**Vaporization dynamics of a quantum liquid**

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We study the heating dynamics of a Luttinger liquid, upon suddenly coupling it to a dissipative environment. Within the Lindblad equation, the environment couples to local currents and heats the quantum liquid up to infinite temperatures. The fermionic single particle density matrix, and many other correlators, retain the initial Luttinger liquid correlations in space but decay exponentially in time with a rate that depends on the strength of the interaction. The spectrum of the time evolved density matrix is gapped, which collapses gradually as $-\ln(t)$. The von Neumann entropy crosses over from the early time $-t\ln(t)$ behaviour to a $\ln(t)$ growth for late times. The early time dynamics is captured numerically by performing simulations on spinless interacting fermions, using several numerically exact methods. Our results establish the validity of bosonization for the early time dynamics of 1D lattice models, and could be observed experimentally in bosonic Luttinger liquids.

**Introduction.** While dissipation is traditionally viewed as detrimental due to causing decay and randomization of phase, recent years have witnessed a tremendous progress both in experiment and theory, as a result of which dissipation can now be considered as a useful tool or probe. Coupling to environment, combined with the ability to create and manipulate quantum systems in a controlled manner, has provided us with unique states of matter, where dissipation plays a major role. These include dark states, topological order as well as dissipative preparation of entangled states.

Coupling to environment also allows for controlling the temperature and offers a unique knob to study vaporization. By changing the temperature adiabatically, one can map out the phase diagram of conventional fluids. This reveals that the liquid and gas phases possess the same symmetry and are adiabatically connected by bypassing the critical end point. Considering quantum liquids provides intriguing twists to this story. For example, heating up a quantum spin liquid yields qualitatively distinct situation and the occurrence of a phase transition. The vaporization process of liquid He reveals the underlying quantum processes. Finally, the quantum effects near the event horizon of a black hole would give rise to the celebrated Hawking radiation and eventually to black hole evaporation.

A prototypical quantum liquid is realized in one dimension by a Luttinger liquid (LL). In such a phase, the non-interacting Fermi gas becomes unstable due to interaction and turns into LL, characterized by bosonic collective modes. This phase of matter is realized for spin, fermions, bosons, anyons etc, and can be simulated and probed by a variety of experiments.

The properties of LLs are thoroughly studied both in and out of equilibrium at zero and finite temperatures for closed systems. Here we focus on the evaporation dynamics of such systems by coupling it to a dissipative environment, modeled by the Lindblad equation. We find that the fermionic single particle density matrix retains its initial LL correlations in space but the amplitude is reduced in time due to dephasing. The von Neumann entropy crosses over from $-t\ln(t)$ for early times to $\ln(t)$ growth for late times. The early time dynamics is benchmarked numerically with dissipative interacting fermions. Our results are also relevant for dissipative interacting relativistic field theories, such as the massless Thirring model. Dissipation in the interacting Luttinger model. The low-energy behavior of one-dimensional systems is described by the Luttinger model whose Hamiltonian reads

$$H = \sum_{q>0} \omega_q (b_q^+ b_q + b_{-q}^+ b_{-q}) + g_q (b_q^+ b_{-q} + b_{-q}^+ b_q)$$

where $\omega_q = v|q|$, $g_q = g_2|q|$ and $b_q$ annihilates a bosonic excitation. Here $v = v_0 + g_4$ is the sound velocity, where $v_0$ is the bare sound velocity and $g_2$ and $g_4$ describes forward scattering between fermions with different and same chiralities, respectively. Since the Hamiltonian is quadratic in the bosonic operators, it can be diagonalized by the Bogoliubov transformation, yielding

$$H = E_{GS} + \sum_{q>0} \tilde{\omega}_q (d_q^+ d_q + d_{-q}^+ d_{-q})$$

where $E_{GS} = \sum_{q>0} (\tilde{\omega}_q - \omega_q)$ is the ground state energy and $\tilde{\omega}_q = v|q|$ is the spectrum of elementary excitations.
with the renormalized sound velocity \( \hat{v} = \sqrt{v^2 - g_2^2} \).

