Research on anti crack mechanism of bionic coupling brake disc

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Abstract. According to the biological function of fatigue resistance possessed by biology, this study designed a Bionic Coupling Brake Disc (BCBD) which can inhibit crack propagation as the result of improving fatigue property. Thermal stress field of brake disc was calculated under emergency working condition, and circumferential and radial stress field which lead to fatigue failure of brake disc were investigated simultaneously. Results showed that the maximum temperature of surface reached 890°C and the maximum residual tensile stress was 207 Mpa when the initial velocity of vehicle was 200 km/h. Based on the theory of elastic plastic fracture mechanics, the crack opening displacement and the crack front J integral of the BCBD and traditional brake disc (TBD) with pre-cracking were calculated, and the strength of crack front was compared. Results revealed the growth behavior of fatigue crack located on surface of brake disc, and proved the anti fatigue resistance of BCBD was better, and the strength of crack resistance of BCBD was much stronger than that of TBD. This simulation research provided significant references for optimization and manufacturing of BCBD.

1. Introduction
Friction pair was comprised of brake disc and friction pad, and played the function of transforming kinetic energy of vehicle into heat energy. About 95% of heat energy was absorbed by brake disc, which caused a serious problem such as hot band, hot spot, thermal plastic deformation, friction stability, thermal mechanical fatigue, and so on. With the development of high speed and large loading of vehicle, thermal mechanical fatigue of brake disc has become one of popular issues for many scholars. Dufrénoy and Weichert[1] found that thermal mechanical fatigue crack could be concluded by two type kinds, one was surface craquelures, the other was fractured crack which was more harmful to the safety of vehicle. Bagnoli and Vallet[2,3] reveled most fractured crack appeared on surface in radial direction, and the ratio of length to depth of crack was among 6 to 8, the contour of crack in depth direction was semiellipse. Methods of solving thermal mechanical fatigue of brake disc were mainly focused on two aspects: one was optimizing the structure of heat radiation and convention with the purpose of reducing thermal tension/compression stress as the result of reducing thermal mechanical fatigue damage; the other was optimizing chemical compositions of brake disc in the way of developing new kinds materials of metal based or carbon based composites which could inherently improve the ability of fatigue crack resistance of brake disc. For example, Pan[4] studied the flow distribution, temperature field, and convection coefficient of brake discs with different heat dissipation structures. Through finite volume analysis, convection coefficient was improved 7.4% by changing heat dissipation structure of brake disc, the ability of heat dissipation was improved 17.2% by adding short rib between long ribs among. Galindo-López[5] suggested the heat dissipation could be improved
10% by optimizing the structure of heat rib and assembling parameters of disc brake according to the results of flow velocity, flow distribution, and convection coefficient of brake disc. Rajagopal[6] compared the heat dissipation ability of different brake discs with cylinder, wedge, and rhombus heat rib, and found the cylinder heat rib one was the best in improving heat dissipation ability of brake disc as the result of controlling flow velocity, flow distribution, and efficient of heat transferring. Yamabe[7] studied the thermal fatigue performance of gray cast iron disc, and found the fatigue life of brake disc mainly depended on the time of crack propagation. The content of micro graphite was higher the crack resistance was better. They also found the thermal mechanical performance could be improved by adding elements Ni and Ce into the microstructure of gray cast iron. Lim[8] investigated different chemical compositions of cast iron, and found thermal fatigue life was improved 172% to 214% by modifying the contents of Ni, Cr, and Mo. Yang[9] revealed the mechanism of crack initiation and propagation of composites materials SiCp/A356, and found microstructure defects affected greatly on crack initiation. Zhou[10] studied the thermal plastic elastic properties of particle reinforce composites SiCp/A356, and revealed the relationship between residual stress and thermal fatigue life by presenting the function of strain and stress among temperature 20 °C~300 °C.

