Stabilization for switched linear systems: Hybrid observer-based method

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Abstract
This paper investigates interval observer-based controller design for switched linear systems involving additional disturbance and measurement noise, whose modes need not to be cooperative. First, by giving the upper bound and the lower bound of the disturbance and the measurement noise, we construct hybrid interval observer for the considered switched linear system by means of a switched coordinate transformation, which can transform the observer error systems into cooperative ones. The interval observer can supply certain state information at any instant. Compared with the interval observer design for switched systems with arbitrary switching sequences or dwell time switching based on common Lyapunov function, the difficulty consists in characterizing the jump of the multiple Lyapunov functions. Then, by using the multiple Lyapunov functions method and average dwell time scheme, some sufficient conditions are derived and applied to build the interval observer-based state feedback controller. Finally, we provide an example to illustrate the validity of the derived results.

1 | INTRODUCTION

Switched systems exist widely in many practical systems [1–3], such as circuit systems [4], traffic control systems [5], etc. Stability analysis is a fundamental problem of switched systems. When all the subsystems are stable, the switching signal may not guarantee the stability of the whole switched system, that is, the overall stability is not equal to the simple superposition of each single model stability. Research on stability of switched systems has been a hot topic and many achievements have been obtained [6–16]. For example, the global finite-time stabilization of a class of switched nonlinear systems with the powers of positive odd rational numbers was studied in [14]. Work in [15] provided multiple Lyapunov function-based small-gain theorems for switched interconnected nonlinear systems. Long and Zhao [16] discussed $H_{\infty}$ control of switched nonlinear systems in $p$-normal form by using multiple Lyapunov functions. Note that all the above-mentioned results focused on the stability properties and control synthesis of the switched system with synchronous switching signals. However, in practice, there is often a lag between the system and its associated controller, so the asynchronous switching signal is inevitable. Under asynchronous switching, the robust observer design problems, the output-feedback control, the adaptive output feedback fuzzy stabilization for switched systems were discussed in [17–19], respectively.

State estimation is an important method for state feedback of dynamic systems with unmeasurable states. Interval observers, as a new method of state estimation, mainly deal with the state estimation of dynamic systems with large uncertainties. Since...
the interval observer method was first proposed in reference [29], it has attracted a lot of attention and has been applied in vehicle positioning [21], fault detection [22], etc. Unlike the traditional observers, the interval observer realizes the state estimation by providing the upper and the lower bounds of the state of the observed system at any time and brings part of the uncertain information into the process of the observer architecture as part of the design, which makes the observer have great tolerance for uncertainties [23–25]. Due to the dynamic complexity of switched systems, the interval observer-based controller design for switched systems is much more complicated than usual single model systems. Therefore, there are few literatures about the design of interval observers for switched systems. Huang et al. [26] discussed the problem of interval observer design, which makes the observer have great tolerance for uncertainties [23–25]. Due to the dynamic complexity of switched systems, the interval observer-based controller design for switched systems is much more complicated than usual single model systems. Therefore, there are few literatures about the design of interval observers for switched systems. By monotony method, Huang et al. [26] discussed the problem of interval observer design for discrete-time switched systems. For a class of switched systems with additional disturbance, Wang et al. [28] studied the design of hybrid interval observers for switched linear time-invariant systems. In [29], based on the condition that \( A_{i} L_{i} C_{i} \) are both Metzler and Hurwitz, an interval observer was designed for linear switched systems. But in many practical cases, \( A_{i} L_{i} C_{i} \) do not always meet the above conditions.

Motivated by the above-mentioned concerns, the present work studies the design of hybrid interval observer-based controller for continuous-time switched linear systems with disturbance and measurement noise under asynchronous switching, whose modes need not be cooperative. First, by giving the upper bound and the lower bound of the disturbance and the measurement noise, we construct hybrid interval observer for the considered switched linear system by means of a coordinate transformation, which can transform the observer error systems into cooperative ones. Second, we study the asynchronous control of switched systems based on the improved interval observer by allowing the switching delay to be time-varying. Third, we derive the sufficient condition to guarantee the stability of the switched system when the switching signals satisfying an average dwell time scheme, and further establish the interval observer-based state-feedback controller gains. And last, we provide an example to illustrate the validity of the derived results.

The contribution compared with the literature can be summarised as the following aspects: (1) Unlike the literatures [26, 27], which focused on synchronous and asynchronous interval observers for switched discrete-time systems and did not deal with the controller design, we study the asynchronous control based on interval observers for switched continuous-time systems, that is, the asynchronous phenomenon occurs between the controller and the subsystems. (2) In [28], a switched time-varying coordinate transformation method was used for switched time-invariant systems, and the derived sufficient conditions for the construction of interval observers are related to the switching times \( t_{s} \), which makes it difficult to check these conditions. Compared to the system considered in [28], the system considered in this paper is more extensive. Besides, the interval observer constructed in this paper is time-invariant, and the sufficient conditions for its existence are time-independent and easier to be checked. (3) The interval observers encountered in [29, 30] were designed for cooperative error systems. The interval observers available in the paper are made for the systems, which may not be cooperative. (4) To avoid the switched system being transformed into a hybrid system, Ethab et al. [31] designed the interval observers in the original basis ‘\( x \)’ and only required that the upper and the lower bounds for the initial states make the errors \( E_{q}^{+}(0) \) and \( E_{q}^{-}(0) \) are nonnegative. But in fact, regardless of the use of the original basis ‘\( x \)’, the coordinate has been used to ensure the cooperativity, thus it has to make sure that the errors \( E_{q}^{+} \) and \( E_{q}^{-} \) are non-negative at each switching time, which would certainly require the interval observer system be a hybrid system. (5) In [32], the hybrid framer of the switched system was designed to satisfy the upper and the lower bounds, i.e. \( x \leq x \leq \tilde{x} \), but the framer was not necessarily asymptotically stable. Besides, the conditions for the construction were derived based on common Lyapunov function. This paper adopts multiple Lyapunov functions method for the asympotical stability of the hybrid framer, so the sufficient conditions given in the paper are less conservative.

