Greybody factor for D3-branes in B field

Y.S. Myung, Gungwon Kang, and H.W. Lee

Department of Physics, Inje University, Kimhae 621-749, Korea

Abstract

We calculate the effect of noncommutative spacetime on the greybody factor on the supergravity side. For this purpose we introduce a system of D3-branes with a constant NS $B$-field along their world volume directions $(x_2, x_3)$. Considering the propagation of minimally coupled scalar with non-zero momentum along $(x_2, x_3)$, we derive an exact form of the greybody factor in $B$ field. It turns out that $\sigma_l^{B \neq 0} > \sigma_l^{B = 0}$. This means that the presence of $B$-field (the noncommutativity) suppresses the potential barrier surrounding the black hole. As a result, it comes out the increase of greybody factor.
Recently noncommutative geometry has attracted much interest in studying on string and M-theory in the $B$-field \[1–7\]. For simplicity, we consider supergravity solutions which are related to D3 branes with NS $B$ fields. According to the AdS/CFT correspondence \[8\], the near horizon geometry of $D=7$ black hole solution can describe the large $N$ limit of noncommutative super Yang-Mills theory (NCSYM). We take a decoupling limit to isolate the near horizon geometry from the remaining one. It turns out that the noncommutativity affects the ultra violet(UV) but not the infra red(IR) of the Yang-Mills dynamics. The NCSYM is thus not useful for studying the theory at short distances. It is well known that an NCSYM with the noncommutativity scale $\Delta$ on a torus of size $\Sigma$ is equivalent to an ordinary supersymmetric Yang-Mills theory (OSYM) with a magnetic flux if $\Theta = \Delta^2 / \Sigma^2$ is a rational number \[9\]. The equivalence between the NCSYM and the OSYM can be understood from the T-duality of the corresponding string theory. Hence the OSYM with $B$-field is the proper description in the UV region, while the NCSYM takes over in the IR region. Actually, the noncommutativity comes from the $B \to \infty$ limit of the ordinary theories \[5,6,10\].

On the other hand, it turns out that the total number of physical degrees of freedom for the noncommutative case at any given scale coincides with the commutative case \[5\]. All thermodynamic quantities of the NCSYM including the entropy are the same as those of the OSYM. However, in the next order correction of the $\alpha'$-expansion, the entropy decreases in the NCSYM \[11\]. We remind the reader that aside the entropy, there exists an important dynamical quantity “the greybody factor(absorption cross section)” for the quantum black hole \[12,13\]. Hence it is very important to check whether there is or not a change in the greybody factors between the commutative and the noncommutative cases. On the string side, there was a calculation for the absorption of scalars into the noncommutative D3-branes \[16\].

In this paper we wish to study the quantum aspects of D3-brane black hole in $B$-field background using a minimally coupled scalar. This may belong to the off-diagonal gravitons polarized parallel to the brane ($h_{ab}$, $a, b = 0, 1, 2, 3$). We will derive an exact form of absorp-
tion cross section in $B$-field on the supergravity side. However, it is not easy to obtain the absorption cross section on the gauge theory side because the graviton can turn into pairs of gauge bosons, scalars, or fermions in the world volume approach. In the absence of $B$-field, adding up these rates leads to the absorption cross section of $h_{ab}$ on the superstring side [12].

We start with the D=7 extremal black hole solution for D3-branes in a $B_{23}$-field on the D=10 string frame [5]

$$ds_{str}^2 = f^{-\frac{1}{2}} \left\{ -dx_0^2 + dx_1^2 + h(dx_2^2 + dx_3^2) \right\} + f^{\frac{1}{2}} \left( dr^2 + r^2 d\Omega_5^2 \right),$$

where

$$f = 1 + \frac{R_B^4}{r^4}, \quad h^{-1} = \sin^2 \theta f^{-1} + \cos^2 \theta,$$

$$B_{23} = \tan \theta f^{-1} h, \quad e^{2\phi} = g^2 h, \quad F_{01} = \frac{1}{g} \sin \theta \partial_r (f^{-1}), \quad F_{0123} = \frac{1}{g} \cos \theta h \partial_r (f^{-1}).$$

Here the asymptotic value of $B$-field is $B_{23}^\infty = \tan \theta$ and the parameter $R$ is defined by

$$\cos \theta R_B^4 = 4\pi g N \alpha'^2$$

with $N$ (the number of D3-branes). Here we define $R_0^4 = 4\pi g N \alpha'^2$ for $B = 0$, $R_\infty$ for $B \to \infty$ and $R_B$ for an arbitrary $B$. We note that $R_B > R_0$. And $g = g_\infty$ is the string coupling constant. It is obvious that for $\theta = 0$, one recovers the ordinary D3-brane black hole with the standard $\text{AdS}_5 \times S^5$ in the near horizon. In this case the dilaton background($\phi$) is constant. But for $\theta = \pi/2$ one finds the D3-brane black hole in the $B \to \infty$ limit. This actually reduces to the black D-string with the smeared coordinates $x_2$ and $x_3$. In latter case, one finds a deviation from $\text{AdS}_5 \times S^5$ in the near horizon.

