Bohmian particle trajectories contradict quantum mechanics

Michael Zirpel

March 23, 2009

Abstract

The Bohmian interpretation of quantum mechanics adds particle trajectories to the wave function and ensures that the probability distribution of the particle positions agrees with quantum mechanics at any time. This is not sufficient to avoid contradictions with quantum mechanics. There are correlations between particle positions at different times which cannot be reproduced with real particle trajectories. A simple rearrangement of an experimental test of the Bell-CHSH inequality demonstrates this.

Keywords: Quantum mechanics, Bohmian interpretation, Bohmian mechanics, particle trajectories, Bell inequality, CHSH inequality, Stern-Gerlach experiment

Introduction

The Bohmian interpretation [1] of quantum mechanics (also called Bohmian mechanics [2]) supplements the quantum mechanical wave function with particle trajectories. These trajectories are thought to be real in the following sense:

1. Each particle moves along a trajectory and has at any time $t$ a definite position $Q(t)$ on its trajectory.

2. The result of a position measurement at time $t$ is the particle position $Q(t)$.

In an ensemble of equally prepared particles with the same wave function $\psi$, each particle moves along a randomly chosen trajectory. The quantum equilibrium hypothesis and Bohm’s law of motion [2] guarantee that the quantum mechanical probability distribution of the particle positions

$$\rho(r,t) = |\psi(r,t)|^2$$
is maintained for all times $t$ by the Bohmian theory.

However, the real particle movement along trajectories entails correlations between particle positions at different times. As R. F. Streater noticed in [3] (p. 108) this must contradict quantum mechanics in certain cases because particle positions at different times are represented by non-commuting operators in the Heisenberg picture. Accordingly a Bell-type inequality could be established between certain functions of the particle position at different times, which is fulfilled by trajectories but violated by quantum mechanics.

A simple way to demonstrate this contradiction is to change the standard experimental setting for the test of the CHSH inequality [4], [5] so that all measured quantities are represented by different particle beams in one setting. The Bohmian particle trajectories follow some of these beams so that all quantities will have definite values in each run of the experiment. This implies compliance with the CHSH inequality, which is however violated by quantum mechanics in certain cases.

**CHSH test with Stern-Gerlach apparatuses**

The basic experimental setting was already discussed by D. Bohm in his 1951 text book [6] in the context of the EPR paradox. The source $S$ emits pairs of neutral spin $\frac{1}{2}$ particles in opposite directions, where Stern-Gerlach apparatuses are used to measure spin components: The particle beams are split by inhomogeneous magnetic fields $M_A, M_B$ in different directions. Detection screens $S_A, S_B$ complete the measurement. If a particle is detected in the 'upper' $^1$ half of the screen the corresponding observables $A$ and $B$, respectively have the value $+1$, if the particle is detected in the 'lower' half, the corresponding observables have the value $-1$.

![Figure 1: CHSH test experiment with Stern-Gerlach apparatuses](image)

$^1$The terms 'upper', 'lower' are related to the diagram and not to the real direction.
In a CHSH test experiment the magnetic fields are switched between two different directions on each side so that in total four observables $A, A', B, B'$ are measured. With appropriate directions of the magnetic fields and a suited initial state of the particle pair the expectation value of $A B + A' B' + A' B - A B'$ will be

$$\langle A B + A' B' + A' B - A B' \rangle \approx 2\sqrt{2}$$

which exceeds the bound 2 given by the CHSH inequality [7].

**Four observables in one apparatus**

If the detection screens $S_A, S_B$ are removed, the partial beams can be recombined coherently using other magnetic fields so that the original spin state is restored. This is possible in theory, because the evolution without measurement is unitary and therefore reversible as already has been noticed by D. Bohm in his textbook [6] p.605 (the technical difficulties of the experimental realization are discussed in [8]). After the coherent recombination another pair of magnetic fields $M_{A'}, M_{B'}$ can be used to split the beam again to measure the observables $A', B'$.

