Research regarding stiffness optimization of wires used for joints actuation from an elephant’s trunk robotic arm

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Abstract. Elephant’s trunk robotic arms driven by wires and pulley mechanisms have issues with wires stiffness because of the entailed elastic deformations that is causing errors of positioning. Static and dynamic loads from each joint of the robotic arm affect the stiffness of driving wires and precision positioning. The influence of wires elastic deformation on precision positioning decreases with the increasing of wires stiffness by using different pre-tensioning devices. In this paper, we analyze the variation of driving wires stiffness particularly to each wire driven joint. We obtain optimum wires stiffness variation by using an analytical method that highlights the efficiency of pre-tensioning mechanism. The analysis of driving wires stiffness is necessary for taking appropriate optimization measures of robotic arm dynamic behavior and, thus, for decreasing positioning errors of the elephant’s trunk robotic arm with inner actuation through wires/cables.

1. Introduction

Robotic structures with joints actuated by wire pulleys have solely the possibility of wires pulling, not pushing. Therefore, the actuating wires are constantly in a tension that increases elastic deformations. Wire pulleys are used for actuating specific robotic arms, flexible positioning and high workspace with respect to their size. The structural forces and joints torques moments which change while robotic arms are operating, lead also to tension forces and affect wire stiffness. Elephant’s trunk robotic arms with wire-driven joints and pulley mechanism have stiffness issues of the structure because of wires elastic deformations that affect positioning precision. The influence of elastic deformations on positioning precision is generally reduced by different devices along each actuating wire of joints to optimize tension distribution and, thus, to increase the robotic arm stiffness [1, 2, 3].

In this paper, we consider a robotic arm with 5 degrees of freedom, serial connected joints and the performed motions similar to the elephant’s trunk. In the analysis of wires elastic deformations, we consider no device for automatically control of driving wires stiffness. We obtain variation of wires stiffness depending on the particular wire tensions from each actuated joint. The tensile forces applied to the actuating wires comprises of the total loads given by: moments of resistance, frictions between driving wires and surfaces from joints interior, and rotated masses by each joint. The optimum wires stiffness is analytically determined for driving wire used to actuate each of five joints from the elephant’s trunk robotic arm.

We use an analytical method to optimize wires stiffness by pre-tensions. The stiffness of wires used for joints driving mechanism from elephant’s trunk robotic arms influence dynamic behavior of the
arm structure and has been of interest for different research personnel. Some research teams have suggested stiffness-oriented tension resolution algorithms or wire tension state sensing components to optimize positioning precision [4, 5]. However, all these control devices of wires stiffness make the robotic arm to be heavier, robust and, thus, inappropriate for manipulation in narrow spaces.

Inner actuation with wires and pulley mechanism is advantageous because of miniaturization possibility of robotic arms diameter to achieve thinner robotic structure for manipulation in narrow spaces. For the specific robotic arm structure, we suggest stiffness optimization by pre-tensioning the actuating wires. The obtained results of wire stiffness influence on elastic deformations are necessary for taking suitable measures of dynamic behavior optimization and prove method viability.

2. Tension analysis of driving wires from an elephant’s trunk robotic arm

The particular structure of elephant’s trunk robotic arm enables joints actuation by pulley mechanisms with tensioned wires. Driving system consists of AC servomotors to actuate the joints in a closed loop control. Elastic deformations of driving wires causes positioning errors of elephant’s trunk robotic arm that are outside of the feedback control system. Therefore, tension analysis of driving wires is necessary to evaluate variation of elastic deformations that also affects stiffness of wires and, of motion transmission system. The Hooke’s law given in equation (1) defines elastic deformations \( \Delta l_i \) of driving wires.

\[
\Delta l_i = \left( \frac{T_i \cdot l_i}{E \cdot A_w} \right)
\]

We noted: \( T_i \) - tensile load of the wire that drives joints \( i \), \( l_i \) - length of the wire that drives joints \( i \) before loading, \( E \) - Young modulus of the driving wires made of stainless steel with Teflon coating, \( A_w \) - cross sectional area of driving wire with diameter of 0.38 mm.

From Hooke’s law results that we can decrease elastic deformation either by decreasing the length of wire or of tensile loads, either by increasing wires cross sectional area. We consider in the structure designing stage a minimum length of driving wires according to the dimensional limitations of structure components. Diameter size of driving wire is restricted to the lowest possible value to achieve minimum arm diameter and flexible manipulation in very narrow spaces.

Moreover, from the variation law of wires stiffness to bending given by parameter \( R_f \) in equation (2) results that simultaneously with the increase of wires diameter \( d_w \) we have an increase of wires stiffness too. However, robotic arm structure entails that driving wire diameter allowing the winding on circular surfaces from structure interior and this requirement unfits the increase of driving wire diameter.

\[
R_f = E \cdot I = E \cdot (\pi \cdot d_w^4)/64
\]

In equation (2) we noted \( I \) as the inertia moment of wire cross section about neutral axis and \( d_w \) as the driving wire diameter.

