DECELERATING UNIVERSES OLDER THAN THEIR HUBBLE TIMES †

by

J.C. Jackson* and Marina Dodgson
Department of Mathematics and Statistics
University of Northumbria at Newcastle
Ellison Building
Newcastle upon Tyne NE1 8ST, UK

ABSTRACT

Recent observations suggest that Hubble’s constant is large, and hence that the Universe appears to be younger than some of its constituents. The traditional escape route, which assumes that the expansion is accelerating, appears to be blocked by observations of Type 1a supernovae, which suggest that the Universe is decelerating. These observations are reconciled in a model in which the Universe has experienced an inflationary phase in the recent past, driven by an ultra-light inflaton whose Compton wavelength is of the same order as the Hubble radius.

Key words: cosmology – observations – theory – dark matter.

† Mon. Not. R. Astron. Soc. 297 (1998) 923
* e-mail: john.jackson@unn.ac.uk
1 INTRODUCTION

From time to time, cosmologists have had to face the possibility that the Hubble time $1/H_0$ is shorter than the ages of the oldest Galactic globular clusters (the so-called age crisis), and hence that the Universe appears to contain objects older than itself. This certainly seemed to be the case when determinations of Hubble’s constant using extra-galactic Cepheids (Pierce et al. 1994; Freedman et al. 1994; Tanvir et al. 1995) produced values of $80 \pm 17$, $87 \pm 7$ and $69 \pm 8$ km sec$^{-1}$ Mpc$^{-1}$ respectively, with a best estimate based upon these figures of $79 \pm 5$ km sec$^{-1}$ Mpc$^{-1}$ (see also van den Bergh 1995). However, subsequent recalibration of the Cepheid period-luminosity zero-point, using data from the Hipparcos astrometry satellite (Feast & Catchpole 1997), has indicated that these figures should be reduced by 10%, giving $H_0 = 71.3 \pm 4.5$ km sec$^{-1}$ Mpc$^{-1}$; the corresponding Hubble time is $t_{H_0} = 13.8 \pm 0.9$ Gyr (see, however, Madore & Freedman 1997).

The existence of globular clusters older than 15 Gyr was believed to be well established (see for example Sandage 1982), but more recent re-evaluations have given ages of $15.8 \pm 2.0$ Gyr based on main-sequence turnoff (Bolte & Hogan 1995), and $13.5 \pm 2.0$ Gyr based on giant-branch fitting (Jimenez et al. 1996), with a best estimate based upon these figures of $14.7 \pm 1.4$ Gyr (see also Chaboyer et al. 1996). However, the aforementioned Cepheid recalibration has implications here too, via a corresponding recalibration of RR Lyrae distances to these clusters, which indicates for example that the main-sequence turnoff stars are brighter and younger than previously supposed; Feast & Catchpole (1997) suggest an age reduction of 23%. In similar vein, cluster distances established by main-sequence fitting, to a sample of local subdwarfs with Hipparcos parallaxes, has yielded ages between 12 and 13 Gyr (Gratton et al. 1997; Reid 1997).

On the basis of these revised figures it is tempting to say that there is no age crisis, but we are not convinced that this is so. A formation time must be added to cluster ages, and there is abundant evidence that the Universe contains some dark matter, enough to reduce its age significantly compared with the Hubble time, for example $t_0 \leq 0.845 t_{H_0}$ if $\Omega_0 \geq 0.2$, in which case our best estimate of $H_0$ gives $t_0 \leq 11.7 \pm 0.8$ Gyr. In any case, low cluster ages are not indicated by all of the most recent data. Renzini et al. (1996) have presented Hubble Space Telescope observations of white dwarfs in the local neighbourhood and in the cluster NGC 6752; matching the two cooling sequences yields a distance modulus for the latter, and an age of $15.0 \pm 1.5$ Gyr (however, Hipparcos paralaxes have yet to be used in this context). Finally thorium nucleocosmochronometry applied to a metal-poor halo star (Cowan et al. 1997) has yielded an age of $17 \pm 4$ Gyr.

