CRANKING IN ISOSPACE - TOWARDS A CONSISTENT MEAN-FIELD DESCRIPTION OF $N = Z$ NUCLEI

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Excitation spectra $\Delta E_T$ of $T = 0, 1, 2$ states in even-even (e-e) and odd-odd (o-o) $N = Z$ nuclei are analyzed within a mean-field based model involving isovector and isoscalar pairing interactions and the iso-cranking formalism applied to restore approximately isospin symmetry. It is shown that $T = 0$ states in o-o and $T = 1$ states in e-e nuclei correspond to two-quasiparticle, time-reversal symmetry breaking excitations since their angular momenta are $I \neq 0$. On the other hand the lowest $T = 2$ states in e-e and $T = 1$ states in o-o nuclei, which both are similar in structure to their even-even isobaric analogue states, are described as e-e type vacua excited (iso-cranked) in isospace. It appears that in all cases isoscalar pairing plays a crucial role in restoring the proper value of the inertia parameter in isospace i.e. $\Delta E_T$.

1. Introduction

The theoretical treatment of the generalized pairing problem is interesting and clearly nontrivial. Though its fundamentals in the form of generalized BCS (or HFB) theory were laid down almost thirty years ago in a number of papers by different groups [1] there are still many open problems. They cover a broad range of questions starting from the form of effective pn-pairing interaction and structure of effective pn-Cooper pairs, to problems related to symmetries and symmetry restoration, problems of interplay between the quasiparticle and isospin degrees of freedom to the role played by higher-order effects like quartetting or $\alpha$-type condensation. It includes also the fundamental question concerning the experimental fingerprints of pn-collectivity.

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Thus far, the strongest indications of \( pn \) (isoscalar) collectivity come from: (i) shifts of crossing frequencies in gsb of some e-e \( N = Z \) nuclei [the best studied case is \(^{72}\text{Kr}\)] and (ii) the mass defect in \( N \sim Z \) nuclei commonly known as the Wigner energy problem. The latter effect can be locally restored by enforcing collective isoscalar \( pn \)-pairing \([2]\). Based on such a model we aim here to look into the impact of collective \( pn \) correlations upon the structure of \( T = 0, 1, 2 \) excitations in \( N = Z \) nuclei. The goal is to reveal the interplay between quasiparticle and isospin excitations in \( N = Z \) nuclei and role played by isoscalar \( pn \)-collectivity. The present paper supplements our previous letters \([3, 4]\).

2. The paradigm of the Wigner energy

Traditional mass models based on the mean-field approach strongly underbind \( N \sim Z \) nuclei \([5, 6, 7]\). This additional binding energy which is known as the Wigner energy, is usually parametrized as \([8]\):

\[
E_W = W(A)|N-Z|+d(A)\delta_{NZ}\pi_{pn} \quad \text{where} \quad \pi_{pn} = \begin{cases} 
1 & \text{for odd-odd nuclei} \\
0 & \text{otherwise}
\end{cases}
\]

and \( W(A) \sim 47/A \, \text{MeV} \) \([7]\). A microscopic explanation of the Wigner energy within the mean-field approach is still lacking. The 'congruence' energy mechanism proposed in \([9]\) is in its nature a geometrical concept [i.e. independent on a specific form of nuclear interaction]. There one assumes an enhanced particle-hole (ph) interaction at the \( N \sim Z \) line which, however, is not at all present in traditional spherical Skyrme-HFB calculations, see Fig. 1. This is basically due to the pairing interaction which evenly distributes particles over spherical subshells thus strongly averaging over the properties of neighbouring nuclei. One would expect the effect to show up [and it indeed does] in deformed microscopic calculations. However, as shown in Fig. 1 for a particular set of SIII+BCS mass calculations of Ref. \([10]\) it is seen only in light nuclei and accounts for at most 20% of the empirical Wigner energy strength \( W(A) \). Even in the limit of single-particle (sp) Skyrme-HF calculations one can account for at most 30% of the empirical value.

