Minimal Length Effects on Schwinger Mechanism

Benrong Mu\textsuperscript{a,\textit{b}}, Peng Wang\textsuperscript{b}, and Haitang Yang\textsuperscript{b}

\textsuperscript{a}School of Physical Electronics, University of Electronic Science and Technology of China, Chengdu, 610054, China and

\textsuperscript{b}Center for Theoretical Physics, College of Physical Science and Technology, Sichuan University, Chengdu, 610064, PR China

Abstract

In this paper, we investigate effects of the minimal length on the Schwinger mechanism using the quantum field theory (QFT) incorporating the minimal length. We first study the Schwinger mechanism for scalar fields in both usual QFT and the deformed QFT. The same calculations are then performed in the case of Dirac particles. Finally, we discuss how our results imply for the corrections to the Unruh temperature and the Hawking temperature due to the minimal length.
I. INTRODUCTION

Using the proper-time method, Schwinger[1] calculated the effective action of a charged particle in an external electromagnetic field. He found that the action has an imaginary part for a uniform electric field, which leads to the vacuum decay through pair production. Due to its purely non-perturbative nature, this quantum field theoretical prediction is of fundamental importance. The Schwinger mechanism sheds lights on topics as diverse as the string breaking rate in QCD[2, 3] and on black hole physics[4].

On the other hand, various theories of quantum gravity, such as string theory, loop quantum gravity and quantum geometry, predict the existence of a minimal length[5–7]. The generalized uncertainty principle(GUP)[8] is a simply way to realize this minimal length. An effective model of the GUP in one dimensional quantum mechanics is given by[9, 10]

\[ k(p) = \frac{1}{\sqrt{\beta}} \tanh \left( \sqrt{\beta} p \right), \quad (1) \]
\[ \omega(E) = \frac{1}{\sqrt{\beta}} \tanh \left( \sqrt{\beta} E \right), \quad (2) \]

where the generators of the translations in space and time are the wave vector \( k \) and the frequency \( \omega \), \( \beta = \frac{\beta_0}{m_p^2} \), \( m_p \) is the Planck mass and \( \beta_0 \) is a dimensionless parameter marking
quantum gravity effects. We set $c = \hbar = G = 1$ in the paper. The quantization in position representation $\hat{x} = x$ leads to

$$ k = -i\partial_x, \omega = i\partial_t. \tag{3} $$

Therefore, the low energy limit $p \ll m_p$ including order of $\frac{p^3}{M_f^3}$ gives

$$ p = -i\partial_x \left(1 - \frac{\beta}{3}\partial_x^2\right), \tag{4} $$

$$ E = i\partial_t \left(1 - \frac{\beta}{3}\partial_t^2\right). \tag{5} $$

From eqn. (11), it is noted that although one can increase $p$ arbitrarily, $k$ has an upper bound which is $\frac{1}{\sqrt{\beta}}$. The upper bound on $k$ implies that that particles could not possess arbitrarily small Compton wavelengths $\lambda = 2\pi/k$ and that there exists a minimal length $\sim \sqrt{\beta}$.

In this paper, we investigate scalars and fermions pair production from a static classical electric field using the deformed QFT which incorporates the minimal length via eqn. (4) and eqn. (5). The organization of this paper is as follows. In section II based on the usual and the minimal length modified field theoretic considerations, Schwinger’s mechanism is derived in the case of spinless particles. We then show how the same calculations can be performed in the case of Dirac particles in section III. Section IV is devoted to our discussions and conclusions.

II. SCALAR PAIR PRODUCTION

In this section, we first derive the formula for the scalar pair production rate in the framework of QFT. We then use the formula to calculate pair creation rate in usual QFT and the minimal length modified QFT. As shown in ref. [11], the scalar field theoretic vacuum to vacuum amplitude can be written as

$$ \langle vac \to vac \rangle \propto \int [d\phi] \exp \left( i \int dx^4 \mathcal{L} \right), \tag{6} $$

where $\mathcal{L}$ is the Lagrange density and $\phi$ is the corresponding scalar field. Assume $\mathcal{L}$ is given by $\mathcal{L} = \phi^+ \mathcal{O}_s \phi$ where $\mathcal{O}_s$ is some differential operator. Defining eigenfunctions $\phi_n$ with eigenvalues $\lambda_n$,

$$ \mathcal{O}_s \phi_n = \lambda_n \phi_n, \tag{7} $$
we expand
\[ \phi = \sum_n a_n \phi_n. \] (8)

