Acoustoelectric current in graphene due to electron deformation potential and piezoelectric phonon couplings

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Abstract

Recent studies strongly indicate that graphene can be used as a channel material for converting surface acoustic waves to acoustoelectric current, which is a resource for various exciting technological applications. On the theoretical side, studies on phonon amplification/attenuation and acoustoelectric current at low temperatures in graphene have reported approximate analytical results under exceedingly simplifying conditions using the Boltzmann transport equation. Overcoming the earlier simplifying assumptions, we investigate both numerically and analytically the governing kinetic equations for amplification/attenuation and acoustoelectric current, taking into account the piezoelectric and deformation potential electron phonon coupling mechanism in the semi classical Boltzmann transport formalism approach, and obtain analytical results that are in reasonable agreement with the reported experimental results.

1. Introduction

The coupling of acoustic phonons with electrons is a phenomenon that serves both as a source of exciting physics and as a resource for novel device applications that emerge from time to time [1]. Phonons or phonon based mechanisms are an integral part of many modern technologies in which phonons can be manipulated in materials for novel device applications, including the promising functionalities for carrying and processing quantum information [1–8]. This has given rise to a competing fast emerging field of phononics in parallel to electronics and photonics. A significant effect related to the interaction of phonons with electrons in materials discovered in the early ’50s is the acoustoelectric effect (AE), which has been of much practical importance. The AE is observed both in bulk and nano specimen where a dc voltage (if open-circuited) or a dc electric current develops in the direction of propagation of the acoustic wave. The AE occurs by transferring some part of momentum by the acoustic wave to the electron gas in the process of electronic absorption [9, 10].

Apart from the bulk acoustic waves, surface acoustic waves (SAW) have immense use in diverse novel device applications. SAW has the advantage that it travels along the surface of a material and thus is accessible all along its path of propagation. The SAW modulates the electron density and exchanges energy with the electrons via the Deformation potential (DP) and Piezoelectric (PE) interactions. In the case of piezoelectric materials, SAW is accompanied by an electric field through which it can strongly interact with electrons within the adjacent surface and excite electric current in it. Nowadays, SAW is induced by comb-like metallic structures, called interdigital transducers (IDTs), deposited on the surface of the piezoelectric substrate. The piezoelectric effect causes the high frequency input signal at the transmitting IDT to stimulate the SAW. Applications of SAW are too numerous to quote and have been categorically mentioned in research papers, review articles, and monographs [1–14]. One latest application of SAW finds use in life sciences in the process of biosensing, cell monitoring and manipulation, and also in microfluidics for sensing and fluid mixing [15]. A whole new field of high frequency sono-processing for caveat-free acoustic material synthesis processing and manipulating is fast emerging [16]. In a recent review article, the snapshot of applications and future roadmap of SAW devices has been elaborated [17].
The advent of graphene on the material landscape has enriched the scope of the acoustoelectric phenomenon through the occurrence of SAW in this novel material [18]. Monolayer graphene (MLG) is a single two-dimensional hexagonal sheet of carbon atoms that are sp² hybridized, giving rise to three in-plane σ bonds and a π orbital perpendicular to the plane. While the strong σ bonds are the cause of the robust rigidity of the graphene sheet, the out-of-plane π bonds are behind the enhanced electrical and thermal mobility of the charge carriers. The zero bandgap in MLG limits its use as a switch in Logic circuits, but now AE current in MLG can enable overcoming this inherent limitation through the construct of graphene based AE transducer that can function as a logic switch [25]. The AE of the SAW photo amplification can be utilized in the creation of an opto-acoustoelectric device on graphene-piezoelectric structures for collecting, amplifying, and detecting super weak sources of THz-radiation photons [26]. The amplitude of the SAW is affected when target molecules are dropped onto the propagation path of a SAW. This property is anticipated for use in the simultaneous detection of charge and mass by combining Graphene Field-Effect Transistors and SAW sensors [27].

