$B_c^+$ formation from random charm and anti-bottom quarks in the quark-gluon plasma

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We study the $B_c^+$ production in Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. In the quark-gluon plasma (QGP) produced in heavy-ion collisions, heavy quarks make random motions with the energy loss. We employ the Langevin equations to study the non-equilibrium distributions of heavy quarks and the Instantaneous Coalescence Model (ICM) to study the hadronization process. Due to abundant charm and bottom quarks in the QGP, their coalescence probability is significantly enhanced compared with the situations in proton-proton collisions. We find that the final production of $B_c^+$ is increased by the coalescence process, which makes the nuclear modification factor ($R_{AA}$) of $B_c^+$ larger than unit. Our model explains the experimental data well at semi-central and central collisions. The observation of $R_{AA}(B_c^+) > 1$ is regarded as an evident and strong signal of the existence of the deconfined medium generated in heavy-ion collisions.

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The extremely hot QCD matter called “quark-gluon plasma” (QGP) is believed to be generated in the relativistic heavy-ion collisions [1,2]. The abnormal suppression of $J/\psi$ production in nucleus-nucleus (AA) has been regarded as one of the clean signals of this deconfined matter [3]. In the QGP, primordially produced $J/\psi$ in the hadronic collisions are dissociated by the color screening effect and the inelastic scatterings with thermal partons [4–11]. Both of these effects reduce the final production of $J/\psi$, which make the nuclear modification factor ($R_{AA}$) of $J/\psi$ smaller than 1. $R_{AA}$ is defined as the ratio between $J/\psi$ production in AA collisions and the yield in proton-proton (pp) collisions scaled with the number of binary collisions $N_{coll}$. At the Relativistic Heavy-Ion Collider (RHIC) and the Large Hadron Collider (LHC), multiple heavy-quark pairs are produced in parton hard scatterings. They combine to form new heavy quarkonium states at the hadronization of the medium [12–16]. This new process is called coalescence, which can increase the nuclear modification factor of $J/\psi$ at the LHC collision energies [17–19].

For charmonium and bottomonium, their nuclear modification factors are usually smaller than unit [20–23]. Recently, experiments have measured the $B_c^+$ nuclear modification factors at $\sqrt{s_{NN}} = 5.02$ TeV Pb-Pb collisions [24]. Its value is larger than the unit. This is a new strong and clear signal of the existence of QGP. In the coalescence process, the $B_c^+$ nuclear modification factor is proportional to the number of binary collisions, $\frac{N_{coll}}{N_{coll}} \propto N_{coll}$. The phenomenon of $R_{AA}(B_c^+) > 1$ is explained with the coalescence of random $b$ and $c$ quarks in QGP. This coalescence probability will be significantly enhanced when the numbers of charm and bottom pairs become large in the medium. As heavy quarks, especially bottom quarks, are hard to reach kinetic equilibrium due to their large masses, we will employ the Langevin equations to study the realistic dynamical evolutions of heavy quarks in the QGP and the Instantaneous Coalescence Model (ICM) to study the reaction of $\bar{b} + c \rightarrow B_c^+ + g$ (where $g$ represents a gluon) [25–28]. Considering both the cold nuclear matter effect (such as the shadowing effect) and hot medium effects on heavy quarks and $B_c^+$ mesons, we calculate the final spectrum of $B_c^+$ mesons in Pb-Pb collisions and compare with the recent experimental data.

In vacuum, heavy quark potential is parametrized with the Cornell potential. The binding energies and the wave functions of the eigenstates can be obtained by solving time-independent Schrödinger equation $H(r)\psi_{nl}(r) = E_{nl}\psi_{nl}(r)$, where $E_{nl}$ is the radial part of the total wave function. The angular part of the wave function is taken as spherical harmonics $Y_{lm}(\theta, \phi)$, where $(n, l, m)$ are the quantum numbers of the eigenstates. The form of Cornell potential is

$$V_{Cornell}(r) = -\frac{\alpha}{r} + \sigma r,$$

where the parameters are $\alpha = \pi/12$ and $\sigma = 0.2$ GeV$^2$ [3].

