Photoproduction of the P-wave excited $B_{c}^{**}$ meson at the LHeC

He Kai, Bi Huan-Yu*, Zhang Ren-You, Li Xiao-Zhou and Ma Wen-Gan,
Department of Modern Physics, University of Science and Technology of China (USTC),
Hefei 230026, Anhui, People’s Republic of China

Abstract

As an important sequential work of the S-wave $B_{c}^{(*)}(^{1}S_{0}(^{3}S_{1}))$ meson production at the Large Hadron Electron Collider (LHeC), we investigate the production of the P-wave excited $B_{c}^{**}$ states ($^{1}P_{1}$ and $^{3}P_{J}$ with $J = 0, 1, 2$) via photoproduction mechanism within the framework of nonrelativistic QCD (NRQCD) at the LHeC. Generally, the $e^{-} + P \rightarrow \gamma + g \rightarrow B_{c}^{**} + b + \bar{c}$ process is considered as the main production mechanism at an electron-proton collider due to the large luminosity of the gluon. However, according to our experience on the S-wave $B_{c}^{(*)}$ meson production at the LHeC, the extrinsic production mechanism, i.e., $e^{-} + P \rightarrow \gamma + c \rightarrow B_{c}^{**} + b$ and $e^{-} + P \rightarrow \gamma + \bar{b} \rightarrow B_{c}^{**} + \bar{c}$, could also provide dominating contributions at low $p_T$ region. A careful treatment between these channels is performed and the results on total and differential cross sections, together with main uncertainties are discussed. Taking the quark masses $m_b = 4.90 \pm 0.40$ GeV and $m_c = 1.50 \pm 0.20$ GeV into account and summing up all the production channels, we expect to accumulate $(2.48^{+3.45}_{-1.55}) \times 10^4$ $B_{c}^{**}(^{1}P_{1})$, $(1.14^{+1.49}_{-0.82}) \times 10^4$ $B_{c}^{**}(^{3}P_{0})$, $(2.38^{+3.39}_{-1.74}) \times 10^4$ $B_{c}^{**}(^{3}P_{1})$ and $(5.59^{+7.84}_{-3.93}) \times 10^4$ $B_{c}^{**}(^{3}P_{2})$ events at the $\sqrt{S} = 1.30$ TeV LHeC in one operation year with luminosity $L = 10^{33}$ cm$^{-2}$s$^{-1}$. With such sizable events, it is worth to study the properties of excited P-wave $B_{c}^{**}$ states at the LHeC.

*email: bihy@mail.ustc.edu.cn
I. INTRODUCTION

The doubly heavy meson physics has aroused great interests due to its nature, which can be studied in the framework of nonrelativistic QCD (NRQCD) [1]. The production of the doubly heavy meson can be factorized into a hard production of two heavy quark pairs which can be described by perturbation QCD (pQCD), and a soft term related to the nonperturbative binding of them. Thus, it is a good laboratory for testing NRQCD, pQCD and QCD potential models.

Among the doubly heavy mesons, the $B_c$ meson is especially interesting for its unique properties, which is the only observed meson composed of a heavy quark and a heavy antiquark of different flavors. Unlike the charmonium and bottomonium states, which have ‘hidden flavor’, the $B_c$ meson is made of a charm quark and a bottom antiquark, and the production of $B_c$ meson must be accompanied by additional heavy quarks especially in hadronic production [2]. For example, production of a color-singlet $(c\bar{c})_1$ and a color-octet $(c\bar{c})_8$ quarkonium states are allowed for the channel $\gamma + g \rightarrow |(c\bar{c})_1/8\rangle + g/\gamma$. Therefore, compared with the production cross sections of hidden flavor quarkonia, the cross section of $B_c$ meson is suppressed by not only the phase space but also the higher order in the coupling constants of leading-order diagrams. Due to the small production rate and the low colliding luminosity, the $B_c$ meson was not found at LEP despite of careful searches [3–5]. At hadron colliders, the background is extremely serious. With much time and effort, the ground-state $B_c$ meson was finally observed by the CDF at Tevatron in 1998 [6, 7]. Now we also need more data to understand the properties of $B_c$ meson, such as mass spectrum, lifetime and decay. Thus, the researches on various production mechanisms at different platforms are required to study the properties of $B_c$ meson.

The hadronic production of $B_c$ meson was amply studied directly via gluon-gluon fusion and etc, or indirectly via top quark, $W$ or Higgs boson decays [8–30]. These studies indicated that at a hadron collider, such as the Large Hadron Collider (LHC) or Tevatron, sizable $B_c$ events can be produced due to the powerful colliding energy and high luminosity. The $B_c$ meson production at electron-positron colliding platforms, such as super $Z$-factory and International Linear Collider (ILC) were discussed in [31–38]. These lepton platforms have more clean background, hence are more suitable for precision measurement. For example, the authors in Refs. [31, 32] are interested
in the forward-backward asymmetry in the production of doubly heavy-flavored hadron at the Z factory. For an electron-proton collider, which combines the advantages of hadron collider and lepton collider, may also be a good platform to study doubly heavy-flavored hadrons. Thus, we have studied the $B_c^{(*)}$ meson and doubly heavy baryon production \cite{39,40} at the Large Hadron Electron Collider (LHeC) \cite{41} and Future Circular Collider-based electron-proton collider (FCC-$ep$), and we found these platforms are very helpful for study doubly heavy-flavored hadrons.

