Higgs Production
in $SU(2)_c$ Symmetric Interactions

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Abstract

We study the sensitivity of Higgs production and decay processes to the $SU(2)_c$ symmetric couplings $O_W$ and $O_{UW}$. Remarkable results are obtained in the case of $\gamma\gamma \rightarrow H$ and for certain ratios of Higgs decay widths. We also discuss and complete previous results on unitarity constraints for such couplings.

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1 Introduction

Future $e^+e^−$ and pp colliders [1, 2, 3] offer many possibilities for testing the bosonic sector of the electroweak interactions through vector boson pair production. Such possibilities have been studied in [4, 5, 6, 7], where guided by the principle of $SU(2)_C$ symmetry [8], supported by present tests at LEP1 and at low energy experiments, we considered the New Physics (NP) effects generated by the operator

$$O_W = \frac{1}{3!} \left( \bar{W}_\mu^\nu \times \bar{W}_\nu^\lambda \right) \cdot \bar{W}_\lambda^\mu = -\frac{2i}{3} \langle W^{\nu\lambda} W_{\nu\mu} W_{\mu\nu} \rangle \ ,$$

(1)

with a coupling $\lambda_W$, and the operator

$$O_{UW} = \langle (UU^\dagger - 1) W^{\mu\nu} W_{\mu\nu} \rangle \ ,$$

(2)

with a coupling called $d$. In these operators the definitions

$$U = (\bar{\Phi}, \Phi) \sqrt{2} \overline{v}$$

(3)

are used, where $\Phi$ is the standard Higgs doublet, $v$ its vacuum expectation value, $\bar{\Phi} = i\tau_2 \Phi^*$, and $\langle A \rangle \equiv Tr A$. Thus, the first of these operators involves gauge bosons only, whereas the second one includes Higgs bosons also. We have found that the coupling $\lambda_W$ affects $e^+e^−$ and $q\bar{q}$ annihilation processes to gauge boson pairs, as well as boson-boson fusion through 3-boson and 4-boson vertices. From the analysis of gauge boson pair production it was concluded [9, 4] that a sensitivity on $|\lambda_W|$ of 0.002 (0.01) could be reached at $e^+e^−$ (pp) colliders. Correspondingly for the $d$ coupling, we have found that the $O_{UW}$ contribution to the gauge boson pair production arises only through Higgs exchange diagrams in boson fusion processes, thereby leading to much weaker sensitivities on $|d|$, i.e. 0.02 (0.1) from $e^+e^−$ (pp), [7, 4].

The first purpose of this paper is to show that the $O_{UW}$ sensitivity can be improved by looking also at Higgs production and decay processes. Assuming that the Higgs particle is sufficiently light to be actually produced, one expects a particularly high sensitivity in those processes where the standard contribution is suppressed. Such processes are those determined by the $H\gamma\gamma$ and $H\gamma Z$ couplings, which receive standard contributions only at 1-loop, whereas their contributions from $O_{UW}$ already arise at tree level. Since one would expect large $d$ effects in amplitudes involving these couplings, we pay a special attention to Higgs production via $\gamma\gamma$ collisions. In addition we also consider the other main $H$ production mechanisms, namely through gauge boson fusion ($i.e.,$ WW, ZZ, $\gamma\gamma$ and $\gamma Z$ collisions) and through associate production in $e^+e^− \rightarrow Hz$ and $q\bar{q} \rightarrow HW$ at pp colliders. Finally, the $d$ sensitivity achievable by measuring the Higgs decay modes $H \rightarrow \gamma\gamma$ and $H \rightarrow \gamma Z$, is also studied.

The second aim of the present work is to compare the aforementioned sensitivities, to more theoretical constraints on $\lambda_W$ and $d$. One indirect way of deriving such constraints,
is by using LEP1 measurements in 1-loop considerations involving Higgs exchange diagrams \[\text{(10)}\]. In a different approach, very stringent constraints may be derived in a purely theoretical way, by using unitarity \[\text{(3)}\]. These arise because of the high dimensionality of the operators $\mathcal{O}_W$ and $\mathcal{O}_{UW}$, which necessarily leads to violation of unitarity above a certain scale. Thus, an assumption on the magnitude of the scale below which no strong interactions appear (i.e. no unitarity saturation), immediately implies an upper bound on $|\lambda_W|$ and $|d|$. These limits were established in \[\text{(5)}\] using the gauge boson-gauge boson scattering amplitudes given in \[\text{(8)}\], and the Higgs involving amplitudes presented here. The later are required for establishing the unitarity bounds on $d$, and they can also be useful for estimations of the Higgs production rates. Moreover, here we confirm the results of the previous letter \[\text{(5)}\], where the unitarity constraints on $d$ were derived on the basis of the $J = 0$ partial waves amplitudes only, by looking also at the $J = 1$ partial waves.

The contents of the paper is the following. Sect.2 is devoted to Higgs production from photon-photon collisions using laser backscattering in a high energy $e^+e^-$ collider. In Sect.3 we consider Higgs production through gauge boson fusion processes at $e^+e^-$ colliders, using the Weiszäcker-Williams approximation, and in Sect.4 the associate production processes $e^+e^- \rightarrow HZ$ and $e^+e^- \rightarrow H\gamma$ are studied. The $d$ sensitivity to the Higgs decay modes is studied in Sect.5, and the unitarity constraints in Sect.6. Finally the last Sect.7 resumes the experimental and theoretical prospects about the $d\mathcal{O}_{UW}$ coupling. Appendix A and B give the explicit expressions for the $\lambda_W\mathcal{O}_W$ and $d\mathcal{O}_{UW}$ contributions to the single and double Higgs production amplitudes at an energy above 1TeV.

## 2 Higgs production in photon-photon collisions from laser backscattering

The Standard Model (SM) contribution to the $H\gamma\gamma$ coupling is rather weak as it only appears through 1-loop \[\text{(11)}\]. Nevertheless owing to the large $\gamma$ luminosities that may be available for double laser backscattering on the high energy $e^\pm$ beams, a copious Higgs production would be expected in $\gamma\gamma$ collisions \[\text{(2)}\]. Such a production must be very sensitive to the $d\mathcal{O}_{UW}$ interaction, which contributes to it at tree level.