However, optimizing the heat dissipation structure of brake disc was meaningful to improve heat dissipation ability, but the effects on resisting crack initiation and propagation was little. Although modifying chemical compositions of brake disc or developing new kinds materials were significant to improve the mechanical behavior of microstructures, the research investment were much more, and research time cost longer. Therefore, traditional methods of improving fatigue performance of brake disc need high investment and lead to a worse result, and might affect other properties of brake disc. Nature has already provided the most efficient and economical solution to many engineering problem[11-16]. Many creatures that possess the most ingenious morphology, coupling structure, and economical materials of multi-phase, perform remarkable capabilities which are significant to the hardest issue that troubles brake disc in its thermal mechanical fatigue performance, such as shell, dragonfly, and lamina[17-20]. The microstructures of these creatures consist of hard and soft phase which could deviate crack growth direction, and prolong crack path as the result of improving their ability of resisting crack growth. Based on this mechanism of crack resistance, this paper designed a BCBD with considering materials, structure, and surface morphology. Crack growth behavior and crack resistance mechanism of the BCBD and TBD were revealed by comparing their temperature, stress, and fracture intensity factor.

2. Finite element model and boundary condition

2.1. Thermal mechanical model of brake disc

2.1.1. Solid models of TBD and BCBD. The structure parameters of TBD are: $R_1=128$ mm, $R_2=32.5$ mm, $H_1=12.5$ mm, $\theta=64.5^\circ$, $R_3=125$ mm, $R_4=77$ mm, $H_2=14.5$ mm, as shown in Figure 1. Due to the symmetric structure of brake disc, a 2D thermal mechanical finite element model was established in the following investigation.

$R_1$ was the out radius of brake disc, $R_2$ was the inter radius of brake disc, $H_1$ was thickness of brake disc, $\theta$ was the cover angle of pad and disc, $R_3$ was out the radius of friction pad, $R_4$ was the inter radius of friction pad, and $H_2$ was the thickness of friction pad.
Based on previous studies that related with anti fatigue and wear of brake disc\cite{17,21,22}, annular bionic unit was designed on frictional surface of brake disc, which formed the surface of BCBD as shown in Figure 1(b). Bionic unit was fabricated by laser alloying with self fluxing powder of Fe50, as shown in Figure 2. Bionic units located at the radius of $R_5=121$ mm and $R_6=107$ mm, both the width and depth of bionic unit was 2 mm. Due to the hardness increasing after laser alloying, the top surface of bionic unit was lower than the surface of brake disc in order to avoid changing frictional performance of brake disc and friction pad. Meanwhile, a 2D thermal mechanical finite element model of BCBD was established because of the symmetric structure.

2.1.2. Materials properties of brake disc and bionic unit. Materials of brake disc was gray cast iron HT200, the chemical compositions were: C 3.250 %, Si 1.570 %, Mn 0.920 %, P 0.060 %, S 0.059 %, balanced Fe. Physical parameters of HT200 at different temperature were showed in Table 1, and elastic modulus was 101.027 Gpa, poison ratio was 0.27. The chemical compositions of bionic unit was tested by energy dispersive spectrometer, and the results were: Cr 17 %, Si 3.5 %, Ni 3 %, balanced Fe. Comparing the chemical compositions of bionic unit with the existing table of known metal, it was found the chemical compositions of bionic unit was very close to that of AISI403. Thus, we picked the physical properties of AISI403 as that of bionic unit, as shown in Table 2.

![diagram](image_url)

**Fig. 1** Solid models of brake discs

![diagram](image_url)

**Table 1** Materials properties of brake disc at different temperature

| $T$ | $\alpha$ (K) | $\rho$ (kg/m³) | $C$ (J·kg⁻¹·K⁻¹) | $\lambda$ (W·m⁻¹·K⁻¹) |
|-----|--------------|----------------|------------------|----------------------|
| 25  | 0.0          | 7293           | 488              | 53.2                 |
| 20  | 13.1         | 7243           | 563              | 47.1                 |
| 40  | 13.7         | 7182           | 631              | 39.1                 |
| 60  | 13.9         | 7122           | 743              | 33.7                 |
| 70  | 14.2         | 7089           | 918              | 31.2                 |
| 90  | 15.2         | 7012           | 625              | 28.1                 |
As the material of brake disc was gray cast iron, the model behavior of material in ANSYS code should be chosen carefully. It was well known that GCI was characterized by graphite lamellas which dispersed into the ferrous matrix. The quantity, distribution, and morphology of the free graphite all affect the degree to which the steel matrix was weakened. Moreover, this characteristic of microstructure led to a substantial difference in tension and compression. In tension, the material was more brittle with low strength and cracks formed due to the graphite flakes. In compression, no cracks form, the graphite flakes behaved as incompressible media. Downing had already discussed about the mechanical properties and monotonic behaviors of GCI, as shown in Fig. 3.

