Spectral appearance of the planetary surface accretion shock: Global spectra and hydrogen line profiles and fluxes

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(Received —; Revised —; Accepted —)

Submitted to ApJ

ABSTRACT

Hydrogen-line emission, thought to come from an accretion shock, has recently been observed at planetary-mass objects, and current and upcoming dedicated instruments should reveal many more sources. Previous work predicted the shock spectrum in the case of a shock on the circumplanetary disc. However, no extensive investigation has been done on the planet-surface shock. Our main goals are to calculate the global spectral energy distribution (SED) of an accreting planet by combining our model emission spectra with photospheric SEDs, and to predict the line-integrated flux for different hydrogen lines, focusing on Hα but including also Hβ, Paα, Paβ, Paγ, Brα, and Brγ. We apply our non-equilibrium emission calculations to the surface accretion shock for the relevant large parameter space of accretion rate M and mass Mp. In contrast to our previous model, fits to formation calculations provide radii and effective temperatures. We do not consider extinction by the preshock material in this work. We find that the Hα line luminosity increases monotonically with M and Mp, ranging from 10⁻⁸ to 10⁻⁴ L⊙, depending mostly on the accretion rate and weakly on the mass for the relevant range of parameters. We apply the result to the observed planets around PDS70 and demonstrate that the estimated accretion rate is consistent with previous studies. The Lyman, Balmer, and Paschen continua can be visible above the photosphere. The Hβ line ranges between 0.1 and ≈ 1 times the Hα flux, whereas other lines are weaker (~ 0.001–0.1). Based on spectroscopic observations, a shock on the planetary surface and on the CPD surface can be distinguished at very high spectral resolution. The planet surface shock however likely dominates in total intensity if both are present. These predictions of the luminosity in Hα and other lines serve as a baseline prediction or conversely as a tool for interpreting observations of accreting planets.

Keywords: Exoplanet formation (492); Accretion (14); Shocks (2086); Hydrogen lines [H α line emission] (690); H alpha photometry (691); Direct imaging (387); High resolution spectroscopy (2096)

1. INTRODUCTION

Recent instrumental improvement enables to observe forming planets (e.g. Kraus & Ireland 2012; Quanz et al. 2013; Currie et al. 2015; Wagner et al. 2018). In particular, detection of Hα is important because it is expected to give us information on how planetary mass grows.

In the context of accreting protostars (e.g., T-Tauri stars: TTSs), Hα is known as a good indicator and used to estimate mass accretion rate (e.g. Gullbring et al. 1998). The Hα from TTSs is brighter than the photospheric continuum by a few mag and has large width (≥ 200 km s⁻¹). The magnetospheric accretion model (Uchida & Shibata 1984; Königl 1991) can explain these characteristic features when the accretion funnel is hot enough to emit Hα (e.g. Hartmann et al. 1994; Muzerolle et al. 2001). Furthermore, since the Hα luminosity (LHα) or spectral width (∆λHα) shows a correlation to the mass-accretion rate (or accretion luminosity, Lacc) estimated with modeling of continuum emission (e.g. Valenti et al. 1993; Calvet & Gullbring 1998), Hα is used to estimate the accretion rate for the protostars that are too far to observe continuum emission (e.g. Gullbring et al. 1998; Herczeg & Hillenbrand 2008; Fang et al. 2009; Rigliaco et al. 2012; Alcalá et al. 2014, 2017; Natta et al. 2004).
As with the stellar cases, the Hα excess was reported from protoplanets, and the observed luminosity was used to estimate the accretion rate by applying the results of the TTS observations (Sallum et al. 2015; Wagner et al. 2018; Haffert et al. 2019). However, there is no guarantee that relationships between $L_{\text{acc}}$ and $L_{\text{H} \alpha}$ or $\Delta L_{\text{H} \alpha}$ given in TTSs are valid for protoplanets. Thanathibodee et al. (2019) applied the stellar Hα emission model of Muzerolle et al. (2001) to a planetary-mass object (PDS 70 b) and argued the $L_{\text{H} \alpha} - L_{\text{acc}}$ relationship shows a different trend from that of protostars (Ingleby et al. 2013; Rigliaco et al. 2012; see also Szulágyi & Ercolano 2020 and Close (2020) and the discussion of their work in Aoyama et al. (submitted)).

Planetary gas accretion is qualitatively different from the stellar one in some points. An important characteristic feature is that protoplanets and their surrounding gaseous disk (circum-planetary disk, CPD) are embedded in the stellar surrounding disk (protoplanetary disk, PPD). On the way of gas accreting towards the protoplanet, the gas preferentially enters the planetary gravitational sphere in high altitudes above the disk midplane (e.g. Tanigawa et al. 2012). When the gas falling from the PPD to CPD vertically hits the CPD surface, it yields a strong shock, which can be hot enough to emit Hα (Szulágyi & Mordasini 2017). Aoyama et al. (2018) constructed a model of shock-heated gas with cooling, chemical reactions, and radiative transfer, estimated hydrogen line luminosity depending on the gas velocity and density, and estimated the $L_{\text{H} \alpha}$ depending on the shock properties.

On the other hand, the magnetospheric accretion may occur even in the planetary accretion, bringing about a strong shock also on the planetary surface. If protoplanets have strong dipole magnetic fields enough to control the gas dynamics, vertical accretion can occur directly onto the planetary surface (Batygin 2018). While the accretion shock on the TTS surface is too strong and makes gas too hot to emit Hα (see e.g., Hartmann et al. 2016), the weak gravity of protoplanets leads to moderate free-fall velocity (~ 100 km s⁻¹) and to emitting a significant amount of Hα. In contrast, in the CPD surface shock model, only a small fraction (≤ 1%) can contribute to the Hα emission, because most gas hits the CPD far from the planet (Aoyama et al. 2018). Also, in the magnetospheric accretion-funnel model, while the heating mechanism is still an open question (Muzerolle et al. 2001), the funnel gas can be too cool to emit Hα because of the low mass of protoplanets.

Therefore, in this study, we model the hydrogen line emission coming from the planetary surface shock, considering a wide range of parameters. We focus on Hα first and then explore other hydrogen line emission. We combine these results with models of the photospheric emission and discuss when the shock lines are visible above the photosphere emission. Note that part of the planetary surface shock model presented here was used already in Aoyama & Ikoma (2019) for the case of PDS 70 b and c. A more extensive investigation is done in this study.

The paper is organized as follows: In Section 2 we discuss the planetary surface shock properties for the whole relevant parameter space and review our numerical model, which was introduced in Aoyama et al. (2018). In Section 3 we present emission spectra of accreting gas giants for a large grid of models, before applying this in Section 4 to a few objects. In Section 5 we explore other observational aspects, including line strengths for other lines. Finally, we present a critical discussion in Section 6 before summarizing in Section 7. The appendices present further material: a discussion of our approach compared to Storey & Hummer (1995) (Appendix A), the inverse relationship between the shock-microphysical and planet-formation parameters (Appendix B), a map of $L_{\text{H} \alpha}(M, M_\text{p})$ for the cold-start radius fits (Appendix C), and the calculation of the Hα luminosity in Wagner et al. (2018) (Appendix D).

2. OVERVIEW OF THE SHOCK PROPERTIES AND SHOCK MODEL

2.1. Components of the spectral energy distribution

We model the spectral energy distribution (SED) of an accreting gas giant with a surface accretion shock. The radiation from the accreting gas giant is mainly composed of two parts, namely the photospheric radiation and the shock excess. As the photospheric radiation model, we use the CIFIST2011_2015 BT-Settl models, which calculated spherical radiative transfer in atmospheres with solar metallicity¹ (Allard et al. 2012; Baraffe et al. 2015). The shock excess is calculated from the 1D radiation-hydrodynamic model developed by Aoyama et al. (2018) and Aoyama & Ikoma (2019) with incorporation of results from the Bern planet formation model (e.g., Alibert et al. 2005; Mordasini et al. 2015). We outline these models in the following sections. Note that we contrast our model of the shock emission to Storey & Hummer (1995) in Appendix A.

In this study, we focus on the shock-heated gas on the planetary surface. We treat only the emission from the photosphere and shock-heated gas but not from the CPD, whose temperature is lower than those of the photosphere and the shock. Continuum emission from a simplified CPD model has been calculated in Zhu (2015), Eisner (2015), and Szulágyi et al. (2019), and the line emission from the shock on the CPD has been calculated in Aoyama et al. (2018).

In this work, extinction by material between the shock surface and the observer is not considered. To what extent the gas or dust surrounding a forming planet may weaken the shock signal is a relevant and important question, as the recent obser-

¹ From https://phoenix.ens-lyon.fr/Grids/BT-Settl/CIFIST2011_2015/.
vational results and theoretical modeling in Hashimoto et al. (2020) and Sanchis et al. (2020) highlight. Given that it adds an entire level of complexity and brings many uncertainties with itself, however, we deal with extinction by the gas and the dust in a dedicated paper (Marleau et al. submitted).

Locally, a strong shock converts most of the mechanical energy into thermal energy, and the gas temperature increases by orders of magnitude compared to the preshock value. However, since the temperature much exceeds the radiative equilibrium temperature at that point, the shock-heated gas in the postshock region cools rapidly (compared to the postshock flow time) by emitting radiation. We define the shock-heated gas as the gas hotter than the photosphere; under the photosphere, the gas becomes optically thick and radiative cooling stops.

2.2. Fitting of planetary properties

The input parameters for our combined spectra of the accretion shock and the photosphere are mass accretion rate $\dot{M}$, planet mass $M_p$, planet radius $R_p$, filling factor $f_{\text{full}}$ of the shock on the planet surface, and photospheric effective temperature $T_{\text{eff}}$. However, taking them as independent would result in an impractically large parameter space and may lead to unlikely combinations (e.g., small radius and mass but large luminosity). Therefore, only the accretion rate, the planet mass, and the filling factor will be considered as free parameters.

2.2.1. Radius fit

The radius is obtained by a fit to results from existing formation calculations. For definiteness, we use the results from a specific planet formation model, the Bern model (Alibert et al. 2005; Mordasini et al. 2012b,a, 2017; Emsenhuber et al. 2020a,b), with the fits $R_p(M, M_p)$. However, it is clear that this is not meant as a final answer and one could repeat this study with e.g. the $R_p(M, M_p)$ relations of Ginzburg & Chiang (2019), which find (much) larger radii at a given mass.

In principle, the main parameter space for our calculations is $(M, M_p, R_p, f_{\text{full}})$ along with the choice of “cold-start” or “hot-start” accretion (high or low radiation efficiency of the accretion energy; Marleau et al. 2017, 2019). Here, we take for the sake of definiteness the $R_p(M, M_p)$ relations in the cold- and hot-start populations of Mordasini et al. (2012b), using data from all time snapshots. We use here the populations CD752 (hot) and CD753 (cold), described and analysed in Mordasini et al. (2012b,a, 2017) (Generation Ib). Recently, the Generation III population syntheses of the Bern model was released (Emsenhuber et al. 2020a,b; Schlecker et al. 2020), which all assume warm accretion. We verified that the distribution of points in $(\dot{M}, M_p, R_p)$ space is very similar between the 1- and the 100-embryo-per-disk simulations NG73 and NG76, respectively, on the one hand, and CD752 on the other. Two small differences are that the accretion rates reached are not quite as high as in Generation Ib (note however that there are fewer synthetic planets in the region of interest), and that in NG76 the radii can be higher at a given $\dot{M}$ and $M_p$, likely due to interactions between the embryos. Since the radii are overall similar, we will keep using the Generation Ib populations in order to cover also high accretion rates, as could be relevant for instance to outbursts.

These planet structure models were calculated assuming that the planet is at all times convective. Recent work suggests that forming planets may be in fact in part radiative (Berardo et al. 2017; Berardo & Cumming 2017; Cumming et al. 2018) and thus have a different radius. Nevertheless, the relations $R_p(M, M_p)$ from the population synthesis provides a reasonable bracket and reduce the dimensionality of the large parameter space $(\dot{M}, M_p, R_p, \eta, f_{\text{full}}, \ldots)$.

After some experimentation, we arrived at the following relatively simple form for the fitting function:

$$R_p(\dot{M}, M_p) = a_0 + b_0 \log M_2 + c_0 e^{d_0 \log M_2} \quad (M_p - 1)$$

$$+ (a_1 + b_1 \log M_2 + c_1 e^{d_1 \log M_2}) (M_p - 1)^2,$$

where, here only, $R_p$ is in $R_1$ and $M_p$ in $M_1$, and we defined for brevity of notation $\log M_2 \equiv \log_{10}(M/10^{-2} \text{ M}_\oplus \text{yr}^{-1})$. The fits were performed through gnuplot’s built-in fit routine. Only planets with $\dot{M} > 10^{-5} \text{ M}_\oplus \text{yr}^{-1}$, $1 M_1 < M_p < 20 M_1$, and $R_p < 4 R_1$ were used to obtain the fits. We used for each planet a statistical weight inversely proportional to its radius to have a more accurate fit at lower radii, for which $v_0$ is higher and thus the accretion signatures a priori stronger.

