Parallel Trends and Dynamic Choices

Philip Marx
Louisiana State University

Elie Tamer
Harvard University

Xun Tang
Rice University

Difference in differences is a common method for estimating treatment effects, and the parallel-trends condition is its main identifying assumption: the trend in mean untreated outcomes is independent of the observed treatment status. In observational settings, treatment is often a dynamic choice made or influenced by rational actors, such as policy makers, firms, or individual agents. This paper relates parallel trends to economic models of dynamic choice. We clarify the implications of parallel trends on agent behavior and study when dynamic selection motives lead to violations of parallel trends. Finally, we consider identification under alternative assumptions that accommodate features of dynamic choice.

I. Introduction

Difference in differences (DID) is a popular research design for causal inference with panel data or repeated cross-section data. Though

We thank the editor and three referees for their helpful comments. We also thank Aureo de Paula, Ismaël Mourifié, Jonathan Roth, Pedro Sant’Anna, and participants at the 2023
econometricians have estimated panel-data regressions for decades, the causal interpretation of coefficients from these regressions is subtle and forms the basis of this paper. On the one hand, and with nonexperimental data, structural choice models with optimizing agents provide causal interpretations. These depend on what one assumes about how these choices (about treatment, inputs, or prices) were made. On the other hand, and without a behavioral model, identifying assumptions are used to provide causal interpretations that allow for evaluations of various treatments. Chief among them is an assumption of parallel trends, namely that the untreated trend is independent of the realized sequence of treatment. Such assumptions indirectly restrict the kinds of choice behavior that agents are allowed to have. This is especially important in the dynamic settings where these data and designs are mainly used.

In this paper, we clarify the connections between the central DID causal identifying assumption (i.e., parallel trends) and dynamic choice behavior. We do so mainly by highlighting through simple examples the kinds of choice behaviors that are or are not compatible with the assumption. As a second contribution, we consider identification under alternative assumptions that accommodate various features of our discussion on dynamic choice.

A complicating factor that raises various potential selection issues in DID models is time. This is especially so when treatments are indexed by time in observational data. In that case, a variety of dynamic considerations, such as future discounting, option values, learning, and anticipation, may all play an important role in determining choices. While these selection and information issues and their role in empirical models are well known to econometricians, the purpose of this paper again is to shed light on exactly how these behaviors intersect with commonly used assumptions in DID models and on what classes of economic choice models of optimizing behavior are consistent with the behaviorally agnostic DID model.

For example, we show that some dynamic features, such as discounting or time-varying costs of treatment, need not lead to violations of parallel trends per se. Other phenomena, however, such as learning, optimal stopping, or repeated Roy-model-like behavior, can potentially violate parallel trends. We also elucidate how and why parallel-trend assumptions among some subsequences of observed treatment may be more robust to dynamic selection concerns than others. Finally, we translate these observations into alternative identification assumptions and strategies. These include considering weakened parallel-trend assumptions, time stationarity assumptions that can circumvent dynamic selection concerns, and partially identifying assumptions motivated by our structural models of choice.

Northwestern University Econometrics Day, the 2022 Southeastern Economics Conference, and Sciences Po. All errors are our own. This paper was edited by Christian Hansen.
We contribute primarily to a recent yet already substantial literature clarifying aspects of DID assumptions and estimators in settings that extend the basic two-period DID model with no treatment in the initial period. Much of this work has focused on (i) interpreting typical regression-based slope parameters and what these estimate when heterogeneity in effects is allowed over multiple time periods, and (ii) deriving estimators that target a causal parameter of interest under parallel trends.\(^1\) One takeaway from this literature is that a variety of identification and estimation complications arise in the empirically common settings where the basic DID model is enriched to fuzzy designs (where some individuals are treated in the “pretreatment” period) or multiple time periods. Yet, it is precisely also in these richer settings that we might expect richer dynamic selection considerations to arise. Thus, we complement this literature by focusing on the relationship between parallel trends and structural models of dynamic choice.

Our exercise is closest in spirit to two recent papers. In concurrent work, Ghanem, Sant’Anna, and Wütrich (2022) study the relationship between selection and the parallel-trends assumption. They consider the basic design where no unit is treated initially, and selection on untreated outcomes in the following period can be a function of an individual fixed effect or time-varying errors in the untreated outcome equation. We extend the types of selection mechanisms under consideration in two ways. First, we consider selection in fuzzy designs, which allows us to further explore the dynamics of selection with two periods of treatment. In addition, we explicitly model selection under imperfect information, which allows us to consider models with a temporal resolution of uncertainty and learning. Finally, we consider alternative identification approaches in the case where the parallel-trends assumption remains in question.

In other related work, Fudenberg and Levine (2022) study how learning by decision-makers affects the identification and interpretation of treatment effects. They consider a model where decision-makers choose an amount of effort in each period that affects both the treatment and the outcome. Decision-makers learn about the returns to effort from treatment assignment, and outcomes depend on effort but not treatment per se. In their model, the DID estimator identifies the sum of a preference effect and a learning effect but does not separate the two effects. In our learning example of subsection IV.B, we model treatment as the choice variable and past outcomes as potentially informative signals, and we investigate when learning does (not) lead to violations of the parallel-trends assumption.

---

\(^1\) See, for instance, de Chaisemartin and d’Haultfoeuille (2018, 2020), Borusyak, Jaravel, and Spiess (2021), Callaway and Sant’Anna (2021), Goodman-Bacon (2021), Sun and Abraham (2021), and Athey and Imbens (2022) to name a few. With the exception of the first two papers, most of this work considers staggered designs where the sequence of realized treatment is monotonic across time.
More broadly, our work fits into a literature that investigates and offers alternative identification methods for when parallel trends fails. Earlier investigations into modeling selection in panels include Ashenfelter and Card (1985), Heckman and Robb (1985), and Chabé-Ferret (2015). Other work has studied the identified features of the model under weaker assumptions by deriving bounds on parameters of interest (Manski and Pepper 2018; Rambachan and Roth 2019; Ban and Kédagni 2022). Abadie (2005) considers identification under covariate-conditional parallel trends and offers related examples for when such a conditional approach may be more credible. In the presence of pretrends, Dobkin et al. (2018) consider extrapolations of pretrends as an alternative to parallel trends, while Freyaldenhoven, Hansen, and Shapiro (2019) propose an alternative identification approach based on the existence of covariates that relate to the policy of interest only through the pretrend confounder. Similar in spirit to our work, Freyaldenhoven et al. (2021) also consider the economic content of various identifying assumptions in an event-study framework and in the presence of an unobserved confounder. Complementary to this work, our discussion of identification focuses on alternatives that do not require any additional data, such as pretrends, proxies, or instrumental variables. In this sense, our approach is also similar yet complementary to the recent doubly robust approach to identification in panel-data models of Arkhangelsky and Imbens (2021).

In addition, we draw on a large literature in applied economics that explicitly models dynamic decisions capturing incentives, option values, and/or learning. See for instance Heckman (1976), Rust (1994), and Keane and Wolpin (1997). These models are central to the literature on structural estimation of dynamic decision processes.\(^2\) See also the work of Taber (2001), Heckman and Navarro (2007), and Abbring (2010). This structural literature embeds within it causal parameters because it builds a model of behavior that is based on economic optimizing agents. Hence, causal relations are provided by construction through model specification that can be used to address counterfactuals and policy effects. The literature on DID, however, focuses on identifying parameters that are given a causal interpretation under some identifying restrictions without the need for full specification of a behavioral model. We think this is a strength of the approach and one that is worth highlighting. However, we also point out that care should be taken in particular setups when these “design” assumptions come at odds with simple notions of dynamic selection that may be relevant in data applications.

Finally, the exercise that we undertake is familiar to econometricians.\(^3\) In standard static causal setups, Vytlačil (2002) provided a class of choice

\(^2\) These models are directly related to the literature on reinforcement learning and causal bandits. See Igami (2020) for more on this connection.

\(^3\) For a similar exercise on the role theory plays, more precisely general equilibrium, in formulating empirical models, see Acemoglu (2010).
models that are consistent with the local average treatment effect (LATE) assumptions of Imbens and Angrist (1994). In particular, Vytlacil (2002) showed that a separable choice model with a univariate unobservable is (observationally) equivalent to the monotone causal model underlying LATE. A univariate unobservable determining choices (or monotonicity) may well be a reasonable model of behavior in many applications. Still, the existence of such a choice model clarifies the types of behavior under which causal interpretations are warranted. While our paper does not attempt a choice-theoretic characterization of the DID model, it takes an otherwise similar approach in the context of dynamic panel/DID regressions.

The paper proceeds as follows. In section II, we review the two-period fuzzy DID model and the parallel-trends assumption that the mean untreated trend is independent of the entire realized sequence of treatment. In section III, we begin by introducing a simple but useful equivalent formalization of the “full” parallel-trends assumption as a set of conditional or “partial” assumptions on subsequences of treatment. In section IV, we use this to study a sequence of choice models and their compatibility with parallel trends. In section V, we then provide some other suggestive approaches that either weaken parallel trends or replace it with more selection-robust or selection-motivated assumptions. The emphasis throughout is on stylized models that retain key features of dynamic selection in a simple framework and allow one to focus on exact channels of interest. Section VI concludes. Additionally, appendix A collects the proofs, appendix B provides a summary list of guidelines and heuristics that highlight when parallel trends is or is not plausible in an empirical setting, and appendix C discusses these various heuristics in an example based on Ashenfelter and Card (1985).

II. Model

For each of $N$ individuals indexed by $i$ in a pair of time periods indexed by $t \in \{0, 1\}$, a researcher observes a sequence of realized treatments $D_t \in \{0, 1\}$ and realized outcomes $Y_t \in \mathbb{R}$. We restrict our analysis to two time periods to clearly highlight linkages and for ease of exposition. We assume potential outcomes for each individual are indexed by that individual’s treatment in the present period only, and individuals are randomly sampled from a population. We index this assumption by zero because we maintain it throughout most of our investigation of selection in treatment and its relationship to parallel trends.

Assumption 0 (Limited dependence on treatment and random sampling). Potential outcomes at time $t$ depend only on own treatment at time $t$, that is, $Y_t(d_1, \ldots, d_N) = Y_t(d_i)$ for $t \in \{0, 1\}$ and $d_i = (d_{i0}, d_{i1})$, with $i = 1, \ldots, N$. Individuals are randomly sampled from a population
model \((Y_0(\cdot), Y_1(\cdot), \mathbf{U})\), where \(Y_t(\cdot) = (Y_t(0), Y_t(1))\) denotes the pair of potential outcomes in period \(t\), and \(\mathbf{U}\) denotes a random vector that contains decision-relevant variables that may also be indexed by the time period or the sequence of treatment.

In addition to imposing stable unit treatment value assumption, assumption 0 requires that potential outcomes in a period depend only on treatment in that period. This rules out dynamic treatment effects and some forms of anticipation but suffices for exploring the dynamics of selection into treatment.\(^4\) We henceforth work in terms of the population variables and suppress the individual index \(i\) to simplify notation.

The central condition we investigate is that the means of the untreated outcomes, that is, \(Y_t(0)\)’s, satisfy a parallel-trends condition across each treatment sequence \((D_0, D_1)\). This is the key assumption maintained by the DID literature.

**Parallel Trends (PT).** The mean untreated trend is constant across all realized sequences of treatment:

\[
E[Y_t(0) - Y_t(0)|D_0 = d_0, D_1 = d_1] = \tau
\]

for some constant \(\tau \in \mathbb{R}\) and all \(d_0, d_1 \in \{0, 1\}\).

Thus expressed, the PT condition is identical to the strong exogeneity condition for two time periods and a given group in de Chaisemartin and d’Haultfoeuille (2020). In the basic case with a pretreatment period, a.k.a. the sharp design, where \(D_0 = 0\) for all individuals, the PT condition only conditions on treatment in period 1. In that case and under this assumption, the causal effect of treatment on the treated in period 1 is identified by the basic DID estimator:

\[
E[Y_1(1) - Y_0(0)|D_1 = 1] = E[Y_1 - Y_0|D_1 = 1] - E[Y_1 - Y_0|D_1 = 0],
\]

where \(Y_t = D_t Y_t(1) + (1 - D_t) Y_t(0)\) is the realized outcome in period \(t\).

A recent literature studies DID estimators that identify weighted averages of heterogeneous treatment effects in more general cases where some units are treated in period 0 (a.k.a. the fuzzy design) or there are more than two periods.\(^5\) Those papers impose versions of the PT condition in combination with further assumptions such as monotonicity of

---

\(^4\) Allowing for anticipation effects, whereby current potential outcomes can depend on future treatments, poses challenges to identification in dynamic choice models. This has received attention in the literature. See, for instance, Abbring and Van den Berg (2003), Heckman and Navarro (2007), Heckman, Humphries, and Veramendi (2016), and, in the context of DID models, Malani and Reif (2015). We discuss how to relax the dependence on contemporaneous treatment alone in subsection IV.E.

\(^5\) See, e.g., de Chaisemartin and d’Haultfoeuille (2018, 2020), Borusyak, Jaravel, and Spiess (2021), Callaway and Sant’Anna (2021), Goodman-Bacon (2021), Sun and Abraham (2021), and Athey and Imbens (2022).
In $t$ (staggered research design) or some versions of PT in treated outcomes.

