Classical solutions in five dimensional induced matter
theory and its relation to an imperfect fluid

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Abstract

We study five dimensional cosmological models with four dimensional hipers-
ufaces of the Bianchi type I and V. In this way the five dimensional vacuum field
equations $G_{AB} = 0$, led us to four dimensional matter equations $G_{\mu\nu} = T_{\mu\nu}$ and
the matter is interpreted as a purely geometrical property of a fifth dimension.
Also, we find that the energy-momentum tensor induced from the fifth dimension
has the structure of an imperfect fluid that has dissipative terms.

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1 Introduction

The problem of higher dimensional cosmologies has received much attention as one attempt to unify interactions. Most of the people who has worked this kind of cosmologies has done use of the standard dimensional reduction of the Kaluza-Klein type\(^1\). A crucial point for the theory to be realistic is to explain why the “internal” space is so small that one cannot observe it, and why the “ordinary” space is so large. An interesting attempt to this problem is the so called cosmological dimensional reduction, according to this reduction the large discrepancy of two sizes is regarded as a consequence of the dynamical evolution of the higher-dimensional universe. Nevertheless there have been attempts in which the extra dimensions are not required to be compact\(^2-4\), but they are associated to the geometrical four-dimensional properties of the matter\(^5\) by means of scalar fields\(^6\).

The problem of generating an effective four-dimensional stress-energy tensor from five-dimensional vacuum field equations (Induced Matter Theory) is not solved, but we take the ideas of Davidson and Owen\(^3\), Ponce de Leon and Wesson\(^7,8\) (see references in [1], ) in this way, the four-dimensional field equations, with their respective stress-energy tensor \(G_{\mu\nu} = T_{\mu\nu}\), can be obtained, there is, of course, an extra field equation due to the presence of the fifth dimension, namely \(G_{44}\), but this equation is functionally equivalent to \(G_{0i}^0 + G_{i}^i = 0, i = 1, 2, 3\) or \(\rho_{e1} - 3p_{e1} = 0\) and \(T_{\mu\nu}\) is a function of just the scale factor of the extra dimension. The idea of an effective stress-energy tensor in the three-space due to the evolution of the extra dimensions has been considered by other authors\(^9-14\).

In this work we analyze a specific class of five-dimensional metrics, that have the following form,

\[
ds^2 = g_{AB}dx^A dx^B = g_{\mu\nu}(x^\alpha) dx^\mu dx^\nu + \phi^2(x^\alpha)dw^2, \tag{1.1}\]
where the capital latin indices take values on the five dimensions and the greek indices take their values on the four dimensions of ordinary spacetime. Notice that none of the metric coefficients depends on the fifth coordinate, that is we assume the usual cylinder condition. In particular, \( \sqrt{\gamma_{44}} = \phi \), plays the role of the radius of the “internal” space.

Together with the above metric we assume that the action for the theory of gravity in five dimensions has the following form

\[
I = \int dx^5 \sqrt{-\gamma} g^{(5)} R. \tag{1.2}
\]

The Einstein equations from the above action are

\[
^{(4)} G_{\mu \nu} = ^{(4)} T_{\mu \nu}(\phi) := \phi^{-1} \left[ \phi_{,\mu;\nu} - g_{\mu \nu} \Box \phi \right],
\]

\[
^{(5)} G_{44} = ^{(4)} R = 0, \tag{1.3}
\]

