Optimization of a tuned vibration absorber in a multibody system by operational analysis

F Infante¹, S Perfetto¹, D Mayer¹ and S Herold¹

¹Fraunhofer Institute for Structural Durability and System Reliability LBF, Darmstadt, Germany

E-mail: Francesco.Infante@lbf.fraunhofer.de

Abstract. Mechanical vibration in a drive-train can affect the operation of the system and must be kept below structural thresholds. For this reason tuned vibration absorbers (TVA) are usually employed. They are optimally designed for a single degree of freedom system using the Den Hartog technique. On the other hand, vibrations can be used to produce electrical energy exploitable locally avoiding the issues to transfer it from stationary devices to rating parts. Thus, the design of an integrated device for energy harvesting and vibration reduction is proposed to be employed in the drive-train. By investigation of the dynamic torque in the system under real operation, the accuracy of a numerical model for the multibody is evaluated. In this study, this model is initially used for the definition of the TVA. An energetic procedure is applied in order to reduce the multibody in an equivalent single degree of freedom system for a particular natural mode. Hence, the design parameters of the absorber are obtained. Furthermore, the introduction of the TVA in the model is considered to evaluate the vibration reduction. Finally, an evaluation of the power generated by the piezo transducer and its feedback on the dynamic of the drive-train is performed.

1. Introduction
In the last two decades the intensification of low-power wireless sensors has led to increased studies in the conversion of mechanical vibration into electrical energy. In fact, the use of batteries is not always the most suitable choice, when, in particular, the sensors are not installed in an accessible place or a continuous need of power is required. For rotating systems such studies become even more important, due to the issues to transfer energy from a stationary device to rotating parts.

This interest in using mechanical vibrations results in a conflict with the high number of works previously done in order to absorb the vibrations, avoiding their effects on the ordinary operation of mechanical systems. In this sense, vibration absorbers are usually installed to improve the mechanical performances. These devices are single degree of freedom (SDOF) systems attached to the main structure to control its motion. The motion of the structural mass has reduced, while keeping the relative motion between absorber and main structure within admissible bounds. The first idea was developed by Frahm [1] who suggested a mass spring system without damper. In this way the additional component was able to balance the external load without absorbing energy. Such device presents the higher impedance to the main structure but only in the operating frequency (vibration neutralizer). Afterwards, it was demonstrated by Ormondroyd and Den Hartog [2] that by the introduction of damping was possible both dissipating energy and increasing the frequency range where the device is efficient. This case is sometimes referred to as tuned mass damper (TMD) or...
dynamic vibration absorber. The study of Den Hartog led to the definition of the first optimal formulation in 1928. The criterion fixed the natural frequency of the mass damper in order to have the same amplitude of the displacement in the two peaks of the complete response function. In addition, an optimal level of damping is defined to have a zero derivate at the two peaks. By this optimization was possible to minimize the maximum displacement of the primary mass. Several different optimization criteria were then developed. Krenk [3] defined a technique which aims the minimization of the displacement of the main mass and the relative displacement. Warburton [4] proposed a kinetic energy minimisation of the primary mass over all frequencies, while the optimization studied by Ziletti [5] has as aim the definition of the damping level and natural frequency in order to maximise the power dissipated by the absorber. The entire set of optimizations are applicable to a SDOF undamped main system.

The problem of a TMD attached to a multi degree of freedom (MDOF) main system has also been studied extensively, either to find the optimal parameters and the optimal location. Rana and Soong [6] used an eigenvector normalization method to find the optimal absorber parameters when it is attached to a MDOF system. Xu and Kwok [7] had the same aim in order to minimize the wind excitiation of a tall building. In the work of Lewis [8] it has been demonstrated that the invariant points, discovered by Den Hartog, still exist when the absorber is attached to a two degrees of freedom undamped main system. The work was then extended by Ozer and Royston [9] in order to obtain the necessary expressions that would lead the determination of invariant points in a MDOF system of any size.

All these techniques led to a very high optimal damping value. This result, besides being hardly feasible in reality, does not allow a good extraction of energy from the environmental vibrations.

