Production of Chern-Simons bosons in
decays of mesons

Yuliia Borysenkova, Pavlo Kashko, Mariia Tsarenkova, Kyrylo Bondarenko, Volodymyr Gorkavenko

1Department of Physics, Taras Shevchenko National University of Kyiv, 64 Volodymyrs’ka str. 01601, Kyiv, Ukraine
2École Polytechnique Fédérale de Lausanne (EPFL), CH-1015 Lausanne, Switzerland
3SISSA, International School for Advanced Studies, Via Bonomea 265, I-34136 Trieste, Italy
4IFPU, Institute for Fundamental Physics of the Universe, via Beirut 2, 34151, Trieste, Italy
E-mail: yuliya.borisenckova@gmail.com, kashko.pavlo@gmail.com, ters.mar@gmail.com, kyrylo.bondarenko@gmail.com, gorkavol@gmail.com

Abstract: We consider the effective interaction of quarks with a new GeV-scale vector particle that couples to electroweak gauge bosons by the so-called effective Chern-Simons interaction. We call this particle the Chern-Simons (CS) boson. We construct effective Lagrangian of the CS boson interaction with quarks of two different flavors. This interaction is given by a divergent loop diagram, however, it turns out that the divergent part is equal to zero as a consequence of the CKM matrix unitarity in the SM. Therefore, we are able to predict effective interaction of the CS boson with quarks of different flavors without introducing new unknown parameters to the model, using only parameters of the initial effective Lagrangian. Our result shows that the effective interaction of the CS boson with down-type quarks is sufficiently stronger compared with up-type quarks. Based on our results, we give a prediction for the production of CS bosons in mesons decays. Branching fractions were obtained for the main reactions of the CS production in meson decays. The results obtained will be useful for searching for the long-lived GeV-scale CS boson in intensity frontier experiments.
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1 Introduction

Despite all the successes of the Standard Model (SM), see e.g. [1], there are some phenomena that can not be solved within the SM. These include baryon asymmetry of the Universe (see e.g. [2–4]), dark matter (see e.g. [5–7]), and neutrino oscillations (see e.g. [8–10]). In addition to these well-established phenomena, it should be noted that there is a number of observed parameters that are difficult to explain. For example, the strong CP problem (as to why the degree of CP violation in the QCD is unobservably small, see e.g. [11, 12]), the Higgs hierarchy problem (as to why quantum corrections to the Higgs mass cancel well, see e.g. [13, 14]), stability of the SM vacuum (top quark Yukawa coupling and the Higgs mass are very close to its critical value, see e.g. [15, 16]), the cosmological constant and dark energy (as to why the cosmological constant is so small, see e.g. [17]). Therefore, one can conclude that the SM is an incomplete theory and it requires an extension. Moreover, the existence of ”hidden” sectors with particles of new physics seems plausible.

Many of the aforementioned phenomena can be explained by an extension of the SM by Beyond Standard Model (BSM) particles. The values of their masses can be in very different ranges. For example, small neutrino masses, dark matter, and baryon asymmetry of the Universe can be explained by new particles with masses from the sub-eV scale up to the GUT scale, see e.g. [9, 18]. The fact that we do not observe BSM particles in accelerator experiments has two possible explanations. Either these particles are too massive to be produced at modern accelerators like the LHC, or they feebly interact with SM particles. If BSM particles are heavy enough, then to search for them we need more powerful, more expensive accelerators and energy frontier experiments, see e.g. [19, 20]. But the case of light but feebly interacting BSM particles is very topical for the experimental search for new physics right now. To search for them, we need intensity frontier experiments with high-intensity particle beams and large detectors, see e.g. [21, 22]. Several such intensity frontier experiments have been proposed in recent years: MATHUSLA [23], FACET [24], FASER [25, 26], SHiP [27, 28], NA62 [29–31], DUNE [32, 33], etc.

Searching for new physics, one should keep in mind that among the hidden particles there must be particles that solve the above-mentioned SM problems. However, the hidden sector may also contain particles that are not directly related to the solution of an SM problem. That is why it is important to look for manifestations of new physics particles at all accessible energy scales. In this work we will be interested in GeV-scale feebly interacting new particles.

The properties of the new light particles are not yet known. They can be scalars (e.g. [34–36]), pseudoscalars (axion-like particles), see e.g. [37–40], vectors (e.g. [41–43]), or fermions (e.g. [44–47]). Each of these options requires thorough study.
Discussion of their possible interactions with SM particles (portals) and the available experimental constraints are given e.g. in reviews [23, 28].

In this paper, we will be interested in consideration of the Chern-Simons portal, which, in our opinion, has received insufficient attention especially concerning experimental search for the Chern-Simons particles. In this portal a new vector particle $X_\mu$ couples with the SM gauge fields by the so-called effective Chern-Simons interaction in the form of 4-dimension operators [48]:

$$
L_{CS} = c_\gamma \epsilon^{\mu\nu\lambda\rho} X_\mu Z_\nu \partial_\lambda A_\rho + \{ c_w \epsilon^{\mu\nu\lambda\rho} X_\mu W^-_\nu \partial_\lambda W^+_\rho + h.c. \}, \quad (1.1)
$$

where $A_\mu$ is the electromagnetic field; $W^\pm_\mu$ and $Z_\mu$ are fields of the weak interaction; $\epsilon^{\mu\nu\lambda\rho}$ is the Levi-Civita symbol ($\epsilon^{0123} = +1$) and $c_z, c_\gamma, c_w$ are some dimensionless independent coefficients. Coefficients $c_z$ and $c_\gamma$ are real, but $c_w$ can be complex. As one can see, there is no direct interaction of the Chern-Simons (CS) vector boson $X_\mu$ with fields of the matter. It should be noted that Lagrangian (1.1) does not contain term $\epsilon^{\mu\nu\lambda\rho} X_\mu A_\nu \partial_\lambda A_\rho$ which is not gauge invariant with respect to the electromagnetic $U(1)$ group.

The interest in the Chern-Simons model is due to the fact that the CS interaction is derived by an anomaly cancellation. This way is theoretically attractive because the contribution of extremely heavy fermions (not available for direct search at accelerators) to anomaly cancellation remains unsuppressed at low energies [49, 50]. These heavy fermions can act as mediators and induce observable interaction of the SM particles with new light vector particles from the “hidden” sector. A detailed explanation of the origin of the CS interactions (1.1), including using of a toy UV model, can be found in [48]. The Chern-Simons interactions appear in various theoretical models, including extra dimensions and the string theory, see [51–60]. In addition to the need to study the CS theory as one of the possible extensions of the SM, the presence of the Levi-Civita symbol indicates the possible effects of violation of CP invariance in the CS theory, and perhaps the CS boson will be useful in solving some of the SM problems. On the other hand, if the CS boson is discovered, this will unequivocally indicate the presence of new physics at high energy scales. It is important to note that the CS boson was included in the proposal of the SHiP experiment [28].

In order to start looking for the CS boson in experiments, one needs to know the main channels of the CS boson production and decay. In this paper we will consider only the production of GeV-scale CS bosons. We will limit ourselves only to the case of the CS boson production in the decays of different types of mesons. To do this, we will consider loop interaction of the CS boson with the SM fermions and construct the effective Lagrangian of the CS bosons interaction with two different
quarks. This will allow us to find more effective channels for the production of CS bosons and bring us closer to the experimental search for CS bosons.

The paper is organized as follows: in section 2 we discuss theoretical aspects of the Chern-Simons model. In section 3 we obtain the effective Lagrangian of the CS boson interaction with different quarks; in section 4 we consider the CS boson production in mesons decays. The summary and final discussion are given in section 5. Necessary technical clarifications and details about form-factors used are given in appendices.

2 Chern-Simons model

The simplest case of the SM extension with non-trivial anomaly cancellation that involves the electromagnetic $U_{EM}(1)$ gauge group requires small parameters of the photon mass or millicharge of new particles, see [61–64]. These parameters are strongly restricted, see e.g. [65–68], and suppress any visible effects. We get a similar situation when considering non-trivial anomaly cancellation in the electroweak sector of the SM. The small parameter, in this case, is the sum of the electric charges of the electron and the proton, which is also limited to a very small value [69, 70].

To avoid the need to deal with very small parameters of the models that suppress any visible effects it is interesting to consider extension of the SM by an additional gauge group, see e.g. [71] and references therein. In particular, Lagrangian (1.1) can be obtained in $U_X(1)$ gauge field extension of the SM to the general theory with $SU_C(3) \times SU_W(2) \times U_Y(1) \times U_X(1)$ symmetry. One should also add to the theory heavy new chiral fermions that interact both with the gauge field of $U_X(1)$ and with the SM gauge fields. Herewith the SM fermions are not charged with respect to the $U_X(1)$ group [64].

Lagrangian (1.1) has $U_{EM}(1)$ gauge invariance, but unfortunately we need at least 6-dimension operators [28, 48] to restore $U_Y(1) \times SU_W(2)$ invariance:

\[
\mathcal{L}_1 = \frac{C_Y}{\Lambda_Y^2} \cdot X_\mu (\mathcal{D}_\nu H)^\dagger H \mathcal{B}_\rho \cdot \varepsilon^{\mu \nu \lambda \rho} + h.c.,
\]

\[
\mathcal{L}_2 = \frac{C_{SU(2)}}{\Lambda_{SU(2)}^2} \cdot X_\mu (\mathcal{D}_\nu H)^\dagger F_{\lambda \rho} H \cdot \varepsilon^{\mu \nu \lambda \rho} + h.c.,
\]

where $\Lambda_Y$, $\Lambda_{SU(2)}$ are new scales of the theory; $C_Y$, $C_{SU(2)}$ are new dimensionless coupling constants; $H$ – scalar field of the Higgs doublet; $B_{\mu \nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$, $F_{\mu \nu} = -ig \sum_{i=1}^3 \tau^i V_{\mu \nu}^i$ – field strength tensors of the $U_Y(1)$ and $SU_W(2)$ gauge fields.

It is convenient to rewrite the coefficients before operators of dimension-6 as $C_Y/\Lambda_Y^2 = C_1/v^2$ and $C_{SU(2)}/\Lambda_{SU(2)}^2 = C_2/v^2$, where $C_1 = c_1 + ic_{11}$ and $C_2 = c_2 + ic_{21}$ are di-
dimensionless coefficients, \( v \) is the vacuum expectation value of the Higgs field. In this case Lagrangians (2.1), (2.2) after the electroweak symmetry breaking generate Lagrangian of three fields interactions (1.1) with coefficients in unitary gauge:

\[
c_z = -c_{1i}g' + \frac{c_2}{2}g'^2, \tag{2.3}
\]

\[
c_\gamma = c_{1i}g + \frac{c_2}{2}gg', \tag{2.4}
\]

\[
c_w = \frac{c_2 + ic_{2i}}{2}g^2 \equiv \Theta W_1 + i\Theta W_2. \tag{2.5}
\]

In [28] it was pointed out that from dimension-6 operators (2.1), (2.2) follows \( c_z/c_\gamma = \tan \theta_W \). However, we see from relations (2.3) – (2.5) that ratio \( c_z/c_\gamma \) depends on two unknown parameters \( c_{1i} \) and \( c_2 \). It is evident that the ratio of these quantities has a simple form only in the particular case when the real part of the parameter \( C_2 \) is zero \( (c_z/c_\gamma = -\tan \theta_W) \), or the imaginary part of the parameter \( C_1 \) is zero \( (c_z/c_\gamma = \cot \theta_W) \). As one can see, relations (2.3) – (2.5) allow us to write a relation between the real part of parameter \( C_2 \) and \( c_z, c_\gamma \) that gives \( \Theta W_1 = \cos^2 \theta_W (c_z + \tan \theta_W c_\gamma) \). It is also interesting that the real part of parameter \( C_1 \) (parameter \( c_{1i} \)) is not included into (2.3) – (2.5) and the imaginary part of parameter \( C_2 \) (parameter \( \Theta W_2 \)) is an independent parameter.

