Neutron-Antineutron Oscillations in Nuclei Revisited

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Abstract

An upper limit on $n \rightarrow {\bar n}$ oscillations can be obtained from the stability of matter. This relation has been worked out theoretically and together with data yields $\tau_{n\bar{n}} > (1 - 2) \times 10^8 \text{ sec}$. A recent publication claims a different relation and finds a nuclear suppression of the $n \rightarrow {\bar n}$ oscillations which is two orders of magnitude weaker than the previous evaluations. Using the same approach we find, nevertheless, that the earlier estimates are correct and conclude that future experiments with free neutrons from reactors are capable to put a stronger limit on the $n \rightarrow {\bar n}$ oscillation time than experiments with large amount of neutrons bound in nuclei.

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1. Introduction

Although the proton decay expected in a simple version of grand unified theories has not been observed with a large lower limit on the lifetime, other variants of the theory predict reactions with nonconservation of baryon minus lepton number by two units, like neutron-antineutron oscillations [1] or neutrinoless double-β decay (see e.g. [2]). The former phenomenon has been searched for with free neutrons from reactors in the recent experiment ILL-Grenoble [3] and the lower limit for the oscillation time

\[ \tau_{n\bar{n}} > 0.86 \times 10^8 \text{ sec} \]  (1)

is established. A new experiment planned at ORNL [4] is intended to increase this limit by two orders of magnitude.

Experiments which search for the proton decay are sensitive to \( n \to \bar{n} \) transitions as well, since an antineutron produced inside a nucleus annihilates and blows the nucleus up. Such events are not observed above the background in the experiments [5, 6]. The experiments [5, 6] established quite high lower limits on the lifetime of the nuclei, respectively,

\[ T(O^{16}) > 4.3 \times 10^{31} \text{ yr} \]  (2)

\[ T(Fe^{56}) > 6.5 \times 10^{31} \text{ yr} \]  (3)

These data can be used to deduce a limit on \( \tau_{n\bar{n}} \) provided the nuclear effects are known. Two groups [7] and [8] have calculated the nuclear effects and based on these data have provided close estimates of

\[ \tau_{n\bar{n}} > (1 - 2) \times 10^8 \text{ sec} . \]  (4)

Recently, these calculations have been challenged [9] and via the nonrelativistic diagram technique a value of \( \tau_{n\bar{n}} \) is deduced from Eqs (2)-(3), which is about two orders of magnitude higher than Eq. (4). If the latter calculation were true, this lower limit would be comparable to what is planned for future experiments with free neutrons [4].

In this note we discuss this controversy and conclude that the theory which leads to (4) is correct. We suggest an intuitive physical picture of a possible \( n \to \bar{n} \) transition in a
medium and point out why the approximation used in [9] is not valid. In our calculations we emphasize that at low energy the annihilation radius is quite large which may lead to substantial corrections to (4).

2. Interference effects in \( n - \bar{n} \) oscillations

We assume that the physical neutron (antineutron), i.e. the vacuum eigenstate, contains an admixture of a \( \bar{n} \) (\( n \)) component. Suppose that at time \( t = 0 \) we have a pure \( |n\rangle \) state (neutrons from a reactor, see below), which is a mixture of the physical \( |n\rangle_{phys} \) and \( |\bar{n}\rangle_{phys} \) having different masses. As a result of a phase shift between these states which increases with time a \( |\bar{n}\rangle \) component appears with a probability proportional to the square of time (compare with the well known effect of \( K^0 - \bar{K}^0 \) oscillations),

\[
W_{\bar{n}}(t) \approx (\epsilon t)^2
\]  

(5) (provided that \( t \ll 1/\epsilon \)). Here \( \epsilon = 1/\tau_{n\bar{n}} \) is related to the off diagonal matrix element of the vacuum Hamiltonian, and \( \tau_{n\bar{n}} \) is the \( n \rightarrow \bar{n} \) oscillation time.