The piezoelectric materials have been considerably studied as mechanism to convert ambient vibrations into electrical energy that may be used directly or stored. In addition, the implementation of a conversion system can introduce an additional damping to the system by the electrical circuit connected to it. The most important step to define a practical design in an energy harvesting device is to model correctly the dynamic behaviour of the mechanical system with integrated the electrical load. The double nature of this problem has caused some modelling issues. Most of the models presented in literature focused on simplifying the energy harvesting circuit by a simple resistive load, or they did not consider any feedback from the electrical circuit to the mechanical behaviour. In the real world applications, the interaction between both the mechanical structure and electronic circuit has to be considered and the circuit attached is more complex than a simple resistor. For practical use of the energy generated by a piezoelectric material the conversion of the alternative current (AC) to direct current (DC) is required. The conversion is generally done by a rectifier circuit that yield a more complex model. Several models have been used to determine the power generated, and to study the maximum of it that can be dissipated in a simple resistor or in a combination of linear electrical elements. In the work of Motter [10] a strategy to estimate the power provided by a cantilever beam with piezoelectric material integrated and connected to a rectified circuit is proposed. Clementino [11] some years later, considered the same system of Motter and he obtained in his study the maximum power harvested by numerical optimization of the beam and piezoelectric geometry, parameters of resistive load, capacitive filter and diodes.

The aim of this paper is to evaluate both the vibration reduction of a MDOF system and the power generation using piezoelectric transducers, when the rectifier in considered in the electrical circuit. The model of a drive-train is proposed by comparison with experimental measurements. Due to an energetic procedure, the reduction of the MDOF system to an equivalent SDOF system for one natural frequency is presented. Once the model has been simplified, the optimization of Den Hartog for the TMD can be used. Considering it attached in different positions in the main system, the different effects on the dynamics are simulated, and the most appropriate collocation is selected. The absorber mass is meant connected by an electromechanical beam to the main system. The relative velocity between host and absorber masses causes transverse vibration of the piezoelectric material. By a theoretical model of a full-wave diode bridge (rectifier) directly connected to the transducer, the power generated can be evaluated. A choice of capacitance and resistance loads is provided, and with the implementation of the electric circuit, the optimal damping of the absorber initially estimated is reduced to a more realistic value and the electric feedback on the mechanical behaviour is estimated.
2. Validation of the simulation

2.1 Experimental setup
The test rig consists of a drive-train powered by a four stroke engine. The configuration includes a modular test rig of a 15 kW two cylinder internal combustion motor in V-configuration with a volume of 670 ccm which is connected by a clutch to the drive train.

The MDOF system is composed by a high inertia disc, supported by two self-aligning ball bearings, a planetary gear box and an eddy current brake. The axial, radial and angular compensation are guaranteed by the presence of three different polymer couplings collocated between the main bodies. With four torque transducers, the measurement of the dynamic torque is awarded. Two of these are additionally equipped with rpm sensors, while the flywheel and another small rotating disc of the chain are supplied with teeth and they use two magneto-metric sensors for detecting the rotational speed. Employing the entire set of sensors, the behaviour of the dynamic torque in different stations, as well as the power, dissipation are investigated. In figure 1 the entire setup with its main parts is shown. The control of the throttle is realized by a signal generator, developing a run-up and run-down between 1300 rpm to 3500 rpm and again to 1300 rpm in 60 seconds.

A non-linear behaviour of the polymer couplings in the system are taken into account during the experiment. Phenomena connected with temperature, excitation frequency, relative angular displacement between the two sides and torque amplitude are detected during the working process. These non-linearities involve the generation of a wide band of resonance instead of a single peak and they can be reduced by running in a state where the effects are less influential, without completely avoiding them. Applying a constant braking torque at the eddy current break, this state could be achieved [12].

2.2 Validation of the simulation
For the simulation model, four rigid bodies connected by spring and damper elements are considered. The values of damping and stiffness of the shafts are negligible compared to the characteristics of the polymer couplings. The connection parameters between each body are given by technical data sheets. A CAD model of the MDOF system is required in order to evaluate the inertia of all the components (figure 2), and a Simulink Design Optimization, to define the inertia of the gear box, is used.

Finally the multibody system is considered not connected with the ground and this leads to a first resonance frequency $f_1=0$ Hz (rigid body). All numerical tests and results are described and listed by

---

**Figure 1.** Powertrain setup in the initial configuration.

**Figure 2.** CAD model of the powertrain setup in the initial configuration.
using Matlab and the Simscape toolbox of Simulink. For the experimental results, LMS data acquisition is used.

