Macroeconomic Instability of a Capital Markets Union and Stability of a Fiscal Union in the Euro Area: Keynesian and Kaldorian Two–Country Models*

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Abstract

In this paper, we investigate the effect of fiscal transfers in a fiscal union in relation to a Capital Markets Union (CMU) for the euro area using Keynesian and Kaldorian two–country models with a monetary union and imperfect capital mobility. We find that an increase in capital mobility between countries in a CMU is a destabilizing factor, whereas an increase in fiscal transfers between such countries is a stabilizing factor. Fiscal transfers mitigate both the instability caused by an austerity policy and an increase in capital mobility in the spending and recipient countries in the fiscal transfer mechanism.

Key words: Keynesian and Kaldorian two–country models, Macroeconomic stability, Fiscal union, Capital Markets Union, Post-euro crisis

1 Introduction

The euro crisis of early 2010 was tackled by the monetary policies of the European Central Bank (ECB), such as Long Term Refinancing Operations and Outright Monetary Transactions. However, certain problems persist in the post–euro crisis era, such as the gaps between core and peripheral countries. The average growth rate in real GDP from 2014 to 2016 in Germany, which belongs to the core group, is 1.6%, while it is 0.04% in Greece and 0.5% in Italy, both belonging to the peripheral groups.1) De Grauwe and Ji (2016) estimated the long–term trend in GDP and showed that a decline in the long–term GDP growth rate is particularly prominent in Greece, Ireland, Finland, Spain, Portugal, and Italy. Belke, Domnick, and Gros

Received 16 December 2016, Accepted 25 April 2017, Released online in J-STAGE as advance publication 20 June 2017

* An earlier version of this paper was presented at the 75th Annual Meeting of the Japan Society of International Economics at Chukyo University, Nagoya, Japan, 30 October, 2016. The author would like to thank Prof. Sadayoshi Takaya (Kansai University) for helpful comments at the presentation and anonymous reviewers for insightful comments on this paper. Needless to say, possible remaining errors are solely the responsibility of the author.
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1) It is generally considered in previous studies with regard to the euro area economy that even countries with large economies, such as Spain and Italy, belong to peripheral group (e.g. Belke, Domnick, and Gros (2016)).
(2016) suggested that the core countries increased synchronization among themselves after Q4 2007, whereas the peripheral countries decreased synchronization with regards to the core countries. It is important to adopt expansionary fiscal and monetary policies in the peripheral countries to solve the core-periphery problem. However, it is difficult to adopt an expansionary fiscal policy by peripheral countries alone because they are required to obey fiscal discipline and implement austerity policies. Therefore, it is important to construct a fiscal union with a system of fiscal transfers in the euro area.

The importance of fiscal union in a monetary union has been indicated by many economists in relation to the optimum currency area theory. Kenen (1969) suggested that a supranational fiscal transfer system redistributes funds to a country affected by a negative shock and facilitates the adjustment of such a shock in a monetary union. Krugman (2012) indicated that full integration, including a fiscal union, can automatically compensate for troubled regions. De Graauwe (2016) explained the necessity of a common debt in the euro area as a similar system to a budgetary union. As in the case of expansionary fiscal policy, however, it is difficult to construct a fiscal union.

In this situation, the European Commission (EC) proposed the concept of a Capital Markets Union (CMU) in February 2015 to enlarge the European capital market, which is smaller than the market in the United States (US). In other words, the EC intends to increase private capital mobility in the euro area. However, increasing global capital mobility was a contributing factor to the collapse of the Bretton Woods system with fixed exchange rates. Furthermore, private investment from the core countries to the peripheral countries led to a boom, like the housing bubble that led to the global financial crisis and the euro crisis. In this context, it is possible that the CMU would trigger more problems. Therefore, it is important to consider the impact of a CMU along with a fiscal union in the post-euro crisis era for economic stability. Many explanations about the CMU have been provided to consider the institutional framework. By contrast, no theoretical explanation has been provided to suggest the macroeconomic effect of a CMU using the post-Keynesian two-country model.

In this paper, we investigate the effect of a fiscal union in the euro area in relation to a CMU in the post-euro crisis era in Keynesian and Kaldorian two-country models with a monetary union and imperfect capital mobility. The Mundell–Fleming model developed by Mundell (1963) and Fleming (1962) is useful for analyzing the international economy. The textbook Mundell–Fleming model is premised on the condition of a small country and/or perfect capital mobility.2) However, perfect capital mobility is an unrealistic assumption in general and in the euro area in particular. On this point, we analyze a fiscal union in relation to a CMU in the post-euro crisis era using a Keynesian and Kaldorian two-country model. Kawai (1994) explained the effect of monetary policy and fiscal policy with flexible and fixed exchange rates in a Keynesian two-country model; however, capital mobility is not included in that model. Asada (1997) described an economy with both fixed and flexible exchange rates using a Kaldorian model with a small country and imperfect capital mobility. A Kaldorian model that is due to Kaldor (1940) is a kind of intermediate Keynesian model with variable

2) For example, Asada (2016b) analyzed an international economy of a small country with perfect capital mobility and of a small country with imperfect capital mobility. Romer (2006) explained an economy of a small country with imperfect capital mobility.
capital stocks. Asada (2016a) analyzed an economy with a flexible exchange rate using a Keynesian model with a large country and imperfect capital mobility with a three-dimensional system of nonlinear differential equations. Asada (2004) demonstrated an economy with a fixed exchange rate using a Kaldorian two-country model with a five-dimensional system of nonlinear differential equations. A Kaldorian model (Kaldor (1940)) is an intermediate Keynesian model with variable capital stocks. Asada, Chiarella, Flaschel, and Franke (2003) described the international economy using a Keynes–Metzler–Goodwin dynamic model with a multi-dimensional system of nonlinear differential equations.

Although there are many studies with two-country models as described above, few studies focus on the economy of the current euro area and an economy in a fiscal union in a two-country model. This study primarily aims to analyze the effect of a fiscal union in the euro area in relation to a CMU using two models: (1) a Keynesian two-country model with a monetary union and imperfect capital mobility comprising of a three-dimensional system of nonlinear differential equations in the short-run which is based on Asada (2016a) and (2) a five-dimensional Kaldorian model in the medium-run which is based on Asada (2004). On the basis of this analysis, we suggest the adoption of a counter-cyclical fiscal policy and the construction of a fiscal union in order to create the successful Capital Markets Union. In addition, we use numerical simulation to consider how much the equilibrium national incomes of two countries are affected by the fiscal union.

This paper is organized as follows. In Section 2, we formalize the Keynesian short-run model with fixed capital stocks, which consists of a three-dimensional system of nonlinear differential equations. In Section 3, we analyze the nature of the equilibrium solution of the model formulated in Section 2. In Section 4, we investigate the conditions for local stability of the equilibrium point. In Section 5, we formalize a model in the case of implementation of a fiscal union. In Section 6, we analyze the nature of the equilibrium solution of the model formulated in Section 5. In Section 7, we formalize the Kaldorian medium-run model with variable capital stocks that comprise of a five-dimensional system of nonlinear differential equations and analyze the nature of equilibrium solution of this model. In Section 8, we present the results of some numerical simulations that support the theoretical analysis in Section 7. In Section 9, we discuss the results of models and simulations, offer policy advice, and discuss some problems. Section 10 concludes the study.

2 Formulation of the Keynesian Short-Run Two-Country Model: Current Euro Area Model

In this section, we shall consider the euro area economy using a Keynesian two-country model with a monetary union and imperfect capital mobility, based on Asada (2016a) that examined an economy with a flexible exchange rate using a Keynesian model with a large country and imperfect capital mobility with a three-dimensional system of nonlinear differential equations.

To analyze a monetary union like the euro area, we represent the exchange rate as follows.

$$E = E^* = E = 1,$$  \hspace{1cm} (1)
where $E$ is the exchange rate and $E^e$ is the expected exchange rate of the near future. We can assume that the exchange rate and the expected exchange rate are one because a single currency is used in a monetary union.

Under this assumption about the exchange rate, our model consists of the following system of dynamic equations in the short-run with fixed capital stocks.\(^3\)

(i) Behavioral equations
\[
\dot{Y}_i = \alpha_i [C_i + I_i + G_i + J_i - Y_i] ; \quad \alpha_i > 0 ,
\]
\[
C_i = c_i (Y_i - T_i) + C_{0i} ; \quad 0 < c_i < 1 , \quad C_{0i} \geq 0 ,
\]
\[
T_i = \tau_i Y_i - T_{0i} ; \quad 0 < \tau_i < 1 , \quad T_{0i} \geq 0 ,
\]
\[
I_i = I_i (Y_i, \; \eta_i) ; \quad I_{Yi} = \frac{\partial I_i}{\partial Y_i} > 0 , \quad I_{\eta_i} = \frac{\partial I_i}{\partial \eta_i} < 0 ,
\]
\[
G_i = G_{0i} + \gamma_i (Y_i - \bar{Y}_i) ; \quad \gamma_i > 0 ,
\]
\[
\frac{M_i}{p_i} = L_i (Y_i, \; \eta_i) ; \quad \frac{\partial L_i}{\partial Y_i} > 0 , \quad L_{\eta_i} = \frac{\partial L_i}{\partial \eta_i} < 0 ,
\]
\[
J_i = J_i (Y_1, \; Y_2) ; \quad J_{Y1} = \frac{\partial J_i}{\partial Y_1} < 0 , \quad J_{Y2} = \frac{\partial J_i}{\partial Y_2} > 0 ,
\]
\[
Q_i = \beta \{ n - r_i \} ; \quad \beta > 0 .
\]

(ii) Definitional equations
\[
A_i = J_i + Q_i ,
\]
\[
p_1 J_1 + p_2 J_2 = 0 ,
\]
\[
p_1 Q_1 + p_2 Q_2 = 0 ,
\]
\[
p_1 A_1 + p_2 A_2 = 0 ,
\]
\[
\dot{M}_i = A_i ,
\]
\[
\dot{M} = M_1 + M_2 ,
\]

where the subscript $i$ ($i = 1, 2$) is the index number of a country, and the definitions of other symbols are as follows: $Y_i$ is the real net national income, $C_i$ is the real private consumption expenditure, $I_i$ is the real net private investment expenditure, $G_i$ is the real government expenditure, $\bar{Y}_i$ is the level of real national income that a government determine the counter-cyclical government expenditure (this is not necessarily the natural output), $T_i$ is the real income tax, $T_{0i}$ is the negative income tax (basic income), $M_i$ is the nominal money supply, $p_i$ is the price level, $r_i$ is the nominal rate of interest,\(^4\) $J_i$ is the real net export, $Q_i$ is the real capital account balance.

\(^3\) In the post-Keynesian model used in this paper, there is no need to show a boundary condition and initial value. The reason is that there is no need to choose the converging initial value because the initial value is historically given; therefore, there is no need to set a boundary condition unlike the new-Keynesian Dynamic Stochastic General Equilibrium model (Woodford (2003); Galí (2008)).

