Research Article

One nucleon pick-up reaction $^{58}$Ni(p,d)$^{57}$Ni at 68 MeV
Sadia Afroze Sultana* and Md. Azizur Rahman1
Open School, Bangladesh Open University, Gazipur, Bangladesh

ARTICLE INFO
Article History
Received: 21 May 2021
Revised: 8 June 2022
Accepted: 12 June 2022

Keywords: Double-differential cross section, (p,d) reaction, DWBA analysis, Direct reaction model.

Abstract
The double differential cross-section for the $^{58}$Ni(p,d)$^{57}$Ni reaction has been studied with 68 MeV protons for the 25°, 30°, 35°, 45°, and 60° laboratory angles. The spectra have been calculated using DWBA-based cross-sections including an asymmetric form of the Lorentzian strength function containing energy-dependent spreading widths. The comparison between the measured spectra and the theoretical predictions is accomplished in the direct reaction region. The values of the calculated double differential cross-sections agree with those of the measured cross-sections as far as ~17 MeV deuteron excitation energies; the ejectile deuterons-energy for direct reactions at 68 MeV proton energy.

Introduction
Reliable nuclear structure and reaction data represent the fundamental building blocks of nuclear physics and are of importance in astrophysics research and many other applications (U.S. DEPARTMENT OF ENERGY, 2016). The development of modern nuclear technologies requires a large amount of nuclear data for the conceptual design of different fields of applications, such as the technology of radioactive waste transmutation, power production, radiotherapy, shielding problems, and so on (Grudzevich et al., 2007).

Theoretical models for producing nuclear data are always in demand both for a general understanding of the physical phenomena related to the analyzed data and to estimate the required cross-sections in cases where data are contradictory or not fully available (Ignatyuk, 2013). Ideally, nuclear data are expected to be collected experimentally for nucleons at various incident energies and all kinds of emitted particles over all the energies and the emission angles. However, it is not possible, in reality, to have all necessary nuclear data experimentally as desired for its high cost and all kinds of preparations for the experiments. So, developing a theoretical model, that can produce nuclear data is always indispensable.

Therefore, an approach by Lewis (Lewis, 1975) was suggested to be employed in parallel with the predictive models given by Crawley (Crawley, 1980); Gales et al. (Gales et al., 1988), and Matoba et al. (Matoba et al., 1995 and 1996). These authors had suggested methods using an asymmetric shape of the Lorentzian strength function having energy-dependent spreading widths multiplied by DWBA cross-sections for the theoretical calculations of double differential cross-sections. This model has been successfully applied to the (p,d) reactions (Syafarudin et al., 2002; Sultana et al., 2004; Aramaki et al., 2002; Sultana et al., 2005; Sultana et al., 2009; Sultana and Imtiaz, 2017), then applied to the (n,d) reaction (Sultana et al., 2003; Sultana et al., 2004; Sultana, 2006; Sultana et al., 2010) with a slight modification and with reasonable success.

The continuum spectra of the $^{58}$Ni(p,d)$^{57}$Ni reaction have been analyzed in the present work by

*Corresponding author: <sadia_afroze@yahoo.com>
1Department of Physics, University of Dhaka, Bangladesh
the same method of calculations discussed above to ascertain the reliability of this method of calculations as a global one. This may be mentioned here that the same reaction (p,d) for the same target nucleus ($^{58}$Ni) and with the same method of calculation but at different incident energy (42 MeV) were studied by the author (Sultana and Hossain, 2016; Sultana, 2017). In the present work, the laboratory angles of deuterons emission are 25°, 30°, 35°, 45°, and 60° for the incident proton energy 68.0 MeV. The experimental continuum spectra are well produced by calculation as far as $\sim$17 MeV deuteron excitation energies in the direct reaction region.

**Experiment**

The experiments were performed at the TIARA facility of JAERI. A proton beam of 68 MeV from the AVF cyclotron was led to the HB-1 beam line. The energy distribution of light ions emitted from the target was measured using a AE-E counter telescope, which consisted of two thin silicon AE-detectors and a CsI(Tl) E-detector with a photodiode readout. Details of the experimental procedure and the results have been reported in Harada et al. (2002).

