Achromatic axially symmetric wave plate

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Abstract: An achromatic axially symmetric wave plate (AAS-WP) is proposed that is based on Fresnel reflections. The wave plate does not introduce spatial dispersion. It provides retardation in the wavelength domain with an axially symmetric azimuthal angle. The optical configuration, a numerical simulation, and the optical properties of the AAS-WP are described. It is composed of PMMA. A pair of them is manufactured on a lathe. In the numerical simulation, the achromatic angle is estimated and is used to design the devices. They generate an axially symmetric polarized beam. The birefringence distribution is measured in order to evaluate the AAS-WPs.

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1. Introduction

Recently, optical vortices and axially symmetric polarized beams have been investigated [1–3]. An optical vortex has a unique phase distribution known as a helical mode. The center of the intensity distribution has a singular point, giving the distribution a doughnut shape. Such vortices have nonzero orbital angular momentum. In contrast, axially symmetric polarized beams have different properties than optical vortices. Although an axially symmetric polarized beam is in-phase across the beam area, its polarization states are axially symmetric. Thus its intensity distribution is also doughnut shaped. If a geometric phase is introduced into the axially symmetric polarized beam, it acquires a helical mode just as an optical vortex possesses. Consequently, a Pancharatnam charge is introduced by the geometric phase [1, 2]. Most recently, higher-order Poincaré sphere has been proposed to represent axially symmetric polarized beam with higher-order Pancharatnam-Berry phase and their demonstrations have been experimentally shown [4, 5].

A unique property of an axially symmetric polarized beam is its z-polarization [6]. This polarization is generated along the optical axis of an electric field when an axially symmetric polarized beam, such as a radially polarized beam, is focused through an objective lens having a large numerical aperture. A z-polarization can be used to accelerate electrons [7, 8]. Other proposed applications include laser processing, super-resolution microscopes, and laser trapping [9–11]. To generate such an axially symmetric polarized beam, most published research has proposed using liquid crystals, photonic crystals, nanostructures, optical cavities, or optical fibers [12–21]. Although these ideas are innovative, these techniques have previously been used to fabricate only radial or azimuthal polarizers [12–15, 17–21]. Few proposals have suggested fabricating axially symmetric wave plates (AS-WPs) because their manufacturing process is necessarily complicated. In addition, AS-WPs have technical issues including spatial dispersion, wavelength dependence, and temperature dependence. Retardance errors often arise in liquid crystals due to thermal effects. Commercial axially symmetric wave plates are fabricated as 8 to 30 equally divided segments. Taking into account the measuring and analyzing system, achromatic axially symmetric wave plates are required. Some papers have proposed a measuring and analyzing system using achromatic optical vortices [18, 22–25]. However, it is critical to produce an achromatic axially symmetric wave plate without spatial dispersion.
We here develop achromatic axially symmetric wave plates based on Fresnel reflections. The strong point of our proposal is that our wave plate operates achromatically and continuously. Our technique provides a simplified manufacturing process using a precision lathe. The resulting achromatic axially symmetric wave plates are robust with respect to temperature, in contrast to liquid crystals.

2. Theory behind the achromatic axially symmetric wave plate

The optical configuration of a Fresnel rhomb is shown in Fig. 1. A white-light beam is transmitted through a polarizer oriented at 45° before the Fresnel rhomb. A pair of Fresnel reflections can produce a phase difference of $\Delta$ in the p-s orthogonal polarization states [26–29],

$$
\Delta = 4 \tan^{-1} \left( \frac{\sqrt{n^2(\lambda) \sin^2 \beta - 1}}{n(\lambda) \sin \beta \tan \beta} \right)
$$

(1)
where \( n \) and \( \beta \) are the refractive index and slope angle of the Fresnel rhomb, respectively. Although \( n \) depends on wavelength \( \lambda \), \( \beta \) does not. Therefore, we have to take into account the wavelength dependence of the refractive index, denoted \( n(\lambda) \).

Figure 2 shows the optical configuration of an achromatic axially symmetric wave plate based on Fresnel reflections. A white-light beam is transmitted through a polarizer oriented at 45° prior to the wave plate. The wave plate has a concave conical surface similar to an element rotated about optical axis \( z \) by the Fresnel rhomb. The reflected light forms a cone-shaped beam that reflects on the sloping edge of the wave plate. The beam then becomes ring-shaped because the reflection is omnidirectionally generated along the optical axis. Using the phase difference \( \Delta \), the Mueller and Jones matrices of the axially symmetric wave plate can be expressed in the \( x-y \) coordinate system as

\[
M(x, y) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos^2 \theta + \sin^2 \theta \cos \Delta & \sin 2\theta \cos \theta \sin \Delta & -\sin 2\theta \sin \Delta \\
0 & \sin 2\theta \cos \theta \sin \Delta & \cos 2\theta \sin \Delta & \sin \Delta \\
0 & \sin 2\theta \sin \Delta & -\cos 2\theta \sin \Delta & \sin \Delta \\
\end{bmatrix}
\]  

and

\[
J(x, y) = \begin{bmatrix}
e^{-i\Delta/2} \cos\theta + e^{i\Delta/2} \sin\theta & -i\sin(\Delta/2) \sin 2\theta \\
-i\sin(\Delta/2) \sin 2\theta & e^{i\Delta/2} \cos^2\theta + e^{-i\Delta/2} \sin^2\theta \end{bmatrix}
\]

where \( \theta \) is the angle shown in Fig. 2, \( \theta = \tan^{-1}(y/x) \).
Material: PMMA
Size: $\phi = 30 \text{ mm}$, $L = 60 \text{ mm}$
Angle: $\beta = 90 \degree$

The estimated retardance: $\Delta = 142 \pm 6 \degree$

Fig. 4. Photograph of the achromatic axially symmetric wave plate.

