Scalar Stoponium

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Abstract
We study the decays of a scalar ($\tilde{t}_1\tilde{t}_1^*$) bound state $\sigma_{\tilde{t}_1}$, where $\tilde{t}_1$ is the lighter stop eigenstate. If $\tilde{t}_1$ has no tree-level 2–body decays, the dominant decay modes of $\sigma_{\tilde{t}_1}$ are $gg$ or, if $m_h < m_{\tilde{t}_1} \ll m_{\tilde{t}_2}$, a pair of light scalar Higgs bosons $h$. The best signal for $\sigma_{\tilde{t}_1}$ production at hadron colliders is probably its decay into two photons.
It is by now quite well known that the lighter scalar top (stop) eigenstate $\tilde{t}_1$ is expected to be lighter than the superpartners of the first two generations of quarks, and might even be the lightest colored sparticle $[1, 2]$. There are two reasons for this: Since the top quark is heavy, $m_t \geq 110$ GeV, mixing between the superpartners $\tilde{t}_L$, $\tilde{t}_R$ of left– and right–handed top quarks cannot be neglected, in contrast to the superpartners of light quarks. Furthermore, if we assume all squarks to have the same mass at some very high (GUT, string or Planck) scale, radiative corrections $[3]$ will reduce the masses of $\tilde{t}_L$ and $\tilde{t}_R$ relative to those of the other squarks.

Open $\tilde{t}_1$ pair production at $e^+e^-$ $[4]$ and $\bar{p}p$ $[4]$ colliders has been discussed previously. Here we study possible signals for the production of a scalar ($\tilde{t}_1\tilde{t}_1$) bound state $\sigma_{\tilde{t}_1}$ within the Minimal Supersymmetric Standard Model (MSSM) $[6]$. In ref.$[7]$ it has been pointed out that the branching ratio for $\sigma_{\tilde{t}_1} \to hh$ might be large, where $h$ is the lighter scalar Higgs boson. Very recently it has been claimed $[8]$ that $\sigma_{\tilde{t}_1} \to W^+W^-$ can have a very large branching ratio, which might give rise to interesting signals at hadron supercolliders. This motivated us to compute all potentially large branching ratios of $\sigma_{\tilde{t}_1}$. We basically agree with the results of ref.$[7]$, but were unable to reproduce those of ref.$[8]$. The clearest signal for $\sigma_{\tilde{t}_1}$ production at hadron colliders arises from its 2–photon decay, giving rise to a peak in the $\gamma\gamma$ invariant mass spectrum.

The starting point of our discussion is the stop mass matrix. Following the convention of ref.$[4]$ we write it as (in the basis $\tilde{t}_L$, $\tilde{t}_R$):

$$\mathcal{M}_t^2 = \begin{pmatrix} m_{t_L}^2 + m_{\tilde{t}_1}^2 + 0.35D_Z & -m_t(A_t + \mu \cot\beta) \\ -m_t(A_t + \mu \cot\beta) & m_{t_R}^2 + m_{\tilde{t}_1}^2 + 0.16D_Z \end{pmatrix}. $$

Here, $D_Z = M_Z^2 \cos2\beta$ with $\tan\beta = \langle \tilde{H}^0 \rangle / \langle H^0 \rangle$ as usual $[3]$, $\mu$ is the supersymmetric Higgs(ino) mass parameter, $A_t$ a trilinear soft supersymmetry breaking parameter, and $m_{t_L,R}^2$ the nonsupersymmetric contributions to the squared masses of the $\tilde{t}_L$, $\tilde{t}_R$ current states. The diagonalization of $\mathcal{M}_t^2$ is straightforward. We find for the lighter eigenstate $\tilde{t}_1 \equiv \cos\theta_t \tilde{t}_L + \sin\theta_t \tilde{t}_R$:

$$m_{\tilde{t}_1}^2 = \frac{1}{2} \left[ m_{t_L}^2 + m_{t_R}^2 - \sqrt{(m_{t_L}^2 - m_{t_R}^2)^2 + 4m_{t_L}^2} \right];$$

$$\tan\theta_t = \frac{m_{t_L}^2 - m_{t_R}^2}{m_{t_L}^2},$$

where $m_{t_L,L,R,RR}^2$ refers to the $LL$, $RR$ and $LR$ elements of $\mathcal{M}_t^2$. While the gauge interactions of $\tilde{t}_1$ only depend on $\theta_t$, the couplings of stop squarks to Higgs bosons depend on all parameters entering eq.$(1)$; all these quantities therefore have to be specified before $\sigma_{\tilde{t}_1}$ branching ratios can be computed.

