Cosmic Web-halo Connection between Twin Universes

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Abstract

Both simulation and observational data have shown that the spin and shape of dark matter halos are correlated with their nearby large-scale environment. As structure formation on different scales is strongly coupled, it is tricky to disentangle the formation of a halo from that of the large-scale environment, making it difficult to infer which is the driving force behind the correlation between halo spin/shape and the large-scale structure. In this paper, we use N-body simulations to produce twin universes that share the same initial conditions on small scales but that are different on large scales. This is achieved by changing the random seeds for the phase of those k modes smaller than a given scale in the initial conditions. In this way, we are able to disentangle the formation of halo and large-scale structure, making it possible to investigate how halo spin and shape correspond to the change of environment on large scales. We identify matching halo pairs in the twin simulations as those sharing the maximum number of identical particles within them. Using these matched halo pairs, we study the cross match of halo spin and the correlation with the large-scale structure. It is found that when the large-scale environment changes (eigenvector) between the twin simulations, the halo spin has to rotate accordingly, although not significantly, to maintain the universal correlation seen in each simulation. Our results suggest that the large-scale structure is the main factor to drive the correlation between halo properties and their environment.

Unified Astronomy Thesaurus concepts: Large-scale structure of the universe (902); Cold dark matter (265); Computational methods (1865); Astrostatistics (1882); Galaxy evolution (594)

1. Introduction

In the most widely accepted picture of structure formation, the initial seeds of gravitational instability are inherited from the adiabatic density fluctuations in the Big Bang, which can be well measured from the Cosmic Microwave Background (Planck Collaboration et al. 2014). After the recombination era, the competition between the gravitational force and cosmic expansion leads to the formation of dark matter halos at small scales in the regions where the linear density contrast surpasses the threshold (Gunn & Gott 1972). On large scales, both observations of galaxy surveys, such as the 2dF Galaxy Redshift Survey, (Colless et al. 2003) and the Sloan Digital Sky Survey (York et al. 2000), and numerical simulations (e.g., Davis et al. 1985) have revealed that the matter distribution can be characterized as a cosmic web (e.g., voids, walls, filaments, and clusters). As dark matter halos are residing in the cosmic web, one would expect that properties of dark matter halos are correlated with their surrounding cosmic environments or large-scale structures (hereafter LSS).

The correlation between halo shape/spin and LSS has been extensively investigated in detail (see Tables 1 and 2 in Forero-Romero et al. 2014 and Table 1 in Wang et al. 2018) in the last two decades in both observational and theoretical work. On the theory side, many studies that used N-body and hydrodynamical simulations have almost converged on the conclusion that the major axes of halos/galaxies tend to be parallel with the slowest collapse direction of LSS, and the vector of angular momentum (spin) of halos/galaxies is preferentially either aligned or perpendicular to the LSS, depending on the halo/galaxy mass (e.g., Bailin & Steinmetz 2005; Aragón-Calvo et al. 2007; Hahn et al. 2007a, 2007b; Zhang et al. 2009, 2009; Trowland et al. 2013; Dubois et al. 2014; Codis et al. 2016; Wang & Kang 2017; Codis et al. 2018; Wang & Kang 2018; Wang et al. 2018; Ganeshiaiah Veena et al. 2019; Kraljic et al. 2020; López et al. 2021).

Along with the theoretical efforts, observational studies have focused on testing the results from simulations about the correlations between the shape and spin of a galaxy with the large-scale structure. Using galaxy shape as a proxy for galaxy spin, several studies (e.g., Zhang et al. 2009; Tempel & Libeskind 2013; Tempel & Libeskind 2013; Zhang et al. 2015; Pawa et al. 2016; Motloch et al. 2021; Welker et al. 2020) claimed that the spins of low-mass, spiral galaxies tend to align with the LSS, while high-mass, elliptical galaxies are preferentially perpendicular to the LSS, which is consistent with the simulation results. Some investigations (Zhang et al. 2013) also found that red central galaxies show the strongest alignment signal between galaxy major axes with the host filaments or walls.

