A note on strings in deformed $AdS_4 \times \mathbb{C}P^3$: giant magnon and single spike solutions

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Abstract

In this paper we study the solitonic string solutions of magnon and single spike type in the beta-deformed $AdS_4 \times \mathbb{C}P^3$ background. We find the dispersion relations which are supposed to give the anomalous dimension of the gauge theory operators.

1 Introduction

The string/gauge theory duality has attracted the attention of the high energy community for many years. As an example of such duality AdS/CFT correspondence remains in the focus of the recent studies. A new exciting example of this duality has been added, namely duality between string in $AdS_4 \times \mathbb{C}P^3$ and $\mathcal{N} = 6$ superconformal Chern-Simons theory suggested by $[3]$. The ABJM model $[3]$ is originally defined in M-theory and is believed to be holographically dual to M-theory on $AdS_4 \times S^7/Z_k$. Its reduction to string theory is believed to be dual to two Chern-Simons theories of level $k$ and $-k$, respectively, and each with gauge group $SU(N)$. The two pairs of chiral superfields transform in the bi-fundamental representation of $SU(N) \times SU(N)$ and the R-symmetry is $SU(4)$ as it should be for $\mathcal{N} = 6$ supersymmetry.

The semi-classical string has played an important role in studying various aspects of the $AdS_4/CFT_3$ correspondence $[6]-[25]$. The developments and successes in this particular case suggest the methods and tools that should be used to investigate the new emerging duality. An important role in these studies plays the integrability. Superstrings on $AdS_4 \times \mathbb{C}P^3$ as a coset were first studied in $[6]$ which opens the door for investigation of integrable structures in the theory. Various properties on the gauge theory side and

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1 Certainly we are not able to cite the huge list of papers contributing to the subject, we just quote the necessary minimum.
tests on the string theory side, like rigid rotating strings, pp-wave limit, relations to spin chains, as well as a pure spinor formulation, have been considered [1]-[16]. The complete type-IIA Green-Schwarz string action in AdS$_4 \times$ CP$^3$ superspace has been constructed in [17].

The theories with reduced supersymmetry are important not only conceptually but also for describing realistic physics. A very elegant technique for generating solutions with reduced supersymmetry was proposed by [28]. The method consists in T-duality along an isometry direction, followed by shift with a parameter $\beta$ defining the deformation, and another T-duality called TsT- transformation. The method was further studied and developed in [29]. One should note that the deformed theory has a richer structure of the vacua than its undeformed cousin. It is believed that the AdS/CFT correspondence persists after the deformation, thus providing important information. In this paper we are studying the solitonic string solutions of magnon and single spike type in the beta-deformed AdS$_4 \times$ CP$^3$ background. The dispersion relations are supposed to give the anomalous dimension of the gauge theory operators.

The paper is organized as follows. In the Introduction we give the basic concept of magnon and single spike string solutions and their dispersion relations. In the second section we present the corresponding solutions in the deformed AdS$_4 \times$ CP$^3$ background and their dispersion relations. The summary of our results is presented in Conclusions.

2 About giant magnons and single spikes in AdS$_4 \times$ CP$^3$ in short

In this section we will give very brief review of the giant magnon and single spike string solutions in AdS$_4 \times$ CP$^3$. The purpose is to set up the notations and write the solutions in the undeformed case for comparison to our results.

Let us start with the AdS$_4 \times$ CP$^3$ metric and field content are given by

$$ds_{IIA}^2 = \frac{R^3}{k} \left( \frac{1}{4} ds_{AdS_4}^2 + ds_{CP^3}^2 \right)$$

$$ds_{CP^3}^2 = d\xi^2 + \cos^2 \xi \sin^2 \xi \left( d\psi + \frac{1}{2} \cos \theta_1 d\varphi_1 - \frac{1}{2} \cos \theta_2 d\varphi_2 \right)^2$$

$$\quad \quad + \frac{1}{4} \cos^2 \xi (d\theta_1^2 + \sin^2 \theta_1 d\varphi_1^2) + \frac{1}{4} \sin^2 \xi (d\theta_2^2 + \sin^2 \theta_2 d\varphi_2^2),$$

