Gamma-ray burst prompt emission variability in synchrotron and synchrotron self-Compton light curves

Lekshmi Resmi¹,²⋆ and Bing Zhang¹

¹Department of Physics and Astronomy, University of Nevada, Las Vegas, NV 89154-4002, USA
²Department of Astronomy and Astrophysics, Tata Institute of Fundamental Research, Mumbai 400005, India

Accepted 2012 June 15. Received 2012 June 11; in original form 2011 November 26

ABSTRACT
Gamma-ray burst prompt emission is believed to originate from electrons accelerated in a highly relativistic outflow. Internal shocks, which are a result of collisions between shells ejected by the central engine, are the leading candidates for electron acceleration. While synchrotron radiation is generally invoked to interpret prompt gamma-ray emission within the internal shock model, the synchrotron self-Compton (SSC) process is also considered as a possible candidate for the radiation mechanism. In this case, we would expect a synchrotron emission component at low energies, and the “Naked-Eye Burst” GRB 080319B is considered to be such an example. Considering that the gamma-ray light curve of GRB 080319B is much more variable than its optical counterpart, in this paper, we study the relative variability between the synchrotron and SSC components. We develop a ‘top-down’ formalism using observed quantities to infer the physical parameters, and subsequently to study the temporal structure of the synchrotron and SSC components of a GRB. We complement the formalism with a ‘bottom-up’ approach, where the synchrotron and SSC light curves are calculated using Monte Carlo simulations of the internal shock model. Both approaches lead to the same conclusion. Small variations in the synchrotron light curve can only be moderately amplified in the SSC light curve. Therefore, the SSC model cannot adequately interpret the gamma-ray emission properties of GRB 080319B.

Key words: radiation mechanisms: non-thermal – gamma-ray burst: general.

1 INTRODUCTION
Gamma-ray burst (GRB) prompt emission light curves are complex, with superimposed rapid short-scale variabilities. Variabilities of the order of milliseconds in the prompt phase, detected in gamma-rays, have been known since the discovery of the earliest GRBs. This has led to the subsequent proposal of the internal shock model, where the energy in the relativistic flow is dissipated through multiple collisions within the ejecta.

In the internal shock model (Rees & Meszaros 1994), the ultra-relativistic outflow from the central engine (ejecta) consists of a succession of shells with random Lorentz factors. When a fast moving shell (with Lorentz factor Γ_f) collides with one moving slowly (with Γ_s) ahead of it, a pair of internal shocks develops, which dissipates the kinetic energy in the flow. Each pulse in the burst light curve corresponds to one such collision (Kobayashi, Piran & Sari 1997; Maxham & Zhang 2009). The physical parameters of the dissipation region (i.e. the magnetic field and the electron distribution) depend on the masses and initial Lorentz factors of the colliding shells, and the unknown microphysics of relativistic shocks (Daigle & Mochkovitch 1998). Hence, these vary erratically between collisions, and so does the final flux.

In almost all cases, prompt emission has been observed only in the narrow gamma-ray band, until very recently. This has limited our understanding of the underlying emission process. The observed spectra suggest that the radiative process is non-thermal. The most likely candidate is synchrotron radiation. None the less, the synchrotron self-Compton (SSC) process has also been suggested (e.g. Panaiteescu & Mészáros 2000; Kumar & McMahon 2008). For such models, we would expect a synchrotron component to peak in the lower energy band, and we would expect prompt optical emission. In recent years, a few rapidly responding GRB-dedicated optical telescopes (e.g. RAPTOR, TORTORA, ROTSE) have become instrumental in detecting optical emission simultaneous to the GRB (Vestrand et al. 2005, 2006). Most often, these detections have been limited to a few observations in the entire duration of the burst. Nevertheless, in a few cases, it has been possible to establish a temporal correlation between the optical and the gamma-ray light curves, which indicates that their origin is possibly from the...
same dynamical process (Yost et al. 2007; Page et al. 2007). This improved spectral coverage has led to a better understanding of the prompt emission region (Shen & Zhang 2009).

The optical flash of GRB 080319B, seen in unison with the gamma-ray emission, was exceptionally bright (Racusin et al. 2008). Optical prompt emission was observed throughout the entire duration of the gamma-ray component with remarkable time resolution. The onset was simultaneous in both bands and the overall shapes of the light curves were similar; this indicates that the emission in the two bands was possibly physically related. The flux in the V band was almost four orders of magnitude higher than the extrapolation of the gamma-ray spectrum, implying that the two light curves were likely to have originated from two different emission processes.

The fact that the high- and low-energy emission tracked each other but belonged to two different radiative processes naturally led to the conjecture that the optical prompt emission in GRB 080319B was a result of the synchrotron mechanism in the internal shocks and that these photons were upscattered to the gamma-ray band by the SSC process (Kumar & Panaitescu 2008; Racusin et al. 2008). Despite the advantage of interpreting the rough tracking behaviour between the two bands, this model also has several difficulties. For example, the lag of a few seconds between the two light curves is not straightforwardly expected in this model. From several later calculations, it was found that the emission radius required under this scenario will be much larger if internal shocks were to occur (Kumar & Narayan 2009; Zou, Piran & Sari 2009). Another drawback of this model is the energy crisis that occurs because of the presence of the bright second-order SSC component (Bošnjak, Daighe & Dubus 2009; Piran, Sari & Zou 2009; Zou, Piran & Sari 2009).

The non-detection of this second SSC bump in the prompt emission spectra, as observed by the Fermi Large Area Telescope (LAT; e.g. Abdo et al. 2009; Zhang et al. 2011), also places a great constraint on the synchrotron + SSC model. Alternative models to interpret the rough tracking optical/gamma-ray behaviour of GRB 080319B have been proposed. Fan, Zhang & Wei (2009) advanced the idea of a neutron-loaded fireball, where both optical and gamma-ray emission are synchrotron in origin but from two different electron populations, one being the original electrons in the plasma while the other originates from the β-decay process. Yu, Wang & Dai (2009) suggested that a pair of internal forward and reverse shocks could be responsible for the gamma-ray and optical emission, respectively. Acknowledging the difficulty of the simplest internal shock SSC model, Kumar & Narayan (2009) invoked relativistic turbulence to improve the SSC model (cf. Lazar, Nakar & Piran 2009).

One interesting observational feature of GRB 080319B is that its gamma-ray light curve is much more variable than its optical counterpart (Racusin et al. 2008). The time resolution of optical observation is poorer than gamma-rays, but even if we rebinned the gamma-ray light curve to the same temporal resolution as the optical light curve, the gamma-ray light curve still appears much more variable. This feature would give important constraints on the models. For example, the forward/reverse internal shock model (Yu et al. 2009) would predict a similar variability in both the optical and gamma-ray light curves, so it cannot interpret the above feature. However, the two-zone model (Fan et al. 2009) is more consistent, because the optical emission is expected to occur at a larger radius, where the angular spreading time is longer. The synchrotron + SSC model (Kumar & Panaitescu 2008; Racusin et al. 2008) is more difficult to access because the relative variability between the two emission components has not been studied in the past.

