AN OPTIMAL SETUP COST REDUCTION AND LOT SIZE FOR ECONOMIC PRODUCTION QUANTITY MODEL WITH IMPERFECT QUALITY AND QUANTITY DISCOUNTS

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Abstract. The purpose of this paper concentrates on an economic production quantity model with the factors of imperfect quality and quantity discounts, in which the inspection action occurs during the production stage. There is specific consideration of there being a finite production rate, and the quantity discounts offered by the supplier serves the purpose of stimulating buying greater quantities. This is in contrast to EPQ models that do not take these added factors into consideration. The objective of this paper is to determine the setup cost reduction, which is a function of capital investment, and inventory lot size. An alternative solution procedure was developed that does not employ the Hessian Matrix concavity in the expected total profit. We develop an algorithm to determine the optimal solution for this model. Theoretical results are discussed and a numerical example is proposed. Managerial insights are also examined.

1. Introduction. In the currently highly competitive business environment, companies must make appropriate decisions regarding inventory to maintain their current operations and also sustain growth. While the traditional EPQ models assume production process is to be stationary, manufacturing and maintaining the appropriate product quality levels to meet customer’s requirements has become an important issue for any enterprise. In real life, the above assumption is not completely accurate since the product quality generally depends on the condition of the production status. The manufacture process is under normal control in the beginning stage.
with ideal quality control for items that are being produced. An unchanged production process gradually deteriorates, which consequently leads to the production of defective items that are of sub-standard quality and denomination [16]. Several researchers (e.g. [11, 46]) have investigated inventory models in which production quality may be weak due to there being an unreliable production process. Three relative issues regarding to this paper are discussed as follows.

*Imperfect Inventory Quality Issue:* Salameh and Jaber [17] developed an interesting model in which they extended the traditional EPQ model by applying an inspection process to identify defects among manufactured items. Ever since the quality control was considered by Salameh and Jaber's [4], this has been considered to be more accurate than the traditional EOQ/EPQ models, many researchers (e.g. [25, 30, 8, 26, 6, 39, 44, 15]) integrated the issue of EOQ/EPQ models into the contents of their earlier works to include the factor of imperfect quality control. Recent works by Moussawi-Haidar et al. [34] integrated the factor of screening time into the EPQ model in which they considered a production system with a random supply and an inspection procedure that was administered at the end of the production process. Al-Salamah [42] formulated a finite production system for cases including non-destructive and destructive screening of the sample items in which neither the production process nor the inspection were optimal. These above-mentioned models were developed based on Salameh and Jaber’s [17] work, which assumed that the retailer tests items after an order received. However, in the real life, the items produced is immediately inspected and thus the EPQ cases may not follow the above assumption. Any nonconforming items that were subject to the imperfect manufacturing process were then usually stored in a different warehouse to set them apart from the controlled items to isolate the quantities and tracking costs for the imperfect ones (Paknejad et al. [45]; Lin et al. [11]). Several semiconductor products are categorized in this manner. In view of these considerations, the storage costs for a unit of measurement for approved items per unit time are greater than that for a unit of measurement for imperfect items for the same period. An EPQ model in which the inspecting stage is undertaken during the manufacturing period with there being different storage costs is thus worth examining in order to determine the possibility of additional cost control considerations.

*Quantity Discounts Issue:* For the traditional imperfect quality inventory models, there exists another unrealistic assumption: the direct product cost is irrelevant with obtained lot size. On the other hand, many researchers have recognized that quantity discounts could offer economic advantages, including lower ordering costs, lower per-unit purchasing costs, and the decreased likelihood of shortages for both the buyer and the vendor [6]. Benton and Park [43] examined a comprehensive amount of literature on lot sizing problem given several types of discount policy, and discussing some of the significant contributions to this field. Burwell et al. [28] (1997) formulated an inventory model to decide the optimal lot size and retail price in which the storage cost is a function of the purchasing cost, which also factor in the offers of both all-unit quantity and freight discounts. A modified model on Burwell et al.'s [28] work and the algorithm maximizing the profit, and subsequently determined the exact optimal values for the lot size and retailer selling price was developed by Chang [24]. Wang and Wang [49] examined quantity discount policies for heterogeneous retailers, when each retailer meets demand composing a decreasing function of its retail price. Furthermore, Burnetas et al. [12] and Ebrahim et al. [9] analyzed the inventory decisions of a downstream buyer facing a single period
of stochastic demand in which the incremental and all-unit discounts are provided for single-period supply contracts. Mendoza and Ventura [33] and Munson and Hu [48] adopted incremental and all-units discounts to consider four different cases based on centralized and decentralized purchasing and pricing systems with local distribution. Lin [26] formulated an imperfect quality inventory model for items with imperfect quality under quantity discounts in which the mighty retailer has logistics control over its supplier.

While considering different tracks, some researchers (e.g. [23, 37, 31, 21, 48]) employed quantity discounts as a tool to integrate a two-echelon inventory system with various demand. Some researchers (e.g. [5, 6, 35, 22, 2]) extended quantity discounts to deteriorating and perishable products, and then construct several efficient inventory models. Other extensions of the EOQ/EPQ models with quantity discounts could be found in Lin et al. [11], Mendoza and Ventura [33], Mansini et al. [47], Lee et al. [32], Meena and Sarmah [20], and Venegas and Ventura [51]. All of these researchers recognize that quantity discounts provided by the supplier may significantly influence the buyer’s decision making. Using quantity discounts in the EPQ model with different storage costs, which had not been discussed in previous studies, is therefore worth further study.

