Collective excitation of quantum wires and effect of spin-orbit coupling in the presence of a magnetic field along the wire

Hyun C. Lee

Department of Physics and Basic Science Research Institute, Sogang University, Seoul, Korea

S. -R. Eric Yang∗

Department of Physics, Korea University, Seoul, Korea and School of Physics, Korea Institute for Advanced Study, Seoul, Korea

(Dated: June 19, 2018)

The band structure of a quantum wire with the Rashba spin-orbit coupling develops a pseudogap in the presence of a magnetic field along the wire. In such a system spin mixing at the Fermi wavevectors $-k_F$ and $k_F$ can be different. We have investigated theoretically the collective mode of this system, and found that the velocity of this collective excitation depends sensitively on the strength of the Rashba spin-orbit interaction and magnetic field. Our result suggests that the strength of the spin-orbit interaction can be determined from the measurement of the velocity.

PACS numbers: 73.21.Hb, 71.10.Pm, 72.10-d, 73.21.-b

Keywords: spintronics, Rashba spin-orbit interaction, Luttinger liquid

I. INTRODUCTION

Recently active research is taking place on how to manipulate spin properties of single electrons, and several semiconductor spin devices based on spin-orbit coupling have been proposed.1,2 Among these we focus on a spin filter3 proposed by Štěrda and Šeba. They proposed a spin filter combining strong Rashba spin-orbit interaction (SOI) and the magnetic field parallel to a quantum wire (see Fig. 1). This system has an interesting one-dimensional band structure, (see Fig. 2): a pseudogap is present at zero wavenumber and the orientation of electron spin depends on the wavevector. For the lower band the electron with sufficiently negative $k$ is mostly polarized in the $+z$ direction while that of sufficiently positive $k$ is mostly polarized in the $-z$ direction. When the Fermi energy lies in the pseudogap, the transmission/reflection coefficients of such a wire in the presence of a step potential has been calculated in the presence of electron-electron interaction using poor man’s renormalization group approach.3

The dispersion relation of this mode is

$$
\omega = \left[ v_\theta(q)v_\phi(q) \right]^{1/2} q = v_\phi q.
$$

(1)

$v_\theta(q)$ and $v_\phi(q)$ are defined as follows:

$$
v_\theta(q) = v_F \left( 1 + \frac{V_q}{\pi v_F} - \frac{gV_{2k_F}}{2\pi v_F} \right),
$$

$$
v_\phi(q) = v_F \left( 1 + \frac{gV_{2k_F}}{2\pi v_F} \right),
$$

(2)

where $v_F$ is the Fermi velocity and the renormalization factor of the strength of backscattering is

$$
g = \frac{e^2}{\epsilon_Z^2 + (\eta R_k F)^2}.
$$

3

FIG. 1: The geometry of a quantum wire with a magnetic field along the wire.

*corresponding author, eyang@venus.korea.ac.kr
θ and φ are the phase fields which are basically linear combinations of density operators η_R/L and they are defined in Eq. \[33\]. \(V_q\) is the interaction matrix element, \(\epsilon_z\) is the magnitude of Zeeman coupling, and \(\eta_R\) is a parameter characterizing Rashba SOI (see below). From the expression of \(g\) [Eq. \[33\]] we see that the velocity of this collective excitation depends sensitively on the Rashba SOI and magnetic field. This result differs from the that of an ordinary Luttinger liquid in that the back scattering term \(V_{2k_F}\) is renormalized by a factor \(g\). The physical origin of this factor reflects the different spin mixing of single particle states near the Fermi wavevectors, which are coupled by backscattering.

