OPTIMAL CONSUMPTION WITH REFERENCE-DEPENDENT PREFERENCES IN ON-THE-JOB SEARCH AND SAVINGS

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Abstract. This paper studies a model of on-the-job search and savings under reference-dependent preferences that implies loss aversion in a worker’s consumption behaviors. The model analyzes how loss aversion affects the worker’s consumption decisions in job search. The results demonstrate that the presence of loss aversion will lead to a set of high steady-state consumption levels and the range of steady-state consumption levels is wider if the worker is more loss averse. Nevertheless, we show that there is a unique steady-state consumption level, which is a lower bound of the set, in the absence of loss aversion. In addition, we also find that great loss aversion may reduce consumption level, while small loss aversion not only causes consumption to remain at a high level, but also induces that the worker’s future consumption level goes down when the employment status changes.

1. Introduction. In most current studies of the labor market, traditional job search models view a worker as a rational agent who makes decisions based on current consumption [24, 33, 36]. However, in practice, workers’ consumption decisions in job search depend not only on their current consumption, but also on how their current consumption compared with a reference consumption level [13, 32]. Psychological evidences reviewed in Kahneman et al. [16] and Köszegi & Rabin [20], which are known as reference-dependent preferences, indicate that workers perceive consumption prospects in gains and losses relative to the reference consumption level. Meanwhile, workers’ behaviors exhibit loss aversion, which means losses loom larger than gains of the same magnitude [17]. The reference consumption level and loss aversion of workers’ consumption prospects are two major elements influencing their consumption behaviors in actual job searches. Despite their importance in practice, reference-dependent preferences have received little attention in the job search literature, which does not take into account reference consumption level and loss aversion in consumption decisions.
This paper studies how the worker’s consumption decisions with reference dependent preferences in job search are affected by loss aversion. We develop a model of on-the-job search and savings in which a worker chooses search effort and consumption level based on reference-dependent preferences. In fact, the on-the-job search and savings process is a multi-stage decision problem under the uncertain environment. At any stage, the worker will search for a new job offer. The wage of this new job offer is random and obeys a probability distribution. In addition, the worker also faces an exogenous probability of job destruction. The principle of optimality describes the state transition of multi-stage decision-making process. Therefore, the model can be essentially established as the stochastic dynamic programming. To this end, we consider the reference-dependent consumption utility, which consists of instantaneous consumption utility and gain-loss utility. The instantaneous consumption utility is standard instantaneous utility, and the gain-loss utility function satisfies the properties of reference-dependent preferences value function. To investigate the effects of loss aversion on consumption decisions in job search, we further study the models without and with loss aversion. Since the utility function is nonsmooth for the loss aversion case, subdifferential is introduced to study optimal consumption decisions for the loss-averse worker. By deriving the first order necessary conditions and the Euler equations, the steady-state consumption levels for loss-neutral and loss-averse workers are discussed in the on-the-job search and savings model.

The main findings of our paper are as follows. Firstly, the search and consumption decisions of the worker under loss aversion exhibit a set of high steady-state consumption levels, and the range of steady states is wider when the worker is more loss averse. For the loss-neutral case, such decisions exhibit a unique steady-state consumption level, which is a lower bound of the set for the loss aversion case. Secondly, great loss aversion may cut down consumption level. In particular, we show that if loss aversion is small and the worker’s employment status changes, the future consumption level of both unemployed and employed workers will be lower than the present consumption level. Thirdly, unemployment benefit or reservation wage has a negative impact on search effort. In addition, we find that assets also have a negative effect on search effort of both unemployed and employed workers. This implies that the workers with more assets will search for much less job opportunities. Finally, when the reference consumption level of the loss-neutral worker is equivalent to his own current consumption level, the consumption level of the unemployed decreases with time. In addition, the consumption level of the employed is also decreasing over time when the wage is less than the unemployment benefits.

2. Literature review. There is an increasing interest in job search and savings models where workers can choose consumption and search effort, and accumulate assets by saving behaviors. Lentz & Tranaes [22] studied a risk-averse worker’s optimal savings and job search behavior as he moves back and forth between employment and unemployment. Their models assumed that the objective utility function is separable over time and the wage is constant in the labor market. Browning et al. [9] established a theoretical model of job search in which risk-averse, debt-constrained job seekers can use short-term employment in undesirable jobs as a way to finance consumption during subsequent unemployment search for better jobs. Lammers [21] discussed the interrelations among wealth, reservation wages and search effort theoretically and empirically. Although the model assumes that wage is a
random variable and utility function is separable, it does not consider on-the-job search. Empirical research [23, 26] estimated a lot of job search and savings models and examined the relationship between wealth accumulation and job search dynamics. Although different models and methods [38, 39] have been provided for the job search and savings problems [20], none of them has taken a concept that requires reference-dependent preferences, on-the-job search and random wages in the job search and savings model. Hence this gives us reasonable motivation to research on such a pragmatic problem.

The theory of reference-dependent preferences has a long-standing tradition in psychology and is regarded as an extension of Kahneman & Tversky [17]’s prospect theory. Tversky & Kahneman [34] were the first to propose a theoretical model of reference-dependent preferences. The most prominent feature of such reference-dependent preferences is loss aversion, that is, people care much more about losses relative to their reference levels than gains. Recent research trends in reference-dependent preferences (in particular, loss aversion) investigated the issue of consumption utility and saving behaviors, which assumes that a person’s per-period consumption utility depends on not only instantaneous consumption utility, but also gain-loss utility. Bowman et al. [8] and Siegmann [29] studied a consumption-savings model based on reference-dependence, which implies a fundamental asymmetry in consumption behavior. Psychological evidence discussed in the literature [12, 18, 19, 20] indicated that the larger the perceived gain and loss relative to the reference consumption level, the greater impact on the consumption. Furthermore, an important empirical implication [35] was that negative information regarding future consumption prospects tends to have a greater impact on expected consumption growth than positive information.

Although the theory of reference-dependent preferences is not yet common in the context of job search models, there are many fields to which it (especially, loss aversion) has been successfully applied, such as macroeconomic growth models [14, 27], asset pricing [11, 15, 25] and supply chain pricing decisions [37]. Ruan et al. [28] combined the preference-based index with the $\alpha$-cut idea to present a new method for comparing triangular fuzzy numbers and propose an approach for proportionately allocating limited relief supplies. Apesteguia & Ballester [6] provided an axiomatic characterization of reference dependence and some applications of their work, e.g., status-quo bias and addictive behavior. To investigate the consumption decisions with reference-dependent preferences in the job search dynamic, we quote the work of Popescu & Wu [25] to consider on-the-job search and savings model, without and with loss aversion respectively. Moreover, we study the effects of loss aversion on consumption decisions in on-the-job search and savings by comparing with the loss-neutral case.

The rest of the paper is organized as follows. Section 3 introduces a model of reference-dependent consumption utility. Section 4 presents a model of on-the-job search and savings with reference-dependent preferences. Section 5 focuses on models where the worker is loss neutral. We investigate properties of optimal consumption decisions and the existence of the steady-state consumption level under loss neutral. Section 6 studies the same issues when the worker is loss averse. The properties of optimal consumption decisions and the set of steady-state consumption levels are presented under loss aversion. Finally, conclusions are summarized in Section 7.
3. The reference-dependent consumption utility. In this section, a worker’s reference-dependent consumption utility is described to analyze how the worker maximizes expected lifetime consumption utility based on current consumption level and reference consumption level. The worker’s reference-dependent consumption utility is assumed to be divided into two parts, which are instantaneous consumption utility and gain-loss utility [12, 19, 20]. The former is the standard instantaneous consumption utility, and the latter depends on the difference between the current consumption level and the reference consumption level. The fundamental properties of gain-loss utility underlie the model of reference-dependent preferences [18]. The current consumption level and the reference consumption level are denoted by $c$ and $r$, respectively. The worker’s reference-dependent consumption utility function can be written as

$$u(r, c) = (1 - \alpha)\mu(c) + \alpha v(c - r), \quad (1)$$

where the parameter $\alpha \in (0, 1)$, $\mu(\cdot)$ is the instantaneous consumption utility, which is a twice differentiable function, and $v(\cdot)$ is the gain-loss utility function that satisfies the following assumptions, as stated by Bowman et al. [8].

- **A0**: $v(z)$ is continuous for all $z$, twice differentiable except when $z = 0$, and $v(0) = 0$.
- **A1**: $v(z)$ is strictly increasing in $z$.
- **A2**: If $y > z > 0$, then $v(y) + v(-y) < v(z) + v(-z)$.
- **A3**: $v''(z) \leq 0$ for $z > 0$, and $v''(z) \geq 0$ for $z < 0$. (diminishing sensitivity)
- **A4**: $v'_-(0)/v'_+(0) = \phi$, where $v'_+(0) = \lim_{z \to 0} v'(|z|)$ and $v'_-(0) = \lim_{z \to 0} v'(-|z|)$. If $\phi = 1$, the worker is loss neutral. If $\phi > 1$, the worker is loss aversion.

Assumption A1 implies that $u(r, c)$ is decreasing in $r$. This implication is consistent with the previous assumption. Assumption A2 indicates a steeper utility function in the negative domain than in the positive domain. A3 captures diminishing sensitivity of gain-loss utility. It has two meanings: on the one hand, people are risk averse in the case of gains. On the other hand, people are risk seeking in the case of losses. A4 describes larger losses loom than gains if $\phi > 1$. These are referred to as loss aversion, which means that people have a relative distaste for losses when comparing very small losses to very small gains. There is strong empirical evidence on loss aversion, in which the loss aversion index $\phi$ is estimated between 1.75 and 2.5 [1]. Therefore, assumptions A0-A4 imply that the gain-loss utility function $v(z)$ is an S-shaped function of the reference consumption gap $z = c - r$, with a concave kink at $z = 0$. In particular, assumptions A0 and A1 combining with the properties of reference-dependent preferences generate the following remark.

