Near Space Hypersonic Vehicle Target Tracking
Adaptive Non-Zero Mean Model

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ABSTRACT As a typical maneuver model, the second-order Markov model, which is based on acceleration periodic autocorrelation, is an effective way used for tracking near space hypersonic vehicle (NSHV) target tracking. It is found that, however, the model responds slowly to the target maneuvering and has weak maneuverability. To solve this problem, the adaptive non-zero mean damped oscillation model (ANM-DO), which based on the idea of mean compensation, is proposed. Then the difference of the mean compensation method between the first order Markov model and the two order Markov model is analyzed, and the physical essence of adaptive nonzero mean is discussed from time domain and frequency domain. Furthermore, to further investigate the performance of the ANM-DO model, we deduced the systematic dynamic errors of ANM-DO taking Kalman Filter (KF) as filtering algorithm. On this basis, the superior performance of ANM-DO model is verified in terms of maneuverability. Finally, simulation experiments in different scenarios show that the ANM-DO model shows lower filtering errors tracking near space hypersonic jump gliding targets, and verified the adaptability of the model proposed in this paper.

INDEX TERMS Maneuver model, target track, Markov model, Kalman filter.

I. INTRODUCTION
As a new type of cross-age weapon, the near space hypersonic vehicle (NSHV) weapon has the ability to change the rules of war at the strategic or tactical level. NSHV is different from conventional aerodynamic targets (aircraft, drones, etc.) or ballistic targets. It has high speed, high maneuverability, and electromagnetic Stealth and other advantages have brought huge challenges to the detection and tracking of such targets.

As we all known, the tracking algorithm of NSHV targets has always been focused on. Broadly speaking, target tracking mainly includes two parts: tracking model and filtering algorithm. The key to target tracking is to optimally extract useful information about the target motion state from the measurement information. The tracking model determines the trend and direction of tracking filtering, and most of the existing tracking algorithms are model-based [1], [2]. Up to now, the maneuvering target tracking model is divided into two major categories of ballistic models and dynamics models [1], [3]. The ballistic model was first applied to the tracking of ballistic targets and the prediction of landing points [5], this model analyzes the acceleration generated by different forces [6]. It can be further divided into reentry model and boost phase model according to the forces in different stages of ballistic trajectory, and it is suitable for target tracking with relatively simple forces [7], [8]. While the NSHV is unable to realize transient strong maneuvers due to its high speed, and its control volume is relatively simple in actual guidance and control design [9], [10]. Therefore, the attack angle and the inclination angle model are usually described by simple functions. Moreover, the NSHV can glide in near space for a long period of time, and the force is relatively simple and stable (as opposed to an on-line control target such as an aircraft). The ballistic model can also be applied to near space hypersonic vehicles (NSHV) target tracking due to above factors [11], [12]. The advantage of the ballistic model is that the model contains a large amount of information and can obtain a variety of aerodynamic and kinematic parameters, and the tracking accuracy is high when the model is accurate.
However, it requires more prior information, and the model will be seriously degraded when the priori hypothesis does not match the actual one. The current research on the NSHV jumping gliding target tracking ballistic model focuses on the acceleration expression, identification of guidance and control parameters, etc. [13], [14].

Besides the ballistics model, the dynamic model is another important development direction of the maneuvering model. The dynamics model is the most traditional tracking model. It can be applied to almost all tracking of maneuvering targets (including ballistic targets, on-line control targets such as airplanes). This model directly describes the target motion from the perspective of acceleration, jerk and so on, by a reasonable random process assumption [15]. Furthermore, the dynamic models are divided into two categories: white noise model and colored noise model according to different random process assumptions [16]. The white noise model describes the maneuvering characteristics of the target through Gaussian white noise. And the common white noise models include Constant velocity (CV) model, Constant acceleration (CA) model, and Coordinate turn (CT) model. The white noise model has a simple structure and fast calculation speed. However, it is too simple to characterize the target, so it is not suitable for target tracking with complex motion and strong maneuver. To solve that, the colored noise model was proposed. This model holds that there is a certain correlation between the target maneuvers, and the acceleration is assumed to be a Markov process with specific autocorrelation properties. Moreover, it includes a first-order Markov model and a second-order Markov model according to different orders. The classic first-order Markov models such as the Singer, current statistical (CS) and Jerk model. While the above three first-order Markov models assume that the acceleration correlation decays exponentially, and there are limitations in the description of the periodic motion characteristics of the NSHV [17], [18]. In response to this problem, Wang et al. [19] considered that the NSHV acceleration correlation completely obeys sine/cosine function, and proposes an acceleration sine-wave correlation model (SW). The SW model has a greater influence on the field of NSHV maneuver model. However, we believe that NSHV acceleration has both periodicity and attenuation, and in this way, a more general second-order Markov model is proposed - the damping oscillation model (DO) [1], [4]. The SW and DO model can describe the NSHV’s motion characteristics more accurately, but they have common shortcomings that the target maneuver response has a long delay and the maneuverability is weak.

In order to improve the accuracy of the model's representation of the target motion, we propose an adaptive non-zero mean form based on the DO model. Under the condition of ensuring that the model has both periodic and decaying acceleration autocorrelation assumptions, the maneuverability and convergence speed of the model are further improved. The core of the ANM-DO model is that the acceleration correlation is the same as that of DO (with periodicity and attenuation at the same time), but the acceleration distribution is not a random process with zero mean, so the acceleration needs to be averaged during the modeling process. On this basis, we deduced the ANM-DO model, and verified that the state transition matrix of the model is consistent with the constant jerk motion. Moreover, to further analyze the performance of the model, the system dynamic error of the tracking model was deduced taking KF as filtering algorithm [21], [22]. Through the above-mentioned time domain and frequency domain analysis, we reveal the physical meaning of the adaptive non-zero mean, and verify that ANM-DO has a stronger ability to adapt to the motion of the DO model. Finally, we conducted a series of simulations to verify the validity of the model.

