1}{4} \beta \dot{\phi}^4 + \gamma \phi^2 \dot{\phi} \right] + \nabla_{\mu} \phi \nabla_{\nu} \phi \left[ (1 - g(\phi)) M_P^2 + \beta \dot{\phi}^2 + \gamma (\phi + \sum_{l} (\theta_l + H_l) \phi) \right] - \gamma \left[ (\nabla_{\mu} \nabla_{\lambda} \phi) \nabla^{\lambda} \phi \nabla_{\nu} \phi + \nabla_{\mu} \phi (\nabla_{\nu} \nabla_{\lambda} \phi) \nabla^{\lambda} \phi \right]. \quad (2.6) \]

For homogeneous universe, the energy momentum tensor can be written in the form of an ideal fluid,
\[ T_{\mu \nu} = (\rho + p) u_{\mu} u_{\nu} + pg_{\mu \nu}, \quad (2.7) \]
\[ u_{\mu} \] being the 4-velocity of the isotropic fluid and \( \rho, p \) the respective density and pressure of the fluid. The corresponding scalar field energy density and pressure for the model under consideration becomes
\[ \rho_{\phi} = \frac{1}{2} M_P^2 (1 - g) \dot{\phi}^2 + \frac{3}{4} \beta \dot{\phi}^4 + 3 \gamma H \dot{\phi}^3 + V(\phi), \quad (2.8) \]
\[ p_{\phi} = \frac{1}{2} M_P^2 (1 - g) \dot{\phi}^2 + \frac{1}{4} \beta \dot{\phi}^4 - \gamma \dot{\phi}^2 \dot{\phi} - V(\phi). \quad (2.9) \]

From the Einstein equations could be worked the equation of motion for the scale factor. The temporal field equation gives the 1st Friedmann equation
\[ H^2 = \frac{\rho}{3 M_P^2} + \frac{1}{6} \sum_{i} \dot{\theta}_i^2, \quad (2.10) \]

while the simplification of the spatial component yields the acceleration equation:
\[ \ddot{H} = -\frac{\rho_{\phi} + p_{\phi}}{2 M_P^2} - \frac{1}{2} \sum_{i} \ddot{\theta}_i^2 \text{ and } \ddot{\theta}_i + 3H \dot{\theta}_i = 0. \quad (2.11) \]

Inferring from the constraint equation for \( H \) (2.10) the anisotropic stress energy density could be expressed as \( \sum_{i} \dot{\theta}_i^2 \). The dynamics of whom is according to equation of motion for \( \theta_i \) (2.11). Now solving for \( \dot{\theta}_i \),
\[ \begin{align*}
\frac{d\dot{\theta}_i}{dt} &= -3 \frac{da}{a(t)} \frac{\dot{\theta}_i}{\dot{a}(t)} \\
\Rightarrow \dot{\theta}_i &= M_{\theta, i} \frac{a_B(t)^3}{a(t)^3}, \quad (2.12)
\end{align*} \]

gives the form of the anisotropic energy density, \( \dot{\theta}_i^2 \propto a^{-6} \) responsible for the BKL instability. Here the parameter \( M_{\theta, i} \) is introduced to tune the bounce phase.

2.2 Matter contraction

In this model, the universe undergoes a matter-dominated contraction from negative infinity until the slow Ekpyrotic contraction, beginning at \( -t_E \) and extending into the bounce. The mean scale factor here evolves as a power law contraction, given by,
\[ a(t) \approx a_E \left( \frac{t - t_E}{t_E - t_E} \right)^{2/3}, \quad (2.13) \]

wherein \( a_E \) is the transition mean scale factor. The conformal time in the matter bouncing era i.e. from \( -t \) to \( -t_E \) is:
\[ \tau = \int_{-t}^{-t_E} \frac{dt}{a(t)} = 3(t_E^{1/3} - t^{1/3}). \quad (2.14) \]

and the mean Hubble parameter is
\[ \bar{H}(t) = \frac{2}{3(t - t_E)}. \quad (2.15) \]

