Feedback-Aware Precoding for Millimeter Wave Massive MIMO Systems

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Abstract—Millimeter wave (mmWave) communication is a promising solution for coping with the ever-increasing mobile data traffic because of its large bandwidth. To enable a sufficient link margin, a large antenna array employing directional beamforming, which is enabled by the availability of channel state information at the transmitter (CSIT), is required. However, CSIT acquisition for mmWave channels introduces a huge feedback overhead due to the typically large number of transmit and receive antennas. Leveraging properties of mmWave channels, this paper proposes a precoding strategy which enables a flexible adjustment of the feedback overhead. In particular, the optimal unconstrained precoder is approximated by selecting a variable number of elements from a basis that is constructed as a function of the transmitter array response, where the number of selected basis elements can be chosen according to the feedback constraint. Simulation results show that the proposed precoding scheme can provide a near-optimal solution if a higher feedback overhead can be afforded. For a low overhead, it can still provide a good approximation of the optimal precoder.

I. INTRODUCTION

Communication over millimeter wave (mmWave) frequencies is defining a new era of wireless communication, as it can enable gigabit-per-second data rates because of the large available channel bandwidth. In order to reap the benefits of mmWave communication, however, massive antenna arrays need to be employed, which provide a sufficient beamforming gain to combat the high path loss at mmWave frequencies.

In traditional multiple-input multiple-output (MIMO) systems, fully digital processing is enabled by connecting each antenna to the baseband processor. In a practical mmWave system employing a large number of antennas, i.e., a massive MIMO system, this approach is not feasible anymore due to the high cost and high power consumption of mmWave mixed signal and RF components. Hybrid MIMO architectures [1], [2] overcome this limitation by utilizing only few RF chains and by processing the signals in both the digital and analog domains. The digital baseband processing provides full control over both the phase and the amplitude of the signal, while the analog RF processing enabled by phase shifters (PSs) can only control the phase of the signal, i.e., there is a constant modulus constraint. Despite the lower cost and power consumption of hybrid architectures, they impose a set of constraints on MIMO precoding/combining due to the limitations imposed by analog processing. Several approaches have been proposed for precoding/combining in such architectures [3]–[5]. In [3], the sparse structure of the mmWave channel is exploited to formulate the precoding problem and then the Orthogonal Matching Pursuit (OMP) algorithm is used to approximate the optimal unconstrained (digital) precoder, which maximizes the achievable rate. In [4], hybrid precoding is treated as a matrix factorization problem and alternating minimization (AltMin) algorithm is used to find a solution. The authors of [5] propose several low-complexity solutions that provide different tradeoffs between precoding performance and algorithm complexity.

Any precoding algorithm relies on the availability of channel state information (CSI) at the transmitter. For frequency division multiplex (FDD) systems, which are a popular choice, e.g., for Long-Term Evolution (LTE), and the focus of this paper, channel reciprocity does not hold and hence the CSI can be acquired at the transmitter only via an explicit feedback link. The existing CSI feedback strategies for MIMO systems can be classified into two categories: i) codebook-based feedback, where an index from a known codebook is fed back, and ii) individual quantized feedback, where each CSI value is quantized and send back [6]. Both approaches have shortcomings when it comes to systems with large antenna arrays [7], such as mmWave systems, and thus the limitation of the feedback channel should be taken into account for the design of precoding algorithms [8]–[12]. In [9] and [10], multi-resolution codebooks are proposed to minimize the training power and feedback overhead during channel estimation. The authors of [11] and [12] propose a limited feedback precoding scheme for a multi-user scenario, where the RF precoder is configured by using codebook-based feedback from each user. The baseband precoder is then computed based on the effective channels of the users, which have a much lower dimension than the complete channel matrix, and thus can be quantized and fed back to the transmitter. While the above approaches are trying to optimize and reduce the feedback overhead required for precoder computation at the transmitter, the tradeoff between precoder performance and feedback rate is unclear and not easily adjustable.