The presence of the renormalization factor \(g\) may be exploited to determine the constant \(\eta_R\). There is no simple way to calculate \(\eta_R\) because it depends both on the electric field inside the semiconductor heterostructure and on the detailed boundary conditions at the interface. Instead these spin-orbit coupling constants were measured by electric, optical, and photoelectrical means.\[11,12,13,14\] We suggest that the measurement of the velocity of the collective excitation \(v_0\) may provide another way to determine the value of \(\eta_R\). This measurement can be carried out using tunneling between two parallel wires in the presence of an additional magnetic field \(B_g = \nabla \times \mathbf{A}_f\) along the y-axis.\[15,16\] This method allows one to determine the spectrum of elementary excitations\[17,18\] of momenta much larger than \(2k_F\).\[19\]

This paper is organized as follows. In Sec. II we introduce our model and review the results obtained by Středa and Šeba for the non-interacting case. In Sec. III we incorporate the electron-electron interaction and obtain an effective Hamiltonian for the system. In Sec. IV the dispersion of collective excitation is computed based on the effective action obtained in Sec. III. Sec. V we discuss how our result for the velocity differs from the results of ordinary Luttinger liquids. An experiment is proposed to measure \(\eta_R\).

### II. MODEL FOR SINGLE PARTICLE HAMILTONIAN

In our model confinement potentials are present along the y- and z-axis and quasi-one-dimensional motion of electrons is possible along the x-axis. The widths of the wavefunction along both the y- and z-axis are assumed to be negligible. The lowest subband energies along the y- and z-axis are denoted by \(E_y\) and \(E_z\). A magnetic field parallel to the quantum wire along x-axis is present \(B = -B\mathbf{x}\). The corresponding vector potential can be chosen to be \(\mathbf{A} = -B\mathbf{y}\). \(B > 0\). In our model Rashba electric field is applied along the y-axis (see Fig. 1), and is given by \(\mathbf{E} = +E_0\mathbf{y}\), \(E_0 > 0\). The Rashba spin-orbit interaction\[20,21\] then takes the form

\[
\mathcal{H}_R = \eta_R \left( k_x \sigma_z - k_z \sigma_x \right),
\]

where \(\eta_R = |e| \hbar^2 E_0 / 4 m^* \epsilon_z^2 > 0\). The strength of Rashba SOI can be controlled by changing electric field.\[22,23\]

Note that in quantum wires with electron propagating along the x-axis \(k_y\) and \(k_z\) must be replaced by dynamical momentum operators. The expectation value of \(k_y\), \(k_z + eA_z / \hbar c\) with respect to the lowest subband state wave function of transverse degrees of freedom \((y, z)\) vanish by symmetry considerations.

\[
\mathcal{H}_R = \eta_R k_x \sigma_z.
\]

The bulk Hamiltonian of Dresselhaus SOI is given by

\[
\mathcal{H}_{\text{bulk,D}} = \gamma_c \left[ \sigma_y k_x (k_y^2 - k_x^2) + \sigma_y k_y (k_z^2 - k_x^2) + \sigma_x k_z (k_x^2 - k_y^2) \right].
\]

To obtain the effective Hamiltonian of quantum wire we have to take the average of the above bulk Hamiltonian.
with respect to the ground state wave function of transverse \((y, z)\) degrees of freedom. In our geometry the Rashba electric field is applied in y-direction, and the lateral confining potential enforcing quasi one-dimensional motion is applied in z-direction. Clearly \(\langle k_y \rangle = 0\) since the subband wavefunction along the z-axis has even parity. The subband wavefunction along the y-axis is a real function and therefore the expectation value \(\langle k_y \rangle = 0\), too. But we have to note that \(\langle y \rangle \neq 0\) since the inversion symmetry is lacking in the y-direction. The effective Hamiltonian for quantum wire is then

\[
H_D = \gamma_c \sigma_x k_x (\langle k_y^2 \rangle - \langle k_z^2 \rangle) = \eta_D \sigma_x k_x, \tag{7}
\]

where \(\eta_D = \gamma_c (\langle k_y^2 \rangle - \langle k_z^2 \rangle)\).

