On the orbits of low-mass companions to white dwarfs and the fates of the known exoplanets

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ABSTRACT
The ultimate fates of binary companions to stars (including whether the companion survives and the final orbit of the binary) are of interest in light of an increasing number of recently discovered, low-mass companions to white dwarfs (WDs). In this Letter, we study the evolution of a two-body system wherein the orbit adjusts due to structural changes in the primary, dissipation of orbital energy via tides, and mass-loss during the giant phases; previous studies have not incorporated changes in the primary’s spin. For companions ranging from Jupiter’s mass to $\sim 0.3 \, M_\odot$ and primaries ranging from 1 to 3 $M_\odot$, we determine the minimum initial semimajor axis required for the companion to avoid engulfment by the primary during post-main-sequence evolution, and highlight the implications for the ultimate survival of the known exoplanets. We present regions in secondary mass and orbital period space where an engulfed companion might be expected to survive the common envelope phase (CEP), and compare with known M dwarf+WD short-period binaries. Finally, we note that engulfed Earth-like planets cannot survive a CEP. Detection of a first-generation terrestrial planet in the WD habitable zone requires scattering from a several au orbit to a high-eccentricity orbit (with a periastron of $\sim R_\odot$) from which it is damped into a circular orbit via tidal friction, possibly rendering it an uninhabitable, charred ember.

Key words: stars: AGB and post-AGB – binaries: close – stars: late-type – stars: low-mass – white dwarfs.

1 INTRODUCTION
The ultimate fate of low-mass companions to main-sequence (MS) stars is of interest as substellar and stellar companions to intermediate-mass stars are plentiful (Duquennoy & Mayor 1991; Raghavan et al. 2010; Wright et al. 2011). Observationally, evidence for companions to evolved stars is varied. In the early subgiant phase, a number of giant planets and giant planet candidates have been detected (Johnson et al. 2006, 2011). At the end of post-MS evolution, low-mass companions have been found in post-common-envelope, short-period orbits around subdwarfs and white dwarfs (WDs) (see Maxted et al. 2006; Silvestri et al. 2007; Charpinet et al. 2011; Liu et al. 2012; Rebassa-Mansergas et al. 2012; also Section 3), and in long-period orbits around WDs (Farihi, Becklin & Zuckerman 2005; Farihi, Hoard & Wachter 2006; Farihi et al. 2012).

During post-MS evolution, dynamical interactions induced by radial expansion of the primary, strong mass-loss via stellar winds and tidal interactions can occur. Such processes have been previously studied for two-body systems (Carlberg, Majewski & Arras 2009; Villaver & Livio 2009; Nordhaus et al. 2010; Mustill & Villaver 2012) and many-body systems (Veras et al. 2011; Kratter & Perets 2012; Perets & Kratter 2012; Veras & Tout 2012). In this Letter, we focus on two-body interactions. We improve upon previous studies by including changes in the rotation rates of both bodies due to tidal dissipation and a varying moment of inertia (MoI). During the subgiant and giant phases, the growth of the MoI of the primary can cause companions within a few astronomical unit to switch from stable orbital configurations to being on the unstable side of the inner co-rotation point, thereby facilitating a plunge into their host stars (Spiegel 2012). On the other hand, by tracking the spin-up of the primary, we find that higher mass companions can retard their infall – an effect not incorporated in previous studies.
example, although Mustill & Villaver (2012) include the evolution of the planet’s spin (which is negligible at separations of a few au\(^1\)), they neglect the changes in the primary’s spin that drive the system’s tidal evolution. This is reasonable for the very low mass companions they consider, but calculating changes in the primary’s spin is necessary for following the orbital evolution for higher mass companions.

The structure of this Letter is as follows. In Section 2, we describe the physics and assumptions of our approach. In Section 3, we present the minimum semimajor axis necessary to avoid engulfment as a function of companion mass and zero-age-main-sequence (ZAMS) primary mass. For engulfed companions, we estimate whether or not the companion survives the common envelope and the corresponding orbital period at which it emerges. In Section 4, we comment on the implications of finding a first-generation terrestrial planet in the WD habitable zone in the context of binary evolution. We conclude in Section 5.

2 TIDAL DISSIPATION AND MASS-LOSS

To study the evolution of our two-body system, we employ a tidal interaction model that couples the mass and radius of the primary star with the orbit of a secondary body. Observational constraints on tides are difficult to achieve. Zahn (1977, 1989) proposed a tidal theory based on turbulent viscosity, which we adopt in this work. The theory was tested by introducing a dimensionless parameter \( f \) that was then calibrated using eccentricity measurements of a sample of post-MS binaries (Verbunt & Phinney 1995). Based on the divide between observed circularised and non-circularised systems, it has been argued that \( f \) is constant and equal to unity\(^2\) (Verbunt & Phinney 1995).

