An Improved Energy Balance Approach and Its Application in CHAMP Gravity Field Recovery

XU Tianhe  HE Kaifei

Abstract  An efficient method for gravity field determination from CHAMP orbits and accelerometer data is referred to as the energy balance approach. A new CHAMP gravity field recovery strategy based on the improved energy balance approach is developed in this paper. The method simultaneously solves the spherical harmonic coefficients, daily integration constant, scale and bias parameters. Two 60 degree and order gravitational potential models, XISM-CHAMP01S from the classical energy balance approach, and XISM-CHAMP02S from the improved energy balance, are determined using about one year’s worth of CHAMP kinematic orbits from TUM and accelerometer data from GFZ. Comparisons among XISM-CHAMP01S, XISM-CHAMP02S, EIGEN-G03C, EIGEN-CHAMP03S, EIGEN2, ENIGN1S and EGM96 are made. The results show that the XISM-CHAMP02S model is more accurate than EGM96, EIGEN1S, EIGEN2 and XISM-CHAMP01S at the same degree and order, and has almost the same accuracy as EIGEN-CHAMP03S.

Keywords  Earth gravity; field model; energy balance; CHAMP; kinematic orbit

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Introduction

The German Challenging Minisatellite Payload (CHAMP), launched in July 2000, carries a GPS receiver and a three-axes accelerometer. It is the first satellite that provides orbit and accelerometer data simultaneously. Using these data, a global earth gravity field can be recovered. Many methods, such as, Kaula’s orbit perturbation[1], the acceleration method[2, 4], numerical integration method[2, 5] and energy balance method[6-13], have been studied in detail. At present, the energy balance method is one of the most efficient methods used for CHAMP gravity recovery[7-13]. A key problem in the energy balance method is to compute for the frictional energy using the preprocessed accelerometer data provided by GeoForschungsZentrum (GFZ). A lot of research has shown that the accelerometer data have to be recalibrated by estimating the scale factor and bias when used in CHAMP orbit determination or gravity field recovery. At present, two calibration methods have been developed and are widely used for modeling the system errors in CHAMP accelerometer data. One is to estimate the scale factor and bias by using the unknown gravity field model, and the other, by using the crossover point of CHAMP orbits[4,7-15]. The estimated scale and bias parameters from these methods are influenced by the prior gravity field model more or less[4], and the calibrating procedure should...
be done in advance when estimating potential coefficients. Another problem, which has been paid little attention to, is the computation of the unknown integration constant in the energy conservation equation, because it is always estimated in an approximate way \cite{4, 9, 13}.

In this paper, an improved energy balance approach for CHAMP gravity field recovery is developed. The method does not require pre-calibration of the accelerometer data by using the known gravity field model or the crossover point of CHAMP orbits. It can efficiently estimate the potential coefficients together with the integration constant, scale and bias parameters. The rest of this paper is organized as follows. In Sect. 1, we briefly introduce the classical energy balance approach. An improved energy balance approach for gravity field recovery is developed in Sect. 2. We apply the suggested method to obtain a 60 degree and order gravity field model XISM-CHAMP02S using one year’s worth of kinematic orbits from the Technical University of Munich (TUM) and the accelerometer data from GFZ Potsdam. The comparisons between the new model and other models are made in Sect. 3. Finally, some conclusions are drawn in Sect. 4.

1 Classical energy balance approach

The energy conservation equation for the CHAMP satellite can be expressed in the inertial system as follows \cite{7-10}:

\[
T = \frac{1}{2}v^2 - V_s - V_n - \omega(xv_y - yv_x) - F - E_o - U \tag{1}
\]

where \(T\) is the disturbing potential; \(\frac{1}{2}v^2\) is the kinetic energy per unit mass of the satellite; \(v = (v_x, v_y, v_z)^T\) is the velocity vector; \(V_s\) and \(V_n\) are the tidal potentials of the sun and the moon, respectively; \(\omega\) is the rotation angular velocity of the earth; \(x=(x, y, z)\) is the position vector; \(F\) is the frictional energy resulting from the non-conservative force; \(E_o\) is an unknown integration constant; and \(U\) is the normal potential without the centrifugal term of Geodetic Reference System 1980 (GRS80). Eq.(1) is the basic equation for CHAMP gravity field recovery using the energy balance approach.

The frictional energy \(F\) can be calculated as

\[
F = \int_{t_i}^{t_f} v \cdot a_{fr} dt \tag{2}
\]

where \(a = (a_x, a_y, a_z)^T\) is the acceleration vector due to non-conservative forces measured by the accelerometer onboard.

