Stochastic and nonlinear-based prognostic model

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The prevention of failure and the evaluation of products state are the most important aims for industrialists since nonpredicted failure is very expensive in most cases. These can be done mainly by the evaluation of the “Remaining Useful Lifetime” (RUL) using the prognostic approaches that compensate the inconveniences of classical maintenance strategies. A proposed analytic prognostic methodology based on nonlinear damage laws is developed here to determine the RUL of the system. It permits to ensure a high availability and productivity with less cost for industrial systems. To make this approach more reliable, it is essential to introduce the stochastic description. In the case of the fatigue effect where the damage state is growing from macro-cracks to total failure, \( D(N) \) expresses an increasing scalar damage function in terms of loading cycles \( N \). The RUL is estimated from a predefined threshold of damage \( D_C \). Pipelines tubes, subject to fatigue effects due to pressure–depression alternation, belong to vital mechanical systems in petrochemical industries that serve to transport natural gases or liquids. The prognostic evaluation of their states increases the tubes performance and the availability while minimizing their mission cost.

Keywords: prognostic; nonlinear damage; stochastic; fatigue; pipelines; offshore; lifetime

1. Introduction

The subject of cumulative fatigue damage is extremely complex, and various theories have been proposed (Byington, Roemer, Kacprzynski, & Galie, 2002) to predict the service fatigue life in advance. The crack is allowed to grow up to the point at which it becomes unstable, thereby determining the lifetime of the material under the prescribed stress program.

In our previous analytical model (Abou Jaoude, El-Tawil, Kadry, Noura, & Ouladsine, 2010), damages have been assumed to accumulate linearly using the famous damage law “Palmgren-Miner’s law” widely used in specialized literature for most steel materials even though it is unlikely to be the case of brittle material. The present paper intends to develop a more advanced prognostic tool by exploring the nonlinear side of cumulative damage. Its importance is not only because the case is not very well treated until now, but in addition, our intended stochastic study needs to consider this nonlinearity in cumulative damage.

Hence, a nonlinear cumulative damage is explored (Abou Jaoude & El-Tawil, 2013a) to take into account the nature, the level, and the mode of the applied constraints and influence environment that can accentuate the nonlinear aspect related to some materials behavior subject to fatigue effects.

At the same time, other reasons can disturb the prediction capacity of the prognostic model and which are the fluctuations of some basic parameters; in fact, these factors can be taken into account by adopting a stochastic model for these parameters.

In the present paper, a stochastic analysis is introduced in addition to the nonlinear damage model in order to make the “Remaining Useful Life” (RUL) prediction of industrial systems more accurate. It is done by considering some parameters as random variables. Our aim is to propose a general prognostic tool that can be capable of well predicting the RUL of a system based on both analytical linear and nonlinear damage accumulation in either the deterministic or stochastic context. Knowing that the RUL can be determined as fatigue in terms of various limit values like: the critical crack length \( c_C \), or the critical number of loading cycles \( N_C \), or the material tenacity \( K_{IC} \), from which we can write various limit states or performance criteria.

Figures 1 and 2 represent an example of linear and nonlinear damage accumulation laws (Hashin & Rotem, 1978; Lemaitre & Desmorat, 2005). Where \( n_1 \) and \( n_2 \) are the number of loading cycles, \( N_{R1} \) and \( N_{R2} \) are, respectively, the critical number of cycles for the loading levels \( \Delta \varepsilon_1 \) and \( \Delta \varepsilon_2 \), and \( t \) is the time of loading.

These two figures show the influence of the loading order between linear and nonlinear cases; in fact, when small loading \( \Delta \varepsilon_1 \) precedes high loading \( \Delta \varepsilon_2 \) (upper case) the linear rule (Palmgren–Miner) does not make the difference for this order, whereas the nonlinear rule permits...
to give a convex curve of damage (Figure 2) which can be modeled by a double linear damage rule (DLDR) (Figure 1). When high loading $\Delta \varepsilon_1$ precedes small loading $\Delta \varepsilon_2$ (lower case) also the linear rule is insensible to this order contrarily to the nonlinear rule, where it gives a concave curve of damage (Figure 2) modeled in some methods by a DLDR (Figure 1).

Finally, this paper will propose a new advanced prognostic tool capable to take into account the nonlinear aspect of damage accumulation and at the same time the stochastic effect of some basic parameters. These improvements permit to make a more accurate and realistic prognostic of industrial systems subject to fatigue damage. The proposed model is applied to evaluate the RUL of offshore pipelines system.

2. State-of-the-art: nonlinear damage accumulation

Fatigue damage increases with applied load cycles in a cumulative manner which may lead to a fracture. Cumulative fatigue damage analysis plays a key role in the life prediction of components and structures subject to field load histories. The most widely known and used procedure is the linear cumulative damage LCD rule commonly called the Miner rule. The linear damage rule, which indicates that a summation of cycle ratios is equal to unity, is not completely accurate; however, because of its simplicity and because of its agreement with experimental data for certain cases, it is frequently used in design. If a new method is to replace the linear damage rule in practical design, much of the simplicity of the linear damage rule must be retained. For example, the DLDR, retains much of this simplicity and at the same time attempts to overcome some of the limitations inherent in the conventional linear rule.

One of the limitations of the linear damage rule is that it does not consider the effect of the order of loading. For example, in a two-stress-level fatigue test in which a high load is followed by a low load, the cycle ratio summation is less than 1, whereas a low load followed by a high load produces a cycle ratio summation greater than 1.

The effect of residual stress is also not properly accounted for by the conventional linear damage rule, nor does it consider cycle ratios applied below the initial fatigue limit of the material (Byington, et al., 2002). Since prior loading can reduce the fatigue limit, cycle ratios of stresses applied below the initial fatigue limit should be accounted for (Byington, et al., 2002).

On the other hand, the concept of change in endurance limit due to pre-stress exerted an important influence on subsequent cumulative fatigue damage research. Kommers and Bennett (Christensen, 2007) further investigated the effect of fatigue prestressing on endurance properties, using a two-level step loading method. Their experimental results suggested that the reduction in endurance strength could be used as a damage measure, but they did not correlate this damage parameter to the life fraction. This type of damage models based on endurance limit reduction are nonlinear and able to account for the load sequence effect. Some of these models can also be used for predicting the instantaneous endurance limit of a material, if the loading history is known. None of these models, however, take into account the load interaction effects.

