Reflection coefficient for superresonant scattering

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We investigate superresonant scattering of acoustic disturbances from a rotating acoustic black hole in the low frequency range. We derive an expression for the reflection coefficient, exhibiting its frequency dependence in this regime.

I. INTRODUCTION

Progress in understanding black hole radiance, both in the semiclassical and quantum gravitational regimes, have suffered greatly in the past, due to the lack of experimental feedback. Black hole radiance consists of spontaneous radiation (Hawking radiation) [1] as well as stimulated emission (superradiance) [2], [3], [4], [5]. For astrophysical black holes, the Hawking temperature is invariably smaller than the Cosmic Microwave Background temperature. So the black hole mostly accretes rather than radiates spontaneously. Similarly, superradiant emission is also hard to observe from within a background of x-rays being emitted from accretion processes. Due to these reasons physicists have been looking for alternative physical models of gravity so that experiments can be devised for indirect verification of phenomena that are predicted by the semiclassical theory of gravity. These alternative physical models are called analog models of gravity.

Among the first of such analog gravity models, Unruh’s [6] was the pioneering idea of analog models based on fluid mechanics. The idea of such models arose from a remarkable observation regarding sound wave propagation in a fluid in motion. Unruh showed that if the fluid is barotropic and inviscid, and the flow is irrotational, the equation of motion of the acoustic fluctuations of the velocity potential is identical to that of a minimally coupled massless scalar field in an effective (3+1) dimensional curved spacetime, and is given by,

$$\Box \Psi = \frac{1}{\sqrt{-g}} \partial_{\mu} \left( \sqrt{-g} g^{\mu\nu} \partial_{\nu} \right) \Psi = 0$$

where $g_{\mu\nu}$ is the ‘acoustic’ metric and $\Psi$ is the fluctuation of the velocity potential. The most remarkable fact about this effective spacetime ‘seen’ by the acoustic fluctuations is that the acoustic metric is curved and Lorentzian in signature, even though the fluid flow is governed by non-relativistic physics in a flat Minkowski background spacetime. This particular feature of these models raises the possibility of being able to mimic a gravitational system in its kinematical sector. Since the fluid has barotropic equation of state, the pressure does not generate vorticity. So an initially irrotational flow will remain irrotational. Viscosity of the fluid has been ignored in order to maintain the Lorentzian character of the effective acoustic geometry.

In acoustic analog models of gravity, the acoustic geometry entirely depends on the background fluid motion. So the changes of the acoustic geometry are also related to that of the background fluid motion. Since the background fluid motion is governed by the Euler equation and the continuity equation, not by Einstein’s equation, it may mimic only the kinematical aspects of gravity, and not its dynamical ones. If the flow contains a well-defined region where it is supersonic, that region resembles an ergoregion of a rotating black hole, characterized by the time translation Killing vector turning spacelike. Within this region, there may exist a surface where the inward normal velocity of the fluid exceeds the local speed of sound at every point. Such a surface therefore traps all acoustic disturbances within it, resembling an outer trapped surface, and can therefore serve as the sonic or acoustic horizon of the analogue black hole.

Unruh [6] has used this idea of an acoustic analogue black hole horizon, to show that quantized acoustic perturbations (phonons) may be emitted from it precisely like the emission of photons from a black hole horizon via Hawking radiation. The Hawking temperature is likewise given by the surface gravity at the sonic horizon. This is acceptable provided we continue to ignore the backreaction of the phonon radiation on the background metric, which is a legitimate thing to do in a fixed-metric problem. Unruh’s work has led, in the last couple of years, to several
proposals for the experimental verification of acoustic Hawking radiation. The Hawking temperature for typical acoustic analogues has been estimated to be in the nano-Kelvin range; with improved experimental techniques and equipment, this is not outside the realm of realistic possibilities in the near future.

One slightly unsavory feature of the Unruh approach has to do with the necessity of quantizing linearized acoustic disturbances and the manner in which they are quantized. In a classical fluid, there is no compelling physical reason to quantize acoustic disturbances. So quantizing acoustic perturbations just to demonstrate Hawking radiation from the horizon of acoustic black holes appears somewhat artificial. In a quantum fluid like superfluid helium on the other hand, there are phonon excitations of the fluid background itself. On top of this one may have acoustic perturbations which are quantized in terms of phonons. However, in that case, the division between the background and linear acoustic fluctuations tends to get a bit hazy.

