Gas depletion in primordial globular clusters due to accretion on to stellar-mass black holes

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Abstract
We consider the effect of compact stellar remnants on the interstellar medium of a massive star cluster following the initial burst of star formation. We argue that accretion on to stellar-mass black holes is an effective mechanism for rapid gas depletion in clusters of all masses, as long as they contain progenitor stars more massive than $M_\odot \gtrsim 50$. This scenario appears especially attractive for the progenitor systems of present-day massive globular clusters which likely had masses above $M \gtrsim 10^7 \, M_\odot$. In such clusters, alternative mechanisms such as supernovae and stellar winds cannot provide a plausible explanation for the sudden removal of the primordial gas reservoir that is required to explain their complex chemical enrichment history.

In order to consider different regimes in the rate of gas accretion on to stellar-mass black holes, we consider both the Bondi–Hoyle approximation as well as Eddington-limited accretion. For either model, our results show that the cluster gas can be significantly depleted within only a few tens of Myr. In addition, this process will affect the distribution of black hole masses and, by extension, may accelerate the dynamical decoupling of the black hole population and, ultimately, their dynamical ejection. Moreover, the time-scales for gas depletion are sufficiently short that the accreting black holes could significantly affect the chemistry of subsequent star formation episodes.

The gas depletion times and final mass in black holes are not only sensitive to the assumed model for the accretion rate, but also to the initial mass of the most massive black hole which, in turn, is determined by the upper mass cut-off of the stellar initial mass function. Given that the mass function of ‘dark’ remnants is a crucial parameter for their dynamical ejection, our results imply that their accretion history can have an important bearing on the observed present-day cluster mass-to-light ratio. In particular, we show that the expected increase of the upper mass cut-off with decreasing metallicity could contribute to the observed anticorrelation between the mass-to-light ratio and the metallicity of globular clusters.

Key words: accretion, accretion discs – black hole physics – stars: abundances – stars: formation – globular clusters: general.

1 INTRODUCTION

Over the last few years, observational evidence has accumulated that few, if any, globular clusters (GCs) consist of a single stellar population with a well-defined age and chemical composition. Instead, multiple stellar populations have been observed in a number of massive Galactic GCs (e.g. Gratton, Carretta & Bragaglia 2012). Photometric evidence for multiple populations comes in the form of split sequences in the colour–magnitude diagram during at least one of the various stellar evolutionary phases, i.e. main sequence (MS), sub-giant branch, horizontal branch, etc. (e.g. Piotto et al. 2007). More detailed spectroscopic studies have shown that the different stellar populations within the same GC often show differences in the relative abundances of light elements involved in the proton-capture process such as C, N, O, F, Na, Mg, Al or Si (e.g. Osborn 1971; Kraft 1979; Gratton, Sneden & Carretta 2004).

This can only be explained if the different stellar populations are born from gas with different chemical compositions. A popular scenario to explain the time-variable chemistry of the gas reservoir invokes the ‘pollution’ of the pristine gas by the chemically enriched winds from asymptotic giant branch (AGB) stars of the first
stellar generation (e.g. Gratton et al. 2001; Ramirez & Cohen 2002; Carretta et al. 2010).

This ‘self-enrichment’ scenario, however, is complicated by the fact that the spectroscopic data also demonstrate that the different stellar populations in a GC often show the same abundances of iron peak and other elements that are produced in Type II supernovae (SNe II). The chemical peculiarities also cannot be limited to surface contamination, or they would be diluted after the first dredge-up episode (e.g. Gratton et al. 2012). Currently, no scenario offered to explain the presence of multiple populations in GCs is able to fully explain the peculiar abundance anomalies, at least not without resorting to a non-canonical initial mass function (IMF; e.g. Bekki 2011; Gratton et al. 2012). For instance, the merging of primordial clusters or gas clumps struggles to simultaneously explain both the similarities in the α-element abundances and the differences in the light element abundances (e.g. Bekki 2012). The simplest solution to explain these peculiar abundance patterns is that an efficient mechanism for gas depletion must have operated after the formation of the first generation, but before the birth of the second generation (e.g. Conroy & Spergel 2011; Conroy 2012).

Building on previous work (Cottrell & Da Costa 1981; Smith 1987; Carretta et al. 2010), the following picture for the formation of multiple stellar populations in massive GCs has therefore been proposed by Conroy & Spergel (2011): the most massive members of the first stellar generation quickly reach the end of their lifetime and explode as SNe II. The energy injected into the interstellar medium (ISM) from these explosions can remove the material that is enriched in α-process elements from the GC and can interrupt the star formation. Over time, a second gas reservoir is then created from the ejecta of either AGB stars (Ventura et al. 2001) or rotating massive stars (Decressin et al. 2007), combined with any remaining (and/or additional) pristine gas. After a few hundred Myr, a second generation of stars is born from this mix of processed and unprocessed material. To first order, this scenario could explain the observed abundance variations in the light elements while avoiding differences in the α-element abundances.

However, the question is whether the SNe II can indeed remove (only) the gas component that is enriched in α-elements. Theoretical arguments suggest that this is only feasible in clusters less massive than \(\sim 10^5 \, M_\odot\) (Dopita & Smith 1986; Krause et al. 2012), because more massive clusters contain enough gas that the SNe II cannot plausibly unbind it all. The masses of present-day GCs are likely to be only a small fraction of their original mass (e.g. D’Ercole et al. 2008). It follows that for the most massive Galactic GCs, including ω Cen (e.g. Pancino et al. 2003), NGC 1851 (Bekki & Yong 2012), 47 Tuc (Milone et al. 2012a), NGC 6752 (Milone et al. 2010), M22 (Milone et al. 2012b) and others, it appears likely that the initial gas reservoir had a mass of at least \(\sim 10^4 \, M_\odot\), and probably more. It thus appears doubtful that SNe II were able to efficiently remove the α-enhanced gas from these clusters.

