A Novel Carrier Loop Algorithm Based on Maximum Likelihood Estimation (MLE) and Kalman Filter (KF) for Weak TC-OFDM Signals

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Abstract—Digital broadcasting signal is promised to be a positioning signal in indoors. A novel positioning technology named Time & Code Division-Orthogonal Frequency Division Multiplexing (TC-OFDM) is mainly discussed in this paper, which is based on China mobile multimedia broadcasting (CMMB). Signal strength is an important factor that affects the carrier loop performance of the TC-OFDM receiver. In the case of weak TC-OFDM signals, the current carrier loop algorithm has large residual carrier error. This paper proposes a novel carrier loop algorithm based on Maximum Likelihood Estimation (MLE) and Kalman Filter (KF) to solve the above problem. The discriminator of the current carrier loop is replaced by the MLE discriminator function in the proposed algorithm. The Levenberg-Marquardt (LM) algorithm is utilized to obtain the MLE cost function consisting of signal amplitude, residual carrier frequency and carrier phase, and the MLE discriminator function is derived from the corresponding MLE cost function. The KF is used to smooth the MLE discriminator function results, which takes the carrier phase estimation, the angular frequency estimation and the angular frequency rate as the state vector. Theoretical analysis and simulation results show that the proposed algorithm can improve the tracking sensitivity of the TC-OFDM receiver by taking full advantage of the characteristics of the carrier parameters. Compared with the current carrier loop algorithm, the tracking sensitivity is effectively improved by 3-5 dB in the simulation, and the better performance of the proposed algorithm is verified in the real environment.

Keywords—Carrier loop; MLE; LM; KF; TC–OFDM

I. INTRODUCTION

As main positioning technologies, Global Navigation Satellite Systems (GNSS) have many significant advantages in outdoors, such as large coverage area, high positioning accuracy, and superior navigation performances. However, buildings obstruct, reflect, and diffract the GNSS signals, the positioning performance is limited in indoor environments and cities. Recently, terrestrial radio positioning systems and their enhancements to the GNSS have drawn increasing attention, and the digital broadcasting signal is promised to be a positioning signal in indoors [1]. This paper mainly studies a novel terrestrial radio positioning system called Time & Code Division-Orthogonal Frequency Division Multiplexing (TC-OFDM), which is based on the China Mobile Multimedia Broadcasting (CMMB) system. The TC-OFDM system multiplexes the CMMB signal and pseudorange noise (PRN) codes in the same frequency band, and the positioning performance can be achieved by adding some simple modifications to the deployed CMMB facilities. The positioning part of the TC-OFDM system is a direct-sequence spread spectrum code division multiple access (DSSS-CDMA) system employing binary phase shift keying (BPSK) modulation, and reference [2] describes the TC-OFDM system in detail. Compared with the GNSS, the TC-OFDM system has many potential advantages: the TC-OFDM signal transmission power is stronger, the frequency band is U band. The TC-OFDM receiver demodulates PRN codes for high accuracy positioning, which can overcome the positioning limitations caused by the CMMB single network coverage.

Weak signal strength is the main factor that limits the receiver tracking loop to improve tracking sensitivity. Since the carrier loop is the weakest link in the receiver tracking loop, the superior carrier loop performance can improve the tracking sensitivity of the receiver in weak TC-OFDM signals to a certain degree [3]. The carrier loop is usually divided into frequency-locked loop (FLL) and phase-locked loop (PLL) [4, 5]. The FLL has good dynamic performance, but the carrier phase measurement accuracy is lower due to the wide noise bandwidth [6]. The PLL tracks the received signal more closely and has the high carrier phase measurement accuracy, but it has poor tolerant of dynamic stress [7]. Due to the complex indoor environment, the received signal strength attenuates greatly. Therefore, carrier loop performance needs to be improved. Currently, the methods to improve the carrier loop performance are divided into three categories: the KF-based PLL [8], the algorithms based on the Maximum likelihood estimation (MLE) algorithm [9], and the FLL-assisted PLL [10]. The KF-based PLL includes Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF) algorithms [11, 12]. The observation model in the carrier loop is nonlinear, which means the observed signal vector and the estimated state vector are nonlinear in carrier loop. The EKF and UKF are utilized to expand the nonlinear observation function using Taylor series and keep the linear part of the function while ignoring the higher order nonlinear part. But the EKF and UKF based on the linear minimum mean square error estimation criterion is a suboptimal carrier loop scheme, which still remain a large parameter estimation error for the weak received signal. The method based on FLL-assisted PLL can ensure the dynamic of

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the carrier loop but limited to the high estimation accuracy in weak signal environment [13]. Reference [14] uses the MLE to adjust Doppler frequency in high dynamic receiver carrier tracking loop, but it does not make a detailed analysis for the weak received signal. Iterative and non-iterative MLE are proposed to estimate the signal Doppler frequency and code phase in [15] and [16]. These two algorithms need to deal with real-time IF data, which means a heavy calculation.

