**Abstract**

We consider non-renormalizable $1/M_X$ interaction terms as a perturbation of the conventional neutrino mass matrix. Particular attention is given to the gravitational interaction with $M_X = M_{Pl}$. We find that for the degenerate neutrino mass spectrum, the considered perturbation generates a non-zero $U_{e3}$ which is within reach of the high performance neutrino factories and just on the borderline to be of interest for supernova physics. For the hierarchical mass spectrum this effect is small. For $1/M_X$ interaction terms with $M_X$ about the GUT scale, a detectable $U_{e3}$ term is induced for the hierarchical mass spectra also. Numerical estimates are given for all the above mentioned cases and renormalization effects are considered.

**Introduction**

One of most important issues in neutrino physics is the magnitude of $U_{e3}$ or equivalently of the mixing angle $\theta_{13}$. The actual value of this parameter is of great interest to various aspects of neutrino physics: To theory, to the search for CP violation in the next generation of long baseline oscillation experiments, to interpret supernova neutrino signals, etc. The only solid information we have on this parameter is the upper bound that is based on the CHOOZ experiment [1], that is $\theta_{13} \leq 7^\circ$ at 1 sigma ($13^\circ$ at 3 sigma).

In this paper, we point out that a tentative lower bound on this parameter arises from physics above the grand unified theory (GUT) scale, in particular from the gravitational interaction of neutrinos. Let us describe this idea. Most probably, grand unified dynamics generates the main part of the neutrino mass matrix. However, contributions from other sources are likely to exist. Specifically, gravitational interactions [2, 3, 4] can produce additional terms in the mass of neutrinos, and these can affect the size of $\theta_{13}$.

The relevant gravitational dimension-5 operator for the spinor $SU(2)_L$ isodoublets,$^1$ $\psi_\alpha = (\nu_\alpha, \ell_\alpha)$ and the scalar one, $\varphi = (\varphi^+, \varphi^0)$, can be written with the operators introduced by Weinberg [5] as

$$\mathcal{L}_{\text{grav}} = \frac{\lambda}{M_{Pl}} (\psi_{A\alpha} \epsilon_{AC} \varphi_C) C^{-1}_{ab} (\psi_{B\beta} \epsilon_{BD} \varphi_D) + h.c.,$$

(1)

where $M_{Pl} = 1.2 \times 10^{19}$ GeV is the Planck mass, and $\lambda$ is a number $\mathcal{O}(1)$. In eq. (1), all indices are explicitly shown: the Lorentz indices $a, b = 1, 2, 3, 4$ are contracted with the charge conjugation matrix $C$, the $SU(2)_L$ isospin indices $A, B, C, D = 1, 2$ are contracted with $\epsilon = i\sigma_2$; $\sigma_m$ ($m = 1, 2, 3$) are the Pauli matrices. After spontaneous electroweak symmetry breaking, the

$^1$Here and everywhere below we use Greek letters $\alpha, \beta, ...$ for the flavor states and Latin letters $i, j, k, ...$ for the mass states.
Lagrangian (1) generates additional terms of neutrino mass: $\mathcal{L}_{\text{mass}} = \lambda v^2/M_{Pl} \nu_\alpha C^{-1} \nu_\beta$, where $v=174$ GeV is the vev of electroweak symmetry breaking. We assume that the gravitational interaction is “flavour blind”, i.e. $\lambda$ does not contain $\alpha, \beta$ indices. In this case, the contribution to the neutrino mass matrix is of the order of:

$$\mu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},$$

(2)

where the scale $\mu$ (‘scale of perturbation’) is

$$\mu = v^2/M_{Pl} = 2.5 \times 10^{-6} \text{ eV}.$$  

(3)

In our calculations, we take eq.(2) as a ‘contribution of perturbation’ to the main part of neutrino mass matrix, that is generated by GUT. In other words, our results are normalized to the case $\lambda = 1/2$; more discussion on $\lambda$ follows.

