Solidification of Cu-Water nanofluid in a trapezoidal cavity: A CFD study

R K Sharma¹*, P Ganesan¹, I H Metselaar¹
¹Department of Mechanical Engineering, Faculty of Engineering, University of Malaya, 50603, Kuala Lumpur, Malaysia
E-mail: ¹ravipvb@gmail.com

Abstract. A numerical study has been carried out to investigate the solidification of a binary mixture of water and Cu nanoparticles inside a horizontal trapezoidal cavity of different aspect ratio under specific given boundary conditions for temperature and concentration gradients. The vertical side walls of the cavity are insulated while the top wall temperature is kept lower than that of the bottom wall. The effect of parameters such as the ratio of the cavity length to height (aspect ratio), the cold wall temperature (-5 to -30 °C) and the initial temperature of the nanofluid (0 °C to 16 °C) on solidification time is investigated. The moving solid-liquid interface is obtained using Enthalpy-porosity technique in the model. We found that the solidification time decreases with the increase of the aspect ratio (i.e., a longer trapezoidal cavity) and the decrease of cold wall temperature. Aspect ratio is found to give a prominent effect. However, the initial temperature of fluid does not affect the solidification time much.

1. Introduction

In order to bridge the gap between energy availability and its uses, the latent heat thermal energy storage (LHTES) system is an effective way to store the energy for later use by melting/freezing of Phase Change Materials (PCMs). These devices have applicability in wide range of applications such as buildings, electronic cooling, material processing and thermal management of air and space-craft. PCMs with nano size are preferred than the micro size particles to avoid clogging during flow. In the current study a PCM (water + Cu nanoparticles) is used to investigate the solidification process. A mixture of a base fluid and solid particles of nano size is termed as nanofluid [1]. The solidification process of water-Cu nanofluid of different volume concentration of Cu nano particles was numerically investigated by Khodadadi and Hosseinizadeh [2] and they found that the nanofluid freezes faster than the base fluid. PCMs are encapsulated for LHTES because it provides the larger heat transfer area, reduces PCMs interaction with external environment, and controls the variation in volume change during phase change process. In a pioneer work done by Gau and Viskanta [3], the effect of buoyancy driven flow in gallium filled in a rectangular cavity is experimentally investigated. They found that as the time progresses, convection takes over the melting process over conduction. Later this work was numerically validated by Brent et al. [4] using the Enthalpy-porosity technique. A numerical study carried out by Assis et al. [5] for melting of RT27 (Rubitherm GmbH) filled in spherical enclosure shows that the melting process incorporate the convection phenomenon in liquid phase. Recently an

¹ To whom any correspondence should be addressed.
experimental and numerical study [6] has been carried out for constrained melting of PCMs in spherical shell and observed that with time convection gets strengthened. There has been number of studies considering the three regular geometries namely square/rectangular, cylindrical and spherical. The selection of geometrical shape of cavity mainly depends upon the application [7]. Because of the large surface area normal to the direction of heat transfer, trapezoidal cavity has received significant attention of researchers for encapsulation. This geometry has wide range of application such as solar heater, casting and mold design etc. An experimental study [8] was carried out to investigate the effect of concentration of NH₄Cl (0-19.8%) and wall temperatures (-30 to 0 °C) on the solidification of binary alloy (NH₄Cl+H₂O) filled in trapezoidal cavity and they found that the rate of solidification diminishes with increase of NH₄Cl concentration.

Despite of much experimental and numerical investigation carried out using regular geometries such as square/rectangular, spherical and cylindrical, very little work has been done to investigate the solidification/melting process by the conduction or convection heat transfer of PCM in a trapezoidal cavity. Therefore, the purpose of the present work is to analyze the solidification of water-Cu nanofluid filled in a horizontally placed isosceles trapezoidal cavity using the Ansys-Fluent 13.0 CFD commercial package. We found that the horizontal placement of the cavity saves the solidification time when compared with vertically placed one.

3. Research Methodology

3.1 Mathematical formulation

Fig. 1 shows the two dimensional (2D) geometry considered in the current study. Isosceles trapezoidal cavity placed in horizontal direction with 10 mm² internal area is used. The effect of the ratio of trapezoidal length to height, namely aspect ratio (AR = L/H) is investigated. Different AR, i.e., 1.0, 1.2, 1.4, 1.6, and 1.8, are tested, but the internal area is kept same for all the cavities. The nanofluid within the cavity is Newtonian, laminar, and incompressible. Thermo-physical properties (Table 1) of the nanofluid are assumed to be constant, whereas the density variation in the buoyancy force is based on the Boussinesq approximation. The nanoparticles are assumed to have a uniform shape and size (10 nm diameter). The left lower corner of the cavity was the origin of the coordinate system. Gravity acts in the negative y-coordinate direction, $\ddot{y} = 0$, and $\ddot{x} = -1$.

