Electron system correlated by the zero-point field: physical explanation for the spin-statistics connection

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Abstract. Which is the physical agent behind the antisymmetry of the electron state vectors? With the purpose to find an answer to this key question, we analyze the stationary states of a system containing two noninteracting electrons, using the tools of stochastic electrodynamics. In previous work, the resonant response of two particles to common modes of the random zero-point field has been shown to lead to the nonfactorizability of the composite state vector. Here we extend the analysis to particles with spin. When two electrons constitute a single system, a correlation is established between their dynamical variables through the common relevant modes of the zero-point field, which acts as a mediator. An analysis of the exchange properties of the bipartite state vectors obtained is shown to lead to the connection between spin and symmetry. The conclusion is that due consideration of the vacuum field in first quantization leads to the corresponding statistics for an assembly of electrons.

Keywords: Zero-point radiation field, foundations of quantum mechanics, stochastic electrodynamics, spin-statistics, Pauli Principle

1. Introduction

One of the most fruitful tenets of quantum mechanics is the link between spin and statistics for an assembly of equal particles, and the related exclusion principle for fermions. The spin-statistics theorem constitutes a core support for some of the most successful applications of quantum theory, including the whole of atomic, molecular and nuclear physics, and of course, quantum statistical physics. Yet despite its central importance, it is conventionally taken in quantum mechanics as a given law of nature, an empirical ingredient distinctive of quantum systems. Pauli himself, the author of the first ‘rigorous’ proof of the spin-statistics theorem, expressed his concern and dissatisfaction with the state of affairs on this matter in his Nobel lecture and subsequently [1]-[3]—already over six decades ago. Various explanations have been proposed to justify the postulate, most of which resort to mathematical assumptions that replace the usual arguments with some even more formal ones, such as topological properties of 3D space,[4] or conditions on the state vectors imposed by permissible representations of the permutation group.[5] A particular case of the latter, of relevance for the present work, involves the inclusion of the internal (spin) coordinates among the parameters affected by the exchange operation. [6]

There are numerous other proposals, both relativistic and nonrelativistic (see e.g. Refs. [7]-[14], and additional references in Ref. [6]), but none has gained general acceptance up to now.

Here we tackle the problem of the spin-symmetry relation from of a physical perspective, as opposed to a formal one. The task is accomplished by making recourse to results derived on the
basis of stochastic electrodynamics (SED). [15]-[18] We study in detail the case of a system of two electrons, yet the results are seen to extend naturally to an arbitrary number of particles and to any spin value.

We recall that according to SED as presented extensively in Ref. [18], (nonrelativistic) quantum mechanics emerges as a result of the permanent interaction of otherwise classical particles with the random electromagnetic field at zero temperature (the zero-point field, ZPF). It should be stressed that at the formal level, the final descriptions of SED and usual quantum mechanics are indistinguishable (see Ref. [18]); yet the SED approach serves to disclose key physical elements that are not apparent in the usual formalism, thereby providing interesting clues about the physics behind quantum mechanics. The present work represents a further step in this direction.

A basic finding of SED is that, when the particle-plus-field system has reached a stationary quantum regime, the particle responds linearly and resonantly to a given set of field modes, precisely the modes involved in the transitions between states. As has been shown in Ref. [18], in the two-particle case it is the interaction between the particles mediated by common (transition) ZPF modes what correlates their motion and leads to entangled states. A further key result to be used in this paper, is the emergence of the electron spin as a consequence of the interaction of the particle with polarized modes of the vacuum field, as described in Refs. [18] and [19]. This allows us to address the problem of the two electrons with spin and find the corresponding correlated states. The symmetry properties of the states are thus found to be intimately associated with the particle spin.

The structure of the paper is as follows. We start by briefly recalling the main elements of SED that are needed for subsequent derivations. The description of the random ZPF must now include an angular momentum component, with its corresponding degree of freedom. First we present the solutions for the one-particle stationary states. Next, we show for the two-particle stationary state how the nonfactorizable state vectors result from the correlations generated by the ZPF-mediated interaction between (otherwise noninteracting) particles, due to their common transition frequencies. Finally, an analysis of the exchange properties of these bipartite state vectors is shown to lead to the connection between spin and symmetry. The extension of this result to systems with more than two particles is then discussed. The paper ends with a discussion of the physical implications of the results obtained.

