Inverse solution optimization and research on Trajectory Planning of cleaning manipulator for insulator

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Abstract. Railway catenary insulators are an important component of electrified railway, and it is becoming increasingly necessary to cleaning. For six degree of freedom manipulator cleaning automatically catenary insulators is analysed with D-H method and researched about kinematic and trajectory planning. Motion and trajectory of cleaning manipulator’s mathematical model and theoretical support is established. About the situation of multiple solutions is optimized by energy optimization rule to save energy, and quantitative analysis about Optimized energy consumption. The manipulator’s trajectory on Cartesian Space is simulated and analysed. The Basement is built that the solution of positive and inverse kinematics, trajectory optimization, collision-free trajectory and the actual manipulator’s working space.

1. Introduction

It is well known that railway contact network insulators are the most important component of a railway power supply system. With the rapid development of China's railway industry, the cleaning task of the dirt of the railway contact net insulators is becoming more and more arduous. The traditional cleaning method mainly adopts manual wiping method, which is inefficient and costly [1]. In this regard, the method of using human to wipe the insulator dirt is largely unable to meet the increasingly heavy cleaning task, and it is urgent to study the cleaning robot arm which can replace the manual cleaning of the insulator dirt. The robotic arm is a mechanical device designed according to the principle of bionics. It has high flexibility. It can replace some humans to achieve some repetitive work, which can ensure stability and reduce labor costs [2]. The control object used in this paper is a six-degree-of-freedom robotic arm. The main content of the research is kinematics solution and trajectory planning. The most important part of the robot cleaning insulators is the inverse kinematics solution and trajectory planning. Under the simulation platform, the spatial position of the known insulator contamination is simulated, and then the inverse kinematics solution and trajectory planning are used to bring the robot end effector to the spatial position. Because the inverse solution of the manipulator has multiple solutions, based on this, a target energy evaluation function is proposed to minimize the energy consumption of the arm, and also to make the mechanical body wear less, and to realize the trajectory planning and simulation of the end effector workspace. Lay the foundation for future research.
2. Positive and negative kinematics of robotic arm

2.1. Establishment of mechanical arm structure model

The object studied in this paper is a series of six-degree-of-freedom robots, consisting of six rigid rods connected in series and their corresponding six rotating pairs (joint axes) [3]. The linkage coordinate system is established as shown in Fig. 1. The D-H parameters of each joint are shown in Table 1.

![Figure 1. Coordinate system of mechanical arm](image)

| Link i | \( \theta \) | d | a | \( \alpha \) |
|--------|-------------|---|---|----------|
| 1      | \( \theta_1 \) | 0 | 0 | 0        |
| 2      | \( \theta_2 \) | 0.14909 | 0 | -90      |
| 3      | \( \theta_3 \) | 0.4318 | 0 | 0        |
| 4      | \( \theta_4 \) | 0.43307 | 0.2032 | -90 |
| 5      | \( \theta_5 \) | 0 | 0 | -90      |
| 6      | \( \theta_6 \) | d_6 | 0 | -90      |

The homogeneous coordinate transformation matrix of any adjacent two links is:

\[
J_j^{-1}A(\theta_j, d_j, a_j, \alpha_j) = T_{Rz}(\theta_j)T_{xz}(d_j)T_x(a_j)T_{Rx}(\alpha_j)
\]

It can be expanded to:

\[
J_j^{-1}A = \begin{bmatrix}
\cos \theta_j & -\sin \theta_j \cos \alpha_j & \sin \theta_j \cos \alpha_j & a_j \cos \theta_j \\
\sin \theta_j & \cos \theta_j \cos \alpha_j & -\cos \theta_j \sin \alpha_j & a_j \sin \theta_j \\
0 & \sin \alpha_j & \cos \alpha_j & d_j \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
Among them, the formula (1) contains two rotation matrices and two translation matrices specified by the D-H method, so it is only necessary to substitute the parameters in Table 1 into the equation (1) to obtain the transformation equation between adjacent links.

