Electromagnetic Fields of Slowly Rotating Magnetized Gravastars

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We study the dipolar magnetic field configuration and present solutions of Maxwell equations in the internal background spacetime of a slowly rotating gravastar. The shell of gravastar where magnetic field penetrated is modeled as sphere consisting of perfect highly magnetized fluid with infinite conductivity. Dipolar magnetic field of the gravastar is produced by a circular current loop symmetrically placed at radius $a$ at the equatorial plane.

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The existence of strong electromagnetic fields is one of the most important features of rotating compact stars observed as pulsars and magnetars with surface magnetic field as high as $10^{14} \text{G}$. On the other hand the notion of a compact astrophysical object has been one of the central issues in contemplating general relativity as the theory of astrophysical processes and structures. The search for solutions of Einstein equations with different inputs for the physical sources of gravitational field has been one of the stepping stones on the way to achieving comprehensible picture of the universe. Among solutions found so far one eminent position certainly belongs to black hole solutions with intriguing properties and characteristics.

The recent discovery of accelerating universe had inspired discussion of the existence of dark energy (see, for example, 2,3,4,5) and in turn research for alternative configurations which led to a solution dubbed gravastar, the gravitational vacuum
star of dark energy, by Mazur and Mottola. These spherically symmetric static global solutions to the Einstein equations candidates for highly compact astrophysical objects and in this sense alternatives to black holes evolve from the segment of the de Sitter geometry in the center with the equation of state of dark energy, proceed through a thin vacuum phase transition layer, avoid formation of the event horizon, and swiftly match the exterior Schwarzschild spacetime. The common feature of gravastar realization is the anisotropy of pressure in the outer shell of the object.

Several astrophysically relevant aspects of gravastar solutions such as thermodynamic properties, modes of quasi-normal oscillations, and ergoregion instability were recently discussed in the literature. De Benedictis et al. and Chirenti and Rezzolla investigated the stability of the original model of Mazur and Mottola against axial perturbations, and found that gravastars are stable to these perturbations. Chirenti and Rezzolla also showed that their quasi-normal modes differ from those of a black hole of the same mass, and thus can be used to distinguish a gravastar from a black hole. The aim of this short note is to find interior electromagnetic fields in the shell of gravastar consisting of perfect highly magnetized fluid with infinite conductivity. It is assumed that dipolar magnetic field of the gravastar is produced by a circular current loop symmetrically placed at radius $a$ at the equatorial plane.

Throughout, we use a space-like signature ($- , + , + , +$) and a system of units in which $G = 1 = c$ (However, for those expressions with an astrophysical application we have written the speed of light explicitly.). Greek indices are taken to run from 0 to 3 and Latin indices from 1 to 3; covariant derivatives are denoted with a semi-colon and partial derivatives with a comma.

Metric, which is describing spacetime the spherical symmetric slowly rotating gravastar, can be written in the following form (see, for example, 6711),

$$ds^2 = -A^2(r)dt^2 + A^{-2}(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 - 2\omega(r)r^2 \sin^2 \theta d\phi dt \quad (1)$$

where $r$ is the radial coordinate, $\omega(r)$ is the angular velocity of dragging of inertial frames around slowly rotating gravastar, and here

$$A^2(r) = \begin{cases} 
1 - \frac{2M}{r}, & r > a(\tau), \\
1 - \frac{r^2}{R^2}, & r < a(\tau).
\end{cases}$$

where $r = a(\tau)$ is a timelike hypersurface, at which the infinitely thin shell is located, and $\tau$ denotes the proper time of the thin shell and the constant $R = \sqrt{a^2/2M}$.

A circular current loop with current $I$ and net charge $q$, symmetrically placed at radius $a$ in the equatorial plane of a slowly rotating gravastar, has electric current
density components as in \cite{12}:

\begin{align}
J^i &= \frac{qA}{2\pi a^2} \delta(r - a) \delta(\cos \theta) , \\
J^\phi &= \frac{Ir \sin \theta}{2\pi a^2} \delta(r - a) \delta(\cos \theta) ,
\end{align}

where 'hatted' quantities are the orthonormal components measured by zero angular momentum observers (ZAMO) with four-velocity

\begin{align}
(u^\alpha)_{\text{ZAMO}} &\equiv \frac{1}{A} (1, 0, 0, \omega) , \quad (u_\alpha)_{\text{ZAMO}} \equiv A (-1, 0, 0, 0) .
\end{align}

Gravastar is slowly rotating with 4-velocity:

\begin{align}
\omega^\alpha &\equiv \frac{1}{A} (1, 0, 0, \Omega) , \quad \omega_\alpha \equiv A \left( -1, 0, 0, \frac{\bar{\omega}r^2 \sin^2 \theta}{A^2} \right) ,
\end{align}

where \( \bar{\omega} = \Omega - \omega \), \( \Omega \) is angular velocity of rotation of gravastar.

We now look for an interior solution of Maxwell equations in background spacetime given by (1) assuming that magnetic field of the star is dipolar. To simply the search for a solution we look for separable solutions of Maxwell equations in the form

\begin{align}
B^\alpha(r, \theta) &= F(r) \cos \theta , \\
B^\theta(r, \theta) &= G(r) \sin \theta , \\
B^\phi(r, \theta) &= 0 ,
\end{align}

where functions \( F(r) \) and \( G(r) \) will account for the relativistic corrections due to a curved background spacetime \cite{13}.