**Remark 1.** $v(z) \geq 0$ for $z > 0$, $v(z) \leq 0$ for $z < 0$, and $v(c - r)$ is decreasing in $r$.

By (1) and the above assumptions, we obtain that $u(r, c)$ is a continuous function that is decreasing in $r$ and increasing in $c$. An increase in the reference consumption level $r$, decreases $u(r, c)$ for fixed $c$. However, for any given $r$, an increase in the current consumption level $c$ yields higher total consumption utility. This assumption commonly accords with most models of habit formation [3, 5]. Moreover, if the parameter $\alpha = 0$, the gain-loss utility has no effect on the consumption utility. If the parameter $\alpha = 1$, the consumption utility fully equals to the gain-loss utility.

The consumption implications of reference-dependent preferences are further investigated in Section 6. Models with loss-neutral workers are investigated in Section 5.
4. The on-the-job search and savings model with reference-dependent preferences. Consider a utility-maximizing worker who faces opportunity of new job arrival and risk of job loss in discrete time indexed by $t$. The worker has not only the job search behavior, but also the saving behavior. He derives utility from consumption and disutility from the effort of searching for a new job. In addition, he can smooth consumption over income states by use of savings that carry a rate of return.

Assume an infinite planning horizon for the worker. To maximize expected lifetime utility, he makes his choice of consumption $c_t$ and search effort $e_t$ at time $t$. It is assumed that the objective utility is separable in both time with the instantaneous utility given by $u(r_t, c_t) - s(e_t)$, where $u(r_t, c_t)$ represents consumption utility function with reference-dependent preferences at time $t$, and $s(e_t)$ represents search cost at time $t$. Furthermore, $s'(e_t) > 0$ and $s''(e_t) > 0$ with $s(0) = 0$. This implies that, as search effort increases, the search cost and marginal search cost are raised. Therefore, while greater search effort implies more incentives to move back into employment, it also increases the search cost and consequently raises the marginal search cost. At time $t + 1$, he is able to update his reference consumption level $r_{t+1}$ as he chooses the past consumption $c_t$ and the past reference consumption level $r_t$. Following the reference level formation mechanism [8, 25], we assume that the reference consumption formation equation takes the form of

$$ r_{t+1} = pr_t + (1 - p)c_t, \quad (2) $$

where $p \in [0, 1]$ represents the speed at which the reference consumption changes in response to recent consumption. If $p = 0$, then the reference consumption equals the past consumption. This model is identical to the assumption of the reference consumption level [14]. If $p = 1$, the previous consumption has no effect on the current reference consumption level.

At any time $t$ the worker may be employed or unemployed. He makes consumption decision and searches for a better job both when employed and when unemployed. The probability of new job arrival is $\lambda(e_t)$, which depends positively on the worker’s search effort $e_t$ at time $t$ with $\lambda'(e_t) > 0 \geq \lambda''(e_t)$ and $\lambda(0) = 0$. This means that an increase in the search effort tends to increase the probability of new job arrival and decrease the marginal probability of new job arrival. Upon contacting a job offer, the worker receives a wage $w$ drawn from a known wage distribution $F(w)$ with support $[\underline{w}, \overline{w}]$. When employed, the worker engages in on-the-job search and faces an exogenous probability $\delta$ of job destruction. When unemployed, he receives unemployment benefits $b$. These benefits also include income from a secondary labor market, surplus from household production and so on.

For a given initial reference consumption level $r_0$, the worker faces the following decision problem,

$$ \max \left\{ c_t, e_t \right\}_{t=0}^{\infty} \begin{array}{l}
E_0 \sum_{t=0}^\infty \beta^t \left[ u(r_t, c_t) - s(e_t) \right] \\
\text{s.t. } A_{t+1} = (1 + \rho)A_t + d_t w + (1 - d_t)b - c_t, \quad (3b) \\
r_{t+1} = pr_t + (1 - p)c_t, \quad (3c) \\
c_t \geq 0, e_t \geq 0, \text{ for all } t = 0, 1, \ldots,
\end{array} $$

where $\beta$ denotes the per-period discount factor with $0 < \beta < 1$, $E_0(\cdot)$ stands for the mathematical expectation conditional on the worker’s information set at time $t$.
For any time $t = 0$, and the constraint condition (3b) represents the budget constraint. This constraint can be also referred to as the asset accumulation equation, where $A_t$ is the worker’s asset at time $t$, $\rho$ is the risk free interest rate, $d_t \in \{0, 1\}$ denotes the state of employment at time $t$. $d_t = 0$ means that the worker is unemployed, and $d_t = 1$ means that the worker is employed. In order to ensure that the problem is bounded, the worker’s asset $A_t$ is assumed to be both lower and upper bounded, that is, $A_t \in [A_l, A_u]$. Aiyagari [2] claimed that a lower bound on assets naturally arises from the assumption of nonnegative consumption combined with asymptotic present value budget balance. Lentz & Tranaes [22] pointed out that a lower bound on assets can be justified as a borrowing limit imposed by the capital market. Moreover, the upper bound on assets is a set in order to state a simple fact that individual does not choose an infinite value of assets.

The model can be formulated recursively by the principle of optimality [7]. Let $V_u(A_t, r_t)$ denote the maximum expected present value of being unemployed with the asset $A_t$ and the reference consumption level $r_t$. Similarly, $V_e(A_t, r_t, w)$ denotes the value function associated with employment at the asset $A_t$, the reference consumption level $r_t$ and the wage $w$. The Bellman equations of the model can then be formulated as follows.

$$V_u(A_t, r_t) = \max_{\{A_{t+1}, c_t\}} \left\{ u\left(r_t, (1 + \rho)A_t + b - A_{t+1}^w\right) - s(e_t) + \beta \lambda(e_t) \int_{-\infty}^{\infty} \max\{V_e(A_{t+1}^r, r_{t+1}, x), V_u(A_{t+1}^r, r_{t+1})\} dF(x) \right\},$$

(4)

$$V_e(A_t, r_t, w) = \max_{\{A_{t+1}, c_t\}} \left\{ u\left(r_t, (1 + \rho)A_t + w - A_{t+1}^r\right) - s(e_t) + \beta \lambda(e_t) \int_{-\infty}^{\infty} \max\{V_e(A_{t+1}^r, r_{t+1}, x), V_e(A_{t+1}^r, r_{t+1}, w)\} dF(x) \right\} + \beta \delta V_u(A_{t+1}^r, r_{t+1}) + \beta (1 - \delta - \lambda(e_t)) V_e(A_{t+1}^r, r_{t+1}, w),$$

(5)

The flow value of being unemployed with asset $A_t$ and reference consumption level $r_t$ is given by the utility flow from consumption utility $u(r_t, c_t)$ less the search cost $s(e_t)$ plus the expected change in the value of unemployment. The latter has two components. The first term equals the value of being unemployed by the product of the job arrival rate and the maximum expected present discounted net benefit associated with new job offer arrival. The second part $\beta (1 - \lambda(e_t)) V_u(A_{t+1}^r, r_{t+1})$ equals the value of remaining unemployed by the product of the no job arrival rate and the expected present discounted net benefit relative to unemployment. When the worker is employed, the flow value of employment will additionally change in job destruction by the product of the job destruction rate and the expected present discounted net benefit associated with unemployment.

Recall that $u(\cdot, \cdot)$ is increasing in the second component. According to the theorems in Stokey & Lucas Jr. [31], the following lemma can be derived.

**Lemma 4.1.** For any time $t$, the value function $V_u(A_t, r_t)$ is increasing in the asset $A_t$. And the value function $V_e(A_t, r_t, w)$ is increasing in the asset $A_t$ and the wage $w$.

Existence and uniqueness of the value functions $V_u^*$ and $V_e^*$ solving (4) and (5) respectively can be proved by Blackwell’s sufficient conditions [31] for a contraction.
mapping. Indeed, let \( C(X) \) be the space of all bounded and continuous functions on \( X = [\underline{A}, \overline{A}] \times \{0, 1\} \). Furthermore, introduce a mapping \( T \) from \( C(X) \) to \( C(X) \) defined by

\[
T(V_\varepsilon, V_u)(A_t, 0) = \max_{\{A_{t+1}^u, e_t\}} \{u^1(r_t, A_t, A_{t+1}^u, e_t) + \beta E_t V_u(A_{t+1}^u, r_{t+1}, x)\},
\]

where

\[
u^1(r_t, A_t, A_{t+1}^u, e_t) = u(r_t, (1 + \rho)A_t + b - A_{t+1}^u) - s(e_t)\]

and

\[
E_t V_u(A_{t+1}^u, r_{t+1}, x) = \lambda(e_t) \int \max\{V_\varepsilon(A_{t+1}^u, r_{t+1}, x), V_u(A_{t+1}^u, r_{t+1})\} dF(x)
\]

\[
+ \beta (1 - \psi(e_t)) V_u(A_{t+1}^u, r_{t+1}).
\]

Likewise, \( T(V_\varepsilon, V_u)(A_t, 1) \) equals the right-hand side of (5). Thereby, it is clear that the mapping \( T \) is monotone \( (V_\varepsilon^1 \leq V_\varepsilon^2 \text{ and } V_u^1 \leq V_u^2 \implies T(V_\varepsilon^1, V_u^1) \leq T(V_\varepsilon^2, V_u^2)) \), and has the discounting property \( (\text{for all } \chi \in X, T(V_\varepsilon + d, V_u + d)(\chi) \leq T(V_\varepsilon, V_u)(\chi) + \beta d) \) because of \( 0 < \beta < 1 \). By the contraction mapping theorem, existence and uniqueness of the fixed point \( (V_\varepsilon^*, V_u^*) \) solving (4) and (5) can be also proved.

