Abstract—Full-duplex (FD) radio has been introduced for bidirectional communications on the same temporal and spectral resources so as to maximize spectral efficiency. In this paper, motivated by the recent advances in FD radios, we provide a foundation for hybrid-duplex heterogeneous networks (HDHNs), composed of multi-tier networks with a mixture of access points (APs), operating either in bidirectional FD mode or downlink half-duplex (HD) mode. Specifically, we characterize the network interference from FD-mode cells, and derive the HDHN throughput by accounting for AP spatial density, self-interference cancellation (IC) capability, and transmission power of APs and users. By quantifying the HDHN throughput, we present the effect of network parameters and the self-IC capability on the HDHN throughput, and show the superiority of FD mode for larger AP densities (i.e., larger network interference and shorter communication distance) or higher self-IC capability. Furthermore, our results show operating all APs in FD or HD achieves higher throughput compared to the mixture of two mode APs in each tier network, and introducing hybrid-duplex for different tier networks improves the heterogeneous network throughput.

Index Terms—Heterogeneous networks, full-duplex, half-duplex, self-interference, network interference, stochastic geometry

I. INTRODUCTION

Conventional communication systems operate in half-duplex (HD) such as time-division or frequency-division approaches, which require different orthogonal resources in either temporal or spectral domain for bidirectional communications. As a way of enhancing the spectral efficiency of communication systems, full-duplex (FD) has been introduced to perform bidirectional communications on the same temporal and spectral resources. Thus, FD radios can potentially be employed in heterogeneous networks for increased link capacity, more flexibility in spectrum usage, and improved communication security [1].

Different FD systems have been studied considering the asynchronous transmission and reception [2], the one-way relay transmission [3], the two-way relay transmission [3], [6], the imperfect channel estimation and limited dynamic range in a multiple-input multiple-output (MIMO) system [7], and relays with different self-interference cancellation (IC) capabilities in multi-hop transmission [8]. The achievable rates of HD and FD in MIMO systems have been compared in [9], and the degree-of-freedom of the system with a FD base stations (BSs) with HD users has been analyzed by considering the intra-cell inter-node interference [10]. The FD radios has also been used in jamming techniques for communication secrecy [11] and bidirectional broadcast communications by implementing rapid on-off-division duplex [12]. The hybrid of FD- and HD-relaying schemes have also been presented, which allows a relay to opportunistically switch two modes based on instantaneous [13]–[15] or statistical channel state information (CSI) [15].

The key challenging in implementing a FD radio is the presence of self-interference, received at a node from its own transmission while transmitting and receiving at the same time. Self-IC techniques for FD systems with multiple antennas have been proposed by exploiting the following domains: 1) propagation-domain schemes including antenna separation [16], [17] and directional transmit/receive antennas (e.g., beamforming-based techniques) [18]–[20]; 2) analog circuit-domain including channel-unaware schemes [16], [21] and channel-aware schemes [16], [22]–[24]; 3) digital circuit-domain [14], [15]; and 4) hybrid of analog and digital domains [25], [26]. The self-IC techniques are being researched actively as the current self-IC capability is still challengeable. Recently, the feasibility of single (shared)-array FD transceivers has also been presented in [24], [27], [28]. However, there is no work that considers the self-interference together with the network interference generated from randomly distributed FD-mode nodes for the performance evaluation of FD systems. If more nodes operate in FD, the number of communicating nodes in the network increases, but network interference also increases, which can degrade the communication reliability between nodes. The network interference from FD-mode nodes has been presented in [29], [30], but one tier network was considered with the perfect self-IC assumption.

The performance of heterogeneous networks has been studied [31]–[43] by taking into account the spatial node distribution using the Poisson point process (PPP), which is widely used in wireless networks [44]–[48]. The heterogeneous network throughput has been presented by considering the K-tier spectrum sharing network in downlink [33] and in uplink [34], the BS loads of different tier networks [35], the interference cancellation capability [36], [37], the spectrum sharing methods [38], [39], and the trade-off between traffic

Furthermore, simpler network model was used in [29] by ignoring the intra-cell interference, which can be generated by users accessing the same resource in a cell, and the fixed link distance between a user and its communicating access point (AP) was used in [30].

1Recently, the PPP has also been used to evaluate some advanced techniques such as the coordinated multiple-point (CoMP) with BS-centric [49] and user-centric [50] clustering, and the self-powered transmitters using energy harvesting techniques [51].
offloading and energy consumption of small cells \cite{40}. To solve the load balancing problem in heterogeneous networks with HD systems, the concept of cell range expansion has also been considered to offload users to less loaded networks using a biased cell association rule \cite{41}, \cite{42}. The design of duplex communication modes is also presented by considering the coordinated time-division duplexing (TDD) underlay structure in two-tier networks \cite{52}, and the hybrid division duplexing in the network composed of macro-cells in frequency-division duplexing (FDD) and cognitive femto-cells in TDD \cite{43}. However, most of these works is based on HD and does not explore the effect of FD on network throughput, impeding the efficient duplex mode design for heterogeneous networks.

Motivated by the recent advances in FD radios, we propose the novel idea of hybrid-duplex cell networks for future heterogeneous cellular networks. We consider HDHNs, composed of multi-tier networks with a mixture of APs operating either in bidirectional FD mode or downlink HD mode. We develop a framework for HDHNs in the presence of self-interference and network interference. Specifically, after characterizing the network interference of HDHNs, we define a performance metric, namely the HDHN throughput, to measure the average data rate achieved by APs and users successfully communicating in this network. Based on this metric, we present the effect of network parameters such as the AP spatial density, the network interference, and the self-interference on the HDHN throughput. We then determine the portion of FD-mode APs that maximizes the HDHN throughput based on the system parameters including AP spatial density. Note that this is different from the opportunistic mode switching based on CSI in \cite{13}–\cite{15}. The main contributions of this paper can be summarized as follows:

- we characterize the network interference generating from distributed APs and users in FD-mode cells;
- we introduce and derive the HDHN throughput that accounts for self-interference, spatial AP densities, and transmission power of APs and users; and
- we quantify the HDHN throughput and present the optimal portion of FD-mode APs to maximize the HDHN throughput according to the self-IC capability and network parameters.