![Fig. 2 Bionic unit and the location of bionic unit on brake disc surface](image)

### Table 2 Physical properties of bionic unit

|   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|
| $\rho$ | $\alpha$ | $C$ | $E$ | $\lambda$ | $v$ |
| kg/m$^3$ | $10^{-6}$/K | J·kg$^{-1}$·K$^{-1}$ | Pa | W·m$^{-1}$·K$^{-1}$ |   |
| 7715 | 13.1 | 190 | 180e$^9$ | 490 | 0.27 |

2.1.3. **Boundary condition of thermal mechanical model.** During braking, the kinetic energy was assumed to be converted into heat completely, and was dissipated totally by brake disc, although the practical conversion rate should be around 95%. Due to the object of this paper, the calculating methods were also simplified correspondingly. Heat flux was calculated according to previous acknowledgeable methods and was applied in the area where was the interface between brake disc and friction pad. Constant convection was applied on the surface where brake disc contacts with air, the boundaries applied in the model were shown in Fig. 4. The initial braking conditions which were used for simulation was listed in Table 3.
Fig. 4. Boundary conditions of 2D model established for thermal-mechanical calculation

| Initial conditions                  | Values                      |
|------------------------------------|-----------------------------|
| Initial angular velocity           | 88.46 r/s                   |
| Acceleration                       | 25.86 r/s²                  |
| Braking pressure                   | 3.17 Mpa (uniform)          |
| Friction Coefficient               | 0.45                        |
| Braking time                       | 3.41s                       |
| Cooling time                       | 1296.59s                    |
| Convection                         | 50 w/m²k (constant)         |

In practical condition, pressure was supposed to be uniform on the surface of brake disc, and without considering the effect of third body on thermal problems, thus, the variation of heat flux only depended on the variation of angular rotation under ideal assumption. In addition, the heat energy converted from kinetic energy was assumed to be absorbed totally $\sigma = 1$ by brake disc. And,

$$\dot{E} = dP = VdF_f = \omega \mu \phi_0 r^2 dr$$

$$dS_{disc} = 2 \pi r dr$$

$$q_{disc} (r,t) = \frac{d E_{disc}}{dS_{disc}} = \frac{\phi_0}{2 \pi} \sigma \rho r \mu \omega (t)$$

where: $F_f$ was Friction Force; $r$ was Radius of one point on brake disc surface; $t$ was Time; $E$ was Energy absorbed; $p$ was Pressure; $\sigma$ was The heat partitioning factor; $\mu$ was Friction Coefficient; $\omega$ was Angular Rotation Speed of disc; $\phi_0$ was The contact angle of friction pad.

For the practical working condition of brake disc, normally, the pressure distribution of contact surface was not uniform due to the surface smoothness, wear debris or even thermal deformation of brake disc and friction pad. The practical pressure of local area could be up to 10 times of normal pressure. So, in conservative estimation, 10 times of normal pressure was applied in a local area where there were thermal cracks observed on surface of TBD. The rest braking duties were kept the same with the assumption of uniform pressure distribution simulation.

