THE IMPACT OF MASS SEGREGATION AND STAR FORMATION ON THE RATES OF GRAVITATIONAL-WAVE SOURCES FROM EXTREME MASS RATIO INSPIRALS

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Received 2016 August 11; revised 2016 September 14; accepted 2016 September 18; published 2016 October 3

ABSTRACT

Compact stellar objects inspiraling into massive black holes (MBHs) in galactic nuclei are some of the most promising gravitational-wave (GWs) sources for next-generation GW detectors. The rates of such extreme mass ratio inspirals (EMRIs) depend on the dynamics and distribution of compact objects (COs) around the MBH. Here, we study the impact of mass-segregation processes on EMRI rates. In particular, we provide the expected mass function (MF) of EMRIs, given an initial MF of stellar black holes (SBHs), and relate it to the mass-dependent detection rate of EMRIs. We then consider the role of star formation (SF) on the distribution of COs and its implication on EMRI rates. We find that the existence of a wide spectrum of SBH masses leads to the overall increase of EMRI rates and to high rates of the EMRIs from the most massive SBHs. However, it also leads to a relative quenching of EMRI rates from lower-mass SBHs, and together produces a steep dependence of the EMRI MF on the highest-mass SBHs. SF history plays a relatively small role in determining the EMRI rates of SBHs, since most of them migrate close to the MBH through mass segregation rather than forming in situ. However, the EMRI rate of neutron stars (NSs) can be significantly increased when they form in situ close to the MBH, as they can inspiral before relaxation processes significantly segregate them outward. A reverse but weaker effect of decreasing the EMRI rates from NSs and white dwarfs occurs when SF proceeds far from the MBH.

Key words: galaxies: general – galaxies: kinematics and dynamics – Galaxy: center – gravitational waves – stars: black holes – stars: general

1. INTRODUCTION

Nuclear stellar clusters (NSCs) hosting massive black holes (MBHs) are thought to exist in a significant fraction of all galactic nuclei, including our own (Eisenhauer et al. 2005; Ghez et al. 2005). The dynamics of stars in the dense NSC lead to strong interactions between stars and the MBH. In particular, compact objects (COs), such as white dwarfs (WDs), neutron stars (NSs), and stellar black holes (SBHs), may inspiral onto the MBH and emit gravitational waves (GWs) observable to cosmological distances with next-generation GW detectors. The inspiral of a CO into an MBH (“extreme mass ratio inspiral,” EMRI) is among the main targets of future Evolved Laser Interferometer Space Antenna (eLISA).

The properties of EMRIs and their rates depend strongly on the evolution and dynamics of different stellar populations near MBHs, processes in which mass-segregation processes play a key role (see Freitag et al. 2006; Hopman & Alexander 2006b; Alexander & Hopman 2009; Keshet et al. 2009; Preto & Amaro-Seoane 2010 and Amaro-Seoane & Preto 2011 for a detailed treatment of mass segregation near MBHs). Mass segregation occurs through two-body interactions between less massive and more massive objects. The encounters drive stellar populations of different masses toward energy equipartition, which results in the more massive objects migrating closer to the center, while the less massive ones migrate outward. In particular, mass segregation can increase the density of the more massive COs within the region $r < a_{GW}$, where $a_{GW}$ is the maximal semimajor axis, at which a CO could still inspiral and become an eLISA source (hereafter the critical separation), rather than plunge in on a too radial orbit and spending too short a time in the GW detector band and unlikely to be detected. Hopman & Alexander (2005) derived an analytical order-of-magnitude estimate for the critical separation, given by

$$a_{GW} = r_{h} \left( \frac{d_{r}}{r_{h}} \right)^{3/(3-2p)}$$

where $r_{h}$ is the radius of influence (see Section 3), $d_{r}$ is a length scale, and $p$ is the power law of the distribution function (DF; $f(E, t) \sim E^{p}$; see Section 3 and Bahcall & Wolf 1976).

The event rate of EMRIs has been estimated by several studies (e.g., Hils & Bender 1995; Sigurdsson & Rees 1997; Freitag 2001; Ivanov 2002; Alexander & Hopman 2003; Hopman & Alexander 2006b) but remains rather uncertain, in part because of the slow nature of the inspiral process, which occurs on many dynamical times (see further discussion by Hopman & Alexander 2006b).

