One-dimensional quantum channel and Hawking radiation of the Kerr and Kerr-Newman black holes

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Abstract

In this paper, we review the one-dimensional quantum channel and investigate Hawking radiation of bosons and fermions in Kerr and Kerr-Newman black holes. The result shows the Hawking radiation can be described by the quantum channel. The thermal conductances are derived and related to the black holes’ temperatures.

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1 Introduction

Black holes are not black and radiate thermally [1], which reveals the relation between quantum theory and gravity theory. When first proved it, Hawking described the radiation as a tunnelling process caused by vacuum fluctuations near the black hole horizon. Where a virtual particle pair spontaneously creates near the horizon. The negative energy particle is absorbed by the black hole, while the positive energy particle is left outside the horizon, moves to infinite distance and forms Hawking radiation.

There are several methods to derive Hawking radiation. Hawking’s original derivation is directly physical and dependent on the calculation of Bogoliubov coefficient [1]. The black hole’s background spacetime was seen as fixed one with the emission of particles. In the method of Damour, Ruffini and Sannan [2, 3], the second quantizen is avoided and Hawking radiation is derived by relativistic quantum mechanics in the curved spacetime. There is no need to demand the thermal equilibrium between black holes and the environment outside the black holes and consider black hole’s collapse. The radiation spectrum was obtained and given by the Planck distribution as [1, 2, 3]

\[ N^\pm (\omega) = \frac{1}{e^{\frac{k_B T H}{\omega}} \pm 1}, \]

where + (-) correspond to the fermion (boson), \( k_B \) and \( \omega \) are the Boltzmann constant and the energy of emission particle, respectively. The semi-classical tunnelling method, based on particles in a dynamical geometry, was put forward by Parikh and Wilczek [4]. Their work shown that the radiation spectrum is not purely thermal one and the tunnelling rate is related to the change of Bekenstein-Hawking entropy. This result gives the leading correction to Hawking radiation spectrum. The method based on the
calculation of Hawking fluxes has a long history [5, 6, 7, 8, 9, 10, 11]. The expectation value of energy-momentum tensor in the vacuum state was obtained and connects the black hole’s temperature. In [5], the expectation value with respect to the Unruh vacuum was derived, which gave a elegant description for the radiation. In [10, 11], the fluxes of Hawking radiation were derived by the cancellation of gravity anomaly and gauge anomaly at the horizon. It shown the scalar field theory of a arbitrary \((D + 2)\)-dimensional black hole can be reduced to \((1 + 1)\)-dimensional quantum field theory by a dimensional reduction technology.

On the other hand, the holographic principle [12, 13] shows that a generic physical system in three-dimensions (3D) can be seen as two-dimensional (2D). Considering this case, Bekenstein and Mayo [14] gave a further constriction of dimensions and shown the black hole in 3D behaves as the one-dimensional (1D) channel with the consideration of entropy or information flow. In this work, the entropy flux was researched. The maximum entropy rate was obtained and is the same as that of Pendry’s result [15].

The 1D quantum channel was first put forward in [16] and applied to the calculation of conductance. In [16], the authors described the channel as follows: an ideal channel adiabatically connects two reservoirs. The reservoirs are the electronic equivalent of a radiative blackbody; the electrons coming out of a reservoir are occupied according to the Fermi distribution that characterizes the deep interior of that reservoir. If the channel is narrow enough, only the lowest of the transverse eigenstates in the channel has its energy below the Fermi level. Then the channel can be effectively seen as 1D. Subsequently, people introduced the channel to research on the fluxes of entropy and energy and others topics [17, 18, 19, 20].

