Fermi-type Particle Acceleration from Magnetic Reconnection at the Termination Shock of a Relativistic Striped Wind

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Abstract

An oblique-rotating pulsar generates a relativistic striped wind in a pulsar wind nebula (PWN). The termination shock of the PWN compresses the Poynting-flux-dominated flow and drives magnetic reconnection. By carrying out particle-in-cell simulations of the termination shock of the PWN, we study the shock structure as well as the energy conversion processes and particle acceleration mechanisms. With the recent advances in the numerical methods, we extend the simulations to the ultrarelativistic regime with a bulk Lorentz factor of up to $\gamma_0 = 10^6$. Magnetic reconnection at the termination shock is highly efficient at converting magnetic energy to particle kinetic energy and accelerating particles to high energies. Similar to earlier studies, we find that the resulting energy spectra crucially depend on $\lambda/d_e$ ($\lambda$ is the wavelength of the striped wind and $d_e$ is the relativistic plasma skin depth). When $\lambda/d_e$ is large ($\lambda \gtrsim 40d_e$), the downstream particle spectra form a power-law distribution in the magnetically dominated relativistic wind regime. By analyzing particle trajectories and statistical quantities relevant to particle energization, we find that Fermi-type mechanism dominates the particle acceleration and power-law formation. We find that the results for particle acceleration are scalable as $\gamma_0$ and $\sigma_0$ increase to large values. The maximum energy for electrons and positrons can reach hundreds of TeV if the wind has a bulk Lorentz factor of $\gamma_0 \approx 10^6$ and a magnetization parameter of $\sigma_0 = 10$, which can explain the recent observations of high-energy gamma rays from PWNe.

Unified Astronomy Thesaurus concepts: High energy astrophysics (739); Plasma astrophysics (1261)

1. Introduction

A rotating pulsar creates the surrounding pulsar wind nebula (PWN) by steadily releasing an energetic wind into the interior of the expanding shockwave of a core-collapse supernova explosion (Gaensler & Slane 2006). The wind is composed of magnetized plasma of relativistic electrons and positrons. The wind propagates radially and abruptly transits at the termination shock where the ram pressure balances that of the surrounding medium (supernova remnant or interstellar medium depending on the age and motion of the PWN; see Gaensler & Slane 2006; Torres 2017). The wind can transit from being Poynting-dominated to being particle-dominated at the termination shock. Particles in the pulsar wind will be accelerated at the termination shock, producing a broadband spectrum that can be observed from the radio to X-ray bands. The spectral breaks between the radio and the X-ray band have been found in the synchrotron spectra of PWNe. Some of the spectral breaks suggest emission processes fed by nonthermal particle distributions at the termination shock (Rees & Gunn 1974; Kennel & Coroniti 1984a, 1984b). How the electromagnetic energy is converted into particle energy and how nonthermal particles are efficiently accelerated at the termination shock is not fully understood.

Recently, high-energy gamma-ray emissions have been detected in both young PWNe and mid-aged PWNe. The Crab Nebula (about $10^5$ yr old, and the Crab pulsar has spin-down luminosity $\dot{E} \sim 5 \times 10^{32}$ erg s$^{-1}$) is a prototype of young and energetic PWNe. The origin of the observed photons of energy $E_{ph} > 100$ TeV from the Crab is likely due to the acceleration of leptons in the vicinity of PeV in the Crab Nebula (Abeysekara et al. 2019; Amenomori et al. 2019). The ultra-high-energy electrons and positrons can be produced by particle acceleration in the nebula (Abeysekara et al. 2017). Mid-age pulsars such as Geminga (more than $10^5$ yr old, $\dot{E} \sim 3 \times 10^{34}$ erg s$^{-1}$) are beyond the synchrotron cooling time, but still accelerate electrons to very high energies in the nebulae (Yuksel et al. 2009). In a recent survey (Abeysekara et al. 2020), nine Galactic sources are found to emit above 56 TeV with data from the High Altitude Water Cherenkov (HAWC) Observatory, eight of which are within a degree of the Galactic plane (the ninth source is the Crab Nebula). Those eight inner Galactic plane sources are associated with high spin-down pulsars ($\dot{E} \gtrsim 10^{36}$ erg s$^{-1}$) and remain extended in apparent size above 56 TeV even though the gamma-ray radiating electrons cool quickly. How pulsar winds efficiently accelerate electrons and positrons to high energies is a major puzzle and holds the key of understanding the near-Earth positron anomaly (Yuksel et al. 2009; Accardo et al. 2014; Hooper et al. 2017) as well as gamma rays from the Galactic center (Abdo et al. 2007; Yuksel et al. 2009; Linden & Buckman 2018; Abeysekara et al. 2020).

In the case of an oblique-rotating pulsar, a radially propagating relativistic flow is continuously launched. Near the equatorial plane, toroidal magnetic fields of alternating polarity, separated by current sheets, are embedded in the flow. Such a flow has been modeled as a steady state striped wind, containing a series of drifting Harries current sheets (Coroniti 1990; Kirk & Skjæraasen 2003). Numerical simulations including magnetohydrodynamics (MHD; Kennel & Coroniti 1984b; Porth et al. 2014; Olmi et al. 2015; Porth et al. 2016), particle-in-cell (PIC; Sironi & Spitkovsky 2011), and test-particle simulations (Giacinti & Kirk 2018) have been used to model the termination shock of PWNe. However, how magnetic energy is converted, and the role of the termination shock (Sironi & Spitkovsky 2009; Summerlin & Baring 2011;
Sironi et al. 2013) and relativistic magnetic reconnection (Guo et al. 2014, 2015, 2016, 2019; Sironi & Spitkovsky 2014) in accelerating particles is still unclear (Torres 2017). Magnetic reconnection driven by the termination shock may dissipate the magnetic energy and accelerate particles (Pétrí & Lyubarsky 2007; Sironi & Spitkovsky 2011).

Particle acceleration in relativistic magnetic reconnection has been a recent topic of strong interest (see Guo et al. 2020 for a review). In the case of a spontaneous reconnection, controversy on the role of direct acceleration and Fermi acceleration in producing the power-law particle energy distribution has been extensively addressed (Guo et al. 2014, 2015; Sironi & Spitkovsky 2014). Sironi & Spitkovsky (2014) have suggested that the power law forms as the particles interact with the X-points (diffusion regions with weak magnetic field $|E| > |B|$) through direct acceleration. In contrast, analyses by Guo et al. (2014, 2015, 2019) show that the power-law distributions are produced by Fermi-like processes and continuous injection from the reconnection inflow. In the case of the shock-driven reconnection at the termination shocks of highly relativistic striped pulsar winds, Sironi & Spitkovsky (2011) have proposed that high-energy particles are mainly accelerated at the electric fields at the X-points. However, the role of Fermi-like processes have not been studied in the shock-driven reconnection systems, which is a main focus of this paper.

