Dynamic vibration protection of the railway carriage

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Abstract. The article describes the conditions under which the freight in the car is subject to force impact. The car has two degrees of freedom, and the freight has one. In the general case, if the system parameters satisfy the equation of the system, we have an antiresonance. In particular cases, if Н1 or Н2 or Н3 is zero, the antiresonance mode occurs when conditions are met, respectively. In these cases, the freight is at rest, which is often required to preserve the freight during transportation by rail. The conditions for dynamic damping of freight oscillations are obtained.

1. Introduction

Safety of transported goods is one of the conditions of the contract of carriage, the performance of which is the responsibility of the carrier. The LCA (art. 95) provides for liability for non-preservation of the goods after acceptance for carriage and storage until delivery to the consignee. Integrity also relates to the requirements of the route development, constructions and devices of a cargo economy of the railway stations, to the weight the economy, to public and non-public.

We consider the task of securing the freight placed in the car according to the technical conditions of placement and fastening of freight in cars [1-5], from the effects of shocks and vibrations.

2. Construction of a mathematical model

It had been previously obtained [1, 6-9] that the motion of a system with six degrees of freedom with sufficient accuracy can be represented by a system with two degrees of freedom. Therefore, let us assume that the car body has two degrees of freedom: lateral motion and wabbling. We consider the forced oscillations of a system with three degrees of freedom in Fig. 1.

We take for generalized coordinates $z_k$, $\phi_k$ and $z_w$ – linear, angular movement of the center of masses of the car and movement of the freight, respectively; $m_k$, $I_k$, $m_w$ – mass, moment of inertia of the body and mass of the freight; $c_{11}$ and $c_{12}$ – the reduced stiffnesses of the spring suspension, $c_w$ – the reduced stiffness of the freight fastening elements; $L_1 + L_2$ – the base of the car.
We assume that the generalized disturbing forces are harmonic functions of time, having the same frequency $p$ and different amplitudes $H_1$, $H_2$ and $H_3$, that is:

$$
\begin{align*}
F_1 &= H_1 \sin pt; \\
F_2 &= H_2 \sin pt; \\
F_3 &= H_3 \sin pt.
\end{align*}
$$
Having generated expressions for the kinetic and potential energy [2, 10-15] and using the Lagrange equations of the second kind, we obtain a system of differential equations:

\[
\begin{align*}
    m_k \ddot{z}_k + \left( \dd dot{c} + \dd dot{c}_w + c_w \right) z_k - c_w \dot{z}_w + \\
    + \left( \dd dot{c}_1 L - \dd dot{c}_2 L - c_w L_w \right) \phi_k &= F_1; \\
    m_w \ddot{z}_w + c_w z_w + c_w L_w \phi_k &= F_2; \\
    I_k \ddot{\phi}_k + \left( \dd dot{c}_1 L - \dd dot{c}_2 L - c_w L_w \right) z_k + \\
    + c_w L_w \dot{z}_w + \\
    + \left( \dd dot{c}_1 L^2 + \dd dot{c}_2 L^2 + c_w L^2 \right) \phi_k &= F_3,
\end{align*}
\]

where \( L_w > 0 \) is the distance from the gravity center of the car to the center of gravity of the freight.

To simplify the system (1), we assume that \( A_1 = m_k, \ C_1 = \dd dot{c}_1 + \dd dot{c}_2 + c_w, \ C_2 = C_2 = -c_w, \ C_3 = C_3 = \dd dot{c}_1 L - \dd dot{c}_2 L - c_w L_w, \ A_2 = m_w, \ C_2 = c_w, \ C_3 = C_3 = c_w L_w, \ A_3 = I_k, \)

\( C_3 = \dd dot{c}_1 L^2 + \dd dot{c}_2 L^2 + c_w L^2 \).