Setup Cost Reduction Issue: The topic of setup cost reduction in manufacturing processes has recently received considerable attention (e.g., [1, 27, 41, 18]). These researchers identified that setup costs can be reduced through specialized equipment acquisition, learning effects, and manufacture procedures. Porteus [41] developed a model which contains diversifying the demand rate and the setup cost for investing the setup cost reduction in the EOQ model. Porteus [7] further innovated options into the EOQ model for exploring the production system improvement and setup cost reduction effects on the lot size. Several extensions were built based on Porteus ([41, 7]) ideas. By replacing a function of capital investment, an EPQ model with the setup cost parameter was proposed by Billington [14]. Hong et al. [3], and Hong and Hayya [38] examined the economic benefits of setup cost reduction and the process quality improvement by investing in new manufacture process. Kim et al. [13] and Hou [4] discussed some relationships between the setup cost reduction and the capital investment expenditure. Unlike previous papers, some studies (e.g., [52, 40, 10]) further examined setup cost reduction in multi-stage production inventory models. Other related works on setup cost reduction include research undertaken by Lin et al. [11], Huang et al. [50], Voigt and Inderfurth [19], and Cheung et al. [29], among others found in their references. These literature sources illustrate the setup cost reduction through capital investments, which thereby has a significant influence on decision making regarding cost control.

Based on the above discussion, this paper concentrates on an economic production quantity model with the factors of imperfect quality and quantity discounts, in which the inspection action occurs during the production stage. There is specific consideration of there being a finite production rate, and the quantity discounts offered by the supplier serves the purpose of stimulating buying greater quantities. This is in contrast to EPQ models that do not take these added factors into consideration. Thus, this paper develops a new economic production quantity (EPQ) model for a manufacturing system with imperfect production processes wherein the screening process occurs at the production stage and ordering is based on quantity discounts. The proposed model, assuming each item is screened as it is an output from the production run, is more practical than traditional EPQ model. This
proposed model is especially practical for a manufacturing system with frequently changeover production line (such as a just-in-time manufacturing environment) because the set up cost reduction is considered and each item is screened as it is an output from the production run. Several different considerations pertaining to practicality of the situation were included in this new model to capture the real life situation: (i) different holding costs for good items and defective items, (ii) quantity discounts, (iii) capital investment, and (iv) an imperfect production processes. The traditional EPQ model is a special case of this model in which the nonfinancial holding of good and defective items are equal to the standard financial holding cost with no defects, i.e., when the capital investment is ignored in set up cost reduction and quantity discounts. Some properties and an efficient solution algorithm are proposed to help the manager to make decisions quickly. The different impacts can be reflected as different weighted coefficients in the proposed model. The new EPQ model consists of several parameters that have different effects on the total profit. Sensitivity analysis is also explored to provide some important information for practitioners. The model has both theoretical underpinnings and practical significance in high volume production systems. Obviously, the results are significantly important for expensive products where rework is an important element in the profit improvement of a company. The problem has great economic impacts on many discrete product manufacturing systems such as electronics and heavy machinery.

The remainder of this study is organized as follows. Section 2 shows the notations adopted in this paper. The model environment and mathematical model are developed in Section 3. Section 4 develops an efficient algorithm that determines the optimal production lot size and capital investment. Numerical examples and sensitivity analysis are provided in Section 5. Section 6 indicates the managerial insights.

2. Notation and assumptions.

2.1. Notation. The following notations are used in the development of our mathematical model for an optimal setup cost reduction and lot sizing with imperfect production processes and quantity discounts.

σ demand rate, units/year
M production rate (M > σ), units/year
Q quantities for each manufacture run, units/cycle
S₀ original setup cost for each manufacture run prior to investment, $/cycle
S nominal setup cost, $/cycle
CR unit purchasing cost of rth level corresponding to the cost discount scheme, $/unit
f₉ nonfinancial storage cost rate for a unit of quality controlled item per period, as expressed as a fraction of dollar value and f₉ > f₉, $/unit/year
f₉ nonfinancial storage cost rate for a unit of defective item per period, as expressed as a fraction of dollar value and f₉ < f₉, $/unit/year
i cost of capital
d percentage of imperfect quality items in Q, which follows the probability density function f(d).
p unit selling price of quality controlled items, $/unit
θ unit selling price of imperfect quality items, θ < p, $/unit
x unit inspecting cost, units/year
$t_1$ manufacture run, $t_1 = Q/M$, year/manufacture cycle

$\Omega_s$ capital investment required for setup cost reduction from $S_0$ to $S$, $\$$ / setup cost reduction

$\eta$ fraction of reduction in $S$ per dollar increase in $\Omega_s$, $\eta < 1$

$T$ cycle length, year/cycle

2.2. Assumptions. We employ the following assumptions for the system under study, which could shrink the complex model environment.

1. The demand rate for items is known and constant.
2. The storage cost for good items in another warehouse is higher than that of defective items in a particular warehouse.
3. Each item is screened immediately after it is produced. All of the defective items are detected, and then sold with a salvage value when the production cycle is over.
4. Shortages are not allowed.
5. The relationship between setup cost reduction and capital investment for each cycle time can be treated as a logarithmic investment cost function. The setup cost, $S$, and the life cycle of a production system consists of $N$ units of time. The total amortization cost of the capital invested for setup cost reduction is $\Omega_s$ can be expressed as $\Omega_s = \frac{1}{\eta} \ln \left( \frac{S_0}{S} \right)$, for $0 < s \leq S_0$, where $1/\eta$ is the fraction of the reduction in $S$ per dollar increase in investment. The amortization cost is evenly (uniformly) charged over the life cycle of the system, then the amortization cost per unit of time for the setup cost reduction is $\Omega_s/N$.
6. An all-unit quantity discount scheme is employed, where $c_r$ ($r = 1, 2, \ldots, j$) is the unit procurement cost when the order quantity is in the interval $[Q_r-1, Q_r)$.

