With the new cosmological data gathered over the last few years, the inflationary paradigm has seen its predictions largely unchallenged. A recent proposal, called the ekpyrotic scenario, was argued to be a viable competitor as it was claimed that the spectrum of primordial perturbations it produces is scale invariant. By investigating closely this scenario, we show that the corresponding spectrum depends explicitly on an arbitrary function of wavenumber and is therefore itself arbitrary. It can at will be set scale invariant. We conclude that the scenario is not predictive at this stage.

1 Introduction

The standard cosmological model is nowadays well-established on a firm basis consisting of precise observational data. It supposes in particular a phase of accelerated expansion, i.e. an inflationary epoch during which the total energy density of the Universe would have been driven towards its critical value, the monopole excess diluted and the horizon size enormously increased. Such a process would thus solve the major problems of ordinary cosmology, with a bonus: the spectrum of primordial perturbations it generates, a scale invariant Harrison-Zel’dovich one, matches the distribution of anisotropies in the observed micro-wave background sky and the large scale structure distribution. With the advent of precision data, such a prediction happens to be unchallenged as all previously advocated alternative models have revealed themselves unable to reproduce these data. The so-called inflationary paradigm, even with its own difficulties, thus stands alone on stage.

By evolving the expanding Universe backwards in time, one can show that it must have initiated from a singularity. This fact is nowadays assumed to reflect the current limitation of the understanding of the physics at very high energies at which General Relativity (GR) in particular does presumably not hold. It was suggested, back in the thirties by Tolman, Einstein
and Lemaître, that the present phase of expansion might have been preceded by a collapsing one. Some kind of exotic matter, or violation of GR was needed for that purpose and the idea sank into oblivion.

Avoiding the primordial singularity through a bounce in a four-dimensional theory was revived during the seventies and more recently. It was shown that either some instability develops, in the case of purely hydrodynamical perturbations, or the resulting spectrum is very much model dependent. Another proposal, which is the subject of this paper, was suggested as a viable competitor to inflation, and called the ekpyrotic scenario, includes a collapsing phase that is extremely slow. Such a slowly contracting phase was also shown to have the ability to produce a scale invariant spectrum.

In what follows, we briefly recall the basics of the model, discuss how perturbations are supposed to be produced, and show why they cannot yet be used in observational cosmology: their calculation involves an explicit arbitrary function of scale which may be adjusted in such a way as to reproduce the data. The most difficult part for this model has yet to be done, namely to find a way of evaluating this arbitrary function on the basis of an underlying microphysics.

2 The ekpyrotic scenario

The ekpyrotic scenario is supposedly a five-dimensional superstring inspired model in which two four-dimensional branes move towards each other, collide, and move apart. These phases are viewed, in one of the brane, as a slowly collapsing phase, a singularity which is assumed to provide the Big-Bang origin of time and to produce entropy, followed by a subsequent radiation dominated expansion phase. The process may repeat itself, leading to the so-called cyclic extension. It should be mentioned that many criticisms have been made to this proposal, mostly from the viewpoint of particle physics, but also on the applicability of perturbation theory. We shall not be concerned here with these criticisms and assume that they have somehow been answered.

From the four-dimensional point of view, it is argued that the model reduces essentially to GR together with a scalar field $\phi$, the field evolving in a negative exponential potential, namely

$$S = \int d^4x \sqrt{-g} \left[ -\frac{R}{2\kappa} - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right], \quad V(\phi) = -V_i \exp \left[ \frac{4\sqrt{\pi\gamma}}{m_{\text{Pl}}} (\phi - \phi_i) \right],$$

with $\gamma$ a constant and $\kappa = 8\pi G = 8\pi/m_{\text{Pl}}^2$. The model then yields a power-law collapse up to the point at which the potential is supposed to vanish. From then on, the scale factor goes as $a(\eta) \propto \sqrt{\eta}$, which vanishes when $\eta$, the conformal time, changes sign. Later on, radiation takes over, leading to the standard model with asymptotic behaviour $a \sim \eta$.

Given the four-dimensional model and its matter content thus fixed, we can in what follows concentrate on the calculation of the spectrum of primordial perturbations it generates (using notations and conventions that can be found elsewhere).

3 Perturbations

During the slowly contracting phase, labeled in what follows by a “<” superscript as it refers to negative times, it can be shown that the Bardeen gravitational potential acquires a scale invariant piece in the growing mode. In the long wavelength limit, one gets explicitly

$$\Phi = B_1(k) \frac{\mathcal{H}}{a^2} + B_2(k) \frac{\mathcal{H}}{a^2} \int_0^\eta d\tau \frac{\vartheta}{a^2}, \quad B_1^\leq \propto k^{-3/2}, \quad B_2^\leq \propto k^{-1/2},$$

with $\mathcal{H}$ the Hubble parameter.
with $H \equiv a'/a$, $\theta^{-2} \equiv 2\gamma a^2/3$ and $\gamma = 1 - H'/H^2$. In the specific case of a free scalar field dominated collapse, $a = \ell_0 \sqrt{-\eta}/(2\sqrt{2})$ and $\gamma = 3$, this gives a growing mode $\Phi^<_G \propto B^<_G/\eta^2$ and a decaying mode $\Phi^<_D \propto B^<_D$.