In this paper, we propose a feedback-aware strategy for hybrid precoder design by exploiting the properties of the mmWave channel and the transmitter architecture introduced in [13] that employs two PSs per RF chain for each antenna and is capable of realizing any arbitrary precoder via
hybrid precoding. In particular, we propose to approximate the optimal unconstrained precoder by selecting a variable number of vectors from a basis matrix that is constructed as a function of the transmitter array response, where the number of selected basis vectors can be adapted according to the feedback constraint. We further propose a multi-beam basis matrix that improves the approximation accuracy especially when the affordable feedback overhead is low. Finally, we characterize the total feedback overhead for our proposed scheme and we provide a comparison with state-of-the-art precoder designs. We show that the proposed precoder design offers a flexible tradeoff between performance and feedback overhead. In particular, the optimal precoder can be approximated with high accuracy at the expense of an increased feedback overhead, whereas by limiting the feedback overhead, an acceptable approximation of the optimal precoder can still be maintained.

Notation: Throughout this paper, $\mathbb{C}$ is the field of complex numbers; $A$ is the field of modulo-one complex numbers; bold upper-case letters $A$ denote matrices; bold lower-case letters $a$ denote vectors; lower-case letters $a$ are scalars; $A_{i,j}$ is the entry in the $i$-th row and $j$-th column of $A$; $A^*$ and $A^T$ denote conjugate transpose and transpose of matrix $A$, respectively; $\|A\|_F$, $|A|$, $|a|$, and $\angle a$ are the Frobenius norm of $A$, the determinant of $A$, the magnitude of $a$, and the phase of $a$, respectively; $I_N$ is the $N \times N$ identity matrix; $\mathbf{0}_{N \times M}$ is the $N \times M$ all-zero matrix; $\mathbb{C}N(m, \sigma^2)$ denotes the complex normal distribution with mean $m$ and variance $\sigma^2$; and expectation is denoted by $\mathbb{E}\{\cdot\}$.

II. SYSTEM MODEL AND PROBLEM STATEMENT

In this section, we first introduce the considered system model. Subsequently, we formally present the problem statement for the feedback-aware precoder design.

A. System Model

We consider a narrow-band point-to-point MIMO system consisting of an $M$-antenna transmitter that sends $S$ independent data streams to an $N$-antenna receiver. The input-output channel model is given by

$$y = Hs + z,$$  \hspace{1cm} (1)

where $y \in \mathbb{C}^{N \times 1}$ denotes the received signal vector and $s \in \mathbb{C}^{S \times 1}$ represents the vector of transmitted independent data streams. We assume $\mathbb{E}\{ss^*\} = PS$, where $P$ denotes the transmit power. Moreover, $z \in \mathbb{C}^{N \times 1}$ denotes the additive white Gaussian noise vector at the receiver, i.e., $z \sim \mathcal{C}N(\mathbf{0}_{N \times 1}, \sigma_z^2 I_N)$ where $\sigma_z^2$ is the noise variance at each receive antenna. In $[1]$, $F \in \mathbb{C}^{M \times S}$ represents the precoding matrix satisfying $\|F\|_F = 1$ and $H \in \mathbb{C}^{N \times M}$ denotes the channel matrix, which are both discussed in more detail in the following.

1) Sparse mmWave Channel Model: mmWave channels are expected to have limited scattering due to the high free-space path loss. To incorporate this effect, a clustered channel model, i.e., the Saleh-Valenzuela model, is considered $[15]$. In this model, the channel is assumed to be a superposition of $L$ scattering clusters, each of which contributes $J$ rays (i.e., in total they are $LJ$ paths) as

$$H = \sqrt{\frac{MN}{LJ}} \sum_{k=1}^{LJ} h_k \mathbf{h}_r(\phi_k) \mathbf{h}_t(\theta_k),$$ \hspace{1cm} (2)

where $h_k \in \mathbb{C}$ is the channel coefficient of the $k$-th path and $\mathbf{h}_r(\phi_k)$ ($\mathbf{h}_t(\theta_k)$) denotes the transmitter (receiver) antenna array response vector at angle of departure (AoD) (angle of arrival (AoA)) $\phi_k$ ($\theta_k$), assuming a horizontal beamforming array only. For a uniform linear array, we obtain $\mathbf{h}_r(\phi)$ and $\mathbf{h}_t(\theta)$ as

$$\mathbf{h}_r(\phi) = \frac{1}{\sqrt{M}} \left[1, e^{j \pi d \sin(\phi)}, \ldots, e^{j (M-1) \pi d \sin(\phi)}\right]^T,$$ \hspace{1cm} (3a)

$$\mathbf{h}_t(\theta) = \frac{1}{\sqrt{N}} \left[1, e^{j \pi d \sin(\theta)}, \ldots, e^{j (N-1) \pi d \sin(\theta)}\right]^T,$$ \hspace{1cm} (3b)

where $\lambda$ denotes the wavelength and $d$ is the spacing between the antennas.

