Probabilistic Optimal Power Flow for Day-Ahead Dispatching of Power Systems with High-Proportion Renewable Power Sources

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Received: 1 December 2019; Accepted: 8 January 2020; Published: 9 January 2020

Abstract: With the increasing proportion of uncertain power sources in the power grid; such as wind and solar power sources; the probabilistic optimal power flow (POPF) is more suitable for the steady state analysis (SSA) of power systems with high proportions of renewable power sources (PSHRPSs). Moreover; PSHPRPSs have large uncertain power generation prediction error in day-ahead dispatching; which is accommodated by real-time dispatching and automatic generation control (AGC). In summary; this paper proposes a once-iterative probabilistic optimal power flow (OIPOPF) method for the SSA of day-ahead dispatching in PSHPRPSs. To verify the feasibility of the OIPOPF model and its solution algorithm; the OIPOPF was applied to a modified Institute of Electrical and Electronic Engineers (IEEE) 39-bus test system and modified IEEE 300-bus test system. Based on a comparison between the simulation results of the OIPOPF and AC power flow models; the OIPOPF model was found to ensure the accuracy of the power flow results and simplify the power flow model. The OIPOPF was solved using the point estimate method based on Gram–Charlier expansion; and the numerical characteristics of the line power were obtained. Compared with the simulation results of the Monte Carlo method; the point estimation method based on Gram–Charlier expansion can accurately solve the proposed OIPOPF model.

Keywords: renewable power sources; PSHPRPSs; day-ahead dispatching; OIPOPF; steady state analysis; point estimation; Gram–Charlier expansion; Monte Carlo method

1. Introduction

The steady state analysis (SSA) of power systems is the most fundamental method for the investigation of power system planning and operations. In particular; a calculation method for the steady state operation of power systems is based on power flow. The injection power and voltage of traditional power flow calculation are both determinate quantities; and the bus voltage and line power obtained are also determinate quantities [1,2]. With an increase in proportion of uncertain power sources such as wind and solar power generation in the power grid; there is a corresponding increase in the number of indeterminate variables in the power grid. A grid with a high proportion of uncertain power sources is referred to as a power system with high proportions of renewable power sources. Due to the indeterminate variables; in accordance with the law of probability and statistics,
the probabilistic optimal power flow (POPF) model is suitable for the SSA of power systems with high proportions of renewable power sources (PSHRPSs) [3–7].

After the proposal of the POPF, significant research attention was directed toward the development of the model and calculation method [8–14]. The POPF model mainly includes the direct-current (DC) POPF model [3], the linear alternating-current (AC) POPF model [8], the piecewise linear AC POPF model [9], and the nonlinear AC POPF model [13]. The calculation methods of POPF models include cumulant, point estimation method, and Monte Carlo simulation method. Moreover, a method of combined cumulants and the Gram–Charlier expansion was proposed. For the DC power flow model, the relationship between the target and control variables can be directly obtained, and the operation speed can be improved [15]. The point estimation method was proposed to solve the AC probabilistic power flow model [16]. Literature [17] applies the two-point estimation method to account for the uncertainty in the optimal power flow problem in the context of the competitive power market. Although the Monte Carlo method demonstrates a high accuracy, it is typically employed as a verification method for other probabilistic power flow algorithms, due to the significant computational load required [18]. Literature [19] proposes a new possibilistic probabilistic power flow that exploits artificial neural networks to model the mathematical relationship between input and output variables and solve the non-linear equation set of power flow problems. Literature [20] proposes a novel method that utilizes radial basis functions (RBF) neural networks in order to find the input/output relationship of power-flow nonlinear equation sets for the microgrids, which does not depend on the partial derivatives of equations and Jacobian matrix (JM) inversions. Literature [21] proposes a novel method suitable for calculating the ill-conditioned network power flow, which utilizes neural networks to find the relationship between inputs and outputs of load-flow equations. Furthermore, the POPF is highly applicable to the optimization of grid dispatching control systems and the reduction of the generation cost or line loss [22–24]. In several studies, the use of the cumulants method for the solution of the POPF was proposed [25–28]. Uncertain renewable energy sources such as solar power and wind power increase the uncertain variables in the grid. For grid research with wind or solar power, probabilistic power flow models are commonly used [29–31]. Literature [32] reviews the latest research progress in design, planning, and control problems in the field of renewable and sustainable energy, and concludes that heuristic approaches, Pareto-based multi-objective optimization, and parallel processing are promising research areas. Literature [33] presents the model for finding an optimal joint bid of hydroelectric systems and wind parks on a day-ahead electricity market, which can be combined with hydropower system to make day-ahead plan for the power grid with wind power. In the PSHPRPSs, the power prediction error of uncertain power sources is significantly high. The POPF model of PSHPRPSs should consider the actual situation of significantly high errors in the power generation prediction of uncertain power sources.

The maintenance of power balance is the main objective of grid dispatching control. Because the PSHPRPSs contains a large number of uncertain power sources, only relying on automatic generation control (AGC) cannot balance the power forecast error of day-ahead dispatching. For the safe and steady operation of the power grid, the addition of real-time dispatching links between the day-ahead dispatching and AGC is required, to accommodate uncertain power sources and limit the errors within the range of the AGC [34–38]. Due to the large day-ahead power prediction error of PSHPRPSs, the day-ahead generation plan is inaccurate, and the day-ahead active power prediction error is accommodated by the real-time dispatching and AGC. The reason for the power forecast error of PSHPRPSs is the inherent uncertainty of uncertain renewable energy sources, not the power flow model. Therefore, the use of AC power flow model for day-ahead dispatching of PSHPRPSs cannot improve the calculation accuracy but also complicates the power flow models. However, the probabilistic DC power flow model will lead to large calculation errors. On this basis, in this paper, the once-iterative probabilistic optimal power flow (OIPOPF) model is proposed as the POPF model for the day-ahead dispatching SSA of PSHPRPSs. The power flow model proposed in this paper not only retains the same calculation accuracy as AC power flow, but also has the same operation efficiency as the DC
power flow model. The model was solved in this study using the point estimate method based on the Gram–Charlier expansion, and the numerical characteristics of the target variables were obtained. The OIPOPF mathematical model and its solution algorithm were validated on the modified Institute of Electrical and Electronic Engineers (IEEE) 39-bus test system and modified IEEE 300-bus test system. Moreover, the simulation results verify the accuracy of the model and the effectiveness of the method.