We consider a LL, prepared in the ground state of the interacting Hamiltonian thus no excitations are present. At \( t = 0 \), the coupling between the LL and its environment is switched on, and for \( t > 0 \), the time evolution is governed by the Lindblad equation [24–26]. The coupling to environment is modeled by local current operators, as in Refs. 27–30. Such dissipation arise naturally when considering fluctuating vector potential or gauge field as the environment. The Lindblad equation reads as

\[
\partial_t \rho = -i[H, \rho] + \gamma \int dx \left( [j(x), \rho j(x)] + \text{h.c.} \right)
\]

where \( \rho(t) \) is the density matrix of the system and \( j(x) \) is the current operator playing the role of the jump operator. Using bosonization [16], the current operator is

\[
j(x) = \sum_{q \neq 0} \sqrt{\frac{|q|}{2\pi L}} \text{sgn}(q)e^{-iqx} (b_+ - b_-)
\]

with \( L \) the system size and the spatial integral in Eq. (3) results in

\[
\partial_t \rho = -i[H, \rho] + \gamma \sum_{q \neq 0} \left( [L_q, \rho L_q^+ \gamma] + \text{h.c.} \right)
\]

with \( L_q = \sqrt{|q|} (b_q - b_+^q) \). The spectrum of Eq. (3) is expected to be gapless since the energy scale in both the Hamiltonian and the dissipator \( \sim |q| \). After Bogoliubov transformation, the jump operator is rewritten as \( L_q = \sqrt{|q|} (d_q - d_+^q) \), where \( K = \sqrt{(v - g_2)/(v + g_2)} \) is the Luttinger parameter [16] and \( K < 1 \) for repulsive (attractive) interaction. The presence of the interaction induces a renormalization of the dissipative coupling \( \gamma \rightarrow \gamma/K \). This indicates that dissipation becomes effectively stronger/weaker for repulsive/attractive interaction for the density matrix, respectively.

Based on the Lindblad equation, the expectation values of the occupation number and the anomalous operator are obtained as

\[
n_q(t) = \langle \rho(t)d_q^+d_q \rangle = \gamma |q| t/(\pi K)
\]

\[
m_q(t) = \langle \rho(t)\sqrt{i K} d_q^+d_\pi^q \rangle = \frac{\gamma}{2\pi i K} (e^{2\hat{v}|q| t} - 1)
\]

in accordance with Ref. 31. The linear increase of the occupation number implies that the system heats up to infinite temperatures and the LL eventually evaporates during the Lindblad dynamics. This is also follows from the observation that the jump operator is hermitian.

**Green’s function.** To have a deeper understanding of correlations, we study the time evolution of the single particle density matrix or equal time Green’s function defined as

\[
G(x,t) = \langle \rho(t)\Psi_R^+(x)\Psi_R(0) \rangle
\]

where \( \Psi_R(x) = \frac{1}{\sqrt{2\pi \alpha}} \exp \left[ i \sum_{q>0} \sqrt{\frac{2\gamma}{q}} \left( e^{iqx} b_q + e^{-iqx} b_\pi^q \right) \right] \) is the fermionic field operator of right-moving electrons. By evaluating the trace in Eq. (7), the single particle density matrix is obtained as

\[
\ln \frac{G(x,t)}{G_0(x)} = \sum_{q > 0} \frac{8\pi}{L|q|} \text{Re} m_q(t) - \frac{v}{\hat{v}} n_q(t) \sin^2 \left( \frac{2|x|}{\hat{v}} \right)
\]

where \( G_0(x) = \frac{1}{2\pi \alpha} \exp \left( \frac{\gamma}{\pi \alpha} \right) \left( \frac{K + K^{-1}}{2} \right)^{-1} \) is the initial Green’s function obeying the well-known [16] power-law decay for \( x \gg \alpha \) with the exponent of \((K + K^{-1})/2\). The momentum summation is regularized with the exponential cutoff \( \exp(-\alpha |q|) \) with \( \alpha \) the short distance cutoff.

It is important to note that the time-dependence of the single particle density matrix occurs only through the quantities \( n_q(t) \) and \( m_q(t) \) which have been calculated in Eqs. (3). Substituting these into Eq. (8), the summation over \( q \) is carried out analytically as

\[
\ln \frac{G(x,t)}{G_0(x)} = -\frac{\gamma t K^{-2} + 1}{\pi \alpha} \left( \frac{K}{2} \right)^2 + \frac{\gamma}{\pi \hat{v}} \left( \frac{1}{K^2 - 1} \right) I \left( \frac{\hat{v}}{\alpha}, \frac{x}{\alpha} \right)
\]