It seems therefore, that a microscopic scenario based on isoscalar \( pn \)-pairing \([2]\) is the most promising so far. It requires [within the mean-field model] the isoscalar pairing to be on the average stronger than the isovector. Although there are clear empirical \([7]\) as well as theoretical \([11, 7, 12]\) arguments that the Wigner energy is indeed due to the isoscalar interaction it is not at all settled whether it is due to a static pairing effect. In spite of that, in the following we will enforce the strength of the isoscalar pairing...
Fig. 1. Empirical and theoretical values of $V_{pn} \approx \frac{2B}{N-Z}$ [for both spherical and deformed HFB calculations]. Solid symbols mark $V_{pn}(A)$ for e-e $N-Z = 2, 4$. In these cases $V_{pn}$ probes essentially the quadratic term in the nuclear symmetry energy $E_{sym} \sim (N-Z)^2$ and both empirical and theoretical curves do roughly overlay each other. For e-e $N = Z$ nuclei a strong enhancement of $V_{pn}$ is seen in the empirical data (◦). This apparent Wigner energy effect is not at all seen in the spherical calculations (∆) and only modestly visible in deformed calculations (⋄).

interaction so as to reproduce the Wigner energy and will look into the impact of these correlations on the structure of isobaric excitations in $N = Z$ nuclei.

3. The model

Our model Hamiltonian contains the Woods-Saxon $sp$ potential and a schematic pairing interaction including isovector ($t = 1$) and isoscalar
(t = 0) terms\(^1\) of the form:

\[ H_{\text{pair}} = -G^{t=1}_{\mu} P_{1\mu} P_{1\mu} - G^{t=0}_{00} P_{00} P_{00} \]  

(2)

where \( P_t^\dagger \) create isovector and isoscalar pairs in time-reversed orbits:

\[ P_{1\pm} = a_{\alpha\tau}^\dagger a_{\alpha\bar{\tau}}, \quad P_{10(00)} = \frac{1}{\sqrt{2}}(a_{\alpha n}^\dagger a_{\alpha p}^\dagger \pm a_{\alpha p}^\dagger a_{\alpha n}^\dagger). \]  

(3)

Note that our interaction does not ascribe any specific structure [e.g. deuteron like] to the \( pn \) pairs but only counts their effective number. The Bogoliubov transformation which is used [\( \alpha \) runs over \( sp \) states, \( \tau \) denotes third component of isospin, and \( k \) labels the quasiparticles]:

\[ \alpha_k^\dagger = \sum_{\alpha\tau > 0} (U_{\alpha\tau,k} a_{\alpha\tau}^\dagger + V_{\alpha\tau,k} a_{\alpha\bar{\tau}}^\dagger + U_{\bar{\alpha}\bar{\tau},k} a_{\bar{\alpha}\bar{\tau}}^\dagger + V_{\bar{\alpha}\bar{\tau},k} a_{\bar{\alpha}\tau}^\dagger) \]  

(4)

is the most general one, allowing for an unconstrained mixing of neutron and proton holes and particles. The problem is solved using the Lipkin-Nogami (LN) approximate number projection technique. In this respect the present model follows rather closely the description of Ref. [13]. The use of the LN model allows for mixing of both \( t = 1 \) and \( t = 0 \) phases at the cost of spontaneous isospin symmetry breaking [2, 13]. Generally, the mechanism of spontaneous symmetry breaking is the only mechanism which allows to take into account correlations which are seemingly beyond the mean-field. Obviously, the best example is the spontaneous breaking of spherical symmetry. Without it the mean-field would be capable to describe only a few nuclei. In our opinion the spontaneous breaking of isospin symmetry brings our solution closer to reality and allows for simulation of higher order effects like quartetting or \( \alpha \)-clustering. These effects are present in exact-model solutions [14] which always do mix \( t = 0 \) and \( t = 1 \) phases in contrast to the generalized BCS approximation [15, 16, 17]. Moreover, in the following we will approximately restore this symmetry by applying the isospin cranking mechanism to generate isospin \( T = 0, 1, 2 \) states in \( N = Z \) nuclei.

