Solving a dynamic assignment problem in the socio-economic system

Irina Zaitseva\textsuperscript{1, 2, a)}, Oleg Malafeyev\textsuperscript{3, b)}, Natalya Poddubnaya\textsuperscript{1, c)}, Anna Vanina\textsuperscript{4, d)and Elena Novikova\textsuperscript{4, e)}

\textsuperscript{1}Stavropol State Agrarian University, Zootekhnicheskiy lane 12, Stavropol, 355017, Russia
\textsuperscript{2}Stavropol branch of the Moscow Pedagogical State University, Dovatortsev str. 66 g, Stavropol, 355042, Russia
\textsuperscript{3}St. Petersburg State University, Faculty of Applied Mathematics and Control Processes, Universitetskaya nab., St. Petersburg, 199034, Russia
\textsuperscript{4}North-Caucasus Federal University

\textsuperscript{a)Corresponding author: irina.zaitseva.stv@yandex.ru
\textsuperscript{b)malafeyevoa@mail.ru
\textsuperscript{c)cherkasovanatas@mail.ru
\textsuperscript{d)hnykina_anna@mail.ru
\textsuperscript{e)novikovaelena_nik@mail.ru

Abstract. The deterministic variant of the dynamic assignment problem is considered. The task of finding the path of maximum cost is investigated. Examples of solution the dynamic assignment problem and solving the problem of finding the path of maximum cost are given.

Introduction

Solving the optimal distribution problem of labor resources is one of the main ones in the strategy of Russian innovative development. The solution of such tasks determines the dynamics and the possibility of the development of basic and innovative industries, the diversification of Russian economic structure and its regions. Optimization of the labor market structure requires the construction of a management system that ensures the optimal distribution of labor resources, according to the requirements of the regional economy. That, in turn, requires solving a complex of tasks for developing mathematical models for the optimal distribution of labor resources and corresponding methods. Solving the problem of optimal labor resources management allows us to study the processes of formation and use of labor resources, and also actualizes the investigation of flow characteristics of these processes in the conditions of the market Russian economy within a particular enterprise, industry, region and country as a whole\textsuperscript{[1-5]}.

1. The dynamic assignment problem

1.1. Deterministic version of the dynamic assignment problem

Some text There are $l$ jobs $x_1, ..., x_l$, that can be performed by various employees and $m$ vacancies $y_1, ..., y_m$ to perform these jobs. Suppose that workers differ in their ability to perform individual work
(for example, in productivity). We will assume that these differences can be described by assigning each possible assignment \( x_i \rightarrow y_j \) of a real number \( c_{ij} \), where \( c_{ij} \) is the efficiency of performing a certain work by a certain employee. The optimal name is such an assignment in which vacancies are assigned to all jobs in such a way that the total cost is minimal [6-7].

The assignment process takes place over \( T \) periods of time \( t = 0, 1, 2, ..., T \). In any period of time \( (t, t + 1) \) each work can be in one of a finite number of states \( a_1, ..., a_M \). Workers can do work in one of a finite number of modes \( h_1, ..., h_M \). The state of the system \( S_i \) is determined by the state of work and the state of workers.

Suppose \( S_1, ..., S_N \) are the states of the system. Each state of the process corresponds to a non-empty set of states into which the process can be transferred in the next step.

Let \( r_{ij} \) denote the income received in the transition from state \( S_i \) to state \( S_j \). It can be calculated with the use of the income received in the solution of static problems of optimal assignment at each step. The income on \( T \) steps is equal to the sum of the income on the individual steps.

It is required for a given initial state of the process \( S_0 \) (and, possibly, a given duration \( T \)) to maximize total revenue.

To this problem, you can apply the usual technique of dynamic programming, and specifically, the functional Bellman equations [8-12].

Consider a finite oriented graph \( G(\mathbf{X}, \mathbf{U}) \), the vertices \( x_1, ..., x_N \in \mathbf{X} \) of which are the states of the system, and the arcs \( u_{ij} = (x_i, x_j) \in \mathbf{U} \) are the possible paths of transition from state to state. Each arc \( u_{ij} \) is attributed to cost — income \( r_{ij} \) derived from the appropriate transition.