Recently, a new state has been observed by the ATLAS experiment \cite{42}, whose mass and decay mode are consistent with the theoretical prediction of the second S-wave state $B_c^{\pm}(2S)$. The $B_c^{\pm}(2S)$ state is reconstructed through its decay to the ground state accompanied with two oppositely charged pions, and the $B_c$ ground state is detected through its decay $B_c^{\pm} \rightarrow J/\psi \pi^\pm$. Besides, the excited $B_c$ states can also decay (or in a cascade way) to the ground state through the electric or magnetic dipole transitions. In contrast with the hadronic decay, the feature of the electromagnetic decay of the excited $B_c$ states is that the characteristic product is an additional photon with energy about dozens or hundreds of MeV \cite{43,44,45} rather than pion. The measurements to the characteristic products, i.e., the pion or photon, can be treated as the signals for the discovery of the excited $B_c$ states and the measurements of the ground or excited $B_c$ states can provide the opportunity for extracting information on the mass spectrum of the $(c\bar{b})$ bound states and QCD potential models.

Generally speaking, the excited $B_c$ states shall decay (or in a cascade way) to the ground state via electromagnetic or hadronic interactions with almost 100% probability, besides, its excited states may not be discriminated easily from its ground state in experiments. Thus, it’s necessary to estimate the production rate of excited P-wave $B_c$ states, which will contribute to the production rate of S-wave $B_c^{(*)}$ states. On the other hand, it’s helpful for the discovery of the excited $B_c$ states to give the dynamic distributions of the production. Studies on the production of excited $B_c$ states have been done in the literature \cite{17,18,20,31,33,37}, and they found that the excited P-wave $B_{c}^{(*)}$ states can provide about 14% $\sim$ 17% contributions compared to the S-wave $B_{c}^{(*)}$ states.

As indicated in Ref. \cite{39}, large number of $B_c^{(*)}$ mesons (about $6 \times 10^5$ events per year) can
be produced at LHeC. Motived by the discovery of the second S-wave state $B_c^+(2S)$ and the sizable $B_c^{(*)}$ events at the LHeC, we are interested in whether enough P-wave $B_c^{**}$ events can be accumulated at the LHeC. In this paper, in addition to gluon-induced channel $\gamma + g \rightarrow B_c^{**} + b + \bar{c}$, two extrinsic heavy quark channels $\gamma + c \rightarrow B_c^{**} + b$ and $\gamma + \bar{b} \rightarrow B_c^{**} + \bar{c}$ are included. Although the density of $\bar{b}$ and $c$ quarks are small in proton, the contributions of $\gamma + c$ and $\gamma + \bar{b}$ channels can not be neglected for the larger phase space and lower order in the coupling constants compared to the $\gamma + g$ channel.

The photoproduction of the $B_c$ meson at the LHeC can be divided into three steps, which contain three subprocesses,

\begin{align*}
e^- + P \rightarrow \gamma + g & \rightarrow (c \bar{b})[n] + b + \bar{c} \rightarrow B_c^{**} + b + \bar{c}, \\
e^- + P \rightarrow \gamma + c & \rightarrow (c \bar{b})[n] + b \rightarrow B_c^{**} + b, \\
e^- + P \rightarrow \gamma + \bar{b} & \rightarrow (c \bar{b})[n] + \bar{c} \rightarrow B_c^{**} + \bar{c}.
\end{align*}

(1.1)

First, the photon beams are produced from the electron bremsstrahlung and the partons are radiated from the protons. The density of photon beams can be described by the Weizsäcker-Williams approximation (WWA) \cite{46}, and the parton densities are described by the parton distribution functions (PDFs). Second, photon beams interact with the partons and a diquark state with certain quantum numbers $(c \bar{b})[n]$ is produced, and this step can be calculated by the pQCD. Finally, the $(c \bar{b})[n]$ are bounded together to form the $B_c^{**}$ meson through nonperturbative effect, which can be described by the nonperturbative matrix element, and the matrix element is proportional to the inclusive transition probability of the $(c \bar{b})[n]$ diquark to the bound state $B_c^{**}$. In this work, we only focus on four color-signet diquark states of $(c \bar{b})[n]$, i.e., $(c \bar{b})_1[1P_1]$, $(c \bar{b})_1[3P_0]$, $(c \bar{b})_1[3P_1]$, and $(c \bar{b})_1[3P_2]$.

This paper is organized as follows: we present calculation details in Section II. The numerical results are given in Section III and the summary is presented in Section IV.

