Possible Discrimination between Gamma Rays and Hadrons using Čerenkov Photon Timing Measurements

V. R. Chitnis and P. N. Bhat

Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400 005, India.

Abstract

Atmospheric Čerenkov Technique is an established methodology to study TeV energy gamma rays. However the challenging problem has always been the poor signal to noise ratio due to the presence of abundant cosmic rays. Several ingenious techniques have been employed to alleviate this problem, most of which are centred around the Čerenkov image characteristics. However there are not many techniques available for improving the signal to noise ratio of the data from wavefront sampling observations. One such possible technique is to use the Čerenkov photon arrival times and identify the species dependent characteristics in them. Here we carry out systematic Monte Carlo simulation studies of the timing information of Čerenkov photons at the observation level. We have parameterized the shape of the Čerenkov shower front as well as the pulse shapes in terms of experimentally measurable quantities. We demonstrate the sensitivity of the curvature of the shower front, pulse shape parameters as well as the photon arrival time jitter to primary species and show their efficiency in improving the signal to noise ratio. The effect of limiting the Čerenkov telescope opening angle by using a circular focal point mask, on the efficacy of the parameters has also been studied for each of the parameters. Radius of the shower front, pulse decay time and photon arrival time jitter have been found to be the most promising parameters which could be used to discriminate γ-ray events from the background. We also find that the efficiency of the first two parameters increases with zenith angle and efficiency of pulse decay time decreases with increasing altitude of observation.

Key words: VHE γ-rays, Extensive Air Showers, Atmospheric Čerenkov Technique, Simulations, CORSIKA, Čerenkov photon arrival time studies, gamma-hadron separation
1 Introduction

Atmospheric Čerenkov Technique (ACT), is a well established and a unique method for the astronomical investigation of Very High Energy (VHE, also referred to as $TeV$) $\gamma$-$\gamma$ rays. It is based on the effective detection and study of the Čerenkov light emitted by the secondary particles produced in the extensive air showers initiated by the primary $\gamma$-$\gamma$ ray [1–4].

As is well known, the abundant charged cosmic ray particles generate Čerenkov light akin to that produced by the $\gamma$-$\gamma$ rays as a result of which the $\gamma$-$\gamma$ ray signal is buried in a vast sea of cosmic ray background. This has been a major difficulty in applying the atmospheric Čerenkov technique successfully for $\gamma$-$\gamma$ ray astronomy.

The first generation Čerenkov telescopes addressed the problem of cosmic ray background by matching the telescope aperture to the angular size of the Čerenkov flash [5]. In addition, many of the potential astrophysical sources of VHE $\gamma$-$\gamma$ rays are expected to produce modulated signals (e.g. $\gamma$-$\gamma$ ray pulsars, X-ray binaries, cataclysmic variables etc). Thus techniques based on well established methods of phase sensitive detection were used to enhance the detection of $\gamma$-$\gamma$ rays against the randomly arriving background [6,7].

This technique cannot be applied however for steady sources of $\gamma$-$\gamma$ rays like the Blazars, normal galaxies or unidentified $\gamma$-$\gamma$ ray sources discovered at $GeV$ energies. There are, however, differences in the detailed structure of the detected Čerenkov flash due to the fact that the cosmic ray background protons are spatially and temporally isotropic while the $\gamma$-$\gamma$ rays emanate from the point sources. In addition, the physics of the hadronic cascades initiated by the protons in the earth’s atmosphere is different from the electromagnetic cascade generated by the $\gamma$-$\gamma$ rays. Detailed monte carlo simulations have been employed to study these differences. Techniques based on the shape of the lateral distribution of the Čerenkov light pool to delineate the source direction, have been studied [8,9] despite large fluctuations in the measured Čerenkov photon densities [10,11]. There are several Atmospheric Čerenkov arrays designed precisely to apply these techniques to ground based VHE $\gamma$-$\gamma$ ray astronomy [12–17]. On the other hand the imaging technique has been shown to be successful and also has been demonstrated to work reliably in detecting emission from several $TeV$ $\gamma$-$\gamma$ ray sources [18,19].

The spatial and temporal properties of the Čerenkov photons also contain valuable information on the development and propagation of the EAS in the atmosphere. As a result, systematic studies of these photons as received at the observation level could lead to the development of techniques to distinguish between hadronic or photon primaries. $\gamma$-$\gamma$ ray primaries for example, develop
higher in the atmosphere resulting in compact images as well as narrower pulses. One would therefore expect correlations between different pulse shape parameters and imaging parameters.

Extensive studies have already been carried out in this regard using detailed simulation techniques. Most of these studies were carried out at higher energies with the aim of studying the elemental composition of cosmic rays at these energies [20]. The possibility of applying pulse shape discrimination technique to improve the signal to noise ratio in the data from Crab nebula was first demonstrated by Tümer et al. [21]. Their study was solely based on the presence or absence of kinks and other anomalies like long trailing edges of the Čerenkov pulses indicating hadronic origin. These criteria were not based on any systematic simulation studies nor were they conveniently parameterized so that it could be used by others.

Several characteristics of the hadron- & $\gamma$- ray showers which were suggested as possible discriminators in the past have been met with limited success. Some of these are the presence of penetrating particles among the EAS secondaries [22], the ultraviolet excess in hadron initiated showers due to their proximity to the observation level [23] and time duration [24,25]. On the other hand methods developed for improving the angular resolution technique [26] are still in use and are one of the important methods of discrimination for non-imaging arrays which are generally spread out.

A brief description of the efficacy of the pulse shape discrimination in VHE $\gamma$- ray astronomy was made by Patterson & Hillas [27]. Their study was based solely on the presence of structure on the leading or trailing edge of the Čerenkov pulses as suggested before [21]. However no systematic studies of the rise or decay times, FWHM etc were carried out. Recently Roberts et al. [28], developed a technique based on the temporal Čerenkov pulse shape and showed that the use of rise time at large zenith angles and FWHM at smaller zenith angles are effective discriminators. However these studies are carried out at larger zenith angle ($\geq 35^\circ$) and hence the conclusions are relevant only at larger primary energies ($\geq 40 \text{ TeV}$). On the other hand Cabot et al. [29] suggest possibility of identifying the muonic component, which precedes the electromagnetic component, of the Čerenkov pulse in order to separate the $\gamma$- ray signal from the background. This could be a very efficient technique in experiments based on wavefront sampling technique. But once again this method works only at higher primary energies ($\geq a \text{ few TeV}$) where a significant number of muons are produced.

