Application of Elliptic Equations for the Study of Asymptotic Distributions of Temperature and Moisture Content During Drying by Electromagnetic Radiation

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Abstract. A new approach is proposed to study the processes of heat and moisture propagation during drying by electromagnetic radiation. It consists in the fact that transient processes preceding the constant drying speed mode are excluded from the calculations. Thus, the initial boundary value problem for two related parabolic equations can be replaced by two independently solved boundary value problems for elliptic equations. On this basis, a numerical algorithm was developed to study the temperature and moisture content fields when drying samples of arbitrary shape. It can be used in all cases where transients are not of interest.

1. Introduction
The processes of heat and moisture propagation during electromagnetic drying of wet materials are described by a system of partial differential equations for two, or, depending on the accepted mathematical model, for a larger number of desired functions. These processes are studied using both analytical and numerical methods [1, 2]. According to the experimental data, the drying materials most often happen that after a short initial period when the material observed transients, and its moisture content remains nearly constant, occurs with characteristic signs and the most important from the practical point of view the first period, or period of constant drying rate [3].

The mathematical description of the period during which the bulk of the moisture is removed from the material is simpler than the description of the initial period. The results obtained allow us to determine the effect on drying of each of the numerous parameters included in the mathematical model. Thus, you can solve various optimization problems. For the case when the sample to be dried has a simple geometry (plate, cylinder, ball, rectangle and rectangular parallelepiped), the study of the period of constant drying can be performed by analytical methods. The results obtained in this direction are published by the authors in [4-10].

In this paper, we propose a numerical algorithm for investigating the period of constant drying rate for samples with arbitrary shape.

2. Formulation of the initial boundary value problem
Consider a wet capillary-porous body blown by an air stream and exposed to electromagnetic waves. Let $V$ be the area occupied by the body, $\Sigma$ is the boundary of this area, and $T=T(M, \tau)$ and $U=U(M, \tau)$ are the distributions of temperature and moisture content inside the body and at its boundary as a
function of the variable point $M$ and time $\tau$. Provided that the material is homogeneous, the initial boundary value problem for calculating the $T$ and $U$ fields will have the following form [3, 4, 12]:

$$\frac{\partial T}{\partial \tau} = a_w \cdot \nabla^2 T + \frac{r_\gamma}{c} \frac{\partial U}{\partial \tau} + \frac{1}{\rho} W; \quad M \in V;$$  

(1)

$$\frac{\partial U}{\partial \tau} = a_m \cdot \nabla^2 U + a_n \delta \cdot \nabla^2 T; \quad M \in V;$$  

(2)

$$Q + r(1-\gamma)J = \lambda \frac{\partial T}{\partial n} + S; \quad N \in \Sigma;$$  

(3)

$$J = a_m \rho \frac{\partial U}{\partial n} + a_m \rho \delta \frac{\partial T}{\partial n}; \quad N \in \Sigma;$$  

(4)

$$Q(N, \tau) = \sigma \bar{A} \left[ (T(N, \tau) + T_1)^4 - (T_n + T_1)^4 \right] + \alpha_w [T(N, \tau) - T_n];$$  

(5)

$$J(N, \tau) = \alpha_m \left[ P(T(N, \tau)) - \varphi \cdot P(T_\sigma) \right]; \quad P(T) = 6.03 \cdot 10^{-3} \cdot \exp \left[ \frac{17.3 \cdot T}{T + T_2} \right];$$  

(6)

$$T(M, 0) = T^0(M), \quad U(M, 0) = U^0(M).$$  

(7)

Relations (1), (2) are equations of heat and moisture propagation in the region $V$, relations (3) – (6) are boundary conditions on the boundary $\Sigma$, where $N$ is a variable point on this boundary, and (7) are initial conditions. The coefficients and functions in (1) to(7) have the following meanings: $a_w = \lambda/(ec)$ is the diffusion coefficient of heat ($\lambda$, $c$, $\rho$ respectively the coefficient of thermal conductivity, specific heat and density of dry material); $\gamma$, $a_m$, $\delta$ are the criterion of evaporation, the diffusion coefficient of moisture and the relative coefficient of diffusion of moisture; $r$ – specific heat of vaporization of water; $W=W(M, \tau)$ is the density of internal heat sources caused by the absorption of penetrating electromagnetic radiation; $S=S(N, \tau)$ is the density of surface heat sources caused by the absorption of radiation with a small penetration depth; $Q=Q(N, \tau)$ and $J=J(N, \tau)$ are intensity of heat and mass transfer of the sample surface to the air; $\partial/\partial n$ is the derivative symbol in the direction of the inner normal to the boundary of $\Sigma$; $a$ – Stefan-Boltzmann constant; $\bar{A}$ is coefficient of thermal radiation; $T_\sigma$ and $\varphi$ are the temperature and humidity of the air outside the boundary layer near the point of $N$; $a_w$ and $a_m$ are coefficients of heat and mass transfer of the sample surface with the air medium; $P(T)$ is a function modeling the dependence of the relative partial pressure of saturated water vapor to its temperature $T$ when the total normal pressure; $T_1=273$ °C and $T_2=238$ °C – constant; $T^0(M)$ and $U^0(M)$ are the given functions, determine the distribution of temperature and moisture content at time $\tau=0$.