Due to a failure in one of the electrodes of the accelerometer, large errors may occur in the measurements of the radial component. So the energy dissipation calculated from Eq.(2) will be distorted if the radial component is not dealt with reasonably \cite{4, 8, 13}. An approximate formula is proposed as \cite{8, 10}:

\[
F = \int_{t_i}^{t_f} |v| \cdot a_r dt \tag{3}
\]

where \(|v|\) is the module of the velocity vector. Eq.(3) seems acceptable because the main part of the non-conservative forces is due to air drag, which is measured by the alone-track component \(a_r\) of the accelerometer. This approximation induces an error due to the fact that the velocity vector is not necessarily aligned with the flight direction at all times, however the deviation is at most 1~2 degrees and therefore can be neglected \cite{13}.

Obviously, in order to determine the fractional energy, we need to obtain \(v\) and \(a_r\) with high precision. Although the accelerometer data have been preprocessed by GFZ, there still exist residual system errors which have to be calibrated in orbit determination or gravity field recovery. There are two commonly used methods. One calibration method involves using a known gravity field model \cite{14}. The other uses the crossover points of CHAMP orbits \cite{15}. The former calculates the disturbing potential using a known gravity field model, then estimates the scale factor and bias from Eq.(1). The estimated parameters are obviously influenced by the prior gravity field model, which will influence the recovered gravity field model to some extent. The latter is a more efficient method, which uses the disturbing potential discrepancy of crossover points to estimate the scale factor and bias. Since the crossover point is not always the same as the sampling point and usually the potential is radially continued to a constant orbit height to insure enough crossover points, up or down continua-
tion should have to be done by using a prior gravity field model. Therefore, the recovered gravity field model is also influenced by the prior one, but the extent is either more or less than that of the former. In any case, the calibration methods mentioned above are more or less influenced by the prior gravity field model.

Anticipating the calibration of accelerometer data, $E_0$ becomes an unknown integration constant which has to be determined firstly. As an approximation, the mean of the time series from Eq.(1) is used. This is plausible since in a global sense the disturbing potential has zero mean, and the satellite track just covers most of the Earth with the problem of the pole gap\(^9\). This is obviously an approximate method to compute for $E_0$\(^{13}\).

2 An improved energy balance approach

In order to overcome the problems mentioned above in relation to the classical energy balance approach, an improved energy balance approach for CHAMP gravity field recovery is developed.

Eq.(1) can be rewritten as

$$T + E_0 + F = \frac{1}{2} \int (-V_e - V_n - \omega(xv_e - yv_n)) - U \, dt \quad (4)$$

where the right term can be calculated from CHAMP orbits and accelerometer data. As the introduction says, although the accelerometer data have been pre-processed by GFZ, there still exist residual system errors which have to be calibrated. Let the bias parameter and scale factor be $k_0$ and $k_1$ respectively, the frictional energy formula can then be written as

$$F(t) = \int_0^t |v(t)| \cdot (k_1 a_T + k_0) \, dt \quad (5)$$

Since the above formula is in the integral form, it should be transformed into a discrete form according to the sampling interval of the accelerometer data (such as 10s or 30s). So the frictional energy $t_a$ can be written as

$$F(t_a) = \sum_{n=1}^{t_a} |v(t_n)| \cdot (k_1 a_T (t_n) + k_0) \, \Delta t \quad (6)$$

Eq.(6) is indeed an approximate formula, which indicates that the module of the velocity vector and the alone-track component of the accelerometer should be constants in the sampling intervals 10s or 30s. A more precise mid-point formula can be used

$$F(t) = \frac{1}{2} \left[ |v(t_n)| + |v(t_{n-1})| \right] \left[ k_1 a_T (t_n) + k_1 a_T (t_{n-1}) + k_0 \right] \Delta t \quad (7)$$

