A multiple criteria nominal classification method in a web-based platform: Demonstration in a case of recruitment for the Portuguese Army

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Abstract
Cat-SD (Categorization by Similarity-Dissimilarity) is a recently developed method for handling nominal classification problems in the context of Multiple Criteria Decision Aiding (MCDA). This paper describes the design and implementation of this method, as well as an application dealing with a recruitment process in the Special Forces of the Portuguese Army. In addition, it proposes interaction protocols to elicit the preference parameters of the method to facilitate the construction of a decision model when the analyst guides the decision maker. Cat-SD has been implemented in DECSpace, a user-friendly on-line platform for supporting decision aiding processes using one or more MCDA methods. The study related to the Portuguese Army Special Forces recruitment presents and demonstrates how these protocols and a tool like DECSpace can facilitate the process of applying the method in real-world scenarios.

Keywords: Decision Support Systems, Multiple Criteria Decision Aiding, Classification problems, Web-based platform, Special Forces.

1. Introduction

Multiple Criteria Decision Aiding (MCDA) is a sub-discipline of Operations Research focus on the development and implementation of methods, techniques and tools that explicitly consider multiple criteria to assist a Decision Maker (DM) for dealing with decision situations. Some MCDA methods can be quite complex and some decision situations might require the use of more than one method. Therefore, having a Decision Support System (DSS) computing these methods and make it possible to visualize the results can be of an extreme valuable help. In fact, due to the increasing interest in this area, there has been a growing number of methods and, consequently, the number of DSS tools supporting them has been steadily increasing too. For example, two reviews of that are \cite{Weistroffer2016} and \cite{Mustajoki2017}, which offers not only a comprehensive survey, but also provides insights for future developments of software for this purpose, and \cite{Mustajoki2017}, motivated by the support to environmental planning processes, provide not only an analysis of well-known MCDA software, but also identifies the relevant aspects for good practices and innovative features. In fact, these tools can be challenging for users who are not experts in MCDA. Most of the available software solutions have been developed to meet the requirements of the researchers within this area, usually
making available only one or a few methods, making it very hard for others to use that software. For that reason, requirements related to usability and user friendly issues need to be taken into account in the development of this kind of software solution. This is in line with the trend of filling the gap between MCDA researchers and practitioners (see, e.g., Thokala and Madhavan 2018; Voinov et al. 2016).

In addition, many MCDA methods also might require the support of an analyst specialized in that method (but not forcibly in the tools) to help the DM (who must be knowledgeable about the problem requiring a decision, but not forcibly about the MCDA method or the tool). Techniques and interaction protocols between the analyst and the DM need further research to properly support the application of the MCDA methods. This kind of protocols are useful to guide the dialogue between the analyst and the DM when gathering the information related to the opinion, judgments and preferences of the DM. Such information is then used to assign properly values to the parameters needed for building a preference decision model.

In this paper, we focus on Cat-SD (Categorization by Similarity-Dissimilarity) (Costa et al., 2018), a recently proposed MCDA nominal classification method. The main objective of this paper is to provide tools for supporting the application of this method in real-world cases. With such a purpose in mind, we introduce several protocols to be adopted by the analyst and the DM for eliciting the preference parameters in a co-constructive way (i.e., through a collaborative dialogue between the analyst and the DM), and we present the implementation of the Cat-SD method in a novel web-based platform – DecSpace (Decision Space), which makes use of forefront technology, usability and user interaction techniques (Amador et al., 2018; Barbosa, 2017). While designing this platform, the main requirements were that it should be user-friendly and accessible, since it not only intends to make it possible to efficiently use various MCDA methods, but also to facilitate the understanding of the methods by non-expert users. The application of Cat-SD using DecSpace may help a DM to better understand the decision problem at hand.

The Cat-SD method has been developed to model the preferences of a DM and obtain assignment results when facing with multiple criteria nominal classification problem, while focusing on similarity and dissimilarity concepts. In the presence of a problem of this kind, a set of actions (or alternatives) has to be assigned to a set of categories previously defined in a nominal way (no preference order exists among them), considering a set of criteria and according to the preferences of the DM. For modeling purposes, the method needs data and preference parameters to be used as input for the computations, in order to obtain the categorization of the actions. A per-criterion similarity-dissimilarity function is used to model such concepts considering the differences of performance of two actions on a given criterion. Other preference parameters have to be defined, per category, namely reference actions, weights, possibly criteria interaction coefficients, and likeness thresholds. For eliciting such preference parameters, techniques are required, as proposed in Costa et al. (2018). In this paper, we propose interaction protocols to be used in a co-constructive way between the analyst and the DM to support the elicitation of such parameters.

DecSpace is a web-based platform that aims to provide an easy-to-use and intuitive approach to apply any kind of MCDA method, independently of its complexity. It is inspired in diviz (Bigaret and Meyer, 2010; Meyer and Bigaret, 2012), in the sense that it permits to construct workflows that may use multiple MCDA methods and data, so the outputs of a certain method can be used as inputs to another one, allowing to design complete decision aiding processes. DecSpace is suitable to explore solutions for simple or complex decision situations and aims to be user-friendly to either non-expert users or MCDA expert users, with researching or teaching purposes, or even for consulting purposes.

This paper is organized as follows. Section 2 contains a brief overview of the Cat-SD method.
Section 3 introduces a case study. Section 4 proposes generic interaction protocols for eliciting the preference parameters used in this method. Section 5 describes the interaction between the analyst and the DM, and the preference parameters defined to build the decision model. Section 6 is devoted to the design and application of Cat-SD in the DecSpace platform, and to demonstrate the use of the method by presenting the data and the model built with the intervention of the DM. Section 7 presents the main lessons learned from the application of the proposed protocols and the use of DecSpace while conducting the case study. Section 8 provides some concluding remarks, and proposes lines for future research and developments.

2. Overview of the Cat-SD method

This section is devoted to briefly introduce the Cat-SD method, designed to handle MCDA nominal classification problems. We present the main notation and the steps of the method. Related technical aspects, namely computations and the assignment procedure, are provided in Appendix. Details about the method can be found in Costa et al. (2018).

2.1. Basic notation

Beforehand, several data sets and preference parameters have to be built in order to apply the method. The basic data of a multiple criteria nominal classification problem is presented in the following two sets: $A = \{a_1, \ldots, a_i, \ldots\}$, which is the set of actions (not necessarily completely known a priori); and $G = \{g_1, \ldots, g_j, \ldots, g_n\}$, which is the a coherent family of criteria as defined according to Roy and Bouysson (1993). Let $g_j(a_i)$ denote the performance of action $a_i$ on criterion $g_j$. A performance table can be built. In addition, we consider a set of non-ordered categories, $C = \{C_1, \ldots, C_h, \ldots, C_q, C_{q+1}\}$, also called nominal categories ($C_{q+1}$ is a dummy category that receives actions which are not assigned to the other categories).

The application of the method needs the construction of several preference parameters. For characterizing the categories, we consider $B = \{B_1, \ldots, B_h, \ldots, B_{q+1}\}$ as the set of all reference actions (where $B_{q+1} = \emptyset$), and $B_h = \{b_{h1}, \ldots, b_{h\ell}, \ldots, b_{h|B_h|}\}$ is the set of reference actions considered to define category $C_h$, for $h = 1, \ldots, q$.

In addition, we must consider the following preference parameters.

- $k^h_j$ is the weight of criterion $g_j$ (its relative importance) with respect to category $C_h$, for $j = 1, \ldots, n$ and $h = 1, \ldots, q$;

- $k^h_{je}$ is the mutual-strengthening (or mutual-weakening) coefficient of the criteria pair $\{g_j, g_\ell\}$, with $k^h_{je} > 0$ (or $k^h_{je} < 0$), for $h = 1, \ldots, q$;

- $k^h_{jp}$ is the antagonistic coefficient for the ordered criteria pair $(g_j, g_p)$, with $k^h_{jp} < 0$, for $h = 1, \ldots, q$;

- $\lambda^h$ is the likeness threshold defined for category $C_h$, for $h = 1, \ldots, q$.

2.2. The flowchart of the method

The following flowchart contains three main blocks: one devoted to the input, other to the calculations, and the last is related to the output provided by the method (i.e., the assignment results). They can be briefly described as follows.
1. **Input.** There are basically two types of input:

(a) **Data.** The basic data is composed of the set of criteria $G$, the set of actions $A$, the performance table, and the set of categories $C$;

(b) **Preference parameters.** These are in the second box of the input block, in the flowchart, and they are the set of reference actions $B$, the weights of criteria and the interaction coefficients per category. Per-criterion similarity-dissimilarity functions (SD functions) have to be constructed. In addition, a likeness threshold per category should also be defined. The notation for these preference parameters is provided in the previous subsection;

2. **Calculations.** The calculation phase is composed of a sequence of steps as presented in Appendix, Steps 1-5;

3. **Output.** As output, the method provides the assignment of the actions to the non-ordered categories. An action can be assigned to one or several categories, or not assigned at all (i.e., it is assigned to the dummy category).

![Figure 1: A flowchart of the Cat-SD method](image)

The next subsection presents a case study that help us to show how to proceed to transform the preference information into parameters as proposed in Section 4.

3. **Case study**

This section introduces a case study, comprising its context and the required data to construct a Cat-SD decision model.

The Portuguese Armed Forces are a national institution in charge of guarantee the national independence and the territorial integrity of Portugal. The Portuguese Army, one of three branches (the largest branch), contributes to this mission as landing forces, in cooperation with the Portuguese
Navy and Air Force. The Portuguese Army is also an institution as old as the national foundation, having been always present in great moments of the history of Portugal.

Currently, the Portuguese Army essentially generates forces and capabilities, for projection and employment inside and outside the country’s borders, to support people and their institutions. In order to achieve its mission it needs adequate resources, being the human factor, around twelve thousand military and civilians, its greatest valuable resource and the primary cause of its efficacy.

The soldier is the principal value of the military forces, and, for this reason, the military recruitment should be an essential and permanent concern. Today, the Portuguese military have been operated in demanding theater of operations, namely in Iraq, Afghanistan, and Central African Republic, in order to defend the State interests under the scope of cooperative security. In these operational environments, complex and presenting high risk, it is used Special Forces with advance training and highly qualified. The Special Forces that integrate the Portuguese Army are mainly divided in Commandos, Paratroopers and Special Operations. Included in the Special Operations are the snipers, being elite forces recognized and praised by their high performance.

Accordingly, the process of recruitment and selection of military candidates for Special Forces requires particular attention. Upon completion of the initial military training, a military intending to join the Portuguese Army Special Forces has to fulfill some prerequisites. Depending on the type of Special Forces, different profiles are required, based on a set of features. Those features are evaluated through ability tests related to physical fitness, psychological aspects, and health and medical issues. Interviews are also applied in order to assess personality traits.

In the presented context, we have conducted a study, in collaboration with the Portuguese Army through the Centro de Psicologia Aplicada do Exército (CPAE), which is the military center evolved in the process of military recruitment with the mission of studying, applying and supervising the activities of the Army in the area of psychology. The DM is an expert in the assessment and selection of the candidates to the Army.

