Fault estimation for nonlinear systems with sensor gain degradation and stochastic protocol based on strong tracking filtering

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ABSTRACT
This paper focuses on the fault estimation problem for a class of nonlinear systems with sensor gain degradation and stochastic protocol (SP) based on strong tracking filtering. The phenomenon of the sensor gain degradation is described by sequences of stochastic variables in a known interval. The stochastic protocol (SP) is used to deal with possible data conflicts in multi-signal transmission. The augmented system is constructed by combining the original system state vectors and the related faults into an augmented state vectors. The strong tracking filter (STF) is designed by introducing a fading factor into the filter structure to solve the problem of burst faults. Finally, a simulation example is given to verify the effectiveness and applicability of the proposed filter.

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Nonlinear systems; strong tracking filter; fault estimation; sensor gain degradation; stochastic protocol

1. Introduction
Over the past decades, fault diagnosis has been recognized as a basic issue to improve the reliability and security of modern networked systems, and it has attracted extensive research attention (He et al., 2013; Qin et al., 2016; Shahnazari & Mhaskar, 2018). As an important part of fault diagnosis, fault estimation plays an important role in achieving the final index of system reliability, and it has been reported in a large number of literatures (Dong et al., 2016; Qiu et al., 2018; D. Zhang & Liu, 2019). For example, fault estimation was studied in D. Zhang and Liu (2019) for a class of complex networks with stochastic communication protocols and model uncertainties. In Qiu et al. (2018), the self-triggering fault estimation and fault-tolerant control were studied for networked control systems. In Dong et al. (2016), the problem of $H_{\infty}$ estimation was studied for stochastic faults of nonlinear time-varying systems with fading channels. In Hu et al. (2018), the fault was set as an augmented vector of system states and the joint estimation of states and fault was investigated for time-varying nonlinear systems.

In recent years, the application of nonlinear system filtering in the field of fault estimation has aroused extensive research interest. The scholars have proposed various filtering algorithms such as extended Kalman filtering (EKF) (Hu et al., 2012), unscented Kalman filtering (UKF) (Li et al., 2017), particle filtering (PF) (Xu et al., 2018) and $H_{\infty}$ filtering (Chao et al., 2018). In the fault estimation of various nonlinear systems, EKF has been more widely used. Unfortunately, the robustness of EKF to model uncertainties is very poor. When the system reaches stationary state, EKF will lose the ability to track the abrupt state. Therefore, the strong tracking filter (STF) has attracted researchers’ attention due to its robustness and tracking ability (He et al., 2014; Wei et al., 2019; Xie et al., 1999).

Recently, networked control system has gradually become a research hotspot in control field because of the widespread application of the network in various practical control systems. However, some unfavourable network-induced phenomena (such as random delay Zeng & Sheng, 2018, signal quantization Ding et al., 2017, packet dropouts Sheng et al., 2017, and channel fading W. Liu & Shi, 2019) would degrade the performance of networked control systems. In many existing filters for networked control systems, it is assumed that all sensor nodes are simultaneously connected to the communication network to transmit data. However, due to the limited bandwidth of network, data losses or collisions may occur when multiple nodes try to use the communication channel at the same time. Therefore, communication protocols are applied in networked control systems to prevent data collisions by arranging the transmission order of network nodes, and they include but are not limited to Try-Once-Discard protocol (TOD) (Walsh et al., 2006; J. Zhang & Peng, 2019), Round-Robin protocol (RRP) (Gao et al., 2019; Ugrinovskii & Fridman, 2014) and stochastic protocol (SP) (Sheng et al., 2019; H. Zhang et al., 2019; Zou...
et al., 2018). Although the problems of control and filtering have received much attention for networked control system with SP, the problem of fault estimation for system with SP has not been fully investigated, and this gives rise to the primary motivation of this paper.

In practical engineering, sensors often fail owing to various reasons. In addition to the common sensor failure phenomenon (Peng et al., 2018; B. Zhang et al., 2020), sensor gain degradation (He et al., 2008; Y. Liu et al., 2014, 2016) also occurs. In the networked system, the sensor gain degradation is mainly caused by the long-term ageing of the sensor and the interference of the external environment. The problem of the optimal filtering was investigated in Y. Liu et al. (2014) for networked systems with stochastic sensor gain degradation. By using the linear matrix inequality method, a new state estimation method was presented in He et al. (2008) for time-delay systems with probabilistic sensor gain degradation. The minimum variance filtering problem was discussed in Y. Liu et al. (2016) for sensor network systems with sensor gain degradation. However, the problem of fault estimation for networked nonlinear systems with sensor gain degradation has not been appropriately studied.

Based on the above analysis, the purpose of this paper is to study the strong tracking filtering based fault estimation for networked nonlinear system with sensor gain degradation and stochastic protocol. The main contributions of this paper are summarized as follows: (1) The problem of fault estimation for networked nonlinear system under the stochastic protocol scheduling is studied; (2) Compared with the extended Kalman filter, a new strong tracking filter is designed in this paper, which has good tracking performance for burst fault.