The remainder of this paper is structured as follows. Section 2 presents the mathematical model of OIPOPF and details the solution of the proposed OIPOPF using the point estimation method based on the Gram–Charlier expansion. Section 3 presents the simulation results for the OIPOPF, as applied to the IEEE 39-bus test system, to verify the accuracy of the OIPOPF mathematical model. Thereafter, the conclusions of this study are presented in Section 4.

2. Materials and Methods

2.1. POPF Mathematical Model

Renewable energy sources such as wind power and solar power have uncertainties based on their randomness and intermittent nature. For the PSHPRPSs whose bus variables obey probability and statistics law, it is suitable to use the POPF for grid SSA. In this paper, an OIPOPF mathematical model is used to formulate a day-head dispatching plan for the PSHPRPSs.

For a PSHPRPS with \( n \) buses, the bus active power difference, \( \Delta P \), is defined as expressed by Equation (1):

\[
\Delta P = P_S - P_0^e, \quad (1)
\]

where \( \Delta P \) is a \( 1 \times n - 1 \) dimensional vector; \( P_S \) is the bus active power; and \( P_0^e \) is the initial calculated value of the bus active power. \( P_S = (P_{S,1}, P_{S,2}, \ldots, P_{S,i}, \ldots, P_{S,n}) \), \( P_{S,i} \) is the \( i \)th variable of \( P_S \) vector. If \( P_{S,i} \) is an uncertain renewable energy source, its mean value is \( \mu_{S,i} \), and its standard deviation is \( \sigma_{S,i} \). Moreover, \( P_0^e \) is calculated using AC power flow equations, which can reduce the error in the calculation of \( \Delta P \).

The voltage phase angle error calculated by the modified active power error is more accurate. The grid bus active power difference, \( \Delta P \), and the bus voltage phase angle difference, \( \Delta \theta \), satisfy the functional relationship expressed by Equation (2):

\[
\Delta P = B \cdot \Delta \theta, \quad (2)
\]

where \( \Delta \theta \) is a \( 1 \times n - 1 \) dimensional vector. The bus voltage phase angle difference \( \Delta \theta \) is defined as expressed by Equation (3):

\[
\Delta \theta = \theta_S - \theta_e, \quad (3)
\]

where \( \theta_S \) is the bus voltage phase angle, and \( \theta_e \) is the calculated value of bus voltage phase angle.

In Equation (2), the coefficient matrix, \( B \), is an \( n - 1 \times n - 1 \) dimensional matrix with elements as expressed by Equation (4):

\[
\begin{align*}
B_{ij} &= -\frac{1}{x_{ij}} \\
B_{ii} &= \sum_{j \in i} \frac{1}{x_{ij}} \quad j \neq i
\end{align*} \quad (4)
\]

Given that \( B \) is an invertible matrix, Equation (2) is transformed into Equation (5):

\[
\Delta \theta_I = E \cdot \Delta P_I. \quad (5)
\]

In Equation (5), \( E = B^{-1} \), and \( E \) is a \( n - 1 \times n - 1 \) dimension matrix.

For any branch, \( l \), the from-bus is the \( i \) bus, and the to-bus is the \( j \) bus. The corresponding voltage phase angle of bus \( i \) is \( \theta_i \), and that of bus \( j \) is \( \theta_j \). Moreover, \( \theta_i \) and \( \theta_j \) are the \( i \)th and \( j \)th elements in the
The active power of the line between bus $i$ and bus $j$ is defined as expressed by Equation (6):

$$P_{ij} = V_i^2 y_{i0} + V_i^2 G_{ij} - V_i V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}), \quad (6)$$

where $B_{ij}$ represents the reactance between bus $i$ and bus $j$; $G_{ij}$ represents the conductance between bus $i$ and bus $j$; and $y_{i0}$ represents the admittance between bus $i$ and the zero-potential bus.

For a grid with $L$ lines, the line power is $P_L = (p_{l1}, p_{l2}, \cdots, p_{lj}, \cdots, p_{L})^T$. The day-ahead dispatching of PSHPFRPs considers the distribution of active power in the power grid. Therefore, according to Equation (6), the line power of the OIPOPF is defined as expressed by Equation (7):

$$P_L = f(\theta_1, \theta_2, \cdots, \theta_i, \cdots, \theta_n). \quad (7)$$

The day-ahead dispatching of PSHPFRPs not only needs to set generation plan and reserves accommodation reserve power for the grid, but also needs to reserve line active power margin for the grid to prevent line power congestion during real-time dispatching. Hence, the line power in the day-ahead dispatching plan of PSHPFRPs should be minimized, so as to reserve line power margin. The OIPOPF model should be defined as expressed by Equation (8), to constrain the minimization of line power.

$$\min h(P_i) = \min \sum_{i=1}^{L} P_{li} \quad (8)$$

where $P_i$ represents the power of power sources, and $P_l$ represents the line power.

The OIPOPF mathematical model for the day-ahead dispatching of PSHPFRPs is constrained by power balance constraints, tie-line power constraints, and reserve capacity constraints.

2.1.1. Power Balance Constraints

$$\sum P_d = \sum P_u + \sum P_c \quad (9)$$

Equation (9) is the power balance expression; where $\sum P_d$ represents the total load power of the power grid, $\sum P_u$ represents the total power of the uncertain power generation, and $\sum P_c$ represents the total power generated by conventional power sources such as thermal power and hydropower sources.

2.1.2. Tie-Line Power Constraints

The maximum line power is $P_{l_{\text{max}}}$, and the power of each line in Equation (7) is $P_l$. Moreover, $P_l$ represents the $l$th line power. Equation (7) satisfies the line power constraint expression as shown in Equation (10).

$$P_l \leq P_{l_{\text{max}}} (i = 1, 2, \cdots, L) \quad (10)$$

2.1.3. Reserve Capacity Constraints

The number of power sources in the grid is assumed to be $m$. Moreover, $P_i$ represents the $i$th power source. The power source described in $P_i$ satisfies the reserve capacity constraints in Equation (11).

$$P_{i_{\text{min}}} \leq P_i \leq P_{i_{\text{max}}} (i = 1, 2, \cdots, m) \quad (11)$$
2.2. Point Estimation Method Based on Gram–Charlier Expansion

2.2.1. Moments, Central Moments, and Their Relationship

For a continuous random variable \( X \) of any positive integer \( k \), the \( x^k \) function can be integrated on \((-\infty, +\infty)\) with respect to \( f(x) \). The \( k \) order moment of \( X \) can be expressed by Equation (12) [39]:

\[
\alpha_k = \int_{-\infty}^{+\infty} x^k \cdot f(x) \, dx, \tag{12}
\]

where \( \alpha_k \) represents the \( k \) order moment.

