Overview of AC Motor Sensorless Algorithms: a Unified Perspective

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Abstract—This paper reviews sensorless algorithms for both induction motors and permanent magnet motors using the active flux model, such that any design applicable for non-salient pole ac motors can also be included in the review framework. The proposed review framework classifies all sensorless algorithms following a five-layer hierarchy abbreviated as O-I-M-A-I, resulting in four main categories as i) inherently sensorless position estimation, ii) non-inherently sensorless position estimation, iii) post-position-estimation speed estimation, and iv) speed estimation for indirect field orientation. Various ac motor models are derived by assuming a constant active flux amplitude, based on which seven generic sensorless algorithms are summarized and compared with respect to a natural choice of state (i.e., rotor flux) in the inverse-Γ circuit of induction motor [7], and it has long been found useful in analysis, e.g., of direct torque control (DTC) [8, Eq. (7)]. In salient PM motor context, active flux has other names in literature, such as fictitious PM flux [9], linear flux [10], and extended flux [11].

The contribution of this paper is to propose an overview framework that reviews SAs for both induction motors and PM motors. The overview has two parts depending on how $K_{\text{Active}}$ is modelled. Sec. II–Sec. V reviews SAs with constant $K_{\text{Active}}$ assumption. Sec. VI reviews SAs with time-varying $K_{\text{Active}}$ as in (1). The rest of this section builds the foundation to understand the review with a classification and a tutorial.

A. Proposed Classification

We propose a classification for sensorless algorithms (SAs) that obeys a 5-layer hierarchy, abbreviated as O-I-M-A-I:

1) Output, i.e., position $\hat{\theta}$ or magnetic field speed $\dot{\theta} = \dot{\hat{\theta}}$;
2) Input, i.e., $i$-only, $\hat{i}$-dependency, $\hat{\theta}$-dependency, or ($\int \hat{\theta} dt$)-dependency;
3) Model, i.e., voltage model (VM), 2nd-order current, 4th-order emf, 4th-order disturbed flux, motion dynamics;
4) Algorithm, i.e., stabilized voltage model, disturbance observer, state observer, adaptive observer, etc.;
5) Issues and improvements associated with the algorithm, which forms the outline of Sec. II–V as follows.

- Position Estimation (PE)
  - Inherently sensorless (IS) (i.e., no speed anywhere)
    * Stabilized VM with flux or voltage compensation
      - Amplitude (i.e. current model) based correction
      - Angle (i.e., orthogonality) based correction
    * Disturbance observer (DO) for emf
      - Linear DO (that treats ac emf as dc disturbance)
      - Sliding mode (SM) DO
  1) Improvement in SM control law: milder switching function and dynamic correction.
B. Sensorless Control: A Tutorial

To help understand the classification above, a tutorial on sensorless algorithm (SA) design is now provided.

1) Problem Formulation: Motor’s electrical angular rotor speed \( \omega_r \) is governed by Newton’s second law of motion:

\[
J_s \frac{d}{dt} \omega_r = T_{em} - T_L
\]

where \( J_s \) denotes rotor shaft inertia, \( n_{pp} \) designates pole pair number, \( T_L \) is load torque, and \( T_{em} \) is electromagnetic torque. The sensorless control of system (2) is challenging, because both the inputs \( T_{em}, T_L \) and the output \( \omega_r \) are unknown.

2) The Objective of SAs: The objective of an SA is to estimate \( \omega_r \) and \( T_{em} \). The latter is proportional to the cross product of the stator flux \( \psi_s \) and the active flux \( \psi_{active} \):

\[
T_{em} = \frac{3}{2} n_{pp} L_q \psi_s \cdot (-J \psi_{active}) = \frac{3}{2} n_{pp} K_{active} i_q
\]

where the second equal sign is derived by substituting the following relation between \( \psi_s \) and \( \psi_{active} \) in \( \alpha/\beta \)-frame [1] .

\[
\psi_{active} \triangleq K_{active} \angle \theta_d \equiv K_{active} \begin{bmatrix} \cos \theta_d \\ \sin \theta_d \end{bmatrix} = \psi_s - L_q i
\]

and \( i_q = -i_s \sin \theta_d + i_d \cos \theta_d \) is the q-axis current. The FOC formulation in (3) motivates the concept of active flux [1]. Based on (4), the \( d \)-axis angle \( \theta_d \) can be extracted by

\[
\theta_d = \arctan2(\psi_\beta, \psi_\alpha) \quad \text{or} \quad \theta_d = -\arctan2(e_\alpha, e_\beta) \quad \text{(5a)}
\]

or \( \theta_d = -\arctan2(e_\alpha, e_\beta) \quad \text{(5b)} \)

where \( e_\alpha \triangleq \frac{d}{dt} \psi_{active} \) is the emf due to active flux.

3) The Main Assumption: From (3), the objective of estimating \( T_{em} \) reduces to estimation of active flux angle \( \theta_d \), if we assume the active flux amplitude \( K_{active} \) is a known constant such that both \( \alpha \)-axis and \( \beta \)-axis components of \( \psi_{active} \) are sinusoidal [cf. (4)]. Constant \( K_{active} \) assumption is reasonable for PM motors, and simplifies the design and analysis of SAs for induction motors, which is not well realized in literature.

4) The Models: The estimation of \( \theta_d \) relies on the dynamics of measured \( i \) to reveal \( \psi_{active} \) or its derivative \( \frac{d}{dt} \psi_{active} \). As per Faraday’s law, the stator voltage equation in \( \alpha/\beta \)-frame describes the dynamics among \( i \), \( \psi_{active} \), and \( \psi_s \):

\[
\frac{d}{dt} \psi_s = \frac{d}{dt} (\psi_{active} + L_q i) = e_{active} + L_q \frac{d}{dt} i = u - Ri \quad \text{(6)}
\]

where \( u \) is \( \alpha/\beta \)-frame voltage. Since there is no speed in (6), the SAs based on (6) are called inherently sensorless [12].

The magnetic field speed \( \omega \triangleq \dot{\theta}_d \) begins to appear, if we further: i) include the steady state sinusoidal model for either flux or emf: [note differentiating (7a) yields (7b)]

\[
\frac{d}{dt} \psi_{active} = \omega J \psi_{active} + K_{active} \begin{bmatrix} \cos \theta_d \\ \sin \theta_d \end{bmatrix} \quad \text{(7a)}
\]

\[
\frac{d}{dt} e_{active} = \omega J e_{active} + \frac{d}{dt} \psi_{active} = \frac{d}{dt} E_u; \quad \text{Unmodelled dynamics (7b)}
\]

or ii) cascade disturbed sinusoidal model for flux estimate [13]:

\[
\frac{d}{dt} \psi_s = \omega J \left( \psi_s - D_\psi \right), \quad \frac{d}{dt} D_\psi \approx 0 \quad \text{(8)}
\]

with \( D_\psi \in \mathbb{R}^2 \) the low frequency disturbance in stator flux; or iii) substitute (4) into (6) to derive a nonlinear model:

\[
L_q \frac{d}{dt} i_q = u - Ri - \omega K_{active} \begin{bmatrix} \sin \theta_d \\ -\cos \theta_d \end{bmatrix}, \quad \omega = 0, s \theta_d = \omega \quad \text{(9)}
\]

or iv) apply Park transformation \( P(\theta_d) = \begin{bmatrix} \cos \theta_d & \sin \theta_d \\ -\sin \theta_d & \cos \theta_d \end{bmatrix} \) to (6):

\[
\frac{d}{dt} K_{active} + L_q \frac{d}{dt} i_d = u_d - R_i d + \omega L_q i_q \triangleq e_{d,ss} \quad \text{(10a)}
\]

\[
\omega K_{active} + L_q \frac{d}{dt} i_q = u_d - \dot{R}_i d - \omega L_q i_d \triangleq e_{q,ss} \quad \text{(10b)}
\]

where \( e_{d,ss}, e_{q,ss} \) are steady state emf of \( d \)- and \( q \)-axis. By manipulating inductance, the \( dq \)-model (10) is rewritten as

\[
\frac{d}{dt} K_e + L_e \frac{d}{dt} i_d = u_d - R_i d + \omega L_q i_q \quad \text{or (11)}
\]

Amplitude of the extended emf \( e \)
Transforming (11) to $αβ$-frame gets extended emf $e$’s model

\[
\begin{align*}
L_d \frac{d}{dt} \hat{i} + e + \omega J (L_d - L_q) \hat{i} &= u - R \hat{i} \\
\frac{d}{dt} e &= \omega J e + \text{Unmodelled Dynamics}
\end{align*}
\]

(SA2.2)

The SAs that are based on (7), (8), (9), (10), (11), or (12) are non-inherently sensorless owing to the presence of $ω$.

5) Position SA Designs: The model based $θ_d$ estimation can be achieved by the following generic SAs via (5):

SA1: Disturbance observer (DO) for emf using (6):

\[
L_q \frac{d}{dt} \hat{i} = u - R \hat{i} + f(\hat{i}), \quad \text{with} \quad i \triangleq i - \hat{i}
\]

(SA2.3)

where $f(\cdot)$ is correction term to be designed, and emf information is extracted from $f(\hat{i})$. It is worth pointing out that $f(\cdot)$ can be implemented as dynamic correction.

SA2.1: State observer, e.g., using extended emf model (12):

\[
L_d \frac{d}{dt} \hat{i} = f_1(\hat{i}) + u - R \hat{i} - e - \omega J (L_d - L_q) \hat{i} \\
\frac{d}{dt} e = f_2(\hat{i}) + \omega J e
\]

(SA2.4)

where $f_1(\cdot), f_2(\cdot)$ are corrections to be designed. Similar observer can be constructed for (6) and (7). Note (14) has its reduced-order variant, e.g.,

\[
s \hat{e} = \omega J \hat{e} + f (\hat{s} - s)
\]

(SA2.5)

where $\hat{s} \equiv [u - R \hat{i} - e - \omega J (L_d - L_q) \hat{i}] / L_d$.