The coefficients for the cold-nominal population are

$$a_0 = 1.53; \quad b_0 = 0.11;$$

$$c_0 = 1.06; \quad d_0 = 0.906;$$

$$a_1 = -0.195; \quad b_1 = -0.0307;$$

$$c_1 = 0.0977; \quad d_1 = 0.000695;$$

$$a_2 = -0.250; \quad b_2 = 0.000276;$$

$$c_2 = 0.254; \quad d_2 = 0.000214$$

and for the warm population the coefficients are

$$a_0 = 0.411; \quad b_0 = -0.244;$$

$$c_0 = 3.45; \quad d_0 = 0.762;$$

$$a_1 = -0.489; \quad b_1 = -0.0961;$$

3 The data can be visualized at, and downloaded from, the “Evolution” section of the Data Analysis Centre for Exoplanets (DACE) platform under https://dace.unige.ch.
c1 = 0.652; d1 = 0.353;
a2 = −0.228; b2 = −0.00106;
c2 = 0.226; d2 = 0.000220.

We verified that excluding from the fitting procedure the planets for which the accretion radius \( R_{\text{acc}} < 10 R_p \) (this concerns only a small fraction of the planets) changed neither the relationships nor the quality of the fit significantly. The accretion radius \( R_{\text{acc}} \) is a spherically-averaged estimate of the typical distance from which the gas is effectively falling onto the planet. It is defined through

\[
\frac{1}{R_{\text{acc}}} = \frac{1}{R_{\text{Bondi}}} + \frac{1}{k_{\text{Liss}} R_{\text{Hill}}},
\]

where \( R_{\text{Bondi}} \) and \( R_{\text{Hill}} \) are the Bondi and Hill radii and \( k_{\text{Liss}} = 1/4 \) (and not \( k_{\text{Liss}} = 1/3 \) as in Mordasini et al. (2012b)).

The resulting relations \( R_p(M, M_p) \) for the cold-nominal and the warm populations are shown in Figure 1. The fit is overall excellent, with a match to roughly 10%. For the cold-nominal population, which displays the largest deviations: Only for masses \( M_p \gtrsim 15 M_J \) near \( M = 10^{-3} - 10^{-2} M_\odot \) yr\(^{-1} \) is the function too small, by at most only \( \approx 30\% \), and at \( M \gtrsim 3 \times 10^{-2} M_\odot \) yr\(^{-1} \) for \( M_p \) between 1 and 10 \( M_J \) larger or smaller by at most \( \approx 30\% \). This reflects in part the intrinsic scatter in the population synthesis results. For the warm population, the fitted function yields radii also at most 30% too small but only towards high masses and low accretion rates. Overall the deviation is less than 10%. At lower accretion rates and, in the cold-nominal population, for lower masses than shown, the fit re-increases but this matches rather well overall the data (not shown).

### 2.2.2. Effective temperature

For the photospheric temperature \( T_{\text{eff}} \), we adopt a semi-analytical prescription that ensures that, approximately, the total outgoing flux (shock and photosphere) is equal to the sum of the internal and incoming energy flux.

At the shock, a portion of the incoming energy is converted into hydrogen-line and recombination-continuum emission. The remaining portion travels downward into the atmosphere, where it is expected to be thermalised because most of the energy is in Ly \( \alpha \), which can easily be thermalised. What matters for the structure of the planet is whether this radiation goes deep into the planet, thereby heating it up, or whether only the top layers are heated up and re-emit the radiation, on a timescale that is short compared to the cooling time of the planet. The former outcome corresponds to the “Hot start (accreting)” case of Mordasini et al. (2012b), their Equation (13)), and the latter to their “Cold start” case.

In both cases, the total luminosity just outside the planet\(^4 \) is \( L \approx L_{\text{int}} + L_{\text{acc}} \), where \( L_{\text{int}} \) is the energy coming from the deep interior. However, the spectra differ, as noted by Mordasini et al. (2012b). In the “Hot start” extreme case the spectrum is entirely thermalised, as given by an atmospheric model with \( T_{\text{eff}}^4 = T_{\text{int}}^4 + T_{\text{acc}}^4 \), with

\[
T_{\text{acc}} = \left( \frac{L_{\text{acc}}}{4 \pi R_{\text{acc}}^2 c^2} \right)^{1/4},
\]

and there would be no emission-line spectrum as we have been computing in this work. In the other extreme, in the “Cold start” case the spectrum is given by the sum of an atmosphere at \( T_{\text{eff}} = T_{\text{int}} \) (i.e., not heated up at all by the shock) and the shock spectrum. Clearly both are extreme cases.

Which scenario is likely more accurate? The single-stream, frequency-averaged simulations of Marleau et al. (2017, 2019) suggest that on the net, only a small fraction of the incoming \( L_{\text{acc}} \) will go in deeper, but that this small portion is likely (much) higher than the internal luminosity in the extreme cold starts by Marley et al. (2007). However, for the frequency-dependent calculations presented here, we mentioned that roughly one half of the radiation goes down and one half goes up (Aoyama et al. 2018), which holds in the limit that the emitting region is optically thin. We now quantify this fraction in Figure 2 more precisely, which we will use to derive \( T_{\text{eff}} \).

Figure 2 shows the fraction \( f_{\text{down}} \) of the incoming kinetic energy flux that is present in the downward-moving radiation field at the bottom of the computational domain, where \( T \) reaches \( 10^4 \) K. For this, we have first written

\[
\begin{align*}
    f'_{\text{down}} &= a_0 + b_0 \log n_{12} + c_0 n_{12}^2 \\
    &+ \left( a_1 + b_1 n_{12} + c_1 n_{12}^2 \right) \left( v_0 - 100 \text{ km s}^{-1} \right) \\
    &+ a_2 \left( v_0 - 100 \text{ km s}^{-1} \right)^2,
\end{align*}
\]

\[
f_{\text{down}} = \min \left( \max \left( f'_{\text{down}}, 0 \right), 1 \right),
\]

where \( \log n_{12} \equiv \log_{10} \left( n_0 / 10^{12} \text{ cm}^{-3} \right) \). The second line of Equation (4) ensures that \( f_{\text{down}} \) remains between 0 and 1 and is needed only for a small part of the parameter space, for some models on the grid edge. Using gnuplot’s built-in fit command yielded

\[
\begin{align*}
    a_0 &= 0.703752; \quad a_1 = -0.00527886; \\
    b_0 &= -0.0967987; \quad b_1 = -0.00146833; \\
    c_0 &= -0.0254579; \quad c_1 = -0.000321504; \\
    a_2 &= -9.91492e-06,
\end{align*}
\]

\(^4\)This does not consider the energy recycling in the accretion flow discussed by Marleau et al. (2017, 2019), and ignores any possible absorption of the radiation in the first layers outside the shock, i.e., taking \( \eta_{\text{in}} = 1 \) for this discussion.
which matches very well the model data (not shown). We had included in the first version of the fit log_{12} terms in the \( v_0^2 \) term but their coefficients were consistent with zero. The result of this fit depends only on our grid of models in \((n_0, v_0)\) space and is thus independent of the population. Then, we used the radius fits to obtain \( f_{\text{down}}(M, M_p) \).

We find that the fraction \( f_{\text{down}} \) does vary at low masses but that it covers mainly \( f_{\text{down}} \approx 0.3–0.8 \) between the border of the brown-dwarf region and a few \( M_1 \). The fraction depends only relatively weakly on \( M \) and much more on \( M_p \) (through the \( v_0 \) dependence), reaching \( f_{\text{down}} \approx 1 \) at \( M_p \approx M_1 \). Note that \( f_{\text{down}} \) is affected to some extent by the non-inclusion of low-temperature \((T \lesssim 10^4 \text{ K})\) cooling processes at the bottom of the computation domain, for instance from molecules.

Currently they are not included, so that in a future model iteration \( f_{\text{down}} \) could be different, once the inclusion of helium and metals will accelerate the cooling and thus make it computationally feasible to let the simulations cool down to lower temperatures than \( T = 10^4 \text{ K} \). Nevertheless, Figure 2 already provides some guidance.

Having computed \( f_{\text{down}} \), we use it to write the photospheric temperature as

\[
T_{\text{eff}}^4 = T_{\text{int}}^4 + f_{\text{down}} T_{\text{acc}}^4, \tag{5}
\]

i.e., we assume that the component \( F_{\text{down}} = f_{\text{down}} T_{\text{acc}}^4 \) from the shock (lines and recombination continua) is thermalised and re-emitted. Equation (5) ensures that the total upward-travelling radiative flux from our combined models is \( F = \)
the true photosphere to a larger distance from the planet, leading to a lower $T_{\text{eff}}$ and thus a different contrast to the hydrogen lines. However, they too can be affected by the preshock material, depending on the system parameters and dust properties (Hashimoto et al. 2020; Marleau et al. submitted), so that the final outcome is not clear. Moreover, these classical atmospheres are computed assuming hydrostatic equilibrium, whereas the accretion flow has a very different $P$–$T$ structure, set by free-fall with approximately constant luminosity (Marleau et al. 2017, 2019). It would be interesting to explore, with dedicated radiative transfer calculations, how this modifies the spectral shape.

Note that spectra for the BT-Settl models we use (CIFIST2011_2015) exist only for $T_{\text{eff}} \geq 1200$ K. Therefore, for the case $T_{\text{eff}} < 1200$ K, we use 1200 K.

### 2.3. Shock model

#### 2.3.1. Shock parameter space

Given the thinness of the radiation-emitting shock layer compared to the planetary radius, we assume the shock on the planetary surface to be one-dimensional as in Aoyama et al. (2018). Then, the shock structure is mainly determined by two input parameters, namely the hydrogen proton number density $n_0$ and the velocity $v_0$ before the shock. The preshock temperature of the gas $T_0$ hardly affects the shock properties, as we discuss below.

As in Aoyama et al. (2018) and e.g. Shapiro & Kang (1987), $n_0$ is defined as the preshock number density of hydrogen protons (i.e., contained in H$_2$, H, and H$^+$ taken together). Thus, contrary to the definition, common in the stellar-structure literature (e.g. Hansen et al. 2004), of $n$ as the number density of all particles, our $n_0$ is independent of the dissociation and ionization degrees of hydrogen. This definition implies that $n_0$ is related to the (total) preshock gas mass density $\rho_0$ by

$$X \rho_0 = n_0 m_H,$$

where $X$ is the hydrogen mass fraction and $m_H$ is the mass of a hydrogen atom. We use number ratios given by $\text{H} : \text{He} : \text{C} : \text{O} = 1 : 10^{-1.07} : 10^{-3.48} : 10^{-3.18}$ (Cox 2000). As in Aoyama et al. (2018) we do not consider the other elements, which are negligible. Thus the hydrogen, helium, and metal mass fractions are respectively $X = 0.738$, $Y = 0.251$, and $Z = 1 - X - Y = 0.011$.

In the chemistry module, the abundances of the species are defined by $y_i$ (see also Iida et al. 2001). The quantity $y_i$ is the relative abundance (in number) of species $i$ (of any particle) with respect to the number of hydrogen protons. For example, pure H$_2$ has $y_{\text{H}_2} = 0.5$. Defining the total $y_1 = \sum y_i$, the usual total number density of all particles is given by $n = y_1 n_0$; the number density of particles of species $i$ is $n_i = y_i n_0$. With the mean weight per particle given by $\mu_0 = \sum y_i m_i / y_i$.
for particle masses $m_i m_H$, we have that $\rho_0 = \chi_i \mu_0 m_H n_0$, implying $X = 1/(\chi_i \mu_0)$.

The shock input parameters $(n_0, v_0)$ are related to the macrophysical, planet formation parameters $(\dot{M}, M_p, R_p, f_{\text{fill}})$ by

$$v_0 = \sqrt{\frac{2GM_p}{R_p}}$$  \quad (7)

$$n_0 = \frac{X M}{4\pi R_p^2 f_{\text{fill}} m_H v_0}$$  \quad (8)

$$= \frac{X M}{\sqrt{32G}\mu_0 m_H f_{\text{fill}} \sqrt{M_p R_p^3}},$$  \quad (9)

where $G$ is the gravitational constant, $M_p$ is the planet mass, $R_p$ is the planet radius, $\dot{M}$ is the accretion rate, and $f_{\text{fill}}$ is the filling factor of the shock on the planet surface. Equations (7)–(9) are valid in the limit that the accretion radius $R_{\text{acc}} \gg R_p$, with the gas free-falling from $R_{\text{acc}} \sim R_{\text{Hill}}$ (Equation (2); Bodenheimer et al. 2000), where $R_{\text{Hill}}$ is the Hill radius. Note that our model is applicable when the gas falls from the inner edge of the CPD, but, in such a case, the $v_0$ and the estimated $L_{H\alpha}$ is smaller by a factor of a few than the results in this paper. Inserting typical values for stars and planets, the typical preshock number density is larger in the planetary than in the stellar case by about a factor of 100 (see Equation (A1)) but one should keep in mind that the parameter space is large.

