Chiral gravitational waves from $z = 2$ Hořava-Lifshitz gravity

Yun Soo Myung

Institute of Basic Science and School of Computer Aided Science
Inje University, Gimhae 621-749, Korea

Abstract

We construct the chiral gravitational waves from the $z = 2$ Hořava-Lifshitz gravity with gravitational Chern-Simons term in the de Sitter and Minkowski backgrounds. These gravitational waves which show a feature of the Hořava-Lifshitz gravity may be related to the generalized uncertainty principle. In addition, we find the classical and quantum IR-UV transition rules in the $z = 2$ Hořava-Lifshitz gravity.
1 Introduction

Recently Hořava has proposed a renormalizable theory of gravity at a Lifshitz point [1], which may be regarded as a UV complete candidate for general relativity. At short distances the theory of $z = 3$ Hořava-Lifshitz (HL) gravity describes interacting nonrelativistic gravitons and is supposed to be power-counting renormalizable in 3+1 dimensions. The equations of motion were derived for $z = 3$ HL gravity [2, 3]. Its cosmological implication first appeared in [2], while its black hole solution was found in asymptotically anti-de Sitter spacetimes [3] and black hole in asymptotically flat spacetimes [4].

Even though the $z = 3$ HL gravity was proposed to be a power-counting renormalizable theory in 3+1 dimensions, there are many fundamental issues to be clarified. Renormalizability beyond power-counting has not yet been proven and the renormalization group (RG) flow of various coupling constants has not been studied. Especially, the recovery of general relativity depends critically on the assumption that the parameter $\lambda$ flows to 1 in the IR limit. Since no one insists that the $z = 3$ HL gravity is the final, one may either improve it or modify it. In this sense, we may consider the $z = 2$ HL gravity as an alternative because of the difficulty in working with $z = 3$ HL gravity.

One application of the $z = 2$ HL gravity in 2+1 dimensions, as a candidate membrane world-volume theory was discussed in [5]. In this work, we wish to consider the $z = 2$ HL gravity in 3+1 dimensions as a candidate for the quantum theory of general relativity. The reason is as follows. It was shown that the renormalized Wheeler-DeWitt equation possesses a solution with a $z = 2$ Lifshitz point, but no other $z > 2$ solutions to leading order of strong coupling expansion [6]. This indicates that the quantum Einstein gravity has a $z = 2$ Lifshitz point, but no other higher Lifshitz points. Adding the gravitational Chern-Simons (gCS) term to the $z = 2$ HL gravity leads to the fact that their conclusion remains unchanged. The author in [7] has obtained that the Ricci flow in “d” spatial dimensions is the holographic RG flow to the $(d + 1)$-dimensional $z = 2$ HL gravity with $\lambda = 1/2$. Also, there are some results, supporting that perturbative corrections to $z = 2$ Lifshitz point are promising. These include the quantum Lifshitz model in $d + 1$ dimensions [8], Lifshitz-type scalar field theory in $d = 4$ and $d = 10$ [9], and Lifshitz-type gauge theory [10].

Concerning cosmological implications of the $z = 3$ HL gravity, it has provided a new mechanism of generating scale-invariant cosmological perturbations [2, 11] and regular bounce

\footnote{In this case, the scaling factor takes the form $a(t) \sim t^p (p > 1/3)$. It may be regarded as an alternative to inflation. However, there are a lot of questions. An urgent question is how one does solve the flatness problem without inflation [11].}
solutions in the early universe [12, 13]. Importantly, the authors in [15] have shown that the chiral primordial gravitational waves are generated from the $z = 3$ HL gravity when working the pure de Sitter cosmological background. These circularly polarized modes are generated only when the Cotton tensor $C_{ij}$ is present, making parity violation. Since these modes are composed of higher order spatial derivatives, it may be likely observed if these modes were really present in the very early universe$^3$. Let us study what happens in the $z = 2$ HL gravity. Aside from the fact that the scaling-invariant spectrum is a perturbative feature of the $z = 3$ HL gravity, the remaining two could be achieved when using the $z = 2$ HL gravity. A matter bounce solution can be derived from the $z = 2$ HL gravity with a matter, since there is no essential difference in Friedmann equations between $z = 2$ and $z = 3$ HL gravities. This is so because the Cotton tensor from the gCS term did not contribute to the Friedmann-Robertson-Walker (FRW) universe based on the isotropy and homogeneity, but it contributes to the Mixmaster universe (Bianchi IX) based on the anisotropy and homogeneity [16, 17]. Hence, it is quite interesting to see whether chiral gravitational waves without ghost instability can be generated from the $z = 2$ HL gravity.

