A Reducing Iteration Orthogonal Matching Pursuit Algorithm for Compressive Sensing

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A Reducing Iteration Orthogonal Matching Pursuit Algorithm for Compressive Sensing

Rui Wang*, Jinglei Zhang, Suli Ren, and Qingjuan Li

Abstract: In recent years, Compressed Sensing (CS) has been a hot research topic. It has a wide range of applications, such as image processing and speech signal processing owing to its characteristic of removing redundant information by reducing the sampling rate. The disadvantage of CS is that the number of iterations in a greedy algorithm such as Orthogonal Matching Pursuit (OMP) is fixed, thus limiting reconstruction precision. Therefore, in this study, we present a novel Reducing Iteration Orthogonal Matching Pursuit (RIOMP) algorithm that calculates the correlation of the residual value and measurement matrix to reduce the number of iterations. The conditions for successful signal reconstruction are derived on the basis of detailed mathematical analyses. When compared with the OMP algorithm, the RIOMP algorithm has a smaller reconstruction error. Moreover, the proposed algorithm can accurately reconstruct signals in a shorter running time.

Key words: compressed sensing; signal processing; wireless sensor networks

1 Introduction

The rapid development of Compressed Sensing (CS) [1-4] has given rise to some new situations. On the one hand, CS theories have overcome traditional restrictions that stipulate that sampling rate must satisfy the Nyquist sampling theorem. However, a lot of data is required for sampling, thus resulting in increased resource consumption in the subsequent process. On the other hand, CS is being used in a wide range of applications such as multi-sensors and distributed compressed sensing, Internet of Things [5], compressed sensing radars [6], medical imaging [7], computational biology, wireless communication processing, analog to information conversion [8, 9], geographic data analysis, remote sensing technology, and spectroscopy [10]. Undoubtedly, the CS theory has led to significant developments in numerous fields. The main principle of the CS theory is that if a signal is sparse or compressible by transformation (such as Fourier transform and discrete cosine transform), then a measurement matrix can be designed to measure the signal. Moreover, by solving the optimization problem with the measured value, the original signal can be precisely recovered. Compressed sensing also directly discards unimportant data during sampling and recovers a signal accurately with less but important information. Therefore, this theory can effectively reduce the amount of data required, thus improving performance.

Currently, CS emphasizes three aspects: choice of sparse base, selection of measurement matrix, and design of reconstruction algorithm. Among these, researchers consider reconstruction algorithm design to be the core part of reconstruction processing [11-14]. The current reconstruction methods can be classified into three categories [15]. Convex relaxation, greedy, and combination methods are typically used for recovering signals. Convex relaxation methods such
as Basis Pursuit[^16] (BP)/interior-point, gradient projection, and iterative threshold methods are used to convert a non-convex optimization problem into a convex optimization problem and then obtain the approximation of the signal. On the other hand, greedy method algorithms, including Matching Pursuit (MP), Orthogonal Matching Pursuit (OMP)[^17], Stagewise Orthogonal Matching Pursuit (StOMP), and Regularized Orthogonal Matching Pursuit (ROMP) algorithms select a local optimal solution close to the original signal. Combinatorial methods, such as Fourier sampling, chain tracking, and Heavy Hitters on Steroids pursuit (HHS), require sampling support to rapidly reconstruct the signal by grouping test. Each algorithm has its inherent advantages and disadvantages. Therefore, different reconstruction algorithms are required to achieve more accurate reconstruction results. Convex relaxation methods can reconstruct signals well with a fewer number of observations but involve greater complexity, thus resulting in heavy computation and a long running time[^15]. Therefore, long signals cannot be easily reconstructed. When compared with the convex relaxation method, the observation method is better in combination algorithms, and the running speed during reconstruction is faster. However, for a certain number of observations, these algorithms can rebuild a signal with high probability but not accurately. Greedy methods can be easily implemented with low complexity and require a shorter operation time; however, the reconstruction accuracy is less than that obtained with convex relaxation algorithms. Owing to the continuous improvement of a greedy algorithm, it can now satisfy refactoring requirements. Considering complexity and reconstruction performance, the greedy method is superior in terms of accurate results. However, a bottleneck exists in current degree algorithms, thus limiting their development and application. Specifically, during reconstruction processing, many classical algorithms such as OMP and ROMP algorithms depend on the sparse degree \( M \) as the iteration to update the current estimated value to reconstruct a signal[^12]. However, the sparse degree \( M \) is not a certain number. Therefore, a precise \( M \) cannot be easily chosen. And the complexity of this step cannot be easily reduced. Currently, there is no effective method to solve this problem.

