Entangled graphs: A classification of four-qubit entanglement

Seyed Javad Akhtarshenas *, Masoud Gharahi Ghahi †
Department of Physics, University of Isfahan, Hezar Jerib, Isfahan, 81746-73441, Iran
Quantum Optics Group, University of Isfahan, Hezar Jerib, Isfahan, 81746-73441, Iran

March 16, 2010

Abstract
In this paper we use the concept of entangled graphs with weighted edges and present a classification of four-qubit entanglement. Entangled graphs, first introduced by Plesch et al [Phys. Rev. A 67 (2003) 012322], are structures such that each qubit of a multi-qubit system is represented as a vertex and an edge between two vertices denotes bipartite entanglement between the corresponding qubits. Our classification is based on the use of generalized Schmidt decomposition (GSD) of pure states of multi-qubit systems. We show that for every possible entangled graph one can find a pure state such that the reduced entanglement of each pair, measured by concurrence, represents the weight of the corresponding edge in the graph. We also use the concept of tripartite and fourpartite concurrences as a proper measure of global entanglement of the states. In this case a circle including the graph indicates the presence of global entanglement.

Keywords: Quantum entanglement; Entangled graphs; Four-qubit entanglement

PACS: 03.65.Ud; 03.67.Mn

1 Introduction
Entanglement, first noticed by Einstein, Podolsky, and Rosen [1] and Schrödinger [2], is at the heart of quantum mechanics. The interest on quantum entanglement has dramatically increased over the last two decades due to the emerging field of quantum information theory. It turns out that quantum entanglement provides a fundamental potential resource for communication and information processing [3, 4, 5]. A pure quantum state of two or more subsystems is said to be entangled if it is not a product of states of each components. On the other hand, a bipartite mixed state \( \rho \) is said to be entangled if it can not be expressed as a convex combination of pure product states [6], otherwise, the state is separable or classically correlated. It is, therefore, of primary importance testing whether a given state is separable or entangled. For systems with dimensions \( 2 \otimes 2 \) or \( 2 \otimes 3 \), there exists an operationally simple necessary and sufficient condition for separability, the so called Peres-Horodecki criterion.
Entangled graphs

[7, 8]. It indicates that a state \( \rho \) is separable if and only if the matrix obtained by partially transposing the density matrix \( \rho \) is still positive. However, in higher dimensional systems this is only a necessary condition; that is, there exist entangled states whose partial transpose is positive.

On the other hand it is interesting to ask how strongly the two subsystems are entangled. From several approaches introduced to quantify entanglement, the entanglement of formation, which in fact intends to quantify the resources needed to create a given entangled state [5], has attracted much attention. Remarkably, Wootters [9] has shown that entanglement of formation of a two-qubit state \( \rho \) is related to a quantity called concurrence as

\[
C(\rho) = \max \left\{ 0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4} \right\},
\]

where the \( \lambda_i \) are the non-negative eigenvalues, in decreasing order, of the non-Hermitian matrix \( \rho \dot{\rho} \). Here \( \dot{\rho} \) is the matrix given by \( \dot{\rho} = (\sigma_y \otimes \sigma_y) \rho^* \sigma_y \otimes \sigma_y \) where \( \rho^* \) is the complex conjugate of \( \rho \) when it is expressed in a standard basis such as \( \{ |00\rangle, |01\rangle, |10\rangle, |11\rangle \} \) and \( \sigma_y \) represents Pauli matrix in the local basis \( \{ |0\rangle, |1\rangle \} \). Furthermore, the entanglement of formation is monotonically increasing with the concurrence \( C \), and thus the concurrence itself may be used as a measure of entanglement.

Bipartite pure state entanglement is almost completely understand by means of Schmidt decomposition [10]. Schmidt decomposition allows one to write, by means of local unitary transformation, any pure state shared by two parties A and B in a canonical form, in the sense that all the non-local properties of the state are contained in the positive Schmidt coefficients. Accordingly, any two-qubit pure state \( |\Psi\rangle_2 \in \mathbb{C}^2 \otimes \mathbb{C}^2 \) can be written in the following form [10] (the subscript indicates the number of qubits)

\[
|\Psi\rangle_2 = \alpha |00\rangle + \beta |11\rangle, \quad \alpha, \beta \geq 0, \quad \alpha^2 + \beta^2 = 1.
\]

In this sense any two-qubit pure state is separable if \( \alpha \beta = 0 \), i.e. if one of the Schmidt coefficients \( \alpha, \beta \) is zero, or entangled if \( \alpha \beta \neq 0 \).