On the other hand, one of main ingredients for studying quantum gravity is the generalized uncertainty principle (GUP), which has been argued from various approaches to quantum gravity and black hole physics [18]. We note that the GUP is in the heart of the quantum gravity phenomenology. Certain effects of quantum gravity are universal and thus, influence almost any system with a well-defined Hamiltonian [19]. It seems that the GUP may be related to black holes found in the deformed HL gravity [20, 21]. Furthermore, it was shown that the GUP-corrected tensor propagator takes similar form as that derived from the $z = 3$ HL gravity [22, 23].

We wish to mention why the connection between $z = 2$ HL gravity and GUP will be important to understand the quantum aspects of $z = 2$ HL garvity. The GUP usually satisfies the modified Heisenberg algebra $[x_i, p_j] = i\hbar(\delta_{ij} + \beta p^2\delta_{ij} + 2\beta p_i p_j)$, where $p_i$ is considered as the momentum at high energies. Thus, it can be interpreted to be the UV-commutation relation. On the other hand, introducing IR-canonical variable $p_0$ with

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$^3$ Concerning the observability of chiral gravitational waves, the primordial gravitational waves should be created at high energy in order to be observable. Once they are created, their amplitudes will conserve until the final horizon crossing. On the other hand, the important point is the initial condition. In the presence of the parity violating term, the initial quantum state of gravitational waves has chirality. This is the reason why one has obtained chiral primordial gravitational waves in the parity violating theory [15]. It does not depend on the number of derivatives. Thus, it is conjectured that there is no essential difference between $z = 3$ HL gravity and $z = 2$ HL gravity with gCS term.
$x_i = x_{0i}$ via the replacement $p_i \rightarrow p_{0i}(1 + \beta p_{0i}^2)$, these variables satisfy canonical (IR) commutation relation $[x_{0i}, p_{0j}] = i\hbar \delta_{ij}$. Here $p_{0i}$ is considered as the momentum at low energies. It is easy to show that the UV-commutation is satisfied to $\beta$-order when using the IR-commutation. Hence, the replacement could be used as an “important low-energy window” to investigate quantum gravity phenomenology up to $\beta$-order. Similarly, if the UV-tensor propagator of $z = 2$ HL gravity which carries an information on the renormalizability could be obtained from the IR-UV transition of $p^2 \rightarrow p^2(1 + 2p^2/\omega)$, one insists that the GUP can be realized in the $z = 2$ HL gravity partly. We note that even though the GUP is in the heart of the quantum gravity phenomenology, it reveals a part of quantum gravity effects but not whole of quantum gravity effects.

In this work, we wish to construct chiral gravitational waves from the $z = 2$ HL gravity \cite{5} with the gCS term in the de Sitter and Minkowski backgrounds. Also we suggest that these gravitational waves may be related to the GUP.

## 2 $z = 2$ HL gravity with gCS term

Introducing the ADM formalism where the metric is parameterized

$$ds^2_{ADM} = -N^2dt^2 + g_{ij}(dx^i - N^i dt)(dx^j - N^j dt),$$  \hspace{1cm} (1)

the Einstein-Hilbert action can be expressed as

$$S_{EH} = \frac{1}{16\pi G} \int d^4x \sqrt{\gamma N} \left[K_{ij} K^{ij} - K^2 + R - 2\Lambda\right],$$  \hspace{1cm} (2)

where $G$ is Newton’s constant and extrinsic curvature $K_{ij}$ takes the form

$$K_{ij} = \frac{1}{2N} \left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i\right).$$  \hspace{1cm} (3)

