The Exponentiated Exponential-Inverse Weibull Model: Theory and Application to COVID-19 Data in Saudi Arabia

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1.Introduction

In statistical theory, improvement of classical distribution becomes a common practice. Probability distributions are used to model the phenomenon of natural life, but in many situations, there is a need to propose a new model for the better exploration of the data. The recent development in distribution theory stresses on new approaches for introducing new models. The new approaches depend on modifying the baseline by adding one or more parameters, to generalize the existing family. The aim of these is to provide more flexibility or to obtain better fits to the model compared with related distributions.

Barreto-Souza et al. [1] discussed the beta generalized exponential (BGE) model, Khan [2] investigated the beta inverse Weibull (BIW) model, Kundu and Howlader [3] studied Bayesian inference of the inverse Weibull (IW) model under type II censored schemes. Gusmao [4] discussed the generalized IW (GIW) distribution, modified IW distribution has been studied by Khan and King [5], Hanook et al. [6] introduced beta IW distribution, Abbas et al. [7] studied the Topp–Leone IW distribution. Elbatal and Muhammed [8] proposed Exponentiated generalized inverse Weibull distribution, and Elbatal et al. [9] introduced the beta generalized IW geometric model. Alkarni et al. [10] studied the half logistic IW. Nadarajah [11] studied the exponentiated exponential (EE) model, and also, Alzaghal et al. [12] defined a new family named “exponentiated T-X distribution”. Some of its characteristics and specific instances are examined, and obtained on $t$ is a nonnegative continuous random variable (RVr) $T$ specified as $[0, \infty)$.

In this study, we used the T-X family approach to obtain the EE-IW model. The newly suggested model is formed by combining two models known as the $T-X$ family. The RVr $T$ is the generator of the EE model and IW model. The primary goal of this study is to propose and determine the statistical features of a novel distribution (EE-IW). The hazard function and its many shapes allow it to suit various datasets.

The remainder of the paper is arranged as described in the following. Section 2 introduces the new model (EE-IW) distribution with some important different characteristics such as the probability density function (pdf), the cumulative function (cdf), the hazard function, and graphs of different values for parameters. The $r^{th}$ moment is discussed in Section 3. The MLE estimators are introduced in Section 4. A simulation study is introduced in Section 5. A real dataset is applied in Section 6. Finally, Section 7 concludes this study.
2. The EE-IW Model

In this section, we propose the EE-IW distribution, and we derive density, cumulative, reliability, and hazard functions of the new distribution.

Let \( r(t) \) be the pdf of RV \( r \), then the exponential model of \( t \) is

\[
r(t) = \beta e^{(-\beta t)}, \quad \beta, t > 0.
\]

The cdf and pdf of the RV \( x \) of the IW model are

\[
g(x) = ax^{-\alpha-1}e^{-x^2}; \quad x > 0, \; \alpha > 0,
\]

\[
G(x) = e^{-x^2}; \quad x > 0, \; \alpha > 0.
\]

Using the formula in Alzaghah et al. [12], we define the cdf for the EE-IW model for an RV \( x \) as

\[
f(x) = c\beta g(x) [G(x)]^{(c-1)} (1 - G(x))^{\beta-1}.
\]

Inserting (2) and (3) in (4), we get the pdf EE-IW as

\[
f(x) = ca\beta x^{-a-1}\left(e^{-cx^a}\right)\left[1 - \left(e^{-cx^a}\right)\right]^{\beta-1}; \quad x > 0, \; c, \; \alpha > 0,
\]

where \( c, \alpha, \) and \( \beta \) are the shape parameters. We can expand the above pdf given in (5) using the binomial expansion as follows:

\[
f(x) = ca\beta \sum_{i=0}^{\infty} (-1)^i \binom{\beta - 1}{i} x^{(a+1)i} e^{-ci x^a}.
\]

The corresponding cdf for the EE-IW model given in (5) is

\[
F(x) = 1 - \left[1 - \left(e^{-cx^a}\right)\right]^{\beta}; \quad \beta, \alpha, \; c, \; x > 0.
\]

The corresponding reliability of the EE-IW model has the following form:

\[
R(x) = \left[1 - \left(e^{-cx^a}\right)\right]^{\beta}; \quad \beta, \alpha, \; c, \; x > 0.
\]

The corresponding hazard function of the EE-IW model has the following form:

\[
h(x) = \frac{ca\beta x^{-a-1}\left(e^{-cx^a}\right)\left[1 - \left(e^{-cx^a}\right)\right]^{\beta-1}}{\left[1 - \left(e^{-cx^a}\right)\right]^{\beta}}.
\]

2.1. The Submodels of the EE-IW Distribution. In this section, some special cases of the proposed model are given. Table 1 introduces a brief list of the submodels.