For three-qubit pure state, there is no a straightforward generalization of Schmidt decomposition in terms of three orthogonal product states [11]. A generalization of two-qubit Schmidt decomposition for three-qubit pure states have presented by Acín et al in [12]. They have shown that for any pure three-qubit state there exist a local bases which allows one to build a set of five orthogonal product states in terms of which the state can be written in a unique form as

\[
|\Psi\rangle_3 = \lambda_0 |000\rangle + \lambda_1 e^{i\phi} |100\rangle + \lambda_2 |101\rangle + \lambda_3 |110\rangle + \lambda_4 |111\rangle,
\]

\[
\lambda_i \geq 0, \quad 0 \leq \phi \leq \pi, \quad \sum_i \lambda_i^2 = 1.
\]

They have also presented a canonical form for pure states of four-qubit system as [13]

\[
|\Psi\rangle_4 = \alpha |0000\rangle + \beta |0100\rangle + \gamma |0101\rangle + \delta |0110\rangle + \epsilon |1000\rangle + \zeta |1001\rangle + \eta |1010\rangle + \kappa |1011\rangle + \lambda |1100\rangle + \mu |1101\rangle + \nu |1110\rangle + \omega |1111\rangle.
\]

A multipartite generalization of the Schmidt decomposition for pure states of a general multipartite system have presented by Carteret et al [14].
**Entangled graphs**

Multipartite entanglement refers to nonclassical correlations between three or more quantum particles, and characterization and quantification of them is far less understood than for bipartite entanglement. In the case of multipartite entanglement there are various aspects of entanglement and, in particular, it is no longer sufficient to ask if the particles are entangled, but one needs to know how they are entangled. There are different ways that the multipartite state $\rho$ can be entangled from which the bipartite entanglement, i.e. the entanglement of the reduced density matrix $\rho_{kl}$, is only a certain aspect of the multipartite entanglement properties of $\rho$. For example for three-qubit states, apart from separable and biseparable pure states, there exist also two different types of locally inequivalent genuine tripartite entangled states: the so-called Greenberger-Horn-Zeilinger (GHZ) type [15] and W type [16, 17], with representatives $|\text{GHZ}\rangle_3 = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ and $|\text{W}\rangle_3 = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$, respectively. States belonging to GHZ and W types cannot be transformed into each other by local operations and classical communication (LOCC).

The entanglement properties of a multi-qubit system may be represented mathematically in several ways. In [18], Dür has investigated the entanglement properties of multipartite systems, concentrating on the bipartite aspects of multipartite entanglement, i.e. the bipartite entanglement that is robust against the disposal of the particles. In this sense, the two qubits belonging to a multipartite system are entangled if their reduced density matrix is entangled. He has introduced the concept of entanglement molecules, i.e. structures such that each qubit is represented by an atom while an entanglement between a pair of qubits is represented by a bound. He has shown that an arbitrary entanglement molecule can be represented by a mixed state of a multiqubit system. On the other hand, Plesch et al [19, 20] have introduced the concept of entangled graphs such that each qubit of a multipartite system is represented as a vertex and an edge between two vertices denotes bipartite entanglement between the corresponding qubits. They have shown that any entangled graph of $N$ qubits can be represented by a pure state from a subspace of the whole $2^N$-dimensional Hilbert space of $N$ qubits. Recently a classification of entanglement of three-qubit states based on their three-qubit and reduced two-qubit entanglement has presented [21].

In this Letter we turn our attention to entangled graphs and associate a pure four-qubit state to each graph of order 4. This, naturally, leads to a classification of entanglement in four-qubit systems. Our classification is based on the use of generalized Schmidt decomposition (GSD) of pure states of multi-qubit systems. We show that for every possible entangled graph one can find a pure state such that the reduced entanglement of each pair, measured by concurrence, represents the weight of the corresponding edge in the graph. In order to characterize the global entanglement of the state, we use the concept of tripartite and fourpartite concurrences, introduced in [22], in such a way that a circle including the graph indicates the nonzero global entanglement of the corresponding state.