The quadratic time dependence is a direct manifestation of quantum coherence, a linear dependence would be the result in classical physics. This fact is extremely important for an understanding of the suppression mechanism \( n \rightarrow \bar{n} \) transitions in nuclei. Let us consider a set up where a neutron propagates a distance \( L \) to a counter which can measure the antineutron component. In another set up we have \( N \) such counters each separated by a distance \( L/N \). Each counter filters away the antineutron component. In the first case we have a probability \( W_{\bar{n}}^{I} \approx (\epsilon L/v)^2 \), in the second set up \( W_{\bar{n}}^{II} \approx N (\epsilon L/Nv)^2 \approx W_{\bar{n}}^{I}/N \), to detect the \( \bar{n} \).Here \( v \) is the velocity of the neutron. The second case is closely related to the absorption in a nucleus, where each bound nucleon serves as a detector for the \( \bar{n} \) component.

2.1 The mean lifetime of an antineutron in a medium

The mean free time of propagation of the antineutron through nuclear matter (without
annihilation) is $t_{\text{free}} = 1/(\sigma_{\text{ann}} v \rho)$, where $\sigma_{\text{ann}}$ is the $\bar{n} - N$ annihilation cross section and $\rho$ is the density of nucleons. The smallness of $t_{\text{free}}$ is one important source of suppression of $n \rightarrow \bar{n}$ transitions in a medium. A neutron which propagates through a nuclear medium produces an antineutron which annihilates with a rate

$$\frac{dW_{\bar{n}}}{dt} = \sigma_{\text{ann}} v \rho \epsilon^2 t_{\text{free}}^2,$$  \hspace{1cm} (6)

In the limit of small velocity one can replace $\sigma_{\text{ann}}$ by the imaginary part of the $\bar{n} - N$ scattering length \[9\],

$$v \sigma_{\text{ann}} = \frac{4\pi}{m} \text{Im} a_{\bar{n}N}$$  \hspace{1cm} (7)

The annihilation rate (6) turns out to be inversely proportional the scattering length

$$\frac{dW_{\bar{n}}}{dt} = \frac{m \epsilon^2}{4\pi \rho \text{Im} a_{\bar{n}N}},$$  \hspace{1cm} (8)

This result is quite different from that in \[9\], where the annihilation rate is found to be proportional to $\text{Im} a_{\bar{n}N}$. Below we show that this might be true only in the limit of very small density $\rho$ which is not the case for a nuclear medium.

\section*{2.2 The coherence and formation times}

There is another source of the $n \rightarrow \bar{n}$ suppression. In the presence of an external field which acts differently on $n$ and $\bar{n}$, the transition $n \rightarrow \bar{n}$ is forbidden by energy conservation. It may happen only virtually, with a lifetime of the $\bar{n}$ fluctuation in the neutron,

$$t_c = \frac{1}{\Delta E},$$  \hspace{1cm} (9)

where $\Delta E$ is the energy splitting between $n$ and $\bar{n}$. In order to be detected the $\bar{n}$ fluctuation has to interact (annihilate) during its lifetime.

Note that at the far periphery of a nucleus the neutron is quasi-free, \textit{i.e.} $\Delta E \rightarrow 0$, and one may conclude from (9) and (8) that the $\bar{n}$ fluctuation lifetime has no restriction. This is not true because the neutron cannot stay at the periphery longer than $1/\omega$, where $\omega$ is the
oscillator frequency (the shell splitting). Thus, one must replace effectively $\Delta E \rightarrow \Delta E + \omega$. This is equivalent to inclusion of the formation time effects \[10\].

We conclude that the effect of coherence and formation time becomes the main source of $n \rightarrow \bar{n}$ suppression in the case of low density,

$$\frac{dW_{\bar{n}}}{dt} = \frac{4\pi \rho \text{ Im } a_{nN}}{m (\Delta E + \omega)^2} \quad (10)$$

The (unrealistic) limit of low density is recovered by calculations in \[9\]. There the energy splitting between $n$ and $\bar{n}$ in nuclear medium is taken to be $\Delta E = 8 \text{ MeV}$, the mean binding energy of a bound nucleon. It is assumed that the antineutron appears momentarily and is unable to feel the mean field of the whole nucleus, therefore, it has no binding. This cannot be true because the $\bar{n}$ fluctuation lifetime \[9\] is very long in this case, $t_c = 25 \text{ fm}$.