In order to maximize expected lifetime utility, the unemployed worker must decide on the reservation wage with the exception of making consumption and search effort decisions. Therefore, the unemployed worker would accept any wage offer which is no less than this reservation wage. This reservation wage is unique and equal to the unemployment benefit \( b \). Existence of an unemployed worker’s reservation wage is discussed in the following.

**Proposition 1.** For any value of asset \( A_t \), the reservation wage for the unemployed worker is unique and equal to the unemployment benefit \( b \), that is, \( w^* (A_t) = b \).

**Proof.** See the proof of Proposition 1 in the appendix.

Proposition 1 shows that the reservation wage for the unemployed worker is constant and any wage higher than the unemployment benefit is acceptable. This result is in line with Lise’s conclusion of reservation wage. It also implies a fact that the search cost \( s(e_t) \) and the job arrival rate \( \lambda(e_t) \) do not depend on the worker’s employment status.

5. **Model without loss aversion.** This section mainly discusses the case when the worker is loss neutral about the gain and loss of consumption level. In other words, the gain-loss utility function is smooth and the loss aversion index \( \phi = 1 \). We first examine the effects of unemployment benefit, reservation wage and asset on search effort. Afterwards, the optimal consumption decisions for loss-neutral workers are presented and the unique steady-state consumption level for loss-neutral workers is characterized.

Assume that the gain-loss utility function \( v(z) \) is twice differentiable in \( z \). The slope of the gain-loss utility function at \( z = 0 \) is denoted \( \psi = \frac{dv(z)}{dz} \big|_{z=0} \). Furthermore, suppose that \( \frac{\partial^2 u(r_t, c_t)}{\partial c_t^2} < 0 \). Therefore, \( u(r_t, c_t) \) is strictly concave and increasing in \( c_t \). To derive the following conclusions, we prove the lemma 5.1 according to the proof by Stokey & Lucas Jr. in their chapter 9 and the lemma 4.1.
Lemma 5.1. For any time $t$, the value function $V_u(A_t, r_t)$ and the value function $V_e(A_t, r_t, w)$ are strictly concave and increasing in the asset $A_t$ and the wage $w$.

The optimal consumption decisions and optimal search effort for workers are discussed under the condition of the worker being loss neutral for consumption. The first order necessary conditions associated with optimal consumption and search effort when unemployed and employed are given as follows.

$$\frac{∂u(r_t, c_t^u)}{∂c_t^u} = βλ(e_t) \int_0^w \left[ \frac{∂V_e(A_{t+1}^u, r_{t+1}, x)}{∂A_{t+1}^u} - \frac{∂V_u(A_{t+1}^u, r_{t+1})}{∂A_{t+1}^u} \right] dF(x) + β\frac{∂V_u(A_{t+1}^u, r_{t+1})}{∂A_{t+1}^u},$$

(6)

$$s'(e_t) = β\lambda(e_t) \int_0^w \left[ V_e(A_{t+1}^u, r_{t+1}, x) - V_u(A_{t+1}^u, r_{t+1}) \right] dF(x).$$

(7)

$$\frac{∂u(r_t, c_t^e)}{∂c_t^e} = βλ(e_t) \int_0^w \left[ \frac{∂V_e(A_{t+1}^e, r_{t+1}, x)}{∂A_{t+1}^e} - \frac{∂V_e(A_{t+1}^e, r_{t+1}, w)}{∂A_{t+1}^e} \right] dF(x) + β\frac{∂V_e(A_{t+1}^e, r_{t+1}, w)}{∂A_{t+1}^e} + β(1 - δ)\frac{∂V_e(A_{t+1}^e, r_{t+1}, w)}{∂A_{t+1}^e},$$

(8)

$$s'(e_t) = β\lambda(e_t) \int_0^w \left[ V_e(A_{t+1}^e, r_{t+1}, x) - V_e(A_{t+1}^e, r_{t+1}, w) \right] dF(x).$$

(9)

where $c_t^u$ and $c_t^e$ represent the worker’s decision of consumption in the state of unemployment and employment, respectively. Conditions (6) and (8) clearly show that the marginal utility flow of consumption equals to the expected present discounted utility flow of the marginal assets, when both unemployed and employed. The optimal consumptions for unemployed and employed workers can be chosen by calculating the two equations. As the conclusion of the first order conditions for optimal consumption, if the utility flow of the marginal assets at employment is larger than the the utility flow of the marginal assets at unemployment, the consumption levels of the unemployed worker will be increased. Moreover, if the utility flow of the marginal assets at the next employment is larger than its counterpart at the current employment, the consumption levels of the employed worker will also be increased. Conditions (7) and (9) require the marginal cost of search effort to be equal to the expected change in the present discounted utility associated with a marginal offer arrival rate and an accepted wage offer. The optimal search effort is determined by equating the marginal costs with the marginal benefits of search. An increase in the marginal benefits of search, i.e., $V_e(A_{t+1}^e, r_{t+1}, x) - V_u(A_{t+1}^u, r_{t+1})$ and $V_e(A_{t+1}^e, r_{t+1}, x) - V_e(A_{t+1}^e, r_{t+1}, w)$, will therefore directly increase the search effort. At the same time, an increase in $V_e(A_{t+1}^e, r_{t+1}, x) - V_u(A_{t+1}^u, r_{t+1})$ will decrease the reservation wage $b$. This result follows directly from the definition of the reservation wage, i.e., $V_e(A_{t+1}^e, r_{t+1}, b) = V_u(A_{t+1}^u, r_{t+1})$. This implies that the search effort will be decreasing as the unemployment benefit is increasing, the reservation wage will be increasing. Similarly, (9) shows that the search effort will be decreasing as the current wage increases.

According to the above discussions about the value function and the first order condition, we derive the following proposition.

Proposition 2. The search effort is decreasing with the increase of unemployment benefit or reservation wage.
There is widespread evidence that the assets often influence the difference between the value of employment and the value of unemployment, that is, it often affects the marginal benefits of search. Because of discounting, we know that a sufficient condition for $V_e(A_t, r_t, w) < V_u(A_t, r_t)$ is $1 - \delta \geq \lambda(e_t)(1 - F'(w))$, i.e., the employed worker has a high probability to maintain employment. Suppose that \( \frac{\partial V_e(A_{t+1}, r_{t+1}, e)}{\partial A_{t+1}} < \frac{\partial V_u(A_{t+1}, r_{t+1}, w)}{\partial A_{t+1}} \), \( \forall w \in [b, \bar{w}] \) and \( \frac{\partial V_e(A_{t+1}, r_{t+1}, \hat{x})}{\partial A_{t+1}} < \frac{\partial V_u(A_{t+1}, r_{t+1}, w)}{\partial A_{t+1}} \), \( \forall \hat{x} \in [w, \bar{w}] \). These assumptions imply that the reservation wage $w^r$ is increasing with the asset $A_t$ because of $V_e(A_t, r_t, w^r) = V_u(A_t, r_t)$. Since a higher reservation wage often results in a lower marginal benefits of search, an increase in the asset indirectly decreases search effort. Therefore, the direct effect of asset on search is also negative when the above assumptions hold. Consequently, we obtain the following lemma according to Lentz & Traaen [22] and Lammers [21].

**Lemma 5.2.** Under the assumption that $1 - \delta \geq \lambda(e_t)(1 - F(b))$, it holds that
\[
\frac{\partial}{\partial A_t} \int_b^{\bar{w}} \left[ V_e(A_t, r_t, w) - V_u(A_t, r_t) \right] dF(w) < 0
\]
and
\[
\frac{\partial}{\partial A_t} \int_b^{\bar{w}} \int_w^{\bar{w}} \left[ V_e(A_t, r_t, x) - V_e(A_t, r_t, w) \right] dF(x) dF(w) < 0,
\]
for all $A_t \in [A, \bar{A}]$.

**Proof.** See the proof of Lemma 5.2 in the appendix.

With Lemma 5.2, the following proposition can characterize how the search effort is affected by the asset and recall that the search cost function $s(e_t)$ and the new job arrival function $\lambda(e_t)$ are convex and concave in the search effort $e_t$, respectively.

**Proposition 3.** For any time $t$, the search effort $e_t$ increases as the asset $A_t$ decreases, both when unemployed and employed.

**Proof.** See the proof of Proposition 3 in the appendix.

Proposition 3 indicates that an unemployed worker has less incentive to obtain employment when he has more assets. Since it is assumed that the search cost is not affected by the asset holdings, we conclude that wealthier workers prefer to search for much less job opportunities. Propositions 2 and 3 also hold for the loss aversion case.

### 5.1. Optimal consumption decisions for loss-neutral workers.

To investigate the consumption decisions for loss-neutral workers, this subsection derives two stochastic Euler equations for unemployed and employed workers. We show a monotonic decrease of the consumption level for loss-neutral workers with time.