It should be noted that relations (2.3) – (2.5) and corresponding conclusions are valid only in the assumptions that the Chern-Simons theory is derived only by dimension-6 operators (2.1), (2.2). But it may turn out that the main or a significant contribution to Lagrangian (1.1) comes from higher dimension operators. So, following [28], below we will consider \( c_z, c_\gamma, c_w \) as independent dimensionless parameters.
3 Effective interaction of the CS boson with different quarks

Lagrangian (1.1) gives diagrams of the CS boson loop interactions with two SM fermions presented in figure 1. Since we want to consider the CS boson production in decays of mesons we only need diagrams (c) and (d), where a heavy initial particle decays in a light particle with the production of the CS boson. It means we can consider in Lagrangian (1.1) only the interaction of the CS boson with $W$ bosons and put $c_z = c_\gamma = 0$ for simplicity.

Certainly, one can present four fermion interaction with the CS boson without loop, see figure 2, but this interaction will be suppressed by $G_F^2$ and, as it will be shown in section 3.3, can be neglected compared to the loop interaction of the CS boson with two fermions.

3.1 Interaction of the CS boson with two different down-type quarks

Computation of the loop diagram for the interaction of the CS boson with down-type quarks gives us, see figure 4 and appendix A, the following amplitude of the heavy down-type quark ($d_n$) decay into light down-type quark ($d_m$) and the CS boson in the unitary gauge:

$$M_{fi} = \frac{g^2}{2} \sum_{i=u,c,t} V_{d_i+}^\dagger V_{d_m} \bar{d}_m(p') Y_{(i)}^\mu d_n(p) \epsilon^{\lambda \mu \nu \rho}_f,$$  \hspace{1cm} (3.1)

where $Y_{(i)}^\mu$ has a rather cumbersome form

$$Y_{(i)}^\mu = \hat{\Lambda}_0 \left\{ 2(x + y - 1) \{ \Theta_{W1}(2y - 1) - i\Theta_{W2} \} - 2y \{ \Theta_{W1}(1 - 2x - 2y) + i\Theta_{W2} \} - \frac{x}{M_W^2} \left[ c_w \{ (3x + 3y - 1)m^2 + ym'^2 - 4y(p p') \} \right] - \right.$$

$$- 2\Theta_{W1} \{ (x + y)^2m^2 + y^2m'^2 - 2y(x + y)(p p') \} +$$

$$+ 2i\Theta_{W2} \{ (x + y)m^2 - 2y(p p') \} \right\} p_\lambda^\dagger \gamma'_\rho p_\nu \hat{P}_L \epsilon^{\mu \nu \lambda \rho} +$$

$$- \hat{\Lambda}_0 m' \{ \Theta_{W1}((1 - 2x - 2y)p + (2y - 1)p')_\lambda + i\Theta_{W2}(p - p')_\lambda \} \gamma_\rho \gamma'_\nu \hat{P}_L \epsilon^{\mu \nu \lambda \rho} +$$

$$+ \hat{\Lambda}_1 \left\{ -i\Theta_{W1}\gamma_\rho \gamma_\lambda \gamma'_\nu \gamma'_\lambda + 2iP_\lambda^\dagger \gamma_\rho p_\nu \Theta_{W1}(1 - 3x) \right\} \hat{P}_L \epsilon^{\mu \nu \lambda \rho} +$$

$$+ \hat{\Lambda}_0 \left\{ - (x + y - 1)m \{ \Theta_{W1}[(1 - 2x - 2y)p + (2y - 1)p']_\lambda + i\Theta_{W2}(p - p')_\lambda \} \gamma_\rho \gamma'_\nu +$$

$$+ \frac{x m m'}{M_W^2} \Theta_{W1}(1 - x) + i\Theta_{W2}(2y + x - 1) \right\} p_\lambda^\dagger \gamma_\rho p_\nu \hat{P}_L \epsilon^{\mu \nu \lambda \rho} -$$

$$- \hat{\Lambda}_1 i(p - p')_\lambda \gamma_\rho \gamma'_\nu c_w m \hat{P}_R - c_w m' \hat{P}_L \epsilon^{\mu \nu \lambda \rho},$$  \hspace{1cm} (3.2)
integral operators $\hat{\Lambda}_{0(1)}$ are defined as
\begin{align}
\hat{\Lambda}_0 &= i\frac{\pi^2}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \frac{1}{D(m_i)}, \\
\hat{\Lambda}_1 &= -\frac{\pi^2}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \ln \frac{\Lambda^2 x}{D(m_i)}.
\end{align}

Here
\begin{equation}
D(m_i) = x m_i^2 + (1 - x) M_W^2 + x y (M_X^2 + m^2 - m'^2) - x(1 - x) m^2 - y(1 - y) M_X^2,
\end{equation}
$\hat{P}_{R(L)}$ – projection operators on the right(left)-handed chirality states, $p$ is the 4-momentum and $m$ is the mass of the initial down-type quark ($d_n$), $p'$ is the 4-momentum and $m'$ is the mass of the final down-type quark ($d_m$), $m_i$ is the mass of the virtual up-type $u_i$ quark, $M_W$ and $M_X$ are the masses of the W and CS vector bosons, $\Lambda$ is the regularization parameter that should be set to infinity.

The divergent part of $Y_{(i)}^\mu$ is hidden in the operator $\hat{\Lambda}_1$ (3.4). We can extract in this operator singular and finite parts:
\begin{equation}
\hat{\Lambda}_1 = \hat{\Lambda}_1^{\text{sing}} + \hat{\Lambda}_1^{\text{fin}} = -\frac{\pi^2}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \left\{ \ln \frac{\Lambda^2 x}{\mu^2} - \ln \frac{D(m_i)}{\mu^2} \right\},
\end{equation}
where $\mu$ is some parameter with the dimension of mass. It should be noted that operator $\Lambda_1^{\text{sing}}$ does not depend on the mass of the virtual quark in the loop ($m_i$) and it acts on an expression that does not depend on $m_i$ too. So, after summation over all virtual quarks in (3.1) we get a term proportional to
\begin{equation}
\sum_{i=u,c,t} V_{d_n i}^+ V_{d_m i} \ln \frac{\Lambda^2 x}{M_W^2} = (V^+ V)_{d_m d_n} \ln \frac{\Lambda^2 x}{M_W^2} = 0, \quad m \neq n,
\end{equation}
due to the unitarity of the CKM-matrix in the SM. So, in the expression (3.2) we can replace operator $\hat{\Lambda}_1$ by finite operator $\hat{\Lambda}_1^{\text{fin}}$
\begin{equation}
\hat{\Lambda}_1^{\text{fin}} = \frac{\pi^2}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \ln \frac{D(m_i)}{\mu^2}.
\end{equation}
With the help of the unitary condition for the CKM-matrix, it is not difficult to show that the amplitude of the process does not depend on the value of the parameter $\mu$, see appendix A. In the following, we will put $\mu = M_W$ for the convenience of computations. It should be noted that the established fact of the divergent part of the CS boson loop interaction with quarks of two different flavors being zero (as a consequence of the CKM matrix unitarity in the SM) is consistent with the results of [59, 60].

An explicit form of the $Y_{(i)}^\mu$ can be obtained after applying the operators $\hat{\Lambda}_0$, $\hat{\Lambda}_{fin}^i$ and performing summation over the virtual up-type quarks in (3.2), see appendix B.

The amplitude of a heavy down-type quark (d$_m$) (with mass $m$ and 4-momentum $p$) decay into a light down-type quark (d$_m$) (with mass $m'$ and 4-momentum $p'$) and the CS boson in the unitary gauge (3.1) is convenient to present in the form

$$M_{fi} = \frac{g^2}{32\pi^2} \frac{m_t^2}{M_W^2} V_{dm}^+ V_{td} \overline{d}_m(p') J_{\mu}^{d}(p, p') d_m(p) \epsilon_{\mu}^{\lambda \chi},$$  \hspace{1cm} (3.8)

where $m_t$ is mass of the top quark and

$$J_{\mu}^{d} = \left( a_{dL}^{L} \hat{P}_R \gamma_\rho \gamma_\lambda \gamma_\nu \hat{P}_L \right. +
\left. b_{dL}^{L} \frac{p_\alpha}{M_W^2} \hat{P}_R \gamma_\rho \hat{P}_L +
\left. c_{dL}^{L} \frac{m_1 p_\lambda}{M_W^2} \hat{P}_L \gamma_\rho \gamma_\nu \hat{P}_L +
\left. d_{dL}^{L} \frac{m_2 p_\lambda}{M_W^2} \hat{P}_L \gamma_\rho \gamma_\nu \hat{P}_L \right) \epsilon^{\mu \nu \lambda \rho}. \hspace{1cm} (3.9)$$

Coefficients in (3.9) in the first approximation can be considered independent of the masses of down-type quarks. They can be computed numerically, but we also managed to find the analytical expression for these coefficients with less than 1% difference from their values obtained numerically, see appendix B. The values of the coefficients are given in table 1. Coefficients at $\Theta_{W1}$ are imaginary with sufficient accuracy, but coefficients at $\Theta_{W2}$ are real. Taking into account that near the coefficients $b_{L,R}^{d}$, $c_{L,R}^{d}$, $d_{L,R}^{d}$ there are suppressing factors, one can see that the main contribution to (3.9) comes from the term $a_{L}^{d}$ (if $\Theta_{W1} \neq 0$). It should be noted that coefficient $a_{L}^{d}$ depends only on one parameter ($\Theta_{W1}$) unlike almost all other coefficients depending on both parameters: $\Theta_{W1}$ and $\Theta_{W2}$.

3.2 Interaction of the CS boson with two different up-type quarks

Let us now consider the interaction of the CS boson with up-type quarks. The amplitude of a heavy up-type quark (u$_n$) decay into a light up-type quark (u$_m$) and the CS boson in the unitary gauge is given by

$$M_{fi} = \frac{g^2}{2} \sum_{i=d,s,b} V_{im}^+ V_{un} \overline{u}_m(p') Y_{(i)}^\mu u_n(p) \epsilon_{\mu}^{\lambda \chi}, \hspace{1cm} (3.10)$$
where \( Y^\mu_{(i)} \) is defined in (3.2). However, in this case \( p \) is the 4-momentum and \( m \) is mass of the initial up-type quark \((u_n)\), \( p' \) is the 4-momentum and \( m' \) is mass of the final up-type quark \((u_m)\), \( m_i \) is mass of the virtual down-type quark \((d_i)\).

It should be noted that the mass of the virtual down-type quark \((d_i)\) in \( Y^\mu_{(i)} \) is included only in function \(D(m_i)\) (3.5), but for all virtual down-type quarks \( m_i \ll M_W \). So, in the first approximation function, \( D(m_i) \) can be considered independent of the mass of virtual down-type quark \( m_i \) and can be taken as \( D(m_i) = (1 - x)M_W^2 \). It means that \( Y^\mu_{(i)} \approx Y^\mu \) and

\[
\sum_{i=d,s,b} V_{um_i}^+ V_{in} Y^\mu_{(i)} \approx Y^\mu \sum_{i=d,s,b} V_{um_i}^+ V_{in} = 0, \quad m \neq n. \tag{3.11}
\]

Thus, in the first approximation the amplitude of a heavy up-type quark \((u_n)\) decay into a light up-type quark \((u_m)\) and the CS boson is zero. If we perform accurate calculations and present the amplitude in a form similar to the case of the down-type quarks:

\[
M_{fi} = \frac{g^2}{32\pi^2} \frac{m_i^2}{M_W} V_{st}^+ V_{tb} \bar{u}_m(p') J^{\mu,up} u_n(p) e_{\mu}^{\lambda\lambda}, \tag{3.12}
\]

where \( J^{\mu,up} \) has form (3.9), but with coefficients with superscript \( up \), we get values presented in the table 1.

