Finite element based parametric study of elastic-plastic contact of fractal surfaces

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Abstract

The present investigation utilizes the commercial finite element software ANSYS to study the effect of material properties such as yield strength (Y), modulus of elasticity (E) on the contact parameters like contact area, normal displacement and mean contact pressure in both perfect slip and full stick contact of fractal surfaces. The results indicate the nonlinear relationship among the contact parameters that are affected by the yield strength and the contact conditions; however the same E/Y ratio yields identical contact parameters. Load carrying capacity in full stick contact condition is slightly higher than that in perfect slip contact condition. The results are validated with experimental ones and also compared with single asperity finite element results.

Keywords: Fractal surfaces; Yield strength; Elastic modulus; Elastic-plastic contact; ANSYS

1. Introduction

The research on contact parameters begins with the pioneering work of Hertz [1] who provided the contact analysis of two elastic solids with geometries defined by quadratic surfaces in frictionless condition. Later several authors have made attempt to continue the contact analysis in the elastic-plastic and full plastic ranges. Kogut and Etsion [2], Jackson and Green [3] effectively utilized commercial finite element software ANSYS for the accurate calculation of contact parameters in elastic-plastic as well as plastic contact analysis of a deformable sphere against a rigid flat. Several authors [4-7] then investigated the effect of material properties, contact conditions on the

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Nomenclature

- A: real contact area
- A₀: nominal contact area
- D: fractal dimension
- E: elastic modulus
- G: fractal roughness
- L: sample length
- M: number of superimposed ridges
- n: frequency index
- p: contact pressure
- P: contact load
- x, y: coordinates of point
- Y: yield strength
- z: surface height
- δ: displacement
- γ: scaling parameter
- ϕ: random phase
- ν: Poisson’s ratio

...interfacial parameters during the elastic-plastic contact of a deformable sphere against a rigid flat using ANSYS to make an idea about the tribological properties of the two solid bodies in contact. Greenwood and Williamson [8] proposed an asperity based elastic model assuming a Gaussian distribution of asperity height using Hertz theory. Some elastic-plastic models [9-11] have also been developed based on Greenwood and Williamson assumption. But the characterizations of the statistical rough surfaces are made through the statistical parameters like variances of height, the slope and the curvature although the variance of slope and curvature depends strongly on the resolution of the roughness-measuring instrument or any other forms of filter and hence is not unique [12]. Thus the rough surface should be characterized in a way such that the structural information of roughness of all scales is retained which make it logical characterization to use in contact theory. Several authors [13-15] made it established that such a behavior can be characterized by fractal geometry.

In variety of surfaces, appropriate magnification shows magnified image very similar to the original surface. The analytical fractal models [13-15] ignored the interactions between neighboring asperities and considered small deformation. Finite element analysis is robust enough to consider interaction between asperities as well as bulk deformation. Komvopoulos and Ye [17, 18] performed finite element analysis of head-disc layered interface with fractal topography. Hyun et al. [19] considered a fully three-dimensional finite element analysis for elastic contact between rough surfaces with a range of self-affine fractal scale behavior. Pei et al. [20] extended the findings of elastic contact [19] to study the influence of elastic properties of the material in the elastic-plastic phase considering a wide range of self-affine surface topographies. In the last decade, Sahoo and Ghosh [21] have studied the effect of variation of fractal parameters on contact behavior involving area, load and displacement by means of commercial finite element software ANSYS. Then Sahoo et al. [22] have investigated the effect of strain hardening on the interfacial parameters in the contact of self-affine fractal surfaces under perfect slip contact condition. Recently Buczkowski and Kleiber [23], Pohrt and Popov [24] have undertaken the study of contact stiffness with fractal surfaces for elastic-plastic and elastic contact respectively. But all these simulations [17-24] considered only the perfect slip contact condition.

The motivation for the present study comes from the results found in deformable sphere against the rigid flat with the variation of material yield strength (Y) [3] and the unavailability of elastic-plastic contact analysis of fractal surfaces under full stick contact condition in the literature. In this paper, the elastic-plastic contact analysis of fractal surfaces against the rigid flat is performed to study the influence of yield strength (Y), modulus of elasticity (E) and the surface properties of the fractal surface in both perfect slip and full stick contact conditions.
Commercial finite element software ANSYS 11.0 is the base of our simulation where three-dimensional rough surfaces are generated following a modified two-variable Weierstrass-Mandelbrot function [25].