3. Formulation of the problem and mathematical model. This paper examines an EPQ system with imperfect quality and quantity discounts, where the inspection procedures occur during the manufacture process. At the core of this matter is considering that the production rate is finite, and quantity discounts is provided by the supplier to stimulate the buyer ordering greater lot sizes. Each produced item is screened immediately and the quality controlled items are then stored at a specific warehouse with the storage cost rate $f_g$ (a unit of good item per period), compared to imperfect quality items stored at another warehouse with the storage cost $f_d$ (a unit of imperfect quality item per period), in which $f_g > f_d$. The manufacturer then sells these imperfect quality items for a salvage value in a second market by the end of production stage. The company may devote its effort to decreasing setup costs in their production process. Furthermore, the supplier offers quantity discounts to encourage the buyer ordering a larger quantity. An opportunity cost per unit time occurs because undiscounted costs are included. Several different considerations pertaining to practicality of the situation were included in this new model to capture the real life situation: (i) different holding costs for good items and defective items, (ii) quantity discounts, (iii) capital investment, and (iv) an imperfect production processes. The traditional EPQ model is a special case of this model in which the nonfinancial holding of good and defective items are equal to the standard financial holding cost with no defects. This paper therefore formulates a mathematical model with the total profit maximization objective for the investment cost of changing and the inventory related costs associated with quantity discounts by optimizing both $S$ and $Q$. 
The fluctuation of the manufacturing system with imperfect processes and different storage costs during an inventory cycle for the proposed model are illustrated in Figure 1. In that figure, we know the cycle time is $T$, the time when inventory builds up is $t_1$ (i.e., the production run), the time period when there is no production and the inventory depletes is $t_2$. We note $t_1 = Q/M$, $t_2 = Q \left[ 1 - \frac{d}{M} - \frac{\sigma}{M} \right]$, and $T = t_1 + t_2 = Q(1-d)/\sigma$. Because each lot contains imperfect quality items, each item is inspected as it is output from the production run (i.e., $[0,t_1]$). Therefore, the maximum inventory quantity at the end of $t_1$ is $Q(1-\sigma/M)$ from Figure 1 and the imperfect quality items, $dQ$, are then sold at a secondary market after the production stage is stopped. This leads the inventory level to immediately drop to $Q(1-\sigma/M-d)$. We let $TR_r(Q)$, which includes the revenues from good and defective items, be the total revenue per cycle corresponding to the quantity discounts structure. The revenues are expressed as $TR_r(Q) = pQ(1-d) + \theta Qd$. Alternatively, let $TC_r(Q,S)$, which consists of the set up cost per cycle, inspection cost per cycle, procurement cost, amortized capital cost and storage cost, be the total cost per cycle corresponding to the quantity discounts structure. These costs are measured as follows.

(a) Storage cost:

Figure 1 illustrates the cycle time $T$ for each production lot contains two parts: $t_1$ (production run) and $t_2$ (non-production run), where the production run $t_1$ includes perfect and imperfect quality items and non-production run $t_2$ are all perfect quality items. Besides, the defective items are sold at the terminal time of $t_1$. The storage costs occurring in the duration of $t_1$ and $t_2$ thus show as the following two cases:

Case I: The storage cost occurs in the duration of $t_1$

The items produced in $t_1$ can be classified as perfect quality items and imperfect quality items after the screening processes. The perfect items are then stored at a
specific warehouse compared with the imperfect quality items, stored at a different warehouse, with $p$ proportion. The imperfect items are then sold when the manufacturing run is over. The storage cost, $HC_d(Q)$, for imperfect quality items in the duration of $t_1$ is the triangular area of (beg) with proportion $d$ in Figure 1. It can be obtained by employing a geometric argument using:

$$HC_d(Q) = dc_r(f_d + i) \left[ \frac{Q^2 \pi}{2M} \right], \text{where} \pi = 1 - \left( \sigma/M \right) \text{and} \forall r = 1, 2, \ldots, j$$ (1)

Alternatively, the storage cost for perfect quality items, $HC_{g1}(Q)$, in the duration of $t_1$ is

$$HC_{g1}(Q) = (1 - d)c_r(f_g + i) \left[ \frac{Q^2 \pi}{2M} \right], \forall r = 1, 2, \ldots, j$$ (2)

Case II: The storage cost occurs in the duration of $t_2$

It is noted that all items completely are inspected at the end of $t_1$ and the imperfect quality items are sold at that time in the second market. This implies that all items in the duration of $t_2$ are in perfect quality condition. Thus, the storage cost for perfect quality items, $HC_{g2}(Q)$, in $t_2$ could be computed based on the triangular area of $(fgy)$ in Figure 1.

$$HC_{g2}(Q) = c_r(f_g + i) \left[ \frac{(\pi - d)(Q)^2}{2\sigma} \right], \forall r = 1, 2, \ldots, j$$ (3)

(b) Setup cost: The setup cost for each cycle is $S$

(c) Procurement cost: The procurement cost is some function of the quantity purchased due to the all-units quantity discounts policy offered by the supplier. This implies the procurement cost corresponds to the unit invoice cost $c_r$ is $c_rQ$

(d) Amortized capital cost: This paper employs logarithmic investment function, which has been adopted by many researchers (Porteus [41]; Sarker and Coates [18]; Sarkar and Moon [36]), as the relationship between the reduction in setup cost reduction and capital investment. Suppose that the setup cost is $S$ and the life cycle of a production system consists of $N$ units of time. The total amortization cost of the capital invested for setup cost reduction is $\Omega_s$, can be expressed as

$$\Omega_s = \frac{1}{\eta} \ln \left( \frac{S_0}{S} \right), \text{for} 0 < s \leq S_0 \text{ where} \frac{1}{\eta} \text{is the fraction of the reduction in} S \text{ per dollar increase in investment}.$$ The amortization cost is evenly (uniformly) charged over the life cycle of the system, then the amortization cost per unit of time for the setup cost reduction is $\Omega_s/N$. Because the cycle time is $T = Q(1 - d)/\sigma$ and the cost of capital is $i$, as we take investment to reduce setup cost, an amortized investment cost, $ATCC$, illustrating the economic consequences of the investment per unit of time is shown as follows:

$$ATCC = \frac{i}{\eta} \left[ \ln \left( \frac{S_0}{S} \right) \right] \frac{Q(1 - d)}{\sigma N}$$ (4)

(e) Inspection cost: All items produced in each cycle is inspected when they are manufactured from the production line. Thus, the screening cost is given by $xQ$.

