Massive Black Hole Binary Systems in Hierarchical Scenario of Structure Formation

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The hierarchical scenario of structure formation describes how objects like galaxies and galaxy clusters are formed by mergers of small objects. In this scenario, mergers of galaxies can lead to the formation of massive black hole (MBH) binary systems. On the other hand, the merger of two MBH could produce a gravitational wave signal detectable, in principle, by the Laser Interferometer Space Antenna (LISA). In the present work, we use the Press?Schechter formalism, and its extension, to describe the merger rate of haloes which contain massive black holes. Here, we do not study the gravitational wave emission of these systems. However, we present an initial study to determine the number of systems formed via mergers that could permit, in a future extension of this work, the calculation of the signature in gravitational waves of these systems.

Keywords: massive black hole, structure formation, galaxies

1. Introduction

Recently, observational evidence for the existence of massive black holes in galaxies has been reported in the literature. On the other hand, the hierarchical scenario of structure formation describes how objects like galaxies and galaxy clusters are formed in the early universe. In this way, we can suppose that mergers of galaxies can lead to the formation of the massive black holes (MBH) observed in galaxies and their binary systems. Thus, the main goal of this work is to describe a method to determine the evolution of binary systems of massive black holes in the hierarchical scenario. To do that, we present in Sec. 2 a short review on the Press?Schechter formalism. In Sec. 3 we describe how to obtain the relation between the central black hole and the mass of the host dark halo. We also present the way to calculate the number of massive binary systems in Sec. 3. In Sec. 4 we present the main results, and finally in Sec. 5 we present our conclusions. Our models are obtained using the
following set of cosmological parameters: $\Omega_m = 0.24$, $\Omega_b = 0.04$, $\Omega_{\Lambda} = 0.76$ and $h = 0.73$.

2. Hierarchical scenario of structure formation: Press-Schechter formalism and its extension

The hierarchical scenario of structure formation: the core of Press-Schechter (P-S) formalism is that a dark matter halo leaves the linear regime when the mean density within a given volume is larger than a threshold level $\delta_c$. In particular, Lacey and Cole\cite{Lacey1993} proposed an extension of the P-S formalism based on the Brownian random walk of Bond et al.\cite{Bond1991}. The goal of this extension was to take into account the probability that a dark matter halo (henceforth halo), with mass $M_1$, has to merger with another halo with mass $M_2$, for any redshift $z$, in order to form a new halo with mass $M_f = M_1 + M_2$. Fakhouri and Ma\cite{Fakhouri2007} showed that, using the P-S formalism and its extension, the merger rate of haloes is given by:

$$
\frac{B(M_1, M_f, z)}{f(M_f, z; P-S)} = \sqrt{\frac{2}{\pi}} \frac{1}{\sigma^2(M_1)} \left| \frac{d\sigma(M_1)}{d\ln(M_1)} \right| \left| \frac{d\delta_c}{dz} \right| \left[ 1 - \frac{\sigma^2(M_1)}{\sigma^2(M_f)} \right]^{-3/2}
$$

where $f(M, z; P-S)$ is the P-S mass function of dark haloes, $\sigma(M)$ is the variance and $B(M_1, M_f, z)$ is the merger of dark halos.

3. Binary systems of massive black holes

Wythe & Loeb\cite{Wythe2001} proposed a model for the relation between central black hole and the mass of host dark halo. The authors consider that the central black hole (CBH) stop growing when the accretion reaches the Eddington luminosity. In particular, they consider that the circular velocity is equal the virial velocity, in this case, the mass of dark halo $M_h$, as a function of the CBH is (see Refs\cite{Wythe2001, Loeb2000}):

$$
M_h(M_{BH}) = \varepsilon_0^{-3/5} \left( \frac{\Omega_m}{\Omega_m(z)} \right) \Delta_c \left( 1 + z \right)^{-3/2} \left( \frac{M_{BH}}{10^{12}M_\odot} \right) 10^{12}M_\odot,
$$

where $\Omega_m^0$ is the dark matter parameter at present time, $\varepsilon_0 = 10^{-5.7}$ and $\Delta_c$ is the linear overdensity by virialization of a spherical perturbation “top-hat”-like, that for $\Lambda$CDM is\cite{Ade2013}:

$$
\Delta_c = 18\pi^2 + 82[\Omega_m(z) - 1] - 39[\Omega_m(z) - 1]^2.
$$

If we consider that the fraction $\varepsilon_1$ of dark haloes, at $z < 10$, having a central MBH, then we can obtain, using equations\cite{Wythe2001} (2) and (1), the following equation to the formation rate of massive binary systems:

$$
R(M_{BH,1}, M_{BH,2}, z) = \varepsilon_1 \varepsilon_2 f(a) B(M_h(M_{BH,1}), M_h(M_{BH,2}), z),
$$

where $M_{bh,i}$ ($i = 1, 2$) is the mass of CBH, $a$ is the separation of binary system at $z$ and $f(a)$ is the separation distribution function.
We considered in Eq. (4) that the central MBH quickly forms binary systems and which the fraction $\epsilon_2$ of these systems achieve the gravitational wave regime. On the other hand, for the separation distribution function we have:  
\begin{equation}
 f(a) da = \frac{3}{2} \left[ \left( \frac{a}{\pi} \right)^{3/4} - \left( \frac{a}{\pi} \right)^{3/2} \right] \frac{da}{a}. 
\end{equation}

In the above equation we consider that the MBHs are formed in galaxy clusters. Thus, $\pi$ represents the maximum separation of the components, and $\pi$ is the typical dimension of galaxies clusters.

