Electromagnetic Form Factor of the Nucleon in the Time-like Region*

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Abstract

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We explore the possibility to verify vector meson dominance for the nucleon by measuring the half-off-shell form factor in the vector-meson mass region. Cross sections for the process $\gamma p \rightarrow p + e^+e^−$ be presented.

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There exists by now a vast amount of experimental data on the electromagnetic form factor of the nucleon \[1, 2\]. Most of these data were obtained with electron scattering experiments yielding information on the form factor in the space-like sector. The form factor in the time-like region has been explored by \( p\bar{p} \) annihilation experiments; because of the masses of the particles involved this channel is open only for \( q^2 > 4m_p^2 \approx 3.6 \text{ GeV}^2 \).

To explain the coupling of photons to hadrons vector meson dominance (VMD) has been invoked; in this picture one assumes that the photons couple to hadrons only through the vector mesons. While this picture is well established for the pion, the situation for the nucleon is less obvious \[3\]. The experimentally determined form factors have been shown to follow very well the so-called dipole fit \[4\]

\[
G_E(q) = \left( \frac{m_v^2}{q^2 - m_v^2} \right)^2
\]  

where \( m_v^2 \approx 0.71 \text{ GeV}^2 \). The fact that not a monopole, but a dipole dependence appears here presents a problem for naive vector meson dominance since the form factor should contain the vector meson propagator only linearly. Furthermore, coupling constants often agree only within factors of 2 with the predictions of vector meson dominance \[5, 6\].

There are essentially two classes of explanations for the unexpected \( q \)-dependence of the propagators. In the first one assumes that the observed form factor is the sum of contributions of several vector mesons and that these contributions conspire such that the effective \( q \)-dependence is of dipole type \[1\]. To this class of models we also count the attempts to describe the form factor as a sum of two terms, one describing the coupling of the photon to the meson cloud (vector meson dominance) and a second giving the direct coupling of the photon to the quark core \[5\]. In the
second class of explanations one assumes that the effective form factor is the \textit{product} of the vector meson propagator and a cut-off form factor of monopole type that takes the properties of perturbative QCD at large $q^2$ into account \cite{7}. Both of these models, if extended into the timelike sector, give large, resonance-like contributions at the vector-meson mass $m_v \approx 0.78$ GeV \cite{8} which, of course, have never directly been seen because they lie in the so-called unphysical region which is not accessible in on-shell processes. There the models as well as the dispersion theoretical analyses \cite{9} have to be considered as analytic continuations from the space-like measurements into the time-like region, and, more specifically, to the on-shell point there. The direct experimental proof for VMD for the nucleon, the presence of a clear resonance in its electromagnetic form factor just as for the pion, is thus still missing.

In this letter we explore the possibility to ascertain the resonance behavior of the form factor expected from VMD by determining experimentally the \textit{half-off-shell} form factor of the nucleon in the time-like region just mentioned. If measured, this form factor could then be analytically continued to the on-shell point even at time-like $q$ in the unphysical region. This continuation from a time-like half-off-shell point may supplement and be even more conclusive and more stringent than the continuation from spacelike, on-shell points.

One way to explore the half-off-shell form factor in the time-like region is that of dilepton production in elementary processes. The DLS group at the BEVALAC \cite{10} has recently undertaken such experiments both for $p + p$ and $p + d$ reactions. First theoretical analyses \cite{11,12} indicate a sensitivity of the invariant-mass spectra to the electromagnetic form factor. However, the results of these experiments do not allow for an easy, unambiguous extraction of the form factor, because the strong-interaction vertices affect the cross-section. The same is true for quite early experiments employing the reaction $p + \pi \rightarrow p + e^+ e^-$ \cite{13}. 

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In order to avoid the ambiguities necessarily connected with the presence of strong-interaction vertices and form factors we propose to explore a purely electromagnetic reaction. The process we consider is Compton-scattering of a real photon into the time-like sector, i.e. the process \( \gamma p \rightarrow p + e^+ e^- \).

Real Compton scattering in the energy range considered here, i.e. up to 1 GeV photon energy, has been investigated in refs. [14] and [15]. These authors have given a decomposition of the Compton scattering amplitude

\[
A = A^R + A^B \exp(-C(1 - \cos \vartheta)) \tag{2}
\]

into a resonance contribution \( A^R \) and into a non-resonant Born amplitude \( A^B \).

The Born amplitudes are given in detail in ref. [14], the exponential multiplying it damps the backward-angle scattering and represents an ad-hoc correction that takes neglected effects such as the \( t \)-channel scattering into account; \( \vartheta \) is the photon scattering angle in the c.m. frame.

