LATE-TIME NEAR-INFRARED OBSERVATIONS OF SN 2005df

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ABSTRACT

We present late-time near-infrared (NIR) spectral evolution, at 200–400 days, for the Type Ia supernova SN 2005df. The spectra show numerous strong emission features of [Co ii], [Co iii], and [Fe ii] throughout the 0.8–1.8 μm region. As the spectrum ages, the cobalt features fade as would be expected from the decay of 56Co to 56Fe. We show that the strong and isolated [Fe ii] emission line at 1.644 μm provides a unique tool to analyze NIR spectra of SNe Ia. Normalization of spectra to this line allows the separation of features produced by stable versus unstable isotopes of iron group elements. We develop a new method of determining the initial central density, ρc, and the magnetic field, B, of the white dwarf (WD) using the width of the 1.644 μm line. The line width (LW) is sensitive because of electron capture in the early stages of burning, which increases as a function of density. The sensitivity of the LW to B increases with time, and the effects of the magnetic field shift toward later times with decreasing ρc. Through comparison with spherical models, the initial central density for SN 2005df is measured as ρc = 0.9±0.2 × 10^9 g cm⁻³, which corresponds to a WD close to the Chandrasekhar mass, with MWD = 1.31±0.03 M⊙ and systematic error less than 0.04 M⊙. This error estimate is based on spherical models. We discuss the potential uncertainties due to multi-dimensional effects, mixing, and rotation. The latter two effects would increase the estimate of the WD mass. Within MCh explosions, however, the central density found for SN 2005df is very low for a H-accretor, possibly suggesting a helium star companion or a tidally disrupted WD companion. As an alternative, we suggest mixing of the central region. We find some support for high initial magnetic fields of strength 10^6 G for SN 2005df, however, 0 G cannot be ruled out because of noise in the spectra combined with low ρc. We discuss our findings in the context of mixing by Rayleigh–Taylor instabilities during deflagration burning and a wide variety of explosion scenarios. Observations strongly support a very limited amount of mixing during a deflagration phase and high central densities characteristic of a MCh WD.

Key words: line: identification – magnetic fields – supernovae: individual (SN 2005df)

1. INTRODUCTION

Type Ia supernovae (SNe Ia) are extremely important in astronomy because of their use in measuring cosmological distances and the opportunities they provide to study the physics of flames, instabilities, radiation transport, non-equilibrium systems, and nuclear and high energy physics. SNe Ia are bright time-dependent events which have been seen out to redshifts of z ≈ 2.5 (Graur et al. 2014; Rodney et al. 2014). Although there is some variety in the absolute luminosity of these events, they have been shown to be standardizable by using characteristics of their light curves (Phillips 1993). With increasing numbers of SNe Ia being discovered, differences in spectra can be examined in order to create additional corrections to the standardization of these objects and to further hone our understanding of the progenitor systems, the cause of the explosion, and the propagation of material and energy due to the explosion. For overviews see Branch et al. (1995), Nomoto et al. (2003), Di Stefano et al. (2011), Di Stefano & Kilic (2012), Wang & Han (2012), and Hoeftich et al. (2013).

The consensus is that SNe Ia come from close binary systems and result from a degenerate carbon/oxygen (CO) white dwarf (WD) undergoing a thermonuclear runaway (Hoyle & Fowler 1960). There are multiple potential progenitor systems in addition to multiple explosion scenarios. In the double degenerate (DD) system, the binary system consists of two or more WDs. In the single degenerate (SD) system, the binary system may consist of a single WD and either a main sequence, helium star, or giant star companion (Nomoto et al. 1984; Hernanz et al. 1988; Piersanti et al. 2003, 2009; Lorén-Aguilar et al. 2009; Pakmor et al. 2012; Wang & Han 2012; Hoeftich et al. 2013; Tornambè & Piersanti 2013; Wang et al. 2009a, 2009b). Whether SNe Ia occur from one or likely both of the proposed progenitor systems, the rate of SNe Ia from each channel and observations of either the progenitor system or the post-explosion companion in supernova remnants (SNR) will help solidify the overall picture of SNe Ia. Observations of both SD and DD candidate systems can be found in Greiner et al. (1991), van den Heuvel et al. (1992), Rappaport et al. (1994), Kahabka & van den Heuvel (1997), Ruiz-Lapuente et al. (2004), González Hernández et al. (2009), Kerzendorf et al. (2009), Edwards et al. (2012), and Schaefer & Pagnotta (2012), and, in particular, Foley et al. (2014) and McCully et al. (2014).

Aside from the makeup of the progenitor system, the explosion mechanism is also a topic of debate. In the case of two WDs merging on a dynamical timescale, the explosion is triggered by heat release during the merging process (Iben & Tutukov 1984; Webbink 1984; Benz et al. 1990; Lorén-Aguilar et al. 2009), which is only possible in a DD system. Alternatively, the explosion can be triggered by compressional heat in an accreting WD when it approaches the Chandrasekhar mass limit (MCh). The donor star can be a non-degenerate star or a tidally disrupted WD (Whelan & Iben 1973; Wang & Han 2012; Hoeftich et al. 2013). Scenarios with MCh explosions, whether originating from SD or DD progenitor systems, seem to be favored by observations of light curves and spectra, with
some additional contribution from dynamical mergers (Saio & Nomoto 1985, 1998; Hoeflich & Khokhlov 1996; Shen et al. 2012). Uncertainties about the progenitor system and explosion mechanism result in questions about the WD central density, total mass attained by the WD prior to the explosion, and the density profile of the WD just prior to the explosion.

The telltale indicator between the explosion scenarios is the central density of the WD at the time of the explosion. Within MCh models, the central density is an additional free parameter which depends mostly on the accretion rate and history (Sugimoto & Nomoto 1975; Nomoto et al. 1984; Thielemann et al. 1986; Brachwitz et al. 2000; Hoeflich 2006; Hoeflich et al. 2010; Seitenzahl et al. 2011). Slower accretion rates will lead to a higher central density of the WD just prior to the explosion, \( \rho_c \), because electron conduction is able to remove heat from the WD core and delay ignition of \( ^{12}C \) in the core. In dynamical or violent mergers, the timescale for merging does not allow for cooling and ignition can be triggered at a much lower \( \rho_c \).

Nuclear burning of a CO WD provides the energy to overcome its binding energy and provides the kinetic energy of the explosion. The time scales and the burning products depend mostly on the local density at the time of burning. Hydrodynamical instabilities may cause a redistribution of the burning products. Within the MCh channel, the most likely scenario involves delayed-detonation models (Khokhlov 1991; Yamaoka et al. 1992; Woosley & Weaver 1994; Gamezo et al. 2004; Röpke & Niemeyer 2007; Sim et al. 2013), i.e., models with a transition from a deflagration front to a detonation front (delayed-detonation transition; DDT). Independent of the explosion model, burning of a C/O mixture will produce regions of: iron group elements when burning reaches nuclear statistical equilibrium (NSE); layers of Si/Si/Ar/Ca by incomplete burning; O/Mg/Ne as a product of explosive carbon burning; and some unburned outer layers. In the inner layers, NSE is achieved in all current explosion scenarios. However, only in MCh does burning occur under sufficiently high densities that electron capture can take place on hydrodynamical time scales, which in turn shifts the peak of the NSE from \(^{56}Ni\) to stable isotopes of the iron group.

