Non-monotonic nucleus-nucleus potential and incompressibility of infinite cold nuclear matter

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Abstract. A novel method for the determination of the yet not well-known quantity of nuclear incompressibility, K is presented. Non-monotonic (NM) nucleus-nucleus potentials from the energy-density functional (EDF) theory including the Pauli principle have been considered for K in the range 188-266 MeV. The experimental cross sections of \(^{16}\text{O} + ^{16}\text{O}\) elastic scattering over the 31-350 MeV incident energies have been analyzed in the optical model using the NM potentials. Sensitivity of K on the elastic scattering data is studied and its value for infinite cold nuclear matter deduced to be 222 ± 5 MeV

1. Introduction
Nucleus-nucleus potential plays a pivotal role in a nucleus-nucleus interaction. Knowledge of the ion-ion potential is a key ingredient to the estimation of cross sections of elastic scattering and non-elastic processes including the fusion reactions and fragmentation needed for important applications [1], for example, astrophysical applications [2], space radiation protection [3], heavy-ion radio surgery [4], heavy-ion radiotherapy [5] and so on, also to get an answer to a fundamental question of how heavy elements are formed in the universe. In particular, nuclear incompressibility \(K\) [6] of infinite cold nuclear matter, a fundamental property of equation of state (EOS), is of considerable importance for a complete understanding of the neutron stars.

2. Types of potentials
The widely used phenomenological potentials of Woods-Saxon (WS) [7] and squared WS (SWS) [8, 9] types suffer from discrete ambiguities, as observed by Mohr et al. [10] in addition to the continuous ambiguities [11]. Recently, the WS and SWS potentials are found inconsistent in terms of the volume integral [12] of their real part, which can describe satisfactorily the angular distribution of cross-sections (CS).

3. Microscopic and semi-microscopic
Microscopic and semi-microscopic nucleus-nucleus potentials are derived from the realistic nucleon-nucleon (NN) potential.

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3.1. Potential from double-folding (DF) model:
Most familiar and widely used one is double-folded potential [6, 13] using the either M3Y-Reid or M3Y-
Paris type of effective NN potential. DF potentials are directly obtained by folding the NN potential over
the density distributions of the target and projectile. The present position of the DF potential is reviewed
elegantly by Khoa et al. [6]. We shall not go in detail except mentioning its major drawbacks in spite of
its tremendous successes enjoyed so far.

The DF potentials of $^6$Li and $^7$Li in the framework of the optical model (OM) model yield the elastic
scattering CSs which are typically a factor 1.5-2.0 higher than the experimental values and need an
arbitrary normalization factor of $N_R=0.5-0.7$ [14, 15] for the description of the data. Moreover, OM
calculations using the folding potentials are unable to account for the opposite signs of vector analyzing
powers (VAPs) $iT_{11}$ of $^6$Li and $^7$Li elastic scattering at the same incident energy by $^{58}$Ni at 20 MeV and
$^{120}$Sn at 44 MeV.

Sakuragi and his group [16] demonstrated elegantly that the renormalization may be done away with
through the generation of a repulsive dynamic polarization potential (DPP) using a coupled-discretized
continuum channels (CDCC) method [17] in conjunction with the DF potential. On the other hand,
coupled-channels calculations of Nishioka et al. [18] have shown to generate opposite signs of $iT_{11}$ for
$^6$Li and $^7$Li elastic scattering through generation of their different dynamic spin-orbit potentials.

3.2. Non-monotonic (NM) potential from energy-density functional (EDF) formalism:
Microscopic potential, derived from the energy density functional (EDF) theory [19] which incorporates
the Pauli effect of anti-symmetrization microscopically, has been found to be non-monotonic (NM) and
shallow [20, 21, 22, 23, 24].

Apart from accounting well for the detailed feature of the $\alpha$ elastic scattering [20, 21, 22, 23, 24],
NM potentials have also been found successful in reproducing the correct order of CSs for $\alpha$ inelastic
scattering [25], one- [26], two- [27, 28] and three-nucleon [29] stripping reaction on Si-isotopes.