The density number ($n_{BH}$) of black hole binary systems obeys the conservation equation:
\begin{equation}
 \frac{\partial n_{BH}}{\partial z} \frac{dz}{dt} + \frac{\partial (n_{BH}(da/dt))}{\partial a} = R(M_{bh,1}, M_{bh,2}, z). 
\end{equation}

In a gravitational wave regime, the variation of separation with time is:
\begin{equation}
 \frac{da}{dt} = -\frac{64}{5} \frac{G^3}{c^5} \frac{(M_{BH,1} + M_{BH,2})}{a^3} M_{BH,1} M_{BH,2}. 
\end{equation}

Finally, the number of systems, as a function of $z$ and observed frequency, $\nu_{obs}$, is:
\begin{equation}
 N_{sys} = -n_{BH} \frac{dV}{dz} \frac{da}{d\nu_{obs}}. 
\end{equation}

with:
\begin{equation}
 \frac{da}{d\nu_{obs}} = -\frac{3}{2\pi} \frac{G(M_{BH,1} + M_{BH,2})}{a^{-5/2}(1 + z)} 
\end{equation}

where $dV/dz$ is the comovelandar volume.

4. Numerical results

In this work, we use the Lax-Wendroff scheme\textsuperscript{9} to obtain the numerical solution of equation (5). Figure 1 on the left hand side, shows the number of systems, by frequency by redshift, of binary system formed by black holes of mass $M_{BH,1} = 10^5M_\odot$ and $M_{BH,2} = 0.10M_{BH,1}$ and, on the right hand side, the total number of systems into the mass range $10^4M_\odot \leq M_{BH,1} \leq 10^7M_\odot$ and $0.1M_{BH,1} \leq M_{BH,2} \leq M_{BH,1}$ for different values of $\epsilon_1$ and $\epsilon_2$. In both cases, we assumed that the separation range is $3(r_{sh,1} + r_{sh,2}) \leq a \leq 100(r_{sh,1} + r_{sh,2})$, with $r_{sh,i}$ is the Schwarschild ratio of black hole $i$ and we assumed $\pi = 1.5$ Mpc. Note that the function $N_{BH}$ peaks at $z \approx 2$.

5. Conclusion

We presented a different method to calculate the number of binary systems of massive black holes using the Press-Schechter formalism. It is important to emphasize that the total number of systems obtained here takes into account all systems. This
Fig. 1. The left hand side showed the number of systems as a function of observed frequency and $z$ with $\epsilon_1 = \epsilon_2 = 1$ and black holes with mass $M_{BH,1} = 10^5$ and $M_{BH,2} = 0.1M_{BH,1}$. In the right hand right, is presented the total number of binary systems of massive black holes into the mass range $10^4 M_\odot \leq M_{BH,1} \leq 10^5 M_\odot$ and $0.1 M_{BH,1} \leq M_{BH,2} \leq M_{BH,1}$. On the top it was assumed $\epsilon_1 = \epsilon_2 = 1$ and on the bottom it was considered $\epsilon_1 = 0.01$, $\epsilon_2 = 0.001$, $\epsilon_1 = 0.09$ and $\epsilon_2 = 0.0001$.

is a different result when compared, for example, for the work of Wythe & Loeb [4] who obtained their results only for systems within the LISA range. The numerical method used here is stable and it produces a smooth function for the density of binary systems. The same method was used by Banerjee & Ghosh [6] or the calculus of the formation of binary systems in globular clusters of stars.

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References

1. A. L. Erickcek and M. Kamionkowski and A. J. Benson, MNRAS, (2005).
2. C. Lacey and S. Cole, MNRAS, (1993), 262, 627.
3. J. R. Bond and S. Cole and G. Efstathiou and N. Kaiser, Apj, (1991), 379, 440.
4. J. S. B. Wythe and A. Loeb, Apj, (2003), 590, 691.
5. O. Fakhouri and C. Ma, MNRAS, (2008), 386,577.
6. S. Banerje and P. Ghosh, Apj, (2007), 670,1090.
7. W. A. Hiscock, Apj, (1998),509,L101.
8. W. H. Press and P. Schechter, Apj, (1974) 425.
9. W. H. Press et al., Numerical Recipes in Fortran 77, Vol. 1,2nd edn; (Cambridge, Cambridge Univ. Press, Cambridge, 1992), p. 948.
10. Y. Li and H. J. Mo and F. C. van den Bosh and W. P. Lin, MNRAS, (2007), 379, 689.