Multi-jet physics at high-energy colliders and TMD parton evolution

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Summary. — We discuss implications of the evolution of transverse momentum dependent (TMD) parton distributions on the structure of multi-jet states at high energies. In particular we analyze the theoretical systematics associated with multi-jet merging. We introduce a new merging methodology incorporating TMDs, illustrate its main features and present a comparison of our theoretical results with experimental measurements for $Z$-boson + jets production at the Large Hadron Collider (LHC).

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1. – Introduction

In the last few years, experimental studies of Drell-Yan (DY) lepton pair production [1, 2, 3, 4] and deep inelastic scattering [5, 6] have underlined the role of TMD parton evolution [7]. See e.g. [8] for a recent study of the interplay between perturbative and non-perturbative effects induced by TMD evolution in the DY spectrum at low transverse momenta.

The impact of TMD distributions on the high transverse momentum region and on multi-jet production, on the other hand, is as yet unexplored, and constitutes the subject of the work presented in this article.

Theoretical predictions for multi-jet observables have relied for the past twenty years on “merging” techniques [9, 10, 11, 12, 13] to combine matrix-element and parton-shower event generators. The former describe the underlying hard process with bare partons providing the primary sources for widely separated jets; the latter describe the evolution of partons by radiative processes predominantly at small angles; and the two are sewn together, so as to avoid either double counting or missing events, via a “merging scheme” and merging scale. The choice of the merging scheme and merging scale is one of the main...
theoretical systematics in studies of multi-jet final states at high-energy hadron colliders, investigated at leading order (LO) [13, 14] and next-to-leading order (NLO) [15, 16, 17, 18].

Transverse momentum recoils in the shower evolution may be taken into account through TMD parton distributions [7] and can influence the theoretical systematics associated with combining matrix-element and parton-shower contributions [19, 20], thus affecting the dependence of multi-jet cross sections on the merging scale. Motivated by this observation, in [21] we devise a systematic procedure of multi-jet merging, dubbed “TMD merging”, which extends to the case of TMD parton evolution the familiar merging approach [9, 10, 11, 12, 13]. We use TMD merging to analyze theoretical systematic uncertainties in multi-jet observables and to perform comparisons of theoretical predictions with experimental measurements for Z-boson + jets production at the LHC [22, 23].

The analysis [21] employs the parton branching (PB) formulation of TMD evolution set out in [24]. It constructs a merging at LO level expanding on the MLM matching prescription [11, 12, 13, 14]. A similar construction is possible starting from other approaches, such as CKKW-L [9, 10].

In what follows we begin by discussing parton kT broadening effects due to TMD evolution, and their implications for multi-jet production (Sec. 2). Then we present the TMD merging method (Sec. 3), illustrate a few applications to Z-boson + jets production (Sec. 4), and discuss the associated theoretical systematics (Sec. 5). We finally give concluding remarks (Sec. 6).

2. – kT broadening from TMD evolution

We consider the broadening in the transverse momentum kT of the partonic initial state which results from TMD evolution [21]. For a multi-jet final state characterized by the hard momentum-transfer scale µ, we analyze the contribution to the production of an extra jet with transverse momentum pT < µ from the high-kT tail of the initial state parton distribution, kT > pT. To estimate this, we introduce integral TMD distributions a_j, obtained from the TMD distributions A_j by kT-integration as follows

\[ a_j(x, k^2, \mu^2) = \int \frac{d^2k'}{\pi} A_j(x, k'^2, \mu^2) \Theta(k'^2 - k^2). \]

The distribution a_j evaluated at kT = 0 gives the fully integrated initial-state distribution. We are interested in the fractional contribution to it from the tail arising above transverse momentum kT, with kT of the order of the jet pT. For any flavor j we thus construct the ratio

\[ R_j(x, k^2, \mu^2) = a_j(x, k^2, \mu^2)/a_j(x, 0, \mu^2). \]

In Fig. 1 we illustrate the kT dependence of Eq. (2) by an example showing the integral TMD gluon distribution a_g(x, k^2, \mu^2) normalized to kT = 0 for x = 10^{-2} and various values of \mu, obtained from the TMD fitted in [25] to precision DIS data using xFitter [26] (for other available TMD fits, see the library [27]). We observe, for instance, that for \mu = 100 (500) GeV, there is a 30% probability that the gluon has developed a transverse momentum larger than 20 (80) GeV.