In the following calculations we will freeze the deformation degree of freedom of our Woods-Saxon potential at \( \beta_2 = 0.05 \). We compute \( G^{t=1}(A) \) using the average gap method of Ref. [18] and taking a symmetric cut-off including the lowest \( [A/2] \) neutron and \( [A/2] \) proton \( sp \) states. The strength of isoscalar \( G^{t=0}(A) \) pairing is computed by means of a direct fit to the Wigner energy strength \( W(A) \sim 47/A \text{ MeV} \), see [3] for details. This method assumes that \( W(A) \) is entirely due to the isoscalar pairing field.

\(^1\) Throughout the text small letter \( t \) is reserved to label the type of pairing correlations while capital \( T \) corresponds to the total nuclear isospin.
Since we use here an almost spherical mean-field this assumption seems to be consistent with the conclusions of the preceding section. Deformation effects are expected to result in a rather modest reduction of $G_{T=0}$. Let us mention that the same procedures and parameters are used systematically in all presented calculations and that they exactly follow Refs. [3, 4].

4. Isobaric excitations in $N = Z$ nuclei

The isoscalar pairing field naturally takes into account the mass-excess in $N = Z$ nuclei. The aim of this section is to show that it is also of vital importance to understand the excitation energy pattern, $\Delta E_T$, of the elementary isospin $T = 0, 1, 2$ excitations in even-even (e-e) and odd-odd (o-o) $N = Z$ nuclei. To compute these excitations, which follow a $\Delta E_T \sim T(T+1)$ pattern, we invoke the technique of cranking in isospace in analogy to spatial rotations. The discussion is organized as follows: (i) the pure single-particle model will be succeeded by (ii) the standard, isovector $nn-$ and $pp-$ paired model, and eventually (iii) by the isovector and isoscalar paired model.

4.1. The extreme single-particle model

Let us consider the isospin-cranked $sp$ model: $\hat{H}^{\omega_T} = \hat{H}_{sp} - \hbar \omega_T \hat{t}_x$. Let us further assume for simplicity that $\hat{H}_{sp}$ generates a fixed, equidistant spectrum of 4-fold degenerate levels [including isospin and Kramers degeneracy] $e_i = \delta e i$. The isospin-cranking removes the isospin degeneracy. The quartets of $sp$-states split into two pairs of Kramers degenerate rothians $|\pm\rangle = \frac{1}{\sqrt{2}}(|n\rangle \pm |p\rangle)$. The slope of the pair of $|+\rangle$ ($|-\rangle$) rothians is determined by their $sp$ alignment in iso-space $\langle \pm | \hat{t}_x | \pm \rangle = \pm 1/2$, respectively.

Fig. 2 shows the $sp$ rothians for the lowest $sp$ configurations in even-even (upper part) and odd-odd (lower part) $N = Z$ nuclei. For the case of the even-even vacuum [Fig. 2a] one obtains configuration changes at iso-frequencies: $\omega_T^{(c)} = \delta e, 3\delta e, \ldots, (2n-1)\delta e$. At each crossing a pair of upsloping rothians become empty and a pair of downsloping rothians become occupied. This reoccupation process gives rise to stepwise changes in iso-alignment in units of $\Delta T_x = 2$. Since simultaneously $T_y = T_z = 0$, one obtains a sequence of even-isospin states $T = 0, 2, 4, \ldots, 2n$. The odd-$T$ sequence of states in even-even $N = Z$ nuclei can therefore be reached only for excited states. The lowest particle-hole excitation leading to odd-$T$ states is shown in Fig. 2b. Indeed, the initial iso-alignment of this state is $T_x = 1$ and, as in the case of the e-e vacuum discussed above, it is changed in steps of $\Delta T_x = 2$ but at iso-frequencies $\omega_T^{(c)} = 2\delta e, 4\delta e, \ldots, 2n\delta e$. Simple calculations give the
Fig. 2. Single-particle routhians representative for the following cases: (a) e-e nucleus even-T, (b) e-e nucleus odd-T case, (c) o-o nucleus even-T case, and (d) o-o nucleus odd-T case. Filled circles mark occupied states. Arrows indicate configuration changes versus iso-frequency.