Generally speaking, the spin of a dark matter halo is the composition of the linear collapse of the cosmological density field during the early stages and the nonlinear collapse and mergers with other halos during the late stages. The former part can be well explained by the linear theory of the large-scale tidal field, i.e., tidal torque theory (Peebles 1969; White 1984). However, the later nonlinear part, which can only be studied with the help of N-body simulations, could be more dominant in the final angular momentum acquisition (van Haarlem & van de Weygaert 1993; Maller et al. 2002; Vitvitska et al. 2002). For a long time, the correlations between halo properties and the LSS environment were usually explored in a statistical way using halo populations as whole. If we focus on the merger...
history of every single halo, the individual differences can blur any conclusion that can be drawn. That said, if we want to study the effect of the LSS direction or the environment, another halo sample with different merger histories has to be chosen from the simulation. It is thus difficult to study the effects of the halo and environment separately. The degeneracy of the LSS environment and nonlinear factors originating from small scales weaken our understanding of these halo-LSS conclusions mentioned above. Consequently, it is necessary to set up a controlled N-body simulation in which we can somehow manipulate the large-scale environment of the halo and see how properties of given halos correspond to the changes of their LSS environments. In this work, we implement twin N-body simulations (explained in detail in Section 2) to show how the spin (and major axis) of matching halos evolves when the $e_3$ vector (i.e., the least compressed direction) changes its direction.

This paper is organized as follows. In Section 2 we present the method to produce twin universes using N-body simulation data and how to identify identical or matching dark matter halos. In Section 3 we present the main results of the work. As a first step, we show the halo mass function and the correlations between halo spin/major axis and the $e_3$ vector of the LSS to check if the twin simulations are equal statistically. We then show how the spin/major axis of a halo changes with a change in the LSS environment. Lastly, the influence of LSS on halo shape parameters ($S$, $Q$, and $T$; see their definition in Section 3) will be explored. Conclusions and discussions are presented in Section 4.

2. Methods and Simulations

According to linear theory, the power spectrum of cosmic density perturbations and the two-point correlation function are a Fourier transform pair, which can be expressed as follows:

$$\xi(x) = \frac{1}{(2\pi)^3} \int P(k) e^{i k \cdot x} d^3 k,$$

(1)

where $P(k) = V_u \langle |\delta_k|^2 \rangle$ is the power spectrum, $x = |x|$ is the distance, $k$ is the wavenumber, and $V_u$ is the volume we consider. This relation often holds true for $V_u \rightarrow \infty$, or equivalently for $k \rightarrow 0$. This indicates that the information of the LSS in the late stages is totally contained in the small $k$ of the initial overdensity field $\delta(k)$, just like fingerprints.

In the most favored frame for the generation of ΛCDM initial conditions, the power spectrum and the phase information are encapsulated as a Gaussian white noise field (Salmon 1996), which is particularly convenient to work with numerically. One can expect that the whole LSS pattern will change severely if we replace the random seeds for the phase of those $k$ modes smaller than a given scale, while the formation of dark matter halos on small scales will not be significantly affected. Besides, with the help of an octree basis function (Jenkins 2013), the LSS controlled initial conditions can also be generated in real space (Sawala et al. 2021). This kind of technique paves a new way to manipulate the LSS environment of dark matter halos by a couple of N-body simulations.

In this work, we produce twin N-body simulations L500N1024, L500N1024a, L500N1024b, and L500N1024c (hereafter N1024, N1024a, N1024b, and N1024c). The letters “a/b/c” here mark the three simulations for which the random seeds for small $k$ in the initial conditions have been changed; i.e., we replace the random seeds for the phase of $k$ mode such that:

$$|k| < \frac{2\pi}{L_0}.$$  

(2)

Note that $L_0$ is an arbitrarily selected scale (discussed later) and should not be confused with the box size of the simulation. In addition, we also run an independent simulation, L500N1024r (hereafter N1024r), as a reference, for which the random seed for the phase of all $k$ modes is completely different from that used in the twin simulations. This reference simulation is only used to test the stochastic background of our method.

