Extremely local electric field enhancement and light confinement in dielectric waveguide

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The extremely local electric field enhancement and light confinement is demonstrated in dielectric waveguide with corner and gap geometry. The numerical results reveal the local electric field enhancement in the vicinity of the apex of fan-shaped waveguide. Classical electromagnetic theory predicts that the field enhancement and confinement abilities increase with decreasing radius of rounded corner \( r \) and gap \( g \), and show singularity for infinitesimal \( r \) and \( g \). For practical parameters with \( r = g = 10 \text{ nm} \), the mode area of opposing apex-to-apex fan-shaped waveguides can be as small as \( 4 \times 10^{-3} A_0 \) (\( A_0 = \frac{\lambda^2}{4} \)), far beyond the diffraction limit. This way of breaking diffraction limit with no loss outperforms plasmonic waveguides, where light confinement is realized at the cost of huge intrinsic loss in the metal. Furthermore, we propose a structure with dielectric bow-tie antenna on a silicon-on-insulator waveguide, whose field enhancement increases by one order. The lossless dielectric corner and gap structures offer an alternative method to enhance the light-matter interaction without metal nano-structure, and will find applications in quantum electrodynamics, sensors and nano-particle trapping.

I. INTRODUCTION

Strong light confinement in photonic devices can enhance the light field intensity, and then lead to very strong light-matter interaction. The enhanced light field is essential for a wide range of applications, such as quantum electrodynamics [1, 2], nonlinear optical effect [3], quantum optomechanics [4], optical sensors [5] and nano-optical tweezers [6]. Therefore, the surface plasmon in metal nanostructures is attracting more and more attentions for its unique ability to confine light in the deep subwavelength scale. Extreme strong field enhancements in surface plasmon are mainly attributed to the very small geometry size [7, 8], sharp corners [9–11] and nanoscale gaps [12–14], which has been stated in Ref.[15]. However, intrinsic loss due to internal damping of radiation in metal limits the explosion of practical applications.

For the case of dielectric, light is confined in the wavelength-scale photonic devices, with weak evanescent field interacting with outside. Great efforts have been dedicated to enhance the light field intensity by engineering the dielectric structure. In 2004, Almeida et al. have proposed the dielectric slot waveguide structure, and demonstrated a very effective way to enhance the light field in the void [16, 17]. Very recently, three-dimensional photonic crystal structure has been proposed to guide the light in the wedge-like waveguide with field enhanced at the apex [18]. Actually, the electric field enhancement in the vicinity of the dielectric corner is well known in electrostatics [19]. While in the studies of electromagnetic waves, the corner effect is noticed because it may lead to diffraction at corners and computation difficulties in rectangle-cross-section waveguides [20–22].

In this paper, we numerically demonstrated the extremely local electric field enhancement and light confinement in dielectric waveguide by the corner and gap structures. The waveguide with fan-shaped cross-section is proposed to utilize the corner effect [19–22] to enhance the local electric field, while the opposing apex-to-apex fan-shaped waveguides are studied to apply the gap effect [16, 17]. It is shown that the mode area can be as small as \( 4 \times 10^{-3} \) of diffraction limited area for practical geometry, superior to the confinement in hybrid surface plasmon waveguides [23]. Furthermore, we proposed a structure with dielectric bow-tie antenna on a silicon-on-insulator (SOI) waveguide to enhance the amplitude of electric field by one order. It is worth noting that the very sharp corner and closed gap give rise to singularity behavior of local field enhancement, which calls for theoretical efforts to study the quantum effects of dielectric at atomic scale [24–26].

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II. FIELD ENHANCEMENT BY CORNER

First of all, we proposed a fan-shaped waveguide to study the local field enhancement at the dielectric corner. As schematically illustrated in Fig. 1(a), the radius and angle of the fan are $R$ and $\theta$, respectively. Based on practical situations, the apex of fan is rounded corner with radius of curvature $r$. In our model, the waveguide is made by silicon and embedded in air, with permittivities of waveguide and air cladding being $\epsilon_d = 12.25$ and $\epsilon_c = 1$ at the working wavelength $\lambda = 1550$ nm. All the following results are obtained from the classical electromagnetic theory, which are solved by the Finite Element Method numerically, with commercial available software (COMSOL Multiphysics 4.3).

