The analysis of the accuracy of the wheel alignment inspection method on the side-slip plate stand

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Abstract. The article presents the theoretical basis and the results of the examination of the wheel alignment inspection method on the slide slip plate stand. It is obligatory test during periodic technical inspection of the vehicle. The measurement is executed in the dynamic conditions. The dependence between the lateral displacement of the plate and toe-in of the tested wheels has been shown. If the diameter of the wheel rim is known then the value of the toe-in can be calculated. The comparison of the toe-in measurements on the plate stand and on the four heads device for the wheel alignment inspection has been carried out. The accuracy of the measurements and the influence of the conditions of the tests on the plate stand (the way of passing through the plate) were estimated. The conclusions about the accuracy of this method are presented.

1. Introduction

The vehicle inspection stations are equipped with the slide surface plate stands for the wheels toe-in control in dynamic conditions (figure 1). Test is realized during crossing the plate by the vehicle with small constant speed. The plate is crosswise bearing. The lateral displacement of the plate is proportional to the toe-in of the car wheels.

Figure 1. Side slip tester for control of the car wheels toe-in [2].
The article presents theoretical basis and the results of the examination of that method. The dependence between the lateral displacement of the plate and toe-in of the tested wheels has been shown. The comparison of the toe-in measurements on the plate stand and on the four heads device for the wheel alignment inspection with laser measurement system has been carried out (figure 2). The accuracy of measurements and the influence of the conditions of the investigation on results of tests on the plate stand were estimated. The conclusions about the accuracy of this method are presented.

![Figure 2. Four heads device with CCD camera measurement system.](image)

**2. Base of the measurement – theoretical analysis**

The plate length 1 m can shift in the lateral direction $x$, but doesn’t shift in the longitudinal direction $y$ (figure 3). Friction resistances during lateral movement of the plate are minimalized by special bearing. The car wheel with toe-in or toe-out regulation crosses the plate with the constant small speed. Second wheel of the axle rolls on the stiffness surface of the stand floor. The direction of the right wheel setting and reaction between this wheel and stiff surface of the floor causes the lateral displacement of axle wheels axle and reaction on the left wheel. This wheel together with the crosswise bearing plate moves lateral in relation to the surface of the stand floor. The plate displacement is proportional to the toe-in or toe-out and is measured. The value of toe-in/out is calculated on the base of the plate displacement and evaluated. The analysis was done for few cases of the wheels alignment.
Figure 3. Scheme of the vehicle position in relation to the side slip plate:
1 – slide plate, 2 – vehicle, $\beta_1$, $\beta_2$ – toe-in angles.

I case
The right wheel setting with toe-in angle, the left wheel setting directly (straight ahead), (figure 4).

Right wheel forces the direction of the wheel axle in relation to the stand floor. The right wheel
displacement in the $y$ direction on the length $l$ causes lateral displacement the centre of this wheel by
line segment $\Delta x_1$. If the wheel slip will be omitted (the rolling resistance has very small value) then the
centre of the right wheel displacement on the length $l$ is:

$$\Delta x_1 = l \cdot \tan \beta_1$$  \hspace{1cm} (1)

$\beta_1$ – toe-in angle of the right wheel,
$l$ – length of the plate

This displacement reacts on the car body and the left wheel. If the steering wheel will be positioned
directly (straight ahead), the reaction form the right wheel causes the lateral displacement of the left
wheel centre by line segment $\Delta x_2$ together with the plate. Ignoring the friction resistances in the plate
bearings and slip of the tire, the lateral displacement of the plate will be equal lateral displacement of
the right wheel centre:

$$\Delta x_2 = \Delta x_1$$  \hspace{1cm} (2)

For this case (right wheel with toe-in angle $\beta_1$, left wheel with zero toe-in angle) the toe-in of the
wheels $z_1$ (figure 4) is:

$$z_1 = d \cdot \sin \beta_1$$  \hspace{1cm} (3)

$d$ – diameter of the wheel rim
On the base of figure 4 we can write:

\[ z_1 = \frac{\Delta x_1}{d} \frac{\Delta x_1}{\sqrt{\Delta x_1^2 + l^2}} \] (4)

**Figure 4.** The position of the front wheels of the car at the start and end of the measurement on the side slip plate.