The case study aims at assigning military candidates into a small number of categories, which represent distinct types of Portuguese Army Special Forces. For modelling purposes, the Cat-SD method was applied. For that, with intervention of the DM, in a co-constructive way, a decision model has been built, including a set of criteria and a set of preference parameters reflecting the judgments of the DM. For confidentially reasons, some data used in this study are not real.

In general, a decision aiding process is a long process requiring several interaction sessions between analysts and DMs, to end-up with a decision classification model. However, in this case, conducting this study was immediately well-accepted by the DM, since it is in line with the current Portuguese Army recruitment process. Once the model is built, it can be used in the future and the DM is more open to have long interaction sessions when needed.

In the first meeting, the analyst and the DM formulated the classification problem, representing the decision situation at hand. The DM refereed that the profile of a sniper is very specific and, for that reason, a category should be defined for the sniper profile. Thus, in this study, four (nominal) categories have been considered (five categories when the dummy category, $C_5$, is counted), as described below.

- **Commandos ($C_1$)**: light unshielded combat forces intended for conventional operations of eminently offensive nature, taking advantage of the surprise, speed, violence and precision of the attack. They have great technical and tactical abilities, and high state of readiness;

- **Paratroopers ($C_2$)**: military parachutists (light infantry force) who have the capability of insertion in the theater of operations through parachute jumping, usually in surprise attacks. They
are characterized by a capability of a great speed in action and flexibility, presenting a high state of readiness;

- Special Operations ($C_3$): military that uses unconventional methods and resources to operate in diverse environments. They are generally adaptable and self-reliant;

- Snipers ($C_4$): military who shoot targets from long distances, using high-precision rifles and high-magnification optics, while maintaining effective visual contact with enemy targets from concealed positions. They typically present character traits of patience, attention to detail, perseverance, and physical endurance, and an attitude of determination;

- Unsuitable candidates ($C_5$): military that are not suitable to any previous category according to the constructed decision model.

It should be recalled that the application of Cat-SD provides at least a category for which a candidate should be assigned, meaning that a candidate may be suitable for more than one special force category according to the model. Based on the assignment results, a final decision is taken by the DM, and then each candidate assigned to a given category will be subject to a similar training course.

The following are the criteria considered relevant by the DM for the assignment of the military candidates into the Special Forces. Let us remark that all the criteria are to be maximized.

- Physical fitness ($g_1$) - PF: It consists of the current level of fitness assessed by the performance of the candidate on some exercises, such as wall transposition, swimming, pushing-up, curling-up, ditch jumping and running. The result of each test is converted to a corresponding value within the range $[0, 20.00]$, with exception of the result of the decision tests (e.g., wall transposition), which can be yes or no. For a candidate, the average of her/his numerical results of the tests is calculated, within the range $[0, 20.00]$, while the result in the decision tests is yes. Only candidates with an average score greater or equal to 10.00 are considered. Thus, the performances of this criterion are assessed in a quantitative scale within the range $[10.00, 20.00]$;

- Intelligence ($g_2$) - Intel: It includes intellectual aptitudes of the candidate, while evaluating the competency in logical reasoning, the capacity to understand, plan and solve problems. The performances of this criterion are assessed in a qualitative scale with five levels: insufficient (1), weak (2), satisfactory (3), good (4), excellent (5);

- Numerical reasoning ($g_3$) - NR: It consists of the candidate’s ability to interpret, analyze and draw logical conclusions from numerical data and make reasoned decisions when solving problems. The performances of this criterion are assessed in a quantitative scale, corresponding to percentile scores, within the range $[30, 99]$;

- Spatial ability ($g_4$) - SA: It consists of the candidate’s capacity of the spatial perception, including the ability to understand the spatial relations among objects and to mentally visualize and manipulate them. The performances of this criterion are assessed in a quantitative scale, through the application of a perception-cognitive test, resulting in a percentile score, within the range $[30, 99]$;

- Mechanical reasoning ($g_5$) - MechR: It consists of the practical knowledge of mechanics and physics, that is, the candidate’s capability to understand and apply the concepts and principles
of mechanics within a variety of situations. The performances of this criterion are assessed in a quantitative scale, through the application of a perception-cognitive test, resulting in a percentile score, within the range [30, 99];

- \textit{Velocity perception} \((g_6) - \text{VP}\): It is related to the processing speed of the candidate, a cognitive ability to do a mental task, i.e., the time to capture and react to the information received (the stimulus can be whether visual, auditory or movement). The performances of this criterion are assessed in a quantitative scale, through the application of a perception-cognitive test, resulting in a percentile score, within the range [30, 99];

- \textit{Psychomotor ability} \((g_7) - \text{PmA}\): It consists of the physical movement related to conscious cognitive processing, associated to the motor coordination and manual dexterity. The performances of this criterion are assessed in a qualitative scale with five levels: very low (1), low (2), medium (3), high (4), very high (5);

- \textit{Personality} \((g_8) - \text{Pers}\): It includes some candidate’s personality traits and adaptation abilities in the military environment, such as emotional stability, maturity, adaptability, resilience, teamwork skills and motivation. The performances of this criterion are assessed in a qualitative scale with five levels: very weak (1), weak (2), medium (3), good (4), very good (5);

- \textit{Physical condition (medical)} \((g_9) - \text{Med}\): It consists of the assessment of medical aspects, such as the candidate’s physical constitution, audition and vision. The performances of this criterion are assessed in a qualitative scale with five levels: clearly below average (1), below average (2), average (3), above average (4), clearly above average (5).

A set of candidates to become Special Forces soldiers have been assessed according to the nine predefined criteria. Table 1 displays the performances on all criteria of twenty candidates (the data are not real).

4. Assessing the DM preferences and the modeling of the parameters

As stated in Costa et al. (2018), different approaches can be adopted to assess the preference parameters of our model, starting from a pure learning from data based approach with “training” examples provided by the DM to a full co-constructive approach. In this work, we focus on the latter, in which the analyst and the DM interact and cooperate to assess the preferences and model them in view to construct a preference model. This section is devoted to present interaction protocols, in order to assign appropriate values to the parameters according to the preferences of the DM. This kind of protocols are based on a structured dialogue between the analyst (the questioner) and the DM (the questionee) involving introductory and easy questions (Roy, 1996, Chapter 11).

4.1. The reference actions

Each category is defined by a non-empty set of reference actions. A reference action is a representative action of a given category, i.e., a prototype or a typical element. In order to build the set of reference actions, the analyst can start by asking the DM to choose a category. If the DM is able to identify at least a representative action (e.g., from past decisions, guidelines, holistic judgments, etc.) of such a category, those actions can be used to characterize the considered category. If this is not possible, at least a dummy action must be built. This can be done by considering some adequate performances
levels in the criteria scales. The analyst and the DM should proceed in this way while considering individually each one of the predefined categories. In the end, each category is defined by at least one reference action (with exception of the dummy category).

4.2. The per-criterion functions

A per-criterion similarity-dissimilarity function, $f_j(\Delta_j(a,b))$, is used for each criterion, $g_j$, to model similarity-dissimilarity judgments in the comparison of each ordered pair $(a,b)$ of actions, where $\Delta_j(a,b)$ is the performance difference between $g_j(a)$ and $g_j(b)$. (for more details, see Costa et al., 2018). A general definition of this function is also presented in 8. The per-criterion similarity-dissimilarity functions make use of similarity-dissimilarity thresholds are defined such that $v(g_j(b)) \geq u_j(g_j(b)) \geq t_j(g_j(b)) \geq 0$ and $v_j'(g_j(b)) \geq u_j'(g_j(b)) \geq t_j'(g_j(b)) \geq 0$. If the difference of performances, $\text{diff}\{g_j(a), g_j(b)\}$, is within the range $[−t_j'(g_j(b)), t_j(g_j(b))]$, then there is a positive contribution to the similarity on the criterion $g_j$. If the difference of performances, $\text{diff}\{g_j(a), g_j(b)\}$, is within the ranges $[−\text{diff}\{g_j^{\max}, g_j^{\min}\}, −u_j'(g_j(b))]$ and $[u_j(g_j(b)), \text{diff}\{g_j^{\max}, g_j^{\min}\}]$, then there is a negative contribution to the similarity: a negative dissimilarity when $a$ is strictly less than $b$, and a positive dissimilarity when $a$ is strictly greater than $b$.

The thresholds can be constant thresholds, i.e., when they are invariable along the scale of the criterion, which means that the same value is used to compare two actions and does not depend on $g_j(b)$, while variable thresholds vary along the range of the criterion scale.

In the next subsection we provide a more formal definition of all the types of thresholds mentioned above.

4.2.1. Definition of the similarity and dissimilarity thresholds

The thresholds can be defined as in Definition 1 below.

Table 1: Performance of the candidates to Special Forces on each criterion

| Candidate | PF | Intel | NR | SA | MechR | VP | PmA | Pers | Med |
|-----------|----|-------|----|----|-------|----|-----|------|-----|
| $a_1$     | 17.25 | 4    | 65 | 75 | 70    | 75 | 4   | 4    | 4   |
| $a_2$     | 16.05 | 4    | 85 | 85 | 90    | 80 | 4   | 5    | 5   |
| $a_3$     | 14.91 | 4    | 80 | 75 | 85    | 55 | 4   | 5    | 4   |
| $a_4$     | 15.00 | 3    | 65 | 85 | 80    | 65 | 4   | 4    | 5   |
| $a_5$     | 13.73 | 4    | 75 | 96 | 75    | 70 | 4   | 4    | 3   |
| $a_6$     | 18.28 | 3    | 70 | 75 | 60    | 75 | 4   | 5    | 4   |
| $a_7$     | 12.83 | 5    | 80 | 60 | 75    | 85 | 4   | 4    | 3   |
| $a_8$     | 14.50 | 4    | 75 | 80 | 96    | 85 | 4   | 5    | 5   |
| $a_9$     | 15.75 | 4    | 55 | 65 | 75    | 97 | 5   | 5    | 5   |
| $a_{10}$  | 15.86 | 4    | 90 | 80 | 75    | 80 | 2   | 5    | 4   |
| $a_{11}$  | 19.12 | 3    | 50 | 75 | 65    | 75 | 4   | 4    | 5   |
| $a_{12}$  | 14.35 | 2    | 80 | 85 | 85    | 70 | 4   | 3    | 4   |
| $a_{13}$  | 11.65 | 4    | 75 | 85 | 96    | 65 | 4   | 4    | 4   |
| $a_{14}$  | 16.00 | 5    | 80 | 55 | 65    | 75 | 3   | 4    | 4   |
| $a_{15}$  | 18.00 | 3    | 75 | 70 | 50    | 75 | 4   | 4    | 5   |
| $a_{16}$  | 17.22 | 4    | 60 | 70 | 75    | 85 | 3   | 4    | 5   |
| $a_{17}$  | 13.85 | 4    | 90 | 85 | 80    | 90 | 5   | 5    | 4   |
| $a_{18}$  | 15.10 | 3    | 70 | 90 | 95    | 60 | 5   | 4    | 4   |
| $a_{19}$  | 12.45 | 5    | 80 | 65 | 70    | 70 | 4   | 5    | 4   |
| $a_{20}$  | 14.32 | 4    | 85 | 80 | 85    | 75 | 5   | 5    | 5   |
Definition 1 (Similarity and dissimilarity thresholds). Three different kind of thresholds are defined as follows, for two actions $a$ and $b$, where $a$ is the action to be assessed and $b$ is the reference action.