Notations. \( \mathbb{R}^n \) denotes the \( n \)-dimensional Euclidean space, and \( \mathbb{R}^{m \times n} \) stands for the set of \( m \times n \) real matrices. \( A^T \) stands for the transpose of matrix \( A \). \( \mathbb{E}\{\cdot\} \) depicts the expectation of the stochastic variable. \( \text{Prob}\{\cdot\} \) represents the occurrence probability of the event. \( \text{diag}\{\cdot \cdot \cdot\} \) denotes the diagonal matrix. \( \text{tr}\{\cdot\} \) represents the trace of matrix. \( I \) is the unit matrix with appropriate dimensions. \( \theta(a) \) is the Kronecker function defined as \( \theta(a) = 1 \) if \( a = 0 \) and equals 0 otherwise. For a square matrix \( A \in \mathbb{R}^{n \times n} \), \( \text{sym}(A) \) denotes \( A + A^T \) for simplicity.

2. Problem formulation and preliminaries

2.1. System model

Consider the following nonlinear system with sensor gain degradation:

\[
\begin{align*}
    x_{k+1} & = g(x_k) + F_k f_k + B_k w_k, \\
    y_k & = \Omega_k C_k x_k + \tilde{D}_k v_k,
\end{align*}
\]

where \( x_k \in \mathbb{R}^{n_x} \) is the system state, \( y_k \in \mathbb{R}^{n_y} \) is the measurement output, and \( f_k \in \mathbb{R}^{n_f} \) represents the additive faults. \( w_k \in \mathbb{R}^{n_w} \) and \( v_k \in \mathbb{R}^{n_v} \) are, respectively, the process and measurement noises, which are uncorrelated and obey Gaussian distribution with zero mean. \( f_k, B_k, C_k \) and \( \tilde{D}_k \) are known matrices with compatible dimensions. \( g : \mathbb{R}^{n_x} \rightarrow \mathbb{R}^{n_y} \) is a smooth nonlinear function and its partial derivative exists. The sensor gain degradation phenomenon is described by \( \Omega_k = \text{diag}\{\mu_{1,k}, \mu_{2,k}, \ldots, \mu_{n_y,k}\} \), where the random variable \( \mu_{i,k} \) is uniformly distributed in the interval \( [a_i, b_i] (0 < a_i < b_i < 1) \). Specifically, they are independent with each other and satisfy \( \mathbb{E}\{\mu_{i,k}\} = \beta_{i,k}, \text{Var}\{\mu_{i,k}\} = \eta_{i,k} \), where \( \beta_{i,k} \) and \( \eta_{i,k} \) are exactly known scalars.

In this paper, we need the following assumptions.

Assumption 2.1: The second-order difference of additive fault is assumed to be zero piecewise, i.e.

\[
f_{k+1} = f_k + \Delta f_k,
\]

where \( \Delta f_k \) is a constant amplitude.

Assumption 2.2: The initial state \( x_0 \), the sensor gain degradation is \( \mu_{i,k} \), the noises \( w_k \) and \( v_k \) are mutually uncorrelated and have

\[
\mathbb{E}\{x_0\} = \bar{x}_0, \quad \mathbb{E}\{w_k w_k^T\} = Q_k, \quad \mathbb{E}\{v_k v_k^T\} = R_k,
\]

where \( Q_k > 0 \) and \( R_k > 0 \) are known matrices.

Remark 2.1: In many studies on fault estimation, it is assumed that the \( \text{ith} \)-order derivative of the fault is zero (Gao et al., 2019; D. Zhang & Liu, 2019), and this type of faults include some common faults such as step faults and ramp faults. Moreover, the order of fault can be determined by some prior knowledge of the system in practical applications.

Denoting \( \bar{x}_k = [x_k^T, f_k^T, \Delta f_k^T]^T \), and system (1) can be written as:

\[
\begin{align*}
    \bar{x}_{k+1} = \bar{g}(\bar{x}_k) + \bar{B}_k w_k, \\
    y_k & = \Omega_k \bar{C}_k \bar{x}_k + \tilde{D}_k v_k
\end{align*}
\]

where

\[
\bar{g}(\bar{x}_k) = \begin{bmatrix} g(x_k) + F_k f_k \\ f_k + \Delta f_k \\ \Delta f_k \end{bmatrix}, \quad \bar{B}_k = \begin{bmatrix} B_k \\ 0 \\ 0 \end{bmatrix}, \quad \bar{C}_k = \begin{bmatrix} C_k & 0 & 0 \end{bmatrix}.
\]

