Self-organized criticality in superferromagnets

V N Kondratyev 1,2 and Ph Blanchard 2
1 Nuclear Physics Department, Taras Shevchenko National University, Pr. Acad. Glushkova 2, bdg. 11, Kiev, UA-03022 Ukraine
2 Faculty of Physics, Bielefeld University, D-33501 Bielefeld, Germany

E-mail: vkondra@univ.kiev.ua

Abstract. Superferromagnetic structures are studied accounting for quantum fluctuations due to the discrete level structure and disorder within the randomly jumping interacting moments model. The occurrence of self-organized criticality is found to indicate an existence of spinodal regions and critical points in magnetic state equation and phase diagram. The magnetodynamics show jerky behaviour displayed as erratic stochastic discontinuities for magnetic induction. Magnetic noise correlations are proposed as model-independent analytical tools employed in order to specify, quantify and analyse magnetic structure and origin of superferromagnetism.

1. Introduction
Superferromagnetism (SFM) was invoked to specify the structure of a system containing quantum confined objects, e.g., atomic clusters, quantum dots referred for hereafter as, simply, QDs, see [1-5] and refs. therein. In particular, an assembly of magnetic nanoparticles displays FM long range order at sufficiently large density. Such a property requires to consider the inter-dot magnetic interaction beyond dipolar components [4,5], like quadrupole, superexchange, strain fields etc. The magnetodynamics of realistic supercrystalline heterostructures is substantially determined by the relationship between inter-dot magnetic interaction and the disorder (see [1-3] and refs. therein). Then occurrence of self-organized (SO) criticality represents, perhaps, one of the most interesting and important phenomenon. At such a regime magnetic induction of QD arrays displays erratic stochastic discontinuities with rather wide distribution of jump amplitudes. In the present work we investigate further SFM structure paying particular attention for universal scaling behaviour in magnetodynamics. The correlation properties of noise signal amplitude distributions are explored as an analytical tool for quantitative definition, description and analysis of SFM properties and origin. In sect. 2 we give brief overview of discrete spin model for realistic QD arrays and consider some methods to reveal magnetic structure. Conclusions are given in sect. 3.

2. Magnetodynamics in dot assemblies with disorder
Overall magnetisation of a system composed from Π elements with moments \( m_i \) can be represented as

\[
P = \sum_i m_i / V = \langle m \rangle / V_o \]

with total volume \( V \) and an area occupied by \( i \)th dot \( V_o = V / \Pi \). For SFM with ferromagnetic coupling of a strength \( J \) between the nearest neighbor (nn) dots the respective Ising term \(-J\sum_{ij} m_i m_j\) contributes to the Hamiltonian. Here the sum runs over the nn elements. Besides, inhomogeneity and disorder in the form of defects, grain boundaries, impurities lead to
random crystalline anisotropy and varying interaction strengths in the super-crystalline heterostructure. Such effects can be accounted for by the random fields \( h_i \). We point out also dynamical components of \( h_i \) due to inexactness of the model description with nn interaction. The central limit theorem suggests, thereby, that the random fields obey the Gaussian distribution

\[
W(h) = \exp\left\{-h^2/R^2\right\}/R\sqrt{\pi}
\]

of a width \( R \), which we call the disorder. The total Hamiltonian \( H \) of an array in a field \( H \) can be expressed through an interaction of the dot magnetic moment \( m_i \) with local fields

\[
b_i = H(t) + JV_d \sum_{j=nn} P_j + h_i
\]

as \( H = -\sum_i m_i h_i \). We refer for this model as randomly jumping interacting moments (RJIM) model [2,3]. Hereafter, we consider an array of dots with single discontinuity in magnetic moment response, \( m_i = m \cdot \text{sign}(b_i) \). Due to the ferromagnetic interaction a jumping moment can cause some of the nearest neighbours to jump, which may in turn trigger some of their neighbours, and so on, generating, thereby, (re)magnetizing avalanche. As a consequence some sharp stepwise discontinuity arises on magnetization curves. Such sharp changes of magnetic induction result in a release of magnetic energy with noise signals proportional to avalanche size, similarly to the well-known Barkhausen effect.

