Application of exponential homotopy algorithm in the inverter harmonic elimination

Li Zhang\textsuperscript{1,2,3}

\textsuperscript{1}Data Recovery Key Laboratory of Sichuan Province, Neijiang Normal University, Neijiang 641110, China
\textsuperscript{2}Numerical Simulation Key Laboratory of Sichuan Province, Neijiang Normal University, Neijiang 641110, China
\textsuperscript{3}College of Mathematics and Information Science, Neijiang Normal University, Neijiang 641110, China

e-mail: lizhang0212@126.com

Abstract. Eliminating harmonic pollution and improving power factor are the quite important task in the field of power electronics. From the mathematical point of view, harmonic elimination problems can be translated into nonlinear equations. But it is difficult to directly solve the nonlinear equations because of complexity. For this reason, exponential homotopy method is proposed based on homotopy method in this paper. It has focused on built up homotopy equations by modifying the singularities of Jacobian matrix, and on this basis homotopy equations are transformed into the differential initial value problems. Numerical results show that the new exponential homotopy method has higher precision than other algorithms, and the singularity is improved.

1. Introduction

In recent years, special harmonic elimination technology \textsuperscript{[1]} (SHET) is proposed by Patel H.S and Hoft R.G in 1973, which is a pulse width modulation technology and has been widely used in frequency control devices and other projects \textsuperscript{[2]}. Due to its many advantages, such as high quality output waveform, lower switching frequency, weak loss, etc., it has been a hot concern \textsuperscript{[3-7]}. Since transcendental nonlinear equations often are involved in the harmonic elimination model with more dependence on the initial, iterative methods have poor convergence, even divergent for solving. Based on these reasons, many experts and scholars achieved good results by using homotopy algorithms \textsuperscript{[8-10]}. Despite homotopy algorithm is a very effective method, it is difficult to avoid singularity \textsuperscript{[11-13]}. Here, we propose the exponential homotopy method, which is continuous with large-scale convergence, and Jacobi matrix of new function is nonsingular. Furthermore, the solution of new function is in accordance with equations, and has better effect. Exponential homotopy algorithm not only improves the convergence speed and accuracy, but also has a small amount of calculation. It is a method for solving harmonic elimination equations, which close enough to the real solution. Based on this observation in this paper, we give exponential homotopy method in the next section. In section 3, PWM model has been stated and discussed. Some numerical experiments show feasibility and validity of it.

2. Exponential homotopy method
Let \( D \) be a domain in \( \mathbb{R}^n \), we consider following nonlinear equations:

\[
F(x) = 0
\]  

(1)

where \( F(x) = (f_1(x), f_2(x), \ldots, f_n(x)) \). It is well known that nonlinear mappings \( f_i(x) \) \((i = 1, 2, \ldots)\) are defined in domain \( D \).

We focus on the following form:

\[
H(x, s) = F(x) + (s - 1)P(x) = 0
\]  

(2)

with \( s \in [0, 1] \), \( x \in D \). The solution of equation (2) is \( x_0 \) as \( s = 0 \), and when \( s = 1 \), satisfying \( H(x, 1) = F(x) = 0 \). Taking \( P(x) = G(x)F(x_0) \), where \( G(x) = \text{diag}(e^{-u_i x}) \) is diagonal matrix, which \( e^{-u_i x} \) \((i = 1, 2, \ldots, n)\) are the main diagonal elements. \( u \) is a parameter. Obviously, for any \( x \in \mathbb{R}^n \) is nonsingular. Thus homotopy equation (2) can be rewritten as

\[
F(x) + (s - 1)\text{diag}(e^{-u_i x})F(x_0) = 0
\]  

(3)

Let \( s = 1 - e^{-t} \), we have

\[
F(x) - e^{-t}\text{diag}(e^{-u_i x})F(x_0) = 0
\]  

(4)

with \( t \in [0, +\infty) \), \( x \in D \). Here, Eq. (4) is called exponential homotopy method.

Equation (4) is equivalent with

\[
e^{-t}P(x) = e^{-t}\text{diag}(e^{-u_i x})F(x_0)
\]  

(5)

According to the definition of function Jacobi matrix, we derive that \( P'(x) = -u \text{diag}(e^{-u_i x}) \text{diag} f_i(x_0) \).