The predominant SM contributions to the $H \rightarrow \gamma\gamma$ width, are due to the top and W loops \[\text{(11)}\]. Adding to this the tree level $d\mathcal{O}_{UW}$ contribution, we obtain

$$
\Gamma(H \rightarrow \gamma\gamma) = \frac{\sqrt{2}G_F}{16\pi}m^3_H \left[ \frac{\alpha}{4\pi} \left( \frac{4}{3}F_t + F_W \right) - 2ds^2_W \right]^2 , \quad (4)
$$

where the top and W contributions are respectively determined by

$$
F_t = -2t_t(1 + (1 - t_t)f(t_t)) , \quad (5)
$$

$$
F_W = 2 + 3t_W + 3t_W(2 - t_W)f(t_W) , \quad (6)
$$
in terms of

\[ f(t) = \left( \sin^{-1}(1/\sqrt{t}) \right)^2 \quad \text{if} \quad t \geq 1 , \]

\[ f(t) = -\frac{1}{4} \left[ \ln \left( \frac{1 + \sqrt{1-t}}{1 - \sqrt{1-t}} \right) - i\pi \right]^2 \quad \text{if} \quad t < 1 , \quad (7) \]

where \( t = \frac{4m_T^2/m_H^2}{t_w} \), \( t_w = \frac{4M_W^2/m_H^2}{} \). Using the width \( \Gamma(H \to \gamma\gamma) \), the cross section \( \sigma_{\gamma\gamma}^H \) for the elementary process \( \gamma\gamma \to H \) is then expressed as

\[ \sigma_{\gamma\gamma}^H = \frac{8\pi^2}{m_H} \Gamma(H \to \gamma\gamma) \delta(s_{\gamma\gamma} - m_H^2) . \quad (8) \]

In order to calculate now the corresponding cross section for double laser scattering in \( e^+e^- \) colliders, we need the induced spectral \( \gamma\gamma \) luminosity. For unpolarized laser and \( e^\pm \) beams, this is given by [12]

\[ \frac{d\mathcal{L}_{\gamma\gamma}(\tau)}{d\tau} = \int_{x_{\text{max}}}^{x_{\text{max}}} \frac{dx}{x} f_{\gamma/e}^\text{laser}(x) f_{\gamma/e}^\text{laser}(\frac{x}{\xi}) , \quad (9) \]

where

\[ \tau = \frac{s_{\gamma\gamma}}{s_{ee}} \quad (10) \]

is the ratio of the \( \gamma\gamma \) c.m. squared energy to the \( e^+e^- \) one, \( \xi = 2(1 + \sqrt{2}) \simeq 4.8 \) and \( x_{\text{max}} = \frac{\xi}{1 + \xi} \simeq 0.82 \) [13, 14]. The photon distribution \( f_{\gamma/e}^\text{laser}(x) \) in (6) is obtained by Compton scattering the laser beam on the \( e^\pm \) beams and it is given by [13, 14]

\[ f_{\gamma/e}^\text{laser}(x) = \frac{1}{D(\xi)} \left( 1 - x + \frac{4x}{1 - x} - \frac{4x}{\xi(1 - x)} + \frac{4x^2}{\xi^2(1 - x)^2} \right) , \quad (11) \]

where \( x \) is the fraction of the incident \( e^\pm \) energy carried by the backscattered photon, while

\[ D(\xi) = \left( 1 - \frac{4}{\xi} - \frac{8}{\xi^2} \right) \ln(1 + \xi) + \frac{1}{2} + \frac{8}{\xi} - \frac{1}{2(1 + \xi)^2} . \quad (12) \]

Using the above \( \gamma\gamma \) luminosity, the cross section for Higgs production through double laser scattering is then given by

\[ \frac{d\sigma}{d\tau} = \frac{d\mathcal{L}_{\gamma\gamma}(\tau)}{d\tau} \sigma_{\gamma\gamma}^H , \quad (13) \]
which combined with (8) leads to the integrated cross section

$$\sigma = \mathcal{L}_{\gamma\gamma}(\tau_H) \left( \frac{8\pi^2}{m_H} \right) \frac{\Gamma(H \rightarrow \gamma\gamma)}{s_{ee}} . \quad (14)$$

The correspondingly expected number of events per year is then determined by

$$N = \mathcal{L}_{ee} \sigma , \quad (15)$$
in terms of the integrated $e^+e^-$ annual luminosity $\mathcal{L}_{ee}$ taken to be $10 fb^{-1} year^{-1}$.

The results are presented at variable Higgs mass in Fig.1a-c for a 0.5, 1 and 2 TeV center of mass energy $e^+e^-$ collider. One recognizes the typical structure of the standard predictions with the dips and bumps due to the $t\bar{t}$ and $W^+W^-$ threshold effects in the loop and the interferences of their contributions. As expected the sensitivity to $d$ is very interesting. With the predicted number of events, values of $d$ down to $\pm 0.001$ could even be observed. As one can be see on the figures, the precise value of the observability limit strongly depends on the Higgs mass, which determines the important interferences between the SM and $O_{UW}$ contributions.

3 Higgs production by WW or ZZ fusion

Another standard way of producing the Higgs boson in $e^+e^-$ or pp colliders is through fusion of the vector bosons emitted from the fermions. Let us first discuss the processes $e^+e^- \rightarrow \nu\bar{\nu}(WW) \rightarrow \nu\bar{\nu}H$ and $e^+e^- \rightarrow e^+e^-(V_1V_2) \rightarrow e^+e^-H$. In SM, the first one goes through $W^+W^- \rightarrow H$, while at tree level the second one goes through $ZZ \rightarrow H$. The corresponding SM cross sections have been given in a compact form for $V = W$ or $Z$ in [15]. In order to include the $dO_{UW}$ effect, one has to separate the transverse-transverse (TT) from the longitudinal-longitudinal (LL) WW states. Because of the gauge invariant nature of the $O_{UW}$ operator, only the TT partial width is appreciably affected by it, as one can see from the subsequent expressions for the Higgs partial widths

$$\Gamma(W_TW_T) = \left( \frac{\alpha M_W^2 \beta_W}{2s_W^2 m_H} \right) \left[ 1 - \frac{d}{M_W^2} (m_H^2 - 2M_W^2) \right]^2 , \quad (16)$$