Maxwell equations with the ansatz (7)–(9), yield the following set of equations

\begin{align}
(r^2 F)_{,r} + 2rG/A &= 0 , \\
(rAG)_{,r} + F &= 0 .
\end{align}

Note a first important result in the system of equations (10)–(11). In the case of stationary electromagnetic fields, the general relativistic frame dragging effect and gravitomagnetic charge do not introduce a correction to the radial eigenfunctions of the magnetic fields. In other words, in the case of infinite conductivity and as far as the magnetic field is concerned, the study of Maxwell equations in a slow rotation metric provides no additional information with respect to a non-rotating metric. The dependence from the frame dragging effects is therefore expected to appear at \( \mathcal{O}(\omega^2) \).
The stationary vacuum magnetic field external to an aligned magnetized relativistic star is well known and given by

\[ B^\hat{r}_{\text{ex}}(r,\theta) = -\frac{3\mu}{4M^3} \left[ \ln N^2 + \frac{2M}{r} \left( 1 + \frac{M}{r} \right) \right] \cos \theta, \tag{12} \]

\[ B^\hat{\theta}_{\text{ex}}(r,\theta) = \frac{3\mu N}{4M^2 r} \left[ \frac{r}{M} \ln N^2 + \frac{1}{N^2} + 1 \right] \sin \theta, \tag{13} \]

where \( \mu = \pi a^2 (1 - 2M/a)^{1/2} I \) and \( N = (1 - 2M/r)^{1/2} \) is lapse function.

Interior solution for magnetic field is

\[ B^\hat{r}(r,\theta) = -\frac{3\mu a^3}{4r^3 M^3} \left[ \ln N_a^2 + \frac{2M}{a} \left( 1 + \frac{M}{a} \right) \right] \frac{r}{r_a} \frac{\pi}{\pi_a} - \frac{\arctanh \frac{r}{r_a}}{\arctanh \frac{r}{r_a}} \cos \theta, \tag{14} \]

\[ B^\hat{\theta}(r,\theta) = \frac{3\mu a^2 L N_a}{4r^3 L a M^2} \left[ \frac{a}{M} \ln N_a^2 + \frac{1}{3N_a^2} - \frac{2N_a^2}{3} + \frac{7}{3} \right] \frac{\pi r_a}{\pi a} - \frac{\arctanh \frac{r}{r_a}}{\arctanh \frac{r}{r_a}} \sin \theta, \tag{15} \]

where \( L = (1 - r^2/R^2)^{1/2} \), subscript \( a \) denotes quantities measured at \( r = a \). The values of the integration constant are defined from the continuity of the radial magnetic field across the gravastar surface, i.e. that \([B^\hat{r}]_{\text{in}} = [B^\hat{r}]_{\text{ext}}\), boundary condition for the tangential magnetic field \([B^\hat{\theta}]_{\text{ext}} - [B^\hat{\theta}]_{\text{in}} = 4\pi i \hat{\phi}\), where \( i \) is density of the surface current at \( r = a \). The radial dependence of magnetic field of gravastar \((12)\)–\((15)\) is plotted in Fig. 1.

Interior electric fields

\[ E^\hat{r}(r,\theta) = \frac{\omega r \sin \theta}{c L} B^\hat{\theta}(r,\theta), \tag{16} \]

\[ E^\hat{\theta}(r,\theta) = -\frac{\omega r \sin \theta}{c L} B^\hat{r}(r,\theta). \tag{17} \]

can be found from the condition of the infinite conductivity \( \sigma \) in Ohm law with the explicit components of the conduction current \( j^\alpha \) as

\[ j^\hat{r} = \sigma \left[ E^\hat{r} + A^{-1} \left( v^\hat{\theta} B^\hat{\theta} - v^\hat{r} B^\hat{r} \right) \right], \tag{18} \]

\[ j^\hat{\theta} = \sigma \left[ E^\hat{\theta} + A^{-1} \left( v^\hat{r} B^\hat{r} - v^\hat{\theta} B^\hat{\theta} \right) \right], \tag{19} \]

\[ j^\hat{\phi} = \sigma \left[ E^\hat{\phi} + A^{-1} \left( v^\hat{\phi} B^\hat{\phi} - v^\hat{r} B^\hat{r} \right) \right], \tag{20} \]

and using expressions for velocity of rotation \( v^\hat{\phi} \) and magnetic field \((14), (15)\).

Exterior electric field of slowly rotating gravastar coincides with the electric field of rotating neutron star and given by equations (124)–(126) in the paper \(13\).

In this paper we have derived components of the dipolar magnetic field of the gravastar which is produced by a circular current loop symmetrically placed at radius \( a \) at the equatorial plane. The knowledge of gravastar’s magnetic field can be useful for description of different physical processes in the gravastar.
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2.5
3
3.5
4
\frac{r}{M}

Fig. 1. Dependence of the radial (left) and azimuthal (right) components of the magnetic field of the gravastar from the radius. Interior magnetic field inside of the shell increases up to the border at \( a = 2.1 \) (here \( a/M = 2.1 \)) which is indicated with the vertical dash line. Azimuthal component of magnetic field subjects to the discontinuity across the boundary of gravastar. Exterior magnetic field of gravastar is decaying as \( 1/r^3 \).

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