Leak Isolation in Pressurized Pipelines using an Interpolation Function to approximate the Fitting Losses

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Abstract. The present paper is motivated by the purpose of detection and isolation of a single leak considering the Fault Model Approach (FMA) focused on pipelines with changes in their geometry. These changes generate a different pressure drop that those produced by the friction, this phenomenon is a common scenario in real pipeline systems. The problem arises, since the dynamical model of the fluid in a pipeline only considers straight geometries without fittings. In order to address this situation, several papers work with a virtual model of a pipeline that generates an equivalent straight length, thus, friction produced by the fittings is taking into account. However, when this method is applied, the leak is isolated in a virtual length, which for practical reasons does not represent a complete solution. This research proposes as a solution to the problem of leak isolation in a virtual length, the use of a polynomial interpolation function in order to approximate the conversion of the virtual position to a real-coordinates value. Experimental results in a real prototype are shown, concluding that the proposed methodology has a good performance.

1. Introduction
The hydraulic efficiency of a pipeline system, from a physical perspective, is associated to the capacity of the system to provide the injected fluid to its final destination. In order to preserve this efficiency, it is necessary to point out the importance of early detection, isolation and reparation of leaks. A neglectful treatment of these activities can causes fluid losses, discontinuities in the services and low pressures in the pipeline, which produce an excessive energy consumption in the pumping systems and consequently, the operating costs rise[1].

In the last years, different analytical methods have been proposed, based on Fault Model Approach (FMA) and Fault Sensitive Approach (FSA) algorithms, for instance [2], [3], [4], [5], [6], [7], to cite a few. Such techniques use, in first instance, sensors to monitor certain internal quantities (flow, pressure, temperature, etc.), and in second instance, nonlinear mathematical models [8] deduced from a pair of partial differential equations that describe the fluid dynamics in closed conduits, these equations are known as Water Hammer Equations (WHE).

The nonlinear models are generally used to design an observer that estimates the unmeasurable states using input and output measurements from a real system. In the case where the Extended Kalman Filter is used as an observer, when a leak occurs the leak isolation algorithm begins to estimate its location and magnitude.
The WHE are deduced for straight pipelines without fittings, which in most of real systems is not satisfied. Generally, pipeline systems are composed by different accessories, in order to adapt the installation to topography features. Several devices are also included, as valves for flow control, for example. All of the mentioned above cause energy losses, known as local losses, which are different from those produced by the friction of the pipes. In the actual literature exist an increasing number of studies focused on leak detection and isolation based on FMA and FSA, these studies takes into account the presence of fittings in the analysis performing the leak isolation on a virtual straight pipeline. In order to do that, each fitting of the original pipeline is replaced by a section of straight pipe that presents the same pressure loss that the respective fitting [2], [9], [10]. Under the mentioned conditions, it is said that the leak location is performed in Equivalent Straight Length coordinates (ESL). Several other works say nothing about the coordinates that represent the leak position, even though there is presence of accessories in the pipeline in which the analysis was performed. In the consulted literature, only [2] presents a leak isolation in real length coordinates, however, there is not an explanation of how to arrive to the final result of the research. In another hand, [11] gives a methodology to achieve the leak isolation in pipelines with fittings, in real coordinates, but it introduces proportionality relationships for coefficients of lost owned by each accessory that not always can be reached. Considering the importance of the real-coordinates representation, this paper proposes a new alternative method to isolate a leak in real coordinates, unlike the other approaches found in the literature, where the leak is isolated in equivalent coordinates. This method is performed through the calculation of an interpolation function obtained from the estimation of the pressure at different points of the pipeline. Once the interpolation function is formulated, it is used then as an equivalent-to-real coordinate converter. Results are tested in a prototype and presented in this work, showing a good agreement with the real pipeline conditions.

The paper is organized as follows: Section 2 provides a mathematical model for pipelines and the algorithm of the Extended Kalman Filter (EKF), which is implemented as an observer for Leak Detection and Isolation (LDI) problem. Section 3 presents our interpolation method considering the pressure drops in each pipeline fitting. Section 4 shows the experiments and results obtained. Finally, Section 5 presents some relevant conclusions and discusses the future work.

