Optimal scheduling of the next preventive maintenance activity for a multi-component system

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Abstract

We propose a binary linear optimisation model whose solution may suggest wind turbine owners which components, and when, should undergo the next preventive maintenance (PM). Our scheduling strategy takes into account eventual failure events of the multi-component system, in that after the failed system is repaired, we update the previously scheduled PM plan treating the restored components to be as good as new.

Our model is tested through three numerical case studies. The first study addresses the illuminating case of a single component system. The second study analyses the case of seasonal variations of set-up costs, as compared to the constant set-up cost setting. Among other things, this analysis reveals a dramatic cost reduction achieved by our model as compared to the PM-free strategy. The third case study compares our model with that of [1] which was the major source of inspiration for our work. This comparison demonstrates that our model is accurate and much more effective.

We envisage that the algorithm stemming from the proposed model can be used as a key module in a maintenance scheduling app.

Keywords:
Integer optimisation, discrete optimisation, maintenance scheduling, wind turbine, linear programming, binary variables

1. Introduction

Wind energy is one of the lowest-priced renewable energy technologies available today; see [2]. A large part of the cost of operations is due to the cost of maintaining the wind power equipment, especially for offshore
wind farms. To further reduce the maintenance cost, one can improve the
design of the components, making them more reliable. One can also reduce
the maintenance costs by means of an improved scheduling of the compo-
nent replacements. The latter task is the main motivation for this paper,
which proposes an optimisation model dealing with a single wind turbine, or
more generally with a single multi-component system. Our approach has a
straightforward extension to a group of several multi-component systems (a
farm of wind turbines) with possibly different parameters.

Typically, a maintenance model distinguishes between a corrective main-
tenance (CM) event, when a component should be attended after it breaks
down, and a preventive maintenance (PM) event, when one or several com-
ponents are renewed before they break down; see the recent survey [3]. An
optimal PM scheduling is aimed at reducing the lost production due to the
down-time caused by CM events.

Among the multitude of papers devoted to the optimal PM scheduling
for multi-component systems, see [4], the PMSPIC (Preventive Main-
tenance Scheduling Problem with Interval Costs) model from [1] was the major inspi-
ration for this work. The main feature of the PMSPIC model is a rescheduling
step characterised by a cost function of the planned PM which depends on
the time between two consecutive PM activities.

In Section 2 we introduce the framework of our optimisation model for a
multi-component system in a discrete time setting

\[ t = 0, 1, \ldots, T, \]

where the unit of time can be a day or a month or a year, depending on
the concrete situation. It is assumed that at time 0 all the components of
the system were new and that the system lifespan is \( T \). In the same Section
2 we summarise our main result, Algorithm 1, producing an optimal PM
scheduling for the time period \([s, T]\), starting at some given time \( s \in [0, T-1] \).

The key ingredient of Algorithm 1, the NextPM algorithm, is carefully de-
scribed in Section 3, where we also clearly specify the key differences between
our approach and that of [1].

Section 4 contains several numerical studies that demonstrate the flexi-
bility of our approach, its accuracy and effectiveness, which makes it relevant
as a part of a future app for PM scheduling for farms of wind turbines.

For the motivated reader, we give complete formal presentations of our
linear optimisation models from Section 3.1 and Section 3.4 in Appendix A
and Appendix B, respectively.
2. Optimal rescheduling algorithm

Consider a system composed of $n$ components characterised by different life length distributions. For the component $j$, we will assume that its total life length $L_j$, without any maintenance, has a Weibull distribution with parameters $(\alpha_j, \beta_j)$, so that the corresponding survival function has the following parametric form

$$P(L_j > t) = e^{-\left(\frac{t}{\alpha_j}\right)^{\beta_j}}, \quad t \geq 0, \quad j = 1, \ldots, n;$$

see [5] concerning the use of the Weibull distribution for the modelling of multi-component systems. The means and variances of the component life lengths are computed as

$$\mu_j = \alpha_j \Gamma(1 + \frac{1}{\beta_j}), \quad \sigma_j^2 = \alpha_j^2 \Gamma(1 + \frac{2}{\beta_j}) - \mu_j^2, \quad j = 1, \ldots, n.$$