2.2. Positive kinematics analysis
In the motion control of the robot arm, we always hope that it can quickly reach the specified position and posture, so positive kinematics is particularly important. For a tandem six-degree-of-freedom manipulator, the positive kinematics result is that the angle at which the six joints are rotated causes a change in the spatial position of the end effector. That is to say, the relationship between the joint variable and the position and attitude of the end effector in space is also studied, and the relationship between the links is also involved. From the kinematic transformation equation between the adjacent links above, the equation of motion of the six-degree-of-freedom manipulator is obtained, that is, multiplication between every two adjacent links:

\[ T_{6}^{0} = A_{6}A_{5}A_{4}A_{3}A_{2}A_{1} \]

Obviously, by the formula (3), the posture of the end effector of the arm can be obtained.

\[ T(3) \]

2.3. Inverse kinematics analysis:
The inverse kinematics process is the inverse of positive kinematics. It is the position and direction of the end of the robot arm in the known Cartesian space to find the joint angle of each joint axis. Compared with positive kinematics, the inverse kinematics of the manipulator is the most important basis for the control of the manipulator and its more accurate tasks. At the same time, the inverse kinematics solution will be designed to some complex matrix inverse multiplication, but also consider the inverse and the singular configuration of the solution, so the inverse solution is more complicated, and the computational complexity is much larger than the positive kinematics solution. Inverse kinematics methods generally have algebraic methods, geometric methods and the PIEPER solution proposed by Pieper for three-axis intersection [4]. The inverse solution of the geometric method is small, but it is only for the robotic arm of certain geometrical positions, and it is not universal and versatile. The PIEPER solution requires that the axes of the three arms at the rear of the robot arm intersect at a point, which also has certain limitations. Compared with the former two methods, the algebraic method is relatively large, but its solution is complete, and there is no requirement for the mechanical structure of the mechanical arm. Therefore, the algebraic method is used in this paper to solve the following [5]:

According to formula (3):

\[ T_{6}^{0} = A_{6}A_{5}A_{4}A_{3}A_{2}A_{1} \]

Both ends of the formula (4) are left-multiplied:

\[ A_{i}^{-1} \times \begin{bmatrix} n_{x} & o_{x} & a_{x} & p_{x} \\ n_{y} & o_{y} & a_{y} & p_{y} \\ n_{z} & o_{z} & a_{z} & p_{z} \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_{1}^{-1} T_{6}^{0} = A_{1}^{-1} A_{2} A_{3} A_{4} A_{5} A_{6} \]
According to formula (6):

\[ \begin{bmatrix} n_x c_1 + n_x s_1 & o_x c_1 + o_x s_1 & a_x c_1 + a_x s_1 & p_x c_1 + p_x s_1 \\ n_x & o_x & a_x & p_x \\ n_x s_1 - n_x c_1 & o_x s_1 - o_x c_1 & a_x s_1 - a_x c_1 & p_x s_1 - p_x c_1 \end{bmatrix} = \begin{bmatrix} c_{234}c_5c_6 - s_{234}s_6 & -c_{234}c_5c_6 - s_{234}c_6 & c_{234}s_5 & c_{234}a_4 + c_{23}a_3 + c_2a_2 \\ s_{234}c_5c_6 + c_{234}s_6 & -s_{234}c_5c_6 + c_{234}c_6 & s_{234}s_5 & s_{234}a_4 + s_{23}a_3 + s_2a_2 \\ 0 & 0 & c_5 & 0 \end{bmatrix} \]

According to formula (6):

\[ p_x s_1 - p_y c_1 = 0 \]

Therefore: \[ \theta_i = \arctan\left(\frac{p_x}{p_y}\right), \quad \theta_i = \theta_1 + 180^\circ \]

By analogy, on the basis of the above formula, respectively, by multiplying left \( A_1^{-1}, A_2^{-1}, A_4^{-1}, A_5^{-1} \) and \( A_6^{-1} \), respectively, we can get the matrix equation to obtain \( \theta_2, \theta_3, \theta_4, \theta_5, \theta_6 \).

