Numerical/multiphysical investigation of shrinking hole at notch tip under high electric energy

Thomas Jin-Chee Liu

Department of Mechanical Engineering, Ming Chi University of Technology, Taishan, New Taipei City, Taiwan
E-mail: jinchee@mail.mcut.edu.tw

Abstract. In this paper, the shrinking hole creation at the notch tip is simulated using the finite element method (FEM) and element birth and death (EBD) technique. The removed material, stress, temperature, and electric current density are obtained for estimating the effects of the hole creation. Also, the variation of the shrinking hole size is discussed by the numerical/multiphysical simulation. When the melting material is removed, it shows rational stress values. The stress field is affected by the shrinking hole size and it must be considered in the analysis.

1. Introduction

The Joule heating effect is an electro-thermal coupling phenomenon. According to past studies [1-6], under the electric load, the electric concentration, Joule heating and local hot spot exist at the crack/notch tip in the conductive material. In Fig. 1, the electric current density concentrates at the notch tip and then a hot spot occurs.

Under high electric energy, the local hot spot at the notch tip can melt and then a shrinking hole may produce. In the author’s past studies [7,8], numerical and experimental results present the hot spot, melting area, solidification, shrinking hole and heat affected zone around the notch tip under high electric current. These results provide the concepts for increasing the fatigue life or stopping the crack propagation [7,8]. However, it was found that the production of the shrinking hole is a complicated case. As shown in Fig. 2, the author’s experimental study shows different hole size under the same loading and boundary conditions [8].
In this paper, the shrinking hole creation at the notch tip will be simulated using the finite element method (FEM) and element birth and death (EBD) technique of the ANSYS software. The removed material, stress, temperature, and electric current density will be obtained for estimating the effects of the hole creation. Especially, the variation of the shrinking hole size such like Fig. 2 will be discussed by FEM and EBD.

![Image](image_url)

**Figure 2.** Variation of shrinking hole size at notch tip (applied electric current = 600 A, operating time = 0.3333 s) [8].

2. **Case study**
For this case study, the sample is the same as refs. [7,8]. As shown in Fig. 3, the stainless steel strip has the dimensions $W \times L \times h$. The notch length and notch angle are respectively denoted as $a$ and $\alpha$. The round notch tip radius is $R_t$. The steel strip is subjected to a direct current (DC) $i_0$. The material type of the strip is SUS 304 stainless steel and the material constants are listed in Table 1 [9,10]. The elasto-plastic stress-strain relation with the tangent modulus $E_T=0.4E$ is considered in the FEM.

The geometric properties are $L=0.1$ m, $W=0.008$ m, $a=0.004$ m, $h=0.001$ m, $\alpha=20^\circ$, and $R_t=0.0002$ m. The electric load $i_0$ is 600 A. The following conditions are considered:

1. Three-dimensional analysis
2. Electro-thermo-structural coupled-field analysis
(3) Transient heat transfer (initial condition: 25 °C)
(4) Convection on air/steel interface (natural convection coefficient: 25 W/(m²-K), ambient temperature: 25 °C)
(5) Steady DC/RMS electric current