In this paper, we employ 2D PIC simulations to model the relativistic striped wind interacting with the termination shock near the equatorial plane of obliquely rotating pulsars. We focus on studying the dynamics in a local box near the termination shock of the wind. While it is extremely difficult to model the macroscopic system due to the enormous scale separation between the system size and the skin depth, PIC models provide a reliable and self-consistent description of the shock structure, magnetic reconnection, and particle acceleration. We find that the magnetic reconnection driven by the precursor perturbation from the shock converts the magnetic energy into particle energy and accelerates particles forming a power-law energy spectrum. We examine a wide range of bulk Lorentz factor $10^2 \leq \gamma_0 \leq 10^6$ and magnetization parameter $10 < \sigma_0 \equiv B_0^2/(4\pi \rho_0 n_0 c^2) < 300$ (assuming uniform magnetic field strength $B_0$ and uniform electron+positron density $n_0$ in the upstream) and show the scaling of the particle spectrum. Most of our simulations have large $\gamma_0$ well above the range $3 < \gamma_0 < 375$ used in previous simulations (Sironi & Spitkovsky 2011). This is made possible using the recent improvement (Lu et al. 2020) of the PIC method to overcome the numerical problems. The wide range of $\gamma_0$ and $\sigma_0$ is expected for PWNe with various pulsar spin-down luminosity and age. Our analysis of the particle trajectories and particle energization terms shows that Fermi-type mechanisms by magnetic reconnection (Guo et al. 2019; Lemoine 2019) dominate the particle acceleration and power-law formation.

The rest of this paper is organized as follows. In Section 2, we discuss the numerical methods and the setup of our simulations. In Section 3, we discuss the evolution and structure of the shock-reconnection system for the standard run. In Section 4, we study the particle spectrum and its dependency on parameters $\gamma_0$, $\sigma_0$, and $\lambda$. In Section 5, we give some detailed analyses of the particle acceleration mechanism. We discuss and conclude the paper in Section 6.

![Figure 1](image_url)

**Figure 1.** 2D PIC simulation setup for the termination shock of relativistic striped wind. The upstream incoming flow drifts with bulk Lorentz factor $\gamma_0$ and is composed of in-plane magnetic field $B_0$ in the $+y$ or $-y$ direction, and particles (electrons and positrons). In each current sheet, there is a hot dense component of particles as shown in the density profile. The hot component balances the magnetic pressure and ensures the steady electromagnetic profile.

### 2. Numerical Simulations

We use the 2D version of PIC code EPOCH (Arber et al. 2015) to study the structure and physical processes in the termination shock of a relativistic striped wind. Overcoming numerical problems, especially the numerical Cherenkov instability (NCI; Godfrey 1974), is critical for correctly modeling highly relativistic plasma flows. To improve the numerical stability, we have heavily modified the code to implement the WT (standing for weighting with time-step dependency) interpolation scheme (Lu et al. 2020), a piecewise polynomial force interpolation scheme with time-step dependency. This scheme eliminates the lowest-order NCI growth rate and significantly suppresses growth from the residue resonances of higher orders by reducing time steps.

The spatial profile of the relativistic striped wind in our simulations is shown in Figure 1. The steady electron–positron flow propagates along the $-x$ direction with a bulk Lorentz factor $\gamma_0$ before interacting with the reflected flow. The spatial profile of the electromagnetic field in the simulation frame is

$$B_y = B_0 \tanh \left\{ \frac{1}{\delta} \left[ \alpha + \cos \left( \frac{2\pi (x + \beta_0 ct)}{\lambda} \right) \right] \right\} \quad (1)$$

$$E_z = \beta_0 B_0 \tanh \left\{ \frac{1}{\delta} \left[ \alpha + \cos \left( \frac{2\pi (x + \beta_0 ct)}{\lambda} \right) \right] \right\}, \quad (2)$$

where $\beta_0$ is the velocity of the wind normalized by the speed of light $c$, and $\lambda$ is the wavelength of the stripes in the wind. The
dimensionless parameters $\delta$ and $\alpha$ are such that the half thickness of the current sheet is $\Delta = \delta \lambda / (2\pi)$, and $B_0$ averaged over one wavelength is $(B_0\lambda) = B_0[1 - 2(\arccos(\alpha)/\pi)]$. The background cold plasma in the wind is uniform, with constant density $n_{e,c,0}^\text{cold} = n_{e,0} / 2$ and constant temperature $kT_{e,c,0}^\text{cold} = 0.04e^2c^{-2}$ for both electrons and positrons. The time in our simulations is normalized by $1/\omega_p$, where $\omega_p = \sqrt{4\pi n_{e,0}e^2/(\gamma_0 m_e)}$ is the plasma frequency, and the spatial coordinates in our simulation are normalized by $d_e = c/\omega_p$. A hot electron–positron plasma inside the Harris current sheets balances the magnetic pressure and maintains the steady profile of electromagnetic field. The density of the hot electron–positron plasma in the current sheet in the simulation frame is

$$n_{e,\text{hot}}^\text{c} = \frac{n_{e,0}}{2 \cosh^2 \left[ \frac{1}{2} \alpha + \cos \left( \frac{2 \pi \lambda (\alpha + \beta_0^2 \gamma)}{\lambda} \right) \right]},$$

where $n_{e,0}/n_{e,0} = \eta$ is the overdensity factor relative to the cold particles outside the layer, and is set to be $\eta = 3$ (Kirk & Skjæraasen 2003; Sironi & Spitkovsky 2011, 2014). The drift velocity in the $z$ direction of the hot particles is setup to ensure the steady profile of electromagnetic field, i.e., in the rest frame of the wind $\nabla \times \mathbf{B} = (4\pi/e)\mathbf{J}$ is satisfied everywhere so that the electric field stays zero. The left boundary located at $x = 0$ is reflecting for particles and conducting for electromagnetic fields. The shock is self-consistently generated by the interaction between the reflected flow and the incoming flow. Our simulations are performed in the downstream frame, where the resulting downstream plasma bulk flow is at rest when the shock is well developed and separated from the boundary. The simulation is periodic in the $y$ direction. The length of the simulation box in the $y$ direction is $L_y = 400d_e$ for the standard run S0, which is large enough to hold the largest island in the simulation, and we have tested that a larger length in $y$ direction does not change our main conclusions. In our standard run, we have $\alpha = 0.1$ (i.e., $B_1/\lambda = 0.064B_0$), $\Delta = d_e$, $\lambda = 640d_e$, $\gamma_0 = 1/\sqrt{1 - \beta_0^2} = 10^4$, and $\sigma_0 = B_0^2 / (4\pi \gamma_0 m_e n_e c^2) = 10$, and $d_e$ is resolved with 7.5 computational cells. We use fourth-order particle shape, which significantly reduces the numerical noise even for a relatively small number of particles per cell. Each computational cell is initialized with two electrons and two positrons in the cold wind, and an additional two electrons and two positrons in the current sheets. The parameters for other runs with different $\gamma_0$ and $\sigma_0$ are listed in Table 1, while the resolution and $\alpha$ remain the same for all the runs. We have also performed limited experiments with higher resolutions, obtaining essentially the same results.

