A novel synchronization between two different chaotic systems (Convert Lorenz chaotic system to Chua chaotic system)

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Abstract

In this work, a new synchronization was achieved between two different chaotic systems in behavior (Chua-Lorenz) by coupling them together with a coupling factor \( k \) and it was noted that Lorenz’s behavior completely transformed Chua’s behavior by changing the value of this factor. It is observed that when the values of the coupling factor are zero, the two systems are asynchronous, and when its value is 100, there is partial synchronization, while its value is 100000, so we get perfect synchronization. Where it is possible to use this synchronization in the field of secure communications.

1. Introduction

There are many successful attempts that have been made by researchers working in the field of Chaos world, including Chen, Ikeda, Chua and others[1–7]. Researcher Chua invented his simple electric circuit in 1983[1]. These numerous attempts became theoretically and practically largely supportive of the idea of chaos that began with Researcher Lorenz[2]. Chua proved, through his theory, that chaos is an existing physical phenomenon and not just an illusion resulting from computer approximation errors. Chua has demonstrated through his work that chaotic behavior accompanies all areas of our daily lives [8–9]. He designed his simple electronic circuit that displays chaotic behavior (vibrating irregular behavior) in which the wave does not repeat itself over time. The Chua system is considered a standard model for studying chaos because of its ease of construction. There are many previous studies that were presented to solve many problem[10–13], including the issue of secure communications [14–18], but the most important of these studies conducted now is an attempt to transform the behavior of a chaotic system into another, through linking and synchronizing them.

2. Modling

In this work, the behavior of two different systems, namely Lorenz and Chua, was studied and they were coupling together with a coupling factor \( k \) and the effect of this coupling on the behavior of the Lorenz system was studied.

The Lorenz system is defined by three ordinary differential equations, as follows[1]:

\[
\frac{dx}{dt} = \sigma (y - x)
\]
\[
\frac{dy}{dt} = (x)(r - z) - y (1)
\]
\[
\frac{dz}{dt} = (x)(y) - \beta z
\]

Where the Lorenz parameters system \([r, \sigma, and \beta]\) are real positive number that equal [28, 15, 2.1] respectively and the system exhibit chaotic behavior for these value. The initial conditions of the Lorenz system \([x_1, y_1, z_1]\) were chosen to be [0, 0.1, 0], respectively.
For the Chua system, its electronic circuit can be analyzed using Kirchhoff’s laws. The dynamics of an electronic circuit can be modeled with three nonlinear ordinary differential equations in the variables \(x, y, \) and \(z\), as follows [2].

\[
\begin{align*}
\frac{d}{dt} (x_2) &= a(y_2 - x_2 - g) \\
\frac{d}{dt} (y_2) &= x_2 - y_2 + z_2 \\
\frac{d}{dt} (z_2) &= -b(y_2) + \gamma(z_2) \\
g &= c(x_2) + (0.5)(((d - c)(\text{abs}(x_2 + 1) - \text{abs}(x_2 - 1))))
\end{align*}
\]

Where Chua parameters system are \([a, b, c, d, \) and \(\gamma]\) equal \([15, 25.58, -0.714286, -1.14287,\) and \(0.001]\) respectively and initial conditions \([x_{i2}, y_{i2}, z_{i2}]\) are \([0, 0.1, 0]\) respectively.

To achieve complete synchronization between the Lorenz system and the Chua system, the coupling terms are added to all the equations of the Lorenz system, as shown below:

\[
\begin{align*}
\frac{d}{dt} (x_1) &= \sigma (y_1 - x_1) + k(x_2 - x_1) \\
\frac{d}{dt} (y_1) &= (x_1)(r - z_1) - y_1 + k(y_2 - y_1) \\
\frac{d}{dt} (z_1) &= (x_1)(y_1) - \beta(z_1) + k(z_2 - z_1)
\end{align*}
\]

Where \(k(x_2 - x_1), k(y_2 - y_1),\) and \(k(z_2 - z_1)\) are coupling terms and \(k\) is called coupling factor and is real numbers.

### 3. Results And Discussion

To analyze the results obtained from the mathematical model, the Matlab program was used, where the differential equations of all systems were solved using fourth-degree Runga–Kutta integration, as follows:

Figure (1) represent the time series of Lorenz system at initial conditions equal \([0, 0.1, 0]\), while Figure (2) represent the strange attractor was shown. The shape of the Lorenz attractor, when plotted graphically, it be seen to resemble a butterfly.

Figure (3) represent the time series of Chua system at initial conditions equal \([0, 0.1, 0]\), while Figure (4) represent the strange attractor (double scroll) was shown. The double-scroll system is often described by a system of three nonlinear ordinary differential equations and a 3-segment piecewise-linear equation (Chua’s equations). This makes the system easily simulated numerically and easily manifested physically due to Chua’s circuits’ simple design. The double scroll attractor contains an infinite number of fractal-like layers.
To investigation the coupling between two chaotic systems, Figure (5) represents the correlation between to chaotic system at coupling factor $k = 0$, where (a) represent the relation between $x_2$-dynamic versus $x_1$-dynamic, (b) represent the relation between $y_2$-dynamic versus $y_1$-dynamic, and (c) represent the relation between $z_2$-dynamic versus $z_1$-dynamic. The correlation figure represents the distribution of diffuse points in the axis space and collects around the original point.

