Precision study on $W^-W^+H$ production with subsequent decays at the LHC

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Abstract

The production of $W^-W^+H$ with subsequent $W^\pm \rightarrow l^\pm \nu_l$ and $H \rightarrow b\bar{b}$ decays at the LHC is an important process to test the Higgs gauge couplings of the Standard Model (SM) as well as an irreducible background to some important signal processes. The EW correction to this process is also directly related to the triple and quartic gauge self-couplings. Therefore, a precision study on this process is valuable for probing the EW symmetry breaking mechanism and searching for new physics beyond the SM. In this paper, we calculate the QCD + EW + $q\bar{q} + \gamma\gamma$ correction to the $W^-W^+H$ production at the 14 TeV LHC, and deal with the subsequent decays of Higgs and $W^\pm$ bosons by adopting the MadSpin method. Both integrated cross section and some kinematic distributions of $W^\pm$ and $H$ as well as their decay products are provided. We find that the QCD correction enhances the LO differential cross section significantly, while the EW correction from the $q\bar{q}$ annihilation channel obviously suppresses the LO differential cross section, especially in the high energy phase-space region due to the Sudakov effect. The $q\gamma$- and $\gamma\gamma$-induced relative corrections are positive, and insensitive to the transverse momenta of $W^\pm$, $H$ and their decay products. These photon-induced corrections strongly cancel the EW correction from the $q\bar{q}$ annihilation channel, and becomes the dominant EW contribution as the increment of the $pp$ colliding energy. Thus, besides the QCD correction, the EW corrections from the $q\bar{q}$-initiated and photon-induced channels should be taken into account in precision study on the $W^-W^+H$ production at the LHC. We also investigate the scale and PDF uncertainties, and find that the theoretical error of the QCD + EW + $q\gamma + \gamma\gamma$ corrected integrated cross section mainly comes from the renormalization scale dependence of the QCD correction.

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I. INTRODUCTION

The Higgs boson accounts for the electroweak (EW) symmetry breaking and the masses of fundamental particles [1–5], thus plays an important role in the Standard Model (SM). The Higgs boson has been discovered in 2012 at the CERN Large Hadron Collider (LHC) [6, 7]. One of the main tasks nowadays at the LHC is to detailedly study the Higgs properties, such as spin, CP and couplings. Among them, probing the couplings between Higgs and vector bosons is very important, since it can be used to determine the ratio of the $WWH$ and $ZZH$ couplings [8] and to test the anomalous Higgs gauge couplings [9]. Besides, the EW corrections to $VV'H$ ($V, V' = W$ or $Z$) productions at the LHC are also directly related to the triple and quartic gauge couplings, such as $WWZ$, $WW\gamma$, $WWZZ$, $WWZ\gamma$, $WW\gamma\gamma$ and $WWWW$ couplings. Thus, measuring $pp \to VV'H + X$ processes precisely is a good way to probe the EW symmetry breaking and search for new physics beyond the SM.

Careful and detailed study of Higgs couplings needs accurate theoretical predictions and precise experimental measurements on both the signal and background. At the LHC, the production channel $pp \to W^-W^+H \to W^-W^+bb$ is also an irreducible background to some important processes, such as $pp \to t\bar{t} \to W^-W^+bb$ and $pp \to HH \to W^-W^+bb$. The top pair production with subsequent $t \to Wb$ decay at the LHC has been widely investigated over the past twenty years, and the Higgs pair production and decays into $W^-W^+bb$, $\gamma\gamma bb$ and $\tau^-\tau^+bb$ final states have been studied at the High-Luminosity LHC [10]. The $W^-W^+H$ production at the QCD next-to-leading order (NLO) including parton shower matching has been investigated in Refs. [11–13]. Further study of the precision predictions on the $W^-W^+H$ production should involve the NLO QCD+EW corrections, photon-induced contributions and the subsequent decays of $W^\pm$ and Higgs bosons.

In this work, we study in detail the $W^-W^+H$ production with subsequent $W^\pm \to l^\pm\nu_l$ and $H \to bb$ decays at the LHC, i.e., $pp \to W^-W^+H \to l^+l^-\nu_l\bar{\nu}_{l'}bb + X$ ($l = e$ or $\mu$), including the
QCD correction and the EW corrections from the $q\bar{q}$ annihilation and photon-induced channels. The rest of this paper is organized as follows. In Sec. II we describe in detail the analytical calculation strategy. In Sec. III we present the numerical results of the integrated cross section and some kinematic distributions, and discuss the theoretical uncertainties from the factorization/renormalization scale and parton distribution functions (PDFs). Finally, a short summary is given in Sec. IV.

II. CALCULATION STRATEGY

In this paper, the precision calculation for the $pp \to W^-W^+H+X$ process involves the following partonic channels: (1) quark-antiquark annihilation $q\bar{q} \to W^-W^+H+(g/\gamma)$, (2) real light-quark emission $qg/\gamma \to W^-W^+H+q$, (3) gluon-gluon fusion $gg \to W^-W^+H$ and (4) photon-photon fusion $\gamma\gamma \to W^-W^+H$, where $q$ runs over all five light flavors of quarks. The $q\bar{q}$ annihilation subprocesses are calculated up to the QCD+EW NLO, 

$$\sigma_{q\bar{q}} = \sigma_{q\bar{q}}^{LO} + \Delta\sigma_{q\bar{q}}^{QCD} + \Delta\sigma_{q\bar{q}}^{EW}, \tag{2.1}$$

where $\sigma_{q\bar{q}}^{LO}$, $\Delta\sigma_{q\bar{q}}^{QCD}$ and $\Delta\sigma_{q\bar{q}}^{EW}$ are the $\mathcal{O}(\alpha^3)$, $\mathcal{O}(\alpha^3 \alpha_s)$ and $\mathcal{O}(\alpha^4)$ contributions from the $q\bar{q}$ annihilation subprocesses, respectively. The subprocesses with $qg$ and $q\gamma$ initial states are calculated only at the LO, and the corresponding cross sections are denoted as $\sigma_{qg}$ and $\sigma_{q\gamma}$. It should be noted that the PDF counterterm corrections from the $q \to q + g$, $q \to q + \gamma$, $g \to q + \bar{q}$ and $\gamma \to q + \bar{q}$ parton splittings should be included into $\Delta\sigma_{q\bar{q}}^{QCD}$, $\Delta\sigma_{q\bar{q}}^{EW}$, $\sigma_{qg}$ and $\sigma_{q\gamma}$, respectively, for IR safety. Due to the large gluon density in proton at high energy hadron colliders, the loop-induced channel $gg \to W^-W^+H$ is taken into account in our precision QCD calculation, although the LO contribution of the $gg$ fusion channel is one order of $\alpha_s$ higher than the NLO QCD correction from the $q\bar{q}$ annihilation channel. For the $\gamma\gamma$ fusion

