Color confinement at long distance is one of the fundamental properties of Quantum Chromodynamics (QCD), which is the underlying theory to describe the strong interactions between quarks and gluons. Because of confinement, quarks and gluons produced in hard scattering processes will ultimately become colorless hadrons. The transition from quarks $q$, antiquarks $ar{q}$ and gluons $g$ to hadrons $h$, which occurs at a scale with a small momentum transfer, must be treated non-perturbatively due to the large strong-coupling constant $\alpha_s$. Consequently, the fragmentation function (FF) $D_{q,g,h}(z,\mu_F)$ is used to describe the non-perturbative
long-distance behavior associated with the hadronization process [3]. Here, $\mu_F$ is the factorization scale used to factorize the cross section in terms of the convolution of perturbative hard-part coefficients and non-perturbative fragmentation functions, and is usually set to be the center-of-mass (c.m.) energy ($\sqrt{s}$) in $e^+e^-$ experiments. The term $z = 2\sqrt{p_T^0 c^2 + M_h^2 c^4}/\sqrt{s}$ is the relative hadron energy, where $p_T$ and $M_h$ are the momentum and mass of hadron $h$, respectively. Single inclusive $e^+e^-$ annihilation, $e^+e^- \rightarrow h + X$, where $h$ is an identified hadron and $X$ represents everything else, provides a clean way to study FFs [4]. A typical experimental observable is

$$\frac{1}{\sigma(e^+e^- \rightarrow \text{hadrons})} \frac{d\sigma(e^+e^- \rightarrow h + X)}{dp_h}, \tag{1}$$

where $\sigma(e^+e^- \rightarrow \text{hadrons})$ is the total cross section for $e^+e^-$ annihilation to all possible hadronic final states (referred to as inclusive hadronic events hereafter). At the leading order in $\alpha_s$, the observable can be interpreted as $\sum_q e_q^2 [D_q^h(z, \sqrt{s}) + D_{\bar{q}}^h(z, \sqrt{s})]$, where $e_q$ is the fractional charge of the quark $q$. As summarized in Ref. [4], extensive high-precision measurements of this observable have been made in $e^+e^-$ experiments in the c.m. energy range above 10 GeV, but measurements in the region from 3.6 to 5.2 GeV have statistical uncertainties ranging from 20% to 50% [5, 6].

On the other hand, remarkable progress has been made in the experimental study of nucleon structure by taking advantage of the semi-inclusive deep-inelastic scattering (SIDIS) process. Due to its explicit dependence on both the identified initial and final state hadrons, the SIDIS process plays a crucial role in probing specific quark flavors. In particular, the FFs act as the weights used to perform flavor separation in the initial state. For example, precise kaon FFs might be the key to understand the puzzle of the strange-quark polarization inside a longitudinally polarized proton [7, 8]. The momentum-transfer $Q$ (equivalent to $\sqrt{s}$ in $e^+e^-$ annihilation) for the existing SIDIS data from fixed-target experiments like the ones at Jefferson Lab, COMPASS and HERMES [9] covers the range from 1 to 10 GeV, where only sparse single inclusive $e^+e^-$ annihilation data exist. Moreover, the proposed Electron-Ion Collider (EIC) and the Electron-Ion Collider in China (EicC) [10, 11] would provide very high precision structure function measurements, which, in order to determine precise parton distribution functions (PDFs) for various quark flavors, would put unprecedented requirements on the precision of FFs for the $Q$ value down to 1 GeV and almost complete $z$ coverage from 0 to 1. Therefore, comprehensive investigations on the single inclusive annihilation with identified pion and kaon final states, especially in the region not well covered by previous data, will provide valuable input for the nucleon structure study.

In this Letter, the processes $e^+e^- \rightarrow \pi^0/K^0_S + X$ are studied at six c.m. energies from 2.2324 to 3.6710 GeV, with a $z$ coverage from 0.1 to 0.9. Data sets used in this research were collected with the BESIII detector [12] running at BEPCII. The detector has a geometrical acceptance of 93% of the 4π solid angle for the relatively stable final state particles. It is based on a superconducting solenoid magnet with a main drift chamber (MDC) as a central tracking system, plastic scintillators as a time-of-flight system, CsI crystals as an electromagnetic calorimeter (EMC) and resistive plate chambers as a muon system.