2) Hybrid Precoder: In this paper, we consider the hybrid MIMO architecture introduced in $[13]$ and $[16]$, which is illustrated in Fig. 1. This hybrid architecture consists of $S$ RF chains and $2M$ PSs per RF chain and has the advantage of being able to represent any fully digital precoder. The following lemma provides the structure of $F$ for this hybrid MIMO architecture.

**Lemma 1:** Any arbitrary precoder $F \in \mathbb{C}^{M \times S}$ can be
decomposed as

$$F = RTB,$$  \hspace{1cm} (4)$$

where \(T = [I_s, I_s]^T\). Here, \(B \in \mathbb{C}^{S \times S}\) is the baseband precoder with entries

$$B_{s,s'} = \begin{cases} \frac{1}{2} \max_{1 \leq m \leq M} |F_{m,s}|, & \text{if } s = s' \\ 0, & \text{otherwise}, \end{cases}$$  \hspace{1cm} (5)$$

where \(s, s' = 1, \ldots, S\). Diagonal matrix \(B\) can be implemented by \(S\) RF chains. Furthermore, \(R \in \mathbb{A}_{M \times S}\) is the RF precoder which can be written as \(R = [\hat{R}, \tilde{R}]\), where the entries of \(\hat{R} \in \mathbb{A}_{M \times S}\) and \(\tilde{R} \in \mathbb{A}_{M \times S}\) are given by

$$\hat{R}_{m,s} = \exp \left( j \left( \angle(F_{m,s}) + \cos^{-1} \left( \frac{|F_{m,s}|}{2B_{s,s}} \right) \right) \right),$$  \hspace{1cm} (6a)$$

$$\tilde{R}_{m,s} = \exp \left( j \left( \angle(F_{m,s}) - \cos^{-1} \left( \frac{|F_{m,s}|}{2B_{s,s}} \right) \right) \right).$$  \hspace{1cm} (6b)$$

Matrix \(R\) has modulo-one entries and hence can be implemented by a network of 2MS phase shifters.

Proof: The proof is given in [13, Theorem 1]. We note that the definitions of matrix \(R\) and \(T\) in [13] are slightly different from those in this paper, cf. [13] Eqs. (11)-(15). In fact, for notational simplicity, we decompose matrix \(R\) into \(\hat{R}\) and \(\tilde{R}\). As a result, matrix \(T\) consist of two identity matrices.

\[\blacksquare\]

B. Problem Statement

The hybrid architecture shown in Fig. [1] can achieve the same transmission rate as any fully-digital precoder, while the complexity is highly reduced because only \(S\) RF chains are used. Although any precoder matrix can be realized by the above decomposition strategy, the main problem for the design of general precoders for massive MIMO systems is the feedback overhead from the receiver to the transmitter, which is either the full channel matrix \(H\), i.e., \(MN\) complex-valued numbers, or the designed precoder \(F\), i.e., \(MS\) complex-valued numbers. As this overhead scales with the number of transmit antennas, it becomes the CSI feedback bottleneck for large \(M\). This limitation is addressed in the literature by decomposing the optimal precoder into the baseband and the RF precoders and by sending the codebook indices or quantized values of the precoders, which have much lower dimensions than the optimal precoder, over the feedback link [5], [6], [11], and [12]. Even though this approach reduces the excessive feedback overhead in massive MIMO systems and makes it dependent on the number of transmit RF chains and data streams rather than the number of transmit antennas, a precoder design that can be adapted according to the actual constraint on the rate of the feedback link has not been proposed yet. Therefore, the goal of this paper is to design a precoder whose achievable rate can be traded for the affordable feedback overhead. To this end, we will exploit the special structure of the mmWave channel, as explained in the next section.

III. FEEDBACK-AWARE PRECODER DESIGN

In this section, we design an adaptive precoder for mmWave massive MIMO systems. We first discuss the proposed design methodology and then analyze the corresponding feedback overhead.

