Cosmological Breaking of Supersymmetry

Or

Little Lambda Goes Back to the Future II

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Abstract: It is conjectured that M-theory in asymptotically flat spacetime must be
supersymmetric, and that the observed SUSY breaking in the low energy world must be
attributed to the existence of a nonzero cosmological constant. This would be consistent
with experiment, if the critical exponent $\alpha$ in the relation $M_{SUSY} \sim M_P (\Lambda/M_P^4)^\alpha$
took on the value $1/8$, rather than its classical value $1/4$. We attribute this large
renormalization to the effect of large virtual black holes via the UV/IR correspondence.

Keywords: Cosmological Constant, Holographic Principle.

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1. Introduction

This paper is an expanded version of a short talk I gave at Lenny Susskind’s 60th birthday celebration at Stanford University. It is dedicated to Lenny, who taught me how to think about physics, and whose own recent ideas have profoundly influenced those I am reporting on here. The central message of the paper can be summarized in a few sentences: The Bekenstein-Hawking Entropy of (Asymptotically) DeSitter (AsDS) spaces represents the logarithm of the total number of quantum states necessary to describe such a universe. This implies that the cosmological constant is an input to the theory, rather than a quantity to be calculated. The structure of an AsDS universe automatically breaks supersymmetry (SUSY). From this point of view, the “cosmological constant problem” is the problem of explaining why the SUSY breaking scale is so much larger than that associated in classical supergravity (SUGRA) with the observed value of (bound on ?) the cosmological constant. I suggest that large renormalizations of the classical formula are to be expected on the basis of the UV/IR correspondence.
in M theory. These may be viewed as contributions from virtual black holes. The phenomenologically correct formula $M_{SUSY} \sim (\Lambda M_P^4)^{1/8}$ may be derivable from such considerations.

The implication of these ideas is that SUSY breaking vanishes in the flat space limit, which is consistent with the fact that we have not succeeded in finding a string vacuum with broken SUSY and asymptotically flat spacetime. We will begin the paper with a brief review of the evidence for this.

We then turn to a defense of the contention that an asymptotically DeSitter (AsDS) universe can be described by a finite number of states, given by the Bekenstein-Hawking formula. We discuss the difference between such an AsDS universe and the temporary DeSitter phase of an inflationary universe. We caution the reader that once he accepts these arguments, he will be forced to conclude that the cosmological constant is an input, or boundary condition, rather than a parameter to be calculated. The conventional cosmological constant problem can then be rephrased as: why isn’t the scale of SUSY breaking related to the cosmological constant by the standard classical SUGRA formula, which (without fine tuning) predicts $M_{SUSY} \sim \Lambda^{1/4}$.

We argue that this formula may receive large renormalizations. Indeed, standard field theory calculations predict logarithmically divergent renormalizations (at finite orders in the loop expansion) of the masses of particles in softly broken SUSY theories. Conventionally, these divergences are absorbed into the parameters in the low energy effective field theory. Wilsonian renormalization group arguments suggest that these divergent terms have no dependence on the cosmological constant if this parameter is much smaller than the cutoff scale. We argue that the UV/IR correspondence of M theory suggests a possible source for such a dependence. The highest energy states of the theory are huge black holes with a size of order the DS horizon size. Their spectrum is very sensitive to the value of the cosmological constant. We speculate that it may change the value of the “critical exponent” relating the SUSY breaking scale to the cosmological constant, to $M_{SUSY} \sim (\Lambda M_P^4)^{1/8}$. This formula fits the observational data.

2. Vacuum selection

One of the unfortunate features of M-theory as a theory of the real world, is its plethora of unphysical, exactly SUSic vacua. On the one hand, this aspect of the theory is precisely what has enabled us to get so much mathematical control over its properties. On the other hand, even if one succeeded in finding a SUSY breaking vacuum which precisely describes the real world (we should be so lucky!) one would still have the un-
comfortable task of explaining why the universe does not resemble one of the beautifully SUSic vacua.

From this point of view, it is interesting and exciting that it appears very difficult to break SUSY in a way which leaves us with an approximately flat spacetime. Many candidate SUSY violating vacuum states of M-theory have tachyonic instabilities. Almost all\(^1\) classical candidate vacua generate potentials for their moduli in the quantum theory. The effect of these potentials is either to drive the system into a region of moduli space where we are unable to analyze it (except to conclude that, since it must have a large negative vacuum energy, it cannot describe an asymptotically flat spacetime), or to drive it deep into the weak coupling regime where gravity becomes a free field theory. In neither case do we get an acceptable description of the real world. The weakly coupled system has massless moduli and low energy effective parameters which vary too rapidly with time.