To describe a given graph, we introduce a matrix \( A_{[M][M]} = \{a_{ij}\} \), which we call the arc cost matrix. This matrix is defined as follows:

\[
a_{ij} = \begin{cases} 
  r_{ij}, & \text{if transit is possible from state } S_i \text{ to } S_j; \\
  \phi, & \text{if such a transit is impossible.}
\end{cases}
\]

Suppose there is a set of finite system states that we would like to achieve in a finite number of steps. The problem arises of finding the path of maximum cost for these final states [13-14].

Here are possible options. You can search for a path from a given initial state or from any state to the final one. You can also determine how many steps reach the final state from any initial state and what is the maximum total income [15].

Let \( \mu \) is called a sequence of arcs \( \mu=(x_{i_1}, x_{i_2}), ..., (x_{i_p}, x_{i_q}) \) in which all \( x_{i_1}, ..., x_{i_p} \) are different. We define a path simple if the initial and final vertices coincide. The cost of the simple path is

\[
r(\mu) = \bigoplus_{k=1}^{p} r_{i_k, i_{k+1}}.\]

Let \( M \) is the set of all simple paths from \( x_i \) to \( x_j \). There is a path \( \mu* \in M \) that has a cost \( r(\mu*) \geq r(\mu), \forall \mu \in M \). We introduce the matrix \( A* = \{a*\} \) the matrix of the highest values:

\[
a_{*ij} = \begin{cases} 
  \bigoplus_{\mu \in M} r(\mu), & \text{if } i \neq j; \\
  \phi, & \text{if there are no paths from } x_i \text{ to } x_j; \\
  \phi, & \text{if } i = j.
\end{cases}
\]
In order to solve the problem, it is necessary to solve the equation: \( Y = AY \oplus B \), where is \( A \) the arc cost matrix [16].

According to Carre: if the graph \( G(X,U) \) does not contain cycles \( \gamma \) whose values \( \gamma(\gamma) \leq e \), then this equation has a unique solution \( Y = A \ast B \). Otherwise, the solution is not the only one.

One of the methods for solving this equation is an iterative method - an analogue of the functional Bellman equations in terms of an idempotent semiring. It consists in the following [17].

We choose the initial approximation \( Y^{(0)} \). Form a sequence of approximations according to the formula: \( Y^{(k+1)} = AY^{(k)} \oplus B \). The sequence \( Y^{(k)} \) converges to a solution \( Y = A \ast B \).

According to Carre: if there are no cycles \( \gamma \) in the graph \( G(X,U) \), whose values \( \gamma \) are \( \gamma(\gamma) \leq e \), then the method converges for any initial approximation. Otherwise, convergence for any \( Y^{(0)} \) one cannot be guaranteed [18-20].

1.2 An example of solving a dynamic assignment problem

Consider an example. Suppose there are two jobs and two employees. Each job can be in one of three states (at one of three stages), each employee can be in one of two states (this, for example, affects somehow the performance). It is obvious that the system can be in one of 36 possible states [21-25].

At the same time, the states of the system in which both works are in the third states (at the last stages) will be considered final (most desirable). Let it be: the 9th (both workers are in the first state, both are in the third), the 18th (the 1st worker is in the 1st condition, the 2nd is in the 2nd), the 27th (1st the second worker is in the 2nd condition, the 2nd is in the 1st) and the 36th (both workers are in the second state). These states need to be achieved in a finite number of steps [26-30].

Suppose there are two strategies:

- \( q_1 \) - the first employee is assigned to the first job, the second is to the second;

- \( q_2 \) - the first employee is assigned to the second job, the second is to the first one.

Suppose, having solved static problems of optimal assignment for each state, we have found incomes from each specific purpose, and then incomes obtained from the transition from one state to another. Let there also be known sets of states to which the process can be transferred in the next step for each specific state using a certain strategy [31].