II. OUTLINE OF THE CALCULATION

Based on pQCD, the total cross section of the $B_c^{**}$ meson production can be factorized into the convolution of the parton/photon density functions and the partonic cross section $d\sigma_{\gamma i}(\mu, x_1, x_2)$
as follows:

\[ d\sigma(e^- + P \rightarrow B_c^{**} + X) = \int dx_1 dx_2 \sum_{i=c,b,g} f_{\gamma/e^-}(x_1) f_{i/P}(\mu, x_2) d\tilde{\sigma}_\gamma(\mu, x_1, x_2), \quad (2.1) \]

here we have taken the renormalization scale \( \mu_r \) and the factorization scale \( \mu_f \) to be the same, i.e., \( \mu_r = \mu_f = \mu \). \( f_{i/P} \) are the PDFs and \( f_{\gamma/e^-} \) is the photon density function which is described by the WWA as

\[ f_{\gamma/e^-}(x) = \frac{\alpha_s}{2\pi} \left[ 1 + \frac{(1-x)^2}{x} \ln \frac{Q_{\text{max}}^2}{Q_{\text{min}}^2} + 2m_c^2 x \left( \frac{1}{Q_{\text{max}}^2} - \frac{1}{Q_{\text{min}}^2} \right) \right], \quad (2.2) \]

where \( x \) is the fraction of the longitudinal momentum of the photon to electron beams. \( Q_{\text{min}}^2 \) and \( Q_{\text{max}}^2 \) are the minimum and maximum photon virtuality which can be expressed as

\[ Q_{\text{min}}^2 = \frac{m_c^2 x^2}{1-x}, \quad Q_{\text{max}}^2 = (\theta_c E_e)^2 (1-x) + Q_{\text{min}}^2, \quad (2.3) \]

where \( \theta_c \) is the electron-scattering angle and \( E_e \) is the electron beam energy, which are determined by the collider. In this work we set \( \theta_c = 32 \text{ mrad} \) which is consistent with the choices in Refs. [50, 51] and is satisfied with the requirement of \( \theta_c \ll 1 \text{ rad} \) [52, 53].

To avoid the 'double counting' between \( \gamma + g \) and \( \gamma + q \) channels, the general-mass variable-flavor-number scheme (GM-VFNs) [54–58] is adopted here. The cross section under the GM-VFNs is

\[ d\sigma(e^- + P \rightarrow B_c^{**} + X) = f_{\gamma/e^-}(x_1) f_{g/P}(\mu, x_2) \otimes d\tilde{\sigma}_{\gamma g}(\mu, x_1, x_2) + \sum_{q=c,b} f_{\gamma/e^-}(x_1) [f_{q/P}(\mu, x_2) - f_{q/P}^{\text{sub}}(\mu, x_2)] \otimes d\tilde{\sigma}_{\gamma q}(\mu, x_1, x_2). \quad (2.4) \]

\( d\tilde{\sigma}_{\gamma g} \) contains mass logarithmic terms \( \ln(Q^2/m_q^2) \), and these logarithmic terms originate in the Feynman diagrams which contains initial gluon splitting to a heavy quark pair \( g \rightarrow q\bar{q} \). \( d\sigma \) is the infrared-safe partonic cross section which avoid the logarithmic terms through the subtraction of the term \( f_{q/P}^{\text{sub}}(\mu, x_2) \):

\[ f_{q/P}^{\text{sub}}(\mu, x_2) = \int dx_2 \int \frac{1}{y} f_{q/P}(\frac{x_2}{y}) \frac{\alpha_s(\mu)}{2\pi} \ln \frac{\mu^2}{m_q^2} P_{g\rightarrow q}(y) \frac{dy}{y}, \quad (2.5) \]
where $P_{g \rightarrow q}(y) = \frac{1}{2}(1 - 2y + 2y^2)$ is the $g \rightarrow q\bar{q}$ splitting function.

The partonic hard cross section $d\sigma_{\gamma i}$ can also be factorized into a diquark production $(c\bar{b})[n]$ multiply by the nonperturbative matrix element $\langle O^{B_2^{*+}} \rangle$,

$$d\sigma_{\gamma i} = \frac{\langle O^{B_2^{*+}} \rangle}{4E_\gamma E_i |\vec{v}_\gamma - \vec{v}_i|} \sum |M|^2 d\Phi. \quad (2.6)$$

For the color-singlet $B_2^{*+}$ meson production, $\langle O^{B_2^{*+}} \rangle$ is related to the first derivative of the wave function at the origin of the $(c\bar{b})[n]$ bound state $[1]$, i.e., $\langle O^{B_2^{*+}} \rangle \simeq |R'_{P}(0)|^2/(4\pi)$, and $R'_{P}(0)$ can be calculated from the potential model $[47-49]$. $d\Phi$ stands for the phase-space element and $M$ for the hard scattering amplitudes of $(c\bar{b})[n]$ production which contains the precise spin and orbit information,

$$M^{S=0,L=1} = \varepsilon_\alpha(p_3) \frac{d}{dq_\alpha} T|_{q=0}, \quad (2.7)$$

$$M^{S=1,L=1} = \varepsilon_{\alpha\beta}(p_3) \frac{d}{dq_\alpha} T|_{q=0}. \quad (2.8)$$

