Production of Dirac Particles in External Electromagnetic Fields

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Pair creation of spin-$\frac{1}{2}$ particles in Minkowski spacetime is investigated by obtaining exact solutions of the Dirac equation in the presence of electromagnetic fields and using them for determining the Bogoliubov coefficients. The resulting particle creation number density depends on the strength of the electric and magnetic fields.

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I. INTRODUCTION

After the pioneering works of Sauter [1], Heisenberg and Euler [2] on the particle creation by the strong electromagnetic fields, Schwinger formulated the following pair creation probability per unit volume and time by obtaining the one-loop effective action in a constant and homogeneous classical electric field (in natural units, $\hbar = c = 1$) [3]:

$$\omega = \frac{(eE)^2}{4\pi^3} \sum_{n=1}^{+\infty} \frac{1}{n^2} \exp(-\frac{n\pi m^2}{eE})$$

where $m$ and $e$ are the mass and charge of the electron, $E$ is the electric field, respectively. Since then, this process is called Schwinger mechanism and has become an important problem in the quantum field theory (QFT). Such kind of a classical electric field is assumed to be of order $E \sim 10^{16} V/cm$ [4] which is very difficult to generate by the current technology. Strong fields arising from the collisions between relativistic high energy particles and heavy-ions are called color electric fields and have ability to create particles from the vacuum. These type of collisions are generated at the modern colliders, i.e at CERN. Schwinger mechanism is attributed to the hadronic particle creation and on the base of Color Glass Condensate (CGS), this phase is called Glasma.

The Schwinger mechanism have been studied in the presence of various stationary and non-stationary external fields [2]-[9]. The studies about the Schwinger mechanism in gauge fields having both electric and magnetic field components have revealed that electric field has a dominant influence in creating the particles. Therefore, the pair creation mechanism is totally attributed to the pure electric field [10]. This quantum effect of the classical electromagnetic fields is carried out to the curved spacetime as well [11]-[13].

There is a considerable point in some of the studies in the literature that they support the magnetic field has a reduction effect in the particle creation process. One of the aims of this study is to investigate this phenomena for a particular choice of the electromagnetic gauge field that has both electric and magnetic field components.

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The outline of the paper is as follow: In Section 2 we solve the relativistic HJ equation and obtain the asymptotic behavior of the solutions. In Section 3 we solve the Dirac equation for the considered electromagnetic fields and obtain the asymptotic limits of the solutions to define the vacuum "in" and "out" modes by referring the asymptotic solutions of the HJ equation. We use the Bogoliubov transformation technique to relate the solutions at the boundaries and calculate the particle creation rate as required to define the positive and negative frequency energy states, namely the "in" and "out" mode vacuum solutions. For the motion of the relativistic charged particles moving in an external field, analysis of mode functions as positive and negative frequency solutions is not easy since the Lagrangian of the corresponding system completely depends on space-time coordinates. Namely, particle concept becomes indefinite owing to interaction with the external fields. For this reason we require a condition to define "particle" concept. In the present study we will apply a quasiclassical method. We obtain exact solutions of the Hamilton-Jacobi (HJ) equation and discuss their asymptotic behavior in the infinite past and future. Then, asymptotic behavior of the solutions of the Dirac equation in the neighborhood of the time singularities will be identified. With the help of this analysis and comparison of asymptotic solutions of both HJ and Dirac equations in the infinite past and future, the particle picture will be identified.

We define positive and negative frequency mode functions in such a way that the positive frequency mode function approaches $e^{iS(t)}$ and the negative frequency one $e^{-iS(t)}$ in asymptotic regions [4], where $S(t)$ is the solution of the HJ equation for the presence of a 4-vector electromagnetic potential given as

$$A_\nu = B_0 \tau [1 + \tanh(x/\tau)] \delta^2_\nu - E_0 (\Gamma + \Lambda t) \delta^2_\nu$$

where $\tau$, $\Gamma$, and $\Lambda$ are constants. This new suggested form of the vector potential generates parallel stationary electric [5] and Sauter type magnetic fields [14] that are persuaded in the Glasma flux tube model of high energy heavy ion collisions.

