Improved noise-to-state stability criteria of random nonlinear systems with stochastic impulses

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Abstract
This paper considers noise-to-state stability for random non-linear systems with stochastic impulses. The impulsive random non-linear systems contain three random characteristics: the second-moment processes in continuous dynamics, the sequence of random variables in discrete dynamics, and the number of stochastic impulses obeyed a renewal process. Firstly, the improved criteria of noise-to-state stability are established for random non-linear systems subject to unstable stochastic impulses based on the uniformly asymptotically stable function. Secondly, improved Lyapunov approaches of NSS with unstable stochastic continuous dynamics are accomplished by the uniformly exponentially stable function. Finally, two numerical examples are used to illustrate the feasibility of the proposed methods.

1 | INTRODUCTION
Impulsive random non-linear systems (IRNSs) studied in this paper are formulated by stochastic continuous dynamics and stochastic instantaneous state jumps, which are more suitable to describe the complex stochastic phenomena of many real-world systems. The stochastic continuous dynamics were often represented by stochastic differential equations (SDEs) [1–5], owing to the complete stability theory [6–8] and a wide range of applications [9–11], but SDEs are not suitable for the application to engineering due to the fact that the mean power of white noise is infinite [12]. In [8], the dynamic models subject to second-moment process whose mean power is bounded were described by random differential equations (RDEs) and many results about the random non-linear systems expressed by RDEs were reported; see [13–18]. Accordingly, the IRNSs whose continuous dynamics are expressed by RDEs are different from the impulsive stochastic non-linear systems whose continuous dynamics are described by SDEs. There are two classes of random characteristics in stochastic instantaneous state jumps. One is the stochastic impulsive intensity, that is, the impulses are modeled as a stochastic difference equation. For example, the impulses in [19, 20] contained the random variable sequence so that the impulsive intensity was random. The other is the stochastic impulsive density, that is, the number of impulses is a random variable or the impulses occur at random moments. The number of impulses in [21–23] followed the Poisson distribution, that is, the impulsive intervals obeyed the...
exponential distribution. In addition, the impulsive intensity in [24] was a bounded random variable and the impulsive instants were taken into account to satisfy several cases such as renewal process and semi-Markov chain, but the random characteristics were not considered in continuous dynamics.

The input-to-state stability, whose notion originally proposed in [25], is of such importance in system synthesis as it describes the effects of external signals on a system. In [26], the concept of noise-to-state stability (NSS) was first proposed for SDEs with the unknown covariance of Brown motion being regarded as the external signals. When the second-moment process is regarded as a stochastic input, some new notions of NSS were proposed for RDDEs in [14], which are different from those of [26]. Based on the new notions of NSS proposed in [14], the definitions of NSS and the related stability criteria were presented in [19] for IRNSs. In [20], the problems of NSS were studied for random impulsive and switching neural networks, but the impulsive moments in [19, 20] were a series of increasing real numbers, that is, the impulses only occurred at fixed times. Hence, the assumption of average impulsive interval in [19, 20, 27–29] cannot be directly applied to IRNSs with impulses effect at random times. Although there have been salient results of stability [24, 30–32] for impulsive stochastic non-linear systems, the research on IRNSs with stochastic impulses contained stochastic impulsive intensity and stochastic impulsive density was not found.

As is already known, a suitable Lyapunov function is one of the keys to the Lyapunov stability criterion. Thus, it makes sense to relax restrictions on the Lyapunov function in the stability criterion. As is shown in [33–35], the negativity of the time-derivative for the Lyapunov function in the stability criterion was relaxed by the uniformly asymptotically stable function (UASF). For discrete-time time-varying systems, the improved stability criteria were established in [36, 37] based on the uniformly exponentially stable function (UESF) in the sense that the time-shifts in the criteria can be selected as positive values at some moment. Inspired by the aforementioned analysis, the improved Lyapunov approaches of NSS is investigated for random non-linear systems with stochastic impulses which contain stochastic continuous dynamics, stochastic instantaneous state jumps and stochastic impulsive times. The tasks undertaken lie in the following aspects:

- The continuous dynamics in IRNSs are described by REDs in which the random disturbances are driven by second-order processes with bounded average power, and the energy of the random variables in the instantaneous state jumps is also bounded. The model studied is more suitable for many physical systems with complex stochastic phenomena than those in [4, 21, 22, 27–29].
- The number of impulses is driven by a renewal process, that is, the time intervals between two consecutive impulses are independently identically distributed, which is more general than the Poisson process in [21–23] and the cases in [19, 20] with the impulses to happen at determined moments.
- The criteria of NSS with some easy-to-verify conditions for random non-linear systems with stochastic impulses are established based on the UASF and the UESF, which were introduced in [34] and [36], respectively. The criteria of NSS in [19] can be regarded as the special cases of our proposed results.

The paper is organised as follows: Section 2 is the necessary preliminaries for the proposed model. Section 3 is divided into two subsections. In Section 3.1, the improved criteria of NSS for random non-linear systems with unstable stochastic impulses are established. The improved Lyapunov approaches of NSS for IRNSs with unstable continuous dynamics are given in Section 3.2. Two examples are provided to illustrate the effectiveness of the proposed results in Section 4. The conclusion is given in Section 5.

**Notations:** $[x]$ is the usual Euclidean norm of a vector $x$. For a matrix $F$, $\lambda_{\max}(F)/\lambda_{\min}(F)$ stands for the maximum/minimum characteristic value of $F$. $\Psi$ represents the complement set of a set $\Psi$. $\mathbb{N}^+$ stands for all positive integers and $\mathbb{N} = \{1, 2, \ldots\}$. A non-negative function $\nu(t, \omega)$ is of such importance in system synthesis as it describes the effects of external signals on a system. In [26], the concept of average impulsive interval in [19, 20] cannot be directly applied to IRNSs with impulses effect at random impulsive and switching neural networks, but the impulsive intensity in [4, 21, 22, 27–29].

$\mathbb{K}$ is of such importance in system synthesis as it describes the effects of external signals on a system. In [26], the concept of average impulsive interval in [19, 20] cannot be directly applied to IRNSs with impulses effect at random impulsive and switching neural networks, but the impulsive intensity in [4, 21, 22, 27–29].

2 | PRELIMINARIES

Consider the following IRNSs:

$$\begin{aligned}
\dot{x}(t) &= f(x(t), t) + g(x(t), t)\xi(t), \ t \in [\tau_k, \tau_{k+1}], \\
\dot{x}(\tau_{k+1}) &= F_{k+1}(x(\tau_{k+1}), \xi_{k+1}), \\
x(t_0) &= x_0, \ k \in \mathbb{N},
\end{aligned}$$

where the state $x(t) \in \mathbb{R}^n$ is right continuous and satisfies $x(\tau_{k+1}) = \lim_{\tau_{k+1} \to \tau_k} x(t)$. The $\mathbb{R}^n$-valued stochastic process $\xi(t) \in \mathbb{L}^2$ is defined on a complete filtering space $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$ and it is independent of the $\mathcal{F}_t$-adapted independent random variable $\xi_k \in \mathbb{R}^r$. Non-linear functions $f: \mathbb{R}^n \times [0, \infty) \to \mathbb{R}^n$, $g: \mathbb{R}^n \times [0, \infty) \to \mathbb{R}^{n \times r}$, $g_k: \mathbb{R}^n \times \mathbb{R}^r \to \mathbb{R}^{n \times r}$ meet the locally Lipschitz condition and $f(0, \cdot) = 0$, $g(0, \cdot) = 0$, $F_k(0, \cdot) = 0$.