Simple OM calculations using the EDF-generated potentials can describe the correct CSs without the
need of renormalization and can reproduce the correct signs of VAPs for the $^6$Li and $^7$Li elastic scattering
[30].

4. Outline of EDF potential:
Unlike the DF potentials, EDF-potentials are generated from the total energies of the projectile, target
and the composite nucleus, formed during the collision. The total energy of each of the three nuclei is
computed by folding a realistic NN potential over the respective density distribution function (DDF).
The EDF potential $V(R)$ between the projectile and the target at a separation distance of $R$ is given [31]
by
$$V(R) = E[\rho(\vec{r}, R)] - E_P[\rho_P(\vec{r}, R=\infty)] - E_T[\rho_T(\vec{r}, R=\infty)],$$
where $\rho$ is the density distribution function (DDF) of the composite system. Here, $\rho_P$ and $\rho_T$ are,
respectively, the DDFs for the projectile and the target at $R=\infty$. In the sudden or frozen approximation
[19] the DDF of the composite system is given by
$$\rho(\vec{r}) = \rho_P(\vec{r}) + \rho_T(\vec{r}).$$

Sudden approximation is an ansatz whereby one can reach a region of EOS beyond the saturation
density i.e. one can simulate the repulsive core through the Pauli effect. At higher incident energies,
the projectile has enough energy to penetrate into the nuclear interior where the approximation seems
to violate the Pauli principle due to large overlap between the projectile and target. However, at higher
energies the time of interaction is too small for the composite system to settle to its saturation density.
At lower projectile energies the time of interaction although becomes large, the collision is dominated
at the surface region where total density of the composite system in the sudden approximation does
not exceed its saturation value due to small overlap. Hence the a priori application of the sudden
approximation seems to be justifiable. However, the true justification is found a posteriori, in the results we obtained employing the ansatz while studying a diverse range of nuclear interactions, published in Refs [20, 23, 24, 25, 26, 27, 28, 29, 30].

In our studies, the EDF theory of Brueckner et al. [19] for computing the total energy of a nucleus in its ground state employs a realistic nucleon-nucleon (NN) potential of Brueckner, Gammel and Thaler (BGT) [32]. The BGT potential describes all the observed deuteron properties and two-nucleon scattering data up to the pion-production energy. Moreover, in this EDF theory, the Pauli principle is considered in determining the mean-field in nuclear and nucleonic matter approximation. The starting point of the EDF theory is the theorem that the energy of a system of fermions for a given density distribution \( \rho(r) \) can be expressed [19, 25, 33, 34] as

\[
E = \int e[\rho(\vec{r})] \, d^3\vec{r},
\]

where \( e[\rho(\vec{r})] \), the energy density, is given by

\[
e[\rho(\vec{r}),\xi] = B(\rho, \xi) \rho(\vec{r}) + \left( \frac{\epsilon}{2} \right) \Phi_c(\vec{r}) \rho_p - 0.739e^2 \rho_p^{4/3} + \left( \frac{\hbar^2}{8M} \right) \eta(\nabla \rho)^2
\]

where

\[
B(\rho, \xi) = 0.3 \left( \frac{\hbar^2}{2M} \right) \left( \frac{3\pi^2}{2} \right)^{2/3} \left[ (1 - \xi)^{5/3} + (1 + \xi)^{5/3} \right] \rho^{2/3} + \nu(\rho, \xi)
\]

Here \( B(\rho, \xi) \) denotes the binding energy per nucleon in the nuclear matter; \( \xi = (N-Z)/A \), the neutron excess in the target nucleus; \( M \), the nucleon mass and \( \eta \), the free inhomogeneity-parameter adjusted to reproduce the nuclear mass. The first term in Eq.(5) representing the nucleon kinetic energy \( T(\rho, \xi) \) in nuclear matter is given by

\[
T(\rho, \xi) = 0.3 \left( \frac{\hbar^2}{2M} \right) \left( \frac{3\pi^2}{2} \right)^{2/3} \left[ (1 - \xi)^{5/3} + (1 + \xi)^{5/3} \right] \rho^{2/3}.
\]

\( T(\rho, \xi) \) is proportional to \( \rho^{2/3} \).