Very recently, the energy flux and entropy flux of Hawking radiation of photons in a Schwarzschild spacetime has been seen as 1D Landauer transport process [21]. The energy flux of photons in a individual single-channel was obtained and is identical to the outgoing Hawking flux. Then the authors concluded that Hawking radiation process of photons can be described by a 1D quantum channel and the channel connects two thermal baths with one side being the black hole and another side being the outside thermal environment surrounding the black hole. In their work, the and entropy flux and net entropy production in \((1 + 1)\)-dimensions were also investigated in detail. The derivation of Hawking flux was dependent on the expectation value for the stress-energy tensor in the conformal structure. The expectation value with respect to the Unruh vacuum [5] was derived and the flux seen by an inertial observer at infinity distance was only related to the black hole’s temperature.

Our aim in this paper is to use the Landauer transport model to investigate Hawking radiation of bosons and fermions in Kerr and Kerr-Newman black holes. In the Kerr and Kerr-Newman black holes, Hawking fluxes contains the energy-momentum tensor flux, the angular momentum flux and the charge flux(for the charged particle in the Kerr-Newman spacetime). Meanwhile, due to the existence of dragging effect of coordinate system in the rotating spacetime, the matter field in the ergosphere near the horizon is dragged by the gravitational field with an azimuthal angular velocity, therefore the chemical potential should describes this dragging effect. Meanwhile, in the Kerr-Newman black hole due to the effect of electromagnetic field, the chemical potential should also contain electromagnetic
potential. The magnetic quantum number of the particle is regarded as a general charge in this paper. Both of Hawking radiation of the boson and the fermion are described by a 1D quantum channel in this paper. The thermal conductances of the Kerr and Kerr-Newman black holes are derived by the 1D channel description and related to the Hawking temperatures.

The rest of this paper is outlined as follows. In sect. 2, from the formulae of charge and energy fluxes in the 1D quantum channel, we get the expressions of charge flux and energy flux of a boson and a fermion. In sect. 3, Hawking radiation of the boson and the fermion in the Kerr black hole is obtained and described by the 1D channel. Extending this work to charged rotating black holes, we investigate the Hawking radiation of charged particles in a Kerr-Newman spacetime in sect. 4. Sect. 5 contains some discussions and conclusions.

2 Review the one-dimensional quantum channel

The 1D quantum channel was first put forward in [16]. Subsequently, Rego and Kirczenow gave the formulae of energy flux and charge flux in the channel [17]. One can refer to [16, 17, 18, 19, 20, 21] in detail. In this section, we review the 1D quantum channel. In a two terminal transport experiment two infinite reservoirs are adiabatically connected to each other by a 1D channel. $T$ and $\mu$, which are independent variables, denote temperatures and chemical potentials of the reservoirs, respectively. In case of a reservoir with charged particles, $\mu$ is the electrochemical potential energy and is the combination of chemical potential and electrostatic particle energy governed by the external field. In a signal channel, the currents of charge and energy flowing from the lift (L) and right (R) reservoirs are given by

\[ I_{R(L)} = \frac{q k_B T_{R(L)}}{2\pi \hbar} \int_{-\frac{\mu}{k_B T_{R(L)}}}^{\infty} f(x, g) \, dx, \]  

(2.2)

\[ \dot{E}_{R(L)} = \frac{(k_B T_{R(L)})^2}{2\pi \hbar} \int_{-\frac{\mu}{k_B T_{R(L)}}}^{\infty} f(x, g) \left( x + \frac{\mu}{k_B T_{R(L)}} \right) \, dx, \]  

(2.3)

with the fractional exclusion statistics $f(x, g)$. In this paper, we focus our attention on identical particles system. The distribution function was derived by Wu [22] and for an idea gas of particles it obeys the fractional exclusion statistics $f(x, g) = \left( \omega \frac{x-\mu}{k_B T} + g \right)^{-1}$. The relation between $x$ and $w$ is $\omega^g (1 + \omega)^{1-g} = e^x$, where $g$ denotes the statistical interaction and describes bosons for $g = 0$ and fermions for $g = 1$.

