The Accretion of Dark Energy onto a Black Hole

E.O. Babichev∗, V.I. Dokuchaev† and Yu.N. Eroshenko‡

November 1, 2018

Institute For Nuclear Research of the Russian Academy of Sciences
60th October Anniversary Prospect 7a, 117312 Moscow, Russia

Abstract

The stationary, spherically symmetric accretion of dark energy onto a Schwarzschild black hole is considered in terms of relativistic hydrodynamics. The approximation of an ideal fluid is used to model the dark energy. General expressions are derived for the accretion rate of an ideal fluid with an arbitrary equation of state \( p = p(\rho) \) onto a black hole. The black hole mass was found to decrease for the accretion of phantom energy. The accretion process is studied in detail for two dark energy models that admit an analytical solution: a model with a linear equation of state, \( p = \alpha(\rho - \rho_0) \), and a Chaplygin gas. For one of the special cases of a linear equation of state, an analytical expression is derived for the accretion rate of dark energy onto a moving and rotating black hole. The masses of all black holes are shown to approach zero in cosmological models with phantom energy in which the Big Rip scenario is realized.

1 INTRODUCTION

In recent years, strong observational evidence that the Universe is currently expanding with acceleration has been obtained. In the Einstein theory of gravitation, this positive acceleration is explained by the dominance of dark energy with a negative pressure in the Universe [1-4]. Several theoretical models of dark energy have been suggested: the vacuum energy (the cosmological constant \( \Lambda \)) or such dynamical components as quintessence [5-10] and k-essence [11-13]. Models with dynamical dark energy seem more realistic, since tracker [14, 15], or attractor, solutions are realized in them. Thus, the problem of fine tuning the parameters of the Universe is solved [11-13].

A peculiar property of cosmological models with dark energy is the possibility of a Big Rip [16, 17]: an infinite increase in the scale factor of the Universe in a finite time. The Big Rip scenario is realized in the case of dark energy, the so-called phantom energy (for which \( \rho + p < 0 \)). In the Big Rip scenario, the cosmological phantom energy density tends to infinity, and all of the bound objects are torn apart up to subnuclear scales. It should be noted, however, that the condition \( \rho + p < 0 \) alone is not enough for the Big Rip scenario to be realized [18]. Alam et al. [19] analyzed data on distant supernovas in a model-independent way and showed that the presence of phantom energy with \(-1.2 < w < -1\) in the Universe at present is highly likely. The quantum properties of the phantom energy in curved spacetime were considered in [20]. The entropy of the Universe filled with phantom energy was discussed in [21]. Models with phantom energy are also used to construct mole burrows [22, 23]. The accretion of a scalar field onto a black hole from special potentials \( V(\phi) \) was considered in [24-29]. We use a different
approach to describe the accretion of dark energy onto a black hole; more specifically, we model the dark energy by an ideal fluid with a negative pressure.

In our recent paper [30] (see also [31]), we showed that the masses of all black holes in the Universe with phantom energy gradually decrease, and the black holes disappear completely by the Big Rip. In this paper, we consider in detail the stationary spherical accretion of dynamical dark energy onto a black hole. The dark energy is modeled by an ideal fluid with a negative pressure. The history of research on the accretion of an ideal fluid onto a compact object begins with Bondi's classic paper [32]. A relativistic generalization was made by Michel [33] (see also [34-41] for further generalizations and supplements to Michel's solution). Carr and Hawking [42] considered the accretion of dust and radiation onto a black hole by solving the complete system of Einstein equations and taking into account the back reaction of the surrounding matter (see also [43] for a description of the progress made in this area and for a discussion of fundamental questions). Below, we obtain a solution for the stationary accretion of a test relativistic ideal fluid with an arbitrary equation of state \( p(\rho) \) onto a Schwarzschild black hole. Using this solution, we show that the black hole mass decreases during the accretion of phantom energy. The masses of black holes can decrease during accretion in the case of phantom energy due to the violation of the energy dominance condition \( \rho + p \geq 0 \) that underlies the theorem on the nondecreasing area of the event horizon of a classical black hole [44].

This paper is structured as follows. In Section 2, we derive general equations for the spherical accretion of an ideal fluid and describe basic parameters of the steady energy flux onto a black hole. We consider an arbitrary equation of state, \( w = p/\rho \), where the pressure \( p \) can be positive (for ordinary matter) and negative (for dark energy, including phantom energy \( w < -1 \)). Note that the parameter \( w \) of the equation of state need not be constant in our approach. Accretion causes the black hole mass to change: the mass increases for \( \rho + p > 0 \) and decreases for \( \rho + p < 0 \). The energy flux turns out to be completely determined by the black hole mass \( M \), the dark energy density at infinity \( \rho_\infty \), and the equation of state \( p = p(\rho) \) only if \( 0 < \partial p/\partial \rho < 1 \). In this case, there is a critical point that fixes the flux just as for an ordinary fluid. When the condition \( 0 < \partial p/\partial \rho < 1 \) is violated, the dark energy flux onto a black hole can formally be arbitrary. For \( 0 < \partial p/\partial \rho < 1 \), we describe the method of calculating the fluid parameters at the critical point and the energy flux onto a black hole for given \( M, \rho_\infty, \) and \( p = p(\rho) \).

In Section 3, we consider specific models of the equation of state for dark energy. In the first model, we use a simple equation of state with a linear density dependence of the pressure. We consider the special cases of accretion of several types of ideal fluid: thermal radiation, matter with an ultrahard equation of state, dark energy with \( \partial p/\partial \rho \leq 0 \), and linear phantom energy. The accretion rate of dark energy onto a moving black hole was calculated for the special case of \( \partial p/\partial \rho = 1 \). As the second model, we investigate the accretion of a Chaplygin gas onto a black hole. The evolution of the black hole mass in the Universe with the Big Rip is considered in Section 4. The possibility that the presence of phantom energy will lead the Universe to the Big Rip in the future has been discussed in recent years. The problem of the fate of black holes in this Universe is solved in a rather unexpected way: black holes are not torn apart, but disappear by the Big Rip due to the accretion of phantom energy, irrespective of their initial masses. In Section 5, we discuss the correspondence between the accretion of dark energy modeled by an ideal fluid onto a black hole and the accretion of a scalar field. The results obtained are briefly discussed in Section 6.