### 3.3 Lagrangian of the effective interaction of the CS boson with two different quarks

After analyzing the data in table 1 one can conclude that the production of CS bosons in decays of up-type quarks is substantially suppressed in comparison with the production of CS bosons in decays of down-type quarks.

**Table 1:** Coefficients in the expression for the amplitude of a heavy quark decay into a down-type quark and the CS boson in (3.9). Superscript \( d \) stands for the decay of down-type quark \( d_n \rightarrow d_m + X \), superscript \( up \) is for the decay of up-type quark \( c \rightarrow u + X \).

| coeff. | value | coeff. | value \( \times 10^6 \) |
|--------|-------|--------|-------------------------|
| \( a_{dL}^d \) | \(-0.13i \Theta_{W1} \) | \( a_{dR}^{up} \) | \((-0.98 + 1.13i) \Theta_{W1} \) |
| \( b_{dL}^d \) | \(0.05i \Theta_{W1} + 4 \cdot 10^{-5} \Theta_{W2} \) | \( b_{dR}^{up} \) | \((-6.4 + 7.4i) \Theta_{W1} - (1.1 + 0.98i) \Theta_{W2} \) |
| \( c_{dL}^d \) | \(-0.058i \Theta_{W1} - 0.098 \Theta_{W2} \) | \( c_{dR}^{up} \) | \((0.27 - 0.31i) \Theta_{W1} + (0.57 + 0.49i) \Theta_{W2} \) |
| \( d_{dL}^d \) | \(0.086i \Theta_{W1} + 0.098 \Theta_{W2} \) | \( d_{dR}^{up} \) | \((0.38 - 0.44i) \Theta_{W1} - (0.57 + 0.49i) \Theta_{W2} \) |
| \( b_{uR}^d \) | \(-0.028i \Theta_{W1} \) | \( b_{uR}^{up} \) | \((-0.64 + 0.75i) \Theta_{W1} \) |
| \( c_{uR}^d \) | \(0.086i \Theta_{W1} - 0.098 \Theta_{W2} \) | \( c_{uR}^{up} \) | \((0.87 - i) \Theta_{W1} + (1.1 + 0.98i) \Theta_{W2} \) |
| \( d_{uR}^d \) | \(-0.058i \Theta_{W1} + 0.098 \Theta_{W2} \) | \( d_{uR}^{up} \) | \((-0.23 + 0.25i) \Theta_{W1} - (1.1 + 0.98i) \Theta_{W2} \) |
If parameter $\Theta_{W_1}$ is nonzero, then the main contribution for the CS boson production from down-type quarks comes from the term $a_d^L = -ia\Theta_{W_1}$, see (3.9). Using relation

$$\epsilon^{\alpha\mu\nu\rho} \gamma_\mu \gamma_\nu \gamma_\rho = 6i\gamma^\alpha \gamma^5,$$

one can write the effective Lagrangian of the GeV-scale CS boson interaction with different down-type quarks in the form

$$\mathcal{L}_{\text{quarks}}^{CS} = \sum_{m<n} \Theta_{W_1} \left( C_{mn} \overline{d}_m \gamma^\mu \hat{P}_L d_n X_\mu + C_{nm} \overline{d}_n \gamma^\mu \hat{P}_L d_m X_\mu \right),$$

(3.14)

where the summation occurs over the generations of quarks,

$$C_{mn} = \frac{3a}{2\sqrt{2}\pi^2} G_F m^2 V^*_{dm} V_{dn},$$

(3.15)

and

$$|C_{sb}| = 1.97 \cdot 10^{-4}, \quad |C_{db}| = 4.43 \cdot 10^{-5}, \quad |C_{ds}| = 1.77 \cdot 10^{-6}.$$  

(3.16)

Interaction of the GeV-scale CS boson with up-type quarks can be neglected. These results are consistent with the results of [59, 60].

There are two points to pay attention to. First, Lagrangian (3.14) is valid only for the GeV-scale CS boson interaction with different quarks. The case of the CS boson interaction with the same quarks must be considered separately. In the last case, we can not eliminate divergences in the loop integral using only the condition of the CKM-matrix unitarity. Second, Lagrangian (3.14) is slightly similar to the interaction Lagrangian of quarks with $W$ bosons in the SM. As in the case of the SM the term $\overline{d}_m \gamma^\mu \hat{P}_L d_n X_\mu$ has no symmetry under the separate action of charge conjugation ($\hat{C}$) and parity transformation ($\hat{P}$) operators. But this term will have symmetry under the simultaneous action of charge conjugation and parity transformation operators

$$\hat{C} \hat{P} \overline{d}_m \gamma^\mu \hat{P}_L d_n X_\mu = \overline{d}_n \gamma^\mu \hat{P}_L d_m X_\mu,$$

(3.17)

if we impose a reasonable condition

$$\hat{C} \hat{P} X_\mu = -X_\mu.$$ 

(3.18)

Thus, having analyzed the effective Lagrangian (3.14), we come to the conclusion that the CS boson has no well-defined symmetry under the separate action of charge conjugation ($\hat{C}$) and parity transformation ($\hat{P}$) operators, but is even under the simultaneous action of $\hat{C} \hat{P}$ transformation like $Z$ boson.

As in the case of the interaction Lagrangian of quarks with $W$ bosons in the SM, the effective Lagrangian (3.14) will be $\hat{C} \hat{P}$ invariant only if matrix $C_{mn}$ is real.
4 The CS boson production in decays of mesons

Since the CS boson interaction with up-type quarks is strongly suppressed, we will consider only production of CS bosons from mesons containing $b$ or $s$ quarks. Such lightest mesons are $B$ mesons and $K^0\pi^\pm$, $K^0\pi^0$, $K^0\pi^\mp$, $K^0\eta$, and $K^0\eta'$ mesons.

Dominant reactions of the $B$ meson decay with the CS boson production are two-body decays into pseudoscalar mesons ($K^0\pi^0$, $K^0\pi^\mp$, $K^0\eta$, $K^0\eta'$, $K^0\pi^\pm$); scalar mesons $K^0\eta^\mp(400)$, $K^0\eta^\pm(1430)$, $K^0\eta^\mp(1430)$; vector mesons $K^0(802)$, $K^0(1400)$, $K^0(1680)$; pseudovector mesons $K^0(1270)$ and $K^0(1400)$; tensor final meson state $K^0(1430)$.

For the initial kaon states, the only possible 2-body decay with the CS boson production is the process

$$K \rightarrow \pi + X.$$  \hspace{1cm} (4.1)

There are 3 types of kaons – $K^\pm$, $K^0_L$, $K^0_S$. Although the decay width for each of them is given by the same loop factor ($C_{d\bar{s}}$) the branching ratios differ. The first reason is that these kaons have different decay widths. The second reason is that $\pi^0$ meson is the $CP$-odd eigenstate, $K^0_S$ is approximately the $CP$-even eigenstate, and $K^0_L$ is approximately the $CP$-odd eigenstate. The $CP$ parity of the final state in the reaction $K^0_L \rightarrow \pi^0 X$ is $(-1)^L (C_P)_{\pi^0}(C_P)_X = +1$ (since the total angular momentum of the initial meson is zero, the final particles must have orbital angular momentum $L = 1$). Therefore, the decay width of the reaction $K^0_L \rightarrow \pi^0 X$ is proportional to the CKM $CP$-violating phase [72, 73].

Branching of the two-body meson decay $h \rightarrow h' + X$ is defined as

$$Br(h \rightarrow h'X) = \frac{1}{\Gamma_h} \frac{|M_{h \rightarrow h'X}|^2}{8\pi M_h^2} |\vec{k}|,$$  \hspace{1cm} (4.2)

where

$$|\vec{k}| = \frac{\sqrt{\lambda(M_h^2, M_{h'}^2, M_X^2)}}{2M_h},$$  \hspace{1cm} (4.3)

and $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ is the Källén function [74].

The amplitude of the decay of a containing $d_n$ quark $h$ meson into a containing $d_m$ quark $h'$ meson and the CS boson has form, see (3.14),

$$M_{h \rightarrow h'X} = \Theta_{1W} C_{mn} \langle h'(p')|\bar{d}_n\gamma^\mu \hat{P}_L d_m|h(p)\rangle \epsilon^\alpha\lambda_X,$$  \hspace{1cm} (4.4)

where the absolute values of $C_{mn}$ are given by (3.15). In the case of decay of $K^0_S$, $K^0_L$ mesons, one has to be more careful due to the presence of a certain $CP$ parity in them, namely

$$M_{K^0_L \rightarrow \pi^0 X} = \Theta_{1W} Im[C_{d\bar{s}}] \langle \pi^0(p')|\bar{d}\gamma^\mu \hat{P}_L s|K^0_L(p)\rangle \epsilon^\alpha\lambda_X,$$  \hspace{1cm} (4.5)

$$M_{K^0_S \rightarrow \pi^0 X} = \Theta_{1W} Re[C_{d\bar{s}}] \langle \pi^0(p')|\bar{d}\gamma^\mu \hat{P}_L s|K^0_S(p)\rangle \epsilon^\alpha\lambda_X.$$  \hspace{1cm} (4.6)
where $\text{Re}[C_{ds}] = -1.62 \cdot 10^{-6}$ and $\text{Im}[C_{ds}] = -6.69 \cdot 10^{-7}$.

The average over meson states $\langle h'(p')|\bar{d}_m\gamma^{\mu}\hat{P}_Ld_n|h(p)\rangle$ can be obtained with help of formalism summarized e.g. in [36].

In the rest frame of the initial $h$ meson we have $p = (M_h, 0)$. It decays into the $h'$ meson and the CS boson with momentums

$$p' = (E', -\vec{k}), \quad p_X = (E_X, \vec{k}), \quad p_X = p - p'.$$

where direction of spatial vector $\vec{k}$ is arbitrary, but module $|\vec{k}|$ is defined by (4.3). Let's guide $Z$-axis along the spatial momentum of the CS boson, then 4-vector of the CS boson polarisation is defined as

$$\epsilon^{(\pm)}_X = (0, 1, \mp i, 0) / \sqrt{2}, \quad \epsilon^{(0)}_X = \left(|\vec{k}|, 0, 0, E_X\right) / M_X,$$

where $\epsilon^{(\pm)}_X$ corresponds to the spin projection $\pm 1$ and $\epsilon^{(0)}_X$ corresponds to zero spin projection on the momentum direction.

