Distributed Entanglement as a Probe for the Quantum Structure of Spacetime

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Simultaneity is a well-defined notion in special relativity once a Minkowski metric structure is fixed on the spacetime continuum (manifold) of events. In quantum gravity, however, the metric is not expected to be a fixed, classical structure, but a fluctuating quantum operator which may assume a coherent superposition of two classically-distinguishable values. A natural question to ask is what happens to the notion of simultaneity and synchronization when the metric is in a quantum superposition. Here we show that the resource of distributed entanglement of the same kind as used by Jozsa et al. [Phys. Rev. Lett. 85, 2010 (2000)] gives rise to an experimental probe that is sensitive to coherent quantum fluctuations in the spacetime metric.

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For a given choice of a Minkowski metric structure on the spacetime continuum, simultaneity is a uniquely defined notion in special relativity. Although there is an infinite class of indistinguishable but equivalent Minkowski metrics on the spacetime manifold, the specific metric that is to be used is a philosophical problem that seems to have no consequences for real experiments. In particular, it does not have any computational implications for classical physics \cite{1}. In quantum gravity, however, the metric is not expected to be a fixed, classical structure, but a fluctuating quantum operator. In particular, it is conceivable that the metric can be in a coherent superposition of two classically-distinguishable values. Experimentally, there exist well known protocols to construct the classical metric once a labeling of actual events as spacetime points is carried out. One particular such protocol involves the synchronization of the clocks of two distant observers (Alice and Bob) at rest with respect to each other. Recently, clock synchronisation received renewed interest with the added resource of shared entanglement between Alice and Bob \cite{2}. A natural question to ask is what happens to the notion of simultaneity and synchronization when the metric is in a quantum superposition. Here we show that the resource of distributed entanglement of the same kind as used in \cite{2} gives rise to an experimental probe that is sensitive to coherent quantum fluctuations in the spacetime metric.

In special relativity, given a fixed Minkowski metric $g$ on spacetime $\mathbb{R}^4$, simultaneity is defined as follows: let $u^\alpha$ be the four-vector of an inertial observer, Alice, and let $P$ be an event along the world-line of Alice. Then Alice’s surface of simultaneity at $P$ is the set of all events $Q$ such that the space-like vector (or geodesic) $S^\alpha$ joining $P$ to $Q$ is orthogonal to $u^\alpha$: $g_{\alpha\beta}u^\alpha S^\beta = 0$. This definition is formulated entirely in terms of physically observable quantities, and, given a fixed metric $g_{\alpha\beta}$, is implemented in practice using the Einstein synchronization protocol.

The protocol works as follows: suppose Alice and Bob are separated by a (large) distance $d$. Alice sends a light signal to Bob, who uses a mirror to return the signal immediately. Alice then measures the time interval between the departure at $t_1$ and the arrival at $t_3$ of the signal: $(t_3 - t_1)$, and defines the half-way time $t_2$ through this interval as

$$t_2 \equiv t_1 + \frac{1}{2} (t_3 - t_1) \, .$$

By construction, the spacelike vector joining the event at time $t_2$ on Alice’s worldline to the event of reflection $t_2'$ in Bob’s mirror is orthogonal to Alice’s four velocity. In other words, $t_2$ and $t_2'$ lie on a surface of simultaneity for Alice (and Bob), according to the above definition. Alice now tells Bob that the time of reflection (which Bob recorded, e.g., by measuring the impulse on the mirror) was at $t_2$ on her clock. Bob can then adjust his clock so that his measured time at this event coincides with $t_2$, and we have therefore obtained clock synchronisation in accordance with the above definition.

Notice that this protocol depends crucially on the specific Minkowski metric $g_{\alpha\beta}$ that is fixed from the outset (see Fig. \textsuperscript{3}). In general, there exists an infinite class of distinct Minkowski metrics on the manifold $\mathbb{R}^4$: For any diffeomorphism $\phi : \mathbb{R}^4 \rightarrow \mathbb{R}^4$, the metric $g' \equiv \phi^* (g)$ is an element in that class (where $\phi^*$ denotes the tensorial “pullback” map associated to the diffeomorphism $\phi$) distinct from $g$ unless $\phi$ happens to be a transformation in the Poincare group (i.e. a Lorentz transformation combined with translations). Which specific metric represents the real Lorentz structure is an operational question that can in principle be answered by experiment; nevertheless, since physics is invariant under isometries, these different Minkowski metrics are in any case physically equivalent to each other \cite{4,5}.

So far, the discussion has been purely classical. In quantum mechanics Alice and Bob might share entanglement, and the spacetime metric is generally no longer a fixed background structure, but is subject to quantum
fluctuations. In this paper, we will show how this extra resource of shared distributed entanglement can be used as an experimental probe that is sensitive to coherent quantum fluctuations in the spacetime metric.

