Mimicking black hole event horizons in atomic and solid-state systems

M. Franz\textsuperscript{1, 2} and M. Rozali\textsuperscript{1}

\textsuperscript{1}Department of Physics and Astronomy, University of British Columbia, Vancouver, BC, Canada V6T 1Z1
\textsuperscript{2}Quantum Matter Institute, University of British Columbia, Vancouver BC, Canada V6T 1Z4

(Dated: August 3, 2018)

Holographic quantum matter exhibits an intriguing connection between quantum black holes and more conventional (albeit strongly interacting) quantum many-body systems. This connection is manifested in the study of their thermodynamics, statistical mechanics and many-body quantum chaos. After explaining some of those connections and their significance, we focus on the most promising example to date of holographic quantum matter, the family of Sachdev-Ye-Kitaev (SYK) models. Those are simple quantum mechanical models that are thought to realize, holographically, quantum black holes. We review and assess various proposals for experimental realizations of the SYK models. Such experimental realization offers the exciting prospect of accessing black hole physics, and thus addressing many mysterious questions in quantum gravity, in tabletop experiments.

I. INTRODUCTION

Among the greatest challenges facing modern physical sciences is the relation between quantum mechanics and gravitational physics, described classically by Einstein’s general theory of relativity. Well known paradoxes arise when the two theories are combined, for example when describing event horizons of black holes\textsuperscript{1}. Indeed, those paradoxes arise even though both general relativity and quantum mechanics should be well within their regime of validity\textsuperscript{2}.

Perhaps the most promising avenue to resolving these paradoxes is rooted in the holographic principle\textsuperscript{3–4}, which posits that quantum gravity degrees of freedom in a (d + 1)-dimensional spacetime bulk can be represented by a non-gravitational many-body system defined on its d-dimensional boundary. By now there are many well-understood examples of such “holographic dualities” providing concrete realizations of that principle. Thus, using such holographic dualities, mysterious questions pertaining to gravitational physics in the quantum regime can be addressed in a context which is much better understood, that of quantum many-body physics.

Part of the difficulty in quantum gravity research is the dearth of relevant experiments. For example, experimental tests of the quantum gravity ideas using astrophysical black holes are impractical, as the relevant energy scale is inaccessible. Instead one can turn to holographic quantum matter\textsuperscript{5} – this is a class of strongly interacting systems that occur in certain solid-state or atomic systems, in which interactions between individual constituent particles are so strong that the conventional quasi-particle description fails completely. Rather, these systems exhibit properties of holography, in that they behave effectively as black hole horizons in a higher dimensional quantum gravitational theory. This opens up the exciting possibility of testing quantum gravity ideas experimentally, as holographic quantum matter is more amenable to experimental study, at least in principle.

This review focuses on recent developments aimed at theoretically understanding and realizing in the laboratory a specific class of systems that give rise to holographic quantum matter. They are described by the so-called Sachdev-Ye-Kitaev (SYK) models\textsuperscript{6–8} which are “dual”, in the sense described above, to a 1+1 dimensional gravitating “bulk” containing a black hole. We first review some of the background material on black hole thermodynamics, statistical mechanics and its relation to many-body quantum chaos. We then introduce the SYK model and discuss some of its important physical properties that furnish connections to holography and black holes. Finally we review proposals for the experimental realization of this model using atomic and solid state systems and assess critically their prospects and feasibility. We end by describing the outlook and potential significance of this realization of holographic dualities.

II. BLACK HOLE MATERIALS

The relation of black hole physics, the hallmark of quantum gravity research, to materials science is extremely surprising. The story begins in early studies of quantum gravity in the 1970s. Famously there is some tension between fundamental concepts of General Relativity and Quantum Mechanics, and that tension manifests itself most clearly when black holes of General Relativity are treated as quantum mechanical objects. In classical gravity, black holes are characterized by having a surface of no return, the so-called event horizon. Any information crossing that surface and falling into the black hole is lost to observers who stay safely away from the black hole. However, that defining feature of black hole is lost to observers who stay safely away from the black hole. Indeed, the celebrated work of Hawking\textsuperscript{9} established that quantum mechanically black holes are no longer black, they glow with what came to be called the Hawking radiation.
A. Black Hole Thermodynamics

The discovery of Hawking radiation finalized a conceptual revolution of our understanding of black hole quantum mechanics. Hawking radiation is thermal, like that of a black body, so black holes have a temperature. Earlier Jacob Bekenstein [10] has speculated that the surface area of a black hole behaves analogously to an entropy (for example it always increases); apparently this was not merely an analogy. Surprisingly black holes, as viewed by an outside observer, behave exactly as thermodynamic systems [11]. They have static properties: energy, charge, entropy, temperature. They have dynamical properties such as viscosity and conductivity. An outside observer who does experiments on black holes views them as chunks of materials with collective properties which obey all the usual relations of thermodynamics.

Indeed, there is a wide variety of possibly black holes in various gravitational theories, in various spacetime dimensions and with diverse matter content, and correspondingly a wide variety of black hole “materials”. To explain that diversity, one had to wait for a microscopic theory of quantum black holes.

B. Black Hole Statistical Mechanics

In the mid-1990s string theory, a leading candidate for a theory of quantum gravity, has reached the level of development needed to provide a microscopic statistical mechanics description of a large class of black holes [12]. Within string theory, gravitational interactions emerge naturally within an inherently quantum mechanical framework. It became possible then to discuss, in great quantitative detail, static and dynamical properties of quantum black holes. The flurry of activity in the following decade established a description of quantum black holes which is, perhaps surprisingly, fairly conventional. Within this framework black holes are described as quantum many-body systems, composed of microscopic constituents interacting strongly, to form a quantum liquid whose thermodynamics forms the gravitational description of the system. In this sense gravity arises from coarse-graining, exactly like a thermodynamic description of conventional materials.

The microscopic description makes it clear that black holes obey the rules of quantum mechanics, in particular they avoid Hawking’s information loss paradox [1]. Nevertheless, it is still unclear how the paradox is avoided, in other words how resolution of the paradox can be phrased in a purely gravitational language. Recently this set of questions was sharpened and cast in the language of quantum information processing [2]. Understanding black holes as quantum computers is an exciting current direction in quantum gravity, which is expected to provide valuable clues on how a gravitational description emerges for black holes, while avoiding information loss and related puzzles.

C. Box 1: Gauge/Gravity Duality

The microscopic description of black holes is at its most developed stage in the context of the AdS/CFT correspondence, also known as the gauge-gravity or holographic duality. Here AdS refers to anti-de Sitter space (see below) and CFT denotes conformal field theory. In this set of examples, the black hole is immersed in a gravitational potential (arising due to a negative cosmological constant), which acts to regulate some problematic long distance behaviour unrelated to the questions we are interested in. When thus regulated, it turns out the microscopic description is in terms of well-known many-body concepts, namely gauge theories of the form that constitutes the standard model of particle physics, as well as many other interesting systems appearing in the condensed matter context.

Twenty years and many thousands of detailed calculations later, we now have a very precise understanding of how gauge theories reproduce properties of quantum black holes immersed in AdS spaces. Note that astrophysical black holes are well-described by classical physics, whereas (small, cold) quantum black holes perhaps play a role in the early evolution of the universe, but that avenue of investigation has many uncertainties. Through the gauge/gravity duality we now face the exciting prospect of accessing quantum black holes experimentally, in a new and extremely surprising context.

This precise correspondence and the possibility for experimental realization raise an urgent question, namely: when does a quantum many-body systems describe black holes in a theory of gravity? Which aspects of the physics require a gravitational description? Or, put more prosaically: given some material, what would one measure to discover if it is usefully described as a black hole in a...
(usually higher dimensional) gravitational theory?