Now the one-particle Hamiltonian becomes

\[
H_1 = E_y + E_z + \frac{\hbar^2 k^2}{2m^*} + \eta_R k \sigma_z + \eta_D k \sigma_x - E_Z \sigma_z. \tag{8}
\]

The Dresselhaus term can be absorbed into the Zeeman term \(E_Z = g_0 \mu_B B/2\) \((g_0 \approx 15\) for InAs) in the following way.

\[
\epsilon_Z \equiv E_Z - \eta_D k. \tag{9}
\]

For the sake of completeness we include the Dresselhaus term in the calculation of the band structure. Later we will ignore it in the bosonization procedure. By the diagonalization of the Hamiltonian Eq. (8) the energy eigenvalues and the corresponding normalized eigenvectors are obtained as follows: For the lower band the eigenvalue is \((E_y, E_z)\) put to zero

\[
E_-(k) = \frac{\hbar^2 k^2}{2m^*} - \sqrt{\epsilon_Z^2 + \eta_R^2 k^2} \tag{10}
\]

and the eigenvector is

\[
\xi_- = \begin{pmatrix} u_k^- \\ v_k^-
\end{pmatrix}, \tag{11}
\]

where

\[
u_k^\pm = \frac{e^\mp \epsilon_Z}{\sqrt{\eta_R k + D}^2 + \epsilon_Z^2}, \tag{12}
\]

\[
u_k^\pm = \frac{\eta_R k + D}{\sqrt{\eta_R k + D}^2 + \epsilon_Z^2}. \tag{13}
\]

Here

\[
D \equiv \sqrt{\eta_R k)^2 + \epsilon_Z^2}. \tag{15}
\]

\(u_k^-\) and \(v_k^-\) represents the amplitudes for the spin to point in the \(+z\) and \(-z\) direction, respectively. For the upper band the results are given in Ref. 24.

Quantum wires can be tailor made so that the quantities \(\langle k_y^2 \rangle\) and \(\langle k_z^2 \rangle\) are almost equal. If we assume the harmonic confining potential \(m^* \omega_0^2 z^2/2\) along the z-axis we have \(\langle k_z^2 \rangle = m^* \omega_0^2 / 2\hbar\). For the y-direction the constant Rashba electric field is acting so that the potential is linearly rising. In this case \(\langle k_y^2 \rangle \sim 0.8 \left( \frac{2m^* |E_D|}{\hbar^2} \right)^{2/3}\).

The condition \(\langle k_y^2 \rangle = \langle k_z^2 \rangle\) is satisfied when the value of the electric field is given by \(eE_0 z_0 = 0.49 \frac{\hbar^2}{2m^* \omega_0}\), where \(z_0 = \sqrt{\hbar/m^* \omega_0}\). For this particular value of the electric field the Dresselhaus term is negligible compared to the Zeeman energy and the Rashba coupling. Note that \(E_- (k)\) becomes an even function of \(k\) in this case. Hereafter we assume this. If the Rashba coupling becomes sufficiently strong such that

\[
\eta_R^2 \gtrsim \epsilon_Z \hbar^2 / m^* \tag{16}
\]

then the energy spectrum develops a double minimum at

\[
k = \pm \frac{1}{\eta_R} \left( \frac{m^* \eta_R^2}{\hbar^2} \right)^{1/2}. \tag{17}
\]

The energy at the minimum is given by

\[
E_{\text{min}} = - \frac{m^* \eta_R^2}{2 \hbar^2} - \frac{\hbar^2 \epsilon_Z}{2 m^* \eta_R}. \tag{18}
\]

In such a case \(E_- (0) > E_{\text{min}}\). In our work we assume that \(E_- (0) - E_{\text{min}} = - \epsilon_Z - E_{\text{min}}\) is less than the Fermi energy so that there are only two Fermi wavevectors.