Let $\sigma_k$ be the time interval between two consecutive impulses, that is, $\sigma_k = \tau_k - \tau_{k-1}, k \in \mathbb{N}$; then $\sigma_k$ is a non-negative independent identically distributed random variable defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and $\tau_k = \sum_{i=1}^k \sigma_i, k \in \mathbb{N}^+$ with $\tau_0 = 0$. Let $N_t = \sum_{k=0}^{\infty} I(t_k \leq t < t_k)$ be the number of stochastic impulses over the interval $[t_0, t]$; then it can
be regarded as a renewal process with $N_0 = 0$, which satisfies the following Lemma:

**Lemma 1.** [38] If $N_t$ is a renewal process, then we have

$$\lim_{t \to \infty} \frac{N_t}{t - t_0} = \frac{1}{m_\sigma}, \ a.s.,$$

where $m_\sigma = \mathbb{E}(\sigma_k) > 0$ is the expectation of the impulse waiting times.

**Remark 1.** Based on Lemma 1, we can find that $N_t \to \infty, \ a.s.$ when $t \to \infty$, that is, infinite impulses can only occur in infinite time, and the number of impulses is less than infinite in a finite interval.

**Remark 2.** For $\forall t > 0$ and almost every $\omega \in \Omega$, there is a constant $T_i = T_i(\omega) \in (t_0, \infty)$, such that

$$\left| \frac{N_j}{t - t_0} - \frac{1}{m_\sigma} \right| < \delta, \ \forall t \geq T_i, \ a.s.,$$

that is, for $\forall t \geq T_i$, we have

$$\left( \frac{1}{m_\sigma} - \delta \right)(t_0 - t) \leq N_t \leq \left( \frac{1}{m_\sigma} + \delta \right)(t_0 - t), \ a.s.$$ 

**Remark 3.** For $\forall t_2 > t_1 > T_i > t_0$, the expectation of the impulsive interval and the number of impulses over the interval $[t_1, t_2]$ satisfy the following inequality:

$$\left( \frac{1}{m_\sigma} - \delta \right)(t_2 - t_1) \leq N_{t_2 - t_1} \leq \left( \frac{1}{m_\sigma} + \delta \right)(t_2 - t_1), \ a.s.$$ 

where $N_{t_2 - t_1} = \sum_{k=0}^{\infty} I(t_1 \leq T_k < t_2)$ is the number of impulses over the interval $[t_1, t_2]$.

In order to analyse the stability of the unique global solution for the system (1), the following assumption should be imposed on the stochastic process $\xi(t)$ and random variable $\zeta_k$:

**A1:** For any stochastic process $\xi(t) \in \mathbb{R}^m$ and any sequence of independent random variable $\zeta_k \in \mathbb{R}^n$, we have

$$\sup_{t_0 \leq t \leq T_i} \mathbb{E}[\xi(t)]^2 < \infty, \ \max_{t \in [0, \infty)} \mathbb{E}[\xi(t)]^2 < \infty.$$ 

For any given $b > 0$, we introduce the first exit time from $B_b = \{x : |x| < b\}$ and its limit:

$$\varphi_b = \inf\{t \geq t_0 : x(t) \in B_b\},$$

$$\varphi_\infty = \lim_{t \to \infty} \varphi_b, \ \inf B_b = \infty.$$ 

The following lemma, which can be seen in [19], ensures the existence and uniqueness of the global solution for system (1):

**Lemma 2.** Under assumption A1, for $\forall t \geq t_0$ and $\forall \theta > 0$, $\varphi_\theta$ is defined in (2), if there exist real parameters $\varepsilon > 0$, $d_\theta$ and a function $V : \mathbb{R}^n \to \mathbb{R}^\bullet_+$ such that

$$\lim_{t \to \infty} \inf_{b > 0} V(x(t), t) = \infty,$$ 

$$\mathbb{E}[V(x(t) \wedge \varphi_\theta), t \wedge \varphi_\theta)] \leq e^\varepsilon e^{d_\theta (t - t_0)},$$

then system (1) has a unique global solution $x(t)$ on $[t_0, \infty)$.

Inspired by [19] and [14], we introduce the following two definitions:

**Definition 1.** For a constant $m > 0$, if there are class-$\mathcal{K}$ functions $\gamma_1(\cdot), \gamma_2(\cdot)$ and a class-$\mathcal{KL}$ function $\beta(\cdot, \cdot)$, such that

$$\mathbb{E}[\xi(t)]^m \leq \beta(\|x_0\|, t - t_0) + \gamma_1(\sup_{t \in [0, t]} \mathbb{E}[\xi(t)]^2) + \gamma_2(\max_{t \in [0, \infty]} \mathbb{E}[\xi(t)]^2), \ \forall x_0 \in \mathbb{R}^n,$$

system (1) is said to be noise-to-state stable in the $m$-th moment (NSS-$m$-M).

**Definition 2.** If there exist class-$\mathcal{K}$ functions $\gamma_1(\cdot), \gamma_2(\cdot)$ and a class-$\mathcal{KL}$ function $\beta(\cdot, \cdot)$ such that

$$\mathbb{P}[|x(t)|^m \geq \beta(\|x_0\|, t - t_0) + \gamma_1(\sup_{t \in [0, t]} \mathbb{E}[\xi(t)]^2) + \gamma_2(\max_{t \in [0, \infty]} \mathbb{E}[\xi(t)]^2)] \leq \varepsilon, \ \forall x_0 \in \mathbb{R}^n,$$

where $\varepsilon \in (0, 1)$, system (1) is said to be noise-to-state stable in probability (NSS-P).

The functions in the following two definitions and the lemma will be of great help to our stability results:

**Definition 3.** [34] $\mu(t)$ is said to be a uniformly asymptotically stable function (UASF), if system $\dot{x}(t) = \mu(t)x(t)$ is uniformly asymptotically stable, which is equivalent to

$$\int_{t_0}^{t} \mu(s)ds \leq -\varepsilon(t - t_0) + d_\varepsilon,$$ 

where constants $t_0 \geq 0, \ v > 0$ and $d_\varepsilon \geq 0$.

**Definition 4.** [36] $\mu_k : \mathbb{N} \to (0, +\infty)$ is said to be a uniformly exponentially stable function (UESF), if $x_{k+1} = \mu_k x_k$ is uniformly exponentially stable, which is equivalent to

$$\prod_{j=i}^{k} \mu_j \leq \varepsilon^{\theta^{k-i}}, \ k \geq i \geq 0,$$ 

where constants $\varepsilon > 0$ and $0 < \theta < 1$.

**Lemma 3 (Gronwall-inequality).** [39] For $t \geq t_0$, let $f(t)$ and $b(t)$ be almost everywhere continuous functions integrable over every finite interval; if there exists an absolute continuous function $\chi(t)$ such that the
Theorem 1. Under assumption A1, for any random impulsive interval sequence \( \{\sigma_j\}_{j \in \mathbb{N}^+} \), if there exist a UASF \( \mu(t) \) as expressed in (5), a function \( V(x, t) \in C^{1, 1}([\mathbb{R}^m \times [0, \infty); \mathbb{R}^+]) \), and positive constants \( b_1, b_2, \sigma > 1, m, \delta_x \), \( \delta_t \), \( \epsilon, d_0 \geq 0 \) such that

\[
\begin{align*}
E_{b1}[x(t)]^m &\leq V(x(t), t) \leq E_{b2}[x(t)]^m, \\
V_i + V_x f(x(t), t) + d_x |V_x g(x(t), t)|^2 &\leq \mu(t) V(x(t), t), \\
V'(x(t), t) &\leq \sigma V(x(t), t) + d_t |\xi|^2;
\end{align*}
\]

then (i) system (1) has a unique global solution on \([0, \infty); (ii) system (1) is said to be NSS-m-M for \( m_0 > \frac{\sigma^2 \delta_t}{4 \delta_x} \).

Proof.