The following expression for the total energy

\[ E \equiv E^o + \omega_T T_x = \frac{1}{4} \delta e [1 - (-1)^T_x] + \frac{1}{2} \delta e T_x^2. \]  

(5)

Both odd and even-T bands have therefore the same inertia parameter [reciprocal of the moment of inertia \( a = \mathfrak{I}^{-1} \)] \( a = \delta e \) which is proportional to the average level spacing at the Fermi energy. The bands are shifted by \( \Delta E = E_{T=1} - E_{T=0} = \delta e \) i.e. by a \( ph \) excitation energy.

For the case of o-o nuclei the last quartet is only half-filled. Hence, two different \( sp \)-configurations can be obtained: (i) an aligned one of \( T_x=1 \) [Fig. 2d] and (ii) a non-aligned one of \( T_x=0 \) [Fig. 2c]. Since the reoccupation taking place at high iso-frequency always gives rise to a stepwise change in total isospin in units of \( \Delta T_x = 2 \) the aligned configuration gives rise to an odd-
T sequence of states while the non-aligned one builds up the even-T sequence of states. Observing further that crossings take place at $\hbar \omega_T^{(c)} = 2n\delta e$ [case (i)] and $\hbar \omega_T^{(c)} = (2n - 1)\delta e$ [case (ii)] it is straightforward to compute the total energy:

$$E = -\frac{1}{4}\delta e[1 - (-1)^T_x] + \frac{1}{2}\delta e T_x^2. \quad (6)$$

Two very interesting conclusions arise from this discussion. First of all, the decoupling effect is so strong in this case that it gives rise to a complete degeneracy of $T=0$ and $T=1$ states. This is in nice qualitative agreement with the data since $T=0$ and $T=1$ states are indeed nearly degenerate in o-o but not in e-e nuclei [19, 20]. Secondly, let us observe that the aligned configuration does not break time-reversal invariance while the non-aligned configuration does break it. Again this is in agreement with empirical data since all $T=0$ states have non-zero angular momentum $I \neq 0$ while $T=1$ states have $I = 0$. In other words the theoretical treatment of $T=0$ states requires the explicit breaking of time-reversal invariance. In the presence of pairing correlations it means that $T=0$ states should be treated as two-quasiparticle (2qp) excitations. Treating them on the same footing as the neighbouring e-e vacua would correspond to what is usually known in the literature as the filling approximation. Within the filling approximation pairs of $\alpha\alpha$ type are always accompanied [with the same occupation probability] by pairs of $\bar{\alpha}\bar{\alpha}$ type forming a time-reversal invariant many-body state.

Note also, that the same situation applies for the $T=1$ states in e-e nuclei. These states, within the $sp$-model are $ph$ excitations. Therefore, they do break time-reversal invariance and, in the presence of pairing correlations, correspond to 2qp configurations.

### 4.2. Influence of $T=1$ pairing

Let us investigate next the influence of standard $nn$ and $pp$ isovector pairing correlations on the $sp$ iso-alignment processes and iso-inertias discussed in the preceding subsection. Let us still consider the equidistant level model and assume that the pairing gaps $\Delta_{pp} = \Delta_{nn} = \Delta$ are constant as a function of $\hbar \omega_T$. In the gap-non-self-consistent regime the BCS equations can be solved analytically. The eigenenergies (positive) are $[\tilde{e}_\alpha \equiv e_\alpha - \lambda]$:

$$E_{\alpha,\pm} = \sqrt{[\tilde{e}_\alpha \pm \frac{1}{2}\hbar \omega_T]^2 + \Delta^2}, \quad (7)$$

$$E_{\alpha,\pm} = \sqrt{[\tilde{e}_\alpha \pm \frac{1}{2}\hbar \omega_T]^2 + \Delta^2}, \quad (7)$$
and the associated eigenvectors:

\[
\begin{bmatrix}
U_{\alpha,n} \\
U_{\alpha,p} \\
U_{\alpha,n} \\
U_{\alpha,p} \\
V_{\alpha,n} \\
V_{\alpha,n} \\
V_{\alpha,p} \\
V_{\alpha,p}
\end{bmatrix}
\rightarrow
\begin{bmatrix}
U_{\alpha}^{(+)} \\
-U_{\alpha}^{(+)} \\
0 \\
0 \\
V_{\alpha}^{(+)} \\
V_{\alpha}^{(+)} \\
0 \\
-\bar{V}_{\alpha}^{(+)}
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
U_{\alpha}^{(-)} \\
-U_{\alpha}^{(-)} \\
0 \\
V_{\alpha}^{(-)} \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix},
\tag{8}
\]

