Research Article

Eliminating Boundary Layer Separation on a Cylinder with Nonuniform Suction

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Boundary layer separation negatively influences the performance of aerospace vehicles, for example, by triggering static stall or reducing combustion engine efficiency. Developing effective flow control to delay or eliminate separation is therefore of real use to the field. In this paper, numerical studies were carried out to optimise distributed suction profiles for preventing boundary layer separation on a circular cylinder in the fully laminar regime (Re < 188), with the least control effort. Relationships were found between the Reynolds number, the separation angle of the uncontrolled case, and the uniform suction needed to eliminate separation. It was found that for Re > 20, the uniform suction required to eliminate separation followed a quadratic profile, as a function of Re. Maximum uniform suction effort was needed at Re = 20, requiring a suction coefficient of CQ = 49.14 (as a percentage of the free-stream velocity) to eliminate separation. To resolve the best nonuniform suction profile at Re = 180, a variety of optimisation studies were performed using the coordinate search method. It was determined that the use of six control segments on each half of the cylinder provided the best control and efficient convergence to the optimal solution. 6-segment nonuniform suction eliminated separation at Re = 180 with net suction effort of CQ = 13.26 compared to CQ = 31.25 for the uniform case. These optimal suction profiles were compared using time-dependent simulations to confirm that both methods eliminate separation when introduced to an already unsteady case. Nonuniform suction eliminated separation faster, though uniform suction was more stable.

1. Introduction

Boundary layer (BL) separation occurs in nearly every real-world fluid flow because the conditions for its occurrence are so prevalent [1]. These conditions are (1) the presence of an adverse pressure gradient, (2) no-slip condition with a bounding surface, and (3) insufficient momentum in the flow [2]. The impact of BL separation is felt throughout the aerospace field. For example, static stall of aeroplanes is caused by early separation over the wing and loss of the pressure profile necessary to generate sufficient lift. In turbofan and other turbine engines, separation of the gas flow through the engine can dramatically increase engine losses, reducing its efficiency [3]. Similarly, in supersonic ramjet (scramjet) engines for hypersonic aircraft, control of BL separation may be a necessary requirement to maintain an effective shock train and ensure continual sufficient thrust as its growth can have a substantial effect on performance [4–7]. The aerospace engineer has two methods at their disposal to combat these dramatic effects: (1) careful design and engineering of the shape and composition of the bounding surfaces (the body) or (2) direct control of the fluid flow [8].

While many constraints typically limit the design choices available to the engineer, the second option—that of direct flow control—offers near limitless options to improve a particular flow. Some examples of direct flow control include suction or blowing through the bounding surface, vortex generators, plasma actuators, and synthetic jets [9–12]. Each has their own set of parameters that can be adjusted to optimise the control. However, this large parameter design space can make it very difficult to determine the best or “optimal” control.
Flow control and its optimisation have been the subject of much interest since the beginning of the twentieth century as demonstrated by the number of reviews on the topic since then [8, 13–17]. Rather than defining an optimal control problem for each new flow (a time-consuming process with no guarantee of success), the engineer can benefit from general rules of control and estimations of their potential effectiveness to guide their initial iterations of design. To provide such rules (if only preliminary) for the case of BL separation with suction control is the ambition of this paper.

Since BL separation occurs in nearly every physical flow, it is possible to investigate and optimise flow control for a simpler flow than that of—for example—a three-dimensional wing or a turbine engine. By carrying out this investigation, relationships between the uncontrolled flow, the optimal control parameters, and the final controlled flow may be developed—relationships which can help guide the engineer in their choice and design of flow control for the next generation of aerospace vehicles and engines.

The phenomena exhibited by fluid flow past a circular cylinder have been the subject of extensive research for over a century now. For almost as long, researchers have investigated control methods to improve the characteristics of such flows [18]. Although the problem appears mundane, the flow around a cylinder is extremely rich and complex. By only changing the Reynolds number, Re = ρUD/μ, a wide range of flow regimes can be seen. Because of this complexity, the design of optimal flow control for the circular cylinder continues to be an active field of research [8, 19, 20]. As the least streamlined bluff body without sharp edges, the circular cylinder offers an extreme example of adverse flow to control. If the characteristics of this flow can be properly understood and adequately controlled, the theory and methods used to achieve this end can be extended to more typical bodies seen in engineering applications.