\footnote{There are two types of PDF counterterm corrections from the $q \to q + \gamma$ splitting, which correspond to the $P_{qq}$ and $P_{qg}$ splitting functions, respectively. The EW PDF counterterm correction induced by the splitting function $P_{qq}$ is absorbed by $\Delta\sigma_{q\bar{q}}^{EW}$, while the correction induced by the splitting function $P_{qg}$ as well as the PDF counterterm correction from the $\gamma \to q + \bar{q}$ splitting is absorbed by $\sigma_{q\gamma}$.}
channel, the NLO EW correction is negligible and we consider only its LO contribution to
the $pp \to W^- W^+ H + X$ process, because the density of photon in proton is much smaller
than those of colored partons (i.e., gluon and light quarks). We generate all the Feynman

diagrams and amplitudes for these partonic channels by adopting FeynArts package [13],
and present some representative Feynman diagrams in Fig.1. Then the corrected cross section for
the $pp \to W^- W^+ H + X$ process calculated in this paper is given by

$$
\sigma^{QCD+EW+q\gamma+\gamma\gamma} = \sigma^{LO} + \Delta\sigma^{QCD} + \Delta\sigma^{EW} + \sigma_{q\gamma} + \sigma_{\gamma\gamma},
$$

(2.2)

where the LO cross section, QCD correction and EW correction are defined as

$$
\sigma^{LO} = \sigma_{q\bar{q}}^{LO}, \quad \Delta\sigma^{QCD} = \Delta\sigma_{q\bar{q}}^{QCD} + \sigma_{qg} + \sigma_{gg}, \quad \Delta\sigma^{EW} = \Delta\sigma_{q\bar{q}}^{EW},
$$

(2.3)

and $\sigma_{gg} \sim \mathcal{O}(\alpha^3 \alpha_s^2)$ and $\sigma_{\gamma\gamma} \sim \mathcal{O}(\alpha^3)$ are the lowest order contributions of the $gg$ and $\gamma\gamma$ fusion

channels, respectively. Since the calculation of the NLO QCD correction has been presented in
Refs. [11][13], we describe only the calculation of the EW correction in this section.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{feynman_diagrams.png}
\caption{Representative Feynman diagrams for the partonic processes contributing to the $pp \to W^- W^+ H + X$ process.}
\end{figure}

\begin{itemize}
\item In this paper, we define $\Delta\sigma^{EW}$ as the EW correction from the $q\bar{q}$ annihilation channel in order to show the photon-induced contributions ($\sigma_{q\gamma}$ and $\sigma_{\gamma\gamma}$) more clearly.
\end{itemize}
In the calculation of $\sigma^{LO}$, $\Delta\sigma^{QCD}$, $\Delta\sigma^{EW}$ and $\sigma_{q\gamma}$, we adopt the $G_\mu$ scheme \cite{15,17} (i.e., $\alpha = \alpha_{G_\mu}$) for all the EW couplings. This fine structure constant scheme is suitable for EW correction due to the EW Sudakov logarithm caused by the soft or collinear weak gauge-boson exchange at high energies \cite{18,19}. But in the evaluation of the $\gamma\gamma$ fusion channel, we adopt the mixed scheme, in which the inputs of the fine structure constant are taken as $\alpha = \alpha(0)$ and $\alpha = \alpha_{G_\mu}$ for the electromagnetic and weak couplings, respectively. In the mixed scheme, the mass-singular terms $\ln(m_f^2/\mu^2)$ ($f = e, \mu, \tau, u, d, c, s, b$) from vacuum polarization at the EW NLO can be either canceled between the external photons and the corresponding electromagnetic couplings or absorbed by $\alpha_{G_\mu}$ in genuine weak couplings. Thus, the mixed scheme is more suitable for performing high-order perturbative calculation for the processes with external photon legs.

The ultraviolet (UV) and infrared (IR) divergences appeared in the NLO EW calculation are regularized in dimensional regularization scheme. For the virtual correction, we encounter the complicated 5-point loop integrals and they can be decomposed into 4-point loop integrals by employing the Passarino-Veltman algorithm \cite{20}. The calculation of 4-point loop integrals may introduce numerical instability due to the small Gram determinant, which can be solved by adopting the quadruple precision arithmetic as used in Refs. \cite{16,17,21,25}. The electric charge is renormalized in the $G_\mu$ scheme, and the relevant fields and masses are renormalized in the on-mass-shell renormalization scheme \cite{26}. The real emission (i.e., the real photon emission and real light-quark emission) corrections are handled by adopting the two cutoff phase space slicing (TCPSS) method \cite{27}, and the independence of the cross section on the soft cutoff $\delta_s$ and collinear cutoff $\delta_c$ has been checked in the range of $\delta_s \in [10^{-6}, 10^{-3}]$ with $\delta_c = \delta_s/50$.

As we expect, the EW correction from the $q\bar{q}$ annihilation channel (i.e., the sum of the virtual correction and the real photon emission correction) and the $q\gamma$-induced correction (i.e., the real light-quark emission EW correction) are both UV and IR finite after absorbing the corresponding PDF counterterms.

\footnote{Although the $\gamma\gamma$ fusion channel is calculated only at the LO because the NLO correction is negligible, we still suggest adopting the mixed scheme for this channel.}
The scalar and tensor integrals are calculated by using our developed LOOPTOOLS package \cite{28}. The PDFs are extracted by LHAPDF6 \cite{29}, and the numerical calculation for squared matrix elements and phase-space integration are performed by employing FORMCALC \cite{30}. The subsequent decays of $W^\pm$ and $H$ are handled by using the MADSPIN method \cite{31}. In order to verify the correctness of our calculation, we recalculate the NLO QCD correction to $W^-W^+H$ production with the same input parameters as in Refs. \cite{11,12}, and find that our numerical results are in good agreement with the corresponding ones in Refs. \cite{11,12} within the calculation errors. We also calculate the NLO EW correction to $ZZH$ production by using our program and obtain $\delta_{\text{EW}} \simeq 9\%$, which is consistent with that obtained by using the newly developed MADGRAPH package \cite{32}.