### 3. The evolution equation for a bound neutron

To incorporate all the above effects one should consider the evolution of a $n - \bar{n}$ system in a medium. This is a typical two-channel problem (see e.g. in \[10, 7\]).

$$i \frac{d}{dt} |\Phi(t, \vec{r})\rangle = \hat{H}(\vec{r}) |\Phi(t, \vec{r})\rangle \quad , \quad (11)$$

where the wave function $|\Phi(t, \vec{r})\rangle$ contains the two components $n$ and $\bar{n}$ and depends on time $t$ and the space $\vec{r}$. The evolution operator $\hat{H}(\vec{r})$ has a form,

$$\hat{H}(\vec{r}) = \begin{pmatrix} E_n(\vec{r}) & \epsilon \\ \epsilon & E_\bar{n}(\vec{r}) - \frac{2\pi i}{m} \bar{\rho}(\vec{r}) \text{ Im } a_{nN} \end{pmatrix} \quad (12)$$

Here $\bar{\rho}(\vec{r})$ is the nuclear density modified due to annihilation radius $R_{\text{ann}}$ which may be large,

$$\bar{\rho}(\vec{r}) = \left(\frac{3}{2\pi R_{\text{ann}}^2}\right)^{\frac{3}{2}} \int d^3s \rho(\vec{r} + \vec{s}) \exp \left(-\frac{3s^2}{2R_{\text{ann}}^2}\right) \quad (13)$$

The modified density $\bar{\rho}(\vec{r})$ spreads to larger distances than $\rho(\vec{r})$. 

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The solution of equation (11) for the weight of the antineutron component at long times $t \gg t_c, t_{\text{free}}$ reads,

$$W_{\bar{n}}(t, r) = \frac{\epsilon^2}{[\Delta E(\vec{r}) + \omega]^2 + ((2\pi/m)\bar{\rho}(\vec{r})\text{Im} a_{\bar{n}N})^2}$$ (14)

We corrected here for the oscillator frequency considered in the previous section.

One should average expression (14) over different neutrons which have different binding energies dependent on a shell [7]. However, this difference is much smaller that the binding energy of an antineutron used in [7, 8] which was assumed to be independent of the shell. Therefore, one can safely use the mean value of $\Delta E$ in (14) and then sum over the neutrons.

The annihilation rate of antineutrons produced in a nucleus is given by integral of (14) over the nuclear volume weighted by neutron density and annihilation cross section,

$$\frac{\Gamma_A}{N} = \frac{\epsilon^2}{T_R} = \frac{m\epsilon^2}{A\pi\text{Im} a_{\bar{n}N}} \int d^3r \frac{\rho(\vec{r})}{\bar{\rho}(\vec{r})} \frac{1}{1 + K^2(\vec{r})}$$ (15)

Here $A$ and $N$ are the atomic and neutron numbers of the nucleus, respectively, and

$$K(\vec{r}) = \frac{\Delta E(\vec{r}) + \omega}{(2\pi/m)\bar{\rho}(\vec{r})\text{Im} a_{\bar{n}N}}$$ (16)

The assumption made in [9] corresponds to $K(r) \gg 1$. This could be not a very rough approximation, but $\Delta E = 8 \text{ MeV}$ used in [9] is an order of magnitude smaller than in [7, 8] and here (see discussion at the end of previous section). This is the main reason why $\Gamma_A$ is overestimated by two orders of magnitude in [8].

