Investigation of performance measures of power generating unit of sewage treatment plant

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Abstract
The main objective of present study to investigate the availability and profit of power generation systems established in sewage treatment plants. The sewage treatment plant is an industry in which waste sewage water has been treated and waste is used to generate power. For this purpose, a mathematical model has been proposed by considering constant failure and repair rates. The power generating unit in sewage treatment plant comprises six subsystems as sludge digester, gas holding tank, gas burner, gas scrubber, gas engine and power generation. By using appropriate redundancy technique in model development Chapman-Kolmogorov differential equations has been drawn with the help of the Markov birth death process. Availability and profit have been analysed based on making variation in the failure and repair rates. The numerical and graphical results have been depicted for a particular case. The derived results have been discussed with system designers and found useful.

Keywords: Power Generator Unit, Birth-Death Process, Exponential Distribution, Availability.

1. Introduction
As human civilization entered in techno-friendly world, lot of development has been made in the field of industrial, mechanical, medical and entertainment. But for the operation of such systems, electricity is the basic requirement. Without electricity, no one can expect industrial and economic growth. In a country like India in which a large portion of population lives in the villages and towns where no proper electricity is available. This population faces lots of problems in education, business, and daily survivals because of insufficient electricity supply. To overcome this situation, government, policy makers and scientists constantly trying to develop new techniques and technologies to reduce insufficient electricity problem. In past electricity was generated through thermal plants and steam turbine plants. But during last few decades, some other areas for electricity generation like nuclear energy, biofuels are also identified. The electricity generation through sewage water and sludge is a
new area for electricity generation. And sewage treatment plant can play an important role in this as it treats the raw sewage water and removes all sludge from it. After the complete treatment to generate some biogases which can be used to generate electricity. But the available plants are not sufficient in ratio of existing population. So, there is need to establish new plants and improve performance of existing plants. Hence, present study can help system developers to propose new design to enhance reliability of plants.

In exiting literature, it is observed that researchers studied importance of reliability measures in various areas. Ebeling (2008) suggested some techniques to investigate the reliability and availability approaches with the help of failure and repair data of various complete systems. Fytiti and Zabaniotou (2008) proposed and studied the model for maintenance of sewage treatment plants. Michael et al. (2009) analyzed the availability of a state transition system by using Markov process. Komal and Kumar (2010) suggested a technique and used RAM methodology to analyze the performance of the repairable system. Garg and Sharma (2012) conducted a two-stage approach to analyze the reliability and availability by using particle swarm optimization and some statistical methods. Modgil et al. (2013) optimized the performance of shoe manufacturing by using Markov process and analyzed the long-run availability of the system. Kumar and Malik (2014) analyzed repairable computer system by using the reliability model. Addabo et al. (2016) analyzed to analyze the reliability and availability of multicores control system of a UPS modular system. De Sanctis et al. (2016) suggested a methodology to reduce the environmental problems and to maintain the strategy for this by using Markov process. Mishra et al. (2016) proposed a method to analyze the availability of drum manufacturing system by using the Markov approach. Kumar et al. (2017) suggested a methodology to optimize the casting process by using reliability approach. Kumar et al. (2018) estimated mathematical modelling to analyze the reliability for sugar mill plant. Kumar et al. (2020) carried out reliability, dependability and maintainability analysis of power generating unit established in a STP using RAMD technique.

But as discussed above, it is revealed that not much work has been done by the researchers in sewage treatment plant by using the Markovian approach under constant failure and repair rates. So, this study has been conducted to develop a methodology for increasing the performance of the system.

The whole study has been divided into eight sections from which the first section is the introduction, second is the complete description of the system, third shows assumptions, fourth is notations, mathematical modelling is shown in section five, performance analysis carried out in section six while numerical analysis and result discussion has been made in section seven along followed by eight conclusion section.

