Optimized localisation for gravitational-waves from merging binaries

Zhi-Qiang You\textsuperscript{1,2,3,6}, Gregory Ashton\textsuperscript{2,3,4} *, Xing-Jiang Zhu\textsuperscript{2,3,6}, Eric Thrane\textsuperscript{2,3}, and Zong-Hong Zhu\textsuperscript{1,5,6}

\textsuperscript{1}School of Physics and Technology, Wuhan University, Wuhan 430072, China
\textsuperscript{2}School of Physics and Astronomy, Monash University, Clayton, Vic 3800, Australia
\textsuperscript{3}OzGrav: The ARC Centre of Excellence for Gravitational Wave Discovery, Clayton, Vic 3800, Australia
\textsuperscript{4}Department of Physics, Royal Holloway, University of London, TW20 0EX, United Kingdom
\textsuperscript{5}Department of Astronomy, Beijing Normal University, Beijing 100875, China
\textsuperscript{6}Advanced Institute of Natural Sciences, Beijing Normal University at Zhuhai 519087, China

ABSTRACT

The Advanced LIGO and Virgo gravitational wave observatories have opened a new window with which to study the inspiral and mergers of binary compact objects. These observations are most powerful when coordinated with multi-messenger observations. This was underlined by the first observation of a binary neutron star merger GW170817, coincident with a short Gamma-ray burst, GRB170817A, and the identification of the host galaxy NGC 4993 from the optical counterpart AT 2017gfo. Finding the fast-fading optical counterpart critically depends on the rapid production of a sky-map based on LIGO/Virgo data. Currently, a rapid initial sky map is produced followed by a more accurate, high-latency, fast-fading optical counterpart critically depends on the rapid production of a sky-map based on LIGO/Virgo data. We find this yield up to a 60% reduction in the time required to produce the high-latency localisation for binary neutron star mergers.

Key words: Gravitational waves, Neutron stars, Bayesian inference

1 INTRODUCTION

Transient multi-messenger astronomy is delivering new insights into the nature of compact objects, the cosmological properties of our Universe, and general relativity. During their second observing run (O2), the the Advanced-LIGO (Aasi et al. 2015) and Virgo (Acernese et al. 2015) detectors observed gravitational-wave emission from a binary neutron star merger, GW170817 (Abbott et al. 2017a). This observation coincided with a short gamma-ray burst, GRB170817A (Goldstein et al. 2017) and follow-up campaigns across the electromagnetic spectrum (Abbott et al. 2017b) led to the observation of an optical counterpart AT2017gfo (Valenti et al. 2017; Yang et al. 2017; Perego et al. 2017) and identification of the host galaxy NGC 4993. This multi-messenger view of the event provided critical insights into multi-messenger astronomy, opening a new path by which to study and understand the mergers of neutron star binaries, short gamma-ray bursts, and their optical counterparts.

The LIGO and Virgo observatories, joined by KAGRA (Aso et al. 2013), have now completed their third observing run (Abbott et al. 2021a). During this observing run, open public alerts were issued (see emfollow.docs.ligo.org) enabling numerous follow-up campaigns (see, e.g., Antier et al. (2020); Ackley et al. (2020); Coughlin et al. (2019); Gompertz et al. (2020)). So far, there has yet to be a bonafide multi-messenger observation from the O3 observing run (however, see Graham et al. (2020); Ashton et al. (2020)) for discussion of a speculative connection between the binary black hole merger GW190521 (Abbott et al. 2020b) and an active galactic nucleus flare. In preparation for the fourth observing run (estimated to be June 2022), the detectors are currently being upgraded with a projected binary neutron star inspiral range of 190 Mpc (The LSC-Virgo-KAGRA Observational Science Working Groups 2020). During this observing run, the Advanced-LIGO, Virgo, and KAGRA (HLVK) network is likely to observe tens of transient systems containing a neutron star and hundreds of binary black hole systems. Binary neutron star (BNS) mergers are known to produce a multi-messenger counterparts (Abbott et al. 2017b) known as a “kilonova” (Li & Paczyński 1998). As yet, no unambiguous detection of a neutron-star black-hole (NSBH) binary has been made; but such systems may also produce an electromagnetic counterpart (Lattimer & Schramm 1974; Li & Paczyński 1998) (see also Fernández & Metzger (2016) for a review). As such, mergers containing a neutron star tend to be prioritised by follow-up campaigns, however, as demonstrated by Graham et al. (2020), the next big surprise may instead come from a binary black hole (BBH) merger.

