Holographic Cosmology from BIonic Solutions

Alireza Sepehri 1∗, Mir Faizal 2†, Mohammad Reza Setare 3‡, Ahmed Farag Ali 4,5§

1 Faculty of Physics, Shahid Bahonar University, P.O. Box 76175, Kerman, Iran.
2 Department of Physics and Astronomy, University of Waterloo, Waterloo, ON, N2L 3G1, Canada.
3 Department of Science, Campus of Bijar, University of Kurdistan, Bijar, Iran.
4 Department of Physics, Florida State University, Tallahassee, FL 32306, USA.
5 Department of Physics, Faculty of Science, Benha University, Benha 13518, Egypt.

In this paper, we will use a BIonic solution for analysing the holographic cosmology. A BIonic solution is a configuration of a D-brane and an anti-D-brane connected by a wormhole. A BIonic configuration can form due to a transition of fundamental black strings. After the BIonic has formed, the wormhole in the BIonic will act as a channel for the energy to flow into the D3-brane. This will increase the degrees of freedom of the D3-brane causing inflation. The inflation will end when the wormhole gets annihilated. However, as the distance between the D3-brane and the anti-D3-brane reduces, tachyonic states get created. These tachyonic states will lead to the formation of a new wormhole. This new wormhole will again increasing the degrees of freedom on the D3-brane causing late time acceleration.

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I. INTRODUCTION

In the framework of effective field theories, a low energy effective action for D-branes can be obtained by integrating the massive open string degrees of freedom [1]-[2]. This non-linear action is called the Dirac-Born-Infeld (DBI) action. The BIonic solution depend critically on the non-linearity of this action [3]-[6]. In BIonic solution the F-string end on a point of a D-brane, and the F-string charge gets associated with the world-volume electric flux carried by the D-brane. Thus, F-strings go from being one-dimensional objects to higher-dimensional brane wrapped on a sphere. These phenomena depend critically on the non-linearity of the DBI action. In fact, this phenomena has been used for analysing various aspects of the AdS/CFT correspondence [7]-[9]. For example, at sufficiently large energies, the gravitons which satisfy the BPS bound, could blow up to become giant gravitons, if they are moved on the equator of the five sphere in the AdS5 × S5 background [10]-[11]. The blown up version of the Wilsons loops has been also analysed using various D-branes configurations [12]-[13]. In this paper, we will use the BIonic solutions for analysing the holographic cosmology.

One possible interpretation of the holographic principle is that the number of degrees of freedom in a region of space is equal to the number of degrees of freedom on the boundary surrounding that region of space. Using this holographic principle, it has been argued that the universe expands due to a difference between the degrees of freedom between a region of space and the boundary surrounding it [14, 15]. This proposal is based on the thermodynamic description of the Einstein equation, in which Einstein equation is viewed as an emergent equation which has its origin in the Clausius relations [16]. To be precise, the Einstein equations follows from the assumption that the Clausius relation holds for all the local Rindler causal horizons through each spacetime point. This thermodynamic description of the Einstein equation has been used to motivate the existence of the entropic force [17]. It is the existence of this entropic force that forms the basis of the holographic cosmology.

It may be noted that Friedmann equations in Gauss-Bonnet gravity, and even more general Lovelock gravity, have been analysed using this holographic cosmology [18]. Furthermore, the brane world models [19], scalar-tensor gravity [20], and $f(R)$ [21], have been studied using the thermodynamic approach. As the holographic cosmology is based on the thermodynamic approach, holographic brane world models, holographic scalar-tensor cosmology, and holographic $F(R)$ cosmology, have been constructed using the thermodynamic description of these theories [22]. A generalization of the original proposal for the holographic cosmology has been used for deriving Friedmann equation corresponding to the Friedmann-Robertson-Walker universe with an arbitrary spatial curvature [23]. The Friedmann equation for Gauss-Bonnet gravity, and more general Lovelock gravity, with an arbitrary spatial curvature, have also been derived

∗ alireza.sepehri@uk.ac.ir
† f2mir@uwaterloo.ca
‡ rezakord@ipm.ir
§ afali@fsu.edu ; ahmed.ali@fsu.edu.eg
using this generalization proposal \[24\]. However, it has been demonstrated that such a generalization to non-flat universe is only valid if the aerial volume is used instead of the proper volume \[25\].

