Supersymmetric configurations in Euclidean Freedman-Schwarz model

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ABSTRACT

We study Euclidean $D = 4$, $N = 4$ gauged $SU(2) \times SU(1, 1)$ supergravity theory which has been obtained from dimensional reduction of $N = 1$, $D = 10$ supergravity on $S^3 \times AdS_3$. We obtain supersymmetric configurations like domain wall, electro-vac type of solutions with geometries $E^2 \times S^2$, $E^2 \times AdS_2$ and axio-vac type $E^1 \times S^3$ solution in this Euclidean Freedman-Schwarz (EFS) model. We also show that the Euclidean gravitational instantons with nontrivial (anti)self-dual $U(1)$ gauge fields are stable vacua preserving one fourth of the original supersymmetry.

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1 Introduction

Recently there has been renewed interest in the study of supergravity theories and in particular gauged supergravities due to the AdS/CFT correspondence [1] as well as the domain-wall/QFT correspondence [2]. Extended supergravity theories with $N$ extended local supersymmetries have a global $SO(N)$ invariance. Gauged supergravity theories arise when a subgroup of the $R$-symmetry group or the automorphism group of the supersymmetry algebra is gauged by the vector fields in the graviton supermultiplet. There are also other ways to obtain gauged models, like gauging the isometries of the vector multiplet moduli space as well as the hypermultiplet moduli space. The procedure of gauging does not change the particle content of the theory, but it introduces new terms proportional to the square of the gauge coupling constant in the action. So gauging necessarily induces either a cosmological constant (for $N \leq 3$) in which case, the supersymmetric ground state is an AdS space or in the presence of scalar fields (for $N \geq 4$), a scalar potential which is unbounded from below and it may or may not have critical points. Though the potential is inverted and unbounded from below, in the presence of gravity, this becomes a perfectly consistent theory having stable ground state configurations. If a background has killing spinors, then one can show that it is a stable background by applying Witten’s positivity of energy argument [3]. Therefore, it is important to understand the nature of the ground states of these theories as well as the relationship between gauged supergravities and consistent compactifications/truncations of higher dimensional supergravity theories.

In four dimensions, there are two versions of $N = 4$ supergravity theories, one with a global $SO(4)$ symmetry [4] and the other one with a global $SU(4)$ symmetry [5]. The equations of motion as derived from the two versions are equivalent by using field redefinition and duality transformations. However, when one considers the gauged models corresponding to the respective local internal symmetries, one finds that the two versions are inequivalent. The $N = 4$, $SO(4)$ gauged model [4] has one coupling constant and the scalar potential which is generated by gauging, is unbounded from below. On the other hand, one can consider the Freedman-Schwarz model (FS) [7], where one considers gauging a $SU(2) \times SU(2)$ subgroup of $SU(4)$ internal symmetry with two independent gauge coupling constants. Here the scalar potential is again unbounded from below and has no critical points. But in both the cases, there exist stable vacuum configurations preserving some amount of supersymmetry. The electro-vac solutions [4] in gauged $SU(2) \times SU(2)$ supergravity is one such well known example. Other backgrounds in the FS model, like
domain walls, strings, pure axionic gravity etc have also been recently obtained and they preserve either half or one fourth of the supersymmetry \[9\]. Nonabelian solitons \[10\] and black holes \[11\] as stable vacuum configurations were also shown to exist. In related work on strings in curved backgrounds \[12\], exact supersymmetric solutions of $D = 4$ gauged supergravities have been constructed by using the techniques of conformal field theory and the connection between gauged supergravities and non-critical strings have been discussed.