The structure of the paper is as follows: Section 2 describes preliminaries and problem formulation. In Section 3, the main results are given. Simulation results are given to illustrate the effectiveness of the proposed methods in Section 4. Section 5 concludes the paper.

**Notations.** \( \mathbb{R}^{n} \) denotes the n dimensional Euclidean space. For a matrix \( P \), \( P > 0 \) \((P < 0) \) means that \( P \) is positive definite (negative definite); \( \lambda_{\max}(P) \) and \( \lambda_{\min}(P) \) denote the maximum and minimum eigenvalues of the matrix \( P \), respectively. \( I \) and \( 0 \) denote the identity matrix and zero matrix with appropriate dimensions, respectively. \( E \) denotes the vector whose elements are all ones. For any two vectors \( x_{1}, x_{2} \in \mathbb{R}^{n} \), the relations \( x_{1} \leq x_{2} \) are understood elementwise, i.e. if \( x_{1} = [x_{11} \ldots x_{12}]^{T} \leq x_{2} = [x_{21} \ldots x_{22}]^{T} \), then \( x_{11} \leq x_{21} \) and \( x_{12} \leq x_{22} \). \( P^{T} \) and \( P^{-1} \) denote the transpose and the inverse of a square matrix \( P \). A matrix \( M \in \mathbb{R}^{m \times n} \) is said to be Metzler if each off-diagonal entry of it is non-negative. Euclidean norm for a vector \( u \in \mathbb{R}^{n} \) will be denoted as \( |u|_{E} \). For a measurable and locally essentially bounded function \( u : \mathbb{R}^{+} \rightarrow \mathbb{R}^{n} \), the symbol \( ||u|_{1}||_{E} \) and \( ||u|_{2}||_{E} \) denote the upper and lower bounds, respectively. \( E \) denotes the vector whose elements are all ones. For any two vectors \( x_{1}, x_{2} \in \mathbb{R}^{n} \), the relations \( x_{1} \leq x_{2} \) are understood elementwise, i.e. if \( x_{1} = [x_{11} \ldots x_{12}]^{T} \leq x_{2} = [x_{21} \ldots x_{22}]^{T} \), then \( x_{11} \leq x_{21} \) and \( x_{12} \leq x_{22} \). \( P^{T} \) and \( P^{-1} \) denote the transpose and the inverse of a square matrix \( P \).

**2 | PRELIMINARIES AND PROBLEM FORMULATION**

Consider the following switched linear system

\[
\begin{align*}
\dot{x}(t) &= A_{\xi} x(t) + B_{\xi} u(t) + w_{q}(t) \quad \forall \theta_{0} \geq 0, \\
\dot{y}(t) &= C_{\xi} x(t) + v_{q}(t)
\end{align*}
\]  

(1)

where \( x(t) \in \mathbb{R}^{n} \) is the system state; \( x(\theta_{0}) = x_{0} \) is the initial state vector and assumed to be bounded by the known...
bound: $|x_0| \leq \varsigma E$, where $\varsigma > 0$ is a scalar constant; $u(t) \in \mathbb{R}^q$ is the input; $y(t) \in \mathbb{R}^p$ is the measurable output vector. $\sigma(t) : \mathbb{R}^+ \rightarrow M = \{1, 2, \ldots, m\}$ is a piecewise constant function of time $t$ and called switching signal, $m$ is the number of subsystems. Corresponding to $\sigma(t)$, it has the switching sequence $\{(t_0, t_0), \ldots, (t_k, t_k), \ldots\}$ with $t_k \in [t_k, t_{k+1})$, $k \in \mathbb{N}$. For any $i \in \{ M, A_i, B_i, C_i \}$ are known real constant matrices of appropriate dimensions, $w_i(t)$ and $v_i(t)$ are the disturbance and the measurement noise, respectively. We assume that the state of the system does not jump at the switching instants and that only finitely many switches can occur in any finite interval. In addition, for any $i \in M$, we assume that the pair $(A_i, C_i)$ is observable.

The description of the main result requires the following definitions and lemmas.

**Definition 1.** Consider a switched system

$$\dot{x} = f_k(t, x(t), w(t))$$

with $x \in \mathbb{R}^n$, $w \in \mathbb{R}^l$ and with $f_k, g \in M$ of class $C^1$. The disturbance $w_i$ is Lipschitz continuous and such that there exist two known bounds $w_i(t), w_i(t) \in \mathbb{R}^l$, Lipschitz continuous, and such that, for all $t \geq 0$

$$w_i(t) \leq w_i(t) \leq w_i(t).$$

Moreover, the initial condition $x(0) = x_0$ is assumed to be bounded by two known bounds:

$$-\varsigma_0 \leq x_0 \leq \varsigma_0.$$  

Then, the dynamical system

$$Z = \varphi(x, x, 0)$$

associated with the initial condition $Z_0 = G(t_0, x_0, x_0) \in \mathbb{R}^{n\times n}$, $\tilde{v}(t) = (w(t), 0) \in \mathbb{R}^l$, the switched signal $\sigma$, and bounds for the solution $x$:

$$-\varsigma_0 \leq x \leq \varsigma_0.$$

Then the dynamical system

$$\dot{Z} = \varphi(x, x, 0)$$

with $\varphi_i(x) \in M$, $H_0, H_i, G$ Lipschitz continuous of appropriate dimension, is called an interval observer (resp. an exponentially stable interval observer) for (2) if

(i) for all Lipschitz continuous function $\tilde{w}(t)$, all the solutions of (5) are defined over $[0, +\infty)$;

(ii) for any vectors $\varsigma_0, \varsigma_0, \varsigma_0$ in $\mathbb{R}^n$ satisfying (4), the solutions of (2), (5) with respectively $x_0, Z_0 = G(t_0, x_0, x_0)$ as an initial condition at $t = t_0$, denoted respectively $x(t), Z(t)$, are defined, for all $t \geq t_0$, and satisfy, for all $t \geq t_0$, the inequalities

$$-\varsigma_0 \leq H_i(t, Z) \leq \varsigma_0 \leq H_i(t, Z) = \tilde{z}_i$$

(iii) system (5) is globally uniformly asymptotically stable (resp. globally uniformly exponentially stable) under the switching signal $\sigma$ when $\tilde{w}$ is identically equal to zero.