2. Leak modeling and isolation

The equations that describe the dynamics of the flow in a pipeline, in transient state, are known as the Water Hammer Equations (1) and (2). Considering the principles of mass and momentum, let the pipeline be straight, without slope, and with duct wall slightly deformable; also consider that the fluid is slightly compressible, the convective velocity changes are negligible and let the pipeline cross section area and fluid density be constants [12]. Then, the continuity equation is defined as follows:

$$\frac{\partial H(z,t)}{\partial t} + \frac{b^2}{gA} \frac{\partial Q(z,t)}{\partial z} = 0,$$

(1)

and the momentum equation takes the form:

$$\frac{\partial Q(z,t)}{\partial t} + gA \frac{\partial H(z,t)}{\partial z} + f \frac{Q(z,t)|Q(z,t)|}{2DA} = 0,$$

(2)

where, $H$ is the pressure head [m], $Q$ is the flow rate [$m^3/s$], $z$ is the length coordinate [m], $t$ is the time coordinate [s], $g$ is the gravity acceleration [$m/s^2$], $A$ is the cross section area [$m^2$], $b$ is the pressure wave speed in the fluid [$m/s$], $D$ is the internal diameter [m] and $f$ is the Darcy-Weisbach friction factor [dimensionless].

The pressure wave speed in the fluid is given by:
\[ b = \sqrt{\frac{\kappa}{1 + \frac{D\kappa}{Ee}}} \]  

(3)

where \( E \) is the Young’s modulus of elasticity for the conduit walls [Pa], \( \kappa \) and \( \rho \) are the bulk modulus [Pa] and the density of the fluid [kg/m\(^3\)], respectively, and finally, \( e \) is the thickness of the pipe wall [m].

The boundary conditions in this work are taken as the pressure heads at the extremes of the pipeline, which are measurable parameters denoted by:

\[ H(x = 0, t) = H_{in}(t), \quad H(x = L, t) = H_{out}(t). \]  

(4)

2.1. Friction factor

One of the most commonly used equations, due to its property to give good values of the friction factor \( f \), is the Coolebrok-White equation:

\[ \frac{1}{\sqrt{f}} = -2.0 \log \left( \frac{\epsilon/D}{3.71} + \frac{2.51}{Re\sqrt{f}} \right), \]  

(5)

where, \( D \) is the internal diameter [m], \( \epsilon \) is the roughness coefficient of the material [m] and \( Re \) is the Reynolds number calculated as follows: \( Re = QD/\nu A \), where \( \nu \) is the kinematic viscosity \([m^2/s]\), \( Q \) is the flow \([m^3/s]\) and \( A \) is the cross section area \([m^2]\).

The general form of the Coolebrok-White equation requires iterative calculations, which can be hard to implement or computationally expensive, therefore several authors have proposed explicit equations for the friction factor based on the Coolebrok-White equation. A set of the most accurate and efficient in the zone of complete turbulence [13], are presented below:

- **Buzzelli.** High accuracy, relative percentage error 0.005:

\[ \frac{1}{\sqrt{f}} = B_1 - B_2 \left( \frac{\epsilon/D}{3.71} + \frac{2.51}{Re\sqrt{f}} \right), \]  

(6)

where:

\[ B_1 = \frac{0.774 \ln(Re)}{1 + 1.32\sqrt{f}}, \quad B_2 = \frac{\epsilon}{3.7D} Re + 2.51B_1. \]

- **Haaland.** Medium accuracy, relative percentage error 0.0373 and applicable range of \( 4 \times 10^3 \leq Re \leq 1 \times 10^8 \), \( 1 \times 10^{-6} \leq \epsilon/D \leq 5 \times 10^{-2} \):

\[ \frac{1}{\sqrt{f}} = -3.6 \log \left( \frac{6.9}{Re} + \left( \frac{\epsilon}{3.7D} \right)^{1.11} \right). \]  

(7)

- **Swamee and Jain.** Medium accuracy, relative percentage error 0.478 and applicable range of \( 5 \times 10^3 \leq Re \leq 1 \times 10^8 \), \( 1 \times 10^{-6} \leq \epsilon/D \leq 5 \times 10^{-2} \):

\[ f = \frac{0.25}{\log_{10} \left( \frac{5.74}{Re^{0.8}} + \frac{\epsilon/D}{3.7} \right)^2}. \]  