There is however an alternative phenomenon which does not necessitate quantization of acoustic disturbances for it to occur, although a full quantum description is possible in principle. This is the phenomenon called Superresonance by us [7] which occurs in a curved Lorentzian acoustic geometry if the acoustic spacetime is that of a rotating black hole. Recall that such a black hole spacetime admits a region - the ergoregion - containing the event horizon, where the time translation Killing vector turns spacelike. The existence of this region is crucial for the Penrose process of energy extraction from the black hole, at the cost of its rotational energy. Superresonance is simply an acoustic wave version of the Penrose process, wherein a plane wave solution of a massless scalar field in the black hole background is scattered from the ergoregion with an amplification of its amplitude. The energy gained by the wave is at the cost of the rotational energy of the black hole.

In the earlier paper [7], superresonance has been shown to occur in a certain class of analogue 2+1 dimensional rotating black holes. Linear acoustic perturbations in such a background are shown to scatter from the ergoregion with an enhancement in amplitude, for a restricted range of frequencies of the incoming wave. However, while the plausibility of superresonance was established in that earlier work through the analysis of the Wronskian of the radial equation of the acoustic perturbations, the actual frequency dependence of the amplification factor was not worked out. In this paper this gap is filled. We present a more detailed quantitative analysis of this phenomenon, as also a somewhat different way of arriving at the results of the previous paper. An explicit expression is derived for the reflection coefficient as a function of the frequency of the incoming wave for a certain low frequency regime. This expression is expected to be useful for possible future experimental endeavors to observe superresonance.

The plan of the paper is as follows: in section II, certain aspects of the acoustic black hole analogue we work with are reviewed. The demonstration of superresonance is repeated with a choice of coordinates different from those used in the earlier work [7]. In section III, detailed quantitative analysis of the reflection coefficient is presented, explicitly exhibiting its frequency dependence, albeit within a certain restricted low frequency regime. We conclude in section IV with a sketch of the outlook on this class of problems.

II. DRAINING VORTEX FLOW AND SUPERRESONANCE

In order to demonstrate superresonance, a certain choice is to be made of the velocity potential of the fluid, which functions as a suitable acoustic analogue of a Kerr black hole. In other words, the flow must have a well-defined (ergo)region of transonic flow containing a sonic horizon as discussed in the Introduction. In 2+1 dimensions, a velocity profile with these properties is the ‘draining vortex’ flow with a sink at the origin, given by [8]

$$\vec{v} = \frac{A}{r} \hat{r} + \frac{B}{r} \hat{\phi},$$

(2)

where $A, B$ are real and positive and $(r, \phi)$ are plane polar coordinates. The metric corresponding to this effective geometry is

$$ds^2 = \left( \rho_0 \right)^2 \left[ \left( c^2 - \frac{A^2 + B^2}{r^2} \right) dt^2 + \frac{2 A}{r} dr \, dt - 2B \, d\phi \, dt + dr^2 + r^2 \, d\phi^2 \right]$$

(3)

A two surface, at $r = \frac{A}{c}$, in this flow, on which the fluid velocity is everywhere inward pointing and the radial component of the fluid velocity exceeds the local sound velocity everywhere, behaves as an outer trapped surface in this acoustic geometry. This surface can be identified with the future event horizon of the black hole. Since the fluid velocity (2) is always inward pointing the linearized fluctuations originating in the region bounded by the sonic horizon cannot cross this boundary. As for the Kerr black hole in general relativity, the radius of the boundary of the ergosphere of the acoustic black hole is given by vanishing of $g_{00}$, i.e., $r_e = \sqrt{A^2 + B^2}/c$. If we assume that the background density of the fluid is constant, it automatically implies that background pressure and the local speed of
sound are also constant. Thus we can ignore the position independent pre-factor in the metric because it will not effect the equation of motion of fluctuations of the velocity potential.