$\varepsilon_\alpha(p_3)$ is the polarization vector of the angular momentum triplet $^1P$ diquark and $\varepsilon_{\alpha\beta}(p_3)$ stands for the polarization tensor of spin triplet $P$-wave $^3P$ diquark. The summation over the polarizations obey the following relations:

$$\sum \text{polarizations} \varepsilon_\alpha \varepsilon_{\alpha'}^* = \Pi_{\alpha\alpha'},$$

$$\varepsilon_0 \varepsilon_0^* = \frac{1}{3} \Pi_{\alpha\beta} \Pi_{\alpha'\beta'},$$

$$\sum \text{polarizations} \varepsilon_{\alpha\beta} \varepsilon_{\alpha\beta'}^* = \frac{1}{2} (\Pi_{\alpha\alpha'} \Pi_{\beta\beta'} - \Pi_{\alpha\beta} \Pi_{\alpha'\beta'}),$$

$$\sum \text{polarizations} \varepsilon_{\alpha\beta} \varepsilon_{\alpha\beta'}^* = \frac{1}{2} (\Pi_{\alpha\alpha'} \Pi_{\beta\beta'} + \Pi_{\alpha\beta} \Pi_{\alpha'\beta'}) - \frac{1}{3} \Pi_{\alpha\beta} \Pi_{\alpha'\beta'}, \quad (2.9)$$

where

$$\Pi_{\alpha\beta} = -g_{\alpha\beta} + \frac{p_{3\alpha} p_{3\beta}}{M^2}, \quad (2.10)$$

$M$ is the mass of $B_2^{*+}$ meson and $T$ is related to the Feynman diagrams. For example, the $\gamma(p_1) + g(p_2) \rightarrow (c\bar{b})[n](p_3) + \bar{c}(p_4) + b(p_5)$ channel can be expressed in a general form as

$$T \propto \bar{u}(p_4, s_4, c_4) \cdots v(\frac{p_3}{2} - q, s_3, c_3) \times \bar{u}(\frac{p_3}{2} + q, s'_3, c'_3) \cdots v(p_5, s_5, c_5), \quad (2.11)$$
where $s$ and $c$ represent the spin and color indices of the spinors, respectively. Then the color and spin projection operation should be applied to form the color-singlet spin-singlet (or spin-triplet) diquark. To implementate the color projection, each amplitude should be multiplied by a factor $\delta_{c_3,c'_3}/\sqrt{3}$. For the spin projection operator, one can replace $\sum_{s_3,s'_3} u\left(\frac{p_3}{2} - q, s_3, c_3\right) \times \bar{u}\left(\frac{p_3}{2} + q, s'_3, c_3\right)$ by the operator

$$
\Pi(p_3) = \sqrt{M} \left(\frac{m_b\, p_3 - q - m_b}{2m_b}\right) \Gamma\left(\frac{m_c\, p_3 + q + m_c}{2m_c}\right),
$$

where $\Gamma = \gamma^5$ for the spin singlet state, and $\Gamma = \gamma^\beta$ for the spin triplet states. $q$ is the relative momentum among the quarks inside $(c\bar{b})[n]$, which can be set as zero after the derivation of the amplitudes.

The Feynman diagrams and amplitudes are generated by FeynArts [59] and further simplification on the amplitudes are handled by FeynCalc [60] and FeynCalcFormLink [61]. Numerical calculations are performed by FormCalc [62].

**III. NUMERICAL RESULTS**

The derivative of the wave function at the origin $|R'_{P,0}|^2 = 0.201 \text{ GeV}^5$ is taken from Ref. [47]. The P-wave $B_{c^*}$ mesons mass $M$ is taken as same with the S-wave $B_{c}^{(*)}$ mesons, which is the requirement for the NRQCD formalism and are explained in Ref. [18], i.e., $M = m_b + m_c$ with $b$-quark mass $m_b = 4.90 \text{ GeV}$ and $c$-quark mass $m_c = 1.50 \text{ GeV}$. The electron mass $m_e$ is taken as $0.51 \times 10^{-3} \text{ GeV}$ and the fine-structure constant is chosen as $\alpha = 1/137$. The renormalization and factorization scale are set to be the transverse mass of the $B_{c^*}$ meson $\mu = \mu_r = \mu_f = M_T = \sqrt{p_T^2 + M^2}$. We use CT10nlo [63] as default and the $\alpha_s$ is extracted from the PDFs.