This paper proposes a novel carrier loop algorithm based on MLE and KF for weak TC-OFDM signals. With the novel algorithm, the MLE discriminator function replaces the current carrier loop discriminator. The Levenberg-Marquardt (LM) algorithm is utilized to obtain the MLE cost function consisting of signal amplitude, residual carrier frequency and carrier phase, and the MLE discriminator function is derived from the corresponding MLE cost function. The KF is used to smooth the MLE discriminator function results, which takes the carrier phase estimation, the angular frequency estimation and the angular frequency rate as the state vector. Theoretical analysis and simulation results show that the proposed algorithm can improve the tracking sensitivity of the TC-OFDM receiver by taking full advantage of the characteristics of the carrier parameters. Compared with the current carrier loop algorithm, the tracking sensitivity is effectively improved by 3-5 dB in the simulation and the better performance of the proposed algorithm is verified in the real environments.

The following of this paper is organized as follows. Section II presents the signal model. The proposed carrier loop algorithm is introduced in Section III. Section IV presents the simulation and real data test results of the comparison between different carrier loop algorithms. Finally, Section V concludes the work.

II. SIGNAL MODEL

The CMMB signal and pseudorange noise (PRN) codes are multiplexed in the same frequency band. Fig. 1 shows the structure of the TC-OFDM signal. The duration of each frame of the CMMB signal is 1s, and each frame consisted of 40 time slots. The duration of each time slot is 25ms. The CMMB system uses orthogonal frequency division multiplexing (OFDM) to transmit the broadcasting signal. Two kinds of pseudo codes are superimposed on the CMMB signals to provide high accuracy positioning services, which is named short codes and long codes respectively. The beginning of the frame has the TxID and two synchronization signals. The TxID is empty in the practice CMMB signal, so the short codes are superimposed on the TxID to distinguish different base stations by using different pseudo codes signals. Thanks to the TxID is empty, the short codes and the CMMB signals have the same transmission power. In order not to affect the normal communication of the CMMB signal, the power of long codes is lower than the CMMB signal by 20dB [2]. In the positioning, the receiver uses short codes to achieve acquisition and long codes to achieve stable tracking.

The TC-OFDM signal of the nth time slot can be expressed as:

\[ s(n) = \sum_{k=-\infty}^{\infty} d^{(n)}(t) s^{(n)}_k(t) \]

where \( d(t) \) denotes the positioning-assisted information. The signal transmitted by the ith base station is:

\[ S^{(n)} = s^{(n)}(t) \cos(2\pi f_c t + \varphi_0(t)) \]

where \( f_c \) is the carrier frequency, \( \varphi_0(t) \) is the initial phase.

The signal from the ith base station is received by the receiver radio frequency (RF) antenna. The output signal of the RF antenna can be written as:

\[ r(t) = \sum_{n=0}^{N} A^{(n)} \Delta^{(n)}(t-\tau_n) \cos(2\pi(f_c + f_d) t + \varphi_0(t)) + \omega(t) \]

where \( N \) denotes the signal received from \( N \) different base stations. \( A^{(n)} \) is the signal amplitude, \( \tau_n \) is the incoming signal delay, \( f_c \) is the carrier frequency, \( f_d \) is the incoming Doppler shift, \( \varphi_0(t) \) is the initial phase, \( \omega(t) \) is the additive Gaussian white noise (AWGN) with zero mean and variance \( \sigma_w^2 \).

After the receiver RF front-end, down-conversion, low-pass filter and analog-digital conversion module (ADC), the digital intermediate frequency (IF) signal is expressed as:
where $T_s$ is the sampling duration, $A_{IF}$ is the IF signal amplitude, $\tau_i$ is the incoming signal delay, $f_{dr}$ is the IF frequency, $f_{ds}$ is the Doppler shift, $\phi_0$ is the initial carrier phase, $\sigma(n)$ is the additive Gaussian white noise (AWGN) with zero mean and variance $\sigma_n^2$.