Despite this subject’s long history, the bulk of research has been focussed on either reducing the drag of the body or eliminating the vortex shedding—which occurs in certain ranges of Re and generates undesirable lateral forces on the cylindrical body. This narrow focus can be seen clearly in the discussion on this topic in the recent review by Choi et al. [8]. On the other hand, one parameter of the flow that is disproportionately underreported in the literature is the angle of separation, θ. The angle of separation marks the point around the cylinder at which the boundary layer detaches from the wall and flow reversal commences. Furthermore, separation of flow marks the onset of the body wake and is the cause of pressure drag on a body. Pressure drag is by far the major contributor to total drag coefficient at the Re commonly encountered in engineering applications (Re > 1000). Delaying or eliminating separation, therefore, can drastically reduce the drag on a body. Conversely, in some cases, it is desirable to have early separation. For example, it is desirable to develop a strong recirculatory region and promote mixing such as in heat exchangers. It is clear then that the separation angle is a critical parameter in describing the nature of the flow around a cylinder and should be investigated thoroughly.

The separation angle may be underrepresented in the literature because it can be more difficult to measure experimentally than, say, the drag on a body. Due to the unsteady nature of flow around a cylinder for a large range of Reynolds number, it can be hard to clearly identify the separation angle at any moment or even measure a time-averaged value. A 2004 paper by Wu et al. collated the existing experimental and numerical data on the separation angle from the literature for Re < 400 and found an up to 10° discrepancy between their values [21]. Modern measurement techniques such as particle image velocimetry (PIV) and the improved capability of computational fluid dynamics (CFD) software have made investigation of the separation angle much more accessible in recent years, making it a more viable value for measurement and parameter for use in control systems.

One of the many methods for controlling the flow over a cylinder is the use of suction. This control method has a long history, with Prandtl carrying out experiments on a cylinder with suction through a slot to test his boundary layer theory in 1904 [18]. Nevertheless, this control method continues to offer new opportunities for improved control, and our knowledge of its effects is by no means comprehensive. Suction helps reduce drag because it delays flow separation, reducing the pressure drag. Wall suction removes decelerated fluid elements near the wall, causing higher momentum particles from the free stream to reinvigorate the boundary layer [2]. Suction on a cylinder has been investigated to a great depth in the literature, both experimentally and numerically. The reader is directed to reviews by Rashidi et al. and Choi et al. for some examples [8, 22]. Suction can be applied at a bounding wall either through slots (slot injection) or through distributing continuously across regions with the use of a porous wall. The latter case is of most interest in this research as it allows the possibility of arbitrarily varying suction profiles.

The most notable examples of experimental investigation into uniform suction on a cylinder are those by Pankhurst et al. [23] and by Fransson et al. [24], though there have been many others [25–27]. The experiments by Pankhurst et al. were some of the earliest, while those carried out by Fransson et al. were comprehensive in the turbulent wake regime. Pankhurst et al. determined that sufficient suction brings the flow close to that predicted by potential flow theory. Fransson et al. developed an effective Reynolds number which can be used to map the uncontrolled flow characteristics to the expected controlled flow characteristics using the Strouhal number of the resulting wake. However, this cannot be employed if the aim is to eliminate separation entirely (and thereby the vortex street too) or when there is no oscillatory wake—as it is often the case in aerospace applications.

In addition to uniform suction across the whole cylinder, some researchers have investigated the possibility of nonuniform suction. Notable for the present paper is a study by Li et al. which used an adjoint-based method to determine the optimal suction and blowing profile on a cylinder at Re < 200 to minimise drag [28]. Again, however, separation was not eliminated. The advantage of nonuniform suction profiles is that the total suction effort required might be reduced.
by focusing the control where it is most effective. For suction, this is typically over the rear-half of the cylinder [28].