If $\langle h'(p')|\bar{d}_n\gamma^{\mu}\hat{P}_Ld_n|h(p)\rangle$ depends only on $p^\mu$ or $p'^\mu$, then only the longitudinal component of the CS boson polarisation gives contribution into the amplitude of the reaction because $\epsilon^{(\pm)}_X|p| = \epsilon^{(\pm)}_X|p'| = 0$. It will be useful to write also

$$\epsilon^{(0)}_X|p| = \epsilon^{(0)}_X|p'| = |\vec{k}| \frac{M_h}{M_X}, \quad \epsilon^{(0)}_X|p_X| = 0.$$

Results for the branchings for the corresponding reactions are presented in table 2, where for the decays of the $B$ mesons we took into account only the lightest final excited $K$ meson states from its scalar, pseudoscalar, vector, pseudovector and tensor states. The branching dependencies on the CS boson mass for decays of charged $K$ mesons and neutral $K^0_L, K^0_S$ mesons are shown in figure 3(a). The branching dependencies on the CS boson mass for decays of charged $B$ mesons are shown in figure 3b, where, for clarity, we have summed up the contributions over the (pseudo)scalar, (pseudo)vector and tensor channels. Because of the inverse quadratic divergence of the amplitude of meson decay at small masses of the CS boson (it originates from its longitudinal component), values of the reaction branchings are presented for quantity $\lim_{m_X \to 0} \left( \frac{m_X}{1\text{GeV}} \right)^2 \frac{Br(m_X)}{\theta_w^2}$. Main contribution to the uncertainties in the presents results follows from uncertainties in meson transition form-factors. Our results are consistent with the results of [60], where some of the reactions we considered had been calculated.
Table 2: Properties of the main production channels of the CS boson from kaons and $B$ mesons. First column: decay channels; Second column: type of final mesons; Third column: branching ratios of 2-body meson decays evaluated at $m_X \to 0$ and normalized by $\theta_{W1}^2$. For $B$ mesons the numerical values are given for $B^\pm$ mesons; in the case of $B^0$ meson all the given branching ratios should be multiplied by a factor of 0.93 that comes from the difference in total decay widths of $B^\pm$ and $B^0$ mesons [70]; Fourth column: closing mass, i.e. the difference between the masses of the initial and the final mesons; Fifth column: a reference to the appendix with details about form-factors used.

| Process | final meson | $\lim_{m_X \to 0} \left( \frac{m_X}{1\text{GeV}} \right)^2 \frac{\text{Br}(m_X)}{\theta_{W1}^2}$ | Closing mass [GeV] | appendix |
|---------|-------------|-------------------------------------------------|-------------------|----------|
| $K^+ \to X\pi^+$ | pseudo scalar | $2.49 \cdot 10^4$ | 0.35 | C.1 |
| $K^0_L \to X\pi^0$ | pseudo scalar | $1.56 \cdot 10^4$ | 0.36 | C.1 |
| $K^0_S \to X\pi^0$ | pseudo scalar | $1.61 \cdot 10^{-1}$ | 0.36 | C.1 |
| $B^+ \to X\pi^+$ | pseudo scalar | $2.37 \cdot 10^2$ | 5.14 | C.1 |
| $B^+ \to XK^+$ | pseudo scalar | $7.73 \cdot 10^4$ | 4.79 | C.1 |
| $B^\pm \to XK^{*\pm}(700)$ | scalar | $1.43 \cdot 10^4$ | 4.46 | C.2 |
| $B^\pm \to XK^{*\pm}(892)$ | vector | $9.14 \cdot 10^3$ | 4.39 | C.3.1 |
| $B^\pm \to XK^{*\pm}(1270)$ | pseudo vector | $1.72 \cdot 10^4$ | 4.01 | C.3.2 |
| $B^\pm \to XK^{*\pm}(1400)$ | pseudo vector | $2.34 \cdot 10^2$ | 3.88 | C.3.2 |
| $B^\pm \to XK^{*\pm}(1410)$ | vector | $3.99 \cdot 10^3$ | 3.86 | C.3.1 |
| $B^\pm \to XK^{*\pm}(1430)$ | scalar | $1.85 \cdot 10^4$ | 3.85 | C.2 |
| $B^\pm \to XK^{*\pm}(1430)$ | tensor | $6.03 \cdot 10^3$ | 3.85 | C.4 |
| $B^\pm \to XK^{*\pm}(1680)$ | vector | $2.53 \cdot 10^3$ | 3.56 | C.3.1 |

5 Summary

The goal of our study was to describe the production of the new GeV-scale vector particle $X_\mu$ (Chern-Simons boson) in the decays of mesons starting from the effective interaction of this boson with electroweak gauge bosons (1.1).

We consider the loop interaction of the CS boson with quarks of different flavors through $W$ bosons. In this case, see figure 1(c) and figure 1(d), we have the CS boson interaction with two down-type or two up-type quarks. We have shown that the divergent terms in the calculation of loop diagrams are automatically canceled out due to the unitarity of the CKM matrix, see appendix A. Unfortunately, this result is valid only for the CS boson interaction with quarks of different flavors. The case of the interaction of the CS boson with two identical quarks or with leptons requires a separate, more accurate consideration and some renormalization scheme, which is the subject of further research.

We have shown that the interaction of the CS boson with up-type quarks is sufficiently suppressed compared with down-type quarks. This is due to the fact that
Figure 3: Dependence of the branching $Br = \theta_{W_1}^{-2} \left( \frac{m_X}{1\text{GeV}} \right)^2 Br(h \to h'X)$ of reactions of the CS boson production on the CS boson mass: a – reaction of the CS boson production in two-body decays of charged $K$ mesons and neutral $K^0_L$, $K^0_S$ mesons (values of the branching for the reaction $K^0_S \to \pi^0 + X$ are multiplied by $10^2$); b – reaction of the CS boson production in two-body decays of charged $B$ mesons (contributions over the (pseudo)scalar, (pseudo)vector and tensor channels of the $B$ meson decays are summed up).

When the up-type quarks interact with the CS boson, the masses of the virtual down-type quarks in the loop are small compared to the $W$ boson mass. As a result, in the first approximation, there is no dependence of the loop terms on the mass of virtual down-type quarks and interaction of up-type quarks is suppressed by the condition of the unitarity of the CKM matrix.

We construct the effective Lagrangian of the CS boson interaction with two different down-type quarks (3.14). It turned out that the effective Lagrangian depends only on one of two unknown couplings ($\Theta_{W_1}$) of the CS boson interaction with $W$ boson, see (1.1). It should be noted that in the interaction of the CS boson with quarks (3.14), the CS boson behaves like a CP even particle, see (3.18).

We consider the production of the CS boson in decays of mesons. Since CS bosons interact effectively only with down-type quarks, they can be produced in decays of mesons containing $b$ or $s$ quarks. Such lightest mesons are $B$ mesons and $K^\pm$ mesons. Due to the contribution of longitudinal polarization of the CS boson, the decay width of the reactions increases with decreasing boson mass as $M_X^{-2}$. So, for convenience, we computed quantity $\theta_{W_1}^{-2} \left( \frac{m_X}{1\text{GeV}} \right)^2 Br(h \to h'X)$.

We consider the production of the CS bosons in decays of charged $K$ mesons and neutral $K^0_L$, $K^0_S$ mesons. In the case of decays of $K^0_L$, $K^0_S$ mesons, we took into account that these particles have certain CP parity and the CS boson is a CP even particle. As a result, we found that reaction $K^0_L \to \pi^0 + X$ has the greatest branching among the reactions of the CS boson production in decays of kaons. The branching of the reaction $K^0_S \to \pi^0 + X$ is suppressed in $\sim 10^3$ times compared to the reaction $K^0_L \to \pi^0 + X$, see table 2 and figure 3(a).
We consider dominant reactions of the CS bosons production in two-body decays of $B$ meson into pseudoscalar mesons ($K$ and $\pi$ mesons); scalar mesons $K^{0*}(700)$ and $K^{0*}(1430)$; vector mesons $K^*(892)$, $K^*(1410)$, $K^*(1680)$; pseudovector mesons $K_1^*(1270)$ and $K_1^*(1400)$; tensor final meson state $K_2^*(1430)$. Results for the branching of the corresponding reactions are presented in table 2 for a very small mass of the CS boson. Dependence of the branching of the CS boson production on its mass is presented in figure 3(b), where, for clarity, we summed up the contributions over the (pseudo)scalar, (pseudo)vector and tensor channels. As one can see, channels of the CS boson production with the greatest branching are decays of $B$ mesons into pseudovector, vector and scalar mesons for the CS boson mass up to $m_X \simeq 3$ GeV, into pseudovector and vector mesons for $3$ GeV $\lesssim m_X \lesssim 4$ GeV, vector and pseudoscalar mesons for $4$ GeV $\lesssim m_X \lesssim 4.37$ GeV, and pseudoscalar mesons for $4.37$ GeV $\lesssim m_X \lesssim 4.79$ GeV. The greatest branching of the CS boson production up to $m_X \simeq 3.8$ GeV is that of channel $B^\pm \rightarrow X K_1^\pm(1270)$, but other channels are also important and can not be neglected.

The results of our research will be helpful for the construction of the sensitivity region and the search for the GeV-scale long-lived CS boson at intensity frontier experiments such as MATHUSLA [23], FACET [24], FASER [25, 26], NA62 [29–31], DUNE [32, 33], etc. Especially considering that the CS boson was already included in the SHiP experiment proposal [28].

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A Computation of the loop diagram for CS boson interaction with different down-type quarks

\[ \begin{array}{c}
 d_n \\
p-k \\
d_p \\
w-
\end{array} \quad u_i \quad w-
\begin{array}{c}
p' \\
X \\
w-
\end{array} \quad X \\
p-k \\
X \\
p_m
\]

**Figure 4:** Interaction of the CS boson with two down-type quarks via loop diagram.

The amplitude of a heavy down-type quark \((d_n)\) decay into a light down-type quark \((d_m)\) and the CS boson \(X_\mu\) in the unitary gauge can be presented as

\[ M_{fi} = \frac{g^2}{2} \sum_{i=u,c,t} V_{dn}^+ V_{dn} \bar{T}_m(p') \hat{P}_R I^\mu_{(i)} \hat{P}_L d_m(p) \epsilon^\lambda \chi, \quad (A.1) \]

where \( \hat{P}_R \) (left-handed chirality states) and in the unitary gauge \( I^\mu_{(i)} \) has form

\[ I^\mu_{(i)} = Z^d_4 k (2\pi)^4 \frac{1}{4} \gamma^\alpha G(p - k) D_{\alpha\rho}(k - p X) \left[ c_\omega(k - p X)_\lambda + c_\omega^* k_\lambda \right] \gamma^\beta \epsilon^{\mu\nu\lambda\rho} = \]

\[ \hat{A}(k) \gamma^\alpha \left[ m_i + (p - k) \right] \left[ g_{\alpha\rho} - \frac{(k - p X)_\alpha(k - p X)_\rho}{M_W^2} \right] \left[ c_\omega(k - p X)_\lambda + c_\omega^* k_\lambda \right] \times \]

\[ \left[ g_{\beta\nu} - \frac{k_{\beta\nu}}{M_W^2} \right] \gamma^\beta \epsilon^{\mu\nu\lambda\rho}, \quad (A.2) \]

where \( m_i \) is the mass of the \( u_i \) quark and integral operator \( \hat{A} \) is defined as

\[ \hat{A}(k) = \int \frac{d^4k}{(2\pi)^4} F(k), \quad F(k) = \left[ m_i^2 - (p - k)^2 \right] \left[ M_W^2 - (k - p')^2 \right] \left[ M_W^2 - k^2 \right] . \quad (A.3) \]

Relation (A.2) can be further simplified with the help of the following identities:

\[ \hat{P}_R(a + b\gamma_i) \hat{P}_L = b \hat{P}_R \gamma_i \hat{P}_L, \quad \hat{P}_R \gamma_i \gamma_j \hat{P}_L = 0. \quad (A.4) \]

