Experimental investigation of a five-spring vibration isolator

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Abstract. In order to suppress low frequency vibration more effectively, a quasi-zero stiffness vibration isolation device is designed by using five parallel linear springs. The stiffness characteristic of the system is analyzed, and the quasi-zero stiffness (QZS) condition of the system is given. The dynamic behavior analysis of the system is analyzed by harmonic balance method, and the force transmissibility is obtained. The effects of damping ratio and excitation force on the system transfer rate are analyzed. In order to verify the vibration isolation performance of QZS vibration system, a series of experiments were carried out. The experimental results show that the low frequency vibration isolation performance of the QZS vibration system is much better than that of the corresponding linear system.

1. Introduction
The traditional method of vibration isolation is to reduce the vibration by reducing the natural frequency of the system, but reducing the natural frequency of the system means reducing the rigidity of the system and reducing the load capacity of the system. In order to solve this contradiction, many scholars have put forward a vibration isolation system with QZS characteristics through the combination of positive and negative stiffness. Under small vibration, the system has the characteristics of high static stiffness and low dynamic stiffness, which ensures that the natural frequency of the system is small. The theory and application fields of quasi-zero stiffness vibration isolation system are introduced in reference [1]. In reference [2-10], the steady-state response of a quasi-zero stiffness vibration isolation system is analyzed. Robertson et al. [11] used the concept of high static stiffness and low dynamic stiffness to achieve low frequency vibration isolation by using magnet spring. Platus [12] designed a QZS structure by using longitudinal bending beams and linear springs which exhibit negative stiffness under axial loads.

Existing research work is mainly about the motion mechanism and theoretical analysis of QZS vibration system. Few papers involve experimental research in this field. In order to evaluate its actual isolation performance, a simple quasi-zero stiffness isolation device is designed by using five linear springs in parallel, and a series of experiments are carried out. Firstly, the model is analyzed theoretically to show its static and dynamic characteristics. Then through experimental investigation, the actual isolation performance of QZS model and its corresponding linear system under harmonic excitation is compared, and the actual isolation effect is evaluated. The experimental results show that
the five-spring vibration isolation system has lower natural frequencies than the equivalent linear system, and can exert good vibration isolation performance at the balanced position.

2. Structural design
The structure of the five-spring vibration isolation system is shown in Fig. 1. The vertical spring provides positive stiffness and a certain load carrying capacity for the system. Two pillars are fixed on the support plate. The oblique springs are hinged to the strut and the other end are hinged to the connector. The four oblique springs act as a negative stiffness structure, which acts to counteract the positive stiffness provided by the vertical spring while preventing the intermediate load bearing platform from rotating. The guide rod can slide freely in the hole of the connecting rod. The load bearing platform is fixed at the top of the connecting rod for mounting the vibration isolation mass.

Fig. 1. The structure of the five-spring vibration isolation system.
1. load bearing platform; 2. connecting rod; 3. oblique spring; 4. guide rod; 5. vertical spring; 6. connector; 7. pillar

3. Theoretical analysis

3.1. Static analysis
The horizontal position is taken as the initial position and the vertical downward direction is taken as the positive direction. The original length of the oblique spring is \( L \). The length of the oblique spring at the static equilibrium position is \( l \). The stiffness of four oblique springs is both \( K \). The stiffness of the vertical spring is \( K_v \). The damping coefficient of the system damping is \( C \). \( x \) is the displacement of the intermediate mass \( M \) from the static equilibrium position in the vertical direction.

The shape variable of the oblique spring is:

\[
\Delta \delta_b = \sqrt{x^2 + l^2} - L
\]  

(1)

The shape variable of the vertical spring is:

\[
\Delta \delta_v = \frac{(\sqrt{x^2 + l^2} - L)x}{\sqrt{x^2 + l^2}}
\]  

(2)

The force-displacement characteristics can be expressed as:

\[
f = K_v x + 4K_b \left[ \frac{\left( \sqrt{x^2 + l^2} - L \right)x}{\sqrt{x^2 + l^2}} \right]
\]  

(3)

The stiffness-displacement characteristics can be expressed as follow:
Introducing the non-dimensional parameters \[ \hat{F} = f / (K_v x), \quad \hat{x} = x / x_s, \quad \hat{K} = K_h / K_v. \]

Equation (3) and equation (4) become:

\[
\hat{F} = \hat{x} + 4\hat{k} \left( 1 - \frac{1}{\sqrt{\hat{x}^2 (1 - \hat{l}^2) + \hat{l}^2}} \right) \hat{x}
\]

\[
\hat{K} = (1 + 4\hat{k}) - \frac{4\hat{k}\hat{l}^2}{(\hat{x}^2 (1 - \hat{l}^2) + \hat{l}^2)^{\frac{3}{2}}}
\]

The quasi-zero stiffness condition of the system can be obtained by making the stiffness of the system zero at the static equilibrium position. Inserting \( \hat{x} = 0 \) and \( \hat{K} = 0 \) into equation(6), then \( \hat{l} \) should be satisfied that:

\[
\hat{l}_{qs} = \frac{4\hat{k}}{4\hat{k} + 1}
\]

