Modelling surface magnetic field evolution on AB Doradûs due to diffusion and surface differential rotation

G.R. Pointer\textsuperscript{1}, M. Jardine\textsuperscript{1} \textsuperscript{*}, A. Collier Cameron\textsuperscript{1} and J-F. Donati\textsuperscript{2}

\textsuperscript{1}School of Physics and Astronomy, University of St Andrews, St Andrews, Fife, KY16 9SS, Scotland
\textsuperscript{2}Laboratoire d’Astrophysique, Observatoire Midi-Pyrénées, F-31400 Toulouse, France

ABSTRACT
From Zeeman Doppler images of the young, rapidly-rotating K0 dwarf AB Doradûs, we have created a potential approximation to the observed radial magnetic field and have evolved it over 30 days due to the observed surface differential rotation, meridional flow and various diffusion rates. Assuming that the dark polar cap seen in Doppler images of this star is caused by the presence of a unipolar field, we have shown that the observed differential rotation will shear this field to produce the observed high-latitude band of unidirectional azimuthal field. By cross-correlating the evolved fields each day with the initial field we have followed the decay with time of the cross-correlation function. Over 30 days it decays by only 10%. This contrasts with the results of Barnes et al. (1998) who show that on this timescale the spot distribution of He699 is uncorrelated. We propose that this is due to the effects of flux emergence changing the spot distributions.

Key words: stars: activity – stars: imaging – stars: individual: AB Dor – stars: magnetic fields – stars: rotation – stars: spots

1 INTRODUCTION
AB Doradûs is a K0 dwarf, originally discovered as a bright X-ray source by Pakull (1981) and subsequently observed by ROSAT (Kürster et al. 1997) and BeppoSAX (Maggio et al. 2000). Its photometric variability is believed to be due to starspots (Anders 1990, Innis et al. 1988) and this, combined with its brightness ($V \approx 6.8 - 7.0$) and rapid rotation ($P=0.12514$) have made it an attractive candidate for Doppler imaging (Kürster, Schmitt & Cutispoto 1994, Collier Cameron & Unruh 1994, Collier Cameron 1995, Unruh, Collier Cameron & Cutispoto 1995, Donati & Collier Cameron 1997, Donati et al. 1999, Collier Cameron et al. 1999). From observations of the lithium line at 6708 Å, Rucinski (1982) suggested it was a post-T Tauri star. According to HIPPARCOS data it is 14.94±0.12 pc away. Collier Cameron & Foing (1997) inferred an age of $\sim$2-3$\times$10$^7$ year and its common-proper-motion companion, the M dwarf Rst 137b.

AB Dor is of interest for a variety of reasons. The most important, for the purposes of this paper, is that Zeeman Doppler images have been obtained in 3 consecutive years: 1995 Dec 7-13 (Donati & Collier Cameron 1997); 1996 Dec 23-29 (Donati et al. 1999); and 1998 Jan 10-15. These studies reveal that the radial field has at least 12 regions of opposite polarities at intermediate to high latitude, which are approximately regularly spaced in longitude together with a unidirectional ring of azimuthal field at 70-80° indicating an underlying large-scale toroidal field (Donati et al. 1999).

$\textsuperscript{*}$ E-mail: moira.jardine@st-and.ac.uk
The purpose this paper is to investigate the effect of diffusion and differential rotation on the evolution of AB Doradus’s magnetic field. We aim to find out whether shearing at the edge of a unipolar cap can produce the observed ring of unidirectional azimuthal field. We also seek to determine the lifetimes of surface magnetic features subject to diffusion and differential rotation.

2 TEMPORAL EVOLUTION OF THE MAGNETIC FIELD

We are using a code originally developed by van Ballegooijen, Cartledge & Priest (1998) to study the formation of filament channels on the Sun. It can also be used to study the field of AB Doradus because we have high-resolution magnetic maps (2" at the equator) and the differential rotation is similar to the solar value. The code takes the observed surface radial component of the field and calculates a potential field from this, and then evolves the calculated magnetic field due to the effects of differential rotation and diffusion.