We use the technique of \( \alpha \) (Schwinger) representation, see e.g. [75], namely relations

\[ \frac{1}{m^2 - k^2 - i\varepsilon} = i \int_0^\infty d\alpha e^{\alpha(k^2 - m^2 + i\varepsilon)}, \quad \varepsilon \to 0, \quad (A.5) \]
\[\int_{-\infty}^{\infty} d^4 k \; e^{i(Ak^2 + 2Bk)} = \frac{\pi^2}{i} \cdot \frac{1}{A^2} \; e^{-\frac{1}{4A^2} \frac{B^2}{\pi}}, \quad (A.6)\]

\[\int_{-\infty}^{\infty} d^4 k \; e^{i(Ak^2 + 2Bk)} k^\mu = \frac{\pi^2}{i} \cdot \frac{1}{A^2} \; e^{-\frac{1}{4A^2} \frac{B^2}{\pi} \left[ -\frac{B^\nu}{A} \right]}, \quad (A.7)\]

\[\int_{-\infty}^{\infty} d^4 k \; e^{i(Ak^2 + 2Bk)} k^\nu k^\mu = \frac{\pi^2}{i} \cdot \frac{1}{A^2} \; e^{-\frac{1}{4A^2} \frac{B^2}{\pi} \left[ \frac{2B^\nu B^\mu + iA g^\mu\nu}{2A^2} \right]}, \quad (A.8)\]

\[\int_{-\infty}^{\infty} d^4 k \; e^{i(Ak^2 + 2Bk)} k^\nu k^\mu k^\lambda = \frac{\pi^2}{i} \cdot \frac{1}{A^2} \; e^{-\frac{1}{4A^2} \frac{B^2}{\pi} \times \left[ -\frac{4B^\mu B^\nu B^\lambda}{A^3} + 2iA \left[ g^{\mu\nu} B^\lambda + g^{\mu\lambda} B^\nu + g^{\nu\lambda} B^\mu \right] \right]} \]. \quad (A.9)\]

With the help of this technique, we can get the following relations:

\[K^{(0)} = \hat{A} \cdot 1 = \frac{i\pi^2}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \frac{1}{D(m_i)}, \quad (A.10)\]

\[K^{(1)}_{\alpha} = \hat{A} k_\alpha = \frac{i\pi^2}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \frac{(xp + ypX)_\alpha}{D(m_i)}, \quad (A.11)\]

\[K^{(2)}_{\alpha \beta} = \hat{A} k_\alpha k_\beta = \frac{i\pi^2}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \left[ \frac{(xp + ypX)_\alpha (xp + ypX)_\beta - g_{\alpha \beta}}{2 D(m_i)} \right], \quad (A.12)\]

where \(\Lambda\) is some constant with dimension of mass (it should be put to infinity in the end of the computation, \(\Lambda \to \infty\)) and function \(D(m_i)\) is defined by (3.5).

If we introduce integral operators \(\hat{A}_{\theta(1)} \) (3.3), (3.4) and notation \(P = xp + ypX\), we can rewrite relation (A.10) – (A.12) as

\[K^{(0)} = \hat{A}_0, \quad K^{(1)}_i = \hat{A}_0 P_i, \quad K^{(2)}_{i\lambda} = \hat{A}_0 P_{\lambda P_i} + \frac{i}{2} g_{\lambda i} \hat{A}_1. \quad (A.13)\]

It will be useful also to give the relation

\[K^{(3)}_i = \hat{A} k^2 k_i = \hat{A}_0 P^2 P_i + 3i \hat{A}_1 P_i. \quad (A.14)\]

Using the relations obtained above, we can get

\[I_{(i)}^\mu = \hat{A}_0 \left\{ \gamma_\rho (P - \not{p}) \gamma_\nu \left\{ \Theta_{W1}(p_X - 2P)_\lambda + i\Theta_{W2}p_{X,\lambda} \right\} + \frac{p_X \lambda}{M_W^2} P_\nu \gamma_\rho \left[ \epsilon^*_w \{ 2(pP) + \not{p}_X P - \not{p}_X \not{p} \} - 2\Theta_{W1} P^2 + 2i\Theta_{W2} \not{p} P \right] \right\} \epsilon^{\mu \nu \lambda \rho} + \hat{A}_1 \left\{ -i\Theta_{W1} \gamma_\rho \gamma_\nu + \frac{p_X \lambda}{M_W^2} \left[ \frac{1}{2} \epsilon^*_w \gamma_\rho \not{p}_X \gamma_\nu - 6i\Theta_{W1} \gamma_\rho P_\nu - \Theta_{W2} \gamma_\rho \not{p}_\nu \right] \right\} \epsilon^{\mu \nu \lambda \rho}. \quad (A.15)\]
The divergent part of this relation is hidden in the integral operator $\hat{\Lambda}_1$, see (3.4), but we can distinguish in this operator singular and finite parts (3.6) and replace operator $\hat{\Lambda}_1$ by finite operator $\hat{\Lambda}_1^{fin}$ (3.7), see section 3.1.

With help of the unitary condition for the CKM matrix, it is not difficult to show that the amplitude of the process does not depend on the value of $\mu$. Indeed, for reaction $b \to s + X$ we have $V_{su}^+ V_{ub} = -(V_{st}^+ V_{tb} + V_{ct}^+ V_{cb})$ and

$$
\sum_{i=u,c,t} V_{si}^+ V_{ib} \hat{\Lambda}_1^{fin} F(x, y, \{p\}) \sim \int_0^1 dx \int_0^{1-x} dy \sum_{i=u,c,t} V_{si}^+ V_{ib} \ln \frac{D(m_i)}{\mu^2} F(x, y, \{p\}) =
$$

$$
= \int_0^1 dx \int_0^{1-x} dy \left[ V_{st}^+ V_{tb} \left( \ln \frac{D(m_s)}{\mu^2} - \ln \frac{D(m_u)}{\mu^2} \right) + V_{ct}^+ V_{cb} \ln \frac{D(m_c)}{\mu^2} \right] F(x, y, \{p\}) =
$$

$$
\int_0^1 dx \int_0^{1-x} dy \left[ V_{st}^+ V_{tb} \ln \frac{D(m_s)}{D(m_u)} + V_{ct}^+ V_{cb} \ln \frac{D(m_c)}{D(m_u)} \right] F(x, y, \{p\}). \quad (A.16)
$$

In the following, we will put $\mu = m_W$ for the convenience of computations.

Instead of momentum $p_X$, it is better to use the momentums of quarks $p_X \to p - p'$. It will allow us to use relations $\tilde{d}_m(p') \lambda' = m' \tilde{d}_m(p')$ and $\tilde{d}_n(p) = m \tilde{d}_n(p)$ and, in particular, to get

$$
\begin{align*}
\tilde{d}_m(p') \hat{P}_R \gamma_\rho^0 \tilde{\gamma}_\rho^0 \hat{P}_L d_n(p) &= \tilde{d}_m(p') \hat{P}_R [2 \gamma_\rho^0 \rho_\rho^0] \hat{P}_L d_n(p) - \tilde{d}_m(p') \hat{P}_R [m_b \gamma_\rho^0 \rho_\rho^0] \hat{P}_R d_n(p), \\
\tilde{d}_m(p') \hat{P}_R \gamma_\rho^0 \tilde{\gamma}_\rho^0 \hat{P}_L d_n(p) &= \tilde{d}_m(p') \hat{P}_R [2 \gamma_\rho^0 \rho_\rho^0] \hat{P}_L d_n(p) - \tilde{d}_m(p') \hat{P}_L [m_s \gamma_\rho^0 \rho_\rho^0] \hat{P}_R d_n(p), \\
\tilde{d}_m(p') \hat{P}_R \gamma_\rho^0 \tilde{\gamma}_\rho^0 \hat{P}_L d_n(p) &= \tilde{d}_m(p') \hat{P}_R [2 m_b p_\rho^0] \hat{P}_R d_n(p) - \tilde{d}_m(p') \hat{P}_L [m_s m_b \gamma_\rho^0 \rho_\rho^0] \hat{P}_R d_n(p),
\end{align*}
$$

where $m$ is the mass of the initial down-type quark ($d_n$) and $m'$ is the mass of the final down-type quark ($d_m$). After this substitution, we finally get the relation $Y_{(i)}^\mu = \hat{P}_R I_{(i)}^\mu \hat{P}_L$ (3.2).

B Coefficients in the amplitude of the CS boson interaction with different down-type quarks

Let us perform summation over virtual quarks and integration over the variables $x$ and $y$ in (3.1), (3.2). For simplicity of notation we consider only decay of the $b$ quark into $s$ quark and the CS boson. First, it is more convenient to compute detached terms that are included in (3.1) such as

$$
\begin{align*}
A_0 &= \sum_{i=u,c,t} V_{si}^+ V_{ib} \hat{\Lambda}_0, & A_{0f} &= \sum_{i=u,c,t} V_{si}^+ V_{ib} \hat{\Lambda}_0 f, \\
A_1 &= \sum_{i=u,c,t} V_{si}^+ V_{ib} \hat{\Lambda}_1^{fin}, & A_{1f} &= \sum_{i=u,c,t} V_{si}^+ V_{ib} \hat{\Lambda}_1^{fin} f.
\end{align*}
\quad (B.1)
$$
To do it analytically, we will make an estimation, taking into account that $M_W^2 \gg m^2, m'^2, M^2$, and simplify relation (3.5) to the form:

$$D(m_i) = \begin{cases} D_{uc} = (1 - x)M_W^2 & \text{for } i = u, c \text{ quarks,} \\ D_t = (1 + (t_w - 1)x)M_W^2 & \text{for } i = t \text{ quark,} \end{cases}$$

where $t_w = m_t^2/M_W^2$. Then one can easily obtain quite simple relations e.g.

$$A_0 = \frac{i\pi^2}{(2\pi)^4} \frac{1}{M_W^2} \left\{ V_{su}^+ V_{ub} + V_{sc}^+ V_{cb} + V_{st}^+ V_{tb} \frac{1 - t_w + t_w \ln t_w}{(t_w - 1)^2} \right\};$$

$$A_{0x} = \frac{i\pi^2}{(2\pi)^4} \frac{1}{2M_W^2} \left\{ V_{su}^+ V_{ub} + V_{sc}^+ V_{cb} + V_{st}^+ V_{tb} \frac{t_w^2 - 2 t_w - 1 - 2 t_w \ln t_w}{(t_w - 1)^3} \right\};$$

$$A_1 = -\frac{1}{4} \frac{\pi^2}{(2\pi)^4} \left\{ V_{su}^+ V_{ub} + V_{sc}^+ V_{cb} - V_{st}^+ V_{tb} \frac{2 t_w^2 \ln t_w - 1 + 4 t_w - 3 t_w^2}{(t_w - 1)^2} \right\}.$$