A. Proposed Precoder Design

We start by examining the structure of the optimal unconstrained precoder. Let \(H = U \Sigma V^*\) denote the singular value decomposition (SVD) of the channel matrix \(H\), where \(U\) and \(V\) are unitary matrices containing the left and right singular vectors, respectively, and \(\Sigma\) is a diagonal matrix containing the singular values. Thereby, the optimal linear precoder that maximizes the mutual information between \(s\) and \(y\) is \(F_{opt} = [\alpha_1 v_1, \ldots, \alpha_s v_s]\), where column vector \(v_s \in \mathbb{C}^{M \times 1}\) is the right singular vector corresponding to the \(s\)-th largest singular value of \(H\) and \(\alpha_s\) is the power allocation factor for the \(s\)-th stream that has to satisfy \(\sum_s \alpha_s^2 = P\) and can be obtained with the water-filling algorithm [17]. Additionally, given the clustered channel model in (2), we can give a compact representation \(H = \hat{H}, \hat{H}^H\), where \(\hat{H}\) is a diagonal matrix containing the coefficients of all rays/path in all clusters, and \(H_r = [h_r(\theta_1), \ldots, h_r(\theta_{LJ})] \in \mathbb{C}^{N \times LJ}\) and \(H_c = [h_c(\phi_1), \ldots, h_c(\phi_{KL})] \in \mathbb{C}^{M \times LJ}\) are the receiver and transmitter array response matrices for corresponding AoAs and AoDs, respectively. Given the previous observations, it can be shown that the optimal precoder can be written as \(F_{opt} = \sum_{k=1}^{LJ} h_r(\theta_k)g_k^s\), where \(h_k\) is the \(k\)-th channel AoD and \(g_k^s \in \mathbb{C}^{S \times 1}\) is a corresponding appropriate combining vector [3].

Based on the above, we propose to approximate the optimal precoder as follows

$$\begin{align*}
\min_{\phi \in \mathbb{C}_s^{K \times 1}, \Psi \in \mathbb{C}^{M \times S}} & \|F_{opt} - \Psi(\phi)G\|_F, \\
\text{subject to:} & \|\Psi(\phi)\|_F = 1,
\end{align*}$$  \hspace{1cm} (7)$$

where \(G \in \mathbb{C}^{K \times S}\) is the combining matrix and \(\Psi(\phi) \in \mathbb{C}^{M \times K}\) is the basis matrix whose columns are the transmitter array response evaluated at the discrete AoAs in \(\phi = [\phi_1, \ldots, \phi_K]^T \in \mathbb{C}^{K \times 1}\), i.e.,

$$\Psi(\phi) = [h_r(\phi_1), \ldots, h_r(\phi_K)].$$  \hspace{1cm} (8)$$

Moreover, \(C_\phi\) denotes the set of discrete angles used to approximate the AoDs \(\phi_k\), \(\forall k\), and \(K\) is a design parameter and determines the number of AoDs used for approximating the optimal precoder. Since in general the allowed feedback overhead is small, we choose \(K \ll LJ\). Therefore, the problem in (7) is equivalent to the sparse recovery problem in compressive sensing. Thereby, OMP is a widely-adopted algorithm, which can efficiently find the best \(K\) elements of basis matrix \(\Psi(\phi) \in \mathbb{C}^{M \times |C_\phi|}\) to approximate \(F_{opt}\) [18]. Once \(\phi\) is derived, the optimization problem in (7) reduces to the well-known least square problem for \(G\) whose closed-form solution is known. Algorithm 1 summarizes the proposed
Algorithm 1 Feedback-Aware Precoding via OMP

Require: $F^{\text{opt}}, C_\phi$
1: $\phi = \text{Empty vector}$
2: $F_{\text{res}} = F^{\text{opt}}$
3: for $i = 1 : K$ do
4: $Q = \Psi^*(C_\phi) F_{\text{res}}$
5: $\hat{\phi} = \arg\max_{l \in [1 : |C_\phi|]} \|Q^* l\|_1$
6: $\phi = [\hat{\phi}, \hat{\phi}]$
7: $G = (\Psi^*(\hat{\phi}) \Psi^*(\hat{\phi}))^{-1} \Psi^*(\hat{\phi}) F^{\text{opt}}$
8: $F_{\text{res}} = \frac{F^{\text{opt}} - \Psi^{\hat{\phi}} G}{\|\Psi^{\hat{\phi}} G\|_{F}}$
end for
9: $\phi^{\text{opt}} = \phi$ and $G^{\text{opt}} = \frac{G}{\|\Psi^{\phi^{\text{opt}}} G\|_{F}}$

Return $\phi^{\text{opt}}, G^{\text{opt}}$

precoder design based on OMP and returns solutions $\phi^{\text{opt}}$ and $G^{\text{opt}}$.