**Definition 2.** For any $0 \leq t \leq T$, let $N_0(t)$ denote the switching numbers of $\sigma(t)$ over $(t, T)$. If $N_0(t) \leq N_0 + \frac{T-t}{\tau_a}$ holds for $\tau_a > 0$ and $N_0 \geq 0$, then $\tau_a$ is called average dwell time and $N_0$ is called a chattering bound. Denoted by $N_{av}[\sigma, N_0]$ the class of switching signals with average dwell time $\tau_a$ and chattering bound $N_0$.

**Lemma 1.** Consider the system

$$\dot{x} = Ax + w(t),$$

with $x \in \mathbb{R}^n$, $w \in \mathbb{R}^q$, where $A$ is Metzler and $w \geq 0$ is a Lipschitz continuous function, then $x(0) \geq 0$ implies $x(t) \geq 0$ for all positive time $t$.

**Lemma 2.** Let $x \in \mathbb{R}^n$ be a vector satisfying $-\varsigma \leq x \leq \varsigma$ and $A \in \mathbb{R}^{n \times n}$ be a constant matrix, then $A^T \varsigma - A \varsigma \leq A \varsigma - A \varsigma = A \varsigma - A \varsigma$.

**Lemma 3.** Let $\delta > 0$ be a scalar and $S \in \mathbb{R}^{n \times n}$ be a symmetric positive definite matrix, then $2x^T \leq \frac{\delta}{\delta} x^T + \delta y^T y, x, y \in \mathbb{R}^n$.

## 3 | MAIN RESULT

In the section, we will give the design of the interval observer-based controller for the system (1) under asynchronous switching.

### 3.1 | Hybrid interval observer

We suppose that the function $w_i, i \in M$ is external disturbance with known bounds.

**Assumption 1.** The disturbance and the measurement noise are assumed to be unknown but bounded with priori known bounds such that $-\tilde{w} \leq w_i(t) \leq \tilde{w}$, $|v_i(t)| \leq 9 E$ are verified for $\forall t \in \mathbb{R}^+$ and $\forall i \in M$, where $\tilde{w} \in \mathbb{R}^n$ and $\tilde{v}$ is a positive scalar.

**Lemma 4.** Consider matrices $T_i, i \in M$ such that $A_i = T_i(A_i - L_i C_i) T_i^{-1}$ are Metzler. Under Assumption 1, if there exist matrices $N_i > 0, W_i$, and scalars $\varepsilon_i > 0, i \in M$ such that

$$A_i^T N_i - C_i^T W_i^T + N_i A_i - W_i C_i + \frac{5}{\varepsilon_i} N_i < 0$$

then an asymptotically interval observer for the subsystem $i$ of (1) is given by:

$$\dot{x}_i = (A_i - L_i C_i) x_i + B_i u(t) + L_i y + \Theta_i + \Omega_i$$

$$\dot{z}_i = (A_i - L_i C_i) x_i + B_i w(t) + L_i y - \Theta_i - \Omega_i$$
\[
\dot{x} = R_i^+ T_i^+ \delta - R_i^- T_i^0 \delta
\] (13)
\[
\dot{x} = R_i^+ T_i^+ \delta - R_i^{-} T_i^0 \delta
\] (14)
\[
\lambda_{t_0} = R_i(T_i^+ \delta - T_i^- \delta)
\] (15)
\[
\lambda_{t_0} = R_i(T_i^+ \delta - T_i^- \delta)
\] (16)
where \( \Theta_i = T_i^{-1}(T_i^+ \delta + T_i^- \delta) \), \( \Omega_i = T_i^{-1}|T_i|L_i|\delta E_i \), and \( R_i = T_i^{-1} \), the observer gain \( L_i \) is given by \( L_i = N_i^{-1} W_i \).

Proof. The proof of the lemma is similar to the proof of Theorem 5 in [31].

Remark 1. For the single subsystem, we adopt the interval observer framework in [31]. However, for the interval observer of switched systems, we wish to point out that: To avoid the switched system being transformed into a hybrid system, Ethabat et al. [31] designed the interval observers in the original basis ‘x’ and only required that the upper and the lower bounds for the initial states make the errors \( E_{\bar{y}} (0) \) and \( E_{\bar{y}}^-(0) \) non-negative. But in fact, regardless of the use of the original basis ‘x’, the coordinate has been used to ensure the cooperativity, thus it has to make sure that the errors \( E_{\bar{y}}^{+} \) and \( E_{\bar{y}}^- \) are non-negative at each switching time, which would certainly require the interval observer system to be a hybrid system.

