Observation of Partially Suppressed Shot Noise in Hopping Conduction

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We have observed shot noise in the hopping conduction of two dimensional carriers confined in a p-type SiGe quantum well at a temperature of 4K. Moreover, shot noise is suppressed relative to its “classical” value 2eI by an amount that depends on the length of the sample and carrier density, which was controlled by a gate voltage. We have found a suppression factor to the classical value of about one half for a 2 μm long sample, and of one fifth for a 5 μm sample. In each case, the factor decreased slightly as the density increased toward the insulator-metal transition. We explain these results in terms of the characteristic length (≈ 1μm in our case) of the inherent inhomogeneity of hopping transport.

Shot noise, which is a manifestation of the particle nature of the electric current, has lately received much attention because it can yield information complementary to that obtained from conductance measurements. It is most pronounced when the current is formed by statistically independent charges tunneling through a single potential barrier of low transparency, in which case the noise power spectral density, S, is equal to the Schottky, or classical, value of current direction is smaller than the inelastic scattering length, it is

F = 1/N. When, like in hopping, tunneling occurs between single electron states, depending on their occupancy, shot noise suppression can be a different function of N. Since the equivalent resistances of the various hops are exponentially different from each other and only the most resistive hop (“bottle neck”) determines the current, it could be argued that effectively N = 1. However, in real quasi one-dimensional hopping, in which there is a maximum resistance obtainable (hard hop), the effective N should be the number of hard hops along the sample length, as in the case of identical barriers.

In a 2D system, hopping conduction can be seen as occurring through a network of one-dimensional chains connected to each other at certain nodes, as is normally done in percolation theory, where the network is modeled by resistors of exponentially different values, out of which the most conductive subnetwork (critical percolation subnetwork) is selected. The characteristic size of this subnetwork is the length beyond which the sample is homogeneous, and its nodes are such that each chain contains only one resistor with the largest resistance (hard hop). Even this simpler subnetwork is still complicated enough as to make it difficult to guess, let alone to calculate, what the effective F will be.

To answer experimentally this question we chose a 2D hole system confined in a modulation-doped SiGe well.
The heterostructure, grown by molecular beam epitaxy on a n-Si substrate, consists (from the substrate up) of 4300 Å of Si boron-doped at $1 \times 10^{18}$ cm$^{-3}$, 225 Å of undoped Si, 500 Å of Si$_{0.8}$Ge$_{0.2}$ (quantum well), 275 Å of undoped Si, and finally 725 Å of Si boron-doped at $1 \times 10^{18}$ cm$^{-3}$. The 2D hole density and the in-plane resistivity were controlled by a voltage $V_g$ applied to an Al Schottky gate deposited on the top layer. The heterostructure was processed into samples with gate width of 50 μm and length (along the current direction) of either 2 μm or 5 μm. The noise measurements were done with the samples immersed in liquid He. At $T = 4K$ even for the smallest $V_g$ used the samples had resistance per square much larger than the quantum resistance $h/2e^2$, so that they were always in the insulating regime.

Current through the sample was produced by applying a dc bias and a small ac signal, $V_{in}$, to a 1 MΩ load resistor. The voltage drop across the sample, $V_{sd}$, was measured simultaneously with the ac signal and the noise, using a lock-in technique and a spectrum analyzer. The $1/f$-noise contribution was reduced by doing the measurements at high frequencies up to 100 kHz, which demanded minimizing lead capacitance and required placing a preamplifier inside the cryostat and very close to the sample. The preamplifier, with an output resistance of about 100 Ω, was a commercial low power switching MOSFET, connected in source follower configuration, as in [1]. To avoid heating, the dc current through the preamplifier was kept at 1 mA, enough to get an amplification coefficient of 0.9. The input impedance (defined by the resistance of the sample in parallel with the load resistor and parasitic capacitances) of the preamplifier was proportional to the preamplifier’s transfer function, $TF$ ($V_{out}/V_{in}$), whose real and imaginary components were determined with the ac signal applied to the load.