The cross sections for all the production channels with four collision energies at two electron-proton colliders are presented in Table I, i.e., for the LHeC $\sqrt{S} = 1.30, 1.98 \text{ TeV}$ [11] which corresponds to two beam energy sets designs as $E_e = 60, 140 \text{ GeV}$ and $E_P = 7 \text{ TeV}$. For the FCC-ep we take $\sqrt{S} = 7.07, 10.00 \text{ TeV}$ [64] which correspond to two beam energy sets as $E_e = 250, 500 \text{ GeV}$ and $E_P = 50 \text{ TeV}$ separately. Here, we use $\sigma_{g\gamma}, \sigma_{c\gamma}$ and $\sigma_{\bar{b}\gamma}$ to denote the cross sections of $\gamma + g, \gamma + c$ and $\gamma + \bar{b}$ channels, respectively. Summing up all the contributions of
three channels and four P-wave $B_{c}^{**}$ states, we find that the contribution from P-wave $B_{c}^{**}$ states can be 19.63%, 19.90%, 19.82% and 20.13% of the S-wave $B_{c}^{(*)}$ state production \cite{39} for above four colliding energies cases. That shows these ratios are larger than the corresponding ones at the Z-factory, LHC ($\sqrt{S} = 14$ TeV) and Tevatron ($\sqrt{S} = 1.96$ TeV), where the ratios are about 17.40%, 16.20% and 14.93%, respectively \cite{18,31}. We can conclude that the LHeC and FCC-ep colliding experiments might have advantages on the study of the P-wave $B_{c}^{**}$ states, thus it is worth to study the excited states at these two platforms. By summing up the four P-wave $B_{c}^{**}$ states, We see also that the $\gamma + \bar{b}$ channel makes the largest contributions to the P-wave $B_{c}^{**}$ state production, while the contributions from the $\gamma + g$ and $\gamma + c$ channels are at the same order which only provide about 5% and 1% to the total cross section for various electron-proton colliding energies. Besides, by summing up all the production channels, the four $B_{c}^{**}$ states, i.e., $1P_1$, $3P_0$, $3P_1$ and $3P_2$, provide individually about 21%, 10%, 21% and 48% contributions to the total cross section, respectively. The total cross section are similar at different colliding energies, and in the following, we focus on the $B_{c}^{**}$ meson production at the $\sqrt{S} = 1.30$ TeV LHeC.

Table I: The cross sections (pb) for $B_{c}^{**}$ production at two electron-proton colliders. Four electron-proton colliding energies are adopted, i.e., $\sqrt{S}$=1.30 and 1.98 TeV for LHeC, and $\sqrt{S}$=7.07 and 10.0 TeV for FCC-ep

| $\sqrt{S}$ | $\sigma_{1P_{1}}$ | $\sigma_{1P_{3}}$ | $\sigma_{3P_{0}}$ | $\sigma_{3P_{1}}$ | $\sigma_{3P_{2}}$ | Total |
|-------------|------------------|------------------|------------------|------------------|------------------|-------|
| LHeC 1.30   | 0.049            | 0.013            | 0.027            | 0.071            | 2.300            | 1.098 |
|            | 0.073            | 0.019            | 0.041            | 0.107            | 3.804            | 1.806 |
| 1.98        | 0.168            | 0.047            | 0.096            | 0.253            | 11.828           | 5.554 |
| FCC-ep 7.07 | 0.214            | 0.058            | 0.123            | 0.322            | 16.185           | 7.585 |
|            | 0.214            | 0.058            | 0.123            | 0.322            | 16.185           | 7.585 |
| 10.0        | 0.049            | 0.013            | 0.027            | 0.071            | 2.300            | 1.098 |
|            | 0.073            | 0.019            | 0.041            | 0.107            | 3.804            | 1.806 |
|             | 0.168            | 0.047            | 0.096            | 0.253            | 11.828           | 5.554 |
|             | 0.214            | 0.058            | 0.123            | 0.322            | 16.185           | 7.585 |

The $B_{c}^{**}$ transverse momentum ($p_T$) distributions of all the production channels at the LHeC are shown in Figs.1(a,b). From the figures we can see that the contribution from $\gamma + \bar{b}$ channel dominates in the low $p_T$ regions, but drops down more drastically than that from the $\gamma + g$ channel. Although the contribution of the $\gamma + g$ channel is suppressed in the low $p_T$ region, it becomes the main contribution when the $p_T$ goes up to large value range. We can see if one wants to select the P-wave $B_{c}^{**}$ production signal from the $\gamma + g$ production channel at the LHeC, one can simply accept the events by imposing proper high lower $p_T^{\text{lower}}$ cut ($p_T > p_T^{\text{lower}}$),
and then the heavy quark initiated P-wave $B_{c}^{**}$ production events can be suppressed. We find also from both Figs.1(a) and (b) that each differential cross section curve for the P-wave $B_{c}^{**}$ meson from the $\gamma + \bar{b}$ production channel, has a peak around 1 GeV, which is quantitatively exceeded one or two order of the those for the $\gamma + g$ and $\gamma + c$ production channels. Thus, if the $p_{T}$ acceptance range of the $B_{c}^{**}$ mesons is limited all around 1 GeV, it is advantageous for the investigation of the P-wave $B_{c}^{**}$ mesons from the $\gamma + \bar{b}$ production channel.

![Figure 1](image.png)

Figure 1: The transverse momentum distributions for the $B_{c}^{**}$ meson at the $\sqrt{S} = 1.30$ TeV LHeC. (a) For the processes of $e^{-} + P \rightarrow \gamma + i \rightarrow B_{c}^{**}(1P_{1},3P_{0})$. (b) For the processes of $e^{-} + P \rightarrow \gamma + i \rightarrow B_{c}^{**}(3P_{1},3P_{2})$.

We present the rapidity ($y$) distributions of $B_{c}^{**}$ mesons in Figs.2(a,b). There the asymmetry rapidity distributions indicate that the dominate contributions are located in the region around $y = 1$, due to the colliding energies of the incoming particles being not equal and the majority of the photons radiated from electron beams carrying less energies than the partons in the protons. Figs.2(a,b) show that the $B_{c}^{**}$ meson rapidity distributions of all the three channels drop sharply when $y$ increases from 2 to 3, while go down gently if $y$ decreases from -4.

