Strongly interacting particles on an anisotropic kagome lattice

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Abstract. We study a model of strongly interacting spinless fermions and hard-core bosons on an anisotropic kagome lattice near 2/3-filling. Our main focus lies on the strongly anisotropic case in which the nearest-neighbor repulsions $V$ and $V'$ are large compared to the hopping amplitudes $|t|$ and $|t'|$. When $t = t' = 0$, the system has a charge ordered insulating ground state where the charges align in striped configurations. Doping one electron or hole into the ground state yields an anisotropic metal at $V' > V$, where the particle fractionalizes along the $V'$-bonds while propagates along the $V$-bonds in a one-body like manner. The sixth order ring exchange processes around the hexagonal unit of the lattice play a crucial role in forming a bound state of fractional charges.

1. Introduction

Triangular and kagome lattice systems are well known examples of geometrically frustrated lattice systems. Both consists of triangles which share edges in the former and corners in the latter case. The classical Ising antiferromagnet on these lattices has an extensive ground-state degeneracy and thus a finite zero temperature entropy, which amounts to $S_{tr} = 0.3383R$ [1, 2] for a triangular lattice and $S_{kag} = 0.50183R$ [3] for a kagome lattice which is more than half of the maximum paramagnetic value. This indicates that the model remains classically disordered throughout the whole temperature range [1, 3]. Usually, such disordered classical states are highly sensitive to the quantum fluctuations which lift the degeneracy. Small perturbations lead to a high density of low-lying excitations, and interesting physical effects can emerge. In $S = 1/2$ quantum spin system on a kagome lattice, it is still a controversial issue what kind or order or lack or order is realized (see [4] and citations therein).

In the present paper, we deal with a "partially" frustrated kagome lattice with anisotropic interactions. In particular, we consider the case where the interactions along bonds in one direction are larger than in the other two directions. In such case, we find a residual ground-state degeneracy in the classical Ising antiferromagnet which is smaller as in the case of the regular kagome geometry. We analyze the anisotropic kagome lattice near 2/3-filling model by means of a strong coupling expansion. We show that quantum fluctuations in the strongly interacting limit lead here to confined fractionalized excitations and a dimensional tuning. The physical mechanism can be compared to the one we found recently on the anisotropic triangular lattice near half filling [5].
2. Model

We introduce a basic $t$-$V$ model Hamiltonian which can apply to both fermionic and bosonic systems as,

$$
H_{t-V} = \sum_{\langle ij \rangle} \left( -t_{ij} c_i^{\dagger} c_j + \text{h.c.} + V_{ij} n_i n_j \right).
$$

Here, $c_j$ are annihilation operators of fermions/bosons and $n_j = c_j^{\dagger} c_j$ are number operators. The interactions act only between nearest-neighbor (nn) pairs of sites indicated as $\langle ij \rangle$. The anisotropy of the hopping amplitudes and repulsion strengths are given by $t_{ij} = t / V_{ij} = V$ for diagonal bonds and $t_{ij} = t' / V_{ij} = V'$ for vertical bonds as shown in Fig. 1. We consider an $N_x \times N_y$ lattice with $N = 6N_x N_y$ sites.

3. Ground state at 2/3-filling

Let us focus on the 2/3-filled case with $V' > V$ and consider the classical limit $t = t' = 0$. Each of the corner sharing triangles consists of two diagonal and one vertical bond. The interaction energy is minimized if we place two particles on each triangle avoiding any connection along the vertical bond. The particles form stripes along the $V$-bonds, as shown in Figs. 1(a)-(c). We have two different choices at each traversed triangle and thus the resulting ground-state degeneracy is $2^{N_x}$. This can be compared to case of the corresponding anisotropic triangular lattice [5]. These stripes are stable against quantum fluctuation, so that in case of small but finite hopping $t, t' \ll V, V'$, the system is an insulator.

Now, we introduce a small but finite $t = t' \neq 0$, $t/V' = t'/V' \rightarrow 0$, and consider the perturbation effect in terms of $t/V'$-terms. We find that the afore mentioned degeneracy is not lifted up to fifth order. The leading correction are the sixth order perturbation processes along the hexagons as shown in Fig. 1(d); there are four particles per hexagon, and the configurations are classified into two types, (A) and (B). The contribution from these processes including both the clockwise and anti-clockwise ones are summed up as,

$$
E(A) = -(-)^f t^2 t'^4 \left( \frac{1}{3V' - 2V} \right)^2 \left( \frac{1}{V'} + \frac{1}{2V' - V} \right)^2 \left( \frac{1}{V'^2} \right) = -\frac{1}{4V'^4(2V' - V)}
$$

$$
E(B) = -(-)^f t^2 t'^4 \frac{9}{V'^3(3V' - 2V)(2V' - V)}
$$
Figure 2. (a) Energy difference between A and B hexagons of the sixth order perturbative ring exchange, $E_A - E_B$. (b) Phase diagram of the strong coupling limit, $V'/V \gg t, t'$.

