Interaction among neighbouring rectangular finite strike slip faults in a linear viscoelastic half space representing lithosphere-asthenosphere system

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(Received 10 June 2019, Accepted 30 April 2020)

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ABSTRACT. There are seismically active regions consisting of fault system with a number of neighbouring earthquake faults. A movement across any one of them may affect the nature of stress accumulation near the others. Mathematical models may be developed to study these interactions and the pattern of interseismic stresses during the aseismic period in between two consecutive seismic events. In this paper, the lithosphere-asthenosphere system is being represented by a linear viscoelastic half space. The material of the half space is expected to possess the properties of both Maxwell and Kelvin type materials. It is assumed that the system is under a steady shear stress generated by some tectonic phenomena. For obtaining the solution for displacement, strain and stresses from the resulting boundary value problem, Integral transform, Green’s function technique and correspondence principle have been used. Appropriate estimates of the model parameters were used in carrying out the numerical computations for investigating the nature of interactions among the faults.

Key words – Linear viscoelastic lithosphere-asthenosphere system, Strike slip fault, Aseismic period, Stress accumulation, Interseismic stress, Earthquake prediction.

1. Introduction

Occurrence of an earthquake is a cyclic phenomena, two major seismic events are usually separated by a comparatively long aseismic periods of the order of a few decades or so. To understand the mechanism of earthquake process it is necessary to develop models of both the earthquake phenomena producing seismic disturbance lasting for a short period of time, as well as the slow ground deformation observed during the aseismic period, which may exist over several tens of years.

The small ground deformation during the aseismic period attracts the attention of the seismologists because of the fact that this aseismic period may be looked upon as the preparatory period for the next major seismic event. Some theoretical models have been developed in the lithosphere-asthenosphere system during aseismic period in a seismically active regions by Savage and Prescott (1978); Mukhopadhyay and Mukherjii (1979, 1984, 1986); Cohen (1980a,b); Cohen (1984) and others. In the present paper, we are developing a mathematical model highlighting the essential features of such ground
deformation and interseismic and post seismic stress during the aseismic period which typically precedes and follows two major seismic events in a seismically active region in the presence of earthquake faults.

Most of the seismically active regions usually consist of faults systems comprising of a number of faults which are closely located within a specific region. For example, in North America, San Andreas fault system, the Hayward and Calaveras faults are roughly parallel and are closed to the main San Andreas fault. A movement across any one of them is likely to affect the pattern of stress accumulation near the others. Some theoretical models with interacting fault system during aseismic period have been developed by Mukhopadhyay et al. (1979c, 1988); Mukhopadhyay and Mukherji (1984, 1986); Ghosh et al. (1992); Debnath and Sen (2014a, 2014b, 2015); Manna and Sen (2017). Most of the earlier work dealt with elastic/viscoelastic layer in welded contact with a viscoelastic half space or in a viscoelastic half space of Maxwell type followings Mukhopadhyay et al. (1978c, 1978, 1978b, 1979a, 1979b, 1986); Fred (1992). Post seismic relaxation effects in a linear viscoelastic Maxwell type material was discussed by Pollitz. However, the properties of the material in the lithosphere-asthenosphere system indicates that different other types of viscoelastic material may also be relevant. We therefore introduce linear viscoelastic half space in order to represent the lithosphere-asthenosphere system featuring the properties of both Maxwell and Kelvin-Voigt type.

Stresses are accumulated in the model under the action of tectonic phenomena including mantle convection. Now by the effects of interseismic stress it accumulated exceeds the local cohesive and frictional forces which keep the fault locked, the fault starts moving. Depending upon the local rheological nature of the region, the movement across the fault may be sudden in nature leading to an earthquake or alternatively, a creeping movement across the fault which releases the accumulated stress near it.

In most of the earlier studies the faults were assumed to be quite long compared to its depth, so that the problem reduced to a 2D model. In view of the fact that there are numerous other faults which are not so long compared to their depth, a 3D model is imminent.

In view of this we consider two neighbouring strike slip faults of finite length situated in a linear viscoelastic solid, one of which is taken to be surface breaking while the other is buried.

2. Formulation

We are considering two rectangular vertical strike-slip faults $F_1$ and $F_2$ of length $2L_1$ and $2L_2$ $(L_1, L_2$ finite) supposed to be situated in a linearly viscoelastic half space. Let $D_1$ and $D_2$ be their respective widths.