(i) Let \( \{\mathbb{E}_k\} \) represent the active region of \( k \)-th subsystem as follows:

\[
\begin{align*}
\dot{x}(t) &= f(x(t), t) + g(x(t), t)\xi(t), \quad t \in [\tau_{k-1}, \tau_k), \\
x(\tau_k) &= F_k(x(\tau_{k-1}), \xi_k), \\
x(\tau_{k-1}) &= x_0 \in \mathbb{R}^m, \quad k \in \mathbb{N}.
\end{align*}
\]

then we can get that \( \bigcup_{k=1}^{\infty} \mathbb{E}_k = \mathbb{R}^m \) with \( \mathbb{E}_i \cap \mathbb{E}_j \neq \emptyset \), and it is assumed that \( \bigcup_{k=1}^{\infty} \mathbb{E}_k = \mathbb{R}^m \) with \( \mathbb{E}_k \) being the interior of \( \mathbb{E}_k \). The impulsive moments can be represented as

\[
\tau_k = \inf \{\tau_{k-1} < t < \varphi_\infty : |\Phi_k(t, \tau_{k-1}) \neq x_0| \in \mathbb{E}_k\} \quad (11)
\]

and \( \emptyset = \varphi_\infty \), where \( \Phi_k(t, \tau_{k-1}) \) is the flow of the \( k \)-th subsystem.

Let the upper right-hand derivative of \( V \) be \( D^+ [V'(x(t), t)] = \limsup_{t \to \tau \downarrow \tau_{k-1}} \frac{1}{4d_x} \left[ V'(x(t + \Delta t), t + \Delta t) - V'(x(t), t) \right] \). For any \( t \in [\tau_k, \tau_{k+1}) \), it follows from (8) that

\[
\begin{align*}
D^+ [V'(x(t), t)] &= V'_i + V'_x f(x(t), t) + g(x(t), t)\epsilon(x(t), t) \\
&\leq V'_i + V'_x f(x(t), t) + \frac{d_x}{4d_t} [V'_x g(x(t), t)]^2 + \frac{1}{4d_t} |\epsilon(x(t), t)|^2 \\
&\leq \mu(t) V'(x(t), t) + \frac{1}{4d_t} |\epsilon(x(t), t)|^2.
\end{align*}
\]

On the basis of Lemma 3 and (5), yields that

\[
V'(x(t \wedge \varphi_\infty), t \wedge \varphi_\infty) \leq \int_{\tau_k}^{\tau_{k+1}} \left[ \int_{t}^{\tau} \mu(s) ds \right] dt + \int_{\tau_k}^{\tau_{k+1}} \epsilon(t) ds \\
\leq \int_{\tau_k}^{\tau_{k+1}} \sigma V'(x(t \wedge \varphi_\infty), t \wedge \varphi_\infty) dt + \epsilon(t) ds
\]

which, combining with (9), gives

\[
\begin{align*}
V'(x(t \wedge \varphi_\infty), t \wedge \varphi_\infty) &\leq \sigma \int_{\tau_k}^{\tau_{k+1}} \epsilon(t) ds + \int_{\tau_k}^{\tau_{k+1}} \frac{1}{4d_t} \left[ \epsilon(t) \right]^2 dt \\
&\leq \sigma \int_{\tau_k}^{\tau_{k+1}} V'(x(t \wedge \varphi_\infty), t \wedge \varphi_\infty) dt + \int_{\tau_k}^{\tau_{k+1}} \frac{1}{4d_t} \left[ \epsilon(t) \right]^2 dt \\
&\leq \sigma \int_{\tau_k}^{\tau_{k+1}} V'(x(t \wedge \varphi_\infty), t \wedge \varphi_\infty) dt + \int_{\tau_k}^{\tau_{k+1}} \frac{1}{4d_t} \left[ \epsilon(t) \right]^2 dt
\end{align*}
\]

