The contribution of strange quarks to the proton magnetic moment

G.Dillon and G.Morpurgo

Università di Genova and Istituto Nazionale di Fisica Nucleare, Sezione di Genova. 1

Abstract. We show that from the e.m. magnetic moments of all octet baryons one cannot determine $\mu_s^p$ (the strange proton magnetic moment), contrarily to a result of Jido and Weise. Using the general QCD parametrization (GP) we clarify why one can, instead, obtain $\mu_s^p$ from the Z magnetic moment (or Sachs $G_M^Z(0)$) of the proton. This is due to the Trace terms in the GP of the magnetic moments and specifically to the fact that the coefficient of $\text{Tr}[Q^sP^s]$ cannot be extracted from the octet ($\gamma$) moments ($Q^s$’s are the $u,d,s$ electric charges and $P^s$ is the $s$ quark projector). To measure (or put an upper limit to) $\mu_s^p$ by the electroweak $e-p$ scattering experiments, one exploits the small terms proportional to $\text{Tr}[Q^Z]$ and $\text{Tr}[Q^ZP^s]$ ($Q^Z$ are the electroweak charges of $u,d,s$). We relate $\mu_s^p$ in the Beck-McKeown formula to the Z Trace terms. We estimate a 3-gluon exchange factor reducing the order of magnitude of $|\mu_s^p|$.

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1. Introduction

The $s\bar{s}$ contribution $\mu_s^p$ to the proton magnetic moment has been the subject, till 5 years ago (compare [1]), of at least 20 calculations producing values (with both signs) of $\mu_s^p$ from 0.8 down to 0.003 in units $\mu_N$; a very recent QCD lattice calculation [2] gives $\mu_s^p = -0.046 \pm 0.019 \mu_N$. The experimental data - based on the e.m.-weak interference in e-p scattering at low $q^2$ - are at present unable to reach this precision. 2

In a most recent calculation on the subject [3] it is asserted that $\mu_s^p$ can be obtained from a group theoretical $SU_3$ analysis of the octet baryon magnetic moments; the result is (fit 2) $\mu_s^p = +0.155 \pm 0.022$ or (with a more complete calculation - fit 3) $\mu_s^p = +0.161 \pm 0.028$ in units $\mu_N$. However the method is incorrect. Even a perfect experimental knowledge of the magnetic moments of all octet baryons would not allow to deduce -contrarily to what is implied in Ref.[3]- the contribution of the $s\bar{s}$ to the proton magnetic moment. We consider this here, because the general QCD parametrization (GP) reveals some interesting aspects. The main argument, to be developed below, goes as follows: The treatment in [3] ignores the circumstance that the $\text{Tr}[Q^Z]$ of the electric charge matrix of $u,d,s$ is zero whereas the $\text{Tr}[Q^Z]$ of the electro-weak charge

1 e-mail: dillon@ge.infn.it ; morpurgo@ge.infn.it

2 The last published value from the SAMPLE experiment ([4]) at $q^2 = 0.1(GeV^2)$ is : $G_M^p(q^2 = 0.1)(p) = 0.37 \pm 0.20 \pm 0.26 \pm 0.07$. For the results of other continuing experiments (HAPPEX, GO and MAINZ) compare [5].
$Q^2$ of $u,d,s$ is different from zero; this explains why, to find $\mu_s^p$, one had to embark in the difficult $\gamma-Z$ interference experiments mentioned above.

We will proceed as follows. In Sect.2 we recall the basis of the GP; in Sect.3 we re-discuss the most general QCD expression of the octet baryon moments; indeed, to extract the contribution of the $s\bar{s}$ loops in the proton, one must focus on the Trace terms in the above expression. Note the following: A Trace term $\text{Tr}[Q^\gamma P^s]$ -that, in principle, might contribute to $\mu_s^p$- appears in the GP of the octet baryon magnetic moments ($P^s$ is a projector on the quark s). But (Sect.3), once the octet magnetic moments of $p,n,\Lambda,\Sigma^\pm,\Xi^0,-\Sigma_0 \rightarrow \Lambda\gamma$ transition element are parametrized, an identity relating the coefficients of the various terms precludes to extract the coefficient $g_0$ in front of $\text{Tr}[Q^\gamma P^s]$ from the magnetic moments. Thus $\mu_s^p$, cannot be related (via the term $\text{Tr}[Q^\gamma P^s]$), to the experimental values of the magnetic moments. To determine $\mu_s^p$, one needs (Sect.4) the trace terms $\text{Tr}[Q^Z]$ and $\text{Tr}[Q^Z P^s]$ from the $Z$ moments. To show this we will re-derive (Sect.4) by the GP the Beck-McKeown “key formula” (Eq.(28) of [1]) for the extraction of $\mu_s^p \equiv G_s^q(0)$.

