The Complex Torque Coefficient Applied in Subsynchronous Oscillation Caused by HVDC

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Abstract. Subsynchronous oscillation is an unstable state in power system, which may damage rotors of turbine generators and break the steady operation of the power system. In China, some accidents of subsynchronous oscillations had happened in the power stations where there are HVDC projects nearby. So that it is necessary to study the Subsynchronous oscillation caused by HVDC. After analysis of the principle of subsynchronous oscillation, the mathematical calculation model of the system containing HVDC is built and analysed in method of complex torque coefficient. And the simulation model is established in PSCAD/EMTDC to prove the validation of the method of complex torque coefficient used for the system containing HVDC, which combines calculation and simulation of the method for the system.

1. Introduction

According to the IEEE Committee Report by the Subsynchronous Resonance Working Group of the System Dynamic Performance Subcommittee[1], subsynchronous resonance (SSR) is an abnormal state of power system, where the power system exchanges energy with turbine generators at one or more frequencies below the synchronous frequency. Besides the interaction of the turbine generators and the series capacitor compensated power system network, subsynchronous oscillation (SSO) also results from the interactions with other sources, including high voltage direct current (HVDC), static var compensator (SVC), power system stabilizer (PSS).

In China, HVDC meets the large demand of transmission capacity and distance, thus becoming the powerful complement to AC power transmission. By far, China has the largest number of HVDC projects and the highest voltage level around the world[2,3]. Generally, SSO occurred the turbine generators near the HVDC projects, which may cause the torsional vibration, out of steady state and even shaft breakage threatening the safety of power system operation. In 1977, The first SSO caused by HVDC happened in Square Butte project of the United States[4]. The research finally indicated that the torsional vibration was caused by the interaction of power oscillation of the turbine generator with the HVDC. The similar accidents also happened in China several times. In October of 2008, the Gaoling back-to-back HVDC caused SSO of the turbine generators in Suizhong power plant, which connects the northeast grid and the north grid of China[5]. In September of 2010, the HVDC project...
from Hulun Buir to Liaoning caused SSO of the turbine generators in Yimin power plant\cite{6}. Therefore, it is urgent to research the SSO characteristic between the HVDC and turbine generators.

The analysis methods fall into two categories: qualitative methods and quantitative methods. Qualitative methods are applied to roughly analyse the possibility of SSO in power system and find out the risky generators. The qualitative methods need a small amount of original data and is easy to calculate, but the error of calculation results is considerable. The qualitative methods include UIF and frequency scanning. The quantitative methods are applied to concretely analyze the SSO characteristic, with detailed original data for more precise research. The qualitative methods include eigenvalue analysis, complex torque coefficient and the time domain simulation. Eigenvalue analysis has a clear theory and high analysis accuracy. But it is difficult to apply in big power system networks, because the factors matrix of the model is too complicated to be calculated. Although the complex torque coefficient also needs detailed turbine generators’ rotors parameters as eigenvalue analysis, its calculation is easier. It is feasible for applying in big power system networks. The time domain simulation can evaluate the power system stability by the laws of changes of variables with time in electromagnetic transient simulation application. It has high speed and accuracy, however the mechanism of SSO cannot be presented.

In conclusion, the complex torque coefficient is suitable to study the subsynchronous oscillation caused by HVDC, which can both ensure the accuracy of big power system network and the feasible calculation.

Firstly, the mechanism of SSO caused by HVDC is studied. Then the mathematical models of turbine generators and HVDC are established for analysis by the complex torque coefficient. After that simulation model is built in PSCAD/EMTDC to verify the validation of the method combining theory calculation and simulation, which is applied to SSO caused by HVDC.

2. The mechanism of SSO caused by HVDC

The common reason that HVDC caused SSO is the fast control of HVDC, in which the regulating effect provide feedback to the turbine generator to torsional vibration, causing unstable oscillation.

The simplified network configuration of the effect of HVDC on SSO is shown in Figure 1., in which the left part is the grid and the right part is HVDC. In the left part, the aim turbine generators are separated from the grid to focus on the SSO happening. In the right part, \( u \) and \( i \) are voltage and current of the AC voltage of commutation bus, \( x_T \) is the commutation reactance, and the \( U_d \) and \( I_d \) are the voltage and current of the DC.

\[
U_d = \frac{3\sqrt{2}}{\pi} U \cos \alpha - \frac{3}{\pi} x_T I_d
\] (1)

When the voltage of the turbine generator rotor changes, the voltage of stator accordingly changes leading to the change of the amplitude and the phase of the voltage of the AC voltage of commutation bus. According to Equation (1), the changes of the voltage amplitude of the AC voltage of commutation bus \( (U) \) and the firing angle \( (\alpha) \) can result in the change of DC voltage \( (U_d) \), thus the DC current and power changing.

