Descriptive and Computer Aided Drawing Perspective on an Unfolded Polyhedral Projection Surface

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Abstract. The aim of the herby study is to develop a method of direct and practical mapping of perspective on an unfolded prism polyhedral projection surface. The considered perspective representation is a rectilinear central projection onto a surface composed of several flat elements. In the paper two descriptive methods of drawing perspective are presented: direct and indirect. The graphical mapping of the effects of the representation is realized directly on the unfolded flat projection surface. That is due to the projective and graphical connection between points displayed on the polyhedral background and their counterparts received on the unfolded flat surface. For a significant improvement of the construction of line, analytical algorithms are formulated. They draw a perspective image of a segment of line passing through two different points determined by their coordinates in a spatial coordinate system of axis x, y, z. Compared to other perspective construction methods that use information about points, for computer vision and the computer aided design, our algorithms utilize data about lines, which are applied very often in architectural forms. Possibility of drawing lines in the considered perspective enables drawing an edge perspective image of an architectural object. The application of the changeable base elements of perspective as a horizon height and a station point location enable drawing perspective image from different viewing positions. The analytical algorithms for drawing perspective images are formulated in Mathcad software, however, they can be implemented in the majority of computer graphical packages, which can make drawing perspective more efficient and easier. The representation presented in the paper and the way of its direct mapping on the flat unfolded projection surface can find application in presentation of architectural space in advertisement and art.

1. Introduction

Perspective as a method of a visual representation embraces a wide variety of fields, such as architecture, interior design, industrial design, engineering design and fine arts among others. The history of perspective is very old. At first perspective was regarded as a practical method of representation of a scene from a specific view point by an artist [1]. However, only the development of descriptive geometry as a branch of mathematics in seventeenth century enabled further study of the theory of perspective as a method of projection [2,3,4]. Perspective drawing has been much discussed as a medium for design and for communication in architecture, both during preliminary architectural work and at an advanced stage [5,6].

Much work has been done in the field of perspective analyses and perspective construction of the architectural environment onto a flat projection plane [3]. Most researches concentrate on a descriptive construction of linear perspective of different types.
In this paper, we present a new approach to rectilinear perspective. Contrary to the commonly applied methods of perspective representation onto a flat picture plane, we consider perspective projection onto a non-flat projection surface. The projection surface is composed of several flat elements and it has a prism polyhedral shape. The idea of drawing perspective is to compose a perspective image directly on an unfolded projection surface. The methods of perspective construction developed in the paper are both descriptive - presented in section 3, and with computer aid - presented in section 4.

2. Geometrical aspects of perspective projection onto a polyhedral surface

According to its descriptive geometry definition, perspective is a central projection from a real point onto a projection plane/surface, and it is subjected to projective geometry rules. The linear perspective system creates the illusion of depth onto a two-dimensional plane by use of so-called ‘vanishing points’ to which all parallel lines converge at the level of horizon, which is an eye level. The base elements which define linear perspective are: a picture plane, a station point, the depth of sight, a horizon line, and a ground line.

The apparatus of perspective projection onto a polyhedral surface is received from an apparatus of a vertical linear perspective onto a flat projection plane by replacing the flat projection plane with the polyhedral surface. Due to this fact, the apparatus of the considered representation is composed of a polyhedral projection surface $\tau$, a viewpoint $S$ and a base plane $\pi \perp \tau$, figure 1a. Defining the apparatus of perspective projection in this way, a perspective image of any point $F$ is a pair of two points $(F^S, F^{OS})$, figure 1b.

![Figure 1. Perspective on a polyhedral surface: a - a structure of an apparatus, b - a perspective image $(F^S, F^{OS})$ of a point $F$, c- a perspective image of a straight line $m$](image)

The point $F^S$ is a central projection of $F$ onto $\tau$ from $S$, whereas $F^{OS}$ is a central projection of $F^O$ onto $\tau$. $F^O$ is an orthogonal projection of $F$ onto a base plane $\pi$. Both points $F^S$ and $F^{OS}$ are included in the same vertical line $\ell$ being a projection of a vertical line $t$ from $S$ onto $\tau$. The point $F^S$ is a main projection, whereas the point $F^{OS}$ is a supplemental projection enabling restitution, figure 1b.

The perspective main projection of a straight line depends on its location towards the apparatus. It can be a point when a line is projecting, a vertical line when a line is vertical, a horizontal angular line when a line is horizontal. However, the image of a straight line $m$ not particularly situated towards any
elements of the projection apparatus is an angular line $m^S$ contained in a projection surface $\tau$, figure 1c.

3. **Descriptive construction of perspective on an unfolded projection surface**

Let us consider a simple architectural building which location towards a projection surface is given in the figure 2.

![Figure 2. The location of a model building towards projection surface: a - a pictorial drawing, b - orthographic projections](image)

The starting point in our construction process is establishing base elements of perspective such as a horizon line, a station point, the depth of sight, and a ground line, as well as defining a model building by its metric properties.

The metric properties of the presented building and of the projection apparatus can be read from their horizontal and vertical views. The height of the building was assumed as 9 m and the height of horizon was assumed as 8.57 m.

