Single-field inflation, anomalous enhancement of superhorizon fluctuations and non-Gaussianity in primordial black hole formation

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Abstract. We show a textbook potential for single-field inflation, namely the Coleman–Weinberg model can induce double inflation and formation of primordial black holes (PBHs), because fluctuations that leave the horizon near the end of first inflation are anomalously enhanced at the onset of second inflation when the time-dependent mode turns into a growing mode rather than a decaying mode. The mass of PBHs produced in this mechanism with an appreciable density are distributed at certain intervals depending on the model parameters. We also calculate the effects of non-Gaussian statistics due to higher-order interactions on the abundance of PBHs, which turns out to be small.

Keywords: CMBR theory, cosmological perturbation theory, inflation, physics of the early universe
Contents

1. Introduction 2
2. Chaotic new inflation model 3
3. Enhancement of curvature perturbation 4
   3.1. Evolution of curvature perturbation 5
   3.2. Power spectrum of curvature perturbation in the chaotic new inflation model 6
4. Abundance of primordial black holes 7
5. Non-Gaussian correction to PBH abundance 8
   5.1. Three-point correlation functions 8
   5.2. Correction to PBH abundance from three-point correlation functions 10
   5.3. Estimation of correction to PBH abundance in chaotic new inflation model 12
6. Parameter search 13
7. Discussion 15
Acknowledgment 16
References 16

1. Introduction

The primordial perturbation generated in the inflationary epoch [1, 2] is believed to be the origin of large scale structure observed in the universe today. We have accurate information on the primordial perturbation by observing anisotropy of the cosmic microwave background (CMB) with the help of cosmological perturbation theory [3]. However, the CMB observation provides us with the information on the perturbations on a limited range of scales. Therefore, we cannot say, a priori, anything on the primordial perturbation at the smaller scales from CMB data. If the perturbation at these scales is of the order of unity, primordial black holes (PBHs) can be produced when the scale of the overdensed region crosses the horizon\(^4\) [4]. The typical mass of these black holes is given by the horizon mass at the horizon crossing\(^5\):

\[
M_{\text{BH}} = \frac{4\pi}{3} \rho (H^{-1})^3 = \frac{4\pi M_G^2}{H},
\]

where \(M_G = (8\pi G)^{-1/2} = 2.4 \times 10^{18} \text{ GeV}\) is the reduced Planck mass\(^6\). Though the PBH evaporates through the Hawking radiation process [6], those with mass greater than \(10^{15} \text{ g}\)

\(^4\) In this paper, we use the term ‘horizon’ to refer to the ‘Hubble radius’, \(H^{-1}\).

\(^5\) Strictly speaking, the mass of a black hole depends on the density perturbations \(\delta\) and black holes with a small fraction of horizon mass are also produced [5]. Since the contributions of these black holes are small compared to black holes with horizon mass, we ignore this dependence for simplicity.

\(^6\) We use units for which \(c = 1\).
Primordial black hole formation can remain until the present time [7, 8]. These PBHs can be the origin of intermediate-mass black holes [9] or dark matter in the universe [7, 10].

It is difficult to produce appreciable numbers of PBHs in the simple single-field slow-roll inflation models, which predict a nearly scale-invariant power spectrum of curvature perturbations because the amplitude of fluctuations on small scales cannot be much different from that normalized by CMB observation on large scales. One exception among others [11] is the chaotic new inflation model [12] where double inflation is realized with a single field. In [12], however, calculation of curvature fluctuation was done using the slow-roll formula in which only the time-independent mode has been taken into account. In the present paper, we solve the evolution equation of each Fourier mode of fluctuations properly and find an anomalous growth due to the temporal deviation from slow-roll evolution between two inflationary stages. This provides the first realistic example of the anomalous growth of perturbation in the superhorizon regime discussed in [13]. As a result we obtain a power spectrum highly peaked on some scales depending on the values of the model parameters and PBH formation is more easily realized than concluded in [12]. We search for the values of parameters with which appreciable numbers of PBHs are produced under the observational constraints [14]. We also analyze the effects of non-Gaussianity generated in this model on the abundance of PBHs.

The organization of this paper is as follows. In section 2, we introduce the chaotic new inflation model and explain the background evolution in this model. In section 3, a mechanism of the enhancement of the perturbation is explained and the power spectrum in the chaotic new inflation model is given. In section 4, we give an expression of PBH abundance resulted from a peaked power spectrum. In section 5, we estimate the non-Gaussian correction to PBH abundance. In section 6, we calculate the PBH abundance with various values of parameters and give a relation between mass and PBH abundance. Section 7 contains the conclusion of this paper. In this paper, we use curvature perturbation in the comoving gauge $\zeta$ as a degree of freedom of scalar perturbation.

2. Chaotic new inflation model

We consider a single-field inflation model with the Coleman–Weinberg potential [15]:

$$V(\varphi) = \frac{\lambda}{4} \varphi^4 \left( \ln \left| \frac{\varphi}{v} \right| - \frac{1}{4} \right) + \frac{\lambda}{16} v^4. \tag{2}$$

We shall set the initial field value to be large (super-Planckian) so that the field is energetic enough to achieve both chaotic [16] and new inflation [17]. Phase space arguments also show that this is an appropriate choice of initial condition [18].

