Constraints on Lorentz Invariance Violating Quantum Gravity and Large Extra Dimensions Models using High Energy Gamma Ray Observations

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Abstract

Observations of the multi-TeV spectra of the nearby BL objects Mkn 421 and Mkn 501 exhibit the high energy cutoffs predicted to be the result of intergalactic annihilation interactions, primarily with infrared photons having a flux level as determined by various astronomical observations. After correction for this absorption effect, the derived intrinsic spectra of these multi-TeV sources can be explained within the framework of simple synchrotron self-Compton emission models. Stecker and Glashow have shown that the existence of such annihilations via electron-positron pair production interactions up to an energy of 20 TeV puts strong constraints on Lorentz invariance violation. Such constraints have important implications for Lorentz invariance violating (LIV) quantum gravity models as well as LIV models involving large extra dimensions. We also discuss the implications of observations of high energy γ-rays from the Crab Nebula on constraining quantum gravity models.

Key words: gamma-rays; BL Lac objects; background radiation; infrared; Lorentz invariance; quantum gravity

1 Introduction

Absorption of high energy γ-rays from extragalactic sources occurs via interactions of these photons with low energy photons of intergalactic radiation [1], [2]. These low energy photons are produced by stellar radiation and the reemission of such radiation by interstellar dust in galaxies. These photons then leave the galaxies in which they were produced, escaping into intergalactic space. The absorption is expected to produced by the annihilation of high
energy $\gamma$-rays with low energy photons into electron-positron pairs in intergalactic space. Stecker and Glashow [3] have shown that the very existence of such interactions puts quantitative constraints on Lorentz invariance violation. We show here that high energy $\gamma$-ray observations can also be used to place significant constraints on proposed extra dimensions and quantum gravity scenarios.

2 Observations of the Blazars Mkn 501 and Mkn 421

The highest energy extragalactic $\gamma$-ray sources in the known universe are the active galaxies called ‘blazars’, objects that emit jets of relativistic plasma aimed directly at us. Those blazars, known as X-ray selected BL Lac objects (XBLs), or alternatively as high frequency BL Lac objects (HBLs), are expected to emit photons in the multi-TeV energy range, but only the nearest ones are expected to be observable at TeV energies, the others being hidden by intergalactic absorption [4].

Extragalactic photons with the highest energies yet observed originated in a powerful flare coming from the giant elliptical active galaxy known as Mkn 501 [5]. Its spectrum is most easily understood and interpreted as manifesting the high energy absorption to be expected from $\gamma$-ray annihilation by extragalactic pair production interactions. The analyses of de Jager and Stecker [6] and Konopelko et al. [7] indicate the presence of the absorption effect in agreement with that predicted by calculating the expected energy dependent opacity inferred from the background light spectral energy distributions derived in reference [8] and reference [6]. This absorption is the result of electron-positron pair production by interactions of the multi-TeV $\gamma$-rays from Mkn 501 primarily with intergalactic infrared photons. Intrinsic absorption by pair production interactions within $\gamma$-ray sources such as Mkn 421 and Mkn 501 is expected to be negligible because such giant elliptical galaxies contain little dust to emit infrared radiation and because BL Lac objects have little gas (and therefore most likely little dust) in their nuclear regions. It also appears that $\gamma$-ray emission in blazars takes place at superluminal knots in their jets, downstream of the radio cores of these active galaxies and therefore downstream of any putative accretion disks [9].

The spectrum of Mkn 501 in the flaring phase extends to an energy of at least 24 TeV [5]. The calculations of de Jager and Stecker [6] predict that intergalactic absorption should strongly suppress the spectra of these sources at multi-TeV energies. The intrinsic spectrum of Mkn 501 was derived by by these authors, correcting for the opacity calculated for $z = 0.03$ as a function of energy, based on models of MS01, extended into the optical and UV range.
Konopelko, et al. [7] have reexamined the spectra of both Mkn 421 and Mkn 501 corrected for absorption and have found that they fit synchrotron self-Compton (SSC) emission models from the radio to TeV $\gamma$-ray range. Both the X-ray and intrinsic TeV spectra of Mkn 421 peak at lower energies than those of the flaring spectrum of Mkn 501, which can be understood if electrons were accelerated to higher energies in the April 1997 flare of Mkn 501 than in Mkn 421. The intrinsic spectrum of Mkn 501 with the absorption effect removed actually peaks at multi-TeV energies, rather than falling off in this energy range as observed [5], [10]. Thus, it appears that the dropoff in the observed $\gamma$-ray spectrum of Mkn 501 above $\sim 5$ TeV is a direct consequence of intergalactic absorption. We will therefore interpret the Mkn 501 data as evidence for intergalactic absorption with no indication of Lorentz invariance violation (see next section) up to a photon energy of $\sim 20$ TeV. This conclusion can be substantiated by observations of other extragalactic $\gamma$-ray sources at differing redshifts [2].

