It is advisable that parts whose shape is complex and which are made from solid or hollow blanks should be made by means of transverse and combined radial-longitudinal extrusion. The variation of manufacturing modes, tool configurations (in the form of chambers and rounding of the transitional sections of matrices) requires an adequate preliminary assessment of the force regime and the features of part shape formation. This paper proposes a curvilinear kinematic module of the trapezoidal form for modeling radial-longitudinal extrusion processes in the presence of matrix rounding. Given the impossibility of using a quarter-circle boundary for the kinematically assigned possible velocity field, it has been proposed to use approximate curves in the form of \( z_1(r) \) and \( z_2(r) \). Taking into account the slightest deviation in the length of the arc of the approximate curve \( z_1(r) \) and the area of the curvilinear trapezoid bounded by it relative to a quarter of the circle (not exceeding 0.8 % for any ratio), it has been recommended using this particular replacement. We have performed calculations of the value of the reduced deformation pressure inside the kinematic module with rounding taking into consideration the power of cutting forces at the border with adjacent kinematic modules. An example, the devised module with rounding embedded in the estimation scheme of radial extrusion was analyzed. A significant impact of friction conditions on the force mode and the corresponding optimal value of the rounding radius have been identified. The resulting kinematic module makes it possible to expand the capabilities of the energy method for modeling cold extrusion processes involving the tools of complex form according to new deformation schemes. That could contribute to preparing recommendations and more active industrial implementation of these processes.

**Keywords:** simulation of combined extrusion processes, tool configuration, kinematic module, energy method

1. Introduction

Under current conditions, the engineering industry plays one of the key roles in ensuring the development of the economy. In this context, the priorities are determined by the search, development, and implementation of state-of-the-art science-intensive technologies, as opposed to the classic techniques of part shape formation by removing chips. In modern realities, the variety of methods for processing metals by pressure (PMP) renders relevance to the task of choosing the most rational and economically one defined in [1–9]. Examples of successful solutions of specific practical issues [2–5]...
are inextricably linked with the development and constant improvement of finite-element modeling methods, statistical data processing [4–6]. That makes it possible to expand the boundaries of applying the selected deformation technique by determining the optimal manufacturing modes, tool configuration, as well as the conditions that provide for the quality of resulting articles [3, 7]. Theoretical advancements in the research of a wide range of processes relate to the development and improvement of methods for describing the shape of bodies, searching for criteria and procedures for simplifying the assessment of force regimes, and the shape formation of semi-finished articles [2, 6, 7].

Promising resource-saving technologies now include the PMP methods based on cold plastic deformation. That makes it possible to obtain blanks whose dimensions and quality approach the required finished parts so that the additional machining by cutting is reduced or completely excluded [1, 7–11]. The persistent tendency to expand the range and materials of parts that are made by using cold extrusion processes indicates the need for in-depth research of factors that could contribute to extending the technological capabilities of these processes.

2. Literature review and problem statement

Theoretical and experimental studies of recent years, primarily to obtain an assessment of the force regime and identify the peculiarities of the deformation process, contributed to the expansion of the manufacturing capabilities of cold extrusion processes. Analysis of force and deformation modes of processes of combined (combined or consistent) extrusion was performed mainly experimentally, based on a finite-element method (FEM) and the method of the upper estimate. The techniques of «matrix-free extrusion» in the manufacture of deep hollow vessels achieve a significant decrease in the specific forces of deformation compared to the use of reverse extrusion processes [12]. The force mode of the process of consistent radial-direct extrusion was investigated in work [13] where the authors analyzed the influence of structural and manufacturing parameters. They verified the reliability of the results acquired from FEM by using experimental data on deformation efforts. However, the specified processes were simulated within the selected limits of variation parameters; no analytical solutions were obtained, which reduces the value of the reported results and conclusions. Paper [14] investigated the influence of the structural geometrical parameters of the process of consistent radial-direct extrusion on load fluctuations. However, the conclusion about a minor effect exerted on the formation of the load on the punch and counterpunch by the radii of the rounding of the transitional sections of the tool is debatable. The geometry of the tool and the radii of transitional areas of matrices are especially important when designing dies when these elements affect not only the active deformation forces but also the reactive forces of opening the detachable matrices [15–17]. All this allows us to assert that it is advisable to develop theoretical methods for predicting part shape formation and evaluating the force regime depending on the tool configuration.

