Neutron-$^3$H scattering above the four-nucleon breakup threshold

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The four-body equations of Alt, Grassberger and Sandhas are solved for the neutron-$^3$H scattering at energies above the four-nucleon breakup threshold. The accuracy and practical applicability of the employed complex energy method is significantly improved by the use of integration with the special weights. This allows to obtain fully converged results with realistic nuclear interactions. A satisfactory description of the existing neutron-$^3$H elastic scattering data is obtained.

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The quasi-singular factor \((x_0^+ + iy_0 - x^n)^{-1}\) is separated and absorbed into the special integration weights \(w_j(n, x_0, y_0, a, b)\). The set of \(N\) grid points \(\{x_j\}\) where the remaining smooth function \(f(x)\) has to be evaluated is chosen the same as for the standard Gaussian quadrature. However, while the standard weights are real \([24]\), the special ones \(w_j(n, x_0, y_0, a, b)\) are complex. They are chosen such that for a set of \(N\) test functions \(f_j(x)\) the result (8) is exact. A convenient and reliable choice of \(\{f_j(x)\}\) are the \(N\) spline functions \(\{S_j(x)\}\) referring to the grid \(\{x_j\}\): their construction and properties are described in Refs. [24] [28]. The corresponding special weights are

\[
w_j(n, x_0, y_0, a, b) = \int_a^b \frac{S_j(x)}{x_0^+ + iy_0 - x^n} dx,
\]

where the integration can be performed either analytically or numerically using sufficiently dense grid. This choice of special weights guarantees accurate results for quasi-singular integrals (8) with any \(f(x)\) that can be accurately approximated by the spline functions \(\{S_j(x)\}\).

In the integrals over the momentum variables one has \(n = 2\), \(a = 0\), and \(b \to \infty\). For example, when solving the Lippmann-Schwinger equation [3] the integration variable in Eq. (8) is the momentum \(k_y\) with \(x_0^+ = 2\mu_{\alpha\beta}(E - k_y^2/2\mu_{\alpha\beta} - k_z^2/2\mu_{\alpha\beta})\) and \(y_0 = 2\mu_{\alpha\beta}\varepsilon\). Alternatively, the quasi-singularity can be isolated in a narrower interval \(0 < a < b < \infty\) and treated by special weights only there.
TABLE I. Elastic phase shifts (in degrees) and inelasticities in selected partial waves for $n^3\text{H}$ scattering at 22.1 MeV neutron energy. Results for INOY04 potential obtained using different sets of $\varepsilon$ values ranging from $\varepsilon_{\text{min}}$ to $\varepsilon_{\text{max}}$ (in MeV) are compared. In the last line the predictions with $\varepsilon = 1.4$ MeV without extrapolation are given.

| $[\varepsilon_{\text{min}}, \varepsilon_{\text{max}}]$ | $\delta^{(1)S_0}$ | $\eta^{(1)S_0}$ | $\delta^{(1)P_0}$ | $\eta^{(1)P_0}$ | $\delta^{(1)P_2}$ | $\eta^{(1)P_2}$ |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $[1.0, 2.0]$                    | 62.63           | 0.990           | 43.03           | 0.959           | 65.27           | 0.950           |
| $[1.2, 2.0]$                    | 62.60           | 0.991           | 43.04           | 0.959           | 65.29           | 0.951           |
| $[1.4, 2.0]$                    | 62.67           | 0.991           | 43.03           | 0.958           | 65.27           | 0.950           |
| $[1.2, 1.8]$                    | 62.65           | 0.992           | 43.03           | 0.959           | 65.28           | 0.950           |
| 1.4                            | 73.37           | 0.916           | 44.77           | 0.840           | 67.38           | 0.933           |

Other numerical techniques for solving the four-nucleon AGS equations are taken over from Ref. [9]. They include Padé summation of Neumann series for the transition operators $U_\alpha$ and $U_{\beta\alpha}$ using the algorithm of Ref. [30] and the treatment of permutation operators (basis transformations) using the spline interpolation. The specific form of the permutation operators leads to a second kind of quasi-singular integrals with $n = 1$, $a = -1$, $b = 1$, and the integration variable $x = k'_y \cdot k_y$ or $k'_z \cdot k_z$ being the cosine of the angle between the respective initial and final momenta.

