ABSTRACT

We show that Fermi repulsion can lead to cored density profiles in dwarf galaxies for sub-keV fermionic dark matter. We treat the dark matter as a quasi-degenerate self-gravitating Fermi gas and calculate its density profile assuming hydrostatic equilibrium. We find that suitable dwarf galaxy cores of size $\gtrsim 130$ pc can be achieved for fermion dark matter with mass in the range 70 eV – 400 eV. While in conventional dark matter scenarios, such sub-keV thermal dark matter would be excluded by free streaming bounds, the constraints are ameliorated in models with dark matter at lower temperature than conventional thermal scenarios, such as the Flooded Dark Matter model that we have previously considered. Modifying the arguments of Tremaine and Gunn we derive a conservative lower bound on the mass of fermionic dark matter of 70 eV and a stronger lower bound from Lyman $\alpha$ clouds of about 470 eV, leading to slightly smaller cores than have been observed. We comment on this result and how the tension is relaxed in dark matter scenarios with non-thermal momentum distributions.

Key words: Cosmology: theory, dark matter, elementary particles – galaxies: dwarf.

1 INTRODUCTION

The cold dark matter paradigm ($\Lambda$CDM) provides a remarkably good description of cosmology and astrophysics. However, observations connected with small scales might be in tension with this framework. In particular, structure formation simulations to date assuming $\Lambda$CDM predict a greater number of galaxy satellites and suggest that the density profiles of dwarf spheroidal galaxies (dSphs) should exhibit cusps, in contrast to observations (de Blok 2009; Weinberg et al. 2013). Some cores might be explained by baryonic feedback e.g. (Weinberg & Katz 2001; Mashchenko et al. 2008; Pontzen & Governato 2012; Nipoti & Binney 2015), but this is still undecided (Sellwood 2003; Dubinski et al. 2009; Jardel & Sellwood 2008; Penarrubia et al. 2012; Marinacci et al. 2014), indicating the possibility of a more fundamental discrepancy. It is of interest to look at potential resolutions from dark matter effects beyond $\Lambda$CDM, such as warm dark matter (Dalcanton & Hogan 2000), self-interactions (Spergel & Steinhardt 2000), or boson degeneracy (Ji & Sin 1994; Hu et al. 2000; Goodman 2000; Peebles 2000; Hui et al. 2016).

An interesting alternative resolution was proposed by Destri et al. (2012) as well as Domcke & Urbano (2014) (see also Alexander & Cormack, 2016), who observed that a sub-keV fermion dark matter particle could resolve the core-cusp problem when quantum pressure due to Fermi repulsion provides the dominant support in the central regions of dwarf galaxies. In this scenario, the cores of dwarf galaxies are highly analogous to white dwarfs or neutron stars, both of which are supported by degeneracy pressure. This proposal would have the remarkable implication that the structure of dwarf galaxies arises due to a quantum effect on a cosmic scale.

However, dark matter composed of a thermal relic lighter than a few keV would erase small scale density perturbations, in conflict with observations (Bode et al. 2001; Bolton et al. 2004; Viel et al. 2013). Thus the prospect of Fermi pressure playing a role in the structure of dwarf galaxies is lost for dark matter at the baryonic temperature. However, this scenario might be realized when dark matter is not in thermal contact with baryons so it is cooler – when dark matter is decoupled and is not reheated by the Standard Model, for example. The dark matter would then be non-relativistic earlier than a thermal relic, mitigating free streaming bounds. For models that realize such a scenario see for instance: Feng et al. (2008); Berezhiani et al. (1995).

Another possibility is that the lower limit on the dark matter mass is relaxed by a late-entropy release, as with “Flooded Dark Matter” (FDM) (Randall et al. 2013). In this class of models the Standard Model undergoes late-time reheating via entropy injections from long-lived heavy states decaying to the visible sector. We provide a brief outline of the minimal model of Flooded Dark Matter in Appendix B1. We recast the warm dark matter mass bounds derived from Lyman-$\alpha$ data into constraints on models such as this one.
with colder temperature and find the following mass limit for Flooded Dark Matter

\[ m_{\text{FDM}} \gtrsim 470 \text{ eV} N_f^{1/4}, \tag{1} \]

in terms of \( N_f \) ‘flavors’ of Majorana fermion dark matter. This value is borderline for resolving the core-cusp problem but might work well with baryonic feedback in explaining the variability in core sizes that is observed so far. The full set of constraints we present is:

- **Lower mass bound from fit to cores:** Domcke & Urbano (2014) find a preference for a narrow dark matter mass range \( 100 \text{ eV} \lesssim m \lesssim 200 \text{ eV} \). Once we include the effects of a thermal envelope these constraints reduce to \( m \gtrsim 50 \text{ eV} \), with no upper bound on the mass.

- **Upper mass bound from core size:** Both Walker & Peñarrubia (2011) and Amorisco et al. (2013) find a two sigma constraint of \( R_c(2\sigma) > 200 \text{ Parsec} \), which would imply \( m \lesssim 310 \text{ eV} \). When we partially redo their analysis with a Burkert profile for dark matter we obtain smaller preferred values \( R_c = 0.63^{+0.17}_{-0.13} \text{ kpc} \). Our limited analysis does not recover their result but when we fit their data using a Fermi core with two scales (core radius and scale radius), we obtain an even smaller core \( R_c = 0.45^{+0.11}_{-0.11} \text{ kpc} \), consistent with a larger fermion mass of \( m \lesssim 280 \text{ eV} \). We can conclude, based on comparing our one-scale and two-scale results, that the profile with the Fermi-degenerate core (and hence two scales) should reduce the lower bound on the core size and increase the upper bound on the mass of the fermion to \( m < 400 \text{ eV} \).

- **Lower mass bound from largest allowed core size:** Amorisco et al. (2013) also put an upper bound on core size at 2\( \sigma \) level \( R_c(2\sigma) < 2.6 \text{ kpc} \) for the Fornax dSph which implies \( m \gtrsim 70 \text{ eV} \) lower mass bound.

- **Lower mass bounds from black hole formation:** We show that \( m \gtrsim 0.1 \text{ eV} \) suffices to avoid forming black holes out of degenerate Fermi gas cores of dwarf galaxies.

- **Lyman-\( \alpha \):** For constant \( n_\alpha \), Baur et al. (2015) found \( m_{\text{FDM}} > 2.93 \text{ keV} \), corresponding to \( m \gtrsim 530 \text{ eV} \) as we will show in eq. (2). The most conservative, even weaker, bound for warm dark matter was \( m_{\text{WDM}} > 2.5 \text{ keV} \), corresponding to \( m \gtrsim 470 \text{ eV} \). We later argue that using a flavored dark matter model we can relax the Lyman-\( \alpha \) bounds arbitrarily, at the cost of more complicated dark matter model. Alternatively, while the Lyman-\( \alpha \) constraint \( (m \gtrsim 470 \text{ eV}) \) is in tension with the mass range for which appropriately large cores are obtained \( (m < 400 \text{ eV}) \), this bound can be reconciled if the dark matter has a non-thermal momentum distribution. We will discuss one way to achieve such a non-thermal scenario.