A number of alternative scenarios for the ‘purging’ of the gas reservoir in massive stellar clusters can be found in the literature. For example, Herwig et al. (2012) invoke Galactic plane passages and the associated ram pressure stripping of the gas content to explain the enrichment history of ω Cen, thus elaborating on earlier work by Tayler & Wood (1975). However, in order for this scenario to explain the multiple stellar populations in an arbitrary GC, some fine-tuning between their age difference and the orbital period of the cluster may be required. In many cases, the orbital period could be altogether too long for this mechanism to be viable, particularly for clusters at large Galactocentric radii. Another alternative was proposed by Spergel (1991) who invokes the relativistic winds of millisecond pulsars as a catalyst for gas expulsion. Lastly, the feedback from X-ray binary jets on to the ISM could also contribute (e.g. Fender, Maccarone & van Kesteren 2005; Justham & Schawinski 2012).

Although the precise mechanism for gas depletion in primordial GCs is currently unknown, the effects on the subsequent cluster evolution can be dramatic. For example, Marks, Kroupa & Baumgardt (2008) showed that rapid gas expulsion can unbind a significant fraction of stars in the cluster outskirts. It can even result in complete cluster dissolution, especially in the absence of primordial mass segregation (e.g. Marks & Kroupa 2010; Kroupa 2012). Thus, in addition to accounting for the peculiar abundance anomalies, gas depletion could play a crucial role in deciding the present-day relative numbers of first- and second-generation stars in GCs hosting multiple populations (e.g. D’Ercole et al. 2008; Conroy 2012).

In this paper, we consider yet another mechanism for the removal of gas after an initial burst of star formation, namely the accretion of the ISM onto compact stellar remnants such as BHs and neutron stars (NSs). This scenario is appealing because it can significantly deplete the gas reservoir even for very large initial cluster masses and does not require any specific orbital conditions. We will show that, in principle, the most massive stellar BHs with parent stars at the high end of the IMF can be very effective in depleting the gas within the cluster. This can significantly modify the final BH mass distribution, and therefore accelerate the phase of BH–BH ejections that, as theoretical studies suggest, should remove most BHs from the cluster (e.g. Phinney & Sigurdsson 1991; Downing et al. 2010; Banerjee, Baumgardt & Kroupa 2010).

The high-mass end of the stellar IMF of primordial GCs and, by extension, the number of BH progenitors are highly uncertain (e.g. Maeder 2009). In open clusters and star-forming associations, however, it has been shown that the maximum IMF mass correlates with the total cluster mass, and stars sufficiently massive to be the progenitors of BHs should form in clusters more massive than \(\sim 10^3 \, M_\odot\) (e.g. Kirk & Myers 2011, 2012). Thus, we expect that at least some BHs should have formed in a typical primordial GC. In support of this, there now exists compelling observational evidence in favour of BHs residing in present-day GCs. Maccarone et al. (2007) confirmed the first such BH in the giant elliptical galaxy NGC 4472 in the Virgo Cluster. More recently, Strader et al. (2012) reported two flat-spectrum radio sources in the Galactic globular cluster M22, which the authors argue are accreting stellar-mass BHs. This result suggests that the dynamical ejection of BHs from clusters may not be as efficient as previously predicted, and that a cluster such as M22 could contain of the order of \(\sim 5−100\) stellar-mass BHs. Observational evidence has also been reported in favour of an intermediate-mass black hole (IMBH) existing in the globular cluster M54. Ibata et al. (2009) first reported the detection of stellar density and kinematic cusps in M54, which they interpreted as being due to the presence of a central BH with a mass of \(\sim 9400 \, M_\odot\). More recently, however, Wrobel, Greene & Ho (2011) used Chandra and Hubble Space Telescope astrometry to rule out the existence of an X-ray counterpart to the proposed IMBH, and placed an improved upper limit on its luminosity of \(L(8.5 \, GHz) < 3.6 \times 10^{29}\) erg s\(^{-1}\). Nevertheless, the observational evidence supports the presence of at least some BHs in present-day GCs. Assuming that BHs are efficiently ejected during the dynamical evolution of the cluster, it is likely that a more substantial population of stellar-mass BHs once existed in primordial GCs.

Following this introduction, we describe in Section 2 our assumptions for the initial conditions, i.e. the stellar-mass function and dynamical state of the cluster at the time of BH formation. Using an analytic closed-box model for the evolution of the
remnant mass function due to accretion from the ISM, we consider two theoretical models for accretion of ISM on to a BH, namely the Bondi–Hoyle approximation and the Eddington limit. These models, which are described in more detail in Section 3, represent two different mass-dependencies for the accretion rate. The results of our analysis, which are presented in Section 4, show that the accretion of gas on to compact stellar remnants can have a significant effect on subsequent episodes of star formation, and even present-day mass-to-light ratios. We discuss the implications of our findings on models of GC formation in Section 5, and summarize our work in Section 6.

2 INITIAL CONDITIONS

In this section, we present an analytic model for the formation of a massive star cluster from its parent giant molecular cloud.