In this paper, we study the relative variability within the framework of the synchrotron + SSC model. We approach the problem using two complementary methods. In the ‘top-down’ method, we use the observed optical light curve as the input synchrotron component, we derive the fluctuations in the underlying physical parameters and we self-consistently calculate the SSC light curve. In the ‘bottom-up’ method, we follow the standard formalism to simulate the light curves in the framework of the internal shock model. We generate a set of basic physical parameters using Monte Carlo simulations, we calculate both the synchrotron and SSC light curves and we compare the fluctuations. The aim is to compare the relative variability between the two light curves and then to address whether the observational features of GRB 080319B can be interpreted.

In Sections 2 and 3, we describe our methods and results from the two approaches mentioned above, respectively.

2 LIGHT-CURVE CALCULATION IN THE TOP-DOWN METHOD

We first construct a top-down method where physical parameters are expressed in terms of the observed optical luminosity. This approach enables us to reconstruct the temporal fluctuations of the physical parameters from the structure of the observed synchrotron light curve (Beskin et al. 2010). We use the above physical parameters to estimate the corresponding variability that would appear in the SSC component.

The synchrotron component is the input V-band light curve itself. We have to consistently estimate the SSC component that would have arisen from the optical photons and the electrons that have produced them. This would require knowledge of the bulk Lorentz factor Γ of the outflow, the distance R of the emission region from the centre of the explosion, the comoving magnetic field (B′) in the dissipation region, the electron distribution N(γe) and the ratio γ of SSC to synchrotron luminosity. The temporal structure of the observed optical light curve is the combined effect of the time evolutions of all these parameters. It is difficult, but possible, to disentangle each of these parameters from the optical light curve alone with some simplifications and assumptions.

2.1 Magnetic field in the internal shock region

In the internal shock scenario, magnetic fields are generated by the shocks in the dissipation region (e.g. Medvedev & Loeb 1999). Because this process is poorly understood from theoretical considerations, empirical methods are followed, where the magnetic energy density is assumed to be proportional to the dissipated thermal energy measured in the comoving frame. If Ls is the luminosity of the wind from the central engine, δt is the typical variability time-scale and η is the efficiency of energy dissipation, then the internal energy in the comoving frame can be expressed as (ηLs δt)/Γ, where Γ is the bulk Lorentz factor of the final shell after collision, which enters the expression through frame transformation. Assuming that the shells are of equal mass, Γ can be written as Γ = √ωΓ, where ω = 1/Γ. If the dissipation region is at a distance R from the central engine, the comoving volume can be written as 4πR2 ΔR, where ΔR is the comoving width of the shell. This width can be approximated as ΔR = RT, and the variability time-scale can be expressed as

\[ dt = \frac{R}{2 \Gamma c} \left( \frac{a_s^2}{a_e^2} - 1 \right). \]
This leads to the final expression of the comoving magnetic energy density

$$u_B' = \frac{B'^2}{8\pi} = \epsilon_B \frac{1}{8\pi} \frac{a_B^2 - 1}{a_B^2} \frac{L_w}{\Gamma_1^2 R_c^2}$$

(Zhang & Mészáros 2002).

From theoretical considerations, the bolometric luminosity is essentially related to the luminosity of the outflow (wind) from the central engine, \(L_w\). An amount \(\eta L_w\) of the original wind luminosity is dissipated as internal energy via internal shocks, which is carried mostly by protons. Depending on the interaction between the protons and electrons in the plasma, a fraction \(\epsilon_e\) of this thermal energy is transferred to the random kinetic energy of the electron pool, of which a fraction \(\kappa\) is radiated away. In the fast cooling regime, it is valid to assume that all the kinetic energy available to the electron pool is converted to radiation (\(\kappa = 1\)). Hence, the bolometric luminosity can be written as

$$L_{bol} = \epsilon_e \eta L_w.$$  \hfill (1)

Using equation (1) to replace \(\eta L_w\) in the expression of \(u_B'\), we can express the comoving magnetic field strength \(B'\) as

$$B' \simeq 193 \left(\frac{\epsilon_B}{e}\right) \frac{1}{L_{bol,52}} \frac{a_B^2 - 1}{a_B} \frac{R_{16}}{\Gamma_{300}^2}.$$  \hfill (2)

where \(R_{16}\) is \(R\) in units of \(10^{16}\) cm, \(\Gamma_{300}\) is \(\Gamma/300\) and \(L_{bol,52}\) is the bolometric luminosity in units of \(10^{52}\) erg s\(^{-1}\).

We need to know the bolometric luminosity \(L_{bol}\) in order to calculate \(B'\). In Section 2.2, we describe how \(L_{bol,52}\) can be written in terms of the observed optical specific luminosity \(L_{\nu}\) and other physical parameters.

### 2.2 From observed optical specific luminosity to bolometric luminosity

The bolometric luminosity includes radiation emitted via both synchrotron and SSC processes. The luminosity of the first-order inverse Compton (IC) component can be expressed as \(L_{IC,1} = \gamma_1 \nu L_{\nu}\), and that of the second-order IC component can be written as \(L_{IC,2} = \gamma_2 \nu L_{\nu}\) (Sari, Narayan & Piran 1996; Kobayashi et al. 2007). Here, \(\gamma_1\) is the Compton parameter for the first-order IC scattering, defined by \(L_{IC,1}/L_{\nu}\), and \(\gamma_2\) is the Compton parameter for the second-order IC scattering, defined by \(L_{IC,2}/L_{\nu}\).

Compton scattering between electrons of Lorentz factor \(\gamma\) and synchrotron photons of frequency \(\nu_{syn}\) (measured in the comoving frame of the relativistic ejecta) can be treated in the Thomson regime if \(\nu(\nu_{syn}/m_e c^2) < 1\). Using the characteristic frequency, \(\nu = (\epsilon/2\pi m_e c^2) B' y^2\), of a synchrotron photon emitted by an electron of Lorentz factor \(\gamma\), this threshold leads to a limiting Lorentz factor \(\gamma_{KN} \sim 3500 (B'/1000\ G)^{-1/3}\), above which Klein–Nishina (KN) corrections to the scattering cross-section become important. Alternatively, any SSC photon above a limiting frequency \(\nu_{KN}\), defined as \(\gamma_{KN} m_e c^2\), has undergone the scattering process that took place in the KN regime. The KN limiting frequency for the first-order SSC is \(\sim 300\) GeV \((B'/1000\ G)^{-1/3}\). Hence, the SSC scattering leading to the soft gamma-ray emission can safely be assumed to be in the Thomson regime in the rest frame of the electrons. As a result, we have \(\gamma_1 = \gamma_{TH}\). However, following the same argument, \(\gamma_{KN,2}\) of the second-order SSC scattering between electrons and the first-order SSC photons is \(117.2 (B'/1000\ G)^{-1/3}\) and \(\gamma_{KN,2}\) is \(\sim 18\) GeV \((B'/1000\ G)^{-1/3}\). We find that \(\nu_{KN,2}\) could fall below \(\nu_{in}\) of the second-order IC component, which means that a fraction of the scattering events will require KN correction. As a result, the KN correction could very well be applicable for GRB 080319B, and we are likely to have \(\gamma_2 = \gamma_{KN} < \gamma_{TH}\). Hence, the bolometric luminosity can be written as

$$L_{bol}(t) = (1 + \gamma_{TH} + \gamma_{TH} \gamma_{KN}) L_{\nu}(t).$$  \hfill (3)