We are introducing a rectangular Cartesian coordinate system $(Y_1, Y_2, Y_3)$ is used by the midpoint $O$ of the upper edge of the fault $F_1$, which is taken to be surface-breaking, as the origin, the strike of the fault as the $Y_1$ axis, $Y_2$ axis perpendicular to the fault and $Y_3$ axis pointing towards the downwards direction so that the fault $F_1$ is given by $F_1 : (-L_1 \leq y_1 \leq L_1, y_2 = 0, 0 \leq y_3 \leq D_1)$. For convenience, we introduce another rectangular system $(Z_1, Z_2, Z_3)$ for the second fault $F_2$ which is taken to be buried as shown in [Fig. 1]. Fault $F_2$ is given by $F_2 : (-L_2 \leq z_1 \leq L_2, z_2 = 0, 0 \leq z_3 \leq D_2)$, $d$ being the depth of the upper edge of $F_2$ below the free surface. The relationships between $(Y_1, Y_2, Y_3)$ and $(Z_1, Z_2, Z_3)$ are given by:

\[ z_1 = y_1, \quad z_2 = y_2 - D_1, \quad z_3 = y_3 - d \]

2.1. Constitutive equations (stress-strain relations)

For the linear viscoelastic type materials the constitutive equations may be taken as:

\[
\tau_{11} + \eta \frac{\partial}{\partial t} (\tau_{11}) = \mu \frac{\partial u}{\partial y_1} + 2\eta \frac{\partial}{\partial t} \left( \frac{\partial u}{\partial y_1} \right) \tag{1}
\]

\[
\tau_{12} + \eta \frac{\partial}{\partial t} (\tau_{12}) = \mu \left( \frac{\partial u}{\partial y_2} + \frac{\partial v}{\partial y_1} \right) + 2\eta \frac{\partial}{\partial t} \left( \frac{\partial u}{\partial y_2} + \frac{\partial v}{\partial y_1} \right) \tag{2}
\]

\[
\tau_{13} + \eta \frac{\partial}{\partial t} (\tau_{13}) = \mu \left( \frac{\partial u}{\partial y_3} + \frac{\partial w}{\partial y_1} \right) + 2\eta \frac{\partial}{\partial t} \left( \frac{\partial u}{\partial y_3} + \frac{\partial w}{\partial y_1} \right) \tag{3}
\]
\[ \tau_{22} + \frac{\eta}{\mu} \frac{\partial}{\partial t} (\tau_{22}) = \mu \left( \frac{\partial v}{\partial y_2} + 2 \eta \frac{\partial}{\partial t} \left( \frac{\partial v}{\partial y_2} \right) \right) \]  

(4)

\[ \tau_{23} + \frac{\eta}{\mu} \frac{\partial}{\partial t} (\tau_{23}) = \mu \left( \frac{1}{2} \left( \frac{\partial v}{\partial y_3} + \frac{\partial w}{\partial y_2} \right) \right) + 2 \eta \frac{\partial}{\partial t} \left( \frac{\partial v}{\partial y_3} + \frac{\partial w}{\partial y_2} \right) \]  

(5)

\[ \tau_{33} + \frac{\eta}{\mu} \frac{\partial}{\partial t} (\tau_{33}) = \mu \left( \frac{\partial w}{\partial y_3} + 2 \eta \frac{\partial}{\partial t} \left( \frac{\partial w}{\partial y_3} \right) \right) \]  

(6)

where, \( \eta \) is the effective viscosity and \( \mu \) is the effective rigidity of the material.

2.2. Stress equation of motion

The equation of motion (explained underneath) for quasistatic deformation is satisfied by these stresses; but, in that case, the inertia terms are neglected.

\[ \frac{\partial}{\partial y_1} (r_{11}) + \frac{\partial}{\partial y_2} (r_{12}) + \frac{\partial}{\partial y_3} (r_{13}) = 0 \]  

(7)

\[ \frac{\partial}{\partial y_1} (r_{21}) + \frac{\partial}{\partial y_2} (r_{22}) + \frac{\partial}{\partial y_3} (r_{23}) = 0 \]  

(8)

\[ \frac{\partial}{\partial y_1} (r_{31}) + \frac{\partial}{\partial y_2} (r_{32}) + \frac{\partial}{\partial y_3} (r_{33}) = 0 \]  