The above analysis is based on weakly coupled string theory and semiclassical SUGRA. Similar conclusions follow from an analysis of SUSY breaking in Matrix Theory\(^2\)[\(^3\)]. In this nonperturbative formulation of M-theory in a variety of asymptotically flat, SUSY, spacetimes, asymptotic spacetime (more precisely the configuration space of multiparticle asymptotic states propagating on the spacetime) arises as the moduli space of a SUSY quantum system. Breaking SUSY collapses spacetime.

By analogy with the AdS/CFT correspondence for asymptotically AdS SUSY vacua, one might try to find a nonsupersymmetric version of this correspondence (with an AdS space with curvature much less than the Planck scale) by searching for conformal field theories with certain properties. In particular, they should have a large gap in dimensions between the stress tensor, and all but a small number of other operators in the theory. The stress tensor is the primary field corresponding to gravitons in AdS, while other operators correspond to states with mass of order the string or Planck scale. The gap in dimensions indicates a large ratio between the AdS curvature scale and the Planck scale. In SUSY examples this gap is ”guaranteed” by SUSY nonrenormalization theorems combined with an hypothetical scaling law for the dimensions of nonchiral operators in the large \(g_s N\) limit. The dimension of the stress tensor is always protected, but in the absence of SUSY we do not expect to have lines of fixed points and there is no obvious parameter which could tune the gap to be asymptotically large.

Another feature of a large AdS space which would have to be reproduced by our hypothetical conformal field theory, would be the existence of multigraviton excitations.

\(^1\)The exceptions are the models of \([1]\). These have no potential for moduli at one and two loops. I am suspicious of these models because they appear to have an infinite fermi-bose degeneracy, despite the absence of SUSY. I suspect that this degeneracy is lifted at some order of perturbation theory, at which point a potential is generated.
Even in supersymmetric examples this property is not well understood. That is, it is understood only in the regime of $AdS_5 \times S^5$ moduli space where the 't Hooft expansion is applicable. And in this regime, multiparticle excitations exist even when the curvature of AdS space is large. There should be a purely field theoretical argument which would prove the existence of multiparticle excitations in regimes where the AdS curvature is large, independently of the dimension of the space or the existence of a weakly coupled string regime. Only when we understand this could we hope to check whether a SUSY violating conformal field theory really represented a large AdS space.

Finally, we would have to show that the theory contained metastable excitations corresponding to black holes with size much bigger than the Planck scale but much smaller than the AdS radius. So far, there is no evidence for nonsupersymmetric CFTs with these properties. Indeed, we have little understanding of either cluster decomposition and multiparticle structure, or metastable flat space black holes, in the supersymmetric versions of AdS/CFT.

In summary, all the extant evidence indicates the absence of asymptotically flat M-theory vacua with broken SUSY. There are no solid examples, though the models of [1] may yet turn out to fulfill their design criteria.

3. The entropy of DeSitter space

The results reviewed in the preceding section suggest (but certainly not very strongly) that SUSY breaking in asymptotically flat spacetime may be impossible in M-theory. There is certainly a well known relation between the breaking of SUSY and the Ricci scalar of spacetime. Namely a generic nonsupersymmetric quantum field theory generates a cosmological constant of order at least the SUSY breaking scale. Conversely, a positive cosmological constant is incompatible with SUSY.

The well known problem with this relation is the relative scale of the two effects. The cosmological constant is bounded from above by a number of order eighty percent of the critical density, while the scale of SUSY breaking is bounded from below by several hundred GeV. Without fine tuning of parameters, and using the methods of effective field theory, this leads to a cosmological constant about 60 orders of magnitude larger than the observational bound. We normally think about this problem by doing quantum field theory in flat spacetime and then calculating the corrections to the spacetime background. SUSY breaking “causes” a large cosmological constant which then makes the flat spacetime a bad approximation. I would like to suggest that we have been thinking about this problem the wrong way around. The flat space computation counts the zero point energy of the degrees of freedom in spacetime. We have been learning that the number and properties of the degrees of freedom in M-theory depends
crucially on our specification of the boundary conditions on spacetime [4]. Asymptotically Anti-DeSitter spaces of various dimensions have very different kinds of high energy degrees of freedom and further they all differ drastically from asymptotically flat spaces. Remarkably, the semiclassical Bekenstein-Hawking formula consistently gives the right answer for the extreme high energy entropy. This is an example of the UV/IR connection. High energy states are associated with large, low curvature (outside the horizon) geometries, whose gross properties are encoded in general relativity.