\(^4\) In this paper, for the sake of simplicity, public bonds and a stocks are treated as perfect substitute goods. In considering the Capital Markets Union more accurately, it needs to be introduced such as
and $A_i$ is the real total balance of payments. The dots above the symbols represent derivatives with respect to time.

Eq. (2) is the disequilibrium quantity adjustment process in the goods market. The parameter $\alpha_i$ represents adjustment speed of goods market; Eq. (3) is the Keynesian consumption function indicating the behavior of the consumer; Eq. (4) is the standard tax function; Eq. (5) is the standard Keynesian investment function; Eq. (6) is the government expenditure function. The parameter $\gamma_i$ represents the degree of counter-cyclical fiscal policy. The larger $\gamma_i$, the larger is the counter-cyclical government expenditure; Eq. (7) is the LM equation that represents the equilibrium condition in the monetary market; Eq. (8) is the real net export function of country 1; Eq. (9) is the real capital account balance function of country 1 in the model with imperfect capital mobility. The parameter $\beta$ indicates the degree of mobility of international capital flows. The larger $\beta$, the higher is the degree of mobility of international capital flows. The model of perfect capital mobility is a special case in which $\beta$ is infinite, and the following equation is always established in a case of the fixed exchange rate system: $r_1 = r_2$; Eq. (10) is the definitional equation of the real total balance of payments of country 1; Eqs. (11), (12) and (13) imply that net export surplus, capital account balance surplus and the total balance of payments surplus of a country must be accompanied by the same amounts of the current account deficit, capital account balance deficit, and the total balance of payments deficit of another country, respectively; Eq. (14) means that nominal money supply of country 1 increases (decreases) according to the total balance of payment surplus (deficit) of country 1; and Eq. (15) indicates that a total nominal money supply of two countries are fixed by the ECB.

Furthermore, we assume a fixed price economy.

$$p_1 = p_2 = 1.$$  \hspace{1cm} (16)

In this paper, to simplify the analysis, we focus on a fixed price economy in the short-run. This assumption eliminates price fluctuations. Therefore, we do not deal with the issues of inflation and deflation.

Fixing real government expenditure $G_{0i}$, marginal tax rate $\tau_i$, and the total nominal money supply of two countries $M$ as policy parameters, the system of Eqs. (2)-(16) determines the dynamics of $Y_i, C_i, T_i, I_i, J_i, Q_i, A_i, M_i,$ and $p_i (i = 1, 2)$.

Then, we can transform this system into a more compact system. We obtain the following LM equation by solving Eq. (7) with respect to $r_i$.

$$r_i = n_i(Y_i, M_i), \quad r_i \frac{\partial n_i}{\partial Y_i} = - \frac{L_i}{L_i}, \quad r_i \frac{\partial n_i}{\partial M_i} = \frac{1}{L_i} < 0. \quad (17)$$

Given policy parameters $M, G_i$ and $\tau_i (i = 1, 2)$, we can obtain the following three-dimensional system of nonlinear differential equations by substituting Eqs. (3), (4), (5), (6), (8), (11), (15), (16) and (17) into Eq. (2), and Eqs. (8), (9), (10), (15), (16), and (17) into Eq. (14).

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Tobin three-asset model.
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\[ \dot{Y}_1 = \alpha_1 \{ c_1(1 - \tau_1)Y_1 + C_{01} + c_1T_{01} + G_{01} + \gamma_1(\dot{Y}_1 - Y_1) + I_1(Y_1, r_1(Y_1, M_1)) \]
\[ + J_1(Y_1, Y_2) - Y_1 \} \]
\[ = F_1(Y_1, Y_2, M_1; \alpha_1, \gamma_1), \quad (18) \]
\[ \dot{Y}_2 = \alpha_2 \{ c_2(1 - \tau_2)Y_2 + C_{02} + c_2T_{02} + G_{02} + \gamma_2(\dot{Y}_2 - Y_2) + I_2(Y_2, r_2(Y_2, M - M_1)) \]
\[ - J_2(Y_1, Y_2) - Y_2 \} \]
\[ = F_2(Y_1, Y_2, M_1; \alpha_2, \gamma_2), \quad (19) \]
\[ \dot{M}_1 = J_1(Y_1, Y_2) + \beta \{ r_1(Y_1, M_1) - r_2(Y_2, M - M_1) \} = F_3(Y_1, Y_2, M_1; \beta). \quad (20) \]

3 Nature of the Equilibrium Solution: Current Euro Area Model

The equilibrium solution \((Y_1^*, Y_2^*, M_1^*)\) of the system Eqs. (18)-(20) is determined by the following system of equations.

\[ c_1(1 - \tau_1)Y_1 + C_{01} + c_1T_{01} + G_{01} + \gamma_1(\dot{Y}_1 - Y_1) + I_1(Y_1, r_1(Y_1, M_1)) + J_1(Y_1, Y_2) - Y_1 = 0, \quad (21) \]

\[ c_2(1 - \tau_2)Y_2 + C_{02} + c_2T_{02} + G_{02} + \gamma_2(\dot{Y}_2 - Y_2) + I_2(Y_2, r_2(Y_2, M - M_1)) - J_1(Y_1, Y_2) - Y_2 = 0, \quad (22) \]

\[ J_1(Y_1, Y_2) + \beta \{ r_1(Y_1, M_1) - r_2(Y_2, M - M_1) \} = 0. \quad (23) \]

Under the assumption that \(\beta\) is sufficiently small, we obtain the following equations by solving Eq. (23) with respect to \(M_1\).

\[ M_1 = \dot{M}_1(Y_1, Y_2, M); \quad \dot{M}_1 = \frac{\partial \dot{M}_1}{\partial Y_1} = -(j_{h1} + \beta r_1^{(h)}) r_1^{(h)} > 0, \]

\[ \dot{M}_1 = \frac{\partial \dot{M}_1}{\partial Y_2} = -(j_{h2} - \beta r_2^{(h)}) r_1^{(h)} + r_2^{(h)} < 0, \]

\[ \dot{M}_1 = \frac{\partial \dot{M}_1}{\partial M} = r_2^{(M - M_1)}(r_1^{(h)} + r_2^{(h)}) > 0. \quad (24) \]

Then, we have the following functions \(U_1\) and \(U_2\) by substituting Eq. (24) into Eqs. (21) and (22).

\[ U_1(Y_1, Y_2) = c_1(1 - \tau_1)Y_1 + C_{01} + c_1T_{01} + G_{01} + \gamma_1(\dot{Y}_1 - Y_1) \]
\[ + I_1(Y_1, r(Y_1, M_1(Y_1, Y_2, M_1)) + J_1(Y_1, Y_2) - Y_1 \]
\[ = 0, \quad (25) \]
\[ U_2(Y_1, Y_2) = c_2(1 - \tau_2) Y_2 + C_{02} + c_2 T_{02} + G_{02} + \gamma_2(Y_2 - Y_1) \]
\[ + I_2(Y_1, r_2(Y_2, \bar{M} - \bar{M}(Y_1, Y_2, \bar{M}))) - J_1(Y_1, Y_2) - Y_2 \]
\[ = 0. \]  
(26)

where \( U_i \) is the function of \( Y_1 \) and \( Y_2 \) in the case of \( M_1 = \bar{M}_1 \).

Totally differentiating Eq. (25) and (26), we have the following relationships.

\[
\frac{dY_2}{dY_1} \bigg|_{Y_2 = 0} = -\frac{U_{11}}{U_{12}} = \left\{ \begin{array}{l}
1 - c_1(1 - \tau_1) - I_{1h} - I_{1r}^1 \bar{r}_{1h}^\prime - I_{1r}^1 \bar{r}_{1d}^\prime \bar{M}_1 - J_1^h + \gamma_1 \\
(+) 
\end{array} \right. \\
\left( -)(-)(+)(-) \right) \\
\left( -)(-)(+)(+) \right)
\]  
(27)

\[
\frac{dY_2}{dY_1} \bigg|_{Y_2 = 0} = -\frac{U_{21}}{U_{22}} = \left\{ \begin{array}{l}
I_{2h} r_{2d}^2 \bar{M}_1 - J_2^h \\
(+) 
\end{array} \right. \\
\left( -)(-)(-) \right) \\
\left( + \right)
\]  
(28)

where \( U_{11} = \partial U_i/\partial Y_1 = -\{1 - c_1(1 - \tau_1)\} + I_{1h} + I_{1r}^1 \bar{r}_{1h}^\prime + J_1^h - \gamma_1 \), \( U_{12} = \partial U_i/\partial Y_2 = J_1^h \), \( U_{21} = \partial U_2/\partial Y_1 = J_2^h \), \( U_{22} = \partial U_2/\partial Y_2 = -\{1 - c_2(1 - \tau_2)\} + I_{2h} + I_{2r}^2 \bar{r}_{2h}^\prime + J_2^h - \gamma_2 \).

Now, let us assume as follows.

**Assumption 1.**

\[
\frac{1 - c_1(1 - \tau_1)}{(+) - I_{1h} - I_{1r}^1 \bar{r}_{1h}^\prime - I_{1r}^1 \bar{r}_{1d}^\prime \bar{M}_1 - J_1^h + \gamma_1 < 0,}
\]

\[
\frac{1 - c_2(1 - \tau_2)}{(+) - I_{2h} - I_{2r}^2 \bar{r}_{2h}^\prime - I_{2r}^2 \bar{r}_{2d}^\prime \bar{M}_1 - J_2^h + \gamma_2 < 0.}
\]

**Assumption 2.**

\[
I_{1r}^1 \bar{r}_{1d}^\prime \bar{M}_1 + J_1^h < 0, \\
(+-)(-) (+) 
\]

\[
I_{2r}^2 \bar{r}_{2d}^\prime \bar{M}_1 - J_2^h > 0. \\
(+-)(-) (-)
\]

**Remark 1.**

Assumptions 1 and 2 mean that the absolute value of a negative term \((-I_{1h}\)) is relatively larger than the value of positive terms. Assumption 2 means that the sensitivity of export with respect to the change of national income is sufficiently large in country 1 and it is sufficiently
small in country 2. Thus, Assumptions 1 and 2 automatically imply that we have the following inequality:

\[
\frac{dY_2}{dY_1} \bigg|_{U_2=0} > 0,
\]

\[
\frac{dY_2}{dY_1} \bigg|_{U_2=0} < 0.
\]

Using Assumption 1, we can obtain equilibrium national incomes \((Y_1^*, Y_2^*)\) from the point on the plane \((Y_1, Y_2)\) at the intersection of \(U_1(Y_1, Y_2) = 0\) with \(U_2(Y_1, Y_2) = 0\), given the fiscal policy parameters \((G_{01}, \tau_1, \gamma_1, G_{02}, \tau_2, \text{and } \gamma_2)\), and the monetary policy parameter \((M)\). Further, substituting \((Y_1^*, Y_2^*)\) into Eq. (23), we have the equilibrium money supply of country 1 \((M_1^*)\).