Observations during the photospheric phase allow investigation of the layers of incomplete burning and the extent of the \(^{56}Ni\) region, which determines the peak brightness, \( M_{B,V} \), and brightness decline rate, \( \Delta m_{15} \). However, observations of these regions provide hardly any information on the central region. The isotopic structure of the central region can be probed during the nebular phase by spectral line profiles. To use line profiles we need an emission feature of iron that is unblended. Optical and most near-infrared (NIR) features are dominated by blends of multiple bound–bound transitions of various elements. However, the 1.644 \( \mu m \) feature of [Fe \( \text{ii} \)] is a sensitive tool to separate stable and radioactive isotopes (Hoeflich et al. 2004; Motohara et al. 2006; Maeda et al. 2010b; Sadler 2012; Penney & Hoeflich 2014). To produce an emission line, we need both the element and energy input by radioactive decay. After 200–300 days, the energy deposition is dominated by positrons, which cannot excite the central low-velocity non-radioactive regions. The low-velocity emissivity produces “flattopped” or “stubby” profiles, which are a signature of MCh explosions without mixing. Up to about 200 days, the line profiles are peaked because gamma-rays contribute significantly to the energy input and excite the central, non-radioactive iron. Time evolution of the line profiles after 2–3 years is sensitive to positron transport effects and therefore magnetic fields. Time evolution of the spectrum is needed in order to separate density, asymmetry in chemistry, and magnetic field effects on the line profiles.

For the first time, a late-time evolutionary sequence of an SN Ia is presented in the NIR that includes the 1.644 \( \mu m \) line of [Fe \( \text{ii} \)]. Details of SN 2005df are presented in Sections 2 and 3. Reference models and rationale are outlined in Section 4. We develop general methods for comparing SNe Ia with the reference models as well as discuss the interpretation of the SN 2005df data in terms of our reference models and the implications of the analysis in Section 5. We compare the entire spectrum between 0.82–1.8 \( \mu m \) to show the viability and uniqueness of the 1.644 \( \mu m \) [Fe \( \text{ii} \)] feature. Finally, we discuss the implications of our results in Section 6.

2. OBSERVATIONS OF SN 2005df AND DATA REDUCTION

SN 2005df was discovered on 2005 August 4, by Evans & Gilmore (2005a, 2005b). Mid-infrared (MIR) observations of this SN Ia were previously analyzed by Gerardy et al. (2007), focusing on the [Co \( \text{iii} \)] emission line at 11.89 \( \mu m \) and the [Ar \( \text{ii} \)] and [Ar \( \text{iii} \)] emission lines at 6.99 and 8.99 \( \mu m \), respectively. Reviewing the initial published photometry and spectroscopy of SN 2005df, in addition to the range of distance estimates to the host galaxy NGC 1559, that analysis seemed consistent with a somewhat subluminous SN Ia (Gerardy et al. 2007). Light curve data for SN 2005df in the ultraviolet and optical were analyzed by Brown et al. (2010) and Milne et al. (2010). The supernova’s peak date in B-band was 2005 August 15. Milne et al. (2010) found that the light curve and \( \Delta m_{15} \) value of SN 2005df were consistent with a normal bright SN Ia, with a distance to the host galaxy larger than that used in the Gerardy et al. (2007) analysis.

For our analysis, observational epochs are given in relation to the estimated explosion date of 2005 August 2, to maintain consistency with Gerardy et al. (2007). Four epochs of late-time NIR spectroscopy, ranging from 198 to 380 days after explosion, were observed using the Gemini Near Infrared Spectrograph (GNIRS) at the Gemini South Observatory. All spectra were taken in cross-dispersed (XD) mode, with the last observation also including spectra taken in long-slit (LS) mode. The spectra taken in the XD mode are spread out into different spectral orders; however, only orders 3–8 have significant signal and are used in this analysis. The data from the 198 days observation were combined with the 199 days observation data, because they lack accompanying standard star calibration images. The data were reduced following the standard procedure outlined in the Gemini IRAF package, with an exception for variance handling. Because of the low signal-to-noise ratio of these data, the variance was propagated for each step in the reduction using a custom script, assuming the original variance determined by the Gemini IRAF package and Poisson distribution of noise. This variance propagation deviates from the standard procedure during the ncombine and nsextract steps in the reduction.

The telluric standard stars we use for calibration purposes are SAO 248743, used with the 198 and 199 days observations, and SAO 249308, used with the 217 and 380 days observations. Both of these stars have a spectral type of A0, with visual magnitudes of 8.3 and 7.3 mag, respectively (Ochsenbein...
et al. 2000). We use Kurucz' model for an A0V star along with the reduced telluric standard star spectra to determine the atmospheric and detector response corrections for the data (Kurucz 1993). The resolution for the theoretical standard star is at levels of 20, 50, and 100 Å in the 1.0–1.6, and >1.6 μm regions, respectively. The 198 and 199 days observations are corrected to be at the same relative fluxing as the 217 and 380 days observations and then normalized to the 217 days observation’s [Fe II] peak at 1.644 μm. Absolute flux calibrating is not possible because photometric observations were not included in the observations. Fluxes have been normalized to the peak of the emission feature at 1.64 μm for this analysis.

3. PRELIMINARY ANALYSIS

The reduced spectra span a wavelength range of ≈0.8–2.4 μm. The spectra are dominated by broad emission features formed by line blends. Lines in the spectra are identified based on the models described in Section 4. Most of the identifiable emission is due to forbidden line transitions of iron and cobalt. Large regions of atmospheric absorption and low filter transmission obscure the spectra in the ranges 1.35–1.48 μm and 1.82–1.95 μm. The full spectra are split up into regions corresponding roughly to standard band-passes: 0.8–1.1 μm (see Figure 1), J at 1.1–1.35 μm (see Figure 2), H at 1.48–1.8 μm (see Figure 3), and K at 1.9–2.4 μm. The XD order 3 data, corresponding roughly to K-band, are not shown because of the extremely low signal, especially in the 380 days observation, which prohibits any clear line identification in that region.

The XD orders 6, 7, and 8 data (see Figure 1 and Table 1) have features that are blends of many lines. Strong features are present at ≈0.86, 0.89, 0.95, and 1.02 μm, which are dominated by forbidden transitions of the iron-group elements and sulfur. In addition, many forbidden and allowed transitions contribute both to the features and to the overall "quasi-continuum" flux. The spectral evolution reflects the transition from a cobalt- to an iron-dominated regime. We note that this region poses some problems related to the atmospheric corrections. The spectra of the telluric standard star flux reference, an A0V star (Kurucz 1993), includes features of strong and narrow Paschen lines of hydrogen. In combination with the low wavelength resolution in the telluric reference spectrum discussed in Section 2, the correction produces artifacts in the reduced spectra of SN 2005df. In the lower section of Figure 1, we show the correction factors applied for the three orders that make up this wavelength region. The overall sharp-edged shape of the strongest feature at 0.9–1.0 μm is likely caused by the rapid variations of the correction factors. Similarly, the narrow strong line at ≈0.951 μm is likely artificial. Its width corresponds to the detector resolution, ≈400 km s⁻¹, and it coincides in wavelength and size with a feature in the correction factors.

The XD order 5 data (see Figure 2 and Table 2) correspond roughly to J-band. In addition to the emission feature at

Figure 1. Spectra of SN 2005df in the region 0.8–1.1 μm at 198/9, 217, and 380 days. The spectra are dominated by forbidden transition emission lines of iron, nickel, cobalt, and sulfur. Line identifications and the intrinsic line width are based on spectra from the reference model and its 56Ni distribution, respectively, as discussed in Figure 4. Note that all of the features are heavily blended. In the lower plot, the relative correction factors are given for the atmosphere and the detector response for the XD orders 8, 7, and 6.

Figure 2. Same as Figure 1 but for the region 1.1–1.35 μm. Note that all of the features are heavily blended. In the lower plot, the relative correction factors are given for the atmosphere and the detector response for the XD orders 7, 6, and 5.
and this Species Term J response for the XD orders 5 and 4. Note the small variability of the correction functions for SN 2005df (0.85–1.1 μm).