### III. Setup and a Simple Example

Our first goal is to explore when the PT condition is (in)consistent with treatments that are determined by forward-looking and rational dynamic choices. In doing so, we focus on models that satisfy assumption 0. This illustrates when individual motives leading to treatment selection are compatible with the PT condition. We discuss generalizations in subsection IV.E.

To start, notice that the PT condition can be equivalently expressed as a set of pairwise equalities, which will be useful in analyzing subsequent examples.

**Observation 1.** The PT condition is equivalent to the two joint statements below:

\[
E[Y_i(0) - Y_{0}(0) | D_0 = d_0] \text{ is constant in } d_0; \quad (1)
\]

\[
E[Y_i(0) - Y_{0}(0) | D_0 = d_0, D_1 = d_1] \text{ is constant in } d_1 \text{ for each } d_0. \quad (2)
\]

By construction, the two constants mentioned in (1) and (2) must be the same. In turn, each of these two equalities can be rearranged and reinterpreted as a stationarity condition in the magnitude of selection.

**Observation 2.** Conditions (1) and (2) above are respectively equivalent to

\[
E[Y_i(0) | D_0 = 1] - E[Y_i(0) | D_0 = 0] \text{ is constant in } t; \quad (3)
\]

\[
E[Y_i(0) | D_0 = d_0, D_1 = 1] - E[Y_i(0) | D_0 = d_0, D_1 = 0] \text{ is constant in } t \text{ for each } d_0. \quad (4)
\]

Conditions (3) and (4) clarify how the PT condition allows for flexible selection on the untreated outcome in a given period. This is evident because the two constants mentioned in (3) and (4) are allowed to be different. At the same time, the conditions require the magnitude of this selection (in terms of the mean untreated potential outcome) to be fixed across periods $t$ given a treatment history.

It will also be instructive to illustrate PT in a parametric setting where potential outcomes, $Y_{it}(d)$ with individual subscript $i$, are determined as in a panel-data model:

\[
Y_{it}(d) = \alpha_i + \delta_i + \beta_id + \epsilon_{it} \quad \text{for } d \in \{0, 1\}. \quad (5)
\]

Then the PT condition holds if
Thus, PT holds if the evolution of time trends $\delta_1 - \delta_0$ and the difference between transitory errors $\varepsilon_{i1} - \varepsilon_{i0}$ are mean-independent of observed treatments. This shows that PT allows for some correlation between transitory errors and observed treatments and hence allows for some selection on treatment, as long as (6) holds.\(^6\)

For the rest of this section and section IV, we investigate the limits of endogenous selection into treatment allowed under PT. We begin by revisiting a special case of selection on past outcomes. In a classic example, Ashenfelter (1978) finds that participants in job-training programs typically experienced a decline in earnings in the year prior to training. As he observes, “In retrospect this is not very surprising since the Department of Labor was instructed to enroll unemployed workers in the MDTA [training] programs in this period and it is just such workers who would be most likely to want to enter a training program.”

The example below presents a stylized version for models of this phenomenon in Ashenfelter and Card (1985) and Abadie (2005); we focus on a two-period case where no one is treated in period 0 (i.e., there is a pretreatment period), potential outcomes are binary, and there is perfect selection on the past (untreated) outcome in period 1. With a pretreatment period, the PT condition reduces to the familiar condition that $E[Y_1(0) - Y_0(0)|D_0 = d_0]$ is constant in $d_0$. In this example, the PT condition fails, except for the extreme case where the potential outcome without treatment does not vary over time.

**Example 1 (Selection on past outcomes).** Potential outcomes are binary, that is, $Y_t(0), Y_t(1) \in \{0, 1\}$. At $t = 0$, everyone is untreated, that is, $D_0 = 0$. At $t = 1$, there is perfect selection on the previous period’s realized outcome, that is, $D_1 = 1 - Y_0$. Assume $P(Y_0(0) = 0) \in (0, 1)$. Then the PT condition holds if and only if the potential outcome with no treatment does not vary over time:

$$Y_0(0) = Y_1(0) \text{ almost surely.}$$

We present proofs of all claims in the examples in appendix A. Because the equality of untreated potential outcomes over time is a strong and often falsifiable condition, this simple example confirms that PT are likely to fail when there is backward-looking dynamic selection: the propensity to enroll in a training program in the current period depends on employment in the previous period. More generally, when current decisions depend directly (or indirectly) on past outcomes, PT are likely to fail.

---

\(^6\) Abadie (2005) makes a similar observation for the case with sharp designs when $D_0 = 0$. 
In the next section, we provide an economic framework that is helpful in framing the PT condition within a dynamic discrete-choice model that is familiar to economists.

IV. PT under Dynamic Treatment Choices

We now introduce a dynamic model where treatment decisions are made by rational, forward-looking individuals who maximize the present value of expected utility over time. This provides a unifying framework for the remaining examples of the section.

To define the decision-makers’ maximization problem, we introduce two components. First, preferences over treatment histories \( d_t = (d_0, \ldots, d_t) \) are summarized by a flow of utilities \( V_t(d_t) \) in each period \( t \), whose net present value is computed using a discount factor \( \beta \in (0, 1) \) that may vary across individuals. In turn, per-period utility \( V_t(d_t) \) is separable into the potential outcome \( Y_t(d_t) \) in that period and an additional preference element \( K_t(d_t) \) capturing the possibly history-dependent costs of (no) treatment. The costs \( K_t = (K_t(d_t))_{t, d_0} \) are allowed to vary with potential treatment history, which accommodates staggered designs or switching costs. Second, the information available to the decision-maker consists of \( I_0 \) in period 0 and \( I_1(d_0) \) in period 1; the latter is allowed to depend on past treatment \( d_0 \in \{0, 1\} \).

We assume \( I_0 \subseteq I_1(d_0) \). That is, the information available to the decision-maker accrues over time. For simplicity, let the preference components be known to the decision-maker initially, that is, \( \beta, K \in I_0 \).

**Dynamic Utility Maximization (DUM).** Given initial information \( I_0 \) at the start of period 0 and accrued information \( I_1(d_0) \) at the start of period 1, the treatment decisions in periods 1 and 0 maximize the sum of expected discounted utility:

\[
D_1(d_0) = 1 \{E[V_1(d_0, 1) - V_1(d_0, 0) | I_1(d_0)] \geq 0\},
\]

\[
D_0 = 1 \{E[V_0(1) - V_0(0) + \beta(W_1(1) - W_1(0)) | I_0] \geq 0\},
\]

where \( 1\{\cdot\} \) denotes the indicator function, and

\[
W_t(d_0) = \max_{d_1 \in \{0, 1\}} E[V_t(d_0, d_1) | I_1(d_0)],
\]

with \( V_t(\cdot) \) being the per-period utility, which is equal to a deterministic function of the realized outcome net an individual-varying cost of treatment\(^7\)

\[
V_t(d_t) = Y_t(d_t) - K_t(d_t).
\]

\(^7\) It is straightforward to generalize our results by letting the utility from outcomes vary using a function \( f(Y_t(d_t)) \); we refrain from doing so in order to simplify notation.
Preference components are known to the decision-makers initially, that is, \( \beta, K_i(\cdot) \in \mathcal{I}_0 \).

To present our analysis, it is helpful to write \( \mathcal{I}_0 = \{U_0\} \) and \( \mathcal{I}_1(\mathbf{d}_0) = \{U_0, U_1(d_0)\} \). Thus, \( U_0 \) denotes the initial information, and \( U_1(d_0) \) denotes the incremental information accrued before period 1, which may be specific to past treatment \( d_0 \). In the notation of assumption 0, this means \( \mathbf{U} = (U_0, U_1(0), U_1(1)) \). Furthermore, we let \( U_1(d_0) = Y_0(d_0) \) in our examples below. This is only for the purpose of simplifying exposition; our results extend readily to the cases where \( Y_0(d_0) \) is a subvector of \( U_1(d_0) \).

The econometrician observes the realized sequence of outcomes \( (D_0, D_1, Y_0, Y_1) \). Thus, the costs \( K \) can also capture unobserved states in dynamic discrete-choice models (e.g., Rust 1994). The dependence of information on counterfactual treatment histories allows for information-acquisition motives. We assume the decision-makers rationally form and update beliefs given knowledge of the joint distribution of unobservables and potential outcomes.

Consider two applications that fit in this setup. The first is the entry and exit of firms. For example, Gentzkow, Shapiro, and Sinkinson (2011) use a first-difference approach, with state and year fixed effects, to estimate the effect of newspaper entry and exit on voter turnout in local markets. The second is the effect of union membership on wages. This is commonly estimated in panel data by regressing worker wages on union membership, with worker and time fixed effects used to control for unobserved differences in worker productivity (e.g., Jakubson 1991). In this case, workers may decide to join a union if the union wage premium exceeds costs of union membership, and the rational, forward-looking workers may take into account uncertainties about potential outcomes, particularly those from untested options.\(^8\)

Next, we investigate how the PT condition can be consistent with such endogenous selection of treatment in the DUM context.

### A. Sufficient Initial Information

We begin with a benchmark case of this model that satisfies the PT condition. In the subsequent section, we show how relaxing certain attributes of this benchmark case leads to violations of the PT condition.

For several examples in this and subsequent sections, we will maintain that the trends in the untreated potential outcomes is mean-independent from the initial information:

\[
E[Y_1(0) - Y_0(0) \mid U_0] = \tau \text{ for some constant } \tau \in \mathbb{R}. \tag{11}
\]

\(^8\) Another example is Vella and Verbeek (1998), who estimated a dynamic Roy-type model, where workers locate each period in their preferred sector.
Without this condition, one could expect the PT condition to fail. For example, if the treatment selection is static (e.g., when \( D_0, D_1 \) are determined by the initial information \( U_0 \) alone), then the PT condition can fail as
\[
E[Y_1(0) - Y_0(0) \mid U_0] \text{ varies over the support of } U_0 \text{ partitioned by realizations of } (D_0, D_1). 
\]
Nonetheless, our goal is to investigate the PT condition under dynamic endogenous selection (when rational forward-looking individuals self-select into treatments). Therefore, we maintain (11) in several examples below so that the role of these dynamic motives can be isolated and manifest.

Example 2 (No learning about potential outcomes). Suppose DUM and (11) hold. Assume the initial information \( U_0 \) is sufficient for mean outcomes in the following sense:
\[
E[Y_1(d_1) \mid U_0, Y_0(d_0)] = E[Y_1(d_1) \mid U_0] \text{ for all } d_0, d_1 \in \{0, 1\}. \tag{12}
\]
Then the PT condition is satisfied.

Condition (12) posits that the initial information \( U_0 \) is sufficient for mean potential outcomes. In other words, once conditional on \( U_0 \), observing the realized past outcome \( Y_0(d_0) \) adds no new information regarding the mean of \( Y_1(d_1) \). Also, recall the initial information subsumes the sequence of treatment costs \( K_t(\cdot) \) (e.g., tuition for a job-training program is predetermined and publicly announced as part of \( U_0 \)). Therefore, under (12), the treatment \( D_t \) in each period \( t \) is a function of \( U_0 \) alone.

The proof of example 2 shows that (11) is in fact stronger than necessary for PT. It is also worth mentioning that the sufficient conditions for PT in example 2 are consistent with selection on individual fixed effects in \( U_0 \).

B. Learning and Experimentation

We now investigate whether PT can hold if the restrictions on information in example 2 are relaxed to allow for dynamic learning and rational forward-looking behavior in treatment choices. That is, selection into treatment is allowed to depend on information accrued over time, such as past outcomes \( Y_0(d_0) \). This information is valuable to decision-makers if it is informative about the mean potential outcome under (no) treatment.

Two effects related to learning are introduced in this subsection. First, forward-looking individuals internalize the value of experimentation while choosing treatment in period 0. Second, upon observing information from the past outcome in period 0, individuals update beliefs before choosing treatment in period 1.

It is helpful to conceive of an individual’s status in each period (treated/untreated) as “arms” (risky/safe) in a multiarmed bandit model. There is some learning about the mean returns from pulling these arms.
That is, the decision-makers update beliefs on the basis of past realized outcomes.

We start with a restriction that there is only learning from and about the treated arm. In other words, the mean return of the untreated arm is known to decision-makers at the outset, and pulling the untreated arm yields no information about the treated arm. This corresponds to a simple two-armed bandit model with a safe and a risky arm. The absence of treatment represents the status quo, and the treatment represents a new action (e.g., productivity in a new job, firm entry into a new market, or the effects of a new policy or a new drug). Note that in the case of a pretreatment period, this example would assume there is essentially no learning from the past outcome.

Example 3 (Learning on the treatment arm). Suppose \textsc{dum} and (11) hold. Assume

(i) There is no learning across different arms or from the untreated arm:

\[
E[Y_1(d_1) \mid Y_0(d_0), U_0] = E[Y_1(d_1) \mid U_0] \quad \text{when } d_0 \neq d_1 \text{ or } d_0 = d_1 = 0. \tag{13}
\]

(ii) The treated outcome provides no information about the untreated outcome in period 0, conditional on initial information:

\[
E[Y_0(0) \mid Y_0(1), U_0] = E[Y_0(0) \mid U_0]. \tag{14}
\]

Then the PT condition is satisfied.