Previous studies of EMRI rates in NSCs typically considered only populations of single-mass SBHs (with mass $m_{*} = 10 M_{\odot}$); however, theoretical studies, and the recent GW detection of a merger of two ~30 M, suggest a potentially wide range of SBH masses. The effect of such non-trivial SBH population on the rates of GWs from a merger of binary-SBH (detectable by aLIGO) was considered by O’Leary et al. (2009). Here, we focus on the implications for a different type of GW sources, namely, inspirals of COs on MBHs, producing EMRIs. Future EMRI detections could potentially probe the mass function (MF) of inspiraling SBHs. However, the mutual interactions between SBHs and stars of different masses could significantly alter their distributions near MBHs, and thereby the EMRI rate from SBHs of different masses. Therefore, translating the EMRI MF into the original MF of SBHs requires understanding the non-trivial evolution and mass-segregation processes in NSCs. Finally, we note that not only the MF of SBHs change their distribution, but potentially the star formation (SF) history and build-up of the NSC (see...
Antonini 2014; Perets & Mastrobuono-Battisti 2014; Aharon & Perets 2015; Aharon et al. 2016).

In this Letter, we explore for the first time the expected mass function of EMRIs (with the main focus on SBHs), and its relation and translation to the general MF of SBHs. Furthermore, we consider both the rates from relaxed NSCs, as well the role of the build-up and SF history in affecting the EMRI properties and rates.

2. ANALYTIC DERIVATION OF EMRI RATES IN RELAXED NUCLEAR CLUSTERS

In relaxed NSC systems, the distribution of different stellar populations can typically achieve a steady state. Studies of simple stellar populations (composed of four populations: solar-mass main-sequence (MS) stars, 1.4 \( M_{\odot} \) NSs, 0.6 \( M_{\odot} \) WDs, and 10 \( M_{\odot} \) SBHs), using Fokker–Planck (FP; see also below), Monte Carlo, or \( N \)-body simulations showed them to be distributed with power-law density profiles, generally consistent with analytic estimates of mass-segregation effects near MBH by Bahcall & Wolf (1977). Later studies (Alexander & Hopman 2009) pointed out that the mass-segregation solution for the steady-state distribution of stars around an MBH has two branches: a weak-segregation solution (described by Bahcall & Wolf 1977) and a different, strong-segregation solution. They found that their properties depend on the heavy-to-light stellar mass ratio \( M_{H}/M_{L} \) and on the unbound population number ratio \( N_{H}/N_{L} \), through the relaxational coupling parameter

\[
\Delta = 4N_{H}M_{H}^{2}/[N_{L}M_{L}^{2}(3 + M_{H}/M_{L})],
\]

where systems with \( \Delta \leq 1 \) reside in the strong mass-segregation regime. In the strong mass-segregation regime, the massive objects can achieve a much steeper density profile compared with the Bahcall & Wolf (1977) weak mass-segregation regime. Alexander & Hopman (2009) mainly focused on cases that can be generally divided between a small population of massive objects (\( M_{H} \sim 10 M_{\odot} \), SBHs) and a large population of low-mass objects (MS stars, NSs, and WDs with \( M_{L} \sim 1 M_{\odot} \)) in the \( \Delta < 1 \) regime. However, a more complex situation arises when the high-mass population is composed of a range of masses. For example, in the case of two massive populations (with \( M_{H1}, M_{H2} \) and \( N_{H1}, N_{H2} \)), the calculated \( \Delta \) parameter could be below unity when comparing each of the high-mass populations to the low-mass one, while \( \Delta \) could be above unity when comparing the two massive-star populations (i.e., treating \( M_{H2} \) as a low mass compared with \( M_{H1} \)).

Keshet et al. (2009) used analytic tools and generalized the derivation for such non-trivial MF cases. Generally, one needs to compare each population with the dominant population. For example in our cases of 1, 10, and 30 \( M_{\odot} \) stellar populations, we can compare the 30 \( M_{\odot} \) population with the 1 \( M_{\odot} \) population to find that scattering of 1 \( M_{\odot} \) leads to the strong mass segregation of the 30 \( M_{\odot} \). One should note that the addition of the 10 \( M_{\odot} \) population does affect the distribution of the 30 \( M_{\odot} \) (compared with the case of taking population 1 \( M_{\odot} \) stars), but only mildly (not shown).

