Research and verification of similar laws in valveless piezoelectric micropumps

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Abstract
Due to the constraints of actual conditions, large-scale research and analysis of micropumps are difficult to achieve. In this paper, the micropump is divided into a vibrating part and a flowing part through a similar theory research method, and the two parts of the mathematical model are simplified and dimensionless-processed. Then, the similar relationship between the prototype machine and the model machine is proposed, the conversion relationship between the prototype machine and the model machine is obtained. In order to verify the correctness of the theoretical analysis, this paper uses ANSYS CFX analysis software to perform numerical analysis on a pair of prototype machines and model machines when the angle of diffusion and contraction is 30°. The size ratio of the prototype and model is 1/2. At the same time, experiments were used to verify the accuracy of numerical analysis by comparing the experimental value and the simulated value of the micropump outlet flow when the frequency is 100 Hz and the voltages are 25 Vpp, 50 Vpp, 75 Vpp, 100 Vpp, and 125 Vpp. The results obtained by numerical simulation and experimental methods are consistent with theoretical analysis, thus verifying the accuracy of similar theories. The research results can provide the theoretical basis for the experimental research of the piezoelectric pump and also provide guidance for the design of the piezoelectric pump.

1 | INTRODUCTION
As the key microfluid components of the microfluid controlling system, various types of micropumps have been developed in recent 30 years [1]. The valveless piezoelectric micropump is a kind of vibrating diaphragm pumps, which conveys the working fluids based on the converse piezoelectric effect of piezoelectric material [2]. It has been found to have many advantages, such as small size, low power consumption, no electromagnetic interference and insensitivity of fluid viscosity, ionic strength, acidity and basicity [3–6]. With the development of MEMS technology, the valveless piezoelectric micropumps are widely applied in numerous fields including drug detection, biomedicine, μ-TAS (micro-total analysis system) and microelectronic chip cooling systems [7–10].

The study of the valveless piezoelectric micropump began in the 1990s. In 1993, Stemme of the university of Sweden and others first proposed a valve-less piezoelectric micropump based on the Coanda effect in 1993. [11] In 1995, Forster from the University of Washington and others developed a piezoelectric micropump based on a Tesla-tube that improved the output characteristics of the micropump [12]. In recent years, new structures of piezoelectric micropumps have emerged endlessly. For example, Zhao Da et al. designed a valve-less piezoelectric micropump with the crescent-shaped structure in 2019. [13] Bian R.Q. et al. proposed a novel valveless piezoelectric micropump with a blunt-body based on the Coanda effect in 2019. [14] In addition to using the test method to study the performance of the micropump, simulation experiments with models are also a convenient, effective and low-cost method. Some scholars have studied the part of the piezoelectric vibrator by means of mathematical modelling. They regard the vibrator as a round sheet in which the load is concentrated on the centre [15–17]. In 2003, Bu proposed a bilayer piezoelectric oscillator displacement model with constant excitation voltage [18], and experience shows that this model is more consistent with the actual oscillator displacement [19]. The mathematical analysis method can directly analyse the movement of the micropump, but it is...
difficult to describe the flow state in micropump because of the complex conditions inside the micropump. However, similarity analysis is a decent method to study fluid flow conditions under time domain response. [20–22] With the wide application of micropumps in the fields of chemistry, medicine, biology, and microelectronics, micropumps of different sizes are required to provide sufficient power in a variety of microfluidic systems. Similarity analysis of micropumps makes it easier to design micropumps of different sizes. This paper will explore the similarity of valveless piezoelectric micropumps and use numerical simulation and experimental methods to study. By studying this similarity, a simple and convenient method is provided: when studying micropumps with the same microchannel structure, a model can be established for simulation through this similarity theory, thereby saving manufacturing costs and experimental resources.

This paper is divided into three parts: First, through the dimensionless treatment of the Navier-Stokes equations and the oscillator equation, the conditions for the similarity of the flow field are obtained and the conversion method of the prototype machine and the model machine is given. Second, experiment with prototype machine to verify the feasibility of simulation method to study. By studying this similarity, a simple and convenient method is provided: when studying micropumps with the same microchannel structure, a model can be established for simulation through this similarity theory, thereby saving manufacturing costs and experimental resources.