(9)

where, \((-\infty < y_1 < \infty, -\infty < y_2 < \infty, y_3 \geq 0, t \geq 0\)

2.3. Boundary conditions

The boundary conditions are assumed as, with \( t = 0 \) representing a suitable instant when the medium remains in a seismic state:

\[ \lim_{y_1 \to L_4-} r_{11}(y_1, y_2, y_3, t) = \lim_{y_1 \to L_4+} r_{11}(y_1, y_2, y_3, t) = \tau_{L_4} \]  

\[ y_2 = 0, 0 \leq y_3 \leq D_4, t \geq 0 \]  

(10)

\[ \lim_{y_1 \to L_4-} r_{11}(y_1, y_2, y_3, t) = \lim_{y_1 \to L_4+} r_{11}(y_1, y_2, y_3, t) = \tau_{L_4} \]  

\[ y_2 = 0, 0 \leq y_3 \leq D_4, t \geq 0 \]  

(11)

where, \( \tau_{L_4} \) is the constant stress maintained at the tips of the fault along the \( Y_1 \) axis. It is likely that its magnitude will be small enough to exclude the possibility of any further extension along the \( Y_1 \) direction.

Similarly for the fault \( F_2 \):

\[ \lim_{y_1 \to L_2-} r_{11}(y_1, y_2, y_3, t) = \lim_{y_1 \to L_2+} r_{11}(y_1, y_2, y_3, t) = \tau_{L_2} \]  

\[ y_2 = 0, d \leq y_3 \leq D_2, t \geq 0 \]  

For the stresses we assume:

\[ r_{12}(y_1, y_2, y_3, t) \rightarrow \tau_{\infty} \text{ as } y_2 \rightarrow \infty, -\infty \leq y_1 \leq \infty, \]  

\[ y_3 \geq 0, t \geq 0 \]  

(12)

On the free surface \( y_3 = 0, ( -\infty \leq y_1, y_2 \leq \infty, t \geq 0) \)

\[ r_{13}(y_1, y_2, y_3, t) = 0 \]  

(13)

\[ r_{23}(y_1, y_2, y_3, t) = 0 \]  

(14)

\[ r_{33}(y_1, y_2, y_3, t) = 0 \]  

(15)

Also, as \( y_3 \rightarrow \infty, ( -\infty \leq y_1, y_2 \leq \infty, t \geq 0) \)

\[ r_{13}(y_1, y_2, y_3, t) = 0 \]  

(16)

\[ r_{23}(y_1, y_2, y_3, t) = 0 \]  

(17)

\[ r_{33}(y_1, y_2, y_3, t) = 0 \]  

(18)

\[ \tau_{22}(y_1, y_2, y_3, t) = 0 \text{ as } y_2 \rightarrow \infty, -\infty \leq y_1 \leq \infty, t \geq 0 \]  

(19)

[where, \( \tau_{\infty}(t) \) is the shear stress maintained by mantle convection and other tectonic phenomena far away from the fault].
2.4. Initial conditions

Let \((u_0, v_0, w_0, \tau_{1j}, e_{ij})\) \(i, j = 1, 2, 3\) be the values of \(u, v, w, \tau_{ij}, e_{ij}\) respectively at time \(t = 0\). They are functions of \(y_1, y_2, y_3\) and satisfy the relations (1) to (19).

3. Displacements, stresses and strains in the absence of any fault movement

When there is no fault movement, the displacements, stresses and strains are assumed to be continuous throughout the system. For obtaining displacement, strain and stresses, we are introducing Integral transform like Laplace transform of (1) to (19) are taken with respect to \(t\). On taking inverse Laplace transform we get the solutions.

\[
(y_1, y_2, y_3, t) = (u_0) e^{\frac{\mu t}{2\eta}} + y_1 \tau_{\eta} \left(1 - e^{\frac{\mu t}{2\eta}}\right) - y_2 e^{\frac{\mu t}{2\eta}}
\]

\[
v(y_1, y_2, y_3, t) = (v_0) e^{\frac{\mu t}{2\eta}} + \frac{y_1 \tau_{\eta}}{\mu} \left(1 - e^{\frac{\mu t}{2\eta}}\right)
\]

\[
w(y_1, y_2, y_3, t) = (w_0) e^{\frac{\mu t}{2\eta}}
\]