To solve this problem, Li and Jilkov [20] pointed out that the performance of adaptive non-zero mean model is generally better than zero-mean model, and it is more adaptable. In this paper, on the basis of the DO model, the adaptive non-zero mean damped oscillation model (ANM-DO) is proposed to track the NSHV jumping gliding target. Firstly, the ANM-DO is constructed as the modeling mechanism. And then we analyzed the difference of the mean compensation method between the first order Markov and the second Markov model. the adaptability of DO and ANM-DO models to different movements was discussed. Finally, we conducted a series of simulations to verify the validity of the model.

II. GUIDELINES FOR MANUSCRIPT PREPARATION

A. ANM-DO CONTINUOUS TIME MODEL

The key to the establishment of the maneuver model is the design of autocorrelation function for acceleration. And it is designed as exponential function, such as the Singer, CS, SCT and Jerk model. However, it is well known in reality for NSHV that acceleration along the height coordinate direction is oscillatory. The model constructed based on the exponential function is not very suitable for such acceleration. To solve this problem, Wang et al. [19] proposed the SW model based on the longitudinal motion characteristics of the NSHV gliding target. And the SW model thinks that the autocorrelation function of the acceleration obeys the periodicity, this assumption can describe the periodic characteristics of NSHV better. But it is too ideal that the correlation of acceleration completely conforms to the law of positivity/cosine variation.

According to the NSHV motion characteristics, it can be seen that the NSHV acceleration-related properties should embody periodicity and attenuation. Therefore, we believe that attenuation is the basic law of general affairs, and assuming that the target acceleration self-correlation is a zero-mean stochastic process that simultaneously exhibits periodicity and attenuation. On the basis of this assumption, the acceleration autocorrelation of the DO model may be described as the damped oscillation function as Eq.(1) [4]. And for clarity and convenience of brevity some key notations
TABLE 1. Symbolic description.

| Symbol   | Definition                                                                 | Symbol   | Definition               |
|----------|-----------------------------------------------------------------------------|----------|--------------------------|
| \( a(t) \) / \( \dot{a}(t) \) / \( \ddot{a}(t) \) | Target acceleration/ target jerk/second derivative of acceleration       | \( \alpha \) | Damping coefficient       |
| \( a_0(t) \) | Acceleration disturbance at time \( t \)                               | \( \delta(\cdot) \) | The Dirac function     |
| \( \bar{a}(t) \) / \( \dot{\bar{a}}(t) \) / \( \ddot{\bar{a}}(t) \) | Target mean acceleration/ target mean jerk/second derivative of mean acceleration at time \( t \) | \( Z(t) \) | Measurement vector at time \( t \) |
| \( R_{a_0}(\tau) \) | Acceleration autocorrelation function                                       | \( H(t) \) | Measurement matrix at time \( t \) |
| \( \tau \) | Time difference                                                              | \( V(t) \) | Measurement noise at time \( t \) |
| \( \Delta T_e \) | Target oscilulation period                                                   | \( s \) | The Laplace operator     |
| \( \sigma^2_{a_0} \) | Acceleration variance                                                       | \( I \) | The unit matrix          |
| \( \omega(t) \) | Mean zero Gaussian white noise at time \( t \)                              | \( \bar{X}(t) \) | Target state vector \( k \) including position, velocity, acceleration and jerk |
| \( \bar{X}(t) \) | Target state vector \( t \) including position, velocity, acceleration and jerk | \( \dot{\bar{X}}(t) \) | Tracking sampling interval |
| \( F(T,\alpha,\beta) \) | State transition matrix                                                     | \( \tilde{X}(t) \) | Target state estimation \( t \) including position, velocity, acceleration and jerk |
| \( T \) | Tracking sampling interval                                                  | \( K(t) \) | KF gain matrix at time \( t \) |
| \( X(k) \) | Target state vector at time \( k \) including position, velocity, acceleration and jerk | \( \tilde{X}_d(s) \) | Dynamic estimator vector including position, velocity, acceleration and jerk |
| \( W(k) \) | Process noise at time \( k \)                                               | \( \tilde{X}_e(s) \) | Error estimator vector including position, velocity, acceleration and jerk |
| \( \Phi(T) \) | State transition matrix of the ANM-DO model                                | \( \sigma^2_e \) | Distance error square    |
| \( Q(k) \) | The covariance of the process noise at time \( k \)                       | \( \sigma^2_o \) | Elevation error square   |
| \( a_{\text{max}} \) | Maximum acceleration                                                       | \( \omega(t) \) |                                    |

and instructions are listed in TABLE 1.

\[
R_{a_0}(\tau) = \sigma^2_{a_0} e^{\alpha |\tau|} \cos(\beta \tau) \tag{1}
\]

We can get the differential equation of Eq.(1).

\[
\ddot{a}(t) + 2a\dot{a}(t) + \left(\alpha^2 + \beta^2\right) a(t) = \dot{\omega}(t) + \sqrt{\alpha^2 + \beta^2} \omega(t) \tag{2}
\]

The essence of the non-zero mean model is to introduce the acceleration mean so that the acceleration distribution moves with the mean. This paper constructs the ANM-DO model based on the DO model. Assume that the target acceleration is a combination of the mean acceleration and the associated disturbance:

\[
a(t) = \bar{a}(t) + a_0(t) \tag{3}
\]

Non-zero mean time correlation model was constructed based on Eq.(3).