The coupling between charge carriers in thin films placed on a piezoelectric (PE) substrate with SAW is induced mainly by the presence of PE potential accompanying the propagating SAW. In a PE crystal, a mechanical strain produces an electric field proportional to the strain, and in a non-PE crystal, a deformation of the lattice produces a change in potential energy of the conduction electrons proportional to the strain. In figure 1(a) is depicted the schematics of a Graphene sheet placed on a PE substrate between two IDT devices. A SAW is generated by a high-frequency signal input to an IDT1 formed on a piezoelectric substrate that reaches IDT2, where it is converted back into a high-frequency signal for detection. When a SAW propagates in graphene, an acoustoelectric current (IAE) flows between the two attached electrodes. (b) The ratio of the difference between the complete and approximate Fermi–Dirac functions at $\omega_0 = 1$ THz and $k = 2 \times 10^8$ m⁻¹.
We present a theoretical investigation on the AE current \( I_{AE} \) generated by acoustic phonons interacting with electrons through piezoelectric (PE) and deformation potential (DP) electron phonon coupling in a MLG using the Boltzmann transport equation (BTE) approach. In a few theoretical studies, the investigations on attenuation/absorption coefficient/ rate and acoustoelectric current in the BTE approach have been reported \([36–39]\). The results have been obtained in the hyper sound regime where \( q' \gg 1 \). The mentioned studies have reported analytical expressions for the said quantities under simplifying assumptions of Fermi–Dirac (FD) distribution function, which remain valid only at temperatures far below the room temperature, where the approximations utilized to derive the analytical results are valid. The difference in the FD functions used at lower and higher temperatures has been illustrated in figure 1(b), where a plot of the ratio of the difference between the complete and approximate FD functions, \( f(K) - f(K') / \text{Appr.}(f(K) - f(K')) \) at \( \omega_q = 1 \) THz and \( k = 2 \times 10^8 \text{ m}^{-1} \). Clearly, the ratio is 1 in the low temperature limit, and above \( (T > 30 \text{ K}) \), the ratio begins to decline. So the reported results under this approximation at higher temperatures may not represent the accurate picture and hence are suspect. This warrants a complete evaluation of the parameter dependencies for phonon amplification and of the AE current, so that a better correspondence with experiments could be established.

The paper is organized as follows—the section 2 describes the formalism using the Boltzmann transport equation and the obtained analytical results for electron-DP and PE couplings. In section 3, the numerical evaluation of the complete expression and their comparison with analytical results are discussed. Finally, in section 4, the study is concluded.

2. Formalism and analytical results

The peculiar linear energy dispersion relation of graphene is given by \( \varepsilon_{\pm}(\vec{k}) = \pm \hbar v_f |\vec{k}| \) and the charge carrier is described by the two-component wavefunction \( \Psi_{\pm,\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}(1, \pm e^{i\varphi}) / \sqrt{2} \). Here, \( \pm \) refers to electronic and hole charge carriers, \( v_f \) is the Fermi velocity, \( \varphi \) is the angle between \( \vec{k} \) and \( \vec{r} \) directions, \( \vec{r} = (x, y) \) is the position vector in two dimensions, \( \vec{k} = (k_x, k_y) \) is the wavevector for a carrier along the graphene sheet.

In tight-binding approximation, the effective electron phonon Hamiltonian for MLG is written as \( H = H_0 + H_{DP}, \) where \( H_0 \) represents the two-component Hamiltonian near the \( K \) points that corresponds to free electron and \( H_{DP} \) is the electron phonon interaction and is given as \( H_{DP} = \sum_{\vec{q} \neq 0} M_{dq_{\vec{q}}}^{DP} \Psi_{\uparrow, \vec{q}_{\vec{q}}}^{\dagger} \Psi_{\downarrow, \vec{q}_{\vec{q}}} \), where \( M_{dq_{\vec{q}}}^{DP} = M_{dq_{\vec{q}}}^{DP}(q) \) is the electron-phonon coupling strength for DP transition, \( (\Psi_{\uparrow, \vec{q}_{\vec{q}}}^{\dagger} \Psi_{\downarrow, \vec{q}_{\vec{q}}}) \) is the electron-phonon interaction, \( \theta \) is the angle between scattering in and out wave vectors \( k \) and \( k' \). The transition rate induced in the system due to electron phonon interaction is given by Fermi’s golden rule, \( \Gamma_{k'k}^{S} = \frac{2\pi}{\hbar} \sum_{\omega_{q'}} |k'^{S} \rangle H_{DP} |k \rangle \delta(E_k \pm \hbar \omega_{k'k} - E_{k'}). \)

Considering that the scattering mechanism involves both the absorption and the emission process, the square of the matrix element of the electron-phonon interaction evaluates out as \( \langle k' | H_{DP} | k \rangle |F_{XX}^{S} \delta(E_k \pm \hbar \omega_{k'k} - E_{k'}) = |M_{dq_{\vec{q}}}^{DP} \rangle \langle \Psi_{\uparrow, \vec{q}_{\vec{q}}}^{\dagger} \Psi_{\downarrow, \vec{q}_{\vec{q}}} \delta(E_k \pm \hbar \omega_{k'k} - E_{k'}) \rangle + \langle N_{\omega_{q'}} | \delta(E_k \pm \hbar \omega_{k'k} - E_{k'}) \rangle, \) where \( \langle N_{\omega_{q'}} = 1 / \exp(\beta \omega_{q'})) \) is the Bose–Einstein (BE) distribution function for phonons at temperature \( \beta = 1 / k_B T \) \( (T \) is the temperature and \( k_B \) is the Boltzmann constant). The delta function \( \delta(E_k \pm \hbar \omega_{k'k} - E_{k'}) \) ensures energy conservation for the inelastic scattering processes by the absorption and emission of phonons of wavevector \( q = k - k' = 2k \) sin \( (\theta / 2) \). The square of the matrix element for the transition rate from \( k \) to \( k' \) for electron acoustic phonon is given as,