Figure 1(a) shows the measured output voltage noise for a 2 μm long sample as a function of gate voltage, in the absence of any in-plane current (zero bias) and for three different frequencies (20 kHz, 50 kHz, and 80 kHz). The origin of this voltage is thermal noise. As expected, the voltage noise spectral density follows the real part of the impedance and of the transfer function, shown in Fig. 1(b). The background preamplifier noise, seen at small $V_g$ (small sample resistance) in Fig. 1a, has a $1/f$ dependence. Taking into account this noise, about $6nV/\sqrt{Hz}$ for $f_0 = 80kHz$, we confirmed that the measured noise is indeed thermal noise at $T = 4K$ (insert in Fig. 1b). When the sample resistance is much larger than the load resistance (at high gate voltage) the TF saturates. From the saturation value we determined a parallel to the sample capacitance of about 2$pF$, which is mainly the gate-drain capacitance of the preamplifier.

The transfer function exhibits small but noticeable oscillations at $V_g \approx 0.5V$, as illustrated in Fig. 1(b). Their presence suggests that the sample is close to the mesoscopic size, where conductance is not self-averaged but depends on a particular spatial configuration of the fluctuation potential. In the language of percolation theory, we can say that those oscillations reveal that the length of the sample and the size of the critical subnetwork are comparable [2] [3].

![FIG. 1. The output voltage noise for a 2μm long sample (a) and the real part of the transfer function (b) as a function of gate voltage, at three frequencies. The insert depicts the power spectral density noise, obtained from (a), as a function of the input impedance determined from (b). Three curves are superimposed on the graph, one for each of the three frequencies at which the voltage noise was measured. For each frequency, the maximum resistance at which the voltage noise was measured is indicated by an arrow, starting with the highest frequency on the left.](image-url)
the highest voltage the in-plane electric field is still weak enough as not to modify the critical percolation network, and indicates that the measured shot noise is not sensitive to possible field-induced variations of the hopping percolation paths. From that proportionality, a value \( F = 0.59 \) is obtained for the Fano factor.

![Image](image.png)

**FIG. 2.** Current noise spectral density as a function of the current in a 2\( \mu \text{m} \) long sample subjected to a 0.5V gate voltage, showing the proportionality between current and noise for the entire current range. From the slope of the curves a Fano factor \( F = 0.59 \) is deduced. The three superimposed curves are for measurements at the same three frequencies of Fig.1. The arrows point to the hump that appears for a current of about -10 nA, which is most pronounced at the lowest frequency. Th insert shows the current-voltage characteristic, measured simultaneously with the noise.

It is noticed in Fig.2 that at \( I = -10nA \) there is a hump in the noise spectral density, which is larger at lower frequency. This is a signature of random telegraph noise in mesoscopic structures [14], also seen before in hopping transport [15].

The Fano factor does not depend very strongly on gate voltage, as shown in Fig.3 (top curves). When \( V_g \) decreases from \( V_g = 0.55V \) to 0.2V \( F \) drops from 0.61 to 0.43, which is still roughly one half of the classical value in this range of \( V_g \). Since the smaller the gate voltage the larger the in-plane conductance, for \( V_g = 0.2V \) thermal noise dominates at low current, thus the curvature in the noise characteristic observable in Fig. 3. The transition from thermal to shot noise occurs above 2\( kT/e \).

![Image](image.png)

**FIG. 3.** Dependence of shot noise on gate voltage for samples whose length was either 2\( \mu \text{m} \) or 5\( \mu \text{m} \). For each set of curves (marked by an ellipse) the gate voltages were 0.2V and 0.5V. The straight lines superimposed on the curves of the 2\( \mu \text{m} \) set give Fano factors of 0.61 and 0.43 for \( V_g = 0.55V \) and 0.2V, respectively.

