Inductive Program Synthesis over Noisy Datasets using Abstraction Refinement Based Optimization

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We present a new synthesis algorithm to solve program synthesis over noisy datasets, i.e., data that may contain incorrect/corrupted input-output examples. Our algorithm uses an abstraction refinement based optimization process to synthesize programs which optimize the tradeoff between the loss over the noisy dataset and the complexity of the synthesized program. The algorithm uses abstractions to divide the search space of programs into subspaces by computing an abstract value that represents outputs for all programs in a subspace. The abstract value allows our algorithm to compute, for each subspace, a sound approximate lower bound of the loss over all programs in the subspace. It iteratively refines these abstractions to further subdivide the space into smaller subspaces, prune subspaces that do not contain an optimal program, and eventually synthesize an optimal program.

We implemented this algorithm in a tool called Rose. We compare Rose to a current state-of-the-art noisy program synthesis system [Handa and Rinard 2020] using the SyGuS 2018 benchmark suite [Alur et al. 2013]. Our evaluation demonstrates that Rose significantly outperforms this previous system: on two noisy benchmark program synthesis problem sets drawn from the SyGus 2018 benchmark suite, Rose delivers speedups of up to 1587 and 81.7, with median speedups of 20 (out of 54) and 4 (out of 11) more benchmark problems than the previous system. Both Rose and the previous system synthesize programs that are optimal over the provided noisy data sets. For the majority of the problems in the benchmark sets (272 out of 286), the synthesized programs also produce correct outputs for all inputs in the original (unseen) noise-free data set. These results highlight the benefits that Rose can deliver for effective noisy program synthesis.

Additional Key Words and Phrases: Program Synthesis, Machine Learning, Noisy datasets, Abstraction Refinement

1 Introduction

Program synthesis has been successfully used to synthesize programs from examples, for domains such as string transformations [Gulwani 2011; Singh and Gulwani 2016], data wrangling [Feng et al. 2017], data completion [Wang et al. 2017b], and data structure manipulation [Feser et al. 2015; Osera and Zdancewic 2015; Yaghmazadeh et al. 2016]. In recent years, there has been interest in synthesizing programs from input-output examples in presence of noise/corruptions [Handa and Rinard 2021, 2020; Peleg and Polikarpova 2020; Raychev et al. 2016]. This line of work aims to tackle real world datasets which contain noise and corruptions. These techniques can synthesize the correct programs, even in presence of substantial noise [Handa and Rinard 2020].

Noisy program synthesis has been formulated as an optimization problem over the program space and the noisy dataset [Handa and Rinard 2021, 2020]. The optimization problem is parameterized with three functions: a loss function, which measures by how much a program’s output differs from the output in the given noisy data set, a complexity measure, which measures the complexity of a candidate program, and an objective function, which combines these both of these scores to rank programs [Handa and Rinard 2021, 2020]. Given a search space and a set of noisy input/output pairs, the task of the noisy program synthesis is to synthesize a program which minimizes the objective function over the noisy input/output pairs.

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Similar to noise-free program synthesis, noisy program synthesis is effectively a search problem. Working with noisy datasets further complicates the search: synthesizing a program which simply maximizes the number of input-output examples it satisfies (is correct on) may not optimize the objective function over the entire dataset. Moreover, recent research has demonstrated that noisy program synthesis can often synthesize the correct program even when all input/output examples are corrupted [Handa and Rinard 2020].

Current solutions to noisy program synthesis, either fall into enumeration based search techniques [Peleg and Polikarpova 2020] or version space (finite tree automaton) based techniques [Handa and Rinard 2020]. Both of these techniques reduce the search space by partitioning the space of programs based on their execution behaviour on the given inputs. Both exploit the property that programs which produce the same output values on the given input values will have the same loss on a given dataset. If we restrict our search space to a single partition (instead of all programs within the given program space) then the simplest program (based on the complexity measure) will be the optimal solution to the noisy program synthesis problem. This is due to the fact that all programs within this space have the same loss value. Therefore the program which minimizes the complexity measure, minimizes the objective function. If the simplest program within this partition is not the optimal program (i.e, there exists another program in the search space which further reduces the value of the objective function) then we can safely conclude no other program within this partition is the optimal program. With this knowledge, iterating over all partitions and comparing their simplest program allows these techniques to synthesize the optimal program. The performance of these techniques is determined by the number of partitions that are created, given a dataset.

This partitioning approach has also been used in traditional noise-free program synthesis settings [Gulwani 2011; Wang et al. 2017b] and horn-clause verification [Kafle and Gallagher 2015]. A potential solution to reducing the number of partitions was proposed by Kafle at al. in the context of horn clause verification using tree automata [Kafle and Gallagher 2015]. Wang et al introduced this technique to noise-free program synthesis [Wang et al. 2017a]. The technique uses abstract output values to partition the program space, instead of concrete output values. An abstract value is a compact representation of a set of concrete values. Both of these techniques associate partitions of their search space to abstract values. For example, [Wang et al. 2017a] represents a partition with an array of abstract output values and contains all programs which, given input values, maps these inputs to an array of concrete outputs, where each output value is an element of the corresponding abstract output value. Abstract values allow these techniques to reduce the number of partitions and hence decrease the running time of the synthesis algorithm.

Our work applies the abstract value based partitioning approach to create an abstraction refinement based algorithm for solving the noisy program synthesis problem. Our technique uses abstractions to partition the program space, with the abstract value of each partition enabling us to soundly and approximately estimate a lower bound on the loss value of a partition, i.e., the minimum loss of any program within that partition. This lower bound on the loss value, attached with the simplest program within a given partition, allows us to effectively synthesize candidate optimal programs. Given a candidate optimal program, our technique can effectively prune out partitions, which even with the minimum possible loss value, will fail to contain the optimal program. The remaining partitions, based on their lower bound loss value, may contain programs which better fit the noisy dataset compared to our candidate optimal program (i.e., the remaining partitions may contain programs that the object function ranks above the candidate optimal program). The synthesis algorithm then refines these abstractions in order to further refine the abstraction-based partitioning and improve its estimate of the minimum possible loss value. The algorithm guarantees that it will eventually synthesize the optimal program.
We have implemented our algorithm in the *Rose* synthesis tool. *Rose* can be instantiated to work in different domains by providing suitable domain specific languages, abstract semantics, and concrete semantics of functions within the language. *Rose* is parameterized over a large class of objective functions, loss functions, and complexity measures. [Handa and Rinard 2020] highlights that this flexibility is required to synthesize correct programs for datasets which contain a large amount of noise.

We *Rose* to a current state-of-the-art noisy program synthesis system [Handa and Rinard 2020] using the SyGuS 2018 benchmark suite [Alur et al. 2013]. Our evaluation demonstrates that *Rose* significantly outperforms this previous system: on two noisy benchmark problem sets drawn from the SyGus 2018 benchmark suite, *Rose* delivers speedups of up to 1587 and 81.7, with median speedups of 20.5 and 81.7. *Rose* also terminates on 20 (out of 54) and 4 (out of 11) more benchmark problems than the previous system. Both systems synthesize programs that are optimal over the provided noisy data sets. For the majority of the problems in the benchmark sets (272 out of 286), both systems also synthesize programs that produce correct outputs for all inputs in the original (unseen) noise-free data set. These results highlight the significant benefits that *Rose* can deliver for effective noisy program synthesis.

**Contributions:** This paper makes the following key contributions:

- **New Abstraction Technique:** It presents a new program synthesis technique for synthesizing programs over noisy datasets. This technique uses the abstract semantics of DSL constructs to partition the program search space. For each partition, the technique uses the abstract semantics to compute an abstract value representing the outputs of all programs in that partition. The abstract value also allows the technique to soundly estimate the minimum possible loss value over programs all programs in each partition.

- **New Refinement Technique:** It presents a new refinement technique that works with the sound approximation of the minimum loss values to refine the current partition, then discard partitions that cannot possibly contain the optimal program. Iteratively applying this refinement technique delivers the program with the optimal loss function over the given input/output examples.

- **Rose Evaluation:** It presents the *Rose* synthesis system, which implements the new abstraction and refinement techniques presented in this paper. Our experimental evaluation shows that *Rose* delivers substantial performance improvements over a current state of the art noisy program synthesis system [Handa and Rinard 2020] (Section 5). In comparison with this previous system, these performance improvements result in substantially smaller overall program synthesis times and many fewer synthesis timeouts.

## 2 Preliminaries

We first review the noisy program synthesis framework (introduced by [Handa and Rinard 2021, 2020]), the concepts associated with this framework, and the conditions that qualify a program to be the correct solution to a synthesis problem. We also discuss the tree automata based noisy program synthesis technique proposed by [Handa and Rinard 2020].

### 2.1 Finite Tree Automata

**Finite Tree Automata** are a type of state machine which accept trees rather than strings. They generalize standard finite automata to describe a regular language over trees.

**Definition 2.1 (FTA).** A (bottom-up) Finite Tree Automaton (FTA) over alphabet $F$ is a tuple $\mathcal{A} = (Q, F, Q_f, \Delta)$ where $Q$ is a set of states, $Q_f \subseteq Q$ is the set of accepting states and $\Delta$ is a set of transitions of the form $f(q_1, \ldots, q_k) \rightarrow q$ where $q, q_1, \ldots, q_k$ are states, $f \in F$. 

Every symbol \( f \) in the set \( F \) has an associated arity. The set \( F_k \subseteq F \) is the set of all \( k \)-arity symbols in \( F \). 0-arity terms \( t \) in \( F \) are viewed as single node trees (leaves of trees). \( t \) is accepted by an FTA if we can rewrite \( t \) to some state \( q \in Q_F \) using rules in \( \Delta \). We use the notation \( \mathcal{L}(\mathcal{A}) \) to denote the language of an FTA \( \mathcal{A} \), i.e., the set of all trees accepted by \( \mathcal{A} \).

**Example 2.2.** Consider the tree automaton \( \mathcal{A} \) defined by states \( Q = \{ q_T, q_F \} \), \( F_0 = \{ \text{True}, \text{False} \} \), \( F_1 = \text{not} \), \( F_2 = \{ \text{and} \} \), final states \( Q_F = \{ q_T \} \) and the following transition rules \( \Delta \):

\[
\begin{align*}
\text{True} & \rightarrow q_T & \text{False} & \rightarrow q_F & \text{not}(q_T) & \rightarrow q_F & \text{not}(q_F) & \rightarrow q_T \\
\text{or}(q_T, q_T) & \rightarrow q_T & \text{or}(q_F, q_T) & \rightarrow q_T & \text{and}(q_T, q_T) & \rightarrow q_F & \text{and}(q_T, q_F) & \rightarrow q_F & \text{and}(q_F, q_F) & \rightarrow q_F
\end{align*}
\]

The above tree automaton accepts all propositional logic formulas over True and False which evaluate to True.

**2.2 Domain Specific Languages (DSLs)**

The noisy program synthesis framework uses a Domain Specific Language (DSL) to specify to the set of programs under consideration. Without loss of generality, we assume all programs \( p \) are of the form \( \lambda x. e \) (i.e., they take a single input \( x \) ), where \( e \) is a parse trees in a Context Free Grammar \( G \). The internal nodes of this parse tree represent function and the leaves represent constants or variable \( x \). \( \llbracket p \rrbracket x_j \) denotes the output of \( p \) on input value \( x_j \) (\( \llbracket \cdot \rrbracket \) is defined in Figure 1).

All valid programs (which can be executed) are defined by a DSL grammar \( G = (T, N, P, s_0) \) where:

- \( T \) is a set of terminal symbols. These include constants and input variable \( x \). We use the notation \( \mathcal{T}_C \) to denote the set of constants in \( T \).
- \( N \) is the set of non-terminals that represent subexpressions in our DSL.
- \( P \) is the set of production rules of the form \( s \rightarrow f(s_1, \ldots, s_n) \), where \( f \) is a built-in function in the DSL and \( s, s_1, \ldots, s_n \) are non-terminals in the grammar.
- \( s_0 \in N \) is the start non-terminal in the grammar.

We assume that we are given a black box implementation of each built-in function \( f \) in the DSL. In general, all techniques explored within this paper can be generalized to any DSL which can be specified within the above framework.