$$\Gamma(W_LW_L) = \left( \frac{\alpha \beta_W}{4s_W^2 m_H} \right) \left[ \frac{(m_H^2 - 2M_W^2)^2}{4M_W^2} + 4d^2 M_W^2 - 2d(m_H^2 - 2M_W^2) \right] , \quad (17)$$

where $\beta_W = \sqrt{1 - 4M_W^2/m_H^2}$ is the velocity of the final state W's. We note that the total $H \rightarrow W^+W^-$ width

$$\Gamma_{tot}(WW) =$$
\[
\left( \frac{\alpha \beta_W}{4s^2_W m_H} \right) \left[ 2M^2_W + \frac{(m^2_H - 2M^2_W)^2}{4M^2_W} + 4d^2 \left\{ \frac{(m^2_H - 2M^2_W)^2}{2M^2_W} + M^2_W \right\} - 6d(m^2_H - 2M^2_W) \right] ,
\]

being dominated by the LL state, is relatively less affected by \( d \), than the TT part given in (16).

The \( e^+e^- \rightarrow e^+e^- (V_1V_2) \rightarrow e^+e^- H \) case is more delicate. As mentioned above in SM it is dominated by \( ZZ \rightarrow H \), but when \( dO_{UW} \) effects are included, contributions from the \( \gamma \gamma \rightarrow H \) and \( \gamma Z \rightarrow H \) fusion processes should be added. In the spirit of the Weiszäcker-Williams approximation used here, we need therefore the corresponding Higgs partial widths to these three channels. For the later two, we keep and 1-loop SM contributions, together with tree level \( dO_{UW} \) effects.

The width for \( H \rightarrow \gamma \gamma \) is already given in (4). The corresponding expression for \( H \rightarrow \gamma Z \), based on the standard 1-loop (top and W) amplitudes \([11]\) and the tree level \( O_{UW} \) contribution, is

\[
\Gamma(H \rightarrow \gamma Z) = \frac{\sqrt{2}G_F m^3_H}{32\pi} (1 - \frac{M^2_Z}{m^2_H})^3 \left( \frac{\alpha}{2\pi} (A_t + A_W) + 4d s_W c_W \right)^2 ,
\]

\[
A_t = \frac{(-6 + 16s^2_W)}{3s_W c_W} [I_1(t_f, l_t) - I_2(t_t, l_t)] ,
\]

\[
A_W = -\cot \theta_W [4(3 - \tan^2 \theta_W) I_2(t_w, l_w) + [(1 + \frac{2}{t_W})\tan^2 \theta_W - (5 + \frac{2}{t_W})] I_1(t_w, l_w)] ,
\]

where \( t_t = 4m^2_t/m^2_H, t_w = 4M^2_W/m^2_H \) as before, and \( l_t = 4m^2_t/M^2_Z, l_w = 4M^2_W/M^2_Z \). In (20,21) the definitions

\[
I_1(a, b) = \frac{ab}{2(a-b)} + \frac{a^2b^2}{2(a-b)^2} [f(a) - f(b)] + \frac{a^2b}{(a-b)^2} [g(a) - g(b)] ,
\]

\[
I_2(a, b) = -\frac{ab}{2(a-b)} [f(a) - f(b)] ,
\]

where \( f(t) \) is given in (7) and

\[
g(t) = \sqrt{t - 1} \sin^{-1} \left( \frac{1}{\sqrt{t}} \right) \quad \text{if} \quad t \geq 1 ,
\]

\[
g(t) = \frac{1}{2} \sqrt{t - 1} \left[ \ln \left( \frac{1 + \sqrt{1-t}}{1 - \sqrt{1-t}} \right) - i\pi \right] \quad \text{if} \quad t < 1 .
\]
Finally the TT and LL components of the $H \rightarrow ZZ$ width are given by

$$\Gamma(Z_T Z_T) = \left( \frac{\alpha \beta_Z M_W^2}{4 m_H s_W^2 c_W} \right) \left[ 1 - d \frac{c_W^4}{M_W^2} (m_H^2 - 2 M_Z^2)^2 \right]^2 , \quad (25)$$

$$\Gamma(Z_L Z_L) = \left( \frac{\alpha \beta_Z}{8 s_W^2 m_H} \right) \left[ \frac{(m_Z^2 - 2 M_Z^2)^2}{4 M_W^2} + 4 d^2 M_W^2 - 2d(m_H^2 - 2 M_Z^2) \right] . \quad (26)$$

with $\beta_Z = \sqrt{1 - 4 M_Z^2 / m_H^2}$. As in the WW case it is mainly the TT part which is affected by $d$, while the total $H \rightarrow ZZ$ width given by

$$\Gamma_{tot}(ZZ) = \left( \frac{\alpha \beta_Z}{8 s_W^2 m_H} \right) \left[ \frac{2 M_W^2}{c_W^2} + \frac{(m_Z^2 - 2 M_Z^2)^2}{4 M_W^2} + 4 d^2 \left\{ c_W^4 \frac{(m_H^2 - 2 M_Z^2)^2}{2 M_W^2} + M_W^2 \right\} - 6d(m_H^2 - 2 M_Z^2) \right] . \quad (27)$$

is less sensitive.

Results are presented in Fig.2a-c and 3a-c. In the standard case the cross section of $e^+ e^- \rightarrow \nu \bar{\nu}(WW) \rightarrow \nu \bar{\nu}H$ is about 10 times larger than the one of $e^+ e^- \rightarrow e^+ e^- (V_1 V_2) \rightarrow e^+ e^- H$. However as soon as $|d|$ reaches values of the order of 0.01, because of the large contributions from the $H \gamma \gamma$ and $H \gamma Z$ couplings, both processes $e^+ e^- \rightarrow \nu \bar{\nu}H$ and $e^+ e^- \rightarrow e^+ e^- H$ become of the same order of magnitude. The level of sensitivity to $d$ that can be roughly expected is $\sim 0.01$. It is weaker than the one from laser $\gamma \gamma$ collisions because of the suppressed Weiszecker-Williams luminosities \[15\], and the fact that WW and ZZ fusion already occurs at tree level in SM.

