Models of Universe with a Delayed Big-Bang singularity

III. Solving the horizon problem for an off-center observer

Marie-Noëlle CÉLÉRIER

Département d’Astrophysique Relativiste et de Cosmologie, Observatoire de Paris-Meudon,
5 place Jules Janssen, 92195 Meudon Cédex, France

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Abstract. This paper is the third of a series dedicated to the study of the Delayed Big-Bang (DBB) class of inhomogeneous cosmological models of Lemaître-Tolman-Bondi type. In the first work, it was shown that the geometrical properties of the DBB model are such that the horizon problem can be solved, without need for any inflationary phase, for an observer situated sufficiently near the symmetry center of the model to justify the “centered earth” approximation. In the second work, we studied, in a peculiar subclass of the DBB models, the extent to which the values of the dipole and quadrupole moments measured in the cosmic microwave background radiation (CMBR) temperature anisotropies can support a cosmological origin. This implies a relation between the location of the observer in the universe and the model parameter value: the farther the observer from the symmetry center, the more homogeneous the observed large scale temperature anisotropies can support a cosmological origin. This article is the third in a series dedicated to the study of a new cosmological application of the inhomogeneous Lemaître-Tolman-Bondi (Lemaître, 1933; Tolman, 1934; Bondi, 1947) spherically symmetrical dust models.

In a first work (Célérier & Schneider, 1998, hereafter referred to as CS), a subclass of these models which solves the standard horizon problem without need for any inflationary phase has been identified. This subclass exhibits spatial flatness and a conic Big-Bang singularity of “delayed” type. In this preliminary approach, the observer has been assumed located sufficiently near the symmetry center of the model as to justify the “centered earth” approximation. The horizon problem has thus been solved using the properties of the null-geodesics issued from the last-scattering surface and propagating in a matter-dominated region of the universe, as seen from a centered observer.

However, we stressed in CS a potential difficulty, namely the observer at the center. Although such a location is not forbidden by scientific principles, it does not account for the observed large scale temperature anisotropies of the cosmic microwave background radiation (CMBR).

The dipole moment in the CMBR anisotropy is usually considered to result from a Doppler effect produced by our motion with respect to the CMBR rest-frame (Partridge, 1988). In the second work of the series (Schneider & Célérier, 1999, hereafter referred to as SC), we assumed that the measured CMBR dipole and quadrupole moments can have, totally or partially, a cosmological origin, and we studied to which extent they can be reproduced, in

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1. Introduction

The large scale homogeneity of the universe has been recently questioned in an increasing number of works. For instance, the controversy over whether the universe is smooth on large scale or presents an unbounded fractal hierarchy is not yet ended, and its final resolution requires the next generation of galaxy catalogs (Martinez, 1999). From another point of view, a direct test of the Cosmological Principle on our past light cone, up to redshifts approaching unity, has been recently proposed, using type Ia supernovae as standard candles (Célérier, 2000). If such tests were to exclude the universe homogeneity assumption up to such large scales, and even beyond, we would need alternative inhomogeneous models to describe its evolution.

Correspondence to: Marie-Noelle.Celerier@obspm.fr
a peculiar example of our Delayed Big-Bang (DBB) class of models, with no a priori assumption of the observer’s location. We have shown that this implies a relation between this location and the model parameter value, namely the increasing rate \( b \) of the Big-Bang function. The farther the observer is from the symmetry center, the smaller is the model. Its line-element, in comoving coordinates (\( r, \theta, \varphi \)) is

\[
ds^2 = c^2 dt^2 - R^2(r, t) d\theta^2 - R^2(r, t) (d\theta^2 + \sin^2 \theta d\varphi^2). \tag{1}
\]

An appropriate choice of the radial coordinate \( r \) yields

\[
R(r, t) = \left( \frac{9GM_0}{2} \right)^{1/3} \left( r - t_0(r) \right)^{2/3}. \tag{2}
\]

The Big-Bang function, \( t_0(r) \), verifies

\[
\begin{align*}
t_0(r = 0) &= 0, \\
t'_0(r) &= 0 \quad \text{for all } r, \\
5t'_0(r) + 2rt''_0(r) &= 0 \quad \text{for all } r, \\
r|t'_0|_{r=0} &= 0. \tag{3}
\end{align*}
\]

The physical singularity of the model - i.e. the first surface, encountered on a backward path from “now”, where the energy density and the invariant scalar curvature go to infinity - is the shell-crossing surface, represented in the \((r, t)\) plane by the curve:

\[
3t - 3t_0(r) - 2rt'_0(r) = 0. \tag{4}
\]

As the energy density increases approaching this surface, radiation becomes the dominant component in the universe, pressure can no longer be neglected, and the LTB model no longer holds. The region between the Big-Bang surface \( t = t_0(r) \) and the shell-crossing one is thus excluded from the part of the model retained to describe the matter dominated region of the universe, which we discuss here.