In this model, inflation can occur twice [19]. First, chaotic inflation occurs. After chaotic inflation, the inflaton field oscillates between the two minima of the potential. If the parameter $v$ is appropriately chosen, the inflaton field moves slowly in the neighborhood of the origin after the oscillation and new inflation commences there. For example, the number of e-folds of new inflationary expansion, $N_{\text{new}}$, with $|\dot{H}| < H^2$ is larger than 10 for $v = 1.103 M_G - 1.132 M_G$, and it satisfies $N_{\text{new}} \gtrsim 60$ for $v = 1.114 M_G - 1.122 M_G$. According to the number of oscillation cycles, the parameters where new inflation occurs are distributed at certain intervals. In the above example, $\varphi$ settles to the positive potential minimum $\varphi = v$ if $v \geq 1.119 M_G$ and to the negative
Figure 1. The evolution of Hubble parameter (left) and inflaton field (right) with the values of the parameters \((\lambda, v) = (5.4 \times 10^{-14}, 0.355\,139\,M_G)\). \(a_i\) is a value of the scale factor at the initial time.

potential minimum \(\varphi = -v\) if \(v \leq 1.118M_G\) without oscillation. New inflation with \(N_{\text{new}} > 10\) occurs after a half-cycle of oscillation for \(v = 0.5340M_G - 0.5350M_G\) and after a cycle of oscillation for \(v = 0.35510M_G - 0.35524M_G\) and so on. We show the evolution of Hubble parameter \(H\) and the inflaton field \(\varphi\) with the values of the parameters \((\lambda, v) = (5.4 \times 10^{-14}, 0.355\,139\,M_G)\) in figure 1, which shows that the inflaton field moves slowly in the neighborhood of the origin after a cycle of oscillation, and new inflation occurs. With these values of the parameters, \(\varphi\) settles to the negative potential minimum.

In describing the evolution of the inflaton field during inflation, it is convenient to introduce the following Hubble slow-roll parameters:

\[
\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\varphi}^2}{2M_G^2 H^2}, \tag{3}
\]

\[
\eta \equiv \frac{\dot{\epsilon}}{H \epsilon}, \tag{4}
\]

where dots denote differentiation with respect to the cosmic time \(t\). We show the evolution of these slow-roll parameters in figure 2. The values of the model parameters are the same as those employed in figure 1. We can see that slow-roll conditions

\[
\epsilon \ll 1, \quad |\eta| \ll 1, \tag{5}
\]

are not satisfied while the inflaton field is oscillating between the two minima. As we see in section 3, the existence of this period is important for enhancement of curvature perturbation.

3. Enhancement of curvature perturbation

Curvature perturbation is enhanced in the models where slow-roll conditions are temporarily broken as stated in [13]. In this section, we briefly describe the mechanism of the enhancement and give a power spectrum of curvature perturbation in the chaotic new inflation model.
3.1. Evolution of curvature perturbation

Curvature perturbation in the comoving gauge $\zeta$, in terms of which the amplitude of perturbation in the intrinsic spatial curvature of the comoving slicing $R_c$ is written as

$$R_c = \frac{4}{a^2} \nabla^2 \zeta,$$ (6)

evolves according to an equation [20]:

$$\frac{d^2 \zeta_k}{dN^2} + (3 - \epsilon + \eta) \frac{d\zeta_k}{dN} + \left(\frac{k}{aH}\right)^2 \zeta_k = 0,$$ (7)

where $N$ is the number of e-folds and $\zeta_k$ is the Fourier transform of $\zeta$:

$$\zeta_k \equiv \int d^3x \zeta(t, x) e^{-ik \cdot x}.$$ (8)

In the slow-roll inflation regime, the coefficient of the second term in equation (7) is positive and therefore the solutions of equation (7) in the long-wavelength regime, where the last term is negligible, are a constant mode and a decaying mode. The time derivative $d\zeta_k/dN$, which corresponds to the decaying mode, diminishes in proportion to $a^{-2}$. If we neglect the last term in equation (7), $d\zeta_k/dN$ diminishes as $a^{-3}$. However, the last term decreases as $(k/aH)^2 \zeta_k \propto a^{-2}$. As a result the second term in equation (7) soon becomes comparable to the last term in equation (7), and $d\zeta_k/dN$ diminishes as $a^{-2}$. Therefore $\zeta_k$ soon becomes constant after horizon crossing and the power spectrum of curvature perturbation is given by the squared amplitude of the vacuum fluctuation$^7$

$$|\zeta_k|^2 = \frac{1}{2\epsilon M_G^2} \frac{k^{-1}}{a^2}$$ (9)

$^7$ In subhorizon scales, $\zeta_k$ is given by the quantum fluctuation in the vacuum state. Therefore, $\zeta_k \pi \xi_k$ has the minimum value 1 under the uncertainty relation $\zeta_k \pi \xi_k \geq 1$. Here, $\pi \xi_k$ is the momentum conjugate to $\zeta_k$ and expressed as $2a^3 \epsilon M_G^2 \xi_k$. Then, approximating $\zeta_k \sim (k/a)\xi_k$, we obtain the amplitude (9).
at the time the mode crossed the horizon:

\[ P_\zeta(k) = \frac{k^3 |\zeta_k|^2}{2\pi^2} \bigg|_{k=aH} \]

\[ = \frac{1}{\epsilon} \left( \frac{H}{2\pi M_G} \right)^2 \bigg|_{k=aH}. \]  

(10)

The power spectrum given by equation (10) is well approximated by the form \( P_\zeta(k) \propto k^{n-1} \) where \( n-1 = -2\epsilon - \eta \), namely, a nearly scale-invariant spectrum with small value of slow-roll parameters. Since the observed amplitude is small and single-field inflation models generically give a spectral index \( n < 1 \) [21], this power spectrum leads to a small number of primordial black holes.

