Application of Activity-on-arrow Network in Product Development Management

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Abstract: Product development management is the core work of product preparation, which is restricted by many factors and requires regular adjustment of plans and real-time dynamic management. Taking the general process of product development management as an example, this paper adopts activity-on-arrow network in operational research to develop product management, and then draws and calculates the composition, key circuit and time parameters. Afterwards, the construction period is shorten and the consumption of cost is reduced through adopting the activity-on-arrow network in developing product management to achieve the design optimization and calculation of product development management, so as to save cost and improve economic efficiency of enterprises.

1. Introduction
Activity-on-arrow is also known as “arrow diagram method”, which can make the time limit of each work well arranged, and make the beginning and ending time connect with each other in product development management, so that the product development organization and work preparation are more precise and accurate, and the execution time is more strictly planned. The most important goal of the diagram is to save labor, reduce product development cost, and shorten the process time, thereby improving work efficiency. The use of activity-on-arrow is conducive to ensuring that enterprise can profit and compensate the enterprise expense on time. It is beneficial to effectively coordinate enterprise resources, and enable the resources to be used at any time. Therefore, it helps to predict the level of funds and resources in different times, and is beneficial to meet strict deadlines. Therefore, activity-on-arrow plays an important role in product development management.

2. The composition of activity-on-arrow network
Activity-on-arrow network is mainly composed of arrows and nodes[1]. The nodes can be numbered by adding Arabic numerals in circles. One work should have only one arrow line and one pair of nodes. The number of arrow tail nodes is smaller than the number of the arrowhead nodes. The node numbers are arranged in order from small to large, which cannot be continuous, and also the same node is strictly prohibited to repeat. The arrow line indicates the event, which is the sign of the beginning or ending point of the job. The event does not consume resources, represented by one Arabic numeral in a circle. The node indicates the connection point between the production times. The left and right ends of the arrow line are numbered (i, j), in which, i indicates the beginning time, and j indicates the ending time. The upper part of the arrow shows work name, and the lower part indicates work duration. Addition, the arrow node refers to the end time j of the work, and the arrow tail node indicates beginning time i. there is only one start node or end node in activity-on-arrow network. It is strictly
forbidden to have line loop, arrows without arrow and line, two-way arrows, arrow lines without beginning or ending node. (Shown in Figure 1)

\[ \text{Figure 1. Nodes and work of activity-on-arrow network} \]

3. Time code calculation of activity-on-arrow diagram

The activity-on-arrow network indicates the arrangement of scheduled projects and expenses and the relationship between each management links. Based on this, the system analyzes the time parameters required for each process, and infers the key lines according to the longest time required by the line, then finally uses the network map to optimize the process duration and cost. There are many planned tasks that are closely linked in one process duration, if any of the work processes delays upon the expected completion time, it eventually result in cost loss and reduction. How to save time and expense based on above phenomena is an issue to be studied in product development management.

The following is an in-depth study of how to use activity-on-arrow network to rationally arrange the logical relationship between work duration and cost in product development management. According to Table 1, the logic relationship of activity processes can be grasped at a glance from the logic between each process and required time of product project. Therefore, the initial activity-on-arrow network 2 of the activity can be conveniently and quickly drawn based on the theory.

| Activity | Activity Content    | Close Front Time | Required Time (day) |
|----------|---------------------|------------------|---------------------|
| A        | Market research     | /                | 4                   |
| B        | Design concept      | /                | 8                   |
| C        | Product approval    | A                | 2                   |
| D        | Production preparation | A            | 2                   |
| E        | Sample trial        | B, C             | 2                   |
| G        | Production and processing | B, C        | 2                   |
| H        | Product evaluation  | D, E             | 6                   |
| I        | Customer trial      | D, E             | 4                   |
| J        | Mass production     | H, G             | 6                   |

\[ \text{Table 1. Logical relationship of activities} \]

\[ \text{Figure 2. Initial activity-on-arrow network} \]

3.1. Critical lines

The lines on the activity-on-arrow network refer to the lines from the starting node of the arrow in direction from left to right. In this process, the ending node passes through many intermediate nodes; the route trace is called the line. The sum of all work durations is the length of the line. Critical lines consist of critical works from beginning to the end, in another word, the largest sum of time used on all lines and with the longest total work duration are called critical lines. An activity may have more than one critical line, and there may be many lines. The critical lines are shown as double arrow lines.
and thick solid lines in activity-on-arrow network. The works on critical lines are called critical works [1]. The critical line is based on time or process parameters, calculated by various time parameters in computer network, and continuously optimize to obtain the shortest duration and minimum cost by time differences. According to Figure 2, logical process relationship concludes that there are two key lines with the longest total work durations: B-E-H-J and B-E-I. According to the estimated time, the two critical lines indicate that the maximum time required to complete the product development is 19 days.