Monte Carlo (MC) simulations based on GEANT4 software [13], which includes the geometric description of the BESIII detector and implements the interactions between the final-state particles and the detector, are used to optimize the event selection criteria, estimate the number of residual background events, and determine the correction factors accounting for the efficiency loss and initial-state radiation (ISR) effects. The inclusive hadronic events are simulated with the LUARLW generator [14–16], where among others the signal processes $e^+e^- \rightarrow \pi^0/K^0_S + X$ are contained. Background MC samples of the processes $e^+e^- \rightarrow e^+e^-\mu^+\mu^-$, and $\gamma\gamma$ are generated by BABAYAGA3.5 [17]. At $\sqrt{s} = 3.6710$ GeV, the $e^+e^- \rightarrow \tau^+\tau^-$ process is simulated via the KKMC [18] generator, in which the decay of $\tau$ lepton is modeled by EVTVGEN [19]. The background events from the two-photon processes are simulated by the corresponding dedicated MC generators, as detailed in Ref. [16]. In addition, the beam-associated events are estimated by a side-band method.

In this analysis, the inclusive hadronic events are selected first, from which the $e^+e^- \rightarrow \pi^0/K^0_S + X$ events are identified by reconstructing a $\pi^0$ or $K^0_S$ meson. We begin by removing the dominant background processes, i.e., the $e^+e^- \rightarrow e^+e^-\pi^0$ and $e^+e^- \rightarrow \gamma\gamma$ events. In each remaining event, the good charged hadronic tracks are identified with the same selection criteria as described in Ref. [16], and events with at least two good tracks are kept for further analysis. For events with two or three good charged tracks, additional requirements on the charged tracks and the showers in the EMC are implemented to further suppress the background related to the quantum electrodynamics process. Events with more than three good charged tracks are retained directly without any additional requirements. The details of the selection of the inclusive hadronic events are presented in Ref. [16]. The numbers of inclusive hadronic ($N_{\text{had}}^{\text{tot}}$) events and residual background ($N_{\text{bkg}}$) events, as well as the integrated luminosity of each data sample, are summarized in Table I.

A $\pi^0$ candidate is reconstructed via the decay $\pi^0 \rightarrow \gamma\gamma$. Photon candidates are identified using showers in the EMC. The deposited energy of each shower must be greater than 25 MeV in the barrel region ($|\cos \theta| < 0.80$) and 50 MeV in the end-cap region ($0.86 < |\cos \theta| < 0.92$), where $\theta$ is the polar angle defined with respect to the symmetry axis of the MDC. The shower is required to be separated by more than 10 degrees from the closest charged track to eliminate those produced by the charged particles. The difference between the EMC time and the
event start time has to be within [0, 700] ns to suppress electronic noise and showers unrelated to the event. All combinations of two photons are used to form π⁰ candidates. To suppress background due to the miscombination of photons, requirements are made on the polar angle of one photon in the helicity frame of the π⁰ candidate (θπ⁰). For π⁰ candidates with momentum less than 0.3 GeV/c, |cosθπ⁰| is required to be less than 0.8, while for those with momentum larger than 0.3 GeV/c, it must be less than 0.95. Each π⁰ candidate in an event is counted as a candidate of the separate inclusive π⁰ event, and the fraction of the observed hadronic events containing more than one π⁰ meson varies from 42% to 50% at the c.m. energies from √s = 2.2324 to 3.6710 GeV.

A K_S⁰ candidate is formed by combining a pair of oppositely charged tracks. The two tracks are required to satisfy |cosθ| < 0.93. Due to the relatively long lifetime of the K_S⁰ meson, the distance of closest approach of these charged tracks from the interaction point must be less than 30 cm along and 10 cm perpendicular to the beam direction. As a result of the different requirements, the charged track here is not necessarily one of the good charged tracks identified previously. To increase the number of events, no particle identification is applied. Further, to select K_S⁰ signal events, the production and decay vertices are reconstructed, and the decay length between these two vertices is required to be at least twice its uncertainty. Each K_S⁰ candidate in an event is counted as a candidate of the separate inclusive K_S⁰ event. In this measurement, fewer than 1% observed hadronic events contain more than one K_S⁰ meson.

