Optimization of the task of forming a management system of hierarchical multilevel complex organizational systems

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Abstract. A hierarchical configuration procedure in the process of analyzing the functioning of organizational structures using algorithms for solving problems using discrete programming methods is considered in detail in this paper. On the basis of the research, the authors propose a methodology for the combined application of an initial approximation algorithm with certain properties and an algorithm for a recursive procedure for a directed search for the optimal hierarchy, which, in the future, solving the traveling salesman problem in a minimum time much more efficient than using the methods of dynamic programming of branches and boundaries.

1. Introduction

The presentation of the organizational structure in the form of a hierarchical configuration of the system goals will make it possible to more efficiently carry out the process of assessing the functioning and, if necessary, take effective control actions.

The process of forming an optimal hierarchical structure is described in the article “Formation of the structure of hierarchical multilevel organizational systems” [1]. Within the framework of this work, key is to optimize the process of forming a control system for hierarchical multilevel systems.

The considered approach to dividing the control system of hierarchical multilevel complex organizational systems (CS HMCOS) and representing it in the form of a graph G allows creating algorithms for solving problems using discrete programming methods. This problem arises when allocating resources over the structure of CS HMCOS with the determination of the efficiency of using these resources [1, 2, 3, 4, 5].

The formation of a hierarchical model using the structure of a higher educational institution as an example is considered. Let the graph G (V, E) determine the technology for performing a complex of scientific research works (R&D) by various departments of the university.

For the university, an information model of the UCO system has been determined, which is shown in figure 1.
Figure 1. Information model of the UCO. BI\(_0\) - information about the results of the lesson control; BIn - information of regulatory documents; BId - additional information; PI - providing information; EP - educational process; ECP – extra-curricular process; GB - governing bodies; CO - control objects.

We believe that the vertices of the graph G identify the events of the execution of R&D tasks by the performers at certain hierarchical levels of CS HMCOS. The efficiency of using performers to complete tasks at each \(\lambda\) stage is set by the values \(W_{\lambda}(\rho)\). We believe that it is necessary to determine the distribution of tasks by stages of their implementation, \(\sigma^* = (V_1^*, V_2^*, ..., V_L^*)\) so that function (1) is maximal [6].

\[
H_L(\sigma) = \sum_{\lambda=1}^L \sum_{\rho \in V_{\lambda}} W_{\lambda}(\rho)
\]  

(1)

2. Discussion

The algorithm of the initial approximation, which has certain properties and the algorithm of the recursive procedure of directed search for the optimal hierarchy, make it possible to partition a partially ordered set of objects of CS HMCOS into levels obtained by dividing into HMS. These methods can be used to generate a certain ordering on the set of solutions [7, 8] of convex discrete optimization problems, which will subsequently help to use heuristic procedures to solve this problem.

An ordered set of pairwise disjoint subsets \(V_\subseteq V\) that form the collection of the set \(V\):

\[
V_i \cap V_j = 0 \text{ by } i \neq j, \quad \bigcup_{i=1}^n V_i = V.
\]

A real vector function \(q(v) = (q_1(v), ..., q_n(v))\) is defined on \(V\), where \(|v| = N\) is the number of elements of \(V\).

This vector function determines the fact that if the element \(v \in V\) during the partition belongs to the class \(v_k\), then the value of the objective function is increased by \(\alpha(v)\).

Therefore, we assume that a mapping \(w: \{v \times \sigma \} \rightarrow \mathbb{R}\) is given, where \(\sigma\) is the set of all partitions, \(\mathbb{R}\) is the set of real numbers; \(w(v, \sigma) = \rho(\sigma)\), where \(k\) is such that \(v \in V_k\). It is clear that a real function \(w(v, \sigma)\) is defined, the value of which for each element \(v\), with a certain structure of its functioning.