For the switched system (1), we construct the following hybrid dynamical systems

\[
\dot{x}_u = (A_{\epsilon_i} - L_{\epsilon_i} C_{\epsilon_i}) x_u + B_{\epsilon_i} u(t) + L_{\epsilon_i} x + \Theta_{\epsilon_i} + \Omega_{\epsilon_i}, \quad t \in \xi_i
\] (17)
\[
\lambda_{t_0} = R_i(T_i^+ x - T_i^- x)
\] (18)
\[
\dot{x}_i = (A_{\epsilon_i} - L_{\epsilon_i} C_{\epsilon_i}) x_i + B_{\epsilon_i} u(t) + L_{\epsilon_i} x - \Theta_{\epsilon_i}, \quad t \in \xi_i
\] (19)
\[
\lambda_i = R_i(T_i^+ x - T_i^- x)
\] (20)
\[
\dot{z}_i = R_i^+ T_i^+ \delta - R_i^- T_i^0 \delta, \quad t \in \xi_i
\] (21)
\[
\dot{z}_i = R_i^+ T_i^+ \delta - R_i^- T_i^0 \delta, \quad t \in \xi_i
\] (22)
\[
\lambda_{t_0} = R_i(T_i^+ \delta - T_i^- \delta)
\] (23)
\[
\lambda_{t_0} = R_i(T_i^+ \delta - T_i^- \delta)
\] (24)
where \( \Theta_i = T_i^{-1}(T_i^+ \delta + T_i^- \delta) \), \( \Omega_i = T_i^{-1}|T_i|L_i|\delta E_i \), and \( T_i, i \in M \) are chosen as in the following theorem and \( R_i = T_i^{-1} \).

Remark 2. In [28], under average dwell time scheme, a switched time-varying interval observer was established for switched linear time-invariant systems. However, the derived sufficient conditions for the construction of interval observer are related to the switching times \( \tau_i \), which makes it difficult to check these conditions. Compared to the system considered in [28], the system considered in this paper is more extensive. Besides, the interval observer constructed in this paper is time-invariant, and the sufficient conditions for its existence are time-independent, so they are easier to be checked.

Remark 3. In [32], the hybrid frame of the switched system was designed to satisfy the upper and the lower bounds, i.e. \( \bar{x} \leq x \leq \bar{x} \), but the frame was not necessarily asymptotically stable. Besides, the conditions for the construction were derived based on common Lyapunov function. In the paper, multiple Lyapunov functions method is adopted for the asymptotic stability of the hybrid frame, so the sufficient conditions given in the paper are less conservative.

Remark 4. In [27], the interval observer design problem for discrete-time switched systems with asynchronous switching law was investigated. It was assumed that the switching signal of the interval observer is asynchronous with the one of the subsystems. Unlike the literature, we study the asynchronous control based on interval observer for switched continuous-time systems, that is, the asynchronous phenomenon occurs between the controller and the subsystems.

### 3.2 Interval observer-based control

Suppose that the systems (17)–(20) process the same switching signal with the system (1). Let \( \bar{x} = x_0 - x \) and \( \bar{e} = x - x_0 \) be the observation errors, and \( \bar{e} = [\bar{z}_i \bar{x}_i \bar{x}_i] \), then, the error system can be established as

\[
\dot{i} = \bar{A}_i \bar{e} + \bar{b}_i \bar{e}, \quad t \in \xi_i
\] (25)
\[
\dot{e}(t) = \bar{\Psi}_i(\bar{z}(t^-), \bar{x}(t^-), \bar{x}(t))
\] (26)
where

\[
\bar{A}_i = \begin{bmatrix} A_i - L_i C_i & 0 \\ 0 & A_i - L_i C_i \end{bmatrix}, \quad \bar{b}_i = \begin{bmatrix} L_i y_i + \Theta_i + \Omega_i - w_i \\ -L_i y_i + \Theta_i + \Omega_i + w_i \end{bmatrix}
\]

The control signal going into the plant is of the form

\[
u(t) = \hat{K}_i \bar{e}(t)_{\tau_d(t)} x_0 + \bar{K}_i \bar{e}(t)_{\tau_d(t)} x_i
\] (27)
where \( \hat{K}_i \) and \( \bar{K}_i \) are the controller gains of the subsystem \( i \), \( \tau_d(t) \) is the uncertain switching delay, satisfying \( 0 \leq \tau_d(t) \leq \tau_i \). Here we assume that the maximal switching delay \( \tau_i \) is known a priori without loss of generality, and \( \tau_i \leq t_{i+1} - t_i, i \in N \).

In the paper, we set \( \sigma_i(t) = \sigma_i(t), \sigma_2(t) = \sigma_i(t - \tau_d(t)) \) and adopt the merging switching signal \( \sigma' = [\sigma_i(t), \sigma_2(t)] \) to study the asynchronous switching signals in a unified framework. The merging action means that the set of switching times of \( \sigma' \) is the union of the sets of switching times of \( \sigma_i \) and of \( \sigma_2 \).

We introduce two lemmas.
Lemma 5. \cite{[33]} Let $\sigma_1(t) \in S_{\text{ave}}[\tau_a, N_0]$ and $\sigma_2(t) = \sigma_1(t - \tau_d(t))$. Then, it has

$$\sigma_2(t) \in S_{\text{ave}} \left[ \frac{\tau_a + \tau_d}{\tau_a} t, N_0, \frac{\tau_a + \tau_d}{\tau_a} t \right], \sigma'(t) \in S_{\text{ave}} \left[ \frac{\tau_a + \tau_d}{2}, 2N_0, \frac{\tau_a + \tau_d}{2} \right].$$

Lemma 6. \cite{[33]} Let $\sigma_1(t) \in S_{\text{ave}}[\tau_a, N_0]$ and $\sigma_2(t) = \sigma_1(t - \tau_d(t))$ for an interval $(t_0, t)$. Let $m(t_0, t)$ be the total amount of time for which $\sigma_1(t) = \sigma_2(t)$, and let $m(t_0, t) = t - t_0 - m(t_0, t)$. If

$$\tau_d(\alpha + \beta) \leq (\alpha - \bar{\alpha}) \tau_d$$

for some positive constants $\alpha$, $\beta$, and $\bar{\alpha} \in [0, \alpha]$, then

$$-\alpha m(t_0, t) + \beta \overline{m}(t_0, t) \leq c - \alpha (t - t_0) \quad \forall t \geq t_0 \geq 0 \quad (29)$$

where, $c = (\alpha + \beta)(N_0 + 1) \tau_d$.