we will apply these field equations to spaces that are trivial generalizations of Bianchi type I and V in the sense that the four dimensional part of the metric \( g_{\mu \nu} \) are of the Bianchi types I and V. For these two particular examples it is possible to isolate terms that come from the fifth dimension and associate them with the effective “density” \( \rho_{\text{eff}} \), and the effective “pressure” \( p_{\text{eff}} \) that appear in the stress-energy tensor of a perfect fluid in four-dimensions. These models have been studied in other contexts, by using supergravity\textsuperscript{15,16} N=2, D=5. In those models, they found exact solutions and the existence of singularities is considered. In Sec. 2 we present the exact solution for the Bianchi type I, and in Sec. 3 the solution for the Bianchi type V. In Sec. 4 we obtain the stress-energy tensor associated with the general models when we assume the above decomposition of the metric and reduce it to four dimensions. It turns out that in the more general case the effective energy-momentum tensor has the structure of an imperfect fluid under some assumptions as have been found in other theories with a scalar field. In this case, along with besides of the energy density and the pressure we have now viscosity, heat flux and anisotropic stress. This thermodynamic quantities can be
useful to explain several features of the evolution of the universe. Since dissipative effects can counteract the collapse a different scenario of the early stages of the universe appears. Also the generation of entropy can be accounted for by the dissipative processes. The above identification can be useful in the following way: to generate new solution in general relativity with an imperfect fluid from a solution in the theory that we are considering here.

Some exact solutions with viscosity have been found for isotropic and anisotropic models, for instance, Murphy\textsuperscript{17} found the solution for flat Friedmann model and Banerjee and Santos\textsuperscript{18} extended the analysis to the case with curvature. Belinskii and Khalatnikov\textsuperscript{19,20} have considered the qualitative aspects in anisotropic Bianchi type-I models and Banerjee\textsuperscript{21,22} et al. gave some exact solutions for the Bianchi type-I and II models. Bradley and Sviestins\textsuperscript{23} have found exact solution of Bianchi type VIII with heat flow, while Nayak and Sahoo\textsuperscript{24} found solutions of Bianchi type V with an imperfect fluid. Koppar and Patel\textsuperscript{25} found an exact solution of Bianchi type II with viscosity and heat flux.

With these ideas, we write the energy-momentum tensor as

$$T_{\mu\nu} = \rho u_\mu u_\nu + 2q_{(\mu}u_{\nu)} + ph_{\mu\nu} + \pi_{\mu\nu},$$

where $\rho$ is the energy density of the fluid, $u_\mu$ the velocity, $q_\mu$ the heat flux vector, $p$ the pressure, $\pi_{\mu\nu}$ the anisotropic stress tensor and $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ is the projection orthogonal to the velocity. Finally, Sec. 5 is devoted to some final remarks.

## 2 Cosmological Model Bianchi type I

We start with Bianchi type I metric in five dimensions

$$ds^2 = -dt^2 + e^{2A} dx^2 + e^{2B} dy^2 + e^{2C} dz^2 + \phi^2 dw^2,$$
where all function $A$, $B$, $C$ and $\phi$ depend only to time $t$. The Einstein field equations for this model are

\begin{align*}
\dot{A} \dot{B} + \dot{A} \dot{C} + \dot{B} \dot{C} + (A + B + C)\ddot{\phi} + \frac{\dot{\phi}}{\phi} = 0, \\
\dot{B} + (\dot{B})^2 + \dot{B} \dot{C} + (\dot{C})^2 + \ddot{C} + (B + C)\frac{\dot{\phi}}{\phi} + \frac{\phi}{\dot{\phi}} = 0, \\
\dot{A} + (\dot{A})^2 + \dot{A} \dot{C} + (\dot{C})^2 + \ddot{C} + (A + C)\frac{\dot{\phi}}{\phi} + \frac{\phi}{\dot{\phi}} = 0, \\
\dot{A} + \dot{B} + \dot{C} + (\dot{A})^2 + (\dot{B})^2 + (\dot{C})^2 + \ddot{A} + \ddot{B} + \ddot{C} = 0.
\end{align*}

The solution for this set of equations is:

\[
\phi(t) = \phi_0 t^{k/(1+k)}, \quad \frac{k}{1+k} \neq 0,
\]  

\[
A = \ln \left( A_0 t^{p_1/(1+k)} \right), \quad B = \ln \left( B_0 t^{p_2/(1+k)} \right), \quad C = \ln \left( C_0 t^{p_3/(1+k)} \right),
\]

with

\[
k \neq 0, \quad \sum_{i=1}^{3} p_i = 1, \quad \sum_{i=1}^{3} p_i^2 = 1 + 2k.
\]