To ensure that the onset of reconnection is independent of numerical effects, we have extensively tested the stability of current sheets in a double-periodic simulation without the reflecting wall. The results using the WT scheme (Lu et al. 2020) are summarized in Appendix A. They show that the case with time step $\Delta t = 0.2\Delta t_{\text{CFL}}$ can ensure that the onset of reconnection is much later than $\omega_p t = 2000$, which is roughly the longest time it takes for the shock to compress a current sheet in our simulation. Therefore, we choose $\Delta t = 0.2\Delta t_{\text{CFL}}$ for all runs in this paper (see Table 1).

**Figure 2.** The evolution of density profile averaged over the $y$ direction. The variable plotted is $\log_{10}(\langle n_e \rangle / n_{e,0})$. The magenta dotted line marks the location of the fast MHD shock (or the precursor perturbation). The expression for the location of the fast shock can be fitted as $x = \alpha (1 - \omega_p t/1000)^2$. The transition region of the slow (main) shock is between the black dashed line and the blue dotted–dashed line. The expression for the transition region is $(ct - 10000 \omega_p t) / 3 < x < (ct - 8000 \omega_p t) / 3$.

**Table 1.** Parameters for Each Run in This Work

| Run | $L_y/d_e$ | $\lambda/d_e$ | $\gamma_0$ | $\Delta/d_e$ | $\sigma_0$ |
|-----|----------|---------------|-------------|--------------|------------|
| S0  | 400      | 640           | $10^4$      | 1            | 10         |
| A1  | 400      | 640           | $10^2$      | 1            | 10         |
| A2  | 400      | 640           | $10^3$      | 1            | 10         |
| A3  | 400      | 640           | $10^5$      | 1            | 10         |
| A4  | 400      | 640           | $10^6$      | 1            | 10         |
| B1  | 400$\sqrt{3}$ | 640$\sqrt{3}$ | $10^6$     | $\sqrt{3}$     | 30         |
| B2  | 400$\sqrt{10}$ | 640$\sqrt{10}$ | $10^6$      | $\sqrt{10}$     | 100        |
| B3  | 400$\sqrt{30}$ | 640$\sqrt{30}$ | $10^6$      | $\sqrt{30}$     | 300        |
| C1  | 400      | 160           | $10^6$      | 1            | 10         |
| C2  | 400      | 40            | $10^6$      | 1            | 10         |
| C3  | 400      | 20            | $10^6$      | 1            | 10         |
| C4  | 400      | 10            | $10^6$      | 1            | 10         |
| C5  | 400      | 5             | $10^6$      | 1            | 10         |

**Note.** The parameters listed are the transverse box size $L_y$, the wavelength of the striped wind $\lambda$, upstream bulk Lorentz factor $\gamma_0$, the thickness of the current sheets $\Delta$, and the upstream magnetization $\sigma_0$. The lengths are in units of skin depth $d_e$.

**3. Shock Formation and Evolution**

In Figure 2, we show the evolution of density profile averaged over the $y$ direction. In Figure 3, we show the 2D spatial distribution of density $\log_{10}(\langle n_e \rangle / n_{e,0})$. Magnetic energy density over particle energy density $\log_{10}(E_B/E_p)$, averaged particle kinetic energy $\gamma/\gamma_0$, and particle energization rate $\mathbf{J} \cdot \mathbf{E}$ at $\omega_p t = 2000$. The shock-reconnection system forms and evolves self-consistently as the reflected plasma interacts with the incoming flow. During this initial process we observe a precursor perturbation, which is actually a fast shock, that propagates toward the upwind side of the simulation box and compresses and decelerates the incoming current sheets. This drives the magnetic reconnection because the perturbed and
compressed current sheets are subject to rapid growth of the tearing instability. Magnetic reconnection rapidly converts magnetic energy into particle kinetic energies as current sheets evolve. This is further facilitated by the compression at the main shock layer, characterized by a huge density increase. The general shock and reconnection dynamics of our standard run S0 are similar to the previous study (Sironi & Spitkovsky 2011), where a fast shock (the precursor perturbation) and a slow hydrodynamic shock (main shock) are identified.

The fast shock travels close to the speed of light initially and slows down when interacting with the incoming flow afterwards, as tracked by the magenta dotted trajectory in Figure 2. The location of the precursor can be fitted as \( x = c t (1 - \omega_{bd}^2)/103^2 \), with the region with larger \( x \) being unperturbed. The jump at the fast shock is sharp in the beginning and becomes smoother after it encounters more current sheets. The incoming current sheets are slowed down and compressed as they pass through the fast shock. As shown in Figure 3(b), the ratio of magnetic energy density to particle energy density \( \varepsilon_B/\varepsilon_P \) in the regions between current sheets increases right after passing through the fast shock, indicating strong compression. As a result of the slowdown and weakening of the fast shock, a current sheet reaching the fast shock later gets slowed down less than the one reaching the fast shock earlier. More importantly, the fast shock compresses the current sheets, triggering the onset of fast reconnection (also see the discussion below). There is a huge density jump at the main shock around \( x_{sh} \approx 354c/\omega_p \) at \( \omega_{bd} = 2000 \). The jump moves at a constant speed of approximately \( c/3 \) (consistent with the jump condition of the ultrarelativistic shock) in the +\( \hat{x} \) direction as shown in Figure 3(a). This density jump can also be seen in Figure 2 between the black dashed line and the blue dotted–dashed line, i.e., \((ct - 1200c/\omega_p)/3 < x_{sh} < (ct - 800c/\omega_p)/3\).

Inside the transition region between the precursor and the slow shock, i.e., between the magenta dotted line and the black dashed line in Figure 2 or \( 354c/\omega_p < x < 1623c/\omega_p \) in Figure 3, the adjacent regions of opposite magnetic field polarity are pushed toward each other and reconnect. The current sheets continuously break into a series of magnetic islands separated by X-points. Strong energy conversion is associated with magnetic reconnection, resulting in small \( \varepsilon_B/\varepsilon_P \) in the islands, as shown in Figure 3(c). The islands coalesce, grow to larger sizes as they continuously move toward the downstream, and further grow after passing the main shock front. In the downstream, the average \( \varepsilon_B/\varepsilon_P \) is around 0.3, much less than its initial value \( \varepsilon_B/\varepsilon_P = 5 \), indicating that most of the magnetic energy is converted to particle kinetic energy. As shown in Figure 3(c), the average particle Lorentz factor is \( \langle \gamma \rangle \sim \gamma_0(1 + \sigma_0) = 11\gamma_0 \) in the downstream. The particles in

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**Figure 3.** 2D spatial profile of different quantities for the standard run S0 with \( \sigma_0 = 10, \gamma_0 = 10^4, \alpha = 0.1 \) at time \( \omega_{bt} = 2000 \). (a) logarithm of particle number density normalized by \( n_{ic} \), (b) logarithm of the ratio of magnetic energy density to particle energy density, (c) logarithm of average \( \gamma/\gamma_0 \), and (d) energy conversion rate \( \mathbf{J} \cdot \mathbf{E} \).

