Superconducting phase coherence in striped cuprates

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We study the problem of phase coherence in doped striped cuprates. We assume the stripes to form a network of one-dimensional Luttinger liquids which are dominated by superconducting fluctuations and pinned by impurities. We study the dynamics of the superconducting phase using a model of resistively shunted junctions which leads to a Kosterlitz-Thouless transition. We show that our results are consistent with recent experiments in Zn-doped cuprates. We also explain the scaling of the superconducting critical temperature $T_c$ with the incommensurability as seen in recent neutron scattering experiments and predict the behavior of $H_{c2}$.

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It is already well established that cuprates have a strong tendency towards phase separation [1]. Macroscopic phase separation has been observed in $\text{La}_2\text{CuO}_{4+\delta}$ [2]. In other materials, however, phase separation is frustrated, and one observes the formation of domain walls or stripes. Stripes have been experimentally in $\text{La}_{2-x}\text{Sr}_x\text{NiO}_4$ [3]. Magnetic susceptibility measurements, nuclear quadrupole resonance and muon spin resonance [4] indicate formation of domains in $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$. This picture is not inconsistent with neutron scattering experiments in $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ [5].

More recently a direct evidence for stripe formation was given in neutron scattering in $\text{La}_{1.6-x}\text{Nd}_{0.4}\text{Sr}_x\text{CuO}_4$ [6]. Phase separation can be frustrated by the long range Coulomb repulsion between the holes [7,8] or disorder induced by dopants [9].

Superconducting cuprates are naturally disordered because the charge carriers have their origin on doping. In this case holes and impurities have opposite charge which leads to hole-impurity attraction. Localization of holes close to O-impurities has been seen in $\text{La}_2\text{CuO}_{4+\delta}$ [2]. Ab initio calculations seem to imply that a large percentage of the holes can be localized in these materials [10]. Another way to localize charges in the CuO$_2$ planes has antiferromagnetic origin. Zn, for instance, when substituted on the Cu sites, hybridizes poorly with O atoms. Thus Zn breaks local antiferromagnetic bonds. In this case the holes can take advantage of the smaller number of bonds and localize close to the Zn sites. Therefore, while phase separation can lead to stripe formation, impurities can cause stripe pinning. Furthermore, one expects strong magnetic distortions around the impurities [11,12].

In our picture the stripes are quasi-one-dimensional regions of the CuO$_2$ planes where the holes are segregated. We have recently proposed a model [13] that explains the dependence of the antiferromagnetic-paramagnetic phase transition on doping [14] and recent neutron scattering experiments by Yamada et al. [15] in $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$. In these experiments a commensurate-incommensurate transition is observed as a function of doping. It is well known that the incommensurate magnetic peaks are seen at $(\pi/a \pm \epsilon, \pi/a)$ and $(\pi/a, \pi/a + \epsilon)$ where $\epsilon$ depends on doping. In the stripe picture one has $\epsilon = \pi/\ell$ where $\ell$ is the inter-stripe distance [16,17].

Since stripes are one-dimensional objects they cannot show true long range superconducting order [18]. This is only possible if the stripes interact with each other by exchanging Cooper pairs. There are a few ways the stripes can interact. One of them is by a direct exchange of Cooper pairs via stripe fluctuations. This is a dynamical Josephson effect [19]. Another possibility is due to stripe crossing in the presence of impurities. In this paper we propose a scenario where the holes are localized close to the impurities and these “lakes” of holes are connected among themselves by “rivers” or stripes forming a network. See Fig. 1. We show that stripes carry a Josephson and a normal current. While the normal current dissipates, the Josephson current can lead to the exchange of Cooper pairs via the lakes of holes. Moreover, since there is an accumulation of holes close to the impurities, one expects charging effects to play a role in the problem. This scenario leads naturally to the problem of coupled resistively shunted junctions which has been studied in the literature in different contexts [20]. Our arguments, therefore, follow the ones used in the problem of granular superconductors [21], and one can show that the network of stripes undergoes a Kosterlitz-Thouless (KT) phase transition towards a superconducting state at a critical temperature $T_c$. We also show how doping with Zn changes this picture in order to drive the system towards a superconducting-insulating phase transition at an universal value of the resistance given by $R_Q = h/(4e^2) \approx 6.45k\Omega$.