In this work, we shall concentrate on the Euclidean Freedman Schwarz (EFS) model in $D = 4$ which has recently been obtained by Volkov \[13\]. The two theories (FS and EFS) are different as they are obtained from compactification of the ten dimensional theory on different group manifolds and it is to be noted that they are not just related by analytic continuations. $D = 4$, $N = 4$ gauged $SU(2) \times SU(2)$ FS model can be embedded into $N = 1$ supergravity in ten dimensions as an $S^3 \times S^3$ compactification with the group manifold being $SU(2) \times SU(2)$ \[14\]. Previously also, a Kaluza-Klein (KK) interpretation for the $SU(2) \times SU(2)$ gauged supergravity was given in \[15\], where the model was identified as part of the effective $D = 4$ field theory for the heterotic string theory on $S^3 \times S^3$. These two KK interpretations are essentially the same up to consistent truncations. One can also consider another reduction of the $N = 1$ ten dimensional theory on the group manifold $SU(2) \times SU(1, 1)$ so that the geometry of the internal space-time is $S^3 \times AdS_3$ with the signature $(+, +, +, +, +, -)$ and the corresponding four dimensional theory becomes an Euclidean theory. As the scalar curvature of $S^3$ is positive and that of $AdS_3$ is negative, the dilaton or equivalently the scalar potential in the corresponding four dimensional theory becomes proportional to $g_1^2 - g_2^2$, where, $g_1$ and $g_2$ are the gauge coupling constants corresponding to $SU(2)$ and $SU(1, 1)$ respectively. Since the potential is proportional to the square of the difference of the gauge couplings, one can consider a variety of cases, where the potential can be positive, negative or zero. The dimensional reduction on the above group manifold is consistent in the sense that for a given four dimensional configuration which satisfies the four dimensional equations of motion and supersymmetry variations, the corresponding uplifted version also satisfies the ten dimensional equations of motion as well as the supersymmetry variations.

The paper is organized as follows: In section 2, we discuss the four dimensional gauged EFS model as obtained from the dimensional reduction of the corresponding ten dimensional theory. In section 3 and 4, we explicitly obtain the new background solutions and illustrate that they preserve either half or one fourth of the original supersymmetry. The Euclidean solutions we have obtained, include the interesting cases of domain wall,
$E^2 \times S^2$, $E^2 \times AdS_2$, $E^1 \times S^3$ where $E^1$ and $E^2$ denote one and two dimensional Euclidean spaces. We also show that the four dimensional gravitational instanton solutions [16] like Eguchi-Hanson is a solution of the EFS model with nontrivial (anti)self-dual abelian gauge fields belonging to the $U(1)$ of $SU(2)$ and the noncompact $SU(1,1)$ groups. In section 5, we summarize our results.

2 The Euclidean Freedman-Schwarz model

In this section we set our notations and briefly review some necessary aspects of the EFS model [13] which will be necessary for our analysis. The field content of the EFS model is same as that of the FS model. In the EFS model, the four dimensional gravity multiplet contains the graviton $E^m_\mu$, four majorana spin $\frac{3}{2}$ gravitinos $\Psi^I_\mu (I = 1, \ldots, 4)$, three nonabelian vector fields $A^a_\mu (a = 1, 2, 3)$ belonging to $SU(2)$ with gauge coupling $g_1$, three nonabelian pseudovector gauge fields $\dot{A}_a^\mu$ belonging to $SU(1,1)$ group with gauge coupling constant $g_2$, four majorana spin $\frac{1}{2}$ fields $\chi^I$, the axion $a$ and the dilaton $\Phi$. Here the Greek indices $\mu, \nu, \ldots$ refer to the base space indices and latin indices $m, n, \ldots$ refer to the tangent space indices. The bosonic part of the ten dimensional theory contains the metric $\hat{g}_{MN}$, the three form antisymmetric tensor $\hat{H}_{MNP}$ and the dilaton $\hat{\Phi}$. The fermionic field contents are the ten dimensional gravitino $\hat{\Psi}_M$ and the gaugino $\hat{\chi}$. We consider vanishing spinor background fields, however their supersymmetric variations do not vanish and they are important for our considerations.

The bosonic part of the ten dimensional action corresponding to $N = 1$ supergravity is given by,

$$S_{10} = \int \sqrt{-\hat{g}} \; d^{10}x \left( \frac{1}{4} \hat{R} - \frac{1}{2} \partial_M \hat{\Phi} \partial^M \hat{\Phi} - \frac{1}{12} e^{-2\hat{\Phi}} \hat{H}_{MNP} \hat{H}^{MNP} \right)$$

(1)

where $\hat{R}$ is the curvature scalar in $D = 10$. The equations of motion following from this action are given by,

$$\hat{\nabla}_M \hat{\nabla}^M \hat{\Phi} + \frac{1}{6} e^{-2\hat{\Phi}} \hat{H}_{MNP} \hat{H}^{MNP} = 0$$

(2)

$$\hat{\nabla}_M (e^{-2\hat{\Phi}} \hat{H}^{MNP}) = 0$$

(3)

$$\hat{R}_{MN} - 2 \partial_M \hat{\Phi} \partial_N \hat{\Phi} - e^{-2\hat{\Phi}} \hat{H}_{MPQ} \hat{H}^{PQ}_N + \frac{1}{12} e^{-2\hat{\Phi}} \hat{g}_{MN} \hat{H}_{PQS} \hat{H}^{PQS} = 0$$

