Possible explanation of $Zbb$ vertex anomaly
in one-family technicolor models

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Abstract

In technicolor theories a large radiative correction to the $Zbb$ vertex generally emerges, because of the large top quark mass and $SU(2)_L$ gauge symmetry. The correction can give us an explanation for the deviation of the experimental value of $R_b \equiv \Gamma_b/\Gamma_h$ from the standard-model prediction. However, generally the $T$ parameter becomes unacceptably large. We show that in the one-family technicolor model recently proposed by the authors all the $S$, $T$, and $U$ parameters can be consistent with the experimental bounds while the observed anomaly of $R_b$ is naturally explained.

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The measurement of the quantity $R_b \equiv \Gamma_b/\Gamma_h$ at LEP shows a large deviation from the prediction of the standard model. The measured value $R_b = 0.2202 \pm 0.0020$ deviates at 2-\(\sigma\) level from the standard-model prediction $R_b = 0.2157$ ($m_t = 175$ GeV) \cite{1,2}. The recent preliminary experimental value $R_b = 0.2219 \pm 0.0017$ shows the deviation at 3.6-\(\sigma\) level \cite{3}. This may be a signature of new physics.

It has been pointed out that the “sideways” gauge boson of extended technicolor (ETC) theories generates a significant correction to the $Zbb$ vertex \cite{4}. The reason is that the relatively light sideways gauge boson associated with the top quark mass generation must couple with the left-handed bottom quark according to the $SU(2)_L$ symmetry. The “diagonal” gauge boson which appears in most ETC models also generates a correction to the $Zbb$ vertex \cite{5}. The magnitude of this diagonal contribution is comparable with the sideways contribution and the sign is opposite \cite{6}. The sideways and the standard-model contributions make $R_b$ small, while the diagonal contribution makes it large. Therefore, if the diagonal contribution is large enough to cancel out the other two contributions, the LEP result can be explained. It has been shown that the diagonal contribution can naturally explain the LEP result of $R_b$ in some ETC models \cite{7}.

However, the contribution of the diagonal ETC boson to the $T$ parameter \cite{8} generally becomes large in comparison with the experimental bound, because of the large weak isospin violation in the couplings between the diagonal ETC boson and the right-handed fermions (top and bottom quarks and techni-fermions). It has been shown that it is almost impossible in the naive ETC theory to explain the LEP result of $R_b$ without conflict with the experimental bound on the $T$ parameter \cite{9}. Some mechanisms which generate a negative contribution to the $T$ parameter are needed to explain the experimental value of $R_b$.

We stress here the fact that the large contributions to both the $Zbb$ vertex correction and the $T$ parameter are not peculiar to a specific ETC theory but generally result in any technicolor scenario with some underlying physics at TeV scale for the fermion mass generation. To generate the heavy top quark, at least the effective four-fermion interaction
\[ \mathcal{L}^{UR} = \frac{1}{\Lambda^2} (\bar{Q}_L U_R)(\bar{t}_R \psi_L) + \text{h.c.} \quad (1) \]

must be emerged, where \( Q_L = (U_L D_L)^T \) and \( \psi_L = (t_L b_L)^T \) are the \( SU(2)_L \) doublets, and \( U \) and \( D \) are the techni-quarks. If we consider the QCD-like one-family technicolor model, the scale \( \Lambda \simeq 260\text{GeV} \) with \( m_t = 175\text{GeV} \). Here, we used the relation \( \langle \bar{U}U \rangle \simeq 4\pi F_\pi^3 \) (taking the number of the technicolor \( N_{TC} = 3 \) \[11\]), and set \( F_\pi \simeq 125\text{GeV} \) (\( F_\pi \) is the decay constant of the composite Nambu-Goldstone boson). Namely, some unknown dynamics has to emerge at the scale of order \( \Lambda \simeq 260\text{GeV} \).