The green dotted vertical lines are boundaries of regions for more detailed studies, region I for \( 0 < x/d_e < 197 \), region II for \( 573 < x/d_e < 804 \), and region III for \( 1275 < x/d_e < 1497 \).
For runs S0, B1, B2, and B3 and A4, time downstream with peak at standard run S0. The dashed line is the relativistic thermal distribution for the $J_{\dot{\gamma}}$ the downstream can form thermal and nonthermal distributions, which will be discussed in Section 4. The energization rate $\mathbf{J} \cdot \mathbf{E}$, as plotted in Figure 3(d), is concentrated in the current sheets and islands. More details about the energization will be discussed in Section 5.

4. Particle Spectrum

As the shock forms and interacts with the current sheets embedded in the striped wind, the electrons and positrons are heated and accelerated. In Figure 4(a), we show the energy distribution function $f(\gamma)$ (related to the number distribution function $n(\gamma)$ by $f(\gamma) \equiv n(\gamma)/\gamma$) in different regions for the standard run S0 at $\omega = 2000$. Before reaching the fast shock, i.e., for $1600 < x/\omega < 2000$, $f(\gamma)$ has a peak at the bulk Lorentz factor $\gamma_0 = 10^4$. As the flow slows down, i.e., for $400 < x/\omega < 1600$, the peak of the spectrum shifts to a lower energy. As the current sheets reconnect, the tail of the spectrum becomes harder as the flow gets closer to the main shock. According to the jump condition, the averaged kinetic energy due to the energization in the downstream region is $\gamma \approx \gamma_0(1 + \sigma_0)$. For instance, in the standard run S0 ($\gamma_0 = 10^4$ and $\sigma_0 = 10$), we have $\langle \gamma \rangle \approx 1.1 \times 10^5$, which is consistent with the energy density for $0 < x/\omega < 400$ in Figure 4(a). However, the actual downstream particle spectrum is distinctly different from the thermal distribution. The particles with energies between $\gamma = 2 \times 10^7$ and $\gamma = 4 \times 10^5$ follow a power-law distribution $f(\gamma) \propto \gamma^{-p+1}$ with $p = 1.5$. For comparison, we also plot the distribution of a thermal plasma with $\langle \gamma \rangle = 1.1 \times 10^5$ as the dashed line.

How do the resulting energy spectra depend on $\gamma_0$, $\sigma_0$, and $\lambda$? Earlier numerical simulations (Sironi & Spitkovsky 2011) are limited to regimes with $\gamma_0 \lesssim 375$. However, global models (Kirk & Skjærseth 2003; Kirk & Giacinti 2017) have suggested a much larger $\gamma_0$ and wider range of $\sigma_0$. Thanks to our advances in the numerical scheme (Lu et al. 2020), which substantially reduces the growth of numerical instabilities, we are able to explore a much larger range of parameters. For the runs with different $\gamma_0$ and $\sigma_0$ in the range of $10^2 \leq \gamma_0 \leq 10^4$ as shown in Figure 4(b) and in the range of $10 < \sigma_0 < 300$ as shown in Figure 4(c), the downstream particle energy distributions are similar to the one in the standard run. We confirmed that the power-law energy distribution is scalable for different $\gamma_0$ and $\sigma_0$. The particles with energy $0.2 < \gamma/\gamma_0 < 4$ follow a power-law distribution with $p = 1.5$. For different values of magnetization parameter $\sigma_0$ and initial bulk flow Lorentz factor $\gamma_0$, the spectra are similar if one plots the spectrum against $\gamma/\gamma_0$ and the variables $\lambda/(\sqrt{d_{\gamma}/\omega_p})$ and $\alpha$ are kept the same for different runs, as shown in Figure 5(a). Despite the wide ranges of $\gamma_0$ and $\sigma_0$, the dimensionless particle-field equations and the initial conditions are the same, as derived in Appendix B. Thus the solutions, i.e., including the particle and field distributions, between different $\gamma_0$ and $\sigma_0$ are scalable. The particle spectrum depends on the value of $\lambda/(\sqrt{d_{\gamma}/\omega_p})$, as shown in Figure 5(b). While the mean energy per particle does not appreciably vary with $\lambda$ as shown by the red line in Figure 5(b), smaller $\lambda$ results in narrower spectrum and softer high-energy tail. The ratio $\gamma_{\text{max}}/\gamma_{\text{min}}$ increases as $\lambda$ increases and is significantly larger than unity for $\lambda \gtrsim 40_{\text{c}}$. The power-law spectrum forms for the runs with $\lambda \gtrsim 40_{\text{c}}$. This is similar to the previous study (Sironi & Spitkovsky 2011) and will be further explained in Section 5.

The power-law distribution is stable as long as the shock is well developed and separated from the boundary. The lower bound energy $\varepsilon_{\text{lb}}$ and the maximum energy $\varepsilon_{\text{max}}$ scale with $\gamma_0$ and $\sigma_0$ as $\varepsilon_{\text{lb}} \approx 0.2 \gamma_0 \sigma_0$ and $\varepsilon_{\text{max}} \approx 4 \gamma_0 \sigma_0$. The maximum energy for electrons and positrons can reach hundreds of TeV ($\gamma > 2 \times 10^8$) for the run with $\gamma_0 \approx 10^6$ and $\sigma_0 = 10$. The power-law distribution of the particles suggests that an acceleration mechanism is very efficient at energizing particles, which will be analyzed and discussed in Section 5.
5. Particle Acceleration Mechanism

In order to understand the particle acceleration mechanism at the termination shock of the striped wind, we adopt several techniques for analyzing the results of the standard run, including the particle trajectory, decomposing the particle energization \( J \cdot E \) term, and the acceleration rate binned by particle kinetic energy for distinguishing different acceleration mechanisms. With these analyses, we find that the major acceleration mechanism is a Fermi-type acceleration. In the past, the diagnostics for understanding particle acceleration often relies on a few hand-selected particles. Our analysis not only includes particle trajectories but also statistically evaluates the acceleration rate in a quantitative way, so we can evaluate competitive processes without biases.

In Figure 6, we show the trajectories for two typical tracer particles in the simulation. Many of the trajectories we get from the standard run are Fermi-like especially for those accelerated to very high energies, i.e., particles bouncing back and forth and gaining energy. The most important acceleration location is not at the reconnection X-points, but by the relativistic flows generated during magnetic reconnection. As shown in Figure 3(d), the energization rate \( J \cdot E \) near the X-points is relatively small compared to that in the reconnection islands, which indicates that Fermi acceleration is more important than direct acceleration at the X-line. Once a particle travels into the downstream region of the shock it rarely travels back into the upstream region, indicating that the Fermi mechanism in the reconnection islands is also much more efficient than the diffusive shock acceleration at least for the parameter regime we explore in this paper. The acceleration pattern from particle trajectories is distinctly different from earlier work by Sironi & Spitkovsky (2011), who concluded that direct acceleration by the nonideal electric field at X-points is the main acceleration mechanism.