If there is an age crisis, the way out is to suppose that the Universe has passed through an accelerating phase, traditionally effected by a positive cosmological constant $\lambda$, which allows $t_0 > t_{H_0}$; particularly fashionable have been those models which maintain spatial flatness by assuming the appropriate admixture of material and vacuum energy, $\Omega_0 + \Lambda_0 = 1$ (where $\Lambda_0 = \lambda/3H_0^2$), an idea due originally to Peebles (1984); as a precautionary measure this escape route has been reopened (Ostriker & Steinhardt 1995). The problem with such a scenario is that it implies that the expansion of the Universe now should be seen to be accelerating, whereas very recent evidence, in the form of a traditional Hubble diagram based upon Type 1a supernovae, suggests that this is not the case, and that the deceleration parameter satisfies $0.14 < q_0 < 0.79$, with a central value of 0.47 (Perlmutter et al. 1996, 1997a,b); the precise figures (but not the sign) depend upon the value of $\lambda$, here taken to be zero. Of course the classical Hubble diagram based upon first-ranked cluster galaxies always did suggest a positive value for $q_0$ (Sandage 1988; Sandage, Kron & Longair...
1995), but faith in this conclusion was never very strong, owing to the uncertain role of evolutionary effects; the case for a non-negative deceleration parameter now seems much stronger, and is one of the prime factors motivating this work.

It is certainly not our intention to insist that $q_0 > 0$, and that the oldest stars have ages which are longer than the Hubble time, but at face value the observations allow both statements as more than a possibility, and to insist that one of them must be wrong (McGaugh 1996) is to risk falling into a teleological trap. We feel that the addition of an appropriate model to the cosmological compendium is long overdue, that is one which has decelerating phases in which $t_0$ is greater than $t_{H_0}$; the purpose of this work is to examine what reasonable options might be on offer in this context. In Section 2 we discuss models which are dominated by a rolling homogeneous scalar field, and in the Section 3 we confirm that such a model can account for the supernova observations reported by Perlmutter et al.

2 LATE-STAGE INFLATION

A suitable model must retain some of the features of those discussed by Peebles (1984) and Ostriker & Steinhardt (1995), but with an added degree of flexibility. Models in which the current epoch is dominated by a homogeneous scalar field have been considered before in the context of timescales and missing matter, by Olson & Jordan (1987), Peebles & Ratra (1988), and particularly Ratra & Peebles (1988), who introduce explicitly the idea of an ultra-light boson as a CDM candidate; this possibility has been rediscovered recently by Frieman et al. (1995), who examine how such a boson might arise naturally, according to modern theories of fields and particles. The model which we have adopted is based upon a simplified version of their ideas. Although much of what we have to say is implicit in earlier work, to our knowledge the key points which we wish to bring out have not been made before. The basic idea is to adopt the inflationary model of the very early Universe, adapted to describe recent cosmological history by rescaling the appropriate length/mass scales by a factor of approximately $10^{60}$.

The idea that the dynamics of the very early Universe were dominated by a homogeneous isotropic scalar field, with effective classical Lagrangian $\mathcal{L} = \dot{\phi}^2/2 - V(\phi, T)$, has dominated cosmological thinking for almost two decades. In so-called new inflation (Linde 1982), there is an initial period in which the Lagrangian is dominated by the potential term $V(\phi, T)$, during which the Universe inflates and cools below a transition temperature $T_c$; after this event the field $\phi$ begins to move from its false-vacuum value of zero, towards a non-zero value $\phi_0$ at which $V$ has a minimum, about which value $\phi$ undergoes decaying oscillations, effected by dissipation due to coupling to other matter fields. As originally conceived the inflationary process finishes after a time of typically several hundred Planck times, and is thus shielded for ever from the possible embarrassment of direct observation. However, this need not be the case; a trivial example would be when the true vacuum (minimum) value of $V$ is not exactly zero, which would constitute a cosmological constant $\lambda$ whose effects might be observable now. More generally, we might expand $V$ as a Taylor series about $\phi_0$:

\[ V(\phi) = V(\phi_0) + \frac{1}{2} V''(\phi_0)(\phi - \phi_0)^2 + ... \] (1)
For $\phi$ close to $\phi_0$, we retain just these terms and move the origin of $\phi$ to $\phi_0$, giving

$$V(\phi) = \frac{\lambda}{8\pi G} + \frac{1}{2}\omega_c^2\phi^2$$

(2)

where $\omega_c$ is the Compton freqency of the associated Nambu-Goldstone boson. In chaotic inflation (Linde 1983) the potential has the form (2) ab initio, and inflation is attributed to initial conditions $\omega_c\phi \gg 1/\dot{\phi}$ over a suitably large patch of the primordial chaos. The mass $\hbar\omega_c$ is usually very large, typically of order $10^9$ Gev, with corresponding frequency $\omega_c \sim 10^{34}$ sec$^{-1}$. Here we examine the possibility that the current dynamics of the Universe are dominated by a scalar field corresponding to a very light boson, with mass such that $\omega_c$ and $H_0$ are of the same order.

The basic scene is thus a re-run of chaotic inflation, in which the scalar field mimics a positive cosmological constant in a slow-roll phase until the Hubble time exceeds $\omega_c^{-1}$, after which it undergoes damped oscillations (that is Hubble damping; we assume that the scalar field does not couple to ordinary matter). It is during the period immediately after inflation that such a model would exhibit the behaviour we are seeking to illustrate. Corresponding to the potential (2), we have effective density $\rho_\phi = (\dot{\phi}^2 + \omega_c^2\phi^2)/2$ and pressure $p_\phi = (\dot{\phi}^2 - \omega_c^2\phi^2)/2$, and the Friedmann equations for the scale factor $R(t)$ become

$$\ddot{R} = -\frac{4\pi}{3}G(\rho_\phi + 3p_\phi + \rho_m)R = -\frac{4\pi}{3}G(2\dot{\phi}^2 - \omega_c^2\phi^2 + \rho_m)R,$$

(3)

$$\dot{R}^2 = \frac{8\pi}{3}G(\rho_\phi + \rho_m)R^2 - kc^2,$$

(4)

where we have introduced ordinary pressure-free matter with density $\rho_m$ and set the true cosmological constant $\lambda = 0$. The scalar field is governed by the equation

$$\ddot{\phi} + 3H\dot{\phi} + \omega_c^2\phi = 0$$

(5)

where $H = \dot{R}/R$ is Hubble's 'constant' (see for example Peebles 1993). We introduce the usual density parameter $\Omega_m = 8\pi G\rho_m/3H^2$, and similar parameters associated with the scalar field, $\Omega_{\rho_\phi} = 8\pi G\rho_\phi/3H^2$ and $\Omega_{p_\phi} = 8\pi Gp_\phi/3H^2$. Equation (3) and (4) then give the deceleration and curvature parameters as

$$q = (\Omega_{\rho_\phi} + 3\Omega_{p_\phi} + \Omega_m)/2$$

(6)

and

$$K = \Omega_{\rho_\phi} + \Omega_m - 1.$$  

(7)

To solve the system of equations (3) to (5), we introduce variables $u = \sqrt{G}\phi$, $v = \dot{u}$, $x = R/R(0)$, $y = \dot{x}$, and measure time in units of $\omega_c^{-1}$, when these equations are re-cast in the form of an autonomous non-linear dynamical system:

$$\dot{u} = v, \quad \dot{v} = -3yv/x - u,$$

(8)

$$\dot{x} = y, \quad \dot{y} = -4\pi x(2v^2 - u^2)/3 - \frac{1}{2}\omega_m(0)/x^2$$