2. The general parametrization: Some elements

Because the general QCD parametrization (GP) has been described in detail ([6], [7]), we note here only its basis, needed to clarify the expressions of the baryon magnetic moments in the following sections. The name “general QCD parametrization” recalls that the procedure is derived exactly from the QCD Lagrangian exploiting only a few general properties. 3

The GP applies to a variety of QCD matrix elements or expectation values. By integrating on all internal $q\bar{q}$ and gluon lines, the method parametrizes exactly such matrix elements. Thus hadron properties -like hadron masses and magnetic moments- are written exactly as a sum of some spin-flavor structures each multiplied by a coefficient. Each structure (term) has a maximum of three indices. The coefficients of the various structures decrease with increasing complexity of the structure, giving rise to a hierarchy of the coefficients. This ”hierarchy” -see Sect.5 for more details- explains why the non relativistic quark model (NRQM), that keeps only the simplest additive (one index) terms, works fairly well; More generally the GP clarifies the relationship of QCD to constituent quark models.

To exemplify, we write the GP magnetic moment of a baryon $B$:

$$\langle \psi_B | \mathbf{M} | \psi_B \rangle = \langle \phi_B | V^\dagger \mathbf{M} V | \phi_B \rangle$$

(1)

In Eq.(1) $\mathbf{M}$ is the exact QCD magnetic moment operator, which, in the baryon rest

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3Because the script symbols for the quark fields used in the QCD Lagrangian in [6] and in some other papers may be confusing, standard symbols $u,d,s$ were adopted from the first paper cited in [7] on.
system, is:

$$M = (1/2) \int d^3r (r \times j(r))$$ (2)

Here

$$j_\mu(x) = e\bar{q}(x)(1/2)[\lambda_3 + (1/\sqrt{3})\lambda_8]q(x)$$ (3)

is the e.m. current expressed in terms of the quark \((u, d, s)\) fields \(q(x)\); \(|\psi_B\rangle\) is the exact eigenstate of the QCD Hamiltonian for \(B\) at rest; \(|\phi_B\rangle\) is an auxiliary three body state of \(B\), factorizable as

$$|\phi_B\rangle = |X_{L=0} \cdot W_B\rangle$$ (4)

into a space part \(X_{L=0}\) with orbital angular momentum zero and a spin unitary-spin factor \(W_B\). The unitary transformation \(V\) -applied to the auxiliary state \(|\phi_B\rangle\) - transforms the latter into \(|\psi_B\rangle\). Integrating on the space variables, Eq.(1) becomes:

$$\langle \psi_B|M|\psi_B\rangle = \langle \phi_B|V^\dagger M V|\phi_B\rangle = \langle W_B|\sum_\nu g_\nu G_\nu(s, f)|W_B\rangle$$ (5)

where the \(G_\nu\)'s are operators depending only on the spin-flavor variables of the three quarks in \(\phi_B\) and the \(g_\nu\)'s are a set of parameters. The meaning of \(V\) -that generates \(|\psi_B\rangle\) from the \(|\phi_B\rangle\) is [6] that \(V\) dresses the auxiliary state with \(q\bar{q}\) pairs and gluons; it also introduces configuration mixing (in other words, \(\phi_B\) has \(L=0\), but \(\psi_B\) is a superposition of states with different \(L\)'s). The choice of a factorized structure (space×spin-flavor) of the auxiliary state \(|\phi_B\rangle\) (compare Eq.(4)) allows the elimination of the space coordinates in the GP (the second step in Eq.(5)). A final remark: The GP is, of course, compatible with chiral theories (compare on this the recent work by Durand and Ha [8]) provided that the chiral Lagrangians considered have the general QCD properties used by the GP. Chiral theories of baryons incompatible with the correct QCD Lagrangian do not lead to the GP (see e.g. Sect.VII of the 2nd paper in [7]).