### III. MODEL FOR MANY-BODY HAMILTONIAN

Let \(a_k\) and \(b_k\) be the quasiparticle operators corresponding to \(E_- (k)\) and \(E_+ (k)\), respectively. They can be explicitly expressed in terms of electron operators as follows:

\[
b_k = \epsilon_k u_k^\dagger + c_k^\dagger v_k^+, \quad a_k = \epsilon_k u_k^- + c_k v_k^+, \quad c_k^\dagger = b_k u_k^\dagger + a_k^\dagger v_k^-,
\]

\[
\xi_+ = \begin{pmatrix} u_k^\dagger \\ v_k^\dagger
\end{pmatrix}, \tag{11}
\]

\[
u_k^\dagger = \frac{\epsilon_k}{\sqrt{\eta_R k + D}^2 + \epsilon_Z^2}, \tag{12}
\]

\[
u_k^\dagger = \frac{\eta_R k + D}{\sqrt{\eta_R k + D}^2 + \epsilon_Z^2}. \tag{13}
\]

Here

\[
D \equiv \sqrt{\eta_R k)^2 + \epsilon_Z^2}. \tag{15}
\]

\(u_k^-\) and \(v_k^-\) represents the amplitudes for the spin to point in the \(+z\) and \(-z\) direction, respectively. For the upper band the results are given in Ref. 24.

Quantum wires can be tailor made so that the quantities \(\langle k_y^2 \rangle\) and \(\langle k_z^2 \rangle\) are almost equal. If we assume the harmonic confining potential \(m^* \omega_0^2 z^2/2\) along the z-axis we have \(\langle k_z^2 \rangle = m^* \omega_0 / 2\hbar\). For the y-direction the constant Rashba electric field is acting so that the potential is linearly rising. In this case \(\langle k_y^2 \rangle \sim 0.8 \left( \frac{2m^* |E_D|}{\hbar^2} \right)^{2/3}\).

The condition \(\langle k_y^2 \rangle = \langle k_z^2 \rangle\) is satisfied when the value of the electric field is given by \(eE_0 z_0 = 0.49 \frac{\hbar^2}{2m^* \omega_0}\), where \(z_0 = \sqrt{\hbar/m^* \omega_0}\). For this particular value of the electric field the Dresselhaus term is negligible compared to the Zeeman energy and the Rashba coupling. Note that \(E_- (k)\) becomes an even function of \(k\) in this case. Hereafter we assume this. If the Rashba coupling becomes sufficiently strong such that

\[
\eta_R^2 \gtrsim \epsilon_Z \hbar^2 / m^* \tag{16}
\]

then the energy spectrum develops a double minimum at

\[
k = \pm \frac{1}{\eta_R} \left( \frac{m^* \eta_R^2}{\hbar^2} \right)^{1/2}. \tag{17}
\]

The energy at the minimum is given by

\[
E_{\text{min}} = - \frac{m^* \eta_R^2}{2 \hbar^2} - \frac{\hbar^2 \epsilon_Z}{2 m^* \eta_R}. \tag{18}
\]

In such a case \(E_- (0) > E_{\text{min}}\). In our work we assume that \(E_- (0) - E_{\text{min}} = - \epsilon_Z - E_{\text{min}}\) is less than the Fermi energy so that there are only two Fermi wavevectors.
$K_0$ is the modified Bessel function and $w$ is the cutoff length scale which is the order of the width of the quantum wire. $\epsilon$ is the bulk dielectric constant. For the short range interaction such as screened Coulomb interaction the matrix element $V_q$ can be taken to be independent of the momentum transfer $q$. Projecting the Hamiltonian Eq. (21) to the $a$-band with the use of Eq. (20), we obtain

\[
\mathcal{H}_{\text{int}} = \frac{1}{2} \sum_{k_1, k_2, q} \langle k_1, k_2 | \hat{V} | k_1 - q, k_2 + q \rangle \times a_{k_1}^\dagger a_{k_2}^\dagger a_{k_2+q} a_{k_1-q}, \tag{23}
\]

where

\[
\langle k_1, k_2 | \hat{V} | k_1 - q, k_2 + q \rangle = V_p [\xi^\dagger (k_1) \xi_- (k_1 - q)] [\xi^\dagger (k_2) \xi_- (k_2 + q)] \tag{24}
\]

is the projected interaction matrix element in the low energy Hilbert space. The explicit expression of eigenvector $\xi^-$ is given by Eqs. (11,12).