2.1 Progenitor mass function

We consider a molecular cloud of total mass \( M \) that condenses to form a star cluster at time \( t = 0 \). We assume a star formation efficiency (SFE) of 25 per cent, so that, after the cloud is born, the total mass in stars and gas are \( M_s = 0.25M \) and \( M_g = 0.75M \), respectively. While this value for the SFE agrees with available theoretical constraints (e.g. McKee & Ostriker 2007), we point out that our results are insensitive to the precise value of the SFE, as long as it is neither extremely low or extremely high. In the first limit, the stellar mass of the infant cluster would be too small for many massive stars to form, and hence the number of BHs would be limited. At the other extreme, an extremely high SFE would leave little gas, and hence the BHs could not grow significantly in mass. Our chosen value for the SFE stays safely away from these regimes.

We assume an IMF for the stellar population according to

\[
f_\alpha(m) = \frac{dN}{dm} = \beta m^{-\alpha},
\]

where \( N \) is the number of stars with a given stellar mass \( m \), and \( \alpha \) and \( \beta \) are constants. We assume a power-law slope of \( \alpha = 2.3 \) (Salpeter 1955). This choice minimizes the number of high-mass stars relative to other IMFs used in the literature, so that the estimates we derive in Section 4 for the number of stellar remnants and the amount of gas accreted by them are conservative.

As a technical point, the parameter \( \beta \) is needed to ensure that the correct total stellar mass is preserved when integrating the IMF. It is determined by normalizing equation (1) using the total stellar mass:

\[
M_s = \int_{m_{\text{min}}}^{m_{\text{max}}} f_\alpha(m) m \, dm,
\]

where \( m_{\text{min}} \) and \( m_{\text{max}} \) are the minimum and maximum stellar masses, respectively. Integrating equation (2) with respect to \( m \) and solving for \( \beta \) yields

\[
\beta = \frac{M_s (2 - \alpha)}{m_{\text{max}}^{-\alpha} - m_{\text{min}}^{-\alpha}}.
\]

Plausible values for the upper and lower mass cut-off at low metallicity are \( m_{\text{min}} = 0.08 \, M_\odot \) and \( m_{\text{max}} = 150 \, M_\odot \) (e.g. Dabringhausen et al. 2012). However, we consider several different values for \( m_{\text{max}} \) in order to test the sensitivity of our results to this assumption. The mass cut-off values are chosen to be conservative for very low metallicity since, as we will show, a higher initial maximum BH mass translates into a shorter gas depletion time-scale. It follows that our results correspond to upper limits for the gas depletion time-scale due to accretion on to stellar-mass BHs.

2.2 Progenitor velocity distribution

In order to calculate the relative velocity between the accretor and the gas (which affects the accretion rate) as well as the retention fraction of stellar remnants after they experience a potential natal kick, the velocity distribution of the progenitor population is required.

In this respect, we assume that the core radius \( r_c \) is initially comparable to the half-mass radius \( r_h \), since primordial clusters are expected to be more extended than their (dynamically evolved) present-day counterparts (e.g. Spitzer 1987; Heggie & Hut 2003). We further assume primordial mass segregation, i.e. that all remnants form inside \( r_c \), and that energy equipartition has been achieved within \( r_c \) (we will come back to these assumptions in Section 5). We therefore adopt a Maxwell–Boltzmann velocity distribution for the progenitors at every stellar mass (e.g. Binney & Tremaine 1987):

\[
f_\sigma(m, v) = N(m) e^{-\frac{v^2}{2\sigma(m)^2}},
\]

where \( N(m) \) is the number of stars with mass \( m \), \( \sigma(m) \) is the velocity dispersion for stars of mass \( m \), and \( v \) is the stellar velocity. For the velocity dispersion \( \sigma(m) \), we assume

\[
\sigma(m) = \sqrt{\frac{\dot{m}}{m}} \sigma_0,
\]

where \( \sigma_0 \) is the central velocity dispersion of the average stellar mass \( \dot{m} \). For a King model, this is (Binney & Tremaine 1987)

\[
\sigma_0 = \left( \frac{4\pi G}{9} \right) \left( \rho_c + \rho_0 \right)^{1/2},
\]

where \( r_c \) is the core radius, \( \rho_c \) is the average stellar-mass density in the core and \( \rho_0 \) is the average gas mass density in the core.

2.3 Progenitor lifetimes

For the progenitor lifetime \( \tau_\text{p} \), we assume that the MS lifetime \( \tau_{\text{MS}}(m) \) provides a good approximation, provided the progenitor mass is \( m \geq 18 \, M_\odot \). This seems justified because the MS lifetimes of low-mass stars greatly exceed that of every other evolutionary phase, typically by several orders of magnitude (e.g. Clayton 1968; Iben 1991; Maeder 2009). Note that \( \tau_{\text{MS}}(m) \) is a function of only the stellar mass. In other words, we are ignoring any metallicity dependence, since metallicity should only weakly affect the total stellar lifetime. For the MS lifetime, we assume (Hansen & Kawaler 1994)

\[
\tau_{\text{MS}}(m) = \tau_0 \left( \frac{m}{M_\odot} \right)^{-2.5},
\]

with \( \tau_0 = 10^{10} \) yr. For progenitor masses \( m \geq 18 \, M_\odot \), we impose a fixed total lifetime of 7 Myr (note that equation 7 yields the same MS lifetime of 7 Myr for \( m = 18 \, M_\odot \)). This is in rough agreement with stellar evolution models (Rob Izzard, private communication), which predict a near-constant lifetime for massive stars at low metallicity (e.g. Iben 1991; Hurley, Pols & Tout 2000; Maeder 2009). Thus, our final estimate for the total progenitor lifetime is

\[
\tau_\text{p} = \max(\tau_{\text{MS}}(m), 7 \, \text{Myr}).
\]