Based on this, a set \(\{w(v, \sigma); v \in V_k\}\) appears, which determines the quantitative characteristics of the partition \(\sigma\). The quality of CS HMCOS partitioning is characterized by the following functionality:

\[
H(\sigma) = f(\{w(v, \sigma); v \in V\})
\]

(2)

If \(f\) is a linear function then (2) will look like:
The problem of maximizing the functional is considered in more detail (3). Let us find a partition $\sigma^*$ that will allow finding the maximum of functional (3) using the algorithm below:

$$V_{k+1}^* = \{ v \in V_k^* : \sigma_{k+1}(v) = \max_{\alpha \geq k} \sigma_{\alpha}(v) \},$$

for $k=1, 2, 3, ..., n-1$, where $V_n^* = V \setminus V_1^* \setminus V_2^* \setminus ... \setminus V_{k+1}^*$.

For discrete problems that allow determining the partition $\sigma$, it is necessary to define constraints [9]. On the set $V$, a real function is defined that determines the measures of the elements $\mu(v)$, and the admissible values of the parameters of the subsets $v$. For such a case, the restrictions take the form:

$$i_k \leq \sum_{v \in V_k} \mu(v) \leq L_k, \quad k = 1, 2, ..., n.$$

Of particular interest is the assignment problem [9, 10], on the basis of which it is possible to determine $\sigma_k(z)$, that is, determine the degree of compliance of the executor with the $vk$ job so that for the $vk$ task, $\mu(v) \equiv 1$, in such a way that $\sum_{v \in V} \mu(v) = |V_k|$.

The constraints on the subset of the partition in this case are defined in the form $l_k \leq |V_k| \leq L_k$, that is, this determines the number of performers who perform the $k$ task by more $l_k$ and less $L_k$ workers.

Workers should be allocated in such a way as to maximize the efficiency of work performance:

$$H(\sigma) = \sum_{k=1}^{n} \sum_{v \in V_k} \sigma_{k}(v).$$

In general, the assignment task can be described by the expression:

$$H(X) = \sum_{i=1}^{n} \sum_{k=1}^{n} c_{ik} x_{ik},$$

where $c_{ik}$ is the matrix that determines the efficiency of the $i$ performer of the $k$ task, and $x_{ik} = 1$ if the $i$ performer is assigned to the $k$ task.

Comparing (5) and (6), it can be noted that $\sigma_{k}(v) = c_{ik}$, and the subsets $V_k, k=1, ..., n$, correspond to the tasks of the executor. Consequently, the graph $G(V,E)$ can be uniquely identified by some effective set of partitions $\sigma^e$.

3. Materials and methods

These problems are solved by the algorithm for determining the initial approximation, which allows finding a lot of solutions to this problem. Certain solutions correspond to the maximum values of the elements of the matrix $c_{ik}$. It is impossible to completely form a system of $n$ maximal, independent elements (i.e., independent zeros); therefore, it is necessary to form chains of replacement and addition of marked independent zeros at each step [11, 12].

To form the graph $G(V,E)$ its fragments must be built from chains replaced by marked zeros.

The algorithm on the basis of which it is possible to form the graph $G(V,E)$ in an explicit form suggests the need to identify the connections between the vertices that represent the set of tasks to the corresponding vertices $v_1, v_2, ..., v_n$, the correspondence of the performer $v_1$ to the execution of tasks $k$ with an assessment of the effectiveness of these connections, and values of the matrix $c_{ik}$. It is assumed
that in the graph $G (V, E)$ for performers $v$ there are at most $h$ arcs. The algorithm required for constructing the graph $G (V, E)$ is given below.