However, there might be some other interactions of the form (1) with a scale $M_X$ less than $M_{Pl}$, e.g. from string compactification, non-perturbative dynamics, flavor physics or simply higher GUT scales. These interactions are not necessarily flavor blind. This implies not only a different scale of perturbation $\mu = v^2/M_X$, but also a modified structure for the matrix of perturbation, that we denote by $\lambda_{\alpha\beta}$ henceforth. In the case of quantum gravity, $\lambda_{\alpha\beta} = \lambda$.

In this paper, we evaluate the effects of such operators on the conventional neutrino mass matrix, given for example by the seesaw mechanism [5, 6, 7, 8]. We demonstrate that the gravitational perturbation generates a non-zero $U_{e3}$ even if the unperturbed mass matrix has $U_{e3} = 0$. This non-zero value of $U_{e3}$ can be considered as a lower bound imposed by gravitational effects and can be translated into a lower bound for the angle $\theta_{13}$. The operators with the scale $M_X < M_{Pl}$, but higher than the GUT scale (string scales etc.) provides much stronger effects which are of potential interest to present and future observations. For reference and for definiteness we recall that the supersymmetric unification scale is $M_{GUT} \sim 2 \times 10^{16}$ GeV. However the considerations outlined below are not tied to any particular value of $M_{GUT}$.

Before passing to the calculations, we discuss in more detail the coefficient $\lambda$ in eq.(1). The operator in eq.(1) is a term of the effective Lagrangian produced in quantum gravity, thus the coefficient $\lambda$ could be calculated if the details of this theory were fixed; but this is not the case at present. There is another way to see that the coefficient $\lambda$ is defined with considerable uncertainty. Indeed, an $SU(2)$ Fiertz transformation yields $(\psi^t \sigma_2 \bar{\psi})(\psi^t \sigma_2 \bar{\psi}) = -2(\psi^t \sigma_2 \varphi)(\psi^t \sigma_2 \varphi)$. Thus, eq.(1) can be recast in the form

$$\mathcal{L}_{\text{grav}} = -\frac{1}{2} \times \frac{\lambda}{M_{Pl}} (\psi^t \epsilon \bar{\sigma} \varphi) C^{-1} (\psi \epsilon \bar{\sigma} \varphi) + h.c.,$$

(4)

(suppressing all indices except those in flavor space). Here, the reader readily recognizes the operator introduced in Ref.[3], but with a factor $-1/2$ in front of it. In other words, if we knew that quantum gravity yielded the first operator with a coefficient equal to 1, the second one would be generated with a coefficient $-1/2$, not 1. In summary, we have to live with an uncertainty in the coefficient $\lambda$, and this in turn reflects into an uncertainty $\mathcal{O}(1)$ in our results for $U_{e3}$.
Perturbative expansion

A natural assumption is that the unperturbed (0th-order) mass matrix $M$, 

$$M = U^* \text{diag}(M_i) U^\dagger,$$  \hspace{1cm} (5) 

(where $U_{ai}$ is the usual mixing matrix, and $M_i$ the neutrino masses) is generated by grand unified dynamics.\(^2\) Most of the parameters related to neutrino oscillations are known, the major exception is given by the mixing element $U_{e3}$. We adopt the usual parameterization: 

$$|U_{e2}/U_{e1}| = \tan \theta_{12}, \quad |U_{\mu2}/U_{\mu3}| = \tan \theta_{23} \quad \text{and} \quad |U_{e3}| = \sin \theta_{13},$$ 

or equivalently $\theta_{12} = \omega, \theta_{23} = \psi$ and $\theta_{13} = \phi$. Note that in our approach $M_i$ are real and non-negative and we include all possible phases in the mixing matrix:

$$U = \text{diag}(e^{i a_i}) R(\theta_{23}) \Delta R(\theta_{13}) \Delta^* R(\theta_{12}) \text{diag}(e^{i a_i}).$$ \hspace{1cm} (6) 

The phase $\delta$ appearing in $\Delta = \text{diag}(e^{i \delta/2}, 1, e^{-i \delta/2})$ is the one that affects oscillations. $a_i$ are the so called Majorana phases and $f_i$ are usually considered as a part of the definition of the neutrino field. It is possible to rotate away the phases $f_i$, if the mass matrix (5) is the complete mass matrix. However, since we are going to add another contribution to this mass matrix, the phases $f_i$ of the zeroth order mass matrix have an impact on the complete mass matrix and thus must be retained. By the same token, the Majorana phases which are usually redundant for oscillations have a dynamical role to play now.

