Comparing the Aerodynamic Performance of Floating Offshore Wind Turbines with Optimised and Linearised Blades

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Abstract. Wind turbine blade designs that are aerodynamically optimised for maximum efficiency are known to have a large chord and twist at the inboard sections. Manufacturers have adopted simpler blade designs having a linear chord distribution at the inboard regions. While this has a small effect on the efficiency loss of wind turbines with fixed foundations, the effect of such blade linearization on the aerodynamic performance of floating wind turbines has not been sufficiently dealt with in open literature. This study applies a free-wake vortex model to compare the power and thrust characteristics of floating wind turbines having linearized blades with those having optimised blades. Three- and two-bladed rotors are assumed for different platform surge motions that are representative of different metocean conditions. The study showed that the loss in efficiency through blade linearization is small and similar in magnitude to that encountered in fixed rotors.

1. Introduction
Floating offshore wind turbine (FOWT) technology has advanced significantly over the recent years with multiple full-scale prototypes being deployed successfully. Maximising the aerodynamic power coefficient of FOWTs will remain fundamental to ensure cost effective exploitation of the vast offshore wind resources in deep water sites. Various studies have shown that FOWTs are known to exhibit a more complex aerodynamic behaviour when compared to turbines on fixed foundations [1, 2, 3]. The platform motion induced by wave hydrodynamics loads are known to exhibit a considerable influence on the rotor power and thrust characteristics. Various theoretical works have been published to determine the maximum or ideal efficiency of a wind turbine. The upper limit of 16/27 has been determined by Betz [4] in his published work in 1920 through the application of the simple Rankine-Froude theory for a rotor disc. As documented by van Kuik [5], the same limit had also been established independently by Lanchester and Joukowski in the first decades of the past century, thus in reality meriting to be named the Lanchester–Betz–Joukowsky limit. Other theoretical approaches for an optimal rotor include those of Glauert [6], Goldstein [7] and Joukowski [8]. Recent theoretical analysis aimed at understanding upper limits of wind turbine efficiency has been presented by Okulov et al. [9]. Yet no theory to-date has confirmed that the Betz limit can be surpassed. Such approaches have only assumed a fixed (non-moving) rotor and have formed the basis of numerical optimisation techniques for designing wind turbine blades.
Manwell *et al* [10] presents two approaches for developing the aerodynamically optimised blade designs for wind turbines. Both approaches apply the Blade-Element Momentum (BEM) theory and ignore the drag coefficient as well as the tip/root losses. The first approach, which may be referred to as the Betz optimization approach, assumes a uniform axial induction factor of 1/3 across the rotor disc and ignores wake rotation. The second approach accounts for wake rotation and derives the optimal blade chord and inflow angle from the partial derivative of the integral of the power coefficient $C_p$ with respect to the inflow angle and equating it to zero. Both approaches yield very similar blade inflow angle and chord distributions for the outer regions, typically for $r>0.3R$, while the Betz optimisation approach results in a larger chord and twist in the blade regions close to the root region. Yet the geometry of both approaches is too complex to manufacture, primarily due to the large chord and twist at the inboard sections of the blade. The wind turbine industry has adopted linearized blade designs that have a linear chord variation at the inboard sections, resulting in a blade design that is simpler and more cost-efficient to manufacture. Yet the effect of this linearization on the aerodynamic performance of FOWTs has not been well documented in open literature.

2. Objectives
The main objective of this paper is to assess the influence of blade linearization on the aerodynamic behaviours of FOWTs under the influence of platform surge motion. The study compares the rotor power and axial thrust predictions from a free-wake vortex model (FWM) of three- and two-bladed floating rotors having optimised and linearized blades.