Online gravitational-wave searches are able to identify events in the data and analyse their significance in a timescale of $O(s)$. Following the identification, the event is automatically vetted and published in a GCN notification in a timescale of $O(10 s)$. Optical telescopes then perform (often automated) searches for the rapidly-fading, timescales of $O(hrs)$, electromagnetic transient. The ability to identify the transient is highly dependent on the three-dimensional (3D) source-localisation [here, we parameterise in terms of the right ascension (RA), declination (DEC), and luminosity distance $d_L$]. In the first three observing runs, results from two methods for producing localisations were published. First, the low-latency BAYESTAR
(Singer & Price 2016; Singer et al. 2016) algorithm produces a 3D localisation in $O$(minutes) utilizing the maximum-likelihood template from the search pipelines. Second, the high-latency LALInference (Veitch et al. 2015) algorithm, which uses stochastic sampling to construct the full posterior probability distribution and produces results in a timescale of $O$(>12 hrs). While for the majority of systems, differences between the low-latency and high-latency localisation are anticipated to be small (Abbott et al. 2018a), the high-latency localisation is preferable since it can include an improved physical description of the signal and noise. Moreover, a systematic study (Morisaki & Raymond (2020)) comparing the BAYESTAR algorithm with a stochastic-sampling based approach (discussed below), demonstrates that BAYESTAR can over-estimate the localisation uncertainty when the best-fit parameters of the signal are outside the online-detection-pipeline template bank. This can happen, for example, if the online pipeline does not include the spin-precession of the source.

Constructing the full posterior probability distribution is a computationally challenging task. Stochastic sampling methods such as Markov-Chain Monte-Carlo (MCMC) (Metropolis et al. 1953; Hastings 1970) and Nested Sampling (Skilling 2006), require $\sim 10^6$–$10^8$ evaluations of the likelihood to analyse gravitational-wave signals (Christensen & Meyer 1998; Veitch & Vecchio 2008). Typically, each evaluation of the likelihood is dominated by the time required to model the source. This time varies between a few milliseconds to several tens of seconds depending on the signal duration and sophistication of the waveform model. As such, the wall time required to draw a sufficient number of samples to approximate the posterior can be between several hours, to many tens of days. A number of advances have been made in reducing the wall time:

(i) Reduced-order-quadrature (ROQ) methods (Antil et al. 2012; Canizares et al. 2013, 2015; Smith et al. 2016; Qi & Raymond 2021) interpolate the likelihood to high accuracy and can speed up evaluation times by factors of several hundred. While ROQ-based method have enjoyed considerable success (see, e.g., Abbott et al. 2020c), they do require that the ROQ basis be pre-constructed, often at significant computational cost. As such, their utility can be limited for online production of the 3D localisation if the pre-constructed basis set does not cover the required parameter space. Morisaki & Raymond (2020) recently demonstrated that so-called focused-ROQ (FROQ), in which many bases covering narrow ranges of the parameter space offer greater speed-ups still: with gains of up to $10^4$ seen for low-mass systems.

(ii) Heterodyned likelihoods (Cornish 2010, 2021), also known as the relative-binning method (Zackay et al. 2018; Finstad & Brown 2020), exploit the computation for likelihoods of similar waveforms, whose phases and amplitudes differ smoothly with frequency, by pre-computing frequency-binned overlaps of the best-fit waveform with the data. These methods do not require a pre-computation step and offer speed-ups of up to $\sim 10^4$ in likelihood evaluation times. The accuracy of these approaches depends on the expansion order: just a few terms are required to sufficiently approximate the likelihood. Demonstrations of this method are very promising, but work is still needed to verify the accuracy and limitations of the method against the full likelihood.

(iii) Then, there is brute-force parallelisation. Parallelisation can be done at the level of the likelihood itself (Talbot et al. 2019), the stochastic sampler, or using multiple independent stochastic samplers. The latter two aspects have been generously employed in standard inference packages (Veitch et al. 2015; Ashton et al. 2019; Biwer et al. 2019) using the few tens of cores available on typical central processing units, while Smith et al. (2020) demonstrated the capacity to scale to the many hundreds of cores available in high-performance computers using the dynesty (Speagle 2020) nested sampling algorithm. Such approaches are useful as they do not require pre-computation and make no requirements about the waveform itself. Unlike the other techniques to reduce wall time, this technique is not “free” (achieved through clever design)—it requires additional computing resources.

(iv) The RIFT family of stochastic samplers (Pankow et al. 2015; Lange et al. 2018) employs aspects of brute-force parallelisation (with extensions to graphical processing units (Wysocki et al. 2019)) alongside pre-computing aspects of the waveform in order to carry out inference with iterative fitting.

(v) Significant speed-ups may be realised by the use of machine-learning based approaches. Such approaches do not apply the standard principles of stochastic sampling; instead, the algorithm is pre-trained on example of signals in noise and can then produce posterior samples within a few seconds (Gabbard et al. 2019; Green et al. 2020; Green & Gair 2021). Such algorithms present a significant opportunity as they could handle non-Gaussian noise and arbitrarily complex waveforms by developing realistic training sets. For these approaches, the optimizations discussed herein are not directly applicable, in the sense that they do not consider a specific prior or waveform model during the analysis. However, the lessons learned can be applied in selecting the complexity of training data.