**Example 2.3.** The following DSL defines expressions over input \( x \), constants 2 and 3, and addition and multiplication:

\[
\begin{align*}
n & := x \mid n + t \mid n \times t; \\
t & := 2 \mid 3;
\end{align*}
\]

We use the notation \( p[x] \) to denote \( \llbracket p \rrbracket x \). Given a vector of input values \( \mathbf{x} = \langle x_1, \ldots, x_n \rangle \), we use the notation \( p[\mathbf{x}] \) to denote the vector \( \langle p[x_1], \ldots, p[x_n] \rangle \).
2.3 Loss Functions

Given a noisy input-output example \((x, y)\) and a program \(p\), a Loss Function \(L(p[x], y)\) measures how incorrect the program is with respect to the given example. The loss function only depends on the noisy output \(y\) and the output of the program \(p\) on the input \(x\). Given a noisy dataset \(\mathcal{D} = \langle \bar{x}, \bar{y} \rangle\) (where \(\bar{x} = \langle x_1, \ldots, x_n \rangle\) and \(\bar{y} = \langle y_1, \ldots, y_n \rangle\)), we use the notation \(L(p[\bar{x}], \bar{y})\) to denote the sum of loss of program \(p\) on individual corrupted input-output example, i.e.,

\[
L(p[\bar{x}], \bar{y}) = \sum_{i=1}^{n} L(p[x_i], y_i)
\]

**Definition 2.4. 0/1 Loss Function:** The 0/1 loss function \(L_{0/1}(p[x], y)\) returns 0, if \(p\) agrees with an input-output example \((x, y)\), else it returns 1:

\[
L_{0/1}(p[x], y) = 1 \text{ if } (y \neq p[x]) \text{ else } 0
\]

Given a noisy dataset \(\mathcal{D}\), \(L_{0/1}\) counts the number of input-output examples where \(p\) does not agree within the data set.

**Definition 2.5. 0/∞ Loss Function:** The 0/∞ loss function \(L_{0/\infty}(p[x], y)\) is 0 if \(p\) matches the output \(y\) on input \(x\) and \(\infty\) otherwise:

\[
L_{0/\infty}(p[x], y) = 0 \text{ if } (y = p[x]) \text{ else } \infty
\]

**Definition 2.6. Damerau-Levenshtein (DL) Loss Function:** The DL loss function \(L_{DL}(p[x], y)\) uses the Damerau-Levenshtein metric [Damerau 1964] to measure the distance between the output from the synthesized program and the corresponding output in the noisy data set:

\[
L_{DL}(p[x], y) = L_{p[x], y}(|p[x]|, |y|)
\]

where, \(L_{a,b}(i, j)\) is the Damerau-Levenshtein metric [Damerau 1964].

This distance metric counts the number of single character deletions, insertions, substitutions, or transpositions required to convert one input string into another. We use this loss function to work with input-output examples containing human-provided text, as more than 80% of all human misspellings are reported to be captured by a single one of these four operations [Damerau 1964].

2.4 Complexity Measure

Given a program \(p\), a Complexity Measure \(C(p)\) scores a program \(p\) based on their complexity. This score is independent of the noisy dataset \(\mathcal{D}\). Within the noisy program synthesis framework, complexity measure is used to trade off performance on the noisy data set vs. complexity of the synthesized program. Formally, a complexity measure \(C(p)\) maps each program \(p\) to a real number.

As standard in noisy program synthesis literature [Handa and Rinard 2020], we work with complexity measure of the form \(\text{Cost}(p)\), which computes the complexity of given program \(p\) represented as a parse tree recursively as follows:

\[
\text{Cost}(t) = \text{cost}(t)
\]

\[
\text{Cost}(f(e_1, e_2, \ldots e_k)) = \text{cost}(f) + \sum_{i=1}^{k} \text{Cost}(e_i)
\]

where \(t\) and \(f\) are terminals and built-in functions in our DSL respectively. A popular example of a complexity measure, expressed in this form, is Size\((p)\), which assigns the cost of a terminal and function as 1 (\(\text{cost}(t) = \text{cost}(f) = 1\)).

Given an FTA \(\mathcal{A}\), we can synthesize the minimum complexity parse tree (as measured by complexity measures of the form \(\text{Cost}(p)\)) accepted by \(\mathcal{A}\) using a dynamic programming based algorithm proposed by [Gallo et al. 1993] (see Section 6 in [Wang et al. 2017a] for more details).
2.5 Objective Functions

An **Objective Function**, within the noisy program synthesis framework, combines the complexity score of a program and its loss over a noisy dataset, to be a combined score. The objective function is used to compare different programs in the DSL and synthesize the program which *best-fits* the noisy dataset. Formally, an objective function $U$ maps loss and complexity tuples $\langle l, c \rangle$ to a totally ordered set, such that, for all $l$ and $c$, $U(\langle l, c \rangle)$ is **monotonically non-decreasing** with respect to $c$ and $l$.

**Definition 2.7. Tradeoff Objective Function:** Given a tradeoff parameter $\lambda > 0$, the tradeoff objective function $U_\lambda(\langle l, c \rangle) = l + \lambda c$. This objective function allows us to trade-off between complexity and loss, directing the synthesis algorithm to search for a simpler program, in lieu of increasing the loss over the dataset.

**Definition 2.8. Lexicographic Objective Function:** A lexicographic objective function $U_L(\langle l, c \rangle) = \langle l, c \rangle$ maps $l$ and $c$ into a lexicographically ordered space, i.e., $\langle l_1, c_1 \rangle < \langle l_2, c_2 \rangle$ if and only if either $l_1 < l_2$ or $l_1 = l_2$ and $c_1 < c_2$. This objective function first minimizes the loss, then the complexity.

We will use the notation $\langle l_1, c_1 \rangle \leq_U \langle l_2, c_2 \rangle$ to denote $U(\langle l_1, c_1 \rangle) \leq U(\langle l_2, c_2 \rangle)$.

2.6 Program Synthesis over Noisy Datasets

Given a noisy dataset $D = (\vec{x}, \vec{y})$, the noisy programs synthesis aims to find a program $p$ which *best-fits* the dataset $D$, where the *best-fit* is defined by the loss function $L$, complexity measure $C$, and objective function $U$. Formally, given a DSL $G$, we wish to find a program $p^* \in G$ which minimizes the objective function $U$ (parameterized by $L$ and $C$), i.e.,

$p^* \in \arg\min_{p \in G} U(L(p[\vec{x}], \vec{y}), C(p))$

The above condition is equivalent to:

$\forall p \in G. \langle L(p[\vec{x}], \vec{y}), C(p) \rangle \leq_U \langle L(p[\vec{x}], \vec{y}), C(p) \rangle$

**Concrete Finite Tree Automata:** The noisy synthesis algorithm introduced by [Handa and Rinard 2020] builds upon the concept of a Concrete Finite Tree Automaton (CFTA). Given a domain specific language $G$ and inputs $\vec{x}$, a Concrete Finite Tree Automaton is a tree automaton which accepts all abstract syntax trees representing DSL programs. A CFTA is constructed using the rules in Figure 2. The states of the CFTA is of the form $q_\vec{s}$, where $s$ is a symbol in $G$ and $\vec{s}$ is vector of values. The existence of a state $q_\vec{s}$ implies there exists a sub-expression in $G$, starting from symbol $s$, which maps inputs $\vec{x}$ to output $\vec{s}$. There exists a transition $f(q_{s_1}^{\vec{v}_1}, \ldots, q_{s_k}^{\vec{v}_k}) \rightarrow q_s^{\vec{s}}$ in the CFTA, only if, for all $i = [1, |\vec{s}|].f(v_{1i}, \ldots, v_{ki}) = v_i$.

The Var Rule states that if we have an input symbol $x$, we construct a state $q_{\vec{x}}^x \in Q$, where $\vec{x}$ is the input vector. The Const Rule states that for any constant (non variable terminal), we construct a state $q_{t}^{\vec{s}} \in Q$, where $\vec{s}$ is a vector of size equal to $\vec{x}$ with each entry as $[1]$. The Final Rule states that, given a start symbol $s_0$, all states with symbol $s_0$ are added to $Q_f$. The Prod Rule states that if we have a production $s \rightarrow f(s_1, \ldots, s_n) \in P$, and there exists states $q_{s_1}^{\vec{v}_1}, \ldots, q_{s_k}^{\vec{v}_k} \in Q$, then we add a state $q_s^{\vec{s}} \in Q$, where for all $i = [1, |\vec{s}|].f(v_{1i}, \ldots, v_{ki}) = v_i$. We also add a transition $f(q_{s_1}^{\vec{v}_1}, \ldots, q_{s_k}^{\vec{v}_k}) \rightarrow q_s^{\vec{s}}$ in the transition set $\Delta$.

Given a CFTA $(Q, Q_f, \Delta)$ (constructed using rules in Figure 2) and a state $q_{s_0}^{\vec{s}} \in Q_f$, a tree representing program $p$ is accepted by automaton $(Q, \{q_{s_0}^{\vec{s}}\}, \Delta)$ if and only if $p[\vec{x}] = \vec{s}$ ([Handa and Rinard 2020]).
We next introduce our synthesis algorithm which builds upon the concept of a Finite Tree Automaton which uses abstract values instead of concrete values, as used in Figure 3. The algorithm then minimizes the objective function, i.e., the program which optimizes the objective function.

The algorithm first constructs a Concrete Finite Tree Automaton (line 2) based on the rules presented in Figure 2. Given this Concrete Finite Tree Automaton (line 3-4). Given an accepting state of the form \(\langle \circ_0, \ldots, \circ_k \rangle\), the following statement is true:

\[
P[\circ_0] = \arg\min_{p \in G[\circ \mapsto \circ]} C(p)
\]

where \(G[\circ \mapsto \circ] = \{p | p \in G, p[\circ] = \circ\}\).

The algorithm then finds an accepting state \(q^0 \in Q_f\), such that, for all accepting states \(q^0 \in Q_f\),

\[
\langle L(\circ^*, \circ), C(P[q^0]) \rangle \leq_U \langle L(\circ, \circ), C(P[q^0]) \rangle
\]

Since, for all programs \(p \in G\), there exists an accepting state \(q^0\), such that, \(p\) is accepted by \((Q, \{q^0\}, \Delta) ([Handa and Rinard 2020])\), the following statement is true:

\[
\langle L(\circ^*, \circ), C(P[q^0]) \rangle \leq_U \langle L(\circ, \circ), C(P[q^0]) \rangle \leq_U \langle L(\circ, \circ), C(p) \rangle
\]

Therefore,

\[
P[q^0] \in \arg\min_{p \in G} U(L(p[\circ], \circ), C(p))
\]

In general, the rules in Figure 2 may result in a FTA which has infinitely many states. [Handa and Rinard 2020] handles this by only adding a new state within the constructed FTA if the height of the smallest tree it will accept is less than some threshold height.

**Synthesis Algorithm:** Figure 3 presents a simplified version of the noisy synthesis algorithm presented in [Handa and Rinard 2020]. The algorithm, given a noisy dataset \(D = (\bar{x}, \bar{y})\), a DSL \(G\), an objective function \(U\), a loss function \(L\), and a complexity measure \(C\), synthesizes a program which minimizes the objective function, i.e., the program \(p^*\) returned by the algorithm satisfies the following constraint:

\[
p^* \in \arg\min_{p \in G} U(L(p[\bar{x}], \bar{y}), C(p))
\]

Theorem 3.1 \([Handa and Rinard 2020]\) states that for all programs \(p \in G\), there exists an accepting state \(q^0\), such that, \(p\) is accepted by \((Q, \{q^0\}, \Delta) ([Handa and Rinard 2020])\), the following statement is true:

\[
\langle L(\circ^*, \circ), C(P[q^0]) \rangle \leq_U \langle L(\circ, \circ), C(P[q^0]) \rangle \leq_U \langle L(\circ, \circ), C(p) \rangle
\]

Therefore,

\[
P[q^0] \in \arg\min_{p \in G} U(L(p[\bar{x}], \bar{y}), C(p))
\]

### 3 Noisy Program Synthesis using Abstraction Refinement Based Optimization

We next introduce our synthesis algorithm which builds upon the concept of a Finite Tree Automaton which uses abstract values instead of concrete values, as used in Figure 3. The algorithm then uses an abstraction refinement based optimization technique to direct the algorithm towards the program which optimizes the objective function.
procedure Synthesize(D, G, U, L, C)
  input: Noisy Dataset D = (x, y), DSL G.
  input: Objective Function U, Loss Function L, and Complexity metric C.
  output: A program p*, such that, \(\forall \ p \in G, \langle L(p[x, \bar{y}], C(p)) \rangle \leq_U \langle L(p[x, \bar{y}], C(p)\rangle).