The optical depth of the universe to Thomson scattering is approximated by a step function (see SC). The last-scattering surface is thus defined, in the local thermodynamical equilibrium approximation (see CS), by its temperature, \( T = 4000 \, \text{K} \), as is the now-surface, \( T = 2.73 \, \text{K} \), where the observer is located. The equal temperature surfaces verify

\[
t = t_0(r) + \frac{r}{3} t'_0(r) + \frac{1}{3} \sqrt{r^2 t''_0(r) + \frac{3S(r)}{2\pi G\beta L_0 n_b T^3}}. \tag{5}
\]

The value of the entropy function \( S(r) \) is assumed to be constant with \( r \). The shell-crossing surface is thus asymptotic to every (monotonically increasing with \( r \)) \( T = \text{const.} \) curve.

An observer located at a distance from the center sees an axially symmetrical universe in the center direction. In the geometrical optics approximation, the light travelling from the last-scattering surface to this observer follows null-geodesics, which is thus legitimate to consider in the meridional plane. The photon path is uniquely defined by the observer’s position \((r_p, t_p)\) and the angle \( \alpha \) between the direction from which the light ray comes and the direction of the center of the universe, \( C \).

The microwave radiation observed in the direction of this symmetry center follows a light-cone issued from point \( D \) on the last-scattering surface, then passes through the center \((r = 0)\) and reaches the earth at point \( O \). Observed

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1 An interesting example, studied in CS and SC, is the subclass with \( t_0(r) = br^n, b > 0, n > 0 \).
in the opposite direction, it starts from point $E$, and travels on the $EO$ null-geodesic to the observer (see Fig.2).

The radial null-geodesic equation, as established in CS, is

$$\frac{dt}{dr} = \pm \frac{1}{3} \left( \frac{9GM_0}{2} \right)^{1/3} \frac{3t - 3t_0(r) - 2rt_0'(r)}{|t - t_0(r)|^{2/3}}. \quad (6)$$

It is easy to see that, with a function $t_0(r)$ verifying conditions (3), for a fixed $t$, $|\frac{dt}{dr}|$ decreases with increasing $r$, and thus: $r_D < r_E$.

If observed with an angle $\alpha$ in the inward direction, a light beam issued from point $A$ on the last-scattering surface approaches $C$ to a comoving distance $r_{\text{min}}$, then proceeds toward $O$. In the outward (opposite) direction, it follows the $BO$ geodesic (see Figs.1 and 2).

The corresponding null-geodesics are solutions of the system of differential equations established in SC (in units $c = 1$):

$$\frac{dt}{d\lambda} = k^t, \quad (7)$$

$$\frac{dr}{d\lambda} = \pm \frac{1}{R'} \left[ (k^t)^2 - \left( \frac{R_p \sin \alpha}{R} \right)^2 \right]^{1/2}, \quad (8)$$

$$\frac{dk^t}{d\lambda} = -\frac{R'}{R} (k^t)^2 + \left( \frac{R'}{R} - \frac{\dot{R}}{R} \right) \left( \frac{R_p \sin \alpha}{R} \right)^2. \quad (9)$$

with a plus sign in Eq.(8) from $O$ to $r_{\text{min}}$ ($\frac{dr}{d\lambda} > 0$), and a minus sign from $r_{\text{min}}$ to $A$ ($\frac{dr}{d\lambda} < 0$). The equations corresponding to the observer looking outward (OB curve) require a minus sign.

If one considers Eq.(8), for a fixed $t$ and for a same variation $d\lambda$ of the $\lambda$ affine parameter, the absolute value of the radial coordinate variation $dr$ is smaller with $\alpha \neq 0$ or $\pi$ than with $\alpha = 0$ or $\pi$. It follows that $r_D < r_A < r_B < r_E$.

With $\alpha$ taking every value between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, and considering the two opposite directions of sight, the CMBR is observed from $O$ as coming from a set of points each located on a 2-sphere belonging to the subset $\{r = \text{const.}, r_D < r < r_E\}$ on the last-scattering surface $T = 4000K$.

To prove that this set of points can be causally connected, it is sufficient to show that there is at least one forward radial light-cone, i.e. issued from a point $(r = 0, t > 0)$ including the $DE$ subset.
3. Solving the horizon problem

An inspection of Eqs. (4) and (6), as done in CS, shows that the shell-crossing singularity surface is null: it cannot be crossed by any ingoing null geodesic.