2.4 Initial remnant mass function

For the initial remnant masses, we use equations (4), (5), and (7) from Fryer et al. (2012) for progenitors in the mass ranges
\[ 7.2 \leq m/M_{\odot} < 11, \quad 11 \leq m/M_{\odot} < 30, \quad \text{and} \quad m \geq 30 \, M_{\odot}, \text{respectively.} \]

In that order, the initial remnant masses are
\[ M_i = 1.35 M_{\odot}, \quad (9) \]
\[ M_i = 1.1 + 0.2e^{(M_p - 11.0)/4.0} - 2.01e^{0.6(M_p - 26.0)}, \quad (10) \]
\[ M_i = \min(33.35 + 4.76(M_p - 34.0), M_p - 0.1(1.3M_p - 18.35)), \quad (11) \]

where \( M_p \) and \( M_i \) are the progenitor and remnant masses, respectively, in units of solar masses, and we have used \( Z = 0.01 \, Z_{\odot} \), with \( Z_{\odot} \) denoting solar metallicity. This choice of metallicity roughly agrees with the average for Milky Way GCs (e.g. Harris 1996, 2010 update) and should be appropriate for a star cluster composed of Population II stars.

For each progenitor mass range, two equations are given in Fryer et al. (2012) for the initial-final mass relations, one assuming a delayed supernova explosion and the other assuming a rapid explosion. The delayed explosion model predicts considerably greater kick velocities that are roughly consistent with what is observed for pulsars in the Galactic field (Hobbs et al. 2005). Therefore, we assume a delayed supernova explosion in choosing our initial-final mass relations.

However, we note that our results are not sensitive to this choice. This is because the gas accretion is completely dominated by the most massive BHs, specifically those with masses \( \geq 50 \, M_{\odot} \). In this high-mass regime, the differences in the final remnant mass functions between the delayed and rapid explosion models of Fryer et al. (2012) are negligible. For the same reason, we do not discuss gas accretion on to NSs and white dwarfs further: they do not affect the results presented here. Nevertheless, in order to check the plausibility of our assumptions, we compare the retention fractions of NSs to previous estimates in the literature in Section 4.3.

### 2.5 Remnant velocity distribution

The accretion rate depends on the relative velocity between the gas and the accretor. Thus, it is important to obtain a realistic final remnant velocity distribution from the initial progenitor velocity distribution.

The primary factor that determines the initial remnant velocity distribution is natal kicks. These are imparted to most NSs, and some (low-mass) BHs, at the time of their formation. The precise physical mechanism responsible is unknown, but appears to be related to asymmetries in the collapse of the progenitor core, or the subsequent supernova explosion (e.g. Pfahl, Rappaport & Podsiadlowski 2002a). Typical kick speeds range from 100 to 200 km s\(^{-1}\), although radio pulsars with speeds in excess of 1000 km s\(^{-1}\) have been observed (e.g. Lyne & Lorimer 1994; Cordes & Chernoff 1998; Hobbs et al., 2005), and kick speeds \( \leq 50 \, \text{km s}^{-1} \) are also thought to be possible (e.g. Blaauw 1961; Pfahl et al., 2002b; Pfahl, Rappaport & Podsiadlowski 2002b; Podsiadlowski et al., 2004; Smits et al., 2006; Paolillo et al., 2011).

We adopt different assumptions for the kick speed for different progenitor mass ranges. These assumptions are based on the physical processes thought to be driving the kicks. Here, we present only our assumed kick velocities for each mass range and refer the interested reader to Fryer et al. (2012) for more details. Specifically, we assume that no kick occurs for progenitor masses \( < 11 \, M_{\odot} \) (e.g. Podsiadlowski et al. 2004). For progenitors in the mass range \( 11 \leq m/M_{\odot} < 30 \) (Fryer & Kalogera 2001), we adopt a kick velocity \( \nu_{\text{kick}} \) by randomly sampling from the distribution of observed velocities reported by Hobbs et al. (2005) for radio pulsars in the field of our Galaxy. For progenitors with masses \( 30 \leq m/M_{\odot} < 40 \), we adopt a constant kick velocity of 50 km s\(^{-1}\), since the amount of fallback is thought to be less sensitive to the explosion mechanism in this range of progenitor masses (e.g. Fryer et al. 2012). Finally, for progenitors with \( m/M_{\odot} \geq 40 \), we assume prompt collapse to a BH without a supernova explosion, and thus without any kick.

In order to calculate the initial velocity distribution for our remnant population, we add the kick velocity \( \nu_{\text{kick}} \) to a randomly drawn velocity \( \nu_0 \) from equation (4) for the appropriate progenitor mass. This is done using randomized vector addition, since there is no reason to expect that the direction of the kick should be correlated with the direction of motion of the progenitor through its host cluster. The resulting velocity \( \nu_{\text{rel}} \) is then compared to the escape velocities from both the core and the cluster. The latter quantities are calculated using the relations
\[ \nu_{\text{esc,c}} = \sqrt{\frac{2G M_c}{r_c}} \quad (12) \]
\[ \nu_{\text{esc}} = \sqrt{\frac{2GM}{r_c}}, \quad (13) \]

where \( M_c \) is the total (stellar and gas) mass of the core, \( M \) is the total (stellar and gas) cluster mass and \( r_c \) is the core radius (recall that we have assumed that \( r_c \approx r_i \) initially).