From Table 1, it can be noticed that the EE-IW reduces to the exponentiated IW (E-IW) model when \( \beta = 1 \). For \( \beta = c = 1 \), it becomes the standard IW exponentiated (SIWE) model. For \( \beta = \alpha = 1 \), it reduces to the exponentiated standard inverted exponential (ESIE) distribution. For \( \alpha = -1 \), it becomes the EE model. For \( c = 1 \), we get the exponentiated Frechet (EF) distribution.

Figures 1–4 illustrate the plots of the pdf, cdf, hazard, and reliability functions, respectively.

Figure 1 shows various shapes of the pdf for various values of the parameters, such as unimodal right-skewed.

Figure 2 shows the cdf curves for various values of some selected parameters.

Figure 3 shows the \( h(x) \) curves of the EE-IW model with various values of the shape parameters, and as the shape parameter increases, the \( h(x) \) first increase and then decrease.

Figure 4 shows the \( R(x) \) curves for different values of the parameters for distribution, and as the shape parameter increases, the \( R(x) \) decreases.

3. Basic Properties

This section investigated some important basic properties of the EE-IW model.

3.1. The Noncentral Moment. The \( r^{th} \) moment about zero of the EE-IW model is provided by

\[
\mu'_r = E(X^r) = A_r \Gamma \left(1 - \frac{r}{\alpha}\right), r < 1,
\]

where

\[
A_r = \left\{ c\beta \sum_{i=0}^{\infty} (-1)^i \binom{\beta - 1}{i} \frac{1}{[c(i+1)]^{1+\alpha}} \right\}, \quad r = 0, 1, \ldots.
\]

Let \( r = 1 \) in equation (10), we get the expected value or the first moment:

\[
\mu'_1 = \mu = E(X) = A_1 \Gamma \left(1 - \frac{1}{\alpha}\right).
\]

For \( r = 2 \) in equation (10), we get the second moment:

\[
\mu'_2 = E(X^2) = A_2 \Gamma \left(1 - \frac{2}{\alpha}\right).
\]

For \( r = 3 \) in equation (10), we get the third moment:

\[
\mu'_3 = E(X^3) = A_3 \Gamma \left(1 - \frac{3}{\alpha}\right).
\]

For \( r = 4 \) in equation (10), we get the fourth moment:

\[
\mu'_4 = E(X^4) = A_4 \Gamma \left(1 - \frac{4}{\alpha}\right).
\]

The variance of the EE-IW distribution is obtained by using both equations (12) and (13) as follows:

\[
\sigma^2 = \text{Var}(X) = A_2 \Gamma \left(1 - \frac{2}{\alpha}\right) - \left[A_1 \Gamma \left(1 - \frac{1}{\alpha}\right)\right]^2, \alpha > 0.
\]

We can define the coefficient of variation of EE-IW distribution by using both equations (12) and (13):
Table 1: The subdistribution from the EE-IW.

| $c$ | $\alpha$ | $\beta$ | Model CDF References |
|---|---|---|---|
| $\sqrt{1}$ | 1 | 1 | Exponentiated inverted Weibull (E-IW) distribution $F(x) = (e^{-cx})$ Flaih et al. [13] |
| 1 | $\sqrt{1}$ | Standard inverted Weibull exponentiated (SIWE) distribution $F(x) = (e^{-x})$ Flaih et al. [13] |
| $\sqrt{1}$ | 1 | Exponentiated standard inverted exponential (ESIE) distribution $F(x) = (e^{-cx})$ Flaih et al. [13] |
| 1 | $\sqrt{1}$ | Exponentiated Frechet (EF) distribution $F(x) = 1 - [(1 - e^{-x})]^\beta$ Badr [14] |
| $\sqrt{1}$ | $\sqrt{1}$ | Exponentiated exponential (EE) distribution $F(x) = [(1 - e^{-x})]^\beta$ Gupta and Kundu [15] |

Figure 1: pdf of the EE-IW model.