### 4 Associate production in $e^+ e^- \rightarrow HZ$ and $e^+ e^- \rightarrow H\gamma$

The process $e^+ e^- \rightarrow HZ$ is the main one \[17\] planed to be used for Higgs search in $e^+ e^-$ collisions beyond the Z peak at LEP2 or a 0.5 TeV Collider. The cross section for this process decreases with energy, but the rate is still acceptable at NLC; see \[15\]. The relevant interactions are given by the $HZZ$ coupling obtained by adding the tree SM and $\mathcal{O}_{UW}$ contributions, and the $H\gamma Z$ coupling derived by adding the 1-loop SM and tree level $\mathcal{O}_{UW}$ results. In order to discuss the $d$ sensitivity it is sufficient to have the tree level result

$$\sigma(e^- e^+ \rightarrow HZ) = \frac{\pi \alpha^2 M_W^2 \beta_H}{s_W^2} \left\{ (1 + \frac{s \beta_H^2}{12 M_Z^2}) \frac{a_Z^2 + b_Z^2}{c_W(s - M_Z^2)^2} \right\}$$

$$+ 2d \frac{(s + M_Z^2 - m_H^2)}{M_W^2} \left\{ \frac{a_Z^2 + b_Z^2}{(s - M_Z^2)^2} - \frac{a_Z s_W}{c_W s(s - M_Z^2)} \right\}$$

$$16.$$
+4d^2\left\{\left(\frac{s + m_W^2 - m_h^2}{2M_W^2}\right)^2 - \frac{s^2\beta_H^2}{12m_W^4}\right\} \frac{[s_Z^2 + b_Z^2]c_W^4}{(s - m_Z^2)^2} - \frac{2a_Zc_Ws_WM_H^2}{s(s - m_Z^2)} + \frac{c_W^2s_W^2}{s^2}\right\}, \tag{28}

where \(a_Z = (-1 + 4s_W^2)/(4s_Wc_W)\), \(b_Z = -1/(4s_Wc_W)\) and \(\beta_H = 2p_H/\sqrt{s}\) with \(p_H\) denoting the momentum of the final Higgs.

The sensitivity to \(d\) comes from the linear term associated to the \(HZZ\) coupling and from the quadratic terms associated to both \(HZZ\) and \(H\gamma\gamma\) couplings. The result is shown in Fig.4a-c. Because of the rather low rate at high energy, one cannot expect this process to compete with the laser \(\gamma\gamma\), especially for high Higgs masses. Nevertheless a \(d\) sensitivity of \(\pm 0.005\) may be reached this way for low \(m_H\), in \(e^+e^-\) collisions at NLC \(0.5\text{TeV}\). The corresponding result at LEP2 \((190\text{ GEV})\) with an integrated luminosity of \(500pb^{-1}\) is \(\pm 0.01\) for \(M_H = 80\text{GeV}\).

We have also looked at the process \(e^+e^- \rightarrow H\gamma\), which receives no tree SM contribution. At 1-loop, this process goes through photon exchange due to the \(H\gamma\gamma\) coupling and \(Z\) exchange due to the \(HZ\gamma\) coupling. In addition gauge boson box diagrams also contribute \([1]\). The resulting SM cross section beyond the \(Z\) peak is too low to be observable. However when the tree level \(H\gamma\gamma\) and \(HZ\gamma\) couplings due to \(O_{UV}\) are included it becomes significant. This later tree level result is

\[
\sigma(e^+e^- \rightarrow H\gamma) = \frac{2\pi\alpha^2s^2}{3s_W^2M_W^2}\beta_H^2d^2\left\{\left(\frac{a_Z^2 + b_Z^2}{s - m_Z^2}\right)^2 - \frac{2a_Zc_Ws_WM_H^2}{s(s - m_Z^2)} + \frac{s_W^4}{s^2}\right\}, \tag{29}
\]

on the basis of which only a few events would be expected at LEP2 if \(|d| > 0.05\). At a higher energy this \(|d|\) limit may reach 0.01.

Similar types of processes are available at pp colliders. For example the \(qq' \rightarrow WH\) cross section is given by

\[
\sigma(qq' \rightarrow HW) = \frac{N_c\pi\alpha^2m_W^2}{4s_W^4(s - M_W^2)^2}\beta_H^2[(1 + \frac{s\beta_H^2}{12m_W^4})
+2d\frac{s + m_W^2 - m_h^2}{m_W^2} + 4d^2\left(\frac{s + m_W^2 - m_h^2}{2m_W^2}\right)^2 - \frac{s^2\beta_H^2}{12m_W^4}] . \tag{30}
\]

Here the bare sensitivity to \(d\) is similar to the one in \(e^+e^- \rightarrow HZ\), however the experimental accuracy and the presence of large backgrounds can certainly not allow to reach the values obtained above.

5 Tests with ratios of Higgs partial decay widths

We now turn to another way of testing \(d\). Once the H is discovered and its mass known, using any copious way of producing it one can study how the ratios of various decay modes are sensitive to \(d\). This is exactly how one should check that a newly discovered scalar particle is a candidate for a Higgs boson. One has to verify that all its couplings
agree with the standard prediction that they should be proportional to the mass of the particle to which it couples.

We have already given in (4,19,27,18) the $H$ decay widths to $\gamma\gamma$, $\gamma Z$, $ZZ$ and $W^+W^-$ respectively, including SM and $O_{UUW}$ contributions. If the Higgs mass is smaller than twice the gauge boson mass (but larger than one boson mass) we need to compute also the decay into one real and one virtual gauge boson. The results for SM have been given in [18]. Including also $O_{UUW}$ contributions we get

$$\Gamma(H \rightarrow W^*W) = \frac{3\alpha^2 m_H}{32\pi s_W^4} [D_{SM}(x) + dD_1(x) + 8d^2D_2(x)]$$ ,

(31)

$$\Gamma(H \rightarrow Z^*Z) = \frac{\alpha^2 m_H}{128\pi s_W^4 c_W^4} \left(7 - \frac{40s_W^2}{3} + \frac{160s_W^4}{9}\right) [D_{SM}(x) + dc_W^2 D_1(x) + 8d^2 c_W^4 D_2(x)]$$ ,

(32)

where

$$D_{SM}(x) = \frac{3(20x^2 - 8x + 1)}{\sqrt{4x - 1}} \cos^{-1}\left(\frac{3x - 1}{2x^{3/2}}\right)$$

$$- (1 - x) \left(\frac{47x}{2} - \frac{13}{2} + \frac{1}{x}\right) - 3(2x^2 - 3x + 1)\ln x$$ ,

