Abstract
Rational estimation of the mechanical state of a competent rock layer is of prime importance for structural and support design in longwall coal mining. This paper analytically examines the stability of a voussoir-structured stiff rock by considering it acting as a symmetric flat arch with discontinuities at the abutments and midspan. Three typical failure modes including snap through, midspan crush and sliding failure are inspected. The solution assumes that the shape of the compressive arch obeys a parabolic variation as the voussoir beam deflects downwards with large-deformation. The crush failure due to stress concentration in the beam hinge is predicted by relating the squeezing area with the beam rotation, which is absent in previous studies. The proposed approach can examine the voussoir beam stability without cumbersome iteration. Primarily, this work facilitates the evaluation of the support requirements to reinforce roof beds in longwall panels. After accounting for the total loads of the overlying rock layers, the stability assessment developed in this paper can further be utilized to predict the span limit of the main roof above a longwall panel and to estimate the critical width which governs the magnitude of mining subsidence.

KEYWORDS
competent rock stratum, stability analysis, Voussoir beam

1 | INTRODUCTION

Stratified rocks are frequently encountered during the mining process in underground coal mines with longwall layouts. Among the laminated rock layers, a competent rock stratum with high strength and stiffness, also termed “key stratum”\(^1\)\(^,\)\(^2\) carries the loads of the overlying strata with more compliant properties. The mechanical behavior of this competent rock stratum governs the ground pressure, strata movement, and surface subsidence. Particularly, the sudden breakage of the competent rock stratum generates dynamic impacts, which lead to catastrophic disasters to longwall panels.\(^3\)\(^,\)\(^4\)\(^,\)\(^5\) Therefore, reasonable assessment of the mechanical response of the competent rock strata to underground excavation is crucial for ground control in mining engineering.

As coal seam extraction proceeds, the competent rock beam fails in tension as its span reaches a critical value, which will be referred to as limited span limit. The beam remains stable and bears load since transition from continuous elastic structure to voussoir arch commonly occurs. Analytical attempts have been made to resolve the indeterminate features of the voussoir beam structure. Evans\(^7\) attributed the bearing capacity to an invisible arch
structure developed in the beam and examined the failure modes of the voussoir beam based on static analysis. The formulation between vertical deflection and lateral thrust of Evans\textsuperscript{7} has been later reformulated by several researchers.\textsuperscript{8–14} The thickness and shape of the compression arch are commonly simplified for overcoming the instinct indetermination of voussoir beams. Brady and Brown\textsuperscript{9} conjectured a bilinear variation for the shape of the compressive arch, which was later critiqued by Diederichs and Kaiser\textsuperscript{11} who suggested a parabolic distribution is more appropriate. However, the solution scheme of Diederichs and Kaiser\textsuperscript{11} implicitly assumed that the beam undergoes small vertical deflection, and involves a step-wise iterative process whereas large deflection is more frequently observed in situ.\textsuperscript{2,3,15} On the other hand, the work of Sofianos\textsuperscript{10} allows large downward deflection of the beam. Nevertheless, the thickness of the compressive arch is unreasonably low.\textsuperscript{16,17} An analytical solution to consider both large beam deflection and reasonable compression arch geometry is required.

Experimental studies have been performed to capture the mechanical response of the voussoir beam structure. Wright and Mirza\textsuperscript{18} reported that the abutment stresses were much higher than the solution of Evans\textsuperscript{7} and the thrust acting length much smaller than half the beam thickness. Sterling and Nelson\textsuperscript{19} investigated rock beams with varying geometries subjected to vertical loading. Typical failure modes were observed, including elastic buckling or snap through for thin beams with high span to thickness ratios, crush failure at the beam center or abutments for midrange of span to thickness ratios, sliding or shear failure at the abutment joints for low span to thickness. Ran et al.\textsuperscript{20} stated that if the angle between the joints plane and the normal to lamination plane was less than one-third to one-half of the effective friction angle, then voussoir beam solution was applicable. It has been frequently observed that the beam upon vertical loading will gradually rotate and the hinges will crush at the midspan.\textsuperscript{19–21} However, the beam rotation and the area alteration due to midspan crush has been rarely considered.