Figure 2: cdf of the EE-IW model.
\[ CVar = \frac{\sqrt{A_2 \Gamma(2 - 1/\alpha) - [A_1 \Gamma(1 - 1/\alpha)]^2}}{A_1 \Gamma(1 - 1/\alpha)}. \]

\[ (17) \]

The skewness for EE-IW is \( \gamma_3 \) which can be obtained by referring to the moments by using equations (12)–(14) as:

\[ \gamma_3 = \frac{A_4 \Gamma(1 - 3/\alpha) - 3A_2A_3 \Gamma(1 - 1/\alpha) \Gamma(1 - 2/\alpha) + 2\left[ A_4^2 \Gamma(1 - 1/\alpha)^3 \right]}{\left\{ A_2 \Gamma(1 - 2/\alpha) - \{A_1 \Gamma(1 - 1/\alpha) \}^2 \right\}^{3/2}}, \alpha > 0. \]

\[ (18) \]

The kurtosis for EE-IW is \( \gamma_4 \) which can be obtained by referring to the moments by using equations (12)–(15) as follows:

\[ \gamma_4 = \frac{A_4 \Gamma(1 - 4/\alpha) - 4A_3A_2 \Gamma(1 - 3/\alpha) \Gamma(1 - 1/\alpha) + 6A_2^2A_1 \Gamma(1 - 1/\alpha) \Gamma(1 - 2/\alpha) - 3\left[ A_4^2 \Gamma(1 - 1/\alpha)^4 \right]}{\left\{ A_2 \Gamma(1 - 2/\alpha) - \{A_1 \Gamma(1 - 1/\alpha) \}^2 \right\}^2}, \alpha > 0. \]

\[ (19) \]

Figure 5 shows the mean, variance, skewness, and kurtosis curves of the EE-IW model with \( c = 2 \) and for various values of \( \alpha \) and \( \beta \).
When the values of $\gamma_4$ and $\gamma_3$ are constant for various values of $\beta$, the mean, MD, MO, and SD will be decreased, but $c_4$ and $c_3$ are decreasing.

4. The Maximum Likelihood Estimators

In this section, the MLE of the unknown parameters is introduced.

Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample from the EE-IW model which has parameters $c, \alpha$, and $\beta$. The likelihood function (LLF) is $L(\theta|x) = \prod_{i=1}^{n} f(x_i)$ where $f(.)$ is reported in (5) and $\theta = (c, \alpha, \beta)$. By calculating the logarithm of LLF, we have the following:

$$l = n \ln c + n \ln \alpha + n \ln \beta - (\alpha + 1) \sum_{i=1}^{n} \ln x_i - c \sum_{i=1}^{n} x_i^{-\alpha} + (\beta - 1) \sum_{i=1}^{n} \ln[1 - (e^{-cx_i^{-\alpha}})].$$  \hspace{1cm} (23)

Differentiate (23) in regard to $c, \alpha$, and $\beta$ and correspondingly we have

$$l_j = \frac{\partial \ln L}{\partial \theta_j} = \frac{1}{L} \frac{\partial L}{\partial \theta_j}, \quad j = 1, 2, 3.$$  \hspace{1cm} (24)

$$l_1 = \frac{\partial l}{\partial c} = -c - \sum_{i=1}^{n} x_i^{-\alpha} + (\beta - 1) \sum_{i=1}^{n} x_i^{-\alpha} \left\{ \frac{x_i^{-\alpha}}{1 - (e^{-cx_i^{-\alpha}})} \right\},$$  \hspace{1cm} (25)

$$l_2 = \frac{\partial l}{\partial \alpha} = -\sum_{i=1}^{n} \ln x_i + c (\ln x_i) \sum_{i=1}^{n} x_i^{-\alpha} - c (\beta - 1) (\ln x_i) \sum_{i=1}^{n} x_i^{-\alpha} \left[ e^{-cx_i^{-\alpha}} \right],$$  \hspace{1cm} (26)

$$l_3 = \frac{\partial l}{\partial \beta} = \frac{n}{\beta} + \ln[1 - (e^{-cx_i^{-\alpha}})].$$  \hspace{1cm} (27)

By setting the previous two equations (24) and (25) equal to 0 and solving them simultaneously yield the MLEs $(\hat{c}, \hat{\alpha})$ of parameters $(c, \alpha)$.

