Study on Darcy-Forchheimer Flow and MHD Boundary Layer Flow with Heat Transfer Characteristics of Williamson Nanofluid Over Curved Stretching Surface.

S Sharma
Research Scholar Mathematics, Career Ponit University, Hamirpur, Himachal Pradesh, India.
sonikasharmablp@gmail.com

Abstract
Principle targets of this paper are to consider the Darcy-Forchheimer and magneto hydrodynamic limit layer stream with warmth and mass exchange of Williamson nanofluid over a bended extending surface. To create the stream, a non-straight bended extending surface is utilized. Likewise the capacities thermophoretic dispersion and irregular movement are created in the stream. The pertinent overseeing limit layer fractional differential conditions are changed into common differential conditions by utilizing existing comparability changes. These nonlinear coupled normal differential conditions, subject to the fitting limit conditions, are then addressed by utilizing bvp4c strategy. The impacts of the actual boundaries on the stream, heat move and nanoparticle fixation qualities of the issue are introduced through diagrams and are talked about in detail. The skin rubbing coefficient, neighborhood Nusselt number and Sherwood numbers are figured. In light of these plots the ends are given, and got results are tried for their exactness.

Keywords: Williamson nanofluid, Darcy-Forchheimer flow, curved stretching surface, Nanoparticles, convective heat and mass conditions, MATLAB, bvp4c solver.

1. Introduction
During the time spent creation, the extending of sheet unmistakably affects the nature of completed items. Stream because of extending surface has acquired plentiful importance in the modern and mechanical cycles, for example, materials fabricated by polymer expulsion, gem developing, paper creation and glass fiber, wire drawing and hot moving, extending of plastic film, tempering and tinning of copper wires, cooling of electronic chips or metallic sheets, and some more. The eventual outcome of wanted attributes relies on the pace of the way toward extending and cooling altogether these cases. The marginal layer stream and warmth move over a straight or nonlinear stretch sheet, surface or level plate were examined by numerous scientists however a next to no writing is accessible on the liquid stream past over a bended extending surface. Pioneer work on stream brought about by direct extending of the sheet was finished by Crane [1]. In his work, he built up a precise answer for the stream produced by direct extending of the surface which is significantly seen in Navier Stoke’s answer of liquid powerful conditions. Notwithstanding the expected actual attributes, including warmth and mass exchange along level plate, infusion in vertical course, and some more, his work has been proceeded from various perspectives. A few investigations have investigated Crane’s work by thinking about various parts of stream and warmth move qualities by extending the surface directly from the current writing [2, 3, 4]. The stream with attractions and blowing over an extending surface was finished by Gupta and Gupta [5]. They recommended that the direct extending of the surface may not be sensible, prompting the beginning of non-straight extending, which has made various abstract commitments starting today. Logical and mathematical answers for non-Newtonian force law liquid past extending surface are inspected by Anderson and Kumaran [6]. Bank [7] found a mathematical answer for gooey liquid stream over power-law extending. Jalil et al. [8] gave a mathematical model to the progression of the force law liquid over the force law extending of the level surface. The force law considered of the structure $u_\omega(x) = cx^\theta$
[9] broke down the issue of the two dimensional stream created by the non-straight surface. The investigation of thick liquid stream past the bended surface has once in a while existed. Ahmad and Asghar [10] discovered the numerical and analytical solution for the flow and heat transfer over the hyperbolic stretching surface. Sajid et al. [11] looked out the numerical displaying of the gooey liquid stream past surface. They found that the drag in liquid proceeding onward a level surface builds more than that of bended sheet. They stressed the meaning of pressuring factor variety and, obviously, the application which can be viewed as helpful in bending the jaw in the assembling cycle of machines. Abbas et al. [12] analyzed the warmth move and liquid stream within the sight of attractive field over a bended extending surface. Further, Abbas et al. [13] additionally explored stream and warmth move in limit layer over a bended extending sheet with heat age and warm radiation within the sight of electrical conductivity. Rosca and Pop [14] investigated precarious stream over a permeable bended extending/contracting sheet. Hayat et al. [15] considered the compound response and warm radiation impacts in magneto hydrodynamics convective liquid stream past a bended surface. Imtiaz et al. [16] investigated homogeneous-heterogeneous responses in convective ferro liquid stream over a bended stretchable sheet. Ali et al. [17] discussed the flow of Jeffrey fluid through a vibratory stretching surface. Hayat et al. [18] extracted a numerical model for the boundary layer flow of Jeffrey fluid using a homotopic method. Sanni et al. [19] investigated the flow analysis with nonlinear stretching velocity through a curved stretching sheet. Kumar et al. [20] evaluated the synchronized solutions for the magnetohydrodynamics flow of Williamson fluid through a curved sheet with a irregular heat source/sink. They analyzed that the curvature parameter enhances the velocity field, while the reverse trend is noticed for the Williamson fluid and the magnetic field parameter.