Then the system (1) takes the form:

\[
\begin{align*}
    A_{11} \ddot{z}_k + C_{11} \dot{z}_k + C_{12} \dot{z}_w + C_{13} \phi_k &= F_1; \\
    A_{22} \ddot{z}_w + C_{22} \dot{z}_k + C_{22} \dot{z}_w + C_{23} \phi_k &= F_2; \\
    A_{33} \ddot{\phi}_k + C_{31} \dot{z}_k + C_{32} \dot{z}_w + C_{33} \phi_k &= F_3.
\end{align*}
\]

We consider the possibility of using this system in the form of a dynamic damper suppressing the oscillations of the freight.

Particular solutions of the system (2), characterizing forced oscillations, will be sought in the form:

\[ z_k = A_1 \sin pt, \quad z_w = A_2 \sin pt, \quad \phi_k = A_3 \sin pt \]  

Substituting (3) into (2), we obtain an inhomogeneous algebraic system of equations for the oscillation amplitudes \( A_1, A_2, A_3 \). The solution of this system of equations is:

\[
\begin{align*}
    A_1 &= \frac{H_1 \left[ C_{22} - A_2 p^2 \right] \left[ C_{33} - A_3 p^2 \right] - C_{23} C_{32} + H_2 \left[ C_{32} C_{13} - C_{32} \left( C_{33} - A_3 p^2 \right) \right] + H_3 \left[ C_{22} C_{13} - C_{22} \left( C_{32} - A_2 p^2 \right) \right]}{\Delta}, \\
    A_2 &= \frac{H_2 \left[ C_{23} C_{13} - C_{22} \left( C_{33} - A_3 p^2 \right) \right] + H_2 \left[ C_{31} - A_1 p^2 \right] \left( C_{33} - A_3 p^2 \right) - C_{31} \Delta + H_3 \left[ C_{22} C_{13} - C_{22} \left( C_{31} - A_1 p^2 \right) \right]}{\Delta}, \\
    A_3 &= \frac{H_3 \left[ C_{21} C_{32} - C_{22} \left( C_{32} - A_2 p^2 \right) \right] + H_2 \left[ C_{22} C_{13} - C_{22} \left( C_{31} - A_1 p^2 \right) \right] + H_3 \left[ C_{22} - A_2 p^2 \right] \left( C_{23} - A_3 p^2 \right) - C_{33} \Delta}{\Delta},
\end{align*}
\]

where \( \Delta = \begin{vmatrix} C_{11} - A_1 p^2 & C_{12} & C_{13} \\ C_{21} & C_{22} - A_2 p^2 & 0 \\ C_{31} & 0 & C_{33} - A_3 p^2 \end{vmatrix} \).

The combination of the values of \( A_1, A_2, A_3 \) determines the form of forced oscillations. The determinant \( \Delta \) vanishes when the frequency of the driving force coincides with any of the natural frequencies. When the frequencies coincide, a resonance takes place [3, 16-20].

The opposite cases are also possible, when at certain values of the frequency of the driving force \( p \), the amplitudes \( A_1, A_2, A_3 \) of the corresponding coordinates \( z_k, z_w, \phi_k \) are equal to zero, which indicates the absence of oscillations along these coordinates.

The solution found for the freight amplitude can be represented as:
\[
H_{AA} \left[ \frac{C_{23}C_{13}}{A_{31}} - C_{12} \left( \omega_i^2 - p^2 \right) \right] + H_{AA} A_{33} \left[ \left( \omega_i^2 - p^2 \right) \left( \omega_i^2 - A_3 p^2 \right) - \frac{C_{13}C_{13}}{A_{11}A_{33}} \right] + H_{AA} \left[ \frac{C_{13}C_{13}}{A_{11}} - C_{23} \left( \omega_i^2 - p^2 \right) \right],
\]

where \( \omega_1 = \sqrt{\frac{C_{11}}{A_{11}}} \), \( \omega_2 = \sqrt{\frac{C_{22}}{A_{22}}} \), \( \omega_3 = \sqrt{\frac{C_{33}}{A_{33}}} \) are partial frequencies.