2.2. Stochastic protocol

According to the spatial distribution, suppose that the sensors of system (4) are supposed to \( n_y \) sensor
nodes. Then, the output $y_k$ can be rewritten as $y_k = [y^T_{1,k}, y^T_{2,k}, \ldots, y^T_{n,k}] \in \mathbb{R}^{n_y}$ where $y^T_{i,k}, i \in \{1, 2, \ldots, n_y\}$ is the measurement of the $i$th sensor node before it is transmitted. In order to overcome the data conflicts in a constrained network, the data transmission is scheduled by using the stochastic protocol in this paper. Let $\tau_k \in \{1, 2, \ldots, n_y\}$ be the selected sensor node obtaining permission to send message through network at time $k$. Generally, $(\tau_k)_k \geq 0$ can be viewed as a sequence of stochastic variables, and assume it is independent of other noise signals. The occurrence probability of $\tau_k = i, i \in \{1, 2, \ldots, n_y\}$ is

$$\text{Prob}(\tau_k = i) = p_i,$$  \hspace{1cm} (5)

where $p_i > 0$ denotes the occurrence probability of selecting node $i$ to send data via the communication channel, and $\sum_{i=1}^{n_y} p_i = 1$. The measurement output received by the filter is denoted by

$$\bar{y}_k = [\bar{y}^T_{1,k}, \bar{y}^T_{2,k}, \ldots, \bar{y}^T_{n_y,k}] \in \mathbb{R}^{n_y},$$

$$k > 0, i \in \{1, 2, \ldots, n_y\},$$ \hspace{1cm} (6)

where the updating rule of $\bar{y}_{i,k}$ due to the stochastic protocol scheduling is

$$\bar{y}_{i,k} = \begin{cases} y_{i,k}, & \text{if } i = \tau_k, \quad k \geq 0, \\ 0, & \text{otherwise}. \end{cases}$$ \hspace{1cm} (7)

Therefore, we have

$$\bar{y}_k = \begin{cases} \Upsilon_{\tau_k} y_k, & \text{if } k \geq 0, \\ 0, & \text{otherwise}, \end{cases}$$ \hspace{1cm} (8)

where $\Upsilon_{\tau_k} = \text{diag}[\theta(\tau_k - 1), \theta(\tau_k - 2), \ldots, \theta(\tau_k - n_y)]$ ($1 \leq i \leq n_y$), and $\theta(i) \in \{0, 1\}$ is the Kronecker delta function. According to the definition of $\Upsilon_{\tau_k}$, one has

$$\Upsilon_{\tau_k} = \sum_{i=1}^{n_y} \theta(\tau_k - i) \Upsilon_i,$$ \hspace{1cm} (9)

where $\Upsilon_i = \text{diag}[\theta(i - 1), \ldots, \theta(i - n_y)]$. Moreover, we have $\mathbb{E}(\theta(\tau_k - i)) = \sum_{i=1}^{n_y} p_i \theta(i - i) = p_i$ and

$$\theta(\tau_k - i) \theta(\tau_k - j) = \begin{cases} 0, & \text{if } i \neq j, \\ \theta(\tau_k - i), & \text{if } i = j. \end{cases}$$ \hspace{1cm} (10)

3. Main results

In this section, the recursive filter will be first designed for the nonlinear systems with sensor gain degradation and stochastic protocol in the least mean square sense. Then, the strong tracking filter with sensor gain degradation and stochastic protocol will be designed by introducing a fading factor in the prediction error covariance. This filter can estimate faults more quickly.

**Lemma 3.1:** Let $Q = [q_{ij}]_{n \times n}$ be a real-valued matrix and $S = \text{diag}(s_1, s_2, \ldots, s_n)$ be a diagonal matrix with random variables. Then

$$\mathbb{E}\{S Q S^T\} = \begin{bmatrix} \mathbb{E}\{s_1^2\} & \mathbb{E}\{s_1 s_2\} & \cdots & \mathbb{E}\{s_1 s_n\} \\ \mathbb{E}\{s_2 s_1\} & \mathbb{E}\{s_2^2\} & \cdots & \mathbb{E}\{s_2 s_n\} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{E}\{s_n s_1\} & \mathbb{E}\{s_n s_2\} & \cdots & \mathbb{E}\{s_n^2\} \end{bmatrix} \circ Q,$$

where $\circ$ is the Hadamard product.

3.1. Design of the recursive filter

For model (4), the one-step prediction of $\hat{x}_k$ at time $k$ is

$$\hat{x}_{k+1|k} = \tilde{g}(\hat{x}_{k|k}),$$ \hspace{1cm} (11)

and the estimate of $\hat{x}_k$ is

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} y_{k+1},$$ \hspace{1cm} (12)

where

$$y_{k+1} = \bar{y}_{k+1} - \Upsilon_{\tau_{k+1}} \bar{y}_{k+1} + \tilde{g}(\hat{x}_{k+1|k}),$$ \hspace{1cm} (13)

is the residual signal, the $K_{k+1}$ is the filter gain to be designed and $\tilde{g}(\hat{x}_{k+1|k})$ is the nonlinear function $\tilde{g}(\hat{x}_k)$. By using the Taylor series expansion around $\hat{x}_{k|k}$, the non-linear function $\tilde{g}(\hat{x}_k)$ can be linearized as

$$\tilde{g}(\hat{x}_k) = \tilde{g}(\hat{x}_{k|k}) + \tilde{A}_k \hat{x}_{k|k},$$ \hspace{1cm} (15)

where

$$\tilde{A}_k = \left[ \frac{\partial \tilde{g}(\hat{x}_k)}{\partial \hat{x}_k} \right]_{\hat{x}_k = \hat{x}_{k|k}} = \begin{bmatrix} 0 \\ \hat{x}_{k+1|k} - \hat{\theta}_{k+1|k} F_k \\ \hat{x}_{k+1|k} - \hat{\theta}_{k+1|k} \end{bmatrix},$$

$$H = \begin{bmatrix} 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$  

Following from (14)–(15), $\hat{x}_{k+1|k}$ can be transformed into

$$\hat{x}_{k+1|k} = \hat{A}_k \hat{x}_{k|k} + \tilde{B}_k w_k.$$ \hspace{1cm} (16)