For convenience, the asset accumulation equation can be defined as $A_{t+1} = g(A_t, c_t)$, where $g(A_t, c_t) = (1 + \rho)A_t + d_tw + (1 - d_t)b - c_t$. Off corners, the value

\footnote{Algan et al. [4] considered a quantitative dynamic equilibrium search model calibrated on French data, and found that the value of unemployment is increasing faster in assets than the value of employment, the reservation wage is increasing in the assets.}
functions $V_u(A_t, r_t)$ and $V_e(A_t, r_t, w)$ are differentiable in $A_t$ with
\[
\frac{\partial V_u(A_t, r_t)}{\partial A_t} = (1 + \rho) \frac{\partial u(r_t, c_t^u)}{\partial c_t^u} + \beta \lambda(e_t) \frac{\partial g(A_t, c_t)}{\partial A_t} \int_w \left[ \frac{\partial V_e(A_{t+1}^u, r_{t+1}, x)}{\partial A_{t+1}^u} \right] \, dF(x) + \beta \frac{\partial g(A_t, c_t)}{\partial A_t} \frac{\partial V_u(A_{t+1}^u, r_{t+1})}{\partial A_{t+1}^u}.
\]

Substituting (12) and (13) into the first order necessary condition (6) gives the stochastic Euler equation for the unemployed worker,
\[
\frac{\partial V_e(A_t, r_t, w)}{\partial A_t} = (1 + \rho) \frac{\partial u(r_t, c_t^e)}{\partial c_t^e}.
\]

Application of the Benveniste-Scheinkman formula gives
\[
\frac{\partial V_u(A_t, r_t)}{\partial A_t} = (1 + \rho) \frac{\partial u(r_t, c_t^u)}{\partial c_t^u}.
\]
\[
\frac{\partial V_e(A_t, r_t, w)}{\partial A_t} = (1 + \rho) \frac{\partial u(r_t, c_t^e)}{\partial c_t^e}.
\]

Substituting (12) and (13) into the first order necessary condition (6) gives the stochastic Euler equation for the unemployed worker,
\[
\frac{\partial u(r_t, c_t^u)}{\partial c_t^u} = (1 + \rho) \frac{\partial u(r_{t+1}, c_{t+1}^u)}{\partial c_{t+1}^u}.
\]

Substituting (12) and (13) into the first order necessary condition (6) gives the stochastic Euler equation for the unemployed worker,
\[
\frac{\partial V_e(A_t, r_t, w)}{\partial A_t} = (1 + \rho) \frac{\partial u(r_t, c_t^e)}{\partial c_t^e}.
\]

Substituting (12) and (13) into the first order necessary condition (6) gives the stochastic Euler equation for the unemployed worker,
\[
\frac{\partial u(r_t, c_t^u)}{\partial c_t^u} = (1 + \rho) \frac{\partial u(r_{t+1}, c_{t+1}^u)}{\partial c_{t+1}^u}.
\]

Substituting (12) and (13) into the first order necessary condition (6) gives the stochastic Euler equation for the unemployed worker,
\[
\frac{\partial V_e(A_t, r_t, w)}{\partial A_t} = (1 + \rho) \frac{\partial u(r_t, c_t^e)}{\partial c_t^e}.
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\frac{\partial V_e(A_t, r_t, w)}{\partial A_t} = (1 + \rho) \frac{\partial u(r_t, c_t^e)}{\partial c_t^e}.
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Substituting (12) and (13) into the first order necessary condition (6) gives the stochastic Euler equation for the unemployed worker,
\[
\frac{\partial V_e(A_t, r_t, w)}{\partial A_t} = (1 + \rho) \frac{\partial u(r_t, c_t^e)}{\partial c_t^e}.
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Substituting (12) and (13) into the first order necessary condition (6) gives the stochastic Euler equation for the unemployed worker,
\[
\frac{\partial V_e(A_t, r_t, w)}{\partial A_t} = (1 + \rho) \frac{\partial u(r_t, c_t^e)}{\partial c_t^e}.
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Substituting (12) and (13) into the first order necessary condition (6) gives the stochastic Euler equation for the unemployed worker,
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\frac{\partial V_e(A_t, r_t, w)}{\partial A_t} = (1 + \rho) \frac{\partial u(r_t, c_t^e)}{\partial c_t^e}.
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Substituting (12) and (13) into the first order necessary condition (6) gives the stochastic Euler equation for the unemployed worker,
\[
\frac{\partial V_e(A_t, r_t, w)}{\partial A_t} = (1 + \rho) \frac{\partial u(r_t, c_t^e)}{\partial c_t^e}.
\]

Substituting (12) and (13) into the first order necessary condition (6) gives the stochastic Euler equation for the unemployed worker,
\[
\frac{\partial V_e(A_t, r_t, w)}{\partial A_t} = (1 + \rho) \frac{\partial u(r_t, c_t^e)}{\partial c_t^e}.
\]
to the reference consumption formation model (2), i.e., \( r_t = c_t \). Due to \( v'(0) = \psi \), (15) can be rewritten as

\[
\mu'(c_t^u) - \beta(1+\rho)\mu'(c_{t+1}^u) = \beta(1+\rho)\lambda(e_t) \int_b^\infty \left[ \mu'(c_{t+1}^x(x)) - \mu'(c_{t+1}^u) \right] dF(x) \\
- \frac{\alpha}{1-\alpha} [1 - \beta(1+\rho)] \psi.
\]

The unemployed worker’s marginal instantaneous consumptions at current time and next time are denoted by \( \mu'(c_t^u) \) and \( \beta(1+\rho)\mu'(c_{t+1}^u) \), respectively. The right hand side of (16) consists of the marginal instantaneous consumption surplus of search at next time and the difference value of marginal reference-dependent consumption. The latter value implies that the difference between the marginal gain-loss utility at next time and that at current time, where \( \frac{\alpha}{1-\alpha} \) represents the relative weight as (1) is defined. Because the unemployed worker is loss neutral in the difference between the current consumption and the reference consumption, the marginal reference-dependent consumption of this stochastic Euler equation is constant.

Substituting (12) and (13) into the first order necessary condition (8) yields

\[
\frac{\partial u(r_t, c_t^e)}{\partial c_t^e} - \beta(1+\rho) \left[ \frac{\partial u(r_{t+1}, c_{t+1}^u)}{\partial c_{t+1}^u} + (1-\delta) \frac{\partial u(r_{t+1}, c_{t+1}^e)}{\partial c_{t+1}^e} \right] \\
= \beta(1+\rho)\lambda(e_t) \int_b^\infty \left[ \frac{\partial u(r_{t+1}, c_{t+1}^x(x))}{\partial c_{t+1}^x(x)} - \frac{\partial u(r_{t+1}, c_{t+1}^e)}{\partial c_{t+1}^e} \right] dF(x).
\]

This stochastic Euler equation for the employed worker is similar to (14) except for the difference that the employed worker is likely to be unemployed. The first term on the left hand side is still the employed worker’s marginal consumption utility \( \frac{\partial u(r_t, c_t^e)}{\partial c_t^e} \) at current time. The second term \( \beta(1+\rho) \left[ \frac{\partial u(r_{t+1}, c_{t+1}^u)}{\partial c_{t+1}^u} + (1-\delta) \frac{\partial u(r_{t+1}, c_{t+1}^e)}{\partial c_{t+1}^e} \right] \) represents the expected marginal consumption utility at next time when considering the probability \( \delta \) of job destruction. Similarly for the expression form of the Euler equation for the unemployed worker, the right hand side of the stochastic Euler equation is also equal to the marginal consumption surplus of continuing the job search at next time.

Therefore, it follows from \( \frac{\partial u(r_t, c_t)}{\partial c_t} = (1-\alpha)\mu'(c_t) + \alpha v'(c_t - r_t) \) that

\[
\left[ \mu'(c_t^e) + \frac{\alpha}{1-\alpha} v'(c_t^e - r_t) \right] - \beta(1+\rho) \left\{ \mu'(c_{t+1}^e) + \frac{\alpha}{1-\alpha} v'(c_{t+1}^e - r_{t+1}) \right\} \\
+ (1-\delta) \left[ \mu'(c_{t+1}^e) + \frac{\alpha}{1-\alpha} v'(c_{t+1}^e - r_{t+1}) \right] \}
\]

\[
= \beta(1+\rho)\lambda(e_t) \int_b^\infty \left[ \mu'(c_{t+1}^x(x)) + \frac{\alpha}{1-\alpha} v'(c_{t+1}^x(x) - r_{t+1}) - \mu'(c_{t+1}^e) \right] \]

\[
- \frac{\alpha}{1-\alpha} v'(c_{t+1}^e - r_{t+1}) \] dF(x).
\]

By the same token and without regard to the reference consumption formation model, we consider that the reference consumption level \( r_t \) is equivalent to the
current consumption level \(c_t\) at time \(t\). Since \(\nu'(0) = \psi\), it results that

\[
\mu'(c_t^0) - \beta(1 + \rho) \left[ \delta \mu'(c_{t+1}^0) + (1 - \delta) \mu'(c_{t+1}^0) \right] = \beta(1 + \rho) \lambda(c_t) \int_w^\infty \left[ \mu'(c_{t+1}^0(x)) - \mu'(c_{t+1}^0) \right] dF(x) - \frac{\alpha}{1 - \alpha} [1 - \beta(1 + \rho)] \psi.
\]

(19)

The left hand side is still equal to the difference between the employed worker’s marginal instantaneous consumption \(\mu'(c_t^0)\) at current time and the expected marginal instantaneous consumption \(\beta(1 + \rho) \left[ \delta \mu'(c_{t+1}^0) + (1 - \delta) \mu'(c_{t+1}^0) \right]\) at next time. On the right hand side, the first part represents the marginal instantaneous consumption surplus of searching a better job at next time. The second term also implies the change value of current constant marginal gain-loss utility and future marginal gain-loss utility.