Table 1  
| λ (μm) | Species | Term | J−J′ |
|--------|---------|------|------|
| 0.85020 | [Ni II] | 3P−1D | 2–2 |
| 0.86193 | [Fe II] | aF−aP | 9/2–5/2 |
| 0.87312 | [Fe II] | aP−1D | 2–3 |
| 0.88406 | [Fe II] | aP−1D | 2–2 |
| 0.89444 | [Fe II] | aF−aP | 7/2–3/2 |
| 0.90544 | [Fe II] | aF−aP | 7/2–5/2 |
| 0.90711 | [S III] | 3P−1D | 1–2 |
| 0.92291 | [Fe II] | aF−aP | 5/2–3/2 |
| 0.92701 | [Fe II] | aF−aP | 3/2–1/2 |
| 0.93357 | [Co II] | aF−bF | 4–3 |
| 0.94467 | [Fe II] | H−aG | 5–4 |
| 0.95332 | [S III] | 3P−1D | 2–2 |
| 0.96113 | [Fe II] | H−aG | 4–4 |
| 0.96418 | [Co II] | aF−bF | 3–2 |
| 0.97045 | [Fe II] | H−1I | 6–6 |
| 1.0191 | [Co II] | aF−bF | 4–4 |
| 1.0248 | [Co II] | aF−bF | 3–5 |
| 1.0280 | [Co II] | aF−bF | 2–2 |
| 1.0290 | [S II] | 2P−3P | 3/2–3/2 |
| 1.0323 | [S II] | 2P−3P | 5/2–3/2 |
| 1.0339 | [S II] | 2P−3P | 3/2–1/2 |
| 1.0463 | [Ni II] | 2P−1P | 7/2–5/2 |
| 1.0507 | [Fe II] | aP−1D | 0–1 |
| 1.0718 | [Ni II] | 2D−3F | 5/2–7/2 |

1.02 μm described above, a strong feature is apparent at 1.28 μm. Because the spectra are normalized to the [Fe II] line at 1.644 μm and this 1.28 μm feature is dominated by blends of [Fe II], it shows little evolution with time. Even though some of these emission lines are quite strong, the proximity of lines and the presence of the [Co II] line, which is decaying over time scales much shorter than that of the iron lines, make this feature difficult to use for probing the central chemical structure of the supernova.

The XD order 4 data and the LS data (see Figure 3 and Table 3) correspond roughly to H-band. The dominant emission features are at 1.54, 1.64, 1.73, and 1.82 μm. The second feature is dominated by one transition, an [Fe II] line at 1.644 μm, and shows very little evolution in shape over the observed epochs. Additional [Fe II] emission lines sit in both the blue and red wings of this main [Fe II] line, but their contributions are far enough out in the wings and low enough in strength that we can actually describe what is happening to that single emission line. The observed line center has a Doppler shift of less than 1200 km s⁻¹ after correcting for the host galaxy’s radial velocity. In contrast to this, the neighboring features are blends with significant evolution (Penney & Hoefflich 2014). The line center of the blended [Fe II]/[Co II] feature at 1.54 μm shifts toward the blue (iron-line dominated side) in the 380 days spectrum, which is consistent with the
cobalt line fading but the iron line remaining strong. The blended [Co m] feature at 1.74 μm has completely faded in the 380 days spectrum. The strongly blended [Fe m]/[Co m]/ [Ni m]/[Ni IV] feature at 1.82 μm, with wings starting just at the red edge of Figure 3, does not seem to change between the three epochs. However, these lines are just at the edge of a low transmission region due to filter overlap and atmospheric corrections. The small variation in this feature is indicative of stable isotopes, namely iron and nickel, with the latter also being observed as [Ni II] and [Ni IV] in the MIR as isolated lines in Gerardy et al. (2007) and Telesco et al. (2015).

Over the entire spectral range, the extracted spectra show evolution of cobalt emission lines but minimal evolution of iron emission lines, consistent with the decay from cobalt to iron. The region of most interest for our analysis is around 1.5–1.8 μm, which includes a mixture of both iron and cobalt emission lines. In Section 5, the evolution of the [Fe II] emission line at 1.644 μm is used to put limits on the central density of the WD and the strength of the magnetic field in the progenitor by comparing with models produced by Hoeﬂich & Penney (Hoeﬂich et al. 1998; Penney 2011; Penney & Hoeﬂich 2014).

4. MODELS

Based on the detailed spherical models for supernovae discussed below, we analyze the spectral features of SN 2005df and their evolution. The models are consistent with respect to the explosion, light curves, and spectra; namely, the free parameters describe the initial condition of the WD and the time of the deflagration to detonation transition. The free parameters are the amount of burning prior to the DDT, which we characterize by a transition density where the DDT occurs, ρtr, the central density of the WD prior to the explosion, ρc, which is determined by the accretion history of the system, and the zero-age main sequence mass of the progenitor, MMS, and the metallicity of the progenitor, Z, which in combination determine the chemical structure of the WD (Hoeﬂich et al. 1998). We emphasize that the modeled time evolution of the NIR spectrum follows without additional free parameters, given the initial WD model and parameterized properties for nuclear burning.

4.1. Methods

For the calculations of explosions and spectra, we use our code for radiation hydrodynamics (HYDRA; Hoeﬂich 1990, 2003, 2009) which solves for the hydrodynamics using the explicit Piecewise Parabolic Method (Colella & Woodward 1984), detailed nuclear and atomic networks (Kurucz 1994; Hoeﬂich et al. 1998; Seaton 2005; Cyburt et al. 2010), transport for low-energy and gamma photons and positrons by variable Eddington Tensor solvers and Monte Carlo Methods (Stone et al. 1992; Hoeﬂich et al. 1993; Mihalas & Mihalas 1999; Hoeﬂich 2003, 2009; Penney & Hoeﬂich 2014). For this study, atomic data for forbidden line transitions have been updated using Quinet (1996), Quinet et al. (1996), Quinet (1998), De et al. (2010), Friesen et al. (2014), Kramida et al. (2014), and van Hoof (2014).

4.2. Model Selection Criteria

In the simulations, we use a spherical delayed detonation model. Spherical geometry implies suppression of mixing during the deflagration phase. Varying the amount of burning prior to the DDT produces a wide range of values for the 56Ni mass produced and the corresponding brightness of the SN, spanning ≈3 mag. It shifts the characteristic chemical pattern in velocity space (see Figure 3 in Hoeﬂich et al. 2003). Spherical DDT models have previously been used with success in reproducing the optical light curves, IR light curves, and spectra of a Branch-normal and several sub-luminous SNe Ia, in addition to the statistical properties of the SN Ia class (Hoeﬂich et al. 2002, 2010; Howell et al. 2006; Marion et al. 2006, 2009; Quimby et al. 2007; Maund et al. 2010; Patat et al. 2012; Sim et al. 2013; Dessart et al. 2014).

In the following, we describe our procedure for selecting the model for a Branch-normal SNe Ia. The main free parameter for the models is the amount of burning prior to the DDT, which in spherical models is commonly characterized by the transition density, ρtr. Varying ρtr causes changes of the 56Ni mass by an order of magnitude when keeping the same initial conditions of the WD. The previously investigated 5p0z22-series of models in Hoeﬂich et al. (2002) give MV between −17.21 and −19.35 mag. We choose the model with ρtr that produces about 0.60 M⊙ of 56Ni, which is consistent with SN 2005df. This model burned about 0.3 M⊙ during the deflagration phase.

The other free parameters result in variations in the brightness and light curves on the order of 0.2 mag (Hoeﬂich et al. 2010). The maximum brightness and the early light curves, up to about one to two weeks after maximum light, are hardly affected by the central region of the WD because the diffusion time scales are comparable or larger than the expansion time scales (Hoeﬂich 1995a; Hoeﬂich et al. 1998, 2013; Sadler 2012; Sadler et al. 2013). Using the early light curve of SN 2005df from Milne et al. (2010), we produced a series of models with very similar outer layers by keeping the amount of deflagration burning constant at 0.3 M⊙. As ρtr varies, the total production of 56Ni also varies, which is consistent with the findings of Hoeﬂich (2006) and Krueger et al. (2010). For compactness, in plots we will use the dimensionless quantity ρtr = ρtr/(0.5 g cm−3).