When increasing the coupling factor $k$ to be 100, Figure (6) represent the time series of two different chaotic systems, Lorenz system is blue line and Chua system is red line, (a) $x$-dynamic, (b) $y$-dynamic, (c) $z$-dynamic. While Figure (7) represent the attractor of two different chaotic systems where the Lorenz system is blue line and red line of Chua system (a) $y$-dynamic versus $x$-dynamic, (b) $z$-dynamic versus $x$-dynamic, (c) $z$-dynamic versus $y$-dynamic. Where we notice that the attractor is identical in ($x$-$z$) plane for both systems, with the difference of the attractor in ($x$-$y$) plane and ($y$-$z$) plane. This means that the system at this value of the coupling factor will force the Lorenz system to follow the behavior of the Chua system for dynamics $x$ and $z$. This is clearly evident through the correlation relationship, as in Figure (8), where a straight line was obtained that makes an angle of $45^0$ with the $x$-axis of the dynamic $x$ and $z$, and this confirms the state of synchronization in these two dynamics, while the dynamic $y$ remains in the asynchronous state of two system.

To achieve full synchronization between the two chaotic systems, the value of the coupling factor is changed to 100000, as shown in Figure(9), which represents the time series of the two systems, where notice a complete transformation of the Lorenz system into the Chaotic Chua system with the directions of the three dynamics $x$, $y$ and $z$. The fact that the two chaotic systems are identical, or rather, the complete transformation of the behavior of the Lorenz system into the behavior of the Chua system, is the attractor and the corrselion, as in Figs. 10 and 11. As Figure (10) shows the congruence of the strange attractor for all dynamics, as well as the appearance of the correlation in straight lines and at an angle of $45^0$ for all dynamics.

4. Conclusion

Through the results, we can conclude that it is possible to change the behavior of any chaotic system to a different behavior when it is coupling to another system that is completely different from it, through the coupling factor.

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Figures
Figure 1

Time series of Lorenz system at initial conditions $x_i=0, y_i=0.1, \text{ and } z_i=0$, (a) $x_1$-dynamic, (b) $y_1$-dynamic, $z_1$-dynamic.

Figure 2

Attractor of Lorenz system at initial conditions $x_i=0, y_i=0.1, \text{ and } z_i=0$, (a) $y_1$ vs. $x_1$, (b) $z_1$ vs. $x_1$, (c) $z_1$ vs. $y_1$.

Figure 3

Time series of Chua chaotic system at initial conditions $x_i=0, y_i=0.1, \text{ and } z_i=0$, (a) $x_2$-dynamic, (b) $y_2$-dynamic, $z_2$-dynamic.

Figure 4

Attractor of Chua chaotic system at initial conditions $x_i=0, y_i=0.1, \text{ and } z_i=0$, (a) $y_2$ vs. $x_2$, (b) $z_2$ vs. $x_2$, (c) $z_2$ vs. $y_2$.

Figure 5

correlation between two chaotic system at coupling factor $k=0$, (a) $x_2$ vs. $x_1$, (b) $y_2$ vs $y_1$, (c) $z_2$ vs $z_1$.

Figure 6

time series of two different chaotic systems at $k=100$, Lorenz system is blue line and Chua system is red line, (a) $x$-dynamic, (b) $y$-dynamic, (c) $z$-dynamic.

Figure 7
Attractor of two different chaotic systems at \( k=100 \), Lorenz system is blue line and Chua system is red line, (a) \( y \) vs. \( x \), (b) \( z \) vs. \( x \), (c) \( z \) vs. \( y \)

**Figure 8**

correlation between two chaotic system at coupling factor \( k=100 \), (a) \( x_2 \) vs. \( x_1 \), (b) \( y_2 \) vs \( y_1 \), (c) \( z_2 \) vs \( z_1 \)

**Figure 9**
time series of two different chaotic systems at \( k=100000 \), Lorenz system is blue line and Chua system is red line, (a) \( x \)-dynamic, (b) \( y \)-dynamic, (c) \( z \)-dynamic

**Figure 10**

Attractor of two different chaotic systems at \( k=100000 \), Lorenz system is blue line and Chua system is red line, (a) \( y \) vs. \( x \), (b) \( z \) vs. \( x \), (c) \( z \) vs. \( y \)

**Figure 11**
correlation between two chaotic system at coupling factor \( k=100000 \), (a) \( x_2 \) vs. \( x_1 \), (b) \( y_2 \) vs \( y_1 \), (c) \( z_2 \) vs \( z_1 \)