This paper is aimed at determining to what extent this is true in the investigated flow regime and how the separation angle is affected. This research was motivated by the deficiency of research in the present literature on the effect of flow control on the angle of separation over a cylinder. Since the separation angle is the determining characteristic on the pressure drag of a body and an important characteristic to control, the present research was aimed at filling this gap in the literature by studying the effect on separation angle of flow around a cylinder controlled with uniform and nonuniform suction.

To this end, numerical simulations were used to model the flow around a circular cylinder with nonuniform suction in the fully laminar regime. The objective of the study was to determine whether suction could eliminate boundary layer separation on a cylinder entirely and investigate what control parameters could achieve this goal with the least effort, i.e., the minimum suction to prevent separation. Simulations with uniform suction were performed at a variety of Re in the range 4 < Re < 188, and the effect of changing the uniform suction rate at Re = 80 was investigated. Initially, the aim was to compare results for nonuniform suction with those found for suction and blowing by Li et al. who determined an optimal profile for drag reduction at Re = 80 [28]. The parameters of that study could not be compared easily; however, so it was decided to carry out the study of nonuniform suction at Re = 180 instead. This Re marks the most extreme end of the 2D vortex shedding regime, so control of this flow should result in more interesting control characteristics. Although the range of Re investigated is not typically experienced by bodies in the real world, it offers a simplified starting point to build up numerical models and optimisation methods for flow control by nonuniform suction. Once these methods have been successfully tested, they can be adjusted for further investigation in more physically appropriate regimes, such as the prevention of separation over airfoils, or promoting separation and mixing in hypersonic combustion engines.

2. Model and Numerical Method

2.1. Geometry, Boundary Conditions, and Governing Equations. The geometry for the numerical simulations can be found in Figure 1(a). The domain was based on that used by Wu et al. for the numerical portion of their paper and has also been used successfully by the present authors [21, 29]. The dimensions are dictated by the diameter of the cylinder, D = 0.1 m. Not visible in the figure is a circular curve with diameter 1.0002D concentric with the cylinder. This curve was used for identifying the angles of separation (detailed in Section 3.2). The length of the domain aft of the cylinder was found to be sufficient for vortex shedding to fully develop. The domain and geometry remained the same for all studies; the Reynolds number was adjusted by scaling the inlet velocity.

Uniform flow conditions were imposed at the inlet (left boundary) with u = U = Re v/D, v = 0. The upper and lower boundaries were defined as no-slip moving walls with the same velocity as the inlet. By applying this condition to the upper and lower boundaries, the blockage ratio is effectively reduced to zero. A pressure outlet condition with zero relative pressure was imposed on the right-hand boundary. For the cylinder walls, an outlet condition was applied with the normal outflow velocity constrained, un = w, ui = 0, where un is the fluid velocity normal to the cylinder surface, ui is the tangential velocity, and w is the suction velocity. The nondimensional suction coefficient typically used in the literature, cQ = w/U × 100, was used as the control parameter (in this paper, cQ indicates the local suction coefficient, while CQ indicates the net suction coefficient). For the case of nonuniform suction, the suction velocity profile was altered to be a function of the angle from the trailing edge (TE) of the cylinder, i.e., w = w(θ) (further details in Section 3.1). When there is no suction applied, the boundary is treated as a no-slip wall condition.

Fluid properties for water at 20°C were used to generate an incompressible two-dimensional isothermal flow. The flow was governed by the Navier-Stokes equations, which will not be detailed here for the sake of conciseness.
2.2. Computational Mesh and Time-Dependent Parameters. The mesh used for the studies can be seen in Figures 1(b) and 2. A structured grid with quadrilateral elements was used throughout the domain. The mesh was refined significantly around the cylinder in order to accurately capture the flow details and for precise measurement of the angle of separation. Table 1 displays the key characteristics of the final mesh used, which produced mesh convergent results for the transient case.

2.3. Solver Methods. The numerical analysis of this study was carried out using COMSOL Multiphysics® to solve the incompressible two-dimensional Navier-Stokes equations. The laminar flow module was used, assuming incompressible flow. No turbulence model was employed as the uncontrolled Re range under investigation is fully laminar.