In the above \( t_E \) is introduced as an integration constant to match the mean Hubble parameter at transition, \( H_E \).
\[ \bar{H} \simeq a_E \frac{2}{3(t - t_E)}. \quad (2.16) \]

Hence (2.13) allows (2.12) to be rewritten as,
\[ \dot{\theta}_i(t) = M_{\theta, i} \frac{a_B(t)^3}{a(t)^3} \bar{H}(t)^2. \quad (2.17) \]

It can be integrated to get the anisotropy factors,
\[ \dot{\theta}_i(t) = -M_{\theta, i} \frac{2a_B(t)^3 \bar{H}(t)^2}{3a(t)^3 H_E^2}. \quad (2.18) \]

Here, the mean scale factor during bounce is normalized to unity.
Now, within the set up of this matter contracting bouncing universe, we first only consider a test massless scalar background field \( \chi(x, t) \) which includes the small fluctuations around the background field \( \chi_I(x, t) \) and then obtain the power spectrum by computing \( \langle \chi(x, t) \chi(y, t) \rangle \) correlations. We do so via two techniques described in the next section in details.

### 3 Power spectrum in matter dominated contracting phase

To calculate the power spectrum in the matter-dominated contracting universe and search for the existence of the duality in the first-order correction, the first approach that we follow is the perturbative one. In this, we first approximate the anisotropic metric with a fictitious isotropic metric by considering the deviation from isotropy to be very small. That isotropic metric is the background metric representing the matter-dominated contracting universe. Since the deviation from the isotropy is very small, we can write the scale factors of the anisotropic metric as a small perturbation to the scale factor of the fictitious isotropic metric. Then we define the anisotropy parameter and use it to separate out the scale-invariant part of the power spectrum and the deviation caused by the anisotropy in that. The obtained results are then shown in Table 1, Figs. 1 and 2.

The second approach discussed below is a direct one in which we consider a free massless test field \( \chi(x, t) \) in the anisotropic background, which is still represented as a small perturbation to the fictitious metric. Then we proceed to solve the field equation numerically in the Fourier space and obtain the power spectrum by working out the correlation function \( \langle \chi(x, t) \chi(y, t) \rangle \). Though via this method, we get a complete power spectrum that includes the scale-invariant part of the power spectrum as well as the effect of anisotropy on that. Then we separate out the imprints of the anisotropy and represent those in Fig. 3.

#### 3.1 Perturbative approach

As is discussed in [31], introducing the anisotropy in the background manifests as direction dependency in the power spectrum. Using the primordial density perturbations \( \delta(k) \), the power spectrum is defined as

\[
\langle \delta(k) \delta(q) \rangle = P(k) \delta^3(k - q). \tag{3.1}
\]

Here \( P(k) \) may change the form to \( P'(k) \) assuming the presence of broken rotational invariance during the inflationary era and can be written in parametric form as,

\[
P'(k) = P(k) \left( 1 + g(k)(\hat{\mathbf{k}} \cdot \mathbf{n})^2 \right), \tag{3.2}
\]

with the line element for the universe characterised by a bidirectional isotropy given by

\[
ds^2 = dr^2 - d'(t)^2 [dx^2 + dy^2] - b^2(t)dz^2. \tag{3.3}
\]

Now with the model under consideration as the background, the respective scale factors evolve as

\[
a'(t) = \tilde{a}(t)e^{\theta_1(t)}, \quad b(t) = \tilde{a}(t)e^{\theta_2(t)} \tag{3.4}
\]

and the Hubble parameters become

\[
H_a = \frac{\dot{a}'}{a'}, \quad H_b = \frac{\dot{b}}{b}.
\]

The line element written in (3.3) can be approximated using a fictitious isotropic metric and is given by

\[
ds^2 = dr^2 - \tilde{a}(t)^2 [dx^2 + dy^2 + dz^2],
\]

wherein the average scale factor \( \langle \tilde{a}(t) \rangle \) evolves as in matter dominated contracting universe case. The average Hubble parameter is \( \bar{H} = \dot{a}/a \) and the deviation from isotropy is parametrized using \( \epsilon_H \) as

\[
\epsilon_H = \frac{2(H_b - H_a)}{3H} \approx -\frac{2}{3} \frac{\bar{H}}{a_E^3 H_E^2}.
\]