Substituting Eq.(3) into Eq.(2) on the basis of the DO model, the acceleration differential equation of the ANM-DO model is:

\[
\ddot{a}(t) + 2a\dot{a}(t) + \left(\alpha^2 + \beta^2\right) a(t) - \bar{a}(t) - 2a\dot{\bar{a}}(t) - \left(\alpha^2 + \beta^2\right) \ddot{\bar{a}}(t) = \dot{\omega}(t) + \sqrt{\alpha^2 + \beta^2} \omega(t) \tag{4}
\]

The continuous time state equation of the ANM-DO model is:

\[
\frac{d}{dt} \begin{bmatrix} x(t) \\ \dot{x}(t) \\ \ddot{x}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \\ \ddot{x}(t) \end{bmatrix} + \begin{bmatrix} -\alpha^2 - \beta^2 \\ 2\alpha \end{bmatrix} \dot{a}(t) + \begin{bmatrix} \alpha^2 - \beta^2 \\ 0 \end{bmatrix} \ddot{a}(t) + \begin{bmatrix} 1 \\ \sqrt{\alpha^2 + \beta^2} \end{bmatrix} \omega(t) \tag{5}
\]

Eq.(5) can be further written as follow:

\[
\dot{X}(t) = A_{\text{ANM-DO}} X(t) + U_1 \dot{a}(t) + U_2 \ddot{a}(t) + U_3 \dddot{a}(t) + B \omega(t) \tag{6}
\]

B. ACCELERATION MEAN COMPENSATION

According to Eq.(6), we can get the state transition matrix as follow.

\[
F(T,\alpha,\beta) = e^{A_{\text{ANM-DO}} T} = \begin{bmatrix} 1 & T & f_1(T) & f_2(T) \\ 0 & 1 & f_3(T) & f_4(T) \\ 0 & 0 & f_5(T) & f_6(T) \\ 0 & 0 & f_7(T) & f_8(T) \end{bmatrix}

f_1(T) = \frac{\left(3\alpha^2 - 3\beta^2\right) \sin \beta t + \left(3\alpha^2 \beta - \beta^3\right) \cos \beta t}{\beta \left(\alpha^2 + \beta^2\right)^2}
\]
The derivation of Eq.(5) implies a condition that part of the mean compensation part, that is discrete form of

\[
\begin{align*}
\dot{\bar{a}} (t) &= \dot{\bar{a}} (t) + \int_0^T \dot{\bar{a}} (k) \, dk, \\
\bar{a} [(k + 1) T] &= \bar{a} (k T) + \int_0^T \dot{\bar{a}} (k) \, dk \\
\end{align*}
\]

Since \( \dot{\bar{a}} (\xi) \) is not constant to zero, the compensation effect of \( \dot{\bar{a}} (\xi) \) on \( \bar{a} [(k + 1) T] \) must be considered when the mean is compensated. However, there is no \( \bar{a} (\xi) \) mean value in the CS model, and it is the reason that the CS model mean compensation method cannot be used directly in the second-order Markov model.

\[
\begin{align*}
\bar{U}_1 (T) \dot{\bar{a}} (k) + \bar{U}_2 (T) \dot{\bar{a}} (k) + \bar{U}_3 (T) \dot{\bar{a}} (k) \\
= \int_0^{(k+1)T} \bar{U}_1 (\xi) \dot{\bar{a}} (k) \, d\xi \\
\end{align*}
\]

\[
\begin{align*}
\int_0^{(k+1)T} \bar{U}_1 (\xi) \dot{\bar{a}} (k) \, d\xi \\
\int_0^{(k+1)T} \bar{U}_2 (\xi) \dot{\bar{a}} (k) \, d\xi \\
\int_0^{(k+1)T} \bar{U}_3 (\xi) \dot{\bar{a}} (k) \, d\xi \\
= \bar{U}_1 (T) \dot{\bar{a}} (k) + \bar{U}_2 (T) \dot{\bar{a}} (k) + \bar{U}_3 (T) \dot{\bar{a}} (k) \\
\end{align*}
\]

In the Eq.(10), part I is the discretization of the mean value of acceleration \( \bar{U}_1 \dot{\bar{a}} (t) \). And part II is the \( \dot{\bar{a}} (t) \) compensation for the \( \bar{a} (t) \). Part III is the discretization of the \( \dot{\bar{a}} (t) \). Part IV is the discretization of \( U_3 \bar{a} (t) \), and the result is zero. And the part I is:

\[
\begin{align*}
\bar{U}_1 (T) \dot{\bar{a}} (k) \\
= \int_0^{(k+1)T} \bar{U}_1 (\xi) \dot{\bar{a}} (k) \, d\xi \\
= \left[ \bar{a}^{1} (T), \bar{a}^{2} (T), \bar{a}^{3} (T), \bar{a}^{4} (T) \right]_{-0}^{T} \dot{\bar{a}} (k) \\
\end{align*}
\]

\[
\begin{align*}
\left\{ \\
\bar{u}_1^{1} (T) = \left[ \begin{array}{c}
-\alpha^3 + 3\alpha^2 \beta^2 \, e^{-\alpha^2 T} \sin \beta T \\
-\alpha^2 \beta - \beta^3 \, e^{-\alpha^2 T} \cos \beta T \\
2 \alpha \beta \left( \alpha^2 + \beta^2 \right) T^2 - 2 \alpha \beta T + \frac{3 \alpha^2 \beta - \beta^3}{\alpha^2 + \beta^2} \\
\end{array} \right] \\
\bar{u}_1^{2} (T) = \left[ \begin{array}{c}
\alpha^2 - \beta^2 \, e^{-\alpha^2 T} \sin \beta T \\
\beta \left( \alpha^2 + \beta^2 \right) T - 2 \alpha \beta \left( \alpha^2 + \beta^2 \right) \\
- \alpha e^{-\alpha^2 T} \sin \beta T \\
\end{array} \right] \\
\bar{u}_1^{4} (T) = \left[ \begin{array}{c}
\frac{\alpha^2 + \beta^2}{\beta} e^{-\alpha^2 T} \sin \beta T \\
\beta e^{-\alpha^2 T} \sin \beta T + 2 \alpha \beta \, e^{-\alpha^2 T} \cos \beta T + T^2 \\
\end{array} \right] \\
\end{align*}
\]