Due to the assumption that \(\frac{\partial u(r_t, c_t)}{\partial c_t} > 0\) and \(\frac{\partial^2 u(r_t, c_t)}{\partial c_t^2} < 0\), it is apparent that \(\mu'(c_t) > -\frac{\alpha}{1 - \alpha} \psi\) and \(\mu''(c_t) < 0\) when \(c_t = r_t\). Assume that both the unemployed and employed workers are impatient in that the subjective rate of time preference exceeds the risk free interest rate, i.e., \(\frac{1}{\beta} - 1 > \rho\). Through the analysis of the two stochastic Euler equations, the following proposition provides the consumption decisions for unemployed and employed workers.

**Proposition 4.** Given that \(\beta(1 + \rho) < 1\) and the reference consumption level is equivalent to the current consumption level under loss-neutral, the consumption level of the employed worker is decreasing in time \(t\). When the wage is less than the unemployment benefits, i.e., \(w < b\), the consumption level of the employed worker is also decreasing in time \(t\).

**Proof.** Since \(\mu'(c_t)\) is strictly decreasing in \(c_t\) due to the assumption that \(\mu''(c_t) < 0\), it follows from (16) that

\[
\left( \mu'(c_t^0) + \frac{\alpha}{1 - \alpha} \psi \right) - \beta(1 + \rho) \left( \mu'(c_{t+1}^0) + \frac{\alpha}{1 - \alpha} \psi \right) = \beta(1 + \rho) \lambda(c_t) \int_b^\infty \left[ \mu'(c_{t+1}^0(x)) - \mu'(c_{t+1}^0) \right] dF(x) < 0.
\]

Therefore, we obtain that \(\mu'(c_t^0) + \frac{\alpha}{1 - \alpha} \psi < \beta(1 + \rho) \left( \mu'(c_{t+1}^0) + \frac{\alpha}{1 - \alpha} \psi \right)\). On account of \(\beta(1 + \rho) < 1\), it holds that \(\mu'(c_t^0) < \mu'(c_{t+1}^0)\). Since \(\mu'(c_t)\) is strictly decreasing in \(c_t\), we have \(c_t^0 > c_{t+1}^0\).

Similarly, it follows from (19) that

\[
\left( \mu'(c_t^0) + \frac{\alpha}{1 - \alpha} \psi \right) - \beta(1 + \rho) \left[ \delta \mu'(c_{t+1}^0) + (1 - \delta) \mu'(c_{t+1}^0) + \frac{\alpha}{1 - \alpha} \psi \right] = \beta(1 + \rho) \lambda(c_t) \int_w^\infty \left[ \mu'(c_{t+1}^0(x)) - \mu'(c_{t+1}^0) \right] dF(x) < 0.
\]

Consequently, we derive that \(\mu'(c_t^0) + \frac{\alpha}{1 - \alpha} \psi < \beta(1 + \rho) \left[ \delta \mu'(c_{t+1}^0) + (1 - \delta) \mu'(c_{t+1}^0) + \frac{\alpha}{1 - \alpha} \psi \right]\). When \(w < b\), we have \(c_{t+1}^0 < c_{t+1}^0\) according to the inequality \((1 + \rho)A_{t+1} + w - A_{t+2} < (1 + \rho)A_{t+1} + b - A_{t+2}\). Considering that \(\beta(1 + \rho) < 1\) and \(c_{t+1}^0 < c_{t+1}^0\), we have \(\mu'(c_t^0) < \mu'(c_{t+1}^0)\). Furthermore, we come to the conclusion that \(c_t^0 > c_{t+1}^0\). □
Proposition 4 shows that when the reference consumption level equals the current consumption level, the unemployed worker’s consumption level drops under a longer time $t$ of unemployment. Moreover, if the employed worker’s wage does not exceed the unemployment benefits, the worker’s consumption level will also come down with the time advances.

5.2. Steady state for loss-neutral workers. This subsection proves the existence and uniqueness of the steady-state consumption level for loss-neutral workers, and further studies some properties of the steady-state consumption level.

Define a steady-state consumption level for unemployed and employed workers as $c^{u*}$ and $c^{e*}$, respectively. To investigate when the steady state exists, we start out from a situation where $c^{u*}_t = c^{u*}_{t+1}$ and $c^{e}_t = c^{e}_{t+1} = c^{e*}$. Furthermore, if the steady states $c^{u*}$ and $c^{e*}$ exist, the reference consumption level also has a steady state $r^*$. Thereby, it follows from (2) that $r^* = c^{u*} = c^{e*}$. The following theorem insures that such a steady state for unemployed and employed workers exists and can be uniquely determined by a simple equation.

**Theorem 5.3.** The problem without loss aversion has a unique steady-state consumption level $c^* = c^{u*} = c^{e*}$, which is determined by

$$\mu'(c^*) = -\frac{\alpha}{1-\alpha} \psi.$$  \hfill (20)

Furthermore, $c^*$ is increasing in $\psi$ and $\alpha$, respectively.

**Proof.** See the proof of Theorem 5.3 in the appendix.

![Figure 1. Steady-state consumption levels under loss-neutral](image)

Theorem 5.3 demonstrates that the unemployed worker’s steady-state consumption $c^{u*}$ is equivalent to the employed worker’s steady-state consumption $c^{e*}$. This implies that a small increase in the marginal gain-loss utility $\psi$ for loss-neutral workers leads to an increase in the steady-state consumption level $c^*$. The parameter $\alpha$ is between zero (no reference-dependent utility) and one (pure reference-dependent utility). As the parameter $\alpha$ is increasing, that is, a worker’s consumption utility is larger dependent on the reference-dependent utility, the steady-state consumption level for loss-neutral workers will be increasing. These results are illustrated with
the example in Fig. 2. We consider that the marginal gain-loss utility $\psi$ can be chosen in the set \{0.5, 1.5, 2\}. Assume that the instantaneous consumption utilities are $\mu(c) = -c^2 + 21c$ and $\mu(c) = -c^3/3$, respectively.

6. Model with loss aversion. In this section, the optimal consumption decisions and steady-state consumption levels of unemployed and employed workers are investigated when workers are loss aversion in the reference-dependent consumption. Assumptions A0-A4 are satisfied throughout this section. In contrast to section 5, the gain-loss utility function is nonsmooth and the loss aversion index $\phi > 1$.

Since $v(z)$ is a piecewise and continuous utility function which weights losses in consumption heavier than gains, it is apparent that $v(z)$ is twice differentiable about $z$ except for $z = 0$ and the loss aversion index $\phi > 1$. For convenience, we assume that $v'_+(0) = \eta$ and $v'_-(0) = \eta\phi$. An illustration is provided in Fig. 2. To study the effects of the asymmetry between gains and losses on the consumption decisions, the subdifferential of $v(z)$ at $z = 0$ is defined as the set $[\eta, \eta\phi]$, where $\eta$ and $\eta\phi$ are the one-sided limits $\eta = \lim_{z \to 0^+} \frac{v(z)}{z}$ and $\eta\phi = \lim_{z \to 0^-} \frac{v(z)}{z}$, respectively. Similarly to the model without loss aversion, assume that $\frac{\partial u(r_t, c_t)}{\partial c_t} > 0$ and $\frac{\partial^2 u(r_t, c_t)}{\partial c_t^2} < 0$.

Recall that $\frac{\partial u(r_t, c_t)}{\partial c_t} = (1 - \alpha)\mu'(c_t) + \alpha \partial v(c_t - r_t)$, where

$$\partial v(c_t - r_t) = \begin{cases} v'(c_t - r_t), & \text{if } c_t \neq r_t \\ [\eta, \eta\phi], & \text{if } c_t = r_t. \end{cases} \quad (21)$$

Consequently, we have $\mu'(c_t) > -\frac{\alpha}{1-\alpha} \partial v(c_t - r_t)$ and $\mu''(c_t) < -\frac{\alpha}{1-\alpha} \partial^2 v(c_t - r_t)$. From assumption A3 in the section 3, we further have $\mu''(c_t) < 0$ when $c_t \leq r_t$.

6.1. Optimal consumption decisions for loss-averse workers. As discussed, we can also derive the first order conditions for loss-averse workers associated with optimal consumption and search effort. Along an optimal consumption decision, where $c_t^n - r_t$ and $c_{t+1}^n - r_{t+1}$ are different from zero, the stochastic Euler equation
for the unemployed worker is given by

\[
\left[ \mu'(c^u_t) + \frac{\alpha}{1-\alpha} \partial v(c^u_t - r_t) \right] - \beta(1+\rho) \left[ \mu'(c^u_{t+1}) + \frac{\alpha}{1-\alpha} \partial v(c^u_{t+1} - r_{t+1}) \right] = \beta(1+\rho) \lambda(e_t) \int_b^\infty \left[ \mu'(c^u_{t+1}(x)) + \frac{\alpha}{1-\alpha} \partial v(c^u_{t+1}(x) - r_{t+1}) - \mu'(c^u_{t+1}) \right] \alpha \partial v(c^u_{t+1} - r_{t+1}) \right] dF(x).
\]

Like in the case of the unemployed and loss-neutral worker, (22) is consistent with (15). However, the difference in this Euler equation is that \( v(x) \) is not differentiable at \( z = 0 \) because of the presence of loss aversion. Since the subdifferential \( (22) \) also holds with equality in time periods where in the case \( r_t = c_t \). Indeed, substituting \( r_{t+1} = c_{t+1} \) and \( r_t = c_t \) into (2), we obtain

\[ c_{t+1} = pc_t + (1-p)c_t = c_t. \]