On the other hand, the coefficient of the second term in equation (7) can be negative in the model where slow-roll conditions are temporarily broken such as the chaotic new inflation model, in which we find \( \epsilon \simeq 3 \) and \( \eta < 0 \) near the end of the oscillatory phase so that \( 3 - \epsilon + \eta < 0 \). In this case, equation (7) has a growing mode solution instead of the decaying mode solution outside the horizon. Therefore \( \zeta_k \) grows even after horizon crossing and its amplitude is enhanced. This enhancement of the perturbation enables even a single-field inflation model to produce a large number of PBHs.

3.2. Power spectrum of curvature perturbation in the chaotic new inflation model

We have estimated the power spectrum of curvature perturbation in the chaotic new inflation model by solving equation (7) numerically. In the integration of equation (7), we have imposed the initial conditions that the solution is purely a positive-frequency solution when the mode is far inside the horizon\(^8\) where \( k/aH \gg 1 \), which means we have chosen the Bunch–Davies vacuum state [22]. In figure 3, we show the power spectrum which is normalized to the amplitude observed by WMAP [3], \( 2 \times 10^{-9} \) at \( k = 0.002 \) Mpc\(^{-1} \), by choosing \( \lambda \) appropriately. The values of the model parameters are the same as those employed in figures 1 and 2.

In figure 3, it is observed that the power spectrum deviates from the one estimated by using the slow-roll formula (10) and the enhancement of the perturbation has occurred. The power spectrum has a peak with amplitude \(~6.2 \times 10^{-3}\) and results in formation of a large number of PBHs with mass corresponding to the scale of the peak. The scale of the peak corresponds to the scale which crossed the horizon near the end of chaotic inflation. The non-constant mode for the scales which crossed the horizon earlier is a decaying mode during slow-roll inflation and becomes exponentially small as \( a^{-2} \). Therefore, even if it turns to a growing mode temporarily after the first inflation, only the modes which left the horizon in the late stage of the first inflation are enhanced to a significant level. The scale and the amplitude of the peak vary with the parameter \( v \). For increasing number of oscillation cycles of the inflaton field, a larger amplitude is obtained. We investigate the abundance of PBHs resulting from the peaked spectrum in the following sections.

\(^8\) For the modes which cross the horizon twice, we have imposed the initial condition before the horizon crossing during chaotic inflation.
Figure 3. Power spectrum of curvature perturbation (solid line). This spectrum is calculated under the parameters \((\lambda, v) = (5.4 \times 10^{-14}, 0.355139 M_G)\). We show also a power spectrum estimated by using the formula (10), which is used for a slow-roll inflation model (dashed line).

4. Abundance of primordial black holes

In this section, we give an expression of the PBH abundance resulting from a strongly peaked power spectrum.

We estimate the PBH abundance based on the Press–Schechter method [23]. In this method, a PBH with mass greater than \(M\) is formed when the perturbation which is smoothed on scale \(R_M\) corresponding to \(M\) exceeds the threshold \(\zeta_{\text{th}}\). The smoothed perturbation \(\zeta_{R_M}\) is defined by

\[
\zeta_{R_M}(x) \equiv \int d^3 x' W(|x' - x|/R_M) \zeta(x'),
\]

where \(W(x/R)\) is a window function. The fraction of the energy density of the universe collapsing into PBHs with mass \(M < M_{\text{BH}} < M + \Delta M\) at the time they form is given by

\[
\beta(M; \Delta M) \equiv \frac{\rho_{\text{BH}}(M; \Delta M)}{\rho_{\text{tot}}} = -2 \int_M^{M+\Delta M} dM \int_{\zeta_{\text{th}}} d\zeta \frac{\partial P_{R_M}}{\partial M},
\]

where the prefactor 2 is due to Press–Schechter’s prescription and \(P_{R_M}\) is the probability distribution of \(\zeta_{R_M}(x)\). \(P_{R_M}\) is independent of \(x\) because of homogeneity of the universe. Therefore, we omit the argument \(x\) in what follows. For the moment, we assume \(\zeta_k\) to be Gaussian distributed, which is the case to the lowest order of perturbation, and consider the non-Gaussian correction in section 5 to test the validity of this assumption. Then \(P_{R_M}\) is Gaussian with variance

\[
\sigma_{R_M}^2 \equiv \int \frac{dk}{k} W(kR)^2 \mathcal{P}_\zeta,
\]
Primordial black hole formation

where $\tilde{W}(kR)$ is the volume-normalized Fourier transform of the window function $W(x/R)$. We have used a top-hat function as $\tilde{W}(kR)$.