Past experiments have yielded a range of ratios from 0.7 to 2.2 for uniaxial loadings, resulting in failure predictions erring just slightly on the side of nonconservative to more than the double for a conservative prediction (Shigley & Mischke, 1989). For the biaxial loadings, a Miner’s summation of 0.19 was found in these experiments.
(Shigley & Mischke, 1989), indicating thus extremely non-conservative results as it is so far from the failure point (equal to 1.0). This proves the dependence of Miner’s law on load directions.

Various methods have been proposed as alternatives to the linear damage rule. None overcomes all the deficiencies, and many introduce additional complexities that either preclude or make their use extremely difficult in practical design problems.

However, LCD is widely acknowledged to be inadequate. This is partially on the basis of its empirical nature and partly upon its prediction of unsatisfactory results (Manson, Freche, & Ensign, 1966). Miner’s rule assumes that damage contribution from each cycle of the loading history is independent from the other cycles. Therefore, the damage inflicted by n stress cycles with defined magnitude

\[ S \] is given by: \[ D = \frac{n}{N}. \]  

(1)

For all stress levels, this damage rules yields (Miner, 1945):

\[ D = \sum_{i=1}^{m} d_i = \sum_{i=1}^{m} \frac{n_i}{N_i}, \]  

(2)

where N denotes the cycles to failure at S from the constant-amplitude S–N curve (Wöhler curve); ni is the number of cycles having amplitude Si.

In the LCD, the measure of damage is simply the cycle ratio with a basic assumption of constant work absorption per cycle, and characteristic amount of work absorbed at failure. The energy accumulation, therefore, leads to a linear summation of cycle ratio or damage.

The main deficiencies with LCD are its load-level (σi, where i = 1, 2, 3) independance, the load sequence independance and the lack of load-interaction accountability. However, due to the inherent deficiencies of the LCD, no matter which version is used, the life prediction is based on this rule is often unsatisfactory. Experimental evidence under completely reversed loading condition often indicates that \( \sum d_i > 1 \) for a low-to-high (L–H) loading sequence, and \( \sum d_i < 1 \) for a high-to-low (H–L) loading sequence. To remedy the deficiencies associated with the LCD, some authors like in reference (Christensen, 2007) introduced the concept of damage curves and speculated that these curves ought to be different at different stress levels (Figure 3).

Then, the first nonlinear load-dependent damage theory was proposed by Marco and Starkey (Christensen, 2007), it is represented by a power relationship \( D = \sum d_i^{\alpha_i} \) where \( \alpha_i \) is a variable quantity related to the ith loading level. The plots of these curves are shown in Figure 3. In this figure, a diagonal straight line represents the Miner rule which is a special case of the above Equation (2) with \( \alpha_i = 1 \). As illustrated by Figure 3, life calculations based on Marco–Starkey theory would result in \( \sum d_i > 1 \) for the L–H load sequence, and in \( \sum d_i < 1 \) for the H–L load sequence.

3. State-of-the-art: stochastic fatigue modeling

There is a significant interest in improving our understanding of fatigue related to damage and to the prediction of the useful residual life of components experiencing fatigue damage. One of the principal tools for modeling fatigue damage is linear elastic fracture mechanics, and the resulting models have facilitated the design of fatigue-resistant mechanical and aerospace structural components (Abou Jaoude & El-Tawil, 2013b). In fact, decision tools for failure prognostics must have the capability to incorporate material damage under both normal and peak operating conditions (Abou Jaoude & El-Tawil, 2013b; Hess, Calvello, Frith, Engel, & Hoitsma, 2005).

The science and technology of prognosis and structural health management offer the potential for significant enhancements in the safety, reliability and availability of high-value resources (Larsen & Christodoulou, 2004; Mohanty, Das, Chattopadhyay, Peralta, & Willhauck, 2008). This concept is based on a closed-loop process whose successful implementation depends on the integration of several multi-disciplinary elements including (Christodoulou & Larsen, 2004):

1. Onboard sensing of operational parameters and material damage states;
In the study on lives of turbine engines (Christodoulou & Larsen, 2004), enhancements were added to the DARWIN code to enable the type of analyses required for prognosis:

1. Establishment of interface with engine sensor data;
2. Adding of the fatigue crack initiation analysis to existing fatigue crack propagation analysis;
3. Incorporates the integration of crack initiation and propagation algorithms; including correlation effects between the two damage processes;
4. Adding a damage-based load filtering method to reduce computational time;
5. Capability to analyze a large number of inspections (or interrogation – up to once per flight cycle) to simulate continuous monitoring with an on-board sensor.

Although DARWIN contains several probabilistic solutions methods, the analyses in reference Christodoulou and Larsen (2004) were performed using Monte Carlo simulation.

Other models have been proposed to describe the random behavior of fatigue crack growth in metals. In Yang and Manning’s stochastic model (Enright et al., 2003; Yang & Manning, 1990), a simple second-order approximation of a deterministic crack growth model is used with a random component. An experimental study was conducted by Yang and Manning (2003) and Wu and Ni (2003) using this concept, which confirmed the practical applications of Yang and Manning’s model. Other applicable models based on discrete continuous random processes were proposed by Sobczyk and Spencer (Wu & Ni, 2004). Sobczyk and Spencer (1992) and Bogdanoff and Kozin (1985) explored the Markov chain theory and utilized it to create discrete and continuous fatigue crack growth models. In earlier studies, Lin proposed a Fokker–Planck equation that relates the continuous Markov process (Kozin & Bogdanoff, 1989). The Yang and Manning model is used in reference Zhu, Lin, and Lei (1992) to analyze the variable type loading because of its versatile functionality. This model utilizes only the crack growth rate and crack length data; the information about loading and material is not employed into the model and is accounted for in the random component and model parameters.

For instance, with transitional loading, the model parameters will vary as the fatigue damage propagates. The model parameter variability was taken into account in the data driven part of the analytical crack exceedance probability, which is the probability that the crack length will exceed a number of cycles, with the respective load period. To directly account for the variance in the crack growth rate, the random component is assumed to follow a lognormal distribution (Hong, Xing, & Wang, 1999; Willhauck, Mohanty, Chattopadhyay, & Peralta, 2008; Yang, Manning, Rudd, & Artley, 1987).
A significant part of main pipelines is subject to external cracking, which is a serious problem for the pipeline industry like, for example, in Russia (Moreno, Zapatero, & Dominguez, 2003), in USA, and in Canada (Sergeeva & Bolotov, 1996). Identification of external cracks is achieved using different Nondestructive Evaluation methods. If cracks are revealed during inspection, their influence on the remaining life (RUL) of the pipeline should be assessed in order to choose what maintenance action should be used: do nothing/repair/replace.