Fig. 3. Algorithm for noisy program synthesis using Finite Tree Automaton.

3.1 Abstractions

We construct an abstract version of the Finite Tree Automaton by associating abstract values with each symbol. We assume that the abstract values are represented as conjunctions of predicates of the form \(f(s) \text{ op c}, \) where \(s\) is a symbol in the given DSL, \(f\) is a function, \(\text{op}\) is an operator, and \(c\) is a constant.

**Universe of predicates:** Given a DSL, our algorithm is parameterized by a universe \(U\) of predicates, that our algorithm uses to construct abstractions for our synthesis algorithm. The universe \(U\) is specified using a family of function \(F\), a set of operators \(O\), and a set of constants \(C\), such that, all predicate in the universe \(U\) can be written as \(f(s) \text{ op c}, \) where \(f \in F, \) \(\text{op} \in O, \) \(c \in C, \) and \(s\) is a symbol in the DSL (except predicates true and false). We assume that \(F\) contains the identity function, \(O\) contains equality, and \(C\) includes the set of all values that can be computed by any sub-expression within the DSL \(G\).

**Notation:** Given predicates \(P \subseteq U\) and an abstract value \(\varphi \in U\), we use \(\alpha^P(\varphi)\) to denote the strongest conjunctions of predicates in \(P\), such that \(\varphi \Rightarrow \alpha^P(\varphi)\). Given a vector of abstract values \(\vec{\varphi} = (\varphi_1, \ldots, \varphi_n), \) \(\alpha^P(\varphi)\) denotes the vector \((\alpha^P(\varphi_1), \ldots, \alpha^P(\varphi_n))\). As standard in abstract interpretation literature [Cousot and Cousot 1977], we use the notation \(\gamma(\varphi)\) to denote the set of concrete values represented by the abstract value \(\varphi\).

**Abstract semantics:** In addition to concrete semantics for each DSL construct, we are given abstract semantics of each DSL construct in the form of symbolic post-conditions over the universe of predicates \(U\). Given a production \(s \rightarrow f(s_1, \ldots, s_n)\), we use the notation \([f(\varphi_1, \ldots, \varphi_k)]=\varphi\) to represent the abstract semantics of function \(f\), i.e., \([f(\varphi_1, \ldots, \varphi_k)]=\varphi\) if the function \(f\) returns \(\varphi\) (for symbol \(s\)), given abstract values \(\varphi_1, \ldots, \varphi_k\) for arguments \(s_1, \ldots, s_k\). We assume that the abstract semantics are sound, i.e.,

\[
[f(\varphi_1, \ldots, \varphi_k)]=\varphi \text{ and } v_1 \in \gamma(\varphi_1), \ldots, v_k \in \gamma(\varphi_k) \Rightarrow [v_1, \ldots, v_k]\in \gamma(\varphi)
\]

However, we do not require the abstract semantics to be precise, i.e., formally:

\[
[f(\varphi_1, \ldots, \varphi_k)]=\varphi, \text{there may exist a } v \in \varphi, \text{s.t., } \exists v_1 \in \varphi_1, \ldots, v_k \in \varphi_k, [f(v_1, \ldots, v_k)] = v
\]

There may exist concrete value \(v\) in the abstract output \(\varphi\), such that, no concrete input parameters \(v_1, \ldots v_k\) in the abstract inputs \(\varphi_1, \ldots \varphi_k\) exist, for which \(f(\varphi_1, \ldots, \varphi_k) = v\).
Given a loss function, the abstract semantics of a loss function allows us to find the minimum possible loss value for a given abstract value, i.e., given a loss function $L$, noisy output $y$, and an abstract value $\varphi$:

$$L(\varphi, y) = \min_{z \in y(\varphi)} L(z, y)$$

### 3.2 Abstract Finite Tree Automaton

An Abstract Finite Tree Automata (AFTA) generalizes an Concrete Finite Tree Automaton by replacing concrete values by abstract values while constructing automaton states. This allows us to compress the size of an FTA, as multiple states with concrete values can be represented by a single state with abstract value.

Given predicates $\mathcal{P}$, DSL $G$, and inputs $\vec{x}$, Figure 5 presents the rules for constructing an AFTA $(Q, Q_f, \Delta)$. States in an AFTA are of the form $q^\varphi_s$, where $s$ is a symbol and $\varphi$ is a vector of abstract values. If $q^\varphi_s \in Q$, then there exists an expression $e$, starting from symbol $s$, such that $[e]^{\mathcal{P}} \vec{x} = \varphi$. If there is a transition $f(q^\varphi_{s_1}, \ldots, q^\varphi_{s_k}) \rightarrow q^\varphi_s$ in the AFTA then

$$\forall j = [1, |\varphi|].f(\varphi_{1j}, \ldots, \varphi_{kj}) \implies \varphi_j$$

The Var Rule states constructs a state $q^\varphi_x \in Q$ for variable symbol $x$, where $\varphi = a^\mathcal{P}((x = x_1), \ldots (x = x_n))$. The Const Rule constructs a state $q^\varphi_t \in Q$ for each constant terminal $t$, where $\varphi$ is a vector of size equal to $\vec{x}$ with each entry as $a^\mathcal{P}(t = [t])$. The Final Rule adds all states with symbol $s_0$ are added to $Q_f$, where $s_0$ is the start symbol. The Prod Rule constructs a state $q^\varphi_s \in Q$, if there exists a production $s \rightarrow f(s_1, \ldots, s_n) \in P$ and there exists states $q^\varphi_{s_1}, \ldots, q^\varphi_{s_k} \in Q$ (where $\forall i = [1, |\varphi|].f(\varphi_{i1}, \ldots, \varphi_{ki}) \implies \varphi_i$). We also add a transition $f(q^\varphi_{s_1}, \ldots, q^\varphi_{s_k}) \rightarrow q^\varphi_s$ in $\Delta$.

**Theorem 3.1.** (Structure of the Tree Automaton) Given a set of predicates $\mathcal{P}$, input vector $\vec{x} = \langle x_1, \ldots, x_n \rangle$, and DSL $G$, let $\mathcal{A} = (Q, Q_f, \Delta)$ be the AFTA returned by the function ConstructAFTA$(\vec{x}, G, \mathcal{P})$. Then for all symbols $s$ in $G$, for all expressions $e$ starting from symbol $s$ (and height less than bound $b$), there exists a state $q^\varphi_e \in Q$, such that, $e$ is accepted by the automaton $(Q, \{q^\varphi_e\}, \Delta)$, where $\varphi_e = [e]^{\mathcal{P}} x_1, \ldots [e]^{\mathcal{P}} x_n$. 

![Fig. 4. Abstract Execution semantics for program p.](image-url)
We present the implementation of procedure

\[ \varphi = \alpha^P (x = x_1, \ldots, x = x_n) \]  

(VAR) \[ q_x^\varphi \in Q \]  

(FINAL) \[ \Delta \]

3.3 Synthesis Algorithm

We present our synthesis algorithm in Figure 6. The Synthesize procedure takes a noisy dataset \( \mathcal{D} \), a DSL \( G \), a threshold \( \epsilon \geq 0 \), initial predicates \( \mathcal{P} \), a universe of possible predicates \( \mathcal{U} \), objective function \( U \), loss function \( L \), and a complexity measure \( C \). We assume that true, false \( \in \mathcal{P} \).

The synthesis algorithm consists of a refinement loop (line 3-10). The loop first constructs a Abstract Finite Tree Automaton (line 4) with the current set of predicates \( P \) (line 5). The algorithm maintains the program \( p \), which is the best candidate program out of all the candidate programs \( p^* \) generated.

If the distance between the current program \( p^* \) and the best possible program in the DSL \( G \) is less a tolerance level \( \epsilon \) (Distance function, line 8), the algorithm returns the best candidate program \( p^* \). Note that

\[ \langle L(p^* [\bar{x}], \bar{y}), C(p^*) \rangle \leq_U \langle L(p^* [\bar{x}], \bar{y}), C(p^*) \rangle \]

Otherwise, the algorithm refine our AFTA to either improve our estimation of the best possible program or synthesize a better candidate program. To refine our AFTA, the algorithm first picks an input-output example \((x, y)\) from dataset \( \mathcal{D} \), on which we can improve the candidate program \( p^* \) (line 9). Given an input-output example \((x, y)\), the procedure OptimizeAndBackpropagate constructs the constraints required to improve the AFTA and then returns the set of predicates which will allow the algorithm to build a more refined AFTA.

We discuss each of these sub-procedures in detail next.

3.4 Minimum Cost Candidate

We present the implementation of procedure MinCost in Figure 7. Given an AFTA \((Q, Q_f, \Delta)\), noisy dataset \( \mathcal{D} = (\bar{x}, \bar{y}) \), objective function \( U \), loss function \( L \), and complexity measure \( C \), MinCost returns a program \( p_{\min} \) which minimizes the abstract objective function, where, given a program \( p \), the abstract objective function is defined as

\[ U(L(\|p\|^P \bar{x}, \bar{y}), C(p)) \]
Therefore, for all programs \( p \in G \):

\[
\langle L(\|p\|_P \bar{x}, \bar{y}), C(p) \rangle \leq_U \langle L(\|p\|_P \bar{x}, \bar{y}), C(p) \rangle
\]

Note that, since for all programs \( p \), \( L(\|p\|_P \bar{x}, \bar{y}) \leq L(p[\bar{x}], \bar{y}) \), the following statement is true:

\[
\langle L(\|p\|_P \bar{x}, \bar{y}), C(p) \rangle \leq_U \langle L(p[\bar{x}], \bar{y}), C(p) \rangle
\]

Hence, for programs \( p \in G \):

\[
\langle L(\|p\|_P \bar{x}, \bar{y}), C(p) \rangle \leq_U \langle L(p[\bar{x}], \bar{y}), C(p) \rangle
\]

The procedure first finds the least complex program (i.e., program which minimizes the complexity measure) for each accepting state \( q \in Q_f \) (line 2-3). Given an accepting state of the form \( q^\phi_{s_0} \), a program \( p \in G \) is accepted by the automaton \((Q, \{q^\phi_{s_0}\}, \Delta)\) if and only if \( \|p\|_P \bar{x} = \bar{\phi} \). Given an
accepting $q^*_s$, $P[q^*_s]$ is the least complex program which maps input vector $\vec{x}$ to outputs $\vec{\phi}$, i.e.,

$$P[q^*_s] \in \arg\min_{p \in G[\vec{x} \rightarrow \vec{\phi}]} C(p)$$

where $G[\vec{x} \rightarrow \vec{\phi}] = \{p | p \in G, \|p\|^P \vec{x} = \vec{\phi}\}$.

The algorithm then finds an accepting state $q^* \in Q_f$, such that, for all accepting states $q^*_s \in Q_f$,

$$\langle L(\vec{\phi}, \vec{y}), C(P[q^*_s]) \rangle \leq_U \langle L(\vec{\phi}, \vec{y}), C(P[q^*_s]) \rangle$$

**Theorem 3.3.** Given predicates $\mathcal{P}$, DSL $G$, noisy dataset $\mathcal{D} = (\vec{x}, \vec{y})$, objective function $U$, loss function $L$, complexity measure $C$, and $\mathcal{A} = \text{ConstructAFTA}(\vec{x}, G, \mathcal{P})$, if $p^* = \text{MinCost}(\mathcal{A}, \mathcal{D}, U, L, C)$ then

$$p^* \in \arg\min_{p \in G} U(L(\|p\|^P \vec{x}, \vec{y}), C(p))$$

i.e., $p^*$ minimizes the abstract objective function.

We present the proof of this theorem in the appendix A.1 (Theorem A.4).