**Theoretical Calculations**

**The Glimpses of the Theoretical Basis**

Generally, the spectra of the emitted particles from one nucleon transfer reactions can be divided into three types, direct, pre-equilibrium, and evaporation processes, because of different mechanisms and the time lengths of events involved. The spectrum of the evaporation process results from three processes. These are absorption of the incident particles by the target nucleus, formation of a compound nucleus and emission of nucleons, or light particles from a highly excited state of the compound nucleus, or the decay of the compound nucleus. The direct reaction process occurs with a low momentum transfer condition through interactions between the incident particles and surface nucleons in the target nucleus. Here, the energy of the emitted particles is higher, and the residual nucleus remains in lower excitation energy. A relatively flat spectrum, in the pre-equilibrium region is observed between the evaporation and direct reaction regions. The former results from multi-step, direct and/or compound nuclear reactions. The pre-equilibrium processes progress step by step through the interactions between incident nucleons and nucleons within the target nucleus.

The theoretical calculations of the double differential cross-sections, a function both of solid angle and incident energy, have been performed, considering a direct reaction model as an incoherent summation of the direct reaction components. The double differential cross-section, $\frac{d^2\sigma}{d\Omega dE}$, consists of three factors (Sultana and Rahman, 2021), a normalization constant, spin-weighted spectroscopic factor $C^2S_{lj}(E)$, where the energy dependence is taken care of by an asymmetric Lorentzian function, $f_{ij}(E)$ (Matoba et al., 1995 and 1996; Mahaux and Sartor, 1989 and 1991) and the $\frac{d\sigma}{d\Omega}$, the DWBA based cross-section calculated by the code DWUCK-4 (Kunz, unpublished). The sum rule of the spectroscopic factors of nucleon orbits for $T \pm 1/2$ isospin states is estimated with a simple shell model prescription (French and Macfarlane, 1961): 

$$C^2S_{lj} = \frac{n_p(l,j)}{2T+1}$$  \hspace{1cm} (1)

Here, $n_p(l,j)$ is the number of protons for each $l,j$ orbit, and $T$ is the target isospin.

The strength function, $f_{ij}(E)$ is of the form

$$f_{ij}(E) = \frac{n_e}{2E} \frac{\Gamma(E)}{(E-E_F-E_{lj})^2+\Gamma^2/4}$$  \hspace{1cm} (2)

where, $n_e$ is the renormalization constant. The Fermi energy $E_F$ is calculated by an empirical formula given by Hisamochi et al. (Hisamochi et al.,1993). The sum rule of the spectroscopic factor and the
centroid energies \((E_{lj})\) for \(j = l \pm \frac{1}{2}\) shell orbits have been estimated using a BCS calculation (Bardeen et al., 1957). The required single particle energies are calculated following Bohr and Mottelson (Bohr and Mottelson, 1996). The strength function \(f_{lj}(E)\) integrates over the energy integrand \(dE\) to unity (Sultana and Rahman, 2021), starting from a certain initial to a final energy of the ejectile, deuterons. The spreading width \(\Gamma(E)\) is expressed by a function proposed by Brown and Rho (Brown and Rho, 1981) and also by Mahaux and Sartor (Mahaux and Sartor, 1989 and 1991) as:

\[
\Gamma(E) = \frac{\epsilon_0 (E - E_F)^2}{(E - E_F)^2 + E_0^2} + \frac{\epsilon_1 (E - E_F)^2}{(E - E_F)^2 + E_1^2}
\]

where, \(\epsilon_0, \epsilon_1, E_0,\) and \(E_1\) are constants and describe the effects of nuclear damping in the nucleus (Matoba et al., 1995). The estimated values of the constants are taken from Matoba et al. (1995).

**Results and Discussion**

Double differential cross-sections (DDXs) have been analyzed for the \(^{58}\text{Ni}(p,d)^{57}\text{Ni}\) reaction at 68 MeV incident proton energy, as shown in Figs. 1-5. The laboratory angles are 25°, 30°, 35°, 45°, and 60° for this work. The data are binned in 500 keV intervals. Experimental and theoretical results are given by the circles and lines, respectively. Three global potentials (Becchetti and Greenlees, 1969; Koning and Delaroche, 2003; Menet et al., 1971) are used here for protons, while for deuteron, an adiabatic potential from Koning and Delaroche (Koning and Delaroche, 2003; Menet et al., 1971) are used for the peak production and shape at 30° angle. But for Becchetti and Greenlees’s (Becchetti and Greenlees, 1969) potential, the calculated DDXs are somewhat underestimated compared with the experimental one.