where:

\[
U_{\alpha}^{(\pm)} = \frac{1}{2}\sqrt{1 \pm \frac{\tilde{e}_{\alpha} \pm \frac{1}{2}h\omega_{r}}{E_{\alpha,\pm}}}
\quad \text{and} \quad
V_{\alpha}^{(\pm)} = \frac{1}{2}\sqrt{1 \mp \frac{\tilde{e}_{\alpha} \pm \frac{1}{2}h\omega_{r}}{E_{\alpha,\pm}}}
\tag{9}
\]

It is interesting to observe that the solutions of the \(sp\) model \(|\alpha;\pm\rangle\) form, in this case, the canonical basis. Indeed, see (8), the quasiparticle operators take the following structure:

\[
\alpha_{\alpha,\pm}^\dagger = \sqrt{2}U^{(\pm)}a_{\alpha,\pm}^\dagger + \sqrt{2}V^{(\pm)}\bar{a}_{\alpha,\mp}^\dagger \quad \text{where} \quad a_{\alpha,\pm}^\dagger = \frac{1}{\sqrt{2}}(a_{\alpha,n}^\dagger \pm a_{\alpha,p}^\dagger).
\tag{10}
\]

Let us further observe that this solution does also preserve iso-signature \([R_r = \exp^{-i\pi\tau_r}]\) as a self-consistent symmetry.

It is relatively simple to derive an analytical expression for the kinematical iso-moment of inertia \(\Im = T_z/\omega_r\). In case of the symmetric basis cut-off it reads:

\[
\Im = \frac{1}{2\omega_r} \sum_{i=1}^{A/4} \left\{ \frac{(2i - 1)\delta e + h\omega_r}{\sqrt{\frac{1}{4}(2i - 1)\delta e + h\omega_r}^2 + \Delta^2}} - \frac{(2i - 1)\delta e - h\omega_r}{\sqrt{\frac{1}{4}(2i - 1)\delta e - h\omega_r}^2 + \Delta^2}\right\}
\tag{11}
\]

In the low-frequency limit one obtains:

\[
\Im \sim \frac{1}{2\omega_r} \sum_{i=1}^{A/4} \frac{\Delta^2}{\frac{1}{4}(2i - 1)\delta e^2 + \Delta^2}^{3/2} \sim \frac{1}{\delta e} \left\{ \begin{array}{c}
0.80 \quad \text{for} \quad \Delta = \frac{1}{2}\delta e \\
0.96 \quad \text{for} \quad \Delta = \delta e \\
0.90 \quad \text{for} \quad \Delta = 2\delta e
\end{array} \right\}
\tag{12}
\]

i.e. \(\Im \sim \Im_{sp}\) almost independently [within reasonable range] on the magnitude of pairing correlations, see also Fig. 3 showing the numerical results of the exact formula (11). Taking into account the self-consistency of the pair
Fig. 3. Iso-alignment (a) and iso-MoI (b) calculated using Eq. (11) for very weak $\Delta = 0.001\delta_e$, weak $\Delta = 0.5\delta_e$, intermediate $\Delta = \delta_e$ and strong $\Delta = 1.5\delta_e$ pairing as a function of iso-frequency.

gap does not change this conclusion as it was demonstrated within the LN approximation in [3].

The isovector $nn$- and $pp$-pairing introduces a kind of collectivity on top of the discrete $sp$ solutions. In order to take into account quantum fluctuations in a similar way as for spatial rotations, the cranking constraint in our calculations corresponds to $T_x = \langle \hat{I}_x \rangle = \sqrt{T(T+1)}$.