### 3.5 Termination Condition and Tolerance

Given a candidate program $p^*$, predicates $\mathcal{P}$, noisy dataset $\mathcal{D} = (\vec{x}, \vec{y})$, and a loss function $L$, the Distance function returns the difference between the concrete loss of program $p^*$ over noisy dataset $\mathcal{D}$ and the abstract loss (given predicates from $\mathcal{P}$) over noisy dataset $\mathcal{D}$. Formally:

$$\text{Distance}(p, (\vec{x}, \vec{y}), \mathcal{P}, L) := L(p[\vec{x}], \vec{y}) - L(\|p\|^P \vec{x}, \vec{y})$$

The algorithm terminates if the distance is less than equal to the tolerance level $\epsilon$. Note that if $\epsilon = 0$, then the algorithm only terminates when $L(p^*[\vec{x}], \vec{y}) = L(\|p^*\|^P \vec{x}, \vec{y})$, and since for all program $p \in G$:

$$\langle L(\|p\|^P \vec{x}, \vec{y}), C(p) \rangle \leq_U \langle L(p[\vec{x}], \vec{y}), C(p) \rangle$$

is true, the following statement is also true:

$$p^* \in \arg\min_{p \in G} U(L(p[\vec{x}], \vec{y}), C(p))$$

In general, the previous work in noisy program synthesis has maintained a very strict version of correctness, i.e., they generally synthesize a program $p^*$ which minimizes the objective function. This leaves out any speedups which can be achieved by relaxing the requirement to synthesizing a program which is close to the optimal program but may not be one of the optimal programs.

To capture this relaxation, we introduce the concept of $\epsilon$-correctness. Given DSL $G$, a noisy dataset $\mathcal{D} = (\vec{x}, \vec{y})$, an objective function $U$, a loss function $L$, a complexity measure $C$, and set of programs $G_{\rho} \subseteq G$, let $B_\epsilon(G_p)$ be a set of programs in $G$, such that, $p^* \in B_\epsilon(G_p)$ if and only if there exists a program $p \in G_p$, such that,

$$\langle L(p^*[\vec{x}], \vec{y}), C(p^*) \rangle \leq_U \langle L(p[\vec{x}], \vec{y}) + \epsilon, C(p) \rangle$$

i.e., $p^*$ will be a better fit the dataset $\mathcal{D}$ compared to program $p$, if the loss of $p$ over dataset $\mathcal{D}$ was increased by $\epsilon$.

**Definition 3.4.** ($\epsilon$-correctness) Given a noisy dataset $\mathcal{D} = (\vec{x}, \vec{y})$, a DSL $G$, an objective function $U$, a loss Function $L$, and a complexity measure $C$, a program $p_\epsilon \in G$ is $\epsilon$-correct if and only if:

$$p_\epsilon \in B_\epsilon(\arg\min_{p \in G} U(L(p[\vec{x}], \vec{y}), C(p)))$$
A program $p_r \in G$ is \textit{\(\epsilon\)-correct} if and only if there exists a $p^* \in G$, such that,
\[
\langle L(p_r, [x], \bar{y}), C(p_r) \rangle \leq_U \langle L(p^*, [x], \bar{y}), C(p^*) \rangle
\]
\[
\forall p \in G. \langle L(p^* [x] - \epsilon), C(p^*) \rangle \leq_U \langle L(p[x], \bar{y}), C(p) \rangle
\]

Note that, for $\epsilon = 0$, the above condition reduces to
\[
\forall p \in G. \langle L(p_r, [x], \bar{y}), C(p_r) \rangle \leq_U \langle L(p[x], \bar{y}), C(p) \rangle
\]

Therefore, for $\epsilon = 0$, $p_r \in \text{argmin}_{p \in G} U(L(p[x], \bar{y}), C(p))$.

\textbf{Theorem 3.5. (Soundness)} Given a dataset $\mathcal{D}$, a DSL $G$, tolerance $\epsilon \geq 0$, universe of predicates $\mathcal{U}$, initial predicates $\mathcal{P}$, objective function $U$, loss function $L$, and the complexity measure $C$, if Algorithm 6 returns the program $p^*$, then $p^*$ satisfies the $\epsilon$-correctness condition (Definition 3.4).

\textbf{Proof.} Let us assume that the algorithm terminates on the $i^{th}$ iteration. Let $\mathcal{A}_i = (Q, Q_f, \Delta)$ be the AFTA when the algorithm terminates. Let $p_i$ be the program returned by MinCost on the $i^{th}$ iteration.

From Theorem 3.3, for all programs $p \in G$,
\[
\langle L([p], \mathcal{P}, \mathcal{P}, [x], \bar{y}), C(p_i) \rangle \leq_U \langle L(p[x], \bar{y}), C(p) \rangle
\]

When the algorithm terminates, the following condition is true:

Distance($p_i$, $\mathcal{D}$, $\mathcal{P}$, $L$) \leq \epsilon

which implies:
\[
L(p_i[x], \bar{y}) - L([p_i], \mathcal{P}, [x], \bar{y}) \leq \epsilon
\]

Therefore, for all programs $p \in G$,
\[
\langle L(p_i[x], \bar{y}) - \epsilon, C(p_i) \rangle \leq_U \langle L(p[x], \bar{y}), C(p) \rangle
\]

and the following is true for the synthesized program $p_r$,
\[
\langle L(p_r[x], \bar{y}), C(p_r) \rangle \leq_U \langle L(p_i[x], \bar{y}), C(p_i) \rangle
\]

Hence, if the algorithm 6 returns a program $p_r$, then $p_r$ satisfies the $\epsilon$-correctness condition. \qed

\subsection{3.6 Abstraction Refinement based Optimization}

Given a dataset $\mathcal{D}$ and predicates $\mathcal{P}$, the program $p^*$ (returned by MinCost) minimizes the \textit{abstract} objective function, i.e., for all programs $p \in G$:
\[
\langle L([p^*], \mathcal{P}, [x], \bar{y}), C(p^*) \rangle \leq_U \langle L([p], \mathcal{P}, [x], \bar{y}), C(p) \rangle \leq_U \langle L(p[x], \bar{y}), C(p) \rangle
\]

Since, the algorithm did not terminate, Distance($p^*$, $\mathcal{P}$, $\mathcal{D}$, $L$) > $\epsilon$.

Let us consider the case when $\epsilon = 0$. Since Distance($p^*$, $\mathcal{P}$, $\mathcal{D}$, $L$) > 0, the concrete loss of program $p^*$ over dataset $\mathcal{D}$ is greater than the abstract loss of $p^*$ over $\mathcal{D}$. Formally,
\[
L(p^* [x], \bar{y}) > L([p^*], \mathcal{P}, [x], \bar{y})
\]

This means that
\[
\langle L([p^*], \mathcal{P}, [x], \bar{y}), C(p^*) \rangle <_U \langle L(p^* [x], \bar{y}), C(p^*) \rangle
\]

At this point, even though, for all programs $p \in G$, the following is true:
\[
\langle L([p^*], \mathcal{P}, [x], \bar{y}), C(p^*) \rangle \leq_U \langle L(p[x], \bar{y}), C(p) \rangle
\]

We cannot prove that $p^*$ is the optimal function, i.e.,
\[
\langle L(p^* [x], \bar{y}), C(p^*) \rangle \leq_U \langle L(p[x], \bar{y}), C(p) \rangle
\]

And therefore, just using predicates $\mathcal{P}$, we cannot prove that $p_r$ is the optimal function.
Similarly, if $\epsilon > 0$, for all programs $p \in G$, the following is true:

$$\langle L(\llbracket p^* \rrbracket^P \tilde{x}, \tilde{y}), C(p^*) \rangle \leq_U \langle L(p_x), C(p) \rangle$$

But we cannot prove that the following statement is true:

$$\langle L(p^*[\tilde{x}] - \epsilon, \tilde{y}), C(p^*) \rangle \leq_U \langle L(p_x), C(p) \rangle$$

And therefore, just using predicates $P$, we cannot prove that $p^*$ is $\epsilon$-correct.

Therefore, in order to find the optimal program and prove its optimality, we have to expand the set of predicates $P$.

To achieve this goal, the algorithm first selects an input-output example from the noisy dataset $D$ on which can improve the difference between the abstract loss and the concrete loss of programs using procedure PickDimension. Given an input-output example $(x, y)$, the idea here is to expand the set of predicates $P$ to $P'$, such that:

$$L(\llbracket p^* \rrbracket^P x, y) < L(\llbracket p^* \rrbracket^{P'} x, y) \leq L(p^*[x], y)$$

Thus improving our estimation of the abstract loss function for programs in $G$.

The algorithm allows us to plug any implementation of the procedure PickDimension, assuming it satisfies the following constraint:

$$\langle x_i, y_i \rangle = \text{PickDimension}(p, (\tilde{x}, \tilde{y}), P, L) \implies L(p_{x_i}, y_i) > L(\llbracket p \rrbracket{P} x_i, y_i)$$

Since $L(p^*[\tilde{x}], \tilde{y}) - L(\llbracket p^* \rrbracket^P \tilde{x}, \tilde{y}) > \epsilon$ (as $\text{Distance}(p^*, D, P, L) > \epsilon$), there exists at least one $i \in [1, n]$, such that,

$$L(p^*[x_i], y_i) > L(\llbracket p \rrbracket^P x_i, y_i)$$

If multiple input-output examples exist for which the abstract loss is less than the concrete loss, an implementation of PickDimension can return any one of them and our synthesis algorithm will use that example to optimize the automaton.

Given the input-output example $(x, y)$, the algorithm uses the procedure OptimizeAndBackPropagate to expand the set of predicates to $P'$, such that

$$L(\llbracket p^* \rrbracket^P x, y) < L(\llbracket p^* \rrbracket^{P'} x, y) \leq L(p^*[x], y)$$

Figure 8 presents the OptimizeAndBackPropagate procedure. The procedure tries to find the strongest formula $\psi^*$, such that, $(s_0 = p[x]) \implies \psi^*$ and:

$$L(\llbracket p^* \rrbracket^P x, y) < L(\llbracket p^* \rrbracket^P x \land \psi^*, y)$$

Note that since $(s_0 = \llbracket p^* \rrbracket x) \implies \psi^*$ (line 7):

$$L(\llbracket p^* \rrbracket^P x \land \psi^*, y) \leq L(p^*[x], y)$$

**Theorem 3.6.** Given expression $e = f(e_1, \ldots, e_n)$, input $x$, abstract value $\psi_p$ (assuming $s = \llbracket e \rrbracket x$) $\implies \psi_p$, predicates $P$, and universe of predicates $U$, if the procedure BackPropagate($e, x, \psi_p, P, U$) returns predicate set $P_r$, then:

$$\llbracket e \rrbracket^{P \cup P_r} x \implies \psi_p$$

We present the proof of this theorem in the appendix A.1 (Theorem A.5).