Indeed, in a recent paper Fukuzumi et al. [22] studied the temperature behavior of the resistivity of single crystals of $\text{YBa}_2(\text{Cu}_{1-z}\text{Zn}_z)_3\text{O}_{7-\delta}$ and $\text{La}_{2-x}\text{Sr}_x\text{Cu}_{1-z}\text{Zn}_z\text{O}_4$. It was shown that these materials undergo a universal superconducting-insulator transition as a function of the Zn doping, and indeed the superconducting critical temperature in the underdoped samples seems to vanish very close to the universal value $R_Q$. This behavior is reminiscent of the behavior in granular superconductors where the resistivity, instead of decreasing close to the superconducting transition, actually increases if the sample resistance is greater than $R_Q$ [9].
At zero temperature we assume the network to be in a superconducting state which is characterized by a gap $|\Delta|$ and a superconducting phase $\Phi$. We also assume that the connection between the stripes and the lakes is a perfect interface. The electronic system on the stripes is described in terms of a Luttinger liquid [13]. The Luttinger liquid is described in terms of right, $R$, and left, $L$, moving fermions with spin $\sigma = \uparrow, \downarrow$ which are created (destroyed) by operators $\psi^\dagger_{R,L,\sigma}(x)$ ($\psi_{R,L,\sigma}(x)$). These fermions can be bosonized via the transformation $\psi_{R,L,\sigma}(x) = \sqrt{F_{R,L,\sigma}} \phi_{R,L,\sigma}(x)$. The bosonic modes $\phi$, and phase, $\theta$, modes as $\phi_{R,L,\sigma}(y) = \phi_{\alpha}(y) \mp \theta_{\alpha}(y)$. In turn these bosonic fields can be written in terms of charge and spin bosonic modes, $\phi_{\rho,s}(y) = (\phi_{\uparrow} \pm \phi_{\downarrow})/\sqrt{2}$ and $\theta_{\rho,s} = (\theta_{\uparrow} \pm \theta_{\downarrow})/\sqrt{2}$, and it is easy to show that the Euclidean Lagrangean density of the system can be written as (with the units $\hbar = k_B = 1$),

$$\mathcal{L}_S = \sum_{i = \rho,s} \left\{ \frac{g_i}{2v_i} \left[ (\partial_x \phi_i)^2 + v_i^2 (\partial_x \phi_i)^2 \right] \right\}. \tag{1}$$

gs and $g_s$ are the Luttinger parameters for spin and charge respectively, and $v$s and $v_p$ are their velocities.

It is believed that the stripes fluctuate in the superconducting phase. Thus, the electrons on the stripe will undergo strong backscattering (by corners, for instance) which can lead to localization. Since we assume that the stripes are metallic, we have to rely on a strong attractive interaction between the electrons in order to get delocalization. This attraction could be provided, for instance, by the surrounding antiferromagnet [21,22] and the phase coherence by the mechanism described in this paper. Renormalization group studies of disordered Luttinger liquids show that if $g_p$ is smaller than a critical value $g_c$ (for singlet pairing $g_s^2 = 1/3$), the Luttinger liquid delocalizes. In this case it was shown in ref. [23] that the temperature dependence of the resistance is given by,

$$R(T) \approx T^{1+\gamma}. \tag{2}$$

where $\gamma = 1/g_p - 1/g_c$. This result is also consistent with the presence of a Cooper-pair gap in the normal phase of these materials. Our picture is the one where in the normal state of these materials the electrons on the stripe are paired but there is no superconducting phase coherence. As it was explained by Emery and Kivelson [1], this is possible in one dimension because pairing and phase coherence have completely different origins. This could be an explanation of the so-called “spin gap” [24] seen in some cuprates. Since the conductance occurs within the stripes, the only effect of Zn is to add a residual zero temperature resistance, $R_0$ (besides the effects it can have on a d-wave order parameter [25]). This leads to a total resistance, $R_s(T) = R_0 + aT^{1+\gamma}$ where $A$ is a non-universal coefficient which does not depend on Zn doping but on the stripe fluctuations. Indeed in the experiments the linear part of resistance is insensitive to Zn doping [26].