We are ready to state and prove the following result.

Theorem 1. Consider matrices $T_{i, i} \in M$ such that $F_i = T_i(A_{i} - L_i C_i) T_i^{-1}$ are Metzler. Under Assumption 1, for the gain matrices $L_{i, i} \in M$ given by Lemma 4 and known positive constants $\alpha$, $\beta$, and $\mu > 1$, if there exist matrices $P_i > 0$, $Q_i > 0$, $S_i > 0$, $i \in M$ such that

$$\begin{bmatrix}
\sum_{ij} \Delta_{ij} & \Delta_{ij} & \Gamma_{ij} S_i - \Gamma_{ij}^T Q_i + \mu Q_i \\
\sum_{ij} \Delta_{ij}^T & \Gamma_{ij}^T S_i & \Gamma_{ij}^T Q_i - \mu Q_i \\
\sum_{ij} \Delta_{ij}^T & \Gamma_{ij}^T S_i & \mu S_i \\
\end{bmatrix} \geq 0 \quad (30)
$$

holds, which implies

$$n \leq n \leq \bar{n} \quad (35)$$

is true for all $t \in \xi_{i-1}$ by Lemma 2 and (21) and (22). We will show that for any $t \in \xi$, (35) is true.

In fact, since $F_{i, i}$ is assumed to be Metzler, consider (17)–(22), by Lemma 4, it holds that $n \leq n \leq \bar{n}$ for all $t \in \xi$.

Step 2. Due to the switching delay, at the switching instant $t_i$, the controller $K_{i, j}$ is still active for the time $\tau_d(t_i)$ after the subsystem $\ell_i$ has been switched to the subsystem $\ell_j$.

Thus, substituting controller (27) into (17)–(20) and letting $\hat{x} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \\ \hat{x}_4 \end{bmatrix}$, we have, for all $\ell_i, \ell_j \in M, \ell_i \neq \ell_j$,

$$\begin{aligned}
\dot{\hat{x}} &= \hat{A}_{\ell_i, \ell_j} \hat{x} + \hat{B}_{\ell_i, \ell_j} \hat{y} + \hat{v}_{\ell_i, \ell_j}, t \in [t_i, t_i + \tau_d(t_i)] \\
\bar{x}(t) &= \Phi_{\ell_i, \ell_j} \bar{x}(t), t \in [t_i, t_i + \tau_d(t_i)] \\
\bar{z}(t) &= \bar{A}_{\ell_i, \ell_j} \bar{x}(t) + \bar{B}_{\ell_i, \ell_j} \bar{y} + \bar{v}_{\ell_i, \ell_j}, t \in [t_i + \tau_d(t_i), t_i + \tau_d(t_i) + \tau_d(t_i)]
\end{aligned} \quad (36)-(39)$$

where

$$\begin{aligned}
\Phi_{\ell_i, \ell_j} &= \begin{bmatrix} R_{\ell_i, \ell_j} & T_{\ell_i, \ell_j} & S_{\ell_i, \ell_j} & 0 \\
R_{\ell_i, \ell_j}^T & T_{\ell_i, \ell_j}^T & 0 & 0 \\
0 & 0 & I & 0 \\
0 & 0 & 0 & I \\
\end{bmatrix} \\
\bar{A}_{\ell_i, \ell_j} &= \begin{bmatrix} A_{\ell_i, \ell_j} + B_{\ell_i, \ell_j} K_{\ell_j} & B_{\ell_i, \ell_j} K_{\ell_j} \\
B_{\ell_i, \ell_j} K_{\ell_j} & A_{\ell_i, \ell_j} + B_{\ell_i, \ell_j} K_{\ell_j} \\
0 & 0 \\
0 & 0 \\
\end{bmatrix} \\
\bar{B}_{\ell_i, \ell_j} &= \begin{bmatrix} -L_{\ell_i, \ell_j} C_{\ell_i} & 0 \\
0 & L_{\ell_i, \ell_j} C_{\ell_i} \\
\end{bmatrix}
\end{aligned}$
Let \( \xi = \hat{\xi} \), then, the compact form of (25) and (26) and (36)–(38) can be written as
\[
\begin{align*}
\dot{\xi} &= A_{\ell, \ell, \ell} \xi + \bar{\nu}_{\ell, \ell, \ell}, t \in [t_i, t_i + \tau_d(t_i)) \quad (40) \\
\dot{\xi} &= A_{\ell, \ell, \ell} \xi + \bar{\nu}_{\ell, \ell, \ell}, t \in [t_i + \tau_d(t_i), t_{i+1}) \quad (41) \\
\xi(t) &= G(\hat{x}(t^-), \hat{x}(t^+), x(t)) \quad (42)
\end{align*}
\]
where
\[
G(\hat{x}(t^-), \hat{x}(t^+), x(t)) = \begin{bmatrix}
\Phi_{\ell, \ell, \ell} (\hat{x}(t^-), \hat{x}(t^+), x(t)) \\
\Psi_{\ell, \ell, \ell} (\hat{x}(t^-), \hat{x}(t^+), x(t))
\end{bmatrix},
\]
\[
\begin{bmatrix}
A_{\ell, \ell, \ell} & B_{\ell, \ell, \ell} \\
0 & A_{\ell, \ell, \ell}
\end{bmatrix},
\]
\[
\begin{bmatrix}
A_{\ell, \ell, \ell} & B_{\ell, \ell, \ell} \\
0 & A_{\ell, \ell, \ell}
\end{bmatrix} \bar{\nu}_{\ell, \ell, \ell} = \frac{\bar{\psi}_{\ell, \ell, \ell}}{\bar{\psi}_{\ell, \ell, \ell}}.
\]