3 High Energy Consequences of Breaking of Lorentz Invariance

It has been suggested that Lorentz invariance (LI) may be only an approximate symmetry of nature [11] - [13]. A simple and self-consistent framework for analyzing possible departures from exact LI was suggested by Coleman and Glashow [14], who assume LI to be broken perturbatively in the context of conventional quantum field theory. Small Lorentz noninvariant terms are introduced that are renormalizable, being of mass dimension no greater than four and having dimensionless coupling constants in the kinetic part of the Lagrangian. These terms are also chosen to be gauge invariant under $SU(3) \times SU(2) \times U(1)$ (“the almost standard model”). It is further assumed that the Lagrangian is rotationally invariant in a preferred frame which is presumed to be the rest frame of the cosmic microwave background.

Consequent observable manifestations of LI breaking can be described quite simply in terms of different maximal attainable velocities of different particle species as measured in the preferred frame. This is because the small LI violating terms modify the free-field propagators so that the maximum velocities of various particles are not equal to $c$. It follows that if LI is violated the maximum attainable velocity of an electron need not equal the velocity of light in vacuo, i.e., $c_e \neq c_\gamma$. The physical consequences of this violation of LI depend on the sign of the difference between $c_e$ and $c_\gamma$ [14]. Following the discussion of Stecker and Glashow [3], we define

$$ c_e \equiv c_\gamma (1 + \delta) , \quad 0 < |\delta| \ll 1 , \quad (1) $$
and consider the two cases of positive and negative values of $\delta$ separately.$^1$

If $c_e < c_\gamma$ ($\delta < 0$), the decay of a photon into an electron-positron pair is kinematically allowed for photons with energies exceeding

$$E_{\text{max}} = m_e \sqrt{2/|\delta|}.$$  \hfill (2)

The decay would take place rapidly, so that photons with energies exceeding $E_{\text{max}}$ could not be observed either in the laboratory or as cosmic rays. Since photons have been observed with energies $E_\gamma \geq 50$ TeV from the Crab nebula [15], we deduce for this case that $E_{\text{max}} \geq 50$ TeV, or that $|\delta| < \frac{2}{3} \times 10^{-16}$ [3]. Stronger bounds on $\delta$ can be set through observations of very high energy (TeV) photons. The detection of cosmic $\gamma$-rays with energies greater that 50 TeV from sources within our galaxy would improve the bound on $\delta$.

If, on the other hand, $c_e > c_\gamma$ ($\delta > 0$) and electrons become superluminal if their energies exceed $E_{\text{max}}/\sqrt{2}$. Electrons traveling faster than light will emit light at all frequencies by a process of ‘vacuum Čerenkov radiation.’ This process occurs rapidly, so that superluminal electron energies quickly approach $E_{\text{max}}/\sqrt{2}$. Because electrons have been seen in the cosmic radiation with energies up to $\sim 2$ TeV[16], it follows that $\delta < 3 \times 10^{-14}$. This upper limit is about two orders of magnitude weaker than the limit obtained for Case I.

A smaller, but more indirect, upper limit on $\delta$ for the $\delta > 0$ case can be obtained from theoretical considerations of $\gamma$-ray emission from the Crab Nebula. The unpulsed $\gamma$-ray spectrum of the Crab Nebula can be understood to be produced by synchrotron emission up to the $\sim 0.1$ GeV $\gamma$-ray energy range. Above 25 MeV, the synchrotron component falls off very rapidly with energy as expected from theoretical limits on electron acceleration [17]. Emission above 0.1 GeV, extending into the TeV range, can be explained as synchrotron self-Compton emission of the same relativistic electrons which produce the synchrotron radiation. The Compton component, extending to 50 TeV, implies the existence of electrons having energies at least this great in order to produce 50 TeV photons, even in the extreme Klein-Nishina limit. This is, of course, required by conservation of energy. This indirect argument, based on the reasonable assumption that the 50 TeV $\gamma$-rays are from Compton interactions, leads to a smaller upper limit on $\delta$, viz., $\delta < 10^{-16}$. Better observational $\gamma$-ray data for the Crab Nebula at multi-TeV energies, and data on the shape of its $\gamma$-ray spectrum above 50 TeV, will be needed to further test this indirect constraint on $\delta$. One open question involves the possibility of hadronic interactions involving cosmic ray nucleons producing $\pi^0$’s which decay into observed high energy $\gamma$-rays (although there is no present indication of this). Hopefully,

$^1$ Note that the parameter $\delta$ defined here differs from that defined in reference [14] of $\delta_{\gamma e} \equiv c_\gamma^2 - c_e^2 \simeq 2(c_\gamma - c_e)$ by a factor of $-1/2$. 

