Qi-Wa, a problem that has plagued Chinese scrolls for millennia

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Qi-Wa refers to the up curl on the lengths of handscrolls and hanging scrolls, which has troubled Chinese artisans and emperors for as long as the art of painting and calligraphy exists. This warp is unwelcomed not only for aesthetic reasons, but its potential damage to the fiber and ink. Although it is generally treated as a part of the cockling and curling due to climate, mounting procedures, and conservation conditions, we emphasize that the intrinsic curvature incurred from the storage is in fact the main cause of Qi-Wa. The Qi-Wa height is determined by experiments to obey scaling relations with the length, width, curvature, and thickness of the scroll, which are supported by Molecular Dynamics Simulation and theoretic derivations. This understanding helps us come up with plausible remedies to mitigate Qi-Wa. All proposals are tested on real mounted paper and in simulations. Due to the general nature of this warp, we believe the lessons learnt from studying ancient Chinese scrolls can be applied to modern technologies such as the development of flexible electronic paper and computer screen.

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Chinese painting\textsuperscript{[1–4]} has a long history of tradition and styles in arranging human figures, animals, landscape, and plants into one or multiple themes as in symphonies. The collections of painting scrolls in National Palace Museum in Taipei, Taiwan can be dated back to the Six Dynasties (222-589) by Kaizhi Gu and Daozi Wu. The modes of landscape painting started in the Five Dynasties period (907-960) and have enjoyed many variations through the years. For instance, Kuan Fan, Xi Guo, and Tang Li of the Song Dynasty (960-1279) shifted their emphasis from grand mountains and waterfall to more intimate depictions as a result of the political and cultural shift to south China. Catering for the taste of emperors, painters also adjusted their focus on observing nature infused with poetic sentiments to expressing inner feelings. This focus on poetic sentiment led to the combination of painting, poetry, and calligraphy in the same work by the Southern Song (1127-1279). Subsequent movements include the literati\textsuperscript{[5]} painting in the Yuan Dynasty (1279-1368), concentration on personal cultivations and the establishment of separate local schools in the Ming (1368-1644) and Qing (1644-1911) dynasties. In the mean time, western painting techniques brought by the European missionaries have enthralled the Qing emperors and fused with the traditional ones to develop into an unorthodox yet popular style.

The image of hanging scroll paintings\textsuperscript{[6]} ranges from (1) medium square, e.g., 74.1 x 69.2 cm Traveling on a River After Snow, by Zhongshu Guo (?-977) of Song Dynasty, (2) medium long, e.g., 180 x 104 cm Wintry Forests, attributed to Cheng Li (916-967) also of Song Dynasty, to (3) oversize, e.g., 268 x 110 cm Pine Valley and a Cloudy Spring, attributed to Mao Sheng (ca. early 14th c.) of Yuan Dynasty. In contrast to hanging scrolls that are appreciated in its entirety while guiding the eyes through the artwork, handscrolls are intended to be viewed section by section during the unrolling and flat on a table. As a result, handrolls can be exceptionally long, measuring up to ten meters in the famous case of Along the River During the Ching-ming Festival, by Song artist Tse-duan Chang (1235-1305).

The Chinese phrase, Qi-Wa (Qi means showing up and Wa refers to the roof tile), refers to the up curl on the sides of scrolls upon opening (see Fig.\textsuperscript{[1]}. Trying to roll up any paper into a cylinder and give it a few strokes to consolidate the plasticity before spreading, one can easily witness and be convinced that this phenomenon is real and must have existed since the advent of scroll, be it Chinese or Egyptian. It not only presents an ugly appearance to the viewers, but may tear the fiber and cause pigment loss. Taping the midpoints of length to the wall will only make the matter worse because Qi-Wa now shifts to both one- and three-quarter-length positions. The reason why the phenomenon of Qi-Wa lingers for so long was perhaps partly due to the sectarianism in the craft of mounting\textsuperscript{[7]}, where old masters were said to rarely exchange information to protect their own interest. The cultural and linguistic barrier also contributes to intimidating scientists from entering the branch of oriental art conservation and discovering the Qi-Wa problem.

