Fractional Branes, Confinement, and Dynamically Generated Superpotentials

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Abstract

We examine the effects of instantons in four-dimensional $N = 1$ supersymmetric gauge theory by including D0-branes in type IIA brane constructions. We examine instanton-generated superpotentials in supersymmetric QCD and find that they are due to a repulsive force between D4-branes bound to D0-branes ending on NS 5-branes. We study situations where instanton effects break supersymmetry such as the Intriligator-Thomas-Izawa-Yangagida model and relate this to a IIA brane construction. We also argue how confinement due to a condensate of fractional instantons manifests itself in Super Yang-Mills theory using fractional D0 branes, D4 branes, and NS strings.
1. Introduction

There has been much progress in understanding supersymmetric gauge theories using brane constructions. Generalizing a three-dimensional brane construction of [2], the authors of [3] were able to reproduce Seiberg’s $N = 1$ supersymmetric gauge theory duality [4] using brane constructions in type IIA string theory. In [5], it was shown how to obtain the Seiberg-Witten curve describing the exact quantum Coulomb branch of $N = 2$ gauge theories. A configuration of D4-branes and NS 5-branes in type IIA string theory preserving $1/4$ supersymmetry gives the classical $N = 2$ Super Yang-Mills worldvolume field theory. Lifting the IIA configuration to eleven-dimensional M-theory, the brane construction becomes a single 5-brane and the four-dimensional supersymmetric gauge theory on the worldvolume of the D-branes decompactifies into a six dimensional self-dual tensor theory. Remarkably, the M-theory limit provides quantum information about the four-dimensional field theory on the worldvolume of the IIA branes. Results about $N = 1$ Super Yang-Mills were also obtained by considering configurations of D4-branes and NS 5-branes, preserving only $1/8$ the supersymmetry of type IIA string theory. Strings and domain walls were obtained by relating the IIA configuration a single M-theory 5-brane [3].

The topic of obtaining field theory results about $N = 1$ supersymmetric gauge theory from M-theory is known as MQCD and was pursued in a number of directions [7][8][9][10][11]. In this paper we will investigate quantum effects in worldvolume gauge theories by another route. We will construct brane configurations in IIA string theory with 4 supercharges, but instead of lifting the configuration to M-theory, we will remain in type IIA string theory, keeping the gauge theory four-dimensional. To obtain quantum results about the worldvolume gauge theory, we will introduce D0 branes into the D4-brane worldvolume as was done in [12][13]. Because a D0 brane inside a D4 brane is a Yang-Mills instanton [14][15], including D0 branes in the string theory brane constructions should be equivalent to including instanton effects in the worldvolume gauge theory. D0 branes are Kaluza-Klein modes of the compactified eleventh dimension of M-theory, and therefore our approach should in some sense be the “Fourier transform” of the MQCD approach.

In section 2, we argue that fractional instantons are responsible for gaugino condensation and the mass gap in low energy $N = 1$ $SU(N_c)$ super Yang-Mills theory by showing how fractional D0-branes induce a non-zero vacuum energy in the worldvolume theory on the branes. We go on in section 3 to examine the brane construction for supersymmetric $SU(N_c)$ QCD with $N_f$ flavors and show how D0 branes generate the Affleck-Dine-Seiberg
superpotential in the worldvolume theories lifting the degeneracy of vacua for the case of $N_f = N_c - 1$ flavors. In brane language the superpotential manifests itself as a repulsive force between the D0-D4 brane bound state and other D4-branes ending on an NS 5-brane. For the case of $N_f = N_c$ flavors, instantons can be seen to be responsible for the deformed moduli space using D0-branes in the brane constructions. In section 4, we show how instanton effects break supersymmetry in the Intriligator-Thomas model using brane diagrams: The non-zero vacuum comes from a repulsive force between D4-branes as in the superpotential case. The difference here is that the D4-branes are held together by heavier NS 5-branes and cannot run away to a zero energy configuration. Therefore, there is a non-zero vacuum energy and supersymmetry must break. In section 5, we examine the one instanton effect in $N = 2$ super Yang-Mills theories and explain why fractional instantons do not play a role in the Seiberg-Witten solution. Introducing an orientifold plane, in section 6, takes the $N = 2$ brane construction to $N = 1$ Sp($N_c$) gauge theory with an antisymmetric tensor field. The vacuum of this theory has two branches: one with a dynamically generated superpotential and one that is the same as the classical vacua. We explain the dynamically generated superpotential as occurring in the brane construction where the mass gap induced by Euclidean D0-brane charge, momentum across a surface in the eleventh dimension, does not cancel and the branch in moduli space with no superpotential as coming from the brane configuration where the mass gap induced by the D0-branes exactly cancels. In section 7, we will consider a fundamental type IIA string inside a D4 brane. Such a configuration is only BPS if the string spreads out inside the worldvolume of the D4-brane, becoming unconfined electric flux in the 4+1 super Yang-Mills. By building a string from fractional Euclidean D0 branes, we will show that the string’s lowest energy configuration no longer requires it to spread out. Free electric flux therefore becomes confined to a tube. This appears to realize confinement by instantons. We will apply this result to the brane construction of $N = 1$ SU($N_c$) super Yang-Mills and argue that fundamental strings bound to fractional Euclidean D0-branes behave like confined flux tubes.