In order to have a decreasing $\phi(t)$, it is necessary to have $\frac{k}{1+k} < 0$, and using the fact that $\sum_{i=1}^{3} p_i^2 = 1 + 2k > 0$, then $-\frac{1}{2} < k < 0$, therefore when $t \to \infty$, $\phi \to 0$, giving a dynamical compactification in five dimension.

## 3 Cosmological Model Bianchi type V

We begin with the line element

\[
\text{ds}^2 = -e^{2A} d\tau^2 + e^{2A} dx^2 + e^{2B-2Qx} dy^2 + e^{2C-2Qx} dz^2 + e^{2\psi} d\omega^2,
\]  

3
which defines the anisotropic and homogeneous cosmological model Bianchi V, where $A$, $B$, $C$ and $\psi$ are functions of the “time parameter” $\tau$, $Q$ is a constant parameter that give us a measure of the anisotropy of this model, $g_{44} = e^{2\psi}$, where $\phi = e^\psi$ is a massless scalar field associated to the extra fifth dimension $x^4 = \omega$.

The five-dimensional vacuum field equations for this model are:

$$2A' = B' + C', \tag{3.2}$$
$$A'B' + A'C' + B'C' - 3Q^2 = -(A' + B' + C')\psi', \tag{3.3}$$
$$B'' + B'^2 + B'C' + C'^2 + C'' - A'(B' + C') - Q^2 = -\left( (B' + C' - A')\psi' + \psi'^2 + \psi'' \right), \tag{3.4}$$
$$A'' + C'' + C'^2 - Q^2 = -\left( C'\psi' + \psi'^2 + \psi'' \right), \tag{3.5}$$
$$A'' + B'' + B'^2 - Q^2 = -\left( B'\psi' + \psi'^2 + \psi'' \right), \tag{3.6}$$
$$A'' + B'' + C'' + B'^2 + C'^2 + B'C' - 3Q^2 = 0, \tag{3.7}$$

where now, the primes mean derivatives with respect to the “time variable $\tau$”. The above system is a very symmetric one, and in order to solve it, we took advantage of this fact. The solutions for the above set of equations are

$$\psi(\tau) = \psi_0 + \frac{1}{4Q\alpha_1\gamma} \ln \left[ \frac{e^{2Q\tau} - \gamma}{e^{2Q\tau} + \gamma} \right], \tag{3.8}$$

then

$$\phi(\tau) = \phi_0 \left[ \frac{e^{2Q\tau} - \gamma}{e^{2Q\tau} + \gamma} \right]^{\frac{1}{1(\alpha_1\gamma)}}, \tag{3.9}$$

where $\psi_0$ and $\gamma$ are constants of integration.

$$A = \ln \left[ A_0 \left\{ \left( \frac{e^{2Q\tau} - \gamma}{e^{2Q\tau} + \gamma} \right)^{\frac{1}{\alpha_1\alpha_2 - 2\alpha_2 e^{-2Q\tau}}} \right\}^{-\frac{3}{2}} \right], \tag{3.10}$$

$$B = A + \frac{1}{2}\alpha_3\psi + \alpha_4, \tag{3.11}$$
$$C = A - \frac{1}{2}\alpha_3\psi + \alpha_5, \tag{3.12}$$
where $\alpha_4, \alpha_5$ are integration constants and the following relation is satisfied

$$\alpha_1 \alpha_2 = -\frac{3 + \alpha_3^2}{48Q^2}. \quad (3.13)$$

Using $d\tau = e^{-A}dt$ in the time reparametrization $t$, we can get the following

$$t - t_0 = \int_{\tau_0}^{\tau} \left[ A_0 / \left\{ \left( \frac{e^{2Q\tau} - \gamma}{e^{2Q\tau} + \gamma} \right)^{\frac{1}{2Qa_1}} \left( \frac{1}{\alpha_1 e^{2Q\tau} + \alpha_2 e^{-2Q\tau}} \right) \right\}^{\frac{1}{2}} \right] d\tau. \quad (3.14)$$