The evolution of the semimajor axis, \( a \), is as follows:

\[
\frac{da}{dt} = \left( \frac{da}{dt} \right)_{\text{tides}} + \frac{\Delta a}{\Omega^2}\frac{dt}{\text{mass-loss}},
\]

where the first term represents the change in semimajor axis due to tidal dissipation (described below) and the second term represents the adiabatic change due to mass-loss [i.e. \( (\Delta a/\Omega^2)\text{mass-loss} \approx -a M/\dot{M} \).] Note that, while tidal dissipation occurs in both bodies, we assume that only the primary loses mass. Since this material is lost from the system (Spiegel & Madhusudhan 2012), the second term is positive and acts to widen the orbit. We take the orbit to be circular and refer the reader to Socrates, Katz & Dong (2012a) for intricacies in modelling tidal dissipation in eccentric systems.

As the system moves towards synchronization, the change in rotation rate of each body is

\[
\frac{d\Omega}{dt} = \left( \frac{d\Omega}{dt} \right)_{\text{tides}} + \left( \frac{d\Omega}{dt} \right)_{\text{Mol}},
\]

where \( \Omega_c \) and \( \Omega_p \) are the spin rates of the primary and companion, respectively. The first term represents the change in the spin rate of a body due to tidal dissipation in its interior. The second term affects the spin rate due to changes in the MoI. In this work, we assume that the companion’s mass and MoI do not change. For a spin angular momentum loss rate of \( \dot{S}_p = (2/3) M_R \Omega^2 \Omega_p \), from the primary, we have that

\[
\left( \frac{d\Omega}{dt} \right)_{\text{Mol}} = \frac{2}{3\alpha_c} \left( \frac{M_c}{M_p} \right) \frac{\dot{I}_p}{I_p} \frac{\dot{L}_c}{L_c},
\]

Figure 1. The period gap. A companion (blue circle) orbiting just exterior to the critical initial semimajor axis escapes engulfment such that the orbit expands. A companion (red circle) orbiting just interior to the critical initial semimajor axis is engulfed such that the orbit shrinks during the CEP.

3 PERIOD GAPS

Due to the combined effects of tidally induced orbital decay and mass-loss-induced orbital expansion, there should be a period gap in the distribution of low-mass companions to WDs. A cartoon schematic of this gap is presented in Fig. 1. The outer edge of the gap

\[
\text{primary, we have that}
\]

\[
\left( \frac{d\Omega}{dt} \right)_{\text{tides}} = -\frac{12\alpha k_a f}{\tau_{\text{conv}} M_c M_p} \left( \frac{M_c}{M_p} \right) \left( 1 + \frac{M_c}{M_p} \right) \frac{R_a}{a} \left( 1 - \frac{\Omega_a}{n} \right),
\]

where \( n \) is the orbital mean motion, \( k_a \), is the primary’s tidal Love number which we assume to be unity. The convective time of the primary is taken to be \( \tau_{\text{conv}} = (M_c R_p^2/L_p)^{1/2} \) such that \( L_p \) is the luminosity of the giant and \( M_c \) is the mass of the convective envelope. The change in rotation rate of the primary is then

\[
\left( \frac{d\Omega}{dt} \right)_{\text{tides}} = 6 \pi k_a f \left( \frac{M_c}{M_p} \right) \frac{R_a}{a} \left( 1 - \frac{\Omega_a}{n} \right) \frac{\dot{I}_p}{I_p}.
\]

The equation for \( \Omega_c \) can be obtained by reversing the \( \ast \) and c subscripts in equation (5), although at \( a \gtrsim 1 \text{ au} \), \( \Omega_c \) is negligible.

\(^1\) A Jovian companion will experience a change in spin of \( \lesssim 1 \) part in \( 10^3 \times (a/1 \text{ au})^2 \) during a Hubble time due to the stellar tide raised on it, where \( a \) is the orbital separation.

\(^2\) For a detailed description of the relation between \( f \) and the tidal quality factor \( Q^1 \), (Goldreich & Soter 1966) and the implications of the claim that \( f = 1 \) see sections 2.3 and 4.1 of Nordhaus et al. (2010).