2. Gaugino condensation in $N = 1$ super Yang-Mills.

Here we construct $N = 1$ SU($N_c$) super Yang-Mills from branes. Consider a NS 5-brane extending in directions $(x^0, x^1, x^2, x^3, x^4)$ and a NS’ 5-brane extending in directions $(x^0, x^1, x^2, x^8, x^9)$. $N_c$ D4-branes extend in the directions $(x^0, x^1, x^2, x^3, x^6)$.
and end on the NS and the NS’ 5-branes in the direction $x^6$. The coupling constant of the 3+1 $SU(k)$ $N = 1$ super Yang-Mills theory living in $(x^0, x^1, x^2, x^3)$ is

$$\frac{1}{g_3^2} = \frac{L_6 M_s}{g_s} = \frac{L_6}{R_{11}}.$$  \hspace{1cm} (2.1)

Euclidean D0 branes extending in the $x^6$ direction appear as instantons in the four-dimensional gauge theory. This was noticed in [14][15]. The action for the D4-brane worldvolume theory with D0-branes is

$$\mathcal{L} = F^{(2)} \wedge *F^{(3)} + A_{RR}^{(1)} \wedge F^{(2)} \wedge F^{(2)}$$  \hspace{1cm} (2.2)

where $A_{RR}^{(1)}$ is the Ramond-Ramond 1-form of type IIA string theory that couples to the D0 brane in ten dimensions and $F$ is the field strength of the super Yang-Mills gauge field on the D4 brane world volume. Dimensionally reducing the D4-brane to four dimensions along $L_6$, instantons are self-dual solutions of the Yang-Mills field strength,

$$F^{(2)} = *F^{(2)}.$$  \hspace{1cm} (2.3)

Using the fact that the Bianchi identity is identically satisfied $dF^{(2)} = 0$, instantons also satisfy the equations of motion of the Yang-Mills action $d * F^{(2)} = 0$. The action of the instanton is given by equation (2.1). The $1/R_{11}$ in (2.1) we expect since the D0 brane is momentum in the $x^{11}$ direction across a surface in the $x^6$ direction. If we lift the D0-D4 system to M-theory we see that the energy-momentum tensor of the $(0, 2)$ theory is given by

$$T^{6,11} = H^{\mu,\nu} H_{\mu,\nu}.$$  \hspace{1cm} (2.4)

In the far IR, the gauginos condense giving rise to a mass gap. The expectation value of the gaugino condensate goes as

$$\langle \lambda \lambda \rangle = \Lambda^3 = e^{-8\pi^2/N_c g_3^2}.$$  \hspace{1cm} (2.5)

From the exponent in (2.5), this non-perturbative effect has the interpretation as the effect of one fractional instanton. Indeed, $N = 1$ super Yang-Mills has $2N_c$ fermionic zero modes coming from the gauginos. Performing an instanton calculation using the ’t Hooft vertex, one can show that [16]

$$\langle \lambda \lambda (x_1) \lambda \lambda (x_2) ... \lambda \lambda (x_{N_c}) \rangle = \Lambda^{2N_c}.$$  \hspace{1cm} (2.6)

Cluster decomposition then implies that (2.6) decomposed into (2.5). However, instantons by themselves cannot produce (2.5), whereas fractional instantons have just the right amount of zero modes. Therefore, it is natural to expect that fractional instantons are responsible for the gaugino condensate [17].
2.1. Dimensional reduction of four-dimensional $N = 1$ super Yang-Mills to three-dimensional $N = 2$ super Yang-Mills.

To understand equation (2.5) better, let us now consider compactification of the four-dimensional $N = 1$ super Yang-Mills theory to three dimensions. In the brane construction this is done by compactifying the $x^3$ direction and T-dualizing along it yielding a IIB brane configuration. D4-branes become D3-branes and D0-branes become D1-branes. It is now possible to break the D1-branes on the D3-branes such that the D1-branes move between the D3-branes in the $x^0, x^1, x^2$ directions (see fig. 1). The points where the D1-branes meets the D3-branes looks like a monopoles in the worldvolume theory of the D3-branes. We can consider T-dualizing the D1-D3 system back to the D0-D4 system. What becomes of the broken D1-branes? We will argue here that they can be thought of as fractional D0-branes with charge $1/N_c R_{11}$ [18][19]. In the D4-brane worldvolume these are precisely the fractional instantons that we need to explain (2.5). The gauge potential for a meron, a fractional instanton on $\mathbb{R}^4$, is

$$A_{\mu} = \frac{1}{N_c} g \partial_{\mu} g^{-1}. \quad (2.7)$$

Notice that because of the $1/N_c$ out front this is not gauge equivalent to the vacuum.

