Optimum condition of boost switching regulator for charging tiny electric energy to capacitor

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Abstract. Some energy sources are left unused because they can only generate a small amount of energy. In order to utilize the small amount of electric energy generated from these energy sources, an electric power storage system is investigated. The storage system consists of a capacitor and a switching regulator. In order to obtain an analytic solution of the storage system, the system is approximated by an equivalent circuit, i.e., the energy source is assumed to have a linear I-V characteristic, and a linear approximation is introduced for the diode, the FET, and so on. The exact analytic solution for the circuit is obtained by normalized parameters. The optimum duty ratio is obtained as a function of the parameters such as stored voltage and power source voltage.

1. Introduction
Various kinds of energy sources exist, but some of the sources have not been used, because they can generate only a small amount of electric energy. As a result, the energy generated from these sources is wasted. On the other hand, energy sources such as fossil fuel and nuclear power can generate larger amounts of electric energy, but they cause environmental problems such as air pollution, radioactive pollution, and global warming. Usage of the tiny electric energy mentioned above can be a solution to the problem. In addition, the tiny electric energy can contribute to the power supply for rural areas far from power plants.

In order to utilize the tiny electric energy, accumulation of the energy is necessary because it is unsteady for usage. Thus, a storage device and a boost converter are used to accumulate the energy [1]. A capacitor or a rechargeable battery is used commonly as the storage device. A switching regulator is used as the boost converter for small amounts of electric power to large amounts of electric power [2-4]. Since the electric energy generated is small, the system must be designed and controlled to maximize the charging efficiency of stored energy per input energy.

Though the numerical solution of the boost converter gives a precise design of an efficient system [5], the design is discrete and can be adapted to only limited situations. On the other hand, the analytic solution of the boost converter gives a tolerant design of the system which can be adapted to all situations. As for the control of the system, tracking of the power source voltage [6, 7] may give good efficiency, if the system is designed well. On the other hand, an analytic solution can be given to the design of an efficient control which can be adapted to all circuits. Thus, the analytic solution is significant for both the design and control of the system. It also gives a large range of view on the design of the system from a small scale to a large scale, meanwhile numerical simulation gives us a limited range of view.
The purpose of this paper is to obtain an analytic solution of the system. Firstly, we assumed a system consisting of a switching regulator as the boost converter and a capacitor as the storage device. Secondly, we modelled an equivalent circuit of the system, i.e., the energy source is assumed to have a linear I-V characteristic, and a linear approximation is introduced for the diode, the FET, and so on. Thirdly, we obtained the analytic solution of the system. Finally, we obtained the optimum control parameters expressed as a function of circuit parameters from the analytic solution, and discussed the adaptability of the model.

2. Boost switching regulator and its equivalent circuit

Figure 1 shows a typical circuit of the boost switching regulator. The circuit can superimpose the energy charged in the inductor by inputting pulse waveform to the FET. Thus, the circuit can boost the voltage. In order to model a linear equivalent circuit of the boost switching regulator, the resistance component of the inductance, diode, and FET is assumed as linear. As the simplest power source, the power source in Figure 1 is assumed to have I-V characteristics as shown in Figure 2. For the purpose of accumulating tiny energy, the capacitance of the capacitor must be large enough. Therefore, the capacitor can be assumed as a battery with a constant voltage. An equivalent circuit of the boost switching regulator is shown in Figure 3.

$L$, $R_l$, $R_d$, $R_f$, and $R_p$ are the inductance, the resistance of the inductor, the forward resistance of the diode, the ON resistance of the FET, and the internal resistance of the power source, respectively. The circuit parameter is defined as a collective term of resistance and inductance. $V_{in}$ is the open circuit voltage of the power source and $V_{out}$ is the voltage of the capacitor. The voltage gain is defined as $g = V_{out} / V_{in}$. While the FET function as ON, the resistance of the FET is small enough compare to the resistance of the diode and the capacitor. Thus, the current flowed into the capacitor side is negligible.