2. System Description
As shown in (Fig 1) power generating unit of STP comprises six subsystems as sludge digester, gas holding tank, gas burner, gas scrubber, gas engine and power generation. In this unit, power is generated by treating remaining sludge waste after the complete water treatment process. All six subsystems are connected in series. Failure in any subsystem can cause a complete system failure. The detailed explanation of this unit are given below:
i. **Subsystem 'H' (Sludge Digester)**
The sludge digester unit comprise two unit in 2-out-of-2: G connected with subsystem gas holding tank. In this subsystem STP reduce the amount of sludge waste and form biogases.

ii. **Subsystem 'I' (Gas Holding Tank)**
The gas holding tank comprise only one unit connected with subsystem sludge digester, gas burner and gas scrubber. In this subsystem formed gases are stored and it balance the fluctuation in the production of biogases in digester.

iii. **Subsystem 'J' (Gas Burner)**
The gas burner comprise only one unit connected with subsystem gas holding tank. In this subsystem burner disposed excess and unwanted gases from the system.

iv. **Subsystem 'K' (Gas Scrubber)**
The gas scrubber comprise only one unit connected with subsystem gas holding tank and gas engine. This subsystem removes hydrogen sulfide, neutralize harmful components and absorb pollutant from this.

v. **Subsystem 'L' (Gas Engine)**
The gas engine comprise only one unit connected with subsystem gas scrubber and power generation. This subsystem runs on gaseous fuel and heated digester.

vi. **Subsystem 'M' (Power Generation)**
The power generation unit comprise two unit in 2-out-of-2: G connected with subsystem gas engine. In this subsystem power is generated with the help of gas engine and operate all STP units.

3. **Assumptions**
   a) A maintenance policy is available and there is no waiting time for server to repair system.
   b) The system will work as new with full capacity after repair.
   c) Failure rate and repair rates are considered as exponentially distributed.

4. **Notations**
 : System is working with full capacity 
: System is in failure state 
$H, I, J, K, L, M$ : All subsystems are working with full capacity 
$h, i, j, k, l, m$ : Subsystem has failed 
$\Psi_i (1 \leq i \leq 6)$ : Respectively failure rates in subsystems $H, I, J, K, L, M$ 
$\Omega_i (1 \leq i \leq 6)$ : Respectively repair rate in subsystems $H, I, J, K, L, M$ 
$P_0(t)$ : Probability that system is working with full capacity 
$P_i(t), (i = 1, \ldots, 6)$ : Probability of subsystem on $i^{th}$ state at time $t$ 
$S_i, (i = 0, 1, 2, \ldots, 6)$ : State of the subsystem 

Figure 2. State changeover of the system

5. Mathematical modeling of the system
The mathematical model of power generating unit has been developed by using Markov process and supplementary variable technique with the help of state changeover diagram (Fig.-2) where all repair rates are exponentially distributed. This model is described below:

\[
P_0(t + \Delta t) = [(1 - 2\Psi_1 - \Psi_2 - \Psi_3 - \Psi_4 - \Psi_5 - 2\Psi_6)P_0(t) + \Omega_1 P_1(t) + \Omega_2 P_2(t) + \Omega_3 P_3(t) + \Omega_4 P_4(t) + \Omega_5 P_5(t) + \Omega_6 P_6(t)] \Delta t
\]

\[
P_0(t + \Delta t) - P_0(t) = [(2\Psi_1 + \Psi_2 + \Psi_3 + \Psi_4 + \Psi_5 - 2\Psi_6)P_0(t) + \Omega_1 P_1(t) + \Omega_2 P_2(t) + \Omega_3 P_3(t) + \Omega_4 P_4(t) + \Omega_5 P_5(t) + \Omega_6 P_6(t)] \Delta t
\]

Dividing both side by $\Delta t$ and limit $\Delta t \rightarrow \infty$
\[
\lim_{\Delta \to 0} \frac{P_0(t + \Delta t) - P_0(t)}{\Delta t} = \lim_{\Delta \to 0} \left[ (-2\Psi_1 - \Psi_2 - \Psi_3 - \Psi_4 - \Psi_5 - 2\Psi_6)P_0(t) + \Omega_1 P_1(t) + \Omega_2 P_2(t) + \Omega_3 P_3(t) + \Omega_4 P_4(t) + \Omega_5 P_5(t) + \Omega_6 P_6(t) \right]
\]

\[
\left[ \frac{dP_1(t)}{dt} \right] + (2\Psi_1 + \Psi_2 + \Psi_3 + \Psi_4 + \Psi_5 + 2\Psi_6)P_0(t) = \Omega_1 P_1(t) + \Omega_2 P_2(t) + \Omega_3 P_3(t) + \Omega_4 P_4(t) + \Omega_5 P_5(t) + \Omega_6 P_6(t)
\]

\[
\left[ \frac{d}{dt} + \Omega_1 \right] P_1(t) = 2\Psi_1 P_0(t)
\]

\[
\left[ \frac{d}{dt} + \Omega_2 \right] P_2(t) = \Psi_2 P_0(t)
\]

\[
\left[ \frac{d}{dt} + \Omega_3 \right] P_3(t) = \Psi_3 P_0(t)
\]

\[
\left[ \frac{d}{dt} + \Omega_4 \right] P_4(t) = \Psi_4 P_0(t)
\]

\[
\left[ \frac{d}{dt} + \Omega_5 \right] P_5(t) = \Psi_5 P_0(t)
\]

\[
\left[ \frac{d}{dt} + \Omega_6 \right] P_6(t) = 2\Psi_6 P_0(t)
\]

