CGC initial conditions at RHIC and LHC

Javier L. Albacete\textsuperscript{1}, Adrian Dumitru\textsuperscript{2,3}, Yasushi Nara\textsuperscript{4}

\textsuperscript{1} Institut de Physique Théorique - CEA/Saclay, 91191 Gif-sur-Yvette cedex, France.
\textsuperscript{2} Department of Natural Sciences, Baruch College, CUNY, 17 Lexington Avenue, New York, NY 10010, USA
\textsuperscript{3} RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA
\textsuperscript{4} Akita International University, Yuwa, Akita-city 010-1292, Japan

E-mail: nara@aiu.ac.jp

Abstract. Monte-Carlo implementations of $k_T$-factorization formula with both KLN and running-coupling BK unintegrated gluon distributions for nucleus-nucleus collisions are used to analyze recent experimental data on the particle multiplicities from RHIC(Au+Au@200GeV) and LHC(Pb+Pb@2.76TeV). We also compare the predicted transverse energy at midrapidity to new data from ALICE.

1. Introduction

Relativistic Heavy Ion Collider (RHIC) at BNL has started in 2000 and many interesting data have been obtained. Experiments at Large Hadron Collider (LHC) at CERN has just started. It is well known fact that nearly perfect fluid picture works to explain large elliptic flow discovered at RHIC and LHC. However, there is no detailed understanding on the particle production in high energy hadronic collisions. Especially subsequent non-equilibrium evolution of the system (Glasma) created in nuclear collisions is not known well. Without knowing the detailed glue dynamics in early stages of nucleus collision, correct initial conditions for hydrodynamics would not be obtained. First important issue is to understand the gluon production in the first moment of collision. In order to study such gluon production in nucleus-nucleus collision, we shall use the Monte-Carlo implementation of $k_T$-factorization formulation in which fluctuations of the position of nucleon inside a nucleus is taken into account, and we can study nucleus-nucleus collision event-by-event.

2. Theoretical Models

We will use the Monte-Carlo KLN (MC-KLN) model [1] and its extension to running coupling Balitsky-Kovchegov (MCrcBK) [2] for the computation of gluon production in heavy ion collisions. Both model apply the $k_T$-factorized formula [3] in the transverse plane perpendicular to the beam axis locally. The number distribution of produced gluons is given by

$$\frac{dN_g}{d^2r_\perp dy} \sim \kappa_g \frac{4N_c}{N_c^2 - 1} \int \frac{d^2p_\perp}{p_\perp^2} \int d^2k_\perp \alpha_s \phi_A(x_1, k_\perp^2) \phi_B(x_2, (p_\perp - k_\perp)^2),$$

with $N_c = 3$ the number of colors. Here, $p_\perp$ and $y$ denote the transverse momentum and the rapidity of the produced gluons, respectively. The light-cone momentum fractions of the colliding
gluon ladders are then given by \( x_1,2 = p_{\perp} \exp(\pm y)/\sqrt{s_{NN}} \), where \( \sqrt{s_{NN}} \) denotes the center of mass energy. A constant gluon multiplication factor \( \kappa_g = 5 \) is assumed in the MCrcBK model to obtain \( p_{\perp} \)-integrated hadron yields [2]. It does not appear in the corresponding formula for the transverse energy \( dE_{\perp}/dy \) since gluon splitting and hadronization conserves energy.

At each grid point, we compute the thickness function \( T_A(\mathbf{r}_\perp) \) to obtain the local saturation scale and then compute gluon production probability. Within a hard disk nucleon approximation, thickness function is given by

\[
T_A(\mathbf{r}_\perp) = \frac{\text{number of nucleon within } S}{S} \quad (2)
\]

where we assume that the area \( S \) has the same value as the inelastic proton-proton cross section at the incident energy of \( \sqrt{s_{NN}} = 200 \text{ GeV} \) independent colliding energy.

In MC-KLN model, saturation momentum is defined as

\[
Q^2 s;A(x; \mathbf{r}_\perp) = 2 \text{ GeV}^2 \frac{T_A(x; \mathbf{r}_\perp)}{1.53 \text{ fm}^{-2}} \left( \frac{0.01}{x} \right)^{\lambda}, \quad (3)
\]

where \( \lambda \) is a free parameter which is expected to have the range of \( 0.2 < \lambda < 0.3 \) from HERA global analysis. In MC-KLN, we assume the gluon distribution function as

\[
\phi_{A,B}(x, k^2_\perp; \mathbf{r}_\perp) \sim \frac{1}{\alpha_s(Q^2_s)} \frac{Q^2_s}{\text{max}(Q^2_s, k^2_\perp)}, \quad (4)
\]

