The Model Magnetic Configuration of the Extended Corona in the Solar Wind Formation Region

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Abstract. The coupling between small and large scale structures and processes on the Sun and in the heliosphere is important in the relation to the global magnetic configuration. Thin heliospheric current sheets play the leading role in this respect. The simple analytical model of the magnetic field configuration is constructed as a superposition of the three sources: 1) a point magnetic dipole in the center of the Sun, 2) a thin ring current sheet with the azimuthal current density \( j_\phi \sim r^{-3} \) near the equatorial plane and 3) a magnetic quadrupole in the center of the Sun. The model reproduces, in an asymptotically correct manner, the known geometry of the field lines during the declining phase and solar minimum years near the Sun (the dipole term) as well as at large distances in the domain of the superalfvenic solar wind in the heliosphere, where the thin current sheet dominates and \( |B_\phi(\theta)| = \text{const} \) according to Ulysses observations (Balogh et al., 1995; Smith et al., 1995). The model with the axial quadrupole term is appropriate to describe the North-South asymmetry of the field lines. The model may be used as a reasonable analytical interpolation between the both extreme asymptotic domains (inside the region of the intermediate distances \( \sim (1-10)R_\odot \) ) when considering the problems of the solar wind dynamics and cosmic ray propagation theories.

1. INTRODUCTION

The magnetic configuration of the extended solar corona in the solar wind formation region plays an important role when considering questions related to the analyses of observed solar wind plasma characteristics and energetic particles in the heliosphere. There are no direct measurements of magnetic fields in these regions of space, and available indirect data obtained by remote sensing methods are scarce and sometimes contradictory (see, e.g., Bird and Edenhofer, 1990; Suess, 1993). Theoretical models, generally numerical ones are often based on the use of potential, force-free or other MHD approaches without the sufficient attention to real electric currents. Different theoretical approaches are described in the literature (Schatten, 1971; Gleeson and Axford, 1976; Wang, 1995; Zhao and Hoeksema, 1995).

The purpose of the present paper consists in an attempt to fill the existing gap. A simple analytical model is suggested taking into account qualitatively correctly the main observational information about the structure of the magnetic field in the extended solar corona and the heliosphere during solar minimum years, when the dipole component dominates. A model with the quadrupole term is appropriate to describe the North- South asymmetry.

The crucial point in choosing the model is the requirement that it should reproduce asymptotically correct at least the rough geometrical characteristics of the global three-dimensional magnetic field known at present both near the Sun and far from it.

2. SIMPLEST ANALYTICAL MODEL

We consider the simplest model for solar minimum years, which is represented as a superposition of fields from two sources: a point magnetic dipole in the center of the Sun and an extremely thin ring current sheet in the equatorial plane with the surface current density \( j_\phi \sim r^{-3} \). The analysis made earlier (Veselovsky, 1994) showed that it was a such concentrated current in the heliosphere that created the magnetic field \( |B_\phi(\theta)| = \text{const}, B_\theta = 0 \). There emerged all supporting reasons in favor of the present model for the description of the global structure of the magnetic field due to the Ulysses measurements that showed the \( B_\phi(\theta) \) constancy in the broad range of heliolatitudinal angles investigated up to \( \pm 80^\circ \).

One can neglect the solar rotation in the inner heliosphere, where \( \frac{\Omega R}{V} \ll 1 \). Just this domain we will consider.

In the present model the magnetic field is expressed by formulae

\[
B_r = 2\mu r^{-3} \cos \theta + B_0 \left( \frac{r_0}{r} \right)^2 \text{sign}(\theta - \pi/2) \tag{1}
\]

\[
B_\theta = \mu r^{-3} \sin \theta \tag{2}
\]

where \( \mu \) - the dipole moment, \( |B_0| = \frac{2\pi r_0}{e}|j_\phi| \) - value of the magnetic field created by the surface current
\[ j_\varphi = -\frac{e}{2\pi r^2} \delta(\theta - \pi/2)B_0 \] (Veselovsky, 1994). The jump of the field on the discontinuity equals \(2B_0\).

It is convenient to use the physically plausible quantity \(\Phi = B_0 r_0^2\), simply connected with the magnetic flux, and to express through it the typical length \(a = \frac{2\mu}{\Phi}\) arising in Eq. (1) as well as the typical field \(B_a = \Phi a^{-2}\). Then Eqs. (1), (2) will become

\[
B_r = B_a r^{-2} \left[ \frac{1}{\rho} \cos \theta + \text{sign}(\theta - \frac{\pi}{2}) \right] \quad \text{(3)}
\]

\[
B_\theta = \frac{1}{2} B_a \rho^{-3} \sin B \quad \text{(4)}
\]

where \(\rho = \frac{r}{a}\) is the dimensionless length.

