Abstract

In this paper, we propose FedGP, a framework for privacy-preserving data release in the federated learning setting. We use generative adversarial networks, generator components of which are trained by FedAvg algorithm, to draw privacy-preserving artificial data samples and empirically assess the risk of information disclosure. Our experiments show that FedGP is able to generate labelled data of high quality to successfully train and validate supervised models. Finally, we demonstrate that our approach significantly reduces vulnerability of such models to model inversion attacks.

1 Introduction

The rise of data analytics and machine learning (ML) presents countless opportunities for companies, governments and individuals to benefit from the accumulated data. At the same time, their ability to capture fine levels of detail potentially compromises privacy of data providers. Recent research [Fredrikson et al., 2015; Shokri et al., 2017; Hitaj et al., 2017] suggests that even in a black-box setting it is possible to argue about the presence of individual examples in the training set or recover certain features of these examples.

Among methods that tackle privacy issues of machine learning is the recent concept of federated learning (FL) [McMahan et al., 2016]. In the FL setting, a central entity (server) wants to train a model on user data without actually copying these data from user devices. Instead, users (clients) update models locally, and the server aggregates these models. One popular approach is the federated averaging, FedAvg [McMahan et al., 2016], where clients do local on-device gradient descent using their data, then send these updates to the server where they get averaged. Privacy can further be enhanced by using secure multi-party computation (MPC) [Yao, 1982] to allow the server access only average updates of a big group of users and not individual ones.

Despite many advantages, federated learning does have a number of challenges. First, the result of FL is a single trained model (therefore, we will refer to it as a model release method), which does not provide much flexibility in the future. For instance, it would significantly reduce possibilities for further aggregation from different sources, e.g. different hospitals trying to combine federated models trained on their patients data. Second, this solution requires data to be labelled at the source, which is not always possible, because user may be unqualified to label their data or unwilling to do so. A good example is again a medical application where users are unqualified to diagnose themselves but at the same time would want to keep their condition private. Third, it does not provide provable privacy guarantees, and there is no reason to believe that the aforementioned attacks do not work against it. Some papers propose to augment FL with differential privacy (DP) to alleviate this issue [McMahan et al., 2017; Geyer et al., 2017]. While these approaches perform well in ML tasks and provide theoretical privacy guarantees, they are often restrictive (e.g. many DP methods for ML assume, implicitly or explicitly, access to public data of similar nature or abundant amounts of data, which is not always realistic).

In our work, we address these problems by proposing to combine the strengths of federated learning and recent advancements in generative models to perform privacy-preserving data release, which has many immediate advantages. First, the released data could be used to train any ML model (we refer to it as a downstream task or the downstream model) without additional assumptions. Second, data from different sources could be easily pooled, providing possibilities for hierarchical aggregation and building stronger
models. Third, labelling and verification can be done later down the pipeline, relieving some trust and expertise requirements on users. Fourth, released data could be traded on data markets\footnote{https://www.datamakespossible.com/value-of-data-2018/dawn-of-data-marketplace}, where anonymisation and protection of sensitive information is one of the biggest obstacles. Finally, data publishing would facilitate transparency and reproducibility of research studies.

The main idea of our approach, named FedGP, for federated generative privacy, is to train generative adversarial networks (GANs) [Goodfellow et al., 2014] on clients to produce artificial data that can replace clients real data. Since some clients may have insufficient data to train a GAN locally, we instead train a federated GAN model. First of all, user data still remain on their devices. Second, the federated GAN will produce samples from the common cross-user distribution and not from a specific single user, which adds to overall privacy. Third, it allows releasing entire datasets, thereby possessing all the benefits of private data release as opposed to model release. Figure 1 depicts the schematics of our approach for two clients.