According to (4) and (12), $\hat{x}_{k+1|k+1}$ can be written as

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1} - \hat{x}_{k+1|k+1}$$

$$= \hat{x}_{k+1|k} - K_{k+1} y_{k+1}.$$
From (17), we have

\[
\begin{align*}
P_{k+1} &= (I - K_{k+1} \Gamma_{k+1} \Omega_{k+1} \Gamma_{k+1}) \tilde{x}_{k+1|k} \\
&\quad - K_{k+1} \Gamma_{k+1} (\Omega_{k+1} - \tilde{\Omega}_{k+1}) \tilde{c}_{k+1} \tilde{x}_{k+1|k} \\
&\quad - K_{k+1} \Gamma_{k+1} \tilde{d}_{k+1} \tilde{v}_{k+1}.
\end{align*}
\]

(17)

**Theorem 3.1:** Considering (16) and (17), the one-step prediction covariance \(P_{k+1|k}\) is \(\mathbb{E}[\tilde{x}_{k+1|k} \tilde{x}_{k+1|k}^T]\) and filtering error covariance \(P_{k+1|k+1} = \mathbb{E}[\tilde{x}_{k+1|k+1} \tilde{x}_{k+1|k+1}^T]\) are, respectively, given by the following two equations:

\[
P_{k+1|k} = \tilde{A}_k P_{k|k} \tilde{A}_k^T + \tilde{B}_k Q_k \tilde{B}_k^T,
\]

and

\[
P_{k+1|k+1} = P_{k+1|k} + \sum_{i=1}^{n_y} \tilde{p}_i K_{k+1} \Theta_i (\tilde{\Omega}_{k+1})
\]

\[
\times \tilde{C}_k P_{k+1|k+1}^T + \sum_{i=1}^{n_y} \tilde{p}_i K_{k+1} \Theta_i (\tilde{\Omega}_{k+1})
\]

\[
\times \tilde{C}_k P_{k+1|k+1}^T - \text{sym} \left\{ \sum_{i=1}^{n_y} \tilde{p}_i K_{k+1} \Theta_i (\tilde{\Omega}_{k+1}) \right\}
\]

\[
+ \sum_{i=1}^{n_y} \tilde{p}_i K_{k+1} \Theta_i (\tilde{\Omega}_{k+1})
\]

\[
\times \tilde{C}_k P_{k+1|k+1}^T + \sum_{i=1}^{n_y} \tilde{p}_i K_{k+1} \Theta_i (\tilde{\Omega}_{k+1})
\]

\[
\times \tilde{C}_k P_{k+1|k+1}^T - \text{sym} \left\{ \sum_{i=1}^{n_y} \tilde{p}_i K_{k+1} \Theta_i (\tilde{\Omega}_{k+1}) \right\}
\]

\[
(\tilde{C}_k + \tilde{C}_k P_{k+1|k+1}^T) \Theta_i (\tilde{\Omega}_{k+1})
\]

\[
\gamma_i K_{k+1}^T,
\]

where

\[
\tilde{\Omega}_{k+1} = \text{diag}[\eta_{1,k+1}, \eta_{2,k+1}, \ldots, \eta_{n_y,k+1}].
\]

**Proof:** According to (16), it is easy to obtain that

\[
P_{k+1|k} = \mathbb{E}[\tilde{x}_{k+1|k} \tilde{x}_{k+1|k}^T]
\]

\[
= \tilde{A}_k P_{k|k} \tilde{A}_k^T + \tilde{B}_k Q_k \tilde{B}_k^T.
\]

(20)

From (17), we have

\[
P_{k+1|k+1} = \mathbb{E}[\tilde{x}_{k+1|k+1} \tilde{x}_{k+1|k+1}^T]
\]

\[
= \mathbb{E}\left\{ (I - K_{k+1} \Gamma_{k+1} \Omega_{k+1} \Gamma_{k+1}) \tilde{x}_{k+1|k} \tilde{x}_{k+1|k}^T \right\}
\]

\[
\times (I - K_{k+1} \Gamma_{k+1} \Omega_{k+1} \Gamma_{k+1}) \tilde{x}_{k+1|k} \tilde{x}_{k+1|k}^T
\]

\[
+ \mathbb{E}\left\{ K_{k+1} \Gamma_{k+1} (\Omega_{k+1} - \tilde{\Omega}_{k+1}) \tilde{c}_{k+1} \tilde{x}_{k+1|k} \right\}
\]

\[
\times K_{k+1} \Gamma_{k+1} (\Omega_{k+1} - \tilde{\Omega}_{k+1}) \tilde{c}_{k+1} \tilde{x}_{k+1|k}^T
\]

\[
+ \mathbb{E}\left\{ K_{k+1} \Gamma_{k+1} \tilde{d}_{k+1} \tilde{v}_{k+1} \right\}
\]

\[
\times \tilde{B}_k \left\{ K_{k+1} \Gamma_{k+1} \tilde{d}_{k+1} \tilde{v}_{k+1} \right\}^T\gamma_i K_{k+1}^T.
\]

\[
\gamma_i K_{k+1}^T
\]

