Omnidirectional Klein-like tunneling of sound

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Klein tunneling is a counterintuitive quantum-mechanical phenomenon, predicting perfect transmission of relativistic particles through higher energy barriers. This phenomenon was shown to be supported at normal incidence in graphene due to pseudospin conservation. Here I show that Klein tunneling analogue can occur in classical systems, and remarkably, not relying on the specific attributes of graphene’s spinor wavefunction structure. Instead, the mechanism requires a particular form of constitutive parameters of the penetrated medium, yielding transmission properties identical to their quantum graphene counterpart. I demonstrate this result via tunneling of sound in a two-dimensional acoustic metamaterial. More strikingly, I show that by introducing a certain form of anisotropy, the tunneling can be made unimpeded for any incidence angle, while keeping its original Klein dispersion properties. This phenomenon may be denoted by the omnidirectional Klein-like tunneling, and is reported for the first time. The new tunneling mechanism and its omnidirectional variant may pave the way to a lossless and direction-independent transmission of sound and other waves.

Quantum tunneling described by the Klein paradox [1] is a phenomenon, in which Dirac relativistic particles cross a barrier of energy $V_0$, higher than the particle’s energy $E$, Fig. 1a. The fact that this crossing is unimpeded, i.e. has a unity transmission probability, is counterintuitive, as one would expect the transmission probability to decay with an increasing barrier height, as in the non-relativistic scenario. A similar effect was predicted [2], observed [3,4], and analyzed [5,6] for Dirac electrons in graphene between two domains that differ by an electrostatic potential $V_0$. The underlying principle for tunneling in graphene was shown to originate from the two-component structure of its wavefunctions, which resembles Dirac spinors, and features Dirac-like cone dispersion, fig. 1b. At the transmission to the higher potential side the electron is shifted to the lower band, keeping its velocity direction but flipping its momentum $k$. In Figs. 1c,d, the domains with $V(x) = 0$ at $-\infty < x < 0$, and with $V(x) = V_0$ at $0 < x < \infty$ are labeled by 1 and 2, respectively. For a wave $\psi_I$ incident from domain 1, and a wave $\psi_T$ transmitted to domain 2, Fig. 1c, the incidence and transmission angles, $\phi$ and $\theta$ constitute the phases between the two components of the respective wavefunctions, and are associated with sublattice pseudospin. For $\phi > 0$ a reflected wave $\psi_R$ exists. Momentum equity in the $y$ direction in both domains, together with wavefunction continuity at the domain’s interface, referring to pseudospin conservation [5], yields

$$\sin \phi = \frac{E - V_0}{E} \sin \theta, \quad (1a)$$

$$\begin{cases} 1 + R = T, \\ (1 - R) \cos \phi = T \cos \theta, \end{cases} \quad (1b)$$

where $R$ and $T$ are reflection and transmission amplitudes. The relation in (1a) implies that for a given $\phi$, the relative ‘heights’ of $E$ and $V_0$ are translated to the sign of $\theta$ (and
the momentum in domain 2), which is positive for \( E > V_0 \) and negative for the tunneling case \( E < V_0 \). The tunneling effect is manifested in (1b) at normal incidence (\( \phi = 0 \)), where the transmission becomes unimpeded irrespective of the relative values of \( E \) and \( V_0 \), implying \( R = 0 \) and \( T = 1 \), as depicted in Fig. 1d for \( E < V_0 \). The exotic properties of Klein tunneling inspired the search for analogies in other systems, including photonic \([7,8]\) and phononic systems \([9]\), but were exclusively based on mimicking graphene.