The remainder of this paper is organized as follows: Section II describes the HDHN model and provides the statistical characterization of interference from FD-mode cells. Section III analyzes the successful transmission probability of HDHNs, and introduces and analyzes the HDHN throughput. Section IV quantifies the effects of network parameters and self-interference capability on the HDHN throughput and determines the optimal portion of FD-mode APs in HDHNs. Finally, conclusions are given in Section V.

\textbf{Notation:} The notation used throughout the paper is reported in Table I.

\begin{table}[h]
\centering
\caption{Notations used throughout the paper.}
\begin{tabular}{|c|c|}
\hline
\textbf{Notation} & \textbf{Definition} \\
\hline
$\Pi_{a,k}$ & PPP for AP distribution of network $k$ \\
$\Pi_{m,a,k}$ & PPP for $m$-mode AP distribution in network $k$ \\
$\lambda_k$ & Spatial density of APs of network $k$ \\
$\lambda_{m,k}$ & Spatial density of $m$-mode APs in network $k$ \\
$T_s$ & Symbol time \\
$W$ & Communication bandwidth \\
$H_{x,y}$ & Fading level of the link between nodes at $x$ and $y$ \\
$D_{x,y}$ & Distance of the link between nodes at $x$ and $y$ \\
$D_{m,k}^{\alpha}$ & Distance of the link between an user and its associated $m$-mode AP in network $k$ \\
$\alpha_k$ & Pathloss exponent in network $k$ \\
$\mathcal{W}_k$ & Weighting factor of network $k$ \\
$B_{ik}$ & Ratio between association factors, $\mathcal{W}_i/\mathcal{W}_k$ \\
$P_{x,k}$ & Transmission power of a AP in network $k$ \\
$P_{a,k}$ & Transmission power of a user in network $k$ \\
$P_r$ & Transmission power at a receiving node \\
$I_{m,k}$ & Interference from $m$-mode APs in network $k$ \\
$\gamma_{m,k}$ & SIR at $m$-mode node in network $k$ \\
$\tau$ & Target SIR \\
$C_k^{\text{FD}}(P_r)$ & Self-IC capability of $m$-mode node in network $k$ \\
$p^{\text{FD}}_k$ & Portion of FD-mode APs in network $k$ \\
$p^{m}_{S,k}$ & Successful transmission probability of $m$-mode node \\
$\mathcal{S}$ & HDHN throughput [bits/sec/Hz/m$^2$] \\
$S_k$ & HDHN throughput of network $k$ [bits/sec/Hz/m$^2$] \\
$S^c$ & Cell throughput of HDHNs [bits/sec/Hz/cell] \\
\hline
\end{tabular}
\end{table}
We consider HDHNs composed of $K$-th tier wireless networks. The $k$-th tier network consists of APs distributed in space according to a homogeneous PPP $\Pi_{a,k}$ with spatial density $\lambda_k$. Each AP forms a cell and communicates to nodes in either downlink HD mode or bidirectional FD mode. The portion of FD-mode APs in the $k$-th tier network is $\eta^\text{FD}_k$, and the distributions of HD-mode and FD-mode APs also follow PPPs, $\Pi^\text{HD}_{a,k}$ and $\Pi^\text{FD}_{a,k}$, with spatial densities $\lambda^\text{HD}_k = \lambda_k(1 - \eta^\text{FD}_k)$ and $\lambda^\text{FD}_k = \lambda_k \eta^\text{FD}_k$, respectively. All HD-mode cells are in downlink while all FD-mode cells have both uplink and downlink communications. In the HDHNs, users in FD-mode cells and all APs of the $k$-th tier network transmit with power $P_{a,k}$ and $P_{a,k}$, respectively, and generally $P_{a,k} \geq P_{a,k}$. Each channel is used by only one user in a cell to avoid intra-cell interference, and the whole spectrum is utilized in each cell. An example of downlink HDHNs is presented in Fig. 1.

Users are scattered in HDHNs according to a homogeneous PPP $\Pi_{a}$ with spatial density $\mu$, and a node located at $x_0$ connects to an AP in the $k$-th tier networks based on the association rule, presented by [42]:

$$k = \arg \max_{i \in K} \left\{ \max_{x_i \in \Pi_{a,i}} W_i D_{X_i, x_0}^{-\alpha_i} \right\}$$

where $K = \{1, 2, \ldots, K\}$ is the index set of $K$ tier networks; $W_i$ is the weighting factor for the $i$-th tier network; $D_{X_i, x_0}$ is the distance between nodes at $y$ and $x_0$; and $\alpha_i > 2$ is the pathloss exponent in the $i$-th tier network. This association rule can be extended to special cases such as the case that makes nodes associate to the nearest AP, i.e., $W_i = 1$, or to the AP providing the maximum average received power, i.e., $W_i = W_i(1 + U_i)$ where $U_i$ is the association bias of the $i$-th tier network. Let us denote $D_{x_0}^{m}$ as the distance to the $m$-mode AP with the maximum $W_i D_{X_i, x_0}^{-\alpha_i}$ for all $x_i \in \Pi_{a,i}$. Using the association rule and $D_{x_0}^{m}$, the probability that a user is associated to an $m$-mode AP in the $k$-th tier network is given by

$$p_{A,k}^m = P \left\{ \bigcup_{i \in K, m_i \in \{\text{FD,HD}\}} W_i (D_k^m)^{-\alpha_k} > W_i (D_k^m)^{-\alpha_k} \right\}$$