where \( I(y,z) = \arctan(2y) - \sum_{s=\pm} \frac{\arctan(2y - sz)}{2} \). In the scaling limit, when \( (x, \hat{v}) \gg \alpha \), the time evolution of the single particle density matrix is summarized as

\[
G(x,t) = \frac{i}{2\pi \alpha} \frac{1}{x} \frac{K + K^{-1}}{\pi \alpha K} \exp \left( -\frac{(K - 1)}{K} \right) \times \left\{ \begin{array}{ll} \exp \left( \frac{2\gamma}{\hat{v}}(K^{-2} - 1) \right) & \text{for} \ 2\hat{v} |x| \ll 1 \\ 1 & \text{for} \ 2\hat{v} |x| \gg 1 \end{array} \right.
\]

It exhibits two peculiar phenomena: the power law spatial decay of the single particle density matrix is preserved throughout the time evolution with the initial LL exponent of \((K + K^{-1})/2\). In addition, the spatial correlations are uniformly suppressed, exponentially in time, in accord with Ref. 27. The characteristic time scale of the dephasing is set by the dissipative coupling and the interaction strength as \( K \pi \alpha/(\gamma(K + K^{-1})) \), as found numerically in Fig. 1. The decay rate decreases from attractive \((K > 1)\) to repulsive \((K < 1)\) interaction: even though \( \gamma \) itself is renormalized to \( \gamma/K \) in the Lindblad equation, the original bare fermion, \( \Psi_R(x) \) is also dressed by the interaction, thus reverting the trend for the Green’s function. It is rather remarkable that in spite of the gapless spectrum of the Lindbladian, the fermionic Green’s function still decays exponentially in time. On top of this, one may observe a kink in the single particle density matrix which travels with the velocity \( 2\hat{v} \), which is the only light-cone effect, though this is rather minor and is expected to be hardly observable. The behaviour in Eq. (10) is rather generic and occurs for other correlation functions as well [32].
Time evolved density matrix and entropy. Another interesting quantity which characterizes the time evolution governed by the Lindblad equation, is the entropy defined as $S(t) = -\text{Tr} [\rho(t) \ln(\rho(t))]$. After determining the bosonized version of \(\rho(t)\), the trace is evaluated as

\[
S(t) = 2 \sum_{q > 0} \left[ (N_q(t) + 1) \ln(N_q(t) + 1) - N_q(t) \ln N_q(t) \right],
\]

where $N_q(t) = \sqrt{(n_q(t) + \frac{1}{2})^2 - |m_q(t)|^2 - \frac{1}{2}}$. Interestingly, the time-dependence occurs again only through the functions given in Eq. (2). Its early and long time limits are calculated as

\[
S(t) \sim \frac{L}{\pi \alpha} \begin{cases} 
-\gamma t 
& \text{for } \gamma t \ll K \pi \alpha \\
\ln \left( \frac{\gamma t}{K \pi \alpha} \right) 
& \text{for } \gamma t \gg K \pi \alpha
\end{cases}
\]

The early time growth agrees with numerics on dissipative interacting fermions in Fig. 2 while the latter is reminiscent of the behaviour of the entanglement gap in many-body localized systems.

In order to understand more closely the origin of this behaviour, we can evaluate also the eigenvalues of the time evolved density matrix at each time instant, denoted by $\lambda_0 \geq \lambda_1 \geq \lambda_2 \ldots$. Formally, we can also assign an instantaneous Hamiltonian to the time evolved density matrix, $\rho(t) = \exp(-H(t))$, whose spectrum is $-\ln \lambda_i$. We can define the gap in the many body spectrum as $\Delta_\rho = \ln(\lambda_0/\lambda_1)$. This is analogous to the spectrum of the reduced density matrix and the corresponding entanglement Hamiltonian and entanglement gap in closed quantum systems. Since the initial state is pure, the $t = 0$ spectrum is trivial. During the time evolution, the density matrix is brought to diagonal form after an instantaneous Bogoliubov transformation as $\rho(t) \sim \exp(-\sum_q \Omega_q(t) \hat{b}^\dagger_q \hat{b}_q)$, and for each momentum sector, the single particle spectrum is $\Omega_q(t) = \ln \left( 1 + \frac{1}{n_q(t)} \right)$. At $t = 0$, all $N_q(t = 0) = 0$, therefore $\Omega_q(t = 0) = \infty$, and the $\hat{b}_q$ bosons are in their vacuum state, the gap in the spectrum is infinitely large. After switching on the dissipation, the gap in the many body spectrum, which parallels closely to the entanglement gap, starts to decrease slowly for early times as

\[
\Delta_\rho \approx \ln \left( \frac{\pi K \alpha}{\gamma t} \right).
\]