To estimate potential privacy risks, we use our post hoc privacy analysis framework [Triastcyn and Faltings, 2019] designed specifically for private data release using GANs. Our contributions in this paper are the following:

- on the one hand, we extend our approach for private data release to the federated setting, broadening its applicability and enhancing privacy;
- on the other hand, we modify the federated learning protocol to allow a range of benefits mentioned above;
- we demonstrate that downstream models trained on artificial data achieve high learning performance while maintaining good average-case privacy and being resilient to model inversion attacks.

The rest of the paper is structured as follows. In Section 2, we give an overview of related work. Section 3 contains some preliminaries. In Section 4, we describe our approach and privacy estimation framework. Experimental results are presented in Section 5, and Section 6 concludes the paper.

2 Related Work

In recent years, as machine learning applications become a commonplace, a body of work on security of these methods grows at a rapid pace. Several important vulnerabilities and corresponding attacks on ML models have been discovered, raising the need of devising suitable defences. Among the attacks that compromise privacy of training data, model inversion [Fredrikson et al., 2015] and membership inference [Shokri et al., 2017] received high attention.

Model inversion [Fredrikson et al., 2015] is based on observing the output probabilities of the target model for a given class and performing gradient descent on an input reconstruction. Membership inference [Shokri et al., 2017] assumes an attacker with access to similar data, which is used to train a "shadow" model, mimicking the target, and an attack model. The latter predicts if a certain example has already been seen during training based on its output probabilities. Note that both attacks can be performed in a black-box setting, without access to the model internal parameters.

To protect privacy while still benefiting from the use of statistics and ML, many techniques have been developed over the years, including k-anonymity [Sweeney, 2002], l-diversity [Machanavajjhala et al., 2007], t-closeness [Li et al., 2007], and differential privacy (DP) [Dwork, 2006].

Most of the ML-specific literature in the area concentrates on the task of privacy-preserving model release. One take on the problem is to distribute training and use disjoint datasets. For example, [Shokri and Shmatikov, 2015] propose to train a model in a distributed manner by communicating sanitised updates from participants to a central authority. Such a method, however, yields high privacy losses [Abadi et al., 2016; Papernot et al., 2016]. An alternative technique suggested by [Papernot et al., 2016], also uses disjoint training sets and builds an ensemble of independently trained teacher models to transfer knowledge to a student model by labelling public data. This result has been extended in [Papernot et al., 2018] to achieve state-of-the-art image classification results in a private setting (with single-digit DP bounds). A different approach is taken by [Abadi et al., 2016]. They suggest using differentially private stochastic gradient descent (DP-SGD) to train deep learning models in a private manner. This approach achieves high accuracy while maintaining low DP bounds, but may also require pre-training on public data.

A more recent line of research focuses on private data release and providing privacy via generating synthetic data [Bindschaedler et al., 2017; Huang et al., 2017; Beaulieu-Jones et al., 2017]. In this scenario, DP is hard to guarantee, and thus, such models either relax the DP requirements or remain limited to simple data. In [Bindschaedler et al., 2017], authors use a graphical probabilistic model to learn an underlying data distribution and transform real data points (seeds) into synthetic data points, which are then filtered by a privacy test based on a plausible deniability criterion. This procedure would be rather expensive for complex data, such as images. Fiorettro and Van Hentenryck [2019] employ decision trees for a hybrid model/data release solution and guarantee stronger ε-differential privacy, but like the previous approach, it would be difficult to adapt to more complex data. Alternatively, Huang et al. [2017] introduce the notion of generative adversarial privacy and use GANs to obfuscate real data points w.r.t. pre-defined private attributes, enabling privacy for more realistic datasets. Finally, a natural approach to try is training GANs using DP-SGD [Beaulieu-Jones et al., 2017; Xie et al., 2018; Zhang et al., 2018]. However, it proved extremely difficult to stabilise training with the necessary amount of noise, which scales as $\sqrt{n}$ w.r.t. the number of model parameters $n$. It makes these methods inapplicable to more complex datasets without resorting to unrealistic (at least for some areas) assumptions, like access to public data from the same distribution.