Now let us introduce the perturbation analysis to derive the greybody factor. General fluctuations including the dilaton and the RR scalar will be given as the complicated systems of differential equations [17]. A simple equation may be arised from the off-diagonal graviton fluctuation of $h_{01}$ [8,12]. Let us set all other fluctuations to zero and introduce a minimally coupled scalar $\varphi$ which may be $g^{00} h_{01}$. This scalar is a good test field in the Einstein frame because it is minimally coupled to the background spacetime. Then this in the Einstein frame satisfies
\[
\frac{e^{2\phi}}{\sqrt{-g}} \partial_M \left( \sqrt{-g} e^{-2\phi} g^{MN} \partial_N \varphi \right) = 0. \tag{2}
\]

This leads to
\[
\frac{1}{\sqrt{-g}} \partial_M \left( \sqrt{-g} g^{MN} \partial_N \varphi \right) - 2 (\partial_M \varphi) (\partial^M \varphi) = 0, \tag{3}
\]
where \( g_{MN} \) is the string frame metric in (1).

Now let us consider
\[
\varphi(t, x_1, x_2, x_3, r, \theta_i) = e^{-i\omega t} e^{i(k_1 x_1 + k_2 x_2 + k_3 x_3)} Y_l(\theta_1, \theta_2, \cdots, \theta_5) \phi^l(r) \tag{4}
\]
with \( \nabla^2_\theta Y_l(\theta_i) = -l(l+4)Y_l(\theta_i) \). Here \( \phi^l(r) \) is the radial part of the \( l \)-partial wave of energy \( \omega \). We assume the momentum dependence along the world volume directions. This is a crucial step to translate the noncommutativity in the \( x_2, x_3 \) directions (the presence of \( B_{23} \)) into the greybody factor \([1]\). Then Eq. (3) takes the form
\[
\left\{ \frac{\partial^2}{\partial r^2} + \frac{5}{r} \frac{\partial}{\partial r} - \frac{l(l+4)}{r^2} + (\omega^2 - k_1^2) f - (k_2^2 + k_3^2) \left( \cos^2 \theta (f-1) + 1 \right) \right\} \phi^l = 0. \tag{5}
\]

If \( \theta = 0 \) (\( B \)-field is turned off), one finds the ordinary scalar propagation in the D=7 black hole spacetime \([13]\)
\[
\left\{ \frac{\partial^2}{\partial r^2} + \frac{5}{r} \frac{\partial}{\partial r} - \frac{l(l+4)}{r^2} + (\omega^2 - k_1^2 - k_2^2 - k_3^2) \left( 1 + \frac{P_0^4}{r^4} \right) \right\} \phi^l_{B=0} = 0. \tag{6}
\]

On the other hand, in the presence of the \( B \to \infty \) limit (\( \theta \to \pi/2 \)), Eq. (5) leads to
\[
\left\{ \frac{\partial^2}{\partial r^2} + \frac{5}{r} \frac{\partial}{\partial r} - \frac{l(l+4)}{r^2} + (\omega^2 - k_1^2) f - (k_2^2 + k_3^2) \right\} \phi^l_{B=\infty} = 0. \tag{7}
\]

For simplicity, we require \( k_1 = 0, k_2, k_3 \neq 0 \). And the low energy limit of \( \omega \to 0 \) but still \( \omega^2 > k_2^2 + k_3^2 \). Fortunately, the absorption cross section for the \( l \)-partial wave for Eq. (5) can be extracted from the solution to Mathieu’s equation \([13]\),
\[
\sigma^B_{l=0} = \frac{8\pi^2/3}{\omega^5} (l+1)(l+2)(l+3) P_l, \tag{8}
\]
where the absorption probability \( P_l \) takes the form
\[ R_l = \frac{4\pi^2}{[(l + 1)!]^4(l + 2)^2} \left( \frac{\tilde{\omega}R_0}{2} \right)^{8+4l} \sum_{n=0}^{\infty} \sum_{k=0}^{n} b^l_{n,k}(\tilde{\omega}R_0)^{4n}(\log \tilde{\omega}R_0)^k. \] (9)

