Decoherence of Bose-Einstein condensates in microtraps

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We discuss the impact of thermally excited near fields on the coherent expansion of a condensate in a miniaturized electromagnetic trap. Monte Carlo simulations are compared with a kinetic two-component theory and indicate that atom interactions can slow down decoherence. This is explained by a simple theory in terms of the condensate dynamic structure factor.

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The decoherence of atomic de Broglie waves is a key issue for applications in atom interferometry and quantum information processing. It is particularly relevant for integrated atom optics based on miniaturized hybrid electromagnetic surface traps [1, 2] because the atoms couple to a macroscopic, ‘hot’ substrate nearby. Loss processes due to spin flips driven by thermal magnetic near fields have very recently been observed in the laboratory [3], in agreement with predictions made by one of us [4]. In this paper, we discuss a simple decoherence scenario for Bose-Einstein condensed atomic matter waves in a quasi-one-dimensional microtrap. This setup provides a realization of the standard model of environment-induced decoherence [5] featuring two attractive advantages: (i) the coupling to the environment can be microscopically modelled in terms of the magnetic dipole interaction; (ii) due to atomic interactions, the matter wave equation becomes nonlinear and novel features are expected. We compare Monte Carlo simulations for the condensate order parameter to a kinetic theory for the matter wave coherence function and show that already for moderate interaction parameters, a Bose-Einstein condensate is more robust with respect to a fluctuating environment.

We consider an elongated trap similar to those formed above current carrying wires [1]. In the confinement dominated regime, the matter waves can be described in a one-dimensional mean field approximation [2] (units with $\hbar = m = 1$),

$$i \partial_t \psi = -\frac{1}{2} \partial_x^2 \psi + V(x, t) \psi + g |\psi(x, t)|^2 \psi,$$

(1)

where the interaction parameter $g = 2 \Omega_x a/(1 - 1.46 a_0^2)$ depends on the three-dimensional scattering length $a$, the radial confinement frequency $\Omega_x$, and ground state size $a_0$ [7]. The density $|\psi(x, t)|^2$ is normalized to the total number of particles $N$. The potential $V(x, t)$ determines the dynamics in the axial direction. We assume that for $t < 0$, the atoms are confined in a harmonic trap with frequency $\Omega_x$ and occupy all the zero-temperature condensate mode $\phi_0(x)$ [3]. For $t \geq 0$, the axial confinement is switched off, the atoms expand, and we take into account their interaction with magnetic field fluctuations by letting $V(x, t)$ be a random potential. Note that the radial confinement is kept constant. Eq. (1) thus describes the interplay between matter wave interactions and time-dependent noise in an essentially one-dimensional geometry. In contrast to previous work in the field of nonlinear random waves [3, 10], our initial condition does not correspond to a self-contained soliton because we assume repulsive interactions $g > 0$. Current experiments in wire traps have been hampered by the presence of a static field modulation that leads to the fragmentation of the expanding atom cloud [11, 12]. This is neglected here and makes a direct comparison beyond the scope of our model.

If we ignore spin flip processes for simplicity, magnetic noise in a microtrap above a planar substrate translates into a random potential with correlation function [4, 13]

$$\langle V(x, t)V(x', t') \rangle \approx \gamma \delta(t - t') C(|x - x'|),$$

(2)

$$C(|x - x'|) = \frac{l_{corr}^2}{l_{corr}^2 + (x - x')^2},$$

(3)

where $\gamma$ is the noise strength and the spatial correlation length $l_{corr}$ is of the order of the microtrap height [13]. If the potential fluctuated only in time, $\gamma$ would correspond to the phase diffusion rate. Typically, $1/\gamma$ is a few seconds in $\mu m$ sized traps [3, 4]. In the frequency range relevant for our model (up to $\Omega_x$), the noise spectrum is approximately flat [14].

For a single, typical realization of the noise, the evolution of the density $n(x, t) = |\psi(x, t)|^2$ according to Eq. (1) is shown in Fig. 1. A complicated fringe pattern appears due to the interference between the expanding condensate mode and the excitations generated by noise, with the fringe phase depending on the history of the noise. If we average over the evolutions in an ensemble of noise potentials, a smooth average field $\psi_c(x, t) \equiv \langle |\psi(x, t)| \rangle$ with a decaying weight emerges (Fig. 2). This quantity would be revealed in a (as yet hypothetical) homodyne measurement of the Bose field, and we shall call it the ‘coherent field’ in the following. Note the analogy to the condensate order parameter in the symmetry breaking approach to Bose condensation when $N_c \sim N$ [15].

