Model Diagnostics Based on Cumulative Residuals:  
The R-package **gof**  

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Abstract  
The generalized linear model is widely used in all areas of applied statistics and while correct asymptotic inference can be achieved under misspecification of the distributional assumptions, a correctly specified mean structure is crucial to obtain interpretable results. Usually the linearity and functional form of predictors are checked by inspecting various scatterplots of the residuals, however, the subjective task of judging these can be challenging. In this paper we present an implementation of model diagnostics for the generalized linear model as well as structural equation models, based on aggregates of the residuals where the asymptotic behavior under the null is imitated by simulations. A procedure for checking the proportional hazard assumption in the Cox regression is also implemented.  

*Keywords:* model diagnostics, regression, R, cumulative residuals,  

1. Introduction  
The generalized linear model is one of the most widely used classes of statistical models, however, the standard methods of inference relies on distributional and linearity assumptions. The importance of this is sometimes underestimated, to some extent because few tools are available for checking all the aspects of the model. While the distributional assumptions can be relaxed, i.e., by using a sandwich estimator as implemented in the \texttt{sandwich} package (Zeileis 2006), careful attention should be paid to the validity of the specified mean structure. A typical model check involves assessment of various residual plots. As the true variance of individual residuals are  

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unknown it can be difficult to decide whether a residual plot indicates a reasonable specification of the mean or not. In a paper by Su and Wei (1991) it was proposed instead to look at certain aggregates of the residuals, such as the cumulative sum over predicted values or covariates. The key result here is, that the asymptotic distribution of such aggregates can be determined under the hypothesis that the model is correctly specified.

The R environment is one of the most widely used statistics platforms but lacks objective diagnostics tools for many regression models, and in particular methods based on aggregates of residuals, thus motivating the creation of the gof-package described in the following sections.

2. Implementation

The gof package implements diagnostics of the linearity assumptions for the generalized linear model and linear structural equation models. Further similar methods are available for checking the proportional hazards assumption of the Cox regression model for right censored data. The following section describes the theoretical details behind the implementation.

2.1. Generalized linear model

The case of generalized linear models was first examined by Su and Wei (1991). Let $Y$ be the response variable with a distribution from a (natural) exponential family:

$$f(Y = y_i | \theta_i, \phi) = \exp \left\{ \frac{\theta_i y_i - b(\theta_i)}{a(\phi)} + c(y_i, \phi) \right\},$$  \hspace{1cm} (1)

parameterized by $\theta$ (and the dispersion parameter $\phi$) and the known functions $a, b$ and $c$. Direct calculations reveals that the

$$EY_i = b'(\theta_i), \quad \text{Var}(Y_i) = a(\phi)b''(\theta_i).$$  \hspace{1cm} (2)

The mean $EY_i = \mu(\theta_i)$ is related to some covariates, $x_i$, through a link-function (McCullagh and Nelder 1983), $g$,

$$g \{\mu(\theta_i)\} = \beta^T x_i, \hspace{1cm} (3)$$

i.e., $\theta_i = \theta_i(\beta)$. Typically, the canonical link is chosen such that $g \circ \mu = \text{id}$, with the most common regression models being the general linear model, logistic regression and Poisson regression.
Given \( n \) observations \((y_i, x_{1i}, \ldots, x_{pi})_{i=1,\ldots,n}\) the maximum likelihood estimate \( \hat{\beta} \in \mathbb{R}^p \) is obtained by solving the set of score equations:

\[
U(\beta) = \sum_{i=1}^{n} h(\beta^T x_i) x_i \{ y_i - g^{-1}(\beta^T x_i) \},
\]

with \( h = \partial \{ (g \circ \mu)^{-1} \} \). We define the (raw) residuals \( e_i = y_i - g^{-1}(\hat{\beta}^T x_i) \), \( i = 1, \ldots, n \). Our interest is the cumulative sum of the residuals over the \( j \)th covariate [Su and Wei 1991; Lin et al. 2002]:

\[
W_j(x) = n^{-1/2} \sum_{i=1}^{n} 1_{\{x_{ji} \leq x\}} e_i.
\]

In contrast to the distribution of individual residuals, we can determine the distribution (under the null) of this aggregate. For known parameters the asymptotics can be derived as a Brownian bridge [Shorack and Wellner 1986], however, we need to take uncertainty in estimation of \( \hat{\beta} \) into account.