(8)
2.2. Leak model

The equation that describes the behavior of a leak located in an arbitrary point \( z_L \), as seen in the Figure 1, can be formulated using the orifice equation:

\[
Q_L = C_d A \sqrt{2gH_L},
\]

where \( C_d \) is the discharge coefficient \([\text{dimensionless}]\) and \( A \) is the leak cross section area \([m^2]\). Now, defining \( \lambda = C_d A \sqrt{2g} \), the flow in the leak can be expressed as:

\[
Q_L = \lambda \sqrt{H_L}.
\]  

![Figure 1](image-url)  

**Figure 1.** Discretization of the pipeline with a leak located at position \( z_L \).

In order to obtain a representation in the space of states, the equations (1) and (2) are discretized with respect to the spatial variable \( z \), \([12], [14], [15]\) using the following relationships:

\[
\frac{\partial H}{\partial z} \approx \frac{H_{i+1} - H_i}{\Delta z} \quad \forall i = 1, \ldots, n,
\]

\[
\frac{\partial Q}{\partial z} \approx \frac{Q_i - Q_{i-1}}{\Delta z} \quad \forall i = 1, \ldots, n.
\]

The pipeline is discretized in two sections, as shown in Figure 1, where \( \Delta z \) is the distance step, \( z_i, i = 1, 2 \) are the distances from the start of the pipeline to the leak position \((z_L)\) and from leak position to the end of the pipeline \((L - z_L)\), respectively.

Using approximations (11) and (12), and considering \( z_L \) and \( \lambda \) as constant values, the following dynamical system representation is obtained:

\[
\begin{bmatrix}
\dot{Q}_1 \\
\dot{H}_2 \\
\dot{Q}_2 \\
\dot{z}_L
\end{bmatrix} =
\begin{bmatrix}
-\frac{gA}{z_L^2} (H_2 - u_1) - \frac{f(Q_1)}{2DA} Q_1 |Q_1| \\
\frac{gA}{z_L^2} (Q_2 - Q_1 - \lambda \sqrt{H_2}) \\
-\frac{gA}{z_L^2} (u_2 - H_2) - \frac{f(Q_2)}{2DA} Q_2 |Q_2| \\
0
\end{bmatrix},
\]

where \( [u_1 \ u_2]^T = [H_1 \ H_3]^T \) is the input vector and \( y = [Q_1 \ Q_2]^T \) is the output vector.

The model (13) can be written in compact form as:

\[
\dot{x} = \xi(x, u),
\]

where \( x \triangleq [Q_1 \ H_2, Q_2 \ \lambda \ z_L] \) and \( \xi(.) \) is a nonlinear function.
2.3. Leak isolation scheme
In order to develop a leak isolation scheme, it is necessary in first place to obtain a discrete representation for the model (14). According to [9], the Heun’s method is suitable to perform the discretization.

Defining the initial value problem:

$$\dot{x}(t) = \xi(x(t), u(t)), \ x(t_0) = x_0,$$

then, the Heun’s method equation takes the form:

$$x^{i+1} = x^i + \frac{\xi(x^i, u^i) + \xi(x^i + \Delta t \xi(u^i, x^i), u^{i+1})}{2} \Delta t,$$

where $\Delta t$ is the time step.

The model (14) discretized by equation (16) can be written in compact form as:

$$x^{i+1} = \xi(x^i, u^{i+1}, u^i), \ y = Hx^i,$$

where $x^i \triangleq [Q^i_1 \quad H^i_2 \quad Q^i_2 \quad z^i_L \quad \lambda^i]^T$, $\xi(.)$ is a nonlinear function and $H$ is fixed as:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$ 

Once the discretized model is obtained, an Extended Kalman Filter is implemented as a state observer, taking into account the expressions in Table 1 [16]:

| $\hat{x}^i$ is the a-priori estimate of $x^i$: | $P^i$ is the a-priori covariance matrix: |
| $\tilde{x}^i = \hat{x}^i + K^i(y^i - Hz^i)$ | $P^i = J^i P^i_{i-1} (J^i)^T + Q$ |
| $K^i$ is the Kalman gain for the observer: | $P^i$ is the posterior covariance matrix: |
| $K^i = P^i - H^T (HP^i H^T + R)^{-1}$ | $P^i = (I - K^i H) P^i_{i-1}$ |
| $J^i$ is the Jacobian matrix: | |
| $J^i = \frac{\partial \xi(x, u)}{\partial x} \bigg|_{x=\hat{x}}$ |