### 3. FIELD STRUCTURE

The equation for field lines \(\frac{dr}{r d\theta} = \frac{B_r}{B_\theta}\) using Eq. (3) and Eq. (4) gives

\[
\frac{d\rho}{d\theta} = 2\rho \left[ \rho \text{sign}(\theta - \frac{\pi}{2}) + \cos \theta \right] (\sin \theta)^{-1} \quad \text{(5)}
\]

Equation (5) is integrated in a straightforward way. Its solution is

\[
\rho = \sin^2 \theta (C - 2|\cos \theta|)^{-1} \quad \text{(6)}
\]

where \(C\) is an arbitrary constant of the integration, which is the parameter of field lines.

Let us consider the obtained solution for two cases: 1) \(\rho < 0\) \((-\infty < a < 0)\); 2) \(\rho > 0\) \((\infty > a > 0)\), corresponding to the existence of the thin current sheet with one or another direction of the ring current. The special case \(|a| = \infty (\Phi = 0)\) corresponds to the field of the point dipole in the absence of any current sheet.

1) Case \(\rho < 0\) \((a < 0)\).

In this case the dipole moment and the magnetic flux of the ring current have identical signs. Zero points of the field are absent.

Any values of the parameter \(C\) are admissible in Eq. (6), \(-\infty < C < +\infty\).

The closed "dipole-like" field lines \(\rho(\theta)\) are obtained for negative \(C\) values, \(C < 0\). These lines start and finish at the coordinate system origin \(\rho = 0\) going along \(\theta \to 0,\pi\). They asymptotically coincide with the lines of the dipole field for \(\rho \to 0\). The line with a given value \(C\) crosses the equator at the distance \(\rho_0 = \frac{1}{C}\), that means \(\rho_0 < \frac{1}{2}\) in the dimensional units. With \(C < \frac{1}{2}\) the lines become more tightened to the equatorial plane in comparison with the lines of the dipole field, but with \(C > \frac{1}{2}\) they are less tightened. The fracture of the field lines takes place when crossing the equator. The near equatorial region \(\theta \approx \pi/2\), occupied by the closed field lines, has an asymptotic thickness 1 in dimensionless units at large distances from the Sun.

The value \(C = 0\) corresponds to the separator

\[
\rho = -\frac{1}{2} \sin \theta |\tan \theta| \quad \text{(7)}
\]

delimiting regions of closed and open field lines. In the present model, this is a near equatorial boundary of two polar coronal holes. Values \(C > 0\) correspond to the interiors of coronal holes, the domain of open field lines, which are diverging as a fan from near polar regions at the small distances \((\rho \to 0\) at \(\theta \to 0,\pi\) according to the dipole law). At large distances when \(\rho \to +\infty\) the lines go out on to the radial asymptotics \(\theta = \theta_{1,2}\) where \(\theta_{1,2} = \text{arccos}(\pm C/2)\). The way out on to the asymptotics \(\theta = \theta_{1,2}\) takes place from angles closer to the pole.

The last open line, which is closest to the equatorial plane (the boundary of the polar coronal hole) passes along the separator corresponding to the asymptotic angle \(\theta_{1,2} = \pi/2\). Figure 1 shows field lines calculated according to Eq. (6) in this case.

2) Case \(\rho > 0\) \((a > 0)\).

The dipole moment and the magnetic flux of the ring current are oppositely directed to each other. The field has two zero points placed on the polar axis. The coordinates of zero points in the coordinate system \((\rho, \theta)\) are \((1,0)\) and \((1,\pi)\). In the coordinate system \((r, \theta)\) these coordinates are \((a,0)\), \((a,\pi)\). All lines cross the equatorial plane. In this case only positive values of the parameter \(C\) are admissible in Eq. (6), \(0 < C < +\infty\).

The closed "dipole-like" lines correspond to the values \(2 < C < +\infty\). They start and finish at the origin of
coordinates, crossing the equatorial plane at the distance \( \rho_c = \frac{1}{C} \) (in the dimensional units \( r_c = \frac{a}{C} \)). The value \( C = 2 \) corresponds to the separator, delimiting the internal volume with closed lines from the external volume with open lines. The equation for the separator is

\[
\rho = \frac{1}{2} \sin^2 \theta (1 - |\cos \theta|)^{-1}
\]

The separator is a surface of the oval shape around the origin of coordinates and passes at the distance \( r = a \) over the poles and crosses the equatorial plane at \( r = \frac{a}{2} \).

The region outside the separator is occupied by open field lines. The values of the parameter \( 0 < C < 2 \) correspond to these lines. All such lines cross the equatorial plane at the distance \( \rho_c = \frac{1}{C} \) and go to the infinity along the radial asymptotical directions \( \theta_{1,2} = \arccos(\pm C/2) \), approaching these asymptotics at \( r \rightarrow \infty \) from the side of the equator. Figure 2 shows the calculated field lines using Eq. (6) in this case.

**4. MORE COMPLICATED CASE**

Let us consider the potential of the magnetic field

\[
\psi = \sum_{l,m,t \geq 1} C_{lm} r^{-(l+1)} Y_{lm}(\theta, \phi) + \frac{\Phi}{r} \text{sign} \theta(\phi)
\]

where \( Y_{lm}(\theta, \phi) \) are the spherical harmonics, \( \theta(\phi) \) is the conic form of the heliospheric current sheet, \( \Phi \) - the open magnetic flux in the heliosphere.