At zero temperature the system is in a superconducting state and a Josephson coupling develops between the Luttinger stripes via the lakes. At finite temperatures, above the bulk superconducting critical temperature, $T_c$, the electrons propagate in the system via the network of Luttinger liquids but there is no phase coherence. At low temperatures the scattering is dominated by the Luttinger liquid scattering since the quasi-two dimensional scattering is much smaller. Experimentally it has been established for quite some time that the resistivity behaves linearly with temperature. Since the cuprates are very close to a superconductor to insulator transition one has from the above discussion that $\gamma \ll 1$ which is consistent with this linear behavior. Deviations from the linear power have also been observed [24] and can be absorbed into $\gamma$. At low temperatures as the Cooper pairs propagate they are going to feel differences in the superconducting gap in different regions. This is known to lead to Andreev reflections [27]. Therefore one has to add to the Luttinger liquid Lagrangean [1] another term which is related to the presence of the gap. This problem is very similar to the problem of superconducting metals coupled by a Luttinger liquid that has been studied in the context of mesoscopic physics [28]. The Lagrangean associated with pairing is

$$\mathcal{L}_{pair} = |\Delta| \left\{ \cos \left[ \Phi + \sqrt{2\pi}(\theta_p - \phi_s) \right] + \cos \left[ \Phi - \sqrt{2\pi}(\theta_p + \phi_s) \right] \right\}. \tag{3}$$

If $|\Delta|$ is large, the main effect of the cosine term is to pin the value of the fields to the minima of the potential, i.e.,

$$\langle \theta_p \rangle = (\pi n - \Phi) / (\sqrt{2\pi})$$

where $n$ is an integer. This implies that in the regions where the gap is large the bosonic fields are subjected to twisted boundary conditions [29].

We can calculate the current density that flows in the stripes which is given by $j(x) = -2v_p \partial_x \theta_p / (\sqrt{2\pi g_p})$. According to [1] it is given by

FIG. 1. Geometry of the problem. The antiferromagnet (shown with $\uparrow, \downarrow$) in an anti-phase domain configuration together with stripes (as black circles) and impurities (gray squares). Observe that the distance between impurities $L$ is the inter-stripe distance $\ell$. 

L

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1}
\caption{Geometry of the problem. The antiferromagnet (shown with $\uparrow, \downarrow$) in an anti-phase domain configuration together with stripes (as black circles) and impurities (gray squares). Observe that the distance between impurities $L$ is the inter-stripe distance $\ell$.}
\end{figure}
This is the value of the supercurrent density at zero temperature. As expected it depends only on the value of the phase. Moreover, our argument is based on a large value of the gap which is only valid at zero temperature. At finite temperatures the phase fluctuates. In this case we have to add another contribution to the problem which is the normal dissipative current. Thus, if the distance between two stripes is \( \ell \) (see Fig.1) and \( \Phi \) is the phase difference between them we have from (4) that the Josephson current is

\[
I_J(\Phi) = ev_\rho \Phi / (\pi g_\rho \ell)
\]

where \( e \) is the electronic charge. Notice that the energy scale in the problem is given by

\[
T_L = v_\rho / \ell,
\]

and the Josephson energy in this case is

\[
E_J = I_J(2\pi)/(2e) = T_L / g_\rho.
\]

