Comments on Neutrino Tests of Special Relativity

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Abstract

We point out that the assumption of Lorentz noninvariance examined recently by Coleman and Glashow leads to neutrino flavor oscillations which are phenomenologically equivalent to those obtained by assuming the neutrinos violate the principle of equivalence. We then comment on the limits on Lorentz noninvariance which can be derived from solar, atmospheric, and accelerator neutrino experiments.
In a recent paper [1], Coleman and Glashow have proposed several interesting ways to test how well Lorentz invariance is obeyed in nature. If Lorentz invariance is violated, one possible consequence is that the propagation of a free particle will depend on its identity. In the case of massless neutrinos, this may lead to neutrino flavor oscillations because different neutrino species may have different maximum attainable velocities (which are no longer necessarily $c$). For this to happen, it is necessary that the neutrino flavor eigenstates be distinct from their velocity eigenstates, defined to be the energy eigenstates at infinite momentum, so that a flavor eigenstate is a linear superposition of the velocity eigenstates and vice versa.

If one considers the case of two neutrino mixing, say $\nu_e$ and $\nu_\mu$, the $\nu_e$ survival probability is given by

$$P(\nu_e \to \nu_e) = 1 - \sin^2(2\theta) \sin^2(\delta v E L / 2),$$

where $\delta v = v_1 - v_2$ is the difference between the velocities of the velocity eigenstates $\nu_1$ and $\nu_2$, $\theta$ is the mixing angle:

$$\nu_e = \nu_1 \cos \theta - \nu_2 \sin \theta, \quad \nu_\mu = \nu_1 \sin \theta + \nu_2 \cos \theta,$$

$E$ is the neutrino energy and $L$ is the distance traveled by the neutrino.

We would like to point out that the energy dependence described in Eq. (1) is exactly the same as what one will get if one assumes that neutrinos violate the principle of equivalence in a certain way [2], [3]. This is interesting but not totally surprising because general coordinate invariance is violated in both cases. The phenomenology of the case of equivalence principle violation has been studied in some detail over the last several years [4] - [11]. The results of these studies can be straightforwardly translated to set limits on the possible violation of Lorentz invariance. This is what we will discuss in the remainder of this paper.

We shall use the notation of Ref. [10]. It is easy to see that (see, e.g., Eqs. (14) to (16) in Ref. [10]) the parameter $|\delta v|$, which measures the degree of violation of Lorentz invariance, should be compared with $2|\phi \Delta \gamma|$ in Ref. [10], where $\phi$ is the gravitational potential in which the neutrino propagates and $\Delta \gamma$ is a parameter which measures the degree of violation of
the equivalence principle. The two are equivalent for the case of a constant gravitational potential. In addition, the mixing angle $\theta_v$ is equivalent to the mixing angle $\theta_G$ in Ref. [10].

Currently, there are positive indications of neutrino flavor mixing from solar and atmospheric neutrino experiments. These data have been used to obtain allowed regions for the mixing parameters $\sin^2(2\theta_G)$ and $|\phi\Delta\gamma|$ (see Refs. [4], [8] and [10]), which can be translated directly into allowed regions for $\sin^2(2\theta_v)$ and $|\delta v|$.

First of all, due to the specific energy dependence, the solar neutrino data cannot be explained by long-wavelength vacuum oscillations caused by the Lorentz invariance violation. This is because, if the mixing parameters are chosen such that enough $^8\text{B}$ neutrinos are suppressed, there will not be sufficient suppression for the lower energy solar neutrinos, in contradiction to the data. It is therefore necessary to invoke the Mikheyev-Smirnov-Wolfenstein mechanism [12] of matter enhanced transitions in the sun. If we assume $\delta v$ to be constant inside the sun, the situation will be equivalent to the case of constant $\phi$ analyzed in Ref. [10]. Using the solar model of Bahcall and Pinsonneault [13], one would find that a large portion of the mixing parameter space has already been excluded by the solar neutrino data, with two remaining allowed regions: a small mixing angle region for which

$$|\delta v| \sim 6 \times 10^{-19}, \quad 0.002 < \sin^2(2\theta_v) < 0.003,$$

at 90% confidence level, and a large mixing angle region for which

$$4 \times 10^{-22} < |\delta v| < 4 \times 10^{-21}, \quad 0.38 < \sin^2(2\theta_v) < 0.81,$$

also at 90% confidence level. Furthermore, the energy dependence implies that higher energy neutrinos have shorter oscillation length. As a consequence, the higher energy atmospheric neutrino data imply a violation of Lorentz invariance in a small but overlapping parameter region (see Fig. 4 in Ref. [10]). It is quite remarkable that the mixing of two neutrinos is sufficient to account for both the solar neutrino and the atmospheric neutrino data.

Aside from offering a possible resolution to the solar neutrino problem, velocity oscillations of neutrinos may provide the most sensitive tests of Lorentz invariance and the
equivalence principle. Unlike conventional neutrino oscillations, velocity oscillations become more important at higher energies. Presently available accelerator data already provide useful constraints, as mentioned in Ref. [1] (see also Fig. 1 in Ref. [10]). In fact, part of the large angle region allowed by the solar neutrino data may be ruled out by the accelerator neutrino data. Planned long-baseline neutrino oscillation experiments will be able to push the limit on $|\delta v|$ lower by one to two orders of magnitude, thereby limiting possible departures from special relativity or the equivalence principle.

Whether neutrinos have observable masses is a central question of particle physics. It is often said that the observation of neutrino oscillations at accelerators (or their deduction from solar neutrino or cosmic ray experiments) would be conclusive evidence that at least one neutrino is massive. This is not true! Neutrino oscillations can also result from a tiny breakdown of Lorentz invariance and/or the principle of equivalence. More information than mere detection is needed to determine the underlying mechanism of neutrino oscillation. The differing energy dependence between the mass mechanism and the mechanism due to Lorentz noninvariance (or due to equivalence principle violation) suggests that an accurate spectral measurement is required. Super Kamiokande and SNO can accurately measure the solar neutrino spectrum, and, as the analysis in Ref. [8] shows, this measurement will test the viability of the small mixing region. The current atmospheric neutrino data favor the large mixing region and this possibility will be tested by new atmospheric neutrino data from Super Kamiokande which should be available in the very near future.

It is of course a distinct possibility that neutrinos have nondegenerate masses and, at the same time, Lorentz invariance and/or the principle of equivalence is violated. The phenomenology of neutrino oscillations in this case will be much more complicated. As Coleman and Glashow pointed out, and as also discussed in section 2.3 of Ref. [10], for the case of two neutrino mixing there is an additional phase parameter beside the doubling of mixing parameters. This will present a serious challenge to future neutrino experiments.
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