Forecasting the installed wind capacity using a new information priority accumulated nonlinear grey Bernoulli model

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Abstract. Wind energy is an important renewable and clean energy, and the accurate realization of the prediction of the installed wind capacity is urgent and should not be delayed. This paper proposes a novel new information priority accumulated nonlinear grey Bernoulli model and applies it to forecast the installed wind capacity in Europe and South America. The nonlinear parameters of the proposed model are optimized by the Grasshopper Optimization Algorithm. In the application case, the model our proposed has a more satisfactory predictive performance by comparing it with other three classical models.

1. Introduction

With the advent of the energy crisis and the deterioration of environmental pollution problems, renewable energy, and clean energy have developed rapidly. As an important renewable energy source in the current energy structure, the wind is considered to be the key driving force for an effective solution to pollution emissions. Other studies believe capturing 20% of the global potential of wind would satisfy the entire world’s need for energy [1]. Therefore, the development of wind power has attracted the attention of many countries and regions, and the development of wind power is regarded as an important measure to adjust the energy structure, protect the environment and reasonably realize sustainable development. Moreover, the installed wind capacity is a direct reflection of the country’s wind power development potential. For a long time, the wind power market in Europe is growing healthily, and the wind power market in South America has also made some breakthroughs in recent years. Without a doubt, achieving accurate forecasting of the installed wind capacity will facilitate decision-makers.

The grey system is first proposed by professor Deng in 1982 [2], which has been widely recognized by many scholars because of its outstanding ability to deal with the simple sample problems with poor information. Nowadays, the grey system theory has been successfully applied in many fields and many scholars have invested in the improvement and optimization of the classical grey model. Cui et al. [3] first proposed replacing the traditional grey action with the time linear action and proved the effectiveness of this method with numerical cases. Qian et al. [4] designed a novel grey model with time power driven and applied it to forecast the foundation settlement. Later, Chen et al. [5] first developed a novel nonlinear grey Bernoulli model named NGBM, which could obtain more
satisfactory results by controlling a nonlinear parameter. Thereafter, Hsu et al. [6] applied an optimized NGBM to study the integrated circuit industry in Taiwan. And Wu et al. [7] extended the classical NGBM to the fractional grey model.

On the other hand, some researchers believed the construction of accumulated generation operation can also effectively improve the performance of grey models. Wu et al. [8] first proposed fractional accumulated generation operation in 2013. Recently, Ma et al. [9] designed a novel conformable fractional grey model. Although these methods improved the prediction effect of grey models, they are still not always satisfactory in many cases. The reason for this was the failure to incorporate the new information priority principle.

In this paper, the new information priority accumulated generation operation is introduced and a novel new information accumulated nonlinear grey Bernoulli model is proposed. The nonlinear parameters of the proposed model are optimized by formulating the optimization problem based on the Grasshopper Optimization Algorithm. Finally, the model our proposed is applied to forecast the behavior of the installed wind capacity in Europe and South America.

2. The new information priority accumulated nonlinear grey Bernoulli model
In this section, we introduce the modeling mechanism of the new information priority accumulated nonlinear grey Bernoulli model in detail. Firstly, the new information accumulated generation operation and its inverse operation are defined in the first subsection. Then we proposed the new information accumulated nonlinear grey Bernoulli model in the second subsection, and expounded the parameter estimation and the solution of the proposed model. Finally, the universality of the proposed model is discussed in the last subsection.

2.1. The new information priority accumulated generating operation and its inverse operation
The accumulated generation operation is an important theory in the grey system, which has the advantage of making the uncertain problem clear. Besides, the grey system theory holds that the functions of new pieces of information are more significant than those old pieces of information [10], which is called the new information priority.

In general, for an original data sequence. Its new information priority accumulated generation operation and its inverse operation can be defined as follows:

**Definition 1.** Let the sequence \( Y = \{y(1), y(2), \ldots, y(n)\} \) be the new information priority accumulated generating operation of \( X \), where

\[
\begin{align*}
    y(k) &= \lambda y(k-1) + x(k), \lambda \in (0,1) \\
    y(1) &= x(1)
\end{align*}
\]

(1)

It worth noting that the new information priority accumulated generating operation can be expressed as follows using matrix theory:

\[
Y = X \left[
\begin{array}{cccc}
1 & \lambda & \lambda^2 & \cdots & \lambda^{n-1} \\
0 & 1 & \lambda & \cdots & \lambda^{n-2} \\
0 & 0 & 1 & \cdots & \lambda^{n-3} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 1
\end{array}
\right],
\]

(2)

the upper triangular matrix in Eq. (2) is defined as the new information priority accumulated generating matrix, and it tactfully reflects the weight of “new” is larger than the “old” ones.

