The Effect of the non-singular T-stress components on crack tip plastic zone under mode I loading

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Abstract

Theoretical and numerical analysis of the joint influence of the non-singular components of the T-stresses on the crack tip plastic zone estimated by the von Mises yield criterion is carried out under mode I loading in connection with specimen thickness. Calculations are performed for three thicknesses of the CT specimen. The amplitudes of the non-singular terms in the three-dimensional series expansion of the crack front stress field are the terms $T_{xx}$ and $T_{zz}$ which describe in-plane and out-of-plane constraint, respectively. The plastic zone is analyzed for various combinations of the terms $T_{xx}$ to $T_{zz}$ ratios. The plastic zone size distribution is mapped in the specimen thickness direction. It is shown that the plastic zone is affected by the $T_{zz}$-stress, i.e. there is strong effect of out-of-plane constraint on crack tip plastic zones. Crack tip plastic zone in the middle plane of the specimen decreases with the increase of specimen thickness.

Keywords: Plastic zone, $T_{xx}$ and $T_{zz}$-stress, mode I crack

1. Introduction

The different sources of a change in in-plane constraint at the crack tip are associated with crack size, geometry of specimen and type of loading. The source of a change of the out-of-plane constraint is thickness. To describe in-

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plane and out-of-plane constraint effects in fracture analysis, the following parameters can be used, namely, $T_z$ -parameter by Guo (1999), local triaxiality parameter $h$ by Henry and Luxmoore (1997) and nonsingular components of the $T$-stresses ($T_{xx}$ and $T_{zz}$) by Nakamura and Parks (1992). Not emphasize attention on merits and demerits of the above-mentioned parameters of constraint at the crack tip, our attention is concentrated on the nonsingular components of the $T$-stresses at the crack tip. Nazarali and Wang (2011) and Sousa et al. (2012) show that the sign and value of the $T_{xx}$-stresses considerably effect on the shape and size of the plastic zone at the crack tip. At the same time there is no information about influence of the $T_{zz}$ component on the size of the plastic zone in literature.

Theoretical and numerical analyses of the joint influence of the nonsingular $T_{xx}$ and $T_{zz}$-stresses on sizes of the plastic zone around the crack tip of the mode I are analyzed at the present paper.

2. The model of the crack tip plastic zone

The components of the three-dimensional stress field, which take into consideration three-dimensionality of the stress state near mode I crack front in isotropic elastic body, can be represented in the manner of asymptotic formulas given by Nakamura and Parks (1992)

$$
\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1-\sin \frac{\theta}{2} \cdot \sin \frac{3\theta}{2}\right) + T_{xx} + \ldots
$$

$$
\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left(1+\sin \frac{\theta}{2} \cdot \sin \frac{3\theta}{2}\right) + \ldots
$$

$$
\sigma_{zz} = 2\nu \cdot \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} + T_{zz} + \ldots
$$

$$
\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cdot \cos \frac{3\theta}{2} + \ldots
$$

$$
\tau_{yz} = 0 , \tau_{zx} = 0 .
$$

Here, $\sigma_{xx}$, $\sigma_{yy}$, $\sigma_{zz}$, $\tau_{xy}$, $\tau_{yz}$, $\tau_{zx}$ – components of the stress tensor, which define stress state at the arbitrary point near the crack tip; $r$ and $\theta$ – polar coordinates (Fig. 1); $K_I$ is the stress intensity factor (SIF), $\nu$ is Poisson's ratio.

The $T_{xx}$-stress component can be received from equations (1) and (2) as difference between $\sigma_{xx}$ and $\sigma_{yy}$ stresses, which are defined on the line of crack continuation ($\theta = 0$). The value of $T_{zz}$-stress component can be defined as follows from above-mentioned equations

$$
T_{zz} = E \cdot \varepsilon_{zz} + \nu \cdot T_{xx} .
$$

The von Mises yield criterion can be employed for determination of the shape and size of the plastic zone at the crack tip

$$
\left(\sigma_{xx} - \sigma_{yy}\right)^2 + \left(\sigma_{yy} - \sigma_{zz}\right)^2 + \left(\sigma_{zz} - \sigma_{xx}\right)^2 + 6 \left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right) = 2\sigma_y^2 ,
$$

where $\sigma_y$ is the yield strength.
Substitution of asymptotic formulas (1)-(5) into the criterion (7) allows determining size $r_p$ of the crack tip plastic zone at the crack tip

$$\frac{1}{2\pi \cdot r_p} \left( K_i^2 \cdot \left( A_i + \frac{D_i}{K_j} \cdot \sqrt{r_p} \right) \right) = 2\sigma^2,\quad (8)$$

where

$$D_i = \sqrt{\frac{\pi}{2}} \left( \frac{\cos \theta}{2} + 3\cos \frac{5\theta}{2} \right) \cdot T_{xx} - 8\nu \cdot \cos \frac{\theta}{2} \cdot \left( T_{xx} + \frac{T_{yy}}{\nu} \right) + 16\nu \cdot \cos \frac{\theta}{2} \cdot (T_{zz}) \quad (9)$$

$$A_i = (1-2\nu) \cdot (1 + \cos \theta) \cdot \frac{3}{4} \cdot (\cos 2\theta - 1).\quad (10)$$

Solving equation (8), the following equation can be obtained for determination of the angular distribution of the plastic zone around the crack tip of mode I loading

$$\sqrt{r(\theta)}_{p_1,2} = \frac{V \pm \sqrt{V^2 + 4U \cdot W}}{2U},\quad (11)$$

where

$$U = 4\pi \cdot \sigma^2, \quad V = K_i \cdot D_i, \quad W = K_j^2 \cdot A_i.\quad (12)$$

It is not difficult to show that particular solution for plastic zone in the case of plane strain given by Nazarali and Wang (2011) follows from proposed general solution (11).