We note that the above integration method is not sufficient in the vanishing $\varepsilon$ limit since for $n = 1$ and $y_0 = 0$ the result of the integral contains the contribution $f(x_0) \ln[(x_0 + 1)/(x_0 - 1)]$ with logarithmic singularities at $x_0 = \pm 1$. At finite small $\varepsilon$ the result of (7) may exhibit a quasi-singular behavior. However, since the logarithmic quasi-singularity is considerably weaker than the pole quasi-singularity, at not too small $\varepsilon$ it is sufficient to use the standard integration.

We start by applying the complex energy method with special integration weights to the $n^3\text{H}$ scattering below the three-cluster threshold where our previous results obtained at real energies using subtraction technique are available for comparison. In the test calculations at 3.5 and 6.0 MeV neutron energy we find a very good agreement between the two methods, better than 0.05% for all relevant phase shifts and observables. The considered $\varepsilon$ values range between 0.2 and 2.0 MeV, and the number of grid points is not increased as compared to the real-energy calculations.

Next we test the numerical reliability of our technique above the four-nucleon breakup threshold. We use a realistic dynamics, namely, the high-precision inside-nonlocal outside-Yukawa (INOY04) two-nucleon potential by Doleschall [5, 31] that reproduces experimental binding energies of $^3\text{H}$ (8.48 MeV) and $^3\text{He}$ (7.72 MeV) without an irreducible three-nucleon force. We consider a large number of four-nucleon partial waves sufficient for the convergence, i.e., $l_x, l_y, l_z, j_x, j_y, j_y \leq 4$ and $J \leq 5$. Including more partial waves yields only entirely insignificant changes. There are too many numerical parameters (numbers of points for various integration grids) to demonstrate the stability of our calculations with respect to each of them separately. We found that 10 grid points are sufficient for all angular integrations but 30 to 40 grid points are needed for the discretization of Jacobi momenta. The $\varepsilon \to +0$ extrapolation yields stable results only if sufficiently small $\varepsilon$ are considered and at each of them the respective calculations are numerically well converged. We therefore study in Fig. 1 the stability of the $\varepsilon \to +0$ results obtained via extrapolation using different $\varepsilon$ sets ranging from $\varepsilon_{\text{min}}$ to $\varepsilon_{\text{max}}$ with the step of 0.2 MeV. We show the differential cross section $d\sigma/d\Omega$ and neutron analyzing power $A_y$ for elastic $n^3\text{H}$ scattering at $E_n = 22.1$ MeV neutron energy. We find a very good agreement between the results obtained with $[\varepsilon_{\text{min}}, \varepsilon_{\text{max}}] = [1.0, 2.0], [1.2, 2.0], [1.4, 2.0], \text{and } [1.2, 1.8]$ MeV, confirming the reliability of our calculations. In addition, we show in Fig. 1 the predictions referring to $\varepsilon = 1.4$ MeV without extrapolation that don’t have physical meaning. The difference between $\varepsilon \to +0$ and $\varepsilon = 1.4$ MeV results demonstrates the importance of the extrapolation. Furthermore, in Table II we collect the corresponding values for selected phase shifts $\delta$ and inelasticities $\eta$, i.e., we parametrize the elastic S-matrix as $s = \eta e^{2i\delta}$. As already can be expected from Fig. 1 the stability of
In addition to the INOY04 potential we present results proceeding to the comparison with the experimental data. Of magnitude larger inaccuracies than those in Table I. At large $\varepsilon$ integrations weights. We found that they fail completely calculations keeping the same grids but with standard integration methods. The variations are slightly larger in the physical choice as four different $\varepsilon$ values are sufficient to obtain the physical $\varepsilon \to +0$ results with good accuracy. For curiosity, in $J = 0$ states we performed the calculations keeping the same grids but with standard integration weights. We found that they fail completely at $\varepsilon$ values from Table I with the errors of the $\varepsilon \to +0$ extrapolation being up to 10% for phase shifts and up to 25% for inelasticity parameters. On the other hand, at large $\varepsilon > 4$ MeV the two integration methods agree well but the $\varepsilon \to +0$ extrapolation has at least one order of magnitude larger inaccuracies than those in Table I.

