Turbulence in a Self-gravitating Molecular Cloud Core

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Abstract

Externally driven interstellar turbulence plays an important role in shaping the density structure in molecular clouds. Here we study the dynamical role of internally driven turbulence in a self-gravitating molecular cloud core. Depending on the initial conditions and evolutionary stages, we find that a self-gravitating core in the presence of gravity-driven turbulence can undergo constant, decelerated, and accelerated infall, and thus has various radial velocity profiles. In the gravity-dominated central region, a higher level of turbulence results in a lower infall velocity, a higher density, and a lower mass accretion rate. As an important implication of this study, efficient reconnection diffusion of magnetic fields against the gravitational drag naturally occurs due to the gravity-driven turbulence, without invoking externally driven turbulence.

Unified Astronomy Thesaurus concepts: Astrophysical fluid dynamics (101); Interstellar magnetic fields (845); Star formation (1569)

1. Introduction

The interstellar medium is turbulent (e.g., Armstrong et al. 1995; Chepurnov & Lazarian 2010; Xu 2019). The interstellar turbulence plays a significant role in physical processes, including star formation (McKee & Ostriker 2007; Federrath & Klessen 2012), cosmic ray propagation (Scalo & Elmegreen 2004; Xu & Yan 2013; Xu et al. 2016; Xu & Lazarian 2018), dynamo amplification (Beck et al. 1996; Brandenburg & Subramanian 2005; Xu & Lazarian 2016, 2017a, 2017b; Xu et al. 2019a) and turbulent reconnection (Lazarian & Vishniac 1999; Lazarian et al. 2012, 2020; Lazarian 2014) of interstellar magnetic fields, and formation and evolution of interstellar density structure (Padoan et al. 2001; Burkhart et al. 2009; Xu et al. 2019b), accounting for observations, e.g., interstellar scattering of Galactic pulsars (Cordes et al. 1985; Rickett 1990; Xu & Zhang 2017), rotation measure fluctuations (Minter & Spangler 1996; Haverkorn et al. 2008; Xu & Zhang 2016), fluctuations in synchrotron intensity and polarization (Gaensler et al. 2011; Lazarian & Pogosyan 2012, 2016).

Supernova explosions are believed to be a dominant source of turbulent energy on length scales of the order of 10–100 pc (Korpi et al. 1999; Haverkorn et al. 2008). The injected turbulence cascades down toward smaller length scales (Armstrong et al. 1995; Chepurnov & Lazarian 2010; Chepurnov et al. 2010; Qian et al. 2018). On small length scales in contracting dense cores in molecular clouds, an “adiabatic heating” mechanism acts to amplify the internal turbulence due to compression (Robertson & Goldreich 2012), where the gravitational potential energy is converted to the turbulent kinetic energy (Scalo & Pumphrey 1982; Sur et al. 2010). The resulting additional internal turbulent pressure support is expected to affect the dynamics and evolution of collapsing cores, as well as their density and velocity profiles (Lee et al. 2015; Murray & Chang 2015). In this study, we incorporate the gravity-driven turbulence and investigate the dynamical evolution of a spherical self-gravitating core. The self-similar behavior of a collapsing sphere has been extensively studied both analytically and numerically (Larson 1969; Penston 1969; Hunter 1977; Shu 1977; Foster & Chevalier 1993; Fatuzzo et al. 2004; Lou & Shen 2004). Here we follow the analytical approach of Shu (1977) to solve the hydrodynamic equations, but focus on the differences in solutions due to the presence of turbulence, which was not considered in the original formalism.

Apart from its importance in influencing the dynamics of molecular cloud cores, turbulence can also effectively enhance the diffusion efficiency of magnetic fields by enhancing their reconnection efficiency. In this work, we will also discuss the implication of gravity-driven turbulence on reconnection diffusion (RD) of magnetic fields. The paper is organized as follows. In Section 2, by solving the hydrodynamic equations involving the internal turbulent pressure, we analyze the dynamical effect of gravity-driven turbulence on the gravitational collapse of a spherical core. In Section 3, we discuss the implication on the RD of magnetic fields arising from the gravity-driven turbulence. The conclusions are provided in Section 4.