In an earlier work[11] functional fits have been carried out to the spherical shower front to demonstrate that the radii of curvature are equal to the height of the shower maximum at all observation levels. It has also been demonstrated in that paper that the Čerenkov photon arrival time distributions can be well
fitted by a lognormal function. Using these fits, radial and spectral variation of pulse shape parameters have been studied systematically.

In the present work we plan to make a systematic study of the temporal and spatial profile of Čerenkov light from lower energy primaries both from pure electromagnetic cascades as well as hadronic cascades generated by TeV energy primaries. The question we are trying to answer is whether the observed differences in experimentally measurable temporal information could be used to separate electromagnetic component from the hadronic background.

In §2 of this paper details of simulations are given, followed by definition of figure of merit for discrimination between γ− rays and cosmic rays in §3. In §4 we discuss the results from the analysis of the curvature of the Čerenkov front whereas in §5 we present results based on the pulse shape parameters and identify the more sensitive of them. In §6 we present the detailed study of the photon arrival time jitter. The dependence of the parameters on the telescope opening angle, altitude of observation, incident angle of the primary as well as cosmic ray species and the shower core distance are discussed in §7. A brief discussion of the results is presented in §8 and conclusions are summarized in §9.

2 Simulations

A package called CORSIKA (version 560), [30,31] has been used to simulate Čerenkov light emission in the earth's atmosphere by the secondaries of the extensive air showers generated by cosmic ray primaries or γ− rays. This program simulates interactions of nuclei, hadrons, muons, electrons and photons as well as decays of unstable secondaries in the atmosphere. It uses EGS4 code [32] for the electromagnetic component of the air shower simulation and dual parton model for the simulation of hadronic interactions at TeV energies. The Čerenkov radiation produced within the specified band width (300-650 nm) by the charged secondaries is propagated to the ground. The US standard atmosphere parameterized by Linsley [33] has been used. The position, angle, time (with respect to the first interaction) and production height of each photon hitting the detector on the observation level are recorded.

In the present studies we have mainly used Pachmarhi (longitude: 78° 26′ E, latitude: 22° 28′ N and altitude: 1075 m) as the observation level where an array of Čerenkov detectors each of area 39.35 m² is deployed in the form of a rectangular array. We have assumed 17 detectors in the E-W direction with a

1 This is the total reflective area of 7 parabolic mirrors of diameter 0.9 m deployed paraxially on a single equatorial mount.
separation of 25 m and 21 detectors in the N-S direction with a separation of 20 m. This configuration, similar to the Pachmarhi Array of Čerenkov Telescopes (PACT) [12] but much larger, is chosen so that one can study the core distance dependence of various observable parameters. Monoenergetic primaries consisting of $\gamma$-rays, protons and iron nuclei incident vertically on the top of the atmosphere with their cores at the centre of the array have been simulated in the present studies. The showers simulated in this study have a fixed core position which is chosen to be the detector at the centre of the array. The resulting Čerenkov pool is sampled by all the 357 detectors which are used to study the core distance dependence of the parameters studied here. All the telescopes are assumed to have their optic axes aligned vertically.

An option of variable bunch size of the Čerenkov photons is available in the package which serves to reduce the requirement of hardware resources. However since we are interested in the fluctuations of each of the estimated observables, we have tracked single photons for each primary at all energies. Multiple scattering length for electrons and positrons is decided by the parameter STEPFC in the EGS code which has been set to 0.1 in the present studies [34]. Wavelength dependent absorption of Čerenkov photons in the atmosphere is not however taken into account. The present conclusions are expected to be independent of photon wavelengths.

3 Figure of merit of a parameter

Figure of merit of a parameter that can distinguish between VHE $\gamma$-rays and cosmic ray hadrons depends primarily on two factors. Firstly, it should accept most of the $\gamma$-rays and secondly it should be able to reject most of the hadrons. In general, this figure of merit could be a function of primary energy. In the present work we define such a figure of merit which is often called as quality factor, as [28]:

$$q = \frac{N_\gamma^a}{N_T^\gamma} \left( \frac{N_{cr}^a}{N_{T}^{cr}} \right)^{-\frac{1}{2}}$$

(1)

where $N_\gamma^a$ is the number of $\gamma$ rays accepted,

$N_T^\gamma$ is the total number of $\gamma$ rays,

$N_{cr}^a$ is the number of background cosmic rays accepted and

$N_{T}^{cr}$ is the total number of background cosmic rays.
The quality factor thus defined is independent of the actual number of $\gamma-$rays and protons recorded.

In this paper, whenever a pair of distributions of a parameter under study for a hadron and a $\gamma-$ray primary are shown the threshold value of the parameter is indicated by a vertical line. The threshold is chosen such that it yields maximum quality factor subject to the condition that the accepted fraction of $\gamma-$rays is $> 30\%$ and that of protons is $> 1\%$. The errors on the quality factors shown in each case are statistical only.

4 Shower front parameters

It has been shown long ago that the radius of curvature of the Čerenkov light front is strongly correlated with the height of shower maximum from the observation level [35]. This has been found to be true for different species of cosmic rays [11]. For photonic primaries the height of shower maximum is decided by the radiation length in the atmosphere while that for hadronic primaries it is decided by the interaction length which in turn depends on the interaction cross-section in air. Hence the radius of curvature could be species specific. Therefore we have investigated the possibility of using the fitted radius of curvature of the spherical Čerenkov front as a parameter to distinguish between $\gamma-$ray and proton initiated showers. The details of a spherical fit to the mean arrival times of Čerenkov photons at detectors sampling the front at various core distances are described in detail in [11]. Fig. 1 shows the distribution of the fitted radii for different primary species of various energies. The primaries are incident vertically at the top of the atmosphere except wherever mentioned. Mono-energetic $\gamma-$ray and hadron primaries of comparable Čerenkov yield are simulated and compared. We also simulated showers, both for $\gamma-$ rays and protons whose energies are selected randomly from a power law distribution of a differential slope of -2.65. This would also simulate the real data as one would record in an experiment. While the slope of the $\gamma-$ray spectrum could be different from what is chosen here, the quality factor does not depend on the number of showers and hence independent of the spectral slope except when it shows strong energy dependence. The energy bandwidth chosen here are 500 $GeV$ - 10 $TeV$ for $\gamma-$ rays while it is 1 $TeV$ - 20 $TeV$ for protons.