Moreover, the mean value of the continuous random variable \( X \) is \( \mu \), and the \( k \) order central moment of \( X \) can then be defined as expressed by Equation (13):

\[
\beta_k = \int_{-\infty}^{+\infty} (x - \mu)^k \cdot f(x) \, dx, \tag{13}
\]

where \( \beta_k \) represents the \( k \) order central moment.

The \( k \) order moment, \( \alpha_k \), and the \( k \) order central moment, \( \beta_k \), of the continuous random variable \( X \) satisfy the relationship expressed by Equation (14) [40]:

\[
\beta_k = \sum_{j=0}^{k} C_j^k \alpha_{k-j} (-\alpha_1)^j, (k = 2, 3, \cdots, n), \tag{14}
\]

where the zero-order origin moment \( \alpha_0 = 1 \). Moreover, the coefficient \( C_j^k \) satisfies Equation (15):

\[
C_j^k = \frac{k!}{j! \cdot (k-j)!}. \tag{15}
\]

According to Equation (14), the \( k \) order central moment, \( \beta_k \), can be obtained by the \( k \) order moment, \( \alpha_k \). The relationship between the central moment and the moment satisfies the relationship expressed by Equation (16):

\[
\begin{align*}
\beta_1 &= 0 \\
\beta_2 &= \alpha_2 - \alpha_1^2 \\
\beta_3 &= \alpha_3 - 3\alpha_2\alpha_1 + 2\alpha_1^3 \\
\beta_4 &= \alpha_4 - 4\alpha_3\alpha_1 + 6\alpha_2\alpha_1^2 - 3\alpha_1^4 \\
\beta_5 &= \alpha_5 - 5\alpha_4\alpha_1 + 10\alpha_3\alpha_1^2 - 10\alpha_2\alpha_1^3 + 5\alpha_1^5 \\
&\vdots
\end{align*} \tag{16}
\]

2.2.2. Point Estimation Method

For a grid with \( n \) buses, \( P_i \) represents the \( i \)th bus active power; with a mean of \( \mu_i \) standard deviation of \( \sigma_i \). In the point estimation method, for \( n-1 \) input variables \((P_1, P_2, \cdots, P_{i-1}, P_{i+1}, \cdots, P_{n-1})\), each variable contains \( j \) points. From the first variable to the \( n-1 \)th variable, the POPF is calculated when only one point is considered for one variable at a time and the other variables are mean. For example, for the \( i \)th input variable, the POPF calculated at the \( j \)th estimation point is expressed by Equation (17) [41]:

\[
P_{Li,j} = F(\mu_1, \mu_2, \cdots, P_{i,j}, \cdots, \mu_{n-1}). \tag{17}
\]

The relationship between \( P_i \) and \( \mu_i, \sigma_i \) satisfies the relationship expressed by Equation (18).

\[
P_{i,j} = \mu_i + \eta_{i,j} \cdot \sigma_i \tag{18}
\]
In Equation (18), \( \eta_{i,j} \) represents the standard central moments, which are defined as expressed by Equation (19):

\[
\eta_{i,j} = \begin{cases} 
\lambda_{i,3} + (-1)^{3-j} \cdot \sqrt{\lambda_{i,4} - \frac{3}{4} \cdot \lambda_{i,3}^2}, & j = 1, 2 \\
0, & j = 3 
\end{cases}, \tag{19}
\]

In Equation (19), \( \lambda \) is calculated by the ratio of the central moment, \( M(P_i) \), to the standard deviation, \( \sigma_i \), which is defined as expressed by Equation (20).

\[
\lambda_{i,t} = \frac{M_t(P_i)}{(\sigma_i)_t}, \quad (i = 1, 2, 3, \cdots, n), \tag{20}
\]

where \( M(P_i) \) represents the central moment, which is defined as expressed by Equation (13).

After calculating each point in turn, the grid line power, \( P_L \), and the line power, \( P_{Li,j} \), calculated by each point estimation satisfy the relationship expressed by Equation (21).

\[
P_L = \sum_{i=1}^{n-1} \sum_{j=1}^{J} w_{i,j} \cdot (P_{Li,j})^t, \tag{21}
\]

where \( w_{i,j} \) represents the weight coefficient, which is defined as expressed by Equation (22).

\[
w_{i,j} = \begin{cases} 
\frac{(-1)^{3-j}}{n \cdot \eta_{i}(\eta_{i,1} - \eta_{i,2})}, & j = 1, 2 \\
\frac{1}{n^2} - \frac{1}{n \cdot \eta_{i,1} - \eta_{i,3}}, & j = 3 
\end{cases}, \tag{22}
\]

2.2.3. Gram–Charlier Expansion

Based on the Gram–Charlier expansion, the cumulative distribution function (CDF) of a continuous random variable, \( X \), is expressed by Equation (23), and the probability distribution function (PDF) is expressed by Equation (24) [42].

\[
H(x) = \Phi(x) + \xi_1 \Phi'(x) + \frac{\xi_2}{2!} \Phi''(x) + \frac{\xi_3}{3!} \Phi'''(x) + \cdots, \tag{23}
\]

\[
h(x) = \phi(x) + \xi_1 \phi'(x) + \frac{\xi_2}{2!} \phi''(x) + \frac{\xi_3}{3!} \phi'''(x) + \cdots, \tag{24}
\]

where \( \Phi(x) \) and \( \phi(x) \) are the CDF and the PDF of the standard normal distribution, respectively; and \( \xi_i \) is a constant coefficient as shown in Equation (25).

\[
\begin{cases} 
\xi_1 = \xi_2 = 0 \\
\xi_3 = -\frac{\beta_3}{\sigma^3} \\
\xi_4 = \frac{\beta_4}{\sigma^4} - 3 \\
\vdots
\end{cases} \tag{25}
\]

2.2.4. Algorithm Calculation Steps

The steps for calculating the OIPFPF using the point estimation method based on the Gram–Charlier expansion are as follows:

1. The standard central moment, \( \lambda \), of the probability variable is calculated according to Equation (20).
2. The standard location, \( \eta \), of the probability variable is calculated according to Equation (19).
3. The weight, \( w \), of the probability variable is calculated according to Equation (22).
4. The estimated point, \( P_{i,j} \), is calculated according to Equation (18).
5. The OIPFPF, \( P_{Li,j} \), is calculated according to Equation (17) and its constraints.
(6) After calculating the OIPOPF for all the estimated points, the moment, $\alpha$, of each line power is calculated according to Equation (21).

