Application of fuzzy comprehensive evaluation method using entropy weight in groundwater quality evaluation: A case study on Xianyang, China

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Abstract. Groundwater, especially unconfined water, is easy to be polluted in Xianyang of Guanzhong Basin. Therefore, the evaluation of groundwater quality is of great significance for the safety of drinking and control of pollution. In order to find out the water quality, 14 water quality assessment indicators were selected for 16 points. And the fuzzy comprehensive evaluation method based on entropy weight is used to evaluate the groundwater quality. The results show that the water quality level determined by the weighted average principle is better than that by the maximum membership principle. Nearly 75% of the water quality ranks are Class IV and Class V, and evaluation of water not exceed groundwater Class III water standard is solely approximately 25%. Moreover, the water quality in upstream of groundwater flow field is better than that in downstream. The main indicators of groundwater pollution are fluoride from the impact of geological environment and human activities and hexavalent chromium contained in wastewater discharged from industrial development in Xianyang city.

1. Introduction
As the source of life, the basis of production and the foundation of life, water resources play a very important role in the sustainable development of economy and society. Groundwater, as an important part of water resources, accounting for one third of all freshwater withdrawals in the world [1], provides the majority of drinking water in rural and urban areas and supports agricultural and industrial economies, especially in arid and semi-arid regions [2-4]. Therefore, the groundwater level and quality have become the main focus problems. In recent years, with the continuous growth of China's population and the rapid development of the economy, various environmental problems have become increasingly serious, especially in groundwater pollution [5]. Many scholars have evaluated the quality of groundwater. Tao Hong et al. [6] studied the evolution law of groundwater quality in the Weihe Plain, and concluded that the comprehensive assessment of water quality in some areas of Weibei loess plateau is poor or very poor. Gao Kai [7] studied the water quality status of Hanzhong water source based on fuzzy neural network method, which provided a reference for local ecological protection. Zhang Han et al. [8] have evaluated and predicted the nitrate contamination in groundwater in Chengdu Plateau. However, there are very few studies on groundwater quality in the Guanzhong Basin.
Guanzhong Basin is located in the middle of Shaanxi Province, northwestern China. Human activity has left a significant footprint with agricultural area as the main land use, increasing urban area in this region [9]. Groundwater as an important source of drinking water in the Guanzhong Basin, it is extremely major to clear the water quality. As the fastest-growing industrial city in Guanzhong Basin, the water quality of Xianyang has become more and more serious with the economy rapid development. Especially since 1987, the pollution index has increased by an average of 9% per year, showing a worsening trend [10]. Because the variability in groundwater level and quality lead to a combined result of the natural factors and human activities [11,12], the evaluation and control of water quality is important and necessary for human beings.

Water quality assessment is not only an important aspect of water environment research, but also an important basic work of environmental risk analysis, water environmental protection and pollution control. It is extremely important to select appropriate methods to evaluate regional groundwater quality. At present, the groundwater evaluation methods mainly include fuzzy comprehensive evaluation method [13], set pair analysis method [14], Bayesian model method [15] and matter element method [16]. The fuzzy comprehensive evaluation method is a statistical method based on mathematical theory. It is applied to the water quality evaluation through the data information, and it has the characteristics of strong reasoning ability, easy understanding, and reliable evaluation results.

Based on the above reasons, the fuzzy comprehensive evaluation method based on entropy weight was selected to evaluate the water quality of 16 sampling points in Xianyang (Figure 1) to use and protect the groundwater resources rationally, and provide a theoretical basis for strengthening the drinking water safety in this region.

