Dynamical Peccei-Quinn symmetry breaking and the instanton interference effect: axion models without domain wall problem

Mario Reig1,*

1 AHEP Group, Institut de Física Corpuscular – CSIC/Universitat de València, Parc Científic de Paterna.
C/ Catedrático José Beltrán, 2 E-46980 Paterna (Valencia) - SPAIN

We show that a new chiral, confining interaction can be used to break Peccei-Quinn symmetry dynamically and solve the domain wall problem, simultaneously. The resulting theory is an invisible QCD axion model without domain walls. No dangerous heavy relics appear.

1. INTRODUCTION

The appearance of topological defects during spontaneous breaking of symmetries constitutes a clear and profound connection between particle physics and cosmology [1]. As the Universe cools down several phase transition take place and, depending on the homotopy groups of the manifold of degenerate vacua, stable topological defects may form [2].

In particular, the cosmic domain wall problem [3] is a well-known potential issue of axion models [4, 5]. Recently, it has been pointed out that Majoron models can also suffer from domain walls [6]. To solve such a long-standing problem, several mechanisms have been proposed. Being a cosmological-particle physics issue, it is not surprising that one can tackle it from both, cosmology and particle physics sides. A couple of well known solutions are: cosmic inflation and the Lazarides-Shafi mechanism [7]. In the first one the dangerous walls are pushed beyond the horizon, being a clear example of a cosmological solution. In the second case, one associates the spontaneously broken discrete symmetry to a gauge symmetry. This removes the physical degeneracy among the different vacua, which become gauge equivalent. Another interesting solution that has been recently suggested [8] implements the Witten effect to solve the domain wall problem. More exotic post-inflationary solutions involve primordial black holes to perforate the walls, change their topology and destroy them [9].

As noted by Holdom [10], the cosmic domain wall problem seems to be associated to the breaking of symmetries by scalars. One can imagine that the degeneracy of the associated vacua disappears for theories where the breaking of Peccei-Quinn (PQ) symmetry is dynamically triggered by new confining forces. In addition, in usual invisible axion models [11–14], the introduction of a SM singlet condensate \( \langle \sigma \rangle \) breaking PQ symmetry at high energies generates a fine-tuning problem. The bare mass terms of the Higgs isodoublets, \( \mu_{u,d}^2 |H_{u,d}|^2 \), have to cancel almost perfectly (with EW scale precision) quartic couplings of the type \( \lambda |\sigma|^2 |H_{u,d}|^2 \). This is nothing but the standard, well-known hierarchy problem of axion models. We conjecture that the solution of both problems is intimately related. The reason is that dynamical breaking of symmetries by fermion condensates usually brings associated instanton effects, which explicitly break anomalous symmetries.

Dynamical breaking of PQ symmetry has a long history in the context of composite axion models [15] and has gained interest recently [16–22]. New strongly coupled and confining interactions have been also reconsidered recently to rise the QCD axion mass [23–29].

In this Letter we build a minimal model where the breaking of PQ symmetry arises dynamically at a high scale thanks to a new chiral confining force. In addition, we show how the associated instantons implement the "instanton interference effect", solving the domain wall problem. A similar construction has been explored by Barr and Kim [30]. In this reference it was suggested that new confining interactions can solve the domain wall problem. However, despite it avoids the domain wall problem with a \( N_{DW} = 1 \) scenario, it does illustrate the appearance of a phenomenological and cosmological problem, namely the overclosing of the Universe by heavy stable relics. This issue seems almost unavoidable in the context of confining interactions, since they usually bring associated conserved quantum numbers. Baryon number in the Standard Model is the most clear example.

*Electronic address: mario.reig@ific.uv.es
If the lightest of these unconfined bound states, which we will call hyperbaryons, is stable it might overclose the Universe depending on its mass. Following Griest and Kamionkowski [31], this limit is given by:

\[ \Lambda_{HC} \leq 240 \text{ TeV}. \]  

(1)

Dangerous heavy relics can be diluted after a period of cosmic inflation. However, in such a case one might ask why we do not also use inflation to avoid the domain wall problem. We will show below which are the basic ingredients to achieve a phenomenologically successful solution to all these problems.

