Optimized Stress-Strain Ranges for Hyperelastic Constitutive Models Supporting the Simulation of Vertical Stiffness on Airless Tire

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Abstract. The airless tires are developed to overcome a disadvantage of pneumatic tire which lost maneuverability caused of flattening and blasting. The finite element analysis (FEA) is an effective method for development of airless tires which have a complex construction. The material properties which describe mechanical behavior of each tire component are very important to yield accurate result. The high order constitutive model usually obtains the accuracy of FEA results. Unfortunately, it consumes the simulating time. The suitable stress-strain range of Mooney-Rivlin and Ogden model was selected in this research. The vertical stiffness of airless tire model was compared to the experiment. This study found that the accurate result and consuming less time of TWEEL airless tire model depended on an appropriate pair of material properties. There were 100\% of tread and 30\% of spoke stress-strain characteristic based on the Mooney-Rivlin model. Consequently, the study and design of airless tire by finite element method will be carried out efficiently in the further works.

1. Introduction
The TWEEL airless tire 12N16.5 SSL ALL TERRAIN which was developed by Michelin for using on skid steer loader applications was designed to get better function than pneumatic tire without requirement of inflation pressure. The design of supported structure can provide the various advantages. To reduce the analytical development processes, finite element method was widely used to study tire behavior. Namjoo and Golkakhshi [1] had been studied the carcass stiffness of pneumatic tire by finite element method for tire safety development. The Mooney-Rivlin model was used to describe the material properties of tire model. The comparison of tire deformation agreed well with experimental result. The Yeoh model was used to define strain energy of the rubber material property of pneumatic tire model [2]. The 3D finite element model was used to study the tire noise generating mechanisms. The rolling behavior of tire model was very well with the published experimental results [3]. The finite element method had been also performed to investigate the fatigue damage of off-road tire. The Ogden model combining with fatigue crack growth law was used to define tire material properties [4]. The solid tire which was developed for support heavy load also was studied by 3D
finite element method. To develop solid tire model, Phromjan and Suvanjumrat [5] had been varied hyperelastic models for analysis solid tire model deformation. They found that the Ogden model was the most agreement with the experiment. Moreover, the load capability of solid tire was studied by finite element method [6]. The airless tire was studied the mechanical behavior. Jin et.al, [7] had been performed the 3D finite element model of non-pneumatic tires (NPT) to investigate the static and dynamic behavior of various honeycomb spoke structures. The Ogden hyperelastic model and Prony series for polyurethane were used to define NPT material model. In the comparison of deformation, it is confirmed that the simulation results are reliable. Therefore, the NPT model can be used to study the stress distribution and rolling resistance. Rugsaj and Suvanjumrat [8] had been cut the TWEEL tire into material testing specimen according to ASTM standard. The 100% of stress-strain curve from material testing was used to fit with various hyperelastic model. The Ogden model was found to be the most suitable model for defining the material property of TWEEL airless tire. The thickness of spoke was varied. To obtain the same vertical stiffness with same size pneumatic tire, the spoke thickness of TWEEL tire could be reduced. Next, the accurate finite element model of airless tire was used to study the various spoke structure [9]. In the same way, the TWEEL airless tire model was developed by modification of steel belt layer to reduce the complexity of finite element analysis [10]. However, the complex structure of TWEEL tire was consumed a lot of computational resources and time. To optimize the hyperelastic model, this research will be varied the fitting percentage of stress-strain curve to reduce the simulating time and increase the accuracy of simulation results.

2. Experimental

2.1. Material testing
The airless tire, TWEEL 12N16.5 SSL ALL TERRAIN which is developed by Michelin is employed to studied in this research (Figure 1). It composed of rubber tread, shear band, belt layers and spoke. To obtain the material property of each component, the TWEEL airless tire was cut by the water jet cutting technique. The tread component was cut into cylindrical specimen accorded to the compression test standard, ASTM D575. The compression test on cylindrical specimen just performed to 25% of its height with compression speed of 12 mm/min. The spoke was cut into dumbbell specimen accorded to the tensile test standard, ASTM D412. The dumbbell specimen was pulled until it was broken with tensile speed of 500 mm/min. The Universal testing machine, Instron 5969, was carried out to investigate the material property of TWEEL airless tire components.

