Propagation of hyperbolic sinusoidal Gaussian beam in jet engine induced turbulence

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Abstract
This paper investigates beam evolution and beam sizes variation of hyperbolic sinusoidal Gaussian (HSG) beam propagating in jet engine exhaust-induced turbulence. We find the received field by solving Huygens-Fresnel integral with a relevant power spectrum. Our results reveal that HSG loses its initial profile until 100 m. Besides, beam with low displacement parameter value is more resistive against turbulence. In terms of beam size, beams with low Gaussian source size expand less as compared to the beams with larger Gaussian source size. In the light of our findings, optical systems in aircrafts like directed infrared counter measure (DIRCM) and laser designators can be developed.

Keywords Hyperbolic sinusoidal Gaussian beam · Jet engine · Turbulence · Exhaust · Propagation

1 Introduction

There are some non-conventional beams in the literature and hyperbolic sinusoidal Gaussian beam is one of them. HSG beam takes place in the science world in several application areas. Regarding with this, authors emphasize that Gaussian beam size and shape are adjustable with changing crystal structure (Bayraktar 2021a). The size and shape of partially coherent hyperbolic Gaussian beam will affect scintillation according to aperture (1016, j.ijleo.2021.166613. 1016). Elliptic Gaussian beams are generated when coming together and generating concentric circular rings if the coherence length decreases at a close distance (Eyyuboglu Aug 2015). Astigmatic hyperbolic sinusoidal Gaussian beam is better (more robust) to pass through turbulent flows, so that they maintain the shape better than conventional Gaussian beams or even better than non-astigmatic HSG beams (Bayraktar 2021b). In addition, HSG beam conserves its shape more in atmospheric turbulence (1016, j.ijleo.2021.167741. 1016) than the one in
oceanic turbulence (Eyyuboglu November 2013). Similarly, intensity evolution and degree of polarization of partially coherent electromagnetic hyperbolic Sine-Gaussian vortex beam are derived for non-Kolmogorov turbulence (Bayraktar May 2021). As it is statin in Failed (2021), as turbulence strength decreases, difference of radius of curvature among hyperbolic and sinusoidal beams raises. In nonlocal nonlinear medium, beam evolution period is inversely proportional to initial power (1088, 1402–4896, abce36. 1088).

Beam propagation is affected by engine plume. Engine plume causes performance degradation. Depending on jet engine turbulence, laser source coherence length must be small (Huang et al. 2015). Propagation of untraditional beams is another trending topic in the literature and a similar observation is also valid for Gaussian Schell model beams. This type of beam protects its circular shape at longer distances if the coherence parameter is small on the source plane (Eyyuboglu and Ji Oct 2010). Besides, degree of coherence of Gaussian Schell-Model beam array becomes elliptic when propagation distance increases (Shen et al. 2020). Two methods have been done to correct the scintillation index based on the data from a time-correlated single-photon counting laser radar system: the first one, meager count rates and high temporal resolution. Second one is fitting the theoretical density function for intensity fluctuations induced by propagation via turbulence (1016, j.ijle.2020.165454. 1016). An experimental study has been done with a down-scaled jet engine test probe. Operating wavelengths of nanosecond laser pulses are 1.6 μm and 3.5 μm. Cameras recorded the centroid motion while the laser spots were shown on a screen (Ghassemlooy et al. 2017). The authors states (Ding et al. 2020) that jet exhaust induced turbulence shifts beam by 0.4mrad along the propagation axis. Additionally, the standard deviation of radial beam raises gradually while the refractive index structure constant increases (Failed 2020). In (Failed 2012), numerical propagation method is derived based on experimental measurements. Similarly, it is shown in (1140), epjd, e2012-20603-x. (1140) that experimental results are close to theoretical ones with $C_n^2 = 10^{-5}m^{-2/3}$. Lastly, random phase screen approach is adapted to jet engine exhaust turbulence and Gaussian beam is propagated (Sjoqvist et al. 2004).

It is aimed to deceive or lose the threat-seeker function with directed laser on threat. Related with these two review papers are published (Isterling et al. 2005; Henriksson et al. 2008). Depending on the laser power, the Directed Infrared Counter Measure (DIRCM) can achieve jamming, dazzling or damage of the seeker. The laser sources the most critical subsystems which are capable of jamming with purpose of protecting air platforms against passive anti-missile threats. Therefore, laser radiant intensity shall be increased depending on laser output power and on the beam divergence. Untraditional beams help to get higher received power on the transverse receiver plane.