Figure 1(b) shows the electric field ($|E|$) distribution of the fundamental wedge waveguide mode with $R = 280$ nm, $r = 10$ nm and $\theta = 90^\circ$. Clearly, electric field is greatly enhanced in the vicinity of the apex of fan-shaped waveguide. The polarization of the wedge mode is mainly along the axis of symmetry of fan (i.e. $y$ axis), as indicated by the red arrows. Electric and magnetic field components along $x, y, z$ directions are displayed in Figs. 1(c)-(h), where we can find that only electric field component $E_y$ is enhanced in the vicinity of the apex of corner, showing local electric field enhancement.

In the following, we’ll analyze the properties of the wedge modes in detail. Two types of effective mode area of waveguide are introduced to measure the local field enhancement [27]

$$A_e = \frac{\int \int_{\text{all}} W_e(\mathbf{r}) d^2\mathbf{r}}{\max \{ W_e(\mathbf{r}) \}},$$

and spatial light confinement [27]

$$A_e = \frac{\int \int_{\text{all}} W_e(\mathbf{r}) d^2\mathbf{r}}{\left(\int \int_{\text{all}} [W_e(\mathbf{r})]^2 d^2\mathbf{r}\right)^{1/2}}.$$
Here $W_e(r)$ is the electric field energy density,
\[
W_e(r) = \frac{1}{2} \varepsilon(r)|E(r)|^2,
\]
with $|E(r)|$ and $\varepsilon(r)$ being electric field and dielectric permittivity, respectively. $A_e$ is more sensitive to $\max \{W_e(r)\}$, and is usually applied to quantify local field enhancement and estimate the enhanced spontaneous emission due to the Purcell effect [27]. $A_c$ characterizes the spatial extent of the field, which is not sensitive to the local field enhancement.

In Fig. 2(a), the effective mode index $N_{eff}$ and normalized mode areas $A_{e,c}/A_0$ are plotted against the radius of rounded corner $r$, where $A_0 = \lambda^2/4$ denotes the diffraction limited area of vacuum. When $r$ decreases from 100 nm to 0.1 nm, $N_{eff}$ decreases and $A_e$ increases monotonously with opposite trend. Note that $N_{eff}$ also represents the confinement of light field, since strong confinement means more energy confined in dielectric and leads to larger effect mode index. For large $r$, larger $r$ means larger dielectric cross-section area, so the confinement changes a lot with $r$. At very small $r$, the dielectric cross-section area does not change much, so $N_{eff}$ and $A_e$ show saturation when $r < 1$ nm.

On the contrary, $A_e$ increases with decreasing $r$ in the shadow region with $r > 13.5$ nm, similar to $A_c$, since the cross-section area of dielectric increases. But it decreases with $r$ when $r < 13.5$ nm, with an inflection at $r \approx 13.5$ nm. Carefully comparing the field distributions in the two regimes (see Figs. 2(b) and 2(c) with $r = 13$ nm and $r = 14$ nm), we found that the location of the maximum of the electric energy density ($\max \{W_e(r)\}$) is escaping from the core of the waveguide to the corner as $r$ decreases. For $r < 13.5$ nm, $\max \{W_e(r)\}$ at the corner increases with decreasing $r$, which shows very strong local electric field enhancement due to the corner structure. Compared to the case of $r = 10$ nm, the mode area $A_e$ for the sharper corner with $r = 0.1$ nm has been reduced to about 1/10.

In Fig. 3(a), the field confinement is further studied by the energy confinement ratios in silicon ($\eta_1$) and air ($\eta_2$), where $\eta_1(2)$ is defined as the ratio of the electric energy in the silicon (air) to the total electric energy and satisfies $\eta_1 + \eta_2 = 1$. The behavior of $\eta_1$ against $r$ agrees well with that of $N_{eff}$ (Fig. 2(a)), since more energy in silicon leads to larger $N_{eff}$. The curves of $A_e$, $N_{eff}$ and $\eta_1$ indicate that the dielectric corner does not lead to stronger field confinement, but just local electric field enhancement. Insets of Fig. 3(a) are the electric field energy densities for $r = 0.1$ nm and $r = 1$ nm, which demonstrate that the field enhancement is more localized to the corner for smaller $r$. In Fig. 3(b), the normalized electric fields ($|E|)$ along y axis for various $r$ are shown. The field profiles are similar except the enhancement at the corner ($y = 0$), where $|E|$ around the apex decays more drastically for smaller $r$.

