Simple Technique of Projected Lagrange for a Class of Multi-Stage Stochastic Nonlinear Programs

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Abstract

This paper presents a techniq for solving multi-stage stochastic nonlinear programs. The techniq is based on projected lagrange approach which generates the search direction by solving parallelly a set of quadratic programming subproblems with size much less than the original problem at each iteration. Mathematically, can be pointed out that Lagrange’s projection method can solve multi-stage stochastic nonlinear programs.

Keywords: Multi-stage stochastic nonlinear programs; Projected langrange; Scenario analysis; Decomposition

Introduction

Multi-stage stochastic nonlinear programs emerges in there are many practical situation, as production and manpower planning, portfolio’s selection etc.. Have a lot of research that gets contribution to solve Multi-stage stochastic nonlinear programs amongst those methodologies decomposition that is utilized on nonliner’s program also linear [1,2]. Most of that literature in reference to principle decomposition that introduced by Dantzig and Wide [3].

Decomposition method experience development which is with marks sense L-shape decomposition method that enough effective being utilized to troubleshoot multi-stage stochastic nonlinear programs

Several methods the other to troubleshoot multi-stage stochastic nonlinear programs is analyzed among those by Birge [4]. Since all method is gone upon on special structure of programs characters linear stochastics, so is hard generalisation to solve nonlinear stochastic programs, Gongyun Zhao introduces iterasi’s method that bases to solve nonlinear’s program multi’s stochastic phase.

Base research already being done researchers former to solve multi-stage stochastic nonlinear program, in this paper, writer propose a method for solve to program multi-stage stochastic nonlinear programs which is by use of lagrange’s projection method. This method is expected gets to give alternative solution to solve multi-stage stochastic nonlinear programs.

Materials and Methods

Consider the following multi-stage stochastic program with recourse:

\[
\min_{x \in \mathcal{X}} \mathcal{L}(X, x, \xi) + E_{\Omega} Q(x, \xi) \tag{1.1}
\]

Where \( X = \{ x | c_i(x) = 0 \} \subseteq \mathbb{R}^n \), the recourse function \( Q(X, \xi) = \min_{y \in \mathcal{Y}} c_i(x, y, \xi) + E_{\Omega} Q(x, y, \xi) \) Subject to \( c_i(x, y, \xi) = 0 \) (1.2)

And for \( t = 2, ..., T-1 \), recursively we have

\[
Q(x, y_1, y_2, ..., y_t, \xi) = \min_{y_t \in \mathcal{Y}_t} c_i(x, y_1, y_2, ..., y_t, \xi) + E_{\Omega} Q(x, y_1, y_2, ..., y_t, \xi) \)

Subject to \( c_i(x, y_1, y_2, ..., y_t, \xi) = 0 \) (1.3)

\[
Q(x, y_1, y_2, ..., y_T, \xi) = \min_{y_T \in \mathcal{Y}_T} c_i(x, y_1, y_2, ..., y_T, \xi) + E_{\Omega} Q(x, y_1, y_2, ..., y_T, \xi) \)

(1.4)

\( Q_0 = 0, \quad x \in \mathbb{R}^n \) is the deterministic vector, \( \xi \) is the realization of the random vector \( \xi \), \( y \in \mathbb{R}^n \) is the decision vector in the i-th stage, which is generated recursively by \( X, y_1, ..., y_{t-1} \) and \( \xi_1, ..., \xi_{t-1} \), hence represents \( y_0, y_1, ..., y_2, ..., y_T, \xi_1, ..., \xi_T \) actually. \( c_0 \) and \( c_0 \) are real-valued functions on \( \mathbb{R}^n \), \( c_0 \) is random since it is related to \( \xi_0, ..., \xi_T \).

For the discrete random vector \( \xi = (\xi_1, ..., \xi_T) \) if \( c_i \) have finite realizations \( c_i(i = 1, ..., S_i) \), then all these \( c_i \) form the constraint functions on stage \( i \). The details on the formulation of multi-stage stochastic programs can be found, e.g. in Kall and Wallace [7].

