ON THE EQUIVARIANT FORMAL GROUP LAW OF THE EQUIVARIANT COMPLEX COBORDISM RING

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ABSTRACT. For a finite abelian group $G$, we compute the $G$-equivariant formal group law corresponding to the $G$-equivariant complex cobordism spectrum with its canonical complex orientation.

1. INTRODUCTION

The goal of the present paper is to provide an algebraic description of the equivariant formal group law corresponding to the equivariant complex cobordism ring $MU_G$ for a finite abelian group $G$, which is given as Theorem 4.1 and is adapted from the author’s doctoral thesis [1]. The paper [2] of the author and Kriz provides the corresponding algebraic computation of the coefficient ring $(MU_G)_*$, which facilitates this work. As in [2], our description is in terms of pullback diagrams. The computation of [2] follows several other papers which contribute to the present algebraic understanding of the ring $(MU_G)_*$, namely the computations of Sinha [10], Strickland [11], and Kriz [6].

One motivation for pursuing this computation is the conjecture of Greenlees [4, Conjecture 2.4] that the coefficient ring of the equivariant complex cobordism ring classifies equivariant formal group laws in the same way that non-equivariant complex cobordism classifies non-equivariant formal group laws, vis à vis Quillen’s Theorem [8] (c.f.e. [9, Theorem 1.3.2]). This is vague, but will be made precise in Section 3 when we define the notion of an equivariant formal group law. Greenlees [4] shows that the equivariant complex cobordism ring classifies equivariant formal group laws over Noetherian rings, but the general conjecture is still open. The algebraic description of $(MU_G)_*$ in [2] and the corresponding description here of its equivariant formal group law are intended to partially illuminate the questions posed by the paper [4] of Greenlees.

Section 2 below recalls the computation of $(MU_G)_*$ given in [2]. Section 3 defines equivariant formal group laws and provides the relevant background. Section 4 provides the promised computation of the equivariant formal group law of $MU_G$ for $G$ a finite abelian group. The main result of this paper is Theorem 4.1. To illuminate the description, Section 5 treats the example case $MU_{\mathbb{Z}/p^n}$ of finite cyclic $p$-groups.

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2. The Equivariant Complex Cobordism Ring of a Finite Abelian Group

Throughout this paper, fix a finite abelian group $G$. We recall now the computation of $(MU_G)_*$ given in [2], which is given in that paper as the combination of Theorem 1 and Theorem 2. First we need some definitions. Let

$$(2.1) \quad P(G) = \{ \{H_1 \subseteq H_2 \subseteq \cdots \subseteq H_k\} | H_i \subseteq G \text{ is a subgroup for every } i \}.$$ 

That is, elements of $P(G)$ are increasing chains of subgroups of $G$. Now suppose $S = \{H_1 \subseteq H_2 \subseteq \cdots \subseteq H_k\} \in P(G)$, and let $H_0 = \{e\}$, $H_{k+1} = G$. For $0 \leq j \leq k$, let $R_j$ be $G/H_j$-representatives of the nontrivial complex $H_{j+1}/H_j$ representations. We associate to the chain of subgroups $S$ a ring

$${\text{AS}} = MU_*[u_L,u_M^{-1},u_N^{(i)}| i > 0, L \in R_0 \prod \cdots \prod R_k, M \in R_0 \prod \cdots \prod R_{k-1}, N \in R_0],$$

where of course $MU_*$ is the nonequivariant complex cobordism ring, whose algebraic structure was computed by Milnor [7].

We can topologize $AS$ as follows. Say that a sequence of monomials from $AS$

$$\left\langle a_t, L \in R_t, \prod \cdots \prod R_k \right| u_L^{n(L,t)} \right\rangle_{t=1}^\infty$$

with $0 \neq a_t \in MU_*[u_L^{\pm 1}, u_L^{(i)}| i > 0, L \in R_0]$ converges to 0 if there is a $j \in \{1, \ldots, k\}$ such that $n(L,t)$ is eventually constant in $t$ when $L \in R_t$ and $i > j$, and

$$n(L,t) \to \infty \text{ as } t \to \infty$$

for $L \in R_j$. A sequence $\langle p_i \rangle$ of polynomials from $AS$ converges to 0 if and only if every choice of nonzero monomial summands $m_t$ from $p_t$ gives a sequence of monomials $\langle m_t \rangle$ that converges to 0. We can now define a topology $T_S$ on $AS$ by saying that $C \subseteq AS$ is closed with respect to $T_S$ if and only if the limit of every sequence in $C$ convergent in $AS$ is in $C$.