The initial and boundary conditions for the present investigation are follows:

- at the top inclined wall $T = T_i$
- at the bottom inclined wall $T = T_e$
- at left and right vertical surfaces $\frac{\partial T}{\partial y} = 0$
- at all solid boundaries $u = v = 0$
Figure 1. Sketch of the two dimensional trapezoidal.

Table 1. Thermo-physical properties of the base fluid (water) and the Cu nanoparticles.

| Property          | Copper nanoparticles | Base fluid (water) |
|-------------------|----------------------|--------------------|
| ı [l/l = 1]      | 8954                 | 997.1              |
| ı [l/l = λ]       | -                    | 8.9 × 10⁻¹¹        |
| ı, [l/l = h]      | 383                  | 4179               |
| ı [l/l = h]       | 400                  | 0.6                |
| ı [l/l = 1]       | 1.67 × 10⁻¹¹         | 2.1 × 10⁻¹¹        |
| ı [l/l = 1]       | -                    | 3.35 × 10⁻¹¹       |
| ı, -              | 6.2                  |                    |
| ı, 0              | 0.125                |                    |
| ı, [l/l = 1]      | 10⁻¹¹                | -                  |

The mathematical governing equations are as follows:

The continuity, momentum considering the Boussinesq approximation and energy equation for the above mentioned assumptions can be written in the following form:

Continuity

\[ \frac{\partial u}{\partial t} + \frac{\partial v}{\partial y} = 0, \]  

(1)

X-momentum

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho_{nf}} \left( \frac{\partial p}{\partial x} + \mu_{nf} \nabla^2 u + (\rho \beta)_{nf} g_x (T - T_C) \right), \]  

(2)

Y-momentum

\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \frac{1}{\rho_{nf}} \left( \frac{\partial p}{\partial y} + \mu_{nf} \nabla^2 v + (\rho \beta)_{nf} g_y (T - T_C) \right), \]  

(3)

Energy equation

\[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial x} \left[ \frac{(k_{nf} + k_J)}{(\rho c_p)_{nf}} \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{(k_{nf} + k_J)}{(\rho c_p)_{nf}} \frac{\partial T}{\partial y} \right]. \]  

(4)

The density of the nanofluid is given by:
\[ \rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s, \]

The viscosity of nanofluid is given by Brinkman [9]:

\[ \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}, \]

The heat capacitance of the nanofluid and part of the Boussinesq are:

\[ (\rho c_p)_{nf} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_s \]

\[ (\rho \beta)_{nf} = (1 - \phi)(\rho \beta)_f + \phi(\rho \beta)_s \]

The latent heat of the nanofluid is evaluated using

\[ (\rho L)_{nf} = (1 - \phi)(\rho L)_f \]

Where \( L \) is the latent heat and \( \phi \) is the nanoparticle volume fraction.

Thermal conductivity of the quiescent (subscript 0) nanofluid is:

\[ k_{0nf} = k_{0f} + 2k_f - 2\phi(k_f - k_s) \]

\[ k_f = \frac{k_s}{k_f + 2k_s + 2\phi(k_f - k_s)} \]

Whereas the effective thermal conductivity of the nanofluid is:

\[ k_{nf} = k_{nf0} + k_d, \]

And the thermal conductivity enhancement term due to thermal dispersion is given by:

\[ k_d = C(\rho c_p)_{nf} \sqrt{u^2 + \nu^2} \phi \]

The empirically-determined constant \( C \) is evaluated following the work of Wakao and Kaguei (1982).

3.2 Numerical method

The numerical solution of the problem uses the enthalpy-porosity approach. In this technique, the porosity in each cell is set equal to the liquid fraction in that cell and this fraction is computed at each iteration. The SIMPLE method of the commercial code ANSYS FLUENT is used for solving the governing Eqs. 1-4. Orthogonal and uniform grid size of 6500 elements is used. The time step size for all the simulations in this study is 0.5 s and number of iterations for each time step are 800. The under-relaxation factor all the components, such as velocity components, pressure correction, thermal energy etc. is kept at 0.3. Convergence criteria are set at \( 10^{-7} \) for continuity and momentum and \( 10^{-9} \) for thermal energy. The QUICK differencing scheme was used for solving the momentum and energy equations, whereas the Pressure Staggering Option (PRESTO) scheme was used for pressure correction equations. In enthalpy method, the solution is based on a fixed grid and governing equations are modified such that they are valid for both phases. Also the mushy zone constant is set to \( 10^5 \) [kg/m$^3$s].