Here I demonstrate that a tunneling effect with properties identical to (1a)-(1b), can occur in inherently classical systems without a restriction to the particular graphene’s wavefunction and dispersion structure. To this end I consider the system in Fig. 1c, d to represent a continuous two-dimensional acoustic medium defined by a scalar pressure field \( p(x,y,t) \) and a vector flow velocity field \( \mathbf{v}(x,y,t) \). Domain 1 is a uniform acoustic medium of mass density \( m_0 \) and bulk modulus \( b_0 \). Domain 2 is a complex medium, described by dynamical mass density \( m_0 \tilde{M}(\omega) \) and bulk modulus \( b_0 \tilde{B}(\omega) \). \( \omega \) is the sound wave frequency. The constitutive parameters \( \tilde{M}(\omega) \) and \( \tilde{B}(\omega) \) play a crucial role in reproducing Klein-like tunneling in this system. For a time-harmonic dependence \( p_j(x,y,t) = \tilde{p}_j(x,y)e^{-i\omega t} \) and \( \mathbf{v}_j(x,y,t) = \mathbf{V}_j(x,y)e^{-i\omega t} \), \( j \) = 1, 2 indicating domain number, this system is governed by

\[
\nabla \tilde{p}_j(x,y) = i\omega m_0 \tilde{M}_j(\omega) \mathbf{V}_j(x,y),
\]

\[
\nabla \times \mathbf{v}_j(x,y) = i\omega b_0 \tilde{B}_j(\omega) \nabla \times \mathbf{V}_j(x,y).
\]

(2a)

(2b)

Here \( \tilde{M}_1(\omega) = \tilde{B}_1(\omega) = 1 \), while \( \tilde{M}_2(\omega) = \tilde{M}(\omega) \) and \( \tilde{B}_2(\omega) = \tilde{B}(\omega) \). I now consider a pressure wave of amplitude \( P_l \) incident from domain 1 at angle \( \phi \), a reflected wave \( P_r \), and a wave \( P_t \) transmitted to domain 2 at angle \( \theta \), which respectively stand for \( \psi_l, \psi_r \) and \( \psi_t \) in Fig. 1c. Employing horizontal stratification, continuity of pressure and of normal flow velocity along the domain’s interface \([10]\), gives

\[
\sin \phi = (\tilde{M}(\omega) \tilde{B}^{-1}(\omega))^{1/2} \sin \theta,
\]

\[
\begin{align*}
1 + R &= T, \\
(1 - R) \cos \phi &= (\tilde{M}(\omega) \tilde{B}(\omega))^{-1/2} T \cos \theta.
\end{align*}
\]

(3a)

(3b)

These classical concepts of wave propagation between media, Snell’s law of refraction in (3a), and Fresnel’s reflection and transmission coefficients \( R \) and \( T \) in (3b), are strikingly similar to the quantum tunneling properties \((1a) \) and \((1b) \). Using the analogy \( E \leftrightarrow \omega^2 \) due to classical dynamics in (2a)-(2b) being second order versus the first order quantum dynamics, matching (3a)-(3b) to \((1a)-(1b) \) gives

\[
(\tilde{M}(\omega) \tilde{B}^{-1}(\omega))^{1/2} = \frac{\omega^2 - V_0}{\omega^2},
\]

\[
(\tilde{M}(\omega) \tilde{B}(\omega))^{1/2} = 1.
\]

(4a)

(4b)

This determines the mass density and bulk modulus as

\[
\tilde{M}(\omega) = \frac{\omega^2 - V_0}{\omega^2}, \quad \tilde{B}(\omega) = \frac{\omega^2}{\omega^2 - V_0}.
\]

(5)

The particular combination of the parameters in (5) creates acoustic tunneling with properties identical to the tunneling of electrons in graphene, although the underlying continuous fields physics in (2a)-(2b) is fundamentally different from the quantum Dirac physics. In (4a), \( (\tilde{M}(\omega) \tilde{B}^{-1}(\omega))^{1/2} = k_2/k_1 \), with \( k_1 = \omega/c, c = (b_0/m_0)^{1/2} \), is the ratio of domain 1 and 2 wavenumbers. In (4b), \( (\tilde{M}(\omega) \tilde{B}(\omega))^{1/2} = z_2/z_1 \), with \( z_1 = (m_0 b_0)^{1/2} \), is the respective ratio of the specific acoustic impedance \([11]\). (4b) indicates that the impedance of domains 2 and 1 is matched all frequencies, implying that a normally-incident acoustic wave for any \( \omega^2/V_0 \) (and an obliquely-incident wave for the particular case \( \omega^2/V_0 = \frac{1}{2} \)), will penetrate through domain 2 completely free of backscattering, \( R = 0 \), similarly to the quantum tunneling. For all other values of \( \omega^2/V_0 \), at oblique incidence, \( R \neq 0 \). However, impedance matching alone is not enough for the analogy; a particular dispersion, dictated by (4a), is required in domain 2. Here, \( k_2, \tilde{M}(\omega) \) and \( \tilde{B}(\omega) \) in (5), are positive for \( \omega^2 > V_0 \), and negative for \( \omega^2 < V_0 \).