D. Many-Body Quantum Chaos

The defining property of a black hole is its event horizon. To identify quantum black holes in the lab, it is useful to identify universal quantities, universal in that they only depend on the structure of spacetime near the horizon. One such quantity was identified by Shenker and Stanford \[13\] and later employed by Kitaev, in an attempt to define a quantum mechanical analog of the butterfly effect and the associated Lyapunov exponent, which characterize that effect in classical chaos.

The classical butterfly effect quantifies the sensitivity to initial conditions in chaotic systems. Nearby trajectories diverge exponentially quickly from each other, with a rate that is called the Lyapunov exponent. In quantum systems trajectories do not exist, but one can still discuss the influence of an initial perturbation on subsequent measurements. The suggestion was to quantify that dependence by looking at the size of the commutator \([V(0), W(t)]\), where the operator \(V(0)\) provides the initial perturbation and \(W(t)\) measures its influence at subsequent times. Instead of calculating the expectation value of \([V(0), W(t)]\), which defines the linear response of the system, it turns out it is useful to focus on the second moment: \(C(t) = \text{tr}(e^{-\beta H}[V(0), W(t)]^2)\), which is sometimes referred to as the out-of-time-order correlator or OTOC. If we choose \(V, W\) to be initially commuting Hermitian operators, the quantity \(C(t)\) increases exponentially, as the information contained in the perturbation spreads rapidly throughout the system, a phenomenon referred to as “scrambling”. The exponent characterizing this exponential growth was dubbed the quantum Lyapunov exponent \(\lambda_L\).

Various protocols for measuring the quantum Lyapunov exponent have been suggested. Holographic quantum matter is in many ways as strongly interacting as possible, consistent with quantum mechanical unitarity and causality (or more precisely, limits on information propagation \[14\]). As such it is a useful arena to discover limits on what is possible in highly quantum mechanical many-body systems. There are many recent attempts to precisely quantify such limits, which are typically saturated by holographic matter, starting with the observation that the shear viscosity to entropy ratio for a strongly interacting quantum liquid cannot become arbitrarily small \[15\], an observation which played a role in theoretical understanding of relativistic heavy ion collisions. Perhaps the most precise and well-established such bound to date is the bound on chaos: it is rigorously proven that the quantum Lyapunov exponent obeys \(\lambda_L \leq \frac{2\pi}{\beta k_B T}\) for any quantum system \[16\], where \(\beta = 1/k_B T\) is the inverse temperature. Black holes are as strongly coupled as possible, their Lyapunov exponents are as large as is allowed, and they scramble information as fast as is consistent with unitarity and causality.

Indeed, an elegant calculation in the gravitational context reveals that the Lyapunov exponent depends only on the properties of black holes near the event horizon, and is therefore universal in the sense described above, that it takes the same values for all black holes in the context of the AdS/CFT duality. Furthermore, that value is precisely the maximally allowed value, \(\lambda_L = \frac{2\pi}{\beta k_B T}\) for quantum black holes, realizing the intuition that holographic matter is as strongly coupled as possible. This provides an answer to the question posed in Box 1: looking at the manner in which information is scrambled in a quantum system can provide a connection to its gravitational description as a black hole. In particular, saturating the chaos bound is a strong indication of a connection to quantum black holes and gravitational description.

As it turns out, one can explicitly construct a simple model which has that property, that of maximal scrambling. This is the so-called Sachdev-Ye-Kitaev (SYK) model, which we review presently.

III. THE SYK MODEL AND ITS LARGE-\(N\) SOLUTION

The SYK model describes a system of \(N\) interacting Majorana fermions. These are particles identical to their antiparticles originally predicted in the seminal work of Ettore Majorana \[17\] and recently observed in a variety of solid-state platforms \[24\]–\[32\], (also see Box 2). It is defined by the Hamiltonian

\[
\mathcal{H}_{\text{SYK}} = \sum_{i<j<k<l} J_{ijkl} \chi_i \chi_j \chi_k \chi_l.
\]

schematically depicted in Fig. 2. Here \(\chi_j\) with \(j = 1 \cdots N\) represent the Majorana fermion operators that obey the canonical anticommutation relations

\[
\{\chi_i, \chi_j\} = \delta_{ij}, \quad \chi_i^\dagger = \chi_j.
\]

\(J_{ijkl}\) are real-valued random independent variables drawn from a Gaussian distribution with the width

\[
J^2 = \frac{N^3}{3!} \langle J_{ijkl}^2 \rangle.
\]

The \(N^3\) factor is necessary for the total energy of \(\mathcal{H}_{\text{SYK}}\) to scale extensively in the number of particles \(N\), and for the interaction to stay finite in the large \(N\) limit.

Two key properties make \(\mathcal{H}_{\text{SYK}}\) interesting and non-generic: (i) the interactions between Majorana fermions are all-to-all and completely random, and (ii) there is no term bilinear in the fermion operators present in the Hamiltonian. The first property implies that there is no concept of distance between the fermions and the Hamiltonian is therefore zero-dimensional. By power counting it is easy to see that any bilinear term in fermion operators would represent a relevant perturbation to \(\mathcal{H}_{\text{SYK}}\). The second property thus ensures that the model defined
tions for the averaged fermion propagator \( G_{\text{f}}(\omega_n) \) \( \propto 1/\omega \) asymptotically exact in the limit \( \omega \to \infty \). In this review we will focus, for the most part, on the SYK model Eq. (3.1) as it is more straightforward to analyze. We comment on the cSYK model as appropriate – we will see that it might be easier to realize it in a laboratory because it does not require the elusive Majorana particles as basic building blocks.

Perhaps the most remarkable and useful property of the SYK model is that, despite being strongly interacting, it is exactly solvable in the limit of large \( N \). Specifically, it is possible to write down a simple pair of equations for the averaged fermion propagator

\[
G_{\text{c}}(\omega_n) = i\pi^{1/4} \frac{\text{sgn}(\omega_n)}{\sqrt{J_{\omega_n}}}
\]

and the corresponding spectral function

\[
\Lambda_{\omega} = \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{J_{\omega}}}.
\]

By Eq. (3.3) remains in the strong coupling regime all the way to the lowest energies.

A closely related model formulated with complex fermions \[33, 35\], also known as the complex Sachdev-Ye-Kitaev (cSYK) model or Sachdev-Ye (SY) model \[31\], is defined by the second-quantized Hamiltonian

\[
\mathcal{H}_{\text{cSYK}} = \sum_{ij:kl} J_{ijkl} c_i^\dagger c_j c_k c_l - \mu \sum_j c_j^\dagger c_j,
\]

where \( c_j^\dagger \) creates a spinless complex fermion, \( J_{ijkl} \) are zero-mean complex random variables satisfying \( J_{ijkl} = J_{klji} \) and \( J_{ij:kl} = J_{ij:kl} = J_{ij:kl} \) and \( \mu \) denotes the chemical potential. At half filling (\( \mu = 0 \)) the two models exhibit very similar behavior but the cSYK model shows more complexity when \( \mu \neq 0 \). In this review we will focus, for the most part, on the SYK model Eq. (3.1) as it is more straightforward to analyze. We comment on the cSYK model as appropriate – we will see that it might be easier to realize it in a laboratory because it does not require the elusive Majorana particles as basic building blocks.

Perhaps the most remarkable and useful property of the SYK model is that, despite being strongly interacting, it is exactly solvable in the limit of large \( N \). Specifically, it is possible to write down a simple pair of equations for the averaged fermion propagator

\[
G_{\text{c}}(\omega_n) = i\pi^{1/4} \frac{\text{sgn}(\omega_n)}{\sqrt{J_{\omega_n}}}
\]

and the corresponding spectral function

\[
\Lambda_{\omega} = \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{J_{\omega}}}.
\]

where subscript \( c \) denotes the conformal regime \( |\omega| \ll J \). At high frequencies the behavior must cross over to \( 1/\omega \) in both cases. The absence of a pole in \( G_{\omega}(\omega_n) \) indicates the expected non-Fermi liquid behavior of the SYK model. In addition we will see that the characteristic inverse square root singularity of the spectral function could be measurable by various spectroscopies in some of the proposed experimental realizations of the model.