Consider our two observers, Alice and Bob, who initially are co-located and share a singlet state of two qubits whose computational basis states $|0\rangle$ and $|1\rangle$ correspond to nondegenerate (distinct) energy levels $E_0$ and $E_1$ (where we define without loss of generality $E_1 > E_0$). The initial quantum state of the joint system is given by

$$
|\Psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A |1\rangle_B - |1\rangle_A |0\rangle_B). \tag{2}
$$

Throughout this paper, the subscripts $A$ and $B$ Alice and Bob respectively. Suppose this entanglement is now distributed by letting Alice move a large distance $d$ away from Bob. After the distribution, when Bob and Alice are at relative rest again, the state of the system can be written in the form

$$
|\Psi\rangle = \frac{1}{\sqrt{2}} \left( e^{-i\hbar \Omega_0 \tau_A} |0\rangle_A \otimes e^{-i\hbar \Omega_1 \tau_B} |1\rangle_B - e^{-i\hbar \Omega_1 \tau_A} |1\rangle_A \otimes e^{-i\hbar \Omega_0 \tau_B} |0\rangle_B \right), \tag{3}
$$

where $\tau_A$ and $\tau_B$ are the proper times that elapsed in Alice and Bob’s frame during the entanglement transport, and $\hbar \Omega_0$ and $\hbar \Omega_1$ are the ground and excited state energies, respectively. Up to an overall phase, the state Eq. (3) can be rewritten as

$$
|\Psi\rangle = \frac{1}{\sqrt{2}} \left( |0\rangle_A |1\rangle_B - e^{i\Omega (\tau_B - \tau_A)} |1\rangle_A |0\rangle_B \right), \tag{4}
$$

where $\Omega \equiv \Omega_1 - \Omega_0$. When Alice and Bob are at relative rest (comoving: $\tau_A = \tau_B$), $|\Psi\rangle$ is a dark state since its time evolution corresponds to multiplication by an overall phase factor.

Alice and Bob now execute the clock synchronization protocol introduced by Jozsa et al. [3]. First Alice makes a measurement on her qubit in the $\{|\pm\rangle_A\}$ basis:

$$
|+\rangle_A \equiv \frac{1}{\sqrt{2}} (|0\rangle_A + |1\rangle_A),
$$

$$
|-\rangle_A \equiv \frac{1}{\sqrt{2}} (|0\rangle_A - |1\rangle_A), \tag{5}
$$

and communicates the result classically to Bob. Assume that Alice obtains the result $|+\rangle_A$. Bob then knows that his qubit is in the reduced state

$$
|\phi(-)\rangle_B = \frac{1}{\sqrt{2}} \left( |1\rangle_B - e^{i\Omega (\tau_B - \tau_A)} |0\rangle_B \right), \tag{6}
$$

obtained via the projection $P_{|+\rangle_A} |\Psi\rangle$ from the state Eq. (4). This is the same protocol used in the clock synchronization application of [3], and the phase $\Omega (\tau_B - \tau_A)$ is the “Preskill” phase which makes this application difficult to implement in practice [3]. Here we are not interested in a synchronization protocol, however, and the important thing about the state Eq. (6) is that it is a pure state (though an unkown pure state) as Bob can verify experimentally.

Suppose now that the background spacetime on which the above protocol is implemented has a metric subject to quantum fluctuations. Let us assume that the quantum state of the metric $|g\rangle$ is in a coherent superposition of two macroscopically-distinguishable (orthogonal) states $|g_0\rangle$ and $|g_1\rangle$ given by

$$
|g\rangle = \alpha |g_0\rangle + \beta |g_1\rangle, \tag{7}
$$

where $\alpha$ and $\beta$ are complex numbers with $|\alpha|^2 + |\beta|^2 = 1$. Assume, furthermore, that the proper time elapsed during Alice’s trip to her final destination differs for the two metrics $g_0$ and $g_1$ by a (small) time interval $\Delta$. Under these assumptions, after entanglement distribution the total state of the singlet system plus the metric can be written in the form [compare Eq. (3)]

$$
|\Psi, g\rangle = \frac{1}{\sqrt{2}} \left[ |0\rangle_A e^{-i\hbar \Omega_0 \tau_B} |1\rangle_B \otimes \left( e^{-i\hbar \Omega_1 \tau_A} \alpha |g_0\rangle + e^{-i\hbar \Omega_1 (\tau_A + \Delta)} \beta |g_1\rangle \right) - |1\rangle_A e^{-i\hbar \Omega_0 \tau_B} |0\rangle_B \otimes \left( e^{-i\hbar \Omega_1 \tau_A} \alpha |g_0\rangle + e^{-i\hbar \Omega_1 (\tau_A + \Delta)} \beta |g_1\rangle \right) \right]. \tag{8}
$$

From the point of view of Alice and Bob, the quantum state of the gravitational field is inaccessible via any direct observation; therefore, their joint state is described
by tracing over the gravitational part of the wave function Eq. (8):

$$\rho_{AB} = \text{Tr}_g [\Psi, g]/[\Psi, g]$$

$$= \frac{1}{2} \left[ |0\rangle_A (|0\rangle_B \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B |0\rangle_B - W |1\rangle_A (|0\rangle_A \otimes |0\rangle_B |0\rangle_B + |1\rangle_B |0\rangle_B |1\rangle_B ) \right],$$

where $W$ denotes the complex number

$$W \equiv e^{i\Omega (\tau_B - \tau_A)} (|\alpha|^2 + e^{-i\Omega \Delta}|\beta|^2).$$