Here, a dot denotes a derivative with respect to $t$. The action of $z = 2$ HL gravity is given by \cite{5}

$$S_{HL} = \int dt d^3x L_{z=2} = \int dt d^3x \left[\mathcal{L}_K^\lambda + \mathcal{L}_\lambda^V\right],$$  \hspace{1cm} (4)

where the kinetic Lagrangian is given by

$$\mathcal{L}_K^\lambda = \frac{2}{\kappa^2} \sqrt{g} N K_{ij} G^{ijkl} K_{kl} = \sqrt{g} N \frac{2}{\kappa^2} \left(K_{ij} K^{ij} - \lambda K^2\right),$$  \hspace{1cm} (5)

with the DeWitt metric

$$G^{ijkl} = \frac{1}{2} \left(g^{ik} g^{jl} - g^{il} g^{jk}\right) - \lambda g^{ij} g^{kl}$$  \hspace{1cm} (6)
and its inverse metric
\[ G_{ijkl} = \frac{1}{2} \left( g_{ik} g_{jl} - g_{il} g_{jk} \right) - \frac{\lambda}{3\lambda - 1} g_{ij} g_{kl}. \] (7)

The potential Lagrangian is determined by the detailed balance condition as
\[
\mathcal{L}_V^\lambda = -\frac{\kappa^2}{2} \sqrt{g} N E^{ij} G_{ijkl} E^{kl}
= \sqrt{g} N \frac{\kappa^2 \mu^2 (-\Lambda_W)}{8(3\lambda - 1)} \left( R - 3\Lambda_W - \frac{4\lambda - 1}{4\Lambda_W} R^2 + \frac{(3\lambda - 1) R_{ij}^2}{\Lambda_W} \right). \] (8)

Explicitly, \( E_{ij} \) could be derived from the Euclidean 3D gravity
\[
E_{ij} = \frac{1}{\sqrt{g}} \frac{\delta W_{3D}}{\delta g_{ij}}
\] (9)
with
\[
W_{3D} = -\mu \int d^3 x \sqrt{g} (R - 2\Lambda_W). \] (10)

In order to generate chiral gravitational modes, it is necessary to introduce the gravitational Chern-Simon (gCS) term \[24\]
\[
S_{gCS} = \alpha \int dt d^3 x \sqrt{g} \mathcal{L}_{gCS} = \alpha \int dt d^3 x \sqrt{g} N \epsilon^{ikl} \left( \Gamma^m_{il} \partial_j \Gamma^l_{km} + \frac{2}{3} \Gamma^m_{il} \Gamma^l_{jm} \Gamma^m_{kn} \right)
\] (11)
whose variation with respect to \( g_{ij} \) leads to the Cotton tensor \( C^{ij} \)
\[
\alpha C^{ij} = \alpha \epsilon^{ikl} \nabla_k \left( R^j_{\ell} - \frac{1}{4} R \delta^j_{\ell} \right).
\] (12)
Here \( \epsilon^{ikl} \) is a tensor and \( \alpha \) is an arbitrary parameter with scaling dimension \([\alpha] = 2\).

In the IR limit, comparing \( S_{HL} \) with Eq. (2) of general relativity, the speed of light, Newton’s constant and the cosmological constant are given by
\[
c = \frac{\kappa^2 \mu}{4} \sqrt{\frac{\Lambda_W}{1 - 3\lambda}}, \quad G = \frac{\kappa^2}{32\pi c}, \quad \Lambda_{cc} = \frac{3}{2} \Lambda_W. \] (13)

Considering the \( z = 2 \) HL gravity with \([t] = -2, [x^i] = -1\) \([[\mathcal{L}_K^\lambda]] = [[\mathcal{L}_V^\lambda]] = [W_{gCS}] = 5\), UV scaling dimensions are given by
\[
[N] = 0, \quad [\mu] = 1, \quad [\kappa^2] = -1, \quad [\Lambda_W] = 2, \quad [c] = 1. \] (14)