2. Self-similar Collapse of a Self-gravitating Turbulent Sphere

We consider a spherical geometry for a self-gravitating and isothermal sphere. The governing equations include the continuity equation in terms of mass M, the continuity equation in terms of density ρ, and the momentum equation (Shu 1992):

\[ \frac{\partial M}{\partial t} + \frac{\partial \rho}{\partial r} = 0, \frac{\partial M}{\partial r} = 4\pi r^2 \rho, \]  

\[ \frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial r} + \frac{1}{r} \frac{\partial (\rho f v^2)}{\partial r} + \frac{GM}{r^2} = 0, \]

where \( |a| \) is the fluid speed, \( a \) is the sound speed, and \( v \) is the turbulent speed. In the absence of external driving, the turbulence in a contracting gas is amplified via the “adiabatic heating” mechanism, that is, turbulence adiabatically heats during contraction (Robertson & Goldreich 2012). On the other
hand, the turbulence dissipates as the turbulent energy cascades toward smaller scales. Under the effects of “adiabatic heating” and dissipation of turbulence, \( v_t \) follows (Robertson & Goldreich 2012; Murray & Chang 2015):

\[
\frac{\partial v_t}{\partial t} + \frac{u}{\partial r} \left( 1 + \frac{v_t}{u} \right) v_t u = 0,
\]

where the two terms in the brackets correspond to the turbulence driving and dissipation, respectively, and the parameter \( \eta \) represents the efficiency of turbulent energy cascade. Both the turbulent motion driven by gravitational contraction and the thermal motion of gas contribute to the cascade. Both the turbulent motion driven by gravitational contraction and the thermal motion of gas contribute to the cascade.

To solve Equation (1), we follow the analytical approach presented in Shu (1977) and combine the radius \( r \) and the time \( t \) into a dimensionless variable

\[
x = \frac{r}{at}.
\]

We then look for a similarity solution of the form

\[
\rho(r, t) = \frac{\alpha(x)}{4\pi G t^2}, \quad M(r, t) = \frac{\alpha^2 r}{G} M(x),
\]

\[
u(r, t) = v(x), \quad v_t(r, t) = C v(x),
\]

where the dimensionless variables \( \alpha, m, \) and \( v \) are the reduced density, mass, and fluid speed, and \( G \) is the gravitational constant. Besides, [\( |C| \leq 1 \)] is the ratio of \( v_t \) to \( |v| \) and also the ratio of the turbulent eddy-turnover rate to the gravitational contraction rate,

\[
|C| = \frac{v_t/r}{|u|r}.
\]

For the contraction induced turbulence, both simulations and physical considerations suggest that \( v_t \) tracks \( |u| \) and tends to synchronize with \( |u| \) (Robertson & Goldreich 2012; Murray & Chang 2015). Therefore, we consider \( C \) as a constant.

By substituting Equation (4) into Equations (1) and (2), we find

\[
m = x^2 \alpha(x - v),
\]

\[
\left( x - v \right) \left( x - v - 2C^2v \right) \frac{dv}{dx} = \left( x - v \right) \left( \alpha \left( x - v \right) - \frac{2}{x} \left( 1 + C^2 v^2 \right) \right),
\]

\[
\left( x - v \right) \left( x - v - 2C^2v \right) \frac{d\alpha}{dx} = \alpha \left( x - v \right) \left\{ \alpha - \frac{2}{x} \left( x - v - 2C^2v \right) \right\},
\]

\[
(x - v) \frac{dv}{dx} = (1 + \eta C) \frac{v^2}{x}.
\]

The ratio of the gravitational force to the pressure gradient force is

\[
R = \frac{GM}{\rho} \frac{1}{r^2} + \frac{\partial p}{\partial r} \sim \frac{\alpha x (x - v)}{C^2 v^2 + 1},
\]

where the expressions in Equations (4) and 6(a) are used. If the effect of turbulence is negligible, it becomes

\[
R_{\text{the}} \sim \alpha x (x - v).
\]

For the “inside-out” collapse of a singular isothermal sphere considered in Shu (1977), the hydrodynamic signal propagates at the speed of sound. The envelope at \( x > 1 \) can remain in the initial hydrostatic state, while the interior at \( x < 1 \) undergoes gravitational infall. Here we incorporate the effect of self-driven turbulence. In the case of highly supersonic turbulence, i.e., \( C v \gg 1, R \) is approximately

\[
R_{\text{tur}} \sim \frac{\alpha x (x - v)}{C^2 v^2},
\]

which is smaller than \( R_{\text{the}} \).

