Automatic correction of orbital elements using continuous thrust controlled in closed loop

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Abstract. This work aims to study and simulate the control of a spacecraft trajectory in order to correct automatically and simultaneously the orbital elements that define the orbit: semi-major axis, eccentricity, periapse argument, inclination and right ascension of the ascending node. Thus, to perform the control of the trajectory was used a propulsion system able to apply thrust with adjustable magnitude and direction of application. In this study it was considered that the propulsion system is controlled in closed loop, so the adjustments of the magnitude and direction of thrust depends on the error generated by comparing a reference state (position and velocity) and a current state. The reference state is determined according to the final orbital parameters. The current state is estimated at each step of the simulation, therefore, the reference and current states must be determined and compared at each step in order to generate the error signal that is inserted into the trajectory control system. However, the control of the orbital parameters simultaneously can be characterized as a multi-objective problem with conflicting goals. The correction of the semi-major axis causes an eccentricity modification and vice-versa. One possibility to deal with this problem is to define when and where to make adjustments for each of the parameters. Thus, the automatic control seeks the best way to correct each parameter, adjusting each one sequentially. At the end of the process all orbital parameters are automatically adjusted and maintained due to the use of the closed loop control system.

1. Simulation and control of the orbital trajectory
The main objective of this study is to simulate the control of the trajectory described by a spacecraft in order to automatically change the orbital elements. The problem can be defined as follows: control the performance of the propulsion system in order to adjusting the orbital parameters that define the orbit.

The trajectory of the vehicle must be controlled through the application of continuous thrust, in a way that at the end of the simulation the final orbit presents the desired orbital elements. To accomplish this task, it was used a control system able to automatically detect the difference between the current state and the desired state of the vehicle, which is a function of keplerian elements of the final orbit. Thus, the simulation occurs discretely, that is, at every simulation step (sampling time) the state of the vehicle is calculated enabling the correction of the orbital velocity.

The orbital motion can be determined solving the Kepler's equation at each step of the simulation. Thus, given an initial state (position and velocity) and the step of the simulation, the state can be converted to keplerian elements using the classical equations from celestial mechanics [1]:

[1]
\[ M = n(t - T) = u - e \sin u \quad (1) \]
\[ n = \sqrt{\frac{\mu}{a^3}} \quad (2) \]
\[ r = (X^2 + Y^2 + Z^2)^{1/2} \quad (3) \]
\[ v = (\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2)^{1/2} \quad (4) \]
\[ X_1 = A_1 (\cos u - e) + B_1 \sin u \quad (5) \]
\[ \dot{X}_1 = \frac{a_1}{r} (-A_1 \sin u + B_1 \cos u) \quad (6) \]
\[ A_x = aR_{12}; \quad B_x = \alpha \sqrt{1 - e^2} R_{12} \quad (7) \]
\[ \mathbf{R} = R_{11} R_{12} R_{13} \]
\[ R_{11} = \cos \omega \cos \Omega - \sin \omega \sin \Omega \cos i \]
\[ R_{12} = -\sin \omega \cos \Omega - \cos \omega \sin \Omega \cos i \]
\[ R_{13} = \sin \Omega \sin i \]
\[ R_{21} = \cos \omega \sin \Omega + \sin \omega \cos \Omega \cos i \]
\[ R_{22} = -\sin \omega \sin \Omega + \cos \omega \cos \Omega \cos i \]
\[ R_{23} = -\sin i \cos \Omega \]
\[ R_{31} = \sin \omega \sin i \]
\[ R_{32} = \cos \omega \sin i \]
\[ R_{33} = \cos i \quad (9) \]
\[ \vec{h} = \vec{r} \times \vec{v} = (YZ - YZ)i + (ZX - ZX)j + (XY - XY)k \quad (10) \]
\[ \vec{r} = XXX + YYY + ZZZ \quad (11) \]
\[ e = \left( \frac{\vec{r} \cdot \vec{v}}{ma^2} \right)^2 \left( 1 - \frac{r}{a} \right)^2 \quad (12) \]
\[ i = \arctan \left( \frac{h_x^2 + h_y^2}{h_z} \right) \quad (13) \]
\[ \Omega = \arctan \left( \frac{h_y}{-h_x} \right) \quad (14) \]
\[ f = \arcsin \left( \frac{1 - e^2\sqrt{\sin u}}{1 - e \cos u} \right) \quad (15) \]
where $M$ is the mean anomaly; $u$ is the eccentric anomaly; $f$ is the true anomaly; $\mu$ is the gravitational constant; $a$ is the semi-major axis; $e$ is the eccentricity; $i$ is the inclination; $\Omega$ is the right ascension of ascending node; and $\omega$ is the periapse argument.

To simulate and control the trajectory was used the simulation environment STRS (Spacecraft Trajectory Simulator). This simulator is designed to operate in closed loop controlling the trajectory at each instant of time, determined by simulation step defined as one of the input parameters for the simulator [2] [3] [4] [5]. Normally, for correction maneuvers and orbit transfer, it is used a control in open loop commanded by the ground station. However, for maneuvers using continuous thrust, applied during long period of time, the closed loop control of the trajectory becomes advisable, since the errors generated by the non-linearities of actuators and sensors, and further, errors generated by the disturbances could deviate the vehicle from the optimal trajectory. Thus, the control system must be able to detect the deviations from the reference trajectory and produce control signals in order to correct these deviations. A schematic representation of the control system is shown in Figure 1:

**Figure 1.** Control system for the trajectory.

In a closed loop control system the reference signal is compared with the signal representing the current state, to generate an error which is inserted into the controller, which generates the control signal. This signal is sent to the actuator to apply it in the dynamic of the system. The output of the dynamic represents the current state that is compared with the reference state, closing the loop.

