CONSTRAINTS TO THE EOS OF ULTRADENSE MATTER WITH MODEL-INDEPENDENT ASTROPHYSICAL OBSERVATIONS.

G. Lavagetto$^1$, I. Bombaci$^2$, A. D’Ai$^3$, I. Vidaña$^2$ and N. R. Robba$^1$

ABSTRACT

The recent discovery of burst oscillations at 1122 Hz in the x-ray transient XTE J1739-285, together with the measurement of the mass of the binary millisecond pulsar PSR J0751+1807 (2.1 ± 0.2 $M_{\odot}$) can finally allow us to put strong, model-independent observational constraints to the equation of state of compact stars. We show that the measurement of the moment of inertia of PSR J0737+3039A, together with these constraints, could allow to discriminate further the details of the inner structure of neutron stars. Moreover, we show that if XTE J1739-285 is constituted of nucleonic matter, any equation of state allows only a narrow range of very high masses, and this could explain why up to now compact stars spinning faster than a millisecond have been so difficult to detect.

Subject headings: Stars: neutron – X-rays: binaries – binaries: close – relativity.

1. INTRODUCTION

Ever since the discovery of the first neutron star (NS) by [Hewish et al. 1968], these compact objects have been regarded as the ideal experimental test-bed for the theories of the state of matter at supernuclear densities. In particular, the mass, the radius, the attainable spin frequency of a NS strongly depend on the equation of state (EOS) governing matter at these densities.

Different models for the EOS of dense hadronic matter predict a NS maximum mass ($M_{\text{max}}$) in the range of 1.4–2.6 $M_{\odot}$ and a corresponding central density $n_c$ in range of 4–10 times the saturation density ($n_0 = 0.16$ fm$^{-3}$) of nuclear matter [Prakash et al. 1997]. In the case of a star with $M \approx 1.4$ $M_{\odot}$, different EOS models predict a radius in the range of 7–16 km.

Due to the large value of the stellar central densities, various regimes (particle species and phases) of dense hadronic matter are expected in the interiors of NS. Consequently, different types of “neutron stars” (compact stars, CSs) are hypothesized. In the simplest model the core of a neutron star is described as a uniform fluid of neutron rich nuclear matter in equilibrium with respect to the weak interaction: these are the so-called “traditional” NSs. However, due to the large value of the stellar central density and to the rapid increase of the nuclear chemical potentials with density, hyperons ($\Lambda$, $\Sigma^-$, $\Sigma^0$, $\Sigma^+$, $\Xi^-$, and $\Xi^0$ particles) are expected to appear in the inner core of the star. Other exotic phases of hadronic matter such as a Bose-Einstein condensate of negative pion ($\pi^-$) or negative kaon ($K^-$) could be present in the inner part of the star. The core of the more massive NS is also one of the best candidates in the Universe for a phase transition from hadronic matter to a deconfined quark phase to occur. Compact stars which possess a “quark matter core” either as a mixed phase of deconfined quarks and hadrons or as a pure quark matter (QM) phase are called Hybrid Stars (HyS) [Glendenning 1996].

The more conventional NSs in which no fraction of QM is present, are referred to as pure Hadronic Stars (HSs).

Even more challenging than the existence of a quark core in a NS, is the possible existence of a new type of CS consisting completely of a charge neutral deconfined mixture of up, down, strange quarks and electrons, satisfying the hypothesis on the absolute stability [Bodmer 1971, Witten 1984] of strange quark matter (SQM). Such CSs have been called strange stars (SS). The analysis of different type of observational data has given indirect evidence for the possible existence of SS (see e.g. [Li et al. 1999]). We will refer to hybrid stars and strange stars collectively as Quark Stars (QS).

Unluckily, for decades no significant constraint on both the mass and the radius of a single object could be extracted from astrophysical observations. While the determination of the masses of some CSs was made possible thanks to the study of the timing motion of radio pulsars in binary systems, the radius determination, or alternatively the moment of inertia, has been more elusive. Today the measured masses of CSs lie around the average value of 1.35 ± 0.04 $M_{\odot}$ and are roughly distributed according to a Gaussian distribution [Thorsett & Chakrabarty 1999].