The notion of a negative wavenumber, resulting from simultaneously negative constitutive parameters, is a celebrated concept in the research of wave propagation in anisotropic, or metamaterials, in electromagnetic and acoustic engineering \([12–15]\). It indicates an antiparallel phase and group velocity, leading to extraordinary phenomena, unavailable in natural materials. In fact, the expressions in (5) coincide with the matched version of the so-called Drude model \([12]\) in the electromagnetic terminology, and also referred to as left-handed electric networks \([13]\).

Here, I showed that (5) constitutes an exact classical analogue of Klein tunneling, and now propose its realization using an acoustic metamaterial \([16–18]\), as depicted in the schematic in Fig. 2a. Domain 1 is a two-dimensional waveguide of area \( L_x \times h \), consisting of two rigid parallel plates, gapped by a distance \( d \). Domain 2, of area \( L_x \times h \), is a square matrix of \( a \times a \times d \) cuboids. Fig. 2a inset, with elastic membranes (blue circles) of radius \( R \) and stiffness \( B_m \) for a unit cell) mounted in the walls, and an open Helmholtz resonator of length \( l \) and radius \( r \) \([11]\) (red cylinder) at the top. The external walls are sealed, with an array of acoustic actuators (grey circles) at the left wall, producing source waves (black arrow). The membranes create an effective mass density of \( \tilde{M}(\omega) = (\omega^2 - \omega_m^2)/\omega^2 \), where \( \omega_m^2 = B_m/(ma^2d) \) \([10]\). The resonator creates an effective bulk modulus of \( \tilde{B}(\omega) = \omega^2/(\omega^2 - \omega_m^2) \), where \( \omega_m^2 = \pi r^2 c^2/(a^2 dl) \). To satisfy (5), \( \omega_m^2 = \omega_m^2 \) must hold, yielding \( B_m = \pi r b_0/l \) and \( V_0 = \pi r^2 c^2/(a^2 dl) \). The graphene potential \( V_0 \) thus translates into a function of the metamaterial’s constitutive parameters. The particular value of \( V_0 \) depends on the desired ratio \( \gamma = \omega^2/V_0 \). The wavelength in domain 2, \( \lambda_2 = 2\pi/k_2 \), is determined from (4a). For \( a < \lambda_2 \), the collective dynamics of the unit cells turn the metamaterial into a homogenized, ef-
Fig. 3. Schematic of an acoustic metamaterial supporting omnidirectional Klein-like tunneling. Anisotropic design with x direction membranes (blue circles) and resonators (red cylinders) as in Fig. 2a, but different y direction membranes (yellow circles).

Since (4a) and (4b) uniquely determine the metamaterial parameters, an additional degree of freedom in the design is required. This can be obtained by introducing anisotropy [19, 20] to the effective mass density, with $M_x(\omega)$ in the x and $M_y(\omega)$ in the y directions. The system is then described by (2) with two distinct equations in (2a). Continuity of pressure along the domain’s interface gives $1 + R = T$, similarly to (1b). The distinction from the original effect is manifested in the continuity of normal flow velocity. Instead of matching it with the quantum conditions in (1), the substitution $k_{2x} = k_1, M_x(\omega)$ should be used [10]. Adding horizontal stratification $k_{1x} = k_{2y},$ leads to $M_x(\omega)\tilde{B}(\omega) = 1$ and $\tilde{B}(\omega)M^{-1}_y(\omega) = 1.$ To relate to Klein tunneling dispersion (4a) in domain 2, the substitution $\tilde{M}_x(\omega) = \tilde{M}(\omega)$ is necessary, with $\tilde{M}(\omega)$ and $\tilde{B}(\omega)$ defined in (5). This results in

$$\tan \phi = \frac{\omega^2 - V_0}{\omega^2} \tan \theta,$$  

(6a)

$$R = 0, \quad T = 1.$$

(6b)