i) The similarity thresholds, $t_j(g_j(b))$ and $t'_j(g_j(b))$, can be defined as follows:
- Consider $g_j(a) \geq g_j(b)$. The threshold $t_j(g_j(b))$ is the largest performance difference that allows to consider that action $a$ is similar to action $b$ according to criterion $g_j$;
- Consider $g_j(a) \leq g_j(b)$. The threshold $t'_j(g_j(b))$ is the largest performance difference that allows to consider that action $a$ is similar to action $b$ according to criterion $g_j$;

ii) The dissimilarity thresholds, $u_j(g_j(b))$ and $u'_j(g_j(b))$, can be defined as follows:
- Consider $g_j(a) \geq g_j(b)$. The threshold $u_j(g_j(b))$ is the smallest performance difference that allows to consider that action $a$ is dissimilar to action $b$ according to criterion $g_j$;
- Consider $g_j(a) \leq g_j(b)$. The threshold $u'_j(g_j(b))$ is the smallest performance difference that allows to consider that action $a$ is dissimilar to action $b$ according to criterion $g_j$;

iii) The total dissimilarity thresholds, $v_j(g_j(b))$ and $v'_j(g_j(b))$, can be defined as follows:
- Consider $g_j(a) \geq g_j(b)$. The threshold $v_j(g_j(b))$ is the smallest performance difference that allows to consider that action $a$ is totally dissimilar to action $b$ according to criterion $g_j$;
- Consider $g_j(a) \leq g_j(b)$. The threshold $v'_j(g_j(b))$ is the smallest performance difference that allows to consider that action $a$ is totally dissimilar to action $b$ according to criterion $g_j$.

4.2.2. Assessing similarity and dissimilarity thresholds
The assignment of values to these thresholds can be carried out in a constructive way adopting a similar protocol usually used in the elicitation of veto thresholds in ELECTRE methods [Roy et al., 2014]. The analyst can start by using some levels from the criterion scale as reference values to help DM in the assessment of the thresholds values. According to Roy et al. (2014), for discrete scales, if checking all levels of the scale the conclusion is that the value of the threshold is the same, then we are in presence of a constant threshold. Otherwise, the threshold has to be defined for each level of the criterion scale. They should proceed in a analogous way with all thresholds.

Gathering the preference information. For continuous scales, through a co-construction interactive process between the analyst and the DM a way of determining the thresholds can be as follows (see also Figure 2):

![Figure 2](image-url)
1. Consider a criterion, \( g_j \), to be maximized. Choose a reference scale level in the lower part of the criterion scale. Let \( g_j^l (b) \) denote such a level. In general, a “good” choice is such that the chosen level is in between 1/4 to 1/3 of the scale.

2. Consider the two performance scale levels, \( g_j^l (b) \) and \( g_j (a) \). The latter should be closed enough to the reference level. In general, with \( g_j (a) > g_j^l (b) \), but we can start with the equality. The analyst starts by asking the DM whether, \( g_j^l (b) \) and \( g_j (a) \), can be considered similar from her/his point of view:

   (a) If the DM answers negatively, the analyst incrementally decreases \( g_j (a) \), and asks the same question until obtain a positive answer (if any). This may occur only when \( g_j (b) \) and \( g_j (a) \) become equal. Then, \( g_j (a) \) is fixed. Let \( g_j^l (a) \) denote such a performance level.

   (b) If the DM answers positively, the analyst incrementally increases \( g_j (a) \), and asks the same question until she/he gets a negative answer. Then, \( g_j (a) \) is fixed as \( g_j^l (a) \).

   The objective of this step is thus to identify, in a co-constructive way, i.e., through a dialogue between the analyst and the DM, how many (if any) performance levels of action \( a \) must be removed from the performance, \( g_j^l (b) \), to consider that both actions, \( a \) and \( b \), are similar, according to criterion \( g_j \). The performance level \( g_j^l (a) \) is the lowest constructed performance level allowing to conclude about the similarity with respect to \( g_j^l (b) \). Any other performance level lower than \( g_j^l (a) \) leads to a non-similarity situation.

3. An identical procedure to the one presented in the previous step can lead to obtain \( g_j^{l''} (a) \). In this case, consider two performance levels, \( g_j^l (b) \) and \( g_j (a) \), with \( g_j (a) > g_j^l (b) \). Then, in 2(a), the analyst must incrementally increase \( g_j (a) \), and in 2(b), the analyst must incrementally decrease \( g_j (a) \). Figure 3 shows the two co-constructed performance levels, \( g_j^l (a) \) and \( g_j^{l''} (a) \).

   ![Figure 3: Co-constructed performance levels \( g_j^l (a) \) and \( g_j^{l''} (a) \).](image)

4. Consider a performance level in the upper part of the criterion scale, say \( g_j^{u''} (b) \). A procedure as described in Steps 2 and 3 can be applied. This performance level, \( g_j^{u''} (b) \), must be significantly different from \( g_j^l (b) \). In general, a “good” choice for such a level is in between 2/3 to 3/4 of the criterion scale. Figure 4 shows the fixed levels until this step.

   ![Figure 4: Co-constructed performance levels](image)

5. Analogously, an identical procedure as previously described in Steps 1-4 can be applied with the aim of determining how many (if any) levels of action \( a \) must be removed from (and added to) the reference scale level to consider that both actions, \( a \) and \( b \), are dissimilar according to criterion \( g_j \). The performance level \( g_j^{u''} (a) \) is the lowest constructed performance level allowing to conclude about the dissimilarity with respect to \( g_j^{u''} (b) \). Any other performance level lower than \( g_j^{u''} (a) \) leads to a dissimilarity situation.
Finally, a procedure as in Step 5 must be followed to determine the performance levels of action $a$, for which actions $a$ and $b$ are considered totally dissimilar, according to criterion $g_j$.

**Computational aspects.** The analyst must wonder whether the value of each threshold depends on the chosen reference scale level. For that, after application of the proposed interaction protocol, the difference between each fixed performance level and the respective reference level is determined. For instance, when considering two performances, $g^l_j(a)$ and $g^l_j(b)$, we can define the performance difference, $\Delta^l_j(a, b)$, where $\Delta^l_j(a, b) = g^l_j(a) - g^l_j(b)$ for cardinal levels, and $\Delta^u_j(a, b)$ is equal to the number of levels in between the two performance levels, $g^l_j(a)$ and $g^l_j(b)$, for ordinal or discrete levels. When we obtain the same value for the difference of performances whether considering a reference level in the lower or upper part of the scale (e.g., $\Delta^l_j(a, b) = \Delta^u_j(a, b)$), we can conclude that the respective threshold is constant along the scale. For example, in the case of the (non-negative) similarity threshold $t(g_j(b))$, such a value can be determined as follows: $t(g_j(b)) = |\Delta^l_j(a, b)| = |\Delta^u_j(a, b)|$. In an identical way, the values of the other thresholds can be determined. Otherwise, an affine function, $g_j(b)\alpha_j + \beta_j$, can be used to represent the variation of the threshold along the scale. The coefficients of the affine function can be computed from the following system:

\[
\begin{align*}
|\Delta^l_j(a, b)| &= g^l_j(b)\alpha_j + \beta_j \\
|\Delta^u_j(a, b)| &= g^u_j(b)\alpha_j + \beta_j
\end{align*}
\]

This is also valid for the other thresholds.

**Illustrative example.** Let us analyze the following simple numerical example:

1. Consider a criterion, $g_j$, to be maximized and suppose that its scale, $E_j$, is quantitative and continuous, with $E_j = [0, 200]$. Let $b$ represent a reference action, and $g_j(b) = 70$ and $g_j(b) = 140$ be the performances chosen by the as the two representative levels on criterion $g_j$. Analyst asks DM: how much levels can action $a$ be less (and greater) than action $b$ to consider any similarity between these two actions on criterion $g_j$? Suppose that the following information is given by the DM:

- For $g_j(b) = 70$, the maximal “negative difference” and the maximal “positive difference” compatible with similarity between two actions is 10 for both.
- For $g_j(b) = 135$, the maximal “negative difference” and the maximal “positive difference” compatible with similarity between two actions are 20 and 25, respectively.

2. In order to determine the form of the two similarity thresholds, $t_j(g_j(b))$ and $t'_j(g_j(b))$, and taking into consideration the information above, the following two systems can be obtained:

\[
\begin{align*}
10 &= 70\alpha' + \beta' \\
20 &= 135\alpha' + \beta' \\
10 &= 70\alpha + \beta \\
25 &= 135\alpha + \beta
\end{align*}
\]

3. The solutions of Systems (1) and (2) are $\alpha' = \frac{2}{13}$ and $\beta' = -\frac{10}{13}$, and $\alpha = \frac{3}{13}$ and $\beta = -\frac{80}{13}$, respectively. Therefore, we obtain the following variable similarity thresholds:
\[
- t_j'(g_j(b)) = \frac{2}{13} g_j(b) - \frac{10}{13}, \\
- t_j(g_j(b)) = \frac{3}{13} g_j(b) - \frac{80}{13}.
\]

In the same way, the two dissimilarity thresholds, \(u_j(g_j(b))\) and \(u'_j(g_j(b))\), and the two total dissimilarity thresholds, \(v_j(g_j(b))\) and \(v'_j(g_j(b))\), can be determined. Let us only consider the case of dissimilarity thresholds.

1. Analyst ask DM for determining: how much levels can action \(a\) be less (and greater) than action \(b\) to consider any dissimilarity between these two actions on criterion \(g_j\)? Suppose that the following information is provided:

- For \(g_j(b) = 70\), the minimal “negative difference” and the minimal “positive difference” compatible with dissimilarity between two actions is 20 for both.
- For \(g_j(b) = 135\), the minimal “negative difference” and the minimal “positive difference” compatible with dissimilarity between two actions is 40 for both.

2. Therefore, we obtain the following two systems:

\[
\begin{align*}
20 &= 70 \alpha_u' + \beta_u' \\
40 &= 135 \alpha_u' + \beta_u' \\
20 &= 70 \alpha_u + \beta_u \\
40 &= 135 \alpha_u + \beta_u
\end{align*}
\]

3. The solutions of Systems 3 and 4 are obviously the same: \(\alpha_u' = \alpha_u = \frac{4}{13}\) and \(\beta_u' = \beta_u = -\frac{20}{13}\).

Therefore, we obtain the following variable dissimilarity thresholds:

\[
u'_j(g_j(b)) = u_j(g_j(b)) = \frac{4}{13} g_j(b) - \frac{20}{13}.
\]

This means that we have the same dissimilarity thresholds whenever the performance of action \(a\) is greater or less than the performance of action \(b\). The same procedure can be applied to determining the variable total dissimilarity thresholds.