**Definition 2.** Let the original data sequence \( X \) can be generated by the new information priority inversely accumulated generating operation of \( Y \), where
\[
\begin{align*}
\begin{cases}
x(k) = y(k) - \lambda y(k-1), k \geq 2, \\
x(1) = y(1)
\end{cases}
\end{align*}
\]  
(3)

Similarly, the inversely accumulated generating operation also can be expressed as follows:
\[
\begin{bmatrix}
1 & -\lambda & 0 & \cdots & 0 \\
0 & 1 & -\lambda & \cdots & 0 \\
0 & 0 & 1 & \ddots & 0 \\
\vdots & \vdots & \vdots & \ddots & -\lambda \\
0 & 0 & 0 & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
\vdots \\
y_n
\end{bmatrix}
= \begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
\vdots \\
y_n
\end{bmatrix}.
\]  
(4)

It is not difficult to prove that the square matrix in Eq. (2) and Eq. (4) are reciprocal.

**Definition 3.** Let the grey background value sequence \( Z = \{z(2), z(3), \ldots, z(n)\} \) generated by consecutive neighbors of \( Y \), where
\[
z(k) = \frac{1}{2}(y(k) + y(k-1)), k \in {2, 3, \ldots, n}.
\]  
(5)

### 2.2. The new information priority accumulated nonlinear grey Bernoulli model our proposed

Let \( y(k) \) and \( z(k) \) be the same as those defined in Definition 1 and Definition 2, the mathematical form of the new information priority accumulated nonlinear grey Bernoulli model (abbreviated as NIP-NGBM) is:
\[
dy(t)/dt + ay(t) = b(y(t))^r,
\]  
(6)

Eq. (6) is called as the whitening differential equation of the NIP-NGBM our proposed, where power index \( r \) is any real number except one.

In order to realize the solution of system parameters, the grey differential equation of the NIP-NGBM is listed as follows according to Ref. [7].
\[
y(k) - y(k-1) + az(k) = b(z(k))^r
\]  
(7)

the least-square rule is applied to solve the linear system Eq. (7), and the system parameters satisfies \( (a, b)^T = (\Theta^T\Theta)^{-1}(\Theta^T\xi) \), where
\[
\xi = \begin{bmatrix}
y(2) - y(1) \\
y(3) - y(2) \\
\vdots \\
y(n) - y(n-1)
\end{bmatrix}, \Theta = \begin{bmatrix}
-z(2) & (z(2))^r \\
-z(3) & (z(3))^r \\
\vdots & \vdots \\
-z(n) & (z(n))^r
\end{bmatrix}.
\]  
(8)

### 2.3. The time response sequence of the new information priority accumulated nonlinear grey Bernoulli model

Suppose the system parameters of the NIP-NGBM are described in the last subsection, the time response sequence of the NIP-NGBM can be obtained based on the basic theory of differential equations. The detailed derivation of formulas is displayed as follows:

Firstly, multiply \( y(t)^r \) on both sides of the whitening differential equation of the NIP-NGBM, one obtains:
\[
y(t)^r \frac{dy(t)}{dt} + ay(t)^{1-r} = b,
\]  
(9)
The linear differential equations are constructed as follows by performing \( f(t) = y(t)^{1-r} \):

\[
\frac{df(t)}{dt} + a(1-r)f(t) = b(1-r),
\]

then the general solution of Eq. (6) can be written as in the following form by using the method of variation of constants,

\[
f(t) = \left( f(1) - \frac{b}{a} \right) e^{-a(1-r)(t-1)} + \frac{b}{a}.
\]

Therefore, the time response sequence of the NIP-NGBM is given by

\[
y(t) = \left( x(1)^{1-r} - \frac{b}{a} \right) e^{-a(1-r)(t-1)} + \frac{b}{a}^{1/r},
\]

2.4. Universality of the new information priority accumulated nonlinear grey Bernoulli model

The new information priority accumulated nonlinear grey Bernoulli model our proposed is a more universal grey model, the NIP-NGBM reduces to some classical models in some given situations.

