Thermal bistability through coupled photonic resonances

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We present a scheme for achieving thermal bistability based on the selective coupling of three optical resonances. This approach requires one of the resonant frequencies to be temperature dependent, which can occur in materials exhibiting strong thermo-optic effects. For illustration, we explore thermal bistability in two different passive systems, involving either a periodic array of Si ring resonators or parallel GaAs thin films separated by vacuum and exchanging heat in the near field. Such a scheme could prove useful for thermal memory devices operating with transition times ≲ hundreds of milliseconds.

Rapid progress in the synthesis and processing of materials at small length scales has created demand for understanding thermal phenomena in nanoscale systems. Recent interest in harnessing excess heat that is readily available at the nanoscale has culminated in several proposed thermal devices with various functionalities, including thermal rectifiers, thermal memory, thermal transistors, phononic logic gates, and phonon waveguides. In this paper, we propose a scheme to achieve thermal bistability based on the coupling between three or more optical resonances. Our approach complements and builds on recently proposed ideas in several ways, described further below.

A thermal, bistable system can be used as a memory device that stores thermal information by maintaining the temperature of the system in one of two or more possible states. Realizing such temperature bistability requires a nonequilibrium thermal circuit supporting multiple steady states. Such a circuit was first proposed several years ago based on the concept of negative differential thermal resistance (NDTR), which relies on the ability to achieve heat flux rates between objects that decrease with increasing temperature differences. While first proposed in a model system consisting of a lattice of one-dimensional nonlinear mechanical oscillators, recent implementations of NDTR have instead sought to exploit radiative energy transfer between slabs separated by nanometer gaps and heated to very high ≳ 1500K temperatures. Here, we propose a simple and experimentally feasible, all-optical scheme based on a system of three optical resonances that builds and expands on a recently proposed and related scheme which requires materials supporting metal-insulator phase-transitions.

Instead, our approach exploits common materials exhibiting strong thermo-optic effects and relies instead on thermal bistability induced by a resonant mechanism involving three optical resonances—microring cavities supporting travelling-wave resonances or polar–dielectric slabs supporting surface–propagating polaritonic resonances. This work extends previous studies of thermal rectification and NDTR through vacuum, and also parallels recent ideas based on exotic non-volatile memory systems, which have recently been proposed as viable alternatives to traditional electrostatic memory.

Thermal bistability in triply resonant structures.— We begin by briefly describing the main mechanism behind the proposed thermal bistability scheme, leaving quantitative predictions for later. Consider a system of three thermal bodies, shown schematically in Fig. 1(a), two of which are maintained at constant temperatures $T_h$ and $T_c$, with $T_h > T_c$, while the remaining body is thermally isolated from its surroundings and has variable temperature $T_0$. The hot and cold bodies exchange heat with the isolated body through flux rates $J_h$ and $J_c$, respectively, leading to a net heat influx $J_i = J_h - J_c$ and a steady-state temperature satisfying (neglecting losses due to thermal conduction) $\rho c_p V \frac{\partial T_h}{\partial t} = J_i = 0$, where $\rho, c_p, V$ are the density, specific heat capacity, and volume of the body, respectively. For typical heat transfer mechanisms such as conduction or radiation, the
heat flux between any two bodies increases with increasing temperature difference, leading to monotonic $J_{t}(T_0)$ and thereby giving rise to a single steady state, i.e. $J_{t}(T_0) = 0$, as illustrated in Fig. 1(a). As recently illustrated in Ref. [11], NDTR can be realized in the context of radiative heat transfer between bodies exhibiting significant thermo-optic effects: namely, by exploiting the monotonic increase in the frequency of planar resonances with increasing temperature. Here we extend this idea by considering a system of three bodies that support narrow and slightly detuned resonances of frequencies $\omega_j$, with $j \in \{h, 0, c\}$. Consider a situation under which $T_0 = T_c$ and $\omega_0 < \omega_c < \omega_h$. As the temperature $T_0$ is increased from $T_c \rightarrow T_h$, $\omega_0$ sweeps over the frequencies of both the hot and cold resonators, whose temperatures and frequencies are held fixed. As $\omega_0 \rightarrow \omega_c$, the two resonators exchange heat more effectively and hence experience larger overall heat loss, causing $J_t$ to decrease considerably. As $\omega_0$ moves past $\omega_c$ and approaches $\omega_h$, $J_t$ increases again due to increased coupling with the hot resonance, decreasing with increasing $\omega_0$ as it moves past $\omega_h$. Thus, if properly engineered, such a system can lead to three steady states, consistent with zero net heat exchange ($J_t = 0$). Such a situation is illustrated on the right half of Fig. 1(b), wherein the intermediate state (red dot) is unstable while the remaining two (blue dots) are stable, i.e. $\frac{\partial J_t}{\partial \omega_0} < 0$. If, on the other hand, the initial configuration is such that $\omega_0 < \omega_h < \omega_c$ when $T_0 = T_c$, similar arguments imply the existence of a single steady state, as illustrated in Fig. 1(c). While this NDTR scheme can be generalized to any system of resonances, below we consider and quantify the feasibility of observing thermal bistability using this scheme in realizations based on Si photonic resonators and GaAs thin films exchanging heat in the near field. Note that non-linear thermo-optic effects in driven photonic resonators have been shown to lead to optical bistability[24], but their use as ultrafast optical memory devalues their potential as a slow thermal memory. In this work, we focus on passive systems in line with previous implementations of thermal memory, in which case no optical driving mechanisms are employed.