In any case, the coring mechanism based on Fermi degeneracy operates in conjunction with baryonic feedback and it is quite likely that baryonic feedback creates a large core in the Fornax galaxy for example. As a result it may be the case that only smaller cores in systems such as Draco need to be resolved by new physics, in which case the Lyman-\( \alpha \) constraints become irrelevant.

In what follows, we extend and improve the analysis of Domcke & Urbano (2014). The earlier study made the simplifying assumption of a completely degenerate Fermi gas, which is not appropriate for realistic scenarios. We treat the dark matter as a quasi-degenerate Fermi gas surrounded by a thermal envelope. Furthermore, we highlight concerns regarding the criteria used in matching to observed data and clarify the condition under which dSphs would be successfully cored through Fermi pressure.

The structure of this paper is as follows: In Section 2 we outline the physics of self-gravitating Fermi gases and the role of Fermi repulsion in determining the density profile. We argue that even with sharp power law density profiles dwarf galaxies cannot accommodate arbitrarily large central densities and that Fermi repulsion is generally important in the central region. In Section 3 using the empirically motivated criterion that dwarf spheroidal galaxies should feature a core of order a few hundred pc, we determine the parameters that yield appropriate sized cores. We point out essential details, such as the importance of stellar anisotropy degeneracy with reference to Domcke & Urbano (2014). We also supplement with newer data to reproduce observed properties of the classical Milky Way dwarf galaxies. In Section 4 we derive a number of lower bounds on the mass of fermion dark matter. In Section 5 we discuss the lower bound on the mass of fermion dark matter from the requirement that dwarf cores do not collapse into black holes. Appendix A presents certain definitions and derivations that we use in the main text and Appendix B gives more details about the models we discuss in this paper.

## 2 Fermi Repulsion

We first argue that in the absence of a central black hole, sufficiently light fermionic dark matter manifests Fermi repulsion that would be relevant in the central region of dwarf galaxies. Assuming a cuspy profile as a starting point, there would always exist a central region with density high enough that the Fermi velocity would exceed the escape velocity so the density in the central region would be inconsistent with the Pauli exclusion principle and would have to be reduced. We later show that full solutions including Fermi pressure lead to cored density distributions.

A fermionic gas of density \( \rho \) is degenerate at temperatures below

\[ T_{\text{deg}} \sim \frac{h^2}{2\pi m} \left( \frac{\rho}{2m} \right)^{3/2}, \tag{2} \]

where \( m \) is the mass of the fermion. In this regime the highest occupation level is \( p_F \) (see e.g. Landau & Lifshitz (1980))

\[ p_F = m v_F = h \left( \frac{3}{\pi^2 N_f} \frac{\rho}{m} \right)^{1/3}, \tag{3} \]

leading to the Fermi pressure in the non-relativistic limit:

\[ P_F = \frac{8\pi}{3h^3} \int_0^{p_F} dp \left( \frac{p^4}{\sqrt{p^2 + m^2}} \right) = \frac{h^2}{5m^{3/2}} \left( \frac{3}{8\pi N_f} \right)^{2/3} \rho^{5/3}, \tag{4} \]

which at high density and low temperature dominates over the classical pressure (we choose units such that \( k = 1 \))

\[ P_{\text{cl}} = \rho \left( \frac{T}{m} \right), \tag{5} \]

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A typical temperature at any point inside this ball is determined by the virial condition
\[ T(R) = \frac{GM(R)m}{2R}. \] (6)

Therefore, the classical pressure can be estimated to be
\[ P_{cl} = \frac{GM(R)p(R)}{2R}. \] (7)

When \( T \gtrsim T_{\text{deg}} \) the classical pressure term dominates. The pressure of the gas is then
\[ P = \begin{cases} \frac{GM(R)p(R)}{2R} & T \geq T_{\text{deg}} \\ \left( \frac{\rho(R)}{\rho_0} \right)^{n/3} & T \leq T_{\text{deg}} \end{cases}. \] (8)

### 2.1 Power Law Profiles

We now argue that systems described by integrable power-laws will generically become quasi-degenerate. It is useful to parameterize the density as
\[ \rho = \rho_0 \left( \frac{R_0}{R} \right)^n. \] (9)

Integrable profiles satisfy the condition
\[ \frac{d\log \rho}{d\log R} > -3 \quad \text{as} \quad R \to 0. \] (10)

We claim that for an integrable density profile one can always find a radius below which the Fermi velocity is larger than escape velocity \( v_{\text{esc}} \), leading to an inconsistent density profile, because states with \( v > v_{\text{esc}} \) will escape the gravitational potential, thus reducing the central density. Hence, the system would dynamically relax through evaporative cooling to a quasi-degenerate Fermi gas even from an initially non-degenerate configuration.

Observe that eq. (1) implies a mass profile of the form
\[ M(R) = 4\pi \int_0^R R^2 \rho(R') \, dR' = 4\pi \left( \frac{\rho_0 R_0^n}{3 - n} \right) R^{3-n}. \] (11)

It follows that the virial velocity can be written
\[ v_{\text{vir}}^2 \equiv \frac{GM(R)}{R} \approx 4\pi GR^{2-n} \left( \frac{\rho_0 R_0^n}{3 - n} \right). \] (12)

For the given local density \( \rho(R) \), the Fermi surface lies at
\[ p_F = \hbar \left( \frac{3\pi^2 \rho}{m} \right)^{1/3} = \hbar \left( \frac{3\pi^2 \rho_0 R_0^n}{m R^n} \right)^{1/3}, \] (13)

which determines the Fermi velocity
\[ v_F = p_F/m \equiv \frac{\hbar}{m^{1/3}} \left( \frac{3\pi^2 \rho_0 R_0^n}{R^n} \right)^{1/3}. \] (14)

There is some critical radius \( R_\epsilon \) for which \( v_{\text{vir}} = v_F \). This roughly coincides with the point at which \( T < T_{\text{deg}} \). For an integrable profile \( n < 3 \)
\[ R_\epsilon^{(n)} = R_0^{(6-n)} \left( \frac{\hbar^6}{G^3 \rho_0^3 m} \frac{9\pi^3}{4} \right)^{1/(6-n)}. \] (15)

We trade the unknown parameters \( \rho_0 \) and \( R_0 \) for the measured mass \( M_{1/2} \) within a half-light radius \( R_{1/2} \), defined by
\[ 4\pi \left( \frac{\rho_0 R_0^n}{3 - n} \right) = \frac{M_{1/2}}{R_{1/2}^3 n}, \] (16)
in order to obtain
\[ R_{\epsilon}^{(n)} = \left[ \frac{\hbar^6}{m^8} \frac{9\pi^2 (3 - n)^2}{16} \frac{1}{G^3} \left( \frac{R_{1/2}^3}{M_{1/2}} \right) \right]^{1/(6-n)}. \] (17)

Degeneracy effects are important in the region \( R < R_\epsilon \). For example for \( n = 2 \):
\[ R_{\epsilon}^{(2)} \approx 160 \text{ pc} \left( \frac{300 \text{ eV}}{m} \right) \left( \frac{R_{1/2}}{100 \text{ pc}} \right)^{1/4} \left( \frac{10^8 M_\odot}{M_{1/2}} \right)^{1/4}. \] (18)

Similarly for \( n = 0 \):
\[ R_{\epsilon}^{(0)} = 130 \text{ pc} \left( \frac{300 \text{ eV}}{m} \right)^{4/3} \left( \frac{R_{1/2}}{100 \text{ pc}} \right)^{1/2} \left( \frac{10^8 M_\odot}{M_{1/2}} \right)^{1/6}. \] (19)

More generally, for \( 0 \leq n < 3 \) solving consistently gives a core of \( O(100) \) pc for dark matter masses of a few hundred eV, for typical values of \( M_{1/2} \) and \( R_{1/2} \).