Under certain regularity conditions, a Taylor expansion around the true parameter value, \( \beta_0 \), gives us

\[
W_j(x) = W_j(x \mid \beta) = W_j(x \mid \beta_0) + \frac{\partial}{\partial \beta} W_j(x \mid \beta) \bigg|_{\beta = \beta_0} (\hat{\beta} - \beta_0) + o_p(1).
\]

(6)

Let \( I(\hat{\beta}) = E(-\nabla U(\hat{\beta})) \) denote the Fisher information. Now \( (\hat{\beta} - \beta_0) \) is asymptotically normally distributed and asymptotically equivalent with \( I(\hat{\beta})^{-1} U(\hat{\beta}) \):

\[
(\hat{\beta} - \beta_0) = I(\hat{\beta})^{-1} U(\hat{\beta}) + o_p(1).
\]

(7)

It then follows that the process

\[
\hat{W}_j(x) = n^{-1/2} \sum_{i=1}^{n} \left[ 1_{\{x_{ji} \leq x\}} + \eta_j(x \mid \hat{\beta}) I^{-1}(\hat{\beta}) x_i h(\hat{\beta}^T x_i) \right] e_i G_i
\]

(8)
with i.i.d. $G_1, \ldots, G_n \sim \mathcal{N}(0, 1), i = 1, \ldots, n$, and
\[
\eta_j(x \mid \beta) = -\sum_{i=1}^{n} 1_{\{x_j \leq x\}} \frac{\partial g^{-1}(\beta^T x_i)}{\partial \beta},
\] (9)

(see Table 1) converges weakly to the same limiting distribution as the observed process 5 (Lin et al., 2002).

| $g(x)$       | $g^{-1}(z)$       | $\partial (g^{-1})(z)$ |
|--------------|------------------|------------------------|
| $x$          | $z$              | $1/(1 + \exp(-z))$    |
| logit($x$)   | $\exp(-z)/[1 + \exp(-z)]^2$ | $\exp(z)$ |
| log($x$)     | $1$              | $\exp(z)$             |

Table 1: Some link functions and their inverse.

To test the functional form of the $j$th covariate we look at a Kolmogorov-Smirnov (KS) type supremum statistic:
\[
\mathcal{T}_\infty^{(j)}: W_j \mapsto \sup_x |W_j(x)|.
\] (10)

Alternatively tests can be based on the Cramer-von-Mises (CvM) functional:
\[
\mathcal{T}_2^{(j)}: W_j \mapsto \int |W_j(x)|^2 \, dx.
\] (11)

A large number of realizations of $\hat{W}_j$ is generated. The supremum statistic is calculated for each realization and the p-value is estimated from the empirical distribution of these statistics. The residuals can also be cumulated after the predicted values (Lin et al., 2002)
\[
W_{\hat{y}}(t) = n^{-1/2} \sum_{i=1}^{n} 1_{\{g^{-1}(\beta^T x_i) \leq t\}} e_i,
\] (12)

which leads to a test of misspecified link function.

2.2. Structural equation models

The linear structural equation models covers a broad range of models including the general linear model, path analysis and various latent variable models. Diagnostics based on cumulative residuals was examined in this case by Sánchez et al. (2009) building on the work of Pan and Lin (2005) on
Generalized Linear Mixed Models (GLMM) sharing many of the aspects of structural equation models. The basic idea and proof of weak convergence is very similar to the case of GLM.