A comparison between \( \nu_{\text{rel}} \) and \( \nu_{\text{esc}} \) determines the NS and BH retention fractions immediately after their formation. If the final velocity is less than the escape velocity of the core (i.e. \( \nu_{\text{rel}} < \nu_{\text{esc,c}} \)), we leave it unchanged from the velocity previously drawn from the distribution in equation (4). This is because we assume that the remnant will rapidly re-achieve energy equipartition with the rest of the core, where the local relaxation time is much shorter than the global relaxation time (e.g. Heggie & Hut 2003). If the remnant ends up with a velocity greater than the escape velocity from the cluster immediately after it receives its kick (i.e. \( \nu_{\text{rel}} > \nu_{\text{esc}} \)), then it is removed completely from the remnant population. However, if the post-kick velocity exceeds the escape velocity from the core but not that from the cluster (i.e. \( \nu_{\text{esc,c}} < \nu_{\text{rel}} < \nu_{\text{esc}} \)), then the remnant will temporarily leave the core but still remain bound to the cluster. In this case, we calculate the radius to which the remnant is kicked (called the 'kick radius') using conservation of energy. This requires two additional assumptions, namely that the kick is always imparted at a distance \( r_i/2 \) from the cluster centre and that the resulting orbit of the kicked remnant within its host cluster is entirely radial. We then calculate the fallback time \( \tau_{\text{FB}} \), which decides when the remnant will return to the core. This is given by
\[ \tau_{\text{FB}} = 2\nu_{\text{rel}} r_c / GM_{\text{c}}. \quad (14) \]

This assumption is justified given that primordial GCs are thought to have been more extended at birth than they are now. It follows that stars should be relatively unaffected by dynamical friction once outside the core (e.g. Mapelli et al. 2006; Leigh, Sills & Knigge 2011).

If \( \tau_{\text{MS}} + \tau_{\text{FB}} < \tau_i \), where \( \tau_i \) denotes the time from cluster birth until the gas reservoir is depleted, then the remnant will return to the core with enough time to accrete at least some gas from the dense ISM. Once in the core, the remnant quickly re-achieves energy...
equipment with the surrounding population. The remnant’s final velocity is then once again given by equation (4).

### 3 ACCRETION MODELS

The Bondi–Hoyle approximation describes spherically symmetric accretion and is based on the assumption that the forces due to gas pressure are insignificant compared to gravitational forces (Bondi & Hoyle 1944). The background gas is treated as uniform and either stationary or moving with constant velocity relative to the accretor. This assumption gives reasonable accretion rates, provided the properties of the gas are such that the density and total angular momentum are low, at least in the vicinity of the accretor. Numerical studies have confirmed that this is indeed the case, provided the gas is not moving rapidly relative to the accretor (e.g. Fryxell & Taam 1988; Ruffert 1994, 1997; Foglizzo & Ruffert 1999). High accretion rates at least approaching the Bondi–Hoyle rate have also been observed in several high-mass X-ray binaries (e.g. Agol & Kamionkowski 2002; Barnard, Clark & Kolb 2008). It thus appears that, at least in the low-density, low-angular-momentum regime, the Bondi–Hoyle approximation is not unrealistic.

The Bondi–Hoyle approximation should be regarded as a strict upper limit to the true accretion rate. For large accretor masses, the Bondi–Hoyle rate specified in Section 3.1 can become extremely high, and pressure forces could play an important role in reducing the accretion rate. Notwithstanding, theoretical studies have confirmed that super-Eddington accretion is indeed possible for massive BHs. For example, although the accretion rates of super-massive black holes (SMBHs) at high redshift are poorly constrained (e.g. Hopkins & Quataert 2010), some have been observed at redshift \( z \approx 6 \) with masses \( \gtrsim 5 \times 10^9 \, M_\odot \) (e.g. Barth et al. 2003; Willott, McLure & Jarvis 2003). The presence of such massive BHs only \( \approx 10^7 \) yr after the big bang (King & Pringle 2006) is difficult to explain without invoking accretion rates approaching the Bondi–Hoyle limit. This should be kept in mind when discussing accreting BHs in primordial GCs, which were also born soon after the big bang and at similar low metallicity. Additionally, photon trapping can occur at very high accretion rates, which makes accretion discs radiatively inefficient and provides a means of circumventing the Eddington limit (Paczynsky & Wiita 1980). Otherwise, when an accretor is radiating at above the Eddington luminosity, significant amounts of gas can be expelled at high velocities due to the intense winds that are initiated (e.g. King & Pounds 2003).

Eddington-limited accretion applies when pressure forces become important, and the outward continuum radiation force balances the inward gravitational force (Eddington 1926, 1930). The accretion luminosity corresponding to this limit is (Rybičkí & Lightman 1979)

\[
L_{\text{Edd}} = \frac{4\pi G m c}{\kappa},
\]

where \( m \) is the accretor mass, \( c \) is the speed of light and \( \kappa \) is the electron scattering opacity (e.g. King & Pounds 2003). Compared to the Bondi–Hoyle approximation, the Eddington limit provides a much more conservative estimate for the accretion rate in the limit of high gas density. This is due to the linear dependence on the accretor mass \( m \) (see Section 3.1). Additionally, the Eddington rate should be much closer to the true accretion rate if the gas contains significant angular momentum, and accretion proceeds mainly via angular momentum re-distribution within a disc (e.g. King & Pounds 2003).

We assume a constant time-independent background density and velocity for the accreting gas, since a more sophisticated treatment is beyond the scope of this paper. To illustrate the importance of accreting BHs for prolonged star formation in primordial GCs, we use the Bondi–Hoyle and Eddington approximations to explore two different regimes in the gas properties. The Bondi–Hoyle approximation is representative of the gravity-dominated, low-angular-momentum regime, whereas the Eddington approximation is representative of the pressure-dominated or high-angular-momentum regime.