SA2.2: State observer using (6) and (7a):

\[
L_q \frac{d}{dt} \hat{i} = f_1(\hat{i}) + u - R \hat{i} - \omega J \psi_{\text{Active}} \\
\frac{d}{dt} \hat{\psi}_{\text{Active}} = f_2(\hat{i}) + \omega J \hat{\psi}_{\text{Active}}
\]

Note (16) has its reduced-order variant, e.g.,

\[
s \hat{\psi}_{\text{Active}} = \omega J \hat{\psi}_{\text{Active}} + K [\omega J \psi_{\text{Active}} - \omega J \hat{\psi}_{\text{Active}}]
\]

(SA2.6)

where $\omega J \psi_{\text{Active}} = s \psi_{\text{Active}} \equiv u - R \hat{i} - L_q s \hat{s}$ must be substituted, and $K = k_1 I - k_2 \text{sign}(\hat{\omega}) J, k_1, k_2 > 0$.

SA2.3: Extended Kalman filter for the constant speed 4th-order nonlinear model (9) [15], or for model (9) with $s \omega = 0$ being replaced by motion dynamics [2] [16].

SA2.4: As an alternative to SA2.3, transforming (9) into dq-frame results in a 2nd-order open-loop current observer that is disturbed by position error $\hat{\theta}_d = \theta_d - \hat{\theta}_d$:

\[
L_q \hat{s}_d = u_d - R i_d + \omega L_q i_q + K_{\text{Active}} \omega \sin \hat{\theta}_d \\
L_q \hat{s}_q = u_q - R i_q - \omega L_q \hat{i}_d - K_{\text{Active}} \omega \cos \hat{\theta}_d
\]

(SA2.7)

for which the $\hat{\theta}_d = 0$ assumption simplifies the unknown position error terms as $\sin \hat{\theta}_d = \hat{\theta}_d$ and $\cos \hat{\theta}_d = 1$.

SA3: Voltage model with voltage compensation using (9):

\[
\hat{\psi}_s = \hat{\psi}_{\text{Active}} + L_q \hat{i} = \int_0^t (u - R \hat{i} + \hat{D}) \, dt
\]

(SA2.8)

where $\hat{D}$ is the stabilizing voltage yet to be designed.

1The term ‘dynamic’ means the correction $f$ has internal state, implying $f$ involves integral operation. Disturbance observer with dynamic correction is in the form of a state observer with an extended state (similar idea to ESO).

SA4: Flux compensation for VM based on DO using (8):

\[
\begin{align*}
\frac{d}{dt} \hat{\psi}_s &= f_1 (\hat{\psi}_s - \hat{\psi}_s) + \omega J (\hat{\psi}_s - \hat{D}) \\
\frac{d}{dt} \hat{D} &= f_2 (\hat{\psi}_s - \hat{\psi}_s)
\end{align*}
\]

(SA2.9)

where $\hat{\psi}_s$ is obtained by (19), and the estimated flux disturbance $\hat{D}$ is used for building $\hat{D}$ in (19). The final flux estimate is $\hat{\psi}_s - \hat{D}$ rather than $\hat{\psi}_s$.

In summary, SA1 and SA4 are disturbance observers (DOs), and the difference is that SA1 assumes voltage disturbance in the current dynamics while SA4 assumes a flux disturbance as $\psi_s = \hat{\psi} + D$. Note SA4 is an example of DO with dynamic correction $D$. SA2 is state observer, and its difference from DO is that state observer has a model for the unknown state. SA2 and SA4 always rely on a speed estimate $\hat{\omega}$. SA3 is integrator, and its difference from SA4 is that SA3 relies solely on voltage compensation in the integrator input, while SA4 further utilizes flux compensation at the integrator output.

SA1–4 are the multi-input multi-output (MIMO) version of the story that links SA1–4 in Appendix A.

- SA1 is the MIMO version of DO [19, 40] and [41].
- SA2 is the MIMO version of state observer [42] and [43].
- SA3 is the MIMO version of voltage compensation [44].
- SA4 is the MIMO version of flux compensation $D_{\psi}$.

Remark 1: SA4 is a cascaded DO rather than a reduced-order observer. SA4 is also not a CAO because it does not implement speed adaptation law but instead uses $\hat{\omega}$ extracted from the prior flux estimate $\hat{\psi}_s$ [13].

6) Speed SA Designs: The model based $\omega_r$ or $\omega$ estimation can be achieved by the following generic SAs:

SA5: Direct calculation from $dq$-frame emf using the simplified steady state model of [10] or [11]:

\[
\hat{\omega} = \frac{\hat{e}_{d,ss}}{K_{\text{Active}}}, \quad \text{if} \quad \hat{e}_{d,ss} = 0
\]

(SA2.10)

or direct calculation from position estimate by forward difference of $\hat{\omega} = \frac{\hat{\omega}}{\hat{\theta}_d}$, or direct calculation from stator flux and stator emf $e_s$ by:

\[
\hat{\omega} (\hat{\psi}_s^T \hat{\psi}_s) = -(J \hat{\psi}_s^T) e_s.
\]

SA6: Speed adaptation law driven by some output error $\varepsilon$:

\[
\frac{d}{dt} \hat{\omega} = \text{Gain} \times \text{Regressor} \times \varepsilon
\]

(SA2.11)

SA7: Speed observer using [2] corrected by output error $\varepsilon$

\[
\frac{d}{dt} \hat{\omega}_r = n_{pp} J_s^{-1} \left( T_{em} - \hat{T}_L \right) + k \varepsilon
\]

(SA2.12)

or simply

\[
\frac{d}{dt} \hat{\omega}_r = \hat{\alpha} + k \varepsilon
\]

(SA2.13)

where the dynamics of $\hat{T}_L$ and $\hat{\alpha}$ are yet to be designed.

To sum up, SA5 and SA6 are inertia-free, while SA7 often depends on inertia but provides an estimate of load torque; SA6 is often derived by Lyapunov analysis that requires constant $\omega$ assumption to ensure the asymptotical stability of output error $\varepsilon$. (23b) is an inertia-free variant of (23a). A generalized form of SA7 is (46) in Appendix B.
Several tools are proposed and can be combined to build assumption (i) and the orthogonality between \( \alpha \) and assumption (ii) leads to assumption (ii) and (iii) if \( \dot{\omega} \). Both assumption (ii) and (iii) are built upon assumption (i), to position from integral of speed: \( \int \omega dt \). The block diagram of a sensorless control system using SA3/4 and SA5/6 is shown in Fig. 2. In order to achieve the least parameter dependency, the speed is extracted from the stator flux, which is, however, reported in [22] to have much worse dynamic performance compared with the speed extracted from the active flux.

II. Inherently Sensorless Position Estimation (IS-PE)

As shown in Fig. 1(a), in the framework of IS-PE, the output is the \( \theta_d \)-related state (i.e., flux or emf), the input is the measured current \( i \), and the model is \( \dot{i} = 2\text{nd order flux} \frac{d}{dt} \psi_s \text{ and 2nd order current} \frac{d}{dt} i \) that correspond to the two algorithms, SA3 and SA1, respectively. This section reviews the IS versions of SA3 and SA1, and their associated issues and improvements.

A. IS-SA3: Stabilized Voltage Model (VM) of Flux \( \psi_s \)

SA3 applies pure integration to the calculated emf, and the integration is stabilized by voltage compensation \( D \) or flux compensation \( D_\psi \). When there is a drift \( D_\psi = \int_0^t D dt \) in the flux estimate, the resulting errors in position and speed estimation can be derived as in [24] Eqs. (19), (21)). In order to build the stabilizing terms \( D \) or \( D_\psi \), several assumptions are resorted to, including:

(i) the constant \( K_{\text{Active}} \) assumption \( \text{cf. [4]} \), so the active flux trajectory (i.e., the Lissajous curve) is circular;
(ii) the orthogonality between flux and emf: \( e_{\text{Active}} = \omega J_\psi \psi_{\text{Active}} \text{ cf. [7a]} \)
(iii) and the orthogonality between \( \alpha \)-axis and \( \beta \)-axis emfs: \( \dot{e}_{\text{Active}} = \omega J e_{\text{Active}} \text{ cf. [7b]} \).

Both assumption (ii) and (iii) are built upon assumption (i), and assumption (i) leads to assumption (ii) and (iii) if \( \dot{\omega} = 0 \). Several tools are proposed and can be combined to build \( \dot{\psi}_s \).

\[ D_\psi = \frac{1}{2} \left[ \max \left( \hat{\psi}_{\alpha s} \right) + \min \left( \hat{\psi}_{\beta s} \right) \right] \]

should be 0. If \( D_\psi \) in (24) is not null, one can directly use it to produce a final flux estimate as \( \hat{\psi}_s - D_\psi \) [23], or one can use it to construct a voltage compensation as \( D = LPF(s) D_\psi \) [25]. In fact, the voltage compensation error \( D - D_\psi \) can be exactly calculated from \( D_\psi \) by utilizing time information [26].

Remark 2: As discussed in [22, Sec. III], it is better to use active flux to calculate \( D_\psi \) in (24), because stator flux components are not sinusoidal with sudden change in current. For FOC drive, the active flux is obtained from (4), while for DTC drive, the angle between active flux and stator flux can be found by looking up the torque angle [22].

3) Amplitude Correction: Amplitude limiter does not work when the estimated flux amplitude is less than \( K_{\text{Active}} \) [2], so it is natural to replace the limiter with an amplitude correction as follows [2].

\[ D = PI \left( \theta_d \right) \left[ \begin{array}{c} \varepsilon \\ 0 \end{array} \right] = PI \left( s \right) \left[ \frac{\psi_{\text{Active}}}{\left| \psi_{\text{Active}} \right|} \right] PI \left( s \right) \left( \frac{1}{s} + \frac{k_2}{s} \right) \]

where \( \varepsilon = K_{\text{Active}} - \left| \psi_{\text{Active}} \right| \) is the amplitude mismatch. (26) can be interpreted as transforming the \( q \)-axis mismatch \( \varepsilon \) back to \( \alpha \beta \) frame. SA3 plus [26] describes the classical hybrid flux estimator that outputs the sum of high-pass filtered voltage model estimate and low-pass filtered current model estimate, and leads to the classical interpretation that voltage model is used for high speeds and current model is used for low speeds.
as follows to facilitate the stability analysis [36], [37]. In the IS observer framework as shown in Fig. 1(b), this dilemma can be addressed by introducing the IS observer. A simple fix is to use a variable gain \( k \) from the control community also suggests to not implement this fixed gain for slow speed reversal. Coincidentally, a contribution documented in [2], [27], and it is suggested in [2] to set the angle of the flux command can be determined by IFO integrator, i.e., using a different angle for correction in terms of flux command [38], [39]. The difference \( K \) as in [39]. Similarly, the flux command is used in replace of \( \omega \) to force \( \dot{\theta}_d \) can be extracted. The phase delay caused by the LPF should be compensated for different speeds [45]–[47]. Given a constant SM gain, it is suggested that the pole of the LPF should rely on \( \omega \) leading to a non-IS-PE design, as shown in Fig. 3. Additionally, it is proposed to use smaller SM gain and execute the SMDO at a frequency that is 3 times as high as the PWM frequency [45], [48].