On the stand the displacement of the plate \( \Delta x_2 \) is measured. Because \( \Delta x_2 = \Delta x_1 \) then toe-in of the wheels equals:

\[ z = z_1 = \frac{\Delta x_2 \cdot d}{\sqrt{\Delta x_2^2 + l^2}} \] (5)

**II case**

Right wheel P setting directly (straight ahead), left wheel L with toe-in angle \( \beta_2 \) (figure 5)

Left wheel rolls at the angle \( \beta_2 \) to the longitudinal axis of the plate. Right wheel objects to this direction. It rolls on the stiff surface of the stand floor parallel to the longitudinal axis of the slide plate. This wheel forces the direction of the motion of the axle of the wheels. It causes reaction between left wheel and plate and displacement of the plate on left in relation to the left wheel. The lateral slip of the left wheel in relation to the plate is compensated (balanced) by lateral displacement of the plate.

Component \( v_x \) (figure 5) is the speed of the wheel in relation to the slide plate in the x direction. At the same time the plate shifts in the opposite direction with the speed \( -v_x \). Value of the \( v_x \) is:

\[ v_x = v_y \tan \beta_2 \] (6)

During uniform motion of the vehicle on the \( l \) distance the lateral displacement of the plate is:
\[\Delta x_2 = v_x \cdot t = v_y \cdot \tan \beta_2 \cdot t\]
\[t = \frac{l}{v_y}\]
\[\Delta x_2 = l \cdot \tan \beta_2\]

As in previous case the toe-in of the wheels is:

\[z = z_2 = \frac{\Delta x_2 \cdot d}{\sqrt{\Delta x_2^2 + l^2}}\]

**Figure 5.** The orientation of the front left wheel L on the plate and the front right wheel P on the floor of the stand.

**III case**

The right wheel setting with toe-in angle \(\beta_1\), left wheel with toe-in angle \(\beta_2\) (figure 6)

In this case the sum of the reactions from two wheels effects on the plate. The right wheel rolls on the stiff surface and hold its direction of motion at the angle \(\beta_1\). This motion on the length \(l\) causes the lateral displacement of the left wheel together with the plate on the left by section \(\Delta x_1 = lt \tan \beta_1\). Left wheel tries to keep its direction of rolling \(\beta_2\). Right wheel sets against of this direction. It rolls on the stiff surface. It causes reaction between left wheel and plate and displacement of the plate in relation to the wheel on left by \(\Delta x_2\). Displacements \(\Delta x_1\) \& \(\Delta x_2\) have the same directions.

Vectors analysis of the wheels speed allows to set sum of the plate displacement. Axle of the left and right wheels displacements in relation to the stand floor with speed \(v_P\). Component \(v_{x1}\) of this speed is the speed of the left wheel and plate in \(x\) direction. Left wheel is positioned in \(\beta_2\) direction and rolls with speed \(v_L\) (figure 6). Value of the speed \(v_L\) results from value of the component \(v_y\) of the speed \(v_P\). Component \(v_{x2}\) of the speed \(v_L\) is the speed of the left wheel in relation to the plate. On the contrary the plate moves in the reverse direction with speed \(-v_{x2}\). The speed of the plate displacement in relation to the stand floor is the sum of the vectors \(v_{x1}\) and \(-v_{x2}\). If the slip between the wheel and the plate is omitted then the speed value of the plate in relation to the floor of the stand can be described as:
\[ v_x = v_{x1} + v_{x2} = v_y \tan \beta_1 + v_y \tan \beta_2 \]  
\[ \text{(10)} \]

\( v_{x1} \) – component of the speed \( v_p \) in \( x \) direction, it causes shift of the left wheel together with plate on the left,

\( v_{x2} \) – speed of the plate in \( x \) direction in relation to the left wheel

During uniform motion of the vehicle the lateral displacement of the plate is:

\[ \Delta x_2 = v_x \cdot t = v_y (\tan \beta_1 + \tan \beta_2) \cdot t \]

\[ t = \frac{l}{v_y} \]

\[ \text{(11)} \]

On the base of these dependences the displacement of the slide plate is:

\[ \Delta x = l \cdot (\tan \beta_1 + \tan \beta_2) = \Delta x_1 + \Delta x_2 \]

\[ \text{(12)} \]

Toe-in of the wheels is:

\[ z = z_1 + z_2 = \frac{\Delta x_1 \cdot d}{\sqrt{\Delta x_1^2 + l^2}} + \frac{\Delta x_2 \cdot d}{\sqrt{\Delta x_2 + l^2}} \]

\[ \text{(13)} \]

Because \( \Delta x_1 << 1 \) and \( \Delta x_2 << 1 \), after simplification we can describe:

\[ z = z_1 + z_2 = \frac{\Delta x_1 \cdot d}{l} + \frac{\Delta x_2 \cdot d}{l} = \Delta x \cdot \frac{d}{l} \]

\[ \text{(14)} \]

If the wheels are toe-out the sign of the sum \( z_1 + z_2 \) will be reversed. Displacement of the plate will be on right. If one of the wheels will be toe-in positioned and second toe-out then the displacement of the plate will be the difference between \( \Delta x_1 \) and \( \Delta x_2 \).