Using orthogonality of \( \phi_n \), we find
\[ e^{-\xi_s} \equiv \langle \text{vac} \rightarrow \text{vac} \rangle \propto \int da_n da_n^* \exp \left( i \int dx^4 \lambda_n |a_n|^2 \right) \]
\[ \propto \prod_n \frac{1}{\lambda_n} = \frac{1}{\det O_s} = \exp \left( -\text{Tr} \left( \ln O_s \right) \right) = \exp \left( -\sum_n \ln \lambda_n \right). \] (9)

Using the integral
\[ \ln a = -\int_0^{+\infty} ds \frac{e^{-as}}{s} + \text{const.}, \] (10)

one has
\[ \xi_s = \xi_s^E + C, \] (11)

where \( C \) is a constant and we define
\[ \xi_s^E \equiv -\sum_n \int_0^{+\infty} ds \frac{e^{-\lambda_n s}}{s}. \] (12)

The imaginary part of \( \xi_s \) is always infinite due to vacuum energy shift. Here, we are only interested in the real part of \( \xi_s \) which gives the vacuum decay. As shown later in this section, the electric field \( E \) only appears in \( \lambda_n \) and \( C \) is independent of \( E \). To obtain \( C \) we consider the case without the electric field. When there is no electric field, the vacuum is stable and no pairs are produced. In this case, the vacuum to vacuum amplitude is
\[ e^{i\alpha} = e^{-\xi_s} = \exp \left( -\xi_s^0 + C \right) \]
where \( \alpha \) is a phase irrelevant to the vacuum decay and \( \xi_s^0 \) is \( \xi_s^E \) with \( E = 0 \). Thus, one has
\[ C = \xi_s^0 + i\alpha. \] (13)

Plugging eqn. (13) into eqn. (11), we find that the real part of \( \xi_s \) is
\[ \gamma_s \equiv \text{Re} \xi_s = \text{Re} \left( \xi_s^E - \xi_s^0 \right). \] (14)

Squaring \( \gamma_s \), we observe that the total pair production rate per unit volume is simply
\[ \frac{\text{prob}_{\text{pair}}}{VT} = \frac{1}{VT} \left( 1 - e^{-2\gamma_s} \right) \approx \frac{2\gamma_s}{VT}. \] (15)
A. Usual Scalar Field

For the case of a scalar field $\phi$ of mass $m$ and charge $e$ in the presence of an external electromagnetic interaction described by vector potential $A_\mu$, the Lagrangian is given by

$$ \mathcal{L}_s = \phi^* \mathcal{O}_s^{(0)} \phi, $$

where $\mathcal{O}_s^{(0)} = (\partial + ieA)^2 + m^2$. If there is a uniform electric field $\mathbf{E} = E\mathbf{e}_z$, we can utilize the gauge $A = -Et\mathbf{e}_z$. The operator $\mathcal{O}_s^{(0)}$ then becomes

$$ \mathcal{O}_s^{(0)} = \partial_t^2 - \partial_{\perp}^2 - (\partial_z + ieEt)^2 + m^2, $$

where $\partial_{\perp}^2 = \partial_x^2 + \partial_y^2$. Assume that eigenfunction of $\mathcal{O}_s^{(0)}$ takes the form as $\psi = \exp(i k \cdot \mathbf{r}_{\perp} + ik_z z) \sigma(t)$ where $\mathbf{r}_{\perp} = xe_x + ye_y$ and $\sigma(t)$ satisfies

$$ \left( \left( \frac{d}{dt} \right)^2 + k_{\perp}^2 + (k_z + eEt)^2 + m^2 \right) \sigma(t) = \lambda \sigma(t). $$

Performing the Wick Rotation $t \rightarrow -i\tau$ and $E \rightarrow -i\tilde{E}$, we have

$$ \left( -\left( \frac{d}{d\tau} \right)^2 + e^2\tilde{E}^2 \left( \tau - \frac{k_z}{e\tilde{E}} \right)^2 \right) \sigma(t) = (\lambda - k_{\perp}^2 - m^2) \sigma(t). $$