From the unitarity condition $(V^+V)_{sb} = 0$, one can conclude $V_{su}^+ V_{ub} + V_{sc}^+ V_{cb} = -V_{st}^+ V_{tb}$ and write

$$A_0 = -\frac{i\pi^2}{(2\pi)^4} \frac{V_{st}^+ V_{tb} t_w}{M_W^2} \frac{t_w - 1 - \ln t_w}{(t_w - 1)^2};$$

$$A_{0x} = \frac{i\pi^2}{(2\pi)^4} \frac{V_{st}^+ V_{tb} t_w}{M_W^2} \frac{3 + (t_w - 4) t_w + 2 \ln t_w}{2(t_w - 1)^3};$$

$$A_{0y} = -\frac{i\pi^2}{(2\pi)^4} \frac{V_{st}^+ V_{tb} t_w}{M_W^2} \frac{t_w^2 - 2 t_w - 1 - 2 t_w \ln t_w}{4(t_w - 1)^3};$$

$$A_{0y} = -\frac{i\pi^2}{(2\pi)^4} \frac{V_{st}^+ V_{tb} t_w}{M_W^2} \frac{2 t_w^2 - 11}{6(t_w - 1)^4};$$

$$A_{0y} = -\frac{i\pi^2}{(2\pi)^4} \frac{V_{st}^+ V_{tb} t_w}{M_W^2} \frac{1 - 6 t_w + 3 t^2_w + 2 t^3_w - 6 t^4_w}{18(t_w - 1)^4};$$

$$A_{0xy} = -\frac{i\pi^2}{(2\pi)^4} \frac{V_{st}^+ V_{tb} t_w}{M_W^2} \frac{2 + 3 t_w - 6 t^2_w + t^3_w}{12(t_w - 1)^4};$$

$$A_{0x} = -\frac{i\pi^2}{(2\pi)^4} \frac{V_{st}^+ V_{tb} t_w}{M_W^2} \frac{25 - 48 t^2_w + 56 t^2_w - 16 t^3_w}{12(t_w - 1)^5};$$

$$A_{0x} = -\frac{i\pi^2}{(2\pi)^4} \frac{V_{st}^+ V_{tb} t_w}{M_W^2} \frac{-3 - 10 t_w + 18 t^2_w - 6 t^3_w + t^4_w - 12 t^2_w}{24(t_w - 1)^5};$$

$$A_{0y} = -\frac{i\pi^2}{(2\pi)^4} \frac{V_{st}^+ V_{tb} t_w}{M_W^2} \frac{-1 + 8 t_w - 8 t^3_w + t^4_w + 12 t^2_w}{36(t_w - 1)^5};$$

$$A_1 = \frac{\pi^2}{(2\pi)^4} \frac{V_{st}^+ V_{tb} t_w}{M_W^2} \frac{1 - t_w + t_w \ln t_w}{2(t_w - 1)^2};$$

$$A_{1x} = \frac{\pi^2}{(2\pi)^4} \frac{V_{st}^+ V_{tb} t_w}{2(t_w - 1) + (t_w - 3) t_w \ln t_w}. $$
Now we can write relations for coefficients in (3.9):

\[ a_{0x} = -i a_1 \Theta W_1, \quad \text{(B.18)} \]
\[ c_{L}^d = i \left( a_{0y} - 2a_{0y^2} - \frac{a_1}{2} \right) \Theta W_1 - \left( a_{0y} + \frac{a_1}{2} \right) \Theta W_2, \quad \text{(B.19)} \]
\[ d_{L}^d = i \left( 2a_{0xy} - a_{0y} + 2a_{0y^2} + \frac{a_1}{2} \right) \Theta W_1 + \left( a_{0y} - a_1 \right) \Theta W_2, \quad \text{(B.20)} \]
\[ b_{R}^d = -i \left( a_{0x} - a_{0x^2} \right) \Theta W_1 + (a_{0x^2} + 2a_{0xy} - a_{0x}) \Theta W_2, \quad \text{(B.21)} \]
\[ c_{R}^d = i \left( a_0 - a_{0x} + 2a_{0xy} - 3a_{0y} + 2a_{0y^2} + \frac{a_1}{2} \right) \Theta W_1 - \left( a_0 - a_{0x} - a_{0y} + \frac{a_1}{2} \right) \Theta W_2, \quad \text{(B.22)} \]
\[ d_{R}^d = i \left( -a_0 + 3a_{0x} - 2a_{0x^2} - 4a_{0xy} + 3a_{0y} - 2a_{0y^2} - \frac{a_1}{2} \right) \Theta W_1 + \left( a_0 - a_{0x} - a_{0y} + \frac{a_1}{2} \right) \Theta W_2. \quad \text{(B.23)} \]

We also write separately the most cumbersome coefficient

\[ b_{L}^d = i \left[ -8a_{0xy} - 8a_{0y^2} + 2a_{0x} + 8a_{0y} - 2a_0 - 2a_1 + 6a_{1x} + \frac{m_b^2}{M_W^2} \left[ -a_{0xy} + 2a_{0x^2} \right] + \frac{m_b^2}{M_W^2} \left[ 3a_{0x^2} + a_{0xy} - a_{0x} - 2a_{0x^2} - 2a_{0xy} + \frac{M_X}{M_W^2} \left[ 2a_{0xy} - 2a_{0x^2y} - 2a_{0xy^2} \right] \right] \right] \Theta W_1 + \left[ -2a_{0x} - 4a_{0y} + 2a_0 + \frac{m_b^2}{M_W^2} \left[ 2a_{0x^2} + a_{0xy} - a_{0x} \right] + \frac{m_b^2}{M_W^2} \left[ 3a_{0xy} \right] \right] \Theta W_2. \quad \text{(B.24)} \]

The numerical values of these coefficients are presented in table 1.

---

**Table 3**: Numerical values of the coefficients \( a_{0x} \) and \( a_{1x} \).

| \( a_0 \) | \( a_{0x} \) | \( a_{0y} \) | \( a_{0x^2} \) | \( a_{0xy} \) | \( a_{0y^2} \) | \( a_{0x^2y} \) | \( a_{0y^2x} \) | \( a_{1} \) | \( a_{1x} \) |
|---|---|---|---|---|---|---|---|---|---|
| 0.16 | 0.094 | 0.033 | 0.066 | 0.014 | 0.012 | 0.051 | 0.0076 | 0.0042 | 0.13 | 0.066 |
C Decays of mesons with the CS boson production

C.1 Decay into mesons of the same parity

Let us consider here the production of the CS boson in decays of $B$ mesons into $K$, $\pi$ mesons and decays of $K$ mesons into $\pi$ mesons.

In the case of a decay of one meson into another meson with the same parity, we have the following relation for the averaging the quark current over the meson states:

$$2\langle h'(p')|\bar{Q}_i \gamma^\mu \hat{P}_L Q_j|h(p)\rangle = \langle h'(p')|\bar{Q}_i \gamma^\mu Q_j|h(p)\rangle = (p + p')^\mu - \frac{m_P^2 - m_P'^2}{q^2}q^\mu f_{hh'}^{h'}(q^2),$$  \hspace{1cm} (C.1)

where $q$ is the momentum transfer to $h'$ meson, namely $q = p - p'$, $q^2 = M_X^2$.

Using relations (4.4), (4.9) one can get the amplitude of the an $h$ meson decay (pseudomeson, $P$) into an $h'$ meson (with the same parity) and the CS boson:

$$M_{P \rightarrow P'X} = \Theta_{1W} C_{mn} |\vec{k}| \frac{M_h}{M_X} f_{hh'}^{h'}(M_X^2).$$  \hspace{1cm} (C.2)

Thus, we will be interested only in the $f_{hh'}^{h'}$ form-factor. It should be noted that in the case of $K^0_S$, $K^0_L$ mesons with a certain CP parity, we have to use the real or the imaginary part of the coefficient $C_{ds}$, see (4.5), (4.6).

We take form-factors $f_{hh'}^{h'}(q^2)$ for the decays of $B$ mesons from [76], where they were given with the help of pole parametrizations:

$$f_{hh'}^{h'}(q^2) = f_{hh'}^{h'}(0) \left( 1 + \frac{q^2}{m_{hi}^2} \right) + \frac{\lambda_{1}'}{m_{hi}^2} + \frac{\lambda_{2}'}{m_{hi}^2},$$  \hspace{1cm} (C.3)

$$f_{hh'}^{h'}(q^2) = f_{hh'}^{h'}(0) \left( 1 + \frac{q^2}{m_{di}^2} \right) + \frac{\lambda_{1}''}{(1 - q^2/m_{di}^2)^2}.$$  \hspace{1cm} (C.4)

The last relation is used for the case when $m_{di}$ gets too close to $m_1$.

Form-factors for the decay of $K$ meson are discussed in detail in [70].

We use form-factors $f_{hh'}^{h'}(q^2)$ for the decays of $K^\pm$ mesons in the form of a quadratic expansion:

$$f_{hh'}^{h'}(q^2) = f_{hh'}^{h'}(0) \left( 1 + \lambda_{1}' \frac{q^2}{m_{hi}^2} + \frac{\lambda_{2}'}{2} \left( \frac{q^2}{m_{hi}^2} \right)^2 \right),$$  \hspace{1cm} (C.5)

We take values of the parameters $\lambda_{1}'$ and $\lambda_{2}'$ from [70], which are the averaged over the values given in [77–79].

For the decays of $K^0_L$ mesons we take form-factors $f_{hh'}^{h'}(q^2)$ in the form of the quadratic expansion (C.5) too. We take values of the parameters $\lambda_{1}'$ and $\lambda_{2}'$ from [70], which are the averaged over the values given in [80–82], assuming $\mu - e$ universality.
Table 4: Fit parameters for Eqs. (C.3) – (C.5). Here \( m_1 \) is the meson mass in the corresponding channel: \( m_1^\pi = m_{B^*} = 5.32 \) GeV and \( m_1^K = m_{B_{s}^*} = 5.41 \) GeV.

The values of the parameters in (C.3) – (C.5) are summarized in table 4.

For the decays of \( K_0^* \) mesons we take form-factors \( f^{hh'} (q^2) \) in the form of a linear expansion:

\[
 f^{hh'} (q^2) = f^{hh'} (0) \left( 1 + \frac{q^2}{m_{\text{fit}}^2} \right),
\]

where values of the parameters \( f(0) = 0.96 \) and \( \lambda_+ = 3.39 \cdot 10^{-2} \) were taken from [83].

C.2 Decays into mesons of another parity

Let us consider here the production of the CS boson in the decays of \( B \) mesons into \( K_0^* (700), K_0^* (1430) \) mesons.

In the case of a decay of one meson into another meson with different parity we get

\[
 2 \langle h' | \bar{Q}_i \gamma^\mu \hat{P}_L Q_j | h(p) \rangle = - \left[ (p + p')^\mu - q^\mu \right] f^{hh'} (q^2) = -2p^\mu f^{hh'} (q^2),
\]

where we used \( f_+ (q^2) = -f_- (q^2) \) in (C.1), [84].

There is an open question whether hypothetical \( K_0^* (700) \) is a state formed by two or four quarks, see, e.g. [85], discussions in [84, 86] and references therein. In this paper, we will do the same as we did in [36], namely, we assume that \( K_0^* (700) \) is a di-quark state and \( K_0^* (1430) \) is its excited state. There are no experimentally observed decays of \( B \) meson into \( K_0^* (700) \), and therefore there is quite a large theoretical uncertainty in the determination of the form-factors (see the discussion in [87]). We will use [84], where there are results for \( B \to K_0^* (700) \) and \( B \to K_0^* (1430) \), and the results for the latter are in good agreement with the experimental data for \( B \to K_0^* (1430) \eta' \) decay.

Using relations (4.4), (4.9) one can get the amplitude of an \( h \) meson decay (pseudomeson, \( P \)) into an \( h' \) meson (scalar meson, \( S \)) and the CS boson:

\[
 M_{P \to S X} = -\Theta_{1W} C_{mm} |\vec{k}| \frac{M_h}{M_X} f^{hh'} (M_X^2),
\]
We take \( f^{\BK_0^+} (q^2) \) from [84] in the form of a pole-like function:

\[
 f^{\BK_0^+} (q^2) = \frac{F_0^{\BK_0^+}}{1 - a \frac{q^2}{m_B^2} + b \left( \frac{q^2}{m_B^2} \right)^2},
\]

where \( m_B = 5.3 \text{ GeV} \) is the mass of the \( B^+ \) meson. The fit parameters are given in table 5.

\section{C.3 Vector and pseudovector final meson state}

\subsection{C.3.1 Vector}

Let us consider here the production of the CS boson in the decays of \( B \) mesons into vector states \( B \rightarrow K^*(892), K^*(1410), K^*(1680) \). Since the total angular momentum of the \( B \) meson is zero, two final vector particles must have zero total angular momentum.

