Crossover from fast to slow dynamics in quantum Ising chains with long range interactions.

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Quantum many body systems with long range interactions are known to display many fascinating phenomena experimentally observable in trapped ions, Rydberg atoms and polar molecules. Among these are dynamical phase transitions which occur after an abrupt quench in spin chains with interactions decaying as $r^{-\alpha}$ and whose critical dynamics depend crucially on the power $\alpha$: for systems with $\alpha < 1$ the transition is sharp while for $\alpha > 1$ it fans out in a chaotic crossover region. In this paper we explore the fate of critical dynamics in Ising chains with long-range interactions when the transverse field is ramped up with a finite speed. While for abrupt quenches we observe a chaotic region that widens as $\alpha$ is increased, the width of the crossover region diminishes as the time of the ramp increases, suggesting that chaos will disappear altogether and be replaced by a sharp transition in the adiabatic limit.

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I. INTRODUCTION

In the last decades the engineering of long-range interactions in low dimensional quantum many-body systems has been the focus of many experiments, ranging from Rydberg atoms\textsuperscript{14,15} to polar molecules\textsuperscript{16,17} and trapped ions\textsuperscript{18,19}. These systems give the possibility to study a rich variety of phenomena which contrast the non-equilibrium dynamics of systems with long-range interactions to that of short-range ones, such as the violation of Lieb-Robinson bounds and anomalous propagation of information\textsuperscript{20,21}, the localization of kink-like excitations\textsuperscript{22,23} as well as the observation of prethermal phases and dynamical phase transitions\textsuperscript{24,25}.

It is known that dynamical phase transitions are very sensitive to the range of interactions\textsuperscript{26} in fact moving from short range to long range ones new dynamical features emerge. In a recent work\textsuperscript{27} we have shown that for power law decaying interactions ($J(r) \propto 1/r^\alpha$) the dynamical properties are very sensitive to the precise value of the power law exponent $\alpha$. In particular, for quantum Ising chains subject to a quench in the transverse field the dynamical phase transition for $\alpha < 1$ is sharp, while for $\alpha > 1$ the critical point fans out in a crossover chaotic region where the dynamics and the asymptotic state depend sensitively on the system parameters\textsuperscript{28}.

While the overall dynamics is dominated by long-range correlations among spins, the key ingredient to observe chaos appear to be residual short range correlation\textsuperscript{29,30}, which effectively damp the order parameter dynamics. In the chaotic region, the spin system behaves collectively like a tossed coin: initially the magnetization flips periodically between positive and negative values along paramagnetic trajectories, until the damping produces relaxation and localization on stable ferromagnetic trajectories with pseudo-random asymptotic magnetization\textsuperscript{31,32}.

The robustness of this chaotic region can be investigated by taking out of equilibrium the system with a linear ramp. Changing the control parameter with a finite velocity, in fact, can affect the interplay between nonequilibrium dynamics and dissipation suppressing chaos in favour of regular dynamics. Intuitively, while an abrupt quench is analogous to a fast coin toss, varying smoothly the system parameters is like placing slowly a coin on the table, a process that is obviously devoid of uncertainty. In this work we will show that decreasing the ramping speed of the transverse field in an Ising model with long range interactions has the same effect: the chaotic region shrinks in size until the dynamical phase diagram resembles the equilibrium one in the adiabatic limit. We will show this using the cluster mean field theory (CMFT)\textsuperscript{30,31}.

Then we investigate the dynamics when the transverse field is smoothly switched on with a particular focus on the robustness of the chaotic region and on the crossover between randomness and predictability (Sec. III).

II. MODEL AND EQUILIBRIUM PHASE DIAGRAM

Let us first introduce the model. In this paper we study the long range interacting quantum Ising chain, described by the following Hamiltonian

$$H = -\frac{J}{N(\alpha)} \sum_{i \neq j}^{N} \sigma_{i}^{x} \sigma_{j}^{x} \alpha - h \sum_{i}^{N} \sigma_{i}^{z},$$

