Extraction of the neutron charge radius from a precision calculation of the deuteron structure radius

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We present a high-accuracy calculation of the deuteron structure radius in chiral effective field theory. Our analysis employs the state-of-the-art semilocal two-nucleon potentials and takes into account two-body contributions to the charge density operators up to fifth order in the chiral expansion. The strength of the fifth-order short-range two-body contribution to the charge density operator is adjusted to the experimental data on the deuteron charge form factor. A detailed error analysis is performed by propagating the statistical uncertainties of the low-energy constants entering the two-nucleon potentials and by estimating errors from the truncation of the chiral expansion as well as from uncertainties in the nucleon form factors. Using the predicted value for the deuteron structure radius together with the very accurate atomic data for the difference of the deuteron and proton charge radii we, for the first time, extract the charge radius of the neutron from light nuclei. The extracted value reads \( r^2_n = -0.106^{+0.009}_{-0.007} \) fm\(^2\) and its magnitude is about 1.7σ smaller than the current value given by the Particle Data Group. In addition, given the high accuracy of the calculated deuteron charge form factor and its careful and systematic error analysis, our results open the way for an accurate determination of the nucleon form factors from elastic electron-deuteron scattering data measured at the Mainz Microtron and other experimental facilities.

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The tremendous progress in atomic spectroscopy achieved in the last decade led to a series of high-precision measurements of the energy-level shifts in light atomic systems which are important for understanding the structure of light nuclei and their charge distributions. In particular, a series of extremely precise measurements of the hydrogen-deuteron 1S-2S isotope shift (see Ref. [1] for the latest update) accompanied with an accurate theoretical QED analysis up through \( O(\alpha^3) \) resulted in the extraction of the deuteron-proton mean-square charge radii difference [2]

\[
\Delta r^2 \equiv r^2_d - r^2_p = 3.82070(31) \text{ fm}^2.
\]  

Because of its very high accuracy, this difference provides a tight link between \( r_d \) and \( r_p \) and thus is important in connection with the light nucleus charge radius puzzle. For many years, the values for \( r_p \) extracted from electron and muon experiments showed more than a 5σ discrepancy [3]. The very recent atomic hydrogen measurements [4, 5], however, claim consistency with the analogous muonic hydrogen experiments. The recommended value for the proton root-mean-square charge radius has been changed to \( r_p = 0.8414(19) \) fm in the latest CODATA-2018 update [6], and the deuteron charge radius was updated accordingly, by virtue of the difference in Eq. (1). The updated CODATA deuteron charge radius is only 1.9σ larger than the spectroscopic measurement on the muonic deuteron [7] but still 2.9σ smaller than the \( r_d \) value from electronic deuteron spectroscopy [8].

From the nuclear physics perspective, the charge radius of the deuteron provides access to the deuteron internal structure through its structure radius, which is obtained from \( r_d^2 \) by subtracting the contributions from the individual nucleons and the relativistic (Darwin-Foldy) correction,

\[
r^2_{\text{str}} = r^2_d - r^2_p - r^2_n - \frac{3}{4m_p^2}.
\]  

where \( m_p \) is the proton mass and \( r^2_n \) is the neutron mean-square charge radius. Traditionally, this relation is used to determine \( r^2_{\text{str}} \), assuming that \( r^2_d \) and \( r^2_p \) are known. The current value for the neutron charge radius quoted by the PDG is based on measurements of the neutron-electron scattering length in four different experiments carried out in 1973–1997 on \(^{208}\)Pb, \(^{209}\)Bi and other heavy targets. The world average gives \( r^2_n = -0.1161(22) \) fm\(^2\), where the estimated error was increased by a scaling factor of 1.3 [9]. Nevertheless, the spread in the results on Pb and Bi is significantly larger than even the increased uncertainty quoted by the PDG, which suggests that the error for the neutron mean-square charge radius might be underestimated [10].

With the recent advances in chiral effective field theory (χEFT), theoretical analyses of low-energy few-nucleon reactions and nuclear structure enter the precision era [11–13]. In this Letter we demonstrate that by employing the nuclear forces and currents derived up through fifth order in χEFT, a very accurate determination of \( r^2_{\text{str}} \) is becoming possible from the analysis of the deuteron charge form factor (FF). Equipped with this result and using the information from the hydrogen-deuteron isotope shift measurements given in Eq. (1), we use the relation (2) to extract, for the first time, the neutron mean-square charge radius from the lightest atoms.

