TOWARD A QUANTUM THEORY OF OBSERVATION

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Abstract: The program of a physical concept of information is outlined in the framework of quantum theory. A proposal is made for how to avoid the introduction of axiomatic observables. The conventional (collapse) and the Everett interpretations of quantum theory may in principle lead to different dynamical consequences. Finally, a formal ensemble description not based on a concept of lacking information is discussed.

1. INTRODUCTION

In the conventional interpretation of quantum theory, the wave function or state vector describing a physical system evolves in time according to two different laws: (a) the Schrödinger equation, being deterministic in the sense that the wave function is determined completely if given at a certain time, and (b) the “reduction” or “collapse”, which describes the transition of the wave function into one member of a set of mutually orthogonal states, this set (defining an “observable” up to scale transformations) being phenomenologically associated with a certain type of measurement.
The second kind of time evolution has to be taken into account regardless of whether it is called “real” or “due to the increase of information”, in particular since communication is to be considered a physical process.

All attempts to derive the reduction from a Schrödinger equation for the total system including the measurement apparatus have been shown to be doomed to failure [1,2]. Derivations from a master equation seem to be more appropriate but lead to (apparently related) difficulties in deriving the master equation itself [3]. In both cases similar pseudo-arguments are often introduced by using an abstract \textit{a priori} concept of information instead of a physical one.

Everett proposed an interpretation of quantum theory that avoids the reduction of the wave function, and instead postulates the universal validity of the Schrödinger equation [4]. This means that the “other” components (which would disappear according to the reduction postulate) still “exist” after a measurement. However, they are correlated with different states of the apparatus and subsequently with different states of human observers. Therefore Everett has to assume that “we are aware” of only one of these world components, and that the world apparently “splits” into many components whenever a measurement-like process occurs.

Is it not entirely meaningless to postulate the existence of world components that are not observed? Even if the latter were true, it would at least be conventional to regard those things as “existing in reality” which are extrapolated from the observed by means of established laws. It will be demonstrated in Section 3, however, that the different world components \textit{may} interact if the total wave function obeys a Schrödinger equation – as assumed by Everett. Consequently, in spite of the linearity of the equation of motion, the time evolution of the “observed component” might show a dependence on the other components.

Proposals to avoid the reduction by considering several successive measurements as one correlation experiment [5] either leave the process of “preparation” of the initial state conceptually unexplained or have to refer to a universal wave function again. In the latter case, all measurements (in the widest sense) ever performed must be considered as one great correlation experiment (that is still going on). None of these interpretations describes the complete process of observation (including the increase of information) \textit{by means of a Schrödinger equation}. 

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2. PHYSICAL DETERMINATION OF OBSERVABLES

The Everett interpretation in itself is not complete. There are many possibilities of decomposing a wave function into components

$$\psi = \sum \phi_i \Phi^{(i)}$$

(1)

where \( \phi_i \) are orthogonal states of a considered system, while the \( \Phi^{(i)} \) are the corresponding “relative states” of the “remainder of the universe”. Everett suggested a decomposition with respect to different “memory states” without specifying them physically. In analogy to superselection rules, this proposal thus excludes a splitting into superpositions of different memory states. The phenomenological selection of memory states corresponds to the axiomatic introduction of observables in the conventional description of measurements.

An essential property of memory states is their stability with respect to external perturbations, even though they may in turn easily affect their environment [6]. (Consider pointer positions, books, or different DNS chains.) This property will cause a strong correlation of memory properties with the remainder of the world. Dynamical stability (“robustness”) is therefore suggestive as a criterion for states with respect to which the universal wave function splits (or, alternatively, collapses).

Another possibility of specifying the splitting is based on the fundamental subject-object relation of observation. Assume the total system to be conceptually cut into two subsystems, one containing the object, the other one the observer. Then there is an essentially unique decomposition of the total wave function into a single sum of orthonormal states for both subsystems (the “Schmidt canonical form”) [7],

$$\psi(t) = \sum_{i} p_i(t) \phi_i(t) \Phi_i(t)$$

(2)

For dynamical and statistical reasons, this decomposition tends to keep different memory states in different terms of the series, irrespective of the cut’s precise position. (In contrast to Bohr’s epistemology, the cut does here not have to be interpreted as one between quantum and classical concepts.)