Continuing within the iteration from 0 to \( N \) yields that

\[
\begin{align*}
V'(x(t \wedge \varphi_\infty), t \wedge \varphi_\infty) &\leq \sigma N \epsilon(t) + \int_{\tau_k}^{\tau_{k+1}} \frac{1}{4d_t} \left[ \epsilon(t) \right]^2 dt \\
&\leq \sigma N \epsilon(t) + \int_{\tau_k}^{\tau_{k+1}} \frac{1}{4d_t} \left[ \epsilon(t) \right]^2 dt \\
&\leq \sigma N \epsilon(t) + \int_{\tau_k}^{\tau_{k+1}} \frac{1}{4d_t} \left[ \epsilon(t) \right]^2 dt
\end{align*}
\]
It follows from assumption A1 that
\[
V'(x(t \wedge \bar{\xi}), t \wedge \bar{\xi}) \leq \sigma \rho_j(N_j+1) V(x(t_0), t_0) + \frac{d_j^1}{4d_j} \max_{j \in \mathbb{N}_1} \int_0^t \left[ \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right] \left( \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right) dt + \frac{d_j^1}{4d_j} \sup_{t \leq t_0} E\left[ e_0^2 \sigma \rho_j \right] dt
\]
\[\leq \sigma \rho_j(N_j+1) V(x(t_0), t_0) + \frac{d_j^1}{4d_j} \max_{j \in \mathbb{N}_1} \int_0^t \left[ \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right] \left( \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right) dt + \frac{d_j^1}{4d_j} \sup_{t \leq t_0} E\left[ e_0^2 \sigma \rho_j \right] dt
\]
\[\leq \sigma \rho_j(N_j+1) V(x(t_0), t_0) + \frac{d_j^1}{4d_j} \max_{j \in \mathbb{N}_1} \int_0^t \left[ \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right] \left( \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right) dt + \frac{d_j^1}{4d_j} \sup_{t \leq t_0} E\left[ e_0^2 \sigma \rho_j \right] dt
\]
\[\leq \sigma \rho_j(N_j+1) V(x(t_0), t_0) + \frac{d_j^1}{4d_j} \max_{j \in \mathbb{N}_1} \int_0^t \left[ \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right] \left( \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right) dt + \frac{d_j^1}{4d_j} \sup_{t \leq t_0} E\left[ e_0^2 \sigma \rho_j \right] dt
\]
\[\leq \sigma \rho_j(N_j+1) V(x(t_0), t_0) + \frac{d_j^1}{4d_j} \max_{j \in \mathbb{N}_1} \int_0^t \left[ \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right] \left( \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right) dt + \frac{d_j^1}{4d_j} \sup_{t \leq t_0} E\left[ e_0^2 \sigma \rho_j \right] dt
\]
When $t \in [t_0, T]$, according to Remark 1, there exists an integer $N_1 \in (0, \infty)$ such that $N_j < N_1 < \infty$. It follows from (12) that
\[
E[V'(x(t \wedge \bar{\xi}), t \wedge \bar{\xi})] \leq e_0 \sigma \rho_j(N_j+1) V(x(t_0), t_0) + \frac{d_j^1}{4d_j} \max_{j \in \mathbb{N}_1} \int_0^t \left[ \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right] \left( \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right) dt + \frac{d_j^1}{4d_j} \sup_{t \leq t_0} E\left[ e_0^2 \sigma \rho_j \right] dt
\]
\[\leq e_0 \sigma \rho_j(N_j+1) V(x(t_0), t_0) + \frac{d_j^1}{4d_j} \max_{j \in \mathbb{N}_1} \int_0^t \left[ \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right] \left( \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right) dt + \frac{d_j^1}{4d_j} \sup_{t \leq t_0} E\left[ e_0^2 \sigma \rho_j \right] dt
\]
\[\leq e_0 \sigma \rho_j(N_j+1) V(x(t_0), t_0) + \frac{d_j^1}{4d_j} \max_{j \in \mathbb{N}_1} \int_0^t \left[ \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right] \left( \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right) dt + \frac{d_j^1}{4d_j} \sup_{t \leq t_0} E\left[ e_0^2 \sigma \rho_j \right] dt
\]
where $d_j^1 = \max\{d_j^1, d_j^2\}$. According to Lemma 2, system (1) has a unique global solution on $[t_0, \infty)$. (ii) It follows from (5), (11) and Lemma 3 that
\[
V'(x(t), t) \leq \sigma V'(x(t), t) + \frac{d_j^1}{4d_j} \max_{j \in \mathbb{N}_1} \int_0^t \left[ \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right] \left( \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right) dt + \frac{d_j^1}{4d_j} \sup_{t \leq t_0} E\left[ e_0^2 \sigma \rho_j \right] dt
\]
\[\leq \sigma V'(x(t), t) + \frac{d_j^1}{4d_j} \max_{j \in \mathbb{N}_1} \int_0^t \left[ \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right] \left( \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right) dt + \frac{d_j^1}{4d_j} \sup_{t \leq t_0} E\left[ e_0^2 \sigma \rho_j \right] dt
\]
\[\leq \sigma V'(x(t), t) + \frac{d_j^1}{4d_j} \max_{j \in \mathbb{N}_1} \int_0^t \left[ \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right] \left( \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right) dt + \frac{d_j^1}{4d_j} \sup_{t \leq t_0} E\left[ e_0^2 \sigma \rho_j \right] dt
\]
which, combining this with (9), gives that
\[
V'(x(t), t) \leq \sigma V'(x(t), t) + \frac{d_j^1}{4d_j} \max_{j \in \mathbb{N}_1} \int_0^t \left[ \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right] \left( \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right) dt + \frac{d_j^1}{4d_j} \sup_{t \leq t_0} E\left[ e_0^2 \sigma \rho_j \right] dt
\]
\[\leq \sigma V'(x(t), t) + \frac{d_j^1}{4d_j} \max_{j \in \mathbb{N}_1} \int_0^t \left[ \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right] \left( \sum_{j=1}^{N_j} e_0 \sigma \rho_j \right) dt + \frac{d_j^1}{4d_j} \sup_{t \leq t_0} E\left[ e_0^2 \sigma \rho_j \right] dt
\]
On the basis of assumption A1, (16) can be written as
\[ V'(x(t), t) \leq \sigma V'(x(t), t) e^{-\left(t - t_k - 1\right) + 2Î} + \sup_{\xi \in \Omega} \left| \frac{\xi(t)}{\epsilon} \right|^2 \int_{t_k}^t e^\theta ds + \frac{\epsilon^2}{4} \sup_{\xi \in \Omega} \left| \frac{\xi(t)}{\epsilon} \right|^2 \int_{t_{k-1}}^{t_k} e^\theta ds + \epsilon^2 \int_{t_{k-1} - 1}^{t_k} e^{-\left(t - t_k - 1\right)} ds + \frac{1}{4} \int_{t_{k-1} - 1}^{t_k} e^{-\left(t - t_k - 1\right)} ds. \] Continuing within the iteration from 0 to \( N_i \) yields
\[ V'(x(t), t) \leq \sigma V'(x(t), t) e^{-\left(t - t_k - 1\right) + 2Î} \]
\[ + d_k \sup_{\xi \in \Omega} \left| \frac{\xi(t)}{\epsilon} \right|^2 \int_{t_0}^t e^{-\left(t - t_k - 1\right)} ds + \sigma \epsilon^2 \int_{t_{k-1} - 1}^{t_k} e^{-\left(t - t_k - 1\right)} ds + \epsilon^2 \int_{t_{k-1} - 1}^{t_k} e^{-\left(t - t_k - 1\right)} ds + \frac{1}{4} \int_{t_{k-1} - 1}^{t_k} e^{-\left(t - t_k - 1\right)} ds. \]
Taking expectations on both sides of (17), we can obtain that
\[ E[V'(x(t), t) + \int_{t_0}^t e^{-\left(t - t_k - 1\right)} ds + \sigma \epsilon^2 \int_{t_{k-1} - 1}^{t_k} e^{-\left(t - t_k - 1\right)} ds + \epsilon^2 \int_{t_{k-1} - 1}^{t_k} e^{-\left(t - t_k - 1\right)} ds + \frac{1}{4} \int_{t_{k-1} - 1}^{t_k} e^{-\left(t - t_k - 1\right)} ds. \]

On the other hand, we have
\[ -V(x(t), t) \leq \frac{m_0}{1 + m_0} \left( x(t), t \right), \forall t > T_i, \ a.s., \]
which, together with (17), yields that for \( \forall t > T_i, \)
\[ V'(x(t), t) \leq \sigma V'(x(t), t) e^{-\left(t - t_k - 1\right) + 2Î} + \frac{d_k \epsilon^2 + \sigma \epsilon^2}{1 + m_0} \sup_{\xi \in \Omega} \left| \frac{\xi(t)}{\epsilon} \right|^2 \int_{t_0}^t e^{-\left(t - t_k - 1\right)} ds + \frac{1}{4} \int_{t_{k-1} - 1}^{t_k} e^{-\left(t - t_k - 1\right)} ds. \]

When \( t \in [t_0, T_i] \), on the basis of Remark 1, there is an integer \( N_i \in (0, \infty) \) such that \( N_i < N < \infty \); then it follows from (17) that
\[ V'(x(t), t) \leq \sigma V'(x(t), t) e^{-\left(t - t_k - 1\right) + 2Î} + \frac{d_k \epsilon^2 + \sigma \epsilon^2}{1 + m_0} \sup_{\xi \in \Omega} \left| \frac{\xi(t)}{\epsilon} \right|^2 \int_{t_0}^t e^{-\left(t - t_k - 1\right)} ds + \frac{1}{4} \int_{t_{k-1} - 1}^{t_k} e^{-\left(t - t_k - 1\right)} ds. \]
Taking expectations on both sides of (20) yields that

\[
E[V(\mathbf{x}(t), t)I_{t_0 \leq t \leq \tau_1}] \\
\leq \sigma^N \rho_2^N \sigma_0^N \mathbb{E}[V(\mathbf{x}(t_0), t_0)] e^{-\sigma_1 t} + \rho_0^N \\
+ \frac{\max \mathbb{E}[\mathbf{r}_j^2]}{\rho_0^N} \max \mathbb{E}[\mathbf{r}_j^2] + \max \mathbb{E}[\mathbf{r}_j^2] \\
+ \frac{\max \mathbb{E}[\mathbf{r}_j^2]}{\rho_0^N} \max \mathbb{E}[\mathbf{r}_j^2] + \max \mathbb{E}[\mathbf{r}_j^2].
\]

Overall, for \( \forall t \in [t_0, \infty) \), we can obtain that

\[
E[V(\mathbf{x}(t), t)] \\
= E[V(\mathbf{x}(t), t)I_{t_0 \leq t \leq \tau_1}] + E[V(\mathbf{x}(t), t)I_{t > \tau_1}] \\
\leq D_1^* \mathbb{E}[V(\mathbf{x}(t_0), t_0)] e^{-\sigma_1 t} + \rho_0^N \mathbb{E}[\mathbf{r}_j^2] + \max \mathbb{E}[\mathbf{r}_j^2] + \max \mathbb{E}[\mathbf{r}_j^2]
\]

where

\[
D_1^* = 2\sigma^N \rho_2^N \sigma_0^N + \rho_0^N > 0,
\]

\[
M_1 = \max \left\{ \frac{\max \mathbb{E}[\mathbf{r}_j^2]}{\rho_0^N} \right\} > 0,
\]

\[
M_2 = \max \left\{ \frac{\max \mathbb{E}[\mathbf{r}_j^2]}{\rho_0^N} \right\} > 0.
\]