3. Results

We consider the various cases of application of force.

1. Let \( H_1 \neq 0 \), \( H_2 \neq 0 \) and \( H_3 \neq 0 \).

Then the amplitude \( A_2 \) will be equal to zero, if:

\[
H_{AA} A_{33} \left[ \frac{C_{23}C_{13}}{A_{31}} - C_{12} \left( \omega_i^2 - p^2 \right) \right] + H_{AA} A_{33} \left[ \left( \omega_i^2 - p^2 \right) \left( \omega_i^2 - A_3 p^2 \right) - \frac{C_{13}C_{13}}{A_{11}A_{33}} \right] +
\]

or

\[
H_{AA} A_{33} \left( C_{13} C_{23} - C_{12} A_{3} \omega_i^2 \right) + H_{AA} A_{33} \left( \omega_i^2 - C_{13} C_{31} \right) + H_{AA} \left( C_{12} C_{13} - A_{1} C_{12} \omega_i^2 \right) = 0.
\]

Then:

\[
\begin{align*}
\rho_1^2 + \rho_2^2 &= \omega_i^2 + \omega_2^2 - \frac{H C_{23}}{H_2 A_{33}} - \frac{H C_{12}}{H_2 A_{11}}; \\
\rho_1^2 \cdot \rho_2^2 &= \omega_i^2 \omega_2^2 - \frac{H C_{12} \omega_i^2}{H_2 A_{33}} - \frac{H C_{23} \omega_i^2}{H_2 A_{33}} - \frac{C_{13} C_{31}}{A_{11} A_{33}} + \frac{C_{13} \left( H C_{23} + H C_{12} \right)}{H_2 A_{11} A_{33}}.
\end{align*}
\]  

Condition (4) determines the frequency of antiresonance. When the frequency satisfies this condition, there are no oscillations along the coordinate \( z_w \) (the freight is fixed in the vertical direction).

2. Consider a special case \( H_1 = 0 \).

Then the amplitude \( A_2 \) will take the form

\[
H_{AA} A_{33} \left[ \left( \omega_i^2 - p^2 \right) \left( \omega_i^2 - A_3 p^2 \right) - \frac{C_{13} C_{13}}{A_{11} A_{33}} \right] + H_{AA} \left[ \frac{C_{13} C_{13}}{A_{11}} - C_{23} \left( \omega_i^2 - p^2 \right) \right],
\]

and will be zero if

\[
H_{AA} A_{33} \left[ \left( \omega_i^2 - p^2 \right) \left( \omega_i^2 - A_3 p^2 \right) - \frac{C_{13} C_{13}}{A_{11} A_{33}} \right] + H_{AA} \left[ \frac{C_{13} C_{13}}{A_{11}} - C_{23} \left( \omega_i^2 - p^2 \right) \right] = 0
\]

or

\[
H_{AA} A_{33} \left( C_{13} C_{23} - C_{12} A_{3} \omega_i^2 \right) + H_{AA} \left( C_{13} C_{23} - A_{1} C_{12} \omega_i^2 \right) = 0.
\]

Then:

\[
\begin{align*}
\rho_1^2 + \rho_2^2 &= \omega_i^2 + \omega_2^2 - \frac{H C_{23}}{H_2 A_{33}}; \\
\rho_1^2 \cdot \rho_2^2 &= \omega_i^2 \omega_2^2 - \frac{H C_{12} \omega_i^2}{H_2 A_{33}} - \frac{C_{13} C_{31}}{A_{11} A_{33}} + \frac{C_{13} \left( H C_{23} + H C_{12} \right)}{H_2 A_{11} A_{33}}.
\end{align*}
\]  


When this condition is met, the freight is at rest (with $H_1=0$).