We use secondary parameters to optimize the early light curve information using the Carnegie Supernova Project I and II samples, as describe in Sadler (2012) and Hoeﬂich et al. (2013). The result suggests MMS(SN 2005df) ≈ 7 M⊙, though the uncertainties are large. Variations in MMS may change the explosion energy and with it, the resulting expansion velocities by up to 5% while the density profiles remain very similar (Dominguez & Hoeﬂich 2001, Figure 5). Note that high MMS provides the most stringent low limit for the progenitor mass. For Z, we use solar values.

Using the procedure described, the explosion models used in this analysis are based on a WD of solar metallicity with a main sequence mass of 7 M⊙, 7p0z22. For this study, the initial central density of the WD and initial magnetic field have been varied between 0.5 × 109–4.0 × 109 g cm−3 and 109–1010 G, respectively, to provide a range of models to analyze the NIR spectra of SN 2005df. The density at which the DDT occurs has been varied depending on the initial central density of the WD so that the outer chemical structure of the SN, and hence the peak brightness, remains constant and consistent with the observed light curve of SN 2005df. Because of this variation, the total amount of 56Ni produced in our models ranges from 0.57–0.72 M⊙ for ρtr of 4.0 × 109–0.5 × 109 g cm−3. Note that
the resulting early light curve variations are small: $M_V(\delta) \approx 0.04 \pm 0.05$, $m_B(\delta) \approx 0.0715 \pm 0.05$, and $m_V(\delta) \approx 0.03$ mag.  

For our WD, using the best-fit initial central density of $0.9 \times 10^9$ g cm$^{-3}$, as determined in Section 5.4, the model peaks at an absolute magnitude of $M_B = -19.29$ mag and has brightness decline rates of $m_B(\delta) = 1.14$ and $m_V(\delta) = 0.61$ mag.  

These values are comparable to the observed values for SN 2005df of $m_B(\delta) = 1.2$, $m_V(\delta) = 0.63$, and $M_B = -19.23(\pm 0.10)$ mag, assuming a distance modulus of $\mu = 31.81$ as in Milne et al. (2010).  

5. RESULTS  

5.1. Effects of Central Density  

Because of the construction of our models, the $^{56}$Ni abundance profiles and density profiles are very similar in the outer regions of velocity-space (see Figure 4).  

However, the interior profile varies dramatically, with radioactive nickel providing less and less of a low-velocity contribution with increasing central density.  We note that the velocity profile cannot be related directly to the extent of the $^{56}$Ni region in the velocity-space, which here is on the order of 10,000 km s$^{-1}$, but is in fact significantly narrower.  

The profile is determined by the convolution of the density structure and abundance profile in addition to projection effects.  In emission profiles, the contributions come from all directions and show projected velocities.  We note that in contrast, the minimum of the absorption components of P-Cygni profiles resembles the expansion velocity because they are produced by absorption or scattering of photons out of the line of sight of an observer.  

As the central density increases, there is a larger fraction of material at a high enough density for electron capture to occur during the deflagration and produce sizable amounts of stable $^{58}$Ni (Hoeflich 2006).  

The variation of spectral line profiles during the early nebular phase has recently been discussed by Penney & Hoeflich (2014).  

Prior to 200 days, the entire central region will contribute to the emission features because gammarays are still the dominant source of energy and will deposit energy in the central region, resulting in peaked line profiles.  

However, once positrons become the dominant energy source, the “hole” in the radioactive material does not contribute to the emission features in the nebular spectrum until very late times when the positrons have become non-local.  Figure 4 shows the size of the $^{56}$Ni hole increasing with $\rho_c$.  The $^{58}$Ni does not contribute to the emission when positrons are dominant and still locally depositing their energy.  

Because the $^{58}$Ni is
centrally located, there is less emission at low-velocity, corresponding to a broadening of the nickel, cobalt, and iron emission lines with increasing $\mu$. 

5.2. Benefits of the H-band Region

As can be seen from the SN 2005df spectra in Figures 1–3, there are significant overlapping emission lines contributing to most of the features seen in the NIR. The “cleanest” feature in the entire observed region is the $\text{[Fe II]}$ line at 1.644 $\mu$m, whose closest satellite line with moderate strength is 0.333 $\mu$m away from line-center. The entire H-band region from 1.5–1.8 $\mu$m can effectively be modeled using just the contributions from the strongest emission lines listed in Table 3. In this analysis, we will find which of the reference models best fits the data by considering only the 1.644 $\mu$m line. The other NIR features come out naturally.

5.3. Model Comparison

By 200 days after the explosion the energy deposition transitions from a gamma-dominated to positron-dominated regime and becomes more local. As a result “stubby” or “flat-topped” line profiles develop. Later on, transport effects by positrons become increasingly important, starting in the outer high-velocity $^{56}\text{Ni}$ regions which produce the line wings. Eventually stable iron group elements can be excited in the central regions, leading again to peaked profiles. The line width (LW) increases with $\mu$ because the $^{56}\text{Ni}$ distribution shifts toward higher velocities. As a consequence, the effective cross section of every inner mass shell decreases and its energy deposition is reduced. In higher-$\mu$ models, less emission comes from low-velocity regions, resulting in a wider profile. An in-depth discussion of line evolution with magnetic fields is presented in Section 5.5.

5.4. Central Density Fitting

As discussed in Section 5.1, the distribution of iron group elements in the central regions depends on the initial central density of the WD. Therefore, the shape of the line profile, specifically the LW, is sensitive to measuring $\mu$. Our procedure for finding the LW of the 1.644 $\mu$m line is as follows:

1. subtract the free–free continuum produced by the models, re-normalizing to the 1.644 $\mu$m peak, to give the line profile a zero-point;
2. pick fixed $y$-value points (denoted $H$ in plots) in increments of 0.1 or smaller, as signal-to-noise of the data allow;
3. for each of the chosen $y$-values, the corresponding velocity $x$-values are obtained using a linear regression through adjacent data points, with multiple resolution elements used in the case of the data to decrease the effects of noise;
4. for each epoch and magnetic field strength, measure LW from the $+x$-value to the $-x$-value for each of the chosen $y$-values.

Because we consider line profiles and relative flux normalized to the 1.644 $\mu$m line, evolution of the line profile is determined only by drop-out in the wings or core and does not include changes in the line peak. This representation allows for direct comparison to our observational data, in which total flux could not be measured. Figure 6 shows the observational data and the velocity $x$-values we obtain for each $y$-value.

![Figure 6](image-url)

Figure 6. Observed spectra at 198/199, 217, and 380 days for the $\text{[Fe II]}$ line at 1.644 $\mu$m. The plot shows the velocity approximations at different heights ($0.5 \leq H \leq 0.8$) overlaid on the actual data. These values are the ones used to determine line width for the observational data. We limit the range for $H$ because the wings of this feature are contaminated by neighboring iron and cobalt emission lines.

Line profiles depend on the continuum. Our model continua are used to zero-point the SN 2005df data. The continua produced by the reference models are at levels of 9%, 7%, and 2% of the line-height for the 198/9, 217 and 380 days epochs, respectively. Uncertainties in the observed continuum flux will introduce a systematic error to the measurements of LW. Under- and overestimating this continuum flux will increase and decrease LW, respectively. From the observations, line
wings must stay above zero, providing a natural upper limit for the continuum in SN 2005df. Using Figure 3, these values are less than 15\%, 16\%, and 12\% for the 1989, 217, and 380 days, respectively. The systematic error estimates in LW are obtained as follows: use the continuum error to determine the maximum adjusted $H'$; use the original $H$ values to interpolate new velocity values at $H'$; obtain a new estimate for LW$'$, where the systematic uncertainties are given by LW$' - LW$. For our observations, comparing $H = 0.6$ and $H'$, we get $H' = 0.624, 0.636, and 0.64$ for the three epochs. By using this method, we guarantee that we have overestimated the continuum flux, and therefore the actual LW must be larger than the value found. The systematic error due to continuum uncertainty for SN 2005df will affect the LW at a maximum level of 600–700 km s$^{-1}$, with the 217 days observation having the largest possible effect.

