Linear Feedback Control of Chaotic Motion in Gear Transmission System

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Abstract. Considering the internal and external excitation such as backlash, time-varying mesh stiffness (TVMS), transmission error and friction force, a lumped mass model of the spur gear system is established. On this basis, the influences of the support stiffness on dynamic response including bifurcation diagrams, time response diagrams, amplitude–frequency spectrums, phase plane diagrams, Poincaré maps are examined by using numerical method. To avoid the system moving in the unstable region, a linear feedback control method (LFCM) in the gear system is realized. Numerical simulations show that the LFCM is effective and feasible, hence the gear system can be controlled from chaotic motion to stable periodic orbits by using the appropriate control parameter.

1. Introduction

With the development of nonlinear theory, the bifurcations and chaos of the geared system have become the most interesting research areas [1-3]. Chaos motion has a great influence on the stability and reliability of gear system, therefore active control was used in various fields with its good effect and adaptability, and lots of researches were done on gear vibration control [4-6]. Among the control models, chaotic vibration suppressed by using linear and nonlinear system feedback is effectively method, and different methods such as additional periodic signal method, plus load method and phase method was adopted. Since LFCM can achieve good control performance, it has become a new field of engineering applications.

In this paper, the bifurcation and chaos behavior of the gear system are studied, in which the excitations such as backlash, TVMS and friction force are included. Meantime, the influences of support stiffness on dynamic response including bifurcation diagrams, time response maps, amplitude-frequency spectrums, phase plane diagrams, Poincaré maps are tested by numerical method. To avoid the system moving in the unstable region, a LFCM is adopted to control the chaotic motion in the system, and the correctness of LFCM is verified by numerical solution.

2. Gear transmission system model

The developed model is based on a one-stage gearbox [1]. As seen in Figure. 1, gear meshing is described by backlash, stiffness and damping elements along the line–of–action direction. If \( m_n (n=1,2) \) are mass of pinion and gear, \( l_n \ (n=1,2) \) are mass moment of inertia, \( k_n \ (n=1,2) \) are bearing support stiffness, \( c_s (n=1,2) \) are bearing damping, respectively. Meanwhile, \( k(t) \) are time-varying mesh stiffness, \( c_m \) are damping coefficient of gear pair, \( y_e \ (n=1,2) \) are translational displacement of pinion and gear, \( \theta_e \ (n=1,2) \) are rotational displacement. \( T_i \) is drive torque, and \( T_f \) is load torque. If \( F_n \) are meshing forces, \( F_f \) are friction forces, \( \mu(t) \) are friction coefficients, \( L_{dn} \ (n=1,2) \) are time-varying sharing ratios, \( \lambda(t) \) are...
direction coefficient of the friction forces, the value of \( k(t) \) is taken according to Ref.1, then the system model can be obtained as follows

\[
\begin{align*}
\frac{m_1 \ddot{y}_1 + c_{11} \dot{y}_1 + k_{11} y_1}{m_2 \ddot{y}_2 + c_{22} \dot{y}_2 + k_{22} y_2} &= -F_n \\
I_1 \ddot{\theta}_1 + r_{b1} F_n + [L_{11} L_{d1}(t) - L_{21} L_{d2}(t) \dot{\lambda}(t)] \mu(t) F &= T_1 \\
I_2 \ddot{\theta}_2 - r_{b2} F_n - [L_{12} L_{d1}(t) - L_{22} L_{d2}(t) \dot{\lambda}(t)] \mu(t) F_n &= -T_2
\end{align*}
\]

Let \( L_3 = L_{11} L_{d1}(t) - L_{21} L_{d2}(t) \dot{\lambda}(t) \), \( L_2 = L_{12} L_{d1}(t) - L_{22} L_{d2}(t) \dot{\lambda}(t) \), \( y = y_1 + r_{b1} \dot{\theta}_1 - y_2 - r_{b2} \dot{\theta}_2 - e(t) \), then the model above can be derived as

\[
\begin{align*}
&\frac{m_1 \ddot{y}_1 + c_{11} \dot{y}_1 + k_{11} y_1}{m_2 \ddot{y}_2 + c_{22} \dot{y}_2 + k_{22} y_2} = -k(t) f(y) - c_m \dot{y} \\
&\frac{m_1 \ddot{y}_1 + m_1 \ddot{y}_2 + [1 - L(t) \mu(t)] k(t) f(y) + [1 - L(t) \mu(t)] c_m \dot{y}}{m_2 \ddot{y}_2 + m_2 \ddot{y}_2 + [1 - L(t) \mu(t)] k(t) f(y) + [1 - L(t) \mu(t)] c_m \dot{y}} = F_m(t) + F_b(t)
\end{align*}
\]