For an $SU(N_c)$ gauge theory on $\mathbb{R}^4$ the $\nu$ instanton moduli space has dimension $4\nu N_c$. The dimension of instanton moduli space has the interpretation roughly as $\nu$ instantons that in addition to $4\nu$ translational zero modes have modes which correspond to scale size. However, when the instantons becomes point-like they can fractionate. The fractional instanton moduli space is then a space of $\nu N_c$ fractional instantons that can move on
\( \mathbb{R}^4 \) in four different ways. Instead of the dimension of the moduli space being associated with translations and “fatness”, the dimension of fractional instanton moduli space is only associated with translations of point-like objects, as the T-dual D1-D3 system suggests. This fits nicely with the fact that the cohomology of \( \nu \cdot SU(N_c) \) instantons on \( T^4 \) is the same as \( (T^4)^\nu N_c / S_{\nu N_c} \) implying \( \nu N_c \) points on \( T^4 \) \[20\].

In the \( N = 2 \) three-dimensional \( SU(N_c) \) brane construction, \( N_c \) D3-branes can separate in the direction \( x^3 \) breaking the \( SU(N_c) \) gauge theory to \( U(1)^{N_c} \). This corresponds to an vacuum expectation value for the real adjoint scalar field. Although globally the supersymmetry is broken the four real supercharges, in actuality locally where the D3-branes end on the NS 5-brane, eight real supercharges are preserved. Including D1-branes wrapping the \( x^3 \) direction, breaks the supersymmetry locally where the D3-brane meets the NS 5-brane to four real supercharges. Monopole-monopole interactions cancel only in theories with eight supersymmetries, and therefore the D3-branes must repel each other. If we take the \( x^3 \) direction to be compact, then the D3-brane will move around the circle to equally separated points. This was discussed in \[22\] and is consistent with field theory \[23\]. Because the D3-branes are equally distributed around the circle this constrains the D1-branes to break up into equal sized pieces. This is consistent with the fact that solutions of the Yang-Mills equations with fractional topological charge must come in units of \( 1/N_c \) rather than some arbitrary charge as can be seen in \( \ref{eq:2.7} \). The splitting of the D1-branes on the D3-branes Higgses the \( U(1)^{N_c} \) gauge group that is present on the world volume of the D3-branes. We can see that there will be two D1-D3 strings on each fractionated D1-brane. There will be two fermion zero modes coming from these strings. This is very reminiscent of \( \ref{eq:2.3} \). Similar observations have been made in \[24\].

\[ \text{2.2. Return to four-dimensional } N = 1 \text{ super Yang-Mills.} \]

In the T-dual four-dimensional brane construction, the D4-branes look like vortices in the NS 5-brane worldvolume. We expect the vortices to repel each other as do the

\begin{equation}
\exp(-S_I) = \exp(-\infty) = 0.
\end{equation}

However, this is only true in the semi-classical approximation where we expand about the centerpoint of a Gaussian integral \[21\].
monopoles in the three-dimensional case. The mass gap comes about because the D0-brane breaks the supersymmetry of the D4-branes to 1/8 at the point where the D0-D4 bound state intersects the NS’ 5-brane. Like the monopole-monopole interactions in the three-dimensional theory, the vortex-vortex interaction cancel in theories with eight supersymmetries. Therefore we expect for $N_c$ D4-branes in the presence of a D0 brane, there will be a repulsive force between the D4-branes. However, because the NS and NS’ 5-brane are holding the D4-branes in place, the D4-branes have no where to go. This force between the D4-branes produces a mass gap in the theory, consistent with field theory. It would be interesting to understand the origin of the vortex-vortex interaction from the point of view of the IIA NS 5-brane worldvolume theory.

2.3. Fractional D0-branes lifted to M-theory.

From the M-theory point of view fractional D0-branes have a very nice interpretation: Since a D0-brane in $N_c$ D4-branes is momentum in the M5-brane in the eleventh direction. If we think of the $N_c$ M5-branes as a single M5-brane wrapped $N_c$ times around the eleventh direction, then a wavefunction should be periodic in $N_c R_{11}$ rather than $R_{11}$. So rather than being a Kaluza-Klein mode with mass $\frac{1}{R_{11}}$ a fractional D0-brane is a Kaluza-Klein mode with mass $\frac{1}{N_c R_{11}}$. Related observations have been made for D-branes [25] [26] [27] [28]. This point of view also explains why the fractional D0-branes appear as point-like instantons in the D4-brane worldvolume. D0-brane on a single D4-brane have no size corresponding to the fact that the single M5-brane is trivial. A single M5-brane wound $N_c$ times also has a sector where it looks trivial, indicating that the instantons with charge $\frac{1}{N_c}$ also have no size in the $U(N_c)$ gauge theory. Of course, the multiply wound single M5-brane is really non-trivial since different parts of the M5-brane can interact with each other. This is seen in the D0-D4 system from the fact that multply fractional instantons can join together to form instantons that can have non-zero size.