In general, other kind of effective four-fermion interactions also emerge at the scale \( \Lambda \). The effective four-fermion interactions with \( U_R \)
\[ \mathcal{L}_{1UR}^{UR} = \frac{g_1^U}{\Lambda^2} (\bar{\psi}_L U_R)(\bar{U}_R \psi_L), \quad (2) \]
\[ \mathcal{L}_{2UR}^{UR} = \frac{g_2^U}{\Lambda^2} (\bar{U}_R \gamma^\mu U_R)(\bar{U}_R \gamma^\mu U_R), \quad (3) \]
are expected to emerge with the scale \( \Lambda \) and \( g_{1,2}^U \sim 1 \). The interaction of eq.(2) gives the correction to the \( Zb_L b_L \) vertex, and the interaction of eq.(3) gives the contribution to the \( T \) parameter.

The order of the correction to the \( Zb_L b_L \) coupling \( g_L^b \) (at tree-level in the standard model \( g_L^b = g_Z \{-\frac{1}{2} + \frac{1}{3}s^2\} \), where \( g_Z = \sqrt{4\pi \alpha/c^2 s^2} \), and \( s \) and \( c \) denote the sine and cosine of the Weinberg angle, respectively) due to the interaction of eq.(2) is estimated as
\[ |(\delta g_L^b)_A| \simeq \frac{g_Z F_\pi^2}{8 \Lambda^2} |g_1^U| \simeq 0.021|g_1^U|, \quad (4) \]
which should be compared with the experimental bound on the total correction to the coupling \( g_L^b \) from \( R_b \) data alone \[12\]
\[ \delta g_L^b = -0.0004 \pm 0.0019 \quad (5) \]
and the standard-model contribution \( (\delta g_L^b)_{SM} = 0.0037 \) \[2\]. If the magnitude of the coupling \( g_1^U \) is of the order of \( 10^{-1} \) and the sign is negative, the contribution of the standard model can be canceled out and the experimental value of eq.(5) can be explained. In this estimation
we used the method in which the techni-fermion current is replaced by the corresponding current in the low energy effective theory \[4\]. The significantly large correction of eq.(4) comes from the large top quark mass and \(SU(2)_L\) symmetry.

The order of the contribution to the \(T\) parameter due to the interaction of eq.(3) is estimated as

\[
|T_\Lambda| \simeq \frac{g_2^2 N_C^2}{16\alpha} \frac{F_\pi^4}{m_Z^2 \Lambda^2} \left| g_2^U \right| \simeq 17 \left| g_2^U \right|, \tag{6}
\]

where \(N_C = 3\). This value should be compared with the experimental bound \[12\]

\[
T = 0.34 \pm 0.20 \tag{7}
\]

for \(m_t = 175\text{GeV}\) and \(m_H = 1\text{TeV}\). We used also the method of the current replacement with the factorization hypothesis \[3\]. This extremely large contribution to the \(T\) parameter comes from the large weak isospin violation in the mechanism of the fermion mass generation. Even if \(\left| g_2^U \right|\) is of the order of \(10^{-1}\) as we expect for \(\left| g_1^U \right|\), the magnitude of \(|T_\Lambda|\) is still much larger than the experimental bound.

Therefore, having large contributions to the \(Zb_L b_L\) vertex correction and the \(T\) parameter is the generic in the technicolor scenario, if there are no special cancelations or suppressions. If the large deviation of the experimental value of \(R_b\) from the the standard-model prediction is naturally explained in the technicolor scenario with the appropriate value of \(g_1^U\), some mechanisms to generate the negative contribution to the \(T\) parameter and/or some mechanisms which suppress the magnitude of \(g_2^U\) are needed.