Non-GUT effects related to a larger mass scale $M_X > M_{GUT}$ will add other contributions to the mass matrix and in particular will affect the magnitude of $U_{e3}$. Thus, let us assume that the mass matrix is modified as:

$$M \rightarrow M + \mu \lambda,$$ \hspace{1cm} (7) 

with $\mu = \frac{v^2}{M_X}$ and $\lambda$ being a matrix of dimensionless terms as discussed in the introduction \(^2\). The impact of the new terms on the mixing can be seen by forming the hermitian matrix $(M + \mu \lambda)^\dagger(M + \mu \lambda)$, which is the matrix relevant for oscillation physics. To first order in the small parameter $\mu$, the above matrix is $M^\dagger M + \mu \lambda^\dagger M + M^\dagger \mu \lambda$. Now by using eq.(5) and the fact that this new mass squared matrix must be diagonalized by a new mixing matrix resulting in corrected eigenvalues, one can write,

$$U(M^2 + m^2 M + M m)U^\dagger \equiv U^\dagger M^{2} U^\dagger,$$ 

with $m = \mu U^\dagger \lambda U.$ \hspace{1cm} (8) 

Here $M$ and $M'$ are the diagonal matrices with neutrino masses at 0th and at 1st order in $\mu$. It is clear from eq.(8) that the new mixing matrix can be written as

$$U' = U (1 + i \delta \theta),$$ \hspace{1cm} (9) 

\(^2\)For instance, the complete seesaw formula in minimal SO(10) \(^3\) is: $M = \frac{v^2}{\sqrt{2}} (Y_{\nu} Y^{-1} Y_{\nu}^c + \xi Y)$ where $M_X = V$ is the vev of the singlet contained in the 126-plet, of the order of or somewhat smaller than the supersymmetric grand unification scale. Here $Y_{\nu}$ is the neutrino Yukawa coupling, $Y$ the 126-plet coupling to the fermions and $\xi v^2/V$ is the effective vev of the triplet in the 126-plet (these two give the noncanonical, or type II seesaw).
where $\delta \theta$ is a hermitian matrix that appears at first order in $\mu$.

From eq. (8) one obtains

$$M^2 + m^1 M + M m = M'\bar{2} + [i\delta \theta, M'^2].$$

(10)

Therefore to first order in $\mu$, the mass squared differences $\Delta M^2_{ij} = M^2_i - M^2_j$ get modified as:

$$\Delta M'_{ij}^2 = \Delta M_{ij}^2 + 2 (M_i \text{Re}[m_{ii}] - M_j \text{Re}[m_{jj}]).$$

(11)

and the new contributions to the mixing matrix are:

$$\delta \theta_{ij} = \frac{i \text{Re}[m_{ij}](M_i + M_j)}{\Delta M'^2_{ij}} - \frac{i \text{Im}[m_{ij}](M_i - M_j)}{\Delta M'^2_{ij}}.$$  

(12)

The diagonal elements of $\delta \theta_{ii}$ are undetermined, as follows from the phase invariance of eq. (8). Thus we set them to zero. Putting together eq. (12) and the definition of $m$ in eq. (8), we obtain the contribution to $U_{e3}$ at $O(\mu)$:

$$\delta U_{e3} = \frac{\mu (M_3 + M_1)}{\Delta M'^2_{31}} U_{e1} \text{Re}(U^T \lambda U)_{13} - i \frac{\mu (M_3 - M_1)}{\Delta M'^2_{31}} U_{e1} \text{Im}(U^T \lambda U)_{13} + (1 \to 2).$$

(13)