The stability of the voussoir beam has also been examined numerically with the rapid advancement of the computational technique. Wright,\textsuperscript{22} using FEM (Finite Element Method), stated that triangular shape stress distribution was inaccurate, but adequate for engineering practice. Furthermore, he revealed the stress distribution length in the midspan was larger than that at the ends. Sofianos\textsuperscript{10} revisited the numerical outcome of Wright\textsuperscript{22} and suggested a much lower value of the thickness coefficient varying at 0.1 to 0.3. Diederichs and Kaiser\textsuperscript{11} re-evaluated Brady and Brown\textsuperscript{9} iterative procedures and argued that the thickness coefficient for initial stage balance approximated 0.75, which decreases to about 0.35 when failure is imminent. The discrepancies between Sofianos\textsuperscript{10} and Diederichs and Kaiser\textsuperscript{11} stemmed from different boundary conditions assigned in the DEM (Distinct Element Method) models.\textsuperscript{16,17} Tsesarsky,\textsuperscript{23} based on FLAC (Fast Lagrangian Analysis of Continua) modeling, found the thickness coefficient of the compression arch at the beam abutments varies from 0.3 to 0.4, which is inversely proportional to the joint stiffness.

In this paper, occurrences for typical failure modes, including snap through, crush, and sliding of the voussoir beam, are addressed analytically. The solution considers a competent voussoir beam experiences large vertical deflection. The compressive arch is assumed to be quadratic. The areal reduction due to midspan crush after beam rotation is considered. Mining-induced strata movement and surface subsidence which are dominated by the breakage of the competent rock stratum can be predicted based on the proposed analytical procedures.

## 2 STABILITY ANALYSIS OF THE VOUSSOIR BEAM

### 2.1 Problem statement and background knowledge

As the voussoir beam is horizontally symmetric, half span is analyzed. For half span, a parabolic arch arises with an original length \( L_0/2 \) (Figure 1A). Naturally, to stabilize, a thrust moment \( M_R \) is induced to resist the self-weight (\( gts/2 \)) moment \( M_A \) of the rock beam. The thrust force (\( F \)) acting at the midupper part and the lower abutment is supposed identical with the same contact length \( h = nt, n \) is thickness coefficient (contact length to beam depth ratio), as shown in Figure 1B. The distribution of \( F \) is assumed triangular. The moment equilibrium is expressed by:

\[
M_A = \frac{1}{8} \gamma t s^2 = \frac{1}{2} \sigma_s n t z_1 = M_R
\]

(1)

\[
\sigma_s = \frac{1}{4} \frac{\gamma s^2}{n z_1}
\]

(2)

where, \( \sigma_s \) is the maximum compressive stress, exerted at the bottom edge at the abutment and the top edge at the midpoint; \( z_1 \) is the lever arm of \( F \). Apparently, Equation 1 is essentially undetermined, involving three unknown parameters \( \sigma_s, n \) and \( z_1 \). Classic voussoir beam theory commences from an assumption that the initial moment arm \( z_0 \) before deflection is:

\[
z_0 = t(1 - \frac{2}{3}n)
\]

(3)

The corresponding parabolic arch length is:

\[
L_0 = s + \frac{8\zeta_0^2}{3s}
\]

(4)

Sofianos\textsuperscript{10} and Brady and Brown\textsuperscript{24} adopted a higher order curve \( (L_0 = s + (16\zeta_1^2)/3s) \). Due to downwards deflection, the deformed arch length with moment arm \( z_1 \) is:
Combining Equations (4) and (5), the elastic shortening of the arch is calculated by:

The lever arm \( z_1 \) is:

Equation 7 indicated that \( z_1 \) would be determined if \( \Delta L \) is solved. Assume the rock beam is compressed with an average longitudinal stress \( \sigma_{av} \), then:

where, \( E \) is the elastic modulus of the rock beam. \( \sigma_{av} \) and \( \sigma_x \) can be related linearly through a stress concentration coefficient \( \lambda \), that is:

Substituting \( \sigma_x \) with Equation 2, we have:

The beam vertical deflection \( \delta \) is expressed as:

Introducing Equation 13 into Equation 12, we derive:

Clearly, \( n \) is the key to solve the above equation, based on which the voussoir beam stability can be examined.