Obviously, eqn. (19) describes a one-dimensional harmonic oscillator with its well centered at $\frac{k_z}{e\tilde{E}}$ and a resonant frequency $2e\tilde{E}$. One can express $\frac{d}{d\tau}$ and $\tau$ in terms of ladder operators, $a$ and $a^+$, as

$$ \tau - \frac{k_z}{e\tilde{E}} = \frac{1}{\sqrt{2e\tilde{E}}} (a^+ + a), $$

$$ \frac{d}{d\tau} = -\sqrt{\frac{e\tilde{E}}{2}} (a^+ - a). $$

Thus, the energy levels are quantized as

$$ \lambda_{n,k_{\perp}}^{s(0)} = k_{\perp}^2 + m^2 + (2n + 1) e\tilde{E} $$

where $n = 0, 1, 2 \cdots$. Note that $k_x$ and $k_y$ range over all values from $-\infty$ to $\infty$, but $k_z$ is constrained to be in the range $0 < k_z < e\tilde{E}iT$ in order that the entire range of time is included as $k_z$ is varied, where $T$ is a total interaction time. Thus, the corresponding
degeneracy is $\frac{e E V d k}{(2 \pi)^3}$ where $V$ is the volume. After switching back to $t$ and $E$, eqn. (12) and eqn. (22) yields

$$\xi^E_s = -\frac{e E V T}{(2 \pi)^3} \int d k \int_0^{+\infty} \frac{d s}{s} \exp \left[ - (k_\perp^2 + m^2) s \right] \sum_{n=0}^{\infty} \exp \left[ -i (2n + 1) e E s \right].$$

(23)

With the help of Dirac comb

$$\pi \sum_{k=-\infty}^{\infty} \delta (t - k \pi) = \sum_{n=-\infty}^{\infty} \exp (-i 2n t),$$

(24)

one easily gets

$$\text{Re} \xi^E_s = -\frac{e E V T}{2 (2 \pi)^3} \int d k \int_0^{+\infty} \frac{d s}{s} \exp \left[ - (k_\perp^2 + m^2) s \right] \exp (-i E s) \pi \sum_{k=-\infty}^{\infty} \delta (e E s - k \pi).$$

(25)

When $E$ approaches zero, we have for $\text{Re} \xi^0_s$

$$\text{Re} \xi^0_s = -\frac{V T}{16 \pi^2} \int d k \int_0^{+\infty} \frac{d s}{s} \exp \left[ - (k_\perp^2 + m^2) s \right] \delta (s).$$

(26)

Thus, we find

$$\gamma^{(0)} = \text{Re} \left( \xi^E_s - \xi^0_s \right)$$

$$= \frac{V T}{16 \pi^2} \int d k \int_0^{+\infty} \frac{d s}{s} \exp \left[ - (k_\perp^2 + m^2) s \right] \exp (-i E s) \left[ \sum_{k=-\infty}^{\infty} \delta \left( s - \frac{k \pi}{e E} \right) - \delta (s) \right]$$

$$= \frac{e^2 E^2 V T}{16 \pi^3} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2} \exp \left( \frac{-m^2 k \pi}{e E} \right).$$

(27)

B. Minimal Length Modified Scalar Field

In the presence of an external electromagnetic potential $A_\mu$, eqn. (4) and eqn. (5) can be generalized to

$$p_i = -i D_t \left( 1 - \frac{\beta}{3} D_t^2 \right),$$

(28)

$$E = i D_t \left( 1 - \frac{\beta}{3} D_t^2 \right),$$

(29)

where $D_\mu = \partial_\mu + i e A_\mu$. For a scalar field $\phi$ of mass $m$ and charge $e$ in the external electromagnetic potential $A_\mu$, the Lagrangian incorporating eqn. (28) and eqn. (29) can be written as

$$\mathcal{L}_s = \eta_{\mu \nu} (p_\mu \phi)^+ (p_\nu \phi) + m^2,$$

(30)
where \( p_\mu = (E, p_i) \). After integrating by part, we have

\[
L_s = \phi^+ (\mathcal{O}^{(0)}_s + \delta \mathcal{O}_s) \phi, \tag{31}
\]

where we define

\[
\delta \mathcal{O}_s = -\frac{2}{3} \beta [D^4_t - (D^4_x + D^4_y + D^4_z)] + \mathcal{O}(\beta^2). \tag{32}
\]