The relations (6a)-(6b) resemble (3a)-(3b), yet are essentially different. (6a), which may be considered the modified Snell’s law of refraction, indicates that there is no critical angle for any $\gamma = \omega^2/V_0$ and $\phi,$ yet the refractive angle $\theta$ is positive for $\gamma > 1$ and negative for $\gamma < 1$, as in the original effect [10]. However, (6b) indicates unimpeded transmission to domain 2 for any $\gamma$ and $\phi.$ I denote this effect by omnidirectional Klein-like tunneling. To realize this effect in an acoustic metamaterial, the explicit expression for $M_x(\omega)$ needs to be derived. It turns out that $M_x(\omega)$ can be of any form, provided the overall system is dynamically stable, where the condition $\tilde{B}(\omega)M^{-1}_y(\omega) = 1$ is applied to a specific working frequency $\omega_0$ [10], i.e. $\tilde{M}_x(\omega_0) = \omega_0^2/\left(\omega_0^2 - V_0\right).$ A possible design is illustrated in Fig. 3, which is similar to the one in Fig. 2a, but with y axis membranes (yellow bars) of a different stiffness than the x axis membranes. That is,

$$\tilde{M}_x(\omega) = \frac{\omega^2 - \alpha}{\omega^2}, \quad \alpha = \frac{\omega_0}{1 - \omega_0}V_0,$$

(7)
where $\gamma_0 = \frac{a_0^2}{V_0} < 1$. For $\gamma_0 = \frac{1}{2}$, $M_y(\omega)$ in (7) equals $M(\omega)$ in (4a). This is since in this case only the isotropic system tunneling is unimpeded for any incidence angle, as in the anisotropic system. Implementation of (7) in a single platform for different $a_0$, without the need to refabricate the y axis membranes when $\omega_0$ is changed, is possible with an embedded feedback-based design [21–27], using active, voltage-controlled y axis membranes [10].

The underlying mechanism of the omnidirectional tunneling can be deduced from its dispersion relation, captured by $k_{2y}/k_{1x} = (\omega^2 - V_0)/\omega^2$, where $k_{2y} = k_{1y} = (\omega/c)\sin \phi$. This is the x axis projection of the original Klein dispersion in (4a), indicating that the omnidirectional $k_2$ has the same Klein-like frequency dependence as in the angle-dependent case, just scaled by the positive constant $\cos \phi/\cos \theta$. Contrary to the quantum graphene, for which the dispersion at the vicinity of Dirac points consists of two touching cones both in domains 1 and 2, with a constant shift of $V_0$ in domain 2, Fig.1b, the situation for the omnidirectional acoustic analogue is quite different [10]. In domain 1 the dispersion is a single cone, Fig.4a. With the addition of potential, expressed through the anisotropic constitutive parameters of domain 2, this cone transforms into three surfaces, the form of which depends on $\gamma_0$, as depicted in Figs.4b,c,d for $\gamma_0 = \frac{2}{3}, \frac{1}{2}$ and $\frac{1}{4}$, respectively. The different values of $\gamma_0$ indicate different working frequencies, $\omega_0^2 = \gamma_0 V_0$, with the corresponding wavevector components in both domains respectively highlighted by a purple, green and yellow curve. The potential $V_0$ is kept constant at the value set in (5). For $\frac{1}{2} < \gamma_0 < 1$ and $0 < \gamma_0 < \frac{1}{2}$ in Figs.4b and d, there are three dispersion surfaces. Two of them are elliptic cones, one with a single vertex, and the other with two vertices connected by a straight line. The third surface consists of two tilted Dirac-like cones, with a saddle point at the level of $\omega = \sqrt{V_0}$. At $\gamma_0 = \frac{1}{2}$, the middle surface degenerates, as captured by the semi-transparent sheet in Fig.4c, and is no
longer a part of the solution. The top and bottom surfaces become regular cones with a circular cross-section, touching at the origin. For any $\gamma_0$ the lower surface corresponds to the wave transition to domain 2, similarly to the electron transition from the upper to the lower cone in quantum graphene. At the transition from $\gamma_1 < \gamma_0 < 1$ to $0 < \gamma_0 < \frac{1}{2}$ the major axes switches from $k_{2x}$ to $k_{2y}$.