A great number of $W^-W^+H$ events are from the $W^-tH$ and $W^+\bar{t}H$ associated productions with subsequent top-quark decay $t \rightarrow Wb$, i.e., $pp \rightarrow bg/\gamma \rightarrow W^-tH \rightarrow W^-W^+Hb + X$ and $pp \rightarrow \bar{b}g/\gamma \rightarrow W^+\bar{t}H \rightarrow W^-W^+H\bar{b} + X$ (see Fig.\cite{11}5). These events should be treated as the single top production, and thus should be subtracted carefully from our calculation to avoid double counting and to keep the convergence of the perturbative description of the $W^-W^+H$ production. In this paper, we introduce four schemes to subtract the on-shell $W^-tH$ and $W^+\bar{t}H$ events in handling the $bg/\gamma$- and $\bar{b}g/\gamma$-induced subprocesses. In scheme I, we assume the event with a final $b$-jet can be rejected with 100\% efficiency, so that the $pp \rightarrow bg/\gamma \rightarrow W^-W^+Hb + X$ and $pp \rightarrow \bar{b}g/\gamma \rightarrow W^-W^+H\bar{b} + X$ event samples can be easily excluded \cite{12}. In scheme II, we adopt the diagram subtraction (DS) method \cite{33,34} to subtract the top-resonance effect. This subtraction scheme is defined as a replacement of the Breit-Wigner propagator

$$\frac{|\mathcal{M}|^2(p_t^2)}{(p_t^2 - m_t^2)^2 + \Gamma_t^2 m_t^2} \rightarrow \frac{|\mathcal{M}|^2(p_t^2)}{(p_t^2 - m_t^2)^2 + \Gamma_t^2 m_t^2} - \frac{|\mathcal{M}|^2(m_t^2)}{(p_t^2 - m_t^2)^2 + \Gamma_t^2 m_t^2} \theta(\sqrt{s} - M_W - m_t - M_H), \quad (2.4)$$

where $p_t^2$ is the squared momentum flowing through the intermediate top-quark propagator and $\sqrt{s}$ represents the parton-level colliding energy. In this scheme, the contributions from the squared amplitudes with on-shell top quark are removed point by point over the entire phase space, and the gauge invariance is guaranteed in the limit $\Gamma_t \rightarrow 0$. In scheme III, we adopt
the diagram removal (DR) method \cite{35, 36}, i.e., remove all the top-resonance diagrams at the amplitude level, to subtract the top-resonance effect. This DR method violates gauge invariance. However, the authors in Refs. \cite{35, 36} investigated in detail the gauge dependence for the $Wt$ and squark-pair productions and found that the influence of gauge dependence in the DR scheme can be safely neglected in numerical studies. In scheme IV, we introduce the following subtraction term to remove the contributions from the $W^-tH$ and $W^+\bar{t}H$ productions with subsequent top-quark decay at the cross section level\cite{37},

$$
\sigma_{\text{sub}} = -\left[ \sigma^{\text{LO}}(pp \to bg/\gamma \to W^-tH + X) + \sigma^{\text{LO}}(pp \to \bar{b}g/\gamma \to W^+\bar{t}H + X) \right] \times \text{Br}(t \to Wb). \quad (2.5)
$$

This scheme is a gauge invariant scheme, since there is no diagram removal at the amplitude level. In Refs. \cite{35, 36} the authors suggest applying an invariant mass cut on the $W^+b$ and $W^-\bar{b}$ systems to exclude the $W^-tH$ and $W^+\bar{t}H$ events, respectively. This invariant mass cut is given by

$$
|M_{Wb} - m_t| > \kappa \Gamma_t, \quad (2.6)
$$

where $\kappa$ is an experiment-dependent parameter. However, we do not adopt this scheme in our calculation, because the $pp \to bg/\gamma \to W^-W^+Hb + X$ and $pp \to \bar{b}g/\gamma \to W^-W^+H\bar{b} + X$ subprocesses cannot be properly handled by using the TCPSS method after applying the invariant mass cut in Eq. (2.6).

## III. NUMERICAL RESULTS

### III.1 Input parameters

The Fermi constant and mass parameters are taken from the recent CERN Yellow Report “Handbook of LHC Higgs cross sections: 4. Deciphering the nature of the Higgs sector” \cite{39}:

\[
M_W = 80.385 \text{ GeV}, \quad M_Z = 91.1876 \text{ GeV}, \quad m_t = 172.5 \text{ GeV},
\]

\[
M_H = 125 \text{ GeV}, \quad G_{\mu} = 1.1663787 \times 10^{-5} \text{ GeV}^{-2}.
\]  

\footnote{We assume \text{Br}(t \to Wb) = 100\% for simplicity.}
The top-quark decay width $\Gamma_t = 1.41$ GeV and the fine structure constant in the $\alpha(0)$ scheme $\alpha(0) = 1/137.035999139$ are taken from Ref. [40]. In the $G_\mu$ scheme we obtain

$$\alpha = \alpha_{G_\mu} = \frac{\sqrt{2}}{\pi} G_\mu M_W^2 \left( 1 - \frac{M_W^2}{M_Z^2} \right).$$

(3.2)

The strong coupling constant $\alpha_s$ is taken from the PDFs. The factorization and renormalization scales are set to be equal, i.e., $\mu_f = \mu_r = \mu$, and the central scale is chosen as $\mu_0 = M_T/2$ unless stated otherwise, where $M_T$ is the sum of the transverse masses of final particles. We adopt the LUXqed_plus_PDF4LHC15_nnlo_100 PDFs [41] throughout the LO and NLO calculations as used in Refs. [42, 43]. All leptons and quarks except the top quark are treated as massless particles\(^5\), and the Cabibbo-Kobayashi-Maskawa (CKM) matrix is set to $1_{3 \times 3}$. The $W$-boson decay branching ratio $\text{Br}(W^\pm \rightarrow l^\pm \nu l) = 22.2\%$ is obtained by using the MadSpin program, and the Higgs-boson decay branching ratio $\text{Br}(H \rightarrow b\bar{b}) = 57.5\%$ is taken from Ref. [43].