$$e^{2\psi} = \frac{R^3}{k^3} \quad (2.1)$$

$$C_1 = \frac{k}{2} \left( (\cos^2 \xi - \sin^2 \xi) d\psi + \cos^2 \xi \cos \theta_1 d\varphi_1 + \sin^2 \xi \cos \theta_2 d\varphi_2 \right),$$

$$F_2 = k \left( -\cos \xi \sin \xi d\xi \wedge (2d\psi + \cos \theta_1 d\varphi_1 - \cos \theta_2 d\varphi_2) \right.$$  

$$\quad \quad \quad \quad \quad - \frac{1}{2} \cos^2 \xi \sin \theta_1 d\theta_1 \wedge d\varphi_1 - \frac{1}{2} \sin^2 \xi \sin \theta_2 d\theta_2 \wedge d\varphi_2 \right)$$

$$F_4 = -\frac{3R^3}{8} \sigma_{AdS_4}. \quad (2.2)$$
The general solitonic string solution in this background is very difficult to obtain, however it can be done in a certain subsectors. Let us describe below one of them.

Let us consider the case of $\theta_1 = \theta_2 = \pi/2$ and $\psi = 0$ (see for instance [20]). One can check that this is a consistent truncation and for this subsector the Polyakov Lagrangian has the form

$$L \sim -\tfrac{1}{4} i^2 + \xi'^2 + \tfrac{1}{4} \cos^2 \xi \left( \phi_1'^2 - \phi_1^2 \right) + \tfrac{1}{4} \sin^2 \xi \left( \phi_2'^2 - \phi_2^2 \right)$$

(2.3)

The ansatz for a rotating string soliton is

$$t = \kappa \tau, \quad \xi = \xi(y), \quad \phi_1 = \phi_1 + f_1(y), \quad \phi_2 = \phi_2 + f_2(y), \quad y = c \sigma - d \tau.$$  

(2.4)

The Virasoro constraints plus requirement of existing of a turning point at $\xi = \pi/2$ lead to the equation of motion for $\xi$

$$\xi'^2 = \frac{1}{4 \left( c^2 - d^2 \right)} \left[ (c^2) d^2 \kappa^2 - \frac{C^2}{\sin^2 \xi} - c^2 \left( \phi_1^2 \cos^2 \xi + \phi_2^2 \sin^2 \xi \right) \right],$$

(2.5)

where $\phi_2 C = \kappa^2 d$. The turning points can be obtained from (2.5) with one of them $\xi_\pm = \pi/2$ and the other, $\xi_+$, determined by

$$(\kappa^2 - \phi_2^2 \frac{\left( c^2 - \kappa^2 d^2 \right)}{\phi_2^2}) = 0.$$  

(2.6)

The two roots correspond to magnon and single spike solutions as follows

$$\sin^2 \xi_+ = \frac{d^2}{\left( \phi_2^2 - \phi_1^2 \right) c^2} \text{ (giant magnon)}, \quad \sin^2 \xi_+ = \frac{c^2}{\left( \phi_2^2 - \phi_1^2 \right) d^2} \text{ (spike)}$$

(2.7)

The dispersion relation in the case of $R_t \times \mathbb{CP}^3$ is obtained to be

$$E - J_1 = \sqrt{J_2^2 + \frac{8l}{\sin^2 \xi}},$$

(2.8)

which is half of the standard one obtained from Neumann-Rosochatius integrable system in $R_t \times S^3 \times S^3$ [19]. Here the magnon momentum is identified with the angle difference $\Delta \phi_2$.

In the case of single spike string solution the dispersion relation is obtained in the form

$$J_2 = \sqrt{J_1^2 + 8l \sin^2 \bar{\xi}},$$

(2.9)

where $\bar{\xi} = \pi/2 - \xi_+$. The large energy is combined with the large winding number around $\Delta \phi_2$ and they are related as follows

$$E - \frac{\sqrt{2}}{2} \Delta \phi_2 = 2 \sqrt{2} \left( \frac{\pi}{2} - \xi_+ \right) = 2 \sqrt{2} \bar{\xi}.$$  

(2.10)

