Determination of the neutron star mass-radii relation using narrow-band gravitational wave detector

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Abstract. The direct detection of gravitational waves will provide valuable astrophysical information about many celestial objects. The most promising sources of gravitational waves are neutron stars and black holes. These objects emit waves in a very wide spectrum of frequencies determined by their quasi-normal modes oscillations. In this work we are concerned with the information we can extract from \( f \) and \( p_I \)-modes when a candidate leaves its signature in the resonant mass detectors ALLEGRO, EXPLORER, NAUTILUS, MiniGrail and SCHENBERG. Using the empirical equations, that relate the gravitational wave frequency and damping time with the mass and radii of the source, we have calculated the radii of the stars for a given interval of masses \( M \) in the range of frequencies that include the bandwidth of all resonant mass detectors. With these values we obtain diagrams of mass-radii for different frequencies that have allowed to determine the better candidates to future detection taking in account the compactness of the source. Finally, to determine which are the models of compact stars that emit gravitational waves in the frequency band of the mass resonant detectors, we compare the mass-radii diagrams obtained by different neutron stars sequences from several relativistic hadronic equations of state (GM1, GM3, TM1, NL3) and quark matter equations of state (NJL, MTI bag model). We verify that quark stars obtained from MIT bag model with bag constant equal to 170 MeV and quark matter in color-superconductivity phase are the best candidates for mass resonant detectors.

1. Introduction
The detection of gravitational waves (GWs) will have many implications in Physics and Astrophysics. Besides the confirmation of the general relativity theory, it will allow the investigation of several astrophysical phenomena, such as the existence of black holes and the mass and abundance of neutron stars, thus opening new scientific frontiers. The most promising sources for detection of GWs are neutron stars and black holes. These objects emit waves in a very wide spectrum of frequencies determined by their quasi-normal modes oscillations [1].

With the goal of analyzing the possibility of a future detection of the quasi-normal modes of compact stars by resonant mass detectors (RMDs), we focalize our attention in the region of the spectrum in the range at 0.8-3.4 kHz, which is the operation region of the antennas: ALLEGRO, EXPLORER, NAUTILUS, AURIGA [2], SCHENBERG, and MiniGrail. In particular we will
work with the frequency band of the spherical detectors SCHENBERG and MiniGrail (2.8-3.4 kHz) [3, 4].

The SCHENBERG detector, which is installed at the Physics Institute of the University of Sao Paulo (at Sao Paulo city, Brazil), is the second spherical detector ever built in the world and the first equipped with a set of parametric transducers. It has undergone its first test run in September 8, 2006, with three transducers operational. Recent information on the present status of this detector can be found in reference [5]. It is worth stressing also that among all known GW detectors, the spherical ones are the only ones capable to determine the direction of the incoming wave [6, 7].

In this paper we present an extension of the work made by Marranghello [1], where the authors show results for a restricted band of frequency that include the SCHENBERG and MiniGrail bandwidth. We analyze the case of a possible future detection made by all resonant antennas and compare the mass and radius range obtained from the frequency bands of the GWs modes to the mass and radius calculated with several relativistic equations of state (EoS) models.

In section 2 we introduce the f and p_l-modes, in section 3 we show the mass-radii diagrams for the f and p_l modes and compare them with the ones obtained with different relativistic EoS models, in section 4 we introduce the damping time of the f-mode and its mass-radii diagram and finally, in section 5, we make the last considerations.

2. The quasi-normal modes: f and p_l-modes
The neutron stars have a rich spectrum of frequencies because the fluid perturbation oscillates in many different modes. From the GW point of view the most important quasi-normal modes are the fundamental mode of the fluid oscillation (f-mode), the first pressure mode (p_l-mode), the first GW mode (w_l-mode) [8] and the r-modes that, under certain circumstances, can be an important source of GWs [9].

In this work we concentrate in the f and p_l-modes. The fundamental mode can be described by the density distribution inside the star, while the p-mode is the pressure restoration force. In reference [10], the authors have obtained an empirical formulae for the frequencies of these two modes as a function of the mass and radius using a wide sample of equations of state:

\[ \nu_f = (0.79 \pm 0.09) + (33 \pm 2) \sqrt{\frac{M}{R^3}}, \]  
\[ \nu_p = \frac{1}{M} \left[ (-1.5 \pm 0.8) + (79 \pm 4) \frac{M}{R} \right], \]

where the mass and the radii are given in km (remember that \( M_\odot \approx 1.477 \text{ km} \)), while \( \nu_f \) and \( \nu_p \) are given in kHz. Using the empirical relations (1) and (2), we have calculated the radii \( R \) of the stars for a given interval of masses \( M \) in the range of frequencies that include the bandwidth (0.8-3.4 kHz) of all RMDs in operation. In Table (1) we can see the resonant frequencies of these detectors.

Through these relations we have obtained diagrams for p_l and f-modes that relate GW frequency with masses and radii of the sources. These diagrams allow us to determine the better candidates for a future detection by resonant antennas from the compactness of the star. We can see in figure (1) and (2) the f and p_l-mode’s mass-radii diagrams where the different colors scale identify the different frequencies.
Table 1. Frequency band of the RMDs in operation in the world.

| Antenna   | Location         | Freq.(Hz) | Type      |
|-----------|------------------|-----------|-----------|
| ALLEGRO   | Baton Rouge      | 890-920   | Bar       |
| EXPLORER  | CERN             | 895-920   | Bar       |
| NAUTILUS  | Frascai          | 905-925   | Bar       |
| AURIGA    | Legnaro          | 850-930   | Bar       |
| SCHENBERG | São Paulo        | 3100-3300 | Spherical |
| MiniGrail | Leiden           | 2800-3000 | Spherical |

3. Comparison of the mass-radius diagrams with the ones obtained by relativistic EoS models

To determine which relativistic models of compact stars emit GW in the frequency bands of the RMDs we compare the diagrams of the relations (1) and (2) with some mass/radius neutron star sequences obtained by different relativistic models that generate several equations of state for hadronic matter such as models NP, NPH, NPHQ with and without isovector-scalar δ [11], namely

- the models with parameters set GM1, GM3, NL3, TM1 [12, 13]

and some for quark strange matter as

- Nambu-Jona-Lasinio model (NJL) [14], color-flavor locked phase (CFL) [15] and the MIT bag model with different values of the bag constant [16] and hybrid star EoS.