Quality factors, as defined above, have been estimated for the present sample from the distributions of the estimated radii of curvature for $\gamma-$ray and hadronic primaries as shown in figure 1. These are given in table 1. The threshold values in each case are indicated as vertical lines in figure 1. Fraction of total number of $\gamma-$ray and proton showers that pass this cut, i.e., fraction of showers with radii below the threshold as well as the number of showers...
Table 1
Quality of radius of curvature of the spherical photon front as a discriminating parameter for vertical showers

| Type of primary | Energy of primary (GeV) | Threshold radius of curvature (km) | Fraction of showers accepted (%) | Number of showers simulated (%) | Quality factor |
|-----------------|-------------------------|-----------------------------------|----------------------------------|-------------------------------|---------------|
| γ− rays and protons | 100 | 11.7 | 99 | 200 | 1.13 ± 0.13 |
| and protons | 500 | 8.8 | 98 | 200 | 1.13 ± 0.13 |
| and protons | 1000 | 8.2 | 99 | 100 | 1.11 ± 0.18 |
| and protons | 2000 | 6.3 | 48 | 100 | 4.8 ± 2.6 |
| and Fe nuclei | 10000 | spectrum | 8.3 | 100 | 1.13 ± 0.18 |

simulated in each case are also tabulated. The last row in the table shows the quality factor estimated for primaries chosen from a powerlaw spectrum as mentioned before. This is consistent with that for mono-energetic primaries showing that a change in the height of shower maximum with energy does not change the quality factor significantly. Fig. 1 shows the distributions of the fitted radii of curvature of the shower front for proton primaries and Fe primaries with respect to that for γ−ray primaries of equivalent Čerenkov yield. The vertical lines represent the threshold value with respect to which quality factors are estimated. One can see a rather high degree of overlap between the pair of distributions and hence the quality factors from this parameter are rather modest in value but are almost independent of primary energy. However we will see in §7.1 that the use of a circular mask to limit the telescope opening angle improves the quality factor at all energies. Also the quality factor against Fe primaries improves dramatically suggesting that this could be the ideal parameter to discriminate heavy primaries.

5 Pulse shape parameters
Fig. 1. Distribution of the shower front radii for $\gamma$-rays (continuous line) and protons or Fe nuclei (dashed line) of respective energies as indicated in each panel. The last panel shows similar distributions for $\gamma$-rays & protons of primary energies chosen from a power law spectrum of slope -2.65 (see text for details). The vertical lines indicate the threshold values of parameters.

5.1 Observation level of Pachmarhi

Even though pulse shape parameters contain information on the history of shower development in the atmosphere, their use for identifying the primary species was always in doubt especially at lower primary energies [27]. On the other hand, it was known that the muons generated in hadron initiated showers will reach the observation level several nanoseconds before the light from the electrons. Consequently, the light from muons would give rise to an unmistakable precursor which could be used as a discriminating parameter [29].

Here we tried to investigate the possibility of using pulse shape parameters to discriminate between $\gamma$-ray and hadron showers, using the same data set as was used for shower front parameters. Distributions generated using
pulse shape parameters were derived from predicted lognormal distribution for individual detectors (see [11] for details). It may be recalled here that the decay time of the pulse need not be measured directly for this purpose. If one measures the mean arrival time and the RMS fluctuations in Čerenkov photon arrival times at a detector, it will enable us to derive the pulse shape parameters from the data directly [11]. Among the three pulse shape parameters \textit{viz.} the rise time, the decay time \& the pulse width, decay time seems to be the most sensitive parameter which exhibits species sensitive behaviour at $TeV$ energies. However, Čerenkov photon arrival time distribution yields one set of pulse shape parameters for each of the 357 detectors. So also the arrival time jitter (discussed in §6). Hence for the purpose of estimating the statistical errors on the quality factors, the sample size considered is the product of the number of detectors and the number of simulated showers. In order to take into account the core distance dependence of these parameters (see [11] for details) they are averaged over 16\footnote{The choice of this number is purely arbitrary. We chose this number because the spatial separation of the resulting 22 samples turned out to be reasonably uniform. However this grouping was not done for inclined showers.} consecutive detectors arranged in the order of increasing core distance. The resulting 22 (352/16, excluding the detectors at the shower core and the farthest 4) sets of parameters are treated as independent samples of the parameter under study. The distributions of the these samples of decay time both for $\gamma-$ray and proton primaries of various energies are shown in fig. 2. The number of showers simulated are same as those listed in table 1. The distributions from hadronic primaries are in general characterized by longer tails compared to that from $\gamma-$ray primaries. There is a fair amount of overlap between the distributions of two types of primaries. All the relevant parameters including the fractions of $\gamma-$rays and protons whose decay times are less than the threshold value are listed in table 2. In spite of the modest quality factor, one is able to reject nearly 96\% of protons from the data while loosing nearly two thirds of the $\gamma-$ray signal.

The other pulse shape parameters like the rise time and the pulse width (defined as the full width at half maximum, FWHM) for vertical showers have been found to be quite insensitive to the primary species and hence are not useful parameters to distinguish hadronic events. However as discussed in §7.3 it can be seen that the pulse width could be a good parameter for inclined showers.

6 Timing Jitter

The Čerenkov light emitted at the top of the atmosphere reaches the observer later than that from deeper down. The latter appears at larger angles but
Table 2
Quality of pulse decay time as a discriminating parameter for vertical showers. The number of showers simulated are same as those indicated in table 1.

| Type of primary | Energy of primary (GeV) | Threshold value (ns) | Fraction of showers accepted (%) | Quality factor |
|-----------------|-------------------------|----------------------|---------------------------------|---------------|
| γ− rays and protons | 250 | 3.2 | 37.7 | 1.67 ± 0.02 |
| γ− rays and protons | 500 | 3.7 | 35 | 2.22 ± 0.03 |
| γ− rays and protons | 1000 | 3.9 | 30.4 | 2.41 ± 0.06 |
| γ− rays and Fe nuclei spectrum | 10000 | 4.0 | 32.1 | 2.48 ± 0.06 |
| γ− rays and protons spectrum | 3.9 | 32.5 | 1.57 ± 0.03 |

arrives earlier because the particles travel faster than light in the atmosphere. It was suggested long ago that such small time differences could be useful in discriminating against proton showers [27]. We have seen before, that a majority of the Čerenkov photons originate at around the shower maximum as a result of which at observation level we observe the spherical front centred around that point. However the spread in the arrival time at any core distance is largely decided by the photons emitted elsewhere in the atmosphere. For example, at large core distances a bulk of the photons are emitted at lower atmospheric heights, largely from lower energy electrons undergoing multiple Coulomb scattering. In addition, photons seem to arrive at increasingly larger angles away from the core thus implying larger arrival times. In other words, the spread in the arrival times within a given shower at any core distance has the definite signature of the kinematics of the shower development. Hence we tried to search for species dependent signature in the arrival time spread of Čerenkov photons. We quantify the arrival time spread in terms of the RMS of the photon arrival times at a given detector. The ratio of this RMS to the mean arrival time, called the relative jitter, is used as a discriminating parameter to identify the primary species.
6.1 Core distance dependence