With these results in the subspace of $\theta_1 = \theta_2 = \pi/2$ and $\psi = 0$ we conclude the short review of the solitonic solutions in $AdS_4 \times \mathbb{CP}^3$. In the next section we consider the deformed case.
3 Giant magnons and single spikes in deformed $AdS_4 \times \mathbb{CP}^3$

Let us start with the deformed $AdS_4 \times \mathbb{CP}^3$ metric and the field content \[31\]

$$
\begin{align*}
&d s_{11}^2 = \frac{R^3}{k} \left( \frac{1}{4} d s_{AdS_4}^2 + d s_{\mathbb{CP}^3}^2 \right), \quad e^{2 \phi} = \frac{R^3}{k^3} G \\
&d s_{\mathbb{CP}^3}^2 = d \xi^2 + 4 \sin^2 \xi (d \theta_2^2 + G \sin^2 \theta_2 d \varphi_2^2) + \frac{1}{4} \cos^2 \xi (d \theta_1^2 + G \sin^2 \theta_1 d \varphi_1^2), \\
&\quad + G \cos^2 \xi \sin^2 \xi \left( \frac{d \psi + \frac{1}{2} \cos \theta_1 d \varphi_1 - \frac{1}{2} \cos \theta_2 d \varphi_2}{2} \right)^2, \quad \tilde{\gamma}^2 G \cos^4 \xi \sin^4 \xi \sin^2 \theta_1 \sin^2 \theta_2 d \psi^2,
\end{align*}
$$

where

$$
B = -\tilde{\gamma} G R^3 \frac{\cos^2 \xi \sin^2 \xi}{k} \times \left( \frac{1}{2} \cos^2 \xi \sin^2 \theta_1 \cos \theta_2 \psi \psi + \cos \theta_1 \psi \varphi_1 + \frac{1}{2} \sin^2 \xi \sin^2 \theta_2 \cos \theta_1 \psi \varphi_2 \\
\quad + \frac{1}{4} (\sin^2 \theta_1 \sin^2 \theta_2 + \cos^2 \xi \sin^2 \theta_2 \cos \theta_2 \cos \theta_1) \varphi_1 \varphi_2 \right)
$$

$$
F_2 = k (- \cos \xi \sin \xi d \xi \wedge (2 d \psi + \cos \theta_1 d \varphi_1 - \cos \theta_2 d \varphi_2)
\quad - \frac{1}{2} \cos^2 \xi \sin \theta_1 d \theta_1 \wedge d \varphi_1 - \frac{1}{2} \sin^2 \xi \sin \theta_2 d \theta_2 \wedge d \varphi_2)
$$

$$
F_4 = -\frac{3 R^3}{8} \omega_{AdS_4} + 4 \tilde{\gamma} \cos^3 \xi \sin^3 \xi \sin \theta_1 \sin \theta_2 d \xi \wedge d \psi \wedge d \theta_1 \wedge d \theta_2
\quad - \frac{1}{8} d (\tilde{\gamma} G \cos^2 \xi \sin^2 \xi (\cos^2 \xi \sin^2 \theta_1 - \sin^2 \xi \sin^2 \theta_2)) \wedge d \psi \wedge d \varphi_1 \wedge d \varphi_2,
$$

(3.2)

where $\tilde{\gamma} = (R^3/4k) \gamma$ and

$$
G^{-1} = 1 + \tilde{\gamma}^2 \cos^2 \xi \sin^2 \xi (\sin^2 \theta_1 \sin^2 \theta_2 + \cos^2 \xi \sin^2 \theta_1 \cos^2 \theta_2 + \sin^2 \xi \sin^2 \theta_2 \cos^2 \theta_1).
$$

One can see that the equations (3.1) and (3.2) preserve four supercharges and thus matching the dual $\mathcal{N} = 2$ supersymmetric gauge theory in three dimensions. According to \[27\] the gauge theory superpotential is modified and takes the form

$$
\frac{4 \pi}{k} \text{Tr} (A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1) \rightarrow \frac{4 \pi}{k} \text{Tr} (e^{-i \gamma / 2} A_1 B_1 A_2 B_2 - e^{i \gamma / 2} A_1 B_2 A_2 B_1).
$$

To find solitonic string solutions we employ the following ansatz

$$
t = \kappa \tau; \quad \theta_1 = \theta_2 = \theta = \frac{\pi}{2}; \quad \psi = 0
\quad \xi = \xi (y); \quad \varphi_i = \varphi_i (\tau) + f_i (y)
$$

(3.4)

where $y = c \sigma - d \tau$. 