### 2 GENERAL EQUATIONS FOR SPHERICAL ACCRETION

Let us consider the stationary, spherically symmetric accretion of an ideal fluid that models the dark energy in the special case of a negative pressure onto a black hole. The dark energy density
is assumed to be low enough for the metric to be a Schwarzschild one with a high accuracy:

\[ ds^2 = \left( 1 - \frac{2M}{r} \right) dt^2 - \left( 1 - \frac{2M}{r} \right)^{-1} dr^2 - r^2( d\theta^2 + \sin^2 \theta d\phi^2 ), \]  

where \( M \) is the mass of the black hole, \( r \) is the radial coordinate, and \( \theta \) and \( \phi \) are the angular spherical coordinates. We model the dark energy by an ideal fluid with the energy-momentum tensor

\[ T_{\mu\nu} = (\rho + p) u_\mu u_\nu - pg_{\mu\nu}, \]

where \( \rho \) is the density, \( p \) is the dark energy pressure, and \( u^\mu = dx^\mu / ds \) is the radial 4-velocity component. The pressure is assumed to be an arbitrary function of the density, \( p = p(\rho) \). Integrating the zeroth (time) component of the conservation law \( T^\mu_{\mu\nu} = 0 \) yields the first integral of motion for stationary, spherically symmetric accretion (Bernoulli’s relativistic equation or the energy equation):

\[ (\rho + p) \left( 1 - \frac{2}{x} + u^2 \right)^{1/2} x^2 u = C_1, \]

where \( x = r/M, \ u = dr/ds, \) and \( C_1 \) is the constant determined below. To find the second integral of motion, we use the equation for the component of the energy-momentum tensor conservation law along the 4-velocity

\[ u_\mu T^{\mu\nu}_{\::\nu} = 0. \]

In our case, this equation is [45]

\[ u^\mu \rho_{\mu} + (\rho + p) u^\mu_{\::\mu} = 0. \]  

For the given equation of state

\[ p = p(\rho), \]

the auxiliary function \( n \) can be defined by the relation

\[ \frac{d\rho}{\rho + p} = \frac{dn}{n}. \]

The function \( n \) is identical to the particle concentration for an atomic gas, but it can also be used to describe a continuous medium that does not consist of any particles. In this case, the 'concentration' \( n \) is a formal auxiliary function. For an arbitrary equation of state \( p = p(\rho) \), we obtain a solution for \( n \) from Eq. (14):

\[ \frac{n(\rho)}{n_\infty} = \exp \left( \int_{\rho_\infty}^{\rho} \frac{d\rho'}{\rho' + p(\rho')} \right), \]

Using (8), we find the sought second integral of motion from Eq. (14):

\[ \frac{n(\rho)}{n_\infty} u x^2 = -A, \]

where \( n_\infty \) (the dark energy 'concentration' at infinity) was introduced for convenience. In the case of a fluid flow directed toward the black hole, \( u = (dr/ds) < 0, \) and therefore, the numerical constant \( A > 0. \) From (8) and (14), we can easily obtain

\[ \frac{\rho + p}{n} \left( 1 - \frac{2}{x} + u^2 \right)^{1/2} = C_2, \]
where
\[ C_2 = \frac{\rho_\infty + p(\rho_\infty)}{n(\rho_\infty)}. \tag{11} \]

Let us now calculate the radial 4-velocity component and the fluid density on the event horizon of the black hole, \( r = 2M \). Setting \( x = 2 \), we obtain from Eqs. (9), (10) and (11)
\[ \frac{A}{4} \frac{\rho_H + p(\rho_H)}{\rho_\infty + p(\rho_\infty)} = \frac{n^2(\rho_H)}{n^2(\rho_\infty)}, \tag{12} \]
where \( \rho_H \) is the density on the \( x = 2 \) horizon. Thus, having specified the density at infinity \( \rho_\infty \), the equation of state \( p = p(\rho) \), and the flux \( A \) and using definition \( (7) \) of the concentration, we can calculate the fluid density \( \rho_H \) on the event horizon of the black hole from (12). Given the density on the horizon \( \rho_H \), we can easily determine the radial fluid 4-velocity component on the horizon from (13):
\[ u_H = -\frac{A}{4} \frac{n(\rho_\infty)}{n(\rho_H)}. \tag{13} \]

Below, we will see that the constant \( A \), which defines the energy flux onto the black hole, can be calculated for hydrodynamically stable ideal fluids with \( \partial p/\partial \rho > 0 \). This can be done by determining the fluid parameters at the critical point. Following Michel [33], we find the relationship between the parameters at the critical point:
\[ u^2 = \frac{1}{2 x_*}, \quad V^2 = \frac{u^2}{1 - 3 u^2}, \tag{14} \]
where
\[ V^2 = \frac{n}{\rho + p} \frac{d(\rho + p)}{dn} - 1, \tag{15} \]
Together with (7), this yields
\[ V^2 = c^2_s(\rho), \tag{16} \]
where \( c^2_s = \partial p/\partial \rho \) is the square of the effective speed of sound in the medium. We derive the following relation for the critical point from Eqs. (14), (10), (11), and (10):
\[ \frac{\rho_* + p(\rho_*)}{n(\rho_*)} = \left[1 + 3 c^2_s(\rho_*)\right]^{1/2} \frac{\rho_\infty + p(\rho_\infty)}{n(\rho_\infty)}, \tag{17} \]
which fixes the fluid density at the critical point \( \rho_* \) for an arbitrary equation of state \( p = p(\rho) \). Specifying \( \rho_* \) and using (5), we can determine \( n(\rho_*) \). Accordingly, the quantities \( x_* \) and \( u_* \) can be calculated from (13) and (10). As a result, the numerical constant \( A \) can be calculated by substituting the derived quantities into (8). For \( c^2_s < 0 \) or \( c^2_s > 1 \), no critical point exists beyond the event horizon of the black hole \( (x_** > 1) \), implying that the dark energy flux onto the black hole depends on the initial conditions for an unstable ideal fluid \( c^2_s < 0 \) or a ‘superluminal’ fluid \( c^2_s > 1 \). This result has a simple physical interpretation: the accreted fluid has a critical point if its speed increases from subsonic to supersonic values as it approaches the black hole. In contrast, for \( c^2_s \) or \( c^2_s > 1 \), the critical point either does not exist or is formally within the event horizon of the black hole. It should also be noted that fluids with \( c^2_s < 0 \) are hydrodynamically unstable (see [46, 47] for a discussion).

