Filter Order and Transmission Zeroes Calculation of Cross-coupled Resonator filters

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Abstract. Determination of filter order and transmission zeroes is the precondition of synthesizing cross-coupled filters. Firstly, we propose a method to determine them according to given filter specifications using Genetic Algorithm. The effective objective function is proposed for Genetic Algorithm. We can obtain the least filter order and the optimum locations of transmission zeros to fulfill filter specifications by this method.

1. Introduction
Filters play important roles in transmitters and receivers for various types of radar system. Both analytical and numerical methods for the synthesis of coupling matrices corresponding to cross-coupled filters have been extensively studied. A fundamental analytical theory of cross-coupled resonator bandpass filters was developed in the 1970s by Atia and Williams [1]. A slightly different, widely used, analytical technique based on generating the Chebyshev filtering functions with prescribed transmission zeros was advanced by Cameron [2]. Cameron further proposed “N + 2” coupling matrix synthesis techniques for microwave filters with source/load-multiresonator coupling [3]. These analytical techniques produce a full coupling matrix which must be transformed to a form suitable for realizations by repeated matrix similarity transformations. The main difficulty with these methods is that the sequence of matrix transformations is not known in advance and may be difficult to derive. Numerical optimization was used in synthesizing coupling matrix in recent years [4-10]. These optimization methods can strictly enforce the desired topology; this eliminates the need for similarity transformations of the coupling matrix.

These analytical and numerical methods mentioned above didn’t discuss how to determine the filter order and the transmission zeroes to fulfill the given filter specifications, although determining the filter order and transmission zeros(TZs) is the precondition of coupling matrix synthesis and this is the foundation of the cross-coupled filter design. There are still few papers to discuss this question. Although this question can be solved in [11], [12], there is no the analytical expression for two or more transmission zeros in [11]. The author in [12] proposed a method to determine the filter order and TZs according to the extreme characteristic of the general Chebyshev function and the relationship between the filter order and the number of TZs, but it spends much time on comparison and needs complex formula derivation and solving nonlinear equations.

This paper presents a simple and effective method to obtain the least filter order and optimum TZs to fulfill the given filter specifications by minimizing an objective function of desired variables whose...
values are to be optimized by Genetic Algorithm (GA). This method is much simpler than that given by [11], [12].

2. Determination of filter order and transmission zeroes

A genetic algorithm (GA) uses a process similar to biological evolution based on some genetic operators such as selection, crossover, mutation, and inversion to emulate an evolutionary process. It begins with an initial set of random configurations. The set of configurations is called the population. Each configuration in the population will be a set of designable variables. More introductions about GA in details are in [13, [14] and [15]. GA can be used to solve the minimum value of multivariate function; it is more powerful to search for the global minimum.

Genetic algorithm (GA) was only a tool. Its function was to find the global minimum value of a multivariate function. The key to solve the problem was to propose different objective error functions (multivariate functions) for different problems, and then use GA to minimize the objective error function to solve the problem. An objective function is key point for successful optimization using GA. We construct the objective function for GA according to given filter specifications in order to obtain the least filter order and the optimum locations of TZs.

For any two-port lossless filter network, the transmission function \( S_{21}(\Omega) \) and reflection function \( S_{11}(\Omega) \) may be expressed as:

\[
\left| S_{21}(\Omega) \right|^2 = \frac{1}{1 + \varepsilon^2 C_N^2(\Omega)} , \quad \left| S_{11}(\Omega) \right|^2 = 1 - \left| S_{21}(\Omega) \right|^2 \quad (1)
\]

where, \( \Omega \) is the normalized frequency variable and \( \varepsilon \) is a ripple constant related to the passband return loss \( RL \) by \( \varepsilon = [10^{RL/10} - 1]^{1/2} \). The General Chebyshev filtering function \( C_N(\Omega) \) is given by

\[
C_N(\Omega) = \cosh\left[ \sum_{n=1}^{N} \cosh^{-1}(x_n) \right], \quad x_n = \frac{\Omega - 1/\Omega_n}{1 - \Omega/\Omega_n} \quad (2)
\]

Here, \( s_n = j\Omega_n (n=1, 2 \ldots N) \) is the location of the \( n \)th transmission zero in the complex s-plane, \( N \) is the filter order. Note that \( |C_N(\pm 1)| = 1 \) for all values of \( N \). Also, as all \( N \) of the prescribed TZs approach infinity, \( C_N(\Omega) \) degenerates to the familiar pure Chebyshev function.