We estimate the PBH abundance, (12), resulting from a strongly peaked power spectrum at the scale $k^{-1} = k_{\text{peak}}^{-1}$. For $P_{R_{\text{M}}}$, which is Gaussian with variance $\sigma_{R_{\text{M}}}$, its derivative with respect to $M$ is given by

$$\frac{\partial P_{R_{\text{M}}}}{\partial M} = \frac{1}{\sqrt{8\pi\sigma_{R_{\text{M}}}}} \left( \frac{\sigma_{R_{\text{M}}}}{\sigma_{R_{\text{M}}}} - 1 \right) \exp \left( -\frac{\tilde{\zeta}^2}{2\sigma_{R_{\text{M}}}^2} \right). \quad (14)$$

Since the power spectrum is strongly peaked, we can approximate $d\sigma_{R_{\text{M}}}/d(\log M)$ to be $P_{\tilde{\zeta}}(k_{\text{peak}})$ for $M \sim M_{\text{peak}}$ and 0 otherwise. Here, $M_{\text{peak}}$ is the mass corresponding to the scale of the peak. Therefore, only PBHs whose mass is $M_{\text{peak}}$ form$^9$. In this approximation, its abundance is given by

$$\beta(M_{\text{peak}}) = \frac{1}{\sqrt{2\pi}} \int d\tilde{\zeta} \left( \tilde{\zeta}^2 - 1 \right) e^{-\tilde{\zeta}^2/2} \left( \tilde{\zeta} \equiv \sqrt{P_{\tilde{\zeta}}(k_{\text{peak}})} \right), \quad (15)$$

where $\beta$ is independent of $\Delta M$ under the current approximation. In the peaked spectrum, the amplitude at the scale of the peak can be large while the amplitude at large scales is consistent with the observed value. Therefore, a large number of PBHs are produced from the strongly peaked power spectrum.

5. Non-Gaussian correction to PBH abundance

The amplitude of perturbation producing PBHs is so large that non-Gaussianity of curvature perturbation due to higher-order effects can be important. In the case of slow-roll inflation, this effect on the formation of PBHs has been studied in [25] and shown to be negligibly small, contrary to the intuitive expectation. However, in models where slow-roll conditions are temporarily violated such as the chaotic new inflation model, large non-Gaussianity arises [26] and can modify the PBH abundance obtained above. In this section, we investigate this possibility.

5.1. Three-point correlation functions

Here, in order to estimate the effects of the deviation from Gaussian due to the higher-order interaction in the action, we first evaluate three-point correlation functions.

We sketch the derivation of the three-point correlation functions [27]. The three-point functions are estimated by perturbative expansion with respect to the interaction. For the estimation of the three-point correlation functions, the cubic terms of $\tilde{\zeta}$

\[ S_3 = \int dt \, L_3(\tilde{\zeta}, \dot{\tilde{\zeta}}; t) \]

\[ = M_G^3 \int dt \, d^3x [ae_{\tilde{\zeta}}(\nabla \tilde{\zeta})^2 + a^3 e H^{-1} \tilde{\zeta}^3 - 3a^3 e \tilde{\zeta}^2 \]

\[- \frac{1}{2a} (3\tilde{\zeta} - H^{-1} \dot{\tilde{\zeta}})(\nabla_i \nabla_j \nabla_i \nabla_j \psi - \nabla^2 \psi \nabla^2 \psi) + 2a^{-1} \nabla_i \psi \nabla_i \nabla^2 \psi], \quad (16)\]

$^9$ If we take the width of the peak of the power spectrum, $\Delta(\log k)$, into account, FWHM of the mass distribution is estimated to be $\Delta M = \Delta(\log M) \sim O(0.1)$.

$^{10}$ To be precise, this $\zeta$ is a generalization of $\zeta$ used in the linear perturbation theory [27]. At linear order, two $\zeta$ coincide with each other.
where $\psi$ is defined by

$$\psi \equiv -\frac{\zeta}{H} + a^2 \epsilon \nabla^{-2} \zeta. \quad (17)$$

From these terms, the interaction Hamiltonian up to the third order is

$$H_I(\zeta, \pi_\zeta; t) = -L_3(\zeta, \pi_\zeta/(2a^3 \epsilon); t)$$

$$= -L_3(\zeta, \dot{\zeta}; t)$$

$$= M \int d^3x \left[ -a \zeta (\nabla \zeta)^2 - a^3 \epsilon H^{-1} \zeta^3 + 3a^2 \epsilon \zeta \zeta^2 \right]$$

$$+ \frac{1}{2a} (3 \zeta - H^{-1} \zeta) \left( \nabla_i \psi \nabla_i \nabla_j \psi_j - \nabla^2 \psi \nabla^2 \psi \right) - 2a \zeta \nabla \psi \nabla \zeta \nabla^2 \psi \right]$$

$$= M \int d^3k_1 \frac{a^2 H^2}{(2\pi)^3} \left[ \delta(k_1 + k_2 + k_3) \right] \left[ H^{(1)}(k_1, k_2, k_3) \right]$$

$$+ H^{(2)}(k_1, k_2) \frac{d\zeta_{k_1}}{dt} + H^{(3)}(k_1, k_2, k_3) \frac{d\zeta_{k_1}}{dt} + H^{(4)}(k_1, k_2, k_3) \frac{d\zeta_{k_1}}{dt} \right], \quad (18)$$

where $\pi_\zeta$ is the momentum conjugate to $\zeta$ and the variables with subscript $I$ denote variables in the interaction picture. Coefficients $H^{(i)} (i = 1, 2, 3, 4)$ are given by

$$H^{(1)} = \epsilon \bar{k}^2 \cos \theta - \frac{1}{6} \bar{k}^4 \sin^2 \theta, \quad (19)$$

$$H^{(2)} = -\epsilon \bar{k}^2 (\sin^2 \theta + z \cos \theta) + \frac{\bar{k}^4}{2} \sin^2 \theta, \quad (20)$$

$$H^{(3)} = 3\epsilon + \frac{\epsilon^2}{2} (\sin^2 \theta + 2z \cos \theta) + \frac{\epsilon}{2} \bar{k}^2 (z - 2 \cos \theta) \sin^2 \theta, \quad (21)$$