All these data are recorded in the following table 1.

**TABLE 1. Sets of system states**

| \( q_i \) | \( S_i \) | \( S_j \) | \( r_{ij} \) | \( q_i \) | \( S_i \) | \( S_j \) | \( r_{ij} \) | \( q_i \) | \( S_i \) | \( S_j \) | \( r_{ij} \) | \( q_i \) | \( S_i \) | \( S_j \) | \( r_{ij} \) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 33 | 30 | 1 | 19 | 12 | 20 | 2 | 1 | 21 | 20 | 2 | 19 | 19 | 0 |
| 1 | 2 | 15 | 20 | 1 | 20 | 3 | 10 | 2 | 2 | 21 | 10 | 2 | 20 | 6 | 10 |
| 1 | 3 | 15 | 10 | 1 | 21 | 3 | 0 | 2 | 3 | 3 | 0 | 2 | 21 | 3 | 0 |
| 1 | 4 | 36 | 30 | 1 | 22 | 33 | 20 | 2 | 4 | 24 | 20 | 2 | 22 | 23 | 10 |
| 1 | 5 | 27 | 20 | 1 | 23 | 24 | 10 | 2 | 5 | 24 | 10 | 2 | 23 | 6 | 10 |
| 1 | 6 | 27 | 10 | 1 | 24 | 24 | 0 | 2 | 6 | 6 | 0 | 2 | 24 | 6 | 0 |
| 1 | 7 | 18 | 20 | 1 | 25 | 18 | 20 | 2 | 7 | 27 | 20 | 2 | 25 | 26 | 10 |
| 1 | 8 | 9 | 10 | 1 | 26 | 9 | 10 | 2 | 8 | 27 | 10 | 2 | 26 | 9 | 10 |
| 1 | 9 | 9 | 0 | 1 | 27 | 27 | 0 | 2 | 9 | 9 | 0 | 2 | 27 | 27 | 0 |
| 1 | 10 | 32 | 20 | 1 | 28 | 11 | 10 | 2 | 10 | 35 | 30 | 2 | 28 | 32 | 20 |
| 1 | 11 | 24 | 20 | 1 | 29 | 3 | 10 | 2 | 11 | 35 | 20 | 2 | 29 | 15 | 20 |
Thus, knowing the set of states in which you can transfer the process in the next step for each specific state using a particular strategy, you can choose the optimal solution.

2. The task of finding the path of maximum cost

2.1. Solution of the problem of finding the path of maximum cost

Consider each state of the system as a vertex of the graph. To do this, we write the matrix cost of arcs $A_{36 \times 36} = \{a_{ij}\}$. Each row of this matrix will have two values (according to the number of strategies, the other elements are equal to $\phi$).

Thus, on the graph, the problem arises of finding the path of maximum value from each vertex to one of the four vertices corresponding to the final states.

Let's use the method described earlier. Construct a sequence of approximations $Y^{(k)}$: $Y^{(k)} = AY^{(k-1)} \oplus B$. The construction is finished when $Y^{(k)} = Y^{(k-1)}$.

These vectors will show the magnitude of the total income received on this and previous moves.

\[
Y^{(0)} = Y^{(0)} \oplus B.
\]

As a vector, take a vector: $B = e$

Recall that in the considered idempotent semiring $e = 0$, $\phi = -\infty$. The single elements are on the 9th, 18th, 27th and 36th places, the other elements of the vector are zero. Let us assume that $Y^{(0)} = B$.

1) $Y^{(1)} = AY^{(0)} \oplus B$. At the same time, a solution vector was obtained showing the incomes that are obtained if one of the end states is reached in one step.
2) \( Y^{(2)} = AY^{(1)} \oplus B \). From this vector one can see from which states it is possible to reach the final states in two steps, and what revenues are obtained from this.

3) \( Y^{(3)} = AY^{(2)} \oplus B \). This is the final vector, with subsequent applications of the formula, it will not change.