It has also been possible to use analyse the holographic cosmology using the Blonic solution \[26\]. Thus, the origin of the emergence of space in the holographic cosmology has been explained using this Blonic solution. Here the D3-brane in the Blon corresponds to the universe, and the degrees of freedom on this brane are controlled by the evolution of Blon in extra dimensions. In the configuration of a D3-brane and a anti-D3-brane, if the D3-brane is away from the anti-D3-brane, then the spike of the D3-brane also remains separated from the spike of the anti-D3-brane spike. However, a Blon can form when the spikes of the D3-brane meets the spike of the anti-D3-brane, and this occurs when the distance between the D3-brane and the anti-D3-brane reduces to a critical point \[27\]-\[29\]. Thus, a wormhole is formed when the spike of the D3-brane and meets the spike of the anti-D3-brane. This wormhole can act as a channel for the energy to flow into the D3-brane increasing its degrees of freedom. In this paper, we will analyse the holographic cosmology using a similar Blonic model. Thus, in our model, the Blon will first form due to a transition of fundamental black strings. The D3-brane in this Blon will represent our universe. The wormhole in the Blon will form a channel for the energy to flow into the D3-brane causing inflation. Then as the D3-branes moves away from the anti-D3-brane, the wormhole will get annihilated due to the spike of D3-brane separating from the spike of anti-D3-brane. As the wormhole will get annihilated no further energy will flow into the D3-brane and thus the inflation will end with the annihilation of the wormhole. However, as the D3-brane approaches a anti-D3-brane a transition of fundamental black strings, and use it for analyzing holographic inflationary cosmology. In section \[\text{III}\] we analyse the tachyonic states in theory and use them for explaining the late time acceleration. In the last section we summarize our results and discuss some possible extensions of the results obtained in this paper.

\section{Holographic Blonic Solutions}

In this section, we will analyse the holographic cosmology using Blonic solutions. Thus, we start from the transition of \( k \) fundamental black strings to a Blon at a certain point. Thus, we now write the supergravity solution for \( k \) coincident non-extremal black F-strings lying along the \( z \) direction \[30\],

\[
\begin{align*}
\frac{dM_{F1}}{dz} &= T_{F1}k + \frac{16(T_{F1}k)^{3/2}T^3}{81T_{D3}} + \frac{40T_{F1}^2k^2\pi^3T^6}{729T_{D3}^2}. \\
\frac{dM_{F2}}{dz} &= T_{F2}k + \frac{16(T_{F2}k)^{3/2}T^3}{81T_{D3}} + \frac{40T_{F2}^2k^2\pi^3T^6}{729T_{D3}^2}. \\
\end{align*}
\]

Using this metric, an expression for the mass density along the \( z \) direction can be writing as \[28\],

\[
\frac{dM_{F1}}{dz} = T_{F1}k + \frac{16(T_{F1}k)^{3/2}T^3}{81T_{D3}} + \frac{40T_{F1}^2k^2\pi^3T^6}{729T_{D3}^2}. 
\]

The metric for Blon can be written as \[27\],

\[
ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) + \sum_{i=1}^{6} dx_i^2. 
\]

Choosing the world volume coordinates of the D3-brane as \( \{\sigma^a, a = 0..3\} \) and defining \( \tau = \sigma^0, \sigma = \sigma^1 \), the coordinates of Blon are given by \[27\],

\[
t(\sigma^a) = \tau, r(\sigma^a) = \sigma, x_1(\sigma^a) = z(\sigma), \theta(\sigma^a) = \sigma^2, \phi(\sigma^a) = \sigma^3, 
\]

and the remaining coordinates \( x_{i=2..6} \) are constant. The embedding function \( z(\sigma) \) describes the bending of the brane. Now let \( z \) be a transverse coordinate to the branes and \( \sigma \) be the radius on the world-volume. The induced metric on the brane can then be written as

\[
\gamma_{ab}d\sigma^a d\sigma^b = -d\tau^2 + (1 + z'(\sigma)^2)d\sigma^2 + \sigma^2(d\theta^2 + \sin^2 \theta d\phi^2), 
\]
so that the spatial volume element is \(dV_3 = \sqrt{1 + z'(\sigma)^2} \sigma^2 d\Omega_2\). We impose the two boundary conditions that \(z(\sigma) \to 0\) for \(\sigma \to \infty\) and \(z'(\sigma) \to -\infty\) for \(\sigma \to \sigma_0\), where \(\sigma_0\) is the minimal two-sphere radius of the configuration. For this BIon, the mass density along the \(z\) direction at corresponding point can be obtained \(^{30}\),

\[
\frac{dM_{BIon}}{dz} = T_{F1} k + \frac{3\pi T_{D3}^2 k^2 T^4}{32 T_{D3}^2 \sigma_0^4} + \frac{7\pi^3 T_{F1}^3 k^3 T^8}{512 T_{D3}^2 \sigma_0^8}.
\]  

(6)

Comparing the mass densities for BIon to the mass density for the F-strings, we see that the thermal BIon configuration behaves like \(k\) F-strings at \(\sigma = \sigma_0\). At this corresponding point, \(\sigma_0\) should have the following dependence on the temperature,

\[
\sigma_0 = \left(\frac{\sqrt{k T_{F1}}}{T_{D3}}\right)^{1/2} \sqrt{T} \left[C_0 + C_1 \frac{\sqrt{k T_{F1}}}{T_{D3}} T^3\right],
\]