Table 1. Kalman Filter Equations

$R$ and $Q$ are known as the covariance matrices of measurements and process noises, respectively. Notice that:

$$P^0 = (P^0)^T > 0, \ R = R^T > 0, \text{ and } Q = Q^T > 0.$$ 

3. Locating leaks in pipelines with fittings
Fittings such as elbows, valves, unions and contractions present resistance to the flow, known as local or minor losses; the total of these pressure drops in a pipeline system usually has a value between 5% to 20% of the total pressure drop. In the case of the LDI problem, such minor losses must be known with high precision in order to achieve a good leak isolation.

Pressure losses due to friction in a straight pipeline can be obtained through the Darcy-Weisbach equation:
where \( h_f \) is a friction loss [m], \( f \) is the friction factor [dimensionless], \( \Delta z \) is the length [m] of a straight pipe section and \( Q(t)^2/2gA^2 \) is the head velocity in terms of flow [m].

Pipelines with fittings have additional local losses due to the accessories, calculated as:

\[
h_l = K_f \frac{Q(t)^2}{2gA^2},
\]

where \( h_l \) is the local loss [m], \( K_f \) is a dimensionless coefficient, known as Coefficient of Loss, which depends on: the type of fitting, the Reynolds number and the roughness of fitting material.

For a pipeline with accessories, the total pressure drop is calculated as:

\[
H = h_f - \sum_{i=1}^{N} h_{l(i)},
\]

where \( h_{l(i)} \) represents the local loss in the \( i \)-th accessory. As an example, let \( L_a \) be an arbitrary distance in the pipeline, such that \( L_1, L_2 \) are straight pipes and \( F_1 \) and \( F_2 \) are fittings that belong to the \([0, L_a]\) section. Then, to calculate the pressure drop in the point \( HL_a \) of the \([0, L_a]\) section due to each individual component, the Equation (20) takes the form:

\[
H_{L_a} = H_{in} - f \frac{L_1}{D} \frac{Q^2}{2gA^2} - f \frac{L_2}{D} \frac{Q^2}{2gA^2} - K_{F_1} \frac{Q^2}{2gA^2} - K_{F_2} \frac{Q^2}{2gA^2},
\]

where, \( H_{in} \) represents the pressure at the beginning of the \([0, L_a]\) section, provided by the pump, \( f \frac{L_1}{D} \frac{Q^2}{2gA^2} \) is the pressure drop produced by the \( L_1 \) straight pipe, \( f \frac{L_2}{D} \frac{Q^2}{2gA^2} \) is the pressure drop produced by the \( L_2 \) straight pipe, and finally, \( K_{F_1} \frac{Q^2}{2gA^2} \) and \( K_{F_2} \frac{Q^2}{2gA^2} \) represent the pressure drop added by the fittings \( F_1 \) and \( F_2 \).

Figure 2 shows the piezometric profile of the pipeline described at the previous example. The local losses generate a sharper drop, thus, the piezometric profile is not completely linear.

### 3.1. Equivalent straight length to real length

To tackle the problem of leak isolation using the FMA in pipelines with fittings, different works as ([2], [9] and [10]) propose the calculation of a virtual equivalent straight pipeline, in which a longitudinal compensation is performed for each fitting, in order to obtain a straight pipeline.
with losses that are equivalent to the losses in the original pipeline with fittings. The equivalent
straight length $L_{eq}$ is calculated and introduced in the mathematical model.

Using the Darcy-Weisbach equation (18) and the pressure head measurements, $L_{eq}$ is
calculated as follows:

$$L_{eq} = \frac{\Delta H(t)D^5\pi^2g}{8f(t)Q(t)^2}.$$  \hfill (21)

Using the Equation (21), the position of a leak is isolated in an equivalent straight length,
but from a practical point of view, this result does not represent a complete solution: in fact,
an equivalent-to-real coordinates conversion is necessary to locate the leak in the real pipeline.
Further more, there does not exist a direct conversion from equivalent to real coordinates, since
it can be possible to obtain an infinite number of pipelines with different structures and fittings
that present the same equivalent length.