**Ring resonators:** We first focus on a system of Si ring resonators[21-23] which exhibit both large thermo-optic coefficients and long-lived resonances at mid infrared wavelengths $\sim$ peak thermal wavelength $\lambda_T \sim 10 \mu$m. In particular, we consider three one dimensional arrays of ring resonators shown in Fig. 2(a) with period $\Lambda$, two of which are maintained at fixed $T_h = 800K$ (left) and $T_c = 300K$ (right) while the middle one is suspended on insulating posts and has variable temperature $T_0$. We ignore the negligible interactions between adjacent rings along the array ensured by a sufficiently large $\Lambda$ and obtain the flux rates by considering heat exchange for three coupled resonators shown schematically in the top inset of Fig. 2(a). Such a simplified system can be described via the temporal coupled-mode theory framework[23-25] which provides accurate predictions while circumventing the need for numerically intensive calculations[26]. In this framework, the resonances are described by mode amplitudes $a_j$, normalized such that $|a_j|^2$ are mode energies[26] and have frequencies $\omega_j$ and decay/loss rates $\gamma_j$, where $j = \{h, c, 0\}$. They are subject to thermal noise sources $\xi_j$ described by delta-correlated, Gaussian noise terms satisfying $\langle \xi_j^*(\omega)\xi_j^*(\omega') \rangle = \delta(\omega - \omega') \Theta(\omega_j, T_j)$ where $\langle \cdots \rangle$ denotes the statistical ensemble average and $\Theta(\omega, T) = h\omega/(e^{h\omega/kT} - 1)$ is the Planck function. The three resonators are coupled to one another via coupling coefficients $\kappa_{hc}$ and $\kappa_{ec}$, allowing heat to flow from the hot to the cold resonator as described by the following
coupled-mode equations:

\[ \frac{d a_h}{d t} = i \omega_h a_h - \gamma_h a_h + i \kappa_h a_0 + \sqrt{2 \gamma_h} \xi_h \]  
(1)

\[ \frac{d a_0}{d t} = i \omega_0 (T_0) a_0 - \gamma_0 a_0 + i \kappa_0 a_h + i \kappa_c a_c + \sqrt{2 \gamma_0} \xi_0 \]  
(2)

\[ \frac{d a_c}{d t} = i \omega_c a_c - \gamma_c a_c + i \kappa_c a_0 + \sqrt{2 \gamma_c} \xi_c. \]  
(3)