It follows from (7) and (18) that

\[
\mathbb{E}[\mathbf{x}(t)] \\
\leq \beta(t, t - h) + \gamma_1(\max_{\mathbf{r}_{t} \in \mathcal{N}_{t}} \mathbb{E}[\mathbf{r}_{t}^2])
\]

\[
+ \gamma_2(\max_{\mathbf{r}_{t} \in \mathcal{N}_{t}} \mathbb{E}[\mathbf{r}_{t}^2]),
\]

where class-\( \mathcal{K} \) functions

\[
\gamma_1 = \frac{M_1}{h_1} \max_{\mathbf{r}_{t} \in \mathcal{N}_{t}} \mathbb{E}[\mathbf{r}_{t}^2],
\]

\[
\gamma_2 = \frac{M_2}{h_1} \max_{\mathbf{r}_{t} \in \mathcal{N}_{t}} \mathbb{E}[\mathbf{r}_{t}^2],
\]

and the class-\( \mathcal{KL} \) function

\[
\beta(t, t - h) = \frac{h_2 D_1^*}{h_1} |\mathbf{x}(t)| e^{-\sigma_1 (t-h)}.
\]

According to Definition 1, system (1) is said to be NSS-\( m \)-M, which completes the proof.

Theorem 2. Under assumption A1, for any random impulsive interval sequence \( \{\sigma_k\}_{k \in \mathbb{N}^+} \), if there exist a UASF \( \mu(t) \) as expressed in (5), a function \( V(\mathbf{x}, t) \in C^{1,1}(\mathbb{R}^n \times [t_0, \infty); \mathbb{R}^+), \mathcal{K}_\infty \)-functions \( \alpha_1, \alpha_2 \)

and positive constants \( \sigma > 1, d_1, d_2, \epsilon, d_0 \geq 0 \) such that

\[
\alpha_1(|\mathbf{x}(t)|) \leq V(\mathbf{x}(t), t) \leq \alpha_2(|\mathbf{x}(t)|),
\]

\[
V_i + V_i f(\mathbf{x}(t), t) + d_1 \max_{\mathbf{r}_{t} \in \mathcal{N}_{t}} \mathbb{E}[\mathbf{r}_{t}^2] \leq \mu(t) V(\mathbf{x}(t), t),
\]

\[
V(\mathbf{x}(\tau_k), \tau_k) \leq \sigma V(\mathbf{x}(\tau_k), \tau_k) + d_2 |\mathbf{x}_{\tau_k}|^2;
\]

then (i) system (1) has a unique global solution on \([t_0, \infty)\); (ii) system (1) is said to be NSS--P as \( m_{\sigma} > \frac{|\ln|\epsilon||}{\epsilon} \).

Proof.

(i) As the similar lines of Theorem 1 we can get that system (1) has a unique global solution on \([t_0, \infty)\) and

\[
E[V(\mathbf{x}(t), t)] \leq D_1^* E[V(\mathbf{x}(t_0), t_0)] e^{-\sigma_1 t} + M_1 \max \mathbb{E}[\mathbf{r}_j^2] + M_2 \max \mathbb{E}[\mathbf{r}_j^2]
\]

\[
\leq \frac{e E[V(\mathbf{x}(t), t)]}{\alpha_1} \leq \epsilon.
\]

It follows from (18) that there exist a class-\( \mathcal{K} \mathcal{L} \) function

\[
\beta_2(t, t - h) = \alpha_1^{-1} \left( \frac{2M_2}{\epsilon} \max_{\mathbf{r}_{t} \in \mathcal{N}_{t}} \mathbb{E}[\mathbf{r}_{t}^2] \right) e^{-\sigma_1 (t-h)}.
\]

and class-\( \mathcal{K}_\infty \) functions

\[
\gamma_3(\max_{\mathbf{r}_{t} \in \mathcal{N}_{t}} \mathbb{E}[\mathbf{r}_{t}^2]) = \alpha_1^{-1} \left[ \frac{2M_2}{\epsilon} \max_{\mathbf{r}_{t} \in \mathcal{N}_{t}} \mathbb{E}[\mathbf{r}_{t}^2] \right],
\]

\[
\gamma_4(\max_{\mathbf{r}_{t} \in \mathcal{N}_{t}} \mathbb{E}[\mathbf{r}_{t}^2]) = \alpha_1^{-1} \left[ \frac{2M_2}{\epsilon} \max_{\mathbf{r}_{t} \in \mathcal{N}_{t}} \mathbb{E}[\mathbf{r}_{t}^2] \right]
\]

such that

\[
P(|\mathbf{x}(t)|) \geq \beta_2(t, t - h) + \gamma_3(\max_{\mathbf{r}_{t} \in \mathcal{N}_{t}} \mathbb{E}[\mathbf{r}_{t}^2])
\]

\[
+ \gamma_4(\max_{\mathbf{r}_{t} \in \mathcal{N}_{t}} \mathbb{E}[\mathbf{r}_{t}^2]) \leq \epsilon,
\]

which completes the proof.

Remark 4. When the impulses are unstable, they can be regarded as the disturbances on stable continuous dynamics. Theorems 1
and 2 give the sufficient conditions of NSS for the stable stochastic continuous dynamics subject to stochastic impulses. The condition of \( m_\sigma > \frac{\ln \sigma + c}{\varepsilon} \) gives a lower bound of the random impulsive intervals in the sense of expectation, that is, it requests that the impulses should not affect continuous dynamics too frequent.

Based on the proof of the above two theorems, when the UASF takes a special value, that is \( \mu(t) = -\varepsilon \), we can get the following corollaries:

**Corollary 1.** Under assumption A1, for any random impulsive interval sequence \( \{\sigma_k\}_{k\in\mathbb{N}^+} \), if there exist a function \( V'(x,t) \in C^{1,1}(\mathbb{R}^n \times [0, \infty); \mathbb{R}^+) \), and positive constants \( b_1, b_2, \sigma > 1, m, d_k, d_k', \varepsilon \) such that

\[
\begin{align*}
& b_1 |x(t)|^m \leq V'(x,t), t) \leq b_2 |x(t)|^m, \\
& V_x f(x(t), t) + d_k' \left| V_x g(x(t), t) \right|^2 \leq -c V'(x,t), t), \\
& V(x(t_k), t_k) \leq \sigma V'(x(t_k), \tau_k) + d_k' |\xi_{k\delta}|^2;
\end{align*}
\]

then (i) system (1) has a unique global solution on \([0, \infty)\); (ii) system (1) is said to be NSS-m-M as \( m_\sigma > \frac{\ln \sigma + c}{\varepsilon} \).

**Corollary 2.** Under assumption A1, for any random impulsive interval sequence \( \{\sigma_k\}_{k\in\mathbb{N}^+} \), if there exist a function \( V'(x,t) \in C^{1,1}(\mathbb{R}^n \times [0, \infty); \mathbb{R}^+) \), \( \mathcal{K}_{\infty} \) functions \( \alpha_1, \alpha_2 \) and positive constants \( \sigma > 1, \varepsilon, d_k, d_k' \) such that

\[
\begin{align*}
& \alpha_1(|x|) \leq V'(x,t), t) \leq \alpha_2(|x|), \\
& V_x f(x(t), t) + d_k' \left| V_x g(x(t), t) \right|^2 \leq -c V'(x,t), t), \\
& V(x(t_k), t_k) \leq \sigma V'(x(t_k), \tau_k) + d_k' |\xi_{k\delta}|^2;
\end{align*}
\]

then (i) system (1) has a unique global solution on \([0, \infty)\); (ii) system (1) is said to be NSS-\(P\) as \( m_\sigma > \frac{\ln \sigma + c}{\varepsilon} \).