We know that the total cross section for $B_{c}^{**}$ production should be sensitive to various physical cuts, such as the $p_{T}$ and $y$ cuts on the final mesons. The cross sections by applying different $p_{T}$ and $y$ cuts on $B_{c}^{**}$ mesons are presented in Table II and Table III separately. The data show that if the events are accepted with the condition of $p_{T} > 1.0$ GeV, the cross section via $\gamma + \bar{b}$ channel is about one order larger than that from the $\gamma + g$ channel, which can be demonstrate also form the $p_{T}$ distributions shown in Figs.1(a,b). By summing up the three channels and the four P-wave $B_{c}^{**}$ states, we can get the ratio ($\frac{\sigma_{\text{cuts}}}{\sigma_{\text{No cuts}}}$) as 59.09%, 9.34%, 2.81%.
Figure 2: The rapidity distributions for the $B_{c}^{**}$ meson at the $\sqrt{S} = 1.30$ TeV LHeC. (a) For the processes of $e^- + P \rightarrow \gamma + i \rightarrow B_{c}^{**}(1P_1/3P_0)$. (b) For the processes of $e^- + P \rightarrow \gamma + i \rightarrow B_{c}^{**}(3P_1/3P_2)$.

by applying $p_T$ cuts as 1.0 GeV, 3.0 GeV and 5.0 GeV, respectively.

Table II: The cross sections (pb) for the $B_{c}^{**}$ production at the $\sqrt{S} = 1.30$ TeV LHeC under various $p_T$ cuts.

| $p_T$ | $1P_1$ | $3P_0$ | $3P_1$ | $3P_2$ | $1P_1$ | $3P_0$ | $3P_1$ | $3P_2$ | $1P_1$ | $3P_0$ | $3P_1$ | $3P_2$ |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $\sigma_{\gamma c}$ | 0.045 | 0.011 | 0.024 | 0.065 | 0.026 | 0.006 | 0.011 | 0.034 | 0.012 | 0.002 | 0.004 | 0.013 |
| $\sigma_{\bar{s}b}$ | 1.437 | 0.343 | 1.015 | 3.400 | 0.206 | 0.009 | 0.104 | 0.351 | 0.042 | 0.001 | 0.023 | 0.053 |
| $\sigma_{\gamma g}$ | 0.121 | 0.030 | 0.074 | 0.280 | 0.083 | 0.021 | 0.055 | 0.176 | 0.046 | 0.012 | 0.034 | 0.084 |
| Total | 6.844 | 1.082 | 0.326 |

Table III: The cross sections (pb) for the $B_{c}^{**}$ production at the $\sqrt{S} = 1.30$ TeV LHeC under various $y$ cuts.

| $y$ | $1P_1$ | $3P_0$ | $3P_1$ | $3P_2$ | $1P_1$ | $3P_0$ | $3P_1$ | $3P_2$ | $1P_1$ | $3P_0$ | $3P_1$ | $3P_2$ |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $\sigma_{\gamma c}$ | 0.019 | 0.005 | 0.010 | 0.027 | 0.034 | 0.008 | 0.017 | 0.047 | 0.042 | 0.010 | 0.022 | 0.060 |
| $\sigma_{\bar{s}b}$ | 1.035 | 0.489 | 1.017 | 2.342 | 1.776 | 0.844 | 1.752 | 4.036 | 2.090 | 0.997 | 2.066 | 4.751 |
| $\sigma_{\gamma g}$ | 0.058 | 0.014 | 0.036 | 0.137 | 0.100 | 0.024 | 0.060 | 0.234 | 0.117 | 0.028 | 0.071 | 0.276 |
| Total | 5.184 | 8.932 | 10.530 |

In photoproduction experiments, the study on inelastic $B_c$ events can give information on the gluon distribution in the nucleon [65], and the inelasticity of the photoproduction can be denoted by the variable $z = \frac{p_{Bc} p_P}{p_\gamma p_P}$, where $p_{Bc}$, $p_\gamma$ and $p_P$ denote the momenta of the $B_c$, $\gamma$
and proton, respectively. In the elastic domain, \( z \approx 1 \), and at low \( z \) region, the resolved effect should also make some contributions [66]. Normally one can obtain clean samples of inelastic events in the range of \( 0.3 \lesssim z \lesssim 0.9 \) [66–69]. Figs 3(a,b) show the distributions of variable \( z \), and in Table IV we list the total cross sections at the \( \sqrt{S} = 1.30 \) TeV LHeC for different \( B_c^{**} \) production processes by accepting the events in the \( z \) range of \( 0.3 \leq z \leq 0.9 \).