To distinguish the relative importance of Fermi acceleration and direct acceleration, we present a number of analyses to statistically evaluate the two mechanisms. The Fermi-type acceleration is mainly through the electric field induced by bulk plasma motion, or more generally, the electric field perpendicular to the local magnetic field, whereas the direct acceleration is driven by the nonideal electric field, or parallel electric field if a nonzero magnetic field exists. One can distinguish the two mechanisms by calculating the statistical motion of the charged particles and the energization from different components of electric fields. The generalized Fermi acceleration in the relativistic reconnection layer (Drake et al. 2006; Guo et al. 2014; Lemoine 2019) follows the particle journey through a sequence of local frames where the local electric field vanishes. For such local frames, we can decompose the electric fields based on Lorentz transformation, with detailed derivations given in Appendices C.1 and C.2. The ways by which we decompose the electric fields are generalizations for the nonrelativistic case used extensively in previous studies (Guo et al. 2014, 2015, 2019; Li et al. 2018). Such generalization is critical in this work because the motion of the bulk flow in our simulations can be highly relativistic. We also calculate the integral of the energization from each term over \( 0 < y < 1500c/\omega_p \) and \( 0 < y < 400c/\omega_p \) and compare it with the integral of total energization \( P = \vec{J} \cdot \vec{E} \).

First, we distinguish the electric field associated with bulk plasma motion (motional electric field) from the one that is not (nonideal electric field). As described in Appendix C.1, we use the following procedures: (1) make a Lorentz boost transforming the electromagnetic field \( \vec{E} \) and \( \vec{B} \) in the simulation frame to \( \vec{E}' \) and \( \vec{B}' \) into the local comoving frame (the frame moving at \( \beta_c \), where \( \beta_c \) is the speed of the fluid motion), (2) set \( \vec{E}' = 0 \) in the comoving frame, (3) transform the magnetic field \( \vec{B}' \) back into the simulation frame as \( \vec{E}_m \) and \( \vec{B}_m \), where \( \vec{E}_m \) is the electric field due to plasma motion and the remaining part \( \vec{E}_n = \vec{E} - \vec{E}_m \) is the nonideal electric field. In the nonrelativistic limit, one obtains \( \vec{E}_n \approx -\beta_n \times \vec{B} \) and \( \vec{E}_m \approx \vec{E} + \beta_n \times \vec{B} \) (Guo et al. 2019). The spatial profiles of \( \vec{J} \cdot \vec{E}_m \) and \( \vec{J} \cdot \vec{E}_n \) are shown in Figures 7(b) and (c). The profiles of \( \vec{J} \cdot \vec{E} \) and \( \vec{J} \cdot \vec{E}_m \) have similar features, both showing different signs on different sides of each island, while \( \vec{J} \cdot \vec{E}_n \) has a smaller value but opposite polarity compared with \( \vec{J} \cdot \vec{E}_m \). We compare \( P_m = |\vec{J} \cdot \vec{E}_m| \) with \( P = |\vec{J} \cdot \vec{E}| \) and find \( P_m \approx 0.56P \), which indicates that the motional electric field has a larger contribution to the particle energization than the nonideal electric field.

The second way to decompose the electric field finds the generalized perpendicular and parallel electric field through a series of frame transformations. As described in Appendix C.2, we use the following procedures: (1) make a Lorentz boost transforming the electromagnetic field \( \vec{E} \) and \( \vec{B} \) in the simulation frame to \( \vec{E}' \) and \( \vec{B}' \) in a local frame where \( \vec{E}' \) and \( \vec{B}' \) are...
parallel—generally this local frame is also the guiding center frame; (2) set \( E' = 0 \) in that frame; and (3) transform the magnetic field \( B' \) back into \( E_g \) and \( B_g \) in the simulation frame. Since \( E_g \cdot B_g = 0 \) and \( (E - E_g) \parallel B_g \), we have the generalized perpendicular electric field \( E_\perp = E_g \) and parallel electric field \( E_\parallel = E - E_g \). In the nonrelativistic limit we have \( E_\perp \approx E - (E \cdot B/B^2)B \) and \( E_\parallel \approx (E \cdot B/B^2)B \) (Guo et al. 2014, 2015, 2020; Li et al. 2018). The spatial profiles of \( J \cdot E_\perp \) and \( J \cdot E_\parallel \) are shown in Figures 7(d) and (e). The spatial profile and amplitude of \( J \cdot E_\perp \) are similar to that of \( J \cdot E \) in Figure 7(a), while \( J \cdot E_\parallel \) is negligible compared to \( J \cdot E_\perp \). The integral over \( 0 < x < 1500c/\omega_p \) for \( J \cdot E_\perp \) is \( P_\perp = \int \lvert J \rvert \cdot E_\perp = 0.84P \). The fact that \( P_\parallel = P - P_\perp = 0.16P \) is small indicates that particles gain energy mainly through Fermi-type acceleration.

We also compare the energization rate in the \( \lvert E \rvert < \lvert B \rvert \) region and the \( \lvert E \rvert > \lvert B \rvert \) region in Figures 7(f) and (g) as suggested by Sironi & Spitkovsky (2011, 2014). Since \( \lvert E \rvert^2 - \lvert B \rvert^2 \) is a Lorentz invariant, the decomposition into the \( \lvert E \rvert < \lvert B \rvert \) region and the \( \lvert E \rvert > \lvert B \rvert \) region are consistent among all the reference frames. Similar to the \( J \cdot (E - E_g) \) term, the energization term in the \( \lvert E \rvert > \lvert B \rvert \) region is negligible. By integrating \( J \cdot E \) over subregions where \( \lvert E \rvert < \lvert B \rvert \), we find that \( P_L = \int \lvert E \rvert < \lvert B \rvert J \cdot E = 0.93P \). The fact that \( P - P_L = 0.07P \) is small confirms that the energy conversion from the X-points is much

**Figure 6.** Particle trajectories for two different particles. The left panels (a) and (c) show particle Lorentz factor (normalized by \( \gamma_0 \)) vs. \( x - x_{sh} \) coordinate where the location of the shock jump is approximately at \( x_{sh} \approx (ct - 1000c/\omega_p) / 3 \), and the right panels (b) and (d) show particle Lorentz factor (normalized by \( \gamma_0 \)) vs. \( y \) coordinate. Panels (a) and (b) are for tracer particle \#1, and panels (c) and (d) are for tracer particle \#2. The red line is the trajectory where the particle is in the upstream, and the red line is the trajectory where the particle is in the downstream. The black dots are for particles at \( t = 0 \), and the red dots are for particles at \( \omega_p t = 2000 \). The main acceleration process is the Fermi-like bounces and particles gain a significant amount of energy during each bounce.
Figure 7. 2D spatial profile for $J \cdot E$ terms in the standard run S0 with $\sigma_0 = 10$, $\gamma_0 = 10^4$, and $\alpha = 0.1$ at time $\omega_p t = 2000$. (a) Total energization rate $J \cdot E$, (b) $J \cdot E_m$, where $E_m$ is the motional electric field, (c) $J \cdot E_n$, where $E_n$ is the nonideal electric field, (d) $J \cdot E$, where $E$ is the generalized perpendicular electric field, (e) $J \cdot E_p$ where $E_p$ is the generalized parallel electric field, (f) $J \cdot E$ for $|E| < |B|$, and (g) $J \cdot E$ for $|E| > |B|$. All the subfigures have the same color scale. $J \cdot E_m$ has a smaller value but opposite polarity compared to $J \cdot E_m$, $J \cdot E_p$, and $J \cdot E(|E| < |B|)$ are greater than $J \cdot E_1$ and $J \cdot E(|E| > |B|)$. 