The assumption here is that all of the accreting gas is available for a shock, whether this turns out to produce H\,\alpha or not. In reality, some fraction of the accreting gas could be added to the planet through boundary layer accretion (BLA; e.g., Kenyon & Hartmann 1987; Kley 1989), which does not feature supersonic radial velocities. In this scenario, the temperature in the boundary layer would not be high enough for H\,\alpha to be emitted. Thus converting an observed H\,\alpha luminosity to planetary parameters such as accretion rate and mass needs to assume something about the fraction of the incoming gas that is able to produce H\,\alpha. Put differently, a measured H\,\alpha luminosity yields an estimate of the H\,\alpha-emitting accretion rate, while the total accretion rate could be higher. However, it is not clear how likely BLA is in the planetary case; Owen & Menou (2016) argue for BLA but this is based, through the Christensen et al. (2009) scaling, on magnetic field strengths appropriate of old and faint planets ($B \approx 0.03–0.06$ kG), not forming or young, high-luminosity (Mordasini et al. 2017) objects ($B \approx 1$ kG; Katarzyński et al. 2016). Thus, assuming that all the gas can undergo a shock seems plausible, but high-resolution studies are required to help settle the question.

In Figure 4 we situate the planet accretion shock in the input parameter space of preshock velocity $v_0$ and hydrogen proton density $n_0$. We use the Bern population synthesis values to give some context. Typical values are $v_0 = 30–200$ km s$^{-1}$ and $n_0 = 10^{11–10^{14}}$ cm$^{-3}$. The latter corresponds to $\rho_0 \sim 10^{-13}–10^{-10}$ g cm$^{-3}$. As a comparison, in the Alcalá et al. (2017) sample of young stellar objects of mass $M \approx 10–180 M_\odot$ and radius $R \approx 2–30 R_\odot$, the preshock velocity is $v_0 \approx 150–500$ km s$^{-1}$. There is however no overlap between

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**Figure 3.** Approximate effective temperature of accreting planets (Equations (4) and (5), using $T_{\text{int}} = 1000$ K; black contours). This is compared to contours of constant $T_{\text{acc}}$. We fix $f_{\text{fill}} = 1$. This is based, through the Christensen et al. (2009) scaling, on magnetic field strengths appropriate of old and faint planets ($B \approx 0.03–0.06$ kG), not forming or young, high-luminosity (Mordasini et al. 2017) objects ($B \approx 1$ kG; Katarzyński et al. 2016). Thus, assuming that all the gas can undergo a shock seems plausible, but high-resolution studies are required to help settle the question.
the region occupied by planets and the Alcalá et al. (2017) sample, with their closest point near (10 R\textsubscript{J}, 250 km s\textsuperscript{-1}).

Finally, for reference, the preshock state of the hydrogen is indicated in Figure 4b: for f\textsubscript{fill} = 1, the hydrogen is molecular at low densities or velocities and atomic above this. In a few high-velocity cases, it arrives at the shock significantly ionized. Overall, according to the (M, M\textsubscript{p}, R\textsubscript{p}) combinations found in the Bern population synthesis, for large filling factors f\textsubscript{fill} \approx 1 the gas reaches the planet in an atomic form in the majority of cases.

Figure 5 displays contours of n\textsubscript{0} and v\textsubscript{0} in the M–M\textsubscript{p} plane using the R\textsubscript{p}(M, M\textsubscript{p}) fit (Equation (1)) for the warm and the cold population. (The inverse relations are shown in Figure 14.) We use f\textsubscript{fill} = 1. The v\textsubscript{0} contours depend on M because of the dependence of the radius on M. This leads even to a non-monotonic behaviour of v\textsubscript{0} with M, with, in the warm (cold) population, a maximum around M \approx 3 \times 10\textsuperscript{-5} M\textsubscript{E} yr\textsuperscript{-1} (M \approx 10\textsuperscript{-3} M\textsubscript{E} yr\textsuperscript{-1}). The maximum preshock velocity is for both populations roughly v\textsubscript{0} \approx 100 km s\textsuperscript{-1} (v\textsubscript{0} = 180 km s\textsuperscript{-1}) for M\textsubscript{p} \approx 5 (M\textsubscript{p} \approx 15–20 M\textsubscript{J}). Since the radii are smaller in the cold population, the velocities are slightly higher at a given mass but only by some tens of kilometers per second. The curving of the v\textsubscript{0} contours at high v\textsubscript{0} (see Figure 5) means that a measurement of the preshock velocity v\textsubscript{0}, for instance through the Doppler broadening of the emission lines, can be explained only by a limited range of accretion rates, assuming that a rough upper limit on the mass exists (e.g. from imaging or dynamical arguments).

2.3.2. Shock conditions

We assume that at the shock, the preheated gas (Marleau et al. 2019) undergoes a hydrodynamical shock before cooling down radiatively. This corresponds effectively to the Zel’dovich spike (see Vaytet et al. 2013), and our actual computations begin directly after the hydrodynamical shock, i.e., roughly at the tip of the Zel’dovich spike. To obtain this immediate postshock state, we use the classical Rankine–Hugoniot shock jump conditions, which reflect mass, momentum, and energy conservation. This is valid because the time needed for the gas to cross the hydrodynamical shock thickness, of the order of a particle mean free path, is much less than the cooling timescale of the gas. In Marleau et al. (2017, 2019), “the shock” referred to both the hydrodynamic jump and the postshock cooling region; here we refer by “shock” only to the hydrodynamic jump, with the postshock cooling region.
the focus of this work. The Rankine–Hugoniot relations read:

\[
\rho_1 = \frac{(y + 1) M^2}{(y - 1) M^2 + 2 \rho_0} \quad (10)
\]

\[
v_1 = \frac{\rho_0 v_0}{\rho_1} \quad (11)
\]

\[
P_1 = \frac{2 y M^2 - (y - 1)}{y + 1} P_0 \quad (12)
\]

\[
= \frac{2}{y + 1} P_{\text{ram}} - \frac{y - 1}{y + 1} P_0, \quad (13)
\]

with the subscript “0” denoting the preshock and “1” the postshock state. The ram pressure is given by \( P_{\text{ram}} \equiv \rho_0 v_0^2 \). The upstream Mach number is \( M = v_0 / c_s \), with \( c_s = \sqrt{\Gamma_1 k_B T_0 / (\mu_0 m_H)} \), where \( \Gamma_1 = (\partial \ln P / \partial \ln \rho)_s \) is the first adiabatic index and \( s \) the entropy. Across the hydrodynamic jump (but not below, in the main part of our computations), we assume that the abundances and thus \( \mu \) and \( \Gamma_1 \) remain constant. This is justified if the gas undergoes the jump (a few mean free paths thick) on a timescale shorter than the chemical reaction time. For simplicity, we take \( \Gamma_1 = \gamma \), where \( \gamma \) is the ratio of specific heats, and we specify its value below (see Equation (20)).

The high-Mach number limits for \( \rho_1 \) and \( P_1 \) are

\[
\rho_1 = \frac{\gamma + 1}{\gamma - 1} \rho_0 \quad (14)
\]

\[
P_1 = \frac{2 y M^2}{y + 1} P_0 \quad (15)
\]

\[
= \frac{2}{y + 1} P_{\text{ram}}, \quad (16)
\]

which implies, still in the limit \( M \gg 1 \),

\[
T_1 = \frac{\mu_0 m_H}{k_B} \frac{2(y - 1)}{(y + 1)^2} v_0^2 \quad (17)
\]

\[
\approx 4 \times 10^3 \left( \frac{M_p}{10 \, M_1} \right) \left( \frac{2 \, R_b}{R_p} \right) \, \text{K}, \quad (18)
\]

taking \( \mu = 1.23 \) and \( \gamma = 1.43 \) (appropriate for the incoming gas) for the second line. Expressions for the case of different \( \gamma \) and \( \mu \) values left and right of the shock can be found in Equation (4.17ff) of Drake (2006). This is the same physics as for stars (see Equation (4) of Hartmann et al. 2016). Note that \( T_1 \) is a non-equilibrium temperature, which holds over only a very small temporal and spatial scale relative to any other relevant scale. (In the example in Figure 6, this is \( \sim 10^{-2} \) s and \( \sim 10^{-6} \, R_J \).) It is thus in no way an effective temperature \( T_{\text{eff}} \) nor an equilibrium gas temperature. This non-LTE effect usually cannot be resolved in full radiation-hydrodynamical simulations because of the vast differences in scales.
The preshock temperature $T_0$ is needed to set the preshock pressure $P_0$ and the preshock Mach number $M$, as well as the chemical abundances before and therefore also directly below the shock. Using grey radiation transfer, Marleau et al. (2017, 2019) found that the radiation and the gas ahead of the shock were able to equilibrate. This is due to the sufficiently high Planck opacity of the gas or, for lower shock temperatures, of the dust. Therefore, we calculate the preshock temperature from $(\rho_0, v_0)$ by

$$T_0 = \sqrt{\frac{\rho_0 v_0^2}{2\sigma}}, \quad (19)$$

where $\sigma$ is the Stefan–Boltzmann constant. Because it has $\sigma$ and not $ac = 4\sigma$ on the denominator, where $a$ is the radiation constant and $c$ the speed of light, this expression is higher by a factor of $4^{1/4} \approx 1.4$ than the equilibrium shock temperature obtained analytically and numerically by Marleau et al. (2019, see their Equation (6))\(^5\). It also ignores the negligible contribution of the internal luminosity in setting the planet’s surface temperature; see e.g. Equation (32) of (Marleau et al. 2019). However, since the shock is strong (i.e., the Mach number is large; Marleau et al. 2019), both $M$ and $P_0$ barely affect the postshock quantities, as Equations (14) and (16) show. Thus it is inconsequential that Aoyama et al. (2018) assumed a constant $T_0 = 200$ K. Also, the initial postshock composition does not affect the radiative fluxes by more than $\sim 1\%$, as we have verified (not shown) by varying $T_0$ even by a factor of ten.

Finally, the adiabatic index for the mixture is given by

$$\frac{1}{\gamma - 1} = \sum \frac{y_i/y_1}{\gamma_i - 1}, \quad (20)$$

with $\gamma = 5/3$ for H, He, C, and O, and $\gamma = 7/5$ for H$_2$. It enters into the Rankine–Hugoniot equations and in the time-dependent energy equation through $P = (\gamma - 1)E$, where $P$ is the pressure and $E$ the internal energy (see Equation (7) of Aoyama et al. (2018)).

2.3.3. Postshock flow

We assume a time-independent plane-parallel one-dimensional flow after the shock. Then, the gas flows with conserved mass flux and momentum flux, implying

$$\rho v = \rho_1 v_1 \quad (21)$$

$$\rho v^2 + P = \rho_1 v_1^2 + P_1. \quad (22)$$

Chemical reactions including electron level transitions are the external energy source. Therefore, the internal-energy volume density of the gas $E$ is not conserved but evolves according to

$$\frac{dE}{dt} = (\Gamma - \Lambda) + \left[ \frac{P + E}{\rho} \frac{dp}{dt} \right], \quad (23)$$

where $\Gamma$ and $\Lambda$ are the heating and cooling rates per unit volume, respectively. Note that, in a 1-D flow, temporal differentiation is easily converted into spatial differentiation with flow velocity $v$.

In this study, the coolants are the dissociation of molecular hydrogen, the collisional excitation and ionization of atomic hydrogen, and the emission of radiation by CO, OH, and H$_2$O. The heat sources are the formation of molecular hydrogen as well as the collisional de-excitation and collisional recombination of atomic hydrogen. For detailed expressions, see Aoyama et al. (2018).

2.3.4. Radiative transfer

We consider electron level transitions between ten levels of neutral hydrogen and the ionized state. We numerically calculate the radiative transfer of 45 lines and ten recombination continua with de-exciting transitions. To integrate the flux, we use the two-stream approximation, assuming a plane-parallel 1-D flow. We iterated the hydrodynamic simulation and the radiative transfer until the Hα flux converges. Note that since the Ly α still changes when the other lines converge and the iteration stops, the Ly α intensity is less reliable, in this model. Detailed expression and equations are given in Aoyama et al. (2018).

Finally, given the assumed geometry described in Section 2.3.1, the luminosity of hydrogen lines and recombination continua emitted from the shock-heated gas is given by

$$L = 4\pi R_p^2 f_{\text{fill}} \mathcal{F}, \quad (24)$$

where $\mathcal{F}$ is the photon energy flux at the shock, which is the result of the radiative transfer in the postshock gas flow.

3. THEORETICAL SPECTRA OF FORMING GAS GIANTS

We now turn to results from the methods described above. We look first at one representative example in detail (Section 3.1; Aoyama et al. 2018 showed three other cases) and then survey a large part of the relevant $(\dot{M}, M_p)$ parameter space (Section 3.2).