$$= 2\pi \lambda_k^m \int_0^\infty x \exp \left\{ -\pi \sum_{i \in K, m_i \in \{\text{FD,HD}\}} \lambda_i^m B_{ik}^2 x^{2\alpha_k/\alpha_i} \right\} dx$$

where (a) is from Lemma 4 in [42], and $B_{ik} = W_i/W_k$ is the ratio between the association factor. Using (2), for a node associated to $m$-mode AP in the $k$-th tier network, the probability distribution function (PDF) of the link distance to the associated AP, $D_{k}^m$, is given by [42]:

$$f_{D_k^m}(x) = \frac{2\pi \lambda_k^m}{p_{A,k}^m} \exp \left\{ -\pi \sum_{i \in K} \lambda_i B_{ik}^2 x^{2\alpha_k/\alpha_i} \right\}$$

$$= \frac{x}{\pi} \exp \left\{ -\pi \sum_{i \in K} \lambda_i B_{ik}^2 x^{2\alpha_k/\alpha_i} \right\}$$

Note that $D_{k}^m$ depends not on the spatial density $\lambda_k$, but on the sum of scaled spatial densities, i.e., $\sum_{i \in K} \lambda_i (W_i/W_k)^{2\alpha_k/\alpha_i}$.

All users and APs have a single antenna, and they are transmitting and receiving at the same time in FD mode [27], [28]. A node in FD mode receives self-interference from its transmitted signal, and performs IC for the self-interference. Since the amount of self-interference depends on the transmission power at the receiver $P_r$, [27], we define the residual self-interference power after performing cancellation as [14], [15], [19]

$$C_k^m(P_r) = P_r R_k$$

for all $k \in K$ and $m \in \{\text{FD,HD}\}$. Here, $R_k = |h_{R,k}|^2$ shows the self-IC capability of nodes where $h_{R,k}$ is the residual self-interfering channel of a node in the $k$-th tier network. In [4], $C_k^m(P_r) = 0$ denotes perfect self-IC, and $C_k^m(P_r) = 0$ since HD-mode nodes are not transmitting while receiving data.

The residual self-interfering channel gain $H_{R,k}$ in [4] needs to be characterized according to cancellation algorithms. For instance, after a digital-domain cancellation, $h_{R,k}$ can be presented as $h_{R,k} = h_{S,k} - h_{I,k}$, where $h_{S,k}$ and $h_{I,k}$ are the self-interfering channel and its estimate as the self-interference is subtracted using its estimate [14], [19], [53], [63]. Then, $H_{R,k}$ can be modeled as a constant value such as $H_{R,k} = \sigma_e^2$ for the estimation error variance $\sigma_e^2$ [14], [19], [53], [63]. However, for other cancellation techniques such as analog-domain schemes [16], [21]–[24], propagation-domain schemes [17]–[20], and combined schemes of different domains [25], [26], the modeling of $H_{R,k}$ is still a challenging problem. Hence, the parameterization of the self-IC capability in [4] can make the analysis more generic. We consider $H_{R,k}$ as a constant value in this paper, but note that the analysis can be easily extended for the case of random $H_{R,k}$ within our framework.

B. Network Interference Characterization

In the $k$-th tier HDHN, the signal-to-interference ratio (SIR) received by a node at $x_o$ from a transmitter at $y_o$ for a propagation channel model with pathloss and Rayleigh fading is defined as

$$\gamma_k^m = \frac{P_r H_{x_o, y_o} D_{x_o, y_o}^{-\alpha_k}}{C_k^m(P_r) + \sum_{i \in K} (\lambda_i^{\text{HD}} + \lambda_i^{\text{FD}})}$$

where $P_r$ is the transmission power at the transmitter, $P_r$ is the transmission power at the receiver, and $H_{x_o, y_o}$ is the i.i.d. fading channel gain of the link, i.e., $H_{x_o, y_o} \sim \exp(1)$. In (5),

Note that if channels are not always used in every cells, it only affects the spatial density of interfering nodes in the same framework of this paper.
when a user at \( x \) associates to an AP at \( y \), \( I^\text{HD}_i \) and \( I^\text{FD}_i \) are the aggregate interference received from HD-mode cells and FD-mode cells in the \( i \)th-tier network, given by
\[
I^\text{HD}_i = \sum_{z \in \Omega^\text{HD}_i \setminus \{y\}} P_{a,i} H_{x,z} d^{-\alpha_i}_{x,z} \tag{6}
\]
\[
I^\text{FD}_{0,i} = \sum_{z \in \Omega^\text{FD}_0 \setminus \{y\}} P_{a,i} H_{x,z} d^{-\alpha_i}_{x,z} + P_{u,i} H_{x,z+N(z)} d^{-\alpha_i}_{x,z+N(z)} \tag{7}
\]
where \( N(z) \) is the relative location of a user to its associated AP at \( z \). Note that in (7), interference from a FD-mode cell consists of the interference from an AP and a user.

Let us consider a user at \( x \), its associating AP at \( y \), the other APs at \( z \in \Omega^\text{FD}_0 \setminus \{y\} \), and their associated users at \( z + N(z) \). Generally, the distance between \( x \) and \( z \) is greater than the distance between \( z \) and \( z + N(z) \), i.e., \( \norm{x-z} \gg \norm{N(z)} \).