There are two different kinds of $\sigma_{\tilde{t}_1}$ decays: Single stop decays and annihilation decays. In the first case either $\tilde{t}_1$ or $\tilde{t}_1^*$ decays, leaving the other squark behind. We assume that the gluino is too heavy to be produced in these decays. In general we then have to consider the following decay modes:

$$\tilde{t}_1 \to b\tilde{W}_i, i = 1, 2;$$

$$\tilde{t}_1 \to t\tilde{Z}_i, i = 1, \ldots, 4;$$

$$\tilde{t}_1 \to c\tilde{Z}_i, i = 1, \ldots, 4,$$

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where \( \tilde{W}_i \) (\( \tilde{Z}_i \)) denotes a generic chargino (neutralino) state. The decays (3a,3b) occur at tree level and with full gauge or top Yukawa strength. It has been shown in ref.\[2\] that (3c) is the dominant \( \tilde{t}_1 \) decay mode if the first two modes are kinematically forbidden. However, this last decay only occurs at 1–loop level and necessitates flavor mixing; it is therefore suppressed relative to the tree–level decays by a factor \(|\epsilon|^2 \approx 10^{-7}\) \[2\]. We will see below that the mode (3c) can therefore be neglected in the discussion of \( \sigma_{\tilde{t}_1} \) decays. The widths of (3a,3b) can be computed using the couplings of refs.\[6\]; the decay width of \( \sigma_{\tilde{t}_1} \) is twice that of \( \tilde{t}_1 \).

In annihilation decays \( \tilde{t}_1 \) and \( \tilde{t}_1^* \) annihilate into a flavor and color singlet final state; this kind of decay is by far dominant for the familiar (\( c\bar{c} \)) and (\( b\bar{b} \)) bound states (quarkonia). We computed the widths for the following modes:

\[
\begin{align*}
\sigma_{\tilde{t}_1} &\rightarrow gg; \quad (4a) \\
\sigma_{\tilde{t}_1} &\rightarrow W^+W^–; \quad (4b) \\
\sigma_{\tilde{t}_1} &\rightarrow ZZ; \quad (4c) \\
\sigma_{\tilde{t}_1} &\rightarrow Z\gamma; \quad (4d) \\
\sigma_{\tilde{t}_1} &\rightarrow \gamma\gamma; \quad (4e) \\
\sigma_{\tilde{t}_1} &\rightarrow h\bar{h}; \quad (4f) \\
\sigma_{\tilde{t}_1} &\rightarrow b\bar{b}; \quad (4g) \\
\sigma_{\tilde{t}_1} &\rightarrow t\bar{t}; \quad (4h) \\
\sigma_{\tilde{t}_1} &\rightarrow \tilde{Z}_i\tilde{Z}_j, i, j = 1, \ldots, 4. \quad (4i)
\end{align*}
\]

We computed the corresponding branching ratios using “Method 2” described in the Appendix of ref.\[10\]. Since \( \sigma_{\tilde{t}_1} \) is a scalar (\( s–wave \)) state, we only need the \( \tilde{t}_1 \) velocity \( v \rightarrow 0 \) limit of the \( \tilde{t}_1\tilde{t}_1^* \) annihilation amplitudes leading to the final states of eqs.(4). In this limit, reactions (4a,4d,4e) proceed via \( t–channel \) \( \tilde{t}_1 \) exchange as well as via 4–point “butterfly” interactions; (4b) proceeds via \( \tilde{t}_2 \) exchange, a 4–point interaction as well as scalar Higgs exchange in the \( s–channel \), while (4c) involve \( \tilde{t}_1 \) or \( \tilde{t}_2 \) exchange in the \( t–channel \), a 4–point coupling and Higgs exchange.† Processes (4g,4h) involve \( s–channel \) scalar Higgs exchange and \( t–channel \) chargino or neutralino exchange; note that the corresponding matrix elements are proportional to the final state quark masses, so that the width for (4g) is very small unless \( \tan\beta \gg 1 \). Finally, (4i) proceeds via \( s–channel \) Higgs exchange or \( t–channel \) top exchange.