The paper is organized as follows. In section 2 we present the tripartite concurrence and give a classification of the tripartite pure states. Section 3 is devoted to present a classification of four-qubit pure states. We conclude the paper in section 4, with a brief conclusion.

## 2 Classification of three-qubit entanglement

In this section we associate a three-qubit pure state to a weighted entangled graph of order 3. Entangled graphs are structures such that each qubit is represented as a vertex and an edge between two vertices denotes bipartite entanglement between the corresponding qubits. Following the paper [21] we use a circle including the three vertices in order to indicate
full tripartite entanglement. We show that for every possible entangled graph one can find a pure state such that the reduced entanglement of each pair, measured by concurrence, represents the weight of the corresponding edge in the graph. In order to characterize the full entanglement of the state we use the tripartite concurrence defined by [22]:

$$C_{123}(\rho) = \left( C_{1(23)}C_{2(13)}C_{3(12)} \right)^{\frac{1}{3}},$$

where the bipartite concurrence $C_{A(BC)}$ is defined as $C_{A(BC)} = \sqrt{2(1 - \text{Tr}(\rho_A^2))}$ where $\rho_A$ is the reduced density matrix obtained from the whole density matrix $\rho$ by tracing over the two subsystems B and C. Now, our classification is as follows:

1. **Fully separable:** A pure state $|\psi\rangle$ is fully separable or unentangled if it can be written as $|\phi_1\rangle \otimes |\phi_2\rangle \otimes |\phi_3\rangle$. The corresponding graph has three vertices without any edge. All bipartite entanglement as well as the global entanglement of this state is zero, i.e. $C_{12} = C_{13} = C_{23} = 0$ and $C_{123} = 0$. These states can be represented by a graph of order 3, without any edge (see Fig. (1a)). Any state which can not be written in fully separable form is entangled and belongs to one of the following categories.

2. **Biseparable:** A pure state $|\psi\rangle$ is biseparable if it is not fully separable and that it can be written as $|\phi_{ij}\rangle \otimes |\phi_k\rangle$ for one $k \in \{1, 2, 3\}$. A representative of this kind of states is given by

$$|\psi\rangle = (\alpha|00\rangle + \delta|11\rangle) |0\rangle.$$  

In this case we find for bipartite concurrences $C_{12} = 2\alpha\delta$, $C_{13} = C_{23} = 0$. This state has not global entanglement and therefore we have $C_{123} = 0$. The corresponding graph has three vertices with only one edge connecting two vertices $i$ and $j$ (see Fig. (1b)).
3. **Fully inseparable**: A pure state $|\psi\rangle$ is fully inseparable if it is not separable or bisep-
parable. The fully inseparable states have nonzero global entanglement, i.e. $C_{123} \neq 0$. This category includes the following classes (see Figs. (1c)-(1f))

(a) **Three vertices without any edge but a circle includes the graph (Fig. (1c))**: A pure representaive of this class is as follows

$$|\psi\rangle = \alpha|000\rangle + \lambda|111\rangle. \quad (7)$$

Surprisingly, similar to fully separable states, all bipartite entanglement of this state is zero, i.e. $C_{12} = C_{13} = C_{23} = 0$, but the state has nonzero global entanglement, i.e. $C_{123} = 2\alpha\lambda$. This state is a GHZ type state.

(b) **Three vertices with one edge and a circle includes the graph (Fig. (1d))**: This class has the following representative

$$|\psi\rangle = \alpha|000\rangle + \delta|110\rangle + \lambda|111\rangle. \quad (8)$$

In this case for bipartite concurrence we find $C_{12} = 2\alpha\delta$, $C_{13} = C_{23} = 0$ and for global entanglement we get $C_{123} = 2\alpha(\lambda(\delta^2 + \lambda^2))^{1/2}$.