Magnetic current emerging is found to be

$$J_\nu = \frac{1}{4\pi} \frac{2(|B|)}{\tau} \tanh(x/\tau) \delta^2_\nu$$

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number density for fermions in Section 4. Finally, in Section 5 we discuss the results we obtained. Throughout the paper the natural units, $\hbar = c = 1$ are used.

II. SOLUTIONS OF THE HAMILTON-JACOBI EQUATION

The relativistic HJ equation for the action $S$ is given by [11]:

$$
\zeta \frac{\partial S}{\partial x^\mu} - e A_\mu \frac{\partial S}{\partial x^\mu} - e A_0 + m^2 = 0
$$

(4)

where $\zeta = (1, -1, -1, -1)$ is the Minkowski metric, $m$ is the mass of the particle and $A_\mu$ is the 4-vector electromagnetic potential.

The electromagnetic potential satisfy the Lorentz gauge and the Lorentz invariants are determined from the electromagnetic field tensor as follows

$$
F^\mu_\nu F^\nu_\mu = 2(B^2 - E^2) = 2B_0^2 \text{sech}^2(x/\tau) - E_0^2 \Lambda^2
$$

(5)

and

$$
F^\mu_\nu F^\nu_\mu = 4E \cdot B = 4 E_0 B_0 \text{sech}^2(x/\tau)
$$

(6)

Because of the space-time dependence of the considered electromagnetic field, the solution of the HJ equation can be separated as follow:

$$
S(t, x^\mu) = P(x) + Q(t) + (yk_y + zk_z)
$$

(7)

where $k_y$ and $k_z$ can be viewed as the conserved momenta that exist given the symmetries chosen for the electromagnetic gauge (2). By using (7) in Eq. (4) we obtain

$$
\dot{Q}^2 - \dot{P}^2 - [k_z + eE_0(\Gamma + \Lambda t)]^2 - [k_y - eE_0 \tau (1 + \tanh(x/\tau))]^2 + m^2 = 0
$$

(8)

where dot and acute denote derivatives with respect to $t$ and $x$, respectively.

We obtain two first order differential equations as follows:

$$
\dot{Q}^2 - [eE_0(\Gamma + \Lambda t)]^2 - 2k_z eE_0(\Gamma + \Lambda t) + m^2 - k_z^2 = v^2
$$

(9)

and

$$
\dot{P}^2 + [eE_0 \tau (1 + \tanh(x/\tau))]^2 - 2k_y eE_0 \tau (1 + \tanh(x/\tau)) = v^2
$$

(10)

where $v^2$ is the constant of separation.

Time-dependent external fields cause unstable vacuum and this results in the pair creation by vacuum. For this reason the dynamics involving spatial coordinates effect the solutions only by a constant and we obtain the solution of the HJ equation for electromagnetic gauge (2) as follow

$$
S(\rho, x^\mu) = S_0(0, x^\mu)
$$

$$
+ \frac{eE_0}{\Lambda} \int_0^\epsilon \sqrt{\rho^2 + \frac{2k_z eE_0}{\rho} + \left(\frac{k_z^2 + v^2 - m^2}{e^2E_0^2}\right)} d\rho
$$

$$
= \left(\frac{geE_0 + k_z}{2\Lambda}\right) \sqrt{\rho^2 + \frac{2k_z eE_0}{\rho} + \left(\frac{k_z^2 + v^2 - m^2}{e^2E_0^2}\right)}
$$

$$
+ \frac{v^2 - m^2}{2eE_0\Lambda} \ln \left\{2\rho + \frac{2k_z eE_0}{\rho} + \left(\frac{k_z^2 + v^2 - m^2}{e^2E_0^2}\right)\right\} + S_0(0, x^\mu)
$$

(11)

where $\rho = (\Gamma + \Lambda t)$.

The dependence of the solution on time is derived by $\psi \rightarrow e^{iS(t)}$ and we arrive the following expressions for the asymptotic behavior of the relativistic wave function:

$$
\psi(t \rightarrow \pm \infty) = \frac{e^{iS(t)}}{\epsilon^{-(\frac{2k_z E_0}{m} + (\frac{2 - m^2}{m}) \ln(2\Lambda t))}}
$$

(12)

where the upper and lower signs represent the negative and positive-frequency states, respectively.

