Numerical modelling of Rayleigh-Benard convection and heat transfer in normal 3He near the critical point

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Abstract. New numerical results on thermal gravity-driven convection in a layer filled with near-critical 3He and heated from below are presented. Corrections of conditions for convection onset are discussed. The heat transfer calibrations near the critical point are tested using experimental data. Stratification effects are analysed. As found, space environment that suppresses the strong density gradients near the critical point may provoke the enhancement of convection compared to the terrestrial conditions.

1. Introduction
The unique experiments on heat transfer and convection onset in 3He near the critical point and fundamental overview were done by A.B. Kogan and H. Meyer [1]. More recently, a number of the papers devoted to the analysis of these data were published (see [2-5] and the references within). However some correction of the theoretical basis is necessary for complete analysis of these precise experimental data. In this paper which is based on our recent results [6-9] we study thermal gravity-driven convection in a layer filled with near-supercritical 3He and heated from below with the use of physical properties corresponding to the conditions of the experiments [1]. This paper is focused on the convection onset and multi-cell modeling on the basis of the linearized as well as full Navier-Stokes equations for a high compressible fluid obeying the state equation of van der Waals or a perfect gas. By introducing some corrections into the formula for the adiabatic temperature gradient, the convection onset in Jeffrey’s form for a perfect gas is generalized for non-perfect compressible gases. The correction of the one cell approach for 2D case using multi-cell solutions is done. Recently, a three-dimensional case was studied in [9].

2. Statement of the problem. Steady-state roll structure above the stability threshold
We consider a fluid flow in a layer with the length L and the height H confined to two differentially heated horizontal boundaries (T₂ > T₁) near the critical point of 3He. The theoretical approach is based on the Navier-Stokes equations of compressible media with the van der Waals state equation using analytical and numerical technique (see details in [6-7]). Instead of the Boussinesq approach, the problem is characterized not only by the Rayleigh number Ra=ΘgH³/(T₁Dν), the Prandtl number Pr=ν/D and the aspect ratio L/H but also non-Boussinesq numbers: the temperature drop θ=(T₂−T₁)/T₁, the compressibility number Cₚ=gH/γRTₖ, the ratio of the heat capacities γ, and the reduced temperature ε=(T−Tₖ)/Tₖ. Note that in experiments [1], L/H~57. Because of numerical difficulties in

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simulation of long layers, the one-cell approach is usually applied [2, 4, 6, 7]. Calculations for Boussinesq fluids show that the difference of the Nusselt number in a real layer and one-cell model is less than 1%. In figure 1, the first result on multi-roll steady-state 2D non-Boussinesq convection at L/H =10 using technique [7, 8] is shown.

Figure 1. Multi-cell convection structure in the near critical $^3$He above the threshold of equilibrium stability at $Ra=11.9$ ($Ra_s=2.44 \times 10^3$), $\varepsilon=0.01$, $Pr=1.19$, $\theta=4.19 \times 10^{-7}$, $C_F=3.46 \times 10^{-7}$, L/H=10, $\gamma=1.67$. Stream function lines (a), isotherms (b) and isodensity lines (c)

3. Conditions and criteria of convection onset

New corrected conditions of convection onset in the Rayleigh–Benard problem for a viscous heat-conducting medium which use the modified Rayleigh number (Jeffrey’s condition) are obtained in [8]. The linearized equations in the case of stationary perturbations for the perfect and van der Waals gases are analyzed. The analytical formulas indicate the dependence of the horizontal convection scale on the temperature difference at the boundaries and the hydrostatic compressibility. The values of the threshold parameters are verified by analytical solutions of the linearized equations and numerical solutions of the complete Navier–Stokes equations. Following [8], the condition of stability for a perfect gas seems as $\rho_m^2 Ra_c (1-K) = Ra_s^8$, and for the van der Waals gas is

$$\rho_m^2 w(\gamma, T_m, \rho_m) Ra_c (1-K) = Ra_c^8$$ \hspace{1cm} (3.1)

Here $Ra_c = Re_c^2 C_P Pr \beta |\Delta T|$ is the critical Rayleigh number, $K = |dT_e^a/dz_e|/|\Delta T|^{-1}$ is the Schwarzschild number. The function $w(\gamma, T_m, \rho_m)<1$ everywhere.

The formula (3.1) makes it possible to obtain the criterion of Jeffrey for small parameter of hydrostatic compressibility in a perfect gas. Note, this criterion was not obtained by Jeffrey for the van der Waals case. Such kind of corrections may be important for near-critical experiments in very close vicinity of the critical point and makes it possible to find the exact critical values Ra_c^8.

The commonly used adiabatic temperature gradient obtained from the Schwarzschild condition corresponds to the temperature gradient in a perfect gas in equilibrium. In the case of the van der Waals gas, the adiabatic gradient differs from that obtained from the Schwarzschild condition. The new corrected ratio between adiabatic gradients in the van der Waals and perfect gases was found in [8] (see figure 2(a)). Generally, the ratio between adiabatic gradients is a function of the coordinate x and depends on the coordinate of the position of the critical point inside the volume $x_0$ and the parameters $\gamma$ and $C_F$. It is important at large values of the hydrostatic compressibility parameter as shown in figure 2(a) for $C_F=0.1$. 