Condition (13) relaxes the sufficiency of initial information in (12) by allowing

\[
E[Y_1(1) \mid Y_0(1), U_0] \neq E[Y_1(1) \mid U_0].
\]

This introduces net continuation value \(W_1(1) - W_1(0) \neq 0\) into the decision-maker’s problem even when treatment costs \(K_t(\cdot)\) are a function of the contemporary choice \(d_t\) alone. Condition (14) requires mean independence between the outcomes from two treatment arms within period \(t = 0\) conditional on initial information \(U_0\). This mean independence is conditional on \(U_0\) and does not rule out unconditional correlation in the arms across individuals.\(^9\) For example, in the case of treatment effects of a new drug, the condition requires the patient’s period 0 outcome under the new drug to be mean independent of the period 0 outcome under the status quo, conditional on the patient’s (private) initial information.

\(^9\) For the canonical case with a pretreatment period (when \(P(D_0 = 0) = 1\)), (14) is not needed for the PT condition.
Our next (counter-)example illustrates how learning about the untreated (control) arm leads to violation of PT. To focus on the selection issue, we consider the case where potential outcomes are binary, and there is a pretreatment period $P(D_0 = 0) = 1$.\footnote{By observation 1, the same kind of violation of the PT condition would arise conditional on the treatment choice in period 0 if we allowed for a fuzzy design.}

To highlight the effect due to relaxing (13), we continue to maintain (11), that is, $Y_1(0) - Y_0(0)$ is mean independent from $U_0$. Also, with $D_0 = 0$, the treatment cost at $t = 1$ is indexed only by $d_1$; we let $\bar{K}_1 = K_i(1) - K_i(0)$ denote the net cost of treatment in period 1.

Let $\Omega_{vl}$ denote the set of “valuable learners,” that is, the subset of the sample space whose optimal decision in period 1 depends counterfactually on the realized outcome in period 0:

$$\Omega_{vl} = \{ \omega \in \Omega : E[Y_1(0)|U_0(\omega), Y_0(0) = 0] \leq E[Y_1(1) - \bar{K}_1|U_0(\omega)]$$

$$< E[Y_1(0)|U_0(\omega), Y_0(0) = 1] \},$$

where $U_0(\omega)$ is parameterized by elements of the sample space. Thus, the decision-makers in $\Omega_{vl}$ could learn about the return from the control arm by observing past outcomes. In turn, such learning is valuable in period 0 only if it affects decisions in period 1.

The following example also relaxes the restrictions on learning in example 3, that is, (13), by allowing learning on the control arm. In this case, the PT condition essentially rules out the possibility of valuable learning.

**Example 4 (Learning on the control arm).** Suppose DUM and (11) hold. Assume there is a pretreatment period, that is, $P(D_0 = 0) = 1$, and potential outcomes are binary, $Y_i(d_i) \in \{0, 1\}$. Replace (13) with the following conditions:

(i) There is no correlated learning across no treatment and treatment arms by decision-makers:

$$E[Y_1(1)|Y_0(0), U_0] = E[Y_1(1)|U_0].$$

(ii) The past outcome of the control arm provides an informative signal about the mean return of that arm in the next period:

$$E[Y_1(0)|U_0, Y_0(0) = 0] \leq E[Y_1(0)|U_0] \leq E[Y_1(0)|U_0, Y_0(0) = 1].$$

Then the PT condition holds if and only if either

1. there is zero probability of valuable learning, $P(\Omega_{vl}) = 0$, or
2. valuable learning occurs where untreated outcomes are identical over time almost surely (i.e., $P(Y_0(0) = Y_1(0) | \Omega_{vl}) = 1$), and the
mean trend of untreated potential outcomes conditional on initial information is zero (i.e., $\tau = 0$ in [11]).

If learning about the control arm from past outcomes is valuable to a decision-maker, then past realizations by definition affect future decisions. Yet, conditioning on past untreated outcomes introduces backward-looking dynamic selection, similar to what we saw in example 1. Thus, the PT condition is violated if there is a positive probability of valuable learning on a subset of the sample space where untreated outcomes are not identical over time almost surely.

It is noteworthy that a partial PT condition as defined in (1) holds in period 0 even when there is learning in period 1 and selection into treatment in period 0, that is, $P(D_0 \neq 0) > 0$. Namely, in this case, the optimal decision rule in period 0,

$$
D_0 = \mathbf{1}\{E[V_0(1) - V_0(0) + \beta(W_1(1) - W_1(0)) \mid U_0] \geq 0\},
$$

is still a function of initial information $U_0$. Therefore, combining this rule with (11), similar reasoning as in subsection IV.A yields

$$
E[Y_1(0) - Y_0(0) \mid D_0 = d_0] = E[E[Y_1(0) - Y_0(0) \mid U_0] \mid D_0 = d_0] = \tau.
$$

This implies partial PT (1) from the standpoint of period 0.

Intuitively, decision-makers can anticipate the expected value of future information and even internalize the endogenous motive for learning in their period 0 decisions, which in turn can affect the selection of initial information $U_0$ into treatment in period 0 relative to the case where there is no initial uncertainty about the control arm. Nevertheless, the important distinction is that the period 0 decisions cannot condition differentially on period 0 versus period 1 potential outcomes, because both outcomes are identically uncertain at the time of decision-making. In contrast, future decisions can condition on past outcomes and may optimally do so when past outcomes convey valuable information about the future (which would generally invalidate the PT condition). In summary, learning about the control arm in our example violates PT because of selection on past untreated outcomes rather than the forward-looking value of strategic experimentation.

C. Selection on Present Outcomes

Next, we explore the case when treatment is a function of present outcomes. To begin, consider a Roy model with treatment-invariant uncertainty, which is subsumed in our DUM context, with $K_0(d^*) = 0$, and $U_1(0) = U_1(1) \equiv U_1$. In other words, such a model allows incremental information $U_1(d_0)$ but either rules out experimentation (i.e., $Y_0(d_0)$ is not
subsumed in $U_1(d_0)$, because $U_1(d_0)$ does not vary with $d_0$ or requires $Y_0(d_0)$ to be independent from, and thus not indexed by, potential treatment in period 0.

Throughout this subsection, we no longer maintain the mean independence of untreated trends $Y_1(0) - Y_0(0)$ from the initial information $U_0$ as in (11). This is because our focus in this subsection is on the endogenous selection of treatment due to present potential outcomes rather than the evolution of information.

**Example 5 (Static, repeated Roy model).** Recall the model in DUM with $\mathcal{I}_0 = \{U_0\}$ and $\mathcal{I}_1(d_0) = \{U_0, U_1(d_0)\}$. Suppose $Y_1(d_1) \in \{0, 1\}$ for $d_1 = 0, 1$, and $P(Y_1(1) \geq Y_1(0)) \in (0, 1)$ for $t = 0, 1$. Assume

(i) The initial information $U_0$ subsumes $Y_0(\cdot)$, and $K_t(\cdot) = 0$ for $t = 0, 1$.

(ii) The incremental information accrued before period 1 consists of both potential outcomes in period 1, that is, $U_1(d_0) = U_1 = \{Y_1(0), Y_1(1)\}$ for $d_0 = 0, 1$.

Under these conditions, there is no net continuation value, because $W_1(1) = W_1(0)$. Therefore, the optimal treatment rule is reduced to $D_t = 1\{Y_1(1) \geq Y_1(0)\}$.

The PT condition holds if and only if the following conditions hold jointly:

1. Untreated outcomes are stationary: $E[Y_0(0)] = E[Y_1(0)]$.
2. Untreated outcomes are degenerate at 1 among the ever-untreated:

$$P(Y_0(0) = Y_1(0) = 1|D_0D_1 = 0) = 1.$$ 

The claim in example 5 follows from the simple treatment rule. This example shows that the PT condition restricts the distribution of potential outcomes when the treatment is determined by a comparison of present potential outcomes. For example, to be consistent with PT in this example, the probability of strict switches in potential outcomes across time must be zero:

$$P(Y_t(0) > Y_t(1), Y_{t-1}(0) < Y_{t-1}(1)) = 0 \text{ for } t = 0, 1.$$

Such requirements rule out nontrivial independence of potential outcomes over time.

Next, we generalize the treatment rule to an arbitrary function of present information. We focus on the case where selection in each period is based on serially uncorrelated factors. In this case, the PT condition is reduced to strict exogeneity of untreated outcomes.
Proposition 1. Suppose in each period, decision-makers choose treatment $D_t$ as a deterministic function $f_t(\cdot)$ of history-invariant information $U_t$ about present outcomes $Y_t(\cdot)$:

$$D_t = f_t(U_t).$$

(18)

If potential outcomes and information are independent over time,

$$(U_0, Y_0(\cdot)) \indep (U_1, Y_1(\cdot)),$$

(19)

then a necessary and sufficient condition for PT is strict exogeneity of untreated outcomes

$$E[Y_t(0) | D_0 = d_0, D_1 = d_1] = \mu_t$$

(20)

for some constants $\mu_t \in \mathbb{R}$ and for all $t$, all $(d_0, d_1)$ occurring with positive probability.

The combination of (18) and (19) is testable by comparing observed untreated outcomes across treatment sequences. For example,

$$E[Y_0 | D_0 = 0, D_1 = 0] = E[Y_0 | D_0 = 0, D_1 = 1].$$

The role of PT is then to extrapolate the equality to mean untreated potential outcomes $Y_t(0)$ in period 1.

D. Staggered Designs

A common DID setting involves staggered designs, where treatments are irreversible, that is, $D_1 \geq D_0$. We investigate the viability of PT under two micro-founded models of staggered designs. The first revisits the repeated Roy setup in example 5 but with prohibitive costs for treatment reversal $K_t(1, 0) = \infty$. This introduces dynamic motives for treatment selection (with nonzero net continuation value) so that the staggered design arises endogenously within the DUM context. The second model rationalizes the sequence of irreversible treatments as a solution to an optimal stopping problem.

We relate these models to the repeated Roy model in example 5 and the models with learning and experimentation in example 3 and example 4. Our goal is to understand how these staggered designs alter the implications of dynamic selection for PT.

1. DUM with Prohibitive Costs for Treatment Reversal

In the repeated Roy model in subsection IV.C, we abstracted away from history-dependent treatment costs $K_0(d_0)$ and $K_t(d_0, d_1)$ by assuming that these are finite and subsumed in the initial information $U_0$ and by
normalizing them to zero. This partially led to the lack of history-dependent dynamics of information in example 5.

In contrast, the next example rationalizes irreversible treatments by using prohibitive costs for a reversal in treatment (i.e., when $K_t(1, 0)$ is large) and examines its implications on the PT condition.

**Example 6 (Roy model with prohibitive costs for treatment reversal).**

Recall the model in DUM. Suppose that $I_0 = I_1(d_t) = \{U_0\}$, with $U_0$ subsuming $Y_t(\cdot)$ and $K_t(\cdot)$ for $t = 0, 1$, and that $Y_t(d_t) \in \{0, 1\}$ for $d_t = 0, 1$. Let $K_t(1, 0) = \infty$ (infinite cost for treatment reversal) while $K_t(d_t') = 0$ otherwise.

Under these conditions, the optimal decision rule at $t = 0$ is\(^{11}\)

$$D_0 = 1\{Y_0(1) - Y_0(0) + \beta \min\{Y_t(1) - Y_t(0), 0\} \geq 0\}. \quad (21)$$

Suppose $P(D_0 = D_1 = 0), P(D_0 < D_1) \in (0, 1)$. Then the PT condition holds if and only if conditions 1 and 2 of example 5 hold jointly.\(^{12}\)

Unlike in the Roy model of example 5, the prohibitive cost for treatment reversal in example 6 introduces dynamic considerations through the nonzero net continuation value:

$$W_t(1) - W_t(0) = \min\{Y_t(1) - Y_t(0), 0\} \leq 0, \quad (22)$$

which measures the forgone option value of choosing $D_t = 0$. Thus, in contrast to the examples with learning, expectations about future outcomes may influence (and, in the present case, discourage) treatment in period 0 even without information-acquisition considerations.

To relate the logic behind example 6 to that of example 5, it is useful to decompose the population in terms of its members’ (now counterfactual) decisions without the prohibitive costs for treatment reversal. Those who would have been movers out of treatment ($D_0 > D_1$) when the treatment reversal cost is zero now become either always-treated or never-treated. Those who become never-treated as a result of the treatment reversal costs are those with a relatively high net value of no treatment in period 1. Those who would have been never-treated even without the treatment reversal costs remain such and have a zero untreated trend by the same logic of example 5. Because movers into treatment have a high value of no treatment in period 0, PT between the (constrained) never-treated and movers into treatment is satisfied if and only if the untreated trend is zero and untreated outcomes among the ever-untreated are degenerate at 1.

\(^{11}\) At $t = 0$, the payoff from $D_0 = 0$ is $Y_0(0) + \beta \max\{Y_t(1), Y_t(0)\}$ while that from $D_0 = 1$ is $Y_0(1) + \beta Y_t(1)$. (The latter expression is simplified because $K_t(1, 0) = \infty$ implies that if $D_0 = 1$, then the optimal continuation value cannot involve $D_0 = 0$.) Taking the difference of these two expressions leads to the optimal treatment rule in (21).