3. Numerical Modelling

3.1 Blade Design Optimisation and Linearisation
The blade design optimisation approach presented by Manwell et al [10] which accounts for wake rotation was applied in this study. The optimisation procedure is governed by the two following relations that define the blade inflow angle and chord distributions:

$$\phi = \left(\frac{2}{3}\right) \tan^{-1}\left(\frac{1}{\lambda_r}\right) \tag{1}$$

$$c = \frac{8\pi r}{N C_L} \left(1 - \cos \phi\right) \tag{2}$$

Where $\phi$ and $c$ are the optimised inflow angle and chord, respectively, at blade element of radius $r$. $\lambda_r$ is the local tip speed ratio while $N$ and $C_L$ are the rotor number of blades and design lift coefficient. The operational angle of attack of the aerofoil at a given radial location is based on minimizing the drag-to-lift ratio ($C_D/C_L$). Burton *et al.* [11] propose the use of a linearized chord profile by extrapolating a straight line passing through the 70 to 90 percent span regions of the blade. This produces a blade which employs much less material especially at the proximity of blade root. Since most of the power is produced in the outer regions, the removal of material next to the root would not significantly impact the power output. A similar power coefficient to the optimal one is obtained. However, such studies have thus far been restricted to fixed rotors and it is not known whether a similar power coefficient would also be observed for floating rotors that face large motions caused by wave action on support platforms.

3.2 Free-wake Vortex Modelling
The open-source Wake Induced Dynamic Simulator (*WInDS*) is the free-wake vortex model code used in this study to perform the simulations for the FOWT rotors with optimised and linearised blades. The code, developed by Sebastian [12], is able to model the aerodynamics of floating rotors by being able to prescribe the motion at the tower base as a function of time. A lifting line model is used to represent the blades while the wake is modelled using helical vortex sheets consisting of a mesh of vortex filaments emerging from each blade. The Biot-Savart law is applied to the individual vortex filaments in conjunction with a time-marching numerical integration algorithm implementing a predictor-corrector scheme to model the vorticity in the turbine wake at each rotor time step. The Lamb-Oseen vortex core
model was used to account for viscous effects around vortex filaments while the Ramasamy–Leishman (RL) model was used to model the vortex core growth with time [12, 13].

4. Methodology
The optimised blade profiles for a two- and a three-bladed rotor were generated using equations (1) and (2) for a design tip speed ratio \( \lambda_d \) of 8.55. The rotor tip and root diameters were set to be equal to 126 m and 3 m, respectively. The designed rotors were assumed to have the same aerofoil type (NACA64_A17) across all radial stations, with a design lift coefficient \( C_l \) equal to 1.01, corresponding to an angle of attack of 5°. The linearised blade profiles were produced by extrapolating a straight line for the chord distribution passing through the 70 to 90 percent span regions of the blade, as proposed by Burton et al [11]. However, the linearized blades were assigned a twist distribution equal to that of the optimised blades. The blade twist distributions for the three- and two-blade rotors are presented in Figure 1 while the chord distributions for the optimised and linearised blades are presented in Figure 2.

The four rotors were modelled in the FWVM code as illustrated in Figures 3 and 4. The blade tip pitch angle was set to 0°. The blades were discretised into 37 lifting line elements, with a cosine distribution for the element length applied at the blade tip and root regions. The incremental azimuthal step for the free-wake time-marching solution was set to 10° while the number of rotor revolutions was selected such that the free-wake extended by at least 7 rotor diameters downstream. To account for the effect of floating platform motion on rotor aerodynamics, a sinusoidal motion along the surge degree of freedom was applied to the turbine:

\[ X = A \sin(2\pi f) \]  

Where \( A \) and \( f \) are the amplitude and frequency of the surge motion, respectively. Three sea states were considered, each having a different wave height \( (H_w) \) and period \( (T_w) \), as shown in Table 1. In view of the fact that the present study was intended to compare the influence of rotor blade geometry on the aerodynamic performance, all four rotor variants were prescribed the same platform motion amplitude and frequency for a given sea state. The estimates for the platform surge amplitude were obtained from a recent study involving the modelling of a 5MW floating wind turbine installed on a semi-submersible platform using the open source code FAST [14].

The free wind speed was assumed to be steady and equal to 11.4 m/s. The presence of wind shear and turbulence were ignored. Three different rotor tip speed ratios were considered (\( \lambda = 4, 8, 12 \)), resulting in a total of 36 FWVM simulations as summarised in Table 2. Figure 5 shows typical wake plots generated by the FWVM at the different rotor tip speed ratios. Platform motion causes the rotor to experience a time varying rotor power and thrust as shown in Figure 6. The number of surge oscillations was selected to be large enough for the sinusoidal variation for the power and thrust values to stabilise with time. These were used to determine (1) the percentage deviation in the time-averaged rotor power and thrust from that obtained for a fixed (non-floating) rotor and (2) the percentage ratio of the amplitude in power and thrust to the corresponding time-averaged values. These two parameters are expressed mathematically through the relations:

\[ \delta_\lambda = \frac{Z_{\text{Mean}, F} - Z_{\lambda F}}{Z_{\lambda F}} \times 100\% \]  

\[ \Delta_\lambda = \frac{Z_{\text{Max}} - Z_{\text{Min}}}{2Z_{\text{Mean}, F}} \times 100\% \]