It is reasonable to think that the time $\tau$ has a behavior similar to time $t$, because, when $\tau \to \infty$, the integral goes to $\infty$ too. On the other hand, taking a look to Eq. (3.9), when $\tau \to \infty$, the function $\phi \to \phi_0 = \text{const.}$, to have dynamical compactification we chose a small value for $\phi_0$. Moreover, as can be seen from Eqs. (3.10-3.12), our model isotropize for $\tau \to \infty$.

### 4 Stress-energy Tensor in four dimensions

In the set of equation (2.2) to (2.6) for the Bianchi type I, the terms in which the scalar field $\phi$ appears can be associated to the components of an imperfect fluid stress-energy tensor in four dimensions in the following sense,

$$-(A + B + C)\frac{\dot{\phi}}{\phi} = 8\pi \rho = T_{00}^0, \quad (4.1)$$

$$-(B + C)\frac{\dot{\phi}}{\phi} + \frac{\ddot{\phi}}{\phi} = 8\pi p_1 = T_{11}^1, \quad (4.2)$$

$$-(A + C)\frac{\dot{\phi}}{\phi} + \frac{\ddot{\phi}}{\phi} = 8\pi p_2 = T_{22}^2, \quad (4.3)$$

$$-(A + B)\frac{\dot{\phi}}{\phi} + \frac{\ddot{\phi}}{\phi} = 8\pi p_3 = T_{33}^3. \quad (4.4)$$

In a similar way, for the Bianchi type V, Eqs. (3.3) to (3.6), become

$$-(A + B + C)\dot{\psi} = 8\pi \rho = T_{00}^0, \quad (4.5)$$

$$(B + C)\dot{\psi} + \psi^2 + \ddot{\psi} = 8\pi p_1 = T_{11}^1, \quad (4.6)$$
\[(A + C)\dot{\psi} + \dot{\psi}^2 + \ddot{\psi} = 8\pi p_2 = T_{2}^2,
\]
\[(A + B)\dot{\psi} + \dot{\psi}^2 + \ddot{\psi} = 8\pi p_3 = T_{3}^3.
\]

For these models, in our case, if we identify Eqs. (2.2-2.5) and (3.3-3.6) in four dimension with a matter tensor \(T_{\mu\nu}\), the components of this tensor obey the relation \(T^\mu_{\mu} = 0\) when we use the constraints equations (2.6) and (3.7), which imply that the matter is ultrarelativistic or radiation like. The anisotropy represented by \(T_{11}^1 \neq T_{22}^2 \neq T_{33}^3\) is due physically to the inequality of the pressures. Thus, although the fluid (scalar field) doesn’t have viscosity it has an energy-momentum tensor that differs from the conventional one

\[T_{\mu\nu} = [(p + \rho)u_\mu u_\nu + pg_{\mu\nu}],\]

for perfect fluids. However, in those cases we can derive an equation of state by using the technique of the references [3,8] in which they identify \(-T_{00}^0 = \rho_{\text{eff}}\) and \(\frac{1}{3}T_{i}^i = p_{\text{eff}}\), in order to have \(\rho_{\text{eff}} = 3p_{\text{eff}}\). In the following subsection we show that the analogy with an imperfect fluid is more general.

### 4.1 Structure of the imperfect fluid Energy-Momentum Tensor

We recall here the explicit field equations for this theory,

\[G_{\mu\nu} = T_{\mu\nu}(\phi) := \phi^{-1}(\phi_{\mu\nu} - g_{\mu\nu}\Box\phi).\]

\[\Box\phi = 0,
\]

where \(G_{\mu\nu}\) is the Einstein tensor.