In the one-family technicolor model recently proposed by the authors \[13\] we have a mechanism for generating large negative \(T\) parameter contributions while both the \(S\) and \(U\) parameters are consistent with the experimental bounds. The one-family technicolor model has the separate structure of the technicolor gauge interactions \(SU(3)_T^{Q} \times SO(3)_T^{L} \times U(1)_B-L\), where \(SU(3)_T^{Q}\) couples only with the techni-quarks in the triplet representation, \(SO(3)_T^{L}\) couples only with the techni-leptons in the triplet representation, and \(U(1)_B-L\) denotes the techni-\((B-L)\) symmetry which is spontaneously broken. This structure realizes...
the minimum value of the number of technicolors \(N_{TC}\) (the smaller \(N_{TC}\) means the smaller \(S\) parameter) while the techni-leptons belong to the real representation of the technicolor gauge group, which is needed to have the gauge invariant Majorana mass of the right-handed techni-neutrino. The Majorana mass can generate the negative contribution to the \(T\) parameter.

In the following we estimate how large the correction to \(g_L^t\) may be keeping the values of the \(S\), \(T\), and \(U\) parameters consistent with the experimental bounds in this one-family technicolor model. We consider the effective model of the fermion mass generation in ref. [7], since it can naturally explain the observed anomaly of \(R_b\). Although the effective model is inspired by an ETC theory, it is not itself an ETC model. We do not always restrict the fermion mass generation mechanism to the ETC theory, but imagine some underlying physics at TeV scale for the fermion mass generation.

We obtain the effective model of the fermion mass generation in the following way. First we consider the naive ETC scenario with no weak isospin violation, for example, \(SU(4)^Q_{ETC} \times SU(4)^L_{ETC} \times U(1)^{ETC}_{B-L} \rightarrow SU(3)^Q_{ETC} \times SO(3)^L_{ETC} \times U(1)^{ETC}_{T^F_{B-L}} \) broken. Then we replace the gauge coupling of the massive sideways and diagonal ETC bosons by the effective couplings so that the weak isospin violation is included. Therefore, this effective model is not a gauge theory, but we assume that the massive particles like the sideways and diagonal ETC bosons in this model emerge due to the underlying physics at the TeV scale. In other words, we assume that a part of the low-energy effective interactions of the underlying theory is well described by the following effective model.

The exchanges of some massive particles like the sideways ETC gauge boson give the effective four-fermion interactions

\[
\mathcal{L}_1^S = -\frac{g_t^2}{M_S^2} \left( \bar{Q}_L \gamma^\mu \psi_L \right) \left( \bar{t}_R \gamma^\mu U_R \right) + \text{h.c.},
\]

\[
\mathcal{L}_2^S = -\frac{(g_t \xi_t)^2}{M_S^2} \left( \bar{Q}_L \gamma^\mu \psi_L \right) \left( \bar{\psi}_L \gamma^\mu Q_L \right),
\]

where \(M_S\) denotes the typical mass of the exchanged particles, and \(g_t \xi_t\) and \(g_t / \xi_t\) are the effective coupling constants of the left-handed current and the right-handed current with \(t_R\),
respectively. We assume that the perturbative condition
\[ \frac{(g_t \xi_t)^2}{4\pi} < 1 \quad \text{and} \quad \frac{(g_t / \xi_t)^2}{4\pi} < 1 \] (10)
should be satisfied. The possible range of \( \xi_t^2 \) is determined by this condition, when the value of \( g_t^2 \) is given. The mass of the top quark is generated through the interaction of eq. (8) as
\[ m_t \approx \frac{g_t^2}{M_S^2} 4\pi F_\pi^3 \sqrt{\frac{N_C}{N_{TC}}}, \] (11)
where the relation \( \langle \bar{U} U \rangle \approx 4\pi F_\pi^3 \sqrt{N_C/N_{TC}} \) is used (from the naive dimensional analysis \cite{10} and the leading \( 1/N \) behavior). We can determine the value of \( g_t^2 \), when the value of \( M_S \) is given (\( N_{TC} = 3 \) in our model). The exchanges of some massive particles like the diagonal ETC gauge boson give
\[ \mathcal{L}^D = -\frac{1}{M_D^2} J_D^\mu J_{D\mu}, \] (12)
where
\[ J_{D\mu} = \bar{g}_t \xi_t \sqrt{\frac{N_{TC}}{N_{TC} + 1}} \left( a \bar{\psi}_L \gamma_\mu \psi_L - b \frac{1}{N_{TC}} \bar{Q}_L \gamma_\mu Q_L \right) \]
\[ + \frac{g_t}{\xi_t} \sqrt{\frac{N_{TC}}{N_{TC} + 1}} \left( a \bar{t}_R \gamma_\mu t_R - b \frac{1}{N_{TC}} \bar{U}_R \gamma_\mu U_R \right) \]
\[ + \frac{g_t}{\xi_b} \sqrt{\frac{N_{TC}}{N_{TC} + 1}} \left( a \bar{b}_R \gamma_\mu b_R - b \frac{1}{N_{TC}} \bar{D}_R \gamma_\mu D_R \right), \] (13)
\( M_D \) denotes the typical mass of the exchanged particle, and \( g_t / \xi_b \) is the effective coupling constant of the right-handed current with \( b_R \). The constants \( a \) and \( b \) equal unity and \( M_D = M_S \) in the naive ETC model based on the one-family technicolor theory. In general, the constants \( a \) and \( b \) are expected to be of the order of unity and \( M_D \simeq M_S \).