In the case of \( \lambda = 1(\Lambda_W < 0) \), the \( z = 2 \) HL potential Lagrangian with gCS term takes the form
\[
\mathcal{L}_{V}^{\lambda=1} = \sqrt{g} N \frac{2\kappa^2}{\kappa^2} \left( R - 3\Lambda_W - \frac{3}{4\Lambda_W} R^2 + \frac{2}{\Lambda_W} R_{ij}^2 + 2\sqrt{-\frac{2}{\Lambda_W} \mathcal{L}_{gCS}} \right), \] (15)
where we recover the general relativity with cosmological constant in the limit of $\Lambda_W \to \infty$. This Lagrangian is useful to study cosmological implications. It is obvious that $\mathcal{L}_V^{\lambda=1}$ does not satisfy the detailed balance condition because the last term is present. Here, we choose

$$\alpha = \frac{4c^2}{\kappa^2} \sqrt{\frac{2}{\Lambda_W}} = \frac{\kappa^2 \mu^2}{4} \sqrt{\frac{\Lambda_W}{2}} = \mu c,$$

(16)

to ensure that ghost-free chiral gravitational waves are propagating in the pure de Sitter background. We could not obtain ghost-free chiral gravitational waves unless $\alpha = \mu c$.

We would like to mention that the IR vacuum of this theory is anti-de Sitter (AdS) spacetimes. Hence, it is interesting to take a limit of the theory, which may lead to a Minkowski vacuum in the IR sector. To this end, one may deform the theory by adding “$\mu^3 R$” ($\tilde{\mathcal{L}}_V^{\lambda} = \mathcal{L}_V^{\lambda} + \sqrt{g}N\mu^3 R$) and then, taking the $\Lambda_W \to 0$ limit [4]. We call this the “deformed $z = 2$ HL gravity”. This does not alter the UV property of the theory, while it changes the IR property. That is, there exists a Minkowski vacuum, instead of an AdS vacuum. In the IR limit, the speed of light and Newton’s constant are determined by

$$c^2 = \frac{\kappa^2 \mu^3}{2}, \quad G = \frac{\kappa^2}{32\pi c}.$$  

(17)

For $\lambda = 1$, the deformed $z = 2$ HL gravity with gCS term takes the form

$$\tilde{\mathcal{L}}_V^{\lambda=1} = \sqrt{g}N\frac{2c^2}{\kappa^2}\left(R + \frac{3}{4\omega} R^2 - \frac{2}{\omega} R_{ij} + 2\sqrt{2/\omega} \mathcal{L}_{gCS}\right),$$

(18)

where an important parameter $\omega$ [4] and $\alpha$ are given by

$$\omega = \frac{16\mu}{\kappa^2}, \quad \alpha = \mu c = 2\sqrt{2/\omega},$$

(19)

with scaling dimension $[\omega] = 2$. We note that a choice of $\alpha = \mu c$ is essential to find ghost-free chiral gravitational waves propagating on the Minkowski background. In the limit of $\omega \to \infty$, we recover the general relativity. Comparing $\mathcal{L}_V^{\lambda=1}$ with $\tilde{\mathcal{L}}_V^{\lambda=1}$, these become the same form when replacing $-1/\Lambda_W$ by $1/\omega$ up to cosmological constant $-3\Lambda_W$.

3 Cosmological implications

First of all, we look for cosmological equation to $z = 2$ HL gravity with gCS term by introducing the FRW metric

$$ds_{FRW}^2 = -c^2 dt^2 + a(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right].$$

(20)
where $\bar{k} = 1, 0, -1$ correspond to a closed, flat, and open universe, respectively. For vacuum solution without matter ($p = \rho = 0$), the first Friedmann equation takes the form

$$H^2 = \frac{\Lambda_W}{2} - \frac{\bar{k}}{a^2} \left[ 1 - \frac{1}{2\Lambda_W a^2} \right], \quad (21)$$

where scaling dimensions are $[\bar{k}] = 2$ and $[H^2] = 2$ with $H = \frac{\dot{a}}{ca}$. Here, we observe a replacement for the IR-UV transition

$$\frac{\bar{k}}{a^2} \to \frac{\bar{k}}{a^2} \left( 1 - \frac{1}{2\Lambda_W a^2} \right). \quad (22)$$

However, we note that for the $\bar{k} = 0$ case, there is no contribution from higher order curvature terms $R^2$ and $R_{ij}^2$. Also, there is no contribution from the Cotton tensor originated at gCS term because the FRW metric (20) is based on the isotropy and homogeneity. Thus, we obtain the same Friedmann equation even if one considers the $z = 3$ HL gravity. In the presence of a matter, there were bounce solutions to Friedmann equations \cite{13, 25, 26}. Also, classical solutions to the IR limit of Hořava-Lifshitz gravity can mimic general relativity plus cold dark matter \cite{27}.