(vi) Finally, there are many other approaches to speeding up stochastic sampling such as the use of machine-learning coupled nested sampling (Williams et al. 2021), adaptive frequency-banding (Morisaki 2021), and representing the signals in the time-frequency domain (Cornish 2020).

In the fourth observing run, one (or many) of these approaches may be used to reduce the latency of full parameter-estimation results. However, the wall time they require (or the training time in the case of machine-learning based approaches) still depends on the choice of waveform model and the astrophysical prior. In this work, we study how to optimize the choice of model and prior to reduce the wall time. We will develop these ideas in the context of ROQ-methods and parallelisation, but the ideas apply equally to many of the methods listed above.

This paper is organized as follows. In Section 2, we describe the optimization of the prior then in Section 3, we validate which of these optimizations produces acceptably small bias and calculate the improvement in wall time. In Section 4, we discuss optimization of the waveform model. In Section 5, we conclude with a discussion on how these choices can be used during the next observing run to minimize the time to produce localisation.

2 Optimized choices for priors

A compact binary coalescence (CBC) signal is described by up to 10 intrinsic parameters and 7 extrinsic parameters. The intrinsic parameters are: chirp mass $M$, mass ratio $q$, component spin magnitude ($a_1$ for the primary heavier-mass object and $a_2$ for the secondary lighter-mass object), and four angles describing the spin orientation ($\theta_1, \theta_2, \phi_{12}$, and $\phi_{3L}$). For systems containing one or more neutron stars, the intrinsic parameters also include the neutron star tidal deformability $\Lambda_1$ and $\Lambda_2$. The extrinsic parameters are: the 3D localisation coordinates (RA, DEC, $d_L$), the polarization angle of the source $\psi$, the GPS reference time $t_{\text{geo}}$ of the merger, the angle between the total angular momentum and the line of sight $\theta_{\text{NN}}$, and the binary phase $\phi$ (defined at a fixed reference frequency). (See...
Table E1 of Romero-Shaw et al. (2020) for a detailed description of these parameter and alternative parameterisations. For stochastic samplers, the number of likelihood evaluations (and hence the wall time) depends on the complexity of the posterior. As a general rule of thumb, the number of parameters is a good leading-order description of the complexity: more parameters require more likelihood evaluations and hence a longer wall time. Optimizing the choice of prior can help minimize the wall time while ensuring the result remains unbiased.

First, we discuss different options for the spin prior. In an aligned-spin prior, the source model is restricted to solutions excluding precession (Schmidt et al. 2015). In a low-spin model, the magnitudes of the spin are restricted (typically to a dimensionless spin of 0.05 (Abbott et al. 2018b)). The upper end of the low-spin prior corresponds to a conservative limit on the effective spins of pulsars in known Galactic double neutron stars that are capable of merging within a Hubble time (Zhu & Ashton 2020). Further, one can neglect the effects of spin entirely. It was shown by Farr et al. (2016) that the 3D localisation for low-spin binary neutron star (BNS) sources is unbiased given a zero-spin prior. However, black-hole binaries can sustain significant dimensionless spin (see, e.g., Abbott et al. (2021c)).

Second, we examine the priors for tidal parameters. For BNS and NSBH binaries, the tidal parameter are relatively poorly measured (Abbott et al. 2018b, 2020c). A straightforward prior choice is to neglect tidal parameters (in effect, we set $\Delta_1 = 0$ and $\Delta_2 = 0$, i.e. that both components are black holes). The logic here is that, if we cannot accurately measure the tides, the tidal parameters probably do not have a strong affect on the sky localisation.

Third, we consider the prior for mass ratio. For the two confident BNS events in Gravitational-wave Transient Catalog 2 (GWTC-2; Abbott et al. (2021a)), the masses of the two components are nearly equal. Under the astrophysical low-spin mass prior, the mass ratio is constrained to be between 0.7 – 1 and 0.8 – 1 for GW170817 and GW190425 respectively (Abbott et al. 2018b, 2020c). This suggests another potential prior optimization: restricting to equal-mass systems $q = 1$.

We investigate these optimization strategies. In Table 1, we list four priors distributions. A precessing, but low-spin astrophysical prior, captures our broad expectation for the typical population parameters of neutron star binaries; it is from this prior that we draw simulation parameters. The remaining three priors combine increasing restrictions on the spin and mass-ratio. For all three choices, we assume zero tidal deformability: this prior optimization was already used during the third observing run for high-latency 3D localisation.

### 3 VALIDATION OF PRIOR OPTIMIZATION

The $a_i < 0.05$, $a_i = 0$, and $a_i = 0$, $q = 1$ priors defined in Table 1 increasingly constrain the astrophysical properties of the source. Almost certainly, these over-constrain the source properties. For example, GW190425 shows some support for spin effects and is unlikely to be equal mass while the data from GW170817 does constrain the tidal deformability. The question is, do these unphysical prior constraints bias the 3D sky localisation? To answer this question, we perform tests on simulated data and look at the localisation of detections during previous observing runs that have the potential for electromagnetic counterparts.