Besides the component survival parameters $(\alpha_j, \beta_j)$, our model of a multi-component system (which we sometimes call an $n$-system) requires the following parameters describing the various costs associated with the maintenance of the $n$-system:

- $d_t$, the time-dependent set-up cost for either a PM or CM activity,
- $b_j$, the CM cost of the component $j$,
- $c_j$, the PM cost of the component $j$,

where time $t = 0, \ldots, T$ is discrete and $j = 1, \ldots, n$. To summarise, the full set of the model parameters is

$$\{d_1, \ldots, d_T, (\alpha_1, \beta_1, b_1, c_1), \ldots, (\alpha_n, \beta_n, b_n, c_n), \lambda\},$$

including an extra parameter $\lambda$ introduced in Section 3.2 by formula (4). Notice that we will extend the definition of $d_t$ to the continuous range of the variable $t$ by setting $d_t = d_{\lfloor t \rfloor}$.

Suppose that the $n$-system is observed at some discrete time $s \in [0, T - 1]$, and we are given the last maintenance times $t_j \in [0, s]$ for each of the components $j = 1, \ldots, n$, so that at the time $s$ the $n$ components have the effective ages $(s - t_1, \ldots, s - t_n)$. Our NextPM optimisation model described in Section 3 has the input $(t_1, \ldots, t_n, s, r)$, where $r \in [s + 1, T]$ is the end of the current planning period. The output of NextPM is a PM plan specifying the time $\tau \in [s + 1, r + 1]$ of the next PM event, as well as which of the
components $\mathcal{P} \subset \{1,\ldots,n\}$ should be maintained at the time $\tau$. The $\tau = r + 1$ implies $\mathcal{P} = \emptyset$ which means that no PM should be scheduled during the planning period $[s + 1, r]$.

The NextPM is the key module of the following algorithm for PM scheduling until the time $T$ when the $n$-system will be dismantled. The algorithm relies on a rescheduling procedure, where each NextPM step covering $r - s$ units of the planning time is accompanied by a NextOM module. The latter is a modification of NextPM, see Section 3.4, which addresses the possibility of a component failure before the planned PM, followed by opportunistic maintenance (OM) activities.

**Algorithm 1 Optimal rescheduling algorithm**

Input $t_1,\ldots,t_n, s, r$

Start

Solve NextPM{$t_1,\ldots,t_n, s, r$}

Output $\tau, \mathcal{P}$, where $\mathcal{P} \subset \{1,\ldots,n\}$

If $\tau < T$

If a failure during the period $(s, \tau]$ damages component $i$ at time $u_i$

Set $u := \lfloor u_i \rfloor$

Solve NextOM{$i, t_1,\ldots,t_n, u$}

Output $\mathcal{O} \subset \{1,\ldots,n\}$

Perform CM of component $i$ at time $u + 1$

Perform PM of each component $j \in \mathcal{O}$ at time $u + 1$

Update $r := \min(u + 1 + r - s, T)$, $s = u + 1$

Update $t_j := u + 1$, $j \in \mathcal{O} \cup \{i\}$

Else

Perform PM of each component $j \in \mathcal{P}$ at time $\tau$

Update $r := \min(\tau + r - s, T)$, $s := \tau$, $t_j := s$, $j \in \mathcal{P}$

End

Go to Start

Else

Stop

End

Comments:

$\mathcal{P}$ is the set of components that should undergo PM at time $\tau$,

$\mathcal{O}$ is the set of components that should undergo OM at time $u+1$. 

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3. An optimal plan for the next preventive maintenance

In this section we describe NextPM, the key ingredient of Algorithm 1 summarised in Section 2. The purpose of the NextPM model is to produce an optimal PM plan for the period \([s + 1, r]\), where the planning timespan \(r - s\) is chosen so that it is reasonable to expect at most one PM event during time \(r - s\).