Hence, due to the difficulty in obtaining the exact characteristics of \( I^\text{FD}_i \), we assume that the distance between a user at \( x \) and a user at \( z + N(z) \) can be approximated to the distance between a user at \( x \) and the unassociated AP at \( z \).

\[
D_{x,x+N(z)} \approx D_{x,z} \tag{8}
\]

Using the approximation in (8), the interference received from FD-mode cells can be presented as
\[
I^\text{FD}_i = \sum_{z \in \Omega^\text{FD}_i \setminus \{y\}} G_i D^{-\alpha_i}_{x,z} \tag{9}
\]
where \( G_i \) is given by
\[
G_i = P_{a,i} H_{x,z} + P_{u,i} H_{x,z+N(z)} \tag{10}
\]

In (10), if \( P_{a,i} = P_{u,i} \), \( G_i \) is the sum of two exponential random variables all with same rate \( P_{a,i}^{-1} \) and it follows the Erlang distribution \( \text{Erlang}(2) \). On the other hand, if \( P_{a,i} \neq P_{u,i} \), \( G_i \) is the sum of two exponential random variables with different rates \( P_{a,i}^{-1} \) and \( P_{u,i}^{-1} \) and it follows the hypo-exponential distribution \( \text{HypoExp}(2) \). Hence, the PDF of \( G_i \) is given by \( \text{Erlang}(2) \) \( \text{HypoExp}(2) \).

\[
p_{G_i}(x) = \begin{cases} \frac{1}{P_{a,i}^2} x e^{-x/P_{a,i}}, & \text{if } P_{a,i} = P_{u,i}, \\ e^{-x/P_{a,i}} - e^{-x/P_{u,i}}, & \text{otherwise} \end{cases} \tag{11}
\]

\footnote{From the law of cosines, we have \( \norm{x-(z+N(z))}^2 \approx \norm{x-z}^2 \) for \( \norm{x-z} \ll \norm{N(z)} \).}

and we have \( \text{E} G_i \{ G_i^\delta \} \) for \( \delta > -1 \) and the gamma function \( \Gamma(\cdot) \). With the approximation in (8), we now obtain the Laplace transform of \( I^\text{FD}_i \) as follows.

Lemma 1: The Laplace transform of the approximated interference received from FD-mode cells in the \( i \)th-tier network, \( I^\text{FD}_i \) in (9), is given by (13) (on top of the page) where \( \hat{D} \) is the minimum distance to an interfering node and \( I_1(x,y,z,v) \) is defined by
\[
I_1(x,y,z,v) = \int_0^\infty t^{\nu - 1} e^{-v t} \Gamma(z, v t) dt = \frac{\nu^\nu \Gamma(x+z)}{x(y+v)^{x+z+2}} F_2 \left( 1, x+z; x+1; \frac{y}{y+v} \right) \tag{14}
\]
for all constants \( v + y > 0, y > 0, \) and \( x + z > 0 \), \( \Gamma(\cdot, \cdot) \) is the upper incomplete function, and \( F_2(\cdot, \cdot; \cdot; \cdot) \) is the hypergeometric function.

Proof: See Appendix A.
An example of the Laplace transform of $P_{\text{FD}}$ is presented in Fig. 2 when the association policy in (1) is applied. For other parameters, the values presented in Table II are used. Fig. 2 shows a good match between the cases with and without the approximation in (8), especially for dense networks, i.e., large $\lambda_{\text{FD}}^m$.

Note that the Laplace transform of interference from FD-mode cells in Lemma 1 and the following analytical results related to FD mode can also be used when each FD-mode node has two antennas, one for transmitting and the other for receiving. In this case, the self-IC capability will be determined differently to the case of single antenna.

III. HYBRID-DUPLEX HETEROGENEOUS NETWORK THROUGHPUT

In this section, we analyze the successful transmission probability of HDHNs, and define and derive the HDHN throughput as a new performance measurement for HDHNs.

A. Successful Transmission Probability

In this subsection, we analyze the successful transmission probability of HDHNs. We present the successful transmission probability of a $m$-mode node in the $k$th-tier network as $p_{\text{s,k}}^m(P_t, P_r; \tau) = \mathbb{P}\{\gamma_{\text{s,k}}^m \geq \tau\}$, where $\tau$ is the target SIR value. Users and APs may have different target data rates such as $R_u$ and $R_a$, respectively.

In this case, the target SIRs of user and AP can be set to $\tau_u = 2R_u/W - 1$ and $\tau_a = 2R_a/W - 1$, respectively, where $W$ is a communication bandwidth. The $p_{\text{s,k}}^m(\tau)$ is derived as follows.

**Theorem 1:** In HDHNs, the successful transmission probability of a $m$-mode node ($m \in \{\text{HD, FD}\}$) in the $k$th-tier network is given by

$$ p_{\text{s,k}}^m(P_t, P_r; \tau) = 2\pi \sum_{i \in \mathcal{K}} \lambda_i B_{\text{FD}}^2/\alpha_i \left[ C_{\text{m}}(P_t; \tau) - \sum_{i \in \mathcal{K}} 2^{\alpha_i/\alpha_i} 2\pi \lambda_i \phi_{\text{ik}}(P_t; \tau) \right] \exp \left( - \frac{C_{\text{m}}(P_t; \tau)}{P_t} - \sum_{i \in \mathcal{K}} 2^{\alpha_i/\alpha_i} 2\pi \lambda_i \phi_{\text{ik}}(P_t; \tau) \right) $$

(15)

where $\phi_{\text{ik}}(\alpha_i, P_t)$ is given by

$$ \phi_{\text{ik}}(\alpha_i, P_t) = \frac{B_{\text{FD}}^2}{2} + (1 - P_{\text{FD}}^i) \phi_{\text{ik}}(\alpha_i, P_t) + P_{\text{FD}}^i \phi_{\text{ik}}(\alpha_i, P_t) $$

(16)

Note that $L_{\text{FD}}(s)$ can be extended to the case with transmission power control for $P_{\text{a,i}}$, or $P_{\text{a,i}}$, such as [34] by taking expectation to the exponential in (33) according to the power distribution.
For $\alpha = 4$, by substituting $r^2$ with $t$ in (21), we have
\[
p_{S,k}^{FD}(P_t, P_r, \tau) = \pi \sum_{i \in K} \lambda_i B_{ik}^{2/\alpha} \int_0^{\infty} \exp \left\{ -t^2 C_k^i(P_t) \tau \right\} dt
\]
which results in (20) by [56, eq. (3.322)].