In this study, intensity evolution and beam size variation of HSG beam are studied. Receive field after jet exhaust induced turbulence is calculated Huygens-Fresnel integral. Effect of source beam parameter on beam intensity profile and beam size is discussed in detail. It provides and insight that the results obtained will be useful for future practical implementations of countermeasure or laser designator systems.
2 Derivation of received field

On transverse source plane, average intensity of hyperbolic sinusoidal beam is written as

\[
I_s(s) = u_x(s_1) u_x^*(s_2) = u_x(s_{x1}, s_{y1}) u_x^*(s_{x2}, s_{y2}) = \sinh (a_{s1}) \sinh (b_{s1}) \exp \left( -\frac{s_{x1}^2 + s_{y1}^2}{w^2} \right) \sinh (a_{s2}) \sinh (b_{s2}) \exp \left( -\frac{s_{x2}^2 + s_{y2}^2}{w^2} \right)
\]

where \( u_x \) denotes source field, \( * \) indicates complex conjugate, \( w \) is Gaussian source size, \( s_{1,2}(s_{x1,2}, s_{y1,2}) \) are transverse source plane coordinates, and \( a \) and \( b \) refer to displacement parameters, \( \langle \rangle \) indicates ensemble average. Benefiting from Huygens-Fresnel integral, average intensity on transverse receiver plane is calculated as

\[
I_r(r_x, r_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} ds_1^2 ds_2^2 I_s(s) \exp \left( -\frac{jk}{2L} (s_1 - r_1)^2 + \frac{jk}{2L} (s_2 - r_2)^2 \right) xexp\left[ \psi^*(r_1, s_1, L) + \psi(r_2, s_2, L) \right]
\]

where \( r = (r_x, r_y) \) being transverse receiver plane coordinates, \( L \) refers to propagation distance, \( k \) indicates wave number. It is equal to \( 2\pi/\lambda \) where operating wavelength \( \lambda = 4.55 \mu m \). Lastly, \( \exp[\psi^*(r_1, s_1, L) + \psi(r_2, s_2, L)] \) indicates random phase fluctuations due to exhaust turbulence and this term can be written as in Eq. 3 below.

\[
\exp[\psi^*(r_1, s_1, L) + \psi(r_2, s_2, L)] = T_1 + T_2
\]

Terms \( T_1 \) and \( T_2 \) are taken from Shen et al. (2020) as

\[
T_1 = \exp\left( -\frac{\pi^2 k^2 L A}{3 \beta^2_x} \left[ (s_{x1} - s_{x2})^2 + (s_{x1} - s_{x2})(r_{x1} - r_{x2}) + (r_{x1} - r_{x2})^2 \right] \right)
\]

\[
xexp\left( -\frac{\pi^2 k^2 L A}{3 \beta^2_y} \left[ (s_{y1} - s_{y2})^2 + (s_{y1} - s_{y2})(r_{y1} - r_{y2}) + (r_{y1} - r_{y2})^2 \right] \right)
\]

Here, \( \beta^2_x \) and \( \beta^2_y \) are anisotropic scaling factors, and \( A \) is in form of

\[
A = 0.033 C^2_n (\mu_x, \mu_y)^{5/6} \left[ \frac{5}{6} + \frac{6}{\kappa^2_{m} L_0^2} \right] \exp \left( \frac{1}{\kappa^2_{m} L_0^2} \right) \Gamma \left( \frac{1}{6}, \frac{1}{\kappa^2_{m} L_0^2} \right) - \frac{3}{5} \left( \frac{1}{L_0^2} \right)^{1/6}
\]

Here, \( C^2_n = 10^{-9} m^{-2/3} \) is the refractive index structure constant, \( \Gamma \) indicates Gamma function, \( L_0 \) is outer scale, \( \kappa_m \) refers to a parameter related with inner scale, and \( \mu_x \) and \( \mu_y \) are scaling factors. Similarly, other component of power spectrum is given as

\[
T_2 = \exp\left( -\frac{\pi^2 k^2 L B}{3} \left[ (s_{x1} - s_{x2})^2 + (s_{x1} - s_{x2})(r_{x1} - r_{x2}) + (r_{x1} - r_{x2})^2 \right] \right)
\]

\[
xexp\left( -\frac{\pi^2 k^2 L B}{3} \left[ (s_{y1} - s_{y2})^2 + (s_{y1} - s_{y2})(r_{y1} - r_{y2}) + (r_{y1} - r_{y2})^2 \right] \right)
\]

where \( B \) being
\[
B = 0.033 C_n^2 Q \left[ \frac{5 + \frac{24 \pi^2}{k^2 L_s^2}}{10 \left( \frac{1}{k_s^2} \right)^{1/6}} \exp \left( \frac{4 \pi^2}{k_s^2 L_s^2} \right) \Gamma \left( \frac{1}{6} \left( \frac{4 \pi^2}{k_s^2 L_s^2} \right) \right) - \frac{3}{5} \left( \frac{4 \pi^2}{L_s^2} \right)^{1/6} \right]
\]

