State-of-Charge estimation of Li-ion battery at different temperatures using particle filter

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Abstract: State-of-Charge (SOC) estimation is one of the fundamental functions undertaken by Battery Management System (BMS) in an Electric Vehicle (EV) to assess the residual service time of the battery during operation. Thus, an accurate model of the battery that efficiently describes its dynamic characteristics is necessary for precise SOC estimation. The variation in temperature affects battery parameters, and consequently, the estimation of SOC is subject to change in temperature. In this paper, the identification of parameters of battery model is considered as an optimisation problem and solved using meta-heuristic Ageist Spider Monkey Algorithm (ASMO) under the influence of varying temperature. The developed model is used for SOC estimation using three Recursive Bayesian filtering based adaptive filter algorithms. Further, the efficiency of the implemented adaptive filter algorithms is compared in terms of solution quality and computation time required for evaluation of SOC.

Introduction
The annual increase in anthropogenic Green House Gas (GHG) emission since last one decade has gained immense attention and concern around the world. Energy production (47%), Industry consumption and life by protecting it from any physical damage. BMS can monitor the measured voltage, current, and temperature to assess the state of the battery such as State-of-Charge (SOC), State-of-Health (SoH), and State-of-Power (SoP). SOC reflects the residual capacity of the battery and is considered as one of the essential factors to alleviate the risk of over/under charging/discharging during operation. Mathematically, SOC can be defined as the ratio between the remaining charge to the total charge of the battery.

\[
\text{SOC}(t) = \text{SOC}(t_0) - \frac{1}{Q_0} \int_{t_0}^{t} \eta(I(t)) dt
\]

Here, \(\text{SOC}(t_0)\) represents the initial value of SOC, \(I\) represent the instantaneous battery terminal current which is considered positive during discharging and negative during the charging process; \(t\) is time, and \(Q_0\) stands for the nominal capacity of the battery.

In EVs, function of estimating battery SOC is analogous to fuel gauge in the conventional vehicles, which indicates how much energy is remaining inside the battery to power the powertrain in an EV. But no sensor has been developed till now which can calculate a value of SOC directly [2]. BMS utilises measured voltage, current and temperature values for estimation of SOC of the battery. Estimation of SOC is a challenging task as the performance of the battery is influenced by variation in temperature, number of charge/discharge cycles and aging of the battery.

In literature, various methods for estimation have been studied by researchers in recent years. Primarily these methods can be classified as follows [2]: (i) Conventional methods (ii) Adaptive Filter Algorithms (iii) Learning Algorithms (iv) Non-linear observers, depending upon techniques used for estimation purpose. The conventional methods utilise measurements such as current, voltage and resistance for estimation. These methods are easy to implement and require less computation power but provide inaccurate estimate due to the presence of high uncertain disturbances during evaluation [3]. A model-based assessment has been carried out using an adaptive filter algorithm which depends on the accuracy of the model of battery [4, 5]. A significant amount of data is required for training and testing purpose while implementation of learning algorithms [6]. The demand for high computation power and memory space makes these methods unsuitable for online estimation of SOC. The nonlinear observers are designed to handle high non-linearity in the systems [7].

In this paper, first-order RC equivalent battery model is considered to predict the behaviour of the battery under dynamic conditions. Identification of battery parameters is considered as an optimisation problem and solved using Ageist Spider Monkey Optimization (ASMO). Next, SOC estimation is performed by three model-based adaptive filter algorithms viz. the non-linear derivative of Kalman Filter (KF) (both Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF)) and with Particle Filters (PF). In Section II, the equivalent circuit of Li-ion battery is established for implementation of SOC estimation techniques. Merits and demerits of estimation techniques with detailed description for PF implementation for SOC estimation are discussed in Section III. Comparison of EKF, UKF, and PF in terms of error and execution time is done in Section IV. The performance of estimation techniques is concluded in section V.

2 Battery modelling
The primary function of BMS is to estimate the SOC of the battery for which accurate model of battery is required when estimation is done using adaptive filters algorithms. Different forms of the
battery models are developed and studied by researchers in the literature. In ref. [8], the comparison is made between the different order of equivalent circuit and conclusion is made that first order RC battery model can achieve good accuracy with minimal complexity.