(7) The central moment, $\beta$, of the line power is calculated according to Equation (14) based on the moment, $\alpha$, of each line power.

(8) The Gram–Charlier series coefficient, $\xi$, is calculated according to Equation (25) based on the central moment, $\beta$, of each line power.

(9) The CDF and PDF of the power of each line are calculated according to Equations (23) and (24).

A flow chart for the calculation of the OIPOPF based on the Gram–Charlier expansion point estimation method is presented in Figure 1.

![Flow chart](image)

**Figure 1.** Flow chart for the calculation of the once-iterative probabilistic optimal power flow (OIPOPF) based on the Gram–Charlier expansion point estimation method. CDF—cumulative distribution function, PDF—probability distribution function.

### 3. Results

In this study, the OIPOPF mathematical model of PSHPRPs was applied to the modified IEEE 39-bus test system and the modified IEEE 300-bus test system for verification. In this section, a comparison of the OIPOPF calculation results with those of the DA power flow model and AC power flow model is presented, to verify the accuracy of the OIPOPF mathematical model. The OIPOPF model proposed in this paper is solved by the point estimation method based on the Gram–Charlier expansion.
The results were compared with those obtained using the Monte Carlo method. The simulations were conducted using MATLAB 8.3.0.532 (R2014a, TheMathWorks, Inc, Natick, MA, USA) on a personal computer with an i3-4170 CPU, 3.70 GHz processor, and 4.0 GB of random-access memory (RAM).

3.1. Modified Power Test System Raw Data

3.1.1. Modified IEEE 39-Bus Test System

The modified IEEE 39-bus test system diagram is presented in Figure 2. As shown in Figure 2, the modified IEEE 39-bus test system adds uncertain renewable energy sources such as wind and photovoltaic power.

![Figure 2. Modified Institute of Electrical and Electronic Engineers (IEEE) 39-bus test system diagram.](image)

The IEEE 39 bus test power system contains 10 power sources. In the modified test system, four of the power sources were replaced by uncertain renewable energy sources. The numerical characteristics such as the mean and standard deviation of each uncertain renewable energy sources prediction error are shown in Table 1.

| Bus | Mean   | Standard Deviation |
|-----|--------|--------------------|
| 34  | 0.1524 | 0.9652             |
| 35  | 0.1374 | 1.374              |
| 36  | 0.058  | 0.87               |
| 37  | 0.01   | 0.1                |
3.1.2. Modified IEEE 300-Bus Test System

The IEEE 300 bus test power system contains 69 power sources. In the modified test system, 28 power sources replaced were by uncertain renewable energy sources. The numerical characteristics such as the mean and standard deviation of each uncertain renewable energy source prediction error are shown in Table 2.

Table 2. Numerical characteristics of uncertain renewable energy sources prediction errors of the modified IEEE 300-bus test system.

| Bus | Mean  | Standard Deviation | Bus | Mean  | Standard Deviation |
|-----|-------|--------------------|-----|-------|--------------------|
| 84  | 0.048 | 0.475              | 213 | 0.037 | 0.372              |
| 91  | 0.026 | 0.255              | 220 | 0.02  | 0.2                |
| 92  | 0.039 | 0.39               | 221 | 0.055 | 0.55               |
| 98  | 0.017 | 0.168              | 222 | 0.035 | 0.35               |
| 108 | 0.022 | 0.217              | 227 | 0.04  | 0.403              |
| 124 | 0.034 | 0.34               | 7001| 0.057 | 0.567              |
| 141 | 0.038 | 0.381              | 7002| 0.072 | 0.723              |
| 143 | 0.08  | 0.796              | 7011| 0.033 | 0.334              |
| 146 | 0.018 | 0.184              | 7012| 0.047 | 0.472              |
| 147 | 0.032 | 0.317              | 7017| 0.043 | 0.43               |
| 149 | 0.02  | 0.203              | 7023| 0.029 | 0.285              |
| 152 | 0.047 | 0.472              | 7024| 0.051 | 0.51               |
| 153 | 0.032 | 0.316              | 7071| 0.022 | 0.216              |
| 198 | 0.052 | 0.524              | 9054| 0.015 | 0.15               |

3.2. Comparison of Simulation Results with DC Power Flow Model Results

The day-ahead dispatching of PSHPRPSs considers the distribution of active power in the power grid. In this study, the once-iteration power flow model, DC power flow model, and AC power flow model were used to determine the active power of power lines in the power grid, and the results were compared to verify the accuracy of the OIPOPF. In order to verify the accuracy of the calculation results of the power flow model proposed in this paper, the calculation results of the OIPOPF model and the DC power flow model are compared with those of the AC power flow model. The relative error of the simulation results of the two models relative to the AC power flow model is defined by Equation (26).

\[ \text{error} = \frac{|P_{\text{AC}}^L - P_{\text{1}}^L|}{P_{\text{AC}}^L} \times 100\%, \quad (26) \]

where \( \text{error} \) represents the percentage error of the simulation results; \( P_{\text{AC}}^L \) represents the simulation results of the AC power flow model; \( P_{\text{1}}^L \) represents the simulation results of the OIPOPF model or DC power flow model.

Table A1 (Appendix A) presents the percentage errors of the OIPOPF mathematical model and AC power flow simulation results for the modified IEEE 39-bus test system. Table A2 (Appendix A) presents the percentage errors of the DC power flow model and AC power flow model simulation results for the modified IEEE 39-bus test system.