3.2. Calculation of node time parameters
A node is one circle within a number with arrow head and tail of each arrow line. The circle is called node. The number inside is the node number. One node indicates the end of previous process and also the beginning to the next.

3.2.1. The earliest starting time of the node
The earliest time of the node is calculated by possible earliest time of the process, as at before time there might lack of start conditions, in another word, the earliest time of start node is 0, which shall be calculated from the beginning node of activity-on-arrow network, and calculated each node from left to right along the direction of the arrow line of the activity-on-arrow network. Its formula: ESj = max{ESi+tij}, ESj is the arrow node, j is the earliest end time; ESi is the arrow tail node, i is the earliest start time; tij is work time of the process i-j. The total duration is the earliest start time of the end node of activity-on-arrow network, which is the maximum sum of all the value of process durations from start to end node in all the lines of network diagram.

3.2.2. The latest ending time of the node
The latest ending time of the node refers to the time that the process of the node must be completed. If not, the next work will not start on time. The latest ending time of the node should be gradually calculated from the ending point against arrow line toward the start node. The end time of the end node is normally its earliest start time, which is the total duration. Its formula: LFj = max{LFj+tij}, where LFj is the latest start time of the tail node i; LFj is the latest end time of the arrow node j; tij is the work time of the process i-j.

3.2.3. Calculation of process time parameters
(1) The earliest start time of the process = the earliest start time of the previous node. Formula: ESij = ESi
(2) The earliest end time of the process = the earliest start time + the work time of the process. Formula: EFij = ESij+tij
(3) The latest end time of the process = the latest end time of the node afterwards. Formula: LFij = LFj
(4) The latest start time of the process = the latest end time of the process - the work time of the process. Formula: LSij = LFij-tij
(5) Time difference refers to the maneuver time available for an activity without affecting the completion time of the entire task.
(6) Node time difference = the latest end time of the node - the earliest start time, in which the node time difference=0 is the key node.
(7) The total time difference means free and maneuverable time between close front and after activity without affecting the total work duration. Formula: TFij = LSij-ESij = LFij-EFij
(8) The free time difference is free and maneuverable time when the work is started at the earliest time without affecting the work. Formula: EFij = ESj-ESi-tij
Table 2. Calculation of activity time parameter

| Activity code | Earliest start time ESi | Earliest end time EFj | Latest start time LSi | Latest end time LFj | Total time difference TFij | Free time difference FFij |
|---------------|-------------------------|-----------------------|-----------------------|---------------------|---------------------------|--------------------------|
| A             | 0                       | 0+4=4                 | 6-4=2                 | 8-2=6               | 2-0=2                     | 6-4=2                    |
| B             | 0                       | 0+8=8                 | 8-8=0                 | 13-5=8              | 0-0=0                     | 8-8=0                    |
| C             | 0+4=4                   | 4+2=6                 | 8-2=6                 | 13-5=8              | 6-4=2                     | 8-6=2                    |
| D             | 0+4=4                   | 4+2=6                 | 13-2=11               | 19-6=13             | 11-4=7                    | 13-6=7                   |
| E             | 0+8=8                   | 8+5=13                | 13-5=8                | 19-6=13             | 8-8=0                     | 13-13=0                  |
| G             | 0+8=8                   | 8+6=14                | 15-6=9                | 19-4=15             | 9-8=1                     | 15-14=1                  |
| I             | 8+5=13                  | 13+6=19               | 19-6=13               | 15+4=19             | 13-13=0                   | 19-19=0                  |
| J             | 13+2=15                 | 15+4=19               | 19-4=15               | 15+4=19             | 15-15=0                   | 19-19=0                  |

4. Production time of product Development—cost optimization

The so-called optimization is the process of seeking an optimal network plan by continuously adjusting according to a certain target under the established constraints[1]. The ideal measure includes various factors such as expense, production time and resources. How to revile the three optimization of expense, production time and resources, which means the optimization of balancing the expense with limited production time; the optimization of shortest production time with limited expense; the optimization of the best production time and lowest cost, and optimization of the production time and cost discussed in this paper.

4.1. Production time-cost optimization plan

The production time-cost optimization plan mainly study the shortest production time, the low cost, and settle the problem of the shortest production time and lowest cost under the condition of guaranteed established completion period or minimum cost and under limited fees. Total cost = direct cost + indirect cost. Direct costs include direct surcharges, tools and equipment, worker wages, etc. Indirect costs include the salaries and administration expenses. As is shown in Figure 3, direct costs increase as the production time decreases, and indirect costs decrease as the total production time decreases.