4.0 Results

4.1 Validation of the model

Experimental results of Gau and Viskanta (1984), numerical predictions by Brent et al. (1988) and Khodadadi and Hosseinizadeh (2007) for melting of solid gallium in a rectangular cavity are compared with the current numerical data in Fig 2. A two dimensional rectangular cavity of 8.89 cm × 6.35 cm size completely filled with solid gallium was considered and qualitatively, the trends are agreeable among all four approaches, whereas the present computational results are more close to the experimental data and previous numerical predictions.
3.2 Effect of aspect ratio on solidification time

Fig. 3 shows the variation in the heat flux in the nanofluid of $\phi = 20\%$ filled in square cavity (AR = 1.0) and trapezoidal cavity (AR = 1.2-1.8) until 1600 sec (or complete solidification). Note that the tilt angle is kept 0° for these cases. The increasing heat flux with increasing aspect ratio is seen in the figure. As the solidification progresses, layer of solidified nanofluid increases which enhances the thermal resistance, resulting, decrease of heat flux from cold wall. As the aspect ratio increases, the surface area of the cold wall normal to the direction of heat flow increases which increases the heat flux at this wall. Increasing heat flux indicates the rapid solidification of nanofluid in the cavity.
Fig. 4 shows the total solidification time of nanofluid in trapezoidal cavity for different aspect ratios and placed at 0° tilt angle. The figure shows that the solidification time significantly reduces with the increase of the aspect ratio (a longer trapezoidal cavity). This implies that for a specific internal area cavity, the solidification process can be expedited by changing the geometrical structure. Since the internal area (10 mm²) remains constant for all aspect ratios, the amount of PCM filled in the cavity is also equal, so, by changing the aspect ratio, we change the surface area of cold wall normal to the direction of heat transfer, which increases the heat transfer rate. Almost 8.5% increment was seen in the cold wall surface area when the aspect ratio was increased from 1.0 to 1.8 and because of this increment the solidification time of PCM was decreased by more than 17%. Therefore, the aspect ratio of this cavity can be utilized to control the solidification time.

3.3 Effect of wall and fluid temperatures

Fig 5 shows the effect of cold surface temperature on the total solidification time of nanofluid (ϕ = 20%) filled in trapezoidal cavity of aspect ratio 1.4. The effect of five different cold surface temperature, Th = -5°, -10°, -15°, -20°, -25°, and -30° is investigated in simulation Cases 15-20. It was found, as expected, that decreasing the cold surface temperature (Tc) increases the heat transfer rate, resulting, decreases the solidification time. It was found that when Tc was decreased from -5 to -10 °C, the reduction in solidification time was more than half (from 2540s to 1160s) while further reducing the wall temperature from -10 to -15 °C, and so on does not decrease the solidification time this much. This is because, in the first step the temperature was reduced by exact half (-5 to -10 °C) but from -10 to -15 °C and onwards the reduction is less than half so as the saving in solidification time.

Fig 6 shows the effect of cavity aspect ratio on the nanofluid’s solidification time for initial temperature Ti = 0°, 4°, 8°, and 16°. Simulation Cases 26-45 are used to investigate the effect of these parameters on solidification time. This shows that aspect ratio plays a significant role in controlling the solidification time but initial temperature does not affect the total solidification time and it is nearly same for all aspect ratio.
4. Conclusions
The solidification phenomenon of PCM in the horizontal trapezoidal cavity is studied using CFD. The effect of various aspect ratios, cold wall temperature, PCM’s initial temperature, cavity tilt angle, and Grashof number on heat transfer rate in PCM is investigated. The results of this numerical study lead to the following conclusions:

1. Solidification time is significantly decreased with the increase of the aspect ratio; e.g., the trapezoidal cavity with an AR=1.8 requires almost 17% lesser solidification time than that of the square cavity (AR = 1.0) having the same internal area. Therefore cavity aspect ratio can be used as a controlling parameter to improve the heat transfer rate.

2. The decrease of the cold surface temperature ($T_c$) decreases the solidification time significantly (nearly 50%) for the case where $T_c$ was reduced from -5 to -10 °C but relatively less significant for other cases in the square cavity as well as trapezoidal cavity.

The theoretical prediction in this paper is hoped to be a useful guide for experiments dealing with the study of effectiveness of aspect ratio of trapezoidal cavity filled with PCM in improving the heat transfer rate.
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