Vector Quantization by Minimizing Kullback-Leibler Divergence

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Abstract. This paper proposes a new method for vector quantization by minimizing the Kullback-Leibler Divergence between the class label distributions over the quantization inputs, which are original vectors, and the output, which is the quantization subsets of the vector set. In this way, the vector quantization output can keep as much information of the class label as possible. An objective function is constructed and we also developed an iterative algorithm to minimize it. The new method is evaluated on bag-of-features based image classification problem.

Keywords: Vector quantization, Kullback-Leibler Divergence, Bag-of-features, Image Classification

1 Introduction

Vector quantization is a problem to quantize the continuous signal vectors into a discrete dictionary. The vectors are usually quantized into several discrete quantization subsets, and then the index of the subset will be used as its new representation. It should compress the original vectors, and also be helpful for the effective representation of these vectors. An example is the bag-of-features based image representation. In this problem, the local image features are quantized to a visual dictionary. The quantization is usually conducted in an unsupervised way, assuming that the class labels are not available. But it’s better to learn it by combining the class labels in a supervised way. The ideal quantization we are seeking is the one that can contains all the information needed for the classification.

In this paper, we present a new quantization algorithm so that the class label can be used. The algorithm is developed by first modeling the distribution
of the class labels over the quantization input (the original vectors), and the distribution of the class labels over the quantization output (the quantization subsets), and then minimizing the Kullback-Leibler divergence between the two distributions \([33,12,29,10,4,31,19,11,11]\). We build an objective function to this end, which involves the quantization of the vectors. This idea is shown in Fig. 1.

Our method is motivated by Wang et al.’s work [39], which learned the codebook for vector quantization in an supervised way. However, the differences are of two-folds:

- Wang et al. [39] used the class label to define the margin and then maximize it, while we directly minimize the Kullback-Leibler divergence between the class label distributions over the quantization input and output.
- Wang et al. [39] implement the vector quantization by using a codebook, while we directly quantize the vectors into subsets without using a codebook.

Fig. 1. Minimizing the distribution of class labels over the original vectors and the quantization subsets.

The rest part of this paper is organized as follows: Section 2 introduce the proposed algorithm in details. Section 3 shows the evaluation of the proposed algorithm bag-of-features image classification. Finally, Section 6 concludes the paper.

2 Method

We first introduce the distribution of the class label over the vectors and the quantization subsets in Section 2.1 and then given the algorithm to minimize the Kullback-Leibler divergence between these distributions to quantize the vectors in Section 2.2.
2.1 Class label distributions

Assume we have a set of \( N \) labeled training samples, denoted as \( \{(x_i, y_i)\}_{i=1}^N \). \( x_i \in \mathbb{R}^d \) is the \( d \)-dimensional vector of the \( i \)-th sample, and \( y_i \in \mathcal{Y} \) is its corresponding class label. The classification problem is to predict the class label of a sample from its vector. To estimate the class label distribution over a sample vector, we define the \( p(y|x) \) as the conditional distribution of class label \( y \) over a sample vector \( x \), where \( y \in \mathcal{Y} \), and \( x \in \{x_i\}_{i=1}^N \).

**Class label distribution over input** Given an input sample \( x_i \) and a class label \( y \), to estimate \( p(y|x_i) \), we first find its nearest neighbors \( N_i[9,8,20,40,36] \)

\[
N_i = k \cdot \text{armin}_{x_j} ||x_i - x_j||^2
\]  

(1)

and then

\[
p(y|x_i) = \frac{1}{|N_i|} \sum_{x_j \in N_i} I(y_j = y)
\]

(2)

where

\[
I(y_j = y) = \begin{cases} 
1, & \text{if } y_i = y \\
0, & \text{else}
\end{cases}
\]

(3)

**Class label distribution over output** Then we quantize the vectors \( \{x_i\}_{i=1}^N \) into \( M \) non-overlapping quantization subsets, denoted as \( \{S_m\}_{m=1}^M \), as shown in Fig. 2. Given subset \( S_m \), and a class label \( y \in \mathcal{Y} \), the conditional distribution for \( y \) over \( S_m \) is defined as

\[
p(y|S_m) = \frac{1}{|S_m|} \sum_{x_i \in S_m} I(y_j = y)
\]

(4)

**Kullback—Leibler divergence** The Kullback—Leibler divergence between the two distributions \( p(y|S_m) \) and \( p(y|x_i) \) are defined as

\[
D(p(y|x) \parallel p(y|S)) = \sum_{y \in \mathcal{Y}} \left\{ \sum_{m=1}^M \left[ \sum_{x_i \in S_m} p(y|x_i) \log \left( \frac{p(y|x_i)}{p(y|S_m)} \right) \right] \right\}
\]

(5)

We try to have a quantization output \( \{S_m\}_{m=1}^M \) from \( \{x_i\}_{i=1}^N \) so that the the quantization can minimize the Kullback—Leibler divergence

\[
\min_{S_1, \ldots, S_M} \sum_{y \in \mathcal{Y}} \left\{ \sum_{m=1}^M \left[ \sum_{x_i \in S_m} p(y|x_i) \log \left( \frac{p(y|x_i)}{p(y|S_m)} \right) \right] \right\}
\]

(6)

To solve this problem, we adapted an iterative algorithm.
2.2 Interactive algorithm

In this algorithm, we repeat a two-step procedure for many times:

- In this first step, we fix the quantization subset as $S_{1}^{old}, \ldots, S_{M}^{old}$, and compute the class label distribution over the quantization output as:

$$p_{new}^{new}(y|S_{m}) = \frac{1}{|S_{m}^{old}|} \sum_{x_{j} \in S_{m}^{old}} I(y_{j} = y)$$

(7)

- In the second step, we fix the distribution and update quantization subset by (8).

$$S_{m}^{new} = \left\{ x_{i} \| m = \arg \min_{m' = 1, \ldots, M} \sum_{y \in Y} p(y|x_{i}) \log \left( \frac{p(y|x_{i})}{p_{new}(y|S_{m})} \right) \right\}$$

(8)
When a new sample $\mathbf{x}$ not in the training set is given, it is quantized to the quantization subset as

$$S_{m^*} \leftarrow \mathbf{x}$$

where

$$m^* = \arg\min_{m=1,\ldots,M} \sum_{y \in \mathcal{Y}} p(y|x) \log \left( \frac{p(y|x)}{p(y|S_m)} \right)$$

3 Experiment

Experimental evaluation is given in this section on a real data set with a bag-of-features based image classification problem. We use the Fifteen Natural Scene Categories database in this experiment [13]. There are 15 classes and for each class, there are around 200 or 300 images. We use the proposed vector quantization algorithm to represent the image as a quantization histogram under the bag-of-features framework.

Fig. 4 shows the results for the Fifteen Natural Scene Categories data set. The kmeans algorithm is used as a baseline quantization method [6,45,3,25,55,44]. As in Fig. 4, the proposed method outperforms the baseline method.

4 Conclusion

In this paper, the problem of vector quantization is investigated. The idea is motivated by the supervised quantization dictionary learning method proposed by Wang et al. [39]. We try to keep all information about the class label from the quantization input to the output. It is implemented by minimizing Kullback-Leibler Divergence between the class label distributions over quantization input and output. This method can also be applied to social media data analysis [21,27,24], user recognition of mobile [26], transportation prediction [23,22], web news extraction [13], malicious websites detection [4,19,39,52,46,58,53].

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Fig. 3. Images in the Fifteen Natural Scene Categories database.

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