For the comparison between simulation and experimental results, spectrograms in different sections are performed. Figures 3 and 4 represent the amplitude spectra of dynamic torque as functions of the engine rotational speed. For simplicity, only the results in two sections at the flywheel and at the eddy-current brake are presented. The first three natural resonances of the MDOF system are excited from the 0.5th, 1.0th and 1.5th orders of the engine during the run down (the rigid mode is not visible in the plots). It is possible to see that the two resonances are quite well separated, and not all bodies are excited in the same way at the same frequencies. The second mode interests principally the eddy current brake, while the amplitude of the flywheel present a peak at the third resonance frequency. Comparing the results, significant similarities between experiments and simulations are observed. Both natural frequencies and dynamic torque amplitudes can be well predicted. In particular, the flywheel presents a peak of resonance of 10 Nm at 89 Hz, frequency excited by the 1.5th order of the engine around 3200 rpm (figure 3 a and b). Around 1600 rpm, the second mode at 14 Hz is excited by the 0.5th order and affects principally the eddy current brake (figure 4 a and b):

![Figure 3. Spectrogram for flywheel: measured (a), simulated (b)](image)

![Figure 4. Spectrogram for eddy current brake: measured (a), simulated (b)](image)

### 2.3 Mode shape of the MDOF main system

Considering the 4DOF model, a modal analysis to define the equations of motion (EOM) using eigenvalues and eigenvectors is performed. With this technique the physical coordinates, are transformed to modal coordinates and for structural dynamics they can be interpreted as response amplitude of orthonormalized vibration modes. The EOM in physical coordinate for the MDOF forced and damped systems in compact notation are expressed in equation (1)
\[ J\dot{\theta} + C\theta + K\theta = t, \]  

where \( J \), \( C \) and \( K \) are the inertia, damping and stiffness matrix respectively, \( t \) and \( \theta \) are the torque vector generated by the engine and the rotational displacement. A dot above a variable represents differentiation with respect to time. Supposing the damping matrix removed, natural frequencies and mode shapes of the unforced undamped system are obtained by the eigenproblem defined in equation (2):

\[ K\theta = \omega^2 J\theta. \]  

Let \( \Phi \) be the modal matrix defined by the orthonormalized mode shapes respect to the inertia matrix \( \phi_j \), and denoting \( \eta \) the array of modal amplitudes \( \eta_j \) (where \( j \) is the number of modes), the rotational displacement can be expressed in modal coordinates as shown in equation (3):

\[ \theta = \Phi\eta. \]  

Replacing it in equation (2), and premultiplying by \( \Phi^T \), the EOM in modal coordinates are obtained in equation (4)

\[ \Phi^T J\Phi\ddot{\eta} + \Phi^T K\Phi\eta = \Phi^T t. \]  

Since \( \Phi \) is built after the normalization respect to the inertia matrix, it is possible to demonstrate the identities:

\[ \Phi^T J\Phi = I, \quad \Phi^T K\Phi = \text{diag}[\omega^2]. \]  

There is a major difficulty, regarding the damping matrix neglected in the beginning of the analysis. Since it is not diagonal generally, the modal EOM cannot be decoupled. Nevertheless, a study of the dynamic behaviour can be predicted by the analysis of the mode shapes and of the resonance frequencies. By the model with four rigid bodies, the same number of natural frequencies is obtained. In table 1 the results of the analysis are summarized, and the bodies mainly excited for each resonance are defined. Figure 5 shown the mode shapes of the system, where the dynamic behaviour, defined by the eigenvectors, is clearly visible.

| Table 1. Results of modal analysis. |
|-------------------------------------|
| **Frequency** | **Body excited** |
| First mode | 0 Hz | Rigid body resonance |
| Second mode | 14.50 Hz | Eddy-current brake |
| Third mode | 89.55 Hz | Gear box |
| Fourth mode | 287.90 Hz | Flywheel |

3. Degrees of freedom reduction

In this section the reduction of the 4DOF system to an equivalent SDOF is shown. The Den Hartog optimization can be used only considering an undamped, or slightly damped, SDOF main system. By a total kinetic energy formulation for the main system an equivalent inertia \( J_{eq} \) and an equivalent torsional stiffness \( k_{eq} \) for a given natural mode \( k \) are defined. The motion of each body defined is given by the vector \( \theta \), described in the previous section, and the absorber is considered coupled to the inertia \( J^* \). The damping of the main system is neglected. From the modal analysis, the natural frequencies \( f_j \) and the modal shape, as a vector of the vibration rotational velocities for each natural vibration \( \phi_i \), are given.