Based on the percentage error, the average error was calculated. The average error \( \overline{\text{error}} \) is defined as expressed by Equation (27), and \( N_L \) represents the total number of percentage errors.

\[ \overline{\text{error}} = \frac{\sum \text{error}}{N_L} \quad (27) \]

According to Table 3, the average relative error of the AC power flow and DC power flow calculation result for the modified IEEE 39-bus test system is 4.45%; the average relative error of the AC power flow and the OIPOPF model calculation result for the modified IEEE 39-bus test system is...
0.64%. The average relative error of the AC power flow and DC power flow calculation result for the modified IEEE 300-bus test system is 11.91%; the average relative error of the AC power flow and the OIPOPF model calculation result for the modified IEEE 300-bus test system is 0.90%. Regardless of whether the modified IEEE 39-bus test system or the modified IEEE 300-bus test system was used, the simulation results show that the results of the optimal power flow model proposed in this paper are more accurate than those of the DC power flow model, and similar to those of the AC power flow model. Furthermore, in the PSHPRPSs, the uncertain power source accounts for a relatively high proportion, and its power prediction error is generally 25–40% [43]. The relative average error of the OIPOPF and AC power flow models for the modified IEEE 39-bus test system and the modified IEEE 300-bus test system are 0.64% and 0.90%, respectively, which is significantly less than the power prediction error of PSHPRPSs. Therefore, the simulation results prove the accuracy of the OIPOPF model proposed in this paper.

### Table 3. Average errors of simulation results for the modified IEEE 39-bus test system and the modified IEEE 300-bus test system.

| Scale of Test Power System | Once-Iterative Probabilistic Optimal Power Flow (OIPOPF) Model | Direct Current (DC) Power Flow Model |
|----------------------------|-------------------------------------------------------------|-----------------------------------|
| IEEE 39-bus test system    | 0.64%                                                       | 4.45%                             |
| IEEE 300-bus test system   | 0.90%                                                       | 11.91%                            |

In order to verify that the probabilistic power flow model proposed in this paper can improve computing efficiency, the running time of the OIPOPF mathematical model, the DC power flow model and the AC power flow model are compared. The comparison results are shown in Table 4.

### Table 4. Simulation operation time of the modified IEEE 39-bus test system and the modified IEEE 300-bus test system.

| Scale of Test Power System               | OIPOPF  | DC Power Flow | AC Power Flow |
|------------------------------------------|---------|---------------|---------------|
| Operation time of IEEE 39-bus test system (s) | 0.012   | 0.009         | 0.15          |
| Operation time of IEEE 300-bus test system (s) | 0.037   | 0.031         | 0.55          |

As shown in Table 4, regardless of whether the modified IEEE 39-bus test system or the modified IEEE 300-bus test system was used, the operation time of the OIPOPF model is similar to that of the DC power flow, and shorter than that of the AC power flow. Therefore, the simulation results show that the proposed model can maintain high operation efficiency.

To sum up, using the OIPOPF mathematical model to set the day-ahead dispatching plan of PSHPRPSs can not only maintain high calculation accuracy but also have high operation efficiency.

### 3.3. Simulation Results of OIPOPF Model

In the modified IEEE 39-bus test system and the modified IEEE 300-bus test system, point estimation methods based on Gram–Charlier expansion and Monte Carlo simulation method were employed for the calculation of the OIPOPF model proposed in this paper. The Monte Carlo simulation method was found to be more accurate, and it is generally used to verify the accuracy of the simulation results of other models or algorithms. In this study, the simulation results of the two abovementioned methods were compared to verify that the point estimation method based on Gram–Charlier expansion can be used to solve the proposed OIPOPF model. The mean and standard deviation of the line power calculated using these two simulation methods are shown in Table A3 (Appendix B).
The simulation results of the point estimation method based on Gram–Charlier expansion are represented by $A$, and the simulation results of the Monte Carlo simulation method are represented by $A_{mcs}$. The relative error, $\varepsilon$, of the simulation result is expressed by Equation (28):

$$\varepsilon = \frac{|A - A_{mcs}|}{|A_{mcs}|}.$$  

(28)

The relative error, $\varepsilon$, can represent the mean relative error, $\varepsilon_\mu$, or the standard deviation relative error, $\varepsilon_\sigma$.

To verify the accuracy of the point estimation method based on Gram–Charlier expansion, the cumulative distribution average root mean square error (CDARMSE) was defined as expressed by Equation (29).

$$\text{CDARMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (A_i - A_{mcs,i})^2},$$  

(29)

where $N$ represents the number of point estimates; $A_i$ represents the cumulative distribution value obtained using the point estimation method based on Gram–Charlier expansion for the $i$th point estimation; and $A_{mcs}$ represents the cumulative distribution value obtained using the Monte Carlo simulation method for the $i$th point estimation.

Table A4 (Appendix B) presents the relative errors and CDARMSE values of the line power for the modified IEEE 39-bus test system calculated using the Gram–Charlier point estimation and Monte Carlo simulation methods. As seen in Table A4, the CDARMSE values of the two methods were generally less than 0.001. Hence, the point estimation method based on Gram–Charlier expansion can be used to solve the proposed OIPOPF model. Table A4 indicates that the CDARMSE of the line power between busses 31 and 6 was the highest. The PDF and CDF of the line power between busses 31 and 6, as obtained using the two abovementioned methods, are shown in Figures 3 and 4. As shown in Figures 3 and 4, PECCS represents the simulation results of the point estimation method based on Gram–Charlier expansion; MC represents the simulation results of the Monte Carlo method. Although the CDARMSE of the line power between busses 31 and 6 was the highest, the simulation results of the two methods were very similar; thus, verifying the accuracy of the point estimation method based on Gram–Charlier expansion.

![PDF of line with the largest cumulative distribution average root mean square error (CDARMSE) for the modified IEEE 39-bus test system.](image)

Figure 3. PDF of line with the largest cumulative distribution average root mean square error (CDARMSE) for the modified IEEE 39-bus test system.
This paper uses $2m + 1$ point estimation method based on Gram–Charlier expansion to calculate the OIPOPF mathematical model proposed in this paper. In order to obtain the calculation result, $2m + 1$ simulation calculations are required, where $m$ is the number of indeterminate variables. The modified IEEE 39-bus test system has four indeterminate variables, and the modified IEEE 300-bus test system has 28 indeterminate variables.