When the switch is on 1, the current flows through the inductor and FET. The energy is stored in the inductor. When the switch is on 2, the energy is released for accumulation in the capacitor. $T_1$ and $T_2$ are the time interval when the switch is on 1 and 2, respectively. $T = T_1 + T_2$ is the switching period. The duty ratio of the switching regulator is defined as $\bar{d} = T_2 / T$. The control parameter is defined as a collective term of the switching period and the duty ratio. The state of current in one switching cycle depends on the number of switching. After a sufficient number of switching, the state of the current reaches a steady state in Figure 4. Applying the Kirchhoff’s law of the voltage to Figure 3 gives

$$V_{in} = L \frac{di}{dt} + R_l i$$  \hspace{1cm} (switch on 1) \hspace{1cm} (1)

$$V_{in} - V_{out} = L \frac{di}{dt} + R_d i$$,  \hspace{1cm} (switch on 2) \hspace{1cm} (2)

where $R_l = R_{in} + R_d$ and $R_d = R_l + R_d + R_p$. Equation (1) and Equation (2) are solved for $i$ under the condition of a continuous current. Subsequently, the current can be expressed as

**Figure 1.** Boost switching regulator.

**Figure 2.** I-V characteristic of the power source.
Figure 3. Equivalent circuit.

Figure 4. Current and switching period.

where

\[ \bar{\gamma} = R_2 / R_1, \quad \bar{T} = R_1 T_1 / L, \quad \bar{T} = R_2 T_2 / L, \quad \text{and} \quad \bar{T} = \bar{T}_1 + \bar{T}_2. \]

The open circuit voltage of the power source \( V_{in} \) and the circuit parameters are determined when the circuit is designed. The voltage of capacitor \( V_{out} \) increases with increasing charge in the capacitor during the operation. Therefore, the voltage gain is a given parameter. The optimum condition is defined as the condition which maximizes the electric charge stored in the capacitor per unit time. In order to obtain the optimum condition, the control parameters should be optimized according to the circuit parameters and the voltage gain.

3. Derivation of optimum condition

The charge accumulated in the capacitor in one switching cycle is

\[ q = \int_{t_1}^{t_2} i dt. \]  

The optimum control parameters can be obtained by maximizing

\[ \frac{q}{T} = \frac{V_{in}}{R_2} \left\{ \frac{1}{\bar{T}} \left( \frac{1}{1 - e^{-\bar{T} \bar{\gamma}}} \right) \left[ \frac{1 - e^{-\bar{\gamma} \bar{T}}}{1 - e^{-\bar{\gamma} \bar{T}} - \bar{\gamma} \bar{T}} \right] \right\}. \]  

The optimum duty ratio \( \bar{\alpha} \) should be chosen so that

\[ \frac{\partial(q/T)}{\partial \bar{\alpha}} = 0 \]  

is satisfied. Equation (7) leads to

\[ \bar{\gamma} = 1 - \bar{\gamma} + \frac{\bar{\gamma}^2}{\bar{T} - e^{-\bar{T} \bar{\gamma}} + \bar{\gamma} e^{-\bar{T} \bar{\gamma}} + 2 \left( 1 - \bar{\gamma} \right) e^{-\bar{T} \bar{\gamma}} + \bar{\gamma} e^{-\bar{T} (2 - \bar{T} \bar{\gamma})} - e^{-\bar{T} \bar{\gamma} - 2 \bar{T} \bar{\gamma}} \right) \]  

Since \( \partial^2(q/T)/\partial \bar{\alpha}^2 \) is always negative, equation (8) is the optimum condition. As shown in equation (8), the optimum duty ratio can be obtained as a function of the switching period, the circuit parameters, and the gain. In order to monitor the relation between the gain and the optimum duty ratio, equation (8) is plotted in Figure 5 with given parameters of \( \bar{\gamma} = 1 \) and \( \bar{T} = 1, 5, 10 \). We can plot Figure 5 independent of the specific value of each circuit parameter because the circuit parameter is normalized. The gain \( \bar{\gamma} \) increases with increasing voltage of the capacitor. The optimum duty ratio is obtained according to the voltage of the capacitor.
Using the optimum duty ratio, equation (6) is a function of $\bar{T}$ and $\bar{r}$. In equation (4), $\bar{T}$ appears in the term

$$f(\bar{T}, \bar{r}) = \frac{1}{\bar{T}} \left( 1 - e^{-\bar{d} \bar{r}} \right) \left( 1 - e^{-\bar{r} \bar{d}} \right).$$