| Table 1. Material data of SUS 304 stainless steel [9,10]. |
|---------------------------------|--------|--------------|--------------|--------------|--------|
| Temperature (°C) | Young’s modulus E (GPa) | Yielding strength S_y (MPa) | Coefficient of thermal expansion α (1/°C) | Thermal conductivity k (W/m°C) | Specific heat C_p (J/kg°C) | Resistivity ρ (Ω-m) |
|-----------------|------------------|---------------------|-------------------|---------------------|----------------|-----------------|
| 20              | -                | -                   | 16.3·10⁻⁶         | 16.2                | 456            | 7.2·10⁻⁷       |
| 27              | 193              | 290                 | -                 | -                   | -              | -              |
| 90              | -                | -                   | -                 | -                   | -              | -              |
| 93              | 192              | -                   | -                 | -                   | -              | -              |
| 100             | -                | -                   | 16.3·10⁻⁶         | 16.2                | -              | -              |
| 149             | 187              | 182                 | -                 | -                   | -              | -              |
| 200             | 183              | -                   | -                 | -                   | -              | -              |
| 260             | 175              | 150                 | -                 | -                   | -              | -              |
| 300             | -                | -                   | 17.1·10⁻⁶         | -                   | -              | -              |
| 318             | 177              | -                   | -                 | -                   | -              | -              |
| 320             | -                | -                   | -                 | -                   | -              | -              |
| 371             | 170              | 134                 | -                 | -                   | -              | -              |
| 400             | -                | -                   | 17.6·10⁻⁶         | -                   | -              | -              |
| 427             | 166              | -                   | -                 | -                   | -              | -              |
| 430             | -                | -                   | -                 | -                   | -              | -              |
| 482             | 160              | 125                 | -                 | -                   | -              | -              |
| 500             | -                | -                   | 18.0·10⁻⁶         | 21.5                | -              | -              |
| 538             | 155              | -                   | -                 | -                   | -              | -              |
| 540             | -                | -                   | -                 | -                   | -              | -              |
| 593             | 150              | 113                 | -                 | -                   | -              | -              |
| 600             | -                | -                   | 18.3·10⁻⁶         | -                   | -              | -              |
| 649             | 145              | -                   | -                 | -                   | -              | -              |
| 650             | -                | -                   | -                 | -                   | -              | -              |
| 700             | -                | -                   | 19.0·10⁻⁶         | -                   | -              | -              |
| 704             | 141              | 95                  | -                 | -                   | -              | -              |
| 760             | 124              | -                   | -                 | -                   | -              | -              |
| 800             | -                | -                   | 20.0·10⁻⁶         | -                   | -              | -              |
| 816             | 125              | 68                  | -                 | -                   | -              | -              |
| 870             | -                | -                   | -                 | -                   | -              | -              |

Poisson’s ratio ν = 0.29, density β = 8000 kg/m³, melting point = 1400 °C.

3. Numerical methods
The finite element equations of the thermo-electro-structural coupled-field analysis are listed as follows [11]:

\[
\begin{bmatrix}
M & 0 & 0 & \tilde{U}
\end{bmatrix} + \begin{bmatrix}
C & 0 & 0 & U
\end{bmatrix} + \begin{bmatrix}
K & K_{\text{ut}} & 0 & U
\end{bmatrix} = \begin{bmatrix}
F
\end{bmatrix}
\]

where \( U, T, V, F, Q \) and \( I \) are the vector forms of the displacement, temperature, electric potential, force, heat flow rate and electric current, respectively. The material constant matrices \( M, C, C', C'' ', K, K', K'' \) and \( K''' \) are the structural mass, structural damping, thermal specific heat, thermo-structural damping, structural stiffness, thermal conductivity, thermo-structural stiffness and electric conductivity, respectively. The coupled heat flow matrix \( Q \) contains the effects of the thermal loading.
and electrical Joule heating. $C_{tu}$ and $K_{tu}$ are thermo-structural coupled terms. Equation (1) is a directly coupled nonlinear equation which is solved using the Newton-Raphson iterative method. As shown in Fig. 4, the finite element model is established using ANSYS software. The solid element SOLID226 is adopted to construct the numerical mesh. The nodal degrees of freedom of SOLID226 are the displacements, temperature and electric potential.

Figure 4. Finite element model.

Figure 5. Element birth and death (EBD) technique. (a) Concept, (b) Assumption of $R_h$.

Figure 6. Loading history.

The element birth and death (EBD) technique of the ANSYS [11] is adopted to simulate the shrinking hole creation. The concept of EBD is plotted in Fig. 5. The circular area (radius=$R_h$) of the material is removed to simulate the shrinking hole creation. The time of hole creation is at the end time of the electric load. After that, the electric load is removed and the structure is subjected to the cooling condition. Figure 6 shows the loading history of this study. The applied electric current $i_0= 600$ A. The operating time is $t_p= 0.3333$ s.
Referring to the photos in Fig. 2, it is difficult to define the accurate shrinking hole radius $R_h$. As shown in Fig. 5(b), the relation $R_h=KR_m$ is assumed, where $K$ and $R_m$ are the scale factor and the radius of the melting circle, respectively. $R_m$ must be defined from the finite element results.