Both time-dependent and steady-state simulations were performed for this research. The PARDISO solver algorithm was used for both cases. For the time-dependent case, an implicit Backward Differentiation Formula (BDF) time-stepping method was employed, with variable order time stepping to automatically adjust the stepping parameters to achieve convergence. For the time-dependent simulations, this time-stepping condition was set to “intermediate” so that COMSOL must solve at least one time step within each interval specified. This prevents numerical smearing that might stop the development of instabilities that induce vortex shedding.

2.4. Mesh Independence and Time Parameters. An independence study was carried out at Re = 80 and Re = 180 to determine the most appropriate mesh and time-scale parameters (step size, simulation length) to accurately capture the vortex shedding expected. As a time-dependent simulation is necessary to capture the periodic flow behaviour, the time parameters must be scaled with the minimum mesh element size to satisfy the CFL condition. It was determined that a time step of \( \frac{1}{20} \) was sufficient for the instability to be fully developed after 400 time steps, where \( x_0 \) is the distance from the inlet to the centre of the cylinder as shown in Figure 1.

Because the mesh is symmetric along the flow path, a disturbance to the flow must be introduced for the vortex shedding to develop. To achieve this, we used a steady-state solution was found for an unstructured mesh under the same conditions. The solution to this was then used as the initial conditions for the following time-dependent study. This proved a reliable method for the solver to converge to the Bénard-von Kármán street flow quickly. To improve the simulation time, a coarse time step was used over a long period to fully develop the wake instability, followed by finer time stepping to get precise measurements of the vortex shedding—usually about \( \frac{1}{30} \).

3. Optimisation Methods

3.1. Objective Variables. The variables altered by the optimisation algorithm were the local suction coefficients, \( c_s \), used to define the suction velocity over defined regions of the cylinder wall. To allow nonuniform suction and impose any arbitrary profile, the suction coefficient at any point on the cylinder wall was made the output of a piecewise function. The function specifies the suction coefficient in any defined arc on the cylinder, which is then used as independent control variables. With a piecewise constant function, sharp transitions at the boundaries of the suction profile often lead to small separation bubbles. To minimise this occurrence, continuous second derivative smoothing between each data point was used, as in Delaunay and Kaiktsis [26]. This provides a continuous distributed suction profile across the entire cylinder surface.

With this method, the suction profile over the cylinder wall can be broken into as many discrete segments as desired without altering the geometry or boundary conditions—only redefining the piecewise function. Figure 3 gives an example of the cylinder divided into 36 arcs by the piecewise function and an arbitrary suction profile. To avoid the suction profile imposing any asymmetry in the cross-flow direction, it was imposed that the suction profile on the lower half of the cylinder always mirrored that on the upper along the central \( x \)-axis. Therefore, a scenario with the cylinder broken into 36 segments, as shown in Figure 3, really only has 18 control parameters and will be referred to as an 18-segment control case.

3.2. Optimisation Objectives and Method. The overall objective of the control is to prevent separation over the cylinder.

![Figure 2: Close-up view of the mesh around the cylinder surface with 42 × 80 P1 + P1 elements in each quartile.](image)

| Region | Mesh resolution \((x \times y)\) | Number of elements | Average quality | Minimum quality |
|--------|-------------------------------|---------------------|-----------------|-----------------|
| A      | 20 × 20                       | 400                 | 1.000           | 1.000           |
| B      | 80 × 20                       | 1600                | 1.000           | 1.000           |
| C      | 20 × 60                       | 1200                | 1.000           | 1.000           |
| D      | 20 × 80                       | 1600                | 1.000           | 1.000           |
| E      | (42 × 80) × 4                 | 3360                | 0.881           | 0.500           |
| F      | 60 × 80                       | 4800                | 1.000           | 1.000           |
| Total  |                               | 26240               | 0.9298          | 0.500           |
with the least suction possible. To this end, two objective functions were defined and are described in

\[
\text{Obj}_1 = \theta_s = \frac{1}{2} (\theta_{\text{upper}} + \theta_{\text{lower}}),
\]
\[
\text{Obj}_2 = C_Q = \frac{1}{n} \sum_{i=1}^{n} c_{Q_i}.
\]