In the hot dense medium, both the coulomb term and the linear term in the Cornell potential are screened by the thermal partons due to the color screening effect. This screening effect becomes more vital at the larger distance and the higher temperature. In-medium heavy quark potential has been studied by complex-valued potential model [31] and lattice QCD calculations [32–33]. The realistic potential is between two limits: the free energy $F$ and the internal energy $U$. In this work, we don’t intend to determine the exact formula of in-medium heavy quark potential. Instead, the potential is used to estimate the parameters in the Wigner functions which will be used in the coalescence process of $B_c^+$. The color...
screened potential is parametrized with the formula [33],
\[
F(T, r) = -\frac{\alpha}{r} (e^{-\mu r} + \mu r) - \frac{\sigma}{2^{3/4} \Gamma[3/4]} \left( \frac{T}{\mu} \right)^{1/2} K_{1/4}((\mu r)^2) \\
+ \frac{\sigma}{2^{3/2} \mu \Gamma[3/4]},
\]
where \(\alpha\) and \(\sigma\) are the same as in the Cornell potential Eq. (1). The \(\Gamma\) and \(K_{1/4}\) are the Gamma function and the modified Bessel function respectively. The screening mass \(\mu \equiv \mu(T)\) from Lattice results is parametrized as,
\[
\frac{\mu(T)}{\sqrt{\sigma}} = s T + a_1 T \sqrt{\pi} \left[\text{erf}(-b + \frac{\sqrt{T}}{2\sigma}) - \text{erf}\left(\frac{b - T}{\sqrt{2} \sigma}\right)\right]
\]
where \(T \equiv T/T_c\), and \(s=0.587, a=2.150, b=1.054, \sigma = 0.07379\). \(\text{erf}(z)\) is the error function. With the increase of the temperature, heavy quark potential inside quarkonium is screened. The mean radius of bound states increases with \(T\) and approach to infinity at a certain temperature where the bound state is totally screened [34]. In Fig. 1 we plot the mean radius \(\langle r \rangle_{Bc^+}(T)\) of \(B_c^+\) at different temperatures when taking the potential to be \(V = F(r, T)\). At temperatures around the critical phase transition, the mean radius of \(B_c^+\) is around 0.4 fm and increase to \(0.8\) fm at \(1.2T_c\). The exact values of \(B_c^+\) in-medium geometry size depend on the choice of in-medium potentials. If we approximate the \(B_c^+\) wave function as a Gaussian function in a simple situation, the width of the Wigner function is determined with the relation \(\kappa = 2T^2/D_s\). The value of \(D_s\) is around \(D_s(2\pi T) \approx 5\) in the QGP [33]. The drag term \(\eta_D(p)\) is determined with the fluctuation-dissipation relation, \(\eta_D(p) = \kappa/(2TE)\). Here \(E = \sqrt{m^2 + |p|^2}\) is the energy of heavy quarks.

The medium-induced gluon radiation term \(f_g = -d\mathbf{p}_g/dt\) contributes the recoil force on heavy quarks when they move through QGP [37]. \(\mathbf{p}_g\) is the momentum of emitted gluon. The number of radiated gluons in the time interval \(t \sim t + \Delta t\) is,
\[
P_{rad}(t, \Delta t) = \langle N_g(t, \Delta t) \rangle = \Delta t \int dx dk_T^2 \frac{dN_g}{dx dk_T^2 dt}
\]
when the time interval \(\Delta t\) is sufficiently small, the radiated gluons become smaller than the unit. Then \(P_{rad}\) can be interpreted as the probability to radiate a gluon in this time interval. \(x = E_g/E\) is the ratio of the emitted gluon energy and the heavy quark energy. \(k_T\) is the transverse momentum of the radiated gluon. \(dN_g/dx dk_T^2 dt\) is the spectrum of emitted gluons per unit time. It is adopted from the higher-twist calculation for the medium-induced gluon radiation in perturbative QCD, where the spectrum of gluons radiated from massive heavy quarks is introduced by Ref. [37]. Gluon radiation term dominates the energy loss of heavy quarks at high \(p_T\) bins. At low \(p_T\) bins, the contribution of gluon radiation becomes small, and the drag and random terms dominate the energy loss of heavy quarks. Relative magnitudes of two energy loss processes are compared in Ref. [25]. Both collision process and the radiative process have been considered in the below figures.