Approximating the correlation function in series of \( \epsilon_H \), by treating it as a small perturbation [32], we can write

\[
\langle \chi(x, t) \chi(y, t) \rangle \equiv \langle \chi_I(x, t) \chi_I(y, t) \rangle + \int_0^t dt' \langle \{H_I(t'), \chi_I(x, t) \chi_I(y, t) \} \rangle \tag{3.5}
\]

and the interaction picture field as a Fourier series in ladder operators as [32],

\[
\chi_I(x, t) = \int \frac{d^3k}{(2\pi)^3} \left( e^{ik \cdot x} \chi(t)a_k + e^{-ik \cdot x} \chi^*(t)a_k^\dagger \right). \tag{3.6}
\]

The Fourier transform of the two-point function (3.5) results in the power spectrum (of the form (3.2)),

\[
\langle \chi(x, t) \chi(y, t) \rangle = \int \frac{d^3k}{(2\pi)^3} e^{ik \cdot (x-y)} \left( P(k) + (\hat{k} \cdot \mathbf{n})^2 \Delta P(k) \right). \tag{3.7}
\]

The interaction picture Hamiltonian for a massless scalar field would be,

\[
\hat{H}_I = \int \frac{d^3k}{(2\pi)^3} e^{ik \cdot x} \chi(t) \hat{H}_k + \text{h.c.}
\]
\[
\frac{\Delta_1(k)}{\kappa} \text{ vs } k \text{ using the perturbative method at four different values of } \tau
\]

\[
\frac{\Delta_2(k)}{\kappa} \text{ vs } k \text{ for four different values } \tau \text{ on log scale via perturbative technique}
\]

\[
H_I(t) = \int d^3x \frac{1}{2} \left[ (b - \bar{a})(d_\chi x_\perp)^2 + \left( \frac{a'^2}{b} - \bar{a} \right) (d_\chi x^3)^2 \right]
\]

\[
\mathcal{L}_\chi = -\frac{1}{2} \sqrt{-g} g^{\mu \nu} \partial_\mu \chi \partial_\nu \chi,
\]

(3.8)
in which we employ the procedure of finding the conjugate momentas for the foreground and background metrics, \(\Pi(\chi) = \Pi(\chi) = \partial \mathcal{L}(\chi, \partial_\mu \chi)(\partial_\mu \chi)\) and then the respective Hamiltonian densities \(\mathcal{H}(\chi, \Pi) = \hat{H} - \mathcal{L}_\chi\). The interaction part Hamiltonian density, \(\mathcal{H}_I\) is the difference in the background from the perturbative densities.

Using (3.6) and (3.8) in (3.5), and furthermore transforming into the conformal time of the background metric we obtain \(P(k) \simeq |\chi_k(\tau)|^2\), and

\[
\Delta P(k) \simeq -6k^2 \int_{3/2}^\tau dt' \hat{H} a^2(t') e_H(t') \left( \frac{1}{3} - \frac{
abla}{2} \right)^2 \times \left[ \chi_k(0) \chi_k(0) \right] - (\chi_k(0) \chi_k(0))^2, 
\]

(3.9)

where

\[
\chi_k(0) = \frac{\bar{H}}{\sqrt{2k}} e^{i k \tau} \left[ \tau - \frac{i}{k} \right]. 
\]

becoming,

\[
\Delta P(k) \simeq \frac{6k^2}{2} \left( \frac{2}{3} \hat{H} \right)^{-2/3} \times \int_{3/2}^\tau dt' \hat{H} a^2(t') e_H(t') \left[ \left( \frac{1}{3} - \frac{
abla}{2} \right)^3 - iE + \frac{2}{3} \hat{H} \right]^{2/3} \times \left( \frac{1}{3} - \frac{
abla}{2} \right)^5 \times \left( \frac{1}{3} - \frac{
abla}{2} \right)^5.
\]