Since there are many available momenta close the cutoff $1/\alpha$, all these together give the dominant contribution also to the von Neumann entropy for early times, and are responsible for the $S(t) \sim -t \ln(t)$ growth. We note that it is far from being obvious that bosonization accounts properly for the scaling of this gap. It originates from high energy modes at around the cutoff, while bosonization is designed to describe low energy, long wavelength excitations. As we demonstrate later in Fig. 6, the predicted $\gamma t$ dependence of $\Delta_\rho$ is surprisingly correct.

Interacting fermions within the Lindblad equation. To illustrate our findings and check their validity in lattice models, we have investigated one dimensional spinless fermions in an open tight-binding chain with nearest neighbour interaction at half filling using several numerical techniques. The closed system is equivalent to the 1D Heisenberg XXZ chain after a Jordan-Wigner transformation. The Hamiltonian is

\[
H = \sum_{m=1}^{N} \left[ \frac{J}{2} (c^\dagger_{m+1} c_m + c^\dagger_m c_{m+1}) + J_z n_{m+1} n_m \right],
\]

where $c$’s are fermionic operators, $n_m = c^\dagger_m c_m$ and $J_z$ denotes the nearest neighbour repulsion, $N$ the number of lattice sites and the model hosts $N/2$ fermions. This model realizes a LL for $|J_z| < J$ and the strength of the interaction is characterized by the dimensionless LL parameter $K = \pi/2[\pi - \arccos(J_z/J)]$ from the Bethe Ansatz solution of the model. Due to the bounded spectrum of Eq. (14), the bosonization results are only applicable for early times, before the whole band is populated during heating.

The lattice version of the current operator in Eq. 3. The early time scaling of the Green’s function for various $x$ values, obtained using three distinct numerical methods. The Green’s function decays with the same interaction dependent exponent at each spatial separation, $x$. Top panel: $J_z/J = 0.3$, $\Gamma/J = 0.04$ and $N = 22$ (thick solid line) using the quantum jump method with ED and PBC and 6000 averages over quantum trajectories and for $N = 14$ (thin dashed line) using ED with PBC for the Lindblad equation. Bottom panel: $J_z/J = -0.5$, $\Gamma/J = 0.4$ and $N = 41$ using TDVP (thick solid line) with OBC and for $N = 14$ (thin dashed line) using ED with PBC for the Lindblad equation. The agreement between various methods indicates that the data is relatively free from finite size effects. Here, $x = 1, 3, 5, 7, 9, 11, 13$ (blue, red, black, green, magenta, gold and light blue, respectively), but not all $x$’s are shown.
We consider a many-body system in a dissipative environment, with a dissipative Hamiltonian and a Lindblad equation. The Lindblad equation is given by:

$$\dot{\rho} = -i[H, \rho] + \sum_j \gamma_j (\rho J_j - J_j^\dagger \rho - \rho J_j^\dagger J_j)$$

where $H$ is the Hamiltonian of the system, $J_j$ are the Lindblad operators, and $\gamma_j$ are the decay rates.

In the context of Luttinger liquids, the dynamics of the single particle density matrix is important. The von Neumann entropy is used to quantify this, and is given by:

$$S(\rho) = -\text{Tr} (\rho \ln \rho)$$

For the single particle density matrix, the entropy converges fast to its maximal value on longer times.

With the knowledge of the time dependent density matrix, the dynamics of the von Neumann entropy is evaluated. For early times, it follows the expected $-\Gamma t \ln(\Gamma t)$ early time growth, and obeys the scaling form predicted by bosonization, as shown in Fig. 2. Here we had to account for the mild interaction dependence of the cutoff by slightly renormalizing the value of the rate $\Gamma \to \Gamma/\delta$.

Distinct cutoff dependent physical quantities may require slightly different interaction dependence of the cutoff. For late times, the entropy converges fast to its maximal value on the lattice $\sim N \ln(2)$ and the ln(t) late time growth of the LL is not reproduced due to the small local Hilbert space dimension (i.e. 2) for fermions. We speculate that this late time growth could possibly show up in bosonic realization of LLs, where the local Hilbert space is much bigger.