**Definition 1:** The HDHN throughput is defined by
\[
S = \frac{1}{W |A|} E \left\{ \sum_{k=1}^{K} \sum_{x \in \Pi_0^{m,t}(A)} R_a \mathbb{I}_{T_k^m}(X, U(X)) \right\}
\]
where $A$ is a bounded space with area $|A|$, $U(x)$ is the associating user to an AP at $x$, and
\[
\mathbb{I}_{T_k^m}(x, y) = \begin{cases} 1, & \text{if } (x, y) \in T_k^m \\ 0, & \text{otherwise} \end{cases}
\]
Here, $T_k^m$ is a random set of transmitter-receiver pairs $(x, y)$ that a transmitter at $x$ and its corresponding receiver at $y$ communicates successfully with higher received SIR than a threshold value $\tau$, i.e., $(x, y) \in T_k^m$ when $T_k^m = \{(x, y) \in \mathbb{R}^d : \gamma_k^m \geq \tau\}$.

The HDHN throughput measures the average data rate achieved by nodes (e.g., APs and users) communicating successfully in the network, and its unit is bits/sec/Hz/m$^2$. One can also define the **cell HDHN throughput** by normalizing the total HDHN throughput achieved over the network, $|A| S$, with respect to the average number of cells in HDHNs, $\sum_{i \in K} |A| \lambda_i$, as
\[
S^c = \frac{S}{\sum_{i \in K} |A| \lambda_i}.
\]
This shows the average data rate per cell in this network and its unit is bits/sec/cell. Now, we derive the HDHN throughput.

**Lemma 2:** The HDHN throughput is given by
\[
S = \frac{1}{W} \sum_{k=1}^{K} \lambda_k \left\{ (1 - p_k^{FD}) R_a \mathbb{E} \left\{ \mathbb{I}_{T_k^m}(x, y) \right\} \right.
\]
\[+ \left. \sum_{i \in K} \lambda_i \mathbb{E} \left\{ \mathbb{I}_{T_k^m}(x, y) \right\} \right\}
\]
by Campbell’s theorem and the stationarity of a homogeneous PPP [57].

**Corollary 3:** For $\alpha_i = \alpha > 2$, $\forall i \in K$, and the perfect self-IC, i.e., $C_k^{FD}(P_r) = 0$, the HDHN throughput is given by
\[
S = \frac{1}{W} \sum_{k=1}^{K} \lambda_k \left\{ (1 - p_k^{FD}) R_a \mathbb{E} \left\{ \mathbb{I}_{T_k^m}(x, y) \right\} \right.
\]
\[+ \left. \sum_{i \in K} \lambda_i \mathbb{E} \left\{ \mathbb{I}_{T_k^m}(x, y) \right\} \right\}
\]
(31) (on top of the next page).

**Proof:** It is obtained by substituting (23) into (29).
TABLE II
PARAMETER VALUES IF NOT OTHERWISE SPECIFIED

| Parameters | Values | Parameters | Values |
|------------|--------|------------|--------|
| $\alpha_k, \forall k$ | 4 | $\lambda_1, \lambda_2$ [nodes/m$^2$] | $10^{-3}$ |
| $T_a$ [sec] | $10^{-4}$ | $R_a$ [bits/sec] | $10^4$ |
| $W$ [Hz] | $10^4$ | $L_{\text{db},1}$ [dB] | $-40$ |
| $P_{h,1}$ [W] | $30$ | $P_{h,2}$ [W] | $6$ |
| $B_{ij}, \forall i, j$ | 1 | $p_{k}^{\text{FD}}$ | 0 (HD mode) |

not clear how to determine the portion of FD-mode cells to maximize the throughput of each tier network. From Corollary 3, we obtain the optimal portion of FD-mode cells, $p_k^{\text{FD}}$, for the perfect self-IC case as follows.

**Corollary 5:** For $\alpha_k = \alpha > 2$ with $R_u = R_a$, when the self-IC in the FD mode is perfect, i.e., $C_k^{\text{FD}}(P_r) = 0$, the optimal portion of FD-mode APs in the $k$th-tier network that maximizes the throughput of the network is $p_k^{\text{FD}} = 1, \forall k \in K$.

**Proof:** See Appendix C

**Remark 1:** Corollary 5 shows that, when the self-IC is perfect, in spite of the degradation of successful transmission probability, having more communicating nodes by operating more cells in FD enhances the network throughput. Therefore, in this network, the network throughput is maximized by operating all APs in FD mode regardless of network parameters such as transmission power or AP spatial density.

### IV. Numerical Results

In this section, we evaluate the throughput of two tier HDHNs consisted of network 1 and network 2 (except for Fig. 9 that considers three tier network), and present the effect of network parameters on the HDHN throughput. Specifically, we first show the HDHN throughput of network 1 in the presence of interference from network 2 as well as network 1 to explore the environment that FD achieves better throughput compared to HD. We then show how to determine the portions of FD-mode APs in two (or three) networks to maximize the HDHN throughput. Note that we use the self-IC capability of nodes in the $k$th-tier network as $C_k^{\text{FD}}(P_r) = P_r \cdot 10^{L_{\text{db},k}/10}$, where $L_{\text{db},k}$ [dB] is the ratio of the residual self-interference after IC to the transmission power at the receiver. Unless oth-

![Fig. 3](image-url)

**Fig. 3.** HDHN throughput of network 1 $S_1$ in bits/sec/Hz/m$^2$ as a function of the self-IC capability $L_{\text{db},1}$ in dB for FD mode ($p_1^{\text{FD}} = 1$) and HD mode ($p_1^{\text{FD}} = 0$) in network 1 and different AP spatial densities of network 2, $\lambda_2$, in nodes/m$^2$ when $P_{h,1} = 30$ W. Simulation results are marked by filled circles.