With all the mentioned above requirements we consider optimization of driving wires stiffness using a pre-tensioning device for reducing tensile loads \( T_i \) that are causing elastic deformations of driving wires along their length on different sections \( l' \), \( l'' \), \( l''' \) between points A, B, C and D from figure 1 and figure 2. The actuation of joints by pulley mechanical with wires for motion transmission supposes several tensile loading \( T_i \) of driving wires at the lower ends. Tensile loads \( T_i' \) at the upper ends of wires that drives joints \( i \) are smaller than tensile loads \( T_i \), equation (3). Parameter \( \eta \) defines the coefficient of efficiency for each transmission mechanism.

\[
T_i' = \eta^{-1} \cdot T_i
\]

Stiffness optimization of active wires, used for driving the joints from elephant’s trunk robotic arm, consists in using a pre-tensioning mechanism, which reduces elastic deformations so that positioning
errors are at minimum values while operating. After applying a pre-tension load $T_{0,i}$ to the active wire of joint $i$ there is an additional elastic deformation $\Delta I_{s,i}$ as shown in figure 2. While the robotic arm is operating, an extra tension load $T_{a,i}$ is causing elastic deformation $\Delta I_{a,i}$ besides pre-tensioning load.

In addition, efficiency coefficient of the transmission mechanism (wire-pulley) affects tension loads from the two ends of driving wire as given in equation (4) for pre-tension loads and in equation (5) for extra tension loads.

$$T_{0,i} = \eta^{-1} \cdot T_{0,i}$$  \hspace{1cm} (4)

$$T_{a,i} = \eta^{-1} \cdot T_{a,i}$$  \hspace{1cm} (5)

From equations (4-5) we find elastic deformations caused by pre-tension loads and extra loads mentioned above, equations (6-7).

$$\Delta I_{s,i} = \eta^{-1} \cdot \Delta I_{s,i}$$  \hspace{1cm} (6)

$$\Delta I_{a,i} = \eta^{-1} \cdot \Delta I_{a,i}$$  \hspace{1cm} (7)

![Figure 1. Tension loads of active wire that drives joint $i$.](image1)

![Figure 2. Elastic deformations at active wire ends.](image2)

The values of tension loads $T_i$ are known from features dimensioning of transmission mechanism as given in table 1. We have specific features of the driving wire, $E=252523 \cdot 10^6$ N/m and $A_w=0.114$ mm$^2$, and using equation (1) we obtain maximum elastic deformations caused by tension loads $T_i$ as given in table 1.

| $i$ | $T_i$ (N) | $l_i$ (mm) | $\Delta l_i$ (mm) |
|-----|-----------|------------|-------------------|
| 1   | 4         | 200        | 0.000358          |
| 2   | 3         | 250        | 0.000336          |
| 3   | 2         | 300        | 0.000269          |
| 4   | 1         | 350        | 0.000156          |
| 5   | 0.02      | 400        | 0.000036          |
Elastic deformations $\Delta l_i$ of wires that actuate the five joints from elephant’s trunk robotic arm vary as shown in the diagram from figure 3.

![Figure 3. Elastic deformations before pre-tensioning of active wires.](image)

The ratio between tension loads $T_i$ and elastic deformations $\Delta l_i$ define stiffness of the wire that actuates joint $i$, given as $\tan \gamma_i$. From the variation diagram of elastic deformation, figure 3, we find that wire stiffness decrease as we go from the first joint to the fifth joint because the length of driving wires $l_i$ increase and the manipulated masses decrease, tension loads $T_i$ respectively.

3. Stiffness optimization of driving wires by pre-tensioning

Stiffness optimization of wires that actuates five joints from an elephant’s trunk robotic arm by pre-tensioning aims to reduce elastic deformations, caused while robotic arm operates, and systematic positioning errors respectively. The method consists in analytical determination of the optimum value given for pre-tension loads $T_{0,i}$ to achieve optimum stiffness of the wire that actuates joint $i$, figure 4. Pre-tension loads $T_{0,i}$ and $T'_{0,i}$ from the lower and the upper ends of active wires, figure 4.a and figure 4.b respectively, cause additional elastic deformations $\Delta l_{a,i}$ and $\Delta l'_{a,i}$ given by equation (8) and equation (9) respectively.

