ON STRUCTURE OF SUPERON-GRAVITON MODEL (SGM)

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Abstract

The fundamental action of superon-graviton model (SGM) for space-time and matter is written down explicitly in terms of the fields of the graviton and superons by using the affine and the spin connection formalisms, alternatively. Some characteristic structures including some hidden symmetries of the gravitational coupling of superons are manifested (in two dimensional space-time) with some details of the calculations. SGM cosmology is discussed briefly.

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1. Introduction
It seems inevitable to introduce new particles yet to be observed and new (gauge) symmetries for exploring the new physics and the new framework for the unification of space-time and matter beyond the standard model (SM). Supersymmetry (SUSY)\cite{ref1} \cite{ref2} may be the most promising gauge symmetry beyond SM, especially for the unification of space-time and matter. Nambu–Goldstone (N-G) fermion\cite{ref2} \cite{ref3} would appear in the spontaneous SUSY breaking and plays essentially important roles in the unified model building.

Here it is useful to distinguish the qualitative differences of the origins of N-G fermion. In O’Raifeartaigh model N-G fermion stems from the symmetry of the dynamics (interaction) of the linear representation (multiplet) of SUSY, while in Volkov-Akulov model N-G fermion stems from the symmetry of space-time and gives the nonlinear (NL) representation of SUSY\cite{ref4}. As demonstrated in supergravity (SUGRA) coupled with Volkov-Akulov model it is rather well understood in the linear realization of SUSY (L SUSY) that N-G fermion is converted to the longitudinal component of spin 3/2 field by the super-Higgs mechanism and breaks local linear SUSY spontaneously by giving mass to gravitino\cite{ref5}. N-G fermion degrees of freedom become unphysical in the low energy, so far.

While, speculating that the chirality (, i.e. the massless fermionic state) of SM may indicate the N-G fermion nature of matter behind SM, it may be interesting to survey other possibilities concerning how SUSY is realized and where N-G fermion has gone (in the low energy).

In the previous paper \cite{ref6} we have introduced a new fundamental constituent with spin 1/2 superon and proposed superon – graviton model (SGM) equipped with NL SUSY as a model for unity of space-time and matter. In SGM, the fundamental entities of nature are the graviton with spin-2 and a quintet of superons with spin-1/2. They are the elementary gauge fields corresponding to the ordinary local GL(4,R) and the global nonlinear supersymmetry (NL SUSY) with a global SO(10), i.e. N=10 V-A model, respectively. Interestingly, the quantum numbers of the superon-quintet are the same as those of the fundamental representation 5 of the matter multiplet of SU(5) GUT\cite{ref7}. All observed elementary particles including gravity are assigned to a single irreducible massless representation of SO(10) super-Poincaré (SP) symmetry and reveals a remarkable potential for the phenomenology, e.g. they may explain naturally the three-generations structure of quarks and leptons, the stability of proton, various mixings, ..etc\cite{ref6}. And in SGM except graviton they are supposed to be the (massless) eigenstates of superons of SO(10) SP symmetry \cite{ref4} of space-time and matter. The uniqueness of N=10 among all SO(N) SP is pointed out. The arguments are group theoretical so far.

In order to obtain the fundamental action of SGM which is invariant at least under local GL(4,R), local Lorentz, global NL SUSY transformations and global SO(10), we have performed the similar geometrical arguments to Einstein general relativity.
theory(EGRT) in the SGM space-time, where the tangent (Riemann-flat) Minkowski space-time is specified by the coset space SL(2, C) coordinates (corresponding to Nambu-Goldstone(N-G) fermion) of NL SUSY of Volkov-Akulov(V-A) in addition to the ordinary Lorentz SO(3,1) coordinates, which are locally homomorphic groups. As shown in Ref. the SGM action for the unified SGM space-time is naturally the analogue of Einstein-Hilbert(E-H) action of GR and has the similar concise expression. And interestingly it may be regarded as a kind of a generalization of Born-Infeld action. (The similar systematic arguments are applicable to spin 3/2 N-G case.)

In this article, after a brief review of SGM for the self contained arguments we expand SGM action in terms of the fields of graviton and superons in order to see some characteristic structures of our model and also show some details of the calculations. For the sake of the comparison the expansion is performed by the affine connection formalism and by the spin connection formalism.

Finally some hidden symmetries and a potential cosmology, especially the birth of the universe are mentioned briefly.

2. Fundamental action of superon-graviton model(SGM)

In Ref., SGM space-time is defined as the space-time whose tangent(flat) space-time is specified by SO(1,3) Lorentz coordinates $x^a$ and the coset space SL(2, C) coordinates $\psi$ of NL SUSY of Volkov-Akulov(V-A). The unified vierbein $w_a^\mu$ and the unified metric $s^{\mu\nu}(x) \equiv w_a^\mu(x)w_a^\nu(x)$ of SGM space-time are defined by generalizing the NL SUSY invariant differential forms of V-A to the curved space-time. SGM action is given as follows

$$L_{SGM} = -\frac{c^3}{16\pi G} |w|(\Omega + \Lambda),$$  

(1)

$$|w| = \det w_a^\mu = \det(e_a^\mu + t_a^\mu), \quad t_a^\mu = \frac{\kappa}{2i} \sum_{j=1}^{10} (\bar{\psi}_j \gamma_\rho \psi_j(x)) \partial^\rho \bar{\psi}_j \gamma_\alpha \psi_j,$$  

(2)

where $\kappa(= \kappa_{V-A})$ is an arbitrary constant up now with the dimension of the fourth power of length, $e_a^\mu$ and $\psi_j(j = 1, 2, ..10)$ are the fundamental elementary fields of SGM, i.e. the vierbein of Einstein general relativity theory(EGRT) and the superons of N-G fermion of NL SUSY of Volkov-Akulov, respectively. $\Lambda$ is a cosmological constant which is necessary for SGM action to reduce to V-A model with the first order derivative terms of the superon in the Riemann-flat space-time. $\Omega$ is a unified scalar curvature of SGM space-time analogous to the Ricci scalar curvature $R$ of EGRT. SGM action (1) is invariant under the following new SUSY transformations

$$\delta \psi^j(x) = \zeta^j + i\kappa(\bar{\zeta}^j \gamma_\rho \psi_j(x)) \partial_\rho \psi_j(x),$$  

(3)

$$\delta e_a^\mu(x) = i\kappa(\bar{\zeta}^j \gamma_\rho \psi^j(x)) D_{[\rho} e_a^{\mu]}(x),$$  