4.2.3. Modeling and determining the strength of similarity-dissimilarity

In this elicitation part, we focus on determining the intensities of the similarity and dissimilarity between two actions, according to a given criterion. The preference information leading to compute such intensities is be modeled through a procedure based on a deck of cards technique as the one in Figueira and Roy (2002). Indeed, we aim at determining the form of each one of the four components of the SD function on criterion \(g_j\), i.e., \(f^1_j\), \(f^2_j\), \(f^3_j\), and \(f^4_j\), which are defined in the following intervals, respectively:

- \(f^1_j: ]-v'_j(g_j(b)), -u'_j(g_j(b)]\); \\
- \(f^2_j: ]-t'_j(g_j(b)), 0[\); \\
- \(f^3_j: [0, t_j(g_j(b)]\); \\
- \(f^4_j: ]u_j(g_j(b)), v_j(g_j(b)]\).
Gathering the preference information. For determining the strength of similarity and dissimilarity judgments on a given criterion, we propose that the analyst applies a simple interaction protocol with the DM. Let us consider a criterion \( g_j \) to be maximized, expressed on a discrete or continuous scale. A reference scale level in the central part of the criterion scale, \( g^c_j(b) \), must be chosen and, taking into account the thresholds previously defined, the four intervals are determined. For each one of these intervals, let \( \Delta^1_j \) denote the value of the lower bound of the interval and \( \Delta^p_j \) the value of its upper bound. For instance, for \( f^3_j \), \( \Delta^1_j = 0 \) and \( \Delta^p_j = t_j \). Now, we must consider some values in between these two (i.e., some performance differences of scale levels and \( g^c_j(b) \)), allowing thus to form a sequence of ordered values representing the differences of two performance levels, \( \Delta^1_j, \ldots, \Delta^k_j, \ldots, \Delta^p_j \).

The number \( p \) to be considered depends on the scale. The deck of cards technique works as follows, through interaction between the analyst and the DM:

1. A set of \( p \) cards, with values \( \Delta^1_j, \ldots, \Delta^k_j, \ldots, \Delta^p_j \), is provided to the DM. If the she/he considers that some (consecutive) values of the performance difference are equally similar or dissimilar, then the she/he must place the corresponding cards in the same position in the ranking, building a subset of cards. At this point, the DM has a total order of subsets, say \( S^1, \ldots, S^k, \ldots, S^r \).

2. A large enough set of blank cards is provided to the DM. The similarity-dissimilarity of two successive positions of the cards (or two subsets of cards) in the ranking can be more or less close. These blank cards are used to model the more or less “closeness” of the positions. Indeed, the DM is asked to introduce the blank cards in such a way that the greater the difference between two consecutive positions, the greater the number of blank cards: no blank card means that the difference would be minimal; one blank card means twice the minimal difference; two blank cards means three times the minimal difference, and so on.

The proposed technique must be applied to each one of the four considered intervals. Thus, we obtain the necessary preference information to compute the values of the similarity and dissimilarity intensities. Then, we can determine the form of each one of the four components.

Computational aspects. The similarity and dissimilarity intensities take values between \(-1\) and \(1\). While for components \( f^2_j \) and \( f^3_j \), similarity intensity values are assigned to the levels, within the range \([0, 1]\), for \( f^1_j \) and \( f^4_j \), dissimilarity intensity values are assigned, within the range \([-1, 0]\). In the case of a discrete scale, when we compute the intensities, a discrete SD function is then obtained. In the case of a continuous scale, even though various forms can be assumed by these functions, in what follows, we will only consider the linear case. For assigning the intensities using the judgments of the DM obtained through the application of the proposed deck of cards technique, we introduce a procedure based on the one proposed in Bottero et al. (2018) for building interval scales. We present a step-by-step procedure to compute the intensities as follows.

1. Consider the two subsets \( S^1 \) and \( S^r \), and assign intensity values to them:

   (a) Similarity cases \((s_j(a,b))\):
   
   i. \( f^2_j : f_j(S^1) = 0 \) and \( f_j(S^r) = 1 \);
   
   ii. \( f^3_j : f_j(S^1) = 1 \) and \( f_j(S^r) = 0 \);

   (b) Dissimilarity cases \((d_j(a,b))\):

   i. \( f^1_j : f_j(S^1) = -1 \) and \( f_j(S^r) = 0 \);
ii. \( f_j^4 : f_j(S^1) = 0 \) and \( f_j(S^r) = -1 \).

2. Let \( e^k \) denote the number between two consecutive subsets, \( S^k \) and \( S^{k+1} \), for \( k = 1, \ldots, r - 1 \). Consider the following ranking:

\[
S^1 e^1 S^2 e^2 \ldots S^k e^k S^{k+1} \ldots S^{r-1} e^r S^r.
\]

3. Compute the unit, \( \alpha \), as follows:

\[
\alpha = \frac{1}{h},
\]

where

\[
h = \sum_{i=1}^{r-1} (e^i + 1).
\]

(the number of units between subsets \( S^1 \) and \( S^r \)).

4. Let \( \Delta_j^k \) denote a value belonging to the subset \( S^k \), and compute the intensity for each performance difference, \( f(\Delta_j^k) \), for \( k = 1, \ldots, r \), as follows:

(a) \( f_j^1 \) and \( f_j^2 \):

\[
f_j(\Delta_j^k) = f_j(S^1) + \alpha \left( \sum_{i=1}^{k-1} (e^i + 1) \right), \text{ for } k = 2, \ldots, r.
\]

(b) \( f_j^3 \) and \( f_j^4 \):

\[
f_j(\Delta_j^k) = f_j(S^1) - \alpha \left( \sum_{i=1}^{k-1} (e^i + 1) \right), \text{ for } k = 2, \ldots, r.
\]

All levels belonging to a given subset, \( S^k \), for \( k = 1, \ldots, r \), will have the same intensity value.

Finally, we assign the intensities values of the considered performance differences to the function of the respective differences between each level and \( g_j^c(b) \), i.e., \( f(\Delta_j^k) = f(g_j(a) - g_j^c(b)) \), for \( k = 1, \ldots, r \). For instance, for the extreme values of the interval in which component \( f_j^3 \) is defined, we have: \( f_j(0) = 1 \), and \( f_j \left( t_j(g_j^c(b)) \right) = 0 \).

Therefore, we assume that the function \( f_j \) has the same form of the obtained function when considering the particular reference level \( g_j^c(b) \). Then, we generalize that to the difference between the performance of two actions, \( g_j(a) \) and \( g_j(b) \), and obtain the SD function, \( f_j(\Delta_j(a,b)) = f_j(g_j(a) - g_j(b)) \).

In the case of continuous scales, the proposed procedure can also be applied. The value of the intensities on a given sub-interval of performance differences, between \( \Delta_j^a \) and \( \Delta_j^b \), with \( \Delta_j^a < \Delta_j^k < \Delta_j^b \), can be defined by linear interpolation as follows:

\[
f_j(\Delta_j^k) = f_j(\Delta_j^a) + \frac{\Delta_j^k - \Delta_j^a}{\Delta_j^b - \Delta_j^a} \left( f_j(\Delta_j^b) - f_j(\Delta_j^a) \right).
\]
Hence, a piecewise-linear interpolation function is obtained. It should be remarked that other type of interpolation can be used, when it is considered more adequate according to the preferences of the DM.

4.3. The weights of criteria

The relative importance of a given criterion is modeled by a criterion weight. These weights can be different for each category. They can be determined by using the revised Simos procedure proposed in Figueira and Roy (2002). It should be mentioned that, at this stage, the relative importance of a criterion must be analyzed ignoring any potential interaction between criteria that could exist. Such interaction effects will be considered later in Subsection 4.4.

There are two main steps for assigning the weights of criteria, presented in the next paragraphs.

**Gathering the preference information.** The preference information can be obtained through the following interaction protocol:

1. The analyst provides a set of cards with the identification of each criterion (e.g., name, code) on each card and some additional information when necessary (e.g., scale unit, short description of the criterion);

2. The analyst asks the DM to place the cards in a ranking, from the more important to the last important one (in the case of a tie, the DM has to place the cards in the same position of the ranking);

3. The analyst provides to the DM an enough set of blank cards and asks her/him to insert blank cards between successive positions in the ranking previously obtained. If she/he considers that a greater importance difference exists between two consecutive positions (the more the number of cards, the more the difference of importance between the subsets of criteria);

4. Finally, the analyst asks the DM to say how many times the most important subset of criteria are more important than the least important one. This is called ratio $z$.

The above protocol should be individually applied for each predefined category when the DM considers that the relative importance of criteria differs among categories. Then, the non-normalized weights are obtained according to the algorithm described in next paragraph.

**Computation aspects.** In order to determine the values of the criteria weights, compute as presented below:

1. Let $S^1, \ldots, S^k, \ldots, S^r$ represent the subsets of criteria cards in the same rank position in the ranking (a subset may contain only one card), where $S^1$ is the rank position with the least important criterion (or criteria). Let $k(S^1), \ldots, k(S^k), \ldots, k(S^r)$ represent the non-normalized weights of the respective subsets. Assume that $k(S^1) = 1$;

2. Let $e^k$ denote the number of blank cards placed between the rank position $S^k$ and $S^{k+1}$. Set $e$ as follows:

$$e = \sum_{i=1}^{r} (e^i + 1).$$
3. Compute each weight as follows:

\[ k(S^k) = 1 + u \sum_{i=1}^{k-1} e^i \quad \text{with} \quad u = \frac{z-1}{e}. \]

All criteria in the same rank position have the same weight. Then, \( k_j = k(S^k) \) for \( j = 1, \ldots, n \) and \( k = 1, \ldots, r \). These computations must be done for each category when distinct rankings are constructed by the DM. Thus, we obtain all values of \( k^h_j = k(S^k) \), for \( j = 1, \ldots, n \) and \( h = 1, \ldots, q \).

4.4. The interaction coefficients

Three types of interaction effect in pairs of criteria can be taken into account when applying the method (see Costa et al., 2018):

1. Mutual-strengthening effect modeled by a positive strengthening coefficient \( k^h_{j\ell} \), for \( h = 1, \ldots, q \) (with \( k^h_{j\ell} = k^h_{\ell j} \));
2. Mutual-weakening effect modeled by a negative weakening coefficient \( k^h_{j\ell} \), for \( h = 1, \ldots, q \) (with \( k^h_{j\ell} = k^h_{\ell j} \));
3. Antagonistic effect modeled by a negative coefficient \( k^h_{jp} \), for \( h = 1, \ldots, q \).

It should be remarked that it is assumed that an antagonistic effect in a given pair of criteria excludes the mutual interaction effects in such a criteria pair.