![Figure 1. Groundwater flow field and sampling point location in the study area.](image)

2. **Fuzzy comprehensive evaluation method**

2.1. **The principle and selection basis of this method**

The fuzzy comprehensive evaluation method is based on fuzzy mathematics. It is widely used in the evaluation of groundwater quality because of its rigorous theory, simple calculation and objective evaluation. However, the determination of the weight is still somewhat empirical. Therefore, this paper chooses the fuzzy comprehensive evaluation method based on entropy weight [17] to evaluate the groundwater quality. The fuzzy comprehensive evaluation method transforms the measured values reflecting various water pollution problems into quality values reflecting the water quality through functional relations. It describes the fuzzy water quality classification boundaries by membership degree.
2.2. The calculation steps of this method

According to the groundwater quality assessment standard and the actual detection value, the fuzzy comprehensive evaluation method uses the fuzzy transformation to quantify the uncertain factors in the groundwater through the membership measurement, and then obtain the scientific evaluation results. There are seven steps for specific calculation.

1. Establishing an indicator set: \( U = \{ x_1, x_2, \cdots, x_n \} \), \( n \) represents the number of indicators used for groundwater quality evaluation. In this paper, the value of \( n \) is 13.

2. Establishing an evaluation set: \( V = \{ \text{I}, \text{II}, \text{III}, \text{IV}, \text{V} \} \). This matrix is determined based on the classification of water quality in the national standard for groundwater quality assessment.

3. Establishing membership function. The membership function is the basis for the fuzzy evaluation of each evaluation index, and is also the key steps for the fuzzy comprehensive evaluation. The specific steps as follows:

   Membership function for classification \( \text{I} \):
   
   \[
   \gamma_{ij} = \begin{cases} 
   \frac{s_{i(j+1)} - x_i}{s_{i(j+1)} - s_{ij}} & s_{i1} < x_i < s_{i2} \\
   1 & x_i < s_{i1} \\
   0 & x_i > s_{i2} 
   \end{cases}
   \]

   Membership function for classification \( \text{II}, \text{III}, \text{IV} \):
   
   \[
   \gamma_{ij} = \begin{cases} 
   \frac{s_{i(j+1)} - x_i}{s_{i(j+1)} - s_{ij}} & s_{ij} < x_i < s_{i(j+1)} \\
   \frac{x_i - s_{i(j-1)}}{s_{i(j-1)} - s_{ij}} & x_i < s_{i(j-1)} \text{ or } x_i > s_{i(j+1)} \\
   0 & x_i < s_{i(j-1)} \text{ or } x_i > s_{i(j+1)} 
   \end{cases}
   \]

   Membership function for classification \( \text{V} \):
   
   \[
   \gamma_{ij} = \begin{cases} 
   \frac{x_i - s_{i4}}{s_{i5} - s_{i4}} & s_{ij} < x_i < s_{i5} \\
   0 & x_i < s_{i4} \\
   1 & x_i > s_{i5} 
   \end{cases}
   \]

4. Establishing a fuzzy matrix \( R \). The fuzzy matrix \( R \) is determined by the standard membership function.

   \[
   R = \begin{bmatrix}
   \gamma_{11} & \cdots & \gamma_{15} \\
   \vdots & \ddots & \vdots \\
   \gamma_{n1} & \cdots & \gamma_{n5}
   \end{bmatrix}_{n \times 5}
   \]

5. Determining the weight set \( W \). The weight is calculated using the entropy weight method [17].

6. Establishing fuzzy comprehensive evaluation model \( M \).

   \[
   M = W \times R
   \]

7. Determining the water quality classification. There are two methods for determining the water quality classification. One is the Maximum Membership Principle (MMP) and the other is the Weighted Average Principle (WAP). The MMP is based on the probability set \( M \), and the one with the highest probability is the corresponding water quality classification. The WAP quantifies the water quality classification, and the ranks of five levels are 1, 2, 3, 4, 5, respectively. After weighted summing the components of \( M \) with the ranks of the levels, the final value via rounding is equal to the water quality evaluation. The fuzzy evaluation results based on the weighted average principle are more credible, and the formula is as follows:

   \[
   B = \left( \sum_{i=1}^{5} m_i^k \cdot i \right) / \sum_{i=1}^{5} m_i^k
   \]

   Where, \( k \) is the undetermined coefficient, which can be 1 or 2. The value in this paper is 2.
3. An application example

3.1. Study area
Xianyang City (Figure 2), located in the middle of Guanzhong Basin with an annual mean precipitation of 648.3 mm, and an annual average atmospheric temperature of 13.6 °C, is the origin of China's land, and its total area is around 10,246 km². The water resources in Xianyang are mainly composed of river runoff and groundwater, which mainly supplied by atmospheric precipitation. Unconfined water in southern (mainly study area) is the main part of groundwater, which is shallowly buried, making groundwater vulnerable to pollute.