The total current from one stripe to another has three different contributions: the Josephson current, a normal dissipative current, and a current through a capacitor with capacitance \( C \) which corresponds to the charge accumulation at the lakes. First we consider the problem of two stripes connected by one lake. Henceforth we introduce the phase as a quantum mechanical operator which is conjugated to the number of Cooper pairs ([\( \Phi, n \] = i). The Josephson effect is described by the potential \( V(\Phi) = E_J U(\Phi) \) where \( U(\Phi) = \Phi^2 / 2 \) (module 2\( \pi \)) is the potential associated with (4). The simplest way to mimic the dissipative part of the Luttinger-stripe is to introduce a set of decoupled harmonic oscillators following Caldeira-Leggett. The Lagrangean is

\[
\mathcal{L} = \frac{M}{2} \left( \frac{d\Phi}{dt} \right)^2 + E_J U(\Phi) - \frac{1}{2c} \frac{d\Phi}{dt} \sum_i \lambda_i x_i + \sum_i \left( \frac{p_i^2}{2m_i} + \frac{1}{2} m_i \omega_i^2 x_i^2 \right),
\]

where \( M = C / (2e)^2 \). This Lagrangean describes the motion of a fictitious particle with mass \( M \) moving in a periodic potential \( U \) coupled to a heat bath. As was shown before (29), the properties of this system depend only on the spectral function \( J(\omega) = \frac{\pi}{2} \sum_i (\lambda_i^2 \omega_i / m_i) \delta(\omega - \omega_i) \). By requiring that the voltage between the lakes is given by the resistance \( R_S \) times the normal current we find that this spectral function is uniquely given by \( J(\omega, T) = \omega / R_S(\ell) \). It is well established that the model described by (5) has a zero temperature phase transition as a function of the parameter \( \alpha(T) = R_Q / R_S(\ell) \). It has been shown (29) that for \( \alpha(0) < 1 \) the junction has a finite resistance and for \( \alpha(0) > 1 \) the junction is in a superconducting state with zero resistance. This model has a duality symmetry (29) which allows the calculation of the resistance as a function of temperature at low temperatures \( T \ll T_s = 1 / \sqrt{AC} \). One has

\[
\frac{R(T)}{R_Q} \approx \frac{\Gamma[\alpha(T)] \pi^{2\alpha(T) + 1/2}}{2^\alpha \Gamma[\alpha(T) + 1/2]} \left( \frac{E_J}{\gamma(T)} \right)^2 \left( \frac{T}{\gamma(T)} \right)^{2\alpha(T) - 1}
\]

where \( \gamma(T) = 1 / (R_s(T)C) \) and \( \Gamma[z] \) is a Gamma function.

In the absence of Zn we have \( R_0 \approx 0 \) (29). This implies that the resistance behaves like \( R(T) \approx 2R_Qe^{-2T_0/T \ln(T_s/T)} \), where \( T_s = R_Q/A \) and is rapidly suppressed at low temperatures showing the growth of superconducting fluctuations in the system. However, superconductivity is obtained only at zero temperature because we have only two connected stripes. In the Zn-doped case one can substitute for \( \alpha(T) \) its zero temperature value at low temperatures, \( \alpha_0 = R_Q / R_0 \), and one finds that \( R(T) \sim T^{2\alpha_0 - 1} \). Therefore, for \( \alpha_0 > 1 \) \( (R_0 < R_Q) \) the resistance goes to zero and superconductivity is obtained. For \( \alpha_0 < 1 \) \( (R_0 > R_Q) \) the resistance becomes very large at low temperatures indicating a transition to an insulating state. Moreover, for \( R_0 \approx R_Q \) one finds a logarithmic behavior. These results are consistent with the available data on Zn doped cuprates (29).

In order to explain the finite temperature phase transition one has to rely on the geometric structure of the array of lakes and Luttinger stripe rivers. In order to do that we generalize the Lagrangean (8) to \( \mathcal{L} = \sum_{a,b} \mathcal{L}_{ab}(\Phi_{ab}) \), where, \( \mathcal{L}_{ab} \) is given in (8) and \( a,b \) label each two stripes linked by one lake. The calculation of the partition function for the problem is analogous to the one for two stripes. However we have to introduce the disordered distribution of lakes with different sizes and stripes with different lengths. One can show that this is a problem of a stack of X-Y models coupled in the imaginary time direction with random couplings (29). We can then coarse grain the imaginary time direction and reduce the system to one effective classical X-Y model with renormalized coupling constants. When this is done, one finds a partition function for a classical model given by