2. Fractal surface modeling

Repeated magnifications of a rough surface unveil increasing details of roughness even right down to nanoscales. A realistic multiscale roughness description can only be done using scale-independent fractal parameters. A 3D fractal surface topography has been generated with the help of modified (truncated) two-variable Weierstrass-Mandelbrot function [25], expressed in the following form

$$z(x, y) = L \left( \frac{G}{L} \right)^{(\beta-3)} \left( \ln \frac{y}{M} \right) \frac{L}{M} \sum_{n=1}^{N_{\max}} \sum_{m=0}^{M-1} \left( \frac{2\pi}{L} \right)^{2} \left( x \cos \left( \frac{2\pi}{\beta} \frac{x^{2} + y^{2}}{L} \right) \cos \left( \frac{2\pi}{\beta} \frac{y}{L} \right) + \phi_{n,m} \right)$$

Here $z(x, y)$ is surface function, $L$ is the sample length; $G$ is the fractal roughness, $D$ represents the fractal dimension ($2<D<3$), $\gamma (\gamma>1)$ and $M$ are used to express the scaling parameter and the number of superimposed ridges applied during the construction of the rough surface. The term “n” describes the frequency index. The upper limit of frequency index, $n_{\max} = \text{int} \left[ \log \left( \frac{L_{s}}{L} \right) / \log \frac{L}{L_{s}} \right]$, where $L_{s}$ is the cut-off length and $\phi_{n,m}$ denotes the random phase. The scaling parameter $\gamma$ controls the density of frequencies in the surface profile. $\gamma=1.5$ [25] provides both the phase randomization and spectral density based on surface flatness and frequency distribution density. The frequency index, $n$, has a lower value of zero for a truncated series of the height function. Thus the scaling property is approximate, i.e. scaling is satisfied within a small additive term [26]. Hence the surface function expressed by equation (1) possesses a scale invariant (fractal) behavior [27] only within a finite range of length scales, outside of that, the surface topography can be represented by a deterministic function. The smallest wavelength corresponds to the cut-off length $L_{s}$, which is of the order of about six lattice distances. The fractal roughness $G$ is a height scaling parameter independent of frequency within the scale range, wherein fractal power law behavior exists. Physically, higher $G$ values signify rougher (less dense) surface topography. The magnitude of the fractal dimension $D$ describes the contribution of high and low frequency components in the surface function $z(x, y)$. Thus, high values of $D$ indicate that high-frequency components are more dominant than low-frequency components in the surface topography profile. The physical significance of $D$ is the extent of space occupied by the rough surface, i.e. larger $D$ values denote denser profile (smoother topography). The surface height function provided by the equation (1) is continuous, non-differentiable, scale-invariant within the range determined by the upper and lower wavelengths used in the truncated series and self-affine asymptotically according to the analysis of Blackmore and Zhou [28]. The self-affinity indicates that as the surface is repeatedly magnified, more and more surface features appear and the magnified image shows a close resemblance to that of the original surface obtained at different scale. These properties make the function given by equation (1) suitable for construction of rough surfaces possessing topographies closely resembling the actual surfaces with the same fractal parameters $D$ and $G$.

3. Finite element modeling

Commercial finite element software ANSYS 11.0 is used to simulate the finite element contact analysis. The surface heights $z(x, y)$, generated in MATLAB from equation (1) as per the supplied x and y values, are imported to ANSYS as keypoints. Connection of those keypoints in an ordered manner generates the surface; finally the surface is made solid to use as a 3-D model for the contact analysis. The upper surface of the solid is identified as the CONTACT surface; whereas a rigid surface is made to just touch the contact surface from the top. This rigid top surface is set as the TARGET surface. The rough deformable solid body is meshed with 3D solid elements SOLID185. TARGE 170 represents the 3D ‘target’ surface for the associated contact element (CONTA174). The contact elements themselves overlie the solid elements describing the boundary of a deformable body and are potentially in contact with the target surface, defined by TARGE 170. In order to ensure that the results are accurate within the conceptual framework used to analyze the present problem, mesh convergence must be
satisfied. The mesh density is iteratively increased until the contact force and contact area differ by less than 1% between iterations. Depending on the fractal parameters and expected region of contact, the number of elements in resulting mesh varies. The resulting mesh consists of at least 17496 SOLID185, 2916 CONTA174, and 10 TARGE170 elements. The figure 1 shows a meshed model of a fractal surface with \( D=2.4 \), \( G=1.36 \times 10^{-10} \)m. Each side of the cube is taken as 1\( \mu \)m. The base nodes of the deformable solids are restricted to move in all the x, y, and z directions and the target surface is allowed to move in the z direction only. In our simulation, a force is applied through a pilot node, which controls the movement of the entire target surface, to move incrementally downward and the interferences of the rigid plane are found from the nodal solution. The contact with rigid plane is modeled with surface-to-surface contact elements that use the Augmented Lagrangian contact algorithm. The static large deformation analysis is performed with 100 sub steps. Computation took 3-4 hrs for each solution in a 1.6 GHz PC. The stick contact condition prevents the contact point of the deformed surface with the rigid flat from the relative displacement in the radial direction.

