The resources vertical aggregation model in the construction and installation works planning

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Abstract. The exchange of information between the global and local tasks occurs in the aggregated nomenclature, which reduces the bandwidth requirements of inter-layer communication channels. In the horizontal aggregation scheme, a global task is formed on the basis of non-aggregated information of local systems. The resources vertical aggregation model was obtained in the construction production planning, which differs by taking into account the restrictions on the number of teams used for the works functional dependencies.

Introduction
One of the main tasks of resource management at the construction organization level is the task of resource allocation. The minimum resources consuming technological unit is work. In the classical formulation of the distribution problem, each production method is characterized by the resource expenditure volumes with a unit intensity of the method, and the total costs of the i-th resource \( a_i \) are defined as:

\[
a_i = a_{is} \cdot x_{is}, i \in I_s,
\]

where:
- \( I_s \) – is plenty of resources;
- \( a_{is} \) – is the amount of costs of the i-th resource at a unit intensity \( s \)- production method;
- \( x_{is} \) – is the required intensity of the s-th production method.

We assume that the technological structure of the consumption of the i-th resource is discrete in nature and is described by a combination of technologies for resource consumption (T). Each technology, in turn, consists in the use of a set of methods of resource consumption (C). The way of spending the resource \( j \) is the terminal concept of the model and is characterized by the volume of expenses \( d_j \) resource at a single intensity of the method, and the total consumption of the resource within the s-th production method is defined as:

\[
a_i = \sum_{j \in I_s} d_j x_j, i \in I_s
\]

(2)
where: $J_i$ – defines many ways to expend i-th resource; $x_j^\phi$ – is the desired intensity of the j-th method of resource consumption.

**Materials and methods**

A universal measure of the intensity of methods for different resources is the time interval for using the method. In addition to this, let’s call it production, the intensity of the method introduces financial intensity $\Phi_j$, the cost expression of resource costs by the j-th method. Production and financial intensity of the method associated with the condition of its belonging to the r-th technology of resource use [1,2]. Thus, the intensity of the production method is represented as a spending resources ways intensities vector:

$$x_S = \{(x_j, x_j^\phi) \mid j \in J_i, i \in I_s\},$$

(3)

the components of which are related by the production stabilization relations. All the relationships are divided into intra-technological:

$$f_k(x_j, x_j^\phi) \leq 0, j \in J_r, r \in R_s,$$

(4)

Inter-technological:

$$F_k(\sum_{j \in J_i} x_j, \sum_{j \in J_i} x_j^\phi) \leq 0, r \in R_s,$$

(5)

and inter-resource types:

$$\Phi_k(\sum_{j \in J_i} x_j, \sum_{j \in J_i} x_j^\phi) \leq 0, i \in I_s.$$

(6)

The structure of PS and relations (4) - (6) will be called the model of a non-rigid production method. A system based on non-rigid PS will be called a non-rigid production system. Software resources are the frames (I’), as well as n passive resources. For the “frames” resource, where the method corresponds to the one position performer use [3], the relations of the type (4) are as follows:

$$\omega_r x_j - x_j^\phi \leq 0, j \in J_r, r \in R_s, s \in S;$$
$$x_j^\phi - \overline{\omega}_r x_j \leq 0, j \in J_r, r \in R_s, s \in S;$$
$$\sum_{j \in J_i} \left( x_j^\phi - \frac{\overline{\omega}_j - \omega_j}{2} \cdot x_j \right) = 0, r \in R_s, i \in I'_s, s \in S,$$

where \(\overline{\omega}_j, \omega_j\) - are the upper and lower limits of official salaries, and the last condition determines the limit on the average wage.

For passive resources, the relations like (4) can take the form of restrictions on the average cost of the costs per unit time for this technology methods:

$$\omega_r \sum_{j \in J_r} x_j - \sum_{j \in J_r} x_j^\phi \leq 0, r \in R_s, s \in S;$$
$$\sum_{j \in J_r} x_j^\phi - \overline{\omega}_r \sum_{j \in J_r} x_j \leq 0, r \in R_s, s \in S.$$
\[
B_r \sum_{j=0}^{r} \sum_{j=0}^{r} x_j - (1 - B_r) \sum_{j=0}^{r} x_j \leq 0, r \in R, i \in I_s, s \in S
\]

where: \( B_r \) - is a standard minimum level. Denote for further \( B^- = B_r, B^* = (1 - B_r) \).