\[
|M_{dq_{\vec{q}}}^{DP}(q)|^2 \Gamma_{k'k}^{S} = \frac{V^{DP} \hbar \omega_{q}^{S} \left(1 + \cos \theta \right)}{2\pi A \rho_d \nu^{DP}^{S}} + \frac{V^{PE} \nu^{PE} \rho_{e} \left(1 + \cos \theta \right) q_{x} q_{y}^{S} \left(1 + \cos \theta \right)}{2\pi A \rho_{e} \nu^{PE}^{S}}
\]

Where \( V^{DP}, V^{PE} \) is the coupling constant for electron-DP (electron-PE) phonon interaction, \( A \) is the area of graphene specimen, \( \rho_d(q) \) is the graphene areal mass density (substrate mass density), \( \nu^{DP}(\nu^{PE}) \) is the velocity of sound in material for DP(PE) interaction, \( \rho_{e} \) is a constant arising out of the elastic properties of GaAs substrate, \( d \) is the distance between graphene and PE substrate. As can be seen from the above matrix elements for the dispersion relation, \( \omega_{q} = \nu^{DP}^{S} q \) the DP interaction Hamiltonian is proportional to \( \sqrt{q} \), whereas, in contrast, the PE interaction vertex is independent of the magnitude of \( q \) but depends on its orientation. Comparing the
relative magnitude of PE and DP phonons matrix elements, \( |M^{\text{PE}}_{kk'}(q)|/|M^{\text{DP}}_{kk'}(q)| = 1.3 \times 10^7/\text{q m}^{-1} \) suggests that both these scattering mechanisms are important in providing important channels for energy relaxation.

The interaction between a SAW and the two-dimensional electron gas in a doped graphene layer placed over the substrate induces a linear acousto electric direct current density, \( J_{AE} \). This \( J_{AE} \), as stated above, develops in a closed circuit in the specimen in direction of propagation of acoustic waves due to the transmission of momentum to the electron gas (electronic absorption) by the acoustic phonon population \( N_q \) characterizing the acoustic wave. When the system is in equilibrium (i.e., the external driving field is absent) the distribution of phonons is given by BE distribution of acoustic phonons at the lattice temperature. The momentum transferred to the electrons is \( \frac{dn}{dt} = -h \frac{dn_0}{dt} \) where \( \frac{dn_0}{dt} \) is the number of phonons per unit time absorbed or emitted by the electrons. The energy transferred to the electrons is \( \frac{dn}{dt} = -h \frac{dn_0}{dt} \). The absorption coefficient is \( \Gamma = -\frac{1}{\phi} \frac{dn}{dt} \) is just the energy transferred per unit time divided by the incident energy flux \( \Phi \). This relation was originally derived by Weinreich in the absence of external dc electric and magnetic fields [40]. The kinetic equation for the acoustic phonon population \( N_q \) in the medium is governed by the following equation [35],

\[
\frac{dN_q}{dt} = \frac{1}{\tau_q} N_q
\]

Where \( \tau_q \) is the electron-acoustic DP(PE) phonon relaxation time. This time for electron-phonon scattering can vary widely depending on the nature of electron-phonon interaction mechanism and the electron energy distribution. Typical electron-phonon scattering times are in general of the order tens of picoseconds. Denoting, \( \tau_q = R_q^{-1} \) where \( R_q \) is the phonon amplification/attenuation rate (\( R_q > 0 \), amplification occurs and when \( R_q < 0 \), absorption or attenuation occurs),

\[
R_q = \frac{1}{N_q} \frac{\partial N_q}{\partial t}
\]

The total rate of phonon absorption and emission from the perturbation theory can be expressed as,

\[
\frac{\partial N_q}{\partial t}(t) = (2\pi/\hbar) g_s g_v \sum_{k,k'} |M^{\text{DP(PE)}}_{kk'}(q)|^2 \{ (N_q + 1) C_{kk'} \delta_{k,k'} \\
\times f(K)(1 - f(K')) \delta(E_k - E_k' - h\omega_q) - N_q f(K')(1 - f(K)) \}
\]