For the N-body simulations, the initial conditions are set at redshift $z = 127$. At this initial time, the dark matter particles that later settle into a typical halo (with a mass around $10^{13} M_\odot$ at $z = 0$) occupied an initial volume of about several cubic megaparsecs. After some simple tests, we choose $L_0 = 20 h^{-1}$ Mpc. This scale is selected to produce significant changes of large-scale environments for most halos, but ensures halo identities (more explicit definition is in Section 2.1) are not severely affected. Here we remind the reader that, in terms of statistical properties, all twin simulations are identical. Unless otherwise specified, all figures in this paper are plotted in a symmetric way; i.e., the conclusions do not change if we switch the N-body simulations. That is why we call them twin simulations.

Our simulations were run using the P-Gadget-3 code, a TreePM code based on the publicly available code Gadget-2 (Springel 2005), and they all contain 1024$^3$ dark matter particles in a periodic box of 500 $h^{-1}$ Mpc. The cosmological parameters are chosen from the WMAP7 data (Komatsu et al. 2011), namely: $\Omega_{\Lambda} = 0.7274$, $\Omega_m = 0.2726$, $h = 0.704$, and $\sigma_8 = 0.821$. The mass of a single particle in our simulation is $8.8 \times 10^9 h^{-1} M_\odot$, and 136 snapshots are stored from $z = 127$ to $z = 0$. We identify dark matter halos at each snapshot using the standard friend-of-friend (FoF) algorithm (Davis et al. 1985) with a linking length that is 0.2 times the mean interparticle separation. To ensure relatively robust results of halo spin and shape, and to avoid the spurious matches to the same halo purely by chance, the main halos containing at least 100 particles are chosen for further analysis. For each FoF halo, we identify subhalos using the SUBFIND algorithm (Springel et al. 2001) and construct merger trees for N1024 and N1024c, respectively. Around $\sim 568,000$ main halo candidates were identified from each simulation (see Table 1).

### 2.1. Match Ratio and Matching Halos

There is no unique way to match halos across N-body simulations. For example, Sawala et al. (2021) use the halo’s 50 most bound particles and a mass criterion to select matching halos. Here, we simply define the match ratio between halos from the twin N-body simulations as Equation (3). Supposing
main halo $A$ from N1024 contains $a$ dark matter particles, main halo $B$ from N1024a contains $b$ particles, and they have $c$ common particles according to the particle ids; then, the match ratio is defined as:

$$R(A, B) = \frac{c^2}{ab}.$$  \tag{3}

Generally speaking, when the random seeds for the phase of small $k$ are changed, a dark matter halo in N1024 may go through different evolutionary paths, such as splitting into a few FoF groups, merging with other halos, or simply disappearing. In other words, main halo $A$ may be linked to multiple corresponding halos in N1024a. Each of them has a match ratio with main halo $A$ according to Equation (3), and the one with the largest ratio will be chosen as the matching halo of $A$ in N1024a (and vice versa). To ensure that there are enough matched halos for a statistical analysis, we set an arbitrary threshold to $R(A, B) \geq 0.4$ for matching halo pairs, which roughly corresponds to each halo containing more than 65% of its matching halo. After this selection, 157,748 matching halo pairs were identified from the N1024 and N1024a simulations at $z=0$. More information about the number of identified matching halo pairs is listed in Tables 1 and 2. Moreover, the numbers of main halos containing at least 100 particles at $z=2$ and $z=1$ are also listed.