Equation (10), together with (6), (8), and (11), defines the accretion rate onto a black hole. These equations are valid for an ideal fluid with an arbitrary equation of state \( p = p(\rho) \), in particular, for a gas of massless particles (thermalized radiation) and a gas of massive particles. For a gas of massive particles, the system of equations (8) and (11) reduces to a similar system of equations found by Michel [33]. It should be noted, however, that Eqs. (6), (8), (9), and (10) are also valid for dark energy, including phantom energy with \( \rho + p < 0 \). In these cases, the concentration \( n(\rho) \) is positive for any \( \rho \), while the constant \( C_2 \) in Eq. (10) is negative.
The rate of change in the black hole mass (the energy flux onto the black hole) through accretion is
\[ \dot{M} = -4\pi r^2 T_0 r. \]
Using (9) and (10), this expression can be rewritten as [30]
\[ \dot{M} = 4\pi AM^2 [\rho_\infty + p(\rho_\infty)]. \] (18)
It follows from Eq. (18) than the mass of the black hole increases as it accretes the gas of particles when \( p > 0 \), but decreases as it accretes the phantom energy when \( p + \rho < 0 \). In particular, this implies that the black hole masses in the Universe filled with phantom energy must decrease. This result is general in nature. It does not depend on the specific form of the equation of state \( p = p(\rho) \); only the satisfaction of the condition \( p + \rho < 0 \) is important. The physical cause of the decrease in the black hole mass is as follows: the phantom energy falls to the black hole, but the energy flux associated with this fall is directed away from the black hole. If we ignore the cosmological evolution of the density \( \rho_\infty \), then we find the law of change in the black hole mass from (18) to be
\[ M = M_i \left( 1 - \frac{t}{\tau} \right)^{-1}, \] (19)
where \( M_i \) is the initial mass of the black hole, and \( \tau \) is the evolution time scale:
\[ \tau = 1/ \left\{ 4\pi AM_i [\rho_\infty + p(\rho_\infty)] \right\}. \] (20)

3 ANALYTICAL ACCRETION MODELS

3.1 Model of a Linear Equation of State
Let us consider the model of dark energy with a linear density dependence of pressure [30]:
\[ p = \alpha (\rho - \rho_0), \] (21)
where \( \alpha \) and \( \rho_0 \) are constants. Among the other cases, this model describes an ultrarelativistic gas (\( p = \rho/3 \)), a gas with an ultrahard equation of state (\( p = \rho \)), and the simplest model of dark energy (\( \rho_0 = 0 \) and \( \alpha < 0 \)). The quantity \( \alpha \) is related to the parameter \( w = p/\rho \) of the equation of state by \( w = \alpha (\rho - \rho_0)/\rho \).

An equation of state with \( w = \text{const} < 0 \) throughout the cosmological evolution is commonly used to analyze cosmological models. The matter with such an equation of state is hydrodynamically unstable and can exist only for a short period. Our equation of state (21) for \( \alpha > 0 \) does not have this shortcoming. For \( \alpha > 0 \), it also allows the case of hydrodynamically stable phantom energy to be described, which is not possible when using an equation of state with \( w = \text{const} < -1 \). In the real Universe, the equation of state changes with time (i.e., \( w \) depends on \( t \)). Therefore, Eq. (21) has the meaning of an approximation to the true equation of state only in a limited \( \rho \) range. From the physical point of view, the condition \( \rho > 0 \) must be satisfied for any equation of state in a comoving frame of reference. In particular, the state of matter with \( \rho = 0 \), but \( p \neq 0 \), is physically unacceptable. The corresponding constraints for the equation of state (21) are specified by conditions (29) and (30) given below.

For \( \alpha < 0 \), there is no critical point for the accreted fluid flow. For \( \alpha > 0 \), using (14) and (16), we obtain the parameters of the critical point
\[ x_* = \frac{1 + 3\alpha}{2\alpha}, \quad u_*^2 = \frac{\alpha}{1 + 3\alpha}. \] (22)
Note that the parameters of the critical point (22) in the linear model (21) are determined only by \( \partial p/\partial \rho = \alpha \) and do not depend on \( \rho_0 \), which fixes the physical nature of the fluid under consideration: a relativistic gas, dark energy, or phantom energy. Note also that no critical point exists beyond the event horizon of the black hole for \( \alpha > 1 \) (this corresponds to a nonphysical situation with a superluminal speed of sound).

Let us calculate the constant \( A \), which defines the energy flux onto the black hole. We find from (8) that

\[
\frac{n}{n_\infty} = \left| \frac{\rho_{\text{eff}}}{\rho_{\text{eff},\infty}} \right|^{1/(1+\alpha)},
\]

where we introduced the effective density

\[
\rho_{\text{eff}} \equiv \rho + p = -\rho_0 \alpha + (1 + \alpha) \rho.
\]

Using (17), we obtain

\[
\left( \frac{\rho_{\text{eff},\infty}}{\rho_{\text{eff}} \rho_{\text{eff},\infty}} \right)^{\alpha/(1+\alpha)} = (1 + 3\alpha)^{1/2},
\]

where \( \rho_{\text{eff}} \) and \( \rho_{\text{eff},\infty} \) are the effective densities at the critical point and at infinity, respectively. Substituting (24) into (23) and using (9), we obtain for the linear model

\[
A = \frac{(1 + 3\alpha)^{1+3\alpha}/2\alpha}{4\alpha^{3/2}}.
\]