2. THE MODEL

Let us consider a model based on the symmetry:

\[ G \times U(1)_{PQ} \times SM, \]  

(2)

with the SM factor being the local symmetry of the Standard Model, \( SU(3)_C \times SU(2)_L \times U(1)_Y \), and \( G \) a new confining gauge group. \( U(1)_{PQ} \) is the global, anomalous PQ symmetry. All SM particles are \( G \) singlets. For simplicity, we assume that the PQ charges for SM fermions are given by fermion chirality, this is +1 for left-handed (LH) fermions and −1 for right-handed (RH) fermions. This assignment is compatible with the original PQWW axion model [33–35]. In the \( G \) sector, we assume one of the fermions has PQ charge +1 while the other has no PQ charge. The reason for this will become clear later. Therefore, the fermion content of the model is given by:

\[
\begin{align*}
q_L^i &\sim (1, 3, 2, 1/6)_1, \quad l_L^i \sim (1, 1, 2, −1/2)_1, \\
\nu_R^i &\sim (1, 3, 1, 2/3)_{−1}, \quad d_R^i \sim (1, 3, 1, −1/3)_{−1}, \\
\psi_1 &\sim (R, 1, 1, 0)_1, \quad \psi_2 \sim (R, 1, 1, 0)_0, \\
\end{align*}
\]  

(3)

where the first quantum number stands for \( G \) and the subscript is the PQ charge. The distinction between \( \psi_1 \) and \( \psi_2 \) requires the coupling to different scalar fields.

It can be seen that \( U(1)_{PQ} \) symmetry is anomalous under both, QCD and \( G \). This is a reasonable assumption, since it seems rather artificial to protect an anomalous symmetry from anomalies of another gauge group. The first question that arises is which kind of groups are appropriate for \( G \). Many possibilities emerge. However, one has to deal with the limitations coming from the \( G \) triangle anomaly. As we will see below, \( G = SO(4N + 2) \), for \( N \geq 2 \), suggest themselves as the most natural choice. Strikingly enough, they admit complex, chiral representations and are anomaly free. The use of spinor representations will also be important to solve the heavy relic problem. From now on, we will assume the \( G \) gauge group is given by the well known group:

\[ G = SO(10)_{HC}, \]  

(4)

and we will refer to it as hypercolor (HC). The HC fermions are, then, a couple of \( SO(10)_{HC} \) spinors, \( \psi_1 \sim (16, 1, 1, 0)_1 \) and \( \psi_2 \sim (16, 1, 1, 0)_0 \). The scalars required to make a difference between them can be HC vectors, \( H_{1,2} \sim (10, 1, 1, 0)_{2,0} \). Therefore, the Yukawa lagrangian of the HC sector is given by:

\[ \mathcal{L}_{HC} = y_1 \psi_1 H_1 \psi_1 + y_2 \psi_2 H_2 \psi_2. \]  

(5)

Additionally, we assume that \( H_2 \) has an inverted phase transition as in [30]. This inverted phase transition is characterized by a non-zero vacuum expectation value (vev), \( \langle H_2 \rangle \neq 0 \), above a critical cosmic temperature \( T \geq T_c \). Below this temperature, \( T \leq T_c \), the vev vanishes \( \langle H_2 \rangle = 0 \) and a chiral symmetry associated to \( \psi_2 \) emerges. It was shown by Weinberg [36] that this kind of phase transitions can exist. We also require that the HC confinement scale is larger than the critical temperature, \( \Lambda_{HC} \geq T_c \gg \Lambda_{QCD} \) so that PQ and the chiral symmetry of \( \psi_2 \) do not coexist as classical symmetries of the lagrangian. As we will see later, \( \psi_1 \) also gets a non-zero mass from HC dynamics.