$$H^{(4)} = -\epsilon - \frac{\epsilon^2}{2} \sin^2 \theta, \quad (22)$$

where we have defined

$$\bar{k} \equiv \frac{\sqrt{k_1 k_2}}{a H}, \quad \cos \theta \equiv \frac{k_1 \cdot k_2}{k_1 k_2}, \quad z \equiv \frac{k_3^2}{k_1 k_2}, \quad (23)$$

for simplicity of expression. For an equilateral triangle, $k_1 = k_2 = k_3 = k$, the values of these quantities are

$$\bar{k} = \frac{k}{a H}, \quad \cos \theta = -\frac{1}{2}, \quad z = 1.$$

In calculating the three-point correlation functions, the in–in formalism [28] is used since we want to calculate an expectation value with respect to the vacuum state at $t \to -\infty$:

$$\langle \zeta_{k_1}(t) \zeta_{k_2}(t) \zeta_{k_3}(t) \rangle = \langle U_{t, t_0}^{-1} \zeta_{k_1}(t) \zeta_{k_2}(t) \zeta_{k_3}(t) U_{t, t_0} \rangle$$

$$\left( U_{t, t_0} \equiv T e^{-i \int_{t_0}^t H(t) dt} \right), \quad (24)$$

where $H(t)$ is the Hamiltonian.
where \( \epsilon_0 \) is a positive infinitesimal constant\(^{11}\). To leading order of \( H_I \), the three-point correlation function is given by

\[
\langle U_I^{-1} \zeta_{k_1}I(t) \zeta_{k_2}I(t) \zeta_{k_3}I(t) U_I \rangle = -i \int_0^t dt' \langle [\zeta_{k_1}I(t) \zeta_{k_2}I(t) \zeta_{k_3}I(t), H_I(\zeta_{k_1}I, \pi_{\zeta_{k_1}I}; t')] \rangle
\]

\[
= 2 \int_0^t dt' \text{Im} \langle (\zeta_{k_1}I(t) \zeta_{k_2}I(t) \zeta_{k_3}I(t) H_I(\zeta_{k_1}I, \pi_{\zeta_{k_1}I}; t')) \rangle,
\]

(25)

where the \( t' \) integration contour is deformed so that both bra and ket are projected onto the vacuum state at \( t \rightarrow -\infty \). According to the behavior of \( \zeta_{kI} \), the integrals in equation (25) can be split into three parts, an integral over the region inside the horizon, the region around the horizon crossing and the region outside the horizon. In the former two parts the deviation from the slow-roll inflation models is small, since the third term in equation (7) is dominant in these parts. The deviation arises in the last part, because \( \zeta_k \) grows outside the horizon in the chaotic new inflation model and not in the slow-roll inflation models. In the slow-roll inflation models, the contribution from the part outside the horizon is negligible since each term in the interaction (18) includes \( \dot{\zeta}_I \) or \( k/aH \) which have small values outside the horizon and, furthermore, the commutator of \( \zeta_k \)'s or its time derivatives vanish as \( a^{-\nu} (\nu \geq 2) \).\(^{12}\) The three-point correlation functions are suppressed by the slow-roll parameters estimated at horizon crossing [27]. In contrast, in the chaotic new inflation model, \( \dot{\zeta}_I \) has a non-negligible value outside the horizon and the integral over the region outside the horizon contributes to the three-point correlation functions. The reason for obtaining large non-Gaussianity is different from that in [26]. In the chaotic new inflation model it is the growth of the perturbation outside the horizon, while in [26] it is the characteristic behavior of the slow-roll parameters near or inside the horizon.

### 5.2. Correction to PBH abundance from three-point correlation functions

Here, we give an expression of the PBH abundance resulting from the perturbation with the three-point correlation functions.

The probability distribution \( P_{R_M} \) can be expressed as

\[
P_{R_M}(\zeta_{R_M}) = \frac{1}{2\pi} \int d\eta_{R_M} \Phi(\eta_{R_M}) e^{-i\eta_{R_M} \zeta_{R_M}},
\]

(26)

where \( \Phi \) is defined by

\[
\Phi(\eta_{R_M}) \equiv \langle e^{i\eta_{R_M} \zeta_{R_M}} \rangle.
\]

(27)

\( \Phi \) can be expanded by cumulants of \( \zeta_{R_M} \), \( \langle \zeta_{R_M}^n \rangle_c \):

\[
\Phi(\eta_{R_M}) = \exp \left( \sum_{m=0}^{\infty} \frac{(\eta_{R_M})^m}{m!} \langle \zeta_{R_M}^m \rangle_c \right).
\]

(28)

\(^{11}\) In an abuse of language, we use the same symbol \( \zeta_k \) for denoting a quantized variable as a classical one.

\(^{12}\) If \( \zeta_k(t) \) is constant \( \zeta_{kI}(t) = \zeta_{kI}^{\text{const}} \), the three-point functions vanish because a term \( \zeta_{k_1}I(t) \zeta_{k_2}I(t) \zeta_{k_3}I(t) \zeta_{k_1}I(t') \zeta_{k_2}I(t') \zeta_{k_3}I(t') \) becomes a real number \( \langle \zeta_{k_1}^{\text{const}} \zeta_{k_2}^{\text{const}} \zeta_{k_3}^{\text{const}} \rangle^2 \) in this case. Then the leading term of the commutator of \( \zeta_k \)'s has a decaying mode in \( \zeta_k \)'s, and decreases as \( a^{-2} \). With time derivatives of \( \zeta_k \)'s, the commutators decreases more rapidly.
The cumulants of $\zeta_{RM}$ can be expressed by the connected part of the correlation functions of $\zeta_k$. With the formula (28), we can estimate the correction to $P_{RM}$ from higher-order correlation functions.