Note that the transmitter only needs to know $\phi^{\text{opt}}$ and $G^{\text{opt}}$ to construct the precoder $\tilde{F} = \Psi(\phi^{\text{opt}}) G^{\text{opt}}$. Next, the transmitter can fully implement this precoder based on Lemma 1 for the hybrid MIMO architecture introduced in Section II. The structure of precoder $\tilde{F}$ implies that the required feedback scales with $K(1 + S)$, which is not only independent of the number of transmit antennas $M$, but can also be controlled with the proposed design parameter $K$. Therefore, the solution of optimization problem (7) ensures that the set of the best $K$ vectors for spanning the channel’s row space and their corresponding combining matrix are fed back to the transmitter. Consequently, the optimal precoder can be better approximated by choosing a larger $K$ at the cost of increased feedback overhead, while choosing a smaller $K$ can lower the overhead when the feedback link bandwidth is limited. We will study the effect of parameter $K$ on the precoding performance in Section IV.

B. Extension to Multi-Beam Basis Elements

The columns of the matrix $\Psi(\phi)$ defined in (8) are the transmitter array response vectors at the angles in $\phi$. Recall that the angles in $\phi$ are selected from the channel AoD codebook $C_\phi$, where the transmitter sector is partitioned into sub-sectors by the codebook resolution and the middle angle of each sub-sector is naturally selected as a codebook element. Therefore, the higher the codebook resolution is, the better the beams of $\Psi(C_\phi)$ can approximate all possible AoD realizations. However, if the codebook resolution is forced to be low due to the limited affordable feedback overhead, the beams of $\Psi(C_\phi)$ cannot cover all the AoD realizations in the transmitter beam sector and thus the performance of $\tilde{F}$ can be severely degraded. Although this is a fundamental limitation of any precoder design scheme, it can be remedied for the proposed precoder.

Note that the MIMO architecture presented in Section II can implement any precoder matrix and hence the decomposition in (7) does not impose any implementation constraint on the basis matrix $\Psi(\phi)$. Therefore, the structure of $\Psi(\phi)$ only needs to be agreed to by the transmitter and the receiver. Motivated by the above, we propose to redefine $\Psi(\phi)$ in (7) and Algorithm 1 as

$$\Psi(\phi) = \frac{1}{\sqrt{\Gamma}} \sum_{\gamma=1}^{\Gamma} h_\gamma \left( \frac{\phi_\gamma - \Delta \phi}{2} + \frac{\gamma \Delta \phi}{\Gamma + 1} \right),$$

where $\Delta \phi$ is the resolution of the AoDs in $C_\phi$ and $\Gamma$ is a design parameter. This proposition implies that the beam candidate of each sub-sector is a superposition of $\Gamma$ beams of the angles in that sub-sector. For $\Gamma = 1$, the beam candidate is the array response vector at the middle angle of the corresponding sub-sector and thus (9) reduces to (8). Note that the proposition in (9) does not impose any extra feedback overhead as $\Gamma$ is agreed to by the transmitter and the receiver.

Examples of beam patterns obtained with the proposed approach are shown in Fig. 2 where $M = 128$, $|C_\phi| = 16$, and a sector covering $\phi \in [-\pi/6, \pi/6]$ are assumed. In particular, in Fig. 2 we show $g_k(\phi)$ versus $\phi$ where $g_k(\phi)$ is defined as

$$g_k(\phi) = \frac{\int_{\phi} |h_k^*(\phi) \Psi(\phi)|^2 d\phi}{\int_{\phi} |h_k^*(\phi) \Psi(\phi)|^2 d\phi},$$

characterizing the normalized power radiated in direction $\phi$ when $\Psi(\phi_k)$ is adopted as the RF beamformer. As can be observed from Fig. 2 for $\Gamma = 1$, the power distribution across the sub-sector significantly varies with the angle and there are even several nulls within the sub-sector. This leads to a severe performance degradation when the actual AoDs are at or close to these nulls. On the other hand, the power distribution across the sub-sector becomes more uniform when $\Gamma$ is increased. Nevertheless, by increasing $\Gamma$, the maximum achievable beamforming gain decreases and the interfering
side lobes increase, too. Therefore, \( \Gamma \) is also a design parameter that should be properly chosen for a given AoD codebook resolution \( |C_\phi| \). We will study the effect of this parameter on precoder performance in Section IV.