One observes that it is the atmospheric neutrino mass difference $\Delta M'^2_{31}$ that enters into the final expression for $\delta U_{e3}$. The solar neutrino mass difference has no role in determining the magnitude of this effect. If the corrected mass squared difference $\Delta M'^2_{31}$ is almost the same as the original mass squared difference $\Delta M^2_{31}$ (as is true for Planck scale effects), then the above formula simplifies to the one given below:

$$\delta U_{e3} = \mu U_{e1} \left( \frac{\text{Re}(U^T \lambda U)_{13}}{M_3 - M_1} - i \frac{\text{Im}(U^T \lambda U)_{13}}{M_3 + M_1} \right) + (1 \to 2).$$

(14)

Eq. (13) should be used to calculate the contribution for scales less than the Planck scale. Eqs. (13) and (14) are our main results. They describe the contribution to $U_{e3}$ coming from a perturbation above the GUT scale.

From eqs. (11) and (12) one can also obtain the size of the deviations of $\theta_{12}$ and $\theta_{23}$ from maximal values and study the stability of the $\Delta M^2_{ij}$ under effects due to physics above the grand unified scale. In this paper however, we focus on $U_{e3}$.

**Numerical results**

To estimate $\delta U_{e3}$ numerically from eq. (14) we need to know the mixing terms $U_{\alpha j}$, the mass squared differences $\Delta M'^2_{ij}$ and the absolute neutrino masses $M_1$, $M_2$ and $M_3$. While the former two can be taken either from experimental data (except for the phases that at present remain unknown) or from a specified theoretical model, the ‘absolute’ neutrino masses cannot be obtained from oscillation experiments. See for instance [12]. We take the solar neutrino mass difference $\Delta M^2_{21} = 7.1 \times 10^{-5}$ eV$^2$ and the solar mixing angle $\theta_{12} = 34^\circ$. The atmospheric neutrino mass
difference is taken to be $\Delta M^2_{31} = 2.8 \times 10^{-3}\text{eV}^2$ and the mixing angle $\theta_{23}$ to be $45^\circ$. The usual CP violating phase can be taken as $\delta = 0$.

To get a numerical estimate for $\delta U_{e3}$ we should consider what we know on ‘absolute’ neutrino masses. More precisely there are a certain number of masses:

1) The mass of the lightest neutrino $M$, equal to either $M_1$ or to $M_3$ for the normal and the inverted hierarchy respectively.

2) The “cosmological” mass $M_{\cosm} = M_1 + M_2 + M_3$, which can be determined from the distribution of matter on large-scales and the anisotropy of the cosmic microwave background radiation. From the first paper in [9], we quote $M_{\cosm} < 0.71\text{ eV}$ at 95 % CL. This bound is valid under the simplest cosmological assumptions and depends on them rather crucially. See e.g. [10].

3) The effective mass of the electron neutrino coming from the tritium experiment $M^2_{\nu_e} = \sum |U^2_{ei}|M_i^2 < (2.2\text{ eV})^2$ at 95 % CL [11].

4) Neutrinoless double beta decay constrains $m_{ee} = |\sum U^2_{ei}M_i| < 0.38\text{ eV}$ at 95 % CL [12] ($h = 0.6 - 2.8$ quantifies the uncertainty in nuclear matrix elements). If the Majorana phases are the same, e.g. zero, there is no cancellation of the 3 contributions and the bound implies $M_1 \approx M_{\nu_e} \approx M_{\cosm}/3 < 0.38\text{ eV}$. If the Majorana phases produce the largest possible cancellation, this relaxes to $< 1.2\text{ eV}$ [13].

The introduction of these unknown (although constrained) quantities results in uncertainties in the values of $U_{e3}$ that we obtain in our framework. In particular, we see that the second term between brackets in eq.\((14)\) decreases when the scale $M$ increases, while the first term becomes instead larger. In other words the contribution of new physics at the scale $M_X$ to $U_{e3}$ can be rather large in the case of degenerate neutrinos, although it is possible to diminish this effect by certain choices of the phases. It should be noted that the relative phases between the GUT contribution (phases $f_i$) and the perturbative contribution influence the magnitude of $U_{e3}$.