Here, $(-)^p$ denote the fermionic exchange sign when we consider the fermions. Therefore, the sign of both types of energy correction becomes minus when we consider the hard core bosonic case. If we fill all the hexagons with (A) configuration, the horizontal stripe state emerges while for (B) we have diagonal stripes. Mixing (A) and (B) configurations yields non-regular stripes as shown in Fig. 1(c).

Let us estimate the energy corrections as a function of $V'/V$. Fig. 2(a) shows that at around $V'/V = (V'/V)_c \sim 1.473$ there is a change of sign in the energy difference, $E_A - E_B$. Since $E_A$ and $E_B$ represents the energy of horizontal and diagonal stripes, respectively, $(V'/V)_c$ corresponds to the phase transition point. The schematic phase diagram in the strong coupling limit is shown for the hard-core bosonic and fermionic cases in Fig. 2(b). Note that we excluded other quantum states that might be dominant at $V' \sim V$; the most possible candidate is the plaquette state, which is the state where half of the hexagons are fully occupied, and the third order exchange along the hexagon will mix the manifold of these configurations to gain energy [6]. Even if these states violate the stripe-based fractionalized state near $V' \sim V$, for larger $V'/V$, we confirm that the horizontal stripe remains dominant compared to any other states due to the anisotropy energy of $V'$s.

The competition of a plaquette state and stripes reminds us of the similar case found in the triangular lattice system, where the pinball liquid state[7] and the super solid state[8] are dominant at $V' \sim V$ for the fermions and bosons, respectively. These states retain symmetry of three directions in the lattice structure. When $V' \gg V$, these states are replaced by ordered stripes.

4. Particle-doping

Now, let us restrict ourselves the case of extremely large anisotropy, $V' \gg V \gg t, t'$. First we consider the case of doping one particle to 2/3-filling. Since all neighboring sites are completely filled, it is not possible to move the doped particle itself. The only possible action allowed is to move the nearest-neighbor particle in the vertical direction, which does not change the interaction energy. Once the neighboring one particle moves, its neighboring particle can move one site further. Thus, the particles along the doped chain can move one by one collectively. Let us focus on the two vertical interacting bonds added by the doping which are marked in bold lines in Fig. 3(a). The above mentioned propagation corresponds to the next-nearest neighbor hopping of these bonds. If we consider the fermionic case, these bonds each carry $e/2$-charges and are regarded as fractional charges. As for the bosons, it corresponds to dispersing spinons.

Next, by removing one particle (doping a single hole), the same kind of fractionally charged
Figure 3. Schematic description of the state where a single (a) particle or (b) hole is doped to the $2/3$-filled horizontal striped ground state on the anisotropic kagome lattice. Shaded hexagons correspond to those replaced from hexagon A to hexagon B when two fractions of vertical bonds separate in the vertical direction.

excitations occur. In addition, the doped hole itself can propagate freely along the stripes when it is not fractionalized. The situation is very similar to the case of the anisotropic triangular lattice. Note that in the triangular lattice system, both particle and hole-dopings showed coexistent fractional and free-particle propagations, while in the kagome lattice only the holes are allowed for two different types of propagation.

We now remind that in the triangular lattice system, the fractional charges are confined by a linear potential terms that originates from fourth order perturbation processes. In the present kagome lattice, a similar confinement takes place in sixth order perturbation. When the bonds separate to distance $d$ in unit of lattice spacing, the $4d$-hexagons are replaced from A to B for fermions and B to A for bosons. Therefore the energy rises by $4|E_A - E_B|d$. This energy correction, even though very small in order, works as a linear potential to these fractionalized bonds, so that the distance between two fractions, $d$, remains finite.

5. Summary
We find an ordered ground state with stripe configurations of the ‘partially’ frustrated anisotropic kagome lattice at $2/3$-filling in the strong coupling region. This is induced by the quantum fluctuation from the classical disorder. The ring-exchange processes along the hexagonal units plays a key role here. When a particle is doped it fractionalizes into two degrees of freedom and move separately in one direction. This is regarded as a collective many body behavior, while the system remains insulating in the other directions. If we dope a hole, an additional propagation of the doped charges as free particles along the stripes is found. The two different types of dynamics are regarded as dimensional tuning as discussed in the similar behavior of the fermionic system on a triangular lattice [7]. We expect an extention of the present idea to the cases of other partially frustrated lattices as well.

References
[1] Wannier H Phys. Rev. 79(1950)357
[2] Houtappel R M F Physica Amsterdam 16 (1950) 425
[3] Kano K Naya S Prog. Theor. Phys.10 (1953) 158
[4] Misguich G Sindzingre P J. Phys. Condens. Matter 19 (2007) 145202
[5] Hotta C Pollmann F Phys. Rev. Lett. 100 (2008) 186404
[6] Moessner R Sondhi S L Phys. Rev. B 63 (2001) 224401
[7] Hotta C Furukawa N Phys. Rev. B 74 (2006) 193107
[8] Wessel S Troyer M Phys. Rev. Lett. 95 (2005) 127205; Heidarian D Damle K Phys. Rev. Lett. 95 (2005) 127206; Melko R G Parameswaran A Burkov A A Vishwanath A Sheng D N Balents L Phys. Rev. Lett. 95 (2005) 127207