For DS space, the Bekenstein-Hawking formula predicts a finite entropy. More precisely, any observer in an AsDS space only sees a finite portion of the universe, bounded by a cosmological event horizon. One quarter of the area of this of this event horizon (in Planck units) is the finite Bekenstein-Hawking entropy. I would like to interpret this number as the logarithm of the total number of quantum states necessary to describe the universe as seen by this observer.

There are three arguments for this. The first is simply an analogy with black hole physics (according to the holographic principle): event horizons may be viewed as holographic screens on which all information about “what is going on on the other side of the horizon” is encoded for the benefit of observers “on this side”. All of the arguments in favor of this holographic view of black hole horizons apply equally well to DS space.

The second argument is by far the most convincing. Imagine an observer inside DS space trying to contradict our contention by collecting as much entropy as she can. As long as she works on scales smaller than the DS radius of curvature, she can do this most efficiently by forming flat space black holes, whose entropy is bounded by their area. The black hole size is bounded by something of order the horizon size so there is no way to violate our bound. Put another way, a system with an entropy larger than the DS horizon size would simply not evolve into an AsDS spacetime with the assumed value of the cosmological constant.

The third argument is more technical. While few people believe any longer that quantum gravity is described by an Euclidean functional integral over metrics, this paradigm does seem to provide helpful and correct hints about the quantum physics of black holes and AdS spaces[5]. Euclidean DS space is a sphere, a compact geometry. The rules for Euclidean quantum gravity (c.f. perturbative world sheet physics in string theory) tell us that all diffeomorphisms, including the DS group of isometries are gauge transformations and should be integrated over. All physical information is invariant. This is in marked contrast to asymptotically flat or AdS universes, where the isometries act nontrivially on the nonfluctuating boundary geometry. In these cases, the isometries are large gauge transformations and physical states need not be invariant under them.
Now consider quantum field theory in DS spacetime, defined by analytic continuation of Euclidean Green’s functions on the sphere. Long ago, constructive field theorists showed [6] for a large class of superenormalizable theories, that these Green’s functions have a Hilbert space interpretation in terms of the Hilbert space of an observer living in the static patch of Lorentzian DS space. The state defined by these Green’s functions is the thermal state of the static patch Hamiltonian, at the Hawking temperature. These rigorous results are the generalization of the observations of [7] for free field theory and the perturbation expansion around it. To obtain the field theory in the full DS space one uses DS isometries to copy the Green’s functions from one static patch to another. According to the argument of the previous paragraph, this procedure just produces gauge copies of the original system. Thus, from this point of view it would be wrong to introduce independent physical degrees of freedom for each static patch.

It is important to examine several situations which appear to contradict the idea that AsDS spaces have a finite number of degrees of freedom. One such argument is based on considering a spacetime which is DS in the remote past. At early times, the volume of space is very large, and one can easily impose initial conditions which have a larger entropy than the DS maximum. However, most of these initial conditions will not lead to an AsDS spacetime (with the same value of the cosmological constant). Einstein’s equations (with appropriate conditions on the stress tensor) will not allow a violation of the Bekenstein-Fischler-Susskind-Bousso (BFSB) bound[8].

The “approximately DeSitter” spacetimes of inflationary cosmology are confusing only so long as we forget the nature of the holographic principle. There is no cosmological event horizon in these spacetimes (unless things settle down into a DS phase much later in the history of the universe), so the horizon size of the inflationary DS phase is at best a temporary measure of the maximal entropy in the experience of local observers. When the inflationary phase ends, the horizons of these observers expand. The proper holographic screen on which all the information in these universes can be encoded depends on their evolution after the end of inflation.

By taking a limit in which the number of e-folds of inflation becomes infinite, we can generate a paradoxical situation. If we admit the possibility of independent information in different static patches of DS space (as we have for any finite number of e-foldings) then we obtain AsDS spacetimes with entropy larger than the horizon area. These are essentially the time reverse of the spacetimes we encountered two paragraphs ago. Of course, if we extrapolate these expanding geometries back into the past, we inevitably encounter a spacelike singularity. Thus, the proper description of these spacetimes is a Big Bang singularity which evolve to DS space in the future. Note that no local observer in such a universe will ever encounter more entropy than is allowed by the bound. The confusion lies in the fact that there are many ways of cutting the space
up into regions observed by independent local observers. I believe that the confusion engendered by this example is connected to initial conditions at the singularity, and propose that a proper quantum treatment of cosmology will never lead to spacetimes of this type. In particular, I suspect that general initial conditions at the singularity for a number of degrees of freedom larger than the DS entropy will not evolve into the postulated DS space. The particular solutions described above will involve extreme fine tuning of initial conditions at the singularity, and might not exist at all in a quantum mechanical treatment.