(c) **Three vertices with two edges and a circle includes the graph (Fig. (1e))**: This class is represented by the following state

$$|\psi\rangle = \alpha|000\rangle + \beta|100\rangle + \delta|110\rangle + \lambda|111\rangle. \quad (9)$$

The state has two nonzero bipartite concurrences as $C_{12} = 2\alpha\delta$ and $C_{23} = 2\beta\lambda$. The nonzero global entanglement of the state is given by

$$C_{123} = 2 \left( \sqrt{\alpha^2(\delta^2 + \lambda^2)} \sqrt{\alpha^2(\delta^2 + \lambda^2)} + \beta^2\lambda^2 \sqrt{\lambda^2(\alpha^2 + \beta^2)} \right)^{1/2}. \quad (10)$$

(d) **Three vertices with three edges and a circle includes the graph (Fig. (1f))**: The last class of this category has the following representative

$$|\psi\rangle = \alpha|000\rangle + \gamma|101\rangle + \delta|110\rangle. \quad (11)$$

All bipartite concurrences of this state are nonzero as $C_{12} = 2\alpha\delta$, $C_{13} = 2\alpha\gamma$ and $C_{23} = 2\gamma\delta$. The state has also a nonzero global entanglement as $C_{123} = \left( \sqrt{\alpha^2(\delta^2 + \lambda^2)} \sqrt{\delta^2(\alpha^2 + \gamma^2)} \sqrt{\gamma^2(\alpha^2 + \delta^2)} \right)^{1/2}$. This state is a W type state.

### 3 Classification of four-qubit entanglement

In this section we associate a four-qubit pure state to a weighted entangled graph of order 4. In order to characterize the global entanglement of the state we use the fourpartite concurrence defined by [22]:

$$C_{1234}(\rho) = (C_{1(234)}C_{2(134)}C_{3(124)}C_{4(123)}C_{12(34)}C_{13(24)}C_{14(23)})^{1/2}, \quad (12)$$

where $C_{A(BCD)}$ and $C_{A(BCD)}$ are defined as $C_{A(BCD)} = \sqrt{2(1 - \text{Tr}(\rho_A^2))}$ and $C_{A(BCD)} = \sqrt{\frac{1}{2}(1 - \text{Tr}(\rho_{AB}^2))}$, respectively. The classification is as follows:
Entangled graphs

Figure 2: Four-qubit entangled graphs and classification of entanglement in four-qubit system. (a) Fully separable, (b) tri-separable, (c)-(f) biseparable, (g)-(q) fully inseparable.

1. **Fully separable**: A pure state $|\psi\rangle$ is fully separable if it can be written as $|\phi_1\rangle \otimes |\phi_2\rangle \otimes |\phi_3\rangle \otimes |\phi_4\rangle$. All bipartite entanglement as well as the tripartite and global entanglement of this state are zero, i.e. $C_{ij} = 0$, $C_{ijk} = 0$ and $C_{1234} = 0$. The corresponding graph has four vertices without any edge (see Fig. (2a)). Any state which can not be written in fully separable form is entangled and belongs to one of the following categories.

2. **Tri-separable**: A pure state $|\psi\rangle$ is tri-separable if it is not fully separable but it can be written as $|\phi_{ij}\rangle \otimes |\phi_k\rangle \otimes |\phi_l\rangle$ for $k,l \in \{1, 2, 3, 4\}$. A representative for this type of states is as

$$|\psi\rangle = (\alpha |00\rangle + \lambda |11\rangle) |00\rangle. \quad (12)$$

Only one bipartite concurrence of this state is nonzero, i.e. $C_{12} = 2\alpha\lambda$ and $C_{13} = C_{14} = C_{23} = C_{24} = C_{34} = 0$. This state has also a zero global entanglement $C_{1234} = 0$. The corresponding graph of this category has four vertices with only one edge (see Fig. (2b)).
3. **Biseparable** A pure state $|\psi\rangle$ is bi-separable if it is not three-separable and that it can be written as $|\phi_{ijk}\rangle \otimes |\phi_l\rangle$ for one $l \in \{1, 2, 3, 4\}$ or as $|\phi_{ij}\rangle \otimes |\phi_{kl}\rangle$ for one $kl \in \{12, 13, 14, 23, 24, 34\}$. For the former, the global entanglement is zero and the following classes are defined (see Figs (2c)-(2f)).