The conformal structure implied by Eqs. (3.15) has also been instrumental in extracting the quantum-chaotic properties of the SYK model. It allows for the calculation of the out-of-time-order correlator \( C(t) \) which shows the characteristic exponential growth \[7, 8\] with the maximal permissible Lyapunov exponent \( \lambda_L = \frac{2\pi}{\sqrt{J}} \), thus confirming the intuition that the SYK model is indeed holographically connected to a black hole.

Finally we mention various important extensions of the SYK model developed in the recent literature. These include models showing unusual spectral properties \[36–39\], supersymmetry \[40\], quantum phase transitions of an unusual type \[41, 42\], quantum chaos propagation \[43–45\], and patterns of entanglement \[46, 47\].

A. Box 2: Majorana zero modes and the Kitaev chain

Majorana fermions have been originally introduced in the context of particle physics as special solutions of the celebrated Dirac equation \[48\] describing relativistic quantum mechanics of spin-\(1/2\) particles. Unlike ordinary fermions (e.g. electrons or protons) Majorana particles lack the distinction between particle and antiparticle \[17\]. In the elementary particle physics the leading candidate for a Majorana fermion is the neutrino but the experimental evidence remains inconclusive at present \[23\].

In condensed matter physics Majorana fermions can appear as emergent particles in various many-body systems. The most prominent of these are topological superconductors in which Majorana particles often appear as topologically protected zero modes \[19, 23\]. A canonical example of such a topological superconductor is the Kitaev chain \[49\] which we now briefly review to illustrate the emergence of Majorana zero modes in a simple setting.
χ

for each \( j \) and \( j' \) associated with site \( j \) of the chain. We assume that \( \Delta \) is real and consider a simplest model of a superconductor in such a setting is described by the Hamiltonian

\[
\mathcal{H} = \sum_j \left[ -t(c^\dagger_j c_{j+1} + \text{h.c.}) - \mu c^\dagger_j c_j + (\Delta c^\dagger_j c^\dagger_{j+1} + \text{h.c.}) \right],
\]

(3.8)

where \( c^\dagger_j \) creates a fermion on site \( j \) and satisfies the usual fermionic anticommutation relation

\[
\{c^\dagger_i, c_j\} = \delta_{ij}.
\]

(3.9)

t and \( \Delta \) represents the nearest-neighbour hopping and pairing amplitudes, respectively and \( \mu \) denotes the chemical potential. We assume that \( \Delta \) is real and consider a chain with \( N \) sites and open boundary conditions.

Following Kitaev \[49\] we now express each Dirac fermion \( c_j \) in terms of two Majorana fermions \( \chi_j \) and \( \chi^\prime_j \),

\[
c_j = \frac{1}{\sqrt{2}}(\chi_j + i\chi^\prime_j), \quad c^\dagger_j = \frac{1}{\sqrt{2}}(\chi_j - i\chi^\prime_j).
\]

(3.10)

It is easy to check that if the Majorana operators obey the anticommutation algebra given by Eq. \( (3.5) \) then Eq. \( (3.9) \) is satisfied and the transformation is thus canonical.

In the Majorana representation the Hamiltonian \( (3.8) \) becomes

\[
\mathcal{H} = i\sum_j \left[ (\Delta + t)\chi^\prime_j \chi_{j+1} + (\Delta - t)\chi_j \chi^\prime_{j+1} - \mu \chi_j \chi^\prime_j \right].
\]

(3.11)

To appreciate the physical content of this Hamiltonian it is useful to consider a special limit in which \( \mu = 0 \) and \( \Delta = t \). We then have simply

\[
\mathcal{H} = 2it \sum_{j=1}^{N-1} \chi^\prime_j \chi_{j+1},
\]

(3.12)

where we restored the summation bounds.

Equation \( (3.12) \) is remarkable for what it is lacking: notice that two operators, \( \chi_1 \) and \( \chi^\prime_N \), do not participate in the Hamiltonian. They therefore constitute exact “zero modes” of the system. If we construct a new

\[
\Sigma = \begin{tikzpicture}
  \draw[thick] (0,0) node [circle, fill=black, inner sep=1pt] (A) {} -- (1,0) node [circle, fill=black, inner sep=1pt] (B) {} -- (2,0) node [circle, fill=black, inner sep=1pt] (C) {} -- cycle;
  \draw[thick] (0,0) -- (0,1) node [circle, fill=blue, inner sep=1pt] (D) {} -- (1,1) node [circle, fill=blue, inner sep=1pt] (E) {} -- (2,1) node [circle, fill=blue, inner sep=1pt] (F) {} -- cycle;
  \draw[thick] (0,0) -- (0,-1) node [circle, fill=red, inner sep=1pt] (G) {} -- (1,-1) node [circle, fill=red, inner sep=1pt] (H) {} -- (2,-1) node [circle, fill=red, inner sep=1pt] (I) {} -- cycle;
\end{tikzpicture}
\]

FIG. 3. The Kitaev chain. Blue and red dots represent Majorana modes \( \chi_j \) and \( \chi^\prime_j \) associated with site \( j \) of the chain. a) In the topological phase a bond forms between \( \chi^\prime_j \) and \( \chi_{j+1} \) for each \( j \), leaving \( \chi_1 \) and \( \chi^\prime_N \) unpaired. b) In the trivial phase \( \chi_j \) and \( \chi^\prime_j \) pair up on the each site leaving behind no unpaired end modes.

Consider spinless fermions in a 1D lattice. The simplest model of a superconductor in such a setting is described by the Hamiltonian

\[
\mathcal{H} = \sum_j \left[ -t(c^\dagger_j c_{j+1} + \text{h.c.}) - \mu c^\dagger_j c_j + (\Delta c^\dagger_j c^\dagger_{j+1} + \text{h.c.}) \right],
\]

(3.8)

where \( c^\dagger_j \) creates a fermion on site \( j \) and satisfies the usual fermionic anticommutation relation

\[
\{c^\dagger_i, c_j\} = \delta_{ij}.
\]

(3.9)

t and \( \Delta \) represents the nearest-neighbour hopping and pairing amplitudes, respectively and \( \mu \) denotes the chemical potential. We assume that \( \Delta \) is real and consider a chain with \( N \) sites and open boundary conditions.

Following Kitaev \[49\] we now express each Dirac fermion \( c_j \) in terms of two Majorana fermions \( \chi_j \) and \( \chi^\prime_j \),

\[
c_j = \frac{1}{\sqrt{2}}(\chi_j + i\chi^\prime_j), \quad c^\dagger_j = \frac{1}{\sqrt{2}}(\chi_j - i\chi^\prime_j).
\]

(3.10)

It is easy to check that if the Majorana operators obey the anticommutation algebra given by Eq. \( (3.5) \) then Eq. \( (3.9) \) is satisfied and the transformation is thus canonical.

In the Majorana representation the Hamiltonian \( (3.8) \) becomes

\[
\mathcal{H} = i\sum_j \left[ (\Delta + t)\chi^\prime_j \chi_{j+1} + (\Delta - t)\chi_j \chi^\prime_{j+1} - \mu \chi_j \chi^\prime_j \right].
\]

(3.11)

To appreciate the physical content of this Hamiltonian it is useful to consider a special limit in which \( \mu = 0 \) and \( \Delta = t \). We then have simply

\[
\mathcal{H} = 2it \sum_{j=1}^{N-1} \chi^\prime_j \chi_{j+1},
\]

(3.12)

where we restored the summation bounds.

