Numerical simulation of tsunami propagation with Finite Difference Method and Runge-Kutta 4th Order Method (study case: south coast of Java island)

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Abstract. Tsunamis are disasters that cause so much damage. In the history of the tsunami, Indonesia has been hit by tsunami several times, based on data from the National Disaster Management Agency (BNBP), as of 1629-2007, Indonesia was recorded 184 times affected by large and small tsunami disasters. Based on these data, it is very important for Indonesia to increase security against the tsunami disaster. Also, the tsunami in Indonesia occurred due to earthquakes, volcanic eruptions, and landslides in the sea. This study discussed about the tsunami with a location located in the southern of Java, precisely in the area of Central Java, considering that in 2017 there was friction between the Indies and Eurasian plates which caused an earthquake measuring 6.9 magnitude, but did not cause a tsunami. In this study, a tsunami wave propagation simulation was carried out with the aim of knowing how long it would take the waves to the shoreline and how high the waves would be when they were on the shoreline. The model used is a tsunami model created by Imamura by solving differential equations using finite difference and Runge-Kutta 4th order. Based on the simulation results with the initial determination of a 5 m tsunami wave centered at a distance of 25 km from the shoreline, Tsunami waves with a height of 8.7 m will arrive at the shoreline at 140 seconds. With wave height is 8.7 m at the shoreline, the water will enter the land or can also be called the tsunami disaster. At that time, residents are expected to have been evacuated from the coastal area. In addition, based on numerical simulations that have been carried out, information is obtained that the computation time of the method is faster than the computation time of the Runge-Kutta method with differences in altitude results that are quite small and can be tolerated. This shows that the Finite Difference method can be applied to tsunami disaster mitigation software because it has a fairly fast computing time compared to the Runge-Kutta 4th order method.

1. Introduction

Indonesia is a country that has 3 active plate movements, namely the Hinda-Australia plate, the Eurasian plate, and the Pacific plate. Java Island is the most populated in Indonesia. Based on the data there are 2 plate movements in Java Island, in the north there is the Pacific plate and in the south there is the Indian-Australian plate. If there is friction or collision in both, the potential for a tsunami is very large [1].

Tsunami can travel at speed of 500 to 1000 km per hour in the deep ocean but near the shore it can slow down until 10 km per hour, different with speed, the high wave of tsunami in deep ocean
only a few meter but when the wave arrived in the shore line, the wave high increase until ten meters and can higher [2]. Tsunamis are disasters that cause so much damage. In the history of the tsunami, Indonesia has been hit by tsunami several times, based on data from National Disaster Management Agency (BNBP), as of 1629 - 2007, Indonesia was recorded 184 times affected by large and small tsunami disasters. Based on these data, it is very important for the Indonesian Government to increase security against the tsunami disaster.

This study discussed about the tsunami incident with the location located in the south of the island of Java, precisely in the area of Central Java, considering that in 2017 there was friction between the Indian and Eurasian plates which caused a 6.9 magnitude earthquake, but did not cause a tsunami [3].

Research on tsunamis was carried out to save human life, one Japanese researcher has modeled the tsunami, by reducing the mass equation and momentum models [4]. Based on the mathematical model that has been formed by Imamura, there will be a discretization process with two methods, namely the Finite Difference method and the Runge Kutta 4th Order methods. Furthermore, from the results of discretization, numerical simulations will be carried out related to the propagation of tsunami waves. In the simulations carried out, given the initial value of the tsunami height and wavelength, as well as the distance of the wave source from the shoreline. From the numerical simulation results, obtained the time and height of the wave when it reached the shoreline and the effectiveness of the discretization method related to computing time.

2. Tsunami
Tsunamis come from Japanese, namely tsu (port) and name (wave). A tsunami is a very large wave propagation caused by a variety of factors, including a loose motion of the earth, an eruption of an earth's volcano, or a fall of a meteor to the sea. The height of the tsunami wave was not too large at first, but the height had become tens of meters when approaching the shoreline. Inversely proportional to altitude, the initial speed of the source of the tsunami is very high but when approaching the shoreline the speed will decrease.