2. Stationary SED solutions. The single-particle case

Let us briefly recall that linear SED studies the dynamics of a charged particle embedded in the random ZPF and acted on by an arbitrary external conservative (binding) force \( f(x) \). [15], [18] The standard starting point is the Abraham-Lorentz equation,

\[
\dot{m}\vec{x} = f(x) + m\tau\dot{\vec{x}} + e\vec{E}(x,t) + \frac{e}{c}c\times\vec{B}(x,t),
\]

where \( m\tau\dot{\vec{x}} \) (with \( \tau = 2e^2/3mc^3 \)) represents the radiation reaction force, and the Lorentz force is due to the random ZPF. The electric component of this field is written as

\[
\vec{E}(t) = \sum_{k,\lambda} \tilde{E}(\omega)\epsilon_\lambda^\alpha a^0(\omega)e^{i\omega t-k\cdot x} + c.c.,
\]

where the value of the (real) coefficient \( \tilde{E}(\omega) \) corresponds to a field having an energy \( \mathcal{E}(\omega) = \hbar\omega/2 \) per mode, and the \( a^0(\omega) \) are complex coefficients of the form

\[
a^0(\omega) = e^{i\xi},
\]

A more detailed knowledge of Ref. ([18]) is convenient for a full understanding of the present paper.
with statistically independent random phases \( \zeta \) corresponding to different realizations of the field. As usual, \( \lambda \) stands for the polarization index, \( \epsilon^\lambda \cdot \mathbf{k} = 0 \) and \( |\mathbf{k}| \equiv k = \omega/c \).

Since electrons are known to interact with electromagnetic field modes of circular polarization, it is convenient to decompose the ZPF into such modes, which means that the index \( \lambda \) refers to the two possible circular polarizations, left and right (or clockwise and counter clockwise), associated with the spin angular momentum of the electromagnetic field. We may therefore write the factor multiplying the coefficient \( \tilde{E}(\omega) \) as

\[
a(\omega, \phi, t) = a^0(\omega)e^{i(\omega t - \mathbf{k} \cdot \mathbf{x} + \gamma \phi)}.
\]

In the long-wavelength (dipole) approximation \( \exp i(\mathbf{k} \cdot \mathbf{x}) \simeq 1 \) —which has been shown to be sufficient to establish contact with quantum mechanics, see [18]— this reduces to

\[
a(\omega, \phi, t) = a^0(\omega)e^{i\omega t}e^{i\gamma \phi}.
\]

As for the rotation angle \( \phi \), notice that it changes by \( 2\pi \) during a time interval equal to \( 2\pi/\omega \) so that

\[
\gamma = \pm 1
\]

for the full rotation to take place in a full period (or along a full wavelength \( \lambda = 2\pi/k \)), the plus and minus signs referring to the left and right polarizations, respectively.

In previous work [15], [18] it has been shown that as a result of the interplay between the action of this field and the particle’s own radiation reaction, as described by Eq. (1), the system reaches eventually a condition of energy balance. In this regime the particle attains any one of a set of possible stationary states (say, state \( \alpha \)), described by the stationary solutions of Eq. (1), namely

\[
x_\alpha(t, \phi) = \sum_\beta x_{\alpha \beta}a_{\alpha \beta} = \sum_\beta x_{\alpha \beta}a^0_{\alpha \beta}e^{i\omega_{\alpha \beta}t}e^{i\gamma_{\alpha \beta} \phi},
\]

which indicates that the particle responds linearly to a well-defined set of (polarized) field modes \{\( \alpha \beta \)\}, with amplitudes given by

\[
a_{\alpha \beta} = a^0_{\alpha \beta}e^{i\omega_{\alpha \beta}t}e^{i\gamma_{\alpha \beta} \phi},
\]

where \( \beta \) denotes any other state attainable by means of a transition from \( \alpha \). This linear response to the field is the reason for the name linear SED (or LSED, for short) given to the theory. The corresponding relevant frequencies \( \omega_{\alpha \beta} \) turn out to be given precisely by the frequencies of transition between states \( \alpha \) and \( \beta \) according to Bohr’s formula, [15]

\[
\omega_{\alpha \beta} = \Omega_\alpha - \Omega_\beta = (\mathcal{E}_\alpha - \mathcal{E}_\beta)/\hbar,
\]

\( \mathcal{E}_\alpha \) (\( \mathcal{E}_\beta \)) being the energy of the particle in state \( \alpha \) (\( \beta \)). The response of the particle becomes extremely selective, as it leaves out all the modes contained in Eq. (2) except for those appearing in Eq. (7). Equation (7) is but a particular case of the general expression for a dynamical variable \( F_\alpha(t) \) in state \( \alpha \), namely (using 1D notation) [18]

\[
F_\alpha(t, \phi) = \sum_\beta F_{\alpha \beta}a_{\alpha \beta} = \sum_\beta F_{\alpha \beta}a^0_{\alpha \beta}e^{i\omega_{\alpha \beta}t}e^{i\gamma_{\alpha \beta} \phi}.
\]