The part II is:

\[
\begin{align*}
\int_0^{(k+1)T-\xi} \bar{U}_1 (\xi) \dot{\bar{a}} (k) \\
= \int_0^{(k+1)T-\xi} \bar{U}_1 (\xi) \dot{\bar{a}} (k) \, d\xi \\
= \left[ \bar{u}_1^{1} (T), \bar{u}_1^{2} (T), \bar{u}_1^{3} (T), \bar{u}_1^{4} (T) \right]_{-0}^{T} \dot{\bar{a}} (k) \, d\xi \\
= \frac{\dot{\bar{a}} (k)}{\beta \left( \alpha^2 + \beta^2 \right)} \left[ \bar{u}_1^{1} (T), \bar{u}_1^{2} (T), \bar{u}_1^{3} (T), \bar{u}_1^{4} (T) \right] \\
\end{align*}
\]
where

\[
\begin{align*}
\dot{u}_1^2 (T) &= \alpha^4 - 6\alpha^2\beta^2 + \beta^4 e^{-\alpha T} \sin \beta T \\
&+ \frac{4\alpha\beta (\alpha^2 - \beta^2)}{(\alpha^2 + \beta^2)^2} e^{-\alpha T} \cos \beta T \\
&+ \frac{\beta (\alpha^2 + \beta^2)}{\alpha^2 + \beta^2} T - \alpha T^2 \\
&+ \frac{3\alpha^2\beta - \beta^3}{\alpha^2 + \beta^2} T + \frac{4\alpha\beta (\alpha^2 - \beta^2)}{(\alpha^2 + \beta^2)^2} e^{-\alpha T} \\
&\times \cos \beta T + \frac{\beta (\alpha^2 + \beta^2)}{\alpha^2 + \beta^2} T^2 - 2\alpha\beta T + \frac{3\alpha^2\beta - \beta^3}{\alpha^2 + \beta^2} T^2 \\
&\times e^{-\alpha T} \cos \beta T + \frac{2}{\alpha^2 + \beta^2} e^{-\alpha T} \sin \beta T \\
&+ \frac{2}{\alpha^2 + \beta^2} e^{-\alpha T} \sin \beta T \\
&+ 2\alpha\beta e^{-\alpha T} \cos \beta T + 0T^3 + \beta (\alpha^2 + \beta^2) T - 2\alpha\beta \\
&\text{(14)}
\end{align*}
\]

The part III is:

\[
\begin{align*}
U_2^A (T) \hat{a} (k) &= \int_{\xi \in \mathbb{R}} e^{A_{\text{ANM-DO}}(k+1)T - \xi} U_2 (\xi) \hat{a} (k) \, d\xi \\
&= \frac{2\alpha}{\alpha^2 + \beta^2} \begin{bmatrix} \hat{u}_1^4 (T), \hat{u}_1^2 (T), \hat{u}_1^3 (T), \hat{u}_1^4 (T) \end{bmatrix}^T \hat{a} (k) \\
&\text{(15)}
\end{align*}
\]

From the above three parts we can get the ANM-DO model discrete-time state prediction:

\[
\begin{align*}
\hat{X} (k+1|k) &= F (T, \alpha, \beta) \hat{X} (k|k) + U_1^A (T) \hat{a} (k) \\
&\quad + U_2^A (T) \hat{a} (k) \\
&\text{(16)}
\end{align*}
\]

where \( \hat{a} (k) = \hat{\xi} (k|k), \hat{a} (k) = \hat{\xi} (k|k) \)

\[
\begin{bmatrix}
\dot{\hat{x}} (k+1|k) \\
\dot{\hat{\xi}} (k+1|k) \\
\dot{\hat{x}} (k+1|k) \\
\dot{\hat{\xi}} (k+1|k)
\end{bmatrix}
= \begin{bmatrix}
\dot{\hat{x}} (k|k) \\
\dot{\hat{\xi}} (k|k) + T \hat{\xi} (k|k) \\
\dot{\hat{x}} (k|k) + T \hat{\xi} (k|k) + \frac{T}{2} \hat{\xi}^2 (k|k) \\
\dot{\hat{x}} (k|k) + T \hat{\xi} (k|k) + \frac{T}{2} T \hat{\xi}^2 (k|k) + \frac{1}{8} T^3 \hat{\xi}^3 (k|k)
\end{bmatrix}
\]

\[
\text{(17)}
\]

According to Eq.(17) that the ANM-DO model state prediction is:

\[
\begin{bmatrix}
\dot{\hat{x}} (k+1|k) \\
\dot{\hat{x}} (k+1|k) \\
\dot{\hat{x}} (k+1|k) \\
\dot{\hat{x}} (k+1|k)
\end{bmatrix}
= \Phi (T) \hat{X} (k|k)
\]

\[
\text{(18)}
\]

The physical nature of the adaptive non-zero mean model is shown in Eq.(18), and the state prediction of the ANM-DO model becomes a basic constant jerk model. The constant jerk model is abbreviated as a CJ model comparing to the CA model.

**C. ADAPTIVE PROCESS NOISE COVARIANCE**

See Literature [4] for the detailed solution process of \( Q (k) \). However, the calculation of \( \sigma^2_{a_0} \) in \( Q (k) \) is also very important. Usually, in the zero-mean model such as Singer and Jerk model, a fixed acceleration or jerk variance is set, and most of the tracking algorithms based on non-zero mean model adopt the acceleration variance form of the CS model. This paper also uses this idea to achieve the adaptation of \( Q (k) \).

\[
\sigma^2_{a_0} (k) = \frac{4 - \pi}{\pi} [a_{max} - |a (k)|]^2 
\]

\[
\text{(19)}
\]

The characteristic of the Eq.(19) is that when the target is approximately uniform motion, the difference between \( a_{max} \) and \( |a (k)| \) is large, and a large bandwidth is reserved for the filtering system, but the filtering efficiency is low; When the target is maneuver \( a_{max} \) and \( |a (k)| \) are close, the system bandwidth becomes small and the filtering efficiency is high. So, the Eq.(19) can reserve the maximum filtering bandwidth that the acceleration may change in real time, and can quickly respond to the target acceleration change.

**D. PHYSICAL MEANING OF MEAN COMPENSATION**

As mentioned before, the state prediction of the second-order Markov model after the mean compensation is consistent with the equation of motion of the CJ model. That is, the state transition matrix is only related to the time interval \( T \). It is well known that Markov time-correlated models, such as the first-order or the second-order Markov models. Their essence is the assumption of acceleration as a stationary random process with certain correlations. The physical meaning of mean compensation for the DO model, more precisely, in the time domain, this way ensures that the target state transition matrix is only affected by the sampling interval \( T \). In the frequency domain, it is represented as a priori hypothesis parameter \( \alpha, \beta \) cancellation in matrix \( A \), making it appear as a standard 0-1 matrix, so that the filter in the frequency domain is a linear system, and enhanced the ANM-DO model adapts to different maneuvering styles.