\( \delta_\lambda \) is the percentage deviation in a time-averaged performance characteristic \( Z \) (representing power coefficient \( C_p \) or thrust coefficient \( C_T \)) between floating (F) and non-floating (NF) conditions. \( \Delta_\lambda \) is the percentage amplitude in terms of the mean value taken with respect to time of a performance characteristic \( Z \) when subjecting the turbine to surge motion.

| Table 1. Modelled sea states |
|-----------------------------|
| Sea State | \( H \) (m) | \( A \) (m) | \( T_w \) (s) | \( f \) (Hz) |
| S1         | 3.66        | 0.75        | 9.50        | 0.10526     |
| S2         | 6.40        | 1.85        | 11.65       | 0.08584     |
| S3         | 9.14        | 3.31        | 13.6        | 0.07353     |
Figure 1. Spanwise twist variation for all the designed rotors with $\lambda_d = 8.55$.

Figure 2. Spanwise chord variations for all the designed rotors.

Figure 3. Three- and two-bladed rotors with optimised blades as implemented in WInDS.
Figure 4. Three- and two-bladed rotors with linearised blades as implemented in WInDS.

Figure 5. Wake evolution for three-bladed linearised rotor operating under sea state S3.

Table 2. Test Conditions

| Sea-States | Tip Speed Ratios | Number of Blades | Geometric Profiles |
|------------|------------------|------------------|--------------------|
| S1         | 4                | 2                | Linearized         |
| S2         | 8                | 3                | Optimized          |
| S3         | 12               |                  |                    |

Total Number of Simulations: 36
5. Results and Discussion

5.1 Percentage Deviation Analysis
Figures 7 and 8 present the results obtained using Equation (4) for comparing the percentage deviations in the time-averaged power and thrust coefficients for floating conditions with those obtained for fixed conditions. It is noted that the deviations are small (< 2.5%) and within the order of magnitude of the numerical errors resulting from spatial and temporal discretization of the FWVM solution. Thus, the impact of platform surge motion on rotor time-averaged power and thrust values is expected to be marginal for both the optimised and linearized blades. This trend is observed for both the 3-bladed and 2-bladed rotors across all modelled tip speed ratios and sea states. It is noted that for the majority of cases, the values of $\delta C_P$ and $\delta C_T$ are somewhat larger for the optimised rotors, implying that the rotors with linearized blades are likely to experience smaller deviations in time-averaged power and thrust when compared with those obtained for non-floatable conditions. Yet the difference is very small.

5.2 Percentage Amplitude Analysis
Figures 9 and 10 present the results for the percentage amplitude for the four different rotors, computed using Equation (5). An increase in the operational tip speed ratio leads to a corresponding increase in the average peak to peak values. At $\lambda = 8$ and $\lambda = 12$, the values of $\Delta C_P$ generated by all four rotors is much greater than that of $\Delta C_T$. This was anticipated since, according to the momentum theory, the thrust coefficient has a quadratic relationship with the axial induction factor, while the power coefficient varies cubically. As it would be expected, the increase in the percentage amplitude is shown to increase for larger sea state amplitudes. At $\lambda = 4$, the $\Delta C_P$ and $\Delta C_T$ values are all shown to fall below 4%. Such small amplitudes indicate that the power and thrust is quasi-steady with time, even at the most extreme sea state $S3$. However, the percentage amplitude increases rapidly at higher tip speed ratios. The highest percentage amplitude values in power and thrust are reached while operating at $\lambda = 12$, under sea state $S3$. At these conditions, when considering the three-bladed optimized rotor, percentage amplitudes in the power and thrust coefficients are found to be equal to 46.1% and 18.0%, respectively. These indicate a significant number of fluctuations in the power output and blade loadings on the turbine. The values of the percentage amplitudes obtained by both the linearized and optimised rotors are very similar to each other, for both the two-bladed and the three-bladed variants. The optimized rotors are shown to generate slightly higher percentage amplitudes than the linearized rotors, however such a difference is found to be marginal, especially while operating at $\lambda = 12$. 