#### 3.1 Calculating the accretion rates

The Bondi–Hoyle accretion rate is given by equation 2 of Maccarone & Zurek (2012), which has been re-scaled from the formula of Ho, Terashima & Okajima (2003):

\[
\dot{M}_{\text{BH}} = 7 \times 10^{-9} M_\odot \text{yr}^{-1} \left( \frac{m}{M_\odot} \right)^2 \left( \frac{n}{10^6 \text{ cm}^{-3}} \right) \left( \frac{c_s^7 + v^2}{10^8 \text{ cm s}^{-1}} \right)^{-3}
\]

\[
= A \left( \frac{m}{M_\odot} \right)^2,
\]

where \( m \) is the accretor mass, \( n \) is the central gas density, \( c_s \) is the sound speed, and \( v \) is the relative velocity between the gas and accretor. We assume \( n \approx 10^6 \text{ cm}^{-3} \) with central temperatures of a few thousand K, which translates into \( c_s \approx 10^5 \text{ cm s}^{-1} \). These values were argued by D’Ercole et al. (2008) and Maccarone & Zurek (2012) to be representative of the cores of primordial GCs, yielding \( A \approx 10^{-9} M_\odot \text{ yr}^{-1} \) for small accretor masses and velocities.

The Eddington-limited accretion rate, in turn, is given by (e.g. King & Pounds 2003)

\[
\dot{M}_{\text{Edd}} = \frac{4\pi G m c}{\eta \kappa c},
\]

where \( c \) is the speed of light, \( \eta \kappa^2 \) is the accretion yield from unit mass and \( \kappa \) is the electron scattering opacity. We adopt \( \eta = 0.1 \) for the accretion efficiency.

#### 3.2 Evolving the remnant mass function

The most massive stars in the cluster are the first to reach the end of their lives due to stellar evolution. If gas is still present in significant quantities, the most massive stellar remnants (i.e. BHs) should begin accreting from the surrounding ISM almost immediately after their formation, since they do not experience kicks.

If a kick is imparted, there is a possibly indefinite delay before significant accretion begins, because the remnant can be expelled from the core. However, if the remnant remains bound to the cluster and the fallback time \( \tau_{\text{FB}} \) is sufficiently short, then the remnant will eventually begin accreting from the ISM after it returns to the central cluster regions. Thus, accretion is assumed to occur only in the core,
and to begin after a total time $\tau_i = \tau_{\text{MS}} + \tau_{\text{FB}}$, where $\tau_{\text{FB}} = 0$ if the remnant never leaves the core. Once returned to the core, we assume that the remnant remains there. This is not unreasonable since the time-scale for accretion to occur is comparable to the crossing time of the core, even for accretion rates lower than the Eddington rate. Therefore, the accretor velocity should be significantly reduced within a single core crossing time.

The post-accretion mass of a given remnant is calculated as follows. The total time spent accreting from the ISM can be written as

$$\tau_{\text{acc}} = \int_{\tau_i}^{\tau_f} \frac{dm}{\dot{m}} dt,$$

(18)

where the upper limit of integration $\tau_f$ corresponds to the time at which the post-accretion remnant mass is calculated, and $\tau_i = \tau_{\text{MS}} + \tau_{\text{FB}}$. Rewriting this in terms of the accretion rate $\dot{m} = dm/dt$ gives

$$\tau_{\text{acc}} = \int_{m_i}^{m_f} \frac{dm}{\dot{m}},$$

(19)

where $m_i$ and $m_f$ are the initial and final remnant masses, respectively. We then integrate and solve for $m_f$ in order to obtain the final remnant mass. Using equation (16) for the accretion rate as an example, this yields the final remnant mass:

$$m_f = m_i \left(1 - \frac{m_i}{A \tau_{\text{acc}}} \right).$$

(20)

In our analysis, we have neglected the evolution of the remnant velocity distribution during the accretion process even though, in principle, conservation of momentum causes the BHs to slow down as they accrete mass. In the Eddington-limited case, this does not affect the accretion rate, since equation (17) is independent of the accretor velocity. In the Bondi–Hoyle case, however, a reduction in the accretor velocity increases the accretion rate, and hence the rate of gas depletion. Neglecting the BH deceleration, therefore, leads to a lower limit for the accretion rate in the Bondi–Hoyle approximation, and properly accounting for these effects would only strengthen our conclusions on the efficiency of the gas depletion.

A possibly more important consequence of a lower average BH velocity is that it could lead to a more efficient migration of massive BHs towards the cluster centre, and thus could accelerate the onset of the dynamical BH ejection phase (e.g. Phinney & Sigurdsson 1991; Downing et al. 2010) or the rate of BH–BH mergers near the cluster centre, as discussed in more detail in Section 5.1. Both effects caused by gas accretion – deceleration and mass growth – should therefore be taken into account when constructing dynamical models of the BH population in gas-rich environments.

4 RESULTS

4.1 Black hole depletion times

The results of our analysis are summarized in Figs 1 and 2 which show the time evolution of the relative mass fraction of gas, stars and remnants for Bondi–Hoyle and Eddington-limited accretion, respectively. In both cases, the first BHs form 7 Myr after the birth of their progenitor population, and immediately begin accreting from the ISM.