**III. MANEUVER MODEL ADAPTABILITY ANALYSIS**

The overall performance of the maneuvering target tracking algorithm depends on the interaction of the maneuver model and the filtering algorithm. In order to analyze the performance improvement of the ANM-DO model, the KF filter is used to verify advantage of the mean compensation. We assume the continuous time measurement equation can be written as [24]:

\[
Z (t) = H (t) X (t) + V (t)
\]

\[
\text{(20)}
\]
We can get the state estimate according to the KF algorithm:

\[
\dot{\hat{X}} (t) = A\dot{\hat{X}} (t) + K (t) \left[ Z (t) - \hat{Z} (t) \right]
\]

\[
= [A - K (t) H (t)] \dot{\hat{X}} (t) + K (t) Z (t)
\] (21)

As we all know the filter gain corrects the state prediction based on the residual information. With the KF gain, when the filter reaches a steady state, it can accurately describe the distribution information of the measurement noise, and it is close to constant value. For convenience, only one dimension measurement of position data is considered. Let \( H (t) = [1, 0, 0, 0]^T \), then the Laplace transform of Eq.(21) can be explained as follows (assuming the initial condition is zero):

\[
s\hat{X} (s) = (A - KH) \hat{X} (s) + KHZ (s)
\]

\[
\Rightarrow \dot{\hat{X}} (s) = \frac{(sE - A + KH)^{-1} KHZ (s)}{\dot{\hat{X}} (s)}
\]

\[
= \frac{(sE - A + KH)^{-1} KHX (s)}{\dot{\hat{X}} (s)} + \frac{(sE - A + KH)^{-1} KHL [V (t)]}{\dot{\hat{X}} (s)}
\] (22)

Therefore, the system error can be further written as:

\[
\hat{X} (s) = X (s) - \hat{X} (s)
\]

\[
= \left[ E - (sE - A + KH)^{-1} KH \right] X (s)
\]

\[
\Rightarrow \dot{\hat{X}} (s)
\]

\[
= \frac{(sE - A + KH)^{-1} KHX (s)}{X (s)} + \frac{(sE - A + KH)^{-1} KHL [V (t)]}{X (s)}
\] (23)

\( \hat{X} (s) \) and \( \dot{\hat{X}} (s) \) are the system dynamic error and system random error, respectively. Since the system random error cannot be controlled, so we put more attention to the system dynamic error of the DO model. In this case,

\[
KH = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \\ 0_{3 \times 4} \end{bmatrix}
\]

when the filter reaches steady state.

If the maneuver of the target is three different accelerations, including the unit pulse function, the unit step function, and the unit ramp function, the steady state values of the dynamic errors in the two model status updates can be obtained.

Case I: \( \ddot{x} (s) = 0, \dot{x} (s) = 1, x (s) = 1/s, x (s) = 1/s^2 \),

Case II: \( \ddot{x} (s) = 1, \dot{x} (s) = 1/s, \dot{x} (s) = 1/s^2, x (s) = 1/s^3 \),

Case III: \( \ddot{x} (s) = 1/2s, \dot{x} (s) = 1/2s^2, x (s) = 1/2s^3 \),

A. DO MODEL DYNAMIC ERROR STEADY STATE

Substituting \( A = \begin{bmatrix} 0_{4 \times 3} & f_{3 \times 3} \\ 0 & - (\alpha^2 + \beta^2) - 2\alpha \end{bmatrix} \) into \( \hat{X} (s) \). We get the corresponding system dynamic error of the DO model.

\[
\begin{bmatrix} \ddot{x}_d (s) \\ \dot{x}_d (s) \\ \dot{x}_d (s) \\ x_d (s) \end{bmatrix}
\]

\[
= \begin{bmatrix} s^4 + 2a s^3 + (a^2 + b^2)^2 s^2 \\ det(J - A + KH) \end{bmatrix} \\begin{bmatrix} x (s) \\ s^4 + (2a + k_1) s^3 + (a^2 + b^2 + 2k_1 a + k_2) s^2 + (2k_1 a + k_2) \end{bmatrix} \hat{x} (s)
\]

\[
= \begin{bmatrix} \ddot{x} (s) \\ \dot{x} (s) \\ x (s) \end{bmatrix}
\] (24)

where

\[
M_1 = s^4 + (2a + k_1) s^3 + (a^2 + b^2 + 2k_1 a + k_2) s^2 + (2k_1 a + k_2) \alpha + k_2 \beta
\]

\[
\det (sI - A + KH) = s^4 + (2a + k_1) s^3 + (a^2 + b^2 + 2k_1 a + k_2) s^2 + (2k_1 a + k_2) \alpha + k_2 \beta
\]

Then the system dynamic error steady state of the DO model is:

\[
\begin{bmatrix} \dot{x}_d (t) \vert_{t \to \infty} \\ \ddot{x}_d (t) \vert_{t \to \infty} \\ \dddot{x}_d (t) \vert_{t \to \infty} \\ \dddot{x}_d (t) \vert_{t \to \infty} \end{bmatrix}
\]