We consider the following piecewise Lyapunov function candidate \( V(t) = V_{\sigma^*}(t) \) for the system (40)–(42) with
\[
V_{\sigma^*}(t) = \xi^T(t) \hat{P}_{\sigma^*} \xi(t) \quad (43)
\]
where
\[
\sigma^* = \begin{cases}
\ell, \ell, \ell, & t \in [t_i, t_i + \tau_d(t_i)] \\
\ell, \ell, j, & t \in [t_i + \tau_d(t_i), t_{i+1}]
\end{cases}, \ell_i \neq \ell_j
\]
and
\[
P_{\ell, \ell, \ell} = P_{\ell, \ell, j} = \text{diag}(P_{\ell, j}, F_{\ell, j}, Q_{\ell, j}, S_{\ell, j})
\]
\[
P_{\ell, \ell, \ell} = \text{diag}(P_{\ell, j}, F_{\ell, j}, Q_{\ell, j}, S_{\ell, j}).
\]

Along the trajectories of (40), by Lemma 3, we deduce, for \( \forall \bar{\ell} \in \{t_i, t_i + \tau_d(t_i)\} \),
\[
\begin{align*}
\dot{V}_{\ell, \ell, \ell}(t) &= \beta V_{\ell, \ell, \ell}(t) \\
&= \xi^T(t) \begin{bmatrix}
\bar{A}_{\ell, \ell, \ell} P_{\ell, \ell, \ell} + P_{\ell, \ell, \ell} A_{\ell, \ell, \ell} - \beta P_{\ell, \ell, \ell} \\
\bar{A}_{\ell, \ell, \ell} P_{\ell, \ell, \ell} + P_{\ell, \ell, \ell} A_{\ell, \ell, \ell} - \beta P_{\ell, \ell, \ell}
\end{bmatrix} \xi(t) \\
&+ \bar{\nu}_{\ell, \ell, \ell}^T(t) P_{\ell, \ell, \ell} \xi(t) + \xi^T(t) \begin{bmatrix}
\bar{A}_{\ell, \ell, \ell} P_{\ell, \ell, \ell} + P_{\ell, \ell, \ell} A_{\ell, \ell, \ell} - \beta P_{\ell, \ell, \ell} \\
\bar{A}_{\ell, \ell, \ell} P_{\ell, \ell, \ell} + P_{\ell, \ell, \ell} A_{\ell, \ell, \ell} - \beta P_{\ell, \ell, \ell}
\end{bmatrix} \xi(t) \\
&+ \bar{\nu}_{\ell, \ell, \ell}^T(t) P_{\ell, \ell, \ell} \bar{\nu}_{\ell, \ell, \ell}(t).
\end{align*}
\]
From (30), we can derive that
\[
\dot{V}_{\ell, \ell, \ell}(t) < \beta V_{\ell, \ell, \ell}(t) + \gamma \| \bar{\nu}_{\ell, \ell, \ell}(t) \|_	ext{2},
\]
where
\[
\gamma(\bar{\omega}) = \max_{\ell, \ell, j \in M} \{ \lambda_{\text{max}}(P_{\ell, \ell, j}) \bar{\omega}^2, | \bar{\nu}_{\ell, \ell, \ell}(t) |^2 = \max_{\ell, \ell, j \in M} \{ | \bar{\nu}_{\ell, \ell, \ell}(t) |^2 \}.
\]
From (43), it holds
\[
V_{\ell,\ell'}(t^-) = x_T(t^-) P_{\ell,\ell'} x_T(t^-) + x_T(t^-) F_{\ell,\ell'} x_T(t^-) + \left[ x_T(t^-) - x_T(t^-) \right] Q_{\ell,\ell'} x_T(t^-) - x_T(t^-),
\]
\[
+ \left[ x_T(t^-) - x_T(t^-) \right] S_{\ell,\ell'} x_T(t^-) - x_T(t^-).
\]
Substituting (21) and (22) into (50), from (31) and (51), we can derive that
\[
V_{\ell,\ell'}(t^-) - \mu V_{\ell,\ell'}(t^-)
\]
\[
= Y_T(t^-) \left[ \Theta_{\ell,\ell'} P_{\ell,\ell'} \Theta_{\ell,\ell'}^T + \Theta_{2,\ell'} F_{\ell,\ell'} \Theta_{2,\ell'}^T + \Theta_{3,\ell'} S_{\ell,\ell'} \Theta_{3,\ell'}^T \right] Y_T(t^-)
\]
\[
= \left[ \begin{array}{cc}
\mu P_{\ell,\ell'} + Q_{\ell,\ell'} & 0 \\
0 & \mu S_{\ell,\ell'} + F_{\ell,\ell'} & \mu Q_{\ell,\ell'} + S_{\ell,\ell'}
\end{array} \right] Y_T(t^-)
\]
\[
< 0,
\]
where
\[
Y(t) = \begin{bmatrix}
x_T(t) \\
x_T(t)
\end{bmatrix}, \quad \Theta_{\ell,\ell'} = \begin{bmatrix}
\Gamma_T & -\Gamma_T \\
-\Gamma_T & \Gamma_T
\end{bmatrix}, \quad \Theta_{2,\ell'} = \begin{bmatrix}
-\Gamma_T \\
\Gamma_T
\end{bmatrix}, \quad \Theta_{3,\ell'} = \begin{bmatrix}
\Gamma_T \\
-\Gamma_T
\end{bmatrix}.
\]
Let \( T_{1}, ..., T_{N_{\ell,\ell'}(t,b)} \) denote the switching times of \( \sigma' \) in \((t_0,b)\), and \( T_0 = t_0, T_{N_{\ell,\ell'}(t,b)+1} = t^- \) by convention. Since \( \sigma' \) is constant for \( t \in [T_k, T_{k+1}] \), from (45) and (47)–(49), we have
\[
V(t) \leq e^{-\Omega_{\sigma'}(t,t^-)} \mu V(t^-) + \int_{T_{k+1}}^{t^-} e^{-\Omega_{\sigma'}(t,t^-)} Y(\|\tilde{v}_\ell(t)\|_1) dt
\]
\[
\leq \mu^{N_{\ell,\ell'}(t,b)} e^{\sum_{j=0}^{N_{\ell,\ell'}} -\Omega_{\sigma'}(T_j, T_{j+1} - T_j)} V(t_0)
\]
\[
+ \sum_{k=0}^{N_{\ell,\ell'}} \int_{T_k}^{T_{k+1}} \mu^{N_{\ell,\ell'}(t,b)} e^{-\Omega_{\sigma'}(T_k, T_{k+1} - T_k)} \gamma(\|\tilde{v}_\ell(t)\|_1) dt
\]
\[
\leq \mu^{N_{\ell,\ell'}(t,b)} \beta^{m(t,b)} \alpha M(t,b) V(t_0)
\]
\[
+ \sum_{k=0}^{N_{\ell,\ell'}} \int_{T_k}^{T_{k+1}} \mu^{N_{\ell,\ell'}(t,b)} \beta^{m(t,b)} \sigma M(t,b) \gamma(\|\tilde{v}_\ell(t)\|_1) dt.
\]
By using Lemma 5, it follows that
\[
\mu^{N_{\ell,\ell'}(t,b)} \beta^{m(t,b)} \alpha M(t,b) V(t_0)
\]
\[
< \frac{2\lambda \mu}{\tau_a} < \lambda < \alpha + \beta \tau_a
\]
which gives (28) in Lemma 6. Therefore, from Lemma 6, it is clear that
\[
V(t) < \frac{2\lambda}{\tau_a} \|\tilde{v}_\ell(t)\|_1 \gamma(\|\tilde{v}_\ell(t)\|_1) dt
\]
\[
< \frac{2\lambda}{\tau_a} \|\tilde{v}_\ell(t)\|_1 \gamma(\|\tilde{v}_\ell(t)\|_1).
\]
Let \( q = \frac{2\lambda}{\tau_a} \). From (55) we have \( q < 0 \) and
\[
V(t) < \frac{2\lambda}{\tau_a} \gamma(\|\tilde{v}_\ell(t)\|_1).
\]
By the definition of \( V(t) \), it is clear that
\[
a \xi(t) \leq V(t) \leq \beta \xi(t)
\]
where \( a = \min \{ \lambda_{\min}(P_{\ell,\ell'}), b = \max \{ \lambda_{\max}(P_{\ell,\ell'})) \} \).