Consequently, this situation will lead to the steady state condition. An unemployed and loss-averse worker will find it optimal to choose \( c^u_t - r^u_t = c^u_{t+1} - r^u_{t+1} = c^u_{t+1} - r^u_{t+1} = 0 \) if the following inequality holds

\[ \beta(1+\rho) \beta(1+\rho) \eta - \frac{\alpha}{1-\alpha} \eta \phi \leq B^* \mu'(c^u) - D^* \mu'(c^e) \leq \beta(1+\rho) \frac{\alpha}{1-\alpha} \eta \phi - \frac{\alpha}{1-\alpha} \eta, \]

where \( B^* = 1 - \beta(1+\rho) + \beta(1+\rho) \lambda(e_t)(1-F(b)) \) and \( D^* = \beta(1+\rho) \lambda(e_t)(1-F(b)) \). To derive (23), start out from a situation where \( c^u_t = c^u_{t+1} = c^u, c^e_t = c^e_{t+1} = c^e \) and \( \partial v(0) = [\eta, \eta \phi] \). As shown in (23), neither a small reduction nor a small increase in \( c_t \) can increase utility in the case \( c^u_t = c^u_{t+1} = c^u \) and \( c^e_t = c^e_{t+1} = c^e \). Since the loss aversion index \( \phi > 1 \) and \( \beta(1+\rho) < 1 \), it is apparent that a positive range of index values must exist such that (23) holds if \( \phi > \frac{1}{\beta(1+\rho)} \). Furthermore, the following proposition holds if \( 1 < \phi \leq \frac{1}{\beta(1+\rho)} \).

**Proposition 5.** If the loss aversion index \( 1 < \phi \leq \frac{1}{\beta(1+\rho)} \), the consumption level of the unemployed worker in the period of unemployment is higher than the future consumption level in the period of employment because of the presence of loss aversion.

**Proof.** When \( 1 < \phi \leq \frac{1}{\beta(1+\rho)} \), the right hand side of the inequality (23) is less than or equal to zero identically, i.e., \( \frac{\alpha}{1-\alpha} \eta \beta(1+\rho) \phi - 1 \leq 0 \). From (23), it follows that \( B^* \mu'(c^u) - D^* \mu'(c^e) \leq 0 \), where \( B^* = 1 - \beta(1+\rho) + D^* \). Because of \( \beta(1+\rho) < 1 \),

\[ D^* \mu'(c^u) < [1 - \beta(1+\rho) + D^*] \mu'(c^u) \leq D^* \mu'(c^e). \]

Therefore, we have \( \mu'(c^u) < \mu'(c^e) \). Since \( \mu'(c_t) \) is strictly decreasing in \( c_t \), it is obvious that \( c^u_t > c^e_t \).
For optimal consumption decision of the employed worker, when \( c_t^e - r_t \neq 0 \) and \( c_{t+1}^e - r_{t+1} \neq 0 \), the stochastic Euler equation is derived by
\[
\beta(1+\rho)(1+\alpha)\frac{\alpha - \alpha}{1-\alpha} - \frac{\alpha - \alpha}{1-\alpha} \eta [c_t^u + c_t^e - r_t] = 0.
\]
(24)

The stochastic Euler equation for loss-averse workers has a similar expression as for loss-neutral workers. Though \( v(z) \) is not differentiable at \( z = 0 \), (24) still holds with equality in time periods where in the similar case \( r_t = c_t \). The employed and loss-averse worker will find it optimal to choose \( c_t^e - r_t = c_{t+1}^e - r_{t+1} = c_t^u - r_t = 0 \) if the following inequality holds
\[
\beta(1+\rho)\frac{\alpha - \alpha}{1-\alpha} - \frac{\alpha - \alpha}{1-\alpha} \eta [c_t^u + c_t^e - r_t] = 0.
\]
(25)

where \( c_t^u = c_{t+1}^u = c_t^e \) and \( c_t^e = c_{t+1}^e = c_t^e \). As shown in the proof of Theorem 5.3 in the appendix, we know that \( R = 1 - \beta(1+\rho)(1-\delta) \) and \( Q = \beta(1+\rho)\delta \). The weak inequalities in (25) follow from the definition of subdifferential. When the loss aversion index \( \phi > \beta(1+\rho) \) and \( \beta(1+\rho) < 1 \), (25) shows that the right hand side of the inequality is greater than zero. It implies that \( R\mu'(c^e) - Q\mu'(c^u) \) may be equivalent to zero by using the intermediate value theorem. When the loss aversion index \( 1 < \phi \leq \frac{1}{\beta(1+\rho)} \), we obtain the following proposition.

**Proposition 6.** If the loss aversion index \( 1 < \phi \leq \frac{1}{\beta(1+\rho)} \), the consumption level of the employed worker in the period of employment is higher than the future consumption level in the period of unemployment because of the presence of loss aversion.

**Proof.** Clearly, the right hand side of the inequality (25) is identically less than or equal to zero if \( 1 < \phi \leq \frac{1}{\beta(1+\rho)} \), that is, \( \alpha \beta(1+\rho) - \eta [\beta(1+\rho)\phi - 1] \leq 0 \). Consequently, it holds that \( R\mu'(c^e) - Q\mu'(c^u) \leq 0 \), where \( R = 1 - \beta(1+\rho) + Q \). Because of \( \beta(1+\rho) < 1 \),
\[
Q\mu'(c^e) < [1 - \beta(1+\rho) + Q] \mu'(c^e) \leq Q\mu'(c^u).
\]
Hence, it is apparent that \( \mu'(c^e) < \mu'(c^u) \). Since \( \mu'(c_t) \) is strictly decreasing in \( c_t \), it follows from \( \mu'(c^e) < \mu'(c^u) \) that \( c^e > c^u \).

Synthesizing both cases of unemployed and employed workers when \( \phi > \frac{1}{\beta(1+\rho)} \), we obtain that there exist \( c^{u^1} \) and \( c^{e^1} \) such that \( B^*\mu'(c^{u^1}) = D^*\mu'(c^{e^1}) \) and \( R\mu'(c^{e^1}) = Q\mu'(c^{u^1}) \) by the intermediate value theorem. Consequently, it is apparent that \( \mu'(c^{u^1}) = \mu'(c^{e^1}) = 0 \). According to the above discussions, the following proposition holds.
Proposition 7. If the loss aversion index \( \phi > \frac{1}{\beta(1 + \rho)} \), the consumption level of workers in the period of employment may be equivalent to the consumption level in the period of unemployment, i.e., there exists \( c^1 = c^{u1} = c^{e1} \) satisfies \( \mu'(c^1) = 0 \).

Under the condition \( \phi > \frac{1}{\beta(1 + \rho)} \), the consumption level of workers who show high loss aversion may converge to the steady state, implying that greater loss aversion maybe give rise to stronger consumption smoothing and lower consumption level. Clearly, \( \mu'(c^1) = 0 > -\frac{\alpha}{1-\alpha} \psi = \mu'(c^*) \). Since \( \mu'(c_t) \) is strictly decreasing in \( c_t \), it is obvious that \( c^1 < c^* \).

6.2. Steady state for loss-averse workers. We extend the results obtained in the section 5 for the loss-neutral case to propose existence and properties of the steady state for loss-averse workers. The steady-state consumption level for workers under loss aversion resembles that for the loss-neutral case, except that there is a range of steady states.

Define a steady-state consumption level for unemployed and employed workers under loss aversion as \( c^{ua*} \) and \( c^{ea*} \), respectively. To study the existence of steady state, assume that \( c^{u1} = c^{u1+1} = c^{ua*} \) and \( c^{e1} = c^{e1+1} = c^{ea*} \). Furthermore, if the consumption levels of unemployed and employed workers exist steady states \( c^{ua*} \) and \( c^{ea*} \), respectively, the reference consumption level also has a steady state \( r^{a*} \).

Therefore, it follows from (2) that \( r^{a*} = c^{ua*} = c^{ea*} \). With (22) and (24), we give the existence of steady state for unemployed and employed workers as proposed in Theorem 6.1.

Theorem 6.1. The problem under loss aversion has a set of steady-state consumption levels \( c^{a*} = c^{ua*} = c^{ea*} \), which are determined by

\[
\mu'(c^{a*}) = -\frac{\alpha}{1-\alpha} [\eta, \eta \phi].
\]

Furthermore, \( c^{a*} \) is increasing in \( \eta \) and \( \alpha \), respectively.

Proof. See the proof of Theorem 6.1 in the appendix.
the above results when $\mu(c) = -c^2 + 21c$ and $\mu(c) = -c^3/3$, $\eta = 1$ and the loss aversion index $\phi$ can be chosen in the set $\{1.5, 2, 2.5\}$.

7. **Conclusion.** The main contribution in this paper is to incorporate reference-dependent consumption utility into the on-the-job search and savings model. Accordingly, we develop a model of on-the-job search and savings in the presence of reference-dependent preferences, in which a worker’s utility depends on not only his current consumption level, but also how his current consumption compares to a reference consumption level. We further derive the implications for consumption behaviors in on-the-job search and savings under loss neutral and loss aversion. Results demonstrate that the search effort is decreasing with unemployment benefit or reservation wage, and the search effort also decreases with asset. The latter implies that wealthier workers will search for much less job opportunities. If workers are loss neutral, we show that the consumption level remains in a unique steady state. Furthermore, the consumption level of the unemployed worker is decreasing over time when the reference consumption level of the unemployed worker is his own
current consumption level, and the consumption level of the employed worker is also decreasing over time when the wage is below the unemployment benefits and the reference consumption level of the employed worker is his own current consumption level. If workers are loss aversion, we find that loss aversion leads to a set of steady-state consumption levels. This implies that the set of steady states remains in high consumption level and if workers are more loss averse, the range of steady states should be wider. Importantly, the results further show that if loss aversion is great, the consumption level may be reduced. Moreover, if loss aversion is small, the consumption level will be higher than the future consumption level both when employed and when unemployed.

