Majorana Fermions Under Uniaxial Stress in Semiconductor-Superconductor Heterostructures

Ming Gong$^{1,2}$, Li Mao$^{1,2}$, Sumanta Tewari$^3$, and Chuanwei Zhang$^{1,2}$

$^1$Department of Physics, the University of Texas at Dallas, Richardson, TX, 75080 USA
$^2$Department of Physics and Astronomy, Washington State University, Pullman, WA, 99164 USA
$^3$Department of Physics and Astronomy, Clemson University, Clemson, SC, 29634 USA

Spin-orbit coupled semiconductor nanowires with Zeeman splitting in proximity contact with bulk s-wave superconductivity have recently been proposed as a promising platform for realizing Majorana fermions. However, in this setup the chemical potential of the nanowire is generally pinned by the Fermi surface of the superconductor. This makes the tuning of the chemical potential by external electrical gates, a crucial requirement for unambiguous detection of Majorana fermions, very challenging in experiments. Here we show that tunable topological superconducting regime supporting Majorana fermions can be realized in semiconductor nanowires using uniaxial stress. For n-type nanowires the uniaxial stress tunes the effective chemical potential, while for p-type systems the effective pairing may also be modified by stress, thus significantly enhancing the topological minigap. We show that the required stress, of the order of 0.1%, is within current experimental reach using conventional piezo crystals.

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FIG. 1: (Color online). Engineering Majorana fermions via uniaxial stress. The stretching direction is along $z$ in model A (a) and $z$ in model B (b). The stress is assumed to be provided by conventional piezo crystals, (c) shows the pinning of the chemical potential in semiconductor by the Fermi surface of the s-wave superconductor. However, the total band structure of the semiconductor can still be shifted by uniaxial stress.

Contact with the superconductor which has an extremely high carrier density. The pinning of the chemical potential of the nanowire by the Fermi surface of the superconductor thus poses a major challenge in experiments. In the recent Delft experiments it has been found that very large electric gate voltages ($\sim (10^4 - 10^5)\Delta$) are required to tune the topological quantum phase transition, which likely provides strong evidence for the pinning of the nanowire chemical potential. The main motivation of the present work is to provide an alternative method to overcome this experimental difficulty which may greatly facilitate future experimental searches of MFs in semiconductor-based heterostructures.
which can be generated and controlled even by conventional piezo crystals. The uniaxial stress can modify the band structure of the nanowires slightly after the heterostructure system is fabricated, remarkably leading to a topological transition from the trivial to the topological superconducting state with MFs at the wire-ends. With the experimentally accessible strength of the uniaxial stress, the effective chemical potential can be tuned about 42 meV for electrons, and 5 - 20 meV for hole levels. Moreover, for the p-type systems, the uniaxial stress may also significantly enhance the minimum topological energy gap (minigap) that protects the MFs from thermal excitations. The newly added elements for generating the uniaxial stress can be effectively integrated into the design of semiconductor devices using modern nanotechnology. Therefore, our proposed scheme can go a long way in facilitating the realization and detection of Majorana fermions in semiconductor quantum wire heterostructures and the eventual implementation of topological quantum computation.

Our basic setup for experiments is illustrated in Fig. 1a and 1b. The semiconductor nanowire (e.g., InSb, InAs, etc.) is in proximity contact with an s-wave superconductor. The uniaxial stress applied on the semiconductor nanowires can be generated using nano-ferroelectric materials or by simply gluing the nanowire tightly to the surface of piezoelectric crystals such as the piezoelectric lead zirconate titanate (PZT) ceramic stack. The stretching direction of the piezo crystal can be chosen either along x (model A) or along z direction (model B). The strain tensor can be determined as,