\[
\tau_{11}(y_1, y_2, y_3, t) = (\tau_{11})_0 e^{\frac{\mu t}{\eta}} + \tau_{\eta} \left(1 - e^{\frac{\mu t}{\eta}}\right)
\]

\[
\tau_{12}(y_1, y_2, y_3, t) = (\tau_{12})_0 e^{\frac{\mu t}{\eta}} + \tau_{\eta} \left(1 - e^{\frac{\mu t}{\eta}}\right)
\]

\[
\tau_{13}(y_1, y_2, y_3, t) = (\tau_{13})_0 e^{\frac{\mu t}{\eta}}
\]

\[
\tau_{22}(y_1, y_2, y_3, t) = (\tau_{22})_0 e^{\frac{\mu t}{\eta}}
\]

\[
\tau_{23}(y_1, y_2, y_3, t) = (\tau_{23})_0 e^{\frac{\mu t}{\eta}}
\]

\[
\tau_{33}(y_1, y_2, y_3, t) = (\tau_{33})_0 e^{\frac{\mu t}{\eta}}
\]

From the above expressions we find that as \(t \to \infty, \tau_{11} \to \tau_L, \tau_{12} \to \tau_e\), and all the others stress components \(\tau_{13}, \tau_{22}, \tau_{23}, \tau_{33} \to 0\). However the rheological behaviour of the material near the fault \(F_1\) is assumed to be capable of withstanding stress of magnitude \((\tau_e)\), called critical value of the stress where \((\tau_e)\) is less than \(\tau_c\). When the accumulated stress \(\tau_{12}\) becomes large enough near the fault and exceeds this threshold value \((\tau_e)\), a sudden slip across \(F_1\) occurs after a time \(t = T_1\) and thereby releasing the accumulated stress to a lower value.

Following Deb Nath and Sen (2013) the magnitude of slip is likely to satisfy the following conditions:

(C1) Its magnitude will be maximum near the middle of the fault on the free surface.

(C2) Its magnitude will gradually decrease to zero at the tips of the fault \((y_1 = \pm L_1, y_2 = 0, 0 \leq y_3 \leq D_1)\) along its length.

(C3) The magnitude of the slip will decrease downwards with \(y_3\) and ultimately tends to zero near the lower edge of the fault \((y_1 = \pm L_1, y_2 = 0, y_3 = D_1)\).

If \(f(y_1, y_3)\) be the slip function, it should satisfy the above conditions.

It is assumed that the critical value of the stress, say \((\tau_e)\), near \(F_2\) is greater than \((\tau_e)\), so that when \(F_1\) slips, \(F_2\) remains locked.

4. Solutions after the commencement of the fault movement across \(F_1\) and after the fault slip across \(F_2\)

We consider first the movement across \(F_1\), while \(F_2\) remains locked.

Let us suppose that after a time \(T_1\), the stress component \(\tau_{12}\), the main driving force behind the strike-slip motion of the fault \(F_1\), exceeds the critical value \((\tau_e)\), \(F_1\) slips, which is characterized by the following dislocation condition:

\[
(u)_{F_1} = U_{f_1}(y_1, y_3)H(t - T_1)
\]

where,

\[
(u)_{F_1} = \text{The discontinuity in } u \text{ across } F_1 \text{ and } H(t - T_1) \text{ is Heaviside unit step function.}
\]
Similarly, we are considering the fault slips commence across $F_2$ after time $T_2$ and calculating in a similar way.

The resulting boundary value problem can be solved by using a modified Green's function technique and correspondence principle, following Maruyama (1964, 1966); Rybicki (1971) and this technique has been explained by Mukhopadhyay et al. (1980a) and we get the final solution for displacements, strains and stresses as:

$$u(y_1, y_2, y_3, t) = (u)_p e^{-\frac{\mu t}{2\eta}} + y_1 \left[ \tau_{L} \left( 1 - \frac{1 - e^{-\frac{\mu t}{2\eta}}}{2\mu} \right) \right] + \frac{U_1}{2\pi} H(t - T_1) \phi_1 (y_1, y_2, y_3)$$

$$v(y_1, y_2, y_3, t) = (v)_p e^{-\frac{\mu t}{2\eta}} + \frac{U_1}{2\pi} H(t - T_1) \phi_1 (y_1, y_2, y_3)$$