III..2 Integrated cross sections

In Table I, we present the LO and QCD + EW + $q\gamma + \gamma\gamma$ corrected integrated cross sections and the QCD, EW, $q\gamma$-induced and $\gamma\gamma$-induced corrections ($\Delta\sigma_{QCD}^{LO}$, $\Delta\sigma_{EW}^{LO}$, $\sigma_{q\gamma}$ and $\sigma_{\gamma\gamma}$) for the $W^-W^+H$ production at the 14 TeV LHC by employing the four different subtraction schemes mentioned above. The corresponding relative corrections are defined as

$$\delta_{QCD} = \frac{\Delta\sigma_{QCD}^{LO}}{\sigma_{LO}}, \quad \delta_{EW} = \frac{\Delta\sigma_{EW}^{LO}}{\sigma_{LO}}, \quad \delta_{q\gamma} = \frac{\sigma_{q\gamma}}{\sigma_{LO}}, \quad \delta_{\gamma\gamma} = \frac{\sigma_{\gamma\gamma}}{\sigma_{LO}}.$$  

(3.3)

As we expect, $\sigma_{LO}$, $\Delta\sigma_{EW}$ and $\sigma_{\gamma\gamma}$ are independent of the subtraction scheme, because the subtraction of the $W^-tH$ and $W^+tH$ events reduces only the contributions of the $bg/\gamma$ and $\bar{b}g/\gamma$ scattering channels, respectively. The $q\gamma$-induced correction, which is insensitive to the subtraction scheme, is significant ($\Delta\sigma_{q\gamma} \simeq 0.59$ fb, $\delta_{q\gamma} \simeq 6.1\%$), and just compensates the negative EW correction from the $q\bar{q}$ annihilation channel ($\Delta\sigma_{EW} = -0.58$ fb, $\delta_{EW} = -6.0\%$). The contribution from the $\gamma\gamma$ fusion channel is sizable ($\sigma_{\gamma\gamma} = 0.28$ fb, $\delta_{\gamma\gamma} = 2.9\%$), and thus

\(^5\)In this paper, the bottom-quark mass is set to zero in the calculation of the $pp \rightarrow W^-W^+H + X$ production process, but is kept to be nonzero when considering its subsequent Higgs-boson decay $H \rightarrow bb$. 
should be taken into account in precision EW calculation, especially when $\delta_{\text{EW}} + \delta_{q\gamma} \sim 0$. Then the full EW relative correction, defined as $\delta^{(\text{full})}_{\text{EW}} = \delta_{\text{EW}} + \delta_{q\gamma} + \delta_{\gamma\gamma}$, is obtained as $\delta^{(\text{full})}_{\text{EW}} = 2.7\%$ by adopting scheme I. The QCD corrections in scheme I, II and III are almost the same ($\delta_{\text{QCD}} = 30–31\%$), while the QCD correction in scheme IV is obviously overestimated since we adopt the narrow-width approximation to subtract the $W^-tH$ and $W^+\bar{t}H$ events in scheme IV (see Eq. (2.5)). Since the difference between scheme I, II and III are tiny and the $b$-jet veto can be easily implemented, we adopt only scheme I to deal with the $pp \rightarrow bg/\gamma \rightarrow W^-W^+Hb + X$ and $pp \rightarrow \bar{b}g/\gamma \rightarrow W^-W^+\bar{H}b + X$ subprocesses in the following discussion.

| Subtraction scheme | $\sigma^{\text{LO}}$ | $\Delta\sigma_{\text{EW}}$ | $\sigma_{\gamma\gamma}$ | $\sigma_{q\gamma}$ | $\Delta\sigma_{\text{QCD}}$ | $\sigma_{\text{QCD}+\text{EW}+q\gamma+\gamma\gamma}$ |
|--------------------|----------------|--------------------------|----------------------|-----------------|-----------------|---------------------------------|
| I                  | 0.56           | 2.99                     |                      |                 |                 | 12.90                           |
| II                 | 9.65           | -0.58                    | 0.28                 | 0.59            | 2.94            | 12.88                           |
| III                | 0.59           | 2.95                     |                      |                 |                 | 12.89                           |
| IV                 | 0.59           | 3.83                     |                      |                 |                 | 13.77                           |

Table I: LO and QCD+EW+qγ+γγ corrected integrated cross sections (in fb) for the $W^-W^+H$ production at the 14 TeV LHC in subtraction scheme I, II, III and IV.

The factorization/renormalization scale dependence of the LO and QCD + EW + qγ + γγ corrected integrated cross sections as well as the corresponding QCD, EW, qγ-induced and γγ-induced corrections for the $W^-W^+H$ production at the 14 TeV LHC are shown in Table II. To estimate the theoretical error from the factorization/renormalization scale, we define the scale uncertainty at a given scale $\mu_0$ as

$$
\varepsilon_{\text{scale}}(\mu_0) = \frac{1}{\sigma(\mu_0)} \max \left\{ \sigma(\mu) - \sigma(\mu') \left| \mu, \mu' \in [\mu_0/2, 2\mu_0] \right. \right\}.
$$

(3.4)

We adopt two typical central scales for comparison: (1) $\mu^{(1)}_0 = M_T/2$ and (2) $\mu^{(2)}_0 = M_F/2$

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*The scale uncertainties of $\Delta\sigma_{\text{QCD}}$, $\Delta\sigma_{\text{EW}}$, $\sigma_{q\gamma}$ and $\sigma_{\gamma\gamma}$ listed in Table II are normalized by $\sigma_{\text{QCD}+\text{EW}+q\gamma+\gamma\gamma}$, since $\Delta\sigma_{\text{QCD}}$, $\Delta\sigma_{\text{EW}}$, $\sigma_{q\gamma}$ and $\sigma_{\gamma\gamma}$ are regarded as the correction components of the corrected cross section.*
\[ M_F = 2M_W + M_H \], which are dependent and independent of the final-state phase space, respectively. The scale uncertainties at these two scales are denoted as \( \varepsilon_{\text{scale}}^{(1)} \) and \( \varepsilon_{\text{scale}}^{(2)} \). As we know, \( \sigma^{\text{LO}}, \Delta \sigma^{\text{EW}}, \sigma_{q\gamma} \) and \( \sigma_{\gamma\gamma} \) depend only on the factorization scale, while \( \Delta \sigma^{\text{QCD}} \) (and thus \( \sigma^{\text{QCD+EW+q\gamma+\gamma\gamma}} \)) depends on both the factorization scale and the renormalization scale, because the running strong coupling constant \( \alpha_s(\mu_r) \) is only involved in the QCD correction.