**Planck scale effects**

When we focus on Planck scale effects, we assume that $\lambda_{\alpha\beta} = \lambda$ for each $\alpha$ and $\beta$. Let us set in the unperturbed mass matrix the mixing $\theta_{13} = 0$ and also $f_i = a_i = 0$. From eq.\((14)\) one observes that the surviving contribution to $U_{e3}$ is given by the first term. All our results can be easily modified to include a specific GUT model for the unperturbed mass matrix $M$, or for different experimental inputs and phases. The dimensional factors $\mu(M_3 + M_1)/\Delta M^2_{31}$ and $\mu(M_3 + M_2)/\Delta M^2_{32}$ are equal with good accuracy and they are multiplied by $\cos \theta_{23} + \sin \theta_{23} \approx \sqrt{2}$ [14]. Note also that the dependence on the solar mixing angle in this case disappears. Finally from eq.\((14)\) we get the range of values for $U_{e3}$:

$$U_{e3}|_{\text{Planck}} = 7 \times 10^{-5} - 6 \times 10^{-3},$$ \hspace{1cm} (15)

where the lower limit corresponds to the normal hierarchy for $M_1 = 0$ ($\Delta M^2_{32} \approx \Delta M^2_{31} = 2.8 \times 10^{-3}\text{ eV}^2$ [14]) and the upper limit corresponds to the kinematical limit from tritium decay search, $M < 2.2\text{ eV}$. This corresponds to what is usually called the “quasi degenerate” spectrum, where the common mass scale is much higher than the splittings between the masses.

Some remarks are in order:

(i) For the case of the inverted hierarchy, the numbers are practically the same as that for the normal hierarchy. (ii) If one takes the cosmological limit on neutrino mass, $M < 0.71\text{ eV}$ into
Figure 1: Impact of the Majorana phases on calculated $\sin^2 2\theta_{13}$ values for Planck scale effects and a degenerate (tritium mass) spectrum, for $a_1, a_2 = 0 - 180^\circ$. For illustration we show the contours where the value is: $1 \times 10^{-5}$, dotted line; $4 \times 10^{-5}$, dashed line; $1 \times 10^{-4}$, long dashed line. The value at the peak (innermost region) is $1 \times 10^{-4}$. The value at the lower left corner (where $a_1 = a_2 = 0$) is $3 \times 10^{-6}$.

account, then the upper limit becomes $5 \times 10^{-4}$, which is a order of magnitude lower than the one in eq.(15). (iii) The range in eq.(15) can be alternatively written as:

$$\sin^2 2\theta_{13} = 2 \times 10^{-8} - 1 \times 10^{-4}, \text{ or as } \theta_{13} = 0.004^\circ - 0.3^\circ. \tag{16}$$

(iv) When the cosmological constraint is used, the upper limit of $\sin^2 2\theta_{13}$ is $1 \times 10^{-6}$, corresponding to an angle $\theta_{13}$ of $0.03^\circ$.

These numbers can be regarded as lower limits to $U_{\nu 3}$ from Planck scale physics. They are admittedly small for a hierarchical spectrum, but large for degenerate neutrinos.

**The impact of phases**

For arbitrary values of the left and the right phases these numbers will change. Note that $a_3$ can always be taken as 0. Putting arbitrary values of phases for the hierarchical spectrum gives values ranging between $1.6 - 1.9 \times 10^{-8}$ for $\sin^2 2\theta_{13}$. So one observes that the phases have only a mild influence on the value of $U_{\nu 3}$.

For a degenerate spectrum, phases can affect the bound significantly. For illustration in fig.[1] we give $\sin^2 2\theta_{13}$ as a function of the right phases $a_1$ and $a_2$ as a contour plot, for fixed arbitrary values of $f_1 = 66^\circ$, $f_2 = 20^\circ$ and $f_3 = 48^\circ$. Observe that the maximum value of $1 \times 10^{-4}$ given in eq.(16) is reached for some combination of the phases. We also see that some particular choice of phases can heavily suppress $\sin^2 2\theta_{13}$ and make it vanishingly small.