3. Can one extract \(\mu_p^s\) from the magnetic moments of the octet baryons?

To derive the Beck-McKeown formula for \(\mu_p^s\) we need only the GP expressions of the \(p\) and \(n\) magnetic moments. But the paper by Jido and Weise [3] is based on the magnetic moments of \(all\) octet baryons. Thus we must write them below. It is:

$$M_z(B) = \langle W_B|\sum_{\nu=0}^7 g_\nu(G_\nu)_z|W_B\rangle = \langle W_B|\sum_{\nu=1}^7 \tilde{g}_\nu(G_\nu)_z|W_B\rangle$$ (6)

The fact that Eq.(6) appears as two identical sums, one with eight, the other with seven terms will be explained in a moment. First we clarify the notation and use of Eq.(4). The symbols are as follows: To first order in flavor breaking (but -because \(P^s = [P^s]^n\) for any integer \(n\)- the trace term remains the same at all orders in flavor breaking), the
The sums over i,j,k extend to the three quarks in the \( W_B \) factor of the auxiliary state \( |\Phi_B \rangle \) of the baryon \( B \), \( Q_i^\gamma \) is the electric charge of the \( i \)-th quark and \( \sigma_i \) its (Pauli) spin; \( P_s^i \) is a projection operator (=1 if the \( i \)-th quark is strange, =0 if it is \( u \) or \( d \)). From Eq.(6) it appears that the magnetic moments are the expectation values of the \( G_\nu \) in the spin-flavor states \( W_B \) of the baryons.

The equality of the two sums (with 8 and 7 parameters) in Eq.(6) is due to the following identity (Ref.[9]) relating the \( G_\nu \)\(^\prime\)s, that holds for their expectation values in the \( |W_B\rangle \)\(^s\):

\[
G_0 = \frac{1}{3} G_1 + \frac{2}{3} G_2 - \frac{5}{6} G_3 + \frac{5}{3} G_4 + \frac{1}{6} G_5 + \frac{1}{6} G_6 + \frac{2}{3} G_7
\]  

(8)

The magnetic moments of the octet baryons expressed [Eqs.(6),(7)] in terms of the \( \tilde{g}_\nu \)\(^\prime\)s are (the baryon symbol indicates the magnetic moment):

\[
p = \tilde{g}_1
\]

\[
n = -(2/3)(\tilde{g}_1 - \tilde{g}_3)
\]

\[
\Lambda = -(1/3)(\tilde{g}_1 - \tilde{g}_3 + \tilde{g}_2 - \tilde{g}_5)
\]

\[
\Sigma^+ = \tilde{g}_1 + (1/9)(\tilde{g}_2 - 4\tilde{g}_4 - 4\tilde{g}_5 + 8\tilde{g}_6 + 8\tilde{g}_7)
\]

\[
\Sigma^- = -(1/3)(\tilde{g}_1 + 2\tilde{g}_3) + (1/9)(\tilde{g}_2 - 4\tilde{g}_4 + 2\tilde{g}_5 - 4\tilde{g}_6 - 4\tilde{g}_7)
\]

\[
\Xi^0 = -(2/3)(\tilde{g}_1 - \tilde{g}_3) + (1/9)(-4\tilde{g}_2 - 2\tilde{g}_4 + 4\tilde{g}_5 - 8\tilde{g}_6 + 10\tilde{g}_7)
\]

\[
\Xi^- = -(1/3)(\tilde{g}_1 + 2\tilde{g}_3) + (1/9)(-4\tilde{g}_2 - 2\tilde{g}_4 - 8\tilde{g}_5 - 2\tilde{g}_6 - 2\tilde{g}_7)
\]

and:

\[
\mu(\Sigma\Lambda) = -(1/\sqrt{3})(\tilde{g}_1 - \tilde{g}_3 + \tilde{g}_6 - \tilde{g}_7)
\]  

(10)

This Okubo (Ref.[10]) equation, relating \( \mu(\Sigma\Lambda) \) to the other magnetic moments -correct to first order in flavor breaking- is reobtained, of course, in the GP [6].

