Promoting middle school students’ abstract-thinking ability through cognitive apprenticeship instruction in mathematics learning

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Abstract. The aim of this study is to get an in-depth understanding of students’ abstract-thinking ability in mathematics learning. This study was an experimental research with pre-test and post-test control group design. The subject of this study was eighth-grade students from two junior high schools in Bandung. In each schools, two parallel groups were selected and assigned into control and experimental groups. The experimental group was exposed to Cognitive Apprenticeship Instruction (CAI) treatment, whereas the control group was exposed to conventional learning. The results showed that abstract-thinking ability of students in experimental group was better than that of those in control group in which it could be observed from the overall and school level. It could be concluded that CAI could be a good alternative learning model to enhance students' abstract-thinking ability.

1. Introduction
Abstract-thinking ability in mathematics learning is very important for students to develop. Mathematics abstraction is a process of the building a continuous mathematical knowledge from concrete to abstract. Abstraction is a fundamental process both in mathematics and in mathematics education [1]. The abstraction is a natural process with full awareness as mind focuses on several aspects of circumstances to make a decision and process of making such concrete situations by finding new meaning in order to establish interconnections within the different elements of the whole [2] [3]. In fact, junior high school students’ abstract-thinking ability in Bandung is still relatively low [4] [5].

The indicators of abstract-thinking ability in this study are: (1) the ability to transform problems into symbolic form; (2) the ability to construct an equation; (3) the ability to state relationship between the concept of geometry shapes and linear equations; (4) the ability to make generalizations; and (5) the ability to make equations according to the given situation [4].

According to Yusepa, teachers have an important role in learning process and improving national education and become guidance. Furthermore, Yusepa states that in order for learning to be meaningful, the teacher should be able to teach students with varied models or learning approaches [6][7].

The effective of teacher characteristics are caring, assertive, modeling and enthusiastic, and has high expectations. Teacher's awareness in organizing the classroom or succeeding in all teaching sections...
should be done earnestly and be able to accommodate students' learning styles. Assertiveness means the ability to improve students' responsibilities and apply those responsibilities to their actions. Modeling and enthusiasm in question are that teacher's belief in teaching and learning is communicated through modeling. By showing enthusiasm, teachers communicate their real-world interest in the topic [8].

Cognitive Apprenticeship Instruction (CAI) is one of learning models that is parallel to the characteristics of student-centered learning and emphasizes on improving students' abstract-thinking abilities. CAI involves real-world problems and contextual problems. To solve problems related to the real world, students need to communicate their ideas through mathematical abstraction. Research shows that if the CAI model is combined with other learning areas (content, method, sequence, and sociology) students will have the ability to apply what they have learned in other learning situations [9]. The components of CAI in this study are (1) modeling; (2) coaching; (3) scaffolding (4) articulation; (5) reflection; (6) exploration.

2. Methodology
This study is an experimental research with a pre-test and post-test control group design. The subject in this study are eighth-grade students from two different types of schools, one of which was categorized as a high-quality school and the rest was a moderate-quality school in Bandung. In each schools, two parallel groups were assigned into experimental and control groups. The experimental group was exposed to Cognitive Apprenticeship Instruction (CAI) treatment while the control group was exposed to conventional learning (PKv).

3. Result and Discussion
The prior ability of students' mathematical abstraction as a whole, based on school level and Mathematical Prior Knowledge, both students who received CAI and PKv did not differ significantly. For more details, the results of statistical data analysis of students' mathematical abstraction abilities are presented in Table below.

**Table 1. Normality Test and Comparison Test Results of Students’ Abstract-Thinking Abilities (level of sig = 0.05)**

| Variable                     | Learning   | Normality Test | Homogeneity | Comparison Test | Conclusion                                  |
|------------------------------|------------|----------------|-------------|-----------------|---------------------------------------------|
| Prior Abstract-Thinking Abilities | CAI        | 0.000 Not Normal | -           | Mann-Whitney    | 0.120 There is no difference of prior mathematical abstraction abilities |
|                              | PKv        | 0.000 Not Normal | -           |                 |                                             |
| The Achievement of Abstract-Thinking Abilities | CAI        | 0.622 Normal    | 0.604 Homogeneous | t-test          | 0.000 There is a difference of the achievement mathematical abstraction abilities |
|                              | PKv        | 0.053 Normal    | 0.604 Homogeneous |                 |                                             |

Based on Table 1, it is found that: 1) there is no difference in abstract-thinking abilities between students in experimental group and control group, and 2) there is a difference in the achievement of abstract-thinking abilities between students who were exposed to CAI and students were exposed to PKv.