It is noticed that the ordinary liquids like ethylene glycol, propylene glycol and water and so forth have helpless warm conductivity. Subsequently, different procedures for improving liquid warm conductivity have been proposed. Due to their capacity to upgrade heat move, nanofluids have acquired a lot of consideration lately. In customary warmth move liquids, the expansion of nanosized particles like gold, copper, aluminium, iron, or their oxides expands the warm conductivity of such liquids. The nanofluids have beneficial application in motor coolants, drag reductions, temperature control, transformer cooling, atomic frameworks cooling, oil motor exchange, electric diesel generator like coat of water coolant, microwave tubes, high force lasers, cooling and warming of structures, welding cooling, warm capacity and warming of sun powered water [21]-[24]. Choi and Eastman [25] were the principal who thought about "Nanofluid". Warmth move examination of nanofluid was finished by Xuan and Li [26]. Attributes of $Al_2O_3$, $TiO_2$ and $Fe_3O_4$ nanoparticles in water based parts are proposed in the writing [27, 28]. Boungiorno [29] has proposed a numerical model for convective vehicle of nanofluid. He uncovered that the Brownian movement and thermophoresis are the main slip component. Tiwari and Das [30] have examined the improvement of nanofluids heat move in a two sided top driven warmed square depression. The magneto hydrodynamic streams past extending surface assumes a noticeable part in oil designing, projecting, metal working, surface cooling inside an atomic reactor regulation vessel. MHD possesses huge applications in the field related with drug conveyance, attractive cell detachment, decrease of blood during medical procedures, therapy of some blood vessel infection and hyperthermia. Makinde and Chinyoka [33] investigated the MHD stream of dusty liquid between two equal surfaces with variable thermophysical conduct. Jalilpour et al. [31] researched the non-symmetrical stagnation point MHD stream of nanofluid within the sight of warm radiation over the extending sheet. Sandeep [32] considered the effects of the adjusted attractive field on the fluid flimsy film stream of attractive nanofluids coordinated with graphene nanoparticles. Sucharitha et al. [34] investigated the impacts of joule warming and divider adaptability on the peristaltic transport of magnetohydrodynamic nanofluids. The Williamson liquid with shear diminishing properties is highlight of a non-Newtonian liquid model. Williamson [35] recommended this model for clarifying the qualities and stream conduct of pseudo plastic liquids. Nadeem and Hussain [36] analyzed the two-dimensional progression of Williamson liquid over an extending sheet and talked about the impacts of nanoparticles on Williamson fluid. Krishnamurthy et al. [37] examined the effect of substance responses on the limit layer stream of MHD just as the dissolving heat move of Williamson nanofluid in permeable media.