**Figure 6.** The orientation and speed of the left wheel on the plate and right wheel on the floor of the stand.
The analysis shows that measurements of the plate displacement allows to determine the sum of the toe-in for the data wheel rim diameter. Analysis was performed for the ideal conditions of measurement and state of the plate stand: the friction in bearings was disregarded, the slip of the wheels was ignored, during crossing the plate the driving or braking forces doesn’t act, the crossing the plate is parallel to the plate axis. During real tests these factors can influence the measurements of the plate displacement. The influence of these factors and the examination the accuracy of that method were realized by verification tests.

3. Influence of the exploitation parameters on the results of the toe-in measurement

The parallel direct during crossing the plate can be not maintained (figure 7). It causes the change of the angle of the left wheel during crossing the plate by angle $\alpha$ in relation to parallel drive. Also right wheel crosses the floor with angle $\alpha$. For the wheels with toe-in regulation as in figure 7 and for small angle $\alpha$ can be described:

$$\Delta x_1 = l \cdot \tan (\beta_1 + \alpha)$$
$$\Delta x_2 = l \cdot \tan (\beta_2 - \alpha)$$
$$\Delta x = \Delta x_1 + \Delta x_2 = l \cdot \left( \tan (\beta_1 + \alpha) + \tan (\beta_2 - \alpha) \right) = l \cdot \left( \frac{\tan \beta_1 + \tan \alpha}{1 - \tan \beta_1 \tan \alpha} + \frac{\tan \beta_2 - \tan \alpha}{1 + \tan \beta_2 \tan \alpha} \right)$$

(15)

For the small angles $\alpha$ the product $\tan \beta \cdot \tan \alpha$ is near zero. If we take it into consideration, it can be described:

$$\Delta x = \Delta x_1 + \Delta x_2 = l \cdot (\tan \beta_1 + \tan \beta_2)$$

(16)

It shows that for the small angles $\alpha$ the result of the toe-in measurement isn’t disturbed by not parallel drive by the plate. The tests confirmed above theoretical analysis (table 1). During tests the toe-in of the vehicle was regulated up to manufacturing data 0°0” (0mm).
Table 1. Results of the toe-in measurements on the plate stand depending on the angle $\alpha$ of the drive in the plate

$\alpha = 3^\circ$ right

| nr of the test | toe-in value [mm] |
|---------------|-------------------|
| 1             | 0                 |
| 2             | 0                 |
| 3             | 0                 |

$\alpha = -3^\circ$ left

| nr of the test | toe-in value [mm] |
|---------------|-------------------|
| 1             | 0                 |
| 2             | 0                 |
| 3             | 0                 |

Another factor which can influence the result of the measurement is the change of the car speed during crossing the plate. If the vehicle accelerates or brakes the tangent forces act between plate and tire. The influence of this parameter was analysed in the small range of speed changes possible in practice ($\Delta v \approx 5$ km/h). Before the tests the toe-in of the wheels was regulated up to manufacturing data $0^\circ0''$ (0mm). Tests didn’t show the influence of the speed changes on the results of toe-in measurements (table 2).

Table 2. The results of the toe-in measurements for the front wheels on the plate during acceleration and braking on the plate

| nr of the test | toe-in for acceleration [mm] | toe-in for braking [mm] |
|---------------|-----------------------------|-------------------------|
| 1             | 0                           | 0                       |
| 2             | 0                           | 0                       |
| 3             | 0                           | 0                       |

The next controlled factor was changing of the drive direction during crossing the plate. The driver turned the steering wheel about $\pm 15^\circ$. Before the tests the toe in value was regulated up to manufacturing data $0^\circ0''$ (0mm). The results are shown in table 3.

