Influence of oceanic turbulence on the spectral switches of partially coherent pulsed beams

Chaoliang Ding¹, Xiaotong Feng, Pei Zhang, Haixia Wang and Yongtao Zhang
Department of Physics and Henan Key Laboratory of Electromagnetic Transformation and Detection, Luoyang Normal University, Luoyang 471934

¹Email: dingchaoliang2006@126.com

Abstract. According to the theory of non-stationary optical field, the spectral properties of Gaussian Schell-model (GSM) pulsed beams propagating in oceanic turbulence are considered. The spectral switches of GSM pulsed beams in oceanic turbulence are investigated. And the effect of three oceanic turbulence parameters, i.e., temperature-salinity balance parameter $\nu$, mean square temperature dissipation rate $\chi_T$ and energy dissipation rate per unit mass $\varepsilon$ on the spectral switches of GSM pulsed beams upon propagation is given. It is shown that the spectral switches would disappear when the $\nu$ and $\chi_T$ are big enough or the $\varepsilon$ is small enough. The results obtained have potential application in optical under-water communications and imaging.

1. Introduction

In the past decades, the spectral switch phenomenon [1], which was closely related to the singular optics of physical optics [2], has gained considerable attention because of its potential applications in the optical communications, information transmission [3], and lattice spectroscopy [4]. Diffraction-induced and correlation-induced spectral switches of fully/partially coherent beams have been studied comprehensively in various media or optical systems [5-8]. And, scattering-induced spectral switches of plane wave scattered by some random media or particles have been presented [9, 10], which extend the application field of spectral switches. With the development of ultra-short pulse technology, the spectral switches of fully/partially coherent ultrashort pulsed beams in free space [11, 12] and in atmospheric turbulence medium [13, 14] have been investigated. In particular, it has been demonstrated that atmospheric turbulence plays an important role on the spectral switches of fully/partially coherent ultrashort pulsed beams.

On the other hand, the oceanic turbulence is another random medium which might significantly affect propagation properties of a random beams [15]. Due to some practical applications in optical underwater communications, imaging and sensing [16], the investigation of propagation properties of a random beam in oceanic turbulence becomes important, and attracts many researchers’ consideration [17-26]. In the aforementioned literature, the evolution of intensity, coherence and polarization, beam quality, beam wander, beam scintillation and spectral changes of various stationary beams are investigated in detailed. Recently, Liu et al. has studied the spectral shifts of fully coherent Gaussian pulse in oceanic turbulence [27]. However, up to now, there is no report about the spectral switches of fully/partially coherent pulsed beams in oceanic turbulence. Because of some inherent advantages of pulsed laser in underwater propagation [28], an interesting question appears: Are there some spectral switches of partially coherent pulsed beams in the oceanic turbulence propagation? What is the influence of oceanic turbulence on the spectral switches of partially coherent pulsed beams?
In this paper, we consider the spectral properties of GSM pulsed beams propagating in oceanic turbulence. And the effect of temperature-salinity balance parameter $\sigma$, mean square temperature dissipation rate $\chi_T$ and energy dissipation rate per unit mass $\epsilon$ on the spectral switches of GSM pulsed beams upon propagation is emphasized. In Section 2, the analytical expression for the spectral intensity of GSM pulsed beams is obtained, and used to study the spectral switches of GSM pulsed beams in oceanic turbulence. Section 3 gives numerical calculation results to illustrate the dependence of spectral switches on the temperature-salinity balance parameter $\sigma$, mean square temperature dissipation rate $\chi_T$ and energy dissipation rate per unit mass $\epsilon$. Finally, in Section 4, a brief summary of the main results is presented.

2. Theory

Let us consider two-dimensional spatially and temporally partially coherent GSM pulsed beams propagating in oceanic turbulence, which is located plane $z=0$ at points $x'_1$ and $x'_2$. According to coherent theory of non-stationary field, the mutual coherence function of GSM pulsed beams in the space-time domain at the initial plane $z=0$ can be given [29]

$$\Gamma(x'_1, x'_2, 0, t_1, t_2) = \exp[-\frac{x'_1^2 + x'_2^2}{w_0^2} - \frac{(x'_1 - x'_2)^2}{2\sigma_0^2} - \frac{t_1^2 + t_2^2}{2T_c^2} - i\omega_0(t_1 - t_2)],$$