Table IV: The cross section (pb) in the range of \( 0.3 \leq z \leq 0.9 \) for different \( B_c^{**} \) production processes at the \( \sqrt{S} = 1.30 \) TeV LHeC.

| \( \sigma \) (pb) | \( ^1P_1 \) | \( ^3P_0 \) | \( ^3P_1 \) | \( ^3P_2 \) | Total |
|----------------|-------------|-------------|-------------|-------------|-------|
| \( \sigma_{\gamma g} \) | 0.009 | 0.001 | 0.049 | 0.199 | 0.504 |
| \( \sigma_{\gamma b} \) | 1.390 | 0.680 | 1.399 | 3.260 | 6.729 |
| \( \sigma_{\gamma c} \) | 0.033 | 0.004 | 0.010 | 0.037 | 0.085 |

Figure 3: Differential cross sections \( d\sigma/dz \) versus \( z \) for the \( B_c^{**} \) production at the \( \sqrt{S} = 1.30 \) TeV LHeC. (a) For the processes of \( e^− + P \rightarrow \gamma + i \rightarrow B_c^{**}(^1P_1/3P_0) \). (b) For the processes of \( e^− + P \rightarrow \gamma + i \rightarrow B_c^{**}(^3P_1/3P_2) \).

For the scale uncertainty, we present the results for \( \mu = 0.75M_T, M_T \) and \( 1.25M_T \) in Table V separately. For the \( \gamma + c \) and \( \gamma + g \) production channels, the cross sections decrease slightly when the scale becomes larger, while for the \( \gamma + \bar{b} \) channels, the situation is opposite. The scale uncertainties for the cross section (defined as \( \sigma(\mu = \mu') - \sigma(\mu = M_T) \) with \( \mu' = 0.75, 1.25M_T \)) via the \( \gamma + c, \gamma + \bar{b} \) and \( \gamma + g \) channels are \(-6\% \sim 8\%\), \(-79\% \sim 31\%\) and \(-11\% \sim 15\%\), respectively. By summing up all the three channels and the four P-wave \( B_c^{**} \) states, the total cross section still increases with the increment of the scale owing to the large contributions from the \( \gamma + \bar{b} \) channels. Such a large scale dependence could be reduced by involving higher-order QCD corrections or improved by taking a proper QCD scale value [70][71].
Table V: The cross sections (pb) for the $B_{c}^{* *}$ meson production at the $\sqrt{S} = 1.30$ TeV LHeC for $\mu = 0.75 M_T$, $M_T$ and $1.25 M_T$ separately.

| $\mu$       | $\sigma_{\gamma c}$ | $\sigma_{\gamma b}$ | $\sigma_{\gamma g}$ | Total          |
|-------------|----------------------|----------------------|----------------------|----------------|
| 0.75 $M_T$  | 0.053                | 0.014                | 0.029                | 0.077          |
| $M_T$       | 0.049                | 0.013                | 0.027                | 0.071          |
| 1.25 $M_T$  | 0.046                | 0.012                | 0.025                | 0.067          |

Finally, we discuss the uncertainties from different choices of quark masses. We take $m_c = 1.50\pm0.20$ GeV and $m_b = 4.90\pm0.40$ GeV into account. The $m_c$ is fixed as its center values when discussing the uncertainty from $m_b = 4.90 \pm 0.40$ GeV and vice versa. The cross sections under different value of $m_c$ and $m_b$ are presented in Table VI and Table VII, respectively. From the tables we can see that for most of the channels, the cross sections decrease when the $c$ or $b$-quark mass becomes larger. The only exception is for the $\gamma + \bar{b}$ channel, whose cross sections increase when the $b$-quark mass becomes larger. The cross sections are much more sensitive to $m_c$ than to $m_b$, however, the $\gamma + \bar{b}$ channel provides most of the contributions, thus we conclude that the total cross section decreases when the $c$-quark mass becomes larger or the $b$-quark mass becomes smaller. Besides, it is clear that the cross sections of P-wave states are more sensitive to quark masses than the S-wave states as declared in Ref. [33, 37]. By summing up the cross sections of all production channels and their mass uncertainties, we obtain the total cross sections as

$$\sigma_{LHeC}^{Total} = 11.584^{+15.745}_{-6.060} \text{ pb}, \text{ for } m_c = 1.50 \pm 0.20 \text{ GeV},$$  \hspace{1cm} (3.1)

$$\sigma_{LHeC}^{Total} = 11.584^{+3.659}_{-5.274} \text{ pb}, \text{ for } m_b = 4.90 \pm 0.40 \text{ GeV},$$ \hspace{1cm} (3.2)

and by adding the errors from two mass uncertainties in quadrature, we finally obtain

$$\sigma_{LHeC}^{Total} = 11.584^{+16.165}_{-8.634} \text{ pb}, \text{ for } m_b = 4.90 \pm 0.40 \text{ GeV and } m_c = 1.50 \pm 0.20 \text{ GeV}. \hspace{1cm} (3.3)$$

That means the contribution from total cross section of the P-wave $B_{c}^{* *}$ states can be $20\% \sim 26\%$ of that from the S-wave $B_{c}^{(*)}$ state production [39] in the range of the uncertainties of heavy quark masses. So these excited state contributions should be taken into consideration, especially for the future high energy and high luminosity colliders.
Table VI: The cross sections (pb) for $B_c^{**}$ production by taking different value of $m_c$ and fixing $m_b = 4.90$ GeV at the $\sqrt{S} = 1.30$ TeV LHeC.