(a) **Four vertices without any edge, but a circle includes three of vertices (Fig. (2c))**: A pure state representative of this class is as follows

$$|\psi\rangle = |1\rangle (\epsilon|000) + \omega|111\rangle. \quad (13)$$

All bipartite concurrences of this state are zero, i.e. $C_{ij} = 0$ and $C_{1234} = 0$, but the system has entanglement through three vertices 2,3,4, i.e. $C_{234} \neq 0$.

(b) **Four vertices with only one edge and a circle includes three of vertices (Fig. (2d))**: The pure state representative of this class is as

$$|\psi\rangle = |1\rangle (\epsilon|000) + \nu|110) + \omega|111\rangle. \quad (14)$$

This state has zero value for all but one bipartite concurrences as $C_{23} = 2\epsilon\nu$. Also the global entanglement of the state is zero but the entanglement through three vertices 2,3,4, is nonzero, i.e. $C_{1234} = 0$, $C_{234} \neq 0$.

(c) **Four vertices with two edges and a circle includes three of vertices (Fig. (2e))**: The pure state representative of this class is as

$$|\psi\rangle = |1\rangle (\epsilon|000) + \lambda|100) + \nu|110) + \omega|111\rangle. \quad (15)$$

The nonzero bipartite concurrences of this state are $C_{23} = 2\epsilon\nu$, $C_{34} = 2\lambda\omega$. The state has also zero global entanglement, i.e. $C_{1234} = 0$, but nonzero tripartite entanglement $C_{234} \neq 0$.

(d) **Four vertices with three edges and a circle includes three of vertices (Fig. (2f))**: The pure state representative of this class is as

$$|\psi\rangle = |1\rangle (\epsilon|000) + \mu|101) + \nu|110\rangle. \quad (16)$$

The nonzero bipartite concurrences of this state are as $C_{23} = 2\epsilon\nu$, $C_{24} = 2\epsilon\mu$, $C_{34} = 2\mu\nu$. This state has again a zero global entanglement $C_{1234} = 0$ and a nonzero tripartite entanglement $C_{234} \neq 0$.

4. **Fully inseparable** A pure state $|\psi\rangle$ is fully inseparable if it is not separable, trio-separable or bi-separable. This category has nonzero global entanglement, i.e. $C_{1234} \neq 0$, and has the following classes (see Figs. (2g)-(2q)).

(a) **Four vertices without any edge, but a circle includes the graph (Fig. (2g))**: A pure state representative of this class is as follows

$$|G\rangle = \alpha|0000) + \omega|1111\rangle. \quad (17)$$

All bipartite concurrences of this state are zero, i.e. $C_{ij} = 0$, but the state has nonzero global entanglement $C_{1234} = 2\alpha\omega$. This state is a GHZ type state.
(b) *Four vertices with one edge and a circle includes the graph (Fig. (2h))*: This class has the following representative

$$|\psi\rangle = \alpha|0000\rangle + \kappa|1011\rangle + \mu|1101\rangle. \quad (18)$$

The only nonzero bipartite concurrence is $C_{23} = 2\kappa\mu$. Also the state has nonzero global entanglement $C_{1234} \neq 0$.

(c) *Four vertices with two edges and a circle includes the graph (Fig. (2i))*: This class has the following representative

$$|\psi\rangle = \alpha|0000\rangle + \kappa|1011\rangle + \mu|1101\rangle + \eta|1010\rangle + \kappa|1011\rangle + \mu|1101\rangle. \quad (19)$$

In this case two nonzero bipartite concurrences are $C_{13} = 2\alpha\eta$ and $C_{23} = 2\kappa\mu$. The state has also a nonzero global entanglement $C_{1234} \neq 0$.

(d) *Four vertices with two edges and a circle includes the graph (Fig. (2j))*: This class has the following representative

$$|\psi\rangle = \alpha|0000\rangle + \kappa|1011\rangle + \mu|1101\rangle + \eta|1010\rangle + \omega|1111\rangle. \quad (20)$$

The state has two nonzero bipartite concurrences as $C_{13} = 2\alpha\eta$ and $C_{24} = 2\eta\omega$. The state has also a nonzero global entanglement $C_{1234} \neq 0$.