Figure 6. Power and thrust with time for an optimised 3-bladed rotor at sea state $S2$ and $\lambda = 8$
Figure 7. Percentage deviation in $C_p$ and $C_T$ for 3-bladed rotor designed at $\lambda_d=8.55$.

Figure 8. Percentage deviation in $C_p$ and $C_T$ for 2-bladed rotor designed at $\lambda_d=8.55$. 
Figure 9. Percentage amplitude in $C_P$ and $C_T$ for 3-bladed rotor designed at $\lambda_d=8.55$.

Figure 10. Percentage amplitude in $C_P$ and $C_T$ for 2-bladed rotor designed at $\lambda_d=8.55$. 
5.3 Analysis of the Time-averaged Power Coefficient

Figure 11 shows the time-averaged power coefficient for all four rotors for different tip speed ratios and sea states. State $S0$ represents the fixed, non-floating condition. The rotors with optimised blades generate marginally higher power than those with linearized blades at all sea states and tip speed ratios. Yet the amount of loss in the power coefficient across all the modelled sea states is small and has the same order of magnitude as for the fixed rotor condition ($<5\%$).

When the rotors were modelled at high frequency sea states while operating close to the design tip speed ratio, the instantaneous power coefficient was estimated to exceed the Betz limit of 59.3%. It should be noted that this limit originates from steady flow assumptions in which the air velocities at any point in the flow field and the rotor axial thrust are invariant with time. The flowfield around a surging rotor is unsteady and this it is possible for the instantaneous power coefficient to exceed the Betz limit. As indicted by Wen et al. [15] the instantaneous increase in $C_p$ is believed to occur due to the presence of a time advance or delay between the maximum power available and the power being extracted by the rotor. Whether a time advance or time delay is achieved depends highly on the operational tip speed ratio. Yet, the present study has shown that, even though the power coefficient did instantaneously exceed the Betz limit, the mean power coefficient, $C_{pm}$, was still below this limit.

5.4 Effect of Blade Pitch Angle

The results presented above have only considered a fixed pitch angle of $0^\circ$. Five equally spaced pitch angle values lying in between $-4^\circ$ and $4^\circ$ were considered for the three-bladed rotors designed at $\lambda_d = 8.55$, while running at $\lambda = 8$. The use of the same rotor and operational tip speed ratio allows for a comparison between the effects of pitch angle variation generated at fixed-based and floating conditions to be drawn. Sea State $S3$ was chosen for this analysis since it has the largest wave motion amplitudes, which would yield more observable results. The effect of pitch angle on the aerodynamic performance of the rotor considered should be well representative to those attainable by the other rotors. The results for the time-averages power ($C_{pm}$) and thrust coefficients ($C_{Tm}$) are presented in Figure 12. It is shown that both parameters are lower for the rotor with linearized blades, yet the differences are small ($<5.7\%$) across the entire blade pitch angle range modelled.

6. Conclusion

The loss in the time-averaged power coefficient resulting from blade chord linearisation is predicted by the free-wake vortex model to be small and has the same order of magnitude of fixed rotors. Furthermore, blade linearization leads to a small decrease in the time-averaged thrust coefficient which would lead to a reduction in the aerodynamic load exerted by the rotor on the floating platform. These trends were also observed for extreme sea states leading to large platform surge motion and over different blade pitch angles. However, linearized two-bladed rotors were found to be more susceptible to a significant performance drop when operating at off-design conditions. While the results presented by the aerodynamic simulation tool are yet to be validated through experiments in controlled conditions, this study indicates that the same linearization approach applied for the aerodynamic design of fixed rotors to simplify blade manufacturing and lower costs may be readily adapted for floating rotors as well without significant detriment to turbine performance.
Figure 11. Time-averaged power coefficient values for all four rotors at $\lambda = 4, 8$ and 12.

Figure 12. Variation of the time-averaged power (left) and thrust (right) coefficient with blade pitch angle for the 3- designed at $\lambda_d=8.55$, while operating at $\lambda=8$ and sea state S3.
7. References

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