The defined energy momentum for the scalar field is covariantly conserved as follows from the Bianchi identities or the field equation for \(\phi\),

\[\nabla_\mu T^{\mu\nu}(\phi) = 0.
\]
We will show that this energy momentum tensor has the structure of an imperfect fluid energy-momentum tensor

\[ T_{\mu\nu} = \rho u_{\mu} u_{\nu} + 2q_{(\mu} u_{\nu)} + ph_{\mu\nu} + \pi_{\mu\nu}, \]  

(4.13)

where \( \rho \) is the energy density of the fluid, \( u_{\mu} \) the velocity, \( q_{\mu} \) the heat flux, \( p \) the pressure, \( \pi_{\mu\nu} \) the anisotropic stress tensor and,

\[ h_{\mu\nu} = g_{\mu\nu} + u_{\mu} u_{\nu} \]  

(4.14)

is the projection orthogonal to the velocity. The following relations are satisfied,

\[ u_{\mu} u^{\mu} = -1; \quad h_{\mu\nu} u^{\mu} = 0; \quad \pi_{\mu}^{\mu} = 0; \quad h_{\mu\nu} u^{\mu} u^{\nu} = 0. \]  

(4.15)

The thermodynamic quantities can be obtained by projections along and orthogonal to the velocity field and taking traces,

\[ \rho = T_{\mu\nu} u^{\mu} u^{\nu}, \quad q_{\alpha} = -T_{\mu\nu} u^{\mu} h_{\nu}^{\alpha}, \]  

(4.16)

\[ \Pi_{\alpha\beta} := ph_{\alpha\beta} + \pi_{\alpha\beta} = -T_{\mu\nu} h_{\alpha}^{\mu} h_{\beta}^{\nu}, \]  

(4.17)

\[ p = \frac{1}{3} \Pi_{\alpha}^{\alpha}, \]  

(4.18)

\[ \pi_{\alpha\beta} = \Pi_{\alpha\beta} - ph_{\alpha\beta}. \]  

(4.19)

Also of interest are the kinematical quantities of the fluid, that appear in the following decomposition of the derivative of the velocity

\[ u_{\alpha;\beta} = -\dot{u}_{\alpha} u_{\beta} + \omega_{\alpha\beta} + \sigma_{\alpha\beta} + \theta h_{\alpha\beta}/3, \]  

(4.20)

with

\[ \dot{u}_{\alpha} = u_{\alpha;\beta} u^{\beta}, \]  

(4.21)

\[ \theta = u^{\alpha}_{;\alpha}, \]  

(4.22)

\[ \sigma_{\alpha\beta} = u_{(\alpha;\beta)} + \dot{u}_{(\alpha} u_{\beta)} - \frac{1}{3} \theta h_{\alpha\beta}, \]  

(4.23)
\[ \omega_{\alpha\beta} = u_{[\alpha;} + \dot{u}_{\alpha} u_{\beta]}, \quad (4.24) \]

which are the acceleration, expansion, shear and rotation of the fluid respectively. In this case we take for the velocity of the fluid the normalized derivative of the field,

\[ u_\alpha := \pm \frac{\dot{\phi}_\alpha}{\sqrt{Z}} \quad (4.25) \]

where \( Z := -\phi_\sigma \dot{\phi}^\sigma \) and we have to choose the most convenient sign in each particular calculation. In order to make sense for the last expression it is necessary that \( \phi_\sigma \) be a timelike vector; this condition could be a serious restriction to the applicability of the generating method. We shall see latter that there are situations of physical interest where the condition is fullfilled.