Finally, the ratio of type (6) can define the various resources use proportions within the production method framework and for the construction organization has the following form:

\[
B^S \sum_{i=0}^{I_s} \sum_{j=0}^{I_s} x^S_j - (1 - B^S) \sum_{j=0}^{I_s} x^S_j = 0, i' \in I', s \in S
\]

\[
\sum_{i=0}^{I_s} b'_i \sum_{j=0}^{I_s} x_j - (1 - B^-) \sum_{j=0}^{I_s} x_j \leq 0, i' \in I', s \in S
\]

where: \( b'_i \) - is a specific rate of personnel time spent on servicing the \( i \)-th resource; \( B^- \) - is the share of the wage fund of the total resource value of the \( s \)-th production method; \( B^S \) - is the norm of the minimum amount of time allocated to the theoretical work of personnel \( s \)-th production method.

Denote: \( B^S = B^S, B^{S+} = 1 - B^S, B^S = 1 - B_S \).

Let be \( c^j \) - the specific efficiency of the \( j \)-th mode of resource consumption. Let us consider the two cases within the resource allocation model.

Case 1. Resource-independent production methods.

Let \( A \) be the general financial fund of a construction company. Then the problem of optimal resource allocation is to maximize the linear criterion:

\[
\sum_{j=0}^{S} \sum_{j=0}^{I_s} c^j x_j \rightarrow \text{max}
\]

(7)

given the restrictions on the production stabilization of the type (4) - (6), and the restriction on the general financial fund:

\[
\sum_{j=0}^{S} \sum_{j=0}^{I_s} x^S_j \leq A
\]

(8)

Case 2. Resource-dependent production methods.

Case 1 correlations add resource constraints on the aggregated resource:

\[
\sum_{q=0}^{Q} \sum_{i=0}^{I_q} x_j \leq x_v, v \in N, I_q \cap I_s = i_q
\]

(9)

where: \( Q \) and \( V \) - are the indexes of intermediate and final aggregated resource; \( x_v \) - limit \( V \)-th resource; restrictions on the resource consumption aggregated method:
\[ \sum_{p \in P_f, j \in J_p} x_j \leq T_f, f \in F, J_p \cap J_s = j_{ps} \]  

(10)

where: \( P \) and \( f \) - are the indexes of intermediate and final aggregated method; 
\( F \) - defines many aggregated methods; 
\( f \) - is the total time limit for using the \( f \)-th aggregated method.

For the construction organizations, a hierarchical production structure is characteristic, which should be taken into account when solving the problem of resource allocation. In this case, the initial mathematical model is divided into detailed models of local systems and an aggregated model of the global system (we confine ourselves to the case of a two-level hierarchy). After that, the solution to the original problem is sought by interconnecting the conditionally optimal solutions of individual models [4].

The iterative aggregation methods developed in recent years [5, 6] suggest that the global system model is built by aggregating models of local systems in accordance with the conditionally optimal solutions obtained for them. On the other hand, the aggregated problem solution is used in local systems in finding its own solutions.

A scheme for solving the resource allocation problem using the methods of iterative aggregation should, on the one hand, ensure the convergence of the solution to the optimal one, and on the other hand, be meaningfully adequate to the hierarchical system under consideration. The concept of meaningful adequacy includes the actual existence of selected local systems and components of the aggregated model, as well as the practical feasibility of local and global criteria [8, 9].

The original problem structure allows to select the \( S \)-th production method as a local system. Let us suppose there are resource-independent production methods defined by the relations (4, 5, 6) [12]. Then the local system problem is as follows:

\[ \sum_{j \in J_i} (\omega_j x_j - x_j^\phi) \leq 0, r \in R_s \]  

(11)

\[ \sum_{j \in J_i} (x_j^\phi - \omega_j x_j) \leq 0, r \in R_s \]  

(12)

\[ B_r \sum_{i \in I_r} x_j - B_r' \sum_{j \in J_i, i \in I_s} x_j \leq 0, r \in R_r, i \in I_s \]  

(13)

\[ B_S^{\phi^r} \sum_{i \in I_s, j \in J_s} x_j^\phi - B_S^{\phi^r} \sum_{j \in J_i, i \in I_s} x_j^\phi = 0, i' \in I' \]  

(14)

\[ \sum_{i \in I_s} b_i \sum_{j \in J_s} x_j - B_S^{\phi^r} \sum_{j \in J_i, i \in I_s} x_j \leq 0, i' \in I' \]  

(15)

\[ \sum_{j \in J_s} x_j^\phi \leq A_s \]  

(16)

\[ -\delta_S \leq \sum_{j \in J_s} x_j^\phi - x_\phi^\phi Z^{(o)}_S \leq \delta_S \]  

(17)

\[ \sum_{j \in J_s} (c_j x_j - Z^{(o)}_S x_j^\phi) \rightarrow \max \]  

(18)

Conditions (11) - (15) describe the \( s \)-th production method. The constraint (16) gives the initial financial limit of the \( s \)-th production method. The ratio is usually:
\[ \sum_{s \in S} A_s > A \]  

(19)

So, for example, if a difficult work acts as a terminal non-rigid production method. Relation (17) is a condition for the connection of local and global problems, as well as the second term in the objective function (18) [7].