(4)
where $\zeta^i, (i = 1, \ldots, 10)$ is a constant spinor parameter, $D_{[\rho e^a_{\mu}]}(x) = D_{\rho} e^a_{\mu} - D_{\mu} e^a_{\rho}$ and $D_{\mu}$ is a covariant derivative containing a symmetric affine connection. The explicit expression of $\Omega$ is obtained by just replacing $e^a_{\mu}(x)$ in Ricci scalar $R$ of EGRT by the unified vierbein $w^a_{\mu}(x) = e^a_{\mu} + t^a_{\mu}$ of the SGM curved space-time, which gives the gravitational interaction of $\psi(x)$ invariant under (3) and (4). The invariance can be easily understood by observing that under (3) and (4) the new vierbein $w^a_{\mu}(x)$ and the new metric $s_{\mu\nu}(x)$ have general coordinate transformations.

\[
\delta \zeta w^a_{\mu} = \xi^\nu \partial_{\nu} w^a_{\mu} + \partial_{\mu} \xi^\nu w^a_{\nu},
\]

(5)

\[
\delta \zeta s_{\mu\nu} = \xi^\kappa \partial_{\kappa} s_{\mu\nu} + \partial_{\mu} \xi^\kappa s_{\kappa\nu} + \partial_{\nu} \xi^\kappa s_{\mu\kappa},
\]

(6)

where $\xi^a = i\kappa (\bar{\zeta} \gamma^a \psi \zeta)$. The overall factor of SGM action is fixed to $\frac{e^3}{16\pi G}$, which reproduces E-H action of GR in the absence of superons (matter). Also in the Riemann-flat space-time, i.e. $e^a_{\mu}(x) \rightarrow \delta^a_{\mu}$, it reproduce V-A action of NL SUSY[2] with $\kappa^{-1}_{V-A} = \frac{e^3}{16\pi G} \Lambda$ in the first order derivative terms of the superon. Therefore our model(SGM) predicts a (small) non-zero cosmological constant, provided $\kappa_{V-A} \sim O(1)$, and possesses two mass scales. Furthermore it fixes the coupling constant of superon (N-G fermion) with the vacuum to $(\frac{e^3}{16\pi G} \Lambda)^{\frac{1}{2}}$ (from the low energy theorem viewpoint), which may be relevant to the birth (of the matter and Riemann space-time) of the universe.

It is interesting that our action is the vacuum (matter free) action in SGM space-time as read off from (1) but gives in ordinary Riemann space-time the E-H action with matter (superons) accompanying the spontaneous supersymmetry breaking.

The commutators of new SUSY transformations induces the generalized general coordinate transformations

\[
[\delta \zeta_1, \delta \zeta_2] \psi = \Xi^\mu \partial_{\mu} \psi,
\]

(7)

\[
[\delta \zeta_1, \delta \zeta_2] e^a_{\mu} = \Xi^\rho \partial_{\rho} e^a_{\mu} + e^a_{\rho} \partial_{\mu} \Xi^\rho,
\]

(8)

where $\Xi^\mu$ is defined by

\[
\Xi^\mu = 2ia(\bar{\zeta}_2 \gamma^a \zeta_1) - \xi_1^\rho \xi_2^\sigma e^a_{\mu} (D_{[\rho \sigma]} e^a_{\mu}).
\]

(9)

We have shown that our action is invariant at least under

\[
[\text{global NL SUSY}] \otimes [\text{local GL}(4, R)] \otimes [\text{local Lorentz}] \otimes [\text{global SO}(N)],
\]

(10)

which is isomorphic to N=10 extended (global SO(10)) SP symmetry through which SGM reveals the spectrum of all observed particles in the low energy[8]. In contrast with the ordinary SP SUSY, new SUSY may be regarded as a square root of a generalized GL(4,R). The usual local GL(4,R) invariance is obvious by the construction.
The simple expression (1) invariant under the above symmetry may be universal for the gravitational coupling of Nambu-Goldstone (N-G) fermion, for by performing the parallel arguments we obtain the same expression for the gravitational interaction of the spin-3/2 N-G fermion [10].

Now to clarify the characteristic features of SGM we focus on N=1 SGM for simplicity without loss of generality and write down the action explicitly in terms of \( t^a_\mu (or \psi) \) and \( g^{\mu\nu} (or e^a_\mu) \). We will see that the graviton and superons (matter) are complementary in SGM and contribute equally to the curvature of SGM space-time. Contrary to its simple expression (1), it has rather complicated and rich structures.

We use the Minkowski tangent space metric \( \frac{1}{2} \{ \gamma^a, \gamma^b \} = \eta^{ab} = (+, -, -, -) \) and \( \sigma^{ab} = \frac{i}{4} [\gamma^a, \gamma^b] \). (Latin (a,b,..) and Greek (\( \mu, \nu, .. \)) are the indices for local Lorentz and general coordinates, respectively.) By requiring that the unified action of SGM space-time should reduce to V-A in the flat space-time which is specified by \( x^a \) and \( \psi (x) \) and that the graviton and superons contribute equally to the unified curvature of SGM space-time, it is natural to consider that the unified vierbein \( w^a_\mu (x) \) and the unified metric \( s^{\mu\nu} (x) \) of unified SGM space-time are defined through the NL SUSY invariant differential forms \( \omega^a \) of V-A [3] as follows:

\[
\omega^a = w^a_\mu dx^\mu, \quad (11)
\]

\[
w^a_\mu (x) = e^a_\mu (x) + t^a_\mu (x), \quad (12)
\]

where \( e^a_\mu (x) \) is the vierbein of EGRT and \( t^a_\mu (x) \) is defined by

\[
t^a_\mu (x) = i\kappa \bar{\psi} \gamma^a \partial_\mu \psi, \quad (13)
\]

where the first and the second indices of \( t^a_\mu \) represent those of the \( \gamma \) matrices and the general covariant derivatives, respectively. We can easily obtain the inverse \( w^a_\mu \) of the vierbein \( w^a_\mu \) in the power series of \( t^a_\mu \) as follows, which terminates with \( t^4 \) (for 4 dimensional space-time):

\[
w^a_\mu = e^a_\mu - t^a_\mu + t^a_\rho t^\rho_\mu - t^a_\sigma t^\sigma_\rho t^\rho_\mu + t^a_\mu t^\sigma_\rho t^\kappa_\sigma t^\kappa_\mu. \quad (14)
\]

Similarly a new metric tensor \( s_{\mu\nu} (x) \) and its inverse \( s^{\mu\nu} (x) \) are introduced in SGM curved space-time as follows:

\[
s_{\mu\nu} (x) \equiv w^a_\mu (x) w^b_\nu (x) = w^a_\mu (x) \eta_{ab} w^b_\nu (x) = g_{\mu\nu} + t_{\mu\nu} + t_{\nu\mu} + t^a_\mu t^a_\nu. \quad (15)
\]

\[
s^{\mu\nu} (x) \equiv w^a_\mu (x) w^{a\nu} (x)
\]

5
By using (12), (14), (15) and (16) we can express SGM action (1) interms of the ordinaly GL(4,R) and under (3) and (4).