In the case of Bondi–Hoyle accretion (Fig. 1), the steep dependence of the accretion rate on BH mass ($\dot{M} \propto m^2$) causes the most massive BH(s) to reach very high accretion rates after only a few Myr. For example, for an IMF upper mass cut-off of 150 $M_\odot$, the most massive BH reaches an accretion rate of $\dot{M} \approx 10^{-4} M_\odot \text{ yr}^{-1}$ after only $\sim 4$ Myr. At this point, it enters a phase of runaway growth and, as shown in Fig. 1, the remnant mass fraction rises asymptotically from only a few per cent to nearly 80 per cent in as little as a few thousand years. Thus, in the Bondi–Hoyle case, the entire gas reservoir is depleted within a few Myr.
of cluster formation, nearly independent of our assumption for the IMF upper mass cut-off. It follows that the gas depletion time will be under 1 Gyr provided the accretion rate \( \gtrsim 10^{-2} M_{\odot}^{-1} \). In the case of Eddington-limited accretion (Fig. 2), in contrast, a runaway accretion phase is never reached, because of the weaker (linear) dependence of the accretion rate on BH mass. Consequently, the remnant mass fraction rises gradually at first over a period of several tens of Myr, reaching 5–10 per cent (depending on the assumed IMF upper mass cut-off) within \( \sim 50 \) Myr of cluster formation. Nevertheless, the accretion rate continues to rise as the BHs grow in mass and, after \( \sim 100 \) Myr, the remnant mass fraction reaches 20–40 per cent. Thus, provided the accretion rate \( \geq 10^{-1} M_{\odot}^{-1} \), accretion on to stellar-mass BHs will significantly contribute to depleting the available gas reservoir within a few hundred Myr.

The key conclusion from these plots is that in both accretion models, stellar-mass BHs cause a non-negligible reduction in the total mass of the gas reservoir within as little as a few tens of Myr. This is the case even if we truncate the IMF upper mass cut-off at \( 60 M_{\odot} \), which is considerably lower than what is theoretically predicted for massive stars at very low metallicity (e.g. Iben 1991; Maeder 2009).

4.2 Black hole growth

The effects of a larger initial BH mass are also demonstrated in Fig. 3, which shows the maximum BH mass as a function of time for three different assumptions for the upper mass cut-off for the IMF, namely 150, 100 and \( 60 M_{\odot} \). In the Bondi–Hoyle case, the upper limit for each mass is obtained by dividing the remaining gas mass by the number of BHs in the highest mass bin (2, 6 and 19 for an IMF upper-mass cut-off of 150, 100 and \( 60 M_{\odot} \), respectively) at the time when runaway accretion sets in. This represents a theoretical upper limit to the maximum final BH mass. We caution that in the Bondi–Hoyle case, such extreme BH masses conflict with current observational upper limits in several massive GCs (e.g. Maccarone & Servillat 2008; Miocchi 2010; Kirsten & Vlemmings 2012; McNamara et al. 2012).

Our results show that the assumed model for the accretion rate and its dependence on accretor mass can have a significant effect on the final distribution of remnant masses. Interestingly, in the Bondi–Hoyle case, the accretion rates can become sufficiently high that if one or a few BHs reach these rates well before their peers, there will be a large discrepancy between the final masses of these BHs and the rest. It is even feasible that one especially massive BH (or IMBH) will completely dominate the accretion ‘race’. The possible implications of this case for the subsequent dynamical evolution of the BH population and the evolving cluster mass-to-light ratio will be discussed in Section 5.

Finally, kicks have a minor effect on BH population size due to the fact that it is only low-mass BHs that receive kicks. For initial cloud masses \( M = 10^5, 10^7 \) and \( 10^8 M_{\odot} \), we find BH retention fractions of roughly 66, 90 and 96 per cent, respectively. Kicks also do not significantly affect the gas depletion times since low-mass BHs have the lowest accretion rates. This assumes, however, that the IMF upper mass cut-off is sufficiently large that the most massive BHs will not receive kicks (see Section 2.5).

4.3 Neutron stars

Due to their relatively small masses, accretion leaves the numbers of NSs largely unchanged for up to \( \sim 100 \) Myr. Even in the Bondi–Hoyle case, many of the first NSs to form will only accrete enough material within this time frame to end up as especially massive NSs, as opposed to low-mass BHs. Thus, the primary factor affecting NS population size in the core is natal kicks. We can therefore consistently compare the NS retention fractions we obtain for our model to previous estimates given in the literature calculated using more sophisticated methods. This is meant to provide a test of the validity of our assumptions. For initial cloud masses \( M = 10^6, 10^7 \) and \( 10^8 M_{\odot} \), we find NS retention fractions of roughly 14, 37 and 73 per cent, respectively. Recently, Ivanova et al. (2008) used Monte Carlo models for GC evolution to look at NS retention due to natal kicks. Our results are in close agreement with theirs for an initial cloud mass of \( M = 10^6 M_{\odot} \) with 25 per cent SFE, which yields a total stellar mass that agrees roughly with the cluster mass considered by Ivanova et al. (2008).

5 DISCUSSION

We now discuss the implications of our results for the dynamical ejection of BHs, subsequent episodes of star formation and the present-day cluster mass-to-light ratio.