The electromagnetic FFs of the deuteron certainly belong to the most extensively studied observables in nuclear physics, see Refs. [14–16] for review articles. A large variety of theo-
retical approaches ranging from nonrelativistic quantum mech
anics to covariant models have been applied to this problem
since the 1960s; see Ref. [17] for an overview. The electromag
etic structure of the deuteron has also been investigated in
the framework of pionless EFT [18] and χEFT [19,25]. It
is therefore crucial to emphasize the essential new aspects of
the current investigation.

- For the first time the calculation of the deuteron charge FF
is pushed to fifth order (N^4LO) in \( \chi EFT \). This is achieved
by (i) using the currently most accurate and precise \( \chi EFT \)
two-nucleon (2N) potentials from Ref. [26] and (ii) taking
into account the short-ranged contribution to the two-body
charge density operator at N^4LO.

- The two-body charge density is regularized consistently
with the 2N potential using the improved approach of
Ref. [26], which maintains the long-range interactions. The
residual cutoff dependence of our results is verified to be
well within the truncation uncertainty.

- We employ the most up-to-date parametrizations of the
nucleon FFs from the global analysis of experimental data [27,28]. To estimate the corresponding systematic uncertainty, we also use the results from the dispersive analyses of Refs. [29,31], which incorporate constraints from unitarity and analyticity and predict the small proton radius consistent with the CODATA-2018 recommended value [6].

- A thorough analysis of various types of uncertainty in the
calculated deuteron FFs and the structure radius is per
formed.

**Framework.** In the Breit frame, the deuteron charge form
factor is expressed in terms of the matrix elements of the
electromagnetic current, convolved with the deuteron wave func
tions as

\[
G_C(Q^2) = \frac{1}{2e} \frac{1}{2P_0} \sum \lambda \langle P', \lambda | J^\mu_B | P, \lambda \rangle ,
\]

\[
\frac{1}{2P_0} \langle P', \lambda | J^\mu_B | P, \lambda \rangle = \int \frac{d^3l_1}{(2\pi)^3} \frac{d^3l_2}{(2\pi)^3} \times \psi^\dagger \left( l_2 + \frac{k}{4} - v_B \right) J^\mu_B \psi \left( l_1 - \frac{k}{4} - v_B \right) ,
\]

where \( e \) is the magnitude of electron charge, \( J^\mu_B \) is the four
vector current calculated in the Breit frame, \( \psi \lambda \) is the deuteron wave function with polarization \( \lambda \) and the deuteron in the
final (initial) state moves with the velocity \( v_B = (0, k) \) with
\( v_B = \frac{k}{\sqrt{k^2 + 4 + m_d^2}} = \frac{k}{\sqrt{1 + \eta}} \) along the
photon momentum. The relativistic corrections to the deuteron wave functions related to the motion of the initial and final deuterons are included along the line of Ref. [32]. Furthermore, denoting the photon momentum \( k = (0, k) \) (with \( Q^2 = -k^2 \geq 0 \)) and the deuteron mass \( m_d \), the deuteron initial and final momenta read \( P = (P_0, -k/2) \) and \( P' = (P_0, +k/2) \), respectively, with \( P_0 = m_d \sqrt{1 + \eta} \) and \( \eta = Q^2/(2m_d)^2 \).

The deuteron charge radius is defined as follows:

\[
r_d^2 = -6 \left( \frac{\partial G_C(Q^2)}{\partial Q^2} \right)_{Q^2=0} .
\]  

The calculation of the deuteron FFs requires two important
ingredients which need to be derived in a consistent manner,

- the nuclear wave functions and the electromagnetic
currents. The employed deuteron wave functions are calculated
from the state-of-the-art \( \chi EFT \) 2N potentials of Ref. [26]
which are among the most precise interactions on the market.
Among many appealing features of these interactions, we
especially benefit from a simple regularization scheme
for the pion exchange contributions which (i) maintains the
long-range part of the interaction, (ii) is applied in momentu
space and (iii) allows for a straightforward generalization
to current operators and many-body forces at tree level.