\(^{12}\) In condition 2, note that the event of being ever-untreated can differ across examples 5 and 6.
2. Optimal Stopping

Next consider a simple, two-period optimal stopping problem. Let $D_t = 1$ denote an irreversible decision to “stop,” which leads to a terminal utility that is normalized to zero. For example, a firm may choose to irreversibly exit a market and only collect the scrap value of its assets.\(^\text{13}\) In each period $t$, the decision to “continue” ($D_t = 0$) incurs a continuation cost $K_t$ and leads to an instantaneous payoff $Y_t(0)$. Let the initial information available to the decision-maker be $U_0 \equiv (K_0, K_1, \xi)$, with $\xi$ denoting any feature that is potentially correlated with the transition of potential outcomes $Y_t(0)$ under continuation.

At the start of period $t = 1$, a decision-maker has accrued information $I_1(0) = \{U_0, U_1(0)\}$ with $U_1(0) = Y_0(0)$ and chooses

$$D_t(0) = 1\{E[Y_t(0) \mid Y_0(0), U_0, D_0 = 0] - K_t \leq 0\},$$

with a value function

$$v_1(Y_0(0)) = \max\{0, E[Y_t(0) \mid Y_0(0), U_0, D_0 = 0] - K_t\}.$$ 

Note that we suppress $U_0$ as an argument in $v_1(\cdot)$ for simplicity. At the start of period $t = 0$, a decision-maker has initial information $I_0 = \{U_0\}$ and chooses

$$D_0 = 1\{E[Y_0(0) + \beta v_1(Y_0(0)) \mid U_0] - K_0 \leq 0\}.$$ 

Thus $D_0$ is determined by $U_0$ while $D_t$ is determined by $Y_t(0)$ and $U_0$.

For the rest of this subsection, we maintain the mean independence condition in (11),

$$E[Y_t(0) - Y_0(0) \mid U_0] = \tau.$$ 

We show that, even with such mean independence in the trend of $Y_t(0)$, the PT condition fails generically because of learning and forward-looking motives in the selection into treatments.

Similar to what we have done for the general DUM model in subsection IV.A and subsection IV.B, we now analyze the conditions for PT in the optimal stopping problem with or without substantive learning from past outcomes.

Case 1 (Sufficient initial information).—Suppose (11) holds. Assume

$$E[Y_t(0) \mid U_0, Y_0(0)] = E[Y_t(0) \mid U_0],$$

\(^\text{13}\) Optimal stopping has a long history in empirical economics. Early examples are the two classic works by Mortensen (1970) and Lippman and McCall (1976). Other important contributions in economics are the works of Pakes (1986), Rust (1987), and Wolpin (1987). More generally, this problem is part of the dynamic Markov decision problem literature that includes reinforcement learning problems.
so that $D_t$ is a function of $U_0$ alone (just as $D_0$ is). By the law of iterated expectation,
\[
E[Y_t(0) - Y_0(0) \mid D_0 = 1] = E[Y_1(0) - Y_0(0) \mid D_0 = 0, D_t = d]
= \tau \text{ for } d = 0, 1.
\]
Therefore, the PT condition holds.

**Case 2 (Learning from past outcomes).**—Suppose (11) holds. Relax (25) so that $E[Y_t(0) \mid U_0, Y_0(0)]$ is nondegenerate in $Y_0(0)$. The choice/treatment rule at $t = 0$ remains the same as in (24), which indicates $D_0$ depends only on $U_0$. Thus, we can write the treatment rule at $t = 1$ in (23) as
\[
D_1(0) = 1\{E[Y_1(0) \mid Y_0(0), U_0] - K_1 \leq 0\}.
\]
For the rest of this subsection, we present and discuss a necessary and sufficient condition for PT in case 2. Let $\mathcal{U}_d$ denote the set of realized values of the initial information $U_0$ that lead through (24) to $D_0 = d$. For any $u \in \mathcal{U}_d$, let $S_u(u)$ denote the set of realized values $y$ for $Y_0(0)$ such that $(u, y)$ induces the event “$D_0 = 0$ and $D_1(0) = d$.”

With treatments determined by optimal stopping, the PT condition is
\[
E[Y_1(0) - Y_0(0) \mid D_0 = 1] = E[Y_1(0) - Y_0(0) \mid D_0 = 0, D_t = d \text{ for } d = 0, 1.
\]
By the mean independence of $Y_t(0) - Y_0(0)$ from $U_0$ in (11),
\[
E[Y_1(0) - Y_0(0) \mid D_0 = 1] = (P_{1, \cdot})^{-1} \times \int_{\mathcal{U}_d} E[Y_1(0) - Y_0(0) \mid U_0 = u]dF_{U\cdot}(u)
= \tau,
\]
where $P_{1, \cdot} = P(D_0 = 1) = P(U_0 \in \mathcal{U}_d)$. Furthermore, for $d = 0, 1$,
\[
E[Y_1(0) - Y_0(0) \mid D_0 = 0, D_t = d] = (P_{0, d})^{-1} \times \int_{\mathcal{U}_d} \delta_d(u)dF_{U\cdot}(u),
\]
where
\[
\delta_d(u) = \int_{S_{u}(u)} \{E[Y_1(0) \mid U_0 = u, Y_0(0) = y] - y\}dF_{Y_1(0) \mid U_0 = u}(y),
\]
and $P_{0, d} = P(D_0 = 0, D_t = d) = P(U_0 \in \mathcal{U}_d, Y_0(0) \in S_{u}(U_0))$. As a result, in case 2, the PT condition is equivalent to the statement that the right-hand sides of (26) equal $\tau$ for $d = 0, 1$. Furthermore, under (11), $\delta_1(u) + \delta_0(u) = E[Y_1(0) - Y_0(0) \mid U_0 = u] = \tau$. It then follows that PT holds if and only if
\[
\int_{\mathcal{U}_d} \delta_0(u)dF_{U\cdot}(u) = \tau \times P_{0, 0}.
\]
This necessary and sufficient condition for PT in case 2 fails generically if there are no further restrictions on the joint distribution of \((Y_1(0), Y_0(0), U_0)\) beyond (11). To see this, consider a simple example \(Y_1(0) = Y_0(0) + \eta\) with

\[
E[\eta \mid Y_0(0) \in S^d(u), U_0 = u] = \varphi_d(u)
\]

for all \(u \in \mathcal{U}^0\). By construction, \(\delta_d(u) = \varphi_d(u)P_d(u)\) for \(u \in \mathcal{U}^0\), where \(P_d(u) = P(Y_0(0) \in S^d(u) \mid U_0 = u)\). In this model, mean independence in (11) imposes

\[
E[\eta \mid U_0 = u] = \varphi_1(u)P_1(u) + \varphi_0(u)P_0(u) = \tau \text{ for all } u
\]

but does not further restrict how \(\varphi_d(\cdot)\) varies between \(d = 0, 1\) over \(\mathcal{U}_0\). As a result, the necessary and sufficient condition for PT in (27) is not guaranteed to hold, because the left- and right-hand sides of (27) differ generically without further restrictions:

\[
\int_{u \in \mathcal{U}^0} \delta_d(u) dF_{\mathcal{U}_0}(u) = \int_{u \in \mathcal{U}^0} \varphi_0(u)P_0(u) dF_{\mathcal{U}_0}(u) \neq \int_{u \in \mathcal{U}^0} \tau P_0(u) dF_{\mathcal{U}_0}(u) = \tau \times P_{0,0}.
\]

This leads to failure of PT to hold in optimal stopping problems when we allow for learning from past outcomes (even while maintaining the mean independence restriction [11]).

E. Discussion: Past Treatment, Anticipation, and Latent Factors

So far, our overarching DUM framework and specific examples have focused on selection into treatment given potential outcomes indexed by treatment in the same period. We now briefly discuss extensions of the outcome model (assumption 0) and their relation to our preceding analysis. We maintain our focus on individual decision problems, thereby abstracting from spillovers across units such as social learning or general equilibrium effects.

One extension of our model relaxes assumption 0 to allow potential outcomes to vary by treatment sequence, that is, \(Y(d_0, d_1)\), thereby introducing the possibility of dynamic effects. In that case, the PT assumption is typically expressed in terms of the trend of never-treated potential outcomes

\[
E[Y_1(0, 0) - Y_0(0, 0) \mid D_0 = d_0, D_1 = d_1] = \tau
\]

for some constant \(\tau \in \mathbb{R}\) and all \(d_0, d_1 \in \{0, 1\}^{14}\)

\[\]

14 In the case of staggered designs \(D_1 \geq D_0\), the treatment sequence (and thus the generalized potential outcome parameterization) is summarized by the timing of first
In our DUM model, the dependence of future outcomes $Y_1(d_0, d_1)$ on past treatment $d_0$ introduces an additional motive for forward-looking behavior and selection into treatment in period 0 based in part on persistent effects in period 1, as in the preceding case of optimal stopping.

The dependence of past outcomes $Y_0(d_0, d_1)$ on future treatment $d_1$ also allows anticipation of (known) treatments to affect potential outcomes. However, typical DID methods require significant restrictions on anticipation in order to identify an average treatment effect on the treated. For example, identification of such an effect among switchers into treatment $D_0 < D_1$ requires assuming no anticipation on average among the switchers,

$$E[Y_0(0, 0)|D_0 < D_1] = E[Y_0(0, 1)|D_0 < D_1],$$

in order to identify the never-treated outcome in period 0 among switchers in (28). Stated as such, no-anticipation is partially an assumption about selection based on future treatment. In our DUM model where $Y_0$ is realized before $D_1$ is chosen, no-anticipation is most sensible in the stronger guise where $Y_0(0, d_1)$ is everywhere constant in $d_1$. Alternatively, anticipation could occur on the basis of uncertain future treatments, as in Malani and Reif (2015). For example, in our DUM model, uncertainty about future treatment arose with learning about expected outcomes from period 0. We embed this case in a more general discussion about our implicit assumptions regarding the determination of potential outcomes below.

More generally, potential outcomes may be affected by other latent, unobserved choices, which in turn may depend on the expected sequence of treatment. For concreteness, consider again the random-coefficient model (5) with direct dependence on the present treatment, but suppose that treated and untreated outcomes are also affected by an additional latent effort $f$ chosen by decision-makers before treatment in each period. That is,

$$Y_{it}(d_{it}) = \alpha_i + \delta_i d_{it} + \gamma_d f_{it} + \rho_d d_{eit} + \varepsilon_{it},$$

in period $G$ with realizations $g \in \{0, 1, \infty\}$, where $g = \infty$ denotes the never-treated sequence $d_0 = d_1 = 0$ in the basic two-period data. In that case, condition (28) coincides with a two-period version of assumption 1 of Sun and Abraham (2021).

It is useful to distinguish this from anticipation in the treatment decision on the basis of expectations about future outcomes and treatments, which is compatible with our baseline specification. Such anticipation in treatments could also induce a kind of anticipation in realized outcomes $Y_0$, which we showed in subsection IV.B need not violate PT per se.

This anticipation effect is certainly present also in macro where current outcomes (and decisions) by the Fed anticipate and are affected by future potential decisions.
where \( f_u \) is an endogenous level of effort also chosen by decision-makers.\(^{17}\) Taking time trends \( \delta_t \) to be nonstochastic, and assuming the effort levels \( F_u \) are unobserved by the researcher, one can write the PT condition as

\[
E[\gamma_1 F_1 - \gamma_0 F_0 + \epsilon_1 - \epsilon_0 | D_0 = d_0, D_1 = d_1]
\]

is constant across \( d_0, d_1 \in \{0, 1\} \). In addition to supposing that the difference in errors \( \epsilon_u \) is mean independent of treatment (6), a sufficient condition for PT is that there is no selection on the difference in untreated returns from effort

\[
E[\gamma_1 F_1 - \gamma_0 F_0 | D_0 = d_0, D_1 = d_1] = \psi
\]

for some constant \( \psi \in \mathbb{R} \) and all \( d_0, d_1 \in \{0, 1\} \). For example, this would hold if the marginal return from effort \( \gamma_u \) is constant across units and time, and the change in effort \( F_1 - F_0 \) is independent of treatments \( (D_0, D_1) \). However, the latter seems especially unlikely when effort also affects the returns from treatment, \( \rho_u \neq 0 \). There is also a further issue of causal interpretation, because even the difference in potential outcomes holding effort fixed at \( f_u = f \) contains the returns from effort, \( Y_u(1) - Y_u(0) = \beta_u + \rho_u f \). If the interest is in identifying (averages of) \( \beta_u \), then this setting requires methods beyond the two-way fixed effects paradigm that is the focus of this paper; and so we do not consider the problem further. For the same reason, we do not consider lagged outcomes studied by a large literature on dynamic panel-data models (e.g., Anderson and Hsiao 1982; Bhargava and Sargan 1983; Arellano and Bond 1991). Instead, we return to the canonical setup (assumption 0) and consider alternative methods for identification when multiperiod PT may be violated.

V. Alternative Identification without Parallel Trends

On the basis of insights from the models in section IV, this section considers alternative approaches for identification when the PT condition fails. In developing these alternative approaches, we emphasize that there is typically not a single solution because different models of the data-generating process yield different results. This is similar to the framework of Manski and Pepper (2018).\(^{18}\)

---

\(^{17}\) This extension is similar in spirit to a classic literature on the estimation of production functions with endogenous inputs (Marschak and Andrews 1944) using panel data (Hoch 1962; Mundlak 1963), except that (30) allows for an interaction term between effort and treatment, and we think of effort as being unobserved. If the additional choice variable is instead observed, the preceding literature suggests more structural proxy-based approaches to control for unit-level heterogeneity (e.g., Olley and Pakes 1996; Levinsohn and Petrin 2003).