![Fig.1. Meshed model of a fractal surface with \( D=2.4 \), \( G=1.36 \times 10^{-10} \)m.](image)

4. Contact simulation and comparison problems

First we have performed a set of simulations with the surface roughness input parameters and the material properties available in the literature to validate the capacity and accuracy of the present model. The contact load (\( P^* \)), contact area (\( A^* \)), displacement (\( L^* \)) and mean contact pressure (\( p^* \)) are normalized as follows:

\[
P^* = \frac{P}{E^* A_0}, \quad A^* = \frac{A}{A_0}, \quad L^* = \frac{\delta}{L}, \quad p^* = \frac{P}{AY}
\]

Where \( P \) is the contact load, \( A \) is the real contact area; \( \delta \) is the interference or displacement of the deformable surface. \( A_0 \) is the nominal contact area (=\( L^2 \)), \( E^* \) is the composite elastic modulus \( \left( E^* = E / \left(1 - \nu^2 \right) \right) \), E, Y and \( \nu \) are the elastic modulus, yield strength and the Poisson’s ratio of the deformable rough surface.

The simulated result of the proposed model is validated using the data and results provided in Kucharski et al. [29]. They studied the contact load, real contact area and the relative approach between steel specimens having
Young’s modulus ($E$) = 200 GPa, tangent modulus ($E_t$) = 20 GPa, Poisson’s ratio ($\nu$) = 0.3 and tensile yield strength ($Y$) = 400 MPa. We have considered fractal dimensions as $D$ = 2.3, $G = 1.36 \times 10^{-11}$ m for the comparative study in the present case from 3D profilometric data available in the study of Kucharski et al. The equivalent elastic modulus $E^*$ is evaluated as $E/(1-\nu^2)$, which is equal to 219.78 GPa and hardness $H$ is taken as 1.12 GPa (2.8 times yield strength). The same material property was used in the finite element solution as well as in the analytical solution of Yan and Komvopoulos [25]. Figure 2 presents the comparison of the present model with the experimental results of Kucharski et al. [29] and the analytical solution [25] for non-dimensional contact area variation with non-dimensional contact load. It is evident from the figure; the present model exhibits favorable agreement between the simulated contact area/load values to that obtained by the experimental results [29]. However a significant variation of the analytical solution persists in comparison with the results of ANSYS and the experimental work owing to the assumption of elastic perfectly plastic material behavior and ignoring the asperity interaction in the analytical analysis.

![Fig. 2. Comparison between FEM (ANSYS) results, analytical results [25] and experimental results [29]](image)

5. Results and discussions

The results of the simulated finite element fractal contact model are presented for variety of contact load in both perfect slip and full stick contact conditions covering elastic, elastic-plastic up to fully plastic range. While the elastic modulus ($E$) and the Poisson’s ratio are held constant at 200 GPa and 0.32, respectively, five different material yield strengths are chosen. These are 0.210, 0.5608, 0.9115, 1.2653 and 1.619 GPa. These yield strength values cover a typical range of steel materials used in engineering applications [3]. Similarly in order to study the effect of roughness, the fractal dimension is varied in the range of $2.2 \leq D \leq 2.5$ and the fractal roughness is varied in the range $1.36 \times 10^{-12}$ m $\leq G \leq 1.36 \times 10^{-10}$ m. The results of the finite element simulations are presented for a variety of dimensionless contact load.

