STRING RADIATIVE BACKREACTION

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Abstract

We discuss radiative backreaction for global strings described by the Kalb-Ramond action with an analogous derivation to that for the point electron in classical electrodynamics. We show how local corrections to the equations of motion allow one to separate the self-field of the string from that of the radiation field. Modifications to this ‘local backreaction approximation’ circumvent the runaway solutions familiar from electrodynamics, allowing these corrections to be used to evolve string trajectories numerically. Comparisons are made with analytic and numerical radiation calculations from previous work and the merits and limitations of this approach are discussed. Finally, we comment on the relevance of this work to simulations of a cosmological network of global or axion strings. These methods can also be applied to describe gravitational, electromagnetic and other forms of string radiative backreaction.

1 Introduction

Global strings arise naturally in theories with a spontaneously broken global $U(1)$ symmetries, such as axion models, superstrings and some GUT models. The Goldstone bosons or axions, associated with the broken symmetry, can be radiated by an oscillating global string. This radiation power can be calculated using perturbed trajectories, but the effect of radiative damping on the actual string motion remains to be adequately understood. This deficiency is a major obstacle impeding the quantitative description of string network evolution. Radiative backreaction is believed to determine the nature of long string small-scale structure and small loop creation sizes and, hence, the amplitude and spectrum of the resulting radiation background.

It is appropriate to begin discussion of the problem of radiative backreaction by considering the point electron in classical relativistic electrodynamics [1,2,3]. In this case, the self and radiation fields of the electron can be distinguished easily since the self-field is of order $1/R^2$, whereas the radiation field fall-off is of order $1/R$. Careful analysis of the equations of motion leads to the renormalisation of the electron mass $M_{\text{ren}} = \frac{4}{3} M_e$ by the Coulomb self-field and the first-order approximation to the radiation force—the Abraham–Lorentz force—is given by

$$F_{\mu}^{\text{rad}} = -\frac{2}{3} \frac{e^2}{4\pi} \left( \ddot{X}_\mu + \dot{X}^2 \dot{X}_\mu \right),$$

(1)
where \( X_\mu(\tau) \) is the position on the electron’s worldline at time \( \tau \). However, the dependence of this force on \( \ddot{X}_\mu \) leads to problems in numerical applications because (1) has exponentially increasing solutions. These unphysical ‘runaway’ solutions can only be suppressed by rewriting the equations of motion as an integro-differential equation. In the following we shall derive the analogue of (1) for a global string using an antisymmetric tensor formalism.

2 Self-field renormalisation

The essential features of global strings in flat space are exhibited in the simple U(1) Goldstone model, with action given by

\[
S = \int d^4x \left\{ \partial_\mu \Phi \partial^\mu \Phi - \frac{1}{4} \lambda (\Phi \Phi - f_a^2)^2 \right\},
\]

where \( \Phi \) is a complex scalar field which can be split into a massive (real component \( \phi \) and a massless (real periodic) Goldstone boson \( \vartheta \). The analytic treatment of global string dynamics is hampered by the topological coupling of the self field of the string to the Goldstone boson radiation field. However, we can exploit the well-known duality between a massless scalar field and a two-index antisymmetric tensor \( B_{\alpha\beta} \) to replace the Goldstone boson \( \vartheta \) in (2) via the relation \( \phi^2 \partial_\mu \vartheta = \frac{1}{2} f_a \epsilon_{\mu\nu\lambda\rho} \partial^\mu B^{\lambda\rho} \). Performing this transformation carefully and integrating over the massive degrees of freedom about the two-dimensional string worldsheet \( X^\mu(\sigma, \tau) \) \[4\], yields the flat-space Kalb–Ramond action \[5,6,7\],

\[
S = -\mu_0 \int \sqrt{-g} d\sigma d\tau - \frac{1}{6} \int \sqrt{-g} d^4x H^2 - 2\pi f_a \int B_{\alpha\beta} V^\alpha\beta d\sigma d\tau,
\]

where \( H_{\mu\alpha\beta} = \partial_\mu B_{\alpha\beta} + \partial_\beta B_{\mu\alpha} + \partial_\alpha B_{\beta\mu} \) is the field strength of \( B_{\alpha\beta} \), the worldsheet metric is \( \gamma_{ab} = g_{\mu\nu} \partial_a X^\mu \partial_b X^\nu \) with \( \gamma = \det(\gamma_{ab}) \), and \( V_{\alpha\beta} = \epsilon^{ab} \partial_\alpha X_a \partial_b X_\beta \) is the antisymmetric vertex operator.