The aggregated task is built by vertical aggregation, i.e. consists of a set of aggregated descriptions of each local system [11]. Based on the requirement of content, adequacy in the global task, the S-th production method is characterized by the production method aggregated financial intensity \( x_{s}^{(\sigma)} \). \( Z_{s} \), where \( x_{s}^{(\sigma)} \) - is the production method aggregated financial intensity, found on the resource consumption methods financial intensity basis, calculated at the iteration \( (\sigma - 1) \):

\[ x_{s}^{(\sigma)} = \sum_{j \in J_s} x_{j}^{(\sigma - 1)} \]  

(20)

\( Z_{s} \) - is a desired corrective multiplier of the aggregated task [10].

Let be \( x_{j}^{(\sigma - 1)} \), \( x_{j}^{(\sigma - 1)} \), \( y_{r}^{\phi(\sigma - 1)} \), \( y_{r}^{\phi(\sigma - 1)} \), \( y_{l}^{\phi(\sigma - 1)} \), \( y_{l}^{\phi(\sigma - 1)} \), \( y_{s}^{(\sigma - 1)} \), \( y_{s}^{(\sigma - 1)} \) - the approximations to the direct and dual solutions of local problems (11) - (18), obtained at iteration \( (\sigma - 1) \). Then the global task has the following form:

\[ \sum_{s \in S} x_{s}^{(\sigma)} \cdot Z_{s} \leq A \]  

(21)

\[- \epsilon_{s} \leq Z_{s} - 1 \leq \epsilon_{s}, s \in S \]  

(22)

\[ Z_{s} \geq 0 \]  

(23)

\[ \sum_{s \in S} c_{s}^{(\sigma)} Z_{s} \rightarrow \max \]  

(24)

where: \( x_{s}^{(\sigma)} \) is calculated by the formula (20), and the aggregated efficiency of the s-th production method \( c_{s}^{(\sigma)} \) is calculated as a spending resources ways components efficiencies weighted sum, taking into account the local constraints estimates:

\[ c_{s}^{(\sigma)} = \sum_{j \in J_s} c_{j} x_{j}^{(\sigma - 1)} - \sum_{r \in R_s} y_{r}^{\phi(\sigma - 1)} \sum_{j \in J_s} (\omega_{r} y_{j}^{(\sigma - 1)} - x_{j}^{(\sigma - 1)}) - \]  

\[ - \sum_{i \in I_s} y_{r}^{\phi(\sigma - 1)} \sum_{j \in J_s} (x_{j}^{(\sigma - 1)} - \bar{\omega}_{j} x_{j}^{(\sigma - 1)}) - \]  

\[ - \sum_{i \in I_s} \sum_{j \in J_s} B_{i} \sum_{j \in J_s} x_{j}^{(\sigma - 1)} - B_{i} \sum_{j \in J_s} x_{j}^{(\sigma - 1)} - \]  

(25)
\[ -y_s^{n(\sigma-1)} (\sum_{i \in J_i} b_i \sum_{j \in J_j} \lambda_j^{(\sigma-1)} - B_s^* \sum_{j \in J_j} \lambda_j^{(\sigma-1)} - \lambda_s^{(\sigma-1)} \sum_{j \in J_j} \varphi_j^{(\sigma-1)}) \]

Denote the global problem direct and dual solution by \( Z_s^{(\sigma)} \) and \( \hat{\lambda}^{(\sigma)} \) \[13\]. Then the next approximation to the dual solution is obtained by the damping:

\[ \lambda^{(\sigma)} = (1-\alpha) \lambda^{(\sigma-1)} + \alpha \hat{\lambda}^{(\sigma)}, \quad \alpha = \frac{1}{\sigma}. \]

(26)

The similar to (26) approximations to direct and dual solutions of problems at iteration are also calculated by the formulas \((\sigma)\) \[14\].

**Summary**

The main advantage of the vertical aggregation scheme is the fact that the exchange of information between the global and local tasks occurs in the aggregated nomenclature, which reduces the bandwidth requirements of the inter-layer communication channels. In the scheme of horizontal aggregation, the global task is formed on the basis of the local systems non-aggregated information, which requires either the horizontal communication channels introduction or increases the load on the vertical (inter-level channels).

Thus, a model of the resources vertical aggregation was obtained when planning the construction production, which differs by taking into account the restrictions on the number of teams used for the functional dependencies between the works, which allows to obtain the dependence of the project duration on additional costs, while their volumes are not related to a rigid functional dependence.

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