A natural vibration \( k \) is selected for the design of the absorer. At this frequency, the total kinetic energy without the application of the absorber is calculated by equation (6):

\[ W_{\text{kin}} k = \frac{1}{2} \sum_{j=1}^{4} J_j \phi_j^2 k. \]
where $\hat{\phi}_{j_0k}$ is the normalization of the modal shape for the absorber in position $j^*$, defined in equation (7):

$$\Phi = (\Phi_1 \ldots \Phi_k \ldots \Phi_4), \quad \hat{\phi}_{j_0k} = 1.$$  

The kinetic energy of the equivalent system with the resonance $k$ is indicated in equation (8), where the velocity $\hat{\phi}_{eqk}$ of the defined system at point $j^*$ is also set to 1, i.e. $\hat{\phi}_{eqk} = \hat{\phi}_{j_0k} = 1$

$$W_{eqk} = \frac{1}{2} J_{eqk} \hat{\phi}_{eqk}^2.$$  

Considering that the kinetic energies of the main system and of the SDOF equivalent system are equal, the equivalent inertia and therefore the equivalent stiffness at the position $j^*$ are defined by equations (9) and (10) respectively:

$$J_{eq} = \sum_{j=1}^{4} J_j \hat{\phi}_{j_k}^2 = 2 W_{kin,k}, \quad (9)$$

$$k_{eq} = J_{eq} (2\pi f_k)^2. \quad (10)$$

For the optimization of the tuned vibration absorber, an equivalent SDOF system defined respect to the third resonance frequency is used ($f_k=89.55$ Hz with $k=3$). In the following, the position $j^*$ indicates the collocation of the absorber in the main system.

3.1 Design of the tuned vibration absorber

The characteristic quantities of the damped dynamic vibration absorber are selected such that the amplitudes of the initial system are as small as possible. For the definition of the SDOF system at the third resonance, the vector $\Phi_3$ is considered. In particular, it is composed by the mode shapes of the four bodies at 89.55 Hz.

By an impedance formulation, the FRF of $\dot{\theta}/t$ of each body for the undamped MDOF main system is defined in figure 6. It is possible to see, at the frequency of interest, the gear box results excited more than the other bodies. Using the above energetic formulation, the inertia and torsional stiffness of the equivalent system is defined.

In literature, typical values of inertia ratio ($\mu$) between the absorber and equivalent system inertia are suggested in the range of 0.15-0.25. For this simple case, a value of $\mu=0.2$ has been chosen and the optimal natural frequency $f_{abs}$ and damping ratio $\varsigma_{opt}$ for the dynamic vibration absorber can be calculated [2]:

![Figure 5](image-url)
\[ f_{\text{abs}} = \frac{1}{1 + \mu} f_k, \quad \zeta_{\text{opt}} = \left( \frac{3\mu}{8(1 + \mu)} \right)^{1/2}. \]  

In figure 7 the full line represents the FRF of the SDOF equivalent rotational speed over excitation. The amplitude value is connected to the collocation of the harmonic excitation in the main system.

The dynamic behaviour of the equivalent SDOF system when it is attached to the TVA is shown with a dashed line. Once the mechanical parameters of the absorber are defined, the optimal location in the MDOF system has to be carefully investigated.

3.2 Optimal position of the tuned vibration absorber

Using the lumped mass simulation is possible to evaluate the effects when the absorber is attached in different positions. Considering the engine not alterable, three different positions for attaching the absorber have been studied. The evaluation is focused on two analyses in the complete frequency range: rotational speed amplitude analysis and kinetic energy reduction analysis.

The first analysis has been performed in order to exclude all the locations that produce a deterioration of the global system in terms of amplitudes. When the TVA is collocated in the remaining positions, its effect is visible only in the small frequency range around the third mode; no effects due to the absorber are detected for the other initial resonances.

The second analysis regards the efficiency of the absorber. Between the cases that satisfy the first analysis, the location that leads the higher kinetic energy reduction, has been chosen. By this comparison, the maximum reduction of the third resonance (without affecting the other resonance frequencies) is obtained when the absorber is attached to the gear box. Therefore, using an optimal damping ratio of 0.25, given by an inertia ratio of 0.2, a vibration reduction of 7.80 dB at the resonance frequency is obtained. In figure 8, the effect of the TVA on the MDOF dynamics is shown in the complete frequency range, while a particular around the third resonance is displayed in figure 9.