As in the \( SU(N) \) family, \( SO(N) \) gauge theories have non-trivial vacuum for \( N \geq 3 \). This can be seen from their non-trivial homotopy group, e.g. \( \pi_3(SO(N)) = \mathbb{Z} \) (with the exception \( \pi_3(SO(4)) = (\mathbb{Z})^2 \)). There are two different \( \theta \)-terms, one for QCD and one for HC. Since we are imposing a unique PQ symmetry it might seem that we are not solving the strong CP problem, as one usually needs the same number of anomalous symmetries than \( \theta \)-terms or confining interactions. However, being a chiral representation, the spinor does not allow a bare mass term. This implies an extra anomalous \( U(1) \) symmetry below \( T_c \), when the mass of \( \psi_2 \) is turned off. Under this

\[ \text{This particular choice is compatible with an underlying } SO(10) \text{ GUT [32].} \]
symmetry, the HC spinor transforms as:
\[ \psi_2 \rightarrow e^{i\alpha} \psi_2. \] (6)

The other particles in the model, see eq. 3, do not transform under this \(U(1)\) symmetry. This works in analogy to the massless quark solution to the strong CP problem, helping to reduce the extra \(\theta\)-term. This fact renders \(\theta_{HC}\) unphysical while \(\theta_{QCD}\) is driven dynamically to zero by the standard PQ mechanism. This situation resembles the one proposed by Barr and Kim in their work [30].

3. THE INSTANTON INTERFERENCE EFFECT

There are two different sources of explicit breaking of PQ symmetry as one can see from \([SO(10)_{HC}]^2 \times U(1)_{PQ}\) and \([SU(3)_c]^2 \times U(1)_{PQ}\) anomalies. QCD and HC instantons break \(U(1)_{PQ}\) explicitly. The breaking is done in the direction of \(Z_{NHC}\) and \(Z_{NQCD}\) discrete subgroups, with \(N_{HC}\) and \(N_{QCD}\) the anomaly coefficients. This may lead to an interesting situation where the explicit breaking is not in the direction of the same subgroup. The residual discrete symmetry unbroken by the combination of non-perturbative effects will be the common subgroup of both, \(Z_{NHC}\) and \(Z_{NQCD}\). This is what we call instanton interference and can be pictorially visualized in figure 1. If the anomaly coefficients \(N_{HC}\) and \(N_{QCD}\) are co-prime numbers, the instanton interference effect solves the domain wall problem, since \(Z_{NHC}\) and \(Z_{NQCD}\) have no common subgroup\(^2\). The anomaly coefficients are given by
\[ N = \sum_R q_R t_R, \] (7)
with \(t_R\) the Dynkin index of the representation \(R\), defined in terms of the group generators as \(\text{Tr}[T^a T^b] = t_R \delta^{ab}\), and \(q_R\) the PQ charge of \(R\). A straightforward calculation give us \(N_{HC} = 2\).

3.1. Scalar potential and \(N_{QCD}\)

The scalar potential of the model described above is relatively simple. Since the model is PQWW-like in the SM sector, only a couple of Higgs isodoublets with opposite PQ charges are present\(^3\):
\[ V(\phi_u, \phi_d) = \mu_u^2 |\phi_u|^2 + \mu_d^2 |\phi_d|^2 + \lambda_u |\phi_u|^4 + \lambda_d |\phi_d|^4 + \lambda_1 |\phi_u|^2 |\phi_d|^2 + \lambda_2 (\phi_u \phi_d)(\phi_u \phi_d) + h.c. \] (8)

The degeneracy of the vacuum is determined by the gauge invariant order parameters. This potential, in combination with the anomaly coefficient computed as dictated by Eq. 8, reveals a \(Z_{NQCD} = Z_3\) symmetry among the different vacua. Since \(N_{HC} = 2\) and \(N_{QCD} = 3\) are co-prime numbers, the instanton interference effect solves the domain wall problem.

3.2. The \(\eta_{HC}\) and the axion

The \(\theta_{HC}\) term is not physical because can be rotated away by a \(\psi_2\) chiral rotation, which is independent of the PQ transformation (see Eq. 6). In the dual description, after confinement, this \(\theta\)-term is relaxed dynamically by the \(\eta_{HC}'\), which acquires a mass of the order \(\Lambda_{HC}\) and decouples from the low-energy theory. Since below \(T_c\) there is a massless fermion in the \(SO(10)_{HC}\) sector, non-perturbative effects of this interaction do not generate an effective potential for the axion field\(^4\) (see for example [37]). The axion potential coming from HC instantons turns off below \(T_c\) and only QCD effects generate a mass for the axion, giving the usual QCD prediction [38]:
\[ m_a^2 \sim \frac{m_u m_d}{(m_u + m_d)^2} \frac{f_a^2 m_a^2}{f_a^2}. \] (9)
As we will see below, \(f_a\) is also identified with \(\Lambda_{HC}\), since the condensate \(\langle \psi_1 \psi_1 \rangle \sim \Lambda_{HC}^3\) breaks PQ symmetry spontaneously. The HC scale, \(\Lambda_{HC} \sim f_a\), is presumably very large making the QCD axion naturally invisible.