Having selected the velocity field we use it in Eqs. (4.19, 4.24) to obtain the thermodynamic and kinematical quantities of the fluid in terms of the scalar field and its derivatives. After a straightforward calculation we obtain

\[ \rho = \frac{Y}{Z\dot{\phi}}, \quad (4.26) \]

\[ p = \frac{1}{3} \frac{Y}{Z\dot{\phi}}, \quad (4.27) \]

\[ q_\alpha = \mp Z^{-\frac{3}{2}} \frac{1}{\dot{\phi}} \left[ \phi^\nu \phi_{\nu \alpha} + \frac{Y \phi_\alpha}{Z} \right], \quad (4.28) \]

\[ \Pi_{\alpha\beta} = \frac{\phi_{\alpha \beta}}{Z} \left[ \frac{Y}{Z\dot{\phi}} \right] + \frac{\phi_{\alpha \beta}}{\dot{\phi}} + 2 \frac{\phi_{(\alpha \beta) \mu} \dot{\phi}^\mu}{Z\dot{\phi}}, \quad (4.29) \]

\[ \pi_{\alpha\beta} = -\frac{1}{3} g_{\alpha\beta} \left[ \frac{Y}{Z\dot{\phi}} \right] + \frac{\phi_{\alpha \beta}}{\dot{\phi}} + \frac{1}{3} \frac{\phi_{\alpha \beta}}{\dot{\phi}} \left[ 2 \frac{Y}{Z^2} \right] + 2 \frac{\phi_{(\alpha \beta) \mu} \dot{\phi}^\mu}{Z\dot{\phi}}, \quad (4.30) \]

\[ \dot{u}_\alpha = \frac{\phi_{\alpha \beta} \dot{\phi}^\beta}{Z} + \frac{Y \phi_\alpha}{Z^2}, \quad (4.31) \]
\[ \theta = \pm \frac{Y}{Z^2}, \quad (4.32) \]

\[ \omega_{\alpha\beta} = 0, \quad (4.33) \]

\[ \sigma_{\alpha\beta} = \pm \left\{ -\frac{1}{3} g_{\alpha\beta} \frac{Y}{Z^{3/2}} + \frac{1}{3} \phi_{\alpha\gamma} \frac{2Y}{Z^{5/2}} + \frac{2}{3} \phi_{(\alpha\mu\beta)} \phi^{\mu} \right\}, \quad (4.34) \]

Here

\[ Y := \phi_{,\lambda\mu} \phi^{,\lambda} \phi^{,\mu}. \quad (4.35) \]

We notice that the fluid that we have obtained is irrotational because the velocity field is proportional to a divergence, it is also a Newtonian fluid, i.e., satisfying the relation

\[ \pi_{\alpha\beta} = -2 \eta \sigma_{\alpha\beta}, \quad (4.36) \]

with the viscosity coefficient given by

\[ \eta = \mp \frac{Z^{1/2}}{2\phi}. \quad (4.37) \]

Also the equation of state is \( p = \frac{1}{3} \rho \) that correspond to a relativistic fluid, but an imperfect one since heat flux and viscosities are present.

### 4.2 Examples

In this section we apply the results obtained above to generate solutions in the anisotropic homogeneous cosmology. In this models the scalar field depends only on the time coordinate and the derivative is a timelike vector. We exhibit the thermodynamic and kinematical quantities obtained for the solutions of Bianchi type I and Bianchi type V.
4.2.1 Bianchi type I solution

Here we consider the solutions (2.7-2.10), substituting these solutions in Eqs.(4.26 - 4.37) we obtain the following thermodynamic and kinematical quantities:

\[ \rho = -\frac{k}{(1 + k)^2 t^2}, \]  
\[ p = -\frac{1}{3} \frac{k}{(1 + k)^2 t^2}, \]  
\[ q_\alpha = 0, \]  
\[ \pi_{ij} = -\frac{k}{6(1 + k)^2} \left(3p_i - 2\right) t^{-\frac{\rho}{1+\kappa}-2} \delta_{ij}, \]  
\[ \eta = -\frac{k}{2(1 + k) t^2}, \]  
\[ \theta = \frac{1}{(1 + k) t}. \]

In order to have an expanding universe (\( \theta > 0 \)) and a positive viscosity coefficient we have to put the restriction \(-1 < k < 0\), being this to agreement with the limits set at the end of Sec. 2, and also the pressure and energy density are positive.