An experimental study and a finite-element analysis, based on the ABAQUS software package, of the three-way direct-reverse-radial extrusion of hollow parts, were reported in work [18]. The distortion of the dividing grid and the distribution of deformations, the influence of the radius of the matrix rounding, the thickness of the gap, and friction conditions on the deformation force mode were analyzed. Further studies into the process of a three-way direct-reverse-radial extrusion of hollow parts in terms of the shape formation features and main control factors were carried out in works [19, 20].

The issues related to estimating the stressed-strained state of the workpiece and tool, defect formation, and the deformation of the metal of blanks during cold combined extrusion were discussed in [21–24]. Papers [21, 22] assessed the stressed-strained state (SSS) and calculated the plasticity resource according to various criteria for the process of combined radial-direct extrusion. The main purpose of the finite-element analysis of the process of radial-direct extrusion in [23] was to study the effect of tool geometry on the workpiece’s SSS and the tool wear. Along with the advantages of cold extrusion processes, one should note the existence of restrictions and disadvantages in the form of defect formation, which significantly reduces the quality of the articles produced. Paper [25] systemized the types of defects characteristic of cold extrusion based on different manufacturing techniques and represented them in the form of a generalized classification. The basic parameters that determine the quality of stamped parts include deviations in the shape and surfaces of parts, non-compliance with size, surface quality, uneven deformed state, cracks (detachments) and destruction. A common defect characteristic of most extrusion techniques is deviations in shape, as well as cracks. Therefore, determining the parameters and ratios that are critical in terms of possible defect formation, technological factors that make it possible to delay or level the emergence of a defect are paramount tasks. That could significantly expand the application scope of cold extrusion technologies, primarily for combined extrusion involving several degrees of freedom of the metal flow [25, 26]. However, most of the reported studies are limited in nature and do not derive engineering formulae for calculating the force regime (especially in the presence of a tool whose shape is complex), which indicates the insufficient suitability of the obtained results for use in production.

The effectiveness of using the energy method of power balance to assess the force regime, the part shape formation in the processes of combined pressing is demonstrated in works [27–33]. The processes of the combined reverse-direct extrusion and two-way extrusion for making internal and external flanges from pipe blanks were modeled in papers [27, 28]. The estimation schemes of these processes demonstrated the possibilities of using the unified kinematic modules of the rectangular and trapezoidal shape. The mathematical models, built on the basis of the simplified estimation schemes of parallel flow using rectangular kinematic modules, make it possible to receive operational solutions to technological tasks. That, in turn, could lead to a significant difference with experimental data on the efforts of deformation and increments of a semi-finished article.

The development of capabilities of the energy method of power balance based on the expansion of the base of kinematic modules of complex configuration was addressed in a series of works [29–36]. Papers [29, 30] suggested techniques for simplifying the power of plastic deformation forces using a minimizing function in a general form. The force characteristics of the process of combined three-way extrusion were determined in work [31] using the planning of the experiment according to modeling data in DEFORM 3D. That allowed the authors, among other things, to acquire data on the shape change in a semi-finished article within
the ratios of the deformation process chosen for the cited study. The main research results reported in [32, 35] make it possible to take into consideration the peculiarities of tool configuration, the shape of the boundaries of the distribution of a metal flow inside the workpiece. The authors identified the complication of the mathematical apparatus when deriving the value of the reduced deformation pressure (in an analytical form) inside the kinematic module of a triangular or trapezoidal shape with slanting boundaries. A series of works attempted to resolve the issues related to searching for techniques to simplify the expressions of the reduced deformation pressure and deriving the appropriate engineering formulae for calculating the force regime of cold extrusion processes [33, 34, 36]. The variation of manufacturing modes, tool configurations in production significantly expands the possibilities of cold extrusion processes, on the one hand. On the other hand, they require an adequate preliminary assessment of the force regime and part shape formation. In turn, devising new kinematic modules whose shape is complex would contribute to preparing appropriate recommendations for their use to derive an assessment of the deformation force regime and determine the optimal configuration of the tool.

3. The aim and objectives of the study

The aim of this work is to design a kinematic module of the trapezoidal shape with rounding, which could make it possible to take into consideration the tool configuration when modeling the processes of radial-longitudinal extrusion.

To accomplish the aim, the following tasks have been set:

- to analyze the effect of the existence of a matrix rounding on the features of radial-longitudinal extrusion processes and its application as a factor to manage the deformation process;
- to design a scheme of the solution of a kinematic module with rounding, which could make it possible to determine, in an analytical form, the reduced deformation pressure;
- to analyze the possibilities and features of using a kinematic trapezoidal module with rounding to optimize the toll shape.