Combining set up cost, inspection cost, procurement cost, amortized capital cost and storage cost, we know the total cost for each cycle is
\[ TC_r(Q, S) = S + c_rQ + c_r (f_g + i) \left\{ \left[ \frac{Q^2 \pi \cdot (1 - d)}{2M} \right] + \left[ \frac{[(\pi - d)Q]^2}{2\sigma} \right] \right\} \\
+ c_r (f_d + i) \left[ \frac{dQ^2 \pi}{2M} \right] + xQ + \frac{i}{\eta} \left[ \ln \left( \frac{S_0}{S} \right) \right] \frac{Q(1 - d)}{\sigma N}, \forall r = 1, 2, \ldots, j \]

The total profit, \( TP_r(Q, S) \), per cycle is the total revenue less the total cost and is given as:

\[ TP_r(Q, S) = TR_r(Q) - TC_r(Q, S) \]

\[ = pQ(1 - d) + \theta Qd - S - c_r (f_g + i) \cdot \left\{ \left[ \frac{Q^2 \cdot \pi \cdot (1 - d)}{2M} \right] + \left[ \frac{(\pi - d)^2}{2\sigma} \right] \right\} \\
- dc_r (f_d + i) \left[ \frac{Q^2 \pi}{2M} \right] - xQ - \frac{i}{\eta} \left[ \ln \left( \frac{S_0}{S} \right) \right] \frac{Q(1 - d)}{\sigma N} - c_r Q, \forall r = 1, 2, \ldots, j \]

When we take the expected value for \( TP_r(Q, S) \) with respect to \( d \), the above equation can be written as:

\[ ETP_r(Q, S) = pQ(1 - E[d]) + \theta E[d] - S - c_rQ - xQ - \frac{iQ(1 - E[d])}{\eta \sigma N} \ln \left( \frac{S_0}{S} \right) \]

\[ - dc_r (f_d + i) E[d] \pi Q^2 = c_r (f_g + i) \left\{ \frac{Q^2 \pi (1 - E[d])}{2M} + \frac{Q^2 \pi^2 - 2E[d] \pi + E[d^2]}{2\sigma} \right\} \]

\[ \forall r = 1, 2, \ldots, j \]

Taking the expected duration of cycle time \( T \) in Eq. (1) with respect to \( d \), \( E[T] \) is written as \( E[T] = Q(1 - E[d]) / \sigma \). Employing the renewal-reward theorem (Ross, 1996), the expected profit per unit time could be formulated as follows.

\[ \lim_{t \to \infty} \frac{ETPU_r(Q, S)}{t} = \frac{ETP_r(Q, S)}{E[T]} \]

\[ = \frac{\sigma p + \sigma \theta E[d]}{E_1} - \frac{S \sigma}{QE_1} - \frac{\sigma c_r + x}{E_1} - \frac{i}{\eta \sigma N} \ln \left( \frac{S_0}{S} \right) \\
- \frac{Q \sigma c_r (f_d + i) E[d]}{2ME_1} - \frac{Qc_r (f_g + i)}{E_1} \left\{ \frac{\sigma \cdot (1 - E[d])}{2M} + \frac{\pi^2 - 2E[d] \cdot \pi + E[d^2]}{2} \right\} \]

\[ \forall r = 1, 2, \ldots, j \]

where \( E_1 = 1 - E[d] \). It needs to be shown that the Hessian Matrix of Eq. (5) is negatively definite maximizing the expected total profit per unit time. Observing Eq. (5), we know it is not easy to determine the concavity of the Hessian Matrix. Therefore, we employ an alternative methodology to obtain the profit function properties.

4. Methodology and algorithm. To analyze the structure of the optimization equation (5), we need the following propositions to determine the unique solution for the expected profit, \( ETP_{PU_r}(Q, S) \).

**Proposition 1**: \( ETP_{PU_r}(Q, S) \) is concave in \( S \) when \( Q \) is fixed.

*Proof*. See appendix A. \( \square \)

**Proposition 2**: The candidate optimal set up cost is less than or equal to \( S_0 \).

*Proof*. See appendix B. \( \square \)
Employing Proposition 1 and 2, the possible optimal value of $S$ can be obtained by letting $\partial ETPU_r(Q,S)/\partial S = 0$. Therefore,

$$\tilde{S}_r(Q) = \min \left( \frac{i E_1}{\eta \sigma N}, S_0 \right), \forall r = 1, 2, \ldots, j$$ (6)

Substituting Eq. (6) into Eq. (5), we have

$$ETPU_r(Q) = \sigma p + \frac{\sigma \theta E[d]}{E_1} - \frac{i}{\alpha} - \frac{\sigma [c_r + x]}{E_1} - \frac{1}{\eta N} \ln \left( \frac{\eta \sigma S_0}{E_1 Q} \right) - Q c_r (f_g + i) \left\{ \frac{\sigma \pi \cdot (1 - E[d])}{2M} + \frac{\pi^2 - 2E[d] \pi + E \left[ E[d]^2 \right]}{2} \right\}$$ (7)

$$- Q c_r (f_d + i) \left[ \frac{\sigma \pi E[d]}{2M} \right], \forall r = 1, 2, \ldots, j$$

Thus, proposition 3 given as following illustrates the candidate optimal value of $Q_r$.