Following the results of the $sp$ model we have calculated $T = 2$ states in e-e nuclei by means of iso-cranking the quasiparticle vacuum to $T_x = \sqrt{6}$. The results of the calculations are shown in Fig. 4. In these calculations we used the static $sp$ spectrum of the weakly $[\beta_2 = 0.05]$ deformed Woods-Saxon potential plus standard isovector $pp$— and $nn$—pairing interaction. The results of the calculations are far below the empirical data. It is rather obvious that this level of disagreement must be related to a physics mechanisms which is beyond our model. First of all the $ph$ isovector terms are
Fig. 4. Empirical (stars) and calculated (filled circles) excitation energies of $T = 2$ states in e-e $N = Z$. These calculations do include only $t = 1$ pairing correlations which give rise to large discrepancies between theory and experiment.

missing. At present we do not have any control over these effects. However it is very unlikely that they can fully cure the problem since the self-consistent mean-field models which do include the isovector $ph$ channel yield:

$$E_{\text{sym}}(A, T) \approx \frac{1}{2} a_{\text{sym}} \frac{T^2}{A} = \frac{1}{2} \left[ \frac{T^2}{T(T+1)} a_{\text{sym}} \right] \frac{T(T+1)}{A}$$

(13)

i.e. provide only $\frac{T^2}{T(T+1)}$ fraction of the empirical symmetry energy$^2$

$^2$ In fact, the empirical data in $N \sim Z$ nuclei are consistent with $T(T+\lambda)$ dependence for the symmetry energy where $\lambda \sim 1.25$ [19].
4.3. The influence of \( t=0 \) pairing

The condensate of \( pn \) Cooper pairs with isospins coupled antiparallel is expected to lower considerably the iso-moment-of-inertia (iso-MoI) in a similar fashion as nuclear superfluidity influences the spatial MoI. Furthermore, in response to iso-rotations, the isoscalar paired nucleus is expected to undergo a phase transition due to the iso-CAP (Coriolis Anti-Pairing effect) similar to the standard CAP effect which is so well documented in high-spin physics. The situation here is probably even simpler as compared to the case of spatial rotations. There, the phase transition is always heavily modified due to the strong dependence of CAP on the orbital angular momentum [Stephens-Simon effect [21]]. In isospace one would expect a phase transition to be of bulk type resembling much closer the Meissner effect known in macroscopic superconductors [22].

4.3.1. The \( T=2 \) states in even-even nuclei

To verify these ideas we have performed a set of microscopic calculations for \( T = 2 \) states in \( 8 < N = Z < 30 \) e-e nuclei using the LN model which includes both isovector and isoscalar pairing correlations. The \( T = 2 \) states were computed by iso-cranking the e-e vacuum to iso-spin \( T_x = \sqrt{6} \), see Fig. 5a. These calculations nicely reveal all features of \( t = 0 \) pairing expected from our intuitive considerations. The iso-MoI are indeed considerably lowered by \( t = 0 \) pairing. At large iso-frequencies, but systematically below \( T_x = \sqrt{6} \), we observe a phase transition from the \( t = 1 \) and \( t = 0 \) paired system to the \( t = 1 \) paired system. The transition is indeed sharp, indicating its bulk character although one can always argue that fluctuations can smear it out, as it is usually the case in finite many-body system. In spite of that one can rather safely state that \( T = 2 \) states in e-e \( N = Z \) nuclei are \( t = 0 \) unpaired [or at most vibrational]. This conclusion is in nice agreement with direct calculations of the ground-states in \( N - Z = 4 \) nuclei. These isobaric analogue states are predicted systematically to be \( pn \)-unpaired by various types of models [15, 16, 12]. The results of the \( \Delta E_{T=2} \) excitation energy calculations are summerized in Fig. 5c. Provided the simplicity of the model the agreement is rather amazing. Let us also bring the reader’s attention to certain details. For example the shell-structure, reflecting proportionality of the \( sp \) iso-inertia to the \( sp \) energy splitting at the Fermi surface, is quite clearly seen. Simultaneously, let us also note that these shell-structure effects are smeared out quite substantially by \( t = 0 \) correlations as compared to calculations which include only \( t = 1 \) pairing, see Fig 4. For more details showing the basic features of these calculations we refere the reader to [3].
4.3.2. The $T=1$ states in even-even nuclei

Let us now turn our attention to $T = 1$ states in e-e nuclei. Guided by the $sp-$model we will treat these states as the lowest 2$qp$ configurations. We block the two lowest quasiparticles in the original Woods-Saxon basis by exchanging:

$$
\begin{pmatrix}
U_K \\
V_K
\end{pmatrix}
\rightarrow
\begin{pmatrix}
V_K^* \\
U_K^*
\end{pmatrix}
$$

(14)

for $[K = 1, 2]$ while solving the HFB(LN) equations. Since at iso-frequency $\omega_\tau = 0$ the lowest 2$qp$ state carries zero iso-alignment we impose the iso-cranking condition $T_x = 1$, as shown in Fig. 5a.