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weaker than that in the reconnection islands (Guo et al. 2015, 2019).

We further study the averaged acceleration rate \( \langle W \rangle \equiv \langle q v \cdot E \rangle \) for particles of different energy in the simulation at \( \omega_{pi} = 2000 \). Note that the curve for \( E - E_{\mu} \) is the absolute value. (b) Same as (a) but for \( 573c/\omega_{pi} < x < 804c/\omega_{pi} \), i.e., region II in Figure 7. (c) Same as (a) but for \( 1275c/\omega_{pi} < x < 1497c/\omega_{pi} \), i.e., region III in Figure 7. \( E_{\mu} \) only contributes to positive acceleration in a small subset of energy ranges especially in low energies compared to the contribution from \( E_{\mu} \).

Figure 8. (a) Acceleration rate by total electric field \( E \), motional electric field \( E_{\mu} \) and nonideal electric field \( E_{\nu} \) for particles at different energies for \( 0 < x < 197c/\omega_{pi} \), i.e., region I in Figure 7, at \( \omega_{pi} = 2000 \). Note that the curve for \( E - E_{\mu} \) is the absolute value. (b) Same as (a) but for \( 573c/\omega_{pi} < x < 804c/\omega_{pi} \), i.e., region II in Figure 7. (c) Same as (a) but for \( 1275c/\omega_{pi} < x < 1497c/\omega_{pi} \), i.e., region III in Figure 7. \( E_{\mu} \) only contributes to positive acceleration in a small subset of energy ranges especially in low energies compared to the contribution from \( E_{\mu} \).

Figure 9. Same as Figure 8 but for the generalized perpendicular electric field \( E_{\perp} \) and parallel electric field \( E_{\parallel} \). \( E_{\parallel} \) only contributes to positive acceleration in a small subset of energy ranges in high energies compared to the contribution from \( E_{\perp} \).

in the downstream and already coalesce and grow to large size), in Figures 8(b) and 9(b) for region II \( (573c/\omega_{pi} < x < 804c/\omega_{pi} \) where the magnetic islands have a large size but have not yet reached the main shock), and in Figures 8(c) and 9(c) for region III \( (1275c/\omega_{pi} < x < 1497c/\omega_{pi} \), where a current sheet just breaks into magnetic islands). Region I is in the downstream of the shock, while regions II and III are pre-shock regions with ongoing magnetic reconnection. These regions are also marked as in Figures 3(d) and 7. The averaged acceleration rate is roughly
proportional to particle energy over a wide range of energies, indicating that the major acceleration mechanism is Fermi acceleration. The nonideal electric field $E_n$ only contributes to positive acceleration in a small subset of energy ranges especially in low energies compared to the contribution from the motional electric field $E_m$. The parallel electric field $E_\parallel$ only contributes to positive acceleration in a small subset of energy ranges in high energies compared to the contribution from the motional electric field $E_m$.

Theoretical analysis (Guo et al. 2014, 2015, 2019, 2020) has shown that the power law of the particle spectrum can be explained by solving the energy continuity equation with injection of particles. While Fermi acceleration does not change the shape of the spectrum for the particles in the initial reconnection layer, the particles injected into the layer can form a power-law distribution with accelerated energy. In the case of the relativistic striped wind, the injection is continuously going on due to the incoming flow from the striped wind and the particles flowing into the downstream of the shock. The first-order Fermi process, where the acceleration rate is proportional to the energy of the particles as confirmed in Figure 8, is accompanied by the escape of particles from the reconnection islands. In the case of $\lambda = 6.4d_\gamma$, the spectrum we get from the simulation has $p = 1.5$ as shown in Figure 4, which is consistent with $1 < p < 2$ as we get from the analytical model (Guo et al. 2014, 2015, 2019). In the case of smaller wavelength $\lambda \lesssim 40d_\gamma$ for the striped wind, there are more X-points as the shock-reconnection system evolves. However, the magnetic islands occupy a larger fraction of the total area for a smaller wavelength for the striped wind, resulting in more particles in the initial reconnection layer and fewer particles injected into the layer. Thus, by applying the theoretical analysis in Guo et al. (2014) the particle spectra are softer and more Maxwellian-like as the wavelength $\lambda$ decreases, as shown in Figure 5(b).

6. Conclusion and Discussion

By carrying out first principles kinetic PIC simulations, we have studied the shock structure and dynamics, magnetic reconnection, and particle energization at the termination shock of a relativistic striped wind. Our parameter regime covers an unprecedented relativistic regime with a large bulk Lorentz factor of up to $\gamma_0 = 10^6$, which is expected in PWNe. The values of $\gamma_0$ and $\sigma_0$ are even uncertain for a single PWN, due to the uncertainties in the density at the light cylinder and the global dissipation mechanism in the nebula (Kirk & Skjæraasen 2003; Kirk & Giacinti 2017). For the observed gamma-ray flares (Abdo et al. 2011; Tavani et al. 2011; Buehler et al. 2012; Weisskopf et al. 2013; Buhler & Blandford 2014) in the Crab Nebula, a sudden drop in the mass loading of the pulsar wind may result in $\sigma_0 \approx 10$ and $\gamma_0 > 10^6$ at the termination shock (Kirk & Giacinti 2017). The maximum energy for accelerated electrons and positrons can reach hundreds of TeV if $\gamma_0 \approx 10^6$ and $\sigma_0 = 10$. Our study is relevant to magnetic energy conversion and particle acceleration in PWNe produced by oblique-rotating pulsars. While a growing body of research using PIC methods focuses on particle acceleration in the spontaneous relativistic magnetic reconnection in the magnetically dominated regime, in this work we extended the study by examining the particle acceleration mechanism in shock-driven magnetic reconnection at the termination shock of relativistic striped wind. While the plasma dynamics in this regime is studied extensively by Sironi & Spitkovsky (2011), we study carefully the particle energization and acceleration. By analyzing PIC simulations, we found that the termination shock is efficient at converting magnetic energy to particle energy and producing nonthermal power-law distributions. The particle distribution is scalable with upstream bulk Lorentz factor $\gamma_0$ and magnetization $\sigma_0$. For sufficiently large $\gamma_0$ and $\sigma_0$, the spectrum we get from the simulation has a power law with $p = 1.5$ in the energy range $0.2 < \gamma/(\gamma_0\sigma_0) < 4$ for the standard run. Detailed analysis shows that the Fermi-type mechanism dominates the particle acceleration and power-law formation. For smaller wavelength ($\lambda \lesssim 40d_\gamma$ for $\sigma_0 = 10$), the spectrum is more Maxwellian-like due to more particles in the initial reconnection layer and fewer particles injected into the layer.

The high-energy particles are known to efficiently produce radiation through a few mechanisms, e.g., synchrotron, bremsstrahlung, and inverse-Compton scattering. The morphology of magnetic fields and the spectra of electron–positron pair plasma we get from the PIC simulations can be post-processed to generate the spectrum (Lyutikov et al. 2019) and polarization (Dean et al. 2008; Forot et al. 2008; Jourdain & Roques 2019) of the resulting radiation and compared to the observations from the Crab and other PWNe, which will be the subject of future reports.