2 | SIMILARITY ANALYSIS

The flow chart in Figure 2 shows the reasoning process of the similarity analysis of the micropump. The specific analysis is divided into the following parts:

2.1 | Geometric similarity

The similarity of the geometric part of the piezoelectric pump can be divided into a fixed geometric part and a vibrating part. For the fixed geometric part, it just need to verify that the length is similar. For the vibrator portion, each point in the vibrating portion is required to be similar at similar time nodes.

a. For fixed geometry, the size ratio of the prototype to the model should follow:

\[
\frac{D_p}{D_m} = \lambda_1
\]

where \(D_p\) is the diameter of the prototype pump cavity and \(D_m\) is the diameter of the model pump cavity, the ratio of \(D_p\) to \(D_m\) is written as \(\lambda_1\).
a. According to the two-layer vibrator model proposed by Bu [18], in Figure 1, the displacement of the vibrator can be obtained by satisfying the following equation:

\[
\omega_1 (r) = \omega_{\text{max}} \left[ 1 + \frac{(b^2 - a^2) r^2}{2a^2 b^2 \ln(a/b)} \right] (0 \leq r \leq a) \tag{2}
\]

\[
\omega_2 (r) = \omega_{\text{max}} \left[ \frac{b^2 + 2b^2 \ln(r/b) - r^2}{2b^2 \ln(a/b)} \right] (a \leq r \leq b) \tag{3}
\]

where \(\omega_{\text{max}}\) is the maximum displacement of the center point of the vibrator; \(r\) is the radial length of the point on the vibrator; \(a\) is the radius of the piezoelectric ceramic, and \(b\) is the radius of the elastic substrate. \(\omega_1\) represents the displacement of the double-layer circular plate in the center of the vibrator; \(\omega_2\) represents the displacement of the single-layer hollow circular plate around the vibrator.

In the time-dependent single-valued condition, since the volumetric deformation is the most influential to the piezoelectric pump, the rate of volume change of the vibrator is considered. According to Equations (2) and (3),

\[
\frac{dV_c(t)}{dt} = \omega_{\text{max}} \frac{\pi^2 f (a^2 - b^2)}{\ln(a/b)} \cos \left( 2\pi \frac{r}{b} \right) \tag{4}
\]

where \(f\) is the vibration frequency in Hz; \(t\) is the time in s.

Take the pump cavity diameter \(D\) as the length feature, and \(f\) is the time characteristic, then, take the following dimensionless numbers:

\[
\omega^* = \frac{\omega}{D}, \quad b^* = \frac{b}{D}, \quad a^* = \frac{a}{D}, \quad r^* = \frac{r}{D}, \quad t^* = \frac{t}{f}; \quad V_c^* = \frac{V_c}{D^3} \tag{5}
\]

Substituting the dimensionless numbers of Equation (5) into Equations (2), (3), and (4), the dimensionless equation for vibratory motion is:

\[
\omega_{1}^* (r) = \omega_{\text{max}} \left[ 1 + \frac{(b^2 - a^2) r^2}{2a^2 b^2 \ln(a/b)} \right] (0 \leq r \leq a) \tag{6}
\]

\[
\omega_{2}^* (r) = \omega_{\text{max}} \left[ \frac{b^2 + 2b^2 \ln(r/b) - r^2}{2b^2 \ln(a/b)} \right] (a \leq r \leq b) \tag{7}
\]

\[
\frac{dV_c^* (t)}{dr^*} = \frac{\omega_{\text{max}}}{D} \frac{\pi^2 f (a^2 - b^2)}{\ln(a/b)} \cos \left( 2\pi \frac{r}{b} \right) \tag{8}
\]

Therefore, as long as the dimensionless constants \(\omega_{\text{max}}^* = \frac{\omega_{\text{max}}}{D} \) [23],

\[
\frac{\omega_{\text{max}}_p}{D_p} = \frac{\omega_{\text{max}}_m}{D_m} = \lambda_1 \tag{9}
\]