In various asymptotic limits, the solution to the coupled Equations 6(b) and (c) has different behaviors. We start with the limit \( x \gg |v| \), i.e., \( r \gg |u| \gg v t \). It is beyond the radius where the hydrodynamic signals carried by turbulence can reach. We consider different cases with small and large initial infall velocities.

Case (1): \( x \to \infty \) \((t \to 0)\), \( v \to 0 \), \( \alpha \ll 1 \). At an initial state, if the infall velocity is sufficiently small, the effect of turbulence is negligible. This initial state can be treated as the case of collapse of a singular isothermal sphere at a large \( x \) considered in Shu (1977), where turbulence was not taken into account. One can easily obtain the solution:

\[
v = -(A - 2)x^{-1},
\]

\[
\alpha = Ax^{-2},
\]

and

\[
m \approx \alpha x^3 = Ax,
\]

where the constant \( A \) should not be smaller than two as \( v(\leq 0) \) is an inward velocity.

Case (2): \( x \to \infty \) \((t \to 0)\), \( C v \gg 1, \alpha \ll 1 \). This initial condition with a large infall velocity allows the generation of supersonic turbulence. Accordingly, Equations 6(b) and (c) can be approximated by:

\[
\frac{dv}{dx} = \alpha - \frac{2C^2 v^2}{x^2},
\]

\[
\frac{d\alpha}{dx} = -2\alpha.
\]

We find the solution

\[
v \approx -v_s, \quad \alpha = \alpha_2 x^{-2},
\]

where \( v_s \) and \( \alpha_2 \) are constants and \( \alpha_2 \leq 2C^2 v_s^2 \). With the same density profile as in Case (1), the asymptotic form of \( m \) is the same as Equation (11),

\[
m \approx \alpha_2 x.
\]
To have a constant $v$, it requires $\eta = -1/C$ (Equation 6(d)).

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With comparable rates of turbulence driving and dissipation, the resulting turbulence has a constant $v$. The above result shows that the system has undisturbed infall motions.

The different initial conditions in Case (1) and Case (2) lead to different behaviors of the subsequent collapse. We next consider the limit $x \ll |v|$, i.e., $r \ll |v|/t$, and rewrite Equations 6(b) and (c) as

$$\frac{dv}{dx} = 1 + C^2 \left( \alpha + \frac{2C^2}{x} v \right).$$

Equation (15a)

$$\frac{d\alpha}{dx} = - \frac{\alpha}{(1 + C^2)} \left[ \frac{1}{x} + \frac{2(1 + 2C^2)}{x} \right].$$

Equation (15b)

The ratio of the two terms in the brackets in Equation 15(a) reflects the relative importance of gravitational contraction and turbulent pressure support (see Equation (9)). In the regime where the turbulent pressure plays a dominant role, i.e., $R_{\text{tur}} \ll 1$, we can further simplify the above equations and have

$$\frac{dv}{dx} = \frac{2C^2}{1 + C^2} v.$$ 

Equation (16a)

$$\frac{d\alpha}{dx} = \frac{2(1 + 2C^2)}{1 + C^2} \alpha.$$ 

Equation (16b)

We derive the solutions as

$$v = -v_1 x^{2C^2},$$

Equation (17a)

$$\alpha = \alpha_1 x^{\frac{1 + 2C^2}{1 + 2C^2}}.$$ 

Equation (17b)

where $v_1$ and $\alpha_1$ are constants. As $|v|$ decreases with decreasing $x$, it shows that the turbulent pressure dominates the dynamics and causes deceleration of the infall. Given the above expressions, $m$ (Equation 6(a)) is approximately

$$m \approx \alpha_1 v_1.$$ 

Equation (18)

By comparing Equation 6(a) with Equation 6(d), we find that the corresponding relation between $\eta$ and $C$ is

$$\eta = -\frac{1 + 3C^2}{C(1 + C^2)}.$$ 

Equation (19)

With $|\eta|C > 1$, the dissipation is more efficient than driving. Thus $v_1$ decreases with decreasing $x$.