III. SOLUTIONS OF THE DIRAC EQUATION

The Dirac equation in external electromagnetic fields is given by [13]

$$
[i\gamma^\nu \partial_\nu + eA_\nu \gamma^\nu - m] \psi = 0
$$

(13)

where $\gamma^\nu$ are Dirac matrices, $A_\nu$ is the 4-vector electromagnetic potential, $m$ is the mass of electron, $e$ is the charge of the electron and $\psi$ is the four-component spinor.

The Dirac equation yields four coupled differential equations for the spinor and usually it is difficult to obtain the exact analytical solutions, in particular for mathematically complicated external fields. This difficulty of the problem has been accomplished by Feynmann and Gell-Mann by considering a two-component form of the Dirac equation in the presence of electromagnetic fields as follow [16]

$$
[(\vec{P} - e\vec{A})^2 + m^2 - e\vec{\sigma} \cdot (\vec{B} + i\vec{E})] \phi = (p_0 - eA_0)^2 \phi
$$

(14)

where $\vec{\sigma}$ are usual Pauli matrices and $\phi = (\phi_1, \phi_2)$ are the solutions of the two-component equation. The four-component spinor can be derived from $\phi$ as follow

$$
\psi = \left(\frac{\vec{\sigma} \cdot (\vec{P} - e\vec{A}) + (p_0 - eA_0) + m\phi}{\vec{\sigma} \cdot (\vec{P} - e\vec{A}) + (p_0 - eA_0) - m\phi}\right)
$$

(15)

Thence, for the purpose of obtaining the analytic solutions we follow up the two-component formalism and consider the electromagnetic gauge (2). Because the given
In order $\chi/f$ to be finite in the range $-\infty \leq r \leq +\infty$, $(a^2 + b^2 - \varepsilon) = 0$ and $(2ab + \Upsilon) = 0$ conditions are necessary [17]. From these conditions we derive the following expressions for $a$ and $b$:

$$a = -\frac{1}{2}[(\varepsilon + \Upsilon)\hat{r} - (\varepsilon - \Upsilon)\hat{\tau}]$$

and

$$b = \frac{1}{2}[(\varepsilon + \Upsilon)\hat{r} + (\varepsilon - \Upsilon)\hat{\tau}]$$

Therefore, keeping these expressions and by introducing $\eta = \frac{1}{2}(1 + \tanh r)$ we arrive

$$\{\eta(1 - \eta)\frac{d^2}{d\eta^2} + [a + b + 1 - 2(b + 1)\eta] \frac{d}{d\eta} + [\Upsilon - b(b + 1)]\} f = 0$$

which is the differential equation satisfied by the hypergeometric functions. The hypergeometric function remaining finite at $\eta = 0$ will provide this equation and solution will be given as [18]

$$f_\eta(\eta) = 2 F_1([b + 1/2] - (\Upsilon + 1/4)\hat{r};
(b + 1/2) + (\Upsilon + 1/4)\hat{r}; a + b + 1; \eta)$$

So, we obtain

$$\chi_s = e^{ra} \cosh^{-b} r F_1([b + 1/2] - (\Upsilon + 1/4)\hat{r};
(b + 1/2) + (\Upsilon + 1/4)\hat{r}; a + b + 1; \eta)$$

Then

$$a = -\Upsilon[(4\Upsilon + 1)\hat{r} - (2n + 1)]^{-1}$$

and

$$b = \frac{1}{2}[(4\Upsilon + 1)\hat{r} - (2n + 1)]$$

The constant of separation $\varpi$ can be easily derived from $(a^2 + b^2 - \varepsilon) = 0$.