\[ \varepsilon_{xx}^{(a)} = -\varepsilon, \quad \varepsilon_{yy}^{(a)} = \varepsilon_{zz}^{(a)} = \frac{2C_{12}}{C_{11}}\varepsilon, \]
\[ \varepsilon_{xx}^{(b)} = -\varepsilon, \quad \varepsilon_{yy}^{(b)} = \varepsilon_{zz}^{(b)} = \frac{2C_{12}}{C_{11}}\varepsilon, \]

where \( \varepsilon = (1 - a/a_0) \) defines the relative changes of the lattice constant along the corresponding crystallographic directions. Here \( a_0 \) and \( a \) are the equilibrium and distorted lattice constants, respectively. \( \varepsilon > 0 \) corresponds to compressive (tensile) stress. In experiments the sign of \( \varepsilon \) can be controlled by the voltage bias across the piezo crystals. \( C_{11} \) and \( C_{12} \) are the elastic stiffness tensors. The superscripts in Eq. (1) represent the two different models shown in Fig. 1. Note that \( \varepsilon \approx P/Y \), where \( P \) is the stress and \( Y \) is Young’s modulus. Using typical values for \( Y \sim 100 \) GPa and \( P \sim 100 \) MPa, we see that \( \varepsilon \sim 0.1\% \). Such a small strain can be provided using conventional piezo crystals. For model A, \( \varepsilon \) up to 0.11\% has already been realized in experiments, and in principle, \( \varepsilon \) up to 0.6\% can be achieved. For model B, it is more suitable to provide compressive stress along the z direction, and there is no limitation on the maximum \( \varepsilon \) because the compressive stress is not limited by the gluing technique. We assume \( |\varepsilon| < 0.3\% \) the most probable regime that can be accessed in experiments. Since the lattice deformation is very small, the uniaxial stress has negligible effects on the superconducting transition temperature as well as the s-wave pairing symmetry and deliver two properties that are crucial for the generation of Majorana fermions.

The effective Hamiltonian for n-type nanowire under uniaxial stress reads as,

\[ H = \left[ \frac{k^2}{2m} + \alpha (p \times s) + V_z \sigma_z + a_c \mathrm{Tr}(\varepsilon) \right] - \mu_F. \]

where \( a_c \) denotes the deformation potential of the conduction band, \( \alpha \) is the spin-orbit coupling strength, and \( V_z \) is the external Zeeman field induced by the magnetic field. \( \mu_F \) is the true chemical potential of the semiconductor that is pinned to the Fermi surface of the superconductor. The uniaxial stress does not change the band structure, but shifts the effective chemical potential to \( \mu = \mu_F - a_c \mathrm{Tr}(\varepsilon) \), through which the topological regime in the experiment.

![FIG. 2: (Color online) (a) Band structure of InSb nanowires. The solid horizontal dash-dotted line represents the possible pinned chemical potential when placed in proximity to the superconductor. (b) The chemical potential as a function of carrier density. \( \alpha \sim 2 \) eV-A, \( m = 0.013 m_0 \), \( \Delta_0 = 0.5 \) meV, \( V_z = 1.0 \) meV. Parameters are from Ref. 38.](image)

The Hamiltonian for p-type nanowires (assuming the sizes along the y and z directions are \( L_y \) and \( L_z \), respectively) under uniaxial stress reads as,

\[ H = H_{KL} + H_{BP} - \mu_F, \]
bands at $E \pm \hbar \omega$ are split by a small Zeeman field superconducting order parameter need be taken into account. Because the uniaxial stress only shifts the effective chemical potential for the n-type of semiconductor, we consider only p-type nanowires. In the nanowire heterostructures, the superconducting order parameter can be induced to the nanowire through proximity effect\textsuperscript{42,43}, yielding $H_{sc} = \sum_{m=\pm} \int d\mathbf{r} \Delta m v_{m}^{\dagger} \psi_{m}^{\dagger} \psi_{m}$. The corresponding BdG Hamiltonian can be written as\textsuperscript{10}

$$H_{\text{BdG}} = \left( \frac{H_{1D}}{\Delta_{4}} - \gamma \frac{1}{\beta} H_{1D}^{{\gamma}} \right)$$