It is easy to see that \( A \geq 4 \) for \( 0 < \alpha < 1 \). \( A = 4 \) for \( \alpha = 1 \) (this corresponds to \( c_s = 1 \)); i.e., the constant \( A \) is on the order of 1 for relativistic speeds of sound. Using (25), we obtain from (20)

\[
\tau = \left[ \pi M_i (\rho_\infty + p_\infty) \frac{(1 + 3\alpha)^{(1+3\alpha)/2\alpha}}{\alpha^{3/2}} \right]^{-1}.
\]

To determine the fluid density on the event horizon of the black hole, we substitute (23) into (12) to yield

\[
\rho_H = \frac{\alpha \rho_0}{1 + \alpha} + \left( \rho_\infty - \frac{\alpha \rho_0}{1 + \alpha} \right) \left( \frac{A}{4} \right)^{(1+\alpha)/(1-\alpha)},
\]

where \( A \) is given by (25). For \( 0 < \alpha < 1 \), the effective density on the horizon \( \rho_{\text{eff},H} \) cannot be lower than \( \rho_{\text{eff},\infty} \). The radial 4-velocity component on the horizon can be found from (13) and (27):

\[
u_H = - \left( \frac{A}{4} \right)^{-\alpha/(1-\alpha)},
\]

The value of \( u_H \) changes from 1 to \( 1/2 \) for \( 0 < \alpha < 1 \).

The linear model (21) describes the phantom energy when \( \rho_\infty/\rho_0 < \alpha/(1 + \alpha) \). In this case, \( \rho + p < 0 \). However, the requirement that the density \( \rho \) be nonnegative should be taken into account. This parameter can formally be negative in the range \( 0 < \alpha \leq 1 \). Such a nonphysical situation imposes a constraint on the linear model (21) under consideration. For a physically proper description of the accretion process, we must require that the density \( \rho \) be nonnegative. We obtain the following constraint on the validity range of the linear model from (27) for hydrodynamically stable phantom energy:

\[
\frac{\alpha}{1 + \alpha} \left[ 1 - \left( \frac{A}{4} \right)^{-(1+\alpha)/(1-\alpha)} \right] < \frac{\rho_\infty}{\rho_0} < \frac{\alpha}{1 + \alpha}.
\]

As follows from (29), at a given \( \alpha \), we can always choose the parameters \( \rho_0 \) and \( \rho_\infty \) in such a way that \( \rho > 0 \) for any \( r > 2M \).
On the other hand, model (21) describes the quintessence (not the phantom energy) for the entire $r$ range only if $p < 0$. Consequently, a physically proper description of the quintessence can be obtained from (27) if

$$\frac{\alpha}{1+\alpha} < \frac{\rho_\infty}{\rho_0} < \frac{\alpha}{1+\alpha} \left[ \frac{1}{\alpha} + \left(\frac{A}{4}\right)^{-(1+\alpha)/(1-\alpha)} \right].$$

(30)

For some of the specific choices of $\alpha$ (more specifically, for $\alpha = 1/3, 1/2, 2/3, 1$), $\rho(x)$ and $u(x)$ can be calculated analytically (see the Appendix for details on these calculations). In Figs. 1 and 2, the radial 4-velocity component, $u$, and the density normalized to the density at infinity, $\rho/\rho_\infty$ are plotted against the coordinate $x = r/2M$.

Figure 1 (left) shows the plots of $u(x)$ for hydrodynamically stable fluids with $c_s^2 > 0$ at $\alpha = 1/3, 1/2, 2/3, 1$ (the curves are arranged from top to bottom, respectively). Figure 1 (right) shows the plots of $u(r)$ for $c_s^2 < 0$ at $\alpha = -1.1, -2, -1/2$ (the curves are also arranged from top to bottom). For this case, we chose the boundary condition $u_H = 1$ on the horizon. Figure 2 (left) shows the plots of $\rho/\rho_\infty$ for a hydrodynamically stable fluid with $\alpha = 1$ for various cases: $\rho_0 = 0$ (the model of neutron star matter); $\rho_0/\rho_\infty = 16/9$ (the linear model of nonphantom dark energy); $\rho_0/\rho_\infty = 7/3$ (the linear model of phantom energy); and $\rho_0/\rho_\infty = 7/3$ (the linear model of phantom energy with $\rho_H = 0$) (the curves are arranged from top to bottom, respectively). Figure 2 (right) shows the plots of $(\rho/\rho_\infty)$ for various cases: $\alpha = -2$, $\rho_0 = 0$, and $A = 4$ (the linear model of phantom energy, the upper curve); and $\alpha = -1/2$, $\rho_0 = 0$, and $A = 4$ (the linear model of nonphantom energy, the lower curve). For this case, we chose the velocity $|u_H| = 1$ on the horizon.

### 3.2 Accretion onto a Moving and Rotating Black Hole

Let us consider the accretion onto a moving and rotating black hole in the special case of a linear equation of state with $\alpha = 1$. The condition $\alpha = 1$ allows an exact analytical expression to be derived for the accretion rate of dark energy onto a black hole.

For $\alpha = 1$, we easily find from (23) that

$$\frac{n}{n_\infty} = \left| \frac{\rho_{\text{eff}}}{\rho_{\text{eff},\infty}} \right|^{1/2}.$$  

(31)
Figure 2: Accreted fluid density normalized to the density at infinity, $\rho/\rho_\infty$, versus radial coordinate $x$ for the linear model (21) (solid curves). The dashed lines indicate the density of the $\Lambda$-term, $\rho/\rho_\infty$.

We obtain the following continuity equation for the particle concentration from (5):

$$(nu^\mu)_{;\mu} = 0.$$ 

We can introduce the scalar field $\phi$ in terms of which the fluid velocity can be expressed as follows (there is no torsion in the fluid):

$$\frac{\rho + p}{n} u_\mu = \phi_{,\mu}. \quad (32)$$

We derive an equation for the auxiliary function $\phi$ by using Eqs. (31) and (32),

$$\phi_{,\mu} = 0. \quad (33)$$

Exactly the same equation arises in the problem of the accretion of a fluid with the equation of state $p = \rho$ [41]. Thus, we reduced the problem of a black hole moving in dark energy with the equation of state $p = \rho - \rho_0$ to the problem of a fluid with an extremely hard equation of state, $p = \rho$. Using the method suggested in [41], we obtain the mass evolution law for a moving and rotating black hole immersed in dark energy with the equation of state $p = \rho - \rho_0$:

$$\dot{M} = 4\pi(r_+^2 + a^2)(\rho_\infty + p(\rho_\infty))|u_{\text{BH}}^0|,$$ 

where

$$r_+ = M + (M^2 - a^2)^{1/2}$$

is the radius of the event horizon for a rotating black hole, $a = J/M$ is the specific angular momentum of the black hole (rotation parameter), and $u_{\infty}^0$ is the zeroth 4-velocity component of the black hole relative to the fluid. Expression (34) for $u_{\text{BH}}^0 = 0$ reduces to (18) for a Schwarzschild ($a = 0$) black hole at rest.