We will consider the completion $(AS)_{T_S}$ of the ring $AS$ with respect to the topology $T_S$. Let $F$ denote the universal formal group law. We define an ideal by

$${IS} = \left( u_{L_1} + F u_{L_2} - \left( \sum_{i=1}^m \right) \left| L_1 L_2 \cong \prod M_i \text{ and there is a } j \in \{1, \ldots, k\} \right. \right.$$

\hspace{1cm} \text{s.t. } \left. L_1, L_2 \in R_j \text{ and } M_i \in R_j \prod \cdots \prod R_k \right).$

Note that the definitions of $AS$, $T_S$, and $IS$ above depend on the group $G$, so when necessary for clarity we will denote these by $A_{G,S}$, $T_{G,S}$ and $I_{G,S}$, respectively.

Theorem 2.1. (Abram and Kriz) For $G$ a finite abelian group, we have

$$(MU_G)_* \cong \lim_{S} (AS)_{T_S} / IS.$$ 

This computation is illuminated somewhat by the following corollary of Theorem 2.1 and the exposition of [2], which corollary appears in [11]. The notation $[k]_{F,x}$ denotes the $k$-fold sum $x + F x + F^{2} x + \cdots + F^{k-1} x$ of $x$ under the nonequivariant universal formal group law.
Corollary 2.2. Let \( u_{[k]} \) denote \( [p^k]_{F_u} \) and
\[
R_k = MU_*[u_j, u_j^{-1}, b_j^{(i)} | i > 0, j \in \{1, 2, \ldots, p^k - 1\}]/([u_{[k]}])/([p^{k-1}]_{F_u_{[k]}},
\]
\[
S_k = MU_*[u_j, u_j^{-1}, b_j^{(i)} | i > 0, j \in \{1, 2, \ldots, p^k - 1\}]/([u_{[k]}])/([p^{k-1}]_{F_u_{[k]}}, [u_{[k]}]^{-1}),
\]
\[
R^n = MU_*[u_j, u_j^{-1}, b_j^{(i)} | i > 0, j \in \{1, 2, \ldots, p^n - 1\}].
\]
Then \( (MU_{Z/p^n})_* \) is the \( n \)-fold pullback of the diagram of rings
(2.2)

\[
\begin{array}{c}
R^n \\
\downarrow \phi_{n-1} \\
R_{n-1} \\
\downarrow \phi_{n-2} \\
\vdots \\
\downarrow \phi_1 \\
R_2 \\
\downarrow \phi_0 \\
R_1 \\
\downarrow \phi_0 \\
R_0 \\
\end{array}
\]

The maps \( \psi_k \) are localization by inverting \( u_{[k]} \), and the maps \( \phi_k \) are determined by the properties of sending \( u_{[k+1]} \) to \( [p]_{F_u_{[k]}} \) and \( b_j^{(i)} u_j \) to the coefficient of \( x^i \) in \( x + F [j]_{F_u_{[k]}} \).

3. EQUIVARIANT FORMAL GROUP LAWS

Let \( G \) be a finite abelian group, and \( E \in GSL \) a \( G \)-equivariant spectrum. Let \( \hat{G} \) denote the character group of \( G \). Cole, Greenlees, and Kriz \[5\] makes the following definition.

Definition 3.1. A \( G \)-equivariant formal group law over a commutative ring \( k \) consists of:

(1) A \( k \)-algebra \( R \) complete at an ideal \( I \) and a cocommutative, coassociative, counital comultiplication

\[
\Delta : R \to R \hat{\otimes} R \quad (R \hat{\otimes} R = (R \otimes R)_{I \otimes R \otimes R \otimes I}),
\]

(2) An \( I \)-continuous map of \( k \)-algebras \( \epsilon : R \to k^G \) compatible with comultiplication:

\[
\begin{array}{c}
R \\
\downarrow \Delta \\
R \hat{\otimes} R \\
\downarrow \epsilon \hat{\otimes} \epsilon \\
k^G \otimes k^G \otimes k^G
\end{array}
\]
(3) A system of elements $x_L \in R$, $L \in \hat{G}$ such that

$$x_L \text{ is regular for each } L \in \hat{G},$$

$$R/(x_L) \cong E_\ast \text{ for each } L \in \hat{G},$$

$$I = \left( \prod_{L \in \hat{G}} x_L \right),$$

and

$$x_L = (\epsilon(L) \otimes 1) \Delta(x_1) \text{ for } L \in \hat{G}.$$ 

One can adapt this definition to the case where $G$ is a compact Lie group by replacing the ideal $I$ above with the system of finite product ideals $(\prod_i x_{L_i})$ (cf. [3]). Greenlees [4] makes the following conjecture.