Furthermore, the system modeling considered by the physics-based prognosis is derived by using physics laws and principles. Crack initiation models must include all the available information about the component and its environment. The crack propagation models can be divided into two main groups: deterministic and stochastic. Deterministic crack propagation models, which usually describe the growth of the crack, are based on Paris’ law (Timashev, Malyukova, & Maltsev, 2005). Stochastic crack propagation involves models with random parameters which can be estimated using Monte Carlo simulations.

As a matter of fact, all previously mentioned parameters are affected by some probability of realization that influences the resulting RUL deduced from $D(N)$. The sampling of the basic parameters for a large number $j$ leads to $j$ curves $D_j(N)$ from which we can compute the mean curve $\bar{D}(N)$ and the standard deviation ($D(N)$). The stochastic nature of these curves derives from the stochastic aspect of the chosen influential parameters ($\alpha_0$, $\Delta\sigma$).

4. Nonlinear-damage-based prognostic
Various approaches to prognostics have been developed that range in fidelity from the simple historical failure rate models to high-fidelity physics-based models (Fatemi & Tangt, 1997). The required information (depending on the type of prognostics approach) include: engineering model and data, failure history, past operating conditions, current conditions, identified fault patterns, transitional failure trajectories, maintenance history, environment of equipment, system degradation and failure modes.

An earlier proposed prognostic procedure (Luo et al., 2003) belongs to the model driven approach. It is based on a physical model and leading to a normalized degradation indicator. It is focused on developing and implementing effective diagnostic and prognostic technologies with the ability to detect faults in the early stages of degradation. Early detection and accurate analysis may lead to better prediction and end of life estimates by tracking and modeling the degradation process.

The idea was to use these estimates to make accurate and precise prediction of the time to failure of components. The chosen failure mode was the fatigue failure formulated mathematically on the basis of analytic damage laws of Paris and Miner. The last law is an LCD model (Figure 1). These laws are very well known in the mechanics of fracture; thus, their use in the present prognostic procedure helps as a support for an example of a degradation expression.

Past research has shown that there is a nonlinear interaction effect between high cycle fatigue (HCF) and low cycle fatigue (LCF) in many engineering materials. This effect has been observed within uniaxial loadings, but is often more pronounced under multiaxial loading, particularly when the loading is nonproportional. An example here is the development of fatigue damage assessment methods for turbine engine materials combining the LCF and HCF cycles.

The nonlinear interaction effect precludes the use of the most common technique for linear damage accumulation. A thorough review of nonlinear cumulative damage (Figure 2) methodologies (Abou Jaoude et al., 2010) show that these techniques have included simple extensions of the linear damage rule to include nonlinear terms. Several nonlinear methods exist, including endurance-limit modification techniques, fracture-mechanics-based approaches, continuum-damage, and life-curve approaches. Traditional methods of damage summation have been shown to provide an inaccurate life prediction when multiple load levels are simultaneously considered. This is due to the effect that one load level has on the other(s).

In the present study, the effect of HCF loading has had a more detrimental effect when coupled with the LCF loadings than predicted by a linear summation rule. Nonlinear damage accumulation theories can account for this influence and have shown an improvement in prediction. The stress levels were chosen to correspond to levels previously tested to failure, resulting in fatigue lives ranging from approximately $10^5$ to $10^7$ cycles. A nonlinear damage summation is required to properly define the fatigue process since the linear summation of damage is often not adequate to predict the service life of a component when subjected to variable-amplitude loadings.

5. Nonlinear cumulative damage proposed model
The damage model proposed in this paper, whose evolution is up to the point of macro-crack initiation, is represented in Figure 4. The state of damage of a specimen at a particular

![Figure 4. Nonlinear law of damage.](image-url)
cycle $N$ during fatigue is represented by a scalar damage function $D(N)$. The magnitude $D_0 = 0$ corresponds to no damage, and $D_C = 1$ corresponds to the appearance of the first macro-crack (total damage).

The following model is chosen for the nonlinear prognostic study. It represents the nonlinear evolution of damage $D$ in terms of the number of cycle $N$ given under the following first-order nonlinear ordinary differential equations (Kulkarni, Sun, Moran, Krishnaswamy, & Achenbach, 2006):

$$\frac{dD}{dN} = \begin{cases} \frac{1}{N_C} \left( \frac{1 - \sigma_0}{\Delta\sigma/2} \right)^m \frac{1}{(1-D)^\alpha} & \text{if } \Delta\sigma/2 > \sigma_0, \\ 0 & \text{if } \Delta\sigma/2 < \sigma_0, \end{cases}$$

(3)

where, $N_C$ is the number of cycles at failure as a normalizing constant, $\Delta\sigma$ the stress range in a loading cycle, $\sigma_0$ the endurance limit, it is a function of the stress mean in a cycle: $\tilde{\sigma}$

$$\sigma_0(\tilde{\sigma}) = \sigma_0(0) \left( 1 - \frac{\tilde{\sigma}}{\sigma_{ult}} \right); \text{ where } \sigma_0 < \frac{\Delta\sigma}{2},$$

$\sigma_{ult}$: the ultimate tensile strength of the material, $m$ and $\alpha$: they are constants depending on the material and the loading condition ($m \approx 2.91$ and $\alpha \approx 2.23$). These constants are defined in reference Kulkarni et al. (2006) as consequences of experimental and empirical studies.

Although Equation (3) seems to be very complicated, it is very useful in the practical engineering field related to material damage. In fact, its origin derives from many experiments (Kulkarni et al., 2006) on the behavior of a material during the process of fatigue damage. The nonlinear final form of this equation is expected as it reflects the real behavior of materials under extreme loading and environmental conditions. These conditions are accentuated by the introduction of stochastic effects. The physical meaning of Equation (3) above can be explained by the fact that it is the increasing rate of damage $D(N)$ in terms of the number of loading cycles $N$. This rate increases nonlinearly in terms of $D(N)$ and the loading $\Delta\sigma$ to reach a critical value of damage considered as the failure point where the RUL is inferred. Hence, the resolution of this equation is too complicated.

This nonlinear ordinary differential Equation (3) needs to be solved in order to find an expression of $D(N)$.