The MLE of the parameter $\beta$, $\hat{\beta}_{MLE}$, can be computed by using (26) as

$$\hat{\beta}_{MLE} = \frac{\sum_{i=1}^{n} \ln \left[ 1 - (e^{-cx_i^{-\alpha}}) \right]}{\sum_{i=1}^{n} x_i^{-\alpha}}.$$  \hspace{1cm} (28)

We computed the asymptotic variance-covariance (VC) matrix by $I_{ij}(\hat{\theta})$, which includes the VC of estimations while
ignoring the expectation of the second partial derivative (SPD) \( I_{ij}(\hat{\theta}) = E(\partial^2 \ln L / \partial \theta_i \partial \theta_j) \).

The SPD of the parameters for the EE-IW model is

\[
\begin{align*}
\frac{\partial^2 I}{\partial c^2} &= \frac{-1}{c^2} \left[ \frac{(\beta - 1)(x^{-a})(e^{-cx})^2}{1 - (e^{-cx})} \right] - \frac{(\beta - 1)(x^{-a})^2 (e^{-cx})^2}{1 - (e^{-cx})^2}, \\
\frac{\partial^2 I}{\partial a^2} &= \frac{-1}{a^2} \left[ c(\beta - 1)x^{-a}(\ln x)^2 (e^{-cx}) - \frac{c^2(\beta - 1)(x^{-a})^2 (\ln x)^2 (e^{-cx})^2}{1 - (e^{-cx})} \right] - \frac{c^2(\beta - 1)(x^{-a})^2 (\ln x)^2 (e^{-cx})^2}{1 - (e^{-cx})^2}, \\
\frac{\partial^2 I}{\partial \beta^2} &= \frac{-1}{\beta^2}, \\
\frac{\partial^2 I}{\partial \beta \partial c} &= \frac{-1}{\beta} \left[ \frac{c(\beta - 1)x^{-a}(\ln x)(e^{-cx})}{1 - (e^{-cx})} \right], \\
\frac{\partial^2 I}{\partial \beta \partial a} &= \frac{-1}{\beta} \left[ \frac{cx^{-a}(\ln x)(e^{-cx})}{1 - (e^{-cx})} \right], \\
\frac{\partial^2 I}{\partial a \partial c} &= \frac{-1}{c\beta} \left[ \frac{c(\beta - 1)x^{-a}(\ln x)(e^{-cx})}{1 - (e^{-cx})} \right].
\end{align*}
\]

(29)

5. Simulation Outcomes

To demonstrate the theoretical outcomes of the estimated issue, simulation experiments were conducted using Mathematica 11.2 software. 1000 random samples of size \( n = 20, 40, 60, 80, \) and 100 were generated from the EE-IW model. The initial value is chosen as \( c = 0.8, a = 0.2, \beta = 0.5 \). The accuracy of the produced parameter estimators has been evaluated in terms of their estimate for the parameters, bias (B) and mean square error (MSEr), where

\[
B = \frac{\sum_{i=1}^{1000}(\hat{\theta}_i - \theta)}{1000}, \quad \text{MSEr} = \frac{\sum_{i=1}^{1000}(\hat{\theta}_i - \theta)^2}{1000}.
\]

(30)

The B and MSEr of the estimators for the parameters for each sample size are computed.

Table 3 shows the values of B and the MSEr for the non-Bayesian estimators when parameters \( \hat{c}, \hat{a}, \) and \( \hat{\beta} \) are unknown based on complete samples, using different sample sizes \( n \).

Table 4 shows the values of B and MSEr for the non-Bayesian estimators for the parameter \( \hat{\beta} \) when \( a \) and \( \beta \) are known based on complete samples, using different sample sizes \( n \).

Table 5 shows the values of B and MSEr for the non-Bayesian estimators for the parameter \( \hat{\beta} \) when \( c \) and \( \beta \) are known based on complete samples, using different sample sizes \( n \).

Table 6 shows the values of B and MSEr for the non-Bayesian estimators for the parameter \( \hat{\beta} \) when \( c \) and \( \alpha \) are known based on complete samples, using different sample sizes \( n \).

From Table 3 The values of B and the MSEr for the non-Bayesian estimators for the parameters \( \hat{c} \) are evaluated when \( a \) and \( \beta \) is known based on complete samples, using different sample size \( n \). we note that

1. The biases of the estimates decrease as the \( n \) increases
2. The MSErs of the estimates decrease as the sample size increases

From Tables 4–6, we note that

1. The B and the MSErs of the estimates decrease as the \( n \) increases
2. As the sample size increases, the MSErs approaches zero

6. Modelling to Real Data

In this section, we choose different distributions of the same family and approximately from the EE-IW distribution such as exponentiated Weibull (EW) [16], EE Bur XII [17], EE [15], and exponentiated Frechet (EF) [14], and it is considered an application to three datasets. In order to choose the best model, we calculate some information criterion (IC), Akaike IC (AIC), corrected AIC (CAIC), and Bayesian IC (BIC) for all competing and subdistribution. We compute the MLEs for the EE, EW Bur XII, EE, and EF models.