### 3.3 Chaplygin Gas

Let us consider a Chaplygin gas with the following equation of state as another example of the solvable model:

$$p = \frac{-\alpha}{\rho}, \quad (35)$$
where $\alpha > 0$. The range of parameters $\rho^2 < \alpha$ represents the phantom energy with a superluminal speed of sound, implying that the phantom energy flux onto the black hole is not fixed by the condition of its passage through the critical point. The case of $\rho^2 > \alpha$ corresponds to dark energy with $\rho + p > 0$ and $0 < c_s^2 < 1$. We can easily find from Eq. (8) that
\[
\frac{n}{n_\infty} = \left| \frac{\rho^2 - \alpha}{\rho^2 - \alpha} \right|^{1/2}.
\]  
(36)

The density at the critical point can be calculated from (17) and (36):
\[
\rho^2_* = 4\rho^2_\infty - 3\alpha.
\]  
(37)

The velocity and the radial coordinate at the critical point are given by
\[
x_* = \frac{2\rho^2_\infty}{\alpha}, \quad u^2_* = \frac{\alpha}{4\rho^2_\infty}.
\]  
(38)

We then find the constant $A$ from Eq. (9):
\[
A = 4 \left( \frac{\rho^2_\infty}{\alpha} \right)^{3/2}.
\]  
(39)

For $0 < c_s^2 < 1$, the constant $A$ cannot be smaller than 4, as in the case of the linear model. The evolution time scale of the black hole mass without any cosmological change in the dark energy density is given by
\[
\tau = \left[ 8\pi M_\text{f} \frac{\rho^2_\infty}{\alpha} (\rho_\infty + p_\infty) \right].
\]  
(40)

Note that Eqs. (36)-(39) are applicable only for dark energy with $\rho + p > 0$ and are invalid for phantom energy. On the black hole horizon,
\[
\rho_H = \frac{A}{4} \rho_\infty, \quad u_H = -\frac{A}{4} \left[ \frac{\rho^2_\infty - \alpha}{(A/4)^2 \rho^2_\infty - \alpha} \right]^{1/2}.
\]  
(41)

For $0 < c_s^2 < 1$, the density on the horizon $\rho_H$ cannot be lower than $\rho_\infty$, and $u_H$ changes from 1 to $1/2$. The Chaplygin gas density distribution can be determined from the general equations (9) and (10):
\[
\rho = \rho_\infty \left( \frac{\rho^2_\infty}{\alpha} \right)^{3/2} \left( \frac{2}{x} - \frac{\alpha}{\rho^2_\infty} \right)^{-1/2} \left[ \frac{16}{x^4} \left( 1 - \frac{\alpha}{\rho^2_\infty} \right) - \left( \frac{\alpha}{\rho^2_\infty} \right)^4 \left( 1 - \frac{2}{x} \right) \right]^{1/2}.
\]  
(42)

The velocity distribution $u(r)$ can be calculated by using Eqs. (9), (36), and (42). In Figs. 3 and 4, the velocity $u$ and the density normalized to the density at infinity, $\rho/\rho_\infty$, are plotted against the coordinate $x = r/2M$. Figure 3 (left) shows the plots of $u(r)$ for nonphantom dark energy at $\rho^2_\infty/\alpha = 3$, 2, and 1.1 (the curves are arranged from top to bottom, respectively). Figure 3 (right) shows the plots of $u(r)$ for phantom energy at $\rho^2_\infty/\alpha = 0.3$, 0.5, and 0.9 (the curves are also arranged from top to bottom, respectively). In this case, the boundary condition $u_H = 1$ is set on the horizon. Figure 4 (left) shows the plots of the normalized density, $\rho/\rho_\infty$, for nonphantom dark energy at $\rho^2_\infty/\alpha = 3$, 2, and 1.1 (the curves are arranged from top to bottom, respectively). Figure 4 (right) shows the plots of the normalized density, $(\rho/\rho_\infty)$, for phantom energy at $\rho^2_\infty/\alpha = 0.9$, 0.5, and 0.3 (the curves are also arranged from top to bottom, respectively). For this case, we chose the boundary condition $u_H = 1$ on the horizon.
4 THE FATE OF BLACK HOLES DURING THE BIG RIP

Let us now consider the evolution of black holes in the cosmological Big Rip scenario, where the scale factor $a(t)$ increases to infinity in a finite time [16, 17]. For simplicity, we take into account only the dark energy and disregard the other forms of energy. In the linear model (21), the Big Rip takes place at $\rho + p < 0$ and $\alpha < -1$. The following relation can be derived from Friedmann’s equations in the case of a linear equation of state:

$$|\rho + p| \propto a^{-3(1+\alpha)}.$$  

Setting, for simplicity, $\rho_0 = 0$, we find the evolution law of the phantom energy density in this Universe:

$$\rho_\infty = \rho_{\infty,i} \left(1 - \frac{t}{\tau}\right)^{-2},$$  

where

$$\tau^{-1} = - \frac{3(1 + \alpha)}{2} \left(\frac{8 \pi}{3} \rho_{\infty,i}\right)^{1/2}.$$  

Here, $\rho_{\infty,i}$ is the initial cosmological phantom energy, and the initial time was chosen in such a way that the Big Rip occurs at time $\tau$. We easily see from Eqs. (20) and (43) that the Big Rip takes place at $\alpha = \partial p/\partial \rho < -1$. In general, the condition $\rho + p < 0$ alone is not enough for the cosmological evolution to be ended with the Big Rip [18].

