Improvement of the failure-assessment diagrams used to check the harmfulness of pipe defects

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In this paper, the principle of the failure-assessment diagram (FAD) is presented including the definitions of the failure-assessment curve and the assessment point. Classical FAD allows computing a safety factor associated with defect geometry and loading conditions. Extension to a probabilistic FAD (PFAD) is used to compute the failure probability. In order to reduce the conservatism associated with the ‘crack-like defect’ procedure, the use of notch-failure-assessment diagram (NFAD) takes advantage of the increase of fracture toughness by reducing the defect acuity. The domain-failure-assessment diagram (DFAD) are pertinent to associate the right fracture-mechanics’ tool with the failure mechanism: brittle, ductile, or plastic collapse. The fatigue-assessment diagram (fAD) use the Wöhler curve as failure-assessment curve and is used to develop a maintenance policy by determining loading conditions to guarantee a life duration with a given probability.

Key words: failure-assessment diagram, safety factor, failure probability.

Failure-assessment diagram

The failure-assessment diagram (FAD) methodology replaces the three fracture-mechanics-parameter relationship (fracture toughness, defect size, and loading) by two parameters, in order to have a plane representation where the non-dimensional crack-driving force $k_r$ and the non-dimensional applied stress $L_r$ are the coordinates.

The non-dimensional crack-driving force is defined as the ratio of the applied stress-intensity factor, $K_{app}$, to the fracture toughness of the material, $K_{ic}$:

$$k_r = \frac{K_{app}}{K_{ic}}$$ (1)

An improvement was made by introducing the $J$-integral or crack opening displacement as:

$$k_r = \sqrt{\frac{J_{app}}{J_{mat}}} \quad \text{or} \quad k_r = \sqrt{\frac{\delta_{app}}{\delta_{mat}}}$$ (2)

where $J_{app}$ and $\delta_{app}$ are the applied $J$-integral and crack opening displacement and $J_{mat}$ and $\delta_{mat}$ are fracture toughness in terms of critical value of the $J$-integral or critical crack-opening displacement of the material.

The non-dimensional stress $L_r$ is described as the ratio of the gross stress $\sigma_g$ over flow stress, chosen as yield stress $\sigma_y$, ultimate stress $\sigma_U$ or classical flow stress $\sigma_0$:

$$L_r = \frac{\sigma_g}{\sigma_0}$$ (3)

The FAD exhibits a failure curve relating the critical non-dimensional crack-driving force $k_{rc}$ to the critical non-dimensional stress or loading parameter $L_{rc}$. The curve $k_{rc} = f(L_{rc})$ is obtained from fracture-toughness data measured from specimens tested under high levels of stress triaxiality (deep crack associated with bending). Such conditions ensure conservative conditions. The local stress distribution ahead of the crack tip is assumed to...
be plane strain with high constraint. However, for real structures, defect-tip constraint is reduced by small thickness, blunt defect, or tensile loading and real fracture toughness increases.

The failure-assessment curve \( k_{rc} = f(L_{rc}) \) delineates a fracture design curve according to the available codes, including SINTAP [1], R6 [2], and RCC-MR[3].

The failure curve for the basic level of the SINTAP procedure is given by Eqn 7. The fracture toughness, yield strength, and ultimate strength of the material are required for this, as shown in Equns 4 and 5 (below).

The assessment point of a component can be highlighted by a point of coordinates \( k_{rc}^* \) and \( L_{rc}^* \). If this point is inside the boundary lines of the diagram which is limited by the failure-assessment curve curve, the structure is safe (Fig.1); if not, failure occurs, and the assessment point is situated outside of the interpolation curve.

In the FAD, the assessment point is denoted A. Due to the definition of the parameter \( k_r \) and \( L_r \), if the crack length remains constant during loading, the loading path is linear from the origin to B. Increasing loading until failure allow the failure-assessment curve at point B to be reached. As illustrated in Fig.1, the safety factor \( f_s \) is defined by the ratio of OB over OA, i.e.:

\[
f_s = \frac{OB}{OA}
\]

According to the codes safety-factor consideration, the assessment point is positioned within the acceptable zone of the FAD and the structure fulfils the required conditions for practical engineering applications.

