Comment on “Spinon Attraction in Spin-1/2 Antiferromagnetic Chains”

In a recent Letter [1], Bernevig, Giuliano, and Laughlin (BGL) conclude that there is an attractive interaction between spinons in the Haldane–Shastry model (HSM) [2]. We wish to point out here that this conclusion is incorrect. Spinons in the HSM actually constitute an ideal half-fermi statistics, as asserted by Haldane [3] and in some sense confirmed by Essler [4], who calculated the spinon scattering matrix using the asymptotic Bethe Ansatz (ABA) and found $S = \pm i$.

BGL attribute the apparent contradiction between their conclusion and the ABA result to the fact that “the interaction between spinons is encoded in the definition of the pseudomomenta” which label the Bethe Ansatz solutions. In other words, they assert that it is a special feature of the ABA technique that the spinon excitations appear to be free, while there is in fact an attractive interaction between them. This line of thought, however, is unsustainable. The pseudomomenta in the ABA solutions label the exact eigenstates of the Hamiltonian. If the spinon scattering matrix $S$ does not depend on them, it does not depend on the true and physical spinon momenta either, whatever they may be. So if the ABA is applicable to the HSM at all, which is not guaranteed a priori as the spin-spin interaction is long-ranged, $S = \pm i$ directly and unambiguously implies that the spinons are non-interacting particles with half-fermi statistics.

Let us now critically re-examine the arguments presented by BGL. The first argument they give in favor of a spinon interaction is based on a plot of $|p_{mn}(e^{i\theta})|^2$ as defined in (18) (the equation numbers here and below refer to [1]) for $m = M$, $n = 0$, as a function of $\theta$. BGL interpret $|p_{mn}(e^{i\theta})|^2$ as probability for finding the spinons at a distance $\theta$ along the circle from each other, and show it to be strongly enhanced at small $\theta$. The problem with the argument is that, as one can easily see from (18), the $p_{mn}(\eta_{\alpha-\beta})$’s are the coefficients in the expansion of the overcomplete basis states $|\Psi_{\alpha,\beta}\rangle$ at fixed $\alpha$, $\beta$ in terms of $|\Phi_{mn}\rangle$. Due to this overcompleteness, the $p_{mn}(\eta_{\alpha-\beta})$’s as functions of $\eta_{\alpha-\beta}$ have no direct physical interpretation [5].

The second argument of BGL is that the last term in their expression (10) for the energy of the two-spinon state (8) represents “a negative interaction contribution that becomes negligibly small in the thermodynamic limit”. The problem here is that BGL have identified the momenta $q_m$ and $q_n$ of the individual spinons according to

$$q_m = \frac{\pi}{2} - \frac{2\pi}{N} \left( m + \frac{1}{2} \right), \quad q_n = \frac{\pi}{2} - \frac{2\pi}{N} \left( n + \frac{1}{2} \right),$$

when interpreting the two preceding terms in (10) as the kinetic energies of the individual spinons. The correct identification of the spinon momenta for $m \geq n$, however, is

$$q_m = \frac{\pi}{2} - \frac{2\pi}{N} \left( m + \frac{3}{4} \right), \quad q_n = \frac{\pi}{2} - \frac{2\pi}{N} \left( n + \frac{1}{4} \right),$$

which implies that the kinetic energy of the spinons is given by all three terms in the square bracket in (10). With $E(q) = \frac{\pi}{2} \left[ \left( \frac{q}{\pi} \right)^2 - q^2 \right]$, one finds

$$E_{mn} = -J \frac{\pi^2}{24} \left( N + \frac{5}{N} - \frac{6}{N^2} \right) + E(q_m) + E(q_n).$$

The alleged spinon interaction term has disappeared. Physically, the relative shift between $q_m$ and $q_n$ by one-half of a momentum spacing $\frac{2\pi}{N}$ is a manifestation of the half-fermi statistics of the spinons. While the allowed values for the total momentum $q_m + q_n$ are those for PBCs, the allowed values for the difference in the momentum $q_m - q_n$ are those for anti-PBCs, i.e., PBCs with the ring threaded by a flux $\pi$.

Finally, BGL claim to prove through a derivation of the Haldane–Zirnbauer formula that the spinon attraction, or more specifically the enhancement of $|p_{mn}(e^{i\theta})|^2$ they find when plotting it as a function of the spinon separation $\theta$, is responsible for the square-root singularity in the dynamical spin susceptibility (DSS). Their reasoning is problematic in several regards. First, the coefficients $|p_{mn}(e^{i\theta})|^2$ cannot be interpreted as a probability as a function of the spinon separation $\theta$, as explained above. Second, it is not $p_{mn}(e^{i\theta})$ as a function of $\theta$ for fixed $m$ and $n$ which enters their derivation of the DSS, but $p_{mn}(1)$ as a function of $m$ and $n$, as can be seen directly from (25).

In response to our considerations, BGL have countered that they merely investigated “the short distance effects” of the “statistical interaction” associated with the half-fermi statistics of the spinons. This is definitely not consistent with the claims they make in their Letter [1].

A more elaborate account of our understanding is given elsewhere [5].

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