**Theorem 3.7.** Let $P_r = \text{OptimizeAndBackPropagate}(p^*, x, y, P, U)$.

$$L(\llbracket p^* \rrbracket^P x, y) < L(\llbracket p^* \rrbracket x, y) \implies L(\llbracket p^* \rrbracket^P x, y) < L(\llbracket p^* \rrbracket^{P \cup P_r} x, y)$$
procedure \textsc{OptimizeAndBackPropagate}(p, x_j, y_j, \mathcal{P}, \mathcal{U}, L) \\
\textbf{input:} Program \( p \), input \( x_j \), noisy output \( y_j \), predicates \( \mathcal{P} \), universe of predicates \( \mathcal{U} \), and Loss Function \( L \). \\
\textbf{output:} A set of predicates \( \mathcal{P}_r \). \\
1: \quad \phi := \llbracket p \rrbracket^\mathcal{P} x_j; \quad \phi := (s_0 = \llbracket p \rrbracket x_j); \\
2: \quad \Phi := \{ q \in \mathcal{U} \mid \phi \implies q \}; \\
3: \quad \Psi := \Phi; \\
4: \quad \textbf{for } i = 1 \ldots m \textbf{ do} \quad \triangleright \text{ Use a maximum of } m \text{ predictates.} \\
5: \quad \Psi := \Psi \cup \{ \psi \land q \mid \psi \in \Psi, q \in \Phi \}; \\
6: \quad \psi^* := \phi; \\
7: \quad \textbf{for } \psi \in \Psi \textbf{ do} \\
8: \quad \textbf{if } \psi^* \implies \psi \text{ and } L(\psi, y_j) - L(\phi \land \psi, y_j) \leq \delta > 0 \text{ then } \psi^* := \psi; \\
9: \quad \triangleright \text{ The abstract loss is increased by atleast } \delta. \\
10: \quad \textbf{return} \text{ \textsc{ExtractPredicates}(\psi^*)} \cup \text{ \textsc{BackPropagate}(p, x_j, \phi \land \psi^*, \mathcal{P}, \mathcal{U})}; \\

Fig. 8. Algorithm for extracting predicates \( \mathcal{P}_r \) to refine the abstract value of \( p \), such that, \\
\( L(\llbracket p \rrbracket^\mathcal{P} x_j, y_j) < L(\llbracket p \rrbracket^{(\mathcal{P} \cup \mathcal{P}_r)} x_j, y_j) \).

procedure \textsc{BackPropagate}(f(e_1, \ldots, e_n), x_j, \psi_p, \mathcal{P}, \mathcal{U}) \\
\textbf{output:} A set of predicates \( \mathcal{P}_r \), such that, \( \llbracket f(e_1, \ldots, e_n) \rrbracket^{(\mathcal{P} \cup \mathcal{P}_r)} x_j \implies \psi_p \). \\
1: \quad \hat{\phi} := \llbracket e_1 \rrbracket^\mathcal{P} x_j, \ldots, \llbracket e_n \rrbracket^\mathcal{P} x_j; \quad \hat{\phi} := \llbracket e_1 \rrbracket^\mathcal{P} x_j, \ldots, \llbracket e_n \rrbracket^\mathcal{P} x_j; \\
2: \quad \hat{\Phi} := \{ \Phi_i \mid \Phi_i := \{ q \in \mathcal{U} \mid \phi_i \implies q \}; \Psi := \hat{\Phi}; \\
3: \quad \textbf{for } i = 1 \ldots m \textbf{ do} \quad \triangleright \text{ Use a maximum of } m \text{ predictates.} \\
4: \quad \Psi_i := \Psi \cup \{ \psi \land q \mid \psi \in \Psi_i, q \in \Phi_i \}; \\
5: \quad \psi^* := \hat{\phi}; \\
6: \quad \textbf{for all } \psi \in \Psi_i \textbf{ do} \\
7: \quad \textbf{if } \forall i = 1, \ldots, n. \psi_i \implies \psi \text{ and } \llbracket f(\psi_1, \ldots, \psi_n) \rrbracket^* \implies \psi_p \text{ then } \psi^* := \psi^*; \\
8: \quad \mathcal{P}_r := \emptyset; \\
9: \quad \textbf{for } i = 1 \ldots n \textbf{ do} \\
10: \quad \mathcal{P}_r := \mathcal{P}_r \cup \text{ \textsc{ExtractPredicates}(\psi^*)}; \\
11: \quad \textbf{if } e_i \notin \mathcal{T} \textbf{ then } \mathcal{P}_r := \mathcal{P}_r \cup \text{ \textsc{BackPropagate}(e_i, x_j, \phi_i \land \psi^*, \mathcal{P}, \mathcal{U})}; \\
12: \quad \textbf{return} \mathcal{P}_r; \\

Fig. 9. Algorithm to back propagate abstract value \( \phi \land \psi^* \) of expression \( e = f(e_1, \ldots, e_k) \), such that, \\
\( \llbracket f(\psi_1, \ldots, \psi_k) \rrbracket^* \implies \psi_p \).

\textbf{Proof.} Let \( \phi = \llbracket e \rrbracket^\mathcal{P} x \) and \( \psi^* \) be the abstract value from which predicates are extracted (line 10, Figure 8). If \( \psi^* \) was assigned by the if condition (line 9), then \\
\[ L(\llbracket p^* \rrbracket^\mathcal{P} x, y) < L(\phi \land \psi^*, y) \] \\
However if \( \psi^* \) was not assigned on line 9, then \( \psi^* = (s_0 = p[x]) \), and the following is true: \\
\[ L(\llbracket p^* \rrbracket^\mathcal{P} x, y) < L(\llbracket p^* \rrbracket x, y) = L(\phi \land \psi^*, y) \] \\
From Theorem 3.6, \\
\[ \llbracket p^* \rrbracket^{\mathcal{P} \cup \mathcal{P}_r} \implies \phi \land \psi^* \]
Theorem 3.5, the returned program will eventually satisfy the $\epsilon$-correctness condition 3.4.

**Proof.** From Theorem 3.8, algorithm 6 will eventually terminate and return a program $p_r$, which will satisfy the $\epsilon$-correctness condition. \qed

4 **Rose Implementation**

We have implemented our synthesis algorithm in a tool called *Rose*. *Rose* is written in Java. The implementation is modular and allows a user to plug-in different DSLs, abstract semantics, loss functions, objective functions, and complexity measures. To support the experiments presented in Section 5, we instantiate the *Rose* implementation with the string-processing domain-specific language from [Handa and Rinard 2020; Wang et al. 2017a].

**Domain Specific Language and Abstractions:** We use the string processing domain specific language from [Handa and Rinard 2020; Wang et al. 2017a] (Figure 10), which supports extracting substrings (using the SubStr function) of the input string $x$ and concatenation of substrings (using the Concat function). The function SubStr function extracts a substring using a start and an end position. A position can either be a constant index (ConstPos) or the start or end of the substring (Pos).

**Universe of Predicates:** We construct a universe of predicates using predicates of the form $\text{len}(s) = i$, where $s$ is a symbol of a type of string and $i$ presents an integer. We also include predicates of the form $s[i] = c$ indicating the $i^{th}$ character of string $s$ is $c$. Besides these predicates,
We define a generic transformer for conjunctions of predicates as follows:

\[
\begin{align*}
\llbracket f(s_1 = c_1, \ldots, s_k = c_k) \rrbracket \equiv & \quad (s = \llbracket f(c_1, \ldots, c_k) \rrbracket) \\
\llbracket \text{Concat}(\text{len}(f) = i_1, \text{len}(e) = i_2) \rrbracket \equiv & \quad (\text{len}(e) = (i_1 + i_2)) \\
\llbracket \text{Concat}(\text{len}(f) = i_1, e[i_2] = c) \rrbracket \equiv & \quad (e[i_1 + i_2] = c) \\
\llbracket \text{Concat}(\text{len}(f) = i, e = c) \rrbracket \equiv & \quad (\text{len}(e) = (i + \text{len}(c)) \land \bigwedge_{j=1}^{\text{len}(c)} e[i + j - 1] = c[j - 1]) \\
\llbracket \text{Concat}(f[i] = c, p) \rrbracket \equiv & \quad (e[i] = c) \\
\llbracket \text{Concat}(f = c, \text{len}(e) = i) \rrbracket \equiv & \quad (\text{len}(e) = (\text{len}(c) + i)) \land \bigwedge_{j=1}^{\text{len}(c)} e[j - 1] = c[j - 1] \\
\llbracket \text{Concat}(f = c_1, e[i] = c_2) \rrbracket \equiv & \quad (e[\text{len}(c_1) + i] = c_2) \land \bigwedge_{j=1}^{\text{len}(c_1)} e[j - 1] = c_1[j - 1] \\
\llbracket \text{Str}(p) \rrbracket \equiv & \quad p
\end{align*}
\]

Fig. 11. Abstract Semantics for String Transformation DSL.

---

we also include predicates of the form \(s = c\), where \(c\) is a value which a symbol \(s\) can take. We also include both true and false. In summary, the universe of predicates, we are using, is:

\[
\mathcal{U} = \{\text{len}(s) = i \mid i \in \mathbb{N}\} \cup \{s[i] = c \mid i \in \mathbb{N}, c \in \text{Char}\} \cup \{s = c \mid c \in \text{Type}(s)\} \cup \{\text{true}, \text{false}\}
\]

**Abstract Semantics:** We define a generic transformer for conjunctions of predicates as follows:

\[
f\left(\bigwedge_{i_1} p_{i_1}, \ldots, \bigwedge_{i_k} p_{i_k}\right) := \bigcap_{i_1} \ldots \bigcap_{i_k} f(p_{i_1}, \ldots, p_{i_k})
\]

This allows us to just define an abstract semantics for every possible combination of atomic predicates, instead of abstract semantics for all possible abstract values. Figure 11 presents the abstract semantics for functions in string processing DSL for all possible combinations of atomic predicates.

**Initial Abstraction:** The initial abstraction set \(\mathcal{P}\) includes predicates of form \(\text{len}(s) = i\), where \(s\) is a symbol of type string and \(i\) is an integer. It also includes true and false.

**Abstractions and Loss Functions:** We present the abstract version of the 0/∞ Loss Function and 0/1 Loss Function below:

\[
L_{0/\infty}(\varphi, y) = 0 \text{ if } y \in \gamma(\varphi), \infty \text{ otherwise and } L_{0/1}(\varphi, y) = 0 \text{ if } y \in \gamma(\varphi), 1 \text{ otherwise}
\]

If \(\varphi \neq \text{false}\) (\(L_{\text{DL}}(\text{false}, y) = \infty\)), the abstract version of the Damerau-Levenshtein is \(L_{\text{DL}}(\varphi, y) = d_{c,y}(|c|, |y|)\), where \(c = \text{ToStr}(\varphi, y)\) and \(d\) is defined below:

\[
d_{c,y}(i, j) = \min\begin{cases} 
  j & i = 0 \\
  i & j = 0 \\
  d_{c,y}(i - 1, j - 1) & i, j > 0 \text{ and } (c[i - 1] = \text{null} \text{ or } c[i - 1] = y[j - 1]) \\
  1 + d_{c,y}(i - 1, j - 1) & i, j > 0 \text{ and } (c[i - 1] \neq \text{null} \text{ and } c[i - 1] \neq y[j - 1]) \\
  1 + d_{c,y}(i - 1, j) & i > 0 \\
  1 + d_{c,y}(i, j - 1) & j > 0 \\
  d_{c,y}(i - 1, j - 1) & i = |y| \text{ and } \varphi \text{ may contain strings of multiple lengths.} \\
  1 + d_{c,y}(i - 2, j - 2) & i, j > 1 \text{ and } (c[i - 1] = \text{null} \text{ or } c[i - 1] = y[i - 2]) \text{ and } (c[i - 2] = \text{null} \text{ or } c[i - 2] = y[i - 1])
\end{cases}
\]
Let $P = \text{ExtractPredicates}(\varphi)$. The procedure ToStr returns an array $c$, such that, if $\text{len}(s) = i \in P$ then $|c| = i$, otherwise it is the maximum of the length of string $y$ or $i$ such that $s[i] = c' \in P$. For all $s[i] = c_i \in P$, $c[i] = c_i$, otherwise it is null.

In addition to the loss functions introduced in Section 2 use the following loss functions:

**Definition 4.1. 1-Delete Loss Function:** The 1-Delete loss function returns 0 if the outputs from the synthesized program and the data set match exactly, 1 if a single deletion enables the output from the synthesized program to match the output from the data set, and $\infty$ otherwise:

$$L_{1D}(z, y) = \begin{cases} 0 & z = y \\ 1 & a \cdot c \cdot b = z \land a \cdot b = y \land |c| = 1 \\ \infty & \text{otherwise} \end{cases}$$

The abstract version of the 1-Delete Loss Function:

$$L_{1D}(\varphi, y) = \begin{cases} 0 & y \in \gamma(\varphi) \\ 1 & a \cdot b = y \land (\exists c. a \cdot c \cdot b \in \gamma(\varphi) \land |c| = 1) \\ \infty & \text{otherwise} \end{cases}$$

**Definition 4.2. n-Substitution Loss Function:** The $n$-Substitution loss function counts the number of positions where the noisy output does not agree with the output from the synthesized program. If the synthesized program produces an output that is longer or shorter than the output in the noisy data set, the loss function is $\infty$:

$$L_{ns}(z, y) = \begin{cases} \infty & |z| \neq |y| \\ \sum_{i=1}^{\min(|z|, |y|)} 1 & \text{if } z[i] \neq y[i] \text{ else } 0 \end{cases}$$

The abstract version of the $n$-Substitution loss function is:

$$L_{ns}(\varphi, y) = \begin{cases} 0 & \varphi = \text{true} \\ \infty & \varphi = \text{false} \\ \infty & \varphi = (s = c) \\ \infty & c_\varphi = \text{ToStr}(\varphi, y) \land |c_\varphi| \neq |y| \\ \sum_{j=1}^{\min(|y|, |c_\varphi|)} 1 & \text{if } c[j] \neq \text{null} \text{ and } c[j] \neq y[i_j] \text{ else } 0 \end{cases}$$

**Incremental Automata Update:** To avoid regenerating the entire FTA at every iteration of the algorithm as in Algorithm 6, our *Rose* implementation applies optimization that incrementally updates parts of the FTA as appropriate at every iteration of the algorithm.