Bottom and charm pairs are produced in parton hard scatterings at the beginning of nucleus-nucleus collisions. Their density is proportional to the number of binary collisions in the overlap area of two nuclei. The initial spatial density of heavy quarks in Pb-Pb collisions is proportional to the thickness functions,
\[
\frac{dN_{Q^Q}^{pp}}{d\mathbf{x_T}} = \sigma_{pp}^{QQ} \times T_A(x_T - b/2)T_B(x_T + b/2)R_s(x_T).
\]

The classical Langevin equation with the gluon radiation is written as,
\[
\frac{d\mathbf{p}}{dt} = -\eta_D(p)\mathbf{p} + \xi + \mathbf{f}_g
\]
where \(\mathbf{p}\) is the momentum of heavy quarks. Neglect the momentum dependence in the white noise, \(\xi\) is determined with the relation,
\[
\langle \xi^i(t)\xi^j(t') \rangle = \kappa \delta^{ij} \delta(t-t')
\]
i and \(j\) represent three dimensions. \(t\) is the time of heavy quark evolutions. \(\kappa\) is the momentum diffusion coefficient of heavy quarks, which does not depend on the momentum of heavy quarks [35]. It is connected with the spatial diffusion coefficient \(D_s\) with the relation \(\kappa = 2T^2/D_s\). The value of \(D_s\) is around \(D_s(2\pi T) \approx 5\) in the QGP [33]. The drag term \(\eta_D(p)\) is determined with the fluctuation-dissipation relation, \(\eta_D(p) = \kappa/(2TE)\). Here \(E = \sqrt{m^2 + |p|^2}\) is the energy of heavy quarks.

When heavy quarks move in the QGP, they lose energy via quasi-elastic scatterings with thermal light partons and the gluon radiation due to multiple scatterings. The random motion of heavy quarks is treated as Brownian motions. Langevin equations have been widely applied to simulate the time evolutions of the momentum distributions of heavy quarks. The classical Langevin equation

FIG. 1. Mean radius of \(B_c^+\) ground state as a function of temperature. The heavy quark potential is taken as the free energy \(V = F(T, r)\). \(T_c\) is the critical temperature of the phase transition.
nucleus A(B), which is defined as the integration of the nucleon density $\rho$ over the longitudinal coordinate $z$. $b$ is the impact parameter. It characterizes the distance between the centers of two nuclei in the transverse plane. At 5.02 TeV pp collisions, the production cross section of charm quarks in the central rapidity ($|y| < 2.3$) has been measured by ALICE Collaboration, $d\sigma_{pp}/dy = 1.165$ nb [35]. Bottom production cross section at 5.02 TeV is extracted with the pp collision data at 1.96 TeV and 2.76 TeV. The differential cross section is fitted to be $d\sigma_{pp}/dy = 47.5 \mu$b in the central rapidity ($|y| < 2.3$) [13]. The initial momentum distributions of charm and bottom quarks are calculated with FONLL model [39]. In the central rapidity of Pb-Pb collisions at 5.02 TeV, shadowing effect $R_{c}(x_T)$ reduces the number of charm pairs by around 25%. We use the EPS09 model to generate the shadowing factor in our calculations [40], which modifies the initial spatial and momentum distributions of charm pairs in Pb-Pb collisions. The shadowing effect in bottom quark distribution becomes weaker due to the larger mass. It is also included in the same way as charm quarks.