Having shown that a integrable profile always leads to Fermi degeneracy (for a sufficiently small mass-dependent region), we now consider non-integrable profiles (which do not satisfy the inequality of eq. (10)). Since these profiles lead to an unphysical infinite total mass in the center of the distribution they must be cut off at some small radius \( R = \epsilon \). Below the cutoff \( \epsilon \) the profile transitions to an integrable profile. Rather than focus on the outer region, we consider the central cutoff region, which we parameterize solely by the mass \( M \) enclosed and the cut-off radius \( \epsilon \).

We determine the minimum radius required to fit a typical dwarf galaxy mass inside a given size as a function of the fermion mass, where we require that the escape velocity is larger than the Fermi velocity everywhere inside the gas. This happens for
\[ v_{\text{esc}}^2 = \frac{GM}{\epsilon} > v_F^2 = \frac{\hbar^2}{m^{1/3}} \left( \frac{3\pi^2 \rho}{m} \right)^{2/3}. \] (20)

The least stringent requirement is found when the matter is evenly distributed over the whole region, generating the lowest maximum density, in which case the system must satisfy:
\[ \frac{GM}{\epsilon} > \frac{\hbar^2}{m^{8/3}} \left( \frac{9\pi M}{4 \epsilon^3} \right)^{2/3}. \] (21)

The above inequality implies a lower bound on the cut off
\[ \epsilon > \frac{\hbar^2}{GM^{1/3} m^{8/3}} \left( \frac{9\pi}{4} \right)^{2/3} \left( \frac{10^8 M_\odot}{M} \right)^{2/3} \left( \frac{300 \text{ eV}}{m} \right)^{8/3}. \] (22)

Therefore for light fermion masses, \( \epsilon \) needs to be similar to, or greater than, the dwarf galaxy size. In other words, the system should be described by an integrable profile over the whole range.

One might suppose that rather than a soft profile below the cutoff of a sharp profile one could encounter a sequence of nested singular profiles. However, ultimately this sequence must be cut off at some \( R = \epsilon \) at which the profile becomes non-singular, and the argument above still applies. Having reduced to the previous case, we conclude that a Fermi degenerate region must always be present in a physically relevant region.
We next solve the hydrostatic equilibrium equations explicitly to derive the density profile accounting for Fermi degeneracy to show that the result is a cored central region.

### 2.2 Static Solutions

We are interested in determining the density profile of a self-gravitating ball of fermions. Such a static density profile has to satisfy three equations:

**i)** hydrostatic equilibrium

$$\frac{dP}{dR} = -\left(\frac{GM(R)}{R^2}\right)\rho(R), \quad (23)$$

**ii)** pressure equality

$$P(R) = P_{\text{cl}} + P_F$$

$$\approx \frac{GM(R)\rho(R)}{2R} + \frac{h^2}{5m^{\frac{3}{5}}} \left(\frac{3}{8\pi N_f}\right)^{\frac{2}{3}} \rho(R)^{\frac{5}{3}}, \quad (24)$$

**iii)** the continuity condition

$$M(R) = 4\pi \int_0^R \rho(r)r^2 dr. \quad (25)$$

Equations (23)–(25) with initial conditions \(\rho(0) = \rho_0\), and \(M(0) = 0\) form a closed system that can be solved to obtain a self-consistent static density profile of a gas of fermions.

First we construct solutions supported by degeneracy pressure alone, by setting \(P_{\text{cl}} = 0\). In this case eqs. (23), (24) reduce to

$$\frac{h^2}{3m^{\frac{3}{5}}} \left(\frac{3}{8\pi}\right)^{\frac{2}{3}} \frac{d}{dR} \left(\frac{R^2}{\rho(R)^{\frac{1}{3}}} \frac{d\rho(R)}{dR}\right) = -4\pi GR^2 \rho(R). \quad (26)$$

Note that for small \(R\) an approximate solution is given by \(\rho = \rho_0[1 - (R/R_c)^2]\), for \(R_c\) an appropriately chosen scale

$$R_c^2 = \frac{h^2}{2\pi Gm^{\frac{3}{5}} \rho_0^{\frac{1}{3}}} \left(\frac{3}{8\pi N_f}\right)^{\frac{2}{3}}, \quad (27)$$

in agreement with the approximate analysis in the previous section. We can also see that this scale arise in eq. (15) for the choice \(n = 0\). This indicates we should expect a constant density core of size \(R_c\) as Fermi pressure flattens the expected cusps of density distributions of dwarf galaxies.

We now look at the full solution. Eq. (26) is an example of a Lane-Emden equation with index \(n = \frac{3}{2}\). The solutions to this differential equation are polytropes with index \(\gamma = \frac{3}{5}\), see e.g. (Jaffe 2006; Domeck & Urbano 2014). Indeed, these solutions are constant as \(R \to 0\), acting as cored profiles. Moreover, the density profile of a fully degenerate Fermi gas has finite extent with density vanishing at

$$R_c^3 = \xi_1^2 \frac{h^2}{8\pi Gm^{\frac{3}{5}} \rho_0^{\frac{1}{3}}} \left(\frac{3}{8\pi N_f}\right)^{\frac{2}{3}}, \quad (28)$$

where \(\xi_1 = 3.65\) is a numerical constant.

Figure 1 illustrates the behavior of the two pressure components for an example with dark matter mass of \(m = 200\) eV and central density \(\rho_0 = 10^{-20}\) kg/m\(^3\). With these parameter values, \(R_c \approx 440\) pc (taking \(N_f = 1\)). We see that close to \(R_c\), the classical pressure is as important as the Fermi pressure.

To increase the range of validity of our density profiles we investigate the full solutions to eqs. (23)–(25) including both \(P_{\text{cl}}\) and \(P_F\). Figure 2 shows a set of solutions to these equations including both classical and degeneracy pressure (we fix the central densities of these solutions such that they reproduce the \(M_{1/2}\) for the Fornax dSph). These solutions are constant for \(R \to 0\) just as those supported by the Fermi pressure alone. However, the full solutions do not vanish at some finite \(R\). In fact \(\rho \sim R^{-2}\) is a solution for \(R \to \infty\) as the classical pressure term takes over.