The first objective is the angle of separation, \(\theta_s\), as measured from the trailing edge of the cylinder. \(\theta_s\) is taken as the average of the upper, \(\theta_{\text{upper}}\), and lower, \(\theta_{\text{lower}}\), separation angles for when any asymmetry develops. To determine the separation angles, a MATLAB function was created and incorporated into the COMSOL model. The function takes the angle and tangential velocity data at the node points along a curve defined just above the cylinder surface. The function identifies the separation points as any point where the tangential velocity changes from +ve to –ve, i.e., where the flow reverses. When this point falls between two node points, linear interpolation is used to estimate its location more precisely. In standard practice, the separation point is typically identified by evaluating or measuring the skin friction along the surface and identifying where it changes from a negative value to a positive one. We choose to use this unconventional method of directly measuring the flow direction just above the wall. The reason for this was that the wildly varying suction profiles imposed by the optimisation process could in certain circumstances create numerical discontinuities or artefacts that might interfere with the conventional evaluation. The reversed flow method was verified against the conventional \(\theta_s\) identification process (using skin friction) for the uncontrolled case and simple suction cases (uniform suction) and returned numerically identical results.

In the case of multiple separation points (e.g., if separation bubbles are present), the function returns the angle of separation closest to the leading edge of the cylinder. \(\text{Obj}_1\) was scaled by a factor of 10 when incorporated into the global objective so that its reduction is favoured over minimising suction effort and the final solution will be unseparated flow. The second objective is the average suction applied. The global objective was to minimise the sum of these objectives. An optimality tolerance of 0.001 was used and the maximum number of simulations limited to 100,000.

For this study, the “coordinate search” method from the COMSOL Optimization Module was used. This is a derivative-free method and attempts to improve the objective function along the coordinate directions of the parameter space. Although it is quick, it is restricted in that it can only move in one coordinate direction each step. The effectiveness and limitations of this optimisation method will be discussed briefly in Results and Discussion.

4. Results and Discussion

4.1. Validation. The results of this study were validated by comparing the separation angles for the no-suction, time-dependent cases with the fit found by Wu et al. [21]. This process was used to evaluate a variety of uncertainties: whether the current time-dependent method can accurately model periodic vortex shedding, whether the steady-state simulations identify accurate separation angles, and if the method of separation detection, itself, is adequate.

As can be seen in Figure 4, the time-dependent results match Wu et al. very closely, while the steady-state simulations predict slightly earlier separation than reality. Note that the error bars of the time-dependent results show the variation in the instantaneous separation angle, whereas the solid markers give the time-averaged value after one period of
vortex shedding. The minimum and maximum instantaneous separation angles also closely align with those from Wu et al. (see Figure 8 of that paper).

Based on these results, we conclude that the time-dependent model and $\theta_s$ detection method are valid. While the steady-state results are off slightly from the time-averaged values, they do follow the same trend as the time-dependent model. The steady-state results closely follow the upper bound of the separation angle. As each steady-state simulation is much faster than the time-dependent cases ($<7$ s vs. $>15$ min) and it provides a conservative estimate of the separation angle, it was deemed appropriate to use the steady-state case for the optimisation procedure. In addition, as suction is known to improve the stability of a given flow, most of the simulations carried out during the optimisation studies would indeed be steady, and not periodic, in reality. For the case where separation is successfully eliminated, there should not be any time-dependent behaviour in the wake. This was verified with full time-dependent simulations, described in Section 4.4. Thus, the steady-state simulations were determined to be sufficiently accurate for optimisation studies, and the optimal scenarios can be verified with confidence using the time-dependent method.