In order to calculate the nuclear modification factor ($R_{AA}$) of $B_c^+$, we also need the $B_c^+$ production cross section in pp collision, which is used in the denominator of $R_{AA}$.

$$R_{AA} = \frac{N_{AA}^{B_c^+}}{N_{coll}N_{pp}^{B_c^+}}.$$  

(8)

The numerator $N_{AA}^{B_c^+}$ is the production of $B_c^+$ from the coalescence of random charm and anti-bottom quarks in Pb-Pb collisions. In the denominator, $N_{pp}^{B_c^+}$ is the production of $B_c^+$ in pp collisions, which is proportional to the production cross section $\sigma_{pp}^{B_c^+}$. This cross section has been only measured at 1.96 TeV pp collisions by CDF Collaboration [11]. At $p_T > 6$ GeV/c and the central rapidity $|y| < 1$, the differential cross section of $B_c^+$ with the branching ratio is $\sigma(B_c^+)B(B_c^+ \rightarrow J/\psi\mu^+\nu) = 0.60 \pm 0.09$ nb. In order to parametrize the values of $\sigma(B_c^+)$ at 5.02 TeV pp collisions, we firstly employ PYTHIA to calculate the values of $\sigma(B_c^+)$ at 1.96 TeV and 5.02 TeV pp collisions respectively, and obtain the ratio $\sigma(B_c^+,5.02\text{TeV})/\sigma(B_c^+,1.96\text{TeV}) = 2.40$. Then $B_c^+$ production cross section at 5.02 TeV is extracted to be $\sigma(B_c^+)B(B_c^+ \rightarrow J/\psi\mu^+\nu) = 2.4 \times 0.6$ nb by scaling with the ratio. The branching ratio $B(B_c^+ \rightarrow J/\psi\mu^+\nu)$ has been calculated with different theoretical models. The value of the branching ratio is predicted to be $B(B_c^+ \rightarrow J/\psi\mu^+\nu) = (1.15 - 2.37)\%$ by QCD sum rules [42], (non-)relativistic constituent-quark models [43,44], QCD relativistic-potential models [45] respectively. While nonrelativistic QCD model predicts the value to be $6.7+2.9^{+3.9}_{-1.4}\%$ [46]. This contributes large uncertainty in the determination of $d\sigma_{pp}/dy$. In this work, we take the branching ratio to be $2.37\%$ and the middle value of two scenarios 4.54%. The production cross section of $B_c^{+}$ at 5.02 TeV is $d\sigma_{pp}^{B_{c}^{+}}|_{p_T>6\text{GeV/c}} = 30.38$ nb (taking $B = 2.37\%$) and $15.86$ nb (taking $B = 4.54\%$) respectively in the central rapidity $|y| < 2$. This production cross section has included the contributions from the decay of $B_c^+$ excited states.

In order to obtain the $B_c^+$ production cross section at other $p_T$ bins, we use PYTHIA to generate the normalized transverse momentum distribution,

$$\frac{dN_{pp}^{B_c^+}}{2\pi p_T dp_T} = \frac{(n-1)}{\pi(n-2)(p_T^{pp})^{2(n-2)}} \left[1 + \frac{p_T^2}{(n-2)(p_T^{pp})^2}\right]^{-n}$$  

(9)

where the mean transverse momentum square of $B_c^+$ in the central rapidity is $(p_T^{pp}) = 25.1$ (GeV/c)$^2$. The value of $n = 4.16$. With this normalized distribution, we extract the production cross section of $B_c^+$ at other $p_T$ bins by scaling with the values of $d\sigma_{pp}/dy$ at $p_T > 6$ GeV/c. We obtain the relation $d\sigma_{pp}^{B_{c}^{+}}|_{p_T>11\text{GeV/c}} = 3.65$ nb (taking $B = 2.37\%$) and 1.90 nb (taking $B = 4.54\%$) respectively. Furthermore, the total cross section of $B_c^+$ in $p_T > 0$ becomes $d\sigma_{pp}^{B_{c}^{+}}|_{p_T>0} = 5.0 \times d\sigma_{pp}^{B_{c}^{+}}|_{p_T>6\text{GeV/c}}$.