Therefore, we propose a novel degree algorithm for the CS theory referred to as the Reducing Iteration Orthogonal Matching Pursuit (RIOMP) algorithm. Based on the OMP algorithm, this algorithm combines sparse degree with the correlation of the current residual value and the measurement matrix as iteration, thus adding threshold for control. The algorithm commences when the iteration number is within \( M \), and the correlation value is then determined. If the correlation value is less than the threshold, then the iteration is stopped. The threshold is a measured parameter that can be obtained by experiment. In this situation, this algorithm not only meets the iteration of sparse degree but also generates a nearly accurate reconstruction signal in a shorter running time.

We can systematically summarize the above mentioned process based on the CS theory as follows. To address the basic problem of iteration, we use the idea of OMP algorithm, which chooses the most correlative column of measurement matrix with the residual value. If the decreasing trend of correlation stabilizes before the iteration reaches the sparse degree, then the latter correlation value will not vary further and the obtained result will not be influenced. We can then stop the iteration and directly choose the current value to obtain the reconstruction signal. On the basis of this method, we propose the RIOMP algorithm, which considers the correlative of the residual value and the measurement matrix as the iteration. In order to validate our proposed RIOMP algorithms, we use real-world speech signal data sets sampled by Kinect sensors in the simulation experiment. Further, we compare the performance of the OMP and the proposed RIOMP algorithms and use AFSNR and AFRE as the evaluation methods.

### 2 RIOMP Algorithm

#### 2.1 Preliminary

A summary of the CS theory is given below. Let us consider an original signal \( x \). If it is sparse in a certain orthogonal basis \( \Phi \), we can use one unrelated sensing matrix \( \Phi \) to project the high dimension of \( x \) to the low dimension space. We can then obtain significantly fewer measurements than unknown signal values. Using the reconstruction algorithm, the optimization problem can then be solved to recover the original signal \( x \) accurately or with a high probability.

In the classic OMP algorithm, the sparse degree \( M \) is considered to be the iteration number for signal reconstruction.
However, the value of $M$ is not certain. If the value of $M$ is selected properly, then depending on the strength of the correlation of the residual error with the columns of the measurement matrix, the original signal can be accurately reconstructed. However, if the value of $M$ is selected improperly and the $M$ value is larger, the correlation tends toward stability and will increase the computational complexity to continue the iteration. On the other hand, a smaller $M$ value indicates that strong correlation columns have not been selected, thus resulting in an inaccurately reconstructed signal. In order to solve this problem, we propose the RIOMP algorithm.

The following notations have been used in the algorithm. Its input may include the original signal, measurement value, etc. Let us suppose that $s$ is an $M$-sparse signal in $\mathbb{R}^d$, and $v$ is an $N$-dimensional measurement vector. The $N \times d$ matrix $\Theta$ indicates that the sensing matrix and the measurement information in $s$ can be collected by $v = \Theta s$, where $v$ is a linear combination of $M$ columns in $\Theta$.

Therefore, in the recovering algorithm, we must determine the columns of $\Theta$ can be chosen in the measurement vector $v$ to obtain the sparse signal $s$, and the algorithm chooses columns in a greedy fashion\textsuperscript{[13]}.

Iteration and threshold are considered to be the core elements and key features of the proposed RIOMP algorithm. This is reasonable and scientific because in each iteration, within the sparse degree $M$, we compare the result of each column with the residual value and select the best one as the atom matrix and then obtain the estimated signal. When the difference between the current and previous correlations is less than the threshold $\theta$, the most correlative columns are considered to have been selected and the iteration is stopped. If the trend is not stable, the iteration is stopped when it reaches the sparse degree $M$, and the algorithm will have identified the recovered signal.

### 2.2 Introduction of the proposed algorithm

The iteration is significantly influenced by the correlation between the residual value and measurement matrix. Therefore, we propose the RIOMP algorithm for the theoretical foundation of the CS theory.