On the other end of spectrum, McMahan et al. [2016] proposed federated learning as one possible solution to privacy issues (among other problems, such as scalability and com-
munication costs). In this setting, privacy is enforced by keeping data on user devices and only submitting model updates to the server. It can be augmented by MPC [Bonawitz et al., 2017] to prevent the server from accessing individual updates and by DP [McMahan et al., 2017; Geyer et al., 2017] to provide rigorous theoretical guarantees.

3 Preliminaries

This section provides necessary definitions and background. Let us commence with approximate differential privacy.

Definition 1. A randomised function (mechanism) $M : D \rightarrow R$ with domain $D$ and range $R$ satisfies $(\varepsilon, \delta)$-differential privacy if for any two adjacent inputs $d, d' \in D$ and for any outcome $o \in R$ the following holds:

$$\Pr [M(d) = o] \leq e^\varepsilon \Pr [M(d') = o] + \delta. \quad (1)$$

Definition 2. Privacy loss of a randomised mechanism $M : D \rightarrow R$ for inputs $d, d' \in D$ and outcome $o \in R$ takes the following form:

$$L_{(M(d)||M(d'))} = \log \frac{\Pr [M(d) = o]}{\Pr [M(d') = o]}.$$

Definition 3. The Gaussian noise mechanism achieving $(\varepsilon, \delta)$-DP, for a function $f : D \rightarrow \mathbb{R}^m$, is defined as

$$M(d) = f(d) + N(0, \sigma^2), \quad (3)$$

where $\sigma > C \sqrt{2 \log \frac{1.25}{\delta}} / \varepsilon$ and $C$ is the $L2$-sensitivity of $f$.

For more details on differential privacy and the Gaussian mechanism, we refer the reader to [Dwork and Roth, 2014].

In our privacy estimation framework, we also use some classical notions from probability and information theory.

Definition 4. The Kullback–Leibler (KL) divergence between two continuous probability distributions $P$ and $Q$ with corresponding densities $p, q$ is given by:

$$D_{KL}(P||Q) = \int_{-\infty}^{+\infty} p(x) \log \frac{p(x)}{q(x)} dx. \quad (4)$$

Note that KL divergence between the distributions of $M(d)$ and $M(d')$ is nothing but the expectation of the privacy loss random variable $\mathbb{E}[L_{(M(d)||M(d'))}]$.

Finally, we use the Bayesian perspective on estimating mean from the data to get sharper bounds on expected privacy loss compared to the original work [Triastcyn and Faltings, 2019]. More specifically, we use the following proposition.

Proposition 1. Let $l_1, l_2, \ldots, l_n$ be a random vector drawn from the distribution $p(L)$ with the same mean and variance, and let $\bar{L}$ and $S$ be the sample mean and sample standard deviation of the random variable $L$. Then,

$$\Pr \left( \frac{\bar{L} - F^{-1}_{n-1}(1-\gamma) \sqrt{\frac{1}{n} S}}{\sqrt{n-1}} \leq \gamma \right), \quad (5)$$

where $F^{-1}_{n-1}(1-\gamma)$ is the inverse CDF of the Student’s $t$-distribution with $n-1$ degrees of freedom at $1-\gamma$.

The proof of this proposition can be obtained by using the maximum entropy principle with a flat (uninformative) prior to get the marginal distribution of the sample mean $\bar{L}$, and observing that the random variable $\frac{\bar{L} - F^{-1}_{n-1}(1-\gamma) \sqrt{\frac{1}{n} S}}{\sqrt{n-1}}$ follows the Student’s $t$-distribution with $n-1$ degrees of freedom [Oliphant, 2006].

4 Federated Generative Privacy

In this section, we describe our algorithm, what privacy it can provide and how to evaluate it, and discuss current limitations.

4.1 Method Description

In order to keep participants data private while still maintaining flexibility in downstream tasks, our algorithm produces a federated generative model. This model can output artificial data, not belonging to any real user in particular, but coming from the common cross-user data distribution.