Varying the action (3) with respect to the worldsheet coordinates and the antisymmetric tensor yields the string equations of motion and the tensor field equations,

\[
\mu_0 \partial_\alpha \left( \sqrt{-\gamma} \gamma^{ab} \partial_b X^\mu \right) = F^\mu = 2\pi f_a H^{\mu\alpha\beta} V_{\alpha\beta},
\]

\[
\partial_\mu H^{\mu\alpha\beta} = 4\pi J^\alpha\beta = 2\pi f_a \int d\sigma d\tau \delta^4(x - X(\sigma, \tau)) V^{\alpha\beta}.
\]

In the Lorentz gauge, \( \partial_\mu B^{\mu\nu} = 0 \), the field equations can be solved using standard Green’s function methods in retarded time,

\[
B_{\alpha\beta}(x) = 2\pi f_a \int d\sigma d\tau D_{\text{ret}}(x - X(\sigma, \tau)) V_{\alpha\beta}(\sigma, \tau),
\]

where \( D_{\text{ret}}(x) = (1/2\pi) \theta(x_0) \delta(x^2) \) is the retarded Green’s function. Defining \( \Delta_\mu = x_\mu - X_\mu(\sigma, \tau) \) and treating \( \bar{\sigma} = \bar{\sigma}(\bar{\tau}) \) one can perform various standard manipulations \[2,8,9\] to obtain the Lienard-Wiechert potential and its derivative

\[
B_{\alpha\beta}(x) = \frac{f_a}{2} \int d\bar{\sigma} \left( \frac{V_{\alpha\beta}}{|\Delta.X|} \right) \bigg|_{\bar{\tau} = \tau_R}, \quad \partial_\mu B_{\alpha\beta}(x) = \frac{f_a}{2} \int d\bar{\sigma} \frac{1}{\Delta.X} \partial_\tau \left( \frac{\Delta_\mu V_{\alpha\beta}}{|\Delta.X|} \right) \bigg|_{\tau = \tau_R},
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some point on the string. As discussed above, this procedure strongly suggests a natural

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The equations of motion (4) are problematic because the field diverges as any point

on the string is approached $x_\mu \rightarrow X_\mu(\sigma, \tau)$, as for the point electron in classical electrodynamics. The electron has a natural scale, the classical electron radius, which allows the renormalisation of the electron mass and the calculation of the first-order approximation to the radiation force (1). In a similar manner for strings, the short distance divergences are cut off by the natural scale of the string width $\delta$. However, the string self-field also diverges at large distances and so we must introduce another arbitrary cut-off scale $\Delta$. Formally, this scale corresponds to the integrals in (5), (6) and (7) running over a region of size $\Delta$ around the point in question, that is, we only consider the effects from neighbouring string points within a distance of $\Delta/2$. Note that we define $\Delta$ in terms of an invariant length of string. Physically, it will correspond to a distance beyond which the long-range string fields become uncorrelated and begin to cancel, so we are essentially approximating the effect of the nearby string with a square or ‘top hat’ window function. Our expectation, then, is that an appropriate choice for $\Delta$ would be near the average curvature radius of the string.

For flat-space string dynamics, the conformal string gauge is usually employed. However, when considering problems in which the string energy decays it is best to use the temporal transverse gauge in which $X_0 = t = \tau$ and $\dot{X}.X' = 0$ with $X^\mu = (t, X)$. The equations of motion for the string (4) and the energy of the string become

$$
\partial_\mu B^{\text{self}}_{\alpha\beta}(x) = \frac{f_a}{4} \int d\bar{\sigma} \left\lfloor \frac{1}{\Delta.\bar{X}} \partial_{\bar{\tau}} \left( \frac{\Delta_\mu V_{\alpha\beta}}{|\Delta.\bar{X}|} \right) \right\rfloor_{\bar{\tau}=\tau_R} + \frac{1}{\Delta.\bar{X}} \partial_{\bar{\tau}} \left( \frac{\Delta_\mu V_{\alpha\beta}}{|\Delta.\bar{X}|} \right) \right\rfloor_{\bar{\tau}=\tau'_R},
$$

where $\Delta^2|_{\bar{\tau}=\tau_R, \tau'_R} = 0$ and $\tau_R < t, \tau'_R > t$.