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better data will determine whether or not there is a significant hadronically
induced component contributing to the multi-TeV spectrum of the Crab.

A further constraint on $\delta$ for $\delta > 0$ ($c_e > c_\gamma$) follows from the modification of
the threshold energy for the pair production process $\gamma + \gamma \to e^+ + e^-$. This
follows from the fact that the square of the four-momentum is changed to give
the threshold condition

$$2\epsilon E_\gamma (1 - \cos \theta) - 2E^2_\gamma \delta > 4m^2_e, \quad (3)$$

where $\epsilon$ is the energy of the low energy photon and $\theta$ is the angle between the
two photons. The second term on the left-hand-side comes from the fact that
c_\gamma = \partial E_\gamma / \partial p_\gamma.

For head-on collisions ($\cos \theta = -1$) the minimum low energy photon energy
for pair production becomes

$$\epsilon_{\text{min}} = \frac{m^2_e}{E_\gamma} + \frac{E_\gamma \delta}{2}. \quad (4)$$

It follows that the condition for a significant increase in the energy threshold
for pair production is $E_\gamma \delta / 2 \geq m^2_e / E_\gamma$, or equivalently,

$$\delta \geq \frac{2m^2_e}{E^2_\gamma}. \quad (5)$$

As discussed in the previous section, there is no indication of LI violation
suppressing the physics of pair production for photons up to an energy of
$\sim 20$ TeV. Thus, it follows from eq. (6) that the Mkn 501 observations imply
the constraint $\delta \leq 2m^2_e / E^2_\gamma = 1.3 \times 10^{-15}$ \cite{3}. This constraint on positive $\delta$
is more secure than the smaller, but indirect, limit given by the Crab Nebula
acceleration model.

4 Quantum Gravity Models

In the absence of a true and complete theory of quantum gravity, theorists
have been suggesting and exploring models to provide experimental and ob-
servational tests of possible manifestations of quantum gravity phenomena.
Such phenomena have usually been suggested to be a possible result of quan-
tum fluctuations on the Planck scale $M_{\text{Planck}} = \sqrt{\hbar c / G} \simeq 1.22 \times 10^{19}$ GeV/c^2,
corresponding to a length scale $\sim 1.6 \times 10^{-35}$ m \cite{18} - \cite{20}. In models involving
large extra dimensions, the energy scale at which gravity becomes strong
can be much smaller than $M_{\text{Planck}}$, with the quantum gravity scale, $M_{QG}$, approaching the TeV scale [21], [22].

In many of these models Lorentz invariance is predicted to be violated at high energy. This results in interesting modifications of particle physics that are accessible to observational tests using TeV $\gamma$-ray telescopes and cosmic ray detectors. An example of such a model is a quantum gravity model with a preferred inertial frame given by the cosmological rest frame of the cosmic microwave background radiation (For an extensive discussion, see the review given in Ref. [23].)

In the most commonly considered of these models, the usual relativistic dispersion relations between energy and momentum of the photon and the electron

\begin{align}
E_{\gamma}^2 &= p_{\gamma}^2 \\
E_{e}^2 &= p_e^2 + m_e^2
\end{align}

(with the “low energy” speed of light, $c \equiv 1$) are modified by a leading order quantum space-time geometry corrections which are cubic in $p \simeq E$ and are suppressed by the quantum gravity mass scale $M_{QG}$. Following Refs. [12] and [20], we take the modified dispersion relations to be of the form

\begin{align}
E_{\gamma}^2 &= p_{\gamma}^2 - \frac{p_{\gamma}^3}{M_{QG}} \\
E_{e}^2 &= p_e^2 + m_e^2 - \frac{p_e^3}{M_{QG}}
\end{align}

We assume that the cubic terms are the same for the photon and electron as in eqs. (9) and (10). More general formulations have been considered by Jacobson, Liberati and Mattingly [24] and Konopka and Major [25].

We note that there are variants of quantum gravity and large extra dimension models which do not violate Lorentz invariance and for which the constraints considered here do not apply. There are also variants for which there are no cubic terms in momentum, but rather terms of the quartic form $\sim p^4/M_{QG}^2$. These terms are suppressed by two orders of the Planck mass and are therefore much smaller and do not violate the constraints derived here.