By introducing a variable $\xi = \sqrt{\frac{2}{eE_0\Lambda}}(eE_0\Lambda t + eE_0\Gamma + k_z)$ we obtain the following equation from Eq.(22)

$$\{\frac{d^2}{d\xi^2} + \frac{1}{4}\xi^2 - \frac{iesE_0\Lambda + k_z^2 - \varpi^2}{2eE_0\Lambda}\} T_s(\xi) = 0$$

Solutions of this differential equation are parabolic cylinder functions [18]

$$T_s(\xi) = \frac{e^{-\frac{\pi\xi}{2}}}{(2eE_0\Lambda)^{1/2}}[D_{-\frac{i\pi}{2}}(e^{i\pi/4}\xi) + D_{-\frac{i\pi}{2}}(e^{i\pi/4}\xi)]$$

where $\tilde{a} = \frac{iesE_0\Lambda + k_z^2 - \varpi^2}{2eE_0\Lambda}$.

Therefore, exact solutions are obtained and all components of the Dirac spinor can be found with the insertion of Eqs. (30) and (35) into Eq.(16).
IV. PARTICLE CREATION VIA BOGOLIUBOV TRANSFORMATION METHOD

Due to difficulty of the direct observation of the pair creation in a constant field [10], because the typical $|eE|$ is smaller than $m^2$, the particle creation will be induced by the time-dependent components of the wave-function (28), namely the parabolic cylinder functions. Two solutions of the Eq. (34) are given as:

$$T_{s_1}(\xi) = \frac{e^{-\frac{\pi}{4}i}}{(2\epsilon E_0 \Lambda)^{\frac{1}{4}}} D_{-i\alpha -1/2}(e^{i\pi/4} \xi)$$ (36)

and

$$T_{s_2}(\xi) = \frac{e^{-\frac{\pi}{4}i}}{(2\epsilon E_0 \Lambda)^{\frac{1}{4}}} D_{i\alpha +1/2}(e^{i\pi/4} \xi)$$ (37)

These are not the only solutions and any of the remaining two-sets can be constructed via Bogoliubov coefficients as follow:

$$\tilde{T}_{s_1}(\xi) = \alpha T_{s_1}(\xi) - \beta^* T_{s_2}(\xi)$$ (38)

and

$$\tilde{T}_{s_2}(\xi) = \alpha^* T_{s_2}(\xi) + \beta T_{s_1}(\xi)$$ (39)

The Bogoliubov transformation method is a technique that associates a canonical commutation relation algebra or a canonical anti-commutation relation algebra into another representation, caused by an isomorphism [19]. In the Minkowskian QFT, eigenfunctions of the field equation, $\psi$, can be written with the help of the mode solutions as [19-20].

$$\psi = \sum_n (a_n \varphi_n + a_n^\dagger \varphi_n^*) = \sum_k (b_k \Theta_k + b_k^\dagger \Theta_k^*)$$ (40)

where we have the relations $(\varphi_i, \varphi_j) = \delta_{ij}$, $(\varphi_i^*, \varphi_j^*) = \delta_{ij}$, $(\varphi_i, \varphi_j^*) = 0$ and $(\Theta_i, \Theta_j) = \delta_{ij}$, $(\Theta_i^*, \Theta_j^*) = \delta_{ij}$, $(\Theta_i^*, \Theta_j) = 0$ for $\varphi$ and $\Theta$ are mode solutions. The $\varphi$ and $\Theta$ can be expanded in terms of each other.

The creation and annihilation operators $a_n^\dagger, b_k^\dagger$ and $a_n, b_k$ are in correlation by the following expressions

$$a_n = \sum_k (\alpha_{nk} b_k + \beta_{nk}^* b_k^\dagger)$$ (41)

$$b_k = \sum_n (\alpha_{kn}^* a_n - \beta_{kn} a_n^\dagger)$$ (42)

$\alpha_{kn}$ and $\beta_{kn}$ are Bogoliubov coefficients determined by $\alpha_{ij} = (\Theta_i, \varphi_j)$, $\beta_{ij} = -(\Theta_i, \varphi_j^*)$. They are related as

$$\sum_i (\alpha_{ni} \delta_{ki} - \beta_{ni}^* \beta_{k_i}) = \delta_{nk}$$ (43)

$$\sum_i (\alpha_{ni} \delta_{ki} - \beta_{ni}^* \beta_{k_i}) = 0.$$ (44)