The IF signal is sent to the baseband processor for acquisition [2], tracking and data demodulation. A rough estimation of carrier frequency and code phase of the received signal is obtained from the acquisition process, which initialize the related parameters of the tracking channel. Then the estimation of these two signal parameters is refined step by step through the tracking loop. The numerically controlled oscillator (NCO) in the tracking loop produces mutually orthogonal sine and cosine signals. After carrier stripped, the coherent integration result of I and Q channel signals by integration and dumping module can be created as:

$$
V_{cor}(n) = A_{mp}s(n)\sin c(\Delta T_{coh})\exp(j(2\pi f_{2,5} f_{ds} + \Delta \phi) + \phi_0(n))
$$

$$
A_{mp} = A_{dp}(n)T_{coh}, \quad \Delta f = f_d - f_{dc}, \quad \Delta \phi = \phi_d - \phi_{dc}
$$

where $V_{cor}(n)$ represents the $n$th coherent integration result, $T_{coh}$ is the coherent integration duration, $s(n)$ denotes the positioning-assisted information, $A_{mp}$, $\Delta f$ and $\Delta \phi$ represent the signal amplitude, carrier and phase residual respectively.

For the convenience of analysis, we assume that the local code is strictly aligned with the received signal. To avoid the bit transition, the coherent integration duration is not exceed one slot length. When the received carrier is stably tracked, the parameters $A_{mp}$, $\Delta f$ and $\Delta \phi$ have small changes in the coherent integration duration. The conventional carrier loop structure is shown in Fig. 2. In the tracking process, $\sin c(\Delta T_{coh}) = 1$ when $\Delta f$ is small enough. Then (6) can be simplified as:

$$
V_{cor}(n) = A_{mp}s(n)\exp(j(2\pi f_d f_{ds} + \Delta \phi) + \phi_0(n))
$$

The coherent integration results of (7) is sent into the carrier discriminator and loop filter. The Carrier NCO is adjusted in the real based on the carrier frequency estimation.

### III. MLE PARAMETER ESTIMATION MODEL

#### A. The principle of MLE

As we known, in the case of the unknown prior distribution of estimated parameters, the MLE method provides the parameter estimation with the smallest error variance. In terms of the linear discrete-time system, linear KF is the optimal filter [17]. Therefore, the MLE combined with the KF can make an accurate estimation of the parameters in the carrier loop, which can improve the tracking loop performance. The MLE is used to nonlinearly estimate the carrier loop parameters of interest, and then KF is applied to smooth the estimated parameters. Therefore, the proposed algorithm of combining MLE with KF is the optimal solution for the TC-OFDM carrier frequency tracking in the weak TC-OFDM environment.

MLE is often used to estimate unknown nonrandom parameters. MLE is defined as the estimator of $\theta$ value that maximizes the likelihood function. And the estimated value is denoted as $\hat{\theta}_{MLE}(x)$. We take the MLE for single-parameter $\theta$ as an example. For unknown nonrandom estimation $\theta$, the probability density function $p(x|\theta)$ of the observation vector (denotes as $x$) is called the likelihood function. The basic principle of MLE can be explained as follows: for a selected $\theta$, MLE consider the probability density function $p(x|\theta)dx$ that $x$ falls in a small area, and take the corresponding $\theta$ whose $p(x|\theta)dx$ is the largest as the estimated $\hat{\theta}_{MLE}(x)$. As shown in Fig. 3, the likelihood function is obtained after $x = x_0$, then we can draw a curve between the likelihood function and the estimator of $\theta$. The value of $p(x|\theta)dx$ for each $\theta$ means the probability that $x$ falls within $dx$ and $x$ centered on $x_0$ in the observation space $R$ for $\theta$. When $x = x_0$, it can be seen that $\theta = \theta_1$ is unreasonable. So we choose $\theta = \theta_0$ as the estimation, and the $\theta$ that maximizes $p(x = x_0|\theta)$ is selected as the estimated value $\hat{\theta}_{MLE}(x)$ within a range allowed by the estimated $\theta$.