(3.10)

To investigate the scale independent part of the power spectrum and the deviation from it, we multiply by \(k^3\) and get:

\[
P'(k)k^3 = P(k)k^3 + (\mathbf{k} \cdot \mathbf{n})^2 \Delta P(k)k^3,
\]

(3.11)

where,

\[
P(k)k^3 = \frac{\bar{H}^2(t)}{2},
\]

\[
\Delta P(k)k^3 \simeq \frac{6k^2}{2} \left( \frac{2}{3} \hat{H} \right)^{-2/3} \times \int_{3/2}^\tau dt' \hat{H} a^2(t') e_H(t') \left[ \left( \frac{1}{3} - \frac{
abla}{2} \right)^3 - iE + \frac{2}{3} \hat{H} \right]^{2/3} \times \left( \frac{1}{3} - \frac{
abla}{2} \right)^5 \times \left( \frac{1}{3} - \frac{
abla}{2} \right)^5.
\]

(3.12)

Here, as can be seen in Eq. (3.12), \(P(k)k^3\) is independent of \(k\) and makes up for the scale-invariant part while \(\Delta P(k)k^3\) in Eq. (3.13) represents the correction caused by the anisotropy. Though the scalar correction is incorporated in \(\Delta P(k)k^3\), the direction dependence is still governed by \((\mathbf{k} \cdot \mathbf{n})^2\), similar to the case of inflation as discussed in [31], hence setting up the aspect of search for the duality between the inflationary
scenario and matter-bounce up to the first order in the power spectrum.

3.2 Direct approach with scalar field in anisotropic background

Another way to obtain the power spectrum is to solve the field equation \( \square \chi = 0 \) in the matter-dominated collapsing background for the test field directly in Fourier space and then working out the correlation function using the solutions obtained. This equation for the modes with wavenumbers along the \( \hat{z} \) direction in the anisotropic background can be written in the Fourier space as

\[
d[2] \chi_k t + 3 \dot{H} d \chi_k t + \frac{k^2}{b(t)^2} \chi_k = 0, \tag{3.14}
\]

which in conformal time becomes

\[
d[2] \chi_k \tau + \frac{1}{Y(\tau)^4} \left[ \frac{2}{3} \frac{3H(\tau)}{Y(\tau)^2} \right] d \chi_k \tau + \frac{k^2}{b(\tau)^2} \chi_k = 0, \tag{3.15}
\]

where,

\[
Y(\tau) = \left( t_E^{1/3} - \frac{\tau}{3} \right),
\]

\[
H(\tau) = \frac{2}{3 \left[ Y(\tau)^3 - t_E + 2/3H_E \right]},
\]

\[
b(\tau) = a_E \left[ \frac{Y(\tau)^3 - t_E + 2/3H_E}{2/3H_E} \right] \times \exp -M_{\theta,2} \left( \frac{2}{3a_E^2H_E^2} \right) H(\tau).
\]

For simplicity, in dealing with scale factors, the parametrization

\[
b(\tau) = \bar{a}(\tau) (1 + \epsilon_H),
\]

has been used.

Solving the Eq. (3.15) for \( \chi_k \) and calculating \( \langle \chi_k \chi_k^* \rangle \) gives us the power spectrum albeit that includes the scale-invariant part and its first order correction with the effect of the direction dependence on the top of that. In the next section, we discuss how we numerically solve Eq. (3.15) by setting up all the parameters for the matter-dominated background and obtain the results for the power spectrum using the perturbative approach (discussed in the previous section) and the direct approach (discussed in this section).