As opposed to the Coleman-Glashow formalism [14], which involves mass dimension four operators in the Lagrangian and preserves power-counting renormalizability, the cubic term which modifies the dispersion relations may be considered in the context of an effective “low energy” field theory, valid for
$E \ll M_{QG}$, in which case the cubic term is a small perturbation involving dimension five operators whose construction is discussed in Ref. [26]. With this caveat, we can generalize the LI violation parameter $\delta$ to an energy dependent form

$$\delta \equiv \frac{\partial E_e}{\partial p_e} - \frac{\partial E_\gamma}{\partial p_\gamma} \simeq \frac{E_\gamma}{M_{QG}} - \frac{m_e^2}{2E_e^2} - \frac{E_e}{M_{QG}}, \quad (10)$$

which is a valid approximation for the energy regime $E_e \gg m_e$. Note that the maximum velocities of particles of type $i$ are reduced by $\mathcal{O}(E_i/M_{QG})$.

For pair production then, with the positron and electron energy $E_e \simeq E_\gamma/2$,

$$\delta = \frac{E_\gamma}{2M_{QG}} - \frac{2m_e^2}{E_\gamma^2} \quad (11)$$

and the threshold condition given by eq.(6) reduces to the constraint

$$M_{QG} \geq \frac{E_\gamma^3}{8m_e^2}. \quad (12)$$

Since pair production occurs for energies of at least 20 TeV, as indicated by our analyses of the Mkn 501 and Mkn 421 spectra [6],[7], we then find the constraint on the quantum gravity scale $M_{QG} \geq 0.3 M_{Planck}$. This constraint contradicts the predictions of some proposed quantum gravity models involving large extra dimensions and smaller effective Planck masses. Previous constraints on $M_{QG}$ for the cubic model, obtained from limits on the energy dependent velocity dispersion of $\gamma$-rays for a TeV flare in Mkn 421 [27] and from $\gamma$-ray bursts [28] were in the much less restrictive range $M_{QG} \geq (5 - 7) \times 10^{-3} M_{Planck}$.

Within the context of a more general cubic modification of the dispersion relations given by eqs. (9) and (10), Jacobson, et al. [29] have obtained an indirect limit on $M_{QG}$ from the apparent cutoff in the synchrotron component of the in the Crab Nebula $\gamma$-ray emission at $\sim 0.1$ GeV. By making reasonable assumptions to modify the standard synchrotron radiation formula to allow for Lorentz invariance violation, they have concluded that the maximum synchrotron photon energy will be given by $E_{\gamma,max} = 0.34 (eB/m_e)(m_e/M_{QG})^{-2/3}$. This reasoning leads to the constraint $M_{QG} > 1.2 \times 10^7 M_{Planck}$.

Future observations of the Crab Nebula with the GLAST (Gamma-Ray Large Area Space Telescope) satellite, scheduled to be launched in 2005, will provide a better determination of its unpulsed $\gamma$-ray spectrum in the energy range above 30 MeV where the transition from the synchrotron emission component
to the Compton emission component occurs. This will provide a more precise determination of the maximum electron energy in the Nebula and therefore provide a more precise constraint on the parameter $M_{QG}$ as we have defined it here. However, this constraint will still be orders of magnitude above the Planck scale.

5 Conclusions

Nearly a century after the inception of special relativity, high energy $\gamma$-ray observations have confirmed its validity up to electron energies of 2 TeV, photon energies of 20 TeV and, indirectly, up to electron energies in the PeV range. These results indicate an absence of evidence for proposed violations of Lorentz invariance as predicted by some phenomenological quantum gravity and large extra dimension models. Thus, high energy astrophysics has provided important empirical constraints on Planck scale physics.

Some models with large extra dimensions [21], [22] are ruled out by the existence of absorption in the very high energy spectra of nearby BL Lac objects. The fact that more distant brighter sources are not seen can also be taken as indirect evidence of intergalactic absorption by pair production interactions [4]. The constraints based on analysis of the Crab Nebula $\gamma$-ray spectrum, discussed in the previous section, imply that the quantum gravity scale is orders of magnitude above the Planck mass scale. This indicates that the class of models considered here with linear Planck scale suppressed terms in the dispersion relations cannot be reflective of physics at the Planck scale. Models such as loop quantum gravity with a preferred inertial frame are ruled out by this line of reasoning. Alternative models to consider might be models with quartic momentum terms with $M_{QG}^2$ suppression in the dispersion relations, Lorentz invariant quantum gravity models, or really new Planck scale physics such as string theory, which preserves Lorentz invariance.

Acknowledgments

I would like to thank Ted Jacobson, Stefano Liberati, David Mattingly and Serge Rudaz for helpful discussions.

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