#### B. MLE cost function

According to the MLE, when the likelihood function $p(x|\theta)$ is known, $\theta_{MLE}(x)$ can be obtained by the following equation:

![Fig. 3. The principle of maximum likelihood estimation.](image-url)
\[ \frac{\partial p(x \mid \theta)}{\partial \theta} \bigg|_{\theta=\theta_{\text{max}}} = 0 \quad (8) \]

\[ \frac{\partial \ln p(x \mid \theta)}{\partial \theta} \bigg|_{\theta=\theta_{\text{max}}} = 0 \quad (9) \]

MLE is considered as a valid parameter estimation algorithm in the tracking loop of the TC-OFDM receiver. According to the above analysis, the MLE of signal parameters is the estimation of signal parameter when the joint conditional probability density of a set of signal observations get maximized. The MLE produces the minimum variance of estimation error, when the statistical distribution of the estimated signal parameters in an uncertain interval is unknown [18]. In order to obtain the MLE of the unknown \( A_{\text{mp}}, \Delta f \) and \( \Delta \phi \) in (7), the joint probability density function of \( N \) consecutive coherent integration results is obtained according to [19]:

\[
p(x_{\text{corr}}, n | A_{\text{mp}}, \Delta f, \Delta \phi) = \frac{1}{(2\pi)^{\frac{N}{2}}} e^{-\frac{1}{2} (V_{x_{\text{corr}}, n} - V_{\text{corr}, n})^T \Lambda^{-1} (V_{x_{\text{corr}}, n} - V_{\text{corr}, n})} \quad (10)
\]

where \( V_{x_{\text{corr}}, n} = [V_{\text{corr}}(0), V_{\text{corr}}(1), \ldots, V_{\text{corr}}(N-1)] \) represents \( N \) consecutive coherent integration results, \( \hat{V}_{\text{corr}, n} \) is the estimated value of \( V_{\text{corr}, N} \), \( W \) is the diagonal matrix which stands for the weight factor of \( V_{\text{corr}, n} \). Superscript \( \Lambda \) denotes the matrix transpose and conjugate operations.

The MLE minimum variance unbiased estimation can be achieved by obtaining the maximum value of (10) to get the values \( A_{\text{mp}}, \Delta f \) and \( \Delta \phi \). The diagonal elements of the diagonal matrix \( W \) are set as \( 1 \) to obtain the log-likelihood cost function of (10) as in [20]:

\[
\begin{align*}
N(\mu | V_{\text{corr}, N}) &= -\frac{1}{2\sigma^2} (V_{\text{corr}, N} - V_{\text{corr}, n})^T \Lambda^{-1} (V_{\text{corr}, N} - V_{\text{corr}, n}) - N\ln(2\pi\sigma^2) \\
&= -\frac{1}{2\sigma^2} [V_{\text{corr}, N} - V_{\text{corr}, n}]^T H^{-1} (V_{\text{corr}, N} - V_{\text{corr}, n}) - N\ln(2\pi\sigma^2) \\
&= -\frac{1}{2\sigma^2} \sum_{n=0}^{N-1} \text{real}(V_{\text{corr}, n}) - A_{\text{mp}} \cos(\theta) - \frac{1}{2\sigma^2} \sum_{n=0}^{N-1} \text{imag}(V_{\text{corr}, n}) - A_{\text{mp}} \sin(\theta) - N\ln(2\pi\sigma^2) \\
&= -\frac{1}{2\sigma^2} \sum_{n=0}^{N-1} [V_{\text{corr}, n} - A_{\text{mp}} \cdot \text{real}(V_{\text{corr}, n}) - 2A_{\text{mp}} \cdot \text{imag}(V_{\text{corr}, n}) - A_{\text{mp}} \cdot \text{imag}(V_{\text{corr}, n})] - N\ln(2\pi\sigma^2) \\
\end{align*}
\]

(11)

where \( V_{\text{corr}, n} = V_{\text{corr}}(n) \), \( V_{\text{corr}, N} = [V_{\text{corr}, 0}, V_{\text{corr}, 1}, \ldots, V_{\text{corr}, N-1}] \) is \( N \) consecutive coherent integration results. \( \mu = [A_{\text{mp}}, \Delta f, \Delta \phi]^T \) represents the signal parameters to be estimated, \( n = 0, \ldots, N-1 \), \( \theta = 2\pi fT_{\text{coh}} + \Delta \phi \), \( \text{real}(\cdot) \) and \( \text{imag}(\cdot) \) represent the real and imaginary part respectively.