In the case of deformed $z = 2$ HL gravity with gCS term, the corresponding Friedmann equation leads to \cite{4}

$$H^2 = -\frac{\bar{k}}{a^2} \left[ 1 + \frac{1}{2\omega a^2} \right], \quad (23)$$

where we find a replacement for the IR-UV transition

$$\frac{\bar{k}}{a^2} \to \frac{\bar{k}}{a^2} \left( 1 + \frac{1}{2\omega a^2} \right). \quad (24)$$

We may call (22) and (24) the “classical” IR-UV transition because we are using Friedmann equations.

In order to generate chiral gravitational waves, we consider the de Sitter inflation by introducing a positive cosmological constant $\bar{\Lambda}$ as

$$H^2 = \frac{1}{2} \left( \bar{\Lambda} - |\Lambda_W| \right), \quad (25)$$

where we choose $\bar{k} = 0$ and $\bar{\Lambda} > |\Lambda_W|$. This leads to the Sitter inflation like $a(t) \sim e^{Ht}$. Then, we introduce tensor perturbations only around de Sitter inflation \cite{15}

$$ds_{tp}^2 = -dt^2 + a^2(t) \left[ \delta_{ij} + h_{ij}(t, x_i) \right] dx^i dx^j, \quad (26)$$

where $h_{ij}$ is a transverse-traceless tensor. At this stage, we note that we choose $N^2 = 1$ for tensor perturbations, but not $N^2 = c^2$ as in Eq. (20) because the former choice makes the cosmological perturbation transparent.
Substituting this metric into the total action with \( \alpha = \mu c \), we find the bilinear action for \( h_{ij} \) as

\[
\delta^2 S = \int dtd^3x \left[ \frac{1}{2\kappa^2} h^i_j h^j_i - \frac{\kappa^2 \mu^2 \Lambda_W}{64a^2} h^i_j \triangle h^j_i + \frac{\alpha}{4a^3} \epsilon^{ijk} h_{il} \triangle h^l_{ij} - \frac{\kappa^2 \mu^2}{32a^4} h^i_j \triangle^2 h^j_i \right] \tag{27}
\]

\[
= \int dtd^3x \frac{a^3c^2}{2\kappa^2} \left[ \frac{\partial h^i_j}{\partial x^0} \frac{\partial h^j_i}{\partial x^0} + \frac{h^i_j \triangle h^j_i}{a^2} + \frac{2}{a^3} \sqrt{-\frac{2}{\Lambda_W}} \epsilon^{ijk} h_{il} \triangle h^l_{ij} + \frac{2}{\Lambda_W a^4} h^i_j \triangle^2 h^j_i \right], \tag{28}
\]

where \( \triangle \) denotes the spatial Laplacian and \( x^0 = ct \) with \([x^0] = -1\).

Tensor \( h_{ij} \) could be expanded in terms of plane waves with wave vector \( k_i \) with comoving momentum \( k = \sqrt{k_i k^i} \) and \([k^2] = 2\) as

\[
h_{ij}(t,x) = \sum_{A=L,R} \int \frac{d^3k}{(2\pi)^3} \psi^A_k(t)e^{ixk} p^A_{ij}, \tag{29}
\]

where \( p^A_{ij} \) is a circularly polarization tensor defined by \( ik_x \epsilon^{rsj} p^A_{ij} = k p_A \rho^r \) \( \rho^L = 1(\rho^R = -1) \) denote for left (right)-handed circularly polarized modes, respectively. Using the variable \( v^A_k = a \psi^A_k \) and the conformal time \( \eta \) defined by \( d\eta/dt = 1/a \), we obtain the Mukhanov-type equation for describing two circularly polarized modes

\[
\frac{d^2 v^A_k}{d\eta^2} + \left[ (k^A_{\text{eff}})^2 - \frac{2}{\eta^2} \right] v^A_k = 0, \tag{30}
\]