Fig. 3(a) presents the dimensionless contact area as a function of dimensionless contact load for the fractal surface $D$ = 2.3, $G = 1.36 \times 10^{-11}$ m in perfect slip contact condition. The dimensionless contact area increases with the decrease in yield strength for the same dimensionless contact load. Fig. 3(b) shows the dimensionless displacement with the increase in dimensionless contact load with same fractal surface properties in perfect slip contact condition. It is clear from the figure that the dimensionless normal displacement decreases with the increase in yield strength of the material under the same contact load. The increment rates of dimensionless contact area and the dimensionless interferences are increasing with the increase in dimensionless applied load in elastic-plastic as well as in plastic range due to the plastic flow of the asperities.
Fig. 4(a) is the plot of the dimensionless mean contact pressure with the variation of dimensionless contact load in perfect slip contact condition for fractal parameter $D=2.3$, $G=1.36 \times 10^{-11}$ m. It is observed that the mean contact pressure is dependent on the value of material yield strength. It is also evident; the dimensionless mean contact pressure attains to a stabilized decreased value after reaching its peak value, the location and the magnitude of peak value and stabilization rate depends on the yield strength. Jackson and Green [3] simulated the results in ANSYS for a deformable sphere against a rigid flat (commonly termed as single asperity contact). The results are in good agreement with the findings of Jackson and Green (Fig. 5). Fig. 4(b) presents the dimensionless contact area against the dimensionless contact load for the fractal surfaces with the fractal parameter, $D=2.3$, $G=1.36 \times 10^{-11}$ m under full stick contact condition. It is observed from the figure that the load area relationship curve follows the same trend as perfect slip contact condition. However load carrying capacity slightly increases under full stick contact condition as the outward displacement is held back by the stick contact condition imposed by rigid flat. This is in conformity with Brizmer et al. [5].

![Figure 3](image1.png)

**Fig. 3.** Contact parameters at perfect slip contact with fractal surface having $D=2.3$, $G=1.36 \times 10^{-11}$ m (a) Dimensionless contact area vs. dimensionless load, (b) Dimensionless displacement vs. dimensionless load

![Figure 4](image2.png)

**Fig. 4.** Contact parameters for fractal surface having $D=2.3$, $G=1.36 \times 10^{-11}$ m (a) Dimensionless mean contact pressure vs. dimensionless load at perfect slip contact, (b) Dimensionless contact area vs. dimensionless load at full stick contact

Fig. 5(a) shows the dimensionless normal displacement versus the dimensionless contact load for the same fractal surface property of Fig. 3(b) under full stick contact condition. Fig. 3(b) and 5(a) follow the same trend for dimensionless normal displacement-dimensionless contact load curve for perfect slip and full stick contact condition except the load range.
Fig. 5(b) presents the dimensionless mean contact pressure as a function of dimensionless contact load for the fractal surface property \(D=2.3, G=1.36 \times 10^{-11} \text{m}\) under full stick contact condition. The mean contact pressure varies non-monotonically, increasing initially up to a maximum value and then falling. The sharpness of decrease of mean contact pressure depends on yield strength. The value of maximum mean contact pressure decreases with the increase in yield strength that conforms to the results of Jackson and Green [3]. The location of maximum mean contact pressure is reached at lower dimensionless contact load with decreasing yield strength due to the early plastic deformation at lower yield strength. The results are in well agreement with the findings of Pei et al [20]. The friction in general caused by the interlocking of the asperities [30] restricts further increase in contact area; thus the maximum value of dimensionless mean contact pressure is slightly higher under full stick contact condition in comparison to perfect slip contact condition.

Fig. 5. Contact parameters at full stick contact with fractal surface having \(D=2.3, G=1.36 \times 10^{-11} \text{m}\) (a) Dimensionless displacement vs. dimensionless load, (b) Dimensionless contact pressure vs. dimensionless load

Fig. 6. Contact parameters for varying \(D\) at \(G=1.36 \times 10^{-11} \text{m}, Y=560.8 \text{MPa}\) with perfect slip contact (a) Dimensionless contact area vs. dimensionless load, (b) Dimensionless displacement vs. dimensionless load

In order to study the effect of surface roughness on the contact parameters, the simulated results of dimensionless contact area against the dimensionless contact load are presented in Fig. 6(a) for various values of fractal dimension \(D\) with fixed surface roughness parameter \(G=1.36 \times 10^{-11} \text{m}\) and yield strength, \(Y=560.8 \text{MPa}\). It is seen that increasing fractal dimension increases the contact area at a particular load in perfect slip contact.
condition. Fig. 6(b) shows the plots of non-dimensional normal displacement of the rigid plane as a function of non-dimensional contact load for fractal roughness parameter $G=1.36 \times 10^{-11} \text{m}$, yield strength $Y=560.8 \text{ MPa}$ and varying fractal dimension $D$ in perfect slip contact condition. It is observed that at a particular load, displacement is small for a higher value of $D$. Present results do concord with the findings of Sahoo and Ghosh [21].