It is obvious from the above general covariant arguments that (1 ) is invariant under the ordinaly GL(4,R) and under (3) and (4).

We can easily show

\[ w_a^\mu w_b^\nu = \eta_{ab}, \quad s_{\mu
u} w_a^\mu w_b^\nu = \eta_{ab}. \]  

(17)

It is obvious from the above general covariant arguments that (1) is invariant under the ordinaly GL(4,R) and under (3) and (4).

By using (12), (14), (15) and (16) we can express SGM action (1) in terms of \( e^a(x) \) and \( \psi^b(x) \), which describes explicitly the fundamental interaction of graviton with superons. The expansion of the action in terms of the power series of \( \kappa \) (or \( t^a \)) can be carried out straightforwardly. After the lengthy calculations concerning the complicated structures of the indices we obtain

\[
L_{SGM} = -\frac{c^3 A}{16\pi G} e|w_{V-A}| - \frac{c^3}{16\pi G} e R + \frac{c^3}{16\pi G} e [2t^{(\mu\nu)} R_{\mu\nu}
+ \frac{1}{2} \{g^{\mu\nu} \partial^\rho \partial_{(\mu\nu)} - t_{(\mu\nu)} \partial^\rho \partial_\rho g^{\mu\nu}
+ g^{\mu\nu} \partial^\rho \partial_{\mu\nu} - 2g^{\mu\nu} \partial^\rho \partial_{\mu\nu} g_{\rho\sigma} - g^{\rho\sigma} \partial^\rho \partial_{\mu\nu} g_{\mu\nu})
+ (t^\mu t^\nu + t^\nu t^\mu + t^{\mu\nu} t\sigma) R_{\beta\mu}
- 2(t^{(\mu\nu)} t^{(\nu\rho)} R_{\mu\nu} + t^{(\mu\nu)} t^{(\nu\rho)} R_{\mu\nu})
+ \frac{1}{2} t^{(\mu\nu)} (g^{\rho\sigma} \partial^\rho \partial_{\mu\nu} - g^{\rho\sigma} \partial^\rho \partial_{\mu\nu}) + \ldots
)\}
+ \{O(t^3)\} + \{O(t^4)\} + \ldots + \{O(t^{10})\}],
\]

(18)

where \( e = \det e^a_{\mu}, \quad t^{(\mu\nu)} = t_{\mu\nu} + t^{\mu\nu} + t^{(\mu\nu)} + t_{\mu\nu}, \) and \( |w_{V-A}| = \det w^a_{\mu\nu} \) is the flat space V-A action containing up to \( O(t^4) \) and \( R \) and \( R_{\mu\nu} \) are the Ricci curvature tensors of GR.

Remarkably the first term can be regarded as a space-time dependent cosmological term and reduces to V-A action [3] with \( \kappa_{V-A}^{-1} = \frac{c^3}{16\pi G} A \) in the Riemann-flat \( \epsilon_a(x) \rightarrow \delta_a \) space-time. The second term is the familiar E-H action of GR. These expansions show the complementary relation of graviton and (the stress-energy tensor of) superons. The existence of (in the Riemann-flat space-time) NL SUSY invariant terms with the (second order) derivatives of the superons beyond V-A model.
are manifested. For example, the lowest order of such terms appear in $O(t^2)$ and have the following expressions (up to the total derivative terms)

$$+ \epsilon^{abcd} \epsilon_{afg} \partial_c t_{b(e)} \partial_f t_{d(g)}.$$  

(19)

The existence of such derivative terms in addition to the original V-A model are already pointed out and exemplified in part in [13]. Note that (19) vanishes in 2 dimensional space-time.

Here we just mention that we can consider two types of the flat space in SGM, which are not equivalent. One is SGM-flat, i.e. $w_a^{\mu}(x) \rightarrow \delta_a^{\mu}$, space-time and the other is Riemann-flat, i.e. $e_a^{\mu}(x) \rightarrow \delta_a^{\mu}$, space-time, where SGM action reduces to $-\frac{c^3_A}{16\pi G}$ and $-\frac{c^3_B}{16\pi G}|w_{V-\Lambda}| - \frac{c^5}{16\pi G}$ (derivative terms), respectively. Note that SGM-flat space-time may allow Riemann space-time, e.g. $t_a^{\mu}(x) \rightarrow -e_a^{\mu} + \delta_a^{\mu}$ realizes Riemann space-time and SGM-flat space-time. The cosmological implications are mentioned in the discussions.

3. SGM in two dimensional space-time

It is well known that two dimensional GR has no physical degrees of freedom (due to the local GL(2,R)). SGM in SGM space-time is also the case. However the the arguments with the general covariance shed light on the characteristic off-shell gauge structures of the theory in any space-time dimensions. Especially for SGM, it is also useful for linearizing the theory to see explicitly the superon-graviton coupling in (two dimensional) Riemann space-time. The expansion holds up to $O(t^2)$ in four dimensional spacetime as well.

3.1 SGM in affine connection formalism

Now we go to two dimensional SGM space-time to simplify the arguments without loss of generality and demonstrate some details of the computations. We adopt firstly the affine connection formalism. The knowledge of the complete structure of SGM action including the surface terms is useful to linearize SGM into the equivalent linear theory and to find the symmetry breaking of the model.

Following EGRT the scalar curvature tensor $\Omega$ of SGM space-time is given as follows

$$\Omega = s^{\beta \mu} \Omega^\alpha_{\beta \mu \alpha}$$

$$= s^{\beta \mu} \left[ \partial_\mu \Gamma^\lambda_{\beta \alpha} + \Gamma^\alpha_{\lambda \mu} \Gamma^\lambda_{\beta \alpha} \right] - \{ \text{lower indices} (\mu \leftrightarrow \alpha) \},$$  

(20)

where the Christoffel symbol of the second kind of SGM space-time is

$$\Gamma^\alpha_{\beta \mu} = \frac{1}{2} s^{\beta \rho} \left\{ \partial_\rho s_{\mu \alpha} + \partial_\mu s_{\beta \rho} - \partial_\rho s_{\mu \beta} \right\}.$$  

(21)