Here derivation to equation (5) at \( t \), we get

\[
F'(x)x'_t - e^{-t}\text{diag}(e^{-u_i x})[-F(x_0) - u \text{diag} f_i(x_0)x'_t] = 0
\]  

(6)

Combining (6) with relation (5), we obtain

\[
F'(x)x'_t + F(x) + u \text{diag} f_i(x)x'_t = 0
\]  

(7)

Furthermore,

\[
x'_t = -[F'(x) + u \text{diag} f_i(x)]^{-1} F(x)
\]  

(8)

Therefore, we get the initial value problem of ordinary differential equation

\[
\begin{cases}
\frac{dx}{dt} = -[F'(x) + u \text{diag} f_i(x)]^{-1} F(x), \\
x(0) = x_0, \quad t \in [0, +\infty).
\end{cases}
\]  

(9)

Generally speaking, when \( F(x) \) is singular or approach singularity, \( u \) can be adjusted. There exists \( u \) such that \( F'(x) + u \text{diag} f_i(x) \) is nonsingular, as long as \( u \) is large enough. Here, we adopt midpoint quadrature formula [14] to solve equation (9), then combining with Newton method, we get the iterative schemes:
\[
\begin{align*}
x_i &= x_0 - h \left[ G(x_0) \right]^{-1} F(x_0), \\
x_{k+1} &= x_k - \frac{1}{2} (x_k - x_{k-1}), \quad k = 1, 2, \ldots, N-1 \\
x_{k+1} &= x_k - h \left[ G(x_{k+1}) \right]^{-1} F(x_{k+1}).
\end{align*}
\] (10)

and
\[
x_{k+1} = x_k - F'(x_k)^{-1} F(x_k), \quad k = N, N+1, \ldots
\] (11)

where \( G(x) = F'(x) + u \text{diag } f_i((x)) \), \( h \) is a step.

3. Harmonic elimination model

Taking the single phase SHEPEM as example, figure 1 is known to be unipolar PWM waveform of switching.

![Figure 1. Unipolar control pulse waveform](image)

Fourier expansions of its output voltage are expressed as
\[
f(\omega t) = \sum_{n=1}^{\infty} \left[ a_n \sin(n\omega t) + b_n \cos(n\omega t) \right]
\] (12)

where
\[
\begin{align*}
a_n &= \frac{1}{\pi} \int_{\omega}^{2\omega} f(\omega t) \sin(n\omega t) d(\omega t), \\
b_n &= \frac{1}{\pi} \int_{\omega}^{2\omega} f(\omega t) \cos(n\omega t) d(\omega t).
\end{align*}
\] (13)

The output voltage waveform in figure 1 is both odd functions and odd harmonics. That is to say, for \( f(\omega t), \frac{\pi}{2} \) is the axis of symmetry in \([0, \pi]\), and \( \pi \) is symmetry point in \([0, 2\pi]\). Then we have
\[
\begin{align*}
f(\omega t) &= -f(\omega t + \pi), \\
f(\omega t) &= f(\omega t - \pi).
\end{align*}
\] (14)

Eq. (14) is substituted in relation (13), we get
\[
\begin{align*}
b_n &= 0, \quad n = 1, 2, 3, \ldots \\
a_n &= 0, \quad n = 2, 4, 6, \ldots \\
a_n &= -\frac{4E}{n\pi} [1 + 2 \sum_{k=1}^{N} (-1)^k \cos(n\alpha_k)] \quad n = 1, 3, 5, \ldots
\end{align*}
\]

where \( E \) is voltage of direct-current generatrix, \( N \) is the number of switching, \( \alpha_k \) is the \( k \)-th switching in \( N \) and \( n \) is frequency of harmonics.