In equation (4), \( g_s = 2, g_v = 2 \) are the spin and valley degeneracies, respectively and \( E_k = E_k + h\omega_q \). The \( f(K) \) is the shifted equilibrium electronic electronic FD distribution function in the linear response regime approximated as \( f(K) = \{ \exp[\beta(h\nu K - e\phi)] \}^{-1} \), where \( K = \sqrt{k^2 - 2kk'd \cos \phi + k_d^2} \) and \( k_d \) is the shift of the electron momentum due to the presence of the drift velocity, \( v_d \), caused by the application of the dc electric field, \( E_d = V_d/L \), with \( V_d \) being the voltage applied along the conducting channel of length \( L \). The \( f(K)(1 - f(K')) \) represents the probability that the initial k state is occupied and the final electron state \( k' \) is empty, while the factor \( f(K')(1 - f(K)) \) represents the initial k state is empty and the final electron state \( k' \) is occupied, in which \( K' = \sqrt{k'^2 - 2k'k_d \cos (\phi + \phi) + k_d^2} \). In equation (4), the summation over \( k \) and \( k' \) can be transformed into two dimensional integrals by \( \sum_{k,k'} = A^2/(2\pi)^2 \int dk dk' \). Taking \( (N_q(t) + 1) = N_q(t) \) and neglecting \( f(K)f(K') \), equation (4) transforms to,

\[
\frac{\partial N_q(t)}{\partial t} = (2\pi/\hbar) g_s g_v |M^{\text{DP(PE)}}_{kk'}(q)|^2 \int_{0}^{\infty} dk \int_{0}^{2\pi} d\phi \int_{0}^{\infty} k'dk' \int_{0}^{2\pi} d\theta (1 + \frac{\cos \theta}{2}) N_q(t)
\]

Using equation (5), \( R_q^{\text{DP(PE)}} \) can be expressed as,

\[
R_q^{\text{DP(PE)}} = \frac{2\pi g_s g_v A^2 |M^{\text{DP(PE)}}_{kk'}(q)|^2}{\hbar} \int_{0}^{\infty} dk \int_{0}^{2\pi} d\phi \int_{0}^{\infty} k'dk' \int_{0}^{2\pi} d\theta (1 + \frac{\cos \theta}{2})
\]

\[
\times \left\{ \frac{1}{1 + \exp[\beta h\nu \sqrt{k^2 - 2kk'd \cos \phi + k_d^2}]} - \frac{1}{1 + \exp[\beta h\nu \sqrt{k'^2 - 2k'k_d \cos (\phi + \phi) + k_d^2}]} \right\} \times \delta(k - k') \frac{-\omega_q}{\nu}
\]

A closed-form solution of equation (6) is not possible because integration over \( \theta \) and \( \phi \) makes it non trivial, and we defer its numerical solution to section 3. To obtain an analytical solution, we evaluate the integral under the simplifying assumption that the electrons and phonons are propagating along the direction of the applied field,
that is, we make $\theta$ and $\phi = 0$. Therefore equation (6) takes the form

$$R_q^{DP,PE} = \frac{\pi g_s g_c A^2 |M_k^{DP,PE}(q)|^2}{2(2\pi)^4 \hbar^2} \int_0^{2\pi} dk \int_0^{\infty} d\phi \int_0^{2\pi} dk' d\theta \left( k - k' - \frac{\omega_q}{\nu_f} \right)$$

$$\times \frac{1}{1 + \exp \left[ \beta \hbar \nu_f (k - k_d - k_f) \right]}$$

$$= \frac{1}{1 + \exp \left[ \beta \hbar \nu_f \left( k - \frac{\omega_q}{\nu_f} - k_d - k_f \right) \right]}$$

(7)

Both the attenuation coefficients for DP and PE interaction can be evaluated from equation (7). The same explicitly for the case of DP coupling reduces to the following equation,

$$R_q^{DP} = \frac{A V^{DP} \omega_q}{2\pi \hbar \nu_f \nu_{DP}^2} \int_0^{\infty} k \left( k - \frac{\omega_q}{\nu_f} \right)$$

$$\times \frac{1}{1 + \exp \left[ \beta \hbar \nu_f (k - k_d - k_f) \right]}$$

$$= \frac{1}{1 + \exp \left[ \beta \hbar \nu_f \left( k - \frac{\omega_q}{\nu_f} - k_d - k_f \right) \right]}$$

(8)

The above equation on integration yields the following solution,

$$R_q^{DP} = \frac{A V^{DP} \omega_q}{2\pi \hbar \nu_f \nu_{DP}^2} \left( \beta \omega_q \hbar \text{PolyLog}[2, -e^{(k_d + k_f)/\nu_f}] \right.$$

$$- \text{PolyLog}[2, -e^{(k_d + k_f)/\nu_f + \omega_q/\hbar}]$$

$$- 2\text{PolyLog}[3, -e^{(k_d + k_f)/\nu_f + \omega_q/\hbar}] + 2\text{PolyLog}[3, -e^{(k_d + k_f)/\nu_f + \omega_q/\hbar}]$$

(9)