3. Supersymmetric QCD.

3.1. SQCD with $N_f < N_c - 1$.

Now we consider $SU(N_c)$ gauge theory with $N_f$ flavors $Q_i$ in the fundamental representation and $N_f$ flavors $\bar{Q}_i$ in the anti-fundamental representation. There are many
reviews of this subject [4]. In the region where \( N_f < N_c \), this theory is known to have a dynamically generated superpotential that has the form

\[
W = C \left( \frac{\Lambda^{b_0}}{\det(M)} \right)^{\frac{1}{N_c - N_f}}
\]

(3.1)

where \( b_0 = 3N_c - N_f \) and \( C \) is a numerical constant. For the case, \( N_f < N_c - 1 \), we can see from field theory that fractional instantons will play a role since there are not enough zero modes for an instanton to generate equation (3.1). Expectation values for the fundamental fields \( N_f \) break the \( SU(N_c) \) gauge group to \( SU(N_c - N_f) \) super Yang-Mills. In the unbroken gauge group, we can have gaugino condensation as was explained in section 2. The low energy superpotential

\[
W_L = \Lambda^3
\]

(3.2)

is related to (3.1) by the matching relation

\[
\Lambda^{b_0 L} = \frac{\Lambda^{b_0}}{\det(M)}.
\]

(3.3)

To see SQCD in string theory, we will use the same construction as in the case for super Yang-Mills involving D4-branes suspended between NS 5-branes and NS' 5-branes, only now we will add D6-branes extending in the directions \((x^0, x^1, x^2, x^3, x^7, x^8, x^9)\). The D4-D6 strings provide fields in the fundamental representation. The D4-branes can now break along the D6-branes and move in the \( x^7, x^8, x^9 \) directions. The relative position of the pieces of D4-brane correspond to expectation values of the meson fields \( M_{i,j} = Q_i \tilde{Q}_j \). Because the s-configuration rule allows for only a single D4-brane to stretch between a D6-brane and a NS 5-brane [2], there will be always be \( N_c - N_f \) D4 branes left unbroken making it possible for a D0 brane to fractionate. Fractional D0-branes induce gaugino condensation in the worldvolume theory in the same way as discussed in section 2.

3.2. SQCD with \( N_f = N_c - 1 \).
Fig. 2: This is the 't Hooft vertex. The dashed lines represent gaugino zero modes. The dotted lines represent quark zero modes, and the solid lines represent squark expectation values. This diagram leads to the Affleck-Dine-Seiberg superpotential giving a mass to the quarks at finite squark vacuum expectation value. This is also the diagram of a D0-brane with D4 and D6 strings ending on it in the brane construction of SQCD with $N_f = N_c - 1$ flavors.

For the case, $N_f = N_c - 1$, the superpotential can be shown to be generated by instantons. An instanton in SQCD has $2N_f$ zero modes coming form the quarks, $\psi_Q$ (the fermionic components of $Q$), and $2N_c$ zero modes coming from the gauginos, $\lambda$. Using the 't Hooft vertex, we attach $2N_f$ quark “legs” and $2N_c$ gaugino legs. Because there is the gauge-invariant term in the action

$$L = ... + \phi_Q \bar{\psi}_Q \lambda$$

we can use it to tie the gaugino legs to the quark legs (see fig. 2). Here $\phi$ is the scalar part of $Q$. The correlator then roughly looks like

$$< \phi^{2N_f} \psi_Q \bar{\psi}_Q > \approx \Lambda^{b_0}.$$  \hspace{1cm} (3.5)

We see that as $\frac{\phi}{\Lambda} \to \infty$, $< \psi \psi > \to 0$. Therefore, there is a supersymmetric vacuum far out along the Higgs branch, but everywhere else there is an unstable vacuum forcing us to infinity. This is in agreement with the Witten index which predicts that SQCD has $N_c$ supersymmetric vacua.

How can we see such an instanton-generated superpotential using brane constructions? Because of the s-configuration, for $N_f = N_c - 1$, there will always be one unbroken D4-brane. Introducing a D0-brane corresponds to considering the one instanton sector of the theory, and we know from above that there should be a dynamically generated superpotential. In fact, the strings ending on the D0 brane are very reminiscent of the “legs” of
the 't Hooft vertex. There is only one unbroken D4 brane providing D0-D4 strings giving
the fermion zero modes $\psi \psi$ where as the other D0-D4 strings are stretched and contribute
a scalar vacuum expectation value $\phi^{2N_f}$. In the presence of a D0 brane, the vortex-vortex
interactions on the NS 5-brane worldvolume do not cancel and there is a repulsive force
between the D4-branes driving the D4-brane pieces off to infinity along the D6-branes in
the $x^8, x^9$ direction. This is exactly what we expect from the form of the dynamically
generated superpotential (3.1).