Here, \( \omega_0 \) depends on the local resonator temperature through the thermo-optic effect \[^{21}\] with \( \omega_0 (T_0) \approx \omega_0 (T_c) + \frac{\partial n}{\partial T} (T_0 - T_c) \), where \( n \) is the effective refractive index and \( \frac{\partial n}{\partial T} \) the thermo-optic coefficient of the resonator. It follows that the spectral flux densities associated with the coupled modes, \( \Phi_{h/c} = 2 \text{Im} \{ \kappa_{h/c} / a_0^* a_0 \} \), are given by:

\[ \Phi_h(\omega) = \frac{4 \kappa_h^2 \omega_h (\kappa_h^2 \gamma_c \Theta_{hc}(\omega) + \gamma_0 |D_h(\omega)|^2 \Theta_{h0}(\omega))}{|D_h(\omega)|^2 |D_0(\omega)|^2 + |D_c(\omega)|^2} \]  
(4)

\[ \Phi_c(\omega) = \frac{4 \kappa_c^2 \gamma_c (\kappa_h^2 \gamma_c \Theta_{hc}(\omega) + \gamma_0 |D_h(\omega)|^2 \Theta_{h0}(\omega))}{|D_h(\omega)|^2 |D_0(\omega)|^2 + |D_c(\omega)|^2} \]  
(5)

where \( \Theta_{jk}(\omega) = \Theta(\omega, T_j) - \Theta(\omega, T_k) \) and \( D_j(\omega) = i(\omega - \omega_j) + \gamma_j, \) for \( j, k \in \{ h, 0, c \} \), and

\[ D_0(\omega) = D_0(\omega) + \frac{\kappa_h^2}{D_h(\omega)} + \frac{\kappa_c^2}{D_c(\omega)} \]  

The net flux rates per unit length, \( J_{h/c} = \frac{1}{\lambda} \int \Phi_{h/c}(\omega) \frac{d\omega}{2\pi} \), are obtained by integrating over all frequencies.

As an illustration, we consider rings of radii \( R_j = \frac{a_j}{2\omega_j} \) for \( j \in \{ h, 0, c \} \), designed to support resonances at \( \omega_c = 3.62 \times 10^{14} \text{rad/s} (5.2 \mu \text{m}) \), \( \omega_h = 7.2 \gamma_c \), and \( \omega_0 (T_c) = \omega_c - 2 \gamma_c, \) with equal decay rates \( \gamma_j = \omega_c / 500. \) Material properties are thermo-optic coefficient \( \frac{\partial n}{\partial T} = 2 \times 10^{-4} \text{K}^{-1} \) and effective refractive index \( n = 3.42. \) With ring radii \( R_j \sim 0.25 \mu \text{m} \), lattice period \( \Lambda = 1 \mu \text{m} \) is chosen to ignore the interactions between neighboring rings along the array and the arrays are placed such that the coupling rates \( \kappa_h = 0.9 \gamma_c \) and \( \kappa_c = 2 \gamma_c. \) Figure 2(b) shows the flux rates \( J_h \) and \( J_c \) per unit length as a function of the temperature \( T_0 \) of the middle resonator. The net flux entering/leaving the ring \( J_l \) shown in Fig. 2(c) leads to two stable steady states at \( T_{s1} = 413 K \) and \( T_{s2} = 700 K \) (blue dots), along with an unstable state at \( T_u = 600 K \) (red dot). Here, we ignore radiative decay into the surroundings as well as conductive losses into the mechanically supporting structures. These extraneous channels of heat transfer can be suppressed by suspending the middle rings on thermally insulating posts to reduce conductive losses (see bottom schematic) while also operating under vacuum to eliminate conductive/convective heat transfer through air, as discussed in Ref. \[^{27}\]. Apart from stability against temperature perturbations, guaranteed here by large temperature gaps between steady states, robustness against flux perturbations will generally depend on the flux barrier and hence net magnetization of the flux rates \( \sim \Theta(\omega, T) / Q, \) guaranteed here by operating with large wavelengths and relatively small \( Q \).

Figure 2(d) illustrates the relaxation of \( T_0 \) from \( T_h, T_c, T_{h+}, T_{c+} \) to the nearest stable steady states \( T_{s1}, T_{s2}, \) assuming \( V \sim 0.1 \mu \text{m}^3 \) where \( V \) is the volume of the middle ring and the temperature-dependent values of \( c_p \) and \( \rho \) given in Ref. \[^{28}\]. While the relaxation time can be increased arbitrarily by setting the initial condition close to \( T_u \), we estimate the characteristic “thermal memory” timescale as the maximum time it takes the middle ring to reach the stable steady states when its starting temperature is taken to be that of either the hot or cold resonators, which are 0.1s and 1s, respectively.