Notice that the stochasticity is contained solely in the random phases of the relevant field variables, \( a^0_{\alpha \beta} = e^{i\zeta_{\alpha \beta}} \). These phases \( \zeta_{\alpha \beta} \) vary from realization to realization, whence the treatment is necessarily statistical. Under the condition of ergodicity, assumed to hold in the stationary regime, the stochastic amplitudes must obey the chain rule \( a^0_{\alpha \beta}a^0_{\beta \gamma} = a^0_{\alpha \gamma} \), which demands that [18]

\[
a^0_{\alpha \beta} = e^{i\zeta_{\alpha \beta}} = e^{i(\phi_\alpha - \phi_\beta)}.
\]
When the polarization of the field variables does not play a relevant role, the dependence of $F_\alpha$ on the internal rotation angle $\phi$ may be omitted and the expression for the dynamical variable becomes simply

$$F_\alpha(t) = \sum_\beta F_{\alpha\beta} a_\beta^o e^{i\omega_\alpha\beta t}.$$ (12)

This description has been sufficient to develop the lSED theory so far; as we shall see below, however, in dealing with spin the dependence on $\phi$ will need to be taken into account.

The (nonrandom) coefficients $F_{\alpha\beta}$ appearing in Eq. (12) turn out to be the matrix elements in the energy representation of the (quantum) operator associated with the variable $F$, and they satisfy the corresponding matrix algebra. In particular, the statistical (ensemble) average of the dynamical variable (12) coincides with the average calculated with the usual quantum formula, i.e.,

$$\overline{F}_\alpha = \sum_\beta F_{\alpha\beta} a_\beta^o \delta_{\alpha\beta} = F_{aa}.$$ (13)

In summary, the description of the particle state in the time-asymptotic, ergodic regime, which is identified as the quantum regime, is given by the formulas and rules familiar from quantum mechanics in its matrix form. Equations such as (12) for the (stochastic) dynamical variable $F(t)$ in state $\alpha$ endow this description with a concrete physical meaning, in which the zpf plays a central role in ensuring that such stationary state is maintained.

For practical reasons, and to establish contact with the usual quantum formalism, it is convenient to introduce an appropriate Hilbert space for the description; this will prove particularly useful for the analysis of the entangled states to be made below. With this purpose we observe that the matrix $\hat{F}(t)$, represented by the set of elementary oscillators $F_{\alpha\beta} e^{i\omega_{a\beta}t}$, can be expanded as

$$\hat{F}(t) = \sum_\beta F_{\alpha\beta} e^{i\omega_{a\beta}t} |e_\alpha\rangle \langle e_\beta| = \sum_\beta F_{\alpha\beta} e^{i(\Omega_\alpha - \Omega_\beta)t} |e_\alpha\rangle \langle e_\beta|$$ (14)

where $\{|e_\alpha\rangle\}$ represents an orthonormal basis spanning the Hilbert space of states of the system. The corresponding matrix elements are then given by

$$F_{\alpha\beta}(t) = \langle e_\alpha| \hat{F}(t) |e_\beta\rangle = \langle \alpha(t) | \hat{F} | \beta(t) \rangle ,$$ (15)

with $\hat{F}$ having matrix elements $F_{\alpha\beta}$ and

$$|\alpha(t)\rangle = e^{-i\Omega_\alpha t} |e_\alpha\rangle .$$ (16)

In particular, the mean value given by (13) can be written according to (15) as $F_{aa} = \langle \alpha | \hat{F} | \alpha \rangle$, which is the usual quantum formula.

3. Inclusion of spin into the description

In recent work (see Refs. [18], [19]) it has been shown that the electron spin emerges as an internal angular momentum of value $\hbar/2$ acquired by the particle, due to its interaction with modes of the zpf of a given circular polarization. In other words, it is precisely the internal (or spin) angular momentum of the polarized modes of the zpf what gives rise to and sustains the spin angular momentum of the electrons.

To include the spin in our analysis we need to use the complete expression for the dynamical variable $F_\alpha(t)$ with the angular factor included as given by (10), instead of (12). In order to establish contact with the usual quantum description, we propose to write

$$\gamma_{\alpha\beta} = \gamma_{\alpha} - \gamma_{\beta} ,$$ (17)
and hence,

$$F_{\alpha}(t, \phi) = \sum_{\beta} F_{\alpha\beta} e^{i\gamma_{\alpha}\phi} e^{i(\gamma_{\alpha} - \gamma_{\beta})\phi} e^{-i(\Omega_{\alpha} - \Omega_{\beta})t},$$

with $\gamma_{\alpha}, \gamma_{\beta}$ taking half-integral, positive or negative values. This is a natural way to include the spin as a contribution to the total angular momentum of the particle into the description.