4
3.1 Equations and solutions in the deformed theory

Let us write first the Lagrangian setting \( \theta_1 = \theta_2 = \theta \)

\[
L \sim \frac{1}{4} \dot{t}^2 - \dot{\xi}^2 + \xi'^2 + G \cos^2 \xi \sin^2 \xi (1 + \gamma^2 \cos^2 \xi \sin^2 \xi \sin^4 \theta)(-\dot{\psi}^2 + \psi'^2)
+ \frac{G}{4} \cos^2 \xi (\sin^2 \theta + \sin^2 \cos^2 \theta)(-\dot{\phi}_1^2 + \phi_1'^2) + \frac{G}{4} \sin^2 \xi (\sin^2 \theta + \cos^2 \xi \cos^2 \theta)(-\dot{\phi}_2^2 + \phi_2'^2)
+ \frac{1}{4} (-\dot{\theta}^2 + \theta'^2) + G \cos^2 \xi \sin^2 \xi \cos \theta (-\dot{\psi}(\dot{\phi}_1 - \dot{\phi}_2) + \psi'(\phi_1' - \phi_2'))
- \frac{G}{2} \cos^2 \xi \sin^2 \xi \cos^2 \theta (-\dot{\phi}_1 \phi_2 + \phi_1' \phi_2')
- 2\gamma G \cos^2 \xi \sin^2 \xi \left[ \cos^2 \xi \sin^2 \theta \cos \theta (-\dot{\psi}(\dot{\phi}_1 + \dot{\phi}_2) + \psi'(\phi_1' + \phi_2'))
+ \frac{\sin^2 \theta}{4} (\sin^2 \theta + 2 \cos^2 \xi \cos^2 \theta)(-\dot{\phi}_1 \phi_2' + \phi_1' \phi_2') \right].
\] (3.5)

Examining the equations of motion for \( \theta \) and \( \psi \) one can see that \( \theta = \frac{\pi}{2} \) and \( \psi = 0 \) are solutions to the equations of motion.

Then we proceed with the choice of \( \theta = \frac{\pi}{2}, \ \psi = 0 \). One can see that with this ansatz the B-field becomes (we set for a moment \( R^3/\kappa = 1 \))

\[
B = -\frac{\gamma G}{4} \cos^2 \xi \sin^2 \xi d\phi_1 \wedge d\phi_2.
\] (3.6)

The metric takes the form

\[
ds^2_{\text{CF3}}(\theta = \frac{\pi}{2}, \psi = 0) = d\xi^2 + \frac{G}{4} \cos^2 \xi d\phi_1^2 + \frac{G}{4} \sin^2 \xi d\phi_2^2,
\] (3.7)

where

\[
G^{-1} = 1 + \gamma^2 \cos^2 \xi \sin^2 \xi.
\] (3.8)

We make further assumptions for the fields making a rotating string solitonic ansatz

\[
t = \kappa \tau, \ \phi_1 = \omega_1 \tau + f_1(y), \ \phi_2 = \omega_2 \tau + f_2(y), \ y = c\sigma - d\tau.
\] (3.9)

The Lagrangian we will work with takes the form

\[
L \sim \frac{1}{4} \dot{t}^2 + (c^2 - d^2) \dot{\xi}^2 + \frac{G}{4} \cos^2 \xi \left( - (\phi_1 - df_1')^2 + c^2 f_1'^2 \right)
+ \frac{G}{4} \sin^2 \xi \left( - (\phi_2 - df_2')^2 + c^2 f_2'^2 \right) - \frac{G}{2} \gamma \cos^2 \xi \sin^2 \xi \left( - (\phi_1 - df_1')c f_2' + (\phi_2 - df_2')c f_1' \right)
\] (3.10)
**Virasoro constraints:** Next step is to elaborate the conditions imposed by the two Virasoro constraints:

a) $T_{r\tau} + T_{r\sigma} = 0$

\[
(c^2 + d^2)\xi'^2 + \frac{G}{4} \cos^2 \xi \left[ (\phi_1 - df_1')^2 + c^2 f_1'^2 \right] + \frac{G}{4} \sin^2 \xi \left[ (\phi_2 - df_2')^2 + c^2 f_2'^2 \right] = \frac{\kappa^2}{4} \quad (3.11)
\]

b) $T_{r\sigma} = 0$

\[-cd\xi'^2 + \frac{G}{4} \cos^2 \xi (\phi_1 - df_1')c f_1 + \frac{G}{4} \sin^2 \xi (\phi_2 - df_2')c f_2 = 0. \quad (3.12)\]

or

\[
\xi'^2 + \frac{G}{4} \cos^2 \xi \left[ f_1'^2 + \frac{\phi_1^2 - 2\phi_1 df_1'}{c^2 + d^2} \right] + \frac{G}{4} \sin^2 \xi \left[ f_2'^2 + \frac{\phi_2^2 - 2\phi_2 df_2'}{c^2 + d^2} \right] = \frac{\kappa^2}{4(c^2 + d^2)}. \quad (3.13)
\]

and

\[-\xi'^2 + \frac{G}{4} \cos^2 \xi \left[ \frac{\phi_1 f_1'}{d} - f_1'^2 \right] + \frac{G}{4} \sin^2 \xi \left[ \frac{\phi_1 f_1'}{d} - f_1'^2 \right] = 0. \quad (3.14)\]

Adding the last two equations we find

\[
\frac{G}{4} \phi_1 \cos^2 \xi \left[ \frac{\phi_1}{c^2 + d^2} + \frac{(c^2 - d^2) f_1'}{d(c^2 + d^2)} \right] + \frac{G}{4} \phi_2 \sin^2 \xi \left[ \frac{\phi_2}{c^2 + d^2} + \frac{(c^2 - d^2) f_2'}{d(c^2 + d^2)} \right] = \frac{\kappa^2}{4(c^2 + d^2)}. \quad (3.15)
\]

**Equations of motion:** In contrast to the undeformed case here one must account for the non-vanishing B-field (see the Lagrangian (3.10)). Then we find

\[
\partial_y \left\{ \frac{G}{2} \cos^2 \xi \left[ (\phi_1 - df_1')d + c^2 f_1' \right] - \frac{G}{2} \tilde{\gamma} \cos^2 \xi \sin^2 \xi \left[ (\phi_2 - df_2')c + cd f_2' \right] \right\} = 0 \quad (3.16)
\]

or

\[
\partial_y \left\{ \frac{G}{2} \sin^2 \xi \left[ (\phi_1 - df_1')d + c^2 f_1' \right] - \frac{G}{2} \tilde{\gamma} \cos^2 \xi \sin^2 \xi \phi_2 c \right\} = 0. \quad (3.17)
\]

Analogously

\[
\partial_y \left\{ \frac{G}{2} \sin^2 \xi \left[ (\phi_2 - df_2')d + c^2 f_2' \right] + \frac{G}{2} \tilde{\gamma} \cos^2 \xi \sin^2 \xi \phi_1 c \right\} = 0. \quad (3.18)
\]

From here we find

\[
(c^2 - d^2) f_1' = \frac{2A_1}{G \cos^2 \xi} - \phi_1 d + \tilde{\gamma} \phi_2 c \sin^2 \xi, \quad (3.19)
\]

\[
(c^2 - d^2) f_2' = \frac{2A_2}{G \sin^2 \xi} - \phi_2 d - \tilde{\gamma} \phi_1 c \cos^2 \xi. \quad (3.20)
\]
Substituting this expression into (3.15) we find
\[ 2A_1\phi_1 + 2A_2\phi_2 = \kappa^2 d. \] (3.21)

The substitution into the first Virasoro constraint gives the equation for \( \xi \)
\[ \xi'^2 = \frac{1}{(c^2 - d^2)^2} \left\{ (c^2 + d^2)\kappa^2 - \frac{4A_1^2}{G\cos^2 \xi} - \frac{4A_2^2}{G\sin^2 \xi} - c^2\phi_1^2\cos^2 \xi - c^2\phi_2^2\sin^2 \xi \right. \\
\left. + 4c\gamma(A_2\phi_1\cos^2 \xi - A_1\phi_2\sin^2 \xi) \right\} \] (3.22)