Finally, using the configuration defined above, a global maximum kinetic energy reduction in the range of interest of almost 7 dB is obtained. In terms of transfer function the evaluation of kinetic energy as function of the square of excitation force amplitude is possible. By exciting the main system with TVA at the third initial resonance, a reduction of 0.1 J/N² respect to the initial kinetic energy calculated for the system without absorber is achieved. To summarise, in table 2 the complete set of mechanical parameters used for the simulation are shown.

4. Introduction of energy harvesting device

The simulation results defined in the previous chapter are obtained with the introduction of an optimal absorber in the system. Such device presents a hardly feasible damping ratio. The main idea for the following evaluation is to consider a more realistic value of the absorber mechanical damping, and to increase the damping effects with another alternative way. When an energy harvesting device is introduced in the TVA, the vibration mechanical energy can be converted in electrical energy, and the feedback of the electrical circuit on the structure can be used to amplify the damping effect. Several models are presented in literature, most of them consider simplified mechanical system or electrical circuits. The SDOF models representative of a real system, are able to show only one resonance frequency. Therefore, the higher frequency modes are not considered. However, they can have effects on the single mode, in particular when they are closely spaced. Although considering the higher modes, more accurate solutions can be generated, they have been developed only with a pure resistive load as electrical circuit. In reality, the circuits connected to the piezoelectric transducers are more complex and their effects on the dynamics is not negligible [13]. In order to consider the influence on the mechanical system given by the electrical circuit, and viceversa, a model of the TVA attached to the MDOF, considering the introduction of the piezoelectric harvesting device coupled with the energy extraction circuit, is implemented.
Figure 6. FRF of $\dot{\theta}/t$ for the different bodies in the undamped MDOF main system.

Figure 7. FRF of $\dot{\theta}_{eq}/t$, without — and with TVA ---.

Figure 8. FRF of $\dot{\theta}_{gearbox}/t$ in the entire frequency range studied.

Figure 9. Particular of the FRF of $\dot{\theta}_{gearbox}/t$ in a frequency range around the third resonance.

Table 2. Mechanical parameters utilized in the simulation.

| Component                  | Inertia [Kg*m²] | Stiffness [Nm/rad] | Damping [Nm*s/rad] |
|----------------------------|-----------------|--------------------|--------------------|
| Engine                     | 0.0650          | -                  | -                  |
| Flywheel                   | 0.0358          | 60000              | 0.90               |
| Gear box                   | 0.0120          | 3400               | 2.30               |
| Eddy Current Brake         | 0.2500          | 1700               | 0.06               |
| Tuned Vibration Abs        | 0.0027          | 594                | 0.63               |
4.1 Model of piezoelectric energy vibration absorber

A framework of the system act to reduce the vibration of the gear box and to produce electric energy using a piezoelectric material is shown in figure 10.

Electrical power is generated when relative rotational velocity between the main system and the TVA mechanically deforms the piezoelectric material. The electric voltage $v(t)$ produced is considered constant over the thickness of the piezoelectric element.

![Figure 10. Schematic draw of the gear box, connected with TVA by spring damper system, and the piezoelectric material.](image)

The equations of motion of the MDOF system modified by the introduction of the energy harvester are:

\[ J\ddot{\theta} + C\dot{\theta} + K\theta + \alpha v = t, \]
\[ -\alpha^T \dot{\theta}_{rel} + C_p \dot{\psi} = -\dot{q}, \]

where $\alpha = \alpha^T$ is the electromechanical coupling matrix; $C_p$ is the piezoelectric capacitance; $\dot{\theta}_{rel}$ is the relative rotational speed between gear box and TVA. Finally, $q$ is the electric charge. The equations defined above can represent a real power harvesting system only if the piezoelectric transducer is connected to a rectifier circuit. In figure 11 such connection is illustrated, and a full wave diode bridge rectifier circuit is considered, with a capacitive filter $C_L$ and a resistive load $R_L$ that represents the external load. The rectifier circuit is necessary to convert the AC generated by the piezoelectric element to DC, in order to charge a battery or to feed directly an electronic device.