In Fig. 1, at a 25° angle, the theoretical DDXs are in excellent agreement with the experimental one with Koning and Delaroche (Koning and Delaroche, 2003) potential and those using the Becchetti and Greenlees (Becchetti and Greenlees, 1969) and Menet et al. (Menet et al., 1971) potentials, the theoretical DDXs are in good agreement. The shape of the calculated spectra is in good agreement with experimental ones for all three potentials (Becchetti and Greenlees, 1969; Koning and Delaroche, 2003; Menet et al., 1971) throughout the direct reaction region, i.e., up to \(\sim 17.79\) MeV excitation energies.

Fig. 2 shows that the calculated DDX spectra are in good agreement with the experimental one using the Koning and Delaroche (Koning and Delaroche, 2003) and Menet et al. (Menet et al., 1971) potentials, both for the peak production and shape at 30° angle. But for Becchetti and Greenlees’s (Becchetti and Greenlees, 1969) potential, the calculated DDXs are somewhat underestimated compared with the experimental one.
Fig. 3 shows that the calculated DDX spectra are in very good agreement with the experimental one with Menet et al. (Menet et al., 1971) and Koning and Delaroche (Koning and Delaroche, 2003) potentials, both for the peak production and shape at 35° angle while for Beccheti and Greenlees (Becchetti and Greenlees, 1969) potential, the calculated DDX is a little bit underestimated at higher excitation energies (∼12 − 17.57 MeV).

It is observed that there is somewhat underestimation of the theoretically calculated DDXs at the peak values of the experimental cross-sections for every angle (25°, 30°, 35°, 45°, and 60°) at the lower excitation energies. This may be due to contributions of some non-pick-up reactions like the multi-step reaction process, which has not been considered in the present calculation.

In Figs. 4 and 5, the shapes of the theoretical DDXs are reproduced well with Menet et al. (Menet et al., 1971) potential as compared with the experimental ones both for 45° and 60° angles, respectively. But, the theoretical DDXs are somewhat underestimated with the Becchetti and Greenlees (Becchetti and Greenlees, 1969) and Koning and Delaroche (Koning and Delaroche, 2003) potentials.

Generally, the shapes of the continuum spectra are well reproduced, and the absolute values of cross sections are in good agreement with and very close to the experimental data in the direct reaction region. Nevertheless, there is still further scope for improvement by using different optical model potentials for proton and deuteron to overcome the minor discrepancies between theoretical and experimental DDXs in the direct reaction region.
Table 1. Optical model parameters were used in the DWBA calculations for the \(^{58}\text{Ni}(p,d)^{57}\text{Ni}\) reaction at 68 MeV.

**Becchetti and Greenlees potential (Becchetti and Greenlees, 1969)**

| Particle | \(V\) (MeV) | \(r\) (fm) | \(a\) (fm) | \(r_c\) (fm) | \(W_v\) (MeV) | \(W_s\) (MeV) | \(r'\) (fm) | \(a'\) (fm) | \(V_{so}\) (MeV) | \(r_{so}\) (fm) | \(a_{so}\) (fm) |
|----------|-------------|-------------|-------------|-------------|---------------|---------------|-------------|-------------|----------------|---------------|---------------|
| Proton   | 35.96       | 1.17        | 0.75        | 1.25        | 12.26         | 0.00          | 1.32        | 0.53        | 6.20           | 1.01          | 0.75          |
| Deuteron | \(a\) 1.17  | 0.78        | 1.25        | \(b\)       | 1.29          | 0.58          | 6.20        | 1.06        | 0.75          |
| Neutron  | \(c\) 1.25  | 0.65        |             |             |               |               |             |             |                |               |               |

\(^a\)\(V = 110.3 - 0.64(E_d/2) + 0.4Z/A^{1/3}\) (MeV)
\(^b\)\(W_v = 0.44(E_d/2) - 4.26\) (MeV), \(W_s = 24.8 - 0.50(E_d/2)\) (MeV), \(E_d\) is the deuteron kinetic energy.