To achieve an IS-SMDO, it is recommended to implement \( f \) as a dynamic correction that puts signum inside an integral. This integral introduces an additional state, resulting in a second-order SMDO, and the second state can be interpreted as an estimate of the emf appearing in the current dynamics. For example, the super-twisting algorithm can be used to estimate the emf [17], [49], [50], and the resulting continuous \( \alpha \)-axis correction is \( f_{\alpha} = k_1|\dot{\alpha}| \frac{2}{\pi} \text{sign}(\dot{\alpha}) + k_2 \int \text{sign}(\dot{\alpha}) \, dt \), where the integral term serves as a continuous estimate of the emf. As a comparison, the second-order SMDO in [51] uses discontinuous \( \alpha \)-axis correction as \( f_{\alpha} = k_1 \text{sign}(\dot{\alpha}) + k_2 \int \text{sign}(\dot{\alpha}) \, dt \). Note the continuity of \( f_{\alpha} \) is decided by the \( k_1 \) term, and also note applying variable gain \( k_1 = \dot{\alpha} \frac{2}{\pi} \text{sign}(\dot{\alpha}) \) does not eliminate chattering because the derivative of \( |\dot{\alpha}| \frac{2}{\pi} \) is infi nite when \( \dot{\alpha} = 0 \). The chattering is reduced because of using smaller \( k_1 \) value and nonzero \( k_2 \).

So far, all SMDOs reviewed above use current error \( \dot{i} \) as the SM surface \( s \)—the argument of \( \text{sign}(\cdot) \). Improvement is expected by further designing SM surfaces. For example, SM surface involving current error integral \( \int i dt \) (see, e.g., [52]) preserves observer robustness during reaching phase [53], [54]. SM surfaces involving current error derivative \( \dot{i} \) promise faster observer convergence (i.e., “convergence in finite time”), including the fast terminal SM manifold [55] and the non-singular terminal SM manifold [46]. Note the current derivative needed in the manifold can be reconstructed using super-twisting algorithm [46].

Finally, the SM gain \( k \) can be designed to be a function of \( S \) [52], which is also known as the reaching law design in SM control theory.

2) Linear Disturbance Observer: By modeling emf as dc disturbance such that \( \frac{d}{dt} \text{sign}(S) = 0 \), Tomita et al. [56] propose to design the correction term as \( f \propto \left( \dot{i} - \hat{i} \right) \) for SA1, where the \( \dot{i} \)-dependent pole placement is mandatory, and is derived by analyzing the \( H_{\infty} \) norm of transfer function from the

\[ \text{sign}(S) = [\text{sign}(S_1), \text{sign}(S_2)]^T. \]
unmodelled dynamics to the emf error. In [57], proportional correction $f \propto i$ is implemented. In [58], proportional-integral (PI) correction $f = \text{PI}(s)i$ is used, resulting in a dynamic correction. The linear DO is deemed to be poorly damped if constant correction gain is used [59]. Frequency domain formulation of the DO with PI correction is proposed in [60], if constant correction gain is used [59]. Frequency domain correction. The linear DO is deemed to be poorly damped.

The constant emf disturbance assumption $\dot{e}_\text{Active} = 0$ is apparently not reasonable [cf. (7b)]. Intuitively, emf in $dq$ frame can be modelled as dc disturbance, but transforming into $dq$-frame leads to non-IS-PE, which is discussed later in Sec. III-B2.

Remark 3: Nonlinear second-order SMDO with dynamic correction can track ac disturbance, while linear DO with PI correction cannot, as is discussed in Appendix A.

C. Magnetic Asymmetry (Saliency) based IS-PE

There are magnetic asymmetry based SAs that can only be applied to certain types of ac motors. Magnetic asymmetry includes rotor slotting and rotor saliency. The former can be exploited for speed detection (see Sec. V-C) and the latter is utilized for IS-PE. For saliency due to main-flux saturation exploited for speed detection (see Sec. V-C) and the latter is utilized for IS-PE. For saliency due to main-flux saturation.

The performance of the saliency based PE is influenced by the saturation induced magnetic cross-coupling, making the inductance of the salient PM motors more suited with higher load [72], and therefore there is also research trying to take self-sensing capability into consideration for motor design [72]–[76].

III. NON-INHERENTLY SENSORLESS POSITION ESTIMATION (NON-IS-PE)

As shown in Fig. 1(b) and 1(c), in the framework of non-IS-PE, the output is the $\theta_d$-related state (i.e., flux or emf), the input is current $i$ and speed $\dot{\omega}$, and the model is [7], [8], [10], [11] or [12]. SA1 and SA3 have their non-IS variants, while SA2 and SA4 are always non-IS. This section will review non-IS-PE as two categories. The first category uses speed as a parameter in the model, while the other uses speed only for tuning estimator coefficients.

A. Non-IS SA2/4 Due to Speed being Part of $\alpha\beta$-Frame Model

Unlike SA1 and SA3, the unknown state, i.e., emf or active flux, is modelled as internal state in SA2 and SA4 such that the speed parameter $\omega$ appears, and $\omega$ is always replaced with a speed estimate $\dot{\omega}$, meaning that this type of non-IS-PE is disturbed by speed error, as shown in Fig. 3. SA2 is state observer for emf or flux, while SA4 is a reduced-order flux observer with flux disturbance estimation.

The key difference between full-order state observer and reduced-order state observer lies in how they treat speed error. SA2 in its full-order form reconstructs a current estimate $i$ and calculates the output error $\dot{\epsilon}$ to tune the speed estimate $\dot{\omega}$ using SA6. This requires that $\dot{\epsilon}$ is sensitive to speed error. On the other hand, since current $i$ is measured, there is no need to reconstruct $\dot{i}$, and by assuming a speed signal $\dot{\omega}$ is available, the unknown state can be reconstructed by the reduced-order variants of SA2. This needs the state estimate to exhibit robustness against speed error via careful observer pole placement [6], [13].

1) EMF Observer (SA2.1): EMF observer can be implemented in its full-order form [20], [61], [70], [77]–[81] or in its reduced-order form [6]. In [70], [77], [80], [81], SM corrections are implemented. Time-varying gain that depends on SM surface is designed in [80] to reduce chattering, and SM emf observer is found to be easier to tune as compared with the extended Kalman filter for emf [80]. Proportional correction is used in [20] and linear system analysis is conducted to tune the observer. The $dq$-frame or more precisely $\gamma\delta$-frame implementation of the emf observer can be found in [78], where included is an interesting study that analyzes the consequences of using constant speed assumption in emf observer. In [61], the influence of dc offset on emf estimation is analyzed. In [79], observer steady-state errors are analyzed considering current/voltage error, parameter uncertainty and filtering.

As for the reduced-order variant of emf observer, the robustness against speed error can be designed through $H_\infty$ norm based pole placement [6].

2) Flux Observer (SA2.2): Flux observer can be implemented in its full-order form [9], [10], [78], [82], or in its reduced-order form [14]. In [78], the flux observer is implemented in $\gamma\delta$-frame, and the speed difference between $dq$-frame and $\gamma\delta$-frame that will result during speed transients is obtained from a cascaded speed observer. In [9], the flux observer is analyzed by Lyapunov stability theory through finding the positive-definite matrix for the Kalman-Yakubovich lemma. In [82], [83], SM state observer is proposed, and pole placement for robustness improvement is detailed in [83].

In [14], reduced-order flux observer [17] is analyzed in the misaligned $\gamma\delta$-frame, and is described by a rotatory differential operator: $P(\theta) s P^{-1}(\theta) = s I + \Omega J$ and the stability analysis of the flux observer holds only if speed is known.

3) Linearized Position Observation by EKF (SA2.3): EKF can be applied to the nonlinear model [9], [15], [84]. The fourth-order extended state model [9] assumes both constant active flux amplitude and constant speed. It is also possible to use stator flux $\psi_s$ as state instead of $i$ [84], where the flux output error is equivalent to the current error that was revealed [84].

Large corrections in the current observer dynamics will reduce the current error’s sensitivity to speed error. In extreme case, the speed error is forced to be zero (e.g., trapped in SM surface), and therefore there is no way to extract speed signal from current error. This fact dis-encourages the idea of speed-adaptive SM state observer, but motivates the idea of cascaded speed-adaptive observer.
in (25). In addition, the constant speed model \( \frac{d}{dt}\omega = 0 \) can be replaced with the motion dynamics \( \frac{d}{dt}i \) and the resulting EKF implementation can be found in [16].

4) Current Observer in \( dq \)-frame (SA2.4): The \( d \)-axis current observer \( \hat{\psi} \) can be understood as an adaptive observer with constant position error parameter \( \hat{\theta}_d \), provided that \( \hat{\omega} \) is available and accurate—which is a typical assumption in non-IS-PE design. The observer can be implemented as an open-loop one \([85, 86]\) Sec. 9.2.2) or a closed-loop one \([87]\).

Since the \( d \)-axis current error contains the information of position error \( \hat{\theta}_d \), PLL can be used to form an estimate of position \([87]\).