It may be recalled that each telescope in the array considered here consists of compactly mounted 7 paraxial parabolic mirrors of diameter 0.9 m. The average arrival time of Čerenkov photons is calculated for each of the seven mirrors of a telescope, for a given shower. Then the mean arrival time for each telescope is calculated which is the average of seven individual averages. Time jitter is the RMS of seven averages for each telescope as mentioned before. Figure 3 shows a radial plot of the mean arrival times (ns, marked by +), time jitter (ns, marked by open triangles) and the relative jitter (which is the ratio of time jitter to the mean, asterisks) both for 1 TeV γ-rays (left) and 2 TeV protons (right). While there is a general increase in the jitter with increasing core distance for both the primaries, it shows a minimum at around the hump region for γ-ray primaries. This is expected since most of the photons here are emitted by the high energy electrons and also from a limited range of atmospheric heights. Such a signature is swamped by kinematic fluctuations in the case of hadronic primaries, in addition to exhibiting an increased time...
Table 3
Quality of jitter as a discriminating parameter. The number of showers simulated is 100 in all cases for each type of primary.

| Type of primary | Energy of primary (GeV) | Threshold value | Fraction of showers accepted (%) | Quality factor ± |
|----------------|--------------------------|-----------------|----------------------------------|----------------|
| γ− rays        | 100                      | 0.23            | 49.8                             | 2.83 ± 0.05    |
| and protons    | 250                      |                 | 3.1                              |                |
| γ− rays        | 1000                     | 0.07            | 67.5                             | 2.42 ± 0.03    |
| and protons    | 2000                     |                 | 7.8                              |                |
| γ− rays        | 1000                     | 0.09            | 91.3                             | 9.13 ± 0.25    |
| and Fe nuclei  | 10000                    |                 | 1.0                              |                |
| γ− rays        | spectrum                 | 0.08            | 80.5                             | 1.85 ± 0.02    |
| and protons    | spectrum                 |                 | 19                               |                |

spread [10].

6.2 Separation of γ - rays from proton primaries

It may be seen from figure 3 that the relative time jitter for both γ− ray as well as proton primaries is almost independent of the core distance. Hence it is chosen as a parameter for γ−hadron separation even though it is not a necessary condition.

Fig. 4 shows the comparative distributions of this parameter for γ−rays and protons (a & b) as well as γ−rays and Fe primaries (c) of equivalent Čerenkov yields. Also shown in the figure (d) are comparative distributions for γ−ray and proton primaries of varying energies selected from a power law distribution of slope -2.65. The energy bandwidths chosen for γ−rays and protons are 0.5 - 10 TeV and 1 - 20 TeV respectively. The results are based on 100 showers in each case for each species.

It is well known that after shower maximum, γ−ray showers attenuate progressively faster with atmospheric depth than do hadronic showers [36]. In addition, the lateral spread also is larger for hadron initiated showers. As a result, the RMS spread in the time of arrival of photons at the observation level is expected to be larger for hadronic showers. Consequently, the distributions in fig 4 are pretty wide with a long tail for hadrons at all primary energies which makes it possible to discriminate them.
Fig. 3. Radial variation of the mean arrival time, ns (+), arrival time jitter, ns (open triangle) and the relative jitter (asterisk) for 1 TeV γ-ray (left) and 2 TeV proton (right) incident vertically at the top of the atmosphere.

Table 3 shows the quality factors derived from relative jitter for vertical showers of γ-rays, protons and iron primaries. From the table it can be seen that using relative jitter as a discriminating parameter one can reject more than ∼80% of protons while retaining more than ∼80% of γ-rays (from spectrum).

7 Sensitivity to various operational parameters

7.1 Effect of telescope opening angle on quality factors

The opening angle of a Čerenkov telescope is often limited by placing a circular mask at the focal point in front of the photocathode. This limits the arrival angle of the photons reaching the photocathode. In the absence of a mask the opening angle is limited by the photocathode diameter. In other words, the limiting mask is expected to reduce the mean arrival angle of photons, reduce
Fig. 4. Distributions of mean relative jitter for γ-rays (solid line) and protons or iron nuclei (dotted line) of different primary energies are shown. (d) shows the distribution for primaries with a power law (differential slope -2.65) distribution of energies in the range 500 GeV - 10 TeV for γ-rays and 1 TeV - 20 TeV for protons. The vertical lines represent the threshold values.

the mean arrival time as well as increase the average production height. Table 4 summarizes the effect on aforesaid shower parameters by placing a 3° (FWHM) mask at the focal point of a detector. This is to demonstrate qualitatively its effect on some of the parameters of shower development. Vertically incident showers are considered here. The changes in the mean angle of arrival and time is around 30% for γ-rays while it is significantly larger for proton primaries.

The mean production height is much less sensitive to the presence of mask which increases the mean production height by about 10% and 14% for γ-ray and proton primaries respectively.

