Intermediate-mass black holes as LISA sources

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Received 27 October 2008, in final form 15 December 2008
Published 20 April 2009
Online at stacks.iop.org/CQG/26/094031

Abstract
Intermediate-mass black holes (IMBHs), with masses in the range $\sim 10^2 - 4 \, M_\odot$, would be unique sources of gravitational waves for LISA. Here we discuss their context as well as specific characteristics of IMBH–IMBH and IMBH–supermassive black hole mergers and how these would allow sensitive tests of the predictions of general relativity in strong gravity.

PACS number: 97.60.Lf

1. Introduction
Conclusive evidence exists for black holes in the stellar-mass ($\sim 3–30 \, M_\odot$) and supermassive (SMBH; $\sim 10^6–10^{10} \, M_\odot$) ranges. This evidence comes from direct measurements of masses from orbital motion in, respectively, binaries and galactic nuclei. In contrast, the range between these masses does not yet provide clear dynamical evidence for black holes of intermediate mass (IMBHs). This is essentially because candidates are rare (hence the nearest possible IMBH in a binary is not easily observed) and have small radii of influence (hence SMBH-like observations of nearby stars are also challenging). There are, however, numerous indirect suggestions of the $\sim 10^2 - 4 \, M_\odot$ mass range from the high fluxes of ultraluminous x-ray sources (ULXs; see [5, 7]), from their relatively low thermal temperatures [20–22], and from aspects of the dynamics of globular clusters [9, 27, 37]. See [24] for a recent summary of the data. The lack of definitive dynamical evidence means that alternate scenarios have also been proposed for ULXs, including beamed emission [14, 35] and super-Eddington emission [3, 36].

Here we assume that IMBHs exist and explore various ways in which their interactions could lead to gravitational radiation detectable in the $\sim 10^{-2}–10^{-2} \, \text{Hz}$ frequency range of the Laser Interferometer Space Antenna (LISA). In section 2, we discuss the broader context of the IMBHs and proposed formation mechanisms. We also give basic formulae for the amplitude and frequency of gravitational waves from binaries, showing why IMBHs could be bridge sources between space-based and ground-based detectors. In section 3.1, we evaluate the prospects for IMBH–IMBH mergers and what they might tell us. In section 3.2,
we discuss IMBH–SMBH mergers and their prospects as uniquely precise testbeds for strong gravity predictions of general relativity. We present our conclusions in section 4.

2. Context and formation of IMBHs

Most formation scenarios of supermassive black holes propose that they pass through a life stage in which they are IMBHs. Therefore, study of IMBHs can yield insight into the early formation of structure in the universe. This is particularly true if the formation of SMBH seeds is in part due to interactions in dense stellar clusters, as this is a leading candidate for how IMBHs form in the current universe. We now discuss proposed formation mechanisms and their implications.

2.1. Suggested formation mechanisms for IMBHs

The reason for setting the lower limit on IMBHs at \( \sim 100 M_\odot \) is that this appears to be in excess of the maximum black hole mass that can form from a solitary star in the current universe (see [17] for a discussion in the context of IMBHs). IMBH existence therefore requires new formation scenarios. The three basic ideas are

- The first generation of stars had negligible metallicity. This reduces radiative opacity and thus the strength of radiation driven winds, hence stars can start more massive and retain more of their mass than current stars [17].
- In young massive stellar clusters that have relaxation times for the most massive stars that are less than the lifetimes of those stars (about 2.5 million years), mass segregation leads to collisions and mergers of massive stars. If the combined objects do not have catastrophic wind mass loss (see [4]), they can, in principle, accumulate up to thousands of solar masses and might then become black holes with similar masses [6, 12, 31, 33].
- If a seed black hole with a mass of more than \( \sim 200 M_\odot \) is formed by the previous process in a dense stellar cluster, subsequent binary–single and binary–binary interactions can allow it to merge and accumulate mass without being vulnerable to ejection by either three-body kicks or recoil from asymmetric gravitational wave emission [10, 11, 25, 26, 28, 29].

Of special interest to LISA observations is that some simulations suggest that if the initial binary fraction in a stellar cluster exceeds \( \sim 10\% \) then more than one IMBH can form in that cluster ([12]; but see [4]). We will explore the consequences of this in section 3.1. In addition, since massive stellar clusters are often found near the nuclei of galaxies that are interacting actively, the clusters themselves can sink to the centers of galaxies where their IMBHs spiral into the galaxy’s supermassive black hole. We discuss this in section 3.2.