### 2. Automatic correction for orbital trajectory

Usually, before the implementation of orbital maneuver, the optimal maneuver is calculated considering the minimization of fuel or time. Generally, obtaining a solution to the problem of optimizing the trajectory is made using numerical methods, which require the determination of the initial values for the variables [6]. However, sometimes, to obtain these initial values is not an easy task. In some cases, the optimal solution cannot be found due to the difficulty of convergence of the
numerical methods. When it is desired to correct only one or two orbital elements, a solution can be easily found. But maybe difficult to obtain a solution when it is desired to simultaneously change all elements. Nevertheless, in a real situation the maneuver must be somehow implemented. Therefore, this work presents an alternative to the orbital maneuvers, using sub-optimal maneuvers but in an automatically way, without requiring the use of numerical methods and initial values.

In this work, besides the control loop used to maintain the vehicle close to the best trajectory, was used a control loop to control the variation of each orbital element utilized to obtain the reference trajectory. That is, the final orbital elements are defined, then control loops for each of these elements determine, at each step of the simulation, the magnitudes and directions of application of thrust necessary to reach the desired orbital elements at the end of the maneuver. The variations of elements occur gradually until the difference between the current and reference signals do not generate errors. Then, the propulsion system is turned off automatically.

3. Results

Two simulations are presented. In the first (Table 1 and Figures 2 to 5) the semi-major axis and the eccentricity were simultaneously adjusted. In the second simulation (Table 2 and Figures 6 to 12), beyond the correction of the semi-major axis and eccentricity, the purpose was the changing of the orbital plane by controlling the inclination and the right ascension of the ascending node.

Table 1. Simulation I: correction of the semi-major axis and eccentricity.

| initial orbit                     | desired final orbit | final orbit                      |
|----------------------------------|---------------------|----------------------------------|
| \(a = 9900000\) m               | \(a = 9910000\) m   | \(a = 9910000.0016562\) m       |
| \(e = 0.25\)                    | \(e = 0.245\)       | \(e = 0.24500001\)              |
| \(i = 10^\circ\)                |                     | \(i = 10^\circ\)                |
| \(\Omega = 55^\circ\)           |                     | \(\Omega = 55^\circ\)           |
| \(\omega = 25^\circ\)           |                     | \(\omega = 25.80062295^\circ\)  |
| \(M = 353.45695^\circ\)         | maneuver time: 10 days | \(M = 351.84629308^\circ\)      |

Figure 2. Semi-major axis (I)  
Figure 3. Eccentricity (I)  
Figure 4. Velocity deviation (I)  
Figure 5. Applied thrust (I)
### Table 2. Simulation II: correction of all keplerian elements.

|                  | initial orbit | desired final orbit | final orbit |
|------------------|---------------|---------------------|-------------|
| semi-major axis  | $a = 9900000$ m | $a = 9910000.4207331$ m | $a = 9910000.4207331$ m |
| eccentricity     | $e = 0.25$    | $e = 0.245$         | $e = 0.245000212269$ |
| inclination      | $i = 10^\circ$| $i = 10.5^\circ$    | $i = 10.50002165^\circ$ |
| argument of periapsis | $\Omega = 55^\circ$ | $\Omega = 55.5^\circ$ | $\Omega = 55.50005465^\circ$ |
| argument of ascending node | $\omega = 25^\circ$ | $\omega = 30^\circ$ | $\omega = 29.99963123^\circ$ |
| mean anomaly      | $M = 353.45695^\circ$ | simulation time: 32 days | $M = 202.07550398^\circ$ |

### Figures

- **Figure 6.** Semi-major axis (II)
- **Figure 7.** Eccentricity (II)
- **Figure 8.** Inclination (II)
- **Figure 9.** Right ascension ascending node (II)
- **Figure 10.** Periapse argument (II)
- **Figure 11.** Velocity deviation (II)
4. Conclusion

This work represents an alternative to control the trajectory of a spacecraft, especially for long-term missions using continuous propulsion. The closed loop control of the reference allowed the simulating of orbital maneuvers, defined automatically, in order to reach a desired final orbit. Therefore, the automatic procedure to correct the trajectory was implemented and tested successfully. The procedure adopted in this work was able to find sub-optimal maneuvers without the numerical methods and determination of initial values for the solution of these methods.

From simulations was verified that the final orbit was reached with great accuracy, although the time spent in the maneuvers was 10 days for the first simulation and 32 days for the second simulation. However a reduction in the required accuracy could provide a significant reduction of time.

The correction of the orbital elements simultaneously can be characterized as a multi-objective problem with conflicting objectives, because the correction of one element causes a modification on the others. In this work was adopt the automatic control to deal whit the multi-objective problem, so the control system seeks the best way to correct each element, adjusting each one sequentially. A more general study of multi-objective problems can be found in [7].

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