The amplitude of the fundamental is

\[
a_1 = -\frac{4E}{\pi} [1 + 2 \sum_{k=1}^{N} (-1)^k \cos\alpha_k]
\]

Let \( U_m \) be the selected amplitude of the fundamental, and the other amplitudes of higher harmonics be zero, PWM model is described by

\[
\begin{align*}
a_1 &= U_m, \\
a_n &= 0. \quad n = 3, 5, \ldots
\end{align*}
\]

To solve relation (17) with \( N \) dimensions nonlinear equations, we arrive at a group switch angles in \([0, \pi/2]\) and satisfy the conditions:

\[
0 < \alpha_1 < \alpha_2 < \alpha_3 < \alpha_4 < \alpha_5 < \alpha_6 < \alpha_7 < \alpha_8 < \cdots < \pi/2
\]

4. Numerical examples

In order to further illustrate the validity of exponential homotopy method in harmonic elimination equations, Newton method, midpoint quadrature formula Newton homotopy method (MQFNM) are compared with exponential homotopy method by C++ software.

4.1 Numerical example 1

The parameters of PWM model are shown in Table 1.

| Number of switch angle (\( N \)) | Initial value (\( \alpha_0 \)) | Error precision (\( \varepsilon \)) |
|---------------------------------|-----------------------------|-------------------------------|
| 8                              | [5, 20, 30, 40, 45, 60, 65, 89] | \( 10^{-6} \) |

The results of iteration solution and final exact solution are compared in Table 2, and the number of iteration is shown in Table 3.

| \( \alpha \) | \( \alpha_0 \) | Newton method | MQFNM solution | Exponential homotopy solution | Exact solution |
|-------------|---------------|---------------|----------------|-------------------------------|---------------|
| \( \alpha_1 \) | 5             | 8.970         | 8.681          | 8.612                         | 8.689         |
| \( \alpha_2 \) | 20            | 20.156        | 20.746         | 20.133                        | 20.023        |
| \( \alpha_3 \) | 30            | 26.425        | 26.440         | 26.505                        | 26.563        |
| \( \alpha_4 \) | 40            | 40.543        | 40.327         | 40.491                        | 40.421        |
| \( \alpha_5 \) | 45            | 44.694        | 44.923         | 44.764                        | 44.725        |
| \( \alpha_6 \) | 60            | 63.346        | 63.774         | 63.243                        | 63.214        |
| \( \alpha_7 \) | 65            | 64.337        | 64.906         | 64.592                        | 64.578        |
\[ \alpha_8 \quad 89 \quad 89.943 \quad 89.725 \quad 89.881 \quad 89.884 \]

Table 3. Comparison of iteration number by three methods

| Method                        | Newton method | MQFNM solution | Exponential homotopy solution |
|-------------------------------|---------------|----------------|-------------------------------|
| Number of iteration           | 4             | 4              | 5                             |

As can be seen from Table 2 and Table 3, it is clear that three methods converge fast to the accurate solution in a few iterations. The reason was that initial values are very close to the accurate values.

4.2 Numerical example 2

The parameters of PWM model are shown in Table 4.

Table 4. The parameters of numerical example

| Number of switch angle (N) | Initial value(\(\alpha_0\)) | error precision(\(\varepsilon\)) |
|----------------------------|-------------------------------|---------------------------------|
| 8                          | [5, 20, 25, 40, 45, 60, 65, 84.452] | 10^{-6}                        |

The results of iteration solution and final exact solution are compared in Table 5, and the number of iteration is shown in Table 6.

Table 5. Comparison of iteration solution by three methods

| \(\alpha_i\) | \(\alpha_0\) | Newton method | MQFNM solution | Exponential homotopy solution | Exact solution |
|-------------|-------------|--------------|----------------|-------------------------------|---------------|
| 5           | 9.970       | 9.681        | 8.444          | 8.472                         |
| 20          | 22.157      | 22.744       | 21.123         | 21.157                        |
| 25          | 38.024      | 38.140       | 37.105         | 37.160                        |
| 40          | 43.551      | 43.238       | 42.193         | 42.249                        |
| 45          | 56.628      | 56.923       | 56.176         | 56.259                        |
| 60          | divergence  | 63.798       | 63.124         | 63.207                        |
| 65          | divergence  | 75.906       | 75.794         | 75.905                        |
| 84.452      | 83.943      | 83.881       | 83.770         | 83.881                        |

Table 6. Comparison of iteration number by three methods

| Method                        | Newton method | MQFNM solution | Exponential homotopy solution |
|-------------------------------|---------------|----------------|-------------------------------|
| Number of iteration           | 13            | 10             | 5                             |

As can be seen from Table 2 and Table 3, we find that there may be divergence phenomena in Newton method, and exponential homotopy method is much better regarding iterations and convergence than Newton method, midpoint quadrature formula-Newton homotopy method. Therefore, results show that exponential homotopy method not only has been improved significantly for efficiency, but also convergence effects are very good.