(1)

where $w_0$ and $\sigma_0$ indicate the beam width and the transverse correlation length, respectively. $T_0$ characterize the pulse duration, and $T_c$ describes the temporal coherence length of the pulse. $\omega_0$ is carrier frequency of pulse. $(x'_1, x'_2)$ is the transversal coordinates of two points at the $z=0$ plane. According to the Fourier transform, the cross-spectral density function of GSM pulsed beams at the initial plane $z=0$ is derived and given by

$$W(x'_1, x'_2, \omega_1, \omega_2) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Gamma(x'_1, x'_2, 0, t_1, t_2) \exp[-i(\omega_1 t_1 - \omega_2 t_2)] dt_1 dt_2$$

$$= W_0 \exp[-\frac{x'_1^2 + x'_2^2}{w_0^2} - \frac{(x'_1 - x'_2)^2}{2\sigma_0^2} - \frac{(\omega_1 - \omega_0)^2 + (\omega_2 - \omega_0)^2}{2\Omega_0^2} - \frac{(\omega_1 - \omega_2)^2}{2\Omega_c^2}],$$

(2)

where

$$\Omega_0 = \sqrt{\frac{1}{T_0^2} + \frac{2}{T_c^2}}, \text{(spectral width)}$$

(3)

$$\Omega_c = \frac{T_0}{T_c} \Omega_0, \text{(spectral coherence width)}$$

(4)

$$W_0 = \frac{T_0}{2\pi\Omega_0}. \quad \text{(5)}$$

Equation (3) and (4) show the relation between the pulse duration $T_0$, temporal coherence length $T_c$, spectral width $\Omega_0$, and spectral coherence width $\Omega_c$. The spectral coherence width $\Omega_c$ indicates the correlations between the different frequency components of the pulse [30].

By substituting $\omega_1=\omega_2=\omega$ and $x'_1=x'_2=x'$ into equation (2), the normalized spectral intensity at the source plane $z=0$ is

$$S^0(\omega) = \exp\left[-\frac{(\omega - \omega_0)^2}{\Omega_0^2}\right].$$

(6)

According to the extended Huygens-Fresnel principle [31], the cross-spectral density function of GSM pulsed beams propagating through oceanic turbulence is expressed as
where \( \psi(x', x, z) \) denotes the complex phase perturbation induced by the refractive-index fluctuations of the random medium between \( x' \) and \( x \). And we assume that an aperture with half width \( a \) is set at the source plane \( z=0 \). If the medium fluctuations are homogeneous and isotropic, the ensemble average \( \langle \psi(x', x, z) \rangle \) in equation (7) is related to the power spectrum \( \Phi(\kappa) \) by formula \([32]\)

\[
\langle \exp[\psi(x', x, z) + \psi^*(x', x, z)] \rangle_M = \exp\left[-4\pi^2 k^2 z \int_0^\kappa \kappa \Phi(\kappa) \left[1 - J_0(\kappa |x' - x|^2)\right] d\kappa d\xi\right],
\]

where \( \kappa \) is the magnitude of the spatial wave number, \( \Phi(\kappa) \) is the spatial power spectrum of refractive-fluctuations \([32]\), and \( J_0(x) \) is the Bessel function of first kind and order zero. If one uses the paraxial approximation, the two first terms of Bessel function \( J_0(x) \) can be expanded, i.e., \( J_0(x)=1-x^2/4 \), thus, the equation (8) reduces to the form

\[
\langle \exp[\psi^*(x', x, z) + \psi(x', x, z)] \rangle_M \approx \exp\left[-\frac{\pi^2 k^2 z}{3} (x' - x)^2 \int_0^\kappa \kappa^3 \Phi(\kappa) d\kappa\right].
\]

The hard-edged aperture function is given by

\[
H(x) = \begin{cases} 
1, & |x| \leq a \\
0, & |x| > a ,
\end{cases}
\]

(10)

where \( H(x) \) can be regarded as a finite sum of the complex Gaussian function \([33]\)

\[
H(x) = \sum_{m=1}^M A_m \exp\left[-\frac{B_m x^2}{a^2}\right],
\]

(11)

where Gaussian coefficients \( A_m \) and \( B_m \) are given in Ref. \([33]\).