The disturbing potential can be represented by a spherical harmonic expansion in the conventional terrestrial system (CTS) as

$$\begin{align*}
T &= \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left( \tilde{C}_{nm} \tilde{U}_n^m + \tilde{S}_{nm} \tilde{V}_n^m \right) \\
\tilde{U}_n^m &= \frac{GM^a_x T_n^m}{r^{n+1}} \sin \varphi \cos m \lambda \\
\tilde{V}_n^m &= \frac{GM^a_y T_n^m}{r^{n+1}} \sin m \lambda
\end{align*} \quad (8)$$

where $a_x$ is the semi-major axis of the earth; $r$ is the geocentric distance of the satellite; $\varphi$ and $\lambda$ are its latitude and longitude; $M$ is Earth’s mass; $G$ is the gravitational constant; $T_n^m (\sin \varphi)$ is the associated Legendre function; $\tilde{C}_{nm}$ and $\tilde{S}_{nm}$ are normalized geopotential coefficients; $n$ and $m$ denote degree and order of Legendre polynomials; $\tilde{U}_n^m$ and $\tilde{V}_n^m$ can be calculated using the algorithm for the recursive computation of orthonormal associated Legendre functions.

When combining Eq.(4), Eq.(7) and Eq.(8), the observation equation for estimating the potential coefficients, integration constant, scale factor and bias can simultaneously be obtained. Its error equation can be written as

$$V = AX + BY - L \quad (10)$$

where $X$ is the vector of potential coefficients which are the global parameters; $Y$ is the vector of the integration constant, scale and bias parameters, which are the local parameters and may be estimated daily; $A$ is the design matrix of the global parameters; $B$ is the design matrix of the local parameters; $L$ is the vector of the observations.

Based on the above observational equation, the potential coefficients can be estimated as well as the integration constant, scale and bias parameters. Every short arc builds a normal equation. All normal equations are combined by estimating the scale and bias parameters for every arc. When combining the nor-
mal equations, the local parameters (scale and bias parameters) can be eliminated and only the global ones are saved. After all the equations are added, the potential coefficients can be estimated.

It should be pointed out that solving Eq.(10) by using least squares (LS) estimation is very time consuming. In order to overcome the problem, this conjugated gradient method can be used. More details about it can be found in literature [7].

3 Computation and analysis

Many studies have shown that the recovered gravity field from the energy balance approach is biased towards a priori gravity knowledge if the computations are based on dynamic or reduced-dynamic orbits [7,10,11]. The use of purely kinematic orbits could help because no prior information about the gravity field would be necessary [10,11]. Two global gravity field models, XISM-CHAMP01S (Xi’an Research Institute of Surveying and Mapping is abbreviated as XISM) using the classical energy balance method, and XISM-CHAMP02S, using the improved energy balance method with degree and order of 60, are recovered using CHAMP kinematic orbits with a sampling rate of 30 seconds provided by TUM. The orbits cover a time period of one year, from January 2002 to February 2003. The three-dimensional accelerometer data are provided by the CHAMP Information System and Data Centre (ISDC). The number of observational equations is about 360 × 2 880 = 1 036 800, and the number of the estimated potential coefficients is 3 717. The non-conservative forces considered in this paper are tides of the Moon and the Sun, the Earth tides, ocean tides, and relativity effects. It is known that only the satellite positions are given in the kinematic orbits. However, besides the satellite positions, high-precision velocities are also needed for the gravity field recovery using the energy balance approach. The method combining Newton’s numerical differential formula and the remove-restore procedure for determining kinematic velocities is used in this paper [11]. The degree variances of the potential coefficient differences among different gravity models are compared. Degree variance is calculated by

\[
\sigma_l = \sqrt{\sum_{m=0}^{l} [(C_{lm} - \bar{C}_{lm})^2 + (S_{lm} - \bar{S}_{lm})^2]}, \quad l = 2, 3, \ldots, 60
\]  

(11)

where \(C_{lm}\) and \(S_{lm}\) are the potential coefficients of reference gravity field EIGEN-CG03C. \(\bar{C}_{lm}\) and \(\bar{S}_{lm}\) are the estimated potential coefficients from CHAMP data; \(l\) and \(m\) are the degree and order.

The degree variances of the potential coefficient differences between EIGEN-CG03C and XISM-CHAMP01S,XISM-CHAMP02S,EIGEN-CHAMP03S, EIGEN2, EIGEN1S and EGM96 are shown in Fig.1. EIGEN-CHAMP03S, with the maximum degree of 140, is a CHAMP-only gravity field model derived from CHAMP GPS satellite-to-satellite and accelerometer data of about three years. The EIGEN-CG03C model is an upgrade of EIGEN-CG01C, completed to degree and order of 360 in 2005. The model

![Fig.1 Degree variances between EIGEN-CG03C and XISM-CHAMP01S, XISM-CHAMP02S, EIGEN-CHAMP03S, EIGEN2, EIGEN1S, EGM96](image-url)
is based on the same CHAMP mission and surface (gravimetry and altimetry) data as EIGEN-CG01C, but takes into account almost twice as much GRACE mission data\cite{16}.