In order to get eigenfunctions and eigenvalues of \( \mathcal{O}^{(0)}_s + \delta \mathcal{O}_s \), we write the eigenfunctions in the form \( \phi = \exp(i k_\perp \cdot r_\perp + i k_z z) \sigma(t) \). For \( \sigma(t) \), \( \delta \mathcal{O}_s \) becomes

\[
\delta \mathcal{O}_s = -\frac{2}{3} \beta \left[ \partial_t^4 - k_\perp^4 - (k_z + eEt)^4 \right] + \mathcal{O}(\beta^2). \tag{33}
\]

Rotating to imaginary time \( \tau \) and \( \tilde{E} \), we can use eqn. (20) and eqn. (21) to write \( \delta \mathcal{O}_s \) in terms of ladder operators

\[
\delta \mathcal{O}_s = -\frac{1}{6} \beta e^2 \tilde{E}^2 \left[ (a^+ - a)^4 - \frac{4k_\perp^4}{e^2 \tilde{E}^2} - (a^+ + a)^4 \right] + \mathcal{O}(\beta^2). \tag{34}
\]

Treating \( \delta \mathcal{O}_s \) as perturbations, we find the first-order correction to \( \lambda^{(0)}_{n,k_\perp} \)

\[
\delta \lambda^{s}_{n,k_\perp} = \langle n | \delta \mathcal{O}_s | n \rangle = \frac{2\beta k_\perp^4}{3}, \tag{35}
\]

where \( |n\rangle \) is \( n \)th eigenstate for the one-dimensional harmonic oscillator. The corresponding degeneracy stays same, namely \( \frac{eEVTd_k}{(2\pi)^3} \). Therefore, eqn. (12) gives to \( \mathcal{O}(\beta) \)

\[
\xi_e^s \approx -\frac{eEVT}{(2\pi)^3} \int dk_\perp \int_0^{+\infty} \frac{ds}{s} \exp \left[ -\left( k_\perp^2 + m^2 + \frac{2\beta k_\perp^4}{3} \right) s \right] \sum_{n=0}^{\infty} \exp \left[ -i (2n + 1) E s \right]. \tag{36}
\]

Note that the above equation is the same as eqn. (23) except the prefactor \( \exp \left( \frac{-2\beta k_\perp^4}{3} s \right) \).

Following calculations for \( \gamma^{(0)}_s \), one obtains to \( \mathcal{O}(\beta) \)

\[
\gamma_s = \text{Re} (\xi_e^s - \xi_e^0) \\
\approx \frac{eEVT}{16\pi^3} \int dk_\perp \sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k} \exp \left[ -\left( k_\perp^2 + m^2 + \frac{2\beta k_\perp^4}{3} \right) \frac{k\pi}{eE} \right] \\
\approx \frac{e^2E^2VT}{16\pi^3} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2} \exp \left( -\frac{m^2\pi k}{eE} - \frac{4\beta eE}{3\pi k} \right). \tag{37}
\]
III. FERMION PAIR PRODUCTION

The procedure in Section II can also be applied to calculations for a fermion field. However, two differences should be noted. First, the Grassmann numbers are used in fermion case. Second, instead of $O_f$, we usually calculate eigenvalues of $\tilde{O}_f \tilde{O}_f$, where $\tilde{O}_f$ is defined as follows. In even dimension, there exist two charge conjugate operators $C_+$ and $C_-$ such that

$$C_\pm \gamma^\mu C_\mp^{-1} = \pm \gamma^\mu T,$$

where $\gamma^\mu$ are Gamma matrices. We then define

$$\tilde{O}_f \equiv C_\mp^{-1} \left( \gamma^0 \left( C_+ O_f C_+^{-1} \right)^* \gamma^0 \right)^T C_-.$$

Since $\tilde{O}_f$ is Hermitian, one finds

$$\det \tilde{O}_f = \det O_f^* = \det O_f^+ = \det O_f.$$

Assume the Lagrangian for a fermion field $\psi$ is give in the form of $L = \bar{\psi} O_f \psi$. Defining eigenfunctions $\psi_n$ with eigenvalues $\lambda'_n$,