(7)

where \(T_{F1} = 4k \pi^2 T_{D3} g_s l_s^2\), and \(C_0, C_1, F_0, F_1, F_2\) are numerical coefficients which can be determined by comparing that the \(T^3\) and \(T^6\) terms in Eqs. (2) and (6). The inflation ends when the wormhole gets annihilated and the width of its throat reaches zero,

\[
\sigma_0 = 0, \bar{C}_0 = -C_0 \to T_{end} = \frac{\bar{C}_0 \sqrt{T_{D3}}}{C_1 k T_{F1}}.
\]

(8)

These calculations show that temperature was infinity at the beginning, and then it decreased with time and tended to \(T_{end}\) at the end of inflation. Consequently, width of the throat of wormhole decreased with time and tended to zero at \(T = T_{end}\).

The wormhole acted as a channel for the energy to flow into the D3-brane, and this lead to inflation. Putting \(k\) units of F-string charge along the radial direction and using equation (11), we obtain \(^{28, 30}\),

\[
z(\sigma) = \int_{\sigma}^{\infty} d\bar{\sigma} \left(\frac{\tilde{F}(\bar{\phi})^2}{\tilde{F}(\sigma_0)^2} - 1\right)^{-\frac{1}{2}}.
\]

(9)

In finite temperature BIon \(F(\sigma)\) is given by

\[
F(\sigma) = \sigma^4 \frac{4 \cosh^2 \alpha - 3}{\cosh^4 \alpha},
\]

(10)

where \(\cosh \alpha\) is determined by following function:

\[
\cosh^2 \alpha = \frac{3 \cos \frac{\delta}{2} + \sqrt{3} \sin \frac{\delta}{2}}{\cos \delta}.
\]

(11)

Here we have defined

\[
\cos \delta = T \left(1 + \frac{k^2}{\sigma^4}\right) \quad T = \left(\frac{9 \pi^2 N}{4 \sqrt{3} T_{D3}}\right) T,
\]

\[
\kappa = \frac{k T_{F1}}{4 \pi T_{D3}}.
\]

(12)

In above equation, \(T\) is the finite temperature of BIon, \(N\) is the number of D3-branes and \(T_{D3}\) and \(T_{F1}\) are tensions of brane and fundamental strings respectively. Attaching a mirror solution to Eq. (9), we construct wormhole configuration. Now \(\Delta = 2z(\sigma_0)\), is the distance separating \(N\) D3-branes from \(N\) anti-D3-branes for a given Blonic configuration. This configuration is thus defined by four parameters \(N, k, T\) and \(\sigma_0\),

\[
\Delta = 2z(\sigma_0) = 2 \int_{\sigma_0}^{\infty} d\bar{\sigma} \left(\frac{\tilde{F}(\bar{\phi})^2}{\tilde{F}(\sigma_0)^2} - 1\right)^{-\frac{1}{2}}.
\]

(13)

In in the limit of small temperatures, we obtain

\[
\Delta = \frac{2 \sqrt{\pi} \Gamma(5/4)}{\Gamma(3/4)} \sigma_0 \left(1 + \frac{8}{27} \frac{k^2 T^4}{\sigma_0^4}\right).
\]

(14)
Let us now construct the holographic idea in thermal BIon. It may be noted that by surface degrees of freedom we mean the the degrees of freedom on the holographic horizon of the universe represented by the D3-brane. So, we need to compute the contribution of the Bionic system to the degrees of the surface degrees of freedom on the holographic horizon and the degrees of freedom inside the universe. So, we write that these degrees of freedom are related to the entropy of BIon. We also write an expression for the mass density along the transverse direction during inflation,

\[ N_{\text{sur}} + N_{\text{bulk}} = N_{\text{BIon}} = N_{\text{brane}} + N_{\text{anti-brane}} + N_{\text{wormhole}} \]

\[ \simeq 4l_p^2 S_{\text{BIon}} \]

\[ = \frac{4T_{\text{D3}}^2}{\pi T^4} \int d\sigma \frac{F(\sigma)}{\sqrt{F^2(\sigma) - F^2(\sigma_0)}} \sigma^2 \frac{4}{\cosh^4 \alpha} \]

\[ N_{\text{sur}} - N_{\text{bulk}} \simeq \int d\sigma \frac{dM_{\text{BIon}}}{dz} \]

\[ = \frac{2T_{\text{D3}}^2}{\pi T^4} \int d\sigma \frac{F(\sigma)}{F(\sigma_0)} \sigma^2 \frac{4 \cosh^2 \alpha + 1}{\cosh^4 \alpha}. \hspace{1cm} (15) \]