From Table III, we can see that whatever the central scale we use, the scale uncertainties of \( \sigma^{\text{LO}}, \Delta \sigma^{\text{EW}}, \sigma_{q\gamma} \) and \( \sigma_{\gamma\gamma} \) are only about 0.4–0.6\%, while the scale uncertainties of \( \Delta \sigma^{\text{QCD}} \) and \( \sigma^{\text{QCD+EW+q\gamma+\gamma\gamma}} \) are much significant and exceed 4\%. It implies that the theoretical error induced by the renormalization scale is roughly one order of magnitude larger than that induced by the factorization scale. We may conclude that the scale uncertainty of the QCD+EW+\( q\gamma+\gamma\gamma \) corrected cross section mainly comes from the renormalization scale dependence of the QCD correction, and the scale uncertainty of the LO cross section is underestimated since the LO cross section does not depend on the strong coupling. The table also shows that the difference between the corrected cross sections at the dynamical scale \( \mu_0^{(1)} \) and the fixed scale \( \mu_0^{(2)} \) is very small (\(~0.7\%)\). The \( q\gamma \)-induced correction can almost be canceled by the EW correction from the \( q\bar{q} \) annihilation channel, and thus \( \delta_{\text{EW}}^{(\text{full})} \approx \delta_{\gamma\gamma} \), at the 14 TeV LHC.

The LO, QCD+EW+\( q\gamma+\gamma\gamma \) corrected integrated cross sections and the corresponding QCD, EW, \( q\gamma \)-induced and \( \gamma\gamma \)-induced (relative) corrections for the \( W^-W^+H \) production at the 13, 14 TeV LHC and a 33 TeV proton-proton collider are provided in Table III. We see that the QCD and photon-induced corrections (i.e., \( \Delta \sigma^{\text{QCD}}, \sigma_{q\gamma} \) and \( \sigma_{\gamma\gamma} \)) are positive and increase as the increment of the pp colliding energy, while the EW correction from the \( q\bar{q} \) annihilation channel is negative and decreases as the increment of the colliding energy. The QCD correction is significant and the relative correction can reach about 37\% at a 33 TeV proton-proton collider. For the EW correction, the \( q\gamma \)-induced and \( \gamma\gamma \)-induced relative corrections increase quickly from 5.5\% to 11.6\% and from 2.7\% to 4.6\%, respectively, while the EW relative correction from the \( q\bar{q} \) annihilation channel is steady at \(-6 \sim -7\%)\, as the pp colliding energy increases from 13 to
Table II: Scale dependence of the LO and QCD + EW + $q\gamma + \gamma\gamma$ corrected integrated cross sections for the $W^-W^+H$ production at the 14 TeV LHC.

|                      | Cross section (fb) | $\varepsilon^{(1)}_{\text{scale}}$ (%) | Cross section (fb) | $\varepsilon^{(2)}_{\text{scale}}$ (%) |
|----------------------|--------------------|----------------------------------------|--------------------|----------------------------------------|
|                      | $\mu_0^{(1)}/2$   | $\mu_0^{(1)}$ | $2\mu_0^{(1)}$ | $\mu_0^{(2)}/2$ | $\mu_0^{(2)}$ | $2\mu_0^{(2)}$ |
| $\sigma^{\text{LO}}$ | 9.61               | 9.65        | 9.63         | 0.41                | 9.65       | 9.71        | 9.71         | 0.62               |
| $\Delta\sigma^{\text{QCD}}$ | 3.36             | 2.99        | 2.75         | 4.73                | 3.39       | 3.04        | 2.74         | 5.00               |
| $\Delta\sigma^{\text{EW}}$ | −0.60            | −0.58       | −0.55        | 0.39                | −0.63      | −0.60       | −0.58        | 0.38               |
| $\sigma_{q\gamma}$       | 0.59              | 0.56        | 0.53         | 0.47                | 0.61       | 0.58        | 0.55         | 0.46               |
| $\sigma_{\gamma\gamma}$   | 0.25              | 0.28        | 0.31         | 0.47                | 0.23       | 0.26        | 0.29         | 0.46               |
| $\sigma^{\text{QCD+EW+q\gamma+\gamma\gamma}}$ | 13.21            | 12.90       | 12.67        | 4.19                | 13.25      | 12.99       | 12.71        | 4.16               |

33 TeV. The ratio of the full EW correction to the QCD correction, $\delta^{(\text{full})}_{\text{EW}} / \delta_{\text{QCD}}$, is about 9% at the 14 TeV LHC and can exceed 25% at a 33 TeV proton-proton collider. It is concluded that the photon-induced correction would be the dominant EW contribution and the full EW correction becomes more and more important with the increment of the $pp$ colliding energy.

PDF is another source of the theoretical error for scattering processes at hadron colliders. In this work we adopt the LUXqed plus PDF4LHC15 nnlo 100 PDFs, which contains $N = 108$ PDF sets. The PDF uncertainties of the LO and QCD + EW + $q\gamma + \gamma\gamma$ corrected integrated cross sections are given by [17,45]

$$
\varepsilon_X^{\text{PDF}} = \frac{1}{\sigma_X} \left[ \frac{1}{N-1} \sum_{i=1}^{N-1} \left( \sigma_i^X - \sigma_0^X \right)^2 \right]^{1/2},
$$

(3.5)

where $X \in \{\text{LO}, \text{QCD + EW + } q\gamma + \gamma\gamma\}$, and $\sigma_i^X$ ($i=0, ..., N-1$) are the corresponding cross sections calculated by using the $i$-th LUXqed plus PDF4LHC15 nnlo 100 PDF set. Then we obtain

$$
\varepsilon^{\text{LO}}_{\text{PDF}} = 0.18\%, \quad \varepsilon^{\text{QCD+EW+q\gamma+\gamma\gamma}}_{\text{PDF}} = 0.17\%.
$$

(3.6)
| √S (TeV) | Cross section (fb) | Relative correction (%) |
|---------|-------------------|-------------------------|
|         | σ^{LO}   | Δσ^{QCD} | Δσ^{EW} | σ_{qγ} | σ_{γγ} | σ^{QCD+EW+qγ+γγ} | δ_{QCD} | δ_{EW} | δ_{qγ} | δ_{γγ} |
| 13      | 8.56     | 2.60    | -0.51  | 0.47   | 0.23   | 11.35         | 30.4    | -6.0  | 5.5   | 2.7   |
| 14      | 9.65     | 2.99    | -0.58  | 0.56   | 0.28   | 12.90         | 31.0    | -6.0  | 5.8   | 2.9   |
| 33      | 33.87    | 12.66   | -2.27  | 3.93   | 1.56   | 49.75         | 37.4    | -6.7  | 11.6  | 4.6   |