In the regime where the gravitational contraction is more important than the turbulent support, i.e., $R_{\text{tur}} \gg 1$, the solution to Equation (15) is

$$v = -\frac{2\alpha_0}{1 + 5C^2} x^{-\frac{1}{2}},$$ 

Equation (20a)

$$\alpha = \alpha_0 x^{-\frac{1}{2}},$$

Equation (20b)

and

$$m \approx \frac{2\alpha_0^2}{1 + 5C^2},$$ 

Equation (21)

where $\alpha_0$ is a constant. The above result has the same scaling as that of a freefall collapse, as expected in the regime dominated by self-gravity. Equation 20(a) indicates the relation

$$\eta \sim \frac{1}{2C},$$

Equation (22)

showing enhanced turbulence toward smaller $x$ with $\eta/C < 1$. The above relations between $C$ and $\eta$ in both regimes with decelerated and accelerated infall, i.e., Equation (19) with $C = -1$ and Equation (22), are consistent with earlier numerical simulations of contracting turbulence (Robertson & Goldreich 2012).

In Case (1) with an initially small infall velocity, the numerical solution to Equations 6(b) and (c) in the entire range of $x$ is presented in Figure 1. Our analytical solutions well describe its asymptotic behaviors. From the envelope to the inner region, $R$ changes from (Equations 8 and 10(b))

$$R_{\text{the}} \sim \alpha x = A$$

Equation (23) to (Equations 9 and 20)

$$R_{\text{tur}} \sim \frac{\alpha x}{C^2 v} = \frac{1 + 5C^2}{2C^2}.$$ 

Equation (24)

Due to the turbulent support, $R$ remains constant, which would otherwise increase as $x \rightarrow 0$ in a freefall regime (Shu 1977). Comparing the cases with different values of $C$, we see that turbulence in the inner region results in a lower infall velocity, a higher $\rho$, and a lower constant rate of mass accretion onto a central mass point

$$M = \frac{\partial M(t, x)}{\partial \tau} = \frac{\alpha_1}{C_1^2 \tau} x^{-\frac{1 + 2C^2}{1 + 2C^2}}.$$ 

Equation (25)

compared to the freefall case.

In contrast, in Case (2) with an initially large infall velocity, the numerical result in the entire range of $x$ is displayed in Figure 2. We see three different regimes with (i) constant infall, (ii) decelerated infall, and (iii) accelerated infall. To better illustrate the asymptotic scalings in the dynamically unstable regimes (ii) and (iii), Figure 3 presents the solutions for $x < |v|$ with the boundary condition given by Equation (17). From regime (ii) to regime (iii), $R$ changes from (Equations 9 and 17)

$$R_{\text{tur}} \sim \frac{\alpha x}{C^2 v} = \frac{\alpha_1 x^{-\frac{1 + 2C^2}{1 + 2C^2}}}{C_1^2 \tau}.$$ 

Equation (26)

which increases with decreasing $x$, to $R_{\text{tur}}$ expressed in Equation (24). The change of $R_{\text{tur}}$ clearly indicates the transition from turbulent pressure- to gravity-dominated dynamics. Besides, from Figures 2(ii) and 3(c), we find that the change of $m$ in regimes (ii) and (iii) is insignificant and it approaches constant toward a small $x$, leading to a flat radial profile of $M$ and a constant $\dot{M}$ as in Case (1).

