Bloch–Siegert oscillations in the Rabi model with an amplitude-modulated driving field

A P Saiko\(^1\), S A Markevich\(^1\) and R Fedaruk\(^2\)

\(^1\) Scientific-Practical Material Research Centre, Belarus National Academy of Sciences, 19 P.Brovka Str., Minsk 220072, Belarus
\(^2\) Institute of Physics, University of Szczecin, 15 Wielkopolska Str., 70-451, Szczecin, Poland

E-mail: saiko@physics.by

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Abstract
We study the coherent dynamics of a qubit excited by an amplitude-modulated electromagnetic field under the Rabi resonance when the frequency of the low-frequency modulation field matches the Rabi frequency in the high-frequency field. Due to the destructive interference of multiple photon processes within the ultrastrong coupling between the qubit and the low-frequency driving field, Rabi oscillations result exclusively from the Bloch–Siegert effect. It is directly observed in the time-resolved coherent dynamics as the Bloch–Siegert oscillation. In this case, triplets in the Fourier spectra of the coherent response are transformed into doublets with the splitting between the lines equal to twice the Bloch–Siegert shift. These unusual properties are demonstrated in the conditions of experiments with a nitrogen vacancy center in the diamond.

Keywords: Rabi model, Bloch–Siegert effect, amplitude-modulated electromagnetic field, qubit, ultrastrong coupling

(Some figures may appear in colour only in the online journal)
the non-RWA terms hinder coherent control of qubits due to complex (multifrequency) Rabi oscillations [17]. The Bloch–Siegert effect appears important also for experiments with doubly dressed states of qubits driven by the strong bichromatic field. In particular, the second low-frequency field with the frequency closed to the Rabi frequency in the first high-frequency electromagnetic field effectively excites transitions between the dressed states of the qubit. This so-called Rabi resonance has been observed in the optical range [18, 19] as well as in electron paramagnetic resonance [20–22] and NMR [23–25]. The Rabi oscillations between doubly dressed states have also been studied [19–22, 25]. Their frequency is determined by the amplitude of the low-frequency driving field. The bichromatic control of Rabi oscillations between doubly dressed states and the prolongation of their coherence can find applications in quantum information processing [26, 27] and open the possibility for the direct and sensitive detection of weak radio-frequency magnetic fields [28]. Since the coupling between the qubit and the low-frequency field can easily be obtained in a way comparable to the Rabi frequency in the high-frequency field, the Bloch–Siegert effect becomes significant under such strong driving [21, 22, 29]. In these time-resolved experiments, the Rabi resonance was realized when the high-frequency field was in resonance with the qubit transition [31]. It was shown [32] that the ultrastrong regime is reached in the experiment resulting in the significant Bloch–Siegert shift of the Rabi frequency of Raman transitions. More recently, in the stronger light–matter coupling, unexplored behaviors of Rabi oscillations for the second-order Raman transition was considered and a method for direct observation of the Bloch–Siegert oscillation was proposed [33]. Filtering of the Bloch–Siegert oscillation from the multiphoton coherent dynamics of an ultrastrong-driven qubit remains unstudied for other techniques of time-resolved coherent spectroscopy.

In the present paper, we demonstrate the possibility for conventional Rabi resonance in the qubit’s states, excited by the amplitude-modulated microwave field to observe the Bloch–Siegert oscillation separately from other oscillating processes in the coherent dynamics.

2. Coherent dynamics of the qubit in an amplitude-modulated driving field

Quantum transitions between the states of a spin qubit are excited by an amplitude-modulated microwave field

\[ V(t) = \Delta_1 \cos(\omega_d t) + 2A \cos(\omega_d t) \sin(\omega t), \]

where \( \cos(\omega_d t) \) and \( \sin(\omega t) \) describe the high- and low-frequency components of the field with the frequencies \( \omega_d \) and \( \omega \), respectively, and the amplitudes of these components: \( \Delta_1 \) and \( A \ll \omega_d \) [31].