From the components of the draining vortex metric it is clear that the (2+1)-dimensional curved spacetime possesses isometries that correspond to time translations and rotations on the plane. The solution of the massless Klein Gordon equation can therefore be written as,

\[ \Psi(t, r, \phi) = R(r) e^{-i \omega t} e^{i m \phi} \] (4)

where \( \omega \) and \( m \) are real and positive. In order to make \( \Psi(t, r, \phi) \) single valued, \( m \) should take integer values. Then the radial function \( R(r) \) satisfies

\[ \frac{d^2 R(r)}{dr^2} + P_1(r) \frac{dR(r)}{dr} + Q_1(r) R(r) = 0 \] (5)

where,

\[ P_1(r) = \frac{A^2 + r^2 c^2 + 2 i A \omega}{r^2 c^2 - A^2} \]

and

\[ Q_1(r) = \frac{2 B m \omega - r^2 \omega^2}{r^2 (r^2 c^2 - A^2)} \]

Observe that the eqn.(5) is different from that used in [7] (equ. 8), since the diffeomorphisms given in equ. (5) of [7] have not been effected in the present case.

Now we introduce tortoise coordinate \( r^* \) through the equation,

\[ \frac{d}{dr^*} = \left(1 - \frac{A^2}{r^2 c^2}\right) \frac{d}{dr} \]

which implies that,

\[ r^* = r + \frac{A}{2 c} \log \left| \frac{r c - A}{r c + A} \right| \] (7)

This tortoise coordinate spans the entire real line as opposed to \( r \) which spans only the half-line. The horizon at \( r = \frac{A}{c} \) maps to \( r^* \rightarrow -\infty \), while \( r \rightarrow \infty \) corresponds to \( r^* \rightarrow +\infty \). Let us now define a new radial function \( G(r) \) as,

\[ R(r) = \frac{H(r)}{\sqrt{r}} \exp \left[ \frac{i}{2} \left( \frac{\omega A}{c^2} \log \left( \frac{r^2 c^2}{A^2} - 1 \right) - \frac{m B}{A} \log \left( 1 - \frac{A^2}{r^2 c^2} \right) \right) \right] \] (8)

Substituting this in equation (5) we observe that \( G(r) \) satisfies the differential equation

\[ \left(1 - \frac{A^2}{r^2 c^2}\right) \frac{d}{dr} \left[ \left(1 - \frac{A^2}{r^2 c^2}\right) \frac{d}{dr} \right] H(r) + \left[ \frac{1}{c^2} \left( \omega - \frac{m B}{r^2} \right)^2 - \left(1 - \frac{A^2}{r^2 c^2}\right) \left\{ \frac{1}{r^2} \left( m^2 - \frac{1}{4} \right) + \frac{5 A^2}{r^4 c^2} \right\} \right] H(r) = 0 \] (9)

Now in terms of tortoise coordinate one obtains the modified differential equation as,

\[ \frac{d^2 H(r^*)}{dr^{*2}} + \left[ \frac{1}{c^2} \left( \omega - \frac{m B}{r^2} \right)^2 - \left(1 - \frac{A^2}{r^2 c^2}\right) \left\{ \frac{1}{r^2} \left( m^2 - \frac{1}{4} \right) + \frac{5 A^2}{r^4 c^2} \right\} \right] H(r^*) = 0 \] (10)

We analyze this differential equation in two distinct radial regions, viz., near the sonic horizon, i.e., at \( r^* \rightarrow -\infty \) and at asymptopia, i.e., at \( r^* \rightarrow +\infty \). In the asymptotic region, the above differential equation can be written approximately as,

\[ \frac{d^2 H(r^*)}{dr^{*2}} + \frac{\omega^2}{c^2} H(r^*) = 0 \] (11)
This can be solved trivially,

\[ H(r^*) = R_{\omega m} \exp \left( i \frac{\omega}{c} r^* \right) + \exp \left( -i \frac{\omega}{c} r^* \right) \]  

(12)

The first term in the above equation corresponds to reflected wave and the second term to the incident wave, so that \( R \) is the reflection coefficient in the sense of potential scattering. Similarly, near the horizon the above differential equation can be written approximately as,

\[ \frac{d^2 H(r^*)}{dr^*} + \left( \frac{\omega - m \Omega_H}{c} \right)^2 H(r^*) = 0 \]  

(13)

where \( \Omega_H \) is the angular velocity of the sonic horizon. We impose the physical boundary condition that of the two solutions of this equation, only the ingoing one is physical, so that one has,

\[ H(r^*) = T_{\omega m} \exp \left\{ -i \left( \frac{\omega - m \Omega_H}{c} \right) r^* \right\} \]  