3.3 Numerical estimates of power spectrum with both approaches

To numerically evaluate the power spectrum and its first order correction, we work in the same parameter regime as [28] and write all relevant parameters and functions in the reduced Planck mass units as

\[
V_0 = 10^{-7} M_p^4, \quad g_0 = 1.1, \quad \beta = 5, \quad \gamma = 10^{-3}, \quad b_V = 5, \quad b_g = 0.5, \quad p = 0.01, \quad q = 0.1,
\]

and then from Hubble parameter vs time plot (Fig. 2) and energy density vs time (Fig. 3) in [28], we get

\[
t_E = -2.486 \times 10^5, \quad H_E = -6.1 \times 10^{-6}, \tag{3.16}
\]

which is required for our model. Also, we set \( a_E = 1 \) and \( \epsilon_H = 10^{-5} \) and the range of the conformal time (\( \eta \)) for the aforementioned parameter regime turns out to be from \(-9.6 \) to \(0 \).

Using all these parameters and the perturbative approach, we get the results shown in Table 1, Figs. 1 and 2. The Table 1 consists the values for isotropic part of the power spectrum i.e. \( k^3 P(k) \) obtained using Eq. (3.12) corresponding to each value of the conformal time (\( \tau \)). As is evident there itself, this is approximately scale invariant with being independent of wavenumber \( k \) as defined in Eq. (3.12) and is of the order of \( 10^{-10} \). This power spectrum is often also modeled as a power law in the form \( P(k) = A_s (k/k_s)^{n_s-1} \) where \( k_s \) is the pivot point on a fiducial scale from which the deviations are measured and \( n_s \) is the spectral index quantifying the ‘tilt’. Here, \( A_s \) is the variance of the perturbations in a logarithmic wavenumber interval centered around \( k_s \). From PLANCK 2018 Results [3], we have \( k_s = 0.05 \) Mpc\(^{-1} \) and \( n_s = 0.967 \) which lead to average \( A_s = 2.64 \times 10^{-10} \) using Table 1 and Eq. (3.12). Though \( A_s \) obtained here is one order smaller than the measured value of \( 2.09 \times 10^{-9} \) from CMB observations [3], the scale invariant feature in the power spectrum still persists.

Next in Fig. 1, we have plotted \( \Delta(k) \) vs \( k \) using Eqs. (3.12) and (3.13) where \( \Delta(k) = \Delta P(k)/P(k) \), which is defined as the ratio of first order correction of the power spectrum to the scale-invariant part, at four different values of conformal time mentioned in Table 1 for the range of \( k \) from \( 10^{-4} \) Mpc\(^{-1} \) to \( 10^{-1} \) Mpc\(^{-1} \) as this is the regime over which the measurement of primordial power spectrum is done by studying the fluctuation and anomalies in CMB [33–38]. Here, \( \Delta(k) \) is found to be approximately within the order of \( 10^{-8} \) and \( 10^{-9} \) implying that the corrections are atmost of this order for the entire range of the conformal time in our model though these significant values are for the modes lying between \( 4 \times 10^{-2} \) Mpc\(^{-1} \) to \( 10^{-1} \) Mpc\(^{-1} \). Then in Fig. 2, we have the results for the smaller \( k \) modes from \( 10^{-4} \) Mpc\(^{-1} \) to \( 10^{-3} \) Mpc\(^{-1} \) on a log scale since those could not be seen effectively in Fig. 1. Here, \( \Delta(k) \) for this range is found to be much smaller than the observed values of the anomalies from CMB and lies in between \( 10^{-23} \) to \( 10^{-18} \).

Via the second technique, we get the results shown in Fig. 3 for the power spectrum. For this, we first proceed as discussed in Sect. 3.2 and then numerically solved Eq. (3.15) for \( \chi_k \),
and then plotted the results for $\Delta(k)$ vs $k$ for the range of $k$ from $10^{-4}$ Mpc$^{-1}$ to $10^{-1}$ Mpc$^{-1}$. The amplitude of $\Delta(k)$ here is found to be of the order of $10^{-7}$, and not much variation can be seen with $k$ ranging from $4 \times 10^{-2}$ Mpc$^{-1}$ to $10^{-1}$ Mpc$^{-1}$ as was the case in the first approach. Though the results from both the approaches do not match exactly, that might be because of the approximation not working for this parameter regime but this helps us in setting up an upper bound of $10^{-7}$ order on $\Delta(k)$. The other difference in the results using the second approach is an oscillatory character, as seen in Fig. 3. This is because, in the first approach, the oscillations are absorbed in the $k \cdot n$ factor, as shown in Eq. (3.11). Since $\tau = 0$ is the final limit of the conformal time we have considered, we get the trivial result of the field equation within the aforementioned range of wavenumber $k$, zero. Hence, our results from both techniques are expected not to match at this value of conformal time, and the same is verified by the numerical results.