2.2 | Bearing capacity analysis

In the following, three common failure modes, namely, elastic buckling, high stress induced crush and shear failure at discontinuities will be discussed.

2.2.1 | Elastic buckling

Rock beams with high span to thickness ratio tend to snap through before crush, which is termed elastic buckling. Buckling failure occurs because the increasing lateral thrust is insufficient to counteract the decreasing lever arm. To retain stability during large deflection of the three-jointed beam, the maximum compressive stress at the bottom abutment (\( \sigma_x \)) is \(^{10}\):

Substituting \( \sigma_x \) with Equation 2, we have:

The beam vertical deflection \( \delta \) is expressed as:

Introducing Equation 13 into Equation 12, we derive:

Clearly, \( n \) is the key to solve the above equation, based on which the voussoir beam stability can be examined.
For a certain beam span, stress concentration coefficient \( \lambda \) is disputable. Evans\(^7\) proposed a simple relationship \( \sigma_{av} \) and \( \sigma_s \) with \( \sigma_{av} = 11\sigma_s/24 \), based on intuitive assumption \( n = 0.5 \). The constant coefficient was followed by Beer and Meek.\(^8\) Brady and Brown\(^24\) disproved previous assumption and suggested a bilinear variation. The average longitudinal stress is given by:

\[
\sigma_{av} = \frac{1}{2} \sigma_s \left( \frac{2}{3} + \frac{n}{2} \right) = \sigma_s \left( \frac{1}{3} + \frac{n}{4} \right)
\]

For Equation 16 is irrational because no entire beam section is under compression if the stress distribution is bilinear. Instead, they assumed that the distribution of stress along the arch centreline varies parabolically. The yielded average stress in the beam is:

\[
\sigma_{av} = \sigma_s \left( \frac{2}{3} + \frac{n}{3} \right) \quad (0.35 \leq n \leq 0.75)
\]

Sofianos\(^10\) claimed that the coefficient should be \( n/0.38 \), with a lower range value of \( n \) from 0.12 to 0.3, that is

\[
\sigma_{av} = \frac{n}{0.38} \sigma_s \quad (0.12 \leq n \leq 0.3)
\]

According to the thickness coefficient \( n \), Sofianos\(^10\), Diederichs and Kaiser\(^11\), Brady and Brown\(^24\) gave a linear correlation with thickness coefficient \( n \), excepted Evans\(^7\) who selected a constant value for convenience. Overall, \( \sigma_s \) in Equation 15 decreases monotonically with \( \lambda \) and \( n \). The resisting moment \( M_R \) maximizes at the minimal value of \( z_1 \), \( \lambda \) and \( n \):

\[
M_R = \frac{1}{2} \sigma_s n t z_1 = \frac{ntE z_0^3}{3\sqrt{3} z_0^2 + z_0^2}
\]

The safety factor against elastic buckling is:

\[
FS_{Bu} = \frac{M_R}{M_A} = \frac{8ntE z_0^3}{3\sqrt{3} z_0^2 + z_0^2 + \frac{2}{3}}
\]

Here, the assumption of Diederichs and Kaiser\(^11\) was adopted to appraise the safety factor against elastic buckling \( FS_{Bu} \), then we have:

\[
FS_{Bu} = \frac{8ntE z_0^3}{3\sqrt{3} z_0^2 + \frac{2}{3} + \frac{2}{3}}
\]