![Figure 1. The airless tire TWEEL 12N16.5 SSL ALL TERRAIN.](image)

2.2. Vertical stiffness testing of airless tire
The tire testing machine (Ektron PL-2003) is employed to perform the compression test on the TWEEL airless tire as shown in Figure 2. The vertical deformation and compressive force were recorded while the measurement table of testing machine was moved to press airless tire which was mounted on the mounting arm of the tire testing machine. The testing result indicated that the vertical stiffness of TWEEL airless tire by the compressive force of 14 kN was 932.88 N/mm.
Figure 2. The airless tire vertical stiffness testing by tire testing machine (Ektron PL-2003).

3. Finite element method
The hybrid formulation is effective for the analysis of rubberlike material in large strain [11]. The strain energy density per unit volume, \( \partial U \), is defined in equation (1).

\[
d \partial U = \partial S \partial e_i
\]

where \( d \partial U \) and \( \partial S \) are incremental potential energy and Piola-Kirchhoff stress which computed only from the displace field. The integral form of total Lagrangian formulation is shown in equation (2).

\[
\int_{V_0} \partial S \partial e_i dV_0 = R
\]

The general form of principle of virtual work which written in the total Lagrangian formulation is given in equation (3).

\[
\delta \left( \int_{V_0} \partial UdV_0 \right) = R
\]

where \( \partial U \) is the incremental potential which can be modified to include the effect of interpolated pressure by adding to the term of displacement based on the total element pressure.

3.1. Hyperelastic model
The material property of rubberlike material is described by hyperelastic constitutive model which was explained by the strain energy function. The well-known hyperelastic model comprise of Neo-Hookean, Mooney-Rivlin, Ogden, Yeoh and Arruda-Boyce model. The Mooney-Rivlin model and Ogden model are most widely used to describe rubberlike material properties. They are written by equation 4 and 5, respectively [5].

\[
U = C_{10} (\bar{T}_1 - 3) + C_{00} (\bar{T}_2 - 3)
\]

\[
U = \sum_{i=1}^{n} \frac{\mu_i}{\alpha_i} (\bar{\lambda}_i^{\alpha} + \bar{\lambda}_i^{\beta} + \bar{\lambda}_i^{\gamma} - 3) + 4.5K(J^{1/3} - 1)^2
\]

where \( \bar{\lambda}_i = J^{1/3} \lambda_i \), \( J = \lambda_1 \lambda_2 \lambda_3 \)
\( C_{ij} \) is material parameter, \( \bar{T}_1 \) and \( \bar{T}_2 \) are the first and second invariants of the deviatoric strain.
\( \lambda_i \) is the deviatoric principle stretches, \( J \) is the Jacobean determinant, \( K \) is the initial bulk modulus and \( \mu_i, \alpha_i \) are material parameter.
3.2. Airless tire finite element model

The finite element model of airless tire was created according to TWEEL airless tire. It had 4 components which were tread, shear band, belt layer and spoke. The thick shell element was used to model spoke and belt layer of airless tire model while the hexagonal element was used to model tread and shear band. Figure 3(a) shows the finite element model of TWEEL airless tire. The airless tire model is combined with a rigid flat plate model to analyze the compression test by moving the flat plate model to press the fixed airless tire model as shown in Figure 3(b). The material property of tread and spoke were set following the constants of each constitutive model range to simulate the stiffness testing of airless tire. The vertical deformation and compressive force were recorded to compare with the experimental results of airless tire stiffness testing.

![Finite element model of TWEEL airless tire](image)

**Figure 3.** The TWEEL airless tire (a) finite element model and (b) boundary condition of stiffness testing.

4. Results and discussions

The stress-strain relation of compression test and tensile test are presented in Figure 4(a) and 4(b), respectively. The stress-strain curves by the material testing of tread and spoke were divided into three ranges of stress-strain curve. The Mooney-Rivlin and Ogden constitutive model were fit to every stress-strain curve section of each airless tire component. Table 1 presents the constants of Mooney-Rivlin model by linear regression method of MSC.Marc software. The constants of Ogden model are presented in Table 2.