\(^{18}\) See also Rambachan and Roth (2019) for recent work in this direction.
We maintain assumption 0 throughout and focus on a single target parameter, the average treatment effect at $t = 1$ among switchers into treatment:

$$E[Y_1(1) - Y_1(0) | D_0 < D_1].$$

(32)

The treatment effect among switchers (32) coincides with the usual treatment effect on the treated in period 1, that is, $E[Y_1(1) - Y_1(0) | D_1 = 1]$, in the case with a pretreatment period where $P(D_0 = 0) = 1$. The parameter defined in (32) provides a natural generalization under fuzzy designs, where $P(D_0 = 1) > 0$. For example, the treatment effect among switchers also serves as a building block for the DIDM estimator proposed in de Chaisemartin and d’Haultfoeuille (2020).

The treatment effect parameter defined in (32) differs qualitatively from the LATE with time as an instrument, despite a seeming resemblance. In typical panel-data settings, time is not randomly assigned or excluded from the potential outcome equation, and treatment may not be monotonic in time. Additionally, treatment is observed in each time period, whereas it is typically observed for only one realization of instrument values. We return to the conceptual relations between the two parameters and their identification in subsection V.2. Throughout, we restrict ourselves to settings where a positive mass of never-treated and switchers into treatment exists, $P(D_0 = D_1 = 0) > 0$, $P(D_1 > D_0) > 0$.

### A. Partial Parallel Trends

One takeaway from the models in section IV is that parallel-trend comparisons across some treatment sequences may be more robust to selection concerns than others. This motivates the question: What can be identified under relaxations of the full PT condition that impose PT only among subsets of the treatment sequence realizations?

First, recall the following partial PT assumption from observation 1, which relaxes full PT by only imposing parallel-trend restrictions between switchers into treatment and the never-treated.

**Assumption 1** (Partial parallel trend between switchers and never-treated).

$$E[Y_1(0) - Y_0(0) | D_0 = 0, D_1 = d_1] = \tau_{0,1} \text{ for } d_1 \in \{0, 1\}.$$
Assumption 1 requires only that the untreated trend is independent of the period 1 treatment conditional on \( D_0 = 0 \); it imposes no restriction on trend-based selection in the initial period 0. (Of course, this assumption is the same as assuming full PT in a setup with a pretreatment period, where there is by definition no selection in period 0.) Assumption 1 is sufficient for recovering the period 1 average treatment effect for switchers into treatment.

**Proposition 2.** Let assumption 1 hold. Then,

\[
E[Y_1(1) - Y_1(0) | D_0 < D_1] = E[Y_1 - Y_0 | D_0 < D_1] - E[Y_1 - Y_0 | D_0 = D_1 = 0].
\] (33)

The result follows immediately from the usual logic with a pretreatment period applied to the subgroup where \( D_0 = 0 \). Furthermore, it is straightforward to show that (33) is one of only two basic treatment effects among the family \( E[Y_r(1) - Y_r(0) | (D_0, D_1) = (d_0, d_1)] \) that are identified even under full PT. In summary, violations of the full PT condition need not lead to failures of point identification of the treatment effect parameter in (32) per se.

Further motivated by the more general learning example in subsection IV.B, we also consider the content of a weakened parallel-trend restriction that conditions only on past realized treatments.

**Assumption 2 (Forward parallel untreated trend).**

\[
E[Y_1(0) - Y_0(0) | D_0 = d_0] = \tau_0 \quad \text{for} \quad d_0 \in \{0, 1\}. \] (34)

As shown in subsection IV.B, such an assumption is consistent with choice environments where decision-makers can learn but have no differential information about untreated outcomes across period 0 and 1 at the time of deciding in period 0. Assumption 2 relaxes multiperiod

---

21 Relating to the existing literature, proposition 2 captures the basic intuition behind the DID, estimator (in turn underlying the DIDn estimator) of de Chaisemartin and d’Haultfoeuille (2020) but shows that it holds under a logically weaker assumption than the full PT condition. This weaker assumption suffices for comparing the evolution of the untreated trend among switchers into treatment (where this trend is counterfactual) and the never-treated (where this trend is observed). A similar intuition also underlies the “building block” estimators of Callaway and Sant’Anna (2021) and Sun and Abraham (2021) in staggered settings. In that case, however, our simple two-period framework does not identify whether the control group \( D_0 = D_1 = 0 \) is never-treated or not-yet treated. For elaboration of the distinction, see Callaway and Sant’Anna (2021). Of course, it is worth noting that in each of these papers, the primary contribution is to provide methods for aggregating over such basic effects.

22 By symmetry, the other is the period 0 average treatment effect among switchers out of treatment,

\[
E[Y_0(1) - Y_0(0) | D_0 > D_1].
\]
PT by imposing PT only conditional on treatment in period 0. This allows for selection on past outcomes in the treatment decision of period 1.23

Unlike the full PT condition in section II and the partial PT condition in assumption 1, assumption 2 does not point identify the average treatment effect among switchers into treatment (and thereby, as a weakening of the PT condition, any other basic treatment effects). Consider the identity

\[ E[Y(1) - Y(0)|D > 0] = E[Y(1) - Y(0)|D > 0] \]

The first term on the right-hand side is identified, and so it remains to identify the second term, namely the untreated trend among the switchers. Decomposing (34) into its constituent treatment sequences, assumption 2 is equivalent to

\[ \sum_{d} P(D = d_1|D = 0)E[Y(0) - Y(0)|D = 0, D_1 = d] \]

Thus, assumption 2 allows us to express the untreated trend in (35) only in terms of other unobserved untreated trends among the always-treated and switchers out of treatment:

\[ E[Y(0) - Y(0)|D > 0] = \frac{P(D_1 = 1|D_0 = 1)}{P(D_1 = 1|D_0 = 0)} E[Y(0) - Y(0)|D_0 = D_1 = 1] + \frac{P(D_1 = 0|D_0 = 1)}{P(D_1 = 1|D_0 = 0)} E[Y(0) - Y(0)|D_0 > D_1] - \frac{P(D_1 = 0|D_0 = 0)}{P(D_1 = 1|D_0 = 0)} E[Y(0) - Y(0)|D_0 = D_1 = 0]. \]

A stark research design that circumvents this issue is one where \( P(D_1 = 0) = 1 \), but in that case there is no selection in period 1, and so full PT is trivially recovered. Next, we consider other assumptions—in some cases nested by the above—that allow at least partial identification of the desired target parameter.

23 A common approach in such settings is to use estimators that condition or match on lagged dependent variables (e.g., Abadie, Diamond, and Hainmueller 2010 and Dehejia and Wahba 1999, respectively); see also Angrist and Pischke (2008, 246) and Ding and Li (2019) for circumstances where the treatment effect is bounded between the DID and lagged dependent variable estimators. However, conditions for consistent estimation with lagged dependent variables may be necessarily strong (e.g., Nickell 1981).
B. Mean Stationarity

We now revisit the role of mean stationarity of untreated outcomes. In section IV, mean stationarity arose as a necessary condition for PT in examples 1, 5, and 6 and as a salient case of the dynamic choice models with (no) learning in examples 2, 3, and 4, where agents learned about the payoffs of fixed bandit arms. Next we show how, when applicable, such mean stationarity can also bypass selection concerns by providing an alternative path to identification. Recall the notion of mean stationarity of untreated outcomes:

**Assumption 3 (Mean stationarity of untreated outcomes).**

\[ E[Y_1(0) - Y_0(0)] = 0. \]

Assumption 3 is not nested by the PT condition. PT does not restrict the trend to zero, and so it does not imply stationarity. Conversely, stationarity makes no assumptions about endogeneity of the realized treatment sequence \((D_0, D_1)\), and so it does not imply any version of PT. In terms of the linear panel-data model for potential outcomes in (5), mean stationarity assumes the untreated (and nonstochastic) time trend is constant, \(\delta_0 = \delta_1\) (under the location normalization that time-varying errors \(\epsilon_{it}\) have zero means), but it makes no assumption about the exogeneity of treatments. Furthermore, this does not typically impose restrictions on the trend in realized outcomes because of selection.

Even though the unconditional stationarity in assumption 3 does not imply PT, it also identifies the average treatment effect on the treated in the basic setting with a pretreatment period.

**Proposition 3.** Suppose \(D_0 = 0\). Under assumption 3,

\[ E[Y_1(1) - Y_1(0)|D_1 = 1] = E[Y_1(1) - Y_1(0)|D_0 < D_1] = \frac{E[Y_1] - E[Y_0]}{P(D_1 = 1)}. \]

The proof is provided in appendix A. Because \(D_1 \geq D_0 = 0\), this appears related to the LATE theorem of Imbens and Angrist (1994) with time as an instrument. However, recall a few important differences from the previous discussion of the target parameter (32). First, the time index violates the exclusion restriction because the potential outcome \(Y_i(d)\) can

---

24 However, when outcomes are affected by an additional unobserved effort \(F_i\), as in subsection IV.E, mean stationarity requires constant realized returns from effort \(E[\gamma_i F_i - \gamma_i' F_i] = 0\), whereas under PT, these returns are allowed to vary over time as in (31), albeit independently of the treatment sequence.

25 A related version of time invariance is considered by Manski and Pepper (2018). The main difference between our assumptions is that theirs is imposed by indexed observation rather than in expectation; in our setting, an element-wise version of time invariance, i.e., \(Y_0(0) = Y_1(0)\), would also imply PT.

26 For previous results interpreting time as an instrument, see also de Chaisemartin and d’Haultfoeuille (2018).
depend on $t$. Time invariance (assumption 3) can be interpreted as a mean exclusion restriction on the untreated outcomes, but no such assumption is imposed on mean treated outcomes. Instead, mean treated outcomes in period 0 are by assumption never realized. Finally, both $D_0$ and $D_1$ are everywhere observed.

A stronger, conditional version of stationarity identifies an average treatment effect on switchers in a fuzzy design, where some units are initially treated so that $P(D_0 = 1) > 0$.

Assumption 4 (Forward mean stationarity of untreated outcomes). The mean untreated outcome is constant across time, conditional on past treatment:

$$E[Y_1(0) - Y_0(0)|D_0 = d_0] = 0 \text{ for } d_0 \in \{0, 1\}.$$  

Assumption 4 is an assumption jointly about stationarity (because it implies assumption 3) and about selection (because it conditions on treatment and implies assumption 2). Thus, it is similar to a mean version of conditional stationarity in Roth and Sant’Anna (2021). However, it differs by conditioning on past treatment in period 0 rather than treatment in period 1. As illustrated by the examples in section IV that violate PT, conditioning on past treatment may be more credible in dynamic choice contexts where decision-makers are less likely to have (and thereby select on) information about the future than information about the past. In particular, under zero trend, assumption 4 is satisfied with learning in example 3, whereas its analog conditioning on treatment in period 1 is not.

Under assumption 4, an analogous identification result is immediate upon applying proposition 3 in the subpopulation where $D_0 = 0$.

**Corollary 1.** Under assumption 4, the treatment effect in period 1 for the switchers into treatment is identified:

$$E[Y_1(1) - Y_1(0)|D_0 < D_1] = \frac{E[Y_1|D_0 = 0] - E[Y_0|D_0 = 0]}{P(D_1 = 1|D_0 = 0)}.$$

It is worth briefly comparing this to the weaker assumption 2 result, which allowed for a nonzero conditional trend $\tau_0 \neq 0$. In that case, identification was not possible without further assumptions.

In the next subsection, we turn to the possibility of partial identification when such assumptions on trends are not viable.

**C. Bounds from Economic Structure**

We now consider partial identification of $E[Y_1(1) - Y_1(0)|D_0 < D_1]$ under assumptions stemming from our modeling of dynamic selection.\(^{27}\) First,
observe that the treated outcome $E[Y_1(1)|D_b < D_t]$ is identified from the data, and so it suffices to focus on bounding the untreated outcome in period 1 among switchers into treatment, $E[Y_1(0)|D_b < D_t]$. Alternatively, expand the treatment effect as

$$E[Y_1(1) - Y_1(0)|D_b < D_t] = E[Y_1(1) - Y_0(0)|D_b < D_t] - E[Y_1(0) - Y_0(0)|D_b < D_t].$$

It suffices to bound the second term on the right, that is, the switchers’ untreated trend.

We can partially identify the average treatment effect on the switchers into treatment ($D_b < D_t$) using some version of monotone treatment on selection assumptions, similar to those introduced by Manski (1997) and Manski and Pepper (2000) but generalized to our multiperiod setting.

Assumption 5 (Monotone treatment selection; MTS). MTS on level and trend of untreated potential outcomes:

(a) Level: $E[Y_1(0)|D_b < D_t] \leq E[Y_1(0)|D_b = D_t = 0]$.

(b) Trend: $E[Y_1(0) - Y_0(0)|D_b < D_t] \geq E[Y_1(0) - Y_0(0)|D_b = D_t = 0]$.

Although one could consider more general MTS assumptions across treatment sequences, we focus on these specific comparisons to the never-treated because they (i) partially identify our target parameter and (ii) have a basis in our model of learning about the control (example 4). The proof of the following identification result is immediate by the preceding discussion.