The correction from the three-point correlation functions can be calculated, retaining terms up to $m = 3$ in equation (28). In the case $|\langle \zeta_{RM}^3 \rangle_c|$ is much smaller than $\sigma_{RM}^3$, we can give a concrete expression of the corrected PBH abundance. In this case, we can expand $\Phi$ with respect to $J$ which is defined by

$$J \equiv \frac{1}{6} \frac{\langle \zeta_{RM}^3 \rangle_c}{\sigma_{RM}^3},$$

and get

$$P_{RM}(\zeta_{RM}) = \frac{1}{\sqrt{2\pi\sigma_{RM}^2}} \left[ 1 + \left( \frac{3 \zeta_{RM}\sigma_{RM}}{\sigma_{RM}^3} - \frac{\zeta_{RM}^3}{2\sigma_{RM}^3} \right) J \right] \exp \left( -\frac{\zeta_{RM}^2}{2\sigma_{RM}^2} \right).$$

$J$ can be expressed by the three-point correlation functions of $\zeta_k$ as

$$J = \frac{1}{6\sigma_{RM}^2} \int d^3k_1 \int d^3k_2 \int d^3k_3 \widetilde{W}(k_1R)\widetilde{W}(k_2R)\widetilde{W}(k_3R)\zeta_{k_1}\zeta_{k_2}\zeta_{k_3}. \quad (31)$$

Substituting the probability distribution $P_{RM}$ given by equation (30) into equation (12), we get the PBH abundance including the correction from the three-point correlation functions.

Because of homogeneity of the universe, the three-point correlation functions of $\zeta_k$ can be written as

$$\langle \zeta_k(t)\zeta_k(t)\zeta_k(t) \rangle_c = A(k_1, k_2, k_3)\delta^3(k_1 + k_2 + k_3).$$

(32)

The isotropy of the universe guarantees that $A(k_1, k_2, k_3)$ is, in fact, a function of ‘shape’ and ‘size’ of the triangle spanned by $k_i$ ($i = 1, 2, 3$) such as the quantities (23) and independent of the direction of the triangle. In the chaotic new inflation model, $\zeta_k$ grows most significantly at $k = k_{peak}$ and $A(k_1, k_2, k_3)$ has the largest value at $k_1 = k_2 = k_3 = k_{peak}$. Therefore, as before, we can approximate $d\langle \zeta_{RM}^3 \rangle_c/dM$ to be $4\pi k_{peak}^6 A_{peak}$ for $M \sim M_{peak}$ and 0 otherwise. Here, $A_{peak}$ represents $A(k_1, k_2, k_3)$ estimated at $k_1 = k_2 = k_3 = k_{peak}$. Under this approximation, the PBH abundance is given by

$$\beta(M_{peak}) = \frac{1}{\sqrt{2\pi}} \int_{\zeta_{th}} d\tilde{\zeta} \left[ (\tilde{\zeta}^2 - 1) - (\tilde{\zeta}^5 - 8\tilde{\zeta}^3 + 9\tilde{\zeta})J_{peak} \right] e^{-\tilde{\zeta}^2/2} \quad (33)$$

$$\left( \tilde{\zeta} \equiv \zeta / \sqrt{P_\zeta(k_{peak})} \right),$$

where

$$J_{peak} = \frac{2\pi}{3} \frac{k_{peak}^6 A_{peak}}{P_\zeta(k_{peak})^{3/2}}. \quad (34)$$

The PBH abundance with some values of $J_{peak}$ is shown in figure 4.
5.3. Estimation of correction to PBH abundance in chaotic new inflation model

With the formulae obtained in the previous subsections, we can estimate the correction to the PBH abundance from the three-point correlation functions in the chaotic new inflation model. We have carried out the integration in equation (25) numerically over the region outside the horizon, where the deviation from the slow-roll inflation models arises. In the other regions the contributions are almost the same as those in the slow-roll inflation models and do not modify the PBH abundance relevantly. Since the calculation is done only outside the horizon, we do not have the problem in implementing the \(i\epsilon\) prescription, the deformation of the \(t'\) integration mentioned below equation (25), numerically.

For the values of the parameters \((\lambda, v) = (5.4 \times 10^{-14}, 0.355 139 M_G)\) with which the amplitude at the peak has the values for producing relevant number of PBHs, \(\sim 6.2 \times 10^{-3}\), the value of \(A_{\text{peak}}\) is estimated to be

\[
k_{\text{6 peak}} A_{\text{peak}} \sim 10^{-9}.
\]  

(35)

On the other hand, the value obtained by estimating the contribution from the region around horizon crossing as done in slow-roll inflation models is calculated to be [27]

\[
k_{\text{6 peak}} A_{\text{peak}} = 48 \pi^7 f_{\text{NL}} P_\zeta^2 \sim 10^{-16},
\]  

(36)

where the value of \(P_\zeta\) is estimated at horizon crossing, \(\sim 10^{-11}\) (see figure 3). \(f_{\text{NL}}\) is an estimator usually used for parameterizing the size of non-Gaussianity observed in the CMB [27] and has the value of the order of the slow-roll parameters estimated at horizon crossing, \(\sim 10^{-1}\) (see figure 2).