3.3. SQCD with $N_f = N_c$.

Now lets look at the SQCD case where $N_f = N_c$. Clearly, the superpotential term
(3.1) doesn’t make any sense since $N_c - N_f = 0$. The instanton calculation shows that there
is no dynamically generated superpotential but rather a quantum mechanical modification
to the classical constraint on the Higgs branch which is

$$\det(M) - B \tilde{B} = \Lambda^{2N_f}$$  (3.6)

We can see how instantons generate (3.6) by adding $2N_f$ quark legs and gaugino legs to
the 't Hooft vertex. Because of (3.4), we can join the two types of fermionic legs together
giving $2N_f$ scalars. This then gives (3.4).

How can we understand the quantum modified constraint (3.6) from the brane con-
figurations? Because there are equal numbers of D6 brane as there are D4-branes, all of
the D4-branes can break on the D6 branes. Therefore, generically there are no unbroken
D4-branes at the origin of moduli space where the D0 brane is. The heavy D4 strings
ending on the D0 brane correspond to $2N_f$ scalar fields with vacuum expectation values.
Because generically there is never an unbroken D4-brane, supersymmetry is generically
locally broken to eight supercharges, and therefore we expect the monopole-monopole in-
teractions to cancel except for the special situation where pieces of a broken D4-brane join
together at the origin. A D4-brane coincident with a D0-brane at the origin generates a
force driving the other broken D4-branes off to infinity. This is just the type of behavior
we expect from (3.6): As one of the meson’s expectation value becomes small, the other’s
must become large.

The D0 brane in the brane configuration also explain why chiral symmetry breaks
in this case. It is not possible for the D4-branes to become coincident at the origin.
Therefore, it is not possible for the D6-branes to split on the NS’ 5-brane enhancing the
global symmetry group from $SU(N_f)$ to $SU(N_f)_L \times SU(N_f)_R$ \[29\].

\[2\] For $U(N_c)$ theories, this would just be $\det(M) = \Lambda^{2N_f}$. 

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3.4. SQCD with $N_f > N_c$.

For SQCD with $N_f > N_c$, there is no quantum modification to the moduli space. It is in these cases that there is a dual description of the physics at the origin of moduli space. This is well described in the brane configuration by exchanging the NS' with the NS in the $x^6$ direction, a continuation past infinite coupling. Since the D4-branes are repelled from the origin by the presence of the D0-branes, this suggests that the lowest energy configuration may be the dual brane configuration.

4. The Intriligator-Thomas-Izawa-Yanagida model.

Now that we understand instanton dynamics in terms of branes, it is natural to look at another interesting example considered in [30][31]. M-theory constructions were also considered in [32]. This is SQCD where $N_f = N_c$, but now also have scalar field $S$ that couple to the fundamental fields

$$W = SQ\tilde{Q}. \quad (4.1)$$

A supersymmetric minimum demands that $Q\tilde{Q} = 0$, on the other hand we know that there is the quantum modified constraint in this case that forces det $M$ to be non-zero (3.6). Therefore, quantum mechanically, there is no supersymmetric minimum. Although this theory is a non-chiral theory, supersymmetry breaking is not violated since the fundamental flavors cannot be given a mass.

We can now consider a brane construction of this theory. We consider adding D6 branes that lie to the left of the NS’ 5-brane. The NS 5-brane lies to the right of the NS’. $N_f$ D4-branes extend between the D6-branes and the NS’ 5-brane and $N_c$ D4-branes extend between the NS’ and the NS 5-brane. An expectation value for the scalar component of the singlet field $S$ corresponds to moving the $N_f$ D4-branes in the $x^8, x^9$ directions. An expectation value for $S$ gives a mass to the fundamental fields, and we recover the SYM case. As in the field theory, the fundamentals scalars cannot acquire vacuum expectation values since breaking the D4-branes on the D6-branes would produce an s-configuration. Adding a D0 brane again induces a force between the D4-branes which would like to move away from each other in the $x^8, x^9$ direction. However, the D4-brane are constrained to lie at the origin by the configuration of D6, NS’, and NS 5-branes. Because the force between the D4-branes does not cancel, there is a non-zero vacuum energy, and supersymmetry must break.

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$^3$ Here we consider the $U(N_c)$ version of the theory which has no baryons.
5. Instantons in theories with $N = 2$ supersymmetry.