(33)

$$D_1(x) = \frac{24(14x^2 - 8x + 1)}{\sqrt{4x - 1}} \cos^{-1}\left(\frac{3x - 1}{2x^{3/2}}\right)$$

$$+ 12(x - 1)(9x - 5) - 12(2x^2 - 6x + 1)\ln x$$ ,

(34)

$$D_2(x) = \frac{54x^3 - 40x^2 + 11x - 1}{x\sqrt{4x - 1}} \cos^{-1}\left(\frac{3x - 1}{2x^{3/2}}\right)$$

$$+ \frac{(x - 1)}{6}(89x - 82 + \frac{17}{x}) - (3x^2 - 15x + \frac{9}{2} - \frac{1}{2x})\ln x$$ ,

(35)

and $x = (M_V/m_H)^2$ with $M_V = M_W$ or $M_Z$. These results are particularly important for $90GeV \leq m_H \leq 140GeV$.

Finally we also need for comparison the fermionic Higgs decay widths, which are not affected by $O_{UUW}$ at the tree level. Let us take $H \rightarrow b\bar{b}$ as a reference, since the $t\bar{t}$ threshold
is probably very high. The $H \to b\bar{b}$ partial width, which is purely standard and particularly important if $m_H < \sim 140 GeV$, is

$$\Gamma(H \to b\bar{b}) = 3\frac{\sqrt{2}G_F m_b^2}{8\pi} \beta_b^3 m_H.$$  \hspace{1cm} (36)

with $\beta_b = \sqrt{1 - 4m_b^2/m_H^2}$.

In Fig.5a-e we show a sample of ratios of Higgs decay widths, as functions of the Higgs mass and the $d$ coupling. Independently of the Higgs mass, a large sensitivity to $d$ exists for the $H \to \gamma\gamma$ decay width, as opposed to $H \to WW(WW^*)$, $H \to ZZ(ZZ^*)$ or $H \to b\bar{b}$ in Fig.5a-d. A sharp and fortuitous SM-$d\mathcal{O}_{UW}$ interference occurs for $d = 0.01$ and $m_H = 0.15 TeV$. $H \to \gamma Z$ is also very sensitive to $d$. On the opposite the WW and ZZ channels are less sensitive as one could guess from (18,27,31,32) and this sensitivity becomes even smaller in the ratio WW over ZZ.

The use of these results for giving limits on $d$ will actually depend on the number of events that will be observed. This test can however be performed independently of the production mode and one can cumulate events from various Higgs boson sources.

6 Unitarity constraints on $d$ from various Higgs production channels

In this Section we give the explicit check that the unitarity constraints on $d$ established in [3] using $J = 0$ partial waves, are still valid when considering the $J = 1$ case also. Generally, because of the $2J+1$ weight factor, higher partial waves give weaker constraints than lower ones. However the point is that for $J = 1$, new channels containing the Higgs are involved that did not contribute to $J = 0$ amplitudes.

Following the same procedure as in [3], we use the high energy expressions of the scattering amplitudes given in [4] as well as the ones involving one or two Higgs states given in Appendix A and B. We have then to separately treat three sets of VV, VH, HH coupled channels, with total charge $Q = 2, 1, 0$.

The $Q = 2, J = 1$ set involves five independent non vanishing $W^+W^+$ scattering amplitudes. It only contains terms linear in $d$. Diagonalizing the relevant $5 \times 5$ matrix and demanding that the largest eigenvalue is less than 2 gives the unitarity constraint

$$|d| \lesssim \frac{24s_W M_W^2}{s\alpha} \simeq 780 \frac{M_W^2}{s}.$$  \hspace{1cm} (37)

The $Q = 1, J = 1$ set involves 13 different $ZW, \gamma W$ and $HW$ states. The corresponding $13 \times 13$ matrix is rather involved and contains both $d$ and $d^2$ terms. It produces a complicated analytical constraint which is numerically approximated by

$$|d| \lesssim 25.4 \frac{M_W^2}{s} + 9.68 \frac{M_W}{\sqrt{s}}.$$  \hspace{1cm} (38)
Finally, the $Q = 0, J = 1$ set a priori involves 26 different $W^+W^−, ZZ, γγ, γZ, Hγ, HZ$ and $HH$ states. However only 12 of them contribute to $J = 1$ namely, $7 W^+W^−, 3 HZ$ and $2 Hγ$. The relevant $12 \times 12$ matrix leads again to a complicated unitarity constraint which is numerically approximated by

$$|d| \lesssim 0.67 \frac{M_W^2}{s} + 27.04 \frac{M_W}{\sqrt{s}}.$$  

One can check that in the TeV range the constraints (34-36) turn out to be less stringent than the one established from the $J = 0$ partial waves in [5]. Only for energies larger than 30 TeV, the constraint (37) coming from the $Q = 2, J = 1$ set, which is linear in $d$, becomes more stringent.

7 Final discussion

In this paper we have shown that limits on the H and W interactions due to the $SU(2)_c$ symmetric operator $dO_{UW}$, can be largely improved by considering direct H production processes. The best process seems to be the Higgs production in photon-photon collisions from laser backscattering at a high energy $e^+e^−$ linear collider. With the expected luminosities, an observability limit for $d$ of the order of ±0.001 can be achieved for a large range of Higgs masses. The other boson-boson fusion processes are less efficient because of the suppressed Weiszecker-Williams luminosities and they would only be sensitive to $d$ values at the level of 0.01. For low and intermediate Higgs masses, the associate production $e^+e^− → HZ$ is an interesting possibility which could lead to a 0.005 limit. Even at LEP2 a 0.01 limit for $d$ should be settled, provided $m_H \sim 80GeV$. The other process $e^+e^− → Hγ$ is too weak to be competitive and can hardly reach 0.01 at NLC and 0.05 at LEP2.

