Dynamic Analysis of a 6-DOF Wheeled Mobile Robot

Zhifei Ji¹,², Xiaodong Yuan¹, Min Lin¹* and Xiaoqing Huang³

¹College of Ocean Equipment and Mechanical Engineering, Jimei University, Xiamen, 361021, Fujian Province, China
²Fujian Province Key Laboratory of Ocean Renewable Energy Equipment, Xiamen, Fujian, 361021, China
³College of Mechanical and Electrical Engineering, Hainan University, Haikou, 570228, Hainan Province, China

* Corresponding author’s e-mail: 20176100009@jmu.edu.cn

Abstract. The mobile robot is a hot research topic in the research field of robotics. Combined robotic arms and wheeled mobile platforms, a novel mobile robot of six degrees of freedom is researched in this work. On the basis of Lagrange formulations, the dynamic model of the robot is developed. By employing the Adams software, the simulations of the direct and inverse dynamics of the robot are completed. The results indicate that under the conditions of constant driven forces or torques, the upper arm, forearm and rotational platform have the same motion period. In the first five seconds, the accelerations of upper arm, forearm and rotational platform research their peak values respectively, when the time equals 4.63s. Influenced by the motions of upper arm and forearm, the translational platform will obtain kinetic energies in the horizontal plane. Exerting periodic motions on the upper arm and forearm, the corresponding drive forces or torques have the same period with the motions of the upper arm and forearm. The driven forces and torques of the upper arm and forearm have multiple extreme points in one motion period. The motion laws of the robot is of positive significance for its design of controller and path planning.

1 Introduction

With the growth of economy and progress of science and technology, considerable potential of the application of mobile robots have been gradually seen in many fields including industry, agriculture, aviation in its bright future. The walking mechanism of mobile robot mainly includes the wheel, crawler- and leg types. Tracked robot can walk on uneven ground, and has good terrain adaptability [1]. Its disadvantage lies in that steering can only be achieved through the speed difference between the two tracks, without steering mechanism, which brings about a large steering resistance. Legged robot has good motion stability and obstacle surmounting ability [2], but its motion control algorithm is more complex due to its large degree of freedom. Due to its good steering flexibility, wheeled robot has been widely used in many aspects including geological exploration, scientific investigation, material handling, etc. At present, wheeled robot has become a research hotspot widely concerned by scholars both at home and abroad [3].

Researches of wheeled robot mainly focuses on the control strategy, gait planning, dynamic analysis and so on. Zhou et al. [4] carried out dynamic modeling and analysis on the broken line welding tracking motion of wheeled robot. Liu et al. [5] equivalent the mobile robot to a rigid body model and studied the high-speed obstacle avoidance algorithm of the robot. Guo et al. [6] proposed a robust path tracking
control method based on the dynamic model of wheeled robot. Zhou and Liu [7] proposed an adaptive ant colony algorithm, and applied the algorithm to solve the path planning problem of wheeled robot. The adaptive ant colony algorithm overcomes the shortcomings of traditional ant colony algorithm, such as easily falling into local extremum, poor search quality and low accuracy, and improvement of the convergence speed and accuracy. Sun [8] designed the control system of wheeled robot based on ARM microcontrollers; Song [9] proposed a model learning algorithm and applied it to the tracking control of wheeled robots. The kinematic performance of wheeled robot is closely related to the accurate dynamic model. The research on the dynamic performance of wheeled robot is of great significance for the gait planning and application of robot.

A type of 6-DOF mobile robot is designed by combining the manipulator with the wheeled mobile platform in this paper. Based on the Lagrange method, the dynamic model of the mobile robot is established; ADAMS software is used to simulate the forward and reverse dynamics of the robot; Based on the simulation, the relationship between the driving force (moment) and the motion of the robot is obtained.

2 Structure description of mobile robot

The three-dimensional model of 6-DOF mobile robot studied in this paper is shown in figure 1. The mobile robot consists of a mobile platform and a series manipulator mounted on the mobile platform. Establish the coordinate system $\{O\}$ fixed on the ground and the coordinate system $\{O_{1}\}$ moving with the car, as shown in figure 2. The $Z_{1}$ axis of $\{O_{1}\}$ is along the wheel axis, the $X_{1}$ axis is along the forward direction of the mobile platform, and the $Y_{1}$ axis is perpendicular to the mobile platform and points to the rising direction of the lifting platform. The direction of each axis of the coordinate system $\{O\}$ can be set arbitrarily.