3.1. NextPM model

For a given planning period \([s + 1, r] \subset [0, T]\), we call an \((s, r)\)-plan any set of vectors \((z, x^1, \ldots, x^n)\) whose components are vectors

\[
z = (z_{s+1}, \ldots, z_{r+1}), \quad x^j = (x^j_{s+1}, \ldots, x^j_{r+1}), \quad j = 1, \ldots, n,
\]

with binary coordinates \(z_t, x^j_t \in \{0, 1\}\), and satisfy the following linear conditions:

\[
\begin{align*}
\sum_{t=s+1}^{r+1} x^j_t &= 1, \quad j = 1, \ldots, n, \\
x^j_t &\leq z_t, \quad t = s + 1, \ldots, r + 1, \quad j = 1, \ldots, n.
\end{align*}
\]

For \(t = s + 1, \ldots, r\), the equality \(x^j_t = 1\) means that we tentatively plan to perform a PM of the component \(j\) at the time step \(t\): whenever a failure of the component occurs during the period \([s + 1, t]\), the plan requires rescheduling of the next PM.

Likewise, \(z_t = 1\) means that we tentatively plan to perform maintenance of at least one of the components at the time step \(t\). Furthermore, we put \(x^j_{r+1} = 1\) if we don’t plan to perform maintenance for the component \(j\) during the time period \([s + 1, r]\). The equality \(z_{r+1} = 1\) means that we do not plan to perform maintenance for at least one of the \(n\) components during the time period \([s + 1, r]\).

Our NextPM optimisation model is built around the objective function

\[
f(z, x^1, \ldots, x^n) = \sum_{t=s+1}^{r+1} \frac{1}{t - s} \left( d_t z_t + c^1_{s,t} x^1_t + \ldots + c^n_{s,t} x^n_t \right),
\]

(1)
where \(c^j_{s,t}\) is defined in Section 3.2. Since \(d_t z_t\) stands for the set-up cost and the sum \(\sum_{j=1}^n c^j_{s,t} x^j_t\) gives the total maintenance cost, the objective function (1) should be viewed as the time-average maintenance cost per time unit according to the \((s,t)\)-plan \((z, x^1, \ldots, x^n)\).

Let \((\bar{z}, \bar{x})\) be the solution to the linear optimisation problem to minimize \(f(z, x^1, \ldots, x^n)\), over all \((s,t)\)-plans subject to the linear constraint

\[
D^j_{s,t} x^j_t \geq 0, \quad t = s + 1, \ldots, r, \quad j = 1, \ldots, n,
\]

where \(D^j_{s,t}\) is defined in Section 3.3 as the PM benefit for the component \(j\) at time \(t\). Then the output of our NextPM algorithm \((\tau, N)\) is given by

\[
\tau = \min \{ \arg \max \bar{x}^j_t \},
\]

\[
N = \left\{ \begin{array}{ll}
\{ j : \bar{x}^j_t = 1, \quad j = 1, \ldots, n \} & \text{if } \tau \leq r, \\
\emptyset & \text{if } \tau = r + 1.
\end{array} \right.
\]

3.2. Definition of \(c^j_{s,t}\)

Here we present our formula for \(c^j_{s,t}\) which will clarify the definition (1) of the objective function for the optimisation model NextPM. We define \(c^j_{s,t}\) as the fixed PM cost \(c_j\) plus the expected additional costs due to eventual failures of the component \(j\) which may occur prior the planned PM activity at time \(t\).

To this end, define \(n\) independent sequences of renewal times with a delay by letting \(U^j_{s,0} = s\),

\[U^j_{s,0} = t_j + L_{1j}, \quad L_{1j} \overset{d}{=} \{ L_j | L_j > s - t_j \},\]

where \(d\) means equality in distribution (notice however, that the above equality involves a conditional distribution), and

\[U^j_{s,i+1} = U^j_{s,i} + L_{ij}, \quad L_{ij} \overset{d}{=} L_j, \quad \text{for } i = 2, 3, \ldots,\]

assuming that the random variables \((L_{ij})\) are mutually independent. Notice that in the important particular case \(s = 0\), this definition simplifies, so that for each \(j\), the sequence \(\{U^j_{0,i}\}_{i \geq 0}\) describes a renewal process without a delay.
Treating $U_{s,1}^j, U_{s,2}^j, \ldots$ as the consecutive failure times, put