At $\omega_\tau = 0$ the 2$qp$ state mixes both $t = 0$ and $t = 1$ pairing phases [both in BCS as well as BCSLN approximations]. However, different to the e-e iso-ground-state-band, subsequent iso-cranking of the 2$qp$ state does not
affect strongly isoscalar but quenches isovector pairing correlations. The $pp$ and $nn$–pairing correlations disappear completely, exactly when the iso-alignment reaches unity. At this point the system becomes trapped and the state of the nucleus does not change till the very high frequency – high enough to destroy isoscalar correlations. Note that, Fig. 5c, the excitation energies of these states $\Delta E_{T=1}$ again agree surprisingly well with empirical data.

Certain clues explaining this seemingly counterintuitive picture can be obtained by going to the canonical basis in which the the density matrix is diagonal. One observes that:

1. For mixed-phases solution [below $T_x = 1$] the blocking of the original Woods-Saxon states is not equivalent to the blocking of the canonical states i.e. all canonical quasiparticles have fractional occupation numbers.

2. With increasing frequency the occupation probability of the two lowest canonical quasiparticles increases gradually and, at $T_x = 1$, reaches exactly unity. In this state one pair decouples from the purely $t = 0$ $pn$–paired core and the isospins are aligned along cranking axis.

3. At high frequency these canonical quasiparticles are built either on symmetric $|+\rangle \sim |n\rangle + |p\rangle$ or asymmetric $|−\rangle \sim |n\rangle − |p\rangle$ combinations of the initial Woods-Saxon $sp$ states.

These observations are illustrated in Fig. 6. The upper panel of the figure clearly shows that the e-e core is inert. Indeed, the entire iso-alignment can be traced back to four canonical quasiparticles emerging from the degenerate [at $\omega_\tau = 0$] quartet of $sp$ states. At low frequencies all quasiparticles forming this quartet contribute to the iso-alignment i.e. they all have fractional occupations, see open and black dots in the lower part of Fig. 6. With increasing $\omega_\tau$, the occupation probability of the energetically favored pair of canonical quasiparticles increase until they reach unity while the energetically unfavored pair becomes empty. Simultaneously, the iso-alignments of the $sp$ canonical states building these quasiparticles approach exactly $\pm 1/2$ i.e. they are built on $\sim |n\rangle \pm |p\rangle$ combinations of the original basis, see the discussion in sect. 4.1. Clearly, the process of iso-cranking restores the isospin symmetry [$T_x=1$] of the blocked 2qp state and, in fact, the symmetry of the whole system which is composed by a $t = 0$ paired o–o core and a single, properly coupled pair of quasiparticles.

Let us further note that the $t = 1$ and $t = 0$ pairing correspond to an entirely different scattering processes. This becomes evident after transforming the pair operators to $|\pm\rangle$ basis:

$$P^\dagger_{n\bar{n}} + P^\dagger_{p\bar{p}} \rightarrow a^+_+a^+_+ + a^+_+a^+_− \quad \text{and} \quad P^\dagger_{t=0} \rightarrow a^+_+a^+_− - a^+_−a^+_−. \quad (15)$$
Fig. 6. Contributions of the core and the quartet of canonical quasiparticles to the total iso-alignment $T_x$ for the case of an e-e nucleus with isospin $T = 1$. The lower part illustrates the contributions of the individual canonical $sp$ and $qp$ states to the total iso-alignment as a function of $\omega_\tau$. See text for details.