Finally, we evaluate the gap in the spectrum of the time evolved density matrix, as discussed above. Its numerically obtained value is shown in Fig. 3 which, in spite of its cutoff dependence, still follows the $-\ln(\Gamma t)$ prediction of bosonization.

Summary. We have studied the vaporization dynamics of Luttinger liquids after coupling to to dissipative environment through the local currents. The single particle density matrix retains the initial spatial power law LL correlations, but their amplitude becomes suppressed exponentially in time. The von Neumann entropy crossed
over from an early time \(-t\ln(t)\) growth to \(\ln(t)\) growth for late times. The former is attributed to the logarithmic time collapse of the instantaneous gap in the time evolved density matrix. The early time features are captured numerically in a dissipative interacting fermionic lattice model. Bosonization, which has proven valuable for closed quantum systems, is also relevant for the dynamics of open quantum systems.

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Time evolution of the single particle density matrix

The single particle density matrix is defined in Eq. (7) of the main text. Following standard steps \[16\], we obtain

\[
G(x,t) = \exp \left( - \sum_{q>0} \frac{4\pi}{Lq} B_q(t) (1 - \cos(qx)) \right),
\]

where

\[
G_{\text{nonint}}(x) = \frac{i}{2\pi(x + i\alpha)}
\]

is the non-interacting single particle density matrix and

\[
n_q^B(t) = \text{Tr} \left[ \rho(t) b_q^+ b_q \right]
\]

is the instantaneous number of \( b \)-bosons which describe the elementary excitations of the non-interacting system. After Bogoliubov transformation, the number of \( b \)-bosons is expressed as

\[
n_q^B(t) = \frac{v}{\bar{v}} n_q(t) - \frac{g_2}{\bar{v}} \text{Re} m_q(t) + \frac{1}{2} \left( \frac{v}{\bar{v}} - 1 \right)
\]

where

\[
n_q(t) = \text{Tr} \left[ \rho(t) d_q^+ d_q \right]
\]

and

\[
m_q(t) = \text{Tr} \left[ \rho(t) d_q^+ d_q^+ \right].
\]

After substituting \( n_q^B(t) \) into Eq. \[18\], the integral of the term with \( \frac{1}{2} \left( \frac{v}{\bar{v}} - 1 \right) \) leads to a power-law function of \( x \). This function (together with \( G_{\text{nonint}}(x) \)) results in the interacting correlation function

\[
G_0(x) = \frac{i}{2\pi(x + i\alpha)} \left( \frac{\alpha}{\sqrt{x^2 + \alpha^2}} \right)^{K+K^{-1}-1}
\]

which also equals the single particle density matrix in the initial state. The time dependence is described in the first two terms of Eq. \[19\] which, after all, end up in Eq. (8) of the main text.

Time evolution of entropy

In this section, the time dependence of the von Neumann entropy is studied. At any time instant, the system consists of two Bose gases for each \( q > 0 \) quantum numbers. Therefore, the entropy is defined as

\[
S(t) = -\text{Tr} \left[ \rho(t) \ln \rho(t) \right] = -2 \sum_{q>0} \left( (N_q(t) + 1) \ln(N_q(t) + 1) - N_q(t) \ln N_q(t) \right)
\]

where

\[
N_q(t) = \text{Tr} \left[ \rho(t) \tilde{b}_q^+ \tilde{b}_q(t) \right]
\]

is the number of bosons \( \tilde{b} \) which diagonalize the instantaneous density matrix. To calculate \( N_q(t) \) and the entropy, we determine

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where the operators $K_{q,+} = K_{q,-} = d^+_q d^-_q$ and $K_{q,0} = \frac{d^+_q d^-_q + d^-_q d^+_q}{2}$ obey the commutation relations of an $su(1,1)$ algebra. Note that all the time dependence is incorporated into the functions $c_q(t)$ and $v_q(t)$. The prefactor is set to $r_q(t) = (\nu_q(t)^2 - |c_q(t)|^2)/(\nu_q(t) + 1)$ in order to ensure the unit trace in each wavenumber channel. It can be shown that the functions are related to the expectation values $n_q(t)$ and $m_q(t)$, which are obtained in Eqs. (6) of the main text, by

$$v_q(t) = \frac{n_q(t)}{n_q(t)^2 - |m_q(t)|^2} \quad c_q(t) = \frac{m_q(t)}{n_q(t)^2 - |m_q(t)|^2}.$$  (23)