**Conjecture 3.2.** For any complex oriented $G$-equivariant spectrum $E$ there is a unique homomorphism of rings $\theta : MU_G^\ast \rightarrow E^\ast$ such that $\theta$ induces maps that send the structures (1), (2), and (3) for the canonical equivariant formal group law corresponding to $MU_G$ to the corresponding structure for $E$.

In [3, Example 11.3(i)] it is shown that a complex orientation on a $G$-equivariant spectrum $E$ over the complete universe $U$ specifies a $G$-equivariant formal group law with $k = E^\ast$ and $R = E^\ast \mathbb{C}P_G^\infty$. Here $\mathbb{C}P_G^\infty$ is the complex projective space on the complete $G$-universe. Our goal is to follow the construction of [3] to compute the equivariant formal group law corresponding to the equivariant complex cobordism spectrum $MU_G$, for the case where $G$ is a finite abelian group. We can do this because $MU_G^\ast$ has a canonical complex orientation. Our goal at present is to describe the equivariant formal group law corresponding to $MU_G$ for $G$ a finite abelian group, with an eye toward Greenlees’ Conjecture 3.2.

4. **The Equivariant Formal Group Law of $MU_G$**

Our description of the equivariant formal group law of $MU_G$ is a direct consequence of Theorem 2.1 and Definition 3.1 and the description is similar in nature to that of Theorem 2.1. Before we can state our main theorem, we need a few definitions. For $S = \{H_1 \subseteq H_2 \subseteq \cdots \subseteq H_k \} \in P(G)$, $H_0 = \{e\}$, and $H_{k+1} = G$, define

$$Q_j = \prod_{L \in H_j} (A_{H_j, S})_{\mathcal{T}H_j, S}/[u_L | L \in R_j]/(u_L + F u_M = u_{LM})[[x_L]]$$

$$T_j = \prod_{L \in H_{j+1}} (A_{H_j, S})_{\mathcal{T}H_j, S}/[u_L | L \in R_j]/(u_L + F u_M = u_{LM})[[x_L]].$$
$Q^k$ is defined similarly, also using the computations in the proof of [2 Theorem 2]. Here $A^* = \text{Hom}(A, S^1)$ and $\mathcal{A} = A - \{0\}$. We then define $\mathcal{N}_S$ to be the diagram

\[
\begin{array}{c}
\xymatrix{ 
& Q^k 
\ar[d]_{\phi_{k-1}} \\
Q_{n-1} \ar[r]_{\psi_{k-1}} & T_{k-1} \\
\vdots & \\
Q_2 \ar[r]_{\psi_2} & T_{k-2} \\
Q_1 \ar[r]_{\psi_1} & T_1 \\
Q_0 \ar[r]_{\psi_0} & T_0, 
}
\end{array}
\]

where the horizontal maps are given by localization by inverting Euler classes and the condition

\[
x_L \mapsto \prod_{M \equiv L \pmod{H_j}} x_M + F(u_L - u_M),
\]

and the vertical maps are determined by sending $u_L^{(i)}$ to the coefficient of $x^i$ in $x + F u_L$ and by sending

\[
x_L \mapsto x_L.
\]