Qi-Wa is generally included in the discussion of cockling and curling due to (1) quality of mounting paper\textsuperscript{[8]} or consistency on paste for the multiple backings\textsuperscript{[9]}, (2) residual tension from the ridding of creases and the lining and drying procedures\textsuperscript{[10]}, (3) climate and conservation conditions such as humidity/moisture\textsuperscript{[11]} and temperature. Although the scroll is considered replaceable in Chinese tradition, any defect in scrolls can still lead to damages on the artwork since they are structurely joined together. The multiple components and materials (silk, paper, and wood) that compose scrolls make them a very complex object to predict. For instance, the warping due...
FIG. 1: (a) Qi-Wa is pronounced on the hanging scroll, which equals the inner layer with a length difference, \( \Delta L \).

While in storage, the outer layer of scrolls is overcurled relieves the strain \( \Delta w \) which raises the total energy by.

FIG. 2: Phase diagram for the U- and V-shaped Qi-Wa with \( 1/\phi = 17 \text{ mm}, \nu = 0.2, \text{ and } t = 0.11 \text{ mm} \). The contour plot illustrates the magnitude of \( 2w/\ell \), which equals 1 for the whole region of V shape.

In the mean time, the density plot denotes the size of Qi-Wa height \( h \). The phase boundary can be approximated by a parabolic function, \( L \sim (\ell^2/8)\sqrt{6\nu t/\ell} \), before leveling off at \( L \sim 3.60/(\nu t)^{1/3}\).

we have purposely separated the strains on the Qi-Wa strips and the middle section. It is easy to imagine \( \Delta w \) to cause Qi-Wa and write down \( w = w \text{ and } r(1-\cos \theta) = h \text{ where } r = 1/(\nu t) \text{ and } \theta \text{ is the angular span of the warp. The latter relation reveals that } h \text{ has a maximum. In reality most warps are mild and } \theta \text{ can be treated as a small number to obtain:

\[
L \approx \sqrt{2h/(\nu t}) \text{.} 
\] (2)

The up curl relieves the strain \( \Delta w \), but pays the price of stretching Qi-Wa strips along the length by roughly \( 2h^2/L \) which raises the total energy by

\[
\frac{K_{S\|}}{2} \left( \frac{2h^2}{L} \right)^2 L \cdot 2w 
\] (3)

where \( K_{S\|} \) denotes the stretching modulus near the side borders (region 1) along the length. In order to suppress this energy, the middle section (region 2) favors remaining flat and compressed with energy:

\[
\frac{K_{S\perp}}{2} \left( \frac{\Delta(\ell - 2w)}{\ell - 2w} \right)^2 L(\ell - 2w) = \frac{K_{S\perp}}{2} (\nu t)^2 L(\ell - 2w) 
\] (4)

where \( K_{S\perp} \) is the modulus perpendicular to the length. Essentially, it is the competition between Eqs. (3) and (4) that determines the optimal \( w \) and \( h \):

\[
h \sim \left( \frac{K_{S\perp}}{K_{S\|}} \right)^{1/4} L \sqrt{\frac{\nu t}{6}} 
\] (5)

where the lack of \( \ell \)-dependence is expected because Qi-Wa affects only a partial width. This scaling formula is not only of academic interest, but useful in practice. Namely, while \( L, \nu, \text{ and } \phi \) are hard to change, the
mounting artisans are advised to weaken the $K_S$ along the transverse ($\perp$) direction in the middle section, while strengthening that along the longitudinal ($\parallel$) direction in the Qi-Wa strips to mitigate the U-shaped warp.

Unlike Eq. (5), $h$ becomes dependent on $\ell$, which is reasonable because $h$ must vanish as $\ell \to 0$, a limit only applicable to $V$ shape. Another feature to note is the inverted ratio of $K_S$ and different exponent from Eq. (5). As opposed to $U$ shape, one needs to weaken the transverse modulus close to the border, while strengthening the longitudinal one in the middle section to suppress $V$-shaped Qi-Wa.