### 2.4. Inverse optimization

When a six-degree-of-freedom robotic arm cleans the railway and insulates dirt, it must carry a portable power source to supply its energy. Because of the portability of the device, the size of the battery is necessarily limited, so that its power is also limited. When the manipulator's boom and arm rotate the same degree, the boom consumes more energy. Therefore, in this paper, we propose a principle of energy optimization during the cleaning process of the robot arm. The arm is rotated as much as possible to avoid the rotation of the boom. The end effector can reach the specified position and consume the minimum energy. In order to standardize the relationship between energy consumption and the rotation angle of each axis, a weighting method is used to balance the energy consumption function. Assuming that the power consumption function is \( W(\theta) \), it can be known that:

\[ W(\theta) = \sum_{i=1}^{6} P_i |\theta_i| \]

Where \( P_i \) is the energy consumption of the unit angle of the i-th joint angle, which is the degree of rotation of the i-th joint angle. Through several experimental tests, the value of the weight factor is determined as follows:

\[ W(\theta) = |\theta_1| + 2.5|\theta_2| + 2|\theta_3| + 2|\theta_4| + 2|\theta_5| + |\theta_6| \]

Generally speaking, there are 8 sets of inverse solutions for the six-degree-of-freedom manipulator. The set of values that minimizes the objective function \( W \) by the above formula is the optimal solution.
3. Trajectory planning

3.1. Joint space trajectory planning

Joint space trajectory planning is a method of describing trajectory generation as a function of joint angle. The trajectory planning for each axis joint is independent of each other, but must have the same actual and end time. Since joint planning does not require a shape of the trajectory between the two ends of the end, the solution becomes simpler [6–7]. When planning, first select several necessary three-dimensional spatial position points at the end, and transform these points into joint space by inverse kinematics, making it an intuitive joint space angle value, and then for these angle values, The fitting curve is made for each joint angle, and the fitting is generally completed by the cubic interpolation fitting method. For the cubic polynomial, in order to fit a smooth curve, it is necessary to constrain \( \theta(0) = \theta_0 \) and \( \theta(t_f) = \theta_f \) at the beginning and end positions, and the starting and ending speeds. To ensure the continuity of the speed, Their values are \( \dot{\theta}(0) = 0, \dot{\theta}(t_f) = 0 \).

The determined cubic polynomial is as follows:

\[
\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3
\]

Therefore:

\[
\begin{align*}
& a_0 = \theta_0 \\
& a_1 = 0 \\
& a_2 = \frac{3}{t_f^2} (\theta_f - \theta_0) \\
& a_3 = -\frac{3}{t_f^3} (\theta_f - \theta_0)
\end{align*}
\]

3.2. Cartesian space trajectory planning

In reality, the robotic arm may be required to move along a specific trajectory due to the difference of the task. In this case, the space Cartesian trajectory planning must be used. The biggest difference from the joint space trajectory planning is that it arbitrarily specifies the motion path of the end. The interpolation algorithm can find the middle point of the track according to some teaching points, and the robot arm can complete the motion according to the desired track shape, so that the task space track has strong flexibility. The downside is that it is computationally intensive compared to joint space planning, because every two points with small intervals on the continuous trajectory need to solve the joint variables through the inverse kinematics area to control the manipulator to complete the interpolation of all points [8–9].