The boundary conditions (3)-(6) are nonlinear (which also determines the nonlinearity of the entire formulated initial boundary value problem), and they cannot be assigned to any of the standard boundary conditions of mathematical physics (Dirichlet, Neumann, or the third kind). The boundary condition of heat transfer takes into account the heat exchange according to Newton's law and the heat exchange by radiation, and the boundary condition of mass transfer is taken as Dalton's law of evaporation.

3. Asymptotic equations for $T$ and $U$ fields

According to the research plan, we will only be interested in the modes of constant drying speed, which are set after the attenuation of all transients. Formally, such modes can be considered as asymptotic, which are realized at $\tau \to \infty$. But the asymptotic solutions of initial boundary value problems for parabolic equations no longer depend on the initial data [13], so we can ignore the initial conditions (7) when constructing such solutions. If, after this, we can somehow find the functions $T$
and \( U \) so that the remaining equations (1)-(6) are satisfied, then, by virtue of the statement of uniqueness [14], these functions will give the desired solution.

As in the mode of constant speed of drying, which can be observed under the condition of stationarity of the density of heat sources, i.e. \( W = W(M) \), \( S = S(N) \), provided the temperature field \( T(M, \tau) \) is close to stationary, and the drying rate \( \partial U/\partial \tau \) is close to stationary homogeneous [3]. So, when constructing asymptotic solutions for partial derivatives of these fields, we can take the following approximations:

\[
\frac{\partial T}{\partial \tau} = 0; \quad \frac{\partial U}{\partial \tau} = \text{const} \equiv \Omega;
\]

It follows from these formulas that the fields \( T \) and \( U \) have the following form:

\[
T = T(M); \quad U = \Omega \tau + \bar{U}(M). \tag{8}
\]

Here the required values are the constant \( \Omega \) and the functions \( T(M) \) and \( \bar{U}(M) \). The equation for \( T(M) \) is obtained from (1) with (8):

\[
\nabla^2 T(M) = -\frac{r_T}{ca_w} \Omega - \frac{1}{\lambda} W(M). \tag{9}
\]

Using the second of the formulas (8) and the resulting expression for the Laplacian \( \nabla^2 T(M) \), we find from (2) the equation for \( \bar{U}(M) \):

\[
\nabla^2 \bar{U}(M) = \delta \left( \frac{1}{\delta a_m} + \frac{r_T}{ca_w} \right) \Omega + \frac{\delta}{\lambda} W(M). \tag{10}
\]

In the original problem, we had a system of two "connected" parabolic equations for the fields \( T(M, \tau) \) and \( U(M, \tau) \) (equations (1) and (2)); if we study only the asymptotic modes, we get separate equations of elliptic type (9) and (10) for the fields \( T(M) \) and \( \bar{U}(M) \). These circumstances significantly simplify the theoretical study of the drying process.

4. The formula for drying rate
Determine the drying rate \( \Omega = \partial U/\partial \tau \). To calculate this constant, we use the integral theorem for the Laplacian [13]

\[
\iiint_V \nabla^2 \varphi(M) \, dV_M = -\iint_{\Sigma} \frac{\partial \varphi}{\partial n}(N) \, d\Sigma_N.
\]

Here \( \varphi \) is an arbitrary sufficiently smooth scalar field considered in the region \( V \) with the boundary \( \Sigma \); \( \partial/\partial n \) is the derivative in the direction of the inner normal to the boundary.