4 Local Stability Analysis: Current Euro Area Model

In this section, we shall assume that there exists a unique equilibrium solution \((Y_1^*, Y_2^*, M_1^*) > (0, 0, 0)\) and analyze the local stability of this equilibrium solution. We can write the Jacobian matrix of the system of Eqs. (18)–(20) that are evaluated at the equilibrium point as follows:

\[
J = \begin{bmatrix}
F_{11} & F_{12} & F_{13} \\
F_{21} & F_{22} & F_{23} \\
F_{31} & F_{32} & F_{33}
\end{bmatrix} = \begin{bmatrix}
\alpha_1 \Gamma_{11} & \alpha_1 \Gamma_{12} & \alpha_1 \Gamma_{13} \\
\alpha_2 \Gamma_{21} & \alpha_2 \Gamma_{22} & \alpha_2 \Gamma_{23} \\
F_{31}(\beta) & F_{32}(\beta) & F_{33}(\beta)
\end{bmatrix},
\]

where

\[
\Gamma_{11} = -\left\{1 - c_0(1 - \tau_1)\right\} + I_{1h}^0 - I_{1h}^1 r_{1h} + J_{1h}^1 - \gamma_1, \quad \Gamma_{12} = J_{1h}^2 > 0, \quad \Gamma_{13} = I_{1h}^2 r_{1h} > 0,
\]

\[
\Gamma_{21} = -J_{2h}^1 > 0, \quad \Gamma_{22} = -\left\{1 - c_0(1 - \tau_2)\right\} + I_{2h}^0 + I_{2h}^2 r_{2h}^2 - J_{2h}^2 - \gamma_2, \quad \Gamma_{23} = -I_{2h}^2 r_{2h}^2 < 0,
\]

\[
F_{31}(\beta) = J_{h1}^1 + \beta r_{1h}^1, \quad F_{32}(\beta) = J_{h2}^1 - \beta r_{2h}^2, \quad F_{33}(\beta) = \beta (r_{1h}^2 + r_{2h}^2 - M_{h1} - M_{h2}) < 0.
\]

We can express the characteristic equation of this system as follows.

\[
f(\lambda) = |\lambda I - J| = \lambda^3 + a_1 \lambda^2 + a_2 \lambda + a_3 = 0,
\]

where

\[
a_1 = -\text{trace}J = -\alpha_1 \Gamma_{11} - \alpha_2 \Gamma_{22} - F_3(\beta) = a(\beta),
\]

\[
a_2 = \text{trace}^2 J - \text{trace}J^2 = \alpha_1 \text{trace} \Gamma_{11} + \alpha_2 \text{trace} \Gamma_{22} + \text{trace} F_3(\beta) = a(\beta).
\]

\[
a_3 = -\text{det} J = -\alpha_1 \text{det} \Gamma_{11} - \alpha_2 \text{det} \Gamma_{22} - \text{det} F_3(\beta) = a(\beta).
\]
\[ a_2 = \alpha_1 \alpha_2 \begin{vmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{vmatrix} + \alpha_1 \begin{vmatrix} \Gamma_{11} & \Gamma_{13} \\ F_3(\beta) & F_3(\beta) \end{vmatrix} + \alpha_2 \begin{vmatrix} \Gamma_{22} & \Gamma_{23} \\ F_3(\beta) & F_3(\beta) \end{vmatrix} \]
\[ = \alpha_1 \alpha_2 (\Gamma_{11} \Gamma_{22} - \Gamma_{12} \Gamma_{21}) + \alpha_1 (\Gamma_{11} F_3(\beta) - \Gamma_{13} F_3(\beta)) + \alpha_2 (\Gamma_{22} F_3(\beta) - \Gamma_{23} F_3(\beta)) \]
\[ = a_2(\beta), \quad (32) \]

\[ a_3 = -\det J = - \begin{vmatrix} \alpha_1 \Gamma_{11} & \alpha_1 \Gamma_{12} & \alpha_1 \Gamma_{13} \\ \alpha_2 \Gamma_{21} & \alpha_2 \Gamma_{22} & \alpha_2 \Gamma_{23} \\ F_3(\beta) & F_3(\beta) & F_3(\beta) \end{vmatrix} \]
\[ = \alpha_1 \alpha_2 \left[ -\Gamma_{11} \Gamma_{22} F_3(\beta) + \Gamma_{11} F_3(\beta) \Gamma_{23} - \Gamma_{12} \Gamma_{23} F_3(\beta) + \Gamma_{12} \Gamma_{23} F_3(\beta) \right] \]
\[ - \Gamma_{13} F_3(\beta) \Gamma_{21} + \Gamma_{13} \Gamma_{22} F_3(\beta) \]
\[ = a_3(\beta), \quad (33) \]
\[ a_1 a_2 - a_3 = a_1(\beta) a_2(\beta) - a_3(\beta). \quad (34) \]

All the roots of the characteristic equation (30) have negative real parts if and only if the following Routh-Hurwitz conditions are satisfied:
\[ a_1 > 0, \ a_2 > 0, \ a_3 > 0, \ a_1 a_2 - a_3 > 0. \quad (35) \]

Then, the equilibrium point of the system (18)-(20) is locally stable.

**Proposition 1.**

(i) Suppose that the parameter \( \beta \) is fixed at any level. Then, the equilibrium point of the system (18)-(20) is locally stable if at least one of the parameters \( \gamma_1 \) and \( \gamma_2 \) is sufficiently large.

(ii) Suppose that the parameters \( \gamma_1 \) and \( \gamma_2 \) are fixed at any level. Then, the equilibrium point of the system (18)-(20) is locally unstable if the parameter \( \beta \) is sufficiently large.

(proof.) See Appendix A.

Proposition 1 indicates that an increase in capital mobility in a monetary union leads to instability. Asada (1997) described the effect of the parameter \( \beta \) on an economics stability using a Keynesian model with a small country. Similarly, Proposition 1 can be interpreted as follows:

5) Gandolfo (2009) pp. 229–240.
(i) Sufficiently small $\beta$: stabilizing factor

\[ Y_1 \downarrow \rightarrow J_1 \uparrow \rightarrow A_1 \uparrow \rightarrow M_1 \uparrow \rightarrow r_1 \downarrow \rightarrow Y_1 \uparrow \]

\[ r_1 \downarrow \rightarrow Y_1 \uparrow \]

(ii) Sufficiently large $\beta$: destabilizing factor

\[ Y_1 \downarrow \rightarrow r_1 \downarrow \rightarrow r_1 < r_2 \rightarrow Q_1 \downarrow \rightarrow M_1 \downarrow \rightarrow r_1 \uparrow \rightarrow Y_1 \downarrow \]

\[ Y_1 \uparrow \]

First, suppose that the parameter $\beta$ is sufficiently small. If $Y_1$ falls below the equilibrium point by an exogenous shock, a decrease in $Y_1$ leads to a decrease in $r_1$. A decrease in $r_1$ brings about an increase in $Y_1$. By contrast, a decrease in $Y_1$ leads to an increase in real net exports $J_1$ through a decrease in imports. An increase in $J_1$ brings about an increase in $M_1$. If the parameter $\beta$ is small, this “current account effect” is relatively strong compared with the counteracting “capital account effect”. Then, an increase in $M_1$ leads to an increase in $I_1$ and $Y_1$ through a decrease in $r_1$ if the parameter $\beta$ is sufficiently small. Thus, a small $\beta$ is a stabilizing factor in a monetary union.\(^6\)

Second, suppose that the parameter $\beta$ is sufficiently large. If real national income of country 1, $Y_1$, falls below the equilibrium point by an exogenous shock, a decrease in $Y_1$ leads to a decrease in nominal rate of interest $r_1$ and leads to a small $r_1$ relative to $r_2$. An increase in capital transfer to country 2 from country 1 brings about a deficit of capital account balance $Q_1$ and a decrease in money supply $M_1$. If the parameter $\beta$ is large, this “capital account effect” is relatively strong compared with the counteracting “current account effect”. However, a decrease in $M_1$ leads to a rapid increase in $r_1$. An increase in $r_1$ restrains real private investment expenditure $I_1$. Then, a decrease in an investment in country 1 leads to a further decrease in $Y_1$. By contrast, a decrease in $r_1$ brings about an increase in $Y_1$. However, the former effect on $Y_1$ is larger than the latter if the parameter $\beta$ is sufficiently large. Thus, a large $\beta$ is a destabilizing factor in a monetary union.

In the case of local instability, a monetary union experiences extreme boom and depression repeatedly. This extreme business cycle is a significant problem in the euro area, especially because certain political factors influence the European Monetary Union (EMU). There is a possibility that the cycle leads to a political cycle and a danger that the EMU would collapse. It is necessary to make the parameter $\gamma_1$ large if policymakers intend to increase capital mobility within the area while maintaining stability and monetary union. This result is contrary to that by Ingram (1973), who stressed the increase of the degree of financial integration as a condition for creating an optimum currency area. An integration of currencies presup-

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6) It is important to note that these mechanisms work under a fixed exchange rate system. In the case of a flexible exchange rate system, a large $\beta$ may have a stabilizing effect (see Asada (1997) and Asada (2016a)).
poses a liberalization of capital mobility in the area. However, free capital mobility for private market participants destabilizes an economy in a monetary union in the absence of the support provided by government expenditure.

Given this theoretical background, the EC intends to increase capital mobility. Figure 1 shows stock traded in the euro area, the European Union (EU), Japan, the United States, and China. It is clear that capital is not as mobile in the euro area as it is in the US because there are barriers of history, culture, and regulation in the euro area. To resolve these problems, the EC proposed the creation of the CMU in February 2015. The EC intends to construct the CMU according to the following principles7):

1. It should maximize the benefits of capital markets for the economy, job creation, and growth;
2. It should create a single market for capital for all 28 member states by removing barriers to cross-border investment within the EU and fostering stronger connections with global capital markets;
3. It should be built on firm foundations of financial stability, with a single rulebook for financial services which is effectively and consistently enforced;
4. It should ensure an effective level of consumer and investor protection; and
5. It should help to attract investment from all over the world and increase EU competitiveness.

However, the CMU will lead to an increase in the parameter $\beta$. The parameter $\gamma_i$ is small in the euro area because each country in the euro area tends to adopt fiscal austerity policies. In this state, creating the CMU is likely to increase instability in the euro area. Thus, if the EC creates the CMU, each country in the euro area must not adopt fiscal austerity policies but adopt a counter-cyclical expansionary fiscal policy. However, it is difficult to adopt the expansionary fiscal policy in each country independently, because periphery countries such as

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7) European Commission (2015) p. 5.
Greece are required to obey the fiscal discipline. To solve this problem, it is important to create a fiscal union.

5 Formulation of the Keynesian Short-Run Two-Country Model: Fiscal Union Model

In this section, we analyze the stability of the equilibrium point in the case of the creation of a fiscal union. The fiscal union should have a mechanism that triggers fiscal transfers from countries experiencing expansions to countries experiencing depressions. Kenen (1969) proposed fiscal transfers as a condition of the optimum currency area. Furthermore, De Grauwe (2016) presented two points about the effect of the fiscal union. First, the fiscal union creates an insurance mechanism that trigger income transfers from countries experiencing expansions to countries experiencing depressions. In doing so, it mitigates the effects of depression in the countries hit by a negative shock. Second, a fiscal union allows consolidation of a significant part of the debts of the national government, thereby, protecting its members from liquidity crises and forced defaults.