### 4.2. Uniform Suction

Steady-state optimisation studies were carried out for the case of uniform suction at a variety of Reynolds numbers (only one $C_Q$ parameter for the whole cylinder). The key results are shown in Figure 5. There is a clear trend in the suction effort required to eliminate separation. At $Re < 20$, the suction effort is very sensitive to the Reynolds Number. This is reasonable as the separation angle for the no-suction case is very sensitive in this region as the wake develops. With increasing $Re$ from $Re = 20$, the suction required decreases smoothly. A quadratic curve with equation $C_Q = 0.0004183 Re^2 - 0.2009 Re + 53.74$ has been fitted to this region as seen in Figure 5(a). In addition, this figure shows that the maximum suction effort is required at $Re = 20$. This is unexpected as $Re = 20$ is in the middle of the symmetric vortex-pair regime and does not mark any significant change in the flow for the nonsuction case.

Figure 5(b) provides additional information. This plot marks the separation angle before control is applied (i.e., on the cylinder with no suction) and the corresponding uniform suction required to eliminate that separation. Since Wu et al. showed that $\theta_s$ in this $Re$ range could be defined solely in terms of $Re$, this plot is really only an alternative mapping of the one seen in Figure 5(a). An uncontrolled cylinder at $Re = 20$ has $\theta_s = 44.21^\circ$. This is very close to $45^\circ$ which is an inflection point on the pressure curve for potential flow around a cylinder. The plot shows that, after the peak at $45^\circ$, the suction effort to prevent separation actually decreases despite the uncontrolled $\theta_s$ moving further from the trailing edge, while the plotted curve suggests that it may continue to decrease and meet the $x$-axis at some higher uncontrolled $\theta_s$. We expect an asymptote to be reached in actuality. It would suggest, otherwise, that there exists some threshold Reynolds number beyond which no suction is required to prevent separation—which is known to be false.

The optimal parameters found by this study were confirmed by performing a variety of time-dependent simulations at $Re = 80$. The simulations were run with a variety of suction coefficients, up to the optimal found here. This was to confirm that the above results are accurate. The results of this verification can be seen in Figure 6. This study confirmed that the optimal parameters found by the above method were correct. An apparently quadratic relationship between the suction applied and the controlled separation angle can be seen in this plot. The figure also shows that the vortex street is eliminated much sooner than boundary layer separation. Vortex shedding is eliminated between $C_Q = 10$ and $C_Q = 15$. Although the vortex shedding was eliminated at $C_Q = 15$, it took a much longer time than with higher suction. It is interesting to note that the angles of separation with these control parameters are $55.5^\circ$ and $50.0^\circ$, respectively, and vortex shedding on an uncontrolled cylinder starts at $Re = 47$ when the angle of separation is $55^\circ$. This and the fact that maximal suction effort is needed at $Re = 20$ when $\theta_s \sim 45^\circ$ suggest that geometric features are tightly related to the separation of the flow and its stability, and these relationships may continue to be significant even when the flow is drastically altered from the base case through control.

Much of the motivation of this study was to investigate whether the separation angle of an uncontrolled flow is more useful in dictating the optimal control parameters than other features, such as the Reynolds number. This would be useful for the translation of knowledge in the field of flow control. Presently, if it is desired to control the flow around a particular body—e.g., an aeroplane fuselage—it is not possible to carry much—if any—quantitative insight from control studies on a different body, e.g., the circular cylinder. Therefore, data for the optimal location of suction or the strength of suction on the cylinder at a particular Reynolds number is not transferable directly to the fuselage. Indeed, even comparing the Reynolds number directly is not straightforward, as exhibited by the potential differences in critical Reynolds number at which turbulence commences. This means that the engineer seeking to improve the flow around the aeroplane fuselage must define and perform their own flow...
control study and optimisation—a costly and time-consuming exercise.

The present results suggest, however, that instead of comparing the Reynolds numbers—and the control parameters for the best performance at these Reynolds numbers—comparing using the separation angle may be more appropriate. Unlike the Reynolds number, which gives no information about the resultant flow (as seen by the same Reynolds number describing both our uncontrolled separated flow and the controlled unseparated flow here), the location of the separation point is both characteristic and accounts for all the static and dynamic effects in the flow. Therefore, the engineer designing control of a fuselage, needs only know the conditions of the baseline uncontrolled flow in order to design a good first attempt at control using results from a flow control study on the circular cylinder. The distance of the separation point to the trailing edge may act as the “zero point” around which to design—regarding the location and strength of suction (or other flow control parameters). It should be stressed that the present paper does not resolutely confirm that this is, or can be, the case. However, the results do show a good dependency between the optimal control parameters and the separation angle of the uncontrolled flow.