In Pb-Pb collisions, hot deconfined matter consisting of light quarks and gluons turns out to be a strong coupling matter. Its dynamical evolutions are described with hydrodynamic equations. We use the well developed MUSIC package to simulate the time evolutions of local temperatures and velocities of the expanding QGP [47]. Equation of state (EoS) is needed to close the hydrodynamic equations. In the deconfined phase where the medium temperature is above the critical temperature $T_c = 170$ MeV, EoS is taken from Lattice QCD calculations. In the hadronic phase, EoS is taken from the Hadron Resonance Gas model (HRG) [48]. Two phases are connected with the crossover phase transition. The initial temperature of the medium can be determined by the final multiplicity of light hadrons measured in experiments. The initial temperature of QGP is fitted as $T(\tau_0, x_T = 0) = 510$ MeV in the most central collisions with the impact parameter $b = 0$, $\tau_0 = 0.6$ fm/c is the time of QGP reaching local equilibrium. The value of $\tau_0$ is determined by fitting the anisotropic flows of light hadrons measured in experiments. Hydrodynamic equations start evolutions from the time $\tau = \tau_0$. Evolutions of heavy flavors in the pre-equilibrium stage have been neglected.

In the QGP, when the local temperature of QGP is higher than the dissociation temperature of $B_c^+$, charm and anti-bottom quarks do not form a bound state, as the thermal medium screens their interaction. Instead, they make Brownian motions in the medium independently. When heavy quarks move to the regions where the local temperature of QGP is smaller than a certain value, heavy quark potential is partially restored. These heavy quarks have a probability to hadronize into a bound
state. As the binding energies of $B_c^+$ states are larger than the values of light hadrons, $B_c^+$ states can survive at $T > T_c$. We take the coalescence temperature of $B_c^+$ to be $T_{coa l} = 1.2 T_c$, where $B_c^+$ wave function at this temperature is close to the situation in vacuum, as shown in Fig. If the coalescence process happens at a higher temperature where $B_c^+$ wave function is significantly modified by the medium, Landau damping effect will dissociate most of the $B_c^+$ generated at the high temperatures. Only those $B_c^+$ generated at relatively low temperatures can survive in the QGP. The coalescence probability is given by the Wigner function $f^W(x_r, q_r)$ of $B_c^+$ mesons, which is connected with $B_c^+$ wave function via Weyl transform. Instead of employing realistic in-medium wave functions of $B_c^+$, we approximate the wave function to be a Gaussian function and get the corresponding Wigner function, $f^W(x_r, q_r) = 8 \exp\left[-\frac{x_r^2}{\sigma^2} - a^2 q_r^2\right]$. $x_r = x_1^{cm} - x_2^{cm}$ and $q_r = \frac{E_{1}^{cm} p_{1}^{cm} - E_{2}^{cm} p_{2}^{cm}}{E_{1}^{cm} + E_{2}^{cm}}$ are the relative position and momentum between two quarks in the center of mass frame (CoM). $x_1^{cm}$ and $p_1^{cm}$ are the positions and momenta of two particles in the CoM-frame. $E_{1,2}^{cm} = \sqrt{m_i^{2} + |\mathbf{p}_{1,2}^{cm}|^2}$ is the energy of the heavy (anti)quark. The width of the Wigner function is determined with the root-mean-square radius of $B_c^+$ via

$$\sigma^2 = \frac{4}{3} \frac{(m_1 + m_2)^2}{m_1^2 + m_2^2} \langle r^2 \rangle_{B_c^+}^{28, 49}. \langle r^2 \rangle_{B_c^+}$$