On substituting from Eqs. (2), (9), (10) and (11) into equation (7) and letting \( x_1=x_2=x \) and \( \omega_1=\omega_2=\omega \), and after tedious integral calculations, the spectral intensity of GSM pulsed beams propagating through oceanic turbulence at the \( z \) plane is given by

\[
S(x, z, \omega) = W(x, z, x, z, \omega, \omega, \omega) = S^0(\omega)M(\rho, z, \omega),
\]

(12)

where

\[
M(\rho, z, \omega) = \frac{z_0}{\omega} \sum_{m=1}^M A_m A_n^* \exp\left[\frac{\alpha_1 + \alpha_2 - 2\alpha_3}{4(\alpha_1\alpha_2 - \alpha_3^2)} \rho^2\right],
\]

(13)

\[
\alpha_1 = 1 + \frac{1}{2}(\beta^2 - 1) + \frac{\pi^2 \omega^2 w_0^2 z}{3\epsilon^2} \frac{I}{\omega_0^2} - i \frac{\omega}{\omega_0} \frac{z_0}{z},
\]

(14)

\[
\alpha_2 = 1 + \frac{1}{2}(\beta^2 - 1) + \frac{\pi^2 \omega^2 w_0^2 z}{3\epsilon^2} \frac{I}{\omega_0^2} + i \frac{\omega}{\omega_0} \frac{z_0}{z},
\]

(15)

\[
\alpha_3 = \frac{1}{2}(\beta^2 - 1) + \frac{\pi^2 \omega^2 w_0^2 z}{3\epsilon^2} \frac{I}{\omega_0^2},
\]

(16)

\[
\alpha_4 = 2i \frac{\omega}{\omega_0} \frac{z_0}{z},
\]

(17)
\[ \beta = \left[ 1 + \left( \frac{w_0}{\sigma_0} \right)^2 \right]^{1/5}, \]  
(18)

\[ z_0 = \frac{\pi w_0^2}{\lambda_0}, \]  
(19)

\[ \rho = \frac{x}{w_0}, \]  
(20)

\[ \delta = \frac{a}{w_0}, \]  
(21)

\[ I = \int_{0}^{\infty} \kappa^2 \Phi(\kappa) \, d\kappa. \]  
(22)

In equation (22), the \( \Phi(\kappa) \) is the spatial power spectrum developed in Ref. [34] for homogeneous and isotropic oceanic turbulence. The model was derived as the linearized polynomial of two variables: the salinity fluctuations and temperature fluctuations, where the combined effects of these two fluctuations are considered, and is valid under the condition that the turbulence is homogeneous and isotropic. Hence, the model is given by

\[ \Phi(\kappa) = 0.388 \times 10^{-8} \varepsilon^{-1/3} \chi_T^{-1/3} \chi_T \left[ 1 + 2.35(\kappa \eta)^{2/3} \right] \left( e^{-\Lambda_0 \delta} + \sigma^{-2} e^{-\Lambda_0 \delta} - 2 \sigma^{-1} e^{-\Lambda_0 \delta} \right), \]  
(23)

where \( \varepsilon \) is the rate of dissipation of turbulent kinetic energy per unit mass of fluid which may vary in range from \( 10^{-1} \) to \( 10^{-10} \) m\(^2\)/s\(^3\) [15], \( \eta = 10^{-m} \) is the Kolmogorov micro scale (inner scale), \( \chi_T \) is the rate of dissipation of mean-square temperature and has the range from \( 10^{-4} \) to \( 10^{-10} \) K\(^2\)/s, \( \Lambda_0 = 1.863 \times 10^{-2} \), \( A_E = 1.9 \times 10^{-4} \), \( A_{T3} = 9.41 \times 10^{-3} \), and \( \delta = 8.284(\kappa \eta)^{4/3} + 12.978(\kappa \eta)^2 \), \( \sigma \) denotes the relative strength of temperature and salinity fluctuations, wherein the ocean water can range from 0 to -5, with -5 and 0 corresponding to dominating temperature-induced and salinity-induced optical turbulence, respectively. And the minus sign denotes that there is a reduction in the temperature and an increase in salinity with depth.

From equation (13)-(23), \( M(\rho, z, \omega) \) denotes the spectral modifier of GSM pulsed beams propagating through oceanic turbulence. Equation (3), (4) and (12)-(23) indicate that the spectral intensity of GSM pulsed beams propagating through oceanic turbulence is a product of the original spectrum \( S^0(\omega) \) and spectral modifier \( M(\rho, z, \omega) \). The original spectrum \( S^0(\omega) \) is dependent on the pulse duration \( T_0 \) and temporal coherence length \( T_c \). The spectral modifier \( M(\rho, z, \omega) \) depends on the oceanic turbulence parameters including temperature-salinity balance parameter \( \sigma \), mean square temperature dissipation rate \( \chi_T \), energy dissipation rate per unit mass \( \varepsilon \), and other parameters such as truncation parameter \( \delta \), spatial correlation parameter \( \beta \), relative transversal coordinate \( \rho \) and propagation distance \( z \).