The change of DC current activates the constant-current control, thus changing the firing angle \( (\alpha) \). Then the active power of generators to HVDC, changing the electrical torque of the generator rotor.
When the phase lag of the rotor shaft speed and the final change in electrical torque on the generator rotor exceed 90°, the unstable torsion oscillation may occur.

3. The complex torque coefficient analysis of SSO caused by HVDC

3.1. The principle of the complex torque coefficient

3.1.1. The turbine generator rotor. If the amplitude of the disturbance on turbine generators rotor is A, the displacement increment on the rotor is

\[ \Delta \delta = A \sin \omega_m t \]

in which \( \omega_m \) is the natural frequency.

This disturbance makes the rotor speed increase

\[ \Delta \omega = \frac{d \Delta \delta}{dt} = A \omega_m \cos \omega_m t \]

The speed increasement causes the according changes on voltage and flux linkage of the generator stator. The voltage increasement of phase a in dq-coordinates is

\[ \Delta u_{a1} = \Delta u_d \cos \omega t - \Delta u_q \sin \omega t + \]

\[ \Delta \delta (\Delta u_d \sin \omega t - \Delta u_q \cos \omega t) - \Delta \delta (u_d \sin \omega t - u_q \cos \omega t) \]

(2)

After calculation with higher order factors ignored, the voltage increasement of phase a is

\[ \Delta u_{a2} = \sqrt{\psi_d^2 + \psi_q^2} \frac{A}{2} (\omega - \omega_m) \sin[(\omega - \omega_m) t + \rho] - \]

\[ \frac{A}{2} (\omega + \omega_m) \sin[(\omega + \omega_m) t + \rho] \]

(3)

in which \( \psi_d \) and \( \psi_q \) are the flux linkages of d-axis and q-axis, and \( \rho = \arctan(\psi_q/\psi_d) \).

Equation (3) shows that the voltage increasement contains subsynchronous \((\omega - \omega_m)\) and supersynchronous \((\omega + \omega_m)\) components.

(1) The subsynchronous component of the voltage increasement causes current increasement with a frequency of \((\omega - \omega_m)\), and the current has the same phase as the voltage increasement, which can be expressed as

\[
\begin{align*}
\Delta i_{a2} &= \Delta I \sin[(\omega - \omega_m) t + \rho] \\
\Delta i_{b2} &= \Delta I \sin[(\omega - \omega_m) t + \rho - 2\pi/3] \\
\Delta i_{c2} &= \Delta I \sin[(\omega - \omega_m) t + \rho + 2\pi/3]
\end{align*}
\]

in which,

\[ \Delta I = \sqrt{\psi_d^2 + \psi_q^2} \frac{A}{2} \frac{\omega - \omega_m}{R} \]

and the R is the resistance of the stator circuit.

The subsynchronous electromagnetic torque of the rotor calculated by current increments in dq-coordinates is

\[ \Delta T_e = \psi_d \Delta i_d - \psi_q \Delta i_q = - \frac{A}{2} \frac{\omega - \omega_m}{R} (\psi_d^2 + \psi_q^2) \cos \omega_m t \]

(4)
(2) The current increasement with a frequency of \((\omega + \omega_m)\) caused by supersynchronous component of the voltage increasement is

\[
\begin{align*}
\Delta i_{q2} &= \Delta I \sin[(\omega + \omega_m)t - \pi/2 + \rho] \\
\Delta i_{q3} &= \Delta I \sin[(\omega + \omega_m)t - \pi/2 + \rho - 2\pi/3] \\
\Delta i_{q4} &= \Delta I \sin[(\omega + \omega_m)t - \pi/2 + \rho + 2\pi/3]
\end{align*}
\]

in which

\[
\Delta I = \sqrt{\psi_d^2 + \psi_q^2} \frac{A(\omega + \omega_m)}{2X}
\]

and \(X\) is the reactance of the stator circuit.

In the similar way, the supersynchronous electromagnetic torque of the rotor is

\[
\Delta T_e = -\psi_d \Delta i_d - \psi_q \Delta i_q = \frac{A}{2} \frac{\omega + \omega_m}{R} (\psi_d^2 + \psi_q^2) \sin \omega_m t
\]

To sum up the equation (4) and (5), the total electromagnetic torque caused by disturbance on the rotor is

\[
\Delta T_e = \Delta T_e^r + \Delta T_e^c = \frac{\omega + \omega_m}{2\omega_m X} (\psi_d^2 + \psi_q^2) \Delta \delta - \frac{\omega - \omega_m}{2\omega_m R} (\psi_d^2 + \psi_q^2) \Delta \omega
\]

3.1.2. The criteria of complex torque coefficient. The concept and judgement of the complex torque coefficient are firstly proposed by Canay in 1982\[7\], which uses two complex torque coefficients to present the electrical and mechanical torques. Then the possibility of occurrence of SSO in the system is evaluated by analyse the torque coefficients.