4. AVOIDING HEAVY STABLE RELICS

New confining interactions can have a non-trivial cosmological impact. For \(SU(N)\) gauge groups, a global

\(^2\) The scenario proposed by Barr and Kim [30] is the particular case of instanton interference with \(N_{HC} = 1\).

\(^3\) We do not consider, at this point, terms involving \(H_1\) and \(H_2\) since they are not relevant for the determination of \(Z_{NQCD}\).

\(^4\) Note, however, that despite the effective potential is not generated PQ symmetry is explicitly broken by HC instantons.
FIG. 1: QCD instantons break $U(1)_{PQ}$ down to a $Z_3$ symmetry (red circles) while HC instantons break it to a $Z_2$ (blue circles). It can be easily seen that they can only coincide in one point (mod $2\pi$), meaning that there is no common discrete subgroup.

$U(1)$ symmetry analogous to baryon number protects the lightest hyperbaryon from decaying. As an example, a $SU(N)$ theory with a fermion in the fundamental $N$ representation has a stable hyperbaryon composed by $N$ fermions. Analogously, $SO(N)$ groups feature a $Z_2$ conserved quantum number. This $Z_2$ symmetry counts the number of indices for the fundamental representation and can protect the stability of the lightest confined state. These group theoretic properties have been used to stabilize dark matter [39]. Another example of exotic matter stabilized by the conserved $Z_2$ symmetry of an $SO(N)$ confining interaction are hyperbaryons in the context of Comprehensive Unification [40].

One can naively think that because $SO(10)_{HC}$ is confining, there will be stable hyperbaryons. However, this is not the case. When the interaction becomes strongly coupled, HC dynamics will form the condensate:

$$\langle \psi \psi \rangle \sim \Lambda_{HC}^3.$$ (10)

As mentioned before, this condensate breaks $U(1)_{PQ}$ spontaneously. Additionally, the condensate has also non-trivial implications in the dynamics of $SO(10)_{HC}$ itself. Here we briefly describe the situation. In the single gauge boson exchange approximation the potential between two $\psi$ is given by:

$$V \sim \frac{g_{HC}^2}{2r} [C_c - C_{16} - C_{16}],$$ (11)

with $C_{16}$ and $C_c$ the Casimir invariants of the spinor and the possible representations of the condensate. Interestingly this condensate does not contain an $SO(10)$ singlet since:

$$16 \times 16 = 16_a + 120_a + 126_s.$$ (12)

Therefore, the condensate must break $SO(10)_{HC}$ gauge symmetry. This is done following the most attractive channel (MAC) rules [41]. Since the only attractive channel is in the 10 representation direction, which contains a $SO(9)$ singlet, the symmetry breaking reads:

$$SO(10) \to SO(9), \quad 16 \to 16.$$ (13)

The strongly coupled $SO(9)$ interaction, again, rearrange the vacuum and form condensates $\langle \psi \psi \rangle$. However, this condensate now contains an $SO(9)$ singlet, since:

$$16 \times 16 = 1_s + 9_s + 36_a + 84_a + 126_s.$$ (14)

The SSB chain of the confining interaction stops at this point and the $SO(9)$ remains unbroken. Interestingly, this condensate introduces a linear term in the potential for $H_1$:

$$V = -y_1 \langle \psi_1 \psi_1 \rangle H_1 + \mu^2 |H_1|^2 + \lambda |H_1|^4.$$ (15)

In the limit $\lambda \ll 1$, the condensate induces a non-zero vev for $H_1$, $\langle H_1 \rangle = y_1 \langle \psi_1 \psi_1 \rangle / \mu^2$, generating a perturbative mass for $\psi_1$ from HC dynamics.