6.1. First Dataset. The following dataset is presented by Almetwally [18]. The data came from a 32-day COVID-19 dataset from Saudi Arabia. The data are as follows: 0.0557, 0.0559, 0.0617, 0.0649, 0.0683, 0.0709, 0.0711, 0.0736, 0.0737, 0.0739, 0.0741, 0.0743, 0.0778, 0.0783, 0.0804, 0.0820, 0.0818, 0.0819, 0.0840, 0.0850, 0.0864, 0.0867, 0.0869, 0.0901, 0.0904, 0.0907, 0.0914, 0.0943, 0.0946, 0.1009, 0.1134.

Table 7 clearly shows that the EE-IW distribution fits better than the EE Bur XII, EF, EE, and EW models for this
dataset. Also, Figure 6 illustrates the fitted empirical pdf for the dataset. Figure 6 shows that the EE-IW distribution is the best-fitting model among all the models tested, and they back up the results.

6.2. Second Dataset. The following dataset is presented by Nichols [19]. The data resulted from breaking stress of carbon fibers (in Gba). The data are as follows: 3.7, 3.11, 4.42, 3.28, 3.75, 2.96, 3.39, 3.31, 3.15, 2.81, 1.41, 2.76, 3.19, 1.59, 2.17, 3.51, 1.84, 1.61, 1.57, 1.89, 2.74, 3.27, 2.41, 3.09, 2.43, 2.53, 2.81, 3.31, 2.35, 2.77, 2.68, 4.91, 1.57, 2.00, 1.17, 2.17, 0.39, 2.79, 1.08, 2.88, 2.73, 2.87, 3.19, 1.87, 2.95, 2.67, 4.20, 2.85, 2.55, 2.17, 2.97, 3.68, 0.81, 1.22, 5.08, 1.69, 3.68, 4.70, 2.03, 2.82, 2.50, 1.47, 3.22, 3.15, 2.97, 2.93, 3.33, 2.56, 2.59, 2.83, 1.36, 1.84, 5.56, 1.12, 2.48, 1.25, 2.48, 2.03, 1.61, 2.05, 3.60, 3.11, 1.69, 4.90, 3.39, 3.22, 2.55, 3.56, 2.38, 1.92, 0.98, 1.59, 1.73, 1.71, 1.18, 4.38, 0.85, 1.80, 2.12, 3.65.

Table 8 clearly shows that the EE-IW distribution fits better than the EE Bur XII, EF, EE, and EW models for this dataset. Also, Figure 7 illustrates the fitted empirical pdf for
the dataset. Figure 6 shows that the EE-IW distribution is the best-fitting model among all the models tested, and they back up the results.

6.3. Third Dataset. The following dataset is presented by Lawless [20]. The data resulted from a test on the endurance of deep groove ball bearings. The data are as follows: 17.88,
Table 6: The B and MSE of the unknown parameter $\theta = (c, \alpha, \beta)$.

| n   | Method | $c = 0.2, \alpha = 0.2$ | $c = 0.4, \alpha = 0.5$ | $c = 0.8, \alpha = 0.7$ |
|-----|--------|-------------------------|-------------------------|-------------------------|
| 20  | MLE    | 0.6321 0.1321 0.01744   | 0.6424 0.1424 0.0203    | 0.7739 0.2739 0.0750   |
| 40  | MLE    | 0.5796 0.0796 0.0063    | 0.6114 0.1114 0.0124    | 0.6004 0.1004 0.0101   |
| 60  | MLE    | 0.5645 0.0645 0.0042    | 0.5497 0.0947 0.0025    | 0.5967 0.0967 0.0094   |
| 80  | MLE    | 0.5637 0.0637 0.0041    | 0.5146 0.0946 0.0022    | 0.5479 0.0979 0.0023   |
| 100 | MLE    | 0.5289 0.0289 0.0008    | 0.5118 0.0118 0.0001    | 0.5280 0.0280 0.0008   |

Table 7: Parameter estimates and values of AIC, BIC, and CAIC, for the 1st data.

