Self-focusing does not occur for few-cycle pulses

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Abstract. A surprise of nonlinear optics of few-cycle waves, such as the disappearance of the phenomenon of high-power radiation self-focusing, is discussed. An expression is derived and analyzed for the conditions under which the concept of a critical power for self-focusing loses its physical meaning due to the dominance of the process of dispersion over diffraction.

1. Introduction
Self-focusing is a fundamental self-action effect in which a light beam comes to a focus as a consequence of the lens it induces in the nonlinear optical medium. The concept of a critical power is a key feature of conventional theories of self-focusing; the critical power determines when the self-induced focusing begins to dominate over diffractive spreading of the light beam. A large number of papers and books have been devoted to this nonlinear phenomenon from 1962 (see, for example, review [1] and references therein).

In the present work, we demonstrate that, for few-cycle wave packets with longitudinal dimension less the transverse size, the concept of critical power of self-focusing can lose its physical meaning because of the dominance of dispersion over diffraction. We present simple formulas for the estimation of the parameters of the field and medium under which self-focusing disappears and illustrate with numerical calculations the changes in the self-action phenomenon of light in this case.

2. Theoretical estimations
The normalized equation describing propagation of a linearly polarized paraxial quasi-monochromatic wave through a nonlinear media has well-known form [2]

$$\frac{\partial \tilde{E}}{\partial z} + \frac{1}{L_w} \frac{\partial \tilde{E}}{\partial \tilde{t}} - \frac{1}{L_{disp1}} \frac{\partial^2 \tilde{E}}{\partial \tilde{t}^2} - \frac{1}{L_{disp2}} \frac{\partial^3 \tilde{E}}{\partial \tilde{t}^3} - \frac{i}{L_{nl1}} \left| \tilde{E} \right|^2 \tilde{E} + \frac{1}{L_{nl2}} \frac{\partial}{\partial \tilde{t}} \left( \left| \tilde{E} \right|^2 \tilde{E} \right) = \frac{i}{L_{difr}} \tilde{\Delta}_\perp \tilde{E},$$ (1)

where $\tilde{E}$ is the normalized envelope of wave electric field, $z$ is the beam propagation direction, $\tilde{t}$ is the normalized time, $\tilde{\Delta}_\perp = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $L_w = V \tau_0$. Dispersion lengths given by $L_{disp1} = \frac{2 \tau_0^2}{\beta_2}$, $L_{disp2} = \frac{6 \tau_0^3}{\beta_3}$; nonlinear lengths given by $L_{nl1} = \frac{1}{\gamma_1 \bar{E}_0^2} = \frac{c}{\omega_0 \Delta n_{nl}}$, $L_{nl2} = \frac{\tau_0 \bar{E}_0^2}{\gamma_2 \bar{E}_0^2} = \frac{c \tau_0}{\Delta n_{nl}}$.
and diffraction length given by \( L_{difr} = \frac{2k_0 r_0^2}{\omega} \) characterize the influence on the character of light propagation of various physical phenomena: dispersion, nonlinearity and diffraction. Here \( E_0 \) is the initial field amplitude, \( \tau_0 \) and \( r_0 \) are the duration and radius of the beam at the entrance of nonlinear medium, \( k(\omega) = \frac{\omega}{c} n(\omega) \) is the wave number, \( V = (\frac{\partial k}{\partial \omega})^{-1} \) is wave velocity, \( \beta_2 = (\frac{\partial^2 k}{\partial \omega^2})_{\omega_0} \), \( \beta_3 = (\frac{\partial^3 k}{\partial \omega^3})_{\omega_0} \), \( \omega_0 \) is the central frequency of radiation, \( k_0 = k(\omega_0) \), \( \Delta n_{nl} = \frac{1}{2} n_2 E_0^2 = n'_2 I \), \( n_2 \) and \( n'_2 \) are coefficients of nonlinear refractive index and \( I \) is the input wave intensity.

Let’s consider a case

\[
L_{nl} = L_{difr},
\]

when the effect of medium nonlinear response and diffraction are close. In this case it is natural to introduce an expression for the self-focusing critical power \( P_{cr} = (\pi r_0^2) \cdot I \) which is trivially deduced for (2) and takes the form:

\[
P_{cr} = R_{cr} \lambda_0^2 \frac{\lambda_0}{8\pi n_0 n'_2},
\]

where \( R_{cr} \) is the parameter of nonlinearity \( [3] \), \( \lambda_0 = \frac{2\pi}{k_0} \) is the central wavelength of radiation and \( n_0 = n(\omega_0) \) is the linear refractive index. This naively deduced expression for critical power (3) coincides with the well-known expression \( [1] \) to within a constant factor \( R_{cr} \). This indicates the accuracy of our nonlinear and diffraction length estimates.

