Estimation on the neutrino masses by using neutrino oscillation parameters

Wen-Jie Wu and Xiang Zhou

Hubei Nuclear Solid Physics Key Laboratory,
School of Physics and Technology, Wuhan University, Wuhan 430072, China

(Dated: July 11, 2018)

Abstract

It has been shown that neutrino masses can be determined under the particle ansatz. In this paper, we give the general formulas of neutrino masses related to the neutrino oscillation parameters which show that there is a mass hierarchy transition. Using the best fit values measured by electron and muon neutrino oscillation experiments, the total neutrino mass is about 0.25 eV. According to the standard neutrino cosmology, the neutrino energy density is about 55 eV/cm$^3$ and $\Omega_m h^2 > \Omega_\nu h^2 \sim 0.0055$. 
Nonzero neutrino masses are beyond the standard model which are confirmed by various
oscillation experiments. The phase difference (PhD) in the oscillation probability derived
under the so called “same energy” or “same momentum” ansatz in the traditional phe-

\[ \Delta \phi_{ji} \approx \frac{m_j^2 - m_i^2}{4E_\nu} L = \frac{\Delta m_{ji}^2}{4E_\nu} L, \]

where \( m_i \) and \( m_j \) are the mass eigenvalues, \( E_\nu \) is the neutrino energy, \( L \) is the neutrino flight
distance. It is only sensitive to the mass square difference \( \Delta m_{ji}^2 \). However, the so called
“particle” ansatz assumes that in the rest frame of a flavor neutrino \( \nu_\alpha \) all momentums of
its mass eigenstates \( |\nu_i\rangle \) are simultaneously zero\[2\]. Under the particle ansatz, the phase
difference \( \Delta \phi_{ji} \) is mass measurable which is

\[ \Delta \phi_{ji} \approx \frac{2m_\nu_\alpha (m_j - m_i)}{4E_\nu} L = \frac{\Delta m_{\alpha ji}^2}{4E_\nu} L, \]

where \( L \) is the distance and \( \Delta m_{\alpha ji}^2 = 2m_\nu_\alpha (m_j - m_i) \). In this paper, we give the general
formulas of the neutrino masses related to the neutrino oscillation parameters, and estimate
the total mass and the neutrino energy density of the Universe.

In the case of 3-neutrino mixing \( U_{\alpha i} \) is the element of the PMNS matrix \( U = [U_{\alpha i}] \) with
three mixing angles \( \theta_{12}, \theta_{23} \) and \( \theta_{13} \) and Dirac phase \( \delta \). The PMNS matrix can be written
as

\[ U_{\text{PMNS}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \]

where \( c_{ij} = \cos \theta_{ij} \) and \( s_{ij} = \sin \theta_{ij} \). \( m_\nu_\alpha \) and \( m_i \) have the relationship as

\[ \begin{pmatrix} m_{\nu_e} \\ m_{\nu_\mu} \\ m_{\nu_\tau} \end{pmatrix} = \begin{pmatrix} |U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2 \\ |U_{\mu1}|^2 & |U_{\mu2}|^2 & |U_{\mu3}|^2 \\ |U_{\tau1}|^2 & |U_{\tau2}|^2 & |U_{\tau3}|^2 \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}. \]

\[ \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} = \begin{pmatrix} U_{e1}^2 & U_{e1}U_{\mu1} & U_{e1}U_{\tau1} \\ U_{e2}^2 & U_{e2}U_{\mu2} & U_{e2}U_{\tau2} \\ U_{e3}^2 & U_{e3}U_{\mu3} & U_{e3}U_{\tau3} \end{pmatrix} \begin{pmatrix} m_{\nu_e} \\ m_{\nu_\mu} \\ m_{\nu_\tau} \end{pmatrix}. \]
where

\[
\begin{pmatrix}
|U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2 \\
|U_{\mu1}|^2 & |U_{\mu2}|^2 & |U_{\mu3}|^2 \\
|U_{\tau1}|^2 & |U_{\tau2}|^2 & |U_{\tau3}|^2
\end{pmatrix}
\begin{pmatrix}
U_{e1}^2 & U_{\mu1}^2 & U_{\tau1}^2 \\
U_{e2}^2 & U_{\mu2}^2 & U_{\tau2}^2 \\
U_{e3}^2 & U_{\mu3}^2 & U_{\tau3}^2
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}.
\] (6)