(22)

Similarly, the second term of (21) can be written as

\[
\mathbb{E}\{K_{k+1} \Gamma_{k+1} (\Omega_{k+1} - \tilde{\Omega}_{k+1}) \tilde{c}_{k+1} \tilde{x}_{k+1|k}\}
\]

\[
\times [K_{k+1} \Gamma_{k+1} (\Omega_{k+1} - \tilde{\Omega}_{k+1}) \tilde{c}_{k+1} \tilde{x}_{k+1|k}^T]
\]

\[
= \mathbb{E}\left\{ \sum_{i=1}^{n_y} \tilde{p}_i K_{k+1} \Theta_i (\tilde{\Omega}_{k+1}) \gamma_i K_{k+1}^T \right\}
\]

(23)

\[
\gamma_i K_{k+1}^T
\]

\[
\gamma_i K_{k+1}^T
\]
\[ (C_{k+1} \hat{x}_{k+1})^T \gamma_{k+1} \]

and the third term of (21) can be written as

\[
E\{K_{k+1} = \gamma_{k+1} \bar{D}_{k+1} + K_{k+1} \}
\]

\[\begin{align*}
&= E\left\{ \sum_{i=1}^{n_y} \theta(t_{k+1} - i) \gamma_i \bar{D}_{k+1} + K_{k+1} \right\} \\
&= \sum_{i=1}^{n_y} \theta(t_{k+1} - i) \gamma_i K_{k+1} \\
&= \sum_{i=1}^{n_y} p_i \gamma_i \bar{D}_{k+1} + K_{k+1}.
\end{align*}\]

(24)

It can be proved that \( \bar{x}_{k+1}, \tilde{x}_{k+1} \) and \( \psi_{k+1} \) are all equal to zero. Therefore, from (21)–(24), \( P_{k+1|k+1|} \) can be calculated by (19). The proof is complete.

Theorem 3.2: The filter gain \( K_{k+1} = E\{X_{k+1}^{T}|Y_{k+1}^{T}\} \}

is given by the following equation:

\[
K_{k+1} = P_{k+1|k} \tilde{C}_{k+1} \tilde{\Omega}_{k+1} \tilde{\gamma} \end{align*}\]

(25)

where

\[
\begin{align*}
\tilde{A}_{k+1} &= \sum_{i=1}^{n_y} p_i \gamma_i (\tilde{C}_{k+1} + P_{k+1|k} \tilde{C}_{k+1}^{T}) \\
&+ \tilde{D}_{k+1} + R_{k+1} \tilde{D}_{k+1}^{T} \gamma_i.
\end{align*}\]

Proof: From formula (13), we have

\[
E\{X_{k+1}|Y_{k+1}^{T}\}
\]

\[\begin{align*}
&= E\left\{ X_{k+1} = \tilde{C}_{k+1} \tilde{\Omega}_{k+1} \tilde{\gamma} \right\} \\
&= \sum_{i=1}^{n_y} p_i \gamma_i (\tilde{C}_{k+1} + P_{k+1|k} \tilde{C}_{k+1}^{T}) \\
&+ \tilde{D}_{k+1} + R_{k+1} \tilde{D}_{k+1}^{T} \gamma_i.
\end{align*}\]

(26)

Defining \( \Lambda_{k+1} = E\{Y_{k+1}|Y_{k+1}^{T}\} \)

we can get

\[
\begin{align*}
\Lambda_{k+1} &= E\{Y_{k+1}|Y_{k+1}^{T}\} \\
&= E\left\{ Y_{k+1} = \tilde{C}_{k+1} \tilde{\Omega}_{k+1} \tilde{\gamma} \right\} \\
&= \sum_{i=1}^{n_y} p_i \gamma_i (\tilde{C}_{k+1} + P_{k+1|k} \tilde{C}_{k+1}^{T}) \\
&+ \tilde{D}_{k+1} + R_{k+1} \tilde{D}_{k+1}^{T} \gamma_i.
\end{align*}\]

(27)

So the filter gain \( K_{k+1} \) can be recursively calculated as follows:

\[
K_{k+1} = E\{X_{k+1}|Y_{k+1}^{T}\} \end{align*}\]

This completes the proof.

3.2. Design of the strong tracking filter

In this section, the STF will be designed to deal with the burst faults of systems. In order to weaken the influence of old data on the current filtering value, a fading factor \( \lambda_{k+1} \geq 1 \) is introduced into the prediction error covariance \( P_{k+1|k} \):

\[
P_{k+1|k} = \lambda_{k+1} A_{k} P_{k|k} A_{k}^{T} + B_{k} Q_{k} B_{k}^{T}.
\]

(28)

The fading factor can be determined by solving the following problem:

\[
\begin{align*}
\text{Minimize} & \quad E\{(\hat{x}_{k+1} - \tilde{x}_{k+1|k+1})(\hat{x}_{k+1} - \tilde{x}_{k+1|k+1})^{T}\} \\
\text{subject to} & \quad E\{|Y_{k+1}|Y_{k+1}^{T}\} = 0, \quad j = 1, 2, \ldots
\end{align*}\]

(29)

(30)

where \( \hat{x}_{k} \) is the residual signal defined in (13).