$$
\partial_\mu B^{\text{rad}}_{\alpha\beta}(x) = \frac{f_a}{4} \int d\bar{\sigma} \left\lfloor \frac{1}{\Delta.\bar{X}} \partial_{\bar{\tau}} \left( \frac{\Delta_\mu V_{\alpha\beta}}{|\Delta.\bar{X}|} \right) \right\rfloor_{\bar{\tau}=\tau_R} - \frac{1}{\Delta.\bar{X}} \partial_{\bar{\tau}} \left( \frac{\Delta_\mu V_{\alpha\beta}}{|\Delta.\bar{X}|} \right) \right\rfloor_{\bar{\tau}=\tau'_R},
$$

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$$
\mu_0 \left( \dot{X} - \frac{1}{\epsilon} \left( \frac{X'}{\epsilon} \right) \right)' = f, \quad \mu_0 \dot{\epsilon} = f^0, \quad E = \mu_0 \int d\sigma \epsilon,
$$

where $\epsilon^2 = \dot{X}^2/(1 - X'^2)$ and $F^\mu = (f^0, f)$.

We will now renormalize the equations of motion (8), though the details of the procedure will be described elsewhere [10] (see also refs. [8,9,11]). For the purposes of this paper, it suffices to note that the expression for the derivatives of the Lienard-Wiechart potentials of the self and radiation fields (7) can be expanded locally about some point on the string. As discussed above, this procedure strongly suggests a natural scale for the otherwise arbitrary renormalisation cut-off $\Delta$ near the average curvature radius of the string. After a detailed set of manipulations which involve use of the gauge
conditions, one can deduce that

\[ H_{\mu\alpha\beta}^{\text{self}} = \frac{f_a}{2X^4} \left[ \frac{1}{\epsilon} \dddot{X}_\rho V_{\alpha\beta} - \frac{1}{\epsilon^3} X''_\rho V_{\alpha\beta} \right] \log(\Delta/\delta) + O(\Delta^2), \]

\[ H_{\mu\alpha\beta}^{\text{rad}} = \frac{f_a}{2X^4} \left[ -\frac{4}{3} \dddot{X}_\rho V_{\alpha\beta} - \frac{1}{2} \dddot{X}_\rho V_{\alpha\beta} + \frac{3\epsilon}{2\epsilon} X''_\rho V_{\alpha\beta} \right] + \left( \frac{2\dot{X} \cdot \dddot{X}}{X^2} - \frac{\epsilon}{2\epsilon} \right) \dddot{X}_\rho V_{\alpha\beta} \Delta + O(\Delta^2), \]

where \( A_{[\mu\alpha\beta]} = A_{\mu\alpha\beta} + A_{\beta\mu\alpha} + A_{\alpha\beta\mu} \). Note that the self-field has no order \( \Delta \) term. Ignoring terms of order \( \Delta^2 \), we can then deduce expressions for the self-force and the radiation backreaction force,

\[ f^{\text{self}} = -2\pi f_a^2 \log(\Delta/\delta) \left( \dddot{X} - \frac{1}{\epsilon} \left( \frac{X'}{\epsilon} \right)' \right), \]

\[ f^{\text{rad}} = -\pi f_a^2 \Delta \left\{ \frac{4}{3} \epsilon \dddot{X} + \left[ 2\epsilon \left( \frac{\dot{X} \cdot \dddot{X}}{1 - \dot{X}^2} \right) + 3\epsilon \right] \dddot{X} - \frac{2}{\epsilon} \left( \frac{X' \cdot \dddot{X}}{1 - \dot{X}^2} \right) X' - \frac{3\epsilon}{\epsilon^2} X'' \right\}, \]

\[ f^{0,\text{self}} = -2\pi f_a^2 \log(\Delta/\delta) \dot{\epsilon}, \]