Let $\{u_1, u_2, \ldots, u_n\}$ be a set of clients holding private datasets $\{d_1, d_2, \ldots, d_n\}$. Before starting the training protocol, the server is providing each client with generator $G_0^t$ and critic $C_0^t$ models, and clients initialise their models randomly. Like in a normal FL setting, the training process afterwards consists of communication rounds. In each round $t$, clients update their respective models performing one or more passes through their data and submit generator updates $\Delta G_t^t$ to the server through MPC while keeping $C_t^t$ private. In the beginning of the next round, the server provides an updated common generator $G_t^t$ to all clients.

This approach has a number of important advantages:

- Data do not physically leave user devices.
- Only generators (that do not come directly into contact with data) are shared, and critics remain private.
- Using artificial data in downstream tasks adds another layer of protection and limits the information leakage to artificial samples. This is especially useful given that ML models can be attacked to extract training data [Fredrikson et al., 2015], sometimes even when protected by DP [Hitaj et al., 2017].

What remains to assess is how much information would an attacker gain about original data. We do so by employing a notion introduced in an earlier work [Triastcyn and Faltings, 2019] that we name Differential Average-Case Privacy.

It is important to clarify why we do not use the standard DP to provide stronger theoretical guarantees: we found it extremely difficult to train GANs with the amount of noise required for meaningful DP guarantees. Despite a number of attempts [Beaulieu-Jones et al., 2017; Xie et al., 2018; Zhang et al., 2018], we are not aware of any technically sound solution that would generalise beyond very simple datasets.

4.2 Differential Average-Case Privacy

Our framework builds upon ideas of empirical DP (EDP) [Abowd et al., 2013; Schneider and Abowd, 2015] and on-average KL privacy [Wang et al., 2016]. The first can be viewed as a measure of sensitivity of posterior distributions...
of outcomes [Charest and Hou, 2017] (in our case, generated data distributions), while the second relaxes DP notion to the case of an average user.

More specifically, we say the mechanism $\mathcal{M}$ is $(\mu, \gamma)$-DAP if for two neighbouring datasets $D, D'$, where data come from an observed distribution, it holds that

$$\Pr\left(\mathbb{E}[\|L(\mathcal{M}(D))\|_1] > \mu \right) \leq \gamma.$$  \hspace{1cm} (6)

For the sake of example, let each data point in $D, D'$ represent a single user. Then, $(0.01, 0.001)$-DAP could be interpreted as follows: with probability 0.999, a typical user submitting their data will change outcome probabilities of the private algorithm on average by $1\%^2$.

4.3 Generative Differential Average-Case Privacy

In the case of generative models, and in particular GANs, we don’t have access to exact posterior distributions, a straightforward DP procedure in our scenario would be the following: (1) train GAN on the original dataset $D$; (2) remove a random sample from $D$; (3) re-train GAN on the updated set; (4) estimate probabilities of all outcomes and the maximum privacy loss value; (5) repeat (1)–(4) sufficiently many times to approximate $\varepsilon, \delta$.

If the generative model is simple, this procedure can be used without modification. Otherwise, for models like GANs, it becomes prohibitively expensive due to repetitive re-training (steps (1)–(3)). Another obstacle is estimating the maximum privacy loss value (step (4)). To overcome these two issues, we propose the following.

First, to avoid re-training, we imitate the removal of examples directly on the generated set $\tilde{D}$. We define a similarity metric $\text{sim}(x, y)$ between two data points $x$ and $y$ that reflects important characteristics of data (see Section 5 for details). For every randomly selected real example $i$, we remove $k$ nearest artificial neighbours to simulate absence of this example in the training set and obtain $\tilde{D}^{-i}$. Our intuition behind this operation is the following. Removing a real example would result in a lower probability density in the corresponding region of space. If this change is picked up by a GAN, which we assume is properly trained (e.g. there is no mode collapse), the density of this region in the generated examples space should also decrease. The number of neighbours $k$ is defined by the ratio of artificial and real examples, to keep density normalised.