The straightforward expression of SGM action [3] in two dimensional space-time, (which is $3^6$ times more complicated than the two dimensional GR), is given as follows

\[ \text{---} \]
\[ L_{2dSGM} = - \frac{c^3}{16\pi G} e^{\{1 + t^a_{\ a} + \frac{1}{2}(t^a_{\ a} t^b_{\ b} - t^a_{\ b} t^b_{\ a})\}}((g^{\beta\mu} - \tilde{t}^{(\beta\mu)} + \tilde{t}^{2(\beta\mu)}) \times \{\frac{1}{2}\partial_{\mu}(g^{\alpha\sigma} - \tilde{t}^{(\alpha\sigma)} + \tilde{t}^{2(\alpha\sigma)})\partial_{\beta}(g_{\phi\delta} + t_{(\phi\delta)}) + t^2_{(\phi\delta)}\} + \frac{1}{2}(g^{\alpha\sigma} - \tilde{t}^{(\alpha\sigma)} + \tilde{t}^{2(\alpha\sigma)})\partial_{\mu}\partial_{\beta}(g_{\phi\delta} + t_{(\phi\delta)}) + t^2_{(\phi\delta)}\})) -\{lower \ indices(\mu \leftrightarrow \alpha)\} \]

\[ + \{\frac{1}{4}(g^{\alpha\sigma} - \tilde{t}^{(\alpha\sigma)} + \tilde{t}^{2(\alpha\sigma)})\partial_{\lambda}(g_{\phi\rho} + t_{(\phi\rho)}) + t^2_{(\phi\rho)}\}) + \{\frac{1}{4}(g^{\alpha\sigma} - \tilde{t}^{(\alpha\sigma)} + \tilde{t}^{2(\alpha\sigma)})\partial_{\lambda}(g_{\phi\rho} + t_{(\phi\rho)}) + t^2_{(\phi\rho)}\}) \times \{\frac{1}{2}\partial_{\mu}(g^{\alpha\sigma} - \tilde{t}^{(\alpha\sigma)} + \tilde{t}^{2(\alpha\sigma)})\partial_{\nu}(g_{\phi\rho} + t_{(\phi\rho)}) + t^2_{(\phi\rho)}\} + \frac{1}{2}(g^{\alpha\sigma} - \tilde{t}^{(\alpha\sigma)} + \tilde{t}^{2(\alpha\sigma)})\partial_{\nu}\partial_{\mu}(g_{\phi\rho} + t_{(\phi\rho)}) + t^2_{(\phi\rho)}\}) \times \{\frac{1}{2}\partial_{\nu}(g^{\alpha\sigma} - \tilde{t}^{(\alpha\sigma)} + \tilde{t}^{2(\alpha\sigma)})\partial_{\mu}(g_{\phi\rho} + t_{(\phi\rho)}) + t^2_{(\phi\rho)}\} + \frac{1}{2}(g^{\alpha\sigma} - \tilde{t}^{(\alpha\sigma)} + \tilde{t}^{2(\alpha\sigma)})\partial_{\mu}\partial_{\nu}(g_{\phi\rho} + t_{(\phi\rho)}) + t^2_{(\phi\rho)}\}). \]

where we have put

\[ s_{\alpha\beta} = g_{\alpha\beta} + t_{(\alpha\beta)} + t^2_{(\alpha\beta)}, \quad s^{\alpha\beta} = g^{\alpha\beta} - \tilde{t}^{(\alpha\beta)} + \tilde{t}^{2(\alpha\beta)}, \]

\[ t_{(\mu\nu)} = t_{\mu\nu} + t_{\nu\mu}, \quad t^2_{(\mu\nu)} = t_{\mu\nu} t_{\nu\mu}, \]

\[ \tilde{t}(\mu\nu) = t_{\mu\nu} + t_{\nu\mu}, \quad \tilde{t}^2(\mu\nu) = t_{\mu\nu} t_{\nu\mu} + t_{\mu\nu} t_{\nu\mu} + t_{\mu\nu} t_{\nu\mu}, \]

and the Christoffel symbols of the first kind of SGM space-time contained in (21) are abbreviated as

\[ \partial_{\mu}g_{\sigma\nu} = \partial_{\mu}g_{\sigma\nu} + \partial_{\nu}g_{\mu\sigma} - \partial_{\sigma}g_{\nu\mu}, \]

\[ \partial_{\mu}t_{\sigma\nu} = \partial_{\mu}t_{(\sigma\nu)} + \partial_{\nu}t_{(\mu\sigma)} - \partial_{\sigma}t_{(\nu\mu)}, \]

\[ \partial_{\mu}k^2_{\sigma\nu} = \partial_{\mu}k^2_{(\sigma\nu)} + \partial_{\nu}k^2_{(\mu\sigma)} - \partial_{\sigma}k^2_{(\nu\mu)}. \]

By expanding the scalar curvature \( \Omega \) in the power series of \( t \) which terminates with \( t^4 \), we have the following complete expression of two dimensional SGM,

\[ L_{2dSGM} = - \frac{c^3}{16\pi G} e^{\|w_{V-A}\|} \]