Proposition 4. Under assumption 5, the identified set for the treatment effect $E[Y_1(1) - Y_1(0)|D_b < D_t]$ is

$$[E[Y_1(D_t > D_b)] - E[Y_1|D_b = D_t = 0]],$$

$$E[Y_1 - Y_0|D_t > D_b] - E[Y_1 - Y_0|D_b = D_t = 0]].$$

We now motivate the identifying assumptions with the model of learning about the control (example 4), where there is a pretreatment period ($D_b = 0$ for all units) and the PT condition was typically violated. We begin by showing that, among decision-makers whose period 1 decision depends on the past realized outcome, those who continue without treatment are oversampled from higher-mean untreated arms. This follows from two observations. First, recalling (17), past untreated outcomes provide further information about the true returns of the bandit arm, and thereby about future outcomes:28

$$E[Y_1(0)|U_0, Y_0(0) = 1] \geq E[Y_1(0)|U_0, Y_0(0) = 0]. \quad (36)$$

28 Recall that $U_0$ summarizes the information at time 0, i.e., $I_0 = \{U_0\}$. In this section, we use the $U_0$ notation to facilitate imposing additional structure.
Second, for initial information sets \( u^* \in \{ U_0(\omega) : \omega \in \Omega_0 \} \) of “valuable learners” in (15) where the period 1 treatment depends on the past outcome, we have the following equality (see eq. A4 in the appendix for details): \( \{ U_0 = u^*, D_0 = 0, D_1 = d_1 \} = \{ U_0 = u^*, Y_0(0) = 1 - d_1 \} \) for \( d_1 \in \{0, 1\} \). (37)

Combining (36) and (37) leads to higher untreated outcomes \( Y_1(0) \) in period 1 among the never-treated than among those who switch into treatment:

\[
E[Y_1(0)|U_0 = u^*, D_0 = 0, D_1 = 0] \geq E[Y_1(0)|U_0 = u^*, D_0 < D_1]. \tag{38}
\]

Additionally, (37) implies that

\[
E[Y_0(0)|U_0 = u^*, D_0 = 0, D_1 = d_1] = 1 - d_1, \tag{39}
\]

and the fact that \( Y_1(0) \) was assumed binary implies

\[
E[Y_1(0)|U_0 = u^*, D_0 = 0, D_1 = d_1] \in [0, 1]. \tag{40}
\]

Combining (39) and (40) yields the opposite direction of inequality on the untreated trend:

\[
E[Y_1(0) - Y_0(0) | U_0 = u^*, D_0 = D_1 = 0] \leq 0 \leq E[Y_1(0) - Y_0(0)|U_0 = u^*, D_0 < D_1]. \tag{41}
\]

The directions of inequality in (38) and (41) are those of assumption 5. However, these inequalities also condition on a period 0 unobservable realization \( U_0 = u^* \), for which the choice of period 1 treatment is a function of the period 0 realized outcome.

To extend and aggregate the inequalities (38) and (41) beyond valuable learners, one solution is to impose (or ideally, derive) additional structure between treatment sequences, untreated outcomes, and unobservables. For example, consider the following assumption to rationalize MTS on the level of untreated potential outcomes (assumption 5a).

**Assumption 6.** The initial information \( U_0 \in \mathbb{R} \) satisfies

(a) For all \( u \) where the conditional expectations exist,

\[
E[Y_1(0)|U_0 = u, D_0 < D_1] \leq E[Y_1(0)|U_0 = u, D_0 = 0].
\]

(b) \( E[Y_1(0)|U_0 = u, D_0 = 0] \) is nondecreasing in \( u \).

(c) \((U_0|D_0 < D_1)\) is first-order stochastically dominated by \((U_0|D_0 = D_1 = 0)\).

Assumption 6a generalizes (38) across all unobservables \( U_0 = u \): selection into treatment in period 1 is indicative of lower untreated outcomes in expectation. Assumption 6b imposes that higher unobservables
correspond to higher untreated outcomes; for example, the unobservable could be an average return from a previous, fixed-length sequence of untreated outcome realizations observed by decision-makers but not the econometrician. Note that this has no content on its own, because unobservables $U_0$ can always be relabeled and arranged such that this condition is satisfied. However, assumption 6c imposes that unconditional selection into treatment oversamples lower unobservables, which by assumption 6b correspond to lower mean expected returns. Under assumption 6, we recover assumption 5a as follows:

$$E[Y_1(0)|D_0 < D_1] = E[E[Y_1(0)|U_0, D_0 < D_1]|D_0 < D_1]$$

$$\leq E[E[Y_1(0)|U_0, D_0 = 0]|D_0 < D_1]$$

$$\leq E[E[Y_1(0)|U_0, D_0 = D_t = 0]|D_0 = D_t = 0]$$

$$\leq E[Y_1(0)|D_0 = D_t = 0],$$

where the equalities follow by the law of iterated expectations, the first and third inequalities follow from assumption 6a, and the second inequality follows from combining assumption 6b and 6c. Alternatively, one could attempt to (partially) identify a marginal group $U_0 = u^*_0$ with other assumptions or an additional source of exogenous variation, such as an instrument. We leave this to future work.

VI. Conclusion

In this paper, we made connections between the commonly used PT assumption and models of dynamic rational choice in economics. In particular, we highlight models with time-varying treatment costs, learning, correlated utilities, and optimal stopping. Our aim is to focus on channels of dynamic behavior that are at work in these models to understand the way that these channels can validate or invalidate the design-based identifying assumption of PT. The examples we provide are deliberately stylized and simple. In cases when PT may be violated, we provide pointers to inference approaches based on simple and familiar economic restrictions that are motivated by economic concerns. These include either relaxing or considering alternatives to PT.

Our hope is that the paper provides a canvas by which further work on the econometrics of treatment or causal inference with observational data can be examined. This is particularly important with dynamic decisions in rich environments where a variety of preference- and information-based dynamic considerations play a role and where it is helpful to relate choice models based on these considerations to examine what behavior is allowed and what is not under the PT condition. We conclude with a sentiment
consistent with our approach in this paper, namely that “models are most useful when they are used to challenge existing formulations, rather than to validate or verify them” (Oreskes, Shrader-Frechette, and Belitz 1994, 644). In that sense, use of choice models to shed light on DID regressions is a common and worthy use of modeling.

Appendix A

Proofs

A1. Proof of Observation 1

The fact that the PT condition implies the set of pairwise equalities (1) and (2) is immediate. Conversely, suppose (1) holds and define \( \tau = E[Y_i(0) - Y_0(0)|D_i = d_0] \), which is constant and does not vary with \( d_0 \). It suffices to show that (1) and (2) imply \( E[Y_i(0) - Y_0(0)|D_0 = d_0, D_i = d_i] = \tau \) for each \( (d_0, d_i) \). Define \( p(d_0) = E[D_i|D_0 = d_0] \). Then for each \( t \), the law of total probability implies

\[
E[Y_t(0)|D_0 = d_0] = p(d_0)E[Y(0)|D_0 = d_0, D_t = 1] + (1 - p(d_0))E[Y(0)|D_0 = d_0, D_t = 0],
\]

and thus

\[
E[Y_i(0) - Y_0(0)|D_0 = d_0] = p(d_0)E[Y_i(0) - Y_0(0)|D_0 = d_0, D_i = 1] + (1 - p(d_0))E[Y_i(0) - Y_0(0)|D_0 = d_0, D_i = 0].
\]

By (2), both differences on the right are equal to the term on the left, which in turn is equal to \( \tau \). QED

A2. Proof of the Claim in Example 1

We first show that \( Y_0(0) = Y_i(0) \) almost surely is necessary for PT. First, observe that \( D_0 = 0, D_i = 1 - Y_0, \) and \( P(D_i = 1) \in (0, 1) \) imply

\[
E[Y_0(0)|D_i = d] = 1 - d. \tag{A1}
\]

Substituting into the deviation from PT,

\[
E[Y_i(0) - Y_0(0)|D_i = 1] - E[Y_i(0) - Y_0(0)|D_i = 0]
= E[Y_i(0)|D_i = 1] - E[Y_i(0)|D_i = 0] + 1.
\]

Because outcomes \( Y_i(0) \) are assumed binary, this deviation is zero only if

\[
E[Y_i(0)|D_i = d] = 1 - d \quad \text{for } d = 0, 1.
\]

Combining with (A1), this in turn requires \( Y_0(0) = Y_i(0) \) almost surely because the potential outcomes are binary. Conversely, \( Y_0(0) = Y_i(0) \) almost surely is sufficient for PT. QED
A3. Proof of the Claims in Example 2

In period 0, the treatment rule for \(D_0\) is a function of initial information \(U_0\) alone by (8). Furthermore, under (12), the treatment rule at \(t = 1\) is

\[
D_t(d_t) = \mathbf{1}\{E[Y_t(1) - Y_t(0) - (K_t(d_t, 1) - K_t(d_t, 0))|U_t, U_t(d_t)] \geq 0\} = \mathbf{1}\{E[Y_t(1) - Y_t(0) - (K_t(d_t, 1) - K_t(d_t, 0))|U_0] \geq 0\}. \tag{A2}
\]

Hence in period 1, the treatment rule for \(D_t(d_t)\) is also a function of \(U_0\) alone. It then follows that the realized treatment at \(t = 1\), that is, \(D_t = D_t(D_0)\), is also a function of \(U_0\) alone. By the law of iterated expectations and (11), we have

\[
E[Y_t(0) - Y_t(0)|D_0 = d_0, D_t = d_t] = E[E[Y_t(0) - Y_t(0)|U_0]|D_0 = d_0, D_t = d_t] = \tau
\]

for all \(d_0, d_t \in \{0, 1\}\). QED

A4. Proof of the Claims in Example 3

The optimal treatment rule (8) in period 0,

\[
D_0 = \mathbf{1}\{E[V_0(1) - V_0(0) + \beta(W_t(1) - W_t(0))|U_0] \geq 0\}, \tag{A3}
\]

is a function of \(U_0\) alone. By the law of iterated expectations and (11),

\[
E[Y_t(0) - Y_t(0)|D_0 = d_0] = E[E[Y_t(0) - Y_t(0)|U_0]|D_0 = d_0] = \tau.
\]

With no treatment in period 0 and because \(U_t(d_0) = Y_t(d_0)\), the optimal treatment rule (7) in period 1 is

\[
D_t(0) = \mathbf{1}\{E[Y_t(1) - Y_t(0) - (K_t(0, 1) - K_t(0, 0))|U_0, Y_0(0)] \geq 0\} = \mathbf{1}\{E[Y_t(1) - Y_t(0) - (K_t(0, 1) - K_t(0, 0))|U_0] \geq 0\},
\]

where the second equality holds because of (13) and because costs are known initially \(K \in \mathcal{I}_0\). With \(D_t(0)\) and \(D_0\) both being functions of \(U_0\) alone, we can apply the law of iterated expectations and use the condition (11) to show

\[
E[Y_t(0) - Y_t(0)|D_0 = 0, D_t = d_t] = E[E[Y_t(0) - Y_t(0)|U_0]|D_0 = 0, D_t(0) = d_t] = E[E[Y_t(0) - Y_t(0)|U_0]|D_0 = 0, D_t(0) = d_t] = \tau
\]

for all \(d_t \in \{0, 1\}\).

With treatment in period 0, the optimal treatment rule (7) in period 1 is

\[
D_t(1) = \mathbf{1}\{E[Y_t(1) - Y_t(0) - (K_t(0, 1) - K_t(0, 0))|U_0, Y_0(1)] \geq 0\}.
\]

Again, by the law of iterated expectations,

\[
E[Y_t(0) - Y_t(0)|D_0 = 1, D_t = d_t] = E[E[Y_t(0) - Y_t(0)|U_0]|D_0 = 1, D_t(1) = d_t] = E[E[Y_t(0) - Y_t(0)|U_0, Y_0(1)]|D_0 = 1, D_t(1) = d_t].
\]

Note that

\[
E[Y_t(0) - Y_t(0)|U_0, Y_0(1)] = E[Y_t(0) - Y_t(0)|U_0] = \tau,
\]
where the first equality is due to (13) and (14), and the second is due to (11). Hence

\[ E[Y_i(0)] - Y_i(0) | D_h = 1, D_t = d_t = \tau \]

for \( d_t \in \{0, 1\} \). It then follows from observation 1 that the PT condition holds.

QED

A5. Proof of the Claims in Example 4

With no one treated initially \((D_h = 0)\) and costs known to decision-makers \((K \in I_0)\), the optimal treatment rule \((7)\) in period 1 becomes

\[ D_t = D_t(0) = 1 \{ E[Y_i(1) | U_0] - E[Y_i(0) | U_0, Y_0(0)] \geq K_i \} \quad (A4) \]

because there is no learning across arms in \((16)\).