In recent years the discovery of kilohertz quasi-periodic oscillations (QPOs) in the power spectra of several accreting CSs has opened new possibilities, as they are phenomena whose timescale corresponds to the dynamical timescale in the very neighborhood of a CSs (see [van der Klis 2006] for a review). These QPOs have peak frequencies in the 200-1200 Hz range and usually appear as two broad peaks (denoted upper and lower kHz QPO), that simultaneously move up and down according to the source accretion state. The lower kHz QPO sometimes becomes undetected as the upper kHz QPO reaches lower frequencies; for low accretion rates both peaks can disappear. While their peak frequencies smoothly change with the accretion rate of the source, their peak separation is roughly constant and equal, within 20%, to the spin frequency of the NS, or, for some sources, to the half of it [Wijnands et al. 2003]. The reason for this discrepancy is still unaccounted for. Most theoretical models of kHz QPOs associate the frequency of the highest peak to the
motion of matter at the last stable orbit of an accretion disk around the CS. If this is true, assuming that matter in the accretion disc is in Keplerian motion, it is possible to derive an upper limit to the density of the CS. This, in turn, puts some limits to the EOS of CSs (see for example Zhang et al. 2006), but still relies on a theoretical interpretation of a phenomenon which is not completely understood.

Another observational constraint comes from the reported gravitational redshift of absorption lines produced in the photosphere of the X-ray binary system EXO 0748-676 (Cottam, Paerels & Mendez 2002). The redshift \( z = 0.35 \) implies a compactness parameter for the CS of \( GM/Rc^2 \approx 0.23 \). However, even this result (which anyway is not able to constrain the EOS significantly if taken alone) is dependent both on the model adopted for the spectral fitting and on the assumptions on the emitting region.

Another astrophysical observable is the radiation radius \( R_{\infty} \) of the emission from isolated CSs: the nearest and better studied isolated CS, RX J1856.5-3754, has a spectrum that can be well fitted with a blackbody of 57 eV temperature. This value sets an upper limit for the \( R_{\infty} \) of this object, which was estimated \( \leq 8 \) km (Drake et al. 2002). This limit, which is quite demanding, can imply an EOS based on QM. However, the determination of this radius is strongly dependent on the distance of the source and on theoretical issues regarding the geometry of the emission and the composition of the atmosphere. Trümper et al. (2004) has recently argued against the constraints found by Drake et al. 2002, claiming that the upper limit is much higher.

The situation is even worse if we try to infer the radius of the CS from the blackbody temperature of an accreting X-ray binary (see e.g. Zhang et al. 2006): in this case even the origin of the emission can be debatable, as the blackbody emission can be originated near the stellar surface as well as from the accretion disc.

A truly model-independent constraint has been found with the measurement of the mass of several millisecond radio pulsars thanks to the detection of post-Newtonian corrections to their orbital motion. In particular, Nice et al. 2005 found that PSR J0751+1807 has a mass of \( 2.1 \pm 0.2 M_\odot \) (1 \( \sigma \) error), and Ransom et al. 2005 found that pulsar I of the globular cluster Terzan 5 has a mass that exceeds \( 1.68 M_\odot \) at 95\% confidence level, showing that CSs can have quite large masses.

Another model independent limit to the EOS at supernuclear densities comes from laboratory experiments on heavy ions collisions (HIC): Klähn et al. 2000 show that some constraints on the EOS can be given by flow data and sub-threshold Kaon production in HIC. They found that these constraints would rule out some of the very stiff EOSs. Grigorian, Blaschke & Klähn (2006) solved the problem by simply assuming that above a certain energy density, a quark core forms, with a phase transition to stiff, color superconducting QM that can alter only the behavior of the EOS at very high densities, making it softer near the maximum mass. In practice, this results in small modifications to the behavior of the EOS that do not alter very significantly the mass-radius relationship of CSs.