$$O_f \psi_n = \lambda'_n \psi_n,$$

we expand

$$\psi = \sum_n \xi_n \psi_n.$$

Thus, the vacuum to vacuum amplitude for $\psi$ is

$$e^{-\xi_f} \equiv \langle \text{vac} \rightarrow \text{vac} \rangle \propto \int d\xi_n d\xi'_n \exp \left( i \int dx^4 \lambda'_n |\xi_n|^2 \right) \propto \prod_n \lambda'_n = \det O_f = \left( \det O_f \tilde{O}_f \right)^{1/2} = \exp \left( \frac{1}{2} \text{Tr} \left( \ln O_f \tilde{O}_f \right) \right) = \exp \left( \frac{1}{2} \sum_n \ln \lambda_n \right)$$

where $\lambda_n$ are eigenvalues of $O_f \tilde{O}_f$. The real part of $\xi_f$ is given by

$$\gamma_f = \text{Re} \xi_f = \text{Re} \left( \xi_f^E - \xi_f^0 \right)$$

where $\xi_f^0$ is $\xi_f^E$ with $E = 0$ and we define

$$\xi_f^E \equiv \frac{1}{2} \sum_n \int_0^{+\infty} ds \frac{1}{s} e^{-\lambda_n s}.$$

Squaring $\gamma_f$, we observe that the total pair production rate per unit volume is simply

$$\frac{\text{prob}_{\text{pair}}}{VT} = \frac{1}{VT} \left( 1 - e^{-2\gamma_f} \right) \approx \frac{2\gamma_f}{VT}.$$
A. Usual Fermion Field

The Lagrangian for a charged spinor of mass $m$ and charge $e$ is

$$\mathcal{L} = i \bar{\psi} D^\mu \gamma_\mu \psi - m \bar{\psi} \psi = \bar{\psi} O_f^{(0)} \psi,$$

(47)

where $O_f^{(0)} = i D^\mu \gamma_\mu - m$. Using eqn. (39), one finds

$$O_f^{(0)} \tilde{O}_f^{(0)} = \kappa_1 + \kappa_2,$$

(48)

where $\sigma^{\mu \nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$, $[D_\mu, D_\nu] = ieF_{\mu \nu}$, $\kappa_1 \equiv D^2 + m^2$ and $\kappa_2 = \frac{e^2}{2} F_{\mu \nu} \sigma^{\mu \nu}$. Here the eigenvalues of $\kappa_1$ in the case of a constant electric field were determined in Section II and are given by

$$\lambda_1 = k_\perp^2 + m^2 + (2n + 1) e \tilde{E}.$$

(49)

In chiral representation, we have

$$\kappa_2 = ieE \begin{pmatrix} -\sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix},$$

(50)

where $A = -E t e_z$ for a uniform electric field $E = E e_z$. Temporarily rotating to imaginary time as before, we find that the eigenvalues of $O_f^{(0)} \tilde{O}_f^{(0)}$ are

$$\lambda_{n,k_\perp}^{(f)} = k_\perp^2 + m^2 + 2ne \tilde{E}$$

for $n = 0, 1, 2 \cdots$, (51)

and the corresponding degeneracies are

$$\frac{2g_n e E VT dk_\perp}{(2\pi)^3},$$

(52)

where $g_0 = 1$ and $g_{n>0} = 2$. Thus one finds

$$\xi^E_f = \frac{eEVT}{(2\pi)^3} \int dk_\perp \int_0^{+\infty} \frac{ds}{s} \exp \left[ - \frac{(k_\perp^2 + m^2)}{s} \right] \sum_{n=0}^{\infty} g_n \exp \left( -2i n eE s \right).$$

(53)

Using Dirac comb, we get

$$\text{Re} \xi^E_f = \frac{eEVT}{(2\pi)^3} \int dk_\perp \int_0^{+\infty} \frac{ds}{s} \exp \left[ - \frac{(k_\perp^2 + m^2)}{s} \right] \pi \sum_{k=-\infty}^{\infty} \delta (eE s - k\pi),$$

(54)

$$\text{Re} \xi^0_f = \frac{VT}{(2\pi)^3} \int dk_\perp \int_0^{+\infty} \frac{ds}{s} \exp \left[ - \frac{(k_\perp^2 + m^2)}{s} \right] \pi \delta (s).$$

(55)

Therefore, we have

$$\gamma_f^{(0)} = \text{Re} (\xi^E_f - \xi^0_f)$$

$$= \frac{e^2 E^2 VT}{8\pi^3} \sum_{k=1}^{\infty} \frac{1}{k^2} \exp \left( -m^2 k\pi \frac{eE}{k} \right).$$