We are now about to show why no dangerous heavy relics emerge in our framework. For $SO(N)$ with $N$ odd there are two conjugacy classes. The conjugacy classes of the spinor and singlet representations are $C = 1$ and $C = 0$, respectively. Since the product of representations decomposes into representations with the same class (mod 2), only products of an even number of spinors can give us $SO(9)$ singlets. Therefore, since $SO(9)$ supports a conserved $Z_2$ quantum number instead of $U(1)$, no stable $SO(9)$ singlet can appear. All the possible hyperbaryons, i.e. $SO(9)$ singlet bound states, are $Z_2$ singlets and decay.
5. HC COUPLING RUNNING AND CONFINEMENT

We have assumed that the HC interaction becomes strongly coupled at high energies. For the sake of completeness, let us study an explicit example quantitatively. If some kind of new physics like supersymmetry or new scalars is responsible of the unification of gauge couplings at high energies, it is attractive to imagine that the HC interaction is also unified with the other SM interactions at around $M_{\text{GUT}} \sim 10^{16}$ GeV. At one loop and neglecting threshold corrections, the evolution of the couplings is governed by:

$$\alpha^{-1}(\mu) = \alpha^{-1}(M) + \frac{1}{6\pi} \ln \frac{M}{\mu} \left[ -11 C_2(G) + 2 T(R) \right],$$

with $C_2(SO(N)) = N - 2$, and $T($spinor$) = 2N/2 - 4$. With two HC spinors and no light scalars, this give us:

$$\alpha_{HC}^{-1}(\mu) = \alpha_{HC}^{-1}(M) - \frac{80}{6\pi} \ln \frac{M}{\mu}. \quad (17)$$

Taking a supersymmetric example, if $\alpha_{HC}(M_{\text{GUT}}) = 1/28$ and $M_{\text{GUT}} = 10^{16}$ GeV, one can estimate the scale $\Lambda_{HC}$ at which the HC coupling becomes strong as:

$$\Lambda_{HC} \approx e^{-27 \times 6\pi / 80} \times 10^{16}. \quad (18)$$

We obtain $\Lambda_{HC} \approx 1.7 \times 10^{13}$ GeV, which lies close to the upper bound of the axion window (see Fig. 2). The HC scale can be lowered if $\alpha(M_{\text{GUT}}) < 1/28$ or if there is extra matter in the HC sector below the GUT scale, making the running of the coupling slower. Then, one can easily obtain a HC scale $\Lambda_{HC}$ inside the QCD axion window.

Obviously, the coincidence of all interaction strengths in one point of the $(\alpha^{-1}, E)$ plane is meaningless unless there is an underlying unified gauge group in the UV. The reason is that if there is no unified group containing HC and the SM, keeping the coupling evolution together above $M_{\text{GUT}}$, the gauge couplings will separate as can be seen in Fig. 2. However, we believe that this example does illustrate the principles of model building.

6. DISCUSSION

Before closing we comment on different aspects of the model that deserve mention:

- In an hypothetical $SO(10)_{HC} \times U(1)_{PQ} \times SO(10)$ theory, the unbroken group by the instanton interference mechanism is a $Z_2$ symmetry that can be automatically associated to the center of both $SO(10)$ groups. Then, the Lazarides-Shafi mechanism [7] can be naturally implemented without adding extra fermions.

- It is attractive to imagine that some sort of interaction with the condensate $\langle \psi \bar{\psi} \rangle$ can generate small neutrino masses. A plausible possibility is to use $\psi$ in a radiative mechanism, together with the appropriate scalars, in close analogy to the mechanism presented in [42].

7. CONCLUSIONS

New chiral confining interactions with fermions in the spinor representation can simultaneously make the axion invisible and solve the domain wall problem. The instanton interference effect has been described in detail. If PQ symmetry remains unbroken during inflation, the instanton interference mechanism suggests itself as a compelling possibility to avoid the domain wall problem, combining different sources of explicit breaking by instantons. Finally, spinor representations of chiral, anomaly free groups turn out to be the crucial ingredient to explain the absence of heavy stable relics. This fact strongly suggest them as compelling candidates for the confining interaction of composite axion models.

Acknowledgments

I am especially grateful to R. Fonseca for helpful discussions about group theory and P. Agrawal for discussions and insightful comments during the early stages of this work. I am also grateful to M. Yamada for enlightening discussions about a preliminary version of the manuscript. I would also like to thank Stockholm University and the organizers of the "Quantum Connections: Axions in Stockholm" workshop, where this work started, for hospitality. This work is supported by the Spanish grants FPA2017-85216-P (AEI/FEDER, UE), SEV-2014-0398 and PROMETEO/2018/165 (Generalitat Valenciana).
FIG. 2: Running of gauge couplings in a supersymmetric scenario. HC gauge coupling corresponds to the dashed black line. See text.