The relationship between the insertion loss and the transmission function \( S_{21}(\Omega) \) is defined as

\[
La = -10\log\left| S_{21}(\Omega) \right|^2 \quad (dB) \quad (3)
\]

Generally, the transmission and reflection coefficients of a lossless filter network are computed by (1) according to the filter order and the locations of transmission zeros, shown in Figure. 1.

We know that the filter has highest stopband rejection, when maximum of \( \left| S_{21}(\Omega) \right|^2 \) within the range of stopband is minimum. The aim is to search for locations of TZs to minimize the maximum of \( \left| S_{21}(\Omega) \right|^2 \) within the range of stopband.

The transmission function in (1) can be abstracted as a multivariate single-valued function \( F \) which is related to the normalized frequency variable \( \Omega \) and the normalized TZs frequency variables \( \Omega_n \) \( (n=1, 2 \ldots N) \) as follows:

\[
\left| S_{21}(\Omega) \right|^2 = \frac{1}{1 + \varepsilon^2 C_N^2(\Omega)} = F(\Omega, \Omega_1 \ldots \Omega_N) \quad (4)
\]

\( N \) is filter order, Cameron has proved that the number of TZs with finite locations \( N_{tz} \) must satisfy
\( N_{\text{tz}} \leq N \), those zeros without finite locations must be placed at infinity. However, the two-port networks without source/load-multiresonator coupling will realize a maximum of \( N-2 \) finite-location zeros [2], [3]. Amari has given a rigorous proof for the maximum number of finite TZs of cross-coupled filters with a given topology [16], [17].

![Image of characteristic of low-pass prototype filter](image)

Figure 1. Characteristic of low-pass prototype filter.

Usually, the information obtained from given filter specifications is as follows:

When \( \Omega > \Omega_{s1} \text{ rad/s} \), the attenuation in the stopband \( L_a \geq L_{s1} \) dB.

When \( \Omega > \Omega_{s2} \) or \( \Omega < \Omega_{s2} \text{ rad/s} \), the attenuation in the stopband \( L_a \geq L_{s2} \) dB.

Where, \( \Omega_{s1} \) and \( \Omega_{s2} \) are normalized stopband edge frequency. \( L_{s1}, L_{s2} \) are stopband attenuation at \( \Omega_{s1} \) and \( \Omega_{s2} \), respectively. So, two functions are constructed as follows:

\[
\varphi_1(\Omega_1,\ldots,\Omega_N) = \max(F(\Omega,\Omega_1,\ldots,\Omega_N)), \quad \Omega \in [\Omega_{s1} + \infty]
\]

(5)

\[
\varphi_2(\Omega_1,\ldots,\Omega_N) = \max(F(\Omega,\Omega_1,\ldots,\Omega_N)), \quad \Omega \in [-\infty, \Omega_{s2}) \text{ or } \Omega \in [\Omega_{s2} + \infty]
\]

(6)

Where, \( \max(\ ) \) means solving maximum of the function \( F \) within the range of the variable \( \Omega \), \( \varphi_1 \) and \( \varphi_2 \) are the function of the normalized TZs frequency variables. We propose the objective function for GA as follow:

\[
\varphi = \varphi_1 + C \cdot \varphi_2
\]

(7)

here \( C \) is the weighting factor determined by the \( L_{s1}, L_{s2} \).

\[
C = 10^{(L_{s2} - L_{s1}) / 10}
\]

(8)

In order to fulfill the given filter specifications, \( \varphi \) must satisfy \( \varphi \leq esp \), where \( esp \) is the target value

\[
esp = 10^{-L_{s1}/10} + C \cdot 10^{-L_{s2}/10}
\]

(9)

\[
\varphi = \varphi_1 + C \cdot \varphi_2 \leq 10^{-L_{s1}/10} + C \cdot 10^{-L_{s2}/10}
\]

(10)

\( \varphi_1 \) and \( \varphi_2 \) must satisfy.
We use GA to search for $\Omega_n (n=1, 2 \ldots N)$ to minimize the objective function $\phi$. When the minimum of $\phi$ is less than $esp$, we can obtain the locations of TZs to meet given specifications. The filter with these TZs has the least order and the highest stopband rejection. This method is much easier to determine the TZs than [5]; this eliminates the need for complex formula derivation and solving nonlinear equations.

3. Examples
In this section, for the verification of the method presented in this paper, it is applied to two examples of filter designs. Coupling schemes of four filters are shown in Figure. 2.