The Hamiltonian of the qubit at such driving can be written as

\[ H_{lab} = \frac{\Delta_1}{2} \sigma^z + \Delta_2 \sigma^x + A \sin(\omega t) \sigma^y, \]

where \( \Delta_2 = \omega_d - \omega \). The dynamics of the system is described by the Liouville equation for the density matrix \( \rho \):

\[ i\hbar \frac{\partial}{\partial \tau} \rho = H_{eff} \rho \]

(in the following we take \( \hbar = 1 \)). Rotating the frame around the \( y \) axis by an angle of \( \theta \), the effective Hamiltonian for the dressed states of the qubit is

\[ H_{eff} = U_1^\dagger H_{lab} U_1 = \frac{\Delta_1}{2} \sigma^x + A \cos \theta \sin(\omega t) \sigma^y, \]

where \( \omega = \sqrt{\Delta_1^2 + \Delta_2^2} \). The dynamics of the system is described by the Liouville equation for the density matrix \( \rho \):

\[ i\hbar \frac{\partial}{\partial \tau} \rho = H_{eff} \rho \]

with \( \rho_2 = U_2^\dagger \rho_2 U_2 \). We obtain the Liouville equation for \( \rho_2 \) with the Hamiltonian

\[ H_2 = U_2^\dagger H_1 U_2 - iU_2^\dagger \frac{\partial U_2}{\partial \tau} = \frac{A}{2i} \cos \theta \left[ \sum_{n=-\infty}^{\infty} J_n(a) e^{-i n \omega t} \right], \]

where \( J_n(a) \) is the Bessel function of the first kind and

\[ a = 2A \sin \theta / \omega. \]

We consider only the conventional Rabi resonance realized at \( \omega_0 = \omega \). This resonance is well observed and has the largest Rabi frequency among others multiphoton Rabi resonances [32]. The Hamiltonian \( H_2 \) contains an infinite sum of oscillating harmonics with the frequencies that are integer multiplies of the frequency \( \omega \). There are no oscillations for \( n = 0 \) and \( n = -2 \). Therefore, the terms of the sum with these \( n \) give the largest contribution and correspond to the RWA. At the strong coupling condition \( 0.1 < A \cos \theta / \omega < 1 \) the other oscillating terms are significant. Their contribution can be taken into account using the Bogoliubov averaging method [34] for constructing the time-independent effective Hamiltonian in the framework of the non-secular perturbation theory. The averaging procedure up to the second order in \( A \cos \theta / \omega \) (see [32]) gives the following effective Hamiltonian:

\[ H_2 \rightarrow H_{eff} = H_2^{(1)} + H_2^{(2)}, \]

where

\[ H_2^{(1)} = \langle H_2(t) \rangle, \quad H_2^{(2)} = \frac{i}{2} \left[ \int_{0}^{T} d\tau \langle H_2(\tau) - \langle H_2(\tau) \rangle, H_2(t) \rangle \right]. \]

Here the symbol \( \langle \ldots \rangle \) denotes time averaging over rapid oscillations of the type \( \exp(\pm im \omega t) \) given by

\[ \langle O(t) \rangle = \frac{1}{2\pi} \int_{0}^{2\pi / \omega} O(t) dt. \]

The upper limit \( T \) of the indefinite integral indicates the variable on which the result of the integration depends. The square brackets denote the commutation operation.
As a result, the effective Hamiltonian can be written as

\[ H_{\text{eff}} = \frac{\omega_{\text{BS}}}{2} \sigma^z + (\Omega/2)(\sigma^+ + \sigma^-) \]

with

\[ \Omega = 2J_1(a)\, A \cos \theta, \quad \omega_{\text{BS}} = A^2 \cos^2 \theta \frac{2\omega}{2a} \left\{ \sum_{n=-2}^{n} \frac{J_n^2 + J_{n+2}}{n+2} + \sum_{n=0}^{\infty} \frac{J_n^2 + J_{n+2}}{n} \right\}, \] (4)

where \( \Omega \) is the Rabi frequency for the quantum transitions when the Rabi resonance condition \( \omega_{\text{0}} = \omega \) is fulfilled and \( \omega_{\text{BS}} \) is the Bloch–Siegert frequency shift caused by the non-resonant rapidly oscillating non-RWA terms. The Bessel function \( J_1(a) \) in equation (4) for \( \Omega \) appears due to virtual multiphoton transitions in which the number of absorbed (emitted) photons exceeds the number of emitted (absorbed) photons by 1. In the equation for \( \omega_{\text{BS}} \), we omit the argument \( a \) of the Bessel functions.