Synchrotron luminosity \(L_{\nu}\) can be estimated from the observed \(\nu\)-band specific luminosity \(L_{\nu}\), once we know which spectral regime the optical band belongs to. A precise estimation of this requirement is what this section is concerned with. In order to keep energy requirements reasonable, electrons emitting in the optical band have to be radiatively efficient. Hence, the optical \(V\) band is expected to be in the fast cooling regime of the synchrotron spectrum. We check the consistency of this assumption using the value of \(B'\) obtained later in this section, and we find that this starting assumption is self-consistent. It is also assumed that the optical band is not self-absorbed. At the end of the procedure, we also check the consistency of this assumption. There are only two possible spectral regimes satisfying this condition: (i) \(\nu_c < \nu < \nu_m\) and (ii) \(\nu_c < \nu_m < \nu_V\). For the former, the optical specific luminosity \(L_{\nu}\) can be written as \(L_{\nu} = L_{\nu} (\nu/\nu_m)^{-3/2}\) and for the latter it is \(L_{\nu} = L_{\nu} (\nu/\nu_m)^{-p/2}\), where \(p_m\) is the specific luminosity at \(\nu_m\). For fast cooling electrons, the electron luminosity \(L_{\nu}\) can be approximated as \(L_{\nu} \sim L_{\nu} \nu/2\), and hence, in terms of \(\nu_c,\nu_m\), it can be written as \(L_{\nu} = L_{\nu} \nu/\nu_m\) for \(\nu < \nu_m < \nu_V\).

Hence, the bolometric luminosity in equation (3) is represented in terms of the two \(\gamma\) parameters \(\gamma_{TH}\) and \(\gamma_{KN}\), \(B'\) and \(\gamma_m\) (the latter two parameters entering the expression through \(L_{\nu}\)) as

$$L_{bol} = \begin{cases} (1 + \gamma_{TH} + \gamma_{TH} \gamma_{KN}) L_{\nu} \sqrt{\nu/\nu_m} & \text{for } \nu_c < \nu < \nu_m \\ (1 + \gamma_{TH} + \gamma_{TH} \gamma_{KN}) L_{\nu} (\nu/\nu_m)^{p/2} & \text{for } \nu_m < \nu < \nu_V \end{cases}.$$  \hfill (4)

From equations (2) and (4), we can see that \(L_{bol}\) and \(B'\) depend on each other. We essentially require \(B'\) in the rest of the formalism. This can be obtained by algebraically solving equations (2) and (4), and it can be expressed in terms of other physical parameters. The final expressions depend on the Compton parameter \(\gamma_m\), which we derive in Section 2.3. The parameter \(\gamma_m\) also appears in the final expressions; we use it as the input parameter (see Section 2.4).

### 2.3 Calculation of \(\gamma\) parameters

In the top-down method, the Compton-\(\gamma\) parameters enter the expression of the comoving magnetic field through \(L_{bol,52}\). They are also required in the calculation of the cooling frequency. Before deriving these parameters, we first introduce the relation between the Compton parameters and the ratio \(\epsilon_e/\epsilon_B\), which is an important equality we use throughout the formalism. It is used when deriving the expressions in Appendix A, and also when obtaining the \(\gamma\) parameters in terms of \(\gamma_m\).
2.3.1 Relation with $\epsilon_e/\epsilon_B$

For the first-order scattering, the Compton $Y$ parameter can be considered as the ratio of the SSC to synchrotron luminosity, which can be estimated as the ratio of the energy in the seed photon field $U_{syn}$ to that in the magnetic field $U_B$. For the second-order scattering, it would be the ratio between the luminosities of the second- and first-order IC components. Here, $Y$ is equivalent to

$$Y = \frac{U_s}{U_B (1 + \gamma + Y^2)}$$

(Sari et al. 1996; Kobayashi et al. 2007) in the limit where first- and second-order SSC scattering are both in the Thomson regime. In the fast cooling regime, where $\kappa$ is nearly unity, this leads to the relation, $\epsilon_e/\epsilon_B = Y(1 + \gamma + Y^2)$. However, if the second-order IC is in the KN regime (as is the case for our scenario in many runs), this expression is modified to

$$\frac{\epsilon_e}{\epsilon_B} = Y_{th}(1 + Y_{th} + Y_{th}Y_{KN}).$$

(5)

2.3.2 For the first-order scattering: $Y_{th}$

We first estimate $Y_{th}$, the Compton-$Y$ parameter in the Thomson regime, valid for the first-order IC scattering. For a fast cooling synchrotron spectrum, if $v_c < v < v_m$, the optical specific luminosity $L_V$ can be expressed in terms of the peak luminosity $L_{\text{max}}$ (at $v = v_m$) as

$$L_V = \sqrt{\frac{v}{v'}} L_{\text{max}},$$

(6)

where $L_{\text{max}} = N_{\text{rad}}(\sqrt{3}e^2B'/m_ee)^2\Gamma$ (Wijers & Galama 1999).

In terms of the total (both synchrotron and SSC) bolometric luminosity $L_{bol}$, $L_V$ can be expressed as

$$L_V = \frac{L_{bol}}{1 + Y_{th} + Y_{th}Y_{KN}} \sqrt{\frac{v}{v'}} v_m,$$

(7)

(here we have made use of equation 4).

After substituting for $L_{\text{max}}$, $v_m = [\epsilon(2\pi m_e c)B'/m_ee]^2$ and $v_c = [\epsilon(2\pi m_e c)B'/m_ee]^{-1}$, we obtain the relation

$$\frac{L_{bol,52}}{B'^2} = 1.95 \times 10^{-11} N_{\text{rad,52}}(1 + Y_{th} + Y_{th}Y_{KN}) Y_m \Gamma_{300^2}.$$  

(8)

Because $L_{bol,52}/B'^2$ can be substituted as $2.68 \times 10^{-5} \sigma_T R_{16}^2 \Gamma_{300}^2$ (using equation 2) and $(1 + Y_{th} + Y_{th}Y_{KN})$ can be replaced by $(1 + Y_{th} Y_{KN})^{1/2}$ (using equation 5), we arrive at

$$Y_{th} = 2.2 \times 10^{-6} \frac{a_g}{a_e} \frac{N_{\text{rad,52}}}{R_{16}} Y_m \gamma_c.$$  

(9)

Using the above expression, an extra parameter can also be obtained for $v_m < v < v'$. We should mention that, in the standard bottom-up approach, we start from the above expression, derived by integrating the scattering cross-section over the electron energy spectrum assuming fast cooling electrons (Pananitsis & Kumar 2000; Kumar & McMahon 2008), we substitute for $Y_m$ in terms of $\epsilon_e$ and $\gamma_c$, in terms of $\epsilon_B$, and we arrive at equation (5).