**Remark 5.** When the impulsive times are certain, the above two corollaries can be reduced to Theorems 1 and 2 in [19]. When the continuous dynamics are not affected by impulses, Corollary 1 will degenerate into Theorem 6 of [14]. Theorems 1 and 2 can be viewed as the extension of [14] in the case when the system is disturbed by impulses occurring at random times.

### 3.2 NSS for IRNSs with unstable continuous dynamics

The sufficient conditions of NSS are proposed in this subsection for system (1) with the unstable continuous dynamics and the stable stochastic impulses.

**Theorem 3.** Under assumption A1, for any random impulsive interval sequence \( \{\sigma_k\}_{k\in\mathbb{N}^+} \), if there exist a UESF \( \mu_k \) as expressed in (6), a function \( V(x,t) \in C^{1,1}(\mathbb{R}^n \times [0, \infty); \mathbb{R}^+) \), and positive constants \( b_1, b_2, \varphi, m, d_k', d_k, \varepsilon \), \( \theta \in (0, 1) \) such that

\[
\begin{align*}
& b_1 |x(t)|^m \leq V'(x,t), t) \leq b_2 |x(t)|^m, \\
& V_x f(x(t), t) + d_k' \left| V_x g(x(t), t) \right|^2 \leq \varphi V'(x,t), t), \\
& V(x(t_k), t_k) \leq \mu_k V'(x(t_k), \tau_k) + d_k' |\xi_{k\delta}|^2,
\end{align*}
\]

then (i) system (1) has a unique global solution on \([0, \infty)\); (ii) system (1) is said to be NSS-\(m-M\) as \( m_\sigma < -\frac{\ln \theta}{\varphi} \).

**Proof.** As \( \tau_k \) is denoted in (10), for any \( t \in [\tau_k, \tau_{k+1}) \), it follows from (22) that the up-right hand derivative can be represented as:

\[
D^+[V'(x,t), t)] = \frac{\partial V'(x,t)}{\partial t} f(x,t) + g(x,t) \xi(t),
\]

Taking expectations on both sides of (24), it follows from \( \mathbb{E}[D^+[V'(x,t), t)] = D^+[\mathbb{E}[V'(x,t), t)] \) that

\[
D^+[\mathbb{E}[V'(x,t), t)] \leq \varphi \mathbb{E}[V'(x,t), t] + \frac{\sup_{\text{sup}_{\xi \in \mathbb{R}^+}} \mathbb{E}[\xi(t)]^2}{4 \delta_k'}. 
\]

In view of Lemma 3, we have

\[
\begin{align*}
& \mathbb{E}[V'(x \land \varphi_r, t \land \varphi_r)] \\
& \leq \mathbb{E}[V'(x(t_k), \tau_k)] \theta^k \delta_k' \int_0^\infty \mathbb{E}[\xi(t)]^2 ds + \frac{\theta^k \delta_k'}{4 \delta_k'} \sup_{\text{sup}_{\xi \in \mathbb{R}^+}} \mathbb{E}[\xi(t)]^2 \\
& \leq \mathbb{E}[V'(x(t_k), \tau_k)] \theta^k \delta_k' \int_0^T \mathbb{E}[\xi(t)]^2 ds + \frac{\sup_{\text{sup}_{\xi \in \mathbb{R}^+}} \mathbb{E}[\xi(t)]^2}{4 \delta_k} \theta^k \delta_k' \\
& \leq \mathbb{E}[V'(x(t_k), \tau_k)] \theta^k \delta_k' \int_0^T \mathbb{E}[\xi(t)]^2 ds + \frac{\sup_{\text{sup}_{\xi \in \mathbb{R}^+}} \mathbb{E}[\xi(t)]^2}{4 \delta_k} \theta^k \delta_k',
\end{align*}
\]

which, combining with (23), gives
Continuing within the iteration from 0 to \(N_t\), it follows from (6) that

\[
\begin{align*}
E[V(t^\vee \varphi_0), t^\vee \varphi_0)] &\leq e^{\theta(N_t-\theta)} E[V'(x(t_0), t_0)] \\
&+ e^{\varphi} \max_{j \in \mathbb{N}} E[\xi_j^2] \left[ \sum_{j=1}^{N_t} \mathbb{E} (\theta^{N_j}) \theta^{N_j-j} \right] \\
&+ \frac{c_{\sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup 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\sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup \sup 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where
\[ D_x^k = e^{\theta t} > 0, \]
\[ \phi^* = \min \{-\phi + \frac{1}{\theta} \ln |\alpha|, \phi\} > 0, \]
\[ M_3 = \max \left\{ \frac{d_2}{\theta(1-\theta)}, \frac{d_2N_2}{\theta(1-\theta)} \right\} > 0, \]
\[ M_4 = \max \left\{ \frac{4d_2}{\theta(1-\theta)}, \frac{4d_2N_2}{\theta(1-\theta)} \right\} > 0. \]

It follows from (21) that
\[ E|x(t)|^m \leq \beta_3(|x_0|, t - \theta) + \gamma_3(\max_{k \in \mathbb{N}^+} E|\xi_k|^2) + \gamma_6(\sup_{k \in \mathbb{N}^+} E|\xi|^2), \]
where the class-\(KL\) function
\[ \beta_3(|x_0|, t - \theta) = \frac{b_2}{b_1} D_2^k E[V'(x(t_0), t_0)] e^{\theta(t-\theta)}, \]
and the class-\(K\) functions
\[ \gamma_3(\max_{k \in \mathbb{N}^+} E|\xi_k|^2) = M_3(\max_{k \in \mathbb{N}^+} E|\xi_k|^2), \]
\[ \gamma_6(\sup_{k \in \mathbb{N}^+} E|\xi|^2) = M_4(\sup_{k \in \mathbb{N}^+} E|\xi|^2). \]

Based on Definition 1, system (1) is said to be NSS-m-M. \( \square \)

**Theorem 4.** Under assumption A1, for any random impulsive interval sequence \( \{\sigma_k\}_{k \in \mathbb{N}^+}\), if there exist a USEF \( \mu_{\theta} \) as expressed in (6), a function \( V(x, t) \in C^{1,1}(\mathbb{R}^x \times [t_0, \infty); \mathbb{R}^+), \) \( K_{\infty}\) functions \( \alpha_1, \alpha_2 \) and positive constants \( \phi, d_1, d_2, \epsilon, \theta \in (0, 1) \) such that
\[ \alpha_1(|x(t)|) \leq V(x(t), t) \leq \alpha_2(|x(t)|), \]
\[ V' + V'_x f(x(t), t) + d_2 V'(x(t), t) \leq \phi V(x(t), t), \]
\[ V'(\sigma(t), \tau_k) \leq \mu_k V'(\sigma(t), \tau_k) + d_1 |\xi_k|^2; \]
then (i) system (1) has a unique global solution on \([t_0, \infty)\); (ii) system (1) is said to be NSS-P as \( m_\theta < -\ln \frac{\phi}{V} \),

**Proof.**

(i) From Theorem 3, the existence of unique solution on \([t_0, \infty)\) can be obtained.