As mentioned in Section 1, once the sample of matching halos between the twin N-body simulations is established, we can study the halo—LSS correlations. We employ the Hessian matrix method used in many previous articles (e.g., Hahn et al. 2007a, 2007b; Libeskind et al. 2009; Kang & Wang 2015) to define the matrix as:

$$H_{ij} = \frac{\partial^2 \rho_i(x)}{\partial x_j \partial x_i}.$$  \tag{4}

This method is based on the smoothed density field $\rho_i(x)$ at the halo position (based on a more accurate and improved algorithm by Wang et al. 2020), which can be given by the Cloud-in-Cell (CIC) technique (MacNeice 1995). The smoothing length $R_s$ is the only parameter of the CIC, which can be regarded as the typical scale of a halo LSS environment identified by the Hessian matrix method. Many previous works (e.g., Aragón-Calvo et al. 2007; Hahn et al. 2007a; Zhang et al. 2009; Codis et al. 2012; Hoffman et al. 2012; Libeskind et al. 2013; Trowland et al. 2013, but see also Libeskind et al. 2014) used a constant smoothing length. To determine which $R_s$ value should be chosen, we test some LSS properties (e.g., the environment and eigenvectors) of matching halos for three fixed $R_s$: 2.5, 5, and 10 h$^{-1}$ Mpc. We find that the halo—LSS correlation, as well as other main conclusions we make below, become stronger as $R_s$ decreases, which is a reasonable result according to our previous discussions. Consequently $R_s = 2.5 h^{-1}$ Mpc is chosen hereafter. The three eigenvalues of the Hessian matrix are marked as $\lambda_1$, $\lambda_2$, and $\lambda_3$, with corresponding eigenvectors $e_1$, $e_2$, and $e_3$. Eigenvalues $\lambda_i$ can be used to define the LSS environment of dark matter halos according to the number of positive eigenvalues (Zel’dovich 1970; Hahn et al. 2007a, 2007b; Libeskind et al. 2018; Zhang & Yang 2019); i.e.,

1. void: $0 < \lambda_1 < \lambda_2 < \lambda_3$
2. wall: $\lambda_1 < 0 < \lambda_2 < \lambda_3$
3. filament: $\lambda_1 < \lambda_2 < 0 < \lambda_3$
4. cluster: $\lambda_1 < \lambda_2 < \lambda_3 < 0$

and the eigenvectors $e_i$ stand for the three compressed directions of the smoothed density field. The $e_3$ vector indicates the least compressed direction, which is a robust and universal definition of the LSS. In this work, we will focus on the alignment of halo spin and shape with the $e_3$ vector; i.e., $\cos \theta_3 = a \cdot e_3$, where $a$ is the halo spin or major-axis vector.

Figure 1 displays a 100 h$^{-1}$ Mpc $\times$ 100 h$^{-1}$ Mpc $\times$ 20 h$^{-1}$ Mpc slice from the twin simulations N1024 and N1024a. It presents a global view of the twin simulations. Over two hundred matching halo pairs are identified in this displayed slice, but here we only mark nine matching halos with hollow circles, with sizes proportional to their virial radii. We mark their main progenitors in the upper panels if and only if they are still matched at $z=1$. We can see that the LSS environment of these samples really changes tremendously after we replaced the random seeds for the small $k$ of the initial overdensity field.

### 3. Results

Our main results, mostly regarding the influence of the LSS environment transformation on the properties of dark matter halos, will be shown in this section. Before that, we show some statistical results, including the halo mass function and correlations between halo properties (spin and major axis) and the LSS environment, to test whether the twin simulations are statistically equal. Then we will focus on the mass dependence of the match ratio $R$ in different LSS environments. Lastly, the influence of LSS on the total spin, major axis, and shape parameters of the matching halos will be explored. Specifically, the shape parameters of dark matter halos, $S$, $Q$, and $T$, are defined by:

$$S = \frac{c}{a}, \quad Q = \frac{b}{a}, \quad T = \frac{a^2 - b^2}{a^2 - c^2},$$  \tag{5}

where $a$ is the major axis of a modeled ellipsoid (by calculating the inertia tensor) of an FoF halo, $b$ is the intermediate axis, and $c$ is the minor axis. The parameters $S$ and $Q$ give the ellipticity of dark matter halos, and $T$ quantifies the triaxiality (Franx et al. 1991;
Warren et al. (1992). Purely prolate halos have $T = 1$ and purely oblate halos have $T = 0$.