At low energy, only the electron states near $-k_F$ and $k_F$ Fermi points need to be considered. Following the usual procedures of g-ology and bosonization method, we can express the interaction Hamiltonian Eq. (23) within g-ology scheme. Forward scattering $g_2$ and $g_4$ process. Backscattering $g_1$ process. We note further that for fermions of a single species (like $a$-quasiparticle here) $g_1$ process is identical with $g_2$ process. In this paper a commensurate filling is not considered, so that the Umklapp processes ($g_3$) can be neglected. From now on we will call electrons with $k < 0$ ($k > 0$) left (right) movers.

$g_4$ process: For instance let us assume that all four momenta $k_1, k_2, k_2+q, k_1-q$ are located near the right Fermi point. The dotted line indicates the matrix element $V_q$. See text for details.

![FIG. 3: A Feynman diagram of the $g_4$ process. All four momenta $k_1, k_2, k_2+q, k_1-q$ are located near the right Fermi point. The dotted line indicates the matrix element $V_q$. See text for details.](image)

\[
\frac{\mathcal{H}_g}{\mathcal{H}_{g_k}} = \frac{1}{N} \sum_{|q| < \Lambda} V_q \rho_R(q) \rho_L(-q) \tag{26}
\]

$g_{1,\parallel}$ processes: In this case the matrix elements play crucial role as can be seen in

\[
[\xi^\dagger (k_1) \xi_- (k_1 - q)] [\xi^\dagger (k_2) \xi_- (k_2 + q)] \approx [\xi^\dagger (-k_F) \xi_- (k_F + q)]
\]

According to the same reason as $g_4$ interaction one can make the approximations in Fig. 4.

![FIG. 4: A Feynman diagrams of the $g_2$ process. See text for details. There is another $g_2$ Feynman diagram in which $R \leftrightarrow L$.](image)
Evidently the dominant momentum transfer \( q \) must be \( 2k_F \). By changing the order of operators (thereby changing overall sign of interaction) and by summing over momenta one arrives at

\[
H_{g_1} = -g V_{2k_F} \int dx \rho_R(x) \rho_L(x),
\]

where

\[
g = \frac{[\xi_\uparrow^\dagger (-k_F) \xi_\downarrow^\dagger (+k_F)] [\xi_\uparrow^\dagger (+k_F) \xi_\downarrow^\dagger (-k_F)]}{\epsilon^2_F + (\eta_R k_F)^2}.
\]

As can be seen from Eq. (28) the coupling constant \( g \) depend on the applied magnetic field and the Rashba SOI as well as Fermi momentum.

We observe that \( g_2 \) Hamiltonian Eq. (26) and \( g_1 \) Hamiltonian Eq. (28) can be combined completely. This is a special feature of fermions of single species. In the presence of other degrees of freedom such as spin a backscattering term \((g_{1,\perp})\) appears which is not of the Luttinger interaction form.

\[
\begin{align*}
\text{FIG. 5: A Feynman diagram of the } g_{1,\parallel} \text{ processes. There exists one more diagram where } R &\leftrightarrow L. \text{ See text.}
\end{align*}
\]

\[\begin{align*}
\text{IV. BOSONIZATION AND COLLECTIVE EXCITATIONS}
\end{align*}\]

The total effective Hamiltonian incorporating interaction is given by

\[
H = H^{(0)} + H_{g_4} + H_{g_2} + H_{g_1}.
\]

The linearized non-interacting Hamiltonian \( H^{(0)} \) is

\[
H^{(0)} = \sum_p \left[ v_F \psi_{R}^\dagger (p) \psi_R (p) - v_F \psi_{L}^\dagger (p) \psi_L (p) \right].
\]