The solutions we are looking for are of folded type and therefore one is to require a turning point at \( \xi = \frac{\pi}{2} \). This condition forces
\[ \xi = \frac{\pi}{2} \Rightarrow A_1 = 0. \] (3.23)

In order \( \xi = \frac{\pi}{2} \) to be a turning point it must be zero of (3.22). This gives (we use also that \( A_1 = 0 \) and that \( 2A_2\phi_2 = \kappa^2\phi_2 \))
\[ \left( \frac{\kappa^2}{\phi_2^2} - 1 \right) \left( \frac{\kappa^2}{\phi_2^2} - \frac{\kappa^2}{d^2} \right) = 0 \Rightarrow \begin{cases} \kappa^2 = \phi_2^2 \text{ magnon} \\
d\kappa^2 = c^2\phi_2^2 \text{ spike} \end{cases} \] (3.24)

Therefore we have two cases: magnon solutions and single spike solutions.

The solutions for \( f'_1 \) and \( f'_2 \) become
\[ (c^2 - d^2)f'_1 = -\phi_1d + \gamma\phi_2c\sin^2 \xi, \] (3.25)
\[ (c^2 - d^2)f'_2 = \frac{2A_2}{G\sin^2 \xi} - \phi_2d - \gamma\phi_1c\cos^2 \xi. \] (3.26)

We are interested in the dispersion relation for the obtained solutions. As it is well known from the \( AdS_5 \times S^2 \) background, we have in general two types interesting soliton solutions - giant magnons and spiky strings. The shape of the solutions depends on the parameters and here we will consider the two cases separately.

**Giant magnon solutions:** Since we already fixed \( A_1 \) to be vanishing, from (3.21) we find
\[ A_2 = \frac{d\kappa^2}{2\phi_2}. \] (3.27)

The magnon type solutions are determined by
\[ \kappa^2 = \phi_2^2. \] (3.28)

The equation for \( \xi \) takes the form
\[ \xi'^2 = \frac{1}{4(c^2 - d^2)^2} \left\{ (c^2 + d^2)\kappa^2 - \frac{4A_2^2}{G\sin^2 \xi} - c^2(\phi_1^2\cos^2 \xi + \phi_2^2\sin^2 \xi) \right. \\
\left. + 4c\gamma A_2\phi_1\cos^2 \xi \right\} \] (3.29)
We substitute $\kappa^2 = \varphi^2$ and $A_2 = d\kappa^2/(2\varphi_2)$ and obtain
\[
\xi'^2 = \frac{c^2\Omega_0^2 \cos^2 \xi}{4(c^2 - d^2)^2 \sin^2 \xi} \left\{ \sin^2 \xi - \sin^2 \xi_0 \right\},
\]
where we used the following notations
\[
\Omega_0^2 = \left( \varphi_2 - \left( \varphi_1 - \frac{d\gamma}{c} \varphi_2 \right)^2 \right), \quad \sin \xi_0^2 = \frac{d\varphi_0^2}{c^2 \Omega_0^2}.
\]

The solution of the equation (3.30) is
\[
\cos \xi = \frac{\cos \xi_0}{\cosh \left( \frac{c\Omega_0 \cos \xi_0}{4(c^2 - d^2)^2} \right)}.
\]

**Single spike solutions:** The single spike type solutions are determined by
\[
d^2\kappa^2 = c^2 \varphi_2^2.
\]

The equation of motion for $\xi$ becomes
\[
\xi'^2 = \frac{c^2\Omega_0^2 \cos^2 \xi}{4(c^2 - d^2)^2 \sin^2 \xi} \left\{ \sin^2 \xi - \sin^2 \xi_0 \right\},
\]
where we used the following definitions
\[
\Omega_0^2 = \left( \varphi_2 - \left( \varphi_1 - \frac{c\gamma}{d} \varphi_2 \right)^2 \right), \quad \sin \xi_0^2 = \frac{c^2 \varphi_0^2}{d^2 \Omega_0^2}.
\]

