Anomaly Mediation and Radius Stabilization by a Boundary Constant Superpotential in a Warped Space

Nobuhito Maru\(^*\), Norisuke Sakai\(^+\) and Nobuhiro Uekusa\(^**\)

\(^*\)Department of Physics, Kobe University, Kobe 657-8501, Japan
\(^+\)Department of Mathematics, Tokyo Woman’s Christian University, Tokyo 167-8585, Japan
\(^**\)Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan

Abstract. We present a very simple model of the radius stabilization in a supersymmetric (SUSY) Randall-Sundrum model with a hypermultiplet and a boundary constant superpotential. A wide range of parameters where the anomaly mediation of SUSY breaking is dominated is found although there are many problematic bulk effects of SUSY breaking. A negative cosmological constant in the radius stabilized vacuum can be cancelled by a localized SUSY breaking. Making use of this localized SUSY breaking also solves the \(\mu\)-problem by Giudice-Masiero mechanism.

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INTRODUCTION

It is well known that the main motivation of introducing extra dimensions is to solve the gauge hierarchy problem. However, there is an alternative motivation in SUSY breaking model building, namely the solution to SUSY flavor problem. In the gravity mediation of four dimensions, we must introduce additional symmetries. Then, the dominant sfermion mass is generated by \(\mu\)-problem interactions for sfermion masses are forbidden by local symmetries. To suppress these interactions in four dimensions, we must introduce additional symmetries.

On the other hand, in higher dimensional case, if the hidden sector and MSSM sector are separated along the extra dimensions, the above mentioned contact interactions for sfermion masses are forbidden by locality. Then, the dominant sfermion mass is generated by anomaly mediation without SUSY flavor problem.

However, this is not the end of the story. The radius of the compactified dimensions must be stabilized. Although the nontrivial radion potential is generated once SUSY is broken, it seems not to be stabilized by only the gravity multiplet in the bulk. Thus, we must introduce additional bulk fields to stabilize the radius. In that situation, we have to check whether these additional bulk fields does not generate flavor violating sfermion masses.

In this talk, we present a very simple model of the radius stabilization and anomaly mediation dominated SUSY breaking.

MODEL

We consider a five-dimensional SUSY model on the Randall-Sundrum background, whose metric is

\[
\begin{align*}
ds^2 = & \ e^{-2\kappa y} \eta_{\mu\nu} dx^\mu dx^\nu + R^2 dy^2, \quad \sigma(y) \equiv k|y|, \\
& \text{where } \eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1). \quad R \text{ is the radius of } S^1, \quad k \text{ is the AdS}_5 \text{ curvature scale, and} \\
& \text{the angle of } S^1 \text{ is denoted by } y \ (0 \leq y \leq \pi).
\end{align*}
\]

As a minimal model to break SUSY and to stabilize the radius, we introduce a single hypermultiplet. In terms of superfields for four manifest SUSY, the single hypermultiplet is represented by chiral supermultiplets \(\Phi, \bar{\Phi}\), and our Lagrangian reads

\[
\mathcal{L}_5 = \int d^4 \theta \frac{1}{2} \phi^+ \phi (T + T^\dagger) e^{-(T + T^\dagger)\sigma} \\
\times (\Phi^\dagger \Phi + \Phi^{\dagger} \bar{\Phi}^{\dagger} - 6M_5^2) \\
+ \int d^2 \theta \left[ \varphi^3 e^{-\frac{3}{2} \sigma} \left( \Phi^\dagger \partial_y \left( \frac{3}{2} - c \right) T \sigma \right) \Phi \\
+ W_{\phi} \right] + \text{h.c.}.
\]

where the compensator chiral supermultiplet \(\phi\) (of supergravity), and the radion chiral supermultiplet \(T\) are denoted as \(\varphi = 1 + \theta^2 F_{\varphi}\) and \(T = R + \theta^2 F_T\), respectively.

The \(\mathbf{Z}_2\) parity is assigned to be even (odd) for \(\Phi(\Phi^\dagger)\). The derivative with respect to \(y\) is denoted by \(\partial_y\), such as \(\sigma^y \equiv \partial_y \sigma/dy\). The five-(four-) dimensional Planck mass is denoted as \(M_5\) (\(M_4\)). A bulk mass parameter for the hypermultiplet is denoted as \(c\).