3. We consider the case when $H_1 = 0$. The amplitude $A_2$ will take the form:

$$A_2 = \frac{H_1 A_{33} \left[ \frac{C_{23} C_{13}}{A_{33}} - C_{12} \left( \omega_3^2 - p^2 \right) \right] + H_2 A_{11} \left[ \frac{C_{12} C_{13}}{A_{11}} - C_{23} \left( \omega_i^2 - p^2 \right) \right]}{\Delta},$$

and will be zero if:

$$H_1 A_{33} \left[ \frac{C_{23} C_{13}}{A_{33}} - C_{12} \left( \omega_3^2 - p^2 \right) \right] + H_2 A_{11} \left[ \frac{C_{12} C_{13}}{A_{11}} - C_{23} \left( \omega_i^2 - p^2 \right) \right] = 0,$$

or

$$H_3 A_{11} C_{23} p^2 + H_1 \left( C_{13} C_{23} - C_{12} A_{33} \omega_i^3 \right) + H_3 \left( C_{12} C_{13} - A_{11} C_{23} \omega_i^3 \right) = 0.$$

Then:

$$p = \sqrt{\frac{H_1 \left( C_{13} A_{11} \omega_i^2 - C_{33} C_{33} \right) + H_1 \left( A_{11} C_{23} \omega_i^2 - C_{23} C_{13} \right)}{H_3 A_{11} C_{23} + H_1 C_{11}}}.$$

When this condition is met, the freight is at rest ($H_2=0$).

4. Let $H_1 = 0$, then the amplitude $A_2$ will take the form:

$$A_2 = \frac{H_1 A_{33} \left[ \frac{C_{23} C_{13}}{A_{33}} - C_{12} \left( \omega_3^2 - p^2 \right) \right] + H_2 A_{11} A_{33} \left[ \left( \omega_i^2 - p^2 \right) \left( \omega_i^2 - A_{33} p^2 \right) - \frac{C_{12} C_{33}}{A_{11} A_{33}} \right]}{\Delta},$$

and will be zero if:

$$H_1 A_{33} \left[ \frac{C_{23} C_{13}}{A_{33}} - C_{12} \left( \omega_3^2 - p^2 \right) \right] + H_2 A_{11} A_{33} \left[ \left( \omega_i^2 - p^2 \right) \left( \omega_i^2 - A_{33} p^2 \right) - \frac{C_{12} C_{33}}{A_{11} A_{33}} \right] = 0,$$

or

$$H_2 A_{11} A_{33} p^4 + \left[ H_{12} A_{33} - H_2 A_{11} A_{33} \left( \omega_i^2 + \omega_3^2 \right) \right] p^2 +$$

$$+ H_1 \left( C_{13} C_{23} - C_{12} A_{33} \omega_i^3 \right) + H_2 \left( A_{11} A_{11} \omega_i^2 - C_{13} C_{33} \right) = 0.$$

Then:

$$\begin{cases}
\quad p_1^2 + p_2^2 = \omega_i^2 + \omega_3^2 - \frac{H_{12}}{H_2 A_{11}}, \\
\quad p_1^2 \cdot p_2^2 = \frac{H_{12} \omega_i^2}{H_2 A_{11} A_{33}} - \frac{C_{13} C_{33}}{A_{11} A_{33}} + \frac{H_{12} C_{12} A_{11}}{H_2 A_{11} A_{33}}.
\end{cases}$$

When this condition is met, the freight is at rest ($H_3=0$).

4. Conclusion

In the general case, if the system parameters satisfy the equation of the system (4), we have an anti-resonance. In particular cases, if $H_1$ or $H_2$ or $H_3$ is zero, the anti-resonance mode occurs when conditions (5)-(7) are met, respectively. In these cases, the freight is at rest, which is often required to preserve the freight during transportation by rail.

The obtained conditions allow us to take into account the features of the rolling stock and the dynamics of movement, and the roughness of the track.

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