3.1. One example

3.1.1. Postshock structure

The postshock structure and hydrogen line emission were detailed by Aoyama et al. (2018). Although the results shown here are basically the same as theirs, we review their findings in this subsection for the reader’s convenience. Also, the

\(^5\)We noticed this difference only at a later stage of this work. Since it barely changes the results, a correction of this factor will be deferred to the next iteration of our models.
Figure 6. Postshock flow structure, beginning immediately after the hydrodynamical jump, for $v_0 = 100 \text{ km s}^{-1}$ and $n_0 = 10^{11} \text{ cm}^{-3}$. This corresponds e.g. to $M = 10^{-8} \text{ M}_J \text{ yr}^{-1}$, $M_p = 5 \text{ M}_J$, and $R_p = 1.7 \text{ R}_J$ with $J_{\text{full}} = 1$. The left (right) axes are the depth below (time elapsed after) the shock surface $\Delta z/\Delta t$. (a): Temperature $T$ (red line, bottom axis) and pressure $P$ (blue line, top linear axis). The preshock $T_0$ (not shown) is $T_0 = 1190 \text{ K}$. The $P$ profile changes inversely to $T$ because the density increases faster than $T$ drops. (b): Cooling rates of H collisional excitation and ionization (red line), dissociation of H$_2$ (black), and OH (brown) and H$_2$O (blue) rotational line emission. Throughout the simulation, molecules hardly affect the cooling because they are minor relative to neutral hydrogen. (c): Number density relative to $n_0$ (see Section 2.3.1) for H$_2$ (purple), H$^{n=1}$ (black), H$^{n=2}$ (red), H$^{n=3}$ (blue), H$^+$ (orange), and e$^-$ (green), with $n$ the principal quantum number. Thin black lines show higher excited states ($n \geq 4$). At intermediate depths, the H ionization fraction approaches unity. (d): Upward energy flux of Ly $\alpha$ (black line), Bal $\alpha$ (H $\alpha$; red), Pa $\alpha$ (blue), Pa $\beta$ (green), Br $\gamma$ (brown), all recombination continua (thin purple), and other lines (thin gray). At intermediate depths where the ionization fraction $\approx 1$, the line fluxes hardly change due to the optical thinness of the lines. Note that the deep region where hydrogen lines steeply increase is well resolved in the simulation, though the details are not readable due to the log scale in these figures.
that the pressure gets slow once around the depth of $2\Delta z$ because of the density enhancement of compression. Notice temperature drops. Although the gas cools, the gas pressure (panel (a)). The gas density (not shown) increases as the $n$ nearly reached unity. Excitation from the ground state to the $\Delta$ state of atomic hydrogen $H$ most abundant, orders of magnitude more than the ground postshock region, the molecular form $H_2$ goes inward also as Ly $\alpha$. In this example, at the shock surface, the upward-travelling Lyman-$\alpha$ flux represents around 76% of the incoming (mostly kinetic) energy, and H $\alpha$ carries only around 1%. The other part of the energy influx travels downward, towards the photosphere (see also Section 2.2.2 for more precise fractions).

In our models, we currently do not include cooling from He or metal lines but this ultimately does not matter. In most regions, hydrogen lines are almost the only coolant, so that when the abundance of neutral hydrogen becomes low enough, cooling by hydrogen becomes inefficient. In Figure 6, this is between $\Delta z \approx 8 \times 10^3$ and $5 \times 10^3$ cm. This leads to a plateau in the temperature, which ends where the hydrogen recombines. In that temperature region (at $T \sim 10^5$ K), cooling by lines of ionic C, O, and He (specifically, the H $\alpha$ line) or other metals lines would be more important (see Figure 3 in Gnat & Ferland 2012) so that there would not be a temperature plateau. Indeed, while the ionisation of C and O is included in the chemistry subroutine, it is not included in the radiation transfer, and neither is the cooling by lines of C and O in the energy equation. For helium, the ionisation is included in the energy equation but, also here, the lines are not. Also, note that for the case presented in Figure 2 of Aoyama et al. (2018), with $(v_0 = 40 \text{ km s}^{-1}, n_0 = 10^{11} \text{ cm}^{-3})$, in the early parts of the flow the electron abundance is higher than the $H_2^+$ abundance. These electrons are coming from ionized helium.

However, the gas in the region of the temperature plateau only contributes to the recombination continuum but not to the hydrogen lines. Therefore, even including helium or metal lines (and thus changing the temperature structure of that region) would not modify the strength of the hydrogen lines.

3.1.2. Radiative properties

Figure 7 shows the entire corresponding SED, including the contribution from the photosphere. For optical or longer wavelengths ($\lambda \gtrsim 4000$ Å), the radiation is dominated by the photospheric contribution (black line) except for some hydrogen lines (red peaks). On the other hand, at shorter wavelength, the dominant component is Ly $\alpha$, stronger by tens of orders of magnitude than the thermal photospheric emission. The other Lyman and Balmer line series also clearly exceed
**Figure 7.** Surface spectral energy distribution of an accreting gas giant, with effective temperature $T_{\text{eff}} = 1800$ K and surface gravity $\log g = 4.0$ (cm s$^{-2}$). This corresponds for example to a $M_p = 1.7$ $M_J$ planet accreting at $M = 10^{-7}$ $M_J$ yr$^{-1}$ with $f_{\text{fill}} = 1$ in the hot-start population. The black line shows the pure photospheric radiation (BT-Settl; Allard et al. 2012) and the red shows the photospheric radiation with the shock excess. The pale grey line is a blackbody at $T_{\text{eff}} = 1600$ K. **Left panel:** Global SED. The continua are plotted darker to indicate that they are less certain. Only the Lyman, Balmer, and Paschen continua are not covered by the photosphere. **Right panel:** Zoom-in on the H$\alpha$ (Balmer-$\alpha$) region, showing (right axis) the transmittance of the SPHERE/ZIMPOL H$\alpha$ Continuum (green), H$\alpha$ Broad (yellow), and H$\alpha$ Narrow (blue) filters (Schmid et al. 2018; see Footnote 6), as well as the MagAO H$\alpha$ filter, normalised to 100%. The red peak around 6560 Å is the H$\alpha$ line coming from the shock radiation. The blackbody is visible as the nearly flat grey line.

In the right panel of Figure 7, we zoom into the H$\alpha$ region and compare with the SPHERE/ZIMPOL H$\alpha$ filters\(^6\) (Schmid et al. 2018) and also show the MagAO filter\(^7\). In this panel, an atmospheric reflective index of 1.000276 (Cid-dor 1996) is assumed for matching the line and filter peaks. The H$\alpha$ line is narrower than filters, i.e., unresolved in all. Therefore, clearly, the high-resolution line profiles shown in Aoyama et al. (2018) and Aoyama & Ikoma (2019) cannot be resolved even with narrow-band filters. Nevertheless, the line in Figure 7b contributes around 99% of the flux in the SPHERE Narrow filter but also in the broad filters (SPHERE or MagAO), clearly dominating over the contribution from the heated photosphere. Also, we note that the narrow filter well contains most of the H$\alpha$ flux even after taking the spec-

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\(^6\) These smooth curves are from J. Milli, private communication; lower-resolution filters available at https://www.eso.org/sci/facilities/paranal/instruments/sphere/inst/filters.html. Note that in Table 6 of Schmid et al. (2018) there is a typographical error: the correct central wavelength of the H$\alpha$ filters is not 656.9 nm as given there but rather $\lambda_c = 656.3$ or 656.5 nm as in Table 5 of Schmid et al. (2017) (H.-M. Schmid, priv. comm.).

\(^7\) See “VisAO Filters” under https://twcl.science/magao/visao/group_reduction_users_guide.html.
tral shifts into account. By design, the continuum filters are securely distant from the H α line.

3.2. Grid of models

We now present results from a large grid in accretion rate and mass,

\[
10^{-8} M_\odot \text{yr}^{-1} \leq \dot{M} \leq 10^{-4} M_\odot \text{yr}^{-1} \quad (25)
\]

\[
0.5 M_\odot \leq M_p \leq 20 M_\odot. \quad (26)
\]

As mentioned above, the radii \( R_p(M, M_p) \) and \( T_{\text{eff}}(M, M_p) \) are given by the relations of Section 2.2.1, where \( T_{\text{eff}} \) is chosen so that the total flux in the SED is equal to the sum of the internal and the incoming kinetic energy flux. We take as the standard case the radius fit from the “accretion hot-start” population and set \( f_{\text{fill}} = 1 \) for simplicity. This range of \( M \) and \( M_p \) is chosen for the following reasons. The lower mass of \( M_p = 0.5 M_\odot \) corresponds to \( v_0 \approx 30 \text{ km s}^{-1} \) (see Figure 4), which is the lower limit of hydrogen line emission when the preshock gas is in molecular form (Aoyama et al. 2018). The mass of \( M_p = 20 M_\odot \) is a rough upper limit on what is often termed a planet and is practical value for computational reasons. The upper accretion rate of \( M \approx 10^{-4} M_\odot \text{yr}^{-1} \) corresponds to a classical value in formation calculations (more precisely, \( 10^{-2} M_\odot \text{yr}^{-1} \approx 10^{-4.5} M_\odot \text{yr}^{-1} \); Marley et al. 2007; Mordasini et al. 2012a; Tanigawa & Tanaka 2016), and \( M = 10^{-8} M_\odot \text{yr}^{-1} \) is, as we shall find, roughly the lower limit to explain the PDS 70 b and c observations of Haffert et al. (2019) towards high masses \( M_p = 20 M_\odot \) (see Figure 9). Also, at too low \( M \) the accretion lines become negligible and the photosphere begins to dominate.

Figure 8 shows a grid of SEDs for \( M_p/M_\odot = 1, 3, 10, 20 \) and \( M/M_\odot \approx (M \text{ yr}^{-1}) = 10^{-8}, 10^{-7}, 10^{-6}, 10^{-5} \). The peak intensities of the H α and other hydrogen lines are significant relative to photospheric emission and increase with \( v_0 \). In all panels except \( (M = 10^{-8} M_\odot \text{yr}^{-1}, M_p = 1 M_\odot) \), the Ly α line at \( \lambda = 1215 \text{ Å} \) has the highest peak value. The effective temperature \( T_{\text{eff}} \) is almost a monotonic function of both \( M \) and \( M_p \) (cf. Section 2.2.2). With increasing planet mass and decreasing accretion rate, the Lyman continuum blueward of \( \lambda \approx 912 \text{ Å} \) becomes stronger relative to the hydrogen lines. As discussed in Section 3.1.1, this means that a large fraction of the hydrogen is ionized and that, in reality, a large fraction of the energy should be converted into He and metal lines instead. However, again, this does not affect the strength of the hydrogen lines.

In all cases shown, Lyman and Balmer lines have significant peaks above the photospheric emission, because the peak of photospheric emission is at a longer wavelength than these lines. Even when we integrated the flux within the SPHERE or Mag-AO broadband filters (not shown), the H α contribution dominates over the photospheric emission.

Figure 8 also shows that the ratio of the Ly α to the H α peaks increases with planet mass, and that for \( M_p \approx 1 M_\odot \), the ratio also increases with decreasing \( \dot{M} \). This is because at high postshock temperatures \( T_1 \) (high \( M_p \); see Equation (17)), hydrogen excitation occurs. This increases the abundance of the absorber of H α, namely electrons in the \( n = 2 \) state, while depopulating the absorber of Ly α, electrons in \( n = 1 \). This leads to a lower \( \text{H} \alpha/\text{Ly} \alpha \) ratio. Towards high postshock densities (high \( M \); see Equation (9)), both \( \text{H} \alpha \) and \( \text{Ly} \alpha \) are more strongly absorbed. However, this absorption occurs in the upper regions (small \( \Delta z \)), where the temperature is high but the excitation degree is low. Normally, hotter gas emits and cooler gas absorbs, but since the hot gas has a low excitation degree, the hot gas can absorb. This is a non-equilibrium (NLTE) effect not captured by a time-independent approach. Therefore, since the lower levels of hydrogen are more populated, Ly α absorption is stronger than H α absorption. This leads to the increase of \( \text{H} \alpha/\text{Ly} \alpha \) with \( M \).

In Figure 8, we also see that longer-wavelength series (e.g., Paschen or Brackett) are embedded in the photospheric signal but tend to emerge towards larger masses and accretion rates. This suggests that high-resolution spectroscopy of strongly accreting or massive planets might be able to detect lines from these other series (see also Sections 5.3 and 5.4), unless infrared excess from dust particles in CPD is significant enough.

Particularly with the hot-start radii, the Mach number is not monotonically proportional to \( M \) because of the non-monotonic dependence of \( M \) on the radius. Although increasing the mass flux of accreting gas \( \dot{M} \) increases the amount of shock-heated (and thus emitting) gas, it is associated with a larger planet radius at the same time. In turn, the larger planet radius leads to a slower free-fall velocity at the planet surface \( v_0 \). Figure 8 reflects this, given that, as we verified, the H α continuum is at least roughly monotonic function of \( T_{\text{eff}} \) at fixed planet mass.