Here we consider a model with a constant (field independent) superpotential localized at the fixed point \(y = 0\)

\[
W_{\phi} \equiv 2M_5^2 w_0 \delta(y),
\]
where $w_0$ is a dimensionless constant.

**Radius stabilization**

The background solutions of equations of motion for the hyperscalars at the leading order of $w_0$ are given by

$$
\phi(y) = N_2 \exp \left( \frac{3}{2} - c \right) R \sigma \right),
$$

$$
\phi'(y) = \hat{\varepsilon}(y) \left( \frac{\phi \dot{\phi}}{6M_5^2} - 1 \right)^{-1} \left( \frac{\dot{\phi} \phi}{6M_5^2} \right)^{5/2 - c} \times \left[ c_1 + c_2 \left( \frac{\phi \dot{\phi}}{6M_5^2} - \frac{1}{2} \right) \left( \frac{\phi \dot{\phi}}{6M_5^2} + \frac{2}{1 - 2c} \right) \right],
$$

(5)

where $c \neq 1/2, 3/2$, and $\varepsilon(y)$ is a sign function of $y$. The solution has three complex integration constants: $c_1, c_2$ are the coefficients of two independent solutions for $\phi'$, and the overall complex constant $N_2$ for the flat direction $\phi$. Two of these three complex integration constants are determined by the boundary conditions. The single remaining constant (which we choose as $N_2$) is determined by the potential minimization.

With the backgrounds (4) and (5), the potential is obtained as [1]

$$
V = \frac{3M_4^2KW_0^2}{2} \times \left\{ \frac{-2(1 - 2c)N + 2 - (3 - 2c)}{(1 - 2c)(e^{2R\pi} - 1) + 2(3 - 2c)} \left( \frac{3}{1 - N} \right) \right\},
$$

(6)

where a dimensionless quantity is defined as $\hat{N} \equiv |N_2|^2/(6M_5^2)$. We need to require the stationary condition for both modes $R$ and $N_2$, namely $\partial V/\partial R = 0$ and $\partial V/\partial \hat{N} = 0$. From these stationary conditions, we find that there is a unique nontrivial minimum with finite values of the radius $R$ and of the normalization $N_2$ for the flat direction $\phi$ provided $c < c_{cr} \equiv 17/12$. To examine the stabilization in more detail, we parameterize $c = c_{cr} - \Delta c$ with a small $\Delta c$. After using the stationary condition solution $\hat{N} = e^{-3(2 - c)/2\Delta c}$, we find that the potential (6) shown in Fig. 1 consists of two pieces at the leading order of $\Delta c \equiv c_{cr} - c$ and $\hat{N}

$$
V \approx \frac{3M_4^2KW_0^2}{2}(V_1 + V_2),
$$

$$
V_1 = \frac{2(2c_{cr} - 1)}{3 - 2c_{cr}} \hat{N} \left( \frac{e^{2\Delta c/3} + 2\Delta c}{2\Delta c} \right),
$$

(8)

The stationary point at the leading order of $\Delta c$ is obtained as

$$
R = \frac{1}{10k} \left( \ln \frac{1}{\Delta c} - 3.4 \right),
$$

(10)

which means that the radius is stabilized with the size of $R > 1/k$ for $\Delta c \ll 10^{-6}$.

The masses of radion and moduli field $N_2$ are found by diagonalizing the kinetic term and mass squared matrix,

$$
m_{\text{radion}}^2 \approx k^2 w_0^2(3.4 + \ln \Delta c)^2 (\Delta c)^{1.7}, \quad m_{\text{moduli}}^2 \approx k^2 w_0^2(0.47) (\Delta c)^{0.70}
$$

(11)

(12)

which are estimated to be

$$
m_{\text{radion}} \approx 1 \, \text{TeV}, \quad m_{\text{moduli}} \approx 100 \, \text{TeV}
$$

(13)

for $kw_0 \approx 10^7$ GeV and $\Delta c \approx 10^{-6}$.