(14)

Now using these approximate solutions of the above differential equation together with their complex conjugates and recalling the fact two linearly independent solutions of this differential equation (10) must lead to a constant Wronskian, it is easy to show that,

\[ 1 - |R_{\omega m}|^2 = \left( 1 - \frac{m}{\omega} \frac{\Omega_H}{\omega} \right) |T_{\omega m}|^2 \]  

(15)

Here \( R_{\omega m} \) and \( T_{\omega m} \) are amplitudes of the reflection and transmission coefficient of the scattered wave respectively. It is obvious from the above equation that, for frequencies in the range \( 0 < \omega < m \Omega_H \), the reflection coefficient has a magnitude larger than unity. This is precisely the amplification relation that emerges in our earlier analysis of superresonance [7]. The demonstration that the final physical result remains the same, despite using a different system of coordinates in the acoustic spacetime, implies that the analogue spacetime indeed exhibits the kind of general coordinate invariance associated with standard general relativity. This relation between reflection and transmission coefficients, however, only establishes the plausibility of superresonance for draining vortex flows. It is not sufficiently detailed to link up with possible forthcoming experimental observations. For that purpose, we need to know the frequency dependence of both the reflection and the transmission coefficients. This requires that we have to solve the differential equation explicitly. In the next section we analyze the radial differential equation in the low frequency limit \( \omega / c^2 A \ll 1 \).

III. THE REFLECTION COEFFICIENT

We begin with defining a new function \( L(r) \) through the relation

\[ R(r) = \frac{r c}{A} \exp \left[ \frac{i}{2} \left\{ \frac{\omega}{c^2} \log \left( \frac{r^2 c^2}{A^2} - 1 \right) - \frac{m B}{A} \log \left( 1 - \frac{A^2}{r^2 c^2} \right) \right\} \right] L(r) \]  

(16)

We next introduce a new variable \( x \equiv \frac{r^2 c^2}{A^2} - 1 \). Then \( L(x) \) satisfies the differential equation ,

\[ \frac{d^2 L(x)}{dx^2} + \left( \frac{1}{x} + \frac{1}{x+1} \right) \frac{dL(x)}{dx} + \left[ Q^2 \frac{1}{x} + \frac{1}{x+1} + (1 - S^2) + \omega^2 \frac{A^2}{c^4} x \right] \frac{L(x)}{4x(x+1)} = 0 \]  

(17)

where,

\[ Q^2 = \left( \frac{\omega}{c^2} - \frac{B m}{A} \right)^2 \]

\[ S^2 = m^2 + \frac{2 B m \omega}{c^2} - \frac{2 \omega^2 A^2}{c^4} \]

In order to find a solution of this differential equation, we adapt a matching procedure employed by Starobinsky [3]; for the region
The most general solution of this differential equation is,

\[
\left(\frac{\omega A}{c^2}\right) x \ll m, \quad \frac{\omega A}{c^2} \ll 1
\]

the above differential equation can be approximately written as,

\[
\frac{d^2 L(x)}{dx^2} + \left(\frac{1}{x} + \frac{1}{x+1}\right) \frac{dL(x)}{dx} + \left[\frac{Q^2}{x} + \frac{1}{x+1} + (1 - S^2)\right] \frac{L(x)}{4x(x+1)} = 0.
\]

(18)

This is the standard Riemann-Papparitz equation with regular singular points at 0, -1, and at \( \infty \). We can convert this differential equation into the Hypergeometric form by the following substitution,

\[
L(x) = x^{\alpha'}(x+1)^{\beta'} G(\tau)
\]

(19)

such that \( G(\tau) \) satisfies the Hypergeometric differential equation given by

\[
\tau(1-\tau) \frac{d^2 G(\tau)}{d\tau^2} + \left[\gamma - (\alpha + \beta + 1)\right] \frac{dG(\tau)}{d\tau} - \alpha \beta G(\tau) = 0
\]

(20)

where

\[
\alpha' = -i \frac{Q}{2}, \quad \beta' = -\frac{1}{2}
\]

\[
\alpha = -\frac{S}{2} - i \frac{Q}{2}, \quad \beta = \frac{S}{2} - i \frac{Q}{2}, \quad \gamma = 1 - i Q
\]