On the other hand, in MCrcBK, \( \phi \) is obtained from the Fourier transform of the numerical results of the running coupling BK (rcBK) evolution equation [7, 8, 9]:

\[
\phi_{A,B}(k, x, b) = \frac{C_F}{\alpha_s(k)^2(2\pi)^3} \int d^2 \mathbf{r} \, e^{-ik\cdot \mathbf{r}} \nabla^2_r N_G(r, Y = \ln(x_0/x), b). \quad (5)
\]

where \( C_F = (N_c^2 - 1)/2N_c \), \( x_0 = 0.01 \), and \( N_G \) is related to the quark dipole scattering amplitude that solves the rcBK equation, \( N \) as follows:

\[
N_G(r, x) = 2N(r, x) - N^2(r, x). \quad (6)
\]

In MC-KLN model, \( x \) dependence is determined by eq. (3), but in rcBK, \( x \) dependence can be obtained from the equation. Therefore, we expect that MCrcBK model has more predictive power than MC-KLN model.

3. Results

3.1. MC-KLN

In the figure 1, the charged particle multiplicities at RHIC and LHC as a function of \( N_{\text{part}} \) from KLN, fKLN, and MC-KLN model are compared to PHOBOS data.

In the KLN model, we plot the equation

\[
\frac{1}{N_{\text{part}}} \frac{dN}{dy} = c \ln \left( \frac{Q^2_s}{\Lambda^2_{\text{QCD}}} \right). \quad (7)
\]

This equation is obtained by picking up the most important contribution of integration in \( k_t \)-factorized formula by using average nuclear saturation momentum. On the other hand, fKLN model [4] does not take into account the effects of fluctuations of nucleons inside a nucleus unlike the MC-KLN model. The main difference between KLN and fKLN model is that fKLN model...
Figure 1. Centrality dependence of charged hadron multiplicity in Au+Au collision from the KLN, fKLN, and MC-KLN model are compared with PHOBOS data [5].

Figure 2. Centrality dependence of charged hadron multiplicity in Pb+Pb collision at \( \sqrt{s_{NN}} = 2.76 \) TeV from the fKLN, and MC-KLN model are compared with ALICE data [6].

3.2. MCrcBK

We now compare in Fig. 3, the centrality dependence of charged hadron with the results obtained from the MCrcBK model in which unintegrated gluon function is taken from the numerical solution of the rcBK equation. The initial condition for the rcBK evolution is assumed to be MV model:

\[
N(r, Y = 0) = 1 - \exp \left[ -\frac{r^2}{4 Q_0^2} \ln \left( \frac{1}{\Lambda r} + c \right) \right],
\]

with the initial saturation scale \( Q_0^2 = 0.2 \) GeV\(^2\) for nucleon and \( \Lambda = 0.2 \) GeV. For the running coupling, we use

\[
\alpha_s(r^2) = \frac{12\pi}{5 \ln \left( \frac{4 C^2}{r^2 \Lambda^2} \right)}
\]

with \( C = 1 \) in solving rcBK equation. rcBK unintegrated gluon function with these parameter set describes the centrality dependence of charged hadron multiplicity for both RHIC and LHC which indicates that most of the gluon is produced at the first impact of the nuclear collision. However, we should check parameter dependence carefully. For example, if one changes the parameter \( C \) in the running coupling, evolution speed changes.
In the bottom-up scenario [10], the number of gluon produced just after the collision will increase by a factor of \( \alpha_s(Q^2)^{2/5} \) during the subsequent evolution of the system. It would be interesting to include such effect into MCrcBK calculations.

### 3.3. Gaussian shape

So far we assumed that nucleus-nucleus collision is described by the incoherent sum of nucleon-nucleon collisions which will occur when transverse distance squared between two nucleons is smaller than the inelastic proton-proton cross section \( \sigma_{\text{NN}} \) divided by \( \pi \):

\[
(x_i - x_j)^2 + (y_i - y_j)^2 \leq \frac{\sigma_{\text{NN}}}{\pi}
\]  

This amounts to assume that nucleon is hard sphere (or disk). However, this approximation may not work in very high energy hadronic collisions. Let us think about the Gaussian shape nucleon in order to take into account the effect of extension of nucleon size as incident energy increases. In this case the thickness function becomes

\[
T_p(r) = \frac{1}{2\pi B} \exp\left[-r^2/(2B)\right],
\]

and the probability of nucleon-nucleon collision \( P(b) \) at impact parameter \( b \) is

\[
P(b) = 1 - \exp(-kT_{pp}(b)), \quad T_{pp}(b) = \int d^2s T_p(s) T_p(s - b).
\]

where (perturbatively) \( k \) corresponds to the product of gluon-gluon cross section and gluon density squared. We fix \( k \) so that integral over impact parameter becomes the nucleon-nucleon inelastic cross section \( \sigma_{\text{NN}} \) at the given energy:

\[
\sigma_{\text{NN}} = \int d^2b (1 - \exp(-kT_{pp}(b))).
\]

In this work, we use \( \sigma_{\text{NN}} = 41.94 \) mb for \( \sqrt{s_{\text{NN}}} = 200 \) GeV and \( \sigma_{\text{NN}} = 61.36 \) mb for \( \sqrt{s_{\text{NN}}} = 2.76 \) TeV. The result of this model is plotted in right hand side of Fig. 3.
underpredicts the multiplicity at peripheral collisions. One possible interpretation may be the following: viscosity is large at large impact parameter, entropy production may be larger as impact parameter becomes large.