### 3.1. Statistical Tests

First, we show the halo mass function as an important statistical test of our twin simulations. Since the amplitude of the initial power spectrum of the twin simulations is exactly the same, one would expect that the difference between their halo mass functions is very small. However, each $N$-body simulation has a finite box size, hence the cosmic variance between different simulations will produce different mass functions. Figure 2 shows the halo mass functions of N1024, N1024a, N1024,b and N1024c. As we can see, all curves are nearly identical except at the high-mass end, which can be explained by the effect of cosmic variance (well within the range of the gray shadows). Because of the same reason, similar figures of the statistical properties of the twin simulations give nearly the same result, so we will not distinguish them hereafter.

In Figure 3 we show another statistical test regarding the mass dependence of correlations between halo spin or halo major axis with the $e_3$ vector for matching halo pairs in the twin simulations. The upper two panels are for halo spin, while the lower two panels are for halo major axis. This figure shows that the correlations between the twin simulations agree with each other within the $1\sigma$ scatter, showing again that the twin simulations are statistically equivalent. In the left panels the halo spin and major axis are calculated using all particles within the virial radius of the halo. The trend of mass dependence of spin–$e_3$ correlation agrees well with previous works (Codis et al. 2012; Pichon et al. 2016; Lee 2019). Note that here we only use halos with more than 100 particles

![Figure 1](attachment:image1.png)  
**Figure 1.** Slices from twin simulations N1024 and N1024a, which are statistically equivalent. A few matching halo pairs found at $z = 0$ are shown in lower panels as colored circles with the radius scaled to the virial radius. Their corresponding progenitors at $z = 1$ are shown in upper panels. For matched halo pairs, their LSS environments are visually different in the twin simulations. Note that the matching halos with green and brown circles disappeared at $z \sim 1$, which means their progenitors are no longer matched at $z \sim 1$.  

![Figure 2](attachment:image2.png)  
**Figure 2.** Halo mass functions of N1024, N1024a, N1024,b and N1024c. As we can see, all curves are nearly identical except at the high-mass end, which can be explained by the effect of cosmic variance (well within the range of the gray shadows). Because of the same reason, similar figures of the statistical properties of the twin simulations give nearly the same result, so we will not distinguish them hereafter.
Figure 2. Comparison of the halo mass function in the twin simulations. The blue dashed line denotes the halo mass with 100 particles, and the gray shaded regions mark the 3σ scatter, i.e., the range from $[N - 3\sqrt{N}]$ to $N + 3\sqrt{N}$, where $N$ is the mean halo count in each mass bin. At all redshifts, the four curves are highly overlapped except at the high-mass end.

($\sim 10^{12} M_{\odot} h^{-1}$), roughly corresponding to the flip mass of the spin–LSS correlation found by previous works (e.g., Wang & Kang 2018, and references within).

In $N$-body simulations, the direction of halo spin is often slowly twisted along the halo radius; i.e., there is a misalignment between the inner spin and the total spin (e.g., Shao et al. 2021). Motivated by the fact that the baryons experience the same tidal forces as the dark matter, the spin of the galaxies embedded in halos usually follows the inner spin of the host halo. To ensure a relatively reliable result, we calculate the spin and major-axis vectors at 0.3 times the virial radius to describe the inner properties of matching halo hereafter. The right two panels of Figure 3 show the mass dependence of inner properties (i.e., spin and major axis at 0.3 times the virial radius). It is seen that, comparing to the alignment of total spin and $e_3$ vector, the correlation of inner spin and $e_3$ is nearly independent of the mass. However, as mentioned by Wang & Kang (2018), the halo spin–$e_3$ correlation is stronger at high redshift. Considering that the inner spin is inherited from the total spin at high redshift, the alignment signal of inner spin and $e_3$ vector is partly lost due to the nonlinear process in the center of high-mass halos at later times.