**Proposition 3:** The candidate optimal lot sizing is given by

$$\bar{Q}_r = \frac{i E_1}{\eta N \left\{ c_r (f_g + i) \left[ \frac{\sigma \pi E_1}{2M} + \frac{\pi^2 - 2E[p] \pi + E \left[ E[p]^2 \right]}{2} \right] + c_r (f_d + i) \left[ \frac{\sigma \pi E[p]}{2M} \right] \right\}}, \forall r = 1, 2, \ldots, j$$ (8)

**Proof.** See appendix C.

Proposition 3 illustrates $Q$ depends on in general, which is similar to Hou’s [4] result. It means the optimal production lot size depends on how costly it is to ensure set up cost reduction. We further know $\bar{Q}_r$ corresponding to the unit-purchasing cost $c_r$ is valid when $Q_{r-1} \leq \bar{Q}_r < Q_r$. Thus, two cases ($Q_r > \bar{Q}_r$ and $\bar{Q}_r \leq Q_{r-1}$) occur because $Q_r$ may not exist at $[Q_{r-1}, Q_r)$.

Case A: $Q_r > \bar{Q}_r$, where $\bar{Q}_r$ is the maximum lot size corresponding to $c_r$.

The buyer employs the lower unit-procurement cost (say $c_b$ and $c_b < c_r$) to obtain the lot size in this case. We then have $ETPU_b(Q) > ETPU_r(Q)$ from Eq. (7). This implies the feasible candidate $Q_r$ corresponding to $c_r$ is not the global optimal solution.

Case B: $Q_r \leq \bar{Q}_r$, where $Q_{r-1}$ is the minimum lot size corresponding to $c_r$.

If we fix $Q$ and $S$ respectively under given $c_j$, in this case, two scenarios occur:

- **Scenario A:** the breakpoint perspective $Q_{r-1}$ corresponding to $c_r$.
  
  Due to the unit-procurement cost depends on the lot size, the candidate optimal lot size may occur at the break point $Q_{r-1}$ with its corresponding candidate optimal setup cost in this scenario. Given $Q_{r-1}$ corresponding to $c_r$, the setup cost, similar to Eq. (6), can be obtained by

$$\tilde{S}_r = \min \left( \frac{i Q_{r-1} E_1}{\eta \sigma N}, S_0 \right), \forall r = 1, 2, \ldots, j$$ (9)

- **Scenario B:** the perspective of fixed $S$

  The candidate optimal solution may occur at $(Q_r^\Delta(S), S)$ satisfying $Q_{r-1} \leq Q_r^\Delta < Q_r$ in this scenario. From Eq. (5) under fixed $S$, we have:
\[
\frac{\partial ETPU_r(Q, S)}{\partial Q} = \frac{S\sigma}{Q^2 E_1} - \frac{c_r (f_g + i)}{E_1} \left(\frac{\sigma \pi E_1}{2M} + \frac{\pi^2 - 2E[d] \pi + E \left[d^2\right]}{2}\right) \leq 0 \quad (10)
\]

\[
\frac{\partial^2 ETPU_r(Q, S)}{\partial Q^2} = -\frac{2S\sigma}{Q^3 E_1} < 0 \quad (11)
\]

Letting \(\frac{\partial ETPU_r(Q, S)}{\partial Q} = 0\), we have:

\[
Q_r^{\Delta}(S) = \min \left(\frac{2S\sigma}{c_r (f_g + i) \left[\frac{\sigma \pi E_1}{M} + \pi^2 - 2E[d] \pi + E \left[d^2\right]\right] + c_r (f_d + i) \frac{\sigma \pi E[d]}{M}}, \forall r = 1, 2, \ldots, j\right) \quad (12)
\]

Plugging Eq. (12) into \(Q_{r-1} \leq Q_r^{\Delta}(S)\), we have the following results:

\[
S_r^{\Delta} = \min \left(\frac{(Q_{r-1})^2 c_r (f_g + i) \left[\frac{\sigma \pi E_1}{M} + \pi^2 - 2E[d] \pi + E \left[d^2\right]\right] + c_r (f_d + i) \frac{\sigma \pi E[d]}{M}}{2\sigma}, \forall r = 1, 2, \ldots, j\right) \quad (13)
\]

The candidate optimal lot size and setup cost reduction corresponding to unit-procurement cost has now been obtained. We therefore develop an algorithm in which the complexity of the proposed algorithm is \(O(n)\) to find the global optimal solution in the next section.

**Algorithm: Finding global optimum**

Step 1. Obtained \(\bar{S}_r(Q)\) from Eq. (6) and \(\bar{Q}_r\) from Eq. (8) for all \(r = 1, 2, \ldots, j\).

Step 2. For \(r = 1\) to \(j\)
- If \(\bar{Q}_r > Q_r\), then \(r = r + 1\)
- End If
- If \(Q_{r-1} \leq \bar{Q}_r < Q_r\), then Do {
  - \{compute \(ETPU_r(Q_r - 1, \bar{S}_r)\) from Eq. (5) and record it\}
  - \{\(r = r + 1\)\}
  - Else {
    - Do {
      - \{obtain \(\bar{S}_r\) from Eq. (9) and compute \(ETPU_r(Q_{r-1}, \bar{S}_r)\) from Eq. (5)\}
      - \{obtain \(S_r^{\Delta}\) from Eq. (13) and compute \(Q_r^{\Delta}(S_r^{\Delta})\) from Eq. (12)\}
      - If \(Q_{r-1} \leq Q_r^{\Delta}(S_r^{\Delta}) < Q_r\) {
        - Compute \(ETPU_r(Q_r^{\Delta}, S_r^{\Delta})\) from Eq. (5)
        - Else {
          - \(ETPU_r(Q_r, S_r^{\Delta}) = -\infty\)
End If
}
{compare ETPU \( \bar{Q}_r, \bar{S}_r \), ETPU \( Q_{r-1}, \bar{S}_r \), ETPU \( Q_r^\Delta, S_r^\Delta \) and then record the maximum one}
{ \( r = r + 1 \) }
End If

Step 3. The maximum expected profit obtained in Step 2 provides the optimal production lot size, setup cost reduction, and the unit-procurement cost.