(4)
The dimensional reduction of the above ten dimensional theory in terms of suitable parametization has been discussed in a recent paper by Volkov \[13\]. The corresponding four dimensional equations of motion for metric, dilaton, axion and gauge fields are respectively given by,

\[
R_{\mu\nu} - 2\partial_\mu \Phi \partial_\nu \Phi + 2e^{-4\Phi} \partial_\mu a \partial_\nu a - 2e^{2\Phi} \left( \eta^{(1)}_{ab} F_{\mu\nu}^a F_{b\nu}^b + \eta^{(2)}_{ab} \hat{F}_{\mu\nu}^a \hat{F}_{b\nu}^b \right) + U(\Phi) = 0
\]

where \( U(\Phi) \) is the dilaton potential given by,

\[
U(\Phi) = -\frac{1}{8} (g_1^2 - g_2^2) e^{-2\Phi}
\]

The structure constants are given by,

\[
f^c_{\ ab} = \eta^{(1)cd} \epsilon_{dab} ; \quad \hat{f}^c_{\ ab} = \eta^{(2)cd} \epsilon_{dab}
\]

\( \epsilon_{abc} \) is the antisymmetric tensor, \( \eta^{(1)}_{ab} \) and \( -\eta^{(2)}_{ab} \) are the cartan metrics corresponding to \( SU(2) \) and \( SU(1, 1) \) respectively, where \( \eta^{(1)}_{ab} = diag(1, 1, 1) \) and \( \eta^{(2)}_{ab} = diag(1, 1, -1) \). The dual field strengths in the four dimensional Euclidean theory are defined as,

\[
\star F_{\mu\nu}^a = \frac{1}{2} \sqrt{g} \epsilon_{\mu\nu\lambda\rho} F_{a\lambda\rho}^a
\]

and,

\[
\star \hat{F}_{\mu\nu}^a = \frac{1}{2} \sqrt{g} \epsilon_{\mu\nu\lambda\rho} \hat{F}_{a\lambda\rho}^a
\]

The above equations of motion can be obtained from the four dimensional Euclidean Freedman-Schwarz action,

\[
S_4 = \int \sqrt{g} \ d^4x \left[ \frac{R}{4} - \frac{1}{4} \partial_\mu \Phi \partial^\mu \Phi + e^{-4\Phi} \partial_\mu a \partial^\mu a - \frac{e^{2\Phi}}{4} \left( \eta^{(1)}_{ab} F_{\mu\nu}^a F_{b\nu}^b + \eta^{(2)}_{ab} \hat{F}_{\mu\nu}^a \hat{F}_{b\nu}^b \right) - \frac{1}{2} \mathbf{a} \left( \eta^{(1)}_{ab} \star F_{\mu\nu}^a F_{b\nu}^b + \eta^{(2)}_{ab} \star \hat{F}_{\mu\nu}^a \hat{F}_{b\nu}^b \right) + \frac{1}{8} (g_1^2 - g_2^2) e^{-2\Phi} \right]
\]
Similarly the ten dimensional spinors can also be consistently reduced and the four dimensional supersymmetry variations are given by,

\[
\delta \chi = \left( \frac{1}{\sqrt{2}} \gamma^\mu \partial_\mu \Phi - \frac{1}{\sqrt{2}} e^{-2\Phi} \gamma_5 \gamma^\mu \partial_\mu a \right) \epsilon + \frac{1}{2} e^\Phi \left( \frac{1}{2} \eta^{(1)}_{ab} \gamma^\alpha \gamma^\beta \tilde{F}^a_{\alpha\beta} - \frac{1}{2} \gamma_5 \eta^{(2)}_{ab} \gamma^\alpha \gamma^\beta \tilde{F}^a_{\alpha\beta} \right) \epsilon + \frac{1}{2} e^{-\Phi} \left( g_1 - g_2 \gamma_5 \right) \gamma^\mu \epsilon,
\]