Initial Conditions:
\[ P_0(0) = 1 \]
\[ P_i(0) = 0, \ i=1 \text{to} \ 7 \]

To calculate long run availability we can take \( \frac{d}{dt} = 0 \) as \( t \to \infty \) and \( P_i(t) = P_i \)

From eq. (1-7), steady state probabilities are:
\[
P_1 = \frac{2\Psi_1}{\Omega_1} P_0 \quad P_2 = \frac{\Psi_2}{\Omega_2} P_0 \quad P_3 = \frac{\Psi_3}{\Omega_3} P_0
\]
\[
P_4 = \frac{\Psi_4}{\Omega_4} P_0 \quad P_5 = \frac{\Psi_5}{\Omega_5} P_0 \quad P_6 = \frac{2\Psi_6}{\Omega_6} P_0
\]

Using normalized condition \( \sum P_i = 1 \)
\[ P_0 = \left(1 + \frac{2\Psi_1}{\Omega_1} + \frac{\Psi_2}{\Omega_2} + \frac{\Psi_3}{\Omega_3} + \frac{\Psi_4}{\Omega_4} + \frac{\Psi_5}{\Omega_5} + \frac{2\Psi_6}{\Omega_6}\right)^{-1} \]

long run availability \((A_r)\)

\[ A_r = P_0 = \left(1 + \frac{2\Psi_1}{\Omega_1} + \frac{\Psi_2}{\Omega_2} + \frac{\Psi_3}{\Omega_3} + \frac{\Psi_4}{\Omega_4} + \frac{\Psi_5}{\Omega_5} + \frac{2\Psi_6}{\Omega_6}\right)^{-1} \]

6. Performance Analysis

In this section, a formula for performance analysis is proposed in which \(K_1\) represent the total revenue taken as per unit time, \(A_r\) represent the long-run availability which we have calculated above and \(K_2\) represented the total repair cost.

Performance = \(K_1A_r - K_2\)

\[ = K_1\left(1 + \frac{2\Psi_1}{\Omega_1} + \frac{\Psi_2}{\Omega_2} + \frac{\Psi_3}{\Omega_3} + \frac{\Psi_4}{\Omega_4} + \frac{\Psi_5}{\Omega_5} + \frac{2\Psi_6}{\Omega_6}\right)^{-1} - K_2 \]

7. Numerical Analysis & Discussion

Here, variation in availability based on changes made in failure rates has been analyzed with respect to the failure rate \(\Psi_2\) range from 0.0009 to 0.0063. The varying failure rates of other subsystems as follows: \(\Psi_1 = 0.005\) to 0.05, \(\Psi_3 = 0.0003\) to 0.003, \(\Psi_4 = 0.0025\) to 0.025, \(\Psi_5 = 0.0007\) to 0.007 and \(\Psi_6 = 0.008\) to 0.08 by considering failure rate of subsystems follows exponential distribution and repair rates are: \(\Omega_1 = 1.75\), \(\Omega_2 = 0.35\), \(\Omega_3 = 0.08\), \(\Omega_4 = 1.2\), \(\Omega_5 = 0.12\) and \(\Omega_6 = 3\). This analysis reveals that availability of the system decreases 4.78% approximately when failure rate \(\Psi_1\) increase from 0.005 to 0.05. Availability of the system decrease 3.19% approximately when failure rate \(\Psi_3\) increase from 0.0003 to 0.003. Availability of the system decrease 1.79% approximately when failure rate \(\Psi_4\) increase from 0.0025 to 0.025. Availability of the system decrease 4.87% approximately when failure rate \(\Psi_5\) increase from 0.0007 to 0.007. Availability of the system decrease 4.47% approximately when failure rate \(\Psi_6\) increase from 0.008 to 0.08.