5 Experimental Results

We use the SyGuS 2018 benchmark suite [Alur et al. 2013] to evaluate *Rose* against the current state-of-the-art noisy program synthesis system presented in [Handa and Rinard 2020]. The SyGus 2018 benchmark suite contains a range of string transformation problems, a class of problems that has been extensively studied in past program synthesis projects [Gulwani 2011; Polozov and Gulwani 2015; Singh and Gulwani 2016]. [Handa and Rinard 2020] use this benchmark suite to benchmark their system by systematically introducing noise within these benchmarks. We recreated the scenarios studied in [Handa and Rinard 2020] and report results for these scenarios.

We run all experiments on a 3.00 GHz Intel(R) Xeon(R) CPU E5-2690 v2 machine with 512GB memory running Linux 4.15.0. We set a timeout limit of 10 minutes for each synthesis task. We
compare Rose with the noisy program synthesis system presented in [Handa and Rinard 2020] (see Section 2), running with a bounded scope height threshold of 4 for all experiments ([Handa and Rinard 2020], Section 9.1). We call this system CFTA.

5.1 Noisy Data Sets

Table 12 presents results for all SyGus 2018 benchmark problems which contain less than ten input/output examples. We omit univ_1, univ_2, and univ_4-6 — these problems time out for both Rose and CFTA (so the rows would contain all -). The first column (Benchmark) presents the name of the SyGus 2018 benchmark. The second column (Number of Input/Output Examples) presents the number of input/output examples in the benchmark problem. The remaining columns present running times, in milliseconds, for Rose and CFTA running with different noise sources and loss functions. A - indicates that the corresponding run timed out without synthesizing a program.

The objective function is the lexicographic objective function. The complexity measure is program size. The noise source cyclically deletes a single character from outputs in the data set, starting with the first character, then wrapping around when reaching the last position in the output. When \( n = 1 \), the noise source corrupts the last output in the set of input/output examples. When \( n = 3 \), the noise source corrupts the last 3 input/output examples in the set of input/output examples. For Rose, the table presents results for each of the \( L^0_0 \), \( L^D_1 \), and \( L^1_1 \) loss functions. For CFTA, we report one running time for each benchmark problem — for CFTA, the running time is the same for all noise source/loss function combinations.

For the benchmarks on which both terminate, Rose runs up to 1957 times faster than CFTA, with a median speedup of 20.5 times over CFTA. This performance increase enables Rose to successfully synthesize programs for 20 more benchmark problems than CFTA — Rose synthesizes programs for 44 of the 54 benchmark problems (timing out on the remaining 10), while CFTA can only synthesize programs for 24 of the 54 benchmark problems (timing out on the remaining 30). These results highlight the substantial performance benefits that Rose delivers.

Every synthesized program is guaranteed to minimize the objective function over the given input/output examples. For \( n = 1 \), all synthesized programs also have zero loss over the original (unseen during synthesis) noise-free input/output examples (i.e., all synthesized programs generate the correct output for each given input). For \( n = 3, 34, 40, \) and \( 40 \) out of 44 synthesized programs, for \( L^0_0 \), \( L^D_1 \), and \( L^1_1 \) respectively, have zero loss over the original noise-free input/output examples. For a given noise source/loss function combination, CFTA and Rose synthesize the same program (unless one or both of the systems times out). These results highlight the ability of Rose to synthesize correct programs even in the face of significant noise.

Table 13 presents results for the SyGus 2018 phone-\( * \)-long-repeat benchmarks running with a noise source that cyclically and probabilistically replaces a single digit in each output string with the next digit (wrapping back to 0 if the current digit is 9). The noise source iterates through each output string in the data set in turn, probabilistically replacing the next character position in each output string with another character, wrapping around to the first character position when it reaches the last character position in the output string. The noise source corrupts 95% of the input-output examples in each dataset.

We report results for two loss functions, \( L^N_0 \) (n-substitution) and \( L^D_1 \) (Damerau-Levenshtein). The objective function is the lexicographic objective function. The complexity measure is program size. There is a row in the table for each phone-\( * \)-long-repeat benchmark; each entry presents the running time (in milliseconds) for the corresponding synthesis algorithm running on the corresponding benchmark problem.

For the benchmarks on which both terminate, Rose runs up to 81.7 times faster than CFTA, with a median speedup of 39.0 times over CFTA. This performance increase enables Rose to successfully
| Benchmark        | No of examples | \( n = 1 \) | \( n = 3 \) | CFTA   |
|------------------|----------------|-------------|-------------|--------|
|                  |                | \( L_0/1 \) | \( L_{DL} \) | \( L_{1D} \) | \( L_0/1 \) | \( L_{DL} \) | \( L_{1D} \) |        |
| bikes            | 6              | 68          | 67          | 65      | 67      | 67          | 68      | 19554   |
| bikes_small      | 6              | 69          | 68          | 65      | 67      | 65          | 65      | 21210   |
| dr-name          | 4              | 402         | 310         | 408     | 359     | 324         | 390     | -       |
| dr-name_small    | 4              | 424         | 312         | 376     | 330     | 302         | 375     | -       |
| firstname        | 4              | 118         | 105         | 114     | 96      | 319         | 367     | 4258    |
| firstname_small  | 4              | 113         | 106         | 118     | 96      | 300         | 393     | 4220    |
| initials         | 4              | 336         | 289         | 353     | 244     | -           | -       | 36188   |
| initials_small   | 4              | 361         | 293         | 343     | 249     | -           | -       | 30920   |
| last-name        | 4              | 124         | 113         | 120     | 97      | 114         | 121     | 175762  |
| last-name_small  | 4              | 122         | 114         | 120     | 97      | 116         | 121     | 178825  |
| name-combine     | 6              | 1288        | 918         | 1609    | 1330    | 984         | 1477    | -       |
| name-combine_short| 6             | 1301        | 875         | 1528    | 1333    | 945         | 1437    | -       |
| name-combine-2   | 4              | 2100        | 1721        | 2162    | 740     | -           | -       | -       |
| name-combine-2_short | 4     | 2120        | 1697        | 2188    | 735     | -           | -       | -       |
| name-combine-3   | 6              | 298         | 287         | 294     | 301     | 289         | 299     | 547447  |
| name-combine-3_short | 6 | 313         | 294         | 291     | 296     | 278         | 290     | 544044  |
| name-combine-4   | 5              | 1863        | 1683        | 1917    | 1875    | 1584        | 1921    | -       |
| name-combine-4_short | 5 | 1888        | 1664        | 1914    | 1815    | 1604        | 1875    | -       |
| phone            | 6              | 66          | 68          | 67      | 66      | 68          | 64      | 943     |
| phone_short      | 6              | 65          | 68          | 66      | 67      | 66          | 68      | 963     |
| phone-1          | 6              | 62          | 62          | 61      | 59      | 63          | 62      | 933     |
| phone-1_short    | 6              | 61          | 64          | 60      | 62      | 63          | 60      | 942     |
| phone-2          | 6              | 83          | 73          | 79      | 80      | 79          | 78      | 953     |
| phone-2_short    | 6              | 117         | 76          | 80      | 80      | 76          | 79      | 943     |
| phone-3          | 7              | 926         | 559         | 692     | 839     | 611         | 648     | -       |
| phone-3_short    | 7              | 789         | 530         | 652     | 786     | 607         | 600     | -       |
| phone-4          | 6              | 2571        | 2067        | 3054    | 2678    | 2202        | 2963    | -       |
| phone-4_short    | 6              | 2637        | 2057        | 2872    | 2608    | 2256        | 3014    | -       |
| phone-5          | 7              | 114         | 101         | 110     | 112     | 98          | 108     | 122     |
| phone-5_short    | 7              | 109         | 99          | 105     | 116     | 98          | 110     | 127     |
| phone-6          | 7              | 171         | 138         | 168     | 175     | 142         | 166     | 3230    |
| phone-6_short    | 7              | 170         | 140         | 169     | 173     | 135         | 163     | 3327    |
| phone-7          | 7              | 165         | 132         | 153     | 159     | 134         | 155     | 2793    |
| phone-7_short    | 7              | 165         | 133         | 162     | 157     | 136         | 154     | 2762    |
| phone-8          | 7              | 158         | 144         | 157     | 162     | 145         | 156     | 3464    |
| phone-8_short    | 7              | 157         | 142         | 154     | 162     | 141         | 155     | 3223    |
| phone-9          | 7              | 28815       | 29672       | 28029   | 28576   | 26465       | 30941   | -       |
| phone-9_short    | 7              | 28658       | 29055       | 28729   | 29115   | 27818       | 28157   | -       |
| phone-10         | 7              | 87772       | 73379       | 66170   | 76963   | 87778       | 69130   | -       |
| phone-10_short   | 7              | 85861       | 77495       | 75849   | 86508   | 81526       | 65247   | -       |
| reverse-name     | 6              | 699         | 645         | 697     | 703     | 666         | 671     | -       |
| reverse-name_short| 6             | 709         | 662         | 811     | 712     | 625         | 685     | -       |
| univ_3           | 6              | 6258        | 4117        | 5994    | 6260    | 3499        | 6068    | -       |
| univ_3_short     | 6              | 6345        | 4364        | 6503    | 6331    | 3510        | 5962    | -       |

Fig. 12. Runtime performance of *Rose* and CFTA on benchmarks with deletion based noise.
synthesize programs for 4 more benchmark problems than CFTA — Rose synthesizes programs for all 11 of the benchmark problems, while CFTA synthesizes programs for 7 of the 11 benchmark problems (timing out on the remaining 4). Once again, these results highlight the substantial performance benefits that Rose delivers.

Every synthesized program is guaranteed to minimize the objective function over the given input/output examples. All synthesized programs have zero loss over the original (unseen during synthesis) noise-free input/output examples (i.e., all synthesized programs generate the correct output for each given input). Once again, these results highlight the ability of Rose to synthesize correct programs even in the face of significant noise.

**Noise-Free Data Sets** We also evaluated the performance of Rose and CFTA by applying it to all problems in the SyGuS 2018 benchmark suite [Alur et al. 2013]. For each problem we synthesize the optimal program over clean (noise-free) datasets. We present the result of all SyGuS benchmark suite in the appendix A.2.

For the benchmarks on which both terminate, Rose runs up to 1831 times faster than CFTA, with the median speedup of 35.7 over CFTA. This enables Rose to successfully synthesize programs for 45 more benchmark problems that CFTA — Rose synthesizes programs for 90 of the 108 benchmark problems (timing out on the remaining 18 problems), while CFTA can only synthesize the correct program for 45 of the 108 benchmark problems (timing out of the remaining problems).

6 Related Work

We discuss related work in the following areas:

**Programming-By-Example/Noise-Free Synthesis**: Synthesizing programs from a set of input-output examples has been a prominent topic of research for many years [Gulwani 2011; Shaw 1975; Singh and Gulwani 2016]. These techniques either require the entire dataset to be noise free, or they try to remove corrupted input-output examples from the dataset before synthesizing the correct program. Instead of removing corrupted examples from the dataset, our technique uses the loss function to capture information from any corrupted examples and uses this information during the synthesis. The experimental results show that, for our set of benchmarks, our approach can synthesize the correct program even in the presence of substantial noise.