(9)

where $\omega_m = 8\pi G\rho_m/3\omega_c^2$. At some arbitrary time $t = 0$ we specify initial values $x(0) = 1$, $H(0)$, $\Omega_m(0)$, $\Omega_{\rho_\phi}(0)$, and $\Omega_{p_\phi}(0) = -\Omega_{\rho_\phi}(0)$, the latter ensuring that the initial value of $\dot{\phi} = 0$, and that the initial effects of the scalar field are inflationary. At the moment an exhaustive survey of this
We choose to concentrate on flat models, in part for mathematical convenience, and in part because the dictates of orthodox inflation still carry some weight. This does not represent a prejudice against open models \((k = -1)\) on our part; the case for an open Universe is in fact quite strong (see for example Coles \\& Ellis 1994). A convenient starting point is \(\Omega_m(0) = 0.999, \Omega_{\rho_0}(0) = 0.001\) (i.e. \(K = 0\) according to equation \(7\)), and we have a one-parameter set of models depending upon \(H(0)\) (equivalent to a choice of \(\omega_c\)). We first run the system backwards to locate the initial singularity, and then forwards to locate the transition from the early matter-dominated phase to the final state in which the scalar field is dominant. Figure 1 shows the scale-factor \(x(t)\) for a typical run, with \(H(0) = 27.2\), showing point A marking the first switch from acceleration to deceleration, and the region A to C where the tangent hits the positive \(t\)-axis, in other words points in this region are older than the corresponding Hubble time. This is quantified in Figure 2, in which \(t - t_H\) (continuous) and \(q\) (dashed) are plotted against \(t\); thus if ‘now’ (point B of Figure 1) corresponds to \(q = 0.5\), we see that \(t_q = 2.81\) exceeds the current Hubble time by 8%. Figure 3 shows \(\Omega_{\rho_0}\) (continuous), \(\Omega_m\) (dashed), and \(\Omega_{\rho_0} + 3\Omega_{\rho_0}\) (dash-dotted), the effective gravitational density of the scalar field; it is the latter’s oscillation between positive and negative values which accounts for the way in which \(q\) changes sign. The matter/scalar field transition at \(t = 0.73\) is quite clear, corresponding to a redshift \(z = 3.1\); \(\Omega_m\) eventually settles down to a value of 0.10. An element of fine tuning is required to achieve acceptable values of the asymptotic value of \(\Omega_m\), but not too fine; \(H(0) > 40\) results in less matter than we see (\(\Omega_m < 0.003\)), whereas \(H(0) < 23\) results in too much dark matter (\(\Omega_m > 0.2\)), in the sense that an epoch in which \(t > t_H\) is precluded. There are also open and closed models which exhibit similar behaviour, including some with long Lemaître-like coasting phases. The fixed non-zero asymptotic values of \(\Omega_m\) are a particularly attractive feature of our flat models, compared with those with \(\Omega_0 + \Lambda_0 = 1\), in which \(\Omega_m \sim 0.1\) is but a passing phase.

3 COMPATIBILITY WITH OBSERVATIONS

A putative cosmological model must expect to run the gauntlet of an increasing number of neo-classical observational tests \((z < 4)\), and of modern tests based upon structure formation and the cosmic microwave background \((z < 1000)\), but in the context of this model a comprehensive survey would seem to be premature until such time as the age crisis has a firmer observational basis. As our universe combines aspects of several models which separately have their advocates, gross contradictions are unlikely. Nevertheless, our ideas will not have achieved their objective if the corresponding magnitude–redshift curves are incompatible with the observations described by Perlmutter et al.; we concentrate on the flat model examined in Section 2, and on an observer at point B of Figure 1, where \(q_0 = 0.5\). It is well-known that the initial (low \(z\)) shape of the \(m - z\) relationship is determined by the value of \(q_0\), and not by the details of how this deceleration is produced (see for example Weinberg 1972), so that compatibility with the preferred matter-dominated model \((q_0 = 0.47)\) is almost guaranteed. However, as the largest redshift \((0.458)\) in the sample is somewhat larger than allowed by the low–\(z\) approximation, we must look at the matter in more detail, particularly to predict the divergences which should be observed at higher redshifts.