The interactions of eq. (11) and (12) generate the correction to \( g_L^b \) (see ref. \cite{7}):
\[ (\delta g_L^b)_\Lambda = \left( \xi_t^2 - ab \frac{2N_C}{N_{TC} + 1} \right) \frac{m_t}{16\pi F_\pi} \sqrt{\frac{N_{TC}}{N_C}} g_L \]
\[ \approx \left( \xi_t^2 - \frac{3}{2} ab \right) \cdot 2.1 \times 10^{-2}, \] (14)
where we have assumed that \( M_D \simeq M_S \) and used eq. (11). The first term in the brackets comes from the interaction of eq. (9), and the second term comes from the interaction of
eq. (12). The interactions of eq. (12) generate large contribution to the $T$ parameter (see ref. [14]):

$$T_A = \frac{1}{16\epsilon^2 s^2} \frac{m_t F_F}{m_Z^2} \frac{N_C}{N_{TC}} + \left( \frac{N_{TC}}{N_C} \right) \left( 1 - \frac{m_b}{m_t} \right) \frac{b^2}{\frac{\xi_b}{\xi_t}} \simeq 0.93 \frac{b^2}{\xi_t^2}. \tag{15}$$

(In most ETC models the massive ETC gauge boson which belongs to the adjoint representation of the technicolor gauge group generates unacceptably large positive contribution to the $T$ parameter. However, we assume that our underlying theory does not have such a dangerous adjoint vector bosons [13].)

In the one-family technicolor model of ref. [13] the value of the $T$ parameter depends on the values of the Dirac masses of the techni-neutrino and techni-electron $m_N$ and $m_E$, respectively. The values of $m_N$ and $m_E$ are dynamically determined, if we fixed the values of the Majorana mass of the right-handed techni-neutrino $M$, the mass and coupling of the $U(1)^{TF}_{B-L}$ gauge boson ($m_{B-L}$ and $\alpha_{B-L}$, respectively), and the coupling of the $SO(3)^L_{TC}$ gauge bosons.

The calculation of the vacuum energy in the one gauge boson exchange approximation shows that the difference between the values of $m_N$ and $m_E$ is about 60GeV, when $M = 250\text{GeV}$, $m_{B-L} = 250\text{GeV}$, and $\alpha_{B-L} = 0.3$. These values of $M$, $m_{B-L}$, and $\alpha_{B-L}$, and the value $\omega = 0.07$ which is the strength of the kinetic mixing between the $U(1)^{TF}_{B-L}$ and $U(1)_Y$ gauge bosons are selected so that the correct electroweak symmetry breaking occurs and the values of the $S$ and $U$ parameters are consistent with the experimental bounds. Both the Majorana mass $M$ and the tree-level kinetic mixing $\omega$ play an important role to get the large negative contribution to the $S$ parameter through the $U(1)^{TF}_{B-L}$ gauge boson exchange. Large kinetic mixing between the $W^3$ and $U(1)_Y$ gauge bosons comes from the loop-level mass mixing between the $W^3$ and $U(1)^{TF}_{B-L}$ gauge bosons due to the Majorana mass, and the tree-level kinetic mixing $\omega$ between the $U(1)^{TF}_{B-L}$ and $U(1)_Y$ gauge bosons.