in the Nambu spinor basis $\Psi = (\psi, \gamma^{\dagger} \psi^{{T}})^{T}$. Here $H_{1D} = \int dy dz \psi^{{*}} \partial_{y} H \psi_{y}$, $\gamma = i(I \otimes \sigma_{y}) \tau_{y}$, $\sigma_{x}$, $I$ and $\tau_{y}$ are Pauli operators, and $\Delta_{4} = \text{diag}(\Delta_{\pm}, \Delta_{\pm}, \Delta_{\pm}, \Delta_{\mp})$. The topological parameter regime for MFs can be obtained by the topological index\textsuperscript{44,45}

$$\mathcal{M} = \text{sign}(\text{PF}(\Gamma(0)) \cdot \text{PF}(\Gamma(\frac{\pi}{a})))$$

where $\text{PF}(\Gamma)$ refers to the Pfaffian of the matrix $\Gamma = -i H_{\text{BdG}}(q) (\tau_{y} \otimes \gamma)$, $a$ is the lattice constant. $\mathcal{M} = +1$ (-1) corresponds to the topologically trivial (nontrivial) superconducting states without (with) MFs. Note that for sufficient large $k$ all the eigenvalues of $\Gamma$ are dominated by the $k^{2}$ terms, yielding $\text{sign}(\text{PF}(\Gamma(q))) = 1$. The Pfaffian at $k = 0$ can be derived analytically, yielding $\mathcal{M} = \text{sign} F$, with $F = f_{0} - f_{1} V_{x}^{2} + \frac{1}{4} V_{y}^{2}$, $f_{0} = (\bar{\mu}^{2} + \Delta_{2}^{2} - \bar{\beta}^{2} - \bar{\gamma}^{2})^{2} + (\Delta_{2} - \Delta_{1})^{2}$, $f_{1} = (10 \bar{\mu}^{2} + 10 \bar{\beta}^{2} + 16 \bar{\mu} \bar{\beta} + 9 \Delta_{1}^{2} + \Delta_{2}^{2} - 6 \bar{\gamma}^{2} / 4)$, $\bar{\beta} = \sqrt{2} \gamma_{2} (L_{z}^{2} - L_{y}^{2} / 2) + Q_{z}$, $\beta, \bar{\beta} = \sqrt{2} \pi^{2} (2 \gamma_{2} L_{y}^{2} / 2 + R_{y})$, $\bar{\mu} = \mu + \gamma_{1} \pi^{2} (L_{z}^{2} + L_{y}^{2} / 2)$\textsuperscript{16}. The boundary for topological phase transition is determined by $F = 0$. Generally, the magnitudes of $\Delta_{\pm}$ and $\Delta_{\mp}$ are not essential for the topological quantum phase transition (but the relative sign is important)\textsuperscript{16}. Henceforth we only consider two different possible cases (I) $\Delta_{\pm} = \Delta_{\mp} = \Delta$ and (II) $\Delta_{\pm} = -\Delta_{\mp} = \Delta$. For other values of $\Delta_{\pm}$ and $\Delta_{\mp}$, the results are similar.

In Fig. 3b and 3c, we plot the boundary between the topological and non-topological superconducting states. In these figures, we assume that without uniaxial stress the chemical potential lies in a regime which requires a large Zeeman field for realizing the topological superconducting state. By applying the uniaxial stress the required critical Zeeman field can be significantly reduced. In case (I) in Fig. 3c, the critical Zeeman field can even approaches zero for model A. We have also verified that for a wide range of parameters $(\mu, L_{z}, L_{y}, \cdots)$ similar features can always be found. For case (II) in Fig. 3b the required Zeeman field can be reduced to around 1 meV ($B_{z} = 0.3$ T for $g_{\mu}^{e} \approx 50$). Generally for case (II), the minimum required $V_{x}^{c} \simeq p \Delta$, where $p = 2 \sqrt{1 + \beta_{1}^{2} / \beta_{2}^{2} / 1 + \sqrt{1 + \beta_{1}^{2} / \beta_{2}^{2}}} \in [2 / 3, 2]$. We see that there are also a wide range of parameters that enable us to achieve the minimum required Zeeman field at $p = 2 / 3$ via uniaxial stress. To further verify that