(1)

where $\sigma_{i}^{\beta}$ are Pauli matrices acting on site $i$, $N(\alpha) = \sum_{r=1}^{N} r^{-\alpha}$ the usual Kac normalization. In Ref.\textsuperscript{32} it was shown that in the 1D case the quantum phase transition is present for all transverse fields but the critical exponent at zero temperature are described by mean field theory only for $\alpha \leq \alpha_c = 5/3$. In turn, a pure mean field description would predict, for all $\alpha$, a sharp equilibrium phase transition from a ferromagnetic to a paramagnetic
phase at $h = 2J$. Mean field theory is in turn exact for $\alpha < 1$, therefore it can be used as a benchmark on the validity of any other approximation. In order to get a feeling of what happens for larger $\alpha$ we decided to use the cluster mean field introduced in Ref. 30, 31 that accounts for the effects of short range correlations. The idea is to divide the system into $N_{\text{cl}}$ clusters of size $\ell$ living in the mean field generated by others and to deduce the physics of the system from the exact physics of one of these clusters. As shown in Ref. 27 the CMFT allows one to write the Hamiltonian as the sum of two parts $H = H_{\text{cl}} + H_{\text{mf}}$, with

$$H_{\text{cl}} = -\frac{1}{N(\alpha)} \sum_{\beta}^{N_{\text{cl}}} \left( \sum_{i<j \in \beta}^{L} \sigma_i^z \sigma_j^z - h \sum_{i \in \beta}^{L} \sigma_i^z \right)$$

$$H_{\text{mf}} = 2 \bar{m} \sigma^{\text{z eff}},$$

where $\beta$ is the cluster index, $\bar{m} = \frac{1}{L} \sum_{i \in \beta}^{L} \langle \sigma_i^z \rangle$ the mean value of the magnetization inside the cluster $\beta$, $J_{\text{eff}} = \frac{1}{N(\alpha)} \sum_{n=1}^{N_{\text{cl}}} \frac{J}{N_{\text{cl}}} \sigma_{\text{eff}}^\beta$ and $\sigma^z = \sum_i \sigma_i^z$. In the whole article we will consider the ferromagnetic case $J = 1$ only and we will set $h = 1$.

To derive the equilibrium phase diagram we solve self consistently for the ground state of the Hamiltonian $H = H_{\text{cl}} + H_{\text{mf}}$ with a precision $\varepsilon = 1e - 5$. We chose the cluster length $\ell = 5$. The result is shown in Fig. 1 where the order parameter $m = \frac{1}{L} \sum_{i<j}^{L} \langle \sigma_i^z \rangle$ is plotted as a function of the transverse field $h$ and the power law exponent $\alpha$. A quantum phase transition from a ferromagnetic to a paramagnetic phase occurs at a critical value $h_c(\alpha)$. What emerges is that for $\alpha < 1$ the critical field coincide with the mean field one while in the other case it assumes a dependence on the power law exponent.

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Now that we have the equilibrium phase diagram to benchmark our non-equilibrium investigations, let us focus on the dynamics following a linear quench of the transverse field. We have already shown in Ref. 27 that the asymptotic state attained after a sudden quench strongly depends on the value of the power law exponent $\alpha$. For $\alpha < 1$ the system exhibits the mean-field dynamical quantum phase transition from a dynamical ferromagnetic to a dynamical paramagnetic phase at $h = J$, as predicted in the literature.\(^{33}\) As soon as $\alpha > 1$ this critical point spreads in a critical region where the system exhibits chaotic behavior and a strong dependence on the initial conditions. To investigate the robustness of this phenomenon we study the post linear quench dynamics, considering the Hamiltonian with a transverse field varied according to

$$h(t) = \begin{cases} 
0 & \text{if } t < 0, \\
h \tanh(\lambda t) & \text{if } t \geq 0.
\end{cases}$$

The limit $\lambda \to \infty$ coincides with the sudden quench dynamics we have already described. Jaschke and collaborators\(^{41}\) have shown that the physics of the Kibble-Zurek mechanism holds despite the long range interactions, thus we expect to find in the adiabatic limit $\lambda \to 0$ the same phase diagram as in in Fig. 1. For intermediate values of the slope we expect to obtain informations on the crossover between the chaotic and regular dynamics.