Jardine et al. (1999) have demonstrated that for latitudes below about 60° the field is well-represented by a potential field due to the effects of differential rotation and diffusion. Jardine et al. (1999) have demonstrated that for latitudes below about 60° the field is well-represented by a potential field due to the effects of differential rotation and diffusion. Since most stellar prominences form at around the corotation radius (2.7 Rs), we know that a significant fraction of the field is closed at that radius. Hence we choose Rs = 5.1 Rs. We then have

\[ a_{lm} r_s^{-l-1} + b_{lm} r_s^{-l-2} = 0, \]

equivalent to

\[ \psi_{lm} = 0. \]

As a second boundary condition we impose the radial field at the surface to be the observed radial field. We can then express the magnetic field in terms of the two-dimensional Fourier coefficients B_{lm}, where

\[ B_{lm}(R_s) = 2\pi \int_0^\pi B_m(\theta) P_{lm}(\theta) \sin \theta d\theta \]

so

\[ B_{lm}(R_s) = -i a_{lm} R_s^{-l-1} + (1 + 1) b_{lm} R_s^{-l-2}. \]

The function B_{lm}(\theta) is derived from a fast Fourier transform performed latitude-by-latitude on the observed radial field B_r(R_s, \theta, \phi).

Once the field is evolved due to diffusion and differential rotation, it is not necessarily potential, although it can still be expressed as a sum of spherical harmonics. The field components are then expressed in terms of the functions

\[ J = \sum_{l=1}^{N} \sum_{m=-l}^{l} J_{lm} P_{lm}(\theta) e^{im\phi}, \]

and

\[ A = \sum_{l=1}^{N} \sum_{m=-l}^{l} A_{lm} P_{lm}(\theta) e^{im\phi}, \]

where

\[ J = \frac{1}{\sin \theta} \left( \frac{\partial}{\partial \theta} (\sin \theta B_\theta) - \frac{\partial B_\phi}{\partial \phi} \right) \]

is the radial component of the current and

\[ A = \frac{1}{\sin \theta} \left( \frac{\partial}{\partial \theta} (\sin \theta B_\theta) + \frac{\partial B_\phi}{\partial \phi} \right) \]

is the 2-dimensional divergence.

2.1 The scalar magnetic potential, \( \psi \)

If we assume that the field is potential, then we can write \( \mathbf{B} \) in terms of a flux function \( \psi \), with

\[ \mathbf{B} = -\nabla \psi, \]

which in spherical co-ordinates gives

\[ B_r = -\frac{1}{r} \frac{\partial \psi}{\partial r}, \]

\[ B_\theta = -\frac{1}{r \sin \theta} \frac{\partial \psi}{\partial \theta}, \]

and

\[ B_\phi = -\frac{1}{r \sin \theta} \frac{\partial \psi}{\partial \phi}. \]

Here \( \psi \) satisfies Laplace’s equation \( \nabla^2 \psi = 0 \) which can be expressed as

\[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \psi}{\partial \phi^2} = 0. \]

A separable solution for \( \psi \) can be found

\[ \psi(r, \theta, \phi) = \sum_{l=1}^{N} \sum_{m=-l}^{l} \psi_{lm}(r) P_{lm}(\theta) e^{im\phi}, \]

where \( P_{lm} \) are the associated Legendre functions and

\[ \psi_{lm}(r) = a_{lm} r^l + b_{lm} r^{-(l+1)}. \]

We chose to truncate the series at \( N = 63 \), corresponding to the maximum resolution of the reconstructed field images. We clearly need two boundary conditions to determine \( \psi \). We chose to specify as one boundary condition that at some distance from the star (the source surface, \( r_s \approx 5.1 R_s \)), the field is radial and so \( B_\phi(r_s) = B_\theta(r_s) = 0 \) (Schatten, Wilcox & Ness 1963). This mimics the stellar wind.