![Stress-strain relation](image)

**Figure 4.** The stress-strain relation of tread and spoke by: (a) compression test and (b) tensile test.

| Curve section | Mooney-Rivlin’s constant | R² |
|---------------|-------------------------|----|
|               | C10 | C01 |
| Tread         |     |     |
| 30% of curve  | 0.99434 | 0.01565 | 0.9983 |
| 60% of curve  | 1.08977 | 0 | 0.9967 |
| 100% of curve | 1.19055 | 0 | 0.9900 |
| Spoke         |     |     |
| 30% of curve  | 0.13031 | 4.95221 | 0.9793 |
| 60% of curve  | 0.53117 | 3.92669 | 0.9579 |
| 100% of curve | 0.8388 | 2.88528 | 0.9721 |

**Table 1.** The constant of Mooney-Rivlin constitutive model.
Table 2. The constant of Ogden constitutive model.

| Curve section | Ogden’s constant | R² |
|---------------|------------------|----|
|               | µ¹ | µ² | a₁ | a² | K |
| Tread         | 30% of curve     | 0.00732 | 15.8447 | 38.8412 | 0.22534 | 9636.99 | 0.9999 |
| Spoke         | 60% of curve     | 0.45942 | 5.48E-08 | 8.25776 | 3.51768 | 9484.52 | 0.9998 |
| Tread         | 100% of curve    | 0.52047 | 30.4738 | 7.43197 | 1.44E-09 | 9670.24 | 0.9998 |
| Spoke         | 30% of curve     | 0.00039 | 566.929 | 7.08047 | 0.03107 | 44037.4 | 0.9842 |
| Spoke         | 60% of curve     | 590.136 | 0.08884 | 0.02825 | 3.38497 | 42433  | 0.9626 |
| Spoke         | 100% of curve    | 5760.18 | 0.20123 | 0.00277 | 2.96849 | 41425.4 | 0.9989 |

The model fitting at the low percentage of stress-strain curve was more precise than the high percentage because of the stress-strain curve trended to be linear in a little deformation for the compression test of tread. The stress-strain curve of spoke was non-linear then the precision is varied. The TWEEL airless tire model which was governed by Mooney-Rivlin and Ogden constitutive model was compressed by the vertical load of 14,000 N. Table 3 presents comparison results of the vertical stiffness between finite element method and experiment.

Table 3. The comparison result of vertical stiffness of airless tire model with experiment.