\[
\delta \Psi = \left( \partial^\mu + \frac{1}{4} \omega^\alpha_{\mu} \gamma^\alpha \gamma^\beta - \frac{g_1}{2} \eta^{(1)}_{ab} \gamma_5 \alpha^a \partial_\mu \alpha^b + \frac{g_2}{2} \eta^{(2)}_{ab} \gamma_5 \alpha^a \partial_\mu \alpha^b + \frac{1}{4} e^{-2\Phi} \gamma_5 \partial_\mu a \right) \epsilon + \frac{1}{2} e^{-\Phi} \left( \eta^{(1)}_{ab} A^b_a + \gamma_5 \eta^{(2)}_{ab} \tilde{F}^a_{\lambda\nu} \gamma^\lambda \gamma^\nu \epsilon \right) + \frac{1}{4} e^{-\Phi} \left( g_1 + g_2 \gamma_5 \right) \gamma^\mu \epsilon,
\]

where \( \epsilon \) is the Majorana spinor corresponding to the supersymmetry transformation parameter. Here, \( \gamma_5 = -\gamma^0 \gamma^1 \gamma^2 \gamma^3 \) and \( \{ \gamma_5, \gamma^a \} = 0 \). \( \gamma^a \) are the four dimensional tangent space gamma matrices satisfying the usual anticommutation relation \( \{ \gamma^a, \gamma^b \} = 2\eta^{ab} \) with \( \eta^{ab} = \text{diag}(+1,+1,+1,+1) \). \( \omega^\alpha_{\mu} \) are the spin connections and \( \alpha^a \) and \( \dot{\alpha}^a \) are the \( 4 \times 4 \) matrices which generate the Lie algebra of the group \( SU(2) \) and \( SU(1,1) \) respectively with the properties,

\[
\alpha^a \alpha^b = -\epsilon^{abc} \eta^{(1)}_{cd} \alpha^c - \eta^{(1)ab} \quad (16)
\]

\[
\dot{\alpha}^a \dot{\alpha}^b = -\epsilon^{abc} \eta^{(2)}_{cd} \dot{\alpha}^c + \eta^{(2)ab} \quad (17)
\]

\[
[\alpha^a, \dot{\alpha}^a] = 0 \quad (18)
\]

The corresponding matrix representation is given by,

\[
\alpha^a = i \tau^a \otimes I_2 \quad (a = 1, 2, 3) \quad \dot{\alpha}^a = I_2 \otimes \tau^a \quad (b = 1, 2) \quad \dot{\alpha}^3 = i I_2 \otimes \tau^3 \quad (19)
\]

where \( \tau^a \) are Pauli matrices.

In the examples below, we choose specific \( U(1) \) directions thereby spontaneously breaking \( SU(2) \) to \( U(1) \) and in the noncompact case, \( SU(1,1) \) to \( U(1) \), similar to Freedman-Gibbons electro-vac solutions with constant dilaton. This corresponds to setting the other two gauge fields of \( SU(2) \) triplets or the \( SU(1,1) \) triplet to zero vacuum expectation value. So whenever we have nonzero gauge fields, they are basically abelian.

### 3. Supersymmetric configurations in EFS model

In the examples below, we choose specific \( U(1) \) directions thereby spontaneously breaking \( SU(2) \) to \( U(1) \) and in the noncompact case, \( SU(1,1) \) to \( U(1) \), similar to Freedman-Gibbons electro-vac solutions with constant dilaton. This corresponds to setting the other two gauge fields of \( SU(2) \) triplets or the \( SU(1,1) \) triplet to zero vacuum expectation value. So whenever we have nonzero gauge fields, they are basically abelian.
3.1 Euclidean domain walls

First we consider the four dimensional Euclidean domain wall obtained by analytically continuing the Lorenztian domain walls [9] with the field configurations,

\[ ds^2 = U(y)(dt^2 + dx_1^2 + dx_2^2) + U^{-1}(y)dy^2, \]
\[ \Phi = \frac{1}{2} \ln U(y), \quad U(y) = m|y - y_0|, \]
\[ A_\mu^a = 0, \quad \dot{A}_\mu^a = 0, \quad a = 0, \]

(20)

This background is singular at \( y = y_0 \). Since there is no other matter field present here other than dilaton, the above configuration represents pure dilaton gravity in Euclidean space. We now study the supersymmetric properties of this background. For \( g_1 \neq 0 \) (in fact the gauge coupling constant and the mass parameter are related by \( g_2^2 = 2m^2 \)) and \( g_2 = 0 \), if we substitute the above background in the supersymmetry equations, we find that the fermionic variations vanish provided the supersymmetry parameters satisfy,

\[ \epsilon = -\gamma_3 \epsilon, \quad \epsilon = U(y)^\frac{i}{4} \epsilon_0 \]

(21)

where \( \epsilon_0 \) is a constant spinor. These conditions break half of the supersymmetry. Thus we see that there exist nontrivial killing spinors preserving \( N = 2 \) supersymmetry for pure dilatonic Euclidean domain wall background.