Equation \( (3.12) \) is remarkable for what it is lacking: notice that two operators, \( \chi_1 \) and \( \chi^\prime_N \), do not participate in the Hamiltonian. They therefore constitute exact “zero modes” of the system. If we construct a new

\[
\Sigma = \begin{tikzpicture}
  \draw[thick] (0,0) node [circle, fill=black, inner sep=1pt] (A) {} -- (1,0) node [circle, fill=black, inner sep=1pt] (B) {} -- (2,0) node [circle, fill=black, inner sep=1pt] (C) {} -- cycle;
  \draw[thick] (0,0) -- (0,1) node [circle, fill=blue, inner sep=1pt] (D) {} -- (1,1) node [circle, fill=blue, inner sep=1pt] (E) {} -- (2,1) node [circle, fill=blue, inner sep=1pt] (F) {} -- cycle;
  \draw[thick] (0,0) -- (0,-1) node [circle, fill=red, inner sep=1pt] (G) {} -- (1,-1) node [circle, fill=red, inner sep=1pt] (H) {} -- (2,-1) node [circle, fill=red, inner sep=1pt] (I) {} -- cycle;
\end{tikzpicture}
\]

FIG. 4. The large-\( N \) solution. Leading Feynman diagrams in the \( 1/N \) expansion of the self energy \( \Sigma(\tau) \). Solid lines represent the fermion propagator \( G(\tau) \) while dashed lines indicate disorder averaging over the pairs of vertices connected by the line.

Dirac fermion operator \( \hat{c} = (\chi_1 + i\chi^\prime_N)/\sqrt{2} \) then it is clear that it costs zero energy to add such a \( \hat{c} \)-particle to the system. Remarkably, the zero mode is fundamentally delocalized between the two ends of the chain as illustrated in figure 3. Kitaev \[49\] noted the possibility of storing quantum information in this delocalized state which would make it resilient to many sources of decoherence. Such a possibility also underlies much of the current interest in Majorana zero modes.

With a little extra work one can demonstrate that he special limit discussed above represents merely a point in the stable topological phase of the Hamiltonian \( (3.8) \). The system exhibits gapped bulk with two Majorana zero modes localized near the ends of the chain whenever \( \Delta \neq 0 \) and \( |\mu| < 2|t| \). When \( |\mu| > 2|t| \) the chain is topologically trivial and zero modes are absent. Various instances of Majorana zero modes observed experimentally in 1D systems \[24–30\] can all be understood as realizations of the Kitaev chain.

B. Box 3: The large-\( N \) solution

The fermion propagator of the SYK model \( (3.5) \) is related to the self energy by the standard Dyson equation,

\[
G(\omega_n) = \left[ -i\omega_n - \Sigma(\omega_n) \right]^{-1},
\]

(3.13)

where we have assumed time-translation invariance and defined \( G(\omega_n) = \int_0^\beta d\tau e^{i\omega_n \tau} G(\tau) \), with Matsubara frequencies \( \omega_n = \pi T(2n + 1) \) and \( n \) integer. The solution exploits the fact that for large \( N \) diagrammatic expansion of the self energy \( \Sigma(\tau, \tau') \) is dominated by a class of the so called melonic diagrams which can be summed up exactly. All other diagrams, including the vertex corrections that typically prevent such exact resummations, are suppressed by factors of \( 1/N \) and can be neglected.

This can be seen from Fig. 4 which illustrates various diagrams that contribute to \( \Sigma \). The leading melonic diagram A has two vertices. Upon averaging over disorder Eq. \( (3.3) \) implies that these contribute a factor \( J_{ijkl}^2 \sim J^2/N^3 \). Summing over the three internal legs contributes a factor \( \sim N^3 \) as each leg can represent one of the \( N \) fermion flavours. The diagram is therefore \( O(1) \). It is easy to check that all diagrams with melonic insertions,
such as the diagram B are $O(1)$. On the other hand diagrams representing vertex corrections, such as diagram C, have fewer internal legs and therefore become $O(1/N)$ and thus negligible at large $N$.

It is straightforward to sum up the $O(1)$ diagrams comprising the self energy – the full series is generated by simply replacing the bare propagator $G_0(\omega_n) = -1/i\omega_n$ by the fully dressed propagator $G(\omega_n)$ in the leading melonic diagram A in Fig.\ref{fig:melonic}. One thus obtains

$$\Sigma(\tau) = J^2 G(\tau),$$

which together with Dyson equation (3.13) determine the form of the propagator at large $N$.

The mean field equations for the bi-local fields $G, \Sigma$ become exact in the large $N$ limit, where we can ignore fluctuations around their saddle point values, which are the solution of the Schwinger-Dyson equations. The property of being mean-field exact is shared by many infinite-ranged models, for example the celebrated Sherrington-Kirkpatrick model of spin glasses \cite{50}. Remarkably, the low energy physics of the SYK model is very different from most such models, as it avoids replica symmetry breaking and instead stays quantum mechanical to the lowest energies.

At sufficiently low frequencies $\omega_n \ll J$ the $i\omega_n$ term in Eq. (3.13) can be neglected and the resulting system of two equations can be seen to acquire time parametrization invariance

$$G(\tau, \tau') \to [f'(\tau)f'(\tau')]^{1/4}G(f(\tau), f(\tau')), \quad \Sigma(\tau, \tau') \to [f'(\tau)f'(\tau')]^{3/4}\Sigma(f(\tau), f(\tau')),$$

for an arbitrary smooth function $f(\tau)$. This emergent conformal symmetry is a key property of the SYK model which hints at its deep connection to black hole physics and holography. In fact, it is described by the same low energy action (the so-called Schwarzian action) as black holes in asymptotically AdS$_2$ \cite{51}. These low-dimensional black holes arise as the near-horizon region of a large class of charged black holes in 3+1 dimensions (see e.g.\cite{52}), thus the Schwarzian action also describes the low energy dynamics of those more realistic black holes.

The relation to black hole physics is explained in greater detail in \cite{63} and many subsequent works. We refer the interested reader to that vast literature for the many fascinating details of that correspondence.

IV. PROPOSED PHYSICAL REALIZATIONS

Given the plethora of interesting phenomena that are thought to be encoded in the SYK model and the rich connections it furnishes between disparate fields of physics, it would be of obvious interest to have its experimental realization. It has been proposed that high-$T_c$ cuprate superconductors and some other strongly correlated “strange metals” may realize various higher-dimensional generalizations of the SYK model \cite{54,60}. This connection is made on phenomenological grounds and although very appealing, it remains speculative because of the difficulties encountered in the attempts to relate microscopic models governing crystalline solids to highly random SYK model (see however Ref. \cite{61} for an SYK-like model without randomness).

Another class of interesting approaches are quantum simulations. An algorithm has been proposed for digital quantum simulation of both SYK and cSYK models using a quantum computer \cite{62}. First experimental steps along this line have already been reported \cite{63}.

In this review we focus on proposals that aim at engineering the cSYK or SYK model bottom up, that is, by designing systems with requisite degrees of freedom and interactions using building blocks that are reasonably well understood and thus afford a high degree of control.

A. Three key challenges

When thinking about the experimental realization of the SYK model several challenges become immediately apparent. The original SYK model employs Majorana fermions so the first challenge is to design a physical system capable of hosting a large number $N$ of these elusive particles. In the language of condensed matter physics one needs Majorana zero modes \cite{19,23} which have been observed to exist in several platforms including semiconductor quantum wires \cite{24,28}, atomic chains \cite{30}, and vortices in topological insulator - superconductor interfaces \cite{31,32}. In these works signatures consistent with individual localized Majorana modes have been reported but manipulating and assembling them into required multi-mode structures remains a considerable challenge.