### Table 1. Variation in availability of the system w.r.t. failure rate \(\Psi_2\)

| \(\Psi_2\) | \(\Psi_1\) | \(\Psi_3\) | \(\Psi_4\) | \(\Psi_5\) | \(\Psi_6\) |
|-----------|---------|---------|---------|---------|---------|
| 0.0009    | 0.795338| 0.927515| 0.9442552| 0.9578216| 0.9278228| 0.9317184|
| 0.0015    | 0.97371 | 0.9272751| 0.9427292| 0.9562515| 0.9263548| 0.9302326|
Variation in availability based on variation in repair rates has been analyzed with respect to the repair rate $\Omega_2$ range from 0.35 to 0.89. The varying repair rates of other subsystems as follows: $\Omega_1 = 1.75$ to 1.79, $\Omega_3 = 0.08$ to 0.12, $\Omega_4 = 1.2$ to 1.6, $\Omega_5 = 0.12$ to 0.16 and $\Omega_6 = 3$ to 3.4 by considering repair rate of subsystems follows exponential distribution and repair rates are: $\Psi_1 = 0.005$, $\Psi_2 = 0.0009$, $\Psi_3 = 0.0003$, $\Psi_4 = 0.0025$, $\Psi_5 = 0.0007$ and $\Psi_6 = 0.008$. This analysis reveals that availability of the system increase 0.012% approximately when repair rate $\Omega_1$ increase from 1.75 to 1.79. Availability of the system increase 0.12% approximately when repair rate $\Omega_3$ increase from 0.08 to 0.12. Availability of the system increase 0.051% approximately when repair rate $\Omega_4$ increase from 1.2 to 1.6. Availability of the system increase 0.14% approximately when repair rate $\Omega_5$ increase from 0.12 to 0.16. Availability of the system increase 0.06% approximately when repair rate $\Omega_6$ increase from 3 to 3.4.

**Table 2.** Variation in availability of the system w.r.t. repair rate $\Omega_2$
Variation in profit based on variation in failure rates has been analyzed with respect to the failure rate $\Psi_2$ range from 0.0009 to 0.0063. The varying failure rates of other subsystems as follows: $\Psi_1 = 0.005$ to 0.05, $\Psi_3 = 0.0003$ to 0.003, $\Psi_4 = 0.0025$ to 0.025, $\Psi_5 = 0.0007$ to 0.007, and $\Psi_6 = 0.008$ to 0.08 by considering failure rate of subsystems follows exponential distribution and repair rates are: $\Omega_1 = 1.75$, $\Omega_2 = 0.35$, $\Omega_3 = 0.08$, $\Omega_4 = 1.2$, $\Omega_5 = 0.12$, and $\Omega_6 = 3$. This analysis reveals that profit of the system decrease 5.32% approximately when failure rate $\Psi_1$ increase from 0.005 to 0.05. Profit of the system decrease 3.54% approximately when failure rate $\Psi_3$ increase from 0.0003 to 0.003. Profit of the system decrease 1.99% approximately when failure rate $\Psi_4$ increase from 0.0025 to 0.025. Profit of the system decrease 5.42% approximately when failure rate $\Psi_5$ increase from 0.0007 to 0.007. Profit of the system decrease 4.98% approximately when failure rate $\Psi_6$ increase from 0.008 to 0.08.
Table 3. Variation in system’s performance w.r.t. failure rate $\Psi_2$

| $\Psi_2$ | $A_0 = \Psi_i = 0.005$ | $A_1 = \Psi_i = 0.05$ | $A_2 = \Psi_i = 0.009$ | $A_3 = \Psi_i = 0.009$ | $A_4 = \Psi_i = 0.009$ | $A_5 = \Psi_i = 0.009$ | $A_6 = \Psi_i = 0.009$ |
|-----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 0.0009    | 3891.237       | 3684.3203      | 3753.4066      | 3813.6723      | 3680.1986      | 3697.5591      |
| 0.0015    | 3884.039       | 3677.7291      | 3746.6168      | 3806.706       | 3673.6192      | 3690.9302      |
| 0.0021    | 3876.863       | 3671.1567      | 3739.8467      | 3799.7602      | 3667.0584      | 3684.3203      |
| 0.0027    | 3869.709       | 3664.603       | 3733.9092      | 3792.835       | 3660.5163      | 3677.7291      |
| 0.0033    | 3862.575       | 3658.0678      | 3726.3653      | 3785.93        | 3653.9927      | 3671.1567      |
| 0.0039    | 3855.464       | 3651.5511      | 3719.6537      | 3779.0454      | 3647.4875      | 3664.603       |
| 0.0045    | 3848.373       | 3645.0528      | 3712.9615      | 3772.1808      | 3641.0007      | 3658.0678      |
| 0.0051    | 3841.304       | 3638.5728      | 3706.2884      | 3765.3364      | 3634.5321      | 3651.5511      |
| 0.0057    | 3834.256       | 3632.111       | 3699.6346      | 3758.5119      | 3628.0816      | 3645.0528      |
| 0.0063    | 3827.228       | 3625.6674      | 3692.9997      | 3751.7073      | 3621.6493      | 3638.5728      |