The separation of $\gamma_{\alpha}\beta$ into individual components $\alpha, \beta$ implies that the full amplitude $a_{\alpha\beta}$ given by Eq. (8) complies with the chain rule, a property that will prove to be fundamental in what follows. In writing $F_{\alpha}(\phi, t)$ in the above form, the dependence on $\phi$ can be transferred to the state vectors, in the same way as was done in the previous section with the time dependence, by means of a unitary transformation applied to (16),

$$|\alpha(\phi, t)\rangle = e^{-i\gamma_{\alpha}\phi} |\alpha(t)\rangle = e^{-i\gamma_{\alpha}\phi} e^{-i\Omega_{\alpha} t} |\alpha\rangle,$$

so that the matrix elements become, in analogy with (15),

$$F_{\alpha\beta}(\phi, t) = \langle e_{\alpha} | \hat{F}(\phi, t) | e_{\beta}\rangle = \langle \alpha(\phi, t) | \hat{F} | \beta(\phi, t) \rangle.$$

The factor $e^{i\gamma_{\alpha}\phi}$, associated originally with the amplitude of the field mode $(\alpha\beta)$, has thus been incorporated into the (particle) state vectors $|\alpha\rangle$, $|\beta\rangle$. Now these vectors include the $\gamma$ space, so they span the product Hilbert space $\mathcal{H}_{\text{field}} \otimes \mathcal{H}_{\gamma}$. As in previous LSED treatments, the ZPF has disappeared altogether from the description; yet the effect of its action on the particle is evident once more, now through the appearance of the angular momentum transferred by the field.

Notice that $\phi$ is the azimuthal angle of rotation with respect to an internal quantization axis, the orientation of which is in general independent of the coordinates in 3D space. It is only when a privileged direction is used for the angular momentum projection — e.g., in the presence of an external magnetic field — that this axis may be made to coincide with the $z$-direction (say) in the configuration space.

### 4. The bipartite system

Let us now consider a system of two noninteracting particles under the action of a common binding potential. Here one has to take into account that if the two particles belong to one and the same system and they are sufficiently near one to another, they are subject to the common background field; i.e., to the same realization of the random ZPF. It is under this condition that we say that the two particles constitute a single system.

From (10), the product of any two (stochastic) dynamical variables pertaining each to one particle is given in general by

$$F_{\alpha}(t, \phi)G_{\alpha'}(t, \phi) = \sum_{\beta, \beta'} F_{\alpha\beta} G_{\alpha'\beta'} a_{\alpha\beta}^{0} a_{\alpha'\beta'}^{0} e^{i\gamma_{\alpha}\phi} e^{i\gamma_{\alpha'}} e^{i\omega_{\alpha} t} e^{i\omega_{\alpha'} t}.$$

On both sides of this expression, the first factor refers always to particle 1 and the second one to particle 2; e.g., $F_{\alpha}(t, \phi)G_{\alpha'}(t, \phi) = F_{1\alpha}(\phi, t)G_{2\alpha'}(t, \phi); a_{\alpha\beta}^{0} a_{\alpha'\beta'}^{0} = a_{1\alpha}^{0} a_{2\alpha'}^{0},$ a.s.o. For simplicity in the reading we shall consistently omit the corresponding subindices 1, 2.

Now it is the composite system that eventually reaches a stationary state, characterized by a total energy $\mathcal{E}_{A}$. Let us write this state as $A = (\alpha, \alpha')$, where $\alpha, \alpha'$ represent stationary (single-particle) states of particles 1 and 2, respectively. Then in concise form, the product $FG$ in state $A$ writes as

$$\langle FG \rangle_{A} (\phi, t) = \sum_{B} \langle FG \rangle_{AB} b_{AB} e^{i(\gamma_{\alpha} - \gamma_{\beta})\phi} e^{i(\gamma_{\alpha'} - \gamma_{\beta'})\phi} e^{i(\Omega_{A} - \Omega_{B})t}$$

$$= \sum_{B} \langle FG \rangle_{AB} e^{i(\gamma_{\alpha} - \gamma_{\beta})\phi} e^{i(\gamma_{\alpha'} - \gamma_{\beta'})\phi} e^{i(\Omega_{A} - \Omega_{B})t}$$

(22)
with \((FG)_{AB}\) the elements of a matrix in the product Hilbert space \(H_1 \otimes H_2\) of the two single-particle Hilbert spaces involved,

\[
(FG)_{AB} = F_{\alpha \beta} G_{\alpha' \beta'}.
\]  