3. Results and discussion

Numerical calculations were presented using equation (12) to illustrate the off-axis spectral switches of GSM pulsed beams in oceanic turbulence and to address the influence of three oceanic turbulence parameters i.e., temperature-salinity balance parameter \( \sigma \), mean square temperature dissipation rate \( \chi_T \), and energy dissipation rate per unit mass \( \varepsilon \) on the spectral switches of GSM pulsed beams.
Figure 1. Normalized spectral intensity $S(\omega)$ for different values of relative transversal coordinate $\rho$ (a) $\rho=16$, (b) $\rho=16.3$, (c) $\rho=16.51$, (d) $\rho=16.7$ and (e) $\rho=17$.

Figure 2. Normalized spectral intensity $S(\omega)$ as a function of $\omega/\omega_0$ and $\rho$ in the region $16 \leq \rho \leq 17$ at the propagation distance $z=10\text{km}$.
Figure 1 gives the normalized spectral intensity $S(\omega)$ for different values of relative transversal coordinate (a) $\rho=16$, (b) $\rho=16.3$, (c) $\rho=16.51$, (d) $\rho=16.7$ and (e) $\rho=17$. The other calculation parameters are $\omega_0=0.05m$, $z=10km$, $T_c=5fs$, $T_o=3fs$, $\delta=0.2$, $\omega_0=2.36rad/fs^{-1}$ and $M=10$. As can be seen that the spectrum is red-shifted for $\rho=16$, where there are two spectrum maxima, and the second maximum is small. The second maximum increases with an increase of $\rho$, and the spectral switch appears for $\rho=16.51$. With the increasing of $\rho$ the second maximum is larger than the first one, and the spectrum becomes blue-shifted. Figure 2 gives the normalized spectral intensity $S(\omega)$ as a function of $\alpha^t\omega_0$ and $\rho$ in the region $16\leq\rho\leq17$ at the propagation distance $z=10km$. From figure 2, one can see the more detailed spectral change and spectral switch (see the arrow) with the change of relative transversal coordinate $\rho$.

The relative spectral shift $\delta\omega/\omega_0$ is defined as $\delta\omega/\omega_0= (\omega_{max}-\omega_0)/\omega_0$, where $\omega_{max}$ denotes the frequency at which the spectral intensity takes the maximum value. Figure 3 shows the relative spectral shift $\delta\omega/\omega_0$ versus the relative transversal coordinate $\rho$ for different values of temperature-salinity balance parameter $\sigma=-2.5$, $\sigma=-1$ and $\sigma=-0.5$, where $\chi_T=10^{10}K^2/s$ and $\epsilon=10^{-1}m^2/s^3$. As can be seen that, in the region $0\leq\rho\leq40$, there are four spectral switches at the critical positions $\rho_c=8.26, 16.51, 24.37$ and $32.55$ for $\sigma=-2.5$, which are termed the first, second, third and fourth spectral switches, respectively. With the increase of temperature-salinity balance parameter $\sigma$, the first spectral switch disappears for the case of $\sigma=-1.0$. And, with the further increase of temperature-salinity balance parameter $\sigma$, the first and second spectral switches disappear for the case of $\sigma=-0.5$. In addition, temperature-salinity balance parameter $\sigma$ affects slightly the critical positions of spectral switch and the spectral transition height, which is defined as the sum of the upward and downward transitions at the spectral switch.

Figure 4 shows the relative spectral shift $\delta\omega/\omega_0$ versus the relative transversal coordinate $\rho$ for different values of mean square temperature dissipation rate $\chi_T=10^{10}K^2/s$, $\chi_T=10^{9}K^2/s$ and $\chi_T=10^{8}K^2/s$, where $\sigma=-2.5$ and $\epsilon=10^{-1}m^2/s^3$. It is shown that there are also four spectral switches in the region $0\leq\rho\leq40$. With the increase of mean square temperature dissipation rate $\chi_T$, the first and second spectral switches disappear for the case of $\chi_T=10^{8}K^2/s$. However, full four spectral switches disappear when mean square temperature dissipation rate $\chi_T=10^{7}K^2/s$. What is more, the critical positions shift opposite the optical axis and the spectral transition height of spectral switch decreases with the increase of mean square temperature dissipation rate $\chi_T$ for the third and fourth spectral switches.
Figure 5. Off-axis $\delta \omega / \omega_0$ versus $\rho$ for different values of $\varepsilon=10^{-1}\m^2/s^3$, $\varepsilon=10^{-3}\m^2/s^3$ and $\varepsilon=10^{-4}\m^2/s^3$, where $\sigma=-2.5$ and $\chi_T=10^{-10}\m^2/K^2/s$.