4.2 Optimization of the energy harvester circuit

The goal of this section is to maximize the power generated ($P_{avg}$) by the piezoelectric harvester. Moreover, since the capacitor is required to collect the energy produced, the maximization of the energy stored ($E_c$) needs to be considered. A small unwanted residual periodical variation of the DC output, due to incomplete suppression of the alternating waveform within the power supply, is represented by the ripple parameter. Because it indicates a characteristic quality voltage DC, it has to be as close as possible to zero and it must be taken into account in the optimization.

![Figure 11. Schematic draw of the gear box, connected with TVA by spring damper system, and the piezoelectric material.](image)
The set of relations that needs to be used for the optimization of the circuit are defined in equations (15)-(16)-(17).

\[
P_{\text{avg}} = \frac{v_{\text{avg}}^2(t)}{R_L}, \quad (15)
\]

\[
E_C = \frac{1}{2} C_L v^2(t), \quad (16)
\]

\[
\text{ripple [\%]} = \frac{v_{\text{rmsAC}}}{v_{\text{avg}}} \times 100, \quad (17)
\]

where \(v_{\text{rmsAC}}\) is the root mean square of the component AC of the signal, and \(v_{\text{avg}}\) the average DC voltage. The ripple calculation is possible only for stationary voltage signals. In this case the root mean square can easily be calculated.

The relative rotational speed between TVA and gear box reaches its maximum at 68.7 Hz. Since the electric power generated is proportional to the relative velocity, this frequency can be used for the optimization procedure. Following the technique developed by Motter [10] for the optimization of the electric circuit, the voltage generated from the transducer due to the mechanical excitation has been represented by a voltage generator. With such procedure, once piezoelectric material and the electrical parameters for the diodes have been selected, the optimal capacitance \(C_L\) and the optimal resistance \(R_L\) can be defined. In table 3 the main parameters used in the simulation are summarize:

Table 3. Electrical parameters utilized in the simulation.

| Parameter         | Value     |
|-------------------|-----------|
| \(V_D\)           | Forward voltage drop diode | 50 mV |
| \(R_D\)           | Internal resistance diode | 280 Ω |
| \(C_P\)           | Piezoelectric capacitance | 20 nF |
| \(k^2\)           | Electromechanical adimensional coupling | 0.07 |
| \(R_L\)           | Optimal load resistance | 40 kΩ |
| \(C_L\)           | Optimal load capacitance | 1 mF |

Figure 12 shows the FRF of \(\dot{\theta}_{\text{gearbox}}/t\) when various absorber damping factors are simulated. The range of frequency where the rotational velocity of the gear box is absorbed increases when the damping ratio progressively approaches the optimal value. However, with the same variation of damping, an opposite behaviour of the electric power generated is detected in figure 13. Can be observed that a small value of damping ratio of the dynamic absorber lead to an higher value of power generated than a more damped system. This is due to the higher relative rotational speed between the two parts connected with the piezoelectric material.
Two extreme working points can be highlighted (figure 14): the frequency where the maximum reduction is obtained and the frequency where the highest power is generated, at 77 Hz and 68.7 Hz respectively. With the increasing of the damping factor, the reduction at the antiresonance (77 Hz) gradually decrease but the absorption effect is visible in a wider range of frequency.

In figure 15 are shown the simulation results for the third resonance frequency (89 Hz). A simulation with and without energy harvesting (EH) is conducted in order to evaluate the feedback of the electric circuit on the mechanical behaviour. The introduction of the electric damping is detected, from the difference between the two curves and a maximum value of reduction of 5.70 dB is obtained introducing the energy harvesting in the system. Regarding the power generated at this frequency, an increasing of the mechanical damping lead to a deterioration of the power generated.

Considering a more reasonable, respect to the optimal value, damping ratio of 0.025 a compromise solution is founded. In this case, a vibration reduction of 4.90 dB and a power generated of 30 μW/Nm is obtained. Variation in the damping value and/or in the working frequency can be used to enhance one of the two effects described.

5. Conclusions and outlook
To validate the simulation model of the drive-train, a set up of the rotating system has been built. The evaluation of the dynamic torque in different positions during the real operation is performed and compared with the simulation results.

Using the modal analysis, the frequency response function for each body of the MDOF system is achieved, and resonance frequencies and modal shapes are detected. Through a kinetic energy approach, the initial system has been reduced to an equivalent SDOF system for a selected resonance. For the equivalent system the Den Hartog method, for the definition of an optimal TVA, can been used and the influence of the absorber position in the MDOF system is evaluated.