A structural equation model is typically divided into two separate parts. For the \( i \)th individual we have a measurement part describing the multivariate outcome \( Y_i \):

\[
Y_i = \nu + \Lambda \eta_i + K X_i + \epsilon_i,
\]

(13)

where \( \eta_i \) are the latent variables and \( X_i \) are covariates, and a structural part describing the latent variables:

\[
\eta_i = \alpha + B \eta_i + \Gamma X_i + \zeta_i
\]

(14)

where \( \nu \in \mathbb{R}^p \), \( \Lambda \in \mathbb{R}^{p \times l} \), \( K \in \mathbb{R}^{p \times q} \), and \( \epsilon_i \sim N_p(0, \Sigma_\epsilon) \). And \( \alpha \in \mathbb{R}^l \), \( B \in \mathbb{R}^{l \times l} \), \( \Gamma \in \mathbb{R}^{l \times q} \), and \( \zeta_i \sim N(0, \Psi) \). Hence, the model is parameterized by some \( \theta \) defining \((\nu, \alpha, \Lambda, K, \Gamma, \Sigma_\epsilon, \Psi) \) with some restrictions to guarantee identification. The conditional moments of \( Y_i \) given \( X_i \), are

\[
\mu_i = \mathbb{E}_\theta(Y_i | X_i) = \nu + \Lambda(1 - B)^{-1}\alpha + \Lambda(1 - B)^{-1}\Gamma + K Y_i,
\]

(15)

\[
\Sigma = \text{Var}_\theta(Y_i | X_i) = \Lambda(1 - B)^{-1}\Psi(1 - B)^{-1T} \Lambda^T,
\]

(16)

and inference on \( \theta \) is usually obtained by MLE [Bollen (1989)].

The residuals can be predicted as the conditional mean given the endogenous variables and covariates. Hence,

\[
\hat{\epsilon}_{ik} = \mathbb{E}(\epsilon_{ik} | Y_i, X_i) = \pi^T_k \Sigma_\epsilon \Sigma^{-1}(Y_i - \mu_i)
\]

(17)

\[
\hat{\zeta}_{ig} = \mathbb{E}(\zeta_{ig} | Y_i, X_i) = \pi^T_g \Psi(1 - B)^{-1T} \Lambda^T \Sigma^{-1}(Y_i - \mu_i)
\]

(18)

where \( \pi^*_s : \mathbb{R}^s \rightarrow \mathbb{R} \) is the projection onto coordinate \( s \). Different local aspects of the structural equation model can now be assessed by examining the cumulative residual processes of either \( \hat{\epsilon}_{ik} \) and \( \hat{\zeta}_{ig} \).

**Misspecified covariate effect on the \( g \)th latent variable** is checked by summing \( \hat{\zeta}_{ig} \) with respect to the \( j \)th covariate, \((X_{ij})\):

\[
W^{(l)}_{X_j}(x) = n^{-1/2} \sum_{i=1}^{n} \mathbb{1}(X_{ij} \leq x) \hat{\zeta}_{ig}
\]

(19)
and as for the GLM we can imitate the behavior of this process under the null of no misspecification by simulation. Misspecified link between gth latent variable and its predictors is checked by summing $\hat{\zeta}_{ig}$ with respect to $E(\eta_{ig} \mid X_i) = \pi_g^T(1 - B)^{-1}(\alpha + \gamma X_i)$. To examine departures from the specified association between an endogenous variable and one of its predictors, we can look at the cumulative process defined by summing $\hat{\epsilon}_{ik}$ with respect to $E(Y_{ik} \mid X_i)$. This can also be used to diagnose for so-called item bias (conditional dependence between a covariate and endogenous variable given latent variables). Finally, misspecified link between an endogenous variable and its linear predictors is checked by summing $\hat{\epsilon}_{ik}$ with respect to $E(Y_{ik} \mid X_i)$.

2.3. Cox’s proportional hazard model

The idea of looking at aggregates of residuals can also be applied as a tool for diagnosing the proportional hazards assumptions used in many survival analyses. We will assume that we have triplet observations $(N_i(t), Y_i(t), X_i(t))$, $i = 1, \ldots, n$ of a counting process, at-risk process and covariate process in the compact time-interval $[0, \tau]$. Using the notation of stochastic integrals we let the Martingale decomposition of the counting process be given by

$$dN_i(t) = \lambda_i(t) \, dt + dM_i(t).$$

Cox’s proportional hazard model assumes intensity takes the form

$$\lambda_i(t) = \lambda_0(t) e^{X_i^T(t) \beta},$$

where $X$ is $p$-dimensional covariates. We denote the cumulative baseline hazard