2.3.3 For the second-order scattering: $Y_{KN}$

For the second-order scattering, which is likely to be in the KN regime, the evaluation of the $Y$-parameter is more complex. This is because, unlike in the Thomson regime, the scattering cross-section depends on the Lorentz factor of each electron involved in the scattering. A full numerical calculation is required to obtain an exact estimate of $Y_{KN}$. In this paper, we adopt an analytical formalism where approximate estimates of the reduction due to the KN effect are used, in both the scattering cross-section and the typical energy gain of the photons. Because, in the SSC regime, the $F_{\nu}$ peak is at $v_{\text{IC}}$ for a fast cooling spectrum, the reduction in the effective scattering cross-section can be scaled down as

$$R_s = \frac{\sigma_{KN}(Y_m)}{\sigma_f}.$$  

(10)

where

$$Y_m = \gamma_m \frac{h v_{\text{IC}}^{1/2}}{300^2 300 m_e c^2}$$

is the normalized energy of a first-order SSC photon in the rest frame of the relativistic electron with Lorentz factor $\gamma_m$. Here, $v_{\text{IC}}$ is calculated as $2\gamma_m^2 v_m$. For $x_{th} \gg 1$, we can approximate $R_s$ to be $(3/8) x_{th}^2 + (1/2)].$

We can write $Y_{KN}$ as $Y_{th} R_s \Delta E_R$, where $\Delta E_R$ is the ratio between the average energy gain for the photon in the second-order scattering to that in the first-order scattering. The average gain in the second-order scattering can be approximated as $(Y_{KN} m_e c^2)/(h v_{\text{IC}}^{1/2})$. We divide this by the typical energy gain in the first-order scattering $Y_{th}$ to obtain $\Delta E_R$. Using the expression $Y_{KN,2} = 117.2(B'1000G)$ from Section 2.3, and substituting $2 \times 2.8 \times 10^8 h \nu_{\text{IC}}$ for $v_{\text{IC}}^{1/2}$, we can reduce $\Delta E_R$ to $4 \times 10^{13} Y_{KN,2}(B'1000G)$. Hence, when $KN$ corrections are strong (i.e. for $x_{th} \gg 1$), the final expression for $Y_{KN}$ is

$$Y_{KN} = Y_{th} \frac{3}{8 x_{th}} \left(\log 2 x_{th} + \frac{1}{2}\right) \times 4 \times 10^{13} Y_{KN,2}.$$  

(11)

In our calculations, we assume that $Y_{KN}$ is $Y_{th}$ if $x_{th} \ll 1$; otherwise, we use the above expression. It is not possible to analytically estimate the correction when the KN effect is moderate ($x_{th}$ is a few), and hence we use the two asymptotic estimates.

Substituting for $Y_{KN}$, we can rewrite equation (5) in terms of $Y_{th}$, $\epsilon_e/\epsilon_B$ and $R_s \Delta E_R$. This is detailed in Appendix A. In Fig. 1, we present the behaviour of the $Y$ parameters.

If the second-order scattering is in the Thomson regime, both the first- and second-order $Y$ parameters will be the same and they will depend only on the ratio $\epsilon_e/\epsilon_B$. However, if the second-order scattering is affected by KN effects, the first-order $Y_{th}$ will depend on $\epsilon_e/\epsilon_B$ and the term $R_s \Delta E_R$. This term signifies the extent of the KN effect. (For scattering in the Thomson regime, it is unity. The higher the value of $Y_m$, the smaller $R_s \Delta E_R$ will be.) We have estimated $Y_{th}$ for a range of $\epsilon_e/\epsilon_B$ and $R_s \Delta E_R$; we find that $Y_{th}$ has a strong dependence on the value of $\epsilon_e/\epsilon_B$, and that the dependence on $R_s \Delta E_R$ is fairly weak. Only in cases of very high $\epsilon_e/\epsilon_B$ does the dependence on the second term become important. In addition, it is easy to note from equation (8) that for a given value of $\epsilon_e/\epsilon_B$, a lower value of $Y_{KN}$ – which implies a stronger KN effect and a larger value of $x(Y_m)$ – will result in a higher value of $Y_{th}$.

2.4 Sequential steps to estimate SSC emission

In order to calculate the SSC component, we require the final expressions of $B'$, $N_{rad}$, $Y_m$, $Y_{th}$, and $Y_{KN}$ (we call these class 1 parameters), in addition to the input parameters (which we call class 2 parameters) $a_g$, $\delta$, $\Gamma$, $p$ (which is required only for the spectral regime case ii) and $\epsilon_e/\epsilon_B$. We can see from the above sections that the five quantities in class 1 are all interconnected. The key is to find a way to disentangle them, to start from one and to arrive at all the remaining quantities.
A random number generator is used to determine the value of \( \Gamma_s \) for a given pulse. We compute \( R_{th} \) using the relation of radius \( R = 2c\delta \Gamma_s^2 a^2_\epsilon / (a^2_\epsilon - 1) \), and we calculate \( \Gamma_{300} \sim \sqrt{a^2_\epsilon \Gamma_s} / 300 \). Across the burst, the typical fractional variation in \( \Gamma_s \) is \( \delta \Gamma / \Gamma_s \sim 1/20 \) to 1/10. In a given simulation run, \( a_\delta \) and \( \delta \tau \) are kept constant, \( \delta \tau \) is kept somewhere in the range of 0.5–2.0 and \( a_\delta \) is fixed around 2–5. In a given run, their values are chosen such that the variation in \( R_{th} \) for the burst could be up to a factor of 3 to 4, to reproduce the range of radii in which shocks occur (Daigne & Mochkovitch 2000). Between various realizations of the simulation, \( \Gamma_s \) varies from 50 to 500. The resultant range of \( \Gamma_{300} \) through multiple runs is \( \sim 1/5 \). Multiple simulations result in a large range of \( R_{th} \), from 0.01 to \( \sim 1 \).

Because \( \gamma_m \) is roughly constant for a given pulse during shock crossing, we use \( \gamma_m \) as the input parameter, and we randomly assign its value for a given pulse. The range of \( \gamma_m \) varied from \( \sim 50 \) to \( \sim 500 \) during the runs. Whenever the optical frequency fell below \( \nu_m \), we moved to another set of parameters. Depending on the other parameters, \( \epsilon_f \) was varied such that \( \gamma_{th} \) was around 10.

Equation (A6) connects \( \gamma_{th} \) with \( B^\prime \) and \( \gamma_m \). However, as we can see from its detailed expression given in Appendix A, \( B^\prime \) depends on \( \gamma_m \) and \( \gamma_{th} \). Other than this, only the quantities in class 1 enter these two expressions. Hence, we can use the expressions of \( B^\prime \) in Appendix A and we can write equation (A6) in terms of \( \gamma_m \) alone. However, we are not presenting this long expression (in fact, one expression each for the two spectral regimes, \( \nu_V < \nu_m \) and \( \nu_V > \nu_m \)). The sequential steps that we follow in estimating the SSC emission are as follows.

Step 1. \( \gamma_{th} \) is obtained from equation (A6) for a given \( \gamma_m \) and class 2 parameters using non-linear root-finding algorithms.