For the vector final state, \( h' = V \), we have [88, 89]

\[
 \langle V(p')|\bar{Q}_i\gamma^\mu\gamma_5Q_j|h(p)\rangle = (M_h + M_V)\epsilon^\mu_{V}(p')A_1(q^2) - \\
 - (\epsilon^\nu_{V}(p') \cdot q)(p + p')^\nu \frac{A_2(q^2)}{M_h + M_V} - 2M_V \epsilon^\nu_{V}(p') \cdot q q^2 (A_3(q^2) - A_0(q^2)),
\]

\[
 \langle V(p')|\bar{Q}_i\gamma^\nu Q_j|h(p)\rangle = \frac{2V(q^2)}{M_h + M_V} \epsilon^{\mu\nu\rho\sigma}_{V,p}(p') p_\rho p'_\sigma,
\]

where \( \epsilon^\nu_{V}(p') \) is the polarization vector of the vector meson, and \( A_i, V \) are the form-factors. The form-factor \( A_3 \) is related to \( A_1 \) and \( A_2 \) as

\[
 A_3(q^2) = \frac{M_h + M_V}{2M_V} A_1(q^2) - \frac{M_h - M_V}{2M_V} A_2(q^2).
\]

The amplitude of an \( h \) meson decay into a vector \( V \) meson and the CS boson has form

\begin{table}
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\( S \) & \( F_0^{BS} \) & \( a \) & \( b \) \\
\hline
\( K_0^+(700) \) & 0.466 & 1.501 & 1.026 \\
\( K_0^+(1430) \) & 0.181 & 4.293 & 6.450 \\
\hline
\end{tabular}
\end{table}
\[
M_{h \rightarrow V \chi}(\lambda_V, \lambda_X) = \Theta W_1 g^2 C_{\mu \nu} (V(p', \lambda_V)|\vec{d}_{m} \gamma^\mu \vec{P}_L |d_m| h(p)) \epsilon_{X, \mu}^{\lambda_X} = \\
= -\Theta W_1 \frac{g^2 C_{\mu \nu}}{2} \left[ \frac{2V(q^2)}{M_h + M_V} \epsilon_{\mu \nu}^{\lambda_V \lambda_X} \epsilon_{V, \nu}^{\lambda_V \lambda_X} p_{p' \sigma} + (M_h + M_V)(\epsilon_{V, \mu}^{\lambda_X} \cdot \epsilon_{X, \nu}^{\lambda_X}) A_1(q^2) - \right. \\
\left. - (\epsilon_{V, \mu}^{\lambda_X} \cdot q)(\epsilon_{X, \nu}^{\lambda_X} \cdot (p + p')) \frac{A_2(q^2)}{M_h + M_V} - 2M_V \frac{\epsilon_{V, \mu}^{\lambda_X} \cdot q}{q^2} (\epsilon_{X, \nu}^{\lambda_X} \cdot q)(A_3(q^2) - A_0(q^2)) \right],
\quad (C.13)
\]

where \(\epsilon_{V, \mu}^{\lambda_X}\) and \(\epsilon_{X, \nu}^{\lambda_X}\) correspond to the polarizations of the vector meson \((V_\mu)\) and the CS boson \((X_\mu)\).

In the rest frame of the \(h\) meson \(p = (M_h, 0)\) we have relations \((4.7)\). Let us guide Z-axis along the spatial momentum of the CS boson \(\vec{k}\), then the other vector particle will move in the opposite direction. In this case, 4-vectors of the polarizations for the CS boson or the vector particle have form
\[
\epsilon_{X(\nu)}^{(\pm)} = \frac{1}{\sqrt{2}} (0, 1, \mp i, 0), \quad \epsilon_{X}^{(0)} = \frac{1}{M_X} (|\vec{k}|, 0, 0, E_X), \quad \epsilon_{V}^{(0)} = \frac{1}{M_V} (|\vec{k}|, 0, 0, -E_V),
\quad (C.14)
\]

where \(\epsilon^{(\pm)}\) corresponds to the spin projection \(\pm 1\) and \(\epsilon^{(0)}\) corresponds to zero spin projection on Z-axis.

Using \((C.14)\) one can get \((4.9)\) and the following relations for different polarizations of vector particles:
\[
\epsilon_{V}^{(0)} \cdot p = |\vec{k}| \frac{M_h}{M_V}, \quad \epsilon_{V}^{(0)} \cdot p' = 0, \\
\epsilon_{V}^{(0)} \cdot \epsilon_{X}^{(0)} = \frac{|\vec{k}|^2 + E_X E_V}{M_X M_V}, \quad \epsilon_{V}^{(+)} \cdot \epsilon_{X}^{(-)} = \epsilon_{V}^{(-)} \cdot \epsilon_{X}^{(+)} = -1,
\quad (C.15)
\]

where module \(|\vec{k}|\) is defined by \((4.2)\).

Consider now the following convolution \(\epsilon_{\mu \nu}^{\nu} \epsilon_{V, \mu}^{\lambda_V \lambda_X} (p' p') p_{p' \sigma}\). Due to the fact that only the 0 and 3 components of the momentums \(p\) and \(p'\) are nonzero, only the 1 and 2 components of polarization vectors can be used. These components are zero for the longitudinal polarization of the vector particles, so the contribution from longitudinal polarizations is absent, but the contribution from transversal opposite polarizations is nonzero:
\[
\epsilon_{\mu \nu}^{\nu} \epsilon_{V, \mu}^{\lambda_V \lambda_X} p_{p' \sigma} = (p_0 p'_3 - p_3 p'_0) \left( \epsilon_{X, \mu}^{(\pm)} \epsilon_{V, \nu}^{(\mp)} - \epsilon_{X, \mu}^{(\pm)} \epsilon_{V, \nu}^{(-)} \right),
\quad (C.16)
\]

namely
\[
\epsilon_{\mu \nu}^{\nu} \epsilon_{V, \mu}^{(\pm)} \epsilon_{V, \nu}^{(\mp)} p_{p' \sigma} = -iM_h |\vec{k}|, \quad \epsilon_{\mu \nu}^{\nu} \epsilon_{V, \mu}^{(\pm)} \epsilon_{V, \nu}^{(\mp)} p_{p' \sigma} = +iM_h |\vec{k}|.
\quad (C.17)
\]
So, we get the following nonzero amplitudes of the reaction:

\[
M_{h \rightarrow VX}(+, -) = -\Theta_{1W} \frac{C_{mn}}{2} \left[ \frac{2M_h|\vec{k}|}{M_h + M_V} V(q^2) - (M_h + M_V)A_1(q^2) \right],
\]

\[
M_{h \rightarrow VX}(-, +) = \Theta_{1W} \frac{C_{mn}}{2} \left[ \frac{2M_h|\vec{k}|}{M_h + M_V} V(q^2) + (M_M V)A_1(q^2) \right],
\]

\[
M_{h \rightarrow VX}(0, 0) = -\Theta_{1W} \frac{C_{mn}}{2} \left[ (M_h + M_V) \frac{\vec{k}^2 + E_X E_V}{M_X M_V} A_1(q^2) - 2 \frac{E^2 M^2}{M_V M_X (M_h + M_V)} A_2(q^2) \right],
\]

and

\[
\sum_{\text{polarizations}} |M_{h \rightarrow VX}|^2 = \Theta_{1W}^2 \frac{|C_{mn}|^2}{4} \left[ \frac{8M_h^2 \vec{k}^2}{(M_h + M_V)^2} V^2(q^2) + 2(M_h + M_V)^2 A_1^2(q^2) + \left( \frac{M_h + M_V}{M_X M_V} \frac{\vec{k}^2 + E_X E_V}{M_X M_V} A_1(q^2) - \frac{2E^2 M^2}{M_V M_X (M_h + M_V)} A_2(q^2) \right)^2 \right].
\]

For the case of the decay of a $B$ meson into $K^*(892)$, we follow [88] and parametrize the form-factor $V$ and $A_0$ as

\[
F(q^2) = \frac{r_1}{1 - q^2/m_R^2} + \frac{r_2}{1 - q^2/m_{fit}^2},
\]

form-factor $A_1$ as

\[
F(q^2) = \frac{r_2}{1 - q^2/m_{fit}^2},
\]

and form-factor $A_2$ as

\[
F(q^2) = \frac{r_1}{1 - q^2/m_{fit}^2} + \frac{r_2}{(1 - q^2/m_{fit}^2)^2}.
\]

The values of the corresponding parameters are given in table 6.

| form-factors | $r_1$ | $r_2$ | $m_R$, GeV | $m_{fit}$, GeV | $q^2 = 0$ |
|--------------|-------|-------|-------------|---------------|------------|
| $V(q^2)$     | 0.923 | -0.511| 5.32        | 49.40         | 0.411      |
| $A_1(q^2)$   | -     | 0.29  | -           | 40.38         | 0.292      |
| $A_2(q^2)$   | -0.084| 0.342 | -           | 52.00         | 0.259      |

Table 6: Values of parameters of the vector form-factors (C.22) – (C.24) for the decay of a $B$ meson into $K^*(892)$ [88].
Table 7: Values of parameters in the vector form-factors (C.25) – (C.28) for the decay of a $B$ meson into $K^*(1410), K^*(1680)$, [90, 91].

| $V$          | $\zeta_\perp (0)$ | $\zeta_{||} (0)$ |
|--------------|-------------------|-----------------|
| $K^*(1410)$  | 0.28              | 0.22            |
| $K^*(1680)$  | 0.24              | 0.18            |

For the case of the decay of a $B$ meson into $K^*(1410), K^*(1680)$, we use another parametrization for the form-factors [90, 91]:

\[
A_0(q^2) = \left(1 - \frac{2M_V^2}{M_h^2 + M_V^2 - q^2}\right) \frac{\zeta_{||}(q^2)}{M_V} \zeta_\perp (q^2), \quad \text{(C.25)}
\]

\[
A_1(q^2) = \frac{2E_V}{M_h + M_V} \zeta_\perp (q^2) = \frac{M_V^2 + M_V^2 - q^2}{M_h(M_h + M_V)} \zeta_\perp (q^2), \quad \text{(C.26)}
\]

\[
A_2(q^2) = \left(1 + \frac{M_V}{M_h}\right) \left[\zeta_\perp (q^2) - \frac{2M_h M_V}{M_h^2 + M_V^2 - q^2} \zeta_{||}(q^2)\right], \quad \text{(C.27)}
\]

\[
V(q^2) = \left(1 + \frac{M_V}{M_h}\right) \zeta_\perp (q^2), \quad \text{(C.28)}
\]

where

\[
\zeta_{\perp/||}(q^2) = \frac{\zeta_{\perp/||}(0)}{1 - q^2/M_h^2}. \quad \text{(C.29)}
\]

The values of the corresponding parameters are given in Table 7.

### C.3.2 Pseudo-vector

For the case of pseudo-vector mesons, $h' = A$, one has to interchange the expressions for the vector and axial-vector matrix elements (C.10), (C.11), see [92, 93]:

\[
\langle A(p')|\bar{Q}i\gamma^\mu Q_j|h(p)\rangle = (M_h + M_A)\epsilon_\mu^A(p')V_1(q^2) - (\epsilon_\mu^A(p') \cdot q)(p + p')^\mu \frac{V_2(q^2)}{M_h + M_A} - 2M_A \epsilon_\mu^A(p') \cdot q \epsilon_\mu^A(V_3(q^2) - V_0(q^2)), \quad \text{(C.30)}
\]

\[
\langle A(p')|\bar{Q}i\gamma^\mu \gamma_5 Q_j|h(p)\rangle = \frac{2A(q^2)}{M_h + M_A} i\epsilon^{\mu\nu\rho\sigma} \epsilon_\nu^A(p') p_\rho p'_\sigma, \quad \text{(C.31)}
\]

with the same relation between $V_i$ as for $A_i$ in the case of vector mesons (C.12).