Now we consider the experimental situation with an NV center when the microwave field excites transitions between the spin sublevels \([0]\) and \([-1]\) of this center, while the level \([1]\) is far detuned [31]. We assume that the two-level spin system is initially in the ground state \([0]\). At the Rabi resonance, the probability to find the system in some moment again in the ground state \( P_{(0)}(t) \) is:

\[ P_{(0)}(t) = \frac{1}{2} (1 + \cos^2 \theta - 2c_1) + e \sin \left[ \omega t - a \cos (\omega t) \right] + c \cos (\Omega t - \varphi_c) \]  

\[ + \frac{1}{2} \sin \theta \left[ b \cos (\Omega t - \varphi_b) \cos \left[ \omega t - a \cos (\omega t) \right] \right] + \frac{1}{2} d \cos (\Omega t - \varphi_2) \sin \left[ \omega t - a \cos (\omega t) \right], \] (5)

where the non-RWA Rabi frequency \( \Omega^* = \sqrt{\Omega^2 + (\omega_{\text{BS}})^2} \) takes into account the Bloch–Siegert shift and the following definitions and notations are used: \( c = (c_1^2 + c_2^2)^{1/2}, b = (b_1^2 + b_2^2)^{1/2}, \) \( d = (d_1^2 + d_2^2)^{1/2}, \) \( \cos \varphi_c = c_1/c, \cos \varphi_b = b_1/b \cos \varphi_a = d_1/d; \) \( c_1 = \frac{1}{2} \left( \frac{\Omega}{\Omega^*} \right)^2 \cos^2 \theta - \frac{\omega_{\text{BS}}}{\Omega^*} \sin 2\theta \sin \left[ \omega t - a \cos (\omega t) \right], c_2 = \frac{\Omega}{\Omega^*} \sin \theta \cos \omega t \cos \theta, \) \( e = -\frac{\Omega}{\Omega^*} \sin \theta \sin \theta \cos \left[ \omega t - a \cos (\omega t) \right], b_1 = \frac{\Omega}{\Omega^*} \sin \theta \cos \theta, b_2 = \frac{\Omega}{\Omega^*} \cos \theta - \frac{\omega_{\text{BS}}}{\Omega^*} \sin \theta \sin \theta \sin \left[ \omega t - a \cos (\omega t) \right], d_1 = \frac{\Omega}{\Omega^*} \cos \theta - \left( \frac{\omega_{\text{BS}}}{\Omega^*} \right)^2 \sin \theta \sin \theta, d_2 = -\frac{\omega_{\text{BS}}}{\Omega^*} \sin \theta \cos \theta. \)

3. Time and spectral manifestations of the Bloch–Siegert effect

Figure 1 shows the dependences of the RWA Rabi frequency \( \Omega \), the non-RWA Rabi frequency \( \Omega^* \) and the Bloch–Siegert shift \( \omega_{\text{BS}} \) on the normalized amplitude of the low-frequency driving field at \( \omega/2\pi = 10.44 \text{ MHz}, \Delta \omega/2\pi = 10 \text{ MHz} \) and \( \Delta z/2\pi = 3 \text{ MHz} \). (b) The frequencies presented in (a) near \( A^*/\omega \) are shown in more detail. This plot is useful to obtain the values of these frequencies for \( \Delta A/\omega \) used in the following figures.
remain the Rabi resonance as a function of the evolution time at\( \omega/2\pi = 10.44 \) MHz, \( \Delta c/2\pi = 10 \) MHz and \( \Delta z/2\pi = 3 \) MHz. The strength of the low-frequency driving field is \( A = A^* + \Delta A \), where \( A^*/\omega = 2.00 \) and \( \Delta A/\omega = 0.25, 0.1, 0, -0.1, -0.25 \). The red line shows the Bloch–Siegert oscillation.

The disappearance of the amplitude modulation in the evolution of the ground state population is evidence that the RWA Rabi frequency vanishes. The non-RWA Rabi frequency \( \Delta A/\omega \) shows the Bloch–Siegert oscillation.

\[
P_{[0]}(t; \Omega = 0) = \frac{1}{2} (1 + \cos^2 \theta) + \frac{1}{2} \sin^2 \theta \cos \left[ (\omega + \omega^{BS})t - a^* \cos(\omega t) + a^* \right].
\]

Figure 2 demonstrates that at \( \Delta A/\omega = 0 \) the amplitude modulation vanishes and the oscillations with the constant amplitude and periodically changing frequency occur. The disappearance of the amplitude modulation in the evolution of the ground state population is evidence that the RWA Rabi frequency vanishes and the non-RWA Rabi frequency becomes equal to the Bloch–Siegert shift. In this case, the Bloch–Siegert oscillation is observed as the frequency modulation of the coherent response and is presented in figure 2 by the red line. The used parameters correspond to the ultrastrong regime with the coupling constant \( Acos\theta/\omega \approx 0.57 \).