Compared to previous implementations based on phase-transition materials \[^{9,10}\] the transition times achieved here are of the same order of magnitude while the range of operating temperatures is wider by an order of magnitude. While the relaxation process can in principle be hastened by exploiting large thermo-optic coefficients and/or larger \( Q \), thus decreasing the operating temperature range, the former are constrained by material choices while the latter lead to decreased flux rates. Aside from careful engineering of the coupling rates and resonator frequencies needed to achieve bistability, a thermal memory based on this setup requires good thermal insulation and suitable choice of materials exhibiting large thermo-optic coefficients for speed and improved stability (reliability).

Thin films: One possible way to increase the speed of such a thermal memory device is to exploit planar polarization materials, which offer enhanced heat flux rates owing to the large number of surface localized resonances they can support. In what follows, we consider one such example, shown schematically in Fig. 3(a), consisting of three GaAs thin films exhibiting heat radiatively in the near field, where the hot and cold films are again held at fixed temperatures \( T_h \) and \( T_c \), while the intermediate film is thermally insulated from its surroundings and hence described by a variable temperature \( T_u \). Such a three-body planar configuration has been studied previously using scattering formulations, with the various heat fluxes obtained through straightforward calculation of the reflection/transmission matrices in this geometry \[^{26}\], as described in detail in Ref. \[^{30}\]. Here, we exploit this approach to consider a full calculation of the flux rates that includes thermo-optic effects in GaAs, obtained from Ref. \[^{31}\] assuming operating temperatures \( T_h = 1100 K \) and \( T_c = 300 K \), 200nm films, and vacuum gaps of \( d_h = 48 \text{nm} \) and \( d_c = 45 \text{nm}. \)

Figure 3(b) shows the computed flux rates per unit area, \( J_h \) and \( J_c \), as a function of \( T_u \), while the net flux entering/leaving the middle film \( J_l \) is shown in Fig. 3(c). As before, the thermo-optic induced NDTR results in two stable steady states at \( T_{s1} = 430 K \) and \( T_{s2} = 900 K \) (blue dots) along with an unstable steady state at \( T_u = 710 K \) (red dot). Figure 3(d) illustrates the relaxation of \( T_0 \) from \( T_h, T_c, T_{h+}, T_{c+} \) to the nearest stable steady states, which is substantially decreased compared...
Another recent work proposed a nanothermomechanical memory where NDTR is achieved at very high temperatures $T \sim 1100\,K$ by exploiting the nonmonotonic dependence of near field heat transfer on the separation between two planar slabs, as actuated by the thermal expansion of a mechanical support. The three-body system explored in this work relaxes some of these stringent operating conditions, allowing for a wide range of operating temperatures (steady state temperatures $\lesssim 1000\,K$) and flux rates. Furthermore, our proposed scheme also offers flexibility with respect to material choices in that it does not rely on phase-change materials and could be realized with a wide range of materials exhibiting strong thermooptic effects, such as chalcogenide glasses, silica, and silicon carbide, among many others. While thermal memory devices based on phase-transition materials offer a smaller operating temperature range (close to room temperature in case of vanadium dioxide) and have also been shown to lead to multistability, our three-body scheme leads to wider temperature differences between the steady states, thereby guaranteeing stability against temperature and flux perturbations.

Concluding remarks: We demonstrated a simple scheme to realize temperature bistability in all-passive systems comprising multiple coupled resonant modes. We provided concrete predictions of expected operating conditions (including transition times of several hundred milliseconds) in realistic designs involving either suspended Si ring resonators or GaAs thin films. Since the underlying mechanism is very general and not restricted to the proposed implementations, one possible direction forward could be to explore other geometries such as nanobeam resonators, multilayered thin films, nanostructured materials, and different choices of materials, where one could potentially observe larger heat exchange. With rapidly advancing nanotechnology, the understanding of this and related thermal phenomena could be important for nanoscale heat management.

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