Relativistic hadronic models have been widely used in order to describe nuclear matter, finite nuclei, stellar matter properties, and recently in the high temperature regime produced in heavy ion collisions [17]. Many variations of the well known quantum hadrodynamic model [18] have been developed and used along the last decades. Some of them rely on density dependent couplings between the baryons and the mesons [19–25] while others use constant couplings [26–28]. Still another possibility of including density dependence on the lagrangian density is through derivative couplings among mesons and baryons [29–31] or the coupling of the mediator mesons among themselves [32–34]. The relativistic model couplings are adjusted in order to fit expected nuclei properties such as binding energy, saturation density, compressibility and energy symmetry at saturation density, particle energy levels, etc. These same relativistic models are extrapolated to higher densities as in stellar matter and the results obtained for the neutron star masses and radii are quite good in comparison with the astronomical observations. In the case of bare quark stars, the strange matter inside the star is usually describe by the MIT bag model, a Fermi gas of free quarks with a vacuum energy known as the bag constant, or by chiral models like the Nambu-Jona-Lasinio (NJL) model that has a dynamical chiral symmetry breaking mechanism that originates mass for the quarks. Recently, the possibility that quarks can be paired at high densities and be in a color superconductive phase has originated new quark matter equations of state, that depending on the pairing interaction, can be quite stiff and produce large stars masses and radii [35–37]. The main feature of quark stars, since they are bound by the strong force and not by gravity, is that they are more compact and have smaller mass to radius ratio than a neutron star. As we will see, it is this fact that strange stars can have small radii, that explains the high frequency GWs modes produced by this type of stars.

We can see in the diagrams (1) and (2) that the frequency band of the RMDs are on the dark region, where it is expected that GWs generate from less compact neutron stars in both diagrams. This fact shows the impossibility of a future detection, by cylindrical antennas, of
relativistic neutron star candidates emitting gravitational wave on $p_f$ or $f$-mode. However, we can see on the diagrams that the spherical detectors bandwidth, MiniGrail and Schenberg, have some candidates near their resonant frequencies. The most probable source would correspond to a very compact object with radius smaller than 10 km. The models that fulfill this condition are models of strange quark stars, as preview in [1]. This fact is confirmed when we compare the compact star sequence generated from the MIT bag model (with bag constant $B^{1/4} = 170$ MeV), NJL model and CFL of quark matter. On the other hand the $p_f$-mode would only be expected to come from less compact neutron stars.

![Figure 1](image)

Figure 1. Mass-radius diagram of the empirical relation 1 obtained by Benhar et al for $f_1$-mode. We compare with some models of relativistic EoS. The different colors identify the different frequencies.

4. The damping time
How can we distinguish the $f$-mode in a putative detection? And how to determine the mass and radius of the star? The damping time is the response for these questions [1]. In [10] the authors obtained an empirical relation for the $f$-mode damping time as function of the radius and mass, described by:

$$\tau_f = \frac{R^4}{cM^3} \left[ (8.7 \pm 0.2) \cdot 10^{-2} + (-0.271 \pm 0.009) \frac{M}{R} \right]^{-1}. \quad (3)$$

Even though the RMDs can not determine the damping time properties with small errors, we use the empirical relation (3) to calculate the damping time given the intervals of radius and mass $(R, M)$ obtained with relation (1). We can get a new mass-radius diagram, but doing a distinction in the damping time. We compare the diagram with models of quark stars, CFL and MIT bag model with bag constant equal to 170 MeV. Results obtained can be seen in figure (3). Through these results we can estimate the masses and radii of the stars solving the inverse problem, as show in [9].
Figure 2. Mass-radius diagram of the empirical relations obtained by Benhar et al for $p_1$-mode. We compare with some models of relativistic EoS. The different colors identify the different frequencies.

Figure 3. Mass-radius diagram of the empirical relations obtained by Benhar et al for damping time of the $f_1$-mode. We compare the diagram with models that describe quark stars, CFL of the quark matter with bag constant 200 MeV and gap 100 MeV and MIT bag model with bag constant equal to 170 MeV. The different colors identify the different damping time.

5. Summary
RMDs bandwidth are on the spectrum regions with a few (or none) neutron star candidates emitting GWs through their $f$ and $p_1$-modes. However, the spherical detectors are on a region where the $f$-modes of very compact objects can be detected. All sequences of neutron stars described by quark matter models are on the region near to MiniGrail and Schenberg bandwidth, but the MIT bag model with bag constant equal to 170 MeV and CFL of the quark
matter constant 200 MeV and the gap 100 MeV are the best candidates for these detectors, as we can see in figures (1) and (3). On the other hand the detection of the f and p $I$-modes of neutron stars by bar detectors is unlikely, because their bandwidth is located in low frequencies.

Acknowledgments

This work was partially supported by FEDER and FCT (Portugal) under the project PTDC/FP/64707/2006. The CHL, MM, RMM thank the financial support given by CAPES through the fellowship 2071/07-0 and the international cooperation program Capes-Grices between Brazil-Portugal.

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