In order to compute the decay widths for the processes (4) we have to know the “wave function at the origin” \( |\psi(0)|^2 \), see ref.\[10\]. Recently the \( m_{\tilde{t}_1} \) dependence of this quantity has been parametrized in ref.\[11\], for 4 different values of the QCD scale parameter \( \Lambda \), using a potential that reproduces the known quarkonium spectrum well; we use their fit for \( \Lambda = 0.2 \) GeV.

In fig.1 we show examples for the branching ratios of processes (4) for relatively small

*We ignore mixing in the \( b \) sector.
†\( \tilde{t}_2 \) exchange has not been included in ref.\[3\], where the reaction (4f) has been studied; this contribution is small compared to the \( \tilde{t}_1 \) exchange term for parameters leading to a sizable \( hh \) branching ratio.
\(m_{i_{L,R}}\) (200 GeV). In addition to the parameters appearing in eq. (4) we have to specify the SU(2) gaugino mass \(M_2\) (we assume \(M_1 = 5/3 \tan^2 \theta_W M_2\) as usual) and the mass \(m_P\) of the pseudoscalar Higgs boson. This then determines all relevant masses and couplings. We have included leading radiative corrections to the scalar Higgs sector involving top–stop loops [12].

In this figure we have assumed \(M_2 = 100\) GeV leading to a mass of about 110 GeV for the lighter chargino. For \(m_{i_{L}} > 115\) GeV the single stop decay (\(3a\)) (not shown) opens up and quickly dominates the total decay width. Indeed, in this region the total width of \(\sigma_{i_{L}}\) is comparable to its binding energy. Our calculations are no longer reliable in this case, since we assume that formation and decay of \(\sigma_{i_{L}}\) occur at very different time scales so that they can be treated independently; one has to use methods developed previously [13] for \((t\bar{t})\) bound states instead. However, we can conclude from fig. 1 that if the single stop decays (\(3a, 3b\)) are allowed the branching ratios for final states that might be detectable at hadron colliders (see below) are very small, less than \(10^{-4}\).

In fig. 2 we have therefore varied \(M_2\) along with \(m_{i_{L}}\), so that the decays (\(3a, 3b\)) remain closed for \(m_{i_{L}} \leq |\mu|\). We have also chosen larger values for \(m_{i_{L,R}}\) with \(m_{i_{L}} > m_{i_{R}}\) as predicted by minimal supergravity models [9]. We see that now the \(Br(\sigma_{i_{L}} \rightarrow hh)\) shoots up very rapidly once this decay becomes kinematically allowed. The reason is that the \(h\tilde{t}_1\tilde{t}_1\) coupling \(\tilde{t}_1\) contains a term which, in the limit \(m_P^2 \gg M_Z^2\), is exactly proportional to the LR element of \(M_i^2\). Obviously this element has to be large if \(m_{i_{L}}\) is to be much smaller than \(m_{i_{L,R}}\). As a result, the amplitude for \(\tilde{t}_1\tilde{t}_1 \rightarrow hh\) is proportional to \(\left(\frac{m_{i_{L,R}}^2}{M_W m_{i_{L}}}\right)^2\) if \(m_{i_{L,R}}^2 \gg m_{i_{L}}^2 \gg m_h^2/2\). This explains the decrease of the \(hh\) branching ratio with increasing \(m_{i_{L}}\), in spite of the increasing phase space.