Note that if we ignore the opportunity cost, quantity discounts, capital investment in setup cost reduction, and imperfect quality items (i.e. \( d = 0 \) and \( h = c_r \cdot f_g = c_r \cdot f_d \)), the lot size in Eq. (12) will be:

\[
Q^* = \sqrt{\frac{2S\sigma}{h(1 - \sigma/M)}}
\]

This is equivalent to the result in the traditional EPQ model. This helps validate the model.

5. **Numerical example and sensitivity analysis.** As indicated in the introduction, a manufacture in the semiconductor industry is considered in this example. All necessary costs are estimated from existing data and listed as follows:

- Production rate, \( M = 192000 \) units/year,
- Demand rate, \( \sigma = 9600 \) units/year,
- Screening cost, \( x = $1/ \text{unit} \)
- Cost of capital, \( i = $0.2/\text{/year} \)
- Original setup cost, \( S_0 = $200/\text{cycle} \)
- Storage cost rate for a unit of good item (a fraction of dollar value) \( f_g = $0.05/\text{/year} \)
- Storage cost rate for a unit of defective item (a fraction of dollar value) \( f_d = $0.025/\text{/year} \)
- Selling price of good quality items, \( p = $50/ \text{unit} \)
- Selling price of imperfect quality items, \( \theta = $25/\text{unit} \)

Besides, \( \eta = 0.000002 \) and the life cycle of a production system consists of 50 units of time. The probability density function \( f(d) \), a uniform distribution over the range \([0, 0.04]\), is:

\[
f(d) = \begin{cases} 
25, & 0 \leq d \leq 0.04 \\
0, & \text{otherwise}
\end{cases}
\]

A price discount schedule provided by the supplier is shown in Table 1.

| \( r \) | \( Q_{r-1} \sim Q_r \) | \( c_r \) |
|---|---|---|
| 1 | \( 0 < Q < 150 \) | \( c_1 = 20.05 \) |
| 2 | \( 150 \leq Q < 400 \) | \( c_2 = 20.04 \) |
| 3 | \( 400 \leq Q < 800 \) | \( c_3 = 20.03 \) |
| 4 | \( 800 \leq Q < 1250 \) | \( c_4 = 20.02 \) |
| 5 | \( Q \geq 800 \) | \( c_5 = 20.01 \) |

We then obtain the optimal lot size, setup cost reduction, and expected total profit by employing the algorithm developed in Section 4, which is shown in Appendix D.
Figure 2 illustrates the expected annual profit as a function of $S$ and $Q$ given procurement cost known as $c_4 = 20.02$. The three-dimensional graph shows that the expected total profit is concave and unique solutions for $S$ and $Q$ exist that maximize the expected total profit. For the given parameter set, the optimal solution set is $Q^* = 858.9$ units, the optimal setup cost reduction per cycle is $\Omega^*_{s}/N = $1318.2 and the expected total profit is $274724.7$, which meets the result obtained in the algorithm that is developed. This implies that the capital investment in setup cost reduction is $S^* = $175.3.

To realize the model parameter effects on optimal policy, a further sensitive analysis of the proposed model is studied. We employ the $2^k$ full factorial design to find the effect of every factor (including secondary factors) and their interactions. One, theoretically, could study the impact of all parameters ($\sigma, M, x, i, f_g, f_d, c_r, p, \theta, S_0, \eta$ and $d$) on optimal policy. However, exploring the impact of these parameters on optimal policy is laborious computational loading, especially if the number of factors or levels is large. To comprehensively handle the effects of important parameters on the optimal solution and avoid complex analysis procedures, we only used five significant parameters (i.e., $\sigma, f_g, M, i, d$), according to the literature and the authors’ experience. All of the five parameters are set at two levels (low and high) and are shown below: $\sigma = (9600, 13440); f_g = (0.05, 0.07); M = (192000, 268800); i = (0.2, 0.28); d = (uniformly distributed over the range [0, 0.04], uniformly distributed over the range [0, 0.056]).$ Excepting for the above five parameters, we remain the other parameters unchanged. Table 2 illustrates the optimal solution under 32 combinations of $\sigma, f_g, M, i$ and $d$. Some findings are obtained from Table 2 and list as follows:

1) $ETPU^*$ and $Q^*$ increase with $\sigma$; while $S^*$ decreases in $\sigma$. It is rather intuitive that the manufacturer employs more quantities per lot to satisfy the customer’s need as the demand rate increases. This result agrees the traditional EOQ model result. Simultaneously, as the quantity per lot increases, the setup cost decreases because the manufacturer invests capital in providing more efficient equipment. These combined effects increase the expected total profit. However, the optimal lot
size \( (Q^*) \) may occur at one of the break points corresponding to each unit purchasing cost in the quantity discount schedule. In this case, \( Q^* \) remains unchanged with \( \sigma \); while \( S^* \) decreases and \( ETPU^* \) increases. This fact shows that the manufacturer enjoys the benefit of quantity discounts coming from the supplier, and thus produces the size up to the breaking point of manufacturing quantity.