The polarization flipping is related to the interplay of the tunnelled wave group and phase velocity directions. This is illustrated by the grey and white arrows in the dynamical simulations of the anisotropic metamaterial, in Figs. 4e,f,g for $\gamma_0 = \frac{2}{3}, \frac{1}{2}, \frac{1}{4}$, respectively. The refraction is negative, where for $0 < \gamma_0 < \frac{1}{2}$ ($\frac{1}{2} < \gamma_0 < 1$) the wavelength in domain 2 is smaller (larger) than in domain 1. The exact wavelength and velocity values are in exact accordance with the Klein-like dispersion $k_{2x}/k_{1x} = (\omega^2 - V_0^2)/\omega^2$. In all the cases the tunneling is unimpeded, although the incidence is oblique. To conclude, this work provided an exact analogue of the quantum Klein tunneling phenomenon in an inherently classical acoustic medium, without mimicking graphene, but by tailoring its constitutive parameters according to (5), via the metamaterial in Fig. 2a. Furthermore, the anisotropic design in Fig. 3, with the tuning parameter in (7), enabled the sound to tunnel independently of incidence angle and frequency-potential ratio, obeying the modified Snell’s law in (6a) and the unique three-surface dispersion in Figs. 4b-d. This new phenomenon can be denoted by the omnidirectional Klein-like tunneling. Due to the general effective medium formalism in (2), this strategy offers a platform for omnidirectional unimpeded wave transmission in diverse classical systems.

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Supplementary material

Acoustic analogue of Klein tunneling, eqs. (3a)-(3b).

The explicit form of the time-harmonic constitutive equations (2a),(2b) in domain 1 and 2 is given by

\[
\begin{align*}
\frac{\partial P_1(x,y)}{\partial x} &= i\omega m_0 V_{1x}(x,y) \\
\frac{\partial P_1(x,y)}{\partial y} &= i\omega m_0 V_{1y}(x,y)
\end{align*}
\]

and

\[
\begin{align*}
\frac{\partial P_2(x,y)}{\partial x} &= i\omega m_0 \tilde{M}(\omega) V_{2x}(x,y) \\
\frac{\partial P_2(x,y)}{\partial y} &= i\omega m_0 \tilde{M}(\omega) V_{2y}(x,y)
\end{align*}
\]

so that the total wavenumbers become

\[
k_1 = \omega/c, \quad k_2 = k_1 \left( \tilde{M}(\omega) \tilde{B}^{-1}(\omega) \right)^{1/2}.
\]

Substituting (S4) into the horizontal stratification condition \(k_{1y} = k_{2y}\), i.e. in \(k_1 \sin \phi = k_2 \sin \theta\), results in (3a). The first part of (3b), \(1 + R = T\) does not depend on the constitutive parameters in domain 2. It is the direct result of continuity of pressure at \(x = 0\), \(P_1(x = 0, y) = P_2(x = 0, y)\), or \(P_1(x = 0, y) + P_R(x = 0, y) = P_T(x = 0, y)\), where \(P_I\), \(P_R\) and \(P_T\) are the incident, reflected and transmitted fields, explicitly defined in the main text as \(P_I(x,y) = P_0 e^{(k_{1x}+k_{1y})x}\), \(P_R(x,y) = P_0 e^{(-k_{1x}+k_{1y})x}\) and \(P_T(x,y) = P_0 e^{(k_{2x}+k_{2y})x}\). The second part of (3b) does depend on \(\tilde{M}(\omega)\) and \(\tilde{B}(\omega)\). The requirement on continuity of normal flow velocity, \(V_{1x}(0,y) = V_{2x}(0,y)\), or \(V_{1x}(0,y) + V_{Rx}(0,y) = V_{Tx}(0,y)\), by (S1) implies

\[
\frac{1}{i\omega m_0} \frac{\partial P_I(x,y)}{\partial x} \bigg|_{x=0} + \frac{1}{i\omega m_0} \frac{\partial P_R(x,y)}{\partial x} \bigg|_{x=0} = \frac{1}{i\omega m_0 \tilde{M}(\omega)} \frac{\partial P_T(x,y)}{\partial x} \bigg|_{x=0}.
\]

Differentiating \(P_I\), \(P_R\) and \(P_T\), and using \(k_{1y} = k_{2y}\), \(k_{1x} = k_1 \cos \phi\) and \(k_{2x} = k_2 \cos \theta\) in (S5), gives

\[
(1 - R) k_1 \cos \phi = \tilde{M}^{-1}(\omega) T k_2 \cos \theta.
\]

Combining (S6) with (S3), (3b) is retrieved.