Fig. 7(a) presents the plots of non-dimensional contact area versus non-dimensional contact load for a fixed fractal dimension $D=2.4$ and varying roughness parameter $G$ from $1.36 \times 10^{-10} \text{m}$ to $1.36 \times 10^{-12} \text{m}$. It is observed that at a particular load, contact area is larger for lower $G$ values. Fig. 7(b) represents the dimensionless displacement of the rigid plane as a function of non-dimensional load for a fixed fractal dimension $D=2.4$ with varying roughness parameter $G$. Since $G$ is a height scaling parameter, higher $G$ values correspond to rougher surface topographies. Thus at a particular load higher $G$ value should produce less contact area at higher normal displacement that is confirmed from this plot.

![Fig. 7](image)

Fig. 7. Contact parameters for varying $G$ at $D=2.4$, $Y=560.8 \text{ MPa}$ with perfect slip contact (a) Dimensionless contact area vs. dimensionless load, (b) Dimensionless displacement vs. dimensionless load

![Fig. 8](image)

Fig. 8. (a) Dimensionless displacement vs. dimensionless load for varying $D$ at $G=1.36 \times 10^{-11} \text{m}$, $Y=560.8 \text{ MPa}$ with full stick contact (b) Dimensionless contact area vs. dimensionless displacement for varying $Y$ at $D=2.4$, $G=1.36 \times 10^{-11} \text{m}$ with full stick and perfect slip contact

Fig. 8(a) provides the dimensionless normal displacement against the dimensionless contact load for the fractal roughness parameter $G=1.36 \times 10^{-11} \text{m}$, yield strength $Y=560.8 \text{ MPa}$ with varying fractal dimension, $D$, from 2.2 to 2.5 under full stick contact condition. The qualitative nature of the normal displacement – load curve under full stick and perfect slip contact conditions follows the same trend and the normal displacement increases with the decrease in fractal dimension at a particular load. Fig. 8(b) provides a comparative study of contact parameters under full stick and perfect slip contact condition for fractal dimension, $D=2.4$ and fractal roughness parameter.
G=1.36×10^{-11}m for various yield strength. It is seen that dimensionless contact area and dimensionless normal displacement curves maintain a generalized relationship irrespective of the value of yield strength, although at the fag end of the contact, to produce the same dimensionless contact area, more dimensionless normal displacement is required in case of full stick contact condition than that with perfect slip contact condition.

To study the combined effect of modulus of elasticity and yield strength, we choose four materials with different modulus of elasticity and yield strength but with same E/Y ratio as described in Table 1. Fig. 9 shows the dimensionless contact load as a function of dimensionless area for fractal dimension, D=2.4 and fractal roughness parameter, \(G=1.36\times10^{-11}m\) under full stick contact condition. It is found from the figure that the dimensionless contact load is identical for different materials having same E/Y ratio, even though their modulus of elasticity and yield strengths are different. Chatterjee and Sahoo [6] also found identical behavior in the contact analysis of a deformable sphere against a rigid flat under full stick contact condition.

| Case          | E (GPa) | Y (GPa) | E/Y |
|---------------|---------|---------|-----|
| Ruthenium\(^a\) | 410     | 3.42    | 120 |
| Steel\(^b\)   | 200     | 1.667   | 120 |
| Titanium alloy\(^b\) | 105 | 0.875 | 120 |
| Gold\(^a\)    | 80      | 0.67    | 120 |

\(^a\)From reference [31]. \(^b\)matweb.com

Fig.9. Dimensionless contact area vs. dimensionless contact load for different materials with E/Y=120, D=2.4, G=1.36×10^{-11}m

6. Conclusions

A finite element based contact analysis of fractal surfaces with various elastic material properties like yield strength, elastic modulus is described for varying fractal surface properties in both perfect slip contact and full stick contact conditions using ANSYS. It is found that the yield strength has strong influence on the contact parameters. The simulated results are in good agreement with that of single asperity contact. Contact area increases with the decrease in yield strength at a particular contact load and the contact load –contact area relationship is non-linear in elastic-plastic contact region in both perfect slip contact and full stick contact conditions. However load carrying capacity in full stick contact condition is slightly higher than that of perfect slip contact condition. Similarly normal displacement also increases with the decrease in yield strength. Mean contact pressure varies non-monotonically, depending on the yield strength. Finally it is found that with the varying elastic properties of the material but with same E/Y ratio, loading in fractal surfaces yields identical results.
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