Define another diagram $\tilde{\mathcal{N}}_S$ as

\[
\begin{array}{c}
\xymatrix{ 
& \tilde{Q}^k 
\ar[d]_{\tilde{\phi}_{k-1}} \\
\tilde{Q}_{n-1} \ar[r]_{\tilde{\psi}_{k-1}} & \tilde{T}_{k-1} \\
\vdots & \\
\tilde{Q}_2 \ar[r]_{\tilde{\psi}_2} & \tilde{T}_{k-2} \\
\tilde{Q}_1 \ar[r]_{\tilde{\psi}_1} & \tilde{T}_1 \\
\tilde{Q}_0 \ar[r]_{\tilde{\psi}_0} & \tilde{T}_0, 
}
\end{array}
\]
where
\[
\bar{Q}_j = \prod_{L \in \mathcal{H}_j} (A_{H_j}S)[T_{H_j,S}/I_{H_j,S}[[u_L|L \in R_j]]/(u_L + F u_M = u_{LM})]\]
\[
\tilde{T}_j = \prod_{L \in \mathcal{H}_{j+1}} (A_{H_j}S)[T_{H_j,S}/I_{H_j,S}[[u_L|L \in R_j]]/(u_L + F u_M = u_{LM})]\]
and \(\bar{Q}^k\) is defined similarly. The maps \(\bar{\phi}_j\) and \(\bar{\psi}_j\) are defined as were \(\phi_j\), and \(\psi_j\), with the additional conditions that
\[
\bar{\psi}_j(y_L) = \prod_{M \equiv L (\text{mod } H_j)} y_M + F (u_L - u_M) \tag{4.5}
\]
and
\[
\bar{\phi}_j(y_L) = y_L \tag{4.6}
\]
Let \(\mathbb{CP}_G^\infty = \mathbb{CP}(U)\) denote the complex projective space on the complete \(G\)-universe. We are now ready to state our main theorem.

**Theorem 4.1.** The equivariant formal group law of \(MU_G\) consists of the following structures:

(a) the commutative ring \(k = MU^*_G\), whose algebraic description is given by Theorem 2.1
(b) the \(k\)-algebra \(R = \text{ho lim } NS\);
(c) the ideal \(I = (\prod_{L \in \mathcal{G}} x_L)\);
(d) the \(k\)-algebra \(R \hat{\otimes} R = \text{ho lim } \tilde{NS}\);
(e) the coproduct \(\Delta : R \to R \hat{\otimes} R\) is determined by maps \(\Delta_S : NS \to \tilde{NS}\) of diagrams, which send \(Q_j \to \bar{Q}_j, T_j \to \tilde{T}_j\), and are determined by the identity maps away from power series variables and by the conditions
\[
x_L \mapsto \prod_{M \equiv L (\text{mod } H_j)} (x_M + F y_N); \tag{4.7}
\]
(f) the map \(\epsilon : R \to (MU^*_G)^G\) is defined by choosing a basepoint \(*_L\) in each connected component of
\[
\mathbb{CP}_G^\infty = \bigoplus_{L \in \mathcal{G}} \mathbb{CP}^\infty,
\]
where the superscript \(G\) denotes fixed points. \(\epsilon\) is the induced map in cohomology of the \(G\)-equivariant map
\[
\prod_{L \in \mathcal{G}} *_L \to G.
\]

**Proof.** This is mostly a matter of chasing definitions, and much of the discussion is accomplished merely by reconciling [3] with Theorem 2.1. Since \(MU^*_G\) is complex stable and complex oriented, we can take \(k = MU^*_G\) and
\[
R = MU^*_G \mathbb{CP}_G^\infty. \tag{4.7}
\]
The elements \(x_L \in \tilde{MU}_G^2 T(\gamma_G \otimes L)\) are Thom classes, computed just as in [3 Section 4], where \(\gamma_G\) is the canonical line bundle on \(\mathbb{CP}_G^\infty\) and \(T\) denotes the Thom space. Let \(x_0 \in \tilde{MU}_G^2 T(\gamma_G)\) be the orientation class. Now let \(\phi : \mathbb{CP}_G^\infty \to \mathbb{CP}_G^\infty\) classify \(\gamma_G \otimes L\), i.e. \(\phi^*(\gamma_G) = \gamma_G \otimes L\). Then we define
\[
x_L = \text{Im}(\tilde{MU}_G^2 T(\gamma_G) \to \tilde{MU}_G^2 T \phi \tilde{MU}_G^2 T(\gamma_G \otimes L)). \tag{4.8}
\]
Let \( U = \bigotimes_{L \in \hat{G}} L \). The ideal \( I \) is

\[
(4.9) \quad I = \left( \prod_{L \in \hat{G}} x_L \right),
\]

where the product on the right is computed by the Thom diagonal

\[
\Delta_1 : T(\gamma_G \otimes U) \to \bigwedge_{L \in \hat{G}} T(\gamma_G \otimes L),
\]

as proposed in Theorem [4, Theorem 11.2].