Once we have an assembly of $N$ Majorana particles the second challenge arises. The leading perturbation in such a situation, typically, will come from the hybridization term

$$\mathcal{H}_2 = i \sum_{i<j} K_{ij} \chi_i \chi_j$$

which is quadratic in Majorana operators, not quartic as required for the SYK model. In Eq. (4.1) real-valued constants $K_{ij}$ arise from the overlap between Majorana fermion wavefunctions and describe tunnelling events much like overlaps between atomic orbitals in molecules or solids which describe electron tunnelling. In the presence of interactions (such as those arising from the Coulomb repulsion between the underlying electron degrees of freedom) the full Hamiltonian of the system of $N$ Majorana particles will be $\mathcal{H} = \mathcal{H}_2 + \mathcal{H}_{SYK}$. For such a Hamiltonian it is known \cite{40} that $\mathcal{H}_2$ is a relevant perturbation to $\mathcal{H}_{SYK}$ meaning that for arbitrary non-zero $K_{ij}$ the SYK fixed point is destroyed and the
ground state of the system becomes a disordered Fermi liquid. Fortunately, vestiges of the SYK physics still survive in the excited states above the crossover energy scale \( E_c \approx K^2/J \) where we have defined the average hybridization strength as
\[
K^2 = N K_{ij}^2. \tag{4.2}
\]
The excited states can be probed by performing experiments at non-zero temperature \( T \) or frequency \( \omega \), such that \( K^2/J < T, \omega < J \). In order to have a reasonably wide range of temperatures or frequencies in which the SYK physics remains observable one therefore needs to ensure that \( K \ll J \). This is the second key challenge facing any experimental realization of the SYK model as a generic system of \( N \) Majorana modes would instead have \( K \gg J \).

The final challenge has to do with ensuring sufficient randomness in coupling constants \( J_{ijkl} \). In order to define the SYK Hamiltonian these constants must be sufficiently close to random independent variables with a Gaussian distribution. As we shall see such randomness can arise from the microscopic disorder that is inherently present in solids or else it must be deliberately engineered in the system through fabrication.

Finally we remark that the first of the challenges discussed above is avoided in proposals that aim at realization of the cSYK model Eq. (3.4) which employs ordinary complex fermions as basic building blocks and not Majorana fermions. Indeed these proposals appear to be simpler to implement and are therefore likely to succeed first. As we mentioned the cSYK model at half filling exhibits the same holographic duality to a black hole as the SYK model and it shows additional interesting behavior away from half filling [64]. In solid state systems complex fermions will be electrons and will thus carry electric charge. For this reason such systems will be amenable to standard transport and spectroscopic measurements. This confers another advantage over Majorana fermions, which are electrically neutral.

**B. cSYK model with ultracold gases**

Experiments on ultracold atomic gases in optical lattices have already succeeded in realizing a number of theoretical models originally introduced in the context of condensed matter physics. Notable examples include the Bose-Hubbard model [65] and the Haldane model [66]. Owing to their high degree of controllability, lack of disorder, and the unique set of physical observables experiments on ultracold gases can often answer many important questions about these models that remain inaccessible to their condensed matter analogs.

A proposal to realize the cSYK model with ultracold gases in optical lattices has been given by Dan-shita, Hanada and Tezuka [67–68]. The authors envision fermionic atoms, such as \(^6\)Li, confined in deep potential wells of an optical lattice. Each well is assumed to have

\[
\text{atoms} \quad \text{photoassociation} \quad \text{photodissociation}
\]

**FIG. 5. cSYK model from cold atoms.** Laser beams are used to couple atoms and molecules such that transitions occur as described by Hamiltonian [4.3] (Figure copied from Ref. [68]).

\( N \) strongly localized atomic states filled with \( Q \) atoms. While the eigenstate wavefunctions spatially overlap in the same well, their orthogonality forbids single particle tunnelings, so that perturbations similar to Eq. (4.1), bilinear in fermion operators, can be ignored. The atoms in separate wells are also assumed to be completely decoupled from each other so we can focus on a single well. In addition, two atoms can come together and form bosonic molecular states through the process of photo-association (PA) stimulated by PA lasers of appropriate frequencies, as illustrated in Fig. 5. The Hamiltonian governing such a system is

\[
\mathcal{H} = \sum_{s=1}^{n_{\text{max}}} \nu_s \hat{m}_s^\dagger \hat{m}_s + \sum_{s'=1}^{n_{\text{max}}} \frac{U_{ss'}}{2} \hat{m}_s^\dagger \hat{m}_s \hat{m}_{s'}^\dagger \hat{m}_{s'} + \sum_{i,j} g_{s,ij} \left( m_i^\dagger c_j c_j^\dagger - m_i c^\dagger_j c^\dagger_j - m_i c^\dagger_j c_j + m_i c^\dagger_j c^\dagger_j \right), \tag{4.3}
\]

where \( m_i^\dagger \) and \( c^\dagger \) represent the creation operators for molecules and atoms, respectively, \( \nu_s \) are molecular state energies, \( U_{ss'} \) are molecular interactions and \( g_{s,ij} \) denote the amplitudes of PA processes. The latter are assumed fully controllable as they depend on frequencies and intensities of the PA lasers.

The molecular states can be integrated out from the model defined by Eq. (4.3) and, using perturbation theory to second order in amplitudes \( g_{s,ij} \), one obtains the effective Hamiltonian for the fermions of the form

\[
\mathcal{H}_{\text{eff}} = \sum_{i,j,k,l} \left( \sum_{s=1}^{n_{\text{max}}} \frac{g_{s,ij} g_{s,kl}}{\nu_s} \right) c_i^\dagger c_j^\dagger c_k c_l. \tag{4.4}
\]

This clearly has the structure of the cSYK model Eq. (3.4) provided that we identify the sum in the brackets with \( J_{ijkl} \). The coupling constants in this realization of the cSYK model are real (as opposed to complex-valued) but the authors [67] argue convincingly that for large \( N \)
this difference does not matter. Also, unlike the canonical cSYK model, $J_{ij;kl}$ defined by Eq. (4.4) have internal structure and are not in general Gaussian distributed random variables. If however $g_{n,ij}$ and $\nu_s$ are taken as random variables then for a large number of molecular states $n_{ms} \gg 1$ the distribution of $J_{ij;kl}$ should approach Gaussian by virtue of the central limit theorem ($J$s are then given by a sum of a large number of identically distributed random variables). In this limit, at least, the system defined by Hamiltonian (4.3) should behave as a canonical cSYK model.

Unfortunately, $n_{ms} \gg 1$ is also the limit which might be difficult to realize in a laboratory. The chief bottleneck is the number of lasers required to drive the PA transitions between the atomic and molecular states. This equals $n_{ms} N(N-1)/2$ which becomes a very large number in the interesting limit when both $n_{ms}$ and $N$ are large. In addition, frequencies of these PA lasers must be rather accurately tuned to achieve reasonable rates of transitions. It is known from numerical studies that $N \geq 10$ is required to start observing spectroscopic signatures characteristic of the cSYK model. It is less clear how large $n_{ms}$ must be for a given $N$ to yield Gaussian distributed $J$s to a good approximation. The cSYK model variant defined by Eq. (4.4) could however exhibit interesting behavior even for small $n_{ms}$ and a theoretical study of this limit would be of interest.

The authors [67], among other things, devised a protocol to measure the out-of-time-order correlator $C(t)$ in this system, following the general procedure outlined in Ref. [69]. The protocol involves coupling the atomic states to a control qubit, annihilating individual atoms at different times according to the qubit state, and finally evolving the whole system forward and backward in time according to the Hamiltonian (4.4). Backward time evolution requires flipping the sign of the Hamiltonian which is achieved by flipping the sign of all $\nu_s$. This in turn can be implemented by changing the detuning of the PA lasers. Measuring OTOC is clearly a challenging proposition but first steps have recently been taken using quantum simulators [70] [71].

C. cSYK model in a graphene flake

This proposed solid-state realization [72] takes its inspiration from the rich physics of quantum Hall liquids [73] [74]. It is well known that application of a strong magnetic field to the 2D electron gas (electrons confined to a thin layer) results in quenching of their quantum kinetic energy and strong enhancement of the interaction effects. This is equivalent to the suppression of two-fermion terms $H_2$ relative to the four-fermion interaction terms contained in $H_{cSYK}$. In a large clean sample one then obtains the family of fractional quantum Hall liquids characterized by a range of spectacular physical properties including topological order as well as the charge and statistics fractionalization [75].