Table 2 The values of \( Q^* \), \( S^* \) and \( ETPU^* \) corresponding to 32 combinations of \( \sigma, f_g, M, i, U(d) \)

| \( \sigma \) | \( f_g \) | \( M \) | \( i \) | \( U(d) \) | \( Q^* \) | \( S^* \) | \( ETPU^* \) |
|---|---|---|---|---|---|---|---|
| 0.05 | 0.2 | 0.04 | 858.9 | 175.3 | 274724.7 |
| | | 0.056 | 866.3 | 175.4 | 275046.5 |
| | 0.28 | 0.04 | 910.9 | 200 | 274036.9 |
| | | 0.056 | 918.8 | 200 | 274358.8 |
| | 0.2 | 0.04 | 846.1 | 163.3 | 274694.8 |
| | | 0.056 | 853.5 | 172.8 | 275016.6 |
| | 0.28 | 0.04 | 897.4 | 200 | 274004.6 |
| | | 0.056 | 905.2 | 200 | 274326.4 |
| 9600 | | 0.2 | 0.04 | 800 | 163.3 | 274570.9 |
| | | 0.056 | 800 | 162 | 274892.8 |
| | 0.28 | 0.04 | 858.9 | 200 | 273906.7 |
| | | 0.056 | 866.4 | 200 | 274228.7 |
| | 0.2 | 0.04 | 800 | 163.3 | 274599.6 |
| | | 0.056 | 800 | 162 | 274862.7 |
| | 0.28 | 0.04 | 846 | 200 | 273872.3 |
| | | 0.056 | 853.5 | 200 | 274194.3 |
| 13440 | | 0.2 | 0.04 | 877.4 | 128 | 385689.5 |
| | | 0.056 | 885 | 128.1 | 386139.7 |
| | 0.28 | 0.04 | 930.5 | 190 | 384839 |
| | | 0.056 | 938.6 | 190.1 | 385289.5 |
| | 0.2 | 0.04 | 858.9 | 125.2 | 385646.9 |
| | | 0.056 | 866.3 | 125.3 | 386097 |
| | 0.28 | 0.04 | 910.9 | 186 | 384779.3 |
| | | 0.056 | 918.8 | 186.1 | 385229.8 |
| | | 0.2 | 0.04 | 812.5 | 118.4 | 385535.8 |
| | | 0.056 | 819.6 | 118.5 | 385986.1 |
| | | 0.28 | 0.04 | 877.4 | 179.1 | 384674.4 |
| | | | 0.056 | 885.1 | 179.2 | 385125.1 |
| | | | 0.2 | 0.04 | 800 | 116.7 | 385493.1 |
| | | | 0.056 | 800 | 115.7 | 385943.3 |
| | | | | 0.2 | 0.04 | 858.9 | 175.3 | 384614.7 |
| | | | | | 0.056 | 866.4 | 175.4 | 385065.3 |

(2) In general, \( S^* \), \( Q^* \) and \( ETPU^* \) all decrease with \( f_g \). This illustrates a higher storage cost for good items leading to higher storage expenditure for the manufacturer, which decreases the number of lot sizes produced. When the lot size is smaller than the original one, the number of setup times increases, which decreases the expected total profit. Simultaneously, the manufacture increases his capital investment to reduce the set up cost. These results lead to the expected total profit decreasing as the storage cost increases.

(3) As expected, \( Q^* \) and \( S^* \) increase with \( i \); while \( ETPU^* \) decreases in \( i \). This is intuitive because \( i \) is the fraction opportunity cost of capital per unit time. The
manufacturer does not employ costly capital investment in set up cost reduction, and thus $S^*$ increases. Simultaneously, the manufacturer produces greater quantity to efficiently utilize the cost of capital. Therefore, the higher the cost of capital, the less the expected total profit.

(4) In general, $Q^*$, $S^*$ and $ETPU^*$ all decrease with $M$. It means the higher the production rate, the less the expected total profit. That is, as the production rate increases, the manufacturer produces more quantities, and thus the storage cost increases. This implies that the manufacturer may employ less or no capital investment in the set up cost reduction. Similar to the findings in (2), the set up cost decreases and thus the investment in setup reduction increases. These results illustrate that $Q^*$, $S^*$ and $ETPU^*$ all decrease with $M$. It is noted that an investment in set up reduction is not always necessary (i.e. $S^* = S_0$) when the machine utility is sufficient to meet requirements. In this case, the optimal lot size ($Q^*$) decreases with $M$. This meets the traditional EOQ model. Therefore, $Q^*$ and $ETPU^*$ decrease with $M$.

(5) $Q^*$, $S^*$ and $ETPU^*$, in general, increase with $U(d)$. That is, the higher the defective item rate, the higher the lot size, set up cost, and expected total profit. Salameh and Jaber (2000) showed if the defective rate increases, the manufacturer needs more quantity per lot to satisfy demand. In the meantime, the set up cost trivially increases. Obviously, the benefit of increasing quantity is larger than that of increasing the set up cost. Therefore, $Q^*$, $K^*$ and $ETPU^*$ increase with $U(d)$. We further know the optimal lot size may occur at one of the quantity break points corresponding to each unit purchasing cost; In this case, $Q^*$ remains unchanged with $U(d)$; while $S^*$ decreases and $ETPU^*$ increases in $U(d)$.

6. Conclusions. This paper demonstrated an EPQ model with imperfect production processes, where the inspection procedures occur during the production duration, and quantity discounts. The decision variables are set up cost reduction and lot sizing in which they thereby maximize the expected total profit. This paper takes several practical situations into accounts and thus has some interesting results different the traditional EPQ models. We further develop the expected total profit function and a solution procedure associated with an algorithm finding the optimal solution. If the storage cost for good and imperfect quality items equal to the traditional storage cost and the percentage of imperfect quality items is zero, then the traditional EPQ model is a special case when the capital investment is ignored in set up cost reduction and quantity discounts. The theoretical results show $Q$ depends on $\eta$ in general, which is similar to Hou’s result. It means the optimal production lot size depends on how costly it is to ensure set up cost reduction. The effects of five important parameters (demand rate, storage cost rate for a unit of quality controlled item, production rate, percentage of defective items, and cost of capital) were also investigated. The numerical results illustrate that (1) the parameter effects including the storage cost rate for a unit of quality controlled items, the production rate and the cost of capital on optimal lot size and set up cost are moderate; while they do not significantly impact the expected total profit; (2) the lowest unit purchasing cost may not guarantee the retailer enjoys the maximum expected total profit under the quantity discount policy; (3) the demand rate effects on optimal lot size, set up cost and the expected total profit are significant; (4) the defective item rate effect on production lot size and expected total profit are not significant; while the impact on the set up cost is trivial.
For future research, one can justify the assumption of the amortized capital cost for setup cost reduction incurring in every cycle and then reformulate the model by considering the capital investment (or the amortized capital cost) for setup cost reduction as a fixed cost charged once or during its finite amortization period. Of course, more practical issues considering to match real world situations such as allowable shortage, incremental quantity discounts, trade credit policy and probability demand are also interesting research topics.