![Coupling schemes of two filters](image)

(a) (b)

- source/load terminals
- resonator
- direct coupling
- Cross coupling

Figure 2. Coupling schemes of two filters: (a) filter 1. (b) filter 2.

### 3.1 Filter 1
The given bandpass filter specifications from [12] are given as:

- The maximum return loss in passband is $RL=20$ dB.
- When $\Omega \geq \Omega_{s1}=1.2300$ rad/s, the minimum attenuation in the stopband $L_{s1}=40$ dB.
- When $\Omega \leq \Omega_{s2}=1.2300$ rad/s, the minimum attenuation in the stopband $L_{s2}=40$ dB.

The filter has characteristics of symmetric frequency response and symmetric TZs. We first determine TZs and the filter order according to given specifications using the method described in Section II. We can get $C=1.0$ and $esp=2.0 \times 10^{-4}$. As long as $\phi$ satisfy $\phi < 2.0 \times 10^{-4}$ when $\Omega \in [1.230 \ \infty )$, TZs obtained by using GA will satisfy the given specifications. The filter order and TZs can be obtained by directly optimizing the objective function in (7). The results obtained from this method and [12] are shown in Table 1.

The frequency responses of filter 1 with the same filter order and different locations of finite TZs, as calculated from filtering function in (1), are shown in Figure. 3.

From Table 1 and Figure 3, we know that the filter 1 with TZs obtained by this paper has higher stopband rejection than [12], and we can obtain the same filter order but the different TZs to fulfill given filter specifications, which can be selected according to an end user’s requirement in practical filter design, this increases the flexibility and selectivity of filter design.
Table 1 The comparison of the least filter order and finite TZs of filter 1 obtained by this method and [12] \((\varepsilon =0.1005)\)

| Method in [12] | Method in this paper |
|----------------|----------------------|
| Filter order \(N=7\) | \(N=7\) |
| Number of finite TZs \(N_{Tz}=4\) | \(N_{Tz}=4\) |
| Locations of finite TZs | \(\pm 1.1892\) | \(\pm 1.2511\) |
| The value of the objective function in (7) | \(1.995 \times 10^{-4}\) | \(5.464 \times 10^{-5}\) |

![Figure 3](image-url)  
Figure 3. Frequency responses of 7th order symmetric filter 1 with different number of finite TZs. Solid line: 4 TZs from [12]. Dashed line: 4 TZs from this paper. Dotted line: 6 TZs from this paper.

3.2 Filter 2

The given bandpass filter specifications from [12] are given as:

- The maximum attenuation in the passband is \(L_p=0.5\)dB.
- When \(\Omega \geq \Omega_{s1}=1.3604\)rad/s, the minimum attenuation in the stopband \(L_{s1}=54\)dB.
- When \(\Omega \leq \Omega_{s2}=2.2746\)rad/s, the minimum attenuation in the stopband \(L_{s2}=80\)dB.

The filter has characteristics of asymmetric frequency response and asymmetric TZs. We first determine TZs and the filter order according to given specifications using the method described in Section II. We can get \(C=400\) and \(esp=8.0 \times 10^{-6}\). As long as \(\phi\) satisfy \(\phi < 8.0 \times 10^{-6}\) with the range of stopband, TZs obtained by using GA will satisfy the given specifications. The filter order and TZs can be obtained by directly optimizing the objective function in (7). The results obtained from this method and [12] are shown in Table 2.

From Table 2 and Figure 4, we know that the filter 2 with TZs obtained by this paper has higher stopband rejection than [12].
Table 2 The comparison of the least filter order and finite TZs of filter 1 obtained by this method and [12] (ε = 0.34931)

| Method in [12]      | Method in this paper |
|---------------------|----------------------|
| Filter order        | N=5                  |
| Number of finite TZs| Ntz =3               |
|                     | 1.3948               |
| Locations of finite TZs | 2.3796      |
|                     | 3.8501               |
| The value of the objective function in (7) | 8.054×10⁻⁶ |

Figure 4. Frequency responses of 5th order asymmetric filter 2 with different number of finite TZs. Solid line: 3 TZs from [12]. Dashed line: 3 TZs from this paper. Dotted line: 4 TZs from this paper

4. summary
The effective objective function has been proposed for GA to determine the least filter order and the optimum locations of transmission zeros according to given filter specifications. We can obtain the locations of TZs to meet given specifications. The filter with these TZs has the least order and the highest stopband rejection.

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