4. Shape formation of parts with a flange in the processes of combined radial-longitudinal extrusion by a tool whose configuration is complex

4.1. The processes of combined radial-longitudinal extrusion involving a tool whose configuration is complex

Varying the shape of the tool (a punch and a matrix) by the introduction of such structural features as the chamfers of different angles of inclination and the rounding of different radii expand the possibilities of combined extrusion processes (Fig. 1). As regards processes with one degree of freedom of a metal flow, this helps reduce the force parameters of the deformation mode and create more favorable stressed-strained state in the contact area between the tool and workpiece. As regards the processes of combined extrusion with several degrees of freedom of a metal flow, in addition to the above advantages, the radius of the rounding can be considered as a controlling factor of the part’s shape formation. Examples of the effect of tool configuration on the shape formation of a semi-finished article in the processes of combined radial-longitudinal extrusion are shown in Fig. 2.

The presence of rounding on the matrix in the processes with one degree of freedom of a metal flow helps reduce the force parameters of the process and form a more favorable stressed-strained state in the contact area between the workpiece and tool. As regards the processes of combined extrusion with several degrees of freedom of a metal flow, in addition to the above advantages, the radius of the rounding can be considered as a controlling factor of the part’s shape formation. Examples of the effect of tool configuration on the shape formation of a semi-finished article in the processes of combined radial-longitudinal extrusion are shown in Fig. 2.
4.2. Designing a scheme of the solution of a kinematic module of the trapezoidal shape with rounding

As part of the application of the energy method of power balance, an important step is to split the volume of a workpiece into a set of kinematic modules that make up the estimation scheme of the process. The possibility to build a huge number of different sets of kinematic modules with different kinematically possible velocity fields (KPVF) is the advantage of a given method. However, this requires the development of recommendations on the feasibility (or not) of using a kinematic module and, subsequently, a particular estimation scheme of the process. The complex shape of the boundary of contact between the tool and workpiece requires the use of kinematic modules with a curvilinear boundary (triangular, trapezoidal, or rounded) [32]. Finding the value of the reduced deformation pressure for the designed kinematic module, taking into consideration the power of deformation, friction, and shear forces in analytical form could be used as part of the general estimation scheme of the process under study. The existence of a wide range of extrusion processes with a developed radial component of the metal flow requires a more detailed study in terms of reflecting the real pattern of the metal flow into the flange zone, taking into consideration the presence of chamfers or rounding on the surface of the contact between the matrix and workpiece in zone Z3 (Table 1).

The most commonly used kinematic modules of zone Z3 or the next adjacent zone are kinematic modules of the rectangular, trapezoidal, and triangular shape. The trapezoidal kinematic module with a boundary of different shapes that can be chosen is the kinematic trapezoidal module Z3, which, in a particular case Z3r, reflects the rounding of the matrix in the zone of the radial flow of metal that forms a flange [32].

The impossibility of using a curve describing a quarter circle \( z(r) = h + R - \sqrt{R^2 - (r - (R + R))^2} \) (rounding of the radius \( R \) matrix) for the assigned KPVF is due to the impossibility to meet the condition \( z'(R_j) \), which makes it impossible to conduct further calculations of the cutting and deformation power. Therefore, to model processes with the contact boundary of the matrix with the workpiece in the form of a quarter circle, it is necessary to analyze the possibilities of using curves approximate in shape.

Consider curves that are close in shape to a quarter of a circle:

\[
z_i(r) = \frac{A}{r-R} + B, \quad \text{where} \quad A = \frac{R^2}{2\pi} \left( \frac{R}{2\pi} \right) \quad \text{and} \quad B = \frac{h_i - \frac{2\pi A}{R}}{R}.
\]

Let us demonstrate the difference between these curves and the quarter of a circle (Fig. 3) by moving to the relative values \( \bar{R} = 1, \bar{R} = R / R, \bar{h} = h / R, \bar{h}_{i+1} = h_{i+1} / R \). The quarter-circle curve is shown by a black line, approximate curves – blue, for \( z_1(r) \), and red, for \( z_2(r) \), dashed lines. The character of the curve change is closely similar but the deviation of the curve \( z_2(r) \) by arc length and area of the curvilinear trapeze from a quarter of a circle is less in relation to the use of approximation in the form of \( z_2(r) \).