If \( r = 0 \) and \( \lambda = 1 \), the NIP-NGBM is reduced to the classical GM (1,1) model [2]:

\[
\frac{dy(t)}{dt} + ay(t) = b.
\]

If \( r = 0 \) and \( \lambda = 1 \), the NIP-NGBM is reduced to the grey Verhulst model [11]:

\[
\frac{dy(t)}{dt} + ay(t) = b(y(t))^2.
\]

If \( r = 0 \) and \( \lambda = 1 \), the NIP-NGBM is reduced to the nonlinear grey Bernoulli model [5]:

\[
\frac{dy(t)}{dt} + ay(t) = b(y(t))^r.
\]

3. The optimization of the nonlinear parameter by the Grasshopper Optimization Algorithm

Although the modelling mechanism and the solution of the NIP-NGBM have been described in the previous section, noticing that it is quite challenging to estimate the nonlinear parameters \( \lambda \) and \( r \) using conventional mathematical methods. To overcome this challenge, a nonlinear optimization problem by the Grasshopper Optimization Algorithm is designed in this section.

3.1. Preparing the metrics for evaluating models

Without loss of generality, the mean absolute percentage error (abbreviated as MAPE) is regarded as the metrics for evaluating models in this paper, which is also employed as the fitness function of the optimization problem. The expression and calculation of MAPE in different situation are shown in the following.

The MAPE during modelling:

\[
\text{MMAPE} = \frac{1}{n-1} \sum_{i=1}^{n} \left| \frac{x(i) - \hat{x}(i)}{x(i)} \right| \times 100.
\]

The MAPE during prediction:

\[
\text{PMAPE} = \frac{1}{N-n} \sum_{i=n+1}^{N} \left| \frac{x(i) - \hat{x}(i)}{x(i)} \right| \times 100.
\]

The overall MAPE:

\[
\text{OMAPE} = \frac{1}{N} \sum_{k=1}^{N} \left| \frac{x(k) - \hat{x}(k)}{x(k)} \right| \times 100.
\]
where \( n \) represents the sizes of the modeling data, \( N \) represents the size of the entire data sequence.

### 3.2. Formulating the optimization problem of the nonlinear parameter

Rearranging the modelling process of the NIP-NGBM, the optimization problem of the nonlinear parameters is formulated as follows:

\[
\min_{\beta, \gamma} \text{MMAPE} = \frac{1}{n} \sum_{k=1}^{n} \left| \frac{x(k) - \hat{x}(k)}{x(k)} \right| \times 100%
\]

\[
\xi = \begin{bmatrix} y(2) - y(1) \\ y(3) - y(2) \\ \vdots \\ y(n) - y(n-1) \end{bmatrix}, \Theta = \begin{bmatrix} -z(2) & (z(2))^\top \\ -z(3) & (z(3))^\top \\ \vdots & \vdots \\ -z(n) & (z(n))^\top \end{bmatrix}
\]

\[
(a, b)^\top = (\Theta^\top \Theta)^{-1} (\Theta^\top \xi)
\]

\[
\hat{y}(k) = \left( x(1)^{(1-r)} - \frac{b \gamma}{a} \right) e^{-a(r-1)(k-1)} + \frac{b \gamma}{a} \]

\[
\hat{x}(k) = \hat{y}(k) - \lambda \hat{y}(k-1)
\]

### 3.3. Optimization of the nonlinear parameter using the Grasshopper Optimization Algorithm

The Grasshopper Optimization Algorithm was first proposed by Shahrzad Saremi in 2017 [12], it solves the optimization problems by simulating the behaviors of grasshopper. And it has been applied in many subject areas for its simple operation and fast convergence.