We have also shown how ratios of Higgs partial decay widths could reflect the same sensitivity to $d$, provided one could accumulate a sufficient number of Higgs events. The resulting panorama of the expected sensitivities for $d$ from all direct and indirect processes involving Higgs and gauge bosons is given in Table 1.

| Table 1: Sensitivity to $dO_{UW}$ |
|------------------------------------|
| process                            | collider      | $|d|$      |
|------------------------------------|----------------|-----------|
| $pp(q\bar{q}, WW$ fusion) $        | LHC            | 0.1       |
| $γγ → WW, ZZ$                       | NLC 0.5-2 TeV  | 0.25-0.02 |
| $γγ → H$                           | NLC 0.5-2 TeV  | 0.001     |
| $WW, ZZ → H$                       | NLC 0.5-2 TeV  | 0.01      |
| $e^+e^− → HZ$                      | NLC 0.5 TeV    | 0.005     |

We observe that a gain of a factor 10 to 100 is obtained for the $d$ sensitivity from direct Higgs boson production, as compared to the sensitivity expected from processes involving...
only gauge bosons in the final state. The reason for this is because the later processes depend on \( d \) only through Higgs exchange diagrams \([4], [10]\).

It is also interesting to compare these results to theoretical expectations. In this paper we have confirmed the validity of the unitarity limits announced in a previous letter. Using the full set of coupled VV, VH and HH channels we have shown that the strongest constraints on \( d \), for an NP scale lying in the TeV range, indeed come from the \( J = 0 \) partial waves and can be written as

\[
|d| \lesssim 17.6 \frac{M_W}{s} + 2.43 \frac{M_W}{\sqrt{s}}.
\]  

(40)

It appears therefore that the level of the achievable sensitivity from the processes studied in this paper is largely within the domain allowed by unitarity. Such tests will therefore produce essential informations on the possible New Physics effects affecting the scalar sector and described by the \( SU(2)_c \) symmetric operator \( O_{UW} \). The 0.01 to 0.001 level of observability expected for the coupling \( d \), is comparable to the one expected for the coupling \( \lambda_W \) describing the \( SU(2)_c \) symmetric interactions among W bosons only \([7]\). This level is the one at which standard electroweak radiative corrections start to contribute. It makes therefore sense to pursue high precision tests of the presence of such interactions. The resulting constraints that will be put on the gauge sector and on the scalar sector of the SM will turn out to be as stringent as those which have been obtained on the fermionic sector at LEP1. The combination of all these information should be essential in order to select the allowed ways to extend the SM and to cure its deficiencies.

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It is a pleasure to thank A. Djouadi for many useful discussions and informations about Higgs boson phenomenology.
Appendix A : Helicity amplitudes for single Higgs production in boson fusion processes at high energy

The $V_1(\lambda)V_2(\tau) \rightarrow HV_3(\mu)$ processes are described by 27 helicity amplitudes $F_{\lambda\tau\mu}(\theta)$, where $\lambda$, $\tau$, and $\mu$ denote the helicities, $\theta$ is the c.m. angle between $V_1$ and $H$, and the normalization is such that the differential cross section writes as

$$\frac{d\sigma(\lambda\tau\mu)}{d\cos(\theta)} = C |F_{\lambda\tau\mu}(\theta)|^2, \quad (A.1)$$

where

$$C = \frac{1}{32\pi^2} \frac{p_H}{p_{12}} \quad (A.2)$$

includes no spin average.

1 Standard, $\mathcal{O}_W$ and $\mathcal{O}_{UW}$ contributions

In the following expressions the last factor of the type $(a,b)$ always refers to $(Z,\gamma)$ production respectively.

$$W^-W^+ \rightarrow HZ, H\gamma$$

$$F_{\pm\mp,\mp} = \pm \left\{ \frac{e^2 d\sqrt{s}}{\sqrt{2M_W}} \frac{\sin(1 + \cos \theta)}{1 - \cos \theta} - \frac{e^2 \lambda_W d^{3/2}}{2\sqrt{2M_W^3}} \sin \theta \right\} \left( \frac{c_W}{s^2_W}, \frac{1}{s_W} \right) \quad (A.3)$$

$$F_{\pm\mp,\pm} = \pm \left\{ \frac{e^2 d\sqrt{s}}{\sqrt{2M_W}} \frac{\sin \theta (\cos \theta - 1)}{2 + \cos \theta} - \frac{e^2 \lambda_W d^{3/2}}{2\sqrt{2M_W^3}} \sin \theta \right\} \left( \frac{c_W}{s^2_W}, \frac{1}{s_W} \right) \quad (A.4)$$

$$F_{\pm\pm,\pm} = \pm \left\{ \frac{e^2 d\sqrt{s}}{\sqrt{2M_W}} \frac{1}{c_W \sin \theta} + \frac{e^2 \lambda_W d^{3/2}}{2\sqrt{2M_W^3}} \sin \theta \right\} \left( \frac{c_W}{s^2_W}, \frac{1}{s_W} \right) \quad (A.5)$$

$$F_{\mp\pm,\mp} = \mp \left\{ \frac{e^2 \lambda_W d^{3/2}}{2\sqrt{2M_W^3}} \sin \theta \right\} \left( \frac{c_W}{s^2_W}, \frac{1}{s_W} \right) \quad (A.6)$$

$$F_{0\mp,\mp} = \frac{e^2 (1 - 4c_W^2 - \cos \theta)}{2c_W s^2_W (\cos \theta - 1)} \left( \frac{s_W}{c_W} \right) \quad (A.7)$$
\[ F_{\pm 0, \mp} = \left\{ \frac{e^2(-1 + 4c_W^2 - \cos \theta)}{2c_W s_W^2 (1 + \cos \theta)} \right\} \left( 1, \frac{s_W}{c_W} \right) \]  
(A.8)

\[ F_{0 \mp, \pm} = \frac{e^2 \lambda_W s}{8M_W^2} (3 + \cos \theta) \left( \frac{c_W}{s_W^2}, \frac{1}{s_W} \right) \]  
(A.9)

\[ F_{\pm 0, \mp} = \frac{e^2 \lambda_W s}{8M_W^2} (3 - \cos \theta) \left( \frac{c_W}{s_W^2}, \frac{1}{s_W} \right) \]  
(A.10)

\[ F_{\pm \mp, 0} = -\frac{e^2}{2s_W^2} \cos \theta (1, 0) \]  
(A.11)

\[ F_{\pm \pm, 0} = -\frac{e^2 \lambda_W s}{4s_W^2 M_W^2} \cos \theta (1, 0) \]  
(A.12)