Acoustic metamaterial realization of the constitutive parameters, eq. (5).

The physics of an acoustic cavity-on-neck resonator, aka Helmholtz resonator, as well as sound wave transmission through an elastic membrane, appears in any acoustic textbook, such as Ref. [11]. However, their collective dynamic behavior in the metamaterial setting, producing eq. (5), requires some derivation. The derivation here includes dissipation that naturally exists in both membranes and cavities. To this end consider the schematic in Fig. S1, which represents a unit cell of length \(a\) in a channel of the metamaterial in Fig. 2a. This channel has a cross-sectional area \(A_c = ad\). The resonator, here closed, can be regarded as an air mass per unit area \(M_h [kg/m^2]\) attached to an air spring per unit area \(B_h [N/m^3]\), with dissipation \(D_h\), where the neck of area \(A_n = \pi r^2\) stands for the mass, the cavity of volume \(Vol\) for the spring, and both are given by

\[
M_h = m_0 \frac{A_c}{A_n}, \quad B_h = \frac{m_0 c^2 \pi r^2}{Vol}.
\]

(S7)
where the connection of the resonator to the tube can be thus represented by a serial connection of a dynamic impedance 
\[ z_b(\omega) = M_b i \omega + D_b + B_b / i \omega \] with the air impedance \( B_0 / i \omega \), \( B_0 = b_0 / a \), leading to the effective bulk modulus of
\[ \tilde{B}(\omega) = \frac{i \omega z_b(i \omega)}{B_0 + i \omega z_b(i \omega)} = \frac{\omega_b^2 - \omega^2 + D_b i \omega}{\omega_b^2 - \omega^2 + D_b i \omega}. \] (S8)

Here, \( \omega_b^2 = B_b / M_b \) and \( \omega_b^2 = \omega_b^2 (1 + B_0 / B_b) = (B_h + B_0) / M_h \). For \( \tilde{B}(s) \) in (S8) to retrieve \( \tilde{B}(\omega) \) in eq. (5) in the time-harmonic regime (for small dissipation), \( \omega_b^2 \) needs to equal zero. This implies that either \( M_h \to \infty \) or \( B_h \to 0 \). The latter may be achieved with \( V_0 l \to \infty \), implying an infinite cavity, or, equivalently, an open neck without a cavity, thus keeping the neck radius \( r \) and length \( l \) finite. This condition retrieves the relation \( \omega_b^2 = B_b / M_b = \pi r^2 c^2 / (a^2 d l) \), which equals the potential \( V_0 \). As for the elastic membrane, it can be regarded as a dynamic air impedance \( z_m(\omega) = M_0 i \omega + D_m + B_m / (A_t i \omega) \) of an air mass \( M_0 = m_0 a \) and a spring of stiffness \( B_m \) [N/m] per channel area \( A_t \), implying an effective mass of
\[ \tilde{M}(\omega) = \frac{z_m(\omega)}{M_0 i \omega} = \frac{\omega_m^2 - \omega^2 + D_m i \omega}{\omega^2}. \] (S9)

For small dissipation, (S9) retrieves eq. (5). The characteristic frequency of the membrane is therefore given by \( \omega_m^2 = B_m / (M_0 A_t) = B_m / (m_0 a^2 d l) \). Equating \( \omega_b^2 \) with \( \omega_m^2 \) yields \( \pi r^2 c^2 / (a^2 d l) = V_0 \), as specified in the main text.

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**Fig. S1.** Isotropic acoustic metamaterial unit cell (closed resonator) in one direction.