The question we are addressing here is the effect of telescope opening angle on each of the quality factors discussed here. Tables 5, 6, 7 & 8 summarize the results relating to the role of mask when radius of curvature, pulse decay time, pulse width and relative jitter are used as parameters to discriminate against cosmic ray hadrons. The same data set is used to compare the results with
Table 4
Effect of adding a mask at the focal point on some of the parameters for vertical showers

| Type of primary | Energy of primary (GeV) | Mask used (FWHM) | Mean arrival angle (degrees) | Mean arrival time (ns) | Mean production height (km) |
|----------------|-------------------------|------------------|-----------------------------|------------------------|-----------------------------|
| γ-ray          | 100                     | No mask          | 1.3 ± 0.27                  | 7.23 ± 1.91            | 11.04 ± 1.15                |
| γ-ray          | 100                     | 3.0°             | 0.9 ± 0.11                  | 5.11 ± 0.66            | 12.09 ± 1.13                |
| Proton         | 250                     | No mask          | 1.64 ± 0.42                 | 12.01 ± 18.29          | 7.83 ± 1.11                 |
| Proton         | 250                     | 3.0°             | 0.93 ± 0.11                 | 7.11 ± 14.41           | 8.91 ± 1.11                 |

and without mask. In the no-mask case all the Čerenkov photons incident on a detector, irrespective of their angle of incidence, are accepted. Pulse width was included in this case to show that if a mask is in use it could be a useful parameter to discriminate against heavy primaries. Its sensitivity improves with increasing primary energy consistent with other such studies. From tables it can be readily seen that the use of a mask (5° FWHM) improves the efficiency of the radius of curvature as a discriminating parameter while the pulse shape parameters show a marked improvement only for heavy primaries. For the relative timing jitter the efficiency as a discriminating parameter reduces when a mask is used. This, even though a marginal effect, is quite understandable. Limiting the photon arrival angles to near vertical direction reduces the RMS fluctuations because of limited range of pathlength differences.

7.2 Altitude dependence of quality factors

As the altitude of observation increases, the shower maximum for a given primary energy comes closer to the observation level. As a result, the lateral distribution of Čerenkov photons has been known to change with the altitude of the observation level. The core distance at which the hump appears as well as the prominence of the hump will be smaller with increasing altitude of observation [37,38]. Since the Čerenkov front is being intercepted at different observation levels during its propagation in the atmosphere, the shower parameters like the average arrival angle, time etc also will be different at different observation levels. As we will see in §7.5 that the species discriminating efficiency of the parameter does vary, even though weakly, with core distance. Hence it would be interesting to see if their role depends on the observation level too. Therefore we studied the role played by the observation altitude in using the various types of parameters studied here.
Table 5
Quality of radius of spherical wavefront as a discriminating parameter for vertical showers when 5° mask is used.

| Type of primary | Energy of primary (GeV) | Threshold radius of curvature (km) | Fraction of showers accepted (%) | Quality factor |
|----------------|-------------------------|-----------------------------------|---------------------------------|----------------|
| γ− rays and protons | 100 | 12.5 | 97 | 1.37 ± 0.16 |
| γ− rays and protons | 250 | 50.5 | | |
| γ− rays and protons | 500 | 9.7 | 96.5 | 1.35 ± 0.16 |
| γ− rays and protons | 1000 | 8.8 | 93 | 1.33 ± 0.22 |
| γ− rays and Fe nuclei | 10000 | 8.7 | 91 | 9.1 ± 4.8 |
| γ− rays and protons | spectrum | 9.5 | 100 | 1.3 ± 0.2 |
| γ− rays and protons | spectrum | | 61 | |

Table 6
Quality of decay time as a discriminating parameter with 5° mask

| Type of primary | Energy of primary (GeV) | Threshold value (ns) | Fraction of showers accepted (%) | Quality factor |
|----------------|-------------------------|---------------------|---------------------------------|----------------|
| γ− rays and protons | 250 | 3.4 | 52.8 | 1.14± 0.01 |
| γ− rays and protons | 500 | 3.5 | 44.9 | 1.84 ± 0.02 |
| γ− rays and protons | 1000 | 3.5 | 40.1 | 1.81 ± 0.03 |
| γ− rays and Fe nuclei | 10000 | 4.8 | 71.4 | 6.98 ± 0.19 |
| γ− rays and protons | spectrum | 3.3 | 35.2 | 1.70 ±0.03 |
| γ− rays and protons | spectrum | | 4.3 | |
Table 7
Quality of FWHM as a discriminating parameter when 5° mask is used.

| Type of primary | Energy of primary value (GeV) | Threshold (ns) | Fraction of showers accepted (%) | Quality factor |
|----------------|-------------------------------|----------------|---------------------------------|----------------|
| γ− rays and protons | 250 | 3.8 | 98.2 | 1.02±0.01 |
| γ− rays and protons | 500 | 92.5 |
| γ− rays and protons | 1000 | 2.0 | 45.7 | 1.08±0.01 |
| γ− rays and protons | 2000 | 18.1 |
| γ− rays and Fe nuclei | 10000 | 2.1 | 39.3 | 1.32±0.02 |
| γ− rays and protons | spectrum | 2.2 | 33.8 | 3.31±0.09 |

Table 8
Quality of jitter as a discriminating parameter when a 5° mask is used.

| Type of primary | Energy of primary value (GeV) | Threshold (ns) | Fraction of showers accepted (%) | Quality factor |
|----------------|-------------------------------|----------------|---------------------------------|----------------|
| γ− rays and protons | 100 | 0.2 | 58.2 | 1.7±0.02 |
| γ− rays and protons | 250 | 12.3 |
| γ− rays and protons | 1000 | 0.04 | 62.1 | 2.1±0.03 |
| γ− rays and protons | 2000 | 9.1 |
| γ− rays and Fe nuclei | 10000 | 0.05 | 89.6 | 8.8±0.2 |
| γ− rays and protons | spectrum | 0.05 | 86.4 | 1.7±0.02 |

Tables 9, 10 & 11 summarize the quality factors for the pulse decay times as well as the relative timing jitter at two different observation levels viz. sea level and 2.2 km above mean sea level (for 500 GeV γ−rays and 1 TeV protons). In comparison with the quality factors for the same parameters in table 2 & 3
it can be seen that the quality factors from pulse decay time improve steadily with decreasing altitude while those from timing jitter are almost independent of altitude. In order to understand this we computed the quality factors, for detectors only around the hump region after taking into account the varying hump distances from the core at three altitudes. It is found that the quality factors so calculated for pulse decay time are respectively 8.1, 6.6, and 3.6 for sea level, 1 km and 2.2 km altitudes. As one would see later (table 16) that the quality factors computed exclusively for pre-hump & post-hump regions are poorer than that for the hump region. Hence the improvement in quality factor at lower altitudes is mainly due to the increased prominence of hump at lower atmospheric depths.