Step 2. Now we know both \( \gamma_m \) and \( \gamma_{th} \), so \( B^\prime \) can be calculated.

Step 3. Knowing \( B^\prime \), \( \gamma_{th} \) and \( \gamma_m \), we can now calculate \( \gamma_{KN} \) from the expression in Appendix A. In the code, the possibility that the second-order scattering is in the Thomson regime is checked by monitoring the value of \( x_{\gamma_m} \), once \( B^\prime \) is estimated. In this case, the quantities are re-estimated.

Step 4. Once we know \( \gamma_{th} \), \( \gamma_{KN} \) and \( B^\prime \), we can calculate \( \gamma_c \). If \( \gamma_c \) falls below unity, we set it to unity. Because we know \( \nu_V \) and \( \nu_m \), we can now carry out a self-consistency check for the synchrotron spectral regime, and we can see whether our initial assumption of fast cooling and the location of the optical frequency is valid or not. If the consistency is violated, we redo the calculations with a different set of parameters (especially \( \gamma_m \) values).

Step 5. Finally, we use equation (9) to estimate the value of \( N_{rad} \) with \( \gamma_m \), \( \gamma_c \) and \( \gamma_{th} \).

Thus, we know all the quantities in class 1, and we can compute the SSC light curve. The characteristic frequencies are estimated as \( \nu_B^c = 2\gamma^2_c \nu_v \) and \( \nu_{mB}^c = 2\nu^2_m \nu_v \), and the spectral normalization at \( \nu = v^B_v \) is estimated as \( L_{\nu,max} \gamma_{th} / (\gamma_{mB} \gamma_c) \). We follow Gupta & Zhang (2007) in calculating the SSC spectrum. We assume that the flux decays following the curvature effect, by \( (t/t_B)^{-\beta} + \sigma \) (where \( \beta \) is the spectral index), after the peak \( t_{\nu_B} \) of each pulse (Fenimore, Madras & Nayakshin 1996; Kumar & Pannitsch 2000; Liang et al. 2006; Zhang et al. 2006).

### 2.5 Measure of variability

An important point to note is that, for GRB 080319B, the time resolution of the Burst Alert Telescope (BAT) light curve is almost \( \sim 20 \) times higher than that of the V-band light curve from TORTORA (Racusin et al. 2008). Many individual pulses and fine features might have been lost in the coarse resolution of the optical light curve. Therefore, a direct comparison between our synthesized SSC light curve and the BAT light curve is not entirely appropriate. We can only make a qualitative study on the extent of amplification possible by the SSC process. Our aim is to see how small-scale fluctuations of the synchrotron light curve would appear in the SSC component, and how variations in the electron distribution can control their appearance.

The template for our analysis is the optical V-band light curve of GRB 080319B. However, because our aim is to see whether the high variability of its gamma-ray light curve is a result of the enhancement of small-scale synchrotron fluctuations by Compton upscattering, we synthesize miniscule fluctuations by adding random Gaussian fluctuations at mid-points after interpolating the light curve. We synthesize multiple light curves using Gaussian distributions with different widths. We first test our method in the original optical light curve from TORTORA, and later we use the new synthesized light curves. In Fig. 2, we present one of the template light curves with synthesized fluctuations along with the original light curve.
There are a total of eight pulses in the light curve, of which three are so minuscule that they would not have been discerned as independent pulses typically. Observations exist during the tail only for the third, fifth, seventh and eighth pulses. To obtain the beginning of the pulse \( t_0 \), we fit the decay part of these pulses with the temporal profile expected from the curvature effect, which requires the tail to fall as \((t - t_0)^{-2}\). For the eighth pulse, which has good sampling over the tail, we could constrain the values of both \( t_0 \) (18.0 ± 14.0) and the index \((-2 - \beta = 2.5 ± 1.1\)). For the other three pulses, we have assumed \( \beta \) to be around 0.5 and we have obtained the value of \( t_0 \). For those pulses for which we could not run a fitting routine, we used the same value of \( t_0 \) as its nearest pulse. It should be noted that the optical spectral index can be much steeper, especially because of the large spectral index observed in gamma-rays.

We tune \( \gamma_m \) in order to control the pulse amplification in SSC. For the above synchrotron light curve, there will be a set of eight \( \gamma_m \) values as input. The spectral regimes where the optical and gamma-ray bands fall play a crucial role in the nature of the output gamma-ray light curve. Given the same distribution of \( \Gamma_{300} \) and \( R_{16} \), we see that if \( \nu_V \) is between \( v_m \) and \( v_p \), and if \( \nu_p \) is between \( v^{IC}_m \) and \( v^{IC}_p \), the two light curves become nearly identical (even with a large variation within the input \( \gamma_m \) set), because of the same functional dependence both the synchrotron and SSC fluxes have on \( \gamma_m \). This is true for the case where \( \nu_m < \nu_V \) and also \( v_m < v_p \). The best contrast is possible if \( \nu_V < \nu_p \), or \( \nu_p < v^{IC}_p \). This is because of the additional dependence that the SSC flux has on \( \gamma_m \).

Using the expression for characteristic frequencies, it is easy to see that \( \nu_V \) will be below \( \nu_m \) if \( B'\gamma^{2}_{IC} > 2 \times 10^{11} \). Similarly, \( \nu^{IC}_m \) will be below 650 keV if \( B'\gamma^{2}_{IC} < 5 \times 10^{9} \). Because in our formalism \( B' \) depends on \( \gamma_m \) and other input parameters (class 2 parameters), this basically reduces to a range of \( \gamma_m \) for given class 2 parameters and the input synchrotron luminosity \( L_p \). However, because the \( \gamma_m \) dependence of \( B' \) comes through the non-linear expression in terms of \( \gamma_m \), we cannot give an analytical expression describing the range.

Amplification or quenching of the SSC pulse will depend on the input \( \gamma_m \) of that pulse. An analytical expression for the \( L_p - \gamma_m \) relation is difficult to obtain because of the non-linear nature of the algorithm. We observe the variation of \( L_p^{SSC} \) for a large range of \( \gamma_m \) values, and we see that the pulse peak \( L_p \) is roughly proportional to \( \gamma_m^{3.3} \). Hence, the ratio between two adjacent peaks \( L_{pi}/L_{pi+1} \propto (\gamma_m^{p+1}/\gamma_m^p)^{3.3} \) if \( \Gamma_{300} \) and \( R_{16} \) do not vary much between pulses. Ideally, any contrast between adjoining pulses can be produced by changing \( \gamma_m \), but the pulse profile will appear unnatural with sharp dips and rises for stark contrasts. The variability of the SSC light curve depends on the distribution of \( \gamma_m \) through pulses. In Fig. 3, we present a sample of gamma-ray light curves that we obtained along with the input optical synchrotron light curves. In the first panel, for comparison, we have also given the output synchrotron light curve along with the input light curve (the only difference is the decay determined by the curvature effect for the synthesized light curves).