Similarly on feeding the matrix element for PE interaction in equation (7), yields the equation as under,

$$R_q^{PE} = \frac{A g_p eV^{PE} \nu_f^2}{64\pi^2 \hbar^2 \nu_f \nu_{PE}^2} \int_0^{\infty} k \left( k - \frac{\omega_q}{\nu_f} \right)$$

$$\times \frac{1}{1 + \exp \left[ \beta \hbar \nu_f (k - k_d - k_f) \right]}$$

$$= \frac{1}{1 + \exp \left[ \beta \hbar \nu_f \left( k - \frac{\omega_q}{\nu_f} - k_d - k_f \right) \right]}$$

(10)

Which on integration gives the result,

$$R_q^{PE} = \frac{A g_p eV^{PE} \nu_f^2}{64\pi^2 \hbar^2 \nu_f \nu_{PE}^2} \left( \beta \omega_q \hbar \text{PolyLog}[2, -e^{(k_d + k_f)/\nu_f}] \right.$$

$$- \text{PolyLog}[2, -e^{(k_d + k_f)/\nu_f + \omega_q/\hbar}]$$

$$- 2\text{PolyLog}[3, -e^{(k_d + k_f)/\nu_f + \omega_q/\hbar}] + 2\text{PolyLog}[3, -e^{(k_d + k_f)/\nu_f + \omega_q/\hbar}]$$

(11)

In previous works using the BTE the FD function has been approximated at low temperature as

$$f(k) = \exp \left( -\beta (\hbar \nu_f k - \varepsilon_f) \right),$$

using which the equation (7) reduces to

$$R_q^{DP} = \frac{A V^{DP} \nu_f^2}{2\pi \hbar^2 \nu_f \nu_{DP}^2} \int_0^{\infty} k \left( k - \frac{\omega_q}{\nu_f} \right)$$

$$\times \left( -1 + e^{\beta \omega_q \nu_f} \right) e^{-\beta \omega_q \nu_f} \left( 1 - e^{\beta \nu_f \varepsilon_f} \right)$$

(12)

The solution to the above equation as reported in [36] is

$$R_q^{DP} = \frac{A V^{DP} q^2 k_f^2}{2\pi \hbar^2 \nu_f \nu_{DP}^2} \left( -q + \frac{q}{k_f} \right) e^{-\beta \nu_f \varepsilon_f} \left( 1 - e^{\beta \nu_f \varepsilon_f} \right)$$

(13)
And the same with a slight modification in assumption is reported in [37] as

\[
R_q^{DP} = \frac{A V^{DP} \omega_q}{2 \pi h \beta \nu q^{DP} \nu_f} \left( 2 - \beta \omega_q \hbar \left( 1 - \frac{\nu_f q}{\nu_f} \right) \right) e^{-\beta \hbar \nu_q \left( 1 - \frac{\nu_f}{\nu_f} \right)} = e^{-\beta \hbar \nu_q \left( 1 - \frac{\nu_f}{\nu_f} \right)} \right)
\] (14)

The \( J_{AE}^{DP(PE)} \) is directly proportional to the acoustic phonon population rate, \( J_{AE}^{DP(PE)} \propto R_q^{DP(PE)} \). However, the expression for \( J_{AE}^{DP(PE)} \) in MLG quoted in [37] as \( J_{AE}^{DP(PE)} = \frac{2 \pi q^{DP(PE)} \nu}{q^{DP(PE)} R_q^{DP(PE)}} \), where \( \tau \) is the relaxation time and \( \Gamma_{q}^{DP(PE)} = \nu q^{DP(PE)} R_q^{DP(PE)} \) is suspect as the expression does not bear the dimension of current density. Another correct phenomenological expression obtained in the classical relaxation model is \( J_{AE}^{DP(PE)} = -\frac{\mu q^{DP(PE)}}{v_{q}^{DP(PE)}} \), in which \( \mu \) is the mobility and \( q \) is the intensity of the SAW wave. Since the prefactors in the above expression, i.e., \( \frac{v_{q}^{DP(PE)}}{v_{q}^{DP(PE)}} \), have the dimension of linear current density (current/length) therefore, we obtain an empirical relation for current density by multiplying \( R_q^{DP(PE)} \) with \( \frac{\mu}{\nu_{q}^{DP(PE)}} \). Hence, \( J_{AE}^{DP(PE)} = -\frac{e^{DP(PE)} \mu}{v_{q}^{DP(PE)}} \), which on substituting the full expression for \( R_q^{DP(PE)} \) from equation (6) becomes \( J_{AE}^{DP(PE)} = \frac{1}{2} \frac{\partial N_q}{\partial t} \). Therefore,

\[
J_{AE}^{DP(PE)} = \frac{2 \pi q \nu q \left[M_{AE}^{DP(PE)}(q)\right]^2}{(2 \pi)^4 h v_f} \int_0^\infty k dk \int_0^{2\pi} d\phi \int_0^{2\pi} k' dk' \int_0^{2\pi} d\theta \left( 1 + \cos \theta \right)
\]