The relative timing jitter, on the other hand seems to be much less sensitive to core distance (see figure 3) and hence doesn't vary significantly with observation altitude.
Table 11
Quality of jitter as a discriminating parameter at two observation levels *viz* sea level & 2.2 km above mean sea level. Monoenergetic $\gamma$-rays of 500 GeV and protons of 1 TeV incident vertically at the top of the atmosphere are considered.

| Observation level above msl (km) | Mask used (°) | Threshold value (FWHM) | Fraction of $\gamma$-rays (%) | Fraction of protons (%) | Quality factor $(\gamma, p)$ |
|---------------------------------|---------------|------------------------|-------------------------------|-------------------------|-----------------------------|
| 0.0                             | None          | 0.08                   | 52.5                          | 2.7                     | $3.18 \pm 0.06$             |
| 1.07                            | None          | 0.09                   | 45.8                          | 1.1                     | $4.48 \pm 0.12$             |
| 2.2                             | None          | 0.09                   | 41.2                          | 1.7                     | $3.16 \pm 0.07$             |
| 0.0                             | 5.0           | 0.05                   | 41.0                          | 3.3                     | $2.27 \pm 0.03$             |
| 1.07                            | 5.0           | 0.05                   | 48.9                          | 3.4                     | $2.67 \pm 0.05$             |
| 2.2                             | 5.0           | 0.05                   | 57.3                          | 4.5                     | $2.7 \pm 0.04$              |

7.3 *Incident angle dependence of quality factors*

At larger zenith angles rising absorption and the increasing distance from the shower maximum raise the energy threshold of the primary that are detected by an atmospheric Čerenkov telescope. After the shower maximum $\gamma$-ray induced showers attenuate progressively faster with atmospheric depth than do the hadronic showers. As a result one would expect a zenith angle dependence on the sensitivity of the parameters studied here. Qualitatively speaking inclined showers at a given altitude behave similar to vertical showers at a lower altitude. As a result, one would expect the quality factor to improve with zenith angle, as it would with a decrease in the altitude of observation. The species sensitive imaging parameters like the azwidth for example, have been shown to be much less sensitive for primaries incident at angles $\geq 30^\circ$ [19]. Hence we wanted to check the possible dependence of the parameters, discussed here, on zenith angle of the primary. We have generated showers initiated by $\gamma$-rays and hadrons incident at the top of atmosphere at a fixed zenith angle of $30^\circ$ to the vertical and uniformly distributed in azimuth.

The quality factors have been derived at one test energy for comparison for all the parameters under consideration in the present study. The results are summarized in the following tables.

Table 12 summarizes the efficiency of the radius of curvature of the light front as a discriminant for inclined showers of energy 500 GeV and 1 TeV for $\gamma$-ray and proton primaries respectively. The results are based on 200 showers each.
Table 12
Quality of radius of spherical wavefront as a discriminating parameter for inclined showers (incident at 30° to the vertical).

| Type of primary | Energy of primary (GeV) | Threshold radius of curvature (km) | Fraction of γ− rays accepted (%) | Fraction of protons accepted (%) | Quality factor |
|-----------------|-------------------------|-----------------------------------|----------------------------------|----------------------------------|----------------|
| γ− rays         | 500                     | 10.9                              | 94                               | 51                               | 1.32 ± 0.16    |
| and protons     | 1000                    |                                   |                                  |                                  |                |
| (without mask)  |                         |                                   |                                  |                                  |                |

| γ− rays         | 500                     | 11.6                              | 93.5                             | 46.5                             | 1.37 ± 0.16    |
| and protons     | 1000                    |                                   |                                  |                                  |                |
| (with 5° mask)  |                         |                                   |                                  |                                  |                |

Table 13
Quality from decay time for showers incident at 30° to the vertical when used to discriminate 500 GeV γ−rays from 1 TeV protons.

| Type of primary | Mask dia. used (FWHM) | Threshold value (ns) | Fraction of γ− rays accepted (%) | Fraction of protons accepted (%) | Quality factor |
|-----------------|------------------------|----------------------|----------------------------------|----------------------------------|----------------|
| γ− rays         | No mask                | 5.4                  | 59.9                             | 11.3                             | 1.79 ± 0.02    |
| and protons     | used                   |                      |                                  |                                  |                |
| γ− rays         | 5.0°                   | 4.6                  | 52.5                             | 8.2                              | 1.83 ± 0.02    |
| and protons     |                        |                      |                                  |                                  |                |

of γ−rays and protons. It can be readily seen from a comparison with table 1 that there is a significant improvement in the quality factors for inclined showers. This could be partly due to an increase in the range of fitted radii of proton showers.

Tables 13 & 14 summarize the results of using two of the pulse shape parameters viz. pulse decay time and pulse width for species identification for showers incident at 30° to the vertical. The results are based on 100 showers each of γ−rays & protons. Keeping in mind that the decay times are not grouped for inclined showers (unlike vertical showers) quality factor for decay time improves for inclined showers.

Table 14 on the other hand has an interesting result. As mentioned in §5.1,
Table 14
Quality of pulse width (FWHM) as a parameter from showers incident at 30° to the vertical when used to discriminate 500 GeV $\gamma$-rays from 1 TeV protons.

| Type of primary | Mask dia. | Threshold value (FWHM) | Fraction of $\gamma$-rays accepted (%) | Fraction of protons accepted (%) | Quality factor |
|----------------|-----------|------------------------|----------------------------------------|----------------------------------|----------------|
| $\gamma$-rays and protons used | No mask | 3.9 | 84.3 | 48.8 | $1.21 \pm 0.01$ |
| $\gamma$-rays and protons | 5.0° | 3.4 | 75.8 | 33.6 | $1.31 \pm 0.01$ |

pulse rise time and pulse width do not exhibit significant sensitivity to primary species for vertical showers. However for inclined showers pulse width becomes a useful parameter as shown in table 14. This is consistent with the results of Roberts et al. [28], who showed that the quality factor for pulse width peaks at around an inclination of $\sim 35°$ to the vertical at a given altitude of observation.

The relative shower to shower fluctuations in pulse shape parameters have been found to be less for inclined showers [11] unlike at higher energies where the trend reverses [28]. Table 15 summarizes the quality factors using the timing jitter as the parameter. It may be seen from a comparison of tables 3 & 15 that the species sensitivity of the timing jitter falls with zenith angle of the primary.