In summary, through the dimensionless processing of the geometric part (vibration part and fixed part) of the valveless piezoelectric pump, the similarity equation of the geometric part is obtained. It is expressed as:

\[
\frac{\omega_{\text{max}}_p}{\omega_{\text{max}}_m} = \frac{D_p}{D_m} = \lambda_1 \tag{10}
\]

### 2.2 Time similarity

The performance of a piezoelectric pump is mainly expressed by measuring the flow rate of the piezoelectric pump in a period. Therefore, the similarity in the time of the piezoelectric pump is mainly considered as a periodic issue. Taking a period as a characteristic time, a similar criterion is obtained:

\[
\frac{t_p}{t_m} = \frac{T_p}{T_m} = \frac{f_m}{f_p} = \lambda_2 \tag{11}
\]

### 2.3 Flow similarity

In a piezoelectric pump, the internal flow state is generally considered to be turbulent, and the N-S equation is chosen here to characterize its internal flow [24]:

\[
\rho \frac{\partial \vec{v}}{\partial t} = -\nabla p + \rho \vec{F} + \mu \nabla^2 \vec{v} \tag{12}
\]

Vector forms is:

\[
\frac{\partial \vec{v}}{\partial t} = -\nabla \vec{p} + \vec{f}_x + \nabla^2 \vec{v} \tag{13}
\]

The volume force in the process is much smaller than the inertia and viscous forces, so the volume force can be neglected, and the convection term is expanded:

\[
\frac{\partial \vec{v}}{\partial t} + \left( \vec{v} \cdot \nabla \right) \vec{v} = -\nabla \vec{p} + \nabla^2 \vec{v} \tag{14}
\]

Since the equation has symmetry in three dimensions, it only needs to be expanded on one direction. Expanding it in the \(X\) direction as:

\[
\frac{\partial \vec{v}}{\partial t} + v_x \frac{\partial \vec{v}}{\partial x} + v_y \frac{\partial \vec{v}}{\partial y} + v_z \frac{\partial \vec{v}}{\partial z} = -\frac{1}{\rho} \frac{\partial \vec{p}}{\partial x} + \mu \left( \frac{\partial^2 \vec{v}}{\partial x^2} + \frac{\partial^2 \vec{v}}{\partial y^2} + \frac{\partial^2 \vec{v}}{\partial z^2} \right) \tag{15}
\]

Take the pump cavity diameter \(D\) as the length feature, take the throat speed \(U\) as the velocity characteristic, take the pressure \(P\) in the throat as the pressure characteristic, take the dimensionless constant as follows:

\[
h^* = \frac{h}{U}; \quad x^* = \frac{x}{D}; \quad p^* = \frac{p}{\rho U^2}; \quad t^* = \frac{t}{D/U} \tag{16}
\]
Bring the dimensionless form into the above Equation (14) and simplify it, and eliminate it to get the following formula:

\[
\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{D}) \cdot \vec{v} = -\nabla p + \frac{v}{DU} \nabla^2 v
\]  

(17)

where \( \nu \) is the viscosity coefficient of the liquid flowing in the micropump, the unit is \( \text{m}^2/\text{s} \).

Get a dimensionless constant that can characterize the flow field:

\[
\Pi = \frac{\nu}{DU} = \nu D^2
\]  

(18)

Since the model machine and the prototype machine use water as the medium, the viscosity \( \nu \) is the same, so satisfying \( D_p^2 f_p = D_m^2 f_m \) can satisfy the flow similarity. Under similar conditions, the corresponding dimensionless numbers will remain the same, thus:

\[
\frac{f_p}{\rho U_p^2} = \frac{f_p^*}{\rho U_m^2} = \frac{f_m}{\rho U_m^2}
\]

\[
\frac{i_p^*}{D_p/U_p} = \frac{i_m^*}{D_m/U_m} = \frac{i_m}{D_m/U_m}
\]

(19)

In summary, when satisfied:

\[
D_p^2 f_p = D_m^2 f_m \quad \frac{\omega_{\max p}}{\omega_{\max m}} = \frac{D_p}{D_m} = \lambda_1
\]  