\(^3\) If the wind is strongly magnetically coupled to the primary, Coriolis torques could render the situation somewhat more complicated than the model presented here.
for each binary configuration is given by the final orbital separation of the companion that escapes engulfment (blue circle in Fig. 1). Since tidal torques drop off as a large negative power of orbital separation, most companions that escape engulfment experience essentially no tidal interactions. Therefore, the outer boundary of the gap can be approximated as $a_{\text{outer}}(M_{\ast, i}/M_{\text{WD}})$, where $a_{\text{outer}}$ is the minimum semimajor axis that escapes engulfment, the primary’s ZAMS mass and $M_{\text{WD}}$ is the mass of the emergent WD. The inner edge is determined by the fate of the engulled companion (red circle in Fig. 1) which will either emerge in a post-CE short-period orbit or be destroyed during the common envelope phase (CEP).

The inner edge of the period gap is presented in Fig. 2. The ‘allowed region’ is marked in magenta. The $a_{\text{inner}}$ boundaries (orange dash–dotted lines) are determined by calculating the semimajor axis where a fraction $a_{\text{CE}}$ of the liberated orbital energy during inspiral is used to unbind the CE. The $a_{\text{shred}}$ boundary marks the location where companions tidally disrupt. The known low-mass-short-period companions to WDs are marked by circles and diamonds in Fig. 2. Attempts have been made to calculate $a_{\text{CE}}$ from 3D simulations (Passy et al. 2012a; Ricker & Taam 2012) and from observations (Zorotovic et al. 2010; De Marco et al. 2011) but the large range of scales involved in a CE inspiral makes this a difficult calculation and there is, as yet, no consensus. In Fig. 2, we present $a_{\text{inner}}$ for $a_{\text{CE}}$ values of 0.25 (Zorotovic et al. 2010) and 1.

Spiegel (2012). This $a_{\text{shred}}$ boundary (yellow dash–dotted line) is shown in Fig. 2. Additionally, the properties of the known short-period, low-mass companions to WDs are marked by circles and diamonds in Fig. 2. The inner edge is determined by the fate of the engulled companion (red circle in Fig. 1) which will either escape engulfment or be destroyed during the common envelope phase (CEP).

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Fig. 3 shows the minimum initial semimajor axis required to escape engulfment as a function of primary and companion masses. Employing the Verbunt & Phinney ($f = 1$) calibration of the Zahn (1977, 1989) tidal formalism leads to very strong tides during the post-MS, with $Q_{\ast}$ values as low as $\sim 10^{1}$–$10^{2}$ at the time of plunge (Nordhaus et al. 2010). The values of the contours in Fig. 3 are sensitive to the maximum radius of the stellar model; changes in the stellar model that influence the maximum radius correspondingly influence $a_{\text{crit}}$. The discovery of companions near the outer edge of the gap would constrain the nature of tidal dissipation and the late stages of stellar evolution.

Note that in Fig. 3, there is a decrease in the semimajor axis required to escape engulfment for companions greater than $\sim 60 M_{J}$. This is because at equal separation from the star, a more massive companion torques the star more strongly in proportion to the square of its mass. As the star ascends the asymptotic giant branch (AGB), a sufficiently massive companion can halt its infall by synchronizing the primary. Therefore, for a massive companion to be engulfed, its initial separation needs to be smaller than that of a less massive companion. Extremely massive companions can tidally transfer enough angular momentum to significantly spin up...
the primary. In such cases, enhanced mass-loss and deformation of the primary may occur. Since these processes are not modelled in our calculations, we exclude this region of parameter space (shown in grey in Fig. 3).

Fig. 4 shows the fates of the known planets with masses greater than or equal to Jupiter’s and host-star masses greater than or equal to the Sun’s (data from the Open Exoplanet Catalogue; Rein 2012). For each system, we calculate the joint stellar and tidal evolution to determine whether the planet will be engulfed in the post-MS stages. For those objects that avoid engulfment, we use the WD initial–final mass relation of Catalán et al. (2008) to determine the post-MS orbit expansion. No known systems will evolve to have circumb-DW Jovian planets within ~6.4 au. Relatively few known exoplanets will survive to orbit a WD. Predictions of the demographics of WD planets will improve with a better accounting of the distribution of MS planets in the 3–10 au range, which should be possible with microlensing studies with Wide-Field Infrared Survey Telescope (Green et al. 2012).

4 ARE THERE HABITABLE EARTHS AROUND WHITE DWARFS?

The annulus around a WD that is amenable to habitable climates (Kasting, Whitmire & Reynolds 1993; Spiegel, Menou & Scharf 2008) might be at an orbital separation of ~1R⊙ (Agol 2011; Fossati et al. 2012; Loeb & Maoz 2013). How could a terrestrial planet end up in such a potentially habitable orbit?

There are two ways that a first-generation planet could end up in a close orbit around a WD. It either (a) survives a common envelope stage with a subgiant, red-giant or AGB star, in which the companion is engulfed and inspirals inwards until its plunge is arrested before tidal disruption very close to the degenerate core (at distances of ≤0.3R⊙) or (b) migrates inwards after the WD has finished forming and expelled its envelope.