Feeding the obtained expressions, equations (8) and (9) for \( R_q^{DP(PE)} \), derived in the approximation that electrons and phonons are propagating along the direction of the applied field, that is \( \theta = 0 \) and \( \phi = 0 \), in \( J_{AE}^{DP(PE)} \) we get the following separate expressions for \( J_{AE}^{DP(PE)} \), \( J_{AE}^{DP(PE)} \), respectively as,

\[
J_{AE}^{DP} = -\frac{e^{DP(PE)} \omega_q}{2 \pi h \beta \nu q^{DP} \nu_f^2} \left( (\beta \omega_q \hbar (\text{PolyLog}[2] - e^{(k_x+k_y)\nu_f \gamma \hbar}) - \text{PolyLog}[2] - e^{(k_x+k_y)\nu_f \gamma \hbar} \right)
-2\text{PolyLog}[3] - e^{(k_x+k_y)\nu_f \gamma \hbar} + 2\text{PolyLog}[3] - e^{(k_x+k_y)\nu_f \gamma \hbar} \right)
\] (16)

\[
J_{AE}^{PE} = -\frac{e^{PE} \omega_q}{2 \pi h \beta \nu q^{PE} \nu_f^2} \left( (\beta \omega_q \hbar (\text{PolyLog}[2] - e^{(k_x+k_y)\nu_f \gamma \hbar}) - \text{PolyLog}[2] - e^{(k_x+k_y)\nu_f \gamma \hbar} \right)
-2\text{PolyLog}[3] - e^{(k_x+k_y)\nu_f \gamma \hbar} + 2\text{PolyLog}[3] - e^{(k_x+k_y)\nu_f \gamma \hbar} \right)
\] (17)

Equations (16) and (17) respectively represent the complete unscreened AE \( J_{AE}^{DP(PE)} \) through coupling with DP and PE phonons in MLG.

The same has been calculated in [37] in low-temperature Bloch-Gruneisen (BG) regime as under,

\[
J_{AE}^{DP} = I_0 \left[ 2 - \beta \omega_q \hbar \left( 1 - \text{exp} \left( -\beta \omega_q \hbar \right) \right) \right]
\] (18)

where

\[
I_0 = -16 \pi q V^{DP} v_i T^4 / \pi \rho h T_{BG}^4 = 2 h v_f \nu_f k_f / k_B
\]

The AE current can also be calculated using the equation (13) as

\[
J_{AE}^{DP} = \frac{e^{DP} q^2 k_f^2}{2 \pi h \beta \nu q^2 (v_f - \frac{q}{k_f} \nu_f^2)} e^{-\beta \hbar \nu_q \left( 1 - \frac{\nu_f}{\nu_f} \right)}
\] (19)

3. Numerical and analytical results and discussions

All the obtained and reported analytical expressions for Amplification coefficient and acoustoelectric current density for the cases of electron acoustic phonon coupling via Deformation potential and Piezoelectric modes have been summarized in table 1.

In the following, the numerical evaluation of the complete expression and the obtained analytical results is carried out. We investigate the role of DP and PE coupling on the generation of AE current as a function of
| Coefficient Name                  | Result                                                                 | Amplification coefficient, $R_{q}^{DP/PE}$ (sec$^{-1}$) | Acoustoelectric current density, $J_{AE}^{DP/PE}$ (Am$^{-2}$) |
|----------------------------------|-------------------------------------------------------------------------|----------------------------------------------------------|---------------------------------------------------------------|
| **Case of electron DP coupling** |                                                                         | $\frac{AV_{DP}^2\omega_{q}T^{q/2}}{\pi\hbar\gamma_{DP}^{1/2}} \left( \frac{\omega_{l}h}{k_{q}T} \right) \times \left( \text{PolyLog}[2, -e^{qk_{l}q_{y}^r + q_{y}^b}] \right)$ | $-\frac{e^{AV_{DP}^2\omega_{q}T^{q/2}}}{\pi\hbar\gamma_{DP}^{1/2}} \left( \frac{\omega_{l}h}{k_{q}T} \right) \times \left( \text{PolyLog}[2, -e^{qk_{l}q_{y}^r + q_{y}^b}] - \text{PolyLog}[2, -e^{qk_{l}q_{y}^r + q_{y}^b + q_{y}^b}] \right)$ |
temperature, frequency, phonon velocity, and electronic density. For the estimation of numerical results we use the following values of parameters: $V^D_P = 9eV$, $v_1^D = 2.1 \times 10^5 \text{ m sec}^{-1}$, $\rho = 6.5 \times 10^{-7} \text{ kg m}^{-2}$, $\tau = 10^{-10} \text{ sec}$, $A = 200 \times 100 \mu \text{m}^3$, $\omega_q = 1.5 \times 10^7 \text{ s}^{-1}$, $v_{PE} = 2.4 \times 10^7 \text{ eV cm}^{-1}$, $\rho_1 = 5.3 g \text{ cm}^{-2}$, $c_{PE} = 4.9$ (known by the elastic properties of GaAs), $v_{3 PE} \approx 0.9v_s \approx 2.7 \times 10^5 \text{ m s}^{-1}$ [36, 37]. The electron-PE acoustic phonon coupling is strongest for surface phonons propagating along the diagonal direction with large distances $d > 5\AA$, so $e^{-2qd} \approx 1$.