Gathering the preference information. Firstly, the non-normalized values of the criteria weights have to be previously determined. Secondly, the analyst has to make sure that the DM has a good understanding of the interaction effects between criteria. Finally, an interaction procedure should be followed in order to identify the possible interaction effects in some pairs of criteria and to assign values to the interaction coefficients associated with each criteria pair (see Figueira et al., 2009, for a procedure for ELECTRE methods, and see Bottero et al., 2015, for a practical application of such a procedure). In a collaborative way, the analyst and the DM may interact as follows to determine those interaction coefficients, while considering individually each category:

1. The analyst should start by questioning the DM about the potential interaction effects between two criteria that may be considered in the decision model. For checking possible interactions in all pairs of criteria, the following analysis may be done:
   (a) Consider criterion \( g_1 \);
   (b) Check whether an interaction effect between \( g_1 \) and another criterion, \( g_2, g_3, \ldots, g_n, \) should be taken into account, identifying the criteria pair, if any;
   (c) Identify the type of interaction (mutual-strengthening, mutual-weakening, or antagonistic effect) for the pairs of criteria identified in (b);

An identical procedure is then adopted, considering in (a) each of the remaining criteria, \( g_2, \ldots, g_{n-1} \). It is expected that the DM identifies a small number of pairs of criteria for which there is an interaction effect.
2. The analyst should explain to the DM how the values can be assigned to the interaction coefficients associated with the respective criteria pairs. With such a purpose, the analyst should remind the DM about the meaning of each interaction effect, for instance, by illustrating that with simple examples (see, for instance, real-world applications of Cat-SD in Costa et al. (2019) or examples for ELECTRE methods in Figueira et al. (2009)).

3. The analyst must check, for each category and each criterion, the non-negative condition (see Equation 6 in [8]). If it is not fulfilled, then the above steps must be revised with the DM, and the final values assigned to the interaction coefficients must verify the non-negative condition.

4.5. The likeness thresholds
An additional preference parameter is needed: a threshold must be defined for each category as the minimum likeness degree judged necessary to say that action \( a \) is alike the set of reference actions \( B_h \), for \( h = 1, \ldots, q \). \( \lambda^h \) denotes the likeness threshold of category \( C_h \), for \( h = 1, \ldots, q \). It takes a value within the range \([0.5, 1]\), and it can be viewed as majority measure.

5. Gathering the preference information: Application of the interaction protocols
In this section, we present the preference information obtained through the intervention of the analyst and the DM applying the protocols proposed in the previous section.

5.1. Reference profiles
In an interaction session, the analyst asked to the DM to define reference soldiers’ profiles to characterize the categories previously defined. On the basis of some related documentation, and according and the experience and knowledge of the DM regarding the performance of the Army Special Forces soldiers during the training period and while they perform their role, the DM empirically constructed a typical and good representative profile per category, as presented in Table 2.

| Category          | Reference soldier | PF  | Intel | NR  | SA  | MechR | VP  | PmA | Pers | Med |
|------------------|-------------------|-----|-------|-----|-----|-------|-----|-----|------|-----|
| Commandos        | \( b_{11} \)      | 17.00 | 3     | 65  | 70  | 70    | 80  | 4   | 5    | 5   |
| Paratroopers     | \( b_{12} \)      | 14.00 | 3     | 60  | 80  | 80    | 70  | 4   | 4    | 4   |
| Special Operations | \( b_{13} \)     | 16.00 | 4     | 70  | 70  | 70    | 75  | 4   | 4    | 4   |
| Snipers          | \( b_{14} \)      | 15.00 | 4     | 80  | 85  | 85    | 85  | 5   | 5    | 5   |

5.2. Per-criterion SD functions
With respect to the construction of the SD functions, we started with the first criterion expressed on a scale with the lowest number of possible performance levels, that is, intelligence \( (g_2) \), \( \text{Intel} \), assessed on a five-level qualitative scale. We had expected that this could be an easy way to the DM begins reflecting about similarities and dissimilarities in pairwise comparison of performance levels. We proceeded as follows:

1. The analyst started by placing a card with level 3 (satisfactory) in the table, and asking the DM if a level “good”, 4 (written in another card) has some similarity with respect to level 3; then, the analyst asked the same for level 5 (excellent), getting a positive answer for both levels;
2. The analyst also asked the same question for level 2 (weak), and level 1 (insufficient). While for level 2, the DM answered that some similarity exists, but being lower when comparing to 3 and 4, for level 1 there is no similarity (total dissimilarity);

3. To make sure and find out whether there was coherency in the preferences of the DM, an analogous procedure was applied with other performance levels pairs.

This dialogue allowed to understand that the DM favors more the similarity for levels above the reference level than for levels below the reference one in pairwise comparisons.

In a quite easy way, the DM assigned values as follows:

- 1 to the similarity when no difference between the two levels exists;
- 0.8 to the positive difference of one level (e.g., \(g_2(b) = 4\) and \(g_2(a) = 5\));
- 0.6 to the positive difference of two levels (e.g. \(g_2(b) = 3\) and \(g_2(a) = 5\));
- 0.4 to the negative difference of one level (e.g., \(g_2(b) = 3\) and \(g_2(a) = 2\));
- \(-0.5\) to a positive difference of at least two levels from the reference one;
- \(-1\) to a negative difference of at least two levels from the reference one, meaning that in this case totally dissimilarity is considered and the candidate is not suitable.

All these preference information are represented in the piecewise function above, \(f_2(\Delta_2(a,b))\) (see also Figure 5). After analyzing the cases of \(PmA, Pers\) and \(Med\), we concluded that the same SD function, \(f_2\), could represent well the similarity-dissimilarity between two candidates on these criteria (all expressed on a five-level qualitative scale). Therefore, \(f_2(\Delta_2(a,b)) = f_7(\Delta_7(a,b)) = f_8(\Delta_8(a,b)) = f_9(\Delta_9(a,b))\).

\[
f_2(\Delta_2(a,b)) = \begin{cases} 
-1, & \text{if } \Delta_2(a,b) \leq -2 \\
0.4, & \text{if } \Delta_2(a,b) = -1 \\
1, & \text{if } \Delta_2(a,b) = 0 \\
0.8, & \text{if } \Delta_2(a,b) = 1 \\
0.6, & \text{if } \Delta_2(a,b) = 2 \\
-0.5, & \text{if } \Delta_2(a,b) > 2
\end{cases}
\]

As for criterion \textit{physical fitness} \((g_1)\), \(PF\), the procedure briefly described below was followed:
1. Starting with the level 15.00 written in a card, the analyst asked the DM, incrementally increasing one unit, 16.00, 17.00 and so on (while presenting the respective new cards), whether each level could be considered somehow similar to 15.00. With a positive answer for 18.00, we got a value for a similarity threshold of 3 (18.00 – 15.00), i.e., \( t_1(g_1(b)) = 3 \);

2. Proceeding in a similar way considering levels above 18.00, and then below 15.00, and generalizing for any criteria pair with the same performance difference, the remaining similarity-dissimilarity thresholds were assigned.

Then, the intensities of the differences were determined based on rankings of cards, as described in Subsection 4.2. For example, we got that a difference of one or two levels could be considered equally similar (the cards were placed in the same rank position). Situations of neutrality were also identified. For levels in between, it was agreed on obtained the SD values by linear interpolation.

The DM refereed that a difference of six levels or more below a given reference level should be considered as a completely dissimilarity situation, while for a difference of five levels or more above, a quite dissimilar situation exists, but a candidate should not necessarily be eliminated for that reason. This was justified by the fact that the criterion is to be maximized, and greater performances than a reference one are easily judged similar rather than lower performances.

In that way, a SD function was constructed, as algebraically presented above by \( f_1(\Delta_1(a, b)) \) and graphically represented in Figure 6.
As for all criteria expressed on a percentile scale (\(NR, SA, MechR\) and \(VP\)), it was proposed by the DM on constructing a common function. The procedure applying the protocols described in Subsection 4.2 was followed.

As the level in the lowest part of the scale the performance level 50 was considered, and as the level in the highest part of the scale the level 80 was used. It was determined that we are dealing with constant thresholds.

Having in mind the standardized percentile scales used in the psychological tests, the DM in a quite easy way identified the SD thresholds and the intensities of similarity-dissimilarity for performance differences. Then, an asymmetrical SD function was built, which is algebraically presented above by \(f_1(\Delta_1(a, b))\) and graphically represented in Figure 6. It should be note that \(f_3(\Delta_3(a, b)) = f_4(\Delta_4(a, b)) = f_5(\Delta_5(a, b)) = f_6(\Delta_6(a, b))\), in accordance with the previous statement.

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f_3(\Delta_3(a,b)) = \begin{cases} 
-1, & \text{if } \Delta_3(a,b) \leq -30 \\
\frac{\Delta_3(a,b)}{10} + 2, & \text{if } -30 < \Delta_3(a,b) \leq -20 \\
0, & \text{if } -20 < \Delta_3(a,b) \leq -15 \\
\frac{\Delta_3(a,b)}{15} + 1, & \text{if } -15 < \Delta_3(a,b) \leq 0 \\
-\frac{\Delta_3(a,b)}{20} + 1, & \text{if } 0 < \Delta_3(a,b) \leq 20 \\
0, & \text{if } 20 < \Delta_3(a,b) \leq 30 \\
-\frac{\Delta_3(a,b)}{10} + 3, & \text{if } 30 < \Delta_3(a,b) \leq 40 \\
-0.5, & \text{if } \Delta_3(a,b) > 40 
\end{cases}

Figure 7: Per-criterion similarity-dissimilarity function for NR, SA, MechR and VP

5.3. Weights, interaction coefficients, and likeness thresholds

During a meeting with the DM, and in the presence of a soldier of each one of the Portuguese Army Special Forces (a commando, a paratrooper and a special operations soldier), the protocol described in Subsection 4.3 was applied. In a cooperative way with these three “experts”, the DM has constructed the rankings of the cards. The personal experience and knowledge of each soldier, specially about the respective category, has been taken into account in this process. Thus, a consensus of opinion among the participants allowed to obtain the rankings and establish the ratio \( z \). Some aspects and comments expressed during the application of the protocol should be highlighted, namely:

- **General aspects**: PF is a criterion with low importance to the candidates assessment and classification, since usually in the special training they improve their physical capabilities in a quite easy and satisfactory way. In contrast, the criteria related to innate abilities (e.g., SA, VP, ...) deserve greater relative weights. The greatest importance was given to Med in all categories, justified by the fact that the soldiers will have physically demanding training and their bodies
must be ready for the challenging, without significant medical issues; otherwise they cannot perform well;

- **Commandos** (C₁): *Med* and *Pers* are the most important criteria, because the physical robustness of a commando, and her/his emotional stability and motivation are crucial. Immediately above is *Intel*, since planning and solving problems in unexpected and complicated situations are important aspects to commandos. All criteria regarding innate abilities has been considered equally important. *PF* is the criterion judged as the least important one. In between consecutive levels the DM placed some blank cards and thus constructed a final cards ranking, as depicted in Figure 8. Since all criteria have a similar relative importance, a rather low value was assigned to the ratio between the weight of *Med* and *Pers* (the most important criteria) and the weight of *PF* (the least important criterion): $z = 4$;