$$\Lambda_0(t) = \int_0^t \lambda_0(s) \, ds.$$  

As the model contains a non-parametric term, $\lambda_0$, inference will be based on the partial likelihood \cite{Cox1972}

$$L(\beta) = \prod_{i=1}^n \prod_t \left( \frac{\exp(X_i^T(t) \beta)}{S_0(t, \beta)} \right)^{\Delta N_i(t)}$$

where

$$S_0(t, \beta) = \sum_i Y_i(t) \exp(X_i^T(t) \beta).$$
with the first and second partial derivatives

\[ S_1(t, \beta) = \sum_i Y_i(t) \exp(X_i^T(t)\beta)X_i(t), \]
\[ S_2(t, \beta) = \sum_i Y_i(t) \exp(X_i^T(t)\beta)X_i(t)^{\otimes 2}, \]

and let \( E(t, \beta) = S_1/S_0(t, \beta) \). The score equation then becomes

\[ U(\beta) = \sum_{i=1}^n \int_0^\tau [X_i(t) - E(t, \beta)] \, dN_i(t). \]

The Nelson-Aalen estimator of the cumulative intensity is

\[ \hat{\Lambda}_0(t) = \int_0^t \frac{1}{S_0(s, \beta)} \, dN_i(s), \]

where \( N_i = \sum_i N_{ti} \). Define

\[ I(t, \beta) = \sum_{i=1}^n \int_0^t \frac{S_2}{S_0^2}(s, \beta) - E(s, \beta)^{\otimes 2} \, dN_i(s) = \int_0^t V(s, \beta) \, dN_i(s), \]

and hence minus the derivative of the score is \( I(\tau, \beta) \) (i.e., the information). The estimated martingales residual process is given by

\[ \hat{M}_i(t) = N_i(t) - \hat{\Lambda}_i(t) = N_i(t) - \int_0^t Y_i(s) \exp\left\{ X_i^T(s)\hat{\beta} \right\} \, d\hat{\Lambda}_0(s) \]
\[ = N_i(t) - \int_0^t Y_i(s) \exp\left\{ X_i^T(s)\hat{\beta} \right\} \frac{1}{S_0(s, \beta)} \, dN_i(s), \]

(with the martingale residuals defined by evaluation in \( \tau \)), and the estimated score process

\[ U(\hat{\beta}, t) = \sum_{i=1}^n \int_0^t X_i(s) \, d\hat{M}_i(s) \]
\[ = \sum_{i=1}^n \int_0^t \left\{ X_i(s) - E(s, \hat{\beta}) \right\} \, dN_i(s), \]

where \( X_i(s) - E(s, \hat{\beta}) \) are the Schoenfeld residuals.
To assess the proportional hazards assumption we will calculate the Kolmogorov-Smirnov and Cramer-von-Mises test statistics of the different coordinates of the observed score process. As in the previous section we can simulate realizations under the null (proportional hazards). The key result is that 
\[ n^{-1/2} \sum_{i=1}^{\infty} \left\{ M_{1i}(t) - I(t, \hat{\beta})I(\tau, \hat{\beta})^{-1} M_{1i}(\tau) \right\}, \tag{32} \]
with
\[ M_{1i}(t) = \int_0^t \{ X_i(s) - e(s, \beta_0) \} \, dM_i(s), \tag{33} \]
where \( e(t, \beta_0) = \lim_{n \to \infty} E(t, \beta_0) \) (see Martinussen and Scheike 2006, Lin et al. 1993), which follows from a Taylor expansion around the true parameter \( \beta_0 \). With the estimates plugged in we get
\[ \hat{M}_{1i}(t) = \int_0^t (X_i(s) - E(s, \hat{\beta})) \, dN_i(s) \]
\[ - \int_0^t (X_i(s) - E(s, \hat{\beta})) \frac{\exp(X_i^T(s) \beta)}{S_0(s, \beta)} \, dN_i(s). \tag{34} \]
Given observed times \( (T_1, \ldots, T_n) \) and death-indicators \( \Delta_i, (X_i = X_i(T_i)) \) we can implement this by
\[ 1_{(T_i \leq t, \Delta=1)} \left\{ X_i(s) - E(T_i, \hat{\beta}) \right\} - \exp(X_i^T \beta) \hat{\Lambda}_0(t) \]
\[ + \exp(X_i^T \beta) \int_0^t S_1/S_0^2(s, \hat{\beta}) \, dN_i(s). \tag{35} \]