Let \( \xi = (\xi_1, ..., \xi_T) \) and assume that \( (\Omega, \theta, P) \) is the associated probability space. Suppose that we have \( S \) scenarios \( \xi_1^{(s)}, ..., \xi_T^{(s)} \), which has a fixed and known probability distribution \( \{p_i, \xi_i \} \). Then (1.1)-(1.4) can be reformulated as the following nonlinear programming problem:

\[
\min \sum_{s=1}^{S} f_s(x^{(s)}) \tag{1.5}
\]

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\[
\begin{align*}
\mathcal{H}(x^{(t)}) &= o, s = 1, 2, \ldots S \\
\sum_{i=1}^{s} A_i(x^{(t)}) &= 0
\end{align*}
\]  
(1.6)

\[
\begin{align*}
z^{(t)} &= (x^{(t)}, y_1^{(t)}, \ldots, y_{s_{1}}^{(t)}) \in \mathbb{R}^n, n = \sum_{i=0}^{s-1} n_i \\
f_i(z^{(t)}) &= p_i(z_i^{(t)}) + \sum_{i=0}^{s} q_i(x^{(t)}, y_1^{(t)}, \ldots, y_{s_{1}}^{(t)}, z_i^{(t)}) \\
h_i(z^{(t)}) &= (c_i(z_i^{(t)}) + z_i^{(t)} + \ldots + c_{s_{1}}(z_{s_{1}}^{(t)}) + z_{s_{1}}^{(t)})
\end{align*}
\]  
(1.7)

Constraints (1.7) are the so-called non-anticipativity constraints, which reflect the fact that scenarios sharing a common history up to any moment of time must have a common decision up to that moment. Readers can refer to Rockafellar and Wets [8] for more details on this reformulation.

Lagrange’s function in common is deep shaped as follows,

\[
\min \mathcal{L}(x, x, \lambda, \rho) \\
s.t. \tilde{A}(x) = Ax
\]

Let say to be given by parameters \(q, r\), therefore form augmented lagrange of equation upon is as follows,

\[
\begin{align*}
L,(x, q, r) &= f(x) + q^T Ax + \frac{1}{2} \|Ax\|^2 \\
&\text{Let say to be given } x, \mu, \lambda, \rho, \text{ so}, \\
\min &\ L(x, x, \mu, \lambda, \rho) \\
&\text{subject to } 0, 1 \leq x \leq u \\
&\text{To a } \tilde{A} = Ax \\
&\text{Therefore objective function for problem to multi-stage stochastic nonlinear programs is as follows,}
\end{align*}
\]

\[
L(x, x, \mu, \lambda, \rho) = f(x) + \mu^T (f - \tilde{f})
\]

Result and Discussion

In this paper, Lagrange’s projection method will be utilized for multi-stage stochastic nonlinear programs. The methods based lagrange augmented modified.

Assume that \(f(x) : \mathbb{R}^n \to \mathbb{R}\) and \(h(x) : \mathbb{R}^n \to \mathbb{R}^m\) are two contin differensiable functions, \(h_i : \mathbb{R}^n \to \mathbb{R} (i = 1, \ldots, m)\) and \(h_i(z^{(t)}) = (h_i(z^{(t)}) \ldots, h_i(z^{(t)}) )^T, \zeta \in \mathbb{R}^{n\times d}, A = (a_{1}, \ldots, a_{d}) \in \mathbb{R}^{d\times m}\) is matrix row with full rank and has special structure.

So equation (1.5) – (1.7) are formulated as forms as follows, \(\min f(x)\)

\[
\begin{align*}
&\text{s.t. } h(x) = 0, 1 \leq x \leq u \\
&\text{and } f(x) = \sum_{i=1}^{s} f_i(z^{(t)}) h_i(z^{(t)}) \text{ and } A(x) = A_i z^{(t)}
\end{align*}
\]

That note function of \(x\) assumed contin differensiable.

With Lagrange’s projection method, objectif’s function takings \(f(x)\) one equal to form commons of Lagrange augmented’s functions,

\[
f(x) = -\lambda^T A(x) + \lambda^T A(x) + \frac{1}{2} \rho^T (f - \tilde{f})^2
\]

\[
\lambda^T \text{ is lagrange’s coefficient vector and } \rho \text{ is penalti’s parameter.}
\]

Linear Approximation from constraints nonlinear is make iterasi along starting point \(x(k)\) from iterasi process followings;

\[
A(x(k + 1)) = A(x(k)) + h(x(k + 1) - x(k))
\]

So algorithm that presented to solve subproblem constraint’s line linear with function objective is modify lagrange augmented and linear approximation \(f(x)\) on the \(x(k)\) are as follows

\[
\begin{align*}
&\min \mathcal{L}(x, x, \lambda, \rho) \\
&\text{s.t. } \tilde{A} = 0, 1 \leq x \leq u \\
\end{align*}
\]

Where is function objective is modify lagrange augmented and \(\tilde{A}\) is aproksimasi \(f(x)\) on the \(x(k)\) and, \(\lambda\) one that constitute Lagrange’s solution and coefficient that correspondence to a subproblem.