Table 3. The results of the toe-in measurements for the front wheels on the plate. Driver turned the steering wheel about $\pm 15^\circ$

| nr of the test | toe-in value [mm] |
|---------------|-------------------|
| 1             | -0.8              |
| 2             | -0.8              |
| 3             | -0.4              |
| 4             | -1.1              |

Scatter of measurements was due to the inaccuracy of setting the steering wheel angle. The measurements showed the clear influence of this parameter in the results of toe-in measurements. It means that the steering wheel ought to be held immovable during crossing the plate.

4. Accuracy of the toe-in measurement on the slide slip plate stand

The investigation were done by comparison of the results on the slide slip plate stand MAHA to the results obtained by four heads device Custor HWA G58 (figure 2).
Accuracy of the plate lateral displacement measurement was 1 mm. Accuracy of the toe-in calculation depends on the diameter of the wheel rim. For the wheel rim 15” the toe-in accuracy is 0.4 mm and for rim 18” the accuracy is 0.5 mm. It corresponds to the angle of toe-in 3’ for rim 15” and about 3.5’ for rim 18”. Four heads device makes possible to measure toe-in with accuracy ±1’ (table 4).

Table 4. Range and accuracy measurements by Custor HWA G59 device [1]

| Parameter                               | Range    | Accuracy |
|-----------------------------------------|----------|----------|
| Front axle                              |          |          |
| Toe-in                                   | ±24°     | ±1’      |
| Camber angle                            | ±10°     | ±1’      |
| Castor axis angle                       | ±20°     | ±2’      |
| King pin axis angle                     | ±20°     | ±2’      |
| Axis out of parallel                    | ±10°     | ±1’      |
| Rear axle                               |          |          |
| toe-in                                  | ±24°     | ±1’      |
| Camber angle                            | ±10°     | ±1’      |
| Angle deviation body axis of symmetry from the geometric axis of drive | ±10° | ±1’ |

The measurements on the plate stand were done for wheels toe-in 0°0’ ±10’ up to manufacturing data and for toe-in -0°20’. The toe-in angles for right and left wheel were regulated symmetrical and next asymmetrical. The tests were repeated three times for each toe-in setting. The results of the tests are shown in table 5. The measurements on the slide plate show less toe-in value than controlled by four heads device. It can result from friction in the bearings of the plate which can increase during exploitation of the stand.

Table 5. The results of the toe-in measurements for front wheels on the plate stand and measured four heads device

| wheels toe-in – four heads device | left wheel toe-in - four heads device | right wheel toe-in - four heads device | toe-in - the slide plate |
|----------------------------------|---------------------------------------|----------------------------------------|--------------------------|
| 0°0’                             | 0 mm                                  | 0°0’                                   | 0 mm                     |
| -0°20’                           | -2.5 mm                                | -0°20’                                 | 0°0’                     |
| -0°20’                           | -2.5 mm                                | 0°0’                                   | -0°20’                   |
| -0°20’                           | -2.5 mm                                | -0°10’                                 | -0°10’                   |

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5. Conclusions

The investigations have shown the usefulness of the side-slip plate stand for control and estimation the wheels toe-in during periodical technical inspection of the vehicle. The side-slip plate stand in all cases of the wheels alignment have shown the proper sign and value of the toe-in.

The accuracy of the toe-in calculation depends on the diameter of the wheel rim. For the rim diameter 15” it is 0,4 mm, for rim 18” it is 0,5 mm. This accuracy is sufficient for the periodic technical inspections but too low for the accuracy of the regulation. The accuracy of measurements on the side-slip plate can be increased by use more precise sensors of the lateral displacement of the plate.

The estimated method isn’t sensitive on the not parallel crossing the plate and on the small changes of the car speed. However changes of the direction by turning the steering wheel during crossing the plate cause the errors which make impossible proper estimation of the wheels toe-in.

The side-slip plate stand can measure less value of toe-in than four heads device. It can be caused by friction resistance in plate bearing and lateral stiffness of the tires. During stand exploitation friction resistance in plate bearing can increase.

References

[1] User manual CUSTOR HWA G58, Anwa-Tech Sp.z o.o. Sulejówek 2011
[2] User manual Side-Slip Tester MAHA, Maschinenbau Haldenwang GmbH 2016
[3] Bocheński C 2000 Badania kontrolne samochodów WKŁ Warszawa
[4] Trzeciak K 2008 Diagnostyka samochodów osobowych WKŁ Warszawa
[5] Sitek K and Syta S 2011 Badania stanowiskowe i diagnostyka WKŁ Warszawa
[6] Linia diagnostyczna Uniline Quantum Materiały firmy UNIMETAL Złotów 2015