5. Conclusion

Through depth studying the inverter harmonic elimination model, we propose an effective method to solve nonlinear equations in model. Of course, there is also a bipolar pulse model, the principle exactly as same as that of single pulse model. Through simulation experiments, the results of specific examples show its efficiency and improving the accuracy of solution. In general, it has good results to solve practical problems of harmonic elimination, and provides a possibility for realization of
real-time harmonic elimination. If the algorithm can be better, it will be a major breakthrough in harmonic elimination field.

6. References

[1] Patel HS; and Hoft RS P 1973 Generalized Technique of Harmonic Elimination and Voltage Control in the Thyristor Inverter: Part I-Harmonic Elimination vol 3 (IEEE Trans. on Industry Application) p 301-317

[2] Li XX, et al 1996 Pulse Width Modulation (Wuhan: Huazhong University of Technology Press)

[3] Tong WM Chen XY and Hu Y et al P 1998 Investigation of Solution to the Nonlinear Equation Set up of SHET in Converters vol 5 (Proceedings of the Csee) p 357-360

[4] Fei WM, Lv ZY and Yao WX P 2003 Research on Selected Harmonic Elimination PWM Technique Applicable to Three-level Voltage Inverters vol 9 (Proceedings of the Csee) p 11-15.

[5] Huang YQ and Xie YX P 2003 Algorithm Research of the Inverse Matrix in the Solution of the Inverter Harmonic Elimination Model vol 11 (Journal of South China University of Technology (Natural Science)) P 1-4

[6] Zhang YL Fei WM and Lv ZM et al P 2004 Research of Selected Harmonic Elimination PWM Technique Applicable to Three-level Voltage Inverters vol 19(Transactions of China Electrotechnical Society) p 16-20

[7] Zhao YC and Zhao ZM P 2007 Multiple Solution for Selective Harmonic Eliminated PWM Applied to Three-Level Inverter vol 22 (Transactions of China Electrotechnical society) p 74-78

[8] Li ZD, Zhou QY and Li H et al P 2004 A Novel Algorithm for Real-time Solution of Nonlinear Surmount SHET Equations vol 22 (Journal of Northwestern Polytechnical University) p 37-39

[9] Li X Liu JP and Yang AM P 2007 Study and Application of Homotopy New Algorithm in Inverter Harmonic Elimination Model vol 25 (Journal of Jiamusi University(Natural Science Edition),) p 381-384

[10] Zhu MJ Zhao P and Peng H et al P 2004 Fast Homotopy Algorithm of the Inverter Harmonic Elimination PWM Model vol 2(Journal of South China University of Technology (Natural Science)) p 28-30

[11] Huang XD, Zeng ZG and Ma YN 2004 The Theory and Methods for Nonlinear Numerical Analysis(Wuhan: Wuhan University Press)

[12] Li QY P 1980 Solution of Systems of Nonlinear Equations by Differential Continuation vol 1(Journal on Numerical Methods and Computer Applications) p45-52

[13] Li SB P 1983 Function Factor Method: A New Method of Passing the Singularities Which Arise in the Continuation Methods for Solving Systems of Nonlinear Equations vol 2 (Mathematica Numerica Sinica) p162-175

[14] Li QY Mo ZZ and Qi LQ 1987 Numerical Methods of Nonlinear Equations (Beijing: Science Press)

Acknowledgments

This work was financially supported by the National Natural Science Foundation of China (11502121), the education department of Sichuan Province university innovation team (13TD0001), the education department of Sichuan Province project (16ZA0310), and the major achievements transformation project of Neijiang Normal University (14CZ02).