The claim that the cosmological constant determines the number of degrees of freedom in an AsDS universe is extremely important if true. Traditionally, we think of the cosmological constant as an effective field theory parameter with no direct connection to the microscopic physics of the world. It is to be calculated in terms of more fundamental quantities. If however it is a direct count of the number of degrees of freedom, then its value is part of the fundamental set up of the quantum theory. The dimension of Hilbert space (if it is finite dimensional) or the number of fundamental canonical degrees of freedom (if the Hilbert space is infinite dimensional) is part of the definition of the theory. We will see below that the possibility of such a direct connection between an apparent low energy parameter and the fundamental dynamics is an expression of the UV/IR relation of M-theory.

We must not attempt to calculate the cosmological constant but rather to postulate its value and derive other observable quantities from it. From this point of view the “cosmological constant problem” is turned on its head. It is not “why is the cosmological constant so small”, but “given the value of the cosmological constant, why is SUSY breaking so large”. Indeed, although I cannot derive this logically from what I have already said, in this context it seems inevitable that one should attribute all breaking of SUSY to the fact that we live in an AsDS universe. This is consistent with the impossibility of defining SUSY in DS space, and also with our failure so far to find SUSY violating asymptotically flat states of M-theory, but it flies in the face of all previous wisdom about SUSY breaking.

The classical formula relating SUSY breaking to the cosmological constant is (without fine tuning)

$$M_{SUSY} \sim (\Lambda)^{1/4}.$$  \hspace{1cm} (3.1)

A formula that fits the data is

$$M_{SUSY} \sim (\Lambda M_p^4)^{\alpha}$$  \hspace{1cm} (3.2)

with $\alpha = 1/8$. I would propose that we describe these formulae with the following slogan: The $\Lambda/M_p^4 \to 0$ limit of M-theory is a critical limit in which the number of
degrees of freedom of the system goes to infinity. In this limit, the SUSY breaking scale goes to zero, and we are trying to calculate the critical exponent for its vanishing. The classical mean field value is $1/4$. Experiment indicates that the correct value is $1/8$.

4. How can this be?

If the scale of SUSY breaking is smaller than the Planck scale, then low energy physics is described by a locally SUSY effective Lagrangian. The breaking of SUSY in this Lagrangian is spontaneous. If the relevant SUSY is $N = 1$, $d = 4$, then the Lagrangian can have DeSitter solutions with spontaneously broken SUSY. The cosmological constant and the scale of SUSY breaking are independent parameters in this Lagrangian. The scalar potential has the form

$$V = e^K [K^{ij} F_i \bar{F}_j - 3|W|^2]$$

(4.1)

Everything has been expressed in Planck units. We will be working near the flat space limit, where the cosmological constant is very small. In that limit, the $F_i$ terms are the order parameters for SUSY breaking in the sense that mass splittings in supermultiplets are proportional to the values of the $F$ terms at the minimum of the potential. Note that both supermultiplet and mass are approximate concepts if the cosmological constant is nonzero. Mathematically, there are no global symmetry generators with which to define these words precisely. Physically, particles cannot be separated from each other by more than a horizon size, and we cannot define scattering amplitudes.

By choosing parameters in the superpotential and Kahler potential, we can arrange a minimum with nonvanishing $F$ terms and arbitrary value for the cosmological constant. However, this is generally considered to be fine tuning, according to the following Wilsonian argument. When we calculate radiative corrections to the effective Lagrangian below the SUSY breaking scale, we find a contribution to the renormalized cosmological constant of order $M_{SUSY}^4$, where $M_{SUSY}$ is the largest splitting in supermultiplets, and is also chosen to be the cutoff in the calculation. This can be cancelled, by adroit choice of the parameters in the Lagrangian, but the latter are thought to represent the effect of integrating out fluctuations at very short spacetime scales. In local field theory, degrees of freedom can be classified by their spacetime extent in an underlying classical metric. Degrees of freedom at short scales see long wavelength degrees of freedom as essentially constant parameters. According to this philosophy, the calculation of the effects of short wavelength degrees of freedom is essentially independent of the value of the cosmological constant, as long as the latter is much smaller.

\footnote{See the section on the fate of the universe, below.}
than (the - 4th power of) the wavelength. Thus, one argues, it is unnatural to imagine a cancellation of the bare cosmological constant against the low energy contribution. Furthermore, in a field theory with spontaneously broken SUSY, in flat spacetime, the very high energy contributions to $\Lambda$ cancel. Similar exact cancellations in string theory with exact SUSY, suggest that this is not just a fluke of the field theoretic approximation.