![Fig. 4](image-url)

**Fig. 4.** HDHN throughput of network 1 $S_1$ in bits/sec/Hz/m$^2$ as a function of the self-IC capability $L_{\text{db},1}$ in dB for FD mode ($p_1^{\text{FD}} = 1$) and HD mode ($p_1^{\text{FD}} = 0$) in network 1 and different AP spatial densities of network 2, $\lambda_2$, in nodes/m$^2$ when $P_{h,1} = 9$ W.
otherwise specified, the values of network parameters presented in Table II are used.

Figures 3 and 4 display the HDHN throughput of network 1 $S_1$ as a function of the self-IC capability $L_{dB,1}$ for different duplex modes in network 1 and different values of AP spatial density of network 2 $\lambda_2$. Here, $P_{u,1} = 30$ W is used for Fig. 3 while $P_{u,1} = 9$ W is used for Fig. 4. Simulation results are marked by filled circles in Fig. 3 and they show a good agreement with the analysis. Note that in Fig. 3 $S_1$ in HD mode is not changed according to $\lambda_2$ as $S_k$ in HD mode is given by

$$ S_k = \frac{\lambda_k R_k B^{2/\alpha}}{2W \zeta(\alpha, P_k)} $$

(32)

which is not affected by $\lambda_i$, $\forall i \neq k$. However, in Fig. 4 $S_1$ in HD mode is altered by $\lambda_2$ as $P_{u,1} \neq P_{u,2}$.

From Fig. 3 it can be seen that for large $\lambda_2$ and low $L_{dB,1}, S_1$ in FD mode is higher than that of HD mode. The effect of low $L_{dB,1}$ on $S_1$ is obvious as it means we have smaller residual self-interference. On the other hand, large $\lambda_2$ affects the network throughput in two aspects: 1) increasing the network interference (negative effect); and 2) making an user associate to closer AP with higher probability (positive effect). As $\lambda_2$ increases, we have large network interference, which makes the effect of self-interference on $S_1$ less in FD mode and the successful transmission probabilities in FD and HD modes relatively similar. Hence, for large $\lambda_2$ or low $L_{dB,1}$, operating APs in FD achieves higher $S_1$ compared to that in HD due to additionally communicating users in FD mode.

From Fig. 3 we can also see that for a fixed $L_{dB,1}, S_1$ in FD mode increases with $\lambda_2$. This is due to the fact that when the self-interference is large, as $\lambda_2$ increases, the increased network interference affects less than the shorter distance to associated AP. However, this results becomes different when the self-interference is small as shown in Fig. 4. In Fig. 4 we can see that $S_1$ in FD mode decreases as $\lambda_2$ increases when $L_{dB,1} < -45$. This can be attributed to the fact that for small self-interference, $S_1$ remains more by the increased network interference than the shorter communication link distance. Hence, in this case, having smaller $\lambda_2$ can enhance $S_1$. This result is also applied for the HD mode case. From Fig. 4 we can see that $S_1$ in HD mode decreases as $\lambda_2$ increases due to the large effect of the increased network interference.

Figure 5 shows the ratio of $S_1$ to the achievable $S_1$ in HD mode, $S_1/S_1^{HD}$, as a function of $p_t^{FD}$ for different duplex modes and different values of $R_{\lambda,ij}$, where $R_{\lambda,ij} = \lambda_i/\lambda_j$ for a given $\lambda_j = 10^{-3}$ nodes/m$^2$.

Fig. 5. Ratio of $S_1$ to the achievable $S_1$ in HD mode, $S_1/S_1^{HD}$, as a function of $p_t^{FD}$ for different duplex modes and different values of $R_{\lambda,ij}$, where $R_{\lambda,ij} = \lambda_i/\lambda_j$ for a given $\lambda_j = \lambda_2 = 10^{-3}$ nodes/m$^2$.

Figure 6 displays $S_1$ as a function of $R_{\lambda,12}$ with $\lambda_2 = 10^{-3}$ for different values of $P_{u,1}$ in W and $L_{dB,1}$ in dB and different duplex modes. Figure 6 shows that $S_1$ increases with $p_t^{FD}$ for high $\lambda_1$ while it decreases for low $\lambda_1$. This can be attributed to the fact that the FD mode achieves higher throughput than the HD mode for large $\lambda_1$ as also shown in Fig. 5. In Fig. 5 $S_1$ either increases or decreases with $p_t^{FD}$ over all range of $p_t^{FD}$. This shows, in terms of the throughput of a tier network, operating all APs either in FD mode or HD mode achieves the maximum throughput compared to the mixture of two mode APs. For example, when $R_{\lambda,ij}$ is greater than 1 in Fig. 5 deploying FD-mode APs in all cells of network 1 achieves the maximum $S_1$.

Figuring 6 displays $S_1$ as a function of $R_{\lambda,12}$ with $\lambda_2 = 10^{-3}$ for different values of $P_{u,1}$ and $L_{dB,1}$ in dB and different duplex modes. Note that $S_1$ increases with $\lambda_1$ as shown in (32). From Fig. 6 it can be seen that for $R_{\lambda,12} < 4$, the HD mode achieves higher $S_1$ than the FD mode when $L_{dB,1} = -30$, but...
it becomes opposite for the perfect self-IC, i.e., \( L_{db,1} = -\infty \). This is also verified in Corollary 5 which shows the optimal portion of FD-mode APs is \( \hat{p}^{FD}_{k} = 1, \forall k \), for \( C^{FD}_r(P_r) = 0 \). From Fig. 6 it can be also seen that, for \( R_{\lambda,12} < 4 \), \( S_1 \) in FD mode increases as \( P_{a,1} \) increases for \( L_{db,1} = -\infty \) while it decreases for \( L_{db,1} = -30 \). This is due to the fact that, for high self-IC capability, the network throughput in FD mode increases with \( P_{a,1} \) since higher \( P_{a,1} \) provides more reliable communication between a user and its associated AP. On the other hand, for low self-IC capability, the self-interference mainly determines the network throughput, so lower \( P_{a,1} \) achieves higher \( S_1 \). From Fig. 6 we can also see that the \( S_1 \) in FD mode with \( L_{db,1} = -30 \) converges to that with \( L_{db,1} = -\infty \) as \( \lambda_1 \) increases. This is due to the fact that as we have large network interference (i.e., large \( \lambda_1 \)), the network interference mainly determines the network throughput while the effect of residual self-interference becomes marginal. Due to the relatively weak effect of self-interference for large \( \lambda_1 \), when \( R_{\lambda,12} > 4 \), \( S_1 \) in FD mode with \( L_{db,1} = -30 \) becomes larger than \( S_1 \) in HD mode as we have additional communicating users in FD mode.