There is a characteristic length scale associated with the transition from constant density core to isothermal behavior. We can define this scale as the radius \(R_P\) at which the \(P_{\text{cl}} = P_F\). We choose to define the core radius \(R_c\) as a radius at which the slope of the density distribution reaches a particular value and we use the definition of (Burkert 2015):

$$\frac{d\log \rho}{d\log R} \bigg|_{R_c} = -\frac{3}{2}. \quad (29)$$

This definition allows us to compare our results with the results of others in a model-independent way. Notice that \(R_0 \sim R_a \sim R_c^{(0)} \sim R_P\), and similarly \(R_c \sim R_0\). This is not surprising as there is only one relevant combination\(^1\) of \(G, h, m\) and \(\rho_0\) with dimensions of length namely eq. (28).

### 3 FITS TO MEASURED DWARF GALAXIES

In the previous section we have shown that including Fermi pressure in the hydrostatic equations for a self-gravitating gas of light fermions flattens their density profiles to constant density cores with mass-dependent characteristic sizes. We also showed that including the classical pressure term extends otherwise finite extent solutions to include isothermal tails. In this section we fit these quasi-degenerate profiles to the kinematic properties of the eight classical dwarf galaxies: Carina, Draco, Fornax, Leo I, Leo II, Sculptor, Sextans, and Ursa Minor, see e.g. (Walker et al. 2009). We first consider the kinematic variables studied in (Domeck & Urbano 2014).

\(^1\) It is in fact possible to construct additional length scales, however, these are many orders of magnitude smaller or larger and hence irrelevant.

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and demonstrate that due to a degeneracy this method does not suffice to determine the core sizes.

3.1 Profile Degeneracy

One of the main observables that is used to obtain kinematic information is the projected velocity dispersion along the line-of-sight (LOS) \( \sigma_{\text{LOS}} \), as defined in (Binney & Tremaine 2008):

\[
\sigma_{\text{LOS}}^2(R) = \frac{2G}{I(R)} \int_{R}^{\infty} \nu(r')M(r')(r')^{3/2}F(\beta, r, R') \, \mathrm{d}r',
\]

(30)

where \( I(r) \) is the projected stellar density, and \( \nu(r') \) is the associated 3D stellar density. Following e.g. (Walker et al. 2009), we use the Plummer profile (Plummer 1911), and we give the forms of \( I(r) \), \( \nu(r') \) and \( F(\beta, r, R') \) in Appendix A. As an example, we show the data reported in (Walker et al. 2009) for the classical dwarf galaxy Fornax in Figure 2.

Figure 2: We treat the dark matter as a quasi-degenerate self-gravitating Fermi gas for constituent fermions of various masses and different central densities (as can be read off from the plot). We take the central densities of these profiles such that they reproduce the \( M_{1/2} \) for the Fornax dSph. Observe for 2 keV fermions the profile is decidedly cuspy, whereas the profiles for 100 eV and 200 eV dark matter exhibits flat cores of a few hundred parsecs. The dashed curves correspond to solutions to eq. (20), and give the distributions corresponding to fully degenerate Fermi gases.

The important feature of Figure 2 is that the simulations, assuming a Fermi gas, demonstrate that due to a degeneracy this method does not suffice to determine the core sizes.

3.2 Beyond \( \beta \)-Degeneracy: Core Sizes

New approaches circumvent the \( \beta \)-degeneracy (Battaglia et al. 2008; Walker & Penarrubia 2011; Agnello & Evans 2012; Amorisco et al. 2013) and can be used to more robustly determine core sizes of dwarf spheroidals in which star subpopulations can be separated. Using these more recent results we show that there is a range of fermion masses with cores of...
the right order of magnitude and which fit the stellar kinematics. The approach of Battaglia et al. (2008), Walker & Penarrubia (2011), Agnello & Evans (2012), Amorisco et al. (2013) is to split the tracer star population chemically into subpopulations and extract the mass enclosed at radii at which the uncertainties due to the stellar orbital anisotropy are the smallest for each subpopulation. The resulting mass profile is only known at two or three radial points, since the subpopulations have different spatial extent, but is nearly insensitive to $\beta$. Studies utilizing this approach have confirmed the presence of flat cores in Fornax and Sculptor and are in broad agreement (Battaglia et al. 2008; Walker & Penarrubia 2011; Agnello & Evans 2012; Amorisco et al. 2013).

Walker & Penarrubia (2011) consider a cored and a cuspy density profile to argue that the dSphs Fornax and Sculptor require core radii of order a few hundred pc at $\sim 95\%$ CL. To match observations we will therefore require a constant density core of magnitude

$$R_c \sim \text{few} \times 100 \text{ pc} \ .$$

(33)

Similarly Amorisco et al. (2013) find that at 2$\sigma$ confidence level $R_c > 200$ pc for the Fornax dSph.

However, both Walker et al. (2009) and Amorisco et al. (2013) considered profiles that have a steep drop-off in density ($1/r^3$) outside their core. However, the solution to the hydrostatic equilibrium equation that we derived has an isothermal envelope that behaves like $1/r^2$ (with presumably a steeper drop-off further out). Since both Burkert and NFW profiles use only one scale radius, fitting the kinematic properties of the outer parts of the galaxy makes the core radii of Walker & Penarrubia (2011) and Amorisco et al. (2013) larger than those obtained using an isothermal profile. The fit is sensitive to both the physics responsible for the core and the physics responsible for the asymptotic behavior of the dwarf galaxy. With only one variable to fit, the derived core radii are some sort of geometrical averages of these two scales.

Because our profile includes an explicit core, our solution requires that we separate the two scales. In order to make this clear we fit the $\sigma_{\text{LOS}}$ data for three distinct stellar subpopulations from Amorisco et al. (2013) to a two scale profile (with one more scale parameter than NFW or Burkert profiles) of the form:

$$\rho^2(r) = \frac{\rho_0}{\left[1 + (r/R_c)^2\right]^{\beta_c} \left[1 + r/R_s\right]^\beta_s} .$$

(34)

This profile behaves like a cored profile up to $r \sim R_c$, transitions into an isothermal profile and then asymptotes to a $r^{-3}$ profile when $r > R_c$. We used the velocity dispersions of three sub-populations of the Fornax galaxy from figure 1 of Amorisco et al. (2013) to constrain a two scale dark matter density profile in Fornax.

We fit the data in a following way: for each combination of $R_c$, $R_s$ we sample the four dimensional space of proposed central density $\rho_0$ of the dark matter density profile and three independent velocity anisotropies of each star subpopulation $\beta_i$. We marginalized over this four-dimensional parameter space (central density, three independent $\beta_i$ for each sub-population) to obtain the best $\chi^2$ for a given $R_c$, $R_s$.