Combining the decision rule \((A4)\) and the assumption on learning in \((17)\), we can partition the sample space into three groups on the basis of the response to (possibly counterfactual) information from the past realized outcome \(Y_0(0):\) those who are always \((a)\) or never \((n)\) treated in period 1 regardless of factual period 0 outcomes, that is, for values of \(Y_0(0)\),

\[ \Omega_a = \{ \omega \in \Omega : E[Y_i(1) | U_0(\omega)] \geq E[Y_i(0) | U_0(\omega), Y_0(0) = 1] \}; \]
\[ \Omega_n = \{ \omega \in \Omega : E[Y_i(1) | U_0(\omega)] < E[Y_i(0) | U_0(\omega), Y_0(0) = 0] \}; \]

and those for whom \(D_t(0)\) depends on \(Y_0(0)\) because there is valuable learning \((vl)\), which was defined in \((15)\) and is restated here for convenience: 29

\[ \Omega_d = \{ \omega \in \Omega : E[Y_i(0) | U_0(\omega), Y_0(0) = 0] \leq E[Y_i(1) - K_i | U_0(\omega)] \]
\[ < E[Y_i(0) | U_0(\omega), Y_0(0) = 1] \}. \]

Even though the partition is defined on the basis of the responses to period 0 outcomes, it is by construction measurable with respect to initial information \(U_0\). Also, the groups are defined in terms of expectations conditional on counterfactual period 0 outcomes, that is, for values of \(Y_0(0)\) that may differ from those realized. 30 We can further partition the “valuable learning” group on the basis of its realized period 0 outcomes:

\[ \Omega_{d}^{0} = \{ \omega \in \Omega _{d} : Y_0(0; \omega) = 0 \}, \]
\[ \Omega_{d}^{1} = \{ \omega \in \Omega _{d} : Y_0(0; \omega) = 1 \}. \]

Let \(P_i^j = P(\Omega_i^j)\) for \(i \in \{a, n, vl\}\) and \(j \in \{0, 1\}\).

29 For the always- and never-treated, learning from past outcome does not affect their future decisions and hence is not considered “valuable.”

30 For simplicity of notation, we assume the expectations are well defined. This is so except where potential outcomes \(Y_i(0)\) are degenerate given \(U_0\). At such values of initial information there is no rational learning, in which case the ill-defined history-conditional expectations \(E[Y_i(0) | U_0, Y_0(0)]\) can be replaced with the well-defined history-unconditional expectations \(E[Y_i(0) | U_0]\).
We can then write
\[
E[Y_t(0) - Y_0(0)|D_t = 1, D_0 = 0] = E[Y_t(0) - Y_0(0)|\Omega_v \cup \Omega_v^d]
\]
\[
= \frac{P_a}{P_a + P^d_v} E[Y_t(0) - Y_0(0)|\Omega_v] + \frac{P^d_v}{P_a + P^d_v} E[Y_t(0) - Y_0(0)|\Omega_v^d]
\]
\[
= \frac{P_a}{P_a + P^d_v} \tau + \frac{P^d_v}{P_a + P^d_v} E[Y_t(0)|\Omega_v^d],
\]
where the last equality follows from the fact that \(\Omega_v\) restricts only the initial information \(U_0(\omega)\), combined with an application of the law of iterated expectation to (11), and from the fact that \(Y_0(0) = 0\) among \(\Omega_v^d\). By analogous reasoning,
\[
E[Y_t(0) - Y_0(0)|D_t = 0, D_0 = 0] = E[Y_t(0) - Y_0(0)|\Omega_v \cup \Omega_v^d]
\]
\[
= \frac{P_n}{P_n + P^d_v} E[Y_t(0) - Y_0(0)|\Omega_v] + \frac{P^d_v}{P_n + P^d_v} E[Y_t(0) - Y_0(0)|\Omega_v^d]
\]
\[
= \frac{P_n}{P_n + P^d_v} \tau + \frac{P^d_v}{P_n + P^d_v} (E[Y_t(0)|\Omega_v^d] - 1).
\]

PT holds if and only if both preceding equations are equal to the same constant. Because \(E[Y_t(0) - Y_0(0) = \tau\) by (11), that constant must be \(\tau\). This latter condition (that both preceding equations are equal to the constant \(\tau\)) holds under one of the following two sufficient conditions: (i) there is zero probability of valuable learning, \(P_v = P^d_v = 0\) or (ii) valuable learning occurs where untreated outcomes are identical over time almost surely, \(P(Y_t(0) = Y_t(0)|\Omega_v) = 1\), and the trend is zero (\(\tau = 0\)).

The sufficiency of condition i is straightforward. For the sufficiency of condition ii, suppose \(P_v > 0\), which in turn can be split into two cases. First, if \(P^d_v > 0\), \(P^d_v > 0\), then PT requires that
\[
E[Y_t(0)|\Omega_v^d] = E[Y_t(0)|\Omega_v^d] - 1 = \tau.
\]
Because \(Y_t(0)\) is binary, \(E[Y_t(0)|\Omega_v^d] \geq 0\) while \(E[Y_t(0)|\Omega_v^d] - 1 \leq 0\). So, the two equalities above are only possible together if \(\tau = 0\), which happens if \(P(Y_t(0) = Y_t(0)|\Omega_v) = 1\), and there is a zero trend, \(\tau = 0\). The sufficiency of condition ii when either \(P^d_v\) or \(P^d_v\) is zero but not both follows from an analogous argument and is omitted for brevity. QED

A6. Proof of the Claims in Example 5

That conditions 1 and 2 are sufficient for the PT condition is immediate from the fact they jointly imply
\[
E[Y_t(0) - Y_0(0)|D_h = d_v, D_t = d_v] = 0
\]

This still allows for learning about the untreated arm. For example, there is initial uncertainty about whether the process is one of two degenerate arms, so that \(E[Y_t(0)|\Omega_v] \in (0, 1)\). However, once the first return is observed, this uncertainty is fully resolved, i.e., \(E[Y_t(0)|\Omega_v, Y_0(0)] \in \{0, 1\}\).
for all \((d_0, d_1)\) occurring with positive probability. Conversely, suppose the Roy model holds, so that
\[ D_t = 0 \Rightarrow Y_t(0) = 1. \]

We consider two cases on the basis of whether there is an interior probability of never-treated observations, \(P(D_t = D_0 = 0) > 0\). First, if there exist never-treated, then (A5) implies
\[ E[Y_t(0) - Y_o(0)|D_h = D_1 = 0] = 1 - 1 = 0. \]

Therefore, the PT condition requires stationarity (condition 1). Again invoking (A5), PT also requires that
\[ E[Y_t(0)|D_h < D_1] = 1, \]
\[ E[Y_t(0)|D_h > D_1] = 1. \]

In turn, binary outcomes then imply that
\[ E[Y_t(0) - Y_o(0)|D_h < D_1] \leq 0 \leq E[Y_t(0) - Y_o(0)|D_h > D_1]. \]

Therefore, the PT condition requires stationarity (condition 1). Combined with the preceding restrictions on degenerate outcomes, this also requires condition 2. QED

A7. Proof of Proposition 1

Sufficiency of strict exogeneity (20) for PT is immediate because then
\[ E[Y_t(0) - Y_o(0)|D_h = d_0, D_1 = d_1] = \mu_1 - \mu_0 \]
for constants \(\mu_0, \mu_1 \in \mathbb{R}\) and all \(d_0, d_1 \in \{0, 1\}\). Conversely, suppose treatment decisions satisfy (18) and \((U_t, Y_t(\cdot))\) satisfy (19). Because treatments \(D_t\) are a function of \(U_t\) by (18), it follows from (19) that \(D_t \perp (Y_{t-1}(0), D_{t-1})\), and therefore \(D_t \perp Y_{t-1}(0)|D_{t-1}\), for \(t \in \{0, 1\}\). In that case, we have
\[ E[Y_t(0)|D_h = d_0, D_1 = d_1] = E[Y_t(0)|D_h = d_1]. \] (A6)

(Note that this is testable evaluating at \(d_t = 0\) and comparing across \(d_{t-1} = 0, 1\).)

If \(E[D_t] \in \{0, 1\}\) for each \(t \in \{0, 1\}\), then the implication (20) is trivial. Therefore, suppose \(E[D_h] \in (0, 1)\). Then for a realized period 1 outcome occurring with positive probability, say \(D_1 = 0\), the PT condition combined with (A6) requires
\[ E[Y_t(0)|D_1 = 0] - E[Y_o(0)|D_h = 0] = E[Y_t(0)|D_h = 0] - E[Y_o(0)|D_h = 1] \]
or
\[ E[Y_o(0)|D_h = 0] = E[Y_o(0)|D_h = 1] = \mu_0 \text{ for some constant } \mu_0 \in \mathbb{R}. \]
Invoking again (A6), it follows that

\[ E[Y_0(0)|D_0 = d_0, D_1 = d_1] = \mu_0 \]

for all realizations \((d_0, d_1)\) occurring with positive probability. An analogous argument holds to show (20) for \(t = 1\). QED

A8. Proof of Example 6

As in the proof of example 5, that conditions 1 and 2 are sufficient for the PT condition is immediate. Conversely, suppose the Roy model with irreversible treatment (henceforth the “constrained” model) holds. Then the optimal treatment rules (7) and (8) reduce to

\[ D_1(0) = I\{Y_1(1) - Y_1(0) \geq 0\}, \quad (A7) \]
\[ D_0 = I\{Y_0(1) - Y_0(0) + \beta \min\{Y_1(1) - Y_1(0), 0\} \geq 0\}. \quad (A8) \]

To connect to the logic of the Roy model with \(K/(d^1) = 0\) (henceforth the “unconstrained” model), it is useful to recall and separately define the (counterfactual) unconstrained choices, \(\tilde{D}_t = I\{Y_t(1) \geq Y_t(0)\}\). By definition, in period 1 we have \(D_t(0) = \tilde{D}_t\);\(^32\) in period 0 we have \(D_0 \leq \tilde{D}_0\), because the lost option value from treatment in period 1 weakly discourages treatment in period 0, that is, \(\beta \min\{Y_1(1) - Y_1(0), 0\} \leq 0\). If \(D_1(0) = 1\), then \(Y_1(1) \geq Y_1(0)\), and so this option value is 0 and \(D_0 = \tilde{D}_0\). It follows from \(D_1(0) = \tilde{D}_1, D_0 \leq \tilde{D}_0, \) and \(D_1(0) = 1 \Rightarrow D_0 = \tilde{D}_0\) that the constrained and unconstrained movers into treatment are the same:

\[ \{D_0 < D_1\} = \{\tilde{D}_0 < \tilde{D}_1\}. \]

From (A5), it follows that among the constrained movers into treatment, we also have \(Y_0(0) = 1\). The constrained never-treated consist of (i) the unconstrained never-treated and (ii) the subset of the unconstrained movers out of treatment \(\{\tilde{D}_0 > \tilde{D}_1\}\) who change their choice in period 0 because of the constraint\(^33\)

\[ \{D_0 = D_1 = 0\} = \{\tilde{D}_0 = \tilde{D}_1 = 0\} \cup \{\tilde{D}_0 > \tilde{D}_0 = \tilde{D}_1\}. \]

Again from (A5), it follows that among the constrained never-treated we have \(Y_1(0) = 1\).

By the logic of the unconstrained strong Roy model (example 5), all unconstrained movers into treatment have a weakly negative untreated trend, all unconstrained never-treated have a zero untreated trend, and all unconstrained movers out of treatment have a weakly positive untreated trend:

\[32\] It is useful to note, however, that we may have \(D_1 \neq \tilde{D}_1\) when the constraint binds and \(D_0 = 1\).

\[33\] The unconstrained movers out of treatment who change their decision in period 0 because of the constraint must have a strictly positive continuation value, which implies \(Y_1(0) > Y_1(1)\). Furthermore, the choice rule (A7) and \(\beta \leq 1\) imply \(Y_1(1) = Y_1(0)\). Thus, the subset of unconstrained movers out of treatment affected by the constraint are intuitively those with a relatively high option value of waiting.
By assumption that constrained movers into treatment and constrained never-treated each exist with positive probability, it follows that the groups have weakly negative and positive untreated trends, respectively:

By assumption that constrained movers into treatment and constrained never-treated each exist with positive probability, it follows that the groups have weakly negative and positive untreated trends, respectively:

Thus, the PT condition requires stationarity (condition 1). The necessity of condition 2 then follows because $Y_0(0) = 1$ among movers into treatment and $Y_1(0) = 1$ among the constrained never-treated. QED

A9. Proof of Proposition 3

We have

The first equality follows because no one is treated in period 0, that is, $D_0 = 0$ almost surely. The fourth equality follows from the law of iterated expectation. The fifth equality follows from assumption 3. The sixth equality follows from the law of iterated expectation and the fact that $D_0 = 0$ almost surely. QED.

Appendix B

Selection and Parallel Trends: Heuristics

As discussed in sections III and IV, a DUM model allows individual decision-makers to self-select into treatment sequences on the basis of their information sets. Such endogenous self-selection could invalidate the PT condition, especially when the information sets are correlated with potential outcomes. In this appendix, we provide a list of heuristic guidelines, which may help empiricists to evaluate
the plausibility of PT in DID analyses. This list is motivated only by our results therefore is by no means exhaustive.

1. Direct selection on past outcomes. As in Ashenfelter (1978), PT is suspect if the treatment decision is a function of past outcome (example 1).

2. Indirect selection on past outcomes through learning. The presence of imperfect information about outcomes at the time of treatment, and thus the possibility of past outcome realizations affecting present treatment through learning, can potentially invalidate PT. In particular, PT is suspect if there is learning about untreated (potential) outcomes (examples 3 and 4).

On the other hand, forward-looking experimentation need not invalidate (partial) PT necessarily, because the treatment decision depends on outcome realizations that decision-makers do not yet know (example 4).