For further research, we can investigate such problems to enhance the applicability of reference-dependence utility in job search models. For instance, an important issue is to consider that the consumption level depends on the worker’s initial endowment. That is, heterogeneity in initial endowment implies that both consumption level and reference consumption level differ across workers, this turns out crucial for workers’ savings behaviors. Also, it would be interesting to study the effects of wage increase on job satisfaction. In practice, a higher wage increase will be more likely to lead to higher job satisfaction. Finally, different types of the perceived gain or loss would be worth studying. When a part of workers are loss-averse and another part of workers are loss neutral or loss seeking, we would expect to examine the effects of loss aversion on the labor market equilibrium.

Appendix. Proof of Proposition [1] Since for any asset $A_t$ at time $t$, the value function of being employed $V_c(A_t, r_t, w)$ is increasing in $w$, which follows directly from the assumption that $u(\cdot, \cdot)$ is increasing in the second component. Of course, an employed worker always accepts any wage higher than his current wage. Since at each asset $A_t$, the value of being unemployed $V_u(A_t, r_t)$ is independent of $w$, there exists a unique reservation wage $w^r(A_t)$ for which $V_c(A_t, r_t, w^r(A_t)) = V_u(A_t, r_t)$. Furthermore, [4] and [5] can be rewritten as follows,

$$V_u(A_t, r_t) = u(r_t, c_t(A_t, b)) - s(e_t(A_t, b)) + \beta \int V_c(A_{t+1}, r_{t+1}, x) dF(x) + \beta V_u(A_{t+1}, r_{t+1}),$$

$$V_c(A_t, r_t, w^r(A_t)) = u(r_t, c_t(A_t, w^r(A_t))) - s(e_t(A_t, w^r(A_t))) + \beta \int V_c(A_{t+1}, r_{t+1}, x) - V_c(A_{t+1}, r_{t+1}, w^r(A_{t+1})) dF(x) + \beta V_c(A_{t+1}, r_{t+1}, w^r(A_{t+1})).$$

Since $V_c(A_t, r_t, w^r(A_t)) = V_u(A_t, r_t)$, we find

$$V_u(A_t, r_t) - V_c(A_t, r_t, w^r(A_t)) = u(r_t, c_t(A_t, b)) - u(r_t, c_t(A_t, w^r(A_t))) + s(e_t(A_t, w^r(A_t))) - s(e_t(A_t, b)) + \beta \int V_c(A_{t+1}, r_{t+1}, x) - V_c(A_{t+1}, r_{t+1}, w^r(A_{t+1})) dF(x) = 0.$$

The solution of the above equation can be directly verified as follows,

$$e_t(A_t, w^r(A_t)) = e_t(A_t, b), \quad c_t(A_t, w^r(A_t)) = c_t(A_t, b), \quad w^r(A_t) = b.$$
Hence, the reservation wage for the unemployed worker is independent of asset and equal to the unemployment benefit.

**Proof of Lemma [5.2]** In Section [H] it follows from the contraction mapping theorem that \((V^*_e, V^*_u)\) is a unique fixed point. By the contraction mapping property of \(T\), it implies that for some closed set \(S_1 \subseteq S\), if \(T(S_1) \subseteq S_2 \subseteq S_1\), then \((V^*_e, V^*_u) \in S_2\). It will be shown that \(T\) maps the closed set of functions \(S_1\) defined by

\[
S_1 = \left\{ (V^*_e, V^*_u) \in C(X) \left| \frac{\partial}{\partial A_t} \int_b^w \left[ V_e(A_t, r_t, w) - V_u(A_t, r_t) \right] dF(w) \leq 0, \forall A_t \right\},
\]

into the set \(S_2\) which be characterized by

\[
S_2 = \left\{ (V^*_e, V^*_u) \in C(X) \left| \frac{\partial}{\partial A_t} \int_b^w \left[ V_e(A_t, r_t, w) - V_u(A_t, r_t) \right] dF(w) < 0, \forall A_t \right\}.
\]

Since the value functions \(V_e(A_t, r_t, w)\) and \(V_u(A_t, r_t)\) are strictly concave in the asset \(A_t\), it is apparent that

\[
\frac{\partial}{\partial A_t} \int_b^w \Psi(A_t, r_t, w) dF(w) = \int_b^w \left[ \frac{\partial V_e(A_t, r_t, w)}{\partial A_t} - \frac{\partial V_u(A_t, r_t)}{\partial A_t} \right] dF(w)
\]

and

\[
\frac{\partial}{\partial A_t} \int_b^w \Phi(A_t, r_t, w) dF(w) = \int_b^w \int_w^w \Psi(A_t, r_t, x, w) dF(x) dF(w),
\]

where \(\Psi(A_t, r_t, w) = V_e(A_t, r_t, w) - V_u(A_t, r_t)\) and \(\Phi(A_t, r_t, w) = V_e(A_t, r_t, w) - V_u(A_t, r_t, w)\).

By utilizing the first order conditions (6) and (8), define the derivatives of the mapping as following,

\[
\frac{\partial T(V_e, V_u)(A_t, 0)}{\partial A_t} = (1 + \rho) \frac{\partial u(r_t, c^u_t)}{\partial c^u_t} = \beta(1 + \rho) \lambda(c_t) \int_b^w \left[ \frac{\partial V_e(A^u_{t+1}, r_{t+1}, x)}{\partial A^u_{t+1}} - \frac{\partial V_u(A^u_{t+1}, r_{t+1})}{\partial A^u_{t+1}} \right] dF(x) \tag{27}
\]

and

\[
\frac{\partial T(V_e, V_u)(A_t, 0)}{\partial A_t} = (1 + \rho) \frac{\partial u(r_t, c^u_t)}{\partial c^u_t} = \beta(1 + \rho) \lambda(c_t) \int_w^w \left[ \frac{\partial V_e(A^u_{t+1}, r_{t+1}, x)}{\partial A^u_{t+1}} - \frac{\partial V_u(A^e_{t+1}, r_{t+1}, w)}{\partial A^e_{t+1}} \right] dF(x) + \beta(1 + \rho) (1 - \delta) \frac{\partial V_e(A^e_{t+1}, r_{t+1}, w)}{\partial A^e_{t+1}} \tag{28}
\]

Assume that

\[
\int_b^w \left[ \frac{\partial V_e(A_t, r_t, w)}{\partial A_t} - \frac{\partial V_u(A_t, r_t)}{\partial A_t} \right] dF(w) \leq 0
\]

and

\[
\int_b^w \int_w^w \left[ \frac{\partial V_e(A_t, r_t, x)}{\partial A_t} - \frac{\partial V_e(A_t, r_t, w)}{\partial A_t} \right] dF(x) dF(w) \leq 0,
\]

for all \(A_t \in [\underline{A}, \overline{A}]\). It will then be shown that it holds that

\[
\int_b^w \left[ \frac{\partial T(V_e, V_u)(A_t, 1)}{\partial A_t} - \frac{\partial T(V_e, V_u)(A_t, 0)}{\partial A_t} \right] dF(w) < 0, \forall A_t \in [\underline{A}, \overline{A}].
\]
To prove the assertion, we consider the following two cases.

Case 1. When $A_{t+1}^c > A_{t+1}^c$, \( \frac{\partial V^c_t(A_{t+1}^c, r_{t+1})}{\partial A_{t+1}^c} < \frac{\partial V^w_t(A_{t+1}^w, r_{t+1})}{\partial A_{t+1}^w} \) according to strict concavity of the value function \( V^c_t \). Subtracting (27) from (28) and integrating over the wage distribution,

\[
\int_b^\pi \left[ \frac{\partial T^c_t(V^c_t, V^w_t)(A^c_t, r_{t+1}) - \partial T^w_t(V^c_t, V^w_t)(A^w_t, 0)}{\partial A^c_t} - \frac{\partial V^c_t(A_{t+1}^c, r_{t+1})}{\partial A^c_{t+1}} \right] dF(w) \\
= \int_b^\pi \left\{ \beta(1 + \rho) \lambda(e_t) \int_w^\pi \left[ \frac{\partial V^c_t(A_{t+1}^c, r_{t+1})}{\partial A^c_{t+1}} - \frac{\partial V^w_t(A_{t+1}^w, r_{t+1})}{\partial A^w_{t+1}} \right] dF(x) \\
+ \beta(1 + \rho) \delta \frac{\partial V^c_t(A_{t+1}^c, r_{t+1})}{\partial A^c_{t+1}} \right\} dF(w) \\
< \int_b^\pi \left\{ \beta(1 + \rho) \lambda(e_t) \int_w^\pi \left[ \frac{\partial V^c_t(A_{t+1}^c, r_{t+1})}{\partial A^c_{t+1}} - \frac{\partial V^w_t(A_{t+1}^w, r_{t+1})}{\partial A^w_{t+1}} \right] dF(x) \\
+ \beta(1 + \rho)(1 - \delta) \left[ \frac{\partial V^c_t(A_{t+1}^c, r_{t+1}, w)}{\partial A^c_{t+1}} - \frac{\partial V^w_t(A_{t+1}^w, r_{t+1}, w)}{\partial A^w_{t+1}} \right] \right\} dF(w) \\
= \beta(1 + \rho) \lambda(e_t) \int_b^\pi \left[ \frac{\partial V^c_t(A_{t+1}^c, r_{t+1})}{\partial A^c_{t+1}} - \frac{\partial V^w_t(A_{t+1}^w, r_{t+1})}{\partial A^w_{t+1}} \right] dF(x) \\
+ \beta(1 + \rho) M \int_b^\pi \left[ \frac{\partial V^c_t(A_{t+1}^c, r_{t+1})}{\partial A^c_{t+1}} - \frac{\partial V^w_t(A_{t+1}^w, r_{t+1})}{\partial A^w_{t+1}} \right] dF(x) \leq 0,
\]