Figure 4. The $\chi^2$ per degree of freedom for fits to the eight classical dwarf galaxies as a function of dark matter mass $m$. In blue (yellow) is the $\chi^2$ associated to fits for a quasi (fully) degenerate Fermi gas. The flat $\chi^2$ exhibited by the blue curve for $m > 200$ eV is a result of the $\beta$-degeneracy.

Figure 5. Correlation between core size $R_c$ and central density $\rho_0$ of the eight classical Milky Way dwarf spheroidal galaxies from different fits. “Burkert’s Dwarfs” (red) and “Burkert’s MMGs” (black) refers to fits identified in Burkert (2015) for dwarfs and more massive galaxies, respectively. The blue/yellow/green points provide fits for the eight classical dwarfs assuming a quasi-degenerate Fermi gas for different dark matter masses. Note that some dwarfs have similar parameter fits and so all eight points are not always distinct. (Burkert 2015) observed that the populations appear to obey the scaling relationship $\rho_0 R_c^2 = 75 \pm 30 \ M_\odot \ pc^{-2}$, this is indicated by the solid/dashed lines.

Figure 6. Contours of $\chi^2$ for fits to data of Amorisco et al. The red bands bounded by the dashed (solid) contours show the 1$\sigma$ (2$\sigma$) fits to with a Burkert profile with a single scale parameter $R_c$ (and is therefore independent of $R_s$). The blue bands correspond to a fit with a profile from eq. (34). The grey area corresponds to parameter space $R_c > R_s$. Although not in principle invalid, these solutions do not correspond to quasi-degenerate fermi gas.

\[\chi^2_{DOF} \]

2 M. G. Walker, private communication.
combination. Our fit indicates that allowing for a second scale relaxes the lower bound on the core radius to below 300 pc as can be seen in Figure 6.

However, when we redo the Amorisco et al. analysis for a Burkert profile, we do not reproduce their results: our fit of the Burkert profile leads to $R_c = 650 \pm 130$ pc instead of $R_c = 1000 \pm 400$ pc from Amorisco et al. (2013). We are unable to resolve this discrepancy without a careful reanalysis of the original data. Our incomplete reanalysis of Amorisco et al. would lead to a stronger bound which is however less reliable than the original bound. However, based on comparing our one-scale and two-scale results, the two-scale profile tends to reduce the lower bound on core size compared to the Burkert profile by about a factor of $\sim 1.5$ cf. Figure 6. Applying this factor, we assume that a full analysis with a two-scale profile would lead to reduced lower bound on the core size somewhere in the proximity of $R_c > 200/1.5 \text{ pc} \sim 130$ pc. This lower bound on core size would correspond to a reduced upper bound of $m < 400$ eV.

We illustrate these results in Figure 7. This plot shows our prediction for the core radius in the Fornax dSph as function of $m$, by demanding that the profile reproduces the observed half-light mass $M_{1/2}$ as given in Walker et al. (2009). The blue band indicates the core size preferred by Amorisco et al. (2013) $(200 \text{ pc} < R_c < 2600 \text{ pc})$. Should these be relaxed by a two scale fit, we also mark a weaker condition $R_c > 130$ pc by a green band. Our fits to the core radius data of Amorisco et al. (2013) favor masses of 70 eV – 320 eV and extend to 400 eV once we consider the effect of a two-scale fit.

Some variation in cores size is expected due to differences between dwarfs, e.g. Fornax has stellar mass $8 \times 10^9 M_\odot$ (de Boer et al. 2012b), while Sculptor has $3 \times 10^7 M_\odot$ (de Boer et al. 2012a). Variations in the core size among dwarfs in this model is a result of differences in their central densities.

4 LOWER MASS BOUNDS

Although Fermi repulsion might improve the density profiles of light fermionic dark matter, too low a mass would lead to overly large cores that would disagree with observed structures of dwarf galaxies. Too light dark matter would also prevent the formation of structure on small mass scales, leading to disagreement with Lyman-$\alpha$ constraints on the matter power spectrum. This latter bound depends on the relative temperature of the dark matter, which leads to model dependence. We now derive both bounds.

4.1 Dwarf Galaxy Lower Mass Constraints

We focus on the dwarf galaxy Fornax, for which the core radius has been measured (albeit with sizable uncertainty) by Amorisco et al. (2013):

$$R_{c,Fornax} = 1^{+0.8}_{-0.4} \text{ kpc}.$$  \hspace{1cm} (35)

Fermion masses that would produce cores in excess of $R_c \sim 2.6$ kpc are ruled out at 95% CL by this measurement.

If the density profile remains flat to large radius, the central density of Fornax can be self-consistently determined from the half-light radius $R_{1/2} = 668 \pm 34$ pc (note that

![Figure 7. Applying the static solution of Section 2 to the Fornax dwarf galaxy, we show the necessary dark matter mass $m$ as a function of the desired core radius $R_c$. The blue part of the curve indicates the preferred set of $R_c$ from Walker & Penarrubia (2011). The green portion of the curve shows a weaker condition $R_c > 130 \text{ pc }$.](image-url)

$R_{1/2} < R_c$ and the mass this encloses is $M_{1/2} = (5.3 \pm 0.9) \times 10^7 M_\odot$ (Walker et al. 2009). Taking the central values gives

$$\rho_0 = \frac{M_{1/2}}{\frac{4}{3} \pi R_{1/2}^3} \approx 3 \times 10^{-21} \text{ kg/m}^3.$$  \hspace{1cm} (36)

With this central density, this scenario predicts a core in excess of 2.6 kpc for dark matter mass $m \lesssim 70$ eV, cf. Fig 7. As a result we obtain a bound

$$m \gtrsim 70 \text{ eV }.$$  \hspace{1cm} (37)

This argument is similar to the reasoning underlying the Tremaine-Gunn bound (Tremaine & Gunn 1979), in which the authors used an isothermal profile in the Milky Way galaxy and other structures to put a lower mass bound on dark matter based solely on phase space density considerations, and a slightly weaker bound based on Fermi repulsion. They too obtained their bound by observing that with too small a mass for the dark matter particle, objects of a given total mass would exceed their observed sizes (this statement can be re-expressed in terms of velocity dispersions). Here we use the cored profile given by Fermi repulsion and find that the radius of Fornax would exceed its measured value unless a minimum mass bound applies. Future observations should provide a better understanding of $R_c, R_{1/2}$ and $M_{1/2}$ in Fornax and other dwarf spheriodals, which will in turn improve the lower mass bound for fermion dark matter.

4.2 Lyman-α bound on light dark matter

Small scale structure growth also gives lower bounds on the dark matter mass. The growth of over-densities in the matter distribution is suppressed when dark matter is relativistic. As a result, if dark matter is too light and relativistic during late cosmic evolution, small scale structure over-densities (the matter power spectrum for large $k$) would be suppressed compared to the standard $\Lambda$CDM scenario. But we know that the matter power spectrum $P(k)$ is not suppressed at large $k \sim \text{ Mpc}^{-1}$ because we observe absorption lines in the corresponding spectra of quasars due to small hydrogen clouds in their lines of sight – the so called Lyman-$\alpha$ forests.
Existence of these lines has been interpreted as a lower bound on the mass of dark matter by studies such as Bolton et al. (2004) and Viel et al. (2013). The current bounds on dark matter mass come from Baur et al. (2015) who require that conventional thermal dark matter has to have $m_{\text{WDM}} > 2.5$ keV at 95% CL.