$$c_{s,t}^j := c_j + E\left(\sum_{i=1}^{\infty} 1\{U_{s,i}^j \leq t\} G_j(U_{s,i-1}^j, L_{ij}, t-s)\right), \tag{3}$$

where the cost functions

$$G_j(s, u, t) = b_j + d_s u - \left(\frac{u}{t}\right) \lambda (c_j + d_{s+t}), \quad 0 \leq u \leq t, \tag{4}$$

involve a new parameter $\lambda > 0$ assumed to be independent of $j = 1, \ldots, n$.

Our definition of the cost function (4) further develops the idea of Section 5.1 in [1]; see Section 3.5 below. It describes the additional cost incurred at the failure time $s + u$, taking place between the starting time $s$ and the time $s + t$ scheduled for the next PM. If $u$ is close to 0, then the failure at time $s + u$ will not change our PM plan, implying that the additional cost $G_j(s, 0, t) = b_j + d_s$ is the sum of the CM cost $b_j$ and the set-up cost $d_s$ at time $s$. On the other hand, if $u$ is close to $t$, then the additional cost $G_j(s, t, t) = b_j - c_j$ is simply the difference between the CM and PM costs. The expression in the righthand side of (4) represents an intermediate additional cost, where the parameter $\lambda$ evaluates to what extent the proximity of $u$ to $t$ reduces the planned PM costs.

3.3. Definition of $D_{s,t}^j$

The constraint (2) arises as a check-up step to ensure that a suggested PM at time $t$ brings some benefit, as compared to a simple strategy when no PM is performed. With the PM-free strategy, the total maintenance cost (including set-up costs) for the component $j$ during the period $[s, T]$ would be

$$E\left[\sum_{i=1}^{\infty} 1\{U_{s,i}^j \leq T\} \left(b_j + d_{U_{s,i}^j}\right)\right].$$

Alternatively, we may plan to perform a PM at time $t$, and then perform replacements of the component $j$ whenever it breaks down. Under the latter scenario the total cost would be

$$c_{s,t}^j + E\left[\sum_{i=1}^{\infty} 1\{t + U_{0,i}^j \leq T\} \left(b_j + d_{t + U_{0,i}^j}\right)\right].$$
Taking the difference between these two total costs

\[ D_{s,t}^j = E \left[ \sum_{i=1}^{\infty} 1_{\{ U_{s,i}^j \leq T \}} (b_j + d_{i U_{s,i}^j}) \right] - c_{s,t}^j - E \left[ \sum_{i=1}^{\infty} 1_{\{ t + U_{0,i}^j \leq T \}} (b_j + d_{t + U_{0,i}^j}) \right], \]

we can claim that the planned PM of the component \( j \) at time \( t \) is justified only if \( D_{s,t}^j \geq 0 \).

3.4. NextOM model

The NextOM part of Algorithm 1 is a specialised version of the NextPM part described below in terms of a given input vector

\[(i, t_1, \ldots, t_{i-1}, t_{i+1}, \ldots, t_n, s).\]

Here, \( s \in [0, T] \) and \( i \) is the label of the component whose failure at some time during \([s, s + 1)\) has triggered the OM planning step. For a pair \( \{s, i\}\), we call an \( \{s, i\}\)-plan any set of vectors \((z, x^1, \ldots, x^n)\) whose components are two-dimensional vectors

\[ z = (z_{s+1}, z_{s+2}), \quad x^j = (x^j_{s+1}, x^j_{s+2}), \quad j = 1, \ldots, n, \]

with binary coordinates \( z_t, x^j_t \in \{0, 1\} \) satisfying the following linear conditions

\[ \sum_{t=s+1}^{s+2} x^j_t = 1, \quad j = 1, \ldots, n, \]

\[ x^{(i)}_{s+1} = 1, \]

\[ z_t \geq x^j_t, \quad t = s + 1, s + 2, \quad j = 1, \ldots, n. \]

Observe that necessarily, \( z_{s+1} = 1 \).