The above results clearly demonstrate the importance of gravity-driven turbulence in affecting the collapse dynamics. Compared with the isothermal collapse with only thermal pressure, turbulence provides additional pressure support against the self-gravity and enables deceleration of the infall. Compared with the adiabatic collapse where the released gravitational energy is absorbed, due to the dissipation of turbulent energy, the gravity-driven turbulence is incapable of halting the gravitational contraction. At a sufficiently small $x$, the solution has the freefall scaling. Figure 4 shows the
numerically solved $-u(r, t)$, the number density of atomic hydrogen $n_H(r, t) = \rho(r, t)/m_H$, where $m_H$ is the mass of hydrogen atom, and $M(r, t)$. As a comparison, we present both cases of a non-turbulent collapse with a quasi-static envelope (Figures 4(a), (c), and (e), corresponding to Figure 1(a) with $C = 0$) and a turbulent collapse with an initially large infall velocity (Figures 4(b), (d), and (f), corresponding to Figures 2(a) and (b) with $C = -1$). Note that we adopt the values of parameters here only for illustrative purposes, but not for detailed comparisons with specific observations. In the former case, we can easily see that the outward moving expansion wavefront separates the freefall regime and the quasi-static regime. While in the latter case the collapse exhibits a more complex behavior. Comparing the two scenarios, despite the different initial conditions and different levels of turbulence, the same scalings of velocity, density, and mass apply to the central region after a sufficiently long time, showing the dominance of self-gravity at the center.

3. Discussion

In the presence of turbulence, the stochastic wandering of magnetic field lines naturally takes place as a result of turbulent energy cascade and turbulent mixing of magnetic fields. Consequently, the reconnection of wandering magnetic fields are much more efficient than the microscopic magnetic reconnection (Lazarian & Vishniac 1999). The latter relies on the resistive diffusion in a conducting fluid or the ambipolar diffusion in a partially ionized medium. On length scales where turbulence exists, it is the turbulent diffusion of magnetic fields that dominates over the above microscopic diffusion processes. The turbulent reconnection of magnetic fields violates flux freezing and allows turbulent diffusion of magnetic fields, which has been termed “reconnection diffusion (RD)” (Lazarian 2005). The diffusion rate only depends on the turbulence properties (Kowal et al. 2009).

To illustrate the effect of RD induced by gravity-driven turbulence, in Figure 5, we present the evolution of magnetic field profile in the decelerated infall regime in Case (2) (see Appendix for the detailed calculations). We see that the RD rapidly balances the gravitational drag and stabilizes the magnetic field profile to have the form consistent with
Equation (32),

\[ B_c(r) = 10 \mu G \left( \frac{r}{0.1 \text{ pc}} \right)^{-1}. \]  

(27)

It suggests that the RD results in an efficient expulsion of magnetic fields and prevents the accumulation of magnetic flux in a collapsing region.

The application of RD to star formation processes (Santos-Lima et al. 2010; Lazarian et al. 2012; Leão et al. 2013; Lazarian 2014; Li et al. 2015; Mocz et al. 2017) demonstrates that RD leads to violation of flux freezing (Eyink et al. 2013) and is indispensable for solving the “magnetic flux problem” (Mestel & Spitzer 1956), accounting for the observed super-critical molecular clouds and cores (Crutcher et al. 2010) and the observed strengths of surface magnetic fields of stars (Johns-Krull et al. 2004). RD also mitigates the magnetic braking “catastrophe” and allows the formation of centrifugally supported circumstellar disks (Santos-Lima et al. 2012, 2013; González-Casanova et al. 2016; see also Gray et al. 2018). Most earlier studies on RD involved externally driven turbulence, e.g., the interstellar turbulence driven by supernova explosions on large scales.³ Differently, here we find that RD naturally occurs during the gravitational collapse without an external source for driving turbulence.