C. Feedback Overhead Quantification

In the following, we quantify the feedback overhead of several benchmark precoder designs reported in the literature and the proposed precoder. Subsequently, for some special cases, we provide insightful results for the required feedback overhead of the proposed precoder.

1) Comparison: In general, the precoder can be computed at the transmitter or the receiver. In the latter case, the precoder obtained at the receiver has to be fed back to the transmitter whereas, in the former case, the CSI estimated at the receiver has to be fed back to the transmitter. In the following, we review the corresponding feedback overhead requirements.

Direct feedback: We consider the case where each element of the channel matrix is quantized and fed back to the transmitter, i.e., individual quantized feedback, and the transmitter computes the precoder based on the acquired CSI knowledge. Let \( C_c \) denote the set of complex numbers used for quantization. Thereby, the overall feedback overhead for this scheme is \( MN \log_2 |C_c| \) bits. Alternatively, the receiver may compute the precoder and quantize each element of the precoder and feed them back to the transmitter, which leads to an overall feedback requirement of \( MS \log_2 |C_c| \) bits. Note that the latter scheme requires lower feedback overhead as \( S \leq N \) holds.

Sparse precoder designs: Exploiting the sparsity of the mmWave channel, precoders of different complexity are designed in [3] and [5] for the fully-connected hybrid MIMO architecture, which employs \( Q \) RF chains where \( Q \geq S \). The overall feedback requirement of these precoder designs is \( Q \log_2 |C_\phi| + QS \log_2 |C_c| \) bits where the terms \( Q \log_2 |C_\phi| \) and \( QS \log_2 |C_c| \) correspond to the analog and digital precoders, respectively. Note that unlike the feedback overhead of the direct schemes, the overhead of the precoder designs in [3] and [5] does not scale with the number of antennas \( M \).

Multi-level channel estimation: mmWave channel acquisition is considered in [8] and [10] and multi-level RF codebooks are designed, which are able to improve the CSI acquisition quality by increasing the adopted number of codebook levels. Thereby, for FDD systems, quantized versions of the \( K \) strongest AoDs, \( \phi_1, \ldots, \phi_K \), and AoAs, denoted by \( \theta_1, \ldots, \theta_K \), and their corresponding channel path coefficients, denoted by \( h_1, \ldots, h_K \), are fed back to the transmitter. The transmitter is then able to reconstruct the channel matrix as \( \hat{H} = \hat{H}_r \hat{H}_t^* \), where \( \hat{H}_r = [h_1(\hat{\theta}_1), \ldots, h_1(\hat{\theta}_K)] \) and \( \hat{H}_t = [h_1(\hat{\phi}_1), \ldots, h_1(\hat{\phi}_K)] \), and \( \hat{H} \) is a diagonal matrix with non-zero elements \( \hat{h}_1, \ldots, \hat{h}_K \). Having the estimated channel \( \hat{H} \), the transmitter computes the desired precoder. The feedback overhead of this scheme is \( 2K \log_2 |C_\phi| + K \log_2 |C_c| \) where we assume the same codebook \( C_\phi \) for both the AoAs and AoDs for simplicity. Note that this scheme has the advantage that the accuracy of CSI acquisition and correspondingly the feedback overhead requirement can be adjusted by parameter \( K \).

Proposed precoder design: Similar to [3] and [5], we exploit the sparsity of the mmWave channel; however, we developed our precoder based on the hybrid MIMO architecture in [13]. The overall feedback requirement of our precoder design is \( K \log_2 |C_\phi| + KS \log_2 |C_c| \) bits. Note that unlike the fixed feedback overhead of the precoder designs in [3] and [5], the feedback overhead of the proposed design can be adjusted by choosing the parameter \( K \). This enables the proposed design to adapt itself to a feedback overhead that can be afforded by the system. Unlike [8] and [10], which compute the precoder at the transmitter, we compute the precoder at the receiver, which is shown in Section IV to be more efficient.

Table I summarizes the overhead for different precoder designs for point-to-point MIMO systems.