\[ - \frac{c^3}{16\pi G} e^{\|w_{V-A}\|}|R| \]
\[ -2\tilde{\tau}^{(\mu\nu)} R_{\mu\nu} \]
\[ + \frac{1}{2} \left\{ g^{\mu\nu} \partial^\rho \partial_{\rho} \xi^{(\mu\nu)} - \xi^{(\mu\nu)} \partial^\rho \partial_{\rho} g_{\mu\nu} \right\} \]
\[ + g^{\mu\nu} \partial^\rho \xi^{(\rho\sigma)} \partial^\sigma g_{\rho\sigma} - 2 g^{\mu\nu} \partial^\rho \xi^{(\rho\sigma)} \partial^\sigma g_{\rho\sigma} - g^{\mu\nu} g^{\rho\sigma} \partial^\rho \xi^{(\rho\sigma)} \partial_{\sigma} g_{\mu\nu} \right\} \]
\[ + \tilde{\tau}^{(\beta\mu)} R_{\beta\mu} \]
\[ + \tilde{\tau}^{(\alpha\alpha)} R_{\mu\alpha\sigma\beta} \]
\[ - \frac{1}{2} \tilde{\tau}^{(\beta\mu)} \left\{ g^{\alpha\sigma} \partial_{\alpha} \partial_{\beta} \xi^{(\sigma\mu)} - \partial^\sigma \partial_{\beta} \xi^{(\sigma\mu)} + \partial_{\mu} \tilde{\tau}^{(\alpha\sigma)} \partial_{\beta} g_{\sigma\alpha} - \partial_{\mu} g^{\alpha\sigma} \partial_{\beta} \xi^{(\sigma\mu)} \right\} \]
\[ + \partial_{\sigma} g^{\alpha\sigma} \partial_{\beta} \xi^{(\sigma\mu)} - \partial_{\alpha} \tilde{\tau}^{(\alpha\sigma)} \partial_{\beta} g_{\sigma\mu} + 2 \partial^\rho \xi^{(\sigma\mu)} \partial^\sigma g_{\beta\rho} \]
\[ - 2 g^{\alpha\sigma} \partial_{\alpha} \xi^{(\sigma\mu)} \partial^\beta g_{\alpha\beta} \]
\[ + g^{\alpha\sigma} \partial_{\alpha} \partial_{\beta} \xi^{(\sigma\mu)} \partial^\beta g_{\rho\alpha} - 2 g^{\alpha\sigma} \partial^\rho \xi^{(\sigma\mu)} \partial_{\beta} g_{\rho\mu} + g^{\alpha\sigma} \partial_{\lambda} \xi^{(\sigma\mu)} \partial^\lambda g_{\mu\beta} \}
\[ - g^{\beta\mu} \partial_{\mu} \left( g^{\alpha\sigma} \partial_{\beta} \xi^{(\sigma\alpha)} + \tilde{\tau}^{(\alpha\sigma)} \partial_{\beta} g_{\sigma\alpha} - \tilde{\tau}^{(\alpha\sigma)} \partial_{\beta} \xi^{(\sigma\alpha)} \right) - \tilde{\tau}^{(\alpha\sigma)} \left( 2 \partial_{\beta} \xi^{(\sigma\mu)} - \partial_{\sigma} \xi^{(\sigma\mu)} \right) \]
\[ + g^{\beta\mu} \partial_{\{ \partial_{\alpha} \xi^{(\sigma\mu)} \} - \partial_{\beta} \xi^{(\sigma\mu)} \} - \tilde{\tau}^{(\alpha\sigma)} \left( 2 \partial_{\beta} \xi^{(\sigma\mu)} - \partial_{\sigma} \xi^{(\sigma\mu)} \right) \}
\[ + 2 \partial^\rho g_{\mu\rho} g^{\beta\mu} \tilde{\tau}^{(\alpha\sigma)} \left( 2 \partial_{\beta} \xi^{(\sigma\rho)} - \partial_{\sigma} \xi^{(\sigma\rho)} \right) + 2 \tilde{\tau}^{(\alpha\sigma)} \partial_{\beta} g_{\sigma\mu} g^{\beta\mu} \right\}
\[ - 2 \tilde{\tau}^{(\alpha\sigma)} \partial_{\beta} g_{\sigma\mu} g^{\beta\mu} \left( \partial_{\alpha} g_{\rho\beta} - \partial_{\rho} g_{\alpha\beta} \right) \}
\[ - 2 \tilde{\tau}^{(\alpha\sigma)} \partial_{\beta} \xi^{(\sigma\mu)} g^{\beta\mu} \left( 2 \partial_{\beta} g_{\rho\beta} - \partial_{\rho} g_{\alpha\beta} g^{\alpha\sigma} \right) \}
\[ - \partial^\rho g_{\sigma\alpha} g^{\sigma\alpha} \left( 2 \partial_{\mu} \xi^{(\mu\rho)} - \partial_{\rho} \xi^{(\mu\rho)} \right) - \partial^\rho \xi^{(\rho\sigma)} \left( 2 \partial_{\sigma} g_{\rho\rho} g^{\sigma\alpha} - \partial_{\alpha} g_{\rho\rho} g^{\alpha\beta} \right) \}
\[ - \tilde{\tau}^{(\alpha\sigma)} \partial_{\beta} g_{\sigma\mu} g^{\beta\mu} \left( 2 \partial_{\beta} \xi^{(\sigma\mu)} - \partial_{\rho} \xi^{(\sigma\mu)} \right) - \tilde{\tau}^{(\alpha\sigma)} \partial_{\beta} g_{\sigma\mu} g^{\beta\mu} \right\}
\[ - \tilde{\tau}^{(\alpha\sigma)} \partial_{\beta} g_{\sigma\mu} g^{\beta\mu} \left( 2 \partial_{\beta} \xi^{(\sigma\mu)} - \partial_{\rho} \xi^{(\sigma\mu)} \right) - \tilde{\tau}^{(\alpha\sigma)} \partial_{\beta} g_{\sigma\mu} g^{\beta\mu} \right\}
\[ + g^{\alpha\sigma} \partial^\rho g_{\alpha\beta} \xi^{(\sigma\mu)} \left( 2 \partial_{\beta} g_{\rho\rho} - \partial_{\rho} g_{\alpha\beta} g^{\alpha\beta} \right) \}
\[ - \tilde{\tau}^{(\alpha\sigma)} \partial_{\beta} g_{\sigma\mu} g^{\beta\mu} \left( 2 \partial_{\beta} \xi^{(\sigma\mu)} - \partial_{\rho} \xi^{(\sigma\mu)} \right) - \tilde{\tau}^{(\alpha\sigma)} \partial_{\beta} g_{\sigma\mu} g^{\beta\mu} \right\}
\[ + \frac{1}{2} \tilde{\tau}^{(\beta\mu)} \left\{ g^{\alpha\sigma} \partial_{\alpha} \partial_{\beta} \xi^{(\sigma\mu)} \right\} \]
\[ - \partial^\rho \xi^{(\rho\sigma)} \partial_{\beta} g_{\sigma\alpha} - \partial_{\alpha} g^{\alpha\sigma} \partial_{\beta} \xi^{(\sigma\mu)} \}
\[ + \partial_{\alpha} g^{\alpha\sigma} \partial_{\beta} \xi^{(\sigma\mu)} - \partial_{\alpha} \tilde{\tau}^{(\alpha\sigma)} \partial_{\beta} g_{\sigma\mu} \}
\[ + 2 \partial^\rho \xi^{(\sigma\mu)} \partial^\sigma g_{\beta\rho} - 2 g^{\alpha\sigma} \partial_{\beta} \xi^{(\sigma\mu)} \partial_{\sigma} g_{\rho\mu} \]
\[ + g^{\alpha\sigma} \partial_{\alpha} \xi^{(\sigma\mu)} \partial_{\beta} g_{\sigma\alpha} \}
\[ + \frac{1}{2} \tilde{\tau}^{(\beta\mu)} \left\{ g^{\alpha\sigma} \partial_{\alpha} \partial_{\beta} \xi^{(\sigma\mu)} - \tilde{\tau}^{(\alpha\sigma)} \partial_{\beta} \xi^{(\sigma\mu)} \right\} + \partial_{\alpha} \tilde{\tau}^{(\alpha\sigma)} \partial_{\beta} g_{\sigma\alpha} \]
\[-\bar{f}^{(\alpha\sigma)} \partial_\alpha (2\partial_\beta t^2_{(\mu\lambda)} - \partial_\sigma t^2_{(\mu\beta)})\]  
\[+ \frac{1}{4} \{ g^{\alpha\sigma} (\partial_\alpha t^2_{(\mu\lambda)} + \partial_\mu t^2_{(\lambda\sigma)} - \partial_\sigma t^2_{(\mu\lambda)})) g^{\lambda\rho} (\partial_\beta t^2_{(\rho\alpha)} + \partial_\alpha t^2_{(\beta\rho)} - \partial_\rho t^2_{(\alpha\beta)}) \]  
\[- g^{\alpha\sigma} (\partial_\alpha t^2_{(\sigma\mu)} + \partial_\mu t^2_{(\lambda\sigma)} - \partial_\sigma t^2_{(\mu\lambda)}) \tilde{f}^{(\lambda\rho)} (\partial_\beta t^2_{(\rho\alpha)} + \partial_\alpha t^2_{(\beta\rho)} - \partial_\rho t^2_{(\alpha\beta)}) \]  
\[+ g^{\alpha\sigma} (\partial_\alpha t^2_{(\mu\lambda)} + \partial_\mu t^2_{(\lambda\sigma)} - \partial_\sigma t^2_{(\mu\lambda)}) \tilde{f}^{(\lambda\rho)} (\partial_\beta t^2_{(\rho\alpha)} + \partial_\alpha t^2_{(\beta\rho)} - \partial_\rho t^2_{(\alpha\beta)}) \]  
\[- \bar{f}^{(\alpha\sigma)} (\partial_\lambda t^2_{(\sigma\mu)} + \partial_\mu t^2_{(\lambda\sigma)} - \partial_\sigma t^2_{(\mu\lambda)}) \tilde{f}^{(\lambda\rho)} (\partial_\beta t^2_{(\rho\alpha)} + \partial_\alpha t^2_{(\beta\rho)} - \partial_\rho t^2_{(\alpha\beta)}) \]  
\[- g^{\alpha\sigma} \partial_\alpha t^2_{(\sigma\mu)} g^{\lambda\rho} (2\partial_\beta t^2_{(\rho\mu)} - \partial_\rho t^2_{(\mu\beta)}) \]  
\[+ g^{\alpha\sigma} \partial_\alpha t^2_{(\sigma\mu)} \tilde{f}^{(\lambda\rho)} (2\partial_\beta t^2_{(\rho\mu)} - \partial_\rho t^2_{(\mu\beta)}) \]  
\[- g^{\alpha\sigma} \partial_\alpha t^2_{(\sigma\mu)} g^{\lambda\rho} (2\partial_\beta t^2_{(\rho\mu)} - \partial_\rho t^2_{(\mu\beta)}) \]  
\[+ \bar{f}^{(\alpha\sigma)} \partial_\alpha t^2_{(\sigma\mu)} g^{\lambda\rho} (2\partial_\beta t^2_{(\rho\mu)} - \partial_\rho t^2_{(\mu\beta)}) \} \]} \]