\( \tau = -x \)

The most general solution of this differential equation is,

\[
G(\tau) = C_1 {}_2F_1(\alpha, \beta; \gamma; \tau) + C_2 \tau^{1-\gamma} {}_2F_1(\alpha + 1 - \gamma, \beta + 1 - \gamma; 2 - \gamma; \tau)
\]

(21)

where \( C_1 \) and \( C_2 \) are arbitrary constants to be determined. Near the horizon, \( x \sim 0 \)

\[
G(x) \rightarrow C_1 x^{\alpha'} + C_2 x^{-\alpha'}.
\]

(22)

Here we have absorbed \((-1)^{1-\gamma}\) in \( C_2 \). The first term represents ingoing wave and the second term represents outgoing wave near the horizon. Since nothing can come out of the horizon, the boundary condition at the horizon implies that \( C_2 = 0 \). Therefore the solution of equ.(18) reduces to

\[
L(x) = C_1 x^{\alpha'}(x+1)^{\beta'} {}_2F_1(\alpha, \beta; \gamma; -x)
\]

(23)

To use the matching procedure we need to know the behavior of this solution in the asymptotic region, i.e., at \( x \rightarrow \infty \). This can be obtained by the transformation \( (\tau \rightarrow 1/\tau) \) for the Hypergeometric function in above equation. Using this rule we obtain,

\[
L(x) = C_1 x^{\alpha'}(x+1)^{\beta'} \left[\frac{\Gamma(\gamma)}{\Gamma(\beta)} \frac{\Gamma(\beta - \alpha)}{\Gamma(\gamma - \alpha)} x^{-\alpha} {}_2F_1\left(\alpha, \alpha + 1 - \gamma; \alpha + 1 - \beta; \frac{(-1)}{x}\right)\right]
\]

\[
+ x^{-\beta} \frac{\Gamma(\gamma)}{\Gamma(\beta)} \frac{\Gamma(\alpha - \beta)}{\Gamma(\gamma - \beta)} {}_2F_1\left(\beta, \beta + 1 - \gamma; \beta + 1 - \alpha; \frac{(-1)}{x}\right)
\]

(24)

So in the region \( x \gg \text{Max}(m^2, Q^2) \), we have

\[
L(x) \rightarrow C_1 x^{\alpha'}(x + 1)^{\beta'} \left[\frac{\Gamma(\gamma)}{\Gamma(\beta)} \frac{\Gamma(\beta - \alpha)}{\Gamma(\gamma - \alpha)} x^{-\alpha} + \frac{\Gamma(\gamma)}{\Gamma(\beta)} \frac{\Gamma(\alpha - \beta)}{\Gamma(\gamma - \beta)} x^{-\beta}\right]
\]

(25)

Hence the approximate form of \( L(x) \) in the region,

\[
\text{Max}(m^2, Q^2) \ll x \ll \frac{m}{\left(\frac{\omega A}{c^2}\right)}
\]
is given by

\[ L(x) = \frac{C_1}{\sqrt{x}} \left[ \frac{\Gamma(\gamma)}{\Gamma(\beta)} \frac{\Gamma(\beta - \alpha)}{\Gamma(\gamma - \alpha)} x^{\frac{\beta - \alpha}{2}} + \frac{\Gamma(\gamma)}{\Gamma(\alpha)} \frac{\Gamma(\alpha - \beta)}{\Gamma(\gamma - \beta)} x^{\frac{\alpha - \beta}{2}} \right] \]

\[ = \frac{1}{\sqrt{x}} \left[ D_1 x^{\frac{\beta - \alpha}{2}} + D_2 x^{\frac{\alpha - \beta}{2}} \right] \quad (26) \]

where,

\[ D_1 = \frac{\Gamma(\gamma)}{\Gamma(\beta)} \frac{\Gamma(\beta - \alpha)}{\Gamma(\gamma - \alpha)} C_1 \]

\[ D_2 = \frac{\Gamma(\gamma)}{\Gamma(\alpha)} \frac{\Gamma(\alpha - \beta)}{\Gamma(\gamma - \beta)} C_1 \]