Here we consider $SU(N_c)$ with $N = 2$ supersymmetry. In $N = 1$ terminology, this is like having a single adjoint chiral field. Therefore there are $2N_c$ zero modes coming from the gauginos and $2N_c$ zero modes coming from the fermion component of the adjoint chiral multiplet. In the 't Hooft vertex, we can again join the legs of the two types of fermion zero modes together giving

$$(\det \phi)^2 = \Lambda^{2N_c}. \quad (5.1)$$

As in the SQCD case with equal numbers of colors and flavors, the origin of moduli space appears to have been removed. In the $SU(2)$ Seiberg-Witten solution there is a monopole at the point $u = \Lambda^2$ and a dyon at the point $u = -\Lambda^2$. In the dual coordinates, this is shown in the low energy effective superpotential valid near the origin

$$W_d = (u + \Lambda^2)\bar{q}q. \quad (5.2)$$

We can see in the brane diagram why this is so. The $N = 2$ brane construction has $N_c$ D4-brane suspended between two NS 5-branes. The D4-branes can move in the $x^4, x^5$ direction: This is the Coulomb branch. By putting D0-branes at the origin, we see that this case is very reminiscent of SQCD with $N_f = N_c$. If a D4 brane is placed on top of the D0-brane, it breaks more supersymmetry allowing the D4-brane to exert a force on its neighboring D4-branes, repelling them. We also see why in this case, as in the cases $N_f \geq N_c - 1$ that fractional instantons don’t play a role. There is never a point on the moduli space where unbroken D4 branes are coincident. Therefore, the D0-branes never have the opportunity to split. This explains why fractional branes never played a role in the Seiberg-Witten solution. We can also consider what happens when we put a D2-brane in the directions $x^0, x^4, x^6$. The D2-brane is a monopole in the 3+1 worldvolume theory. Classically, the mass of the monopole is given by the area of the separation between the D4-brane and NS 5-brane. According to (5.2) including the D0-brane we find that the mass of the monopole must change by an amount proportional to $\Lambda^2$. By adding momentum to the membrane along the eleventh direction we have increased the membrane mass as expected.
6. \( N = 1 \) \( Sp(N_c) \) with an antisymmetric tensor field.

An interesting model to consider is \( N = 1 \) \( Sp(N_c) \) gauge theory with an traceless antisymmetric field \( X \). Expectation values for the \( X = diag(a_1, a_2, \ldots, a_{N_c}) \) break \( Sp(N_c) \) down to \( Sp(1)^N_c \) where \( \sum_i a_i = 0 \). Because \( Sp(1) \simeq SU(2) \), there will be gaugino condensation in each of the \( Sp(1) \) giving rise to a superpotential

\[
W = \pm \Lambda_1^3 \pm \Lambda_2^3 + \cdots + \pm \Lambda_{N_c}^3
\]

(6.1)

where \( \Lambda_i \) is the dynamically generated scale of the \( i \)-th \( Sp(1) \) theory. It is possible to have all of the terms in (6.1) cancel such that \( W = 0 \). The scales \( \Lambda_i \) are related to the size of the vacuum expectation values \( a_i \) and the scale of the unbroken \( Sp(N_c) \) \( \Lambda \) by the matching relation

\[
\Lambda_i^{N_c+2} = \Lambda_k^3 \prod_{i \neq j \neq k} (a_i - a_j).
\]

(6.2)

Plugging these relations into (6.1) we find

\[
W = \Lambda_i^{N_c+2} \left( \frac{\pm (a_1 - a_2) \pm (a_2 - a_3) \cdots \pm (a_{N_c-1} - a_{N_c})}{\prod_{i \neq j} (a_i - a_j)} \right)
\]

(6.3)

Choosing the signs correctly in (6.3) we can have a moduli space of the \( Sp(N_c) \) theory where there is no superpotential [33] [34] [35]. There will be another branch where the terms in (6.3) do not cancel and there is a dynamically generated superpotential.

The brane construction for \( Sp(N_c) \) is the same as the configuration for \( N = 2 \) \( SU(N_c) \) SYM with the introduction of an orientifold 6-plane. The O6-plane projects out the symmetric part of the adjoint leaving an antisymmetric field. Motion of the D4-branes in the \( x^4, x^5 \) direction corresponds to giving a vacuum expectation value to the antisymmetric field breaking \( Sp(N_c) \) to \( Sp(1)^N_c \). Inserting D0-branes into each of the \( N_c \) D4-branes generates a mass gap as discussed in section 2 above. However it is possible to arrange all the D0-branes such that the vacuum energy in the \( Sp(N_c) \) cancels. Therefore, the effects of the D0-branes can cancel leaving a unlifted moduli space. As an example, this cancellation of D0-branes is easiest to see in the case of \( Sp(2) \) broken to \( Sp(1) \times Sp(1) \). Here length of the D0-branes in the D4-branes is the same in the \( x^6 \) direction so the \( x^{11} \) momentum through a surface \( L_6 \) is the same for each of the two D0-D4 bound state. The mass gap generated by the two D0-D4 systems is the same magnitude. We then just take a D0-brane in one of the D4-branes to be momentum in \( -x^{11} \) and the D0-brane in the other D4-brane
to be momentum in $-x^{11}$. The D0-brane effects cancel. However what about $Sp(3)$? How can a combination of three D0-branes and anti-D0-branes add to zero? The solution is that the NS 5-branes bend [5]. The bending make the Euclidean D0-branes longer in the $x^6$ direction causing the mass gap energy per unit volume to decrease. The effect of the D0-brane goes as $\Lambda^0 \simeq e^{-\frac{L_6}{R_{11}}}$. Therefore, the two D0-D4 systems with larger $L_6$ will contribute less to the mass gap energy while the D4-brane in the middle with the shortest $L_6$ contributes the most. The mass gap energies in the three vacua can cancel leaving $\mathcal{W} = 0$. The other branch of the moduli space are given when the D0-brane effects do not cancel. In the case of no cancellation, there is a repulsive force between the D4-branes pushing them to infinity in the $x^4, x^5$ direction.