**Notch -failure-assessment diagram**

The classical fracture toughness \( K_{ic} \) is determined from the cracked specimen and plane-strain conditions. The use of pre-cracked specimens is time consuming and also costly. For some brittle materials like ceramics, it is impossible to pre-crack the specimen due to sensitivity to crack propagation. Therefore, the use of notched specimens is preferable and cheaper. However, it has been seen than the fracture toughnesses measured with this kind of specimen is generally higher than those measured with cracked specimens, and called the notch-fracture toughness \( K_{poc} \). It has been seen [4] that \( K_{poc} \) increases linearly with the square root of the notch radius \( \rho \):

\[
K_{poc} = K_{ic} + \alpha \sqrt{\rho} \quad \text{for } \rho > \rho_c
\]

\[
K_{poc} = K_{ic} \quad \text{for } \rho \leq \rho_c
\]

\( \rho_c \) is the critical radius. Due to the sensitivity of the fracture toughness with notch radius, it is necessary - when the defect is to be assessed by the FAD method -to modify Eqn 1 and replace the fracture toughness \( K_{ic} \) by \( K_{poc} \):

\[
k_{poc} = \frac{K_{poc}}{K_{ic}}
\]

\( K_{poc} \) is the notch stress-intensity factor which governs the stress distribution at the notch tip at distance greater than the distance where the maximum stress

\[
k_{poc} = \frac{1}{\sqrt{1 + L^2_{oc}/2}} \left[ 0.3 + 0.7 \exp \left( -\mu L_{oc}^2 \right) \right] \quad \text{for } 0 \leq L_{oc} \leq 1
\]

\[
k_{poc} = f(L_{oc}) \times L_{oc}^{(N-1/2)} \quad \text{for } 1 \leq L_{oc} \leq L_{oc,max}
\]

\[
L_{oc,max} = 0.5 \left[ \sigma_y + \sigma_{ul}/\sigma_{ul} \right]
\]

\[
\mu = \min \left\{ \left( 0.001 E/\sigma_y \right), 0.6 \right\}
\]

\[
N = 0.31 \left( 1 - \left( \sigma_y/\sigma_{ul} \right) \right)
\]
occurs. The $L_r$ parameter keeps the same definition. It can be seen that the failure-assessment curve is independent of the notch radius and is the same as that of the crack. Therefore, using these assumptions, the notch-failure-assessment diagram (NFAD) can be used to assess a non-crack-like defect with radius $\rho$. One notes that the uses of NFAD needs to obtain the critical notch radius and the notch sensitivity for each material.

In the following, the NFAD is used to compute the safety factor associated with a gouge in a pipe made from API X-52 pipe steel submitted to internal pressure. Here we consider three types of defect in the pipe with a diameter $D = 219$ mm and wall thickness $t = 6.1$ mm. The first is central semi-spherical crack-like defect (ss) with depth $d = t/2$; the second is a central semi-elliptical defect (se) of length $L (d = t/2, d/L = 0.1)$; and the third is a central long blunt notch (ln) of notch radius $\rho (d = t/2, d/L = 0.1, \rho = 0.15$ mm). The defect direction is longitudinal, and the service pressure is equal to 70 bar. The applied notch-stress-intensity factors have been obtained from the volumetric method [6] and reported in Table 1. The stress distribution at the notch tip was computed by a finite-element method and extracted from Ref.10.

### Probabilistic-failure-assessment diagram

Engineers gradually realized the insufficiencies of a safety design, and this awakening brought about the development of the concept of reliability from a probabilistic angle. According to the probabilistic approach, a structure is considered sure if its probability of failure is lower than a conventional value, a value which depends on many factors such as the expected life of the structure, the consequences generated by its ruin, the risks of obsolescence, and certain economic criteria, such as the value of replacement, maintenance costs, etc. The safety factor is then defined as the ratio of the ultimate strength, which corresponds to the mean value of the strength distribution over the admissible stress. The admissible stress is the failure stress associated with a low and conventional probability of failure $P_{r, con}$ ($P_{r, con} = 10^{-4}$ or $10^{-6}$ if there is risk to human life).