The Hamiltonian Eq. (30) can be bosonized straightforwardly:

\[
\begin{align*}
\mathcal{H} &= \pi v_F \int dx \left[ : \rho_R^2 (x) : + : \rho_L^2 (x) : \right] \\
&\quad + \frac{1}{2N} \sum_q V_q \left[ \rho_R (q) \rho_R (-q) + \rho_L (-q) \rho_L (q) \right] \\
&\quad + \frac{1}{N} \sum_q \left[ V_q - g V_{2k_F} \right] \rho_R (q) \rho_L (-q).
\end{align*}
\]

\[\begin{align*}
\phi(x) &= \frac{1}{2} \left[ \phi_R (x) + \phi_L (x) \right], \\
\phi(x) &= \frac{1}{2} \left[ \phi_R (x) - \phi_L (x) \right],
\end{align*}\]

where \( \rho_{R/L} (x) = \frac{1}{2\pi} \partial_x \phi_{R/L} (x) \). In terms of phase fields

\[
\begin{align*}
\mathcal{H} &= \frac{v_F}{2\pi} \int dx \left[ (\partial_x \theta)^2 + (\partial_x \phi)^2 \right] \\
&\quad + \frac{1}{N} \sum_q V_q q^2 \left[ (\theta (q) \theta (-q) + \phi (q) \phi (-q)) \right] \\
&\quad + \left( \frac{1}{2\pi} \right)^2 \frac{1}{N} \sum_q \left[ (V_q - g V_{2k_F}) q^2 \left[ (\theta (q) \theta (-q) - \phi (q) \phi (-q)) \right] \right] \\
&\quad + \frac{v_F}{2\pi} \int dx \left[ (\partial_x \theta)^2 + (\partial_x \phi)^2 \right] \\
&\quad + \frac{1}{(2\pi)^2 N} \sum_q \left[ 2(V_q - g V_{2k_F} / 2) q^2 (\theta (q) \theta (-q) - \phi (q) \phi (-q)) \right] \\
&\quad + g V_{2k_F} q^2 (\phi (q) \phi (-q))
\end{align*}
\]

The Euclidean action is given by

\[
S[\theta, \phi] = \int d\tau \left[ \int dx \frac{i}{\pi} \partial_x \phi \partial_x \theta + H \right].
\]

In matrix form the above can be written as

\[
S = \frac{1}{2\pi} \int d\omega dq \left[ \theta (-q, -\omega) \phi (-q, -\omega) \right] \\
\times \left[ v_\theta (q) q^2 \left( v_\phi (q) q^2 \right) \theta (q, \omega) \right] \\
\times \left[ v_\phi (q) q^2 \left( v_\phi (q) q^2 \right) \phi (q, \omega) \right],
\]

where

\[
\begin{align*}
v_\theta &= v_\theta (q) = v_F \left( 1 + \frac{V_q}{\pi v_F} - g \frac{V_{2k_F}}{2\pi v_F} \right), \\
v_\phi &= v_\phi (q) = v_F \left( 1 + \frac{g V_{2k_F}}{2\pi v_F} \right).
\end{align*}
\]

\[\begin{align*}
\theta \text{ and } \phi \text{ are the phase fields which are basically linear combination of density operators } \rho_{R/L} \text{ and they are defined in Eq. (33).}
\end{align*}\]
The dispersion relation of the collective excitation can be obtained from the kernel of action Eq. (36).

$$\det \left[ v_\theta(q)q^2 \begin{pmatrix} i\omega \\ i\omega \end{pmatrix} v_\phi(q)q^2 \right] = 0$$ (37)

After analytic continuation $i\omega \to \omega$ we find

$$\omega = \left[ v_\theta(q)v_\phi(q) \right]^{1/2} q = v_0 q.$$ (38)

$v_0$ is the velocity of collective excitation. From Eq. (36), one can write

$$v_0 = v_F \left[ 1 + \frac{V_q}{\pi v_F} + \frac{(V_q - gV_{2k_F}/2)(gV_{2k_F}/2)}{(\pi v_F)^2} \right]^{1/2}.$$ (39)

The quantity in the bracket of Eq. (39) represents the renormalization effect due to electron-electron interaction. The velocity of collective excitation can be controlled by band filling, Rashba SOI, and magnetic field through dependence on $v_F$ and $g$. Let us estimate the magnitude of the correction terms. In $v_\theta(q)$ the backscattering term, $gV_{2k_F}/2\pi v_F$, is a factor $g/2$ smaller than the forward term, $V_q/\pi v_F$. In $v_\phi(q)$ the correction term $gV_{2k_F}/2\pi v_F \sim 0.1g$ for the width of the quantum wire $w \sim 100\text{Å}$ and $2k_F \approx 1 \times 10^6\text{cm}^{-1}$. We also note that for the screened short range Coulomb interaction the interaction matrix element $V_q$ is almost independent of momentum transfer $q$, and the backscattering term plays an equally important role as forward the scattering.