We note that such bounds depend on the dark matter temperature, which is critical to determining which states are relativistic when the Lyman-α producing clouds are formed. Allowing for dark matter that is colder than the Standard Model sector and that becomes non-relativistic even at a temperature which is critical to determining which states are relativistic can have a nonthermal spectrum from our model would require $\frac{g_{\text{DM}}}{g_{\text{SM}}} > 1$. This fixes a relationship between the dark matter temperature $T_{\text{DM}}$, mass $m$ and degrees of freedom $g_{\text{DM}}$ as indicated in Appendix B1.

$$\frac{T_{\text{DM}}}{T_{\text{SM}}} = \left( \frac{g_{\text{DM}}}{g_{\text{SM}}} \frac{\Omega_{\text{DM}} m_N}{\Omega_{\text{SM}} m_B} \frac{m}{m_B} \right)^{1/3},$$

with $m_N$ the nucleon mass, and $\eta = m_B/s$. A second condition states that two dark matter models will have the same effective number of relativistic degrees of freedom is the same. One of these is relativistic dark matter with mass $m_{\text{WDM}}$, we can determine whether it is ruled out by Baur et al. (2015).

The effective number of relativistic degrees of freedom at a given $z$ is

$$\Delta N_{\text{eff}}(z) = \frac{3(P(z))}{\rho_{\nu}(z)},$$

where $\rho_{\nu}(z)$ is the energy density of neutrinos and $(P(z))$ is the pressure of the dark matter component defined by

$$P(z) = \int \frac{p^2 dp}{2\pi} f(p, z) \frac{p^2}{3\sqrt{m^2 + p^2}}.$$

To translate the bound of Baur et al. (2015), we compare the contribution to the number of relativistic degrees of freedom from a standard warm dark matter model with mass $m_{\text{WDM}}$

| $N_f$ | Min. $m_{1,\text{DM}}$ | Min. $m_{2,\text{DM}}$ |
|-------|-----------------------|-----------------------|
| 1     | 470 eV                | 530 eV                |
| 4     | 330 eV                | 375 eV                |
| 16    | 235 eV                | 265 eV                |

Table 1. Minimum FDM mass as a function of number of flavors of mass degenerate Majorana fermions. The middle column shows a bound assuming $m_{\text{WDM}} < 2.5$ keV, the right column shows bound for $m_{\text{WDM}} < 2.93$ keV.

at temperature $T_{\text{WDM}} = T_{\text{SM}}$ and another dark matter candidate with mass $m$ and temperature $T_{\text{DM}} < T_{\text{SM}}$ given by Eq. (38). Because each model is described by two scales (temperature and mass) and the overall density is fixed by $\Omega_{\text{DM}}$, the ratio of these scales fully characterizes these models. As long as these ratios are equal, the models have the same $\Delta N_{\text{eff}}$. The equality

$$\frac{m_{\text{WDM}}}{T_{\text{DM}}} = \frac{m}{T_{\text{DM}}},$$

together with the expression for $T_{\text{DM}}$ from eq. (38) allows us to establish a correspondence between thermal dark matter and a dark matter model with a temperature given by (38). Using equations (38), (41) and that $T_{\text{WDM}} = T_{\text{SM}}$ we can show that to get an equivalent effect on Lyman-α power spectrum from our model would require

$$m = \left( m_{\text{WDM}}^3 m_B \right)^{1/4} \left( \frac{g_{\nu}}{g_{\text{DM}}} \frac{\Omega_{\text{DM}}}{\Omega_{\text{SM}}} \right)^{1/4} = 235 \text{ eV} \left( \frac{m_{\text{WDM}}}{1 \text{ keV}} \right)^{3/4} \left( \frac{2}{g_{\text{DM}}} \right)^{1/4}.$$  

(42)

We have verified the statement of eq. (42) by computing the power spectrum for both models using the CLASS software (Blas et al. 2011) for a range of equivalent masses. As a result, the 2.5 keV lower mass bound for conventional thermal dark matter (Baur et al. 2015) is weakened:

$$m_{\text{DM}} > 470 \text{ eV} \left( \frac{2}{g_{\text{DM}}} \right)^{1/4}.$$  

(43)

Observe that the Lyman-α limit is even weaker for larger $g_{\text{DM}}$, i.e. for dark matter with more flavors. Table 1 shows the explicit bounds as a function of number of flavors of $N_f \equiv g_{\text{DM}}/2$ and for two different bounds on warm dark matter from Baur et al. (2015): the middle column shows a bound assuming $m_{\text{WDM}} < 2.5$ keV, the right column shows a bound for $m_{\text{WDM}} < 2.93$ keV.

Note that the fits of Evans et al. (2009) and Amorisco et al. (2013) favor masses of 70 eV to 320 eV (400 eV in case we include the two-scale effect). This mass range is in tension with the Lyman-α bound, and thus resolving the core-cusp problem might require additional effects, such as baryonic feedback. There is, however, a caveat to this conclusion, as we discuss next.

4.3 Skewed Dark Matter Momenta

The Lyman-α bounds of Table 1 are model independent lower bounds on dark matter with a thermal momentum distribution, dependent only on $g_{\text{DM}}$. The reason for this is that one needs to match the relic density, this is determined by eq. (38) via matching to late time cosmological observables.
Although Lyman-\(\alpha\) bounds can be relaxed if \(g_{\text{DM}}\) is large, cf. [43], increasing the internal number of degrees of freedom of the dark matter simultaneously increases the available occupation levels in the Fermi gas and therefore shrinks the Fermi gas core size (cf. eq. (25)). Changing the number of flavors of dark matter weakens the bound by \(N_f^{-1/4}\), but the core radius also scales as \(mN_f^{-1/4}\). Notably, these effects scale identically. Thus for larger \(g_{\text{DM}}\) one needs increasingly lighter dark matter to maintain appreciable cores, and larger cores (\(R_c > 0.2~\text{kpc}\)) remain in tension with Lyman-\(\alpha\).

This conclusion can be circumvented if the dark matter momentum distribution is not thermal, but skewed to lower energies. In this case the bounds from Lyman-\(\alpha\) can be relaxed, potentially permitting lighter dark matter, and hence larger cores. Skewed momenta might arise in models with resonantly produced sterile neutrinos [Shi & Fuller 1999] or preheating [Kofman et al. 1997].