4.3. Nonuniform Suction. Eight optimisation studies were carried out with nonuniform suction at Re = 180 using the coordinate search method. In each study, the number of control segments was altered to provide finer control of the suction profile. One additional study was performed using the Nelder-Mead (N-M) optimisation method to test the effectiveness of the coordinate search method. The key results of these simulations can be seen in Figure 7.

From the results, we can conclude that nonuniform suction is more efficient than uniform suction. Additionally, the total suction required to prevent separation tends to decrease with the increasing number of control segments. The decreasing trend in controller effort appears to reach a floor in effectiveness where further increasing the number of segments (and the resolution of the potential profile) is no longer effective at improving the performance or efficiency. The results from 9- and 18-control segments are almost identical. This would suggest that increasing the number of control segments further would not improve the control, despite being able to manipulate the suction profile more precisely.
As can be seen in Figure 7, the best control is achieved with six control segments and requires an average suction coefficient of $\text{Obj}_2 = C_{\text{Q}} = 13.26$. This is less than half that required for uniform suction. The suction profile and resulting flow for this 6-segment control are shown in Figure 8. It is interesting that the best result is achieved with less control parameters than others investigated. One would expect that the most efficient control would be achieved when the suction can be manipulated most precisely. In this instance, it seems reasonable to conclude that the slightly worse results with 9 and 18 segments are due to the coordinate search method converging to a local minimum. The coordinate search method is a relatively simple optimisation method. As it is one-directional in each step, it is more likely to fail at finding the global optimal when the parameter space is very complex. This is illustrated by the drastic difference in optimised control when the Nelder-Mead (N-M) method was used as for the 4-segment case [30]. This result suggests that a different optimisation algorithm may be more appropriate when the number of control parameters is large.

Regardless of the optimisation approach, guaranteeing that a global minimum is reached is impossible without a thorough analytical proof that is not available for this system. The coordinate search method is effective and quick for most situations investigated here.

Even with the improvement in controller effort when the N-M method was employed, the 4-segment case is still worse than the adjoining 3-segment case, despite the ability for the control to be more discriminate. We suggest that this discrepancy is due to the location of the control boundaries on the cylinder for the 4-segment control—namely, at 45° and 90° from the trailing edge. In all control cases investigated for $\text{Re} = 180$, the optimised suction profile featured maximum suction in the region approximately 45° from the trailing edge (see Figure 9). The 4-segment control can only affect control at this critical angle by manipulating two control parameters in unison, due to the border of control segments being defined at 45°. It is impossible to achieve an effective control with this setup, therefore. This implies two important conclusions: (1) the location of the suction (angle of suction) is of critical importance in the effectiveness of control and (2) the control must be set up to account for this. Indeed, other investigations have verified the importance of the location of suction for delaying separation on the circular cylinder, including the fact that this is dependent on the Reynolds number [29]. Consequently, an ideal control that will be effective over a range of Re must be able to account for the changing location of optimal suction.

The optimal scenarios were tested with time-dependent simulations which confirmed their ability to stabilise the flow and keep it attached over the entire cylinder (see Section 4.4). For the uniform suction case, the true optimality was confirmed using parametric studies as shown in Figure 6. For the nonuniform profiles, it is difficult to determine that the global optimum has been achieved without a more comprehensive search of the parameter space, but this would require very long simulation times. Comparison to similar situations in the literature can verify the results partly, however. Some of the optimal control suction profiles are shown in Figure 9. As can be seen in the figure, the best suction profiles ($N > 5$) all featured maximum suction over the rear-half of the cylinder, particularly in the region $30 < \theta < 90$. This fits well with the results seen in literature for both distributed suction [13] and slot suction [27]. Qualitatively, the optimal suction profile found the 18 segments is similar to that found by Li et al. using the adjoint method at $\text{Re} = 80$ with 18 suction and blowing parameters (see Figure 5 of that paper). Unfortunately, the results cannot be compared quantitatively due to the differences in objectives and control setup.