\[ f^{0,\text{rad}} = \pi f_a^2 \Delta \left\{ \frac{4}{3} \epsilon^2 \left( \frac{\dot{X} \cdot \dddot{X}}{1 - \dot{X}^2} \right) + 2 \left( \frac{X' \cdot \dddot{X}}{1 - \dot{X}^2} \right)^2 + 2\epsilon^2 \left( \frac{\dot{X} \cdot \dddot{X}}{1 - \dot{X}^2} \right)^2 + 3\epsilon \dot{\epsilon} \left( \frac{\dot{X} \cdot \dddot{X}}{1 - \dot{X}^2} \right) - \frac{3\epsilon}{\epsilon \dot{\epsilon}} \left( \frac{\dot{X} \cdot \dddot{X}}{1 - \dot{X}^2} \right) \right\}. \]

The expressions for \( f^{\text{self}} \) and \( f^{0,\text{self}} \), facilitate the well-known renormalisation of the equations of motion (8) and string energy,

\[ \mu(\Delta) \left( \dddot{X} - \frac{1}{\epsilon} \left( \frac{X'}{\epsilon} \right)' \right) = f^{\text{rad}}, \quad \mu(\Delta) \dot{\epsilon} = f^{0,\text{rad}}, \quad E = \mu(\Delta) \int d\sigma \epsilon, \]

with the renormalised string tension \( \mu(\Delta) = \mu_0 + 2\pi f_a^2 \log(\Delta/\delta) \), while the expressions for \( f^{\text{rad}} \) and \( f^{0,\text{rad}} \) represent the finite radiation backreaction force in what we denote as the ‘local backreaction approximation’. The assumptions underlying (11) are that the dominant contribution to the integrals (5), (6) and (7) come from string segments close to the point under consideration and that the long-range fields are uncorrelated beyond the string radius of curvature. We believe these are reasonable assumptions for general physical situations, notably for strings in a realistic interacting network.

3 Modified string equations of motion

The equations of motion for the radiation force (10) and (11) are rather complicated. However, at least for the string solutions considered in ref. [11], that is, closed loops and long periodic strings, we can demonstrate that some of these terms are sub-dominant.
(this has also been verified numerically). In particular, it is possible to approximate $H_{\mu\alpha\beta}^\text{rad}$, $f^\text{rad}$ and $f^0,\text{rad}$ by

\begin{align*}
H_{\mu\alpha\beta}^\text{rad} &\approx -\frac{2f_a\Delta}{3X^4} \bar{X}_{[\mu}V_{\alpha\beta]}, \\
f^\text{rad} &\approx \frac{4\pi f^2_a\Delta}{3} \left[ \bar{X} - \frac{1}{\epsilon} \left( X' \cdot \bar{X} \right) \frac{X'}{1 - \bar{X}^2} \right], \\
f^{0,\text{rad}} &\approx \frac{4\pi f^2_a\Delta}{3} \left[ \frac{\epsilon^2 \bar{X} \cdot \bar{X}}{1 - \bar{X}^2} \right].
\end{align*}

(12)

This simplified form of the equations of motion using (12) still has serious shortcomings because of the presence of the $\bar{X}$ term. The equations have unphysical, exponentially growing or ‘runaway’ solutions which will, for example, plague any potential numerical applications. Furthermore, one would be required to store information at three different timesteps, fundamentally changing the nature of a numerical algorithm. It appears, however, that both these problems can be circumvented by resubstituting the equations of motion, that is, making the approximations $\dddot{X} = X''''/\epsilon^2$ and $\bar{X} = \dot{X}''''/\epsilon^2$ in (12). The equations of motion then acquire an analogue of a viscosity term for which there are only damped solutions.

The reason for the suppression of the exponentially growing solution becomes apparent if we consider simplified one-dimensional equations,

$$
\dddot{X} - X'' = \alpha \dddot{X}, \quad \rightarrow \quad \dddot{X} - X'' \approx \alpha \dddot{X},
$$

(13)

where we have performed the resubstitution assuming that $\alpha$ is small. We now take an approximately periodic solution, $X'' \approx -\Omega^2 X$, and we substitute the ansatz $X \sim e^{\mu t}$.