where \( M = 1 - \delta - \lambda(e_t)(1 - F(b)) \geq 0 \). The first strict inequality follows from strict concavity of \( V^c_t \) and \( V^w_t \), and \( A_{t+1}^c > A_{t+1}^w \). The second weak inequality follows from the assumptions that \( 1 - \delta - \lambda(e_t)(1 - F(b)) \geq 0 \),

\[
\int_b^\pi \left[ \frac{\partial V^c_t(A_t, r_t, x)}{\partial A_t} - \frac{\partial V^w_t(A_t, r_t)}{\partial A_t} \right] dF(x) \leq 0
\]

and

\[
\int_b^\pi \left[ \frac{\partial V^c_t(A_t, r_t, x)}{\partial A_t} - \frac{\partial V^w_t(A_t, r_t, w)}{\partial A_t} \right] dF(x) dF(w) \leq 0,
\]

for all \( A_t \in [A, \overline{A}] \).

Case 2. When $A_{t+1}^c \leq A_{t+1}^w$, we have $c_t^c > c_t^w$ according to the budget constraints

\[
A_{t+1}^c = (1 + \rho) A_t + w - c_t^c, \quad A_{t+1}^w = (1 + \rho) A_t + b - c_t^w
\]

and $w \geq b$. Since \( u(r_t, c_t^c) \) is strictly concave and increasing in \( c_t \), it follows that \( \frac{\partial u_t(r_t, c_t^c)}{\partial c_t^c} < \frac{\partial u_t(r_t, c_t^w)}{\partial c_t^w} \). Thus, by (27) and (28), for all \( A_t \in [A, \overline{A}] \),

\[
\int_b^\pi \left[ \frac{\partial T^c_t(V^c_t, V^w_t)(A^c_t, 1) - \partial T^w_t(V^c_t, V^w_t)(A^w_t, 0)}{\partial A_t} \right] dF(w) \\
= (1 + \rho) \int_b^\pi \left[ \frac{\partial u_t(r_t, c_t^c)}{\partial c_t^c} - \frac{\partial u_t(r_t, c_t^w)}{\partial c_t^w} \right] dF(w) < 0.
\]
Hence, it has been shown that $T(S_1) \subseteq S_2$, and the fixed point of $T$ is able to be characterized by
\[
\frac{\partial}{\partial A_t} \int_b^w \left[ V_e(A_t, r_t, w) - V_u(A_t, r_t) \right] dF(w) < 0
\]
and
\[
\frac{\partial}{\partial A_t} \int_b^w \left[ V_e(A_t, r_t, w) - V_e(A_t, r_t) \right] dF(x) dF(w) < 0, \; \forall A_t \in [A, \bar{A}].
\]

**Proof of Proposition 3.** By taking the partial derivative of (7) and (9) with respect to $A_t$, we can obtain
\[
\begin{align*}
\frac{\partial}{\partial A_t} (A^*_t, b) &= \beta \lambda''(e_t) \frac{\partial c_t(A^*_t, b)}{\partial A^*_t} \int_{A^*_t}^w \left[ V_e(A^*_t+1, r_t+1, x) - V_u(A^*_t+1, r_t+1) \right] dF(x) \\
&+ \beta (1 + \rho) \lambda'(e_t) \int_{A^*_t}^w \left[ \frac{\partial V_e(A^*_t+1, r_t+1, x)}{\partial A^*_t} - \frac{\partial V_u(A^*_t+1, r_t+1)}{\partial A^*_t} \right] dF(x),
\end{align*}
\]
where $\Phi(A^*_t+1, r_t+1, x) = V_e(A^*_t+1, r_t+1, x) - V_u(A^*_t+1, r_t+1, w)$. Moreover, (29) and (30) can be rewritten as follows,
\[
\begin{align*}
\frac{\partial c_t(A^*_t, b)}{\partial A^*_t} &= \beta (1 + \rho) \lambda'(e_t) \int_{A^*_t}^w \left[ \frac{\partial V_e(A^*_t+1, r_t+1, x)}{\partial A^*_t} - \frac{\partial V_u(A^*_t+1, r_t+1)}{\partial A^*_t} \right] dF(x) \\
&= \frac{\beta (1 + \rho) \lambda'(e_t)}{s''(e_t) - \beta \lambda''(e_t)} \int_{A^*_t}^w \left[ V_e(A^*_t+1, r_t+1, x) - V_u(A^*_t+1, r_t+1) \right] dF(x)
\end{align*}
\]

The above strict inequalities follows from the results of Proposition 5.2, the concavity of $\lambda(\cdot)$ and the strict convexity of $s(\cdot)$.

Hence, search effort $e_t$ of unemployed or employed worker is increasing with the decrease of asset $A_t$.

**Proof of Theorem 3.** Assume $c_t = c_{t+1} = \cdots = c_{t+T} = c^u = c' = c_{t+1} = \cdots = c_{t+T} = c^e$ and $e_t = e_{t+1} = \cdots = e_{t+T} = e^*$. By the reference consumption formation model $r_{t+1} = p r_t + (1 - p)c_t$, we have $r^* = c^u = c^e$. Therefore, (16) and (19) can be derived as
\[
\begin{align*}
B \mu'(c^u) - D \mu'(c^e) &= - [1 - \beta (1 + \rho)] \frac{\alpha}{1 - \alpha} \psi, \\
R \mu'(c^* - Q \mu'(c^u) &= - [1 - \beta (1 + \rho)] \frac{\alpha}{1 - \alpha} \psi,
\end{align*}
\]
where $B = 1 - \beta(1+\rho) + \beta(1+\rho)\lambda(e^*)(1-F(b))$, $D = \beta(1+\rho)\lambda(e^*)(1-F(b))$, $R = 1 - \beta(1+\rho)(1-\delta)$ and $Q = \beta(1+\rho)\delta$. This yields the following formulas:

\[
\mu'(c^{ax}) = -\frac{D + R}{BR - QD} [1-\beta(1+\rho)] \frac{\alpha}{1-\alpha} \psi,
\]
\[
\mu'(c^{ax}) = -\frac{B + Q}{BR - QD} [1-\beta(1+\rho)] \frac{\alpha}{1-\alpha} \psi.
\]

Since $D + R = B + Q = 1 - \beta(1+\rho)(1-\delta) + \beta(1+\rho)\lambda(e^*)(1-F(b))$, it results that

\[
\mu'(c^*) = \mu'(c^{ax}) = \mu'(c^{ax}) = -\frac{\alpha}{1-\alpha} \psi.
\]

Since $\mu'(c^*)$ is strictly decreasing in $c^*$ due to the assumption that $\mu''(c^*) < 0$, it holds that $c^*$ is increasing in $\alpha$ and $\psi$, respectively.

**Proof of Theorem 6.1** To prove the existence of steady state, assume $c^u = c^u_{t+1} = \cdots = c^u_{t+T} = c^{ax}_t$, $c^e = c^e_{t+1} = \cdots = c^e_{t+T} = c^{ax}_t$ and $c^e_t = c^e_{t+1} = \cdots = c^e_{t+T} = c^{ax}_t$. Furthermore, it follows from the reference consumption formation model $r_{t+1} = pr_t + (1-p)c_t$ that $r^{ax} = c^{ax} = c^{ax}$. Substituting $\partial v(0) = [\eta, \eta\phi]$ into (22) and (24) yields

\[
B\mu'(c^{ax}) - D\mu'(c^{ax}) = -[1-\beta(1+\rho)] \frac{\alpha}{1-\alpha} [\eta, \eta\phi],
\]
\[
R\mu'(c^{ax}) - Q\mu'(c^{ax}) = -[1-\beta(1+\rho)] \frac{\alpha}{1-\alpha} [\eta, \eta\phi],
\]

where $B = 1 - \beta(1+\rho) + \beta(1+\rho)\lambda(e^*)(1-F(b))$, $D = \beta(1+\rho)\lambda(e^*)(1-F(b))$, $R = 1 - \beta(1+\rho)(1-\delta)$ and $Q = \beta(1+\rho)\delta$. By solving two simultaneous equations,

\[
\mu'(c^{ax}) = -\frac{D + R}{BR - QD} [1-\beta(1+\rho)] \frac{\alpha}{1-\alpha} [\eta, \eta\phi],
\]
\[
\mu'(c^{ax}) = -\frac{B + Q}{BR - QD} [1-\beta(1+\rho)] \frac{\alpha}{1-\alpha} [\eta, \eta\phi].
\]

Since it is apparent that $D + R = B + Q = 1 - \beta(1+\rho)(1-\delta) + \beta(1+\rho)\lambda(e^*)(1-F(b))$, we can obtain

\[
\mu'(c^*) = \mu'(c^{ax}) = \mu'(c^{ax}) = -\frac{\alpha}{1-\alpha} [\eta, \eta\phi].
\]

Since $\mu'(c^*)$ is strictly decreasing in $c^*$ due to the assumption that $\mu''(c^*) < 0$, it results that $c^*$ is increasing in $\alpha$ and $\eta$, respectively.

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