The sensitivity of each of the parameters for inclined showers has been further studied after introducing a focal point mask. The results are again summarized in tables 12-15. The opening angles of inclined showers are smaller than that for vertical showers because, inclined showers reach shower maximum at a higher altitude where the Čerenkov angle is smaller. Further, the lower energy electrons propagate in a lower density medium thus undergoing less frequent Coulomb scattering. Consequently, the Čerenkov emission from inclined showers is more collimated compared to that from a same energy primary in vertical direction. Hence the use of a focal point mask is not expected to change the quality factors significantly since a smaller fraction of photons are obstructed by it. The results in tables 12-15 show no major change in quality factors when focal point masks are introduced, essentially supporting the above argument.

For reasons mentioned above, the differential pathlengths of Čerenkov photons also reduce resulting in a poor sensitivity for timing jitter for inclined showers compared to that for vertical showers. The results of table 15 support this conclusion. Once again the use of a focal point mask has no significant effect
Table 15
Quality of relative jitter as a discriminating parameter for γ−ray (500 GeV) and proton (1 TeV) primaries incident at 30° with respect to the vertical.

| Type of primary | Mask dia. used | Threshold value (if any) | Fraction of γ− rays accepted (%) | Fraction of protons accepted (%) | Quality factor |
|-----------------|----------------|--------------------------|---------------------------------|---------------------------------|----------------|
| γ− rays and protons | No mask used | 0.006 | 66.3 | 37.5 | 1.08±0.01 |
| γ− rays and protons | 5.0° | 0.009 | 99.8 | 98.7 | 1.01±0.01 |

on the quality factor. However at higher primary energies it is observed that the spread in the arrival times of Čerenkov photons increases with the mass of the primary and the zenith angle [28]. This is due to the increased muon production in cascades generated by the heavier primaries.

7.4 Species dependence of quality factors

Each of the three species sensitive parameters under study here were applied to Fe primaries as well. It can be easily seen that in all the cases these parameters are more efficient in discriminating heavy primaries as compared to protons. The radius of the shower front, for example, shows a higher sensitivity for Fe primaries which is a reflection of the fact that, an Fe shower reaches its maximum development at a higher altitude compared to that of a lower Z primary. The efficiency also improves significantly with the use of a mask, a feature similar to that of proton primaries.

The pulse shape parameters on the other hand do not exhibit a large change in sensitivity for high Z primaries. However the use of mask shows a drastic improvement in sensitivity of decay time as the quality factor changes from \(~ 2.5\) (with out mask) to \(~ 7\) (with a 5° mask).

The photon arrival time jitter is particularly sensitive to high Z primaries. This perhaps is quite understandable since the Fe shower if considered as superposition of proton showers is expected to be more spread out laterally and hence the timing jitter will deviate more than that for γ−ray showers.

This demonstrates that the parameters discussed here are sensitive not only to
Table 16
Quality of decay time as a discriminating parameter for $\gamma$–ray (500 $GeV$) and proton (1 $TeV$) primaries at three different core distance ranges: pre-hump, hump & post hump

| Core distance ranges | Mask dia. used (if any) | Threshold value ($ns$) | Fraction of $\gamma$–rays accepted (%) | Fraction of protons accepted (%) | Quality factor |
|---------------------|-------------------------|------------------------|---------------------------------------|----------------------------------|---------------|
| Pre-hump            | No Mask                 | 4.3                    | 88.5                                  | 30.5                             | 1.60 ± 0.05   |
| Hump                | No Mask                 | 3.3                    | 80.5                                  | 1.5                              | 6.57 ± 0.51   |
| Post-hump           | No Mask                 | 8.4                    | 53.0                                  | 14.0                             | 1.42 ± 0.06   |
| Pre-hump 5.0°       | 3.8                     | 73.0                   | 19.5                                  | 1.65 ± 0.06                      |
| Hump 5.0°           | 2.8                     | 91.5                   | 1.0                                   | 9.15 ± 0.85                      |
| Post-hump 5.0°      | 6.0                     | 93.5                   | 41.5                                  | 1.45 ± 0.04                      |

Proton primaries but also to high $Z$ primaries, unlike Hillas’s image parameters which are not very sensitive to heavy primaries of cosmic rays.

7.5 Core distance dependence of quality factors

There are mainly three effects that determine the lateral distribution of Čerenkov light at the observation level: the finite Čerenkov angle, Coulomb scattering of the progenitor electrons and the transverse momentum $p_t$, in hadronic interactions. While the first effect is mainly responsible for the proverbial ‘hump’ in the lateral distribution of $\gamma$–ray initiated showers the third effect is responsible for the observable differences between the two types of primaries and the second effect is responsible for smoothing out the other two effects. As a result, one expects species specific parameters to show a dependence on the core distance of the detectors which measure these parameters. Table 16 summarizes the relative efficiencies of one of the parameters, viz pulse decay time as measured by detectors at pre-hump, hump & post-hump regions of showers. It can be seen that the difference between $\gamma$–ray and proton initiated showers is maximum at around the hump region resulting in maximum quality factor at a given primary energy. At the other two regions, viz, pre-hump & post-hump regions, the quality factors are comparable. However in practice it may not be always possible to detect the hump. Hence it may be advisable to limit the core distance of the triggering events to within the hump region at a given altitude of observation which is quite feasible since hump radius is not very sensitive to primary energy.
8 Discussions

In the present study we did not take into account the effect of detector response on the parameters selected here. It is implicit that in an experiment the pulse shapes have to be measured using high bandwidth electronics to minimize the shape distortion due to instrumental response so that the pulse shape parameters could be effective discriminators. However while applying these techniques to real data, it is inevitable that the theoretical parameters discussed here be suitably corrected for instrumental response used in the system. The threshold values of the parameters have to be derived for a given detection system and then applied to the data in order to reject events of hadronic origin.

The quality factors derived from the current studies have been tested for stability by estimating the same for subsets of the totality of data used. Hence their values are not critically dependent on the sample size. However it must be mentioned we chose to use that value of the parameter averaged over 16 detectors, thus reducing the 357 radial samples to 22. Obviously, the quality factors will improve if one averages over a larger number of detectors, as has been seen in our analysis. This only means that arrays with larger number of detectors can reject hadronic showers with a better efficiency.

The effect of mask has been studied qualitatively here. In order to apply the technique to real data one has to use the actual mask diameter. It is possible, in principle, to optimize the mask diameter for the best hadron rejection efficiency.

8.1 Radius of curvature of the shower front:

From the table 1 it can be seen that the radius of curvature of the Čerenkov front can enable us to reject around 20% of proton events at all energies. The rejection efficiency could improve to 50% if one uses a mask to limit the photon incidence angle. On the other hand this parameter is very sensitive to heavy primaries and hence one can reject up to \( \sim 99\% \) (with or without a 5° mask) of them.