(10)

What will happen when Bob and Alice carry out the protocol described above [Eqs. 3-6]? Alice performs the measurement in her $\{|\pm\rangle_A\}$ basis, and obtains a random string of outcomes $\{+\}$, $\{\pm\}$. She sends this bit string to Bob. In practice, this means that Alice tells Bob which of her enumerated singlet-halfs from a large ensemble are projected onto the $|+\rangle_A$ state. Note that this calculation describes an experiment in which the various ensembles are obtained via repeated application of the entanglement distribution and measurement protocols described above, in each such application the metric being in the same coherent state described by Eq. (7). The state $\rho_{AB}$ in Eq. (9) then collapses to the density matrix

$$\rho_{AB} \rightarrow \frac{P_{++} \rho_{AB} P_{++}}{\text{Tr}(P_{++} \rho_{AB} P_{++})}$$

$$= \frac{1}{2} |+\rangle_A \langle +|_A$$

$$\otimes \left[ |0\rangle_B |0\rangle_B + |1\rangle_B |1\rangle_B - W |0\rangle_B |1\rangle_B - W |1\rangle_B |0\rangle_B \right].$$

(11)

Bob’s reduced state, therefore, is given by

$$\rho_B = \frac{1}{2} \left[ |0\rangle_B |0\rangle_B + |1\rangle_B |1\rangle_B - W |0\rangle_B |1\rangle_B - W |1\rangle_B |0\rangle_B \right].$$

(12)

Contrast Eq. (12) with the pure state Eq. (6). The state Eq. (12) is pure if and only if

$$\text{Tr} \rho^2 = \frac{1}{2} (1 + |W|^2) = 1,$$

(13)

which is possible if and only if $|W| = 1$. But

$$|W|^2 = 1 - 4|\alpha|^2|\beta|^2 \sin^2 \left( \frac{\Omega \Delta}{2} \right)$$

$$\approx 1 - |\alpha|^2|\beta|^2 \Omega^2 \Delta^2,$$

(14)

where the approximate equality assumes $\Omega \Delta \ll 1$. In general, Bob’s state at the end of the protocol is mixed, and if experiment can distinguish this “decoherence” effect from other sources of decoherence, it provides a possible probe for the quantum fluctuations in the spacetime metric.

A similar gravitational decoherence effect could be produced by using a single clock qubit carried by Alice through the region with the fluctuating metric. However, the advantages of using the singlet state as a probe are twofold: First, the singlet state is immune to phase decoherence effects that interact with both qubits in the same way; so it provides a more localized probe than a single qubit would. Second, the singlet state allows the final measurement process to be performed in a region (Bob’s location) arbitrarily distant from the region where the gravitational interaction takes place (Alice’s worldline); thus there is no danger of the final measurement process “collapsing” the state of the metric leading to a false negative result (a pure-state outcome for Bob).

What are the prospects for this probe to be a realistic one? For Planck scale fluctuations in the metric, for which $\Delta \sim 10^{-13}$ sec (the Planck time), an observable effect will be present if $\Omega \sim 10^{43}$ Hz, which corresponds, not surprisingly, to energies $\hbar \Omega$ of the order of the rest energy corresponding to the Planck mass (roughly the mass of a grain of sand). This is a macroscopic amount of energy (difference): the entangled state Eq. (2) would have to be a macroscopic, “Schrödinger’s cat” state and preserve its coherence over the large distance $d$. Since many non-gravitational sources of decoherence would tend to destroy such states rather quickly, the prospects for a successful experiment to probe for Planck-scale geometry fluctuations are not good.

There are, however, recent intriguing suggestions that microscopic black holes could be produced in large hadron colliders currently under construction [11]. According to these suggestions, string theory may present a new length scale much larger than the Planck length at which the gravitational interaction becomes a dominant force, allowing much lower energy thresholds for black hole production. The black holes produced in future hadron colliders may be as light as 1 TeV in mass, corresponding to a length scale on the order of $10^{-17}$ cm. This new mass-length scale involves the geometry of compactified dimensions in string theory, but if these small black holes have gravitational signatures in the non-compactified dimensions that have length scales comparable to $10^{-17}$ cm, or equivalently time scales of $10^{-27}$ sec, then our probe would be sensitive to such fluctuations provided $\Omega \sim 10^{27}$ Hz, which corresponds to energies on the order of $\hbar \Omega \sim 1$ TeV. This is a mesoscopic energy scale, and it is not inconceivable that entangled states of mesoscopic systems, like for example high photon-number path-entangled states [12,13], can be produced and maintained against non-gravitational decoherence just long enough for the protocol we discussed above to be practical. We believe this possibility is intriguing enough to deserve further study.

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