Since the field \( T \) is stationary in the asymptotic mode, there is no dependence on the time \( \tau \) in the initial relations (1)-(6). Integrating both parts (2) over the region \( V \), and converting the integral from Laplacians, we get:

\[
\iiint_V \frac{\partial U}{\partial \tau} (M) \, dV_M = -\iint_{\Sigma} \left( a_m \frac{\partial U}{\partial n}(N) + a_m \delta \frac{\partial T}{\partial n}(N) \right) \, d\Sigma_N.
\]

The expression in parentheses on the right side of this equality, up to a factor of \( \rho \), coincides, according to (4), with the moisture flux density \( J(N) \) on the surface \( \Sigma \), and the integrand function on the left side is constant and equal to \( \Omega \). From here we have:
\[ \Omega \cdot V = - \frac{1}{\rho} \int_{\Sigma} J(N) d\Sigma_N. \]

In this formula, \( V \) is the volume of the region \( V \), and the surface integral on the right side of the formula is the time-constant drying intensity. If this value is determined experimentally, which can be done without difficulty, then the drying rate \( \Omega = \partial U / \partial \tau \) can be found from the obtained ratio.

5. Asymptotic boundary value problems for \( T \) and \( U \) fields

Substituting in (3) the expressions for \( Q \) and \( J \) from (5) and (6), and taking into account that the time \( \tau \) is no longer included in these relations, we get the boundary condition for the temperature field:

\[ \frac{\partial T}{\partial n}(N) + \sigma \tilde{A}(N) + T_1 + \alpha_w T(N) + r(1 - \gamma) \alpha_m \cdot P(T(N)) = . \quad (11) \]

\[ = S(N) + \sigma \tilde{A}(N) + T_1 + \alpha_w T(N) + r(1 - \gamma) \alpha_m \cdot P(T(N)) \quad N \in \Sigma. \]

In this equation, on the left is a given function of an unknown temperature \( T(N) \), and on the right is a given function of a variable point \( N \). Combining equations (9) and (11), we obtain a boundary value problem for finding the temperature field \( T(M) \). This problem, due to its nonlinearity, can only be solved numerically.

Assuming that the above problem is solved, that is, the temperature field \( T(M) \) is already definite, we get the boundary condition for the field \( \hat{U}(M) \). Given that \( \partial U / \partial n(N) = \partial \hat{U} / \partial n(N) \), and considering together equations (3) and (4), we have:

\[ \frac{\lambda}{\partial n} \frac{\partial \hat{U}}{\partial \hat{n}}(N) = \left[ \frac{\lambda}{\alpha_m \rho \delta} - r(1 - \gamma) \right] J(N) - Q(N) + S(N) \quad N \in \Sigma. \quad (12) \]

Here, the functions \( J(N) \) and \( Q(N) \) on the right side of the equation are known and calculated using the formulas (5) and (6).

Combining (10) and (12), we get the boundary value problem for finding the field \( \hat{U}(M) \). This is the Neumann problem for the Poisson equation. Unlike the problem for the temperature field, it is linear, and therefore can be solved not only numerically, but also analytically, for example, by the method of green functions or the method of integral equations.

Thus, when moving to asymptotics, the original non-stationary initial boundary value problem (1)-(7) for two related parabolic equations that simulate electromagnetic drying of an arbitrary shape sample can be replaced with two separate stationary boundary value problems, respectively (9), (11) and (10), (12), for elliptic type equations. Thanks to the "decoupling" of the equations and changing their type from parabolic to elliptical, it is possible to determine the effect on drying of each of the various parameters and functions included in the mathematical model, and the algorithm for numerical research of heat and mass transfer processes is significantly simplified. The results obtained with its help allow visual interpretation and allow solving various optimization problems by simple means.

6. Conclusion

In the framework of the theory of heat and mass transfer, an initial boundary value problem is formulated for calculating the temperature and moisture content fields when drying a sample with a capillary-porous structure and an arbitrary geometry with electromagnetic radiation. A numerical algorithm is proposed for studying the processes of heat and moisture propagation after the end of transients, i.e. in the asymptotic mode. It is based on the fact that the original initial boundary value problem for two related parabolic-type equations is replaced by two separate boundary value problems for elliptic-type equations. This replacement allows you to move from a complex non-stationary problem to two more simple stationary problems, which significantly simplifies the study of heat and mass transfer processes: instead of running the calculation procedure on each layer in time, such a
procedure is launched only once, which reduces computational costs and makes the algorithm convenient for correction and debugging. The results obtained allow us to solve various problems of optimization of drying by electromagnetic radiation by simple means.

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