**Koning and Delaroche potential (Koning and Delaroche, 2003)**

| Particle | \(V\) (MeV) | \(r\) (fm) | \(a\) (fm) | \(r_c\) (fm) | \(W_v\) (MeV) | \(W_s\) (MeV) | \(r'\) (fm) | \(a'\) (fm) | \(V_{so}\) (MeV) | \(r_{so}\) (fm) | \(a_{so}\) (fm) |
|----------|-------------|-------------|-------------|-------------|---------------|---------------|-------------|-------------|----------------|---------------|---------------|
| Proton   | 32.11       | 1.20        | 0.67        | 1.26        | 7.36          | 3.11          | 1.28        | 0.54        | 4.51           | 1.02          | 0.59          |
| Deuteron | \(a\) 1.20  | 0.67        | 1.26        | \(a\)       | 1.28          | 0.54          | \(a\)       | 1.02        | 0.59          |
| Neutron  | \(b\) 1.25  | 0.65        |             |             |               |               |             |             |                |               |               |

\(^a\)Adiabatic potentials with those of (Koning and Delaroche, 2003)
\(^b\)Well depth adjusted to fit the separation energy.

**Menet potential (Menet et al., 1971)**

| Particle | \(V\) (MeV) | \(r\) (fm) | \(a\) (fm) | \(r_c\) (fm) | \(W_v\) (MeV) | \(W_s\) (MeV) | \(r'\) (fm) | \(a'\) (fm) | \(V_{so}\) (MeV) | \(r_{so}\) (fm) | \(a_{so}\) (fm) |
|----------|-------------|-------------|-------------|-------------|---------------|---------------|-------------|-------------|----------------|---------------|---------------|
| Proton   | 38.74       | 1.16        | 0.75        | 1.25        | 7.32          | 1.34          | 1.37        | 0.23        | 6.04           | 1.06          | 0.75          |
| Deuteron | \(a\) 1.16  | 0.75        | 1.25        | \(b\)       | 1.37          | \(c\)         | 6.04        | 1.06        | 0.75          |
| Neutron  | \(d\) 1.25  | 0.65        |             |             |               |               |             |             |                |               |               |

\(^a\)\(V = 99.8 - 0.44(E_d/2) + 0.4Z/A^{1/3}\) (MeV)
\(^b\)\(W_v = 2.4 + 0.18(E_d/2)\) (MeV), \(W_s = 8.40 - 0.10(E_d/2)\) (MeV), \(E_d\) is the deuteron kinetic energy.
\(^a'\)\(a' = 0.74 - 0.008(E_d/2) + 1.0(N-Z)/2A\), \(^d\)Well depth adjusted to fit the separation energy.

For all potentials nonlocality, finite-range parameters and spin-orbit term are shown below:

| Particle | Nonlocality parameters (fm) | Finite-range parameter (fm) | Thomas-Fermi spin orbit term |
|----------|-----------------------------|-----------------------------|-----------------------------|
| Proton   | 0.85                        | 0.621                       | \(\lambda = 25\)            |
| Neutron  | 0.85                        | 0.621                       |                             |
| Deuteron | 0.54                        |                             |                             |
Conclusions
Nuclear reaction double differential cross sections have been analyzed for the proton-induced reactions on $^{58}$Ni at 68 MeV. Three global potentials for proton and deuteron, including the adiabatic potentials for deuteron, are used here to analyze the spectra for the confirmation of the globality of this theoretical model. The calculated spectra show a good agreement with the experimental data both in magnitude and shape. The calculation of theoretical values of spectra without using any arbitrary renormalization of the cross-sections renders this theoretical method more applicable for nuclear data analysis.

Acknowledgment
One of the authors (Dr. S. A. Sultana) is grateful to the Ikeda Laboratory, Department of Applied Quantum Physics and Nuclear Engineering, Kyushu University, Japan, for providing a necessary theoretical base to analyze this work and to JAERI, Japan, for allowing to use the experimental data measured at the TIARA facility.