As one of the most important results of this work, Figure 9 shows the H α line luminosity as a function of \( M \) and \( M_p \). Note that Figure 3 of Aoyama & Ikoma (2019) is similar but for \( R_p = 2 R_J \) given independently of \( M \) and \( M_p \). The H α luminosity \( L_{\text{H} \alpha} \) ranges from \( \sim 10^{-9} L_\odot \) to \( \sim 10^{-3} L_\odot \) over the grid, overall increasing monotonically with both \( M \) and \( M_p \). The contours show that for \( M_p \approx 3 – 5 M_\odot \), the H α luminosity is independent of \( M_p \) and is roughly linearly proportional to \( M \). The first part of the reason for this is that the H α luminosity turns out to be roughly linearly proportional to the incoming kinetic energy flux \( L_{\text{acc}} = G M_p \dot{M}/R_p \).
Figure 8. Grid of SEDs of accreting planets as a function of mass and accretion rate. Black lines show the photospheric emission and red lines include the shock-heated gas emission. We took $f_{\text{fill}} = 1$ and the hot-start relations (see Section 2.2.1) for the planetary radius $R_p$ and effective temperature $T_{\text{eff}}$ (see subpanels), rounding to the nearest model and rounding up to $T_{\text{eff}} = 1200$ K. Hydrogen lines are labelled in Figure 7.

(Aoyama et al. 2018), especially at a fixed mass (see Figure 2 in Section 2.2.2). The second part is a simple one: the mass coordinate is on a linear scale, with only a limited range ($\approx 1.3$ dex) relevant to planetary detections, while the accretion rate axis is logarithmic and covers several orders of magnitude.

Our $L_{\text{H} \alpha} (\dot{M}, M_p)$ relation (Figure 9) is robust to changes in the model choices. We compare in Appendix C the luminosity as obtained with the hot- and the cold-start $R_p$ and $T_{\text{eff}}$ relationships and find very little difference. Similarly, varying $f_{\text{fill}}$ from $f_{\text{fill}} = 1$ to $f_{\text{fill}} = 0.01$ (not shown) changes the $\text{H} \alpha$ fluxes by at most a factor of two$^9$. This is because the incoming gas mass at the shock ($\dot{M}$) is independent of $f_{\text{fill}},$

At extremely low $f_{\text{fill}} \lesssim 10^{-4}$, self-absorption becomes very important. For more moderate values $f_{\text{fill}} \gtrsim 10^{-3}$ as inferred for young accreting stars (Ingleby et al. 2013), self-absorption is not a significant effect.
and Hα emission is almost proportional to the mechanical energy of incoming gas (Aoyama et al. 2018), as mentioned above. See also the discussion in Section 6.2.

Figure 9 is meant as a tool for interpreting Hα detections in terms of fundamental planet parameters. Therefore, we also compare the luminosities with those of a few low-mass objects (labeled contours), which we now discuss in the next section.

As further output from our model, we show in Section 5.3 the Brγ, Paα, Paβ, and Hβ luminosities in a similar fashion.

4. APPLICATION TO THE PDS 70 AND LkCa 15 SYSTEMS

4.1. PDS 70 b and PDS 70 c

Wagner et al. (2018) reported the detection using MagAO of an Hα signal from PDS 70 b, a companion in the gap in the transitional disc around a young (5.4 ± 1.0 Myr; Müller et al. 2018) pre-main sequence O.8-M5.0 star. The stacked signal strength was of 3.9σ and thus below a robust threshold of 5σ, but there is ample evidence for the existence of the planet (Kepler et al. 2018; Müller et al. 2018; Kepler et al. 2019). Then, PDS 70 b was confirmed by Haffert et al. (2019) using VLT/MUSE (Bacon et al. 2010). They also reported the discovery in Hα of PDS 70 c, a companion at the edge of the gap. From new VLT/SINFONI K-band data from Christiaens et al. (2019b), Christiaens et al. (2019a) inferred the presence of a circumplanetary disc around PDS 70 b, the first observational evidence for a disc around a planet in a circumstellar disc. From the NIR SED and the models of Eisner (2015), they derived an accretion rate \( \dot{M} \sim 10^{-7.5} M_J\) yr\(^{-1}\). Also, Wang et al. (2020) observed this system with Keck/NIRC2 and estimated mass accretion rate to be 3–8 \( \times 10^{-7} M_J\) yr\(^{-1}\) by comparing to the luminosity-evolution model of Ginzburg & Chiang (2019). More recently, Stolker et al. (2020) added the first detection of PDS 70 b at 4–5 μm and re-analyzed the other data from 1 to 5 μm, confirming the finding by Wang et al. (2020) that a blackbody fits well the SED. Given their modeling results, they concluded that PDS 70 b is likely surrounded by some dusty material, which nevertheless manifestly lets (some) Hα pass through.

The Hα signal of PDS 70 b has been detected with two different instruments, with different luminosity determinations. Wagner et al. (2018) do not report the luminosity explicitly but we derive it as \( L_{H\alpha} = (1.4 \pm 0.6) \times 10^{-6} L_\odot \) following their data and description (see Appendix D for details); this agrees with \( L_{H\alpha} = (1.3 \pm 0.7) \times 10^{-6} L_\odot \) derived by Thanathibodee et al. (2019). As for Haffert et al. (2019), they obtain \( L_{H\alpha} = (1.6 \pm 0.14) \times 10^{-7} L_\odot \) for PDS 70 b and \( (7.6 \pm 1.3) \times 10^{-8} L_\odot \) for PDS 70 c. Thus the luminosity for PDS 70 b derived under the assumption of no extinction\(^{10}\) is about lower by one order of magnitude than in Wagner et al. (2018).

Hashimoto et al. (2020) improved the data-correction method of the VLT/MUSE data and estimated higher values \( L_{H\alpha} = (3.3 \pm 0.1) \times 10^{-7} L_\odot \) and \( (1.3 \pm 0.1) \times 10^{-7} L_\odot \) for PDS 70 b and c, respectively. The value for PDS 70 b is still lower than in Wagner et al. (2018) by a factor of four. This could be due to intrinsic variability in the Hα emission from PDS 70 b and/or from the known variability of the star in the R band, combined with the way the contrast is measured. However, Haffert et al. (in prep.) report that for a dozen measurements over a period of three months, there is no variability in the Hα flux at the \( \approx 30\% \) level. Thus differences in the data reduction seem to be likely explanation for the differences.

With these Hα luminosities, our model yields \( M_p - \dot{M} \) relations. The observed luminosities are shown in Figure 9 as line contours: blue-dashed (PDS 70 b; Wagner et al. 2018), blue-solid (PDS 70 b; Haffert et al. 2019), and green (PDS 70 c; Haffert et al. 2019), respectively. If \( M_p = 5–9 M_J \) for PDS 70 b (Wagner et al. 2018) and \( M_p = 4–12 M_J \) for PDS 70 c (Haffert et al. 2019), our model implies \( \dot{M} = (8.0 \pm 4.8) \times 10^{-8} \) for PDS 70 b from Wagner et al. (2018), \( \dot{M} = (1.1 \pm 0.3) \times 10^{-8} \) for PDS 70 b from Haffert et al. (2019), and \( \dot{M} = (6.3 \pm 3.1) \times 10^{-9} M_J\) yr\(^{-1}\) for PDS 70 c (Haffert et al. 2019), respectively.

While Hashimoto et al. (2020) estimated the mass accretion rate in the same way, they obtained a different \( \dot{M} \) than here because they assumed the observed Hα is extincted and corrected for this. Namely, from the non-detection of Hβ, they inferred \( A_{H\alpha} > 2.0 \) and \( > 1.1 \) mag for PDS 70 b and PDS 70 c, respectively. This implies \( \dot{M} > 5 \times 10^{-7} \) for PDS 70 b and \( \dot{M} > 1 \times 10^{-7} M_J \) yr\(^{-1}\) for PDS 70 c. Therefore the value estimated above (without extinction) provides a lower limit to the accretion rate. This implies that the true errorbars on \( \dot{M} \) are asymmetrical (larger on the high end than quoted above), coming from the uncertainty on the true input luminosity.

By extrapolating the empirical \( L_{\text{acc}} - L_{H\alpha} \) relationship for Young Stellar Objects (YSOs) from Rigliaco et al. (2012), Wagner et al. (2018) estimated \( \dot{M} \approx 10^{-9} M_J \) yr\(^{-1}\) for PDS 70 b. Also, with the \( M - \text{linewidth} \) relationship of Natta et al. (2004), Haffert et al. (2019) derived \( \dot{M} \approx 2 \times 10^{-8} M_J\) yr\(^{-1}\). Thus, applying stellar accretion models to planetary-mass observations yields a lower mass accretion rate than from our model by a few orders of magnitude. To estimate mass accretion rate, we suggest that our model con-

\(^{10}\) In particular, within the system. The interstellar extinction is negligible according to Lallement et al. (2019, see tool under https://stilism.obspm.fr/).
Non-extincted $H\alpha$ luminosity from the planet-surface shock as a function of accretion rate and planet mass (colorscale). Thin gray lines highlight $\log L_{H\alpha}/L_\odot = -9$ to $-4$ in steps of 1 dex. We extrapolated the model results to $\dot{M} \sim 10^{-9.5} \, M_J \, yr^{-1}$. Shaded bands show $H\alpha$ luminosities with their 1-$\sigma$ errorbar for PDS 70 b (dashed blue line: $10^{-5.9} \, L_\odot$; Wagner et al. 2018), another observation of PDS 70 b (solid blue: $10^{-6.8} \, L_\odot$; Haffert et al. 2019), and PDS 70 c (green: $10^{-7.1} \, L_\odot$; Haffert et al. 2019). The contour for the Hashimoto et al. (2020) value of $L_{H\alpha} = 10^{-6.5} \, L_\odot$ without de-reddening would lie between the two blue bands and is not shown. For reference, the grey band shows the value reported for LkCa 15 b (black: $10^{-4.1} \, L_\odot$; Sallum et al. 2015, but using the new distance determination of Gaia Collaboration et al. 2018) for the less secure protoplanet candidate LkCa 15 b.

Also, Thanathibodee et al. (2019) constructed a model of $H\alpha$ emission focusing on PDS 70 b. They modeled the postshock accreting gas as the source of the $H\alpha$ radiation rather than the postshock region that is the subject of this paper. The accretion rate they estimate, $\dot{M} \approx 10^{-8.0 \pm 0.6} \, M_J \, yr^{-1}$, is larger than the results of empirical $L_{H\alpha} - \dot{M}$ relationships and in agreement with our results within the margin of error. As discussed in Section 6.1, from which of the pre- or postshock region the $H\alpha$ emission originates depends on whether the postshock gas is hot enough to emit $H\alpha$. In fact, for PDS 70 b a contribution from both cannot be excluded (Aoyama et al. submitted).

Finally, the two upper limits on Br $\alpha$ (Stolker et al. 2020) and Br $\gamma$ (Christiaens et al. 2019b) emission are discussed in Section 5.4.

### 4.2. LkCa 15 b

Following the discovery of a companion to LkCa 15 A by Kraus & Ireland (2012), Sallum et al. (2015) reported the infrared detection of further sources in the system using sparse-aperture masking (SAM). Intriguingly, they also measured an $H\alpha$ signal which seemed to originate at the position of LkCa 15 b. On the other hand, Thalmann et al. (2016) analyzed scattered light from the disk and showed that the infrared detections of the planetary candidates around LkCa 15 could be false positives related to features of the disc in scattered light. In addition, observations by Mendigutía et al. (2018) using spectro-astrometry suggest that the $H\alpha$ emission may not be coming from a point source but rather from an extended region similar in size to the orbit of the claimed planet LkCa 15 b. Recently, Currie et al. (2019) conducted the first direct-imaging observations of the LkCa 15 system. They...
provided evidence that there is no point source at the location of the claimed planet (nor of the possible further companions) but that in fact the SAM signal originates from disc emission.

Despite the debate as to its origin, we will briefly analyze the H α signal at the position of a putative companion to LkCa 15 as originating from an accretion shock on the planet surface. The reported H α luminosity is $L_{\text{H}\alpha} = 10^{-4.1\pm0.1} L_\odot$ from Sallum et al. (2015) but using the updated Gaia distance determination of 158.8 pc (Gaia Collaboration et al. 2018). From Figure 9 and assuming $M_p = 10 M_\star$, $M = 4.0^{+2.5}_{-0.1} \times 10^{-6} M_\odot$ yr$^{-1}$. This accretion rate is not implausible for a claimed forming gas giant, especially if it were undergoing an accretion outburst.

Using instead the Rigliacio et al. (2012) approach as in Sallum et al. (2015) and again with $M_p = 10 M_\star$ as an example, yields $M = 3 \times 10^{-7} M_\odot$ yr$^{-1}$ for $R_p = 1.6 R_\star$ as Sallum et al. (2015) assumed. At this $(M, M_p)$, our fits (Section 2.2.1) yield $R_p = 1.9 R_\star$ ($R_p = 1.4 R_\star$) for the hot (cold) population, so that 1.6 $R_\star$ is a reasonable value, albeit perhaps on the small side. The upshot of the comparison is that the $M$ implied by the Rigliacio et al. (2012) relationship is one order of magnitude smaller than derived with our approach. As discussed in Aoyama et al. (submitted), we recommend using our models, which are tailored for the planetary case, instead of extrapolations from the stellar regime.