One way, simple but not very accurate, to return to real length coordinates is to establish a
linear relationship between the real and the equivalent length, as is shown in (22) (for instance,
see Figure 5 in Section 4): however, if the leak occurs near to a fitting, the isolation will be
wrong due to the abrupt pressure drop caused by the mentioned accessory. The linear relation
is expressed as follows:

$$z_r = \frac{L_t}{L_{eq}} z_{eq},$$  \hfill (22)

where $z_r$ is a position in real length, $L_{eq}$ is the total equivalent length of the pipeline, $L_t$ is the
total real length of the pipeline and $z_{eq}$ is the leak position in equivalent length, all expressed
in m.

For the sake of understanding, when a leak is isolated on a equivalent length, it will be
noted that the leak is isolated in equivalent coordinates, otherwise, the leak is isolated in real
coordinates.

3.2. Interpolation function for pressure drop

Due to the existence of an infinite number of pipes with different structures and fittings that
presents the same equivalent length, it is necessary to know the structure of the pipeline under
study in order to achieve the leak isolation in real coordinates. To known the structure of the
pipeline is, in general, an easy task, since usually there are design plans with this information
and the standard length of pipes is known.

Using the EKF algorithm to isolate a leak, the position in equivalent coordinates ($z_{Leq}$) and
the value of the pressure at the leak point ($H_2$) are obtained [9],[2]; this pressure value is the
same in both coordinates systems, the equivalent and real one. This correspondence of pressure
values can be used to find the leak position in real coordinates ($z_{Lr}$).

The pressure drop along the length in a pipeline with fittings is not completely linear, however,
the pressure behaviour can be approximated by a smooth curve. A simple way to obtain a
suitable approximation is to calculate an interpolation function with the Least Square (LS)
technique [17].

In order to use the LS technique, a set of measurements is required to approximate the
behavior of the pressure throughout the pipeline. The most common scenario, as is taken in this
research, is the one in which the measurements of pressure are only available at both extreme
points of the pipeline. Given such restriction, the measurements are complemented by a set of
calculated pressure values, obtained from (20) in different points of the pipeline. Each fitting
produces an inflection in the pressure profile, therefore, to include the local loses caused by a
fitting in a pipe, it is required to estimate the pressure with Equation (20). It is necessary
to perform this estimation in the two connection points of each fitting. Thus, using the LS
technique and having as dependent variable the length ($z$) and as independent variable the pressure ($H$), the interpolation function is expressed as follows:

$$\hat{z} = \beta_1 + \beta_2 H + \beta_3 H^2 + \ldots + \beta_n H^n,$$

(23)

where $\beta_j, j = 1, 2, ..., n$ are the interpolation function coefficients calculated with the LS technique, $H$ is the pressure at a specific point of the pipeline, $z$ is its corresponding length and $n$ is the degree of the polynomial equation. Note that, for $n$ data points, a polynomial of order $n - 1$ can be used to obtain a good adjustment.

Using a set of coordinated pairs of the form $(z_i, H_i)$, it is possible to find the coefficients $\beta_j, j = 1, 2, ..., n$ that fit to the Equation (23) in the best way, through the solution of a quadratic minimization problem, where the objective function $J_c$ is given by:

$$J_c = \|z - H\beta\|^2. \quad (24)$$

The polynomial interpolation must be calculated after the establishment of a constant inflow value, that is, after the transient effect caused by the occurrence of a leak has vanished. Once the $\beta_j$ are determined, Equation (23) allows to know a position in the pipeline for a given pressure. In particular, using the pressure at the leak point given by the EKF ($H_2$) is possible to determine the leak position in real coordinates ($z_r$).

The next flow chart illustrates the above procedure in the grey boxes:

Figure 3. Algorithm to isolate a leak in a pipeline with fittings in real coordinates using the proposed interpolation function.

4. Experimental results

In order to compare the relationship of proportionality (22) against the interpolation function (23), two experiments are performed, using real data acquired from a pipeline prototype.