Figure 6 shows the relation between $\bar{T}$ and $f(\bar{T}, \bar{r})$. In Figure 6, $f(\bar{T}, \bar{r})$ monotonically decreases with $\bar{T}$. In order to maximize $f(\bar{T}, \bar{r})$, $\bar{T}$ should be determined as small as possible. Minimizing the $\bar{T}$ by increasing the frequency of the FET is not efficient because energy loss of the switching of the FET increases. Thus, $\bar{T}$ should be minimized using circuit elements with small resistance and an inductor with large inductance. The optimum period $\bar{T}$ should be determined when the circuit is designed in order to make an efficient circuit.

4. Discussion on optimum duty ratio

In the equivalent circuit, a reverse current of diode might exist because the diode is replaced with the resistor. When the reverse current flows, the assumption of the equivalent circuit breaks down. When $i > 0$ is satisfied, the reverse current does not flow. Thus, the condition which the reverse current does not flow is obtained as

$$\tilde{g} < 1 + \bar{r} e^{-\bar{r} \bar{d}} \left( 1 - e^{-\bar{r} \bar{d}} \right).$$

The optimum duty ratio is needed to satisfy equation (10). Equation 8 shows the relation of the voltage gain and the optimum duty ratio. Equation 10 shows the applicable condition of the equivalent circuit. From the difference $g'$ which is obtained by subtracting the right side of equation (8) from that of equation (10) is

$$g' = e^{-\bar{d} \bar{r}} \left( 1 - e^{-\bar{r} \bar{d}} \right)^2 + \bar{r} e^{-\bar{d} \bar{r}} \left( 1 - e^{-\bar{r} \bar{d}} \right).$$

Equation (11) is always positive. Thus, the equivalent circuit is valid as the model of the system when the optimum duty ratio is used.

From equation (8), a relation between the gain and optimum duty ratio is obtained. Switching the FET is not necessary for charging electricity when the voltage gain is smaller than $1/1+\bar{r}$. When the voltage gain is larger than $1/1+\bar{r}$, the switch is controlled with the optimum duty ratio according to the voltage gain. A value as small as possible for the normalized switching period $\bar{T}$ is determined.
In Figure 5, the optimum duty ratio exists when the gain is smaller than 1. This result suggests that the usage of the switching regulator is more efficient than the direct connection between the power source and also the capacitor, in the case that the capacitor voltage is less than $V_{in}$. The area $A$ in Figure 7 shows the charge stored into the capacitor during one cycle in the case of direct connection, i.e., the charge is obtained as

$$q_A = \frac{V_{in} - V_{out}}{R_2} T.$$  \hspace{1cm} (12)

The area $B$ in Figure 7 shows the stored energy during one cycle in the case of using the switching regulator, i.e., the charge is obtained as

$$q_B = \frac{V_{in}}{R_1} L \left[ 1 - e^{-\frac{T}{T_{d} + \frac{T_{d}}{T}}} \right] \left[ 1 - e^{-\frac{\tilde{g}T}{\tilde{r}}} \right] \left( 1 + \frac{\tilde{g} - 1}{\tilde{r}} \right) + \frac{V_{in} - V_{out}}{R_2} T_2.$$  \hspace{1cm} (13)

The term of $q_B - q_A$ is positive, if the duty ratio is controlled according to equation (8). Therefore, the condition of equation (8) is optimum when the gain is $1/1+\tilde{r} < \tilde{g} < 1$.

5. Conclusion

In this paper, the optimum condition which maximizes the stored energy into the capacitor per unit time is obtained from the analytic solution of an equivalent circuit of the system. The optimum duty ratio is controlled by the voltage of the capacitor. An optimum switching period as small as possible should be determined in the design of the system. The equivalent circuit is valid as the model of the system when the optimum duty ratio is used. The optimum condition can be applied even if the gain is smaller than 1. In making the equivalent circuit, we assumed the diode as an ideal diode and the FET as a switch and ON resistance. For the purpose of obtaining the analytic solution, the non-linear element is neglected. Thus, in future studies, we should take into account the neglected factor such as the effect of non-linear elements when we design and control the actual system.

![Figure 7. Current and stored energy.](image-url)

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