Here \( b^0_{0,0} = 1, b^0_{1,1} = -\frac{1}{6}, b^0_{1,0} = \frac{7}{72}, \tilde{R}_0 = e^\gamma R_0/2 \) with \( \gamma = 0.5772 \) (Euler’s constant), and \( \tilde{\omega} = \sqrt{\omega^2 - k^2 - k^2} \simeq \omega(1 - \frac{k^2}{2\omega^2}) \). In the case of \( l = 0 \) mode (s-wave), one finds

\[ \sigma^B_{0=0} = \pi^4(\tilde{\omega}R_0)^8 \left\{ 1 - \frac{1}{6}(\tilde{\omega}R_0)^4 \log(\tilde{\omega}R_0) + \frac{7}{72}(\tilde{\omega}R_0)^4 + \cdots \right\} \] (10)

in the low-energy approximation of \( \tilde{\omega}R_0 < 1 \). In this case we see that the logarithmic term is greater than the fourth power order term.

On the other hand, for \( l = 0, \rho = \hat{\omega}r \), Eq.(7) leads to

\[ \left\{ \frac{\partial^2}{\partial \rho^2} + \frac{5}{\rho} \frac{\partial}{\partial \rho} + 1 + \frac{(\hat{\omega}R_\infty)^4}{\rho^4} \right\} \varphi^0_{\rightarrow \infty} = 0 \] (11)

with \( \hat{\omega}^2 = \tilde{\omega}\omega \). Let us introduce the new coordinate \( y \) and function \( \psi \) as

\[ y = \frac{(\hat{\omega}R_\infty)^4}{\rho} \quad \text{and} \quad \varphi^0_{\rightarrow \infty}(\rho) = y^4 \psi^0_{\rightarrow \infty}(\rho). \] (12)

Then one finds a dual equation to Eq.(11) which is written in terms of \( y \) as

\[ \left\{ \frac{\partial^2}{\partial y^2} + \frac{5}{y} \frac{\partial}{\partial y} + 1 + \frac{(\hat{\omega}R_\infty)^4}{y^4} \right\} \psi^0_{\rightarrow \infty}(\rho) = 0. \] (13)

Using a trick for an improved matching of inner and outer solutions in Ref. [14], one finds the absorption cross section

\[ \sigma^B_{0=0} = \pi^4(\hat{\omega}R_\infty)^8 \left\{ 1 - \frac{1}{6}(\hat{\omega}R_\infty)^4 \log(\hat{\omega}R_\infty) + O((\hat{\omega}R_\infty)^4) \right\} \] (14)

with \( \hat{\omega} = \sqrt{\tilde{\omega}\omega} = \omega \left( 1 - \frac{k^2 + k^2}{2\omega^2} \right)^{1/4} \simeq \omega \left( 1 - \frac{k^2 + k^2}{4\omega^2} \right) \).

Although \( \sigma^B_{0=0} \) and \( \sigma^B_{0 \rightarrow \infty} \) take the same form up to the leading-correction, there exist some differences to point out. First, because of \( \tilde{\omega} < \hat{\omega} \) and \( \tilde{R}_0 < R_\infty \), one finds \( \sigma^B_{0=0} \) < \( \sigma^B_{0 \rightarrow \infty} \). This implies that in the presence of large \( B \)-field (\( B \rightarrow \infty \)), the height of potential is lower than the case of \( B = 0 \). Actually, we observe from Eq.(7) that the height of the effective potential decreases as \( \theta \) (that is, the strength of \( B \)-field) increases. Hence, in the low energy scattering, one finds the larger absorption cross section \( \sigma^B_{0 \rightarrow \infty} \) than \( \sigma^B_{0=0} \). The
greybody factor of the black hole arises as a consequence of scattering of minimally coupled scalar off the gravitational potential barrier surrounding the horizon; that is, this is an effect of spacetime curvature [15]. Furthermore, we wish to point out that there also exists the difference for “$R$” in the logarithmic terms.