We can explain our results if we assume that there is a characteristic scaling length \( L_0 \approx 1\mu \text{m} \) in 2D hopping for both samples, such that the Fano factor is just the ratio of that length to the length of the sample, \( F = L_0/L \). It is reasonable to assume also that this scale is a characteristic of the homogeneity of the sample, which is the distance between hard hops of the critical percolation subnetwork. Then, the trend for noise suppression on \( V_g \) reflects the fact that hopping becomes more uniform as the sample is driven towards the insulator-metal transition.

To justify these assumptions we can take into account the fact that when an electric field is applied, only hard hops along the field direction are modified (e.g., decrease their resistances) and thus the network is separated into a set of equivalent parallel chains. In this case, the total shot noise would be that of a single chain, which, as we have seen, will have a Fano factor inversely proportional to the number of hard hops in the chain.

The distance between hard hops in the percolation subnetwork can be obtained using [18]:

\[
L_0 = l(T) \left( \frac{T_0}{T} \right)^{\nu+1},
\]

where \( l(T) \) is the characteristic hopping length in the zero-field limit, \( T_0 \) is a characteristic temperature inversely proportional to the density of states at the Fermi level and to the localization radius \( a \), \( d \) is the effective dimensionality of the system (\( d=1 \) for hopping with Coulomb gap and \( d=2 \) for 2D hopping), and \( \nu \) is the critical index of the correlation radius, which is about 1.3 for a 2D system. In turn, the hopping length can be estimated within the percolation model, in which the non-linear conductance \( G(E,T) \) is written [16,18] as
\[ G(E, T) = \frac{I}{Vsd} = G(0, T)\exp\left(\frac{eEl(T)}{kBT}\right), \tag{2} \]

where \( E \) is the electric field. (The other symbols have their usual meaning.) This expression is only valid in the low-field regime, that is, when \( eEA < kBT \). The hopping length depends on temperature as

\[ l(T) = a\left(\frac{T_0}{T}\right)^{1/2}. \tag{3} \]

When Eq. (2) was used in combination with the experimental \( I - Vsd \) characteristics (Fig. 2, insert) a value of \( l \approx 0.08 \mu m \) was obtained for the hopping length of the 2\( \mu m \) sample at \( T = 4K \). The localization radius was estimated to be 100-130\( \AA \) from Eq. (3) and the experimental temperature dependence of the zero-field conductance in the range \( 2K < T < 30K \). The 30\( \AA \) variance of \( a \) reflects the difficulty in discerning experimentally between 1/2 and 1/3 for the exponent in Eq. (3). This estimation of the localization radius is consistent with a fluctuation potential created by interface impurities separated about 200\( \AA \) from each other (surface density of \( 2 \times 10^{11} cm^{-2} \) \cite{24}) and validates our low-field assumption since for \( Vsd = 30mV \) (insert, Fig. 2) it is \( Ea \approx 0.15mV < kBT/e \approx 0.36meV \).

Finally, from Eq.(1) we get either \( L_0 \approx 0.8 \mu m \) or \( \approx 1.2 \mu m \), depending, again, on whether we use \( d = 1 \) or 2. A similar analysis for the data on the 5\( \mu m \) sample yielded a value of 1\( \mu m \) for the inter-node distance. This result tells us that if two nodes were separated by 1\( \mu m \), than there would be two hard hops in a 2\( \mu m \) sample and five such hops in a 5\( \mu m \) sample. Consequently, according to the above discussion, the corresponding Fano factor in the shot noise formula should be 0.5 and 0.2, respectively. These values are in excellent agreement with the experimental results.

Although these results have been obtained for a 2D hole gas, they should be general to any other system in the hopping regime. It also follows from our results that by decreasing the sample length even further one could obtain full shot noise, corresponding to tunneling through only one hard hop along the current direction. Interestingly, a further decrease in length and in temperature should cause a transition to resonant tunneling transport, such as resonant tunneling through impurities \cite{22}. The only calculation available in such a regime for the shot noise is for tunneling through one impurity, done recently \cite{23}, in which \( F = 3/4 \). We hope that the experiments presented here will stimulate calculations in the entire hopping conduction regime.

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