4. Estimations of the governing parameters under a high hydrostatic compressibility

To clarify a concrete role of non-Boussinesq numbers, a more detailed analysis of their range and mutual correlations in the range of parameters are necessary. One can see estimations of some key values of near-critical $^3$He in figure 2 (b)-(d); here, $k = \left( 1 - k^2 \right)$.

The curves in figure 2 (b) correspond to the Earth’s conditions at $C_F=3,10^{-7}$. One can see that for small $\theta$ the value of $k$ changes in a wide range and, for small $\varepsilon$, may be even $k<0$. In this case the actual temperature gradient is less then the adiabatic temperature gradient and convection does not develop. If $\theta<\theta_a$, the value of $k$ is negative for all $\varepsilon$. One can find, that for $\varepsilon=0$, $\theta_a=2C_F/3$.

Figures 2 (c, d) show the modified real Rayleigh numbers $Ra_r^S$ at $\varepsilon=3,01\cdot10^{-7}$ depending on $C_F=3,10^{-7}$ (c) and $C_F$ at $\varepsilon=0,02$ (d) which demonstrate stratification effects. The real Rayleigh number $Ra_r$ in figure 2 (d) does not include hydrostatic compressibility effects. The values of $Ra_r^S$ and $Ra_r$ are calculated by relations [7]. In figure 2 (c), $Ra_r^S$ goes to zero with approaching to the critical point ($\varepsilon \to 0$) and does not exceed the threshold value 1708 corresponding to the convection onset. It means that stratification due to the high compressibility may stabilize a fluid layer near the critical point preventing convective motions.

In figure 2 (d), the number $Ra_r^S$ grows if gravity reduces ($C_F \to 0$) and exceeds the threshold value 1708 at $C_F<10^{-7}$. The behavior of $Ra_r^S$ shows that near the critical point in weightlessness an intensification of convection may occur, in contrast to terrestrial conditions under which convection may be absent due to stabilizing high density stratification (see below, Fig.3b). This paradox calls for
meticulous measurements under controlled space flight conditions, which will help to find the limits of continuum mechanics applicability near the critical point.

5. Heat transfer due to convection in $^3$He above the hydrostatic stability threshold

Figure 3. Modified critical Rayleigh numbers and dependency of heat transfer on the $\varepsilon$.

Figure 3 (a) shows the threshold Rayleigh numbers with and without the stratification effect $Ra_r^*$ (1) and $Ra_r^s*$ (2) respectively; marks “+” indicate the experimental values of $Ra_r^*$ [1]. As was shown in [7] the modified Nusselt number $Nus$ (disregarding the adiabatic temperature gradient) versus $Ra_r^s$ for different $\varepsilon$ correlates with the experimental data on air [10] (solid line) well showing some analogy in heat transfer in high and low compressible fluids. This modification for heat transfer in $^3$He was firstly done in [1] based on experimental data, the parametric numerical analysis and calibration relations were done in [7]. However, the dependency of heat transfer upon the $\varepsilon$ (Fig.3b) wasn’t found before. This study was supported by the Russian Foundation for Basic Research N 06-01-00281 and Leading Scientific School N 2496.2008.8.

6. References
[1] Kogan A B and Meyer H 2001 Heat Transfer and Convection Onset in a Compressible Fluid: $^3$He near the Critical Point Phys. Rev. E 63 (5) 056310
[2] Furukawa K, Meyer H, Onuki A and Kogan A B 2003 Convection in a Very Compressible Fluid: Comparison of Simulation with Experiments Phys. Rev. E 68 (5) 056309
[3] Meyer H and Zhong F 2004 Equilibrium and Other Dynamic Properties of Liquids near Liquid-Vapor Critical Point C. R. Acad. Sci. Paris, Ser. II, M´ecanique 332 328
[4] Accary G, Raspo I, Bontoux P and Zappoli B 2005 Stability of a Supercritical Fluid Diffusing Layer with Mixed Boundary Conditions Phys. Fluids 17 (4) 104105
[5] Meyer H 2006 Onset of the convection in a supercritical fluid Phys. Rev. E 73 016311
[6] Polezhaev V I and Soboleva E B 2003 Thermal gravity-driven convection of near-critical helium in enclosures Low Temperature Physics 29 484
[7] Polezhaev V I and Soboleva E B 2005 Rayleigh–Benard Convection in a Near-Critical Fluid in the Neighborhood of the Stability Threshold Fluid Dynamics 40 (2) 209
[8] Gorbunov A A, Nikitin S A and Polezhaev V I 2007 Condition for Rayleigh-Benard Convection Onset and Heat Transfer in a Near-Critical Medium Fluid Dynamics 42 (2) 704
[9] Polezhaev V I, Gorbunov A A, Nikitin S A and Soboleva E B 2006 Hydrostatic compressibility phenomena: New opportunities for near critical research in microgravity Annals of the New York Academy of Sciences: Proc. Interdisciplinary Transport Phenomena in the Space Sciences vol 1077 (Tomar, Portugal, 7-12 August 2005) ed S S Sadhal (Moston, Massachusetts: Blackwill Publishing) pp 304-327
[10] Jaluria J 1980 Natural Convection (Oxford: Pergamon Press)