Using the above mentioned results, one can find the expected density profile \( n(r) \propto r^{-\gamma} \), where \( \gamma = p + 3/2 \), then for a population of stars with a given mass in a relaxed NSC, given

\[
p(m) \approx m/4M_{0}
\]

where \( m \) is the relevant population mass and \( M_{0} \) is the weight average mass. For MF not strongly dominated by light stars (negligible flow), the relation is linear: \( p \propto m \) (Bahcall & Wolf 1977; Keshet et al. 2009). In order to find the number of GW progenitors, we then need to integrate the number of CO GW progenitors (i.e., inside the critical separation, \( a_{GW} \)):

\[
\Gamma \sim \int_{a_{GW}}^{\infty} r^{-\gamma} d^{3}r \sim a_{GW}^{3/2-p}.
\]

Substituting Equation (1) then gives us the inspiral rate dependence on the CO mass per unit mass:

\[
\Gamma(m)dm \sim a_{GW}^{3/2-p} = \left( \frac{r_{H}(d_{m})}{r_{H}} \right)^{3/[(3-p_{m})]} \frac{1}{1 - \frac{m}{M_{0}}} dm.
\]

Given the GW detector sensitivity on the inspiraling CO mass, C0s with \( M_{1} > M_{2} \) would be detected to distances larger than \( M_{2}/M_{1} \), and taking a homogeneous universe at sufficiently large distances, one expects a \( (M_{2}/M_{1})^{-p} \) enhancement in the detection rate. Given an intrinsic MF of C0s (for simplicity assuming a power-law distribution; \( \xi(m) \propto m^{-3} \)), we can now combine all of the above EQs together and relate the CO intrinsic MF to the MF of detected EMRIs:

\[
N(m) \sim \int \left( \frac{r_{H}(d_{m})}{r_{H}} \right)^{3/[(3-p_{m})]} \frac{1}{1 - \frac{m}{M_{0}}} m^{-3} dm
\]

where the last integral can be numerically solved. Given a sufficient number of EMRI detections, one can use these relations to derive the intrinsic MF of SBHs.

3. NUMERICAL CALCULATIONS USING A FOKKER–PLANCK APPROACH

In order to test the analytic results, we use an FP approach. Our model is based on the classic approach of Bahcall & Wolf (1976) and Bahcall & Wolf (1977) to the problem, and we use our parallelized FP code (as described in Aharon & Perets 2015; Aharon et al. 2016). We simulate the evolution in time, \( t \), of the energy, \( E \), DF—\( f(E, t) \), and the number density of stars in a spherical system around an MBH. The DF represents the distribution of stars in a central few parsecs and, in particular, in the range between the Schwarzschild radius and the radius of influence. To our original code we now added the treatment of multi-mass cases, following the same equations as described in Bahcall & Wolf (1977) and Hopman & Alexander (2006b).

In the following, we briefly recapitulate the main assumptions and discuss our treatment of GW capture.

3.1. Fokker–Planck Analysis

The FP model used consists of a time- and energy-dependent, angular momentum-averaged particle conservation equation. It has the following form:

\[
\frac{\partial f(E, t)}{\partial t} = -AE \frac{\partial F}{\partial E} - F_{LC}(E, T) + F_{SF}(E, T)
\]
We followed the evolution of several types of NSCs with an MBH mass of $4 \times 10^6 M_\odot$, which can be a representative of a typical eLISA source. We first studied primordial NSCs that achieved a steady state and did not experience any SF. Such models correspond to the same type of models considered previously (e.g., Freitag et al. 2006; Hopman & Alexander 2006), other than the difference in stellar population used (see the next section; we also recalculated the same models used by Hopman & Alexander 2006 and verified we got the same results in this case).

For the model with SF we tested two cases: inner SF formation in the range 0.05–0.5 pc and outer SF between 2 and 3.5 pc. For each of these two cases we considered two different scenarios of formation and evolution: (1) in situ formation of MS stars and COs and (2) an initial pre-existing cusp of MS + CO stars evolved with continuous SF MS+CO stars (see Table 1). In the latter scenario, we define the initial MS DF within $r_h$ in the form of $f(E_{r<r_h}, t_0) \propto E^{0.25}$, corresponding to the BW steady-state cusp that has the form of $n \propto r^{-7/4}$. The density profile at $r_h$ is normalized to $4 \times 10^4$ pc$^{-3}$, corresponding to number density at $r_h$ in the Galactic Center (GC), assuming a mean mass of $1 M_\odot$ (Genzel et al. 2003).