(7)

where \( L_s \) and \( Q \) refer to parameters which are selected based on experimental measurements respectively. Used power spectrum to reach Eq. 4–7 is given in Eyyuboglu and Ji Oct \((2010)\). After tedious calculations, the received average intensity is calculated as below

\[
I_r(r_x, r_y, L) = \frac{\pi}{\sqrt{AB}} \exp \left( \frac{L^2 (a^2 + b^2) - k^2 \left( r_x^2 + r_y^2 \right) - 2jkL(ar_x - br_y)}{L(jkw^2 - 2L - 2Lw^2 \rho_t^2)} \right) w^2
\]

(8)

\[
x \left[ \exp(-H_1) \sinh(H_2) \right] \left( \exp(-G_1) \sinh(G_2) - \exp(-G_3) \right) + \left( \exp(-F_1) \sinh(F_2) \right) \left( \exp(-K_1) \sinh(K_2) - \exp(-K_3) \right)
\]

where \( \rho_t^2 \) is the inverse of summation of the constants in front of transverse plane coordinates in Eq. 4 and 6 and newly defined variables are defined as

\[
\exp(-H_1) = -\frac{C_4^2 L^2 + b^2 L^2 E^2 + kE_r(-2jC_4 L - kE_r)}{LE\left(4\rho_t^2 L + 2jkE - \frac{4EL}{\rho_t^2}\right)}
\]

(9)

\[
H_2 = -\frac{2C_4 bL^2 E^2 + 2jkLbE^2 r_y}{LE\left(4\rho_t^2 L + 2jkE - \frac{4EL}{\rho_t^2}\right)}
\]

(10)

\[
\exp(-G_1) = -\frac{C_2^2 L^2 + a^2 L^2 D^2 + kDr_x(-2jC_2 L - kDr_x)}{LD\left(4\rho_t^2 L + 2jkD - \frac{4DL}{\rho_t^2}\right)}
\]

(11)

\[
G_2 = -\frac{2C_5 bL^2 D^2 + 2jkLbD^2 r_x}{LD\left(4\rho_t^2 L + 2jkD - \frac{4DL}{\rho_t^2}\right)}
\]

(12)

\[
\exp(-G_3) = -\frac{C_1^2 L^2 + b^2 L^2 D^2 + kDr_x(-2jaLD - 2jC_1 L - kDr_x) + aL^2 D^2(a + 2C_1)}{LD\left(4\rho_t^2 L + 2jkD - \frac{4DL}{\rho_t^2}\right)}
\]

(13)

\[
\exp(-F_1) = -\frac{C_3^2 L^2 + b^2 L^2 E^2 + kEr_y(-2jC_3 L - kEr_y)}{LE\left(4\rho_t^2 L + 2jkE - \frac{4EL}{\rho_t^2}\right)}
\]

(14)
\[ F_2 = -\frac{2C_3bL^2E^2 + 2jkLbE^2r_y}{LE\left(4\rho_1^2L + 2jkE - \frac{4EL}{\rho_1^2}\right)} \] (15)

\[ \exp(-K_1) = -\frac{C_1^2L^2 + \alpha^2L^2D^2 + kDr_x(-2jC_1L - kDr_x)}{LD\left(4\rho_1^2L + 2jkD - \frac{4DL}{\rho_1^2}\right)} \] (16)

\[ K_2 = -\frac{2C_3bL^2D^2 + 2jkLbD^2r_x}{LD\left(4\rho_1^2L + 2jkD - \frac{4DL}{\rho_1^2}\right)} \] (17)

\[ \exp(-K_3) = -\frac{C_2^2L^2 + \alpha^2L^2D^2 + kDr_x(-2jC_2L - kDr_x)}{LD\left(4\rho_1^2L + 2jkD - \frac{4DL}{\rho_1^2}\right)} \] (18)

\[ C_1 = a - \frac{jkr_x}{L} \] (19)

\[ C_2 = -a - \frac{jkr_x}{L} \] (20)

\[ C_3 = b - \frac{jkr_y}{L} \] (21)

\[ C_4 = -b - \frac{jkr_y}{L} \] (22)

and

\[ D = E = -\left(\frac{2L - jkw^2}{2Lw^2}\right) \] (23)