\[
= \begin{bmatrix} \lim_{s \to 0} s \cdot \frac{s^4 + 2a s^3 + (a^2 + b^2)^2 s^2}{det(J - A + KH)} \dddot{x} (s) \\ \lim_{s \to 0} s \cdot \frac{s^4 + (2a + k_1) s^3 + (a^2 + b^2) + 2k_1 \alpha + k_2 \beta s^2}{det(J - A + KH)} \dddot{x} (s) \\ \lim_{s \to 0} s \cdot \frac{s(M_1 + k_1 + 2k_2 \alpha + k_3 (a^2 + b^2)^2 \dddot{x} (s)}{det(J - A + KH)} \dddot{x} (s) \\ \lim_{s \to 0} s \cdot \frac{s(M_1 + k_1 + 2k_2 \alpha + k_3 (a^2 + b^2)}{det(J - A + KH)} \dddot{x} (s) \\ \lim_{s \to 0} s \cdot \frac{s(M_1 + k_1 + 2k_2 \alpha + k_3 (a^2 + b^2}}{det(J - A + KH)} \dddot{x} (s) \\ \lim_{s \to 0} s \cdot \frac{s(M_1 + k_1 + 2k_2 \alpha + k_3 (a^2 + b^2)}{det(J - A + KH)} \dddot{x} (s) \\ \lim_{s \to 0} s \cdot \frac{s(M_1 + k_1 + 2k_2 \alpha + k_3 (a^2 + b^2)}{det(J - A + KH)} \dddot{x} (s) \end{bmatrix}
\] (25)

According to the Eq.(25), for the Case I, the error steady state is 0. With case II and case III, it is difficult for the DO model to ensure that the steady state error of the state update approaches zero. This result also shows that there is a certain risk in the convergence of state update when using the DO models for Cases II and III. To solve this problem, one hand, the dynamic error size of the model can be limited by reasonable parameter values. On the other hand, the non-zero mean value model can alleviate this problem by mean compensation.

B. ANM-DO MODEL DYNAMIC ERROR STEADY STATE

The ANM-DO model state transition matrices shows the CJ form, then the equivalent continuous-time state differential
TABLE 2. Simulation parameters.

| Parameter                     | Value         |
|-------------------------------|---------------|
| NSHV gliding time            | 300 s         |
| NSHV oscillation period      | $\Delta T = 100s$ |
| Initial Ground range / height| 300km/21km    |
| Maximum cruising speed        | 5.2Ma         |
| Measurement distance error   | $\sigma_r = 100m$ |
| Elevation error              | $\sigma_e = 0.2^\circ$ |
| Sampling interval SW         | 0.1s          |
| DO/ANM-DO                    |               |
| SW                            | $\beta = 0.06\, rad/s$, $\sigma^2 = 10^3$ |
| DO                            | $\alpha = 1/300$, $\beta = 0.06\, rad/s$, $\sigma^2 = 10^2$, $a_{max} = 5g$ |
| Monte Carlo number           | 100           |

\[
\begin{bmatrix}
    \tilde{x}(t) \\
    \tilde{x}(t) \\
    \tilde{x}(t) \\
    \tilde{x}(t)
\end{bmatrix}_{\to \infty} = \begin{bmatrix}
    \lim_{s \to 0} s \cdot \frac{s^4 + s^3 + s^2 + s + 1}{s^4 + s^3 + s^2 + s + 1} \tilde{x}_d (s) \\
    \lim_{s \to 0} s \cdot \frac{s^4 + s^3 + s^2 + s + 1}{s^4 + s^3 + s^2 + s + 1} \tilde{\dot{x}}_d (s) \\
    \lim_{s \to 0} s \cdot \frac{s^4 + s^3 + s^2 + s + 1}{s^4 + s^3 + s^2 + s + 1} \tilde{\ddot{x}}_d (s) \\
    \lim_{s \to 0} s \cdot \frac{s^4 + s^3 + s^2 + s + 1}{s^4 + s^3 + s^2 + s + 1} \tilde{\dddot{x}}_d (s)
\end{bmatrix}
\]

\[
\frac{d}{dt} \begin{bmatrix}
    x (t) \\
    \dot{x} (t) \\
    \ddot{x} (t) \\
    \dddot{x} (t)
\end{bmatrix} = \begin{bmatrix}
    0_{3 \times 3} & I_{3 \times 3} \\
    0_{1 \times 4} & I_{1 \times 4}
\end{bmatrix} \begin{bmatrix}
    x (t) \\
    \dot{x} (t) \\
    \ddot{x} (t) \\
    \dddot{x} (t)
\end{bmatrix}
\]

(26)

It presents a linear system in the frequency domain by substituting $A = \begin{bmatrix}
    0_{3 \times 3} & I_{3 \times 3} \\
    0_{1 \times 4} & I_{1 \times 4}
\end{bmatrix}$ into Eq.(23). The steady-state dynamic error of the ANM-DO models is:

\[
\begin{bmatrix}
    \tilde{x}_d (t) \\
    \tilde{\dot{x}}_d (t) \\
    \tilde{\ddot{x}}_d (t) \\
    \tilde{\dddot{x}}_d (t)
\end{bmatrix}_{\to \infty} = \begin{bmatrix}
    \lim_{s \to 0} s \cdot \frac{s^4 + s^3 + s^2 + s + 1}{s^4 + s^3 + s^2 + s + 1} \tilde{x}_d (s) \\
    \lim_{s \to 0} s \cdot \frac{s^4 + s^3 + s^2 + s + 1}{s^4 + s^3 + s^2 + s + 1} \tilde{\dot{x}}_d (s) \\
    \lim_{s \to 0} s \cdot \frac{s^4 + s^3 + s^2 + s + 1}{s^4 + s^3 + s^2 + s + 1} \tilde{\ddot{x}}_d (s) \\
    \lim_{s \to 0} s \cdot \frac{s^4 + s^3 + s^2 + s + 1}{s^4 + s^3 + s^2 + s + 1} \tilde{\dddot{x}}_d (s)
\end{bmatrix}
\]

(27)

When the acceleration input is Case I, II and III, the steady state dynamic error of the ANM-DO models is always 0. This method of mean compensation eliminates the prior-correlation hypothesis parameters and then re-forms the form of CJ. It shows the increase in the number of zeros of the molecule in the dynamic error steady state, furthermore, it guarantees the convergence of three different acceleration inputs [24], [25].