The nucleonic mean field \( \nu(\rho, x) \) in the second term is determined from the BGT NN potential in the Brueckner-Hartree-Fock theory used in [19]. This theory relates the matrix elements of the potential to those of scattering operator with full consideration of the Pauli principle among the nucleons of the same type in the nuclear and nucleonic matter approximation, i.e., using plane wave for nucleonic wave functions. Hence the nucleon potential in the mean field incorporates the exchange effects. The computed curves of \( B(\rho, \xi) \) using the mean-field with the exchange effects are given in [19] for \( \xi = 0, 0.2, 0.4 \) and 0.6 for the homogeneous nuclear matter. These curves of BCD [19] yield values of the parameters in the mean-field, expressed analytically as

\[
\nu(\rho, \xi) = \lambda_1 \left( 1 + a_1 \xi^2 \right) \rho + \lambda_2 \left( 1 + a_2 \xi^2 \right) \rho^{4/3} + \lambda_3 \left( 1 + a_3 \xi^2 \right) \rho^{5/3}
\]

with \( \lambda_1 = -741.28, \lambda_2 = +1179.89, \lambda_3 = -467.54, a_1 = 0.2, a_2 = +0.316 \) and \( a_3 = +1.646 [35] \) for the nuclear matter incompressibility \( K = 188 \text{ MeV} [36] \). The nuclear matter incompressibility or incompressibility modulus \( K \) at the saturation density [6, 13], defined as

\[
K = 9\rho_0^2 \left( \frac{d^2B}{d\rho^2} \right)_{\rho=\rho_0},
\]

provides a measure of the curvature of \( B(\rho, \xi) \) at the saturation density \( \rho_0 \). The mean-field parameters of BCD in Eq.(7) along with \( \eta = 8.0 \) in the inhomogeneity correction term [25] of Eq.(4) reproduce the binding energies (BEs) of nuclei from \( ^{12}\text{C} \) to \( ^{238}\text{U} [35] \) within 1.5%.
The mean-field for an infinite symmetric ($\xi=0$) nuclear matter (ISNM) with redundancy of $a_1$, $a_2$ and $a_3$ following Eq.(7) is given by

$$v_S(\rho) = \lambda_1 \rho + \lambda_2 \rho^{4/3} + \lambda_3 \rho^{5/3}.$$  

(9)

In our investigation of elastic scattering of symmetric nuclei, we use this simple expression with three parameters $\lambda_1, \lambda_2$ and $\lambda_3$ only.

5. Motivation

Recently, Hossain et al. [24] are profoundly successful in accounting for the Airy structure including ‘exponential-type falloff’ in the nuclear rainbow from the refractive scattering [6, 13, 37] in addition to the usual Fraunhofer diffractive oscillations in the elastic scattering of $\alpha+^{90}\text{Zr}$ in the energy range of 15.0-141.7 MeV using the NM potential in the OM analysis.

The refractive scattering, contributed by the lower partial waves, can serve as a good probe the potential in the nuclear interior [6, 13, 38]. The success of the NM potential in describing the nuclear rainbow structure motivated us to study the sensitivity of $K$ on the elastic scattering data. This is because interaction potential is $K$-dependent.

The $^{16}\text{O}$ nucleus is fairly compact and being spin-0 does not involve the spin-orbit and tensor potentials. Its binding energy can be reproduced within $\approx 1.6\%$ with a simple two-point Fermi (2pF) density-distribution function (DDF) of [39]. The $^{16}\text{O}+^{16}\text{O}$ system bears the Boson-symmetry. The symmetry effect is modulated by the coherent mixture of diffractive and refractive structures at the higher incident energies with the refractive pattern competing better for the heavy $^{16}\text{O}$-ion. Good quality data are available with Airy structures in the angular distributions of the $^{16}\text{O}+^{16}\text{O}$ elastic scattering at energies beyond 60 MeV [40], making the system attractive to probe the interior potential for studying its $K$-dependence.