Remark 3.1: The condition (30) represents the orthogonality principle which has a strong physical significance. It shows that when there is model uncertainty, the gain matrix \( K_{k+1} \) should be adjusted online such that the output residual still has the property of Gaussian white noise,
which means that all effective information in the output residual has been extracted. A simple example to illustrate this orthogonality principle is that when the model matches the actual system, the output residual sequence of Kalman filter (KF) is an uncorrelated Gaussian white noise sequence, so (30) is satisfied. When the state estimation value of the filter deviates from the state of the system due to the influence of model uncertainty, it will be reflected in the mean value and amplitude of the output residual sequence. In this case, if the gain matrix $K_{k+1}$ is adjusted online such that the formula (30) still holds, then the residual sequences are still orthogonal to each other.

**Theorem 3.3:** The residual sequence $\gamma_{k+1}$ satisfies

$$
E\{\gamma_{k+1+j}^T\} = \bar{\gamma} \Omega_{k+j+1} \tilde{C}_{k+j+1} \tilde{A}_{k+j} + (I - K_{k+j} \bar{\gamma} \Omega_{k+j} \tilde{C}_{k+j}) \tilde{A}_{k+j+1} \cdots
$$

$$
\times (I - K_{k+2} \bar{\gamma} \Omega_{k+2} \tilde{C}_{k+2}) \tilde{A}_{k+1}
\times (P_{k+1}|k \tilde{C}_{k+1}^T \bar{\gamma} \Omega_{k+1} \tilde{C}_{k+1} - K_{k+1} V_{0k+1}).
$$

(31)

**Proof:** According to (13), one has

$$
E\{\gamma_{k+1+j}^T\gamma_{k+1}^T\} = \bar{\gamma} \Omega_{k+j+1} \tilde{C}_{k+j+1} \tilde{A}_{k+j} \sum_{i=1}^{ny} \sum_{i=1}^{ny} \pi_i \gamma_{k+j} \gamma_{k+j+1}^T
\times E\{(\tilde{x}_{k+j+1} - \tilde{x}_{k+j+1+k+j})^T\gamma_{k+1}^T\}. \tag{32}
$$

From (16), we have

$$
\tilde{x}_{k+j+1} - \tilde{x}_{k+j+1+k+j} = \tilde{A}_{k+j}(\tilde{x}_{k+j} - \tilde{x}_{k+j+k+j}) + \tilde{b}_{k+j} w_{k+j}.
$$

(33)

Substituting (33) into (32) yields

$$
E\{\gamma_{k+1+j}^T\gamma_{k+1}^T\} = \bar{\gamma} \Omega_{k+j+1} \tilde{C}_{k+j+1} \tilde{A}_{k+j} \sum_{i=1}^{ny} \sum_{i=1}^{ny} \pi_i \gamma_{k+j} \gamma_{k+j+1}^T
\times E\{(\tilde{x}_{k+j} - \tilde{x}_{k+j+k+j})^T\gamma_{k+1}^T\}. \tag{34}
$$

From (17), one has

$$
\tilde{x}_{k+j} - \tilde{x}_{k+j+k+j} = (I - K_{k+j} \gamma_{k+j} \Omega_{k+j} \tilde{C}_{k+j}) (\tilde{x}_{k+j} - \tilde{x}_{k+j+k+j-1})
\times (I - K_{k+j} \gamma_{k+j} \Omega_{k+j} \tilde{C}_{k+j} \tilde{A}_{k+j+k+j-1}
\times K_{k+j} \gamma_{k+j} \Omega_{k+j} \tilde{C}_{k+j} \tilde{A}_{k+j+k+j-1}.
$$

(35)

Implementing (35) into (34) yields

$$
E\{\gamma_{k+1+j}^T\gamma_{k+1}^T\} = \bar{\gamma} \Omega_{k+j+1} \tilde{C}_{k+j+1} \tilde{A}_{k+j} \sum_{i=1}^{ny} \sum_{i=1}^{ny} \pi_i \gamma_{k+j} \gamma_{k+j+1}^T
\times (I - K_{k+j} \bar{\gamma} \Omega_{k+j} \tilde{C}_{k+j}) \tilde{x}_{k+j+k+j-1}
\times E\{(\tilde{x}_{k+j} - \tilde{x}_{k+j+k+j-1})^T\gamma_{k+1}^T\}. \tag{36}
$$

Repeating the operations (32)–(36), we have

$$
E\{\gamma_{k+1+j}^T\gamma_{k+1}^T\} = \bar{\gamma} \Omega_{k+j+1} \tilde{C}_{k+j+1} \tilde{A}_{k+j} (I - K_{k+j} \bar{\gamma} \Omega_{k+j} \tilde{C}_{k+j}) \tilde{A}_{k+j+1}
\times (I - K_{k+2} \bar{\gamma} \Omega_{k+2} \tilde{C}_{k+2}) \tilde{A}_{k+1}
\times E\{(\tilde{x}_{k+j} - \tilde{x}_{k+j+k+j-1})^T\gamma_{k+1}^T\}.
$$