4.2. Production time—cost optimization process

According to the initial activity-on-arrow network of Figure 2 drawn by the product development management process, the normal time in Figure 4 refers to the numbers outside the parentheses below the arrow line; the numbers in parentheses are the minimum duration of the work; and the numbers in
parentheses are the direct fee required for the work to be completed in the shortest duration. The known indirect cost rate = 0.8 million Yuan / day.

\[
\begin{align*}
\text{Figure 4. The initial plan of activity-on-arrow network} \\
(1) \text{As is shown in Figure 5, the production time calculated = 19 days, there are two key lines: } 1\rightarrow3\rightarrow4\rightarrow6 \text{ and } 1\rightarrow3\rightarrow4\rightarrow5\rightarrow6.
\end{align*}
\]

\[
\begin{align*}
\text{Figure 5. Key lines in the initial activity-on-arrow network} \\
(2) \text{The direct cost rate for each job calculation} \\
\Delta C1-2 = (7.4 - 7.0) / (4 - 2) = 0.2 \text{ million Yuan / day} \\
\Delta C1-3 = (11.0 - 9.0) / (8 - 6) = 1.0 \text{ million Yuan / day} \\
\Delta C1-4 = (7.4 - 7.0) / (4 - 2) = 0.2 \text{ million Yuan / day} \\
\Delta C2-3 = 0.3 \text{ million Yuan / day} \\
\Delta C2-4 = 0.5 \text{ million Yuan / day} \\
\Delta C3-4 = 0.2 \text{ million Yuan / day} \\
\Delta C3-5 = 0.8 \text{ million Yuan / day} \\
\Delta C4-5 = 0.7 \text{ million Yuan / day} \\
\Delta C4-6 = 0.5 \text{ million Yuan / day} \\
\Delta C5-6 = 0.2 \text{ million Yuan / day}.
\end{align*}
\]

\[
\begin{align*}
(3) \text{The total activity cost Calculation} \\
\text{1direct fee sum = } Cd = 7.0 + 9.0 + 5.7 + 5.5 + 8.0 + 8.0 + 5.0 + 7.5 + 6.5 = 62.2 \text{ million Yuan;} \\
\text{2The sum of indirect costs = } Ci = 0.8 \times 19 = 15.2 \text{ million Yuan;} \\
\text{3total activity cost = } Ct = Cd + Ci = 62.2 + 15.2 = 77.4 \text{ million Yuan.} \\
(4) \text{Production time-cost optimization requires constant compression of critical work} \\
\text{1) First compression} \\
\text{As is seen in Figure 5, there are two key lines in the activity-on-arrow network. To shorten the total duration of the two key lines at the same time, there are four compression schemes:} \\
\text{Compress B, the direct cost rate is } B = 1.0 \text{ million Yuan/ day;} \\
\text{Compress E, direct expense rate = 0.2 million Yuan / day;} \\
\text{Compress H and I in one time, combined direct cost rate = 0.7 + 0.5 = 1.2 \text{ million Yuan / day;} } \\
\text{Compress I and J in one time, combined direct cost rate = 0.5 + 0.2 = 0.7 \text{ million Yuan / day;} } \\
\text{According to above compression scheme, the direct cost rate of E is the smallest, so E is selected as the compression object. The direct cost rate E = 0.2 million Yuan / day, less than the indirect expense}
\end{align*}
\]
rate $E = 0.8$ million Yuan / day, compression $E$ can reduce the total cost. Compress $E$’s duration time to minimum 3 days, recalculate the production time and critical lines shown in Figure 6. At this point, the key work E is compressed into non-critical work; therefore, 4-day time becomes key work. The first compression is shown in Figure 7. The number in parentheses above the arrow line in the figure is the direct expense rate.

![Figure 6. Work E’s compressed shortest time of critical line](image)

2) Second compression

As is seen from Figure 7, there are three key lines in the dual-code network scheme: $①-③-④-⑥, ①-③-④-⑤-⑥$ and $①-③-⑤-⑥$. To simultaneously reduce the total duration of three critical lines, there are five compression schemes:

① Compress B, the direct cost rate is 1 million Yuan / day;
② Compress E and G in one time, the combined direct cost rate is $0.2 + 0.8 = 1.0$ million Yuan/day;
③ Compress E and J in one time, combined direct cost rate $= 0.2 + 0.2 = 0.4$ million Yuan/day;
④ Compress G, H and J in one time, combined direct cost rate $= 0.8 + 0.7 + 0.5 = 2$ million Yuan/day;
⑤ Compress I and J in one time, combined direct expense rate $= 0.5 + 0.2 = 0.7$ million Yuan/day.

