String creation in D6-brane background

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Abstract

The production of string charge during a crossing of certain oriented D-branes is studied. We compute the string charge in the system of a probe D2-brane and a background D6-brane by use of the equations of motion in the ten-dimensions. We confirm the creation of string charge as inflow from the background D6-brane.

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It was first pointed out in [1] that D3-branes are created when certain oriented NS5-brane and D5-brane cross each other and then the created branes are suspended between these 5-branes. This phenomenon has several cousins related by U-duality. For example, a fundamental string is created when a Dp-brane and a D(8-p)-brane, which are mutually transverse, cross. They have been confirmed from various points of view [2]-[17].

However, it has not been clarified yet how the charge of created brane emerges when the branes cross. In this letter we examine in particular the system of a probe D2-brane and a background D6-brane. We calculate the created string charge by use of the equations of motion derived from the action in the ten-dimensions. This ten-dimensional action itself is obtainable from the eleven-dimensional one with M2-brane source term. The D6-brane background is realized by the Taub-NUT geometry in the eleven-dimensions. The string charge in this particular system turns out to be $Q_{F1} = \pi RT(1 + \cos \theta_0)$, where $\theta_0$ is a parameter of the M2-brane embedding, $T$ is the tension of it and $R$ is the radius of $S^1$ of the eleventh-direction. Before or after the crossing of these branes (equivalently $\theta_0$ is $\pi$ or 0 respectively, as will be explained later), one can see $Q_{F1}$ will be 0 or $2\pi RT$. This means indeed the creation of string charge which is supplied from the background D6-brane.

The original technique used here was given in [18] in which they studied the duality between KK- and H-monopoles in the type II theory by showing the charge inflow of a string into the KK-monopole background.

Our starting point is the eleven-dimensional SUGRA (M-theory) whose bosonic part is [13]

$$S_{11D} = \frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{-G} \left[ R_{11} - \frac{1}{2 \cdot 4!} G_{MNPQ}^2 \right] - \frac{1}{2\kappa_{11}^2 \cdot 6} \int C_3 \wedge G_4 \wedge G_4, \quad (1)$$

where $G_{MN}$ is the eleven-dimensional metric ($M, N, \ldots = 0, 1, \ldots, 10$) and $G_4 = dC_3$ is the field strength of the three-form gauge field $C_3$. The M2-brane has the electric charge of $C_3$. Adding the non-linear $\sigma$-model action as
the source term of M2-brane to the above action, it becomes $S_{11D} + S^{M2}_{source}$ where

$$S^{M2}_{source} = T \int d^{11}x \int d^3\xi \delta^{(11)}(x^M - X^M) \left\{ -\frac{1}{2} \sqrt{\gamma} \gamma^{ij} \partial_i X^M \partial_j X^N G_{MN} + \frac{1}{2} \sqrt{\gamma} + \frac{1}{3!} \epsilon^{ijk} \partial_i X^M \partial_j X^N \partial_k X^P C_{MNP} \right\}. \tag{2}$$

$T$ is the membrane tension and $X^M$ are the space-time coordinates of the M2-brane. The world volume coordinates and the induced metric on the M2-brane are denoted by $\xi^i (i = 0, 1, 2)$ and $\gamma_{ij}$.

Let us consider the dimensional reduction of the actions into the ten-dimensions. They provide the IIA SUGRA theory. The eleven-dimensional metric $G_{MN}$ transmutes into ten-dimensional metric $g_{\mu\nu}$ ($\mu, \nu = 0, 1, \ldots, 9$), one-form gauge potential $A$ and dilaton field $\phi$:

$$G_{MN} = e^{-\frac{2}{3} \phi} \left( g_{\mu\nu} + e^{2\phi} A_\mu A_\nu - e^{2\phi} F_{\mu\nu}^2 \right). \tag{3}$$

$C_3$ provides three-form $C_3^{10}$ and two-form $B_2^{10}$ by the reduction;

$$C_{\mu\nu\rho} = C_{\mu\nu\rho}^{10}, \tag{4}$$
$$C_{\mu\nu10} = B_{\mu\nu}^{10}. \tag{5}$$