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For degenerate neutrinos the value of $U_{\nu 3}$ usually scales as $\mu \times M$. So the results in fig.[1] can be easily scaled to other values of $M$. This applies also when $\mu > v^2/M_{Pl}$—see next item.
| Mass spectrum         | $U_{e3}$ | $\sin^2 \theta_{13}$ | $\theta_{13}$ | $U_{e3}$ | $\sin^2 \theta_{13}$ | $\theta_{13}$ |
|-----------------------|----------|-----------------------|----------------|----------|-----------------------|----------------|
| hierarchical          | $7 \cdot 10^{-4}$ | $2 \cdot 10^{-6}$ | 0.04° | $7 \cdot 10^{-3}$ | $2 \cdot 10^{-4}$ | 0.4° |
| degen. (cosm.)        | $5 \cdot 10^{-3}$ | $1 \cdot 10^{-4}$ | 0.3° | $5 \cdot 10^{-2}$ | $1 \cdot 10^{-2}$ | 2.8° |

Table 1: Impact on $U_{e3}$ of a flavor blind mass matrix of perturbation at the scale $M_X = 10^{18}$ GeV (left) and at the scale $M_X = 10^{17}$ GeV (right), for the two types of mass spectra (first column).

**Effect of scales below $M_{Pl}$**

The lower bounds we obtained increase if the scale of new physics decreases (as it is clear from eq.(14)). In other words any model that needs a scale $M_X$ above $M_{GUT}$ but below the Planck mass, can lead to a potentially interesting contribution to the still unknown mixing parameter $U_{e3}$. In this manner the scale $M_X$ can lead to a significant $U_{e3}$ even for non-degenerate spectra, in contrast to the case for Planck scale effects.

We illustrate this in table 1 for two scales, each successively one order of magnitude less than the Planck scale. Here we set again all the phases to zero. We would also like to point out that for scales lower than the Planck scale and for a degenerate neutrino spectrum, the corrections to the solar neutrino mass difference become large (as can be seen from eq.(11)) and fine tuning of the phases becomes unavoidable to control the corrections. This can be taken to suggest that for scales below the Planck scale, the degenerate spectrum is unnatural for such effects. Equivalently one can also say that if the degenerate spectrum is hinted at by some other phenomena, then it strongly constrains the scale of such flavor blind contributions.

**Comparison with renormalization effects**

The neutrino mass matrix is renormalized from the GUT scale to the electroweak scale $M_Z \sim M_{higgs}$, already due to the couplings of the standard model. This renormalization modifies $U_{e3}$ and possibly contributes to make it non-zero. The purpose of this section is to evaluate quantitatively this effect.

At one-loop, renormalization in the standard model is described by,

$$\mathcal{M}_{\alpha\beta} \rightarrow \eta_{\alpha} \mathcal{M}_{\alpha\beta} \eta_{\beta}, \quad \text{with} \quad \eta_{\alpha} = \eta \cdot \exp \left[ \frac{b}{(4\pi)^2} \int_{M_Z}^{M_{GUT}} y_{\alpha}^2(Q) \frac{dQ}{Q} \right].$$  (17)

The indices $\alpha, \beta$ should be not summed over. The coefficient $\eta$ (mostly due to gauge and top Yukawa couplings) modifies the 3 masses, i.e. $M_i \rightarrow \eta^2 M_i$, but leaves the texture of the mixing matrix untouched since it is flavor blind. The other contribution is different for the 3 flavors. Thus it changes the texture and the mixing angles. It arises since the Yukawa couplings $y_{\alpha}$ of the charged leptons are different and the numerical coefficient $b$ is not zero. For instance, in the standard model $b = 1/2$, while in its supersymmetric version, $b = -1$ [15] (here and below, we consider supersymmetry at the electroweak scale). The largest part of this correction is due to $\tau$ Yukawa coupling $y_{\tau}$. When this renormalization is small we can apply the perturbative
expansion developed above. In fact, including the effect of \( \eta \) in \( \mathcal{M} \) (by redefining \( \eta^2 \mathcal{M} \rightarrow \mathcal{M} \)), so that \( \eta^2 M_i \rightarrow M_i \) and \( U \rightarrow U \) we get an expression similar to eq.(17):

\[
\mathcal{M} \rightarrow \mathcal{M} + \varepsilon \cdot \begin{pmatrix}
0 & 0 & \mathcal{M}_{\tau\tau} \\
0 & 0 & \mathcal{M}_{\mu\tau} \\
\mathcal{M}_{\tau\tau} & \mathcal{M}_{\mu\tau} & 2\mathcal{M}_{\tau\tau}
\end{pmatrix}, \quad \text{when } \varepsilon \equiv \eta/\eta - 1 \ll 1. \tag{18}
\]