The dimensionless force-displacement and stiffness-displacement curves of the system when \( \hat{k} = 0.5 \) are shown in Fig. 2 and 3. As can be seen from Fig. 2, when \( \hat{l} > \hat{l}_0 \), the slope of the dimensionless stiffness-displacement curve is always greater than zero. When \( \hat{l} = \hat{l}_0 = \frac{2}{3} \approx 0.667 \), the curve is very stable near the static equilibrium position and the slope is almost zero. At this time, the system exhibits quasi-zero stiffness characteristics. When \( \hat{l} < \hat{l}_0 \), the slope of the system curve near the static equilibrium position is negative, and the system shows negative stiffness characteristic.

![Fig. 2. The non-dimensional force-displacement curves for some values when \( \hat{k} = 0.5 \)](image-url)
As can be seen from Fig. 3, when \( \hat{l} > \hat{l}_0 \), the curves are above the abscissa. Now the system always exhibits positive stiffness characteristics. The positive stiffness near the static equilibrium position is the smallest, and the farther away from the equilibrium position, the greater the positive stiffness. When \( \hat{l} < \hat{l}_0 \), if \( \hat{l} \) is smaller, the negative stiffness characteristics are more obvious. Therefore, \( \hat{l} \) is an important parameter affecting the negative stiffness characteristics of the system.

![Fig. 3. The non-dimensional stiffness-displacement curves for some values when \( \hat{k} = 0.5 \)](image)

Equation (5) is expanded with the Taylor formula at \( \hat{x} = 0 \)

\[
f_{\text{app}} = \left( \frac{1-\hat{l}^2}{\hat{l}^3} \hat{k} \right) \hat{x}^3 + \left( 1 - \frac{4\hat{k}(1-\hat{l})}{\hat{l}} \right) \hat{x} = \gamma \hat{x}^3 + a \hat{x}
\]

(8)

Where \( \gamma = \frac{1-\hat{l}^2}{\hat{l}^3} \hat{k} \), \( a = 1 - \frac{4\hat{k}(1-\hat{l})}{\hat{l}} \)

The non-dimensional stiffness is obtained by differentiating equation (8)

\[
k_{\text{app}} = \left( \frac{\hat{k}(1-\hat{l}^2)}{\hat{l}^3} \right) 3\hat{x}^2 + \left( 1 - \frac{4\hat{k}(1-\hat{l})}{\hat{l}} \right)
\]

(9)

Inserting \( \hat{k} = 0.5 \) and \( \hat{l} = \hat{l}_0 = \frac{2}{3} \) into equation (8) and equation (9)

\[
f_{\text{app}} = \frac{15}{8} \hat{x}^3, \quad k_{\text{app}} = \frac{45}{8} \hat{x}^2
\]

(10)

The comparison between the exact expression curve and the third-order approximation curve is shown in Fig.4 and Fig.5. It can be seen that the closer to the static equilibrium point of the system, the more consistent the third-order approximate expression curve with the exact curve. Therefore, in the small amplitude range near the equilibrium position, the approximate expression can be used instead of its exact expression.
Fig. 4. Comparison between exact and approximate expressions of non-dimensional force

Fig. 5. Comparison between exact and approximate expressions of non-dimensional force

3.2. Dynamic analysis

From the previous analysis, it can be seen that the stiffness of QZS vibration system is very small near its static equilibrium position. In order to ensure that the oblique spring is compressed to the horizontal position when the vibration isolation equipment is placed on the QZS vibration system. The static load is completely supported by the compressed vertical spring. Then the equipment mass $m$ must satisfy the conditions:

$$m = \frac{K_x x}{g}$$  \hspace{1cm} (11)

Fig. 6. System dynamical model.
The dynamical model of the system under external excitation is shown in Fig. 6, considering the effect of damp $c$, the QZS vibration system is just in the static equilibrium position due to the mass $m$. Under the action of harmonic excitation $f = F \cos \omega t$, the system vibrates slightly near its equilibrium position. If the elastic restoring force is replaced by the third-order approximation, the system's dimensionless equation of motion can be as follow:

$$\ddot{y} + 2\xi \dot{y} + r y^3 = \hat{F} \cos \Omega \tau$$

(12)

Where $\tau = \omega_0 t$, $\omega_0^2 = \frac{k_h}{m}$, $\Omega = \frac{\omega}{\omega_0}$, $\xi = \frac{c\omega_0}{2K_v}$, $\hat{F} = \frac{F}{K_v L}$, $\hat{y} = \frac{y}{L}$.

Applying the harmonic balance method to solve equation (12), ignoring the higher-order harmonic terms, the periodic solution of the system can be sent to:

$$\hat{y} (\tau) = \hat{y} (\tau + T) = A \cos (\Omega \tau + \theta)$$

(13)

where $\theta$ is the response phase of the system, $A$ is the response amplitude of the system.