This presentation aims to report how well the Pauli-distorted EDF theory of BCD can examine the sensitivity of $K$ on the $^{16}\text{O}+^{16}\text{O}$ elastic scattering data and deduce its value for infinite cold nuclear matter. In deriving the NM potentials, the Pauli-distorted EOS for the infinite symmetric nuclear matter (ISNM) with non-Coulombic field is taken from [19].

Figure 1. Equations of state for six $K$-values at the saturation point for symmetric infinite nuclear matter.
Table 1. Mean-field parameters $\lambda$'s for different $K$ values in MeV, and corresponding values of $J_R/256$ in MeV.fm$^3$ and $\eta$.

| $K$  | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\eta$ | $J_R/256$ |
|-----|-------------|-------------|-------------|------|----------|
| 188 | −741.28     | +1179.55    | −467.54     | 8.0  | −88.00   |
| 211 | −709.55     | +1040.18    | −451.07     | 8.38 | −83.32   |
| 230 | −676.40     | +895.80     | −173.80     | 8.74 | −79.20   |
| 240 | −696.21     | +953.98     | −214.36     | 9.05 | −76.12   |
| 252 | −675.34     | +880.19     | −146.81     | 8.70 | −73.81   |

6. Procedure

6.1. Step 1
To determine the NM potentials for different values of $K$, we start with the BCD’s equation of state (EOS) i.e. $B(\rho)$ curve for ISNM and five other simulated harder EOSs with higher values of $K$ with identical features at lower densities up to the saturation point. For estimating $K$ for a given EOS, $B(\rho)$ is expressed as determined using Eq. [8] coupled with

$$B(\rho) = \sum_{i=1}^{4} C_i \rho^i$$

(10)

The coefficients $C_i$ (i=1-4) determined by fitting each of the EOSs. The curvature at the saturation density $\rho_0$ is determined using Eq.(10). Figure 1 shows the EOS of BCD in solid circles and five simulated harder EOSs with the corresponding $K$-values, computed by feeding the estimated curvature into Eq.(8).

6.2. Step 2
For each EOS, the mean-field corresponding to a specific $K$ can be obtained following Eqs.(5) and (6) from

$$v_S(\rho) = B_S(\rho) - T_S(\rho)$$

(11)

using the $B_S(\rho)$-value picked from the BCD’s EOS for $\xi=0$ and the computed nucleonic kinetic energy $T_S(\rho)$ from Eq.(6). The calculated mean-field is then parametrized for the mean-field parameters $\lambda$'s in Eq.(9). The mean-field parameters for different $K$s are listed in table 1.

6.3. Step 3
The deduced mean-field for a particular $K$, is then used in the EDF calculation to generate the interaction potential, specific for the $K$-value for application to the OM analysis. Figure 2 displays the EDF-generated $^{16}$O-$^{16}$O potentials for various $K$-values.

7. Analysis
The OM potential between projectile and target nuclei separated by a distance of $R$ is given by

$$U(R) = V_C(R) + V_N(R) + iW(R).$$

(12)

Here $V_C(R)$, $V_N(R)$ and $W(R)$ denote, respectively, the Coulomb, nuclear real and nuclear imaginary potentials. $V_C(R)$ is assumed to be due to a uniformly charged sphere of radius $R_C$ and is given by

$$V_C(R) = \begin{cases} \frac{Z_1 Z_2 \epsilon^2}{2 R_C} \left[ 3 - \frac{R^2}{R_C^2} \right] & \text{for } R \leq R_C, \\ \frac{Z_1 Z_2 \epsilon^2}{R} & \text{for } R > R_C. \end{cases}$$

(13)
The EDF-derived $V_N(R)$ can be parametrized to include the repulsive core as

$$V_N(R) = -V_0 \left[ 1 + \exp \left( \frac{R - R_0}{a_0} \right) \right]^{-1} + V_1 \exp \left[ -\left( \frac{R - D_1}{R_1} \right)^2 \right].$$  \hspace{1cm} (15)$$

In Eq.(14), $V_1$ and $R_1$ are, respectively, the depth and range of repulsive core which stems from the Pauli Principle included in the mean field. $D_1$ denotes the position of the Gaussian peak for the repulsive core and is usually termed as the shifting parameter for the repulsive core.