The liquid streams and transport measure through a permeable space at high Reynolds number have huge
importance and applications in drug, substance, food and biological system. A load of ecological and mechanical frameworks, like a geothermal force framework and a warmth exchanger plan framework, incorporated the penetrable medium convection stream. The changed type of the traditional Darcian model is a non-Darcian permeable medium that includes inertial and limit impacts. The standard Darcy law applies to a restricted scope of low speed and lower porosity. Current work shows that much thought has been given to displaying and examination through Darcian permeable space. Forchheimer [38] utilized a square speed idea in Darcian speed to break down the latency and limit attributes. Muskat [39] alluded to this segment as the Forchheimer factor. Sadiq and Hayat [40] researched the Darcy-Forchheimer stream of magneto Maxwell fluid limited by convectively warmed sheet. The Darcy-Forchheimer stream of micropolar nanofluid among two plates in the pivoting outline with non-uniform warmth age/retention have investigated by Khan et al. [41]. They inferred that the expanded estimation of Fr rots the speed profile, while expanding the cross over speed and expanding the porosity boundary builds the permeable space that creates opposition in the stream way and diminishes the stream movement. Hayat et al. [42] assessed Darcy-Forchheimer stream by methods for a bended extending surface with Cattaneo-Christov twofold dissemination mathematically. Muhammad et al. [43] dissected the Darcy-Forchheimer stream over a dramatically extending bended surface with Cattaneo-Christov twofold dispersion. Hayat et al. [45] have mathematically examined the Darcy-Forchheimer stream because of a bended extending surface with cattaneo-Christov heat transition and homogeneous-heterogeneous responses. Move through permeable medium has an assortment of commonsense uses and applications, particularly in geophysical liquid elements. Some regular instances of normal permeable media are human lung, sandstone, sandy sea shore, limestone, nerve bladder with stones in little veins and bile conduits. Subtleties of permeable media with different application can be accounted for in the writing. Rasool et al. [46] mathematically explored the age of entropy and the results of parallel compound responses on the MHD Darcy-Forchheimer Williamson nanofluid stream over a two-dimensional non-straight extending surface. Liquid stream, mass and warmth transport, just as the elements of entropy age utilizing the Buongiorno model, were assessed by applying the second thermodynamics law.

The primary motivation behind this work is to dissect the impacts of nanoparticles in two dimensional limit layer stream of gooey liquid because of nonlinear extending of the bended surface. The Navier-Stokes conditions are defined for which the thick term is adjusted that considers the ebb and flow impacts. The overseeing conditions are tackled mathematically by bvp4c strategy.

2. Mathematical formulation

Think about the two-dimensional magneto hydrodynamics (MHD) stream of the gooey nano liquid over a nonlinear curved stretching sheet looped in a circle having span R. Convective warmth and mass conditions are presented on a superficial level. Impact of Brownian dissemination and thermophoresis are additionally thought of. It is likewise expected that the surface is warmed by the hot liquid having temperature \( T_f \) and concentration \( C_f \) that give warmth and mass exchange coefficients \( h_f \) and \( k^* \) individually. Curvilinear directions \((r, s)\) are utilized to define the particular conditions. These directions are presented in such a way that s-pivot is brought the stretchable sheet and r-hub is ordinary to it. Surface has extending \( U_w(s) = a s^m \) with \( a \) as extending rate constant and \( m > 0 \) as power law constant.

The resulting boundary layer expressions describing present flow consideration are:

\[
\frac{\partial u}{\partial r} [(r + R)v] + R \frac{\partial u}{\partial s} = 0
\]

\[
\frac{u^2}{r + R} = \frac{1}{\rho} \frac{\partial p}{\partial r}
\]

(1)

(2)
\[ v \frac{\partial u}{\partial r} + \frac{Bu}{r+R} \frac{\partial u}{\partial s} + \frac{w}{r+R} = \frac{-1}{\rho} \frac{R}{r+R} \frac{\partial p}{\partial r} + \nu \left[ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r+R} \frac{\partial u}{\partial r} - \frac{u}{(r+R)^2} \right] - \frac{\sigma (B_0)^2}{\rho} u + \sqrt{2} v \frac{\partial u}{\partial r} - \frac{1}{(r+R)} u \left[ \frac{\partial^2 u}{\partial r^2} + \frac{1}{r+R} \frac{\partial u}{\partial r} - \frac{u}{(r+R)^2} \right] \]

\[ v \frac{\partial T}{\partial r} + \frac{Bu}{r+R} \frac{\partial T}{\partial s} = \alpha^* \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r+R} \frac{\partial T}{\partial r} \right) + \frac{(\partial c)}{(\partial T)} \left[ D_B \left( \frac{\partial T}{\partial r} \right)^2 + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial r} \right)^2 \right] \]

\[ v \frac{\partial C}{\partial r} + \frac{Bu}{r+R} \frac{\partial C}{\partial s} = D_B \left( \frac{\partial^2 C}{\partial r^2} + \frac{1}{r+R} \frac{\partial C}{\partial r} \right) + \frac{D_T}{T_\infty} \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r+R} \frac{\partial T}{\partial r} \right) \]

Figure 1: Schematic diagram of the model for a curved stretching surface.