(23)

The index \(B\) refers to any other accessible state of the compound system, with total energy \(E_B\), and hence it is, like \(A\), a compound index of the form \(B = (\beta, \beta')\). The coefficient \(b_{AB}\) in (22) is the product of the random variables of the common zpf appropriate for describing the linear response in the bipartite case,

\[
b_{AB} = a^{0}_{\alpha \beta} a^{0}_{\alpha' \beta'}.
\]  

(24)

Because of (11), these coefficients satisfy the following relations

\[
b_{AB}^* = b_{BA}, \quad |b_{AB}|^2 = b_{AA} = b_{BB} = 1, \quad b_{AC} = b_{AB} b_{BC},
\]  

(25)

where \(C\) denotes an arbitrary state accessible from \(A\) (or \(B\)). Further, because of (9), the (composite) transition frequencies are given by

\[
\omega_{AB} = h^{-1}(\mathcal{E}_A - \mathcal{E}_B).  
\]  

(26)

Now the condition of ergodicity implies that the time average of \((FG)_A\), which according to (22) is

\[
\overline{FG}_A = \sum_B (FG)_{AB} b_{AB} e^{i\gamma_{\alpha \beta} \phi} e^{i\gamma_{\alpha' \beta'} \phi}\bigg|_{\omega_{AB}=0},
\]  

(27)

must be independent of the realization, which means that for every \(\omega_{AB} = 0\), the corresponding \(b_{AB}\) is nonrandom. Let us call \(\lambda_{AB}\) these nonrandom coefficients, i.e.,

\[
\lambda_{AB} = b_{AB}|_{\omega_{AB}=0} = \lambda^*_{BA},
\]  

(28)

where for the last equality we used (11) and (24). Then the (statistical) average of (22) is given by

\[
\overline{FG}_A = \sum_B (FG)_{AB} \lambda_{AB} e^{i\gamma_{\alpha \beta} \phi} e^{i\gamma_{\alpha' \beta'} \phi} (\omega_{AB} = 0).
\]  

(29)

If there is no energy degeneracy, i.e., if to every state \(A\) there corresponds a different energy \(\mathcal{E}_A\), the condition \(\omega_{AB} = 0\) is satisfied only for \(A = B\), i.e. \(\alpha = \beta\) and \(\alpha' = \beta'\), and Eq. (29) reduces to the single term representing the product of the diagonal elements of \(F\) and \(G\)

\[
\overline{FG}_A = (FG)_{AA} = F_{\alpha \alpha} G_{\alpha' \alpha'},
\]  

(30)

since by virtue of (25) \(\lambda_{AA} = b_{AA} = 1\). When the two particles are different, their stationary energy levels will be different in general so that Eq. (30) applies.

However, let us consider two identical particles, with different single energy eigenstates \(\alpha, \alpha'\) such that \(\mathcal{E}_\alpha \neq \mathcal{E}_{\alpha'}\) (i.e., nondegenerate single states). The composite states will always be degenerate in this case, because to every state \(A(\alpha, \alpha')\) there corresponds a different state \(B(\alpha', \alpha)\) having the same total energy,

\[
\mathcal{E}_A = \mathcal{E}_B = \mathcal{E}_\alpha + \mathcal{E}_{\alpha'} \quad (\mathcal{E}_\alpha \neq \mathcal{E}_{\alpha'}).  
\]  

(31)

There are therefore two terms in the sum (29) that contribute to \(\overline{FG}_A\), namely the first one with \(\beta = \alpha, \beta' = \alpha'\) (i.e. \(A = B\)), and the second one with \(\beta = \alpha', \beta' = \alpha\) (\(\alpha \neq \alpha'\)), [16]-[18]

\[
\overline{FG}_A = (FG)_{AA} + (FG)_{AB} \lambda_{AB} e^{i\gamma_{\alpha \alpha'} \phi} e^{i\gamma_{\alpha' \alpha} \phi}.
\]  