In this paper, to focus on fiscal transfers as an insurance mechanism in a fiscal union, we add equations about the fiscal transfer mechanism to Eq. (6) and formulate the system as follows:

\[ G_1 = G_{01} + \gamma_1 (\bar{Y}_1 - Y_1) + \mu (\bar{Y}_f - Y_1); \quad \gamma_1 > 0; \quad \mu > 0, \]  
\[ G_2 = G_{02} + \gamma_2 (\bar{Y}_2 - Y_2) - \mu (\bar{Y}_f - Y_1); \quad \gamma_2 > 0; \quad \mu > 0, \]  

where the parameter $\mu$ is the degree of the fiscal transfers and $\bar{Y}_f$ is the level of real national income for which a government in a fiscal union determine fiscal transfers. We assume lop-sided fiscal transfers to peripheral countries from the core countries. In other words, these equations indicate that country 2 (core country) increases government expenditure to make transfers to country 1 (peripheral country) in the event of a depression country 1 experiencing depression ($\bar{Y}_f > Y_1$), whereas country 1 does not make transfers to country 2 in the event of a depression in country 2.

By adopting an approach similar to the one adopted for developing the expression for Eqs. (18)-(20), we formulate the system as follows:

\[ \dot{Y}_1 = \alpha_1 \{c_1 (1 - \tau_1) Y_1 + C_{01} + c_1 T_{01} + G_{01} + \gamma_1 (\bar{Y}_1 - Y_1) + \mu (\bar{Y}_f - Y_1) \]  
\[ + J_1 (Y_1, r_1 (Y_1, M_1)) + J_1 (Y_1, Y_2) - Y_1 \} = H_1 (Y_1, Y_2, M_1; \alpha_1, \gamma_1, \mu), \] (38)

\[ \dot{Y}_2 = \alpha_2 \{c_2 (1 - \tau_2) Y_2 + C_{02} + c_2 T_{02} + G_{02} + \gamma_2 (\bar{Y}_2 - Y_2) - \mu (\bar{Y}_f - Y_1) \]  
\[ + J_2 (Y_2, r_2 (Y_2, M - M_1)) - J_1 (Y_1, Y_2) - Y_2 \} = H_2 (Y_1, Y_2, M_1; \alpha_2, \gamma_2, \mu), \] (39)

\[ \dot{M}_1 = J_1 (Y_1, Y_2) + \beta (r_1 (Y_1, M_1) - r_2 (Y_2, M - M_1)) = H_3 (Y_1, Y_2, M_1; \beta). \] (40)

We investigate a nature of the equilibrium solution ($Y_1^*, Y_2^*, M_1^*$) that satisfies

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8) De Grauwe (2016) p. 17.
\( \dot{Y}_1 = \dot{Y}_2 = \dot{M}_1 = 0 \). The equilibrium solution of Eqs. (38)-(40) is determined as a solution of the following simultaneous equations.

\[
\begin{align*}
  c_1(1 - \tau_1)Y_1 + C_{01} + c_1 T_{01} + G_{01} + \gamma_1(\dot{Y}_1 - Y_1) + \mu(\dot{Y}_1 - Y_1) + I_1(1, r_1(Y_1, M_1)) \\
  + J_1(Y_1, Y_2) - Y_1 = 0, \\
  c_2(1 - \tau_2)Y_2 + C_{02} + c_2 T_{02} + G_{02} + \gamma_2(\dot{Y}_2 - Y_2) - \mu(\dot{Y}_2 - Y_2) + I_2(2, r_2(Y_2, M - M_1)) \\
  - J_1(Y_1, Y_2) - Y_2 = 0, \\
  J_1(Y_1, Y_2) + \beta(r_1(Y_1, M_1) - r_2(Y_2, M - M_1)) = 0.
\end{align*}
\]

However, we leave out developing the equilibrium solution, because we can obtain the equilibrium solution in the case of a fiscal union using an approach similar to that used to obtain the equilibrium solution \((Y_1^*, Y_2^*, M_1^*)\) in Section 3.

6 Local Stability Analysis: Fiscal Union Model

In this section, we shall assume that there exists a unique equilibrium solution \((Y_1^*, Y_2^*, M_1^*) > (0, 0, 0)\) in the case of a fiscal union and analyze the local stability of this equilibrium solution. We can write the Jacobian matrix of the system of Eqs. (38)-(40) that are evaluated at the equilibrium point as follows.

\[
J = \begin{bmatrix}
  H_{11} & H_{12} & H_{13} \\
  H_{21} & H_{22} & H_{23} \\
  H_{31} & H_{32} & H_{33}
\end{bmatrix} = \begin{bmatrix}
  \alpha_1 \Theta_{11} & \alpha_1 \Theta_{12} & \alpha_1 \Theta_{13} \\
  \alpha_2 \Theta_{21} & \alpha_2 \Theta_{22} & \alpha_2 \Theta_{23} \\
  H_{31}(\beta) & H_{32}(\beta) & H_{33}(\beta)
\end{bmatrix},
\]

where

\[
\Theta_{11} = -\left\{c_1(1 - \tau_1) + I_{h}^1 + I_{n}^1 + J_{h}^1 - \gamma_1 - \mu\right\} > 0,
\]

\[
\Theta_{12} = -\left\{c_2(1 - \tau_2) + I_{h}^2 + I_{n}^2 + J_{h}^2 - \gamma_2 - \mu\right\} > 0,
\]

\[
\Theta_{13} = -I_{n}^1 r_{m}^1 > 0,
\]

\[
\Theta_{21} = -\left\{c_1(1 - \tau_1) + I_{h}^1 + I_{n}^1 + J_{h}^1 - \gamma_1 - \mu\right\} > 0,
\]

\[
\Theta_{22} = -\left\{c_2(1 - \tau_2) + I_{h}^2 + I_{n}^2 + J_{h}^2 - \gamma_2 - \mu\right\} > 0,
\]

\[
\Theta_{23} = -I_{n}^2 r_{m}^2 > 0
\]

\[
H_{31}(\beta) = J_{h}^1 + \beta r_{m}^1 > 0,
\]

\[
H_{32}(\beta) = -J_{h}^2 - \beta r_{m}^2 > 0,
\]

\[
H_{33}(\beta) = \beta(r_{m}^1 + r_{m}^2) < 0.
\]

We can express the characteristic equation of this system as follows.

\[
f(\lambda) = |\lambda I - J| = \lambda^3 + b_3 \lambda^2 + b_2 \lambda + b_1 = 0,
\]

where

\[
b_1 = -traceJ = -\left\{\alpha_1 \Theta_{11} - \alpha_2 \Theta_{22} - H_{33}(\beta)\right\} = b(\beta),
\]

(46)
\[ b_2 = a_1 a_2 \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{bmatrix} + a_1 \begin{bmatrix} \Theta_{11} & \Theta_{13} \\ H_{31}(\beta) & H_{33}(\beta) \end{bmatrix} + a_2 \begin{bmatrix} \Theta_{22} & \Theta_{23} \\ H_{32}(\beta) & H_{33}(\beta) \end{bmatrix} \]

\[ = a_1 a_2 (\Theta_{11} \Theta_{22} - \Theta_{12} \Theta_{21}) + a_1 (\Theta_{11} H_{33}(\beta) - \Theta_{13} H_{31}(\beta)) + a_2 (\Theta_{22} H_{33}(\beta) - \Theta_{23} H_{32}(\beta)) \]

\[ = b_2(\beta), \quad (47) \]

\[ b_3 = -\det J = - \begin{bmatrix} a_1 \Theta_{11} & a_1 \Theta_{12} & a_1 \Theta_{13} \\ a_2 \Theta_{21} & a_2 \Theta_{22} & a_2 \Theta_{23} \\ H_{31}(\beta) & H_{32}(\beta) & H_{33}(\beta) \end{bmatrix} \]

\[ = a_1 a_2 \left[ -\Theta_{11} \Theta_{22} H_{33}(\beta) + \Theta_{11} H_{32}(\beta) \Theta_{23} - \Theta_{12} \Theta_{23} H_{31}(\beta) + \Theta_{13} \Theta_{21} H_{33}(\beta) \right] \]

\[ = b_3(\beta), \quad (48) \]

\[ b_1 b_2 - b_3 = b_1(\beta) b_2(\beta) - b_3(\beta). \quad (49) \]

All the roots of the characteristic equation (45) have negative real parts if and only if the following Routh–Hurwitz conditions are satisfied.

\[ b_1 > 0, \quad b_2 > 0, \quad b_3 > 0, \quad b_1 b_2 - b_3 > 0. \quad (50) \]

Then, the equilibrium point of the system (38)–(40) is locally stable.

**Proposition 2.**

(i) Suppose that the parameter \( \beta \) is fixed at any level. Then, the equilibrium point of the system (38)–(40) is locally stable if at least one of the parameters \( \gamma_1, \gamma_2 \) and \( \mu \) is sufficiently large.

(ii) Suppose that the parameter \( \gamma_1, \gamma_2, \) and \( \mu \) are fixed at any level. Then, the equilibrium point of the system (38)–(40) is locally unstable if the parameter \( \beta \) is sufficiently large.

(Proof.) See Appendix B.

Proposition 2 indicates that it is necessary to adopt a counter-cyclical fiscal policy and a fiscal transfer mechanism in a fiscal union to mitigate the instability caused by the CMU, as is indicated in Proposition 1. However, the parameter \( \gamma_i \) is small, because the countries in the euro area tend to adopt austerity fiscal policies. Furthermore, the stability effect of the parameter \( \mu \) does not exist due to the absence of a fiscal union.9)

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9) In Appendix C, we consider the effect of monetary policy and fiscal policy in the case of a fiscal union using a comparative statics analysis.
7 Local Stability Analysis: The Kaldorian Medium–Run Two–Country Model with Fiscal Union

In this section, we shall extend the above analysis to the five-dimensional system included variable capital stocks of both countries. The above three-dimensional model is a short-run model with constant capital stocks. However, the five-dimensional model, called the “Kaldorian model,” is a medium–run model that includes capital stocks. A long-term model includes population growth and technical progress, but we are not concerned with those in this model.