#### Table 1. Definition of the astrophysical, low-spin ($a_i < 0.05$), zero-spin ($a_i = 0$), and zero-spin and equal-mass ($a_i = 0$, $q = 1$) priors investigated in this work. We define $a_i$ and $\Lambda_i$ to be the absolute spin magnitude and tidal deformability of the $i$th component in the binary and $q \leq 1$ to be the mass ratio. For the astrophysical prior and low-spin prior, the spins are fully-precessing but restricted in magnitude. For all other parameters, the priors follow the standard distributions described in Table E1 of (Romero-Shaw et al. 2020).

| Parameter      | $a_i < 0.05$ | $a_i = 0$ | $a_i = 0$, $q = 1$ |
|----------------|-------------|-----------|---------------------|
| $a_1$          | [0, 0.05]   | [0, 0.05] | 0                   |
| $a_2$          | [0, 0.05]   | [0, 0.05] | 0                   |
| $q$            | [0.125, 1]  | [0.125, 1] | 1                   |
| $\Lambda_1$   | [5000]      | 0         | 0                   |
| $\Lambda_2$   | [5000]      | 0         | 0                   |

3.1 Parameter-parameter test

For our first test, we simulate 100 binary neutron stars using the IMRPhenomPv2_NRTidal waveform (Dietrich et al. 2019). The simulated signals have spin magnitudes, mass ratios, and tidal deformability parameters drawn from the astrophysical prior (see Table 1). For the remaining parameters we draw them from the standard astrophysical distributions.

The signals are simulated in 128 s of data using the project O4 sensitivity colored Gaussian noise (The LSC-Virgo-KAGRA Observational Science Working Groups 2020) for the HLV detector network. Signals with a network signal to noise ratio (SNR) less than 12 are discarded, with replacement; this ensures the injection set reflects the population of events from which we are likely to observe electromagnetic counterparts: high-SNR systems with good sky localisation. We analyse the simulated data sets using the IMRPhenomPv2 (Schmidt et al. 2012; Hannam et al. 2014; Khan et al. 2016) waveform model (which excludes tidal deformability), an ROQ-basis (Smith et al. 2016), and the dynesty nested sampling algorithm as implemented in bilby (Ashton et al. 2020). The ROQ basis is limited to chirp mass values of 1.42 to 2.60 solar masses; as such, we limit the prior distribution on chirp mass (and similarly the distribution from which we draw simulation parameters) to this range.

We find that all choices of prior specified in Section 2, the 3D localisation is unbiased. We test this using a PP, or parameter-parameter, test (Cook et al. 2006; Talts et al. 2018) over the 100 simulated signals. In Fig. 1, we show the results of the PP test for the most restrictive zero-spin equal-mass ($a_i = 0$, $q = 1$) prior. Qualitatively, bias manifests in a PP plot as a deviation in the parameter curve from the diagonal: for all parameters in Fig. 1, the curves remain inside the $3\sigma$ uncertainty region. One way to quantify the bias from a PP plot is to calculate the $p$-value expressing the probability that the fraction of events in a particular confidence interval is drawn from a uniform distribution. We calculate the $p$-value using the Kolmogorov-Smirnov test as implemented in scipy (Jones et al. 2001). For the most restrictive $a_i = 0$, $q = 1$ prior the $p$-values are 0.122, 0.551, and 0.710 for DEC, $d_L$, and RA, respectively. That none of these $p$-values are small $\ll 0.05$ indicates that there is no measurable sign of bias.

As a further way to understand the bias, in Table 2, we calculate the fraction of events outside of the 90% and 68% credible intervals for each prior. For an unbiased inference, we would expect the fraction...
out of the 90% and 68% credible intervals to be 10% and 32% respectively. The difference then provides a quantitative estimate of the bias. For the $a_i < 0.05$ prior, this is up to 2% while for the $a_i = 0$, $q=1$ prior, it is up to 11%.

In Appendix A, we show additional two PP plots for the $a_i = 0$, $q=1$ prior for a two-detector network (Fig. A1) and for 500 injections (Fig. A2). Furthermore, in Table A1, we list the p-values and fractions of truth parameters that are out of the 90% credible intervals, for 50, 100, 300 and 500 injections. These illustrate that our results are robust against the choice of detector network configuration and the number of injections.

To compare the wall time between the $a_i < 0.05$ and $a_i = 0$, $q=1$ model, in Fig. 2, we plot the optimal SNR of the simulated signals and the sampling time. As expected for a nested sampling algorithm, the sampling time is correlated with the signal SNR. (The sampling time is generally proportional to the ratio of prior to posterior volume (Speagle 2020); at a fixed prior volume, the posterior volume decreases as the SNR increases leading to longer wall times). The mean sampling time for the $a_i < 0.05$ and $a_i = 0$, $q=1$ priors are ~ 4.5 hours and 3.5 respectively. This demonstrates that, in addition to the low-spin savings already identified by Farr et al. (2016), utilising an equal-mass prior can provide a further ~ 20% performance improvement.