The imaginary part is assumed as

$$W_m(R) = -W_0 \exp \left[ -\left( \frac{R}{R_W} \right)^2 \right] - W_S \exp \left[ -\left( \frac{R - D_S}{R_S} \right)^2 \right].$$  \hspace{1cm} (16)$$

This comprises a volume term with the Gaussian shape and a surface term with the shifted Gaussian one. The latter is found to supplement the volume term at all incident energies.

The volume integrals per nucleon pair $J_R/(A_PA_T)$ and $J_I/(A_PA_T)$ of the real and imaginary potentials, respectively, are defined by the integrals,

$$J_R = \frac{1}{A_PA_T} \int_0^\infty 4\pi r^2 V_N(r) dr$$  \hspace{1cm} (17)$$

and

$$J_I = \frac{1}{A_PA_T} \int_0^\infty 4\pi r^2 W(r) dr.$$  \hspace{1cm} (18)$$

Here $A_p$ and $A_T$ are, respectively, the mass number of the projectile and target nucleus.

Experimental cross section data of $^{16}$O+$^{16}$O elastic scattering at 16 energy points in the energy range of 31.0-350.0 MeV from sources [6, 41, 42, 43, 44] have been analyzed in the framework of the simple OM model. The analyses have been carried out using the code SFRESCO which incorporates the coupled-channels code FRESCO2.5 [45] and the minimization code MINUIT [46].

![Figure 2. K-dependence of the EDF-generated $^{16}$O-$^{16}$O nuclear potential.](image-url)
Figure 3. OM predictions are compared with the $^{16}\text{O}+^{16}\text{O}$ elastic scattering data at $E_{\text{lab}} = 49.0, 75.0$ and 103.1 MeV. The relative performances of the EDF-derived potentials for $K = 188, 230$ and 252 MeV are shown to indicate the $K$ dependence on $J_{R}/256$.

8. Results

8.1. Test of $K$-sensitivity

To examine the $K$-sensitivity, the data for 49.0, 75.0 and 103.1 MeV have been fitted with (i) the EDF-derived real-potentials left unchanged for individual $K$-values and (ii) empirically adjusted imaginary potential parameters.

Considering the relative $\chi^2$ of the fits using the EDF-generated NM potentials for $K = 188, 230$ and 252 MeV at each of the energies and visual inspection on the fits in Fig. 3 suggest that the fits to the data at the three energies are best described by the potential for $K = 230$ MeV.

Thus, $K$-dependence of the elastic scattering data is indicated, although the fits to data are not impressive. The reason is obvious, the EDF-generated potentials being valid for the ‘zero’ excitation i.e. $E_x = 0.0 \left[ E_x = E_{\text{lab}}/2 + 16.5 \right]$ MeV for the $^{16}\text{O}+^{16}\text{O}$ system] configuration of EDF-potential to the fitting the experimental data, we are not benefited much. We have opted for a reverse path from fitting the data to the EDF-calculations i.e. comparison of empirical $J_{R}/256$ of the best potentials parameters with the corresponding EDF-derived values.

8.2. Volume integral versus excitation of the composite nucleus

In our method, we have analyzed in the OM model the experimental data at 16 points in the energy interval 31-350 MeV to obtain remarkable fits with $\chi^2$ of single digits excepting the cases of 124 and
Figure 4. Same as Fig. 3 for the best fits at 16 energies obtained by empirical adjustment of the OM parameters including those for the real part of the potentials.
Table 2. Potential parameters for the $^{16}\text{O}+^{16}\text{O}$ elastic scattering in the energy range 31-350 MeV for $R_C = 5$ fm with $J_{R}/256$-values and $\chi^2$ for the fits. The depths are in MeV; geometry parameters in fm; and $J_{R}/(256)$ in MeV.fm$^3$.