Using Eq. (43), we find the evolution of the black hole mass in the cosmological Big Rip scenario from Eq. (18):

$$M = M_i \left(1 + \frac{M_i}{M_0} \frac{t}{\tau - t}\right)^{-1},$$  

where

$$\dot{M}_0 = (3/2) A^{-1}|1 + \alpha|.$$  

and $M_i$ is the initial mass of the black hole. At $\alpha = -2$ and a typical value of $A = 4$ (which correspond to $u_H = -1$), $M_0 = 3/8$. In the limit $t \to \tau$ (i.e., near the Big Rip), the $t$ dependence of the black hole mass becomes linear, $M \simeq \dot{M}_0 (\tau - t)$. When $t$ approaches $\tau$,
Figure 4: Density normalized to the density at infinity, $\rho/\rho_\infty$, versus coordinate $x$ for model (35) (solid curves). The dashed line indicates the normalized density of the $\Lambda$-term, $\rho/\rho_\infty$.

the rate of decrease in the black hole mass ceases to depend on the initial black hole mass and the phantom energy density: In other words, the masses of all black holes near the Big Rip are approximately equal and approach zero. This implies that the accretion of phantom energy dominates over the Hawking evaporation until the black hole mass decreases to the Planck mass. Formally, however, all black holes in the Universe completely evaporate during the Hawking radiation in the Planck time before the Big Rip occurs.

5 THE ACCRETION OF A SCALAR FIELD

In this section, we compare our calculations of the accretion of an ideal fluid with similar calculations of the accretion of a scalar (nonphantom) field onto a black hole [24-29]. The dark energy is commonly modeled by a scalar field with a potential $V(\phi)$. The approximation of an ideal fluid is rougher, since the scalar field $\phi$ and $\partial_\mu \phi$ cannot be unambiguously reproduced for given $\rho$ and $p$, which characterize an ideal fluid. Despite this difference between the scalar field and the ideal fluid, we will show that our results are in close agreement with the corresponding calculations of the accretion of a scalar field onto a black hole.

The Lagrangian of the scalar field is $L = K - V$, where $K$ is the kinetic term and $V$ is the potential. For the standard choice of the kinetic term

$$K = \phi,\mu \phi^{\mu}/2,$$

the corresponding energy flux onto the black hole is

$$T_{0r} = \phi,_{t} \phi,_{r}.$$

Jacobson [24] found a solution for the scalar field in the Schwarzschild metric for a zero potential, $V = 0$:

$$\phi = \phi_\infty \left[ t + 2M \ln(1 - 2M/r) \right],$$

where $\phi_\infty$ is the scalar field at infinity. Frolov and Kofman [26] showed that this solution is also valid for many scalar fields with a nonzero potential $V()$ under certain conditions. For this solution

$$T_{0r} = -(2M)^2 \phi_\infty^2 / r^2,$$
and, accordingly,
\[ \dot{M} = 4\pi (2M)^2 \dot{\phi}_\infty^2. \]

The energy-momentum tensor constructed using Jacobson’s solution is identical to the energy-momentum tensor for an ideal fluid with an extremely hard equation of state, \( p = \rho \), after the substitution
\[ p_\infty \rightarrow \dot{\phi}_\infty^2/2, \quad \rho_\infty \rightarrow \dot{\phi}_\infty^2/2. \]
This is not surprising, since the theory of a scalar field with a zero potential, \( V(\phi) \), is identical to the model of an ideal fluid [48]. In view of this correspondence, we easily see agreement between our result (18) for \( \dot{M} \) in the case of \( p = \rho \) and the corresponding results from [24, 26].

The Lagrangian of the scalar field that describes the phantom energy must have a negative kinetic term [16, 17], for example,
\[ K = -\phi_{,\mu}\phi^{,\mu}/2 \]
(see [49] for more general cases). In this case, the phantom energy flux onto the black hole has the opposite sign,
\[ T_{0r} = -\phi_r\phi,_{r}, \]
where \( \phi \) is the solution of the same Klein-Gordon equation as that for the standard scalar field, but with the substitution \( V \rightarrow -V \). For a zero potential, this solution is identical to Jacobson’s solution [24] obtained for a scalar field with a positive kinetic term.

However, the Lagrangian with a negative kinetic term and \( V(\phi) = 0 \) does not describe the phantom energy. At the same time, the solution for a scalar field with \( V(\phi) = 0 \) is identical to the solution for a positive constant potential, \( V_0 = \text{const} \), which can be chosen in such a way that
\[ \rho = -\dot{\phi}^2/2 + V_0 > 0. \]
In this case, the scalar field describes the accreted phantom energy with \( \rho > 0 \) and \( p < -\rho \), which leads to a decrease in the black hole mass at the rate
\[ \dot{M} = -4\pi (2M)^2 \dot{\phi}_\infty^2. \]

A simple example of phantom cosmology (but without the Big Rip) is realized by a scalar field with the potential
\[ V = m^2 \phi^2/2, \]
where \( m \sim 10^{-33} \text{ eV} \) [50]. After a short transition period, this cosmological model approaches an asymptotic state with
\[ H \simeq m\phi/3^{1/2}, \quad \phi \simeq 2m/3^{1/2}. \]
In the Klein-Gordon equation (with the substitution \( V \rightarrow -V \) mentioned above), the term \( m^2 \) becomes equal to the other terms only on the scale of the cosmological horizon, implying that, in this case, Jacobson’s solution is also valid. Calculations of the corresponding energy flux onto the black hole yield
\[ \dot{M} = -4\pi (2M)^2 \dot{\phi}_\infty^2 = -64M^2 m^2/3. \]
For \( M_0 = M_\odot \) and \( m = 10^{-33} \text{ eV} \), the effective time of the decrease in black hole mass is
\[ \tau = (3/64)M^{-1}m^{-2} \sim 10^{32} \text{ yr}. \]
DISCUSSION AND CONCLUSIONS