At the bounce time $\eta = 0$, i.e. the singularity, the scale factor vanishes and the Bardeen potential, in other words the perturbation, diverges. Even though after that time the linear theory ceases to make sense, it was nevertheless proposed to use, instead, the comoving density contrast $\epsilon_m = \delta \rho/\rho \propto \Phi/(\rho a^2)$ which is regular at the bounce. This variable is subject to the differential equation

$$\epsilon_m'' + H (1 + 3\epsilon^2 - 6w) \epsilon'_m + \left[9H^2 \left(\epsilon^2_1 + \frac{w^2}{2} - \frac{4w}{3} - \frac{1}{6}\right) + k^2 \epsilon^2_2\right] \epsilon_m = -k^2 w \delta S,$$

where the sound velocity is $c^2_s \equiv dp/d\rho$, the equation of state parameter $\omega = p/\rho$, and $\delta S$, the entropy perturbation, is set to zero for the adiabatic perturbations here considered. These functions are different on both sides of the singularity since entropy is produced there. This second order linear differential equation admits two independent solutions, and its solution may, in full generality, be written as $\epsilon_m = \epsilon_0 D(\eta) + \epsilon_2 E(\eta)$, with $D(\eta) \sim 1 + 3\omega_1 \eta/4 + O(k^2 \eta^2 \ln \eta)$ and $E(\eta) \sim \eta^2 + O(\eta^3)$, where the expansion $w = 1 + \omega_1 \eta + \cdots$ is assumed. Given the relation between $\Phi$ and $\epsilon_m$, and the initial value of the Bardeen potential $\Phi^<$, it is a simple matter to derive the values of $\epsilon^<_1$ and $\epsilon^<_2$ before the singularity as functions of $B^<_1$ and $B^<_2$.

The quantity that can be compared with observational data is, however, the Bardeen potential, written $B^<_G$ (the growing mode of the collapsing phase then becoming an uninteresting decaying mode in the expanding phase). This one is given in terms of $\epsilon^>_0$ and $\epsilon^>_2$, again, after the singularity, and the question arises then automatically as to what are the matching conditions thanks to which one may compute these quantities as functions of the pre-bounce ones? It has been suggested in that regard that the quantities $\epsilon_0$ and $\epsilon_2$ should keep their numerical values on both sides of the singularity: $\epsilon^>_0 = \epsilon^<_0$ and $\epsilon^>_2 = \epsilon^<_2$. With such a choice, the quantity $B^<_G$ ends up being a linear combination of $B^<_1$ and $B^<_2$, thereby acquiring a piece proportional to $B^<_1 \propto k^{-3/2}$, i.e. a scale invariant spectrum as announced.

Now it can be argued, as it was indeed pointed out, that the normalization of the function $E(\eta)$ must be physically irrelevant. This amounts to defining a new function $\tilde{E} = f E$, and renormalize $\tilde{\epsilon}_2 = \tilde{\epsilon}_2/f$, with $f$ an arbitrary function of the background and of the perturbation scale $k$. Using then the matching condition $\tilde{\epsilon}_2 = \tilde{\epsilon}_2$ then gives the final spectrum in the form

$$B^<_G(k) = -\frac{9}{8\ell_0^2} B^<_1(k) \left(\frac{f^>}{f^<} \omega^<_1 - \omega^<_2\right) + B^<_2(k) \left[\frac{f^>}{f^<} + 2 \left(\frac{f^>}{f^<} - \frac{\omega^>_2}{\omega^<_1}\right) \ln 2\right],$$

i.e. a spectrum explicitly depending on the arbitrary function $f$ on both sides of the singularity. It should be mentioned at this point that setting $\tilde{\omega}^>_2 = 0$ in this relation does not change the conclusions since in this case Eq. (3) does not hold and the calculation must be done differently to yield

$$B^<_2 = B^<_2 \frac{f^>}{f^<} + \frac{9\omega^>_2}{8\ell_0^2} B^<_1,$$

and the function $f$ has not cancelled out, contrary to what was claimed in the literature.

4 Conclusions

The inflationary paradigm appears at the present time to be the only way to explain the shape of the spectrum of primordial perturbations. The ekpyrotic proposal was suggested as an alternative that should be confronted with future observations, a task which is currently impossible.
since the model predictions depend on an arbitrary function that can always be made to match the data. As a result, we conclude that, for the time being and unless some new physics is specified that explains the matching conditions as well as sets the arbitrary function, the ekpyrotic scenario is not yet endowed with any predictive power.

Acknowledgments

We would like to acknowledge many illuminating discussions with Robert Brandenberger, Ruth Durrer, David Lyth, Raymond Schutz, and Gabriele Veneziano.

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