(37)

According to (12), (13) and (17), one gets

$$
E\{(\tilde{x}_{k+1} - \tilde{x}_{k+1+k+1})^T\gamma_{k+1}^T\} = E\{(\tilde{x}_{k+1} - \tilde{x}_{k+1+k+1})^T\} - K_{k+1} E\{\gamma_{k+1+k+1}^T\gamma_{k+1}^T\}
=E\{(\tilde{x}_{k+1} - \tilde{x}_{k+1+k+1})^T\}
\times (\Omega_{k+1} - \tilde{D}_{k+1} \bar{\gamma} \Omega_{k+1} \tilde{C}_{k+1} \tilde{x}_{k+1+k+1})
\times - K_{k+1} E\{\gamma_{k+1+k+1}^T\gamma_{k+1}^T\}
= p_{k+1} \tilde{C}_{k+1} \bar{\gamma} \Omega_{k+1} \tilde{C}_{k+1} \tilde{x}_{k+1+k+1} - K_{k+1} \gamma_{k+1+k+1}^T\gamma_{k+1}^T\). \tag{38}
$$

Substituting (38) into (37), and then, the proof of the Theorem 3.3 has been completed.

The suboptimal fading factor can be approximated by the following formula:

$$
\lambda_{k+1} = \left\{
\begin{array}{ll}
\lambda_0, & \lambda_0 \geq 1, \\
1, & \lambda_0 < 1,
\end{array}
\right.
$$

(39)

where

$$
\lambda_0 = \frac{\text{tr}(M_{k+1})}{\text{tr}(N_{k+1})},
$$

(40)

$$
M_{k+1} = \sum_{i=1}^{ny} \pi_i \gamma_{k+1+i} (\Omega_{k+1+i} + (\tilde{C}_{k+1+i} \tilde{A}_{k+1+i} \tilde{C}_{k+1+i}^T)) \gamma_{i},
$$

(41)

$$
N_{k+1} = \gamma_{k+1+k+1}^T\gamma_{k+1}^T
- \sum_{i=1}^{ny} \pi_i \gamma_{k+1+i} (\tilde{C}_{k+1+i} \tilde{A}_{k+1+i} \tilde{C}_{k+1+i}^T) \gamma_{i},
$$

(42)
where $V_{0k+1}$ can be calculated by the following formula in (42)

$$V_{0k+1} = \frac{\gamma_1 \gamma_1^T, \rho V_{0k}, \gamma_{k+1} \gamma_{k+1}^T}{1 + \rho}, \quad k = 0,$$

(43)

where $0 \leq \rho \leq 1$ is a forgetting factor, usually take a value $\rho = 0.95$.

**Proof:** According to the principle of orthogonality, it should be forced to make (30) valid, and from Theorem 3 and (30) that

$$p_{k+1} = C_{k+1}^T Q_{k+1}^{-1} C_{k+1}^T - K_{k+1} V_{0k+1} = 0.$$ 

(44)

By substituting (25) into (44), one has

$$p_{k+1} = C_{k+1}^T Q_{k+1}^{-1} C_{k+1}^T (l - (\Lambda_k + 1)^{-1}) V_{0k+1} = 0.$$ 

(45)

Considering (28), equation (45) is equivalent to

$$\sum_{i=1}^{n} p_i \gamma_i (Q_{k+1}^{-1} C_{k+1}^T) \gamma_i + \sum_{i=1}^{n} p_i \gamma_i (\tilde{C}_{k+1}^T \tilde{K}_{k+1} \tilde{C}_{k+1}^T) \gamma_i = V_{0k+1} = 0.$$ 

(46)

Define $N_{k+1}, M_{k+1}$ as shown in (41) and (42), then the above formula can be simplified to

$$\lambda_{k+1} M_{k+1} = N_{k+1}.$$ 

(47)

Taking trace on both sides of (47) and deforming it, we can get (40). Consequently, the proof of Theorem 3.3 has been completed.

4. Simulation studies

In this section, we take an Internet-based three tank (ITTS) as an example to demonstrate the effectiveness of the proposed estimation algorithm. The three tank system, named DTS200, is manufactured and supplied by German Amila automation company. The fault of ITTS can be estimated by monitoring the three liquid levels in the case of network-induced uncertainty. ITTS is a nonlinear continuous-time system. After modelling and discretization at each sampling instant, the ITTS nonlinear discrete model can be approximated as follows:

$$h_{1,k+1} = h_{1,k} + T_s \frac{1}{s} (-Q_{13,k} + Q_{1,k}) + H_{1,k} w_k,$$

$$h_{2,k+1} = h_{2,k} + T_s \frac{1}{s} (Q_{32,k} - Q_{20,k} + Q_{2,k}) + H_{2,k} w_k,$$

$$h_{3,k+1} = h_{3,k} + T_s \frac{1}{s} (Q_{13,k} - Q_{32,k}) + T_s \frac{1}{s} f_k + H_{3,k} w_k,$$