After imposing the above selection criteria, the residual contributions to the mass spectra of the photon and charged-pion pairs, M(γγ) and M(π⁺π⁻), from lepton-pair production, two-photon processes, and beam-associated events are less than 0.1% in both signal processes. The dominant background is caused by the miscombinations of the corresponding daughter particles, which are reproduced by the inclusive hadronic MC samples and well described by a polynomial.

The M(γγ) and M(π⁺π⁻) spectra are divided into the momentum intervals with a step of ∆p/GeV = 0.1 GeV/c, which is 5 times larger than the corresponding momentum resolutions. Unbinned maximum likelihood fits are performed on the the M(γγ) and M(π⁺π⁻) spectra obtained in each momentum interval to determine the corresponding numbers of events, i.e., N_{obs}^{γγ} and N_{obs}^{π⁺π⁻}. For the e⁺e⁻ → π⁰ + X candidates, the signal is described by a Crystal Ball function [20], while the background is parameterized by a second-order Chebychev polynomial. For the e⁺e⁻ → K_S⁰ + X process, the signal is modeled by a double Gaussian function, and the background is described by a first-order Chebychev polynomial. Figure 1 illustrates the fit results of the π⁰ and K_S⁰ candidates with p_{γγ/π⁺π⁻} ∈ (0.4, 0.5) GeV/c from the data sample at √s = 2.8000 GeV. The summary of N_{obs}^{γγ} and N_{obs}^{π⁺π⁻} obtained in each momentum range at each c.m. energy is given in the Supplemental Material [24].

In practice, the normalized differential cross section for the inclusive production of any identified hadron, namely Eq. (1), is determined with

$$\frac{N_{obs}^{h}}{N_{had}^{obs}} \frac{1}{\Delta p_{h}} f_{h},$$

where N_{had}^{obs} = N_{had}^{tot} − N_{bkg} is the net number of observed hadronic events in e⁺e⁻ annihilation at a given c.m. energy, N_{obs}^{h} is that of the e⁺e⁻ → h + X events within a specific momentum range Δp_{h}, and f_{h} is a correction factor which accounts for the effects caused by the limited detector acceptance, the selection criteria, ISR, and vacuum polarization. Since both N_{had}^{obs} and N_{obs}^{h} are obtained from the same data sample, the integrated luminosity usually used in the cross section measurement cancels.

In this analysis, the correction factor, f_{h}, is obtained from the signal MC sample and is given by

| √s (GeV) | L (pb⁻¹) | N_{had}^{tot} | N_{bkg} |
|---------|--------|-------------|--------|
| 2.2324 | 2.645  | 83227       | 2041   |
| 2.4000 | 3.415  | 96627       | 2331   |
| 2.8000 | 3.753  | 83802       | 2075   |
| 3.0500 | 14.89  | 283822      | 7719   |
| 3.4000 | 1.733  | 32202       | 843    |
| 3.6710 | 4.628  | 75253       | 6461   |

### TABLE I. The integrated luminosities and the total observed hadronic and residual background events at various c.m. energy points.

![Figure 1](image-url)
\[ f_h = \frac{N_h^{\text{tru}}(\text{off})}{N_h^{\text{had}}(\text{off})} \bigg/ \frac{N_h^{\text{obs}}(\text{on})}{N_h^{\text{obs}}(\text{on})} \]  

(3)

where the variable “\( \hat{N} \)” denotes the numbers of events determined from the signal MC sample, either at the detector observed level, similar to the experimental data, with superscript “obs” or at the generation level with superscript “tru”. The terms “on” and “off” in the parentheses indicate that the corresponding quantities are obtained from signal MC samples with and without simulating the ISR effects, respectively. \( N_h^{\text{obs}}(\text{on}) \) is determined by applying the same fit procedure on the signal MC events as data. By definition, \( f_h \) compensates the event lost caused by the limited acceptance of the BESIII detector in the small polar angle region [24]. In this analysis, the correction factor \( f_h \) is by far dominated by the detection efficiency. Calculated results of \( f_h \) in different momentum ranges for \( \pi^0 \) and \( K_S^0 \) are presented in the Supplemental Material [24].