Finally, we will show that the recent discovery of burst oscillations with a frequency of 1122 Hz in XTE J1739-285 (Kaaret et al. 2006), is potentially another important constraint to the EOS of CSs: these oscillations, occurring during type I X-ray bursts, have the same frequency of the spin frequency of the CS (Chakrabarty et al. 2003). This discovery can put a real, model independent limit to the radius of the NS in XTE J1739-285, and thus to its EOS.

This discovery rules out some speculations that were made in order to explain the inability of CSs to spin up to periods below one millisecond. It was argued that the lack of these fast spinning objects was due to the emission of gravitational waves during accretion (Andersson, Kokkotas & Stergioulas 1998; Bildsten 1998) or to the disc-magnetic field interaction in low mass X-ray binaries (see e.g. Andersson et al. 2005). All these theories have been wiped out by this discovery, but the problem has now turned to: why only one CS is observed to spin below one millisecond?

In this letter we first impose the model independent constraints to the EOS of CSs, we discuss how further observations can improve these constraints, then we try to explain the difficulty in finding very fast objects on the basis of our findings on the structure of CSs.

## 2. MODEL-INDEPENDENT CONSTRAINTS

In figure 1 we show in the mass-radius plane the sequences for non-rotating CSs with very different compositions, together with constraints coming from model-independent observational results. First, the sequence should include masses as large as the one of PSR J0751+1807 (horizontal dashed lines) and of pulsar I of Terzan 5 (dotted line). We assume that the maximum non-rotating mass is almost equal to the maximum mass at the spin frequencies of these objects, which is a good approximation given the relatively low spin frequency of both objects. Second, the mass-radius relation should be such to allow the star to spin up to the maximum spin rate observed to date, i.e. to 1122 Hz, as observed in XTE J1739-285.

The maximum rate of rotation \( \Omega_{\text{max}} \) sustainable by a CS strongly depends on the overall stiffness of the EOS of dense hadronic matter. Equilibrium sequences of rapidly rotating CSs have been constructed numerically in general relativity by several groups (Cook, Shapiro & Teukolsky 1994; Datta, Thampan & Bombaci 1998; Bombaci, Thampan & Datta 2000). The numerical results for \( \Omega_{\text{max}} \) obtained for a broad set of realistic EOSs can be reproduced with a very good accuracy using simple empirical formulas which relate \( \Omega_{\text{max}} \) to the mass \( (M_{\text{max}}) \) and radius \( R_0 \) of the non-rotating maximum mass configuration (Salgado et al. 1994). The following simple empirical formula given by Lattimer (Conference "Isolated Neutron Stars, London 2006, see also Lattimer & Prakash 2004) approximately describes the minimum rotation period \( P_{\text{min}} = 2\pi/\Omega_{\text{max}} \) for a star of mass \( M \) and non-rotating radius \( R \)

\[
P_{\text{min}} = 0.96 \left( \frac{R/10 \text{ km}}{M/M_\odot} \right)^{3/2} \text{ ms.}
\]

Using the rotation period \( P = 0.891 \) ms, corresponding to the measured X-ray burst oscillation frequency for XTE J1739-285, we get the following upper limit for the
radius of the compact star in XTE J1739-285

\[ R < 9.52 \left( \frac{M}{M_\odot} \right)^{1/3} \text{ km}. \]  

This condition is plotted in figure as a green line, with another green line indicating the causality limit for the mass-radius relation.