In the simplest axially symmetric case with the only equatorial current sheet \( \theta(\phi) = \text{const} = \pi/2 \) and \( l = 1, m = 0 \), that is in agreement with observed structure of the solar corona in the minimum years. The potential of the magnetic field in the model with a quadrupole term represented as

\[
\psi = \frac{\mu}{r^2} Y_{10} + \frac{\kappa}{r^3} Y_{20} + \frac{\Phi}{r} \text{sign} \theta(\phi)
\]

where \( \kappa \) is the quadrupole moment. The potentials of the magnetic dipole \( \psi_1 \) and the quadrupole \( \psi_2 \) are expressed by formulae

\[
\psi_1 = \frac{\mu \cos \theta}{r^2 - 2}
\]

\[
\psi_2 = \frac{\kappa \theta(\phi)}{r^3}
\]

In Figure 3 we show the global magnetic patterns for dipolar and quadrupolar configurations on the Sun together with the equatorial current sheets \( \theta(\phi) = \text{const} = \pi/2 \) in our model. It is convenient to use for the description of the Figure 3 following quantities: \( \Phi \) - the open magnetic flux, \( \mu \) - the dipole moment, \( \kappa \) - the quadrupole moment, and to express through them the typical independent lengths \( a = -2\mu/\Phi \) and \( b = -2\kappa/\mu \). We consider two additional cases: \( 3) a < 0, b > 0; 4) a > 0, b > 0 \), corresponding to the existence of the thin current sheet with one or another direction of the ring current.

3) Case \( a < 0, b > 0 \).

In this case \( \mu \) and \( \Phi \) have identical signs, and \( \kappa \) is opposite to \( \mu \) and \( \Phi \).

4) Case \( a > 0, b > 0 \).

In this case \( \mu \) and \( \Phi \) are oppositely directed to each other, and \( \kappa \) is opposite to \( \mu \). In Figure 3 the dash line corresponds to the separator, delimiting regions of closed and open field lines.

The quadrupole component decreases more rapidly than the dipole component in the minimum years and at a distance of \( 2R_c \), from the centre of the sun, the dipole component clearly dominates. As an illustration, we have chosen \(|b/a| = |\kappa\Phi/\mu^2| = 0.25 \) in our calculations.

**5. ELECTRIC CIRCUIT**

The existence of an equivalent electric circuit with radial electric currents in the heliosphere was suggested by H. Alfven (1977; 1981). Radial electric currents flowing in the heliospheric current sheet of an order of several \( 10^9 \) A are closed via volume electric currents inside the polar regions.

Axial electric currents in the heliosphere are two orders of magnitude stronger. They are forming a thin ring current sheet around the Sun. The lines of this surface current follow hyperbolic spirals orthogonal to the Archimedean spirals of the magnetic field lines. Inhomogeneous distributions of these surface currents are
FIGURE 3. The magnetic field lines for the case 3 are shown on the left side. In this case the dipole moment $\mu$ and the magnetic flux of the ring current $\Phi$ have identical signs, and the quadrupole moment $\kappa$ is opposite to $\mu$ and $\Phi$. The magnetic lines for the case 4 are shown on the right of the figure. In this case $\mu$ and $\Phi$ are oppositely directed each to other, and the quadrupole moment $\kappa$ is opposite to $\mu$.

manifested as strong heliospheric electrojets or partial ring currents inside streamers (Veselovsky, 1994). Each streamer in the corona contains a ribbon with a strong horizontal current and closure currents flowing around the streamer and partially diverted to the Sun along field lines of the global magnetic field. Field-aligned currents arise in this case in a manner analogous to the situation with the partial ring current in the Earth’s night-side magnetosphere. The streamer belt consists, as a rule, of several strong partial streamers and multiple weaker entities. Hence, the system of inhomogeneous in-flowing and out-flowing field-aligned electric currents appears in both magnetic hemispheres, which should be partially conjugated along global magnetic field lines.

It is important to note, that the main free magnetic energy of the heliosphere is accumulated and available for transformations inside the heliospheric current sheet and its closure currents which are mainly field-aligned.

CONCLUSIONS

We have considered the simple analytical model of the magnetic configuration of the extended solar corona in the solar wind formation region. The model is represented by the superposition of the known asymptotics at the small and the large distances from the Sun. During the years of low solar activity the overall geometry near the Sun in the solar corona is dominated by the large-scale field of the Sun. It is mainly represented by the magnetic dipole slightly or moderately inclined against the solar rotation axis, and the magnetic field of the thin heliospheric current sheet situated near the magnetic equator. The dipole inclination increases with the solar cycle activity phase. Quadrupole and higher harmonics and nonstationary perturbations are especially important during higher solar activity years. The strength of the heliospheric current sheet increases and its geometry is more complicated during this period of time. The overall oblate and curved shape of the minimal corona and more spherical and radial maximal structures are related to the evolution of the electric current inside and outside the Sun with the solar cycle.

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