| Distribution | $\hat{c}$ | $\hat{\alpha}$ | $\hat{\beta}$ | $\hat{m}$ | AIC  | BIC  | CAIC |
|--------------|-----------|-----------------|---------------|----------|------|------|------|
| EE-IW        | 0.0402    | 1.5731          | 5.4312        | —        | 676.93| 676.32| 672.78|
| EW           | 3.5476    | 0.4859          | 2.3996        | —        | 935.08| 939.48| 935.94|
| EE-Bur XII   | 8.2617    | 1.5219          | 27.2299       | 24.6929  | 685.49| 691.35| 686.97|
| EE           | 2.2281    | —               | 85.9425       | —        | 895.72| 900.93| 895.84|
| EF           | —         | 1.3895          | 4.5093        | —        | 717.42| 720.35| 717.83|

Figure 6: The empirical distribution and estimated cdf of the models for the COVID-19 data.

28, 92, 33, 41.52, 42.12, 45.60, 48.4, 51.84, 51.96, 54.12, 55.56, 67.8, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, 173.4.

Table 9 clearly shows that the EE-IW distribution fits better than the EE Bur XII, EF, EE, and EW models for this dataset. Also, Figure 8 illustrates the fitted empirical pdf for the dataset. Figure 6 shows that the EE-IW distribution is the best-fitting model among all the models tested, and they back up the results.

For Table 7, the EE-IW distribution has the lowest AIC, BIC, and CAIC values among all fitted models. Hence, this new distribution can be chosen as the best model for fitting these data sets. Modeling to COVID-19 data demonstrates the model’s flexibility, usefulness, and capability.

For Tables 8 and 9, the EE-IW distribution has the lowest AIC, BIC, and CAIC values among all fitted models. Hence, this new distribution can be chosen as the best model for fitting these data. From Table 8, modeling breaking stress of carbon fibers data demonstrates the model’s flexibility, usefulness, and capability. In Table 9, modeling the data resulted from a test on the endurance of deep groove ball bearings.
Table 8: Parameter estimates and values of AIC, BIC, and CAIC, for the 2nd data.

| Distribution | \( \hat{c} \) | \( \hat{\alpha} \) | \( \hat{\beta} \) | \( \hat{m} \) | AIC | BIC | CAIC |
|--------------|---------|---------|---------|----------|-----|-----|-----|
| EE-IW        | 0.0209  | 6.1461  | 0.0613  | —        | 499.64 | 507.45 | 499.89 |
| EW           | 1.4963  | 0.8243  | 14.1075 | —        | 1053.74 | 1061.55 | 1053.99 |
| EE-Bur XII   | 5.8125  | 2.0665  | 0.0976  | 5.8371   | 505.89 | 516.309 | 506.309 |
| EE           | 2.2281  | —       | 85.9425 | —        | 895.72 | 900.93 | 895.84 |
| EF           | —       | 0.2966  | 5.7863  | —        | 874.72 | 879.93 | 874.84 |

Figure 7: The empirical distribution and estimated cdf of breaking stress of carbon fibers (in Gba).

Table 9: Parameter estimates and values of AIC, BIC, and CAIC, for the 3rd data.

| Distribution | \( \hat{c} \) | \( \hat{\alpha} \) | \( \hat{\beta} \) | \( \hat{m} \) | AIC | BIC | CAIC |
|--------------|---------|---------|---------|----------|-----|-----|-----|
| EE-IW        | 2.30296 | 3.9506  | 0.0028  | —        | 259.979 | 264.521 | 262.201 |
| EW           | 0.0279  | 1.0210  | 4.8075  | —        | 1081.60 | 1085.01 | 1082.86 |
| EE-Bur XII   | 26.8896 | 0.9034  | 0.85713 | 1.0249   | 483.52 | 488.07 | 485.75 |
| EE           | 0.09303 | —       | 111.869 | —        | 900.735 | 903.01 | 901.34 |
| EF           | —       | 2.8880  | 0.08317 | —        | 307.03 | 309.30 | 307.63 |

Figure 8: The empirical distribution and estimated cdf of the models for the deep groove ball bearings data.
7. Conclusion

In this study, the three-parameter exponentiated exponential inverted Weibull distribution (EE-IW) is proposed. Statistical properties of the EE-IW are studied. Maximum likelihood estimators of the EE-IW parameters are obtained. The information matrix and the asymptotic confidence bounds of the parameters are derived. Monte Carlo simulation studies are conducted under different sample sizes to study the theoretical performance of the MLE of the parameters.

Data Availability

The numerical dataset used to perform the study presented in the paper can be acquired from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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