Equation (6) shows that the total electromagnetic torque can be expressed as \(\Delta T_e = K_m \Delta \delta + D_m \Delta \omega\). The equal mechanic coefficient is defined as \(K_m = \Delta T_m / \Delta \delta\), and the equal electric coefficient is defined as \(K_e = \Delta T_e / \Delta \omega\). \(K_m\) and \(K_e\) are complex, which are

\[
K_m(j\eta) = K_m(\eta) + j\eta D_m(\eta)
\]

\[
K_e(j\eta) = K_e(\eta) + j\eta D_e(\eta)
\]

in which \(\eta = 2\pi f\); \(K_m\) and \(D_m\) are the equal mechanic coefficients of elastic and damping, \(K_e\) and \(D_e\) are the equal electric coefficients of elastic and damping, and they are all functions of frequency \(f\).

The damping characteristic of the system can be analyzed though the laws of \(D_m\) and \(D_e\) by \(f\) ranging from 0~50Hz. The criteria of SSO occurrence is

\[
(D_m + D_e) \left|_{K_m + K_e = 0} < 0
\]

Below it is about how to get the equal mechanic coefficients of elastic \((K_m)\) and damping \((D_m)\) and the equal electric coefficients of elastic \((K_e)\) and damping \((D_e)\). Hence it can be evaluated whether the SSO will occur the system by Equation (7), and when \(D_m + D_e < 0\), unstable oscillation may happen in the system.

3.2. Method to get the complex torque coefficient

3.2.1. The mechanic coefficient. The mechanic torque increasement equals to the electric torque after the equations of the rotor linearized and the armature reaction torque and the function of turbine generator governor are ignored, therefore which is

\[
(D_m + D_e) \left|_{K_m + K_e = 0} < 0
\]
\[ K_M(j\eta) = K_m(\eta) + j\eta D_m(\eta) \quad (8) \]

in which \( K_m(\eta) \) is proportional to rotor phase, \( D_m(\eta) \) is proportional to the rotor speed, which are defined as equal mechanic coefficient of elastic and damping.

3.2.2. The electric coefficient. When HVDC is in steady state of Figure 1., there are other equations besides Equation (1)

\[ \begin{align*}
I &= \frac{\sqrt{6}}{\pi} I_d \\
\cos \varphi &= \frac{U_d}{3\sqrt{2} U} \\
\Delta \alpha &= \Delta \alpha_c + \Delta \delta_c \\
\Delta \delta_c &= \Delta \delta_c - \Delta \varphi
\end{align*} \quad (9) \]

in which \( \delta_c \) and \( \delta_l \) are phases of the voltage and current of the AC voltage of commutation bus, \( \Delta \delta_c \) and \( \Delta \delta_l \) are their phase errors, \( \varphi \) is the power factor, \( \Delta \alpha_c \) is the output increasement of the instant-current control.

The turbine generators are set to operate in island, which means \( u_s = U \) and \( Z \rightarrow +\infty \). The voltage of the AC voltage of commutation bus is set to be \( U = U_0 \), and the output apparent power is set to be \( S = P + jQ \). Refer to the calculations in [8], there are these equations of the armatures of direct-axis and quadrature-axis and excitation system by Park mode.

\[ X_G = j\eta \Delta \delta(j\eta I - \begin{bmatrix} H_d^{-1}N_d \\ H_q^{-1}N_q \\ 0 \end{bmatrix}) \begin{bmatrix} H_d^{-1}M_d \\ H_q^{-1}M_q \\ 0 \end{bmatrix} P_U \cdot N_i N_h^{-1} N_0 I_{\beta} \cdot P_i^{-1} N_2^{-1} M_d \]

\[ \begin{bmatrix} H_d^{-1}P_d \\ H_q^{-1}P_q \\ 0 \end{bmatrix} \quad (10) \]

in which \( N_h, P_i, P_l, I_\beta \) are parameters of HVDC, \( H_d, N_d, M_d, P_d \) are parameters of armature of d-axis, \( H_q, N_q, M_q, P_q \) are parameters of armature of q-axis and \( N_0, N_i, N_2 \) are constants matrixes.