In a probabilistic approach, two iso-probability failure curves ($P_r = 1$ and $P_r = 10^{-4}$ or $10^{-6}$) divide the FAD into three zones: the unsafe zone below the failure curve ($P_r = 1$); the safe zone with maintenance $P_r > 10^4$ or $10^6$; and the safe zone without maintenance $P_r < 10^4$ or $P_r < 10^6$, see Fig.2. Any assessment point of the coordinates $(L_r, k_r)$ is situated on an iso-probability curve $P_r^*$ and the safety factor keeps the same definition as for a deterministic FAD.

The criticality of the situation of a structure is evaluated with better accuracy by introducing the real scatter of the parameters defining the crack-extension force on the defect (load, defect size, fracture toughness). It is not based on an empirical and unique safety factor for all materials and situations, but on the

| Defect type | $f^{*}_{r, se}$ |
|-------------|-----------------|
| (ss)        | 3.16            |
| (se)        | 3.13            |
| (ln)        | 3.04            |

*Table 1. Safety factors for three types of defect (ss), (se), and (ln) at a service pressure of 70 bar.*
acceptable risk from an economic and societal definition of the risk.

For conservative reasons, a defect which promotes pipe failure is assumed to be a semi-elliptical or semi-spherical surface crack of depth $a$ and length $2c$. For such a defect, the applied stress-intensity factor is given by the following relationship:

$$ K_{app} = \frac{pR_m}{B} \sqrt{\pi a P} \left( \frac{R_i}{B} \frac{a}{2} \right) \quad (9) $$

where $P$ is the internal pressure, $R_m$ and $R_i$ are respectively the mean internal radius, and $B$ the thickness of the pipe wall; $F$ is a geometry correction which is given in Ref.1.

The internal pressure in a gas pipe fluctuates continuously, and may vary depending on the rate of gas injection into the network and the service of delivery points downstream. Pipeline operators often cannot control these flows. To characterize the pressure of a gas pipeline, one must consider three factors:

- the maximum pressure applied
- the range of fluctuation of the pressure and the minimum pressure
- the rate of pressure change (change almost instantly in some cases, over several days in others).

These fluctuations are commonly expressed by the $R$ ratio which is the ratio of the minimum pressure to the maximum. In this study, the $R$ ratio has been given by the gas company and is equal to $4/7$. The lower limit pressure is 40 bar and an upper limit pressure is about 70 bar. This ratio is kept constant when the maximum pressure fluctuates with a coefficient of variation of $CV=0.1$. The fluctuations are normally distributed.

The Weibull distribution describes the scatter of fracture toughnesses, and the normal distribution describes the scatter of yield stress and ultimate strength. For conservative reason, a lower bound of the coefficient of variation has been used ($CV=0.1$). A summary of the distribution, mean, and coefficient of variation, of the five parameters introduced in probabilistic fracture mechanics is given in Table 2.

Figure 3 gives the results of the failure probability given by these two methods for a given maximum pressure converted into circumferential stress $\sigma_{\text{circ}}$. Two security levels are associated: level 2 associated with a conventional failure probability

| Parameters             | Distribution  | Coefficient of variation | Mean     |
|------------------------|---------------|--------------------------|----------|
| Fracture toughness     | Weibull       | 0.1                      | 116 MPa/m|
| Yield stress           | Normal        | 0.1                      | 410 MPa  |
| Ultimate strength      | Normal        | 0.1                      | 528 MPa  |
| Defect size            | Exponential   | 1                        | 3 mm     |
| Internal pressure      | Normal        | 0.1                      | 55 bar   |
of $10^{-6}$ and risk to human life; and level 1 associated with a conventional failure probability $10^{-4}$ and no risk to human life. One note that, for the assessment point associated with a maximum service pressure of 70 bar or a circumferential stress of 125 MPa, the failure probability is less than $10^{-6}$ and the pipe with a surface defect is working in the security zone.