The wheels of the mobile platform are driven by DC motor. Driven by DC motor, the mobile robot can achieve two degrees of freedom in the horizontal plane. The lifting platform can drive its arm, forearm and rotary platform to realize the lifting movement along $Y_{1}$ direction. The rotary platform can rotate the arm and the forearm around the $Y_{1}$ axis, and the rotation angle is calculated by the $\theta_{1}$. The arm can be connected to the rotary platform through the rotary pair, and the rotation angle of the arm is measured by the $\theta_{2}$. The forearm is connected to the arm through a rotating pair, and the rotation angle of the forearm relative to the arm is measured by the $\theta_{3}$. Since the mobile robot shown in Figure 1 has six independent motions, the mobile robot has six degrees of freedom.

Suppose that the coordinates of the origin of the coordinate system $\{O_{1}\}$ under $\{O\}$ are $(x, z, d)$, $d$ is the height of the mobile platform from the ground, and $D$ is a constant value during the movement of the robot; The coordinates of the center of the rotary platform under $\{O_{1}\}$ are $(0, 0, y)$, and $y$ is the distance between the rotary platform and the mobile platform. Then the motion variables $x, y, z$ can be used, $\theta_{1}, \theta_{2}$ and $\theta_{3}$ description.
3 Dynamic model of the mobile robot

Since mobile robot has six degrees of freedom, six generalized variables are needed to describe the motion of the robot. The generalized variable is \( q = [x, y, z, \theta_1, \theta_2, \theta_3] \). The general equations of the robot dynamics model are as follows:

\[
\frac{d}{dt} \frac{\partial T}{\partial \dot{q}} - \frac{\partial T}{\partial q} + \frac{\partial U}{\partial q} = Q_k
\]

where \( T \) is the kinetic energy of the robot, \( U \) is the potential energy of the robot, and \( Q_k \) is the non-conservative generalized force on the robot.

It can be seen from equation (1), in order to obtain the dynamic equation of the robot, the kinetic, potential energy and generalized force of the robot need to be calculated respectively.

3.1 Kinetic energy

If the length of the arm and the forearm are \( l_2 \) and \( l_3 \) respectively, and the mass is \( m_2 \) and \( m_3 \) respectively, the rotational kinetic energy of the arm relative to the rotary platform is:

\[
E_{kb1} = \frac{1}{2} J_{y2} \omega_2^2
\]

where \( J_{y2} \) is the moment of inertia of the arm around the platform, \( \omega_2 \) is the corresponding rotational angular velocity.

The kinetic energy of the arm rotating around the \( Y_1 \) axis of the coordinate system \( \{O_1\} \) is:

\[
E_{kb2} = \frac{1}{2} J_{y3} \omega_1^2
\]

where \( J_{y3} \) is the moment of inertia of the arm around the \( Y_1 \) axis, \( \omega_1 \) is the corresponding rotation angle speed.

During the movement of the robot, the rotation of the arm can be divided into the rotation around the rotary platform and the rotation around the \( Y_1 \) axis. Therefore, from equations (2) and (3), the rotational kinetic energy of the arm can be obtained as follows:

\[
E_{kb} = \frac{1}{2} \left( J_{y2} \omega_2^2 + J_{y3} \omega_1^2 \right) = \frac{1}{6} \left( m_2 l_2^2 \dot{\theta}_2^2 + m_2 l_2^2 \cos^2 \theta_2 \dot{\theta}_1^2 \right)
\]

Similarly, the rotational kinetic energy of the forearm can be calculated as follows:
\[
E_{kc} = \frac{1}{2} \left( J_{s4} \omega_3^2 + J_{s5} \omega_1^2 \right) \\
= \frac{1}{2} \left( \frac{1}{3} m_2 l_2^2 \dot{\theta}_3^2 + \frac{1}{3} m_3 l_3^2 \cos^2 \theta_3 \dot{\theta}_3^2 + m_1 l_1^2 \cos^2 \theta_2 \dot{\theta}_2^2 + m_1 l_1^2 \cos \theta_2 \cos \theta_3 \dot{\theta}_3^2 \right) \tag{5}
\]

\(J_{s4}\) and \(J_{s5}\) are the moments of inertia of the forearm around the arm and \(Y_1\) axis respectively, \(\omega_3\), \(\omega_1\) is the corresponding rotational angular velocity. In the process of robot motion, the rotary platform only rotates around \(Y_1\) axis, and its rotational kinetic energy is obtained.

\[
E_{kr} = \frac{1}{2} J_{s1} \omega_1^2 \tag{6}
\]
Where \(J_{s1}\) is the moment of inertia of the rotary platform around \(Y_1\) axis, \(\omega_1\) is the corresponding rotation speed.