Solving these equations simultaneously, we obtain,

\[ N_{\text{sur}} \simeq \frac{4T_{\text{D3}}^2}{\pi T^4} \int d\sigma \frac{F(\sigma)}{\sqrt{F^2(\sigma) - F^2(\sigma_0)}} \sigma^2 \frac{4}{\cosh^4 \alpha} \]

\[ + \frac{2T_{\text{D3}}^2}{\pi T^4} \int d\sigma \frac{F(\sigma)}{F(\sigma_0)} \sigma^2 \frac{4 \cosh^2 \alpha + 1}{\cosh^4 \alpha}, \]

\[ N_{\text{bulk}} \simeq \frac{4T_{\text{D3}}^2}{\pi T^4} \int d\sigma \frac{F(\sigma)}{\sqrt{F^2(\sigma) - F^2(\sigma_0)}} \sigma^2 \frac{4}{\cosh^4 \alpha} \]

\[ - \frac{2T_{\text{D3}}^2}{\pi T^4} \int d\sigma \frac{F(\sigma)}{F(\sigma_0)} \sigma^2 \frac{4 \cosh^2 \alpha + 1}{\cosh^4 \alpha}. \hspace{1cm} (16) \]

It can thus be inferred that by decreasing temperature of Bionic system, the degrees of freedom inside the universe increase. This is because that wormhole gets annihilated because of the D3-brane moving away from the anti-D3-brane, and this causes a decrease in the temperature and increase in number of degrees of freedom on the holographic horizon of the universe.

Now we can relate some cosmological parameters like Hubble parameter and energy density with certain properties of the BIon. For example, the number of degrees of freedom on the spherical surface of apparent horizon with radius \( r_A \) is proportional to its area,

\[ N_{\text{sur}} = \frac{4\pi r_A^2}{l_p^2}, \hspace{1cm} (17) \]

where \( r_A = 1/\sqrt{H^2 + \frac{k}{a^2}} \) is the apparent horizon radius for the Friedmann-Robertson-Walker Universe, \( H = \dot{a}/a \) is the Hubble parameter, and \( a \) is the scale factor. Using this equation, we can estimate the Hubble parameter for flat universe,

\[ H_{\text{flat, inf}} \simeq \frac{18\pi k^4 N_{12} T_{14}^{11}}{T_{\text{D3}}^4 g_0^8} T^{12} + \frac{8\pi k^2 N_{10} T_{12}^{11}}{T_{\text{D3}}^4 g_0^4} T^{10}. \hspace{1cm} (18) \]

It may be noted that the expansion of universe is controlled by the number of branes, F-string, the brane tensions, the location of throat of wormhole and temperature of BIon.

On the other hand, using the Friedmann equation of the flat Friedmann-Robertson-Walker Universe, we can calculate the universe energy density,

\[ \rho_{\text{flat, inf}} = \frac{3}{8\pi l_p^2} H_{\text{flat}}^2 \simeq \frac{27\pi k^8 N_{24} T_{28}^{28}}{4l_p^2 T_{\text{D3}}^8 g_0^{10}} T^{24} + \frac{3\pi k^4 N_{20} T_{24}^{24}}{l_p^2 T_{\text{D3}}^8 g_0^{8}} T^{20}. \hspace{1cm} (19) \]

Thus, the energy density depends on the temperature of BIon and reduces to lower values at \( T = T_{\text{end}} \). This is because in course of time, the wormhole vanishes and no channel is left for the energy into our universe, and this causes the energy density to decrease.
Now using (17), and assuming \( r_A = 1/\sqrt{H^2 + \frac{k}{a^2}} \), we can re-obtain Hubble parameter for non-flat universe in terms of temperature,

\[
H_{o/c,inf} \simeq \sqrt{\left( \frac{18\pi k^4 N^{12} T_{\text{F1}}^{14}}{T_{D3}^{14} \sigma_0^8} T_{\text{F1}}^{12} + \frac{8\pi k^2 N^{10} T_{\text{F1}}^{12}}{T_{D3}^{14} \sigma_0^8} T^{10} \right)^2 - \bar{k}/a^2}. \tag{20}
\]

Defining \( T \) as

\[
T = \left( \frac{18\pi k^4 N^{12} T_{\text{F1}}^{14}}{T_{D3}^{14} \sigma_0^8} T_{\text{F1}}^{12} + \frac{8\pi k^2 N^{10} T_{\text{F1}}^{12}}{T_{D3}^{14} \sigma_0^8} T^{10} \right)^2,
\]

and solving this equation, we can write the scale factor for open \((k = -1)\) universe as

\[
a_{o,inf}(t) \simeq \exp - \int dt \left[ T + \ln(t) \right], \tag{22}
\]

and the scale factor for closed \((k = +1)\) universe as

\[
a_{c,inf}(t) \simeq \exp \left[ -i \int dt \left[ T + \ln(t) + \frac{\pi}{2} \right] \right]. \tag{23}
\]