Asada (2004) developed a five-dimensional Kaldorian two–country model and considered a relation between $\alpha$ and $\beta$, whereas we consider five-dimensional Kaldorian two–country model with additional parameters $\gamma_i$ and $\mu$ in this section. This analysis with additional parameters enables considerations of the relation between a fiscal union and the Capital Markets Union. We formulate the systems as follows.

\[
\begin{align*}
\dot{Y}_1 &= \alpha_1 \left( c_1(1-\tau_1)Y_1 + C_{01} + c_1T_{01} + G_{01} + \gamma_1(\bar{Y}_1 - Y_1) + \mu(\bar{Y}_f - Y_1) ight) \\
&\quad + I_1(Y_1, K_1, r_1(Y_1, M_1)) + J_1(Y_1, Y_2) - Y_1 \\
&= V_1(Y_1, K_1, Y_2, M_1; \alpha_1, \gamma_1, \mu), \\
\dot{K}_1 &= I_1(Y_1, K_1, r_1(Y_1, M_1)) = V_2(Y_1, K_1, M_1), \\
\dot{Y}_2 &= \alpha_2 \left( c_2(1-\tau_2)Y_2 + C_{02} + c_2T_{02} + G_{02} + \gamma_2(\bar{Y}_2 - Y_2) - \mu(\bar{Y}_f - Y_2) ight) \\
&\quad + I_2(Y_2, r_2(Y_2, M - M_1)) - J_1(Y_1, Y_2) - Y_2 \\
&= V_3(Y_2, K_2, M_1; \alpha_2, \gamma_2, \mu), \\
\dot{K}_2 &= I_2(Y_2, r_2(Y_2, M - M_1)) = V_4(Y_2, K_2, M_1), \\
\dot{M}_1 &= J_1(Y_1, Y_2) + \beta(r_1(Y_1, M_1) - r_2(Y_2, M - M_1)) = V_5(Y_1, Y_2, M_1; \beta),
\end{align*}
\]

where $K_i$ is capital stock of country $i$.\(^{(10)}\)

We investigate a nature of the equilibrium solution $(Y_1^*, K_1^*, Y_2^*, K_2^*, M_1^*)$ of the system (51)–(55). However, we leave out developing the equilibrium solution of Eqs. (51)–(55), because we can obtain the equilibrium solution in the case of the adoption of a counter–cyclical fiscal policy and a fiscal union using an approach similar to the case of Asada (2004).

We assume that there exists a unique equilibrium solution $(Y_1^*, K_1^*, Y_2^*, K_2^*, M_1^*) > (0, 0, 0, 0, 0)$ in the case of a fiscal union and analyze the local stability of this equilibrium solution using the Kaldorian model. We can write the Jacobian matrix of the system of Eqs. (51)–(55) that are evaluated at the equilibrium point as follows.\(^{(11)}\)

\(^{(10)}\) The capital stock $K_i$ is physical capital. In this model, a capital moving internationally is not physical capital but money capital. The money capital mobility is determined by the difference between the interest rates of the two countries, as shown in Eq. (9).

\(^{(11)}\) The private investment at the equilibrium point ($I_e = 0$) is carried out by the amount of capital depreciation.
Macroeconomic Instability of a Capital Markets Union and Stability of a Fiscal Union in the Euro Area: Keynesian and Kaldorian Two-Country Models

\[
J = \begin{bmatrix}
V_{11} & V_{12} & V_{13} & 0 & V_{15} \\
V_{21} & V_{22} & 0 & 0 & V_{25} \\
0 & V_{32} & V_{34} & V_{35} \\
0 & 0 & V_{34} & V_{44} & V_{45} \\
0 & 0 & V_{53} & 0 & V_{55}
\end{bmatrix} = \begin{bmatrix}
\alpha_1 \Phi_{11} & \alpha_1 \Phi_{12} & \alpha_1 \Phi_{13} & 0 & \alpha_1 \Phi_{15} \\
\alpha_2 \Phi_{21} & \alpha_2 \Phi_{22} & 0 & 0 & \alpha_2 \Phi_{25} \\
0 & \alpha_2 \Phi_{31} & 0 & \alpha_2 \Phi_{33} & \alpha_2 \Phi_{34} \\
0 & 0 & \alpha_2 \Phi_{34} & \alpha_2 \Phi_{35} & 0 \\
V_{51}(\beta) & 0 & V_{53}(\beta) & 0 & V_{55}(\beta)
\end{bmatrix},
\]

where

\[
\Phi_{11} = -\left\{1 - c(1 - \tau_1)\right\} + I_h^1 + I_h^1 r_h^1 + J_h^1 - \gamma_1 - \mu, \quad \Phi_{12} = I_h^1 < 0,
\]

\[
\Phi_{13} = J_h^1 > 0, \quad \Phi_{15} = I_h^1 r_{M-h} > 0, \quad V_{21} = I_h^2 + I_h^2 r_h^2, \quad \Phi_{31} = \mu - J_h^2 > 0,
\]

\[
\Phi_{33} = -\left\{1 - c(1 - \tau_2)\right\} + I_h^3 + I_h^3 r_h^3 - J_h^3 - \gamma_2,
\]

\[
\Phi_{34} = I_h^3 < 0, \quad \Phi_{35} = -I_h^3 r_{M-h} < 0, \quad V_{43} = I_h^4 + I_h^4 r_h^4,
\]

\[
V_{51}(\beta) = J_h^1 + \beta r_h^1, \quad V_{53}(\beta) = J_h^2 - \beta r_h^2, \quad V_{53}(\beta) = \beta (r_h^1 + r_h^1 r_{M-h}) < 0.
\]

Now, let us assume as follows:

**Assumption 3.** The following inequality hold because \(I_h\) is sufficiently large at the equilibrium point.

\[
\left(\begin{array}{c}
I_h^1 > 0 \\
I_h^3 > 0 \\
\end{array}\right)
\]

**Remark 2.**

Assumption 3 implies \(V_{21} > 0\) and \(V_{43} > 0\) at the equilibrium point.

The assumption that \(I_h\) is sufficiently large at the equilibrium point is nothing but the standard hypothesis of Kaldorian business cycle model (cf. Kaldor (1940), Asada (1995), Asada (2004), Asada, Inaba and Misawa (2001)).

Further, let us assume as follows for simplicity.

**Assumption 4.**

\[
\gamma_1 = \gamma_2 = \gamma_f.
\]

We can express the characteristic equation of this system as

\[
f(\lambda) = |\lambda I - J| = \lambda^5 + c_1\lambda^4 + c_2\lambda^3 + c_3\lambda^2 + c_4\lambda + b_5 = 0,
\]

where
\[ c_1 = -\text{trace}J = -\alpha_1 \phi_{11} - \phi_{12} - \alpha_2 \phi_{33} - \phi_{34} - V_{55}(\beta), \]

\[ c_2 = \alpha_1 \phi_{11} \phi_{12} + \alpha_1 \alpha_2 \phi_{11} \phi_{13} + \alpha_1 \phi_{11} 0 + \alpha_1 \phi_{11} \phi_{15} + \alpha_2 0 \phi_{33} + \phi_{12} 0 \phi_{34} + \alpha_2 \phi_{12} \phi_{15} + \alpha_2 \phi_{12} \phi_{33} + \alpha_2 \phi_{12} \phi_{34} + \alpha_2 \phi_{12} V_{55}(\beta) \]

\[ + \alpha_2 \phi_{33} \phi_{35} + \phi_{34} \phi_{35} + \alpha_2 \phi_{33} \phi_{35} + \phi_{34} \phi_{35} + \alpha_2 \phi_{33} \phi_{35} + \phi_{34} \phi_{35} + \alpha_2 \phi_{33} \phi_{35} + \phi_{34} \phi_{35} \]

\[ = \alpha_1(\phi_{11} \phi_{12} - \alpha_2 \phi_{12} V_{55}(\beta)) + \alpha_1 \alpha_2(\phi_{11} \phi_{13} - \phi_{13} \phi_{31}) + \alpha_1 \phi_{11} \phi_{34} + \alpha_2 \phi_{12} \phi_{33} + \phi_{12} \phi_{34} + \phi_{12} V_{55}(\beta) \]

\[ + \alpha_2(\phi_{33} \phi_{34} - \phi_{33} \phi_{43}) + \alpha_2(\phi_{33} V_{55}(\beta) - \phi_{33} V_{55}(\beta)) + \phi_{34} V_{55}(\beta), \]

\[ (59) \]

\[ (60) \]
\[ c_3 = -\alpha_1\alpha_2 \left| \begin{array}{ccc} \Phi_{11} & \Phi_{12} & \Phi_{13} \\ V_21 & \Phi_{12} & 0 \\ \Phi_{31} & 0 & \Phi_{33} \end{array} \right| - \alpha_3 \left| \begin{array}{ccc} \Phi_{11} & \Phi_{12} & 0 \\ V_21 & \Phi_{12} & 0 \\ 0 & 0 & \Phi_{34} \end{array} \right| - \alpha_4 \left| \begin{array}{ccc} \Phi_{11} & \Phi_{12} & \Phi_{15} \\ V_51(\beta) & 0 & V_55(\beta) \end{array} \right| \]

\[ -\alpha_1\alpha_2 \left| \begin{array}{ccc} \Phi_{11} & \Phi_{13} & 0 \\ \Phi_{13} & \Phi_{33} & \Phi_{34} \\ 0 & V_{43} & \Phi_{34} \end{array} \right| - \alpha_1\alpha_2 \left| \begin{array}{ccc} \Phi_{11} & \Phi_{13} & \Phi_{15} \\ \Phi_{31} & \Phi_{33} & \Phi_{35} \\ 0 & V_{53}(\beta) & V_{55}(\beta) \end{array} \right| \]

\[ -\alpha_1 \left| \begin{array}{ccc} \Phi_{12} & 0 & \Phi_{15} \\ 0 & \Phi_{34} & \Phi_{35} \\ V_{53}(\beta) & 0 & V_{55}(\beta) \end{array} \right| - \alpha_2 \left| \begin{array}{ccc} \Phi_{12} & 0 & \Phi_{15} \\ 0 & \Phi_{34} & \Phi_{35} \\ 0 & 0 & V_{55}(\beta) \end{array} \right| \]

\[ -\alpha_2 \left| \begin{array}{ccc} \Phi_{33} & \Phi_{34} & \Phi_{35} \\ V_{43} & \Phi_{34} & \Phi_{35} \\ V_{53}(\beta) & 0 & V_{55}(\beta) \end{array} \right| \]

\[ = -\alpha_1\alpha_2 \left( -\Phi_{12} \Phi_{13} \Phi_{31} - V_21 \Phi_{12} \Phi_{33} + \Phi_{11} \Phi_{12} \Phi_{33} - \alpha_1 \Phi_{34}(V_21 \Phi_{12} + \Phi_{14} \Phi_{12}) \right) \]

\[ = -\alpha_1 \left( -V_21V_{55} \Phi_{12} + V_{55} \Phi_{11} \Phi_{12} - \alpha_1 \alpha_2 \left( -V_{43} \Phi_{11} \Phi_{34} - \Phi_{13} \Phi_{31} \Phi_{34} + \Phi_{11} \Phi_{33} \Phi_{34} \right) \right) \]

\[ = -\alpha_1 \left( -V_{53} \Phi_{11} \Phi_{31} + V_{55} \Phi_{15} \Phi_{31} + V_{55} \Phi_{11} \Phi_{33} - V_{51} \Phi_{15} \Phi_{33} - \Phi_{53} \Phi_{11} \Phi_{35} \right) \]

\[ + V_{51} \Phi_{13} \Phi_{35} - \alpha_1(V_{55} \Phi_{11} \Phi_{34} - V_{51} \Phi_{15} \Phi_{34}) - \alpha_2 \Phi_{12} \left( -V_{43} \Phi_{34} - \Phi_{53} \Phi_{34} \right) \]

\[ -\alpha_2 \left( V_{55} \Phi_{12} \Phi_{33} - V_{53} \Phi_{12} \Phi_{35} - V_{55} \Phi_{12} \Phi_{34} - \alpha_2 \left( -\Phi_{13} V_{55} \Phi_{34} + V_{55} \Phi_{15} \Phi_{34} \right), \right) \]

\[ (61) \]
Then, the Routh–Hurwitz terms $\Delta_i$ ($i = 1, 2, ..., 5$) are defined as follows in this five-dimensional case.