However, we know that in the tracking filtering process, in addition to the state update, there is also the covariance update. The state update uses adaptive non-zero mean compensation to make the description of the target motion conform to the constant jerk motion, but the covariance is still updated through the state transition matrix before compensation, which leads to differences in the filter gains of different models. Moreover, the mean value of the adaptive non-zero mean value model refers to the statistical characteristics of the random process that can characterize the target motion acceleration or jerk change. But in the process of tracking filtering, the physical meaning of this parameter is not clear. We generally use the acceleration or jerk at the previous moment to describe this parameter. This is also an important reason for the differences between different models.

IV. SIMULATION RESULTS AND ANALYSIS

We set up two different scenarios and choose SW, DO and ANM-DO model to compare experimental results. In scenario I, using three different maneuver models to track NSHV jumping gliding targets to verify the superiority of tracking accuracy for the ANM-DO model. Moreover, in scenario II, to verify the maneuverability of ANM-DO models by tracking three typical motion.

A. SCENARIO I

The simulation parameters are shown in TABLE 2. The target periodic gliding trajectory is shown in FIGURE 1. And the filtering result is shown in FIGURE 2.

It has been verified in the literature [17], [18] that the SW and DO models with periodicity can more accurately describe the NSHV jumping gliding target motion characteristics, and the tracking accuracy is higher than that of the Singer and CS models.

However, the SW, DO and ANM-DO are compared in scenario I. In terms of positional errors, as shown in FIGURE 2(a), the error of the three models is slowly increasing, and the overall level is relatively stable. Among them, the SW error is the highest, and the fluctuation is more serious. Further, the error caused by the strong maneuver around 100s and 200s has the largest increase. The DO model considers acceleration-related attenuation and periodicity at the same time. Its overall error level is slightly better than SW, and the error fluctuation is smaller. The overall error level of ANM-DO is stable, the filtering error is slightly smaller than SW and DO. And the error suppression is better than the SW, DO model in strong maneuvering period.

In terms of speed error, as shown in FIGURE 2(b), the SW model has the highest error, its speed error rapidly increases during strong maneuvers and maintains a high error level.
state, and difficult to quickly converge to a lower level. The overall error of the DO model is lower, and there is still an increase in error when the maneuver is strong, but the error curve shows a declining trend after a rapid increase. The ANM-DO overall error is similar to that of the DO. However, in the strong maneuver period, non-zero mean model can quickly describe the target speed changes, corresponding, the error curve convergence rate is the fastest. In order to better compare the tracking performance of each model, the statistical average of position and velocity errors of all models is calculated as shown in TABLE 3. In terms of position estimation accuracy, the performance of ADO relative to the DO and SW models is improved by 4.8% and 16.3%, respectively, while the speed estimation accuracy performance is improved by 6.2% and 35.9% respectively.

(This article stipulates that the statistical average starts from the error convergence).

B. SCENARIO II

The simulation parameters are shown in TABLE 4.

TABLE 4. Simulation parameters.

| Parameter          | Value          |
|-------------------|----------------|
| Measurement distance error | $\sigma = 400m$  |
| Sampling interval SW | $0.1s$        |
| DO/ANM-DO | $\beta = 0.2\,rad/s$, $\sigma^2 = 10^3$ |
|                  | $\alpha = 1/60$, $\beta = 0.2\,rad/s$, $\sigma^2 = 10^3$, $a_{max} = 10g$ |

Previously, we analyzed the physical nature of the ANM-DO model from the time-domain state equations and the system dynamic errors in the frequency domain. And the non-zero mean model has better adaptability to target maneuvers. In simulation scenario II, three types of motion (one-dimensional trajectory) are set to verify the adaptability of the model in this paper: uniform acceleration, pulse acceleration, and sine acceleration. The acceleration of three kinds of movements is shown in FIGURE 3.

FIGURE 3. (a) is a constant acceleration motion (acceleration is $40\,m/s^2$, initial position, velocity, acceleration are $1000\,m$, $100\,m/s$, $0\,m/s^2$ respectively).

FIGURE 3. (b) shows the pulse acceleration motion (acceleration is $80\,m/s^2$, initial position, velocity and acceleration are $1000\,m$, $100\,m/s$, $0\,m/s^2$ respectively).

FIGURE 3. (c) is sine acceleration motion (magnitude is $60\,m/s^2$, period is $2\pi/0.2$, initial position, velocity and acceleration are $1000\,m$, $400\,m/s$, $0\,m/s^2$ respectively).

1) MODEL ADAPTABILITY

When tracking CA movement, as shown in FIGURE 4. The SW model assumes that the target acceleration is completely subject to periodicity, resulting in a larger filtering error, and the position, velocity, acceleration errors are the highest among the three models. The DO model introduces attenuation on the basis of acceleration periodic correlation assumption, and the tracking accuracy is better than the SW model. The ANM-DO model makes the state equation appear as CJ form through mean compensation. So it’s filtering accuracy for CA motion is much higher than that of the SW and DO models. The position estimation performance of the ANM-DO model relative to the SW and DO model are improved by 48.1% and 40.8% respectively, while in terms of velocity, 69.3% and 67.5.

Pulse acceleration motion is mainly used to test the ability of different models to adapt to sudden acceleration. The filtering effect is shown in FIGURE 5.

The performance of the SW and DO models is poor with abrupt changes in 40s and 60s. For the SW model, the position error increases sharply at 40 s, and the error within 40~70 s maintains a high level. Further, the SW has a certain ability to
suppress velocity and acceleration errors, its error curve has a certain drop after the mutation, but its overall error is still high. With the DO model, its position error also increased significantly, while the amplitude was smaller than the SW model, and the duration was shorter. Moreover, the speed and acceleration error curves of the DO model are similar to the SW model, but the error level is slightly lower than the former. The reason for this phenomenon is the presence of the attenuation coefficient, which makes the DO model to some extent have the characteristics of the Singer model (adapt to the motion between constant velocity and constant acceleration movement). The ANM-DO model can quickly describe the target maneuver change, and there is no abrupt change in position error at 40s and 60s, but only a small increase in error. Because the state equations of the ANM-DO model show the CJ form, the effect is similar to that of the CA motion tracking filter at the time of 40s and 60s maneuvers. Also, the speed and acceleration errors rapidly rise to a lower level after a sharp increase in this period. The position estimation performance of the ANM-DO model relative to the SW and DO model are improved by 43.2% and 39.6% respectively, while in terms of velocity, 42.6% and 27.1%.