In order to have a quantitative measure of the variability of the light curves, we estimate the standard deviation of the burst from an average profile that best imitates the burst profile. For this purpose, we construct a trapezoidal function with a rising part, a plateau and a tail. The function is determined by five parameters: normalization \( f_0 \) and nodal points \( a, b, c \) and \( d \). The rising part of the function is given as \( f(t) = f_0(t - a)/(b - a) \), the plateau is the constant \( f(t) = f_0 \) and the tail is \( f(t) = f_0(d - x)/(d - c) \). After each run, we fitted the entire output SSC light curve with the trapezoidal function by varying the five parameters mentioned above, and we estimated the standard deviation from the best-fitting trapezoid. We compare this value with the standard deviation obtained for the optical light curve, which is obtained by following the same method.

For a given distribution of \( \Gamma_{300} \) and \( R_{16} \) across the pulses, the best SSC amplification occurs when \( v_c < \nu_V < \nu_p \) and \( v^{IC}_c < v^{IC}_m < \nu_p \). The spectral regime \( \nu_V < v^{IC}_c < v^{IC}_m \) can also produce similar amplification, but for \( v^{IC}_m \) to be below ~650 keV we require a very small (~1–10 G) comoving magnetic field. Only alternative scenarios involving the quick decay of magnetic fields in the shock downstream (e.g. Pe’er & Zhang 2006) can achieve such low magnetic fields for standard input parameters. Moderate amplification can also be obtained if \( v_c < \nu_p < v^{IC}_p \) and \( v^{IC}_c < v_p < v^{IC}_m \). However, this spectral combination also requires the comoving magnetic field to be too low. We could not find any condition where large
variability amplification is possible. Hence, we conclude that variabilities in the synchrotron light curve can be moderately amplified in the SSC light curve.

3 BOTTOM-UP APPROACH AND SIMULATED LIGHT CURVES

To understand the variability better and also to have a complete picture, we approach the problem from the opposite end. We simulate the burst light curves from the bottom-up method, where the final flux is built up from the input physical parameters of the emitting plasma. We assume the internal shock framework where the burst light curve is the sum of several independent pulses produced from random collisions between shells. We use a toy model where the temporal structure of a single pulse is determined by the evolution of the wind luminosity alone. However, more detailed modelling of the internal shock dynamics, including hydrodynamic simulations, has been done previously (Daigne & Mochkovitch 2000).

We consider an isotropic wind luminosity $L_w(t)$, which varies between $t_0$ and $t_p$ as a shallow (as $t^\delta$ with $\delta$ between 0.5–1.0) function of the observed time $t$ for a given pulse (i.e. for one collision).
Here, $t_0$ is the time of collision and $t_\gamma$ is the shock crossing time (also the pulse peak). The total number of radiating electrons $N_e$ is
\[ N_{\text{tot}} = \frac{L_{\text{w},52}}{450(\delta + 1)\Gamma_{300}^2}. \] (12)

The comoving magnetic field ($B'$) is calculated by assuming that a fraction $\epsilon_B$ of the shock-created thermal energy ($\theta_P m_e c^2$) will be carried by the magnetic field, where $\theta_P$ is the internal Lorentz factor of the shocked shell. The downstream magnetic field density $u_B = (N_e/V')\theta_P m_e c^2$, where $N_e$ is the number of protons ejected, $\sim N_x$, and the comoving volume $V' = 4\pi r^2 \Delta r$, with the thickness $\Delta r$ approximated as $ct \Gamma_{300}$. This leads to
\[ B' = 473 \sqrt{\frac{L_{\text{w},52} \theta_P c}{(\delta + 1)\Gamma_{300}^2}}. \] (13)

The minimum Lorentz factor $\gamma_m$ of the shock-accelerated electron distribution is assumed to be
\[ \gamma_m = \epsilon_e (m_p/m_e) \theta_P, \] (14)
where $\epsilon_e$ is the fractional energy carried by these electrons.

The total number of radiating electrons, $N_{\text{rad}}$, is different from $N_{\text{tot}}$ if the plasma is in ‘severe fast cooling’, where the time-scale ($t_{\gamma,1}$) for electrons to cool down and lose all their kinetic energy is less than the dynamical time-scale. In this case, a non-negligible fraction of electrons would pile up at $\gamma$ of unity and they would not be available in the relativistic pool to radiate via the non-thermal processes. This fraction keeps on increasing as the source ages. An exact estimate requires solving the continuity equation involving the electron injection and radiative losses, which is beyond the scope of this paper. When $t_{\gamma,1} \ll t_{\text{dyn}}$ (severe fast cooling), the fraction of electrons remaining in the power law can be assumed to be $t_{\gamma,1}/t_{\text{dyn}}$. Hence, the total number of radiating electrons, $N_{\text{rad}}$, is
\[ \phi_{\text{PL}} = \min \left( 1, \frac{t_{\gamma,1}}{t_{\text{dyn}}} \right). \] (15)

Here, $t_{\gamma,1}$ is the time-scale measure in the comoving frame for an electron to cool down to $\gamma \sim 1$, which can be roughly expressed as
\[ \frac{6\pi m_e c}{\sigma_T} \left( 1 + \frac{\gamma_{\text{th}}}{\gamma} + \frac{\gamma_{\text{th}}^2}{\gamma} \right) B'. \]

The derivation is given in Appendix B. It is relevant now to derive the expression for the cooling break in the electron spectrum $\gamma_c$, with $\gamma_c \rightarrow 1$ in the severe fast cooling regime. During the normal fast cooling, it can be expressed as $t_{\gamma,1}/t_{\text{dyn}}$, which is same as $\phi_{\text{PL}}$. Hence, we have the following:
\[ \gamma_c = \begin{cases} \phi_{\text{PL}} & \text{if } t_{\gamma,1} \gg t_{\text{dyn}} \text{;} \\ 1 & \text{if } t_{\gamma,1} \ll t_{\text{dyn}}. \end{cases} \]

The Compton $Y$ parameter for the first-order SSC scattering is calculated as $10^{-6} N_{\text{rad},52} \gamma_m \gamma_c / R_{16}^2$ (equation 9) because $N_{\text{rad}}$ and $B'$ are known. The $Y$ parameter for the second-order SSC scattering is estimated by equation (11).

We calculate the light curves in both slow and fast cooling cases, unlike in the previous approach. The synchrotron and SSC spectral breaks and peak flux are now calculated following the standard procedure. The synchrotron and SSC spectrum are calculated as piecewise power laws, as described in Section 2. After the pulse peak (shock crossing time), we calculate the flux decay using the curvature effect.