In an appendix we also consider an alternative scenario in which the dark matter has fewer effective degrees of freedom later in the cosmological evolution than when it was first produced. The model has \(N_f\) flavors but also has small mass splittings that lead the heavier dark matter particles to decay without heating the remaining population. As a result of the large \(N_f\), the temperature of the hidden sector can be lower while maintaining the relic DM density and at the same time appear to have fewer degrees of freedom during core formation and therefore lead to larger cores. If we choose \(N_f = 16\), we can relax the Lyman-\(\alpha\) mass bound by a factor of two down to \(m > 235\ eV\) and completely remove the tension between core sizes and Lyman-\(\alpha\) mass constraints. For more details see Appendix [32].

5 THE CHANDRASEKHAR LIMIT

As one final consideration we look at core collapse into a black hole, and find that it does not further constrain our system. Solving the Lane-Emden equation in the ultrarelativistic limit one obtains the Chandrasekhar bound for a given dark matter mass, as derived in [Domcke & Urbano 2014]

\[
M_{\text{Ch}} = 8\pi^2 \sqrt{\omega_3} \frac{M_\odot}{m^2} = 5 \times 10^{18} \left(\frac{\text{eV}}{m}\right)^2 M_\odot, \tag{44}
\]

where \(\omega_3 \approx 2.018\) is a numerical constant.

Without the thermal envelope the solutions are similar to white dwarfs or neutron stars. These objects expel most of their outer layers during their formation and look like the solutions of [Domcke & Urbano 2011]. The condition for the core of a dwarf galaxy of degenerate Fermi gas to collapse into a black hole is that the degenerate core is more massive than the Chandrasekhar limit for a given fermion mass. The mass enclosed in the core is

\[
M_c \sim \rho_0 R_0^3 \sim \frac{M_\odot \sqrt{\rho_0}}{m^4} \left(\frac{3}{\pi N_f}\right), \tag{45}
\]

where a typical dwarf galaxy central density is of order \(\rho_0 \sim 10^{-20}\ \text{kg/m}^3\). A dwarf galaxy core collapses into a black hole if \(M_c > M_{\text{Ch}}\), i.e. when

\[
m \lesssim \rho_0^{1/4} \approx 0.1\ \text{eV}. \tag{46}
\]

However, for \(m \lesssim 0.1\ \text{eV}\), the mass of the core, \(10^{20}M_\odot\), exceeds the mass of any galaxy and therefore dwarf galaxies with degenerate fermion gas cores are never in danger of collapsing into black holes without additional mechanisms.

6 CONCLUSION

A (quasi) degenerate Fermi gas in dwarf galaxies can alter small scale structure formation. This can provide an elegant and minimal solution to the core-cusp problem. Using the criterion that cores of order a few 100 pc are required to reproduce the observations of dwarf galaxies, we have shown that suitable dwarf galaxy cores can be achieved for fermion dark matter with mass in the range 70 eV – 400 eV. Ultralight fermions arise in many motivated contexts, such as gravitinos, or sterile neutrinos.

Modeling the dark matter as a quasi degenerate Fermi gas provides a well defined profile, eq. (28) for describing the density distribution of dwarf galaxies, as illustrated in Figure [1]. Here we have proposed a two-scale profile for general modeling of dwarf galaxy density profiles. We have argued that the two-scale profile defined in eq. (34) may provide more physically motivated fits. This is because with only a single variable scale, as in Burkert and NFW profiles, fits to the dwarf galaxy profile are sensitive to the physics responsible for the core and the asymptotic behavior at large radial distances. Since the physics of the asymptotic behavior and the coring are likely unrelated it is important to treat these two length scales as independent.

Employing the Burkert profile, it was shown in Burkert (2015) that both large and dwarf galaxies obey a scaling relationship \(\langle \rho_0 R_c \rangle = 75^{+85}_{-45}\ M_\odot\ \text{pc}^{-2}\), see also Donato et al. (2009). The assumption of a quasi-degenerate Fermi gas profile automatically implies a relationship between the characteristic core size \(R_c \sim R_0\). From eq. (28) it follows that \(\rho_0 R_c \propto m^{-1/2} \sim \text{constant}\.\) Interestingly, for dark matter in the mass range to give realistic core sizes, the scaling relationship identified by Burkert (2015) can be accommodated. This scaling relationship is illustrated in Figure [5].

While sub-keV dark matter is typically excluded for a thermal relic, it is a viable possibility for non-thermal dark matter models [Peng et al. 2008; Berezhiani et al. 1995], or Flooded Dark Matter [Randall et al. 2015]. Dark matter can become non-relativistic earlier in these models which would weaken the limits from small scale structure observations compared to thermal relics [Bolton et al. 2004; Viel et al. 2013]. Moreover, we argued that if the dark matter momentum distribution is non-relativistic and skewed to lower energies, then the Lyman-\(\alpha\) bounds are relaxed. In the case that free streaming bounds can be ignored, the leading conservative lower bound is \(m \gtrsim 70\ \text{eV}\), derived here in Section [4].

We have focused on the case that Fermi repulsion is entirely responsible for coring dwarf galaxies, and resolving the core-cusp problem. However in principle the cores in dwarfs could emerge due to a combination of Fermi repulsion and baryonic feedback. In the case that baryonic feedback plays a role, this might allow for appreciable cores for somewhat heavier dark matter. It would be interesting to combine these effects in a numerical simulation.
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APPENDIX A: THE PLUMMER PROFILE

For completeness we give here the profiles and functions used in the definition of $\sigma_{\text{LOS}}$ in eq. (50). The Plummer profile (Plummer 1911) for the projected stellar density $I(r)$ is defined, in terms of the total luminosity $L$, as follows

$$I(r) = \left[1 + \left(\frac{r}{r_{1/2}}\right)^2\right]^{-2} \left(\frac{L}{\pi r_{1/2}^2}\right). \quad \text{(A1)}$$

Given eq. (A1) the associated 3D density is (Binney & Tremaine 2008)

$$\nu(r') = \left[1 + \left(\frac{r'}{r'_{1/2}}\right)^2\right]^{-5/2} \left(\frac{3L}{4\pi r'_{1/2}^3}\right). \quad \text{(A2)}$$

The $\beta$-dependent function which appears in eq. (30) is

$$F(\beta, r, r') = \int_r^{r'} \left[1 - \beta \left(\frac{r'}{r''}\right)^2\right] \left(\frac{r''}{\sqrt{r''^2 - r^2}}\right) \text{d}r''. \quad \text{(A3)}$$

APPENDIX B: MODELS OF DARK MATTER

B1 Flooded Dark Matter

Cosmological bounds on light dark matter can be relaxed if the dark matter sector is colder than the Standard Model sector. This scenario arises in Flooded Dark Matter (Randall et al. 2015) but includes other scenarios as well. We first derive a bound on the temperature ratio that allows for thermal populations to give the correct energy and number densities that applies to any model. We allow for decoupling but assume that dark matter is nonetheless of thermal origin and its density is $n \propto T_{\rm DM}^3$, where $T_{\rm DM}$ is its temperature.