![Figure 3](image-url)
the right regime of each curve in Fig. 4a and 4b are indeed the topological superconducting regime we plot the mini-gap (solid line) and the lowest non-negative energy level (dashed line) as a function of uniaxial stress $\varepsilon$ in Figs. 4a and 4b, respectively. In the topological superconducting state, the zero energy MFs indeed exist with large minigaps around several Kelvin. We have also confirmed that the corresponding wavefunctions of MFs are well localized at the two ends of the nanowire.

The difference between the two types of uniaxial stress in models A and B can be understood by projecting the Hamiltonian to the lowest two HH bands in Refs. 37, 41-46, which yields the effective pairing at $k \rightarrow 0$,

$$\Delta_{\text{eff}} = (\Delta_{0}^{2} - \kappa \Delta_{1}^{2}) / (\kappa + 1)$$

where $\kappa = (1 + \beta_{1} / \beta_{2} - 1) / (\beta_{1} / \beta_{2})^{2} \in (0, \infty)$. When $\varepsilon = 0$, $\kappa$ only depends on the size of the nanowire, thus cannot be tuned. However, when the uniaxial stress is applied, $\kappa$ can be tuned in a considerably wide range. For model A, the off-diagonal term $R_{c} \neq 0$, thus $\beta_{2}$ can approach zero with a properly chosen strain $\varepsilon$. For the parameters used in Fig. 4 we find that the effective pairing increases (decreases) monotonically as a function of $\varepsilon$ for model A (B), thus for model B, we observe significant enhancement of the mini-gap ($\sim 30\%$) in Fig. 4. The maximum increase of the mini-gap can be obtained by optimizing different physical parameters.

Finally, several remarks are in order. First, the same idea discussed above for a single band model can be straightforwardly extended to the multiband case. Using the diameter of nanowire from Refs. 37, 41-46, we estimate the band spacing for electron (hole) is $\sim 6$ ($\sim 3$) meV, with $\sim 4$ bands occupied. Thus we expect that the stress can tune the effective chemical potential of both electrons and holes to the topological regime even though the initial value corresponds to an even number of Fermi surfaces. Second, our proposal here can also be used to engineer MFs in the vortex core of semiconductor quantum wells. For electrons the tuning of the band structures is exactly the same as in Eq. 2. For holes there are some qualitative difference from Fig. 3 since the confinement along the $y$ direction is relaxed. As a consequence, $\beta_{2} = 0$ when $\varepsilon = 0$, thus $R_{c}$ play a more significant role in the determination of the mini-gap of MFs. Third, we have also checked the validity of our proposal for InAs nanowires, and similar features have been found. However, for InAs, we note that $a_{c} \sim -5.17$ and $b \sim 1.00$, which are smaller than their counterparts in InSb, thus a slightly larger stress is required.

To conclude, due to the proximity effect between nanowires and a bulk superconductor, the chemical potential of the nanowire is generally pinned by the Fermi surface of the superconductor. Consequently, tuning the chemical potential of nanowires via electrical gates to bring it in the topological regime is inefficient in this setup. We show that this crucial obstacle can be overcome using experimentally accessible uniaxial stress which modifies the band structure slightly, leading, remarkably, to a transition from non-topological to topological states with MFs. The newly added elements for generating uniaxial stress can be effectively integrated into the design of semiconductor devices using modern nanotechnology. Therefore our scheme can be used for the realization of topological Majorana fermion excitations in semiconductors and the implementation of topological quantum computation.

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* Corresponding Author; Email: chuanwei.zhang@utdallas.edu

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