where

\[
(k^A_{\text{eff}})^2 = c^2 a^2 k^2 \left( 1 - \rho^A \sqrt{-\frac{2}{\Lambda_W}} \frac{k}{a} \right)^2 \tag{31}
\]

with scaling dimensions \([\eta] = -2\) and \([(k^A_{\text{eff}})^2] = 4\). On the other hand, \((k^A_{\text{eff}})^2\) is given for \( z = 3 \) HL gravity \[15\]

\[
(k^A_{\text{eff}})^2 = c^2 a^2 k^2 \left[ 1 - \frac{2}{\Lambda_W a^2} \left( 1 - \frac{2\rho^A k}{\xi^2 \mu a} \right)^2 \right]. \tag{32}
\]

At this stage, we wish to point out that for \( \alpha = \mu c \), two ghost-free circularly polarized waves are generated. In the case of \( \alpha \neq \mu c \), \((k^A_{\text{eff}})^2 > 0\) is not guaranteed, which may induce a ghost instability. In the absence of gCS term, we find the “quantum” IR-UV transition as

\[
\frac{k^2}{a^2} \rightarrow \frac{k^2}{a^2} \left[ 1 - \frac{2}{\Lambda_W a^2} \right]. \tag{33}
\]

Expressing this in terms of physical momentum \( p = k/a \) leads to

\[
p^2 \rightarrow p^2 \left[ 1 - \frac{2}{\Lambda_W p^2} \right]. \tag{34}
\]
4 Circularly polarized gravitational waves

In order to investigate tensor propagations in the Minkowski background, we use the perturbation of (5)+(18). Equivalently, we make substitution of $a \rightarrow 1$, $h_{ij} \rightarrow t_{ij}$, and $-\Lambda W \rightarrow \omega$ in Eq.(28) to derive tensor perturbation. Then, the field equation for tensor modes is given by [28, 22, 23]

\[
\ddot{t}_{ij} - c^2 \Delta t_{ij} + \frac{2c^2}{\omega} \Delta^2 t_{ij} - 2c^2 \sqrt{\frac{2}{\omega}} \epsilon_{ilm} \partial^l \Delta t_{jm} = T_{ij} \tag{35}
\]

with linearized-Cotton tensor $\delta C_{ij} = -\epsilon_{ilm} \partial^l \Delta t_{jm}$ and external source $T_{ij}$. In this case, we could not obtain the Euclidean covariant propagator because of the presence of Cotton tensor. Assuming a massless graviton propagation along the $x^3$-direction with $p_i = (0, 0, p_3)$, then $t_{ij}(x^3)$ can be expressed in terms of polarization components as [29, 30, 31]

\[
t_{ij} = \begin{pmatrix}
t_+ & t_\times & 0 \\
t_\times & -t_+ & 0 \\
0 & 0 & 0
\end{pmatrix}. \tag{36}
\]

Here, the spatial Laplacian $\Delta$ reduces to $\partial_3^2 (-p_3^2)$. Using this parametrization, we find two coupled equations for different polarizations

\[
\ddot{t}_+ - c^2 \Delta t_+ + 2c^2 \sqrt{\frac{2}{\omega}} \partial_3 \Delta t_+ + \frac{2c^2}{\omega} \Delta^2 t_+ = T_+, \tag{37}
\]

\[
\ddot{t}_\times - c^2 \Delta t_\times - 2c^2 \sqrt{\frac{2}{\omega}} \partial_3 \Delta t_+ + \frac{2c^2}{\omega} \Delta^2 t_\times = T_\times. \tag{38}
\]

To find two independent components, we have to introduce the left-right base defined by

\[
t_{L/R} = \frac{1}{\sqrt{2}} (t_+ \pm it_\times) \tag{39}
\]

where $t_L(t_R)$ represent the left (right)-handed modes. After Fourier-transformation, we find two decoupled equations

\[
-\omega^2 t_L + c^2 p_3^2 t_L - 2c^2 \sqrt{\frac{2}{\omega}} p_3^3 t_L + \frac{2c^2}{\omega} p_3^4 t_L = T_L, \tag{40}
\]

\[
-\omega^2 t_R + c^2 p_3^2 t_R + 2c^2 \sqrt{\frac{2}{\omega}} p_3^3 t_R + \frac{2c^2}{\omega} p_3^4 t_R = T_R \tag{41}
\]

with scaling dimension $[\omega] = 2$.