The nuclear electromagnetic charge and current operators
have been recently worked out to N^3LO in \( \chi EFT \) using the
method of unitary transformation [19–25] by our group
and employing time-ordered perturbation theory [36–38] by the
JLab-Pisa group, see also Ref. [39] for an early study along
this line. The derivation of the electromagnetic currents and
nuclear forces is carried out using the Weinberg power coun
ling based on the expansion parameter \( Q = p/\Lambda_\chi \) with \( p \sim M_\chi \) being a characteristic soft momentum scale (with \( M_\chi \) denot
ing the pion mass) and \( \Lambda_\chi \) referring to the breakdown scale
of the chiral expansion. This implies that the contributions to
the charge operators relevant for our study appear at or
ders \( Q^{-3} \) (LO), \( Q^{-1} \) (NLO), \( Q^0 \) (N^2LO), \( Q^1 \) (N^3LO) and
\( Q^2 \) (N^4LO). To the order we are working, the single-nucleon
contribution to the charge density operator in the kinematics
\( N(p) + \gamma(k) \to N(p') \) takes a well-known form (see, e.g.,
Ref. [35] and references therein)

\[
\rho_{1N} = e \left( 1 - \frac{k^2}{8m_N^2} \right) G_E(k^2) + ie \frac{G_{1N}}{4m_N} (\sigma \cdot k \times k_1) ,
\]

where \( k_1 = (p + p')/2 \) and \( G_E(k^2) \) and \( G_{1N}(k^2) \) are the elec
tric and magnetic form factors of the nucleon, \( G_{1N} := 2G_M(k^2) - G_E(k^2) \), and \( m_N \) denotes the nucleon mass. The term \( eG_E \) on the rhs of Eq. (6) emerges at LO, while all
other terms start to contribute at N^2LO. Contributions to the
two-body charge density first appear at N^3LO from one- and
two-pion exchange diagrams, see Ref. [34] for explicit expres
sions. Most of them are of the isovector type and, therefore,
do not contribute to the deuteron FFs. The only N^3LO op
erator relevant for our study, to be denoted as \( \rho_{1N}^{\pi\pi} \), is a rel
ativistic correction to the one-pion exchange. It is proportional
to unobservable phases \( \beta_8 \) and \( \beta_9 \) which parametrize the uni
tarity ambiguity of the long-range nuclear forces and currents
at N^3LO. In contrast to nuclear potentials, observable quan
tities such as e.g. the form factors must, of course, be inde
pendent of the choice of \( \beta_8 \), \( \beta_9 \) and other off-shell param
eters. This can be achieved only by using off-shell consisten
t expressions for the nuclear forces and currents. Spec
ically, to preserve consistency with semilocal 2N potentials
of Ref. [26], the one-pion exchange charge density has to be evaluated using the so-called minimal nonlocality choice with $\beta_8 = 1/4$ and $\beta_9 = -1/4$. Although the pionic contributions to the isoscalar charge density at N$^4$LO have not been worked out yet, the complete expression for the contact operators at N$^4$LO reads [40,41] (the contact term relevant for the quadrupole moment of the deuteron was first derived in Ref. [18])

$$\rho_{2N}^{\text{cont}} = 2 e G_E^{\pi}(k^2) (A k^2 + B k^2 (\sigma_1 \cdot \sigma_2) + C k \cdot (\sigma_1 k \cdot \sigma_2)),$$

where three low-energy constants (LECs) $A$, $B$, and $C$ contribute to the deuteron charge FF in one linear combination only, see Supplemental Material [42] for details.

The isoscalar electric nucleon form factor, $G_E^{\pi}(k^2)$, is included in the two-body operators to account for a non-pointlike character of the NN$\gamma$ vertex. The chiral expansion of the electromagnetic FFs of the nucleon is well known to converge slowly as they turn out to be dominated by the contributions of vector mesons [43,44], which are not included as explicit degrees of freedom in $\chi$EFT. Therefore, to minimize the impact of the slow convergence of the chiral expansion of the nucleon FFs on 2N observables, we employ up-to-date parametrizations of the nucleon FFs from Ref. [28] as well as from several dispersive analyses of Refs. [29,31].

The 2N charge density operators $\rho_{2N}^N$ and $\rho_{2N}^{\text{cont}}$ have to be derived using the same regulator as employed in the 2N potentials. The regularization of the operators with the single pion propagator is worked out in Ref. [26] and can be effectively written as a substitution:

$$\frac{1}{p^2 + M_{\pi}^2} \rightarrow \frac{1}{p^2 + M_{\pi}^2} \exp \left( -\frac{p^2 + M_{\pi}^2}{\Lambda^2} \right),$$

where $\Lambda$ is a fixed cutoff chosen consistently with the employed 2N potential in the range of 400–550 MeV. The prescription for regularizing the squared pion propagator consistent with the approach used in [26] can be obtained from Eq. (7) by taking a derivative with respect to $M_{\pi}^2$. To maintain consistency between $\rho_{2N}^{\text{cont}}$ and the corresponding short-range terms in the 2N potential after regularization, we exploit the fact that both can be generated from the same unitary transformation acting on the single-nucleon charge density and the kinetic energy term, respectively [41].