We get larger three-point correlation functions than those without the enhancement of the perturbation\(^\text{13}\). However, the parameter \(J_{\text{peak}}\) which is an estimator for the correction

\(^{13}\)\(f_{\text{NL}}\) with all \(k_i\) \((i = 1, 2, 3)\) corresponding to the observed scale can be estimated from the fluctuations generated in the chaotic inflationary epoch. Therefore, the value of \(f_{\text{NL}}\) at the observed scale is small and consistent with the observation of non-Gaussianity by WMAP [3].
to the PBH abundance due to the three-point correlation functions is estimated to be

$$J_{\text{peak}} \sim 10^{-6}. \quad (37)$$

This value is too small to modify the PBH abundance appreciably. Though large three-point correlation functions are obtained, since a denominator of the estimator is also large, only a small correction is obtained.

The correction to the PBH abundance from $N$-point correlation functions appears with a factor $P_{\zeta}^{-N/2}$. Therefore the corrections are expected to be small as the correction from three-point functions are. We can therefore use the expression (15) for the PBH abundance safely.

6. Parameter search

In section 3, we have shown the curvature perturbation $\zeta$ is enhanced in the chaotic new inflation model. In this section, we calculate the PBH abundance using equation (15) with various values of the parameter $v$ and search the parameter with which relevant number of PBHs can be produced. For each $v$, the parameter $\lambda$ is fixed by the power spectrum of $\zeta$ observed by WMAP, $P_{\zeta} = 2 \times 10^{-9}$ at $k = 0.002$ Mpc$^{-1}$. In the linear perturbation theory, the power spectrum $P_{\zeta}(k)$ scales as

$$P_{\zeta}(k) \rightarrow \tilde{P}_{\zeta}(k) = \tilde{\lambda} P_{\zeta}(\tilde{\lambda}^{-1/2} k) \quad \text{for } V \rightarrow \tilde{V} = \tilde{\lambda} V. \quad (38)$$

Since the horizon mass at the matter–radiation equality is $\sim 10^{17} M_\odot$, PBHs that can be the origin of intermediate-mass black holes or dark matter are produced in the radiation-dominated epoch. In the radiation-dominated epoch, the threshold value of the density perturbation $\delta_{\text{th}}$ is given by $1/3$ [4,7]. Then, we find the corresponding value of the curvature perturbation $\zeta_{\text{th}} = 0.75$ with the help of the formula in the linear perturbation theory, $\delta_{k=aH} = 4 \zeta_{k=aH}/9$ [24]. A similar value has been obtained by numerical calculation in [29]. Equation (1) relates the mass of PBH produced in the radiation-dominated epoch to the scale of the perturbation as

$$M = 5.4 g \left(\frac{10^{16} \text{ GeV}}{V_*^{1/4}}\right)^2 \left(\frac{a_* R_M}{H_*^{-1}}\right)^2, \quad (39)$$

where the variables with subscript * denote those estimated at arbitrary time in the radiation-dominated epoch. We assume the period of reheating is negligible and have chosen the time at which inflation finished as the time at which the variables with subscript * are estimated. We write these variables with subscript ‘end’.

We have found that abundance of PBHs takes a value close to the observational upper bounds [14] around $v \simeq 0.266 67 M_G$, $v \simeq 0.355 14 M_G$ and $v \simeq 0.355 22 M_G$. In the first case, the inflaton field relaxes to the negative minimum after $3/2$ oscillations, and in the second (third) case, it relaxes to the negative (positive) minimum after one oscillation, respectively. In these values of $v$, $V_{\text{end}}^{1/4} \sim (\lambda v^4)^{1/4} \sim 10^{14}$ GeV and equation (39) gives

$$M \sim 10^4 g \left(\frac{R_M}{R_{\text{end}}}\right)^2, \quad (40)$$

14 The equations in the linear perturbation theory and the initial conditions for $\tilde{V} = \tilde{\lambda} V$ can be written as those for $V$ by defining $(\tilde{N}, \tilde{H}, \tilde{\varphi}, \tilde{k}, \tilde{\zeta}) \equiv (N, \lambda^{-1/2} H, \varphi, \lambda^{-1/2} k, \lambda^{1/4} \zeta)$. Then, the relation (38) is obtained.
where $R_{\text{end}} \equiv 1/(a_{\text{end}} H_{\text{end}})$, which is the comoving scale crossing the horizon at the end of the entire inflation. The PBH abundance $\beta$ is plotted as a function of the mass corresponding to the scale of the peak $M_{\text{peak}}$ in figures 5 and 6 with the observational constraints [14]. The constraint on PBH abundance with mass above $10^{15}$ g is obtained from the condition that the abundance of PBHs is less than that of matter today. The other constraints are obtained from the consistency with nucleosynthesis and $\gamma$-ray observation. In figure 6, we can observe that PBHs with mass $6 \times 10^{20}$ g can constitute a large part of matter in the universe. Though PBHs with some range of mass are observationally excluded from being the dominant component of dark matter [30], there are no strong constraints in mass range $10^{20} - 10^{26}$ g $(10^{-13} - 10^{-7} M_\odot)$. The mass $6 \times 10^{20}$ g are in this range. Therefore, we can obtain PBHs that can be the origin of dark matter in the chaotic new inflation model.