An Everett branching in terms of dynamically stable states has the disadvantage of being conceptually approximate, while that based on the Schmidt representation depends on the position of the cut. Nonetheless, the second
version will be preferred in the following. (One knows from the pragmatic rules of quantum theory that the cut has to be put “far enough” away from the object, that is, close enough to the observer [8]. Therefore it may be chosen to have on the observer’s side only the physical carrier of his consciousness – perhaps the cerebral cortex or parts thereof – thus keeping even his personal memory in the “outside world”. If the cut were placed somewhere in between object and observer, the Schmidt decomposition would be too fine-grained. A reduction to one of the components of (2) would then describe a much greater increase of information than that according to the axiomatic observables.)

3. CAN THE EVERETT INTERPRETATION BE VERIFIED?

Provided the wave function $\psi(t)$ of some total system exists and obeys a Schrödinger equation, the coefficients and components of expansion (2) vary in time according to the nonlinear set of equations [9]

$$i\hbar \frac{d}{dt} \sqrt{p_i} = \text{Im} \langle \phi_i \Phi_i | H | \psi \rangle$$

$$i\hbar \frac{d\Phi_i}{dt} = \sum_{j \neq i} \frac{\sqrt{p_i} \langle \phi_i \Phi_j | H | \psi \rangle - \sqrt{p_j} \langle \psi | H | \phi_j \Phi_i \rangle}{p_i - p_j} \Phi_j$$

$$i\hbar \frac{d\phi_i}{dt} = \sum_{j \neq i} \frac{\sqrt{p_i} \langle \phi_j \Phi_i | H | \psi \rangle - \sqrt{p_j} \langle \psi | H | \phi_i \Phi_j \rangle}{p_i - p_j} \phi_j$$

$$+ \sqrt{p_i} \text{Re} \langle \phi_i \Phi_i | H | \psi \rangle \phi_i$$

(Note added: the asymmetry between the second and third equation can be avoided by choosing a more appropriate, symmetric phase convention for the factor states of (2) – see Ph. Pearle, Int. J. Theor. Phys. 18, 489 (1979).)

The “probability resonances” $1/(p_i - p_j)$ appearing in (3) drop out from the time derivatives of the reduced density matrices,

$$i\hbar \frac{d\rho_{\Phi}}{dt} := i\hbar \frac{d}{dt} \sum_{i,j} p_i \phi_i \Phi_j^* \Phi_i$$

$$= \sum_{i,j} (\sqrt{p_i} \langle \phi_i \Phi_j | H | \psi \rangle - \sqrt{p_j} \langle \psi | H | \phi_j \Phi_i \rangle) \Phi_i \Phi_j^*$$

and correspondingly for $\rho_{\phi} = \sum_i p_i \phi_i \phi_i^*$. Only for vanishing interaction ($H = H_{\phi} + H_{\Phi}$) or when the wave function happens to factorize ($p_i = \delta_{ii_0}$)
does one recover the Schrödinger equation, for example
\[ i\hbar \frac{d\phi_{i\alpha}}{dt} = \sum_{j} \langle \phi_j | H_{\phi}^{(i\alpha)} | \phi_{i\alpha} \rangle \phi_j , \]  
where the effective $i$-dependent Hamiltonian $H_{\phi}^{(i)} = \langle \Phi_i | H | \Phi_i \rangle$ in our phase convention includes the $\Phi_i$-energy (but see Pearle, l.c.). And only for special initial conditions is (4) approximated by an autonomous master equation for the density matrix.

Equations (3) lead to a number of phenomena that are not present in the collapse interpretation. For example, time-reversed branchings (recombinations) may occur. If the other branches are not known, recombinations have to be interpreted as irreproducible events.

It is particularly interesting to study dynamical effects relating states of different “memory”. Memory states $\phi_i$ are here defined as dynamically stable ones in the presence of a “normal” environment (consider the chiral states of a sugar molecule as a simple example). This means
\[ \langle \phi_i \Phi_j | H | \phi_{i'} \Phi_{j'} \rangle \approx \delta_{ii'} \langle \Phi_j | H_{\phi}^{(i)} | \Phi_{j'} \rangle \]  
for all relevant environmental states $\Phi_j$.

