Static buckling analysis of shape memory alloy reinforced composite laminated plate

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Abstract. With high strength and stiffness composites are being widely used as structural materials. The properties of composites can further improve by reinforcing with advanced structural components like shape memory alloys (SMA). In the present study nonlinear buckling analysis of laminated composite plate with layer of shape memory alloys are done. Finite element governing equation based higher order shear deformation theory with principle of virtual work is considered. Properties of composite plates are formulated with micro-mechanical approach. A MATLAB program is developed with nine nodes isoperimetric element. Influence of volume fraction of SMA, recovery stress, with and without SMA layer, with temperature on critical buckling load is revealed. Convergence and validation study presented here shows the exactness and efficiency of the model.

1. Introduction
In the recent decades, laminated composites have found an increasing applications in various engineering fields especially in defence, marine, automobile, space and many more due to its high strength to weight ratio and stiffness. Reliability of the structure is still one of the major issue which can get fulfil with the utilisation of the smart material incorporation. Smart / intelligent materials, are those have one or more properties that can be considerably vary in a skilful fashion by peripheral stimuli, such as moisture, electric or light, stress, temperature, magnetic fields, pH, or chemical compounds. Piezoelectric materials, SMA and shape-memory polymers (SMP), Photovoltaic materials, Halochromic materials, Ferrofluids are the few examples of the smart materials. Generally when SMA wires are incorporated with layered composite materials, these are pre-strained, so that when it get utilised in high temperature application or environment, it regains its original shape. With the utilisation of this shape memory effect, in present paper, tried to show effect over nondimensional critical buckling temperature.

Initially the incorporation of SMA wire with various volume fractions were get utilised to regulate buckling of the composites. Rogers and Bakers [1] used SMA in laminated composites to improve in its natural frequency and control its vibration. Baz et al. [2] studied the control of SMA wires on the vibration and buckling performance on composite material. Abbas et al. [3] performed the study to show the influence of volume portion of composite fibers, plate dimensions, fiber orientation angle and SMA wires,
on the stimulated plate natural frequencies. Prabhu and Dhanaraj [4] analysed the composite laminated plate against the thermal buckling load by first order shear deformation theory (FSDT) to find the effect of modulus ratio, fiber orientation, and aspect ratio over critical buckling load parameter. Ounis et al. [5] utilised the classical laminate plate theory with linear isoparametric element and a high accuracy rectangular Hermitian element, for the buckling analysis. In 2009, Zhang and Yang [6] studied the laminated composite plate with the advancement into the finite element analysis. From the available literatures it is found that, the study on the thermal buckling seems to be nominal related to the investigation on the mechanical buckling. Thus, the utmost works in the literature uses finite elements based on the first order (FSDT) [7–12]. Li et al. [15] presented numerical simulation of the post-buckling nature of composite shells embedded with SMA fibers under thermal loading. Investigated the effects of SMA fiber reinforcements on the characteristics of thermal buckling were carried out through numerical analysis in ABAQUS. Panda and Singh [16] estimated post-buckling behavior of composite laminated panel with and without SMA wire reinforcements through a mathematical model, established based on the HSDT taking Green–Lagrange nonlinearity. Xia and Shen [17] analysed the functionally graded material along with the piezo actuator with HSDT implementing Von Karman kinematics and under thermal atmosphere. Tawfik et al. [18] analysed SMA fibres reinforced laminated panels under the influence of combined thermal with aerodynamic loading through nonlinear FEM based on the von Karman relations.

When composite material with reinforcement of SMA wires subjected to high temperatures, then internal strain is get induced because of excessive recovered strain, which further generates the tension into the structural member and build its performance against thermal buckling and post buckling. From the available study, it is found that, limited study is focused on the implementation of smart shape memory wires embedded in laminated composites with the use of HSDT. Also, this study includes the convergence and validation study to shows the exactness and efficiency of the model. A MATLAB program is developed with nine nodes isoperimetric element. Influence of volume fraction of SMA, recovery stress, with and without SMA layer, with temperature on critical buckling load is revealed.

2. Mathematical Modelling

The displacement method shows distinct displacement field equation used, and fulfill the condition that, over the surface transverse shear stress zero and nonzero at additional places. Begin with the displacement field as, (Gadade et al. 2016),

\[ U = u_1(x, y) + (Y_1 z - Y_2 z^3) \psi_1(x, y) + (-Y_4 z^3) \theta_1(x, y) \]
\[ V = v_1(x, y) + (Y_1 z - Y_2 z^3) \psi_1(x, y) + (-Y_4 z^3) \theta_1(x, y) \]
\[ W = w_1(x, y) \] (1)

In Eq. (1), along x, y, z directions, the displacement occurred in plate given by U, V and W respectively, whereas midplane displacements are \( u_1, v_1, w_1 \), \( \psi_1 \) and \( \psi_2 \) are the rotations, their variation along the thickness of plate depends on the functions \( \kappa_1(z) = (Y_1 z - Y_2 z^3) \) and \( \kappa_2(z) = (-3Y_4 z^3) \) respectively. Slope along x and y directions represented by \( \theta_1 \) and \( \theta_2 \) respectively, also \( Y_1 = 1; \ Y_2 = Y_4 = 4/3h^2 \) in Eq. (1).