As shown in Fig. 1, the first order RC model includes open circuit voltage \( V_0 \), instantaneous voltage drops resistances \( R_0 \), diffusion resistance \( R_1 \) and diffusion capacitance \( C_1 \). Mathematical equation for dynamic model of battery can be expressed by the following equations:

\[
\frac{dI_R(t)}{dt} = - \frac{I_R(t)}{R_0C_1} + \frac{I(t)}{C_1} \tag{2}
\]

\[
V_M(t) = V_d(t) - I(t)R_0 - I_R(t)R_1 \tag{3}
\]

Here, \( I_R(t) \) represents the current following through RC network and \( V_M(t) \) represents the voltage across the battery terminal. The values of battery parameters are not constant and depend upon many operating factors alike C-rate, SOC, temperature variations, and a number of cycles [9]. To evaluate the effect of temperature on the performance of battery, parameters are considered to be dependant on temperature. Fig. 2 shows the plot of relationship between \( V_0 - \text{SOC} \). From the given plot, it can be observed that \( V_0 \) varies with respect to SOC but remains constant for different temperatures.

Hence, value of \( V_0 \) depends on SOC but is independent of temperature.

In [10] author suggested identification of battery parameters values as an optimisation problem. Various optimisation techniques alike Genetic algorithm [11], Particle swarm optimisation [8] were applied for identifications of battery parameters. According to no free lunch theorem, a single optimisation algorithm cannot be considered for solving all optimisation problems. In [12], the author compared six different optimisations and concluded that ASMO is significantly reliable and robust for identification of parameters of the battery. In this paper, battery parameters values under different temperature are identified using ASMO. The objective function for optimisation techniques is to minimise the mean absolute error (MAE) between model terminal voltage \( V_M^b \) obtained using predefined battery model and measured terminal voltage \( V_M^e \) obtained from manufacturer’s catalogue. Mathematically, the objective function can be expressed by the following equation [13]:

\[
f(x) = \frac{1}{n} \sum_{k=1}^{n} |V_M^b - V_M^e| \tag{4}
\]

After obtaining the values of parameters, merits and demerits of the non-linear variant of KF are discussed in the next section, and SOC estimation procedure using PF is studied in detail.

3 SOC estimation techniques

Adaptive filter algorithms perform SOC estimation using the model of the battery under dynamic condition. Adaptive filter algorithms recursively adjust the states of system by minimising the mean of the squared error between current observed output and predicted model output from current input value and previous predicted states. For the implementation of adaptive filter, the battery model defined in the continuous-time domain is converted to discrete-form as shown by the following equation:

\[
x_{k+1} = f(x_k, I_k, v_k)
\]

\[
y_{k+1} = h(x_{k+1}, I_{k+1}, v_{k+1})
\]

Here \( x_k \sim N(\bar{x}, \Sigma_x) \) stands for the unobserved state vector of the system with \( \bar{x} \) mean and \( \Sigma_x \) state covariance matrix, \( I_k \) denotes the observed input vector and \( v_k \) stands for measured output vector at time index \( k \). Functions \( f(\cdot) \) and \( h(\cdot) \) are non-linear state and non-linear measurement functions respectively. \( w_k \sim N(\bar{w}, \Sigma_w) \) and \( v_k \sim N(\bar{v}, \Sigma_v) \) are random variables with Gaussian distribution representing noise in process and measurement respectively. A battery system is described using non-linear stochastic difference equation expressed as:

\[
\begin{bmatrix}
SOC_{k+1} \\
I_{R,k+1}
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 \\
0 & e^{-\frac{\Delta t}{\tau}}
\end{bmatrix}
\begin{bmatrix}
SOC_k \\
I_{R,k}
\end{bmatrix}
\begin{bmatrix}
-\frac{nA(t)}{Q_0} & 0 \\
1 & e^{-\frac{\Delta t}{\tau}}
\end{bmatrix}
\begin{bmatrix}
I_k + w_k \\
v_k
\end{bmatrix}
\tag{5}
\]

\[
V_M^k = V_o - I_R(t)R_o - I_{R,k+1}R_1 + v_k
\tag{6}
\]