3.2 \( E^2 \times S^2 \)

Here we consider the case where dilaton is constant. We also take one of the \( U(1) \) gauge fields of the \( SU(2) \) part to be nonvanishing and the geometry as that of \( E^2 \times S^2 \). Corresponding field configurations are given by,

\[ ds^2 = d\psi^2 + d\chi^2 + \frac{1}{B}(d\theta^2 + \sin^2 \theta d\phi^2) \]
\[ F^a = \delta^{a3}Q \sin \theta d\theta \wedge d\phi \quad ; \quad \Phi = \Phi_0 = \text{constant} \]
\[ \dot{F} = 0, \quad a = \text{constant}, \quad g_1 = 0 \]

(22)

where \( \frac{1}{\sqrt{B}} \) is the constant radius of the two sphere. This configuration satisfies the equations of motion with \( Q = \frac{1}{g_2} \). The above solution is analogous to the magnetic solution in the charged Nariai black hole background [17], but here we are in the Euclidean space with \( F^2 = 2B^2Q^2 \) which is strictly positive.
Next, we discuss the supersymmetric property of this background. The only nonvanishing spin connection is \( \omega_{\phi}^{23} = -\cos \theta \). The components of the killing spinor equations are given by,

\[
\begin{align*}
\partial_\psi \epsilon &= 0 \\
\partial_\chi \epsilon &= 0 \\
\partial_\theta \epsilon + \frac{1}{2} \gamma_5 \gamma_2 \epsilon &= 0 \\
\partial_\phi \epsilon - \frac{1}{2} \cos \theta \gamma_2 \gamma_3 \epsilon + \frac{1}{2} \sin \theta \gamma_5 \gamma_3 \epsilon &= 0
\end{align*}
\]

with the condition

\[
\left( \gamma_5 \gamma_2 \gamma_3 \alpha^3 - 1 \right) \epsilon = 0.
\]

The complete set of killing spinors which are the solution to the above equations are

\[
\epsilon = e^{-\frac{1}{2} \theta \gamma_5 \gamma_2} e^{-\frac{1}{2} \phi \gamma_3 \gamma_2} \left( \frac{\gamma_5 \gamma_2 \gamma_3 \alpha^3 + 1}{2} \right) \epsilon_0
\]

where \( \epsilon_0 \) is some constant spinor and \( \alpha^3 \) is in \( SU(2) \). The operator \( [\gamma_5 \gamma_2 \gamma_3 \alpha^3 + 1] \) acts as a projection operator, hence breaks \( \frac{1}{2} \) supersymmetry.

### 3.3 \( E^2 \times AdS_2 \)

Here we consider the geometry \( E^2 \times AdS_2 \) with constant dilaton and the nonvanishing \( U(1) \) gauge field corresponding to the noncompact part of \( SU(1, 1) \). This choice of gauge field is necessary so that the background equations of motion are satisfied.

The field configurations are given by,

\[
\begin{align*}
ds^2 &= d\psi^2 + d\chi^2 + \frac{1}{B} \left( r^2 dt^2 + \frac{dr^2}{r^2} \right) \\
\hat{F}^{t a} &= \delta^{a3} Q dt \wedge dr , \quad \Phi = \Phi_0 = constant \\
F &= 0 , \quad a = constant , \quad g_2 = 0
\end{align*}
\]

where \( \frac{1}{\sqrt{B}} \) corresponds to the radius of the AdS space. The above background fields satisfy the equations of motion with \( Q = \frac{1}{g_1} \). The only nonzero spin connection is given by \( \omega_t^{23} = r \). Considering that the fermionic supersymmetry variations vanish, one gets the following equations for the \( \psi, \chi, t \) and \( r \) components for the killing spinor:

\[
\partial_\psi \epsilon = 0
\]
$\partial_\chi \epsilon = 0 \quad (30)$

$\partial_t \epsilon + \frac{r}{2} \gamma_2 \gamma_3 \epsilon + \frac{1}{2} r \gamma_2 \epsilon = 0 \quad (31)$