3. Roy-style selection on present outcomes. If the treatment in each period is a rational choice based on factors that are correlated with the potential outcomes, and if these factors and outcomes are insufficiently correlated over time, then PT is likely to be violated because of selection (example 5).

4. Staggered designs and irreversible treatments. These introduce inevitable forward-looking considerations on the basis of the foregone option value of delaying the irreversible choice until next period. Similar to the case of experimentation, such considerations may change the margin of treatment without violating PT per se. However, previous concerns about learning and selection on present outcomes remain (example 6 and sec. IV.D).

5. Anticipation and latent choices. As in the study by Malani and Reif (2015), if the expectation of future treatments affects present potential outcomes, then this can lead to violations of PT. More generally, if potential outcomes are indexed by other latent choices, these may lead to violations of PT (sec. IV.E).

Appendix C

Illustration with Ashenfelter and Card (1985)

We illustrate by linking our overall framework based on DUM and the relevant examples of section IV to the simplified setting of the job-training application in Ashenfelter and Card (1985). There, the sample consists of individuals \(i = 1, 2, ..., N\) and two periods \(t \in \{0, 1\}\). At time \(t\), each individual \(i\) receives a binary treatment \(D_i^t \in \{0, 1\}\), with \(D_i^1 = 1\) if \(i\) joins the job-training program at time \(t\). In the examples below, we may let \(Y_i^t(\cdot)\) be either discrete (e.g., employment status for \(i\) at time \(t\)) or continuous (e.g., wage compensation for \(i\) at time \(t\)).

In this case, our assumption 0 posits that an individual \(i\)'s potential outcome \(Y_i^t(d_i)\) depends only on \(i\)'s own contemporaneous treatment \(d_i\).

Again, note that the expectation of future treatments can affect present realized outcomes through treatment decisions with forward-looking motives without necessarily violating PT.
Example 1 (Selection on past outcomes).—Suppose the potential outcome of interest is an individual $i$'s employment status at time $t$, that is, $Y_i(0), Y_i(1) \in \{0, 1\}$. Let there be a pretreatment period, that is, no one joins the job-training program initially so that $D_{it} = 0$ for all $i$. Also, suppose $D_{it} = 1 - Y_{it}$, that is, only those unemployed at $t = 0$ choose to join the job-training program at $t = 1$. In this case, the PT condition holds if and only if the potential employment status of $i$ without participating in the training program does not change over time $t = 0, 1$. That is, $Y_{i0}(0) = Y_{i0}(0)$ for all $i$.

To set the stage for further examples, we fit the Ashenfelter and Card (1985) application within our unifying framework, under which the sequence of treatments is determined endogenously by DUM. We suppress individual subscripts $i$ from notation in what follows. Let the initial information $I_0 = \{U_0\}$ subsume (time-invariant) individual heterogeneity—say, $\omega$, in equation (1) in Ashenfelter and Card (1985)—as well as the discount factor $\beta$ and predetermined fees for program participation,

$$K_i(0) = 0 \text{ and } K_i(1) = k \text{ for } t = 0, 1. \quad \text{(C1)}$$

Let (potential) incremental information $U_t(d_t)$ consist solely of realized past outcomes, that is, $I_t(d_t) = \{U_0, U_t(d_t)\}$ with $U_t(d_t) = Y_t(d_t)$. Let the net (static) payoff at time $t$ depend only on contemporary treatment status, that is, $V_t(d_t) = Y_t(d_t) - K_t(d_t)$. Then the decision rules in (7) and (8) are simplified to

$$D_t(d_t) = 1 \{E[Y_t(1) - Y_t(0) \mid U_0, Y_t(d_t)] \geq k\}, \quad \text{(C2)}$$

$$D_0 = 1 \{E[Y_0(1) - Y_0(0) + \beta (W_t(1) - W_t(0)) \mid U_0] \geq k\}, \quad \text{(C3)}$$

where

$$W_t(d_t) = \max \{E[Y_t(1) \mid U_0, Y_t(d_t)] - k, E[Y_t(0) \mid U_0, Y_t(d_t)]\}$$

is the optimal continuation value.

Example 2 (No learning about potential outcomes).—Suppose the potential outcomes $Y_t(d_t)$ are wage compensations, and the sequence of decisions to join the job-training program are made by rational, forward-looking individuals as under DUM. Assume that an individual’s heterogeneity in the initial information set does not affect the average trend in his wage without any training, as stated in (11). Furthermore, assume that once conditional on such individual heterogeneity, the past wages $Y_t(d_t)$ are never informative about average wages $Y_t(d_t)$ in the future, regardless of how the potential treatment status varies over time (i.e., regardless of whether $d_t = d_t$ or not), as stated in (12). Then the PT condition holds.

Example 3 (Learning only under treatment).—As in example 2, suppose the potential outcomes $Y_t(d_t)$ are wage compensations, and the sequence of decisions to join the job-training program is made by rational, forward-looking individuals as under DUM. Suppose an individual’s heterogeneity in the initial information set does not affect the average trend in his wage without any training. In addition, suppose that once conditional on individual heterogeneity in $U_{it}$, an individual can learn about his expected wage with training in the future $Y_t(1)$ only if he observes past wages with training $Y_t(1)$; no other scenarios of learning beyond the influence of initial heterogeneity $U_0$ is possible. Furthermore, suppose the past
wage under training $Y_0(1)$ is not informative about the mean of past wage without training $Y_0(0)$, once controlling for the initial heterogeneity $U_0$. Then the PT condition holds.

Example 4 (Learning under control).—As in example 1, suppose the potential outcome is an individual $i$’s employment status at time $t$, that is, $Y_t(0), Y_t(1) \in \{0, 1\}$. As in example 2, let the sequence of decisions to join the job-training program be made by rational, forward-looking individuals under DUM. For simplicity, suppose there is a pretreatment period, that is, no one joins the job-training program initially so that $D_0 = 0$ for all individuals. If the past employment status without training $Y_0(0)$ is informative about future employment status without training $Y_1(0)$ for the individual (i.e., $Y_0(0)$ affects the distribution of $Y_1(0)$ even after controlling for initial heterogeneity $U_0$), then the PT condition is typically violated—unless such learning is never valuable (in that it affects the enrollment decision in period 1) or where it is valuable it is also perfect (in that valuable learners become fully aware of their future employment status in period 1 if they were to remain unenrolled). Note that the learning channel would invalidate PT even if the treatment decision in period 1 were not a direct function of employment status in period 0, that is, recruiting for job training among the unemployed.

Example 5 (Selection on present outcomes: static, repeated Roy).—As in example 1, suppose the potential outcome is an individual $i$’s employment status at time $t$, that is, $Y_t(0), Y_t(1) \in \{0, 1\}$. Also, as in example 2, let the sequence of decisions to join job training be made by rational, forward-looking individuals under DUM. For simplicity, suppose that opportunity costs of the training program are known and negligible $k = 0$ and that individuals know their potential employment status in each period. Suppose individuals enroll in job training in period $t$ if it causes employment, $Y_t(1) > Y_t(0)$.$^{35}$ Then the PT condition holds only if there is no aggregate job growth without job training, $E[Y_0(0)] = E[Y_1(0)]$, and those receiving job training in either period would have otherwise been unemployed in both periods, $Y_0(0) = Y_1(0) = 0$.

References

Abadie, A. 2005. “Semiparametric Difference-in-Differences Estimators.” Rev. Econ. Studies 72:1–19.
Abadie, A., A. Diamond, and J. Hainmueller. 2010. “Synthetic Control Methods for Comparative Case Studies: Estimating the Effect of California’s Tobacco Control Program.” J. American Statis. Assoc. 105 (490): 493–505.
Abbring, J. H. 2010. “Identification of Dynamic Discrete Choice Models.” Ann. Rev. Econ. 2:367–94.
Abbring, J. H., and G. J. van den Berg. 2003. “The Nonparametric Identification of Treatment Effects in Duration Models.” Econometrica 71 (5): 1491–517.
Acemoglu, D. 2010. “Theory, General Equilibrium, and Political Economy in Development Economics.” J. Econ. Perspectives 24 (3): 17–32.
Anderson, T., and C. Hsiao. 1982. “Formulation and Estimation of Dynamic Models Using Panel Data.” J. Econometrics 18 (1): 47–82.

$^{35}$ In the original example 5, we consider a deterministic choice rule that breaks ties in favor of treatment, but the results are essentially unchanged for the deterministic choice rule that breaks ties in favor of no treatment.
Angrist, J. D., and J.-S. Pischke. 2008. *Mostly Harmless Econometrics: An Empiricist’s Companion.* Princeton, NJ: Princeton Univ. Press.

Arellano, M., and S. Bond. 1991. “Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations.” *Rev. Econ. Studies* 58 (2): 277–97.

Arkhangelsky, D., and G. W. Imbens. 2021. “Double-Robust Identification for Causal Panel Data Models.” Working Paper no. 28364 (January), NBER, Cambridge, MA.

Ashenfelter, O. 1978. “Estimating the Effect of Training Programs on Earnings.” *Rev. Econ. and Statis.* 60 (1): 47–57.

Ashenfelter, O., and D. Card. 1985. “Using the Longitudinal Structure of Earnings to Estimate the Effect of Training Programs.” *Rev. Econ. and Statis.* 67 (4): 648–60.

Athey, S., and G. W. Imbens. 2022. “Design-Based Analysis in Difference-in-Differences Settings with Staggered Adoption.” *J. Economometrics* 226 (1): 62–79.

Ban, K., and D. Kédagni. 2022. “Generalized Difference-in-Differences Models: Robust Bounds.” https://arxiv.org/abs/2211.06710v1.

Bhargava, A., and J. Sargan. 1983. “Estimating Dynamic Random Effects Models from Panel Data Covering Short Time Periods.” *Econometrica* 51 (6): 1635–59.

Borusyak, K., X. Jaravel, and J. Spiess. 2021. “Revisiting Event Study Designs: Robust and Efficient Estimation.” https://arxiv.org/abs/2108.12419.

Callaway, B., and P. H. Sant’Anna. 2021. “Difference-in-Differences with Multiple Time Periods.” *J. Economometrics* 225 (2): 200–30.

Chabé-Ferret, S. 2015. “Analysis of the Bias of Matching and Difference-in-Difference under Alternative Earnings and Selection Processes.” *J. Economometrics* 185 (1): 110–23.

de Chaisemartin, C., and X. d’Haultfoeuille. 2018. “Fuzzy Differences-in-Differences.” *Rev. Econ. Studies* 85 (2): 999–1028.

———. 2020. “Two-Way Fixed Effects Estimators with Heterogeneous Treatment Effects.” *A.E.R.* 110 (9): 2964–96.

Dehejia, R. H., and S. Wahba. 1999. “Causal Effects in Nonexperimental Studies: Reevaluating the Evaluation of Training Programs.” *J. American Statis. Assoc.* 94 (448): 1053–62.

Ding, P., and F. Li. 2019. “A Bracketing Relationship between Difference-in-Differences and Lagged-Dependent-Variable Adjustment.” *Polit. Analysis* 27 (4): 605–15.

Dobkin, C., A. Finkelstein, R. Kluender, and M. J. Notowidigdo. 2018. “The Economic Consequences of Hospital Admissions.” *A.E.R.* 108 (2): 308–52.

Freyaldenhoven, S., C. Hansen, J. Pérez Pérez, and J. M. Shapiro. 2021. “Visualization, Identification, and Estimation in the Linear Panel Event-Study Design.” Working Paper no. 29170 (August), NBER, Cambridge, MA.

Fudenberg, D., and D. K. Levine. 2022. “Learning in Games and the Interpretation of Natural Experiments.” *American Econ. J. Microeconomics* 14 (3): 353–77.

Gentzkow, M., J. M. Shapiro, and M. Sinkinson. 2011. “The Effect of Newspaper Entry and Exit on Electoral Politics.” *A.E.R.* 101 (7): 2980–3018.

Goodman-Bacon, A. 2021. “Difference-in-Differences with Variation in Treatment Timing.” *J. Economometrics* 225 (2): 254–77.

Heckman, J. J. 1976. “A Life-Cycle Model of Earnings, Learning, and Consumption.” *J.P.E.* 84 (4, pt 2): S9–S44.
Rust, J. 1987. “Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zurcher.” *Econometrica* 55 (5): 999–1033.
———. 1994. “Structural Estimation of Markov Decision Processes.” In *Handbook of Econometrics*, vol. 4, edited by R. F. Engle and D. L. McFadden, 3081–143. Amsterdam: North-Holland.
Sun, L., and S. Abraham. 2021. “Estimating Dynamic Treatment Effects in Event Studies with Heterogeneous Treatment Effects.” *J. Econometrics* 225 (2): 175–99.
Taber, C. R. 2001. “The Rising College Premium in the Eighties: Return to College or Return to Unobserved Ability?” *Rev. Econ. Studies* 68 (3): 665–91.
Vella, F., and M. Verbeek. 1998. “Whose Wages Do Unions Raise? A Dynamic Model of Unionism and Wage Rate Determination for Young Men.” *J. Appl. Econometrics* 13 (2): 163–83.
Vytlacil, E. 2002. “Independence, Monotonicity, and Latent Index Models: An Equivalence Result.” *Econometrica* 70 (1): 331–41.
Wolpin, K. I. 1987. “Estimating a Structural Search Model: The Transition from School to Work.” *Econometrica* 55 (4): 801–17.