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Appendix A

Test Problem with a Double-periodic Box

To make sure the relativistic reconnection onset in our simulations is not due to numerical effects, we carry out several test runs for a current sheet moving in a double-periodic box. The simulation setup is similar to run S0, with $\sigma_0 = 10$, $\alpha = 0.1$, $L_y = \lambda = 640d_\gamma$, $L_x = 400d_\gamma$, and $\Delta = d_\gamma$, except that the $x$ direction is periodic and only covers one wavelength of the striped wind. Therefore, the simulation tests the stability of the striped wind. We use the force interpolation scheme called the
WT (standing for weighting with time-step dependency) scheme proposed in a recent development (Lu et al. 2020) to overcome the NCI. The WT scheme requires a small time step to improve the numerical stability of the simulations. We test three cases with different time steps $\Delta t = 0.7 \Delta t_{\text{CFL}}, \Delta t = 0.3 \Delta t_{\text{CFL}},$ and $\Delta t = 0.2 \Delta t_{\text{CFL}},$ where $\Delta t_{\text{CFL}} = \frac{1}{c_2 \sqrt{1/\Delta x^2 + 1/\Delta y^2}}$.

The results for the test problem in Figure A1. According to the growth rate of tearing instability given in a previous study (Galeev et al. 1986; Daughton et al. 2011), the current sheets in the test runs should not reconnect when $\omega_p t$ is a few thousand. The reconnection of any current sheets before that time is caused by numerical problems. For the runs with $\Delta t = 0.7 \Delta t_{\text{CFL}}$ and $\Delta t = 0.3 \Delta t_{\text{CFL}}$, the reconnection happens before $\omega_p t < 1000$ and the islands have significantly grown in size at $\omega_p t = 2000$, which is roughly the time it takes for the rightmost current sheet to travel before it reaches the main shock. Thus, the results for the runs with $\Delta t = 0.7 \Delta t_{\text{CFL}}$ and $\Delta t = 0.3 \Delta t_{\text{CFL}}$ are not consistent with the theoretical prediction. The theoretical prediction (Galeev et al. 1986; Daughton et al. 2011) is verified using the runs in the comoving frame of the flow. For the run with $\Delta t = 0.2 \Delta t_{\text{CFL}}$, the current sheets have not significantly changed their morphology from the initial condition until $\omega_p t = 2000$. Since the run with $\Delta t = 0.2 \Delta t_{\text{CFL}}$ is the most stable run in Figure A1, and it can ensure the current sheet is stable before it interacts with the fast shock, we use the small time step $\Delta t = 0.2 \Delta t_{\text{CFL}}$ in all the production runs shown in the main context.

**Appendix B**

**Scaling Equations for Relativistic Magnetized Plasmas**

We show that the particle-field equations for relativistic PIC modeling in the dimensionless form with proper normalization. The equations for relativistic PIC modeling in
where $s$ stands for $x$th quasi-particle and $w_s$ is the weight of $x$th pseudo-particle. We introduce the following normalization

$$t = \sigma_0^{-1/2} \omega_p^{-1} t, \quad x_s = \sigma_0^{-1/2} (c/\omega_p) \tilde{x}, \quad u_s = \sigma_0 \gamma_0 \tilde{u}_s, \quad n_s = m_s \tilde{n}_s, \quad q_s = e \tilde{q}_s, \quad \tilde{w}_s = n_s \tilde{n}, \quad E = \sqrt{4\pi n_0 m_e c^2 \sigma_0} \tilde{E}, \quad B = \sqrt{4\pi n_0 m_i c^2 \sigma_0} \tilde{B},$$

where $\omega_p = \sqrt{4\pi n_0 e^2/(\gamma_0 m_e)}$, then we obtain the dimensionless equations

$$\frac{d\tilde{u}}{dt} = \tilde{q} \tilde{E} + \tilde{v} \times \tilde{B}, \quad \frac{d\tilde{v}}{dt} = \frac{\tilde{v}}{c}, \quad \frac{\partial \tilde{E}}{\partial t} = \tilde{\nabla} \times \tilde{B} - \sum_s \tilde{w}_s \tilde{q}_s \tilde{v}_s, \quad \frac{\partial \tilde{B}}{\partial t} = -\tilde{\nabla} \times \tilde{E},$$

and

$$\tilde{v} = \frac{1}{\sqrt{1 + 1/(u^2 \gamma_0^2 \sigma_0^2)}} \tilde{u} = \left(1 - \frac{1}{2\tilde{u} \gamma_0 \sigma_0} + \mathcal{O}\left(\frac{1}{\tilde{u} \gamma_0 \sigma_0}\right)\right) \tilde{u} \tilde{u}^{-1}$$

As long as $\gamma_0 \sigma_0 \tilde{u} \approx 1$, Equation (B4) can be well approximated by $\tilde{v} = \tilde{u} / \tilde{u}$, then the dimensionless equations (Equation (B3)) are the same for different $\gamma_0$ and $\sigma_0$. For the simulations in this work, in the upstream flow, we have $\tilde{u} = u / (\gamma_0 \sigma_0 c) \approx 1/\sigma_0 \ll 1$, $\tilde{E} \approx B = 1$, and the normalized wavelength $\tilde{\lambda} = \lambda / (\gamma_0 \sigma_0 c \omega_p)$. As long as the $\tilde{\lambda}$ remains the same, no matter what $\sigma_0$ and $\gamma_0$ we have, the initial conditions and the solution for Equation (B3) are scalable. One can then use the solution of Equation (B3) to obtain scalable dimensional results using normalization factors from Equation (B2).

### Appendix C

**Decomposing Electromagnetic Fields**

The generalized Fermi acceleration (Lemoine 2019) follows the particle journey through a sequence of local frames where local electric field vanishes. To find such local frames, we can decompose the electric fields based on Lorentz transformation. We generalize the decomposition in previous studies (Guo et al. 2014, 2015, 2019; Li et al. 2018) to the relativistic case, since the bulk flow in this work can be highly relativistic. In Appendix C.1, by removing the electric field in the fluid comoving frame, we distinguish the electric field associated with bulk plasma motion (motional electric field) from the one that is not (nonideal electric field). In Appendix C.2, by removing the electric field in the frame where the electric and magnetic fields are parallel, we get the generalized parallel and perpendicular electric fields.