4. Simulation results and analysis

Use the MATLAB robot toolbox to create a simulation model of the six-degree-of-freedom manipulator [10] and name it Docbot, as shown in Figure 2. Its joint angle control slider panel is shown in Figure 3. The initial position of the end of the arm is set to \((74.00, -10.00, 24.00)\), and the spatial position of the target dirt is \((0.31, 25.96, 79.36)\). The multiple sets of inverse solutions solved by inverse kinematics are shown in Table 2. From the optimization rule mentioned above, it is known to rotate the arm as much as possible to avoid the possibility of rotating the boom to minimize energy consumption. According to formula (9), the solutions of each group in the case of solution are substituted, and the optimal solution
can be obtained. The third set of solutions is calculated to meet the energy-optimized expectation, that is, the required energy is the smallest.

In joint space planning, the selected simulation time step is: \( t = [0:0.05:2] \), the starting joint angle \( \mathbf{q}_1 = [0, 0, 0, 0, 0, 0] \), and the joint angle \( \mathbf{q}_2 = [1.5902, 1.5514, 1.2411, 1.3187, 1.3963, 1.5902] \). Figure 4 and Figure 5 show the relationship between the joint angle and time and the joint angular velocity and time in the planning process. It can be seen that the joint angle changes continuously and smoothly during the movement. The same is true for the joint angular velocity, indicating the planning process. It does not cause a sudden change in the movement of the arm, thereby avoiding damage to the mechanical body. Figure 6 shows the Cartesian space trajectory of the end effector. Perform trajectory planning in Cartesian space, specify initial point and end point according to actual needs, make circular interpolation between two points, and then use kinematics to solve the joint angle value according to a series of points on the arc, thus making the machine the arm completes the planning. Figure 7 shows the linear trajectory planning in Cartesian space. It can be seen that the singularity of the mechanical arm does not occur during the interpolation process. The planning process has achieved ideal results, and its feasibility is illustrated by the forward and inverse kinematics. Planning and obstacle avoidance have laid the foundation.

### Table 2. Multi-inverse solution

| Number | \( \mathbf{q}_1 \) | \( \mathbf{q}_2 \) | \( \mathbf{q}_3 \) | \( \mathbf{q}_4 \) | \( \mathbf{q}_5 \) | \( \mathbf{q}_6 \) |
|--------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1      | -2.1236         | 1.7264          | 2.3589          | 1.8745          | 1.9574          | 0               |
| 2      | No solution     |                 |                 |                 |                 |                 |
| 3      | 1.5902          | 1.5514          | 1.2411          | 1.3187          | 1.3963          | 1.5902          |
| 4      | No solution     |                 |                 |                 |                 |                 |
| 5      | 0               | 1.6949          | 2.3848          | -2.3694         | 1.0325          | 1.8251          |
| 6      | 1.2654          | -1.7895         | -1.9845         | -1.2968         | 2.3649          | 0               |
| 7      | No solution     |                 |                 |                 |                 |                 |
| 8      | 1.6797          | 0.9587          | -1.9865         | 2.0234          | -1.5685         | 1.3644          |

As shown in Figure 8, the inverse solution proposed for the optimization simulation analysis shows. It can be seen that for the five inverse solutions existing, quantitative analysis and comparison of the energy consumption values before and after optimization are made, and it is obvious that the optimized energy consumption value is less than that before optimization.
Figure 4. Angle change of joint

Figure 5. Change of angular velocity of joint

Figure 6. Space trajectory

Figure 7. Descartes linear planning

Figure 8. Comparison of energy consumption before and after optimization
5. Conclusion
In this paper, the D-H method is used to establish a simulation model for the six-degree-of-freedom manipulator. The forward and inverse kinematics analysis is used to obtain the inverse solution of completeness. It is proposed to use the inverse solution energy optimization method to select multiple sets of inverse solutions to obtain the optimal solution, so that the mechanical arm consumes the least energy, which is of great significance in practical applications. Joint space trajectory planning and Cartesian space trajectory planning for the trajectory of the manipulator, and no singular points appear in the Cartesian space planning process, indicating its feasibility. The simulation analysis of the inverse solution optimization of the six-degree-of-freedom cleaning manipulator proposed in this paper has achieved good results.

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