As we said before, there exist various possible classes of CSs. We considered several EOSs in order to match them with observations, and constructed non-rotating sequences varying the central density of the star. In figure we report the mass-radius sequences for several EOSs. The curves relative to NSs are colored in black, those for hyperon stars in brown. In blue we show the sequences for HySs, and in red those for SSs. In particular, we report results for the following classes of systems: i) pure neutron matter CSs (L) calculated within a relativistic field theoretical approach in the mean field approximation with meson exchange (Arnett & Bowers 1977). This outdated and schematic model, is considered as an example of an extremely stiff EOS; ii) nucleonic stars (label BBB2) built with an EOS calculated using the microscopic Brueckner-Hartree-Fock (BHF) many-body approach with the Paris nucleon-nucleon interaction plus nuclear three-body forces (Baldo et al. 1997); iii) nucleonic stars (KS) built with an EOS derived from the Dirac-BHF approach using the Bonn B nuclear interaction. iv) nucleonic stars (GM1) calculated within a relativistic field theoretical approach in the mean field approximation (Glendenning & Moszkowki 1991); v) hyperon stars (Hyp) calculated with the previous GM1 EOS with the inclusion of hyperons; vi) Hybrid star sequences calculated using the same GM1 EOS for hyperonic matter to describe the hadronic phase, and the MIT bag model EOS (Farhi & Jaffe 1984) to describe the deconfined quark phase. For the masses of the three quark flavors we took \( m_u = m_d = 0 \) and \( m_s = 150 \text{ MeV} \). For the value of the bag constant we took \( B = 208.24 \text{ MeV/fm}^3 \) (Hyb1) and \( B = 136.63 \text{ MeV/fm}^3 \) (Hyb2). The curve (Hyb) refers to hybrid stars constructed using a different parametrization for the EOS by Glendenning & Moszkowki (1991) (GM3) for the hadronic phase and using \( B = 80 \text{ MeV/fm}^3 \); vii) Hybrid stars (Hyb(CFL), Alford et al. 2003) whose core is a mixed phase of nuclear matter and color-superconducting quark phase in the so-called "color flavor locked" (CFL) phase; viii) strange star sequences (B85, B70) built with the MIT bag model with \( B = 85 \text{ MeV/fm}^3 \) and \( B = 70 \text{ MeV/fm}^3 \) respectively (\( m_u = m_d = 0 \) and \( m_s = 150 \text{ MeV} \)), and sequence (B60) with \( B = 60 \text{ MeV/fm}^3 \) and massless quarks; ix) CFL strange stars (label SS(CFL)) (Lugones & Horvati 2003) within the bag model EOS with \( B = 70 \text{ MeV/fm}^3 \), \( m_u = m_d = 0 \) and \( m_s = 150 \text{ MeV} \) and quark pairing gap \( \Delta = 100 \text{ MeV} \).

Looking at the figure we see that, although almost all classes of EOSs are still allowed (apart probably from Hyperon stars), only the stiffest SSs are allowed, and that while soft HSs are excluded by the mass constraint, some of the stiffest ones (i.e. Hyb1 and Hyb2, L) are excluded by the rotational limit. Other EOSs are only marginally compatible with the constraints (BBB2, GM1, B60 and CFL) and would require exceptional conditions (e.g. a mass almost equal to the limiting mass for gravitational collapse) to meet the observational constraints.

3. DISCUSSION AND CONCLUSIONS

The exceptional discovery of a CS spinning with a period below one millisecond allowed us to put a new model-independent constraint to the EOS of matter above nuclear density. Combining this with the constraint given by the highest masses measured in millisecond radio pulsars, we are able to narrow the classes of EOSs that could describe compact objects. As it is clear from figure we are left with two main classes of EOSs that comply with our constraints: very stiff quark matter EOSs and some quite stiff nucleonic matter EOSs. These two classes have completely different characteristics.

SSs that satisfy both constraints have radii smaller than \( \sim 10.5 \text{ km} \) for a mass of 1.337 \( M_\odot \) (the mass of the ultrarelativistic pulsar PSR J0737-303A). Using the formula by Beiger & Haensel (2002), the moment of inertia of a SS with a mass of 1.4 \( M_\odot \) and a radius of 10.5 km is, being \( x = M/R \) (\( M_\odot/\text{km} \))

\[ I = \frac{2}{5}(1 + x)MR^2 = 1.32 \times 10^{45} \text{ g cm}^2. \]  

On the other hand, HSs that comply to both constraints have radii in the range 12 – 13.5 km for a mass of 1.4 \( M_\odot \), and consequently larger moment of inertia: using the formula by Beiger & Haensel (2002) for HSs, assuming \( R \approx 13 \text{ km} \) we obtain

\[ I = \frac{2}{9}(1 + 5x)MR^2 = 1.52 \times 10^{45} \text{ g cm}^2. \]  

There is a 13% discrepancy between the two values, so that the measurement of the moment of inertia of PSR J0737-3039A with the uncertainty of 10% - quoted as reachable by Lattimer & Schutz (2003) - could tell us if this pulsar is a SS or a very stiff HS, although it will not be possible to discriminate between a NS and a HyS, as Klähn et al. (2006b) have shown. We have to note that if the HS is one of the less stiff ones \( R \approx 11 \text{ km} \), its moment of inertia would be virtually indistinguishable from the one of a stiff SS. However, soft NSs (like EOS BBB2 of figure 1) are strongly disfavored by the mass constraint.