The battery system is a highly non-linear system hence non-linear variant of KF is utilised for estimation of states of the battery. In EKF, Gaussian Random Variable (GRV) used for approximating state distribution and then these GRV is propagated analytically through the non-linear system linearised using first-order Taylor Series expansion. Generally, EKF linearisation using the first-order approximation works relatively efficiently and requires small computation power for a slightly non-linear system. However, with the highly non-linear system, linearisation introduces significant error in the value of the true posterior mean and covariances of the transformed GRV. Sometimes linearisation might cause the sub-optimal solution and create instability (divergence) in the filter [13]. If EKF is linearised using Jacobian analysis to increase accuracy, then the computation power required also increases. The UKF uses unscented transformation to address the approximated linearisation issues of the EKF. True mean and covariance of the
GRV captured by a minimal set of cautiously determine sample points known as sigma points. Subsequently, these Sigma points propagated through the actual non-linear system, captures the posterior mean and covariance of GRV with accuracy up to 3rd order Taylor series expansion with slightly increased computational complexity. Hence no explicit calculation of Hessians or Jacobians are considered to solve state estimation problem for the high non-linearity systems with non-Gaussian noises \([15]\). In this next subsection, detailed description of PF for estimation of SOC is discussed.

### 3.1 Particle filter (PF)

PF is a recursive Bayes filter based on Monte Carlo techniques used for estimation of states of the system. PF represents the required state probability distribution by a set of random samples known as particles with associated weights. As a number of particles are significant, Monte Carlo characteristics represent the probability distribution of states and the solution approaches to solve the problem of the Bayesian estimation.

Algorithms for implementation of PF are as follow:

1. **Initialisation**: Randomly draw \(N\) initial state particles \(x_i(1, 2,\ldots N)\) from probability distribution \(p(x,\Sigma)\) and initial weight for particular particles is specified as \(1/N\). Number of particles depend upon type of system and computation cost. Threshold for resampling can be initialised as \(N_{\text{thr}} = \frac{2}{3}N\). For \(k = 1, 2\ldots\)

2. **Importance sampling**: Update the value of states using state equation and update the weight of the particle according to the following equation

\[
w_k = w_i \times \frac{1}{\Sigma_i} e^{-\frac{(x_k-\bar{x})^2}{2\Sigma_k^2}}
\]

3. **Normalisation**: The overall weight of all samples is normalised to one using the following equation:

\[
w_k^* = \frac{w_k}{\sum_i w_i}
\]

4. **Resampling**: The threshold of resampling is defined as the effective sample \(N_{\text{eff}}\) which can be defined as:

\[
N_{\text{eff}} = \frac{1}{\sum_i w_i^*^2}
\]

If the effective sample is below the given \(N_{\text{thr}}\) then the particle is resampled to get a new particle with weight \(1/N\).

5. **State estimation**: Calculate the states as

\[
x_{k+1} = \sum_i w_i x_i
\]

The algorithm is executed iteratively, and estimation of SOC can be performed. In the next section, comparison is performed between EKF, UKF and PF techniques for estimation of SOC.

### 4 Simulation results

A 18650-cylindrical type commercial Li-ion (LiFePO4) battery of nominal capacity 2.3 Ah, and nominal voltage 3.3 V is considered for estimation of SOC \([16]\). The typical driving cycle is applied to the battery in the laboratory, for simulating its dynamic discharge behaviour during operation of EVs. Federal Urban Drive Schedule (FUDS) cycle profile representing driving conditions inside the city whose one cycle is of 1369 seconds duration in which a light vehicle will travel for 7.45 miles with a speed of 19.59 mph is considered for the analysis. FUDS is repeated serval times when the fully charged battery (100% SOC) is discharged till the battery reaches to end-of-discharge voltage (2 V). FUDS cycle is applied to the battery for estimation of SOC with varying values of temperatures. Combined process of charging and discharging takes place during one cycle of FUDS. This process is repeated under different conditions of temperatures and corresponding values of terminal voltage and current is measured. Fig. 3 shows a plot for terminal voltage for the given current profile at room temperature of FUDS cycle. For validation purpose, the value of SOC of battery under different temperature is evaluated using current integration based Coulomb Counting method which is considered as reference for the estimated SOC values. The value of SOC calculated using an accurate current sensor with an initial value of SOC set as 100% is shown in Fig. 4 for different values of temperature.