Another condensate definition, which is also applicable in $U(1)$ covariant theories, is based on long-range order in the single-particle density matrix [16]. This quantity corresponds to our problem to the coherence function $\rho(x, x', t) \equiv \langle \psi^*(x, t)\psi(x', t) \rangle$. Its decay with increasing separation $s = |x' - x|$ defines the coherence length $l_{coh}(t)$, a key concept of decoherence theory [5, 17]. We introduce the spatial average

$$\Gamma(s, t) = \int dx \rho(x, x + s, t),$$

(4)
FIG. 1: Expansion of a self-interacting Schrödinger field in a noisy potential, single realization. The normalized spatial density is plotted for different expansion times given in the inset. Parameters in Eq. (1): interaction strength $gN = 10$, noise strength $\gamma = 1$, correlation length $l_{\text{corr}} = 0.1$. Harmonic oscillator units with respect to the initial confinement frequency $\Omega$ are used: $t \to \Omega t$, $x \to (\hbar/m\Omega)^{1/2} x$. The numerical solution uses a discrete space-time grid with time step $dt = 0.1$, and $2^{14}$ space points spaced $dx = 0.0294$ units.

FIG. 2: Normalized density profiles of the coherent (noise-averaged) field for different expansion times. Symbols: Monte Carlo results, dashed lines: gaussian approximate solution to Eq. (5), $|\psi_c(x,t)|^2 = (N/((\sqrt{2\pi}u(t))) e^{-\gamma t} \exp[-x^2/(2u^2(t))]$ with $u(t)$ solving Eq. (10). Inset: coherent fraction (relative particle number). Crosses (open circles): Monte Carlo results for $\gamma = 1$ ($\gamma = 0.1$). Dashed (solid) line: exponential decay with decoherence rate $\gamma$ (renormalized rate $\gamma_{\text{eff}} = 0.82 \gamma$). Units and all other parameters as in Fig. 1.

whose Fourier transform with respect to $s$ is the momentum distribution, averaged over many realizations. This leads to $l_{\text{coh}} \delta p \sim 1$ where $\delta p$ is the width in momentum. The reduction of the coherence length ("decoherence") is borne out in the results plotted in Fig. 2. Long-range coherence is also visible: a fraction of the bosonic wave field is coherent across the full cloud size. We shall see that this fraction can be identified with the coherent field $\langle \psi(x,t) \rangle$, reinforcing the analogy between the condensate order parameter and the noise-averaged nonlinear Schrödinger field.

In the two-component model of Bose-Einstein condensation, the condensate evolves according to a nonlinear Schrödinger equation including loss terms and interactions with the non-condensed density. The non-condensed component is described by a suitable kinetic theory [16, 18]. We have adapted this model to our problem by replacing the average with respect to the field’s density operator by the average over the evolutions in an ensemble of random potentials. For a similar approach, see [19]: the noninteracting case has been treated in [20]. An essentially analytical solution has been obtained with two approximations: (i) we describe the fluctuations around the coherent field by the free space dispersion relation; (ii) we neglect in the nonlinear Schrödinger equation the interaction between the coherent field and its fluctuations. The resulting equations are

$$i\partial_t \psi_c = -\frac{1}{2} \partial_x^2 \psi_c + g |\psi_c(x,t)|^2 \psi_c - \frac{i\gamma}{2} \psi_c$$ \hspace{1cm} (5)

for the coherent field, and the Boltzmann-type equation

$$(\partial_t + p \partial_x) W_i(x, p, t) = \int dp' S_V(p - p') (W_i(x, p', t) + W_i(x, p', t) - W_i(x, p, t))$$ \hspace{1cm} (6)

for the Wigner representation $W_i$ of the ‘incoherent’ field. This is the Wigner transform of the fluctuating part of the coherence function $\rho_i(x, x', t) = \langle \psi^*(x, t) \psi(x', t) \rangle - \psi_c^*(x, t) \psi_c(x', t)$. In Eq. (6), $W_c$ is the Wigner transform of $\psi_c^*(x, t) \psi_c(x', t)$, and the ‘collision integral’ involves the ‘cross section’ [21]

$$S_V(p - p') = \gamma \int \frac{d(x - x')}{2\pi} C(x - x') e^{-i(p-p')(x-x')},$$ \hspace{1cm} (7)

where $C(x - x')$ is the normalized noise correlation function [3]. When Eq. (6) is approximated by a Fokker-Planck equation and $W_c = 0$, we essentially recover the decoherence model discussed by W. H. Zurek [3].