Expression (C.21) in this case takes form

\[
\sum_{\text{polarizations}} |M_{h \to AX}|^2 = \Theta_{1W}^2 \left|C_{\text{mn}}\right|^2 \frac{8M_h^2 k^2}{(M_h + M_A)^2} A^2(q^2) + 2(M_h + M_A)^2 V_1^2(q^2) + \\
+ \left(M_h + M_A\right) \frac{k^2 + E_X E_A}{M_X M_A} V_1(q^2) - \frac{2M_h^2 k^2}{M_A M_X (M_h + M_A)} V_2(q^2)^2. \quad \text{(C.32)}
\]
Table 8: Form-factors for $B \to K_{1A}, K_{1B}$ transitions are fitted to the 3-parameter form in (C.40), see [93].

We will consider decays of $B$ mesons in two lightest pseudo-vector resonances $K_1(1270)$, $K_1(1400)$, each of which is a mixture of unphysical $K_{1A}$ and $K_{1B}$ states [92],

\[
\begin{pmatrix}
|K_1(1270))angle \\
|K_1(1400))angle
\end{pmatrix} = \begin{pmatrix}
\sin(\theta_{K_1}) & \cos(\theta_{K_1}) \\
\cos(\theta_{K_1}) & -\sin(\theta_{K_1})
\end{pmatrix} \begin{pmatrix}
|K_{1A})angle \\
|K_{1B})angle
\end{pmatrix}.
\]

The form-factors $V_i^{BK_1}$ and $A_i^{BK_1}$ can be related to the appropriate form-factors of the $K_{1A}$ and $K_{1B}$ states as

\[
A_{BK1}(q^2) = \sin(\theta_{K_1})\frac{m_B + m_{K_1(1270)}}{m_B + m_{K_{1A}}} A_{K_{1A}}(q^2) + \cos(\theta_{K_1})\frac{m_B + m_{K_1(1270)}}{m_B + m_{K_{1B}}} A_{K_{1B}}(q^2),
\]

\[
B_{BK1}(q^2) = \cos(\theta_{K_1})\frac{m_B + m_{K_1(1400)}}{m_B + m_{K_{1A}}} B_{K_{1A}}(q^2) - \sin(\theta_{K_1})\frac{m_B + m_{K_1(1400)}}{m_B + m_{K_{1B}}} B_{K_{1B}}(q^2),
\]

| $F$ | $F(0)$ | $a$ | $b$ | $F$ | $F(0)$ | $a$ | $b$ |
|-----|--------|-----|-----|-----|--------|-----|-----|
| $V_1^{BK_{1A}}$ | 0.34 | 0.635 | 0.211 | $V_1^{BK_{1B}}$ | -0.29 | 0.729 | 0.074 |
| $V_2^{BK_{1A}}$ | 0.41 | 1.51 | 1.18 | $V_2^{BK_{1B}}$ | -0.17 | 0.919 | 0.855 |
| $A^{BK_{1A}}$ | 0.45 | 1.60 | 0.974 | $A^{BK_{1B}}$ | -0.37 | 1.72 | 0.912 |

where we take $\theta_{K_1} = -34^\circ$, $m_{K_{1A}} = 1.31$ GeV, $m_{K_{1B}} = 1.34$ GeV and the momentum dependence of all form-factors is parameterized as

\[
F(q^2) = \frac{F(0)}{1 - a\frac{q^2}{m_B^2} + b\left(\frac{q^2}{m_B^2}\right)^2}.
\]

The values of all relevant parameters are given in table 8.

- 27 -
C.4 Tensor final meson state

Let us consider here the production of the CS boson in the decays of \( B \) mesons into tensor state \( K_2^*(1430) \). For the tensor meson, \( h' = T \), the expansions of the matrix elements are similar to (C.10), (C.11), see [94, 95],

\[
\langle T(p')|\bar{Q}_s\gamma^\mu\gamma_5 Q_j|h(p)\rangle = (M_h + M_T)\epsilon^{s,\mu}_{\mu'}(p')A_{T1}(q^2) -
\]

\[
- \epsilon^{s}_{\mu}(p') \cdot q (p + p')^\mu \frac{A_{T2}(q^2)}{M_h + M_T} - 2M_T \frac{\epsilon^{s}_{\mu}(p') \cdot q}{q^2} q^\mu (A_{T3}(q^2) - A_{T0}(q^2)),
\]

(C.41)

\[
\langle T(p')|\bar{Q}_s\gamma^\mu Q_j|h(p)\rangle = \frac{2T(q^2)}{M_h + M_T} \gamma_{\mu}^{\mu\rho\sigma} \epsilon^{s,\rho}_{\nu}(p')p_{\nu}p_{\sigma},
\]

(C.42)

with the same relation between \( A_{Tj} \) as for \( A_i \) in the case of vector mesons (C.12).

So, the amplitude of the \( h \) meson decay into a tensor \( T \) meson and the CS boson has form like (C.13):

\[
M_{h\rightarrow TX}(s, \lambda_X) = \Theta_{1W} C_{mn} \langle T(p', s)|\bar{d}_n\gamma^\mu\hat{P}_L d_m|h(p)\rangle \epsilon^{s,\lambda}_{X,\mu} =
\]

\[
= -\Theta_{1W} C_{mn} \frac{M_h + M_T}{2} \frac{2T(q^2)}{M_h + M_T} \gamma_{\mu}^{\mu\rho\sigma} \epsilon^{s,\rho}_{X,\nu} p_{\nu} p'_{\sigma} + (M_h + M_T)(\epsilon^{s,\nu}_{X,\mu} A_{T1}(q^2) -
\]

\[
- (\epsilon^{s,\nu}_{X,\mu} (p + p')) \frac{A_{T2}(q^2)}{M_h + M_T} - 2M_T \frac{\epsilon^{s,\nu}_{X,\mu}}{q^2} q^\nu (\epsilon^{s,\lambda}_{X,\sigma} \cdot q)(A_{T3}(q^2) - A_{T0}(q^2)) \bigg],
\]

(C.43)

where \( \epsilon^{s,\lambda}_{X,\mu} \) corresponds to the CS boson polarization, \( s = \pm 2, \pm 1, 0 \) are the polarization states of the tensor meson and \( \epsilon^{s}_{\mu}(p') \) is a vector defined by

\[
\epsilon^{s}_{\mu}(p') = \frac{1}{M_h} \epsilon^{s}_{\mu\nu}(p) p'_{\nu},
\]

(C.44)

where \( \epsilon^{s}_{\mu\nu} \) is the polarization tensor of \( T \) meson satisfying conditions \( p_{\mu} \epsilon^{\mu\nu s}(p) = 0 \) and \( \epsilon^{\mu\nu s} = \epsilon^{\nu\mu s} \), \( \epsilon^{s}_{\mu} s = 0 \). Tensor \( \epsilon^{s}_{\mu\nu} \) can be constructed with the help of a spin-1 polarization vectors:

\[
\epsilon^{(\pm 2)}_{\mu\nu} = \epsilon^{(\pm 1)}_{\mu\nu}, \quad \epsilon^{(\pm 1)}_{\mu\nu} = \frac{1}{\sqrt{2}} [\epsilon^{(\pm)}_{\mu\nu} + \epsilon^{(\pm)}_{\nu\mu}],
\]

\[
\epsilon^{(0)}_{\mu\nu} = \frac{1}{\sqrt{6}} [\epsilon^{(+)}_{\mu\nu} - \epsilon^{(-)}_{\mu\nu}] + \sqrt{\frac{2}{3}} \epsilon^{(0)}_{\mu\nu}.
\]

(C.45)

In the case of the rest frame of an \( h \) meson we have \( p = (M_h, 0) \) and relations (4.7).

We can guide the CS boson to move along \( Z \)-axis and the tensor meson to move in the opposite direction. In this case we can choose spin-1 polarization vectors for the tensor meson as

\[
\epsilon^{(0)}_{\mu} = \frac{1}{M_T} (|\vec{k}|, 0, 0, -E_T), \quad \epsilon^{(\pm)}_{\mu} = \frac{1}{\sqrt{2}} (0, 1, \mp i, 0),
\]

(C.46)
Table 9: Values of the form-factors’ parameters for transition $B \to K^*_2(1430)$, [94].

| $F$ | $F(0)$ | $a$ | $b$ | $A$ | $A(0)$ | $a$ | $b$ |
|-----|--------|-----|-----|-----|--------|-----|-----|
| $T^{BK^*_2}$ | 0.29 | 2.17 | 2.22 | $A_0^{BK^*_2}$ | 0.23 | 1.23 | 0.74 |
| $A_1^{BK^*_2}$ | 0.22 | 1.42 | 0.50 | $A_2^{BK^*_2}$ | 0.21 | 1.96 | 1.79 |

then we get

$$ \epsilon^\pm_{T\mu} = 0, \quad \epsilon^{+\pm}_{T\mu} = \frac{|\vec{k}|}{\sqrt{2}M_T} \epsilon^{(\pm)}_{\mu}, \quad \epsilon^0_{T\mu} = \sqrt{\frac{2}{3}} \frac{|\vec{k}|}{M_T} \epsilon^{(0)}_{\mu}, \quad (C.47)$$

where module $|\vec{k}|$ is defined by (4.2).

So, the amplitude of the reaction is zero for $s = \pm 2$. Using relations (C.15), (C.17), (C.47) we get

$$ M_{h\to TX}(+, -) = -\Theta_{1W} \frac{C_{mn}}{2} \frac{|\vec{k}|}{\sqrt{2}M_T} \left[ \frac{2M_h|\vec{k}|}{M_h + M_T} T(q^2) - (M_h + M_T)A_{1T}(q^2) \right], \quad (C.48)$$

$$ M_{h\to TX}(-, +) = \Theta_{1W} \frac{C_{mn}}{2} \frac{|\vec{k}|}{\sqrt{2}M_T} \left[ \frac{2M_h|\vec{k}|}{M_h + M_T} T(q^2) + (M_h + M_T)A_{1T}(q^2) \right]. \quad (C.49)$$

$$ M_{h\to TX}(0, 0) = -\Theta_{1W} \frac{C_{mn}}{2} \sqrt{\frac{2}{3}} \frac{|\vec{k}|}{M_T} \left[ (M_h + M_T) \frac{\vec{k}^2 + E_XE_T}{M_XM_T} A_{1T}(q^2) - 2 \frac{\vec{k}^2 M_h^2}{M_T M_X M_h + M_T} A_{2T}(q^2) \right], \quad (C.50)$$

and

$$ \sum_{\text{polarizations}} |M_{h\to TX}|^2 = \Theta_{1W}^2 \frac{|C_{mn}|^2}{4} \frac{\vec{k}^2}{M_T^2} \left[ \frac{1}{2} \left( \frac{8M_h^2 \vec{k}^2}{(M_h + M_T)^2} T^2(q^2) + 2(M_h + M_T)^2 A_{1T}(q^2) \right)^2 + \frac{2}{3} \left( (M_h + M_T) \frac{\vec{k}^2 + E_XE_T}{M_XM_T} A_{1T}(q^2) - 2 \frac{M_h^2 \vec{k}^2}{M_T M_X (M_h + M_T)} A_{2T}(q^2) \right)^2 \right]. \quad (C.51)$$

The parametrization of the form-factors $T$ and $A_{1T}$, $A_{2T}$ is taken from [94, 95]

$$ F^{XT}_0(q^2) = \frac{F^{HT}_0}{1 - a_T \frac{q^2}{M_h^2} + b_T \left( \frac{q^2}{M_h^2} \right)^2}, \quad (C.52)$$

where the corresponding parameters are given in table 9.
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