According to (43), (57), and (58), we have
\[
\|\xi(T)\|_1 \leq \sqrt{\frac{\beta_0}{a} \xi^2(\|\tilde{v}_\ell(t)\|_1)}_1 + \sqrt{\frac{\beta_0}{a} \gamma(\|\tilde{v}_\ell(t)\|_1)}.
\]
which implies that the system (40)–(42) are globally uniformly exponentially stable under the switching signal \( \sigma' \), when \( \tilde{w} \) and \( \tilde{v} \) are identically equal to zero. Now, we exploit the matrix inequalities feasibility problem in Theorem 1.

**Theorem 2.** Consider matrices \( T_{i}, i \in M \) such that \( F_i = T_i(A_i - L_i C_i)^{-1} \) are Metzler. Under Assumption 1, for the gain matrices \( L_i, i \in M \) given by Lemma 4 and known positive constants \( \alpha, \beta, \) and \( \mu > 1, \) if there exist matrices \( P_i > 0, F_i > 0, D_i > 0, \tilde{y}_i > 0, i \in M \) and \( H_i, H_j, f \in M \) such that
\[
\begin{bmatrix}
\Sigma_{ij} & H_i^T B_j + B_i H_j - L_i C_i \tilde{y}_j & 0 \\
* & \Sigma_{ij} & 0 \\
* & * & \Sigma_{ij}
\end{bmatrix} < 0
\]
\[
\begin{bmatrix}
\Delta_{ij} & \Delta_{ij} & \Gamma_{ij} S_j - \Gamma_{ij} Q_j + \mu Q_j \\
* & \Delta_{ij} & \Gamma_{ij} Q_j - \Gamma_{ij} S_j + \nu S_j \\
* & * & Q_j - \nu S_j - \mu Q_j - \mu S_j \\
\end{bmatrix} < 0
\]
(61)

where

\[
\begin{align*}
\Sigma_{ij} &= \bar{P}_j A_j T_i + H_i^T R_i T_j + A_i \bar{P}_j + B_i H_j + \bar{P}_j + \delta_j \bar{P}_j \\
\Sigma_{ij} &= F_j A_j T_i + H_i^T R_i T_j + A_i F_j + B_i H_j + F_j + \delta_j F_j \\
\Sigma_{ij} &= (Q_j A_j T_i - \bar{Q}_j C_j^T L_i^T) + (A_i Q_j - L_i C_i Q_j) + \bar{Q}_j + \delta_j \bar{Q}_j \\
\Sigma_{ij} &= \bar{S}_j A_j T_i - \bar{S}_j C_j^T L_i^T + A_i \bar{S}_j - L_i C_i \bar{S}_j + \bar{S}_j + \delta_j \bar{S}_j \\
\end{align*}
\]