After establishing the reliability of our calculations we proceed to the comparison with the experimental data. In addition to the INOY04 potential we present results derived from the CD Bonn potential [34] that underbinds the $^3\text{H}$ nucleus by 0.48 MeV. In Fig. 2 we show the differential cross section for elastic neutron-$^3\text{H}$ scattering at 14.1, 18.0, and 22.1 MeV neutron energy. Except for the minimum around 115 degrees, the predictions are insensitive to the choice of the potential. At $E_n = 14.1$ MeV the new data set by Frenje et al. [22] is described very well. Other existing data at this energy are consistent with Ref. [22] but have larger error bars; we only show the data by Debertin et al. [32]. At 18.0 MeV the data sets by Debertin et al. [32] and Seagrave et al. [33] are inconsistent with each other around the minimum while the theoretical predictions lie in the middle. The results at $E_n = 22.1$ MeV are compared with the data taken at 21 and 23 MeV by Seagrave et al. [33]. The predictions lie between the two data sets except for the minimum region. However, given the agreement between the [22] and [32] data and disagreement between the [32] and [33] data, one may question the reliability of the data by Seagrave et al. in the minimum region. Thus, new measurements are needed to resolve this discrepancy.

In Fig. 3 we present the neutron analyzing power for elastic neutron-$^3\text{H}$ scattering. INOY04 and CD Bonn predictions at $E_n = 22.1$ MeV are compared with the data from Ref. [32]. INOY04 results at 14.1 MeV and 18.0 MeV are also shown.

![FIG. 2. (Color online) Differential cross section for elastic neutron-$^3\text{H}$ scattering at 14.1, 18.0, and 22.1 MeV neutron energy. Results obtained with INOY04 (solid curves) and CD Bonn (dashed-dotted curves) potentials are compared with the experimental data from Refs. [22, 32, 33]. INOY04 and CD Bonn predictions at $E_n = 14.1$ MeV are also shown.](image)

![FIG. 3. (Color online) Neutron analyzing power for elastic neutron-$^3\text{H}$ scattering. INOY04 and CD Bonn predictions at $E_n = 22.1$ MeV are compared with the data from Ref. [32]. INOY04 results at 14.1 MeV and 18.0 MeV are also shown.](image)
TABLE II. $^3$H elastic $\sigma_e$, breakup $\sigma_b$, and total $\sigma_t$ cross sections (in mb) at selected neutron energies (in MeV).

| $E_n$ (MeV) | $\sigma_e$ (mb) | $\sigma_b$ (mb) | $\sigma_t$ (mb) |
|-------------|----------------|----------------|----------------|
| 14.1        | 928            | 19             | 947            |
| 18.0        | 697            | 41             | 738            |
| 22.1        | 536            | 61             | 597            |

TABLE III. $^3$H breakup $\sigma_b$, and total $\sigma_t$ cross sections (in mb) at selected neutron energies (in MeV).

| $E_n$ (MeV) | $\sigma_b$ (mb) | $\sigma_t$ (mb) |
|-------------|----------------|----------------|
| 14.1        | 928            | 19             |
| 18.0        | 697            | 41             |
| 22.1        | 536            | 61             |

TABLE IV. Total $\sigma_t$, elastic $\sigma_e$, and breakup $\sigma_b$ cross sections (in mb) at selected neutron energies (in MeV).

| $E_n$ (MeV) | $\sigma_t$ (mb) | $\sigma_e$ (mb) | $\sigma_b$ (mb) |
|-------------|----------------|----------------|----------------|
| 14.1        | 928            | 19             | 947            |
| 18.0        | 697            | 41             | 738            |
| 22.1        | 536            | 61             | 597            |

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