For a boson, the formulae of charge current and energy current are expressed as

\[ I_{R(L)} = \frac{q k_B T_{R(L)}}{2\pi \hbar} \int_{-\frac{\mu}{k_B T_{R(L)}}}^{\infty} \frac{1}{e^x - 1} \, dx, \]  

(2.4)
\[ \dot{E}_{R(L)} = \frac{(k_B T_{R(L)})^2}{2\pi \hbar} \int_{-\frac{1}{k_B T_{R(L)}}}^{\infty} \frac{1}{e^x - 1} \left( x + \frac{\mu}{k_B T_{R(L)}} \right) dx. \] 

Due to the particularity of the boson, it is difficult to get values of the above equations. However, photon is a special case and has no rest mass and chemical potential. So there is only the energy current for the photon, which is derived as

\[ \dot{E}_{R(L)} = \frac{\pi k_B^2 T_H^2}{12\hbar}. \]  

For a fermion, the formulae of charge current and energy current are written as

\[ I_{R(L)} = \frac{q k_B T_{R(L)}}{2\pi \hbar} \int_{-\frac{1}{k_B T_{R(L)}}}^{\infty} \frac{1}{e^x + 1} dx = \frac{q \mu_{R(L)}}{2\pi \hbar}, \] 

\[ \dot{E}_{R(L)} = \frac{(k_B T_{R(L)})^2}{2\pi \hbar} \int_{-\frac{1}{k_B T_{R(L)}}}^{\infty} \frac{1}{e^x + 1} \left( x + \frac{\mu}{k_B T_{R(L)}} \right) dx = \frac{\mu_{R(L)}^2}{4\pi \hbar} + \frac{\pi k_B^2 T_H^2}{12\hbar}. \]

Now we give some physical explication on the above equations. Due to the particularity of the boson system, it is difficult to derive the value of energy current and charge current. However, photons are a simple case, so the energy current is easily obtained. For the fermion, Eq. (2.6) shows that the charge current is related to the charge and the corresponding chemical potential of transmission particles. Eq. (2.7) tells us that the energy flux flowing through the 1D system contains two independent components: the first component is the flux of particles, which is determined by the chemical potential \( \mu \); the second component is only related to the temperature \( T \) of the reservoirs.

In the following, we use the 1D channel to describe Hawking radiation of bosons and fermions in the curved spacetimes.

### 3 Hawking radiation in the Kerr spacetime

The thermodynamic property of black holes is an important topic and attracts much attention. The reason is that the research on black holes concerns quantum theory and gravity theory, while the gravity theory of quantum has not been solved commendably now. Therefore it helpful to solve this theory by the research on black holes. In this section, we investigate Hawking radiation of a boson and a fermion in the Kerr background spacetime \[23\]. There are several derivations of Hawking radiation. One derivation is focused on the investigation of the Hawking fluxes, which can be obtained from the expectation value of energy-momentum tensor in the Unruh vacuum or from the cancellation of anomaly. In this paper, the Hawking flux is directly obtained from the radiation spectrum. The thermal spectrum in the Schwarzschild background spacetime was gotten in eq.(1.1) with the Hawking temperature \( T_H \). For the 4D Kerr spacetime, which is given by
With the outer (inner) horizons \( r_H = M + \sqrt{M^2 - a^2} \quad (r_- = M - \sqrt{M^2 - a^2}) \). We first investigate the Hawking fluxes of the boson. Due to the axisymmetrical property, the Hawking fluxes contains two parts: angular momentum flux and energy-momentum tensor flux. Using eqs. (3.10) and (3.11), we derive the fluxes of angular momentum and energy-momentum tensor as

\[
F_\alpha = m \int_0^\infty \frac{N^\pm_m(\omega)}{2\pi h} d\omega = \frac{m}{2\pi h} \int_0^\infty \frac{1}{e^{\omega/m_{BH}} - 1} d\omega = \frac{mk_B T_H}{2\pi h} \int_0^\infty \frac{1}{e^x - 1} dx, \tag{3.12}
\]

\[
F_M = \int_0^\infty \frac{N^\pm_m(\omega)}{2\pi h} \omega d\omega = \frac{1}{2\pi h} \int_0^\infty \frac{\omega}{e^{\omega/m_{BH}} - 1} d\omega = \left(\frac{k_B T_H}{\pi t_H}\right)^2 \int_0^\infty \frac{1}{e^x - 1} \left(x + \frac{m\Omega_H}{k_B T_H}\right) dx. \tag{3.13}
\]