FIGURE 3. Three types of acceleration (a. Constant acceleration motion, b. Pulse acceleration motion, c. Sinusoidal acceleration motion).

FIGURE 4. Comparison of tracking results of constant acceleration motion (a. Comparison of position RMSE, b. Comparison of velocity RMSE, c. Comparison of acceleration RMSE).
The tracking results of sinusoidal accelerated motion are shown in FIGURE 6.

The error of position, velocity and acceleration of the SW and DO models fluctuates periodically. Among them, the SW model has high tracking accuracy for sine/cosine acceleration motions. The filtering result of the DO model converges faster than the SW, and the error mean value is slightly higher due to the existence of the attenuation coefficient. The ANM-DO model can quickly adapt to the target maneuvering change by mean compensation, the fluctuations of the error are smaller and the curve is more stable. However, the state equations of the ANM-DO model show the CJ form, so the overall error level of the ANM-DO model is slightly higher than that of the SW and DO models. More precisely, the error of the SW and DO model valley points are much smaller than ANM-DO, but the error of the ANM-DO is more stable. Therefore, when tracking the sinusoidal acceleration motion, the ANM-DO would be a safer solution. And the SW and DO model filter error valley points have higher accuracy potential, it can be used in the IMM to build a new multi-model structure.
For random parameters, the conditions for estimating the state error consistency in scenario II is shown in TABLE 5.

### Table 5: Statistical average of position, velocity and acceleration errors in tracking different movements.

| Type       | Constant acceleration motion | Pulse acceleration motion | Sinusoidal acceleration motion |
|------------|------------------------------|---------------------------|-------------------------------|
|            | Position error/m | Velocity error/(m/s) | Acceleration error/(m/s²) | Position error/m | Velocity error/(m/s) | Acceleration error/(m/s²) | Position error/m | Velocity error/(m/s) | Acceleration error/(m/s²) |
| SW         | 193              | 124                        | 42                          | 192              | 89                        | 26                          | 118              | 82                        | 25                          |
| DO         | 169              | 117                        | 28                          | 164              | 70                        | 24                          | 123              | 83                        | 29                          |
| ANM-DO     | 100              | 38                         | 12                          | 109              | 51                        | 15                          | 127              | 86                        | 27                          |

tracking this form of movement, to pursue more accurate state estimation. The statistical average of the filter error in scenario II is shown in TABLE 5.

2) STATE ERROR CONSISTENCY

For random parameters, the conditions for estimating the state error consistency in scenario II is shown in TABLE 5.

\[
\lim_{k \to \infty} E \left\{ \hat{X}(k) - X \right\}^T \left\{ \hat{X}(k) - X \right\} = 0 \quad (28)
\]

Target tracking is equivalent to estimating the state of a dynamic system, and it is generally difficult to realize that the estimated value converges to the true value. We believe that when the sample is infinite, the expectation of the filter estimation error should match the covariance of the state estimation error.

\[
P(k|k) = E \left\{ \left( X(k) - \hat{X}(k) \right)^T \left( X(k) - \hat{X}(k) \right) \right\} |Z^k \quad (29)
\]

As the state estimation is unbiased and it meet Eq.(29), the state estimation of the filter is considered to be uniform. Let the squared error of the normalized state estimate be:

\[
\gamma(k) = \left( X(k) - \hat{X}(k) \right)^T P^{-1}(k|k) \left( X(k) - \hat{X}(k) \right) \quad (30)
\]

If it is assumed that the filter estimate is consistent, then \(\gamma(k)\) follows a \(\chi^2\) distribution with a degree of freedom of 3. We conducted a total of 100 Monte Carlo simulations, and the sample mean is:

\[
\tilde{\gamma}(k) = \frac{1}{100} \sum_{i=1}^{100} \gamma^i(k) \quad (31)
\]

The confidence interval for the consistency assumption with a confidence level of 0.95 (the two horizontal lines in FIGURE 7). And \(\tilde{\gamma}(k)\) of each model as shown in FIGURE 7.

As shown in FIGURE 7, when tracking different forms of motion, none of the three models have state error consistency. For uniform accelerated motion and pulse accelerated motion, the consistency of the state error of the ANM-DO model is relatively stable and slightly better than the SW and DO models. When tracking sinusoidal acceleration motion, the carves of the SW and DO models fluctuate greatly, the carve of the ANM-DO model is steady. Therefore, the non-zero-mean model maneuverability has certain advantages from the results of tracking three different movements, and it is a relatively safe and secure option when tracking non-cooperative goals.

V. CONCLUSION

Aiming at the tracking problem of hypersonic targets in nearby space, based on the DO model, combined with the adaptive non-zero mean value of the auto-adaptability, an ANM-DO model is proposed. Then, the difference between ANM-DO and the first-order time correlation model adaptive non-zero mean value model is analyzed, and the dynamic error of ANM-DO model is derived through Kalman filtering, which verifies the mobile adaptability of the model.

Simulation results the tracking accuracy of ANM-DO model is higher than that of DO model when tracking near-space supersonic jump gliding targets, and the superiority of this model in maneuver adaptation is verified by analyzing different motion modes. However, this model still has the following problems for further study.

1) This paper derives the dynamic error of ANM-DO state update through Kalman filtering, and separates the steady state of dynamic error. In this process, the influence of process noise covariance is not considered. In fact, the setting of process noise covariance has a great influence, especially at the beginning of filtering, so the influence of non-zero mean compensation on process noise covariance needs further analysis.

2) This article analyzes and explains the physical meaning of Markov model non-zero mean compensation from the two perspectives of state transition matrix and dynamic error. In practical applications, we generally use the previous jerk instead of the mean jerk. However, in a strict sense, the mean value of jerk represents the statistical distribution of the target maneuver, and its physical meaning is not clear.

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F. Li et al.: Near Space Hypersonic Vehicle Target Tracking Adaptive Non-Zero Mean Model