Since with increasing $\omega_\tau$ the canonical basis approach $|\pm\rangle$ states, the contribution to the $t = 1$ pair field from the lowest $qp$ states is $\propto U_+ V_+^* \rightarrow 0$. In other words, with increasing $\omega_\tau$ blocking is more and more effective in the $t = 1$ channel. Simultaneously, contributions of the blocked $qp$ state to the $t = 0$ pair field are: $\propto U_- V_-^* + U_+ V_+^* \rightarrow 1$. It means that $t = 0$ pairing is rather stable with increasing $\omega_\tau$. Only at very high $\omega_\tau$, when the cranking energy will overcome the coherence of $t = 0$ pairing, the phase transition to the $t = 1$ paired system will take place [see discussion in subsect. 4.3.1].

4.3.3. The $T=0$ and $T=1$ states in odd-odd nuclei

Let us finally discuss briefly $N = Z$ o-o nuclei. The lowest $T = 0$ state ($T = 0$ ground state) in o-o nuclei cannot be treated on the same footing
as the ground state of the neighbouring $N = Z$ e-e nuclei, see also [23].
The major argument stems from the simple empirical fact that all $T = 0$
ground-states in o-o nuclei have $I \neq 0$. Therefore, their treatment requires
the explicit breaking of time-reversal symmetry as it was already discussed
within the $sp-$model. This can be achieved only by treating this state as
a $2qp$ configuration. In contrast, the lowest $T = 1$ states in o-o nuclei are
expected to be seniority-zero states. The basic argument comes from the
isobaric symmetry. These $I = 0$ states form a triplet of isobaric analogue
states together with the ground states of e-e $N - Z = \pm 2$ nuclei. Hence, they
all should have similar structure. A simple calculation scheme [see Fig. 5b]
emerges from these considerations: (i) The $T = 0$ states should be treated
as $2qp$ configurations i.e. seniority-two states typical for all o-o nuclei. (ii)
The $T = 1$ states are HFB(LN) e-e like vaccua [false vaccua] excited in
isospace by means of iso-cranking. Therefore they are seniority-zero states
similar to e-e nuclei.

The relative excitation energy $\Delta E = E_{T=1} - E_{T=0}$ resulting from our
calculations is shown in Fig. 5c. Since $2qp$ excitations are almost as costly
in energy as iso-cranking to $T = 1$, therefore, it is not surprising that both
states stay nearly degenerate. What surprises is that the model predicts not
only the near-degeneracy but also certain details like the inversion of $T = 1$
and $T = 0$ excitations taking place around $f_{7/2}$ ($A \sim 40$) sub-shell nuclei.
Let us mention that in our approach the isoscalar and isovector phases are
present both in $T = 1$ and $T = 0$ states. It is therefore evident that the
near-degeneracy of $T = 1$ and $T = 0$ states cannot be used as an argument
to rule out isoscalar pairing as it was done by the Berkeley group [24]. The
key lays in the understanding of the underlying structure, which can be
achieved by means of a microscopic model only.

5. Summary

We have shown that mean-field based models which incorporate both
t = 0 and t = 1 pairing correlations and, at least in approximate manner,
number- and isospin projection are in principle capable to treat consistently
$N \sim Z$ nuclei. The number projection is treated here at the level of the
Lipkin-Nogami approximation [25, 13] while isospin is restored using the
isospin cranking formalism [26, 3]. Within the model the e-e vacuum is a
mixed $t = 0$ and $t = 1$ phase state due to the spontaneous symmetry breaking
introduced by number projection [2, 13]. The $T = 2$ state in e-e nuclei is
an e-e vacuum calculated at the iso-cranking frequency $\omega_T$ corresponding
to $T_x = \sqrt{6}$ while $T = 1$ state in e-e nuclei is described as a $2qp$ state at
which $T_x = 1$. The $T = 0$ states in o-o nuclei are 2qp excitations while
$T = 1$ states in o-o nuclei are e-e-like vaccua [false vaccua] computed at $\omega_T$
corresponding to $T_x = \sqrt{2}$. In all cases $t = 0$ superfluidity plays a crucial role in restoring the correct iso-MoI and in turn, the excitation energies $\Delta E_T$. In fact, thanks to the simplicity of the iso-cranking approximation, both the role and response of the $t = 0$ phase against iso-rotations can be very simply and intuitively understood by a number of beautiful analogies to well studied phenomena of high-spin physics.

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