Based on data analysis, this study showed abstract-thinking abilities of students who were exposed to CAI is better than those who were exposed to PKv. It was analyzed by considering all students from high- and moderate-quality schools; and from the high-, moderate- and low- category of MPK. Overall,
the average of the achievement of students’ abstract-thinking abilities who received CAI is higher than that of students who received PK. The achievement of students’ abstract-thinking abilities who received CAI is significantly better than that of students who received PK. The average of students’ achievement of abstract-thinking abilities at high- and moderate-quality school who received CAI is higher than students who received PK. The achievement of students' abstract-thinking abilities at high- and moderate-quality school who received CAI is better than that of students who received PK.

The results of this study also provide an illustration that there is no interaction between students who were exposed to CAI and PK, school quality (high and middle) to the abstract-thinking abilities. Because based on school quality (high and moderate) who received CAI, the achievement is higher than the achievement of those who received PK, it can be concluded that CAI can be applied to students who are at high-quality and moderate-quality schools. In other words, the learning factor does not depend on the school quality.

Other factors that influenced the differences in post-test scores of students' abstract-thinking abilities are also seen in each of the MPK categories (high, middle and low). The average of the achievement of students' mathematical abstraction in the high, middle and low category of MPK who received CAI is higher than that of the students who received PK. The achievement of students' abstract-thinking abilities in the high and middle categories of MPK who received CAI is significantly better than that of the students who received PK. Whereas, for the low category of MPK, the achievement of mathematical abstraction ability of students who received CAI is similar to that of students who received PK. The difference in mathematical abstraction achievement of students who received CAI and students who received PK can be seen from its average. The average achievement of students' abstract-thinking abilities in a higher category of MPK is higher than the middle and low category of MPK, the average of students' achievement of abstract-thinking abilities in middle category of MPK is higher than the low category of MPK. The higher the prior mathematical knowledge, the higher the average of the abstract-thinking abilities.

The results of this study also provide an illustration that there is no interaction between students who received CAI and PK, MPK categories (high, middle and low) with students' abstract-thinking abilities. This can be seen from the results of performed statistical tests. Because based on MPK categories (high, middle, and low) who received CAI, the achievement is higher than that of students who received PK, it can be concluded that CAI can be applied to students with high, moderate or low mathematical abilities. In other words, the learning factor does not depend on the category of MPK.

4. Conclusions
Based on the results of this study, it can be concluded that abstract-thinking abilities of students who were exposed to Cognitive Apprenticeship Instruction (CAI) is better than students who were exposed to conventional learning (PK) viewed from overall, school quality, and prior mathematical knowledge (MPK). The abstract-thinking abilities of students who received CAI in the high, middle and low category of KAM is in middle qualification. Thus it can be concluded that CAI can be a good alternative learning model to enhancement students' mathematical abstraction abilities.

References
[1] M Mitchelmore, P White 2004 Abstraction In Mathematics and Mathematics Learning. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* pp 329
[2] E Gray, D Tall 2007 Abstraction as a Natural Process of Mental Compression *Mathematic Education Research Journal* 19 pp 23
[3] J Monahagan, M F Ozmantar 2007 A Dialectical Approach to the Formation of Mathematical Abstractions *Mathematics Education Research Journal* 19 pp 89
[4] B G P Yusepa 2016 Kemampuan Abstraksi Matematis Siswa Sekolah Menengah Pertama (SMP) Kelas VIII *Symmetry: Pasundan Journal of Research in Mathematics Learning and Education* 1 pp 54
[5] Tata 2005 *Peningkatan Kemampuan Pemodelan dan Abstraksi Matematis serta Motivasi Belajar Siswa Sekolah Menengah Pertama Melalui Pembelajaran Kontekstual Kolaboratif* (Disertasi SPS UPI Bandung)

[6] B G P Yusepa 2016c Kesanpan Pembelajaran Matematika Realistik dengan Pembelajaran Matematika Konstektual *Prosiding Seminar Nasional Pendidikan Matematika Universitas Siliwangi* pp 168

[7] B G P Yusepa 2016d Analisis Per-bandlingan Kurikulum Pendidikan Indonesia dan Inggris untuk Meningkatkan Kompetensi Pe-dagogik dan Kompetensi Pro-fesional Guru Matematika. *Prosiding Seminar Nasional Matematika dan Pendidikan Matematika: Strategi Mengembangkan Kualitas Pembelajaran Matematika Berbasis Riset Unswagati* pp 346

[8] A D Jacobsen, P Eggen, D Kauchak 2009 *Methods for Teaching: Promoting Student Learning in K-12 Classrooms* (Yogyakarta: Pustaka Pelajar)

[9] A Collins 2005 Cognitive Apprenticeship. *The Cambridge Handbook of the Learning Sciences* 4 pp 47