There are five independent conditions for \(U_{ai}^2\) which are

\[
\begin{align*}
U_{e1}^2 + U_{\mu1}^2 + U_{\tau1}^2 &= 1, \\
U_{e2}^2 + U_{\mu2}^2 + U_{\tau2}^2 &= 1, \\
U_{e3}^2 + U_{\mu3}^2 + U_{\tau3}^2 &= 1, \\
U_{e1}^2 + U_{e2}^2 + U_{e3}^2 &= 1, \\
U_{\mu1}^2 + U_{\mu2}^2 + U_{\mu3}^2 &= 1.
\end{align*}
\] (7)

Therefore, there are only four independent elements in the matrix \(\bar{U}_{ai}^2 = [\bar{U}_{ai}^2]\). In this paper, \(\bar{U}_{e1}^2, \bar{U}_{\mu1}^2, \bar{U}_{e2}^2\) and \(\bar{U}_{\mu2}^2\) are chosen.

The total neutrino mass \(M\) satisfies

\[
M = m_{\nu_e} + m_{\nu_\mu} + m_{\nu_\tau} = m_1 + m_2 + m_3.
\] (8)

If the coefficients \(k = m_{\nu_e}/M\) and \(\epsilon = m_{\nu_\mu}/m_{\nu_e}\) are introduced, the masses of flavor neutrinos can be written as

\[
\begin{align*}
m_{\nu_e} &= kM, \\
m_{\nu_\mu} &= \epsilon kM, \\
m_{\nu_\tau} &= (1 - k - \epsilon k)M,
\end{align*}
\]

where \(0 < k < 1, 0 < \epsilon k < 1\) and \(0 < 1 - k - \epsilon k < 1\). For neutrino experiments a traditional mass square term in the PhD under the “same energy” or “same momentum” ansatz can be transformed to its corresponding mmPhD which are listed in Table I. The value of \(\epsilon = m_{\nu_\mu}/m_{\nu_e} = |\Delta m_{\mu32}^2|/|\Delta m_{e32}^2|\) can be obtained by spectral analyses under the particle ansatz.

\(\Delta m_{e21}^2\) and \(\Delta m_{\mu32}^2\) can be rewritten as

\[
\begin{align*}
\Delta m_{e21}^2 &= 2[(\bar{U}_{e2}^2 - \bar{U}_{e1}^2)k^2 \\
&\quad + (\bar{U}_{\mu2}^2 - \bar{U}_{\mu1}^2)\epsilon k^2 \\
&\quad + (\bar{U}_{e1}^2 + \bar{U}_{\mu1}^2 - \bar{U}_{\mu2}^2 - \bar{U}_{e2}^2)(k - k^2 - \epsilon k^2)]M^2,
\end{align*}
\] (9)
TABLE I. The correspondence of mass square differences between the “same energy” ansatz (or “same momentum” ansatz) and the ”particle ansatz”.

| Probability | Mass square term in the “same energy” ansatz or “same momentum” ansatz | Mass square term in the particle ansatz |
|-------------|---------------------------------------------------------------------------|--------------------------------------|
| $P(\nu_e \rightarrow \nu_e)$ | $\Delta m_{e21}^2$ | $\Delta m_{e21}^2 = 2m_{\nu_e}(m_2 - m_1)$ |
| $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ | | $= 2m_{\bar{\nu}_e}(m_2 - m_1)$ |
| $P(\nu_\mu \rightarrow \nu_\mu)$ | $|\Delta m_{32}^2|$ | $|\Delta m_{\mu32}^2| = 2m_{\nu_\mu}|m_3 - m_2|$ |
| $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)$ | | $= 2m_{\bar{\nu}_\mu}|m_3 - m_2|$ |
| $P(\nu_e \rightarrow \nu_e)$ | $|\Delta m_{32}^2|$ | $|\Delta m_{e32}^2| = 2m_{\nu_e}|m_3 - m_2|$ |
| $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ | | $= 2m_{\bar{\nu}_e}|m_3 - m_2|$ |