The systematic uncertainties of the normalized differential cross section measurements mainly originate from the differences between the signal MC and data samples, the reconstruction efficiencies of the \( \pi^0 \) and \( K_S^0 \) candidates, the fits to the events in the \( M(\gamma\gamma) \) and \( M(\pi^+\pi^-) \) bins, and the MC simulation model of the inclusive hadronic events. To estimate the uncertainty caused by the imperfect simulation of various kinematic variables of the signal events, all the selection criteria are separately varied to be larger or smaller than their nominal values by one time their resolutions, and the maximum changes of the normalized differential cross sections are taken as the systematic uncertainties. The uncertainty in the photon detection efficiency is estimated to be 1% per photon [21], therefore 2% is taken as the systematic uncertainty due to the \( \pi^0 \) reconstruction efficiency where the complete correlation between the detection of the two photons is assumed. The momentum-dependent systematic uncertainties due to the \( K_S^0 \) reconstruction efficiency are obtained by applying a dedicated weighting procedure [22], that incorporates weights due to the charged particle tracking efficiency and the decay length requirement.

The uncertainties due to the fits on the events in the \( M(\gamma\gamma) \) and \( M(\pi^+\pi^-) \) intervals are examined by using alternative signal and background shapes. The alternative signal shape of \( e^+e^- \rightarrow \pi^0 + X \) is taken as the shape from the signal MC sample, while that for \( e^+e^- \rightarrow K_S^0 + X \) is chosen as a single Gaussian function. The alternative background shapes are obtained by varying the order of the Chebychev polynomials. The relative differences from the original differential cross sections are taken as the corresponding systematic uncertainties.

The dominant source of systematic uncertainty is the MC simulation model of the inclusive hadronic events. The generation fractions of the exclusive processes containing \( \pi^0 \) and \( K_S^0 \), which make up the inclusive process, directly affect the correction factors \( f_{x^0} \) and \( f_{K_S^0} \). To address the corresponding uncertainty, the HYBRID generator, which was developed in Ref. [23] and improved in Ref. [16], is used as an alternative MC model to generate the inclusive hadronic events. In the HYBRID generator, much knowledge of the allowed exclusive processes in the c.m. energy region of BESIII has been incorporated, including measured cross sections and production mechanisms. In addition, a different simulation scheme of the ISR process is adopted [16]. The changes of the correction factors \( f_{x^0} \) and \( f_{K_S^0} \) are assigned as the systematic uncertainties. All these individual systematic uncertainties are regarded as uncorrelated with each other and are summed in quadrature.

![Normalized differential cross sections of the \( e^+e^- \rightarrow \pi^0 + X \) process. The points with error bars are the measured values, where the uncertainties are the quadrature sum of the corresponding statistical and systematic uncertainties. The bands or curves in red, green, blue, magenta, and orange denote the NNFF, MAPFF, AKRS, ARS, and DSS calculations, respectively, where only the former two cover \( \pm 1\sigma \) limits. Normalized differential cross sections as function of \( z \) are shown in the Supplemental Material [24].](image)

The normalized differential cross sections for the inclusive \( \pi^0 \) and \( K_S^0 \) production in \( e^+e^- \) annihilation at the six c.m. energies are tabulated in the Supplemental Material [24] and shown in Fig. 2 and Fig. 3, respectively. Figure 2 also shows various theoretical predictions extrapolated from different FFs determined from existing world data [4]. The FFs are obtained with slightly different assumptions and show the sensitivity of the prediction to assumptions about the behavior at low-\( z \) and different \( \sqrt{s} \). ARS [25], AKRS [26] and NNFF1.0 [27] are all obtained from
In summary, we have measured the normalized differential cross sections of the $e^+e^- \rightarrow \pi^0/K_S^0 + X$ processes, using data samples collected from $\sqrt{s} = 2.2324$ to 3.6710 GeV. The results obtained in this work help to fill the region with $\sqrt{s} < 10$ GeV where precision $e^+e^-$ annihilation data have been rarely reported. The results provide broad $z$ coverage from 0.1 to 0.9 with precision of around 3% at $z \sim 0.4$. These results provide new ingredients for FF global data fits, in which almost no single inclusive annihilation data measured in this special energy region has been included.

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