$\partial_r \epsilon + \frac{1}{2r} \gamma_3 \epsilon = 0 \quad (32)$

The projector is given by $[\gamma_5 \gamma_2 \gamma_3 \dot{\alpha}^3 + 1]$ where $\dot{\alpha}^3$ is along the noncompact direction in $SU(1,1)$. The killing spinors are

$$\epsilon = \frac{1}{2} r^{\frac{1}{2}} \left( \gamma_5 \gamma_2 \gamma_3 \dot{\alpha}^3 + 1 \right) \epsilon_- + \frac{1}{2} \left[ r^{-\frac{1}{2}} - r^{\frac{1}{2}} \gamma_2 t \right] \left( \gamma_5 \gamma_2 \gamma_3 \dot{\alpha}^3 + 1 \right) \epsilon_+ \quad (33)$$

where $\epsilon_\mp$ are constant spinors and they satisfy the conditions $\gamma_3 \epsilon_\mp = \mp \epsilon_\mp$. So as before, this background preserves one half of the supersymmetry.

These last two EFS solutions are analogous to the electro-vac solution in FS model. The later one, $E^2 \times AdS_2$, can be mapped to the Lorentzian section by applying the transformations as discussed in [13].

### 3.4 $E^1 \times S^3$

Here, we would like to obtain a background analogous to the pure axionic gravity solution in FS model [4]. So, we take nontrivial axion field while the dilaton as well as gauge fields are vanishing. The field configuration is given by,

$$ds^2 = d\psi^2 + \frac{1}{Q} \left( d\chi^2 + \sin^2 \chi \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right)$$

$$a = \pm \sqrt{Q} \psi; \quad \Phi = 0$$

$$A = 0; \quad \dot{A} = 0; \quad g_1 = 0 \quad (34)$$

The equations of motion of these background fields are consistent with $Q = \frac{g_2^2}{8}$. To study the supersymmetry properties, we need to calculate the spin connections on three sphere. The nonzero components are given by,

$$\omega^{21}_\theta = \cos \chi; \quad \omega^{31}_\theta = \cos \chi \sin \theta; \quad \omega^{32}_\phi = \cos \theta \quad (35)$$

The projector is given by,

$$\epsilon = -\gamma_0 \epsilon, \quad (36)$$

and the components of the killing equations are given by,

$$\partial_\psi \epsilon = 0 \quad (37)$$
\[ \partial_\chi \epsilon + \frac{1}{2} \gamma_5 \gamma_1 \epsilon = 0 \] (38)
\[ \partial_\theta \epsilon + \frac{1}{2} \cos \chi \gamma_2 \gamma_1 \epsilon + \frac{1}{2} \sin \chi \gamma_5 \gamma_2 \epsilon = 0 \] (39)
\[ \partial_\phi \epsilon + \frac{1}{2} \cos \chi \sin \theta \gamma_3 \gamma_1 \epsilon + \frac{1}{2} \sin \chi \sin \theta \gamma_5 \gamma_3 \epsilon + \frac{1}{2} \cos \theta \gamma_3 \gamma_2 \epsilon = 0 \] (40)

The complete solution of the killing spinor equation is given by,
\[ \epsilon = e^{-\frac{1}{2} \chi \gamma_5 \gamma_1} e^{-\frac{1}{2} \theta \gamma_2 \gamma_1} e^{-\frac{1}{2} \phi \gamma_3 \gamma_2} \left[ \gamma_0 + \frac{1}{2} \right] \epsilon_0 \] (41)

Hence this choice of field configurations breaks \( \frac{1}{2} \) of the supersymmetry.

We find that \( E^3 \times AdS_3 \) background can also be a solution but then one has to consider imaginary axion field to solve of the background equations of motion.

### 4 Gravitational Instantons

In this section we consider gravitational instantons which are solutions in Euclidean gravity. It has been noted in [13] that with vanishing dilaton, axion, gauge fields and for \( g_1 = g_2 \), the flat gravitational instantons (cosmological constant being zero) are vacua of EFS model. Here we show that even in the presence of (anti)self-dual gauge fields, the Eguchi-Hanson instanton [16] satisfying the flat space Einstein equations is a consistent background of this EFS model and it preserves certain fraction of the supersymmetry.