| $m_c$ (GeV) | 1.30   | 1.50   | 1.70   |
|------------|--------|--------|--------|
| $\sigma_{\gamma c}(^1P_1)$ | 0.063  | 0.049  | 0.038  |
| $\sigma_{\gamma c}(^0P_3)$ | 0.015  | 0.013  | 0.011  |
| $\sigma_{\gamma c}(^1P_3)$ | 0.034  | 0.027  | 0.021  |
| $\sigma_{\gamma c}(^2P_3)$ | 0.096  | 0.071  | 0.054  |
| $\sigma_{\gamma b}(^1P_1)$ | 5.632  | 2.300  | 1.052  |
| $\sigma_{\gamma b}(^0P_3)$ | 2.523  | 1.098  | 0.531  |
| $\sigma_{\gamma b}(^1P_3)$ | 5.479  | 2.273  | 1.054  |
| $\sigma_{\gamma b}(^2P_3)$ | 12.547 | 5.219  | 2.427  |
| $\sigma_{\gamma g}(^1P_1)$ | 0.146  | 0.127  | 0.112  |
| $\sigma_{\gamma g}(^0P_3)$ | 0.055  | 0.031  | 0.019  |
| $\sigma_{\gamma g}(^1P_3)$ | 0.155  | 0.077  | 0.042  |
| $\sigma_{\gamma g}(^2P_3)$ | 0.593  | 0.299  | 0.165  |
| Total        | 27.338 | 11.584 | 5.524  |

IV. SUMMARY

In this paper, we studied the P-wave excited $B_c^{**}$ ($^1P_1$ and $^3P_J$ with $J = 0, 1, 2$) meson photoproduction at the LHeC and three photoproduction channels, i.e., $e^- + p \rightarrow \gamma + g \rightarrow B_c^{**} + b + \bar{c}$, $e^- + p \rightarrow \gamma + c \rightarrow B_c^{**} + b$ and $e^- + p \rightarrow \gamma + \bar{b} \rightarrow B_c^{**} + c$ are considered here. It is found that the production of the P-wave $B_c^{**}$ states can contribute about 20% of the S-wave $B_c^{(*)}$ production at the LHeC and FCC-ep, if considering the fact that almost all of the P-wave $B_c^{**}$ states decay to the ground state $B_c(1S_0)$. Therefore, for the studying the production of $B_c$ meson, the P-wave excited states should also be included. Taking the most prominent errors from the heavy quark masses, $m_b = 4.90 \pm 0.40$ GeV and $m_c = 1.50 \pm 0.20$ GeV, into account, we would expect to accumulate about $(2.48^{+3.45}_{-1.55}) \times 10^4 B_c^{**}(^1P_1)$, $(1.14^{+1.49}_{-0.82}) \times 10^4 B_c^{**}(^3P_0)$, $(2.38^{+3.39}_{-1.74}) \times 10^4 B_c^{**}(^3P_1)$ and $(5.59^{+7.84}_{-3.93}) \times 10^4 B_c^{**}(^3P_2)$ events at the LHeC in one operation year with $\sqrt{S} = 1.30$ TeV collision energy and the luminosity $\mathcal{L} = 10^{33}$ cm$^{-2}$ s$^{-1}$. We find the dominating contributions are comes from the small $p_T$ region of $\gamma + \bar{b}$ channel, and with the abilities of small $p_T$ tagging technology, it is possible to directly measuring the P-wave $B_c^{**}$ state experimentally at the LHeC or FCC-ep and thus is helpful in understanding the mass spectrum of the $(c\bar{b})$ bound states and testing the potential models.
Table VII: The cross sections (pb) for $B_{c}^{**}$ production by taking different value of $m_b$ and fixing $m_c = 1.50$ GeV at the $\sqrt{S} = 1.30$ TeV LHeC.

| $m_b$ (GeV) | 4.50 | 4.90 | 5.30 |
|------------|------|------|------|
| $\sigma_{\gamma c}^{(1)P_1}$ | 0.074 | 0.049 | 0.033 |
| $\sigma_{\gamma c}^{(0)P_3}$ | 0.021 | 0.013 | 0.008 |
| $\sigma_{\gamma c}^{(1)P_3}$ | 0.041 | 0.027 | 0.018 |
| $\sigma_{\gamma c}^{(2)P_3}$ | 0.107 | 0.071 | 0.050 |
| $\sigma_{\gamma b}^{(1)P_1}$ | 1.081 | 2.300 | 3.133 |
| $\sigma_{\gamma b}^{(0)P_3}$ | 0.493 | 1.098 | 1.470 |
| $\sigma_{\gamma b}^{(1)P_3}$ | 1.030 | 2.273 | 3.099 |
| $\sigma_{\gamma b}^{(2)P_3}$ | 2.465 | 5.219 | 7.044 |
| $\sigma_{\gamma g}^{(1)P_1}$ | 0.439 | 0.127 | 0.085 |
| $\sigma_{\gamma g}^{(0)P_3}$ | 0.045 | 0.031 | 0.021 |
| $\sigma_{\gamma g}^{(1)P_3}$ | 0.106 | 0.077 | 0.058 |
| $\sigma_{\gamma g}^{(2)P_3}$ | 0.410 | 0.299 | 0.223 |
| Total       | 6.310 | 11.584 | 15.243 |

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