5. FURTHER OBSERVATIONAL ASPECTS

We now discuss to what extent high-mass and high-$M$ planets can be distinguished (Section 5.1) and the planet surface shock from the CPD shock (Section 5.2), before presenting the line strengths and line ratios for accretion-generated hydrogen lines other than H α (Section 5.3). Finally, we discuss what information may be obtained from combining observations of several lines for the same object (Section 5.4).

5.1. Distinguishing massive planets and strongly accreting planets from the line profile?

When characterizing gas giants from their H α luminosity, their mass and mass accretion rate are degenerate because the luminosity depends on their product. However, this degeneracy can be lifted by spectroscopic observations of H α, which was demonstrated by Aoyama & Ikoma (2019) in the case of PDS 70 b and c.

Recall that the preshock velocity $v_0$ mainly depends on $M_p$, while the number density $n_0$ is mainly set by $M$ (see Figure 5). Figure 10 shows H α line shapes for several values of $v_0$ and $n_0$. The preshock velocity $v_0$ sets the shock strength—and thus the temperature just after the shock (Equation (17))—but barely the line width, which is mainly set by Doppler broadening. This is because the gas that becomes ionized and then recombines is the main source of H α, and not the gas immediately after shock, even if there is some amount of excited hydrogen there (see down to $\Delta t \approx 10^{-3}$ s in Figure 6).

The line profile can be divided into three parts: a narrower Gaussian, a broader Gaussian, and a Lorentzian profiles, with the latter visible further from the line center. The layers that emit the two Doppler profiles are separated by the highly ionized region (at $\Delta t \approx 10^{-3}$–10 s in Figure 6). The H α intensity coming from the deeper layers is much larger. Consequently, even the width of the line where the energy density is 10% of the maximum ($W_{10}$ in e.g. Thanathibodee et al. 2019) reflects only the narrower Doppler component, coming from the hydrogen-ion recombination region at low temperatures, in the deep layers. The gas temperature at which hydrogen recombination begins barely depends on $v_0$, because it corresponds to the hydrogen ionization energy of 13.6 eV. We can see the thermal broadening of the optically thinner gas just after the shock only far from the line center, at $\lambda \gtrsim 6564$ Å and $\lesssim 6562$ Å (i.e., $|\Delta \lambda| \gtrsim 50$ km s$^{-1}$ away from the shock).

Since the hot gas immediately below the shock has a high velocity and is travelling away from the observer, the red half of the line is more broadened by this mechanism than the blue half. However, as seen in the right panel, the resolution of MUSE is not sufficient to distinguish this.

As shown in the right panel of Figure 10, $n_0$ changes the width of the normalized line dramatically. However, increasing $n_0$ hardly broadens the H α line because the pressure broadening is negligible relative to the natural broadening. The normalization of the vertical axis makes the line with a higher $n_0$ look broader. A high $n_0$ leads to H α self-absorption in the postshock gas (in the top part of the flow), which flattens the line peak. Since we normalized the line flux at the peak, the self-absorbed line looks broader (see Aoyama et al. 2018 for the non-normalized profile). However, the lines for higher $n_0$ are brighter than lower ones in spite of the absorption. This effect becomes significant for $n_0 > 10^{11}$ cm$^{-3}$ in the right panel.

As shown in Figure 10, current MUSE spectral resolution is not enough to distinguish the profiles clearly, while it barely resolves the spectral profile for higher density (Eriksson et al. 2020). However, it is not possible to determine in general what minimum spectral resolution is required for distinguishing high accretion rates from high masses because it depends on the relative uncertainty in the flux as well as on the planet properties through the dependence of the line profile on $(n_0, v_0)$.

5.2. Distinguishing planetary-surface and CPD-surface shocks?

Hydrodynamic simulations report that gas accreting toward proto-gas-giants goes through multiple shocks (e.g. Kley 1999; Tanigawa et al. 2012). The gas that falls onto the CPD yields a shock, which can emit H α near the planet. However, far from the planet, the shock is not strong enough to emit H α, and only the part of the shock close to the planet can
Figure 10. Spectral profile of Hα for v0 = 100, 150, 200 km s⁻¹ with n₀ = 10¹¹ cm⁻³ (left panel) and for n₀ = 10⁹, 10¹⁰, 10¹¹, 10¹³ cm⁻³ with v₀ = 100 km s⁻¹ (right panel), typical for planetary masses (see Figure 4). The vertical axis is normalized with the peak value of each line. For reference, the Doppler shift velocity is shown at top of the panels, even though some features come from natural broadening rather than Doppler broadening. The red bar corresponds to MUSE spectral resolution of R = 2516 at the wavelength (Eriksson et al. 2020). For the relevant range of parameters (see Figure 5), the Hα line peak depends on n₀ more than on v₀ because high n₀ leads to Hα self-absorption around the line peak.

The left panel of Figure 11 displays the global SEDs. The red and blue lines correspond to the SED assuming planetary surface shock and CPD surface shock, respectively. The black line corresponds to the SED without shock excess for reference. For the surface-shock case, \( M = 10^{-7} M_J \text{yr}^{-1} \) and \( M_p = 8 M_J \), while the CPD-shock case has \( 9 M_J \) and \( M = 10^{-3} M_J \text{yr}^{-1} \). In both cases, we set \( R_p = 1.7 R_J \) and \( T_{\text{eff}} = 1600 \text{K} \) (Section 2.2). For the surface-shock case, \( f_{\text{fill}} = 1.0 \) is also assumed. These parameters are chosen to have the Hα luminosities be the same, with \( L_{\text{Hα}} \approx 1.5 \times 10^{-6} L_\odot \).

In the Lyman and Balmer continua, namely for \( \lambda \lesssim 3000 \text{Å} \), the two SEDs differ by more than a factor of ten. This comes from the difference in gas temperature after the two shocks. For the CPD surface shock, the regions far from the planet dominate the shock excess emission because of their large emitting area. This is a relatively weak shock, associated with a low temperature. This temperature difference can be also seen in the Hα profile in Figure 11b: the profile is narrower in the CPD surface-shock case than in the planet surface-shock case when taking the different heights into account (i.e., looking at the full width at half-maximum (FWHM)). The difference is small but might be detectable in the future with high-resolution observations. The narrower Hα and the weaker recombination continua in the CPD case means a weaker shock, which can also occur at the surface of the less massive planet. However, in such a case, the density should higher than the CPD case, and one can distinguish these two cases. Note that our model does not include some continuum.

Contribute to the Hα emission (Aoyama et al. 2018). If the gas ultimately joining the planet passes firstly through a shock at the surface of the CPD and secondly through the planetary surface shock, the former is negligible for Hα emission, because most of the gas hits the CPD at the far region. There, the free-fall velocity is too small for significant Hα emission. For example, when the CPD is truncated near the planet (and the gas is accreting by magnetospheric accretion or it is falling directly onto the planet from the PPD), the planetary surface shock dominates the emission. However, when most gas passes through boundary layer accretion rather than a planetary surface shock, the CPD surface shock becomes significant. Thus it would be desirable to distinguish the source of shock excess.

In Figure 11, we compare the SEDs from the planetary surface shock and the CPD surface shock. The shock excess from the latter highly depends on the gas accretion model. Here, guided by the results of a 3D hydrodynamic simulation (Tanigawa et al. 2012), we assume the following:

1. All gas accretes vertically from the protoplanetary disk onto the CPD with a free-fall velocity set by the protoplanet’s gravity, \( v_f(r) = \sqrt{2GM_p/r} \), where \( r \) is the radial distance from the protoplanet’s center.
2. The mass accretion flux onto the CPD is constant within 0.1 Hill radius, and zero outside of this.
3. The orbital axis is 22.6 au and the central-star mass is 0.82 \( M_\odot \), which are values appropriate for PDS 70 b.
sources such as a heated photosphere (e.g. Königl 1991; Calvet & Gullbring 1998) or, if present, the boundary layer (e.g. Kenyon & Hartmann 1987), which are well modeled in the stellar accretion context. Such continuum sources can change the spectral appearance, but it is unfortunately difficult to say how important this would be.

In summary, for a given Hα luminosity, the resolution of MUSE is not sufficient to distinguish an accretion shock on the planetary surface from the one on the CPD, but high-resolution spectroscopy might be able to do so.

5.3. Predictions for hydrogen lines other than Hα

The Hα line on which we have focused so far is only one of the 55 hydrogen lines we model. Recently, Eriksson et al. (2020) reported an Hβ flux for the ≈ 10-M_☉ companion Delorme 1 (AB)b with the MUSE instrument on the VLT. Also, the upcoming HARMONI instrument on the ELT is expected to observe other lines, e.g., Paβ and Brγ with a resolution R ≈ 17′000, the HIRES/EELT can observe Balmer and Paschen lines with the high spectral resolution of R ≈ 100′000, and the University of Tokyo Atacama Observatory (TAO) should be able to detect Paα thanks to its location at 5′640 m (Yoshii et al. 2010). Finally, the Keck Planet Imager and Characterizer (KPIC) (Jovanovic et al. 2019) aims at obtaining R = 35′000 spectroscopy in the K, L, and M bands (≈ 2−5 μm). Clearly, it is timely to extend the luminosity predictions to lines other than Hα.

In Figure 12, we show the line luminosity of Hβ, Paα, Paβ, and Brγ for the same grid of (M, M_p) as in Figure 9. The luminosities range from L_β/ν ≈ 10^{-12} to 10^{-4} L_☉, increasing with M and M_p, as for Hα and with the same qualitative shape of a very weak mass dependence for M_p ≥ 3−5 M_☉. The contours are very similar using the fit to the hot- or cold-start populations. We show upper limits for a few objects but discuss them below in Section 5.4.

Next, Figure 13 shows the intensity ratio of Hβ, Paα, Paβ, and Brγ relative to Hα as a function of the Hα flux at the surface of the object. (Absolute fluxes for Hβ, Paα, Paβ, and Brγ as a function of (n_i, v_0) can be found in Aoyama et al. (2018).) The ratios span Hβ/Hα ≈ 0.1−2 to Brγ/Hα ≈ 0.001−0.05, with typically X/Hα ≈ 0.03−0.3 for X =Hβ, Paα, Paβ.

All four ratios are more or less flat within 0.5 dex at low Hα flux (within 0.7 dex for Brγ/Hα) but start to increase around an Hα flux F_{Hα} ≈ 10^7 erg s^{-1} cm^{-2}. For Paα the rise is only moderate. This change of slope occurs because the Hα saturates due to self-absorption in the shock-heated gas as v_0 increases, while the other lines do not saturate. Thus towards high F_{Hα}, v_0 must increase faster along the x axis, which leads to stronger other lines (and thus ratios) since they hardly display self-absorption. Also, when the starting level of the transition is high (i.e., Paβ or Brγ: n =

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11 See http://harmoni-web.physics.ox.ac.uk/Specifications/spectral.asp.
12 See http://obswww.unige.ch/~wildif/publications/2014_9147-75.pdf.

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**Figure 11.** SEDs for two different models of shock-heated gas emission: planetary photospheric emission plus planetary-surface shock (red) and CPD surface-shock (blue). The photosphere SED in the surface-shock case is also shown for reference (black). The M, M_p, R_p values (see figure and text) are chosen such that L_α = 1.5 x 10^{-6} L_☉ in both cases. **Left panel:** Global SED. **Right panel:** Hα line profile against Doppler shift velocity from the line center.
Figure 12. As in Figure 9 but for Hβ (λ 0.4861 μm), Pa α (λ 1.875 μm), Pa β (λ 1.282 μm), Pa γ (λ 1.094 μm), Br α (λ 4.050 μm), and Br γ (λ 2.166 μm). The contours are labelled by \( \log_{10}(L_{\lambda}/L_\odot) \) for each line \( \lambda_i \). The color map and the solid contour lines show the results from the radius fit to the warm population, and the dashed contour lines use the cold population. In the Hβ panel, the black contour is for the 5-sigma upper limit for PDS 70b of \( \log_{10}(L_{\text{H}\beta}/L_\odot) < -7.03 \) (Hashimoto et al. 2020), in the Br α panel, the black contour is for the detection of the continuum, where the Br α line is expected be embedded, of \( \log_{10}(L_{\text{Br}\alpha}/L_\odot) < -6.36 \) (Stolker et al. 2020), in the Br γ panel, the black contour shows the 5-sigma upper limit for PDS 70b of \( \log_{10}(L_{\text{Br}\gamma}/L_\odot) < -7.48 \) (Christiaens et al. 2019b), and in the Pa β panel, the black contour is for the 5-sigma upper limit for a putative planet in TW Hya of \( \log_{10}(L_{\text{Pa}\beta}/L_\odot) < -9.20 \) at 25 au (Uyama et al. 2017). Other lines are available upon request.
Also, our model predicts that the Ly α is much stronger than all lines and carries most of the incoming shock energy, which has been converted to radiation (see Figures 6 and 8). This is an important input for models of CPD chemistry (Cleeves et al. 2015; Rab et al. 2019). However, interstellar extinction is too strong for Ly α to be detected, excepting a few young stellar systems such as TW Hydra. However, other lines induced by planetary Ly α might be detected as for some accreting stars (e.g., fluorescent molecular hydrogen lines: Herczeg et al. 2004).