\(^4\)The Eq. holds for any traceless \( 3 \times 3 \) diagonal matrix \( Q \) -not necessarily \( Q^\gamma \).
If written with the $g_{\nu}'s$ rather than with the $\tilde{g}_{\nu}'s$, using the equations (11) below - obtained from the identity (8) - the above formulas (9) are all changed in the same way, by the addition of $(-1/3)g_0$ to the r.h.s. of all expressions (e.g. $n = -(1/3)g_0 - (2/3)(g_1 - g_3)$, etc.): Eq.(10) stays unchanged.

$$
\begin{align*}
\tilde{g}_1 &= g_1 - (1/3)g_0 ; \\
\tilde{g}_2 &= g_2 + (2/3)g_0 ; \\
\tilde{g}_3 &= g_3 - (5/6)g_0 \\
\tilde{g}_4 &= g_4 + (5/3)g_0 ; \\
\tilde{g}_5 &= g_5 + (1/6)g_0 ; \\
\tilde{g}_6 &= g_6 + (1/6)g_0 \\
\tilde{g}_7 &= g_7 + (2/3)g_0 \\
\end{align*}
$$

The $\tilde{g}_{\nu}'s$ obtained from the magnetic moments of $p, n, \Lambda, \Sigma^{\pm}, \Xi^{-,0}$ are:

$$
\begin{align*}
\tilde{g}_1 &= 2.793 ; \\
\tilde{g}_2 &= -0.934 ; \\
\tilde{g}_3 &= -0.076 ; \\
\tilde{g}_4 &= 0.438 \\
\tilde{g}_5 &= 0.097 ; \\
\tilde{g}_6 &= -0.147 ; \\
\tilde{g}_7 &= 0.154
\end{align*}
$$

We now come back to the term $G_0 = Tr [QP^s] \sum_i \sigma_i$ in Eq.(6) which is the only quantity related to $\mu^s_p$ in the octet magnetic moments (as we shall see below). From the Eqs.(9) above it results that its coefficient $g_0$ cannot be determined from the octet moments. More precisely: The experimental values of the magnetic moments determine only the seven $\tilde{g}_{\nu}'s$ -see Eq.(9)- not $g_0$.

This fact alone shows why the treatment and results of Ref.[3] are not valid. This statement applies to the fit 2 of Ref.[3], where the five terms involved correspond to five of our eight terms $g_{\nu}$, and also to their fit 3 because the $g_0 Tr [Q^s P^s]$ term discussed above is the only possible Trace term. Note: A QCD fit to the octet magnetic moments (correct at least to first order in flavor breaking) requires seven parameters, after the use of the identity (8). If (8) is ignored and a fit is performed with a smaller number of parameters, one may find incorrectly any value for a misidentified $g_0$.

To complete the argument we must state why $g_0$ would be the only coefficient of interest for an (impossible) purely electromagnetic determination of $\mu^s_p$. The answer is straightforward: In the result of any QCD field theoretical calculation of the magnetic moments, Trace terms implying a charge are associated to $q\bar{q}$ loops; $Tr [Q^s P^s]$ is the only Trace term appearing in the $\gamma$ magnetic moments of the octet baryons and therefore the only term that might be related to a loop $s\bar{s}$.

So far we considered only the magnetic moments, but the treatment can be easily extended to the $q^2$ dependent Sachs form factors $G_M(q^2)$ of the baryons. As shown in [11], it is sufficient to operate in the Breit frame and let the coefficients $g_{\nu}'s$ depend on $q^2$, the square of the four-momentum transfer. Thus, although here we will refer to the magnetic moments $G_M(0)$, the same treatment applies to any $G_M(q^2)$.

4. The determination of $\mu^s_p$ from the electroweak interference.

The determination of $\mu^s_p$ in the $e - p$ scattering experiments (measuring the parity non conserving amplitude) is discussed in the GP rewriting the current as:

$$
j_{\mu}(x) = e\bar{q}(x)Qq(x)
$$

(13)
where \( Q \) has a structure similar for the \( \gamma \) or \( Z \) exchange between \( e \) and \( p \):

\[
Q = c_0 \lambda_0 + c_3 \lambda_3 + c_s \lambda_s \equiv Q_u P^u + Q_d P^d + Q_s P^s
\]  

(14)

Here the \( c \)'s are some (real) coefficients, \( \lambda_0 \) is the unit matrix \( \text{Diag}[1,1,1] \equiv 1 \); \( \lambda_0, \lambda_3, \lambda_8 \) commute, as required for a GP calculation in QCD. In the r.h.s. of Eq. (14) \( P^u, P^d, P^s \) are projectors on the quark fields \( u, d, s \). The unit matrix is there to include the case of \( Q^Z \), where \( Tr[Q^Z] \neq 0 \).