The last piece of basic physics has to do with gravitational waves themselves. For a circular binary with a total mass \( M = m_1 + m_2 \) and a reduced mass of \( \mu = m_1 m_2 / M \) at a distance \( d \) from us small enough that redshifts are unimportant, orbiting at a frequency \( f_{\text{orb}} \) so that the dominant gravitational wave frequency is \( f_{\text{GW}} = 2 f_{\text{orb}} \), the angle-averaged dimensionless strain amplitude that we observe is

\[
h = 6 \times 10^{-21} (f_{\text{GW}}/1 \text{ Hz})^{2/3} (M_{\text{ch}}/10^3 M_\odot)^{5/3} (1 \text{ Gpc}/d). \quad (1)
\]

Here \( M_{\text{ch}} \) is the ‘chirp mass’, defined as \( M_{\text{ch}}^{5/3} \equiv \mu M^{2/3} \). The maximum frequency of orbits that evolve relatively slowly is often approximated by the frequency at the innermost stable circular orbit (ISCO), although technically the ISCO concept is strictly valid only when there are no mechanisms for angular momentum loss (i.e., for test particles in geodesic orbits). For
a nonrotating spacetime, this maximum frequency is

\[ f_{\text{GW, max}}(\text{ISCO}) = 4.4 \text{ Hz}(10^3 M_\odot / M). \]  

From this expression we see that IMBHs in the entire mass range of \( M \sim 10^{2-4} M_\odot \) are potential LISA sources, but also that toward the low end of the masses they might be sources for ground-based detectors, which focus on \( f_{\text{GW}} > 10 \text{ Hz} \).

3. Gravitational waves from mergers of IMBHs with other black holes

We will focus on mergers of IMBHs with other IMBHs or with SMBHs, because the rate at which LISA will detect the coalescence of stellar-mass black holes with IMBHs is negligibly low \([39]\), although such mergers may be detected by next-generation ground-based instruments such as Advanced LIGO or Advanced Virgo \([18]\).

3.1. IMBH–IMBH mergers

As demonstrated by \([8]\), if more than one IMBH forms in a young stellar cluster, as may happen if the primordial binary fraction in the cluster exceeds \( \sim 10\% \) \([12]\), the subsequent coalescence of the IMBHs can be visible out to large distances. Specifically, Fregeau \([8]\) found that a comparable-mass binary with a total rest-frame mass of \( 1000 M_\odot \) would have a coalescence visible with LISA out to a redshift \( z \approx 1 \). Given that the star formation rate increases dramatically from \( z = 0 \) to \( z = 1 \), if these mergers occur within a few tens of millions of years after the formation of the clusters then LISA observations of such events could be unique probes of star formation and cluster dynamics.

To explore this further, Amaro-Seoane \([2]\) performed detailed N-body simulations of two IMBHs in a cluster. They started with equal-mass IMBHs (either \( 300 M_\odot \) or \( 1000 M_\odot \) each) at a separation of 0.1 pc, in a cluster of \( 3.2 \times 10^4 \) stars either all at 1 \( M_\odot \) or selected from a Kroupa \([16]\) initial mass function. They then followed the inspiral of the IMBHs until the black holes eventually formed a very hard binary. At that point, they passed off the properties of the binary to a three-body scattering program, where the speeds and masses of the interacting stars were drawn from the appropriate external distribution. Their three major conclusions are