According to the above compression scheme, since the combined direct cost rate of work E and work J is the smallest, E and J should be selected as compression targets. The combined direct cost rate of work E and work J is 0.4 million Yuan/day, which is less than the indirect cost rate of 0.8 million Yuan/day, indicating that compress work E and work J in one time can reduce the total project expense. Since the compression time of work E last only one day, the duration of work J can also be compressed for one day. After the duration of work E and J is compressed simultaneously for one day, the work duration and critical line are re-determined. Meanwhile, the key lines are changed from three before compression to two: $①-③-④-⑥$ and $①-③-⑤-⑥$. The original key work H was passively turned into non-critical works without compression. The network plan after the second compression is shown in Figure 8. At this point, the critical work E has the shortest duration and can no longer be compressed, so its direct cost rate becomes infinity.

![Figure 7. Network plan after first compression](image)
3) Third compression

As is shown in Figure 8, work E cannot be compressed. In order to shorten the total duration of the two key lines ①-③-④-⑥ and ①-③-⑤-⑥, there are only three compression schemes:

Compress work B, the direct expense rate is 1 million Yuan / day;
Compress G and I in one time, combined direct cost rate is 0.8 + 0.5 = 1.3 million Yuan / day;
Compress I and J in one time, combined direct cost rate: 0.5 + 0.2 = 0.7 million Yuan/ day.

In the scheme above, since the combined direct cost rate of I and J is the smallest, I and J are selected as the compression objects. The combined direct expense rate of work I and J is 0.7 million Yuan, which is less than the indirect expense rate of 0.8 million Yuan, indicating that the total cost of work I and J is reduced when compressed in one time. Work J can only be compressed for one day, and I can only compress it for one day afterward. When I and J are compressed one day in one day, the duration and critical lines are recalculated. At this time, the key lines are still two: ①-③-④-⑥ and ①-③-⑤-⑥. The network diagram after the third compression is shown in Figure 9. At this time, the critical work has the shortest duration and cannot be compressed any more, so the direct cost rate becomes infinity.

4) Fourth compression

As seen from the figure, E can no longer be compressed, meanwhile, shorten the total duration of the critical lines ①-③-④-⑥ and ①-③-⑤-⑥, there are two options:

① compression B, direct cost rate = 1.0 million Yuan / day;
② compression work G and I in one time, combined direct cost rate = 0.8 + 0.5 = 1.3 million Yuan.

In the scheme above, the direct cost rate of B is the smallest, and B is selected as compression object. B’s direct cost rate = 1.0 million Yuan / day, greater than the indirect rate 8 million Yuan / day, B will increase the total cost. Therefore, there is no need to compress B, and it is known from the optimization scheme that the optimized network plan is as shown in Figure 10. The number in parentheses above the arrow line is the direct costs of the work.
(5) Calculation of the total optimized activity costs

① The sum of direct cost = Cd0 = 7.0 + 9.0 + 5.7 + 5.5 + 8.4 + 8.0 + 5.0 + 8.0 + 6.9 = 63.5 million Yuan

② The sum of indirect cost = Ci0 = 0.8 × 16 = 12.8 million Yuan

③ The total activity cost = Ct0 = Cd0 + Ci0 = 63.5 + 12.8 = 76.3 million Yuan

Table 3. production time-cost optimization plan statistics

| Compression time | Compressed Work code | Compressed work name | Direct cost rate (Million Yuan/day) | Rate difference (Million Yuan/day) | Shorten time (day) | Cost added value (Million Yuan) | Total production time (day) | Total cost (Million Yuan) |
|-----------------|----------------------|----------------------|-------------------------------------|-----------------------------------|--------------------|--------------------------------|----------------------------|--------------------------|
| 0               | —                    | —                    | —                                   | —                                 | —                  | —                              | 19                         | 77.4                     |
| 1               | 3-4                  | E                    | 0.2                                 | -0.6                              | 1                  | -0.6                           | 18                         | 76.8                     |
| 2               | 3-4                  | E, J                 | 0.4                                 | -0.4                              | 1                  | -0.4                           | 17                         | 76.4                     |
| 3               | 4-6                  | I, J                 | 0.7                                 | -0.1                              | 1                  | -0.1                           | 16                         | 76.3                     |
| 4               | 1-3                  | B                    | 1.0                                 | +0.2                              | —                  | —                              | —                          | —                        |

According to the table analysis above, after several optimization schemes are designed and calculated, the total duration of the activity is reduced by 3 days, and the total cost is saved by 1.1 million Yuan, thereby reducing the cost of product development management and further improving the economic benefits of the enterprise.

5. Conclusions

Through mapping out the logical relationship of activity-on-arrow network and the calculation of time parameters, the key lines are obtained, and the best solution to increase economic benefits for the enterprise can be achieved through continuously network plan to optimize the production time and cost of product development. Taking full use of activity-on-arrow network in product development management can effectively save the production time and cost, and it has high application value in product management, in the meantime, to achieve good results in time and schedule planning. Therefore, the developed new product needs to seize the market opportunity to quickly win the purchasing power of consumers. At this moment, it is significantly necessary to shorten the production time and accelerate product development.

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