- **Paratroopers** (C₂): *Med* is more important than the remaining criteria. With a medical problem they cannot do the parachute jumping. *Pers* is the second more important. A group of criteria has been considered equally important to paratroopers. They are related to personal abilities: *NR*, *SA*, *MechR* and *VP*. Drawing logical conclusions based on numerical data, having a great spatial ability, anticipating movements, and acting with a great speed are relevant capabilities to successfully jump with a parachute. Then, *PF* and *PmA* appear with the same importance. *Intel* is in the last position of the ranking. The DM placed a certain set of blank cards between consecutive levels, the ranking was defined, as in Figure 9. As for the value to the ratio $z$, there was a hesitation between 6 and 8. They argued that there is clearly a significant difference between the weight of most important criterion and the least one. For this reason, initially the value 8 was assigned. After some reflection, they have considered the value 6 more adequate: $z = 6$;

- **Special Operations** (C₃): *Med* is the most important criterion, followed by *Pers*, which is crucial when they operate. Resilience and adaptability, among other relevant characteristics, should be present in these soldiers. *Intel*, *MechR* and *VP* are equally important: solving problems, anticipating movements and acting with great speed are important abilities to special operations soldiers. With lower importance, *PF*, *SA* and *PmA* were placed in the ranking. The last position was occupied by *NR*. Then, a set of blank cards were adequately placed. The final ranking is illustrated in Figure 10. The value established to the ratio $z$ is 6, with some initially hesitation around the value 7. Thus, in the end, $z = 6$;

- **Snipers** (C₄): *Med* and *Pers* are the most important criteria. A sniper can act alone in the ground, therefore she/he should be apt to operate in an intelligent way. Without a good physical condition snipers cannot operate well (e.g., they must have a great vision and physical endurance). *NR*, *SA*, *MechR* and *PmA* were considered criteria with a relative great relevance, being given the same importance for all of them. *VP* was placed in the above position, and *PF* in the last position of the ranking. Blank cards were placed according to the preferences of the DM, as shown in Figure 11. The DM mentioned that all criteria have a relative closed importance to a sniper. So, the value established to the ratio $z$ is 5, with some hesitation in choosing the value 4 instead. Then, $z = 5$.

Firstly, we used the DCM-SRF method available in DecSpace¹ (currently in the pre-alpha release)

¹DecSpace Pre-Alpha is available at [http://app.decspacedev.sysresearch.org](http://app.decspacedev.sysresearch.org)
to obtain the values of the criteria weights, taking into account the various values of $z$ considered by the DM. The Deck Cards Method (DCM) is one of the early current methods supported by DecSpace, mainly designed for determining the criteria weights using the revised Simos’ procedure (“SRF method”). Secondly, we confronted the DM with all sets of weights to select the most adequate one for each category (with exception of category $C_1$, for which a value of $z$ was well defined, and
consequently required only a final validation of the obtained values). Finally, after the validation of the DM, the sets presented in Table 3 were established.

In order to know whether interaction in pairs of criteria could be considered in the model, the analyst began by giving a brief explanation to the DM about the main idea of the possible interaction effects between two criteria. The analyst made use of examples of interactions in a scenario related
Table 3: Criteria weights per Special Forces category ($k^h_j$)

| Category         | PF  | Intel | NR  | SA  | MechR | VP  | PmA | Pers | Med |
|------------------|-----|-------|-----|-----|-------|-----|-----|------|-----|
| Commandos        | 1   | 3.4   | 2.2 | 2.2 | 2.2   | 2.2 | 2.2 | 4    | 4   |
| Paratroopers     | 1.83| 1     | 3.08| 3.08| 3.08  | 3.08| 1.83| 4.33 | 6   |
| Special Operations| 2.5 | 3.5   | 1   | 2.5 | 3.5   | 3.5 | 2.5 | 5    | 6   |
| Snipers          | 1   | 5     | 3.4 | 3.4 | 3.4   | 1.8 | 3.4 | 5    | 5   |

to the evaluation of cars, as presented in Corrente et al. (2016). Once a clear understanding was expressed by the DM, through a systematic analysis, as described in Subsection 4.4, the following possible interactions were identified, being considered valid for all categories:

a) Mutual-strengthening effects:
- $PF$ ($g_1$) and $PmA$ ($g_7$)
- $PF$ ($g_1$) and $Med$ ($g_9$)

b) Mutual-weakening effects:
- $MechR$ ($g_5$) and $PmA$ ($g_7$)
- $VP$ ($g_6$) and $PmA$ ($g_7$)

Making sure that the DM clearly understood the idea behind the interaction coefficients, values were assigned to the four cases of interaction. For the category commandos, the following reasoning was considered to define the final values:

- Taking into account the sum of the weights of the criteria $PF$ ($g_1$) and $PmA$ ($g_7$), $k^1_1 + k^1_7 = 1 + 2.2 = 3.2$, the DM assigned a weight of 4.2 to the coalition of $g_1$ and $g_7$. Then, the value of this strengthening coefficient is $1 (k^1_{17} = 4.2 - 3.2 = 1)$;

- Taking into account the sum of the weights of the criteria $PF$ ($g_1$) and $Med$ ($g_9$), $k^1_1 + k^1_9 = 1 + 4 = 5$, the DM assigned a weight of 7 to the coalition of $g_1$ and $g_9$. Then, the value of this strengthening coefficient is $2 (k^1_{19} = 7 - 5 = 2)$;

- Taking into account the sum of the weights of the criteria $MechR$ ($g_5$) and $PmA$ ($g_7$), $k^1_5 + k^1_7 = 2.2 + 2.2 = 4.4$, the DM assigned a weight of 3.4 to the coalition of $g_5$ and $g_7$. Then, the value of this weakening coefficient is $-1 (k^1_{57} = 3.4 - 4.4 = -1)$;

- Taking into account the sum of the weights of the criteria $VP$ ($g_6$) and $PmA$ ($g_7$), $k^1_6 + k^1_7 = 2.2 + 2.2 = 4.4$, the DM assigned a weight of 4 to the coalition of $g_6$ and $g_7$. Then, the value of this weakening coefficient is $-0.4 (k^1_{67} = 4 - 4.4 = 0.4)$.

The DM argued that the same values of the interaction coefficients could be considered for the remaining three categories. Initially, the non-negativity condition (see Appendix, Equation 6) was not fulfilled for criterion $PmA$ ($g_7$), since the initial values assigned to the weakening coefficients were very low. Then, those coefficients were revised for all categories, and finally they were defined as presented above. Accordingly, we have $k^h_{17} = 1$, $k^h_{19} = 2$, $k^h_{57} = -1$, and $k^h_{67} = -0.4$, for $h = 1, ..., 4$.

Afterward, the analyst asked the DM for considering the assessment of a given candidate as an overall likeness degree with respect to a reference profile with a value between 0 and 1 (maximum
likeness degree). Thus, while considering individually each category, we asked the DM how much should be such a degree to consider that the candidate is suitable to the category under analysis. We mentioned that this value defines a likeness threshold within the range $[0.5, 1]$, being considered as majority of votes in favor of likeness of the candidate and the reference soldier profile. The DM refereed that: (i) for paratroopers the minimum likeness degree is enough; (ii) for commandos and special operations there is a relative demand on having similar soldiers to the reference profile of the respective category (the candidates should present the required overall performance in order to be prepared for the initial intensive training); and (iii) for snipers there is a greater requirement in terms of corresponding to the desired profile, guaranteeing that they can fulfill their function. Accordingly, the DM defined the values of the likeness thresholds as follows:

- Commandos ($C_1$): $\lambda^1 = 0.65$;
- Paratroopers ($C_2$): $\lambda^2 = 0.50$;
- Special Operations ($C_3$): $\lambda^3 = 0.65$;
- Snipers ($C_4$): $\lambda^4 = 0.80$.

6. The Cat-SD in the DecSpace

In this section, we describe the design and use of Cat-SD in the DecSpace. We present first the main features and functionalities of DecSpace. Then, we illustrate how to make use of the available implementation of Cat-SD while presenting the model constructed in the case study introduced in Section 2.

6.1. Overview of DecSpace

DecSpace is an innovative web-based platform to explore MCDA methods, conceived to be user-friendly and to offer an intuitive graphical user interface (in any web browser), designed while a certain expertise in MCDA from the user is not necessarily required. It was designed to make available several methods and to make it possible to add new ones without much programming effort. This platform is intended for use in teaching and researching on MCDA methods, as well as for professional use as a decision support systems (DSS) for supporting decisions using those methods. DecSpace is currently in Pre-Alpha, meaning that the platform has been in development, and this is an early release.

In terms of infrastructure, DecSpace consists of the web browser utilized by the user (where the client tier is deployed); a dedicated server that executes most of the necessary computations; and an external database that keeps all the data secure. Accordingly, the platform consists of a three-tiers architecture:

1. Client tier: It implements the user interface and sends user requests to the application tier;
2. Application tier: It confines most of the platform’s complexity, i.e., where most of the computational work is performed, and carries out connections with the other two tiers;
3. Data tier: It stores and retrieves all the data and replies to any data requests sent by the application tier.
A relevant inherent characteristic of DecSpace is that it is a web platform accessible from devices with an Internet connection and a browser. Its interface is thus optimized for different types of devices, including mobile devices as tablets and smartphones. Figure 12 displays the DecSpace homepage.

To start using DecSpace with all the features available, a registration process is mandatory. It is also possible to use it without registration, as an anonymous user, but with restrictions. Any registered user owns a project area. Each project has its own information, including the privacy setting, public or private. The public project area provides access to projects that were shared by other users, which can be opened, but modifications are only allowed when the project is duplicated as a private project. Private projects are solely available for the own user.

The workspace area, as presented in Figure 13, is intended to support the building and running of workflows. The MCDA methods modules can be chosen from the available methods list, and by doing that, the corresponding “boxes” show up in the workspace. The methods can be locally implemented by developers, or be available as remote methods, as for example from the diviz server (Meyer and Bigaret, 2012). The user is able to manually enter data into the MCDA methods modules or to import data files (data modules) and connect them to the method modules. Moreover, the input data can be imported in the format of Comma-Separated Values (CSV) and JavaScript Object Notation (JSON). Importing a .zip with several files and a .zip with other workflows is also possible. In order to get the results of the method modules, the workflow must be executed.

DecSpace currently presents the following main features and related functionalities:

- Types of Users: There are four types of users with different permissions:
  1. Developers: They can implement and add new MCDA methods;
2. Administrator: She/he manages all users and projects, having permission to modify or delete any object;

3. Anonymous users: They can test the platform and explore it with the restriction of having temporary projects (they cannot be saved);

4. Registered users: They have an account and their work is persistent, that is, the users can create their own projects, having access to all the features that DecSpace offers.