Suitable boundary conditions for this flow problem are as follows:

\[ u = a s^m; \ v = 0; \ -\kappa \frac{\partial T}{\partial r} = h_f (T_f - T); \]
\[ -D_B \frac{\partial C}{\partial r} = \kappa_m^* (C_f - C); \ at \ r = 0, \]
\[ u \to 0; \quad \frac{\partial u}{\partial r} \to 0; \quad C \to C_\infty \quad \text{when} \quad r \to \infty. \quad (7) \]

\[ u \text{ and } v \text{ depicts velocities in the } r \text{ and } s \text{ directions respectively while } v(= \frac{\mu}{\rho}), \mu, \text{ and } \rho_f \text{ denote the kinematic viscosity, dynamic viscosity and density of base liquid respectively, } p \text{ represents the pressure, } \alpha^* = \frac{K}{\rho_f}, \kappa, (\rho c)_f \text{ and } (\rho c)_p \text{ stand for thermal diffusivity, thermal conductivity, heat capacity of liquid and effective heat capacity of nanoparticles respectively, } T \text{ represents the temperature, } D_B \text{ the Brownian diffusivity, } h_f = h s^\frac{m-1}{2} \text{ the variable heat transfer coefficient, } \kappa^*_m = \kappa^* s^\frac{m-1}{2} \text{ the variable heat transfer coefficient, } C \text{ the concentration, } D_T \text{ the thermophoretic diffusion coefficient, } K^* \text{ denotes permeability of porous space, } F = \frac{C_2}{\delta K^{\frac{m}{2}}} \text{ stands for non-uniform inertia coefficient of porous medium, } C_b \text{ the drag coefficient and } T_\infty \text{ and } C_\infty \text{ the ambient fluid temperature and concentration respectively. We take } \]

\[ u = as^m f'(\eta), v = \frac{R}{r+r_b} (\text{avsm}^{-1})^2 \left[ \frac{m+1}{2} f(\eta) + \frac{m-1}{2} \eta f'(\eta) \right], \]
\[ \theta(\eta) = \frac{T-T_\infty}{T_\infty}, ph(\eta) = \frac{C-C_\infty}{C_f-C_\infty}, \eta = \left( \frac{as^{m-1}v}{v} \right)^{\frac{1}{2}}, r, p = \rho a^2 s^2 m P(\eta). \quad (8) \]

where \( f(\eta) \) are the dimensionless velocity and prime the differentiation with respect to \( \eta \). Eq.(1) is automatically satisfied while Eqs. (2)-(7) yield

\[ \frac{\partial P}{\partial \eta} = \frac{f'^2}{\eta + K} \quad (9) \]

\[ \frac{(m-1)K}{2(\eta+K)} \frac{\partial P}{\partial \eta} + \frac{2mK}{\eta + K} P = \frac{f'''}{\eta + K} + \frac{f''}{(\eta + K)^2} - \frac{f'}{\eta + K} + \frac{1}{2} \frac{f''(\eta + K)^2}{(\eta + K)^2} + \frac{(m+1)K}{2(\eta + K)^2} f f' + \frac{(m+1)(m+2)K}{2(\eta + K)^2} \frac{f f''}{(\eta + K)^2} - 2 \frac{f f'''}{\eta + K} + \frac{f f'''}{(\eta + K)^2} - \frac{f f'''}{(\eta + K)^3} \quad (10) \]

\[ \theta'' + \frac{\partial \theta}{(\eta + K)} + \frac{m+1}{(\eta + K)} \frac{M f^2 + N f \theta' + Nt f \theta'' + Pr f \theta'''}{Pr f + Nb Pr \theta' + Nt Pr (\theta'')^2} = 0 \quad (11) \]