(32)
where according to (23),
\[(FG)_{AB} = F_{\alpha\alpha'}G_{\alpha'\alpha}\]  
and from (24) and (28),
\[\lambda_{AB} = a^0_{\alpha\alpha'}a^0_{\alpha'\alpha}.\]  
Analogously we find for \(\overline{FG}_B\), where \(B\) is the compound state having the same energy as \(A\) but with the states \(\alpha, \alpha'\) interchanged,
\[\overline{FG}_B = (FG)_{BB} + (FG)_{BA}\lambda_{BA}e^{i\gamma_{\alpha\alpha'}\phi}e^{i\gamma_{\alpha'\alpha}\phi}.\]  
Let us now use the standard notation for the matrix elements (as before the first factor corresponding consistently to particle 1 and the second one to particle 2)
\[(FG)_{AA} = \langle A | \overline{FG} | A \rangle = \langle \alpha | \langle \alpha' | \overline{FG} | \alpha \rangle | \alpha' \rangle\]  
and
\[(FG)_{AB} = \langle A | \overline{FG} | B \rangle = \langle \alpha | \langle \alpha' | \overline{FG} | \alpha' \rangle | \alpha \rangle.\]  
Eqs. (32) and (35) take then the form
\[
\overline{FG}_A = \sqrt{2}e^{i\gamma_{\alpha\phi}}e^{i\gamma_{\alpha'}\phi} \langle A | \overline{FG} | \psi_{AB} \rangle, \quad (36a)
\overline{FG}_B = \sqrt{2}\lambda_{BA}e^{i\gamma_{\alpha'}\phi}e^{i\gamma_{\alpha}\phi} \langle B | \overline{FG} | \psi_{AB} \rangle, \quad (36b)
\]
with
\[
|\psi_{AB}\rangle = \frac{1}{\sqrt{3}} \left( e^{i\gamma_{\alpha\phi}}e^{i\gamma_{\alpha'}\phi} |A\rangle + \lambda_{AB}e^{-i\gamma_{\alpha'}\phi} |\alpha\rangle + \lambda_{AB}e^{-i\gamma_{\alpha}\phi} |\alpha'\rangle |\alpha\rangle \right). \quad (37)
\]
By combining the two expressions (36a), (36b) we obtain for the average \(\overline{FG}\) in the degenerate state with \(\mathcal{E}_A = \mathcal{E}_B \ (A \neq B)\)
\[
\langle \overline{FG} \rangle_{AB} = \langle \psi_{AB} | \overline{FG} | \psi_{AB} \rangle = \frac{1}{2} \left[ \overline{FG}_A + \overline{FG}_B \right], \quad (38)
\]
with \(|\psi_{AB}\rangle\) given by (37).

The vector \(|\psi_{AB}\rangle\), describing the state of the bipartite system with energy \(\mathcal{E}_A = \mathcal{E}_B \ (A \neq B)\), represents an entangled state. Crucial for this entanglement is the presence of the parameter \(\lambda_{AB}\), which according to (34) is different from zero. Entanglement appears therefore as an unavoidable consequence of the intervention of ZPF modes that are common to the two particles, with frequency \(\omega_{\alpha\alpha'} = -\omega_{\alpha'\alpha}\) equal to the frequency of transition between the two single-particle states. This is to say that the resonant response of both particles to modes of the ZPF of the same frequency is what correlates the motions of the particles.\(^2\) In the usual quantum formalism, the possibility of adding two (or more) state vectors to construct a third one (which is key for the existence of entanglement) is taken as a (formal) consequence of the linearity of the Schrödinger equation. Here we have identified a physical origin and a need for this (linear) combination, as a consequence of the mediation of the common modes of the background field. This is an additional example of the contributions offered by the SED formulation of quantum mechanics to a deeper physical understanding of the theory.

\(^2\) As of recently, a distinction is made between entanglement and nonfactorizability in the case of identical particles. This distinction, which departs from the traditional use of the term entanglement since its introduction by Schrödinger in 1935, \([20]\) is still under debate; for a discussion and an extensive list of references see, e.g., Ref. \([21]\). Since here such distinction is irrelevant, we shall use freely the term entanglement as equivalent to nonfactorizability.
5. Spin-symmetry relation

In order to explore the connection between spin and symmetry, let us analyze the properties of the entangled state vector $|\psi_{AB}\rangle$ given by Eq. (37) under different permutations. We should first perform an exchange of particles, i.e., $(1, 2) \rightarrow (2, 1)$, which means putting particle 1 in the position of particle 2 and vice versa. When performing this exchange operation one must apply it to the entire state vector, including the angular factors; this means that in switching the positions of particles 1 and 2 in the equation, one must take care that the rotations that take particle 1 to the azimuthal position of particle 2 and vice versa, are both made in the same sense — say clockwise — as discussed e.g. in Ref. ([6]).