Our NextOM optimisation model uses a modified objective function

\[ f_i(z, x^1, \ldots, x^n) = \sum_{t=s+1}^{s+2} \frac{1}{t-s} \left( d_t z_t + \sum_{j \neq i} c_{s,t}^j x^j_t \right), \]

where \( c_{s,t}^j \) is defined in Section 3.2. Let \((\bar{z}, \bar{x})\) be the solution to the linear optimisation problem to

\[ \text{minimise} \quad f_i(z, x^1, \ldots, x^n) \]
over all \{s, i\}-plans subject to the linear constraint
\[
D^j_{s, s+1} x^j_{s+1} \geq 0, \quad j = 1, \ldots, i - 1, i + 1, \ldots, n,
\]
where \( D^j_{s, t} \) is defined in Section 3.3. The output of the NextOM is given by the set
\[
O = \{ j : \bar{x}^j_\tau = 1, \quad j = 1, \ldots, i - 1, i + 1, \ldots, n \},
\]
consisting of the labels of the components which will be opportunistically maintained along with the component \( i \) undergoing a CM activity.

3.5. Comparison with the PMSPIC optimisation model

Here we present the optimisation model of [1] in terms similar to the current setting and compare the PMSPIC model with our own approach in the particular case when \( s = 0 \) and the set-up costs \( d_t \equiv d \) are constant over time.

For the planning period \([0, T]\) of the PMSPIC model, define the set of paired time points
\[
\mathcal{I} = \{(u, t) : 0 \leq u < t \leq T + 1 \},
\]
and call an \( \mathcal{I} \)-plan any vector \((z, x^1, \ldots, x^n)\) composed by a vector and \( n \) triangular arrays
\[
z = (z_1, \ldots, z_T), \quad x^j = \{ x^j_{ut} : (u, t) \in \mathcal{I} \}, \quad j = 1, \ldots, n.
\]
It is assumed that the binary components \( z_t, x^j_{ut} \in \{0, 1\} \), satisfy the following linear conditions
\[
z_t \geq \sum_{u=s}^{t-1} x^j_{ut}, \quad t = 1, \ldots, T, \quad j = 1, \ldots, n,
\]
\[
\sum_{t=1}^{T+1} x^j_{st} = 1, \quad j = 1, \ldots, n,
\]
\[
\sum_{u=0}^{t-1} x^j_{ut} = \sum_{v=t+1}^{T+1} x^j_{tv}, \quad t = 1, \ldots, T + 1, \quad j = 1, \ldots, n.
\]
For \((u, t) \in \mathcal{I}\), we write \( x^j_{ut} = 1 \) if we plan to maintain the component \( j \) both at the time step \( u \) and time step \( t \) but not in between. We write \( z_t = 1 \) if we
plan to perform maintenance of at least one of the components at the time step $t$. The last constraint mentioned is the counterpart of the flow balance constraint from [6].

The PMSPIC model minimises the objective function

$$F(z, x^1, \ldots, x^n) = \sum_{t=1}^{T} d_t z_t + \sum_{(u,t) \in I} \sum_{j=1}^{n} c^j_{t-u} x^j_{ut},$$

representing the total maintenance cost of the $I$-plan $(z, x^1, \ldots, x^n)$. Here the term $c^j_t$ given by

$$c^j_t = c_j + E \left( \sum_{i=1}^{\infty} 1_{U^j_{0,i} \leq t} g_j(U^j_{0,i}, t) \right),$$

where

$$g_j(u, t) = b_j + d - \left( \frac{u}{t} \right)^\lambda (c_j + d)$$

should be compared with the term $c^j_{0,t}$ defined by [3] and [4], which in the particular case of $s = t_1 = \ldots = t_n = 0$ and $d_t \equiv d$ takes the form

$$c^j_{0,t} = c_j + E \left( \sum_{i=1}^{\infty} 1_{U^j_{0,i} \leq t} g_j(L_{ij}, t) \right).$$

Comparing the two expressions for $c^j_t$ and $c^j_{0,t}$, we see that the key difference is between the terms $g_j(U^j_{0,i}, t)$ and $g_j(L_{ij}, t)$. We claim that the formula $g_j(U^j_{0,i}, t)$ for $i \geq 2$ is not compatible with the meaning of the cost function $g_j(u, t)$ explained earlier for [4]. Indeed, the term $g_j(U^j_{0,i}, t)$ assumes that the component $j$ has age $U^j_{0,i}$, while actually it is supposed to be restored at the time $U^j_{0,i-1}$ of the previous failure.