Thus, the problem can be solved in no more than three periods of time for any initial state. This solution method makes sense with a small one \( T \).

When \( t \to \infty \), since the graph \( G(X,U) \) is finite, some arcs in the path \( \mu \) will occur multiple times. Then there will be a question about the existence of a cycle with a maximum cost followed by movement along it.

Consider the graph \( G(X,U) \). Let be \( \mathcal{C} \) a set of circuits, i.e. simple oriented cycles in a graph.

To characterize a particular circuit \( C \), we will consider the number \( \lambda(C) = \frac{1}{|C|} \sum_{(x_i,x_j) \in C} r_{ij} \), the average cost of the arc entering the circuit. Here \( |C| \) is the length of the circuit, i.e. the number of arcs entering it. It is necessary to find a circuit for which: \( \lambda(C*) = \max_{C \in \mathcal{C}} \lambda(C) \).

The transition from the initial state to this circuit and movement along it, and then, if necessary, the transition to the final state, will be optimal. You can use a modification of the Howard method.

Choose any circuit \( C^0 \). Denote \( \lambda^0 = \lambda(C^0) \). For all arcs of the graph we recalculate the cost: \( \bar{r}_{ij} = r_{ij} - \lambda^0 \), for \( \forall u_{ij} = (x_i,x_j) \in U \). We construct a subgraph \( G' = (X,U') \) in which exactly one arc emanates from each vertex and is the only circuit \( C^0 \). We assume that the vertices belonging to the cycle are preceding everything else. Choose some \( x_{i0} \in C^0 \) and set \( \varphi_{i0} = 0 \).

For all \( u_{ij} = (x_i,x_j) \in U' \) we set:
\[
\varphi_j = \varphi_j \oplus \bar{r}_{ij} \tag{1}
\]

For all \( x_i \) and all \( u \in U_{i}^{-} \), where \( U_{i}^{-} \) is the set of arcs emanating from the vertex \( x_i \), check the following condition: if there is an arc \( u_{ij} = (x_i,x_j) \in U_{i}^{-} \) for which it does not:
\[
\varphi_i \oplus [\varphi_j \oplus \bar{r}_{ij}] = \varphi_i , \tag{2}
\]
then two options are possible:

1) \( x_j \) is not preceded \( x_i \). Then in \( G' \) it is necessary to replace the arc emanating from the \( x_i \) by the arc \( u \), according to (1), recalculate \( \varphi \) and repeat the test;

2) \( x_j \) precedes \( x_i \). Then the path \( x_j \to x_i \to \ldots \to x_j \) is a circuit whose characteristic is greater than \( \lambda^0 \). Take this circuit for the initial one and return to the beginning of the algorithm.

Since the graph \( G(X,U) \) is finite, the algorithm will converge in a finite number of steps. As a result, we obtain the circuit with the highest average cost of the arc. The optimal solution is to transfer the system from the initial state to the state that is the top in this circuit, and further movement along it, and then, if necessary, transfer the system to the final state. (Possible variation of the task with a given duration of the transition) [32-34].

2.2. An example the problem solving of finding the path of maximum cost

Suppose there are two jobs, each of which can be in one of two states and two workers who can work in one of two modes. The state of the system is determined by the status of the work. Accordingly, the system can be in one of the four states shown in Table 2.
TABLE 2. System States

| System status | The status of the 1st work | The status of the 2nd work |
|---------------|---------------------------|---------------------------|
| 1             | 1                         | 1                         |
| 2             | 1                         | 2                         |
| 3             | 2                         | 1                         |
| 4             | 2                         | 2                         |

In accordance with the modes of workers schedule, we have four strategies:

1) $q_1$ - both works in the 1st mode;
2) $q_2$ - the 1st work in the 1st mode, the 2nd one - in the second;
3) $q_3$ - the 1st work in the 2nd mode, the 2nd one - in the 1st mode;
4) $q_4$ - both works in the second mode.