![Figure 2. Location map of the study area.](image)

3.2. Construction of evaluation index system
In recent years, serious exploitation of groundwater results in significant water pollution, due to the acceleration of urbanization. The imbalance between water supply and water demand becomes more and more prominent. In order to better reflect the actual status of groundwater in this area, this paper selected groundwater samples of Xianyang as the basis for evaluation. There were 16 monitoring points and 13 monitoring indicators at each point (Table 1). This study was based on the "Groundwater Quality Standards" (GB/T 14848-2017).

| Index     | I      | II     | III    | IV     | V     |
|-----------|--------|--------|--------|--------|-------|
| TDS       | ≤300   | ≤500   | ≤1000  | ≤2000  | >2000 |
| Total hardness | ≤150  | ≤300   | ≤450   | ≤650   | >650  |
| SO₄²⁻     | ≤50    | ≤150   | ≤250   | ≤350   | >350  |
| Na⁺       | ≤100   | ≤150   | ≤200   | ≤400   | >400  |
| Fe        | ≤0.1   | ≤0.2   | ≤0.3   | ≤2     | >2    |
| NO₃⁻      | ≤2     | ≤5     | ≤20    | ≤30    | >30   |
| NO₂⁻      | ≤0.01  | ≤0.1   | ≤1     | ≤4.8   | >4.8  |
| F⁻        | ≤1     | ≤1     | ≤1     | ≤2     | >2    |
| As        | ≤0.001 | ≤0.001 | ≤0.01  | ≤0.05  | >0.05 |
| Cr⁶⁺      | ≤0.005 | ≤0.01  | ≤0.05  | ≤0.1   | >0.1  |
| Pb²⁺      | ≤0.005 | ≤0.005 | ≤0.01  | ≤0.1   | >0.1  |
| Cd²⁺      | ≤0.0001| ≤0.001 | ≤0.005 | ≤0.01  | >0.01 |
| Hg²⁺      | ≤0.0001| ≤0.0001| ≤0.001 | ≤0.002 | >0.002|
The groundwater quality evaluation matrix was calculated by Matlab software, substituting the evaluation factor weight coefficient value $W$ into the fuzzy comprehensive evaluation $R$. The probability values of each level of 16 monitoring points are shown in Table 2.

**Table 2.** The probability of comprehensive water quality for all samples with entropy weights.

| Sampling points | I  | II | III | IV | V   |
|-----------------|----|----|-----|----|-----|
| x2              | 0.0438 | 0.1318 | **0.3761** | 0.3071 | 0.1411 |
| x3              | 0.003 | 0.066 | 0.101 | 0.394 | **0.436** |
| x4              | 0.011 | 0.0669 | 0.0157 | 0.0179 | **0.8885** |
| x5              | 0.0446 | 0.1549 | 0.218 | **0.5224** | 0.06 |
| x8              | 0.0172 | 0.0722 | 0.0288 | 0.2909 | **0.5908** |
| x9              | 0.002 | 0.207 | 0.208 | **0.294** | 0.288 |
| x12             | 0.0427 | 0.0271 | 0.2077 | 0.3027 | **0.4198** |
| x14             | 0.0289 | 0.26 | 0.2618 | **0.33** | 0.1193 |
| x17             | 0.0103 | 0.0693 | 0.2738 | 0.1268 | **0.5198** |
| x18             | 0.024 | 0.25 | 0.278 | **0.339** | 0.108 |
| x19             | 0.0207 | 0.0813 | **0.368** | 0.2276 | 0.3024 |
| x20             | 0.0298 | 0.2358 | 0.1881 | 0.1248 | **0.4332** |
| x23             | 0.044 | 0.2588 | **0.4051** | 0.2343 | 0.0579 |
| x36             | 0.049 | 0.099 | 0.041 | 0.223 | **0.588** |
| x37             | 0.0148 | 0.0168 | 0.1246 | 0.2427 | **0.6011** |
| x39             | 0.0973 | 0.4397 | **0.4303** | 0.0327 | 0.0 |

Figure 3. Percentage of water quality ranks for groundwater in Xianyang by two determination methods.