Table 1

| Scenario | CO Formation Rate (yr$^{-1}$) | $\Gamma_{NS}$ (Gyr$^{-1}$) | $\Gamma_{WD}$ (Gyr$^{-1}$) | $\Gamma_{BHout}$ (Gyr$^{-1}$) | $\Gamma_{BHout}$ (Gyr$^{-1}$) |
|----------|-------------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| Pre-existing cusp outer formation without 30 $M_\odot$ BH | $10^{-4}$ | 14 | 79 | 232 | ... |
| Pre-existing cusp outer formation | ... | 11 | 71 | 92 | 265 |
| Pre-existing cusp inner formation | ... | 15 | 73 | 87 | 252 |
| Outer in situ formation | ... | 7 | 22 | 58 | 273 |
| Inner in situ formation | ... | 39 | 62 | 74 | 312 |
| Inner in situ formation | ... | 32 | 112 | 62 | 288 |
| Pre-existing cusp inner formation | ... | 8 | 67 | 15 | 97 |
| Relaxed cusp | ... | 7 | 73 | 89 | 273 |
| Relaxed cusp without 30 $M_\odot$ BH (based on Hopman & Alexander 2006b) | ... | 6 | 33 | 252 | ... |

where

$$A = \frac{32 \pi^2}{3} G^2 M_*^2 \ln(\Lambda).$$

The term $F = F[f(E), E]$ is related to the stellar flow and plays an important role in the evolution of the stellar cluster. It describes the merging of stars in energy space due to two-body relaxation; it is defined by

$$F = \int dE' \left( f(E') \frac{\partial f(E', t)}{\partial E'} - f(E', t) \frac{\partial f(E, t)}{\partial E} \right) \times (\max(E, E'))^{-2/3}.$$  

We note that similar to the work of Hopman & Alexander (2006b), we neglect here the effect of resonant relaxation (RR; Rauch & Tremaine 1996; Rauch & Ingalls 1998), which was suggested to increase the EMRI rate by up to an order of magnitude (Hopman & Alexander 2006a; Merritt 2015). However, a more recent analysis of Bar-O & Alexander (2016, who also considered Merritt’s work; B. Bar-O, private communication) suggests a relatively negligible effect, supporting our approach on this issue.

3.2. NSC Models

We adopt the following parameters taken from the GC values: $\sigma_r = 75 \text{ km s}^{-1}$ and $r_h = 2 \text{ pc}$ (Genzel et al. 2003). We consider the DF at $r > r_h$ to have a Maxwellian distribution: $f(E_{r>r_h}, t) \propto e^{E/\sigma}$ (Bahcall & Wolf 1976).

In order to account for SF, following our previous work in Aharon & Perets (2015), we added the source term that represents the stars and CO formation in the following form:

$$F_{SF}(E, t) = \frac{\partial}{\partial t} (\Pi(E)E_0E^{-\alpha}),$$

where $\Pi(E)$ is a rectangular function, with boundaries that correspond to the region where new stars are assumed to form; $E_0$ is the source term amplitude; and $F_{SF}$ is a power-law function with a slope $\alpha$, defining the SF distribution in phase space.

3.3. Types of Stellar Objects

Similar to Hopman & Alexander (2006b), we consider four basic populations. The first consists of MS stars, assumed here to be of solar mass. MSs do not contribute to the GW inspiral rate since they are tidally disrupted before spiraling in, but they do contribute dynamically. The other populations represent WDs ($M_{WD} = 0.6 M_\odot$), NSs ($M_{NS} = 1.4 M_\odot$), and SBHs. We considered cases that include stellar populations of 10 $M_\odot$, SBHs (similar to previous studies), but also considered cases with both 10 $M_\odot$, and 30 $M_\odot$, SBHs, in order to probe the effect of more than a single type of massive objects. Considering the work of Kroupa (2001), we used the fraction ratios of the four populations as $C_{MS}:C_{WD}:C_{NS}:C_{BHout} = 0.72:0.26:0.014:2.3 \times 10^{-5}:2.2 \times 10^{-4}$, typical for continuously star-forming populations. The MF of SBHs is not well understood; however, for SBHs up to $\sim 40 M_\odot$ theoretical models of direct collapse suggest an almost linear relation between progenitor mass and final SBH mass (Belczynski et al. 2016), and we therefore assume an SBH MF that goes like $\propto m^{-2.3}$.