can be calculated from the Schrödinger equation with in-medium potential. We treat it as a parameter in the model and take different values (e.g. $\langle r^2 \rangle_{B_c^+} = 0.3, 0.5, 1.0$ fm respectively) to consider the effects of different in-medium potentials on the $B_c^+$ production. The mean coalescence probability of one charm and one anti-bottom quark in the hot medium is given by the Instantaneous Coalescence Model (ICM) \cite{28},

$$\langle P_{bc \rightarrow B_c^+} \rangle_{\{x_M, p_M\}} = \frac{dN_{c} dx_{1} dx_{2}}{(2\pi)^{3}} \frac{d^2 N_1}{d^2 p_1} \frac{d^2 N_2}{d^2 p_2} f^W(x_r, q_r) \times \delta^3 \left(\mathbf{P}_M - \mathbf{P}_1 - \mathbf{P}_2\right) \delta^3 \left(\mathbf{x}_M - \frac{x_1 + x_2}{2}\right), \tag{10}$$

where $\langle P_{bc \rightarrow B_c^+} \rangle$ is the ensemble averaged probability of one random $b$ quark combining with one $c$ quark at the coalescence temperature in QGP. Here $dN_i/dx_i dp_i$ is the phase space distribution of one particle. $x_M$ and $p_M$ are the positions and momentum of the formed meson. $x_{1,2}$ and $p_{1,2}$ are the positions and momentum of heavy (anti-)quarks respectively. In the coalescence reaction $b + c \rightarrow B_c^+ + g$, the momentum of the gluon has been neglected in the momentum conservation equation, $\mathbf{p}_{B_c^+} = \mathbf{p}_b + \mathbf{p}_c$, which is represented by the delta function in Eq. [10]. The energy conservation in the reaction has been ignored in the present framework which will be included later \cite{29, 30}. Both $B_c^+$ and $B_c^+ (2s)$ are observed in experiments \cite{50}. They are spin singlets, and it is natural to assume that their corresponding spin triplets exist. The radial excited states will decay into the ground state. We neglect the differences between all these states and count all of them together in $B_c^+$ production, resulting in $g_d = 2/9$, where the factor 2 comes from the radial excited states, and the factor 1/9 comes from the color factor. Charm and anti-bottom quarks are randomly generated according to initial spatial and momentum distributions given before. Their event-by-event stochastic evolutions in QGP are described with Langevin equations, and the Wigner function controls the coalescence probability. The yield of $B_c^+$ in Pb-Pb collisions become,

$$N_{AA}^{B_c^+} = \int d\mathbf{x}_M \frac{d\mathbf{p}_M}{(2\pi)^{3}} \langle P_{bc \rightarrow B_c^+} \rangle_{\{x_M, \mathbf{p}_M\}} \times (N_{AA}^{c\bar{c}} N_{AA}^{bb}), \tag{11}$$

The final production of $B_c^+$ is proportional to the number of charm and anti-bottom quarks in the QGP. The number of heavy quarks $N_{AA}^{QQ}$ in Pb-Pb collisions are obtained with Eq. [7].

With above Langevin equation plus ICM, we can describe the non-equilibrium distributions of heavy quarks in QGP, and their hadronization into hidden heavy flavor hadrons. This model has explained the experimental data of $J/\psi$ spectrum in 5.02 TeV Pb-Pb collisions \cite{28}. Due to the similarity between $J/\psi$ and $B_c^+$ coalescence process, we apply this approach to the $B_c^+$ study. With event-by-event numerical simulations, we obtain the final spectrum of $B_c^+$ produced in Pb-Pb collisions after considering both cold and hot nuclear matter effects. Now, we compare our theoretical results with the experimental data recently measured by CMS Collaborations.