2.1 Tsunami wave mathematical model
Tsunamis can be modeled into mathematical models. The Tsunami Matrix Model is written in a differential equation, which starts from the mass and momentum equation as follows [4]:

\[
\frac{\partial \eta}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}
\]

\[
\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} + \frac{1}{\rho} \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \right) = 0 \tag{2}
\]

\[
\frac{\partial v}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} + \frac{1}{\rho} \left( \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} \right) = 0 \tag{3}
\]

\[
g + \frac{1}{\rho} \frac{\partial \rho}{\partial z} = 0 \tag{4}
\]

Where \(x\) and \(y\) are the horizontal axes and \(z\) is the vertical axis, \(t\) represents the time, \(h\) is the flow of water, \(\eta\) is the height of the wave above the surface of water, \(u, v\) and \(w\) are the velocity in the direction of the \(x, y\) and \(z\) axes, \(g\) is the speed of gravity, by example and decline by Imamura, 2006, obtained by mathematical models, 2-D tsunami waves are as follows [4]:

\[
\frac{\partial \eta}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0 \tag{5}
\]

\[\text{x - axis :} \]

\[
\frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left( M^2 \right) + \frac{\partial}{\partial y} \left( MN \right) + gD \frac{\partial \eta}{\partial x} + \frac{g n^2 M \sqrt{M^2 + N^2}}{D^3} = 0 \tag{6}
\]

\[\text{y - axis :} \]
\[
\frac{\partial N}{\partial t} + \frac{\partial}{\partial x} \left( \frac{MN}{D} \right) + \frac{\partial}{\partial y} \left( \frac{N^2}{D} \right) + gD \frac{\partial h}{\partial y} + g\frac{n^2N\sqrt{MN^2 + N^2}}{D^2} = 0
\]  \(7\)

\(x, y\): coordinate fields

\(M\): Discharge the wave on the x-axis

\(N\): Discharge the wave on the y-axis

\(\eta\): The height of the wave

\(D\): water depth above the source field

The initial condition used in the simulation of a tsunami wave propagation model is the Gaussian Bell function \([5]\). The Bell Gaussian function is formulated as follows:

\[
\eta_0(x, y) = A e^{\left( -\frac{1}{2\sigma_x^2}(x-x_0)^2 - \frac{1}{2\sigma_y^2}(y-y_0)^2 \right)}
\]  \(8\)

Where \(A\) is the amplitude of the initial tsunami wave, \(x_0\) and \(y_0\) are the centers of the initial wave of the tsunami, \(\sigma_x\) and \(\sigma_y\) are the width of the Bell Gaussian function \([4]\). Furthermore, the initial conditions for discharge flux are formulated as follows:

\[
M_0(x,y) = 0 \ , \ N_0(x,y) = 0
\]  \(9\)

3. Discretization of the Tsunami Mathematical Model

In this section, will be explained the discretization with two methods, namely the Finite Difference Methods and Runge Kutta 4th Order methods.

3.1 Finite difference methods

The finite difference method is a numerical method commonly used to solve technical problems and mathematical problems of a physical phenomenon. Important applications of finite difference methods are in numerical analysis, especially in ordinary differential equations and partial differential equations. The principle is to replace derivatives that exist in differential equations with finite difference discretization based on Taylor series. Physically, the Taylor series can be interpreted as the magnitude of the review in a space and time (space and time of review) can be calculated from the magnitude itself in a certain space and time that has a small difference with the space and time of review. In this study, advanced differences for time discretization and central difference are used for discretization of space. The following are the finite difference equations \([6]\):

1. Forward Difference :

\[
\frac{\partial u}{\partial t} = \frac{u(t_1 + \Delta t) - u(t_i)}{\Delta t}
\]  \(10\)

2. Center Difference :

\[
\frac{\partial u}{\partial x} = \frac{u(x_i + \Delta x) - u(x_i - \Delta x)}{2\Delta x}
\]  \(11\)

Furthermore, discretization of tsunami wave models with different and advanced methods for time dimensions and center to center dimensions for space dimensions was obtained, so that it was obtained:

\[
\frac{\eta_{i,j}^{t+1} - \eta_{i,j}^t}{\Delta t} + \frac{(M_{i+1,j}^t - M_{i-1,j}^t)}{2\Delta x} + \frac{(N_{i,j+1}^t - N_{i,j-1}^t)}{2\Delta y} = 0
\]
\[
\begin{align*}
\frac{M_{i,j}^{t+1} - M_{i,j}^t}{\Delta t} &+ \frac{2M_{i,j}^t(M_{i+1,j}^t - M_{i-1,j}^t)}{2\Delta x} \left( \frac{D_{i,j}^t}{D_{i,j}^t} \right)^2 - \left( \frac{M_{i,j}^t}{2\Delta x} \right)^2 \left( D_{i+1,j}^t - D_{i-1,j}^t \right) \\
&\quad + \frac{\left( M_{i,j+1}^t - M_{i,j-1}^t \right)}{2\Delta y} N_{i,j}^t + M_{i,j}^t \left( \frac{N_{i,j+1}^t - N_{i,j-1}^t}{2\Delta y} \right) \left( D_{i,j}^t \right)^2 - M_{i,j}^t N_{i,j}^t \left( \frac{D_{i+1,j}^t - D_{i-1,j}^t}{2\Delta y} \right) \\
&\quad + gD_{i,j}^t \left( \eta_{i,j+1}^t - \eta_{i,j-1}^t \right) + g\eta_{i,j}^t \left( M_{i,j}^t \right)^2 + (N_{i,j}^t)^2 = 0 \\
\frac{N_{i,j}^{t+1} - N_{i,j}^t}{\Delta t} &+ \frac{\left( M_{i,j+1}^t - M_{i,j-1}^t \right)}{2\Delta x} N_{i,j}^t + M_{i,j}^t \left( \frac{N_{i,j+1}^t - N_{i,j-1}^t}{2\Delta x} \right) \\
&\quad - M_{i,j}^t N_{i,j}^t \left( \frac{D_{i+1,j}^t - D_{i-1,j}^t}{2\Delta x} \right) \\
&\quad + \frac{2N_{i,j}^t(N_{i,j+1}^t - N_{i,j-1}^t)}{2\Delta y} \left( \frac{D_{i,j}^t}{D_{i,j}^t} \right)^2 - \left( \frac{M_{i,j}^t}{2\Delta y} \right)^2 \left( D_{i,j}^t \right)^2 + gD_{i,j}^t \left( \eta_{i,j+1}^t - \eta_{i,j-1}^t \right) \\
&\quad + g\eta_{i,j}^t \left( M_{i,j}^t \right)^2 + (N_{i,j}^t)^2 = 0
\end{align*}
\]

So obtained:

\[
\eta_{i,j}^{t+1} = \left( \frac{M_{i+1,j}^t - M_{i-1,j}^t}{2\Delta x} \right) \Delta t + \eta_{i,j}^{t-1} \tag{12}
\]

\[
M_{i,j}^{t+1} = \left( -\frac{2M_{i,j}^t(M_{i+1,j}^t - M_{i-1,j}^t)}{2\Delta x} \frac{D_{i,j}^t}{D_{i,j}^t} + \left( \frac{M_{i,j}^t}{2\Delta x} \right)^2 \left( D_{i+1,j}^t - D_{i-1,j}^t \right) \right) \\
&\quad - \frac{\left( M_{i,j}^t + M_{i,j}^t \right)}{2\Delta y} N_{i,j}^t + M_{i,j}^t \left( \frac{N_{i,j+1}^t - N_{i,j-1}^t}{2\Delta y} \right) \left( D_{i,j}^t \right)^2 \\
&\quad - gD_{i,j}^t \left( \eta_{i,j+1}^t - \eta_{i,j-1}^t \right) - \frac{g\eta_{i,j}^t \left( M_{i,j}^t \right)^2 + (N_{i,j}^t)^2}{\left( D_{i,j}^t \right)^2} \Delta t + M_{i,j}^t \tag{13}
\]

\[
N_{i,j}^{t+1} = \left( -\frac{2M_{i,j}^t(M_{i+1,j}^t - M_{i-1,j}^t)}{2\Delta x} \frac{D_{i,j}^t}{D_{i,j}^t} + \left( \frac{M_{i,j}^t}{2\Delta x} \right)^2 \left( D_{i+1,j}^t - D_{i-1,j}^t \right) \right) \\
&\quad - \frac{2N_{i,j}^t(N_{i,j+1}^t - N_{i,j-1}^t)}{2\Delta y} \left( \frac{D_{i,j}^t}{D_{i,j}^t} \right)^2 - \left( \frac{M_{i,j}^t}{2\Delta y} \right)^2 \left( D_{i,j}^t \right)^2 - gD_{i,j}^t \left( \eta_{i,j+1}^t - \eta_{i,j-1}^t \right) \\
&\quad + g\eta_{i,j}^t \left( M_{i,j}^t \right)^2 + (N_{i,j}^t)^2 \Delta t + N_{i,j}^t \tag{14}
\]
3.2 Runge Kutta 4th Order

Runge Kutta is a numerical method that is useful for solving the differential equations, so that an approach to the exact value (original) is obtained. One of the runge kutta methods that is often used is the Runge Kutta Order-4 method, the general equivalent of this method is [6]:

\[ y_{t+1} = y_t + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\Delta x \]

with

\[ k_1 = f(x_t, y_t) \]  
\[ k_2 = f(x_t + \frac{1}{2}\Delta x, y_t + \frac{1}{2}\Delta x k_1) \]
\[ k_3 = f(x_t + \frac{1}{2}\Delta x, y_t + \frac{1}{2}\Delta x k_2) \]
\[ k_4 = f(x_t + \Delta x, y_t + \Delta x k_3) \]

\[ k_1, k_2, k_3 \] and \[ k_4 \] are connected to each other, this is one of the advantages of the accuracy of the Runge Kutta method.