Appendix A
Proof for Proposition 1. We take the first and second partial derivatives of the expected profit \(ETPU_r(Q, S)\) with respect to \(S\) and have
\[
\frac{\partial ETPU_r(Q, S)}{\partial S} = -\frac{\sigma}{\sigma r E_1} + \frac{1}{\eta NS}, \forall r = 1, 2, \ldots, j
\]
\[
\frac{\partial^2 ETPU_r(Q, S)}{\partial S^2} = -\frac{1}{\eta NS^2} < 0, \forall r = 1, 2, \ldots, j
\]
Therefore, for fixed \(Q\), \(ETPU_r(Q, S)\) is concave in \(S\), for \(r = 1, 2, \ldots, j\). This completes the proof of Proposition 1. □

Appendix B
Proof for Proposition 2. It is intuitive that if capital investment is made, the set up cost should be reduced. Alternatively, if the set up cost increases with capital investment, which is a paradox in this experience, the producer gives up the investment. This implies that the candidate optimal set up cost is less than or equal to the original set up cost, \(S_0\). □

Appendix C
Proof for Proposition 3. Upon taking the first and second partial derivatives of Eq. (7) with respect to \(Q\), we have
\[
\frac{\partial ETPU_r(Q)}{\partial Q} = \frac{i}{\eta N Q} - \frac{c_r (f_g + i)}{E_1} \left\{ \frac{\sigma \pi E_1}{2M} + \frac{\pi^2 - 2E[p] \pi + E[p^2]}{2} \right\}
\]
\[
\text{(C1)}
\]
\[
= \frac{-i c_r (f_d + i) \left[ \frac{\sigma \pi E[p]}{2M} \right]}{\eta N Q^2}, \forall r = 1, 2, \ldots, j
\]
\[
\frac{\partial^2 ETPU_r(Q)}{\partial Q^2} = -\frac{i}{\eta N Q^2} < 0, \forall r = 1, 2, \ldots, j \quad \text{(C2)}
\]
Equations (C1) and (C2) illustrate that the candidate optimal lot size \(Q_r^*\) corresponding to each procurement price \(c_r\) exists and is unique. Therefore, we let \(\frac{\partial ETPU_r(Q)}{\partial Q} = 0\) and then the candidate optimal value of \(Q\) is obtained as:
\[
\tilde{Q}_r = \frac{i E_1}{\eta N \left\{ \frac{c_r (f_g + i)}{E_1} \left[ \frac{\sigma \pi E_1}{2M} + \frac{\pi^2 - 2E[p] \pi + E[p^2]}{2} \right] + c_r (f_d + i) \left[ \frac{\sigma \pi E[p]}{2M} \right] \right\}}, \forall r = 1, 2, \ldots, j
\]
This completes the proof of Proposition 3. □

Appendix D
Illustration for Example: Computing optimal set up cost, lot size, and maximum expected total profit.
Step 1: Employing Eq. (6) and Eq. (8) for \(r = 1, 2, \ldots, 5\), we have
\( \bar{Q}_1 = 857.6, \bar{Q}_2 = 858.0, \bar{Q}_3 = 858.5, \bar{Q}_4 = 858.9, \bar{Q}_5 = 859.3 \)

\( \bar{S}_1 = 175.0, \bar{S}_2 = 175.1, \bar{S}_3 = 175.2, \bar{S}_4 = 175.3, \bar{S}_5 = 175.4 \)

Step 2:
(1) Because \( \bar{Q}_1 > 150, \bar{Q}_2 > 400, \bar{Q}_3 > 800 \), This implies the optimal solution may not occur at these results. Therefore, no further computation will be done.
(2) Because \( 800 \leq \bar{Q}_4 < 1250 \), substitute \( \bar{Q}_4 = 858.9 \) and \( \bar{S}_4 = 175.3 \) into Eq. (5) and thus obtain \( ETPU_4(858.9, 175.3) = 274724.7 \).
(3) Because \( \bar{Q}_5 < 1250 \) from Eq. (9), then
\[ \bar{S}_5 = 175.3 \text{ and } ETPU_5(1250, 175.3) = 274609.0 \]
(4) Obtaining \( S^\Delta_5 \) from Eq. (13) and \( Q^\Delta_5 \) from Eq. (12), we have \( Q^\Delta_5 (S^\Delta_5) = 917.5 \).
(5) Because \( \bar{Q}_5 < 1250 \) and not \( Q^\Delta_5 (S^\Delta_5) = 917.5 > 1250 \), the optimal candidate solution does not exist.

Step 3:
Compare the results obtained in Step 2, we obtain the maximum expected total profit is \( ETPU_4(858.9, 175.3) = 274724.7 \). This illustrates the optimal setup cost reduction per cycle is \( \Omega^*_S/N = $1318.2 \) (i.e. \( S^* = $175.3 \)) and the optimal lot size is 858.9 units, which corresponding to the procurement cost \( c_4 = 20.02 \).

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