Let $|0_n\rangle$ and $|0_0\rangle$, are two states of vacuum in the Fock space and are related to each particle notion in (30). They are represented for all $n$ and $k$ as

$$|0_n\rangle: a_n|0_n\rangle = 0$$ (45)

$$|0_0\rangle: b_k|0_0\rangle = 0$$ (46)

If $|0_0\rangle$ is introduced as the usual vacuum, then $|0_n\rangle$ is regarded as a many-particle state. Therefore, the number of $\Theta_n$-mode particles in the state of $|0_n\rangle$ is

$$\langle 0_n | b_k^\dagger b_k | 0_n \rangle = \sum_n |\beta_{kn}|^2$$ (47)

If the $\varphi_n(x)$ are defined as positive frequency modes and the $\Theta_n(x)$ modes are linear uniformization of them, then $\beta_k = 0$. Then, $b_k|0_n\rangle = 0$ and $a_k|0_n\rangle = 0$. Hence, $\varphi$ and $\Theta$ modes have a common vacuum state. If $\beta_k \neq 0$, then $\Theta_k$ contain a combination of positive-$\varphi_k$ and negative-$\varphi_k^*$ frequency modes.

Therefore, we can define the positive- and negative-frequency solutions in order to find the Bogoliubov coefficients. Asymptotic expansion of the parabolic cylinder functions is given by [21]

$$D_{\nu}(z)\big|_{z \to +\infty} \approx z^\nu e^{-z^2/4}, |argz| < \frac{3\pi}{4}$$ (48)

Taking into account this relation for Eqs. (36),(37) in the limit $t \to +\infty$ (namely, $\xi \to +\infty$) and comparing the asymptotic expansion of them with Eq. (12), we see that the positive and negative-frequency mode solutions will be as follows respectively,

$$T_{s_1}(\xi) \approx (\sqrt{2\epsilon E_0 \Lambda} |t|)^{-1/2} e^{(-i\epsilon E_0 \Lambda t^2/2 - i\alpha \ln(\sqrt{2\epsilon E_0 \Lambda} |t|))}$$ (49)

and

$$T_{s_2}(\xi) \approx (\sqrt{2\epsilon E_0 \Lambda} |t|)^{-1/2} e^{(i\epsilon E_0 \Lambda t^2/2 + i\alpha \ln(\sqrt{2\epsilon E_0 \Lambda} |t|))}$$ (50)

We conclude that the solutions behave as $T_{s\pm} \approx e^{\pm iS(t)}$. For $t \to -\infty (\xi \to -\infty)$, the solutions are in the form

$$T_{s_1}(\xi) = \frac{e^{-\frac{\pi}{4}i}}{(2\epsilon E_0 \Lambda)^{\frac{1}{4}}} D_{i\alpha +1/2}(e^{i\pi/4} \xi)$$ (51)

and

$$T_{s_2}(\xi) = \frac{e^{-\frac{\pi}{4}i}}{(2\epsilon E_0 \Lambda)^{\frac{1}{4}}} D_{-i\alpha -1/2}(e^{i\pi/4} \xi)$$ (52)

so that their asymptotic behavior should be $T_{s\pm} \approx e^{\pm iS(t)}$. It is clear that the solutions are different in the asymptotic regions and this is the nature of the particle creation. Therefore, the solutions for $t \to +\infty$ belong to vacuum "out" mode whereas are vacuum "in" mode for
function given as \[21\] between the parabolic cylinder function and Whittaker
be derived easily by taking the advantage of the relation
where \( \alpha \)
where ˘
we obtain the Bogoliubov coefficients
α
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coefficients
Then, we find the below expression for the Bogoliubov
isfied.
computed as follow
the below formula \[21\]
Expanding the left side of this expression according to
the below formula \[21\]

\[
D_\nu(z) = \left[ e^{-i\pi\nu} D_\nu(-z) + \frac{\sqrt{2\pi}}{\Gamma(-\nu)} e^{-i\pi(\nu+1)/2} D_{-\nu-1}(iz) \right]
\] (54)

and using the result \([D_\nu(z)]^\dagger = D_{-\nu-1}(-iz)\) that can
de be derived easily by taking the advantage of the relation
between the parabolic cylinder function and Whittaker
function given as \[21\]