In the existing tsunami mathematical model, the temporal variable was discreted using the -4 runge kutta method as follows:

Variable of height wave:

\[ \eta_{i,j}^{t+1} = \eta_{i,j}^t + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\Delta t \]

with:

\[ k_1 = F(t_{i,j}^t, \eta_{i,j}^t) \]  
\[ k_2 = F(t_{i,j}^t + \frac{1}{2}\Delta t, \eta_{i,j}^t + \frac{1}{2}\Delta t k_1) \]
\[ k_3 = F(t_{i,j}^t + \frac{1}{2}\Delta t, \eta_{i,j}^t + \frac{1}{2}\Delta t k_2) \]
\[ k_4 = F(t_{i,j}^t + \Delta t, \eta_{i,j}^t + \Delta t k_3) \]

Variable of debit flow in \( x \) – axis:

\[ M_{i,j}^{t+1} = M_{i,j}^t + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\Delta t \]

with:

\[ k_1 = F(t_{i,j}^t, M_{i,j}^t) \]  
\[ k_2 = F(t_{i,j}^t + \frac{1}{2}\Delta t, M_{i,j}^t + \frac{1}{2}\Delta t k_1) \]
\[ k_3 = F(t_{i,j}^t + \frac{1}{2}\Delta t, M_{i,j}^t + \frac{1}{2}\Delta t k_2) \]
\[ k_4 = F(t_{i,j}^t + \Delta t, M_{i,j}^t + \Delta t k_3) \]

Variable of debit flow in \( y \) – axis:

\[ N_{i,j}^{t+1} = N_{i,j}^t + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\Delta t \]

with:

\[ k_1 = F(t_{i,j}^t, N_{i,j}^t) \]  
\[ k_2 = F(t_{i,j}^t + \frac{1}{2}\Delta t, N_{i,j}^t + \frac{1}{2}\Delta t k_1) \]
\[ k_3 = F(t_{i,j}^t + \frac{1}{2}\Delta t, N_{i,j}^t + \frac{1}{2}\Delta t k_2) \]
\[ k_4 = F(t_{i,j}^t + \Delta t, N_{i,j}^t + \Delta t k_1) \]
3.2.1 Discretization of spatial variables

The spatial variable of the model is discretized with different and advanced methods, obtained:

\[
\frac{\partial M}{\partial t} + \frac{2M_i^t(M_{i+1,j}^t - M_{i,j}^t)}{\Delta x} - \frac{(M_{i,j}^t)^3(D_{i+1,j}^t - D_{i,j}^t)}{\Delta x} + \frac{(M_{i+1,j}^t - M_{i,j}^t)}{\Delta y} N_{i,j}^t + \frac{(N_{i,j+1}^t - N_{i,j}^t)}{\Delta y} = 0
\]

\[
\frac{\partial N}{\partial t} + \frac{(M_{i+1,j}^t - M_{i,j}^t)}{\Delta x} N_{i,j}^t + M_{i,j}^t \frac{(N_{i,j+1}^t - N_{i,j}^t)}{\Delta x} - \frac{(M_{i+1,j}^t - M_{i,j}^t)}{\Delta y} = 0
\]

3.2.2 Discretization of temporal variables

The temporal variable will be discretized using the Runge Kutta 4th order method, obtained as follows:

\[
P(t, \eta) = -\left( \frac{M_{i+1,j}^t - M_{i,j}^t}{\Delta x} + \frac{N_{i,j+1}^t - N_{i,j}^t}{\Delta y} \right)
\]

\[
Q(t, M) = -\left( \frac{2M_i^t(M_{i+1,j}^t - M_{i,j}^t)}{\Delta x} \right)
\]

\[
R(t, N) = -\left( \frac{(M_{i+1,j}^t - M_{i,j}^t)}{\Delta x} N_{i,j}^t + M_{i,j}^t \frac{(N_{i,j+1}^t - N_{i,j}^t)}{\Delta x} - \frac{(M_{i+1,j}^t - M_{i,j}^t)}{\Delta y} N_{i,j}^t + \frac{(N_{i,j+1}^t - N_{i,j}^t)}{\Delta y} = 0
\]
4. Numerical Simulation