4. Numerical evaluation

Following to [7] we assume that the $\bar{n}$ binding energy depends on $r$ in the same way as $\rho(\vec{r})$. Therefore, (16) is approximately a constant $K(r) \approx K(0)$ at $r < R_A$, where $R_A \approx r_0 A^{1/3}$ is the nuclear radius. The ratio $\rho(\vec{r})/\bar{\rho}(\vec{r})$ in (15) is unity at $r < R_A$, but then steeply falls down. Using these observation we can perform a very simple (but rough) evaluation of (15),
$$\frac{\Gamma_A}{N} \approx \frac{4 m e^2 r_0^3}{3 \text{Im} a_{\bar{n}N} (1 + K^2(0))}$$  \tag{17}

A spectacular observation which follows from this result is $A$-independence of the nuclear annihilation width. Of course, better calculations (see below) may lead to a weak $A$-dependence. The claim in [7] that annihilation rate gets main contribution from the neutrons on the surface might be misleading. In fact their result is also nearly $A$-independent, what supports our observation that all the nuclear volume contributes (see also in [9]).

The $\bar{p}p$ scattering length was determined in [11] using the LEAR data on $\bar{p}p$ atoms, $|\text{Im} a_{\bar{p}p}| \approx 0.7 - 1.2 \text{ fm}$. Following [11] we use the same value for $\bar{n}n$ and $\bar{n}p$. With this value we have,

$$\frac{2\pi}{m} \bar{\rho}(0) |\text{Im} a_{\bar{n}N}| \approx 30 - 50 \text{ MeV} ,$$  \tag{18}

where we use the central density $\bar{\rho}(0) = 0.16 \text{ fm}^{-3}$.

Expression (18) corresponds to the imaginary part of the antineutron nuclear potential in [7]. The real part of the antineutron potential in the center of the nucleus can be only guessed since the data on antineutron atoms are sensitive only to the surface of the nucleus. This is the reason of diversity of antineutron potentials used in [7, 8], which we consider as very unreliable. An advantage of [8] and our approach is that we use experimental information on $n\bar{n}$ scattering length, rather than guess a value of imaginary part of the nuclear potential. For a simple evaluation we choose the real part to be of the same order as the imaginary one, \textit{i.e.} that $K(0) \approx 1 \ (\Delta E(0) \approx 40 \text{ MeV})$. This leads to the estimate for the nuclear factor $T_R$ in (13), $T_R \approx 0.23 \text{ fm}^{-1}$, which is quite close to the results of exact integration in (15): $T_R = 0.20 \text{ fm}^{-1}$ for $^{16}O$ and $T_R = 0.18 \text{ fm}^{-1}$ for $^{56}Fe$. For these calculations we used the oscillator parameter $\omega$ from [12] and 40 MeV for (18). Following [7, 8] we assumed that $\Delta E(r) \propto \rho(r)$. We use the realistic Woods-Saxon parameterization for the nuclear density. Dependence of the result on the annihilation radius turns out to be quite weak. We use $R_{\text{ann}} = 1 \text{ fm}$.

We take the upper bound for $\Delta E(0)$ to be 200 MeV which exceeds most of model values
used in \([7, 8]\). Corresponding values of \(T_R\) are 1.2 \(fm^{-1}\) for \(^{16}O\) and 1.3 \(fm^{-1}\) for \(^{56}Fe\).

This estimation for nuclear effects combined with the experimental limits for the lifetime of the nuclei leads to the following restrictions for the oscillation time \(\tau_{n\bar{n}} = 1/\epsilon\),

\[
\tau_{n\bar{n}}^{(16}O) > (0.6 - 1.5) \times 10^8 \text{sec} \tag{19}
\]
\[
\tau_{n\bar{n}}^{(56}Fe) > (0.75 - 1.9) \times 10^8 \text{sec} \tag{20}
\]

These results are very close to the low bound (11) found in experiment \([3]\) with free neutrons and are compatible with (2) - (3) as well.

The uncertainty of the lower limit (20) is still conventional since it depends on the guessed real part of the antineutron potential inside the nucleus.

Concluding, we do not see how the lower limit for \(\tau_{n\bar{n}}\) can be pushed up significantly by using the stability of matter and one may have to turn again to experiments with free neutrons with long flight path and small velocity.

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