These relations relate the behaviour of open and closed universes to the temperature. Obviously, the scale factor of open universe is almost zero at the beginning \((T = \infty)\) and grows with decreasing temperature and tends to larger values at the end of inflation. On the other hand, the scale factor of closed universe oscillates during inflation. Now we can calculate the energy density for open and closed universes during this inflation,

\[
\rho_{o/c,inf} = \frac{3}{8\pi l_P^2} \left( \frac{H_{o/c}^2}{8\pi l_P^2} + \frac{k}{a^2} \right).
\]

\[
\simeq \frac{3}{8\pi l_P^2} H_{\text{flat}}^2,
\]

\[
\simeq \frac{3}{8\pi l_P^2} \left( \frac{22\pi k^8 N^{24} T_{\text{F1}}^{28} T^{24}}{4l_P^2 T_{D3}^{28} \sigma_0^{16}} + \frac{3\pi k^4 N^{20} T_{\text{F1}}^{24}}{l_P^2 T_{D3}^{24} \sigma_0^8} \right) T^{20}
\]

\[
= \rho_{\text{flat,inf}}. \tag{24}
\]

The equality indicates that the energy density of all three flat, open and closed universes are the same. This is because that the energy density originates due to some evolution in a BIon system and does not depend on type of universe. In fact, it is the flow of energy from the BIon into our universe which causes inflation.

It may be noted that when the inflation ends, all wormhole parameters vanishes and the mass distribution along \( z \)-direction is absent at this stage. Thus, at this stage the surface degrees of freedom on the holographic horizon and the bulk degrees of freedom inside the universe can be expressed as

\[
N_{\text{sur}} - N_{\text{bulk}} \simeq \int_{\sigma_0}^{\sigma_0} d\sigma \frac{dM_{\text{Blon}}}{dz} = 0. \tag{25}
\]

So, we can write

\[
N_{\text{sur}} = N_{\text{bulk}}. \tag{26}
\]

This equation indicates that degrees of freedom on the the universe horizon will be equal to the degrees of freedom of bulk at the end of universe expansion. This means that inflation evolves by the the phenomenological events and ends at \( \sigma_0 = 0 \).

III. TACHYONIC STATES

In this section, we will analyse the effect of tachyonic states in this theory. These states are created when the distance separating the D3-brane from the anti-D3-brane decreases. These tachyonic states create a new wormhole, and this new wormhole causes the number of degrees of freedom on the universe horizon to increased. Thus, the
universe evolved from non-phantom phase to phantom one and consequently, phantom-dominated era of the universe accelerates and ends up in Big Rip singularity.

To construct non-phantom model, we consider a set of D3-branes and anti-D3-branes in the background \[33\], which are placed at points \(z_1 = 1/2\) and \(z_2 = -1/2\) respectively. This causes the separation between the brane and antibrane to become of order one. The action for a single D3-branes and a single anti-D3-branes, with open string tachyon, can be written as \[34\]–\[36\],

\[
S = -\tau_3 \int d^6 \sigma \sum_{i=1}^{2} V(TA, l)e^{-\phi(\sqrt{-det A_i})},
\]

\[
(A_i)_{ab} = (g_{MN} - \frac{TA^2 i^2}{Q} g_{ab} g_{zN}) \partial_a x_i^M \partial_b x_i^M + F_{ab}^i + \frac{1}{2Q} ((D_a TA)(D_b TA)^* + (D_a TA)^*(D_b TA))
\]

\[
+ il(g_{az} + \partial_a z_i g_{zi})(TA(D_b TA)^*) - TA^*(D_b TA) + il(TA(D_a TA)^* - TA^*(D_a TA))
\]

\[
\times (g_{bz} + \partial_b z_i g_{zi}) \left( 1 - \frac{\pi^2 NT^4}{6T_{D3}} \right).
\]

where

\[
Q = 1 + TA^2 i^2 g_{zz},
\]

\[
D_a TA = \partial_a TA - i(A_{2,a} - A_{1,a})TA, V(TA, l) = g_s V(TA) \sqrt{Q},
\]

\[
e^\phi = g_s \left( 1 + \frac{R^4}{z^4} \right)^{-\frac{1}{2}}.
\]

Here \(\phi\) is the dilaton field, \(A_{2,a}\) is the gauge field, and \(F_{ab}^i\) is the field strengths on the world-volume of the non-BPS brane. Furthermore, \(TA\) is the tachyon field, \(\tau_3\) is the brane tension and \(V(TA)\) is the tachyon potential. The indices \(a, b\) denote the tangent directions of D-branes, while the indices \(M, N\) run over the background ten-dimensional space-time directions. The Dp-brane and the anti-Dp-brane are labeled by \(i = 1, 2\), respectively. Then the separation between these D-branes is denoted by \(z_2 - z_1 = l\). In writing the above expressions, we have used the convention \(2\pi \alpha^\prime = 1\). Now we can write a commonly used potential as \[37\]–\[38\],

\[
V(TA) = \frac{\tau_3}{\cosh \sqrt{\pi T}A}.
\]