(21)

It can be determined that the model machine is similar to the prototype machine. According to the similarity theory, the similarity of the flow and pressure between the prototype machine and the model machine can be derived. The similar conclusion is:

\[
\frac{f_p}{f_m} = \left( \frac{D_m}{D_p} \right)^2, \quad \frac{Q_p}{Q_m} = \frac{D_p}{D_m}
\]  

(22)

### 3 | STRUCTURE OF MICROPUMP

The sketch of micropump with conventional diffuser/nozzle elements is shown in Figure 3. The specific structural dimensions of the micropump are shown in Table 1. The throat width \( d \) and the depth of the channel \( H \) are both \( 200 \mu\text{m} \); The connection between the micropump and the diffusion shrink tube has a circular arc transition, the radius of the Fillet is \( 0.1 \text{ mm} \); the diameter of the inlet and outlet pipes is the same, both are \( D_1 = 1 \text{ mm} \); due to the radius \( a \) of PZT is \( 4.5 \text{ mm} \) and the radius \( b \) of the elastic plate is \( 5 \text{ mm} \), so the diameter \( D \) of the pump cavity is \( 10 \text{ mm} \); the aspect ratio \( L/d \) of diffusion shrink tube is \( 15 \); the width \( d_1 \) and length \( d_2 \) of the buffer chamber are \( 3 \) and \( 6 \text{ mm} \), respectively.

### 4 | EXPERIMENT

#### 4.1 | Experiment equipment

Figure 4 shows the micropump used in the experiment, the same size as the prototype pump in the numerical simulation,
with a diffusion angle of 30°. Figures 5 and 6 show the workbench used in the test, which mainly includes an oscilloscope, generator, amplifier, electronic balance, iron stand, beaker, stopwatch etc. The voltage of the micropump is provided by a signal generator, amplified by a power amplifier, and input to the piezoelectric vibrator of the micropump. The oscilloscope monitors this sinusoidal signal.

### 4.2 Experiment process

According to the vibrator displacement experiment [19], the amplitude should be 1.789, 3.07, 4.46, 5.74, 7.13 μm when the frequency is 100 Hz and the pump is applied with voltages of 25V_pp, 50V_pp, 75V_pp, 100V_pp, 125V_pp. After the micropump is stable, the beaker is used to collect the fluid flowing out from the outlet while timing with a stopwatch. To ensure the accuracy of the measurement, the fluid mass in the culture dish is measured after five minutes and the average flow rate is calculated. And the flow rate of each voltage is measured five times, then take the average of the five measurements as the final value.

### 4.3 Experimental results and analysis

The comparison of experimental and numerical simulation flows is shown in Figure 7. The general trend of the results of simulations and experiments is increasing. It is worth noting that below the voltage 85V_pp, the experimental flow is slightly smaller than the simulated flow, and above 85V_pp, the experimental flow begins to overtake the simulated flow. The reasons may be the amplitude of the piezoelectric vibrator is smaller when the voltage is lower than 85V_pp, so the adhesion force between the piezoelectric vibrator and the pump cavity has a greater impact on the amplitude; when the voltage is higher than 85V_pp, the piezoelectric vibrator gets a larger driving force which reduces the effect of adhesion. When the voltage is 125V_pp, there is a large error between the experimental value and the simulated value of the net flow rate at the outlet of the micropump, the maximum error value is 9.46%. Therefore, this method of numerical simulation is still very reliable.