6. Solution of the differential equation of degradation

The solution of the differential Equation (3) is presented as follows:

$$\frac{dD}{dN} = \begin{cases} \frac{1}{N_C} \left( \frac{1 - \sigma_0}{\Delta\sigma/2} \right)^m \frac{1}{(1-D)^\alpha} & \text{if } \Delta\sigma/2 > \sigma_0, \\ 0 & \text{if } \Delta\sigma/2 < \sigma_0, \end{cases}$$

$$\Rightarrow \frac{D(x)}{D_0} = \int_{D_0}^{D(x)} \frac{1}{N_C} \left( \frac{1 - \sigma_0}{\Delta\sigma/2} \right)^m \frac{1}{(1-D)^\alpha} dD = \int_{N_0}^{N} \left( \frac{1 - \sigma_0}{\Delta\sigma/2} \right)^m \frac{1}{(1-D)^\alpha} dN$$

$$\Rightarrow (1 - D(N))^{\alpha+1} = (1 - D_0)^{\alpha+1}$$

$$\Rightarrow D(N) = 1 - \left[ (1 - D_0)^{\alpha+1} - \frac{N - N_0}{N_C} \left( \frac{1 - \sigma_0}{\Delta\sigma/2} \right)^m \frac{1}{(1-D)^\alpha} \right]^{1/(\alpha+1)},$$

(4)

where, $D(N_0) = D_0$ is the damage at $N = N_0$ cycles corresponding to an initial crack length $d_0$.

We choose an equivalent damage parameter, to be measured by structural health monitoring. The plotting of expression (4) of $D(N)$ is presented in Figure 5.

Particular case:

Take $D_0 = 0$ for $N = N_0$,

$$\Rightarrow D(N) = 1 - \left[ 1 - \left( \frac{N - N_0}{N_C} \left( \frac{1 - \sigma_0}{\Delta\sigma/2} \right)^m \frac{1}{(1-D)^\alpha} \right)^{1/(\alpha+1)} \right]^{1/(\alpha+1)}; \text{ where } \tilde{\sigma} = \frac{\Delta\sigma}{2}.$$
Failure case:

At failure, we have \( N = N_C \) and \( D(N_C) = 1 \), then Equation (5) gives the following:

\[
1 = 1 - \left[ 1 - \left( \frac{N_C - N_0}{N_C} \right) \left( 1 - \frac{\sigma_0}{\bar{\sigma}} \right)^m \right]^{1/(\alpha+1)}
\]

\[
\Rightarrow \left( \frac{N_C - N_0}{N_C} \right) \left( 1 - \frac{\sigma_0}{\bar{\sigma}} \right)^m = \frac{1}{\alpha+1},
\]

Assume that \( N_0 = 0 \):

\[
\Rightarrow \left( 1 - \frac{\sigma_0}{\bar{\sigma}} \right)^m = \frac{1}{\alpha+1}
\]

\[
\Rightarrow 1 - \frac{\sigma_0}{\bar{\sigma}} = \frac{1}{\left( \alpha + 1 \right)^{1/m}}.
\]

Therefore, we have

\[
\sigma_0 = \bar{\sigma} \times \left( 1 - \frac{1}{\left( \alpha + 1 \right)^{1/m}} \right). \tag{6}
\]

7. Relation between \( D \) and \( N \) at a specific cycle \( N_1 \)

Let us study the relation between the degradation \( D \) and the cycle of stress \( N \). To do that easily, let us integrate the relation of degradation between cycle 1 and cycle 2 assuming that failure occurs at cycle 2.

From Equation (3), it can be deduced that

\[
(1 - D)^{\alpha} \, dD = \frac{1}{N_C} \left( 1 - \frac{\sigma_0}{\Delta \sigma/2} \right)^m \, dN
\]

\[
\Rightarrow \int_{D_1}^{D_{N+1}} (1 - D)^{\alpha} \, dD
\]

\[
= \int_{N_i}^{N} \frac{1}{N_C} \left( 1 - \frac{\sigma_0}{\Delta \sigma/2} \right)^m \, dN
\]

\[
\Rightarrow 1 - \frac{N_i}{N_C} = \left( 1 - \frac{\sigma_0}{\Delta \sigma/2} \right)^{-m} (1 - D_1)^{\alpha+1}
\]

\[
\Rightarrow \frac{N_i}{N_C} = 1 - \left[ 1 - \frac{\sigma_0}{\Delta \sigma/2} \right]^m (1 - D_1)^{\alpha+1}
\]

It can be inferred also

\[
\Rightarrow 1 - D_1 = \left[ (\alpha + 1) \left( 1 - \frac{N_i}{N_C} \right) \left( 1 - \frac{\sigma_0}{\Delta \sigma/2} \right) \right]^{1/(\alpha+1)}.
\]

Then:

\[
D_1(N_i) = 1 - \left[ (\alpha + 1) \left( 1 - \frac{N_i}{N_C} \right) \left( 1 - \frac{\sigma_0}{\Delta \sigma/2} \right) \right]^{1/(\alpha+1)}.
\tag{7}
\]

8. Recursive relation of nonlinear damage \( D \)

To construct a recursive relation for the sequence of \( D \), the procedure is as follows:

\[
(1 - D)^{\alpha} \, dD = \frac{1}{N_C} \left( 1 - \frac{\sigma_0}{\Delta \sigma/2} \right)^m \, dN
\]

\[
\Rightarrow \int_{D_n}^{D_{n+1}} (1 - D)^{\alpha} \, dD = \int_{N_n}^{N_{n+1}} \frac{1}{N_C} \left( 1 - \frac{\sigma_0}{\bar{\sigma}} \right)^m \, dN; \quad \text{where} \quad \bar{\sigma} = \frac{\Delta \sigma}{2}
\]

\[
\Rightarrow \left[ \frac{1}{\bar{\sigma}} \frac{\left( 1 - \frac{\sigma_0}{\bar{\sigma}} \right)^m}{N_C} \right]_{N_n}^{N_{n+1}}
\]

\[
(1 - D)^{\alpha+1} = \frac{1}{N_C} \left( 1 - \frac{\sigma_0}{\bar{\sigma}} \right)^m
\]

\[
\Rightarrow (1 - D_{N+1})^{\alpha+1}
\]

\[
= (1 - D_{N})^{\alpha+1} - \frac{(\alpha + 1)}{N_C} \left( 1 - \frac{\sigma_0}{\bar{\sigma}} \right)^m
\]

\[
\Rightarrow D_{N+1} = 1 - \left[ (1 - D_N)^{\alpha+1}
\]

\[
- \frac{(\alpha + 1)}{N_C} \left( 1 - \frac{\sigma_0}{\bar{\sigma}} \right)^m \right]^{1/(\alpha+1)}.
\tag{8}
\]

The previous recursive relation leads to a sequence of values \( D_N \) whose limit is \( D_C = 1 \):

\[
D_0, D_1, D_2, \ldots, D_N, D_{N+1}, \ldots, D_C = 1. \tag{9}
\]

And as the stress-load is expressed in terms of time \( t \), then we can plot the curve of degradation \( D \) in terms of time \( t \).