Computation of the appropriate \(m - z\) curve is quite straightforward within the framework developed
in Section 2; we read off the parameters $\Omega_m(B)$, $\Omega_{\rho_b}(B)$, $x(B)$ and $H(B)$, which are deemed to be current values; $x$ and $t$ are then rescaled, $x \rightarrow x/x(B)$, $t \rightarrow H(B)t$, and for the spatially flat model in question, standard lore then gives the luminosity distance as

$$d_L(t(z)) = \frac{c}{H_0} \int_0^{t(z)} \frac{dt}{x(t)}.$$  \hspace{1cm} (10)

The integration is performed numerically, over the table of $x(t)$ values established in Section 2, with $z(t) = 1/x(t) - 1$. A specific value for $\omega_c$ can be derived by comparing $H(B) = 0.380$, which is Hubble’s constant in terms of the Compton frequency, with $H_0$ in conventional units, giving $\omega_c / H_0 / H(B) = 8.53h \times 10^{-18}$ sec$^{-1}$, and a corresponding mass of $5.61h \times 10^{-33}$ ev ($h = 1 \Rightarrow H_0 = 100$ km sec$^{-1}$ Mpc$^{-1}$). The corresponding $\alpha - z$ curve is the continuous one in Figure 4, to be compared with the dashed curve, which is the canonical $\Omega_0 = 1$, $q_0 = 0.5$ matter-dominated case. The curves are virtually indistinguishable until $z$ exceeds 0.5, after which the divergence becomes quite pronounced; if these supernovae fulfill their promise as standard candles, this divergence would be detectable when objects in the redshift range 0.5 to 1.0 are observed.

Figure 4 also shows (dotted) a popular flat accelerating model, $\Omega_0 = 0.1$ (i.e. the asymptotic value in our scalar universe) and $\Lambda_0 = 0.9$; as would be expected the scalar curve veers towards the accelerating case at high redshifts. We have examined this effect in the context of a neo-classical test which involves much higher redshifts than those associated with supernovae, namely gravitational lensing of quasars by intervening galaxies. The integrated probability $\tau(z)$ that a quasar at redshift $z$ is multiply imaged by a galaxy along the line of sight (the gravitational lensing optical depth) is particularly simple in spatially flat universes, being given by

$$\tau(z) = \frac{F}{30} d_M(z)^3,$$

where $d_M(z) = d_L(z)/(1 + z)$ is the so-called proper-motion distance (Weinberg 1972); $F$ is a dimensionless measure of the lensing effectiveness of the galaxies, here represented by a non-evolving population of randomly-distributed singular isothermal spheres (for further details see Turner, Ostriker & Gott 1984; Gott, Park & Lee 1989; Turner 1990). Thus our work on the Hubble diagram enables curves of $\tau(z)/F$ to be produced easily, and the three cases considered above are shown in Figure 5. As is well-known, the lensing optical depth in the $\Lambda$-dominated case is typically an order of magnitude larger than that in the matter-dominated one; the scalar model falls between the two.