\[ \phi'' + \frac{\phi'}{(\eta + K)} + \frac{m+1}{(\eta + K)} \frac{Sc f \phi' + Nt f \theta'' + \frac{Pr}{\eta + K}}{Pr f + Nb f \phi' + Nt f \phi''} = 0 \quad (12) \]

with boundary conditions given by

\[ f(0) = 0; \quad f'(0) = 1; \quad \theta'(0) = -\gamma_1 (1 - \theta(0)); \quad \phi'(0) = -\gamma_2 (1 - \phi(0)) \quad (13) \]

\[ f'(\infty) \to 0; \quad f''(\infty) \to 0; \quad \theta(\infty) \to 0; \quad \phi(\infty) \to 0. \quad (14) \]

Here \( K \) stands for curvature parameter, \( \gamma_1 \) for thermal Biot number, \( \gamma_2 \) for Concentration biot number, \( Nt \) for thermophoresis parameter, \( Nb \) for prandtl number, \( Sc \) for schmidt number, \( M \) for magnetic fluid parameter, \( \lambda \) for williamson fluid parameter, \( A \) for porosity parameter and \( F \) for Forchhiernum number. These nondimensional numbers can be written as:
\[ K = \sqrt{\frac{\alpha s^{m-1}}{\nu}} R, M = \frac{g B^2}{\rho a s^{m-1}}, Sc = \frac{v}{a}, Pr = \frac{\nu}{\alpha}, \lambda = \sqrt{2} \sqrt{\frac{s^{3a^{m-1}}}{\nu}}, A = \frac{\nu}{\alpha s^{m-1}}, \]
\[ F = F^* S, y_1 = h \sqrt{\frac{v}{a}}, y_2 = \kappa \sqrt{\frac{v}{a}} N b = \frac{(\rho c) D_B (\xi f - C_\infty)}{(\rho c) f v}, N t = \frac{(\rho c) D_B (T_f - T_\infty)}{(\rho c) f v T_\infty}. \] (15)

Now, eliminating pressure term from Eqs. (9) and (10), we have

\[
\begin{align*}
t^4 v + \frac{2 f^\prime \prime \prime}{(\eta + K)^2} - f^\prime + \frac{f}{(\eta + K)^2} + \frac{(m+1)K}{2(\eta + K)} f f^\prime + \frac{(m+1)K}{2(\eta + K)^2} f f^\prime - \frac{(m+1)K}{2(\eta + K)^3} f f^\prime - \frac{(3m-1)K}{2(\eta + K)^2} f^\prime f^\prime + \lambda f^4 f^\prime f^\prime + f f^\prime + 2 \frac{f^\prime f^\prime}{(\eta + K)^2} - 2 \frac{f^2}{(\eta + K)^2} - M f^\prime \left( f^\prime + \frac{f^\prime}{(\eta + K)} \right) - A \left( f^\prime + \frac{f^\prime}{(\eta + K)} \right) & = 0 \end{align*}
\] (16)

The non-dimensional forms of coefficient of skin friction and local Nusselt and Sherwood numbers are

\[ Re \frac{1}{2} \frac{G_f}{s} = \left[ f^\prime (0) - \frac{1}{K} \right] \left[ 1 + \frac{2}{K} f^\prime (0) - \frac{1}{K} \right], Re \frac{1}{2} N u_s = -\theta^\prime (0), Re \frac{1}{2} S h_s = -\phi^\prime (0) \] (17)

where \( Re = \frac{as^{m+1}}{\nu} \) represents the local Reynolds number.