Therefore, our prognostic model in the nonlinear case is given by

\[
D_{N+1} = 1 - \left[ (1 - D_N)^{\alpha+1} - \frac{(\alpha + 1)}{N_C} \left( 1 - \frac{\sigma_0}{\bar{\sigma}} \right)^m \right]^{1/(\alpha+1)}.
\tag{9}
\]

9. Stochastic modeling

The stochastic modeling (Abou Jaoude, Noura, El-Tawil, Kadry, & Ouladsine, 2012; Lemaître & Chaboche, 1990) aims at considering some influent parameters as random variables and hence, the Paris’ law becomes a stochastic crack propagation law. The diagnostic data permit to consider the initial crack length \( \sigma_0 \) as the main random variable, where the second variable is the stress loading. Many other parameters can be also considered as random...
and the stochastic prognostic model can be expressed by the following general function:

\[ \tilde{D}(a) = P_{rob}(a) = \text{fct}(\tilde{a}_0, \text{loading } \tilde{\sigma}, \text{ thickness } \tilde{e}, \text{dimensions}, \tilde{C}, \tilde{m}, \ldots). \]

The degradation indicator \( D \) variant from 0 to 1 gives us instantaneously the RUL in terms of time, or cycle, or distance, depending on the type of the concerned device.

A probabilization of the basic parameters leads to a probabilistic trajectory \( \tilde{D}(a) \). Therefore, a bundle of curves \( D(a) \) is obtained for which a mean value and a standard deviation can be deduced. Hence, a characteristic curve can be computed in terms of a fractal \( a\% \) that depends on the level of the acceptable risk. The characteristic RUL is then deduced from \( D(a) \).

All previously mentioned basic parameters are affected by some probability of realization that influences the resulting RUL deduced from \( \tilde{D}(a) \). Contrary to the deterministic-based prognosis, the RULs concluded in stochastic-based prognosis are related to the probabilistic aspect.

These relevant basic parameters must be modeled stochastically using convenient well known probability distribution laws. For example, the initial crack length \( a_0 \) can be modeled by either a normal or a lognormal distributions, the loading \( \sigma \) is modeled by a normal distribution.

10. Stochastic RUL

The last parameters must be modeled stochastically using convenient probability distribution laws. When this is not taken into consideration, the prognostic results may not reflect really the evaluated lifetime of a device.

The estimated RUL is then no longer deterministic, but affected by some risk percentage in order to be realized. Hence a bundle of RULs trajectories can be plotted. Knowing that the RUL can be expressed by various forms like, for example, in fatigue by: crack length \( a_{IC} \), or critical number of cycles \( N_{IC} \), or material tenacity \( K_{IC} \) depending on the chosen limit states: service limit state \( (a \leq a_{IC}) \), or lifetime limit state \( (N \leq N_{IC}) \), or strength limit state \( (K \leq K_{IC}) \). The RUL adopted in this work is the lifetime limit state: \( N_{IC} - N \) which is expressed in terms of the number of loading cycles.

11. Reliability evaluation of damage state

Each of the limit states cited above is a function of random variables that makes them also random functions in their turn. For this reason, they occur with a certain probability.

The evaluation of these probabilities is the main goal of this section. This can be done by many reliability methods.

The term reliability is the probabilistic evaluation of a limit state performance on a domain of basic variables. In other words, it is obtained by the computation of the failure probability toward a criterion or a limit state.

The methodology is as follows:

1. Identify the limit states that govern the lifetime of the structure.
2. Identify the basic parameters intervening in these limit states.
3. Deduce their probability density functions (PDFs).
4. Compute the failure probability that quantifies the risk of non-satisfaction of these limit states.

Many types of methods exist: the Monte Carlo simulation, the approximate method FORM (first-order reliability method), and SORM (second-order reliability method).

The Monte Carlo simulation method is based on a large number of simulations, it is a time-consuming tool and we must use \( N \) simulations when we want to evaluate a probability of order of \( 10^{-(N+1)} \) (i.e. for a very small probability of failure, a huge simulation number is needed).

The approximate method FORM is an iterative procedure that allows calculating an index of reliability (denoted \( \beta \)). The index \( \beta \) is the distance between the origin and the limit state equation \( G(t) = 0 \) in a standard space. Once we have calculated \( \beta \) we can deduce the failure probability:

\[ P_{rob} = \Phi(-\beta). \]

In FORM approximation, the real limit state (usually non-linear) is replaced by its tangent plane at a specific point called the most probable failure point. This point is the closest point on the curve: \( G(t) = 0 \) from the origin.

The limit state \( G(t) \) divides the space into two regions:

- The first region where \( G(t) > 0 \), called safe region.
- The second region where \( G(t) \leq 0 \), called failure region.

Other methods aim to evaluate the probability of success of performance by means of the reconstruction of the system response PDF under an analytic form.

The limit states are the functions of performance or of satisfaction of some criteria. In our model, we are interested in the criteria of a lifetime; in the fatigue case, the serviceability limit state is usually used.

The serviceability limit state governs the crack length \( a(N) \) at cycle \( N \), in order to be under the allowable limit \( a_C \). This function is given by

\[ G = a_C - a(N) = a_C - a_N. \]

The probability of failure is

\[ P_{rob}(G \leq 0) = P_{rob}(a_N \geq a_C) = \int_{a_C}^{\infty} f_N(a_N) \, da_N. \quad (10) \]

The probability of success is

\[ P_{rob}(G > 0) = P_{rob}(a_N < a_C) = \int_{-\infty}^{a_C} f_N(a_N) \, da_N, \quad (11) \]

With: \( a_C = \Delta K^2 / (Y^2(a) \cdot \pi \cdot \sigma_{\text{max}}^2) \).
12. Stochastic basic parameters

12.1. The initial crack width \(a_0\)

The measurements of the initial crack length \(a_0\) derived from sensors output are treated as realizations of a random variable \(\tilde{a}_0\). Based on previous studies (Wu & Ni, 2003; Yang & Manning, 1990, 2003), we consider a PDF for \(a_0\) that follows a lognormal distribution (Figure 6),

\[
\text{Figure 6. PDF of the crack length.}
\]

\[
f_0(a_0) = \frac{1}{a_0 \cdot \xi \cdot \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\ln(a_0) - \lambda}{\xi} \right)^2 \right], \tag{12}
\]

with \(\xi\) is the standard deviation of the variable \(\ln(a_0)\) which is the equivalent normal distribution, \(\lambda\) is the mean of the variable \(\ln(a_0)\), expectation of \(a_0\):

\[
\text{E}(a_0) = \exp \left( \lambda + \frac{\xi^2}{2} \right),
\]

variance of \(a_0\):

\[
\text{V}(a_0) = \exp \left( 2\lambda + \xi^2 \right) \left( \exp(\xi^2) - 1 \right).
\]

