EMC effect and nuclear structure functions

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We analyze experimental data of nuclear structure function ratios \( F_A^2/F_D^2 \) for obtaining optimum parton distribution functions (PDFs) in nuclei. Then, uncertainties of the nuclear PDFs are estimated by the Hessian method. Parametrization of nuclear parton distribution is investigated in the leading order of \( \alpha_s \). The parton distribution are provided at \( Q^2 = 1 \text{GeV}^2 \) with a number of parameters, which are determined by a \( \chi^2 \) analysis of the data on nuclear structure function. From the analysis, we propose parton distributions at \( Q^2 = 1 \text{GeV}^2 \) for nuclei from deuteron to heavy ones with a mass number \( A \sim 208 \).

1. Introduction

The structure function \( F_2 \) for a bound nucleon, measured in deep inelastic scattering (DIS) of leptons, differs from that for a free nucleon. Unpolarized parton distribution in the nucleon are now well determined in the region from very small \( x \) to large \( x \) by using various experimental data. There are abundant data on electron and muon deep inelastic scattering. In addition, there are available data from neutrino reactions, Drell-Yan processes, \( W \) production, direct photon production, and others.

In this paper we determine the unbounded parton distributions and structure function for free nucleon. For this purpose, we use the Bernstein average of moments, as we used in \footnote{MINUTE [8].} \( [1,2,3,4] \). The polynomials are chosen so that the range of \( x \) for which the experimental values of \( F_2^D \) are not determined, make only a small contribution to the averages. These experimental Bernstein averages are then fitted in sec. 2, using CERN subroutine

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\[ + (\alpha_n g(n, Q_0^2) - \frac{\alpha_n(1 - \alpha_n)}{\beta_n}) \sum (n, Q_0^2) e^{-d_x s} \]

(1)

Here \( \alpha_n \) and \( \beta_n \) and associate anomalous dimension are as following

\[
\alpha_n = \frac{d_n - d_x}{d_x - d_x}, \\
\beta_n = \frac{d_{gg} - d_x}{d_x - d_x}, \\
d_+ = \frac{1}{2}(d_{NS} + d_{gg} + \Delta), \\
d_- = \frac{1}{2}(d_{NS} - d_{gg} - \Delta), \\
\Delta = \sqrt{(d_{NS} - d_{gg})^2 + 4d_{gg}d_{gg}}, \\
d_{NS} = \frac{1}{3\pi b} \left( 1 - \frac{2}{n(n+1)} + \frac{1}{\sum_{j=2}^{n} 1} \right), \\
d_{gg} = -2 \left( \frac{2 + n + n^2}{3\pi b n(n+1)(n+2)} \right), \\
d_{gg} = \frac{-f}{2\pi b} \left( \frac{1}{12} + \frac{1}{n(n-1)} + \frac{1}{(n+1)(n+2)} \right) - \frac{f}{18} \sum_{j=2}^{n} \frac{1}{j},
\] (2)

the parameter \( s \) and \( b \) in above equations are defined as

\[
s = \ln \frac{Q^2}{Q_0^2}, \\
b = \frac{33 - 2f}{12\pi}.
\] (3)

If we back to Eq. (1), we need to know all of the moments of parton at \( Q^2 = Q_0^2 \).

In phenomenological investigations of structure functions, for a given value of \( Q^2 \), only a limited number of experimental points, covering a partial range of values of \( x \), are available. Therefore, one cannot directly determine the moments. A method devised to deal with this situation is to take averages of the structure function weighted by suitable polynomials. We can compare theoretical predictions with experimental results for the Bernstein averages. After extracting some unknown parameters which exist in the functional form of input parton distribution in moment space, we are able to determine parton distributions in \( x \)-space. The parameterizations of the parton densities at input scale of \( Q_0^2 = 1 \text{GeV}^2 \) are determined as

\[
u u_v(x, Q_0^2) = 0.462x^{0.291}(1-x)^{3.219} \\
(1 - 0.959\sqrt{x} + 19.423x), \\
x d_v(x, Q_0^2) = 0.655x^{0.424}(1-x)^{4.093} \\
(1 - 0.634\sqrt{x} + 7.315x), \\
x \Sigma(x, Q_0^2) = 0.534x^{0.176}(1-x)^{3.286} \\
(1 + 0.790\sqrt{x} + 8.698x + 11.530x^2), \\
x g(x, Q_0^2) = 2.113x^{0.039}(1-x)^{6.403} \\
(1 - 2.639\sqrt{x} + 9.358x). \] (4)