4. Numerical results

4.1 Typical results
Based on the loading history in Fig. 6, the numerical results under $K=1$ are shown in this section. The condition $K=1$ expresses that the melting area ($T>1400{^\circ}C$) is the same as the shrinking hole size. In Figs. 7 and 8, the history of the electric current density and temperature filed are plotted around the notch tip. After the time $t=0.3333$ s, the circular region is removed to simulate the production of the shrinking hole. The heat source at the notch tip is induced by the electric concentration and Joule heating.

$$t = 0.2 \text{ s} \quad t = 0.3333 \text{ s} \quad t = 0.5 \text{ s}$$

**Figure 7.** History of electric current density (units: A/m$^2$).

$$t = 0.2 \text{ s} \quad t = 0.3333 \text{ s} \quad t = 0.5 \text{ s}$$

**Figure 8.** History of temperature filed (units: $^\circ$C).

$$t = 0.3333 \text{ s} \quad t = 0.4333 \text{ s}$$

**Figure 9.** von Mises stress (units: Pa).
In Fig. 9, the von Mises stresses at \( t = 0.3333 \) s and \( t = 0.4333 \) s are plotted. At \( t = 0.3333 \) s, the highest stress value is not rational because the melting material is not removed. However, it shows rational results when the shrinking hole is considered at \( t = 0.4333 \) s.

4.2 Shrinking hole size

Figure 10 shows the maximum von Mises stresses around the shrinking hole with \( K=0.5\sim1 \) when \( t = 0.4333 \) s. The stress value of \( K=0.5 \) is higher than others. It means that the stress field is affected by the shrinking hole size and it must be considered in the analysis.

From the results in Fig. 10, the smaller \( K \), i.e. smaller shrinking hole, induces higher stress value. This phenomenon is due to the thermal deformation and stress. The condition of smaller \( K \) means that more material is kept around the shrinking hole and it makes higher thermal stress field.

5. Conclusions

The real case of the melting/cooling process is very complicated so that the numerical simulation cannot consider complete situations of the shrinking hole. However, the EBD method provides a better solution to analyze the shrinking hole at the notch tip. When the melting material is removed, it shows rational stress values. The stress field is affected by the shrinking hole size and it must be considered in the analysis. The smaller shrinking hole induces higher stress value.

References

[1] Parton V Z and Kudryavtsev B A 1988 *Electromagnetoelasticity* (New York: Gordon and Breach)
[2] Fu Y M, Bai X Z, Qiao G Y, Hu Y D and Luan J Y 2001 *Mater. Sci. Tech.* **17** 1653
[3] Qin Z, Librescu L and Hasanyan D 2007 *J. Therm. Stress.* **30** 623
[4] Cai G X and Yuan F G 1999 *Int. J. Fract.* **96** 279
[5] Liu T J C 2011 *Eng. Fract. Mech.* **78** 666
[6] Liu T J C 2014 *Eng. Fract. Mech.* **123** 2
[7] Liu T J C, Cheng C D, Tseng J F, Chen L W and Chen P H 2016 *Procedia CIRP* **42** 679
[8] Liu T J C, Tseng J F and Cheng C D 2018 *MATEC Web of Conferences* **151** 01004
[9] MatWeb LLC 2018 *Online Materials Information Resource – MatWeb* (http://matweb.com/)
[10] American Iron and Steel Institute 2006 *High-Temperature Characteristics of Stainless Steels*
[11] ANSYS Inc 2011 *ANSYS 14.0 Mechanical APDL Theory Reference*

Acknowledgements

The author would like to thank the Ministry of Science and Technology in Taiwan for the financial support under contract numbers MOST 104-2221-E-131-030 and MOST 105-2221-E-131-014.