We will abbreviate $C_3^{10}$ and $B_2^{10}$ to $C_3$ and $B_2$. Inserting (3), (4) and (5) into the actions, we obtain $S^{bulk}_{10D} + S^{source}_{10D}$ where

$$S^{bulk}_{10D} = \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} \left( R_{10} + 4(\partial \phi)^2 - \frac{1}{2} \frac{H_{\mu\nu\rho}^2}{3!} \right) \right\} - \frac{1}{2} F_{\mu\nu}^2 - \frac{1}{2} F_{\mu\nu\rho}^2 - \frac{1}{2} \int dC_3 \wedge dC_3 \wedge B_2, \tag{6}$$

and

$$S^{source}_{10D} = T \int d^{10}x \int d^3\xi \delta^{(10)}(x^M - X^M) \left\{ \frac{1}{2} \sqrt{\gamma} - \frac{1}{2} \sqrt{\gamma} \gamma^{ij} e^{-\frac{2}{3} \phi} (h_{ij} - e^{2\phi} V_i V_j) \right\} + \frac{1}{3!} \epsilon^{ijk} \partial_i X^\mu \partial_j X^\nu \partial_k X^\rho C_{\mu\nu\rho} + \frac{1}{2} \epsilon^{ijk} \partial_i X^\mu \partial_j X^\nu (V_k - \partial_k X^\rho A_\rho) B_{\mu\nu}. \tag{7}$$
In (3), \( F_2 = dA_1, H_3 = dB_2 \) and \( G_4 = dC_3 + A_1 \wedge H_3 \). We take \( 2\kappa_{10}^2 = 1 \).

The source term of M2-brane reduces to (7), where

\[
V_i = \partial_i X^{10} + \partial_i X^\mu A_\mu, \quad h_{ij} = \partial_i X^\mu \partial_j X^\nu g_{\mu\nu}.
\]

The equations of motion which we need are those for the fields \( C, B \) and \( A \).

They can be read from (3) and (7) as follows;

\[
C : \quad \partial_\sigma (\sqrt{-g} G^{\sigma\mu\nu\rho} + \frac{1}{2! \cdot 3!} \epsilon^{\mu\nu\rho \ldots \ldots} \partial C \ldots B.)
\]

\[
= - T \int d^3 \xi \delta^{(10)} (x^\mu - X^\mu) \epsilon^{ijk} \partial_i X^\mu \partial_j X^\nu \partial_k X^\rho,
\]

B : \quad \partial_\mu (\sqrt{-g} e^{-2\phi} H^{\mu\nu} + \sqrt{-g} G^{\mu\nu\sigma} A_\sigma) + \frac{1}{2! \cdot 3! \cdot 3!} \epsilon^{\mu\nu\rho \ldots \ldots} \partial C \ldots \partial C \ldots C
\]

\[
= - \int d^3 \xi \delta^{(10)} (x^\mu - X^\mu) \epsilon^{ijk} \partial_i X^\mu \partial_j X^\nu (V_k - A_k),
\]

A : \quad \partial_\nu (\sqrt{-g} F^{\nu\mu}) - \frac{1}{6} \sqrt{-g} G^{\mu\nu\rho\sigma} H_{\mu\nu\rho\sigma}
\]

\[
= T \int d^3 \xi \delta^{(10)} (x^\mu - X^\mu) \sqrt{-g} \gamma^{ij} \epsilon^{4\phi} \partial_i X^\mu V_j.
\]

Our next task is to describe a D6-brane background in the IIA theory. It is well known that the action (1) admits the eleven-dimensional KK-monopole solution and in the IIA limit it becomes the D6-brane [20]. Thus we consider the KK-monopole in the eleven-dimensions which is described by Taub-NUT metric (plus the flat seven-dimensions),

\[
ds^2 = \eta_{mn} dx^m dx^n + U \left[ dx^{10} + A_\phi d\phi \right]^2 + U^{-1} (dr^2 + r^2 d\Omega_2^2),
\]

where \( m, n = 0, 1, \ldots, 6 \). To avoid the singularity at \( r = 0 \), \( x^{10} \) must have periodicity \( 2\pi R \), where R is the radius of the circle of the eleventh-direction.

The potential terms \( A_\phi \) and \( U(r) \) are

\[
A_\phi^S = \frac{R}{2} (1 + \cos \theta),
\]

\[
U(r) = \left(1 + \frac{R}{2r}\right)^{-1}.
\]
We denote the coordinates regular at the south pole (θ = π) and the north pole (θ = 0) respectively by the indices $S$ and $N$. The metric (10) defined by eq.(11) is non-singular at the south pole but has a coordinate singularity at the north pole ($\theta = 0$). The singularity can be removed by

$$x_N^{10} = x_S^{10} + R\phi,$$

(13)

then the direction of Dirac string is changed,

$$A_N^\phi = \frac{R}{2}(-1 + \cos \theta).$$

(14)

This transformation of coordinates plays particularly important role when we discuss the creation of string charge. Comparing the metric to eq.(3), we take $U = e^{\frac{4}{3}\phi}$.