The branching ratios for \(W^+W^-\) and \(ZZ\) also increase quickly just beyond threshold. However, they do not reach the level of the \(hh\) branching ratio; their amplitudes are at best \(\propto \left(\frac{m_{i_{L}}}{M_W}\right)^2\), if \(m_{i_{L,R}}^2 \gg m_{i_{L}}^2 \gg M_W^2\). This can be understood from the equivalence theorem [14], which states that amplitudes involving longitudinal gauge bosons are equal to corresponding ones involving pseudoscalar Goldstone bosons \(G\), if the energy of the process is \(\gg M_W\). There is no diagonal \(G\tilde{t}_1\tilde{t}_1\) coupling; a \(G\tilde{t}_1\tilde{t}_2\) coupling with strength similar to the \(h\tilde{t}_1\tilde{t}_1\) coupling does exist, but it only affects \(\sigma_{i_{L}}\) decays via diagrams involving a heavy stop propagator. The amplitude for \(\sigma_{i_{L}} \rightarrow Z_LZ_L\) is therefore suppressed by a factor \(\left(\frac{m_{i_{L}}}{m_{i_{L}}^2}\right)^2\) compared to the \(hh\) amplitude. Similar arguments apply for the \(W^+_LW^+_L\) amplitude. Transverse \(W\) and \(Z\) bosons are at best produced with ordinary (weak) gauge strength, and their couplings can even be suppressed by \(\tilde{t}\) mixing. Unlike ref. [8] we therefore never find the width into \(WW\) to exceed the one into gluons. However, the authors of ref. [8] neglected \(\tilde{t}\) mixing, and assumed that \(m_{i_{L}}\) can be varied independently of the mass of the left–handed sbottom \(\tilde{b}_L\). Since \(\tilde{b}_L\) and \(\tilde{b}_L\) reside in the same SU(2) doublet, this introduces a new source of explicit gauge symmetry breaking, which renders the theory nonrenormalizable.

The curves of fig. 2 show a lot of structure around \(m_{i_{L}} = 250\) GeV. This is because for the given choice of parameters the mass of the heavy scalar Higgs \(H\) is just above 500 GeV; the s–channel \(H\) exchange diagrams therefore become resonant, greatly enhancing the matrix elements for \(t\tilde{t}, \bar{b}b, hh\) and \(Z\tilde{Z}\). The enhancement of the \(W^+W^-\) and \(ZZ\) final states is much weaker, since the \(HHV\) couplings \((V = W, Z)\) are small for \(m_H^2 \gg M_Z^2\). If \(2m_{i_{L}}\) is very
close to $m_H$ our treatment again breaks down; in this case the $(\tilde{t}_1\tilde{t}_1^*)$ bound states mix with $H$.

Finally, the curves of figs. 1,2 exhibit numerous minima resulting from destructive interference between different contributions to the matrix elements. In particular, for the $W^+W^-$, $ZZ$ and $hh$ final states the $t-$channel and 4–point coupling diagrams always contribute with opposite signs. Since the size of the $t-$channel contributions increases with increasing ratio $m_{\tilde{t},H}/m_{\tilde{t}}$, the interference minima move further away from threshold as $m_{\tilde{t},H}$ are increased. The $hh$ branching ratio in fig. 2 has a second minimum due to interference with the $s-$channel $H$ exchange contribution. The $t\bar{t}$, $Z_1Z_1$ and $Z_1\bar{Z}_2$ final states also show destructive interference between $s-$ and $t-$channel diagrams. It is important to note that (at least in the limit $\nu \to 0$) usually only a single partial wave is accessible in $\sigma_{\tilde{t}_1}$ decays; if in addition only a single combination of final state helicities can be produced, destructive interference can lead to a vanishing total amplitude even far above threshold.

Clearly one could in principle learn a lot about the MSSM parameters by studying $\sigma_{\tilde{t}_1}$ branching ratios. In practice, however, even discovery of $\sigma_{\tilde{t}_1}$ may not be trivial. We focus here on $pp$ supercolliders. The total cross section for $\sigma_{\tilde{t}_1}$ production at the SSC is shown by the solid line in fig. 3. Hadronic final states ($gg, bb, t\bar{t}$) will be useless for the discovery of $\sigma_{\tilde{t}_1}$ at such colliders, due to the enormous backgrounds. In ref. [8] the use of the $W^+W^-$ final state was advocated. However, we have seen above that $SU(2)$ gauge invariance implies a rather small rate for this final state; besides, it is not clear to us how the $W^+W^-$ invariant mass will be measured, since both $W$ bosons will have to decay leptonically in order to suppress QCD backgrounds. The $ZZ$ final state is very clean if both $Z$ bosons decay leptonically, but then the rate will be very small even at the SSC ($< 5$ events/year).