![Figure 4. Elastic deformations of the wire that drives joint $i$ located at: a. Lower wire end, b. Upper wire end.](image)
From equations (8-9) we find that the stiffness \( i_{o,j} \) of active wire at lower end, equation (10), is equal to the stiffness \( i_{o,j} \) of active wire at upper end, equation (11).

\[
\Delta l_{s,i} = \left( T_{o,i} \cdot L_i \right) / (E \cdot A_u) \tag{8}
\]
\[
\Delta l'_{s,i} = \left( T'_{o,i} \cdot L_i \right) / (E \cdot A_u) \tag{9}
\]
\[
\tan \gamma_{o,i} = \Delta l_{s,i} / \Delta l_{s,i} \tag{10}
\]
\[
\tan \lambda_{o,i} = T'_{0,i} / \Delta l_{s,i} \tag{11}
\]

Transposition of the two diagrams from figure 4 until to the overlapping of pre-tension loads \( T_{0,i} \) and \( T'_{0,i} \) comes out in diagram from figure 5, which shows elastic deformations of the wire that drives joint \( i \). If the active wire is pre-tensioned with load \( T_{0,i} \) and joint \( i \) is rotating under tension load \( T_i \), then it lengthens additionally with elastic deformation \( \Delta l_{s,i} \) and the pulley-joint assembly contracts with the same length because of pre-tension load decrease at an amount given by load \( T_{r,i} \).

![Diagram showing elastic deformations of active wire](image)

**Figure 5.** Elastic deformations diagram of active wire that actuates joint \( i \).

When the elephant’s trunk robotic arm operates, the total tension load of active wire from joint \( i \) becomes \( T_i \), equation (12).

\[
T_i = T_{0,i} + T_{a,i} = T_{r,i} + T_i \tag{12}
\]

As given by equations (8-11), we have same variation of wire stiffness at its two ends given in figure 5 by angle \( \gamma_{0,i} \) and, thus, it follows the equality \( T_{a,i} = T_{p,i} = T_{0,i} + T_{r,i} \). Assuming that we know tension load \( T_i \) applied at lower end of driving wire and that it is the sum between extra-tension load \( T_{a,i} \) and tension load \( T_{p,i} \), then, as a consequence of pre-tensioning of driving wire, we find the equality \( T_{a,i} = T_i / 2 \), shown in diagram from figure 5. The material elasticity of wire used for actuating the joints from elephant’s trunk robotic arm allows the extension and contraction with the same amount \( \Delta l_{s,i} \) while operating and finally the driving wire remains with elastic deformation \( \Delta l_{s,i} \). In case of pre-tension load \( T_{0,i} \) absence, elastic deformations from driving wire decrease its stiffness at the value given by tangent of angle \( \gamma_i \) as shown in diagram from figure 6.
Figure 6. Variation of elastic deformation before and after pre-tensioning.

In diagram from figure 6, $\Delta l_{0,i}$ defines elastic deformation of active wire with stiffness $\tan \gamma_{i,j}$. From diagram we find stiffness variation of active wire before pre-tensioning, depending upon pre-tension load $T_{0,i}$, that is $\tan \gamma_{i,j} = T_{0,i}/\Delta l_{0,i}$. From figure 5 we find the equality $T_{0,i}/\Delta l_{i,s} = T_{a,i}/\Delta l_{a,i}$ that is equivalent with $\Delta l_{i,s}/\Delta l_{a,i} = T_{0,i}/T_{a,i}$. Thus, using equation (10) we determine analytically the stiffness optimization of pre-tensioning on active wire, equation (13).

$$\delta = 100 \cdot \left(\tan \gamma_{i,j} - \tan \gamma_{0,i,j}\right)/\tan \gamma_{j} = 100 \cdot \left(1 - \Delta l_{a,i}/\Delta l_{0,i}\right)$$

(13)

From figure 5 we know that $\Delta l_{i,s} > \Delta l_{a,i}$, and from figure 6 we find the elastic deformations dependence $\Delta l_{0,i} > 2 \cdot \Delta l_{a,i}$, that determine amount of quantity $\delta > 100 \cdot \left(1 - 0.5 \cdot \Delta l_{a,i}/\Delta l_{i,s}\right)$. With these assumptions and with ratio $\Delta l_{i,s}/\Delta l_{a,i} \cong 1$, we obtain that quantity $\delta$ is higher than 50%.

4. Conclusion

In this paper, we highlighted the need of stiffness optimization of active wires that actuate joints from an elephant’s trunk robotic arm through structure interior. Tension loads, manipulated masses of each joint, inherent resistance moments and friction loads, cause elastic deformations of active wires. Length variation of driving wires directly affects the stiffness of motion transmission system and, implicitly, the precision positioning. The total elastic deformations cumulated because of the mentioned loads require compensation methods to reduce them with pre-tensioning mechanism or by electronic adjustments made in programming algorithm of the robotic arm. We determined by an analytical method that a pre-tension mechanism used for optimizing the active wire stiffness goes to minimum 50% stiffness increase. The analytically obtained results are necessary for feature experimental determinations of positioning precision from elephant’s trunk robotic arm.

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