Hysteresis Modeling and Synchronization of a Class of RC-OTA Hysteretic-Jounce-Chaotic Oscillators

Leonardo Acho

CoDAlab, Departament de Matemàtica Aplicada III, Escola Universitària d’Enginyeria Tècnica Industrial de Barcelona, Universitat Politècnica de Catalunya, Comte d’Urgell, 187, 08036 Barcelona, Spain

*Corresponding Author: leonardo.acho@upc.edu

Abstract

A class of RC-OTA hysteretic-chaotic oscillators has been previously reported using electronics; therefore, hysteresis is realized by an electronic circuit. To obtain a mathematical model of this RC-OTA chaotic-electronic device, hysteresis modeling turns an important issue. Here, we develop a new mathematical hysteretic model proposing a new jounce-chaotic oscillator. Chaosity test is proved using Poincaré theory. After that, a synchronization scheme is granted to synchronize our new jounce-chaotic oscillator (the transmitter) to a dynamics second-order system (the receiver).

Keywords
Chaos, Synchronization, RC-OTA oscillator

1 Introduction

During recent years, many chaotic oscillators have been proposed. Some of them are based on hysteresis feedback ([11], [9]). However, these oscillators have been designed from electronic point of view. Nevertheless, hysteresis modeling is an important issue in mechanical and structural systems ([7]). Some hysteresis models are developed invoking physical laws. Meanwhile, others are heuristic ones. Moreover, [8] reported chaotic behavior in structures with hysteresis, in which hysteresis is governed by the well-known Bouc-Wen model. However, this Bouc-Wen model is not appropriate for the class of RC-OTA chaotic oscillators because it has more parameters than needed. Following this line of engineering modeling, we propose a new dynamic-hysteretic model which is able to capture hysteresis behavior for a class of RC-OTA hysteretic-chaotic oscillators designed, for instance, in [9] (where a pioneer study is given in [2]).

In mechanics, a jerk function is the time derivative of acceleration which, actually, is a third-order dynamic system. According to [10], some forms of jerk functions present chaos. And the time derivative of a jerk function might be called jounce ([10]), which is a fourth-order dynamic system. Using our dynamic-hysteretic model, and inspired by the RC-OTA architecture, we propose a fourth-order chaotic system named jounce-chaotic oscillator, and whose chaosity test is realized thought Poincaré theory.

By the other hand, chaos synchronization has gained an important attention among scientist (see, for instance, [1], [4], and [13], among others). This because some fields of engineering issues, e.g. secure communications, use chaos synchronization ([13], [1], and [3]). So far, many of the synchronization systems developed are granted to achieve chaos synchronization between two identical chaotic systems. The case of two different systems seems not to be complete developed ([13]). We also present a synchronization scheme where our jounce-chaotic system is synchronized with a second order system.

The structure of the paper is as follows. Section two presents a review of the simple mathematical model of a class of RC-OTA hysteretic-chaotic oscillators. Our new dynamic-hysteretic model is presented in Section three together with numerical experiments. Our new jounce-chaotic system is commented too. In Section four, our synchronization design is granted. Finally, Section five presents the conclusions.

2 RC-OTA hysteretic chaotic systems

According to [9], a dimensionless dynamic model of a class of RC-OTA hysteretic-chaotic oscillator is given by:

\[ \ddot{x} - 2\delta \dot{x} + x = ph(x), \]
where $\delta$ and $p$ are the system parameters. The hysteresis function $h(x)$ is shown in Fig. 1. This system presents chaos with $\delta = 0.05$, and $p = 1$ ([9]).

**Figure 1.** Normalized hysteretic function.

### 3 Hysteresis modeling and a jounce-chaotic system

Hysteresis behavior is recognized as a system with memory. One way to capture hysteresis is by using a dynamic system. For instance, the new hysteretic system:

$$\dot{z} = \alpha(-z + b\text{sgn}(x + a\text{sgn}(z))),$$

(2)

can reproduce the hysteretic behavior shown in Fig. 2, where $a$ and $b$ are the hysteresis curve parameters. The speed transition between $b$ and $-b$ is governed by the positive parameter $\alpha$; $\text{sgn}(\cdot)$ is the signum function. For instance, if $a = b = 1$ and $\alpha = 10$, the system (2) is:

$$\dot{z} = 10(-z + \text{sgn}(x + \text{sgn}(z))).$$

(3)

Using $x = 10\sin(t)$ and $z(0) = 0$, the simulation result of system (3) is shown in Fig. 3.

Next, we program a chaotic oscillator equivalent to (1):

$$\ddot{x} = -0.1\dot{x} + x = z$$

$$\dot{z} = 10(-z + \text{sgn}(x + \text{sgn}(z))).$$

Fig. 4 shows the simulation result. The obtained chaotic attractor is the same as that shown in [9, Fig. 6].