The equations (9) could become simpler in the Taub-NUT background. The term in the left hand side of the third equation in (9), $\partial_\mu (\sqrt{-g} F^{\mu\nu})$, trivially vanishes. The terms in the eqs.(9), which originate from Chern-Simon-like term in the action (6), turn out to be irrelevant as will be shown. Dropping out these terms, the equations of motion (9) become

$$C : \partial_\sigma (\sqrt{-g} G^{\sigma\mu\nu}) = \sqrt{-g} j_{D2}^{\mu\nu},$$

$$B : \partial_\mu h^{\mu\nu} = \sqrt{-g} j_{F1}^{\mu\nu},$$

$$A : G^{\mu\nu\rho\sigma} h_{\nu\rho\sigma} = 6T \int d^3 \xi \delta^{(10)} (x^\mu - X^\mu) \epsilon^{ijk} \partial_i X^\mu \partial_j X^\nu \partial_k X^{10} + \partial_\mu (\sqrt{-g} G^{\mu\nu\phi} A_\phi),$$

(15)

where $h^{\mu\nu} \equiv \sqrt{-g} U^{-\frac{2}{3}} H^{\mu\nu}$, $j_{D2}$ and $j_{F1}$ are the D2-brane and the fundamental string currents. We take p-brane charge as $Q = \int_{S^{8-p}} * F_{p+2} = \int_{V^{9-p}} * j_{p+1}$. Here $F_{p+2}$ is the $(p + 2)$-form field strength and $j_{p+1}$ is the p-brane current. The definition of the Hodge dual contain a dilaton dependent factor to correctly have a generalized electric-magnetic duality.
Now, let us consider the following embedding of a probe M2-brane in the eleven-dimensions in order to check the consistency of eqs. (15),

\[
X^{10} = 2\pi R \xi^1, \\
X^t = \xi^0, \\
X^r = \xi^2, \\
X^\theta = 0, \\
X^\phi = 0, \\
X^m = 0 \quad m = 1 \ldots 6.
\] (16)

This embedding describes the configuration of an M2-brane wrapping exclusively around the eleventh-direction. Therefore, in the ten-dimensions, it becomes a string which is extending towards the \( r \)-direction.

With this embedding the eqs. (15) become

\[
C : \quad \partial_\sigma (\sqrt{-g} G^{\sigma \mu \nu \rho}) = 0, \\
B : \quad \partial_{\rho} h^{\rho \mu \nu} = \sqrt{-g} j^{\mu \nu}_F \\
\quad = \partial_\phi (\sqrt{-g} G^{\rho \mu \nu \phi} A_\phi) - 2\pi RT \int d^3 \xi \delta^{(10)}(x^\mu - X^\mu) \epsilon^{ij1} \partial_i X^\mu \partial_j X^\nu, \\
A : \quad G^{\mu \nu \rho \sigma} h_{\nu \rho \sigma} = 0.
\] (17)

Let \( G^{\mu \nu \rho \sigma} = 0 \). The current which couples to the NS-NS 2-form \( B \) can be read as

\[
\sqrt{-g} j^{ir}_F = RT \delta(\theta) \delta(\phi) \delta^{(6)}(x).
\] (18)

Thus We have the charge of the fundamental string,

\[
Q_{F1} = \int d^6 x d\theta d\phi \sqrt{-g} j^{ir} = 2\pi RT.
\] (19)

This is in agreement with the relation of string charge and M2-brane charge. Hence the equations (15) provide the desired result for the above embedding.
We would like to consider the system of a probe D2-brane in the D6-brane background both of which are mutually transversed. We will show the creation of string charge in view of the IIA theoretical standpoint. For this purpose we examine the following embedding of an M2-brane in the eleven-dimensions.

\[
\begin{align*}
X^t &= \xi^0, \\
X^r &= \xi^2, \\
X^\theta &= \theta_0 (\text{const}), \\
X^\phi &= 2\pi \xi^1, \\
X^m &= 0, \\
X_{S}^{10} &= 0.
\end{align*}
\]

Obviously, the M2-brane has no winding around the eleventh-direction in the coordinate system regular at the south pole. On the other hand, it can wind around the direction, i.e. \(X_N^{10} = 2\pi \xi^1\), in the coordinate system regular at the north pole obtained by the gauge transformation (13). This seems to imply string charge creation in the ten-dimensional viewpoint. We will investigate concretely this phenomenon by use of the equations of motions (15).