It follows from the Splitting Theorem [3] Theorem 4.3] of Cole that \( R = MU_G^* \mathbb{C}P_G^\infty \) is complete at \( I \), and

\[
(4.10) \quad R \hat{\otimes} R \cong MU_G^*(\mathbb{C}P_G^\infty \times \mathbb{C}P_G^\infty).
\]

The comultiplication \( \Delta \) is induced by the map classifying the tensor multiplication of line bundles:

\[
\mu : \mathbb{C}P_G^\infty \times \mathbb{C}P_G^\infty \to \mathbb{C}P_G^\infty,
\]

i.e. for line bundles \( \epsilon = f^*\gamma_G, \omega = g^*\gamma_G, \epsilon \otimes \omega = (\mu(f \times g))^* \). Choosing basepoints \( u_1 \) in each connected component of \( (\mathbb{C}P_G^\infty)^G \), the description of the map \( \epsilon \) given as Theorem [4, Theorem 4.1(e)] follows from [3].

All of the above is documented in the note [5] of Kriz. Our goal now is to understand better the algebraic structure of the ring \( MU_G^* \mathbb{C}P_G^\infty \). By Theorem 4.3 of [3], we have

\[
(4.11) \quad MU_G^* \mathbb{C}P_G^\infty \cong MU_G^*\{\{x_0, x_{L_1}, x_{L_1L_2}, \ldots \}\},
\]

where \( L_1 \oplus L_2 \oplus \cdots \) is any splitting of the complete \( G \)-universe \( U \). Thus \( x_0, x_{L_1}, x_{L_1L_2}, \ldots \) are a flag basis of the complete universe \( U \), and \( MU_G^*\{\{x_0, x_{L_1}, x_{L_1L_2}, \ldots \}\} \) denotes

\[
(4.12) \quad \left\{ \sum_{i=0}^{\infty} a_i x_0 x_{L_1} \cdots x_{L_i} \mid a_i \in MU_G^* \right\}.
\]

We define

\[
(4.13) \quad x_{L_1} \oplus x_{L_2} \oplus \cdots x_m = \prod x_{L_i},
\]

and now the right hand side of (4.11) is well-defined.

Now the splitting map \( MU \to MU_G \) induces an isomorphism

\[
(4.14) \quad \pi^G_*(F(EG_+, MU)) \cong \pi^G_*(F(EG_+, MU_G)),
\]

and it follows that

\[
(4.15) \quad \pi^G_*(F(EG_+, MU_G)^* \mathbb{C}P_G^\infty) \cong MU_*\{[u_L|L \in \widehat{G^G}] / (u_L + F u_M = u_{LM})[[x]]\}.
\]

We are now able to give a better description of the elements \( x_L \). Clearly,

\[
(4.16) \quad x_0 = x \in MU_*\{[u_L|L \in \widehat{G^G}] / (u_L + F u_M = u_{LM})[[x]]\},
\]

while

\[
(4.17) \quad x_L = x_0 + F u_L.
\]

Greenlees [4, Theorem 11.2] gives

\[
(4.18) \quad \Phi^G MU_G^* \mathbb{C}P_G^\infty \cong \prod_{L \in \hat{G}} \Phi^G MU_G^*[x_L] = \prod_{L \in \hat{G}} \Phi^G MU_G^*[x + F u_L].
\]

Now the description of \( R \) given as Theorem 4.1(a) is a formal consequence of the above. Theorem 4.1(b) concerning \( R \hat{\otimes} R \) is obtained by a direct computation, and the description
of the coproduct $\Delta$ in Theorem 4.1(c) follows from the definition of $\Delta$ above and from the work of [2] as summarized in Section 2. Conditions (1)-(3) of Definition 3.1 are guaranteed by [3] and [5], since the definitions of the algebras, maps, and elements of the equivariant formal group law given here are direct consequences of the example of complex stable, complex oriented cohomology theories and equivariant formal group laws as given in those papers.

\[ \square \]

5. The Case $G = \mathbb{Z}/p^n$

There is intricate structure hiding beneath the surface of our description of the equivariant formal group law for $MU_G$ in the previous section. To illuminate some of this hidden structure, we present here the description of $MU_{\mathbb{Z}/p^n}$ as given by Theorem 4.1.