4. RESULTS

4.1. Distribution of Compact Objects Populations in NSCs

We followed the evolution of the studied NSCs for 10 Gyr corresponding to the Hubble timescale. Note that this timescale is also comparable with the relaxation time for a GC-like NSC, but likely lower than the relaxation time for NSCs hosting more massive MBHs. We present the density profile of the studied
The colors of its density profile. The maximal semimajor axis for each CO population where it experiences “successful inspiral.” For each population, the colors of the dashed–dotted lines correspond to the colors of its density profile. The relaxational coupling parameter for the 30 $M_\odot$ is $\Delta \approx 0.87$. The power laws for each population within its $\alpha_{GW}$ are $\gamma_\text{NS} = 1.4$, $\gamma_\text{WD} = 1.3$ and $\gamma_{10M_\odot,BH} = 1.9$. Right panel: similar to NSC, with the addition of a stellar population composed of 30 $M_\odot$ SBHs. The relaxational coupling parameter for the 30 $M_\odot$ is $\Delta \approx 0.08$, and the power laws are $\gamma_\text{NS} = 1.3$, $\gamma_\text{WD} = 1.3$, $\gamma_{10M_\odot,BH} = 1.4$ and $\gamma_{30M_\odot,BH} = 2.1$.

The distribution of the 10 $M_\odot$ SBHs in NSCs that include 30 $M_\odot$ SBHs is very similar to the NS distribution, where in NSCs without 30 $M_\odot$ SBHs, the distribution of 10 $M_\odot$ has the steeper slope compared to the other populations. In other words, in the absence of 30 $M_\odot$ SBHs, the 10 $M_\odot$ SBHs distribution is borderline between the weak and strong mass-segregation regimes. The existence of the 30 $M_\odot$ population quenches the effects of strong mass segregation on the 10 $M_\odot$, and only the 30 $M_\odot$ SBHs are strongly segregated.

As expected from the effects of strong mass segregation, the density profile of the SBHs in most scenarios is much steeper than the approximately solar-mass MS/CO stars. Consequently, the rates of EMRI of SBHs is the highest compared with NSs and WDs. In the inner regions of the NSC, SBHs can become the most frequent stellar species, where the exact region where they dominate depends on the specific model explored.
that experience inner SF. The slopes calculated in our numerical simulations are consistent with the analytical study of Keshet et al. (2009). We emphasize that we also tested an EMRI rate model with a range of SBH masses in order to verify our analytic derivation of the rates presented in Section 2 and found comparable results.

4.2. EMRI Rates

Following the obtained NSC stellar distribution, we integrated the number of COs up to the critical separation in order to quantify the number of expected EMRIs. We summarize the results in Table 1. The highest EMRIs are obtained in NSCs that evolve from a pre-existing cusp that also experiences strong SF in the outer region. The lowest overall rates are obtained with inner SF that builds up the NSC. For comparison with previous studies, the last row in the table is based on Hopman & Alexander (2006b), as described in Section 4.1.

5. SUMMARY

In this work, we studied the rates of GWs from EMRIs, but considered two novel aspects that were minimally, or not, considered before in this context. We study (1) the impact of wide mass-spectrum SBHs (as suggested by theoretical work and the recent detections of high-mass SBHs by aLIGO; Abbott et al. 2016; Belczynski et al. 2016) on EMRI rates, and (2) the role of in situ formation of stars and COs during the build-up and/or formation of NSCs.

Our main findings are as follows:

1. We find that strong mass segregation produces a steep power-law density profile for the most massive SBHs, but at the same time quenches the migration of less massive SBHs close to the MBH. We quantify this effect and provide a translation between the intrinsic MF of CO, in particular SBHs, and the (future) observable MF of the EMRI GW sources, showing the latter to be strongly biased toward high-mass COs.

2. We find that SF plays a relatively small role in eventually determining the EMRI rates for SBHs (and its MF), since most of them migrate close to the MBH through mass segregation rather than form in situ. However, the rate of EMRI of NSs can be significantly increased when they form in situ close to the MBH. In this latter case, NSs can inspiral before relaxation processes significantly segregates them outward farther away from the MBH. A reverse but weaker effect of decreasing the EMRI rates from NSs and WDs occurs when SF proceeds far from the MBH. In this case, the mass-segregation processes due to the SBHs somewhat quench the newly formed NSs/WDs from diffusing into the inner regions and lower their EMRI rates.

We would like to thank Clovis Hopman for the use of the basic components in his FP code for developing the FP code used in our simulations. We acknowledge support from the I-CORE Program of the Planning and Budgeting Committee and The Israel Science Foundation grant 1829/12, as well support form the Asher Space Research Institute in the Technion.

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