### 3.2 Implications for BNS and NSBH localisation

To date, the LIGO and Virgo detectors have observed two BNS events, GW170817 (Abbott et al. 2017a) and GW190425 (Abbott et al. 2020c), and two NSBH events, GW200105 and GW200115 (Abbott et al. 2021d). These systems are likely to be accompanied by a rapidly fading electromagnetic counterpart and are hence prioritised for follow-up by optical telescopes (Fernández & Metzger 2016). As such, we have the most to gain in optimizing the production of the high-latency 3D localisation.

To study our optimized priors for BNS signals, we re-analyse the public data (Abbott et al. 2021b) for the BNS events GW170817 and GW190425 using the $a_i = 0$, $q=1$ prior and the IMRPhenomPv2 waveform model. In Fig. 3 and Fig. 4, we plot the sky localisation uncertainty from our reanalysis, the O3 low-latency (BAYESTAR), and the O3 high-latency (LALInference) results for GW170817 and GW190425, respectively.

High-latency results better-constrain the sky-localisation: this can be seen in Table 3 where we report the area coverage and was demonstrated systematically by Morisaki & Raymond (2020). Table 3 demonstrates that the $a_i = 0$, $q=1$ prior provides an equivalent constraint on the sky area to the O3 high-latency results for BNS systems. We conclude then that for BNS, any method attempting to reduce the wall time of high-latency localisation results (e.g. the FROQ method Morisaki & Raymond (2020)), could further gain from an optimized choice of prior.

The obvious concern in using an optimized prior with zero-spin and equal-mass is the induced bias if the detectors observe an event which is highly spinning or of unequal mass ratio. We have controlled

### Table 2. The fraction of the truth value out of 90%, 68% credible interval for $a_i < 0.05$ and $a_i = 0$, $q=1$ priors for the 100 simulated BNS discussed in Section 3.1.

| Model           | Fraction     | DEC (%) | RA (%) | $d_L$ (Mpc) |
|-----------------|--------------|---------|--------|-------------|
| $a_i < 0.05$    | out-of-90%   | 9       | 12     | 10          |
|                 | out-of-68%   | 32      | 33     | 31          |
| $a_i = 0$, $q=1$| out-of-90%   | 13      | 12     | 9           |
|                 | out-of-68%   | 37      | 35     | 21          |
for this, to some extent, in our PP tests (Section 3.1) by allowing our simulated signals to be drawn from the “astrophysical prior” (cf. Table 1). In particular, we consider mass ratios down to 1/8. In comparison, the mass ratios for two recent NSBH events are around 1/5 (Abbott et al. 2021d). The spin measurements are dominated by the contribution from the more massive BHs: whereas the dimensionless BH spin magnitude $a_i$ is constrained to less than 0.23 in GW200105 at the 90% credible level, $a_1$ could be as high as 0.5 (though with a large uncertainty and still being consistent with zero) for GW200115 (Abbott et al. 2021d).

This suggests the $a_i=0, q=1$ prior may be appropriate for NSBH systems, provided the mass ratio is not too extreme and that the black hole does not have significant spin. Nevertheless, a real signal which falls outside of our astrophysical prior may be one of the most exciting opportunities for electromagnetic follow-up. As such, while proposing the use of the $a_i=0, q=1$ prior, we also recommend parallel computations be made with a broad non-informative prior. In most instances this will lead to a small amount of wasted computation as the localisation will be essentially identical. But, this provides a safeguard against “over-optimization.”

### 3.3 Implications for BBH localisation

Having studied prior-optimization for the localisation of BNS and NSBH events, we now turn to BBH events. BBH systems have significant spin (Abbott et al. 2021c) and have been observed with highly asymmetric masses (Abbott et al. 2020d,h). This suggests the optimized priors may perform poorly. To test this, we apply the $a_i=0$ prior to the public data (Abbott et al. 2021b) for the GW190814 (Abbott et al. 2020d) and GW190412 (Abbott et al. 2020a) events. We report the credible intervals in Table 3. For both events, we find that the $a_i=0$ prior constrains the posterior to a region almost as large as the O3 low-latency results.

In Fig. 5 and Fig. 6, we show the sky-localisation for the $a_i=0$ model for GW190412 and GW190814 and compare these to the high- and low-latency results. Here we see in detail that the $a_i=0$ prior performs about as well as the low-latency localisation (finding an extra mode not present in the high-latency result, and overall a much broader area). From this we conclude that optimizing the prior for black hole systems (which can exhibit significant spin and mass asymmetry) does not perform as well as a high-latency analysis with a complete prior specification.