- User registration and login: To use the platform a user can register, providing an e-mail address and a password (a username is created based on the user’s e-mail), or login to access the her/his personal area, if she/he is already registered. It is possible to enter as a guest, by clicking on “Try it now!”;

- My Projects: In this area, a registered user can create projects (with a click on the “New Project” button), choose whether the project is public or private, and manage her/his projects, using a set of functionalities. Each created project has associated the following: Name, Privacy, Last Update, Created and Actions. Besides the functionality of creating new projects, the following actions are available for each project: Open Project (go to the “Workspace”), Duplicate Project (create a project equal to the selected project), and Delete Project (permanently eliminate the project, after confirmation by the user). In addition, the default order of visualizing the list of projects can be changed by inverting the alphabetical order of the project name, and the number of rows per page can be chosen among the four options (5, 10, 25, All);

- Settings: In this area, it is possible to change the username and the password (exclusively available in the personal area of registered users);
Public Projects: This area contains all public projects that are shared by users (available to everyone), with the following information for each project: Name, Owner, Last Update and Created. The same possible Actions as in “My Projects” are available (open, duplicate and delete), as well as the possibility of changing the project order by inverting the alphabetical order by “Name” or “Owner”, and choosing the number of rows per page;

Methods: All the available methods are in this area. Each method has a short description, an example and a step-by-step explanation, intended to provide some helpful information to the user;

FAQ: It contains commonly asked questions and the respective answers related to some features of the platform;

Workspace: This is the area where the users construct workflows by dragging and dropping, and properly connecting MCDA methods modules and data modules (“boxes”) in an intuitive graphical user interface. The following actions are available: Execute Workflow, Save Project, Import Data, Method Selection, Delete Workflow and Project Menu (see Figure 13). The data and the preference information can be manually provided in the methods modules. It is possible to import and export CSV and JSON files. A .zip file can also be imported, containing a workflow that was already used. The workflows can be executed, saved and deleted.

6.2. Building a CAT-SD workflow

In order to use CAT-SD in DECSPACE, the user must previously create a project. When opening such a project, it is displayed the workspace, where a decision model can be constructed as a workflow. The “CAT-SD” must be selected among the current available methods (“Method Selection”), clicking on “Add method”, and immediately a method module, “CAT-SD1”, appears in the workspace area, as depicted in Figure 13. All required data and preference parameters need to be linked to such a method module. This can be done by manually inserting the data, after clicking on the boxes modules, or by uploading correctly structured CSV or JSON files, which appear as data modules, to be then adequately connected to the method module. Figure 13 shows all modules connections. At any time, data can be changed in the workspace area (changed data files can be exported later on).

As presented in Subsection 2.2 (see Figure 1), the input include the main data (criteria, actions and performance table) and the preference parameters (SD functions, reference actions, weights, interaction coefficients and likeness thresholds), which are clearly separated in the method module. Only the interaction coefficients are not necessarily required, since in some cases they are not present. The remaining data are mandatory to be possible to execute the constructed workflow and then obtain the results (method output), which appear as a data module in the workspace. By clicking on such a module (“box”), the corresponding results can be viewed and analyzed by the user. The tables presented in the separated spreadsheets are interconnect in such a way that when input data are provided (e.g., criteria), the necessary related information appear in the tables that depend on such data.

In detail, the user can provide the following information:

a) Data

- Criteria: In this table, seven columns appear by default:
  1. Name: It is the name of the criterion;
2. Description: It is the respective criterion description or some related information (it is not mandatory);
3. Direction: It corresponds to the preference direction (the user must choose one of the two possible options: “Maximize” or “Minimize”);
4. Scale Type: It is related to the kind of data of the criterion performance levels (the user must choose “Ordinal” or “Cardinal”);
5. Min: If the scale type is “Cardinal”, then a minimum value for the performance levels should be provided;
6. Max: If the scale type is “Cardinal”, then a maximum value for the performance levels should be provided;
7. Num Levels: If the scale type is “Ordinal”, then the total number of scale levels should be provided;

Actions: Data related to the potential actions only include:
1. Name: It is the name of the action;
2. Description: It is the respective action description (it is not mandatory);

Performance Table: The rows correspond to the actions names and the columns corresponds to the criteria names. Performance levels on each criterion must be provided for each action (the platform verify that they fulfill the criteria scales characteristics);

b) Preference information

Reference Actions: For each action, the following information is needed: the name of the category to be considered (Category); the name of the reference action (Name) and then a performance levels have to be fulfilled in each criterion column;

SD Functions: For each criterion a set of rows with a value within the range $[-1, 1]$ (SD Value) and the respective performance differences for which the function takes the SD Value must be provided in a form of a mathematical condition (Condition);

Weights: For each category and each criterion, a value of the criterion weight must be given, i.e., a set of weights per category;

Interaction Coefficients: Firstly, the user can choose a category among the predefined categories that appear as the possible options (Category). Secondly, for such a category, a first criterion can be chosen among the options, i.e., the previously defined criteria (Criterion 1), and then a second criterion, among the remaining criteria has to be selected (Criterion 2). Thirdly, the type of interaction can be chosen among the three options (Type). Finally, a value for the interaction coefficient must be provided (Value). This procedure has to be followed for all interaction coefficients considered in the model. Alternatively, the data can be previously organized in a file, and then imported and adequately connected, as for the input data and the rest of preference parameters. The platform checks the non-negative condition (see Equation 6 in [8]) and notifies the user, displaying an alert message box, in case of the values do not fulfilled the condition;

Likeness Thresholds: For each predefined category (Category) a likeness threshold must be defined with a value within the range $[0, 1]$ (Value).

The constructed workflow can be executed when all connections are properly done and the required data are provided (DecSPACE algorithms validate that and make the needed calculations for obtaining
the assignment results). The results of a CAT-SD workflow are available in a data box that contains a table summarizing the classification of the actions into the considered categories or possibly into the “Non-assigned” category (it appears by default). Intermediate calculations and results are also available to the users (e.g., Maximum Likeness Degrees Per Category, Maximum Likeness Degrees Per Reference Action). The workflow, including the input files and results, can be exported (it can only be saved by a registered user) as a .zip file containing .csv files.

Once we have designed a decision model, with all preference parameters defined and validated by the DM, we used the data of the case study in the construction of the corresponding CAT-SD workflow in DecSpace. Figure 13 presents such a workflow. Data about actions, performance table and SD functions were previously organized in spreadsheets (.csv files). Then, they were imported and properly connected to the method module. The remaining data seemed easier and more user-friendly to be inserted directly into tables in the workspace. Indeed, the CAT-SD module has been designed with that purpose, offering a graphical user interface to provide data.

6.3. Results and discussion

We have obtained the assignment of the twenty candidates (dummy, since actual data are classified) into five categories (categories representing four Special Forces and the category unsuitable candidates), according to the constructed model. It is worth mentioning that the model was revised twice with the DM, and here we only present the final model and the respective associated results. Table 4 shows the obtained results.

Table 4: Assignment of the candidates to Special Forces

| Candidate | Commandos | Paratroopers | Special Operations | Snipers | Unsuitable Candidates |
|-----------|-----------|--------------|--------------------|---------|-----------------------|
| a1        | ✓         | ✓            | ✓                  |         |                       |
| a2        | ✓         | ✓            | ✓                  |         | ✓                     |
| a3        | ✓         | ✓            | ✓                  |         | ✓                     |
| a4        | ✓         | ✓            | ✓                  |         |                       |
| a5        | ✓         | ✓            | ✓                  |         |                       |
| a6        | ✓         | ✓            | ✓                  |         | ✓                     |
| a7        | ✓         | ✓            | ✓                  |         |                       |
| a8        | ✓         | ✓            | ✓                  |         | ✓                     |
| a9        | ✓         | ✓            | ✓                  |         |                       |
| a10       | ✓         | ✓            | ✓                  |         | ✓                     |
| a11       | ✓         | ✓            | ✓                  |         |                       |
| a12       | ✓         | ✓            | ✓                  |         |                       |
| a13       | ✓         | ✓            | ✓                  |         |                       |
| a14       | ✓         | ✓            | ✓                  |         |                       |
| a15       | ✓         | ✓            | ✓                  |         |                       |
| a16       | ✓         | ✓            | ✓                  |         |                       |
| a17       | ✓         | ✓            | ✓                  |         |                       |
| a18       | ✓         | ✓            | ✓                  |         |                       |
| a19       | ✓         | ✓            | ✓                  |         |                       |
| a20       | ✓         | ✓            | ✓                  | ✓       | ✓                     |

We can observe there are candidates only suitable for one specific category, others are considered apt for more than one category, and others are not suitable at all for the Special Forces (unsuitable candidates).
As one can expect, a few number of candidates (only three out of twenty) are considered suitable to be assigned to category snipers, since it is the most demanded Special Forces category considered in the model. Moreover, these candidates are adequate for all four Special Forces categories, that is, a soldier apt as a sniper is also apt to be any of the remaining forces. This is an expected result, according to the DM. On the contrary, a large number of candidates (fifteen out of twenty) are assigned to category paratroopers, which is the category judged as less demanded. Consequently, candidates are easily accepted as soldiers adequate to paratroopers.

Only two out of twenty military candidates are non-assigned to a Special Forces category, that is, their profiles, in a general way, are not considered adequate to these forces, according to the constructed model. In particular:

- \( a_7 \) is excluded (non-assigned), albeit there is some likeness degree with the reference profiles. Her/his performance of 3 in criterion Med could contributed for that;

- \( a_{10} \) is excluded because she/he has a performance of 2 in Pers, although she/he has a high performance on the majority of criteria. The DM refereed that this is in line with the Army requirements, i.e., a candidate with a personality assessed as a low level cannot be an Army Special Forces soldier.

It should be refereed that soldier \( a_{17} \) is not nearly considered suitable to be a sniper. She/he has a likeness degree of 0.78 with respect to \( b_{41} \) and it is necessary a degree of 0.8 (likeness threshold for snipers). This results specially for the performance of 4 (and not 5 as the reference profile) in Med, criterion with the highest weight, and consequently with the highest contribution to the likeness degree.

According to the opinion of the DM, these results are satisfactory and they are coherent with the requirements for the recruitment of the Portuguese Army Special Forces soldiers. The constructed model takes into account the perspective and perception of the DM about this issue, and the obtained results are aligned with the expectations of the DM.