(a) MMP; (b) WAP
According to Table 2, the final water quality level was determined by the MMP and the WAP, and the evaluation levels of 16 points are shown in Table 3. It can be seen from the results in Table 3 that only the water quality of x2, x23, and x39 meet the requirements of the class III water with "Groundwater Quality Standards" (GB/T 14848-2017) under the two water quality determination principles (MMP and WAP). Water quality in other places was poor, which were class IV and class V. Rate of acceptable water quality was solely 25% using the MMP, while the rate was 31.25% through the WAP (Figure 3). Moreover, the change in water quality basically showed a tendency to deteriorate in the direction of water flow (Figure 1 and Figure 4). The rapid economic development has caused serious pollution of groundwater. Also, it is related to the shallow depth of groundwater, making it vulnerable to pollution.

![Figure 4. Groundwater quality distribution map of Xianyang City.](image)

The groundwater quality was generally poor in Xianyang City. In addition, the groundwater quality was deteriorating along the flow direction of the groundwater and Wei River. The reason for this change is mainly due to the deterioration of the water quality of the Wei River concentrated a number of industrial enterprises. So much polluted water emission came from industrial, agricultural and domestic has a great influence on water quality of Wei River [18]. Also, the non-point source pollution of agriculture and the reduction of the self-purification ability of the Wei River are another important reason for the increase of pollution [10]. The Wei River is the main surface runoff in the urban area of Xianyang and one of the main sources of groundwater supply. The ions will continue to enrich in the direction of the water flow, which explains why the downstream water quality is worse than the upstream water quality.

Although the water quality of the Wei River has improved under the joint management of various departments in recent years, the monitoring of water quality and the protection of the water environment are still not optimistic. Because of the close relationship between surface water and groundwater, the improvement of surface water quality is also improving groundwater to a large extent, thus providing protection for human production and life.

### 3.3. Determination of major pollution indicators

The average value for each evaluation index was compared with the value of groundwater Class III water standard to determine the main indicators of groundwater pollution (Table 4). From Table 4, the content of F\(^-\) and Cr\(^{6+}\) exceeded the class III water standard, and the over-standard rates were 3.55 and 1.76, respectively. Therefore, the main pollutants of groundwater in Xianyang City were F\(^-\) and Cr\(^{6+}\).

Excess fluoride (F\(^-\)) was mainly caused by geological environment factors. Fluoride is widely distributed in nature. It ranks 13 among the various elements that make up the earth's crust, accounting for 0.077% of total crust [19]. With the rapid development of industry, fluoride has been widely used as a raw material in all kinds processes such as various chemical manufacturing processes. A large
amount of fluorine-containing substances in the waste water of the production process were continuously entering people's living environment, polluting water sources.

Table 3. The comprehensive water quality assessment in the Xianyang City.

| Number | Sampling points | P(MMP) | Class | P(WAP) | Class |
|--------|----------------|--------|-------|--------|-------|
| 1      | x2             | 0.3761 | III   | 3.4107 | III   |
| 2      | x3             | 0.436  | V     | 4.476  | IV    |
| 3      | x4             | 0.8885 | V     | 4.9815 | V     |
| 4      | x5             | 0.5224 | IV    | 3.7204 | IV    |
| 5      | x8             | 0.5908 | V     | 4.7656 | V     |
| 6      | x9             | 0.294  | IV    | 3.82   | IV    |
| 7      | x12            | 0.4198 | V     | 4.4022 | IV    |
| 8      | x14            | 0.33   | IV    | 3.2617 | III   |
| 9      | x17            | 0.5198 | V     | 4.5061 | V     |
| 10     | x18            | 0.339  | IV    | 3.279  | III   |
| 11     | x19            | 0.368  | III   | 3.7953 | IV    |
| 12     | x20            | 0.4332 | V     | 4.1302 | IV    |
| 13     | x23            | 0.4051 | III   | 2.9682 | III   |
| 14     | x36            | 0.588  | V     | 4.775  | V     |
| 15     | x37            | 0.6011 | V     | 4.7899 | V     |
| 16     | x39            | 0.4303 | III   | 2.4571 | II    |