$$|\Delta m_{\mu32}^2| = 2|(1 - \bar{U}_{e1}^2 - 2\bar{U}_{e2}^2)\epsilon k^2$$
$$+ (1 - \bar{U}_{\mu1}^2 - 2\bar{U}_{\mu2}^2)\epsilon^2 k^2$$
$$+ (\bar{U}_{e1}^2 + 2\bar{U}_{e2}^2 + \bar{U}_{\mu1}^2 + 2\bar{U}_{\mu2}^2 - 2)(\epsilon k - \epsilon^2 k^2 - \epsilon^3 k^3)|M|^2.$$  \hspace{1cm} (10)

It is easy to show that if $\epsilon = 1$, $k = 1/3$ and then $M \rightarrow \infty$. Thus the masses of three flavor neutrinos $\nu_\alpha$ can not be all the same. $\eta$ is defined as the ratio of $\Delta m_{e21}^2$ and $\Delta m_{\mu32}^2$ which is

$$\eta \equiv \frac{\Delta m_{e21}^2}{|\Delta m_{\mu32}^2|} = \frac{A}{|B|},$$  \hspace{1cm} (11)

where

$$A \equiv \bar{U}_{e1}^2 + \bar{U}_{\mu1}^2 - \bar{U}_{e2}^2 - \bar{U}_{\mu2}^2$$
$$+(2\bar{U}_{e2}^2 - 2\bar{U}_{e1}^2 + \bar{U}_{\mu2}^2 - \bar{U}_{\mu1}^2)$$
$$+(2\bar{U}_{\mu2}^2 - 2\bar{U}_{\mu1}^2 + \bar{U}_{e2}^2 - \bar{U}_{e1}^2)\epsilon k$$

$$B \equiv (\bar{U}_{e1}^2 + 2\bar{U}_{e2}^2 + \bar{U}_{\mu1}^2 + 2\bar{U}_{\mu2}^2 - 2)\epsilon$$
$$+((3 - 2\bar{U}_{e1}^2 - 4\bar{U}_{e2}^2 - \bar{U}_{\mu1}^2 - 2\bar{U}_{\mu2}^2)\epsilon$$
$$+(3 - 2\bar{U}_{\mu1}^2 - 4\bar{U}_{\mu2}^2 - \bar{U}_{e1}^2 - 2\bar{U}_{e2}^2)\epsilon^2)k$$

Combining the results of mixing angles and mass square differences, $k$ and $M$ can be
expressed as Eq. (12) and Eq. (13),

\[
k = - \frac{(U_{e1}^2 + U_{\mu1}^2 - U_{e2}^2 - U_{\mu2}^2 \mp (U_{e1}^2 + 2U_{e2}^2 + U_{\mu1}^2 + 2U_{\mu2}^2 - 2)\eta\epsilon)}{(2U_{e2}^2 - 2U_{e1}^2 + U_{\mu2}^2 - U_{\mu1}^2 + (2(1 \pm \eta)U_{\mu2}^2 - (2 \mp \eta)U_{\mu1}^2 + (1 \pm 4\eta)U_{e2}^2
- (1 \mp 2\eta)U_{e1}^2 \pm 3\eta)\epsilon \mp (3 - 2U_{\mu1}^2 - 4U_{\mu2}^2 - U_{e1}^2 - 2U_{e2}^2)\eta^2)},
\]

(12)

\[
M = \frac{(2(U_{e2}^2 - U_{e1}^2)k^2 + (U_{\mu2}^2 - U_{\mu1}^2)k^2 + (U_{e1}^2 + U_{\mu1}^2 - U_{e2}^2 - U_{\mu2}^2)(k - k^2 - \epsilon k^2)]/[\Delta m_{e21}^2]^{-\frac{1}{2}}},
\]

(13)

where the upper operator in \(\mp\) or \(\pm\) is for \(B > 0\) and the lower operator for \(B < 0\). If the parameters of the PMNS matrix are known and \(\epsilon = m_{\nu_e}/m_{\nu_e}\) is obtained by measuring the ratio of \(m_{\nu32}\) and \(m_{\nu32}\) from neutrino experiments, then the fraction factor \(k = m_{\nu_e}/M\) and the total flavor neutrinos mass \(M\) can be calculated from Eq. (12) and Eq. (13). Consequently all neutrino masses of flavor and mass eigenstate can be obtained. Fig. 1 shows that there is a mass hierarchy transition with \(\theta_{12} = 34.7^\circ, \theta_{13} = 8.43^\circ, \theta_{23} = 30^\circ\) and \(\delta = \pm 90^\circ\).