Let us consider the only \(\sigma\) dependence of the tachyon field \(TA\), and for simplicity and set the gauge fields to zero. In this case, the action \[39\] in the region that \(r > R\) and \(TA' \sim constant\) simplifies to

\[
L \simeq -\frac{\tau_3}{g_s} \int d\sigma \sigma^2 V(TA)(\sqrt{D_{1,TA}} + \sqrt{D_{2,TA}}) \left( 1 - \frac{\pi^2 NT^4}{6T_{D3}} \right),
\]

where

\[
D_{1,TA} = D_{2,TA} \equiv D_{TA} = 1 + \frac{l'(\sigma)^2}{4} + TA^2 - TA'^2.
\]

We have assume that \(TA' \ll TA\). Now, we can study the Hamiltonian corresponding to above Lagrangian. To derive this we need the canonical momentum density \(\Pi = \frac{\partial L}{\partial TA}\) associated with the tachyon,

\[
\Pi = \frac{V(TA)TA}{\sqrt{1 + \frac{l'(\sigma)^2}{4} + TA^2 - TA'^2}} \left( 1 - \frac{\pi^2 NT^4}{6T_{D3}} \right).
\]

So, the Hamiltonian can be written as

\[
H_{DBI} = 4\pi \int d\sigma \sigma^2 \Pi TA - L..\]
By choosing $\dot{T}A = 2TA'$, we obtain

$$H_{DBI} = 4\pi \int d\sigma \sigma^2 [\Pi(TA - 1/2TA') + \frac{1}{2} TA \partial_\sigma (\Pi \sigma^2)] - L.$$  (34)

In this equation, we have in the second step integrated by parts the term proportional to $\dot{T}A$, indicating that tachyon can be studied as a Lagrange multiplier imposing the constraint $\partial_\sigma (\Pi \sigma^2 V(TA)) = 0$ on the canonical momentum. Solving this equation, we obtain

$$\Pi = \frac{\beta}{4\pi \sigma^2},$$  (35)

where $\beta$ is a constant. Now substituting (34) in (30), we obtain

$$H_{DBI} = \int d\sigma V(TA) \left( \sqrt{1 + \frac{l'(\sigma)^2}{4} + TA^2} - TA^2 \right) F_{DBI},$$

$$F_{DBI} = \sigma^2 \sqrt{1 + \frac{\beta}{\sigma^2} \left( 1 - \frac{\pi^2 NT^4}{6T_D^3} \right)}.$$  (36)

The resulting equation of motion for $l(\sigma)$, calculating by varying (36), is

$$\left( \frac{l' F_{DBI}}{4 \sqrt{1 + l'(\sigma)^2}} \right) = 0.$$  (37)

Solving this equation, we obtain

$$l(\sigma) = 2 \left( \frac{l_0}{2} - \int_\sigma^\infty d\sigma \left( \frac{F_{DBI}(\sigma)}{F_{DBI}(\sigma_0)} - 1 \right)^{-\frac{1}{2}} \right).$$  (38)

This solution, for non-zero $\sigma_0$ represents a wormhole with a finite size throat. This equation indicates that the separation distance between the D3-brane and anti-D3-brane is $l_0$, at the birth of wormhole ($\sigma_0$), decreased with time and shrink to zero at larger values of throat. On the other hand, to obtain the explicit form of tachyon, we use of the equations of motion obtained from action (36)

$$\left( \frac{1}{\sqrt{D_{TA}}} TA'(\sigma) \right)' = 1 \sqrt{D_{TA}} \left( \frac{[V(TA)F_{DBI}]}{F_{DBI}(\sigma) F_{DBI}(\sigma_0)} (D_{TA} - TA'(\sigma)^2) \right).$$  (39)

Solving this equation, we obtain

$$TA \sim \sqrt{\frac{\sigma_0^2}{\sigma_0^2 - \sigma^2} \left( \frac{1}{1 + \frac{\pi^2 NT^4}{6T_D^3}} \right)}.$$  (40)

This equation shows that tachyon is zero before the birth of wormhole ($\sigma_0 = 0$), and with decreasing temperature and increasing the throat of wormhole grows and achieves to large values.