Finally, we have tensor propagators

\[
t_{L/R} = \frac{T_{L/R}}{\omega^2 - c^2 p_3^2 \left(1 \mp \sqrt{\frac{2}{\omega}} p_3\right)^2} \tag{42}
\]
which shows clearly that there is no ghost for two circularly polarized modes $t_{L/R}$. It is noted from Eq. (40) that for $\alpha \neq \mu c$, one could not make all spatial momentum terms positive definite, implying that a ghost state appears for the left-handed mode. Hence, the choice of $\alpha = \mu c$ is essential to obtain two ghost-free circularly polarized waves.

In the absence of gCS term, the tensor propagators take the same form

$$t_{L/R} = -\frac{\mathcal{T}_{L/R}}{\omega^2 - c^2p^2\left[1 + \frac{2}{\omega}p^2\right]},$$

where the “quantum” IR-UV transition is observed as

$$p^2 \rightarrow p^2\left[1 + \frac{2}{\omega}p^2\right]$$

with $p^2 = p_ip^i$.

### 5 GUP-corrected propagators

The GUP satisfies the modified Heisenberg algebra [32, 33, 34]

\[
[x_i, p_j] = i\hbar\left(\delta_{ij} + \beta p^2\delta_{ij} + \beta' p_ip_j\right),
\]

\[
[x_i, x_j] = i\hbar\left(2\beta - \beta'\right) + \left(2\beta + \beta'\right)\beta p^2 \frac{1}{1 + \beta p^2} \left(p_ix_j - p_jx_i\right),
\]

\[
[p_i, p_j] = 0,
\]

where $p_i$ is considered as the momentum at high energies and thus, (45) can be interpreted to be the UV-commutation relations. Here, we choose $\beta' = 2\beta$ for achieving the commutativity with UV scaling dimension $[\beta] = -2$. In this case, the minimal length which follows from the modified Heisenberg algebra is given by

$$\delta x_{\text{min}} = \hbar \sqrt{5\beta}.$$  

The presence of the minimal length represents a feature of the GUP. On the other hand, introducing IR-canonical variable $p_{0i}$ with $x_i = x_{0i}$ through the replacement

$$p_i \rightarrow p_{0i}\left(1 + \beta p_{0i}^2\right),$$

these variables satisfy canonical commutation relations

$$[x_{0i}, p_{0j}] = i\hbar\delta_{ij}, \quad [x_{0i}, x_{0j}] = [p_{0i}, p_{0j}] = 0.$$  

Here $p_{0i}$ is considered as the momentum at low energies (IR region) with $p_{0i}^2 = p_{0i}p_{0i}$. It is easy to show that Eq. (45) is satisfied to $\beta$-order when using Eq. (48). Hence, the replacement (47) could be used as the “low-energy window” to see quantum gravity phenomenology.
up to $\beta$-order. However, at $\beta^2$-order, we observe that the translation invariance is broken because coordinate $x_i$ becomes noncommutative. Thus, it is not guaranteed that the replacement (47) works up to $\beta^2$-order. Accordingly, the following replacement is suggested for making a connection between GUP and $z = 2$ HL gravity

$$p^2 \to p_0^2 \left(1 + 2\beta p_0^2\right). \quad (49)$$

At high energies, we assume that the UV-propagator has

$$G_{UV}(\omega, p^2) = \frac{1}{\omega^2 - c^2 p^2}, \quad (50)$$

whereas at low energies, the IR-propagator takes the conventional form

$$G_{IR}(\omega, p_0^2) = \frac{1}{\omega^2 - c^2 p_0^2}. \quad (51)$$

Making use of (49), the UV-propagator (50) can be rewritten as

$$G_{UV}(\omega, p_0^2) = \frac{1}{\omega^2 - c^2 p_0^2 \left(1 + 2\beta p_0^2\right)}. \quad (52)$$

The GUP-corrected tensor propagator is determined by

$$t_{ij}^{GUP} = -G_{UV}(\omega, p_0^2) T_{ij} = -\frac{T_{ij}}{\omega^2 - c^2 p_0^2 \left(1 + 2\beta p_0^2\right)}. \quad (53)$$

This is the same form as the UV-tensor propagator (43) when using the replacement of $\beta \to 1/\omega$ and $p^2 \to p_0^2$ for the $z = 2$ HL gravity.