The solutions for (13) are given respectively by the roots of the following polynomials in $m$,

$$
f(m) = \alpha m^3 - m^2 - \Omega^2, \quad \rightarrow \quad g(m) = -m^2 - \alpha \Omega^2 m - \Omega^2.
$$

(14)

If we rewrite $f(m) = (m^2 + Am + \Omega^2 + B)(\alpha m - C)$, then we see that $A = \alpha \Omega^2 + \mathcal{O}(\alpha^2)$, $B = \mathcal{O}(\alpha^2)$ and $C = 1 + \mathcal{O}(\alpha^2)$. If we ignore terms $\mathcal{O}(\alpha^2)$, then the solutions of $g(m) = 0$ are approximately solutions of $f(m) = 0$. However, the real positive solution of $f(m) = 0$, corresponding to the exponentially growing solution, is not a solution of $g(m) = 0$.

It remains to recast the simplified resubstituted equations of motion into a first-order form accessible to numerical solution. Defining $\bar{\alpha} = X' - \epsilon \bar{X}$ and $\bar{\beta} = X' + \epsilon \bar{X}$, the equations of motion can be rewritten as

\begin{align*}
\mu(\Delta) \left[ \dddot{\alpha} + \left( \frac{\bar{\alpha}}{\epsilon} \right)' \right] &= -\frac{1}{2} \left( f^\text{rad} + f^{0,\text{rad}} \right), \\
\mu(\Delta) \left[ \dddot{\beta} - \left( \frac{\bar{\beta}}{\epsilon} \right)' \right] &= \frac{1}{2} \left( f^\text{rad} + f^{0,\text{rad}} \right), \\
\mu(\Delta) \dot{\epsilon} &= f^{0,\text{rad}},
\end{align*}

(15)

Using the above, one can then evolve string trajectories simply by modifying a total variation non-increasing (TVNI) algorithm [12,13] which has already been well-tested for string network evolution in an expanding universe.
Figure 1: Decay of $\varepsilon$ using the radiative backreaction force (dotted line) and numerical field theory simulations (solid line) for (a) a sinusoidal perturbation, (b) a helicoidal perturbation with unequal left- and right-moving amplitudes, and (c) a pure left moving helicoidal perturbation. Note the excellent quantitative agreement for all three cases.

4 Numerical and analytic comparisons

The power due to the radiation backreaction force can be found by differentiating the expression for the energy in (11)

$$P = -\frac{dE}{dt} = -\frac{4\pi f_a^2 \Delta}{3} \int d\sigma \frac{\varepsilon^2 \dot{X} \cdot \ddot{X}}{1 - X^2}.$$  \hspace{1cm} (16)

For closed loops of length $L$, we take the integration over the range $0 < \sigma < L$. Since the loop oscillates relativistically with a period $T = L/2$, one can estimate $\dot{X} \sim O(1)$, $\ddot{X} \sim O(L^{-2})$. If $\Delta \sim L$, then power per unit length is proportional to $L^{-1}$, which recovers the well known result that the power loss from a loop is independent of its size $L$.

More caution has to be exercised in comparisons with long string configurations because of the possibility of periodicity or other global correlations interfering and suppressing radiation power. The generic result for a long string solution parameterized by its wavelength $L$ and amplitude to wavelength ratio $\varepsilon = 2\pi A/L$, where $A$ is the amplitude, is $\dot{X} \sim O(\varepsilon)$, $\ddot{X} \sim O(\varepsilon L^{-2})$. Therefore the power per unit length is $dP/dl = \beta \varepsilon^2 / L$. The effect of this power loss can be calculated by analogy to the simple backreaction
model presented in ref. [11]; it will lead to an exponential decay of the amplitude and oscillation energy per unit length,

\[
\varepsilon = \varepsilon_0 \exp \left( -\frac{\beta t}{2\alpha \mu L} \right), \quad E/L = \mu + \alpha \mu \varepsilon_0^2 \exp \left( -\frac{\beta t}{\alpha \mu L} \right). \tag{17}
\]

where \(\alpha, \beta\) are constants dependent on the string configuration, defined as in ref. [11].