(ii) For \( \forall \epsilon \in (0, 1) \), on the basis of Markov's inequality and (31), we have
\[ P \left\{ \alpha_1(|x(t)|) \leq \frac{I_{\sigma}}{\epsilon} \alpha_2(|x_0|) e^{\theta(t-\theta)} \right\} \]
\[ \geq P \left\{ V(x(t), t) \leq \frac{I_{\sigma}}{\epsilon} \alpha_2(|x_0|) e^{\theta(t-\theta)} \right\} \]
\[ \geq 1 - \frac{\phi V(x(t), t)}{M_2} \]
\[ \geq 1 - \epsilon, \]

where \( M_2 = D_2^k E[V'(x(t_0), t_0)] e^{\theta(t-\theta)} + \frac{M_3}{\epsilon} \sup_{\xi \in \mathbb{N}^+} E|\xi|^2 \]
\[ + \frac{M_4}{\epsilon} \sup_{\xi \in \mathbb{N}^+} E|\xi|^2. \]

Based on (27), we can find a class-\(KL\) function \( \beta_4(|x_0|, t - \theta) = \alpha_1^{-1} \left[ \frac{2M_3}{\epsilon} \sup_{\xi \in \mathbb{N}^+} E|\xi|^2 \right], \)
and class-\(K\) functions
\[ \gamma_7(\max_{\xi \in \mathbb{N}^+} E|\xi|^2) = \alpha_1^{-1} \left[ \frac{2M_4}{\epsilon} \sup_{\xi \in \mathbb{N}^+} E|\xi|^2 \right], \]
\[ \gamma_8(\sup_{\xi \in \mathbb{N}^+} E|\xi|^2) = \alpha_1^{-1} \left[ \frac{2M_5}{\epsilon} \sup_{\xi \in \mathbb{N}^+} E|\xi|^2 \right] \]

such that
\[ P[|x(t)| < \beta_4(|x_0|, t - \theta) + \gamma_7(\max_{\xi \in \mathbb{N}^+} E|\xi|^2) + \gamma_8(\sup_{\xi \in \mathbb{N}^+} E|\xi|^2)] \geq 1 - \epsilon. \]

Thus, from Definition 2, we complete the proof. \( \square \)

**Remark 6.** The unstable continuous dynamics with the stable impulses can be regarded as the impulsive controller to stabilize the unstable continuous dynamics, such as the Synchronous control \([40]\) and distributed control \([41, 42]\). It follows from Theorems 3 and 4 that an impulsive controller which can stabilize the unstable continuous dynamics in system (1), the expectation of random impulsive intervals should satisfy \( m_\theta < -\ln \frac{\phi}{V} \), which gives a upper bound of random impulsive intervals, that is, the frequency of impulsive controller effect on the unstable continuous dynamics cannot be too low.

When the USEF \( \mu_{\theta} = \theta < 1 \) or \( \prod_{k=1}^j \mu_k \leq \theta^{-i} < 1 \), for any \( j \geq i \geq 0 \), the following two corollaries can be easily obtained.

**Corollary 3.** Under assumption A1, for any random impulsive interval sequence \( \{\sigma_k\}_{k \in \mathbb{N}^+}\), if there exist a function \( V(x, t) \in C^{1,1}(\mathbb{R}^x \times [t_0, \infty); \mathbb{R}^+), \) \( K_{\infty}\) functions \( \alpha_1, \alpha_2 \) and positive constants \( \phi, d_1, d_2, \epsilon, \theta \in (0, 1) \) such that
\[ b_1 |x(t)|^m \leq V(x(t), t) \leq b_2 |x(t)|^m, \]
\[ V' + V'_x f(x(t), t) + d_2 V'(x(t), t) \leq \phi V(x(t), t), \]
\[ V'(\sigma(t), \tau_k) \leq \theta V'(\sigma(t), \tau_k) + d_1 |\xi_k|^2; \]
then (i) system (1) has a unique global solution on \([t_0, \infty)\); (ii) system (1) is said to be NSS-m-M as \( m_\theta < -\ln \frac{\phi}{V} \).

**Corollary 4.** Under assumption A1, for any random impulsive interval sequence \( \{\sigma_k\}_{k \in \mathbb{N}^+}\), if there exist a function \( V(x, t) \in C^{1,1}(\mathbb{R}^x \times [t_0, \infty); \mathbb{R}^+), \) \( K_{\infty}\) functions \( \alpha_1, \alpha_2 \) and positive constants \( \phi, d_1, d_2, \epsilon, \theta \in (0, 1) \) such that
\[ \alpha_1(|x(t)|) \leq V(x(t), t) \leq \alpha_2(|x(t)|), \]
\[ V' + V'_x f(x(t), t) + d_2 V'(x(t), t) \leq \phi V(x(t), t), \]
\[ V'(\sigma(t), \tau_k) \leq \theta V'(\sigma(t), \tau_k) + d_1 |\xi_k|^2; \]
then (i) system (1) has a unique global solution on $[0, \infty)$; (ii) system (1) is said to be NSS-P as $m_\sigma < -\frac{\ln \theta}{\theta}$.

**Remark 7.** It is obvious that the requirements of random impulsive intervals are the same in Theorem 3 and Corollary 3 (or Theorem 4 and Corollary 4), but the restrictions on the time-shifts of the Lyapunov function are relaxed in Theorem 3 and Theorem 4 by the UESF, which was introduced in [37].

### 4 SIMULATION EXAMPLES

Two examples are given in this section: Example 1 shows the NSS-$m$-M for the random non-linear systems with stochastic impulses based on the UASF, and example 2 demonstrates the NSS-P of IRNSs. Furthermore, the Remark 7 can be verified by comparison in example 2.

**Example 1.** Consider system (1) with $n = 2$, and the non-linear functions are described as following:

$$
\begin{align*}
\dot{x}(t) &= \left(\frac{\sin t^2}{4} - \frac{\sin t^2}{4} - \frac{\sin t^2}{4} - \frac{\sin t^2}{4}\right) + g(x(t), t), \\
g(x(t), t) &= (\sin t, \cos t)^T, \\
F_{\xi k+1}(x(\tau_{k+1}^-), x_{k+1}^-) &= \left[2x_1(\tau_{k+1}^-) + \frac{\sin(\tau_{k+1}^-)}{2} x_{k+1}^-ight] \\
&\quad + \left[2x_2(\tau_{k+1}^-) + \frac{\cos(\tau_{k+1}^-)}{2} x_{k+1}^- \right]
\end{align*}
$$

The stochastic process $\xi(t) \in \mathbb{R}$ produced by $2\xi(t) = -\xi(t) + w(t)$, and $\xi(0) = 0$, where $w(t) \in \mathbb{R}$ is the zero-mean white noise with the spectral function of being 1. The random variable sequence $\{\xi_{k+1}\}$ is uniformly distributed on the interval $[0,4]$. The random disturbance $\xi(t)$ and random variable $\xi_k$ can be seen in Figure 1.

We assume that the random impulsive interval $\sigma_k$ follows the Gamma distribution $G(\alpha, \beta)$ with parameters $\alpha = \beta = 1.5$, that is the expectation of $\sigma_k$ satisfies that $m_\sigma = 1$. The impulsive times and the corresponding impulsive intensity of stochastic impulses can be seen in Figure 2.

**Example 2.** Assume that the random impulsive interval $\sigma_k$ follows the Gamma distribution $G(\alpha, \beta)$ with parameters $\alpha = \beta = 1.5$, that is the expectation of $\sigma_k$ satisfies that $m_\sigma = 1$. The impulsive times and the corresponding impulsive intensity of stochastic impulses can be seen in Figure 2.