The solution (26) is in a form where it is not obvious which combinations of the complex coefficients \( D_1, D_2 \) correspond to the ingoing and outgoing modes. This information can be retrieved from the energy flux per unit time radially across an arbitrarily chosen surface at a constant radial coordinate \( r \) in the region \( \text{Max}(m^2, Q^2) \ll x \ll m/\left(\frac{\omega A}{c^2}\right) \). This is obtained by the surface integral

\[ FL = -\int_{0}^{2\pi} T_{\mu\nu} l^\mu k^\nu \sqrt{-g} \, d\phi \]

\[ = i \pi S \omega \left( D_1^* D_2 - D_1 D_2^* \right) \]

\[ = \frac{\pi \omega S}{2} \left( |D_1 + i D_2|^2 - |D_1 - i D_2|^2 \right) \quad (27) \]

where \( T_{\mu\nu} \) is the energy momentum tensor for \( \Psi \), \( l^\mu \) is the normal to a constant \( r \) surface, \( k^\mu \) is the time translational killing vector and \( g \) is the determinant of the acoustic metric. Given that the outgoing mode of the wave corresponds to a positive value of radial component of the flux, and the ingoing mode just the opposite, one concludes that the amplitudes of the incident and reflected wave are proportional to \( (D_1 - i D_2) \) and \( (D_1 + i D_2) \) respectively.

One can now write down an explicit expression for the amplification factor (AF); this is given by,

\[ AF = 1 - |R_{\omega m}|^2 = 1 - \left| \frac{D_1 + i D_2}{D_1 - i D_2} \right|^2 \]

\[ = \frac{2 i (D_1 D_2^* - D_1^* D_2)}{|D_1 - i D_2|^2} \]

\[ = \frac{2 Q |C_1|^2}{S |D_1 - i D_2|^2} = \frac{2 \omega A}{S c^2} \left| \frac{|C_1|^2}{|D_1 - i D_2|^2} \right| \left( 1 - \frac{m \Omega_H}{\omega} \right) \quad (28) \]

It follows from this that the reflection coefficient exceeds unity in the frequency range \( 0 < \omega < m \Omega_H \). Comparing this with the equation (15), we can identify \( T_{\omega m} \) in this frequency range as,

\[ |T_{\omega m}|^2 = \frac{2 \omega A}{S c^2} \left| \frac{|C_1|^2}{|D_1 - i D_2|^2} \right|^2 \quad (29) \]

Equations (28-29) exhibit the frequency dependence of both the reflection and transmission coefficients and hence of the amplification factor in the low frequency regime. If vortex motion with a sink is realized in the laboratory, our result can be used to provide an order of magnitude estimate for the amplification factor. According to our assumption these relations are valid in the frequency range where \( \frac{m}{\omega} \ll 1 \), i.e., when wave length of the sound wave is much larger than the radius of the horizon. Clearly, this frequency range does not cover the entire superresonant range \( 0 \) to \( m \Omega_H \). It is interesting and of importance to find the behavior of the reflection coefficient near the critical point around \( \omega = m \Omega_H \) where the reflection coefficient is expected to cross unity. We hope to report on this in the near future.
IV. OUTLOOK

The link with possible experimental observation of superresonance is yet incomplete, despite the explicit expressions we have derived for the reflection and transmission coefficients. One needs to input into these results typical characteristics of draining vortex flows in fluids like superfluid helium which obey our assumptions of barotropicity, irrotationality and zero viscosity. These characteristics are to be derived from the macroscopic quantum mechanics of superfluid helium, and novel features like flux quantization are expected to play a non-trivial role. In particular, flux quantization may well lead to the phenomenon of \textit{discrete} amplification discussed in \cite{7}, which would perhaps facilitate observation. We should mention that ergoregions in superfluid helium have been considered in ref. \cite{9}, \cite{10}, although from a somewhat different standpoint. These authors do not consider vortex flows with a drain, and as such, their acoustic spacetime does not share all characteristics of black holes, like an outer trapped surface.

Another feature which we have ignored in the foregoing analysis is the existence of shock fronts at the interface of normal and transonic flows. Indeed, in astrophysical black holes, analysis of these shock fronts constitutes an important component of models used in observational work on these objects. The existence of these shock fronts are in part responsible for the non-observation of superradiant scattering in such black holes. The issue of shocks in acoustic black hole analogues thus assumes special significance for future work on this topic.

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