Remark 4: Both SA2.4 and SA1 use 2nd-order \( \frac{d}{dt}i \) dynamics. The difference is that for SA1, a correction is necessary, while SA2.4 can be implemented as open-loop observer. △

5) Frequency-Adaptive System (SA4): The idea of SA4 is to remove the flux disturbance in the sinusoidal flux estimate prior obtained from SA3. Therefore, SA4 is called frequency-adaptive DO in literature \([13, 34]\), and is in fact a cascaded flux observer using \([3]\). The robustness of the cascaded flux estimate \( \hat{\psi} \) against speed error is analyzed in \([13\) Fig. 2]. Note the observer tuning or pole placement in \([13]\) is also dependent on \( \hat{\omega} \). Alternatively, we can understand the overall system of voltage model estimator (SA3) and its DO (SA4) as a “single tune integrator” \([88, 89]\), whose performance relies on the speed estimate. SA4 is typical non-IS-PE with speed dependency in both model and tuning, as shown in Fig. 2.

The readers are referred to \([59]\ Table 4\) for a review of the frequency adaptive observers for eliminating emf harmonics.

B. Non-IS Variants of SA1/3

Even though the original implementation of SA1 and SA3 is IS, it is possible to introduce speed-dependency by i) modeling a different disturbance emf for SA1, ii) using a speed-dependent orthogonal condition, or iii) implementing SA1 and SA3 in \( dq \)-frame. Those non-IS variants are less recommended, though one benefit is that the ac emf disturbance becomes \( dc \) disturbance in \( dq \)-frame.

1) Speed-Dependent Disturbance Model for SA1: In \([13]\), the disturbance is the emf due to active flux. However, one can also implement SA1 (e.g., \([41]\)) using the extended emf model in \([12]\), resulting in a non-IS SMDO as is done in \([68]\).

2) \( dq \)-Frame Variant of SA1: In Sec. II-B2, the linear DO method assumes the dynamics of the ac emf to be zero. This assumption becomes more reasonable if one transforms \([13]\) into \( dq \)-frame, such that \( dq \)-frame emf is modelled as \( dc \) disturbance and the correction \( \hat{f} \) becomes \( Pf \). In \([71, 90, 91]\), \( Pf \) is implemented as a PI law, resulting in a dynamic correction. In \([92]\), a classical DO (that uses LPF) is proposed to estimate the extended emf \( e \) in \( dq \)-frame.\(^6\) In \([69, 93]\), SMDO is proposed in \( dq \) frame. In \([69]\), the \( dq \)-frame correction \( Pf \) is implemented as a combination of proportional correction and SM correction. In \([93]\), a speed-dependent SM surface that consists of current error and its integral is selected, and the correction \( Pf \) is a dynamic one.

3) Speed-Dependent Orthogonal Condition for SA3: Recall that the dot product scalar orthogonality condition \( e_s \cdot J \hat{\psi}_s = 0 \) does not involve speed signal, but one can derive a speed-dependent vector orthogonality condition as \( e_s - \hat{\omega} J \hat{\psi}_s = 0 \), based on the assumption (ii) from Sec. II-A Voltage compensation \( D \) based on \( e_s - \hat{\omega} J \hat{\psi}_s = 0 \) is proposed in \([94, 95]\).

4) \( dq \)-Frame Variant of SA3: Voltage model \([19]\) is IS because it is in \( \alpha\beta \) frame. Transforming \([19]\) into \( dq \) frame makes the flux estimator coupled with speed estimation, resulting in a non-IS-PE. The current error in \( 25 \) that is based on the assumption (i) from Sec. II-A can be used to build a speed-adaptive voltage model in \( dq \)-frame \([67, 72]\).

C. Speed Only being Used for Tuning

Since the SA2/4 have speed in the model, it is natural to also use speed for tuning (e.g., pole allocation). On the other hand, IS-SA1/3 have the potential to be implemented being free of speed, but the IS property is lost if speed is used for tuning in SA1/3.

1) Non-IS Tuning for SA1: We have already addressed in Sec. II-B that speed-dependent tuning is often adopted for SA1, e.g., the linear DO whose pole placement is dependent on \( \hat{\omega} \), and the first-order SMDO with non-dynamic correction whose switching gain or ensuing LPF is dependent on \( \hat{\omega} \). There are also examples of speed-dependent tuning for second-order SMDO with dynamic correction, see e.g., \([50, 101]\). In other words, some practical implementation of SA1 requires speed-dependent tuning, resulting in non-IS-PE.

This section will now focus on the speed-dependent tuning that makes SA3 non-IS.

2) Non-IS Tuning for SA3: In order to stabilize the pure integration in SA3, a high pass filter (HPF) can be added to the output of the voltage model \([96]\), which is equivalent to replace the integrator with an LPF as follows:

\[
\hat{\psi}_s = \frac{\omega_{LPF}}{s + \omega_{LPF}} e_s, \quad e_s = u - Ri \tag{28}
\]

which is IS if a fixed \( \omega_{LPF} \) is used. In fact, it is reported that placing the LPF pole, \( -\omega_{LPF} \), to be close to zero is sufficient for zero speed\(^7\) operation of induction motor \([97]\), but it is often recommended to adopt the non-IS tuning \( \omega_{LPF} = k|\hat{\omega}| \) to adjust observer damping with respect to speed \([5, 98]\). In order to compensate the gain and phase shift introduced by the LPF, a compensation term is introduced as follows \([5, 98-102]\):

\[
\hat{\psi}_s = \frac{1}{s + k|\hat{\omega}|} e_s - J k \text{sign}(\hat{\omega}) e_s + \left[ I - k J \text{sign}(\hat{\omega}) \right] e_s \tag{29}
\]

Two low pass filtered signals add up

There is only one filter \([103]\)

which is called statically compensated voltage model (SCVM) \([5, 21]\) because the compensation is only exact at steady state when \( \omega \) is constant. It is suggested in \([100]\) to apply the static compensation to the stator emf \( e_s \) first before going through the LPF, as indicated by the text under the equations in \([29]\).

See \([5, 100, 101]\) for different advice for choices of \( k \) in

\(^7\)Zero speed \( \hat{\omega} = 0 \) does not mean \( \omega = 0 \) for induction motor with load.
In $^{29}$, three speed-dependent cascaded LPFs are used to recover the frequency response of integrator. Noting the presence of skew-symmetric matrix $J$ in $^{29}$ means, e.g., $\beta$-axis emf is used for compensating $\alpha$-axis dynamics, and this practice implicitly assumes an orthogonality relation between $\alpha$- and $\beta$-axis, i.e., $\int_{t_0}^{t} \epsilon_{\alpha}^T c_{\alpha} \, dt = 0$ $^{[8]}$, which is the assumption (iii) from Sec. II-A. However, if some signal phase shift network is introduced, the compensation can be accomplished within the same axis, and therefore even elliptical trajectory of stator flux vector can be tracked $^{[104]}$.

IV. POST-POSITION ESTIMATION (PE) SPEED ESTIMATION

Post-PE speed estimation extracts speed signal from an assumedly-accurate flux/emf estimate or an erroneous current estimate $\hat{i}$ that is disturbed by $\hat{\omega}$, as shown in Fig. 1(a), 1(b), and 1(c). For post-PE speed estimation, the output is $\hat{\omega}$; the input is $\hat{\theta}_d$-related states or current error $\hat{i}$; and the model is constant speed model $\frac{d}{dt} \hat{\omega} = 0$, steady state model of $^{(10)}$, or motion dynamics $^{[2]}$. The generic algorithms are SA5–7.

A. Post-PE-SA5: Direct Calculation

1) Direct Calculation from Position: Speed estimate can be calculated by the forward difference of the flux angle $\hat{\theta}_d$ $^{[10]}, \hat{\theta}_d$, and an additional LPF is embedded to reduce the amplified noise in $^{[23]}$.

2) Direct Calculation from Orthogonality between Flux and EMF: Acknowledging the fact that speed signal exists in the dynamics of the derivative of stator flux, it can then be calculated as $\hat{\omega}(\hat{\psi}_s^T \hat{\psi}_s) = -(J \hat{\psi}_s)^T s \hat{\psi}_s$, where the derivative of flux can be substituted with the calculated stator emf as $e_s = u - R \hat{i}$ $^{[98], [99], [105]}$, the SMDO emf estimate $^{[55], [106], [107]}$, or the forward difference of flux $^{[108]}. In^{(109)},$ the speed can be calculated as $\frac{d}{dt} \hat{\omega} = \frac{\epsilon_s}{\epsilon_L}$ using the flux $\hat{\psi}_s$.

3) Direct Calculation from Emf: Speed is simply the emf amplitude divided by the flux amplitude $^{[46], [49]}$.

B. SA6: Model Reference Adaptive System (MRAS)

An adaptive observer is established based on the fact that the output error $\epsilon$ is measurable and can be used to drive the speed adaptation law $^{[22]}$. Therefore, for sensorless ac motors, the output error $\epsilon$ should always be the current error $\hat{i}$, and the adaptive observer is of full-order. From the perspective of model reference adaptive system (MRAS), the actual model is the reference model, and the full-order adaptive observer is the adjustable model.

Now consider an MRAS, where the reference model is an IS-PE, and then one can implement a reduced-order adaptive observer as the adjustable model. Such adjustable model is called cascaded adaptive observer (CAO) in this paper.

Full-order adaptive observer has an interconnected structure between PE and speed estimation, while the CAO is designed to be a cascaded speed estimation sub-system that comes after the PE sub-system. However, if the reference model is a non-IS-PE, then the resulting MRAS (of the non-IS-PE and CAO) is interconnected.

$^{[29]}.$ If $K_{\text{Active}}$ and $\omega$ are constant, then $\int_{t_0}^{t} \frac{d}{dt} e_{\alpha, \text{Active}} e_{\beta, \text{Active}} \, dt = 0$ $^{[8]}$ is valid, and it is also true for stator emf if $d$- and $q$-axis currents are constant.