2) Scaling Order of the Feedback Overhead for the Proposed Precoder Design: We first present the scaling order of the required feedback overhead of the proposed precoder needed to approach the performance of an ideal system, where no feedback constraint is applied, on the implementation of the optimal precoder. We further provide the scaling overhead of the required feedback for asymptotic case when \( M \to \infty \).

Lemma 2: To approach the performance of the ideal system, the feedback overhead requirement for the proposed precoding scheme is on the order of \( LJ(\log_2 |C_\phi| + \log_2 |C_c|) \) bits assuming that \( |C_\phi| \) and \( |C_c| \) are sufficiently large.

Proof: For the non-asymptotic regime of \( M \), the columns of the optimal unitary matrix \( F^opt \) may depend on the array response of all existing AoAs. Therefore, we choose \( K = LJ \). Thereby, \( \hat{F} \) approaches \( F^opt \) assuming that \( |C_\phi| \) and \( |C_c| \) are sufficiently large. This leads to an overhead requirement of \( LJ(\log_2 |C_\phi| + \log_2 |C_c|) \) bits and concludes the proof.

The above lemma states that as long as the mmWave channel contains limited scattering, i.e., \( LJ \ll M \), the proposed scheme can effectively approach the performance of the ideal system with a limited feedback requirement that linearly scales with \( LJ \).

Lemma 3: Asymptotically as \( M \to \infty \), the feedback overhead requirement for the proposed precoding scheme to approach the performance of the ideal system is on the order of \( S(\log_2 |C_\phi| + \log_2 |C_c|) \) bits assuming that \( |C_\phi| \) and \( |C_c| \) are sufficiently large.

Proof: As \( M \to \infty \), the columns of matrix \( H_t \) become orthogonal [19]. Thereby, the columns of the optimal matrix \( F^opt \) become identical to a scaled version of the first \( S \) columns of \( H_t \), where the scaling factors are the power

| Precoding scheme | Total feedback overhead [bits] |
|------------------|-------------------------------|
| Feeding back elements of \( H \) | \( MN \log_2 |C_c| \) |
| Feeding back elements of \( F \) | \( MS \log_2 |C_c| \) |
| Sparse precoder design [3], [5] | \( Q \log_2 |C_\phi| + QS \log_2 |C_c| \) |
| Multi-level channel estimation [8], [10] | \( 2K \log_2 |C_\phi| + K \log_2 |C_c| \) |
| Proposed precoder design | \( K \log_2 |C_\phi| + KS \log_2 |C_c| \) |

TABLE I. FEEDBACK OVERHEAD COMPARISON
Fig. 3: a) Achievable rate $I(s;y)$; and b) BER of the proposed precoding scheme with $K = 6$, 8, and 16 in comparison with the ideal system with the optimal precoder $F_{\text{opt}}$, the spatially sparse precoding scheme presented in [3], and the precoder based on the multi-level channel estimation presented in [8] and [10].

allocation variables. Therefore, assuming $K \geq S$, matrix $F_{\text{opt}}$ can be fully reconstructed from (7) by choosing $K = S$ and $G_{\text{opt}} = \text{diag}(\alpha_1, \ldots, \alpha_S)$. Thereby, assuming that $|C_p|$ and $|C_c|$ are sufficiently large, the feedback overhead is on the order of $S(\log_2|C_p| + \log_2|C_c|)$ bits. This completes the proof. 

The above lemma states that asymptotically as $M \to \infty$, simple beam steering along the S dominant AoA is optimal.

IV. COMPARISON AND SIMULATION RESULTS

In this section, we demonstrate the performance of the proposed precoder to illustrate the impact of the design parameters $K$, $\Gamma$, and $|C_p|$.

A. Simulation Setup and Benchmarks

We assume $M = 128$ transmitter antennas, $N = 16$ receive antennas, and $S = 4$ independent data streams. We consider the channel model described in [2] with $L = 12$ clusters and $J = 20$ rays per cluster. The channel coefficients $h_k$ are independent and identically distributed normal random variables $CN(0,1)$ and the AoDs (AoAs) within each cluster $\phi_k(\theta_k)$ are distributed according to a Laplacian distribution with a uniformly-random mean within a 90°-wide transmitter (360°-wide receiver) sector. We also assume a uniform linear array with $d = \lambda/2$ at both the transmitter and receiver.