One of the advantages of the method we have developed is that there is not one specific measurement that must be made. The LW can be measured at a variety of different $H$-values to ensure that the results found are self-consistent, and noise in the data in small wavelength regions will not affect the overall result. The $H$-value of 0.6 has been chosen for display in Figures 7 and 11 even though each $H$-value we tested produces very similar results. At $H = 0.6$ the level of contamination from the neighboring satellite lines to $\mu = 1.644 \mu m$ [Fe II] line, which increases with decreasing $H$, is small. Similarly, the uncertainty when finding the corresponding velocity $x$-values, which increase with increasing $H$, is small because of the slope of the line in that region.

Using LW to determine the initial central density of the WD for SN 2005df, we find that $\rho (SN 2005df) = 0.9 (\pm 0.2) \times 10^9 g cm^{-3}$. The systematic error due to continuum uncertainties is just over the $1\sigma$-level but can only make LW wider and therefore the central density higher. This central density is high enough that SN 2005df would have a moderately sized region of stable $^{58}$Ni at low velocity; however, $\rho (SN 2005df)$ is not high enough to produce the characteristic “flat-topped” line profiles seen in SN 2003du (Hoeftich et al. 2004; Motohara et al. 2006) and SN 2003hv (Motohara et al. 2006). The best-fit reference model is shown plotted with the SN 2005df data in Figure 8. The line-center for the [Fe II] line at 1.644 $\mu m$ is shifted by less than 1200 km s$^{-1}$, which could be attributed to the peculiar velocity of the SN in the host galaxy possibly in combination with an asymmetry in the explosion. The main feature is very well reproduced both in the core and wings of
the line profile. There are some discrepancies in the relative fluxes for the neighboring features around $-20,000 \text{ km s}^{-1}$ (corresponding to $1.54 \mu\text{m}$) and $18,000 \text{ km s}^{-1}$ (corresponding to $1.74 \mu\text{m}$). However, these may be attributed to the treatment of super-levels in the reference models or in the simplistic treatment of continuum subtraction.

5.5. Effects of Magnetic Fields

By looking at the initial line profile and its evolution, we can put a lower limit on the strength of a magnetic field embedded in the SN ejecta. Once positrons dominate the energy deposition, the energy deposition becomes more local with $B$. The effective distance of travel decreases because the positron spirals around the field lines and for high $B$ positrons may be trapped locally depending on the morphology of the field (Penney & Hoeflisch 2014). In order to probe this effect, we have included a range of field strengths for a magnetic field embedded in the SN ejecta: $B = 10^3, 10^4, 10^5, \text{ and } 10^6 \text{ G}$.

The morphology of the magnetic field can also affect the line profile shape and evolution (Penney 2011; Penney & Hoeflisch 2014). As discussed below, viewing angle effects will become large after about 2–3 years, which would allow detangling of the morphology. For the epochs considered in this analysis, morphology effects remain small, and thus we used dipole fields and angle-averaged emission profiles. A discussion of the effect of orientation is presented below and shown in Figure 10. In order to look at the effects of magnetic field strength, a low-$B$ and a high-$B$ case are shown in Figure 9 at 300 days (top panel) and 500 days. Although the energy deposition is dominated by positrons at 300 days, the positrons are still localized and even the most extreme magnetic field, $B = 10^9 \text{ G}$, only affects the flux to a minor extent. By 500 days, the magnetic field effect on the flux is within a measurable range for the high-$B$ spectrum, with the extreme $B = 10^9 \text{ G}$ spectrum retaining its “flat-topped” profile.

In most of the reference models, the difference between the flux when viewed from a polar orientation versus an equatorial orientation is smaller than the resolution limit for our SN 2005df observations and will be negligible for our purposes. In cases of large central densities, however, the difference between flux for the normalized line can be as large as $\Delta F = 0.2$, and this effect should be considered along with other variations in the line profile. Two cases are shown in Figure 10, both at 500 days: $\rho = 0.9 \times 10^9 \text{ g cm}^{-3}$ with $B = 10^6 \text{ G}$ and $\rho = 4.0 \times 10^9 \text{ g cm}^{-3}$ with $B = 10^9 \text{ G}$.

A rough cut of central densities can be made using LWs. Positron transport starts to influence line profiles between 200 and 300 days. In general the LW increases with $B$ because positrons keep energy deposition out of the core of the ejecta. However, note that at 200 days intermediate-sized dipole fields produce a slightly narrower profile because those fields funnel positrons toward the center without strongly restricting the effective distance traveled by the positrons. With the SN 2005df observations, the two effects are indistinguishable because of the noise in the data and the limited epochs of observation, as can be seen in Figure 11. A non-zero magnetic field is only favored by a $1\sigma$-level. Figure 11 shows that an observation at 400 days post-explosion or later, in addition to an early nebular phase observation, is needed in order to obtain both $\rho$ and magnetic field strength.

5.6. Overall Spectral Comparison for the Wavelength Range 0.8–1.35 $\mu\text{m}$

In this section we compare the short wavelength region, 0.8–1.35 $\mu\text{m}$, of SN 2005df with our best-fit reference model.
The late time observation of SN 2005df may favor $B \approx 10^6 \text{G}$, but only by a $1\sigma$-level. (see Figure 12). The fluxes have been normalized to the 1.644 $\mu$m [Fe II] line without further tuning. Most of the features seen in SN 2005df can be reproduced, which allows for the identification of the main spectral features as given in Tables 1 and 2. At both 200 and 400 days, the spectra are dominated by iron group elements, and evolution of the spectra can be understood by the transition from a cobalt-dominated to an iron-dominated regime. This is evident by the rapid evolution of the broad feature at 0.95 $\mu$m compared to the slow changes in the iron-dominated features at 0.86, 1.03, and 1.26 $\mu$m, which are produced by blends of [Fe II], [S II], and [Fe II], respectively. Both the width and flux ratios of features are reproduced in the model. We note, however, that each of the main features is strongly blended, emphasizing the uniqueness of the [Fe II] emission line at 1.644 $\mu$m used in the previous sections for central density and magnetic field strength analyses.

By 200 days after the explosion the envelope is mostly transparent. Most of the radioactive $^{56}\text{Ni}$ and $^{56}\text{Co}$ has decayed to $^{56}\text{Fe}$, and stable nickel and iron are fully exposed. The models show, and we want to emphasize, that normalization of spectra to this line allows us to separate whether features are dominated by stable or unstable isotopes.

Some discrepancies are obvious. The flux between below 0.92 $\mu$m is too low by a factor of 3. The problem may be related to the use of super-levels and the lack of atomic data with the reference model. However, tests with various cross sections from literature produce changes of only 20%–30%. Alternatively, this inconsistency may be related to the atmospheric corrections during data reduction, since this region sits on the edge of two of the orders in the XD spectrum.

Additionally, a narrow feature is present at 0.95 $\mu$m with a width corresponding to the instrumental resolution of $\approx 400$ km s$^{-1}$. As discussed in Section 2, we use the fluxes calibrated by the energy distribution of an A0V standard star with a resolution of 20–50 $\AA$, which is larger than the detector resolution in the wavelength range shown in Figure 12. Additionally, A0V stars have strong lines in the Paschen series in this region, and the correction factors vary by about 2, which may produce an overcorrection of the SN 2005df flux. Moreover, a strong, narrow feature at 0.95 $\mu$m is unexpected based on the models and would imply a component with high emissivity in a very localized region, $\approx 400$ km s$^{-1}$, in the envelope. In the line list, this feature could be [S II], but it is difficult to imagine a scenario where sulfur can be excited at high levels without a corresponding narrow feature in cobalt. Therefore, we regard the atmospheric correction as the likely source, especially considering the coincidence of the feature with a Paschen line in the A0V telluric standard star.