Table III: LO, QCD + EW + qγ + γγ corrected integrated cross sections and the corresponding (relative) corrections for the $W^-W^+H$ production at the 13, 14 TeV LHC and a 33 TeV pp collider.
It shows that the PDF uncertainty of the QCD + EW + $q\gamma + \gamma\gamma$ corrected cross section is almost the same as the PDF uncertainty of the LO cross section, which implies that the PDF uncertainties from the QCD, EW, $q\gamma$-induced and $\gamma\gamma$-induced corrections are negligible. The PDF uncertainties from those corrections are

$$\varepsilon^{\text{PDF}}_{\text{QCD}} = 3.5 \times 10^{-4}, \hspace{1em} \varepsilon^{\text{PDF}}_{\text{EW}} = 1.0 \times 10^{-4}, \hspace{1em} \varepsilon^{q\gamma}_{\text{PDF}} = 6.7 \times 10^{-5}, \hspace{1em} \varepsilon^{\gamma\gamma}_{\text{PDF}} = 3.5 \times 10^{-5}. \hspace{1em} (3.7)$$

We also employ the NNPDF23_nlo_as_0119_qed PDFs in the initial-state parton convolution for comparison, and find that the PDF uncertainties obtained by using the LUXqed PDFs are much smaller than the corresponding ones by using the NNPDF23 PDFs. The feature of small photon PDF uncertainty of the LUXqed PDFs is also discussed in Refs. [41, 43]. Compared to the scale uncertainty, the PDF uncertainty of the integrated cross section is much smaller, especially at the QCD+EW NLO. Thus, we do not consider the PDF uncertainty in estimating the theoretical error for the $W^-W^+H$ production at the LHC.

### III..3 Kinematic distributions

In this subsection, we present the LO and QCD + EW + $q\gamma + \gamma\gamma$ corrected kinematic distributions of final $W^\pm$ and Higgs bosons as well as their decay products for the $W^-W^+H$ production at the 14 TeV LHC.

#### III..3.1 Distributions for $pp \rightarrow W^-W^+H + X$

The LO and QCD + EW + $q\gamma + \gamma\gamma$ corrected invariant mass distributions of the $W$-boson pair are depicted in Fig.2. The corresponding QCD, EW, $q\gamma$-induced and $\gamma\gamma$-induced relative corrections are provided in the lower panel. Both the LO and QCD + EW + $q\gamma + \gamma\gamma$ corrected invariant mass distributions of the $W$-boson pair reach their maxima in the vicinity of $M_{W^-W^+} \sim 200$ GeV, and then drop down approximately logarithmically with the increment of $M_{W^-W^+}$. The QCD correction enhances the LO $W$-boson pair invariant mass distribution significantly in the whole plotted $M_{W^-W^+}$ region, and the corresponding QCD relative correction increases
from 29% to 38% as the increment of \( M_{W-W^+} \). For the EW correction, the contribution from the \( q\bar{q} \) annihilation channel suppresses the LO \( W \)-boson pair invariant mass distribution and the relative correction \( \delta_{\text{EW}} \) decreases from 0 to \(-19\%\), while the \( q\gamma \)-induced and \( \gamma\gamma \)-induced contributions enhance the LO \( W \)-boson pair invariant mass distribution and the corresponding relative corrections \( \delta_{q\gamma} \) and \( \delta_{\gamma\gamma} \) increase rapidly from 3% to 37% and from 0 to 51%, respectively, as \( M_{W-W^+} \) increases from \( 2M_W \) to 1.1 TeV. It clearly shows that the full photon-induced correction, given by \( \sigma_{\gamma\text{-induced}} = \sigma_{q\gamma} + \sigma_{\gamma\gamma} \), is larger than the QCD correction in the high \( M_{W-W^+} \) region. Thus, the LO \( W \)-boson pair invariant mass distribution is mainly enhanced by the QCD correction in the low \( M_{W-W^+} \) region, but mainly enhanced by the photon-induced corrections in the high \( M_{W-W^+} \) region. The very large \( \gamma\gamma \)-induced correction at high invariant mass can also been seen in the invariant mass distribution of the \( W^-W^+Z \) system for \( pp \rightarrow W^-W^+Z+X \) at the LHC [15]. The considerable negative EW correction from the \( q\bar{q} \) annihilation channel in the high \( M_{W-W^+} \) region is due to the well-known Sudakov double logarithm arising from the exchange of a virtual massive gauge boson in the loops. [15, 17]. This large Sudakov virtual correction is canceled by the photon-induced corrections (\( \sigma_{q\gamma} \) and \( \sigma_{\gamma\gamma} \)) obviously, and the full EW correction enhances the LO \( W \)-boson pair invariant mass distribution. Therefore, the photon-induced channels should be considered for precision predictions at high energy colliders, particularly in the high energy region.

The LO and QCD + EW + \( q\gamma + \gamma\gamma \) corrected transverse momentum distributions of the \( W^- \)-boson as well as the corresponding relative corrections are presented in Fig.3. Both the LO and QCD + EW + \( q\gamma + \gamma\gamma \) corrected distributions reach their peaks at \( p_{T,W^-} \sim 45 \) GeV and then decrease consistently as the increment of \( p_{T,W^-} \). The QCD relative correction varies in the range of \([29\%, 40\%]\) as \( p_{T,W^-} \in [0, 400] \) GeV. As \( p_{T,W^-} \) increases from 0 to 400 GeV, the EW relative correction from the \( q\bar{q} \) annihilation channel decreases from about \(-3\%\) to \(-20\%\), while the relative corrections from the \( q\gamma \) and \( \gamma\gamma \) scattering channels are relatively stable, and vary in the ranges of \([5\%, 6\%]\) and \([1\%, 4\%]\), separately. The total production cross section is dominated
Figure 2: $W$-boson pair invariant mass distributions and the corresponding relative corrections for $pp \rightarrow W^-W^+ H + X$ at the 14 TeV LHC. The scale marked on the left side of the lower panel corresponds to $\delta_{EW}$, $\delta_{\gamma q}$ and $\delta_{\gamma\gamma}$, and that on the right side of the lower panel is for $\delta_{QCD}$.

by the contribution from the low $p_{T,W^-}$ region. In the region around $p_{T,W^-} = 150$ GeV, the EW correction from the $q\bar{q}$ annihilation channel is strongly canceled by those from the photon-induced channels.