While for the resonance amplitude the \( u \)-channel is neglected, the only contribution to this amplitude is the \( s \)-channel [15]

\[
A_{\lambda \mu} = \frac{k_{0}}{k} \frac{2\gamma^\lambda \gamma^\mu W}{M_r^2 - W^2 - iW \Gamma_r} e^{i\delta_r} \tag{3}
\]

with

\[
k^2(q^2, W^2) = \left( \frac{W^2 - m_r^2 + q^2}{2W} \right)^2 - q^2
\]

\[
k_{0}^2 = k^2(q^2 = 0, W^2 = M_R^2).
\]

Here \( M_r \) is the mass of the resonance, \( \Gamma_r \) its width and \( W \) the invariant energy. The phases \( \delta_r \) are introduced to take the relative phases between the various resonances.
and the Born amplitude into account. For later purposes we have written eq. (3) in a form also correct for off-shell photons \((q^2 \neq 0)\). Essential for our purposes here is the structure of the elastic photon width \(\gamma^\lambda\) in eq. (3) which is given by

\[
\gamma^\lambda = \gamma_0^\lambda \left( \frac{k}{k_0} \right)^{j_0} F_\gamma(q^2 = 0, W^2) \tag{4}
\]

with the form factor

\[
F_\gamma(q^2, W^2) = \left( \frac{k_0^2 + X^2}{k^2 + X^2} \right)^{j_0/2}.
\]

Here \(j_0\) is the spin of the resonance and \(\gamma_0^\lambda\) the photon coupling constants of the resonance. For all further details see Wada et al. [15]. The analysis includes all resonances up to a mass of about 2 GeV; in the following we use the parameters extracted there from fitting Compton scattering data in the energy range up to 1.2 GeV.

In order to use this information on the electromagnetic vertices of the nucleon for our purpose of investigating the electromagnetic form factor of the nucleon in the time-like sector, we now consider the general structure of the electromagnetic vertex appearing in the \(\gamma + p \rightarrow p + e^+ e^-\) process. We first note that both electromagnetic vertices are half-off shell, at the entrance point for an on-shell photon and at the exit point for a virtual time-like photon. The general structure of the electromagnetic half-off-shell vertex has recently been investigated in detail by Tiemeijer and Tjon [16] and Naus and Koch [17]. These authors have shown that the general electromagnetic form factor for a half-off-shell vertex depends on the squares of the fourmomentum of the photon, \(q^2\), and of the off-shell nucleon, \(W^2\), \(F = F(q^2, W^2)\). The reduction of the original 6 different form factors to only 1 is possible when using gauge-invariance, particle-antiparticle symmetry [18] and the experimentally well established equality.
of electric and magnetic form factors in the space-like region.

In order to explore the sensitivity of the dilepton invariant mass spectra on a possible VMD form of the form factor we now extend the electromagnetic form factor (4) into the region $q^2 \neq 0$ and multiply a VMD based factor $F_{VMD}$ to it. We thus make the ansatz

$$F(q^2, W^2) = F_{VMD}(q^2) F_{\gamma}(q^2, W^2) \quad (5)$$

with

$$F_{VMD}(q^2) = \frac{m_V^2}{m_V^2 - q^2}$$

for the form factor in the timelike region. The singularity at $q^2 = m_V^2$ is avoided by implementing the decay width of the vector mesons in the denominator of $F_{VMD}$ in (5). For simplicity we assume the same form factor for all the hadrons in the timelike region. This is supported by the analysis of Stoler that shows that the spacelike form factors of all the resonances agree within a few percent for $q^2 > -1 \text{ GeV}^2$ [19].

It is now straightforward to generalize the amplitude-based description of Compton scattering of ref. [15] to virtual photons. This is done by using in eqs. (3) and (4) the momentum of a massive photon created in the resonance decays. In addition to the real Compton amplitude the photon vertex $\gamma^\lambda$ is replaced

$$\gamma^{(\lambda = \lambda_f - \mu_f)} = J(\mu_f) \cdot \epsilon^{*(\lambda_f)} \quad \Rightarrow$$

$$\begin{cases}
(J(\mu_f) \cdot \epsilon^{*(+1)}) & (J^{e^+e^-} \cdot \epsilon^{(+1)}) \\
(J(\mu_f) \cdot \epsilon^{*(-1)}) & (J^{e^+e^-} \cdot \epsilon^{(-1)}) \\
(J(\mu_f) \cdot \epsilon^{*(0)}) & (J^{e^+e^-} \cdot \epsilon^{(0)}) \frac{M^2}{k^2} \frac{1}{M^2}
\end{cases} \quad ; \quad (6)$$

the superscripts in parenthesis give the helicity quantum numbers, $M$ is the invariant
mass of the dilepton pair, $1/M^2$ the massive photon’s propagator and $J^{e^+e^-}$ the dilepton current.