| Case No. | Material | Mooney-Rivlin | Ogden | Simulation time (min.) | %error |
|----------|----------|---------------|-------|------------------------|--------|
|          |          | 30% | 60% | 100% | 30% | 60% | 100% |                      |        |
| 1        | Tread    | ●   | ●   | ●   | 32.12 | 15.88 |
| Spoke    | ●        |      |      |      |      |      |
| 2        | Tread    | ●   | ●   | ●   | 34.92 | 19.72 |
| Spoke    | ●        |      |      |      |      |      |
| 3        | Tread    | ●   | ●   | ●   | 34.15 | 24.82 |
| Spoke    | ●        |      |      |      |      |      |
| 4        | Tread    | ●   | ●   | ●   | 31.04 | 15.27 |
| Spoke    | ●        |      |      |      |      |      |
| 5        | Tread    | ●   | ●   | ●   | 30.58 | 19.18 |
| Spoke    | ●        |      |      |      |      |      |
| 6        | Tread    | ●   | ●   | ●   | 28.46 | 24.37 |
| Spoke    | ●        |      |      |      |      |      |
| 7        | Tread    | ●   | ●   | ●   | 31.60 | 14.64 |
| Spoke    | ●        |      |      |      |      |      |
| 8        | Tread    | ●   | ●   | ●   | 34.78 | 18.61 |
| Spoke    | ●        |      |      |      |      |      |
| 9        | Tread    | ●   | ●   | ●   | 34.26 | 23.85 |
| Spoke    | ●        |      |      |      |      |      |
| 10       | Tread    | ●   | ●   | ●   | 29.75 | 19.26 |
| Spoke    | ●        |      |      |      |      |      |
| 11       | Tread    | ●   | ●   | ●   | 31.10 | 20.35 |
| Spoke    | ●        |      |      |      |      |      |
| 12       | Tread    | ●   | ●   | ●   | 32.00 | 21.07 |
| Spoke    | ●        |      |      |      |      |      |
| 13       | Tread    | ●   | ●   | ●   | 33.18 | 20.41 |
| Spoke    | ●        |      |      |      |      |      |
| 14       | Tread    | ●   | ●   | ●   | 33.68 | 21.45 |
| Spoke    | ●        |      |      |      |      |      |
| 15       | Tread    | ●   | ●   | ●   | 34.65 | 22.17 |
| Spoke    | ●        |      |      |      |      |      |
| 16       | Tread    | ●   | ●   | ●   | 34.07 | 20.33 |
| Spoke    | ●        |      |      |      |      |      |
| 17       | Tread    | ●   | ●   | ●   | 31.86 | 21.39 |
| Spoke    | ●        |      |      |      |      |      |
| 18       | Tread    | ●   | ●   | ●   | 32.39 | 22.09 |
| Spoke    | ●        |      |      |      |      |      |
The lowest error of vertical stiffness simulation occurred at the case number 7. The simulation time of this case was 31.60 min was not much when compared to the others and was less than simulation time of the case number 9 and 18 which were the 100% of fitting. Total results of 18 simulating cases were found that the accurate result of TWEEL airless tire was depended on the appropriate pair of material properties. Figure 5 shows the airless tire deformation. The color contour presented the value of displacement of TWEEL airless tire model. The yellow color was the maximum value of displacement while blue was minimum. The simulation result presented the most deformation of TWEEL airless tire which occurred at the tread and spoke while they were compressed. The spoke was bended distinctly when it rolled to be normal with the horizontal floor.

![Figure 5](image_url)

**Figure 5.** The total displacement of TWEEL airless tire model by simulation of case number 7.

5. Conclusion
The TWEEL airless tire components which were composed of tread, shear band, belt later and spoke had been carried out to test the material property. The stress-strain relation of these components was fit with Mooney-Rivlin and Ogden hyperelastic model. The constitutive models of each stress-strain range were implemented to the stiffness testing of TWEEL tire model. The accurate result of TWEEL airless tire was depended on an appropriate pair of material properties. It was found that the pair of 100% of tread and 30% of spoke stress-strain curve fitting based on the Mooney-Rivlin model provided the lowest error of 14.64% while the simulation time was 31.60 mins. The shape of spoke was designed to be deformed when the spoke rolled to be normal with the floor. The method of constitutive model selective to develop airless tire model in this research can be used to study and design the airless tire by finite element analysis under the appropriate accuracy and time in future work.

6. References
[1] Namjoo M and Golbakhshi H 2015 *J. Cent. South Univ.* **22** 189
[2] Palanivelu S, Narasimha R and Ramarathnam K K 2015 *Mech. Syst. Signal Process* **64-65** 385
[3] Kindt P, Sas P and Desmet W 2009 *Opt. LasersEng.* **47** 443
[4] Nyaaba W, Frimpong S and Anani A 2019 *Int. J. of Fatigue* **119** 247
[5] Phromjan J and Suvanjumrat 2018 *Engineering journal* **22**(2) 141
[6] Phromjan J and Suvanjumrat C 2018 *Key Eng. Mater.* **777** 416
[7] Jin X, Hou C, Fan X, Sun Y, Ly J and Lu C 2018 *Compos. Struct.* **187** 27
[8] Rugsaj R and Suvanjumrat C 2019 *Int. J. Automot. Technol.* **20**(4) 801
[9] Rugsaj R and Suvanjumrat C 2020 *Mech. Based Des. Struc.* DOI:10.1080/15397734.2020.1777875
[10] Phromjan J and Suvanjumrat C 2020 *IOP Conf. Ser-Mat Sci.* **773**(1) 012047
[11] Bathe K J 1997 *Finite Element Procedures* (London: Prentice Hall)