5.7. Implications of the Central Density for the Possible Progenitor and Explosion Scenario of SN 2005df

As discussed in Section 1, different progenitor systems and explosion scenarios may be distinguished by $\rho_l$. The pressure in the WD is dominated by a degenerate electron gas. With increasing density, this Fermi gas becomes increasingly relativistic degenerate, and $\rho_l$ increases rapidly when approaching $M_{\text{Ch}}$. As a consequence, $\rho_l$ is a steep function of the WD mass, $M_{\text{WD}}$ (see Figure 13). We note that the equation of state determines the $M_{\text{Ch}}$ for the WD and depends on the metallicity, composition, and accretion. However, in practice, a WD cannot reach this limit because densities become high enough that the time scales for electron capture become shorter than the hydrodynamical time scales, which reduces pressure in the WD core and causes an accretion-induced collapse (AIC). In Figure 13, our reference mass for $M_{\text{Ch}}$ is taken when $\rho_l \approx 7.0 \times 10^9$ g cm$^{-3}$ and assumes low accretion rates. For SN 2005df, we obtain $\rho_l = 0.9(\pm 0.2) \times 10^9$ g cm$^{-3}$ in Section 5.4. Our systematic error due to continuum uncertainty is at about the $1\sigma$-level. Despite the rather large error range in
thought to cool the WD center and delay the onset of explosive C burning (Paczyński 1972; Barkat & Wheeler 1990). This would mean that $M_{\text{Ch}}$ scenarios should transition into an explosion only at central densities in excess of $3 \times 10^9$ g cm$^{-3}$. However, this notion was recently debated. It was noted that neutrino emission also has the opposite effect of slowing down the convection and thus heating the core (Stein et al. 1999). Cooling and heating due to the Urca process may cancel out (Stein & Wheeler 2006), and our results lend support to this conclusion.

5.8. Results in Context of General Explosion Scenarios and Physical Assumptions

The measurement standard for realistic SNe Ia simulations is observation. The limitations in determining realistic simulations are in part due to the diversity of possible explosion scenarios and to the approximations used within models, producing variations in the density structure and spatial distribution of elements. Each of these points will be briefly addressed below. Our method of diagnostics and the analysis of SN 2005df are based on spherical delayed-detonation models, which certainly puts restrictions on applicability and conclusions for individual SNe Ia. Note that line profiles measure the overall density structure in the central region, which is similar for a wide variety of explosion models, combined with the effect of $\rho_c$ on the $^{56}$Ni-free region. Therefore, the line profiles are sensitive to models with $\rho_c$ above $\approx 10^8$ g cm$^{-3}$. Our LW analysis does not allow separation between a low-$\rho_c$ $M_{\text{Ch}}$ system and the merger of two low-mass WDs. Note that rotation of the WD will reduce the effective gravity and thus increase the estimated WD mass based on line profiles.

It is beyond the scope of this paper to present an in-depth discussion of the various progenitors and explosion scenarios, beyond what is said in Section 1. As a starting point, we refer to Section 1 and the recent reviews by Wang & Han (2012) and Hoefliech et al. (2013) and the references therein. We will restrict our discussion to the limitations of our approach with respect to our analysis and note that “first principle simulations” are beyond the current state of the art. Consequently, new realizations may be recognized in the future.

As argued in Section 1, we regard delayed-detonation and pulsating delayed-detonation models as the most commonly realized scenarios. However, from observations of progenitor systems and supernovae light curves and spectra, there is strong evidence that SNe Ia come from an inhomogeneous population (Saio & Nomoto 1985, 1998; Woosley & Weaver 1986; Mochkovitch & Livio 1990; Hoefliech & Khokhloev 1996; Hachinger et al. 2006; Howell et al. 2006; Quimby et al. 2006; Di Stefano & Kilic 2012; Edwards et al. 2012; Schaefer & Pagnotta 2012; Shen et al. 2012; Wang et al. 2014). For detailed discussions see Di Stefano et al. (2012) and the recent review by Maoz et al. (2014).

As discussed in Section 1, SN Ia explosion models produce burning products of the iron group elements, S/Si/Ar/Ca, and O/Mg/Ne. Observations strongly support an overall layered structure. Explosion scenarios may be separated by their amounts and spatial distributions of the various burning products. For individual supernovae, additional constrains must be used when using our method. Note that from the observations, there is a lack of large density asymmetries in SNRs and continuum polarization (Howell et al. 2001;
Hoeßl 2006; Fesen et al. 2007; Maund et al. 2010; Patat et al. 2012). Moreover, the presence of MIR Ni lines at late epochs suggests high-density burning in several cases, namely in SN 2005df and SN 2014J (Gerardy et al. 2007; Telesco et al. 2015).

Dynamical Mergers: as discussed in Section 1, this explosion scenario involves two CO WDs in a binary system. The WDs merge once enough angular momentum has been lost due to gravitational radiation. The thermonuclear detonation is triggered by the heat of the merging process. However, the end-result of this type of explosion is unclear: an SN Ia; an AIC to a neutron star; a surviving WD with high magnetic fields due to the large angular momentum involved in the merger (Penney & Hoeßl 2014). The rates for binary systems comprised of two WDs also need to be taken into account. Reviews of this scenario, including expected rates, can be found in Webbink (1984), Isern et al. (2011), and Wang & Han (2012). Simulations of dynamical mergers have been done by many groups, including Iben & Tutukov (1984), Benz et al. (1990), Rasio & Shapiro (1994), Segretain et al. (1997), Yoon et al. (2007), and Loren-Aguilar et al. (2009). As a common feature, simulations show peaked central densities and a polytropic density structure in the inner region, but with a reduced $\rho$ when compared to a hydrostatic WD of similar mass. In general, $\rho$ is well below the limits for electron capture, and $^{56}\text{Ni}$ is produced in the central region. Thus, we expect to see peaked NIR line profiles. However, density profiles in the wings depend mostly on the angular momentum distribution (Eriguchi & Müller 1993), and lines may have narrow or broad wings depending on whether they are viewed from a polar or equatorial angle. Unless viewed from an extremely polar angle, they will show high continuum polarization generally not seen (Hoeßl 1991; Howell et al. 2001; Patat et al. 2011).

Violent mergers are a variation of dynamical mergers where two WDs collide (Raskin et al. 2009; Rosswog et al. 2009) or a detonation triggered directly in the merger (Pakmor et al. 2011). This class of models may produce a strong asymmetric $^{56}\text{Ni}$ distribution showing a bow-shock structure. The NIR line profiles should be asymmetric and may appear broad or narrow, depending on the viewing angle. Asymmetries in the $^{56}\text{Ni}$ distribution in violent mergers modeled by Pakmor et al. (2011) are similar to cases considered in Hoeßl (1995b). For those, we would expect polarization of the order of 1% and a flip in polarization angle which has never been observed. To distinguish dynamical and violent merger models from $M_{\text{Ch}}$ models, discussed below, we need fits of the entire line profile or polarization measurements.