This result is in agreement with the exponential decay already shown to occur for general configurations with unequal left- and right-moving modes [11]. However, the periodic solutions discussed in ref. [14,11] were shown to have a weaker power law fall-off, \(dP/dl = \beta \varepsilon^4/L\), and some pure left-moving (or right-moving) solutions are known to propagate indefinitely along a straight string solution with no decay [15]. The resolution of this apparent contradiction lies in noticing that the renormalization procedure ignores any global correlations of the long-range fields outside the cut-off scale \(\Delta\). For example, the non-radiating left-moving solution has an accompanying left-moving perturbation in the long-range Goldstone field which is (unphysically) correlated out to infinity [15]. Accordingly, if we suppress the Goldstone field correlations at the curvature radius of the string perturbations, then we should recover the exponential decay anticipated in (17).

We have extensively tested our ‘local backreaction approximation’, using the modified Nambu equations of motion (15), by comparing with field theory simulations of radiating strings in the Goldstone model (2) (see ref. [11]). Here, we shall only briefly report on the remarkable correspondence of the radiative decay results, leaving further details for a longer publication [10]. Note that in the field theory simulations we evolve initial trajectories for which Goldstone field correlations are suppressed by gaussian smoothing for transverse distances greater than the string curvature radius. The massless field of a perturbed string approaches that of a straight string at large distances from the core, as we would expect in a general physical context for random string small-scale structure.

Fig. 1 illustrates the excellent quantitative agreement for three different long string configurations, including a standing sinusoidal perturbation, a helicoidal perturbation with unequal left- and right-moving amplitudes, and a pure left-moving helicoidal perturbation. In all three cases, the decay is seen to be exponential. The agreement persists for the longest time for the (generic) unequal left- and right-moving configuration because exponential decay is predicted even after field correlations have relaxed at large distances. By comparison to the simple backreaction model for exponential decay (17) one can use the numerical field theory results to estimate \(\Delta \approx 0.1L\). We had anticipated that \(\Delta\) should be normalized to a distance near the string radius of curvature \(R\), which for a sinusoidal perturbation is \(R \approx L/4\). The fact that \(\Delta\) is smaller than \(R\) validates taking only the lowest order terms in the expansion (10) for the radiative backreaction force.

Fig. 2 compares the evolution of a sharp kink in the local backreaction approximation with a field theory simulation. The results are almost indistinguishable except for the computational advantages of the former which, in this case, saved a factor of \(10^2\) in cpu time and \(10^4\) in allocated memory. Backreaction leads to a substantial rounding of the kink, in agreement with the intuitive picture described in ref. [11]. A more thorough investigation of this kink rounding, including spectral analysis, will be presented in ref. [10].
Figure 2: Decay of kink perturbation ($\varepsilon_0=0.9$) for using (a) the radiation backreaction force and (b) numerical field theory. Notice the visible rounding of the kink in both cases.

5 Discussion and conclusions

The local backreaction approximation which we have formulated appears to provide a good description of radiative damping in a realistic context, notably that of an evolving string network with uncorrelated left- and right-movers. In physical situations when the long-range string fields are only correlated out to distances comparable with the string curvature radius, numerical simulations confirmed the expected rate of exponential decay. This approach also appears to satisfactorily describe loop decay. Of course, our local approximation does not apply for solutions with global correlations in the long-range fields, but our expectation is that such solutions are not of physical relevance. These methods can also be applied to gravitational and electromagnetic backreaction which we will discuss, along with more detailed tests, in a longer publication on this subject [10].

These methods should prove useful for phenomenologically describing radiative effects in simulations of string networks. The inclusion of such backreaction terms may make tractable a number of outstanding cosmic string problems, enabling us to understand the true nature and existence of the ‘scaling’ solution, as well as the amplitude and spectrum of axion and gravitational wave backgrounds produced by radiating strings. Reformulated, this basic approach to describing string radiation might also find application in non-relativistic physical contexts, such as describing the radiation sound waves by vortex-lines in condensed matter systems.
Acknowledgments

We are grateful for helpful discussions with Brandon Carter, Atish Dabholkar, Sun-Hong Rhie, David Bennett and Alex Vilenkin. The algorithm used for flat space Nambu string evolution was developed by EPS in a collaborative project with Bruce Allen (see ref. [13]). We both acknowledge the support of the Science and Engineering Research Council, in particular the Cambridge Relativity rolling grant (GR/H71550) and Computational Science Initiative grants (GR/H67652 & GR/H57585).

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