It was pointed out in the introduction that the approximations made in obtaining the earlier reported analytical result stands correct in the low temperature limit ($T \leq 30 \text{ K}$), as then the numerical factor of 1 in comparison to exponential factor in the denominator of the FD function, $f(K) = \{\exp[(\beta(\hbar v k - \varepsilon_f)] + 1\}^{-1}$ is small, but for higher temperatures the same is not true as the term $\exp[\beta(\hbar v k - \varepsilon_f)]$ begins to decline fast with rising temperature, ($T > 30 \text{ K}$). The same can be clearly observed from the figure 1(b), where a plot of the ratio of the difference between the complete and approximate FD functions, $f(K) - f(K')/\text{Appr.}(f(K) - f(K'))$ at $\omega_q = 1 \text{ THz}$ and $k = 2 \times 10^8 \text{m}^{-1}$ is depicted. Noticeably, the ratio is 1 in the low temperature limit, and above ($T > 30 \text{ K}$), the ratio begins to decline. So the reported results under this approximation at higher temperatures may not represent the accurate picture and hence are suspect. Next, we consider the numerical estimation of the obtained analytical results for AE current for DP and PE phonon coupling and their comparison with full numerical computation of the governing equation (15). In figure 2(a), we plot the less restrictive approximate analytical result from equation (16) for three different temperatures $T = 95 \text{ K}, 200 \text{ K}, 300 \text{ K}$ at $v_1^D = 2.1 \times 10^5 \text{ ms}^{-1}$. The magnitude of current at $\omega_q = 2.7 \times 10^8 \text{Hz}$ is about 18(nA) at $T = 300 \text{ K}$. We find that there is a slight decrease in the current with an increase in temperature, which further increases with increasing frequency. Figure 2(b) gives a comparative plot of the current, $I_{AE}^D$ versus $\omega_q$ at $T = 100 \text{ K}$ at $v_1^D = 2.1 \times 10^4 \text{ms}^{-1}$, where curve-A corresponds to the full numerical result using equation (15), curve-B from equation (16), and curve-C using equation (14) after feeding it into the definition of $I_{AE}^D = -eB_q^D$. Curve-B is five times more in magnitude than curve-C at $2 \times 10^{11} \text{ Hz}$. Clearly, the reported analytical result underestimates the amplitude coefficient. Overall from figure 2(a), the increasing trend of acoustic current with frequency and temperature and the order of the obtained magnitude of the current in nA for frequencies in hundreds of MegaHertz [28], and in mA from figure 2(b) are in excellent agreement with the reported values.

Figure 3(a) depicts the approximate analytical result for $I_{AE}^D$ from equations (16) and (13), respectively up to 0.2 GHz at $T = 20 \text{ K}$ and $v_1^D = 2.1 \times 10^4 \text{ms}^{-1}$. The temperature dependence of the same is plotted in figure 3(b). The loglogplot of the same with the temperature at $\omega_q = 1 \text{THz}$ and $v_1^D = 2.1 \times 10^4 \text{ms}^{-1}$, where curve-A is from equation (19), curve-B from equation (16). The curve-A from earlier reported result equation (16) has weaker temperature dependence at higher frequencies, while our obtained result incorporating the full FD functions exhibits stronger temperature dependence at lower temperatures (<100 K). This can be understood from figure 1(b), where a plot of the ratio of the difference between the complete and approximate FD functions, i.e., $f(K) - f(k')/\text{Appr.}(f(K) - f(k'))$ at lower and higher temperatures, has been illustrated. Clearly, the ratio is 1 in the low-temperature limit, and above ($T > 30 \text{ K}$), the ratio begins to
So as mentioned in the introduction, the reported results under this approximation at higher temperatures may not represent the accurate picture and hence are suspect. That is why the AE current shown in figure 3 (b) exhibits a strong temperature dependence rather. Physically also at higher temperatures, there is more excitation, and therefore absorption of SAW increases, as also has been reported in\textsuperscript{28} and elsewhere.