![Four observables in one apparatus](image)

In this new setting all four observables $A, A', B, B'$ are represented by different partial beams in one apparatus. They are functions of the particle positions, which are in one-to-one correspondence with some particle spin components. By the placement of the detection screens the experimenter decides which pair of observables is measured. The expectation value of $A B + A' B' + A' B - A B'$ will be the same as above (1).

**Bohmian trajectories fulfill the CHSH inequality**

Bohmian trajectories in a Stern-Gerlach apparatus with packet-like wave function are analyzed in [9], [10]. Nearly all trajectories follow the particle beams.
Where the beam splits, a part of the trajectory set follows one partial beam, the rest follows the other partial beam.
In a single run of the experiment a pair of Bohmian particles follows a randomly chosen pair of trajectories, one in direction $A$, one in direction $B$. The particle moving in direction $A$ moves along its trajectory either through half-plane $A = +1$ or half-plane $A = -1$ and afterwards either through half-plane $A' = +1$ or half-plane $A' = -1$. The same is true for the $B$ particle and the quantities $B, B'$.

Figure 3: A Bohmian trajectory pair with $A = -1, A' = 1$ and $B = 1, B' = 1$

So for each particle pair emitted by the source all four quantities $A, A', B, B'$ will have a definite value. According to proposition 2) these definite values will be measured, when detection screens are placed into the appropriate positions. Definite values, however, always fulfill the CHSH inequality: Let $a_k, a'_k, b_k, b'_k \in \{-1, +1\}$ be the values of the four quantities $A, A', B, B'$ in the $k$-th run of the experiment. A simple valuation table shows that always

$$a_k b_k + a'_k b'_k + a'_k b_k - a_k b'_k = \pm 2$$

So the mean value in $n$ runs of the experiment is bounded by 2 for all $n > 0$

$$\left| \frac{1}{n} \sum_{k=1}^{n} (a_k b_k + a'_k b'_k + a'_k b_k - a_k b'_k) \right| \leq 2 \quad (2)$$

The quantum mechanical expectation value (1) violates this inequality.

**Conclusion**

This demonstrates a contradiction between a quantum mechanical probability statement about particle positions and the consequences of the Bohmian theory. As long as propositions 1) and 2) are thought to be valid, this contradiction cannot be removed. So we must conclude:

*Bohmian particle trajectories contradict quantum mechanics.*
Acknowledgements

Thanks to Prof. Bernhard Lauth and Gerhard Zoubek for many stimulating discussions and constant encouragement.

References

[1] D. Bohm: A suggested interpretation of the quantum theory in terms of ‘hidden’ variables, I and II. Physical Review 85, 166-193 (1952)

[2] D. Dürr, S. Goldstein, R. Tomulka, N. Zanghì: Bohmian Mechanics. http://arxiv.org/abs/0903.2601v1 (2009)

[3] R. F. Streater: Lost Causes in and beyond Physics. Springer-Verlag (2007) http://www.mth.kcl.ac.uk/~streater/lostcauses.html#XI

[4] J. S. Bell: On the Einstein-Podolsky-Rosen paradox. Physics 1, 195-200 (1964)

[5] J. F. Clauser, M. A. Horne, A. Shimony, R. A. Holt: Proposed experiment to test local hidden-variable theories. Phys. Rev. Letters 23 (1969)

[6] D. Bohm: Quantum Theory. Prentice-Hall, Inc. (1951)

[7] J. S. Bell: Bertlmann’s socks and the nature of reality. Journal de Physique, 42 C2 (1981)

[8] B. G. Englert, J. Schwinger, M. O. Scully: Is Spin Coherence like Humpty-Dumpty? I. Found. Phys. Vol. 18, No. 10 (1988)

[9] Dewdney, Holland, Kyprianidis. Phys. Lett. A, 119, 259-267 (1986)

[10] P. R. Holland: The quantum theory of motion. Cambridge University Press (1993)