**Neural Program Synthesis/Machine-Learning Approaches**: Researchers have investigated techniques that use machine learning/deep neural networks to synthesize programs [Balog et al. 2016; Devlin et al. 2017; Raychev et al. 2016]. The techniques primarily focus on synthesizing
programs over noise-free datasets. These techniques require a training phase and a differentiable loss function and provide no guarantees that the synthesized program will minimize the objective function. Our technique, in contrast, does not require a training phase, can work with arbitrary loss functions including, for example, the Damerau-Levenshtein loss function, and comes with a guarantee that the synthesized program will minimize the objective function over the provided (noisy) input/output examples.

**Tree Automata/VSA based synthesis algorithms:** [Polozov and Gulwani 2015; Singh and Gulwani 2016; Wang et al. 2017b] all construct either version spaces or tree automata to represent all programs (within a bounded search space) which satisfy a set of input-output examples. These techniques build the version space in one shot before synthesis and require the dataset to be either be correct or pruned to remove any corrupt input/output examples.

Our technique also works with version spaces (as represented in tree automata), but extends the approach to work with noisy data sets. It also introduces an abstraction based optimization process which can iteratively expand and improve the version space during synthesis (instead of building it in one shot as in previous research).

**Abstraction-Refinement based Synthesis Algorithms:** There has been work done on using abstraction refinement/refinement types to synthesize programs [Guo et al. 2019; Polikarpova et al. 2016; Wang et al. 2017b]. Given a noise free dataset and a program, checking if a program is correct or incorrect simply checks if the synthesized programs satisfy all input/output examples. To refine an abstraction, these techniques construct a proof of incorrectness. Each abstraction identifies a set of programs, some of which may be correct and others of which may be incorrect. Refinement first identifies a program that does not satisfy one or more of the input/output examples, then generates constraints that refine the abstraction to eliminate this program. Iterative refinement eventually produces the final program.

Abstraction in our noisy program synthesis framework, in contrast, works with an abstraction that approximates the loss function over a set of programs. The refinement step selects a program within the abstraction space, computes its loss, then uses this computed loss to refine the loss approximation to bring this approximation closer to the actual loss. This refinement step, in expectation, reduces the inaccuracy in the approximated loss function of the programs identified by the abstraction. In contrast to previous approaches, which work with abstractions based on program correctness and refinement steps that eliminate incorrect programs, our approach works with abstractions that maintain a sound, conservative approximation of the minimum loss function over the set of programs identified by the abstraction and refinement steps that eliminate programs based on the loss of the programs.

One key difference is that refinement steps in previous techniques rely on the ability to identify correct and incorrect programs. Because our technique works with noisy data sets, it can never tell if a candidate program has minimal loss without comparing the program to all other current candidate programs (unless the loss happens to be zero). It instead uses abstract minimum loss values to bound how far off the optimal loss any candidate program may be. Instead of working with correct or incorrect programs, our technique works by iteratively improving the accuracy of the minimum loss function estimation captured by the abstraction.

Our technique therefore combines abstract tree automata with an abstraction-based optimization process. Our approach, in contrast to previous approaches that use abstract tree automata, enables us to synthesize programs that optimize an objective function over a set of noisy input/output examples, including synthesizing correct programs that may disagree with one, some, or even all of the provided input/output examples.

[Wang et al. 2017a] uses abstract tree automata and abstraction refinement for program synthesis. Because their refinement strategy prunes any program that does not satisfy all of the provided
input/output examples, their algorithm requires the dataset to be noise free. This pruning is necessary as this allows their technique to effectively capture constraints to prune large part of the search space.

**Noisy Program Synthesis:** Handa and Rinard formalize a noisy program synthesis framework and present a technique that uses finite tree automata to synthesize the program which best fits the noisy dataset based on an objective function, a loss function, and a complexity measure [Handa and Rinard 2020]. We present a simplified version of the presented technique in Section 2. This paper presents a new abstraction/refinement technique to solve the noisy program synthesis problem. Our experimental results show that this new technique significantly outperforms the technique presented in [Handa and Rinard 2020].

Handa and Rinard also formalize a connection, in the context of noisy program synthesis, between the characteristics of the noise source and the hidden program that together generate the noisy data set and the characteristics of the loss function [Handa and Rinard 2021]. Specifically, the paper identifies, given a noise model, a corresponding optimal loss function as well as properties of combinations of noise models and loss functions that ensure that the presented noisy synthesis algorithm will converge to the correct program given enough noisy input/output pairs. This work is complementary to the work presented in this paper. All of the guarantees established in [Handa and Rinard 2021] also apply to the algorithm presented in this paper.

**Best Effort Synthesis:** [Peleg and Polikarpova 2020] presents an enumeration-based technique to synthesize programs from input-output datasets containing some incorrect outputs. Their technique returns a ranked list of partially valid programs, removing programs which are observationally equivalent. Their technique uses a fixed fitness function to order these partial results. [Peleg and Polikarpova 2020] uses a specific loss function and a specific complexity measure to rank candidate programs. Given this loss function and complexity measure, our technique will synthesize the exact same program. Our technique also supports the use of a large class of loss functions, complexity measures, and objective functions. [Handa and Rinard 2020] has showcased how crafting suitable loss functions is essential and allows one to synthesize the correct program even when some or even all input/output examples are corrupted.

## Conclusion

We present a new technique to synthesize programs over noisy datasets. This technique uses an abstraction refinement based optimization process to search for a program which best-fits a given dataset, based on an objective function, a loss function, and a complexity measure. The algorithm deploys an abstract semantics to soundly approximate the minimum loss function over abstracted sets of programs in an underlying domain-specific language. Iterative refinement based on this sound approximation produces a program whose loss value is within a specified tolerance level of the program with optimal loss over the given noisy input/output data set. We provide a proof that the technique is sound and complete and will always synthesize an $\epsilon$-correct program.

We have implemented our synthesis algorithm in the Rose noisy program synthesis system. Our experimental results show that, on two noisy benchmark program synthesis problem sets drawn from the SyGus 2018 benchmarks, Rose delivers speedups of up to 1587 and 81.7 over a previous state-of-the-art noisy synthesis system, with median speedups of 20.5 and 81.7 over this previous system. Rose also terminates on 20 (out of 54) and 4 (out of 11) more benchmark problems than the previous system. Both Rose and the previous system synthesize programs that are optimal over the provided noisy data sets. For the majority of the problems in the benchmark sets (272 out of 286 for Rose), both systems also synthesize programs that produce correct outputs for all inputs in the original (unseen) noise-free data set. These results highlight the significant benefits that Rose can deliver for effective noisy program synthesis.
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\section{Appendix}

\subsection{Appendix: Additional Theorems}

\textbf{Theorem A.1.} Given $\mathcal{P}$ and $\mathcal{P}^*$, such that, \{true, false\} $\subseteq \mathcal{P} \subseteq \mathcal{P}^* \subseteq \mathcal{U}$ is true, then for any abstract value $\varphi$, the following statement is true:

$$\alpha^{\mathcal{P}^*}(\varphi) \implies \alpha^\mathcal{P}(\varphi)$$

\textbf{Proof.} Proof by contradiction. Assuming $\alpha^{\mathcal{P}^*}(\varphi) = \varphi_1$ and $\alpha^\mathcal{P}(\varphi) = \varphi_2$, such that, $\varphi_1 \not\implies \varphi_2$.

Note that:

$$\varphi \implies \varphi_1 \land \varphi_2, \varphi_1 \land \varphi_2 \implies \varphi_1, \text{ and } \varphi_1 \land \varphi_2 \implies \varphi_2$$

$\varphi_1 \land \varphi_2$ can be expressed by using predicates in $\mathcal{P}^*$, and $\varphi_1 \land \varphi_2$ is stronger than $\varphi_1$. Hence $\varphi_1 \neq \alpha^{\mathcal{P}^*}(\varphi)$. Therefore, by contradiction, the above theorem is true. \qed

\textbf{Theorem A.2.} Given a set of predicates $\mathcal{P} \subseteq \mathcal{U}$ and $\mathcal{P}^* \subseteq \mathcal{U}$, such that \{true\} $\subseteq \mathcal{P} \subseteq \mathcal{P}^*$, then for any expression $e$ (starting from symbol $s$) and any input value $x_i$, the following statement is true:

$$(s = [e]_{x_i}) \implies [e]^{\mathcal{P}^*}_{x_i} \text{ and } [e]^{\mathcal{P}^*}_{x_i} \implies [e]^{\mathcal{P}}_{x_i}$$

\textbf{i.e., adding more predicates will make the abstract computation more precise.}

\textbf{Proof.} We prove this using induction over height of expression $e$.

\textit{Base Case:} The height of $e$ is 1, i.e., $e$ is a terminal $t$. Since true $\in \mathcal{P}$, using definition of $\alpha^\mathcal{P}$:

$$(s = [t]_{x_i}) \implies \alpha^{\mathcal{P}^*}(s = [t]_{x_i})$$

From Theorem A.1:

$$\alpha^{\mathcal{P}^*}(s = [t]_{x_i}) \implies \alpha^\mathcal{P}(s = [t]_{x_i})$$

\textit{Induction Hypothesis:} For all expressions $e$ of height less than equal to $n$, the following statement is true:

$$(s = [e]_{x_i}) \implies [e]^{\mathcal{P}^*}_{x_i} \text{ and } [e]^{\mathcal{P}^*}_{x_i} \implies [e]^{\mathcal{P}}_{x_i}$$

\textit{Induction Step:} Consider an expression $e$ of height $n+1$. Without loss of generality, we can assume $e$ is of the form $f(e_1, \ldots e_k)$, where height of sub-expressions $e_1, \ldots e_k$ is less than equal to $n$. Therefore, for all $j \in [1, n]$

$$e_j_{x_i} \in \gamma([e_j]^{\mathcal{P}^*}_{x_i}) \subseteq \gamma([e_j]^{\mathcal{P}^*}_{x_i})$$

Let $\alpha^\mathcal{P}([f([e_1]^{\mathcal{P}^*}_{x_i}, \ldots [e_k]^{\mathcal{P}^*}_{x_i}])^n) = \varphi^*$ and $\alpha^\mathcal{P}([f([e_1]^{\mathcal{P}^*}_{x_i}, \ldots [e_k]^{\mathcal{P}^*}_{x_i}])^n) = \varphi$. Note that:

$$[f([e_1]_{x_i}, \ldots [e_k]_{x_i})] \in \gamma(\varphi^*) \subseteq \gamma(\varphi)$$

Therefore,

$$(s = [e]_{x_i}) \implies \varphi^* \text{ and } \varphi^* \implies \varphi$$

Hence, by induction, the above theorem is true. \qed

\textbf{Theorem A.3.} \textbf{(Structure of the Tree Automaton)} Given a set of predicates $\mathcal{P}$, input vector $\vec{x} = \langle x_1, \ldots x_n \rangle$, and DSL $G$, let $\mathcal{A} = \langle Q, Q_f, \Lambda \rangle$ be the AFTA returned by the function ConstructAFTA($\vec{x}, G, \mathcal{P}$). Then for all symbols $s \in G$, for all expressions $e$ starting from symbol $s$ (and height less than bound $b$), there exists a state $q^\varphi$s $\in Q$, such that, $e$ is accepted by the automaton $(Q, \{q_{s}^{\varphi}\}, \Lambda)$, where $\varphi = \langle [e]^{\mathcal{P}}_{x_1}, \ldots [e]^{\mathcal{P}}_{x_n} \rangle$. 
Proof. We prove this theorem by using induction over height of the expression $e$.

**Base Case:** Height of expression $e$ is 1. This implies the symbol is either $x$ or a constant. According to Var and Const rules (Figure 5), there exists state $q_i^\phi \in Q$ for terminal $t$, where $\mathcal{\bar{\varphi}} = \langle [t]^p x_1, \ldots [t]^p x_n \rangle$ and $t$ is accepted by automaton $(Q, \{q_i^\phi\}, \Lambda)$.

**Inductive Hypothesis:** For all symbols $s$ in $G$, for all expressions $e$ starting from symbol $s$ of height less than equal to $n$, there exists a state $q_i^\phi \in Q$, such that, $e$ is accepted by the automaton $(Q, \{q_i^\phi\}, \Lambda)$, where $\mathcal{\bar{\varphi}} = \langle [e]^p x_1, \ldots [e]^p x_n \rangle$.