The purpose this paper is to investigate the effect of diffusion and differential rotation on the evolution of AB Doradus’s magnetic field. We aim to find out whether shearing at the edge of a unipolar cap can produce the observed ring of unidirectional azimuthal field. We also seek to determine the lifetimes of surface magnetic features subject to diffusion and differential rotation.
Modelling surface magnetic field evolution on AB Doradus

Figure 1. This shows how the magnetic energy varies as the field evolves for 1998. The left hand diagram is for the case when $\eta=250\text{km}^2\text{s}^{-1}$, with differential rotation (solid line) and without (dotted line), and the right hand diagram compares the case where there is no diffusion, i.e. $\eta = 0$ (solid line) and where $\eta=250\text{km}^2\text{s}^{-1}$ (dotted line).

Figure 2. Cross-correlation of calculated radial field with the observed radial field for 1995, latitude $30^\circ$ (left) and $60^\circ$ (right). The solid line represents $\eta = 250\text{km}^2\text{s}^{-1}$, the dotted line $\eta = 350\text{km}^2\text{s}^{-1}$, the dashed line $\eta = 450\text{km}^2\text{s}^{-1}$ and the dot-dash line $\eta = 550\text{km}^2\text{s}^{-1}$.

We should not expect a cross-correlation of exactly 1 at day 0 since we are correlating an observed field with a calculated one.

and Ohm’s Law

$$\mathbf{j} = \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B}),$$

where $\sigma$ is the conductivity we get the induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times \mathbf{E}'.$$  \hspace{1cm} (2)

Here $\mathbf{E}'$ is given by

$$E'_r = \frac{\eta}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (B_\phi \sin \theta) - \frac{\partial B_\theta}{\partial \phi} \right],$$

$$E'_\theta = \frac{\eta}{r \sin \theta} \frac{\partial B_r}{\partial \phi},$$

$$E'_\phi = - \frac{\eta}{r} \frac{\partial B_r}{\partial \theta},$$

with $\eta = 1/\mu \sigma$ being the magnetic diffusivity. \hspace{1cm} (3)

Here $\lambda \equiv \frac{\pi}{2} - \theta$ is the latitude, $\lambda_0$ gives the latitude above which the meridional flow is zero, $\lambda_0 = 75^\circ$ and $u_0 = 11\text{m s}^{-1}$ which is close to the predicted value \hspace{1cm} (Rüdiger 1999). The values of $B_{lm}$, $A_{lm}$ and $J_{lm}$ are evolved using the induction equation according to the meridional flow, the observed differential rotation and using various values of the magnetic diffusion, ranging from 250 to 550 km$^2$s$^{-1}$ (cf. the solar value of 450 km$^2$s$^{-1}$).

The differential rotation is of the form

$$\Omega(\theta) = 12.2434 - 0.0564 \cos^2 \theta \text{ rad d}^{-1}$$

where $\Omega$ is the rotation rate \hspace{1cm} (Donati & Collier Cameron 1997). We have assumed that $\eta$ is uniform across the surface, although there is evidence that this may not be the case for the Sun \hspace{1cm} (Berger et al. 1998).

2.3 Calculating the evolved field

The third stage is to take the evolved coefficients $B_{lm}$, $A_{lm}$ and $J_{lm}$ and the associated Legendre functions $P_{lm}$ and calculate the three components of the magnetic field- $B_r$ (radial), $B_\phi$ (azimuthal) and $B_\theta$ (meridional)- from them. These will be given by

$$B_r(r, \theta, \phi) = \sum_{l=1}^{N} \sum_{m=-l}^{l} B_{lm} P_{lm}(\theta) e^{im\phi}.$$
This diagram shows the azimuthal and radial components of the field in 1996. The top row gives the observed radial (left) and azimuthal (right) fields for Dec 23/25. The middle row gives the observed radial (left) and azimuthal (right) fields for Dec 29. The bottom row gives the calculated radial (left) and azimuthal (right) fields for Dec 29, assuming that $\eta = 450 \text{km}^2 \text{s}^{-1}$.