Inversely, we have also the following:

\[
\lambda = \ln\left( \text{E}(a_0) \right) - \frac{1}{2} \ln \left( 1 + \frac{\text{V}(a_0)}{\left( \text{E}(a_0) \right)^2} \right), \tag{13}
\]

\[
\xi^2 = \ln \left( 1 + \frac{\text{V}(a_0)}{\left( \text{E}(a_0) \right)^2} \right) \Rightarrow \xi = \sqrt{\ln \left( 1 + \frac{\text{V}(a_0)}{\left( \text{E}(a_0) \right)^2} \right)}.
\]

The allowable value of the crack length \((a_C)\) is fixed when the number of cycles reaches the critical value \((N_C)\) (Figure 7).

The probability of fatigue failure is given by

\[
\text{P}_\text{rob}(a_N > a_C) = \int_{a_C}^{\infty} f_N(a_N) \, da_N,
\]

where \(f_N(a_N)\) is the PDF of the crack width \(a_N\) at cycle \(N\).

It can be assumed that \(a_C = e/8\) (Miner, 1945), where \(e\) is the device dimension in the crack direction (Figure 8).

12.2. PDF of the initial damage \(D_0\)

We have the relation between the initial crack length \(a_0\) and the initial damage \(D_0\) as follows:

\[
D_0 = \frac{a_0}{a_C - a_0} \Rightarrow a_0 = \frac{a_CD_0}{1 + D_0}
\]

\[
\text{Figure 7. Pre-crack fatigue damage.}
\]

\[
\text{Figure 8. Probabilistic crack growth.}
\]
The probabilistic transformation theory gives
\[ f(D_0) = f(a_0) \times \left| \frac{d a_0}{d D_0} \right|. \]

As:
\[ \frac{d a_0}{d D_0} = \frac{a_c}{(1 + D_0)^2} \geq 0 \Rightarrow f(D_0) = f(a_0) \times \frac{a_c}{(1 + D_0)^2}. \]

If the proposed law for \( a_0 \) is lognormal, then the law of \( D_0 \) is also lognormal with the following PDF function:
\[ f(D_0) = \frac{1}{a_0 \cdot \xi \cdot \sqrt{2\pi}} \exp \left[ -\frac{1}{2 \cdot \xi^2} (\ln(a_0) - \lambda)^2 \right] \times \frac{a_c}{(1 + D_0)^2}. \]

As \( a_0 = a_c D_0/(1 + D_0) \) and \( a_c = e/8 \),

Then we can write the PDF as follows:
\[ f(D_0) = \frac{1}{\xi \sqrt{2\pi}} \exp \left[ -\frac{1}{2 \cdot \xi^2} \left( \ln \left( \frac{e \cdot D_0}{8(1 + D_0)} \right) - \lambda \right)^2 \right] \times \frac{1}{D_0(1 + D_0)}. \] (15)

After that we have determined the PDF of \( a_N \) which is \( f_N(a_N) \), we can calculate the probability of failure by the following serviceability criterion: \( a_C < a_N \).

13. **Stochastic nonlinear cumulative damage**

The nonlinear cumulative damage demonstrated earlier is given at each cycle \( N \) by the following:
\[ D(N) = 1 \left[ (1 - D_0)^{\alpha + 1} - \frac{N - N_0}{N_C} \right] \left( 1 - \frac{\sigma_0}{\Delta \sigma / 2} \right)^m (\alpha + 1)^{1/(\alpha + 1)}. \] (16)

The growth of \( D(N) \) at the end of each cycle \( N \) in terms of the crack width \( a(N) \) is given by the following relation:
\[ D(N) = \frac{a(N)}{a_C - a_0}, \] (17)

where, \( \Delta \sigma / 2 \) is the stress amplitude in one cycle, this parameter is generated as an input load whose mean is taken to be equal to 280 MPa, \( \sigma_0 \) is the fatigue limit (is the endurance limit stress of material) taken to be equal to 180 MPa.

Here two random variables are considered (loading and initial crack width \( \tilde{a}_0 \)).

The stochastic nonlinear prognostic model can be written as follows:
\[ \tilde{D}(N) = 1 - \left[ (1 - \tilde{D}_0)^{\alpha + 1} - \frac{N - N_0}{N_C} \right] \left( 1 - \frac{\sigma_0}{\Delta \sigma / 2} \right)^m (\alpha + 1)^{1/(\alpha + 1)}. \]

\[ \tilde{D}_0 = \frac{\tilde{a}_0}{a_C - \tilde{a}_0}. \] (18)

In the following sections, we will apply the proposed prognostic model (Equations (17) and (18)) to industrial systems like petrochemical pipelines.

The life prognostic of petrochemical pipelines is vital in their domain since their availability has crucial consequences. Fatigue failure is their main failure cause due to internal pressure–depression variation along time. Usually, three situations for these pipes exist: unburied, buried and under sea water (offshore pipes). Each one of these situations requires different physical parameters like: corrosion, soil pressure and friction, water and atmosphere pressure. In this paper, we will study only the offshore pipes.

14. **Application to the pipeline systems**

We study the prognostic of the pipeline system by taking into account the nonlinear damage law for the stochastic case of variables (Abou Jaoude et al., 2012). The study is done for two random variables: the internal pressure \( P \) and the initial crack length \( a_0 \).

The pipes are cylindrical thin tubes since their thickness \( e \) to radius ratio is \( (\text{Miner}, 1945): e/R \leq 1/10 \).

In this case, the stresses due to internal pressure \( P \) are of the membrane type without any bending forces except near the junctions and points of load application. The stresses are circumferential (hoop stress) \( \sigma_\theta \) and longitudinal (axial stress) \( \sigma_L \) (Figure 9). They are given by the following:
\[ \sigma_\theta = \frac{P \cdot R}{e}; \quad \sigma_L = \frac{P \cdot R}{2 \cdot e}. \] (19)

The critical position of cracks is longitudinal which is perpendicular to the direction of maximal stresses \( \sigma_\theta \). The crack has a depth (or length) \( a \) measured in the thickness direction (Figure 10). Generally, the following ratio interval can be considered: \( 0.1 \leq a/e \leq 0.99 \).