For the e.m case one inserts in Eq. (13) \( Q = Q^\gamma = (1/2)[\lambda_3 + (1/\sqrt{3})\lambda_8] \), that is the diagonal matrix \( \text{Diag}[Q^\gamma_u, Q^\gamma_d, Q^\gamma_s] \equiv [2/3, -1/3, -1/3] \). It is \( Tr[Q^\gamma] = 0 \) (we limit to the quarks \( u, d, s \)).

For the exchange of a \( Z \) the electro-weak charges \( Q^Z \) of the quarks in Eq. (13) are:

\[
\begin{align*}
Q^Z_u &= k_W [1 - (8/3) \sin^2 \theta_W] \\
Q^Z_d &= k_W [-1 + (4/3) \sin^2 \theta_W] \\
Q^Z_s &= k_W [-1 + (4/3) \sin^2 \theta_W]
\end{align*}
\]  

(15)

where \( k_W = [2 \sin(2\theta_W)]^{-1} \); that is, \( Q^Z \) in Eq. (13) is the matrix \( \text{Diag}[Q^Z_u, Q^Z_d, Q^Z_s] \); both \( Tr[Q^Z] \) and \( Tr[Q^Z P^s] \) do not vanish.

Now we will obtain by the GP the basic formula (Eq. (28) of [1]) giving \( \mu_p^Z \) in the electro-weak interference experiments. We will show that such formula works due the fact that \( Tr[Q^Z] \) and \( Tr[Q^Z P^s] \) do not vanish; the sum of their coefficients \( g_0 + \hat{g}_0 \) is what the experiments are trying to measure.

The current [13], similar for \( \gamma \) and \( Z \) exchange, makes the GP procedure simple. From now on we consider only \( p \) and \( n \). From Eq. (5) their magnetic moments are:

\[
\begin{align*}
M^Q(p,n) = g_1 \sum_i Q_i \sigma_i + g_3 \sum_{i \neq k} Q_i \sigma_k + g_0 \text{Tr}[Q] P^s \sum_i \sigma_i + \hat{g}_0 \text{Tr}[Q] \sum_i \sigma_i
\end{align*}
\]  

(16)

In Eq. (16) \( Q \) is the charge given in Eq. (14); to reproduce the \( \gamma \) magnetic moments of \( p, n \)-(Sect.3)- one inserts for \( Q \) the electric charge diagonal matrix \( \text{Diag}[2/3, -1/3, -1/3] \). To obtain the magnetic moments measured in the parity non conserving electro-weak \( e - p \) scattering, one inserts in Eq. (16) the electro-weak charge \( Q(Z) \) [Eq. (15)].

We write separately the contributions of \( u, d, s \) to \( M^Q \) (for \( p, n \)); from Eq. (16) it is:

\[
M^Q = M^u + M^d + M^s
\]  

(17)

where the \( M^u, M^d, M^s \) are written below [from Eq. (16)] in terms of the projectors

\footnote{The trace term \( \hat{g}_0 \text{Tr}[Q] \sum_i \sigma_i \), that now appears in the Eqs. (16), was not present in the \( \gamma \) determination of the magnetic moments, because \( Tr[Q^\gamma] \) vanishes if only the \( u, d, s \) quarks are considered. [We also saw that \( Tr[Q^\gamma P^s] \) does not play a role for the e.m. moments]
Clearly, from the Eqs. (20,21) multiplying each by its $Q$ emerges that, to determine $\tilde{g}$ which can only be read in the Trace terms, must be extracted measuring $\tilde{g}$ very difficult.