As the mobile platform and lifting platform are made of lightweight materials, the weight of the mobile platform and lifting platform can be ignored compared with the rotary platform, and its arms and forearms. If the mass of the rotary platform is \(m_1\), the translational kinetic energy of the robot is:

\[
\begin{align*}
E_{k1} &= \frac{1}{2} (m_1 + m_2 + m_3) \dot{x}^2 \\
E_{k2} &= \frac{1}{2} (m_1 + m_2 + m_3) \dot{y}^2 \\
E_{k3} &= \frac{1}{2} (m_1 + m_2 + m_3) \dot{z}^2
\end{align*} \tag{7}
\]

Among them, \(E_{k1}\), \(E_{k2}\) and \(E_{k3}\) are translational kinetic energy of \(X\), \(Y\) and \(Z\) axis respectively. From equations (4)-(7), the kinetic energy of the robot can be obtained as follows:

\[
T = \frac{1}{6} \left[ \left( m_2 l_2^2 \cos^2 \theta_2 + m_3 l_3^2 \cos^2 \theta_3 \right) \dot{\theta}_3^2 + \left( m_2 l_2^2 \dot{\theta}_2^2 + m_3 l_3^2 \dot{\theta}_3^2 \right) \right] + \\
\frac{1}{2} \left[ \left( l_2 \cos \theta_2 + l_3 \cos \theta_3 \right) m_1 l_1 \cos \theta_2 \dot{\theta}_2^2 + (m_1 + m_2 + m_3) (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) \right] \tag{8}
\]

### 3.2 Potential energy

Selecting the plane where the wheel center of the mobile platform is located as the zero potential energy surface, the potential energy of the rotary platform can be calculated as follows:

\[
E_{p1} = \frac{1}{2} m_1 g l_1 \tag{9}
\]

The potential energy of the arm is:

\[
E_{p2} = m_2 g l_1 + \frac{1}{2} m_2 g l_2 \sin \theta_2 \tag{10}
\]

The potential energy of the forearm is:

\[
E_{p3} = m_3 g l_1 + m_3 g l_2 \sin \theta_2 + \frac{1}{2} m_3 g l_3 \sin \theta_3 \tag{11}
\]

The total potential energy of the system can be obtained from equations (9)-(11):

\[
U = \left( \frac{1}{2} m_1 + m_2 + m_3 \right) g l_1 + \left( \frac{1}{2} m_2 + m_3 \right) g l_2 \sin \theta_2 + \frac{1}{2} m_3 g l_3 \sin \theta_3 \tag{12}
\]
3.3 Dynamic equations

It is assumed that the friction between its forearm and arm, the arm and the rotary platform, the rotary platform and the lifting platform is zero, and the friction coefficient between the mobile platform and the ground is $\mu$. Then the non-conservative generalized force on the robot can be shown as the following:

$$Q_k = \begin{bmatrix} F - \mu(m_1 + m_2 + m_3) \cos \alpha \\ F - \mu(m_1 + m_2 + m_3) \sin \alpha \\ F_z \\ \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}$$  \hspace{1cm} (13)

$F$ is the driving force exerted by the motor on the mobile platform; $\alpha$ is the angle between the forward direction of the wheel and the x-axis; $F_Z$ is the driving force of the lifting platform; $\tau_1$, $\tau_2$ and $\tau_3$ are the driving torques applied to the slewing platform, and its arm and forearm respectively. By substituting equations (8), (12) and (13) into equation (1), the dynamic equation of the robot can be obtained as follows:

$$M\ddot{q} + H\dot{q}^2 + F = 0$$  \hspace{1cm} (14)

Where $M$ is the matrix of $6 \times 6$, which reflects the influence of system inertia on dynamics; $H$ and $F$ are matrices of $6 \times 1$, respectively. $M$, $H$ and $F$ are as follows:

$$M = \begin{bmatrix} M_{11} = M_{22} = M_{33} = m_1 + m_2 + m_3, & M_{12} = M_{13} = M_{14} = M_{15} = M_{16} = 0 \\ M_{24} = \frac{1}{3}(m_2l_2^2 \cos \theta_2 + m_3l_3^2 \cos \theta_3) + (l_2 \cos \theta_2 + l_3 \cos \theta_3) m_3l_2 \cos \theta_2 \\ M_{55} = \frac{1}{3}m_2l_2^2, & M_{66} = \frac{1}{3}m_3l_3^2, & M_{21} = M_{23} = M_{24} = M_{25} = M_{26} = 0 \\ M_{31} = M_{32} = M_{34} = M_{35} = M_{36} = M_{41} = M_{42} = M_{43} = M_{45} = M_{46} = 0 \\ M_{51} = M_{52} = M_{53} = M_{54} = M_{56} = M_{61} = M_{62} = M_{63} = M_{64} = M_{65} = 0 \end{bmatrix}$$  \hspace{1cm} (15)

$$H = \begin{bmatrix} H_{11} = H_{21} = H_{31} = H_{41} = 0 \\ H_{51} = \frac{1}{6}m_2l_2^2 \sin \theta_2 + \frac{1}{2}m_3l_3^2 \sin \theta_2 \\ H_{61} = \frac{1}{6}m_3l_3^2 \sin \theta_2 + \frac{1}{2}m_3l_3^2 \cos \theta_2 \sin \theta_3 \\ F_{11} = -\left[F - \mu(m_1 + m_2 + m_3)\right] \cos \alpha \\ F_{21} = -\left[F - \mu(m_1 + m_2 + m_3)\right] \sin \alpha \\ F_{31} = -F_z \\ F_{41} = -\tau_1 \\ F_{51} = \left(\frac{1}{2}m_2 + m_3\right)gl_2 \cos \theta_2 - \tau_2 \\ F_{61} = \frac{1}{2}m_3gl_3 \cos \theta_3 - \tau_3 \end{bmatrix}$$  \hspace{1cm} (16)