5.4. Combining the information from multiple accretion-line measurements

We briefly discuss the few current application of these predictions in the planetary-mass regime. Firstly, the 5σ Br γ upper limit for PDS 70 b from Christiaens et al. (2019b), \( \log L_{Br \gamma}/L_\odot < -7.48 \), is shown in Figure 12. Taken at face value, this implies that \( M < 2 \times 10^{-7} M_\odot \) if \( M_p \geq 8 M_\odot \), but is much less constraining at lower \( M_p \). Also, Stolker et al. (2020) detected PDS 70 b for the first time in the Br α filter (NB4.05) of NACO/VLT. The Br α line can be used as an accretion tracer (Komarova & Fischer 2020), but in this case the flux is consistent with the blackbody emission matching the global SED. Namely, given the observed H α flux, the Br α is expected to be embedded in the continuum (see Figure 13), and the upper limit of the Br α emission from the shock itself is the observed line-integrated luminosity of \( L = 4.3 \times 10^{-7} L_\odot \). This implies that \( M < 1 \times 10^{-6} M_\odot \) if \( M_p \geq 8 M_\odot \). These accretion rate upper limits are consistent with the results from Figure 9, \( M \approx 8 \times 10^{-8} M_\odot \) yr\(^{-1}\) from the Wagner et al. (2018) measurement or \( M \approx 1 \times 10^{-8} M_\odot \) yr\(^{-1}\) from Haffert et al. (2019).

For comparison, Hashimoto et al. (2020) derived from their re-analysis of archival MUSE data an H β flux upper limit of \( 2.3 \times 10^{-16} \text{ erg s}^{-1} \text{ cm}^{-2} \) for PDS 70 b, corresponding to \( L_{H\beta} < 9 \times 10^{-8} L_\odot \) (3σ). This H β is inconsistent with the H α result (see Figure 9). The same is true for PDS 70 c. From this, Hashimoto et al. (2020) concluded that there must be extinction. We emphasise that our results are without taking extinction into account, which should be done separately, as in Marleau et al. (submitted). In general, extinction can also be expected to affect the Br γ measurement, and future tighter constraints on Br γ may yield an upper limit inconsistent with the H α measurement; this would then be like for the H β and would confirm that extinction is important.

In the same class as CT Cha b (Schmidt et al. 2008), one of the only low-mass putative accretor for which other accretion lines have been detected is Delorme 1 (AB)b, for which Eriksson et al. (2020) measured H β (and also He i lines and upper limits on the infrared Ca ii triplet). They infer a mass \( M_p \approx 12 M_\odot \) and radius \( R_p = 1.6 R_\odot \) from (hot-start) evolutionary models by combining their results.
to the photometry of Delorme et al. (2013). They report \( \log \left( \frac{L_{H\alpha}}{L_\odot} \right) = -7.05 \pm 0.06 \), from which they infer \( M \approx (0.8-3.0) \times 10^{-8} M_\odot \) yr\(^{-1} \) by combining their \( L_{H\alpha} \) with measurement with different models. Combined with ours (Aoyama et al. 2018; Aoyama & Ikoma 2019; this work), they derive \( M = 1 \times 10^{-8} M_\odot \) yr\(^{-1} \) and \( M_p = 11 M_\oplus \), using the radius \( R_p = 1.6 R_\oplus \) suggested by the photometry. From Figure 12, the predicted \( H\alpha \) luminosity is \( L_{H\alpha} \approx 4 \times 10^{-8} L_\odot \). However, Eriksson et al. (2020) measured \( L_{H\alpha} = 10^{-8} L_\odot \), which is a factor four lower than the model prediction. This suggests that as for PDS 70 b, extinction is affecting the measurement of accretion tracers. The interesting difference is that there is up to now no evidence for an accretion disc around De- lorme 1 (AB)b, but the constraints are not clear (as reviewed by Eriksson et al. 2020). Thus either a detection of or upper limit on a disc around Delorme 1 (AB)b, as well as more observational information on other hydrogen lines would be useful. On the theory side, predictions for the other currently available lines (He i and the infrared Ca ii triplet) would be welcome.

Finally, in Figure 12, we show the 5-\( \sigma \) upper limit on \( Pa\beta \) emission for the TW Hya disc in its gap at 25 au, \( \log \left( \frac{L_{Pa\beta}}{L_\odot} \right) < -9.20 \), derived by Uyama et al. (2017) using Keck. At 95 au, where there is an other gap, the upper limit is \( \log \left( \frac{L_{Pa\beta}}{L_\odot} \right) < -9.79 \) (not shown). This is consistent with the mass constraints of \( M_p \leq 0.5 M_\oplus \) from van Boekel et al. (2017) for TW Hya. Looking at Figure 12, the interesting implication is that Keck at Pa\beta is sensitive to planets with a relatively low mass or accretion rate, at least for the nearest protoplanetary discs.

6. DISCUSSION

We now take a critical look at different aspects of the model and results presented here. Some caveats about the model were already discussed in Section 4.2 of Aoyama et al. (2018) and we do not repeat them here. We compare with other recent models of \( H\alpha \) emission from accreting planets (Thanathibodee et al. 2019; Szulágyi & Ercolano 2020) and discuss the validity of their approach in a different work (Aoyama et al. submitted). Appendix A already details why the physical assumptions behind Storey & Hummer (1995) do not apply to the planetary surface shock.

6.1. Emission by the preshock gas

Our model only treats the shock-heated gas. Several previous studies focusing on stellar-mass objects considered the preshock gas flow as the source of observed \( H\alpha \) excess, assuming unknown (rather than shock) heating there (e.g. Hartmann et al. 1994). Since the shock on the accreting stars makes the gas too hot (> \( 10^5 \) K) to emit hydrogen lines, the shock-heated gas is negligible for hydrogen-line emission. On the other hand, the planet-surface shock emits significant hydrogen lines, while the preshock gas should be cooler and emit weaker hydrogen lines compared to the stellar cases (see also the discussion in Aoyama et al. submitted). This is the reason why we neglect hydrogen line emission from the region.

Even if the preshock gas is too cool to emit \( H\alpha \), the warm gas plays a significant role in excess emission other than hydrogen lines. In the T Tauri-star context, Calvet & Gullbring (1998) found that a Balmer recombination continuum is emitted by the preshock rather than the postshock gas. The hydrogen recombination continua are well modeled and compared with observational results in the stellar accretion context. As seen in Figures 7 and 8, the planetary continua are predicted to be several (up to dozens of) orders of magnitude stronger than the contribution from the planetary photosphere. Therefore, detecting in the planetary context a hydrogen continuum much stronger than the photospheric emission would lend support to our emission models.

6.2. Effect of the accretion geometry: spherical versus magnetospheric

The predicted \( H\alpha \) luminosity (see Figure 9) was derived explicitly in the context of spherical accretion onto the protoplanet’s surface. However, it also represents the signal expected for any accreting planet, regardless of the accretion geometry, i.e., spherical or magnetospherical. In the case of magnetospheric accretion with a filling factor \( f_{\text{fill}} \leq 1 \), relative to spherical accretion, the kinetic-energy flux locally is higher by \( 1/f_{\text{fill}} \) but the accreting area is smaller by \( f_{\text{fill}} \). These effects cancel each other out to a large extent over most of parameter space. However, more precisely, spherical accretion yields an upper limit to the \( H\alpha \) intensity because of \( H\alpha \) self-absorption in the shock-heated gas. To emit more intense \( H\alpha \) by avoiding self-absorption, the gas needs to be less dense, which, at a given \( M \), will be the case for higher \( f_{\text{fill}} \). Note that in a realistic situation, the infalling (preshock) gas and dust could absorb a part of the flux emitted at the planet surface. This is explored in Marleau et al. (submitted).

Also, in the scenario of accretion onto a circumplanetary disk, the \( H\alpha \) emission is at most roughly 1% of the \( H\alpha \) coming from the planet surface for a similar planet mass, at least for the (simple) disc model and scaling assumed in Aoyama et al. (2018). In Section 5.2 we had compared the line shapes at fixed total luminosity.) Therefore, when a strong shock occurs on the planetary surface, regardless of the geometry, the CPD surface shock is negligible.

6.3. Helium and metal lines

For accreting stars, helium and metal lines (He i, Ca ii, Na i, O i, etc.) are also detected and used as indicators of stellar accretion as for hydrogen lines (e.g. Kastner et al. 2002). For very-low mass objects with \( M_p \approx 30 M_\oplus \), the only non-hydrogen line detections of which we are aware are at Delorme 1 (AB)b (Eriksson et al. 2020) and CT Cha B/b.
At the outer edge of the planetary range, $M_p \approx 20 M_J$ leads to $v_{\text{eff}} \approx 200$ km/s for $R_p = 2R_J$ (see Figure 5), so that the postshock gas temperature can exceed $T_J \approx 10^6$ K. In that case, metal lines instead of hydrogen lines are responsible for the dominant emission processes at UV wavelengths.

Our estimate of the hydrogen recombination continua would change when including metal lines. When hydrogen ionization proceeds and neutral hydrogen is minor (e.g. at $T \approx 10^5$ K in Figure 6), the gas should cool through hydrogen recombination continua and/or metal lines. Presently, because we do not include metal lines, almost all the thermal energy is converted into the continua, so that they are overestimated. The hydrogen recombination continua are mostly used as accretion indicator of protostars (e.g. Calvet & Gullbring 1998). For planetary accretion, they are currently expected to highly exceed photospheric emission at UV wavelengths, as in the example in Figure 7, but this should be re-assessed once more complete models are available.

6.4. On the physical size of the cooling region

Towards high $v_0$, the line-forming region can be at a depth of order of the planet size, which is clearly unrealistic. By contrast, in Figure 6 it is at $\Delta z \approx 3 \times 10^3$ cm, which represents 0.002$R_p$ and is much smaller than the planet size and thus reasonable. The reason for the large extent of the cooling zone in some of the other cases is that (i) there is no cooling by helium nor atomic metals (as opposed to CO, H$_2$O, and OH, which are included) and (ii) the geometry is assumed to be plane parallel. The latter approximation is an issue only because of the first; if we included cooling by helium and metals, curvature effects would be unimportant as the cooling region would be thin. Nevertheless, currently, the neglect of gravity is a good approximation because the increasing density compensates for the longer cooling timescale by decreasing back the thermal collision timescale. In any case, we remind that we always resolve the line-forming region because of adaptive time-stepping, which keeps the relative change in temperature at 10% per step.

Despite the unrealistically large extent of the cooling region in some cases, the hydrogen-line fluxes should be relatively accurate within the other model assumptions. This is because the hydrogen-line emission occurs in a spatially thin region (as in the example in Figure 6). Where exactly this region is located (i.e., possibly at an unrealistic depth) does not matter of its emission properties. Also, towards high $n_0$ and $v_0$, absorption in upper layers of the flow becomes important. Here, the distance between the emission and absorption regions does not matter because the intervening region is optically thin. Including metal-line calculations will lead to thinner cooling regions only as a side effect but the aim will be to obtain the fluxes. The presence of the heated photosphere (as in Calvet & Gullbring 1998) is more likely to affect the thermal structure of the postshock region as well as the continuum emission but an exploration of this is beyond the scope of this work.

7. SUMMARY AND CONCLUSIONS

Motivated by recent detections of accretion signatures at young planets or very-low-mass objects (Kepler et al. 2018; Haffert et al. 2019; Eriksson et al. 2020; Haffert et al., in prep.), we have extended the NLTE shock emission model of Aoyama et al. (2018) to the case that the planet surface, as opposed to the circumplanetary disc, is the origin of the hydrogen lines. We compare to other recent models for the H$\alpha$ emission at accreting planets (Thanathibodee et al. 2019; Szulágyi & Ercolano 2020) in a companion publication Aoyama et al. (submitted). We emphasise that the postshock gas cannot reach equilibrium as it cools in the postshock region (Aoyama et al. 2018). Therefore, the equilibrium predictions of Storey & Hummer (1995), which are commonly used in models for Classical T Tauri Stars (CTTS), are not appropriate for hydrogen-line luminosity predictions for the planet surface shock (see discussion in Appendix A).

The macrophysical parameter space is mostly defined by the accretion rate $\dot{M}$, the planet mass $M_p$, the planet radius $R_p$, and the filling factor of the accreting region $f_{\text{fill}}$. To provide guidance, we have fit the radius of forming planets as a function of $(\dot{M}, M_p)$ from the results of detailed planet structure calculations (Section 2.2.1). The photospheric effective temperature $T_{\text{eff}}$ was derived approximately self-consistently from the energy transport from the shock model (Section 2.2.2).

Fixing $f_{\text{fill}} = 1$, we have scanned the large parameter space and combined the shock emission spectra to the spectral energy distribution (SED) of photospheric emission to provide global SEDs of forming planets (Figure 8). We have shown the line luminosities as a function of $\dot{M}$ and $M_p$ for individual hydrogen lines, focusing on H$\alpha$ as well as H$\beta$, Pa$\alpha$, Pa$\beta$, Pa$\gamma$, Br$\alpha$, and Br$\gamma$ (Figures 9 and 12) and discussing their ratios (Figure 13).