The iteration is significantly influenced by the correlation between the residual value and measurement matrix. Therefore, we propose the RIOMP algorithm for the theoretical foundation of the CS theory.

This process is theoretically described specifically as follows:

Let us consider an original signal $x = [x_1, x_2, \cdots, x_N]^T$, where $N$ is the length of the discrete signal. We can then find one vector base $\psi (\psi = [\psi_1, \psi_2, \cdots, \psi_N], \psi$ is a matrix of $N \times N$) such that it can represent every $N$-dimensional vector. If this vector base is orthogonal, we can express the original signal $x$ as follows:

$$x = \psi \alpha$$  \hspace{1cm} (1)

Here, $x$ is the signal represented in the time domain, and $\alpha$ is the signal represented in the $\psi$ domain. If $\alpha$ only has $K$ values ($K \ll N$) and other $N - K$ values are zero, we can say that $x$ is $K$-sparse and $\psi$ is the sparse base.

In the case where the signal is sparse, we can design one measurement matrix $\phi$ unrelated with the vector base to measure the signal. Namely, we can project the signal $x$ to the measurement matrix $\phi$ and obtain the measurement value as follows:

$$y = \phi x$$  \hspace{1cm} (2)

Here, the measurement matrix is an $n \times N$ ($n \ll N$) matrix, and $y$ is an $n$-dimensional vector. By combining Eqs. (1) and (2), we obtain

$$y = \phi x = \phi \psi \alpha = \Theta \alpha$$  \hspace{1cm} (3)

Here, $\Theta = \phi \psi$ is a sensing matrix by $n \times N$. The measurement process is non-adaptive, i.e., $\Theta$ is a random matrix and does not depend on the signal $x$.

In the reconstruction process, our method involves solving Eq. (3). Therefore, the required solution formula can be transformed to the minimum norm problem as follows:

$$\min \|s\|_0, \text{ s.t. } y = \phi x = \phi \psi \alpha$$  \hspace{1cm} (4)

We introduce the RIOMP algorithm to solve it with a greedy algorithm. Suppose that we have a measurement matrix $\Theta$ and a measurement vector $y$, then we can use the sparse base to estimate the sparse degree $M$ in the original signal. In addition, residual $r_0 = v$, index set $A_0 = \phi$, iteration number $t = 1$, and the selected matrix atom $\phi_0 = \Theta$.

The correlation of the residual error and measurement matrix can then be calculated to determine the index $\lambda_t$.

$$\lambda_t = \arg \max_{j=1,2,\cdots,d} |\langle r_{t-1}, \phi_j \rangle|$$  \hspace{1cm} (5)

Here, $d$ denotes the column number of the measurement matrix. After determining the maximum and selecting the most correlative column, the index set and atomic matrix are added as follows:

$$A_t = A_{t-1} \cup \{\lambda_t\} \text{ and } \phi_t = [\phi_{t-1} \phi_{\lambda_t}]$$  \hspace{1cm} (6)
We can then solve the least squares problem and estimate the new signal as follows:

\[ x_t = \arg \min_x \|v - \phi_t x\|_2 \]  

(7)

The new approximation and residual can then be expressed as follows:

\[ \alpha_t = \phi_t x_t \quad \text{and} \quad r_t = v - \alpha_t \]  

(8)

The next step is the most important one. In the proposed RIOMP algorithm, we first calculate the correlation between the new residual and measurement matrix as follows:

\[ \xi_t = \arg \max_{j=1,2,...,d} |\langle r_t, \phi_j \rangle| \]  

(9)

Thereafter, the difference between the current and former correlations is calculated as follows:

\[ C(\xi_t, \xi_{t-1}) = |\xi_t - \xi_{t-1}| \]  

(10)

Subsequently, the difference is compared with the threshold, and if \( C(\xi_t, \xi_{t-1}) < \theta \), the iteration is stopped and the loop is terminated. Otherwise, \( t \) is incremented, i.e., \( t = t + 1 \), and the loop continues if \( t < M \). The flow diagram of the RIOMP algorithm is shown in Fig. 1.

Finally, we obtain the estimated signal \( \hat{s} \) where the position of the nonzero entry corresponds to the element in the index set \( \Lambda_M \) and the amplitude is the index value \( \lambda_j \) in dictionary, which corresponds to the \( j \) value in the least squares solution \( x_t \). The reconstruction signal \( \tilde{x} = \psi \hat{s} \) can be obtained after obtaining the estimated signal \( \hat{s} \).