We give a useful description of the elements $x_j$ arising from the diagram (2.2). Of course $x_0 = x \in MU_*[[u]]/(p^n u)[[x]]$, and $x_j = x_0 + F[j] u$. Let $R_k, S_k, 0 \leq k \leq n - 1$, and $R^n$ be as in Corollary 2.2 and refer to that result for notation. Then the element $u_j b_j^{(i)}$ of $R^n$ maps to an element of $S_{n-1}$ that does not include the term $u_{n-1}^{[n-1]}$, so this element really lives in $R_{n-1}$. For $0 < k < n$, the resulting element of $R_k$ maps to an element of $S_{k-1}$ which does not include the term $u_{k-1}^{[k-1]}$, so it really lives in $R_{k-1}$.

This allows us to map the elements $u_j b_j^{(i)}$ of $R^n$ to $R_0 = MU_*[[u]]/(p^n u)$; call this map $\phi$. Then there is an implied map $\phi : MU_*[u_j b_j^{(i)}][i > 0, 1 \leq j \leq p^n - 1][[x]] \to MU_*[[u]]/(p^n u)[[x]]$. Since $u_j b_j^{(i)}$ maps to the coefficient of $x^i$ in $x + F[j] u$, $x_j$ is the image under $\phi$ of the element

\[
\sum_{i=0}^{\infty} u_j b_j^{(i)} x^i.
\]

We would also like a nice description of the $MU_{\mathbb{Z}/p^n}^*$-algebra

\[ R = MU_{\mathbb{Z}/p^n}^* (\mathbb{C}P_{\mathbb{Z}/p^n}^\infty) = MU_{\mathbb{Z}/p^n}^* \{ \mathcal{U} \} \]

as a product. Greenlees [4, Theorem 11.2] gives us the following:

\[ \Phi^{\mathbb{Z}/p^n} MU_{\mathbb{Z}/p^n}^* \{ \mathcal{U} \} = \prod_{j=0}^{p^n-1} \Phi^{\mathbb{Z}/p^n} MU_{\mathbb{Z}/p^n}^*[[x_j]] = \prod_{j=0}^{p^n-1} \Phi^{\mathbb{Z}/p^n} MU_{\mathbb{Z}/p^n}^*[[x + F[j] u]]. \]

Moreover, we obtain $R$ as an $n$-fold pullback, using Corollary 2.2. The various powers of the Euler class which are invertible on the diagram (2.2) allow for certain product decompositions of the ring $R = MU_{\mathbb{Z}/p^n}^* (\mathbb{C}P_{\mathbb{Z}/p^n}^\infty)$. Let $R^n, S_k, R_k$ stand for the cohomology rings now, rather than homology. Then $R$ is the pullback of the following diagram of rings:
The horizontal maps, as implied, are induced by the maps $\psi_k$ and the condition $x_j \mapsto \prod_{r \equiv j \pmod{p^k}} (x_r + F(j-r)F_u[k])$. The vertical maps are induced by the maps $\phi_k$ and the condition $x_j \mapsto x_j$ for all $j$.

There is a similar description of $R \otimes R$ as a pullback:

The maps are determined by the maps of (5.2) and the corresponding conditions for $y_r$. Namely, under the horizontal maps, $y_r \mapsto \prod_{s \equiv r \pmod{p^k}} (y_s + F(r-s)F_u[k])$. Under the vertical maps, $y_r \mapsto y_r$.

We now specify the coproduct $\Delta : R \rightarrow R \otimes R$ on the terms of the diagrams (5.2) and (5.3). The map $\prod_{k \in (\mathbb{Z}/p^n)^*} R_j[[x_k]] \rightarrow \prod_{k,r \in (\mathbb{Z}/p^n)^*} R_j[[x_k, y_r]]$ is determined by the identity map on $R_j$ and the condition $x_k \mapsto \prod_{k_1 + k_2 = k}(x_{k_1} + F y_{k_2})$, where $k_1 + k_2 = k$ we of course mean $k_1 + k_2 \equiv k \pmod{p^j}$. The map on the top right of the diagrams is defined similarly. The map

\[ \prod_{k \in (\mathbb{Z}/p^n)^*} S_{j-1}[[x_k]] \rightarrow \prod_{k,r \in (\mathbb{Z}/p^n)^*} S_{j-1}[[x_k, y_r]] \]
is determined by the identity map on $S_{j-1}$ and the condition
\[ x_k \mapsto \prod_{k_1+k_2=k} x_{k_1} + x_{k_2}. \]

Having nothing to add to the description of the map $\epsilon$ as given for general finite abelian groups $G$, this completes our description of the equivariant formal group law corresponding to $MU_{\mathbb{Z}/p^n}$.

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