We can control some parameter at input scale in above equations by these constrains

\[
\int_0^1 u_v(x, Q_0^2) dx = 2, \\
\int_0^1 d_v(x, Q_0^2) dx = 1, \\
\int_0^1 (x \Sigma(x, Q_0^2) + x g(x, Q_0^2)) dx = 1. \] (5)

By taking the moments of the above equation and inserting them in the Eq. (1), we access to all of the parton distributions in moment space. Then, by using the inverse Mellin technique as

\[
f(x, Q^2) = \frac{1}{\pi} \int_0^{5+10/ln\frac{Q^2}{Q_0^2}} dz \text{Im}[e^{i\varphi}x^{-c-ze^{i\varphi}}] M(n = c + ze^{i\varphi}, Q^2) \] (6)

we can obtain the parton distribution in \( x \)-space for free proton. In above analysis, we chose the \( \Lambda = 0.22 \text{GeV} \) and \( c=1.1 \) for non singlet part and \( c=2.1 \) for singlet part.
available nuclear data on the structure function \( F_2^A \) are taken in fixed-target experiments at this stage, and they are shown in Fig. 1 as a function of \( Q^2 \) and \( x = Q^2/(2 m \nu) \), where \( \nu \) is the transferred energy to the target, \( m \) is the nucleon mass, and \( Q^2 \) is given by \( Q^2 = -q^2 \) with the virtual photon momentum \( q \). The initial nuclear parton distributions are provided at a fixed \( Q^2(\equiv Q_0^2) \), and they are taken as 

\[
f_i^A(x, Q_0^2) = w_i(x, A, z)f_i(x, Q_0^2),
\]

where \( f_i^A(x, Q_0^2) \) is the parton distribution with type \( i \) in the nucleus \( A \) and \( f_i(x, Q_0^2) \) is the corresponding parton distribution in the nucleon. We call \( w_i(x, A, z) \) a weight function, which takes into account the nuclear modification. This functional form should be considered by:

\[
w_i(x, A, Z) = 1 + \frac{(1 - \lambda x^2)}{(1 - x)^{\lambda^2}}(a_i + a_i \sqrt{x} + b_i x + c_i x^2 + d_i x^3 + e_i x^4).
\]

Because the valence-quark distributions in a nucleus are much different from the ones in the proton, we should be careful in defining the weight function \( w_i(x, A, Z) \).

For nuclear \( A \) at \( Q_0^2 \) we have

\[
u^A(x, Q_0^2) = \frac{Z u_v(x, Q_0^2) + N d_v(x, Q_0^2)}{A}, \quad d_v^A(x, Q_0^2) = \frac{Z d_v(x, Q_0^2) + N u_v(x, Q_0^2)}{A},
\]

\[
\Sigma^A(x, Q_0^2) = w_\Sigma(x, A, Z)\Sigma(x, Q_0^2), \quad g^A(x, Q_0^2) = w_g(x, A, Z)g(x, Q_0^2),
\]

(9)

4. Summery

In the theoretical calculations, the nuclei are assumed as: 

\( ^4\text{He}, ^7\text{Li}, ^9\text{Be}, ^{12}\text{C}, ^{14}\text{N}, ^{27}\text{Al}, ^{40}\text{Ca}, ^{56}\text{Fe}, ^{63}\text{Cu}, ^{107}\text{Ag}, ^{118}\text{Sn}, ^{131}\text{Xe}, ^{197}\text{Au}, \) and \( ^{208}\text{Pb} \)

The initial nuclear distributions are provided at \( Q_0^2 = 1 \text{ GeV}^2 \) with the parameters in Eq. 8. To obtain some unknown parameters which appeared in Eq. 8, we can use the ratios \( R_{F_2}^A = F_2^A(x, Q^2)/F_2^P(x, Q^2) \) to calculate

\[
\chi^2 = \sum_j \frac{(R_{F_2,j}^A - R_{F_2,j}^{\text{theory}})^2}{(\sigma_{j}^{\text{data}})^2},
\]