Assume first that $\phi_2 > \phi_1$, then $\phi_1$ transforms into $\phi_2$ (if the rotation is made in the clockwise sense) and $\phi_2$ transforms into $2\pi + \phi_1$, i.e., with $\Delta \phi = \phi_2 - \phi_1$:

\[
\phi_1 \rightarrow \phi_2 = \phi_1 + \Delta \phi.
\]

\[
\phi_2 \rightarrow 2\pi + \phi_1 = \phi_2 + 2\pi - \Delta \phi.
\]

Therefore, as a result of the exchange applied to Eq. (37) we get

\[
|\psi_{AB}\rangle_{1\leftrightarrow 2} = \frac{1}{\sqrt{2}} \left( e^{-i2\pi \gamma_\alpha} e^{-i\gamma_\alpha \phi} e^{-i\gamma_\alpha' \phi} |\alpha\rangle |\alpha\rangle + \lambda_{BA} e^{-i2\pi \gamma_\alpha} e^{-i\gamma_\alpha \phi} e^{-i\gamma_\alpha' \phi} |\alpha\rangle |\alpha\rangle \right).
\]

Now $\gamma_\alpha$ and $\gamma_\alpha'$ have always a half-integral value, as discussed in Sect. 3. Therefore $e^{-i2\pi \gamma_\alpha} = e^{-i2\pi \gamma_\alpha'} = (-1)^{2\gamma_\alpha}$ regardless of the value of $\gamma_\alpha, \gamma_\alpha'$, and one may take out this common factor from the parentheses in the above expression:

\[
|\psi_{AB}\rangle_{1\leftrightarrow 2} = \frac{1}{\sqrt{2}} (-1)^{2\gamma_\alpha} \left( e^{-i\gamma_\alpha \phi} e^{-i\gamma_\alpha \phi} |\alpha\rangle |\alpha\rangle + \lambda_{BA} e^{-i\gamma_\alpha \phi} e^{-i\gamma_\alpha \phi} |\alpha\rangle |\alpha\rangle \right).
\]

If instead, $\phi_2 < \phi_1$, then $\phi_2$ transforms into $\phi_1$, $\phi_1$ transforms into $2\pi + \phi_2$, and the exchange applied to Eq. (37) leads to

\[
|\psi_{AB}\rangle_{1\leftrightarrow 2} = \frac{1}{\sqrt{2}} \left( e^{-i2\pi \gamma_\alpha} e^{-i\gamma_\alpha \phi} e^{-i\gamma_\alpha \phi} |\alpha\rangle |\alpha\rangle + \lambda_{BA} e^{-i2\pi \gamma_\alpha} e^{-i\gamma_\alpha \phi} e^{-i\gamma_\alpha \phi} |\alpha\rangle |\alpha\rangle \right),
\]

which gives the same result, Eq. (41). Therefore we have in both cases

\[
|\psi_{AB}\rangle_{1\leftrightarrow 2} = \lambda_{AB}^* (-1)^{2\gamma_\alpha} |\psi_{AB}\rangle.
\]

Since the particles are identical, their interchange should have no effect whatsoever; stated explicitly, the interchange of particle positions denoted by $(1, 2) \rightarrow (2, 1)$ should have no consequence on the state vector. This means that $\lambda_{AB}^* (-1)^{2\gamma_\alpha} = 1$. Further, given that the sign of $(-1)^{2\gamma_\alpha}$ is determined solely by the (half-integer) contribution due to the spin $\sigma$, we may write $(-1)^{2\gamma_\alpha} = (-1)^{2\sigma}$, whence

\[
\lambda_{AB} = \lambda_{AB}^* = (-1)^{2\sigma} = -1.
\]

Let us now take the state vector given by Eq. (37) and apply to it a different operation, consisting of the interchange of states $A \leftrightarrow B$ — which is equivalent to replacing $(\alpha, \alpha')$ by $(\alpha', \alpha)$. Since the states $A$ and $B$ are different by construction, the state vector is in principle
not invariant under this interchange. Indeed, the state vector (40) transformed by the operation \( A \leftrightarrow B \) (or \( \alpha \leftrightarrow \alpha' \)) is readily seen to be

\[
|\psi_{BA}\rangle = \lambda_{AB}^* |\psi_{AB}\rangle,
\]

which indicates that its transformation property depends on the sign of the entanglement parameter \( \lambda_{AB} \), given by Eq. (44). Introducing this value into Eq. (45) we obtain the well-known result

\[
|\psi_{BA}\rangle = -|\psi_{AB}\rangle,
\]

and Eq. (37) becomes the antisymmetric state vector

\[
|\psi_{AB}\rangle = \frac{1}{\sqrt{2}} \left( e^{-i\gamma_{\alpha}\phi} e^{-i\gamma_{\alpha'}\phi} |A\rangle - e^{-i\gamma_{\alpha'}\phi} e^{-i\gamma_{\alpha}\phi} |B\rangle \right).
\]

We recall that by construction, the states \( A, B \) must be different for the two particles to constitute a single system. If we were to take \( A = B \), Eq. (47) would reduce to \( |\psi_{AA}\rangle = 0 \), which is just an expression of the well-known Pauli exclusion principle.