### 5 NUMERICAL SIMULATION

#### 5.1 Parameter setting

The micropump shown in the above Figure 3 is used as a prototype machine, the dimensions of the prototype are shown in Table 1, and the micropump of twice the size is used as a model machine for numerical simulation. Derived from Equation (21) the frequency of the model machine is 1/4 of the prototype machine, and the amplitude of the model machine is twice that of the prototype machine. Therefore, the parameter settings of the prototype and the model are shown in Table 2.
TABLE 2  The parameter settings of micropump

| Object                  | Prototype       | Model            |
|------------------------|-----------------|------------------|
| Frequency (f)          | 100 Hz          | 25 Hz            |
| PZT radius (a)         | 4.5 mm          | 9 mm             |
| Elastic plate radius (b)| 5 mm            | 10 mm            |
| Piezoelectric displacement (ω<sub>max</sub>) | 1.798 μm, 3.07 μm, 4.46 μm, 5.74 μm, 7.13 μm | 3.596 μm, 6.14 μm, 8.92 μm, 11.48 μm, 14.26 μm |
| Piezoelectric displacement equation |                    |                  |
|                         | ω<sub>1</sub>(r) = ω<sub>max</sub> \[\frac{1 + \left(\frac{r^2 - a^2}{2a^2\ln(a/b)}\right)^2}{2a^2\ln(a/b)}\] (0 ≤ r ≤ a) | |
|                         | ω<sub>2</sub>(r) = ω<sub>max</sub> \[\frac{r^2 + 2a^2\ln(a/b) - r^2}{2a^2\ln(a/b)}\] (a ≤ r ≤ b) | |

FIGURE 8  The computational domain of the micropump

5.2  Simulation

The CFD software CFX is used for the simulations, and the schematic diagram of the computational domain is shown in Figure 8. The SST model (shear–stress transport $k-\omega$ model) is adopted in the simulation because of the accurate prediction of flows with strong adverse pressure gradients and boundary layer separation.

(The prediction accuracy of the $k$-epsilon model under the reverse pressure gradient is low, the prediction of the point of occurrence of flow separation is poor and the degree of flow separation is underestimated, and the flow in the valveless piezoelectric pump involved in this paper has a large reverse pressure gradient. [25]). Dynamic mesh method is applied to simulate the deflection of the piezoelectric vibrator, the displacement formula of the piezoelectric vibrator adopts Equations (2) and (3), and the pressure boundary conditions are applied on the inlet and outlet. The working medium is water of 25 °C, and the kinetic viscosity is 0.8937 × 10⁻⁶ m²/s, the initial state is stationary. No-slip conditions are applied to the walls.

The analysis of mesh and sensitivity to time step is carried out. The mesh at the throat is dense due to the high velocity. The results indicated that the mesh number of 0.6 million is sufficient to meet the requirements of the simulation. The grid numbers of the prototype and model machines are 601360 and 597837 respectively. An unstructured tetrahedral mesh is used. What is more, the relative error is only 0.18% when the time step is 1 / (50f). Hence, the time step setting at 1 / (100f) is enough in this paper.

5.3  Results and discussions

5.3.1  Flow field contrast

Figure 9 is a comparison of the flow field between the prototype machine and the model machine at a diffusion angle of 30°. (The picture above is a prototype machine, the picture below is a model machine). Figure 9 mainly captures the internal flow field of the micropump at four-time points of 0.25T, 0.5T, 0.75T, and T. The prototype machine and the model machine have frequencies of 100 and 25 Hz, respectively, and the amplitude is respectively 4.46 and 8.92 μm. It can be seen from the figure that the flow trend of the prototype machine and the model machine are basically the same. It can be seen from the image that the streamline of the model machine is denser than that of the prototype machine, which also verifies the conclusion of Equation (22). It can be seen from the coordinate values in the figure that the flow rate of the prototype machine is about twice that of the model machine. Since the velocity is proportional to the flow rate and inversely proportional to the area, it is derived according to Equation (22):

\[
\frac{u_p}{u_m} = \frac{Q_p/Q_m}{S_p/S_m} = \frac{D_m}{D_p}
\]

(23)

where $u_p$ and $u_m$ are the flow rate of the prototype machine and the model machine respectively, the unit is mL/min. This means that the flow field in the micropump is indeed similar under the condition that the Equation (22) is satisfied.