The last equal signs in eqs. (3.12) and (3.13) are obtained with a transformation \( x = \frac{\omega - m\Omega_H}{k_B T_H} \). In \[21\], the Hawking fluxes in the scalar field were derived by cancelation of gravity anomaly and gauge anomaly. Here the expression of the Hawking fluxes are gotten, but it is difficult to get the value of the integral. The reason is the particularity of the bosonic system. However, it does not affect our final result. In the case of a photon, its rest mass and the magnetic quantum number are zero. Therefore the angular momentum flux (3.12) is zero and the energy-momentum tensor flux (3.13) is reduced to

\[
F_M = \frac{\pi k_B^2 T_H^2}{12h}. \tag{3.14}
\]

In the following, we investigate the Hawking fluxes of the fermion. In \[10\], the authors shown that the flux for thermal radiation of massless bosons is 2 times that of massless fermions. The same result was also found in \[25\]. Subsequently this phenomena is explained as that a single massless bosonic field is equivalent to the fermionic field with a massless particle plus its antiparticle \[26\]. Therefore the
Hawking fluxes of fermions should contain the contributions of fermions and anti-fermions. For the fermion, using eqs. (3.10) and (3.11), the total angular momentum flux and the energy-momentum tensor flux are gotten as

\[
F_a = m \int_0^\infty \frac{(N^+_m(\omega) + N^{-}_m(\omega))}{2\pi \hbar} d\omega = \frac{m}{2\pi \hbar} \int_0^\infty \left( \frac{1}{e^{\omega - m\Omega_H} + 1} + \frac{1}{e^{\omega + m\Omega_H} + 1} \right) d\omega = \frac{m^2 \Omega_H}{2\pi \hbar}, \tag{3.15}
\]

\[
F_M = \int_0^\infty \frac{(N^+_m(\omega) + N^{-}_m(\omega))}{2\pi \hbar} \omega d\omega = \frac{1}{2\pi \hbar} \int_0^\infty \left( \frac{1}{e^{\omega - m\Omega_H} + 1} + \frac{1}{e^{\omega + m\Omega_H} + 1} \right) \omega d\omega = \frac{m^2 \Omega_H^2}{4\pi \hbar} + \frac{\pi k_B^2 T_H^2}{12\hbar}. \tag{3.16}
\]

Hawking radiation of the photon in the Schwarzschild spacetime was described by the 1D quantum channel in [21]. In this section, we introduce this view to investigate Hawking radiation of the boson and the fermion. The related parameters are given as follows. The black hole temperature \(T_H\) was derived in eq. (3.11) and the thermal environment surrounding the black hole is seen as \(T_E = 0\) [21]. We know the chemical potential equals that the charge multiplies the corresponding potential. For the Kerr black hole, due to the dragging effect of the coordinate system, the matter field in the ergosphere near the horizon must be dragged by the gravitational field with an azimuthal angular velocity. Therefore the chemical potential of the particle in this spacetime should contains this effect. In this paper, the magnetic quantum number of the particle is regarded as a general/gauge charge. So its chemical potential equals that the magnetic quantum number multiplies the corresponding angular velocity, namely \(\mu = m\Omega_H\). While its chemical potential \(\mu_E\) is regarded as zero in the outside environment surrounding the black hole.