The root cause for the above behavior is Landau quantization: at the non-interacting level magnetic field reorganizes the electron bands into a discrete set of massively degenerate dispersionless Landau levels (LLs). The effective single-particle Hamiltonian for an electron in a given Landau level is then simply $H_2 = 0$. The proposal of Ref. [72] uses the Landau level as the starting point to construct the cSYK model. The key remaining challenge, then, is to produce coupling constants $J_{ij;kl}$ that would be all-to-all and sufficiently random. To achieve this one must include disorder effects. Under normal circumstances, however, microscopic disorder tends to broaden LLs, effectively reintroducing nonzero $H_2$. For this reason ordinary 2D electron gas with disorder does not work as a platform to build the cSYK model.

The idea is to use instead a flake of graphene with a highly irregular boundary serving as a source of disorder, see Fig. 6. Electrons in graphene are well-known to possess “relativistic” energy dispersion described at low energies by a massless Dirac Hamiltonian [76]. Landau levels of such a Hamiltonian occur at energies

$$E_n \simeq \pm n eB/\hbar c$$

(4.5)

with the characteristic Fermi velocity $v$ and $n = 0, 1, \ldots$. The $n = 0$ Landau level, often referred to as LL0, is special in terms of how it responds to disorder. According to the famous Aharonov-Casher construction [77] the exact degeneracy of quantum states in LL0 is protected against any disorder that respects the chiral symmetry of graphene. This is most simply explained by inspecting the tight-binding model describing electrons in the graphene honeycomb lattice,

$$H_0 = -t \sum_{r, \delta} (a^\dagger_r b^\dagger_{r+\delta} + h.c.)$$

(4.6)

Here $a^\dagger_r$ ($b^\dagger_{r+\delta}$) denotes the creation operator of the electron on the sublattice A (B) of the honeycomb lattice. $r$ extends over the sites in sublattice A while $\delta$ denotes the 3 nearest neighbour vectors (inset Fig. 6), $t = 2.7$ eV is the nearest-neighbor tunneling amplitude. The chiral symmetry $\chi$ is generated by setting $(a_r, b_r) \rightarrow (-a_r, b_r)$
for all \( r \) which has the effect of flipping the sign of the Hamiltonian \( H_0 \rightarrow -H_0 \).

Making the shape of the flake irregular introduces randomness that respects \( \chi \). The wavefunctions \( \Phi_j(r) \) of the states belonging to \( \text{LL}_0 \) acquire random spatial structure but, importantly, Aharonov-Casher construction \cite{77} guarantees that their energies remain perfectly degenerate. Under these circumstances one expects the Coulomb interaction \( V(r) \) to produce random and all-to-all coupling constants \( J_{ij;kl} \) between the electrons in \( \text{LL}_0 \). This is because \( J_{ij;kl} \) are given as matrix elements of \( V(r) \) between random-valued wavefunctions \( \Phi_j(r) \) which, according to simulations \cite{72}, have support everywhere in the flake.

Characteristic signatures of the cSYK physics could be observed in this system by a host of spectroscopic and transport probes. Scanning tunneling spectroscopy measures the local density of states which is directly related to the spectral function and should thus exhibit the characteristic inverse square root divergence Eq. \ref{eq:3.7} as a function of bias voltage. Two-terminal conductance through the system and local compressibility, measured via charge sensing techniques, should likewise show signatures of the non-FL behavior. It is currently not known how to measure out-of-time-ordered quantities, such as \( C(t) \) in this context and this remains a challenge for all solid-state realizations of cSYK and SYK models.

D. SYK model with semiconductor quantum wires

Semiconductor quantum wires with strong spin-orbit coupling, such as those made of InSb, have emerged as the prime platform for investigating Majorana zero modes. When such wires are “proximitized” (that is, made superconducting by placing them in contact with a superconducting material, such as Al) and placed in magnetic field, Majorana zero modes are predicted to appear at their ends \cite{78, 79}. There now exists ample experimental evidence for this effect \cite{24, 29} which realizes the Kitaev chain paradigm discussed in Box 2.

Chew, Essin and Alicea \cite{80} proposed to engineer the SYK Hamiltonian by coupling \( N \) such wires to a disordered 2D quantum dot as illustrated in Fig. \ref{fig:7}. In such a setup Majorana zero modes can be shown to spread out from the tips of the individual wires into the quantum dot so that their wavefunctions are essentially delocalized across its entire area. Once delocalized Majorana wavefunctions overlap in real space and even perfectly local interactions produce all-to-all matrix elements \( J_{ijkl} \) precisely as required by the SYK Hamiltonian. The fact that the quantum dot is disordered means that the wavefunctions acquire random spatial structure which in turns translates to randomness in \( J_s \).

Two features of this proposed setup deserve note. First, Majorana zero modes avoid hybridization (that is, formation of bilinear terms in the Hamiltonian that would be detrimental to SYK) by virtue of approximate artificial time-reversal symmetry \( \mathcal{T} \) present in the system. This may be illustrated most easily on the example of the Kitaev chain. Its Hamiltonian Eq. \ref{eq:3.8} respects an antunitary symmetry \( \mathcal{T} \) which sends \( c_j \rightarrow c_j \) and \( i \rightarrow -i \). In the Majorana representation this symmetry is expressed as

\[
\mathcal{T} : \chi_j \rightarrow \chi_j^\dagger, \quad \chi_j^\dagger \rightarrow -\chi_j, \quad i \rightarrow -i, \tag{4.7}
\]

and it is easy to check that Hamiltonian \ref{eq:3.10} is indeed invariant. If we now assume that from each of \( a = 1 \ldots N \) wires only the left-most Majorana mode \( \chi_{1}^a \) is coupled to the dot, and that the entire system continues to respect \( \mathcal{T} \), then bilinear terms of the form \( iK_{ab}\chi_{1}^a\chi_{1}^b \) are prohibited as they would be odd under the symmetry. Of course \( \mathcal{T} \) is not a true microscopic symmetry of the physical system so strictly speaking \( K_{ab} \) is not zero. Ref. \cite{80} however argued that it is an approximate symmetry which implies that \( K_{ab} \) are likely to be small compared to the interaction energy scale.

The second feature has to do with the hybridization of the Majorana modes with the electronic levels present in the dot itself. Such hybridization would also be harmful to the prospects of constructing the SYK model. Here the authors show that a remarkable instance of level repulsion takes place and protects the Majorana modes. Random matrix theory considerations imply that a generic model describing the situation depicted in Fig. \ref{fig:7} has the the spectrum composed of the zero-mode manifold (\( N \) states) separated by \( \epsilon \approx N\delta_{\text{typ}}/\pi \) from all other energy levels. Here \( \delta_{\text{typ}} \) is the typical energy level spacing in the dot before it has been coupled to the wires.

The key advantage of this proposal is that Majorana zero modes are now routinely observed in proximitized semiconductor quantum wires. Assembling a large number of such wires to form a device in Fig. \ref{fig:7} is clearly a significant challenge. However, recent history of this field has proven that even tough challenges can be met when the goal is deemed sufficiently important.
At neutrality massless Dirac fermion in 2D, familiar from the physics protected surface state in a TI is described as a single TI to the so-called neutrality point. The topologically exactly like ˜an extra symmetry at this special point [83] that acts ex-

port identification of the emergent black hole physics is more straightforward in principle but by no means free of challenge.

Despite these technical hurdles, the key conceptual advances achieved over the past two years outline a clear roadmap which, if successfully followed, should lead to experimental insights to some of the most fundamental mysteries facing modern physics. The realization that a simple, solvable quantum mechanical model exhibits fundamental connections to quantum black holes has brought quantum gravity into the realm of being potentially experimentally testable. This exciting prospect

V. CONCLUSIONS AND OUTLOOK

In this review we focussed on proposals for experimental realizations of the cSYK or SYK model. All such proposals currently on the table face significant challenges. These include hurdles in materials synthesis, device fabrication, reproducibility and control. Nevertheless the field evolves rapidly and this provides hope that some version of the theoretically proposed devices could be experimentally realized in the near future.