The different scaling relations in Eqs. (5) and (8) are verified in Fig. 4 by experiments and Molecular Dynamics (MD) Simulation[13]. A direct verification of their $K_S$ ratios, which provides the most practical turning knob to minimize the warp, is achieved in Fig. 4 by adding extra layers to region 1. This raises the $K_{S\parallel}$ for U-shaped Qi-Wa in Eq. (5) without affecting $t$ because the latter originates from Eq. (4) for region 2. Similar technique works for V-shaped Qi-Wa. But now the $t$ in Eq. (8) belongs to region 1, and we need to renormalize $h$ by $t^{-\gamma}$ in Fig. 4(b) to extract the $1/8$ exponent of $K_{S\parallel}/K_{S\perp}$.

![FIG. 3: Experimental and simulation results for the dependence of Qi-Wa height $h$ on the length $L$, width $\ell$, and intrinsic curvature $\phi$. Straight lines are fittings by the scaling relation $h \sim L^\alpha \ell^\beta \phi^\gamma$ with exponents: (a) $\alpha=0.9$, $2\beta$, and $0.84$ for PET film, copy paper, and simulation, respectively; (b) $\beta=0.069$, $-0.133$, and $-0.013$; (c) $\gamma=0.6$, $0.5$, and $0.4$ for U-shaped Qi-Wa. In contrast, for V-shaped Qi-Wa (d) $\alpha=0.53$, $0.36$, and $0.45$; (e) $\beta=0.96$, $0.9$, and $1.06$; (f) $\gamma=0.47$, $0.43$, and $0.51$. The default size of PET film, copy paper, and simulation ($L=90$, $\ell=60$, $1/\phi=60$, $40$, $10$) are (all in mm), and ($150$, $60$, $12.5$) (all in mm), and ($150$, $80$, $10$) for $U$-shape data, as opposed to ($120$, $10$, $6$), ($150$, $20$, $12.5$), and ($150$, $40$, $10$) for V-shape.](Image 63x507 to 290x681)

![FIG. 4: (a) By adding layers to the U-shape Qi-Wa strips, the exponent of $K_S$ ratio in Eq. (5) is measured to be $0.24$ and $0.24$ for PET ($L=60$, $\ell=40$, $1/\phi=15$, and $w=10$ mm) and copy paper ($L=90$, $\ell=60$, $1/\phi=25$, and $w=15$ mm). Simulation data are obtained by directly enhancing $K_S$ in the Qi-Wa strips ($L=150$, $\ell=80$, $1/\phi=10$, and $w=20$), which gives an exponent of $0.27$. (b) Similar procedures determine the exponent of $K_S$ ratio in Eq. (8) as $0.19$, $0.18$, and $0.16$ for PET ($L=80$, $\ell=15$, and $w=5$ mm), copy paper ($L=90$, $\ell=20$, and $w=5$ mm), and simulation ($L=150$, $\ell=40$, $1/\phi=10$, and $w=10$) for V-shaped Qi-Wa. The value of curvature is not specified because it varies sensitively with the change of thickness, which effect is removed by renormalizing $h$ by $\phi^{1/4}$.](Image 326x346 to 553x504)

Plugging Eq. (4) to Eq. (2) allows us to estimate how narrow a scroll needs to be in order to change from $U$- to $V$-shaped Qi-Wa:

$$\ell \leq \sqrt[4]{8L \left( \frac{t}{6w\phi} \right)^{1/4}}. \quad (6)$$

The competing role of Eq. (2) is now played by the end segments (of width $w$) of the V, which straighten themselves to lower $h$ and lessen its pull on the Qi-Wa strips. The relevant energies are

$$\frac{K_{S\perp}}{2}(\nu\phi)^2L \cdot 2w + \frac{K_{S\parallel}}{2} \left( \frac{2h^2}{L} \right)^{2} L(\ell - 2w) \quad (7)$$

where the down-graded stretching energy through modulus $K_{S\parallel}$ has been neglected compared to the first term. The maximum height $h'$ of the middle section that follows the induced curvature $\nu\phi$ obeys $\ell/2 - w \approx \sqrt{2h'/(\nu\phi)}$ similar to Eq. (2) and $h/h'/w = h'/(\ell/2 - w)$ from similar triangles. By use of these conditions, Eq. (4) can be minimized to give:

$$h \sim \left( \frac{K_{S\parallel}}{K_{S\perp}} \right)^{1/8} \sqrt{\frac{t}{2^{7/4}}} (\nu\phi)^{3/4}(t/3)^{1/4}\ell. \quad (8)$$
The roller used to roll up the scroll was attached on the left, which means the left section is originally in the inner core of the scroll. This helped us correlate the larger intrinsic curvature with a more pronounced Qi-Wa. Second insight came from our initial failure in MD Simulations to observe Qi-Wa on a monolayer with build-in curvature. It was not until we added a second layer with a slightly longer bond length to simulate the effect of $\phi$ that Qi-Wa and all its scaling properties finally emerged.