Our interest is the string charge that is the topological charge associated with the winding around the eleventh-direction. Hence we do not need to take care of supersymmetry. However, before calculating the string charge, it is worth considering the correspondence between the embedding (20) and that of \([13, 14]\) which preserve supersymmetry manifestly. In those papers string creation was shown by investigating geometrically the shape of an M2-brane but not the winding around the eleventh-direction. They introduce \([21]\) the following holomorphic coordinates \((v, y)\) to describe the embedding of the M2-brane.

\[
\begin{align*}
y &= e^{-\frac{1}{2}(r \cos \theta + ix_{10}^s)} \sqrt{r(1 - \cos \theta)}, \\
v &= \frac{2}{R} r \sin \theta e^{i\phi}.
\end{align*}
\]
M2-brane embedded holomorphically in the KK-monopole background is given by

\[ y = \text{const.} \]  \quad (23) 

The shape of the M2-brane described by this curve looks a parabolic-like surface, see Fig. 1(a). The embedding (20) means that we approximate this surface by a cone in the eleven-dimensions, see Fig. 1(c). Even though this approximation is not compatible with complex structure of the Taub-NUT space, we may use the approximation to study the NS-NS charge since the embedding to the eleventh-direction will play the role in the determination of the charge and it can be still captured by this approximation. The KK-monopole is located in the distance of the right (left) infinity in the eleven-

\[ 3 \] Although the configuration considered in [13] is M5-brane with a KK-monopole, the same relation is also applicable to our case, an M2-brane, because the condition of holomorphy is for the two-dimensional part of branes. In our case the topology of the M2-brane world volume is \( R^3 \times \Sigma \), where \( \Sigma \) is holomorphically embedded Riemann surface in Taub-NUT space.
dimensions before (after) string creation in ten-dimensions [13, 14]. Here we set the position of the KK-monopole the origin by a parallel shift of the coordinates. The configuration in Fig 1(a), which describes the created string in the ten-dimensions, corresponds to the case of $\theta_0 = 0$. The M2-brane should be pulled by the KK-monopole when it moves, since it is attached to the M2-brane in the embedding (20). Hence when the KK-monopole moves to the right direction in Fig 1, $\theta_0$ should become $\pi$. Therefore the parameter $\theta_0$ should be $\pi$ or 0 before or after the branes cross, respectively.

We now start to calculate string charge by inserting the embedding (20) (or (13)) to (15). As the probe brane is a point-like object in view of the space of the D6 brane world volume, it is natural to take spherical coordinates, $l$ and $\varphi_i$ ($i = 1, \ldots, 5$) where $l$ is the radial direction and $\varphi_i$’s are the angular ones in the space. We assume that the indices run over only the $t, r, \theta, \phi, l$ directions.

As we mentioned before, the parts in eqs.(9) from the Chern-Simon term automatically vanish under the assumption. We find the following solution,

$$\sqrt{-g}G^{tr\phi} = -T \delta(\theta - \theta_0) \Theta(l) \delta^{(5)}(\varphi^i),$$

$$h^{tr\theta} = 2RT(1 + \cos \theta_0) \Theta(\theta - \theta_0) \delta(l) \delta^{(5)}(\varphi^i) - \alpha(r, \theta_0) \delta(x^l) \cdot \theta,$$

$$h^{trl} = \alpha(r, \theta_0) \delta(x^l),$$

$$h^{t\theta l} = -\partial_r \alpha(r, \theta_0) \delta(x^l) \cdot \theta,$$

otherwise = 0, \quad (24)$$

where $\alpha(r, \theta) = -4\pi R r^2 \sin \theta(1 + \cos \theta) \sqrt{|\gamma|} \gamma^{11}$ and we used the fact that $\Theta(x)\delta(x) = \frac{1}{2}\delta(x)$. Then the non-vanishing currents of the D2-brane and the string become

$$\sqrt{-g}J^{tr\phi}_{D2} = T \delta(l) \delta^{(5)}(\varphi^i),$$

$$\sqrt{-g}J^{tr}_{F1} = \frac{RT}{2}(1 + \cos \theta_0) \delta(\theta - \theta_0) \delta(l) \delta^{(5)}(\varphi^i).$$

We should note that the form of these equations does not depend on the selection of coordinate patches. The charge of the D2-brane and the string
Thus we finally obtain string charge creation, $Q_{F1}$ becomes $2\pi RT$ from 0 as $\theta_0$ going to 0 from $\pi$. This gives correct relation between the string tension and that of D2-brane. The net charge must be conserved in the whole space, therefore we conclude that the electric NS-NS charge must be supplied from the D6-brane. Note that $Q_{D2}$ does not depend on $\theta_0$. This means the D2-brane always exists on the process of string creation.