References
Aramaki F, Syafarudin, Wakabayashi G, Uozumi Y, Ikeda N, Matoba M, Sakae T and Koori N. $^{100}$Mo(p,d)$^{99}$Mo reaction at 50 MeV and direct reaction analysis. In: Proceedings of the 2002 Symposium on Nuclear Data. 2002; JAERI-Conf. 2002-006: 178-182.
Bardeen J, Cooper LN and Schrieffer JR. Theory of superconductivity. Phys. Rev. 1957; 108(5): 1175-1204.
Becchetti FD, Jr and Greenlees GW. Nucleonnucleus optical-model parameters, A>40, E<50 MeV, Phys. Rev. 1969; 18(4): 1190-1209.
Bohr A and Mottelson BR. Nuclear Structure. Volume 1; Single-Particle Motion, Appendix 2D: Model for Strength Function Phenomena. W. A. Benjamin, Inc., New York. 1996; 302-307.
Brown GE and Rho M. The giant Gamow-Teller resonance. Nucl. Phys. A. 1981; 372: 397-417.
Crawley GM. Proceeding of the International Symposium on Highly Excited States in Nuclear Reactions, Osaka, Japan. 1980; 590-611.
French JB and Macfarlane MH. Isobaric-spin splitting of single-particle resonances. Nucl. Phys. 1961; 26: 168-176.
Gales S, Stoyanov Ch and Vdovin AI. Damping of high-lying single-particle modes in heavy nuclei, Phys. Rep. 1988; 166 (3): 125-193.
Grudzevich O, Martirosyan J and Yavshits S. Complete files of neutron- and proton-induced nuclear data to 1 GeV for 208Pb target. Proceedings of the International Conference on Nuclear Data for Science and Technology, EDP Sciences. 2007; 2: 1213-1215.
Harada M, Watanabe Y, Tanaka Y, Matsuoka Y, Shin K, Meigo S, Takada H, Sasa T, Iwamoto O, Fukahori T, Chiba S and Tanaka S. Light charged-particle production in proton-induced reactions on $^{12}$C, $^{27}$Al, $^{58}$Ni, $^{90}$Zr, $^{197}$Au, and $^{209}$Bi at 42 and 68 MeV. J. Nucl. Sci. Technol. (Suppl. 2) 2002; 39: 393-396.
Hisamochi K, Iwamoto O, Kisanuki A, Budihardjo S, Widodo S, Nohtomi A, Uozumi Y, Sakae T, Matoba M, Nakano M, Maki T, Matsuki S and Koori N. Hole strengths and spreading widths observed in reaction at 65 MeV. Nucl. Phys. A. 1993; 564: 227-251.
Ignatyuk AV. Statistical theory of nuclear reactions. ICTP-IAEA Joint Workshop on Nuclear Data for Science and Technology: Medical Applications. 2013.
Koning AJ and Delaroche JP. Local and global nucleon optical models from 1 keV to 200 MeV. Nucl. Phys. A. 2003; 713: 231-310.
Kunz, P.D., 1974. DWBA code DWUCK- 4, University of Colorado (Unpublished).
Lewis MB. Effects of spreading widths upon the direct nuclear reaction continuum. Phys. Rev. C. 1975; 11: 145-153.
Mahaux C and Sartor R. From scattering to very deeply bound neutrons in 208Pb: Extended and improved moment approaches. Nucl. Phys. A. 1989; 493: 157-223.
Mahaux C and Sartor R. Single particle motion in nuclei. In: Advances in Nuclear Physic. Negele JW, Vogt E. (eds). Vol. 20, Springer, Boston, MA, 1991.
Sultana and Rahman/J. Bangladesh Acad. Sci. 46(1); 57-63: June 2022

Matoba M, Iwamoto O, Uozumi Y, Sakae T, Koori N, Ohgaki H, Kugimiya H, Ijiri H, Maki T and Nakano M. Fragmentation of neutron-hole strengths in $^{59}$Ni observed in the $^{60}$Ni(p,d)$^{59}$Ni reaction at 65 MeV. Nucl. Phys. A. 1995; 581: 21-41.

Matoba M, Kurohmaru K, Iwamoto O, Nohtomi A, Uozumi Y, Sakae T, Koori N, Ohgaki H, Ijiri H, Maki T, Nakano M and Sen Gupta HM. Analysis of the $^{59}$Ni(p,d)$^{58}$Ni reaction on $^{60}$Ni at 65 MeV. Phys. Rev. C. 1996; 53(4): 1792-1803.