Besides frequency and temperature, the $IAE$ also depends on phonon velocity and carrier density. To investigate the dependence of AE current with velocity and carrier density, we plot in figure 4 (a) the approximate analytical result from equations (16) and (19), respectively as a function of $\frac{\nu_{m \text{ss}}}{DP}$ at $T = 100$ K and $\nu_{d} = 1 \text{THz}$. And in figure 4 (b), the ratio of the currents from equations (16) and (19) as a function carrier concentration at $\omega_q = 1 \text{THz}$ and $T = 100$ K is shown. From figure 4 (a), we notice that the two curves A and B from equations (16) and (19), respectively, show a contrasting behavior with phonon velocity. The amplification of acoustic phonons and thereby the acoustic current takes place when the drift velocity of electrons exceeds the phonon velocity which is also consistent with experiments. Now for a fixed electric field and thereby a fixed drift velocity, the amplification will decrease as the phonon velocity increases, which is what our un-approximated result in figure 4(a) is portraying. As regards the increasing curve from the reported highly approximated analytical result, the behavior is suspect and cannot be relied upon because of the crude approximations. The same type of behavior from the two results is seen also seen with carrier density, as
plotted in figure 5(b). Initially, curve-A from equation (16) dominates, but after a cross-over near $3.5 \times 10^5 \text{cm}^{-2}$, the curve-B begins to dominate.

To check the relative significance of PE and DP coupling currents, we plot the ratio of $I_{DP}/I_{PE}$ using equations (16) and (17) as a function of frequency $\omega$ (GHz) in figure 5(a). This ratio is independent of temperature, and near 11 GHz, the ratio is equal. So we can infer that the PE contribution remains dominant below 11 GHz, and above it, the DP forms the major source of acoustoelectric current. A plausible explanation for this based on the study of SAW absorption by graphene is as follows: The SAW absorption in a material depends on the number of transition channels. As the band structure of graphene is gapless and the carriers massless, the charge carriers in graphene can absorb sound waves and are excited into higher energy states via both intra-band and inter-band transitions. In graphene, the SAW absorption occurs from two competing mechanisms. First, as $\omega$ increases, more electrons are excited from lower energy states to higher energy states through SAW absorption scattering, and this can happen in the low-frequency regime according to the laws of momentum and energy conservations. However, the momentum or wave vector for SAWs increases rapidly with increasing frequency $\omega$. This can limit the number of transition channels required by the momentum and energy conservation laws at higher frequencies, and the absorption intensity decreases with $\omega$. Second, it is known that the Fermi energy at a fixed temperature in graphene increases with an increase in electron density. This allows for the occupation of higher energy states by electrons as the density is increased, which further demands that a larger momentum or energy exchange is needed for electronic transitions. Therefore, the intensity of SAW absorption should decrease with increasing electron density. However, though both the PE and DP scattering depend sensitively on density but the matrix element for DP interaction has a direct dependence on carrier density and phonon velocity as it decreases on decreasing the carrier density and increasing the phonon velocity.

In figure 5(b) is plotted the analytical $I_{PE}$ versus $\omega$ (GHz) for different sets of carrier densities, phonon velocities and temperatures. The analytical $I_{PE}$ with $\omega$ (Hz), top merging curves-A & B are at $T = 20 \text{K}$ and 100K respectively, for $n_e = 1 \times 10^{10} \text{cm}^{-2}$ and $v_{SP_{PE}} = 2.7 \times 10^3 \text{m/s}$, middle curve-C at $T = 100 \text{K}$ and $n_e = 1 \times 10^{12} \text{cm}^{-2}$, $v_{SP_{PE}} = 2.7 \times 10^4 \text{m/s}$, lowest curve-D at $T = 100 \text{K}$ and $n_e = 1 \times 10^{12} \text{cm}^{-2}$, $v_{SP_{PE}} = 2.7 \times 10^4 \text{m/s}$. From this figure, we observe that by increasing temperature $T = 20 \text{K}$ and 100K keeping other variables fixed, the current remains almost the same. But the current has a stronger dependence on carrier density and phonon velocity as it decreases on decreasing the carrier density and increasing the phonon velocity.

4. Conclusions

We investigated both numerically and analytically the governing kinetic equations on amplification/attenuation coefficient and acoustoelectric current in monolayer graphene due to piezoelectric and deformation potential electron phonon couplings in the Boltzmann transport formalism approach by overcoming the earlier simplifying assumptions covering the low and high temperature range and obtain analytical and numerical results. We also investigated the role of temperature, frequency, phonon velocity and electronic density besides the effect of DP and PE coupling on AE current. Our results also differ from the earlier reported results in the

![Figure 5](image-url)
following sense; (i) the obtained results show a much stronger dependence (about five times) on frequency, (ii) it shows a relatively stronger dependence on temperature at higher frequencies, (iii) and with phonon velocity and carrier density as it shows a contrasting behavior with the earlier reported result, (iv) at higher frequencies the DP contribution becomes significant, and above 11 GHz, the DP dominates the PE contribution and forms the major source of acoustoelectric current. We find that the results obtained are in reasonable conformity with the reported experimental results. Further, the obtained magnitude of acoustoelectric current for very high frequencies (10^{12} Hz) frequencies at room temperatures augurs well for graphene to be used as a very high frequency SAW device.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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