1) CAO Based on Prior $\hat{\theta}_d$-related States: CAO assumes that an accurate estimate of the unknown states, e.g., position $\hat{\theta}_d$, emf $\hat{\epsilon}$ or flux $\hat{\psi}_s$, is available, and an speed-adaptive reduced-order observer based on output error $\epsilon = \hat{\theta}_d - \hat{\theta}_d$, $\epsilon = \hat{\epsilon} - \hat{\epsilon}$ or $\epsilon = \hat{\psi}_s - \hat{\psi}_s$ is further reconstructed for observing the “estimate of unknown states” as an effort to extract the hidden speed information by a speed update law $^{[22]}$. EMF-type CAO, as shown in Fig. 3, is implemented in $^{[6], [43], [44], [52], [54], [57], [61]}$. Flux-type CAO can be found in $^{[62]}$, but what is much more often to see is to use the angle of the flux estimate to build a position-type CAO—which is also known as phase-locked loop (PLL).

PLL is widely used for post-PE speed estimation. During formulation of PLL, various types of position error signals can be exploited, such as the $q$-axis voltage $^{[101]}$, the angle of $\gamma\delta$-frame extended emf $^{[71]}$, $d$-axis current error from $^{[18a], 87]}$, and the forward difference of high frequency components of $\alpha\beta$-frame current $^{[110]}$. PLL for induction motors would need to add an additional slip relation $^{[5]}$. Speed error during speed transients is inevitable because typical PLL is 2nd-order system that assumes constant speed $^{[111]}$, which is a type-2 system $^{[13]}$. The transfer function from actual position to estimated position can be found in $^{[92], Eq. (18)}$ and $^{[71], Eq. (39)}$. The speed error from PLL during speed dynamics is analyzed in $^{[58]}$. In $^{[29]}$, an additional PI term driven by torque error is further added to the PLL based speed estimate. PLL can be generalized for higher order to track time-varying speed $^{[14], Eq. (24)}$, and see also $^{[35]}$ and $^{[92]}$ for an application of type-3 PLL for tracking ramp speed signal.

Generally speaking, when designing for post-PE speed estimation, a position signal $\hat{\theta}_d$ is assumed to be available and accurate, meaning that no correction for $\hat{\theta}_d$ is done during post-PE speed estimation stage. However, the reality is that non-IS observer is disturbed by speed error and that the emf estimate is probably obtained from an LPF, thus lagged response of $\hat{\theta}_d$ with respect to $\theta_d$ is expected. A remedy is proposed in $^{[60]}$ to compensate the delay in the estimated emf during the post-PE speed estimation stage using PLL.

2) Speed Adaptation Law Using Prior Current Error $\epsilon = \hat{i}$: The speed adaptation law $^{[22]}$ is driven by the estimated current error $\hat{i}$. The regressor depends on how speed appears in the adopted model, and typical choices of regressors are stator flux $^{[69]}$ and extended emf $^{[20]}$. Particularly, in $^{[185]}$, the regressor is $K_{\text{Active}}^{-1}$. Speed adaptation treats the speed as a constant parameter, but one can design an inertia-dependent speed-adaptation law that includes an additional torque term $^{[87], [108]}$.

C. SA7: Speed Observer

1) Observer with Constant Load Torque Assumption $^{(23a)}$: Speed observer (SA7) uses the motion dynamics $^{[2]}$ to extract speed information from the prior position estimate, resulting in a 3rd-order state observer for position, speed, and load torque $^{[40], [78], [90], [110], [112]}$. Fig. 9. SA7 often assumes a constant load torque, and has various names, such as extended Luenberger observer (ELO) $^{[113], Sec. 4.5.3.5}$ or extended state observer (ESE) $^{[47], [114], [116]}$. Particularly, nonlinear...
correction is used in [115], [116]. See (45) in Appendix B for a discussion of those speed observer variants.

Alternatively, one can also design a 3rd-order state observer using q-axis current, speed and load torque as states, which can be understood as a reduced-order implementation of the original natural speed observer [13] for either PM motors [2] or induction motors [105], and can also be understood as an ELO implementation of the q-axis current observer [18b] [87]. The difference is that the natural speed observer uses motor active power error as the scalar output error \( \epsilon \), which, however, makes observer tuning to be dependent on q-axis voltage [2].

2) Inertia-Free Variant of Speed Observer [23d]: In [70], [77], an extended Kalman filter type speed observer is proposed for a 3rd-order system of position, speed and acceleration, without needing inertia parameter \( J_s \).

V. SPEED ESTIMATION FOR INDIRECT FIELD ORIENTATION (IFO)

If the angle used in Park transformation \( P \) is an integral of a speed estimate, the torque control is said to be dynamic and based on IFO. In this sense, IFO is a special kind of non-IS-PE with the simplest dynamics: \( \dot{\theta}_d = \frac{1}{L_s} \hat{\omega} \). The interesting idea is that now we can construct the speed estimation based on the IFO position estimate \( \frac{1}{L_s} \hat{\omega} \), rather than assuming some accurate position estimate is available, which is a completely different design philosophy from post-PE speed estimation because of this special input, as shown in Fig. [1]d).

A. General IFO-PE

Generally speaking, we can add an additional IFO integrator to the output of any post-PE speed estimation, to achieve general IFO-PE, as shown in Fig. [1]a, [1]b, and [1]c). The IFO integrator provides an additional position estimate as \( \frac{1}{L_s} \hat{\omega} \) that can be used in e.g., SA3 [38], as is discussed in Sec. I-A3 to design voltage compensation \( D \) for induction motors.

B. IFO-SA5: Closed-loop Direct Calculation from EMF

This sub-section reviews the special IFO-PE designs whose speed estimation do not rely on prior position estimate. The input is \( \int \hat{\omega} dt \), the output is \( \dot{\omega} \), the model is [10], and the generic algorithm is SA5.

The key to implement (21) is to design a feedback loop to force \( \dot{\hat{e}}_{d,ss} = 0 \). In other words, if \( \dot{\hat{e}}_{d,ss} \neq 0 \), the speed estimate \( \dot{\omega} \) must be updated to make \( \dot{\hat{e}}_{d,ss} = 0 \). So the practical implementation of (21) is [85] Sec. III [117]

\[
\dot{\hat{\omega}} = k_1 \frac{\hat{e}_{q,ss}}{K_{Active}} + \text{sign}(\hat{e}_{q,ss}) \frac{k_1 s + k_2}{s} \hat{e}_{d,ss} \tag{30}
\]

Special choices are \( k = 0 \) [3], and \( k_2 = 0 \) [21], [118]. It is pointed out in [5] that the introduction of an additional LPF will resolve the algebraic loop caused by (30), because \( \hat{e}_{q,ss} \) is a function of \( \dot{\omega} \), so \( \dot{\omega} \) appears on both side of (30).

A variant of (30) is to replace \( \hat{e}_{q,ss}, \hat{e}_{d,ss} \) with integral of q-axis and d-axis current error, \( \hat{\tau}_d, \hat{\tau}_q \) [85] Sec. IV:

\[
\hat{e}_q = k_1 \frac{1}{s} \left[ i_q - \frac{1}{L_{q,q}} (\hat{e}_{q,ss} - \hat{e}_q) \right], \tag{31a}
\]

\[
s\hat{\theta}_d = \dot{\hat{\omega}} = \frac{\hat{e}_q}{K_{Active}} + \frac{k_1 s + k_2}{s} \left[ \hat{i}_d - \frac{1}{L_{q,q}} \hat{e}_{d,ss} \right], \tag{31b}
\]

with \( [\hat{i}_d, \hat{\tau}_q]^T = P(\hat{\theta}_d) \hat{i} \),

where \( k_2 \) is originally set to zero in [85]. Note the current observer is embedded in (31) as \( L_{q,q} \hat{i}_{q,d} = \hat{e}_{q,ss} - \hat{e}_q \) and \( L_q \hat{i}_d = \hat{e}_{d,ss} \). As a result, the steady state assumption of \( \hat{s}_d = \hat{s}_q = 0 \) for (21) can be removed.

We believe it is one of the most interesting observations in the field of sensorless control, that (when the \( K_{Active} \) is constant,) the open-loop q-axis current error in (31a) contains speed error information, and the open-loop d-axis current error in (31b) contains position error information [85] Eq. (9.26). This is further generalized for time-varying \( K_{Active} \) in [5] Eq. (13) as “basic relations for sensorless flux estimation”.

C. Magnetic Asymmetry (Rotor Slot Harmonics) Detection

For straight slot induction motors of certain stator and rotor slot combinations [119], the rotor slotting will induce harmonics in \( \hat{e}_d, \hat{e}_q \), and rotor speed can be detected from the rotor slot harmonics in \( \hat{e} \) [119], [121], [122]. This kind of speed estimation can detect speed independently from the rotor position and has almost no parameter dependency [121].

VI. LOSE THE CONSTANT \( K_{Active} \) ASSUMPTION

So far, thanks to the constant \( K_{Active} \) assumption, all the SAs reviewed above (except the ones relying on the magnetic asymmetry) can be applied to both induction motors and PM motors [10]. However, \( K_{Active} \) is by definition a time-varying parameter as long as (\( L_d - L_q \) \( i_d \) varies, and a change in \( K_{Active} \) leads to the unmodelled dynamics \( E_u \) in (7).

Sometimes a time-varying \( K_{Active} \) is beneficial, e.g., for efficiency improvement or for better dynamic performance. Specifically, at high speeds, faster speed dynamic process can be achieved if the flux amplitude is first weakened such that more dc bus voltage can be used for producing torque current, which is a result of a multi-step optimization control [123].

This section briefly reviews compensation for the disturbance due to time-varying \( K_{Active} \) in salient motors in Sec. VI-A and then the rest of this section focuses on the derivation of induction motor model and its SA designs. In fact, most literature of sensorless induction motors depends on the time-varying \( K_{Active} \) model.

A. Compensation for Time-Varying \( K_{Active} \) in Salient Motors

In [11], an angle compensation that takes into account the unmodelled dynamics in (7a) when \( K_{Active} \) varies is proposed. In [9], \( \hat{i}_d \) in the unmodelled dynamics \( E_u \) in (7a) is compensated by its estimate.