5.1. The many-electron system

The above discussion can be extended to a system composed of more than two electrons with spin, subject again to a common ZPF. To determine the resulting state of the system, in which the total energy is again a constant, we must consider the various possible configurations \( A, B, C, \ldots \) of stationary states of the individual particles corresponding to the same total energy \( E_A \). Specifically, we are interested in the question whether in general the symmetrization principle finds an explanation in terms of the correlations established by the ZPF between the electronic motions.

Direct application of the above procedure to a larger number of electrons becomes rather cumbersome, as the degeneracy of the composite state increases as \( N! \) with the number of particles \( N \). A convenient approach would be to apply the above procedure firstly to any pair of electrons of the system, say those in states \( \alpha, \alpha' \). By taking successively every possible pair, all relevant frequencies of resonance will be accounted for, and all the respective symmetries will thus be included.[6] Since as a result no pair of electrons can be in the same (single-particle) state, the state of the entire system will be described by a totally antisymmetric, multiply entangled state vector built of different single-particle states that carry the factor \( \frac{1}{\sqrt{2}} \) in front of each term, where \( k \) stands for the number of transpositions needed to reach the corresponding exchanged state, starting from the initial state, and \( \sigma = 1/2 \). If two states were made to coincide (say, \( \alpha' = \alpha \)) the entire state vector would reduce to zero; we have again a direct expression of the exclusion principle, here applied to more than two electrons.

6. Final comments

The results presented here allow us to give an answer to two important questions that have remained open in quantum mechanics along the years. The first one refers to the physics behind the structure of the state vectors for a bipartite or multipartite system of electrons: When its components resonate to a common frequency of the common background field, it is no longer possible to describe the state of the compound system as if every constituent particle were isolated and independent from the others. Particles and field constitute a physical unit of interconnected parts, which explains why this state is represented by a nonfactorizable vector, that peculiar quantum entity that entails correlations between the particles’ dynamical variables. It is the permanent presence of the vacuum field, resonantly and coherently connecting all parts among themselves, that unveils the mystery of the spin-statistics association in matter-plus-field behavior. The corollary is that the consideration of the complete physical system in first quantization is sufficient to explain the statistics of assemblies of electrons.
The second question is how it is that in almost ninety years of quantum mechanics no definitive physical argument has been advanced (to our knowledge) to account for the restriction to antisymmetric states for fermions. An answer to this question is provided by just the above discussion: the existence of the ZPF is responsible for the correlations, but this field is outside the scope of usual quantum mechanics. In ignoring the ZPF, quantum mechanics misses the possibility in principle to delve into the source of this (anti)symmetry.

It is important to stress that the procedure followed here to determine the symmetry properties of the two-electron state vector differs in a subtle but essential way from usual quantum mechanics, where the interchange of identical (or ‘indistinguishable’) particles is considered equivalent to the interchange of states \( \alpha, \alpha' \leftrightarrow \alpha', \alpha \). According to the present discussion, by contrast, whereas the interchange of identical particles means an interchange of labels and should therefore leave the state vector unchanged, the interchange of individual states takes the compound state from \( A(\alpha, \alpha') \) to a new state \( B(\alpha', \alpha) \); although the respective state vectors are physically equivalent and yield the same values for the observables, they happen to differ by an overall minus sign. This minus sign comes from the value of the parameter \( \lambda \) (determined in its turn by the half-integral nature of the spin variable) that correlates the motions of the two electrons via the common ZPF modes.

We should like to add a comment on the extension of the present treatment to particles with different intrinsic angular momentum values. The fact that the sign of the correlation parameter \( \lambda_{AB} = (-1)^{2\sigma} \) is determined by the integral or half-integral value of \( \sigma \), suggests that the results of Sect. 5 may be applied not just to fermions, but equally well to bosons or higher-spin particles.

Finally, a brief comment on the difference between identity and indistinguishability is in place. The notion of distinguishability implies our capacity to distinguish, and thus involves a subjective element. By contrast, in dealing with identical particles, one must consider the indifference between them, in the statistical context of the quantum description. The point may seem irrelevant for some, but it acquires importance for at least two reasons. One is that, contrary to indistinguishability, the notion of statistical indifference does not contain unphysical elements. The second is more technical and refers to classical physics. In the whole of thermodynamics, classical particles are treated as distinguishable, yet a radical change is made in the derivation of the Maxwell–Boltzmann law, which requires to consider them as indistinguishable in order to avoid contradictions such as Gibbs’ paradox. [23] Introducing the notion of statistical indifference when the statistical behavior of classical particles is being considered leads to the correct results without putting at stake the objectivity of the treatment.

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