3.2. Match Ratio and Mergers

In this section we focus on the matching halo pairs between the twin simulations. First, in Figure 4, we show the mass dependence of match ratio $R$ in six twin simulation pairs (i.e., matched halos between N1024 and N1024a, N1024 and N1024b, N1024 and N1024c, and N1024a and N1024b, N1024a and N1024c, and N1024b and N1024c) at $z = 0$ and $z = 2$. The shaded region marks the maximum and minimum median match ratio, which roughly illustrate the systematic errors introduced by cosmic variance. For each twin simulation pair, the mass dependence of match ratio $R$ is for the first simulation. If the mass bins are sorted by the second one, as we mentioned in Section 2, the conclusions are still the same. The lower limit of the y-axis is set to 0.4, which corresponds the arbitrary threshold imposed to select matching halo pairs. It can be seen that the match ratio of low-mass halos is relatively higher, and comparing the same line of both panels, we also find that halos at high redshift usually have higher match ratios. For massive halos at lower redshift, the frequent mergers or mass accretion along the large-scale structure decrease the match ratio between the twin simulations. The yellow and red lines are for matching halos located in the different environments in both simulations. The median match ratio $R$ of halos in filaments decreases much faster compared to those in clusters, indicating that halos in filaments are more influenced by mass accretion (along the filament) than in clusters.

Here, we briefly explain how the value of match ratio is reduced in minor merger events. Suppose that halo $A$ merges with another halo $b$ in N1024, while the matching halo $A'$ and $b'$ in N1024a are not involved in any merger event. At the end of the minor merger, the dark matter particles of $A$ and $b$ mix together. The information about the matching halo pair $b$ and $b'$ in the later snapshots is completely lost, and the match ratio of halo $A$ and $A'$ will be reduced because of the external particles from halo $b$.

3.3. The Evolution of Spin with LSS

As we mentioned in Section 2.1, the $e_3$ obtained from the Hessian matrix indicates the direction of the LSS around a halo. For matching halo pairs in the twin simulations, we calculate the rotational angle of the corresponding eigenvectors between the twin simulation as,

$$\cos(\beta) = \frac{e_3 \cdot e'_3}{|e_3||e'_3|},$$

where $e_3$ and $e'_3$ are the corresponding eigenvectors of matching halo pairs. In Figure 5 the red histogram shows the distribution of the rotation angle between the $e_3$ vector in the twin simulation pair N1024 and N1024a. It is found that there is a higher probability to have halo pairs for which the rotation angle of $e_3$ is not significant, which suggests that although the power spectrum on large scales ($>20$ Mpc h$^{-1}$) is totally different in the twin simulations, the nearby local environment smoothed on 2.5 Mpc h$^{-1}$ is more or less maintained by the matched halo pairs. The black histogram shows a comparison with a larger smoothing length with $R_s = 5.0$ Mpc h$^{-1}$. In this case the rotation angle of the large-scale environment becomes larger, but still maintains some level of a parallel signal.

In this work we are mainly interested in how halo properties, spin or shape, correspond to the change of the large-scale environment, in particular, the direction of the LSS given by $e_3$. To clearly select matching halo pairs with a significant change of their large-scale environments, we select halo pairs from the twin simulations that have:

$$|\cos(\beta)| \leq 0.707 \quad \text{or} \quad \beta \geq 45^\circ.$$

In this way we are looking at halo pairs with significant rotation of their $e_3$ vectors. Of course, we can select halo pairs with larger rotation angles to increase the signal, but that will reduce the sample size.