Once the selection of the optimal collocation for the absorber has been made, the model complexity has been increased by the introduction of the piezoelectric transducer.

The dual effect of the transducer as device for the power generation and as additional damping to the structure has been discussed. The damping ratio defined by the Den Hartog theory, in fact, is not feasible with the materials commonly used in the automotive fields, for example steel and aluminium. Using a more reasonable value, its effect can be increased by the introduction of an electric damping. It is due to the flowing into an electrical circuit of the current generated from the piezoelectric materials, in response to an applied mechanical stress. The circuit considered in the model is composed by a full wave diode bridge rectifier for the conversion of AC in DC connected to a capacitor and a resistor in parallel in order to simulate the energy storage system.

The capacitance and resistance can be optimized by numerical evaluation of the energy stored, of the ripple parameter and finally of the average power generated. When the complete electrical circuit
is modelled, the simulation of the MDOF system connected to the energy harvesting TVA can be implemented.

The simulation results shown the existence of one working frequency where the vibration reduction of the host structure is maximum (28 dB for the undamped system at 77 Hz) and another one where the power generated is maximum (370 $\mu$W/Nm for the undamped system at 68 Hz). Higher the damping, lower the two maxima.

Considering the simulation results at the third resonance frequency for different damping ratios, an intersection point between the curves of vibration reduction and power generation can be observed. The reduction resulted using a damping ratio of 0.025 is comparable with the reduction obtained using the optimal damping factor (4.90 dB instead of 5.20 dB). This result, is due to the feedback generated from the mechanical-to-electrical energy conversion. With a power generation of 30 $\mu$W/Nm the intersection point represents the best compromise between dynamic behaviour and electric power generation.

More work is in progress in order to further investigate the behaviour of the rotational energy harvesting vibration absorber. In particular, the rotordynamic effects due to the introduction of the TVA in the MDOF system needs to be investigated. These are not considered in this paper. Moreover, additional studies needs to be carried out in order to predict the optimal electric parameters of the storage system.

Finally, after the prototyping of the energy harvesting tuned vibration absorber, the experimental testing and validation in the drive-train is planned.

References
[1] Frahm H 1909 Device for damping vibrations of bodies U.S. Patent No. 989958
[2] Den Hartog J P 1956 Mechanical Vibrations (4th Edition McGraw-Hill Editor)
[3] Krenk S 2005 Frequency analysis of the tuned mass damper ASME J. Applied Mechanics 72 pp 936-942
[4] Warburton G B 1982 Optimum absorber parameters for various combinations of excitation parameters J. of Earthquake Engineering and Structural dynamics 10 pp 381-401
[5] Ziletti M 2012 Optimisation of dynamic vibration absorbers to minimise kinetic energy and maximise internal power dissipation J. of Sound and Vibration 331 pp 4093-4100
[6] Rana R 1998 Parametric study and simplified design of tuned mass damper J. Structural Engineering 20 pp 193-204
[7] Xu Y L 1994 Semi-analytical method for parametric study of tuned mass dampers J. Structural Engineering 120 pp 747-764
[8] Lewis F M 1955 The extended theory of the viscous vibration damper ASME J. Applied Mechanics 22 pp 377-382
[9] Ozer M B, Royston T J 2005 Extending Den Hartog’s vibration absorber technique to multi-degree-of-freedom systems ASME J. of Vibrations and Acoustics 127 pp 341-350
[10] Motter D, Lavarda J V and Da Silva S 2012 Vibration energy harvesting using piezoelectric transducer and non-controlled rectifiers circuits J. of the Braz. Soc. Of Mech. Sci. & Eng. 34 pp 378-385
[11] Clementino M A, Brennan, M J and Da Silva S 2014 Optimization of the electrical and mechanical parameters of a vibration energy harvesting Blucher Mechanical Engineering Proceedings
[12] Infante F, Perfetto S, Mayer D and Herold S 2015 Modelling of drive-train using a piezoelectric energy harvesting device integrated with a rotational vibration absorber
[13] Elvin N G and Elvin A A 2009 A general equivalent circuit model for piezoelectric generators J. of Intelligent Material Systems and Structures 20 pp 3-9

Acknowledgments
The authors gratefully acknowledge the European Commission for its support of the Marie Sklodowska-Curie program through the ITN EMVeM project (GA 315967).