(56)
B. Minimal Length Modified Fermion Field

For this case, the Lagrangian incorporating eqn. (28) and eqn. (29) for a charged spinor \( \psi \) of mass \( m \) and charge \( e \) can be written as

\[
\mathcal{L}_f = \overline{\psi} (i p^\mu \gamma^\mu - m) \psi, \tag{57}
\]

where \( p^\mu = (E, p_i) \). We then find

\[
\mathcal{L}_f = \overline{\psi} \mathcal{O}_f \psi = \overline{\psi} \left( \mathcal{O}_f^{(0)} + \delta \mathcal{O}_f' \right) \psi, \tag{58}
\]

where we define

\[
\delta \mathcal{O}_f' = -\frac{i}{3} \beta D^3_\mu \gamma^\mu + \mathcal{O} (\beta^2). \tag{59}
\]

Eqn. (39) and eqn. (59) gives the product of \( \mathcal{O}_f \) and \( \tilde{\mathcal{O}}_f \) to \( \mathcal{O} (\beta) \)

\[
\mathcal{O}_f \tilde{\mathcal{O}}_f = \mathcal{O}_f^{(0)} \tilde{\mathcal{O}}_f^{(0)} + \delta \mathcal{O}_f, \tag{60}
\]

where we find

\[
\delta \mathcal{O}_f = \delta \mathcal{O}_s - e \beta F^\mu_\nu D^2_\mu \sigma^{\mu\nu}. \tag{61}
\]

The first term in eqn. (61) is just \( \delta \mathcal{O}_s \) given in eqn. (33), while one can express the second term in terms of ladder operators after rotating to imaginary time. In fact, using eqn. (20) and eqn. (21) gives

\[
F^\mu_\nu D^2_\mu \sigma^{\mu\nu} = E^2 \begin{pmatrix} -\sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix} (a^2 + a^2), \tag{62}
\]

which doesn’t contribute to the first-order correction to eigenvalues \( \lambda^{(0)}_{n,k_\perp} \) of the leading operator \( \mathcal{O}_f^{(0)} \tilde{\mathcal{O}}_f^{(0)} \) in eqn. (60) since \( \langle n \mid F^\mu_\nu D^2_\mu \sigma^{\mu\nu} \mid n \rangle \) is zero. Thus, the eigenvalues of \( \mathcal{O}_f \tilde{\mathcal{O}}_f \) to \( \mathcal{O} (\beta) \) are

\[
\lambda^f_{n,k_\perp} \approx k_\perp^2 + m^2 + 2ne\tilde{E} + \frac{2\beta k_\perp^4}{3} \text{ for } n = 0, 1, 2 \cdots, \tag{63}
\]

where we use eqn. (35). The corresponding degeneracies are also

\[
\frac{2g_neETVdk_\perp}{(2\pi)^3}, \tag{64}
\]

where \( g_0 = 1 \) and \( g_{n>0} = 2 \). Therefore, one finds for \( \xi^E_f \)

\[
\xi^E_f \approx \frac{eETV}{(2\pi)^3} \int dk_\perp \int_0^{+\infty} ds \exp \left[ -\left( k_\perp^2 + m^2 + \frac{2\beta k_\perp^4}{3} \right)s \right] \sum_{n=0}^{\infty} g_n \exp \left(-2ne\tilde{E}s\right). \tag{65}
\]
Using Dirac Comb, we find that

\[ \Re \xi_{E}^{f} \approx \frac{eEVT}{(2\pi)^{3}} \int dk_{\perp} \int_{0}^{+\infty} \frac{ds}{s} \exp \left[ - \left( k_{\perp}^{2} + m^{2} + \frac{2\beta k_{4}^{4}}{3} \right) s \right] \pi \sum_{k=-\infty}^{\infty} \delta (eEs - k\pi) , \quad (66) \]

\[ \Re \xi_{f}^{0} \approx \frac{VT}{(2\pi)^{3}} \int dk_{\perp} \int_{0}^{+\infty} \frac{ds}{s} \exp \left[ - \left( k_{\perp}^{2} + m^{2} + \frac{2\beta k_{4}^{4}}{3} \right) s \right] \pi \delta (s) . \quad (67) \]