3.1. **An indirect descriptive method of perspective construction**

The perspective drawing is realized on the unfolded projection surface on the basis of the orthographic projections.

The first step is establishing vanishing points for two horizontal and mutually perpendicular lines of the building, as well as establishing the piercing points of these lines with the projection surface, see figure 3.
Figure 3. Establishing vanishing points and piercing points of two mutually perpendicular line directions of the building

The construction of a perspective image in each strip of a flat surface is similar to a classical construction of perspective on a flat projection plane. However, due to the fact that some lines are projected onto the different strips of the surface, their perspective images are not straight lines but angular lines, figure 4.

Figure 4. An indirect construction of perspective projection of a building on the basis of its orthogonal projections

3.2. A direct descriptive method of perspective construction
In order to construct perspective in a direct way, it is necessary to find the graphical connection between a perspective image of a point included in a folded projection surface and the perspective image of this point received on an unfolded surface. Similar attempt was presented in [7] in the case of perspective projection on a flat projection plane.
Figure 5. Central projection of the line $t$ onto $\tau$ and the line $\hat{t}$ onto $\pi$ with characteristic points included in them.

Let us consider a vertical straight line $\hat{t}$ being a perspective projection of a line $t$ onto $\tau$ and its central projection $S^t$ from $S$ onto $\pi$, figure 5. It can be stated that cross ratio of quadruple of collinear points $(F^0, S, W, F)$ included in $\hat{t}$ is preserved during this projection. Similarly, considering the perspective projection of the line $t$ from the point $S$ onto $\tau$, it can be stated that cross ratio of quadruple of collinear points $(F^0, H, F, W)$ is invariant during projection.

Projective relations between a range of points included in $\hat{t}$ and a proper range of points included in $S^t$ permit to establish a graphical connection between a range of points included in $\hat{t}$ and a range of points included in an equivalent line $\hat{t}$ on an unfolded/developed surface. Due to this fact, placing the development of the surface directly on a horizontal view, as it is shown in the figure 6, it is easy to find a perspective image of this part of the building which is included in the base plane $\pi$. In order to construct a perspective image of the rest of the building it is convenient to act in the similar way as in the figure 4.

Figure 6. Construction of a perspective image directly on a development of the projection surface linking it with a horizontal view of the building and the projection surface.
4. Drawing perspective with computer aid

A quick development of CAD technology enables creation of perspective representations with computer aid. A computer program usually uses linear algebra, and in particular a matrix multiplication, to create a perspective image. It enables finding the corresponding coordinates of points of a model to the coordinates of points on a screen. The problem of estimating a perspective projection matrix is the subject of the considerable number of publications of which some important results are presented in [8, 9].

Compared to other perspective construction methods that use information about points, for computer vision and the CAD system, it was more convenient for us to utilize data about lines. Due to this fact, we place a Cartesian coordinate system of axis $x, y, z$ and a Cartesian coordinate system of axis $v, d$ as shown in the figure 5 and 7a. The projective relations between points established earlier, permit creation of the analytical algorithms for drawing a perspective image of the segment of a line passing through two different points. These points are established by their coordinates given in a spatial coordinate system of axis $x, y, z$. The line is drawn as a plot of function $d(v)$ in the Cartesian coordinate system of axis $d, v$ located on the unfolded surface, figure 5, 7a. The plot is realized by Mathcad software. As a starting point for any computer aided construction, it is necessary to determine distances $r_{d1}, r_{d2}...r_{dn}$ of the station point to each strip of the surface, figure 7a. It is also necessary to establish angles $a_1, a_2, ... a_n$ between edge projection rays for each strip, figure 7a.

Possibility of drawing lines in considered perspective enables drawing an edge perspective image of an architectural object. As the example of the perspective image drawn as a Mathcad plot, it is shown the perspective image of the same building, which is presented by descriptive constructions in section 3, see figure 7b.

![Figure 7. The perspective image of the building: a - assumptions, b - a Mathcad plot.](image-url)
5. Results and discussion

It is possible to construct perspective on an unfolded polyhedral projection surface in a descriptive way basing partly on the rules of the classical linear perspective onto flat projection surface. However, computer aided drawing makes construction of perspective much easier. The elaborated algorithms for drawing perspective on an unfolded prism polyhedral surface have been formulated and tested in Mathcad software. However, they can be implemented in the majority of graphical packages. Thanks to the application of the changeable base elements of perspective (the horizon height and the station point location) defining perspective projection, the algorithms are universal for drawing a perspective image. In the case when a polyhedral projection surface is a regular prism, we achieve a full panoramic image, which can be an approximation of a cylindrical panoramic image. Such a case will be a subject of the further consideration.

6. Conclusions

In this paper, we attempted to elaborate on the method of descriptive and computer aided drawing perspective on an unfolded polyhedral projection surface. The analytical algorithms elaborated by us and implemented in any graphical package can make drawing polyhedral perspective efficient and convenient. Therefore, they can find application in presentation of architectural space in advertisement and art when drawings are displayed on the polyhedral surfaces.

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