\[
Z = \int D\phi \exp\left\{-\sum_{a,b} \kappa_{ab}(\phi_a - \phi_b)^2/2\right\}
\]

where at low temperatures in the pure case \( \alpha(T) \to \infty \) as \( T \to 0 \) we have \( \kappa(T) \approx E_J / T \). This leads naturally to a KT transition at some critical value of the coupling, \( \kappa_c \) which depends on the detailed structure of the lattice (30). If \( \kappa < \kappa_c \) then the system is disordered with an exponentially decaying correlation function. For \( \kappa > \kappa_c \) the correlation function decays as a power law, and we have quasi-long range order. Since \( \kappa(T) \) diverges at low temperatures for \( \alpha_0 > 1 \), the system has a transition to quasi-long range order at some critical temperature \( T_c \), which is given by (using (8) and (7)),

\[
T_c = T_L / (g_\rho \kappa_c) = v_\rho / (g_\rho \kappa_c \ell).
\]

This critical temperature signals the transition to a superconducting state with vanishing resistance. Observe
that $\kappa_c$ depends on the impurities. Highly disconnected lattices (which can be caused by Zn doping) have large $\kappa_c$ that decreases substantially $T_c$.

One of the most interesting results of Yamada et al. [14] is the observed relation between $T_c$ and the incommensurability $\epsilon$ which is found to be $T_c(K) \approx 181a(\AA^{-1})$ for a wide range of doping. In our picture one can relate $T_c$ to the inter-stripe distance using $\epsilon = \pi/\ell$. Thus, from the experiments one concludes that

$$T_c \approx 569/\ell. \quad (11)$$

By comparing (11) and (11), we find perfect agreement between experiment and theory. Moreover, from (11), assuming $g_p = 1/3$ (which gives $R_s(T) \propto T$) and $\kappa_c \approx 0.35$ (for the square lattice) [13] one finds (restoring $\hbar$ and $k_B$) $\hbar v_p \approx 6.10^{-2}eV\AA$. We point out that indications of KT transition have been observed in YBa$_2$Cu$_3$O$_7-\delta$ and LaBaCuO [22]. Another non-trivial but straightforward prediction of this theory is the behavior of superconducting coherence length $\xi_s$ with doping. Since the maximum value of the phase gradient in the theory is $\pi/(2\ell)$ a simple Ginzburg-Landau argument gives $\xi_s \sim \ell$ [33]. Thus, in the clean limit one expects the upper critical field $H_{c2}$ to behave like $H_{c2} \propto 1/\ell^2$. Comparing this prediction with neutron scattering data in La$_{2-x}$Sr$_x$CuO$_4$ [4] one finds $H_{c2} \propto T^2$ over a large doping region $(0.06 \leq x \leq 0.2)$. In particular for small doping levels $(0.06 \leq x \leq 0.12)$ one finds $H_{c2} \propto x^2$. These predictions can be tested in new measurements of $H_{c2}$ at low temperatures in high magnetic fields [23].

In conclusion, we propose a new scenario for the problem of phase coherence and superconductivity in striped cuprates which is based on the assumption of a network of Luttinger-stripes and lakes of holes in the CuO$_2$ planes. In our model the stripes are pinned by impurities and a Josephson current is transferred from stripe to stripe via the network. The same network also carries a normal current which dissipates and is responsible for the superconducting-insulator transition seen in these materials. In our case the superconducting transition is a two-dimensional KT transition and our model is consistent with many existing experimental data, especially the recent neutron scattering experiments by Yamada et al. [14]. It also explains the recently discovered universal behavior of the superconducting-insulator transition in the presence of Zn impurities [24]. Moreover, we predict the behavior of $H_{c2}$ as a function of $T_c$ and doping, $x$.

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Note: After this paper was submitted I became aware of the experimental paper in ref. [25] where the increase of disconnectivity of the network was observed in $\mu$SR under Zn doping and a related work on Luttinger liquid networks [26].

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