In recent years, the concept of dark energy has been accepted and extensively discussed in cosmology. The possible existence of dark energy with a negative pressure leads to new cosmological scenarios, including the exotic model of the Universe in which all of the bound objects are destroyed and which dies itself as a result of the Big Rip. To determine the fate of black holes in this cosmological scenario, we considered the spherically symmetric, stationary accretion of dark energy modeled by an ideal fluid onto a black hole. We derived general equations for the accretion of an ideal fluid with the equation of state \( p = p(\rho) \) onto a Schwarzschild black hole. In particular, these equations can be used to describe the accretion of thermal radiation, dark energy, and phantom energy. We also considered the accretion onto a moving and rotating black hole in the special case of an extremely hard equation of state, \( p = \rho \). We calculated the change in the black hole mass through accretion. The black hole masses for \( \rho + p > 0 \) were found to increase, as in the usual case. However, a qualitatively new result was obtained for phantom energy, i.e., for a medium with \( \rho + p < 0 \). We found that the black hole masses decrease in this situation. Using this result, we solved the problem of the fate of black holes in a universe that undergoes the Big Rip. It turns out that all black holes in this Universe must decrease their masses and disappear completely by the Big Rip. We also considered the correspondence between the accretion of dark energy in the model of an ideal fluid and the accretion of a scalar field.

Acknowledgments: This work was supported in part by the Russian Foundation for Basic Research, project nos. 02-02-16762a, 03-02-16436a, and 04-02-16757a and by the Ministry of Science of the Russian Federation, grants 1782.2003.2 and 2063.2003.2.

Appendix: Analytical Solutions for \( \rho(x) \) and \( u(x) \)

In the model under consideration (Section 3), analytical solutions can be found for the dependence of the dark energy density and accretion rate on radius \( r \). Using Eq. (23) for the concentration and Eq. (11) for the constant \( C_2 \), we derive the following equation for \( \rho_{\text{eff}} \) from Eqs. (9) and (10)

\[
\left( \frac{\rho_{\text{eff}}}{\rho_{\text{eff},\infty}} \right)^{2\alpha/(1+\alpha)} \left[ 1 - \frac{2}{x} + \frac{A^2}{x^4} \left( \frac{\rho_{\text{eff}}}{\rho_{\text{eff},\infty}} \right)^{-2/(1+\alpha)} \right] = 1. \tag{47}
\]

Defining

\[
y \equiv \left( \frac{\rho_{\text{eff}}}{\rho_{\text{eff},\infty}} \right)^{2/(1+\alpha)}, \tag{48}
\]

we obtain the following equation from (47):

\[
y \left( 1 - \frac{2}{x} \right) - y^{1-\alpha} + \frac{A^2}{x^4} = 0, \tag{49}
\]

which can be solved analytically for certain values of \( \alpha \). For \( \alpha = 1/3 \), Eq. (49) reduces to a cubic equation:

\[
z^3 \left( 1 - \frac{2}{x} \right) - z^2 + \frac{A^2}{x^4} = 0, \tag{50}
\]

where \( z = y^{1/3} \). Solving this equation yields the fluid density distribution for \( \alpha = 1/3 \):

\[
\rho = \frac{\rho_0}{4} + \left( \rho_\infty - \frac{\rho_0}{4} \right) \left[ z + \frac{1}{3(1-2x^{-1})} \right]^2, \tag{51}
\]
where
\[ z = \begin{cases} 
2\sqrt{\frac{\pi}{3}} \cos \left( \frac{2\pi}{3} - \frac{\beta}{3} \right), & 2 \leq x \leq 3, \\
2\sqrt{\frac{\pi}{3}} \cos \left( \frac{\beta}{3} \right), & x > 3,
\end{cases} \tag{52} \]

\[ \beta = \arccos \left[ \frac{b}{2 (a/3)^{3/2}} \right] \tag{53} \]

and
\[ a = \frac{1}{3(1 - 2/x)^2}, \quad b = \frac{2}{27(1 - 2/x)^3} - \frac{108}{(1 - 2/x)^4}. \tag{54} \]

This solution corresponds to a thermalized photon gas in which the photon mean free path is much smaller than the radius of the black hole horizon, \( \lambda_{fp} \ll 2M \). In this situation, the photon gas may be treated as an ideal fluid. In the opposite case, \( \lambda_{fp} \gg 2M \), the photons are free particles, and their accretion rate is determined by the well-known cross section for the gravitational capture of relativistic particles by a black hole. The corresponding accretion rate is
\[ \dot{M} = 27\pi M^2 \rho_\infty. \]

The case of \( \alpha = 2/3 \) is similar to the case considered above. We obtain the following equation instead of (50):
\[ z^3 \left( 1 - \frac{2}{x} \right) - z + \frac{A^2}{x^2} = 0, \tag{55} \]

where again \( z = y^{1/3} \). The fluid density distribution in this case is
\[ \rho = \frac{2}{5} \rho_0 + \left( \rho_\infty - \frac{2}{5} \rho_0 \right) z^{5/2}, \tag{56} \]

where \( z \) is given by
\[ z = \begin{cases} 
2\sqrt{\frac{\pi}{3}} \cos \left( \frac{2\pi}{3} - \frac{\beta}{3} \right), & 2 \leq x \leq 9/4, \\
2\sqrt{\frac{\pi}{3}} \cos \left( \frac{\beta}{3} \right), & x > 9/4,
\end{cases} \tag{57} \]

\( \beta \) is defined by Eq. (53), and
\[ a = \frac{1}{1 - 2/x}, \quad b = \frac{-2187 \sqrt{3}}{128 (1 - 2/x)^2}. \]

For \( \alpha = 1/2 \), (49) is a quadratic equation and has a simple analytical solution:
\[ \rho = \frac{\rho_0}{3} + \left( \rho_\infty - \frac{\rho_0}{3} \right) z^{3/2}, \tag{58} \]

where
\[ z = \begin{cases} 
\frac{1}{2} \left\{ 1 - \left[ 1 - 3125 (1 - 2/x) (16 x^4)^{-1} \right]^{1/2} \right\} (1 - 2/x)^{-1}, & 2 \leq x \leq 5/2, \\
\frac{1}{2} \left\{ 1 + \left[ 1 - 3125 (1 - 2/x) (16 x^4)^{-1} \right]^{1/2} \right\} (1 - 2/x)^{-1}, & x > 5/2.
\end{cases} \tag{59} \]