If we take $m_N = 340\text{GeV}$ and $m_E = 400\text{GeV}$, we have $S_{TC} \simeq -0.0092$, $T_{TC} \simeq -0.21$, and $U_{TC} \simeq 0.022$. These values of $S_{TC}$ and $U_{TC}$ are consistent with the experimental bounds of $S = 0.068 \pm 0.20$ and $U = -0.41 \pm 0.50$ with $m_t = 175\text{GeV}$ and $m_H = 1\text{TeV}$ [12]. Although a smaller difference of $m_N$ and $m_E$ gives a smaller value of the $T$ parameter,
\( m_E - m_N < 60 \text{GeV} \) is rather hard to reconcile with \( M = 250 \text{GeV} \).

Now we estimate how large the value of \(- (\delta g^b_L)_\Lambda \) can be in this one-family technicolor scenario. A Negative contribution to \( g^b_L \) corresponds to a positive contribution to \( R_b \) which is suggested from the experiment. By using the experimental bound of eq.(4), we can obtain the upper bound on the parameter \( b^2 / \xi^2 \) as

\[
T_\Lambda + T_{TC} = \frac{1}{16c^2s^2}\frac{m_t F_\pi}{m_Z^2} \frac{N_C + 1}{N_{TC} + 1} \left(1 - \frac{m_b}{m_t}\right)^2 \frac{b^2}{\xi^2} - 0.21 < 0.34 + 0.20. \tag{16}
\]

We have \( b^2 / \xi^2 < 0.80 \). From this bound we obtain the lower bound on the \((\delta g^b_L)_\Lambda\):

\[
(\delta g^b_L)_\Lambda > \left( \frac{b^2}{0.80} - ab \frac{2N_C}{N_{TC} + 1} \right) \frac{m_t}{16\pi F_\pi} \sqrt{\frac{N_{TC}}{N_C}} g_Z \simeq b(\frac{b}{0.80} - \frac{3}{2} a) \cdot 0.021. \tag{17}
\]

In the case of \( a = b = 1 \), we have \((\delta b^b_L)_\Lambda > -0.0053\), or \((\delta R_b)_\Lambda < 0.0069\). The perturbative condition eq.(10) is satisfied in this case with \( M_S = 1\text{TeV} \). Then we obtain the bound on the total correction as \( \delta g^b_L = (\delta g^b_L)_{SM} + (\delta g^b_L)_\Lambda > -0.0016 \). Therefore the experimental value of eq.(5) can be explained in this model keeping the \( S, T, \) and \( U \) parameters consistent with the experimental bounds. Both the Majorana mass \( M \) and the tree-level kinetic mixing \( \omega \) play the important role of generating the large negative contribution to the \( S \) parameter through the \( U(1)^{TF}_{B-L} \) gauge boson exchange. The Majorana mass \( M \) is needed to have the mass mixing between the \( U(1)^{TF}_{B-L} \) gauge boson and \( W^3 \), and the tree-level mixing \( \omega \) makes this contribution large. If there is no negative contribution to the \( T \) parameter, namely \( T_{TC} = 0 \), the experimental value of eq.(5) can not be explained, because we have the bounds \((\delta g^b_L)_\Lambda > 0.0047\) and \(\delta g^b_L > 0.0084\) in this case. As long as the model dependent parameters \( a \) and \( b \) satisfy the condition \( b(\frac{b}{0.80} - \frac{3}{2} a) < 0 \), we obtain the negative contribution to \( g^b_L \) (the positive contribution to \( R_b \)) which is favored by the experiment.

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