In this way we take into account that the massive photon can be polarized in longitudinal direction. For the longitudinal contribution we assume

$$J^{(\mu_f)} \cdot \epsilon^{*0} = \gamma^{(-\mu_f)} ;$$

(7)

thus we can use for this amplitude also the photon coupling constant $A_{1/2}$ from ref. [15]. The contribution from the nucleon poles is calculated exactly. In this way we retain the experimentally determined half-off-shell vertex for real photons and extend it to the time-like photon vertex. We also ensure by construction that the cross-section for dilepton production reduces at the photon point to the experimentally correct Compton scattering value.

Within this description we are able to calculate the amplitude for the Feynman diagrams shown in Figs. 1a and 1b. For the creation of virtual photons we also have to add contributions to the amplitude from the so called Bethe-Heitler diagrams (Fig. 1c); for the corresponding space-like form factor we use the measured one. Because of the pole-like behaviour of the electron propagator this contribution dominates in any integrated cross section. Therefore, we look only at a five-times differential cross section, very similar to what was done in [20]. In this symmetric kinematical situation (Fig. 2) it was shown that the influence of the Bethe-Heitler diagrams decreases with increasing angle $\theta$ (ref. [20]). The invariant mass of the dilepton pair is a simple function of only this opening angle.

It is possible in this kinematical situation that a given invariant mass corresponds to two different angles, one less and one larger than $90^\circ$. For $\theta < 90^\circ$ we show only the invariant mass spectra above 400 MeV, because for smaller masses (=smaller angles)
the contribution from the Bethe-Heitler diagrams dominates the spectrum. Even for larger invariant masses the Bethe-Heitler contribution is the main contribution to the cross section (Fig. 3) if we switch off the VMD form factor for the hadrons by setting $F_{VMD} = 1$. Since the contribution of the nucleons (Fig. 3: dotted line) is dominated by the Bethe-Heitler diagrams, it shows only a weak resonance shape. But a resonance behaviour is definitely seen for the contribution of the nucleon resonances which is, now including the VMD, larger than the Bethe-Heitler contribution. Because of the weak sensitivity of the cross section to the nucleon form factor it may be possible to get information on the electromagnetic form factor for the resonances if the detection angle of the leptons is less than 90 degree, but the signal sits on a steeply falling background.

If the angle is larger than 90° (Fig. 4), we find a strong effect of the VMD on the nucleon as well as the resonance contribution, because now the influence of the Bethe-Heitler diagrams is negligible. For this kinematic situation we see in Fig. 4 that the resonance contribution is now larger than the nucleonic one by roughly a factor of two.

After fixing the resonance form factors $F^R$ in the forward region it might be possible to extract from the cross section in backward kinematics the form factors $F^N_i$ for the nucleon. Because the resonances are propagated with $q^2_R > 3 \text{ GeV}^2$, necessary to get dilepton invariant masses well above the vector meson region, many of them are excited and it may not be possible to disentangle their form factors separately. However, on the basis of Stoler’s analysis [19], which shows that the spacelike form factors of all resonances roughly agree in the momentum range considered here, we expect that at least an average form factor can be extracted which is not too different from the individual ones.

Our calculations thus show that indeed sizeable effects from time-like electro-
magnetic form factors on the dilepton mass spectra are to be expected in $\gamma + N \rightarrow N + e^+ e^-$ experiments. Due to the different effects in forward and backward scattering of the proton it is possible to obtain information on the form factors for the nucleon as well as for some of the resonances in the time-like sector in a momentum range where so far no other information is available.

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Figure captions

**Fig. 1** Feynman diagrams photoproduction of dileptons in the s-channel (a), in the u-channel (b) and the Bethe-Heitler diagrams (c).

**Fig. 2** Special symmetric kinematic to reduce the influence of the Bethe-Heitler diagrams (see ref. [20]).

**Fig. 3** The differential cross section for $\omega_{lab} = 1.2 \, GeV$ and $\theta < 90^0$. Here $\Omega_+(\theta_+, \phi_+)$ denotes the angle between the incoming photon and the outgoing virtual photon, $\Omega_-(\theta_-, \phi_-)$ that between the relative momentum of the $e^+e^-$ pair and that of the outgoing virtual photon. The calculation was performed for the cm angles $\theta_+ = 0^0$ and $\theta_- = 90^0$. The solid line gives the results of the full calculation including the effects of VMD, while the dash-dotted line represents only the full calculation without the VMD. The individual contributions (including the VMD) of the nucleon pole and the resonances are given by the dotted and the dashed line, respectively.

**Fig. 4** The differential cross section for $\omega_{lab} = 1.2 \, GeV$ and $\theta > 90^0$. For further details see Fig. 3, but now using $\theta_+ = 180^0$. 

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