The simulation is done with the help of the Matlab 2013 program. The initial input of the differential model of the tsunami model is the initial wave height is 5 m and the distance of the wave source with the shoreline is 25 km. Here is the initial representation of the tsunami wave condition in 0 seconds:

![Figure 1. Early Representation of Tsunami Wave Conditions](image1)

At 0 seconds, initially it is assumed that when a plate shift occurs at a distance of 25 km from the shoreline it will cause the wave height at that location to be 5 m. The location of the initial wave height is the location of the starting point of the earthquake that caused the tsunami. In Figure 1, the initial location is shown at coordinates (0,0). Furthermore, to find out tsunami wave propagation, it is represented in the following 2 dimensions:

![Figure 2. Tsunami wave propagation at 30 seconds](image2)

In Figure 2 it can be seen that at 30 seconds, the wave height of the waveform decreases slowly and the maximum altitude value is around 3.4 m which is 21 km from the shoreline. The maximum height difference from the two methods at 30 seconds is 0.0001 m. Next will be shown a representation of the tsunami wave propagation in the 90 second:
In Figure 3 it can be seen that at 90 seconds, the height of wave propagation changes with the maximum altitude value of about 3.9 m which is 12 km from the shoreline. The maximum height difference from the two methods at 90 seconds is 0.00052 m. In Figure 3, it can be seen that the wave height above sea level or the entire wave height on the y axis is almost the same. So that for the simulation representation then it is shown by a 1 dimensional representation assuming the wave height on the y axis is the same. Here is a representation of 1-dimensional tsunami wave propagation in the 120 second:

In Figure 4 it can be seen that at 120 seconds, the wave propagation height changes with the maximum altitude value being around 7 m which is 4 km from the shoreline and waves as high as 2 m on the shoreline. Next will be shown a representation of tsunami wave propagation in the 140 second:

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**Figure 3.** Tsunami wave propagation at 90 seconds  
(a) Finite difference  
(b) Runge Kutta 4th Order

**Figure 4.** Tsunami wave propagation at 120 seconds  
(a) Finite difference  
(b) Runge Kutta 4th Order

**Figure 5.** Tsunami wave propagation at 140 seconds  
(a) Finite difference  
(b) Runge Kutta 4th Order
In Figure 5 it can be seen that at 140 seconds, the height of the tsunami wave propagation that has reached the shoreline is around 8.79 m. With a wave height of 8.7 m on the shoreline, the water will enter the land or can also be called the tsunami disaster.

5. Discussion
Based on the results of numerical simulations carried out, we obtained comparisons of computation time and difference in mean height values as follows:

| Simulation | Time of computation | Wave heights in the shoreline |
|------------|---------------------|-----------------------------|
|            | Finite difference   | Runge Kutta 4th Order       | Finite difference   | Runge Kutta 4th Order |
| 30 s       | 0.004510 unit time  | 0.011627 unit time          | 2×10⁻⁷ Km          | 2×10⁻⁷ Km             |
| 90 s       | 0.005368 unit time  | 0.013429 unit time          | 0.0002333 Km       | 0.0002333 Km          |
| 120 s      | 0.005774 unit time  | 0.014287 unit time          | 0.0026294 Km       | 0.0026294 Km          |
| 130 s      | 0.005905 unit time  | 0.014569 unit time          | 0.0051016 Km       | 0.0051016 Km          |
| 140 s      | 0.006038 unit time  | 0.014854 unit time          | 0.0087667 Km       | 0.0087667 Km          |

From the table above, it is found that the average computation time of finite difference was faster than the computational time of the Runge Kutta 4th Order method. The average mean difference (η) of the two methods increased according to the increase in wave propagation time. Based on Table 1 above, it is found that the average height difference from the two methods was zero 0. Based on the numerical simulation results obtained, it can be concluded that the application of the estimation and prediction of tsunami wave propagation is better to use the finite difference method because it has a much faster computation time.

6. Conclusion
Numerical simulations of tsunami wave propagation according to a mathematical model formed by Imamura were completed by two methods of discretization, namely the Finite Difference methods and Runge Kutta 4th Order methods. Based on the simulation results with the initial determination of a 5 m tsunami wave centered at a distance of 25 km from the shoreline, Tsunami waves with a height of 8.76 m will arrive at the shoreline at 140 seconds.

In addition, based on numerical simulations that have been carried out, it is obtained information that the computation time of the finite difference method differs faster than the computation time of the Runge Kutta 4th Order method. This shows that the finite difference method can be applied to tsunami disaster mitigation software because it has a fairly fast computing time compared to the Runge Kutta 4th Order method.

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