Therefore, the increasement of electromagnetic torque is presented as

\[ \Delta P_e = (\psi_{d0} \Delta i_d - \psi_{q0} \Delta i_d) + (i_{q0} \Delta \psi_d - i_{d0} \Delta \psi_q) \]

\[ = P \cdot X_G \quad (11) \]

in which \( P = \begin{bmatrix} -x_q i_{q0} - \psi_{q0} & x_a i_{q0} & x_a i_{q0} & x_q i_{d0} + \psi_{d0} & -x_{aq} i_{d0} & -x_{aq} i_{d0} & 0 & 0 \end{bmatrix} \), the reactance, current and flux linkage are variable of stator, and the subscript ‘0’ presents the original state.

Hence the equal electric complex torque coefficient is

As

\[ K_E(j\eta) = K_e(\eta) + j\eta D_e(\eta) \quad (12) \]

therefore the equal electric torque coefficient is

\[ K_e(\eta) = \text{Re}\{K_E(j\eta)\} \quad (13) \]

and the equal mechanic complex torque coefficient is
\[
D_r(\eta) = \frac{\text{Im}\{K_e(j\eta)\}}{\eta}
\]  

(14)

Above all, by calculating the complex torque coefficients, the equal elastic and damping complex torque coefficients of electric and mechanic are gained to evaluate the possibility of SSO occurrence in the system through Equation (7).

4. Simulation verification

In this chapter, a simulation model in PSCAD/EMTDC is established to verify the validation of equations of complex torque coefficients.

4.1. Calculation of complex torque coefficients

In the model, the HVDC section is the same as the CIGRE Benchmark model for HVDC controls and the generator is the same as in the IEEE first Benchmark case for subsynchronous resonance studies. The capacity of the generator is 892.4MVA, the voltage of the output of the generator is 26kV, and the mechanic complex torque is 0. According to the parameters of the model, complex torque coefficients can be calculated by Equation (9) ~ (14), of which the laws show the SSO characteristic.

Set a disturbance at time of 5s with the whole simulation time of 10s. The commutation failure occurs the HVDC at the disturbance and then the system goes back to steady operation. The steady equal electric complex torque coefficient \((D_e)\) changes by frequency \((f)\) as Figure 2.

![Figure 2. \(D_e\) (in calculation)](image)

Figure 2. shows that there is obvious electrical negative damping in the subsynchronous frequencies range. As the mechanical damping of complex torque is 0, the negative damping of electric equals the system unstable, thus verifying to be able to analyse the SSO characteristic of the model system by damping coefficients \((D_m\) and \(D_e)\).

4.2. Simulation analysis

In the model, equal electric damping coefficient can be scanning by frequency ranging from 0~50Hz, which is shown in Figure 3. When equal mechanic complex coefficient is 0, the equal elastic and damping coefficients of mechanic are 0.

![Figure 3. \(D_e\) (in frequency scanning)](image)

Comparing Figure 2 and 3, the SSO frequency range is wider in the calculation result than in the simulation. Meanwhile the damping coefficient is less negative in the calculation result than in the simulation. Nonetheless the calculation and simulation results both demonstrate the complex torque
coefficient can estimate the occurrence of SSO with the error making little effect. In other words, when the equal electric damping is negative in the model, unstable oscillation on the generator will happen. The waveforms of rotor speed and electromagnetic torque in total simulation time are shown in Figure 4., in which SSO can be seen.

![Waveform of Rotor Speed and Electromagnetic Torque](image)

(a) the rotor speed

(b) the electromagnetic torque

**Figure 4.** the SSO occurred

The waveforms in simulation are coincident with the complex torque coefficient results, which the SSO occurred the model system. From the above consequence, the complex torque coefficient can effectively evaluate the possibility of a system containing HVDC. The calculation and simulation both verify that when $K_m$, $D_m$, $K_e$ and $D_e$ meet the condition of Figure (7), unstable oscillation may happen in the system containing HVDC.

5. Conclusion

Through the complex torque coefficient, the subsynchronous oscillation caused by HVDC is analysed. The control of HVDC caused the electrical negative damping on the turbine generator’s rotors, bringing about torsional vibration and SSO. Not only can the complex torque coefficient analyse the big power system in detail, but also it ensures the feasible calculation amount. Calculation and simulation results together prove that the complex torque coefficient can evaluate whether unstable oscillation will occur in the system containing HVDC. When the equal elastic and damping complex torque coefficients of electric and mechanic ($K_m$, $D_m$, $K_e$ and $D_e$) meet the condition of $(D_m + D_e)\mid_{K_e + K_m = 0} < 0$, it can be judged that SSO may happen in the system containing HVDC.

6. References

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