In Fig. 4, the corner effect is studied for different waveguide size $R$ and corner angle $\theta$. For very small $R$ or $\theta$, $A_e$ and $A_c$ are both very large due to the weak confinement of light allowing for the small cross-section area of waveguide. When $R$ or $\theta$ increases, $N_{eff}$ increases monotonously, while $A_e$ and $A_c$ decrease first and then slowly increase as the
cross-section area increases. The shadow regions in Fig. 4 correspond to the max\(\{W(r)\}\) located inside the dielectric, while the white regions correspond to the local electric field enhanced around the apex of corner. For increasing \(\theta\), the wedge mode of fan-shaped waveguide is converted to the channel mode, as shown by the insets in Fig. 4(b).

III. FIELD ENHANCEMENT BY GAP

Gap effect for light enhancement and confinement, caused by large discontinuity of the electric field at high index-contrast interfaces, was studied in Refs. [16, 17], where low-index slot is embedded between two high-index rectangle waveguides. Similar to dielectric slot structures, we can expect further field enhancement in the double fan waveguide poited as apex-to-apex with an air gap \(g\) (Fig. 5(a)). As an example shown in Fig. 5(b), we see the very strong field confinement at the gap between the apexes with \(g = 10\) nm.

Dependences of the modal characteristics of the gap mode on \(g\) are displayed in Figs. 5(c) and 5(d). \(N_{eff}\) increases monotonously for decreasing \(g\), because the low-index air surrounding the apex is replaced by high-index dielectric. The trends of \(N_{eff}\) with different \(r\) are the same, and show saturation for very small gap \(g < 1\) nm. For a fixed \(g\), sharper corner gives rise to weaker light confinement in dielectric, which is consistent with the results of single fan-shaped waveguide (Fig. 2(a)). For the mode areas (5(d)), both \(A_e\) and \(A_c\) show great reduction when reducing the gap. These reveal that the slot structure leads to strong spatial confinement, as well as local field enhancement. However, \(A_c\) is saturated when \(g < 1\) nm for \(r \geq 10\) nm, while \(A_e\) is saturated only for \(r = 100\) nm. This indicates that the gap effect of apex-to-apex structure also depends on the sharpness of corners.
FIG. 4: Dependence of the effective mode index and normalized mode areas on \( R \) (a) and \( \theta \) (b), respectively. Insets in Fig. 4(b) are electric energy density \( W_e \) for \( \theta = 320^\circ \) and enlarged view of the \( \max \{ W_e(r) \} \) for \( \theta = 320^\circ \).

Due to the combined effects of corner and gap structure, we found that \( A_e \) can be as small as \( 10^{-5} A_0 \) for \( g = 0.1 \) nm and \( r = 0.1 \) nm. Compared to the best light confinement \( A_e' \approx 5 \times 10^{-3} A_0 \) in the hybrid dielectric-metal surface plasmon waveguide [23], \( A_e \) is 500 times smaller. It is notable that the maximum of electric field is always located at the corner of waveguide, which is very suitable for light to interact with matters outside the dielectric. The extreme light confinement and local electric field enhancement in the dielectric corner and gap would benefit various applications, such as the waveguide quantum electrodynamics, nanoparticle trapping, and bio-sensors.

IV. QUANTUM LIMITATIONS

From the tendency in Fig. 5(d), we can predict that the smaller effective mode area can be obtained by smaller gap or sharper corners. However, when the size of gap or corner structures is reduced to the atomic scale (sub-nanometer), the classical electromagnetic theory will break down and the quantum mechanical effects appear in three aspects:

1. The quantum size effect. The object with geometry smaller than atomic scale is meaningless. The sub-nanometer-scale geometry should be treated as atom cluster instead of regular corner or straight boundary. In the atomic scale, the optical response of atomic clusters can not be deduced by the classical dielectric constant of bulk.

2. The non-local effect. The atomic-scale wave-function of electrons can spread out from the boundary. Therefore, the response of the electron to electromagnetic wave is no longer local.

3. The quantum tunneling. When the gap between two apexes is sub-nanometer, the electrons may tunnel across the gap. Therefore, the electromagnetic field theory for dielectric with zero conductivity is not valid.