Finally, we estimate scalar spectral index $n$ and tensor-to-scalar ratio $r$ and check the consistency with the observational data for the models with the tensor modes from WMAP5 alone [3]:

$$n = 0.986 \pm 0.022, \quad r < 0.43 \ (95\% \text{CL}) \quad \text{at} \ k = 0.002 \ \text{Mpc}^{-1}. \quad (41)$$
Figure 6. PBH abundance $\beta$ as a function of mass corresponding to the scale of the peak $M_{\text{peak}}$ associated with the values of parameter $v = 0.355136 M_G - 0.355141 M_G$ (solid line) and $v = 0.355121 M_G - 0.355223 M_G$ (dashed line). The straight track represents an upper bound of PBH abundance. The constraint is obtained from the condition that the abundance of PBHs is less than that of matter today in the range above $10^{15}$ g.

In each parameter region, the scalar spectral index $n$ and tensor-to-scalar ratio $r$ is estimated to be

$$n = 0.931 - 0.939, \quad r = 0.33 - 0.37 \quad \text{for } v = 0.266669 M_G - 0.266674 M_G,$$

$$n = 0.922 - 0.928, \quad r = 0.38 - 0.42 \quad \text{for } v = 0.355136 M_G - 0.355141 M_G,$$

$$n = 0.924 - 0.931, \quad r = 0.37 - 0.41 \quad \text{for } v = 0.355121 M_G - 0.355223 M_G,$$

especially

$$n = 0.925, \quad r = 0.40 \quad \text{for } v = 0.355140 M_G \ (M_{\text{peak}} = 6 \times 10^{20} \text{ g}).$$

To estimate $n$ and $r$, we have used the slow-roll formulae

$$1 - n \simeq 2\epsilon + \eta, \quad r \simeq 16\epsilon,$$  \hspace{1cm} (42)

since the slow-roll conditions, equation (5), are satisfied when these values are estimated.

At the first parameter region, $n$ and $r$ are consistent with three-year WMAP data at the confidence level of 99.9%, but lie outside the preferred range of five-year data. At the other regions, $n$ and $r$ also lie outside the preferred range of five-year data.

In the chaotic new inflation model, the scales observable by CMB exit the horizon in the regime closer to the end of the chaotic inflation as the new inflation lasts longer, resulting in larger values of the slow-roll parameters than those of the original chaotic inflation model. Therefore, this model predicts smaller $n$ and larger $r$.

7. Discussion

In this paper, we have shown that the enhancement of the perturbation occurs and a large number of PBHs are produced in the chaotic new inflation model. A growing mode plays an important role in the enhancement. Due to the growing mode, there
is a contribution to the three-point functions from the region outside the horizon. We have estimated this contribution and its effect on the PBH abundance. As a result, we have found that non-Gaussian correction to the PBH abundance due to the higher-order effect is small. Further, we have calculated the PBH abundance with various values of the parameter $v$. We have shown that appreciable amounts of the number of PBHs are formed for $v \approx 0.266 \, 67 M_G$, $v \approx 0.355 \, 14 M_G$ and $v \approx 0.355 \, 22 M_G$. In the second parameter region, the produced PBHs can constitute a large part of the dark matter in the universe. In the first parameter region, though the produced PBHs cannot be observed, we can observe the enhancement of the perturbation with gravitational waves generated from scalar perturbations through nonlinear couplings [31], which far exceeds first-order stochastic tensor perturbation characterized by the tensor-to-scalar ratio $r$. The scale of the peak which gives mass $M = 1.4 \times 10^{13}$ g corresponds to GW frequency $^{15} \sim 100$ Hz, and the amplitude of the induced GW is estimated to be $^{16} \sim \sqrt{P_\zeta^2} \sim 10^{-2}$. which is larger than GWs of astronomical origins [32]. Therefore, the induced GW can be detected by detectors such as GEO600 [33], LIGO [34], TAMA [35] and VIRGO [36].

In both parameter regions, however, the values of spectral index and tensor-to-scalar ratio are not in the preferred region of the observational data. This is because, in the chaotic new inflation model, the observed perturbations are produced at the chaotic inflationary epoch with a potential close to the quartic one, which is not preferred by the observational data. In this paper we adopted the Coleman–Weinberg potential as a simple model to drive inflationary dynamics without specifying particle physics background. In reality, we expect supergravity corrections to the scalar potential are important, especially for $\phi > M_G$. Renormalization group analysis would also flatten the potential. If such corrections modify the potential in an appropriate way, this scenario could be a feasible one. Another way to rescue it is to adopt a smaller self-coupling $\lambda$ using the scaling law (38), so that it predicts smaller amplitudes of both scalar and tensor fluctuations on large scales observed by CMB, etc., assuming that the observed scalar perturbations on these scales were created through a different mechanism, say, curvaton [37] or modulated reheating [38]. Since the peak we have found is so prominent one could produce an observable amount of PBHs even in such a case with smaller $\lambda$.

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$^{15}$ The power spectrum of the induced GW has the peak at $k_{\text{peak}}/\sqrt{3\pi}$ [31].

$^{16}$ In [31], the energy density of the induced GW is calculated to be $\Omega_{\text{GW}} \sim 10^{-17} A^4$, where $A$ is the amplitude of the scalar perturbation at the scale of the peak relative to the observed amplitude ($\sim 10^{-5}$). At $M = 1.4 \times 10^{13}$ g, $A$ is estimated to be $\sim 10^3$ and $\Omega_{\text{GW}} \sim 10^{-5} \, \gamma$. This value is comparable to the nucleosynthesis bound $\Omega_{\text{GW}} \sim 10^{-5}$ [32]. However, the amplitude of the scalar perturbations is so large that the effects neglected in [31] such as backreactions to the scalar perturbations from the tensor perturbations should be properly taking into account.
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