Similarly for the neutron we have:

$$M^u = g_1 \sum_i Q_i P^u_i \sigma_i + g_3 \sum_{i \neq k} Q_i P^u_i \sigma_k + \tilde{g}_0 Tr[QP^u] \sum_i \sigma_i.$$  

$$M^d = g_1 \sum_i Q_i P^d_i \sigma_i + g_3 \sum_{i \neq k} Q_i P^d_i \sigma_k + \tilde{g}_0 Tr[QP^d] \sum_i \sigma_i.$$  

$$M^s = (\tilde{g}_0 + g_0) Tr[QP^s] \sum_i \sigma_i.$$  

These equations hold both for the $\gamma$ and Z contributions to $M^u, M^d, M^s$; that is in Eqs. (18) any $Q$ can be either $Q^\gamma$ or $Q^Z$. Note that $\sum_i \sigma_i = 2J$; of course $2J_z = 1$ if the baryon spin is up along the z axis.

The contributions of the quarks $u, d, s$ to the $\gamma$ or $Z$ magnetic moment of the proton are the expectation values $\mu_f^p = \langle p | M_f^p | p \uparrow \rangle$ where the upper $f$ in $M_f^p$ stays for $u$ or $d$ or $s$. Using the Eq. (18) and writing also:

$$\tilde{g}_0 \equiv g_0 + \tilde{g}_0$$

one obtains:

$$\mu_u^p / Q_u = \frac{4}{3} g_1 + \frac{2}{3} g_3 + \tilde{g}_0 \equiv \frac{4}{3} \tilde{g}_1 + \frac{2}{3} \tilde{g}_3 + \tilde{g}_0$$

$$\mu_d^p / Q_d = -\frac{2}{3} g_1 + \frac{4}{3} g_3 + \tilde{g}_0 \equiv -\frac{2}{3} \tilde{g}_1 + \frac{4}{3} \tilde{g}_3 + \tilde{g}_0$$

$$\mu_s^p / Q_s = \tilde{g}_0$$

Similarly for the neutron we have:

$$\mu_u^n / Q_u = -\frac{1}{3} g_1 + \frac{4}{3} g_3 + \tilde{g}_0 \equiv -\frac{1}{3} \tilde{g}_1 + \frac{4}{3} \tilde{g}_3 + \tilde{g}_0$$

$$\mu_d^n / Q_d = \frac{2}{3} g_1 + \frac{2}{3} g_3 + \tilde{g}_0 \equiv \frac{2}{3} \tilde{g}_1 + \frac{2}{3} \tilde{g}_3 + \tilde{g}_0$$

$$\mu_s^n / Q_s = \tilde{g}_0$$

The Eqs. (20), (21) refer to the $\gamma$ magnetic moments (on inserting there $Q = Q^\gamma$) or to the $Z$ magnetic moments (if $Q = Q^Z$). Our $(\mu_u^p / Q_u)$ (and the similar quantities for $d, s$) are the Sachs form factors [at $q^2=0$] $G_M^u(0), G_M^d(0), G_M^s(0)$ of Ref. [1].

In the Eqs. (20, 21) the right hand side of the expressions for $[\mu_u^p / Q_u, [\mu_d^p / Q_d, [\mu_u^n / Q_u, [\mu_d^n / Q_d]$ have been written in terms of the tilded parameters $\tilde{g}_1, \tilde{g}_3$ using the Eqs. (11). However recall that the tilded parameters extracted from the $\gamma$ magnetic moments, give no information on $g_0$ nor, of course, on $\tilde{g}_0$. Thus the interpretation of the Eqs. (20) and (21) is the following: The $s\bar{s}$ contributions to the magnetic moments, which can only be read in the Trace terms, must be extracted measuring $\tilde{g}_0$ by the very difficult $Z$ exchange experiments (as stated $Tr[Q^Z]$ and $Tr[Q^Z P^s]$ do not vanish). Clearly, from the Eqs. (20, 21) (multiplying each by its $Q_u, Q_d, Q_s$ and summing) it emerges that, to determine $\tilde{g}_0$, $Tr(Q) \equiv (Q_u + Q_d + Q_s)$ has to be different from zero. From the Eqs. (20) and (21) the $\gamma$ and $Z$ moments of $p$ and $n$ follow immediately.