Again, we obtain \( \gamma_{f} \) to \( \mathcal{O} (\beta) \)

\[ \gamma_{f} = \Re (\xi_{E}^{f} - \xi_{f}^{0}) \]

\[ \approx \frac{eEVT}{8\pi^{3}} \sum_{k=1}^{\infty} \frac{1}{k} \int dk_{\perp} \exp \left[ - \left( k_{\perp}^{2} + m^{2} + \frac{2\beta k_{4}^{4}}{3} \right) \frac{k\pi}{eE} \right] \]

\[ \approx \frac{e^{2}E^{2}VT}{8\pi^{3}} \sum_{k=1}^{\infty} \frac{1}{k^{2}} \exp \left( -m^{2} \frac{k\pi}{eE} - \frac{4\beta eE}{3\pi k} \right) . \quad (68) \]

**IV. DISCUSSION AND CONCLUSION**

First, it is noted that the problem of scalar particles pair creation by an electric field in the presence of a minimal length is also studied in ref. [13]. The authors considered another GUP of form

\[ x_{i} = x_{0i}, \quad (69) \]

\[ p_{i} = p_{0i} \left( 1 + \beta p^{2} \right) , \quad (70) \]

where \( x_{0i} \) and \( p_{0i} \) satisfy the canonical commutation relations. Using Bogoliubov transformations, they found

\[ \gamma_{s} \sim \exp \left[ -m^{2} \frac{\pi}{eE} \left( 1 + \frac{\beta m^{2}}{4} \left( 1 - \frac{e^{2}E^{2}}{m^{4}} \right) \right) \right] , \quad (71) \]

where their minimal length corrections depend on the mass of scalar particles while our results don’t, at least to \( \mathcal{O} (\beta) \).

In the case of electron–positron pairs, the pair production is irrelevant for laboratory electric fields, let alone the minimal length corrections. However, the Schwinger mechanism has to do with the Unruh effect which predicts that an accelerating observer will observe a thermal spectrum of photons and particle–antiparticle pairs at temperature \( T = \frac{a}{2\pi} \), where \( a \) is the acceleration [14]. We now investigate how our results imply for the corrections to the Unruh temperature due to the minimal length. Considering a free particle of charge \( e \) and
mass \( m \) moving in a static electric field \( E \), one finds the particle have acceleration \( a = \frac{em}{E} \). Keeping only the leading term, the pair production probability per unit volume per unit time is

\[
\frac{\text{prob}_{\text{pair}}}{VT} \sim \exp \left( -m^2 \frac{\pi}{6eE} - \frac{4\beta eE}{3\pi} \right). \tag{72}
\]

Identifying the reduced mass \( \frac{m^2}{2} = K \) as the energy associated with the pair production process, we find that the production probability can be written in the form

\[
\text{prob}_{\text{pair}} \sim \exp \left[ -\frac{K}{a/2\pi} \left( 1 + \frac{4\beta a^2}{3\pi^2} \right) \right]. \tag{73}
\]

The minimal length modified Unruh temperature can be read from eqn. (73), which gives

\[
T_u \sim \frac{a}{2\pi} \left( 1 - \frac{4\beta a^2}{3\pi^2} \right), \tag{74}
\]

where \( a \) is the acceleration of the observer.

The Unruh effects can be relevant to the phenomenon of black hole decay due to pair creation, the Hawking radiation\[15\]. Consider a Schwarzschild black hole with the black hole’s mass, \( M \). The event horizon of the Schwarzschild black hole is \( r_h = 2M \). Noting that the gravitational acceleration at the event horizon is given by

\[
a = \frac{M}{r_h^2} = \frac{1}{4M}, \tag{75}
\]

one finds from eqn. (74) that the minimal length modified Hawking temperature is

\[
T_h \sim \frac{1}{8\pi M} \left( 1 - \frac{\beta}{12\pi^2 M^2} \right). \tag{76}
\]

Using the first law of the black hole thermodynamics, we find the corrected black hole entropy is

\[
S = \int \frac{dM}{T} \sim \frac{A}{4} + \frac{\beta}{3\pi} \ln \left( \frac{A}{16\pi} \right), \tag{77}
\]

where \( A = 4\pi r_h^2 = 16\pi M^2 \) is the area of the horizon. The logarithmic term in eqn. (77) is the well known correction from quantum gravity to the classical Bekenstein-Hawking entropy, which have appeared in different studies of GUP modified thermodynamics of black holes\[16-18\]. A careful reader might not be satisfied with the above heuristic handwaving argument relating the Schwinger mechanism to the Unruh temperature and the Hawking temperature.
However, using the minimal length modified Hamilton-Jacobi Method incorporating eqn. (28) and eqn. (29), we found in ref. [19] that the corrected Hawking temperature for the Schwarzschild black is given by

$$T_h \sim \frac{1}{8\pi M} \left( 1 - \frac{\beta}{6M^2} \right)$$

which is almost same as eqn. (76) except the numerical factor in front of $\beta$.