In order to evaluate the precision and reliability of the recovered models, the standard deviation (St.d) of the differences of 1°×1° grids of point geoid undulations between XISM-CHAMP01S, XISM-CHAMP02S and the EIGEN-CG03C for various spectral bands at 30, 40, 50, 60 degrees is calculated. The area covers latitude: −60°N ~ 60°S, longitude: −150°N ~ 150°S, and has about 36421 gridding points. The results are shown in Table 1.

On the basis of the above results, the following conclusions can be drawn.

(1) The XISM-CHAMP01S, XISM-CHAMP02S, EIGEN2 and EIGEN-CHAMP03S models have good agreement at low degrees and orders. With an increase in degree, the XISM-CHAMP02S and EIGEN-CHAMP03S models are more accurate than the other models.

(2) The XISM-CHAMP02S model is obviously more accurate than the EGM96, EIGEN1S and EIGEN2, XISM-CHAMP01S models in various spectral bands. For the EIGEN1S, EIGEN2 and XISM-CHAMP01S models, the maximum degrees prior to the EGM96 model are 30, 45 and 55, respectively.

(3) The standard deviations of geoid differences between XISM-CHAMP02S and EIGEN-CG03C for spectral bands at 30, 40, 50 and 60 degrees are 2.23 cm, 3.05 cm, 6.95 cm and 13.15 cm, respectively, which are obviously superior to those of EGM96, EIGEN1S, EIGEN2 and XISM-CHAMP01S at the same degrees, and has almost the same accuracy as EIGEN-CHAMP03S.

(4) On the whole, XISM-CHAMP02S is more accurate than XISM-CHAMP01S, which shows the validity of the improved energy balance method proposed by this paper.

4 Conclusions

In this paper, an improved energy balance approach for CHAMP gravity field recovery is developed. It can estimate the potential coefficients together with

| Model               | Degree | Max / cm | Min / cm | St.d / cm |
|---------------------|--------|----------|----------|-----------|
| XISM-CHAMP01S       | 30     | 15.34    | −17.33   | 3.08      |
|                     | 40     | 26.78    | −28.67   | 6.62      |
|                     | 50     | 58.29    | −42.54   | 12.53     |
|                     | 60     | 128.36   | −139.79  | 20.75     |
| XISM-CHAMP02S       | 30     | 9.26     | −10.34   | 2.23      |
|                     | 40     | 19.56    | −17.87   | 3.05      |
|                     | 50     | 29.73    | −33.95   | 6.95      |
|                     | 60     | 88.58    | −94.88   | 13.15     |
| EGM96               | 30     | 46.73    | −41.50   | 11.25     |
|                     | 40     | 113.16   | −103.77  | 16.57     |
|                     | 50     | 153.82   | −155.50  | 22.20     |
|                     | 60     | 199.62   | −220.67  | 26.13     |
| EIGEN1S             | 30     | 40.78    | −33.82   | 7.80      |
|                     | 40     | 95.67    | −74.77   | 19.31     |
|                     | 50     | 217.02   | −267.61  | 40.49     |
|                     | 60     | 566.53   | −506.70  | 67.61     |
| EIGEN2              | 30     | 10.56    | −11.47   | 2.99      |
|                     | 40     | 28.67    | −23.18   | 6.59      |
|                     | 50     | 108.96   | −105.59  | 17.66     |
|                     | 60     | 431.84   | −401.05  | 44.85     |
| EIGEN-CHAMP03S      | 30     | 6.12     | −4.86    | 1.06      |
|                     | 40     | 10.38    | −8.84    | 2.63      |
|                     | 50     | 22.69    | −19.28   | 5.24      |
|                     | 60     | 48.90    | −42.41   | 10.72     |
the integration constant, scale and bias parameters. A global gravity field model XISM-CHAMP02S with degree and order of 60 is recovered using one year’s worth of CHAMP kinematic orbits provided by TUM, which is superior to XISM-CHAMP01S from the classical energy balance approach in that it only includes the gravity field information measured by CHAMP without the influence of any prior gravity field. The results show that the XISM-CHAMP02S model is more accurate than EGM96, EIGEN1S, EIGEN2 and XISM-CHAMP01S at the same degree and order, and has almost the same accuracy as EIGEN-CHAMP03S.

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