[1] A. Vilenkin, “Cosmic Strings and Domain Walls,” Phys. Rept. 121 (1985) 263–315.
[2] T. W. B. Kibble, “Topology of Cosmic Domains and Strings,” J. Phys. A9 (1976) 1387–1398.
[3] Ya. B. Zeldovich, I. Yu. Kobzarev, and L. B. Okun, “Cosmological Consequences of the Spontaneous Breakdown of Discrete Symmetry,” Zh. Eksp. Teor. Fiz. 67 (1974) 3–11. [Sov. Phys. JETP40,1(1974)].
[4] P. Sikivie, “Of Axions, Domain Walls and the Early Universe,” Phys. Rev. Lett. 48 (1982) 1156–1159.
[5] A. Vilenkin and A. E. Everett, “Cosmic Strings and Domain Walls in Models with Goldstone and PseudoGoldstone Bosons,” Phys. Rev. Lett. 48 (1982) 1867–1870.
[6] G. Lazarides, M. Reig, Q. Shafi, R. Srivastava, and J. W. F. Valle, “Spontaneous Breaking of Lepton Number and Cosmological Domain Wall Problem,” arXiv:1806.11198 [hep-ph].
[7] G. Lazarides and Q. Shafi, “Axion Models with No Domain Wall Problem,” Phys. Lett. 115B (1982) 21–25.
[8] R. Sato, F. Takahashi, and M. Yamada, “Unified Origin of Axion and Monopole Dark Matter, and Solution to the Domain-wall Problem,” Phys. Rev. D98 no. 4, (2018) 043535, arXiv:1805.10533 [hep-ph].
[9] D. Stojkovic, K. Freese, and G. D. Starkman, “Holes in the walls: Primordial black holes as a solution to the cosmological domain wall problem,” Phys. Rev. D72 (2005) 045012, arXiv:hep-ph/0505026 [hep-ph].
[10] B. Holdom, “DOMAIN WALLS. 1. AXION MODELS,” Phys. Rev. D27 (1983) 332–338.
[11] A. R. Zhitnitsky, “On Possible Suppression of the Axion Hadron Interactions. (In Russian),” Sov. J. Nucl. Phys. 31 (1980) 260. [Yad. Fiz.31,497(1980)].
[12] M. Dine, W. Fischler, and M. Srednicki, “A Simple Solution to the Strong CP Problem with a Harmless Axion,” Phys. Lett. 104B (1981) 199–202.
[13] J. E. Kim, “Weak Interaction Singlet and Strong CP Invariance,” Phys. Rev. Lett. 43 (1979) 103.
[14] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, “Can Confinement Ensure Natural CP Invariance of Strong Interactions?,” Nucl. Phys. B166 (1980) 493–506.
[15] J. E. Kim, “A COMPOSITE INVISIBLE AXION,” Phys. Rev. D31 (1985) 1733.
[16] M. Yamada, T. T. Yanagida, and K. Yonekura, “Cosmologically safe QCD axion without fine-tuning,” Phys. Rev. Lett. 116 no. 5, (2016) 051801, arXiv:1510.06504 [hep-ph].
[17] F. Wilczek and G. Moore, “Superheavy Light Quarks and the Strong P, T Problem,” arXiv:1601.02937 [hep-ph].
[18] M. Redi and R. Sato, “Composite Accidental Axions,” JHEP 05 (2016) 104, arXiv:1602.05427 [hep-ph].
[19] M. K. Gaillard, M. B. Gavela, R. Houtz, P. Quilez, and R. Del Rey, “Color unified dynamical axion,” Eur. Phys. J. C78 no. 11, (2018) 972, arXiv:1805.06465 [hep-ph].
[20] H.-S. Lee and W. Yin, “Peccei-Quinn symmetry from a hidden gauge group structure,” arXiv:1811.04039 [hep-ph].
[21] P. Anastasopoulos, P. Betzios, M. Bianchi, D. Consoli, and E. Kiritsis, “Emergent/Composite axions,” arXiv:1811.05940 [hep-ph].
[22] M. B. Gavela, M. Ibe, P. Quilez, and T. T. Yanagida,
“Automatic Peccei-Quinn symmetry,”
arXiv:1812.08174 [hep-ph].