In Fig.2 we calculate the $R_{AA}(B_c^+)$ as a function of $N_p$ at $6 < p_T < 11$ GeV/c in the rapidity range $1.2 < |y| < 2.3$. In the central collisions, both theoretical and experimental results are above unit, which means that QGP can enhance the production of $B_c^+$ compared with the production in pp collisions scaled with $N_{coll}$. This is due to the coalescence of random $b$ and $c$ quarks in QGP. The final production of $B_c^+$ is proportional to the densities of charm and bottom quarks in the medium. With more heavy quarks in QGP, charm quarks have a larger probability to meet with another anti-bottom quark, before they hadronize into D mesons. As most of primordially produced $B_c^+$ are dissociated by the QGP in central collisions, the final production of $B_c^+$ is dominated by the coalescence process. The phenomenon of $R_{AA} > 1$ in central and semi-central collisions indicate the existence of QGP and the coalescence mechanism. In peripheral collisions $N_p < 100$, the temperatures of QGP become lower. As $B_c^+$ can survive in the temperature region $T_c < T < T_d$ in QGP due to the large binding energy, when the temperature of QGP is slightly above $T_c$, some of primordially produced $B_c^+$ can survive in the medium. Meanwhile, the production of $B_c^+$ from the coalescence is reduced as heavy quark pairs are pro-
around 20% in the case of $D_s/(2\pi T) = 4.0$ than the case of $D_s/(2\pi T) = 7.0$. Note that the values of $D_s/(2\pi T)$ in Fig.2 are still within the uncertainties of $D_s$ determined by $D$ meson nuclear modification factors at RHIC and LHC collision energies [25, 51].

In Fig.3 the $p_T$ spectrum of $B_c^+$ in centrality 0-90% is plotted. In the low $p_T$ region, the production of $B_c^+$ in Pb-Pb collisions mainly comes from the coalescence process. As heavy quarks are strongly coupled with the QGP, they lose energy when moving through the medium. Heavy quarks with large $p_T$ will be shifted to the moderate and low $p_T$ bins due to the energy loss. Then the $B_c^+$ generated in the reaction $b + c \rightarrow B_c^+ + g$ are mainly located in low and moderate $p_T$ bins. Therefore, $B_c^+$ final production is dominated by the coalescence process in the low $p_T$ bins. This results in $R_{AA} \gg 1$ at $p_T \lesssim 11$ GeV/c in Fig.3. In the 11 $< p_T < 35$ GeV/c, theoretical results is slightly smaller than the experimental data. This is because we do not include the primordial production which becomes more important in higher $p_T$ bins. The competition between the primordial production and the regeneration from coalescence process at different $p_T$ bins and centralities have already been observed in $R_{AA}$ of $J/\psi$.

In above calculations, one of important ingredients in coalescence process is the Wigner function. It is connected with the geometry size of $B_c^+$ at the coalescence temperature. When taking a strong in-medium potential, the value of root-mean-square radius of $B_c^+$ becomes smaller. In the situation of $\langle r^2 \rangle_{B_c^+} = 0.3$ fm, the $B_c^+$ production is similar to the case of 0.5 fm in Fig.2 with-

**FIG. 2.** $B_c^+$ nuclear modification factor $R_{AA}$ as a function of the number of participants $N_p$ in the transverse momentum $6 < p_T < 11$ GeV/c in 5.02 TeV Pb-Pb collisions. Root-mean-square radius of $B_c^+$ is taken as $\sqrt{\langle r^2 \rangle_{B_c^+}} = 0.5$ fm in both upper and lower panel. The spatial diffusion coefficients satisfy the relation $D_s/(2\pi T) = 5.0$ (upper panel) and $D_s/(2\pi T) = 4.0, 7.0$ (lower panel), respectively. Theoretical bands correspond to the centrality bins: cent.0-20%, 20-40%, 40-60%, 60-80% respectively. The lower and upper limit of the theoretical bands correspond to a larger and smaller value of $d\sigma_{pp}^{B_c^+}/dy$ respectively. Experimental data is from CMS Collaboration [24].