Even though SA1 does not explicitly require \( K_{Active} \) to be constant, but the flux or voltage compensation often relies on the constant \( K_{Active} \) assumption. For example, the flux offset \( D_q \) in (24) does not equal to 0 when flux amplitude varies.
B. Model of Induction Motors with Time-Varying $K_{\text{Active}}$

For induction motors, the unmodelled dynamics $E_u$ satisfy

$$E_u = P^{-1}(\theta_d) \begin{bmatrix} K_{\text{Active}} \\ 0 \end{bmatrix} = R_{\text{req}} \begin{bmatrix} i - \psi_{\text{Active}} & \frac{R_{\text{req}}}{L_d - L_q} \end{bmatrix} - \omega_d \mathbf{J} \psi_{\text{Active}}$$

in which (1) and (2) have been substituted, and $\omega_d = \frac{R_{\text{req}}}{K_{\text{Active}}} \hat{\omega}_{\text{sl}}$ is the slip relation. The model with stator current and active flux as states can be derived from (6), (7a) and (32):

$$L_q \frac{d}{dt} i = u - R \frac{d}{dt} \psi_{\text{Active}} \quad \frac{d}{dt} \psi_{\text{Active}} = -\frac{R_{\text{req}}}{L_d - L_q} \psi_{\text{Active}} + R_{\text{req}} i + (\omega - \omega_d) \mathbf{J} \psi_{\text{Active}}$$

which is exactly the inverse-$\Gamma$ circuit induction motor model. There are two things that make induction motors unique:

1) Active flux amplitude $K_{\text{Active}}$ is not constant and is maintained by stator excitation for non-PM motors.
2) There is a slip speed $\omega_{\text{sl}}$ difference between field speed $\omega$ and rotor speed $\omega_r$, i.e., $\omega_r = \omega - \omega_{\text{sl}}$

In other words, the induction motor is not a PM motor nor synchronous. These two facts correspond to the two unique features of the SAs dedicated for induction motors:

1) There is a chance for estimated flux amplitude $\hat{K}_{\text{Active}}$ to collapse [124], and there is a chance for the change in flux amplitude being mis-interpreted as a change in flux angle [5], such that unstable sensorless operation results.
2) The current model of induction motor (33b) provides angle information, while for synchronous motors, the angle of current model flux can not be utilized [5] (10)]. Therefore, in the context of PM motors, the term “current model” is no more than the active flux parameter $K_{\text{Active}}$.

C. SAs Based on $(i, \psi_{\text{Active}})$ Model (33)

1) IS-PE Design: IS-PE is achieved by designing 4th-order DO for the unknown term $\frac{R_{\text{req}}}{L_d - L_q} i + \omega \mathbf{J} \psi_{\text{Active}}$ in (33) using SM correction [106], [107], [109], [125]–[127] or PI law dynamic correction (a.k.a. ESO) [128]. This results in an IS-PE design with a redundant observer as is discussed in the Appendix A. The redundant observer gives an estimate of flux for direct calculation of $\omega_r$ by [109] Eq. (23) [129] Eq. (26), and see also [129] Sec. V] for a cascaded variant of redundant observer. Alternatively, one can design an IS SMDO whose correction $f$ is in replace of the unknown term $\omega \mathbf{J} \psi_{\text{Active}}$, or $\omega_r \mathbf{J} \psi_{\text{Active}}$, depending on the choice of state variables [130].

2) Non-IS-PE Design: Using current and flux as states, speed-adaptive observer can be designed in $\alpha \beta$-frame [83], [108], [131]–[137] and dq-frame [138]. This is referred in literature as the “full-order” observer of induction motors, which has attracted a lot of research attention. Active flux (or rotor flux) is often chosen as the state, while stator flux $\psi_s$ can also be selected as state [108]. Linear correction is often used, but SM correction can also be found in literature [83], [108], [136].

As a special case of non-IS-PE, the motion dynamics can be further utilized such that speed is treated as a state and inertia is needed. In $\alpha \beta$-frame, a 6th-order natural speed observer is proposed in [18] with the stator current, active flux, speed and load torque as states, and the key feature is to use motor active power error as the scalar output error $\varepsilon$. In dq-frame, it is stated in [139] that the speed estimation can be achieved through the $d$-axis subsystem, and a 5th-order SM state observer consisting of $d$-axis current, $d$-axis rotor flux, rotor position, rotor speed and load torque is proposed.

3) Challenges at Low Speed Regeneration: It is very challenging to stabilize the $(i, \psi_{\text{Active}})$ model based full-order observer in low speed regeneration and slow zero frequency crossing, because the stable speed adaptation law derived from Lyapunov function [133] depends on the unknown flux error, and the hyper-stability analysis in [131] depends on the assumption that the ratio between flux error norm and current error norm has finite upper bound such that the unknown flux error can be replaced by current error. Careful observer gain designs based on the linearized model (see, e.g., [134], [137], [140], [141]) and the positive real property [142] are proposed for improved stability. It is also effective to re-design the speed adaptation law [135], [143], [144] [19] Eq. (26)]. It is shown in [145] that in order to find a Lyapunov function for the full-order observer, the observer coefficients must be dependent on the actual speed, implying global stable design does not exist.

D. Change of States for Global Stability

To attack the regeneration instability challenge, an ideal solution is to find a globally stable speed-adaptive observer design. In the literature of observer design [146]–[150], observer is often proposed for a class of systems that are in Brunovsky observer form [151] [12] which does not describe the induction motor dynamics (33). We will soon see that the key to attain global stability is to use a different state variable other than active flux. In the following, we will use $(y, x)$ to denote the output state and the internal state of the new model.

1) Model $(L_q i, -e_{\text{Active}})$: The dynamics are [19]

$$sy = u - Ri + x$$

$$sx = -R_{\text{req}} \left[ si + \frac{1}{L_d - L_q} x \right] + \omega_r \mathbf{J} x$$

where $si$ can be obtained using a state variable filter [19].

2) Model $(L_q i, R_{\text{req}} i - e_{\text{Active}})$: The dynamics are [19]

$$sy = u - (R + R_{\text{req}}) i + x$$

$$sx = - \left[ R_{\text{req}} \frac{L_d - L_q}{L_d} i - \omega_r \mathbf{J} \right] (x - R_{\text{req}} i)$$

which avoids the term $si$ in (34).

3) Model $(L_q i, L_q i + \psi_{\text{Active}})$: This yields

$$sy = u - (R + R_{\text{req}}) i + \left( R_{\text{req}} \frac{L_d - L_q}{L_d} i - \omega_r \mathbf{J} \right) (x - y)$$

$$sx = u - Ri$$

To simply put, the following system form is desired:

$$\frac{dy}{dt} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ x \end{bmatrix} + \text{Regressor} \times \text{Parameter} + \ldots$$

11The slip speed causes an algebraic loop when using [30] for sensorless q-axis current control [5].
whose key property is that there is no unknown variable $\omega_r$ in the dynamics of stator flux $x$. The research based on this model is summarized in the monograph \[152\].

4) Model $(L_q i_q, \left[ \frac{R_{req}}{L_q} I - \omega_r J \psi_q \right])$: This yields \[153\]

$$sy = x - \left( \frac{R_{req}}{L_q} L_d + R \right) i + \omega_r J y + u$$

$$sx = \left( \frac{R_{req}}{L_q} I - \omega_r J \right) (u - R i) \quad (37)$$

5) Model $(L_q i_q, \left[ \frac{R_{req}}{L_q} I - \omega_r J \psi_q, -s[\omega_r J \psi_{Active}] \right])$: If speed and load torque are treated as system states, a sixth-order system results, and a state transformation is proposed to extend the system \[37\] with a third $\mathbb{R}^2$ vector state that is the time derivative of $-\omega_r J \psi_{Active} \ [154]$.  

6) Model $(L_q i_q, \left[ \frac{R_{req}}{L_q} I - \omega_r J \psi_q, \omega_r, T_L \right])$: A more straightforward model is to extend \[37\] with two scalar states $\omega_r, T_L$, that is, no state transformation for $\omega_r, T_L \ [150]$.  

7) Adaptive Observer Form: In \[147\], a class of system that allows globally stable adaptive observer design is said to be in the adaptive observer form, which is translated into two requirements on induction motor model, i.e., Brunovsky form and known regressor for speed. Even though only model \[37\] is in adaptive observer form, the existence of globally stable speed-adaptive observer is an established fact \[155\], \[156\].

8) Advances in High Gain Observer Design: The known regressor for speed requirement is removed in the high gain observer design \[149\]. Application of this high gain observer to \[37\] is studied in \[157\], and it can also be applied to \[34\] or \[35\]. A simulation study in \[105\] shows the high gain observer design proposed in \[154\] can identify resistances even in speed transients. The requirement on the partitioned matrix needed in \[149\], \[154\] is later removed in \[150\], which allows one to further design high gain observer for the sixth-order induction motor system with two scalar states $\omega_r$ and $T_L$, instead of the $\mathbb{R}^2$ vector $-s[\omega_r J \psi_{Active}]$ \[150\]. The observer design in \[150\] allows certain nonlinear term like $\omega_r x$ to appear, and note both $\omega_r$ and $x$ are states, with $sx$ defined in \[37\].

E. Model Reference Adaptive System (MRAS) for IFO-PE

In the field of sensorless induction motor drive, MRAS is a jargon for a system with the voltage model (VM) as reference model and the current model \[336\] as adjustable model \[11\].

The pioneer work on MRAS in \[96\] (and also the follow-up work \[129\], \[158\]) is a good example showing the spirit of IFO-PE that an accurate estimate of flux/emf is not needed, by using \[28\] as the reference model and using high-pass filtered $i$ for current model as the adjustable model. There are also attempts to compensate for the gain and lag introduced in \[28\] (see, e.g., \[159\]). The convergence of MRAS based flux estimate needs an analysis of the error dynamics of the flux error and the mismatch between VM and current model \[160\]. In \[12\], the VM is transformed to an estimated dq-frame to derive a generalized slip relation in terms of the VM correction gains. In \[161\], the correction in VM is replaced with a super-twisting based dynamic correction. The dual reference observer proposed in \[130\] is also an MRAS, and is an example of implementing the amplitude correction \[26\] in its current error form \[25a\] in a time-varying $K_{Active}$ model. The MRAS implementation in \[27\] has made it clear that the amplitude mismatch is used to stabilize VM and the angle mismatch is used to tune $\hat{\omega}$ used in current model. In \[129\], the speed adaptation law is implemented as an SM control law.