The observational situation is very confusing; until 1993 the frequency of lensing events was believed to be very low, with a sample of 402 quasars showing no such events (Boyle, Fong, Shanks & Peterson 1990), and one comprising 4250 objects showing at most 9 examples (Hewitt & Burbidge 1987, 1989). On the basis of these figures, and an estimate of the lensing strength $F = 0.15$, Turner (1990) concluded that among the conventional $k = 0$, $\Omega_0 + \Lambda_0 = 1$ models, those with $\Omega_0 \sim 1$ were very much favoured. A weakened version of this conclusion survived a downward revision of the lensing strength, to $F = 0.047$ (Fukugita & Turner 1991; Fukugita et al. 1992), which authors also considered selection biases due to finite angular resolution and magnification, which can be large but fortunately have opposing effects and tend to cancel. However, the advent of the Hubble Space Telescope Snapshot survey (Bahcall et al. 1992; Maoz et al. 1992, 1993a,b, hereafter referred to generically as Maoz et al.) has radically changed the situation; this survey reports a lensing frequency almost a factor of 10 higher than earlier ones, 3 to 6 examples in a sample of 502 quasars,
and in this context at least low $\Omega_0$–high $\Lambda_0$ models appear to be supported (see also Chiba & Yoshii 1997). The discrepancy is believed to be due to inadequate angular resolution in the case of pre-HST surveys. We shall compare predicted lensing frequencies with the HST survey, using Fukugita & Turner's value $F = 0.047$. We assume following Maoz et al. that the HST angular resolution is sufficient to preclude any selection bias against small angular separations; however, the tendency of lensed quasars to be over-represented in a magnitude-limited sample due to their increased apparent brightness must be allowed for, and we adopt a fixed magnification-bias factor of 2.5, as suggested by Fukugita & Turner for deep surveys. Opinion is now fairly firm (Maoz & Rix 1993; Torres & Waga 1996) that of the six lensing events mentioned by Maoz et al., one is spurious and another is due to an intervening cluster of galaxies, rather than a single galaxy; thus we take 4 out of 502 as the observed lensing frequency. The predicted frequencies for the canonical CDM, scalar and $\lambda$ models are respectively 1.3, 3.9 and 8.8, with respective binomial probabilities of 0.03, 0.196 and 0.04. It is gratifying to see our model performing so well in this context, but in view of the various uncertainties we cannot regard these results as anything more than illustrative.

4 CONCLUSIONS

Radical measures are contemplated here, of seemingly epicyclic proportions, but there is a growing body of observational evidence that such measures might be called for. (Indeed as this conclusion is being written we note that the $H_0$ pendulum is swinging back towards higher values, according to a re-evaluation of the extragalactic Cepheid distance scale in which metallicity is considered (Kochanek 1997); figures from 69 km sec$^{-1}$ Mpc$^{-1}$ to 90 km sec$^{-1}$ Mpc$^{-1}$ are mentioned.) These measures are based upon a simple and natural generalization of the concept of vacuum energy, represented by equation (2), which we believe to be the minimal form necessary, to account for the putative observational paradox which has motivated this work. We have considered two neo-classical cosmological tests, with regard to which the behaviour of our scalar universe is satisfactory, and falls between the two extremes defined by the standard matter-dominated model and a $\lambda$-dominated one; in this context we believe this to be generally true.
REFERENCES