According to Table 5, the number of simulation calculations of different sizes of test systems is related to the number of uncertain variables.

| Scale of Test Power System | Variables | PECCS Simulation Times | MC Simulation Times |
|---------------------------|-----------|------------------------|---------------------|
| Operation time of IEEE 39-bus test system | 4         | 9                      | 1000                |
| Operation time of IEEE 300-bus test system | 28        | 57                     | 1000                |

This paper compares the simulation results of the point estimation method based on Gram–Charlier expansion and Monte Carlo simulation method in the modified IEEE 300-bus test system to verify that the point estimation method based on Gram–Charlier expansion can calculate the OIPOPF model in large test power systems. In the simulation results of the modified IEEE 300-bus test system, the CDARMSE of the line power between busses 123 and 124 was the highest. The PDF and CDF of the line power between busses 123 and 124, as obtained using the two abovementioned methods, are shown in Figures 5 and 6. As shown in Figures 5 and 6, the simulation results of the two methods were very similar. Therefore, it is verified that the point estimation method based on Gram–Charlier expansion can calculate the OIPOPF model in large test power system.
3.4. Discussion

Because the PSHPREPs contain a high proportion of uncertain renewable energy sources, the prediction error of the day-ahead power is large, and the day-ahead dispatching plan cannot be accurate. Therefore, using the probabilistic AC power flow model not only cannot effectively improve the calculation accuracy, but will also obviously reduce the calculation efficiency. The use of probabilistic DC power flow model can improve the calculation efficiency but reduce the calculation accuracy. In order to verify that the probabilistic power flow model proposed in this paper can not only maintain high calculation accuracy but also maintain high calculation efficiency, this section compares the OIPOPF calculation results with those of the DC power flow model and AC power flow model.

According to the simulation results in Section 3.2, regardless of whether the modified IEEE 39-bus test system or the modified IEEE 300-bus test system was used, the calculation accuracy of OIPOPF model is similar to that of AC power flow model, and higher than that of DC power flow model; the calculation efficiency of OIPOPF model is similar to that of DC power flow model, and higher than that of AC power flow model. To sum up, the OIPOPF model proposed in this paper not only maintains high calculation accuracy, but also maintains high calculation efficiency when setting the day-ahead dispatching plan for the PSHPREPs.

In this paper, the point estimation method based on Gram–Charlier expansion is used to solve the OIPOPF mathematical model. In order to verify that the point estimation method based on Gram–Charlier expansion can solve the mathematical model of probabilistic optimal power flow
proposed in this paper, the simulation results of point estimation method based on Gram–Charlier expansion and Monte Carlo simulation method are compared. According to the simulation results in Section 3.3, regardless of whether the modified IEEE 39-bus small test system or the modified IEEE 300-bus large test system was used, the CDARMSE of the two calculation methods are generally less than 0.001. The figures were drawn by selecting the line power corresponding to the maximum error of the simulation results in the modified IEEE 39-bus small test system and the modified IEEE 300-bus large test system. As shown in the figures in Section 3.3, the simulation results of point estimation method based on Gram–Charlier expansion are similar to those of Monte Carlo simulation method. In summary, by comparing the simulation results of the two algorithms, it is verified that the point estimation method based on Gram–Charlier expansion can accurately solve the OIPOPF mathematical model.

Based on the comparison of the simulation results, the accuracy and efficiency of the proposed OIPOPF model of PSHPRPSs was verified. Moreover, based on the comparison of the calculation results obtained using the two methods, it was included that the point estimation method based on Gram–Charlier expansion can accurately calculate the proposed OIPOPF model.

4. Conclusions

Given that PSHPRPSs contain a large number of uncertain power sources, the day-ahead power prediction error is significantly high. During the dispatch cycle, it is difficult for the day-ahead generation plan to meet the actual active power demand of the power grid. To maintain the power balance of the power grid, the day-ahead power prediction error is accommodated by real-time dispatching and AGC. Due to the significant uncertainty of the power generated by the uncertain power sources, the conventional power flow calculation is not suitable for the SSA of PSHPRPSs. Based on the above reasons, this paper proposes the OIPOPF mathematical model as the power flow model for the SSA of PSHPRPSs. In this paper, the OIPOPF and AC power flow models were applied to a modified IEEE 39-bus test system and modified IEEE 300-bus test system. The simulation results verify that the OIPOPF model can not only ensure the accuracy of power flow results, but also simplify the power flow model. The OIPOPF mathematical model was solved using the point estimation method based on the Gram–Charlier expansion. Compared with the Monte Carlo simulation method, the results indicate that the point estimation method based on Gram–Charlier expansion can be employed to accurately calculate the proposed OIPOPF model. Furthermore, the simulation results verify the accuracy of the OIPOPF model and algorithm.

Author Contributions: Conceptualization, Y.C. and Z.G.; methodology, Y.C.; software, Y.C.; validation, Y.C., and Z.G.; formal analysis, Y.Y.; investigation, H.L.; resources, G.W.; data curation, Y.H.; writing—original draft preparation, Y.C.; writing—review and editing, A.T.T.; visualization, Y.H.; supervision, H.L.; project administration, G.W. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded in part by Key R&D Projects in the Hebei Province of China (grant number 19212103D), the Innovation capability improvement project in the Hebei Province of China (grant number 19962113D), and the National Natural Science Foundation of China (grant number 51877047).

Conflicts of Interest: The authors declare no conflict of interest.

Nomenclature

POPF Probabilistic optimal power flow
SSA Steady state analysis
PSHRPS Power systems with high proportions of renewable power sources
AGC Automatic generation control
OIPOPF Once-iterative probabilistic optimal power flow
DC Direct current
AC Alternating current
CDF Cumulative distribution function
PDF Probability distribution function
CDARMSE Cumulative distribution average root mean square error
Appendix A

Appendix A presents the simulation results of OIPOPF mathematical model compared with DC power flow model for the modified IEEE 39-bus test system (see Section 3.2 for details). Table A1 presents the percentage errors of the OIPOPF mathematical model and AC power flow simulation results for the modified IEEE 39-bus test system. Table A2 presents the percentage errors of the DC power flow model and AC power flow model simulation results for the modified IEEE 39-bus test system.