The KamLAND experiment gave values of \(\Delta m_{21}^2\) and \(\theta_{12}\) by measuring long baseline reactor neutrinos. The fit values with only KamLAND data are \(\Delta m_{21}^2 = (7.54^{+0.19}_{-0.06}) \times 10^{-5}\) eV\(^2\) and \(\tan^2\theta_{12} = 0.481^{+0.092}_{-0.081}\). The latest Daya Bay results based on 1230 live days of data gave \(\sin^22\theta_{13} = 0.0841 \pm 0.0027 \pm 0.0019\) and \(\Delta m_{32}^2 = (2.45 \pm 0.06 \pm 0.06) \times 10^{-3}\) eV\(^2\) for normal hierarchy (NH) and \(\Delta m_{32}^2 = -(2.56 \pm 0.06 \pm 0.06) \times 10^{-3}\) eV\(^2\) for inverted


| Parameters under “same energy” ansatz | Source | Parameters under particle ansatz |
|---------------------------------------|--------|----------------------------------|
|            |        |                                  |
| $\theta_{12} = 34.7^\circ$           | KamLAND[3] | $\theta_{12} = 34.7^\circ$          |
| $\theta_{13} = 8.43^\circ$           | Daya Bay[4] | $\theta_{13} = 8.43^\circ$          |
| $\Delta m^2_{21} = 7.54 \times 10^{-5} \text{ eV}^2$ | KamLAND[3] | $\Delta m^2_{e21} = 7.54 \times 10^{-5} \text{ eV}^2$ |
| $\Delta m^2_{32} = 2.45 \times 10^{-3} \text{ eV}^2$ (NH) | Daya Bay[4] | $\Delta m^2_{e32} = 2.45 \times 10^{-3} \text{ eV}^2$ (I) |
| $\Delta m^2_{32} = -2.56 \times 10^{-3} \text{ eV}^2$ (IH) |       | $\Delta m^2_{e32} = -2.56 \times 10^{-3} \text{ eV}^2$ (II) |
| $\Delta m^2_{32} = 2.67 \times 10^{-3} \text{ eV}^2$ (NH) | NO\[5\] | $\Delta m^2_{\mu32} = 2.67 \times 10^{-3} \text{ eV}^2$ (I) |
| $\Delta m^2_{32} = -2.72 \times 10^{-3} \text{ eV}^2$ (IH) |       | $\Delta m^2_{\mu32} = -2.72 \times 10^{-3} \text{ eV}^2$ (II) |
| $\Delta m^2_{\mu32}/\Delta m^2_{e32}$ | This work | $\epsilon = 1.09$ (I) |
| $\Delta m^2_{e21}/\Delta m^2_{\mu32}$ | This work | $\eta = 0.0282$ (I) |
| $\Delta m^2_{e21}/\Delta m^2_{\mu32}$ | This work | $\eta = 0.0277$ (II) |

Recently, NO\[5\] made a precise measurement of $\nu_\mu$ disappearance and gave $\Delta m^2_{32} = (+2.67 \pm 0.11) \times 10^{-3} \text{ eV}^2$ for NH, and $\Delta m^2_{32} = (-2.72 \pm 0.11) \times 10^{-3} \text{ eV}^2$ for IH[5]. Based on the parameters derived under the “same energy” or “same momentum” ansatz, the center values of corresponding parameters under particle ansatz are listed in Table II. There are two sets of parameters due to the mass hierarchy under the “same energy” or “same momentum” ansatz which are distinguished using label (I) and (II) under particle ansatz. In addition, $\epsilon$ and $\eta$ can be obtained and results are listed in Table II. With $\theta_{12}$, $\theta_{13}$, $\epsilon$ and $\eta$ in Table II, Fig. 2 gives $k$ and $M$ of the parameter set (I) and (II) when $\delta$ runs from $-180^\circ$ to $180^\circ$ and $\theta_{23}$ runs from $30^\circ$ to $60^\circ$ according to Eq. (12) and Eq. (13).