\[
\begin{align*}
\Gamma_{ij} &= R_i T_i^+ K_i T_i + R_i T_i^- K_i T_i \\
\Gamma_{ij} &= R_i T_i^+ K_i T_i + R_i T_i^- K_i T_i \\
\Delta_{ij} &= \Gamma_{ij} F_j T_i + \Gamma_{ij} P_j T_i^+ + \Gamma_{ij} ^T P_j T_i - \mu P_j - \mu Q_j \\
\Delta_{ij} &= -\Gamma_{ij} F_j T_i - \Gamma_{ij} P_j T_i^+ + \Gamma_{ij} ^T P_j T_i - \mu P_j - \mu Q_j \\
\Delta_{ij} &= \Gamma_{ij} F_j T_i + \Gamma_{ij} P_j T_i^+ + \Gamma_{ij} ^T P_j T_i - \mu P_j - \mu Q_j \\
\end{align*}
\]

and \( \delta_{ij} = \left\{ \begin{array}{ll}
-\beta, & i \neq j \\
\alpha, & i = j \\
\end{array} \right\} \), \( P_j = F_j^{-1}, F_j = F_j^{-1}, Q_j = Q_j^{-1} \).

\[
S_j = S_j^{-1}, \text{ then with the interval observer-based controller (27), when } \bar{w} \text{ and } \bar{v} \text{ are identically equal to zero, the switched system (1) is globally uniformly exponentially stable for any switching signal with average dwell time satisfying}
\[
\tau_d > \tau_d^* = \frac{2 \ln \mu + (\alpha + \beta) \tau_d}{\alpha}.
\]
(63)

Moreover, the interval observer-based state-feedback controller gains are given by \( \hat{K}_j = H_j P_j^{-1} \) and \( \hat{K}_j = H_j F_j^{-1} \).

**Proof.** Let \( \bar{K}_j \bar{P}_j = H_j P_j^{-1} \) and \( \bar{K}_j \bar{F}_j = H_j F_j^{-1} \), multiplying both sides of (30) by

\[
\text{diag}(\bar{P}_j, \bar{Q}_j, \bar{S}_j)
\]

with \( \bar{P}_j = P_j^{-1}, \bar{F}_j = F_j^{-1}, \bar{Q}_j = Q_j^{-1}, \bar{S}_j = S_j^{-1} \), we know that (30) is equivalent to (60).

### 4 Numerical Example

In this section, we consider regulator systems used in semiconductor technology [34]. In the system shown in Figure 1, Sw1 is a bipolar transistor and Sw2 is a diode. The regulator has two switching modes: mode 1, Sw1 is closed and Sw2 is off; mode 2, Sw1 is off and Sw2 is closed. Selecting the state variable \( x(t) = [x_1 \ x_2]^T \), control input \( u(t) = V_i \), where \( x_1 \) is the inductance current \( L_i \), \( x_2 \) is the capacitor voltage \( V_i \) and \( V_i \) is the power supply voltage. Under the different modes \((\sigma(t) = 1, 2)\), the system matrices of the buck/boost regulator are given by

\[
\sigma(t) = 1 : A_1 = \begin{bmatrix} -\frac{R_i}{L} & 0 \\ 0 & -\frac{1}{R_i C} \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\]

\[
\sigma(t) = 2 : A_2 = \begin{bmatrix} 0 \\ -\frac{1}{C} \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.
\]

Here, we choose the system parameters \( L = 1H, C = 1F, R_i = 0.5\Omega, R_c = 2\Omega \), and suppose the other system matrices are

\[
C_1 = \begin{bmatrix} 1 & -0.2 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0.5 \end{bmatrix},
\]

\[
w_1 = w_2 = \begin{bmatrix} 0.15 \\ 0.3 \\ 1 + t \end{bmatrix}, v_1(t) = v_2(t) = \frac{\sin(t)}{(2 + t)^2}
\]

with \( \bar{v} = \frac{1}{4} \). Solving the LMIs in Theorem 2, we obtain the observer gain matrices

\[
L_1 = \begin{bmatrix} -0.9615 \\ 0.1923 \end{bmatrix}, L_2 = \begin{bmatrix} 0.7077 \\ -1.1394 \end{bmatrix}.
\]

Note that \( T_i \) are derived to satisfy \( F_i = T_i(A_i - L_i C_i)T_i^{-1} \) \((i = 1, 2)\) are Metzler.

\[
T_1 = \begin{bmatrix} 0.95 & -0.1905 \\ 0.1905 & 0.95 \end{bmatrix}, T_2 = I.
\]

For the parameters \( \alpha = 0.1, \beta = 0.11, \mu = 15 \), solving the LMIs in Lemma 4, we obtain the interval observer-based state-feedback controller gains

\[
\hat{K}_1 = \begin{bmatrix} -2.5869 \\ -2.5558 \end{bmatrix}, \hat{K}_1 = \begin{bmatrix} -2.5763 \\ -2.5458 \end{bmatrix},
\]

\[
\hat{K}_2 = \begin{bmatrix} -3.5733 \\ -3.4199 \end{bmatrix}, \hat{K}_2 = \begin{bmatrix} -3.5574 \\ -3.4298 \end{bmatrix}
\]

and minimum average dwell time \( \tau_d^* = 54.4760 \).
The simulation results of $x_1$ and $x_2$ are shown in Figures 2 and 3.

5 1 CONCLUSION

This paper has investigated asynchronous interval observer-based controller for switched systems with additional disturbance and measurement noise. By giving the upper bound and the lower bound of the disturbance and the measurement noise, we have constructed hybrid interval observer for the considered switched linear system by means of a coordinate transformation, which can transform the observer error systems into cooperative ones. Then, based on multiple Lyapunov functions method and average dwell time scheme, some sufficient conditions have been derived and applied to build the interval observer-based state feedback controller. The design of the event-triggered interval observer-based controller is of great significance which deserves further study.

**DATA AVAILABILITY STATEMENT**

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

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**CONFLICT OF INTEREST STATEMENT**

The authors declare that they have no conflicts of interest.

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