\footnote{Compare their Eqs. (26), (27) that define $G_M^\gamma$ and $G_M^Z$.}
They are (the $\gamma$ moments coincide of course with those of Eq.(9)):

$$\mu^\gamma_p = \tilde{g}_1 \quad \mu^\gamma_n = -(2/3)(\tilde{g}_1 - \tilde{g}_3)$$  \hspace{1cm} (22)

$$\mu^Z_p = k_W \left[ ((5/3) - 4 \sin^2 \theta_W)\tilde{g}_1 - (2/3)\tilde{g}_3 - \tilde{g}_0 \right]$$

$$\mu^Z_n = k_W \left[ -(5/3) + (8/3)\sin^2 \theta_W \right] \tilde{g}_1 + ((2/3) - (8/3)\sin^2 \theta_W)\tilde{g}_3 - \tilde{g}_0 \right]$$  \hspace{1cm} (23)

The first Eq.(23) is identical to the Eq.(28) of [1] (transcribed below):

$$G^Z_M = (1 - 4 \sin^2 \theta_W)G^{\gamma,p}_M - G^{\gamma,n}_M - G^s_M$$  \hspace{1cm} (24)

as can be seen recalling that, in the notation of [1], $\mu^Z_p / k_W \equiv G^{Z,p}_M$, $\mu^\gamma_p \equiv G^{\gamma,p}_M$, $\mu^\gamma_n \equiv G^{\gamma,n}_M$, $\tilde{g}_0 \equiv G^s_M$.

The novel feature of this GP derivation of the Beck-McKeown equation is that $\mu^s_p$ is related directly to the Trace terms in a QCD plus electroweak field theoretical description.

5. A GP estimate of the 3-gluon exchange factor reducing $\mu^s_p$

The GP hierarchy of the coefficients means (Sect.2) that the coefficient of each sum of terms (characterizing the hadron property under study) decreases when the number of different indices in the terms under consideration increases or when -at equal number of indices- flavor breaking factors are present. To exemplify, each $G_\nu$’s of the baryon magnetic moments in Sect.3 is a sum of terms. Other examples, among the many ones, are the octet and decuplet baryon masses [6,7] or the electromagnetic mass differences of baryons [12].

The order of magnitude of the reduction above- of the coefficients is $\approx 0.33$ for the reduction due to a flavor breaking factor and $0.33 \pm 0.05$ for an index due to the exchange of one gluon between two “constituent” quark lines. Here, to estimate $\mu^s_p$, we are interested in the reduction factor due to the three gluons forming a colour singlet state with $J=1$. If, for each gluon in Fig.1, the value of the reduction factor were the same ($\approx 1/3$) as that for gluon exchange between constituent quarks inside a hadron, one would have, approximately, a reduction factor of the order $(1/3)^3$; however, for this case (Fig.1), an independent estimate is appropriate.

This estimate is possible applying the GP analysis [13] of the vector meson $\gamma$ decays to the $\phi \rightarrow \pi\gamma$ and $\omega \rightarrow \pi\gamma$. One sees from the table I of [13] that the rate of the $\phi \rightarrow \pi\gamma$ is determined by the deviation $\Delta\theta_V$ of the vector mixing angle $\Delta\theta_V$ from its ideal value ($35.3^\circ$) plus two terms $\sqrt{2/9} \cdot \Gamma_5$ and $\sqrt{2/3} \cdot \Gamma_7$ produced by a three gluon ($J=1$) exchange between $\phi$ and $\pi^0$. Although $\Delta\theta_V$ is itself due to a three gluon $J = 1$ color singlet exchange, below we will focus on these terms $\Gamma_5$ and $\Gamma_7$.

The uncertainty arising from the poor knowledge of the form factor in the $\phi \rightarrow \pi\gamma$ decay can be partially eliminated by considering the ratio between the $\phi \rightarrow \pi\gamma$ and the $\omega \rightarrow \pi\gamma$ decay rates, because the two decay momenta (379 and 501 Mev) are not...
Figure 1: The Z interacts with a s-quark loop (in the e − p parity violating scattering) that must be connected by at least three gluons to an internal quark line q of the proton. Note: The three gluons may also reach different internal quark lines, not the same as in the figure.