**Omnidirectional Klein-like tunneling by anisotropic design, eqs. (6a)-(6b).**

The time-harmonic constitutive equations of an anisotropic medium in domain 2 take the form
\[
\begin{align*}
\frac{\partial P_2(x, y)}{\partial x} &= i \omega m_0 \tilde{M}_i(\omega) V_{2x}(x, y) \\
\frac{\partial P_2(x, y)}{\partial y} &= i \omega m_0 \tilde{M}_j(\omega) V_{2y}(x, y) \\
i \omega P_2(x, y) &= b_0 \tilde{B}(\omega) \left[ \frac{\partial V_{2x}(x, y)}{\partial x} + \frac{\partial V_{2y}(x, y)}{\partial y} \right],
\end{align*}
\] (S10)

and the total wave equation becomes
\[
c^2 \left[ \tilde{B}(\omega) \tilde{M}_i^{-1}(\omega) \frac{\partial^2 P_2(x, y)}{\partial x^2} + \tilde{B}(\omega) \tilde{M}_j^{-1}(\omega) \frac{\partial^2 P_2(x, y)}{\partial y^2} \right] = -\omega^2 P_2(x, y). \] (S11)

With \( P_2(x, y) \propto e^{i(k_{2x} x + k_{2y} y)} \), (S11) yields the dispersion relation
\[
c^2 \tilde{B}(\omega) \left( \tilde{M}_i^{-1}(\omega) k_{2x}^2 + \tilde{M}_j^{-1}(\omega) k_{2y}^2 \right) = \omega^2. \] (S12)

Continuity of normal flow velocity at \( x = 0, \) \( V_{1x}(0, y) = V_{2x}(0, y) \), results in
\[
(1 - R) k_{1x} = \tilde{M}_i^{-1}(\omega) T k_{2x}. \] (S13)

For angle-independent perfect transmission it is then required to set
\[
k_{2x} = k_{1x} \tilde{M}_i(\omega), \] (S14)
which implies $1 - R = T$. Together with continuity of pressure requirement, which does not change in the anisotropic regime and yields $1 + R = T$, (6b) is retrieved. Now, substituting (S14) into the dispersion relation (S12), and using Snell’s law $k_2 = k_1$, gives

$$c^2 k_{1x}^2 \tilde{B}(\omega) \tilde{M}_x(\omega) + c^2 k_{1y}^2 \tilde{B}(\omega) \tilde{M}_y^{-1}(\omega) = \omega^2.$$  \hspace{1cm} (S15)

This needs to retrieve the uniform dispersion relation in domain 1, which leads to the conditions

$$\tilde{M}_x(\omega) \tilde{B}(\omega) = 1, \quad \tilde{B}(\omega) \tilde{M}_y^{-1}(\omega) = 1.$$  \hspace{1cm} (S16)

On the other hand, using the explicit form of $k_2 = k_1$,

$$\omega^2 \sin^2 \phi = c^2 k_2 \sin^2 \theta = c^2 \left[ k_{2x}^2 + k_{2y}^2 \right] \sin^2 \theta = \left[ c^2 k_{2x}^2 + \omega^2 \sin^2 \phi \right] \sin^2 \theta,$$

and solving for $k_{2x}$, gives

$$c^2 k_{2x}^2 \sin^2 \theta = \omega^2 \sin^2 \phi (1 - \sin^2 \theta) = \omega^2 \sin^2 \phi \cos^2 \theta, \quad \rightarrow \quad c^2 k_{2x}^2 \tan^2 \theta = \omega^2 \sin^2 \phi,$$  \hspace{1cm} (S17)

which together with (S14) and $k_1 = \omega/c \cos \phi$, results in

$$\tan \phi = \tilde{M}_x(\omega) \tan \theta.$$  \hspace{1cm} (S19)

Substituting $\tilde{M}_x(\omega) = \tilde{M}(\omega) = (\omega^2 - V_0)/\omega^2$, (S19) retrieves (6a). As illustrated in Fig.S2a, the critical angle in the original refraction law (3a)-(3b) is manifested by flattening of the $\theta$ surface for $\frac{1}{2} < \gamma$, which does not exist in the modified law (6a)-(6b), Fig.S2b.