Figure 5. Variation in system’s performance w.r.t. failure rate $\Psi_2$

Variation in profit based on variation in repair rates has been analyzed with respect to the repair rate $\Omega_2$ range from 0.35 to 0.89. The varying repair rates of other subsystems as follows: $\Omega_1 = 1.75 to 1.79, \ Omega_2 = 0.08 to 0.12, \ Omega_3 = 1.2 to 1.6, \ Omega_4 = 0.12 to 0.16$ and $\Omega_6 = 3 to 3.4$ by considering repair rate of subsystems follows exponential distribution and repair rates are: $\Psi_2 = 0.0001, \ Psi_3 = 0.0009, \ Psi_4 = 0.0003, \ Psi_5 = 0.0025, \ Psi_6 = 0.0007$ and $\Psi_6 = 0.008$. This analysis reveals that profit of the system increase 0.014% approximately when repair rate $\Omega_3$ increase from 1.75 to 1.79. Profit of the system increase 0.13% approximately when repair rate $\Omega_2$ increase from 0.08 to 0.12. Profit of the system increase 0.056% approximately when repair rate $\Omega_4$ increase from 1.2 to 1.6. Profit of the system increase 0.16% approximately when repair rate $\Omega_5$ increase from
0.12 to 0.16. Profit of the system increased by approximately 0.068% when repair rate $\Omega_6$ increased from 3 to 3.4.

Table 4. Variation in system’s performance w.r.t. repair rate $\Omega_2$

| $\Omega_2$ | $P_0 = \Omega_1 = 1.75$ | $P_1 = \Omega_1 = 1.79$ | $P_3 = \Omega_1 = 1.75$ | $P_4 = \Omega_1 = 1.75$ | $P_5 = \Omega_1 = 1.75$ | $P_6 = \Omega_1 = 1.75$ |
|------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.35       | 3891.237        | 3891.7743       | 3896.4995       | 3893.4284       | 3897.3776       | 3893.8772       |
| 0.41       | 3892.82         | 3893.3576       | 3898.0859       | 3895.0128       | 3898.9647       | 3895.4619       |
| 0.47       | 3894            | 3894.5373       | 3899.268        | 3896.1934       | 3900.1472       | 3896.6427       |
| 0.53       | 3894.912        | 3895.4504       | 3900.1829       | 3897.1071       | 3901.0624       | 3897.5566       |
| 0.59       | 3895.64         | 3896.1779       | 3900.9119       | 3897.8351       | 3901.7917       | 3898.2848       |
| 0.65       | 3896.233        | 3896.7714       | 3901.5065       | 3898.429        | 3902.3865       | 3898.8787       |
| 0.71       | 3896.726        | 3897.2646       | 3902.0007       | 3898.9226       | 3902.8809       | 3899.3724       |
| 0.77       | 3897.143        | 3897.681        | 3902.418        | 3899.3393       | 3903.2983       | 3899.7892       |
| 0.83       | 3897.499        | 3898.0373       | 3902.775        | 3899.6958       | 3903.6555       | 3900.1458       |
| 0.89       | 3897.807        | 3898.3456       | 3903.0839       | 3900.0044       | 3903.9645       | 3900.4544       |

Figure 6. Variation in system’s performance w.r.t. repair rate $\Omega_2$

8. Conclusion

The proposed model is very useful to improve performance of power generating units in sewage treatment plants. The similar kind of models can be developed and utilized to enhance performance of other industries also. The detailed outcome of the system has been shown in tables 1-4 which reveal the effect of variation in availability and performance of the system is highly affected by variation in failure and repair rates. From the above result, it is observed that there is significant variation from 1.79% to 4.78% variation in availability with respect to variation in various failure rates. The gas engine is a highly sensitive subsystem with approximately 4.87% variation in the availability in the comparison of other subsystems. So, the subsystem gas engine is needed highly consideration to
improve the performance of the system. It is also observed that repair rates are less sensitive with variation from 0.012% to 0.14%. The outcomes of the study are shared with the plant employees and found the results significant to improve the plant performance.

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