$$w(y_1, y_2, y_3, t) = (w)_p e^{-\frac{\mu t}{2\eta}}$$

$$\tau_{11}(y_1, y_2, y_3, t) = (\tau_{11})_p e^{-\frac{\mu t}{\eta}} + \tau_{L} \left( 1 - e^{-\frac{\mu t}{\eta}} \right) + \frac{\mu U_1}{2\pi} H(t - T_1) \left( 1 + e^{-\frac{\mu t}{2\eta}} \right) \phi_2 (y_1, y_2, y_3)$$

$$\tau_{12}(y_1, y_2, y_3, t) = (\tau_{12})_p e^{-\frac{\mu t}{\eta}} + \tau_{L} \left( 1 - e^{-\frac{\mu t}{\eta}} \right) + \frac{\mu U_1}{2\pi} H(t - T_1) \left( 1 + e^{-\frac{\mu t}{2\eta}} \right) \phi_3 (y_1, y_2, y_3)$$

$$\tau_{13}(y_1, y_2, y_3, t) = (\tau_{13})_p e^{-\frac{\mu t}{\eta}} + \frac{\mu U_1}{2\pi} H(t - T_1) \left( 1 + e^{-\frac{\mu t}{\eta}} \right) \phi_4 (y_1, y_2, y_3)$$

$$\tau_{22}(y_1, y_2, y_3, t) = (\tau_{22})_p e^{-\frac{\mu t}{\eta}}$$

$$\tau_{23}(y_1, y_2, y_3, t) = (\tau_{23})_p e^{-\frac{\mu t}{\eta}}$$

$$\tau_{33}(y_1, y_2, y_3, t) = (\tau_{33})_p e^{-\frac{\mu t}{\eta}}$$

$$e_{11}(y_1, y_2, y_3, t) = (e_{11})_p e^{-\frac{\mu t}{2\eta}} + \frac{\mu U_1}{2\pi} H(t - T_1) \left( 1 + e^{-\frac{\mu t}{2\eta}} \right) \phi_2 (y_1, y_2, y_3)$$

$$e_{12}(y_1, y_2, y_3, t) = (e_{12})_p e^{-\frac{\mu t}{2\eta}} + \frac{\mu U_1}{2\pi} H(t - T_1) \phi_3 (y_1, y_2, y_3)$$

$$e_{22}(y_1, y_2, y_3, t) = (e_{22})_p e^{-\frac{\mu t}{2\eta}}$$

$$e_{23}(y_1, y_2, y_3, t) = (e_{23})_p e^{-\frac{\mu t}{2\eta}}$$

$$e_{33}(y_1, y_2, y_3, t) = (e_{33})_p e^{-\frac{\mu t}{2\eta}}$$
where,
\[
\psi_1(y_1, y_2, y_3) = \int_{L_1}^{L_2} \int_{-L_1}^{L_1} f_1(x_1, x_3) \left[ \frac{y_2}{(y_1-x_1)^2 + y_2^2 + (y_3-x_3)^2} \right]^{\frac{3}{2}}
\]

Similarly \(\psi_2, \psi_3, \psi_4\) and \(\phi_1, \phi_2, \phi_3, \phi_4\) are obtained from \(\psi_1, \psi_2, \psi_3, \psi_4\) by substituting \(y_1 = z_1, y_2 = z_2 + D, y_3 = z_3 + d\) and \(D_1\) by \(D_2\).

5. Results and discussion

Numerical computations have been carried out to compute the following quantities:

\(i)\quad E_{12}(y_1, y_2, y_3, t) = e_{12}(y_1, y_2, y_3, t) - (e_{12})_0 e^{-\frac{\mu t}{\eta}}\)

\[= \frac{\tau_\mu}{\mu} \left( 1 - e^{-\frac{\mu t}{\eta}} \right)\]

\(=\) The strain before the fault movement due to the tectonic forces.

\(ii)\quad (T_{12})_0(y_1, y_2, y_3, t) = \tau_{12}(y_1, y_2, y_3, t) - (\tau_{12})_0 e^{-\frac{\mu t}{\eta}}\)

\[= \frac{\mu U_1}{2\pi} H(t-T_1) \left[ 1 + e^{-\frac{\mu t}{\eta}} \right] + \psi_3(y_1, y_2, y_3)\]

\[+ \frac{\mu U_2}{2\pi} H(t-T_2) \left[ 1 + e^{-\frac{\mu t}{\eta}} \right] \phi_3(y_1, y_2, y_3)\]

\(=\) The stress due to the fault movement across \(F_1\) and \(F_2\) at some particular point.