Second, we relax the worst-case privacy loss bound in step (4) by the expected-case bound, in the same manner as on-average KL privacy. This relaxation allows us to use a high-dimensional KL diversity estimator [Pérez-Cruz, 2008] to obtain the expected privacy loss for every pair of adjacent datasets $(\tilde{D} \text{ and } \tilde{D}^{-i})$. There are two major advantages of this estimator: it converges almost surely to the true value of KL divergence; and it does not require intermediate density estimates to converge to the true probability measures. Also since this estimator uses nearest neighbours to approximate KL divergence, our heuristic described above is naturally linked to the estimation method.

Finally, having obtained sufficiently many sample pairs $(\tilde{D}, \tilde{D}^{-i})$, we use Proposition 1 to determine DAP parameters $\mu$ and $\gamma$. This is an improvement over original DAP, because this way we can get much sharper bounds on expected privacy loss.

4.4 Limitations

Our approach has a number of limitations that should be taken into consideration.

First of all, existing limitations of GANs (or generative models in general), such as training instability or mode collapse, will apply to this method. Hence, at the current state of the field, our approach may be difficult to adapt to inputs other than image data. Yet, there is still a number of privacy-sensitive applications, e.g. medical imaging or facial analysis, that could benefit from our technique. And as generative methods progress, new uses will be possible.

Second, since critics remain private and do not leave user devices their performance can be hampered by a small number of training examples. Nevertheless, we observe that even in the setting where some users have smaller datasets overall discriminative ability of all critics is sufficient to train good generators.

Lastly, our empirical privacy guarantee is not as strong as the traditional DP and has certain limitations [Charest and Hou, 2017]. However, due to the lack of DP-achieving training methods for GANs it is still beneficial to have an idea about expected privacy loss rather than not having any guarantee.

5 Evaluation

In this section, we describe the experimental setup and implementation, and evaluate our method on MNIST [LeCun et al., 1998] and CelebA [Liu et al., 2015] datasets.

5.1 Experimental Setting

We evaluate two major aspects of our method. First, we show that training ML models on data created by the common generator achieves high accuracy on MNIST (Section 5.2). Second, we estimate expected privacy loss of the federated GAN and evaluate the effectiveness of artificial data against model inversion attacks on CelebA face attributes (Section 5.3).

Learning performance experiments are set up as follows:

1. Train the federated generative model (teacher) on the original data distributed across a number of users.

Table 1: Accuracy of student models trained on artificial samples of FedGP compared to non-private centralised baseline and CentGP. In parenthesis we specify the average number of data points per client.

| Setting | Dataset | Baseline | CentGP | FedGP |
|---------|---------|----------|--------|-------|
| i.i.d.  | MNIST (500) | 98.10% | 97.35% | 79.45% |
| MNIST (1000) | 98.55% | 97.39% | 93.38% |
| MNIST (2000) | 98.92% | 97.41% | 96.23% |
| non-i.i.d. | MNIST (500) | 97.31% | 83.26% |
| MNIST (1000) | 98.78% | 95.89% |
| MNIST (2000) | 98.76% | 96.88% |
2. Generate an artificial dataset by the obtained model and use it to train ML models (students).

3. Evaluate students on a held-out test set.

We choose two commonly used image datasets, MNIST and CelebA. MNIST is a handwritten digit recognition dataset consisting of 60000 training examples and 10000 test examples, each example is a 28x28 size greyscale image. CelebA is a facial attributes dataset with 202599 images, each of which we crop to 128x128 and then downscale to 48x48.

In our experiments, we use Python and Pytorch framework. For implementation details of GANs and privacy evaluation, please refer to [Triastcyn and Faltings, 2019]. To train the federated generator we use FedAvg algorithm [McMahan et al., 2016]. As a sim function introduced in Section 4.3 we use the distance between InceptionV3 [Szegedy et al., 2016] feature vectors.