Now we may make a connection between GUP and $z = 2$ HL gravity with gCS term (circular polarized tensor modes). To this end, we introduce doubly special relativity which suggests modification of commutators [35]

$$\begin{bmatrix} x_i, p_j \end{bmatrix} = i\hbar \left[ \delta_{ij} - \sqrt{\beta} \left(p\delta_{ij} + \frac{p_ip_j}{p^2}\right) + \beta \left(p^2\delta_{ij} + 3p_ip_j\right) \right], \quad (54)$$

with $[x_i, x_j] = [p_i, p_j] = 0$. Then, let us define

$$x_i = x_{0i}, \quad p_i \to p_{0i} \left(1 - \sqrt{\beta} p_0 + 2\beta p_0^2\right), \quad (55)$$

where $x_{0i}$ and $p_{0j}$ satisfy the canonical commutation relations $[x_{0i}, p_{0j}] = i\hbar \delta_{ij}$. It could be shown that Eq.(54) is satisfied at $\beta$-order. From Eq.(55), we have

$$p^2 \to p_0^2 \left(1 - 2\sqrt{\beta} p_0 + 4\beta p_0^2\right) \quad (56)$$

up to $\beta$-order. However, this is not the case of left-handed mode in Eq.(42). Hence, the connection between GUP and circularly polarized tensor modes is not clearly defined.
6 Discussions

We have constructed the chiral gravitational waves from the $z = 2$ Hořava-Lifshitz gravity with gCS term in the de Sitter and Minkowski backgrounds, as the $z = 3$ Hořava-Lifshitz gravity did provide them [15]. It turns out that the gCS term plays an essential role in making circularly polarized gravitational waves in the $z = 2$ and $z = 3$ Hořava-Lifshitz gravity theories. Particularly, its coefficient should be chosen as $\alpha = \mu c$ to obtain ghost-free chiral gravitational waves.

Also, we observe classical and quantum IR-UV transition rules which show the feature of the $z = 2$ HL gravity: (22) and (24) for classical rules and (34) and (44) for quantum rules. As is shown in Eq.(49), quantum transition rule may be explained from the GUP when replacing $-\frac{1}{\Lambda_W}$ and $\frac{1}{z}$ by $\beta$. The modified Heisenberg commutation (45) is satisfied to $\beta$-order. Thus, the replacement (49) was used to derive the GUP-corrected propagator (53) which is the same form as the UV-tensor propagator (43). Hence, the $z = 2$ deformed HL gravity without gCS term is well interpreted by the GUP. We note that $-\frac{2\Lambda_W}{a^3}$ in the Mukhanov-type equation (31) and $\frac{2p^4}{\Lambda_W a}$-term in the tensor propagator (43) come solely from $R_{ij}^2$-term in Eqs. (15) and (18), respectively. Therefore, $R_{ij}^2$-term is responsible for showing the quantum aspects of the $z = 2$ HL gravity.

On the other hand, classical IR-UV transition rules remain unchanged in the presence of gCS term. However, quantum transition rules do not work. Even though a modified commutation (54) was introduced, the connection between circularly polarized tensor modes and GUP is not clearly established.

In conclusion, the $z = 2$ HL gravity may be considered as a candidate for the quantum Einstein gravity. It provides matter bounce solution and chiral gravitational waves, as obtained from the $z = 3$ HL gravity. It was shown that the renormalized Wheeler-DeWitt equation possesses a solution with a $z = 2$ Lifshitz point, but no other $z > 2$ solutions to leading order of strong coupling expansion [6]. Considering the $z = 2$ HL gravity with the gCS term, their conclusion remains unchanged. Moreover, the perturbative corrections to $z = 2$ Lifshitz point are more attractive than the $z = 3$ Lifshitz point.

Acknowledgement

This work was supported by Basic Science Research Program through the National Research Foundation (KRF) of Korea funded by the Ministry of Education, Science and Technology (2009-0086861).
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