We define the compatibility of mass hierarchy so that the mass hierarchy is compatible when the mass hierarchy under the different ansatzs are the same, and incompatible when the mass hierarchy under the different ansatzs are different. It can be shown that the mass hierarchy of the parameter set (I) is compatible but the mass hierarchy of the parameter set (II) is incompatible. The estimation of the neutrino masses are only done for the mass hierarchy compatible parameter set (I). NO\[5\] gave two statistically degenerate values of $\sin^2 \theta_{23}$ and $\delta$ both in the NH: $\sin^2 \theta_{23} = 0.404$, $\delta = 1.48\pi$ and $\sin^2 \theta_{23} = 0.623$, $\delta = 0.74\pi$[6].
FIG. 2. The effects of Dirac phase $\delta$ and mixing angle $\theta_{23}$ on the coefficient $k$ and the total neutrino mass $M$. The mass hierarchy of both parameter set (I) and (II) are normal in the whole $(\delta, \theta_{23})$ parameter space.

(a) Parameter set (I), $k(\delta, \theta_{23})$
(b) Parameter set (I), $M(\delta, \theta_{23})$
(c) Parameter set (II), $k(\delta, \theta_{23})$
(d) Parameter set (II), $M(\delta, \theta_{23})$

The latest T2K results show that $\sin^2 \theta_{23} = 0.532$ and $\delta = -1.791$ for NH[7]. Combining the knowledge of $\theta_{23}$ and $\delta$ with parameter set (I), the absolute neutrino masses can be estimated and shown in Table III. According to the standard neutrino cosmology, the ratio of neutrinos to photons is $3/11$. Therefore, the contribution to the total energy density of the Universe from neutrinos is given by $\rho_\nu = m_{\text{tot}} n_\nu = m_{\text{tot}} (3/11) n_\gamma$, where $n_\gamma = 412 \text{ cm}^{-3}$, $m_{\text{tot}} = \sum_\nu (g_\nu/2) m_\nu$, $g_\nu = 4$ for neutrinos with Dirac masses and $g_\nu = 2$ for Majorana
TABLE III. The estimation on the neutrino masses by using neutrino oscillation parameters.

| P.S. | \( \theta_{23} \) (NO\( \nu \)A) | \( \delta \) (NO\( \nu \)A) | MH | k | M (eV) | \( m_1 \) (eV) | \( m_2 \) (eV) | \( m_3 \) (eV) | \( \rho_\nu \) (eV/cm\(^3\)) | \( \Omega_\nu h^2 \) |
|------|------------------|------------------|---|---|-------|-------|-------|-------|-------------|-------------|
| (I)  | 39.5° 1.48\( \pi \) rad | NH | 0.310 | 0.235 | 0.0722 | 0.0728 | 0.0896 | 52.8 | 0.00499 |
| (I)  | 52.1° 0.74\( \pi \) rad | NH | 0.318 | 0.286 | 0.0904 | 0.0908 | 0.104 | 64.3 | 0.00609 |
| P.S. | \( \theta_{23} \) (T2K) | \( \delta \) (T2K) | MH | k | M (eV) | \( m_1 \) (eV) | \( m_2 \) (eV) | \( m_3 \) (eV) | \( \rho_\nu \) (eV/cm\(^3\)) | \( \Omega_\nu h^2 \) |
| (I)  | 46.8° -1.791 rad | NH | 0.316 | 0.265 | 0.0833 | 0.0837 | 0.0984 | 59.6 | 0.00564 |

neutrinos. The physical density of neutrinos in units of the critical density \( \Omega_\nu = \rho_\nu / \rho_c \) can be written as \( \Omega_\nu h^2 = m_{\text{tot}} / (94 \text{ eV}) \), where \( h \) is the Hubble constant in units of 100 km\( \cdot \)s\(^{-1} \)\( \cdot \)Mpc\(^{-1} \)\[8\]. The neutrino energy density of the Universe with the Dirac neutrino assumption are listed in Table III. The results show that \( \Omega_m h^2 > \Omega_\nu h^2 \sim 0.0055 \).