Finally \( n^{-1/2} M_{1i}(t) \) is asymptotically equivalent to \( n^{-1/2} \sum_{i=1}^{n} \hat{M}_{1i}(t)G_i \) where the \( G_i \)'s are i.i.d. \( N(0,1) \).

2.4. Software

The described methods are implemented in the R-package \texttt{gof} available from the Comprehensive R Archive Network (R Core Team 2012).

The package has been designed to work directly on \texttt{lm}, \texttt{glm} and \texttt{coxph} objects (Therneau and original R port by Thomas Lumley 2013). Additionally, various aspects of latent variable models, fitted via the \texttt{lava}-package (Holst and Budtz-Jørgensen 2012), can be diagnosed.
The simulation routine is computational intensive and to obtain better computing efficiency, the resampling routines was written in C++. The implementation uses the Scythe Statistical Library [Pemstein et al., 2011] which among other things offers operator overloaded matrix operations making the (linear) algebraic computations in the program close to self-documenting.

3. Examples

In the following section the {gof} package will be demonstrated in generalized linear models, a structural equation model and a Cox regression model.

3.1. Generalized linear models

First we define a simple function that allows us to simulate data from Binomial and Poisson regression models with link function $g$, and covariates $X, Z \sim N(0, 1)$

$$g(\mathbb{E}[Y \mid X, Z]) = f(X, Z). \quad (36)$$

R> sim1 <- function(n,f=sum,family=binomial("logit")) {
  x <- rnorm(n)
  z <- rnorm(n)
  if (is.character(family)) family <- do.call(family,list())
  eta <- family$linkinv(apply(cbind(x,z),1,f))
  y <- switch(family$family,
               binomial= (eta>runif(n))*1,
               poisson= rpois(n,eta),
               eta)
  return(data.frame(y,x,z))
}

We first simulate binomially distributed observations and use a complementary log-log link:

$$\log (- \log [1 - \mathbb{E}(Y \mid X, Z)]) = X + Z \quad (37)$$

R> d <- sim1(n=1000,family=binomial("cloglog"))
Next we fit both the correct model and the model with canonical link:

```r
R> l1 <- glm(y~x+z,d,family=binomial("cloglog"))
R> l2 <- glm(y~x+z,d,family=binomial("logit"))
```

Using the `cumres` method, we calculate the cumulative residual process ordered by the predicted values and simulate 1,000 processes from the null:

```r
R> library("gof")
R> (g1 <- cumres(l1,R=1000,variable="predicted"))
```

Kolmogorov-Smirnov-test: p-value=0.466  
Cramer von Mises-test: p-value=0.333  
Based on 1000 realizations. Cumulated residuals ordered by predicted-variable.  
---

Kolmogorov-Smirnov-test: p-value=0.466  
Cramer von Mises-test: p-value=0.333  
Based on 1000 realizations. Cumulated residuals ordered by predicted-variable.  
---

```r
R> (g2 <- cumres(l2,R=1000,variable="predicted"))
```

Kolmogorov-Smirnov-test: p-value=0.014  
Cramer von Mises-test: p-value=0  
Based on 1000 realizations. Cumulated residuals ordered by predicted-variable.  
---

Kolmogorov-Smirnov-test: p-value=0.014  
Cramer von Mises-test: p-value=0  
Based on 1000 realizations. Cumulated residuals ordered by predicted-variable.  
---

There are clear indications, by both the supremum and CvM test, of mis-specification of the link function in model 12. To plot the observed process and realizations from under the null (the number of realization can be changed in the `cumres` call with the argument `plots`), we can use the `plot` method.
R> par(mfrow=c(1,2))
R> plot(g1,title="Model 'l1'"); plot(g2,title="Model 'l2'"))

Figure 1: Cumulative residual processes of model 11 and 12 with residuals ordered by the predicted response. The gray curves are 50 realizations from the null model. The transparent blue area defines a 95% prediction band for all the simulated processes.