The swarming behavior of grasshopper is mainly affected by social action \( (S_i) \), gravity \( (G_i) \) and wind advection \( (A_i) \), which can be expressed as following mathematical model:

\[
X_i = r_i G_i + r_s A_i + r_s S_i,
\]

where \( X_i \) is the \( i \)-th grasshopper’s current position, \( r_i \) (\( i = 1, 2, 3 \)) are random numbers. And

\[
\begin{align*}
A_i &= u \vec{e}_u \\
G_i &= -g \vec{e}_g \\
S_i &= \sum_{j=1, j \neq i}^{N} s(x_j - x_i)
\end{align*}
\]

where \( \vec{e}_u \) and \( s \) represent the unit vector and the social interaction function respectively, \( u \) and \( g \) are constants, the mathematical model form of the Grasshopper Optimization Algorithm can be further expressed as

\[
X_i^d = c \left( \sum_{j=1, j \neq i}^{N} e \frac{ub_j - lb_j}{2} s(x_j - x_i) \right) + \hat{T}_d,
\]

where \( \hat{T}_d \) is the current optimal solution, \( ub_j \) and \( lb_j \) represent the upper and lower boundary of the \( d \)-dimensional space respectively. And \( c \) is a shift coefficient, which effectively promotes the convergence of the Grasshopper Optimization Algorithm and it can be updated by:

\[
c = c \max - \frac{c \max - c \min}{L}.
\]

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In conclusion, the general framework of the Grasshopper Optimization Algorithm can be summarized as the following five steps:

**Step 1:** Initial the grasshopper population and other tunable parameters.

**Step 2:** Calculating the fitness of each grasshopper to evaluate the modelling performance of models and setting \( T \) is the best fitness.

**Step 3:** Determining if the maximum number of iterations has been reached and exit the iteration if so.

**Step 4:** Updating the location of grasshopper and calculating the updated fitness, and the updated fitness replaces \( T \) if the updated fitness outperforms the current \( T \).

**Step 5:** Updating the number of iterations and return to Step 3.

4. Applications in the installed wind capacity

In this section, the installed wind capacity development in Europe and South America is discussed, the raw data about the installed wind capacity are collected from the *Statistical Review of World Energy 2019* [13]. And the data from 2008 to 2015 are employed to constructed prediction models, the remaining data from 2016 to 2018 are employed to test the predictive performance of models.

![Figure 1. The prediction results of the installed wind capacity in South America.](image1)

![Figure 2. The prediction results of the installed wind capacity in Europe.](image2)

Then we construct the NIP-NGBM to describe the behaviours of the installed wind capacity in Europe and South America, and the results of the proposed model are compared with other existing classical models including GM [2], FGM [8] and NGBM [5]. The Figure 1 and Figure 2 show the results from different models and compared them with the actual data, and it can be noticed that the NIP-NGBM always shows more satisfactory results.

| GM   | FGM  | NGBM | NIP-NGBM |
|------|------|------|----------|
| MMAPE| 3.8826| 2.6102| 2.6109| **2.3505** |
| PMAPE| 10.7349| 3.5442| 3.8834| **1.7117** |
| OMAPE| 5.7514| 2.8650| 2.9579| **2.1763** |

| GM   | FGM  | NGBM  | NIP-NGBM |
|------|------|-------|----------|
| MMAPE| 0.7662| 0.3884| 0.4498| **0.2006** |
| PMAPE| 5.0855| 0.7529| 3.3647| **0.6017** |
| OMAPE| 1.9442| 0.4878| 1.2448| **0.3100** |

Furthermore, the metrics in previous section are utilized to evaluate the prediction models. All the error comparison results between the prediction models and the actual value are displayed in Table 1 and Table 2, and it worth mentioning here that the numeric in bold stand for the best results.

As can be seen from Table 1 and Table 2, the NIP-NGBM the proposed model significantly outperforms other classical models in describing the behaviors of the installed wind capacity in Europe and South America, both in modeling and prediction. And the same results are against demonstrated in Figure 1 and Figure 2. In summary, the NIP-NGBM our proposed can noticeable boost the prediction accuracy than the classical GM, the FGM and the classical NGBM model. Thereby we have sufficient reasons to believe that the NIP-NGBM can be applied as an effective method to predict the installed wind capacity in other regions.

5. Conclusions
In our research work, a novel new information priority accumulated nonlinear grey Bernoulli model is proposed, which is abbreviated as the NIP-NGBM. The model our proposed makes up for the new information priority of the classical nonlinear grey Bernoulli model, and it effectively improves the predictive ability of the model. Besides, the Grasshopper Optimization Algorithm is used to optimize the nonlinear parameters. Then the NIP-NGBM is applied to forecast the installed wind capacity, and in comparison with the classical GM, FGM and NGBM, the model our proposed can describe the behavior of the installed wind capacity more accurately.

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