Figure 5 presents the relative spectral shift $\delta \omega / \omega_0$ versus the relative transversal coordinate $\rho$ for different values of energy dissipation rate per unit mass $\varepsilon=10^{-1}\m^2/s^3$, $\varepsilon=10^{-3}\m^2/s^3$ and $\varepsilon=10^{-4}\m^2/s^3$, where $\sigma=-2.5$ and $\chi_T=10^{-10}\m^2/K^2/s$. As can be seen that, with the decrease of energy dissipation rate per unit mass $\varepsilon$, the first spectral switch disappears for the case of $\varepsilon=10^{-3}\m^2/s^3$. And, the first and second spectral switches disappear for the case of energy dissipation rate per unit mass $\varepsilon=10^{-4}\m^2/s^3$.

Further analysis and evaluating to the effect of three oceanic turbulence parameters $\sigma$, $\chi_T$ and $\varepsilon$ on the spectral switches indicate that the mean square temperature dissipation rate $\chi_T$ plays a more important role on the off-axis spectral switches.

4. Conclusions

In this work, the spectral properties of GSM pulsed beams propagating in oceanic turbulence are considered. The analytical expression for the spectral intensity of GSM pulsed beams is obtained, and used to study the spectral switches of GSM pulsed beams in oceanic turbulence. And the influence of three oceanic turbulence parameters, i.e., temperature-salinity balance parameter $\sigma$, mean square temperature dissipation rate $\chi_T$ and energy dissipation rate per unit mass $\varepsilon$ on the off-axis spectral switches of GSM pulsed beams upon propagation is given. It has been shown that the spectral switches of GSM pulsed beams appear in the oceanic turbulence propagation. The spectral switches would disappear when the $\sigma$ and $\chi_T$ are big enough or the $\varepsilon$ is small enough. The results obtained in this paper are useful for optical under-water communications and imaging, in situations where spectral information encoding and spectral diversity techniques are employed. And I would like to point out that there are some advantages of a partially coherent pulsed laser over a partially coherent continuous laser beam, i.e., the spectrum of pulsed laser are broader than that of continuous laser. Thus the phenomenon of spectral switch is more noticeable. What’s more, partially coherent pulsed laser have more controllable parameters such as pulse duration, temporal coherent length and chirped parameter, which can provide more freedom to modulate the spectral switches.

Acknowledgments

This work was financially supported in part by Natural Science Foundation of China (61575091, 61675094, 11474143), and Education Department of Henan Province (17A140001).