\[
B_\theta(r, \theta, \phi) = \sum_{l=1}^{N} \sum_{m=-l}^{l} \lambda_l \left[ -A_l(r) \frac{dP_l}{d\theta} + iJ_m \frac{mP_l}{\sin \theta} \right] e^{im\phi}
\]

\[
B_\phi(r, \theta, \phi) = \sum_{l=1}^{N} \sum_{m=-l}^{l} \lambda_l \left[ -A_l(r) \frac{i m P_l}{\sin \theta} - J_m \frac{dP_l}{d\theta} \right] e^{im\phi}
\]

where \( \lambda_l = \frac{1}{l(l+1)} \).

3 RESULTS

As an initial consistency check we computed the evolution of the magnetic energy in the field at the surface, calculating the ratio of magnetic energy in the evolved case to the original.

A magnetic field has energy $B^2/2\mu$ per unit volume, so the total energy is

\[
W = \int_{\text{volume}} \frac{B^2}{2\mu} dV
\]

Fig 1 shows this for the 1998 field. The left-hand panel shows the degree of diffusive decay of the field energy over 30 days. The right-hand panel shows the increase in energy that would occur in the absence of diffusion, due to the winding-up of the field by the differential rotation.

3.1 Long-term evolution of the surface field

We took the observed field for the 1995 and 1998 observations and evolved it over 30 days according to the induction equation (4). This allowed us to study the variation with time of the cross-correlation of the radial component observed on the first night with that calculated for subsequent nights (Fig 2). We chose two latitudes: 30 and 60° north. The results for 1998 are qualitatively similar. In all cases, the cross-correlation function decays by approximately 10% over 30 days. Although choosing a higher value for the diffusivity does cause a more rapid decay of the field and hence a faster decay of the cross-correlation function, it is still not enough to explain the complete lack of correlation found by Barnes et al. (1998) for He699. It appears that for AB Doradus, if diffusion and differential rotation were the only processes causing the field to evolve, that even after one month there should still be a good correlation.
3.2 Short-term evolution

For comparison, we can look at the field evolution over a much shorter period of time. This has the advantage that we can compare our results with the observed evolution of the field during a single observing run. Here we use data from the 1996 run, in which we secured two sets of magnetic maps, separated by 5 nights (Fig. 3). Our aim is to determine whether the observed magnetic elements retain their identities over a period of 5 nights in the presence of diffusion and differential rotation.

We began by investigating the effect of varying the diffusivity. We evolved the 1996 Dec 23/25 field forward in time using various values of the diffusion coefficient. In each case we cross-correlated the resulting radial field map with the observed radial field for Dec 29 (Fig 4). We ensured that the cross-correlation only involved those longitudes that were well-observed, viz. 18-180° (Jardine et al. 1999). The cross-correlation function was computed for each of a set of latitudes between 0 and 80° in the visible hemisphere, and the amplitude of the strongest peak in the ccf at each latitude is plotted in Fig. 4. It can be seen that the five-day span of these observations is too short for differences in the value chosen for diffusivity to have much effect.

We also considered the case where the field was allowed to evolve under the influence of diffusion only, i.e. with the differential rotation switched off. We found that over the 5-day span of the 1996 December observations, the influence of the differential rotation was negligible (Fig. 5).

From these we see that altering the value of the diffusion and removing the differential rotation have little effect on the cross-correlation over 5 nights. We would need observations over a longer timescale, say a month, to be able to look for meaningful results.

4 THE HIGH-LATITUDE AZIMUTHAL FIELD

Jardine et al. (1999) demonstrated that the high-latitude azimuthal band of field was not reproduced by modelling the field as potential. Here we investigate whether the differential rotation could produce this band by taking the 1998 radial field map and adding in a cap of unipolar radial field extending from the pole to latitude 80°, well within the dark polar spot seen in the stellar surface-brightness map. An identical cap of opposite polarity was added to the unobservable hemisphere to conserve flux. We emphasize that the polarity and strength of the dark polar cap cannot be determined directly from the observations, since the low surface brightness and strong foreshortening suppress the Zeeman signal from this part of the star. For each of a range of plausible polar field strengths and polarities we evolved the field...
forward in time for 5 days, then compared the mean value of \( B_\phi \) at each latitude with the observed value.