In a weakly ionized and magnetized core, besides RD, the ambipolar diffusion (AD) of magnetic fields due to ion–neutral drift also takes place. The comparison between the rates of AD and RD shows

\[ \frac{\omega_d}{\nu_t / r} = \frac{\xi_n V_A^2}{6\nu_{ni} \nu_t r} = \frac{\xi_p \nu_t}{6\nu_{ni} r} \]

= \[3.2 \times 10^{-4} \left( \frac{v_t}{0.1 \text{ km s}^{-1}} \right) \left( \frac{r}{0.1 \text{ pc}} \right)^{-1} \]

\[ \left( \frac{n_H}{10^4 \text{ cm}^{-3}} \right)^{-1} \left( \frac{n_e/n_H}{10^{-5}} \right)^{-1}. \]  

(28)

³ In most MHD simulations of the interstellar turbulence, turbulence is continuously forced at a large driving scale to simulate the externally driven turbulence for a system on scales smaller than the driving scale of turbulence. In the simulations with decaying turbulence, additional turbulence can be internally driven after the gravitational contraction of the system initiates.
where $\xi_n = \rho_n / \rho$ is the neutral fraction with the neutral mass density $\rho_n$ and the total mass density $\rho$, $\nu_{ri} = \gamma_d \rho_i$ is the neutral–ion collision frequency with the drag coefficient $\gamma_d = 3.5 \times 10^{13} \text{cm}^3 \text{g}^{-1} \text{s}^{-1}$ (Draine et al. 1983; Shu 1992) and the ion mass density $\rho_i$, and $n_i$ and $n_n$ are number densities of the atomic hydrogen and electrons. Here we also assume $\nu_i = V_A$, where $V_A$ is the Alfvén speed, and $m_i = 2.3 m_{\text{HI}}$ as the mean molecular mass of ions and neutrals in a core (Shu 1992), where $m_{\text{HI}}$ is the hydrogen atomic mass. Evidently, in the presence of turbulence, AD is subdominant compared to RD.

### 4. Conclusions

The effect of turbulence on gas dynamics varies with the length scale of interest. For the interstellar turbulence with a driving scale $\sim 50$–$100$ pc, shear Alfvénic motions and compressive motions in supersonic turbulence play an important role in shaping the density structures within the inertial range of the interstellar turbulence (Padoan et al. 2001; Federrath et al. 2010; Xu & Zhang 2016, 2017; Mocz & Burkhart 2018; Robertson & Goldreich 2018). For the gravity-driven turbulence in a contracting core considered here, the driving scale is small, and thus the internal turbulent motions provide pressure support for the surrounding density shells. We found that the gravity-driven turbulence can slow down the gravitational infall and mass accretion.

Compared with the Kolmogorov scaling $v_i \propto r^{1/3}$ (e.g., Qian et al. 2018) or the Larson’s scaling $v_i \propto r^{1/2}$ (Larson 1981; Myers 1983) in the inertial range of externally driven turbulence, we found that the gravity-driven turbulence can give rise to various velocity dispersion profiles, $v_i \propto r^{\alpha}$ with $0 \leq \alpha \leq 1$ in the outer region of a dynamically contracting core at an early time of its evolution, and $v_i \propto r^{-1/2}$ toward the center in a quasi-statically contracting core or a dynamically contracting core at a late time of its evolution (Figure 4). Our analytical scalings are consistent with earlier numerical results in the parameter space of the simulations (Robertson & Goldreich 2012). Observations suggest that the non-thermal line width-size relation of massive cores is flatter than that of low-mass cores (Caselli & Myers 1995). Plume et al. (1997) found no statistically significant line width-size relationship or
a positive correlation between line width and density for very massive cores. Observations by Traficante et al. (2018a, 2018b, 2020) indicate non-thermal motions driven by gravity in massive star formation. These unexpected observational findings support the theoretical picture of gravity-driven turbulence in molecular cloud cores (see also Murray & Chang 2015). More detailed and quantitative comparisons will be carried out in our future work.

As an important implication of the current study, the gravity-driven turbulence not only influences the dynamics of a collapsing core, but also enables an efficient diffusion of magnetic fields. At the balance between the gravitational drag...
and diffusion, a stationary radial profile of magnetic field can be reached, with the slope depending on the fraction of gravitational potential energy converted to the turbulent kinetic energy.

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Equation 17

\[ B_i(r) = B_f \left( \frac{r}{r_f} \right)^{1/2}, \]

where \( B_i(r) \) is the field strength at a reference radius \( r_f \). We see that \( B_i(r) \) does not depend on the functional form of \( u \) as it is canceled out in Equation (31), but only depends on \( C \).

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