By finding the maximum value of (11) to get the estimated value of \( \mu \), which can be expressed as:

\[
\frac{\partial N(\mu | V_{\text{corr}, N})}{\partial \mu} = 0
\]

(12)

Deriving the partial derivative of \( \mu \):

\[
\frac{\partial N}{\partial A_{\text{mp}}} = -\frac{1}{\sigma^2} \sum_{n=0}^{N-1} \text{real}(V_{\text{corr}, n}) \cos(\theta) + \text{imag}(V_{\text{corr}, n}) \sin(\theta) \quad (13)
\]

When \( \frac{\partial N}{\partial A_{\text{mp}}} = 0 \), the estimation value of \( A_{\text{mp}} \) is obtained:

\[
A_{\text{mp}} = \frac{1}{N} \sum_{n=0}^{N-1} \text{real}(V_{\text{corr}, n}) \cos(\theta) + \text{imag}(V_{\text{corr}, n}) \sin(\theta) \quad (14)
\]

Since the items of \( |V_{\text{corr}, n}|^2, A_{\text{mp}}^2 \) and \( N\ln(2\pi\sigma^2) \) in (11) do not affect the partial derivative of \( \Delta f \) and \( \Delta \phi \) respectively, we can remove such irrelevant items and the cost function is simplified as:

\[
M = \sum_{n=0}^{N-1} \text{real}(V_{\text{corr}, n}) \cos(\theta) + \text{imag}(V_{\text{corr}, n}) \sin(\theta) \quad (15)
\]

Thus \( A_{\text{mp}} \) can be acquired by (14), the rest is to solve the \( \Delta f \) and \( \Delta \phi \). Solving the problem of \( \Delta f \) and \( \Delta \phi \) in (15) can be regarded as the process of solving a two-dimensional optimal problem. The MLE solution of \( \Delta f \) and \( \Delta \phi \) is transformed to solve the two-dimensional optimal solution by (15).

C. LM algorithm

The LM known as the damped least-squares (DLS) method, is one of the effective methods to solve the nonlinear least squares problems, which has both the advantages of gradient descent method and Gauss-Newton algorithm (GNA). The LM uses gradient to find the maximum (minimum) value. The LM obtains the optimal solution through iterative convergence. The (non-negative) damping factor \( \lambda \) is adjusted at each iteration. When \( \lambda \) is used as a smaller value, bringing the algorithm closer to the GNA, whereas if an iteration gives insufficient reduction in the residual, \( \lambda \) can be increased, giving a step closer to the gradient-descent direction. The LM algorithm is used to solve the two-dimensional optimal solution in (15). The iterative optimal solution is given by the following equation:

\[
\hat{\mu}_{i+1} = \hat{\mu}_i + (H_i + \lambda)^{-1} G_i, i = 0,1,2,\ldots
\]

(16)

where subscript \( i \) represents the number of iterations, \( \hat{\mu}_i \) is an \( 2 \times 1 \) state vector including \( \Delta f \) and \( \Delta \phi \), \( \lambda \) is an \( 2 \times 2 \) diagonal matrix, used to ensure that \( H_i + \lambda \) is positive and adjust the iterative convergence rate. \( H_i \) and \( G_i \) is \( 2 \times 2 \) pseudo-Hessian
matrix and 2×1 gradient vector respectively. $H_i$ and $G_i$ can be written as:

$$H_i = \begin{bmatrix} \frac{\partial^2 M}{\partial \mu^2} \end{bmatrix}_{\mu=\mu_i}$$ (17)

$$G_i = \begin{bmatrix} \frac{\partial M}{\partial \mu} \end{bmatrix}_{\mu=\mu_i}$$ (18)

where

$$\frac{\partial M}{\partial \mu} = \begin{bmatrix} \frac{\partial M}{\partial \mu} \frac{\partial M}{\partial \Delta} \frac{\partial M}{\partial \Delta \phi} \end{bmatrix}^T$$ (19)

Specific partial derivative results of (19) can be derived as:

$$\frac{\partial^2 M}{\partial \mu^2} = 2\pi T \sum_{n=0}^{\infty} n(-\text{real}(V_{\text{corr}})) \sin(\theta) + \text{imag}(V_{\text{corr}}) \cos(\theta))$$

$$\frac{\partial M}{\partial \Delta} = \sum_{n=0}^{\infty} n(\text{real}(V_{\text{corr}})) \cos(\theta) + \text{imag}(V_{\text{corr}}) \sin(\theta))$$

$$\frac{\partial^2 M}{\partial \Delta \phi} = -4\pi^2 T \sum_{n=0}^{\infty} n^2(\text{real}(V_{\text{corr}})) \cos(\theta) + \text{imag}(V_{\text{corr}}) \sin(\theta))$$

$$\frac{\partial M}{\partial \Delta \phi \Delta} = -2\pi T \sum_{n=0}^{\infty} n(\text{real}(V_{\text{corr}})) \cos(\theta) + \text{imag}(V_{\text{corr}}) \sin(\theta))$$