too different and the \( \omega \rightarrow \pi \gamma \) rate is reasonably well known. The \( \rho \rightarrow \pi \gamma \) is presently affected by much larger errors. From the table I of [13] cited above and writing an equation similar to the Eq.(68) of [13] (but now with the numerator referring to \( \phi \) and the denominator to \( \omega \)) we have \(^7\):

\[
\frac{\Gamma(\phi \rightarrow \pi^0\gamma)}{\Gamma(\omega \rightarrow \pi^0\gamma)} = \frac{\sin \Delta \theta_V \cdot \mu_1 f_1(k_\phi) + \sqrt{\frac{2}{3}} \mu_5 f_5(k_\phi) + \sqrt{\frac{2}{3}} \mu_7 f_7(k_\phi)}{\mu_1 f_1(k_\omega) + \frac{2}{3} \mu_5 f_5(k_\omega)} \cdot \frac{\sqrt{3}}{2} \frac{k_\phi}{k_\omega} \tag{25}
\]

Note [13] that the two terms proportional to \( f_5 \) and \( f_7 \) in the numerator of Eq.(25) correspond to a three gluon transition of an ideal \( \phi \) (a pure \( s\bar{s} \)) state into an ideal \( \omega \) followed by the \( \pi_0 + \gamma \) decay of the latter. The ratio \( x \) (to be introduced below) is therefore related to the transition of an \( s\bar{s} \) state into a \( u\bar{u} + d\bar{d} \) state both with \( J = 1 \); it gives some estimate (though in a different situation) of the 3-gluon exchange factor reducing \( \mu_\omega^3 \). We now proceed with the calculation of the r.h.s. of Eq.(25).

In Eq.(25) the term \( \mu_5 \) in the denominator can be neglected. Assuming equal form factors for \( \phi \) and \( \omega \) and defining \( x \):

\[
x \equiv \left[ \sqrt{\frac{2}{3}} \mu_5 + \sqrt{\frac{2}{3}} \mu_7 \right] / \mu_1 \tag{26}
\]

we obtain:

\[
\frac{\Gamma(\phi \rightarrow \pi^0\gamma)}{\Gamma(\omega \rightarrow \pi^0\gamma)} = 2.3 \sin \Delta \theta_V + x^2 \tag{27}
\]

where the factor 2.3 is the ratio \( [k_\phi/k_\omega]^3 \). \(^7\) Because \( \frac{\Gamma(\phi \rightarrow \pi^0\gamma)}{(2.3) \cdot \Gamma(\omega \rightarrow \pi^0\gamma)} = \)

\(^7\)In the table I of [13] all terms proportional to \( \sin \Delta \theta_V \) were ignored except for that of the direct \( \phi \rightarrow \pi^0\gamma \) decay.
\[ (3.4 \pm 0.2) \cdot 10^{-3} \] it is:

\[ \left| \sin \Delta \theta_V + x \right|^2 = (3.4 \pm 0.2) \cdot 10^{-3} \]  \hspace{1cm} (28)

Setting \( \sin \Delta \theta_V = 0 \) (in the linear description - implied by the GP- \( \sin \Delta \theta_V \) is small \( \approx 1.7 \cdot 10^{-2} \)), the right hand side of Eq. (28) gives for the three gluon reduction factor \(|x|\) of interest\(^8\):

\[ |x| \approx 5.8 \cdot 10^{-2} \] \hspace{1cm} (29)

Note that \(|x|\) is expressed in units of the \( p \) magnetic moment \((2.79 \mu_N)\). To estimate the correct order of magnitude of \(|\mu^p_s|\) one should know, however, in addition to \(|x|\), two quantities: a) The factor arising from the various ways of connecting the three gluons from the loop to the quark lines in the proton; b) The factor arising because the three gluons originate from a \( s\bar{s} \) loop, different from the situation intervening in the above calculation of \( x \). We are unable to calculate these factors but we felt that the strong reduction \(|x|\) might be of some interest in itself. To give an idea of the number in Eq. (29), note that if the product of the factors a), b) above produces a further reduction by 2 or 3, we are in the region of values of \(|\mu^p_s|\) given in [2]; a region where the experiments are -perhaps optimistically- called in [2] “tremendously challenging”.

\(^8\)On the other hand a value of \( \sin \Delta \theta_V \) corresponding to quadratic mixing \((\theta_V = 38.7^\circ)\) might give rise to a vanishingly small \(|x|\).
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