Refutation of multi-dimensional interpretations of Presburger arithmetic in itself

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Abstract: We evaluate Visser's conjecture in the multi-dimensional case as not tautologous. In other words, a theorem is supposed to be used to disprove another theorem. This denies the Visser conjecture and refutes multi-dimensional interpretations of Presburger arithmetic in itself. These results form a non tautologous fragment of the universal logic VL4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

Let:

\( \sim \) Not, \( \neg \); \(+\) Or, \(\lor, \cup\); \(-\) Not Or; \(\&\) And, \(\land, \cap\); \(\Leftrightarrow\) Equivalent, \(\equiv, :=, \leftrightarrow, \Leftrightarrow, \approx, \eq, \simeq\); \(\not\) Not Equivalent, \(\neq, \not\equiv\);
\(\%\) possibility, for one or some, \(\exists, \exists!, \diamond, M\); \(\#\) necessity, for every or all, \(\forall, \Box, L\);
\(z=z\) T as tautology, \(\top\); \(s=s\) (z@z) F as contradiction, \(\emptyset, \text{Null}, \bot\); \(\%z>\#z\) N as non-contingency, \(\Delta, \text{ordinal 1}\); \(\%z<\#z\) C as contingency, \(\nabla, \text{ordinal 2}\);
\(~(y < x) (x \leq y) (x \equiv y) (x \subseteq y) (A=B) (A\sim B)\).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Pakhomov, F.; Zapryagaev, A. (2020). Multi-dimensional interpretations of Presburger arithmetic in itself. arxiv.org/pdf/2004.03404.pdf

5 Visser’s Conjecture in Multi-Dimensional Case

Our goal is to prove Theorem 1.2. In order to prove that all multi-dimensional interpretations of \(PrA\) in \((\mathbb{N},+)\) are isomorphic to \((\mathbb{N},+)\), we use the same argument as in one-dimensional case: an interpretation of a non-standard model would entail an interpretation of a non-scattered order, which is impossible by Theorem 1.3.

Thus we have

\[ h(g(x) - 1) < 2h(x) \leq h(g(x)). \]  \hspace{1cm} (5.1.1)

Let \(p, q, r: g, h, x\).

\[ \sim((q&(p\&r))<((q&((p\&r)-(%s>\#s)))<((%s<\#s)\&(q\&r)))) = (s=s); \]

\( \begin{array}{cccc}
\text{T} & \text{T} & \text{T} & \text{T} \\
\text{T} & \text{T} & \text{T} & \text{T} \\
\text{T} & \text{T} & \text{T} & \text{T} \\
\text{T} & \text{T} & \text{T} & \text{T} \\
\end{array} \) \hspace{1cm} (5.1.2)

Remark 5.1.2: Eq. 5.1.2 as rendered is not tautologous. However, it should be to disprove the assumption that \(h\) has the degree \(k\geq2\), which is in fact impossible, by way of contradiction. In other words, a theorem is used to disprove another theorem. This denies Visser's conjecture in the multi-dimensional case, and refutes multi-dimensional interpretations of Presburger arithmetic in itself.