It is necessary to check the following condition before determination of plastic zone sizes.
\[ V^2 + 4U \cdot W^2 \geq 0 \quad \text{and} \quad U \neq 0 . \] (13)

If above-mentioned inequalities (13) is satisfied, plastic zone sizes can be estimated by the following equations

\[ r(\theta)_{p1} = \frac{V + \sqrt{V^2 + 4U \cdot W}}{2U}, \quad r(\theta)_{p2} = \frac{V - \sqrt{V^2 + 4U \cdot W}}{2U}. \] (14)

As a result, the plastic crack zone is determined as follows

\[ r(\theta) = \left[ \text{positive}(r_{p1}, r_{p2}) \right]^ \frac{1}{2} \] (15)

It should be noted that the value of \( r_{p1} \) is positive in the wide range of coefficients \( U, V, W \). At the same time, the value of \( r_{p2} \) has a negative sign.

3. Results and discussion

The calculation results of the angular size distribution of the plastic zone around the crack tip in compact specimen with various relations between specimen thickness \( B \) and specimen width \( W \) are presented below (Fig. 2).

Fig. 2. Analytical calculation of the angular distribution of crack tip plastic zone sizes for the middle plane in the compact specimen.

The stress intensity factor \( K_I \) is assumed to be constant independently on specimen thickness and equal to 66 MPa m\(^{\frac{1}{2}}\). Corresponding values of the \( T \)-stress components are presented in Table. 1. According to slight variation of the \( T_{xx} \)-stresses during specimen thickness change, the value of \( T_{xx} \)-stresses is assumed to be \( T_{xx} = 182 \) MPa. Sizes of the plastic zone around the crack tip with provision for the spatial stress state (3D Analysis) are surrounded by sizes of the zones corresponding to two limit conditions, namely, Plane Stress and Plane Strain. Moreover, when
specimen thickness increases, the shape and size of the plastic zones tend to zones which are typical for plane strain conditions. Thus, the results clearly show that triaxiality of the stress state around the crack tip should be taken into account by means of both non-singular $T_{xx}$-stress and $T_{zz}$-stress according to equation (11).

Table 1. T-stress components for studied compact specimen.

| T-stress components | B/W = 0.25 | B/W = 0.40 | B/W = 0.50 |
|---------------------|------------|------------|------------|
| $T_{xx}$, MPa       | 186.59     | 182.36     | 176.28     |
| $T_{zz}$, MPa       | -159.47    | -106.81    | -84.97     |

To demonstrate the validity of analytical equations for calculation of the plastic zone around the crack tip, finite element analysis (FEA) is carried out. Loading conditions of the compact specimen are given in Table 2. Details of FEA and estimation of the influence of components of the T-stress on the crack tip plastic zone by means of special algorithm are presented by Matvienko and Pochinkov (2013).

Analytical and numerical results are presented in Fig. 3.

Table 2. Loading conditions of the compact specimen.

| Loading parameters | B/W = 0.25 | B/W = 0.40 | B/W = 0.50 |
|--------------------|------------|------------|------------|
| $P$, kN            | 6.0        | 9.6        | 12.0       |
| $K_{Ic}$, MPa m$^{1/2}$ | 66.0      | 66.0       | 66.0       |

Fig. 3. Comparison between analytical and numerical results of the plastic zone.

It follows from Fig. 3 that the deviation between analytical and numerical results does not exceed 20% in the angular range (0°, 30°…45°) and (90°…100°, 135°…145°) and the deviation can reach 35% in the angular range (30°…45°, 90°…100°). In the angular range (135°…145, 180°), the deviation becomes maximal and exceeds 40%, that is related to the specific features of the algorithm used to process and analyze the equivalent stress computed by
finite element around the crack tip. The large deviations between the results of the numerical analysis and the analytical calculation in the angular range \((0^\circ -145^\circ)\) can be explained by the fact that the \(T\)-stress components around the plastic zone are not constant and depend on angle \(\theta\) as reported by Matvienko and Pochinkov (2013).

4. Conclusions

The theoretical analysis of the joint influence of the nonsingular components of the \(T\)-stresses \((T_{xx} \text{ and } T_{zz})\) on the plastic zone around the crack tip of mode I is carried out with attraction of asymptotic formulas, which take into account triaxiality of the stress state at the crack tip, and the von Mises yield criterion.

The size of the plastic zone around the crack tip at the middle plane of the CT specimen decreases with the increase of specimen thickness. It is confirmed that the plastic zone is affected by the \(T_{zz}\)-stress, i.e. there is strong effect of out-of-plane constraint on crack tip plastic zones.

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