For Equation 22, it is reasonable to omit 1 in \( (\frac{2}{3} + \frac{2}{3}) \), because \( \frac{3}{s_0} \gg 1 \) if the span to thickness ratio outnumbers two. This also increases \( FS_{Bu} \), turning the rock beam to a safer side. By replacing \( z_0 \) with Equation 6, Equation 22 is reformulated as:

\[
FS_{Bu} = \frac{3\sqrt{3} z_0^2 + \frac{2}{3} + \frac{2}{3}}{64\gamma^3 E n (1 - \frac{2}{3} n^3)}
\]

Denote \( f(n) = \frac{n(1 - n)}{1 + n} \), the maximum value of \( f(n) \) is solved at 0.477 corresponding to \( n = 0.3 \), as illustrated by Figure 2. The critical vertical deflection \( \delta \) is:

\[
\delta = 0.338t
\]

This value is slightly larger than that of\(^10\) who proposed a relatively conservative \( \delta = 0.25t \) as the lower bound displacement for elastic buckling, while it agrees closely with Sofianos and Kapenis\(^25\) that proposed a critical \( \delta \) for snap through is between 0.35-0.4. Introducing 0.477 into Equation 23, we have:

\[
FS_{Bu} = \frac{1.96\gamma^3 E}{S^4}
\]

Apparently, a rock beam under the load of its own weight with high span to thickness ratio easily undergoes elastic buckling, which was frequently observed experimentally.\(^19,20,26\) The span limit is calculated when \( FS_{Bu} = 1 \):

\[
S_{Bu} = \sqrt{\frac{1.96\gamma^3 E}{\gamma}}
\]

Figure 3 illustrates the safe against unsafe regions for a certain rock beam with properties of \( E = 20 \text{ GPa} \), \( \gamma = 30 \text{ kN/m}^3 \).

2.2.2 Crush at midspan

Crush occurs when the compressive stress at the contact edges exceeds the uniaxial strength of the rock mass (\( \sigma_c \)).
Although Diederichs and Kaiser\textsuperscript{11} and Brady and Brown\textsuperscript{12} offered a simple formula comparing the value of $\sigma_x$ and $\sigma_c$ for a certain rock beam, complex iteration was required. We propose a new solution based on the geometric approximation of Miao.\textsuperscript{27} Figure 4 shows the left half of the whole voussoir beam. During the beam rotation, crush is allowed at the high stress area, that is the abutments and the midcrown. We assume abutment point A is immovable. By considering the geometric variation, the rolling out triangle $\Delta CDE$ should be absorbed equally by the crush regions in the abutment and midspan. This phenomenon was consistent with the observations in the photo-elastic experiments on the mechanical behavior of the voussoir beam.\textsuperscript{27} Thus, we have:

$$2h = C'D = t - \frac{s}{2} \sin \alpha$$

(27)

where $h$ is the contact length in the midspan, and $\alpha$ is the beam rotation angle (Figure 4).

$$h = \frac{1}{2}(t - \frac{s}{2} \sin \alpha)$$

(28)

Then,

$$n = \frac{h}{t} = \frac{1}{2}(1 - \frac{s}{2t} \sin \alpha) < 0.5$$

(29)

$$F(t - \frac{2}{3} h - \delta) = \frac{1}{8} \gamma ts^2$$

(30)

For very small $\delta \approx \frac{s}{2} \sin \alpha$, thus:

$$F = \frac{3\gamma ts^2}{8(2t - s \sin \alpha)}$$

(31)

The maximum compressive stress $\sigma_x$ is expressed as:

$$\sigma_x = \frac{2F}{h} = \frac{3\gamma ts^2}{(2t - s \sin \alpha)^2}$$

(32)

The factor of safety against crush failure is:

$$FS_{Cr} = \frac{\sigma_c}{\sigma_x} = \frac{\sigma_c(2t - s \sin \alpha)^2}{3\gamma ts^2}$$

(33)

Let $FS_{Cr} = 1$, the critical span for crush is:

$$s_{Cr} = \frac{2t}{\sin \alpha} + \sqrt{\frac{3\gamma t}{\sigma_c}}$$

(34)