Based on Figure 3, halos in twin simulations show the same spin–LSS correlation. We can then ask what will happen if the original spin (of a halo in, for example, N1024) does not change: can it still maintain a similar correlation with the new $e_3$ from other twin simulation? If not, it means that the spin needs to be changed systematically to match the correlations (as shown in Figure 3). This can prove that the correlation between spin and LSS is mainly affected by large-scale structures.
In Figure 6 we plot the distribution of the spin−LSS correlation at three different redshifts for halo pairs with larger rotation angles given by Equation (7). The red shaded areas are the probability distribution functions (PDFs) of halos whose spin and $e_3$ are from the same simulation N1024a. The gray histograms are the cross correlations of the halo spin−$e_3$ correlation, where halo spin is from N1024, but the $e_3$ vector is from N1024a. For matching halo pairs, the spin−LSS correlation in the same simulation, N1024a, is similar to that shown in Figure 3 and this is not surprising, as the selected halos are a subsample of those shown in Figure 3. It is interesting to note that the cross correlations (black histograms) are reversed, showing a different trend from the red histograms. This result shows that the rotation of $e_3$ vectors will lead to proper rotation of the halo spin to maintain the original halo spin−$e_3$ correlation seen in one simulation. It indicates that the halo spin is mainly affected by the orbital angular momentum of accreted mass from the LSS along the $e_3$ vector.

3.4. Matching Halos and $e_3$ Rotation Angle

From Section 3.3 we learn that the spin/major axis will adjust its direction appropriately when the $e_3$ vector changes across twin simulations. In this subsection we will explore this reaction in more detail. To maintain the spin−LSS correlation, one may intuitively expect that the rotation angle of halo spin−major axis will depend strongly on the rotation angle $\beta$ (see Section 3.3, Equation (6)). More specifically, we expect that the rotation angle of spin is larger for halos with larger $\beta$ so as to maintain the spin−$e_3$ relation. However, if we check the
correlation mentioned above, we will find that the rotation of spin/major axis is nearly independent of angle $\beta$. This is a bit surprising and it is related to the 3D space configuration of halo spin and the LSS. Even in 2D space, it is easy to configure a case where the $e_3$ vector changes significantly while the halo spin/major axis does not need any adjustment to maintain the same spin—LSS correlation. The situation is more complicated in the 3D case. To better show how this could happen, we introduce the cross-product between $e_3$ vector and spin/major axis as the normal vector of the halo—LSS system:

$$n = e_3 \times a,$$

(8)

where $a$ is the halo spin or major-axis vector. Under this definition, the rotation of the $e_3$ vector will lead to the corresponding rotation of the $n$ vector in most cases. In Figure 7 we show the relationship between the rotation angle of $e_3$ and the normal vector defined above. Here, we can see that a strong positive proportional relationship between the rotation angle $\beta$ and $n$ vectors for all redshifts; i.e., the change of $e_3$ vector will lead to a corresponding rotation of the whole halo—LSS system, in which the correlation between spin/major axis and $e_3$ vector always holds. This gives us a picture of how halo spin/major axis reacts to the change of the LSS. That said, the proportional relationship in both panels is nearly independent of the redshift (only the $n$ vector from the major axis shown in the right panel has a weak evolution from $z = 2$ to the present day). This implies a universality for the picture described above.

### 3.5. Halo Shape Parameters with the LSS

Some previous works (e.g., Hahn et al. 2007b; Morinaga & Ishiyama 2020) have revealed the correlations between halo shape and the LSS environment. In this work we also explore the impact of the LSS environment on the ellipticity and triaxiality of halos ($M_{h} > 10^{12} h^{-1} M_{\odot}$). In Figure 8 the mass dependence of three shape parameters $S, Q,$ and $T$ at $z = 0$ are shown in three panels, respectively. We note that the results are nearly independent of redshift; therefore, we only show the $z = 0$ results. The red and green lines are for halos in different twin simulations. The solid lines are for halos in specific LSS environment (0 stands for...
Figure 6. The probability distribution of the absolute cosine value of the angle between halo spin and $e_3$ vector in the twin simulations at different redshifts. For the red histograms, both halo spin and $e_3$ are from the same simulation ($N_{1024a}$), and the black histograms show the cross correlation between halo spin from $N_{1024}$ and the $e_3$ vector from $N_{1024a}$. If halo spin does not respond to a change of the LSS, the spin–LSS correlation cannot be maintained.