References

[1] Kandpal H C 2001 Experimental observation of the phenomenon of spectral switch J. Opt. A: Pure Appl. Opt. 3 296-299
[2] Soskin M S and Vasnetsov M V 2001 Singular optics in Progress in Optics (Amsterdam: Elsevier) 42 219
[3] Yadav B K, Raman S and Kandpal H C 2008 Information exchange in free space using spectral
switching of diffracted polychromatic light: possibilities and limitations J. Opt. Soc. Am. A 25 2952-9
[4] Han P 2009 Lattice spectroscopy Opt. Lett. 34 1303-1305
[5] Pan L Z and Lü B D 2004 Spectral switches of polychromatic Gaussian beams passing through an astigmatic aperture lens Opt. Commun. 234 13-22
[6] Pan L Z, Zhao Z G, Ding C L and Lü B D 2009 Effect of polarization on spectral switches in the diffraction of stochastic electromagnetic beams Appl. Phys. Lett. 95 181112
[7] Yadav B K and Kandpal H C 2011 Multi-level information encoding through spectral switches and multiple FSO link formation in the phase singularity domain J.Opt. 13 075403
[8] Han P 2012 All optical spectral switches Opt. Lett. 37 2319-2321
[9] Du X Y 2013 Spectral shifts and spectral switches produced by the scattering system of two anisotropic particles in different distance Opt. Express 21 22610-22616
[10] Wang T, Hay D, Wu H and Shi Z 2017 Spectral shift of a light wave on scattering from an ellipsoidal particle with arbitrary orientation J. Opt. Soc. Am. A 34 493-497
[11] Jana S and Konar S 2006 Tunable spectral switching in the far field with a chirped cosh-Gaussian pulse Opt. Commun. 267 24-31
[12] Han P 2009 Spectral Switches of Gaussian Pulse from Adjustable Tilted Mirror in Far Field Jpn. J. Appl. Phys. 48 042404
[13] Ji X L 2008 Influence of turbulence on the spectrum of diffracted chirped Gaussian pulsed beams Opt. Commun. 281 3407-3413
[14] Ding C L, Pan L Z and Lü B D 2009 Effect of turbulence on the spectral switches of diffracted spatially and spectrally partially coherent pulsed beams in atmospheric turbulence J. Opt. A: Pure Appl. Opt. 11 105404
[15] Thorpe S A 2005 The Turbulent Ocean (Cambridge: Cambridge University Press)
[16] Johnson L J, Green R J and Leeson M S 2014 Underwater optical wireless communications: depth-dependent beam refraction Appl. Opt. 53 7273-7277
[17] Lu W, Liu L R and Sun J F 2006 Influence of temperature and salinity fluctuations on propagation behaviour of partially coherent beams in oceanic turbulence J. Opt. A: Pure Appl. Opt. 8 1052
[18] Shchepakina E, Farwell N and Korotkova O 2011 Spectral changes in stochastic light beams propagating in turbulent ocean Applied Physics B 105 415
[19] Tang M and Zhao D 2014 Spectral changes in stochastic anisotropic electromagnetic beams propagating through turbulent ocean Opt. Commun. 312 89-93
[20] Huang Y, Zhang B, Gao Z, Zhao G and Duan Z 2014 Evolution behavior of Gaussian Schell-model vortex beams propagating through oceanic turbulence Opt. Express 22 17723-17734
[21] Lu L, Ji X L and Baykal Y 2014 Wave structure function and spatial coherence radius of plane and spherical waves propagating through oceanic turbulence Opt. Express 22 27112-27122
[22] Huang Y, Huang P, Wang F, Zhao G and Zeng A 2015 The influence of oceanic turbulence on the beam quality parameters of partially coherent Hermite-Gaussian linear array beams Opt. Commun. 336 146-52
[23] Liu D, Wang Y and Yin H 2015 Evolution properties of partially coherent flat-topped vortex hollow beam in oceanic turbulence Appl. Opt. 54 10510-10516
[24] Baykal Y 2016 Scintillations of LED sources in oceanic turbulence Appl. Opt. 55 8860-8863
[25] Wu Y, Zhang Y, Li Y and Hu Z 2016 Beam wander of Gaussian-Schell model beams propagating through oceanic turbulence Opt. Commun. 371 59-66
[26] Peng X, Liu L, Cai Y and Baykal Y 2017 Statistical properties of a radially polarized twisted Gaussian Schell-model beam in an underwater turbulent medium J. Opt. Soc. Am. A 34 133-139
[27] Liu D, Wang Y, Wang G, Yin H and Wang J 2016 The influence of oceanic turbulence on the spectral properties of chirped Gaussian pulsed beam Opt. Laser Technol. 82 76-81
[28] Jaffe J S 2015 Underwater Optical Imaging: The Past, the Present, and the Prospects IEEE J.
Ocean. Eng. 40 683-700

[29] Lajunen H, Vahimaa P and Tervo J 2005 Theory of spatially and spectrally partially coherent pulses J. Opt. Soc. Am. A 22 1536-1545

[30] Pääkkönen P, Turunen J, Vahimaa P, Friberg A T and Wyrowski F 2002 Partially coherent Gaussian pulses Opt. Commun. 204 53-58

[31] Mandel L and Wolf E 1995 Optical Coherence and Quantum Optics. (Cambridge: Cambridge University) 239

[32] Andrews L C and Phillips R L 2005 Laser Beam Propagation through Random Media (Bellingham, Washington: SPIE Publications) 58

[33] Wen J J and Breazeale M A 1998 A diffraction beam field expressed as the superposition of Gaussian beam J. Acoust. Soc. Am. 83 1752-1756

[34] Nikishov V V and Nikishov V I 2000 Spectrum of Turbulent Fluctuations of the Sea-Water Refraction Index Int. J. Fluid Mech. Res. 27 82-98