\[ F_{00, \mp} = \pm \frac{e^2 d\sqrt{s}}{2\sqrt{2} M_W} \sin \theta \left( \frac{c_W}{s_W^2}, \frac{1}{s_W} \right) \]  
(A.13)

\[ F_{0 \mp, 0} = \pm \frac{e^2 d\sqrt{s}}{\sqrt{2}s_W^2 M_W} \sin \theta (1 + \cos \theta) (1, 0) \]  
(A.14)

\[ F_{\pm 0, 0} = \pm \frac{e^2 d\sqrt{s}}{\sqrt{2}s_W^2 M_W} \sin \theta (1 - \cos \theta) (1, 0) \]  
(A.15)

\[ F_{00, 0} = \frac{e^2(1 - 10c_W^2 - \cos^2 \theta(1 - 2c_W^2))}{4e_W^2 s_W^2 (\cos^2 \theta - 1)} \cos \theta (1, 0) \]  
(A.16)

\[ W^- Z, W^- \gamma \rightarrow HW^- \]

\[ F_{\pm \mp, \mp} = \mp \left\{ \frac{4e^2 d\sqrt{s}}{\sqrt{2} M_W} \sin \theta - \frac{e^2 \lambda_W d s^{3/2}}{2\sqrt{2}M_W^3} \right\} \left( \frac{c_W}{s_W^2}, \frac{1}{s_W} \right) \]  
(A.17)
\[ F_{\pm \tau, \pm} = \pm \left\{ \frac{e^2 d \sqrt{s}}{\sqrt{2} M_W} \sin \theta \left( \frac{1 - \cos \theta}{1 + \cos \theta} \right) + \frac{e^2 \lambda_W d s^{3/2}}{2 \sqrt{2} M_W^3} \right\} \left( \frac{c_W}{s_W^2}, \frac{1}{s_W} \right) \] (A.18)

\[ F_{\pm \tau, \mp} = \pm \left\{ \frac{e^2 d \sqrt{s}}{\sqrt{2} M_W} \sin \theta \left( \frac{1 + \cos \theta}{1 - \cos \theta} \right) + \frac{e^2 \lambda_W d s^{3/2}}{2 \sqrt{2} M_W^3} \right\} \left( \frac{c_W}{s_W^2}, \frac{1}{s_W} \right) \] (A.19)

\[ F_{\mp \tau, \mp} = \pm \frac{e^2 \lambda_W d s^{3/2}}{2 \sqrt{2} M_W^3} \sin \theta \left( \frac{c_W}{s_W^2}, \frac{1}{s_W} \right) \] (A.20)

\[ F_{0 \pm, \pm} = \frac{e^2 (1 - 4c_W^2)(1 + \cos \theta) + \cos \theta - 3}{4c_W s_W^2 (1 - \cos \theta)} \left( 1, \frac{s_W}{c_W} \right) \] (A.21)

\[ F_{\mp \tau, 0} = - \frac{e^2 (1 - c_W^2 (1 + \cos \theta))}{2c_W s_W^2} \left( 1, \frac{s_W}{c_W} \right) \] (A.22)

\[ F_{0 \pm, \mp} = \frac{e^2 \lambda_W s^{3/2}}{8 M_W^2} (3 + \cos \theta) \left( \frac{c_W}{s_W^2}, \frac{1}{s_W} \right) \] (A.23)

\[ F_{\pm \pm, 0} = - \frac{e^2 \lambda_W s^{3/2}}{4 M_W^2} \cos \theta \left( \frac{c_W}{s_W^2}, \frac{1}{s_W} \right) \] (A.24)

\[ F_{\pm 0, \pm} = - \frac{e^2 (3 - \cos \theta)}{2 s_W^2} (1, 0) \] (A.25)

\[ F_{0 \pm, \pm} = \frac{e^2 \lambda_W s c_W}{8 s_W^2 M_W^2} (3 - \cos \theta)(1, 0) \] (A.26)

\[ F_{00, \pm} = \pm \frac{e^2 d \sqrt{s}}{2 \sqrt{2} s_W^2 M_W} \sin \theta (1, 0) \] (A.27)

\[ F_{0 \pm, 0} = \pm \frac{e^2 d \sqrt{s c_W}}{\sqrt{2} s_W^2 M_W} \sin \theta (1, 0) \] (A.28)
\[ F_{\pm 0,0} = \pm \frac{e^2 d \sqrt{s}}{\sqrt{2} s_W^2 M_W (\cos \theta - 1)} \sin \theta (1, 0) \quad \text{(A.29)} \]

\[ F_{00,0} = \frac{e^2 (3 - \cos \theta) [\cos \theta (1 - 4 s_W^2 - c_W^2 \cos^2 \theta)]}{4 c_W^2 s_W^2 (\cos^2 \theta - 1)} (1, 0) \quad \text{(A.30)} \]

**Appendix B : Helicity amplitudes for double Higgs production in boson fusion processes at high energy**

These \( V_1(\lambda) V_2(\tau) \rightarrow HH \) processes are described by 9 helicity amplitudes \( F_{\lambda \tau}(\theta) \). \( \theta \) is the angle between \( V_1 \) and \( H \) and the normalization is such that the differential cross section writes

\[ \frac{d\sigma(\lambda \tau)}{d\cos(\theta)} = C |F_{\lambda \tau}(\theta)|^2 , \quad \text{(B.1)} \]

where

\[ C = \frac{1}{32 \pi s} \frac{p_H}{p_{12}} , \quad \text{(B.2)} \]

includes no spin average.