- In the simulations, and by extension in real clusters with more stars, the IMBH merger has only a minor effect on the cluster in general. Superficially, it might seem that the effect could be substantial, because the binding energy of an IMBH binary at the point of the last scattering with a star can exceed the total binding energy of the cluster by a significant factor. Given that interactions with stars are what harden the binary to this point, it might thus appear that the cluster could be disrupted. The reason that this does not happen is that the energy extracted from three-body binary hardening is only put back into the cluster if the star emerges with a speed less than the escape speed of the cluster. In contrast, high-speed ejections share very little energy with the cluster because the relaxation time (which is needed to alter energies significantly) is orders of magnitude greater than the time to leave the cluster. Since the binding energy of the IMBH binary is much less than the binding energy of the cluster at the point when stars are ejected from the cluster, the net effect on the cluster is small.
- The duration of merger is indeed tens of millions of years or less, and as expected is dominated by the phase in which the binary is hard (because the cross section for interactions is less than when the binary is wider). In none of the dozens of runs done in \([2]\) did this phase take more than \( 10^8 \) years. As a result, these mergers will indeed serve as good snapshots of star formation.
In all other categories of comparable-mass black hole mergers, the expected eccentricity upon entry into the frequency band of the relevant detector (LISA for supermassive black holes; Virgo or LIGO for stellar-mass holes) is likely to be so small as to be basically negligible (see [1] for a discussion; for potential exceptions for stellar-mass holes, see [38] and [30]). For IMBH–IMBH coalescences in young clusters, however, Amaro-Seoane [2] find that at the LISA lower frequency limit of $f_{GW} \approx 10^{-4}$ Hz the eccentricity is commonly $e \sim 0.1–0.3$. As they demonstrate, this is detectable and will have to be included in algorithms that characterize the inspiral.

In figure 1, we show two sample evolutions of eccentricity versus the semimajor axis for runs similar to those of [2]. The systematic increase in eccentricity down to separations of $a \sim$ few AU is consistent with the results of Quinlan [34], who found that a massive binary interacting with much less massive stars commonly increases its eccentricity. The decrease at smaller semimajor axes is the result of circularization by gravitational radiation dominating over increases in eccentricity caused by three-body interactions.

### 3.2. IMBH–SMBH mergers

When IMBHs merge with the central SMBHs in galaxies, the resulting gravitational waveform contains uniquely precise information about the spacetime around a rotating black hole. The basic reason is that the mass ratio is extreme enough (typically $\sim 1000 : 1$) that approximate techniques can be used to follow the inspiral, without the need to resort to computationally expensive numerical relativity simulations. These are, therefore, similar to the more familiar
extreme mass ratio inspirals (EMRIs), in which a stellar-mass black hole of typically $\sim 10 M_\odot$ coalesces with a SMBH (see [1] for a review). However, when the secondary is an IMBH with a mass of $\sim 1000 M_\odot$ the amplitude of the waves at a given luminosity distance is two orders of magnitude greater than for a standard EMRI, hence the waveform and comparisons of it with general relativistic predictions can be obtained with far greater precision.

Miller [23] showed that for favorable cases (e.g., $M_{\text{SMBH}} = 10^{5.5} M_\odot$, $M_{\text{IMBH}} = 10^3 M_\odot$ and an angular diameter distance of 3 Gpc) the signal to noise near the end of the inspiral can be great enough that the source would be detectable with LISA in a standard power density spectrum, without the need for matched filtering. More specifically, if one took a power density spectrum over the optimal period of time, in which the frequency drift during this period is equal to the frequency resolution of the spectrum (which equals the reciprocal of the period of integration), then the signal to noise of the single peak occupied by the inspiral during this period would be $S/N = \text{tens}$. As a result, it would be possible to string together such detections and connect the phases of the inspiral, thus building up an empirical waveform without the need to assume that general relativity is correct. Hence, even a single detection of such an event would provide a uniquely powerful test of the properties of massive spinning black holes.

The rate of IMBH–SMBH mergers depends on various details of the dynamics of IMBHs in dense stellar clusters. The basic idea is that although IMBHs not in galactic centers cannot by themselves spiral to the core within a Hubble time (because dynamical friction on such light objects is too weak), if they form in massive clusters within tens to possibly hundreds of parsecs of the center then the cluster will sink as a unit within a few billion years. The cluster itself is eventually disrupted by the tidal field of the galaxy and SMBH, leaving the now solitary IMBH much closer than before and able, in principle, to spiral in to merger (see [23]). This has been proposed as one mechanism to shepherd the young, massive $S$ stars observed near the center of our Galaxy [13]. Matsubayashi et al and Portegies et al [19, 32] examined this process using N-body simulations, and concluded that the rate of LISA detections of these events could be tens per year, depending on how efficiently IMBHs form in clusters.