$M_{\text{Ch}}$ Models: in these simulations, we may have pure deflagrations, as in the W7 model (Nomoto et al. 1984), and delayed-detonation models, as discussed here. In pure deflagration models, a significant amount of matter remains unburned. Line profiles will have a reduced expansion velocity when compared to the profiles produced by our models, but will otherwise be very similar. Within delayed-detonation models, there are prompt DDTs and those where the DDT is triggered by mixing, resulting in a pulsational phase (pulsational delayed detonation; PDD). In PDDs the detonation is triggered by mixing of the inner layers. The overall distribution of $^{56}\text{Ni}$ in the outer layers has been found to be very similar to classical DDT models, however the expansion velocity is slightly lower. PDDs have been successfully used to reproduce a sub-class of SNe Ia (Khokhlov 1993; Hoeßl & Khokhlov 1996; Quimby et al. 2006; Dessart et al. 2014). For large-amplitude pulsations a strong shell is formed (Khokhlov 1993). The telltale sign of a PDD is a plateau in the photospheric velocity, with a significant amount of unburned C/O at high velocities and a lack of high-velocity Si wings commonly observed in SNe Ia (Quimby et al. 2006). In Khokhlov (1993), full mixing is assumed, resulting in peaked but broad profiles as discussed in Hoeßl et al. (2004). Subsequently, we found that full mixing is not required to trigger the DDT (Hoeßl & Khokhlov 1996), and it may be constrained to spatially limited mixing, as discussed below. Our method will produce similar results but with slightly underestimate $M_{\text{Ni}}$.

We want to mention one main problem of spherical DDT models and suppression of mixing. Simulations from both SD and DD systems have been successful in producing thermonuclear explosions in $\approx M_{\text{Ch}}$ WDs. However, Rayleigh–Taylor (R–T) instabilities and mixing, which are inherent to all current multi-dimensional simulations (Khokhlov 1995; Niemeyer & Hillebrandt 1995; Livne 1999; Reinecke et al. 1999; Gamezo et al. 2003; Röpke et al. 2006), produce observables which are at odds with data as discussed in Hoeßl (2006). R–T instabilities break the layered chemical structure expected from spherical models and observations, namely the narrowness of the brightness decline relation, the lack of iron group elements in typical SNe Ia, and the line polarization in typical SNe Ia and SNR. Layering of the outer ejecta may be present due to a detonation front (Gamezo et al. 2004; Röpke et al. 2012), but the central regions where deflagration burning occurred show mixing of iron group elements. An additional problem is that plumes may transport energy from the center of the WD into the outer regions of the ejecta. One extreme model is “confined detonation” where a single plume rises to the surface of the WD (Plewa et al. 2004; Plewa 2007). Although modifications to this model are found to be small in general, and intermediate stages resemble the spherical density profiles (Gamezo et al. 2003, 2004), more deflagration burning is needed to obtain a similar pre-expansion when compared to spherical models. As with the discussion of mixing in Section 5.7, our method can be expected to underestimate $\rho$ when multi-dimensional effects are included. Some of the problems may be overcome by modifying the outer ejecta structure which, for example, may allow fitting of the spectra at maximum light (Kasen et al. 2008; Seitenzahl et al. 2011, 2013; Sim et al. 2013).

New observations strongly suggest a process that partially suppresses the dominant role of R–T instabilities in multi-dimensional models. These observations include (1) post-maximum spectra in normal-bright and subluminous SNe Ia (Wheeler et al. 1998; Hoeßl et al. 2002, Figures 10–14.), including

1. stubby or flat-topped line profiles 1–2 years after the explosion, indicating stable isotopes produced at low velocities after the initial phase of burning (Motohara et al. 2006; Maeda et al. 2010a; Stritzinger et al. 2015). Mixing would result in narrower, peaked profiles (Hoeßl et al. 2004); and
2. narrow late-time emission lines in the MIR of SN 2005df and SN 2014J, which have been identified as Ni (Gerardy et al. 2007; Telesco et al. 2015). Because of the short lifetime of $^{56}\text{Ni}$, any Ni emission at these late epochs must originate from stable Ni. Because the lines are
and (2) direct imaging of S-Andromeda, which shows a Ca-free core indicative of high-density burning and limited mixing (Fesen et al. 2007). We note that the classification of S-Andromeda is debated. The historical light curve data are consistent with an SN Ib (de Vaucouleurs & Corwin 1985; Branch 1986). However, as discussed in Fesen et al. (2007), the time of explosion is uncertain and the LC may be consistent with a subluminous SN Ia, and the chemical structure, namely the wide velocity-range of Ca from 4000–11,000 km s$^{-1}$, suggests a thermonuclear explosion. We note that recent high-resolution observations of this SNR show some evidence of limited mixing in the $^{56}$Ni distribution, which may be consistent with highly suppressed R–T instabilities confined to regions well within the Ca layers (Fesen et al. 2015, in preparation).

Moreover, the trouble is not limited to signatures of individual SN observations. The brightness decline rate plays a key role in cosmology (Phillips 1993; Phillips et al. 1999; Goldhaber & Perlmutter 1998). Although $\Delta m_{15}$ is well-understood as an opacity and expansion effect for similar structures, as in spherical delayed-detonation models, a tight relationship breaks down for variable mixing (Hoeijmakers et al. 1996; Nugent et al. 1997; Umeda et al. 1999; Kasen et al. 2008; Baron et al. 2012).

6. DISCUSSIONS AND CONCLUSIONS

We have shown the time evolution of the NIR spectrum of SN 2005df, specifically focusing on the $[\text{Fe} \text{II}]$ emission line at 1.644 $\mu$m. We found that this line provides a unique tool to analyze NIR spectra. Normalization to this line allows separation of features produced by stable versus unstable isotopes of iron group elements because the envelope is mostly optically thin by 200 days.

We have developed a methodology for using late-time NIR spectra of SNe Ia to determine the initial central density and strength of embedded magnetic field of the WD. Time-series observations in the nebular phase are of great importance because they shed light on the interior structure of the burning products in the SN. There is currently a scarcity of observations showing late-time NIR results for SNe Ia and the time evolution of the nebular spectra of SNe Ia.

The $[\text{Fe} \text{II}]$ emission line at 1.644 $\mu$m is relatively “clean” compared to most of the NIR region, as was shown in Sections 3 and 5.2, especially when compared with the optical spectra of nebular SNe Ia. The neighboring features to the 1.644 $\mu$m line are blended features, with multiple iron and cobalt emission lines contributing. Therefore, other NIR lines are not as well-suited to analysis of the evolution of line profiles that has been presented in this work. The $[\text{Fe} \text{II}]$ 1.644 $\mu$m emission line is ideal because of the relatively little line contamination from nearby features.

An early nebular epoch, somewhere around 100–200 days past explosion, is needed to pin down the central density of the WD before the magnetic fields in the ejecta start affecting the LW curves. As demonstrated by the models, the LWs of the $[\text{Fe} \text{II}]$ 1.644 $\mu$m line converge when gamma-rays still dominate the emission in the nebular phase, independent of the magnetic field strength embedded in the ejecta. Multiple epochs in the nebular phase are needed in order to probe the evolution of the $[\text{Fe} \text{II}]$ 1.644 $\mu$m emission line and set a lower limit on the magnetic field strength, which plays a role once positron emission dominates.

Line profiles are stable against uncertainties in absolute calibrations and distance because they measure the distribution of $^{56}$Ni in the velocity-space, which may be used as an indicator for the total mass of the WD. Absolute fluxes provide an independent absolute calibration for $^{56}$Ni. As discussed in Section 4, our method has been developed based on spherical explosion models because these models produce structures consistent with many observations and, in particular, seem to be applicable to SN 2005df. Although uncertainties in the distance and the lack of accurate IR photometry of SN 2005df do not allow a direct comparison of the absolute flux of the 1.644 $\mu$m $[\text{Fe} \text{II}]$ line, several indicators suggest that the models give the right NIR flux level. Both the early optical LCs and the late-time NIR fluxes—measured with the Spitzer Space Telescope (Gerardy et al. 2007)—of SN 2005df agree with the models.