To diagonalize the exponent of (22), first we rewrite the product of the three exponentials in a single exponential by using the commutation rules of the $su(1,1)$ algebra [50, 51].

$$\rho(t) = \prod_{q>0} r_q(t) e^{\frac{\nu_q}{\sqrt{1 - |\nu_q|^2}} (s_q K_{q,-} + 2 K_{q,0} + s_q^* K_{q,+})}.$$  (24)

where

$$\Omega_q = \left| \text{acosh} \left( \frac{1}{2(n_q(t) + n_q(t)^2 - |m_q(t)|^2)} + 1 \right) \right|.$$  (25)

and

$$s_q = -\frac{2m_q(t)}{1 + 2n_q(t)}.$$  (26)

are both time dependent. Since the exponent of the density matrix is quadratic in the bosonic annihilation and creation operators, it can be diagonalized by the Bogoliubov transformation

$$\begin{bmatrix} \hat{b}_q(t) \\ \hat{b}_{-q}(t) \end{bmatrix}^+ = \begin{bmatrix} u_q(t) & v_q(t) \\ v_q(t)^* & u_q(t) \end{bmatrix} \begin{bmatrix} d_q \\ d_{-q}^* \end{bmatrix}$$  (27)

where

$$u_q(t) = \frac{1}{\sqrt{2}} \sqrt{\frac{1}{\sqrt{1 - |s_q(t)|^2}}} + 1$$  (28)

$$v_q(t) = \frac{s_q(t)^*}{\sqrt{2 |s_q(t)|}} \sqrt{\frac{1}{\sqrt{1 - |s_q(t)|^2}}} - 1$$  (29)

leading to $\rho(t) \sim e^{-\Omega_q(t)(\hat{b}_q(t)^+ \hat{b}_q(t))}$. For the entropy, we have to calculate the expectation value of the number of bosons $\hat{b}$. Substituting the Bogoliubov coefficients, we obtain

$$N_q(t) = \text{Tr} \left[ \rho(t) \hat{b}_q(t)^+ \hat{b}_q(t) \right] = (u_q(t)^2 + v_q(t)^2) n_q(t) + 2 \text{Re}(u_q v_q m_q(t)) + |v_q|^2 = \sqrt{\left(n_q(t) + \frac{1}{2}\right)^2 - |m_q(t)|^2 - \frac{1}{2}}.$$  (30)

**Spin-flip correlation function**

Eq. (14) is equivalent to the Heisenberg XXZ chain and can also be rewritten in terms of hard core bosons [16]. Then, the hard core boson equals time Green’s function or the spin flip correlation function [16, 39] is

$$C(x, t) = \frac{(-1)^x}{2\pi \alpha} \text{Tr} \left[ \rho(t) e^{-i \theta(x)} e^{i \theta(0)} \right]$$  (31)

where

$$\Theta(x) = \int \sum_{q \neq 0} \sqrt{\frac{\pi}{2L |q|}} e^{i q x} (b^+_q - b^-_q).$$  (32)

and the density matrix is given by Eqs. (22) and (23). Evaluating the trace, we obtain

$$\ln \frac{C(x, t)}{C_0(x)} = -\sum_{q>0} \frac{4\pi \sin^2 \left( \frac{|q|}{K} \right)}{LKq} (n_q(t) + \text{Re} m_q(t))$$  (33)

where

$$C_0(x) = \frac{(-1)^x}{2\pi \alpha \left( \frac{\alpha}{\sqrt{x^2 + \alpha^2}} \right)^\frac{1}{2\pi}}.$$  (34)

is the initial correlation function.

Using the results in Eqs. (6) of the main text, the sum over wavenumbers can be carried out analytically as

$$\ln \frac{C(x, t)}{C_0(x)} = -\frac{\gamma}{2\pi K^2} \left( \frac{\alpha^2 \vec{v} t}{\alpha^2 + x^2} + I \left( \frac{\vec{v} t}{\alpha} \right) \right)$$  (35)

where $I(y, z)$ is defined after Eq. (9) in the main text. In the scaling limit, i.e., when $2\vec{v} t \gg \alpha$ and $x \gg \alpha$,

$$C(x, t) = C_0(x) e^{-\frac{x}{2\pi \alpha \gamma}} \left\{ e^{-\frac{\gamma}{4\pi \alpha \gamma}} \right\}$$  (36)

decays exponentially with time.