Other phenomena related to Qi-Wa can also be explained by our model: (1) Had we spreaded the sheet by pressing at the four corners, another pair of Qi-Wa would appear in the widths. They result directly from $\Delta L$ without invoking Eq. (1) and so obey

$$h \sim \left( \frac{K_{S2\perp}^4}{K_{S1\perp}} \right)^{1/4} \left( \frac{t}{\phi \ell} \right)^{1/6}$$

similar to Eq. (5) but with $K_{S1\perp}$ and $K_{S2\perp}$ reversed, $L$ replaced by $\ell$, and $\nu$ deleted. (2) If we buckle the sheet into a convex arc, the Qi-Wa at $L/2$ will gradually diminish and reappear at both $L/4$ and $3L/4$ positions. (3) If the sheet is turned upside down, there is no Qi-Wa. (4) If the sheet is not tightly rolled so that $\phi$ becomes nonuniform, the Qi-Wa peak will shift towards the side that is originally in the inner circle where $\phi$ is the largest.

Since mounting is an old trade with many traditions and strict aesthetic standards, any suggestion for mitigating Qi-Wa cannot deviate too much from the existing techniques. Our following recipe are intended for U- and V-shaped Qi-Wa cannot deviate too much from the existent mounting traditions and aesthetic standards, we propose practical modifications to alleviate these relations are obtained by heuristic theoretic models for U- and V-shaped Qi-Wa, respectively. Having respect for the existent mounting traditions and aesthetic standards, we propose practical modifications to alleviate Qi-Wa. Finally, due to the general nature of this warping phenomenon, we believe the lessons from studying ancient Chinese paintings are relevant to modern technologies such as the development of (1) computer and cellphone screens that can be folded away or rolled up for storage,[15, 16] (2) any area-expanding devices of extraordinary expansion ratios since they require the use of either rolling-up or folding mechanisms of polymer films. For example, the solar cell screen[17] attached in satellites, the wings of air-borne objects, etc., (3) polymer-based flexible electronic paper[18], and (4) manufacturing special shape and curl for membranes by adjusting the length of their underlayer. They may even add a wrinkle or two to tectonics[19] on the effect of nonuniform Poisson ratio in different layers when plates are deformed under compression.

We gratefully acknowledge funding support from National Science Council, technical supports by Hong Tan and Jofan Chien, and hospitality of the Physics Division of National Center for Theoretical Sciences in Hsinchu and the National Palace Museum in Taipei, Taiwan. We also thank Itai Cohen, Xiang Chen and Yenchih Lin for allowing us to use Fig.1(a,b) and Pai-Yi Hsiao and MD Simulations are employed to find consistent scaling behavior for the Qi-Wa height, while analytic forms of these relations are obtained by heuristic theoretic models for U- and V-shaped Qi-Wa, respectively. Having respect for the existent mounting traditions and aesthetic standards, we propose practical modifications to alleviate Qi-Wa. Finally, due to the general nature of this warping phenomenon, we believe the lessons from studying ancient Chinese paintings are relevant to modern technologies such as the development of (1) computer and cellphone screens that can be folded away or rolled up for storage,[15, 16] (2) any area-expanding devices of extraordinary expansion ratios since they require the use of either rolling-up or folding mechanisms of polymer films. For example, the solar cell screen[17] attached in satellites, the wings of air-borne objects, etc., (3) polymer-based flexible electronic paper[18], and (4) manufacturing special shape and curl for membranes by adjusting the length of their underlayer. They may even add a wrinkle or two to tectonics[19] on the effect of nonuniform Poisson ratio in different layers when plates are deformed under compression.

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