1. The graph $G (V, E)$ is defined based on the values of the matrix $C_{ik}$ with at most $h$ arcs outgoing from $V$.
2. According to the criterion of maximum efficiency, tasks are distributed among performers $v_k, k = 1, ..., n$
   \[ V^0_k = \{ v_i : \sigma_k(v_i) = \max_j \sigma_j(v_j) \}. \]

3. A queue of empty subsets $V^e_j, j \in \{ 1, ..., n \}$ is formed.

\[ \text{Figure 2. Algorithm for the formation of the graph } G (V, E). \]
4. The performer $v_i$ at $V^d_i$ is assigned if there is an arc $(v_i, V_j)$, and the following condition is satisfied:

$$\delta_i = \max_{j \neq i} \{\sigma_j(v_i) - (\sigma_k_1(v_i) - \max_{e \neq i} \sigma_k(e))\}. \quad (8)$$

5. If $V^d_i$ is empty then there is an end of the algorithm, otherwise go to 3.

Problems with restrictions on the partition structure $\sigma$ are often used. In such problems, a partially ordered set can be formed between certain pairs of elements of the set $V$, i.e. directed graph $G(V, E)$ without contours for given relations between pairs of elements of this set.

If $v_i$ precedes the element $v_2 (v_1, v_2)$, then in this case the arc $(v_i, v_2) \in E$ and in this case it is required to find an extreme path on the graph $G(V, E)$.

The problem of finding paths that can arise from a fixed vertex $v_i$, graph $G$, to all other vertices is considered. For this, it is possible to use Ford algorithm [13].

For a fixed vertex $v_i$, vertices are selected based on recalculating their indices, which determine their length for a certain shortest path. Consequently, a subset $\sigma_j$ is formed for fixed vertices $v$ that belong to one of the subsets $V_i, V_2, ..., V_k$ or one of the possible options for the formation of subsets from the remaining vertices.

In this case, the constraints on subsets must be specified by the necessary options for moving along the graph $G$. For a graph $G$, having a set of vertices $v_k$, can be determined from the vertices that will be located in a ring on a plane at a distance of no more than $i_k$ from $v_i$, taking into account the elements of the subset $V_i, V_2, ..., V_k$.

In this case, the indices are recalculated only for the vertices of the set $V_k$ and a number of previous subsets, with the need to select all vertices that are connected with the vertices of the subset $V_k$ and some associated vertices. For once, when the indices change, the rings $V_i, V_2, ..., V_k$ are adjusted based on the determined distances from the vertex $v_i$ to the required vertices. After finding the ring $V_k$, we pass to the definition of the next $V_{k+1}$.

In the general case, for graphs $G(V, E)$, the set $V_k$ is determined using its vertices, which are a direct mapping of the subset $V_k$. First, the indices for the vertices of the set $V_k$ are recalculated. Next, the vertices that are in the direct display of the vertices with corrected indices are recalculated.

Further steps to partition the graph are carried out when the vertex indices stop changing and then a new partition $V_i, V_2, ..., V_k$ should be defined. Depending on the analysis of the subject area, the properties of the graph $G$ will be different and therefore, it is required to determine other heuristic approaches to the formation of the partition $V_i, V_2, ..., V_k$, and to develop algorithms for recalculating indices that can be developed by analogy with the algorithms used in artificial intelligence systems [14, 15, 16].

4. Conclusion

Formation of a graph partition for HMS using the commercial traveler problem will be carried out with the least computational cost. Any of the subsets $V_k$ is associated with one element. The set $V_k$ is formed on the basis of the vertices of the graph, which are a direct reflection of the subsets $V_k$ with the need to reduce the dimension of $V_k$ as much as possible using trees of shortest paths between the elements of the set $V$, constructed using the distance matrix $d_{ij}$.

$V_k$ includes a vertex that has a connection with a vertex $V_{k,j}$ of another minimum length. In the event that the partition has a vertex $V_k$, then it is necessary to change this partition, excluding previously the arcs that were used in the previous partitions.

Thus, on the basis of the above research, it is assumed that for plane graphs, the joint application of the initial approximation algorithm with certain properties and the algorithm of the recursive procedure of directed search for the optimal hierarchy will be more efficient than the use of dynamic programming of branches and boundaries, which will allow solving the traveling salesman problem of dimension more
than one hundred vertices in a minimum time.

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