At higher primary energies (> 20 TeV), the height of shower maximum has been shown to be directly proportional to the slope of the Čerenkov photon lateral distribution [39]. This has been suggested to be a good parameter to distinguish between \( \gamma \)- rays and hadrons. The present results show that the height of shower maximum, measured as the fitted curvature of the light front, is a reasonably good parameter even at lower energies and the sensitivity improves for inclined showers. As already mentioned, it is particularly useful
to discriminate against heavy primaries. It can be readily seen from tables 1 & 5 that the quality factor is sensitive to photons arriving at large incidence angles. This is particularly so for heavy primaries.

However the conventional method of measuring the height of shower maximum at higher primary energies is by measuring the pulse width [40]. This technique seems to work only at large core distances and at primary energies $\geq 10^{15}$ eV as the two parameters do not show any correlation at primary energies studied here.

### 8.2 Pulse shape parameters

There are not many results published in the literature regarding the usefulness of the pulse shape parameters to discriminate hadronic showers from electromagnetic showers at lower primary energies. Rodríguez-Fríos et al. [41] reported that the pulse width could be used as a discriminating parameter at primary energies $\geq 100$ TeV and their results show that sensitivity of this parameter is too poor at 10 TeV. This conclusion is consistent with the present results.

Patterson & Hillas [27] too have concluded that the pulse widths of simulated proton showers are not very different from those for electromagnetic showers and hence unlikely to be useful as a discriminating parameter consistent with the conclusions of the present study. However we find that it could be a useful parameter for distinguishing inclined hadronic showers.

The present results show that the pulse width and pulse decay time to be good discriminating parameters when used with a mask. However our study shows no sensitivity to pulse rise time in contrast to the conclusions of Roberts et al. [28]. Roberts et al. on the other hand do not use fall time to achieve discrimination against hadrons for they find that optimum quality factor was achieved only by rejecting an excessive fraction of $\gamma$–ray cascades, unlike in the present studies. They also find that rise time cut discriminates best against heavy primaries while we find that the decay time discriminates best against heavy primaries. The $\gamma$–ray acceptance fractions when decay time is used as a discriminating parameter is in the range 30-38% in contrast to the results of Roberts et al. [28].

### 8.3 Timing jitter

Cabot et al. [29] have estimated the relative mean times of arrival of Čerenkov photons at various core distances. The present results shown in fig. 3 (shown
Fig. 5. Variation of mean relative jitter for $\gamma$-rays (left) and protons (right) as a function of primary energy. The primary energies were selected randomly from power law distribution (differential slope -2.65). The energy band widths are 500 GeV - 10 TeV for $\gamma$-rays and 500 GeV - 20 TeV for protons. The fitted slopes are indicated for each species.

as +) agree well with theirs. The mean arrival times are fairly independent of the primary species. However the RMS fluctuations in arrival times depend on the range of differential pathlengths which in turn depend on the details of the interaction kinematics. Hence they bear the signature of the primary radiation.

One can ask the question how does the relative jitter vary with primary energy. It could be seen from figures 4a, 4b & 4d that the distributions of relative jitter become narrower with increasing energy for both $\gamma$-ray and proton primaries while average value of the relative jitter falls with increasing primary energy far more rapidly for proton primaries than for $\gamma$-rays. Figure 5 shows the primary energy dependence of relative jitter averaged over all core distances. Hence the quality factor for $\gamma$-ray and proton primaries is expected to fall with increasing primary energy as could be seen from table 3.
In an experiment it is not readily obvious how to measure the mean arrival time and the timing jitter. Normally most of the experiments use discriminators which trigger at known thresholds. Knowing that a typical Čerenkov pulse can be well described by a lognormal function (modulated by the phototube response) it is possible to estimate its coefficients knowing when the pulses from individual elements cross the preset threshold as well as the pulse height. The coefficients of the lognormal functions are related to the mean arrival time and its RMS value. It is therefore essential to have independent multiple sampling of the threshold crossing time and photon density from a single Čerenkov telescope. Further details are beyond the scope of this paper.

9 Conclusions

Three different measurable parameters based on Čerenkov photon arrival times at detectors at different core distances at the observational level have been found to be useful in distinguishing between electromagnetic and hadronic showers. The sensitivity of these parameters peaks at around the hump region and hence best discrimination would be obtained by limiting the measurements from detectors around the hump region. The sensitivity of these parameters also seems to increase with the zenith angle of the primary at the top of the atmosphere. This is a useful property in contrast to the imaging technique which is most sensitive at small zenith angles [19]. Similarly, the use of a circular mask at the focal point also increases the quality factor in some cases, especially for the curvature of the shower front. However the last two conclusions are not true to relative timing jitter in which case the sensitivity falls with increasing angle of incidence with respect to the vertical and marginally by the use of a mask.

Although the quality factors for individual parameters are not very large by themselves, by applying them in tandem would greatly augment the detection sensitivity of ground based VHE $\gamma$-ray telescopes designed to exploit the wavefront sampling techniques. Table 17 demonstrates the improvement in the quality factor when the decay time and the timing jitter are applied successively to the same sample of showers. The dramatic improvement in the quality factor exhibits the orthogonal nature of the parameters.

The separation efficiency seems to decrease with increasing altitude of observation mainly because of the decreasing prominence of the hump in the case of electromagnetic showers. This, once again supports the earlier conclusion that the Čerenkov photons around the hump region are more sensitive to photonic primaries. Hence sampling of photons from this region of the light pool is recommended for better discrimination of hadronic primaries.
Table 17
Table showing the improvement in quality factor when two parameters (viz. photon timing jitter and pulse decay time) are used in tandem to discriminate against hadronic events. For this 100 showers each of γ— rays and protons of energy 500 GeV and 1 TeV respectively as sampled at sea level are used.

| Parameter Type | Threshold Value | Fraction of γ— rays accepted (%) | Fraction of protons accepted (%) | Quality factor |
|----------------|-----------------|----------------------------------|----------------------------------|----------------|
| Pulse Decay Time | 3.2 ns          | 32.9                             | 1.4                              | 2.82±0.07      |
| Photon Timing Jitter | 0.08            | 52.5                             | 2.7                              | 3.18±0.06      |
| Decay Time & Timing Jitter | 3.2 ns        | 26.8                             | 0.05                             | 12.6±1.6       |

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