![Fig. S2. Refraction angle $\theta$ as a function of $\gamma = \omega^2/V_0$ and incidence angle $\phi$. a. The original law of refraction (3a)-(3b), with flat surfaces demonstrating existence of critical angle. b. The modified law (6a)-(6b) demonstrating no critical angle for any $\gamma$ and $\phi$.](image)

With $\tilde{M}_x(\omega) = \tilde{M}(\omega) = \tilde{M}_x^{-1}(\omega)$, $\tilde{M}_x(\omega) = (\omega^2 - \alpha)/\omega^2$, and $\alpha = V_0 \gamma_0/(1 - \gamma_0)$, as defined in eq. (7), the dispersion relation (S12) takes the form

$$\gamma^3 - \left( \frac{\alpha}{V_0} + 2 + \frac{\alpha}{V_0} \left( k_{2x}^2 + k_{2y}^2 \right) \right) \gamma^2 + \left( 1 + 2 \frac{\alpha}{V_0} + \frac{\alpha}{V_0} \left( k_{2x}^2 + k_{2y}^2 \right) \right) \gamma - \frac{\alpha}{V_0} = 0,$$  \hspace{1cm} (S20)

where $\gamma = \omega^2/V_0$ for the general frequency $\omega$, and $\gamma_0 = \omega_0^2/V_0$ for the specific working frequency $\omega_0$. The relation in (S20) represents the dispersion plots in Figs.4b,c,d.

Creating the anisotropic medium with real-time feedback control.

The voltage-controlled membranes in the $y$ direction behave as dipole acoustic sources $f_y$, and are operated in a real-time feedback loop, as illustrated in Fig. S3. When the voltage is off, $f_y = 0$, these membranes are equal to their $x$ direction
counterparts, i.e. have a stiffness $B_m$ and create an effective mass density $\tilde{M}(ω)$ in the homogenized system, retaining the isotropic metamaterial in Fig. 2a. The actual implementation is discrete. Here, the homogenized formulation is presented. The constitutive equations of domain 2 in Laplace domain $s$, a control-ready formulation, then take the form

$$\begin{cases} 
\frac{∂p_2(x,y,s)}{∂x} = -m_0s\tilde{M}(s)v_{2x}(x,y,s) \\
\frac{∂p_2(x,y,s)}{∂y} = -m_0s\tilde{M}(s)v_{2y}(x,y,s) + f_y(x,y,s) \\
sp_2(x,y,s) = -b_0\tilde{B}(s) \left[ \frac{∂v_{2x}(x,y,s)}{∂x} + \frac{∂v_{2y}(x,y,s)}{∂y} \right]
\end{cases}$$  \hspace{1cm} (S21)

The control input $f_y$ is based on measurements of the pressure field difference across the membrane, which are processed through a reprogrammable electronic controller $H$,

$$f_y(x,y,s) = -H(s)\frac{∂}{∂y}p_2(x,y,s).$$  \hspace{1cm} (S22)

The closed loop constitutive equation in the $y$ direction then becomes

$$\frac{∂p_2(x,y,s)}{∂y} = -m_0s\tilde{M}(s)\frac{M(s)}{1 + H(s)}v_{2y}(x,y,s).$$  \hspace{1cm} (S23)

Setting $H$ to

$$H(s) = \frac{\tilde{M}(s)}{M_y(s)} - 1,$$  \hspace{1cm} (S24)

creates in closed-loop a homogenized active matter described by

$$\frac{∂p_2(x,y,s)}{∂y} = -m_0s\tilde{M}_y(s)v_{2y}(x,y,s),$$  \hspace{1cm} (S25)

where $\tilde{M}_y(s)$ is the target acoustic mass density in the $y$ direction. The time-harmonic regime $s = iω$ is retained when the closed-loop system converges to steady-state, provided the system is dynamically stable. For $\tilde{M}_y(ω)$ as in eq.(7), the controller takes the form

$$H(s) = \frac{V_0 - \alpha}{s^2 + \alpha}.$$  \hspace{1cm} (S26)

The closed-loop system in the $y$ direction mimics regular membranes, but with a different stiffness than in the $x$ direction. For the specific case $α = V_0$, the loop opens ($H = 0$). This means that the controller itself produces $f_y = 0$, indicating that no change needs to be applied to the isotropic system to obtain unimpeded tunneling.