### 3.2 The dispersion relations

Let us start with the conserved charges. In the case of giant magnons the momenta are given by
\[
P_{\phi_1} = -\frac{\cos^2 \xi}{2(1 - v^2)} [\varphi_1 - \tilde{\gamma} v \varphi_2], \quad P_{\phi_2} = \frac{\kappa}{2} - \frac{\varphi_2 \cos^2 \xi}{2(1 - v^2)} [\varphi_1 - \tilde{\gamma} v \varphi_2], \quad P_t = \frac{\kappa}{2}.
\]

For the sting configuration of single spike type we have large winding number which is combined with the energy to produce finite result. The momentum along $\phi_1$ is the very same
\[
P_{\phi_1} = -\frac{\cos^2 \xi}{2(1 - v^2)} [\varphi_1 - \tilde{\gamma} v \varphi_2],
\]
while the second momentum is
\[
P_{\phi_2} = \frac{\varphi_2 \cos^2 \xi}{2(1 - v^2)}.
\]

Note that both momenta are finite.

Once we have the expressions for the conserved quantities, it is easy to obtain the corresponding dispersion relations.
Giant magnon case: The finite combination is

\[ P_t - P_{\phi_2} = \frac{\phi_2 \cos^2 \xi}{2(1 - v^2)} [\phi_1 - \tilde{\gamma} v \phi_2]. \] (3.39)

Integrating the above expression we obtain

\[ E - J_2 = \sqrt{t} \frac{\phi_2 \cos \xi_0}{\Omega_0}. \] (3.40)

The finite spin \( J_1 \) is

\[ J_1 = \frac{\sqrt{t} \cos \xi_0}{\pi} (\phi_1 - \tilde{\gamma} v \phi_2). \] (3.41)

From here we easily obtain the dispersion relation

\[ E - J_2 = \sqrt{J_1^2 - \frac{l}{\pi^2} \sin^2 \left(\frac{p}{2} - \pi \beta\right)}, \] (3.42)

where we made the identification \( \cos \xi_0 = \sin(p/2 - \pi \beta) \).

The angle deficit is

\[ \frac{\Delta \phi_2}{2} = \left(\frac{\pi}{2} - \xi_0\right) - \tilde{\gamma} \sqrt{\frac{\phi_2^2 - \Omega_0^2}{\Omega_1}} \cos \xi_0. \] (3.43)

Single spike case: The momenta \( J_1 \) and \( J_2 \) are finite. Integrating (3.37) and (3.38) and using the relations between them we find

\[ J_2 = \sqrt{J_1^2 - \frac{l}{\pi^2} \sin^2 \left(\frac{p}{2} - \pi \beta\right)}, \] (3.44)

where we made the identification \( \cos \xi_1 = \sin(p/2 - \pi \beta) \).

Both, the energy and the angle deficit are divergent, but the combination \( E - \Delta \phi_2 \) is finite and is given by

\[ E - T \Delta \phi_2 = \frac{\sqrt{t}}{\pi} \left(\frac{\pi}{2} - \xi_1\right) - \tilde{\gamma} \sqrt{\frac{\phi_2^2 - \Omega_0^2}{\Omega_1}} \cos \xi_1. \] (3.45)

We conclude by stressing on the analogy with the undeformed case, which can be recovered by taking \( \tilde{\gamma} \to 0 \).

4 Conclusions

In this short note we studied the existence of giant magnon and single spike string solutions in beta-deformed \( AdS_4 \times \mathbb{CP}^3 \) background. We used the representation of the background
an $U(1)$ fibration over $S^2 \times S^2$ with a fiber coordinate $\psi$ and the two spheres described by $\phi_i, \theta_i, i = 1, 2$. The deformation via solution generating TsT (T-duality, shift, T-duality) technique is known and is usually used to construct theories with less supersymmetry. In this paper we find giant magnon and single spike classical string solutions in the deformed background We find their dispersion relations which are supposed to describe the anomalous dimensions of certain class gauge theory operators. The complete map between the results obtained from string side and gauge theory deserves further careful study.

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