V. DISCUSSIONS AND SUMMARY

It is instructive to compare this result with the velocities of phase fields of collective excitation of ordinary Luttinger liquids. For spinless fermions it is given by

$$v_\theta(q) = v_F \left[ 1 + \frac{V_q}{\pi v_F} - \frac{V_{2k_F}}{2\pi v_F} \right],$$

$$v_\phi(q) = v_F \left[ 1 + \frac{V_{2k_F}}{2\pi v_F} \right].$$ (40)

In Eq. (40) this corresponds to $g = 1$, which implies absence of spin-orbit coupling and one type of spin, either up or down. For the Luttinger liquids of spinful fermions the velocity of charge mode is given by

$$v_{\theta,\phi}(q) = v_F \left[ 1 + \frac{2V_q}{\pi v_F} - \frac{2V_{2k_F}}{2\pi v_F} \right],$$

$$v_{\phi,\phi}(q) = v_F \left[ 1 + \frac{2V_{2k_F}}{2\pi v_F} \right].$$ (41)

$\theta_\tau$ and $\phi_\tau$ are the phase fields in the charge sector. The spinful velocity is recovered with the replacement $V_q \to 2V_q$ and $g = 1$ in Eq. (2). The velocity of the spin mode is

$$v_{\theta,\phi}(q) = v_F \left[ 1 - \frac{V_{2k_F}}{2\pi v_F} \right],$$

$$v_{\phi,\phi}(q) = v_F \left[ 1 + \frac{V_{2k_F}}{2\pi v_F} \right].$$ (42)

$\theta_s$ and $\phi_s$ are the phase fields in the spin sector. This corresponds to $V_q = 0$ and $g = 1$ in Eq. (4).

The dispersion relation of the collective mode may be measured by adding another quantum wire parallel to the original wire in the presence of a second magnetic field $\vec{B}_t$ along the y-axis. When the first wire is located at $z = 0$ and the second wire at $z = z_0$ the single particle energy dispersion of the second wire is $E(k) = \hbar^2(k - k_0)^2/(2m)$, where $k_0 = eB_z z_0/\hbar c$, $m$ is the electron mass in the second wire, and the Landau gauge $A_t = (zB_t, 0, 0)$ is used. Wave-number selectivity due to momentum-resolved tunneling between them, $E_-(k) = E(k)$, allows a mapping of the dispersion. Even in the presence of electron interactions this technique allows direct measurement of the collective excitation spectrum.

Acknowledgments

This work was supported by grant No. R01-2005-000-10352-0 from the Basic Research Program of the Korea Science and Engineering Foundation and by Quantum Functional Semiconductor Research Center (QSRC) at Dongguk University of the Korea Science and Engineering Foundation. This work was completed during a stay at KIAS.