This is one of the examples, where we keep both the gauge coupling constants and we take them to be equal, \( g_1 = g_2 \). We take nonzero \( U(1) \) gauge field belonging to the noncompact part of \( SU(1,1) \) and the nonvanishing \( U(1) \) gauge field of the \( SU(2) \) part could be any of the triplet (let us choose \( A^3 \) to be nonzero). The field configuration is given by,
\[ ds^2 = \frac{dr^2}{1 - \frac{a^4}{r^4}} + \frac{r^2}{4} \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) + \frac{r^2}{4} \left( 1 - \frac{a^4}{r^4} \right) \left( d\psi + \cos \theta d\phi \right)^2 \]
\[ F = \frac{2}{r^4} \left( e^3 \wedge e^0 + e^1 \wedge e^2 \right) = \tilde{F} \]
\[ \Phi = 0; \quad a = 0; \quad g_1 = g_2 \] (42)

where the vierbeins are
\[ e^0 = \frac{dr}{\sqrt{1 - \frac{a^4}{r^4}}}; \quad e^1 = \frac{r}{2} \left( \sin \psi d\theta - \sin \theta \cos \psi d\phi \right) \]
\[ e^2 = \frac{r}{2} \left( - \cos \psi d\theta - \sin \theta \sin \psi d\phi \right); \quad e^3 = \frac{r}{2} \sqrt{1 - \frac{a^4}{r^4}} \left( d\psi + \cos \theta d\phi \right). \] (43)
One can immediately note that the gauge field strengths in (42) are anti-self-dual. The spin connections which are also anti-self-dual can be calculated from (43) and these are

\[
\begin{align*}
\omega_{\theta}^{10} &= \omega_{\theta}^{23} = \frac{1}{2} \sqrt{1 - \frac{a^4}{r^4}} \sin \psi ; \\
\omega_{\phi}^{10} &= \omega_{\phi}^{23} = -\frac{1}{2} \sqrt{1 - \frac{a^4}{r^4}} \sin \theta \cos \psi \\
\omega_{\theta}^{20} &= \omega_{\theta}^{31} = -\frac{1}{2} \sqrt{1 - \frac{a^4}{r^4}} \cos \psi ; \\
\omega_{\phi}^{20} &= \omega_{\phi}^{31} = -\frac{1}{2} \sqrt{1 - \frac{a^4}{r^4}} \sin \theta \sin \psi \\
\omega_{\psi}^{30} &= \omega_{\psi}^{12} = \frac{1}{2} (1 + \frac{a^4}{r^4}) ; \\
\omega_{\phi}^{30} &= \omega_{\phi}^{12} = \cos \theta \omega_{\psi}^{30}.
\end{align*}
\] (44)

With the above choice of background fields, the supersymmetry variations give the projector conditions,

\[
\begin{align*}
\left(1 - \gamma_5\right) \epsilon &= 0 \\
\left(\alpha^3 + \dot{\alpha}^3\right) \epsilon &= 0
\end{align*}
\] (45)

With these projectors, the killing spinor equations are really simplified:

\[
\partial_\mu \epsilon = 0.
\] (46)

So the killing spinors are independent of \(r, \theta, \phi, \psi\). Because of the twin supersymmetric conditions, the solution preserves \(\frac{1}{4}\) of the supersymmetry. However, once the gauge field backgrounds are switched off the second condition in eq.(45) will drop out and the pure gravitational instanton background will become half supersymmetric.

5 Summary

In this work, we have obtained stable vacuum configurations in \(D = 4, \, N = 4\) \(SU(2) \times SU(1,1)\) gauged supergravity theory (EFS model) which has been obtained from dimensional reduction of \(D = 10, \, N = 1\) supergravity on \(S^3 \times AdS_3\). We have obtained stable domain wall solutions preserving half of the supersymmetry. We have then considered vacua like \(E^2 \times S^2, \, E^2 \times AdS_2\) with constant dilaton and nontrivial \(U(1)\) gauge fields which are analogous to the electrovac solutions in FS model preserving half of the supersymmetry. We also obtained geometry like \(E^1 \times S^3\) with nontrivial axion and vanishing dilaton field preserving half of the supersymmetry. Finally we obtained Euclidean gravitational instanton, namely Eguchi-Hanson instanton with nontrivial abelian gauge fields.
and vanishing dilaton and axion fields as a consistent vacuum configuration breaking one fourth of the supersymmetry. The dilaton potential in this case vanishes as the two gauge coupling constants are equal. So our findings of a rich variety of vacua for the EFS model makes the theory more interesting and worth exploring further.
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