We see from Fig. 6 that a high-latitude azimuthal band of negative polarity is produced when we shear the image at the observed differential rotation rate with a polar cap having \( B_\phi < 0 \) and vice versa. The results confirm that the observed differential rotation is capable of producing a high-latitude negative azimuthal band, as is seen in the observations from all three observing seasons (Fig. 7). This predicts that the magnetic polarity of the dark polar region was predominantly negative in all three seasons.

The latitude of the maximum in \( B_\phi \) occurs at the edge of the imposed unipolar cap in Fig. 6. The azimuthal field is localised here because most field lines originating in the north polar cap are connected to high northern latitudes just outside the cap. The direction of these field lines depends strongly upon the distribution of radial field at mid to high latitudes. Changing the strength of the polar cap has little effect on field lines at low latitude. This corresponds to our result that \( \sum B_\phi \) varies little at low latitude as the strength of the cap is altered (as seen in Fig. 6).

The observed azimuthal band plotted in Fig. 7 is more diffuse than the model, having a broader peak between latitudes 65° and 80° in all three seasons’ data. This corresponds roughly to the edge of the dark polar region, which extends to latitude 70° or so. The breadth of the peak suggests a more gradual fall-off in the polar field than we imposed on the model. The apparent decrease in field strength at latitudes above 70° can probably be ascribed to suppression of the Zeeman signal within the dark polar region.

The strength of the azimuthal field after only 5 nights is substantially less than that observed. This is not surprising, since the timescale on which shear can generate an azimuthal field with a strength comparable to the radial and meridional field will be of order the equator-pole lap time of 110 days. The diffusion time for length scales comparable to the size of the individual magnetic regions in the images is also of this order. Once an equilibrium is established between diffusion and differential rotation, we would expect the azimuthal field strength to be an order of magnitude greater than that produced after 5 days, in agreement with the observations. On a 100-day timescale, however, we expect the picture to be complicated further by the emergence of new flux, making a direct comparison with the observations problematic.

5 CONCLUSIONS

We have modelled the evolution of the magnetic field of AB Doradus due to the effects of differential rotation and diffusion. We use as a starting point the Zeeman Doppler images obtained on three consecutive years and assume that the field is initially potential, but evolves away from this state as a function of time.

Over a timescale of 20 to 30 days we have determined, as a function of time, the cross-correlation of our model radial magnetic field with the observed radial component on the first night. We find that over one month, the cross-correlation function decays by about 10%. Observations of He699 by Barnes et al. (1998) show however that cross-correlating the observed spot distributions over this timescale gives much more rapid decrease of the cross-correlation function. This result suggests that the evolution of AB Doradus’s surface magnetic field is not governed solely by diffusion and differential rotation. We conclude that these results are more likely to be due to the effects of flux emergence changing the spot distribution than the effects of diffusion or differential rotation.

This result is independent of the assumed degree of field diffusion. We have compared the effects of values of \( \eta \) ranging from 250 to 450 km s\(^{-1}\) and found the results to be qualitatively the same. The presence of some diffusion is of course necessary (and we have confirmed that the magnetic energy grows monotonically with time in the absence of diffusion). The exact value of \( \eta \) seems however to have little effect on the five-day timescale of a typical observing run. Since diffusion has little effect on the flux distribution, the differential rotation acts simply to advect the field. Consequently, although at each latitude the peak of the cross-correlation function may be at a different longitude (Donati & Collier Cameron 1997), its actual value is virtually unchanged by the effects of differential rotation.