| $E_{lab}$ | $V_0$ | $R_0$ | $a_0$ | $V_1$ | $R_1$ | $D_1$ | $J_{R}/256$ | $\chi^2$ |
|-----------|-------|-------|-------|-------|-------|-------|-------------|--------|
| 31.0      | 29.9  | 5.735 | 0.653 | 50.0  | 2.84  | 0.0   | −79.2       | 0.29   |
| 41.0      | 31.2  | 5.652 | 0.742 | 103.0 | 2.34  | 0.0   | −79.1       | 1.80   |
| 49.0      | 76.0  | 5.418 | 0.864 | 158.0 | 3.662 | 0.0   | −78.7       | 1.60   |
| 59.0      | 95.0  | 5.216 | 0.785 | 228.0 | 2.30  | 1.360 | −77.8       | 7.90   |
| 75.0      | 79.0  | 5.800 | 0.575 | 107.0 | 2.11  | 2.827 | −76.3       | 3.60   |
| 80.6      | 100.0 | 6.170 | 0.544 | 107.5 | 3.586 | 2.325 | −75.6       | 5.10   |
| 87.2      | 108.0 | 5.695 | 0.526 | 146.0 | 1.633 | 3.480 | −75.0       | 6.20   |
| 92.4      | 37.4  | 6.526 | 0.385 | 104.0 | 0.715 | 3.936 | −74.0       | 7.70   |
| 94.8      | 45.6  | 6.150 | 0.505 | 102.0 | 0.835 | 3.83  | −73.7       | 6.00   |
| 98.6      | 87.0  | 5.820 | 0.550 | 162.0 | 1.372 | 3.326 | −73.2       | 4.70   |
| 103.1     | 49.0  | 6.050 | 0.520 | 141.0 | 0.595 | 4.00  | −72.4       | 6.50   |
| 115.9     | 105.0 | 6.090 | 0.480 | 123.0 | 1.924 | 3.838 | −70.5       | 8.20   |
| 124.0     | 83.0  | 5.292 | 0.660 | 130.0 | 0.800 | 4.208 | −69.0       | 18.8   |
| 145.0     | 79.0  | 4.964 | 0.790 | 353.0 | 0.290 | 3.480 | −75.0       | 8.20   |
| 250.0     | 180.0 | 5.040 | 0.673 | 170.0 | 1.700 | 3.860 | −32.5       | 18.4   |
| 350.0     | 151.0 | 5.040 | 0.670 | 162.0 | 1.783 | 3.742 | +20.3       | 4.20   |

Table 3. Same as table 2 for the imaginary part of the potential with associated volume integral per nucleon-pair $J_I/256$.

| $E_{lab}$ | $W_S$ | $D_S$ | $R_S$ | $W_0$ | $R_W$ | $J_I/256$ | $\chi^2$ |
|-----------|-------|-------|-------|-------|-------|-----------|--------|
| 31.0      | 2.00  | 8.10  | 0.13  | 20.0  | 3.2   | −15.8     | 0.29   |
| 41.0      | 3.00  | 7.48  | 0.11  | 29.0  | 3.2   | −22.3     | 1.80   |
| 49.0      | 2.30  | 7.20  | 0.50  | 32.7  | 3.2   | −28.5     | 1.60   |
| 59.0      | 2.60  | 7.96  | 0.21  | 46.0  | 3.4   | −42.3     | 7.90   |
| 75.0      | 1.80  | 7.11  | 0.15  | 86.0  | 3.40  | −70.5     | 3.60   |
| 80.6      | 1.25  | 8.00  | 0.82  | 90.0  | 3.40  | −82.7     | 5.10   |
| 87.2      | 0.44  | 8.37  | 0.40  | 137.0 | 3.40  | −118.2    | 6.20   |
| 92.4      | 0.40  | 8.95  | 0.30  | 187.0 | 3.40  | −160.7    | 7.70   |
| 94.8      | 0.33  | 8.44  | 0.77  | 200.0 | 3.35  | −165.2    | 6.00   |
| 98.6      | 1.20  | 6.61  | 0.28  | 217.0 | 3.30  | −170.9    | 4.70   |
| 103.1     | 0.34  | 8.94  | 0.15  | 255.0 | 3.35  | −208.9    | 6.50   |
| 115.9     | 1.30  | 6.40  | 0.10  | 260.0 | 3.35  | −213.1    | 8.20   |
| 124.0     | 0.30  | 7.89  | 0.10  | 360.0 | 3.28  | −276.5    | 18.8   |
| 145.0     | 0.50  | 7.14  | 0.11  | 413.0 | 3.14  | −278.4    | 5.50   |
| 250.0     | 0.25  | 6.73  | 0.13  | 1080.0| 3.00  | −634.5    | 18.4   |
| 350.0     | 0.35  | 6.56  | 0.32  | 1258.0| 2.98  | −724.6    | 3.20   |