The solution of the horizon problem, for an off-center observer in a DBB model, can thus proceed from its representation using a Penrose-Carter conformal diagram (see Fig. 3).

The horizon problem is thus solved permanently in this model, a priori, for any location of the observer.

It is worth emphasizing that if the inflationary assumption also solves the horizon problem, it does so only temporarily. In effect, if one considers the horizon problem in a standard FLRW universe, as sketched in Fig. 4, the Big-Bang surface is space-like. It thus implies the existence of a limiting point $L$, in the history of the observer $O$, beyond which the observer sees, on the last-scattering surface, some no causally connected points. The current observer, being located above $L$, is confronted with this horizon problem.

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2 Only half of the diagram is drawn, as permitted by spherical symmetry.

3 As pointed out in Sect. 4 (below), constraints yielded by observational data can be imposed upon the observer location, but solving the horizon problem, in the dust approximation, does not provide any constraint of this kind.
can see non-causally connected points in the CMBR. Inflation thus solves the horizon problem, but only temporarily.

4. Discussion and conclusion

In CS we identified a subclass of the LTB models with a Big-Bang of “delayed” type which solves the standard horizon problem without need for any inflationary phase. In this preliminary approach, the observer was assumed located sufficiently near the symmetry center of the model as to justify the “centered earth” approximation.

Here, we report a further analysis of the properties of the DBB model to show that this model solves the horizon problem even with an off-center observer. The model is thus relieved of a prescription that could be considered as “unnatural”.

The model is also provided with a new free parameter, the spatial location $r_p$ of the earth in the universe, which accounts for the large scale inhomogeneities observed in the CMBR temperature anisotropies. The measured dipole and quadrupole moments of these anisotropies set bounds on the correlated values of this $r_p$ parameter and of the local deviation of the model from homogeneity, accounted for by the slope of the Big-Bang function. A possible cosmological part of these large scale features seen in the CMBR, if once observationally identified, would select an even narrower curve in the parameter space of the model, as shown in SC.

It is of the utmost importance to stress that, as was the case with a centered observer, these results hold for any universe arbitrarily locally close to the FLRW $t_0(r) = \text{const.}$ asymptotic model. The only requirements to be fulfilled are conditions (3) which are obviously compatible with an almost “flatness” of the Big-Bang function up to comoving shells arbitrarily far out the $r_p$ shell where the observer is located. The properties of the light-cones are preserved as long as this function does not reduce to a mere constant. For instance, the subclass retained in SC, with $t_0(r) = br$, reduces to a FLRW model for $b$ equal to zero, but fulfills the conditions (3) for $b$ as small as one wishes, provided $b$ does not vanish. No bound can therefore be a priori inferred on the observer location, as, according to SC, an arbitrarily small value of $b$ corresponds to an arbitrarily large value of $r_p$, and conversely.

A point worth discussing here is the validity of this claim as regards the almost Ehlers-Geren-Sachs (AEGS) theorem (Stoeger et al. 1995). This theorem states that “if all fundamental observers measure the cosmic background radiation to be almost isotropic in an expanding universe region, then that universe is locally almost spatially homogeneous and isotropic in that region.” The U region considered by the AEGS authors is “the region within and near our past light cone from decoupling to the present day”. It is easy to see that small $b$ DBB models fulfills the AEGS prescription, as they can remain “close” to FLRW models for shells located between the center and an arbitrarily large value of the comoving radial coordinate, including the AEGS region. But the further away part of these small $b$ DBB models infinitely diverges from homogeneity. On the contrary, the AEGS theorem does not apply to large $b$ DBB models, implying an observer close to the center. The founding assumption of this theorem, namely the local Copernican principle applied to the U region, is not retained in this case. As was discussed in CS, such a choice is perfectly compatible with all available observational data and scientifically grounded principles.

It is also interesting to note that, contrary to the inflationary assumption which restores causality between the

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4 Such bounds could in fact proceed from further analyses of the model in the light of other theoretical considerations or observational data, but solving the horizon problem does not yield any constraint of this kind.
different points seen in the CMBR, but only temporarily, the DBB model provides a permanent solution to the horizon problem, whatever the position of the observer on his world line.

In the prospect of future observational tests of the large scale (in)homogeneity of the universe, the development of other interesting inhomogeneous models must be regarded as an important issue. However, this presented result is only a first improvement in the release of the simplifying assumptions (retained in CS) for a preliminary study of the properties induced by a “delayed Big-Bang”. Other analyses are still needed, among which the release of the spatial spherical symmetry of the model and of the dust approximation should be considered as priorities.

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