3. Numerical Method: bvp4c Solver

In this portion, we will talk about in detail the usage of the bvp4c procedure for the current issue. The administering PDEs Eqs.(1)-(5) are reduced to a bunch of coupled non-linear ordinary differential conditions in Eqs.(11), (12) and (16) by methods for non-dimensional factors and afterward tackled utilizing limit esteem issue bvp4c solver in MATLAB. The motivation behind lessening ODEs is to simplify it in finding mathematical arrangements. This technique is examined by following advances:

Step 1:

Above all else we will present new factors for the coupled non-linear ODEs in Eqs. (11), (12) and (16):

\[ f = y(1), f^\prime = y(2), f^\prime \prime = y(3), f^\prime \prime \prime = y(4), \]
\[ \theta = y(5), \theta^\prime = y(6), \]
\[ \phi = y(7), \phi^\prime = y(8). \] (18)

Step 2:

Now, we will write these new variables in the first order system of equations as below:

\[
\begin{align*}
f^\prime &= y(2), f^\prime \prime &= y(3), f^\prime \prime \prime &= y(4),
\theta^\prime &= y(5), \theta^\prime &= y(6),
\phi &= y(7), \phi^\prime &= y(8).
\end{align*}
\] (19)
\[ \theta = y(5), \theta' = y(6), \theta'' = \frac{-y(6)}{\eta + K} - \frac{(m+1)K}{2(\eta + K)} Pr y(1) y(6) - Nb Pr y(6) y(8) - Nt Pr \left( y(6) \right)^2 \]

(20)

\[ \phi = y(7), \phi' = y(8), \phi'' = \frac{-y(8)}{\eta + K} - \frac{(m+1)K}{2(\eta + K)} Sc y(1) y(6) - \frac{Nt}{Nt} \left( \frac{y(6)}{\eta + K} - \frac{(m+1)K}{2(\eta + K)} Pr y(1) y(6) - Nt Pr y(6) y(8) - Nt Pr \left( y(6) \right)^2 \right) \]

(21)

**Step 3:**

Transform the boundary conditions in Eqs. (13) and (14) according to new variables introduced:

\[ y_a(1) = 0, \ y_a(2) = 1, \ y_a(6) = -y_1(1 - y_a(5)), \ y_a(8) = -\gamma_z(1 - y_a(7)), \ y_b(2) \to 0, \ y_b(3) \to 0, \ y_b(5) \to 0, \ y_b(7) \to 0. \]

(22)

**Step 4:**

Lastly, we will write first order system of equations in Eqs. (19)-(21) with boundary conditions in Eq. (22) in MATLAB software using bvp4c solver with two different guessing values to obtain the solutions. Using the bvp4c solver, providing an initial guess for the first solution is easy since the solution will converge to the first solution even for poor guesses. This contrary to the initial guesses for the second solution.

4. Results and Discussion

Nearby comparable answers for Eqs.(11), (12) and(16) are portrayed mathematically under limit conditions(13) and(14) by executing the bvp4c strategy in MATLAB. This part has been organized to analyze the effect of different boundaries, for example, Curvature boundary K, Magnetic field boundary M, Williamson liquid boundary \( \lambda \), Forchheimer number \( F \), Local porosity boundary \( A \) and so forth on velocity profile \( f'(\eta) \), temperature \( \theta(\eta) \) and concentration \( \phi(\eta) \). Table 1 presents a near investigation of skin friction coefficient for various estimations of dimensionless power law index \( m \) and radius of curvature \( K \) in a manner that \( \lambda = M = F = A = 0 \) with the current outcomes published by Sanni et al. [19]. The discoveries show an awesome match and along these lines we are certain that our discoveries are precise. Table 2 shows the mathematical calculation of local Nusselt number \( Re_s^{-1/2}Nu_s \) and local Sherwood number \( Re_s^{-1/2}Sh_s \) for different estimations of \( k, m, \gamma_1, \gamma_2, Sc, Pr, Nt \) and \( Nb \). Relative investigation with the current outcomes published by Hayat et al. [44] gives an ideal arrangement. Table 3 displayed the skin friction coefficient, local Nusselt number and Sherwood number corresponding to Williamson liquid boundary \( \lambda \), magnetic field parameter \( M \), porosity parameter \( A \) and Forchheimer number \( F \). It is clearly seen that the skin friction coefficient, local Nusselt number and a Sherwood number show diminishing conduct for higher estimations of \( \lambda \) and \( M \) while the converse pattern is seen for \( A \) and \( F \). The mathematical computations have been completed by setting the estimations of the boundaries as \( Pr = Sc = K = 1.0, Nb = Nt = 0.5, \gamma_1 = \gamma_2 = A = 0.3, \lambda = F = 0.1, m = 1.2, M = 1.5, \) if not characterized in any case.