\begin{align*}
\Delta_1 &= c_1, \\
\Delta_2 &= \begin{vmatrix}
  c_1 & c_3 \\
  1 & c_2
\end{vmatrix} = c_1 c_2 - c_3,
\end{align*}
\[ \Delta_3 = \begin{vmatrix} c_1 & c_3 & c_5 \\ 1 & c_2 & c_4 \\ 0 & c_1 & c_3 \end{vmatrix} = c_1 c_2 c_3 - c_3^2 - c_1^2 c_4 + c_1 c_5, \] (66)

\[ \Delta_4 = \begin{vmatrix} c_1 & c_3 & c_5 & 0 \\ 1 & c_2 & c_4 & 0 \\ 0 & c_1 & c_3 & c_5 \\ 0 & 1 & c_2 & c_4 \end{vmatrix} = c_4 \Delta_3 + c_5(c_4 c_5 - c_2 \Delta_2), \] (67)

\[ \Delta_5 = \begin{vmatrix} c_1 & c_3 & 0 & 0 \\ 1 & c_2 & c_4 & 0 \\ 0 & c_1 & c_3 & c_5 \\ 0 & 1 & c_2 & c_4 \end{vmatrix} = c_5 \Delta_4. \] (68)

All the roots of the characteristic equation (58) have negative real parts if and only if the following Routh-Hurwitz conditions are satisfied.

\[ \Delta_i > 0 \quad \forall i \in \{1, 2, ..., 5\}. \] (69)

Then, the equilibrium point of the system (51)-(55) is locally stable.

**Proposition 3.**

(i) Suppose that the parameter \( b \) is fixed at any level. Then, the equilibrium point of the system (51)-(55) is locally stable if at least one of the parameters \( c_f \) and \( n \) is sufficiently large.

(ii) Suppose that the parameter \( c_f \) and \( n \) are relatively small and inequalities \( \Phi_{11}>0 \) and \( \Phi_{33}>0 \) hold. Then, the equilibrium point of the system (51)-(55) is locally unstable if the parameter \( b \) is sufficiently large.

(Proof.) See Appendix D.

As a result, it is evident from the five-dimensional model as is the case in the three-dimensional model that the increase in parameter \( b \) through the CMU is a destabilizing factor and the increase in the parameters \( c_f \) and \( n \) is a stabilizing factor. In fact, the private investment and capital inflow were active in the euro area before the euro crisis. They contributed to the euro crisis.

8 A Numerical Simulation

In this section, we shall present some numerical simulations that support the theoretical analysis about relations between the CMU and fiscal union in the previous sections. The models in the previous sections made it possible to analyze local stability or instability of the CMU and the fiscal union analytically. By contrast, it is difficult to analyze the global nature using the models. Thus, this section aims to explicate the nature by means of illustrating local dynamics.
Based on Asada (2004), let us assume the following parameter values.

\[ c_i = 0.8, \tau_i = 0.2, T_{oi} = 10, C_{o1} = 20, C_{o2} = 40, \]
\[ G_{o1} = 30, G_{o2} = 60, \bar{M} = 600, \bar{Y}_1 = 240, \bar{Y}_2 = 310. \]

Further, we assume the functional forms of the LM equation, the current account function, and the investment function, as follows:

\[ r_i = 10Y_i - M_i + 160, \tag{70} \]
\[ I_i = 25Y_i - 0.3K_i - r_i + 160, \tag{71} \]
\[ J_i = -0.4Y_i + 0.2Y_2. \tag{72} \]

Asada (2004) assumed that coefficients of \( Y_1 \) and \( Y_2 \) are equal in the export function. Thus, a country that has a larger national income runs current account deficit automatically. However, Germany, which is representative of core countries in the Euro area, runs a current account surplus. Hence, we assume that coefficients of \( Y_1 \) and \( Y_2 \) are not equal in the export function (72).

In this case, the five-dimensional dynamical system (51)-(55) becomes as follows:

\[ \dot{Y}_1 = \alpha_1 \{ -0.76Y_1 + 15\sqrt{Y_1} - 0.3K_1 + M_1 + 0.2Y_2 + \gamma_1(240 - Y_1) + \mu(240 - Y_1) + 58 \}, \tag{73} \]
\[ \dot{K}_1 = 15\sqrt{Y_1} - 0.3K_1 + M_1, \tag{74} \]
\[ \dot{Y}_2 = \alpha_2 \{ -0.56Y_2 + 15\sqrt{Y_2} - 0.3K_2 - M_1 + 0.4Y_1 + \gamma_2(310 - Y_2) - \mu(240 - Y_1) + 708 \}, \tag{75} \]
\[ \dot{K}_2 = 15\sqrt{Y_2} - 0.3K_2 - M_1 + 600, \tag{76} \]
\[ \dot{M}_1 = -0.4Y_1 + 0.2Y_2 + J(Y_1, Y_2) + \beta(10\sqrt{Y_1} - 10\sqrt{Y_2} - 2M_1 + 600). \tag{77} \]

The equilibrium values of Eqs. (73)-(77) depend on the size of the parameter \( \beta, \gamma_1, \gamma_2 \) and \( \mu \).

Now, we shall compute the trajectories produced by Eqs. (73)-(77) by selecting several values of \( \alpha_1, \alpha_2, \beta, \gamma_1, \gamma_2, \) and \( \mu \) and the following initial conditions of the variables:

\[ Y_1(0) = 200, K_1(0) = 1680, Y_2(0) = 280, K_2(0) = 1830, M_1(0) = 940. \]

Figures 2-6 summarize the main results of numerical simulations.

Figures 2 and 3 show the trajectories generated by changing the parameter value \( \beta \) under the common parameter values \( \alpha_1 = 2, \alpha_2 = 2.5, \gamma_1 = 0, \gamma_2 = 0 \) respectively. As these figures indicate, an increase in \( \beta \) destabilizes and collapses the economy of monetary union. However, it is able to mitigate the instability through an increase in \( \beta \) by adopting a counter-cyclical fiscal policy and a fiscal transfer mechanism in a fiscal union, as indicated in Proposition 2. Figure 4 describes the effect of a counter-cyclical policy under the same parameter \( \beta \) in Figure 3. This policy stabilizes the fluctuation generated by an increase in \( \beta \). Further, an additional fiscal expenditure in both countries improves the equilibrium national incomes. Figure 5
shows the effect of the fiscal transfer mechanism in a fiscal union. In country 1, fiscal transfers from country 2 make a fluctuation smaller and stabilize quickly. Meanwhile, fiscal transfers remain in a large fluctuation in country 2 in the beginning but bring stability eventually. The effect of a combination of a counter-cyclical fiscal policy and fiscal transfers appears in Figure 6. The combination has a strong effect in comparison with the case where a counter-cyclical fiscal policy and fiscal transfers are singly implemented.

9 Discussion

In previous sections, we have analyzed the instability of $\beta$ and the stability of $\gamma_i$ and $\mu$
using the Keynesian short-run model and the Kaldorian medium-run model and have illustrated them using simulations. In particular, the euro area becomes fully integrated as Krugman (2012) indicated, by establishing a supranational fiscal union to mitigate the instability of increasing $\beta$ in the CMU.

However, there is a reaction against the burden on core countries in the fiscal union. The EU makes a fiscal transfer to the developing regions as regional policies through the EU budget. Despite the scale of approximately 1% of GDP of the EU as a whole, there is a reaction against the burden on core countries.

The fiscal transfer models in this paper also show that country 2 (core country) bears the finances of country 1 (peripheral country). To be sure, the fiscal transfer decreases the equilib-
Table 1: Equilibrium National Income, Money Supply and Net Export

| Case          | $Y_1^*$ | $Y_2^*$ | $M_1^*$ | $J_2^*$ |
|---------------|---------|---------|---------|---------|
| Figure 2 ($\beta = 1.5, \mu = 0$) | 156.5   | 304.6   | 275.2   | 1.7     |
| Figure 3 ($\beta = 10, \mu = 0$)  | 156.5   | 304.6   | 275.2   | 1.7     |
| Figure 5 ($\beta = 10, \mu = 0.9$) | 196.9   | 264.2   | 288.8   | 25.9    |
| Figure 6 ($\beta = 10, \gamma_1 = 0.1, \gamma_2 = 0.1, \mu = 0.9$) | 201.1   | 279.4   | 287.2   | 24.6    |
cal that the fiscal transfers mitigate the instability caused by austerity and an increase in capital mobility in both a spending country and a recipient country in fiscal transfer mechanism. Thus, it is necessary that countries in monetary union enhance solidarity to implement the fiscal transfer mechanism through a fiscal union.

10 Conclusion

In this paper, we analyzed the effect of fiscal transfers as an insurance mechanism in a fiscal union and its relation to the CMU using a Keynesian and Kaldorian two-country models with monetary union and imperfect capital mobility. The EC intends to increase the size of the capital market to realize the benefits of the CMU. However, the results from this study indicate that an increase in capital mobility between countries in the CMU can be a destabilizing force and have negative consequences for the euro area economy. Therefore, it is important for countries in the euro area to adopt counter-cyclical fiscal policies and create a fiscal union to mitigate the instability. Fiscal transfers can mitigate an instability caused by austerity and an increase in capital mobility in both the spending and recipient countries in the fiscal transfer mechanism.

However, it is difficult for peripheral countries to adopt expansionary fiscal policies because they are required to obey fiscal discipline. Thus, the balance between private capital mobility and support by fiscal policy collapses. As per the theory of the optimum currency area, fiscal transfers allow for shock adjustment between countries. To enlarge a capital market, it is necessary to construct a fiscal transfer mechanism as an insurance mechanism by creating a fiscal union.

One of the limitations of this paper is that we eliminate price fluctuations. In fact, the ECB targets an increase in the inflation rate and expected inflation rate by implementing an expansionary monetary policy. Therefore, a model that includes inflation rate and expected inflation rate should be constructed. However, this study contributes to the literature on economic stability in a fiscal union in that it considers a clear relation between a fiscal union and the CMU, although some challenges remain.

Appendix A  Proof of Proposition 1

Proof.