Three maximal levels of internal pressure \( P \) are considered (Table 1) with a repetition period \( T_p \). At each of these
Figure 10. Cracked pipe section.

Table 1. The three pressure levels.

| Pressure mode   | $P$ (MPa) |
|-----------------|-----------|
| High (mode 1)   | 8         |
| Middle (mode 2) | 5         |
| Low (mode 3)    | 3         |

levels, a degradation trajectory $D(N)$ is deduced in terms of cycle number $N$. When $D(N)$ reaches the unit value, then the corresponding $N$ is the lifetime of the pipe that failed by fatigue.

We simulate three modes of $P$ with the statistical parameters given in Table 1.

15. Equation of the stochastic-based prognostic

In the case of pipes of thickness $e$, the stress ranges are created by the applied internal pressure; hence, the following relation gives the critical hoop stress range $\Delta \sigma_\theta$ in terms of the pressure range $\Delta P$ (Figure 11):

$$\Delta \sigma_\theta = 2 \cdot \Delta \sigma_L = \frac{\Delta P \cdot R}{e}. \quad (20)$$

The simulation of the internal pressure following a triangular law $\tilde{P}$ (Figure 11) generates a sample of stress ranges $\tilde{\sigma}$ following the same triangular law in the equation below. From the following equation:

$$d \tilde{D}_N \equiv \frac{C}{a_C - \tilde{a}_0} (Y(\tilde{a}_N) \cdot \sqrt{\pi \tilde{a}_N} \cdot \Delta \tilde{\sigma})^m. \quad (21)$$

It can be deduced that:

$$d \tilde{D}_N = \frac{C}{(e/8 - \tilde{a}_0)} \left(0.6 \times \frac{1 + 2(\tilde{a}_N/e)}{(1 - \tilde{a}_N/e)^{3/2}} \times \frac{\tilde{P}_j \cdot R}{\tilde{e}}\right)^m, \quad (22)$$

where,

$$Y(\tilde{a}) = 0.6 \times \frac{1 + 2(\tilde{a}/e)}{(1 - \tilde{a}/e)^{3/2}}$$

is the geometric function of the pipes.

16. Generation of internal pressure $P$

The internal pressure $P$ is simulated by the Monte-Carlo method using a triangular distribution over one initial period of pressure $T_P$. The triangular law of the internal pressure is given by the following functions (Figure 12):

The PDF function of $P$:

$$f_P(p) = \begin{cases} 
2(P - a)/(b - a)(c - a) & a \leq P < c, \\
2(b - P)/(b - a)(b - c) & c \leq P \leq b, \\
0 & P < a \text{ and } P > b.
\end{cases} \quad (23)$$

The cumulative density function (CDF) of $P$:

$$F_P(p) = \begin{cases} 
0 & P < a, \\
(P - a)^2/(b - a)(c - a) & a \leq P \leq c, \\
1 - (b - P)^2/(b - a)(b - c) & c < P < b, \\
1 & P \geq b.
\end{cases} \quad (24)$$

The inverse of the CDF function gives a realization $P_j$ for $P$ as follows:

$$P_j = F_P^{-1}(u_j) = \begin{cases} 
a + \sqrt{u_j(b - a)(c - a)} & 0 \leq u_j \leq \theta, \\
b - \sqrt{(1 - u_j)(b - a)(b - c)} & \theta \leq u_j \leq 1.
\end{cases} \quad (25)$$

Figure 12. Triangular PDF function of $P$. 

Figure 11. Triangular pressure law.
where, \( u_j \) is the uniform-based generated value in the interval \([0,1]\),

\[
\theta = \frac{c - a}{b - a}.
\]

The mean value: \( \bar{P} = \frac{(1 - \theta^3)}{6(1 - \theta)} \).

The variance:

\[
V(P) = \frac{1 - \theta \times (1 - \theta)}{18} = \left(\frac{b - a}{18}\right)^2 \times \left\{1 - \frac{(c - a)(b - c)}{(b - a)^2}\right\}.
\]

Here, the simulation of the internal pressure is completed along one period \( T_p \) under a triangular law distribution of mean value \( \bar{P} \):

\[
\bar{P} = \frac{(1 - \theta^3)}{6(1 - \theta)} = \frac{(b - a)^3 - (c - a)^3}{6(b - a)^2(b - c)}.
\]

For the same initial period \( T_p \), each simulation gives a different realization of the PDF; thus, a new value for \( c = P \) is given, keeping always \( a = 0 \) and \( b = T_p \).

We consider the following values for the simulation (Figure 13): \( a = 0 \); \( b = T_p \) (pressure interval); and \( c = P \) (pressure value). Where the period \( T_p \) is a pressure interval that can be taken as a percentage of the maximal pressure.

17. Two random variables (pressure and initial crack length)

Two random variables are considered: the pressure and the initial crack length. We execute a triangular simulation of internal pressure \( P \) for the three modes: high, middle, and low (Table 1). The initial crack length is simulated as a lognormal distribution using the following parameters:

\( \tilde{a}_0 \) : Lognormal Law

\[
\begin{align*}
E(\tilde{a}_0) &= 0.2 \text{ mm}, \\
\sigma(\tilde{a}_0) &= \sqrt{V(\tilde{a}_0)} = 0.002945 \text{ mm}. 
\end{align*}
\]

The equivalent normal parameters for \( a_0 \) are inferred as follows:

\[
\begin{align*}
\lambda &= \ln[E(a_0)] - \frac{1}{2} \ln\left(1 + \frac{V(a_0)}{E(a_0)^2}\right) \\
&= \ln(0.2) - 0.5 \ln\left(1 + \frac{8.673 \times 10^{-6}}{0.04}\right) \\
&= -1.6094 - 0.5 \times 2.168 \times 10^{-4} = -1.6095 \text{ mm} \\
\xi^2 &= \ln\left(1 + \frac{V(a_0)}{E(a_0)^2}\right) \\
&= \sqrt{\ln\left(1 + \frac{8.673 \times 10^{-6}}{0.04}\right)} \\
&= \sqrt{2.168 \times 10^{-4}} = 0.014724 \text{ mm}.
\end{align*}
\]

The initial damage \( D_0 \) is deduced from \( a_0 \) as follows:

\[
D_0 = \frac{a_0}{a_C - a_0} \Rightarrow \tilde{D}_0 = \frac{\tilde{a}_0}{a_C - \tilde{a}_0}
\]

\[
\Rightarrow \tilde{D}_0 : \quad \text{Lognormal Law} \quad \begin{cases} 
E(\tilde{D}_0) = 0.008 \\
\sigma(\tilde{D}_0) = \sqrt{V(\tilde{D}_0)} = 3.784 \times 10^{-4}
\end{cases}
\]