4. Numerical studies

The three case studies presented in this section deal with a wind turbine as an example of the multi-component system. They are all based on the parameter values taken from the paper [7], see Table 1, where the cost unit is 1000 USD and the time unit is 1 month. The lifetime of the wind turbine is assumed to be 20 years, which is the typical case in the industry now,
| Component   | CM cost $b_j$ | PM cost $c_j$ | $\beta_j$ | $\alpha_j$ | $\mu_j$ |
|-------------|--------------|--------------|-----------|-----------|--------|
| Rotor       | 162          | 36.75        | 3         | 100       | 89.9   |
| Main bearing| 110          | 23.75        | 2         | 125       | 110.8  |
| Gearbox     | 202          | 46.75        | 3         | 80        | 71.4   |
| Generator   | 150          | 33.75        | 2         | 110       | 97.5   |

Table 1: key parameters for a four-component system.

It turns out that in the current setting, the constraint (2) can effectively be disregarded so that the optimal PM time $\tau$ is obtained by minimising the objective function (1), which is equivalent to minimising $a_t$ over $t = 1, \ldots, r + 1$.

The left panel of Figure 1 presents a typical profile for the function $a_t$, the inset in the left panel shows that $\tau = 47$ for $d = 10$. The maintenance cost in this case is $a_{47} = 1.9$. Among other things, the graph explains why
4.2. Study 2: seasonal effects

Part A. To address the seasonal effects of the set-up costs \(d_t\), we assume the following set-up costs (in thousands of USD) for different months in a year:

\[
\begin{array}{cccccccccccc}
\text{Jan} & \text{Feb} & \text{Mar} & \text{Apr} & \text{May} & \text{Jun} & \text{Jul} & \text{Aug} & \text{Sep} & \text{Oct} & \text{Nov} & \text{Dec} \\
7.5 & 6.5 & 5.5 & 4.5 & 3.5 & 2.5 & 2.5 & 3.5 & 4.5 & 5.5 & 6.5 & 7.5
\end{array}
\]

so that the average set-up cost is \(\bar{d} = 5\). Table 2 summarises the results produced by our NextPM algorithm applied to the following three settings:

- the winter start scenario with
  \[d_1 = 7.5, d_2 = 6.5, \ldots, d_{12} = 7.5, d_{13} = 7.5, d_{14} = 6.5, \ldots,\]

- the summer start scenario with
  \[d_1 = 2.5, d_2 = 3.5, \ldots, d_{12} = 2.5, d_{13} = 2.5, d_{14} = 3.5, \ldots,\]

- the constant set-up cost scenario with \(d_1 = 5, d_2 = 5, d_3 = 5, \ldots\)
### Table 2: summary of the NextPM results for $\bar{d} = 5$.  

| Component $j$        | 1 | 2 | 3 | 4 | Monthly maintenance cost |
|----------------------|---|---|---|---|---------------------------|
| Winter start         | x | x | 43| x | 4.876                     |
| Summer start         | 48| x | 48|x | 4.863                     |
| Constant set-up cost | 50| 50| 50| 50| 4.964                     |

According to Table 2, in the Winter start setting, the optimal next PM plan suggests a PM activity on month 43 only for the component 3 - the gearbox. The most economic among the three scenarios is to start in the summer time, with the optimal plan being to perform the next PM activity on month 48 by attending the components 1 and 3.