Deviations of \( z_2(r) \) from a quarter of a circle by the arc length \( l_2 \) and area are more significant and depend on the geometric position. Thus, the assigned curve \( z_1(r) \) is rational at \( z_2(r) \) and can be used as an approximate replacement for a quarter of a circle. This replacement provides the possibility of using KPVF in the above form (Table 1), where, unlike a quarter of a circle, the condition of the \( z'(R_j) \) existence is met, which makes it possible to perform the required calculations.

The components of the reduced pressure deformation of the kinematic module Z3r take into consideration the power of deformation forces, the forces of cutting at the surface between the adjacent kinematic modules, and the friction forces on the contact surface of the tool and workpiece. Here is the friction force power on the rounding surface and at the lower boundary of the kinematic module in the form of [32]:

| Generalized estimation scheme of the process of combined extrusion of parts with flange | The scheme and KPVF of the kinematic trapezoidal module Z3 |
|---|---|
| ![Diagram](image) | ![Diagram](image) |
| Module Z3 KPVF: \( V = R h V \) \( z(r) \) \( z_{(r)} \) \( z_{(r)} \) | Where \( z(r) \) is any curve passing points \((R, k)\) and \((R, h_{i+1})\) |
The deformation force power inside the kinematic module Z3r is found in the form of an upper estimate [32]:

\[
N_z < \sigma_s \sqrt{M \cdot W},
\]

where \( M = \int \int \int \epsilon^2 dV \), \( \epsilon \) is the intensity of deformation rates;

\[
V = 2\pi \left[ A \left( R + R \left( 1 - \frac{R}{2\pi} \right) \right) + \frac{B R^2}{R R} \right]
\]

\[
N_a = 4\pi^2 \sigma_a A V h_i,
\]

\[
N_{-acvi} = \frac{\pi \sigma_a R h_i A}{\sqrt{3C^2}}.
\]

The value of the reduced pressure deformation of the kinematic module Z3r, taking into consideration (3) to (6), is expressed by a ratio or is calculated from the following formula:

\[
\beta_{Z3r} = \frac{N_{a} + \Delta N_{a} + \Delta N_{acvi} + N_{-acvi}}{2\pi \sigma_a V R_i h_i},
\]

where expressions:

\[
\Delta N_{a} = |N_{a} - N_{a}^-| \quad \text{and} \quad \Delta N_{acvi} = \begin{bmatrix} N_{acvi}^- - N_{acvi}^- \end{bmatrix}.
\]

4.3. Analyzing the application of a rounded kinematic curvilinear trapezoidal module with rounding in order to optimize the tool shape

A given value takes into consideration the effect of the radius of rounding for all schemes containing a set of these modules. Including the processes of sequential radial-direct extrusion (Fig. 1, a) and the processes of combined radial-reverse extrusion for relatively high blanks in the presence of a disconnected deformation site (Fig. 2, a). Increasing the radius of rounding significantly affects the component of the friction force power on the bordering surface between the workpiece and tool (Fig. 4). Under any friction conditions characteristic of cold extrusion, the reduced deformation pressure curves have a minimum point, which indicates the possibility of optimizing the tool shape based on the radius of rounding. However, the consideration of the reduced deformation pressure only for the Z3r kinematic module and the full value of the entire estimation scheme indicates differences in the optimal value of the rounding radius. For the full estimation value, this optimal value is shifted towards growing and is 0.7 mm relative to 0.2 mm for the same ratios without taking into consideration the adjacent kinematic modules. This is due to changes in the volume of deformation sites Z2a and Z4a, bordering the kinematic module Z3r. Changing the sites’ parameters affects their contribution to the value of the resulting deformation pressure and the degree of the radial flow of the entire estimation scheme.

Therefore, in further studies, all components of the estimation scheme should be taken into consideration, depending on the chosen optimization parameter; in this case, on the radius of rounding. Our analysis of the impact of flange zone thickness and friction conditions on the amount of complete reduced deformation pressure has revealed the following features. Deterioration in the friction conditions at any ratio of geometrical parameters of the process leads to a decrease in the optimal value of the radius of rounding. This is due to an increase in the contact area between the workpiece and tool and, accordingly, an increase in the power of the friction forces.