4.1. Prototype description

The experimental scenarios are implemented in a pipeline prototype built at the Center for Research and Advanced Studies (CINVESTAV) Guadalajara, Mexico. This pipeline prototype is composed by two pressure sensor Promag Propiline 10P and two flow sensor PMP 41, both of them Endress HauserTM. These sensors are placed at the end points of the pipeline. Besides, a temperature sensor PT100 is mounted at the interior of the water supply tank. To distribute the water, the prototype includes a centrifugal pump of 5HP from SiemensTM. The pipeline total length is 68.147m and it has three valves located at 17.045m (valve 1), 33.47m (valve 2) and 49.895m (valve 3) that allow to emulate leaks. Valves 1 and 3 also contain pressure sensors.
from Winters™. These sensors have the objective of validate the head pressure estimations in the leak points. The data logging for sensors is performed by a DAQ module NI US-6229 produced by National Instruments™. Finally, the user interface, which interacts with hardware devices, is developed in Labview™ and Matlab™. The main flow line parameters are shown in the Table 2. For more technical information about the pipeline prototype consult [18].

The architecture of the pipeline prototype is composed by eight joints with metal thread, eleven plastic joins, two plastic elbows, five metal tees and 64.93 m of straight plastic pipe. The coefficients of loss of each fitting are shown in the Table 3. Using the coefficients from Table 3, a pressure in $kgf/m^2$ units is obtained, so, it is convenient to convert the result to a pressure in meters of water column $[mH_2O]$ units.

![Prototype scheme](image)

**Figure 4.** Prototype scheme.

| Parameter             | Symbol | Value    | Units |
|-----------------------|--------|----------|-------|
| Total Length          | $L_r$  | 68.147   | m     |
| Internal diameter     | $D$    | $6.271 \times 10^{-3}$ | m    |
| Wall thickness        | $e$    | $13.095 \times 10^{-3}$ | m    |
| Roughness             | $\epsilon$ | $7 \times 10^{-6}$ | m    |
| Slope                 | $s$    | 0        | %     |

**Table 2.** Pipeline prototype parameters.

| Type of fitting | $K_r$ |
|-----------------|-------|
| Elbow           | 2.0   |
| Plastic join    | 0.25  |
| Metal join      | 0.4   |
| metal tee       | 0.7   |

**Table 3.** Local loss coefficients.
4.2. Experimental results

Each of the tests presented in this section has a duration of 250s, with sampling time of $\Delta t = 0.1 \text{s}$.

For the first experiment, the pressure drop is estimated at nine different points of the pipeline, and the coefficients $\beta_j$ of the interpolation function (23) are determinate using the LS technique. Then, the results obtained using Equation (20) are compared against the measurements obtained at the points where sensors are available (see Figure 5 (a)). The Mean Squared Error (MSE) between these points is $29.68 \times 10^{-2}$ units. Figure 5 (a) shows the result obtained using the equivalent straight length and its conversion to real coordinates through the relationship of proportionality (22). It is easy to note that near to the fittings, the pressure drop significantly differs from the real pressure profile. The MSE between the relation of proportionality and the pressure drop profile has a magnitude of 37.21 units. In Figure 5 (b), the comparison between the pressure profile and the interpolation function (23) is shown. For this case, the MSE has a value of 2.4 units.

For the second experiment, three different leaks coming from valves 1, 2 and 3 are isolated, one at a time. Each leak is localized in equivalent coordinates and then converted to real coordinates with the relationship of proportionality (22) and using the interpolation function (23). For the leaks coming from valves 1 and 3 (Figure 6 and 8, respectively), results are similar for both methods. Nevertheless, for the leak at valve 2, (7), which is the closest to an elbow, a better result is obtained with the interpolation equation, reaching a MSE of 2.009 units against 9.4 units obtained with the relationship of proportionality. The results would be more evident if there were fittings that produce greater pressure drops, such as elbows and sharp contractions, near to the valves.

**Figure 5.** Pressure drop.

**Figure 6.** Leak isolation emulated by valve 1. Real leak position (black), estimation of leak position using polynomial function (blue), estimation of leak position in equivalent coordinates (green) and leak position estimated by proportionality relationship (red).
Figure 7. Leak isolation emulated by valve 2. Real leak position (black), estimation of leak position using polynomial function (blue), estimation of leak position in equivalent coordinates (green) and leak position estimated by proportionality relationship (red).

Figure 8. Leak isolation emulated by valve 3. Real leak position (black), estimation of leak position using polynomial function (blue), estimation of leak position in equivalent coordinates (green) and leak position estimated by proportionality relationship (red).

5. Conclusions
In this paper, an interpolation function that allows to estimate a leak position in real length coordinates is proposed, considering as an approach of low practical value the expression of the leak position in equivalent length coordinates, which is a common scenario in the methods based on Fault Model Approach found in the actual literature.