**FIG. 3.** $p_T$ dependence of the nuclear modification factor $R_{AA}$ of $B_c^+$ in centrality 0-90% in 5.02 TeV Pb-Pb collisions. Root-mean-square radius of $B_c^+$ is taken as $\sqrt{\langle r^2 \rangle_{B_c^+}} = 0.5$ fm. Spatial diffusion coefficient is determined as $D_s/(2\pi T) = 5.0$. Lower and upper limit of the theoretical results correspond to the situations with larger and smaller values of $d\sigma_{pp}^{B_c^+}/dy$ respectively. Experimental data are from CMS Collaboration [24].

The band of the theoretical results is due to the uncertainty of $d\sigma_{pp}^{B_c^+}/dy$ in the denominator of $R_{AA}$. The lower panel of Fig.2 absolute yields of $B_c^+$ in pp collisions scaled by $N_{coll}$ is $dN_{coll}^{B_c^+}/dy$, $N_{coll} = 0.33 - 0.64$ in $6 < p_T < 11.0$ GeV/c in centrality 0-20%. The $B_c^+$ yields in Pb-Pb collisions can be obtained by scaling with the value of $R_{AA}$. In the lower panel of Fig.2 different values of spatial diffusion coefficient are also employed to study the effects of heavy quark thermalization on $B_c^+$ production. In the situation of $D_s/(2\pi T) = 4.0$, bottom and charm quarks are strongly coupled with the hot medium. Their momentum distributions are more close to the equilibrium distribution, which reduces the $B_c^+$ production in high $p_T$ bin (such as in $6 < p_T < 11$ GeV/c) compared with the weak coupling situation of $D_s/(2\pi T) = 7.0$. However, the total production of $B_c^+$ in $p_T > 0$ is enhanced by

In above calculations, one of important ingredients in coalescence process is the Wigner function. It is connected with the geometry size of $B_c^+$ at the coalescence temperature. When taking a strong in-medium potential, the value of root-mean-square radius of $B_c^+$ becomes smaller. In the situation of $\langle r^2 \rangle_{B_c^+} = 0.3$ fm, the $B_c^+$ production is similar to the case of 0.5 fm in Fig.2 with-
out evident difference. If the potential is very weak, we take the value of root-mean-square radius $\sqrt{\langle r^2 \rangle_{B_c^+}} = 1.0$ fm. In the Wigner function, the probability in spatial coalescence condition is enhanced. However, the constraints on the relative momentum between heavy quarks become more strict. The combined effects from spatial and momentum coalescence conditions consistently given by Wigner function reduce the $B_c^+$ production by around 50% compared with the situation of $\sqrt{\langle r^2 \rangle_{B_c^+}} = 0.5$ fm. Another relevant parameter is the coalescence temperature of $B_c^+$. Its value is taken as 1.2$T_c$ according to previous discussions. With a smaller coalescence temperature such as 1.1$T_c$, heavy quarks diffuse into a larger volume of the medium and their momentum distributions become slightly softer. $B_c^+$ production is reduced by around 5% in the case of lower coalescence temperature.

In this work, we employ the Langevin equation to simulate the time evolutions of heavy quarks in QGP, and obtain the non-equilibrium distributions of heavy quarks. The hadronization of $B_c^+$ is studied via the ICM. With multiple charm and bottom pairs making independent Brownian motions in the hot medium, the coalescence probability of random charm and anti-bottom quarks are enhanced. From the theoretical calculations, in central collisions, most of the $B_c^+$ final production is from the coalescence process instead of the primordial production. Furthermore, due to the significant energy loss of heavy quarks when they travel through the medium, the regenerated $B_c^+$ are mainly located in the low and moderate $p_T$ bins. In the high $p_T$ bins, $B_c^+$ production is dominated by the primordial production. Both experimental and theoretical studies show that the nuclear modification factor of $B_c^+$ can be larger than 1.0 in Fig.2B. The observation of $R_{AA}(B_c^+) > 1$ is regarded as an evident signal of the existence of the deconfined medium generated in Pb-Pb collisions.

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