(20)

The entire LM algorithm iterative process is shown in Fig. 4. The first step is to set the initial value of $\mu$ and $\lambda$. Since we assume that the carrier is correctly tracked at the beginning then $\mu_i \approx 0$ and $\lambda$ initialized to an experience value. Next, (17) and (18) are utilized to get $H_i$ and $G_i$. Increasing $\lambda$ to make $H_i+\lambda$ positive definite. Then (16) is used to update $\mu$ and (15) is used to judge whether $M(\hat{\mu}_{i+1}) > M(\hat{\mu}_{i})$ is satisfied. If not, continue to increase $\lambda$. If satisfied, judging the gradient vector $G$ is less than 0.2 or greater than 0.8 (the judgment condition for $G$ is set according to the actual test), then adjust $\lambda$ according to the conditions respectively satisfied. The physical meaning of adjusting $\lambda$ is that if the estimated value $\hat{\mu}$ is closer to the optimal value of iteration, then increase $\lambda$ to slow down the convergence rate, otherwise decrease $\lambda$ to accelerate iterative convergence. The termination condition of the above iterative process is that the value of the gradient vector $G$ falls below predefined threshold (Pre_Thres) or the number of iterations (Iter_Num) exceeds the set maximum (Iter_Max).

### D. KF model

MLE can obtain the minimum error variance of parameters without relying on the prior distribution of signal estimation parameters. However, if we use the estimated frequency directly in weak signal environment, large estimated error which cannot be ignored will occur, and finally cause the low tracking accuracy. Since KF is the optimal filter in linear discrete systems [21]. KF is adopted in this paper to smooth the estimation of the parameters after a non-linear estimation by MLE. Therefore, for the carrier frequency tracking problem of TC-OFDM system, the combination scheme of MLE and KF is the optimal solution. The proposed carrier loop structure is shown in Fig. 5.
The main idea is to use MLE to get the estimated value of $\Delta f$ and $\Delta \phi$, and KF to smooth the MLE parameter estimation error to get more accurate $\Delta f$ and $\Delta \phi$. The output value of the KF is used to adjust the carrier NCO. The MLE uses the LM iteration to get the next value of $\Delta f$ and $\Delta \phi$. In practical application, the form of KF is closely related to the state equation and observation equation.

The state equation describes the behaviour of the state vector. In order to adjust the carrier loop accurately, the state vector in the selected carrier loop is expressed as:

$$X = [\phi \ \omega_d \ \omega_d']$$

where $\phi$ is the carrier phase estimation value, $\omega_d = 2\pi \Delta f$ denotes the angular frequency estimation value, $\omega_d'$ represents the angular frequency rate.

The state equation can be expressed as:

$$X_k = \Phi X_{k-1} + W_{k-1}$$

where $W_{k-1}$ is the input Gaussian white noise with mean zero and variance $Q$. $\Phi$ is the state transition matrix and the form can be written as:

$$\Phi = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}$$

where $T = NT_{coh}$ is the loop update period.

The observation equation describes the relationship between observations and state vectors. The observation equation can be expressed as:

$$Y_k = HX_k + V_k$$

where $H$ is the observation matrix, the form can be written as:

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$V_k$ is the observed noise, the mean is zero and the variance matrix is $R$.

The forms of $Q$ and $R$ can be respectively denoted as [22]:

$$Q = \sigma_w^2 I$$

$$R = \sigma_e^2 I$$

where $I$ is the unit matrix.

The KF algorithm obtains the filter value at the current time according to the state estimation value at the previous moment and the observation value at the current time. The entire carrier loop filtering process can be divided into two parts, the state estimation and state prediction. The output state estimate $X_k$ of KF and the variance matrix $P_k$ of the estimation error can be iteratively calculated by the following equations.