4 Discussion and conclusion

In this paper, we have worked out quantitatively the imprints that the anisotropy can cause on the scale-invariant power spectrum, given the universe had a matter-dominated contracting phase as the background. The argument for considering this as the background comes from [9], in which the author showed that there exists a duality between the two methods of calculating the power spectrum of perturbations for a minimally coupled massless field. The first is the exponentially expanding universe in which the scale factor goes as $a \propto e^{Ht}$, while the second is a matter-dominated collapsing universe with $a(t) \propto (-t)^{2/3}$. The first case has been explored in [31] with detailed discussions on imprints of the anisotropy and breaking of rotational invariance but, the dual to the former one had not been explored yet, thus leading to this work. Using the method described in [28], first we establish the matter-dominated contracting universe as a background for exploration. Then, within the same regime of parameters used in [28], we worked out the power spectrum by studying the correlations for a test massless scalar field in this background and looked at the effects that the anisotropy can cause if it is present from the beginning. To study those imprints, we have used two techniques first, the perturbative approach, while the second technique was the direct approach. The quantitative results for the deviation from the scale-invariant power spectrum were obtained using both methods for the range of $k$ between $10^{-4}$ Mpc$^{-1}$ to $10^{-1}$ Mpc$^{-1}$, as these are the modes of interest based on cosmological observations [34–37]. Using the matter-dominated background, we found the scale-invariant power spectrum to be of the order of $10^{-10}$ which lead to cosmological parameter $A_s = 2.64 \times 10^{-10}$ using $n_s$ and $k^*$ from the Planck results [3] though the value obtained for $A_s$ was one order smaller than $2.09 \times 10^{-9}$ as predicted by the data. Even though the exact value did not match, we found that the feature of scale-invariance is still preserved by the matter-bounce. Moving on to the first order correction to the scale-invariance part and to look for the effect of anisotropy on the power spectrum, we found $\Delta(k)$, ratio of correction to the power spectrum due to the anisotropy to the scale-invariant part to be of order of $10^{-5}$ and $10^{-7}$ via the perturbative approach, while the directly solving the field equation and evaluating the correlation gave us $\Delta(k)$ of the order of $10^{-7}$. The reason for both the values to be different might have to do with the parameters regime under consideration over here. Also, we observed in our results that the $\Delta(k)$ has significant values for the modes lying between $4 \times 10^{-2}$ Mpc$^{-1}$ to $10^{-1}$ Mpc$^{-1}$ via both the approaches. These values can further be constrained if one uses other sets of parameters maintaining the stability of the matter bounce scenario. On exploring more for much smaller modes in the range $10^{-4}$ Mpc$^{-1}$ to $10^{-3}$ Mpc$^{-1}$, we found $\Delta(k)$ varying between $10^{-23}$ to $10^{-18}$, which is much smaller than the observed values of anomalies in CMB. Though here the deviation here from the scale-invariance has upperbound of $10^{-7}$, it is inconsistent with the recent observational value of $10^{-5}$ from Planck data [37] for the correlations. Also, since the value for $\Delta(k)$ differs from the one we get via the inflationary scenario, the work done in this paper can help us in distinguishing between both matter bounce and inflation via looking at just the power spectra, but mismatching of the current results obtained here with the observational values of $A_s$ and the correlations imply that it has become interesting to limit the parameter space of matter bounce by confronting with the current and forthcoming cosmological experiments, also it would be interesting to do a similar study for other bouncing models as well. This work is further left for exploration via CMB constraints and the calculation of other cosmological parameters. We plan to do those in the future.

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