For the execution of estimation of SOC using EKF, UKF, and PF, the value for covariances of process noise and measurement noise are set by a trial-and-error process in such a way that the estimation achieves convergence. Noises are assumed to be white noise, hence mean value of process and measurement noise are set zero also the value for the covariance of process noise \(\Sigma_w\) and measurement noise \(\Sigma_v\) are set as 0.2. Assumed initial value from

![Fig. 3 FUDS cycle profile at room temperature](image)

![Fig. 4 Results for SOC estimation using different estimators under different temperature conditions](image)

**Table 1** Parameters of the battery model under different temperature conditions

| Temperature | \(R_0\)  | \(R_1\)  | \(C_1\)  |
|-------------|---------|---------|---------|
| -10°C       | 0.2607  | 0.0714  | 9.8875  |
| 0°C         | 0.1928  | 0.0821  | 31.3255 |
| 25°C        | 0.1552  | 0.0620  | 99.6597 |
| 40°C        | 0.1515  | 0.0528  | 147.6187|

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using EKF, UKF and PF for −10°C, 0°C, 25°C and 40°C temperatures, respectively. Error plots indicate that corresponding to the low value of SOC, estimated SOC slowly diverges away from reference SOC. Error in measurement of quantities using sensors and error due to Coulombic counting methods is the reason for the difference between values of SOC. From Fig. 4 it can be observed that PF provides less error compared to the EKF and UKF. Error obtained by PF at less value of SOC is about 40% less than the value obtained by EKF and UKF.

Further, the comparison between EKF, UKF and PF is performed by determining Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) between reference SOC value and estimated SOC values. Mathematically, the RMSE can be expressed by the following equation:

\[
RMSE = \sqrt{\frac{1}{N-1} \sum_{k=1}^{N} (SOC^E_k - SOC^M_k)^2}
\] (12)

Mathematically, MAE can be expressed by the following equation:

\[
MAE = \frac{1}{N} \sum_{k=1}^{N} |SOC^E_k - SOC^M_k|
\] (13)

Here, \(SOC^M_k\) stands for measured SOC value from Coulomb Counting method and \(SOC^E_k\) stands for estimated SOC values from EKF, UKF and PF. Fig. 6 shows the value of RMSE and MAE for estimated SOC from EKF, UKF and PF under different temperature conditions. Randomisation in selecting initial particle of the PF from state distribution is the reason of high RMSE values for PF. It can be seen in Fig. 4 also that the value of PF is quite high in the initial phase of the graph. Otherwise the error is low in case of PF which shows the merits of PF over EKF and UKF. PF uses probability density function rather than linearisation for approximation, and thus, enables high accuracy in the estimation of SOC.

Time analysis is also done for comparing the execution time taken by EKF, UKF and PF in estimation of SOC. Fig. 7 shows values of execution time take by EKF, UKF and PF in estimation of SOC under different temperature conditions. PF takes less execution time in comparison to the EKF and UKF and proves to be a reliable technique for online estimation of SOC.

5 Conclusion
SOC estimation is the fundamental task performed by BMS for reliable operation of the battery in EVs. In this paper, three adaptive filter algorithms viz. EKF, UKF, and PF are applied for estimation of SOC. Further, the efficiency of the implemented adaptive filter algorithms is compared in terms of solution quality and computation time required for evaluation of SOC. From the analysis, it can be concluded that EKF gives more accurate results compared to UKF and PF. The reason for the same is that EKF works on the principle of an approximating dynamic process of systems whereas other filters approximate noise distribution of the system states. These filters perform more accurately for highly nonlinear systems whereas battery model considered is this study is mildly nonlinear. When the analysis is performed in terms of computation time, PF filter proved to be faster than other filters which makes it a reliable choice for online estimation of SOC.

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