### 4 OPTIMIZING THE CHOICE OF WAVEFORM MODEL

In Section 3, we demonstrated that for BNS and NSBH signals with small spins and moderate mass ratio’s, the sky localisation is unbiased by a $a_i=0, q=1$ prior. We analysed the data using the IMRPhenomPv2 waveform approximant model which models a fully precessing binary black hole merger. (Waveform approximants allows the generation of a predicted signal to within a few to tens of milliseconds. Their computation time is typically determined by their level of sophistication: approximants which better-model the underlying physics typically are slower to generate). In this section, we aim to investigate if simpler waveform models can be used in place of more physically plausible models for localisation.

In Table 4, we provide a breakdown of the timing of the IMRPhenomPv2 waveform approximant, the IMRPhenomPv2_NRTidal (used to simulate signals in Sec-
Table 3. The 68% credible intervals for the luminosity distance ($d_L$) and sky position from our optimized priors, the publicly available O3 low-latency software (BAYESTAR), and O3 high-latency software (LALInference). Below the event name, we give the credible source classification. For BNS events, we use the optimized $a_i=0, q=1$ prior while we use the $a_i=0$ for events containing a black hole. We note that some of the reduction in the localization of between the low-latency and high-latency results for GW190817 arises from recalibration of the data from the Virgo interferometer (Abbott et al. 2019).

Table 4. Per-likelihood and per-waveform evaluation times. The per-likelihood captures the full cost of computing the likelihood in bilby for simulated data lasting 128s. The difference between the per-likelihood and per-waveform values (which amounts to the bilby data processing overhead) is ~ 40 ms. For the TaylorF2 waveform, the data processing overhead is the dominant contribution to the per-likelihood evaluation. These timing apply only to the standard configuration (without any analytical marginalization). They can be drastically reduced by ROQ and hetrodynem methods as discussed in Section 1. All timings are bench-marked on an Intel Core i7-7820HQ CPU @ 2.90GHz.

Figure 6. Sky location uncertainty for GW190814 (Abbott et al. 2020d). We show posteriors from the $a_i=0$ prior in green, the O3 low-latency BAYESTAR analysis in blue, and the O3 high-latency LALInference analysis in orange. The solid (dashed) line represents the 90% (50%) credible region.

Figure 7. 2D sky localisation of a simulated BNS system comparing the IMRPhenomPv2 and TaylorF2 waveform models with identical $a_i=0, q=1$ priors. The red “cross” indicates the simulated signal value.

tion 3.1), and the non-spinning TaylorF2 (Buonanno et al. 2009) waveform which models only the inspiral of non-spinning point particles. The TaylorF2 is not as physically accurate as the IMRPhenomPv2 waveforms which include the inspiral, merger, and ring-down (and tidal effects in the case of IMRPhenomPv2_NRTidal) of precessing binary systems. This lack of physics translates into a likelihood which can be evaluated ~ 30% faster than the IMRPhenomPv2 waveform. If we are using a non-spinning and equal mass prior, then only remaining difference between TaylorF2 and IMRPhenomPv2 is the physical modelling of the merger and ring-down. However, the localisation is predominantly determined by triangulation from the inspiral signal. This suggests a further optimization: use the TaylorF2 waveform.

To demonstrate that such a waveform optimization does not bias the result, we first look at a fiducial simulated signal. We simulate a spinning BNS (simulation parameters: $q = 0.7, a_1 = 0.04, a_2 = 0.01, A_1 = 1500, A_2 = 750$, and $d_L=150$ Mpc) in 128 s of a two-detector network (HL) assuming O4 design-sensitivity noise. The signal has an a simulated network optimal SNR of ~ 18. We analyse the signal using IMRPhenomPv2 and TaylorF2 waveforms under a $a_i=0, q=1$ prior and plot the 2D sky localisation in Fig. 7. This figure demonstrates that we do not see systematic difference between the two waveforms, despite their differing physical assumptions. (The inferred luminosity distances show a similar level of agreement).

Both the IMRPhenomPv2 and TaylorF2 analyses require a similar number of likelihood evaluation (~ 30 × 10^9), but the TaylorF2 run had an overall wall time 30% less than that of the IMRPhenomPv2 analysis. This confirms that the reduction in per-likelihood evaluations demonstrated in Table 4 translate directly into wall time reductions. For reference, we also analyse the data with the IMRPhenomPv2_NRTidal waveform and a full prior (i.e. using the full range of spins and tidal parameters). Comparing the wall times, the $a_i=0, q=1$TaylorF2 analysis is 60% faster.

Finally, we verify that using the TaylorF2 does not introduce a bias. We repeat the PP-test introduced in Fig. 1, but use the TaylorF2 waveform approximant to analyse the data. We report the results in
be applied to both heterodyning and brute-force parallelisation. Factors of a few hundred. Prior and waveform optimization can reduce the number of required likelihood evaluations and hence can be assured that for systems with significant spins and highly asymmetric masses an updated localisation can be produced.

Fig. 8. Again, the 3D localisation remains inside the 3σ uncertainty region. The individual p-value for DEC, , and RA, are 0.227, 0.592, and 0.203 respectively.