(i) Suppose that the parameter $\beta$ is fixed at any level. $\Gamma_{11}$ and $\Gamma_{22}$ are negative in the case that $\gamma_1$ are sufficiently large. Thus, we can obtain $a_1 > 0$. We can express Eq. (32) as follows.

$$a_2 = a_1 \gamma_1 + b \gamma_1 + c \gamma_2 + d.$$  \hspace{1cm} (A.1)

Then, $a$ is positive because $a_1 a_2 > 0$ holds. Thus, we have $a_2 > 0$, if at least one of the parameters $\gamma_1$ and $\gamma_2$ is sufficiently large. For the same reason, we can obtain $a_3 > 0$, if the parameter $\gamma_1$ and $\gamma_2$ is sufficiently large.

With respect to $a_1 a_2 - a_3$, we can express it as follows if $\gamma_2$ is constant.

$$a_1 a_2 = A \gamma_1^2 + B \gamma_1 + C, \hspace{1cm} (A.2)$$

where
\[ A = -a_1^2(\alpha_2 \Gamma_2 + F_{33}) > 0. \]  
(A.3)

We can also express \( a_1a_2 - a_3 \) as follows in the case that \( \gamma_1 \) is constant.
\[ a_1a_2 - a_3 = D\gamma_2^2 + E\gamma_2 + F, \]  
(A.4)

where
\[ D = -a_1^2(\alpha_1 \Gamma_{11} + F_{33}) > 0. \]  
(A.5)

Therefore, we have \( a_1a_2 - a_3 > 0 \), if at least one of \( c_1 \) and \( c_2 \) is sufficiently large. From the above, the equilibrium point of the system (18)–(20) is locally stable because the Routh–Hurwitz conditions are satisfied.

(ii) Suppose that the parameter \( \gamma_1 \) and \( \gamma_2 \) is fixed at any level. If the parameter \( \beta \) is sufficiently large, we have \( F_{31}(\beta) > 0 \) and \( F_{32}(\beta) < 0 \). \( a_2 \) is written as follows:
\[ a_2(\beta) = a_1a_2(\Gamma_{11} - \Gamma_{22} - \Gamma_{21}) + a_1(\Gamma_{11} - \Gamma_{13}F_{31}(\beta)) + a_2(\Gamma_{22} - \Gamma_{23}F_{32}(\beta)). \]  
(A.6)

Then, we have \( a_2 < 0 \) if the parameter \( \beta \) is sufficiently large. Thus, the equilibrium point of the system (18)–(20) is locally unstable because the Routh–Hurwitz conditions are not satisfied. \( \square \)

Appendix B  Proof of Proposition 2

Proof.
(i) Suppose that the parameter \( \beta \) is fixed at any level. \( \Theta_{11} \) and \( \Theta_{22} \) are negative in the case that \( \gamma_1 \) and \( \mu \) are sufficiently large. Thus, we can obtain \( b_1 > 0 \). If the parameter \( \mu \) is constant, then we can express Eq. (47) as follows:
\[ b_2 = a_1\gamma_1\gamma_2 + b\gamma_1 + c\gamma_2 + d. \]  
(B.1)

Then, \( a \) is positive because \( a_1a_2 > 0 \) holds. If the parameter \( \gamma_1 \) and \( \gamma_2 \) is constant, \( b_2 \) is a quadratic function with respect to \( \mu \). The coefficient of the quadratic term is positive. Thus, we have \( b_2 > 0 \), if the parameter \( \mu \) is sufficiently large. To summarize, we have \( b_2 > 0 \), if the parameters \( \gamma_1 \) and \( \gamma_2 \) or \( \mu \) are sufficiently large. For the same reason, we can obtain \( b_3 > 0 \), if the parameters \( \gamma_1 \) and \( \gamma_2 \) or \( \mu \) are sufficiently large.

With respect to \( b_1b_2 - b_3 \), we can express it as follows if \( \gamma_2 \) and \( \mu \) are constant.
\[ b_1b_2 - b_3 = A\gamma_1^2 + B\gamma_1 + C, \]  
(B.2)

where
\[ A = -a_1^2(\alpha_2 \Theta_{22} + H_{33}) > 0. \]  
(B.3)

Then, we can also express \( b_1b_2 - b_3 \) as follows if \( \gamma_1 \) and \( \mu \) are constant.
\[ b_1b_2 - b_3 = D\gamma_2^2 + E\gamma_2 + F, \]  
(B.4)
where
\[ D = -\alpha_1^2(\alpha_1 \Theta_{11} + H_{33}) > 0. \] (B.5)

Finally, we can express \( b_1, b_2 - b_3 \) as follows if \( \gamma_1 \) and \( \gamma_2 \) are constant.
\[ b_1b_2 - b_3 = G\mu^2 + H\mu + I, \] (B.6)
where
\[ G = -\alpha_1 \alpha_2 (\Theta_{22} + \Theta_{12}) - \alpha_1^2 H_{33} > 0. \] (B.7)

Therefore, we have \( b_1b_2 - b_3 > 0 \), if at least one of \( \gamma_1, \gamma_2 \) and \( \mu \) is sufficiently large.

From the above, the equilibrium point of the system (38)–(40) is locally stable because the Routh–Hurwitz conditions are satisfied.

(iii) Suppose that the parameters \( \gamma_1, \gamma_2 \), and \( \mu \) are fixed at any level. If the parameter \( \beta \) is sufficiently large, we have \( H_{31}(\beta) > 0 \) and \( H_{32}(\beta) < 0 \). \( b_2 \) is written as follows.
\[ b_2(\beta) = \alpha_1 \alpha_2 (\Theta_{12} - \Theta_{11} - \Theta_{22} + \Theta_{21}) + \alpha_1 (\Theta_{11} - \Theta_{13} H_{31}(\beta)) + \alpha_2 (\Theta_{22} - \Theta_{32} H_{32}(\beta)). \] (B.8)
Then, we have \( b_2 < 0 \) if the parameter \( \beta \) is sufficiently large. Thus, the equilibrium point of the system (38)–(40) is locally unstable because the Routh–Hurwitz conditions are not satisfied.

\[ \square \]

Appendix C  Comparative Statics Analysis of Fiscal and Monetary Policies: Fiscal Union Model

We shall consider the effect of monetary and fiscal policies while maintaining economic stability by creating a fiscal union.

Appendix C.1  Comparative Statics Analysis of Monetary Policy

In Eqs. (41)–(43), we investigate the impact of the endogenous variables \( Y_1, Y_2 \) and \( M_1 \) on the equilibrium value if the total monetary supply of both countries \( M \) changes by the amount \( dM \). Totally differentiating Eqs. (41)–(43), we have the following equations.

\[
\begin{bmatrix}
S_{11} & S_{12} & S_{13} \\
S_{21} & S_{22} & S_{23} \\
S_{31} & S_{32} & S_{33}
\end{bmatrix}
\begin{bmatrix}
dY_1 \\
dY_2 \\
dM_1
\end{bmatrix}
=
\begin{bmatrix}
0 \\
-\alpha_2 r^2_{M-M} dM \\
-\beta r^2_{M-M} dM
\end{bmatrix},
\] (C.1)

where
\[ S_{11} = -\left\{ 1 - c_1(1 - \tau_1) \right\} + I^{1}_F + I^{1}_F r^{1}_F + J^{1}_F - \gamma_1 - \mu < 0, \quad S_{12} = J^{1}_F > 0, \quad S_{13} = I^{1}_F r^{1}_M > 0, \]
\[ S_{21} = \mu - J^{1}_F > 0, \quad S_{22} = -\left\{ 1 - c_2(1 - \tau_2) \right\} + I^{2}_F + I^{2}_F r^{2}_F - J^{2}_F - \gamma_2 < 0, \quad S_{23} = -I^{2}_F r^{2}_M - M_i < 0, \]
\[ S_{31}(\beta) = J^{1}_F + \beta r^{1}_F < 0, \quad S_{32}(\beta) = J^{2}_F - \beta r^{2}_F > 0, \quad S_{33}(\beta) = \beta (r^{1}_M + r^{2}_M) < 0. \]

Expressing a coefficient matrix of the left hand side of Eq. (C.1) as \( S \), we have the following det\( S \).

\[
\text{det} S = S_{11}S_{22}S_{33} - S_{11}S_{23}S_{32} + S_{12}S_{23}S_{31} - S_{12}S_{21}S_{33} + S_{13}S_{23}S_{32} - S_{13}S_{22}S_{31}. \tag{C.2}
\]

Now, let us make the following assumption.

**Assumption 5.** \( S_{11} \) and \( S_{22} \) are so sufficiently large that we have \( \text{det} S < 0 \).

The inequality indicated in Assumption 5 is satisfied if the absolute values of \( S_{11} \) and \( S_{22} \) are relatively larger than the absolute values of the other terms. In this assumption, we investigate the change of an endogenous variable in (C.1) using Cramer’s rule:

\[
\frac{dY_1}{dM} = (Y^1_M)^* = \frac{1}{\text{det} S} \left| \begin{array}{ccc}
0 & S_{12} & S_{13} \\
-I^{2}_F r^{2}_M - M_i & S_{22} & S_{23} \\
-\beta r^{2}_M - M_i & S_{32} & S_{33}
\end{array} \right| > 0, \tag{C.3}
\]

\[
\frac{dY_2}{dM} = (Y^2_M)^* = \frac{1}{\text{det} S} \left| \begin{array}{ccc}
S_{11} & 0 & S_{13} \\
S_{22} & S_{22} & S_{23} \\
S_{32} & -\beta r^{2}_M - M_i & S_{33}
\end{array} \right| > 0, \tag{C.4}
\]

\[
\frac{dM_i}{dM} = (M^1_M)^* = \frac{1}{\text{det} S} \left| \begin{array}{ccc}
S_{11} & S_{12} & 0 \\
S_{22} & S_{22} & -I^{2}_F r^{2}_M - M_i \\
S_{31} & S_{32} & -\beta r^{2}_M - M_i
\end{array} \right| > 0. \tag{C.5}
\]

Then, we can establish the following proposition.
Proposition 4. Suppose that at least one of the parameters $\gamma_i$ and $\mu$ is sufficiently large so that the absolute values of $S_{11}$ and $S_{22}$ as a stabilizing factors of the system are larger than the absolute values of the other terms. Then, we have the following relationships.

$$\frac{dY_i}{dM} > 0, \quad \frac{dY_2}{dM} > 0, \quad \frac{dM_1}{dM} > 0.$$  \hfill (C.6)

Proposition 4 implies that the monetary policy of a supranational central bank like the ECB can have an impact on an economy if some additional assumptions are satisfied in the two-country model with a monetary union and imperfect capital mobility. Therefore, the quantitative easing implemented by the ECB has a positive impact for the euro area economy\textsuperscript{12}).