Now, we present the HDHN throughput for two networks. Figure 7 shows the cell HDHN throughput \( S^c \) as a function of \( R_{\lambda,21} \) with \( \lambda_1 = 10^{-3} \) nodes/m\(^2\) for different sets \((m_1, m_2)\) of duplex modes in network 1 \( m_1 \) and network 2 \( m_2 \).

![Figure 7](image)

**Fig. 7.** Cell HDHN throughput \( S^c \) in bits/sec/Hz/cell as a function of \( R_{\lambda,21} \) with \( \lambda_1 = 10^{-3} \) nodes/m\(^2\) for different sets \((m_1, m_2)\) of duplex modes in network 1 \( m_1 \) and network 2 \( m_2 \).

From the figure, it can be seen that the best duplex mode set is (FD, FD) for large \( R_{\lambda,21} \) because the FD mode achieves better throughput than the HD mode for large \( \lambda_0 \). It can be also seen that the \( R_{\lambda,21} \) value that changes the best duplex mode from HD to FD in the network 1 is generally smaller than that in the network 2. This can be attributed to the fact that the network 1 has better self-IC capability and lower \( P_{a,1} \), so the self-interference in the network 1 is smaller than that in the network 2. Hence, the FD mode is preferred to HD mode even for small \( \lambda_2 \) in the network 1. From this figure, it can be seen that the hybrid-duplex mode set can enhance the throughput of heterogeneous network for the \( R_{\lambda,21} \) range of \( M^b = \text{(FD, HD)} \). This is also verified in the following figure.

![Figure 8](image)

**Fig. 8.** HDHN throughput \( S \) in bits/sec/Hz/m\(^2\) as a function of \( p_{1,DF}^D \) and \( p_{2,DF}^D \) (the square and the circle are the points achieving the minimum and the maximum \( S \), respectively).

Figure 8 displays the contour of \( S \) as a function of \( p_{1,DF}^D \) and \( p_{2,DF}^D \). It can be seen that \( S \) is maximized when \( p_{1,DF}^D = 1 \) and \( p_{2,DF}^D = 0 \) (the point marked by a circle in the figure), i.e., (FD, HD), which is the same result for \( R_{\lambda,21} = 1 \) in...
density of APs in HDHNs see that the best duplex mode is determined more by the total throughput of heterogeneous networks, making different tier networks operate in different duplex modes can enhance the throughput. The outcomes of our work provide insights on the efficient design of HDHNs, and opens several issues for future research on HDHNs including the transmission power control for FD- and HD-mode nodes, the throughput of MIMO FD system in the presence of network interference, the effect of network interference cancellation on the HDHN throughput, and the communication secrecy of HDHNs.

APPENDIX

A. Proof of Lemma 7

The Laplace transform $\mathcal{L}_{I^{FD}}(s)$ is given by

$$
\mathcal{L}_{I^{FD}}(s) = \exp \left\{ -2\pi \lambda_i^{FD} \int_0^\infty x \mathbb{E}_{G_i} \left\{ 1 - e^{-sG_i x^{-\alpha}} \right\} \, dx \right\}
$$

where (a) is from the Campbell’s theorem [57] and (b) is obtained by replacing $y$ for $x^{-\alpha}$. In (33), by replacing $z$ for $1/y$, the integral inside of the expectation is represented as

$$
\int_0^\infty y^{2/\alpha_i-1} \left( 1 - e^{-sG_i/y} \right) \, dy = \int_0^{\infty} z^{-2/\alpha_i-1} \left( 1 - e^{-sG_i z} \right) \, dz
$$

Figure 10 shows the cell HDHN throughput $S^c$ for two tier network as a function of the ratio $\lambda_1/\lambda_2$ for different values of $\lambda = \lambda_1 + \lambda_2$ in nodes/m$^2$ and different duplex mode set $(m_1, m_2)$.

From this figure, we can see that the maximum $S^c$ can be achieved by operating all APs in either HD or FD mode, and HD mode achieves higher network throughput than the HPG HD mode. In order to maximize the throughput of a tier network, operating all APs in either HD or FD is better than having two mode APs. On the other hand, in terms of the total throughput of heterogeneous networks, making different tier networks operate in different duplex modes can enhance the throughput. The outcomes of our work provide insights on the efficient design of HDHNs, and opens several issues for future research on HDHNs including the transmission power control for FD- and HD-mode nodes, the throughput of MIMO FD system in the presence of network interference, the effect of network interference cancellation on the HDHN throughput, and the communication secrecy of HDHNs.