\[
D_\nu(z) = 2^{(\nu+1)/2} z^{-1/2} W_{\frac{1}{2}(\nu+\frac{1}{2}), -\frac{1}{4}}(z^2/2)
\] (55)

we obtain the Bogoliubov coefficients \( \alpha \) and \( \beta \) as follows:

\[
\alpha = \sqrt{\frac{2\pi}{\pi} e^{-\pi\tilde{a}/2}} \frac{1}{\Gamma(-i\tilde{a})}
\] (56)

and

\[
\beta = e^{-\pi\tilde{a}}
\] (57)

where \( \tilde{a} = \left( \frac{k_s^2 - m^2}{2eB_0\lambda} \right) \) and \( |\alpha|^2 + |\beta|^2 = 1 \) condition is sat-
sified.

Then, we find the below expression for the Bogoliubov
coefficients

\[
\frac{|\alpha|^2}{|\beta|^2} = \frac{2\pi}{\tilde{a}} e^{\pi\tilde{a}} \frac{1}{|\Gamma(-i\tilde{a})|^2}
\] (58)

By considering the following formula for Gamma func-
tions \[17\]

\[
|\Gamma(iq)|^2 = \frac{\pi}{q \sinh(\pi q)}
\] (59)

the number density of the created particles can be com-
puted as follow

\[
N \sim |\beta|^2 = \left[ \frac{|\alpha|^2}{|\beta|^2} + 1 \right]^{-1} = e^{-2\pi\tilde{a}}
\] (60)

where the parameter \( \tilde{a} \) in terms of the physical constants
of four-vector potential (2) has been given as below.

\[
\tilde{a} = \frac{1}{2eE_0\lambda} \left\{ \frac{4e^2B_0^2(eB_0 - \tau k_y)^2}{\left( -1 - 2n + \sqrt{1 + 4eB_0(s + eB_0\tau^2)} \right)^2} \right. \\
- \left. \frac{1}{4\pi^2} \left( -1 - 2n + \sqrt{1 + 4eB_0(s + eB_0\tau^2)} \right)^2 \right. \\
- \left. (m^2 + k_y^2) - 2eB_0\tau(eB_0\tau - k_y) \right\}
\] (61)

V. CONCLUSION

In this study, we used the two-component formalism for the Dirac equation that is proposed by Feynmann and Gell-Mann. This approach to the problem removes the complexity of obtaining the exact solutions. One of the advantages of working with this form of the Dirac equation is that these solutions are valid for the KleinGordon particles in the case of \( s = 0 \). Thus the results
can be used both for scalar and fermionic particles.

Mechanism of particle production by strong electric
fields is significant in order to figure out the early stages
of the heavy-ion collisions, for example their effect on the
thermalization of quarks and gluons. For the analysis of
our problem we take account a strong constant electric
field and a space-dependent hyperbolic magnetic field.
Exact solutions of the Dirac equation were identified in
terms of the parabolic cylinder and hypergeometric func-
tions.

![Particle Creation Number Density vs E](image)

FIG. 1. Particle creation number density versus electric field strength is depicted. \( m = 1; n = 1; \tau = 1; k_y = 1; k_z = 1; \Lambda = 1 \) and \( B_0 : 0\) (blue), \( B_0 : 0.2\) (red), \( B_0 : 0.4\) (black)

Existence of the strong electric fields cause to unstable
vacuum that is asymptotically static at future. The "in"
and "out" vacuum states were determined with the help
of the asymptotic solutions of relativistic HJ equation.
They were related by the Bogoliubov coefficients that are
used to calculate the particle creation number density in
Eq.\,(60). This expression depends on the parameters of
electric and magnetic fields and is not in Fermi-Dirac
thermal form. As it is seen by analyzing the formula and
also from the Figure 1, selected form of the magnetic field
has a reduction effect on the creation of fermionic parti-
cles. This situation is compatible with previous obtained
results. Also it can be seen from Figure 1, particle cre-
ation rate increases due to electric field strength, \( (E_0\Lambda) \).

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