Bahcall J.N., Maoz D., Doxsey R., Schneider D.P., Bahcall N.A., Lahav O., Yanny B., 1992, ApJ, 387, 56
Bolte M., Hogan C.J., 1995, Nat, 376, 399
Boyle B.J., Fong R., Shanks T., Peterson B.A., 1990, MNRAS, 243, 1
Chaboyer B., Demarque P., Kernan P.J., Krauss L.M., 1996, Sci, 271, 957
Chiba M., Yoshii Y., 1997, 489, 485
Coles P., Ellis G.F.R., 1994, Nat, 370, 609
Cowan J.J., McWilliam A., Sneden C., Burris D.L., 1997, ApJ, 480, 246
Feast M.W., Catchpole R.M., 1997, MNRAS, 286, L1
Freedman, W.L. et al., 1994, Nat, 371, 757
Friedman J.A., Hill C.T., Stebbins A., Waga I., 1995, Phys. Rev. Lett., 75, 2077
Fukugita M., Turner E.L., 1991, MNRAS, 253, 99
Fukugita M., Futamase T., Kasai M., Turner E.L., 1992, ApJ, 393, 3
Gratton R.G., Fusi Pecci F., Carretta E., Clementi G., Corsi C.E., Lattanzi M.G., 1997, ApJ, 491, 749
Gott J.R., Park M.G., Lee H.M., 1989, ApJ, 338, 1
Hamuy M., Phillips M.M., Maza J., Suntzeff N.B., Schommer R., Aviles R., 1995, AJ, 109, 1
Hewitt A., Burbidge G., 1987, ApJS, 63, 1
Hewitt A., Burbidge G., 1989, ApJS, 69, 1
Jimenez R., Thejll P., Jorgensen U.G., MacDonald J., Pagel B., 1996, MNRAS, 282, 926
Kochanek C.S., 1997, ApJ, 491, 13
Linke A.D., 1982, Phys. Lett., B108, 389
Linke A.D., 1983, Phys. Lett., B129, 177
Madore B.F., Freedman W.L., 1998, ApJ, 492, 110
Maoz D., Rix H.-W., 1993, ApJ, 416, 425
Maoz D., Bahcall J.N., Schneider D.P., Doxsey R., Bahcall N.A., Lahav O., Yanny B., 1992, ApJ, 394, 51
Maoz D., Bahcall J.N., Schneider D.P., Doxsey R., Bahcall N.A., Lahav O., Yanny B., 1993a, ApJ, 402, 69
Maoz D. et al., 1993b, ApJ, 409, 28
McGaugh S.S., 1996, Nat, 381, 483
Olson T.S., Jordan T.F., 1987, Phys. Rev., D35, 3258
Ostriker J.P., Steinhardt P.J., 1995, Nat, 377, 600
Peebles P.J.E., 1984, ApJ, 284, 439
Peebles P.J.E., 1993, Principles of Physical Cosmology. Princeton University Press, pp. 394-396
Peebles P.J.E., Ratra, B., 1988, ApJ, 325, L17
Perlmutter S. et al., 1996, Nucl. Phys., S51B, 20
Perlmutter S. et al., 1997a, in Canal R., Ruiz-LaPuente P., Isern J., eds., Thermonuclear Supernovae. Kluwer, Dordrecht, pp. 749-763
Perlmutter S. et al., 1997b, ApJ, 483, 565
Pierce M.J., Welch D.L., McClure R.D., van den Bergh S., Racine R., Stetson P.B., 1994, Nat, 371, 385
Ratra B., Peebles P.J.E., 1988, Phys. Rev., D37, 3406
Reid I.N., 1997, AJ, 114, 161
Renzini A. et al., 1996, ApJ, 465, L23
Sandage A., 1982, ApJ, 252, 553
ACKNOWLEDGMENTS

Marina Dodgson acknowledges receipt of an UNN internal research studentship, 1993-96, during the tenure of which this work was initiated. It is a pleasure to thank Richard McMahon, of the Institute of Astronomy, University of Cambridge, for advice about the rapidly developing observational scene.

FIGURE CAPTIONS

Figure 1. Scale factor $x(t)$; point A marks the first switch from acceleration to deceleration; $q_0 = 0.5$ at point B, and the Hubble time exceeds the age after point C.

Figure 2. Continuous line shows $t - t_H$ as a function of $t$, and the dashed line shows the deceleration parameter $q(t)$; in the region A to C we have $t > t_H$ and $q > 0$.

Figure 3. $\Omega_{\rho_0}$ (continuous), $\Omega_m$ (dashed) and $\Omega_{\rho_0} + 3\Omega_{\rho_0}$ (dash-dotted) as functions of $t$.

Figure 4. Magnitude-redshift relation for the scalar field model discussed in Section 2 (continuous line), for the canonical $\Omega_0 = 1$, $q_0 = 0.5$ matter-dominated model (dashed), and for a flat accelerating model $\Omega_0 = 0.1$, $\Lambda_0 = 0.9$ (dotted). Data points for Type 1a supernovae due to Perlmutter at al. (1997b) and Hamuy et al. (1995).

Figure 5. Gravitational lens optical depth as a function of redshift for the scalar field model discussed in Section 2 (continuous line), for the canonical $\Omega_0 = 1$, $q_0 = 0.5$ matter-dominated model (dashed), and for a flat accelerating model $\Omega_0 = 0.1$, $\Lambda_0 = 0.9$ (dotted).
Figure 1
Figure 2