Table A1. Percentage errors of the OIPOPF and AC power flow simulation results for the modified IEEE 39-bus test system.

| From Bus | To Bus | error | From Bus | To Bus | error |
|----------|--------|-------|----------|--------|-------|
| 1        | 2      | 0.37  | 14       | 15     | 0.48  |
| 1        | 39     | 0.83  | 15       | 16     | 0.09  |
| 2        | 3      | 0.81  | 16       | 17     | 0.01  |
| 2        | 25     | 3.04  | 16       | 19     | 0.03  |
| 2        | 30     | 0.30  | 16       | 21     | 0.02  |
| 3        | 4      | 3.59  | 16       | 24     | 0.14  |
| 3        | 18     | 2.88  | 17       | 18     | 0.59  |
| 4        | 5      | 0.53  | 17       | 27     | 4.91  |
| 4        | 14     | 0.10  | 19       | 20     | 0.14  |
| 5        | 6      | 0.23  | 19       | 33     | 0.09  |
| 5        | 8      | 0.06  | 20       | 34     | 0.01  |
| 6        | 7      | 0.09  | 21       | 22     | 0.14  |
| 6        | 11     | 0.16  | 22       | 23     | 0.50  |
| 6        | 31     | 0.29  | 22       | 35     | 0.11  |
| 7        | 8      | 0.18  | 23       | 24     | 0.09  |
| 8        | 9      | 1.75  | 23       | 36     | 0.09  |
| 9        | 39     | 2.25  | 25       | 26     | 0.06  |
| 10       | 11     | 0.14  | 25       | 37     | 1.10  |
| 10       | 13     | 0.14  | 26       | 27     | 0.48  |
| 10       | 32     | 0.17  | 26       | 28     | 0.02  |
| 12       | 11     | 1.11  | 26       | 29     | 0.03  |
| 12       | 13     | 1.01  | 28       | 29     | 0.08  |
| 13       | 14     | 0.16  | 29       | 38     | 0.03  |

Table A2. Percentage errors of the DC power flow model and AC power flow model simulation results for the modified IEEE 39-bus test system.

| From Bus | To Bus | error | From Bus | To Bus | error |
|----------|--------|-------|----------|--------|-------|
| 1        | 2      | 2.68  | 14       | 15     | 10.30 |
| 1        | 39     | 6.11  | 15       | 16     | 5.63  |
| 2        | 3      | 4.22  | 16       | 17     | 0.87  |
| 2        | 25     | 7.02  | 16       | 19     | 1.93  |
| 2        | 30     | 1.02  | 16       | 21     | 1.57  |
| 3        | 4      | 14.92 | 16       | 24     | 5.73  |
| 3        | 18     | 4.72  | 17       | 18     | 0.83  |
| 4        | 5      | 10.01 | 17       | 27     | 2.61  |
| 4        | 14     | 1.05  | 19       | 20     | 1.56  |
| 5        | 6      | 4.13  | 19       | 33     | 0.46  |
| 5        | 8      | 0.62  | 20       | 34     | 0.50  |
| 6        | 7      | 1.17  | 21       | 22     | 0.72  |
| 6        | 11     | 4.82  | 22       | 23     | 3.67  |
| 6        | 31     | 6.53  | 22       | 35     | 3.49  |
| 7        | 8      | 1.86  | 23       | 24     | 0.03  |
| 8        | 9      | 14.54 | 23       | 36     | 0.25  |
| 9        | 39     | 16.93 | 25       | 26     | 17.11 |
| 10       | 11     | 3.96  | 25       | 37     | 0.31  |
| 10       | 13     | 4.03  | 26       | 27     | 0.61  |
| 10       | 32     | 0.17  | 26       | 28     | 3.23  |
| 12       | 11     | 13.59 | 26       | 29     | 2.60  |
| 12       | 13     | 10.46 | 28       | 29     | 1.08  |
| 13       | 14     | 4.39  | 29       | 38     | 0.63  |
Appendix B

Appendix B presents the simulation results of the point estimation method for the modified test power system (see Section 3.3 for details). Table A3 presents the numerical characteristics of the line power simulation results for the modified IEEE 39-bus test system. Table A4 presents the relative error and CDARMSE of the line power for the modified IEEE 39-bus test system.

Table A3. Numerical characteristics of line power simulation results for the modified IEEE 39-bus test system.