Now using the action (36), we can obtain entropy and mass density along z-direction in tachyonic BIon,

$$S_{tb} = \frac{4T_D^3}{\pi T^4} \int d\sigma V(TA(\sigma)) \frac{F_{DBI}(\sigma)}{\sqrt{F_{DBI}(\sigma) - F_{DBI}(\sigma_0)}} \sigma^3 \frac{\sigma_0}{\cosh^4 \alpha (\sigma_0^2 - \sigma^2)^{1/2}}$$  (41)

$$\frac{dM_{tb}}{dz} = \frac{2T_D^3}{\pi T^4} V(TA(\sigma)) \frac{F_{DBI}(\sigma)}{F_{DBI}(\sigma_0)^3} \sigma^3 \frac{4 \cosh^2 \alpha + 1}{\cosh^4 \alpha} \frac{\sigma_0}{(\sigma_0^2 - \sigma^2)^{3/2}}.$$  (42)

We can consider the effect of tachyonic potential on the number of degrees of freedom on the holographic surface and bulk. This can be done by repeating the analysis of the previous section. Thus, we write again relate these
degrees of freedom to the entropy of BIon. We also write an expression for the the mass density along the transverse direction,

\[
N_{\text{sur}} + N_{\text{bulk}} = N_{\text{BIon}} = N_{\text{brane}} + N_{\text{anti-brane}} + N_{\text{wormhole}}
\]

\[
\simeq \frac{4T_{D3}^2}{\pi T^4} \int d\sigma V(TA(\sigma)) \frac{F_{DBI}(\sigma)}{\sqrt{F_{DBI}^2(\sigma) - F_{DBI}^2(\sigma_0)}} \times \sigma^3 \frac{4}{\cosh^4 \alpha (\sigma^2 - \sigma_0^2)^{3/2}}
\]

\[
N_{\text{sur}} - N_{\text{bulk}} \simeq \int d\sigma \frac{dM_{\text{BIon}}}{dz}
= \frac{2T_{D3}^2}{\pi T^4} \int d\sigma V(TA(\sigma)) \frac{F_{DBI}(\sigma)}{F_{DBI}(\sigma_0)} \frac{\sigma^3 4 \cosh^2 \alpha + 1 \cosh^4 \alpha}{(\sigma^2 - \sigma_0^2)^{3/2}}.
\]

Solving these equations simultaneously, we obtain,

\[
N_{\text{sur}} \simeq \frac{4T_{D3}^2}{\pi T^4} \int d\sigma V(TA(\sigma)) \frac{F_{DBI}(\sigma)}{\sqrt{F_{DBI}^2(\sigma) - F_{DBI}^2(\sigma_0)}} \times \sigma^3 \frac{4}{\cosh^4 \alpha (\sigma^2 - \sigma_0^2)^{3/2}}
\]

\[
+ \frac{2T_{D3}^2}{\pi T^4} \int d\sigma V(TA(\sigma)) \frac{F_{DBI}(\sigma)}{F_{DBI}(\sigma_0)} \times \sigma^3 \frac{4 \cosh^2 \alpha + 1 \cosh^4 \alpha}{(\sigma^2 - \sigma_0^2)^{3/2}},
\]

\[
N_{\text{bulk}} \simeq \frac{4T_{D3}^2}{\pi T^4} \int d\sigma V(TA(\sigma)) \frac{F_{DBI}(\sigma)}{\sqrt{F_{DBI}^2(\sigma) - F_{DBI}^2(\sigma_0)}} \times \sigma^3 \frac{4}{\cosh^4 \alpha (\sigma^2 - \sigma_0^2)^{3/2}}
\]

\[
- \frac{2T_{D3}^2}{\pi T^4} \int d\sigma V(TA(\sigma)) \frac{F_{DBI}(\sigma)}{F_{DBI}(\sigma_0)} \times \sigma^3 \frac{4 \cosh^2 \alpha + 1 \cosh^4 \alpha}{(\sigma^2 - \sigma_0^2)^{3/2}}.
\]

This equation shows that with approaching two branes each other, tachyonic potential increases and tends to infinity and consequently, the number of degrees of freedom becomes large as one trends to the Big Rip singularity. Using this equation and \([17]\), we can estimate the Hubble parameter for flat universe,

\[
H_{\text{flat,ac}} \simeq \left( \frac{1}{V(TA)} \right)^{1/2} \left( \frac{72\pi k^8 N_{14} T_{16}^{14} F_1 T_{14}^{14} + 8\pi k^4 N_{12} T_{14}^{14} F_1 T_{12}^{12}}{T_{D3}^{14} \sigma_0^{16}} \right).
\]

Thus, the Hubble parameter depends on tachyonic potential between D3-brane and anti-D3-brane. As the tachyonic potential increases, Hubble parameter reduces to very small values.