### 2.3 Discussion on the algorithm

As described previously, if the correlation between the residual value and measurement matrix gradually stabilizes, the algorithm exits the loop and the iteration is stopped. An analysis of algorithm performance shows that with the sparse degree as the iteration, the number of iterations can be reduced to the greatest extent, and the signal can be recovered accurately. This improved step shortens the running time and reduces computational complexity.

### 3 Simulation Scenario

#### 3.1 Experimental setup

For the simulation, we selected a speech signal acquired by a Kinect sensor as the original signal. The specific experimental environment is as follows: four Kinect sensors were placed at four corners of one square area,
and speech signals were sampled simultaneously at the center position, as shown in Fig. 2. In this study, the speech signals collected by the Kinect sensor 1 are considered to be the experimental data.

We validate the effectiveness of the proposed algorithm by analyzing processing methods as follows.

**Acquisition of speech signal** Speech signals are time varying and nonstationary signals that can be viewed as short-term stationary signals. Therefore, their characteristic parameters and spectral characteristics remain unchanged within 10–30 ms. Therefore, a fragment of a speech signal must be segmented into some smaller sections. Figure 3 shows the process of the compression and reconstruction of speech signals on the basis of the CS theory.

**Analysis of sparsity** For the CS theory to be applicable, the original signal must be sparse or compressible; therefore, the sparsity of the signal must be validated. To obtain the overall transformation trend, we will expand the Discrete Cosine Transform (DCT) and obtain the Average DCT (ADCT) coefficient, 
\[
\text{ADCT} = \frac{1}{K} \sum_{k=1}^{K} \alpha_k,
\]
where \(\alpha_k\) is the DCT coefficient of \(k\)-th frame, and \(K\) is the total number of frames. The result is shown in Fig. 3.

**Analysis of different thresholds** During the iteration, the RIOMP algorithm uses the threshold to achieve a stable decreasing trend of the correlation. Therefore, the threshold control of the correlation affects the iteration of the algorithm. Consequently, the difference between the former and current correlations is used as the threshold.

We evaluate our method in two aspects as follows:

**Computational complexity** To evaluate the computational complexity of algorithm, we determine the running time.

**Reconstruction quality** The signal reconstruction quality is analyzed via the objective test method using some methods including Signal-to-Noise Ratio (SNR), Relative Error (RE), Bark Spectral Distortion (BSD), and Linear Predictive Cepstral (LPC) distance measurement to measure the quality of a speech signal.

In this context, the SNR and RE are calculated for one frame of the signal as follows:
\[
\text{SNR} = 10 \log \left( \frac{\|x\|_2^2}{\|x - \hat{x}\|_2^2} \right) \tag{11}
\]
\[
\text{RE} = \frac{\|x - \hat{x}\|_2}{\|x\|_2} \tag{12}
\]
Here, \(x\) is the vector of the original signal, and \(\hat{x}\) is the vector of the reconstructed signal. Further, SNR and RE are the measures of the reconstructive accuracy of signal.

However, as mentioned in the previous section, because speech signals have the characteristic of short-term stationarity; we must measure SNR values every 20 ms and average them over a long speech interval, which reflects the quantified quality in different level input section and has a better characteristic when compared with the subjective value. Therefore the Average SNR (ASNR) and Average RE (ARE) are
used to evaluate the reconstruction performance from an objective perspective.

\[
\text{ASNR} = \frac{1}{K} \sum_{k=1}^{K} 10 \log \left( \frac{\|x_k\|_2^2}{\|x_k - \hat{x}_k\|_2^2} \right) 
\]

\[
\text{ARE} = \frac{1}{K} \sum_{k=1}^{K} \frac{\|x_k - \hat{x}_k\|_2}{\|x_k\|_2} \]  

(13)

(14)

Here, \(K\) is the total number of frames. Further, \(x_k\) is the \(k\)-th frame of the original speech signal, and \(\hat{x}_k\) is the \(k\)-th frame of the reconstruction signal.

Therefore, the higher the SNR and ASNR values and the lower the RE and ARE values, the more accurate the reconstructed signal. The advantages of objective assessment methods can be easily calculated, and the result is objective.