Part B. We contrast the results of Part A with the case of doubled set-up costs, when $\bar{d} = 10$ and $d_t$ taking the following values depending on which month represents the discrete time variable $t$:

|                               | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|-------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|                               | 15  | 13  | 11  | 9   | 7   | 5   | 5   | 7   | 9   | 11  | 13  | 15  |

The new results presented in Table 3 are drastically different from the results of Part A. They suggest (as a consequence of higher set-up costs) to perform PM to all four components at a certain time, irrespective of the scenario. Again, the cheapest solution is obtained for the Summer start setting. Notice that in all of our seasonal settings, the proposed PM activities are scheduled for summer months (having lower set-up costs).

Part C. A simple wind turbine maintenance strategy is to ignore the PM option and perform a CM activity whenever a turbine component breaks down. This leads to the question: how much can one save by introducing PM planning? To estimate the total cost associated with the pure CM strategy, we have to take into account the random number of failures over the time
interval $[0, T]$ for all $n$ components of the $n$-system:

$$F(T) = \sum_{j=1}^{n} E \left( \sum_{i=1}^{\infty} 1_{\{V_{ij} \leq T\}}(d_{ij} + b_{j}) \right) = \sum_{j=1}^{n} \int_{0}^{T} (d_{u} + b_{j})dH_{j}(u),$$

where $H_{j}$ are the corresponding renewal functions

$$H_{j}(t) = E \left( \sum_{i=1}^{\infty} 1_{\{V_{ij} \leq t\}} \right), \quad t > 0, \ j = 1, \ldots, n.$$

According to standard renewal theory, for large values of $T$, we have the following approximation formula

$$\frac{F(T)}{T} \approx \sum_{j=1}^{n} \frac{1}{T\mu_{j}} \int_{0}^{T} (d_{u} + b_{j})du = \sum_{j=1}^{n} \frac{\bar{d} + b_{j}}{\mu_{j}},$$

where

$$\bar{d} = \frac{d_{1} + \ldots + d_{T}}{T}.$$

Applying this approximation to our four-component model of the wind turbine, we estimate the monthly maintenance costs for the pure CM strategy to be

7.396 if $\bar{d} = 5$,
7.618 if $\bar{d} = 10$.

We see that compared to the costs produced by our NextPM algorithms in Parts A and B, the PM planning results in roughly 35% cost saving.

4.3. A performance comparison with PMSPIC

Comparing the NextPM model with the PMSPIC model is not a straightforward exercise since the latter produces a maintenance plan for whole lifespan $[0, T]$ of the multi-component system in question. We partially overcome this difficulty by comparing the monthly costs for the first planned PM activity. The following three tables summarise our results for three values of the constant set-up cost $d$:
We see that the main difference between NextPM and PMSPIC lies in the effectiveness of the algorithms reported in the rightmost columns. For example, if \( d = 10 \), then the NextPM optimisation runs 10000 times faster than the PMSPIC optimisation.

For \( d = 5 \), we also performed NextPM calculations with the time unit being three days. The results were somewhat similar to those obtained for the time unit 1 month. We observed the increase in the AMPL time from 0.01 to 0.06 seconds caused by a ten-fold increase of the number of the time steps. The corresponding increase in the AMPL time for the PMSPIC model was much more dramatic: we have stopped running the program after 5 hours.

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| \(d\) | 1 | 2 | 3 | 4 | Monthly maintenance cost | Matlab | AMPL |
|-------|---|---|---|---|---------------------------|--------|------|
| \(d = 1\) | x | x | 43 | x | 4.731 | 49 sec | 0.01 sec |
| NextPM | x | x | 41 | x | 4.749 | 100 sec | 2.25 sec |
| PMSPIC | | | | | | | |

| \(d = 5\) | 1 | 2 | 3 | 4 | Monthly maintenance cost | Matlab | AMPL |
|---|---|---|---|---|---------------------------|--------|------|
| NextPM | 50 | 50 | 50 | 50 | 4.964 | 54 sec | 0.01 sec |
| PMSPIC | 51 | 51 | 51 | 51 | 4.884 | 102 sec | 10.62 sec |