$$X_{k,k-1} = \Phi X_{k-1}$$

$$P_{k,k-1} = \Phi P_{k-1} \Phi^T + Q_{k-1}$$

$$K_k = P_{k,k-1} H_k^T [H_k P_{k,k-1} H_k^T + R_k]^{-1}$$

$$X_{k,k} = X_{k,k-1} + K_k (Y_k - H X_{k,k-1})$$

$$P_{k,k} = [I - K_k H] P_{k,k-1}$$

From (29) can be seen that when $R_k$ is large, the corresponding $K_k$ will be small, then the state estimate calculated by (30) is small; when $Q_k$ is small, one step prediction covariance matrix $P_{k,k-1}$ calculated by (28) will be small, which lead to a smaller state estimate $X_{k,k}$ finally. From the above analysis, it can be seen that every update of the system state by KF is a compromise between the current system state uncertainty and the observation uncertainty. Therefore, in this paper, $R_k$ and $Q_k$ are obtained by real-time statistics of noise on historical observation and current observation to enhance the adaptability of KF to noise.
IV. SIMULATION AND ANALYSIS

According to the previous discussion, a novel carrier loop algorithm based on MLE and KF is designed. In order to further illustrate the feasibility and performance of the proposed algorithm, simulations and real data tests are performed in this section. Monte Carlo simulations are adopted to compare the proposed algorithm with the current methods to make a comprehensive evaluation of the proposed carrier loop algorithm. In addition, the TC-OFDM signals are broadcasted by the modified base stations, a positioning receiver and other related equipment is also utilized to achieve the proposed algorithm and verify the performance of the algorithm. Finally, we choose several points in a test building to test the positioning accuracy for the static receiver.

A. Simulations

In order to verify the performance of the proposed algorithm, Monte Carlo simulation is applied to evaluate the proposed algorithm effectively and comprehensively. To demonstrate the reliability and effectiveness of the novel carrier loop algorithm, comparative tests are performed. The feasibility of iterative estimation of the LM algorithm, the validity of the KF smoothing error and the superiority of the combination the MLE with the KF algorithm are verified respectively. The positioning signal adopts Gold codes and the simulation parameters are listed in Table 1.

| Parameter                          | Value       |
|-----------------------------------|-------------|
| Slot time, \( T_F \)              | 25ms        |
| Sample rate                       | 4.4         |
| Intermediate frequency, \( f_{IF} \) | 0 Hz       |
| Data bit transition               | Random      |
| Coherent integration time, \( T_{coh} \) | 1.6ms      |
| Consecutive coherent integration numbers, \( N \) | 15 |
| Predefined threshold, Pre_Thres | 0.02        |
| Iteration maximum numbers, Iter_Max | 30         |
| Signal to noise ratio, SNR        | -45dB to -20dB |

To test the feasibility and effectiveness of the MLE estimation and the LM algorithm, we set the residual carrier as 25 Hz and the phase as 0.5 rad of the input TC-OFDM signal. The SNR is -25dB. And the convergence curve of frequency and phase using MLE estimation and iteratively by LM algorithm is shown in Fig. 6.

From Fig. 6, after iteration to the 9th, the residual carrier and phase values remain stable and are close to the set value. The final residual carrier value is stable at 25.8Hz and the phase value is stable at 0.53rad, which verifies the feasibility of using MLE to estimate the residual carrier and phase of the carrier loop. For the convenience of analysis, we selected the iteration number fixed at nine in the following tests.

Meanwhile, there is still a large frequency error in adjusting the carrier loop directly using the signal parameters estimated by MLE. To demonstrate the validity of KF smoothing error, the MLE combined with KF loop is compared with MLE loop. The SNR is -25dB. Fig. 7 shows that the error of frequency estimation after KF is obviously decreased in contrast to using MLE only. The validity of KF smooth parameter error is verified. The MLE combined with KE denotes as MLE&KF and the other denotes as MLE in Fig. 7, respectively.

To verify the signal parameter estimation accuracy corresponding to different SNR, we compare the proposed algorithm with conventional second-order frequency-locked assisted third-order phase-locked (FLL&PLL) carrier loop and MLE without KF (MLE). The FLL&PLL contains both the high accuracy of PLL and large dynamic of FLL. And 200 simulations are performed for each SNR with a fixed predetermined frequency error. The average comparison results are shown in Fig. 8.
The results in Fig. 8 show that the frequency estimation errors of the three algorithms all increase with SNR decreasing. Among them, the FLL&PLL has the largest error under low SNR, followed by the MLE without KF (MLE). The proposed algorithm (MLE&KF) can still effectively estimate the frequency, which shows high frequency estimation accuracy and tracking performance in weak TC-OFDM signals. The simulation verifies the superiority of the proposed algorithm for various SNR.