### 3.2 Simulation scenario of experiment

In latter experiments, we used a 65-second TOEFL listening passage as the original speech signal. Considering the characteristic of short-term stability within 10–30 ms, we randomly chose one segment of the speech signal for convenience. The segment was divided into four different frame lengths of 192 sampling points (12 ms), 256 sampling points (16 ms), 320 sampling points (20 ms), and 448 sampling points (28 ms). For comparing the performance of OMP and RIOMP reconstruction algorithms, we used compression ratios of 0.3, 0.4, 0.5, and 0.6, and threshold values of 0.3, 0.35, 0.4, 0.45, and 0.5. In addition, we employed DCT basis as sparse expansion, and the speech quality and computational complexity of the two algorithms were compared.

### 4 Experimental Results

#### 4.1 Sparsity validation

Figure 4 shows the sparse representation with four frames. Figure 4a shows the waveform diagram of four frames of the original signal. Figure 4b shows the average DCT coefficients of this signal segment. As can be seen, most of the coefficients are nearly zero and only few coefficients are large and contain the important information. Additionally, the sparse number of this signal is 15, namely, the sparse degree is 94.14.

This indicates that this segment of the speech signal is approximately sparse and satisfies the sparsity condition.

![Fig. 4 Sparse representation with one segment (four frames).](image)

#### 4.2 Threshold selection

Figure 5 shows the ASNR values of different thresholds with 448 points. The independent variable is the compression ratio \(M/N\), and the dependent variable is the ASNR value. As can be seen in this figure, when the frame length is certain, the threshold value increases as the compression ratio increasing. Moreover, a comparison of the trends of different threshold values shows that these values are almost same, thus indicating that the difference between the different threshold values is minimal. Therefore, within a range of 0.3–0.5, the threshold value has minimal influence on system performance. Hence, we randomly chose 0.45 as the threshold value for the subsequent experiment.

![Fig. 5 ASNR values of different thresholds with 448 points.](image)
4.3 Algorithm comparison

In this study, we compared the performance of OMP and RIOMP algorithms in terms of speech quality and running time. In the aspect of speech quality, Fig. 6 shows the variation in compression ratio with ASNR for a specific frame length for each algorithm. Figure 7 shows the variation in compression ratio with ARE for each algorithm, while Fig. 8 shows the variation in compression ratio with running time for each algorithm for a specific frame length.

In Figs. 6–8, the compression ratio \(\frac{M}{N}\) is the independent variable and ASNR, ARE, and running time are the dependent variables, respectively. Figure 6 shows that when the frame length is certain, as the compression ratio varies, the ASNR of the RIOMP algorithm is slightly lower than that of the OMP algorithm. Figure 7 shows that when the frame length is certain, the ARE of the RIOMP algorithm is a slightly higher than that of the OMP algorithm. Therefore, from the aspect of objective evaluation, the reconstruction error of the RIOMP algorithm is poorer than that of the OMP algorithm.

On the other hand, Fig. 8 shows that for a specific frame length, as the compression ratio varies, the running time of the RIOMP algorithm is shorter than that of the OMP algorithm, because in the RIOMP algorithm, the reconstructed signal is acquired when the correlation value approaches the threshold, thus reducing the number of iterations. However, in the OMP algorithm, the iteration time must meet the certain sparse degree; therefore, the running time of the RIOMP algorithm is shorter than that of the OMP algorithm.

In summary, the RIOMP algorithm shortens the iteration, thus reducing the amount of computation required. However, the reconstruction performance of RIOMP algorithm is slightly lower than that of the OMP algorithm. Overall, the performance of the RIOMP algorithm is better than that of the OMP algorithm.

5 Conclusions

This paper present a novel RIOMP algorithm based on the CS theory. The proposed algorithm uses the correlation between the residual value and the measurement matrix as the iteration condition within the sparse degree \(M\). It not only meets the iteration times with sparse degree, but also accurately reconstructs the signal in a shorter running time. When compared with the OMP algorithm, the proposed RIOMP algorithm reduces computational complexity and running time while delivering the same reconstruction precision. Therefore, the proposed algorithm has practical significance for applications requiring greater precision and time complexity.
In the future, we will further improve the SNR in reconstruction performance.

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