While the distribution is falling off at large kT, we find that for the jet transverse scales observed at the LHC the contribution from the region pT \lesssim kT \lesssim \mu is non-negligible
when compared to that of an extra parton perturbatively emitted through hard-scattering matrix elements. As a result, a merging methodology is needed to avoid the double counting between the extra jet emission induced by the TMD initial-state evolution and that arising from the inclusion of a higher-order matrix element. Such a methodology is developed in [21, 28], and is discussed in the next section.

3. – TMD multi-jet merging and differential jet rates

Current multi-jet merging approaches provide techniques to combine samples of different parton multiplicity showered through emissions in the collinear approximation [9, 10, 11, 12, 13]. The TMD merging approach [21] complements these approaches with the use of the TMD parton branching for the initial state evolution.

The distinctive features of TMD merging, compared to collinear merging, are embodied in three steps: i) for any $n$-jet parton level event, initial-state transverse momenta $k_{T_i}$ are generated according to the TMD distributions obtained as solutions of the PB evolution.
tion equations [24, 29], but rejecting, owing to Sudakov suppression, \( k_T > \mu_{\text{min}} \), where \( \mu_{\text{min}} \) is the minimum energy scale in the \( n \)-jet hard sample; ii) initial state partons of the generated events are showered using the backward space-like shower evolution driven by the PB equations [24, 29], while final state partons are showered using standard time-like showers; iii) a merging prescription, such as MLM [13, 14], is applied between the showered event and the event generated in i) including the \( k_T \) boost. As noted earlier, one may construct a similar procedure by using prescriptions other than MLM, for instance CKKW-L [9, 10].

Fig. 2. – The \( d_{n,n+1} \) spectra for \( n = 0, 1, 2 \) at parton level, where \( d_{n,n+1} \) represents the energy-square scale at which an \((n + 1)\)-jet event is resolved as an \( n \)-jet event in the \( k_T \) jet-clustering algorithm. The dotted curves represent the contributions of the single-multiplicity samples while the solid curve corresponds to their sum. For each panel all jet multiplicities are obtained in exclusive (exc) mode except for the highest multiplicity which is calculated in inclusive (inc) mode.

We next illustrate the TMD merging methodology by computing the differential jet rates (DJR) \( d_{n,n+1} \) at parton level which result from the \( k_T \) jet clustering [30] applied to final states containing a \( Z \)-boson. The \( d_{n,n+1} \) represents the square of the energy scale
at which an $n$-jet event is resolved as an $(n+1)$-jet event. Since the DJRs provide the splitting scales in the jet clustering algorithm, they follow closely the measure used in the definition of the merging scale. Therefore, they are a powerful means to test the merging algorithm defined above.

To do this calculation, we use MadGraph5_AMC@NLO [31] to generate $Z+0, 1, 2, 3$ jet samples at LO with a generation cut $q_{cut} = 15$ GeV in $pp$ collisions at a center-of-mass energy $\sqrt{s} = 8$ TeV. We use the event generator Cascade [32] to generate the TMD backward shower, and Pythia6.4 [33] for the final-state shower. We apply the parton distributions obtained from DIS fits in [25] with $\alpha_s(M_Z) = 0.118$. The nominal value for the merging scale is chosen to be $\mu_m = 23$ GeV.

The results for the DJRs are shown in Fig. 2. The dotted curves represent the $n$-jet sample contributions while the solid curve corresponds to their sum. All the multiplicities are calculated in exclusive mode except for the highest multiplicity which is calculated in inclusive mode. A clear separation between the different jet samples is seen at the merging scale value while the resulting overall prediction remains smooth.

Fig. 3. – Exclusive (left) and inclusive (right) jet multiplicity distributions in the production of a $Z$-boson in association with jets. Experimental measurements by ATLAS [22] at $\sqrt{s} = 13$ TeV are compared to predictions using the TMD merging calculation. Separate contributions from the different jet samples are shown. All the jet multiplicities are obtained in exclusive (exc) mode except for the highest multiplicity which is calculated in inclusive (inc) mode.

4. – $Z$-boson + jets production at the LHC: a case study

In this section we present a few first applications of TMD merging to final states in DY production at the LHC.

We have first tested the method by evaluating the $Z$-boson transverse momentum spectrum. The result has been presented in [21], and compared with the measurements [1]. The merged prediction is found to provide a good description of the data throughout the whole $Z$-boson $p_T$ spectrum, with the $Z+0$ jet sample constituting the main contribution at low transverse momentum $p_T$ and the impact of larger jet multiplicities gradually increasing with increasing $p_T$. Thus the merged prediction [21] retains
the good description of the low-\(p_T\) region already obtained in [34] by applying TMD evolution, and improves the behavior in the high-\(p_T\) region by merging TMD showers with higher multiplicities.