#### C.1. Decomposing Electromagnetic Field in the Bulk Fluid Frame

For convenience, we define the complex vector of electromagnetic field as $\mathbf{F} = \mathbf{E} + i \mathbf{B}$ (Landau et al. 1963). The Lorentz transformation for $\mathbf{F} = \mathbf{E} + i \mathbf{B}$ in the simulation frame $S$ to $\mathbf{F}' = \mathbf{E}' + i \mathbf{B}'$ in fluid comoving frame $S_f$, which moves at speed $\beta c$ with respect to $S$ is (see Chapter 26 in Feynman 1964)

$$\mathbf{F}' = (1 - \gamma_f)(\mathbf{F} \cdot \mathbf{n}_f)\mathbf{n}_f + \gamma_f(\mathbf{F} - i \beta_f \times \mathbf{F}),$$

(C1)

where $\mathbf{n}_f = \beta_f / \beta$ is the direction of $\beta_f$, and $\gamma_f = \sqrt{1 - \beta_f^2}$. If we have a field configuration $\mathbf{F}_m'$ in $S_f$ frame such that the electric field $\mathbf{E}_m'$ is zero and the magnetic field $\mathbf{B}_m'$ is the same as $\mathbf{B}'$, then

$$\mathbf{F}_m' = i \mathbf{B}' = \mathbf{F}_m = (1 - \gamma_f)(\mathbf{F} \cdot \mathbf{n}_f)\mathbf{n}_f + \gamma_f(\mathbf{F} - i \beta_f \times \mathbf{F}).$$

(C2)

Transforming $\mathbf{F}_m'$ back into $\mathbf{F}_m$ in the $S$ frame, we obtain

$$\mathbf{F}_m = (1 - \gamma_f)(\mathbf{F}_m' \cdot \mathbf{n}_f)\mathbf{n}_f + \gamma_f(\mathbf{F}_m' + i \beta_f \times \mathbf{F}_m') = [-\gamma_f^2 \beta_f \times \mathbf{B} - (\gamma_f^2 - 1) \mathbf{E} - (n_f \cdot \mathbf{E})n_f] + i[\gamma_f^2 \beta_f \times \mathbf{E} + \gamma_f^2 (\mathbf{B} - (\mathbf{B} \cdot \mathbf{n}_f)\mathbf{n}_f) + (\mathbf{B} \cdot \mathbf{n}_f)n_f].$$

(C3)

The real part of $\mathbf{F}_m$, i.e., the electric field due to plasma motion, is

$$\mathbf{E}_m = -\gamma_f^2 (\beta_f \times \mathbf{B} + \mathbf{E} - (n_f \cdot \mathbf{E})n_f) + (\mathbf{E} - (n_f \cdot \mathbf{E})n_f)/\gamma_f^2 = (\mathbf{P} \cdot \mathbf{E})\mathbf{P} - \mathbf{P}^2 \mathbf{E} - (\mathbf{E} / \gamma_f)^2 \mathbf{P} \times \mathbf{B}$$

$$= \frac{(\mathbf{E} / c)^2 - \mathbf{P}^2}{(\mathbf{E} / c)^2 - \mathbf{P}^2} \mathbf{E},$$

(C4)

where $\mathbf{P}$ is the momentum density of the fluid and $\mathbf{E}$ is the energy density of the fluid, and the fluid velocity is given by $\beta_f c = \mathbf{P} / E^2 / \mathbf{E}$. In the case where $\beta_f \ll 1$, we have $\mathbf{E}_m \approx -\beta_f \times \mathbf{B}$.

#### C.2. Decomposing Electromagnetic Field in a Frame Where Electric and Magnetic Fields Are Parallel

A frame where the electric and magnetic fields are parallel can be generally found for given electromagnetic field ($\mathbf{E}, \mathbf{B}$) as long as $\mathbf{E}^2 + \mathbf{B}^2 > 0$, even for the $B = 0$ case. The velocity $v_s = \beta_s c$ of such a frame is given by (see Petri 2020 or Section 25 in Landau et al. 1963)

$$\frac{\beta_s}{1 + \beta_s^2} = \frac{\mathbf{E} \times \mathbf{B}}{\mathbf{E}^2 + \mathbf{B}^2},$$

(C5)
where $\beta_x$ and the corresponding Lorentz factor $\gamma_x$ can be solved as
\begin{equation}
\beta_x = \frac{2E \times B}{E^2 + B^2 + \sqrt{(E^2 - B^2)^2 + 4|E \cdot B|^2}} \tag{C6}
\end{equation}
\begin{equation}
\gamma_x^2 = \frac{E^2 + B^2 + \sqrt{(E^2 - B^2)^2 + 4|E \cdot B|^2}}{2\sqrt{(E^2 - B^2)^2 + 4|E \cdot B|^2}}. \tag{C7}
\end{equation}

The electromagnetic field $(E, B)$ in the simulation frame is transformed into the electromagnetic field $(E', B')$ in frame $S'$ through a Lorentz transformation using $v_x$. In frame $S'$, the electromagnetic field $(E', B')$ satisfies $E'\|B'$ and the particle motion follows an accelerating motion in $B'$ direction and a gyromotion perpendicular to the $B'$ direction.

The complex scalar
\begin{equation}
F^2 = F \cdot F = (E^2 - B^2) + 2iE \cdot B = C_R + iC_I \tag{C8}
\end{equation}

is a Lorentz invariant, where $C_R = E^2 - B^2$ and $C_I = 2E \cdot B$ (Landau et al. 1963). The Lorentz transform for $F$ in the simulation frame $S$ to $F' = E' + iB'$ in a frame $S'$ that moves at speed $\beta_x c$ with respect to $S$ is
\begin{equation}
F' = (1 - \gamma_x)(F \cdot n_x)n_x + \gamma_x(F - i\beta_x \times F), \tag{C9}
\end{equation}
where $n_x = \beta_x / \gamma_x$ is the direction of $\beta_x$. If we define a field configuration $F'_a$ in the $S'$ frame such that the electric field $E'_a$ is zero and the magnetic field $B'_a$ is same as $B'$, then by transforming back $F'_a$ into $F_a$ in field frame, we obtain
\begin{equation}
F_a' = \gamma_x[F'_a + i\beta_x \times F'_a] = -\gamma_x^2 \beta_x \times (B - \beta_x \times E) + i\gamma_x^2 (B - \beta_x \times E) = \left(1 - \frac{C_R}{2\sqrt{C_R + C_I}}, \frac{C_I}{2\sqrt{C_R + C_I}} \right)(E + iB) \tag{C10}
\end{equation}
or
\begin{equation}
E'_a = \left(1 - \frac{C_R}{2\sqrt{C_R + C_I}} \right)E - \left(\frac{C_I}{2\sqrt{C_R + C_I}} \right)B \tag{C11}
\end{equation}
\begin{equation}
B'_a = \left(1 - \frac{C_R}{2\sqrt{C_R + C_I}} \right)B + \left(\frac{C_I}{2\sqrt{C_R + C_I}} \right)E.
\end{equation}

Since $E'_a$, $B'_a = 0$ and $(E - E'_a)\|B'_a$, we have the generalized perpendicular electric field $E_\perp = E'_a$ and generalized parallel electric field $E_\parallel = E - E'_a$. In the case where $|E| \ll |B|$ we have $|C_I/C_R| \approx 2E \cdot B/B^2 \ll 1$ and $C_P < 0$, thus the transformation is equivalent to removing the parallel electric field and keeping the perpendicular electric field
\begin{equation}
E_\perp \approx E - E \cdot B/B^2 \tag{C12}
\end{equation}
\begin{equation}
E_\parallel \approx E \cdot B/B^2 \tag{C12}
\end{equation}
\begin{equation}
B_\parallel \approx B. \tag{C12}
\end{equation}