**Figure 1.** Magnetic state equation (Panel A) and phase diagram (Panel B) for SFM composed from QDs with single jump at \( b=0 \) and disorder \( R \).

### 2.1. Magnetic state equation and phase diagram of SFM

The basic features of the nonequilibrium system corresponding to the Hamiltonian Eq. (1) can be analysed by employing the mean-field approach, in which one assumes an equal interaction strength between QDs with the coupling constant \( J_{ij} = J / \Pi \). The local field in Eq. (1) can be then simplified to the form \( b_i^{\text{mf}} = H(t) + JP + h_i \) with an averaged over a sample magnetization \( P \), see above. We see, therefore, that random fields can be viewed as mean-field fluctuations (cf, e.g., [2,3,6]).

In the thermodynamic limit \( \Pi \to \infty \) we calculate the magnetic state equation (MSE) \( P = \int dh W(h) m(b) \) and find for the magnetic susceptibility

\[
\chi = -dP/dH = [\chi_{\Pi J}^{-1} - J]^{-1}
\]

with \( \chi_{\Pi J} = W(b) \) representing the susceptibility of an array without interdot interaction (i.e. \( J=0 \)). The negatively defined susceptibility Eq. (2) yields spinodal region for an array. Figure 1 shows MSE and phase diagram. For coupled QDs the value \( J \chi_{\Pi J} \) measures an average number of induced jumps per single jumping moment. The negatively defined susceptibility yields adiabatic spinodal region for an
array, cf. Eq. (2). Since the number of induced moment jumps exceeds 1 the system favors to evolve in an avalanche spanning almost entire sample with a macro-(de)magnetization discontinuity. Evidently, the relation \( \chi_{NI} \geq J^{-1} \) corresponds to the instability condition. Such spinodal regions are located on \( \{H,R\} \)-plane between the low and up critical fields as indicated in Fig. 1B. The field-lines meet at the critical point \( \{ H_c = b_0 = 0, R_c = J / \sqrt{\pi} \} \). The difference between the number of induced jumps and 1, \( d = J \chi_{NI} - 1 \), provides, therefore, a measure for a vicinity of SO criticality. Since at such conditions the mean number of induced jumps shows an extremum approaching 1, the mean linear size \( l_b \) of the biggest avalanche \( S_b \) can be estimated as \( l_b^{\text{mf}} \approx (1 + d) / 2 \) of the total linear size.

Since within the mean-field treatment an average number of the moments induced to jump by a single jumping moment is site independent for the noise size distribution at \( 1 \ll S \ll \Pi \), one obtains [3]

\[
D_{\text{mf}}(S) \sim S^{-3/2} \exp\left(-Sd^2 / 2\right)
\]  

We perform the simulations for SFM of simple cubic lattice of a size \((30)^3\). The cumulative size distribution \( C(S) = \sum_{N \leq S} D(N) \) integrated over the demagnetizing branch of the hysteresis loop (i.e. for the decreasing magnetic field \( H \)) and averaged over 100 events of, e.g., array samples. One sees a clear transition from the `U' shape distribution at small disorders to an abrupt exponential suppression of large size avalanches at large disorders. At transitional values \( R \) the distribution shows a behavior close to the power law dependence \( C \sim S^{-\tau} \) with an exponent \( \tau \approx 0.85 \) corresponding to 1.85 for \( D(S) \) and being slightly different from the mean-field estimate.

![Figure 2](image)

**Figure 2.** Panel A - Cumulative avalanche size distributions are scaled to the power law with an exponent \( \tau = 0.85 \). Results of the RJIM model for \((30)^3\) simple cubic lattice are shown for disorders \( R=1 - \) solid circles, \( 1.6 - \) solid triangles, \( 3 - \) open circles. Panel B - Mean avalanche size versus the linear size of the biggest avalanche in units of array length. Results of RJIM model at various disorders are shown by dots while dashed-dotted line joins respective average values for each disorder. Solid line displays the prediction of the mean-field approximation in the thermodynamic limit.