In ref. [8] the use of $\sigma_{\tilde{t}_1} \to hh \to \tau^+\tau^+\tau^-\tau^-$ has been proposed. Since $Br(h \to \tau^+\tau^-) \simeq 8\%$ the 4 $\tau$ final state is also relatively rare ($Br < 6.4 \cdot 10^{-3}$). The real problem is that it is not clear how this final state is to be identified experimentally in a hadronic environment. A pair of isolated like–sign leptons would be a rather clean tag; assuming a 30% detection efficiency per lepton one could get more than 50 events/year for $m_{\tilde{t}_1} \leq 100$ GeV if the $hh$ final state dominates. The problem is that the presence of at least 4 neutrinos in the final state makes it impossible to reconstruct the $\sigma_{\tilde{t}_1}$ mass. Given that there are sizable backgrounds (e.g., $\sigma(pp \to ZZX \to \tau^+\tau^+\tau^-\tau^-) \simeq 3 \cdot 10^{-2}$ pb, leading to $\sim 300$ events/year) the identification of the 4 $\tau$ signal might be problematic.

Probably the most promising signal results from the decay $\sigma_{\tilde{t}_1} \to \gamma\gamma$. Searches for intermediate mass Higgs bosons will presumably require good electromagnetic energy resolution for SSC detectors, so the reconstruction of $m(\sigma_{\tilde{t}_1})$ should be rather straightforward. The signal would then be a bump in the $\gamma\gamma$ spectrum on top of the smooth background. The dashed curve in fig. 3 shows the signal cross section for the same parameters as in fig. 2. In ref. [9] the minimal detectable $H \to \gamma\gamma$ cross section has been estimated. The result after 1 year of running can roughly be parametrized as $\sigma_{\tilde{t}_1}^{\min} = 0.05 \text{pb} \cdot (m_H/100 \text{ GeV})^{-1.36}$, and is shown by the dotted curve in fig. 3. The same result obviously also applies for $\sigma_{\tilde{t}_1}$ searches, since the final state is identical. Except for the vicinity of the Higgs pole, where our calculation is not reliable anyway, our signal cross section is always above this minimum. Even though the assumptions of ref. [9] might be somewhat optimistic, it is clear that for most combinations of parameters $\sigma_{\tilde{t}_1}$ should be readily observable via its $\gamma\gamma$ decay already after one year of SSC running.
In summary, we have computed all potentially large branching ratios of a scalar stoponium bound state $\tilde{\sigma}_t$. The dominant decay modes are $gg$, $hh$ or, if $m(\tilde{\sigma}_t) \approx m_H$, $t\bar{t}$. The process $pp \rightarrow \sigma_t^1 X \rightarrow \gamma\gamma X$ should be readily observable at the SSC, provided that $\tilde{t}_1$ has no unsuppressed tree–level 2–body decays.

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Figure Captions

Fig.1 Branching ratios for annihilation decays of $\tilde{\sigma}_{t_1}$ listed in eqs.(4). The range of $m_{\tilde{t}_1}$ values shown results from varying $A_t$ between -310 and -70 GeV. For $m_{\tilde{t}_1} > 115$ GeV single stop decays open up; in this region annihilation decays into charginos and heavier neutralinos are also possible, but they remain small, of order of the $\tilde{Z}_1\tilde{Z}_{1,2}$ modes shown in the figure.

Fig.2 Branching ratios for annihilation decays of $\sigma_{t_1}$ listed in eqs.(4). The range of $m_{\tilde{t}_1}$ values shown results from varying $A_t$ between 440 and 1080 GeV. Unlike in fig. 1 we have increased the $SU(2)$ gaugino mass $M_2$ along with $m_{\tilde{t}_1}$ so that the single stop decays (5a,3b) remain kinematically forbidden.

Fig.3 Cross section for $\sigma_{t_1}$ production at the SSC. The solid line shows the total cross section, and the dashed curve the total cross section multiplied with $Br(\sigma_{t_1} \rightarrow \gamma\gamma)$. The dotted curve is a parametrization of the estimate of ref. [16] for the minimum $\sigma_{t_1} \rightarrow \gamma\gamma$ cross section detectable after 1 year of SSC running; we have extrapolated the result of ref. [16] into the region $m(\sigma_{t_1}) > 200$ GeV.