It is evident from the plot (Figure ??), that the observed process of model 2 is extreme.

Next we simulate data from a Poisson regression model
\[
\log(\mathbb{E}(Y \mid X, Z)) = 0.5 \cdot X^2 + Z
\]  \hspace{1cm} (38)

R> d2 <- sim1(200,f=function(x) 0.5*x[1]^2+x[2],family=poisson())
and we fit a Poisson regression model but with misspecified functional form of the covariate \( X \)

R> l <- glm(y~x+z,family=poisson(),data=d2)

Next we check the link function and functional form of both covariates

R> (g <- cumres(l,R=2000))

Kolmogorov-Smirnov-test: p-value=0.453
Cramer von Mises-test: p-value=0.547

Based on 2000 realizations. Cumulated residuals ordered by predicted-variable.

---

Kolmogorov-Smirnov-test: p-value=0.001
Cramer von Mises-test: p-value=0
Based on 2000 realizations. Cumulated residuals ordered by x-variable.
---
Kolmogorov-Smirnov-test: p-value=0.8185
Cramer von Mises-test: p-value=0.858
---

Based on 2000 realizations. Cumulated residuals ordered by z-variable.
---

Kolmogorov-Smirnov-test: p-value=0.453
Cramer von Mises-test: p-value=0.547
---

Based on 2000 realizations. Cumulated residuals ordered by predicted-variable.
---

Kolmogorov-Smirnov-test: p-value=0.001
Cramer von Mises-test: p-value=0
---

Based on 2000 realizations. Cumulated residuals ordered by x-variable.
---

Kolmogorov-Smirnov-test: p-value=0.8185
Cramer von Mises-test: p-value=0.858
---

and we plot all processes (Figure 2) while changing the color (and alpha blending) of the realizations and prediction-band (setting col or col.ci to NULL will disable either the realizations or the prediction-band)
3.2. Structural equation models

The \texttt{cumres} method is also available for structural equation models fitted via the \texttt{lava} package (Holst and Budtz-Jørgensen 2012). As an example we will examine a simple model, with three outcomes described by the equation

\[ Y_{ij} = \mu_j + \lambda_j \eta_i + \epsilon_{ij}, j = 1, \ldots, 3, \]  

(39)
with \( i = 1, \ldots, n \) individuals and latent variable \( \eta_i \). We also add a structural equation describing the latent variable

\[
\eta_i = \beta_1 \cdot X + \beta_2 Z + \zeta, \tag{40}
\]

with covariates \( X \) and \( Z \). The residual terms \( \epsilon_{i1}, \ldots, \epsilon_{i3}, \zeta \) are normally distributed and independent. In \texttt{lava} we can specify the model as

\begin{verbatim}
R> library(lava)
R> m <- lvm(list(c(y1,y2,y3)~eta,eta~x+z))
R> latent(m) <- ~eta
\end{verbatim}

We simulate 200 observations from a structural equation model like the one defined above, with intercepts set to zero and all other parameters equal to one, but with

\[
Y_{i2} = \eta_i^2 + \epsilon_{i2} \quad \text{and} \quad \eta_i = X + 0.5 \cdot X^2 + Z + \zeta. \tag{41}
\]

\begin{verbatim}
R> m0 <- m
R> functional(m0,y2~eta) <- function(x) x^2
R> functional(m0,eta~z) <- function(x) x+0.5*x^2
R> d <- sim(m0,200)
\end{verbatim}