Next, we consider jet observables measured in association with \(Z\)-boson production, and compare the predictions with the ATLAS measurements [22]. In Fig. 3 we show the results for the exclusive (left) and inclusive (right) jet multiplicities in \(pp\) collisions at \(\sqrt{s} = 13\) TeV. Jets are defined by the anti-\(k_t\) algorithm [35] with radius \(R = 0.4\), and are required to have \(p_T > 30\) GeV and |\(\eta\)| < 2.5. The analysis is performed using RIVET [36].

Fig. 4. – Theoretical systematics studies with TMD merging in DY lepton-pair production with associated jets at the LHC. (top left) \(\phi^*\) distribution of DY lepton pairs; (top right) differential jet rate \(d\sigma_1/\mathrm{d}p_T\); (bottom) leading jet \(p_T\). In each panel, results are shown for three different values of the merging scale, with the solid line giving the default setting at merging scale of 23 GeV.

The very good agreement of the prediction with the experimental measurements in Fig. 3 illustrates that the number of jets which result into the lepton pair \(p_T\) imbalance is well described by the TMD merging calculation. We observe that the agreement holds
up to multiplicities much larger than the maximum number of jets (three) for which the exact LO matrix-element calculation is performed. This underscores the potential benefit of the TMD evolution in better describing hard and non-collinear emissions, compared to the standard collinear evolution.

In Ref. [21] we have further examined the transverse momentum spectra of the associated jets. The comparison of the TMD merging results [21] with the experimental measurements [22] indicates that TMD merging describes well not only the number of jets (as seen in Fig. 3) but also the $p_T$ of the leading jet. Furthermore, one can compare the results of the TMD merging calculation with the results from the collinear merging calculation which is obtained by replacing the initial-state TMD shower evolution implemented in Pythia6, while keeping the same matrix-element and final-state shower in the two calculations. It is found [21] that clear differences emerge in the spectra that are most sensitive to higher-order shower emissions, such as the leading jet $p_T$ distribution in final states with at least 4 jets. The description of the jet $p_T$ improves thanks to TMD with respect to collinear merging at high multiplicities.

5. – Theoretical systematics

We finally turn to the theoretical systematics associated with the multi-jet merging algorithm, and in particular the dependence of theoretical predictions on the merging scale. It is shown in Ref. [21] that the multi-jet rates in $Z$-boson + jets production, computed with TMD merging for different multiplicities with the phase space selection and cuts of [22], have variations of less than 2% for a 10 GeV variation of the merging scale. This systematic uncertainty is significantly smaller than that of standard algorithms of collinear merging, as reported in Ref. [13], where the variation of the jet multiplicity rates is found to be about 10% when a 10 GeV change in the merging scale is considered.

Besides the effects on the rates, in Fig. 4 we present results for differential distributions in $Z$-boson + jets events, obtained by using the TMD merging algorithm and varying the merging scale around the default value. As in the previous calculations, the default value of the merging scale is taken to be 23 GeV, and we consider variations to 20 GeV and 30 GeV. We show results for the $\phi^*$ distribution of lepton pairs [1], the DJR $d_{01}$, the leading jet $p_T$. We observe that the variations in the distributions are localized around the merging scale, and the size of the variations is within 10%.

Our findings indicate that the systematic uncertainties are reduced owing to TMD merging with respect to collinear merging. In general, the merging systematics reflects the mismatch between the matrix-element and parton-shower weights assigned to a given final state. The larger the mismatch, the larger the uncertainty. The phase space regions that are most affected are those describing final states for which the jet multiplicity can vary under small changes of the merging parameters. For instance, this happens if ajet is soft or close to another hard jet. Modeling better the emission probability for such jets by shower evolution, by treating the transverse momentum recoils through TMD distributions, reduces the difference with the weight assigned to these events by the matrix element description, thereby reducing the mismatch and the relative systematics.

6. – Conclusion

We have discussed implications of TMD parton evolution on multi-jet production in high-energy hadronic collisions. We have presented a new multi-jet merging method,
TMD merging, that complements current methods with the use of TMD parton branching for the initial-state evolution.

Compared to standard approaches such as MLM, we find that TMD merging (i) has reduced systematic uncertainties, and (ii) improves the description of higher-order emissions beyond the maximum parton multiplicity considered in the matrix element calculations.

As the TMD broadening grows with increasing evolution scale $\mu$ and decreasing longitudinal momentum fraction $x$, we expect the effects pointed out in this article to become even more relevant in the case of the higher scales probed with jets at higher luminosity [37] and higher energy [38] colliders.

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