For \( \alpha = 1 \), Eq. (49) is linear in \( y \), which gives
\[ \rho = \frac{\rho_0}{2} + \left( \rho_\infty - \frac{\rho_0}{2} \right) \left( 1 + \frac{4}{x^2} \right). \tag{60} \]
References

[1] N. Bahcall, J. P. Ostriker, S. Perlmutter, and P. J. Steinhardt, Science 284, 1481 (1999).
[2] A. Riess, A. V. Filippenko, P. Challis, et al., Astron. J. 116, 1009 (1998).
[3] S. J. Perlmutter, G. Aldering, G. Goldhaber, et al., Astro-phys. J. 517, 565 (1999).
[4] C. L. Bennett, M. Halpern, G. Hinshaw, et al., Astro-phys. J., Suppl. Ser. 148, 1 (2003).
[5] C. Wetterich, Nucl. Phys. B 302, 668 (1988).
[6] P. J. E. Peebles and B. Ratra, Astrophys. J. 325, L17 (1988).
[7] B. Ratra and P. J. E. Peebles, Phys. Rev. D 37, 3406 (1988).
[8] J. A. Frieman, C. T. Hill, A. Stebbins, and I. Waga, Phys. Rev. Lett. 75, 2077 (1995).
[9] R. R. Caldwell, R. Dave, and P. J. Steinhardt, Phys. Rev. Lett. 80, 1582 (1998).
[10] A. Albrecht and C. Skordis, Phys. Rev. Lett. 84, 2076 (2000).
[11] C. Armendariz-Picon, T. Damour, and V. Mukhanov, Phys. Lett. B 458, 209 (1999).
[12] C. Armendariz-Picon, V. Mukhanov, and P. J. Steinhardt, Phys. Rev. Lett. 85, 4438 (2000).
[13] T. Chiba, T. Okabe, and M. Yamaguchi, Phys. Rev. D 62, 023511 (2000).
[14] I. Zlatev, L. Wang, and P. Steinhardt, Phys. Rev. Lett. 82, 896 (1999).
[15] P. Steinhardt, L. Wang, and I. Zlatev, Phys. Rev. D 59, 123504 (1999).
[16] R. R. Caldwell, Phys. Lett. B 545, 23 (2002).
[17] R. R. Caldwell, M. Kamionkowski, and N. N. Weinberg, Phys. Rev. Lett. 91, 071301 (2003).
[18] B. McInnes, J. High Energy Phys. 0208, 029 (2002); M. Bouhmadi-Lopez and J. A. J. Madrid, astro-ph/0404540.
[19] U. Alam, V. Sahni, T. D. Saini, and A. A. Starobinsky, astro-ph/0311364.
[20] S. Nojiri and S. D. Odintsov, Phys. Lett. B 562, 147 (2003).
[21] I. Brevik, S. Nojiri, S. D. Odintsov, and L. Vanzo, hep-th/ 0401073.
[22] M. Visser, S. Kar, and N. Dadhich, Phys. Rev. Lett. 90, 201102 (2003).
[23] P. F. Gonzalez-Diaz, Phys. Rev. D 68, 084016 (2003).
[24] T. Jacobson, Phys. Rev. Lett. 83, 2699 (1999).
[25] R. Bean and J. Magueijo, Phys. Rev. D 66, 063505 (2002).
[26] A. Frolov and L. Kofman, J. Cosmol. Astrophys. Phys. 5, 9 (2003).
[27] W. G. Unruh, Phys. Rev. D 14, 3251 (1976).
[28] L. A. Urena-Lopez and A. R. Liddle, Phys. Rev. D 66, 083005 (2002).
[29] M. Yu. Kuchiev and V. V. Flambaum, gr-qc/0312065.
[30] E. O. Babichev, V. I. Dokuchaev, and Yu. N. Eroshenko, Phys. Rev. Lett. 93, 021102 (2004).
[31] E. O. Babichev, V. I. Dokuchaev, and Yu. N. Eroshenko, Class. Quantum Grav. 22, 143 (2005).

[32] H. Bondi, Mon. Not. R. Astron. Soc. 112, 195 (1952).

[33] F. C. Michel, Astrophys. Space Sci. 15, 153 (1972).

[34] B. J. Carr and S. W. Hawking, Mon. Not. R. Astron. Soc. 168, 399 (1974).

[35] M. C. Begelman, Astron. Astrophys. 70, 583 (1978).

[36] D. Ray, Astron. Astrophys. 82, 368 (1980).

[37] K. S. Thorne, R. A. Flammang, and A. N. Zytkow, Mon. Not. R. Astron. Soc. 194, 475 (1981).

[38] E. Bettwieser and W. Glatzel, Astron. Astrophys. 94, 306 (1981).

[39] K. M. Chang, Astron. Astrophys. 142, 212 (1985).

[40] U. S. Pandey, Astrophys. Space Sci. 136, 195 (1987).

[41] L. I. Petrich, S. L. Shapiro, and S. A. Teukolsky, Phys. Rev. Lett. 60, 1781 (1988).

[42] B. J. Carr and S. W. Hawking, Mon. Not. R. Astron. Soc. 168, 399 (1974).

[43] E. Bettwieser and W. Glatzel, Astron. Astrophys. 94, 306 (1981).

[44] S. W. Hawking and G. F. R. Ellis, The Large Scale Structure of Space-Time (Cambridge Univ. Press, Cambridge, 1973), Chap. 4.3.

[45] C. W. Misner, K. S. Thorne, and J. A. Wheeler, Gravitation (Freeman, San Francisco, 1973).

[46] J. C. Fabris and J. Martin, Phys. Rev. D 55, 5205 (1997).

[47] S. M. Carroll, M. Hoffman, and M. Trodden, Phys. Rev. D 68, 023509 (2003).

[48] V. N. Lukash, Zh. Eksp. Teor. Fiz. 79, 1601 (1980) [Sov. Phys. JETP 52, 807 (1980)].

[49] P. F. Gonzalez-Diaz, Phys. Lett. B 586, 1 (2004).

[50] M. Sami and A. Toporensky, Mod. Phys. Lett. A 19, 1509 (2004).