Scenario (a) – CE survival – is implausible for a companion that is less than ~6MJ (see Fig. 2), because such low-mass companions lack enough energy to unbind even an evolved star’s envelope, a necessary but not sufficient condition for CE survival (Nordhaus et al. 2010; Spiegel 2012). Furthermore, the inspiral accelerates as the companion moves inwards (Nordhaus & Blackman 2006; Nordhaus, Blackman & Frank 2007), which means that for the star to unbind its own envelope right when the companion is at 1R⊙ would require extreme fine tuning. Even if a low-mass companion were to arrest its inspiral at ~1R⊙, the high temperatures that it would encounter deep in the envelope of the primary (~10⁶ K) might pose a severe threat to its subsequent habitability or to its very survival (Villaver & Livio 2007).

Scenario (b) – post-WD orbital evolution – might occur, but could render planets that experience such a process uninhabitable by life as we know it. Indeed, Zuckereman et al. (2010) and others have found evidence of tidally shredded asteroids accreting metals on to WDs, indicating that processes can occur in post-MS planetary systems that excite extreme eccentricities among low-mass particles. Via the Kozai mechanism (Kozai 1962; Fabrycky & Tremaine 2007; Katz, Dong & Malhotra 2011; Naoz, Farr & Rasio 2012; Shappee & Thompson 2013; Socrates et al. 2012b), a massive outer body can drive an inner companion to arbitrarily high eccentricities, such that asteroids tidally disrupt very near the WD. Similar processes certainly might drive an Earth-mass planet from a several-au orbit (far enough out that it avoided being engulfed during the AGB phase) to a high-eccentricity orbit with a periastron of 0.5R⊙, from which tidal friction would damp it to a 1R⊙ circular orbit. However, if this happens, a large amount of orbital energy must be dissipated as heat:

$$\Delta E_{\text{orb}} \sim -3 \times 10^{42} \text{ergs} \times \left( \frac{M_{\text{WD}}}{0.5 \, M_\odot} \right) \left( \frac{M_p}{M_\oplus} \right) \left( \frac{a_{\text{final}}}{R_\odot} \right)^{-1},$$  

where $M_p$ is the planet’s mass. Dissipation of this heat over a circularisation time-scale of $t_{\text{circ}} \sim 10^6$ yr corresponds to a tidal heat flux of $\sim (2 \times 10^7 \, \text{W m}^{-2}) \times (t_{\text{circ}}/1 \, \text{Myr})^{-1}$, or an average cooling temperature of $\sim (4000 \, \text{K}) \times (t_{\text{circ}}/1 \, \text{Myr})^{-1/4}$, which could be catastrophic for the habitability of such a planet (see Barnes et al. 2012 and Barnes & Heller 2012, who considered the adverse effects
Figure 4. The fates of known planetary systems. The red and blue dots indicate current semimajor axes for each of the ∼300 known exoplanets with masses at least Jupiter’s and host-star masses greater than or equal to the Sun’s. A red dot indicates that the planet will be engulfed (according to Fig. 3) while a blue dot indicates that the planet will escape engulfment; green dots indicate final circum-WD separations. The small light red (light blue) dots indicate current periastra of planets that will be (avoid being) engulfed. Current and final orbital semimajor axes of Jupiter are shown with magenta and yellow stars, respectively. Among the currently known planetary systems, none will evolve to have circum-WD Jovian planets within ∼6.4 au.

on habitability of much lower tidal power). Such a migrated terrestrial planet might exist as a charred ember in the habitable zone. Though late-time delivery of volatiles might reintroduce water to a desiccated WDHZ planet, the near-daily cometary impacts on the Sun (Marsden 2005) imply that an Earth at a 1 R☉ orbital radius could be subject to cometary hits at (∼(R⊕/R☉)2 times this rate or once per several decades. These impacts would have ∼100 times the specific energy of what wiped out the dinosaurs. If large, these impactors could sterilize the world; if small, they would deliver very little water.

5 CONCLUSIONS

We have studied the effect of post-MS evolution on the orbits of substellar and M-dwarf companions. For each combination of primary and companion mass, we evolve the system from the ZAMS through the end of the post-MS. The evolution is governed by tidal dissipation, mass-loss from the system and structural changes in the primary. For each binary configuration, we determine the initial semimajor axis required to escape engulfment. We note that terrestrial planets cannot survive engulfment. Therefore, a first generation terrestrial planet in the WDHZ must have experienced an enormous tidal flux and, though formally habitable, might be inhospitable. Interestingly, no currently known Jovian planets will evolve to have circum-WD orbits within ∼6.4 au.

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