Whenever an element of the Schmidt form coincides with one of these stable states, the time derivative of the corresponding coefficient vanishes,
\[ \hbar \frac{d\sqrt{p_i}}{dt} \approx \sqrt{p_i} \text{Im} \langle \phi_i \Phi_i | H | \phi_i \Phi_i \rangle = 0 , \]  
so this component keeps its probability. If the non-negligible other elements are stable, too, their “relative states” $\Phi_i$ can be shown by some rearrangement of the second equation (3) to vary according to
\[ i\hbar \frac{d\Phi_i}{dt} = \sum_{j(\neq i)} \left[ \langle \Phi_j | H_{\phi}^{(i)} | \Phi_i \rangle - \frac{p_j}{p_i - p_j} \langle \Phi_j | H_{\phi}^{(j)} - H_{\phi}^{(i)} | \Phi_i \rangle \right] \Phi_j , \]  
while even the “stable” state itself is bound to change non-trivially in this representation:
\[ i\hbar \frac{d\phi_i}{dt} = \sum_{j(\neq i)} \frac{\sqrt{p_i p_j}}{p_i - p_j} \langle \Phi_i | (H_{\phi}^{(i)} - H_{\phi}^{(j)}) | \Phi_j \rangle \phi_j + p_i \langle \Phi_i | H_{\phi}^{(i)} | \Phi_i \rangle \phi_i . \]  
If $\Phi$ describes a macroscopic system, while $H_{\phi}^{(i)}$ and $H_{\phi}^{(j)}$ differ sufficiently, $\Phi_i$ and $\Phi_j$ must soon differ in many degrees of freedom, and the (partial)
matrix elements $\langle \Phi_i | H | \Phi_j \rangle$ become extremely small for $i \neq j$. The Schmidt states then tend to stick to the dynamically stable states [3].

If a stable state $\phi_i$ occurs as a pure state, $p_i = 1$, its “rate of deseparation” (or entanglement rate – see Eq. (12a) of Ref. 9),

$$A = \sum_{j(\neq i)j'(\neq i)} |\langle \phi_j \Phi_{j'} | H | \phi_i \Phi_i \rangle|^2,$$

vanishes. This quantity $A$ measures the entanglement rate of initially factorizing systems in second order of time. If the initial state happens to be a superposition of stable states, e.g. $\psi = (\phi_i + \phi_j)\Phi/\sqrt{2}$, $A$ can be written in the form $||P_\perp (H^{(i)}_\Phi - H^{(j)}_\Phi) \Phi||^2/2$, where $P_\perp := 1 - |\Phi\rangle\langle\Phi|$ is the projector on to the complement space of $\Phi$.

The resonance terms of (8) and (9) in principle distinguish the Everett version dynamically from the collapse version of quantum theory. If two coefficients $p_i$ of the Schmidt decomposition seem to intersect in time, this leads to a behavior similar to that known from two crossing energy levels when an external parameter is changed. In particular, the coefficients of two world components do not cross unless their Hamiltonian matrix element vanishes exactly. Instead they repel each other, thereby exchanging their states. This appears to be a very drastic effect. However, since the states also exchange their memory, any observer existing in one of the components will “believe” to be the causal successor of an observer in the other Schmidt branch. The event would be felt only during the (in general extremely short) time of resonance when superpositions of memory states may form the Schmidt representation. It is again irreproducible unless the “other” components were distributed in a regular way.

One should keep in mind that the dynamical equations (3) are based on the assumption of a time-independent cut. This may not be entirely realistic for the present purpose.

The resonance terms cannot occur in the collapse version. The latter requires nonlinear and time direction-dependent terms in the equation of motion [10]. In Everett’s interpretation, the absence, in reality, of recombinations (or, more generally, the approximately autonomous dynamics of different world components) can be derived from the assumption of sufficiently small amplitudes for most “other” components, that is, from a statistically improbable cosmological initial condition.
4. INFORMATION CONCEPT IN STATISTICAL PHYSICS

The $\Phi$-system, say, may be further divided into two subsystems. Its Schmidt states may then again be expanded in analogy to (2):

$$\Phi_i = \sum_{\alpha} \sqrt{q_{i\alpha}} \chi_{i\alpha}^{(1)} \chi_{i\alpha}^{(2)}.$$  \hspace{1cm} (11)

For microscopic systems, no further reduction or branching would in general be meaningful with respect to this decomposition, as is well known from many kinds of Einstein-Podolski-Rosen phenomena [11].

Now assume that the states $\chi_{i\alpha}^{(1)}$ contain some dynamically stable degrees of freedom, labeled by an index $n$,

$$\chi_{i\alpha}^{(1)} = \sum_{nm} c_{nm}^{(i\alpha)} \chi_{nm}^{(1)},$$  \hspace{1cm} (12)

where $m$ describes all other degrees of freedom. Assume further that the memory properties $n$ do not only affect the remainder of the world, $\chi^{(2)}$, but may also allow their “observation” (by means of a von Neumann-type interaction) by the $\phi$-system (now regarded as the observer),

$$\phi_i \chi_{nm}^{(1)} \chi_{i\alpha}^{(2)} \rightarrow \phi_{in} \chi_{nm}^{(1)} \chi_{ian}^{(2)},$$  \hspace{1cm} (13)