We have estimated the neutrino masses based on the neutrino oscillation parameters\[4–8\]. The total neutrino mass is about 0.25 eV and the neutrino energy density of the Universe is about 55 eV/cm\(^3\) and then \( \Omega_m h^2 > \Omega_\nu h^2 \sim 0.0055 \). In the future the precise values with their uncertainties will be determined under the particle ansatz by the spectral analyses of neutrino oscillation experiments\[8, 9\].

**Appendix A: Formulas of \( U_{e1}^2 \), \( U_{\mu 1}^2 \), \( U_{e2}^2 \) and \( U_{\mu 2}^2 \).**

\[
U_{e1}^2 = \frac{4 \cos \theta_{12} \cos^2 \theta_{13} \cos \theta_{12} \cos 2\theta_{23} - \cos \delta \sin \theta_{12} \sin \theta_{13} \sin 2\theta_{23}}{\cos \delta \sin 2\theta_{12} \sin \theta_{13} \sin 2\theta_{23} - 3 \cos 2\theta_{13} + 4 \cos 2\theta_{12} \cos 2\theta_{13} \cos 2\theta_{23}}
\]

\[
U_{\mu 1}^2 = (2 \sin^2 \theta_{13} \cos \delta \sin 2\theta_{12} \sin 2\theta_{23} \sin \theta_{13} + 2 \sin^2 \theta_{12} \sin^2 \theta_{13} \cos^2 \theta_{23} + 2 \sin^2 \theta_{23} \cos^2 \theta_{12} - 4 \sin^2 \theta_{12} \cos^4 \theta_{13} \cos^2 \theta_{23})
\] / (\cos \delta \sin 2\theta_{12} \sin \theta_{13} \sin 2\theta_{23} - 3 \cos 2\theta_{13} + 4 \cos 2\theta_{12} \cos 2\theta_{13} \cos 2\theta_{23})

\[
U_{e2}^2 = \frac{-4 \sin \theta_{12} \cos^3 \theta_{13} \cos \delta \sin \theta_{13} \sin 2\theta_{23} \cos \theta_{12} + \sin \theta_{12} \cos 2\theta_{23}}{\cos \delta \sin 2\theta_{12} \sin \theta_{13} \sin 2\theta_{23} - 3 \cos 2\theta_{13} + 4 \cos 2\theta_{12} \cos 2\theta_{13} \cos 2\theta_{23}}
\]

\[
U_{\mu 2}^2 = \frac{2 \sin^2 \theta_{13} \cos \delta \sin 2\theta_{12} \sin \theta_{13} \sin 2\theta_{23} - 2 \sin^2 \theta_{12} \sin^2 \theta_{23} + 4 \cos^2 \theta_{12} \cos 2\theta_{13} \cos^2 \theta_{23}}{\cos \delta \sin 2\theta_{12} \sin \theta_{13} \sin 2\theta_{23} - 3 \cos 2\theta_{13} + 4 \cos 2\theta_{12} \cos 2\theta_{13} \cos 2\theta_{23}}
\]

X. Z. thanks the helpful discussions of Dr. Zhenyu Zhang and Dr. Qian Liu. This work is supported by the Major Program of the National Natural Science Foundation of China (Grant No. 11390381).
[1] M. Thomson, Modern Particle Physics, Cambridge University Press, Cambridge (2013).
[2] X. Zhou, arXiv:1802.09270 [hep-ph] (2018).
[3] A. Suzuki, Eur.Phys.J. C74, 3094 (2014).
[4] F. P. An et al. (Daya Bay Collaboration), Phys. Rev. D 95, 072006 (2017).
[5] P. Adamson et al. (NOνA Collaboration), Phys. Rev. Lett. 118, 151802 (2017).
[6] P. Adamson et al. (NOνA Collaboration), Phys. Rev. Lett. 118, 231801 (2017).
[7] K. Abe et al. (T2K Collaboration), Phys. Rev. Lett. 118, 151801 (2017).
[8] C. Partignai et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update (http://pdg.lbl.gov/2017/).
[9] F. P. An et al. (JUNO Collaboration), J.Phys. G: Nucl. Part. Phys. 43, 030401 (2016).