The amplitude-frequency characteristic equation of the system is obtained by substituting equation (13) into equation (12):

$$\left( \frac{3}{4} y A^3 - A \Omega^2 \right)^2 + 4(\xi \Omega A)^2 = \hat{F}^2$$

(14)

In general, the vibration isolation of the system is evaluated by the transmission rate. Forces transmitted to the foundation through the QZS system get:

$$\hat{F}_t = \sqrt{\left(2\xi \omega A\right)^2 + \left(\frac{3}{4} y A^3\right)^2}$$

(15)

Thus the force transmissibility is given,

$$T = \frac{\sqrt{\left(2\xi \omega A\right)^2 + \left(\frac{3}{4} y A^3\right)^2}}{\hat{F}}$$

(16)

It can be seen from equation (16) that, unlike linear systems, the force transmissibility of QZS system is affected not only by the damping ratio, but also by the change of excitation force. This is because the response amplitude $A$ determined by equation (14) is not linear with the change of excitation force.

When the system parameters $\xi = 0.2$, $\gamma = 1$, the force transmissibility for several values of the damping ratio $F$ are plotted in Fig. 7. It can be seen that with the increase of excitation amplitude, the peak value of transmission rate and resonance frequency increase, and the range of low frequency vibration isolation of the system decreases. When the excitation frequency is large, the curve is close to coincide, and the effect of force transfer rate tends to be consistent. It can be also found from Fig. 7 that compared with the equivalent linear system, the transmission peak value of the QZS system is smaller, the initial isolation frequency is lower, the isolation range is larger, and the isolation performance is better.
Fig. 7. Force transmissibility curve of the system when parameter $F$ is varied

When the system parameters $F = 0.5$, $\gamma = 1$, the force transmissibility for several values of the damping ratio $\xi$ are plotted in Fig. 8. It can be seen that the effect of the damping ratio on the force transfer rate is obvious. When the damping ratio is small, the jumping phenomenon is obvious. As the damping ratio increases, the peak transfer rate and the resonant frequency decrease. The vibration isolation effect of the system in the low frequency band is gradually enhanced, and the vibration isolation effect is gradually weakened in the high frequency band. When increased to a certain extent, the system exhibits a linear characteristic and the jump phenomenon disappears. Compared with the equivalent linear system, the QZS vibration isolation system has a smaller peak transfer rate, a lower initial vibration isolation frequency, and a larger vibration isolation range.

Fig. 8. Force transmissibility curve of the system when parameter $\xi$ is varied

4. Experiments
In order to evaluate the vibration isolation performance of the designed QZS vibration isolation system, a series of experimental work was carried out. The Experimental device of the QZS system is shown...
in Fig.9. When the added mass on the bearing platform is 2.5KG, the oblique springs are in a horizontal state.

Fig. 9. The Experimental device

The natural frequency of vibration isolation system is measured by hammering method. The experimental data are shown in Fig.10-11. By observing its spectrum, we can know that the natural frequency of the equivalent linear system under 2.5KG counterweight is 2.747Hz. The natural frequency of the five-spring vibration system under 2.5KG counterweight is 0.6104Hz, which is 2.1Hz lower than that of the equivalent linear system. It can be concluded that the five-spring vibration system has lower natural frequency and better isolation effect than the equivalent linear system.

Fig. 10. The experimental data of the equivalent linear system under 2.5KG
When the counterweight is 5KG, the load platform is offset downward by 5 mm with respect to the static equilibrium position. The experimental data are shown in Fig. 12-13. The natural frequency of the equivalent linear system is 2.3197Hz. The natural frequency of the five-springs vibration system is 2.655Hz. It can be explained that, deviating from the static equilibrium position, the natural frequency of the five-spring vibration isolation system will increase significantly, and the original excellent vibration isolation performance will be lost.

**Fig. 11.** The experimental data of the five-spring vibration system under 2.5KG

**Fig. 12.** The experimental data of the equivalent linear system under 5KG
Fig. 13. The experimental data of the five-spring vibration system under 5KG

When no counterweight is added, the actual measured position deviates from 7 mm upward relative to the static equilibrium position. The experimental data are shown in Fig. 14-15. The natural frequency of the equivalent linear system is 3.17Hz. The natural frequency of the five-spring vibration system is 3.29Hz. Therefore, the three-spring vibration isolation system has the lowest natural frequency when operating in the equilibrium position. The distance from the equilibrium position is greater, the natural frequency of the system will be greater, and the range of vibration isolation will be narrower.

Fig. 14. The experimental data of the equivalent linear system with no counterweight
5. Conclusions
In this paper, a QZS vibration isolation device consisting of five linear springs in parallel is proposed, and the zero stiffness condition at the static equilibrium position is derived. The force transfer rate expression is obtained by the harmonic balance method. The theoretical analysis shows that with the increase of damping and the decrease of excitation amplitude, the peak value of transfer rate resonance decreases gradually. The vibration isolation characteristics of QZS system are evaluated through experimental study. The results show that the five-spring vibration isolation system has lower natural frequencies than the equivalent linear system, and can achieve good vibration isolation performance.

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