V. CONCLUSION

This paper establishes a foundation for HDHNs accounting for the spatial AP distribution, the self-IC capability, and the network interference. After newly characterizing the network interference generated by FD-mode cells, we define and derive the HDHN throughput. By quantifying the HDHN throughput, we show the effect of network parameters and self-IC capabilities on the HDHN throughput, and present how to optimally determine the duplex mode to maximize the HDHN throughput. Specifically, our results demonstrate that the FD mode achieves higher network throughput than the HD mode for high self-IC capability and large AP density of HDHNs. In order to maximize the throughput of a tier network, operating all APs in either HD or FD is better than having two mode APs. On the other hand, in terms of the total throughput

\[ E_{\Pi} \left\{ \sum_{Y \in \Pi_{A} \cup A} g(Y) \right\} = \lambda_0 \int_A g(y) \, dy \]

where $A$ is a bounded space and $g(y)$ is a bounded measurable function for $y \in \mathbb{R}^d$.
Using the PDF of the AP and where $D_i$ is a typical user distance to nearest unassociated AP, given by $D_i = B_{ik}^{1/\alpha_k} D_{ik}^{\alpha_k/\alpha_i}$ in (41), we have

$$
\mathcal{L}_{\text{P}^{\text{HD}}}(s, \forall s > 0) = \exp \left\{ -2\pi \lambda_i^{\text{HD}} \int_{\tilde{D}} x E_\gamma \left\{ 1 - e^{-s P_{\gamma_i}^{\text{HD}} \gamma_i} \right\} dx \right\}
= \exp \left\{ -2\pi \lambda_i^{\text{HD}} \int_{\tilde{D}} \frac{x}{1 + (s - 1 P_{\gamma_i}^{\text{HD}} \gamma_i)} dx \right\}
$$

Finally, substituting (2), (43) and (44) into (40) results in (15).

10The (44) is obtained when the biased distance to the associated AP from a typical user $W_i D_i^{\gamma_k}$ is smaller than that to any APs. In FD-mode cells, the biased distance from the typical user to a user, who associates to another AP, can be smaller than that to the associated AP. However, we ignore this case since a user generally transmits with smaller power than an AP and for analytical tractability.
C. Proof of Corollary

When $R_a = R_0$, the throughput of $k$th-tier network $S_k$ in (30) is represented by
\[
S_k = \frac{R_i}{2W} \lambda_k \left( \sum_{i \in K} \lambda_i B_{ik}^{2/\alpha} \right) S_k^{\prime}\tag{45}
\]
where $S_k^{\prime}$ is given by
\[
S_k^{\prime} = \frac{1}{\sum_{i \in K} \lambda_i \zeta_{ik}(\alpha, P_{a,k})} + \frac{p_k^{FD}}{\sum_{i \in K} \lambda_i \zeta_{ik}(\alpha, P_{a,k})}\tag{46}
\]
From (16), $\zeta_{ik}(\alpha, P_t)$ in (46) can be represented by
\[
\zeta_{ik}(\alpha, P_t) = \delta_{ik}(P_t) t^{FD} + \delta_{ik}(P_t)\tag{47}
\]
where $\delta_{ik}(P_t)$ and $\delta_{ik}(P_t)$ are given by
\[
\delta_{ik}(P_t) = \delta_{ik}(P_t) \sigma_{ik}(\alpha, P_t) - \sigma_{ik}(\alpha, P_t) + \frac{\gamma_{ik}(P_t)}{2}.
\]
Here, for the equal density $\lambda_0$ of FD-mode and HD-mode APs, $L_{\text{FD}}^{\text{FD}}(s) \leq L_{\text{HD}}^{\text{FD}}(s) \leq L_{\text{HD}}^{\text{HD}}(s)$. Since $L_{\text{HD}}^{\text{DF}}(s) = \exp\left\{-2\pi \lambda_0 r^2 \sigma_{ik}(\alpha, P_t)\right\}$ and $L_{\text{HD}}^{\text{DF}}(s) = \exp\left\{-2\pi \lambda_0 r^2 \sigma_{ik}(\alpha, P_t)\right\}$, we have
\[
\sigma_{ik}(\alpha, P_t) \leq \delta_{ik}(P_t) \leq 2\sigma_{ik}(\alpha, P_t).
\]
Hence, $\delta_{ik}(P_t) > 0$ and
\[
\delta_{ik}(P_t) \leq \tilde{\delta}_{ik}(P_t)\tag{48}
\]
In (46), $\sum_{i \in K} \lambda_i \zeta_{ik}(\alpha, P_{a,k})$ can be presented as a function of $p_k^{FD}$ as
\[
\sum_{i \in K} \lambda_i \zeta_{ik}(\alpha, P_{a,k}) = c_1(P_t) p_k^{FD} + c_2(P_t)\tag{49}
\]
where $c_1(P_t)$ and $c_2(P_t)$ are given by
\[
c_1(P_t) = \lambda_k \delta_{ik}(P_t)\tag{50}
\]
\[
c_2(P_t) = \lambda_k \tilde{\delta}_{kk}(P_t) + \sum_{j \neq k, j \in K} \lambda_j \zeta_{jk}(\alpha, P_t) .
\]
Then, using (49) in (46), we can obtain the first derivative of $S_k^{\prime}$ according to $p_k^{FD}$ as
\[
\frac{\partial S_k^{\prime}}{\partial p_k^{FD}} = \frac{-c_1'(P_t) p_k^{FD} + c_2'(P_t)}{(c_1(P_t) p_k^{FD} + c_2(P_t))^2} + \frac{c_2'(P_t) p_k^{FD} + c_2(P_t)}{(c_1(P_t) p_k^{FD} + c_2(P_t))^2}\tag{51}
\]
for all $i \in K$. Here, $c_1(P_t) \leq c_2(P_t)$ since $\delta_{ik}(P_t) \leq \tilde{\delta}_{ik}(P_t)$ in (48), and
\[
c_1(P_{a,k}) p_k^{FD} + c_2(P_{a,k}) \geq c_1(P_{a,k}) p_k^{FD} + c_2(P_{a,k}), \forall k, \forall p_k^{FD} .
\]
Hence, $c_1(P_t)$ and $c_2(P_t)$ are decreasing function according to $P_t$, and we can see that $\frac{c_1'(P_{a,k}) p_k^{FD} + c_2'(P_{a,k})}{c_1(P_{a,k}) p_k^{FD} + c_2(P_{a,k})}$ in (51) decreases as $P_{a,k}$ decreases (i.e., as both $c_1(P_{a,k})$ and $c_2(P_{a,k})$ increases).

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