(10)

where the experimental error is given by considering the systematic and statistical errors. Information about the used experimental data is given in Table I, where nuclear species, references, and data numbers are listed.

| Nucleus | Experiment | Reference | No. of data |
|---------|------------|-----------|-------------|
| He      | SLAC-E139  | [11]      | 21          |
|         | NMC-95     | [12]      | 18          |
| C       | EMC-88     | [9]       | 9           |
|         | EMC-90     | [10]      | 13          |
|         | SLAC-E139  | [11]      | 17          |
|         | NMC-95     | [12]      | 18          |
| Ca      | EMC-90     | [10]      | 13          |
|         | NMC-95     | [12]      | 18          |
|         | SLAC-E139  | [11]      | 17          |
|         | FNAL-E665-95 | [13]   | 11          |
|         | FNAL-E665-95 | [13]   | 11          |

Table. I Nuclear species-references, and data numbers are listed for the used experimental
data with $Q^2 \gg 1 \text{ GeV}^2$.

With these preparations together with the CERN subroutine MINUIT \[6\], the optimum parameter set is obtained by minimizing $\chi^2$. The experimental data are taken from the publications by the European Muon Collaboration (EMC)\[9\][10] at the European Organization for Nuclear Research (CERN), E139 Collaborations \[11\] at the Stanford Linear Accelerator Center (SLAC), the New Muon Collaboration (NMC) \[12\] at CERN, and the E665 Collaboration \[13\] at the Fermi National Accelerator Laboratory (FNAL). The used data are for the following nuclei: helium (He), carbon (C) and calcium (Ca). In Table II, we present the $\chi^2$/d.o.f for helium, carbon and calcium as an example.

| Nucleus | No.of data | $\chi^2$/d.o.f |
|---------|------------|----------------|
| He      | 39         | 1.12           |
| C       | 68         | 1.49           |
| Ca      | 59         | 3.86           |

Table. II The $\chi^2$ values, resulted from fitting, for different types of nucleus.

In Fig. (2) we present $F_{2e}(x, Q^2)/F_{2D}(x, Q^2)$, $F_{S2}(x, Q^2)/F_{D2}(x, Q^2)$ and $F_{T2}(x, Q^2)/F_{D2}(x, Q^2)$ as a function of $x$ and for $Q^2 = 5 \text{ GeV}^2$. As it can be seen, these ratios are in good agreement to the available experimental data.

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REFERENCES

1. Ali N. Khorramian, A. Mirjalili, S. Atashbar Tehrani, JHEP 0410 (2004)062, hep-ph/0411390
2. Ali N. Khorramian, S. Atashbar Tehrani, A. Mirjalili, AIP Conf. Proc. 717(2004)897.

3. S. Atashbar Tehrani, Ali N. Khorramian and A. Mirjalili, Commun. Theor. Phys. 43(2005)1087.
4. Ali N. Khorramian, A. Mirjalili, S. Atashbar Tehrani, Int. J. Mod. Phys.A 20(2005)1923.
5. M. Gluck, R. M. Godbole, E. Reya, Z. Phys. C41(1989)667.
6. F. James, CERN Program Library Long Writeup D506.
7. M. Glück, E. Reya, and A. Vogt, Z. Phys. C 48 471 (1990)
8. M. Hirai et al., Phys. Rev. D64,(2001)034003; M. Hirai et al., Phys. Rev. C70,(2004)044905
9. EMC, J. Ashman et al., Phys. Lett.B 202, 603 (1988)
10. EMC, M. Arneodo et al., Nucl. Phys. B 333, 1 (1990)
11. SLAC-E139, J. Gomez et al., Phys. Rev. D 49, 4348 (1994)
12. NMC, P. Amaudruz et al., Nucl.Phys.B 441,3 (1995); NMC, M.Arneodo et al.,ibid. B 441, 12 (1995)
13. E665, M. R. Adams et al., Z. Phys. C 67, 403 (1995)