2.2. Crack models of TBD and BCBD

2.2.1. FEMs of crack models. Thermal mechanical crack often occurred on top surface of brake disc, and its contour was semiellipse in depth with the long axis at radial direction of brake disc, shot axis at depth direction. Normally, the ratio of crack length to crack depth was between 6 to 8\[^3\]. Thus, 2D models of TBD and BCBD were established and were showed in Figure 5, the internal and out radius of model were 95 mm and 128 mm, and the thickness was 12.5 mm, and cover angle was 5°. The initial length and depth of crack were 10 cm and 1.25 cm respectively. During braking, the maximum
plastic deformation was much smaller than the size of brake disc. Thus liner elastic fracture theory was applied to calculate the strength of crack. J integral was picked to study crack strength, because it not only available to solve static strength of crack but also suitable for dynamic condition. Meanwhile J integral could be transformed to K and G. To guarantee the accuracy of calculation, the mesh of crack front was made to singularity within 2 mm of crack mesh.

2.2.2. Boundary conditions of crack model. On the semi-model, the displacement of bottom surface was fixed in Y direction which was the same direction with the normal of rubbing surface, and the cross section of the area where there was a crack was supposed to be a symmetrical area, two lines at the edges of the symmetrical area was fixed in X direction which was radial direction, and a nonuniform force was applied at the back surface of the semi-model, which was applied as the driving force for opening crack, as shown in Figure 6.

3. Results and discussions

3.1. Thermal mechanical model
Under the braking condition of initial velocity 200 km/h, braking pressure 3.17 Mpa, and braking time 3.41s, the maximum temperature of TBD and BCBD reached to 890.743 °C and 879.26 °C respectively, their temperature fields were showed in Figure 7(a). From comparing temperature fields, it was clear that the temperature gradient of TBD was smaller than that of BCBD at the same radius, because the heat dissipation ability of BCBD was better than that of TBD as the result of the contact area between disc surface of BCBD and air was larger. After 1300s cooling, thermal compression stress transform to residual tensile stress of which distribution were exhibited in Figure 7(b). It was clear that the maximum tensile stress were around 207 Mpa, the maximum compression stress were 51.7 Mpa and 69.4 Mpa respectively. Because the maximum tensile strength of gray cast iron was 200 Mpa and the maximum compression strength was 400 Mpa. Thus, residual tensile stress which was
generated after braking exceeded the maximum tensile strength of materials which lead to plastic deformation and crack initiation. Through frequency braking, radial crack propagated gradually under circumferential tensile stress and led to the thermal fatigue damage of brake disc. But the radial crack would be stopped or inhibited when crack encounter with bionic unit, as shown in Figure 8, the mechanism of resisting crack referred to 3.2 at following.

![Temperature field](image1)

![Residual thermal stress](image2)

**Fig. 7 Calculation results of TBD and BCBD**

![Direction of crack growth](image3)

**Fig. 8 Radial crack on TBD and BCBD**

### 3.2 Results and discussions of crack model

Figure 9 showed the displacements and J integrals of TBD and BCBD when there was a young crack (initial crack). It was obvious that the maximum displacement of TBD was 0.031 mm, and occurred at the loading surface, as shown in Figure 9(a). The maximum displacement of BCBD was 0 mm, and also occurred at the loading surface, as shown in Figure 9(b). Thus, due to the existing of bionic unit, the strength of brake disc was improved, and the opening tendency of crack was inhibited. J integral of TBD and BCBD were showed in Figure 9(c), it was clear that the variation tendency were similar, but the J integral increased from 10 Kpa·m to 410 Kpa·m and 341 Kpa·m respectively, and reduced gradually. It could be proved that BCBD performed better in resisting crack by comparing their J integral.

![Crack displacement](image4)

![Crack displacement of BCBD](image5)

**Fig. 9 Results of first step**

![J integral](image6)
By studying the variation of J integral, crack growing tendency could be expected in the second step. At the initial status (first step), J integral was largest at the middle of crack, and was smallest at both sides of crack. Thus, crack would grow mainly into depth direction, would grow little at top surface. Here we established the second step crack model according to the results of J integral which could be used to simulate the growing situation of fatigue crack on brake disc.