Domain-failure-assessment diagram

In FAD, the loading path is linear because the two non-dimensional parameters $k_r$ and $L_r$ are proportional to the applied gross stress $\sigma_g$. The angle between this loading path and the $L_r$ axis is called the assessment angle $\theta$. Two particular values of the assessment angle can be defined as $\theta_1$ and $\theta_2$; these two angles determine three failure domains in the NFAD diagram (Fig.3):

- $0 < \theta_1$: brittle fracture
- $\theta_1 < \theta < \theta_2$: elastoplastic fracture
- $\theta > \theta_2$: plastic collapse

Based on Federsen diagram [12] the limit of these three zones is defined conventionally as follows:

- Zone I: $0 < L_r < 0.62 L_{r,y}$
- Zone II: $0.62 L_{r,y} < L_r < 0.95 L_{r,max}$
- Zone III: $0.95 L_{r,max} < L_r < L_{r,max}$

where $L_{r,y}$ is associated with the yield pressure and $L_{r,max}$ is the maximum value of $L_r$. The values of $\theta_1$ and $\theta_2$ are $\theta_1 = 55^\circ$ and $\theta_2 = 22^\circ$.

The use of NFAD is particularly interesting for choosing the appropriate tool for assessing the risk of failure emanating from a pipe defect. Gouges and combined gouges and dents generally fail by elastoplastic fracture, but the use of limit analysis is also possible. Dents are generally assessed by limit analysis.

In the following the NFAD is coupled with a probabilistic approach in order to compute the distribution of the safety factor associated with a defect in a pipe subjected to internal pressure. The pipe exhibits a radial semi-elliptical defect with a notch angle $\Psi = 0^\circ$ and notch radius of $\rho = 0.25$ mm. The notch-stress-intensity factor was calculated by introducing the circumferential maximum stress given in Table 3 and the geometry of the defect. The value of the notch-stress-intensity factor obtained is low compared to the notch fracture toughness, and consequently we are in elastic loading conditions. For this reason, and due to the fact that the notch is sharp and the notch angle is zero, the notch-stress-intensity factor is close to the crack-stress-intensity factor. The pipe is subjected to internal pressure which delivers a hoop stress of 41.8 MPa; the

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Fig. 3. Evolution of the failure probability of a pipe with a semi-spherical surface defect with circumferential stress [6].

Fig. 4. Domain-failure-assessment diagram with three fracture domains.
pipe steel is API 5L X-52, and the data are presented in Table 3.

Within the chosen procedure, the following parameters are treated as random parameters and introduced into the notch-failure-assessment diagram:

- notch fracture toughness $K_{f,c}$
- yield strength $R_y$
- ultimate tensile strength $s_{ult}$
- defect depth $a$
- maximum pressure $p_{max}$

These random parameters are treated as not being correlated with one another. Fracture toughness is assumed as Weibull’s distribution. Yield strength, ultimate, and tensile strength and internal pressure can be mainly assumed as a normal distribution. For the defect, depth $a$ is assumed to follow an exponential distribution. The exponential distribution generally governs the defect-size analysis.

The assessment points lie in the plastic collapse zone. Another way to define the safety factor is to consider that we have plastic collapse only for $\theta = 0$ and the defined associated safety factor $f_s^*$. We have examined the evolution of the safety factor with the angle $\theta$, and it has been shown that the angle is in the range 0-7° for steel. All data are in a narrow scatter band of range $(\mu \pm 3\sigma)$ and in the region of plastic collapse. For this reason the safety factor $f_s^*$ computed from the ultimate pressure done by code ASME B31G is also reported. The mean values and standard deviations for the safety factor of the three materials are reported in Table 4.

### Conclusions

The FAD is a universal tool to appraise the harmfulness of a pipe defect for any kind of failure, from brittle fracture to plastic collapse. This harmfulness is analysed through the comparison of the safety factor associated with pipe geometry and service loading conditions, and a conventional safety factor. This can be done in a deterministic or a probabilistic way. In this case, the probability of failure is comparable to a conventional and admissible probability of failure.

The use of the domain-failure-assessment diagram allows choice of the best fracture-mechanics’ tool according to the position of the assessment point in the domain, or the value of the assessment angle. It can be linear-elastic fracture mechanics, elasto-plastic fracture mechanics, or limit analysis.

For non-conservative reasons, it is not necessary to treat any type of pipe defect as a crack-like defect, but with their real parameters. Therefore, the use of the notch-failure-assessment diagram introduces a supplementary reduction of the conservatism.

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