The analytical solution to Eqs. (5, 6) is derived using previous results for a noninteracting gas [20, 21]. The new ingredient is the nonzero average of the field that enters the collision integral as a source term. The basic idea of the analytical solution is to perform a Fourier transformation with respect to both $x$ and $p$, and to solve the resulting equation with the method of characteristics [22]. For the coherence function $\Gamma$, this yields

$$\Gamma(s, t) = \Gamma_c(s, t) + \Gamma_i(s, 0) e^{-\gamma t[1-C(s) + \gamma C(s) \int_0^t d\tau e^{-\gamma \tau[1-C(s)]} \Gamma_c(s, t - \tau).}$$ \hspace{1cm} (8)
The coherent field contribution, $\Gamma_c(s, t)$, can be found approximately using a time-dependent Thomas-Fermi profile [24] or a gaussian ansatz [25]. We follow the latter method because it simplifies the Wigner transform and get

$$\Gamma_c(s, t) \approx N e^{-\gamma t} \exp \left\{ -\frac{s^2}{2} \left( \frac{1}{\bar{u}(t)} + \dot{\bar{u}}^2(t) \right) \right\},$$

(9)

where the spatial width $u(t)$ is the solution of

$$\ddot{u} = -\frac{\partial}{\partial u} \left( \frac{1}{8 u^2} + \frac{\gamma}{u} \right).$$

(10)

The effective interaction strength is $\tilde{g}(t) = g N e^{-\gamma t}/(4\sqrt{\pi})$, and the initial condition minimizes $U_{\text{eff}}(u) = 1/(8u^2) + \tilde{g}(t)/u + \frac{1}{2}\Omega^2 u^2$ where $\Omega$ is the initial trap frequency.

The result (9) of the kinetic theory splits into a contribution with long-range coherence (the first term $\Gamma_c$ extends across the entire cloud size) and an incoherent part (the second and third terms). For the initial conditions considered here, the second term vanishes so that the incoherent fraction is not coherent beyond the correlation length of the noise potential, as intuitively expected. (Recall that the spatial correlation function $C(s)$ [Eq. (4)] decays on the scale $\xi_{\text{corr}}$.)

The simple kinetic theory outlined above captures qualitatively the features observed in Monte Carlo simulations of Eq. (1), as shown by Figs. 24. We attribute deviations to the approximate treatment of interactions in the theory.

The density profile of the coherent field is not exactly gaussian (Fig. 2) because for the chosen parameters, one already approaches the Thomas-Fermi regime. The coherent fraction of particles $N_c(t)/N = (1/N) \int dx |\psi_c(x, t)|^2$, however, shows an exponential decay as predicted by Eq. (5). We find quantitative agreement with the kinetic theory when a ‘renormalized’ scattering rate $\gamma_{\text{eff}} < \gamma$ is used. Further simulation runs show that the ratio $\gamma_{\text{eff}} / \gamma$ does not change significantly when the noise strength is reduced by an order of magnitude.

A similar renormalization of the coherent field’s loss rate has been observed in simulations of nonlinear pulse propagation with a randomly fluctuating phase mismatch [24]. An analytical approximation for $\gamma_{\text{eff}} / \gamma$ is derived below.

Similarly, the renormalization of the noise correlation length to $l_{\text{corr}} > \xi_{\text{corr}}$ reproduces quantitatively both the short-range and long-range behavior of the coherence function (Fig. 3). The oscillations originate from the Thomas-Fermi density kink at the condensate border that is not captured by our gaussian ansatz.

In a homogeneous condensate, atomic interactions suppress the generation of long-wavelength excitations by a spatially modulated potential. This is described by the vanishing of the dynamic structure factor $S[k; n_c]$ in the limit $k \to 0$ [13]. Let us consider for our problem the condensate as locally homogeneous with a density $n_c(x) = |\psi_c(x)|^2$. Including the structure factor in the scattering cross section, we suggest an improved approximation to the collision integral in Eq. (6)

$$\partial_t W_i(x, k, t) \bigg|_{c-i} = \int dk' S_V(k') S[k'; n_c(x)] W_c(x, k - k', t)$$

FIG. 4: Renormalized decoherence rate $\gamma_{\text{eff}}$ [Eq. (12)] for increasing interaction strength. On the x-axis is plotted the ratio $l_{\text{corr}}/\xi_0 \propto \sqrt{\xi_0}$, where $\xi_0$ is the healing length at the trap center, $\xi_0 = 1/\sqrt{4gn_c(x = 0)}$. Large dots: Monte Carlo data (see text). Small lower (upper) dots: numerical integration of Eq. (12) for a Thomas-Fermi (gaussian) density profile. Dashed lines: asymptotics for a Thomas-Fermi profile at weak and strong interactions [Eq. (14)], thick solid line: interpolation [15]. The decoherence rate is normalized to its value $\gamma$ in an ideal gas.