At the same time, with an increase in the thickness of the flange zone, the optimal value of the radius of rounding also increases along with a decrease in the amount of the reduced deformation pressure. Fluctuations in the optimal value of
the radius of rounding vary from 2.5 mm to 1.2 mm with a deterioration in the friction conditions (Fig. 5), and from 1.7 mm to 0.9 mm for $\mu_s=0.08$, with a decrease in the relative thickness of the flange. In this case, the differences in the reduced deformation pressure for different friction conditions can reach 35–40 %, which indicates a significant contribution of the components of friction force power to the force regime.

Thus, the radius of rounding can be considered as an optimization parameter for processes with one or more degrees of freedom of metal flow with the presence of a radial current in the flange zone. At the same time, for the processes of combined radial-longitudinal extrusion, our calculations can be used for an autonomous (lower) deformation site.

Most of the earlier constructed estimation schemes of combined radial-longitudinal extrusion processes did not take into consideration rounding on the matrix, which led to an overstatement of the projected estimates of the deformation force mode.

The proposed variant of replacing the curve has made it possible to take into consideration the presence of complex areas of the contact surface of the workpiece and tool. We have compared the experimental and theoretically derived data on the deformation effort of billets made from the aluminum alloy AD1. The billet dimensions are as follows: billet’s radius, $R_1=7.5$ mm; inner radius, $R_3=12$ mm; outer radius, $R_2=14$ mm; the flange zone thickness $h$ ranged from 2.2 mm to 5.2 mm; complete punch working run for the process of radial-straight extrusion (Fig. 1, a) was 43 mm. In the Figure, the average value is indicated as data on the experiments carried out on the samples made from AD1 (for three deformed samples for each thickness of the flange $h$ by deformation effort (point data); a corresponding 90 % confidence interval was built, calculated according to the data in Table 3.

To build the theoretical curves of deformation forces, we used the material AD1 strengthening curve, approximated by the index function $\sigma_s=138.4\cdot e^{0.218} \text{ MPa}$ with the average intensity of accumulated deformation, which is equal to the value of the reduced deformation pressure; the friction conditions were set as $\mu_s=0.04$. The curve constructed according to

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the EM-2a scheme, which does not take into consideration the radius of rounding (Fig. 6, blue solid line), is a preliminary rough assessment of the deformation effort. The dashed curve of efforts, calculated according to EM-2c, taking into consideration the radius of rounding, acquired theoretically taking into consideration the adjusting coefficient (which is permissible when using the upper estimation energy method), which depends on the geometrical parameters of the process, makes it possible to derive the predicted value of the effort in the following form:

\[
P' = \left(1 - \frac{h^2 \left( \frac{R_1}{R_2} \right)^2 - 1}{R_2^2} \right) P_0.
\]

(8)

Consideration of curve (8) as an estimated one for deformation efforts is acceptable; the convergence of theoretical and experimental data predetermines the determination coefficient \(R^2 = 0.898\).

| Effort \( P \), kN | Average effort \( P_{\text{ave}} \), kN | \( \Delta P \) | \( P_{\text{ave}} - \Delta P \) | \( P_{\text{ave}} + \Delta P \) |
|-------------------|------------------------|--------|-----------------|-----------------|
| 2.2               | 121                    | 128    | 129             | 126             |
| 3.3               | 123                    | 116    | 121             | 120             |
| 4.3               | 115                    | 123    | 125             | 121             |
| 5.2               | 126                    | 117    | 123             | 122             |

To detect miscalculations given a small number of parallel measurements \((n < 10)\), we additionally used a check according to Romanowski criterion, according to the Q-criterion and R-distribution. Based on the three criteria used for a small number of parallel measurements, no errors were detected, confidence intervals are left in full.

Fig. 6. Comparative analysis of the experimental and theoretically obtained data on deformation efforts

Thus, the consideration of rounding is a significant factor in reducing the estimate of the estimated value of deformation effort; the possibility of using the rounding radius as a varying parameter could make it possible to determine the optimal tool configuration (by the radius of rounding).