| From Bus | To Bus | Monte Carlo Method | Point Estimation Method |
|----------|-------|--------------------|------------------------|
|          |       | Mean               | Standard Deviation      | Mean               | Standard Deviation      |
| 2        | 1     | −1.83              | 10.69                  | 3.12               | 15.32                  |
| 1        | 25    | −1.83              | 10.69                  | 3.12               | 15.32                  |
| 2        | 25    | −7.86              | 51.42                  | −9.90              | 53.49                  |
| 2        | 2     | −10.61             | 61.25                  | −6.78              | 64.98                  |
| 30       | 2     | 0                  | 0                      | 0                  | 0                      |
| 3        | 4     | −4.29              | 24.44                  | −2.76              | 25.84                  |
| 18       | 3     | 4.49               | 27.13                  | 7.14               | 28.23                  |
| 5        | 4     | 0.20               | 2.51                   | −2.24              | 5.40                   |
| 14       | 4     | 4.09               | 23.60                  | 5.00               | 24.57                  |
| 6        | 5     | 0.97               | 5.78                   | 0.83               | 5.94                   |
| 5        | 8     | 0.77               | 5.29                   | 3.08               | 6.89                   |
| 6        | 7     | 0.81               | 5.29                   | 2.65               | 6.38                   |
| 11       | 6     | 1.84               | 10.85                  | 3.49               | 11.53                  |
| 31       | 6     | −0.06              | 1.46                   | 0                  | 0                      |
| 7        | 8     | 0.81               | 5.29                   | 2.65               | 6.38                   |
| 8        | 9     | 1.58               | 10.57                  | 5.73               | 13.24                  |
| 9        | 39    | 1.58               | 10.57                  | 5.73               | 13.24                  |
| 10       | 11    | 1.68               | 9.88                   | 3.17               | 10.49                  |
| 10       | 13    | −1.64              | 9.91                   | −3.17              | 10.49                  |
| 32       | 10    | 0.04               | 2.18                   | 0                  | 0                      |
| 11       | 12    | −0.16              | 0.97                   | −0.31              | 1.04                   |
| 13       | 12    | 0.16               | 0.97                   | 0.31               | 1.04                   |
| 14       | 14    | −1.80              | 10.87                  | −3.49              | 11.53                  |
| 15       | 15    | −4.89              | 34.36                  | −8.49              | 35.88                  |
| 16       | 15    | 5.89               | 34.36                  | 8.49               | 35.88                  |
| 16       | 17    | 25.92              | 147.09                 | 26.29              | 153.49                 |
| 16       | 19    | 10.77              | 93.30                  | 15.24              | 96.52                  |
| 16       | 21    | 12.62              | 98.05                  | 11.94              | 99.97                  |
| 16       | 24    | 8.41               | 62.91                  | 7.60               | 63.71                  |
| 17       | 18    | 4.49               | 27.13                  | 7.14               | 28.23                  |
| 17       | 27    | 21.43              | 120.16                 | 19.15              | 125.65                 |
| 19       | 20    | −10.75             | 93.30                  | −15.24             | 96.52                  |
| 33       | 19    | 0.01               | 0.27                   | 0                  | 0                      |
| 34       | 20    | 10.75              | 93.30                  | 15.24              | 96.52                  |
| 22       | 21    | 12.62              | 98.05                  | 11.94              | 99.97                  |
| 22       | 23    | 0.30               | 67.02                  | 1.80               | 66.54                  |
| 35       | 22    | 12.92              | 134.66                 | 13.74              | 137.40                 |
| 23       | 24    | 8.41               | 62.91                  | 7.60               | 63.71                  |
| 36       | 23    | 8.11               | 88.60                  | 5.80               | 87.00                  |
| 25       | 26    | 11.32              | 61.17                  | 7.78               | 64.02                  |
| 26       | 27    | −21.43             | 120.16                 | −19.15             | 125.65                 |
| 28       | 26    | −16.37             | 90.60                  | −13.46             | 94.80                  |
| 29       | 26    | −16.37             | 90.60                  | −13.46             | 94.80                  |
| 29       | 28    | −16.37             | 90.60                  | −13.46             | 94.80                  |
| 38       | 29    | −32.75             | 181.19                 | −26.93             | 189.60                 |
Table A4. Relative error and cumulative distribution average root mean square error (CDARMSE) of line power for the modified IEEE 39-bus test system.

| From Bus | To Bus | $\varepsilon_{\mu}$ | $\varepsilon_{\sigma}$ | CDARMSE |
|----------|--------|----------------------|------------------------|----------|
| 2        | 1      | 2.70                 | 0.43                   | 0.0027   |
| 1        | 39     | 2.70                 | 0.43                   | 0.0027   |
| 2        | 3      | 0.13                 | 0.04                   | 0.0002   |
| 25       | 2      | 0.36                 | 0.06                   | 0.0004   |
| 30       | 2      | 1.71                 | 0.18                   | 0.2274   |
| 3        | 4      | 0.36                 | 0.06                   | 0.0004   |
| 18       | 3      | 0.59                 | 0.04                   | 0.0005   |
| 5        | 4      | 11.97                | 1.15                   | 0.0053   |
| 14       | 4      | 0.23                 | 0.04                   | 0.0003   |
| 6        | 5      | 0.14                 | 0.03                   | 0.0002   |
| 5        | 8      | 3.00                 | 0.30                   | 0.0025   |
| 6        | 7      | 2.28                 | 0.21                   | 0.0020   |
| 11       | 6      | 0.89                 | 0.06                   | 0.0008   |
| 31       | 6      | 1.00                 | 1.00                   | 0.4965   |
| 7        | 8      | 2.28                 | 0.21                   | 0.0020   |
| 8        | 9      | 2.63                 | 0.25                   | 0.0022   |
| 9        | 39     | 2.63                 | 0.25                   | 0.0022   |
| 10       | 11     | 0.89                 | 0.06                   | 0.0008   |
| 10       | 13     | 0.94                 | 0.06                   | 0.0009   |
| 32       | 10     | 1.00                 | 1.00                   | 0.4957   |
| 11       | 12     | 0.91                 | 0.07                   | 0.0009   |
| 13       | 12     | 0.91                 | 0.07                   | 0.0009   |
| 13       | 14     | 0.94                 | 0.06                   | 0.0009   |
| 14       | 15     | 0.44                 | 0.04                   | 0.0004   |
| 16       | 15     | 0.44                 | 0.04                   | 0.0004   |
| 16       | 17     | 0.01                 | 0.04                   | 0.0002   |
| 19       | 16     | 0.42                 | 0.03                   | 0.0003   |
| 21       | 16     | 0.05                 | 0.02                   | 0.0001   |
| 24       | 16     | 0.10                 | 0.01                   | 0.0001   |
| 17       | 18     | 0.59                 | 0.04                   | 0.0005   |
| 17       | 27     | 0.11                 | 0.05                   | 0.0002   |
| 19       | 20     | 0.42                 | 0.03                   | 0.0003   |
| 33       | 19     | 1.00                 | 1.00                   | 0.4939   |
| 34       | 20     | 0.42                 | 0.03                   | 0.0003   |
| 22       | 21     | 0.05                 | 1.00                   | 0.0001   |
| 22       | 23     | 4.94                 | 0.03                   | 0.0001   |
| 35       | 22     | 0.06                 | 0.02                   | 0.0001   |
| 23       | 24     | 0.10                 | 0.01                   | 0.0001   |
| 36       | 23     | 0.28                 | 0.02                   | 0.0002   |
| 25       | 26     | 0.31                 | 0.05                   | 0.0004   |
| 37       | 25     | 0.42                 | 0.02                   | 0.0002   |
| 26       | 27     | 0.11                 | 0.05                   | 0.0002   |
| 28       | 26     | 0.18                 | 0.05                   | 0.0002   |
| 29       | 26     | 0.18                 | 0.05                   | 0.0002   |
| 29       | 28     | 0.18                 | 0.05                   | 0.0002   |
| 38       | 29     | 0.18                 | 0.05                   | 0.0002   |

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