| \( K \) | \( m = 2 \) | \( m = 3 \) | \( m = 4 \) | \( m = 5 \) |
|---|---|---|---|---|
| | Present Study | Present Study | Present Study | Present Study |
| 5 | 1.4913 | 1.7613 | 1.9938 | 2.2009 |
| 10 | 1.4166 | 1.6904 | 1.9236 | 2.1333 |
| & 1.3818 | 1.6554 | 1.8903 | 2.0986 |
Table 3: Numerical values of skin friction coefficient $-Re_s^{1/2}C_f$, local Nusselt number $Re_s^{1/2}Nu_s$ and local Sherwood number $Re_s^{1/2}Sh_s$ when $K = Pr = Sc = 1.0$, $Nb = Nt = 0.5$, $\gamma_1 = \gamma_2 = 0.3$, $m = 1.2$ for different values of $\lambda$, $M$, $A$, $F$.

| $M$ | $A$ | $F$ | $-Re_s^{1/2}C_f$ | $Re_s^{1/2}Nu_s$ | $Re_s^{1/2}Sh_s$ |
|-----|-----|-----|------------------|------------------|------------------|
| 1.2 | 0.3 | 0.1 | 2.5934           | 0.19335          | 0.18512          |
| 1.2 | 0.3 | 0.1 | 2.1604           | 0.18266          | 0.17912          |
| 1.2 | 0.3 | 0.1 | 1.6662           | 0.17616          | 0.17488          |
| 0.5 | 0.3 | 0.1 | 2.7035           | 0.18102          | 0.18056          |
4.1 Dimensionless velocity profile

Fig. 2(a) shows the variety in speed field $f'(\eta)$ for variable Forchhiemer number $F$. The increment in Forchhiemer number $F$ relates to the lower speed profile $f'(\eta)$. The higher estimation of the inertial coefficient really creates obstruction in the stream way, which in turn eases the movement of the nanofluid. Fig. 2(b) centres around the effects of porosity boundary $A$ on velocity profile $f'(\eta)$. The more noteworthy porosity boundary $A$ shows a decrease in the speed field $f'(\eta)$. Upgrading $A$ builds the permeable space that creates obstruction in the liquid stream and decreases the progression of nanoparticles. In light of the expansion in the consistency of the thermal boundary layers, the permeable surface impact on the limit layers is significant. The increment of the porosity boundary brings about a decrease of porosity. That is the reason $f'(\eta)$ and energy limit layer rots. Fig. 2(c) is given to look at the impact of the Williamson liquid boundary $\lambda$ on $f'(\eta)$. Increment of Williamson liquid boundary $\lambda$ decreases the speed profile. The speed part ways to deal with zero as $n \to \infty$. As we probably are aware, the Williamson liquid boundary is the proportion of unwinding time to explicit interaction time. Thus, the reduction in the particular cycle time expands the Williamson liquid boundary $\lambda$ which shows that both the speed segment and the limit layer thickness decline. Fig. 2(d) shows that the impact of the magnetic field boundary $M$ is to lessen both the speed profile and thermal limit layer thickness. Here decline in speed is dependent upon an increment in the estimations of $M$. It is noticed that the drag power known as the Lorentz force is produced by the utilization of the attractive field. The force can possibly hinder stream over surface. Fig. 2(e) shows that the speed profile is expanded for bigger estimations of curvature parameter $K$. Higher estimations of $K$ demonstrate an improvement in the range of the sheet bringing about an increment in fluid stream.