Now we return to the 1D channel. For the fermion, the total charge current and energy current between the two reservoirs obtained from eqs. (2.7) and (2.8) are

\[
F = I_R - I_L = \frac{q}{2\pi \hbar} (\mu_R - \mu_L), \tag{3.17}
\]

\[
\dot{E} = \dot{E}_R - \dot{E}_L = \frac{1}{4\pi \hbar} (\mu_R^2 - \mu_L^2) + \frac{\pi k_B^2}{12\hbar} (T_R^2 - T_L^2), \tag{3.18}
\]

which are respectively identical to the angular momentum flux (3.16) and the energy-momentum tensor flux (3.17) under the condition that \(T_R = T_H, \ T_L = T_E, \ \mu_R = \mu \) and \(\mu_L = \mu_E\). For the boson, we can also get that the total charge current and energy current obtained from (2.5) and (2.6) are respectively identical to the angular momentum flux (3.12) and the energy-momentum tensor flux (3.13) under the same condition. Photon is a sort of bosons. For a photon, both of the chemical potentials are zero in the black hole and the outside environment surrounding the black hole. The case of the photon has been contained in that of the boson, so we do not investigate it again. Our result shows the Hawking radiation of fermions and bosons in the Kerr spacetime can be described by a 1D quantum channel,
which is in consistence with that of Nation [21], meanwhile gives the proof of the view of Bekenstein and Mayo [14].

Considering an observer is in the dragging field, he/she must move with the dragging field. Then he/she can not feel the flow of angular momentum flux and there is only the flow of the energy-momentum flux for his/her feel. We can also prove the energy current is identical to the energy-momentum tensor flux. The Hawking fluxes in the dragging field have been studied by the cancellation of anomaly in [29]. In this paper, we do not investigate this case.

If the black hole is seen as 1D [14], we can use parameters of the 1D channel to describe that of the black hole. In [17], the 1D thermal conductance was derived as \( \kappa = \frac{\pi k^2 T}{6} \), which is only related to the temperature. Replacing the temperature \( T \) with the Hawking temperature \( T_H \), we get the thermal conductance of a single channel in the Kerr black hole. It should be emphasized here. In the original derivation of energy current and charge current [17], the transmission probability in the 1D channel was ordered to 1; meanwhile, the Hawking fluxes were derived without the consideration of the back reaction. One can investigate the fluxes of energy and charge and the Hawking fluxes with the consideration of the transmission probability and the back reaction at the same times.

4 Hawking radiation in the Kerr-Newman spacetime

For charged particles flowing in a 1D channel, there are not only energy flux but also charge flux. In this section, we investigate this case by Hawking radiation of a charged particle in the Kerr-Newman black hole [28]. We first focus our attention on the Hawking fluxes of the particle. Replacing \( \Delta \) in the metric (3.9) with \( \bar{\Delta} = r^2 - 2Mr + a^2 + Q^2 \), we get the Kerr-Newman metric, which describes a charged rotating spacetime with the electromagnetic potential

\[
A_\mu = A_t dt + A_\phi d\phi = \frac{Qr}{r^2 + a^2 \cos^2 \theta} dt - \frac{Qra \sin^2 \theta}{r^2 + a^2 \cos^2 \theta} d\phi.
\]

The angular velocity \( \Omega_H \) and the Hawking temperature \( T_H \) at the outer horizon can be easily derived and have the same expressions as eq. (3.11) with the different locations of the outer (inner) horizons

\[
r_H = M + \sqrt{M^2 - Q^2 - a^2}, \quad r_- = M - \sqrt{M^2 - Q^2 - a^2}.
\]

There are much work on the thermodynamic property of the Kerr-Newman black hole, and the radiation spectrum of a charged particle with energy \( \omega \), charge \( q \) and magnetic quantum number \( m \) in the Kerr-Newman spacetime was derived [30] as

\[
N_{q,m}^\pm (\omega) = \frac{1}{e^{\frac{\omega - ml_h + \Phi_H}{kT_H}} + 1},
\]

where \( \Phi_H = \frac{Qr_H}{r_H^2 + a^2} \) is the electromagnetic potential at the outer horizon. Due to the existence of the electromagnetic field and dragging field in the black hole, the Hawking fluxes of a charged particle are consisted of three parts: the charge flux, the angular momentum flux and the energy-momentum...
tensor flux. We first derive the Hawking fluxes of the boson. By using eq. (4.21), we can easily derive the fluxes of charge, angular momentum and energy-momentum tensor as