Several measurements designed to make the connection to quantum black holes explicit have been proposed, and they present their own challenges. Measurement of out-of-time-ordered quantities, important for the identification of many-body chaotic behavior characteristic of a black hole, constitutes a huge challenge in the atomic physics setup and remains an unsolved problem for all proposed solid state realizations. Spectroscopic or transport identification of the emergent black hole physics is more straightforward in principle but by no means free of challenge.

E. SYK model at the surface of a 3D topological insulator

Another canonical platform for Majorana zero modes is the so-called Fu-Kane superconductor, which arises from proximitizing the topologically protected surface state of a 3D strong topological insulator (TI). Magnetic vortices in the Fu-Kane superconductor have been predicted to harbor Majorana zero modes. Signatures consistent with this prediction have recently been reported in Bi₂Te₃/NbSe₂ heterostructures [31, 32].

Realization of the SYK model in this platform has been proposed in Ref. [82]. As with the quantum wires the key challenge in this setting is to assemble N Majorana modes in the same spatial region without producing bilinear hybridization terms. This is achieved by engineering a nanoscale hole in the superconducting film on the surface of a TI. The hole serves to trap multiple vortices, each binding a Majorana zero mode. The mechanism behind this trapping has to do with the well known phenomenon of vortex pinning: since the superconducting order parameter Δ is suppressed to zero at the core of a vortex it is energetically preferable to position the core in a region where Δ was already suppressed by other effects (impurities for example). A hole in the superconductor has Δ = 0 and thus serves as a perfect pinning center. When sufficiently large it can trap many vortices, and thus many Majorana zero modes.

Hybridization between the zero modes can be avoided in this setup by tuning the chemical potential μ of the TI to the so-called neutrality point. The topologically protected surface state in a TI is described as a single massless Dirac fermion in 2D, familiar from the physics of graphene [76]. At neutrality μ coincides with the Dirac point. Importantly the Fu-Kane superconductor acquires an extra symmetry at this special point that acts exactly like ˜T defined in Eq. (4.7). This symmetry then excludes any bilinear terms from the effective Hamiltonian describing the zero modes. If interactions are present between the underlying electron degrees of freedom then the leading term in such an effective description is a 4-fermion SYK-like Hamiltonian [81]. Using a combination of analytical and numerical tools Ref. [82] showed that when the hole is engineered to have a highly irregular shape Majorana wavefunctions acquire random spatial structure and the screened Coulomb interaction between electrons produces essentially random couplings J_{ijkl}, as required to implement the SYK model.

In comparison to the quantum wires experimental understanding of the Majorana modes in the Fu-Kane superconductor remains significantly less developed. Results reported in [31, 32] have yet to be reproduced by another group or in another family of materials. This makes it more difficult to ascertain the prospects for realization of the above proposal. On the other hand once the Fu-Kane superconductor has been better understood and characterized, it should be relatively easy to fabricate and probe the device proposed in Ref. [82]. Also, the fact that bilinear terms can be controlled by tuning a single parameter (the chemical potential μ) confers some advantage over the quantum wire realization, where the smallness of such terms relies on an approximate symmetry of the system that cannot be easily manipulated.

FIG. 8. Schematic of the SYK model realization in the Fu-Kane superconductor. A 3D topological insulator surface has been proximitized by depositing on it a thin film of a superconducting material. An irregular-shaped hole in the film serves to trap magnetic flux. For each magnetic flux quantum trapped there is one Majorana zero mode localized in the hole.
advances the decades-long quest, started already by Einstein, to reconcile two fundamental theories of nature, general relativity and quantum mechanics. That deep ideas on the nature of quantum gravity could be ultimately tested in a humble piece of solid replete with disorder and imperfections reinforces the modern belief in the principle of emergence, that indeed “more is different” [84].

Speculatively, but most importantly, once an experiment has produced a system which can be reliably identified as a quantum black hole, one can turn to empirically investigating many subtle questions pertaining to quantum gravity. The effort to formulate such longstanding questions in the context of simple quantum mechanical model is a recent and fast developing subject, see for example [85] for some examples of such attempts to address quantum gravitational questions using the SYK family of models.

ACKNOWLEDGMENTS

The authors are indebted to many colleagues who helped shaped their understanding of the subject. Of these special thanks go to I. Affleck, E. Altman, J. Alicea, L. Balents, M. Berkooz, A. Chen, F. Haelh, A. Kitaev, C. Li, E. Lantagne-Hurtubise, P. Narayan, D. Pikulin, S. Sachdev, J. Simon and M. Tezuka.

[1] S. W. Hawking, “Breakdown of predictability in gravitational collapse,” Phys. Rev. D 14, 2460–2473 (1976).
[2] Ahmed Almheiri, Donald Marolf, Joseph Polchinski, and James Sully, “Black holes: complementarity or firewalls?” Journal of High Energy Physics 2013, 62 (2013).
[3] Leonard Susskind, “The world as a hologram,” Journal of Mathematical Physics 36, 6377–6396 (1995).
[4] G. ’t Hooft, “The Holographic Principle” in Zichichi, A. Basics and highlights in fundamental physics, Subnuclear series 37 (World Scientific, 2001, 2001).
[5] Sean A. Hartnoll, Andrew Lucas, and Subir Sachdev, “Holographic quantum matter,” (2016), arXiv:1612.07324 [hep-th].
[6] Subir Sachdev and Jianwu Ye, “Gapless spin-fluid ground state in a random quantum heisenberg magnet,” Phys. Rev. Lett. 70, 3339–3342 (1993).
[7] A. Kitaev, “A simple model of quantum holography,” in KITP Strings Seminar and Entanglement 2015 Program (2015).
[8] Juan Maldacena and Douglas Stanford, “Remarks on the sachdev-ye-kitaev model,” Phys. Rev. D 94, 106002 (2016).
[9] S. W. Hawking, “Particle Creation by Black Holes,” Euclidean quantum gravity, Commun. Math. Phys. 43, 199–220 (1975) [167(1975)].
[10] Jacob D. Bekenstein, “Black holes and entropy,” Phys. Rev. D7, 2333–2346 (1973).
[11] James M. Bardeen, B. Carter, and S. W. Hawking, “The Four laws of black hole mechanics,” Commun. Math. Phys. 31, 161–170 (1973).
[12] Andrew Strominger and Cumrun Vafa, “Microscopic origin of the Bekenstein-Hawking entropy,” Phys. Lett. B379, 99–104 (1996) arXiv:hep-th/9601029 [hep-th].
[13] Stephen H. Shenker and Douglas Stanford, “Black holes and the butterfly effect,” Journal of High Energy Physics 2014, 67 (2014).
[14] E. H. Lieb and D. W. Robinson, “The finite group velocity of quantum spin systems,” Commun. Math. Phys. 28, 251–257 (1972).
[15] P. Kovtun, Dan T. Son, and Andrei O. Starinets, “Viscosity in strongly interacting quantum field theories from black hole physics,” Phys. Rev. Lett. 94, 111601 (2005), arXiv:hep-th/0405231 [hep-th].
[16] Juan Maldacena, Stephen H. Shenker, and Douglas Stanford, “A bound on chaos,” JHEP 08, 106 (2016), arXiv:1503.01409 [hep-th].
[17] E. Majorana, Nuovo Cimento 5, 171 (1937), english Translation: Soryushiron Kenkyu 63,149 (1981).
[18] F. Wilczek, Nature Physics 5, 614 (2009).
[19] J. Alicea, “New directions in the pursuit of majorana fermions in solid state systems,” Rep. Prog. Phys. 75, 076501 (2012).
[20] C.W.J. Beenakker, “Search for majorana fermions in superconductors,” Annu. Rev. Con. Mat. Phys. 4, 113 (2013).
[21] Martin Leijnse and Karsten Flensberg, “Introduction to topological superconductivity and majorana fermions,” Semiconductor Science and Technology 27, 124003 (2012).
[22] T D Stanescu and S Tewari, “Majorana fermions in semiconductor nanowires: fundamentals, modeling, and experiment,” Journal of Physics: Condensed Matter 25, 233201 (2013).
[23] Steven R. Elliott and Marcel Franz, “Colloquium : Majorana fermions in nuclear, particle, and solid-state physics,” Rev. Mod. Phys. 87, 137–163 (2015).
[24] V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. a. M. Bakkers, and L. P. Kouwenhoven, “Signatures of majorana fermions in hybrid superconductor-semiconductor nanowire devices,” Science 336, 1003–1007 (2012).
[25] Anindy Das, Yuval Ronen, Yonatan Most, Yuval Oreg, Moty Heiblum, and Hadas Shtrikman, “Zero-bias peaks and splitting in an al-inas nanowire topological superconductor as a signature of majorana fermions,” Nat. Phys. 8, 887 (2012).
[26] M. T. Deng, C. L. Yu, G. Y. Huang, M. Larsson, P. Caroff, and H. Q. Xu, “Anomalous zero-bias condutance peak in a nhsnb nanowireb hybrid device,” Nano Lett. 12, 6414–6419 (2012).
[27] Leonid P. Rokhinson, Xinyu Liu, and Jacek K. Furdyna, “The fractional ac josephson effect in a semiconductor-superconductor nanowire as a signature of majorana particles,” Nat. Phys. 8, 795–799 (2012).
[28] A. D. K. Finck, D. J. Van Harlingen, P. K. Mohseni, K. Jung, and X. Li, “Anomalous modulation of a zero-bias peak in a hybrid nanowire-superconductor device,”
O. Bohigas and J. Flores, “Two-body random hamiltonian and level density,” Nat. Phys. 10, 638–643 (2014).