4.4. Verification of Optimal Results. Given that the optimisation study employed steady-state simulations—which were found to be somewhat inaccurate for the uncontrolled case—it was important to verify the results with the validated time-dependent model. For each verification, the final solution for the uncontrolled time-dependent study was used as the initial conditions for the new study. In other words, the flow with the vortex shedding already fully developed was used as the starting point for these simulations. The fine-resolution time stepping from that study was also used here.

To maintain numerical stability, the suction profile was introduced using a ramp profile increasing from 0% at $t^* = 42$ to its full value at $t^* = 142$, where $t^*$ is the nondimensionalised time value $t^* = t(U/x_0)$. The simulations were then run...
until the flow had fully developed and was steady for at least $t^* = 20$. The resulting flow fields for the flow with and without the 18-segment control are shown in Figure 10. A plot of the separation angles and aerodynamic coefficients over time with the optimal uniform suction and 18-segment control is presented in Figure 11.

From the results, it is apparent that both controls eliminate separation of the flow; however, the 18-segment control is slightly faster at achieving this goal. Despite reaching its full profile at $t^* = 142$, the uniform suction control does not eliminate separation entirely until $t^* = 182$, whereas the segmented control does so immediately upon reaching its full profile. On the other hand, the 18-segment control is more sensitive to its parameters as shown by the sudden changes in the separation angle. This is likely because the control is more acutely applied and therefore is more sensitive to flow conditions and its own parameter values. An analogy to dynamic control seems appropriate, with the uniform control similar to an overdamped system, while the 18-segment control is critically damped but potentially unstable. Another interesting feature is that the 18-segment control initially worsens the separation before improving it, despite both controls being activated at the same time step. This is likely due to the more directional profile of the nonuniform suction, making its effect sensitive to the phase of vortex shedding when it is activated.

Despite these differences, both controls stabilise the flow (and stop the vortex shedding) at the same time step, $t^* = 130$. It should be emphasised that the suction to eliminate vortex shedding is less than that required to eliminate separation, as this example shows. The elimination of vortex shedding or the minimisation of drag has usually been the focus of research when investigating flow around cylinders, so further comparison between these two critical controls may be useful.

5. Conclusions

Numerical studies were carried out to determine the optimal suction profiles to prevent boundary layer separation on a circular cylinder in the fully laminar regime ($Re < 180$). The coordinate search method was effective at locating the optimal suction parameters, although the control setup could drastically impact the results. It was determined that both
uniform and nonuniform suction profiles were capable of completely eliminating separation in all flow regimes investigated, though nonuniform suction was more efficient. In the case of uniform suction, clear trends could be seen between the suction required to eliminate separation and the Reynolds number, as well as with the separation angle of the uncontrolled case. Further investigation is necessary to determine the extent of these relationships and if they have any physical justification. The maximum suction coefficient required to prevent separation was \( C_Q = 49.14 \) at \( Re = 20 \). For the case of nonuniform suction, it was determined that 6 control segments resulted in the most efficient control. At \( Re = 180 \), this controller setup required less than half the effort of uniform suction (\( C_Q = 13.26 \) vs. \( C_Q = 31.25 \)). Increasing the number of control segments improved the efficiency of the control initially, but further increasing \( N \) did not result in improvement after a certain point. Whether this is the case for all \( Re \) has not been shown. The optimal results for uniform and nonuniform suction were verified by carrying out time-dependent simulations with their control parameters. For all cases investigated, the suction necessary to eliminate vortex shedding was smaller than that needed to prevent separation. Overall, nonuniform distributed suction was shown to be a very effective method for controlling the angle of separation around a cylinder.

**Data Availability**

Data can be made available on request by emailing the corresponding author, James Ramsay. An alternative email is the institutional one at james.ramsay@pg.canterbury.ac.nz.

**Conflicts of Interest**

The authors declare that there is no conflict of interest regarding the publication of this paper.

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