Finally, using the Friedmann equation for the flat Friedmann-Robertson-Walker Universe, we can calculate the energy density in late time acceleration era,

\[
\rho_{\text{flat,ac}} = \frac{3}{8\pi l_P^2} H^2
\]

\[
\simeq \frac{3}{8\pi l_P^2} \left( \frac{1}{V(TA)} \right) \left( \frac{4900\pi k^{16} N_{24} T_{28}^{28} F_1 T_{14}^{28} + 16\pi k^8 N_{24} T_{28}^{28} F_1 T_{12}^{24}}{T_{D3}^{32} \sigma_0^{20}} \right)
\]

\[
+ \left( \frac{496\pi k^{12} N_{26} T_{28}^{30} F_1 T_{14}^{26}}{T_{D3}^{16} \sigma_0^{16}} \right).
\]

(46)
Thus, the energy density decreases with increasing tachyon and shrink to zero at $TA = \infty$. This is because that D3-brane moves towards the anti-D3-brane, and this creates tachyon states, which in turn increase the radius of universe. This leads to acceleration and decrease in energy density.

Now, we can consider properties of open and closed universes in Bronic system. Doing similar calculations to previous section and using (17), and $r_A = 1/\sqrt{H^2 + k/a^2}$, we can calculate Hubble parameter for non-flat universe during late-time acceleration era,

$$H_{o/c,ac} \simeq \sqrt{\frac{1}{V(TA)}} \times \sqrt{\left(\frac{72\pi^8 N^{14} T^{16}_{F_1} T^{14}}{T^{16}_{D3} \sigma^{10}_0} + \frac{8\pi k^4 N^{12} T^{14}_{F_1} T^{12}}{T^{14}_{D3} \sigma^{6}_0} \right)^2 - \frac{k}{a^2}}. \quad (47)$$

Defining $T_t$ as

$$T_t = \left(\frac{72\pi^8 N^{14} T^{16}_{F_1} T^{14}}{T^{16}_{D3} \sigma^{10}_0} + \frac{8\pi k^4 N^{12} T^{14}_{F_1} T^{12}}{T^{14}_{D3} \sigma^{6}_0} \right)^2, \quad (48)$$

and solving this equation, we can obtain the scale factor for open universe,

$$a_{o,ac}(t) \simeq \exp \left(-\int dt[T_t + \ln(t)]\right), \quad (49)$$

and the scale factor for closed universe,

$$a_{c,ac}(t) \simeq \exp \left(-i\int dt[T_t + \ln(t) + \frac{\pi}{2}]\right). \quad (50)$$

These scale factors depend on the tachyonic potential in addition to temperature. Thus, by increasing tachyons, the scale factor of open universe grows and tends to infinity at $TA = \infty$, however, the scale factor of closed universe oscillates.

The energy density of the open and closed universes can be written as

$$\rho_{o/c,ac} = \frac{3}{8\pi l_p^2} \left(H_{o/c,ac}^2 + \frac{k}{a^2}\right) \simeq \frac{3}{8\pi l_p^2} H_{flat,ac}^2 \simeq \frac{3}{8\pi l_p^2} \left(\frac{1}{V(TA)}\right) \left(\frac{72\pi^8 N^{14} T^{16}_{F_1} T^{14}}{T^{16}_{D3} \sigma^{10}_0} + \frac{8\pi k^4 N^{12} T^{14}_{F_1} T^{12}}{T^{14}_{D3} \sigma^{6}_0}\right)^2 = \rho_{flat,ac}. \quad (51)$$

Similar to inflationary era, the origin of energy density in all three types of the universe is the same and corresponds to energy flowing from BIon into the universe. Also, tachyonic potential plays the main role the late time acceleration as the universe approaches the Big Rip singularity.

IV. CONCLUSION

In this paper, we used a configuration of a D3-brane and an anti-D3-brane connected by a wormhole, for analysing the holographic inflationary cosmology. This configuration was called a BIon. It was obtained by a transition of fundamental black strings. The existence of the wormhole made it possible for the energy to flow into the D3-brane. This caused the degrees of freedom of the D3-brane to increase, which in turn lead to inflation. The inflation ended when the wormhole lost all its energy and got annihilated. However, as the distance between the D3-brane and the anti-D3-brane reduced, tachyonic states were created. Thus, a new wormhole formed due to the presence of these tachyonic states. This again increasing the degrees of freedom on the D3-brane, and caused late time acceleration.

It may be noted that the finite temperature effects for non-extremal self-dual string solution solutions and wormhole solutions interpolating between stacks of M5-branes and anti-M5-branes have also been studied. These solutions define a BIon solution in M-theory \[37, 38\]. This analysis has been done using using the blackfold approach, and self-dual
string solitons solution have been analysed as a three-funnel solution of an effective five-brane world-volume using this approach. It would be interesting to study holographic cosmology using this BIon system. It may be noted that the corrections to Friedmann equations coming from the generalized uncertainty principle has been studied \[39, 40\]. As the extended structure of strings is expected to lead to the existence of a minimum length \[41\], and the existence of a minimum length leads to the generalized uncertainty principle \[42\], it is expected that such corrections will also effect the holographic BIonic cosmology.

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