Readers are referred to \[162\] for a dedicated review of various variants of MRAS that have different output error $\varepsilon$.

VII. CONCLUDING REMARKS

This paper reviews SA designs for both induction motors and PM motors, and a map of the overview is shown in Fig 4. IS-PE is a key concept in this paper, meaning the PE has no speed-dependency, e.g., saliency based methods. If there is two-way coupling between PE and speed estimation, the PE is non-IS. All speed estimation methods are coupled with PE in some way, with an exception being the rotor slot harmonics based speed detection. According to Fig. 4, there are four types of generic PE methods:

1) SA1—the potentially IS DO that does not need the model of the unknown internal states. The key is to design dynamic correction that allows smaller switching gain to reduce chattering for SMDO and reduces lagging when tracking ac disturbance for linear DO.

2) SA2—the non-IS state observer that utilizes the model of the unknown internal states. The key is to decide which path to go: the robust reduced-order state estimation or the speed-adaptive full-order observer. From \[7\], emf observer is disturbed by $\hat{\omega}$ while flux observer is not.

3) SA3/4—the potentially IS stabilized integrator. The key is to design voltage and/or flux compensation based on the three assumptions from prior knowledge, or to implement the integrator as a statically compensated (open-loop) state filter. There is also a trend to design the stabilized voltage model as a non-IS single tune integrator.

4) The IFO integrator $\hat{\theta}_d = \frac{1}{2} \omega$ that enables IFO based speed estimation that can take advantage of $P(\frac{1}{2} \omega)$.

A complete SA scheme consists of the PE and a cascaded or interconnected speed estimation for which the generic algorithms are SA5–7: direct calculation, MRAS, and speed observer, respectively. Now, we are ready to make some general recommendations by topics as follows.

Regeneration stability. There is no regeneration instability issue for PM motors, because classical SA designs for induction motor allows the intermediate state, i.e., the active/rotor flux, to collapse to zero. In fact, with IS flux estimation, there is no need of working-condition-dependent stabilization design for sensorless induction motors anymore. For example, it is mentioned in \[105\] that the original implementation of the non-IS 6th-order natural observer for current, flux, speed, and load torque in \[18\], loses its stability during low-speed regeneration, while a simple modification to a cascaded design with IS flux estimator plus a cascaded speed observer resolves this problem.

Adopt the constant $K_{Active}$ assumption. This is a natural practice for PM motor SA designs. For induction motors, we encourage to design the SA scheme as if $K_{Active}$ is a

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11 This is the reason why we have been using the name CAO instead of MRAS in Sec. IV-B.
constant while calculating $K_{Active}$ using (1). In fact, all SAs reviewed in this paper should be able to be applied to induction motors and there is at least one benefit that the collapse of the estimated flux amplitude can be avoided.

**Time-varying $K_{Active}$ Model.** For PM motors, there is a need for more research of the compensation of the SA scheme when constant $K_{Active}$ assumption is violated. As for induction motors, first, there is also IS-PE design for induction motor model (33a), and an ensuing redundant observer using (33b) shall give the flux angle needed for DFOC (see, e.g., [106], [109]), or this SA design can be easily extended as general IFO-PE; Second, the reality is that for IFOC, only the speed estimate $\hat{\omega}$ is the ultimate goal and this means that even the globally stable speed-adaptive observer (which is non-IS-PE) is only an intermediate step to obtain $\hat{\omega}$. Globally stable design is recommended because the observer tuning freedom is not exhausted for stabilization purpose. However, given the fact that the Lyapunov stability analysis is established to only ensure the asymptotically stability of state estimation through the speed adaptation design, an additional cascaded speed estimation that extracts $\hat{\omega}$ from the globally stable state estimation sounds reasonable. In other words, the speed estimate $\hat{\omega}$ is designed to be disturbed by any parameter uncertainty in the model, as an effort to ensure consistent estimation of the state estimation, and therefore, maybe a post-PE speed estimation from the state estimation could be more accurate than the results from the speed adaptation law.\(^{15}\)

**Try IS algorithm SA1/3 as a starting point.** E.g., SA3 with amplitude correction (i.e., the hybrid of VM and current model) is one of the mostly widely used IS-PE. For SA1 with linear correction, we suggest to implement the dynamic correction $\mathbf{f}$ to higher order (i.e., as GESO or GPIO), as long as $\mathbf{\kappa}$ is within $[0, 1)$. Therefore, it is also beneficial to implement the SMDO to higher order than second order to reduce chattering, but the SM correction involves more coefficients such as $\mathbf{\kappa}_j$, $j = 0, 1, 2$, and the optimal tuning of $\mathbf{\kappa}_j$ along with correction gains is of interest. As a comparison, non-IS-PE introduces two-way coupling between PE and speed estimation and the interconnected structure is more difficult to be analyzed.

**EMF based PE considering harmonics.** First, there is no evidence showing that emf observer is better for PE than flux observer. Also, note the sinusoidal dynamics of emf are disturbed by speed variation $\dot{\omega}$. After emf estimation, a redundant observer can be implemented, which can simply be an integration of the emf, to get a flux estimate for $\hat{\omega}_d$, which attenuates the higher-order harmonics in emf estimate.\(^{16}\)

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\(^{14}\)This even includes the saliency based method, see, e.g., [163].

\(^{15}\)A well known example is that the sensorless controlled induction motor can control accurate torque but can only regulate biased speed when rotor resistance value is erroneous, which implies that active flux angle $\hat{\theta}_d$ is accurate and $\hat{\omega}$ compensates for the uncertainty in rotor resistance [131].

\(^{16}\)EMF based PE considering harmonics includes the following points:

- **EMF based PE considering harmonics:** First, there is no evidence showing that emf observer is better for PE than flux observer. Also, note the sinusoidal dynamics of emf are disturbed by speed variation $\dot{\omega}$. After emf estimation, a redundant observer can be implemented, which can simply be an integration of the emf, to get a flux estimate for $\hat{\omega}_d$, which attenuates the higher-order harmonics in emf estimate.\(^{15}\)

- **Try IS algorithm SA1/3 as a starting point:** E.g., SA3 with amplitude correction (i.e., the hybrid of VM and current model) is one of the mostly widely used IS-PE. For SA1 with linear correction, we suggest to implement the dynamic correction $\mathbf{f}$ to higher order (i.e., as GESO or GPIO), as an effort to reduce the lagging when tracking ac emf disturbance. For SA1 with nonlinear correction $\mathbf{f}$, we would like to emphasize that as long as $\mathbf{\kappa}$ is within $[0, 1)$, there would be chattering. Therefore, it is also beneficial to implement the SMDO to higher order than second order to reduce chattering, but the SM correction involves more coefficients such as $\mathbf{\kappa}_j$, $j = 0, 1, 2$, and the optimal tuning of $\mathbf{\kappa}_j$ along with correction gains is of interest. As a comparison, non-IS-PE introduces two-way coupling between PE and speed estimation and the interconnected structure is more difficult to be analyzed.

- **EMF based PE considering harmonics:** First, there is no evidence showing that emf observer is better for PE than flux observer. Also, note the sinusoidal dynamics of emf are disturbed by speed variation $\dot{\omega}$. After emf estimation, a redundant observer can be implemented, which can simply be an integration of the emf, to get a flux estimate for $\hat{\omega}_d$, which attenuates the higher-order harmonics in emf estimate.\(^{15}\)

\(^{16}\)A well known example is that the sensorless controlled induction motor cannot control accurate torque but can only regulate biased speed when rotor resistance value is erroneous, which implies that active flux angle $\hat{\theta}_d$ is accurate and $\hat{\omega}$ compensates for the uncertainty in rotor resistance [131].
and amplifies the low frequency disturbance in emf estimate. If \( \dot{\omega} \) is available, a single tune integrator that is an integrator only at \( \dot{\omega} \)-frequency can be used to reduce both dc-bias and higher-order harmonics, where the key step is to design a good band-pass filter [59].

**Speed Estimation.** Direction calculation (IFO-SA5) needs very little computational resources thus is recommended for cheap chips, but it is also sensitive to noises. For high dynamic performance, the SA7 (speed observer) should be used instead of SA6 (MRAS). To reject time-varying load torque, higher-order SA7 design such as GPIO and GESO is beneficial.

**APPENDIX A**

**POSITION ESTIMATOR DESIGN BASICS**

Even though ac motor is a multiple-input multiple-output (MIMO) system, but the following single-input single-output (SISO) observer design can be generalized to ac motor.

Consider an SISO system with unknown parameter \( \omega \):

\[
\frac{d}{dt} y_\alpha = f(y_\alpha) + \frac{d}{dt} x_\alpha = f(y_\alpha) + g(y_\alpha, x_\alpha, \omega)
\]

(38)

with two states, i.e., one measurable output state \( y_\alpha \) (i.e., the \( \alpha \)-axis current) and one unknown internal state \( x_\alpha \) (i.e., the \( \alpha \)-axis active flux).

Our target is to reconstruct an estimate of \( x_\alpha \), denoted by \( \hat{x}_\alpha \), or an estimate of \( \hat{x}_\alpha \), denoted by \( \hat{\alpha} \). If we treat emf \( \hat{x}_\alpha \) as dc disturbance, the proportional-integral correction is sufficient to reject the disturbance with \( k_1, k_2 \geq 0 \):

\[
\frac{d}{dt} \hat{y}_\alpha = f(\hat{y}_\alpha) + \hat{x}_\alpha, \quad \hat{x}_\alpha = k_1 \hat{y}_\alpha + k_2 \int \hat{y}_\alpha dt,
\]

(39)

where \( \hat{y}_\alpha \equiv y_\alpha - \hat{y}_\alpha \) is output error, and non-zero \( k_2 \) leads to dynamic correction—meaning that the observer system is extended with an additional state in the correction term \( \hat{x}_\alpha \).