Figure 10 showed the displacements and J integral of TBD and BCBD at the second step. Similar to the results of initial step, the maximum deformation occurred at the loading surface. The maximum crack opening displacements of TBD BCBD were 0.0675 mm and 0.0441 mm respectively, which also proved that bionic units reduced the deformation of brake disc and played as the function of reinforcing brake disc. Results of J integral were different from that of last step as showed in Figure 10(c). J integral of TBD was 1260 Kpa·m at one side of crack (internal radius of brake disc), and reduced gradually until the middle of crack to 370 kpa·m, and increased to 520 Kpa·m at the other side of crack (out radius of brake disc). J integral of BCBD varied similar to that of TBD along the crack, but the J integral was smaller. It reduced from 1070 Kpa·m to 320 Kpa·m, and increased to 490 Kpa·m. Thus, it was also proved that BCBD was reinforced by bionic unit. According to the results of J integral, the third step model of crack growing was established. Here we prolonged the length of crack at top surface, and fixed the depth of crack to simulate the growing tendency of fatigue crack.

Figure 11 showed the results of crack at the third step. The maximum opening displacements were 0.105 mm and 0.0913 mm respectively on TBD and on BCBD. It was also proved that BCBD could resist crack opening. Figure 12 showed opening regularity of crack on both kinds of brake discs. It was easy to find that the resisting effect of bionic unit on crack opening weakened as crack became larger, which indicate bionic unit performed better in resisting crack initiation. The J integral results of TBD and BCBD were different from that of last two steps. Because crack front encountered with bionic unit in the third step, J integral of BCBD was the largest around bionic unit about 1290 Kpa·m, and reduced gradually to 130 Kpa·m where there wasn't bionic unit, and increased suddenly to 470 Kpa·m when crack encountered bionic unit again. But J integral was smaller than that of TBD between two bionic units about 50 Kpa·m ~260 Kpa·m. Thus, bionic unit still reinforced the surface of brake disc. One point to be mentioned that although J integral of BCBD was larger near bionic unit, the facture
strength of bionic unit was much larger than that of gray cast iron\textsuperscript{[21]}. Therefore, the ability of crack resistance of BCBD was still better than that of TBD.

![Figure 12](image1.png)

**Fig. 12** Opening displacement of thermal mechanical crack

![Figure 13](image2.png)

**Fig. 13** Crack growth regularity and mechanism of crack resistance

To sum up, the growth behavior of thermal mechanical crack and the mechanism of BCBD in resisting crack propagation were shown in Figure 13. The initial radial crack was originated by circumferential tensile stress, and grew into the depth of brake disc, and then propagated on surface in radial direction. The contour of thermal mechanical crack on both kinds of brake discs was semiellipse. On the surface of BCBD, bionic unit supported and reinforced the surface of brake disc, and resisted the crack growth when crack front approached to bionic unit.

4. Conclusions

Using finite element calculating methods, this paper studied the temperature fields, stress fields, crack opening condition, and crack intensity factor with considering synthetically the properties variation of materials at different temperature and mechanical behaviors. Crack growth behavior and crack resistance mechanism of surface crack on brake disc were revealed, and the corresponding conclusions as followed:

1) The better heat dissipation ability of BCBD leads to a smaller gradient of surface temperature as the result of reducing thermal strain and thermal stress.

2) Forced by circumferential stress, thermal mechanical crack grew towards to the depth of brake disc when the ratio of crack length to crack depth was 8, and the crack grew on surface when the ratio
of crack length to crack depth was smaller than 2. The contour of thermal mechanical crack on brake disc surface was semiellipse.

3) Annular bionic unit located on surface of BCBD, and played the function of reinforcing brake disc as the result of resisting thermal mechanical crack growth. It was verified that bionic unit could effectively resist the crack initiation and propagation by comparing the opening displacement and J integral of thermal mechanical crack.

Acknowledgements
Special thanks give to Prof. Philippe Dufrénoy for his kindly encouragements and instructions. This research is supported by Science and technology project of Zhejiang Province (Grant No. 2016C31046), National Natural Science Foundation of China (Grant No. 51605125), Opening Project of the Key Laboratory of Bionic Engineering (Ministry of Education) Jilin University (Grant No. K201605), and Zhejiang Provincial Natural Science Foundation (Grant No. LQ14E050011).

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