5.3.2  Pressure comparison

Figure 10 is a pressure comparison cloud of the prototype machine and the model machine under similar conditions when the diffusion angle is 30°. Similar conditions are as follows: the amplitude of the prototype machine and the model machine are 4.46 and 8.92 μm; the frequencies are 100 and 25 Hz. The image above shows the pressure cloud of the model, and the image below shows the pressure cloud of the prototype. Figure 10 mainly captures the internal flow field of the micropump at four-time points of 0.25T, 0.5T, 0.75T, and T. According to Equation (22), the pressure of the model machine should
be a quarter of the prototype machine. This proportional relationship can be seen from the pressure value in the figure (Although the images at some time points are inconsistent, the values of the corresponding legends can prove the similar relationship), and the pressure distribution of the prototype and model machine is also the same. This shows that the law of pressure similarity between the prototype and the model machine is established.

5.3.3 Export flow comparison

When the model machine is twice the size of the prototype, according to Equation (22), the net export flow of the prototype machine should be half that of the model machine. Therefore, in order to make it easier to observe when dealing with the simulation results, twice the outlet net flow of the prototype machine is compared with the model machine. Take twice the result of the prototype machine as the theoretical value, and use the result of the model machine as the numerical value. The frequency of taking the prototype machine is 100 Hz, the amplitude is 1.798, 3.07, 4.46, 5.74, 7.13 μm; according to the Equation (21), the frequency of the model machine is 25 Hz, the amplitude should be 3.596, 6.14, 8.92, 11.48, 14.26 μm. The simulated outlet flow and the theoretical flow obtained by Equation (21) are shown in Figure 11.

As can be seen from the above Figure 11, when the diffusion angle is 30°, the theoretical value has always been slightly larger than the digital value, but the rise of the two graphs is basically the same, the maximum error occurs at the amplitude of 4.46 μm, the difference of theoretical flow and numerical flow is 9.83% and no more than 10%; When the diffusion angle is 20°, when the amplitude is less than 10 μm, the two lines are basically coincident, and when the amplitude is greater than
10 °m, the theoretical flow begins to grow faster, and the maximum error of theoretical flow and numerical flow reaches 9.82% and no more than 10%, which proves the accuracy of the theoretical results. When the fluid enters the pump cavity, a vortex appears near the outlet throat of model machine, causing more fluid to flow in from the outlet, reducing the net flow of the micropump in one cycle. Therefore, the net flow curve of the prototype and model at θ = 20° is more consistent. The reason why the two curves in Figure 11(b) will become larger and larger after the amplitude is 10 μm may be because the vortex in the model machine is increasingly affected by the amplitude, so that the net flow of the model machine is getting less In order to simplify the N-S equation, the effect of the bulk force is neglected when considering the flow, so there is a certain error between the actual value and the theoretical value, which is also consistent with the actual situation.

6 | CONCLUSION

In this paper, the conditions needed in the planar cone-tube valveless piezoelectric micropump prototype machine and the model machine are proposed through the dimensionless of the oscillator equation and the fluid flow equation in the micropump. Thus, the relationship between the pressure, flow rate and driving frequency of the prototype and model machines was introduced. In order to verify the accuracy of the similarity law of micropumps, the prototype and model pumps with a size ratio of \( D_p / D_m = 1/2 \) were numerically simulated. First, the experiment of a valveless piezoelectric micropump with a diffusion contraction tube (with a diffusion angle of 30°) is conducted, and the experimental values are compared with the simulation results to verify the reliability of the simulation. From the simulated micropump internal flow field streamline diagram
and internal pressure cloud diagram, it can be seen that the similarity law of the micropump is established, and then the comparison of the outlet flow rate further confirms the accuracy of the similarity law. This paper not only has reference value for studying the similarity law of micro-pumps of other structures, but also provides guidance for the design and experiment of valveless piezoelectric pumps with flat-tubes. It is of great significance for the research of subsequent valveless piezoelectric micropumps.

ACKNOWLEDGMENTS

Thanks to Shan Chunming for experimental investigation and Yang Song's numerical simulation research. This work was supported by the project of the National Natural Science Foundation of China [grant number: 51276082].

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How to cite this article: He X., Shan C., Yang H.: Research and verification of similar laws in valveless piezoelectric micropumps. Micro Nano Lett. 16, 281–289 (2021). https://doi.org/10.1049/mna2.12041.