We ran a Monte Carlo simulation where the luminosity normalization, temporal index $\delta$, and $\theta_P$ were considered as random variables with a uniform distribution. The luminosity normalization and $\theta_P$ range between the typical values of $1.0$–$15.0$ and $3.0$–$10.0$ respectively. There is no clear idea about the values of $\delta$, either from observations or theory. A time-independent wind luminosity cannot produce a rising, fast-cooling synchrotron pulse, and hence we have chosen a shallow range of $0.5$–$1.0$ for $\delta$. We also consider the shock crossing time (or pulse peak) to be distributed randomly between $0.5$ and $2.0$. Each pulse thus generated is later shifted by a random $\theta_P$ value. The final burst profile is the sum of all these individual pulses (see Fig. 4). As in the top-down formalism, we scan the typical ranges of the parameters: $R_{16}$ is scanned from $0.01$ to $1.0$ and $\Gamma_{300}$ from $0.3$ to $3$. In a single simulation, the radius and $\Gamma$ are changed by a small factor. A range of $0.01$–$0.1$ in $\epsilon_e$ and a range of $10^{-5}$–$5 \times 10^{-4}$ is scanned in $\epsilon_B$. In a single run, these parameters are kept fixed. The value of $\rho$ is fixed at $2.2$. Even though the typical

![Figure 4](https://example.com/f4.png)

**Figure 4.** Typical synchrotron (left) and SSC (right) burst light curves from the simulations. The individual pulses are also shown. The typical time resolution we have is $\sim 0.01$ s in the observed frame, so that is the limit of the sharpest features from the simulation. In this simulation, we have used $\epsilon_e = 0.01$, $\epsilon_B = 10^{-4}$ and $\rho = 2.2$. We have varied $\Gamma_{300}$ from $1.0$ to $2.5$, $R_{16}$ from $0.08$ to $0.1$, $\theta_P$ from $3.3$ to $5.5$ and $\delta$ from $0.5$ to $0.9$. We kept $L_{w,52}$ at $15(t/1\text{ s})^6$. 

© 2012 The Authors, MNRAS 426, 1385–1395 Monthly Notices of the Royal Astronomical Society © 2012 RAS
temporal index for the rising phase of an individual pulse is 3/4, if the observed frequency is between $v_e$ and $v_{\infty}$, a much sharper observed rise in profiles is possible because of the cumulative effect of multiple pulses.

### 3.1 Measure of variability

We estimate the variability following the same method as in the top-down scenario. We obtain the standard deviation of both synchrotron and SSC light curves from the best-fitting trapezoid. We find that the SSC light curves are, at best, only slightly more variable than the synchrotron light curve. With some set of parameters, the SSC variability can even be less than the synchrotron variability. This is because the overall flux variation in the SSC is much less than its synchrotron counterpart. There are a number of extra spikes that, even if present in the SSC, do not contribute much to the estimated variability.

The luminosity normalization and $\theta_p$ have more influence in the amplification of the pulses compared to $\delta$. We made runs by keeping $\delta$ as a constant and also by allowing it to vary. The low variation in both $L_m$ and $\theta_p$ and constant $\delta$ result in nearly identical pulse shapes between synchrotron and SSC components. We did not find any marked difference between the results from slow and fast cooling cases.

### 4 SUMMARY AND DISCUSSION

In this paper, we have carried out a comparative study of the light-curve variability of the synchrotron and SSC components of GRB prompt emission. Starting from a template synchrotron light curve with small-scale fluctuations, we have traced back the magnetic field of the emitting region and we have self-consistently calculated the SSC light curve, using assumptions for the electron distribution. We have investigated how the small-scale fluctuations in the SSC light curve are related to the initial variabilities present in the synchrotron light curve. A multitude of temporal structures can be obtained for the SSC light curve, depending on the parameters of the electron distribution function. The degree of modification of the input fluctuations is different for different spectral regimes within the SSC spectrum, because of the varying sensitivity to $\gamma_m$. Minuscule changes in the electron distribution, which appear as indiscernible pulses in the synchrotron light curve, can become moderately amplified in the SSC light curve. Hence, in general, the SSC light curve can be more variable compared to the synchrotron light curve, but not to a large extent. We have complemented the formalism with a bottom-up approach, where the synchrotron and SSC light curves are calculated using a Monte Carlo simulation of the internal shock model. Only moderate amplification to the variabilities could be obtained in this approach too.

We have applied our method to the naked-eye GRB 080319B. This burst has been interpreted within the framework of the synchrotron + SSC model (Kumar & Panaitescu 2008; Racusin et al. 2008). Several difficulties have been found for this model, including the time lag between the gamma-ray and optical light curves, as well as the energy budget crisis from the second-order SSC. Here, we have applied another criterion, the relative variability between the two components, in order to investigate the validity of the model. We have created a synchrotron template based on the observed optical data of GRB 080319B (with minor modifications to test how small variabilities are amplified), and we have calculated the expected gamma-ray variabilities in various temporal regimes. We have found that the model light curves are all smoother than the observed light curve. We have concluded that the optical/gamma-ray light curves are difficult to account for within the simplest synchrotron/SSC model.

This conclusion is reached based on our analytical formalism presented in this paper. For an analytical treatment to be possible, we have to adopt some approximations and/or make some assumptions. We conclude by listing them here, and commenting on their validity.

- (i) The total bolometric luminosity is a fraction of the power dissipated by the internal shocks, which in turn is a fraction of the wind luminosity from the central engine. Such a treatment has been adopted in all internal shock model calculations in the past.
- (ii) The ratio $\epsilon_e/\epsilon_B$, indicating the fractional energy in electrons and in the magnetic field, respectively, is assumed to be a constant. Numerical simulations have started to derive $\epsilon_e$ and $\epsilon_B$ from the first principle, but none has revealed how these parameters depend on shock parameters. In principle, both values might evolve with shock parameters. However, because little understanding is achieved regarding such an evolution, we take the simplest assumption that the ratio $\epsilon_e/\epsilon_B$ is constant. We note that in afterglow modelling, the assumption of a constant $\epsilon_e$ and $\epsilon_B$ throughout the deceleration phase seems to fit the observed data well.
- (iii) The bulk Lorentz factor $\Gamma$ and the emission radius $R$ are allowed to randomly vary from pulse to pulse. By doing so, we have assumed that multiple collisions occur at different radii between shells of varying Lorentz factor. This variation did not significantly affect the relative variability contrast between the synchrotron and SSC light curves.
- (iv) In both top-down and bottom-up models, we have not considered the evolution of $R$ and $\Gamma$ in one collision.
- (v) In the top-down method, the underlying electron distribution is approximated to be in the fast-cooling regime (i.e. even the cooling time of the electrons with the lowest injection energy is shorter than the dynamical time-scale). Such an approximation is found to be self-consistent given the typical value of $B$ that we derive. In the bottom-up method, both slow and fast cooling conditions are tested.
- (vi) The power-law index of the injected spectrum is assumed to be 2.2, which is a typical value for relativistic shocks. Our results do not significantly depend on this value. Changing it to a different value would not affect our conclusion.
- (vii) The synchrotron spectrum is approximated as a multig畔 broken power law, representing different spectral regimes. The analytical treatment of SSC (Gupta & Zhang 2007) is adopted. In the expression for the peak luminosity of the synchrotron spectrum, we have neglected a correction factor that takes care of the contribution from the power-law electron distribution (Wijers & Galama 1999; Gupta & Zhang 2007). The SSC peak luminosity is assumed to be $3\nu_m$ times the synchrotron peak. All these are standard analytical treatments of synchrotron and SSC emission spectra. More realistic treatments would not affect the variability contrast in the synchrotron and SSC light curves.
- (viii) For the second-order scattering in the KN regime, the $\gamma'$ parameter is approximately estimated by scaling down its value for the first-order scattering in the Thomson regime by a factor invoking the ratio between the KN cross-section and the Thomson cross-section at the electron injection energy.
- (ix) The assumption of a fast cooling spectrum leads to the definition of the bolometric synchrotron luminosity $L_{\text{syn}} = L_{\gamma_m}v_m$. An accurate treatment would introduce a correction factor of the order of unity, but would not affect our conclusion.
ACKNOWLEDGMENTS

We thank the anonymous referees for constructive comments that have improved the quality of the paper. This work is supported by NASA NNX09AO94G, NNX10AD48G and NSF AST-0908362. LR acknowledges support from the French Agence Nationale de la Recherche via contract ANR-JC05-44822.