We have also compared the radial and azimuthal mag-
Magnetic fields generated by our model over 5 nights with those obtained from Zeeman Doppler images on 1996 Dec 23/25 and 29, and found the agreement to be excellent. The evolution of the azimuthal field is of particular interest with regard to the band of high latitude unidirectional azimuthal field seen in the Zeeman-Doppler images. We have investigated whether the shearing effect of the differential rotation is sufficient to generate this band of field. The polarity of this band depends upon the sign of the radial field in the polar cap, whereas its strength depends on the competition between shear and diffusion. Since the diffusion timescale for resolvable features is comparable to the winding time, we expect the azimuthal field to attain a strength comparable to the radial and meridional field near the boundary of the polar cap, as is indeed observed. Our results suggest that the differential rotation could play a major part in the creation and preservation of a high latitude azimuthal band.

6 ACKNOWLEDGEMENTS

We would like to thank Drs. D. Mackay, A. van Ballegooijen and M. Ferreira for useful discussions and assistance during the course of this work. We also thank Drs. G. Hussain and L. Kitchatinov for their careful reading of the manuscript and offering comments. GRP acknowledges the support of a studentship from the University of St Andrews.

REFERENCES

Anders G., 1990, Inf. Bull. Variable Stars, 3437
Barnes J. R., Collier Cameron A., Unruh Y. C., Donati J.-F., Hussain G. A. J., 1998, MNRAS, 299, 904
Berger T. E., Lõddahl M. G., Shine R. A., Title A. M., 1998, ApJ, 506, 439
Collier Cameron A., Foing B. H., 1997, Observatory, 117, 218
Collier Cameron A., Robinson R. D., 1989a, MNRAS, 236, 57
Collier Cameron A., Robinson R. D., 1989b, MNRAS, 238, 657
Collier Cameron A., Unruh Y. C., 1994, MNRAS, 269, 814
Collier Cameron A., Bedford D. K., Rucinski S. M., Vilhu O., White N. E., 1988, MNRAS, 231, 131
Collier Cameron A., Duncan D. K., Ehrenfreund P., Foing B. H., Kuntz K. D., Penston M. V., Robinson R. D., Soderblom D. R., 1990, MNRAS, 247, 415
Collier Cameron A. et al., 1999, MNRAS, 308, 493
Collier Cameron A., 1995, MNRAS, 275, 534
Donati J.-F., Collier Cameron A., 1997, MNRAS, 291, 1
Donati J.-F., Collier Cameron A., Hussain G. A. J., Semel M., 1999, MNRAS, 302, 437
Hussain G. A. J., Jardine M., Collier Cameron A., 2001, MNRAS, 322, 681
Hussain G. A. J., van Ballegooijen A., Jardine M., 2001, in Twelfth Cambridge Workshop on Cool Stars, Stellar Systems, and the Sun. ASP Conference Series, San Francisco, In press
Innis J. L., Thompson K., Coates D. W., Lloyd Evans T., 1988, MNRAS, 235, 1411
Jardine M., Ferreira J. M. T. S., 1996, Astrophysical Letters and Communications, 34, 101
Jardine M., Barnes J., Donati J.-F., Collier Cameron A., 1999, MNRAS, 305, L35
Kitchatinov L., Rüdiger G., 1999, A&A, 344, 911
Kürster M., Schmitt J. H. M. M., Cutispoto G., Dennerl K., 1997, A&A, 320, 831
Kürster M., Schmitt J. H. M. M., Cutispoto G., 1994, A&A, 289, 899
Lim J., White S. M., Nelson G. J., Benz A. O., 1994, ApJ, 430, 332
Maggio A., Pallavicini R., Reale F., Tagliaferri G., 2000, A&A, 356, 627
Pakull M. W., 1981, A&A, 104, 33
Rucinski S. M., 1982, Inf. Bull. Variable Stars, 2203
Schatten K., Wilcox J., Ness N., 1969, Solar Phys., 6, 442
Schmitt J. H. M. M., Cutispoto G., Krautter J., 1998, ApJ, 500, L25
Unruh Y. C., Collier Cameron A., Cutispoto G., 1995, MNRAS, 277, 1145
van Ballegooijen A., Cartledge N., Priest E., 1998, ApJ, 501, 866