An important un-answered question is whether the results of the two spectroscopies could be reconciled in a single theoretical framework that does not abandon the sudden approximation. More broadly, can we understand the wide variety of observed lines shapes in a theoretical framework with a sound microscopic basis and a single set of parameters?

In this Letter, we confront a recent theory of Extremely Correlated Fermi Liquids (ECFL) proposed by Shastry with the above challenge. The new formalism is complex and requires considerable further effort to yield numerical results in low dimensions. In the limit of high enough dimensions, however, a remarkably simple expression for the Green’s function emerges; it is significant in that it differs from the standard Fermi Liquid Dyson form, while satisfying the usual sum rules. We use this simple version of ECFL Green’s function in this Letter, motivated by the attractive spectral shapes produced with very few parameters. In this Letter we show that already the simplest version of the ECFL theory, with very few parameters, is successful to an unprecedented extent in detailed fitting of a wide variety of normal state cuprate ARPES line shapes. Interesting predictions are made for the higher temperature spectral line skew.

Our focus in this Letter is on the data of optimally doped Bi$_2$Sr$_2$CaCu$_2$O$_{8+δ}$ (Bi2212) and La$_{1.85}$Sr$_{0.15}$CuO$_4$ (LSCO) superconductors in the normal state, taken with $\vec{k}$ along the nodal direction connecting (0,0) to ($\pi/a$, $\pi/a$). Most of the data is taken from the published literature, while some original data are also presented (Bi2212 data in Figs. 4,5). Our sample is an optimally doped Bi2212 (T$_c$ = 91 K), grown by the floating zone method at the Brookhaven National Laboratory (BNL), and was measured at the Stanford Synchrotron Radiation Lightsource (SSRL) beam line 5-4 using 25 eV photons. The resolutions are 15 meV (energy) and 0.3° (angle).

**Line shape model:** The ECFL spectral function is given as a product of an auxiliary Fermi Liquid (aux-FL) spectral function $A_{FL}(\vec{k}, \omega)$ and a second frequency dependent “cairaposis” factor $\tilde{G}_{\omega}$:

$$A(\vec{k}, \omega) = A_{FL}(\vec{k}, \omega) \left(1 - \frac{n}{2} + \frac{n^2}{4} \frac{\epsilon^2}{\omega^2} \right)$$

where $n$ is the number of electrons per CuO$_2$ unit cell, $(X)_+ \equiv \max(X,0)$, $\epsilon^{\vec{k}} = (1 - \frac{\pi}{a}) \epsilon(\vec{k})$, where $\epsilon(\vec{k})$ is the bare one-electron band dispersion (see later). Here, $A_{FL}(\vec{k}, \omega) = \frac{1}{\pi} \Im m \omega \epsilon^{\vec{k}}$, with

$$\Im m \Phi(\omega) = \frac{\omega^2 + \tau^2}{\Omega_0} \exp \left(-\frac{\omega^2 + \tau^2}{\omega_0^2}\right) + \eta$$

where $\tau = \pi k_B T$, $T$ is the temperature, and $\omega$ is to be understood as $\omega - i\sigma$. Here, $\omega_0$ is the aux-FL energy scale (i.e. high $\omega$ cutoff), and $\Omega_0$ governs the lifetime, and, by causality, the quasi-particle weight (i.e. the wave function renormalization) of the aux-FL, $Z_{FL} = (1 + \frac{\omega_0}{\omega_0})^{-1}$, as identified from $\Re \Phi(\omega)$. The ECFL energy scale $\Delta_0$ measures the “average intrinsic inelasticity” of the aux-FL. It is given as

$$\Delta_0 = \int_{-\infty}^{\infty} d\omega f(\omega) (A_{FL}(\vec{k}, \omega)(\epsilon^{\vec{k}} - \omega))_{BZ}$$

where $(\cdot)_{BZ}$ denotes averaging over the first Brillouin zone.
are free parameters. For instance, $\Delta_0$ fits determine additionally, in scattering with surface imperfections. Our origin is in impurity scattering as argued in [8], and ad parameter” [7] with respect to the ECFL theory [4]. Its $Z_{\xi}$ can be calculated using the equation for $Z_{FL}$ and Eq. [3] respectively. The parameter $\eta$ in Eq. (2) is an additional “secondary parameter” [7] with respect to the ECFL theory [4]. Its origin is in impurity scattering as argued in [8], and additionally, in scattering with surface imperfections. Our fits determine $\eta \approx 0.03$ eV for laser ARPES and $\eta \approx 0.15$ eV for conventional ARPES. Greater penetration depth in laser ARPES suggests that it should be less sensitive to surface imperfections, thereby yielding a smaller $\eta$. We therefore propose that this parameter summarizes the effective sample quality in different experiments. The difference in line shapes arising from these values of $\eta$ is demonstrated in Figs. [a,b,c].

Our strategy is to fix a common set of intrinsic parameters for all the materials, and allow $\eta$ to be determined separately for each class of data. The most time consuming part is the calculation of $\Delta_0$, the results of which are summarized in Fig. [a].

In our line shape analysis (1) we first set $n = 0.85$, corresponding to the optimal doping. (2) Here $\xi_F$ is taken to be the un-renormalized band dispersion, taken from the literature [5], and then scaled to fit the observed occupied band width, 1.5 eV, of the Bi2212 ARPES result [10, 11]. (3) We choose $Z_{FL} = 1/3$, to account for the dispersion renormalization due to the high energy kink [10, 12], which in this theory is caused by the energy scale $\omega_0$ (cf. Fig. 3). (4) Finally, in all simulations, we include the finite energy resolution effect and the finite angle resolution effect as a combined Gaussian broadening (10 meV FWHM for laser ARPES and 25 meV FWHM for conventional ARPES) in energy [13].