Table 4. Determination of main impact indicators.

| Index   | Average | Class | Max. | Min. | Max. /Min. exceeds multiple |
|---------|---------|-------|------|------|-----------------------------|
| TDS     | 714.45  | III   | 1212 | 310  | 0                           |
| Total hardness | 134.96  | I     | 503  | 54.6 | 0                           |
| SO₄²⁻   | 161.68  | III   | 416.9| 43.2 | 0                           |
| Na⁺     | 197.77  | III   | 354.5| 79.8 | 0                           |
| Fe      | 0.283   | III   | 2.164| 0.033| 0                           |
| NO₃⁻    | 18.15   | III   | 60   | -    | 0                           |
| NO₂⁻    | 0.0258  | II    | 0.12 | -    | 0                           |
| F⁻      | 2.282   | V     | 3.55 | 0.01 | 3.55                        |
| As      | 0.0051  | III   | 0.014| 0.002| 0                           |
| Cr⁶⁺    | 0.0224  | V     | 0.088| 0.002| 1.76                        |
| Pb²⁺    | 0.0034  | I     | 0.008| 0.001| 0                           |
| Cd²⁺    | 0.0032  | III   | 0.008| 0.001| 0                           |
| Hg²⁺    | 0.0005  | III   | 0.0005| 5E-04| 0                           |
Hexavalent chromium ($\text{Cr}^{6+}$) was also the main indicator of groundwater pollution. The chromium in water was mainly in the form of ions with trivalent chromium and hexavalent chromium. Hexavalent chromium is an inhaled and swallowed poison that may cause allergies or skin cancer after skin contact. It may also cause genetic damage and long-term environmental hazard [20]. This pollution mainly came from the emission of chromium-containing industrial wastewater, such as mining, machinery manufacturing, chemical industry, electronics and other industrial production processes. Contamination of chromium was a cumulative process that cannot be broken down and destroyed and should be given sufficient attention.

4. Conclusions

(1) The groundwater pollution is serious in the Xianyang urban area. The rate of unacceptable water quality (Class $\text{IV}$ and Class $\text{V}$) is 68.75%–75% for total samples. The number of samples for acceptable water quality is only 3 to 5, and they are located upstream of the groundwater flow field. So, the change of groundwater quality is closely related to the groundwater flow field, showing a tendency for water quality deterioration with the direction of water flow.

(2) The final evaluation results are different with the same water quality evaluation method yet different determining method. Under the MMP, the rate of acceptable water quality is only 25%. Under the WAP, however, the rate is 31.25%. Therefore, different methods can be used to evaluate the water quality level according to the different requirements.

(3) The main pollutants in the study area are fluoride and hexavalent chromium. Their exceed multiples are up to 3.55 and 1.76, respectively. Fluoride pollution is mainly caused by geological environment and human activities. And the pollution of hexavalent chromium is mainly caused by the development of heavy industry and the discharge of a large amount of wastewater containing heavy metals.

(4) It is imperative to protect water resources and prevent water pollution with economic development, in the case of a shortage of water resources. The main measures that can be taken are: speeding up urban sewage treatment, improving sewage and wastewater treatment capacity; rationally constructing drainage ditch and regularly dredging; improving the quality of residents and beautifying the environment.

Acknowledgment

This study is financially supported by the National Natural Science Foundation of China (Grant No. 41572236 and 41931285). And the contributions of all authors are also gratefully acknowledged.

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