In the basis where the unperturbed mass matrix (the first one in eq.(18)) is diagonal, the components of the term of perturbation (the second term) are,

\[
m_{ij} = \varepsilon \cdot [M_i U_{\tau i}^* U_{\tau j} + U_{\tau i} U_{\tau j}^* M_j]. \tag{19}
\]

The results of eqs.(514) still apply, when we replace the matrix \( m \) introduced in eq.(8) with the one given here (this is the reason why we use the same symbol).

Let us focus now on the case of interest when the mixing \( U_{e3} \) is 0 at the GUT scale. Hence we have \( U_{\tau 1} = \sin \theta_{12} \sin \theta_{23} e^{i\alpha_1}, U_{\tau 2} = -\cos \theta_{12} \sin \theta_{23} e^{i\alpha_2} \) and \( U_{\tau 3} = \cos \theta_{23} \) (from eq.(17) we see that the phases \( f_i \) do not play any role in these considerations and can be set to zero). The contribution to \( U_{e3} \) from renormalization is:

\[
\delta U_{e3} = \frac{\varepsilon}{4} \cdot \sin 2\theta_{12} \sin 2\theta_{23} (f_1 - f_2) \rightarrow \varepsilon \cdot \left( \frac{M}{0.35 \text{ eV}} \right)^2, \tag{20}
\]

where \( f_j = e^{i\alpha_j} [\cos a_j (M_3 + M_j)^2 - i \sin a_j (M_3 - M_j)^2] / \Delta M_{3j}^2 \). A crucial point to be noted is that we use this equation taking the values of \( \theta_{12} \) and \( \theta_{23} \) from low energy data. This is correct when the considered renormalization is a perturbation. The limiting expression given above is obtained with two other simplifying assumptions: Firstly we assume a degenerate mass spectrum \( M_i \sim M_j \sim M \) at the GUT scale and secondly the Majorana phases \( a_1 \) and \( a_2 \) are set to 0.

Note in passing that the contribution to the solar splitting \( \Delta M_{21}^2 \) from \( y_\tau \) renormalization is \( 2\varepsilon \sin^2 \theta_{23} [(M_2^2 + M_3^2) \cos 2\theta_{12} + \Delta M_{31}^2] \). Thus the smallest splitting will get a contribution from the absolute neutrino mass \( M \) [10], unless \( \theta_{12} = 45^\circ \) [17]. This value of \( \theta_{12} \) is however disfavored at \( \sim 3\sigma \) by combined analyses of solar and reactor neutrino data [18]. This suggests a ‘naturalness’ criterion, viz the correction should not exceed \( \Delta M_{21}^2 \), that means: \( \varepsilon < O(1) \cdot (0.01 \text{ eV}/M)^2 \).

In order to describe the impact of renormalization on \( U_{e3} \), the key quantity is \( \varepsilon \), given in eq.(18). To be specific, we consider the case of the supersymmetric model where the grand unification program is most commonly implemented for a variety of reasons. The tau Yukawa coupling is given by \( y_\tau = m_\tau/(v \cos \beta) \) and we take the range \( \beta = 60^\circ - 89^\circ \), to have perturbative

| Mass spectrum       | \( U_{e3} \) | \( \sin^2 2\theta_{13} \) | \( \theta_{13} \) | \( U_{e3} \) | \( \sin^2 2\theta_{13} \) | \( \theta_{13} \) |
|---------------------|--------------|----------------|-------------|--------------|----------------|-------------|
| hierarchical        | 6 \cdot 10^{-6} | 2 \cdot 10^{-10} | 0.0004^\circ | 1 \cdot 10^{-4} | 7 \cdot 10^{-8} | 0.01^\circ |
| degen. (cosm.)      | 3 \cdot 10^{-5} | 5 \cdot 10^{-9} | 0.002^\circ | 7 \cdot 10^{-4} | 2 \cdot 10^{-6} | 0.04^\circ |