In companion papers, we discuss the use of spectrally-resolved line profiles for inferring the physical parameters of planets (Aoyama & Ikoma 2019), study the correlation between $L_{\text{H}\alpha}$ and the accretion luminosity $L_{\text{acc}}$ in the planetary case (Aoyama et al. submitted), and assess the absorption of the H$\alpha$ flux by the infalling gas and dust (Marleau et al. submitted). These models are applied also in Hashimoto et al. (2020) and Eriksson et al. (2020).

Our main results in the present work are the following:

1. The radius of forming planets can be fit by a simple but non-monotonic function of $\dot{M}$ and $M_p$ (Figure 1). At high accretion rates, $R_p$ reaches $\approx 5 R_J$ ($\approx 3 R_J$) in the “warm” (“cold-nominal”) population. While
not definitive, this fit is an improvement over using a constant radius as is often done.

2. Despite the high $T_{\text{eff}}$ of the planet, the shock contribution to the narrow and broad H$\alpha$ filters of SPHERE and MagAO dominates over the photospheric contribution (Figure 7).

3. At the surface of the planet, the Lyman and Balmer series clearly emerge above the photosphere over all parameter space, and the Paschen continuum is visible at low accretion rates. Many lines in other series are visible above the hot photosphere (Figure 8). The details however depend however on the $T_{\text{eff}}$ fit.

4. The H$\alpha$ line luminosity as a function of $\dot{M}$ and $M_p$ is a monotonic function of both and makes it possible to constrain mostly $\dot{M}$ given an observed value (Figure 9). This is one of the main results. Applying this tool to current detections yields reasonable constraints (Section 4). For example, the mass accretion rate of PDS 70 b is estimated as $\dot{M} = (1.1 \pm 0.3) \times 10^{-8} M_J$ yr$^{-1}$ from $L_{H\alpha} = (3.3 \pm 0.1) \times 10^{-7} L_\odot$ (Haffert et al. 2019).

5. Similarly, the line luminosity of other transitions such as H$\beta$, Pa$\alpha$, Pa$\alpha$, Pa$\beta$, Pa$\gamma$, Br$\alpha$, or Br$\gamma$ is a monotonic function of $\dot{M}$ and $M_p$ (Figure 12). We compare this to upper limits for PDS 70 b and the TW Hydra disk. The intensity ratios of these lines to H$\alpha$ range between $10^{-3}$ and $\approx 1$ (Figure 13), and their measurement can yield some constraints on the amount of extinction (Hashimoto et al. 2020).

6. The H$\alpha$ luminosity is a function of planetary mass and accretion rate, and these two are degenerate if only the luminosity of a hydrogen line is observed. The line profile (or spectral width) can give them separately (Aoyama & Ikoma 2019) but the spectral resolution of MUSE, which currently has the highest resolution, can only marginally do it (see also Thanathibodee et al. 2019). Instruments with a higher resolution are needed.

7. If there is an accretion shock both on the planet surface and on a circumplanetary disk, the signal is likely to be dominated by the surface-shock contribution. The two shocks are expected to be spectrally distinguishable with the higher spectral resolution of future instruments (Figure 11).

Continued searches with existing (e.g., VLT/SPHERE, VLT/MUSE, LBT/MagAO, SCExAO/VAMPIRES; Beuzit et al. 2008; Schmid et al. 2018; Bacon et al. 2010; Close et al. 2014a,b; Uyama et al. 2020 and upcoming instruments such as MagAO-X (Males et al. 2018; Close et al. 2018; Close 2020), KPI (Jovanovic et al. 2019), VIS-X (for H$\alpha$ with $R = 15'000$; PI: R. v. Holstein; S. Haffert, priv. comm.) RISTRETTO13, NIRSpec/JWST (but note the low resolution and likely high demand for time), HIRES/ELT ($R = 100'000$ Marconi et al. 2018) are expected not only to reveal more sources, but will hopefully also increase the number of detected lines. Combined with simulations of forming planets, these rich data sets are poised to help constrain observationally the complex accretion geometry and ultimately the origin of gas giants.

ACKNOWLEDGMENTS

We wish to pay tribute to France Allard, who passed away unexpectedly in October 2020. Her world-leading atmospheric models are used widely in the observational and theoretical community, and are an important input in this work too. Her kind nature and expertise will be missed by many. We thank R. van Boekel, M. Keppler, A. Müller, J. Bouwman, B. Husemann, M. Samland, D. Homeier, H. M. Schmid, J. Milli, S. Quanz, G. Cugno, T. Stolker, L. Venuti, B. Stelzberg, Ch. Rab, C. Manara, N. Turner, W. Béthune, S. Kraus, and M. Bonnefoy for useful discussions, insightful questions, and helpful sharing of data. Y. A. was supported by the Leading Graduate Course for Frontiers of Mathematical Sciences and Physics. G-DM acknowledges the support of the German Science Foundation (DFG) priority program SPP 1992 “Exploring the Diversity of Extrasolar Planets” (KU 2849/7-1). G-DM and CM acknowledge support from the Swiss National Science Foundation under grant BSSGI0_155816 “PlanetsInTime”. Parts of this work have been carried out within the framework of JSPS Core-to-Core Program “International Network of Planetary Science (Planet2)” and the NCCR PlanetS supported by the Swiss National Science Foundation. This research was supported in part by the Japanese–German visitor program of PlanetS and by the visitor program of the SPP 1992.

APPENDIX

A. RELATION BETWEEN OUR MODELS AND CASE B

(STOREY & HUMMER 1995)

The work of Hummer & Storey (1987) and Storey & Hummer (1995) is often used to analyze accretion line intensities or their ratios in the stellar context (e.g., Köspál et al. 2011;
Antoniucci et al. 2017; Gutiérrez et al. 2020; see Section 4.3 of Rigliaco et al. 2015), and was recently used by Szulágyi & Ercolano (2020) for the planetary case. Storey & Hummer (1995) assume an equilibrium distribution of electrons. At high densities, this distribution is set by radiative de-excitation balancing collisional excitation. The Case B model used by Storey & Hummer (1995) for radiative recombination and ionization (Baker & Menzel 1938) assumes that the gas is optically thick to photons from the Lyman series but that all other transitions are optically thin.

The main reason why the approach of Storey & Hummer (1995) cannot be used here is an issue of timescales. In the immediate postshock region, the cooling timescale \( t_{\text{cool}} \) is comparable to—and not much longer than—the timescales setting the electron population \( t_{\text{pop}} \), whether this is coming from collisional or radiative processes. Namely, the cooling rate \( \Lambda \) is given by the energy difference between the levels\(^{15} \) \( \Delta E \) divided by the timescale for the transition, and the cooling timescale is set by the internal energy of the gas \( E_{\text{int}} \) divided by the cooling rate. Thus \( t_{\text{cool}} \approx E_{\text{int}}/(\Delta E/t_{\text{pop}}) \), such that \( t_{\text{cool}}/t_{\text{pop}} \approx E_{\text{int}}/\Delta E \), which is around unity for \( \Delta E \approx 10 \text{ eV} \) and the disequilibrium temperatures \( T \approx 10^3 \text{ K} \) found in the cooling region (Figure 6). Thus, no equilibrium, which would require \( t_{\text{pop}} \ll t_{\text{cool}} \), can be reached, making Storey & Hummer (1995) inapplicable to our case. For T Tauri stars the situation is different because the preshock gas is assumed to emit, and the temperature and density of a parcel of gas in the preshock region change on the free-fall (dynamical) timescale, which is sufficiently long for equilibrium to be established.

There is a further, secondary, issue with Storey & Hummer (1995) for the planetary case, concerning the optical depth of the emission lines in the postshock region from where they originate. As we have mentioned in Section 5.1 and shown in Figure 10, in several cases the \( H\alpha \) line is not optically thin in the postshock region, especially towards high \( n_0 \), which is more relevant for planets: from Equation (9), the ratio of the typical number densities is

\[
\frac{n_{0,\text{planet}}}{n_{0,\text{star}}} \approx \frac{10^{-5} M_1 \text{ yr}^{-1}}{10^{-8} M_\odot \text{ yr}^{-1}} \left( \frac{0.5 M_\odot}{5 M_\odot} \right)^3 \left( \frac{2 R_\odot}{2 R_1} \right)^3 \approx 300, \tag{A1}
\]

for the same filling factor. (For reference, the ratio of preshock velocities is

\[
\frac{v_{0,\text{planet}}}{v_{0,\text{star}}} \approx \sqrt{\frac{5 M_1}{0.5 M_\odot} \left( \frac{2 R_\odot}{2 R_1} \right)^2} \approx 0.1 \tag{A2}
\]

\(^{14}\)At low densities, it is radiative recombination (electrons recombining directly or indirectly into the \( n = 3 \) level) that populate the \( n = 3 \) level, but again balanced by radiative de-excitation.

\(^{15}\)In our case, the Ly\( \alpha \) transition is responsible for most of the cooling, such that \( \Lambda \approx \Lambda_{\text{Ly}\alpha} \), where \( \Lambda_{\text{Ly}\alpha} \) is given by Equation (B5) of Iida et al. 2001. Then, \( \Delta E = 10.2 \text{ eV} \). See Figure 6d for an example of \( \Lambda \).

from Equation (7).) Thus a fundamental assumption of Storey & Hummer (1995) is fulfilled in some cases but in several it is not.

A minor third difference is that in the cooling region below the hydrodynamic shock (i.e., in the Zel’dovich spike), the timescale for collisional de-excitation can become smaller (and thus set the population) than for radiative de-excitation. The electron population then tends to the Boltzmann equilibrium distribution and not to the equilibrium assumed in Storey & Hummer (1995). They were concerned mainly with lower densities, in which case the collisional de-excitations do not play a role, whereas they can in our case. These effects change the excitation degree and hence the intensity of the emitted \( H\alpha \).

In summary, there are several reasons why the tables of Storey & Hummer (1995) do not apply to the shock emission of accretion planets. This is why we have developed a time-dependent NLTE radiation-hydrodynamics model for the CPD surface shock (Aoyama et al. 2018) or the planet surface shock (this work).

**B. INVERSE RELATION BETWEEN THE SHOCK-MICROPHYSICAL AND PLANET-FORMATION PARAMETERS**

For completeness, we show in Figure 14 lines of constant \( \dot{M} \) and \( \dot{M}_p \) in the \((n_0, v_0)\) or \((n_0, \rho_0)\) plane. In the warm case, because of the larger radii, the upper right corner (high preshock density and velocity) is not reached, contrary to the cold case. Except for this, however, in both cases the same part of parameter space is covered.

**C. H ALPHA LUMINOSITY AS A FUNCTION OF ACCRETION RATE AND MASS: COLD-START FIT**

In Figure 15 we show contours as in Figure 9 but for the cold-start radii (colour and solid contours). They are very similar to the contours for the hot-start case.

**D. H ALPHA LUMINOSITY DERIVED FROM THE DATA OF WAGNER ET AL. (2018)**

Wagner et al. (2018) do not report the \( H\alpha \) luminosity of PDS 70 b explicitly but they write that they followed the approach of Close et al. (2014a). Thus their luminosity, assuming isotropic emission, should be given by

\[
L_{H\alpha} = 4\pi D^2 \times 10^{As} \times C_{H\alpha} \times 10^{-R_{H\alpha}/2.5} v_0 W_r, \tag{D3}
\]

where \( C_{H\alpha} = (1.14 \pm 0.47) \times 10^{-3} \) is the contrast of the \( H\alpha \) signal at the planet’s position to the signal from the primary star in the adjacent continuum (the \( R \) band; Wagner et al. 2018), \( R_{H\alpha} \) is the extinction in the \( R \) band, \( D = 113.43 \pm 0.52 \) pc is the distance of the system (Gaia Collaboration et al. 2018), \( V_0 = 2.339 \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2} \mu\text{m}^{-1} \) is the zero-point of the Vega magnitude system in the MagAO \( H\alpha \) filter and
Figure 14. Shock parameter space \((n_0, v_0)\) or \((\rho_0, v_0)\) covered by our grid (Equation (25)), using Equations (7)–(9) and the radius fit of Equation 1 for the warm (left panel) and the cold (right) population respectively. Lines of constant \(M\) (dashed) and \(M_p\) (solid) are labeled. We fix \(f_{\text{fill}} = 1\).

\[ W_f = 0.006 \, \mu\text{m} \] is the filter width (Close et al. 2014a), and \(R_A = 11.7 \pm 0.4\) mag is the R band magnitude of PDS 70 (Henden et al. 2015; Wagner et al. 2018), respectively. Taking the case of no extinction \((A_R = 0\) mag; see Footnote 10), we obtain for the companion

\[ L_{H\alpha} = (1.4 \pm 0.6) \times 10^{-6} L_\odot. \] (D4)

The relative error on this \(L_{H\alpha}\) is dominated by the relative uncertainty on the contrast \((C_{H\alpha})\). For the contrast itself we used value from the “combined image”. This value agrees with the value ofThanathibodee et al. (2019), \(L_{H\alpha} = (1.3 \pm 0.7) \times 10^{-6} L_\odot\) assuming the same distance.

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