It is important, we note, that for

\[
L_{disp} < L_{difr}
\]

nonlinearity will compete not with diffraction but with dispersion. Under these conditions, the concept of \( P_{cr} \) will begin to lose its original meaning. The inequality (4) can be expressed in terms of laboratory parameters as:

\[
\frac{l_0}{D_0} < \sqrt{\omega_0 n(\omega_0) \beta_2},
\]

where \( l_0 = 2c\tau_0 \) is longitudinal size of the wave packet and \( D_0 = 2r_0 \) is its transverse size.

For the simplest dispersion formula of \( n(\omega) = N_0 + a\omega^2 \), where \( N_0 \) and \( a \) characterize the medium dispersion, the inequality (5) can be reduced to

\[
\frac{l_0}{D_0} < \sqrt{6N_0 \Delta n_{disp}},
\]

where \( \Delta n_{disp} = a\omega_0^2 \) is the change of the refractive index at the central wavelength due to dispersion.

For example, for optical radiation with a central wavelength of 800 nm propagating through a fused silica with dispersion \( n(\omega) = N_0 + a\omega^2 - bc/\omega^2 \) [4] we find that condition (5) becomes \( l_0/D_0 < 0.18 \). For terahertz radiation with a central frequency of 1.0 THz propagating through a stoichiometric MgO:LiNbO\(_3\) crystal with \( n(\omega) = N_0 + a\omega^2 \) [5] we have \( l_0/D_0 < 0.87 \) according to formulas (5) - (6). It means we can expect that wave packets with longitudinal dimension less than transverse size, the concept of critical power of self-focusing can lose its physical meaning.
3. Numerical illustrations

We consider boundary conditions (the electric field of radiation at the entrance to nonlinear medium) in form of a Gaussian axisymmetric paraxial beam with a small number of oscillations ($N = \tau_0/T_0$)

$$E(0, r, t) = E_0 \exp \left(-\frac{r^2}{r_0^2}\right) \exp \left(-\frac{t^2}{t_0^2}\right) \sin(\omega_0 t),$$

where $r = \sqrt{x^2 + y^2}$, $T_0 = 2\pi/\omega_0$ is central period of electric field oscillations.

Figure 1. The spatiotemporal dynamics of the electric optical field with initially Gaussian transverse profile in bulk fused silica at distances (a) $z = 0$ mm, (b) $z = 1.0$ mm, (c) $z = 1.9$ mm, (d) $z = 3.0$ mm for two different input pulse durations: $2\tau_0 = 13$ fs (left column) and $2\tau_0 = 5$ fs (right column). Here $\tau = t - (c/N_0)z$ is retarded time.

Figure 1 demonstrates typical dependence of light field self-focusing dynamics on the number of oscillations $N$ in the initial pulse. The typical changes in self-focusing dynamics are illustrated for ultrashort pulses of Ti:S laser for central wavelength of $\lambda_0 = 800$ nm with $N = 5$ ($2\tau_0 = 13$ fs) and $N = 2$ ($2\tau_0 = 5$ fs) propagated in bulk fused silica. Here initial pulse parameters are $I = 9 \times 10^{11}$ W/cm$^2$, $\lambda_0 = 800$ nm, $D_0 = 30\lambda_0$. We also used fixed ratio between initial power $P_0$ and critical power $P_{cr}$ for self-focusing $P_0/P_{cr} = 2$. For the case $N = 5$ we get $l_0/D_0 = 0.17$, for $N = 2$ we have $l_0/D_0 = 0.07$. As one can see from Figure 1, reduction of $N$ leads to decreasing of self-focusing efficiency because dispersion starts play a significant role in pulse evolution.
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References
[1] Boyd R W, Lukishova S G and Shen Y 2009 Self-focusing: Past and Present (New York: Springer)
[2] Agrawal G 2013 Nonlinear Fiber Optics 5th ed (New York: Academic Press)
[3] Kandidov V P, Shlenov S A and Kosareva O G 2009 Quantum Electronics 39 205
[4] Kozlov S A and Samartsev V V 2013 Fundamentals of femtosecond optics (Cambridge, UK: Woodhead)
[5] Drozdov A A, Kozlov S A, Sukhorukov A A and Kivshar Y S 2012 Phys. Rev. A 86(5) 053822