Finally, we have discussed some implications of a low-mass WD and put our results into context with progenitor systems and explosion scenarios. The $M_{\text{WD}}$(SN 2005df) we have obtained is close to $M_{\text{Ch}}$, making a violent or dynamical merger scenario unlikely because those can trigger explosions in a wide range of masses. However, the LW alone, as given in Figures 7 and 11, would not be sufficient to exclude dynamical or violent mergers, as discussed in Sections 1 and 5.8. With a fit of the entire line profile, as in SN 2005df, $M_{\text{Ch}}$ is favored. Alternatively, polarization measurements would also be an indicator. Within the scenario of $M_{\text{Ch}}$ explosions, the thermonuclear runaway is triggered by compressional heating due to the accretion of H, He, or C, as discussed in Section 1. A low central density, such as we found for SN 2005df, requires high accretion rates in $M_{\text{Ch}}$ scenarios. For our models, H-accretion would barely be consistent with ignition at our measured $R_{\text{t}}$ for SN 2005df even without Urca process cooling (Barkat & Wheeler 1990; Stein & Wheeler 2006). Our results for SN 2005df may favor He-accretion from a giant companion star in a SD system or C-accretion from a disrupted WD companion in a DD system. However, although line profiles allow us to measure the $M_{\text{WD}}$ for objects close to $M_{\text{Ch}}$, strong mixing must be suppressed during the deflagration phase. Though observational evidence supports this limit, some remaining mixing will make $M_{\text{WD}}$ a lower limit (see Section 5.7). SN 2005df may have some central mixing and originate from a WD even closer to $M_{\text{Ch}}$. Note that MIR spectra allow for investigation of possible mixing and $M_{\text{Ch}}$ mass explosions because of the unblended Ni lines in that region during the first few months when the envelope changes from the optically thick to transparent continua (Telesco et al. 2015).

The methods we have developed for LW determination of central density and magnetic fields is applicable to a large range of SNe Ia. Specifically, SN 2014J has very similar light curve characteristics to SN 2005df, with $M_B = -19.19 \pm 0.10$ and $\Delta m_{15}(B) = 1.12$ (Marion et al. 2014), MIR spectra (Gerardy et al. 2007; Telesco et al. 2015), and $^{56}$Ni mass based on gamma-ray observations (Churazov et al. 2014; Diehl et al. 2014; Isen et al. 2014). These two SNe appear to be very similar in the bulk of the exterior regions of the explosion, but nonetheless, also show some differences and individual peculiarities.
Differential comparison of SN 2005df and SN 2014J should therefore put new constraints on the progenitor system and the explosion scenario. There is some evidence from gamma-ray observations that the very outer layers of SN 2014J show some peculiarities (Diehl et al. 2014). Additionally, Zheng et al. (2014) suggest that SN 2014J is peculiar in the very outer layers based on a broken power-law fit of early LC measurements and upper limits for non-detections. These outer layers would have masses corresponding to $\approx 10^{-2} M_\odot$ (Hoeftich 1995a) and are often modified by the environment, as evidenced by high-velocity Ca features. These outer layers cannot be expected to affect the central layers and because of lack of early LCs for SN 2005df, no direct comparison can be made.

Another difference is related to the inner layers discussed in this paper. From the late-time NIR line profiles we concluded that SN 2005df was close to $M_{\text{Ch}}$ but with a lower central density than expected for a normal SN Ia. This conclusion is supported by light curve analysis of secondary parameters as discussed in Section 4.2. Using light curve data from Milne et al. (2010) and Marion et al. (2014), the visual light curve of SN 2014J is dimmer than SN 2005df by about 0.2 mag about a month after maximum. Using our method of secondary parameters (Hoeftich et al. 2010; Sadler 2012; Hoeftich et al. 2013), this is consistent with a low initial central density, $\rho_c \approx 10^5 g cm^{-3}$, for SN 2005df and is further support against the complete mixing found in current three-dimensional hydrodynamical simulations.

Positron transport allows us to detect magnetic fields. Low $\rho_c$ implies high average densities in the $^{56}$Ni-poor region even without mixing. Thus positron transport effects are delayed compared to other SNe Ia, as discussed in Section 1. Therefore, $B$ effects on the evolution of the line profiles are delayed. We find some support for high initial magnetic fields of strength $10^6 G$ for SN 2005df, however, 0 G cannot be ruled out because of noise in the spectra combined with low $\rho_c$.

Another question relates to the origin of high magnetic fields, such as the ones used in some of our models, and has been discussed recently by Penney & Hoeftich (2014) and Hengeler (2014). Although some WDs have large magnetic fields, others do not show any sign of a measurable field. The production mechanism of the $B$ field, whether before or during accretion of material on the WD or during the thermonuclear runaway, will impact its morphology, the scale-size of the field, and any asymmetries produced (Ghezzi et al. 2004; Stone & Gardiner 2007a, 2007b; Remming & Khokhlov 2014). Suppression of R–T instabilities due to these magnetic fields may be a key factor and is currently being studied in our group.

With the observations of SN 2005df presented in this work, we are able to probe the central density of the WD but are just on the edge of being able to probe the influence magnetic fields have on the spectra. Our models are spherical and, consequently, suppress mixing entirely. Though this does not impair our conclusion that SN 2005df must be close to $M_{\text{Ch}}$, we may underestimate the central density. Close to $M_{\text{Ch}}$, the effect is expected to be small because of the strong variation with $M_{\text{WD}}$, and realistic simulations should be able to predict this effect. Three-dimensional magneto-hydrodynamic simulations in Stone & Gardiner (2007a, 2007b) affect and may suppress of R–T instabilities, but this does not exclude other mechanisms of suppression, and all may be realized in nature. Moreover, we need a statistical sample to examine the diversity of SN Ia and the variety of explosion scenarios. With forthcoming telescopes like the James Webb Space Telescope, the Wide-Field Infrared Survey Telescope, the Giant Magellan Telescope, and the Extremely Large Telescope, many more observations of SNe Ia in the nebular phase will be available at better signal-to-noise. Additionally, the available atomic data needed for modeling is increasing and improving. This type of analysis is crucial to understanding the progenitor systems and explosion scenarios, thereby increasing our ability to standardize and use SNe Ia.

Finally we also want to stress the limits of this analysis, which will be overcome in future. We argue that spherical models are a good approximation to the observation. However, our analysis should be regarded as a starting point and not the final answer, which requires progress on many fronts. Currently, MHD models are being constructed for SNe Ia (D. C. Collins & P. Hoeftich 2015, in preparation). Higher precision in the observations will allow us to probe secondary effects due to variations in the progenitor, the diversity of SNe Ia, and physical effects yet to be included in the current generation of simulations.

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REFERENCES
Barkat, Z., & Wheeler, J. C. 1990, ApJ, 355, 602
Baron, E., Hoeftich, P., Krisciunas, K., et al. 2012, ApJ, 753, 105
Beaudet, G., Petrosian, V., & Salpeter, E. E. 1967, ApJ, 150, 979
Benz, W., Cameron, A. G. W., Press, W. H., & Bowers, R. L. 1990, ApJ, 348, 647
Brachwitz, F., Dean, D. J., Hix, W. R., et al. 2000, ApJ, 536, 934
Branch, D. 1986, ApJL, 300, L51
Branch, D., Livio, M., Yungelson, L. R., Boffi, F. R., & Baron, E. 1995, PASP, 107, 1019
Bravo, E., & García-Senz, D. 2003, in From Twilight to Highlight: The Physics of Supernovae, ed. W. Hillebrandt, & B. Leibundgut (Berlin: Springer), 165
Bravo, E., & García-Senz, D. 2005, in Proc. of IAU Coll. 192, Cosmic Explosions, On the 10th Anniversary of SN1993F, ed. J. M. Marcaide, & K. W. Weiler (Berlin: Springer), 339
Brown, P. J., Roming, P. W. A., Milne, P., et al. 2010, ApJ, 721, 1608
Charalalov, E., Sunyaev, R., Isern, J., et al. 2014, Natur, 512, 406
Colella, P., & Woodward, P. R. 1984, JChPh, 54, 174