Stevan Nadj-Perge, Ilya K. Drozdov, Jian Li, Hua Chen, Sangjun Jeon, Jungplil Seo, Allan H. MacDonald, B. Andrei Bernevig, and Ali Yazdani, “Observation of majorana fermions in ferromagnetic atomic chains on a superconductor,” Science 346, 602–607 (2014).

Jin-Peng Xu, Mei-Xiao Wang, Zhi Long Liu, Jian-Feng Ge, Xiaojun Liu, Canhua Liu, Zhu An Xu, Dandan Guan, Chun Lei Gao, Dong Qian, Ying Liu, Qiang-Hua Jia, Fu-Chun Zhang, Qi-Kun Xue, and Jin-Feng Jia, “Experimental detection of a majorana mode in the core of a magnetic vortex inside a topological insulator-superconductor $\text{bzig}_{\text{1}}/\text{nbs}_{\text{2}}$ heterostructure,” Phys. Rev. Lett. 114, 017001 (2015).

D. Chowdhury, Y. Werman, E. Berg, and T. Senthil, “A candidate Theory for the “Strange Metal” phase at Finite Energy Window,” ArXiv e-prints (2018), arXiv:1802.04293 [cond-mat.str-el].

Y. Huang and Y. Gu, “Eigenstate entanglement in the Sachdev-Ye-Kitaev model,” ArXiv e-prints (2017), arXiv:1709.09160 [hep-th].

Shenming Huang, Jonathan M. Chudnovsky, and Scott Aaronson, “Quantum advantage using Nearly-Short Quantum Advice,” Nature (2017), 547, 213–216.

P. A. Majaranta and Scott Kirkpatrick, “Solvable model of a spin-glass,” Phys. Rev. Lett. 35, 1792–1796 (1975).

The connection between the cSYK models and gravity in $\text{AdS}_{2}$ was first pointed out in [89].

P. Nayak, A. Shukla, R. M. Soni, S. P. Trivedi, and V. Vishal, “On the Dynamics of Near-Extremal Black Holes,” ArXiv e-prints (2018), arXiv:1802.09547 [hep-th].

Richard A. Davison, Wenbo Fu, Antoine Georges, Yingfei Gu, Kristan Jensen, and Subir Sachdev, “Thermoelectric transport in disordered metals without quasiparticles: The sachdev-ye-kitaev models,” Journal of High Energy Physics 2017, 125 (2017).

Xue-Yang Song, Chao-Ming Jian, and Leon Balents, “Quantum criticality, diffusion and chaos in generalized sachdev-ye-kitaev models,” Journal of High Energy Physics 2017, 125 (2017).
Markus Greiner, Olaf Mandel, Tilman Esslinger, Anffany Chen, R. Ilan, F. de Juan, D. I. Pikulin, and Brian Swingle, Gregory Bentsen, Monika Schleier-Smith, Ippei Danshita, Masanori Hanada, and Masaki Tezuka, Gregor Jotzu, Michael Messer, Rémi Desbuquois, Martin Gartner, Justin G. Bohnet, Arghavan Safavi-I. Danshita, M. Hanada, and M. Tezuka, “How to make R. B. Laughlin, “Anomalous quantum hall effect: An
incompressible quantum fluid with fractionally charged
excitations,” Phys. Rev. Lett. 50, 1395–1398 (1983)
[75] Daniel Arovas, J. R. Schrieffer, and Frank Wilczek, “Fractional statistics and the quantum hall effect,” Phys. Rev. Lett. 53, 722–723 (1984)
[76] A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, and A. K. Geim, “The electronic properties of graphene,” Rev. Mod. Phys. 81, 109–162 (2009)
[77] Y. Aharonov and A. Casher, “Ground state of a spin-1/2 charged particle in a two-dimensional magnetic field,” Phys. Rev. A 19, 2461–2462 (1979)
[78] Yuval Oreg, Gil Refael, and Felix von Oppen, “Helical liquids and majorana bound states in quantum wires,” Phys. Rev. Lett. 105, 177002 (2010)
[79] Roman M. Lutchyn, Jay D. Sau, and S. Das Sarma, “Majorana fermions and a topological phase transition in semiconductor-superconductor heterostructures,” Phys. Rev. Lett. 105, 077001 (2010)
[80] Aaron Chew, Andrew Essin, and Jason Alicea, “Approximating the sachdev-ye-kitaev model with majorana wires,” Phys. Rev. B 96, 121119 (2017)
[81] Liang Fu and C. L. Kane, “Superconducting proximity effect and majorana fermions at the surface of a topological insulator,” Phys. Rev. Lett. 100, 096407 (2008)
[82] D. I. Pikulin and M. Franz, “Black hole on a chip: Proposal for a physical realization of the sachdev-ye-kitaev model in a solid-state system,” Phys. Rev. X 7, 031006 (2017)
[83] Jeffrey C. Y. Teo and C. L. Kane, “Topological defects and gapless modes in insulators and superconductors,” Phys. Rev. B 82, 115120 (2010)
[84] P. W. Anderson, “More is different,” Science 177, 393–396 (1972)
[85] Jordan S. Cotler, Guy Gur-Ari, Masanori Hanada, Joseph Polchinski, Phil Saad, Stephen H. Shenker, Douglas Stanford, Alexandre Streicher, and Masaki Tezuka, “Black Holes and Random Matrices,” JHEP 05, 118 (2017) arXiv:1611.04650 [hep-th]
[86] Juan Maldacena, Douglas Stanford, and Zhenbin Yang, “Diving into traversable wormholes,” Fortsch. Phys. 65, 1700034 (2017), arXiv:1704.05333 [hep-th]
[87] Ioanna Kourkoulou and Juan Maldacena, “Pure states in the SYK model and nearly-AdS2 gravity,” (2017), arXiv:1707.02325 [hep-th]
[88] Juan Maldacena and Xiao-Liang Qi, “ Eternal traversable wormhole,” (2018), arXiv:1804.00491 [hep-th]
[89] Subir Sachdev, “Holographic metals and the fractionalized Fermi liquid,” Phys. Rev. Lett. 105, 151602 (2010), arXiv:1006.3794 [hep-th]