The interpolation function method presented in this work can be used in real situations and not only in theoretical or controlled scenarios. The proposed methodology shows a good performance thought experimental results.

Based on the performed analysis and the obtained results, it is possible to emphasize the following conclusions: (i) It is necessary to know the structure of the pipeline and the characteristics of each fitting in order to isolate a leak in a pipeline with fittings with a high degree of exactitude; (ii) the values of local losses coefficients are a fundamental part of the methodology in order to ensure a good level of accuracy for the isolation of a leak using the polynomial interpolation.

As future work, the interpolation function will be tested in different prototypes.

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References

[1] CONAGUA. Manual de incremento de eficiencia física, hidráulica y energética en sistemas de agua potable., 2012.

[2] J. A. Delgado-Aguinaga, G. Besançon, O. Begovich, and J. E. Carvajal. Multi-leak diagnosis in pipelines based on extended Kalman filter. Control Engineering Practice, 49:139–148, 2016.

[3] O. Begovich and G. Valdovinos-Villalobos. DSP application of a water-leak detection and isolation algorithm. In 2010 7th International Conference on Electrical Engineering Computing Science and Automatic Control (CCE), pages 93–98. IEEE, 2010.

[4] Lizeth Torres, Gildas Besançon, Adrian Navarro, Ofelia Begovich, and Didier Georges. Examples of pipeline monitoring with nonlinear observers and real-data validation. In 8th IEEE International Multi-Conference on Signals Systems and Devices, Sousse, Tunisia, 2011.

[5] Tiantian Zhang, Yufei Tan, Xuedan Zhang, and Jinhui Zhao. A novel hybrid technique for leak detection and location in straight pipelines. Journal of Loss Prevention in the Process Industries, 35:157–168, 2015.

[6] Ole Morten Aamo. Leak detection, size estimation and localization in pipe flows. IEEE Transactions on Automatic Control, 61(1):246–251, 2016.

[7] Ignacio Barradas, Luis E. Garza, Ruben Morales-Menendez, and Adriana Vargas-Martínez. Leaks detection in a pipeline using artificial neural networks. In Iberoamerican Congress on Pattern Recognition, pages 637–644. Springer, 2009.

[8] Zdzisław Kowalczyk and Keerthi Gunawickrama. Detecting and locating leaks in transmission pipelines. In Fault Diagnosis, pages 821–864. Springer, 2004.

[9] Jorge Delgado-Aguinaga, Gildas Besançon, and Ofelia Begovich. Leak isolation based on extended Kalman filter in a plastic pipeline under temperature variations with real-data validation. In 2015 23th Mediterranean Conference on Control and Automation (MED), pages 316–321. IEEE, 2015.

[10] G. Espinoza-Moreno, O. Begovich, and J. Sanchez-Torres. Real time leak detection and isolation in pipelines: A comparison between sliding mode observer and algebraic steady state method. In 2014 World Automation Congress (WAC), pages 748–753. IEEE, 2014.

[11] Adrian Navarro, Ofelia Begovich, and Gildas Besançon. Real-time leak isolation based on state estimation with fitting loss coefficient calibration in plastic pipeline. Asian Journal of Control, 19(1):1–11, 2017 (in press).

[12] M Hanif Chaudhry. Applied hydraulic transients. Technical report, Springer, 1979.

[13] Herbert Keith Winning and Tim Coole. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. Flow, turbulence and combustion, 90(1):1–27, 2013.

[14] C. Verde. Multi-leak detection and isolation in fluid pipelines. Control Engineering Practice, 9(6):673–682, 2001.

[15] M. Elena Vázquez-Cendón. Numerical resolution of one-dimensional hyperbolic linear systems. In Solving Hyperbolic Equations with Finite Volume Methods, pages 57–72. Springer, 2015.

[16] Dan Simon. Optimal state estimation: Kalman, H infinity, and nonlinear approaches. John Wiley & Sons, 2006.

[17] John R. Hauser. Numerical methods for nonlinear engineering models. Springer Science & Business Media, 2009.

[18] Ofelia Begovich, Alejandro Pizano, and Gildas Besançon. Online implementation of a leak isolation algorithm in a plastic pipeline prototype. Latin American applied research, 57(6):131–140, 2012.