(25)
where $R_{\mu
u\rho\sigma}$, $R_{\mu
u}$ and $R$ are the curvature tensors of Riemann space and $|w_{V-A}| = \{1 + t^a_a + \frac{1}{2}(t^a_at^b_b - t^a_bt^b_a)\}$ is V-A model in two dimensional flat space. Note that the result is still preliminary, for the multiplication by $|w_{V-A}|$ factorized in (25) should be expanded in the power series in $t$.

### 3.2 SGM in the spin connection formalism

Next we perform the similar arguments in the spin connection formalism for the sake of the comparison. The spin connection $Q^{ab\mu}$ and the curvature tensor $\Omega^{ab\mu\nu}$ in SGM space-time are as follows:

$$Q^{ab\mu} = \frac{1}{2}(w_{a}^\rho \partial_\mu w_{b}\rho - w_{a}^\rho \partial_\rho w_{b}\mu - w_{a}^\rho w_{b}^\sigma \omega_{a\rho} \omega_{b\sigma})$$

and

$$\Omega^{ab\mu\nu} = \partial_\mu Q^{ab\nu} + Q^{c}_\mu Q^{ab\nu}_{c\nu}.$$  (27)

The scalar curvature $\Omega$ of SGM space-time is defined by $\Omega = w_{a}^\mu w_{b}^\nu \Omega^{ab\mu\nu}$. Let us express the spin connection $Q^{ab\mu}$ in two dimensional space-time in terms of $e^a^\mu$ and $t^a^\mu$ as

$$Q^{ab\mu} = \omega^{ab\mu}[e] + T_{ab\mu}^{(1)} + T_{ab\mu}^{(2)} + T_{ab\mu}^{(3)},$$  (28)

where $\omega^{ab\mu}[e]$ is the Ricci rotation coefficients of GR, and $T_{ab\mu}^{(1)}$, $T_{ab\mu}^{(2)}$ and $T_{ab\mu}^{(3)}$ are defined as

$$T_{ab\mu}^{(1)} = \frac{1}{2}(e_{a}^\rho \partial_\mu t_{b}\rho - t^\rho_\mu a \partial_\mu e_{b}\rho - e_{a}^\rho \partial_\rho t_{b}\mu - t^\rho_\mu a \partial_\mu e_{b}\mu - e_{a}^\rho e_{b}^\sigma \omega_{a\rho} \omega_{b\sigma} - e_{a}^\rho e_{b}^\sigma t_{c\mu} \partial_\mu e_{c}\sigma)$  (29)

$$T_{ab\mu}^{(2)} = \frac{1}{2}(-t^\rho_\mu a \partial_\mu e_{b}\rho + t^\rho_\mu a \partial_\mu e_{b}\rho + t^\rho_\mu a \partial_\mu e_{b}\mu - t^\rho_\mu a \partial_\mu e_{b}\mu + e_{a}^\rho e_{b}^\sigma \omega_{a\rho} \omega_{b\sigma} - e_{a}^\rho e_{b}^\sigma t_{c\mu} \partial_\mu e_{c}\sigma - e_{a}^\rho e_{b}^\sigma t_{c\mu} \partial_\mu e_{c}\sigma - e_{a}^\rho e_{b}^\sigma t_{c\mu} \partial_\mu e_{c}\sigma)$$  (30)

$$T_{ab\mu}^{(3)} = \frac{1}{2}(t^\rho_\mu a \partial_\mu e_{b}\rho - t^\rho_\mu a \partial_\mu e_{b}\mu - e_{a}^\rho e_{b}^\sigma \omega_{a\rho} \omega_{b\sigma} - e_{a}^\rho e_{b}^\sigma t_{c\mu} \partial_\mu e_{c}\sigma - e_{a}^\rho e_{b}^\sigma t_{c\mu} \partial_\mu e_{c}\sigma - e_{a}^\rho e_{b}^\sigma t_{c\mu} \partial_\mu e_{c}\sigma)$$  (31)

where $t^a^\mu = e_{b}^a e_{a}^\nu t^\nu_\mu$. Note that $T_{ab\mu}^{(1)}$ and $T_{ab\mu}^{(2)}$ can be written by using the spin connection $\omega^{ab\mu}[e]$ of GR as

$$T_{ab\mu}^{(1)} = e_{a}^\rho D_{\mu} t_{b}\rho + \frac{1}{4} e_{a}^\rho e_{b}^\sigma D_{\mu} t_{[\rho\sigma]},$$  (32)