More recently, Koch and Hansen [15] explored additional effects, such as the interactions of IMBHs with themselves around an SMBH, assuming that the full coalescence process takes longer than the time needed for new clusters and IMBHs to sink to the center. They found that such encounters tend to eject one IMBH (although slowly, so that it will sink back in), and leave the other in an eccentric orbit that decays readily by emission of gravitational radiation. Regardless of the properties or rates of such encounters, any detected by LISA will be valuable probes of strong gravity.

4. Conclusions

The evidence for IMBHs is currently strong but circumstantial, pointing to the need for dynamical mass measurements of binary motion that will establish their existence definitively. Nonetheless, their likely formation mechanisms and dynamical interactions link them to many exciting topics in the current and early universe. As gravitational wave sources they will be unique in several respects: as bridge objects between space-based and ground-based detectors, as comparable-mass binaries with palpable eccentricities, and as the events that potentially will yield the most precise tests of general relativity. Many explorations need to be done, but current results are highly encouraging for their study.
Acknowledgments

We thank Tal Alexander, Pau Amaro-Seoane, Mike Gill, Doug Hamilton, Clovis Hopman, Vanessa Lauburg, Fred Rasio, Derek Richardson and Michele Trenti for stimulating conversations. This work was supported in part by NASA ATPF grant NNX08AH29G.

References

[1] Amaro-Seoane P et al 2007 Class. Quantum Grav. 24 113
[2] Amaro-Seoane P et al 2009 Astrophys. J. at press
[3] Begelman M C 2006 Astrophys. J. 643 1065
[4] Belkus H et al 2007 Astrophys. J. 659 1756
[5] Colbert E J M and Mushotzky R F 1999 Astrophys. J. 519 89
[6] Ebisuzaki T et al 2001 Astrophys. J. 562 L19
[7] Fabbiano G 1989 Annu. Rev. Astron. Astrophys. 27 87
[8] Fregeau J M et al 2006 Astrophys. J. 646 L135
[9] Gültekin K et al 2002 Astrophys. J. 578 L41
[10] Gültekin K et al 2004 Astrophys. J. 616 221
[11] Gültekin K et al 2006 Astrophys. J. 640 156
[12] Gurkan M A et al 2004 Astrophys. J. 604 632
[13] Hansen B M S and Milosavljevic M 2003 Astrophys. J. 593 L77
[14] King A R et al 2001 Mon. Not. R. Astron. Soc. 322 231
[15] Koch F E and Hansen B M S 2008 Astrophys. J. 687 252
[16] Kroupa P 2001 Mon. Not. R. Astron. Soc. 322 231
[17] Madau P and Rees M J 2001 Astrophys. J. 551 L27
[18] Mandel I 2008 Astrophys. J. 681 1431
[19] Matsubayashi T et al 2007 Astrophys. J. 656 879
[20] Miller J M et al 2003 Astrophys. J. 585 L37
[21] Miller J M et al 2004a Astrophys. J. 607 931
[22] Miller J M et al 2004b Astrophys. J. 614 L117
[23] Miller M C 2005 Astrophys. J. 618 426
[24] Miller M C and Colbert E J M 2004 Int. J. Mod. Phys. D 13 1
[25] Miller M C and Hamilton D P 2002a Mon. Not. R. Astron. Soc. 330 232
[26] Miller M C and Hamilton D P 2002b Astrophys. J. 576 894
[27] Noyola E et al 2008 Astrophys. J. 676 1008
[28] O’Leary R M et al 2006 Astrophys. J. 637 937
[29] O’Leary R M et al 2007 Phys. Rev. D 76 061504
[30] O’Leary R M et al 2008 Mon. Not. R. Astron. Soc. submitted arXiv:0807.2638
[31] Portegies Zwart S F et al 2004 Nature 428 724
[32] Portegies Zwart S F et al 2006 Astrophys. J. 641 319
[33] Portegies Zwart S F and McMillan S L W 2002 Astrophys. J. 576 899
[34] Quinlan G D 1996 New Astron. 1 35
[35] Reynolds C S et al 1997 Mon. Not. R. Astron. Soc. 286 349
[36] Ruszkowski M and Begelman M C 2003 Astrophys. J. 586 384
[37] van der Marel R P et al 2002 Astron. J. 124 3255
[38] Wen L 2003 Astrophys. J. 598 419
[39] Will C M 2004 Astrophys. J. 611 1080