The LO and QCD + EW + $q\gamma + \gamma\gamma$ corrected rapidity distributions of $W^-$ and $W^+$ are provided in Figs. 4(a) and 4(b), respectively. The corresponding QCD, EW, $q\gamma$-induced and $\gamma\gamma$-induced relative corrections are shown in the lower panels. Both the LO and QCD + EW + $q\gamma + \gamma\gamma$ corrected rapidity distributions of $W^-$ are slightly larger than the corresponding ones of $W^+$ in the central rapidity region, but a little smaller than those of $W^+$ in the forward-backward rapidity region. The QCD correction enhances the LO $W$-boson rapidity distributions significantly, and the QCD relative correction decreases from 32% to 29% and from 33% to 28% for $W^-$ and $W^+$ rapidity distributions, respectively, as $|y_W|$ increases from 0 to 3. The EW relative correction from the $q\bar{q}$ annihilation channel is negative, and insensitive to the rapidities of $W^-$ and $W^+$. It is steady at about $-6\%$ in the whole plotted $y_W$ region. The $q\gamma$-induced
Figure 3: The same as Fig.2 but for the transverse momentum distribution of $W^-$. The relative correction increases from 4% to 13% and from 5% to 8% for the rapidity distributions of $W^-$ and $W^+$, respectively, with the increment of $|y_{W}|$ from 0 to 3. Compared to the $q\gamma$-induced relative correction, the $\gamma\gamma$-induced relative correction is relatively small, and increases from 1% to 10% and from 1% to 6% correspondingly. The full photon-induced relative correction is sizeable, particularly in the forward-backward rapidity region. We again see the importance of the $q\gamma$ and $\gamma\gamma$ scattering channels and the cancelation between the photon-induced and $q\bar{q}$-initiated EW corrections in the $W^-$ and $W^+$ rapidity distributions.

The Higgs-boson transverse momentum distributions and the corresponding QCD, EW, $q\gamma$-induced and $\gamma\gamma$-induced relative corrections are displayed in the upper and lower panels of Fig.5 respectively. Both the LO and QCD + EW + $q\gamma + \gamma\gamma$ corrected Higgs transverse momentum distributions increase sharply in the low $p_{T,H}$ region ($p_{T,H} < 50$ GeV), reach their maxima at $p_{T,H} \sim 65$ GeV, and decrease approximately logarithmically when $p_{T,H} > 80$ GeV as the increment of $p_{T,H}$. The QCD relative correction is positive, and increases from 27% to 43% as $p_{T,H}$ increases from 0 to 400 GeV. The $q\gamma$-induced relative correction is steady at about 3%
Figure 4: The same as Fig.2 but for the rapidity distributions of $W^-$ and $W^+$. 
in the low $p_{T,H}$ region and 11% in the region of $p_{T,H} > 250$ GeV, respectively, and increases smoothly in the intermediate $p_{T,H}$ region ($p_{T,H} \in [50, 250]$ GeV), while the $\gamma\gamma$-induced relative correction varies in the small range of $[2\%, 3\%]$ in the whole plotted $p_{T,H}$ region. The EW correction from the $q\bar{q}$ annihilation channel always suppresses the LO distribution, and the corresponding EW relative correction decreases from $-2\%$ to $-18\%$ as $p_{T,H}$ varies from 0 to 400 GeV. In the high $p_{T,H}$ region, the $q\bar{q}$-initiated EW correction is sizable and its absolute value is comparable to the QCD correction due to the EW Sudakov logarithm.

**Figure 5:** The same as Fig.2 but for the Higgs transverse momentum distribution.

The LO and QCD + EW + $q\gamma + \gamma\gamma$ corrected Higgs-boson rapidity distributions and the corresponding relative corrections are shown in Fig.6. Analogous to the $W^-$ and $W^+$ rapidity distributions, the QCD relative correction to the Higgs rapidity distribution decreases from 33% to 27% as $|y_H|$ increases from 0 to 3, and the EW relative correction from the $q\bar{q}$ annihilation channel is insensitive to the Higgs rapidity and varies in the range of $[-7\%, -5\%]$ when $y_H \in [-3, 3]$. However, it should be noted that the photon-induced relative corrections to the Higgs rapidity distribution behave quite differently from those to the $W^\pm$ rapidity distributions. The
relative corrections from the $q\gamma$ and $\gamma\gamma$ scattering channels decrease from 7% to 3% and from 4% to 1%, respectively, as $|y_H|$ varies from 0 to 3.

Figure 6: The same as Fig.2 but for the Higgs rapidity distribution.

III.3.2 Distributions for $pp \rightarrow W^-W^+H + X \rightarrow l^+l^-\nu_1\bar{\nu}_1b\bar{b} + X$

Now we turn to the $W^-W^+H$ production with subsequent $W^\pm \rightarrow l^\pm\nu_l$ and $H \rightarrow b\bar{b}$ decays at the 14 TeV LHC. The spin correlation and finite-width effects of the intermediate Higgs and $W^\pm$ bosons are taken into account by adopting the MadSpin method.

In Fig.7 we present the LO and QCD + EW + $q\gamma + \gamma\gamma$ corrected invariant mass distributions of the final charged lepton pair for $pp \rightarrow W^-W^+H + X \rightarrow l^+l^-\nu_1\bar{\nu}_1b\bar{b} + X$. The corresponding relative corrections are plotted in the lower panel. Since the charged leptons are the decay products of $W^\pm$ bosons, the charged lepton pair invariant mass distribution inherits the feature of the $W$-boson pair invariant mass distribution. Both the LO and QCD + EW + $q\gamma + \gamma\gamma$ corrected charged lepton pair invariant mass distributions peak at $M_{l^-l^+} \sim 75$ GeV, and decrease approximately logarithmically as the increment of $M_{l^-l^+}$ in the range of $M_{l^-l^+} > 90$ GeV. In the low $M_{l^-l^+}$ region, the QCD + EW + $q\gamma + \gamma\gamma$ correction is dominated by the QCD contribution,
while in the high $M_{l^-l^+}$ region, the LO charged lepton pair invariant mass distribution is mainly enhanced by the photon-induced corrections. For example, at $M_{l^-l^+} = 900$ GeV, the relative corrections from the $q\gamma$ and $\gamma\gamma$ scattering channels are about 56% and 86%, respectively, while the QCD relative correction is only about 42%. Thus, with the accumulation of events, the photon-induced channels become nonnegligible, especially at future high energy hadron colliders.