We consider the following benchmark schemes:

**Benchmark 1:** As the first benchmark scheme, we adopt the precoder in [3] which was designed to approximate the optimal unconstrained unitary precoder, i.e., $F_{\text{opt}}$ with $\alpha_1 = \cdots = \alpha_S$. Therefore, for a fair comparison, we also use the optimal unitary precoder for the design of our proposed precoder in [7]. Further, we consider $Q = 8$ transmit RF chains for this benchmark scheme. Therefore, from a hardware perspective, the considered architecture in Fig. 1 is simpler than that assumed for this benchmark scheme since it has half the number of RF chains and the same number of PSs, i.e., $8 \times 128$.

**Benchmark 2:** As second benchmark scheme, we consider the case when CSI is estimated using multi-level channel estimators [8] and [10] and is fed back to the transmitter, which computes the precoder. For this benchmark scheme, we assume that an optimal unitary precoder is computed based on the SVD of the fed back CSI, which can be realized using the architecture in Fig. 1.

**Benchmark 3 (Upper bound):** We assume the ideal system for this benchmark, and therefore, we consider the optimal unitary precoder computed based on the exact CSI as a performance upper bound.

Note that we show the results for i) coded transmission in terms of the achievable rate $I(s;y) = \log_2(|I_{yy} + \frac{P}{\sigma_n^2} HH^H|)$, where $I(s;y)$ denotes the mutual information between $s$ and $y$ and ii) uncoded transmission with quadrature phase shift keying (QPSK) modulation and linear minimum mean square error (MMSE) detection in terms of the bit error rate (BER). Due to space constraints, we assume ideal feedback for the amplitudes and focus on the feedback overhead of the angles and the impact of parameters $K$, $\Gamma$, and $|C_p|$.

B. Simulation Results

Fig. 3 shows the performance of the proposed precoding scheme for three feedback parameters $K = 6$, 8, and 16. We plot $I(s;y)$ in Fig. 3a and BER in Fig. 3b vs. the signal-to-noise ratio (SNR) defined as $\frac{P}{\sigma_n^2}$. A sufficiently large resolution for the AoD codebook, i.e., $|C_p| = 2^8$, and $\Gamma = 1$ are assumed, to study the impact of $K$. The results in Fig. 3 indicate that by changing parameter $K$, the precoder performance can be traded with feedback overhead. Specifically, by choosing $K = 8$, the proposed precoder achieves the same performance and
requires the same feedback overhead as the precoder presented in [8]. However, by increasing $K$ to 16 and doubling the feedback overhead, the performance of the proposed precoder is improved by almost 3 dB in terms of BER and 2 dB in terms of achievable rate and closely approaches the performance of the optimal precoder. Furthermore, for $K = 6$, the feedback overhead is decreased at the expense of a lower precoder performance. It is worth noting that, although [10] provides the flexibility of estimating the channel with different numbers of paths and adapting the feedback rate, for similar feedback overhead, the corresponding performance is much lower than that of the proposed precoder design.

In Fig. 4, we evaluate the impact of parameter $\Gamma$ for $K = 16$ on the BER. The figure shows the precoder performance for $|C_0| = 2^4, 2^5, 2^6$, and $2^8$ with $\Gamma = 1$ and $\Gamma = 2$. As can be observed, increasing $\Gamma$ can improve the precoder performance in the low resolution regime, i.e., for $2^4$ and $2^5$, and partially compensate for the corresponding performance loss due the insufficient codebook resolution, while this improvement becomes small for higher resolutions, i.e., for $2^6$ and $2^8$.

V. CONCLUSION

In this paper, we considered a narrow-band point-to-point massive MIMO system with a hybrid architecture that employs two PSs for each coefficient of the analog precoder and is able to realize any arbitrary precoder. Exploiting the degrees of freedom offered by this architecture and the mmWave channel structure, we proposed a precoding scheme that approximates the optimal precoder by selecting a desired number of elements from the transmitter array response basis matrix and a corresponding combining matrix. The proposed precoder design provided a flexible tradeoff between precoding performance and feedback overhead. Simulation results showed that the proposed precoding scheme can provide a near-optimal solution with a higher feedback overhead, while an acceptable approximation of the optimal precoder is maintained when a lower feedback overhead is demanded. Moreover, for a given affordable feedback overhead, the proposed precoder outperforms benchmark schemes from the literature.

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