\(iii)\quad (T_{12})_2(y_1, y_2, y_3, t) = \frac{\mu U_1}{2\pi} H(t-T_1) \left[ 1 + e^{-\frac{\mu t}{\eta}} \right] + \psi_3(y_1, y_2, y_3)\)

\[\text{at } y_2 = 10\text{ km and } -25\text{ km} \leq y_1 \leq 25\text{ km}, 0 \leq y_3 \leq 25\text{ km}\]

\(=\) Stress contour showing the effect of fault movement across \(F_1\) at different points near \(F_2\).

\(iv)\quad (T_{12})_3(y_1, y_2, y_3, t) = \frac{\mu U_2}{2\pi} H(t-T_2) \left[ 1 + e^{-\frac{\mu t}{\eta}} \right] \phi_3(y_1, y_2, y_3)\)

\[\text{at } y_2 = 0\text{ km and } -20\text{ km} \leq y_1 \leq 20\text{ km}, 0 \leq y_3 \leq 20\text{ km}\]

\(=\) Stress contour showing the effect of fault movement across \(F_2\) at different points near \(F_1\).

\(v)\quad \text{Stress contour map after the sudden movement across } F_1 (F_2 \text{ remains locked) on different planes given } y_2 = -10\text{ km, 5 km, 15 km.}\)

\(vi)\quad \text{Stress contour map after the sudden movement across both } F_1 \text{ and } F_2 \text{ on different planes given } y_2 = -10\text{ km, 5 km, 15 km.}\)

Following Catlhes (1975); Aki and Richards (1980) and the contemporary studies on rheological behaviour of crust and upper mantle by Chift et al. (2002), Karato (2010) the values for the model parameters have been chosen for numerical computation.

We consider \(f(x_1, x_3)\) to be

\[f(x_1, x_3) = \left( 1 - \frac{x_1^2}{L_1^2} \right) \left( 1 - \frac{3x_3^2}{D_1^2} + \frac{3x_3^2}{D_2^2} \right) \left( \frac{D_1 - x_3}{D_1} \right)\]

which satisfies all conditions (C1) to (C3) stated above.

\[\mu = 3.5 \times 10^{11}\text{ dyne/sq.cm} \eta = 5 \times 10^{20}\text{ poise}\]

For fault \(F_1\) (Surface breaking)

Length \(2L_1 = 40\text{ km, Width } D_1 = 10\text{ km}\)

For fault \(F_2\) (Buried), length \(2L_2 = 50\text{ km, Width } D_2 = 15\text{ km}\)

\(D = \text{ Horizontal distance between } F_1 \text{ and } F_2 = 10\text{ km}\)

\(\tau_{12} = 20\text{ bar (assumed)}\)

\(\tau_{12} = 40\text{ bar (assuming } 80\%\text{ stress released } \left[ = 80\% \text{ of } \tau_{12} \right] \)
\[ \tau_\infty = 300 \text{ bar}, \ (\tau_c)_1 = 200 \text{ bar}, \ (\tau_c)_2 = 250 \text{ bar} \]

\[ T_1 = \text{The time for fault movement across } F_1 \text{ is found to be 47 years.} \]

\[ T_2 = \text{The time for the second fault movement across } F_2 \text{ is found to be 79 years.} \]

6. **Observations**

6.1. **Strain at the free surface due to the tectonic forces** \( \tau_\infty \)

It is observed from [Fig. 2] that the rate of strain accumulation is \( \sim 8 \times 10^{-6} \text{/year} \) initially with a decreasing rate and reduces approximates to a value \( 6 \times 10^{-6} \text{/year} \) which is in conformity with the observations in seismically active regions.