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**References**

[1] A. Hanany and E. Witten, “TYPE IIB SUPERSTRINGS, BPS MONOPOLES, AND THREE-DIMENSIONAL GAUGE DYNAMICS”, Nucl. Phys. **B492** (1997) 152, [hep-th/9611230](http://arxiv.org/abs/hep-th/9611230).

[2] C. Bachas, M. R. Douglas and M. B. Green, “Anomalous Creation of Branes”, JHEP **9707** (1997) 002, [hep-th/9705074](http://arxiv.org/abs/hep-th/9705074).

[3] U. H. Danielsson, G. Ferretti and I. R. Klebanov, “Creation of Fundamental Strings by Crossing D-branes”, Phys. Rev. Lett. **79** (1997) 1984, [hep-th/9705084](http://arxiv.org/abs/hep-th/9705084).
[4] Y. Imamura, “Born-Infeld Action and Chern-Simons Term from Kaluza-Klein Monopole in M-theory”, Phys. Lett. B414 (1997) 242, hep-th/9706144

[5] I. R. Klebanov, “D-branes and Creation of Strings”, Nucl. Phys. Proc. Suppl. 68 (1998) 140, hep-th/9709160

[6] O. Bergman, M.R. Gaberdiel and G. Lifschytz, “Branes, Orientifolds and the Creation of Elementary Strings”, Nucl. Phys. B509 (1998) 194, hep-th/9705130

[7] S. P. de Alwis, “A note on brane creation”, Phys. Lett. B413 (1997) 49, hep-th/9706142

[8] P. M. Ho and Y. S. Wu, “Brane Creation in M(atrix) Theory”, Phys. Lett. B420 (1998) 43, hep-th/9708137

[9] Y. Imamura, “D-particle creation on an orientifold plane”, Phys. Lett. B418 (1998) 55, hep-th/9710026

[10] N. Ohta, T. Shimizu and J.G. Zhou, “Creation of Fundamental String in M(atrix) Theory”, Phys. Rev. D57 (1998) 2040, hep-th/9710218

[11] U. H. Danielsson and G. Ferretti, “Creation of Strings in D-particle Quantum Mechanics”, Nucl. Phys. Proc. Suppl. 68 (1998) 78, hep-th/9709171

[12] O. Bergman, M.R. Gaberdiel and G. Lifschytz, “String Creation and Heterotic-Type I’ Duality”, Nucl. Phys. B524 (1998) 524, hep-th/9711098

[13] T. Nakatsu, K. Ohta, T. Yokono and Y. Yoshida, “A Proof of Brane Creation via M theory”, Mod. Phys. Lett. A13 (1998) 293, hep-th/9711117

[14] Y. Yoshida, “Geometrical Analysis of Brane Creation via M-Theory”, Prog. Theor. Phys. 99 (1998) 305, hep-th/9711177
[15] C. P. Bachas and M. B. Green, “A Classical Manifestation of the Pauli
Exclusion”, JHEP 9801 (1998) 015, hep-th/9712186

[16] T. Kitao, N. Ohta and J. G. Zhou, “Fermionic Zero Mode and String
Creation between D4-Branes at Angles”, Phys. Lett. B428 (1998) 68,
hep-th/9801135

[17] C. G. Callan, A. Guijosa and K. G. Savvidy, “Baryons and String Cre-
ation from the Fivebrane Worldvolume Action”, hep-th/9810092

[18] R. Gregory, J. A. Harvey and G. Moore, “Unwinding strings and T-
duality of Kaluza-Klein and H-Monopoles”, hep-th/9708086

[19] M. J. Duff, “Supermembranes”, hep-th/9611203

[20] P. K. Townsend, “The eleven-dimensional supermembrane revisited”,
Phys. Lett. B350 (1995) 184, hep-th/9501068

[21] T. Nakatsu, K. Ohta, T. Yokono and Y. Yoshida, “Higgs Branch of
N=2 SQCD and M theory Branes”, Nucl. Phys. B519 (1998) 159, hep-
th/9707258

T. Nakatsu, K. Ohta and T. Yokono, “On the Baryonic Branch Root of
N=2 MQCD”, Phys. Rev. D58 (1998) 26003, hep-th/9712029

T. Nakatsu, “On N=2 MQCD”, hep-th/9805107