**FIGURE 1** Random disturbances $\xi(t)$ and $\zeta_k$

**FIGURE 2** The impulsive intensity of stochastic impulses

**FIGURE 3** The state responses of the system

Let $V = \frac{1}{2}(x_1^2 + x_2^2)$ be the Lyapunov function and $d_g = 1$; then we can get that

$$
\begin{align*}
V' + V'_{f}(x(t), t) + d_g |V_{g}(x(t), t)|^2 \\
&= \frac{t_2^2}{2} x_1 x_2 - 2x_1^2 - 2x_2^2 \leq \left[\frac{t_2^2}{2} - 4\right] V(x(t), t)
\end{align*}
$$

and

$$
\begin{align*}
V'(x(\tau_{k+1}^-), x_{k+1}^-) &= \left[2x_1(\tau_{k+1}^-) + \frac{\sin(\tau_{k+1}^-)}{2} x_{k+1}^- \right]^2 \\
&\quad + \left[2x_2(\tau_{k+1}^-) + \frac{\cos(\tau_{k+1}^-)}{2} x_{k+1}^- \right]^2 \\
&\leq 16V'(x(\tau_{k+1}^-), \tau_{k+1}^-) + \frac{t_2^2}{2} x_{k+1}^2.
\end{align*}
$$

By choosing the UASF $\mu(t) = \frac{t_2^2}{2} - 4$, we have $f_{b_1} \mu(t) dt \leq -4(t - b_1) + \frac{1}{2}$. With $b_1 = \frac{1}{2}$, $b_2 = 1$, $\sigma = 16$, $d_g = 1$, $d_k = 1$, it can be verified that $\frac{\ln x + d_k}{m_\sigma} = 0.82 < m_\sigma$. With the initial values $x_1(0) = 8$ and $x_2(0) = -10$, the state responses can be shown by Figure 3.

Figure 2 shows four different stochastic impulses with different impulsive times and different impulsive intensity. When the four different stochastic impulses are applied to the system, we can get that the states of the system jump to varying degrees.
which can be seen in Figure 3. It follows from Figure 3 that the states of the system have converged to the neighbourhood of the origin although it is disturbed by the unstable stochastic impulses, which demonstrate that the continuous dynamics are still NSS-w-M under conditions of Theorem 1.

Example 2. In this example, we consider the following two systems:

\[
\begin{align*}
\dot{x}_1(t) &= x_1(t) \sin t + \cos t \xi(t), \quad t \in [\tau_k, \tau_{k+1}), \\
x_1(\tau_{k+1}) &= \frac{1}{2} x_1(\tau_{k+1}) \sin(\tau_{k+1}^-) + \xi_{k+1},
\end{align*}
\]

and

\[
\begin{align*}
\dot{x}_2(t) &= x_2(t) \sin t + \cos t \xi(t), \quad t \in [\tau_k, \tau_{k+1}), \\
x_2(\tau_{k+1}) &= \sqrt{\mu_{k+1}} x_2(\tau_{k+1}^-) \sin(\tau_{k+1}^-) + \xi_{k+1},
\end{align*}
\]

where \( \mu_k \) satisfies \( \mu_1 = \mu_2 = 2 \) and \( \mu_j = \frac{1}{2}, \quad j > 2; \) then it meets that \( \prod_{j=1}^{k} \mu_j \leq \frac{4}{3}^{k-2} \). The stochastic process \( \xi(t) \in \mathbb{R} \) is generated by

\[
\xi(t) = \frac{1}{2} \cos(t + \eta), \quad \forall t \geq t_0,
\]

and the random variable \( \eta \) stands for a zero-mean Gaussian white noise with the spectral function being 1. The random variable sequence \( \{\xi_{k+1}\} \) is a uniformly distributed on the interval [0,2]. Figure 4 describes a trajectory of the stochastic process \( \xi(t) \) and random variable \( \xi_k \).

We assume that the random impulsive interval \( \sigma \) follows the Gamma distribution \( G(\alpha, \beta) \) with parameters \( \alpha = 0.6, \beta = 1.2, \) that is the expectation of \( \sigma_k \) satisfies that \( m_{\sigma} = 0.5 \). The impulsive times and the corresponding impulsive intensity for the two systems can be seen in Figure 5.

Choose the Lyapunov functions \( V_1 = x_1^2 \) for system (32), \( V_2 = x_2^2 \) for system (33) and \( \delta_k = 1; \) then we have

\[
\begin{align*}
\frac{\partial V_1(x_1(t),t)}{\partial t} + \frac{\partial V_1(x_1(t),t)}{\partial x_1} f(x_1(t),t) + \delta_k \frac{\partial V_1(x_1(t),t)}{\partial x_1} g(x_1(t),t) &
\leq 4V_1(x_1(t),t), \\
V_1(x_1(\tau_{k+1}),\tau_{k+1}) &\leq \frac{1}{2} V_1(x_1(\tau_{k+1}),\tau_{k+1}) + 2|\xi_{k+1}|^2,
\end{align*}
\]

and

\[
\begin{align*}
\frac{\partial V_2(x_2(t),t)}{\partial t} + \frac{\partial V_2(x_2(t),t)}{\partial x_2} f(x_2(t),t) + \delta_k \frac{\partial V_2(x_2(t),t)}{\partial x_2} g(x_2(t),t) &
\leq 4V_2(x_2(t),t), \\
V_2(x_2(\tau_{k+1}),\tau_{k+1}) &\leq \mu_k V_2(x_2(\tau_{k+1}),\tau_{k+1}) + 2|\xi_{k+1}|^2.
\end{align*}
\]

Let the \( K_{\infty} \)-functions \( \alpha_1(|x|) = \frac{1}{2} |x|^2, \quad \alpha_2(|x|) = 2|x|^2 \) and positive constants \( \varphi = 4, \quad \delta_k = 0.5, \quad \delta_k = \frac{1}{2}, \sigma_k \in (0, 1); \) then it follows from Corollary 4 and Theorem 4 that system (32) and (33) are NSS-P with \( m_{\sigma} < -\frac{\ln \theta}{\varphi} = 0.69 \). The status responses can be shown by Figure 6 when the initial values are chosen as 8, 4, 8, 6, and 6, respectively.

It is obvious that the only difference between system (32) and (33) is the different values of \( \mu_k \), and it is just different in the first two seconds, but just because of the different values of \( \mu_k \), we can get that there is a big difference in impulsive intensity in the first two seconds from Figure 5. In addition, we can find
that the impulsive times are different in the four pictures of Figure 5. In fact, the continuous dynamics in system (32) and (33) are unstable, especially in the first two seconds, the state $x_2$ of system (33) is more divergent than the state of system (32). This is due to the common instability of the continues dynamics and stochastic impulses. In the next time, the states $x_1$ and $x_2$ gradually tend to the neighborhood of the origin because of the action of the stable impulses. From Figure 6, we can get that the system is NSS-P although the continuous dynamics are unstable and the initial values are different.

Remark 8. The system we considered in example 1 is similar to the example 1 in [19] except for the impulsive times, that is, the impulsive times in our paper are random variables and the number of impulse is larger than that in [19], but the simulation results demonstrate that the two systems are all NSS-ne-M.

5 | CONCLUSIONS

The improved criteria of NSS are investigated in this paper on the basis of UASF and UESF for random non-linear systems with stochastic impulses which include three random characteristics: stochastic continuous dynamics described by random differential equations which are driven by second-order processes with bounded average power; stochastic instantaneous state jumps described by stochastic difference equations whose random disturbances are a sequence of stochastic variables with bounded energy; stochastic impulsive moments follow a renewal process. The stability criteria of NSS are established in two cases: stable continuous dynamics with unstable impulses and stable impulses with unstable continuous dynamics.

There are some problems under the current investigation, for example, adaptive tracking control for a random Lagrangian system subject to stochastic impulses, event-triggered impulsive control for random Lagrangian system. In addition, we will extend the result to multi-agent systems or neural networks, combined with some other works, such as [43–45].

ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation of China (No. 61973198), the Research Fund for the Taishan Scholar Project of Shandong Province of China, SDUST Research Fund (No.2015TDJH105), First-class Discipline Special Project of Shandong Province—Control Science and Control Engineering (01010230301). The work of J.H. Park was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. 2020R1A2B5B02002002).

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How to cite this article: Feng L, Park JH, Zhang W. Improved noise-to-state stability criteria of random nonlinear systems with stochastic impulses. IET Control Theory Appl. 2021;15:96–109. https://doi.org/10.1049/cth2.12030