7. Confinement due to Merons.

7.1. Merons

Having explored how instantons generate superpotentials and modify the classical moduli space, let’s see what effect instantons have on confinement in $N = 1$ super Yang-Mills. Although $N = 1$ super Yang-Mills is known to confine, it is not known what the mechanism is. Here we will argue that confinement of electric flux is due to point-like fractional instantons, merons. Merons have been shown to be crucial for understanding confinement in non-supersymmetric Yang-Mills [36]. By placing merons into the interior of a Wilson loop it was shown that the energy becomes proportional to the area of the loop, indicating the onset of confinement. Placing whole instantons into the Wilson loop was not enough for inducing confinement.

7.2. 4+1 SYM with 16 supercharges.

Let’s consider a D2-brane extending in the $x^0, x^1, x^2$ direction and a D4-brane extending in the $x^0, x^1, x^2, x^3, x^4$ directions. When we lift such a configuration to M-theory, the D2-brane remains a membrane and the D4-brane becomes a 5-brane. We therefore have a M2 brane inside a M5 brane. Upon returning to IIA, if the D2-brane in the D4-brane is to remain a BPS object, it must become unconfined magnetic flux, $B_{3,4}$ (assuming that $x^1, x^2$ directions are compact). We can see this from the term in the D4-brane worldvolume action [13]

$$\mathcal{L} = C^{(3)}_{R-R} \wedge F^{(2)} + \ldots$$

(7.1)
From the eleven-dimensional point of view, the M2 brane is H-flux in the M5-brane $H_{0,1,2}$ since

$$\mathcal{L} = C^{(3)} \wedge H^{(3)} + \ldots \quad (7.2)$$

where $C^{(3)}$ is the RR field that couples to the D2-brane.

By similar reasoning to the D2-D4 system, the D0-brane must become magnetic flux inside of the D2-brane. In fact, the D0-D2 system is T-dual to the D2-D4 system. Non-zero magnetic flux in the 2+1 D2-brane worldvolume theory

$$[D_1, D_2] = F_{1,2} \neq 0 \quad (7.3)$$

maps to D-term conditions on the scalar fields of the 0+1 gauge theory in the T-dual D0-brane worldvolume theory, $[X_1, X_2] = A/\nu$ where $A$ is the area of the torus and $\nu$ is the number of D0-branes $[37]$. Therefore, magnetic flux in a D2-brane is T-dual to a membrane built from D0-branes on a non-commuting torus. In the large D0-brane limit we recover commuting space-time.

Now consider D0-branes inside D4-branes. If we have $N_c > 1$ D4-branes, it is not true that D0-branes must be localized since the corresponding instantons can have size. This is the reason that merons, fractionally charged point-like instantons, are so appealing for confinement. Building a membrane out of fractional D0-branes in $N_c$ D4-branes, it follows that the membrane is a localized vortex in the 4+1 SYM.

We can also consider what happens if we have Euclidean D0-branes in the D2-brane worldvolume. Euclidean D0-branes induce electric flux along the D2-brane worldvolume $F_{0,1} \neq 0$. Following the above prescription, we T-dualize the D0-D2 configuration twice along $x^0, x^1$. We now have a membrane built from Euclidean D0-branes where $[X_0, X_1] \neq 0$ implying that space and time generically do not commute. In the large instanton limit we can recover commuting space-time. Again embedding this configuration of D0-branes into a D4-brane, we would expect the D2-brane to appear as a confined flux. It is easy to see by U-duality that we can also build a fundamental string out of Euclidean D0-branes.