5.1 Dynamical BH ejection

Several theoretical studies suggest that BHs which form soon after the formation of a massive star cluster should decouple dynamically from the rest of the stellar population due to their much larger masses. Consequently, if they do not already form in the cluster core, they will rapidly segregate there (e.g. Sigurdsson & Hernquist 1993; Portegies Zwart & McMillan 2000). Inside the core, BH–BH binaries form dynamically through three-body scattering interactions, and most BHs are thought to be eventually ejected from the cluster due to strong gravitational interactions with other BH–BH binaries. However, it is expected to take at least a few Gyr for the bulk of the primordial BH population to be depleted in this way.
is the half-mass relaxation episode of star formation \( \approx 21 \) \( r \) is the initial BH velocity, and \( m \) are the initial (pre-
first \( m \sim \) the birth of a star is removed from the gas reservoir

\begin{equation}
\tau_{\text{seg}}(m) \approx \frac{\bar{m}}{m_{\text{bh}}}.
\end{equation}

Here, \( \bar{m} \) is the average stellar mass and \( t_{\text{bh}} \) is the half-mass relaxation time (which can be used to estimate the global rate of two-body relaxation). Once a star is inside \( r_c \), the local relaxation time can be shorter than the global or half-mass relaxation time by a factor of 100 or more (e.g. Spitzer 1987). The key point is that with or without primordial mass segregation, the stellar remnants of the most massive progenitors are likely to be found in the cluster core very early on, and to be dynamically relaxed.

As a mechanism for gas depletion, accretion on to stellar-mass BHs has the advantage that it can potentially help to explain why stars belonging to the second stellar generations in massive GCs do not always show evidence for enrichment in iron-peak elements (e.g. Gratton et al. 2012). This is because the accreting BHs can remove the polluted material from the SNe II of the first generation before the second generation is born. However, we have not considered dynamical ejections that occur between massive stars while still on the MS, an effect that should be considered in future studies. Finally, the effects of accreting BHs on the first episode of star formation should also be considered, since it may be important if star formation is an extended process. In fact, if the most massive stars reach the end of their lives while star formation is still ongoing, gas accretion on to their BH remnants could reduce the number of first generation stars and affect the shape of the IMF. To summarize, independent of the exact scenario, our results demonstrate that accretion on to BHs could significantly affect the star formation histories of massive GCs.

On the other hand, while the BHs of the first stellar generation offer a potentially attractive mechanism for gas depletion, their continued gas accretion would pose a problem for the formation of the second stellar generation, unless the BHs are efficiently removed from the cluster core after their formation, or sufficiently massive BHs never formed in the first place. This should be considered in any proposed scenario explaining the complex star formation history of GCs.

5.3 Black hole winds

Depleting the gas reservoir via accretion does not necessarily require that the accreted matter end up bound to the accretor(s). If the accretion rate surpasses the Eddington limit, powerful winds could develop that might expel any remaining gas from the cluster (e.g. King & Pounds 2003). It is possible that much of the emission could be in the UV, which could in turn result in photoionization heating of the ISM beyond the escape velocity from the cluster. Properly

2 It is sufficient that they are kicked to sufficiently large cluster radii that the local dynamical friction time-scale exceeds the time-scale for star formation in the core.
quantifying the implications of these effects requires detailed numerical modelling. This technology is currently unavailable in light of the considerable computational difficulties in achieving the necessary resolution (e.g. Hopkins & Quataert 2010).

5.4 The cluster mass-to-light ratio

The considerable uncertainties in the accretion rate on to BHs, the stellar IMF and the total amount of accreted material make it difficult to accurately predict the final mass-to-light ratio of the stellar cluster from our simplistic model.

In general, if the BHs accrete from the surrounding ISM, and are retained by their host cluster (e.g. the BHs merge rather than being dynamically ejected), one may expect an anticorrelation between the cluster metallicity and its mass-to-light ratio. This is because theoretical studies suggest that the IMF upper mass cut-off increases with decreasing metallicity (e.g. Abel, Bryan & Norman 2002), which, in turn, accelerates the mass growth of BHs (see Section 4). Interestingly, such an anticorrelation between the metallicities of GCs and their mass-to-light ratio has indeed been observed by Strader, Caldwell & Seth (2011) in M31.

If, on the other hand, higher accretion rates result in a more efficient dynamical ejection of BHs, as discussed in Section 5.1, such (nearly) all BHs are dynamically ejected, one may even expect a correlation between the cluster metallicity and mass-to-light ratio. We conclude that a more sophisticated model is needed to constrain the dependence of the final mass-to-light ratio on metallicity. The key point arising from our model is that accretion from the ISM on to a population of remnants in a primordial GC can significantly affect its present-day mass-to-light ratio.

6 SUMMARY

In this paper, we have considered accreting BHs as a mechanism to deplete primordial GCs of their gas after the birth of their progenitor population. This mechanism should be effective in clusters of all masses, as long as they are able to form massive stars that turn into BHs.

We have considered both the Bondi–Hoyle approximation and the Eddington limit in order to explore two different regimes in the mass dependence of the accretion rate. Our results suggest that accreting BHs are able to deplete the entire gas reservoir in as little as ~10 Myr, if the Bondi–Hoyle rate is assumed. Even at significantly lower accretion rates of the order of the Eddington limit, BHs can accrete a significant fraction of the total gas reservoir within a few tens of Myr. This time-scale is sufficiently short that accreting BHs would significantly impact the chemistry of any subsequent episodes of star formation. The accretion rates are sensitive to the IMF upper mass cut-off, in the sense that reducing the mass of the most massive BH by a factor of ~2 will result in gas depletion times that are longer by a factor of ~2.

At the end of the accretion period, the average BH mass will have significantly increased. This is likely to accelerate the dynamical decoupling of the BH population from the rest of the cluster system, and thus may result in a more efficient ejection of the BH population from the cluster. We have discussed the implications of this effect on subsequent episodes of star formation.

Lastly, we have pointed out that accretion on to stellar remnants may significantly affect the present-day mass-to-light ratio, although we are currently unable to precisely quantify the magnitude of this effect.

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