If the phase difference $\Delta$ equals $90\degree$, the polarization states of the output beam vary around the ring, as depicted in Fig. 2. The trajectory on the Poincaré sphere corresponds to the polarization states of a rotating quarter-wave plate. In other words, the optical element acts as an achromatic axially symmetric quarter-wave plate. Two such wave plates serve as an achromatic axially symmetric half-wave plate. With four Fresnel reflections, the output beam lies along the original optic axis $z$, but it is necessary to find suitable materials for the wave plates, such as SiO$_2$.

3. Numerical simulations

Using PMMA as the optical material of the axially symmetric quarter wave plate, the slope angle $\beta$ is simulated to obtain the achromatic phase differences between 435 and 675 nm. The phase difference for Fresnel reflections from PMMA are estimated using values for the indices of $n = 1.5027$ ($\lambda = 435$ nm: blue line), $n = 1.4935$ ($\lambda = 555$ nm: green line), and $n = 1.4872$ ($\lambda = 675$ nm: red line). For SiO$_2$, the corresponding indices are $n = 1.463$ ($\lambda = 435$ nm: blue line), $n = 1.458$ ($\lambda = 555$ nm: green line), and $n = 1.456$ ($\lambda = 675$ nm: red line).

Figure 3 plots the wavelength dependence of the phase difference against the slope angle $\beta$. The blue, green, and red lines indicate the phase differences for the indices at 435, 555, and 675 nm, respectively. According to this graph, the phase difference of PMMA is near $90\degree$ from 435 to 675 nm for slope angles between $50\degree$ and $54\degree$. The variation in the phase difference is about $\pm 1.1\degree$ in this region. If one uses materials with smaller index dispersion, the variation becomes smaller. The simulations indicate that BK7 has good performance across the visible spectrum. The variation in the phase difference is within $\pm 0.8\degree$. In the case of SiO$_2$, the achromatic axially symmetric quarter wave plate works coaxially, although the achromatic angle is only $44.3\degree$. It is then necessary that the slope of the wave plate be fabricated to within an angle tolerance of less than $0.1\degree$. The variation in the phase difference for SiO$_2$ is about $\pm 5.0\degree$, substantially larger than that for PMMA or BK7.

4. Experimental results

Figure 4 shows a pair of achromatic axially symmetric wave plates. They are 30 mm in diameter and 60 mm long. Each wave plate was manufactured on a lathe. In the present study, the slope angle of the wave plates was chosen to be $45\degree$ for ease of manufacturing. Due to the end of the drill bit, the center of the optical elements is flat over a 1-mm diameter. According to the simulation results, the phase difference varies between $68\degree$ and $75\degree$ across visible wavelengths. Thus the total phase difference is $142 \pm 6\degree$.

A backlight, two sheet polarizers, and a CCD camera were used to evaluate the optics. Figures 5(a) and 5(b) show experimental and simulated images of the achromatic axially symmetric wave plates through parallel and crossed sheet polarizers, respectively. The yellow arrows indicate the transmission axis of the analyzer. When the analyzer axis was $90\degree$, the pattern varied from “$\times$” to “$\bigcirc$”. The experimental images are in reasonable agreement with the simulated images.
Fig. 5. Comparison of the experimental and simulated images. Right-hand diagrams: experimental images; left-hand diagrams: simulated images.

(a) Parallel polarizers, (b) Crossed polarizers

Fig. 6. Birefringence distributions.

(a) Retardance map, (b) Azimuthal angle map

Figure 6 shows the birefringence distributions of the achromatic axially symmetric wave plates. These distributions were measured using AbrioTM-IM manufactured by Tokyo Instruments. The grey level indicates retardance over the range $0^\circ$ (black) to $273^\circ$ (white). The PMMA exhibits small residual stress. In Fig. 6(b), the colors indicate the azimuthal angle varying over the range of $-90^\circ$ to $+90^\circ$; it is seen that the azimuthal angle varies smoothly. These wave plates can be fabricated without residual stress if one uses glass as the optical material.

7. Conclusion

Achromatic axially symmetric wave plates without spatial dispersion have been designed based on Fresnel reflections. Optical materials such as PMMA, BK7, or SiO$_2$ are used, and the achromaticity of the wave plates is measured. In the case of PMMA, it is estimated that the wavelength dispersion is $1.1^\circ$ in the visible spectrum. A pair of achromatic axially symmetric wave plates has been fabricated on a lathe. The experimental images agree with the simulations. However, some small stress birefringence was found in the optical elements. We anticipate that this stress birefringence can be mitigated by fabricating the optical elements out of a glass such as BK7 or SiO$_2$.

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