[23] V. A. Rubakov, “Grand unification and heavy axion,”
*JETP Lett.* **65** (1997) 621–624, arXiv:hep-ph/9703409
[hep-ph].

[24] A. Hook, “Anomalous solutions to the strong CP problem,”
*Phys. Rev. Lett.* **114** no. 14, (2015) 141801,
arXiv:1411.3325 [hep-ph].

[25] H. Fukuda, K. Harigaya, M. Ibe, and T. T. Yanagida,
“Model of visible QCD axion,” *Phys. Rev. D* **92** no. 1,
(2015) 015021, arXiv:1504.06084 [hep-ph].

[26] T. Gherghetta, N. Nagata, and M. Shifman, “A Visible QCD Axion from an Enlarged Color Group,”
*Phys. Rev. D* **93** no. 11, (2016) 115010, arXiv:1604.01127 [hep-ph].

[27] S. Dimopoulos, A. Hook, J. Huang, and
G. Marques-Tavares, “A collider observable QCD axion,”
*JHEP* **11** (2016) 052, arXiv:1606.03097 [hep-ph].

[28] P. Agrawal and K. Howe, “A Flavorful Factoring of the Strong CP Problem,”
arXiv:1712.05803 [hep-ph].

[29] P. Agrawal and K. Howe, “Factoring the Strong CP Problem,”
*Submitted to: JHEP* (2017),
arXiv:1710.04213 [hep-ph].

[30] S. M. Barr and J. E. Kim, “New Confining Force Solution of the QCD Axion Domain-Wall Problem,”
*Phys. Rev. Lett.* **113** no. 24, (2014) 241301,
arXiv:1407.4311 [hep-ph].

[31] K. Griest and M. Kamionkowski, “Unitarity Limits on the Mass and Radius of Dark Matter Particles,”
*Phys. Rev. Lett.* **64** (1990) 615.

[32] A. Ernst, A. Ringwald, and C. Tamarit, “Axion Predictions in SO(10) × U(1)νQ Models,”
*JHEP* **02** (2018) 103, arXiv:1801.04906 [hep-ph].

[33] R. D. Peccei and H. R. Quinn, “CP Conservation in the Presence of Instantons,”
*Phys. Rev. Lett.* **38** (1977) 1440–1443. [328(1977)].

[34] F. Wilczek, “Problem of Strong p and t Invariance in the Presence of Instantons,”
*Phys. Rev. Lett.* **40** (1978) 279–282.

[35] S. Weinberg, “A New Light Boson?,”
*Phys. Rev. Lett.* **40** (1978) 223–226.

[36] S. Weinberg, “Gauge and Global Symmetries at High Temperature,”
*Phys. Rev. D* **9** (1974) 3357–3378.

[37] T. Schfer and E. V. Shuryak, “Instantons in QCD,”
*Rev. Mod. Phys.* **70** (1998) 323–426,
arXiv:hep-ph/9610451 [hep-ph].

[38] G. Grilli di Cortona, E. Hardy, J. Pardo Vega, and
G. Villadoro, “The QCD axion, precisely,”
*JHEP* **01** (2016) 034, arXiv:1511.02867 [hep-ph].

[39] O. Antipin, M. Redi, A. Strumia, and E. Vigiani,
“Accidental Composite Dark Matter,”
*JHEP* **07** (2015) 039, arXiv:1503.08749 [hep-ph].

[40] M. Reig, J. W. F. Valle, C. A. Vaquera-Araujo, and
F. Wilczek, “A Model of Comprehensive Unification,”
*Phys. Lett.* **B774** (2017) 667–670, arXiv:1706.03116 [hep-ph].

[41] S. Raby, S. Dimopoulos, and L. Susskind, “Tumbling Gauge Theories,”
*Nucl. Phys.* **B169** (1980) 373–383.

[42] M. Reig, D. Restrepo, J. W. F. Valle, and O. Zapata,
“Bound-state dark matter with Majorana neutrinos,”
arXiv:1806.09977 [hep-ph].