5.2 Learning Performance

First, we evaluate the generalisation ability of the student model trained on artificial data. More specifically, we train a student model on generated data and report test classification accuracy on a held-out real set. We compare learning performance with the baseline centralised model trained on original data, as well as the same model trained on artificial samples obtained from the centrally trained GAN (CentGP).

Since critics stay private and can only access data of a single user, the size of each individual dataset has significant effect. Therefore, in our experiment we vary sizes of user datasets and observe its influence on training. In each experiment, we specify an average number of points per user, while the actual number is drawn from the uniform distribution with this mean, with some clients getting as few as 100 data points.

We also study two settings: i.i.d. and non-i.i.d data. In the first setting, distribution of classes for each client is identical to the overall distribution. In the second, every client gets samples of 2 random classes, imitating the situation when a single user observes only a part of overall data distribution.

Details of the experiment can be found in Table 1. We observe that training on artificial data from the federated GAN allows to achieve 96.9% accuracy on MNIST with the baseline of 98.8%. We can also see how accuracy grows with the average user dataset size. A less expected observation is that non-i.i.d. setting is actually beneficial for FedGP. A possible reason is that training critics with little data becomes easier when this data is less diverse (i.e. the number of different classes is smaller). Comparing to the centralised generative privacy model CentGP, we can also see that FedGP is more affected by sharding of data on user devices than by overall data size, suggesting that further research in training federated generative models is necessary.

5.3 Privacy Analysis

Using the privacy estimation framework (see Sections 4.2 and 4.3), we fix the probability γ of exceeding the expected privacy loss bound µ in all experiments to $10^{-15}$ and compute the corresponding µ for each dataset and two settings. Table 2 summarises the bounds we obtain. As anticipated, the privacy guarantee improves with the growing number of data points, because the influence of each individual example diminishes. Moreover, the average privacy loss µ, expectedly, is significantly smaller than the typical worst-case DP loss ε in similar settings. To put it in perspective, the average change in outcome probabilities estimated by DAP is ~1% even in more difficult settings, while the state-of-the-art DP method would place the worst-case change at hundreds or even thousands percent without giving much information about a typical case.

On top of estimating expected privacy loss bounds, we test FedGP’s resistance to the model inversion attack [Fredrikson et al., 2015]. More specifically, we run the attack on two student models: trained on original data samples and on artificial samples correspondingly. Note that we also experimented with another well-known attack on machine learning models, the membership inference [Shokri et al., 2017]. However, we did not include it in the final evaluation, because of the poor attacker’s performance in our setting (nearly random guess accuracy for given datasets and models even on the non-private baseline). Moreover, we only consider passive adversaries and we leave evaluation with active adversaries.

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Table 2: Average-case privacy parameters: expected privacy loss bound µ and probability γ of exceeding it.

| Setting | Dataset | µ       | γ         |
|---------|---------|---------|-----------|
| i.i.d.  | MNIST (500) | 0.0117  | 10^{-15} |
|         | MNIST (1000) | 0.0069  |           |
|         | MNIST (2000) | 0.0021  |           |
|         | CelebA    | 0.0009  |           |
| non-i.i.d. | MNIST (500) | 0.0090  |           |
|         | MNIST (1000) | 0.0044  |           |
|         | MNIST (2000) | 0.0020  |           |

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Table 3: Face detection and recognition rates (pairs with distances below 0.99).

| Setting | Detection | Recognition |
|---------|-----------|-------------|
| Baseline | 25.5% | 2.8% |
| FedGP   | 1.2% | 0.1% |

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Figure 2: Results of the model inversion attack. Top to bottom: real target images, reconstructions from the non-private model, reconstructions from the model trained by FedGP.
considering current research would be achieving differential privacy guarantees for generative models while still preserving high utility of generated data. A step in another direction would be to improve our empirical privacy concept, e.g. by bounding maximum privacy loss rather than average, or finding a more principled way of sampling from outcome distributions.

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