**Induction Step:** For any symbol $s$ in $G$, consider an expression $e = f(e_1, \ldots e_k)$ of height equal to $n + 1$, created from production $s \leftarrow f(s_1, \ldots s_k)$. Note the height of expressions $e_1, \ldots e_k$ is less than equal to $n$, therefore using induction hypothesis, there exists states $q_{s_1}^\phi, \ldots q_{s_k}^\phi \in Q$, such that, $e_i$ is accepted by automaton $(Q, \{q_i^\phi\}, \Lambda)$, where $\mathcal{\bar{\varphi}}_i = \langle [e_i]^p x_1, \ldots [e_i]^p x_n \rangle$. Note based on abstract execution rules (Figure 4):

$$\langle [e]^p x_1 \rangle = \alpha^p (\langle [f(e_1)^p x_1, \ldots [f(e_k)^p x_1] \rangle^p)$$

According to Prod rule (Figure 5), there exists a state $q_i^\phi \in Q$, where $\mathcal{\bar{\varphi}} = \langle [e]^p x_1, \ldots [e]^p x_n \rangle$, and $e$ is accepted by $(Q, \{q_i^\phi\}, \Lambda)$.

Therefore, by induction, for all symbols $s$ in $G$, for all expressions $e$ starting from symbol $s$ (and height less than bound $b$), there exists a state $q_i^\phi \in Q$, such that, $e$ is accepted by the automaton $(Q, \{q_i^\phi\}, \Lambda)$, where $\mathcal{\bar{\varphi}} = \langle [e]^p x_1, \ldots [e]^p x_n \rangle$.

**Theorem A.4.** Given predicates $\mathcal{P}$, DSL $G$, noisy dataset $\mathcal{D} = (\mathcal{\bar{x}}, \mathcal{\bar{y}})$, objective function $U$, loss function $L$, complexity measure $C$, and $\mathcal{A} = \text{ConstructAFTA}(\mathcal{\bar{x}}, G, \mathcal{P})$, if $p^* = \text{MinCost}(\mathcal{A}, \mathcal{D}, U, L, C)$ then

$$p^* \in \arg\min_{p \in G} U(L([p]^p \mathcal{\bar{x}}, \mathcal{\bar{y}}), C(p))$$

i.e., $p^*$ minimizes the abstract objective function.

Proof. Let $\mathcal{A} = (Q, Q_f, \Lambda)$. From corollary 3.2, for each program $p \in G$, there exists a state $q_{s_0}^\phi \in Q_f$, such that, $[p]^p \mathcal{\bar{x}} = \mathcal{\bar{\varphi}}$. Since the algorithm finds an accepting state $q_{s_0}^\phi \in Q_f$, such that, for all accepting states $q_{s_0}^\phi \in Q_f$,

$$\langle L(\mathcal{\bar{\varphi}}, \mathcal{\bar{y}}), C(P[q_{s_0}^\phi]) \rangle \leq_U \langle L(\mathcal{\bar{\varphi}}, \mathcal{\bar{y}}), C(P[q_{s_0}^\phi]) \rangle$$

for all $p \in G$,

$$\langle L(\mathcal{\bar{\varphi}}, \mathcal{\bar{y}}), C(P[q_{s_0}^\phi]) \rangle \leq_U \langle L([p]^p \mathcal{\bar{x}}, \mathcal{\bar{y}}), C(p) \rangle$$

Since $p^* = P[q_{s_0}^\phi]$, $p^* \in \arg\min_{p \in G} U(L([p]^p \mathcal{\bar{x}}, \mathcal{\bar{y}}), C(p))$.

**Theorem A.5.** Given expression $e = f(e_1, \ldots e_n)$, input $x$, abstract value $\psi_p$ (assuming $s = [e]^p x \Rightarrow \psi_p$, predicates $\mathcal{P}$, and universe of predicates $\mathcal{U}$, if the procedure $\text{BackPropogate}(e, x, \psi_p, \mathcal{P}, \mathcal{U})$ returns predicate set $\mathcal{P}_r$ then:

$$[e]^p \cup \mathcal{P}_r \Rightarrow \psi_p$$

Proof. We prove this theorem using induction over height of expression $e$.

**Base Case:** Height of $e$ is 2. This means all sub-expressions $e_1, \ldots e_k$ are terminals. Note that $\mathcal{P}_r \subseteq \text{ExtractPredicates}(\psi_1^*), \text{for all } i \in [1, k]$.

$$[e_i]^p \cup \mathcal{P}_r \Rightarrow \varphi_i \land \psi_i^*$$
and
\[\left\lfloor f(\varphi_1 \land \psi_1^*, \ldots \varphi_k \land \psi_k^*)\right\rfloor^* \implies \psi_p\]

therefore
\[\left\lfloor e\right\rfloor_{P \cup P, x}^r \implies \psi_p\]

**Induction Hypothesis:** For all expressions \(e\) of height less than equal to \(n\), the following is true:
\[\left\lfloor e\right\rfloor_{P \cup P, x}^r \implies \psi_p\]

**Induction Step:** Let \(e = f(e_1, \ldots, e_k)\) be an expression of height equal to \(n+1\). The height of expressions \(e_1, \ldots, e_k\) is less than equal to \(n\).

Note that \(\varphi_1 \land \psi_1^* \implies \left\lfloor e_1\right\rfloor x\) (line-7 and line-9). And since BackPropogate\((e_1, x, \varphi_1 \land \psi_1^*, P, U) \subseteq P_r\), using induction hypothesis:

\[\left\lfloor e_1\right\rfloor_{P \cup P, r} \implies \varphi_1 \land \psi_1^*\]

and
\[\left\lfloor f(\varphi_1 \land \psi_1^*, \ldots \varphi_k \land \psi_k^*)\right\rfloor^* \implies \psi_p\]

therefore
\[\left\lfloor e\right\rfloor_{P \cup P, x}^r \implies \psi_p\]

\(\square\)

**A.2 Appendix: Non Noisy Performance Comparison**
| Benchmark          | No of Examples | Rose | CFTA |
|-------------------|----------------|------|------|
| bikes             | 6              | 67   | 69   |
| bikes-long        | 24             | 116  | 125  |
| bikes-long-repeat | 58             | 196  | 230  |
| bikes_small       | 6              | 67   | 73   |
| dr-name           | 4              | 408  | 349  |
| dr-name-long      | 50             | 501  | 659  |
| dr-name-long-repeat | 150         | 775  | 1123 |
| dr-name_small     | 4              | 411  | 363  |
| firstname         | 4              | 124  | 131  |
| firstname-long    | 54             | 332  | 404  |
| firstname-long-repeat | 204         | 811  | 948  |
| bikes-long        | 6              | 67   | 73   |
| bikes-long-repeat | 24             | 116  | 125  |
| bikes-long-repeat | 58             | 196  | 230  |
| bikes_small       | 6              | 67   | 73   |
| dr-name           | 4              | 408  | 349  |
| dr-name-long      | 50             | 501  | 659  |
| dr-name-long-repeat | 150         | 775  | 1123 |
| dr-name_small     | 4              | 411  | 363  |
| firstname         | 4              | 124  | 131  |
| firstname-long    | 54             | 332  | 404  |
| firstname-long-repeat | 204         | 811  | 948  |
| bikes             | 6              | 67   | 69   |
| bikes-long        | 24             | 116  | 125  |
| bikes-long-repeat | 58             | 196  | 230  |
| bikes_small       | 6              | 67   | 73   |
| dr-name           | 4              | 408  | 349  |
| dr-name-long      | 50             | 501  | 659  |
| dr-name-long-repeat | 150         | 775  | 1123 |
| dr-name_small     | 4              | 411  | 363  |
| firstname         | 4              | 124  | 131  |
| firstname-long    | 54             | 332  | 404  |
| firstname-long-repeat | 204         | 811  | 948  |
| bikes-long        | 6              | 67   | 69   |
| bikes-long-repeat | 24             | 116  | 125  |
| bikes-long-repeat | 58             | 196  | 230  |
| bikes_small       | 6              | 67   | 73   |
| dr-name           | 4              | 408  | 349  |
| dr-name-long      | 50             | 501  | 659  |
| dr-name-long-repeat | 150         | 775  | 1123 |
| dr-name_small     | 4              | 411  | 363  |
| firstname         | 4              | 124  | 131  |
| firstname-long    | 54             | 332  | 404  |
| firstname-long-repeat | 204         | 811  | 948  |

Fig. 14. Runtime performance of Rose and CFTA over noise-free dataset
| Benchmark          | No of Examples | \(L_{0/\infty}\) | \(L_{0/1}\) | \(L_D\) | \(L_{wS}\) | Threshold |
|-------------------|---------------|-----------------|-------------|----------|-----------|-----------|
| phone-5           | 7             | 110             | 116         | 190      | 267       | 98        | 122       |
| phone-5-long      | 100           | 407             | 407         | 485      | 466       | 386       | 683       |
| phone-5-long-repeat | 400         | 1156            | 1153        | 1308     | 1344      | 1123      | 2179      |
| phone-5_short     | 7             | 109             | 115         | 158      | 126       | 101       | 127       |
| phone-6           | 7             | 168             | 170         | 148      | 194       | 119       | 3230      |
| phone-6-long      | 100           | 493             | 503         | 484      | 1047      | 447       | 27566     |
| phone-6-long-repeat | 400         | 1246            | 1295        | 1353     | 1507      | 1234      | 100241    |
| phone-6_short     | 7             | 169             | 178         | 183      | 231       | 121       | 3327      |
| phone-7           | 7             | 152             | 163         | 370      | 179       | 123       | 2793      |
| phone-7-long      | 100           | 458             | 485         | 502      | 655       | 438       | 27770     |
| phone-7-long-repeat | 400         | 1245            | 1253        | 1469     | 1921      | 1230      | 89535     |
| phone-7_short     | 7             | 162             | 164         | 197      | 199       | 120       | 2762      |
| phone-8           | 7             | 155             | 162         | 153      | 266       | 114       | 3464      |
| phone-8-long      | 100           | 460             | 453         | 1490     | 538       | 439       | 22961     |
| phone-8-long-repeat | 400         | 1242            | 1265        | 1483     | 1907      | 1229      | 88932     |
| phone-8_short     | 7             | 156             | 156         | 146      | 183       | 119       | 3223      |
| phone-9           | 7             | 30087           | 28603       | 43063    | 67994     | 12823     | -         |
| phone-9-long      | 100           | 31448           | 33337       | 48872    | 59942     | 13498     | -         |
| phone-9-long-repeat | 400         | 39881           | 41229       | 56789    | 73652     | 18845     | -         |
| phone-9_short     | 7             | 27348           | 31304       | 43994    | 62737     | 12140     | -         |
| phone-long        | 100           | 290             | 285         | 1092     | 329       | 285       | 8592      |
| phone-long-repeat | 400           | 720             | 752         | 806      | 867       | 766       | 28836     |
| phone_short       | 6             | 68              | 66          | 68       | 79        | 68        | 963       |
| reverse-name      | 6             | 783             | 732         | 764      | 1186      | 463       | -         |
| reverse-name-long | 50            | 1075            | 1077        | 2120     | 1341      | 764       | -         |
| reverse-name-long-repeat | 200    | 1682            | 1659        | 1912     | 2590      | 1398      | -         |
| reverse-name_short | 6            | 767             | 774         | 823      | 987       | 440       | -         |
| univ_1            | 6             | -               | -           | -        | -         | -         | -         |
| univ_1-long       | 20            | -               | -           | -        | -         | -         | -         |
| univ_1-long-repeat | 30           | 22101           | 22452       | -        | 54745     | 11928     | -         |
| univ_1_short      | 6             | -               | -           | -        | -         | -         | -         |
| univ_3            | 6             | 6500            | 6362        | 5601     | 11083     | 5770      | -         |
| univ_3-long       | 20            | -               | -           | -        | -         | -         | -         |
| univ_3-long-repeat | 30           | -               | -           | -        | -         | -         | -         |
| univ_3_short      | 6             | 6697            | 6275        | 4506     | 9127      | 5772      | -         |

Fig. 15. Runtime performance of Rose and CFTA over noise-free dataset