**\( W^- W^+, ZZ, \gamma \gamma, \gamma Z \rightarrow HH \)**

\[ F_{\lambda \tau}(\theta) = - (1 - \delta_\tau) (1 - \delta_\lambda) (1, c_W^2, s_W^2, s_W c_W). \]

\[ \left\{ \frac{d^2 g_2^2 s}{2 M_W^2} (1 + 3 \lambda \tau) + \frac{dg_2^2 s}{4 M_W^2} (1 + \lambda \tau) \right\} \quad \text{(B.3)} \]

**\( WH \rightarrow HW, ZH \rightarrow HZ, \gamma H \rightarrow H \gamma, \gamma H \rightarrow HZ \)**

These \( V_1(\lambda) H \rightarrow HV_2(\mu) \) channels are obtained by crossing those above. The helicity amplitudes are now given by
\[ F_{\lambda\mu}(\theta) = -(1 - \delta_{\mu0})(1 - \delta_{\lambda0})(1, c_W, s_W, s_W c_W)(1 + \cos \theta). \]

\[
\left\{ \frac{d^2 g_2^2 s}{4 M_W^2} (1 - 3 \lambda \mu) + \frac{d g_2^2 s}{8 M_W^2} (1 + \lambda \mu) \right\} \tag{B.4}
\]
References

[1] Proc.Workshop on Physics at Future Accelerators, La Thuile, J.H.Mulvey, ed. CERN Report 87-07; Report, Opportunities and Requirements for Experimentation at a Very High Energy $e^+e^-$ Collider, SLAC-329(1928); Proc. Workshops on Japan Linear Collider, KEK Reports, 90-2, 91-10 and 92-16; Physics and experiments with $e^+e^-$ Linear Colliders, Saariselkä 1991, R.Orava, P.Eerola and M.Nordberg eds., World Scientific, Singapore 1992.

[2] Proc. of the Large Hadron Collider Workshop,ed.G.Jarlrskog and D.Rein, CERN 90-10, ECFA 90-133(1990).

[3] P.M.Zerwas, DESY 93-112, Aug.1993; Proc. of the Workshop on $e^+e^-$ Collisions a 500 GeV: The Physics Potential, DESY 92-123A+B(1992), ed. P.Zerwas;

[4] G.J.Gounaris and F.M.Renard, Z. Phys. C59 (1993) 143 ; erratum, p.682.

[5] G.J.Gounaris, J.Layssac and F.M.Renard, preprint PM/93-37, THES-TP 93/11, revised version January 1994.

[6] G.J.Gounaris, J.Layssac and F.M.Renard, preprint PM/93-26, THES-TP 93/8, to be published in Zeit. Phys.C.

[7] G.J.Gounaris and F.M.Renard, preprint PM/94-03, THES-TP 94/01, to appear in Phys.Lett.B.

[8] G.J.Gounaris and F.M.Renard, Z. Phys. C59 (1993) 133 .

[9] M.Bilenky, J.L.Kneur, F.M. Renard and D. Schilknecht, Nucl. Phys. B409 (1993) 22 .

[10] A.De Rujula et al, Nucl. Phys. B384 (1992) 3 . K.Hagiwara et al, Phys. Lett. B283 (1992) 353 ; Phys. Rev. D48 (1993) 2182 . K.Hagiwara et al, Phys. Lett. B318 (1993) 155 .

[11] L.Bergström and G.Hulth, Nucl. Phys. B276 (1986) 744

[12] I.Ginzburg et al, Nucl. Instrum. Methods 205(1983)47; 219(1984)5.

[13] V.Telnov, Nucl. Instrum. Methods A294(1990)72.

[14] J.H.Kühn, E.Mirkes and J.Steegborn, Z. Phys. C57 (1993) 615 .

[15] A. Djouadi, preprint Montreal University 1994.

[16] M.Capdequi Peyranere et al, Z. Phys. C41 (1988) 99 .

[17] The Higgs Hunter's Guide, J.F.Gunion et al, Redwood City (1990).

[18] W-Y Keung and W.J. Marciano, Phys. Rev. D30 (1984) 248 .
**Figure Captions**

Fig.1 Cross sections for Higgs production in $\gamma\gamma$ collisions from laser backscattering at a 0.5 TeV (a), 1 TeV (b), 2 TeV (c) $e^+e^-$ linear collider. Standard prediction (solid line), with $d = +0.01$ (long dashed), $d = -0.01$ (dashed - circles), $d = +0.005$ (short dashed), $d = -0.005$ (dashed), $d = +0.001$ (dashed - stars), and $d = -0.001$ (dashed - boxes). The expected number of events per year for an integrated luminosity of $10 fb^{-1}$ is also indicated.

Fig.2 Cross sections for Higgs production in $e^+e^- \rightarrow H\nu\bar{\nu}$ through $WW$ fusion at a 0.5 TeV (a), 1 TeV (b), 2 TeV (c) $e^+e^-$ linear collider. Standard prediction (solid line), with $d = +0.01$ (short dashed), $d = -0.01$ (dashed - circles), $d = +0.05$ (long dashed), $d = -0.05$ (dashed - circles), $d = +0.1$ (dashed - stars), and $d = -0.1$ (dashed - boxes).

Fig.3 Cross sections for Higgs production in $e^+e^- \rightarrow He^-e^-$ through $\gamma\gamma$, $\gamma Z$ and $ZZ$ fusion at a 0.5 TeV (a), 1 TeV (b), 2 TeV (c) $e^+e^-$ linear collider. Standard prediction (solid line), with $d = +0.01$ (short dashed), $d = -0.01$ (dashed - circles), $d = +0.05$ (long dashed), $d = -0.05$ (dashed - circles), $d = +0.1$ (dashed - stars), and $d = -0.1$ (dashed - boxes).

Fig.4 Cross sections for associate Higgs production in $e^+e^- \rightarrow HZ$ at a 0.5 TeV (a), 1 TeV (b), 2 TeV (c) $e^+e^-$ linear collider. Standard prediction (solid line), with $d = +0.01$ (long dashed), $d = -0.01$ (dashed - circles), $d = +0.005$ (short dashed), with $d = -0.005$ (dashed), $d = +0.001$ (dashed - stars), and $d = -0.001$ (dashed - boxes).

Fig.5 Ratios of Higgs decay widths $\Gamma(H \rightarrow \gamma\gamma)/\Gamma(H \rightarrow b\bar{b})$ (a), $\Gamma(H \rightarrow \gamma\gamma)/\Gamma(H \rightarrow WW)$ (b), $\Gamma(H \rightarrow \gamma\gamma)/\Gamma(H \rightarrow ZZ)$ (c), $\Gamma(H \rightarrow \gamma\gamma)/\Gamma(H \rightarrow \gamma Z)$ (d). Standard prediction (solid line), with $d = +0.01$ (long dashed), $d = -0.01$ (dashed - circles), $d = +0.005$ (short dashed), $d = -0.005$ (dashed), $d = +0.001$ (dashed - stars), $d = -0.001$ (dashed - boxes).