We have seen the effect of $B \to \infty$ on the absorption probability for D3-branes above. Now we may ask how the greybody factor changes in the presence of arbitrary $B$-field. For an arbitrary $B$, we can rewrite the Eq.(5) in the following form:

$$\left\{ \frac{\partial^2}{\partial r^2} + \frac{5}{r} \frac{\partial}{\partial r} - \frac{l(l+4)}{r^2} + \tilde{\omega}^2 \left( 1 + \frac{\tilde{R}_B^4}{r^4} \right) \right\} \varphi_B^{l} = 0, \quad (15)$$

where $\tilde{R}_B^4 = \left\{ 1 + \left( \frac{k^2 + k_2^2}{\tilde{\omega}^2} \sin^2 \theta \right) \right\} R_B^4$. Surprisingly, we find that the above equation has exactly the same form as Eq.(6) with different “$R$”. Then the absorption cross section can be read off from (8) by substituting $R_0$ with $\tilde{R}_B$ as follows

$$\sigma_i^B = \sigma_i^{B=0}(R_0 \to \tilde{R}_B). \quad (16)$$

This is our main result. From (16) one can recover $\sigma_0^{B=0}$ in (11) for $\theta = 0$ as well as $\sigma_0^{B=\infty}$ in (14) for $\theta = \pi/2$. Here we point out several main features. Firstly, for arbitrary $B$-field, one finds $\sigma_i^{B \neq 0} > \sigma_i^{B=0}$ since $\tilde{R}_B > R_0$ and $\tilde{\omega}\tilde{R}_B < 1$. Secondly, since $\tilde{R}_B$ increases as $\tilde{\omega} \to 0$, the wave packet having lower energy is absorbed more easily. Thirdly, the absorption cross section in the presence of $B$-field is exactly same as that in the absence of $B$-field with

$$\tilde{R}_B = \left\{ 1 + \left( \frac{k^2 + k_2^2}{\tilde{\omega}^2} \sin^2 \theta \right) \right\}^{1/4} R_B. $$

It may imply that the scattering wave regards the geometry as being the same with an increased AdS$_5$ radius as long as the absorption cross section is concerned. Furthermore, a recent paper [18] pointed out that the Einstein metric of a D3-brane is not changed by the presence of a $B$-field. This implies that for a minimally coupled scalar, the solution with the $B$-field can be obtained from the solution without the $B$-field by replacing $R_0$ by $\tilde{R}_B$.

We conclude that the main effect of $B$-field (the noncommutativity) suppresses the potential barrier surrounding the black hole. As a result, one finds the increase of greybody factor. Finally we comment on the relationship between the near horizon geometry and
the greybody factor. Taking the decoupling limit \[3\], the near-horizon geometry (a deviation from AdS\(_3\)×S\(_5\) for \(B \neq 0\)) plays an important role in obtaining two-point function of \(\varphi = g^{00} h_{01} : \langle T_{01}(k)T_{01}(-k) \rangle \simeq \exp(-ca|k|)\) with \(c = 1.69, a = \sqrt{bR_B^2}\). In the UV regime, this decays exponentially with the momentum \(k = \sqrt{k_2^2 + k_3^2}\). This is a new feature in the noncommutative geometry. However we remind the reader that in the calculation of the greybody factor we never take the decoupling limit to single the near horizon geometry out. Instead we use the differential equation to find the approximate solution in the whole region. Hence the near horizon geometry is not so sensitive to obtain the greybody factor. In our case, the momentum contributes to the cross section through \(\tilde{R}_B^4 \rightarrow \{1 + \frac{k_i^2}{\omega^2} \sin^2 \theta \} R_B^4\) with a sequence \(\tilde{R}_B^4 > R_B^4 > R_0^4\).

\[\text{Note Added}\]

After our work has been done, we find a related paper \[19\]. The first version of this paper takes over the same subject with the RR scalar (\(C\)), which is non-minimally coupled to the background. Kaya claims that the greybody factor does not change even if the large B-field is turned on. However, we point out that such result comes from the assumption that the scalar field \(C\) does not depend on the noncommuting coordinates \((x_2, x_3)\). In our case, if \(\varphi\) does not depend on \((x_2, x_3)\), equivalently, \(k_2 = k_3 = 0\), we also recover the usual result for ordinary D3-branes except with \(R_B > R_0\). It indicates that, in order to extract the B-field effect (the noncommutativity), the world volume dependence of the scalar field is a key ingredient \[5,18\]. It follows because the propagation of fields does not feel the presence of B-field if the wave does not move on the \(x_2-x_3\) plane \[10\]. Fortunately, Kaya in the third version of ref. \[19\] comment that \(\sigma_0^{B \neq 0} > \sigma_0^{B=0}\) is still valid for the case of the RR scalar by introducing the new parameter \(s = \sqrt{1 - k^2/\omega^2} < 1\). In our notation, \(\tilde{\omega} = s\omega\) and \(\hat{\omega} = \sqrt{s}\omega\). That is, he showed that \(\sigma_0^{B \neq 0} \propto s^4 > \sigma_0^{B=0} \propto s^8\).
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