**Figure 2.** Hysteretic behavior.

**Figure 3.** Simulation result.

**Figure 4.** Chaotic attractor with initial conditions $x(0) = \dot{x}(0) = 0$ and $z(0) = 0.1$.

### 4 Synchronization

Let us introduce the following system:

$$\ddot{y}_1 = y_2,$$

(8)

$$\ddot{y}_2 = z_1 + z_2 + 0.2x_2 - y_1 - 0.1y_2,$$

(9)
where $z_1 := z_1(t)$, $z_2 := z_2(t)$, and $x_2 := x_2(t)$ arrive from the jounce-chaotic system (4)-(7). At this point, the jounce-chaotic system (4)-(7) represents the transmitter and the system (8)-(9) the receiver. Fig 10 gives a schematic representation of our the synchronization design.

It is said that the receiver is synchronized with the transmitter if $y_1(t)$ converges to $x_1(t)$ and $y_2(t)$ converges to $x_2(t)$ as time goes on (and for any initial conditions $y_1(0)$, $x_1(0)$, $y_2(0)$, and $x_2(0)$). This fact can be proved as follows. Consider the signal errors given by

$$
e_1 = x_1 - y_1, \quad e_2 = x_2 - y_2.$$  

Then, after some basic manipulations, we obtain:

$$\dot{e}_1 = e_2, \quad \dot{e}_2 = -e_1 - 0.1e_2.$$  

The above system represents an exponential stable dynamics. Simulation results are shown in Fig. 11.

**Remark 1** On synchronization of chaotic systems, it is used to test the synchronization performance by adding a noisy signal to the lines of the channel communication. This noisy signal is a kind of common noise because it is induced simultaneously on each communication line ([5]). But, from the technological point of view, common noisy signals are easily to remove via Instrumentation Amplifiers. For instance, according to ([5], page 85), the common noise induced

1Note that signals $z_1(t)$ and $z_2(t)$ are bounded for all $t \geq 0$.  

---

**Figure 5.** Jounce-chaotic attractor with initial conditions $x_1(0) = x_2(0) = z_1(0) = 0$ and $z_2(0) = 0.1$.

**Figure 8.** Simulation results: $x_1(t)$ and $x_2(t)$ versus time.

**Figure 9.** High sensitivity test on the initial conditions: blue line with $x_1(0) = 0$ and red line with $x_1(0) = 0.001$.

**Figure 10.** Synchronization block diagram.
on the communication lines can be reduced 140 dB; i.e., it can be attenuate by a factor of $10^{14}$. By the other hand, employing fiber optics, noisy signals are practically unnoticed on our communication system.

Remark 2 The communication scheme shown in Fig. 10 requires three lines of communication. However, it is possible to implement a multiplexing communication technique to drive a single line of communication (Website http://en.wikipedia.org/wiki/Multiplexing).

5 Conclusions

A new hysteretic-jounce-chaotic system has been designed along with a synchronization scheme. According to numerical experiments, chaos synchronization between two different systems can be achieved. This fact was theoretically proved too.

Acknowledgements

This work was supported by grant number DPI2012-32375/FEDER from the Spanish Ministry of Economics and Competitiveness.

REFERENCES

[1] L. Acho, Expanded Lorenz systems and chaotic secure communication systems design, J. of Circuits, Systems, and Computers, Vol. 15, No. 4, 607–614, 2006.
[2] L. Acho, and Y. Vidal, Hysteresis modeling of a class of RC-OTA hysteretic-chaotic generators, 4th International Conference on Physics and Control (PhysCon), León, Spain, 2011.
[3] T.-L. Carroll, and L. M. Pecora, Synchronization chaotic circuits, IEEE Trans. on Circ. Syst., Vol. 38, 453–456, 1991.
[4] G. Chen, and X. Dong, From chaos to order: methodologies, perspectives and applications, World Scientific Pu., Singapore, 1998.
[5] S. Franco, Design with Operational Amplifiers and Analog Integrated Circuits, McGraw-Hill, N.Y., USA, Third Edition, 2002.
[6] H. Fengling, L. Jinhu, Y. C. Xinghuo, and C. Guanrong, Dynamical behaviours of a 3D hysteresis-based system, Chaos, Solitons and Fractals, Vol. 28, 182-192, 2006.
[7] M. Ismail, F. Ikhouane, and J. Rodellar, The hysteresis Bouc-Wen model, a Survey, Arch. Comput. Methods Eng., Vol. 16, 161–188, 2009.
[8] H. G. Li, and G. Meng, Nonlinear dynamics of a SDOF oscillator with BoucWen hysteresis, Chaos, Solitons and Fractals, Vol. 34, 337-343, 2007.
[9] S. Nakagawa, and T. Saito, An RC-OTA hysteresis chaos generator, IEEE Trans. on Circuits and Systems-1: Fundamental Theory and Applications, Vol. 43, No. 12, 1019–1021, 1996.
[10] J. C. Sprott, Some simple chaotic jerk functions, Am. J. Phys., Vol. 65, No. 537, 537–543, 1997.
[11] M. Storace, and M- Parodi. Simple realisation of hysteresis chaos generator, Electronics Letters, Vol. 34, No. 1, 10–11, 1998.
[12] J. J.Thomsen, Vibrations and Stability: Advanced theory, analysis, and tools, Springer, Berlin, Germany, Second Edition, 2003.
[13] H. T. Yau, and J. J. Yan, Chaos synchronization of different chaotic systems subjected to input nonlinearity, Applied Mathematics and Computation, Vol. 197, 775–788, 2008.