5 SUMMARY

We investigate optimization choices for the Bayesian prior and signal model used to produce sky localisation’s from ground-based gravitational-wave observatories. The rapid production of these sky localisation’s is critical to aid in the search for possible counterparts. For BNS systems (where we expect small spins and near equal mass), we demonstrate that a restrictive prior with zero spin and equal mass can reduce the number of required likelihood evaluations and hence wall time by ∼40%. At the same time, we demonstrate that this optimized prior is unbiased, provided the signal does not have extreme spins or mass ratios. We also demonstrate that using physically simpler waveform models provide equivalent sky-maps with up to a ∼30% reduction in wall time. Taken together, for BNS and moderate mass ratio NSBH systems optimized choices of the prior and waveform can reduce the wall time by up to 60%. This efficiency saving can be directly applied to the current stochastic sampling methods used in high-latency (e.g. by implementing the optimized prior in the LALInference or bilby samplers). For BBH systems, we demonstrate that the same prior optimizations do not apply: zero spin and equal mass assumptions produce poorer sky localisation.

We considered these optimization choices in the context of standard stochastic sampling. As discussed in Section 1, a number of new ideas are being developed which offer wall time speed-ups up to factors of a few hundred. Prior and waveform optimization can be applied to both heterodyning and brute-force parallelisation. For ROQ-based methods, a speed-up is achieved by reducing the size of the basis (since it no longer needs to model unequal mass or spinning BNS), resulting in a speed-up of the basis itself. For machine-learning based approaches, the optimizations described herein can be used to simplify the training set used at the learning stage. Before the next observing run, we advocate for comparative head-to-head mock data challenges to ascertain which method is the fastest and most robust. This would include extending the studies of the optimized prior choices herein.

6 ACKNOWLEDGMENTS

We thank Soichiro Morisaki for useful feedback during the development of this work. In analysing GW190425, we made use of the ROQ basis provided by Baylor et al. (2019). This work was supported by the Australian Research Council (ARC) Future Fellowship FT150100281 and Centre of Excellence CE170100004 and also the National Natural Science Foundation of China under Grants Nos. 11633001 and 11920101003. This work was performed on the OzSTAR national facility at Swinburne University of Technology. The OzSTAR program receives funding in part from the Astronomy National Collaborative Research Infrastructure Strategy (NCRIS) allocation provided by the Australian Government. This material is based upon work supported by NSF’s LIGO Laboratory which is a major facility fully funded by the National Science Foundation.

7 DATA AVAILABILITY

This research has made use of data, software and/or web tools obtained from the Gravitational Wave Open Science Center (https://www.gw-openscience.org/), a service of LIGO Laboratory, the LIGO Scientific Collaboration and the Virgo Collaboration. LIGO Laboratory and Advanced LIGO are funded by the United States National Science Foundation (NSF) as well as the Science and Technology Facilities Council (STFC) of the United Kingdom, the Max-Planck-Society (MPS), and the State of Niedersachsen/Germany for support of the construction of Advanced LIGO and construction and operation of the GEO600 detector. Additional support for Advanced LIGO was provided by the Australian Research Council. Virgo is funded, through the European Gravitational Observatory (EGO), by the French Centre National de Recherche Scientifique (CNRS), the Italian Istituto Nazionale di Fisica Nucleare (INFN) and the Dutch Nikhef, with contributions by institutions from Belgium, Germany, Greece, Hungary, Ireland, Japan, Monaco, Poland, Portugal, Spain.
REFERENCES