The disappearance of the RWA Rabi frequency represents a kind of electromagnetically induced transparency. The two-level system may become transparent under its bichromatic driving by the high- and low-frequency field [35, 36]. This effect is based on the destructive interference of excited multiple photon processes. When the non-RWA is taken into account for the low-frequency field, the full electromagnetically-induced transparency cannot be realized because the Bloch–Siegert oscillation remains, even if the RWA Rabi frequency vanishes.

The Fourier spectra \( F(\omega_s) = \int_0^\infty e^{-i\omega_{st}} e^{-\gamma t} P_{[0]}(t) dt \) are shown in figure 3. The decay rate, \( \gamma \), was introduced phenomenologically using its value corresponding to a coherence time of \( 4 \mu s \) [31]. The spectra consist of Lorentzian lines at zero frequency, at the non-RWA Rabi frequency \( \Omega^* \) and a series of triplets are observed at frequencies \( n\omega \) and \( n\omega \pm \Omega^* \), corresponding to the amplitude-modulated oscillations. When \( a \rightarrow a^* \) (or \( A \rightarrow A^* \)), the RWA Rabi frequency \( \Omega \rightarrow 0 \) as well as the coefficients \( c_1, c_2, e \rightarrow 0 \) and only the coefficients \( b_1, b_2, d_1, d_2 \) have non-zero values. In this case the line corresponding to the oscillations at the RWA Rabi frequency vanishes. At \( A = A^* \), the non-RWA Rabi frequency \( \Omega^* = \omega^{BS} \) and only two side lines at the frequencies \( n\omega \pm \omega^{BS} \) remain in triplets, and each triplet is transformed into a doublet. A splitting between the doublet lines becomes exactly equal to \( 2\omega^{BS} \). Note that the second triplet near \( \omega_s/2\pi = 20.9 \) MHz degenerates into a singlet. Using the decomposition expansion of equation (6) by a series of Bessel functions, the time-dependent part of this equation can be written as

\[
\frac{1}{2} \sum_{n=-\infty}^{\infty} J_n(a^*) \exp \left( \{i ((n-1)\omega - \omega^{BS}) t - a^* + n\pi/2) \right) + c.c.
\]

It follows directly from this expression that the line with the the higher frequency in the doublet \( 2\omega - \omega^{BS}, 2\omega + \omega^{BS} \) at \( n = -1 \) and \( n = 3 \) vanishes because its amplitude is \( J_{-1}(a^*) = 0 \). It is an additional indication that the RWA Rabi frequency \( \Omega \) becomes equal to zero and the conditions are realized when the double Bloch–Siegert shift \( 2\omega^{BS} \) can be directly determined from the splitting between the doublet lines.

Figure 2. The state population of the spin level \( |0\rangle \) at the Rabi resonance as a function of the evolution time at \( \omega/2\pi = 10.44 \) MHz, \( \Delta c/2\pi = 10 \) MHz and \( \Delta z/2\pi = 3 \) MHz. The strength of the low-frequency driving field is \( A = A^* + \Delta A \), where \( A^*/\omega = 2.00 \) and \( \Delta A/\omega = 0.25, 0.1, 0, -0.1, -0.25 \). The red line shows the Bloch–Siegert oscillation.

Figure 3. Fourier spectra for \( \Delta A/\omega = -0.25, 0 \) and 0.25. The other parameters are the same as in figure 2.
4. Conclusion

We have studied the coherent oscillations excited by the amplitude-modulated microwave field in the two-level system at the Rabi resonance. It was shown that, in the ultrastrong regime, when the coupling between the qubit and the modulation field exceeds the modulation frequency, the Rabi oscillations are significantly modified by the Bloch–Siegert effect due to multiphoton antiresonant interactions. For the properly chosen parameters of the modulation field, the RWA Rabi frequency can become zero and the Bloch–Siegert oscillation with the frequency $\omega_{BS}$ is directly observed. At these parameters, the amplitude-modulated oscillations in the evolution of the ground state population transform into oscillations of the constant amplitude and periodically changing frequency. In the Fourier spectra of the coherent response, the triplets are transformed into doublets with splitting between the doublet lines equal to $2\omega_{BS}$. We demonstrate this unique possibility for direct measuring of the Bloch–Siegert shift in the conditions of experiments with the NV center in the diamond. The proposed direct observation of the Bloch–Siegert oscillations may be used as a new technique for studying quantum systems at bichromatic and multichromatic driving in the ultrastrong regime. This technique is not limited by spin qubits and can also be realized in quantum optics using an amplitude-modulated light.

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