The density of dark matter today is then

$$\Omega_{\text{DM}} = m_{\alpha}\Omega_{\text{DM}}\gamma_{\text{DM}}T_{\text{DM},0}^3, \quad \text{(B1)}$$

where $T_{\text{DM},0}$ denotes the temperature today. Similarly, for the Standard Model sector

$$\Omega_B = m_{\lambda}\eta_{\lambda}T_{\text{SM},0}^4. \quad \text{(B2)}$$

Here $m_{\lambda}$ is the mass of a nucleon and $\eta = n_B/s = 6.2 \times 10^{-10}$ is the baryon asymmetry. Assuming dark matter froze out while relativistic, the ratios of temperatures of the dark and Standard Model sectors must be related by:

$$\frac{T_{\text{DM}}}{T_{\text{SM}}} = \left(\frac{\gamma_{\text{DM}}}{\eta_{\lambda}}\frac{\Omega_{\text{DM}}}{\Omega_B}\frac{m_{\lambda}}{m_{\alpha}}\right)^{1/3}. \quad \text{(B3)}$$

For dark matter masses above an eV, the dark matter sector can be colder than the Standard Model, in which case the Lyman-$\alpha$ bound on the dark matter mass would be weaker. This relation applies to any model with a thermal population of dark matter. To avoid this conclusion requires a nonthermal distribution.

This is also the conclusion in the Flooded Dark Matter scenario (Randall et al. 2015). Suppose the dark matter constitutes a hidden sector which is essentially decoupled from the visible sector, and that after inflation these sectors are reheated democratically. To reproduce standard cosmology at late time the visible sector must typically receive further entropy injections. A long lived heavy state $\Phi$ which decays only to the visible sector can provide an appropriate source of entropy production. The evolution of the light states and $\Phi$ are distinct. $\Phi$ becomes non-relativistic at early time, at which point its energy density $\rho_{\Phi}$ redshifts as matter. Provided $\Phi$ is sufficiently long-lived, it will come to dominate the energy density of the Universe.

When $\Phi$ decays its energy density is transferred to the Standard Model states. The decay rate required to match the observed dark matter relic density ($\Omega_{\text{DM}} \sim 0.3$) is

$$\Gamma \sim \frac{m_\chi^2}{M_{\text{DM}}} \left(\frac{g_{\text{DM}}}{g_{\text{SM}}}\eta_{\text{DM}}\Omega_{\text{DM}}m_{\alpha}}{\Omega_B m_{\text{DM}}^2}\right) \quad \text{(B4)}$$

The entropy injection from $\Phi$ decays heat up the visible sector and sets the temperature ratio as required by eq. (B3).

Diluting the dark matter sufficiently requires small $\Phi$ decay rates, as dictated by eq. (B4). However successful Big Bang Nucleosynthesis sets a lower bound on the reheating temperature $T_{\text{RH}} \sim \sqrt{\Gamma_{\text{DF}}} \geq 10$ MeV. Moreover, certain baryogenesis scenarios need higher $T_{\text{RH}}$, in particular mechanisms tied to the electroweak phase transition require $T_{\text{RH}} \gtrsim 100$ GeV. See e.g. Morrissey & Ramsey-Musolf (2012) for models and constraints.

B2 Skewed Dark Matter Momenta

Consider $N_f$ dark matter ‘flavors’ of 200-300 eV dark matter with a thermal momentum distribution at $T_{\text{DM}} < T_{\text{SM}}$. To avoid the Lyman-$\alpha$ limit one needs $N_f \sim 16$ (cf. Table 1). Even lighter dark matter can be accommodated with larger $N_f$. Moderate values of $N_f$ could arise due to an adjoint representation of some broken SU($\sqrt{N_f}$), and could be connected to ideas of ‘Flavored Dark Matter’ (Agrawal et al. 2012, Batell et al. 2011). We envisage that the a weakly gauged dark sector gauge group breaks, yielding a low energy theory consisting of $N_f$ closely spaced fermions with small mass splittings $\delta_{ij}$. If the gauge symmetry is broken by high scale operators, cutoff at $\Lambda$, then the induced mass splittings are parametrically

$$\delta_{ij} \equiv \frac{m_{\chi_i} - m_{\chi_j}}{m_{\chi_0}} \sim \frac{\lambda_{\chi}^2}{16\pi^2} \log \left(\frac{m_{\chi_0}}{\Lambda}\right), \quad \text{(B5)}$$

where $\chi_0$ is the lightest state and $\chi_\lambda$ is the gauge coupling.

Eventually the (slightly) heavier states $\chi_i$ decay to light hidden sector states $X$, removing the $\chi_i$ population. In order to relax the Lyman-$\alpha$ bounds, $\chi_i$ decays should occur after the dark matter decouples from the lighter species. Otherwise the $\chi_0$ would receive a compensating energy contribution from the $\chi_i$ decays through interactions with the $X$-bath. As a result there is a reduction of the number of degrees of freedom, but due to the small mass splitting, there is no appreciable heating of the $\chi_0$ population. Accordingly the momentum distribution of the $\chi_0$ states appears with a temperature which is scaled relative to the thermal expectation by a factor of $y^{1/4} = (2N_f)^{1/4}$.
The $\chi_0$ distribution is modified as follows:

$$df(p) = dp^3 \left[ 1 + \exp \left( \frac{2N_p}{T} \right) \right]^{-1}, \quad (B6)$$

where $T$ here can be identified with $T_{\text{DM}}$ in eq. (B3).

As indicated above, in order for the heavier $\chi$ states to decay the model must include light or massless hidden sector states $X$. The energy injected into these radiation states will subsequently redshift away. Since the hidden sector is cold relative to the visible sector the $X$ states will typically not give an appreciable contribution to the number of relativistic species $N_{\text{eff}}$ \cite{Randall et al. 2015}. Generally, this decay should occur with a small coupling to avoid a) thermalization and hence reheating of the remaining light states, b) freeze-out, and c) self-scattering effects which are not ruled out but would greatly alter our core-cusp analysis.

Specifically, for too large $\lambda_X$, dark matter annihilations can be significant and there can be a stage of freeze-out dynamics which would impact the dark matter relic density. Therefore to avoid $\chi_0$ freeze-out the annihilations $\chi_0\chi_0 \rightarrow XX$ must be inactive down to the decoupling temperature: $T \sim m_{\chi_0}$. Given that the annihilation cross section of $\chi_0$ is parametrically $\langle \sigma v \rangle \sim \lambda_X^4/16\pi m_{\chi_0}^2$, which should be insignificant down to the decoupling temperature, we require that

$$\lambda_X \lesssim \left( \frac{8\pi m_{\chi_0}}{M_{\text{Pl}}} \right)^{1/4} \sim 10^{-6} \quad (B7)$$

Furthermore, to avoid significant self scattering we require \cite{Peter et al. 2012}

$$\lambda_X^4/(v^4 m_{\chi_0}^3) \lesssim 0.1 \text{cm}^2/\text{g} \quad (B8)$$

For velocity dispersions of $\sim 10$ km/s, this gives $\lambda_X \lesssim 10^{-9}$.

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