5. Discussion of results of studying the possibilities of using a kinematic module of the trapezoidal shape with rounding

The variation of manufacturing modes and tool configuration in the industrial setting requires an adequate preliminary assessment of the force regime and the features in a part’s shape formation. That necessitates expanding the base of the unified kinematic modules of complex shapes and analyzing their inclusion in estimation schemes. That would contribute to preparing appropriate recommendations for their use to obtain an assessment of the deformation force mode and determine the optimal tool configuration. For the processes of combined extrusion with several degrees of freedom of a metal flow, the introduction of the radius of rounding could be used as a factor in controlling the flow of metal. This, along with changing the friction conditions, could solve the issue of adjusting the increments of semi-finished articles, subject to their deviation from required. To model the processes of radial-longitudinal extrusion, it has been proposed to use approximate curves in the form of \( z_1(r) \) and \( z_2(r) \) as a replacement for a quarter of a circle, which reflects the rounding of the matrix (Fig. 3). It has been established that the deviation of the length of the arc of the approximate curve \( z_2(r) \) in form (1) and the area of the curvilinear trapezoid bounded by it does not exceed 0.8 % for any ratio, which indicates the rationality of using this particular replacement. We have calculated the value of the reduced deformation pressure inside the kinematic module with rounding \( Z_{3r} \) taking into consideration the power of cutting forces at the border with the adjacent kinematic modules in form (7). It has been established that the radius of rounding could be used as a parameter to optimize the tool configuration by the amount of the reduced deformation pressure (Fig. 4, the presence of minimum points). Our comparison of the derived optimal values of the radius of rounding for the reduced deformation pressure (Fig. 4, a) and the full value in form (7) indicates the differences in the obtained values (Fig. 4, b), namely the offset in the direction of increasing the optimal value of the radius of rounding. It is recommended, in order to analyze the possibilities of using the radius of rounding as an optimization parameter, to use data on the full value of the reduced pressure of the entire estimation scheme (Fig. 6). It is also recommended to use, as a component of the full value of the reduced deformation pressure for module 3’, formula (7), and the reduced pressure of known unified adjacent kinematic modules 1, 2, and 3–5, according to the earlier reported studies [32, 34].

The developed kinematic module with rounding \( Z_{3r} \) makes it possible to expand the capabilities of the energy method for modeling cold extrusion processes involving a tool whose shape is complex according to new deformation schemes. In practical terms, it is necessary to include the reduced deformation pressure in form (7) according to the method of kinematic modules. The development of theoretical research may address the construction of similar kinematic modules of the trapezoidal or triangular shape with a lower boundary with rounding. That would make it possible to use our calculations in the new schemes in the future and could contribute to assessing the force regime and a shape change. This would help prepare recommendations for choosing the optimal configuration of the tool (determining the optimal value of the radius of rounding the matrix), in order to extend the possibility to control a part’s shape
formation in processes with several degrees of freedom of flow. This opens up the possibilities of analyzing the shape formation of precision and small-sized parts with thin elements in their geometry, which, taking into consideration their scale, significantly affect the nature of the current and the filling of the matrix cavity. Thus, the issues of determining the growth in a semi-finished article could be resolved, as well as the possibility of adjusting them if necessary, which would contribute to the more active implementation of these processes at an industrial scale.

6. Conclusions

1. The positive effect of the presence of rounding on the matrix on the course of radial-longitudinal extrusion processes in terms of ensuring a more favorable stressed-strained state in the contact area between the workpiece and tool has been detected. For processes with several degrees of freedom of a metal flow, the introduction of rounding is a factor influencing the shape change in a semi-finished article and the character of the metal flow. Thus, the development of kinematic modules, which take into consideration the features of tool configuration, could solve the task of finding effective factors for managing the process of a part's shape formation.

2. When designing a kinematic module of the trapezoidal shape with rounding, it was found that it is impossible to use a quarter of a circle for a given KPVF due to the failure to comply with the condition $\zeta(R)$. As an approximation, a curve in the form of $z_1(r)$ (a deviation of the length of the arc of an approximate curve and the corresponding area of the curvilinear trapezoid does not exceed 0.8% for any ratios) has been recommended. We have managed to derive an expression for the reduced deformation pressure in an analytical form dependent on the radius of the rounding and the geometrical parameters of the kinematic module. This greatly simplifies further analysis to determine the optimal tool configuration (a value of the rounding radius).

3. The possibilities of using the kinematic trapezoidal module $Z_{3r}$ with rounding to optimize the shape of the tool according to the radial extrusion scheme have been analyzed. We established a significant impact exerted by friction conditions on the force regime (differences for varying friction conditions can reach 35–40%); the optimal value of the radius of the rounding has been derived, which also depends to a large extent on the radius of the matrix. The developed kinematic module $Z_{3r}$ with rounding allows expanding the capabilities of the energy method for modeling cold extrusion processes involving a tool whose shape is complex according to new deformation schemes.

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