(e) *Four vertices with three edges and a circle includes the graph (Fig. (2k))*: This class has the following representative

$$|\psi\rangle = \alpha|0000\rangle + \kappa|1011\rangle + \mu|1101\rangle + \nu|1110\rangle, \quad \kappa\mu > \alpha\nu, \quad \kappa\nu > \alpha\mu, \quad \mu\nu > \alpha\kappa. \quad (21)$$

The state has three nonzero bipartite concurrences as $C_{23} = 2(\kappa\mu - \alpha\nu)$, $C_{24} = 2(\kappa\nu - \alpha\mu)$ and $C_{34} = 2(\mu\nu - \alpha\kappa)$. The state has also a nonzero global entanglement $C_{1234} \neq 0$.

(f) *Four vertices with three edges and a circle includes the graph (Fig. (2l))*: This class has the following representative

$$|\psi\rangle = \alpha|0000\rangle + \zeta(|1001\rangle + |1010\rangle + |1100\rangle + |1111\rangle). \quad (22)$$

In this case three nonzero bipartite concurrences are equal and given by $C_{12} = C_{13} = C_{14} = 2\alpha\zeta$. The state has also a nonzero global entanglement $C_{1234} \neq 0$.

(g) *Four vertices with three edges and a circle includes the graph (Fig. (2m))*: This graph has the following pure state representative

$$|\psi\rangle = \alpha|0000\rangle + \kappa|1011\rangle + \lambda|1100\rangle + \mu|1101\rangle + \omega|1111\rangle. \quad (23)$$

Three nonzero bipartite concurrences are as $C_{12} = 2\alpha\lambda$, $C_{23} = 2\kappa\mu$ and $C_{34} = 2\lambda\omega$. Again the state has also a nonzero global entanglement $C_{1234} \neq 0$.

(h) *Four vertices with four edges and a circle includes the graph (Fig. (2n))*: This class has the following representative

$$|\psi\rangle = \alpha|0000\rangle + \zeta|1001\rangle + \kappa|1011\rangle + \lambda|1100\rangle + \mu|1101\rangle. \quad (24)$$

This state has four nonzero bipartite concurrences as $C_{12} = 2\alpha\lambda$, $C_{14} = 2\alpha\zeta$, $C_{23} = 2\kappa\mu$ and $C_{24} = 2\zeta\lambda$. The state has also a nonzero global entanglement $C_{1234} \neq 0$. 

(i) *Four vertices with four edges and a circle includes the graph (Fig. (2o)):*

This class has the following representative
\[ |\psi\rangle = \alpha |0000\rangle + \zeta |1001\rangle + \eta |1010\rangle + \omega |1111\rangle, \]
\[ \alpha \omega \geq 2 \zeta \eta \quad \text{or} \quad \alpha \omega = \zeta \eta. \] (25)

In this case we find four nonzero bipartite concurrences as
\[ C_{13} = 2 \alpha \eta, \quad C_{14} = 2 \alpha \zeta, \quad C_{23} = 2 \zeta \omega \quad \text{and} \quad C_{24} = 2 \eta \omega. \] The state has also a nonzero global entanglement \( C_{1234} \neq 0. \)

(j) *Four vertices with five edges and a circle includes the graph (Fig. (2p)):*

This class has the following representative
\[ |\psi\rangle = \alpha |0000\rangle + \zeta |1001\rangle + \eta |1010\rangle + \omega |1111\rangle, \quad \zeta \eta > \alpha \omega. \] (26)

In this case we have five nonzero bipartite concurrences as
\[ C_{13} = 2 \alpha \eta, \quad C_{14} = 2 \alpha \zeta, \quad C_{23} = 2 \zeta \omega, \quad C_{24} = 2 \eta \omega \quad \text{and} \quad C_{34} = 2(\zeta \eta - \alpha \omega). \] The state has also a nonzero global entanglement \( C_{1234} \neq 0. \)