In this paper, incorporating effects of the minimal length, we derived the deformed Schwinger mechanism for both scalars and fermions in a static uniform electric field. Using handwaving argument, the implications of our results for the Unruh temperature and the Hawking temperature are also discussed.

Acknowledgements.

We would like to acknowledge useful discussions with Y. He, Z. Sun and H. W. Wu. This work is supported in part by NSFC (Grant No. 11005016, 11175039 and 11375121) and SYSTF (Grant No. 2012JQ0039).

[1] J. S. Schwinger, “On gauge invariance and vacuum polarization,” Phys. Rev. 82, 664 (1951).
[2] A. Casher, H. Neuberger and S. Nussinov, “Chromoelectric Flux Tube Model of Particle Production,” Phys. Rev. D 20, 179 (1979).
[3] H. Neuberger, “Finite Time Corrections to the Chromoelectric Flux Tube Model,” Phys. Rev. D 20, 2936 (1979).
[4] R. Brout, S. Massar, S. Popescu, R. Parentani and P. Spindel, “Quantum back reaction on a classical field,” Phys. Rev. D 52, 1119 (1995) [hep-th/9311019].
[5] P. K. Townsend, “Small Scale Structure of Space-Time as the Origin of the Gravitational Constant,” Phys. Rev. D 15, 2795 (1977).
[6] D. Amati, M. Ciafaloni and G. Veneziano, “Can Space-Time Be Probed Below the String Size?,” Phys. Lett. B 216, 41 (1989).
[7] K. Konishi, G. Paffuti and P. Provero, “Minimum Physical Length and the Generalized Uncertainty Principle in String Theory,” Phys. Lett. B 234, 276 (1990).
[8] A. Kempf, G. Mangano and R. B. Mann, “Hilbert space representation of the minimal length uncertainty relation,” Phys. Rev. D 52, 1108 (1995) [hep-th/9412167].
[9] S. Hossenfelder, M. Bleicher, S. Hofmann, J. Ruppert, S. Scherer and H. Stoecker, “Collider signatures in the Planck regime,” Phys. Lett. B 575, 85 (2003) [hep-th/0305262].

[10] S. F. Hassan and M. S. Sloth, “TransPlanckian effects in inflationary cosmology and the modified uncertainty principle,” Nucl. Phys. B 674, 434 (2003) [hep-th/0204110].

[11] B. R. Holstein, “Strong field pair production,” Am. J. Phys. 67, 499 (1999).

[12] J. H. Park, “Lecture note on Clifford algebra”.

[13] S. Haouat and K. Nouicer, “Influence of a Minimal Length on the Creation of Scalar Particles,” Phys. Rev. D 89, 105030 (2014) [arXiv:1310.6966 [hep-th]].

[14] W. G. Unruh, “Notes on black hole evaporation,” Phys. Rev. D 14, 870 (1976).

[15] S. W. Hawking, “Particle Creation by Black Holes,” Commun. Math. Phys. 43, 199 (1975) [Erratum-ibid. 46, 206 (1976)].

[16] A. Bina, S. Jalalzadeh and A. Moslehi, “Quantum Black Hole in the Generalized Uncertainty Principle Framework,” Phys. Rev. D 81, 023528 (2010) [arXiv:1001.0861 [gr-qc]].

[17] P. Chen and R. J. Adler, “Black hole remnants and dark matter,” Nucl. Phys. Proc. Suppl. 124, 103 (2003) [gr-qc/0205106].

[18] L. Xiang and X. Q. Wen, “Black hole thermodynamics with generalized uncertainty principle,” JHEP 0910, 046 (2009) [arXiv:0901.0603 [gr-qc]].

[19] M. Benrong, P. Wang and H. Yang, “Minimal Length Effects on Tunnelling from Spherically Symmetric Black Holes,” accepted to Adv. High Energy Phys.