$$T_{ab\mu}^{(2)} = -t^\rho_\sigma e_{a}^\rho D_{\mu} t_{b}\rho + \frac{1}{2} e_{a}^\rho e_{b}^\sigma t_{c\mu} D_{\mu} t_{e\sigma} - \frac{1}{2} e_{a}^\rho e_{b}^\sigma \partial_\rho(t_{c\mu} t_{e\sigma}) - \frac{1}{2} e_{a}^\rho e_{b}^\sigma \partial_\rho(t_{c\mu} t_{e\sigma}),$$  (33)
where $\hat{D}_\mu t_{ab} := \partial_\mu t_{ab} + \omega_{ab\nu} t^b_\nu$ and $\partial_\mu t_{[j\rho]} := \partial_\mu t_{[\rho\sigma]} + \partial_\sigma t_{(\mu\rho)} - \partial_\rho t_{(\sigma\mu)}$. Then we obtain straightforwardly the complete expression of 2 dimensional SGM action (N=1) in the spin connection formalism as follows; namely,

$$L_{2dSGM} = -\frac{c^3}{16\pi G} e^{|w_{V-A}|} \left[ R - 4t^{\mu\nu} R_{\mu\nu} + 2\epsilon^{a[b} e^{c]} (\hat{D}_c \epsilon_a) \hat{D}_\nu t_{b\rho} + D_\mu (g^{\mu\nu} g^{\rho\sigma} \partial_{\nu} t_{[j\rho]}) + 2(\rho^{\mu} t^{\nu}_\rho + \tau^{\nu} t^{\mu}_\rho + \tau^{\mu} t^{\nu}_\rho) R_{\mu\nu} + t^{(\mu\nu)} t^{(\rho\sigma)} R_{\mu\nu\rho\sigma} - (g^{\rho\mu} g^{[\nu|\nu]} g^{\rho\lambda} + g^{\sigma\mu} g^{[\lambda|\nu]} g^{\rho\lambda} - g^{\sigma\mu} g^{[\lambda|\nu]} g^{\rho\sigma}) e^a_\sigma e^b_\lambda (\hat{D}_\mu t_{ab}) \hat{D}_\nu t_{b\lambda} + g^{\rho\mu} g^{[\nu|\nu]} g^{\rho\lambda} (\hat{D}_\mu t_{ab}) \partial_\nu t_{[\lambda\rho]} + \frac{1}{4} g^{\rho\mu} g^{[\nu|\nu]} g^{\rho\lambda} (\partial_\mu t_{[\rho\sigma]}) \partial_\nu t_{[\lambda\sigma]} - 2(\epsilon^{a[b} e^{c]} \hat{D}_c \epsilon_a) \hat{D}_\nu t_{b\rho} + e^{a[b} e^{c]} \hat{D}_c \epsilon_a \hat{D}_\nu t_{b\rho} - e^{a[b} e^{c]} \hat{D}_c \epsilon_a \hat{D}_\nu t_{b\rho} - e^{a[b} e^{c]} \hat{D}_c \epsilon_a \hat{D}_\nu t_{b\rho} + e^{a[b} e^{c]} \hat{D}_c \epsilon_a \hat{D}_\nu t_{b\rho} - e^{a[b} e^{c]} \hat{D}_c \epsilon_a \hat{D}_\nu t_{b\rho} + e^{a[b} e^{c]} \hat{D}_c \epsilon_a \hat{D}_\nu t_{b\rho} - e^{a[b} e^{c]} \hat{D}_c \epsilon_a \hat{D}_\nu t_{b\rho} + e^{a[b} e^{c]} \hat{D}_c \epsilon_a \hat{D}_\nu t_{b\rho} + e^{a[b} e^{c]} \hat{D}_c \epsilon_a \hat{D}_\nu t_{b\rho}$}
by As in the affine connection case the result is still preliminary, for the multiplication
We can summarize the fundamental idea as follows. In SGM the tangent spacetime
ical field) of SGM spacetime which has a global superGL(4,R) and the ordinary
fundamental invariant actions which have the same features in appearance. However
against the global new NL SUSY and (SGM spacetime) breaks down spontaneously
degrees of freedom are now reducing the initial gravitational (vacuum) energy through the conversion to matter
Gauge parameter, e.g. a local fermionic gauge invariant coupling to SUGRA.

The final results after carrying out the multiplication of \(|w_{V-A}|\) may be rewritten

\[
\begin{align*}
- \frac{1}{2} \left\{ & (g^{\rho \mu} t^{[\alpha \nu]} g \lambda \sigma - g^{\rho \mu} g^{[\lambda \nu]} t^{\sigma \rho}) t^{\kappa} \alpha e_{\alpha \nu} + g^{\rho \mu} t^{[\alpha \nu]} t^{\lambda \sigma} e_{\alpha \nu} \\
- & (g^{\rho \mu} t^{[\alpha \nu]} g \lambda \sigma - g^{\rho \mu} g^{[\lambda \nu]} t^{\sigma \rho}) t_{\alpha \nu} \right\} \partial_{\mu} t^{[\lambda \kappa]} \partial_{\lambda} t^{\alpha \kappa} \\
+ & \left\{ (g^{\rho \mu} t^{[\alpha \nu]} e_{\sigma \beta} - e^{\lambda \mu \nu} t^{[\sigma \rho]} + t^{[\mu | \nu | b]} \epsilon^{\lambda \alpha \beta \gamma} \} (\partial_{\mu} t_{\alpha \nu}) \partial_{\nu} t_{\beta \lambda} \\
- & (e^{\lambda \mu \nu} t^{[\sigma \rho]} - e^{\lambda \mu \nu} t^{[\sigma \rho]} + t^{[\mu | \nu | b]} g^{\lambda \alpha \beta \gamma} \} \right\},
\end{align*}
\]

(34)

where \(\tilde{D}_{\mu} T_{\alpha \beta} := \partial_{\mu} T_{\alpha \beta} + \omega_{\alpha \kappa} T_{\delta \beta} + \omega_{\beta \kappa} T_{\delta \alpha} e_{\alpha \nu} \) and \(D_{\mu} e_{\alpha} := \partial_{\mu} e_{\alpha} + \Gamma_{\mu \nu} e_{\alpha} \lambda \).

As in the affine connection case the result is still preliminary, for the multiplication by \(|w_{V-A}|\) factorized in (34) should be expanded in the power series in \(t\).