![Figure 7: Invariant mass distributions of the charged lepton pair and the corresponding relative corrections for $pp \to W^-W^+H \to l^-l^+\nu\bar{\nu}b\bar{b} + X$ at the 14 TeV LHC. The scale marked on the left side of the lower panel corresponds to $\delta_{\text{EW}}$, $\delta_{\gamma q}$ and $\delta_{\gamma\gamma}$, and that on the right side of the lower panel is for $\delta_{\text{QCD}}$.](image)

Since the transverse momentum distribution of $l^+$ is similar to that of $l^-$, we only depict the transverse momentum distributions of $l^-$ and the corresponding relative corrections in Fig. 8. Both the LO and QCD + EW + $q\gamma+\gamma\gamma$ corrected $p_{T,l^-}$ distributions reach their maxima at $p_{T,l^-} \sim 35$ GeV and then drop down as the increment of $p_{T,l^-}$. The QCD relative correction decreases slightly at the very beginning, and increases from 30% to 49% as $p_{T,l^-}$ increases from 40 to 400 GeV. The EW relative correction from the $q\bar{q}$ annihilation channel decreases...
consistently as the increment of \( p_{T,l^-} \) in the region of \( p_{T,l^-} > 50 \text{ GeV} \), and reaches about \(-25\%\) at \( p_{T,l^-} = 400 \text{ GeV} \) due to the large EW Sudakov effect. The relative corrections from the \( q\gamma \) and \( \gamma\gamma \) scattering channels are insensitive to the transverse momentum of \( l^- \), and steady at about 6\% and 3\%, respectively, in the whole plotted \( p_{T,l^-} \) region.

Figure 8: The same as Fig.7 but for the transverse momentum distribution of \( l^- \).

The LO, QCD+EW+\( q\gamma + \gamma\gamma \) corrected missing transverse momentum distributions and the corresponding relative corrections for \( pp \rightarrow W^-W^+H+X \rightarrow l^+\bar{l}^-\nu_l\bar{\nu}_l\bar{b}\bar{b}+X \) at the 14 TeV LHC are shown in Fig.9. The corrections do not distort the line shape of the LO \( p_{T,\text{miss}} \) distribution, and both the LO and QCD+EW+\( q\gamma + \gamma\gamma \) corrected \( p_{T,\text{miss}} \) distributions reach their maxima at \( p_{T,\text{miss}} \sim 45 \text{ GeV} \). As the increment of \( p_{T,\text{miss}} \) from 0 to 400 GeV, the QCD relative correction increases from 26\% to 54\%, while the EW relative correction from the \( q\bar{q} \) annihilation channel decreases from \(-3\%\) to \(-24\%\). The relative corrections from the \( q\gamma \) and \( \gamma\gamma \) scattering channels are significant, but much smaller compared to the QCD relative correction. They are almost independent of the missing transverse momentum, and steady at around 6\% and 3\%, respectively, in the plotted \( p_{T,\text{miss}} \) region.
Figure 9: The same as Fig.7 but for the missing transverse momentum distribution.

The transverse momentum distribution of the final anti-bottom quark should be the same as that of the bottom quark due to the CP conservation in the $H \rightarrow b\bar{b}$ decay, so that we only study the $b$-quark transverse momentum distribution and discuss the influences of the QCD, EW, $q\gamma$-induced and $\gamma\gamma$-induced corrections on the $p_{T,b}$ distribution in the following. From Fig.10 we see that both the LO and QCD + EW + $q\gamma + \gamma\gamma$ corrected $b$-quark transverse momentum distributions peak at $p_{T,b} \sim 45$ GeV. Analogous to the $p_{T,W^+}, p_{T,H}, p_{T,l^-}$ and $p_{T,\text{miss}}$ distributions, the $b$-quark transverse momentum distribution increases sharply in the low $p_{T,b}$ region and decreases approximately logarithmically after reaching its maximum as the increment of $p_{T,b}$. The QCD relative correction firstly decreases from 32% to 29% and then increases to 44%, while the EW relative correction from the $q\bar{q}$ channel varies in the range of $[-22\%, -4\%]$, as $p_{T,b}$ increases from 0 to 400 GeV. The $q\gamma$-induced relative correction ranges from 4% to about 10% for $p_{T,b} < 200$ GeV, and is steady at around 10% when $p_{T,b} > 200$ GeV. The $\gamma\gamma$-induced correction is extremely insensitive to the $b$-quark transverse momentum, it decreases very slowly from about 3% to 1% as $p_{T,b}$ increases from 0 to 400 GeV.
IV. SUMMARY

The production of $W^-W^+H$ is an ideal channel for determining the Higgs gauge coupling and understanding the EW symmetry breaking mechanism, as well as a background to new physics beyond the SM. In this work, we calculate the QCD correction, the EW correction from the $q\bar{q}$ annihilation channel, and the $q\gamma$- and $\gamma\gamma$-induced corrections to the $W^-W^+H+X$ production with subsequent $W^\pm \rightarrow l^\pm \nu_l$ and $H \rightarrow b\bar{b}$ decays at the 14 TeV LHC. We adopt the MadSpin method to deal with the spin correlation and finite-width effects of the intermediate $W^\pm$ and Higgs bosons, and four different subtraction schemes to subtract the top-resonance effect for comparison. The QCD + EW + $q\gamma + \gamma\gamma$ corrected integrated cross section and some kinematic distributions of $W^\pm$ and $H$ as well as their decay products are provided. The scale and PDF uncertainties of the integrated cross section are also given for estimating the theoretical error. Our numerical results show that the QCD correction enhances the LO differential cross section significantly, especially in the central rapidity and high energy regions, while the EW correction
from the $q\bar{q}$ annihilation channel suppresses the LO differential cross section obviously. The QCD and $q\bar{q}$-initiated EW relative corrections to the integrated cross section are about 31% and −6%, respectively. The relative corrections from the photon-induced channels, $q\gamma \rightarrow W^-W^+Hq$ and $\gamma\gamma \rightarrow W^-W^+H$, are insensitive to the transverse momenta of final products. The $q\gamma$- and $\gamma\gamma$-induced relative corrections to the integrated cross section are about 6% and 3%, separately, which strongly cancel the EW relative correction from the $q\bar{q}$ annihilation channel. Thus, we should take into account the QCD correction and the EW corrections from the $q\bar{q}$ annihilation and photon-induced channels in precision study on the $W^-W^+H$ production at the LHC. The theoretical error of the QCD + EW + $q\gamma + \gamma\gamma$ corrected integrated cross section mainly comes from the renormalization scale dependence of the QCD correction.

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