At the stationary point the potential becomes

$$
V \approx -10^{3(\hat{N}w_0^2(2\Delta c)^{1.2} - (10^{10} \, \text{GeV})^4}
$$

(14)

If we add a spurion supermultiplet $X = F_X \theta^2$ localized at $y = 0$ with the Lagrangian

$$
\mathcal{L}_X = \left[ \int d^4 \theta |\phi|^2 X^\dagger X + \left( \int d^2 \theta \phi^3 m^2 X + \text{h.c.} \right) \right] \delta(y),
$$

(15)

the cosmological constant can be cancelled by an $F$ term contribution

$$
\sqrt{F_X} \approx 10^{10} \, \text{GeV}.
$$

(16)

We comment that this localized $F$-term SUSY breaking can be utilized for solving the $\mu$-problem by Giudice-Masiero mechanism [3]. If two Higgs superfields $H_u, H_d$ in the MSSM are assumed to be localized at $y = 0$, the following Kähler terms are allowed

$$
K = \int d^4 \theta |\phi|^2 \left[ X^\dagger H_u H_u + \frac{X^\dagger X}{M_4^2} H_u H_u + \text{h.c.} \right] \delta(y).
$$

(17)
As in the case of cancellation of the cosmological constant, the VEV of the scalar component for the chiral multiplet $X$ is assumed to be zero. This ensures that equations of motion for auxiliary fields are unchanged. Namely, our successful stabilization mechanism is not affected by addition of (17).

After SUSY breaking, the correct order of $\mu$-term and $B\mu$-term are generated from the first and the second terms, respectively $[1]$

$$\mu^2 \sim B\mu \sim \left(\frac{F_X}{M_4}\right)^2 \sim (100 \text{ GeV})^2 \tag{18}$$

where $\sqrt{F_X} \sim 10^{10}$ GeV, which is required for canceling the cosmological constant, is used. It is very interesting that canceling the cosmological constant and the solution to the $\mu$-problem are realized simultaneously by the same origin of SUSY breaking effect.

### SUSY breaking mass spectrum

We assume that the MSSM fields are localized at $y = \pi$. In our model, the anomaly mediated scalar mass becomes

$$\tilde{m}_{\text{AMSB}} \sim \frac{g^2}{16\pi^2} (F_\phi - F_\sigma) \bigg|_{y=\pi} \sim \mathcal{O}(10^{-4}) \times g^2 k w_0 \sim 100 \text{ GeV} \tag{19}$$

where $g$ is gauge coupling constant for visible sector fields and $g^2 k w_0 \sim 10^6$ GeV is used to obtain the last expression. We can show that soft masses mediated by Kaluza-Klein modes in our model are smaller than those by the anomaly mediation $[2]$. The brane-to-brane mediation of $F_X$ by a bulk gravity (a hypermultiplet), which is tachyonic (flavor dependent), is suppressed enough for $\sqrt{F_X} < 10^{11}$ GeV comparing to the anomaly mediation $[2]$. Therefore our model passes the flavor-changing neutral current constraints.

For gaugino mass, the anomaly mediation is also dominant as long as additional interactions with SUSY breaking gauge singlets are not included in the visible sector gauge kinetic terms.

Finally, the gravitino mass is obtained by solving the equation of motion in the presence of the constant superpotential $w_0$,

$$m_{3/2} \sim 6w_0 k \sim 10^7 \text{ GeV} \tag{20}$$

which is a relatively large gravitino mass specific to the SUSY Randall-Sundrum model $[1]$.

### CONCLUSIONS

We have presented a very simple model of the radius stabilization and anomaly mediation in SUSY Randall-Sundrum model with a massive hypermultiplet and a boundary constant superpotential. We found a range of parameters where other dangerous bulk SUSY breaking mediation effects are suppressed to avoid the SUSY flavor problem. It is interesting that cancellation of the cosmological constant and solving the $\mu$-problem by Giudice-Masiero mechanism are simultaneously realized by the same localized $F$-term SUSY breaking.

A negative slepton problem is still remained unsolved. We are now trying to solve this problem without spoiling our stabilization mechanism $[3]$.

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### REFERENCES

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1 The coefficients of each term in (17) are assumed to be an order unity.