Figure 7. The relationship between the rotation of normal vector of the spin–$e_3$ system and the rotation of the LSS at different redshifts. The rotation of the LSS does effect the spin/major axis of the halo. Shaded regions and solid lines are plotted in the same way as in Figure 4. See the text for more details.

Figure 8. Mass dependence of halo shape parameters ($S$, $Q$, and $T$) at $z = 0$. The subscript denotes the LSS environment of matching halo pairs; i.e., 0 means cluster, 1 means filament, and 0 → 1 stands for the environment transformation from cluster to filament for the matching halo pair. The terms i and j denote the twinned simulations in each twin simulation pair. All the shaded regions, solid lines, and sorted mass bins here are plotted in the same way as in Figure 4. The halo shape is mainly determined by its current large-scale environment.
halos in different LSS environments have some systematic differences; i.e., compared to cluster halos, halos in a filament \( (M_h > 10^{12} h^{-1} M_\odot) \) are much more spherical and more oblate, which agrees with previous results (e.g., Kang & Wang 2015). That said, lines in the same color are quite consistent for all shape parameters (reminding us that there is a symmetry between twin simulations in terms of individual measurements), which proves that the current LSS environment really has a dominant impact on the ellipticity and triaxiality of halos, regardless of their environments in the other symmetric simulation. Combined with results presented in the previous section on the spin--LSS correlation, our results suggest that the current environment dominates the correlation between halo spin/major axis with the LSS.

### 4. Conclusions and Discussion

In this paper, we investigate the effect of the LSS on halo pairs matched between twin N-body simulations. The only difference between the twin simulations is that in one run, the random seeds for the phase of small \( k \) were replaced when the initial conditions are generated. Once the sample of matching halos at different redshifts is established, one can use them to separate the entangled factors of large and small scales in the formation of dark matter halos. In this work we mainly focus on the correlations between the halo properties (spin, major axis, and shape parameters) and the \( \epsilon_3 \) vector of the Hessian matrix, which is thought to be a universal definition of the LSS. Our main results are summarized as follows.

1. A few statistical tests, including the halo mass function and the halo spin--\( \epsilon_3 \) correlation, show that the twin simulations are statistically equal, ensuring that our technique for producing controlled twin simulations is robust and reliable.

2. The ratio of matching halo pairs in the twin N-body simulations, based on a merit function, decreases with increasing halo mass. This is due to the later stronger evolution of massive halos by merger or mass accretion along the large-scale structure, which is different in the twin simulations.

3. The halo spin/major axis will adjust its direction appropriately when the LSS environment changes across twin simulations; i.e., the change of \( \epsilon_3 \) vector will lead to a corresponding rotation of the whole halo--LSS system, to ensure the correlation between spin/major axis and \( \epsilon_3 \) vector remains identical in the twin simulations.

4. Statistically, halos in a filament \( (M_h > 10^{12} h^{-1} M_\odot) \) are much more spherical and more oblate than cluster halos, and the LSS environment really has a dominant impact on the ellipticity and triaxiality of halos.

Since our technique of running twin simulations and establishing the matching halo pairs between them are verified to be robust and reliable, in the future, this technique can be implemented in some larger N-body simulations, or even hydrodynamical simulations, to identify matching galaxies embedded in the center of matching halo pairs. Moreover, multiscale initial conditions for cosmological simulations, e.g., the zoom-in techniques, can also be employed to study how the LSS environment influences galaxy formation and evolution in more detail. That said, one can separate the entangled factors on different scales of halo and galaxy formation by choosing different \( L_0 \) in Equation (2). We conclude that twin simulations with controlled initial conditions on different scales could be useful to entangle the formation of a galaxy/galaxy and the large-scale environment.

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