To this purpose, we simulated the linear quench dynamics using the cluster mean field approach. The system is initially prepared in the ground state of the Hamiltonian $H_0 = \sum_{ij} J_{ij} \sigma_i^z \sigma_j^z$, i.e. all spins polarized along the z direction, and at time $t = 0$ the transverse field $h(t) = h \tanh(\lambda t)$ is turned on. The dynamics has been derived integrating self consistently the Schrödinger equation using an explicit embedded Runge-Kutta-Fehlberg(4,5) step adaptive method.\(^{35}\) We have
The size of the neighborhood $\varepsilon$ of points in the parameter space of the same phase, chaos is defined by the condition $\varepsilon = 0$. In particular, for a fixed value $\alpha$, $\varepsilon(h)$ is the size of the biggest square centered in $h$ containing points with the same sign of the order parameter. In the upper panels of Fig. (3) we plot simulations obtained with increasing resolutions (from the left to the right: $\delta\alpha = \delta h = 0.005, 0.001, 0.0005$) of a portion of the phase diagram ($1.3 < \alpha < 1.5$ and $1.1 < h < 1.3$). In the lower panels we plot the respective normalized neighborhood $\varepsilon(h)/\max_h \varepsilon(h)$ evaluated at $\alpha = 1.4$ (red dotted line). It emerges that for these values of the power law exponent $\varepsilon \to 0$ and the system preserve the chaotic features displayed in the case of the sudden quantum quench. When we move toward smaller values of $\alpha$ we can see that the chaotic region shrinks. This can be observed in the Fig. (3d) where the portion of the phase diagram with $1.1 < \alpha < 1.3$ and $1.4 < h < 1.6$ is plotted as a function of $\alpha$ and $h$. In the bottom panels the quantity $\varepsilon(h)/\max_h \varepsilon(h)$, evaluated along the line $\alpha = 1.15$, is plotted as a function of the final transverse field. It emerges that the region in which $\varepsilon \to 0$ is smaller, sign that chaos is slowly breaking down. From this analysis we can qualitatively argue that the bigger the power law exponent the more robust is chaos. Therefore, we can conclude that the crossover between the chaotic and the regular dynamics will start first from small power law exponents and will move toward the bigger ones. A quantitative analysis of this behavior can be obtained by looking at the critical value $\lambda_c$ of the slope, at fixed $\alpha$, below which the transition is sharp. In Fig. (4) we plot $\lambda_c$ as a function of the power law exponent. What emerges is a linear relation between $\lambda_c$ and $\alpha$. This result confirms the intuition that the bigger the smoother has to be a quench in order to observe a sharp phase transition.

Finally we want to stress that we expect the chaotic region disappears for $\alpha > 2$ in agreement with the expectations short range Ising model can not support dy-
FIG. 3. (color online). a) Upper panels: a portion of the phase diagram \((1.3 < \alpha < 1.5 \text{ and } 1.1 < h < 1.3)\) for three different resolutions (from the left to the right: \(\delta\alpha = \delta h = 0.005, 0.001, 0.0005\)). Bottom panels: normalized neighborhood \(\varepsilon(h)/\max_\varepsilon \varepsilon(h)\) evaluated at \(\alpha = 1.4\) (red dotted line). It emerges that for these values of the power law exponent \(\varepsilon \to 0\) and the system preserve the chaotic features displayed in the case of the sudden quantum quench.

b) Upper panels: a portion of the phase diagram \((1.1 < \alpha < 1.3 \text{ and } 1.4 < h < 1.6)\) for three different resolutions (from the left to the right: \(\delta\alpha = \delta h = 0.005, 0.001, 0.0005\)). Bottom panels: normalized neighborhood \(\varepsilon(h)/\max_\varepsilon \varepsilon(h)\) evaluated at \(\alpha = 1.15\) (red dotted line). It emerges that for this value of the power law exponent the region in which \(\varepsilon \to 0\) shrinks, sign of a regularization of the dynamics.

IV. CONCLUSION

In this paper we have presented the crossover from fast to slow dynamics in the quantum Ising chain with long range interactions. First we used the cluster mean field theory to derive self consistently the equilibrium phase diagram in the limit \(\alpha > 1\) obtaining an hint on how the crossover from long range to short range should occurs. Then we used the same method to simulate the post linear quench dynamics. The results suggest that there are three different regimes of the dynamics that can be observed. The first one, for sharp ramp, exhibits a dynamics that is very close to the post (sudden) quench one and presents the same chaotic features. In the limit of infinite slow quench we are in the adiabatic limit and we recover the equilibrium phase diagram. The last one
FIG. 4. In this figure we show the behavior of $\lambda_c$ as a function of $\alpha$. The blue point are the data extrapolated from the numerical results. The red line is the best fit interpolating them. From this figure we can observe a linear trend confirming the claim that the higher the power law exponent the slower the ramp should be to obtain a sharp phase transition.

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