\[ F_q = q \int_0^\infty \frac{N_{q,m} (\omega)}{2 \pi \hbar} d\omega = \frac{q}{2 \pi \hbar} \int_0^\infty \frac{1}{e^{\frac{-q \Phi_H + \omega}{k_B T_H}} - 1} d\omega = \frac{q k_B T_H}{2 \pi \hbar} \int_0^\infty \frac{1}{e^x - 1} dx, \]  
(4.22)

\[ F_a = m \int_0^\infty \frac{N_{q,m} (\omega)}{2 \pi \hbar} d\omega = \frac{m}{2 \pi \hbar} \int_0^\infty \frac{1}{e^{\frac{-m \Omega_H - q \Phi_H}{k_B T_H}} - 1} d\omega = \frac{m k_B T_H}{2 \pi \hbar} \int_0^\infty \frac{1}{e^x - 1} dx, \]  
(4.23)

\[ F_M = \int_0^\infty \frac{N_{-m} (\omega)}{2 \pi \hbar} \omega d\omega = \frac{1}{2 \pi \hbar} \int_0^\infty \frac{\omega}{e^{\frac{-m \Omega_H - q \Phi_H}{k_B T_H}} - 1} d\omega = \frac{(k_B T_H)^2}{2 \pi \hbar} \int_0^\infty \frac{1}{e^x - 1} x \left( \frac{m \Omega_H + q \Phi_H}{k_B T_H} \right) dx. \]  
(4.24)

The last equal signs in eqs. (4.22)-(4.24) are obtained after we perform transformation \( x = \frac{\omega - m \Omega_H + q \Phi_H}{k_B T_H} \). As it was explained in section 3, it is difficult to get the integral value of the above equations because of the particularity of the bosonic system. However, it is very easy for a photon. For the photon, its rest mass is zero. Thus the Hawking flux only contains the energy-momentum tensor flux, which has the same expression as eq. (3.14) with the different Hawking temperatures.

For the charged fermion, the Hawking fluxes contain the contributions of fermions and anti-fermions \[26\]. So the charge flux, the angular momentum flux and the energy-momentum tensor flux are obtained as

\[ F_q = q \int_0^\infty \frac{d\omega}{2 \pi \hbar} (N_{q,m} (\omega) - N_{-q,-m} (\omega)) = \frac{q (q \Phi_H + m \Omega_H)}{2 \pi \hbar}, \]  
(4.25)

\[ F_a = m \int_0^\infty \frac{d\omega}{2 \pi \hbar} (N_{q,m} (\omega) - N_{-q,-m} (\omega)) = \frac{m (q \Phi_H + m \Omega_H)}{2 \pi \hbar}, \]  
(4.26)

\[ F_M = \int_0^\infty \frac{\omega d\omega}{2 \pi \hbar} (N_{q,m} (\omega) + N_{-q,-m} (\omega)) = \frac{(q \Phi_H + m \Omega_H)^2}{4 \pi \hbar} + \frac{\pi k_B^2 T_H^2}{12 \hbar}. \]  
(4.27)

Now we compare the Hawking fluxes with the charge current and energy current in the 1D channel. In the 1D channel mode, if there are several kinds of charge in the reservoirs, then the transmissions of every type of charge will produce the corresponding fluxes. For the charged fermion, thus the total charge currents and energy current are expressed as

\[ F_q = I_{iR} - I_{iL} = \frac{q_i}{2 \pi \hbar} (\mu_{iR} - \mu_{iL}), \]  
(4.28)
\[
\dot{E} = \dot{U}_R - \dot{U}_L = \frac{1}{4\pi\hbar}(\mu_{iR}^2 - \mu_{iL}^2) + \frac{\pi k_B^2}{12\hbar}(T_R^2 - T_L^2),
\]
(4.29)

where \( i = 1, 2, 3 \ldots \), \( q_i \) and \( \mu_i \) denote the different charges and the corresponding chemical potential, respectively.