Ideally, $\sigma_c$ is the uniaxial compressive strength of the rock. Many laboratory tests showed that $\sigma_c$ at the rock edge is much lower than that of the normal compressive strength. Huang et al.\textsuperscript{28} based on a wide range of physical tests, suggested the reduction factor could be 0.4. This agrees well with the statement in Brady and Brown,\textsuperscript{12} who warned the value of the unconfined compressive strength $\sigma_c$ for the rock mass must be selected in prudence and usually 50\% reduction of the mean laboratory value is necessary. Consequently, the above equation can be updated as:

$$s_{Cr} = \frac{2t}{\sin \alpha} + \sqrt{\frac{7.5\gamma t}{\sigma_c}}$$

(35)

As the beam experiences slight rotation, the angle $\alpha$ is less than $3^\circ$.\textsuperscript{15} Figure 5 describes the relationship between $s_{Cr}$ and rotation angles from $1^\circ$ to $3^\circ$, for the typical strong rock $\sigma_c = 100$ MPa.\textsuperscript{29} The rock beam is prone to fail due to crush for aspect ratios $s_{Cr}/t = 7.5 - 20$. From safety perspective, a conservative prediction of $s_{Cr}$ is suggested to be calculated with $\alpha = 3^\circ$.

### 2.2.3 Shear failure

There is a high possibility for the beam with a low span to thickness ratio to slip along the vertical joint before edge squeezing or elastic buckling.\textsuperscript{20,26} That is to say, the beam may slide along the abutment before achieving stable voussoir beam structure. For a linear stress distribution, the thickness coefficient $n$ is assumed to be 0.75, provided no shear failure occurs.\textsuperscript{30} The critical point for shear failure is:

$$F(t - \frac{2}{3} nt) = \frac{\gamma t^2}{8}$$

(36)

If $n = 0.75$, we have:

$$F = \frac{\gamma t^2}{4}$$

(37)

The maximum resisting force is:

$$F_{re} = F \tan \phi = \frac{\gamma t^2}{4} \tan \phi$$

(38)

where, $\phi$ is the friction angle of the sliding surface. The maximum possible span for sliding corresponding to a factor of safety equal to 1.0 is:

$$s_{Sh} = \frac{2t}{\tan \phi}$$

(39)

Generally, the friction angle $\phi$ ranges from $30^\circ$ to $40^\circ$. Therefore, the ratio of $s_{Sh}/t$ lies between 2.4-3.5, which matches the experimental observations in Sterling.\textsuperscript{21}

We compared the predictions of the proposed model with that of Sofianos.\textsuperscript{16} Table 1 shows that mechanical properties of the competent rock beam. The beam rotation angle is assumed to be $3^\circ$. The span limits at which elastic buckling occurs and slip failure, respectively are close for the two solutions, whereas the difference in span limit for crush possibly resulted from the empirical reduction of rock mass strength.

### 2.2.4 Calculation of the bearing load

The above analysis considers the situation where the rock beam is self-loaded. The simplification is restricted to two scenarios, that is, near-surface underground excavation and that separation occurs between the rock beam of interest and
the upper adjacent layer. Since the competent rock stratum is stiffer, thereby it carries the load of upper layers. 31,32 Given the competent rock beam is the \( m \) layer (Figure 6), the overlying load is 31,32:

\[
q_{N_m} = E_m t_m^3 \sum_{i=1}^{N} \gamma_i t_i / \sum_{i=1}^{N} E_m t_i^3
\]  

(40)

where \( E_m \) is the elastic modulus of the competent rock beam, \( N \) is the total number of the overburden layers, and \( t_i \) is the thickness of any rock layer above the competent stratum. The unit weight \( \gamma \) is replaced as:

\[
\gamma = q_{N_m} / t
\]  

(41)

Rock beam deflection creates a small rotation angle, \( \theta \). Then the effective load is:

\[
\gamma' = q_{N_m} / t \cos \theta
\]  

(42)

As the rotation angle (\( \theta \)) is usually extremely small, it can be omitted.