| \(d = 10\) | 1 | 2 | 3 | 4 | Monthly maintenance cost | Matlab | AMPL |
|---|---|---|---|---|---------------------------|--------|------|
| NextPM | 52 | 52 | 52 | 52 | 5.061 | 55 sec | 0.01 sec |
| PMSPIC | 47 | 47 | 47 | 47 | 5.025 | 101 sec | 13.47 sec |

| \(d = 15\) | 1 | 2 | 3 | 4 | Monthly maintenance cost | Matlab | AMPL |
|---|---|---|---|---|---------------------------|--------|------|
| NextPM | 53 | 53 | 53 | 53 | 5.155 | 57 sec | 0.01 sec |
| PMSPIC | 48 | 48 | 48 | 48 | 5.111 | 105 sec | 14.92 sec |
Appendix A. Complete optimisation model of NextPM

minimize \( f(z, x^1, \ldots, x^n) := \sum_{t=s+1}^{r+1} \frac{1}{t-s} (d_t z_t + c^1_{s,t} x^1_t + \ldots + c^n_{s,t} x^n_t) \),

subject to

\[
\sum_{t=s+1}^{r+1} x^j_t = 1, \quad j = 1, \ldots, n,
\]

\[ z_t \geq x^j_t, \quad t = s + 1, \ldots, r + 1, \quad j = 1, \ldots, n, \]

\[ D^j_{s,t} x^j_t \geq 0, \quad t = s + 1, \ldots, r, \quad j = 1, \ldots, n, \]

\[ z_t \in \{0, 1\}, \quad t = s + 1, \ldots, r + 1, \]

\[ x^j_t \in \{0, 1\}, \quad t = s + 1, \ldots, r + 1, \quad j = 1, \ldots, n. \]

Appendix B. Complete optimisation model of NextOM

minimize \( f(z, x^1, \ldots, x^n) := \sum_{t=s+1}^{s+2} \frac{1}{t-s} (d_t z_t + \sum_{j \neq i} c^j_{s,t} x^j_t) \),

subject to

\[
\sum_{t=s+1}^{s+2} x^j_t = 1, \quad j = 1 \ldots, n,
\]

\[ D^j_{s, s+1} x^j_{s+1} \geq 0, \quad j = 1, \ldots, i-1, i+1, \ldots, n, \]

\[ x^{(i)}_{s+1} = 1, \]

\[ z_t \geq x^j_t, \quad t = s + 1, s + 2, \quad j = 1, \ldots, n, \]

\[ z_t \in \{0, 1\}, \quad t = s + 1, s + 2, \]

\[ x^j_t \in \{0, 1\}, \quad t = s + 1, s + 2, \quad j = 1, \ldots, n. \]

References

[1] E. Gustavsson, M. Patriksson, A.-B. Strömberg, A. Wojciechowski, M. Önnheim, Preventive maintenance scheduling of multi-component systems with interval costs, Computers & Industrial Engineering 76 (2014) 390–400.
[2] Lazard, Lazard’s levelized cost of energy analysis–version 11.0 (2017).

[3] H. Lee, J. H. Cha, New stochastic models for preventive maintenance and maintenance optimization, European Journal of Operational Research 255 (1) (2016) 80–90.

[4] S. Werbúńska-Wojciechowska, et al., Technical system maintenance, Delay-time-based modelling (in rev., Springer) (2019).

[5] H. Guo, S. Watson, P. Tavner, J. Xiang, Reliability analysis for wind turbines with incomplete failure data collected from after the date of initial installation, Reliability Engineering & System Safety 94 (6) (2009) 1057–1063.

[6] D. R. Fulkerson, Flow networks and combinatorial operations research, The American Mathematical Monthly 73 (2) (1966) 115–138.

[7] Z. Tian, T. Jin, B. Wu, F. Ding, Condition based maintenance optimization for wind power generation systems under continuous monitoring, Renewable Energy 36 (5) (2011) 1502–1509.

[8] L. Ziegler, E. Gonzalez, T. Rubert, U. Smolka, J. J. Melero, Lifetime extension of onshore wind turbines: A review covering Germany, Spain, Denmark, and the UK, Renewable and Sustainable Energy Reviews 82 (2018) 1261–1271.