Menet JJH, Gross EE, Malanify JJ and Zucker A. Total reaction cross-section measurements for 30-60 MeV protons and the imaginary optical potential. Phys. Rev. C. 1971; 4(4): 1114-1129.

Sultana SA and Hossain SM. Analysis of continuum spectra for the $^{58}$Ni(p,d)$^{57}$Ni reaction with direct reaction model. Nucl. Sci. Appl., 2016; 25 (1 & 2): 41-44.

Sultana SA and Intiaza A. Study of 27Al(p,d)26Al Reaction at 42 MeV. Nucl. Sci. Appl., 2017; 26 (1&2): 29-32.

Sultana SA and Rahman M A. Study of (p,d) reaction on $^{209}$Bi at 42 MeV. J. Bangladesh Acad. Sci. 2021; 45(2): 197-203.

Sultana SA, Kin T, Wakabayashi G, Uozumi Y, Ikeda N, Watanabe Y and Matoba M. Analysis of continuum spectra for proton induced reactions on $^{27}$Al, $^{58}$Ni, $^{90}$Zr, $^{197}$Au and $^{208}$Bi at Incident energies 42 and 68 MeV-Direct reaction model analysis. In: Proceedings of the 2005 Symposium on Nuclear Data, JAEA-Conference. 2005; 2006-009: 185-189.

Sultana SA, Maki D, Wakabayashi G, Uozumi Y, Ikeda N, Syafarudin, Aramaki F, Kawaguchi T, Matoba M and Sen Gupta HM. The $^{96}$Mo(pd)$^{95}$Mo $^{96}$Mo(p→d)$^{95}$Mo reaction at 50 MeV. Phys. Rev. C. 2004; 70(3): 034612(1)-034612(15).

Sultana SA, Sarker DR, Wakabayashi G, Uozumi Y, Ikeda N and Matoba M. Analysis of $^{197}$Au(pd)$^{196}$Au reaction with direct reaction model. Nucl. Sci. Appl. 2009; 18(1): 24-28.

Sultana SA Syafarudin, Aramaki F, Maki D, Wakabayashi G, Uozumi Y, Ikeda N, Matoba M, Watanabe Y and Sen Gupta HM. Analysis of continuum spectra of (n,d) reactions with direct reaction model. Proceeding of the 2003 Symposium on Nuclear Data of JAERI-Conf. 2004-005, November 27-28, 2003; 133-137.

Sultana SA, Syafarudin, Maki D, Kin T, Wakabayashi G, Uozumi Y, Ikeda N, Watanabe Y, Aramaki F, Matoba M, Kawaguchi T, and Sen Gupta HM. Continuum spectra analysis of (p,d) and (n,d) reactions on Bi in several Tens of MeV energy region. In: Proceeding of 2004 Symposium on Nuclear Data of JAERI-Conference. 2005-003, 2004; 143-147.

Sultana SA, Wakabayashi G, Uozumi Y, Ikeda N and Matoba M. Analysis of the 209Bi(n,d)208Pb reaction with direct reaction model. Nucl. Sci. Appl. 2010; 19(2): 27-30.

Sultana SA. Continuum spectra in one-nucleon transfer reaction- $^{209}$Bi(n,d)$^{208}$Pb reaction, Ep=41.0-53.5 MeV. Nucl. Sci. Appl. 2006; 15(1): 53-57.

Sultana SA. Study of the $^{58}$Ni(p,d)$^{57}$Ni reaction with direct reaction model. Nucl. Sci. Appl., 2017; 26 (1&2): 45-48.

Syafarudin, Aramaki F, Wakabayashi G, Uozumi Y, Ikeda N, Matoba M, Yamaguchi K, Sakae T, Koori N and Maki T. Continuum spectra in one nucleon transfer reactions—(p,d) reactions at medium energy region. J. Nucl. Sci. Technol. (Suppl. 2), 2002; 1: 377-380.

U.S. Department of Energy. White Paper on Nuclear Data Needs and Capabilities for Basic Science. Office of Science, US Nuclear Data Program. 2016, August 10-11; 6.