with different final states of the observer, $\langle \phi_{in} | \phi_{in'} \rangle \approx \delta_{nn'}$. If, as a consequence of the “cosmological assumption” mentioned at the end of Section 3, the Everett decomposition with respect to $i$ is not affected by the further decomposition with respect to $n$ (that is, $\langle \phi_{in} | \phi_{i'n'} \rangle \approx \delta_{nn'} \delta_{ii'}$), the new Schmidt decomposition becomes

$$\psi = \sum_{in} \phi_{in} \Phi_{in},$$  \hspace{1cm} (14)

with not yet normalized though orthogonal relative states

$$\Phi_{in} = \sqrt{p_i} \sum_{am} \sqrt{q_{ia} q_{ia'} c_{nm}^{(i\alpha)} c_{nm}^{(i\alpha')}} \chi_{ian}^{(2)}.$$  \hspace{1cm} (15)

(Note added: Here one assumes that the observer states possess a sufficient memory capacity, that is, low entropy.) The norm of these relative states is

$$\langle \Phi_{in} | \Phi_{in} \rangle = p_i \sum_{aa'm} \sqrt{q_{ia} q_{ia'} c_{nm}^{(i\alpha)} c_{nm}^{(i\alpha')}} \langle \chi_{ian}^{(2)} | \chi_{ian}^{(2)} \rangle \approx p_i \sum_{am} q_{ia} |c_{nm}^{(i\alpha)}|^2.$$  \hspace{1cm} (16)
when assuming random phases. The Schmidt representation now includes the memory states $n$,

$$\psi \approx \sum_{m} p_{n} \sum_{m} q_{i\alpha} |c_{nm}^{(i\alpha)}|^{2} \phi_{i\alpha} \Phi_{in}, \quad (17)$$

where normalized $\Phi_{in}$ are used instead of the relative states $\Phi^{(in)}$.

If the components of the Everett decomposition are weighted by their squared norms [6], the branching ratio between $n$ and $n'$ is

$$\frac{\sum_{\alpha m} q_{i\alpha} |c_{nm}^{(i\alpha)}|^{2}}{\sum_{\alpha m} q_{i\alpha} |c_{n'm}^{(i\alpha)}|^{2}} = \frac{\text{tr}[P_{n}\rho^{(1)}_{i}]}{\text{tr}[P_{n'}\rho^{(1)}_{i}]} \quad , \quad (18)$$

with

$$P_{n} = \sum_{m} \langle \chi_{n m}^{(1)} | \chi_{nm}^{(1)} \rangle \quad \text{and} \quad \rho^{(1)}_{i} = \sum_{\alpha} q_{i\alpha} |\chi_{i\alpha}^{(1)} \rangle \langle \chi_{i\alpha}^{(1)} | \quad . \quad (19)$$

This is equivalent to averaging over an ensemble of pure states $\chi_{i\alpha}^{(1)}$ with weights $q_{i\alpha}$ ($i$ fixed), defining in general a large value of the entropy $S^{(1)}_{i} = -\sum_{\alpha} q_{i\alpha} \ln q_{i\alpha}$. A quantum theory of observation along the lines suggested here therefore leads to (fictitious) ensembles without introducing an a priori concept of information [3,12], in contrast to the Gibbs-Jaynes concept, for example [13]. In the language of d’Espagnat [14], there are no proper mixtures. The constraints describing known expectation values, chosen ad hoc and taken into account by means of Lagrangian multipliers by Jaynes, are here determined physically, controlled by questions of dynamical stability.

The major problem of classical statistical mechanics – How can ensembles be justified? – is thereby reversed: How can microscopic systems be prepared in pure states? This is obviously achieved by correlating the states of a microscopic system completely with the observer system $\phi$ and taking into account the Everett branching (or the reduction). This process describes what in common language is called an increase of information. In practice, it is performed by correlating the micro-system with appropriate memory states of an intervening system (a measuring device). The stability of memory states leads to an “objectivization” of the corresponding measurement results. Microscopic systems are usually reduced to their energy eigenstates by means of the interactions with the electromagnetic quantum field in an expanding and absorbing universe.

Although we are led to describe physical systems by apparent ensembles of states, the special role played by canonical ensembles is not immediately
obvious. In particular, since the $\chi^1$-system, say, is dynamically described by an equation analogous to (4), there is in general no Hamiltonian (not even a time-dependent one) with respect to which the canonical ensemble could be defined. On the other hand, the non-existence of information beyond the mean energy about systems containing no memory states is plausible from dynamical considerations corresponding to Poincare’s theorem concerning the non-analyticity of the constants of motion. The non-separability of quantum systems may then be regarded as a quantum mechanical analog of this classical theorem.

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