![The velocity profile $f'(\eta)$ for deviating values of $K$ ](image)
The velocity profile $f'(\eta)$ for deviating values of $F$,

The velocity profile $f'(\eta)$ for deviating values of $A$,

The velocity profile $f'(\eta)$ for deviating values of $\lambda$. 
4.2 Dimensionless temperature profile

Variety of the porosity boundary $A$ at temperature $\theta(\eta)$ is appeared in Fig.3(a). The more noteworthy estimation of the porosity boundary $A$ produces a more grounded temperature field $\theta(\eta)$ and a higher warm thickness. The more serious porosity calculate results more successful opposition power dynamic in transit of smooth movement. This obstruction power is the purpose for the gradual wonder in the thermal profile and the increment in the thickness of the limit layer. Fig.3(b) shows the conduct of $\theta(\eta)$ for various magnetic field esteem. It is noticed that the temperature profile improves with the magnetic fluid boundary $M$. Because of the Lorentz power, some extra warmth energy will be created in the liquid. Subsequently the consequence of that kind is found. Fig.3(c) presents the expanding estimation of Forchhiemer number $F$ shows an addition in temperature field $\theta(\eta)$. Genuinely, the higher inertial power is because of the concentrated drag coefficient $C_b$. For higher estimations of $C_b$, more grounded inertial power is compelling inside a model that fortifies the impact of liquid bundles and builds the temperature field. Fig.3(d) shows the impact of Williamson liquid boundary $\lambda$ on temperature profile. We see that temperature ascends with ascend in $\lambda$. Fig. 3(e) portrays that an expansion in the bend boundary causes to an upgrade in temperature profile. Higher curvature boundary $K$ prompts more span of sheet which offers high protection from liquid stream.

4.3 Dimensionless concentration profile

Fig.4(a) is outlined to look at the conduct of nearby porosity boundary $A$ on concentration profile $\phi(\eta)$. The expansion in the estimation of the porosity boundary $A$ compares to the increment in the concentration field $\phi(\eta)$ and the related focus layer thickness. Fig.4(b) divulges the conduct of $\phi(\eta)$ for various estimations of Williamson liquid boundary $\lambda$. The impact attractive field boundary $M$ on fixation $\phi(\eta)$ is shown in Fig. 4(c). The nanoparticle of Williamson liquid boundary $\lambda$ expands the nanoparticle volume portion profile. Variety of volume division is found to build the estimation of the attractive liquid boundary $M$ with a blast. This is a direct result of the way that when the estimation of $M$ builds it exits liquid molecule movement which will diffuses quickly into the adjoining liquid layers. Fig.4(d) presents that fixation field $\phi(\eta)$ is an expanding capacity of Forchhiemer number $F$. Fig. 4(e) exhibits that how the focus profile $\phi(\eta)$ is influenced with the variety in curvature boundary $K$. By expanding $K$, an upgrade in the focus field is noticed.
[The temperature profile $\theta(\eta)$ for deviating values of $A$]

[The temperature profile $\theta(\eta)$ for deviating values of $M$]

[The temperature profile $\theta(\eta)$ for deviating values of $F$]
The temperature profile $\theta(\eta)$ for deviating values of $\lambda$.

The temperature profile $\theta(\eta)$ for deviating values of $K$.

Figure 3: The temperature profile $\theta(\eta)$ for different values of $A$, $M$, $F$, $\lambda$ and $K$.

The concentration profile for deviating values of $\phi(\eta)$ for $A$.

[The temperature profile $\theta(\eta)$ for deviating values of $\lambda$]

[The temperature profile $\theta(\eta)$ for deviating values of $K$]

Figure 3: The temperature profile $\theta(\eta)$ for different values of $A$, $M$, $F$, $\lambda$ and $K$.

[The concentration profile for deviating values of $\phi(\eta)$ for $A$]
The concentration profile for deviating values of $\phi(\eta)$ for $\lambda$

The concentration profile for deviating values of $\phi(\eta)$ for $M$

The concentration profile for deviating values of $\phi(\eta)$ for $F$
5. Conclusions
In this work we study the Darcy-Forchheimer stream and MHD stream with heat move attributes of Williamson nanofluid stream over bended extending surface. The primary finish of this examination has summed up as follows:
1) The velocity profile keeps a inverse relationship with Williamson liquid boundary, porosity boundary, attractive field and Forchhiemer number while switch pattern is found in thermal and concentration profiles.
2) Larger curve boundary prompts higher speed, temperature and focus profiles.
3) Skin erosion coefficient, nearby Nusselt number and Sherwood number are upgraded for higher Williamson liquid boundary and attractive liquid boundary however lessens for porosity boundary and Forchheimer number.

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