If we look at figure we can also notice that the mass-radius sequences of the allowed HSs cross the spin limit line for XTE J1739-285 only at very high masses: as an example, the KS EOS allows only stars exceeding 2 \( M_\odot \) to attain such high spins. This is because the radii of stiff
HSs shrink only when approaching the maximum allowed masses. So, not only the moment of inertia of the HSs is large, meaning that it is necessary to accrete more mass to get a star to periods in the order of one millisecond, but also that only a very massive star would be able to reach a period of 0.891 ms, and only at mass shedding. Moreover, the HS should lie in a very narrow range of masses in order to be stable against gravitational collapse: if the star is supramassive (which can be the case of XTE J1739-285), accretion of further matter can easily lead to the collapse to a black hole [Lavagetto et al. 2004].

On the contrary, since SSs are self-bound objects, their radii expand with growing mass, allowing very fast spins even for lower masses (i.e. EOS B60) can attain this period for $M \geq 1.5M_\odot$. This means that it is much easier to attain the period of XTE J1739-285 in this case.

Paradoxically, if - as the discovery of XTE J1739-285 implies - there is no mechanism avoiding the spin up to millisecond periods (see e.g. Chakrabarty et al. 2003), SSs spinning at sub-millisecond periods should be quite common, as accreting $\sim 0.2M_\odot$ could spin the SS to periods below one millisecond [Zdunik et al. 2002]. Even evolutionary arguments proposed by Burderi et al. (2001), that can explain the non-detection of ultrafast radio pulsars do not apply to the detection of burst oscillations, which is thought to be observationally unbiased (see e.g. Chakrabarty 2005). Thus the observational shortage of submillisecond CSs (only one out of 19 X-ray binaries that show coherent pulsations in their lightcurve) would be somewhat puzzling.

On the other hand if CSs are composed of nucleonic matter, the high stiffness required to the EOS by the high measured masses can alone explain the observational shortage of very fast CSs: a stiff EOS implies that a submillisecond HS should both be very massive and reach the mass-shedding regime, which seem difficult given only a few measured NS masses are larger than 2 $M_\odot$ (see e.g. Lattimer & Prakash 2004). Moreover, as the range of masses that allow a stable submillisecond HS is very narrow (i.e. in the case of EOS KS the allowed mass range is $2 \sim 2.2 M_\odot$), and it is even narrower for other EOSs, see figure [I], this shortens further the chances we have to observe a submillisecond HS, as a supramassive HS can either collapse during accretion, or collapse due to braking if accretion is stopped when the star is in the supramassive regime: in general, a supramassive CS is expected to have a very short lifetime [Lavagetto et al. 2004]. We have therefore a strong selection effect against the detection of very fast spinning objects. Again, if there is a transition to deconfined quark matter at very high densities in NSs, as required by the results of HIC experiments [Klähn et al. 2006, Grigorian et al. 2006], this will not change significantly this scenario.

In conclusion, we showed that when the moment of inertia of PSR J0737-3039A would be measured with an accuracy of $\sim 10\%$, we will likely be able, by combining this measurement with the constraints presented here, to assess the nature of CSs using only model-independent observational constraints. In the meanwhile, if no other object spinning below one millisecond is found, this is probably an argument favoring the hypothesis that CSs are composed of nucleonic matter. Anyhow, this possibility challenges our present understanding of particle physics at supernuclear densities. In fact, one should envisage a plausible physical mechanism to avoid the appearance of non-nucleonic degrees of freedom (hyperons, quark matter, etc.) in the dense interiors of neutron stars.