These quantum effects in metal nanostructures have been studied extensively recently both in experiments and theoretical studies [24 26]. In theory, the optical responses of those structures at sub-nanometer scale are studied under quantum model with time-dependent density function theory [24], or under semiclassical model with modified...
b) Electric energy density $W_e$ of the coupled mode in the bow-tie waveguide for $R = 280$ nm, $r = 10$ nm, $\theta = 90^\circ$ and $g = 10$ nm. $N_{\text{eff}}$ (c) and normalized mode areas (d) of the coupled mode as a function of $g$, for $r = 0.1$ nm, $r = 1$ nm, $r = 10$ nm, $r = 100$ nm, respectively.

electromagnetic theory by including hydrodynamic description of conducting electrons [25, 26]. It is demonstrated that the singular behavior is avoided in those theoretical models and experiments. Therefore, we could also expect the singular behavior to be removed by corrected models. However, the property of electrons in dielectric is significantly different from that in metal, so new models should be developed to study the optical response of the atomic scale dielectric structures.

Although the light confinement and local field enhancement may be limited by quantum effects, the apex-to-apex dielectric structure studied here is still superior to the plasmon waveguides: (a) Optical modes in the apex-to-apex fan waveguides are lossless, which is an incomparable advantage over plasmonic waveguides where light confinement is realized at the cost of huge intrinsic loss in the metal. (b) Even for the case of $g = 10$ nm and $r = 10$ nm where the classical Maxwell equations are valid, we get $A_e \approx 4 \times 10^{-3} A_0$ which is smaller than that of hybrid plasmonic waveguide [23].

V. DIELECTRIC BOW-TIE ANTENNA

The corner and gap effects have been demonstrated above for dielectric waveguides with different cross-section geometry, where light propagates perpendicularly to the cross-section. Now, we turn to study the field enhancement in the dielectric bow-tie (DBT) antenna structure, where the corner and gap structures are not uniform along the waveguide. This DBT antenna can be integrated with various photonic structures, such as waveguide and microring, and is very potential for practical applications.

As shown in Fig. 6(a), a dielectric bow-tie antenna is placed on a silicon-on-insulator (SOI) waveguide with rectangle cross-section of 450 nm $\times$ 250 nm. The bow-tie antenna consists of two opposing apex-to-apex silicon isosceles triangles,
the height and thickness of which are denoted as $h$ and $t$, respectively. The vertex angle of the nano-triangle is $100^\circ$, and the radius of the round is $r = 10$ nm. Due to the effects of corner and gap, we can expect a strongly enhanced field in the gap of the DBT antenna. To investigate the field enhancement, the three-dimensional model with 1550 nm fundamental TE-polarized mode loaded in the waveguide is simulated numerically. From the field profiles in Figs. 6(b) and 6(c), the electric field is greatly enhanced in the gap of DBT antenna compared to that in the waveguide. Note that the corner and gap effects can only be expected for the electric field along the axis of symmetry of the corner, and there is no field enhancement for the TM-polarized mode.

In Fig. 6(d), the maximum of $|E|$ at the center of the gap of DBT antenna is plotted as a function of $h$ for $g = 2$ nm and $g = 10$ nm, respectively. The evanescent field of the fundamental TE mode at the top surface of the waveguide without DBT antenna is also shown by the dashed line. We can find that the maximum of $|E|$ is enhanced by 20 and 8 times for $g = 2$ nm and $g = 10$ nm, respectively. In both cases, the enhancement factor is insensitive to $h$. The narrower gap leads to the stronger electric field enhancement, which is consistent with the results in the apex-to-apex fan waveguides. The DBT antenna is easy for fabrication and compatible with CMOS technology, thus it will be useful as building blocks in integrated photonic circuits.

VI. CONCLUSION

In summary, we demonstrate the corner- and gap-enhanced local electric field in dielectric waveguide. As an example, in opposing apex-to-apex fan-shaped waveguides with $r = g = 10$ nm, the mode area of fundamental wedge mode can be as small as $4 \times 10^{-3} A_0$ ($A_0 = \lambda^2/4$), far beyond the diffraction limit. The numerical results indicate
that the field enhancement and confinement abilities increase with decreasing radius of rounded corner \( r \) and gap \( g \), and show singularity for infinitesimal \( r \) and \( g \). The singularity behavior calls for theoretical efforts to study the quantum effects of dielectric at atomic scale. Furthermore, we propose a structure with dielectric bow-tie antenna on a silicon-on-insulator waveguide, the field enhancement of which is improved by one order. Although the waveguide studied in this paper is focused at 1550 nm, we should be aware that the corner and gap effects are broadband. The lossless dielectric corner and gap structures offer an alternative method to enhance the light-matter interaction without metal nano-structure, and will find applications in quantum electrodynamics, sensors and nano-particle trapping.

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