where \( m_y = \frac{I_2}{I_2 r_{b1}^2 + I_2 r_{b2}^2} \), \( L_y(t) = \frac{I_2 r_{b1} L_1 + I_2 r_{b2} L_2}{I_2 r_{b1}^2 + I_2 r_{b2}^2} \), \( F_m(t) = m_v \left( \frac{r_{b1} T_1}{I_1} + \frac{r_{b2} T_2}{I_2} \right) \) and \( F_b(t) = -m_v \ddot{e}(t) \). If the total backlash is \( 2b \), natural frequency \( \omega_n = \sqrt{k(t)/m_v} \), non-dimensional time \( \tau = \omega_n t \), \( w_1 = y_1 / b \), \( w_2 = y_2 / b \), \( w_3 = y / b \), and introducing the following related variables as \( z_1 = y_1, z_2 = \dot{z}_1, z_3 = y_2, z_4 = \dot{z}_2, z_5 = y, z_6 = \dot{z}_5 \), system model can be simplified as

\[
\begin{align*}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= -2 p_{11} z_2 - k_{11} z_1 - 2 p_{12} z_6 - k_{12} f(z_5) \\
\dot{z}_3 &= \dot{z}_4 \\
\dot{z}_4 &= -2 p_{21} z_4 - k_{14} z_3 + 2 p_{22} z_6 + k_{22} f(z_5) \\
\dot{z}_5 &= \dot{z}_6 \\
\dot{z}_6 &= F_m + F_b + z_2 - z_4 - 2 p_{31} [1 - L(\tau) \mu(\tau)] z_6 - k_{31} [1 - L(\tau) \mu(\tau)] f(z_5)
\end{align*}
\]

3. Influence of the support stiffness \( k_1 \) on the nonlinear characteristics

Support stiffness \( k_1 \) is one of the main factors that affect the dynamic responses of the geared system. Figure 2 is the bifurcation diagram by using \( k_1 \) as an independent parameter when the motor speed is
1500 rpm. When $k_1 \in [3.5 \times 10^9, 5.32 \times 10^9]$ or $k_1 \in [6.68 \times 10^9, 9.6 \times 10^9]$, the system can be judged to be in chaotic motion state according to the bifurcation diagram.

It can be seen from Figure 3 that time response diagram shows a non-periodic motion, and amplitude–frequency spectrum is continuous, meantime phase plane diagram is disorder, and Poincaré map shows many discrete points. All these characteristics reveal that the system is in chaos in this district. Figure 4 displays that the system response is the period–1 motion, where time response diagram shows a sine wave, and amplitude–frequency spectrum also has one–peak amplitude, meanwhile phase plane diagram is only one closed circle, and Poincaré map has a single point.

Figure 3. The dynamic characteristics of system when $k_1=7 \times 10^9$ N/m

(a) Time response of $y_1$  
(b) Amplitude-frequency spectrum  
(c) Poincaré map  
(d) Phase plane diagram
4. Linear feedback control of chaotic motion

For the following $n$-dimensional system

$$\dot{X} = F(X(t), t), \quad y = NX$$

If $k$ is the parameter of linear feedback controller, introducing linear feedback controller $G = k(y_1 - y_2)$, then the controlled $n$-dimensional system is

$$\dot{X} = F(X(t), t) - G$$

Apply a controller $K = [k, 0, k, 0, k, 0]^T$, the system is then changed as

$$\begin{align*}
\dot{z}_1 &= z_2 - k(z_1 - z_2) \\
\dot{z}_2 &= -2p_1z_2 - k_1z_1 - 2p_2z_6 - k_{12}f(z_3) \\
\dot{z}_3 &= z_4 - k(z_3 - z_4) \\
\dot{z}_4 &= -2p_1z_4 - k_1z_3 + 2p_2z_6 + k_{22}f(z_3) \\
\dot{z}_5 &= z_6 - k(z_5 - z_6) \\
\dot{z}_6 &= \bar{F}_m + \bar{F}_h + z_2 - z_4 - 2p_{11}[1 - L(\tau)\mu(\tau)]z_6 - k_{11}[1 - L(\tau)\mu(\tau)]f(z_3)
\end{align*}$$

As mentioned above, when $k_1 \in [6.68 \times 10^9, 9.6 \times 10^9]$, the geared system is in chaotic motion. To avoid the system moving in the unstable region, a linear feedback controller is adopted to control chaotic motion in this region. Figure 5 shows the bifurcation diagram by using control parameter $k$ as an independent variable. When $k$ is less than 0.048, the system state is chaotic. With the increasing of $k$, the system enters into period-2 motion until $k = 0.96$. Once $k$ is higher than 0.96, the system is controlled to a stable periodic-1 motion.

Figure 5. Bifurcation diagram of linear feedback control with parameter $k$
5. Conclusions
The influences of the support stiffness $k_1$ on bifurcation and chaos behavior of the gear system are obvious. In order to avoid the geared system moving in the unstable region, it is necessary to use LFCM in the gear system. Numerical simulations show that the LFCM is effective and feasible. The gear system can be controlled from chaotic motion to stable periodic orbits by using the appropriate control parameter $k$.

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