**Line shape fit for laser ARPES:** Fig. 2 shows the fit of the laser ARPES data with the ECFL line shape. These fits were made using a procedure that is somewhat more restrictive than that in the recent work of Casey and Anderson [14, 15], using the equations of Doniach-Sunjic, Anderson-Yuval, and Nozieres-de Dominicis [16] (CADS): we are using global, rather than per-spectrum, fit parameters. However, our fit is somewhat less restricted than other fits shown in this Letter: here we allow a small variation of $\xi_F$ as in Ref. [14]. We find an excellent fit quality, at least comparable to CADS [14]. The gray line in panel (a) shows our calculation for $k > k_F$. Our expectation is that, were the data for $k > k_F$ available, we would find a reasonable fit in this $k$ region as well [17], as for other data sets below.

**Line shape fit for conventional ARPES:** We find that the magnitude of the parameter $\omega_0$ (0.5 eV) determined from the fit of the sharp laser data works very well also for the conventional ARPES data [14]. Thus, all parameters other than $\eta$ are fixed, with one small exception in Fig. 3(d), where a slight change in $\omega_0$ produces a much better fit over a larger energy range for LSCO. Fig. 3 shows our fit of the data in Ref. [18] with a single free parameter $\eta$. The amount of the “extrinsic background” (bg) in ARPES is an issue of importance [20-22], especially when analyzing the conventional ARPES data. Here we fit the bg subtracted data, as well as the raw data (panel d). For subtracting the bg, we use an often-used procedure [22] of equating the background to a fraction (“bg scaling factor”) of the data far be-
Beyond the Fermi surface crossing ($k = k_{10}$ for this data set). The bg scaling factor, $1/2$ for this figure, is determined to be the maximum value for which the resulting intensity is not negative. As shown in the panel d, the ECFL fit remains good by adjusting $\eta$, whether or not the extrinsic background is subtracted. In contrast, we find that the CADS theory, notwithstanding its notable successes [14, 24], cannot cope with even the background subtracted data (Fig. 3c), giving too steep a fall off towards the left. Likewise, the MFL fits [8, 22] have been shown to compare well with the data only after substantial background subtraction [22, 24].

Our own data on Bi2212 data, taken at $T_c$ and well above $T_c$, covering a similar temperature range as the laser data of Fig. 2 can be fit equally well with the same background subtraction procedure, i.e. with the “bg scaling factor” ($1/2$), as shown in Fig. 3.

We also find that the data for a lower-$T_c$ cuprate LSCO can be fit very well with the same intrinsic parameters. Here, we shall discuss only the $k = k_F$ data for brevity. In this case, we determine that the “bg scaling factor” be 1. The subtracted “bg” data [27,28] is shown as the gray curve in Fig. 4c. Given their weak superconductivity features [27, 28], these LSCO data are taken to represent the normal state property even if the temperature is slightly lower than $T_c$. As for the Bi2212 case, the data can be fit well even without the background subtraction, if a somewhat greater $\eta$ value ($\sim 0.17$ eV) is used. It is clear, from Fig. 4c, that the data at a temperature as low as 25 K can be fit very well with the ECFL line shape. In addition, in working with LSCO line shapes, we noticed a steady and rapid rise in intensity beyond $-\omega = 0.25$ eV, a behavior different from that of Bi2212. We leave the full discussion of this non-universal behavior for future work. However, we find it exceptional that the current theory is able to describe the line shape of LSCO up to very high energy, as shown in Fig. 4d).

**Kinks in the spectra:** The two independent energy scales $\omega_0$ and $\Delta_0$ are determined from our fit as $\sim 0.5$ eV and $\sim 0.1$ eV. These are natural candidates for the two main dispersion anomalies in the cuprates [12, 29] as in Fig. 5. Well-defined energy distribution curve (EDC: intensity curve at a fixed $\vec{k}$ value) peaks disappear in a wide energy from $\sim 0.3$ eV to $\sim 1$ eV, as observed experimentally for the high energy kink [10, 12]. As this feature already exists in aux-FL, it cannot be associated with $\Delta_0$ but rather with $\omega_0$. The (numerical) dynamical mean field theory [30] can already account for this feature as can the present ECFL (analytical) theory.

Turning to the low energy ARPES kink at $\sim 70$ meV, Figs. 5c, d, e) illustrate the observed weak dispersion anomaly in the normal state data (c), reproduced in the ECFL theory (d) but not in the aux-FL theory (e). Here we use a visualization method for momentum distribu-
The predicted $T$-dependent asymmetry, predicted even greater for $\eta \approx 0.15$ eV (synchrotron data; not shown), would be interesting to explore in the future.

Further work is necessary to refine the picture suggested in this Letter. For example, as $-\xi_{k,\ell}$ increases, the line shape becomes somewhat too asymmetric. Work is also in progress to apply the theory to two particle response as seen, e.g., in optical conductivity. We have checked that the bubble approximation (conductivity as a product of two $G$’s) shows an agreement in the order of magnitude of the frequency scale and the conductivity.

**Conclusions:** We have shown that it is possible to understand both ARPES data sets (laser or conventional) comprehensively, with identical physical parameters. Work going beyond the nodal cut and the optimal doping value is in progress. The theory is very tolerant of the uncertainty in the background subtraction for the conventional ARPES data. Additionally, the theory satisfies the global particle sum rule, and contains two inter-dependent energy scales ($\omega_0$ and $\Delta_0$) that correspond well to the energy scales of the two kinks. Thus the simplest version of the ECFL theory using a small number of parameters, provides a framework to understand the ARPES line shape data for the normal state of the cuprates: it works extremely well across techniques, samples and temperatures.

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