\section*{Discussion}

Although traditional analysis 7,12 assumed the stress distribution being triangular, numerical 22,23,26,33 and experimental
studies\textsuperscript{21,26,34} showed that the thrust distribution shape at the abutment edge and the upper beam center is not necessarily linear, but is rather a complex geometric function.\textsuperscript{30} Nevertheless, for field practice, triangular area is reasonable.\textsuperscript{33} Besides, the thickness coefficient $n$ assumed identical at the abutments in the classic voussoir beam theory was also approximate for analytical solution. In reality, the value $n_a$ at the abutment and $n_m$ at the span center obeys the following relationship\textsuperscript{23}:

$$n_a < n_m < 1$$ (43)

Correspondingly, the maximum stress $\sigma_x$ at the lower end and the upper midspan is unequal, but with the inverse relation that $\sigma_a > \sigma_m$. For a typical voussoir beam with differing compressive length, namely, $h_a = n_at$ at the abutment, $h_m = n_mt$ at the span center, the triangular distribution of compression is still in use. Eq. (1) is rewritten as:

$$M_A = \frac{1}{8} \gamma ts^2 = \frac{1}{2} \sigma_a n_at_1 = \frac{1}{2} \sigma_m n_mt_1 = M_R$$ (44)

Provided the initial lever arm $z_0$ is given by:

$$z_0 = t - \frac{t}{3}(n_a + n_m)$$ (45)

Equation (14) is updated as:

$$\delta = \left(\frac{3}{8}s^2 + z_0^2\right) \frac{\lambda \gamma s^2}{8E(2z_0 - \delta)(z_0 - \delta)} \left(\frac{1}{n_a} + \frac{1}{n_m}\right)$$ (46)

It indicates for equations in Section 2.1, $n$ is to be replaced with $\frac{n_a + n_m}{2}$, and $\frac{1}{n}$ with $\frac{1}{2}(\frac{1}{n_a} + \frac{1}{n_m})$.

From the initial stability of the voussoir beam to the critical deformation situation where failure is imminent, the rock beam undergoes a dynamical deforming process. The values of thickness coefficients $n_a$ and $n_m$ are changing incessantly. That is to say, acquisition of an exact mathematical expression is hardly possible. Consequently, utilizing numerical methods is required to estimate their values. Mottahed et al.\textsuperscript{26} reported that the stress distribution length for an end joint approximates to half that for the middle joint whereas Tsesarsky\textsuperscript{23} suggested $n_a$ ranges from 0.3-0.4 and $n_m$ varies from 0.4-0.5. Further work is required since no commonly accepted conclusions are achieved.

The competent rock stratum usually exists as the main roof over the longwall panel (Figure 7). Under this circumstance, the rock stratum will not collapse until one of the failure modes is satisfied. Assuming the total height of the immediate roof $\sum t_i$, mining height $H_m$, and rock bulk factor $k_r$ the maximum vertical settlement $\delta_v$ is:

$$\delta_v = \sum_{i=1}^{i} t_i + H_m - \sum_{i=1}^{i} t_i k_i$$ (47)

$\delta_v < \delta$ indicates the caved rock is able to fill the goaf and the voussoir beam keeps stable rather than collapse into the void. Otherwise, the breakage of the strong layer will cause violent dynamic pressure on supporters, which occurs frequently for longwall panels with increasing extraction heights.\textsuperscript{3} The span at beam failure can be treated as the weighting distance of the main roof.

Locating at a middle or higher level of the overburdens, the competent rock stratum whose failure signs that critical width is imminent. When the extraction width is smaller than the limited span, the vertical subsidence is moderate and mining width is subcritical. When the extraction width exceeds the limited span of the voussoir beam, the longwall panel is super-critical. The critical width $w$ is predicted by (Figure 8):

$$w = s + 2d \cot \theta$$ (48)

where, $s$ is the span limit; $d$ is the distance from the panel to the competent stratum; $\theta$ is angle of draw. The maximum vertical displacement ($\delta_m$) roughly equals to the maximum surface subsidence at critical width ($w$).