Suppose that $T$ is large enough. Let we known states in which you can transfer the process in the next step for each specific state when using a particular strategy. Suppose, if we have solved static problems of optimal assignment for each state, we have found incomes from each specific purpose, and then incomes received during the transition.

The source data are summarized in table 3.

TABLE 3. The source data

| $q_i$ | $S_i$ | $S_j$ | $r_{ij}$ |
|-------|-------|-------|----------|
| 1     | 1     | 2     | -17      |
| 2     | 1     | 2     | -23 *    |
| 3     | 1     | 4     | -33      |
| 4     | 1     | 3     | -14      |
| 1     | 2     | 1     | 17 *     |
| 2     | 2     | 1     | 18       |
| 3     | 2     | 4     | -16      |
| 4     | 2     | 3     | 4        |
| 1     | 3     | 2     | 1        |
| 2     | 3     | 1     | 16       |
| 3     | 3     | 4     | -17      |
| 4     | 3     | 4     | -18 *    |
| 1     | 4     | 2     | 18       |
| 2     | 4     | 1     | 34       |
| 3     | 4     | 3     | 17       |
| 4     | 4     | 3     | 15 *     |

Consider a graph $G(X,U)$ whose vertices are the states of the system, and the arcs are possible transitions. Each arc $(x_i, x_j)$ is assigned a cost, that is the income $r_{ij}$ from the transition from state $S_i$ to state $S_j$. 
It is proposed to exclude from the graph multiple arcs. That is, if it is possible to go from state \( S_i \) to state \( S_j \) in several ways (using different strategies), then we choose only the opportunity with high income. (In table 3, the symbol * marks those incomes whose corresponding arcs are excluded from the graph).

In this case, a complete graph is obtained: from any state you can get into any other one in one step. Let's use the method described earlier.

1) Choose an arbitrary circuit \( C^0 = x_i \rightarrow x_1 \rightarrow x_2 \rightarrow x_3 \). We build a subgraph \( G'(X,U') \) containing a circuit \( C^0 \) and an arc \((x_2,x_3)\). Calculate values \( \varphi_1 = 0, \varphi_2 = 18, \varphi_3 = 16, \varphi_4 = 33 \). We are checking. Condition (2) does not hold for:

a) \((x_1,x_2)\), since it does not precede, then it is necessary to replace the arc starting from \( x_i \) to \((x_1,x_2)\);

b) \((x_2,x_3)\), since \( x_2 \) precedes \( x_3 \), it is necessary to consider a new circuit \( x_2 \rightarrow x_3 \rightarrow x_4 \);

c) \((x_1,x_3)\) replace the arc coming from \( x_i \).

d) \((x_3,x_4)\), it is necessary to enter this arc.

2) Consider the new circuit \( C^1 = x_2 \rightarrow x_3 \rightarrow x_4 \). Besides him, in the new subgraph there are two more arcs \((x_2,x_3)\) and \((x_1,x_2)\).

We recalculate \( r_{ij} \), count \( \varphi \) and check condition (2). Now it is not executed only for \((x_1,x_2)\).

Replace with this arc coming from \( x_i \) and recalculate (1) and (2). Since now for all \( i \) and all \( U_i^- \) conditions (2) are fulfilled, the current path \( C^1 \) has the greatest characteristic - the highest average cost of the arc, equal to 2.5. Thus, the optimal policy is to enter the circuit \( x_2 \rightarrow x_3 \rightarrow x_4 \) and then move along it. This circuit assumes to use strategies \( q_4 \) alternately (being in \( S_2 \) ) and \( q_4 \) (being in \( S_3 \)).

If the state \( S_i \) is initial, then, using the strategy \( q_4 \), we move to the state \( S_1 \) and then move along the circuit; if the initial state is \( S_4 \), then, using \( q_2 \), go to \( S_1 \) and next to \( S_3 \).

Conclusion

Thus, a description was given of the game model of the transportation problem, in which the parameters do not depend on time. For this model, an algorithm for finding a compromise point is considered, the application of which is illustrated with a specific example.

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