3.4. Eccentricity and transverse energy

Finally, we plot eccentricity defined by

\[ \epsilon = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle} \]  \hspace{1cm} (14)

in Fig. 4. Since the eccentricity is proportional to the magnitude of elliptic flow, it is important to know the initial value of this quantity. One sees that the incident energy dependence of the eccentricity is very small according to the results from MCrcBK.

Fig. 4. Centrality dependence of eccentricity with respect to reaction plane in Pb+Pb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV and in Au+Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV.

Fig. 5 shows the centrality dependence of the transverse energy at central rapidity for the MV initial condition, and for an initial condition featuring a more rapid fall-off at \( k_\perp > Q_s \);
see ref. [2] for details. We note that $\sim 2.5\%$ $(0.5\%)$ of the energy of the beams is predicted to be deposited initially into the central rapidity region in central collisions. Longitudinal hydrodynamic expansion ($-p \Delta V$ work) may reduce the transverse energy by up to a factor of 2 [13]. In all, the model based on rcBK evolution of the gluon distribution appears to be consistent with the multiplicity and transverse energy data over a good order of magnitude in both $N_{\text{part}}$ and $\sqrt{s_{NN}}$.

4. Summary

Within the MC-KLN and MCrcBK model, we compute the gluon production based on the picture of Color Glass Condensate. Both MC-KLN and MCrcBK model reproduce the centrality dependence of charged hadron multiplicity at RHIC and LHC energies which indicate that the entropy production during the thermalization process may be small as well as the small viscosity after thermalization. However, we need detail systematic study of particle production by looking at different observables together with the parameter dependence.

Acknowledgments

The work of Y.N. was partly supported by Grant-in-Aid for Scientific Research No. 20540276. A.D. acknowledges support by the DOE Office of Nuclear Physics through Grant No. DE-FG02-09ER41620 and from The City University of New York through the PSC-CUNY Research Award Program, grant 63382-0041.

References

[1] H. J. Drescher and Y. Nara, Phys. Rev. C 75, 034905 (2007); 76, 041903(R) (2007).
[2] J. L. Albacete and A. Dumitru, 1011.5161[hep-ph], http://physics.baruch.cuny.edu/node/people/adumitru/res_cgc
[3] D. Kharzeev and M. Nardi, Phys. Lett. B507, 121 (2001); D. Kharzeev and E. Levin, ibid. B523, 79 (2001); D. Kharzeev, E. Levin, and M. Nardi, Phys. Rev. C 71, 054903 (2005); Nucl. Phys. A730, 448 (2004).
[4] A. Adil, H. J. Drescher, A. Dumitru, A. Hayashigaki, and Y. Nara, Phys. Rev. C 74, 044905 (2006).
[5] PHOBOS, B. B. Back et al., Phys. Rev. C65, 061901 (2002), nucl-ex/0201005.
[6] K. Aamodt et al. [The ALICE Collaboration], Phys. Rev. Lett. 105 (2010) 252301; Phys. Rev. Lett. 106 (2011) 032301.
[7] J. L. Albacete and Y. V. Kovchegov, Phys. Rev. D75, 125021 (2007), arXiv:0704.0612 [hep-ph].
[8] J. L. Albacete, N. Armesto, J. G. Milhano, and C. A. Salgado, Phys. Rev. D80, 034031 (2009), 0902.1112.
[9] J. L. Albacete, N. Armesto, J. G. Milhano, P. Quiroga Arias and C. A. Salgado, arXiv:1012.4408 [hep-ph].
[10] R. Baier, A. H. Mueller, D. Schiff and D. T. Son, Phys. Lett. B 502, 51 (2001); Phys. Lett. B 539, 46 (2002); arXiv:1103.1259 [nucl-th].
[11] S. S. Adler et al. [ PHENIX Collaboration ], Phys. Rev. C71, 034908 (2005).
[12] A. Toia for the ALICE Collaboration, talk given at “Quark Matter 2011”, 23 – 28 May 2011, Annecy, France
[13] M. Gyulassy, T. Matsui, Phys. Rev. D29, 419-425 (1984); M. Gyulassy, Y. Pang, B. Zhang, Nucl. Phys. A626, 999-1018 (1997); K. J. Eskola, K. Kajantie, P. V. Ruuskanen, K. Tuominen, Nucl. Phys. B570, 379-389 (2000); A. Dumitru, M. Gyulassy, Phys. Lett. B494, 215-220 (2000).