### 4 CONCLUSIONS

This paper presents an analytical examination of the stability of a voussoir-structured rock stratum. The analytical model is
able to assess the snap through when the voussoir beam deflects downwards with large deflection. The proposed formulation estimating the crush failure at midspan hinge accounts for the dependence between crack area and beam rotation, which has not been considered by previous studies. For a self-loaded rock beam, the span limits for three conventional failure modes are predicted with the following criteria: elastic buckling \( s_{Bu} = \frac{1.96E^2z}{\gamma} \); crush failure \( s_{Cr} = \frac{2t}{\sin a + \sqrt{\frac{t}{\gamma_c}}}, \) and shear sliding \( s_{Sh} = \frac{2t}{\tan \phi} \). Although several assumptions are made in this work, such as the assumption that the shape of the compressive arch follows a parabolic variation and equalizes the thickness coefficient at the abutments to that at the midspan, which is a simplification but adequate for engineering practices. Further experimental and numerical efforts are required to investigate the state of the voussoir beam structure by accounting for a more complex distribution of the thickness coefficients. Natural rock beams are intersected by far more than three discontinuities due to the presence of geological structures, such as faults. Investigating the stability of such multi-jointed voussoir structures is more crucial for preliminary design of support system for underground excavations.

In underground coal mines, the competent rock stratum bears the loads of the overlying thin rock layers. After considering the total weights carried by the competent rock stratum, the proposed solution predicts the span limit of the main roof and facilitates the capacity design of the supporters in the panel. Furthermore, the critical width dominating the vertical settlement of the rock strata can be estimated when the strong rock stratum situates at a higher level above the long-wall panel.

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NOTATIONS

\( q_{nm} \) self-weight load of the \( m \) th rock layer  
\( \phi \) friction angle between sliding surface  
\( D \) distance from the panel to the competent stratum  
\( E \) elastic modulus of the rock beam  
\( E_m \) elastic modulus of the \( m \) th rock layer  
\( F_{Bu} \) safety factor of elastic buckling  
\( F \) force exerted at the bottom abutment  
\( F_{Re} \) maximum resistance force  
\( FSCr \) safety factor of crush  
\( h \) contact length  
\( H_m \) Mining height  
\( k_i \) bulk factor of the \( i \) th rock layer  
\( L_0 \) original parabolic arch of the rock beam  
\( L_1 \) parabolic arch of the rock beam after deformation  
\( M_A \) self-weight moment of the rock beam  
\( m \) number of the competent rock layer  
\( M_R \) thrust moment  
\( n_a \) thickness coefficient at the abutment  
\( n \) mean arch thickness coefficient compared to \( n_a \)  
\( n_m \) thickness coefficient at the midspan  
\( N \) total number of the overburden rock layers  
\( q \) load due to self-weight on the beam  
\( S_{Cr} \) critical span for crush failure  
\( S_{Sh} \) critical span for sliding failure  
\( s \) span limit of rock beam  
\( T \) beam thickness  
\( t_i \) the \( i \) th rock layer thickness  
\( t_m \) thickness of the \( m \) th rock layer  
\( w \) critical width  
\( z_0 \) initial lever arm  
\( z_1 \) Lever arm after deformation  
\( \alpha \) beam rotation angle  
\( \gamma' \) effective unit weight  
\( \gamma_i \) unit weight of the \( i \) th rock beam  
\( \gamma_m \) unit weight of the rock beam  
\( \Delta L \) elastic shortening of the parabolic arch  
\( \delta_m \) maximum vertical deformation  
\( \delta \) vertical deformation  
\( \theta \) Angle of draw  
\( \lambda \) stress concentration coefficient  
\( \sigma_{av} \) Lever average longitudinal stress  
\( \sigma_c \) uniaxial compressive strength of the rock beam  
\( \sigma_{c} \) Maximum compressive stress at the bottom abutment and the midspan

ORCID

Yingchun Li https://orcid.org/0000-0002-0108-3704

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