Definition 1. To \(\rho = 0\), line \([x, \lambda]_x\) convergent, to a \(x(k)\) and \(\lambda\) one that constitute Lagrange’s solution and coefficient that correspondence to a subproblem.

Definition following to give convex requisite to form lagrange’s modification.

Definition 2. To \(\rho = 0\), form modifies Lagrange \(\mathcal{L}(x, x, \lambda, 0)\) is convex.

Solving problem multi-stage stochastic nonlinear programs by use of lagrange’s projection method depends on penalti’s parameter \(\rho\). If \(\rho\) too large therefore will be hard to find solution. On the contrary, if \(\rho\) too little \(x(k)\), one that is expected as solution will go away to reach convergence.

Here after partition \(x\) as form \(x^1 (x linear)\) and form \(x^2 (x nonlinear)\). And partition also as \([B, S, N]\) with matrix \(B\) (basic) is matrix square and nonsingular, \(S\) (super basic) is matrix \(m \times s\) by \(0 \leq s \leq n - m\), and \(N\) (nonbasic) are residues column of matrix \(A\), therefore constraints active becomes as follows;

\[
\begin{bmatrix}
B & S & N \\
I & & \\
\end{bmatrix}
\begin{bmatrix}
x_B \\
x_S \\
-x_N \\
\end{bmatrix} = \begin{bmatrix}
h_B \\
\end{bmatrix}
\]

where \(x_B, x_S, x_N\) called by basics’s variable, superbasics and nonbasics what do accordingly with \([B, S, N]\).

Note: basics’s variable and superbasic is variable one free on bounds.

Theorem following to give that surety nonlinear’s program have solution.

Theorem 3. Let say nonlinear’s program has \(t\) nonlinear’s variable (well on objetkif’s function or constrain even), therefore an optimal solution available on each superbasics’s variable number \(s\) one accomplishes \(s \leq t\).
**Proof.** Let is variable non linear regular on appreciative optimal. Problem is rest is linear program for a basic’s solution whatever available ($s=0$). Its result is trivial if variable nonlinear is regarded as superbasic on early problem. If $s=t$, available variable nonlinear that current on bounds is nonbasic the so called. Therefore has $s \leq t$.

From Theorem 3 secure to mark sense optimal solutions so base 3 get to be made by definition followings.

**Definition 4.** Optimal solution available for number of smaller superbasic variable or equal to nonlinear variable number.

Hereafter been given simple algorithm for multi-stage stochastic nonlinear programs.

Set 1. Let $K=0$, Choose some initial estimates $x_0, y_0$ and $\lambda_0$. Specify a penalty parameter $\rho \geq 0$ and convergence tolerance $\epsilon > 0$.

Set 2. Given $x_k, y_k, \lambda_k$ and $\rho$, solve the linearly constrained subproblem (1.8) to obtain new quantities $x_{k+1}, y_{k+1}$, and $\pi$ (where $\pi$ is the vector of Lagrange multipliers for subproblem).

Set 3. Let $\lambda_{k+1}$ = the first components of $\pi$.

Set 4. Test convergence (see Definition 5). If optimal, exit.

Set 5. Relinearize the constraints at $x_{k+1}$.

Set 6. Let $K= K+1$ and repeat from step 2.

**Definition 5.** The point $(x_k, y_k)$ are solution for problem nonlinear if following condition are satisfied, $(x_k, y_k)$, satisfies the first-order Kuhn Tuckere’s conditions for a solution to the linearized problem.

From the theorems, definitions and algorithm that is given gets to be seen that lagrange projection method can utilized to solve multi-stage stochastic nonlinear programs.

**Conclusions**

A projected Lagrangian method is a very effective approach for solving medium-size nonlinear programming. By using lagrange augmented modified, strategy a for solving a class of multi-stage stochastic nonlinear programs is proposed, which choise of $\rho$ with size much less than the original problem at each iteration. Generalized reduced gradient methods can be introduced to derive the estimates of the dual multiplier associated with the nonanticipativity constraints.

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