The nonlinear cumulative damage, previously demonstrated, is given at each cycle \( N \) by the following:

\[
\tilde{D}(N) = 1 - \left[\left(1 - \tilde{D}_0\right)^{\alpha + 1} - \frac{N - N_0}{N_C}\right] \left(1 - \frac{\sigma_0}{\Delta \tilde{a}_j/2}\right)^{\omega} (\alpha + 1) \right]^{1/(\alpha + 1)}
\]

\[
\tilde{D}_0 = \frac{\tilde{a}_0}{a_C - \tilde{a}_0}.
\]

18. Offshore pipe case

In this case, the situation where the pipes are under sea water (offshore pipeline) serving to transport oil or gas from the marine offshore to the refinery plant is considered (Anderson, 2005; Bai & Bai, 2005; Palmer & King). They are subject, beside internal gas pressure, to external water and atmospheric pressure (Figures 14 and 15).

Consider a pipe (Figure 16) of diameter \( \phi = 480 \text{ mm} \) and of thickness \( e = 8 \text{ mm} \), the external pressure around the offshore pipe is given by the following:

\[
P_{\text{ext}} = P_w + P_{\text{atm}} = \rho_w \cdot g \cdot H + 1 \text{ atm}
\]

\[
\Rightarrow P_{\text{ext}} = 6,163,905 \text{ Pa} = 6.163905 \text{ MPa},
\]
where, the depth of offshore pipe considered here is: $H = 600$ m; atmosphere pressure at sea level $= 1$ atm $= 0.101325$ MPa; the specific weight of seawater is: $\rho_w = 1,030$ kg/m$^3$; the gravitational attraction is: $g = 9.81$ m/s$^2$.

Then, the net maximal stresses in the pipe body are given by Equation (26):

$$\sigma_\theta = \left( P_{int} - P_{ext} \right) \cdot R / e.$$  

The external pressure is here a nonfavorable condition to the crack growth.

19. The simulation of the degradation

Two random variables are considered: the pressure $P$ and the initial crack length $a_0$. We execute a triangular simulation of internal pressure $P$ ($\bar{P}(P); \bar{V}(P)$) for the three modes of pressure $P$: High, Middle, and Low (Table 1). The initial crack length is simulated as a lognormal distribution $a_0$ (Mean $= 0.2$ mm; Standard deviation $= 0.002945$ mm). The simulation of the prognostic Equation (18) permits to draw the degradation trajectory for each level of pressure: high (red), middle (blue), and low (green).
The lifetimes for offshore pipes (Figure 17) show that is nearly 9.25 years for mode 1 (high pressure), 16.41 years for mode 2 (middle pressure), and 28.72 years for mode 3 (low pressure). The degradation evolutions are steeper for the two first modes than the third mode.

The stochastic influence can be seen through the variability over the curve realizations of $D(t)$ obtained by many simulations and not from just one realization. Contrarily to the case of one random variable, the curves are not smooth and the stochastic effects are clearer here.

To develop more these results, a mean curve $\bar{D}(t)$ can be plotted from the mean value of these realizations. The conservative curves are those that give the maximum values. For each mode, a characteristic curve of lifetime can be computed from the mean values, the standard deviation values, and a certain fractal percentage depending on the risk adopted by decision-makers.

20. **Comparison: deterministic – stochastic results (nonlinear damage law)**

To show the stochastic effects, a comparison is done between the deterministic results and the stochastic results (Figure 18).

| Mode   | Deterministic nonlinear | Stochastic nonlinear | Decrease (%) |
|--------|-------------------------|----------------------|--------------|
| Mode 1 | 10.92 years             | 9.25 years           | 15.3%        |
| Mode 2 | 19.11 years             | 16.41 years          | 14.1%        |
| Mode 3 | 33.67 years             | 28.72 years          | 14.7%        |

For all modes of internal pressure, the lifetimes of pipes decrease about 15% from the deterministic case to the stochastic case. These reductions in lifetime are explained by the fact that the dispersions introduced by the random variables influence the prediction of lifetimes in a way that the prediction becomes more realistic and accurate. The stochastic effect is more pronounced and effective for two random variables than for one random variable. The curves for each mode fluctuate and they constitute a bundle of trajectories which are the realizations of many simulations.

21. **Validation of the pipelines lifetimes in stochastic conditions**

The obtained lifetimes values for linear and nonlinear damage rules in stochastic conditions can be verified to be in the range of real lifetimes according to the references (Guan, Jha, & Liu, 2010; Palmer-Jones & Turner, 1998; Xiang & Liu, 2010). In fact, a fatigue life of pipes under good exploitation conditions was found to be 26 years on average which is very close to the results obtained for pipes in mode 3 in the stochastic nonlinear case. Consequently, this proves that our stochastic prognostic model is more accurate and realistic than the deterministic one.
22. Conclusion and perspectives

In this paper, the prognostic model is developed to consider the prognostic study in stochastic conditions. Hence, the model is a general one as it is based on the nonlinear accumulation of damage due to fatigue crack propagation in stochastic situations. These last conditions are taken into account by considering two random variables which are the applied loading and the initial crack length which are considered as the most influential random variables on the degradation function.

The fatigue failure is considered and the damage state of the device is measured by a degradation indicator in terms of the number of loading cycles starting from an initial damage. The lifetimes are concluded from the time reading at each instant on the degradation curve. The RULs at each instant are deduced from the degradation curve by subtracting the current instant from the last predicted instant.

To show the efficiency of this stochastic prognostic model, it is applied to predict the fatigue life of petrochemical pipelines under three modes of internal pressure. Lifetimes results are obtained for nonlinear stochastic cases.

The stochastic results show that the stochastic effects are influential and the curves of degradation are fluctuant and constituted of bundles of trajectories. In this case, the lifetimes are reduced due to the dispersion effects.

As future directions and perspective works, more probabilistic basic variables like the material and the environmental parameters can be considered (tenacity, endurance limit, etc.). Furthermore, additional probabilistic laws for the parameters other than the Normal and the Log-normal laws can be explored. In addition, it is interesting to study other levels of internal pressure and other periods of pressure–depression. Also, it is planned to more explore the variability of the stochastic lifetimes and to deduce a bundle of degradation curves from which a mean curve, a standard deviation curve, and a characteristic lifetime curve can be inferred. The characteristic curve is the one attached to some predefined acceptable risk from which reliability studies can be treated. Moreover, other applications are planned like the buried and unburied pipelines (onshore).

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