The representation of strain vector for the given displacement fields done by,
\[ \{ \varepsilon \} = \{ \varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{xz}, \gamma_{yx} \}^T \] (2)

The resultant strain vector denoted as follows,
\[ \{ \varepsilon \} = \{ \varepsilon_x \} - \{ \varepsilon_{xx} \} - \{ \varepsilon_{vy} \} \] (3)
Where, $\{\varepsilon\}$ is the thermal strain vector, represented as, $\{\alpha_1 \Delta T, \alpha_2 \Delta T, 0, 0, 0\}$

For the $k^{th}$ lamina with composite SMA embedded wires, the relation between strain and stress with the HSDT for arbitrary fibre alignment angle $\theta$, expressed as,

$$\{\sigma\}_k = \left[ \bar{Q} \right] \{\varepsilon\}_k + \{\sigma_{\nu}\}_k \ V_S - \left[ \bar{Q} \right] \ [\alpha] \ V_m \Delta T$$  \hspace{1cm} (4)

Over the stress free state $T$, total stress $\{\sigma\}_k$ vector calculated. Due to temperature variation $\Delta T$, the recovery stresses are generated into the SMA which is termed as $\{\sigma_{\nu}\}_k$ and for $k^{th}$ layer strain $\{\varepsilon\}_k$ vector denoted. In the expression $\left[ \bar{Q} \right]$ and $\left[ \bar{Q} \right]_m$ are the reduced transferred stiffness matrix of composite plate with SMA fiber and composite matrix respectively. $\{\sigma_{\nu}\}_k$ is the recovery stress generated in SMA due to temperature variation $T$ [14]. $V_s$ and $V_m$ are the volume fraction of SMA and matrix respectively.

The buckling equation of the smart composite laminated composite plate expressed as,

$$\left[ K_s \right] - T_{cr} \left[ K_g \right] \{\delta\} = 0$$ \hspace{1cm} (5)

$\left[ K_s \right]$ and $\left[ K_g \right]$ are the global stiffness matrix and global geometrical stiffness matrix, and $T_{cr}$ is the critical buckling temperature (CBT).

3. Result and discussions

In present study, along with the Figure 1, following boundary conditions are utilized for SMA embedded laminated composite plate. Figure 1 illustrates the geometry of the composite laminated plate, having length of plate ‘$a$’, width ‘$b$’ and ‘$h$’ is the total height of plate.

Figure 2 shows the convergence study. Convergence of FEA has been be evaluated with the consideration of square laminated nine noded with seven degree of freedom composite plate of ply orientation (45/-45/45/-45) under the boundary condition of clamp. For the considered plate, it is observed that, from 3x3 graph gets converged and so, for further study 4x4 elements was used for the analysis.

Figure 3 shows the thermal critical buckling temperature (CBT) validation with the literature using layerwise theory. For the validation purpose square laminated composite plate with layer orientation of (45/-45/45/-45) under clamped and simply supported boundary conditions utilised. The obtained result shows the same nature of variation and close agreement with the previous available literature, and it is also observed that, $T_{cr}$ is lowest under clamped condition, than the simply supported boundary condition.

Table 1 shows the effect of variation of SMA fiber volume fraction ($V_s$) over CBT. For the analysis square laminated composite plate [0/S/S/0] considered with clamped all edges. With the increment of SMA volume fraction, the CBT for the plate increases which is validated with the result [13] obtained by FSDT.

From the reference [14] - Fig. 1(a) and (b) show that the Young’s modulus $E_s$ and the recovery stress $\sigma_r$ of SMA vary as the temperature with high nonlinearity. So, in present case Figure 4, with the increment in temperature, the corresponding modulus and recovery stress also get varied. The effect of variation of temperature over nondimensional CBT of clamped square laminated composite plate having aspect ratio of 50 is revealed in Figure 4. For the 5% SMA fiber volume portion, if temperature increases, the critical buckling temperature shows continuous increment of 244% between three layer and two layer, whereas, 20.5% between four layer and three layer respectively.
Figure 1. Schematic view of composite laminated plate with SMA reinforced layer

Figure 2. Convergence of square composite laminated plate with aspect ratio (a/h) of 50
Figure 3. Critical buckling temperature variation with square composite plate aspect ratio

The effect of variation of \((\alpha_2/\alpha_1)\) over CBT of clamped composite plate with stacking sequence of 
\((0/90/90/0)\) and \(a/h=50\), shown in the Figure 5. With the increase in the ratio \((\alpha_2/\alpha_1)\), critical buckling temperature reduces, but with the SMA fiber laminate \((0/S/S/0)\), the value obtained for the CBT is minimum. Along with this the effect of number of layer also plays an vital role, as, these are reduces from four to two, critical buckling temperature is get decreased by 20% and 142% in \((0/S/0)\) and \((0/S)\), with reference to four SMA embedded layer.

| a/h | SMA volume fraction (Vs) |
|-----|--------------------------|
|     | 0% | 5% | 10% | 15% | 20% |
| 100 | Present | 7.1 | 9.5 | 10.5 | 11.5 | 12.6 |
|     | Ref.[13] | 7 | 8.5 | 10 | 11 | 12 |
| 50  | Present | 25.6 | 32.6 | 37 | 41.1 | 44.5 |
|     | Ref.[13] | 20 | 21.6 | 23.6 | 31 | 34 |
| 20  | Present | 106.5 | 127.1 | 153.35 | 178 | 203.3 |
|     | Ref.[13] | 110 | 124 | 140 | 158 | 178 |
4. Conclusions and future scope
Using FEM technique utilising HSDT the thermal analysis of composite laminated plate and composite sandwich plate means SMA embedded plate for buckling analysis is carried out. The effect of temperature variation, aspect ratio and ratio of coefficient of thermal expansion deviation is observed in the current work. From the study, it is witnessed that, as the SMA volume fraction added or increased in composites, the critical buckling load/temperature is increased, in even temperature application. Based on the available literatures, it is clearly observe that, temperature variation is one of the important affecting factor in shape memory effect which is in alloys or in polymers. Based on that, researchers can simulate the shape memory effect in SMA as well as shape memory polymer simultaneously.
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