Aasi J., Abbott B. P., Abbott R., et al. 2015, Classical and Quantum Gravity, 32, 074001
Abbott B. P., Abbott R., Abbott T. D., et al., 2017a, Phys. Rev. Lett., 119, 161101
Abbott B. P., Abbott R., Abbott T. D., et al., 2017b, ApJ, 848, L12
Abbott B. P., Abbott R., Abbott T. D., et al. 2018a, Living Reviews in Relativity, 21, 3
Abbott B. P., Abbott R., Abbott T. D., et al., 2018b, Phys. Rev. Lett., 121, 161101
Abbott B. P., Abbott R., Abbott T. D., et al., 2019, Phys. Rev. X, 9, 011001
Abbott R., Abbott T. D., Abraham S., et al., 2020a, Phys. Rev. D, 102, 043015
Abbott R., Abbott T. D., Abraham S., et. al. 2020b, Phys. Rev. Lett., 125, 101102
Abbott B. P., Abbott R., Abbott T. D., et al., 2020c, ApJ, 892, L3
Abbott R., Abbott T. D., Abraham S., et al. 2020d, ApJ, 896, L44
Abbott R., Abbott T. D., Abraham S., et. al. 2021a, Physical Review X, 11, 021053
Abbott R., Abbott T. D., Abraham S., et. al. 2021b, SoftwareX, 13, 100658
Abbott R., Abbott T. D., Abraham S., et al. 2021c, ApJ, 913, L7
Abbott R., Abbott T. D., Abraham S., et al. 2021d, ApJ, 915, L5
Acerneau F., Agathos M., Agatsuma K., et al. 2015, Classical and Quantum Gravity, 32, 024001
Achley K., et al., 2020, A&A, 643, A113
Antier S., et al., 2020, MNRAS, 497, 5518
Antil H., Field S. E., Herrmann F., Nochetto R. H., Tiglio M., 2012, arXiv e-prints, p. arXiv:1210.0577
Ashton G., et al., 2019, ApJS, 241, 27
Ashton G., Ackley K., Magaña Hernandez I., Piotrzkowski B., 2020, arXiv e-prints, p. arXiv:2009.12346
Aso Y., et al., 2013, Phys. Rev. D, 88, 043007
Baylor A., Smith R., Chase E., 2019, https://doi.org/10.5281/zeno.3478659
Biever C. M., Capano C. D., De S., Cabero M., Brown D. A., Nitz A. H., Raymond V., 2019, PASP, 131, 024503
Buonanno A., Iyer B. R., Ochsner E., Pan Y., Sathyaprakash B. S., 2009, Phys. Rev. D, 80, 084043
Cahillane C., et al., 2017, Phys. Rev. D, 96, 102001
Canizares P., Field S. E., Gair J. R., Tiglio M., 2013, Phys. Rev. D, 87, 124005
Canizares P., Field S. E., Gair J., Raymond V., Smith R., Tiglio M., 2015, Phys. Rev. Lett., 114, 071104
Christensen N., Meyer R., 1998, Phys. Rev. D, 58, 082001
Cook S. R., Gelman A., Rubin D. B., 2006, Journal of Computational and Graphical Statistics, 15, 675
Cornish N. J., 2010, arXiv e-prints, p. arXiv:1007.4820
Cornish N. J., 2020, Phys. Rev. D, 102, 124038
Cornish N. J., 2021, Phys. Rev. D, 103, 104057
Coughlin M. W., et al., 2019, ApJ, 885, L19
Dietrich T., Samajdar A., Khan S., Husa S., Hannam M., Ohme F., Purrer M., Forteza X. J., Bohé A., 2016, Phys. Rev. D, 93, 044007
Dietrich T., Samajdar A., Khan S., Husa S., Hannam M., Ohme F., Purrer M., Forteza X. J., Bohé A., 2016, Phys. Rev. D, 93, 044007
Farr B., et al., 2016, ApJ, 825, 116
Fernández R., Metzger B. D., 2016, Annual Review of Nuclear and Particle Science, 66, 23
Finstad D., Brown D. A., 2020, ApJ, 905, L9
Gabbard H., Messenger C., Heng I. S., Tonolini F., Murray-Smith R., 2019, arXiv e-prints, p. arXiv:1909.06296
Goldstein A., et al., 2017, ApJ, 848, L14
Gompertz B. P., et al., 2020, MNRAS, 497, 726
Graham M. J., et al., 2020, Phys. Rev. Lett., 124, 251102
Green S. R., Gair J., 2021, Machine Learning: Science and Technology, 2, 03LT01
Green S. R., Simpson C., Gair J., 2020, Phys. Rev. D, 102, 104057
Hannam M., Schmidt P., Bohé A., Hageman L., Husa S., Ohme F., Pratten G., Purrer M., 2014, Phys. Rev. Lett., 113, 151101
Hastings W. K., 1970, Biometrika, 57, 97
Jones E., et al., 2001, SciPy: Open source scientific tools for Python, http://www.scipy.org/

Table A1. The p-values and the fraction of the truth value out of 90 credible interval for DEC, $d_L$, RA with the 50, 100, 300, 500 simulated BNS injections.

| N  | DEC | $d_L$ | RA | out-of-90% |
|----|-----|------|----|----------|
| 50 | 0.533 | 0.493 | 0.616 | 18% 8% 12% |
| 100 | 0.122 | 0.551 | 0.710 | 13% 12% 9% |
| 300 | 0.120 | 0.527 | 0.696 | 12% 10% 11% |
| 500 | 0.168 | 0.678 | 0.603 | 13% 9% 11% |

APPENDIX A: ADDITIONAL DETAILS ON THE PP TEST

Here, we present additional details of the PP test we perform for our optimised choice of priors. In Fig. A1, we show the PP plot for the $a_1=0, q=1$ prior for a two-detector (H-L) network. In Fig. A2, we show the PP plot for the $a_1=0, q=1$ prior for the HLV detector network using 500 injections. Furthermore, we list the p-values and the fractions of truth parameter values being out of the 90% credible intervals for the $a_1=0, q=1$ prior, for 50, 100, 300, and 500 injections.
Figure A1. A parameter-parameter (PP) test of the 3D sky localisation parameters of 100 simulated BNS signal using the $a_i=0, q=1$ prior with a two-detector (H-L) network.

Figure A2. As Fig. 1 but for 500 injections.