When a charged particle is emitted in the Kerr-Newman black hole, its motion is affected by both of the electromagnetical field and the dragging field. So the chemical potential of the charged particle in the black hole should reflect the effects of the electromagnetical field and the dragging field. As it was explained in section 3, the magnetic quantum number is regarded as a general/gauge charge in this paper. Therefore the chemical potential is that the electromagnetical potential plus the general/gauge potential corresponded to the magnetic quantum number, namely \( \mu_i = q_i\Phi_H + m\Omega_H \).

The outside environment surrounding the black hole is seen as \( T_E = 0 \) [21] and the corresponding chemical potential of the charged particle in this environment is \( \mu_{iE} = 0 \). Let \( T_R = T_H, T_L = T_E \) and \( \mu_{iR} = \mu_i, \mu_{iL} = \mu_{iE} \), we find the total energy flux (4.29) is identical to the energy-momentum tensor flux (4.27). Meanwhile, when \( q_i \) respectively denotes the charge \( q \) and the magnetic quantum number \( m \), we can also find that the charge fluxes (4.28) are respectively identical to the charge flux (4.25) and the angular momentum flux (4.26). This shows the Hawking radiation of the charged fermion in the Kerr-Newman spacetime can be described by the 1D channel. For the charged boson, if let \( T_R = T_H, T_L = T_E \) and \( \mu_{iR} = \mu_i, \mu_{iL} = \mu_{iE} \), we can also get that the charge currents and energy current in the 1D channel obtained from eqs. (4) and (5) are identical to the charge flux (22), angular momentum flux (23) and energy-momentum tensor flux (24), respectively. This shows the Hawking radiation of the boson in the Kerr-Newman spacetime can be described by the 1D channel.

When an observer is in the dragging field, he/she moves with the dragging field and can not feel the flow of the angular momentum flux, which has been explained in section 3. In the Kerr-Newman spacetime, therefore, he/she only feels the flows of the charge flux and the energy-momentum flux. This case is not discussed in this paper.

If the black hole is seen as 1D [14], we can use parameters of the 1D channel to describe that of the black holes. In section 3, the thermal conductance of the 1D channel has been derived. We use it to describe that of the Kerr-Newman black hole. Thus the thermal conductance of this black hole is \( \kappa = \frac{\pi k_B^2 T_H}{6\hbar} \), which is only related to the Hawking temperature. Now we go on the investigation of the electric conductance. In [16, 17], the electric conductance was derived as \( G = \frac{q^2}{2\pi\hbar} \), which is only related to the charge of the particle.

5 Discussions and Conclusions

In this paper, we have investigated the fluxes of energy and charge in the 1D channel and Hawking radiation in the Kerr and Kerr-Newman black holes. It shown that Hawking radiation of the bosons and the fermions in the Kerr and Kerr-Newman black holes can be described by the 1D channel. Our result is in consistence with that of Nation [21] and gives the proof of the view of Bekenstein and Mayo [14]. In this paper, the magnetic quantum number of the emission particle was regarded as a general
charge, so the corresponding flux was obtained and identical to the charge flux in the 1D channel. In recent work [17, 18, 19, 21], there is some work on the energy flux and entropic flux of particles in the 1D channel. However, considering the particularity of the boson (except for photons), they have not derived values of the flux of energy and entropy. This was shown in the second section. In this paper, we also didn’t get the values, but this have not affected our final result.

If black holes can be seen as 1D, we can use the thermal conductance of the 1D to describe that of the Kerr and the Kerr-Newman black holes. They were derived by the 1D channel and only related to the Hawking temperatures. For the Kerr-Newman black hole, we got the electric conductance, which is related to the charge of the particle. In this paper, the transmission probability in the channel was ordered to 1, and the Hawking fluxes were derived without considering the back reaction. So it is meaningful to investigate the energy flux and the Hawking fluxes with consideration of the transmission probability and the back reaction at the same times.

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