After received field is obtained, the beam size is evaluated benefiting from average intensity as (Sirazetdinov 2008)

\[ \alpha_r^2 = \frac{4\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} r_x^2 u_r(r_x, r_y, L) u_r^*(r_x, r_y, L) dr_x dr_y}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u_r(r_x, r_y, L) u_r^*(r_x, r_y, L) dr_x dr_y} \] (24)

3 Results and discussions

In this section, properties of an HSG beam propagating through jet engine induced turbulence are illustrated by numerical calculations based on formulas derived above. The laser transversal section at the laser waist is defined by two hyperbolic functions respective to
the beam axis. The propagation distance is a measure along the beam axis, and each one of the images shows a transversal cut showing the beam intensity distribution at the values of $L$. The figures show the normalized intensity distribution profiles with a one-dimensional representation of the contour plots of the HSG beams propagating through jet engine exhaust turbulence for two beams with a complimentary beam configuration. Figure 1 is given as model of propagation in jet engine induced turbulence.

To start with, intensity distribution of HSG beam with $a = 10^{-2}m^{-1}$, $b = 10^{-2}m^{-1}$, and $w = 1cm$ at $a L = 10$ m, $b L = 30$ m, and $c L = 100$ m is given in Fig. 2. Closed loop intensity distribution with slight edges and smooth corners attract the attention in Fig. 2 subplot a. Intensity is higher along the edges lying on minima and maxima of y-axis. There is an elliptical hollow in the center.
and this hollow becomes more differentiable and symmetric as propagation distance increases in subplot b and c. It seems that beam enlarges more along y-axis. Similarly, Gaussian beam width is set as 1.5 cm in Fig. 3. Here, observed intensity distribution in subplot a is like in Fig. 2. On the other hand, beam expands less as the propagation distance raises as in subplot b and c. In Fig. 4, we plot intensity distribution of HSG beam with $a = 50m^{-1}$, $b = 10^{-2}m^{-1}$, and $w = 1cm$. We see that closed loop beam profile is broken from negative edge of x-axis despite beam propagates short distance as in subplot a. When propagation distance increases in subplots b and c, beam expands more as compared to the previous settings. As opposed to this, intensity profile of HSG beam with $a = 10^{-2}m^{-1}$, $b = 50m^{-1}$, and $w = 1cm$ along negative y-axis vanishes even it propagates short distance as in subplot a of Fig. 5. When propagation distance becomes 100 m in subplot c, beam evolves and bright part of the intensity is placed along negative y-axis due to the strong refractive index fluctuations.

The difference in atmosphere is due to the change of media indices for optical elements causing the aerosols to lose their homogeneity, and particles and molecules interacting with the laser beam causing light scattering and may undergo some Raman scattering. The atmosphere, which becomes more complex with the movement of aircraft and the vortex structure behind it, makes it difficult for the laser to advance in the medium. With the change of media indices, distortions in the waveform begin. These distortions cause spherical aberration independent of wavelength. The laser hits aerosols of varying media index, causing turbulence.

**Fig. 4** Intensity distribution of hyperbolic sinusoidal Gaussian beam with $a = 50m^{-1}$, $b = 10^{-2}m^{-1}$, and $w = 1cm$ at a $L = 10$ m, b $L = 30$ m, and c $L = 100$ m

**Fig. 5** Intensity distribution of hyperbolic sinusoidal Gaussian beam with $a = 10^{-2}m^{-1}$, $b = 50m^{-1}$, and $w = 1cm$ at a $L = 10$ m, b $L = 30$ m, and c $L = 100$ m
To analyze the properties in a turbulent environment depending on the beam source, we performed the beam size variation as a function of propagation distance in Fig. 6. It is understood from the graph that beam with large Gaussian source size expands less compared to the beams having low Gaussian source size. In other point of view, raise in displacement parameter, regardless of which one is, brings us more divergent beam. To obtain more concentric beam, Gaussian source size should be high and displacement parameter should be low.

4 Conclusions

We analyze the propagation properties of HSG beam in jet engine exhaust-induced turbulence. Intensity evolution and change in beam size are plotted by considering the effect of source beam parameters. Huygens-Fresnel integration is applied to find the received field. Our results indicate that HSG beam with low $a$ and $b$ parameter conserves its shape at longer distances. In addition, raise in displacement parameter brings intensity vanish along the corresponding axis of increased parameter. To reach a smaller beam size, at the receiver plane, a small displacement parameter should be selected on the transverse source plane. The slope of beam size for HSG beam with large Gaussian source size is less as compared to the one having low source size. We anticipate that these results will be beneficial to design laser systems in aircrafts.

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Declarations

Conflict of interest The authors declare there is no conflicts of interests.
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