Table 2: Impact of renormalization on \( U_{e3} \) for the two types of mass spectra (first column) in the supersymmetric standard model. The value of \( \tan \beta \) is equal to \( \sqrt{3} \) in the left part of the table and to 10 in the right part.
Yukawa couplings till $M_{\text{GUT}}$ (note that this Yukawa coupling is always larger than the standard model one, $m_{\tau}/v \sim 10^{-2}$). A numerical integration of the one-loop system of equations as in [19] yields:

$$-\varepsilon = 7 \cdot 10^{-5} - 0.15, \quad \text{when} \quad \beta = 60^\circ - 89^\circ.$$  

(21)

The naive estimate $-\varepsilon \sim y_{\tau}^2(M_Z) \cdot \log(M_{\text{GUT}}/M_Z)/(4\pi)^2$ would give $10^{-4} - 7 \cdot 10^{-2}$.

At this point we have all the ingredients to evaluate the contribution to $U_{e3}$ from eq.(20). The result is given in table (2) for two values of $\beta$. This shows that renormalization effects are not necessarily large, even though they lead us to expect that $U_{e3}$ is non-zero at the electroweak scale. Some remarks are in order:

(a) The Majorana phases affect the result. For simplicity and also for the sake of argument, we focused the discussion on the case $a_1 = a_2 = 0$.

(b) The result depends rather dramatically on the unknown parameter $\beta$.

(c) The naturalness criterion on the absolute mass scale mentioned above is satisfied for the lowest values of $\beta$ and also for the largest value of the common neutrino mass considered.

(d) The largest values of $\beta$ and of $M$ taken together, produce inaccurate estimates in perturbation theory and should be considered only for illustration.

Discussion

A priori, there is no strong reason to believe that $\theta_{13}$ is exactly zero. In the literature there are a number of theoretical models for $U_{e3}$. Some of them have large values, close to the present CHOOZ bound (for instance some minimal $SO(10)$ models or models with $U(1)$ selection rules where $\theta_{13} \sim \theta_C - \theta_C = 13^\circ$ being the Cabibbo angle). More possibilities are reviewed in [20], and there are also works where $U_{e3}$ is generated at the weak scale via radiative corrections to some texture defined at the high scale [21].

However, the experimental indications of an almost maximal mixing angle (i.e. $\theta_{23} \approx 45^\circ$), could suggest the view that the mixing angles take very special values and perhaps $\theta_{13}$ is really very small. More generally we believe that it is appealing to speculate on the possibility that $U_{e3}$ has an anomalously small value, say $\theta_{13} < \theta_C^2 \approx 3^\circ$. In the present paper we have argued that such a small value might be a window for physics above the grand unification scale. In this sense even a negative result from future search of $\theta_{13}$ could be of great interest.

Our analysis concerns the aforementioned case. We assumed that $U_{e3}$ is zero at the GUT scale and we addressed the question on whether the physics above the GUT scale can be responsible for a non-zero value of $U_{e3}$. The outcome is not discouraging, especially if neutrinos are mass degenerate, or if the neutrino masses receive contributions from other scales below $M_{\text{Pl}}$. In fact, a sizable part of the range of values of $\theta_{13}$ obtained here is within reach of the high performance neutrino factories. See for instance [26]. The largest values of $\theta_{13}$ can have an effect on supernova neutrino fluxes via the MSW effect [27]. Indeed, the 'flip probability' in the supernova mantle due to the atmospheric neutrino $\Delta M^2$ is approximatively given by $P_f \approx \exp[-\xi (U_{e3}^2/10^{-5})]$ where $\xi = (15 \text{ MeV}/E_\nu)^{2/3}$. The tentative lower limit we discussed suggest that $\theta_{13}$ could well be at the border of the region of adiabaticity and hence can lead to a distortion in the spectrum of supernova neutrinos [28].
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