Medium field method is applied for studying valence electron behavior in metals. When different wave-vector electrons are attracted at low temperatures, distribution function gets discontinued. As a result, a specific energy gap occurs.

1. Introduction

As it follows from the preceding paper, interacting electron distribution function has the anisotropic effect produced by the energy of interaction of electrons with opposite wave vectors \( k \) and \( -k \), in particular:

\[
\varepsilon_{kk'} = I \delta_{k+k'}.
\]

This means that at low temperatures one of the \( k \) and \( -k \) wave vector states within particular region \( S \) is more likely filled up and another one remains free. This kind of anisotropy is caused by superconductivity [1-3]. But the question is now arises: By what means does an energy gap occur? To clarify the matter we will consider another model Hamiltonian, this paper is devoted to.

As before, we proceed from Fermi–Dirac function mean field approximation equation:

\[
\ln \frac{1-w_k}{w_k} = \beta (\varepsilon_k - \mu),
\]

(1)

where \( w_k \) is probability that the state described by wave function \( \psi_{kn}(r) \chi_\sigma(\xi) \) is filled by one of equilibrium system electrons with wave vector \( k \); \( \beta = (k_B T)^{-1} \), \( T \) – temperature,

\[
\varepsilon_k = \varepsilon_k + \sum_{k'} \varepsilon_{kk'} w_{k'}
\]

(2)

– electron medium energy in one of the Bloch states with wave vector \( k \), \( \varepsilon_k \) is electron energy less interaction with other electrons, \( \varepsilon_{kk'} \) is energy of interaction of electrons with wave vectors \( k \) and \( k' \), \( \mu \) is chemical potential.

2. Model Hamiltonian

We consider the following Hamiltonian [4, 5]

\[
\varepsilon_{kk'} = -J \delta_{k-k'}.
\]

(3)
Here, $\delta_{k-k'}$ is a Kronecker delta, $J$ – electron attraction energy with different wave vectors $k$ and $k' = k$. As provided by this model, those valence electrons moving within their crystal space at equal speeds may be attracted only. Such Hamiltonian is produced by Coulomb electron attraction.

We use electron interaction energy (3) in formula (2) and get the equation as follows:

$$\varepsilon_k = \varepsilon_k - J w_k.$$

(4)

Now, equation (1) takes on the form as follows:

$$\theta \ln \frac{1 - w_k}{w_k} = \varepsilon_k - J w_k - \mu,$$

(5)

where $\theta = k_B T$.

3. Distribution of Attracting Electrons

Equation (5) may be solved by the form as follows:

$$w_k = w(x_k, \tau),$$

(6)

where

$$x_k = \frac{\varepsilon_k - \mu}{J},$$

(7)

$$\tau = \frac{4 \theta}{J}.$$  

(8)

Wherein, relation $w = w(x, \tau)$ is given by the following equation:

$$x = w + \frac{\tau}{4} \ln \frac{1 - w}{w}.$$ 

(9)

For relation diagrams of $w = w(x, \tau)$ regarding various temperature values $\tau$, see Fig. 1.

![Fig. 1](image)

Fig. 1. Electron energy distribution function at various temperature values $\tau$: 

1 – $\tau = 0$; 2 – $\tau = 0.5$; 3 – $\tau = 1$; 4 – $\tau = 2$.

At temperatures $T$ above critical point:

$$T_c = \frac{J}{4 k_B},$$

(10)

($\tau \geq 1$), distribution function $w = w(\varepsilon, \tau)$ has a monotonic decreasing quantity. Should at a particular range ($\varepsilon_1, \varepsilon_2$) of kinetic electron energies $T < T_c$, the distribution function gets three-valued. Naturally, probability
\( w(\varepsilon) \) of filling with electron may gain one of the three possible values only. Actually, electrons are distributed by states provided that electron energy takes on the least value.

4. Real Electron Distribution

Now, we consider different electron equilibrium states at \( T = 0 \). One of the distribution functions, which describe the state of electron system at \( T = 0 \) takes the form as follows:

\[
w(\varepsilon, 0) = \begin{cases} 
1 & \text{at } \varepsilon < \mu + J, \\
0 & \text{at } \varepsilon > \mu.
\end{cases}
\]  

(11)

Other electron system distribution function gets the state described by the following equation:

\[
w(\varepsilon, 0) = \begin{cases} 
1 & \text{at } \varepsilon < \mu, \\
\frac{\varepsilon - \mu}{J} & \text{at } \mu < \varepsilon < \mu + J, \\
0 & \text{at } \varepsilon > \mu + J.
\end{cases}
\]  

(12)

Energy \( E_0 \) of electrons in state (11) is less than energy \( E_1 \) of electrons in state (12) by the following quantity:

\[
E_1 - E_0 = \frac{7N J^2}{16 \varepsilon_F},
\]  

(13)

where \( N \) is number of electrons, \( \varepsilon_F \) – Fermi energy. Thus, state (11) is a basic one of electron system, i.e. the least energy state.

At temperatures \( T < T_c \) state of the electron system which matches the least energy \( E \) is described by the distribution function shown in Fig. 2. This function gets nonremovable discontinuity at the kinetic energy of electron as follows:

\[
\varepsilon = \mu + \frac{J}{2}.
\]  

(14)

![Fig. 2. Real electron energy distribution function at \( \tau = 0.85 \)](image)

5. Energy Gap

Gap width \( \Delta w \) of distribution function may be found by means of equation (9) using \( x = 1/2 \). On making simple calculations we obtain the equation as follows:

\[
\ln \frac{1 + \Delta w}{1 - \Delta w} = \frac{2 \Delta w}{\tau},
\]  

(15)

which describes relation of gap width \( \Delta w \) to temperature. For the relation diagram see Fig. 3.
6. Medium Electron Energy

Medium electron energy is determined by formula (4). As provided by this formula, electron medium energy $\varepsilon_k$, like distribution function $w_k$, is a wave $k$ vector composed function, where kinetic electron energy $\varepsilon_k$ is applied as an intervening variable:

$$\varepsilon(\varepsilon) = \varepsilon - J w(\varepsilon).$$  \hspace{1cm} (16)

For the relation diagram at $\tau = 0, 85$, see Fig. 4.

![Fig. 3. Relation of distribution function gap width $\Delta w$ to temperature $\tau$](image1)

![Fig. 4. Relation of medium electron energy $\varepsilon$ to its kinetic energy $\varepsilon$ at temperature $\tau = 0.85$](image2)

As it seen from Fig. 4, electron energy spectrum has a “discontinuity” which width $2\Delta$ depends on gap width $\Delta w$ of the distribution function and determined by the ratio as follows:

$$2\Delta = J \Delta w.$$  \hspace{1cm} (17)

At temperature $T = 0$ the energy gap acquires the largest width. Should the temperature go up, gap width shall monotonically decrease. At $T = T_c$, the gap shall disappear, since distribution function $w = w(\varepsilon, \tau)$ gets continuous.
7. Conclusion

We hereby have stated that model Hamiltonian (3) together with equation (1) explicitly results in occurrence of an energy gap. This model Hamiltonian is applied for stipulating the state proving availability of a gap only. The real Hamiltonian takes the form as follows [4, 5]

$$\varepsilon_{kk'} = I \delta_{k+k'} - J \delta_{k-k'}.$$  

As provided by the preceding paper, the first term of the formula defines anisotropy and results in superconductivity. As for the second term, it results in occurrence of a gap.

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