REFERENCES

Abdo A. A. et al., 2009, ApJ, 706, L138
Beskin G., Karpov S., Bondar S., Greco G., Guarnieri A., Bartolini C., Piccioni A., 2010, ApJ, 719, L10
Bošnjak Z., Daigne F., Dubus G., 2009, A&A, 498, 677
Daigne F., Mochkovitch R., 1998, MNRAS, 296, 275
Daigne F., Mochkovitch R., 2000, A&A, 358, 1157
Fan Y., Zhang B., Wei D., 2009, PhRvD, 79, 021301
Fenimore E. E., Madras C. D., Nayakshin S., 1996, ApJ, 473, 998
Gupta N., Zhang B., 2007, MNRAS, 380, 78
Kobayashi S., Piran T., Sari R., 1997, ApJ, 490, 92
Kobayashi S., Zhang B., Mészáros P., Burrows D., 2007, ApJ, 655, 391
Kumar P., McMahon E., 2008, MNRAS, 384, 33
Kumar P., Narayan R., 2009, MNRAS, 395, 472
Kumar P., Panaitescu A., 2000, ApJ, 541, L51
Kumar P., Panaitescu A., 2008, MNRAS, 381, L19
Lazar A., Nakar E., Piran T., 2009, ApJ, 695, L10
Li C. et al., 2006, ApJ, 645, 351
Maxham A., Zhang B., 2009, ApJ, 707, 1623
Medvedev M. V., Loeb A., 1999, ApJ, 526, 697
Page K. L. et al., 2007, ApJ, 663, 1125
Panaitescu A., Kumar P., 2000, ApJ, 543, 66
Panaitescu A., Mészáros P., 2000, ApJ, 544, L17
Pe’er A., Zhang B., 2006, ApJ, 653, 454
Piran T., Sari R., Zou Y.-C., 2009, MNRAS, 393, 1107
Racusin J. L. et al., 2008, Nat, 455, 183
Rees M. J., Meszaros P., 1994, ApJ, 430, L93
Sari R., Narayan R., Piran T., 1996, ApJ, 473, 204
Shen R., Zhang B., 2009, MNRAS, 398, 1936
Vestrand W. T. et al., 2005, Nat, 435, 178
Vestrand W. T. et al., 2006, Nat, 442, 172
Wijers R. A. M. J., Galama T. J., 1999, ApJ, 523, 177
Yost S. A. et al., 2007, ApJ, 657, 925
Yu Y. W., Wang X. Y., Dai Z. G., 2009, ApJ, 692, 1662
Zhang B., Mészáros P., 2002, ApJ, 581, 1363
Zhang B., Fan Y. Z., Dyks J., Kobayashi S., Mészáros P., Burrows D. N., Nousek J. A., Gehrels N., 2006, ApJ, 642, 354
Zhang B.-B. et al., 2011, ApJ, 730, 141
Zou Y., Piran T., Sari R., 2009, ApJ, 692, L92

APPENDIX A: DETAILED EXPRESSIONS

For $v_e < v_B < v_m$, 

$$L_{bol,52} \simeq 0.26 L_{V,36}^{4/3} \left( \frac{v_B}{5.45 \times 10^{14}} \right)^{2/3} \left( \frac{a_g^2 - 1}{a_g} \right)^{1/3} \times \frac{\varepsilon_e}{\varepsilon_B} \left( \frac{\gamma_m}{200 \sqrt{R_{IC}} \gamma_{Th}} \right)^{4/3}$$

and

$$B_c \simeq 100G L_{V,36}^{2/3} \left( \frac{v_B}{5.45 \times 10^{14}} \right)^{1/3} \left( \frac{a_g^2 - 1}{a_g} \right)^{2/3} \times \frac{1}{\Gamma_{300}} \left( \frac{\gamma_m}{200 R_{IC} \gamma_{Th}} \right)^{2/3}.$$

For $v_e < v_B < v_m$, 

$$L_{bol,52} \simeq 0.08 L_{V,36}^{20/21} \left( \frac{v_B}{5.45 \times 10^{14}} \right)^{22/21} \times \left( \frac{a_g^2 - 1}{a_g} \right)^{1/3} \epsilon_e \frac{R_{IC}^{2/21}}{\varepsilon_B \gamma_{Th}^{20/21}} \left( 500 \right)^{4/21}$$

and

$$B_c \simeq 53G L_{V,36}^{10/21} \left( \frac{v_B}{5.45 \times 10^{14}} \right)^{11/21} \left( \frac{a_g^2 - 1}{a_g} \right)^{2/3} \times \frac{1}{\Gamma_{300}^{20/21}} \gamma_{Th}^{10/21} \left( 500 \right)^{2/21} \gamma_{Th}^{21/20}. \gamma_{Th}^{16/21}$$

After substituting $500B_c^{-2/3}$ for $\gamma_{KN,2}$ (see Section 2 for details about how we obtained this expression), we can rewrite equation (11) as

$$\gamma_{Th} = 0.3 \left( \frac{B_c}{300} \right)^{-6/5} \left( \frac{\gamma_m}{200} \right)^{-6} \gamma_{Th} \frac{3}{8\gamma_m} \left( \log 2\gamma_m + 1 \right)^{1/2}.$$
\$\gamma_{KN}$ in Section 2.3, and hence we take out the $\gamma$ dependence of the coefficients in equation (B1). We use the magnetic field at the peak of the pulse as a representative value. We can then solve the differential equation under the assumption that the magnetic field is a constant over time, which is not a bad assumption as long as the cooling time-scale is much shorter than the dynamical time-scale:

$$\gamma(t) = \frac{\gamma_0}{1 + 1.3 \times 10^{-9} (1 + Y_{Th} + Y_{Th}Y_{KN}) \gamma_0 B'^2 t}. \quad (B2)$$

Under heavy radiative loss, the term unity in the denominator can be neglected, so that we have

$$\gamma(t) \approx \frac{1}{1.3 \times 10^{-9} (1 + Y_{Th} + Y_{Th}Y_{KN}) \gamma_0 B'^2 t}. \quad (B2)$$

Substituting for $\gamma(t) = 1$, we can then obtain the approximate $t_{\gamma=1} = \frac{1}{[1.3 \times 10^{-9} (1 + Y_{Th} + Y_{Th}Y_{KN}) B'^2 t]^{-1}}$.

This paper has been typeset from a TEX/LATEX file prepared by the author.