A way of seeing why the lowest energy configuration of the D2-brane inside the D4-brane is spread out in the absence of D0-branes and confined in the presence of D0-branes is to consider a U-dual configuration of perpendicular D1-branes and fundamental strings. Consider non-Euclidean D0 branes in $(x^0)$, D2-branes in $(x^0, x^1, x^2)$, and D4-branes in $(x^0, x^1, x^2, x^3, x^4)$. An operation $T_1T_2ST_3$ on this configuration should bring us to a D1-D1-NS1 system where one set of $N_c$ D1-strings are extended along $x^0, x^4$, the other set
of D1 strings and NS strings are extended along $x^0, x^3$. We take $x^4$ to be the vertical direction and $x^3$ to be horizontal. In the absence of fundamental strings along $x^3$, the lowest energy configuration of the D1-branes along $x^3$ and $x^4$ is to be tilted diagonally in $x^3, x^4$. This diagonal configuration of a D1-brane is clearly BPS since it is just a D1-brane in a rotated coordinate system. Because of the tilt, when we T-dualize along either the vertical or horizontal directions, we get magnetic flux on a D2-brane [37]. If we continue to U-dualize, the tilted D1 brane can eventually be related to magnetic flux in a D4-brane. However, going back to the D1-D1 system, if we allow for NS strings along $x^3$ the D1-branes, the NS-strings and the D-strings form a bound state, and it is easy to see that the D1-branes lowest energy configuration will not be the tilted one, but the configuration of vertical D1-branes and horizontal D1-branes. U-dualizing, this configuration transforms it into a membrane of confined magnetic flux in a D4-brane. Notice however that the configuration can be BPS only when the flux has spread out. A D2 brane in a D4-brane that does not spread out is not BPS as can be seen easily from the supersymmetry relations. This is unfortunate but in agreement with the findings of MQCD [4].

7.3. Wilson loops.

![Figure 3](image)

**Fig. 3:** In the figure on the left we have a D4-branes with a fundamental IIA string inside it. The lowest energy configuration is for the string to spread out within the D4-brane. From the point of view of the D4-brane worldvolume the string looks like unconfined electric flux. In the figure on the right, we have put Euclidean D0 branes into the worldvolume of the D4-brane. Because the string ends on the Euclidean D0-brane, the flux must flow through it. One can show that the lowest energy configuration is now for the string to be confined electric flux in the D4-brane worldvolume.

A typical method for demonstrating confinement is to use a Wilson loop. The Wilson loop is created by taking a heavy quark and anti-quark, separating them a distance $y$ for a time $t$ and then having the quarks annihilate. If the energy of the loop as compared to the vacuum goes as the area, $yt$, then the theory is confining.
To create the Wilson loop in the D-branes, we can take two D4-branes separated by a distance $L$. Let us consider two strings joining the two D4-branes. They are oppositely oriented and separated a distance $y$. Typically, when a fundamental string ends on a D-brane, it appears in the worldvolume theory as a charged particle (quark) that couples to the worldvolume gauge field. Gluons are string oscillations on the brane that move away from the point at which the oriented string ends. Classically, there is a Coulomb-like force that attracts the quark and anti-quark. The attractive force will cause the strings to move together joining and then radiating away into the bulk as a closed string. The Wilson loop clearly does not obey an area law.

What happens if we include Euclidean D0-branes, instantons in the worldvolume theory. As we saw above, using point-like Euclidean D0-branes we can build a membrane or fundamental string from the D0-branes. A string built from Euclidean D0-branes is confined electric flux in the D4-brane worldvolume. The charge from the string is then passed from D0-brane to D0-brane and then to the oppositely oriented string forming a flux tube. The quarks are never seen in the worldvolume theory. They are truly confined by the instantons. There is an attractive force between the confined quarks that is linear and depends on the mass of the D0-branes. The string and D0-branes annihilate into the bulk after a time $t$.

7.4. $N=1$ SYM.

Now we return to $N=1$ SYM. At strong coupling $N=1$ SYM is known to confine. Using the brane construction described in section 2, we use Euclidean D0-branes along $x^6$ to build a IIA string along $x^0, x^1$ in the worldvolume of the D4-brane. Having the 5-branes dimensionally reduces the D4-brane in the $x^6$ direction makes the theory 3+1 dimensional, and therefore the IIA strings really have the interpretation as vortices. We can also have a string in the 3+1 worldvolume by building a D2-brane along $x^0, x^1, x^6$ from D0-branes along $x^6$. This string should be confined magnetic flux. There are also two types of instantons in 3+1, electric and magnetic. An electric instanton is the one we have been discussing which has action $\frac{L_6}{R_{11}}$ and is momentum $T^{6,11}$ in the $(0,2)$ theory. A magnetic instantons has action $\frac{R_{11}}{L_6}$ that is momentum $T^{11,6}$. It is interesting that the symmetry between electric and magnetic instantons is related to the fact that the energy momentum tensor is symmetric.

Because the theory of $N_c$ D4-branes is $SU(N_c)$ SYM with only adjoint matter we can think of it as $SU(N_c)/Z_{N_c}$ since the adjoint is invariant under the center of $SU(N_c)$. 16
\[ \pi_1(SU(N_c)/Z_{N_c}) = Z_{N_c} \] and so this theory can have \( Z_{N_c} \) types of topologically stable vortices. This is easy to see from their close relationship to monopoles. We would like to associate each of the \( N_c \) vortices with \( N_c \) fractional instantons. Once the \( N_c \) vortices join together, \( N_c \) fraction instantons have joined together also. The D0 branes can leave the D4-brane as a graviton and so can take the fundamental string with it. In this way, the \( N_c \) types of strings can annihilate.

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