4. Discussions
We can summarize the fundamental idea as follows. In SGM the tangent spacetime is specified by \((x^a, \psi(x))\) and \(w^{a}_{\mu}(x)\) is the unified vierbein, i.e. only one dynamical field) of SGM spacetime which has a global superGL(4,R) and the ordinary local GL(4,R) transformations. In Einstein gravity the tangent spacetime is specified by \(x^a\) and \(e^{a}_{\mu}(x)\) is the vierbein of Riemann spacetime which has the ordinary GL4,R transformation. By the geometrical arguments in each space there appear fundamental invariant actions which have the same features in appearance. However SGM action in SGM spacetime written in terms of \(w^{a}_{\mu}(x)\) is unstable(degenerated) against the global new NL SUSY and (SGM spacetime) breaks down spontaneously to \(e^{a}_{\mu}(x)\)(Riemann metric) + \(t^{a}_{\mu}(x)\)(a specific composite of N-G fermion matter) by reducing the initial gravitational (vacuum) energy through the conversion to matter energy. Note that SGM action possesses two inequivalent flat spaces. The dynamical degrees of freedom are now \(e^{a}_{\mu}(x)\) and \(\psi(x)\) (not the composite \(t^{a}_{\mu}(x)\)) in the ordinary Riemann(Einstein gravity) spacetime and SGM action describes the ordinary GL(4,R) invariant dynamics of \(e^{a}_{\mu}(x)\) and \(\psi(x)\). And (in general as well) it is difficult to eliminate \(\psi(x)\) without introducing a symmetry with a local fermionic arbitrary gauge parameter, e.g. a local fermionic gauge invariant coupling to SUGRA.

We have shown that contrary to its simple expression (1) in unified SGM spacetime the expansion of SGM action possesses very complicated and rich structures describing as a whole(not order by order) gauge invariant graviton-superon interactions.

The final results after carrying out the multiplication of \(|w_{V-A}|\) may be rewritten...
in a simpler form up to total derivative terms. As mentioned above SGM action is remarkably a free action of E-H type action in unified SGM space-time, the various familiar classical exact solutions of GR may be reinterpreted as those of SGM( \( w_a^\mu(x) \) and metric \( s^{\mu\nu}(x) \)) and may have new physical meanings for EGRT.

Concerning the abovementioned two inequivalent flat-spaces (i.e. the vacuum of the gravitational energy) of SGM action we can interpret them as follows.

SGM action \([\mathbb{1}]\) written by the vierbein \( w_a^\mu(x) \) and metric \( s^{\mu\nu}(x) \) of SGM space-time is invariant under (besides the ordinary local GL(4,R)) the general coordinate transformation \([12]\) with a generalized parameter \( i\kappa\bar{\zeta}^\mu\gamma^\nu(x) \) (originating from the global supertranslation \([2]\) \( \psi(x) \rightarrow \psi(x) + \zeta \) in SGM space-time). As proved for E-H action of GR \([15]\), the energy of SGM action of E-H type is expected to be positive (for positive \( \Lambda \)). Regarding the scalar curvature tensor \( \Omega \) for the unified metric tensor \( s^{\mu\nu}(x) \) as an analogue of the Higgs potential for the Higgs scalar, we can observe that (at least the vacuum of) SGM action, (i.e. SGM-flat \( w_a^\mu(x) \rightarrow \delta^a_\nu \) space-time,) which allows Riemann space-time and has a positive energy density with the positive cosmological constant \( \frac{c^2}{16\pi G} \) indicating the spontaneous SUSY breaking, is unstable (i.e. degenerates) against the supertransformation \([3]\) and \([4]\) with the global spinor parameter \( \zeta \) in SGM space-time and breaks down spontaneously to Riemann space-time

\[
    w_a^\mu(x) = e^a_\mu(x) + t^a_\mu(x),
\]

with N-G fermions superons corresponding to

\[
    \frac{superGL(4,R)}{GL(4,R)}.
\]

(Note that SGM-flat space-time allows Riemann space-time.) Remarkably the observed Riemann space-time of EGRT and matter(superons) appear simultaneously from (the vacuum of) SGM action by the spontaneous SUSY breaking.

The analysis of the structures of the vacuum of Riemann-flat space-time (described by \( N=10 \) V-A action with derivative terms similar to \([19]\) ) plays an important role to linearize SGM and to derive SM as the low energy effective theory, which remain to be challenged. The derivative terms can be rewritten in the tractable form \([19]\) up to the total derivative terms. The linearization of the flat-space \( N=1 \) V-A model was already carried out\([14]\). The linearization of \( N=2 \) V-A model is extremely important from the physical point of view, for it gives a new mechanism generating a (U(1)) gauge field of the linearized (effective) theory \([16]\). In our case of SGM the algebra(gauge symmetry) should be changed to broken SO(10) SP(broken SUGRA \([17]\))symmetry by the linearization which is isomorphic to the initial one \([17]\). The systematic and generic arguments on the relation of linear and nonlinear SUSY are
already investigated\[18\]. Recently we have shown that N=1 gauge vector multiplet action with SUSY breaking Fayet-Iliopolos term is equivalent to N=1 flat space V-A action of NL SUSY\[19\]. A U(1) gauge field, though an axial vector field for N=1 case, is expressed by N-G field (and its highly nonlinear self interactions).

Finally we just mention the hidden symmetries characteristic to SGM. It is natural to expect that SGM action may be invariant under a certain exchange between $e^a_\mu$ and $t^a_\mu$, for they contribute equally to the unified SGM vierbein $w^a_\mu$ as seen in (12). In fact we find, as a simple example, that SGM action is invariant under the following exchange of $e^a_\mu$ and $t^a_\mu$\[20\] (in 4 dimensional space-time).

$$e^a_\mu \rightarrow 2t^a_\mu, t^a_\mu \rightarrow e^a_\mu - t^a_\mu,$$
$$e_a^\mu \rightarrow e_a^\mu,$$

or in terms of the metric it can be written as

$$g_{\mu\nu} \rightarrow 4t^\rho_\mu t^\rho_\nu, t_{\mu\nu} \rightarrow 2(t_{\nu\mu} - t_{\rho\mu} t^\rho_\nu),$$
$$g^{\mu\nu} \rightarrow g^{\mu\nu}, t^{\mu\nu} \rightarrow g^{\mu\nu} - t^{\mu\nu}. \quad (38)$$

This can be generalized to the following form with two real(one complex) global parameters\[20\],

$$\begin{pmatrix} e^a_\mu \\ t^a_\mu \\ t^b_\mu e^c_\nu t^c_\nu \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 2(\alpha + 1) & -2(\alpha^2 - \beta) \\ 1 & -(2\alpha + 1) & 2(\alpha^2 - \beta) \\ 1 & -3(\alpha + 2) & 2\alpha(2\alpha + 1) - 3\beta + 1 \end{pmatrix} \begin{pmatrix} e^a_\mu \\ t^a_\mu \\ t^b_\mu e^c_\nu t^c_\nu \end{pmatrix}. \quad (39)$$

The physical meaning of such symmetries remains to be studied. Also SGM action has $Z_2$ symmetry $\psi^j \rightarrow -\psi^j$.

Besides the composite picture of SGM it is interesting to consider (elementary field) SGM with the extra dimensions and their compactifications. The compactification of $w^A_M = e^A_M + t^A_M, (A, M = 0, 1, .., D - 1)$ produces rich spectrum of particles and (hidden) internal symmetries and may give a new framework for the unification of space-time and matter.

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