**Results and discussions.** The calculated deuteron FF at N$^4$LO, $G^{1h}_{C}(Q)$, involves one unknown parameter (a combination of the LECs from $\rho_{2N}^{\text{cont}}$), which is extracted from a fit to the world data for the deuteron charge form factor $G_E^{\pi}(Q)$ from Refs. [45–47]. Here and in what follows, the N$^4$LO results are obtained using the N$^4$LO$^*$ 2N potentials, which include 4 sixth-order contact interactions in F waves and result in a nearly perfect description of 2N data from the Granada 2013 database [48] below the pion production threshold.

The function $\chi^2$ to be minimized in the fit is defined as follows

$$\chi^2 = \sum_i \frac{(G^{1h}_{C}(Q_i) - G^{\text{exp}}_{C}(Q_i))^2}{\Delta G_{C}(Q_i)^2},$$

where following Refs. [49,50] the uncertainty $\Delta G_{C}(Q_i)$ besides the experimental errors also takes into account theoretical uncertainties from the truncation of the chiral expansion estimated using the Bayesian approach and from the nucleon form factors, as given in Refs. [27,28], added in quadrature. Throughout this analysis, we employ the Bayesian model $G^{C_{550}}_{0.5-10}$ specified in Ref. [51] and assume the characteristic momentum scale to be given by $|k|/2$ [22]. The results for the deuteron charge FF from the best fit to data up to $Q = 4$
the structure radius of the deuteron, which reads

\[ r_{\text{str}} = 1.9731^{\pm0.0012}_{-0.0018} \text{ fm}, \]  

with the individual contributions to the uncertainty given in Table I. To make this uncertainty estimate conservatively, the truncation error is actually included twice: (i) by performing the Bayesian analysis for \( r_{\text{str}}^2 \), following the approach of Ref. [51] and (ii) through the statistical uncertainty in the short-range charge density extracted from the fit to \( G_{\text{exp}}^{2S0}(Q^2) \) using Eq. (8). Furthermore, we developed a phase-equivalent version of the 2N potential using a different choice of the unobservable phases \( \beta_8 = \beta_9 = 1/2 \) leading to \( \rho_2^{\text{2N}} = 0 \). Repeating the analysis for this choice of \( \beta_8,9 \), the value of \( r_{\text{str}} \) is found to agree with the one in Eq. (9) to all given figures. The structure radius is also robust with respect to data used in the fit: had we used the parametrization of data by Sick [16] instead of experimental data, we would have arrived at essentially the same result. For the sake of completeness, we also present the results of the order-by-order calculations for \( r_{\text{str}} \) (in units of fm) including the truncation error from the Bayesian analysis, 1.9 ± 0.4 (LO), 1.97 ± 0.03 (NLO), 1.969 ± 0.007 (N²LO), 1.969 ± 0.002 (N³LO), 1.9731 ± 0.0008 (N⁴LO). It is important to keep in mind that these numbers are obtained without relying on the chiral expansion of the nucleon form factors.

Relying on our theoretical prediction for the structure radius, we are now in the position to predict the neutron charge radius from Eqs. (1), (2) and (3), which gives

\[ r_n^2 = -0.106^{+0.007}_{-0.005} \text{ fm}^2. \]  

This value is 1.7σ smaller than the one given by the PDG [9].

In summary, we presented a comprehensive analysis of the deuteron charge form factor up to fifth order in \( \chiEFT \). The only unknown parameter enters the short-range 2N contribution to the charge density operator and is determined from the best fit to the deuteron charge form factor. Equipped with this information, we make a parameter-free prediction for the structure radius of the deuteron and perform a thorough analysis of various kinds of uncertainty. The high-accuracy calculation of the structure radius, together with the high-precision measurement of the hydrogen-deuterium 1S-2S isotope shift [1], have allowed us to extract the neutron charge radius.

Although it is natural to expect that the two-pion exchange contributions to the charge density at N⁴LO, which have not yet been worked out, are largely saturated by the short-range contributions included in this analysis, the complete \( \chiEFT \) calculation at this order would allow for an additional test of the estimated theoretical uncertainty.

The results for the deuteron charge FF presented here pave the way for an accurate determination of the isoscalar nucleon FF by (re)analyzing the experimental data on elastic electron-deuteron scattering at MAMI (see e.g. Ref. [55] for the new measurement of the elastic ed scattering cross section at 0.24 \( fm^{-1} \leq Q \leq 2.7 \text{ fm}^{-1} \) at MAMI), Saclay [56] and other facilities.

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