7. Lessons learned from practice

In this study, we adopted a decision aiding constructive approach that requires an active intervention of the DM, and from which a set of conclusions in terms of lessons learned can be drawn. We highlight the aspects presented below:

a) Meetings: We argue that having several short meetings, aiming at discussing particular points, is more beneficial than having long meetings, with extensive interaction and discussions, which usually involve a great amount of information and require a great cognitive effort from the DM. In addition, we consider that the kind of communication involving interaction with people personally has advantage over alternative ways to communicate (i.e., no presential meetings). Accordingly, the case study was conducted during several face-to-face meetings (interaction sessions), which allowed to understand the perceptions and preferences of the DM with respect to the decision problem at hand, focusing on a particular point in each session. Indeed, it seemed to be effective and efficient, with a high engagement, giving us the chance to easy and quickly clarify concepts, ideas, objectives, etc.;
b) Communication: In general, we think that the analyst should start by introducing some main concepts and terminology related to MCDA and Cat-SD (e.g., criteria, categories, reference actions or profiles), aiming to build a decision model. Even though, we argue that adapting to the background of the DM, and using some terms close to her/his area of knowledge instead of the ones used in MCDA, sometimes can help the conversation. We have proceeded accordingly, and the analyst and the DM talked in a quite natural and easy way;

c) Interaction protocols: The DM expressed that the way in which the interaction protocols were applied was naturally accepted and understood. In addition, the DM recognized that these interactions and discussions allowed to reflect about relevant aspects for the assessment and selection of Special Forces soldiers. This is a positive aspect favoring the application of the method in the context of the case study;

d) Difficulties: At the end of each meeting, the analyst identified with the DM the steps of the protocols easy to understand and apply, and those needing a great effort to be accomplished by the DM. Moreover, the analyst discussed about the difficulties the DM felt along the process of eliciting the preference parameters, namely on proving preference information and assigning numerical values. The main tasks presenting some difficulties to the DM are discussed below, considering the definition of the preference parameters individually:

- Reference profiles: Regarding the definition of the reference soldiers’ profiles, we had the perception that in the beginning the DM had some difficulties providing the profiles in an empirical way, i.e., based on observation and experience, since real data could not be used, but then the DM readily provided the performances for all desired profiles (also somehow supported by existent documentation). It was clear that we are dealing with high demanding soldier’s profiles and, for this reason, in general, the performances on the criteria are high and relatively close among the categories. Still, the DM was able to construct distinct and representative profiles for each category. Besides taking into account well established reference actions, encouraging the DM to reflect about those representative actions based on the people’s experience can be useful, since it expresses somehow the reality, i.e., what is typically observed in practice;

- SD functions: The way in which we proceed allowed to know about norms in the context of the study and get the preferences of the DM. All those information were taken into account and modeled through the SD functions in an adequate way. We observe that the guidance of the analyst during the dialogue with the DM is crucial, but giving some freedom to the DM expresses her/his subjective judgments is also important. Pairwise comparison of some particular performance levels can be useful to generalize the form of the function under construction. Handling cards and observing graphical representations of scales and examples of functions can help the DM to build these functions;

- Weights: The deck cards method for determining the weights was immediately accepted and understood. The reason for that is not only the simplicity associated with the procedure of ranking the cards, but also the fact that the DM is familiarized with methods involving handling cards when performing a competency profile analysis. The \(z\) value required additional explanation from the analyst: “Please consider that the least important criterion (or criteria) has one vote. How much votes should have the most important criterion (or criteria) of the ranking?” The DM understood and assigned the \(z\) values, as described in
Subsection 4.3 Thus, we recommend procedures based on the deck cards method to this aim (not exclusively to this task);

– Interaction coefficients: The explanation using examples proved to be effective, since the DM had no difficulties in understanding the three possible interaction effects that we can model with Cat-SD. Thus, proving some examples (fictitious or real ones) applied in different contexts, avoiding biases, can be an adequate way to explain the interactions between criteria. Although in this case the task of assigning values to the coefficients was performed without presenting difficulties, alternative elicitation protocols can be more adequate in other cases (e.g., using more natural language, classifying each criteria interaction as “low”, “medium” and “strong”, etc.);

– Likeness thresholds: The DM had a well understanding about the likeness thresholds. In this case, the values were easily established by the DM, being clearly different among the categories. This could not happen in other real-world cases, in which the DM could have to deeply reflect about such thresholds, and they can be considered as having the same value for all categories.

It worth to be mentioned that, in a general way, the DM easily understood and accepted our approach, with a prompt attitude towards collaboration to the co-construction of the model. This has facilitated all the interaction between the analyst and the DM for gathering of the preference information and eliciting the parameters.

8. Conclusions

This paper introduces the design and implementation of Cat-SD, an MCDA method recently proposed in the literature, as well as it proposes protocols to elicit the preference parameters used in the method. This paper simultaneously presents a real-world case of application of the method, while focusing on conducting the study through the utilization of the designed protocols, and shows the main features of using Cat-SD in DecSpace.

DecSpace is an innovative user-friendly web-based platform to explore MCDA methods. It allows the construction of workflows, an easy exploration and combination of various methods, while offering a range of visualization features with cutting edge technology. We illustrate how this platform can facilitate the use of the method by providing computational support, as well as providing an intuitive interface, with a good visualization of data and obtained results.

The present case study permitted us to test the support given by Cat-SD to a real-world decision scenario related to the candidates selection process. Moreover, it permits somehow to validate the application of the protocols designed to facilitate the elicitation of the preference parameters, and draw some conclusions in terms of lessons learned. It seems to us that this case constitutes a good example of how the method can help to make informed decisions in this kind of contexts.

Besides the interest and engagement of the DM during this study, the DM has revealed interest in future collaboration. Further research should be done in the context of this study, having the constructed model as basis and performing a deeper analysis of the decision problem. This can involve, for example, a systematic study of the desired profiles for each considered category and a robustness analysis (e.g., sensitivity analysis to changes on some preference parameters). As for Cat-SD in DecSpace, future work relies on improving usability and user experience issues.

From a more generic point of view, this kind of application can be relevant not only in other contexts of recruitment process, but also in contexts facing complex decision situations involving
interconnected problems, with multiple stakeholders. For instance, this approach can be applied to current sharing cities issues, namely urban planning problems, energy issues or other complex economic and environmental problems, while taking into account economic, environmental and social criteria, in order to support decision making on smart cities solutions.

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Appendix

The method was designed based on the concepts of similarity and dissimilarity. The assignment of an action to a given category depends on the way it compares to the reference actions of that category. The pairwise comparison takes into account similarity and dissimilarity (subjective) judgments of the DM, which are modeled through a similarity-dissimilarity function on each criterion. An overall measure of the likeness between such an action and the set of reference actions is then computed. The main steps of the method are presented below.

1. **Per-criterion similarity-dissimilarity function.** A per-criterion similarity-dissimilarity function can be defined as follows, assuming that, the criterion \( g_j \) is to be maximized. An action \( a \) and a reference action \( b \) are compared in terms of performance on \( g_j \). As input consider the difference of performances \( \Delta_j(a, b) = \text{diff} \{g_j(a), g_j(b)\} \) (in cardinal scale, \( \text{diff} \{g_j(a), g_j(b)\} = g_j(a) - g_j(b) \), and in ordinal scales, \( \text{diff} \{g_j(a), g_j(b)\} \) is the number of performance levels in between \( g_j(a) \) and \( g_j(b) \). A per-criterion similarity-dissimilarity function is a real-valued function, \( f_j(\Delta_j(a, b)) \), that has as output a value within the range \([-1, 1]\) and presents the following features. The function \( f_j \) is a non-decreasing function, if \( \Delta_j(a, b) \in [-\text{diff} \{g_j^{\text{max}}, g_j^{\text{min}}\}, 0] \), where \( g_j^{\text{min}} \) and \( g_j^{\text{max}} \) are the lowest and the greatest values of the scale of criterion, respectively; while \( f_j \) is a non-increasing function, if \( \Delta_j(a, b) \in [0, \text{diff} \{g_j^{\text{max}}, g_j^{\text{min}}\} \). It is easy to see that \( f_j > 0 \) iff criterion \( g_j \) contributes to similarity, while \( f_j < 0 \) iff criterion \( g_j \) contributes to dissimilarity. Moreover, we can consider the following: (1) A per-criterion similarity function \( s_j(a, b) = f_j(\Delta_j(a, b)) \), when \( f_j(\Delta_j(a, b)) > 0 \), and \( s_j(a, b) = 0 \), otherwise, and (2) A per-criterion dissimilarity function \( d_j(a, b) = f_j(\Delta_j(a, b)) \), when \( f_j(\Delta_j(a, b)) < 0 \), and \( d_j(a, b) = 0 \), otherwise.

2. **Comprehensive similarity.** A way to compute the comprehensive similarity between two actions, \( a \) and \( b \), is using the function \( s^h(a, b) \) presented in Equation (5). It takes into account the contribution of all criteria to the similarity (i.e., the values of the per-criterion similarity functions, referred above), the criteria weights and the interaction coefficients. It may also take into account some dissimilarity values derived from antagonist effects, if they exist:

\[
s^h(a, b) = \frac{1}{K^h(a, b)} \left( \sum_{j \in G} k_j^h s_j(a, b) + \sum_{(j, \ell) \in M^h} z(s_j(a, b), s_\ell(a, b)) k_{j\ell}^h + \sum_{(j, p) \in O^h} z(s_j(a, b), |d_p(a, b)|) k_{jp}^h \right),
\]

where

\[
K^h(a, b) = \sum_{j \in G} k_j^h + \sum_{(j, \ell) \in M^h} z(s_j(a, b), s_\ell(a, b)) k_{j\ell}^h + \sum_{(j, p) \in O^h} z(s_j(a, b), |d_p(a, b)|) k_{jp}^h
\]

with \( z(x, y) = xy \), for \( h = 1, ..., q \). The set \( M^h \) contains all pairs of criteria, \( g_j \) and \( g_\ell \), for which there is mutual-strengthening or mutual-weakening, and \( O^h \) contains all pairs of criteria, \( g_j \) and \( g_p \) for which there is antagonism. The following condition must be verified.

\[
\sum_{(j, \ell) \in M^h : k_{j\ell}^h < 0} |k_{j\ell}^h| - \sum_{(j, p) \in O^h} |k_{jp}^h| \geq 0, \text{ for all } j \in G; h = 1, ..., q.
\]

3. **Comprehensive dissimilarity.** A way to measure the comprehensive dissimilarity between two actions, \( a \) and \( b \), is using the function \( d(a, b) \) presented in Equation (6). It takes into account the
contribution of all criteria to the dissimilarity between actions $a$ and $b$ (i.e., the values of the per-criterion dissimilarity functions, referred above).

$$d(a, b) = \prod_{j=1}^{n} (1 + d_j(a, b)) - 1. \quad (7)$$

4. **Comprehensive likeness.** The function below assesses the overall degree to which action $a$ is alike to action $b$.

$$\delta(a, b) = s^h(a, b) (1 + d(a, b)). \quad (8)$$

Based on the likeness degree between an action and a set of reference actions, and according to the likeness threshold chosen for a given category, a $\lambda$–likeness binary relation can be defined as follows:

$$aS(\lambda^h)B_h \iff \delta(a, B_h) \geq \lambda^h. \quad (9)$$

where

$$\delta(a, B_h) = \max_{\ell=1, \ldots, |B_h|} \{\delta(a, b_{h\ell})\}.$$

5. **Likeness assignment procedure.** For a $\lambda^h \in [0.5, 1]$, for $h = 1, \ldots, q$, the assignment procedure follows the next steps.

i) Compare $a$ with $B_h$, for $h = 1, \ldots, q$;

ii) Identify $U = \{u : aS(\lambda^u)B_u\}$;

iii) Assign $a$ to $C_u$, for all $u \in U$;

iv) If $U = \emptyset$, assign $a$ to category $C_{q+1}$.

The method provides a set of possible categories (possibly a single one) to which an action $a$ can be assigned to (it may be the dummy category, meaning that action $a$ is not suitable to the remaining categories).
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