If \( \hat{x}_\alpha \) is an ac disturbance of frequency \( \omega \), the proportional-resonant (PR) law that serves as a generalized integrator at a given frequency \( \omega \) can be used to reject the disturbance:

\[
\frac{d}{dt} \hat{y}_\alpha = f(\hat{y}_\alpha) + \hat{x}_\alpha, \quad \hat{x}_\alpha = k_1 \hat{y}_\alpha + k_2 \frac{s}{s^2 + \omega^2} \hat{y}_\alpha
\]

(40)

If \( \hat{x}_\alpha \) is the sum of multiple ac disturbances, or if we acknowledge the fact that \( \omega \) is erroneous, the variable structure correction can be used e.g., with the simplest SM surface \( \hat{y}_\alpha \):

\[
\frac{d}{dt} \hat{y}_\alpha = f(\hat{y}_\alpha) + \hat{x}_\alpha, \quad \hat{x}_\alpha = k_1 \text{sign}(\hat{y}_\alpha) + k_2 \int \text{sign}(\hat{y}_\alpha) dt
\]

(41)

Nonzero \( k_1 \) causes chattering, and nonzero \( k_2 \) leads to dynamic correction. Additionally, one can add a redundant state observer for \( x_\alpha \) to attenuate noises:

\[
\frac{d}{dt} \hat{x}_\alpha = \hat{x}_\alpha, \quad \text{with } \hat{x}_\alpha \text{ defined in (39), (40), or (41)}
\]

Alternatively, if we treat \( x_\alpha \) as a state [17] a Luenberger state observer for \( \hat{x}_\alpha \) is established:

\[
\frac{d}{dt} \hat{y}_\alpha = k_1 \hat{y}_\alpha + g(\hat{y}_\alpha, \hat{x}_\alpha, \hat{\omega}) + f(\hat{y}_\alpha)
\]

\[
\frac{d}{dt} \hat{x}_\alpha = k_2 \hat{y}_\alpha + g(\hat{y}_\alpha, \hat{x}_\alpha, \hat{\omega})
\]

(42)

Remark 2: [45] can be further generalized to fourth-order system with a “\( k_3 \)” term added, which is making the \( T_L = 0 \)

If we treat \( \hat{x}_\alpha \) as a state, another state observer that is in the Brunovsky form can be designed:

\[
\frac{d}{dt} \hat{y}_\alpha = k_1 \hat{y}_\alpha + f(\hat{y}_\alpha) + \hat{x}_\alpha
\]

\[
\frac{d}{dt} \hat{x}_\alpha = k_2 \hat{y}_\alpha + \frac{d}{dt} g(\hat{y}_\alpha, \hat{x}_\alpha, \hat{\omega})
\]

(43)

In (42) and (43), an estimate of \( \hat{\omega} \) is needed, and one can either introduce an adaptive law as \( \frac{d}{dt} \hat{\omega} \propto \hat{y}_\alpha \), or make sure the state estimate is robust against parameter error \( \hat{\omega} = \omega - \hat{\omega} \).

So far, among [39]–[43], [39] and [41] have the potential to avoid \( \hat{\omega} \)-dependency. There exists another \( \hat{\omega} \)-free design, i.e., directly integrate for \( \omega_\alpha = x_\alpha + \hat{y}_\alpha \) (i.e., stator flux):

\[
\frac{d}{dt} \hat{z}_\alpha = \frac{d}{dt}(\hat{x}_\alpha + y_\alpha) = f(y_\alpha) + \hat{D}_\alpha
\]

(44)

where the voltage compensation \( \hat{D}_\alpha \) is included to stabilize the pure integration as there is no \( \omega_\alpha \) on the right hand side. For this SISO system, \( \hat{z}_\alpha \) is unknown, we need to design \( \hat{D}_\alpha \) with some prior knowledge that is solely related to the \( \alpha \)-axis dynamics. For example, the maximum and minimum of \( \hat{z}_\alpha \)-waveform within one electrical cycle should add to zero at steady state. Here, note the electrical cycle can be determined by detecting adjacent zero-crossings of the \( \hat{z}_\alpha \)-waveform. For a MIMO system, we can use even more prior knowledge, e.g., the orthogonality and the flux amplitude, to constraint the behavior of \( \hat{z}_\alpha \), as an effort to stabilize the pure integration.

Note [44] corrects the integration within the integrator dynamics, and we can also correct the integration at its output by \( \hat{z}_\alpha - \hat{D}_\alpha \), where the flux compensation \( \hat{D}_\alpha \) can be either directly calculated by the prior knowledge [23], or can be estimated by a disturbance observer [13].

**APPENDIX B**

**SPEED ESTIMATOR DESIGN BASICS**

Disturbance observer (DO) can be used for extracting speed information from rotor position estimate (denoted by \( \hat{\vartheta} \equiv \hat{\theta}_d \)), and can be described as a 3rd-order system with four terms:

\[
\frac{d}{dt} \hat{\vartheta} = f(\hat{\vartheta}) + k_0 |\hat{\vartheta}|^{\kappa_0} \text{sign}(\hat{\vartheta})
\]

\[
+ k_1 \int |\hat{\vartheta}|^{\kappa_1} \text{sign}(\hat{\vartheta}) dt
\]

\[
+ k_2 \int \int |\hat{\vartheta}|^{\kappa_2} \text{sign}(\hat{\vartheta}) dt dt
\]

(45)

If \( f = k_2 = 0 \), constant speed is assumed. If \( f = 0 \), ramp speed is assumed. If \( f(\hat{\vartheta}) = \frac{2\pi}{T_{em}} \int T_{em}(\vartheta) dt \), [45] is inertia-dependent, and constant load torque is assumed. [45] describes a class of observer designs (with integer \( j = 0, 1, 2 \)):

- If \( \kappa_j \neq 1 \), [45] is nonlinear ESO. Typical choices are \( \kappa_0 = 1, \kappa_1 = \frac{1}{2}, \kappa_2 = \frac{1}{4} \) [164, Ch. 4].
- If \( \kappa_j = 1 \), [45] is linear ESO, proportional-integral observer (PIO), or extended Luenberger observer (ELO).
- If \( \kappa_j = 1, f = 0 \), [45] is EKF in which \( k_0, k_1, k_2 \) are time-varying gains considering the unmodelled noises.
- If \( \kappa_j = 1, k_2 = 0 \), [45] is PLL or adaptive observer.
- If \( \kappa_j = k_1 = k_2 = 0 \), [45] is first-order SMO.
- If \( \kappa_j = k_1 = k_2 = 0, \kappa_0 = \frac{1}{2} \), [45] is second-order SMO with dynamic correction using super-twisting algorithm.

- Remark 5: [45] can be further generalized to fourth-order system with a “\( k_3 \)” term added, which is making the \( T_L = 0 \)
assumption, which is studied in literature as generalized PI observer (GPIO) or generalized ESO (GESO). The speed observer (i.e., SA7) takes the following form:

\[
\dot{\omega} = f(\dot{\theta}) + k_1 \int |\dot{\theta}|^n \text{sign}(\dot{\theta}) \, \text{d}t + k_2 \int |\dot{\theta}|^m \text{sign}(\dot{\theta}) \, \text{d}t
\]  

(46)

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Inherently Sensorless Position Estimation (IS-PE)

I - Introduction

1. Proposed Classification

2. Sensorless Control: A Tutorial

3. Problem Formulation

4. The Objective of SA

5. The Main Assumption

6. The Models

7. Position SA Designs

8. Speed SA Designs

9. Complete Schemes for Estimation of Position and Speed

II - Inherently Sensorless Position Estimation (IS-PE)

1. IS-SA3: Stabilized Voltage Model (VM) of Flux $\psi_{\alpha\beta}$

2. Amplitude Limiter

3. Origin of Flux Vector Trajectory

4. Amplitude Correction

5. Compensation by Orthogonality

6. Disturbance Observer (DO) for EMF

7. Sliding Mode Disturbance Observer (SMDO)

8. Linear Disturbance Observer

9. Magnetic Asymmetry (Salient) based IS-PE

III - Non-inherently Sensorless Position Estimation (Non-IS-PE)

1. Non-IS SA2/4 Due to Speed being Part of $\alpha\beta$-Frame Model

2. EMF Observer (SA2.1)

3. Flux Observer (SA2.2)

4. Linearized Position Observation by EKF (SA2.3)

5. Current Observer in $dq$-frame (SA2.4)

6. Frequency-Adaptive System (SA4)

7. Non-IS Variants of SA2/3

8. Speed-Dependent Disturbance Model for SA1

9. $dq$-Frame Variant of SA1

10. Speed-Dependent Orthogonal Condition for SA3

11. $dq$-Frame Variant of SA3

12. Speed Only being Used for Tuning

13. Non-IS Tuning for SA1

14. Non-IS Tuning for SA3

IV - Post-Position Estimation (PE) Speed Estimation

1. Post-PE-SA5: Direct Calculation

2. Direct Calculation from Position

3. Direct Calculation from Orthogonality between Flux and EMF

4. Direct Calculation from EMF

5. Model Reference Adaptive System (MRAS)

6. CAO Based on Prior $dq$-related States

7. Speed Adaptation Law Using Prior

8. Current Error $e = \dot{\omega}$

9. Torque Assumption

10. Observer with Constant Load

11. Inertia-Free Variant of Speed Observer

V - Speed Estimation for Indirect Field Orientation (IFO)

1. General IFO-PE

2. IFO-SA5: Closed-loop Direct Calculation from EMF

3. Magnetic Asymmetry (Rotor Slot Harmonics) Detection

VI - Lose the Constant $K_{\text{Active}}$ Assumption

1. Compensation for Time-Varying $K_{\text{Active}}$

2. Model of Induction Motors with Time-Varying $K_{\text{Active}}$

3. SAs Based on $(i_f, \psi_{\text{Active}})$ Model

4. IS-PE Design

5. Non-IS-PE Design

6. Challenges at Low Speed Regeneration

7. Change of States for Global Stability

8. Active Model (33)

9. Active Model (34)

10. Active Model (35)

11. Adaptive Observer Form

12. Advances in High Gain Observer Design

13. Model Reference Adaptive System (MRAS) for IFO-PE

VII - Concluding Remarks

Appendix A: Position Estimator Design Basics

Appendix B: Speed Estimator Design Basics

References