In order to further verify the tracking performance of the proposed algorithm, the comparison results of the tracking probability with other two algorithms are shown in Fig. 9. Detection probability is defined as the probability of successful tracking of the total number of Monte Carlo simulations. And tracking sensitivity is defined as the SNR at which tracking probability exceeds 50%. It can be seen that the tracking sensitivity of FLL&PLL is -31dB. Then followed by MLE which is -33dB. The MLE&KF algorithm has the optimal performance than the others, and the tracking sensitivity is -36dB, which is 3dB lower than MLE and 5dB lower than FLL&PLL. The Monte Carlo simulation results prove that proposed algorithm can significantly improve the tracking sensitivity in weak TC-OFDM signals.

B. Real Data Tests

To prove that the proposed algorithm plays a significantly good performance in actual environment, real data tests are conducted. We start our actual tests in the comprehensive test platform, which is shown in Fig. 10 and Fig. 11. The test platform consists of the modified base station and the positioning receiver. Fig. 10 describes each component of equipment of the modified base station in detail. The output frequency of the atomic clock is 10MHz. The time distributor, the counter and the industrial personal computer collaborative work together in order to ensure synchronization between the modified base stations. The synchronization accuracy between the modified base stations is up to 5ns (1σ). Meanwhile, the time distributor generates the positioning data messages containing UTC, air pressure, base station number and base station coordinates which are necessary for positioning. The function of the actuator is to modulate the TC-OFDM signal into RF signal and finally the RF signal is transmitted by the transmitter.

We use the positioning receiver developed by us which is shown in Fig. 11 to test the positioning accuracy. An overview of the internal and external structures of the positioning receiver is shown in Fig. 11a. Fig. 11b shows that the positioning receiver and mobile phone through the Bluetooth protocol to communicate and transmit useful data messages. Then the phone exhibits the final positioning result via the map. An architecture based on FPGA and ARM is adopted in the positioning receiver for baseband processing and data demodulation. The FPGA is used to perform the main logic operations and ARM is responsible for controlling the circuit logic. The IF signal is processed into a zero-digital IF signal for subsequent FPGA processing.
Furthermore, we start the actual test based on the campus and the test environment is shown in Fig. 12. We set up the base station on the roof of four buildings on our campus. Then we selected the 3th floor of another building as test building to start our real test using the positioning receiver. The two-order FLL assisted three-order PLL algorithm is implemented in the receiver to compare with the proposed algorithm. In order to verify the better performance of the proposed algorithm in the weak TC-OFDM signal, we selected 14 test points on the floor of the test building for positioning accuracy test. And the receiver is placed at each test point for one hour. Then we calculated the Root-mean-square error (RMSE) of the test points. The corresponding acquisition and positioning algorithms are utilized for the horizontal positioning. Since the positioning system uses a custom coordinate system for indoor positioning, the output positioning result is compared with the distance between the selected point and the origin of the corresponding floor. The RMSE of the positioning accuracy error results of the two algorithms between the selected point and the corresponding original point are shown in Fig. 13. Fig. 13 shows the horizontal measurement accuracy between the proposed algorithm and the conventional second-order FLL assisted third-order PLL algorithm. It can be seen that the positioning accuracy obtained by the proposed method has smaller positioning error and higher positioning accuracy than the second-order FLL assisted third-order PLL in weak TC-OFDM signals.

V. CONCLUSION

To improve the performance in weak TC-OFDM signals, a novel carrier loop algorithm based on MLE and KF is proposed in this paper. The algorithm derives the MLE discriminator function to replace the existing discriminator in carrier loop. The Levenberg-Marquardt (LM) algorithm is utilized to obtain the MLE cost function consisting of signal amplitude, residual carrier frequency and carrier phase, and the MLE discriminator function is derived from the corresponding MLE cost function. In order to further reduce the estimation error, the KF is used to smooth the MLE discriminator function results. Numerical simulation and real data tests are implemented to verify the performance of the algorithm. The test results show that the proposed algorithm can improve the tracking sensitivity of the TC-OFDM receiver by taking full advantage of the characteristics of the carrier parameters. Finally, compared with the current carrier loop algorithm, the tracking sensitivity is effectively improved by 3-5 dB in the simulation and the positioning accuracy is improved in the real environments. The proposed algorithm is designed for weak TC-OFDM signals. Meanwhile, the method proposed in this paper can be
combined with related multipath suppression or mitigation methods to achieve the effect of suppressing multipath under weak signal conditions. And the method of suppressing multipath has been studied in further research.

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