Appendix C.2 Comparative Statics Analysis of Fiscal Policy

We develop a comparative statics analysis of fiscal policy under Assumption 5. In Eqs. (41)--(43), we investigate the impact of the endogenous variables $Y_1$, $Y_2$ and $M_1$ on the equilibrium value if the fiscal policy parameters $\tau_1$ and $G_{0i}$ change by the amount $d\tau_1$ and $dG_{0i}$, respectively. Totally differentiating Eqs. (41)--(43) in the case of $i = 1$, we have the following equations.

$$\begin{vmatrix}
S_{11} & S_{12} & S_{13} \\
S_{21} & S_{22} & S_{23} \\
S_{31} & S_{32} & S_{33}
\end{vmatrix}
\begin{bmatrix}
\frac{dY_1}{dM} \\
\frac{dY_2}{dM} \\
\frac{dM_1}{dM}
\end{bmatrix} = \begin{bmatrix}
c_1Y_1d\tau_1 - dG_{01} \\
0 \\
0
\end{bmatrix}. \hfill (C.7)
$$

First, we consider the case when $d\tau_1 = 0$ and $dG_{01} \neq 0$. We investigate the change of endogenous variables in (C.1) using Cramer’s rule:

$$\frac{dY_1}{dG_{01}} = (Y_1^*)^* = \left| \begin{array}{ccc}
-1 & S_{12} & S_{13} \\
0 & S_{22} & S_{23} \\
0 & S_{32} & S_{33}
\end{array} \right| = \frac{1}{\det S} (S_{22}S_{23} - S_{23}S_{33}) > 0, \hfill (C.8)
$$

$$\frac{dY_2}{dG_{01}} = (Y_2^*)^* = \left| \begin{array}{ccc}
S_{11} & -1 & S_{13} \\
S_{21} & 0 & S_{23} \\
S_{31} & 0 & S_{33}
\end{array} \right| = \frac{1}{\det S} (S_{21}S_{33} - S_{23}S_{31}) > 0. \hfill (C.9)
$$

\textsuperscript{12} For the details of expansionary monetary policies of the ECB, refer to Nakao (2016).
\[
\frac{dM_i}{dG_{0i}} = (M_{0i})^* = \frac{1}{\det S} \begin{vmatrix} S_{11} & S_{12} & -1 \\ S_{21} & S_{22} & 0 \\ S_{31} & S_{32} & 0 \end{vmatrix} = \frac{1}{\det S} (S_{22}S_{31} - S_{32}S_{21}) < 0.
\] (C.10)

Then, we can establish the following proposition as we have similar relationships when \( i = 2 \).

**Proposition 5.** Suppose that at least one of the parameters \( \gamma \) and \( \mu \) is sufficiently large so that the absolute values of \( S_{11} \) and \( S_{22} \) as stabilizing factors of the system are larger than the absolute values of the other terms. Then, we have the following relationships:

\[
\frac{dY_1}{dG_{01}} > 0, \quad \frac{dY_2}{dG_{01}} > 0, \quad \frac{dM_1}{dG_{01}} < 0,
\] (C.11)

\[
\frac{dY_1}{dG_{02}} > 0, \quad \frac{dY_2}{dG_{02}} > 0, \quad \frac{dM_1}{dG_{02}} > 0.
\] (C.12)

Proposition 5 implies that the government expenditure of one country not only has a positive impact on the national income of that country but also on another country. Therefore, an increase in government expenditure in a peripheral country can increase national income in a core country. Proposition 5 also implies that a decrease in monetary demand of country 1 caused by increased interest rates is larger than an increase of monetary demand of that country caused by increased government expenditure in country 1.

Second, we consider the case when \( d\tau_1 \neq 0 \) and \( dG_1 = 0 \). We investigate the change in endogenous variables in (C.1) using Cramer’s rule:

\[
\frac{dY_1}{d\tau_1} = (Y_1^{*}) = \frac{1}{\det S} \begin{vmatrix} c_1Y_1 & S_{12} & S_{13} \\ 0 & S_{22} & S_{23} \\ 0 & S_{32} & S_{33} \end{vmatrix} = \frac{c_1Y_1}{\det S} (S_{22}S_{33} - S_{32}S_{23}) < 0,
\] (C.13)

\[
\frac{dY_2}{d\tau_1} = (Y_2^{*}) = \frac{1}{\det S} \begin{vmatrix} c_1Y_1 & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & 0 & S_{33} \end{vmatrix} = \frac{c_1Y_1}{\det S} (S_{23}S_{31} - S_{32}S_{21}) < 0,
\] (C.14)
Then, we can establish the following proposition as we have similar relationships when \( i = 2 \).

**Proposition 6.** Suppose that at least one of the parameters \( \gamma_i \) and \( \mu \) is sufficiently large so that the absolute values of \( S_{11} \) and \( S_{22} \) as stabilizing factors of the system are larger than the absolute values of the other terms. Then, we have the following relationships.

\[
\begin{align*}
\frac{dY_1}{d\tau_1} &< 0, \quad \frac{dY_2}{d\tau_1} < 0, \quad \frac{dM_i}{d\tau_1} > 0, \\
\frac{dY_1}{d\tau_2} &< 0, \quad \frac{dY_2}{d\tau_2} < 0, \quad \frac{dM_i}{d\tau_2} < 0.
\end{align*}
\]  

(C.16)

Proposition 6 implies that an increased tax leads to a decrease in national income not only of the peripheral country but also the core country. An increased tax rate in the core country also leads to a decrease in national income in the peripheral country. Then, Proposition 6 also implies that an increase in monetary demand due to a decrease in interest rates is larger than an increase in monetary demand due to increased taxes in the peripheral country.

### Appendix C.3 Comparative Static Analysis of Fiscal and Monetary Policies

We develop a comparative statics analysis of fiscal and monetary policies under Assumption 5. In Eqs. (41)–(43), we investigate the impact of an endogenous variables \( Y_1, Y_2, \) and \( M_i \) on equilibrium value if the total monetary supply of both countries \( \bar{M} \) and the fiscal policy parameters \( \tau \) and \( G_{0i} \) change by the amount \( dM, d\tau, \) and \( dG_{0i} \) respectively.

\[
\begin{align*}
&dY_1 = (Y_{1j})^*dM + (Y_{g1})^*dG_{01} + (Y_{11})^*d\tau_1, \\
&dY_2 = (Y_{2j})^*dM + (Y_{g2})^*dG_{01} + (Y_{22})^*d\tau_1, \\
&dM_i = (M_{1j})^*dM + (M_{g1})^*dG_{01} + (M_{11})^*d\tau_1.
\end{align*}
\]

(C.17) (C.18) (C.19)

In this paper, we consider the following two cases:

(i) \( dM = dG_{01} > 0 \) and \( d\tau_1 = 0 \)

We assume that a supranational central bank purchases national bonds that are issued to finance government expenditure in country 1.

\[
\begin{align*}
&dY_1 = [(Y_{1j})^* + (Y_{g1})^*]dM > 0, \\
&dY_2 = [(Y_{2j})^* + (Y_{g2})^*]dM > 0.
\end{align*}
\]

(C.20) (C.21)
\[ dM_1 = \left[ (M_M^{(+)})^* + (M_G^{(+)})^* \right] d\tilde{M}. \]  

(C.22)

An increase in government expenditure with an issue of national bonds financed by a supranational central bank leads to an increase in the national income of both country 1 and country 2 which is larger than the increase in national income created by an increase in money supply without an increase in government expenditure. Thus, an increase in government expenditure with the issue of national bonds of a peripheral country financed by a supranational central bank leads to an increase in national income not only of the peripheral country but also of the core country. Then, the money supply of country 1 is smaller than the money supply not accompanied by an increase in government expenditure.

(ii) \( d\tilde{M} > 0, dG_{01} = 0 \) and \( d\tau_1 > 0 \)

We assume that country 1 adopts an expansionary monetary policy and increases taxation.

\[ dY_1 = (Y_M^{(+)})^* dM_1 + (Y_{\tau 1}^{(+)})^* d\tau_1, \]  

(C.23)

\[ dY_2 = (Y_M^{(+)})^* dM_1 + (Y_{\tau 2}^{(+)})^* d\tau_1, \]  

(C.24)

\[ dM_1 = (M_M^{(+)})^* d\tilde{M} + (M_{\tau 1}^{(+)})^* d\tau_1 > 0. \]  

(C.25)

A policy mix of monetary expansion by a supranational central bank and increased taxation in country 1 increases national income in both country 1 and country 2 by an amount smaller than that created by a monetary expansion not accompanied by an increased tax rate. Then, the money supply of country 1 is larger than the monetary easing not accompanied by an increased tax. Therefore, a policy mix of monetary easing and increased tax, implemented by the ECB and the peripheral country, has a smaller impact than that of a monetary expansion on the national income of a peripheral country and a core country.

Appendix D  Proof of Proposition 3

Proof.  

(i) Suppose that the parameter \( \beta \) is fixed at any level. If the parameter \( \gamma_f \) is sufficiently large, we have \( c_1 > 0 \). Secondly, if the parameter \( \mu \) is constant, we can express Eq. (47) as follows.

\[ c_2 = A\gamma_f^2 + B\gamma_f + C. \]  

(D.1)

Then, if the parameters \( \gamma_f \) are sufficiently large, we have \( c_2 > 0 \) because of \( A > 0 \). We have \( c_3 > 0, c_4 > 0 \) and \( c_5 > 0 \) in similar way.

Furthermore, \( \Delta_1 \) is a quadratic equation, \( \Delta_2 \) is a cubic equation, \( \Delta_3 \) is a quintic equation, \( \Delta_4 \) is a seventh degree equation and \( \Delta_5 \) is a ninth degree equation with respect to \( \gamma_f \). As a result, we have \( \Delta_i > 0 \) if the parameter \( \gamma_f \) is sufficiently large.

If the parameter \( \gamma_f \) is constant, \( c_2 \) is a linear equation with respect to \( \mu \). The coefficient of \( \mu \) is positive. Thus, we have \( c_1 > 0 \), if the parameter \( \mu \) is sufficiently large. For the same reason, we obtain \( c_2 > 0, c_3 > 0, c_4 > 0, \) and \( c_5 > 0 \), if the parameters \( \mu \) are sufficiently large.
Furthermore, $\Delta_1$ is a linear equation, $\Delta_2$ is a quadratic equation, $\Delta_3$ is a cubic equation, $\Delta_4$ is a quartic equation and $\Delta_5$ is a quintic equation with respect to $\gamma_f$. As a result, we have $\Delta_i > 0$ ($i = 1, 2, ..., 5$) if the parameter $\mu$ is sufficiently large.

From the above, the equilibrium point of the system (51)-(55) is locally stable because the Routh–Hurwitz conditions are satisfied, if at least one of the parameters $\gamma_1, \gamma_2$, and $\mu$ is sufficiently large.

(ii) Suppose that the parameters $\gamma_1$ and $\mu$ are relatively small and inequalities $\Phi_{11} > 0$ and $\Phi_{33} > 0$ hold. If the parameter $\beta$ is sufficiently large, we have $V_{51}(\beta) > 0$ and $V_{53}(\beta) < 0$. Then, $\Delta_2$ is a quadratic equation with respect to $\beta$. The coefficient of $\beta$ squared term is negative. Therefore, $\Delta_2 < 0$ if the parameter $\beta$ is sufficiently large. From the above, the equilibrium point of the system (51)-(55) is locally unstable because the Routh–Hurwitz conditions are not satisfied in the case of sufficiently large $\beta$. □

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