In this particular case the fluid is not only irrotational and Newtonian, the viscosity is also proportional to the energy density and the equation of state is of the barotropic type.

4.2.2 Bianchi type V solution
Here we consider the solutions (3.8-3.12), we obtain the following thermodynamic and kinematical quantities for this model

\[
\rho = \frac{3 e^{4Q\tau} \left(\frac{e^{2Q\tau} - \gamma}{e^{2Q\tau} + \gamma}\right)^{1/(4\alpha_1 \gamma)}}{2 \alpha_1^2 (\alpha_2 + \alpha_1 e^{4Q\tau}) (e^{4Q\tau} - \gamma^2)^2}, \quad (4.44)
\]

\[
p = \frac{1 e^{4Q\tau} \left(\frac{e^{2Q\tau} - \gamma}{e^{2Q\tau} + \gamma}\right)^{1/(4\alpha_1 \gamma)}}{2 \alpha_1 (\alpha_2 + \alpha_1 e^{4Q\tau}) (e^{4Q\tau} - \gamma^2)^2}, \quad (4.45)
\]

\[
q_\alpha = 0, \quad (4.46)
\]

\[
\pi_{11} = \frac{1}{2} \frac{11 - 2Q \left(\alpha_1 e^{2Q\tau} - \alpha_2 e^{-2Q\tau}\right)}{(\alpha_1 e^{2Q\tau} + \alpha_2 e^{-2Q\tau})^2},
\]

\[
\pi_{22} = \frac{1}{2} \frac{11 - 2Q \left(\alpha_1 e^{2Q\tau} - \alpha_2 e^{-2Q\tau}\right) - \alpha_3 e^{-2Q\tau} \phi^\alpha_3}{(\alpha_1 e^{2Q\tau} + \alpha_2 e^{-2Q\tau})^2},
\]

\[
\pi_{33} = \frac{1}{2} \frac{11 - 2Q \left(\alpha_1 e^{2Q\tau} - \alpha_2 e^{-2Q\tau}\right) + \alpha_3 e^{-2Q\tau} \phi^\alpha_3}{(\alpha_1 e^{2Q\tau} + \alpha_2 e^{-2Q\tau})^2}, \quad (4.47)
\]

\[
\eta = \frac{1}{2} \left(\frac{e^{2Q\tau} - \gamma}{e^{2Q\tau} + \gamma}\right)^{1/(4\alpha_1 \gamma)} \left(\frac{1}{\alpha_1 e^{2Q\tau} + \alpha_2 e^{-2Q\tau}}\right)^{3/2}, \quad (4.48)
\]

\[
\theta = -\frac{e^{Q\tau} \left(e^{2Q\tau} - 2\alpha_1 Q e^{4Q\tau} - 2\alpha_1 \gamma^2 Q\right)}{\alpha_1^{3/2} \left(e^{4Q\tau} - \gamma^2\right)^{3/2}}, \quad (4.49)
\]

5 Final remarks

In this work we have considered five dimensional cosmological models that have as four dimensional hypersurface Bianchi type I and V homogeneous spaces. We have found exact solutions where the influence of the extra dimension is encoded in a scalar field that can be considered the radius of that extra dimension. The model that we found are such that for later times the scalar field either vanishes or take a constant very value that
can be chosen small so that we have dynamical compactification of the fifth dimension. The Bianchi type V also isotropize at later times. On the other hand, using the standard dimensional reduction, the energy momentum tensor of the scalar field can be associated with an imperfect fluid that has dissipative effects: heat flux and shear viscosity that can be useful to explain several features of the evolution of the universe as entropy generation. The relation between the pressure and the energy density corresponds to a relativistic fluid, \( p = \rho/3 \). The fluid is also Newtonian.

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