### 2.2. Tools to reveal SFM structure and origin

Making use of the analytical form Eq. (3) we analyze some analytical tools which might be employed in order to specify and analyze SFM systems. Similarly to methods of high energy physics (cf, e.g., [7,8] and refs. therein) the correlations of avalanche size distribution might provide the criticality signals and a tool specifying and quantifying magnetic structure. For certain \( i \)th (re)magnetization event we define the mean noise signal
\[ <p> = \frac{\sum S D(S)}{\sum S D(S)} = \frac{\Pi - S_b}{N_{\text{tot}} - 1} \]  

where the sum runs over the avalanche sizes \( S \) excluding the biggest one, the quantity \( N_{\text{tot}} \) gives the total number of noisy jumps, i.e. avalanches. Substituting Eq. (3) into Eq. (4) the mean avalanche size, i.e. mean value for magnetic emission signals, is evaluated to be \( <p>_{\text{tot}} = |d|^{-1} + \text{const}(d) \), and diverges at critical conditions, i.e. \( d \to 0 \), in the thermodynamic limit \( \Pi \to \infty \).

If the system undergoes a critical behaviour being a precursor of SO criticality in some particular (re) magnetization events, strong correlations will appear in magnetic noise. For instance, in case of magnetodynamics we can study correlations between the strongest signal (i.e., the largest avalanche \( S_b \)) and the mean signal value \( b_{\langle S \rangle} \) for remaining avalanches in this particular event, e.g., from numerical model or experimental data. These correlations are similar to the Campi scatter plots [7,8].

In Fig. 2B we plot the mean avalanche size versus the length of largest avalanche for certain event. The data in Fig. 2B were obtained from RJIM simulations assuming various disorders as partially presented in Fig. 2A. In the figure we can clearly distinguish two branches corresponding to under-critical, i.e., large size of the biggest jump amplitude and small mean value, and over-critical, i.e. small size of the biggest avalanche and small average values. The right branch consists mainly of events with small disorders, while the left branch originates from events having large \( R \). The set of two branches meet in the critical region. The results of the mean field approximation in the thermodynamic limit are in a reasonable agreement with numerical data for overcritical disorders. At sub-critical conditions the mean field approach reproduces only qualitatively the RJIM model simulations.

3. Conclusions
The SFM structure effects in magnetodynamics of QD assemblies were analysed. As is shown the arrays display jerky magnetodynamics with sharp discontinuities in the magnetization process. Magnetic state equation and phase diagram of SFM are demonstrated to exhibit spinodal regions on \{disorder, magnetic field\}-plane and the critical points. Exploring correlations of noise amplitudes represents then convenient analytical tool for quantitative definition, description and study of SFM structure and origin, as well as self-organized criticality. We note that quantum fluctuations due to the dot discrete level structure can bring additional anomalies of magnetic response [9] and new phases [3]. Such features are of great importance for advanced electronic devices, nanoscale storage media, magnetic recording technology, as well as biology and advanced therapy.

4. Acknowledgments
The authors are indebted to G.Reiss, N.M.Kabachnik, F.Schmid, A.Hutten for valuable discussions. V.N.K. thanks the Research Center BiBoS at Bielefeld University for the warm hospitality and the Alexander von Humboldt Foundation for financial support.

References
[1] Kondratyev V N and Lutz H O 1998 Phys. Rev. Lett. 81 4508; 1999 Eur. Phys. J. D 9 483
[2] Kondratyev V N 2002 Phys. Rev. Lett. 88 221101; 2002 JAERI-Research 2001–057
[3] Kondratyev V N 2006 Phys. Lett. A 354 217; 2008 J. Phys.CS 129 012013
[4] Hutten A et al. 2004 J. Biotechnology 112 47
[5] Bedanta S and Kleemann W 2009 J. Phys. D: Appl. Phys. 42 013001
[6] Kondratyev V N 1993 Phys. Lett. A 179 209; 1994 ibid. 190 465; 1996 Z. Phys. B 99 473
[7] Kondratyev V N 1997 AIP Conf. Proc. 416 447; 2008 J. Phys.CS 129 012020; 2009 Bull. RAS Physics 73 491
[8] Kondratyev V N, Lutz H O and Ayik S 1997 J. Chem. Phys. 106 7766
[9] Kondratyev V N 2002 J. Nucl. Sci. Technol. 1 Suppl. 2. 550; 2002 J. Nucl. Radiochem. Sci. 3 205