250 MeV. This has been achieved by adjusting all the parameters of the nuclear potential in Eq.(14) and Eq.(15) including the EDF-generated real part (see table 2) to obtain $J_{R}/256$ at the lower projectile energies, namely in the energy range of 31-59 MeV, very near to those of the EDF-generated potentials and at the higher energies with the $J_{R}/(256)$ values in the same potential-family [47].

Although the OM potential (OMP) parameters are scattered as a result of global searches for the $\chi^2$ of single digits, they are consistent in volume integrals (see tables 2 and 3). The fits are displayed in Fig. 4. Figure 5 shows the plot of $J_{R}/256$ vs $E_x$ curve, where all the points end up on a smooth line to ensure that
Figure 5. Variation of $J_{R}/256$ with $E_{x}$ and the regression curves through the derived points in open circles.

they are in the same potential-family. The uncertainties of the estimated $J_{R}/256$-values are $\leq 1\%$ except for the two highest energies where these are around 10%.

8.3. Volume integral regressed to $E_{x}=0.0$

The energy-variation of $J_{R}/256$ is regressed by a power function $80.9 - 0.0003E_{x}^{2.42}$ (Fig. 5). A polynomial and an exponential function were also tried. The polynomial function fitted the data reasonably well, but when extrapolated to $E_{x} = 0$ dipped to a value of $J_{R}/256$ lower than that near the 31 MeV data point and hence was unacceptable. The exponential fit was unsatisfactory. The intercept term from the power function is very well-determined with the best fit to the $J_{R}/256$ values at all the energy-points, leading to the value $J_{R}/256 (E_{x} = 0) = 80.9 \pm 1.1$ MeV fm$^{3}$. The total error-bar includes the uncertainties from the individual $J_{R}/256$-values, curve-fitting and extrapolation. Considering linearity between the successive values of $K$ and $J_{R}/256$ in the calibrating table I from the EDF-generated potentials, this leads to $K = 222 \pm 5$ MeV.

9. Conclusions

The study concerns the first application of the NM potentials with microscopically built repulsive core, based on EOS of BCD for ISNM, for examining the $K$-sensitivity on the elastic scattering. BCD’s EOS involves a mean-field which is the non-Coulomb potential in nucleonic matter, evaluated in the Hartree-Fock theory. The field is calculated using the Pauli-exchanged K-matrices [48] of a realistic NN potential of [32]. The excellent description of the data with NM potentials calls for a re-analysis of the issue of deep vs shallow OM potentials concerning the S-matrix elements. A careful examination of NM potentials in the model-independent studies such as [38] may shed more light on the issue. Table II shows that the NM potential changes smoothly from overall attractive (negative) $J_{R}/256$-values to a repulsive (positive) one at high energies in line with observations on heavy- [38, 49] and light-ion [50, 51, 52, 53, 54] systems.

Our deduced result $K = 222 \pm 5$ MeV agrees closely with the empirical values obtained in [55]. The intriguing aspect of BCD’s EDF is that its underlying theory applies appropriately the Pauli principle
to EOS for infinite homogeneous nuclear matter. Hence its simple theory, without requiring the transformation from the nuclear incompressibility $K_A$[56, 57] for finite nuclei to $K$, which involves a large uncertainty [56, 60], yields directly the $K$-value of the infinite cold nuclear matter.

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