(k) *Four vertices with six edges and a circle includes the graph (Fig. (2q)):*

This class has the following representative
\[ |\psi\rangle = \alpha |0000\rangle + \zeta |1001\rangle + \eta |1010\rangle + \lambda |1100\rangle. \] (27)

All bipartite concurrences are nonzero and given by
\[ C_{12} = 2 \alpha \lambda, \quad C_{13} = 2 \alpha \eta, \quad C_{14} = 2 \alpha \zeta, \quad C_{23} = 2 \eta \lambda, \quad C_{24} = 2 \zeta \lambda \quad \text{and} \quad C_{34} = 2 \zeta \eta. \] This state has also nonzero global entanglement \( C_{1234} \neq 0. \) This state is a W type state.

4 Conclusions

We have used the concept of entangled graphs with weighted edges and have presented a classification of three-qubit and four-qubit entanglement. In graph representation of pure states each qubit of a multi-qubit system is represented by a vertex and an edge between two vertices denotes bipartite entanglement between the corresponding qubits. The nonzero concurrence of the two-qubit reduced density matrix indicates that the entanglement of the state is robust against the disposal of remaining particles and have represented by an edge connecting the corresponding vertices. In this classification, we have used the generalized Schmidt decomposition and have shown that for every possible entangled graph one can find a pure state such that the reduced entanglement of each pair, measured by concurrence, represents the weight of the corresponding edge in the graph. Using the concept of tri-concurrence and four-concurrence, we have characterized the global entanglement of the tripartite and fourpartite pure states, respectively. The presence of global entanglement of the pure state has been represented by a circle including the corresponding graph.

References

[1] A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. 47, (1935) 777-780.

[2] E. Schrödinger, Naturwissenschaften 23, (1935) 807-812.
[3] C.H. Bennett, S.J. Wiesner, Phys. Rev. Lett. 69, (1992) 2881-2884.

[4] C.H. Bennet, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, W.K. Wootters, Phys. Rev. Lett. 70, (1993) 1895-1899.

[5] C.H. Bennet, D.P. DiVincenzo, J. Smolin, W.K. Wootters, Phys. Rev. A 54, (1996) 3824-3851.

[6] R. F. Werner, Phys. Rev. A 40, (1989) 4277-4281.

[7] A. Peres, Phys. Rev. Lett. 77, (1996) 1413-1415.

[8] M. Horodecki, P. Horodecki, R. Horodecki, Phys. Lett. A, 223 (1996) 1-8.

[9] W.K. Wootters, Phys. Rev. Lett. 80, (1998) 2245-2248.

[10] E. Schmidt, Math. Annalen. 63, (1906) 433-476.

[11] A. Peres, Phys. Lett. A 202, (1995) 16-17.

[12] A. Acín, A. Andrianov, L. Costa, E. Jané, J.I. Latorre, R. Tarrach, Phys. Rev. Lett. 85, (2000) 1560-1563.

[13] A. Acín, A. Andrianov, E. Jané, R. Tarrach, J. Phys. A: Math. Gen. 34, (2001) 6725-6739.

[14] H. A. Carteret, A. Higuchi, A. Sudbery, J. Math. Phys. 41, (2000) 7932-7939.

[15] D. M. Greenberger, M. Horn, A. Zeilinger, Bell’s Theorem, Quantum Theory, and Conceptions of the Universe, edited by M. Kafatos, Kluwer, Dordrecht, 1989. pp. 69-72.

[16] W. Dür, G. Vidal, J. I. Cirac, Phys. Rev. A 62, (2000) 062314 1-12.

[17] A. Acín, D. Bruß, M. Lewenstein, A. Sanpera, Phys. Rev. Lett 87, (2001) 040401 1-4.

[18] W. Dür, Phys. Rev. A 63, (2001) 020303(R) 1-4.

[19] M. Plesch, V. Bužek, Phys. Rev. A 67, (2003) 012322 1-6.

[20] M. Plesch, J. Novotný, Z. Dzuráková, V. Bužek, J. Phys. A: Math. Gen. 37, (2004) 1843-1859.

[21] C. Sabín, G. García-Alcaine, Eur. Phys. J. D 48, (2008) 435-442.

[22] P.J. Love et al., Quantum inf. Process. 6, (2007) 187-195.