$$\approx S_V(k)S[k; n_c(x)]n_c(x, t),$$

(11)

where $S_V(k)$ [Eq. (1)] is the wavevector spectrum of the noise, and the well-known Bogoliubov structure factor is [15] $S[k; n_c] = |k|/(k^2 + 4gn_c)^{1/2}$. In Eq. (11) we have replaced the coherent field’s momentum distribution by a $\delta$-function, assuming a width $u(t) \gg l_{\text{corr}}$. For the ideal gas, $S[k; n_c(x)] = 1$, and we recover Eq. (6). In the general case, we find

$$\gamma_{\text{eff}}(t) = \frac{1}{N_c(t)} \int dx \int dk S_V(k) S[k; n_c(x)] n_c(x, t).$$

(12)

The relevance of interactions is now determined by the competition between the width $1/l_{\text{corr}}$ of $S_V(k)$ and the width $1/\xi(x)$ of the structure factor, involving the local healing length $\xi(x) = (4gn_c(x))^{-1/2}$.

We have computed the integral (12) numerically for gaussian and Thomas-Fermi density profiles. Very similar results are found (see Fig. 4) when the reduction factor $\gamma_{\text{eff}} / \gamma$ is plotted versus the ratio $l_{\text{corr}}/\xi_0$ where the healing length $\xi_0$ is taken at the trap center. It turns out that the time-dependence only appears via $l_{\text{corr}}/\xi_0 \propto 1/\sqrt{u(t)}$. For the Thomas-Fermi profile, the $x$ integral can be evaluated analytically, leading to

$$\frac{\gamma_{\text{eff}}}{\gamma} = \frac{3l_{\text{corr}}}{\bar{u}_0} \int_0^\infty dk e^{-k l_{\text{corr}}} f(k\xi_0),$$

(13)

where $f(z) = z \left[ 1 + z^2 \right] \arctan z$. Asymptotic analysis leads to the limiting behavior (dashed lines in Fig. 4)

$$\frac{\gamma_{\text{eff}}}{\gamma} = \begin{cases} 1 - 0.88 \frac{l_{\text{corr}}}{\xi_0} & \text{for } l_{\text{corr}} \ll \xi_0, \\ 3\pi \frac{\xi_0}{8} \frac{l_{\text{corr}}}{\xi_0} & \text{for } \xi_0 \ll l_{\text{corr}}. \end{cases}$$

(14)

The ideal gas result $\gamma_{\text{eff}} = \gamma$ is recovered for $\xi_0 = \infty$, but the next order correction already comes into play for a moderate
interaction parameter $g$, since $\xi_0 \propto g^{-1/2}$. Very strong interactions, where $\xi_0 \rightarrow 0$, significantly slow down decoherence within our model. In the intermediate regime $\xi_0 \sim l_{\text{corr}}$, the asymptotics (14) is quite inaccurate, but the interpolation

$$\frac{\gamma_{\text{eff}}}{\gamma} = \frac{A}{A + 8 l_{\text{corr}}/(3\pi \xi_0)},$$

(15)

with $A \approx 1.15$ provides good agreement. The decoherence rate extracted from the Monte Carlo results is also fairly well described by Eqs. (13 15). The two data points we have plotted correspond to two values for the healing length: based either on the numerically computed coherent field density or on its Thomas-Fermi approximation.

Let us finally note that for large interactions or long expansion times, the coupling to the incoherent fraction will no longer be negligible compared to the random potential and our approximate solution will lose accuracy. In this limit, we may expect that the noise-induced decoherence is replaced by an ‘open system dynamics’ similar to the models discussed for a condensate at finite temperature [18 27].

To summarize, we have evaluated the impact of weak magnetic field fluctuations on coherent matter wave dynamics in atom chips. A quantum kinetic theory for a degenerate trapped boson gas subject to noise has been worked out in the mean field approximation and solved analytically, neglecting the backaction of excitations onto the coherent field. The comparison to numerical simulations demonstrates that interatomic interactions reduce the decoherence rate relative to the ideal gas. We have suggested an explanation in terms of the structure factor of a quasi-homogeneous system that leads to an accurate agreement with the numerical data. Further investigations will address the renormalization of the noise correlation length due to interactions and the impact of finite temperature in the initial conditions.

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