Next we find the MLE of the first model

\begin{verbatim}
R> (e <- estimate(m,d))
\end{verbatim}

| Measurement       | Estimate | Std. Error | Z-value | P-value |
|-------------------|----------|------------|---------|---------|
| \( y_{2} \leftarrow \eta \) | 2.21830  | 0.24535    | 9.04147 | <1e-12  |
| \( y_{3} \leftarrow \eta \) | 0.99314  | 0.05380    | 18.45847 | <1e-12 |
| \( \eta \leftarrow X \)      | 1.00280  | 0.09820    | 10.21176 | <1e-12 |
| \( \eta \leftarrow Z \)      | 1.08055  | 0.09790    | 11.03729 | <1e-12 |
| \( y_{2} \)                | 2.84517  | 0.48757    | 5.83545 | 5.365e-09 |
| \( y_{3} \)                | -0.05982 | 0.09412    | -0.63558 | 0.5251 |
| \( \eta \)               | 0.50307  | 0.10623    | 4.73553 | 2.185e-06 |
| Residual Variances:       |          |            |         |         |
| \( y_{1} \)              | 0.67836  | 0.15287    | 4.43739 |
| \( y_{2} \)              | 40.57804 | 4.22914    | 9.59488 |
| \( y_{3} \)              | 0.92828  | 0.16468    | 5.63682 |
| \( \eta \)         | 1.57358  | 0.21605    | 7.28351 | 14 |

and as an example we cumulate the predicted residual terms of $Y_3$ and $Y_2$
against $E(\eta_i \mid X_i)$, and the residual term of $\eta_i$ against the two covariates.

\[
R> e.gof <- \text{cumres}(e, \text{list}(y3^\eta, y2^\eta, \eta^x, \eta^z), R=1000)
\]

From the cumulative residual plots (see Figure 3) we clearly see the mis-
specification in the measurement model of the second outcome (with the
observed process also indicating a quadratic form), and also the wrongly
specified functional form of $X$.

For complete flexibility the \texttt{cumres} method can be used with the syntax

\[
\text{cumres(model,y,x,...)}, \text{where y is a function of the model parameters}
\]
returning the residuals of interest, and \texttt{x} can be any vector to order the
residuals by. Typically \texttt{y} will be defined via the \texttt{predict} method of a
\texttt{lvmfit} object (a \texttt{lava} model object).
Selected cumulative residual processes for the structural equation model fit \( \epsilon \). The top row shows the cumulative residuals of \( \epsilon_{3i} \) and \( \epsilon_{2i} \) (see (39)) ordered by \( \mathbb{E}(\eta_i \mid X_i, Z_i) \). The bottom row shows the cumulative processes of the predicted residual term, \( \hat{\zeta}_i \), of the latent variable ordered by each of the two covariates.

3.3. Cox regression - Mayo clinic PBC data

As an example of checking the proportional hazards assumption in a Cox model, we will analyze the Mayo Clinic PBC data. Dickson et al. (1989) suggested a Cox model for analyzing the survival of the liver disease patient with 5 covariates: age, edema status, logarithmic serum bilirubin, logarithmic standardized blood clotting time, and logarithmic serum albumin:

\[
R> \text{library("survival")}
R> \text{data("pbc")}
R> \text{pcbc.cox <- coxph(Surv(time, status==2)~age+edema+log(bili)+log(protime)+log(albumin), data=pcbc)}
\]
To check the proportional hazards assumption, we examine the score process vs. follow-up time:

```R
R> pbc.gof <- cumres(pbc.cox, R=2000)
```
and plot the observed process with realizations from the null

```R
R> par(mfrow=c(2,3))
R> plot(pbc.gof, legend=FALSE)
```

Figure 4: Cumulative score processes for the Cox regression analysis of the PBC data, `pbc.cox`.

There are clear indication of violation of the proportional hazards assumption for blood clotting time (protime), and indication of problems with the edema variable. To remedy the non-proportionality, time-varying covariate effects could be introduced to the model, e.g.,

```R
R> library("timereg")
R> pbc.caalen <- cox.aalen(Surv(time, status==2) ~ prop(age) + prop(edema) +
```
prop(bili) + protime, data=pbc, n.sim=500)

4. Conclusion

The package gof adds a valuable tool to the model diagnostics toolbox and gives an objective method for evaluating the linearity assumptions in the generalized linear model and linear structural equation models. Extensions to other models such as the linear mixed model can be implemented using the C++ interface as used by the cumres method for glm and lvm objects.

5. Acknowledgments

This work was supported by The Danish Agency for Science, Technology and Innovation.

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