---

1. For recent reviews see E. I. Rashba, cond-mat/0507007 and I. Žutić, J. Fabian, and S. Das Sarma, Rev. Mod. Phys. 76, 323 (2004).
2. S. A. Wolf, D. E. Awschalom, R. A. Buhrman, J. M. Daughton, S. von Molnar, M. L. Roukes, A. Y. Chetcheleva, and D. M. Treger, Science 294, 1488 (2001).
3. S. Datta and B. Das, Appl. Phys. Lett. 66, 665 (1990).
4. P. Streda and P. Šeba, Phys. Rev. Lett. 90, 256601 (2003).
5. P. Devillard, A. Crepieux, K. I. Imura, and T. Martin, Phys. Rev. B 72, 041309(R) (2005).
6. G. Eliasson, J. -W. Wu, P. Hawrylak, and J. J. Quinn, Solid State Commun. 60, 41 (1986); V. Cataudella and G. Iadonisi, Phys. Rev. B 35, 7443 (1987); Q. P. Li, and S. Das Sarma, Phys. Rev. B 44, 6277 (1991); S. -R. Eric Yang and G. C. Aers, Phys. Rev. B 46, 12456(1992); L. Wendler and V. G. Grigoryan, Phys. Rev. B 54, 8652(1996).
7. Y. Yu, Y. Wen, J. Li, Z. Su, and S. Chui, Phys. Rev. B 69, 153307 (2004).
8. V. Gritsev, G. Japaridze, M. Pletyukhov, and D. Baeriswyl, Phys. Rev. Lett. 94, 137207 (2005).
9 J. Voit, Rep. Prog. Phys. 58, 977 (1995).
10 A. O. Gogolin, A. A. Nersesyan, and A. M. Tsvelik, Bosonization and Strongly Correlated Systems (Cambridge University Press, Cambridge, England, 1998).
11 W. Knap, C. Skierbiszewski, A. Zduniak, E. Litwin-Staszewska, D. Bertho, F. Kobbi, J. L. Robert, G. E. Pikus, F. G. Pikus, S. V. Iordanskii, V. Mosser, K. Zekentes, and Yu. B. Lyanda-Geller, Phys. Rev. B 53, 3912 (1996).
12 J. B. Miller, D. M. Zumbühlf, C. M. Markus, Y. B. Lyanda-Geller, D. Goldhaber-Gordon, K. Campman, and A. C. Gossard, Phys. Rev. Lett. 90, 076807 (2003).
13 B. Jusserand, D. Richards, G. Allan, C. Priester, and B. Etienne, Phys. Rev. B 51, 4707 (1995).
14 S. D. Ganichev, V. V. Bel’kov, L. E. Golub, E. L. Ivchenko, P. Schneider, S. Giglberger, J. De Boeck, G. Borghs, W. Wegscheider, D. Weiss, and W. Prettl, Phys. Rev. Lett. 92, 256601 (2004).
15 D. Boese, M. Governale, A. Rosch, and U. Zülicke, Phys. Rev. B 64, 085315 (2001).
16 M. Governale, D. Boese, U. Zülicke, and C. Schroll, Phys. Rev. B 65, 140403(R) (2002).
17 D. Carpentier, C. Peca, and L. Balents, Phys. Rev. B 66, 153304 (2002).
18 U. Zülicke and M. Governale, Phys. Rev. B 65, 205304 (2002).
19 O. M. Auslaender, A. Yacoby, R. de Picciotto, K. W. Baldwin, L. N. Pfeiffer, and K. W. West, Science 295, 825 (2002).
20 E. I. Rashba, Fiz. Tverd. Tela (Leningrad) 2, 1224 (1960) [Solid State Ion. 2, 1109 (1960)].
21 Y. A. Bychkov and E. I. Rashba, J. Phys. C 17, 6039 (1984).
22 J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki, Phys. Rev. Lett. 78, 1335 (1997).
23 G. Engels, J. Lange, T. Schäpers, and H. Lüth, Phys. Rev. B 55, R1958 (1997).
24 For the upper band we have $E_+(k) = \frac{\hbar^2 k^2}{2m} + \sqrt{\epsilon_k^2 + \eta_k^2 k^2}$ and $\xi_+ = \left( \frac{u_k^+}{v_k^+} \right)$ where $u_k^+ = \frac{\hbar\sqrt{\eta_k^2 D^2 + \epsilon_k^2}}{\sqrt{(\eta_k^2 D^2 + \epsilon_k^2)^2 + 4\eta_k^2 D^2 \epsilon_k^2}}$ and $v_k^+ = \frac{-\hat{\epsilon}}{\sqrt{(\eta_k^2 D^2 + \epsilon_k^2)^2 + 4\eta_k^2 D^2 \epsilon_k^2}}$.
25 R. Sousa and S. Das Sarma, Phys. Rev. B 68, 155330 (2003).