6.2. **Stress** \( \tau_{12} \) **in the medium against time**

In [Fig. 3], \( \{T_{12}\}_1 \) have been shown at a point given by \( y_1 = 5 \text{ km}, y_2 = 5 \text{ km} \) and \( y_3 = 5 \text{ km} \). We observed that for \( 0 \leq t \leq T_1, \tau_{12} \) increases with a decreasing rate and attain the value near 200 bar at \( t = 47 \text{ years} (=T_1) \). Due to the movement across \( F_1 \) there is a co-seismic stress drop as shown by [Fig. 3]. The stress again increases, but at a lower rate until it reaches the value near \( (\tau_c)_2 \) after at a time \( t = T_2 \) when the second fault \( F_2 \) slips with another co-seismic stress drop. The stress at the point is found to increase further, but with a slow increasing rate and reaches the value 265 bar at \( t = 200 \text{ years} \).
Figs. 6(a&b). (a) Stress $\tau_{12}$: Effect of fault movement across $F_1$ on three different vertical planes given by $y_2 = (i) -10$ km, (ii) 5 km and (iii) 15 km and (b) Stress $\tau_{12}$: Effect of fault movement across $F_2$ on three different vertical planes given by $y_2 = (i) -10$ km, (ii) 5 km and (iii) 15 km
6.3. Stress $\tau_{12}$: effect of fault slip across $F_1$

The stress contour has been shown in [Fig. 4] with $y_2 = 10$ km. The magnitude of the stress varies from -10 bar to 10 bar concentration of the magnitude of the stress have been found to lie on both sides of the middle line $y_1 = 0$ but with opposite direction as is evident from [Fig. 4].

6.4. Stress $\tau_{12}$: effect of fault movement across $F_2$ near $F_1$

It is observed from [Fig. 5] that the effect of fault movement across the fault $F_2$ near the fault $F_1$. It is also observed that accumulation pattern of stress is much distinct in this case and the magnitude of the stress are nearly alike to the previous one.

6.5. Stress $\tau_{12}$: effect of fault movement across $F_1$ [Fig. 6(a)] and $F_2$ [Fig. 6(b)] respectively on three different vertical planes given by $y_2 = -10$ km, 5 km, 15 km

In the [Figs. 6(a&b)] we find that there are certain areas of stress accumulation and certain other areas of stress release. The magnitude of the stress depends upon the value of $y_2$. For small $y_2$ the magnitude of the stresses is found to be large while for higher values of the $y_2$ the magnitudes are found to be comparatively smaller.

7. A prediction of the next slipping movement across $F_1$

We can used the above result suitably for earthquake prediction purposes. In the present work $F_1$ slips after a lapse of 48 years when it reaches a value $\tau_\infty$. The movement across fault $F_1$ affects the nature of stress accumulation in the regions near it. It is assumed that the slipping movement causes 80% of the accumulated stress near $F_1$ to be released. This means the initial value $(T_1^p)$ at the second phase of aseismic period becomes 20% of $\tau_\infty$, that is 40 bars. With the slipping across $F_1$, the stress starts accumulating in a different way. [Fig. 7] shows the accumulation of stress at a point near the fault $F_1$ given by $(y_1 = 1$ km, $y_2 = 0.5$ km, $y_3 = 5$ km) during the second phase of aseismic state. The pink curve shows accumulation of stress under the combined action of $\tau_\infty$ and the slip across $F_1$, while the blue curve indicates the accumulation of stress under the action of $\tau_\infty$ only. The figure shows that after a lapse of another 62 years, the pink curve reaches the threshold level (200 bars). However, in the absence of any movement across $F_1$ the red curve would have reached the threshold level after 42 years. These observations allow us to conclude that the next possible seismic event will likely be delayed by about 20 years due to the movement across $F_1$. Such observations may be expected to prove fruitful in predicting the next seismic event.

(i) Accumulation of $\tau_{12}$ during the first phase of the aseismic state due to the effect of $\tau_\infty$ with $(\tau_{12})_b = 20$ bars.

(ii) Accumulation of $\tau_{12}$ during the second phase of the aseismic state under the action of $\tau_\infty$ only with $(\tau_{12})_p = 40$ bars.

(iii) Accumulation of $\tau_{12}$ under the action of $\tau_\infty$ along with the effect of the fault movement across $F_1$ during the second phase of the aseismic state.

Acknowledgements

One of the authors (Papiya Deb Nath) thanks the Director and Head of the Department of Basic Science and Humanities, Techno International Newtown, a unit of Techno India Group (INDIA), for allowing me to pursue the research, and also thanks the Geological Survey of India; Department of Applied Mathematics, University of Calcutta for providing the library facilities. The contents and views expressed in this research paper/article are the views of the authors and do not necessarily reflect the views of the organizations they belong to.

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