(48)

where

$$Q_{13,k} = a_{13,k} S_n \text{sgn}(h_{1,k} - h_{3,k}) \sqrt{2g|h_{1,k} - h_{3,k}|},$$

$$Q_{32,k} = a_{32,k} S_n \text{sgn}(h_{3,k} - h_{2,k}) \sqrt{2g|h_{3,k} - h_{2,k}|},$$

$$Q_{20,k} = a_{20,k} S_n \sqrt{2g h_{2,k}}.$$ 

Define the following variables and the parameters:

$$Q_i: \text{flow rates (m}^3/\text{s}),$$

$$Q_i: \text{supplying flow rates (m}^3/\text{s}),$$

$$h_i: \text{liquid levels (m)},$$

$$S_o: \text{section of cylinder (m}^2),$$

$$S_n: \text{section of connection pipe (m}^2),$$

$$S_f: \text{section of leak opening (m}^2),$$

$$a_{ij}: \text{outflow coefficients},$$

$$T_s: \text{sampling time (s)},$$

$$g: \text{gravity acceleration (m/s}^2),$$

where $i = 1, 2, 3$ and $(i,j) \in \{ (1,3); (3,2); (2,0) \}$.

The measurement signals under sensor gain degradation and stochastic protocol scheduling are described as follows:

$$y_k = \gamma_{r_k} \Omega_k h_k + v_k,$$

(49)

where

$$\gamma_{r_k} = \text{diag}\{\theta(r_k - 1), \theta(r_k - 2), \theta(r_k - 3)\},$$

$$\Omega_k = \text{diag}\{\mu_{1,k}, \mu_{2,k}, \mu_{3,k}\}.$$ 

The sensor of the system has three sensor nodes and the probability of stochastic protocol scheduling is as follows:

$$\begin{align*}
\text{Prob}(r_k = 1) &= 0.3, \\
\text{Prob}(r_k = 2) &= 0.3, \\
\text{Prob}(r_k = 3) &= 0.4,
\end{align*}$$

and the sensor gain degradation rates $\mu_{1,k}, \mu_{2,k}$ and $\mu_{3,k}$ are uniformly distributed, respectively, over $[0.6, 0.7]$, $[0.7, 0.8]$ and $[0.8, 0.9]$.

**Table 1.** Some technical parameters of ITTS.

| Parameter          | Value   |
|--------------------|---------|
| $S_o$              | 0.00154 m$^2$ |
| $S_n$              | 5 $\times$ 10$^{-5}$ m$^2$ |
| $S_f$              | 0.0095 m$^2$ |
| $a_{12}$           | 0.45    |
| $a_{23}$           | 0.61    |
| $a_{31}$           | 0.48    |
| $T_s$              | 5 s     |
| $g$                | 9.8 m/s$^2$ |
In the simulation experiment, some parameters are set to $Q_{1,k} = 4.5 \times 10^{-5}I$, $Q_{2,k} = 2.5 \times 10^{-5}I$, $H_{1,k} = 0.7$, $H_{2,k} = 0.8$, $H_{3,k} = 0.6$, $Q_k = 2 \times 10^{-12}I$, $R_k = \text{diag}(0.3 \times 10^{-5}, 0.2 \times 10^{-5}, 0.15 \times 10^{-5})$. The initial values are chosen as $\hat{x}_0 = [0.048; 0.023; 0.036]^T$, and $P_{0|0} = 2.5 \times 10^{-5}I$.

The additive fault is assumed as

$$f_k = \begin{cases} -2.5 \times 10^{-5}, & \text{if } k \geq 531, \\ 0, & \text{otherwise.} \end{cases} $$

(50)

Figure 1. Actual state and its estimation for $h_1$.

Figure 2. Actual state and its estimation for $h_2$. 
In this experiment, Figures 1–4 gives the liquid level and fault of three tanks, respectively, and the estimation comparison of corresponding EKF and STF. From these figures, we can see that the STF designed by us can track the real signal well compared with the tracking delay or even failure of EKF in the case of sensor gain degradation and stochastic protocol scheduling. Figure 5 shows the information transmission of different nodes under the stochastic protocol scheduling at each transmission instant, in which time \( k = 600–700 \)
is intercepted. The simulation results above verify the practicability and effectiveness of the proposed filtering scheme.

5. Conclusions

In this paper, a fault estimation algorithm based on Strong Tracking Filter has been proposed for a class of nonlinear systems with sensor gain degradation and stochastic protocol (SP). The phenomenon of the sensor gain degradation has been described by a sequence of stochastic variables. Stochastic protocol has been employed to deal with the data conflicts in multi-signal transmission. New state vectors have been formed by augmenting the original system state vectors and the faults. The STF has been designed for the system with burst faults by introducing a fading factor into the filter structure to estimate the burst states more quickly. The experimental results of three tanks has shown the effectiveness and applicability of the proposed algorithm. It would be interesting to study the following future research topics: (1) development of conditions on the boundedness of the upper bound of filtering error covariance; (2) extension of the results obtained in this paper to nonlinear systems with other network-induced phenomena.

Disclosure statement

No potential conflict of interest was reported by the authors.

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