In formula (17), $g$ is the acceleration of gravity. Equation (14) is the dynamic model of the robot, which describes the motion of the robot under the action of driving force (distance). The relationship
between the driving force and the motion law of the robot can be obtained by solving equation (14). The
dynamic model of the robot lays the foundation for the design of its control system.

4 Numerical simulations
In order to simulate the motion law of the robot, the simulation parameters of the robot dynamics model
are set as shown in Table 1.

| Variable | Value       | Unit  |
|----------|-------------|-------|
| g        | 9.8         | m/s²  |
| l₂       | 0.16        | m     |
| l₃       | 0.16        | m     |
| m₁       | 4.78×10⁻²   | kg    |
| m₂       | 9.16×10⁻²   | kg    |
| m₃       | 9.46×10⁻²   | kg    |
| µ        | 0.1         |       |

In this paper, Adams software is used to simulate the movement of the robot, and the parameters
given in Table 1 are used to establish the solid model of the robot based on Solidworks software. Then
the model is imported into Adams, and the movement of the robot can be analyzed in Adams
environment. This paper mainly analyzes the motion of the robot driven by constant torque. In Adams
environment, the driving torque of the system is set as \( \tau₁ = \tau₂ = \tau₃ = 0.5 \text{ N·m} \). Given that the driving force received by the robot in the horizontal plane is zero, and the force received by the lifting platform in the
vertical direction is \( F_Z = 10.2 \text{ N} \), the motion law of the robot under this working condition can be
obtained, as shown in figure 3 to figure 8.

![Figure 3. The motion law of forearm](image-url)
Figure 4. The movement law of arms

Figure 5. Motion law of lifting platform

Figure 6. Motion law of rotary platform
Comparing Figure 3, 4 and 6, it is clearly seen that the motion laws of arm, forearm and rotary platform are almost the same. The results show that the rotation speed of its arm, forearm and the rotary platform changes periodically, the change period of the arm and the forearm is about 2 s, and the change period of the rotary platform is about 1.75 s. The maximum angular velocity of counter clockwise rotation of arm, forearm and platform is 50°/s. The maximum angular velocity of clockwise rotation is 80°/s. In the first second, the angular velocity and angular acceleration of the forearm and forearm are almost the same. When t=4.63 s, the angular accelerations of its arm, forearm and the platform all reach the peak value, about 4000°/s². As can be seen from Figure 5, the speed and acceleration of the lifting platform show periodic changes, with a period of about 2S. Different from the arm and forearm, the peak velocity of the lifting platform is only 0.55 m/s. As can be seen from Figure 7 and Figure 8, since the robot does not exert driving force in the horizontal plane, the speed of the mobile robot moving along the horizontal direction is small, and the peak value of the robot moving speed in the Z direction is only 0.11 m/s.
5 Conclusion
In this paper, a six degree of freedom mobile robot is analyzed. Based on Lagrange equation, the dynamic model is established. The dynamic simulation of mobile robot is carried out by ADAMS software, and the motion law of the robot is analyzed. The main conclusions are as follows:

(1) Under the constant moment condition, the movement of the big, forearms and the rotating platform is periodic, the movement period of the big and forearms is the same, and the movement period of the rotating platform is slightly smaller than that of the big and forearms. The simulation time is set to 5s, and when t = 4.63s, the acceleration of the large arm, forearm and the rotating platform reaches the peak value.

(2) Under the constant moment condition, the speed and acceleration of the lifting platform also show a certain periodicity, and the cycle is the same as that of the big and forearms. Due to the influence of the swing of the big and forearms, the mobile platform will obtain certain kinetic energy in the horizontal plane. The swing of the robot arm and the movement of the mobile platform are mutually coupled. In the path planning and controller design, the motion coupling between the robot degrees of freedom should be considered comprehensively.

Acknowledgments
The authors gratefully acknowledge the support of the Natural Science Foundation of Fujian Province, China (Grant No. 2018J05088), the Scientific Research Foundation of Jimei University (Grant No. ZQ2017005), Training Program of the National Natural Science Foundation of Jimei University (ZP2020047) and the Young and Middle-aged Teacher Education Scientific Program of Fujian Province (Grant No. JAT200250).

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