There are obvious problems with applying this argument to M-theory. The spacetime metric, which is used to characterize what constitutes long and short wavelength fluctuations, is, in M-theory, an approximate description of fluctuating quantum variables. More importantly, the association of large mass scales with short distances is incorrect in M-theory. This correspondence is valid down to the string scale in weakly coupled string theory. However, the high mass states of string theory are predominantly of large spacetime extent. More generally, above the Planck scale, the high mass excitations are black holes, whose Schwarzschild radius grows with their mass. It is incorrect to say that the dynamics of these objects is unaffected by the cosmological constant. Indeed, black holes with radius larger than the cosmological horizon do not exist in DS space. Thus it is no longer implausible that the low and high energy contributions to $\Lambda$ cancel each other $^3$.

Our identification of the cosmological constant as the (inverse logarithm of) the number of quantum states of an AsDS universe suggests a slightly different point of view. The value of the cosmological constant is now a fundamental parameter (actually a boundary condition - see below) and we should set parameters in our effective Lagrangian to match it. In the low energy effective Lagrangian, this requires us to find a vacuum with spontaneously broken SUSY, but the natural scale of SUSY breaking is set by the cosmological constant. Field theoretic renormalizations will not upset this relation. There are, in Feynman diagrams, logarithmic renormalizations of mass splittings in supermultiplets, but as long as the field theoretic couplings are small, these are not substantial when the cutoff is of order the Planck mass. Furthermore, they do not depend strongly on the cosmological constant. Now however, consider quantum gravity corrections to the mass splittings, first as loops of gravitons in Feynman diagrams. These contribute to logarithmic divergences as well, but there is no longer any small parameter controlling the series in powers of logs. However, there is still no apparent dependence on the cosmological constant. The crucial question now is what cuts off the divergences when we reach the Planck scale. Much has been made of the softness of perturbative string amplitudes at large momentum transfer $^9$. Many people have

$^3$An argument along these lines has been given by L.Susskind in a variety of public and private venues over the last six months.
viewed this as the ultimate cutoff promised by a true theory of quantum gravity. But there is plenty of evidence, both internal to the perturbative analysis[10][9][14] and using D-brane techniques [11] that this is not correct. In [14] it was suggested instead that the ultimate cutoff comes from black hole physics. That is, all high energy high momentum transfer scattering amplitudes, and even the Regge regime, are eventually dominated by black hole production with subsequent decay by Hawking radiation. This is again an invocation of the UV/IR connection. The gross features of the highest energy processes in M-theory are ultimately encoded in General Relativity, because they involve low curvature geometries. We need the microscopic theory to calculate the detailed quantum properties of the states near a black hole horizon, but the level density of the high energy spectrum and many properties of inclusive cross sections can be calculated from semiclassical general relativity.

Thus, I would claim that there is no evidence for suppression of ”diagrams” in which virtual black holes of mass much larger than the Planck scale renormalize the splittings in low energy supermultiplets. The size of these contributions must be estimated from the physics of black holes. In such a calculation it is clear that the DS horizon radius will provide a cutoff on black hole contributions. It is entirely possible that a proper calculation involves the detailed microphysics of black hole states. We will explore a more optimistic scenario below.

It is important to realize that there is no claim being made that the theory with $\Lambda \to 0$ is divergent. We are merely trying to show that various quantities which vanish with the cosmological constant do so more slowly than is indicated by formulae which only take gravity into account classically. What we are claiming is that the theory with vanishing cosmological constant must be supersymmetric. It is reasonable to suppose that the restoration of SUSY will cancel otherwise divergent contributions from virtual black holes.

4.1 Proposal for a thermodynamic calculation

Our proposal implies that a full understanding of the relation between the cosmological constant and the scale of SUSY breaking is possible only if we know something about M-theory at very high energies. Rather than giving up and saying that this puts the problem beyond our powers at the present, I would like to suggest that the UV/IR correspondence may be used to get at least a rough estimate of the size of the effect. According to this principle, high energy physics in M-theory is black hole physics, and some aspects of black hole physics are computable in the semiclassical approximation to SUGRA. We may hope that an estimate of the relation between SUSY breaking and $\Lambda$ may be obtained in the semiclassical approximation.
The first aspect of semiclassical physics in DS space that will be important to us is that the state of the system is a thermal ensemble with respect to the static Hamiltonian of DS space. We consider this relevant, despite our previous remarks that the DS group is a group of gauge transformations. We are contemplating a limit of very small cosmological constant, and trying to describe physics as seen by observers who are unable to discern that space is not asymptotically flat (because they are making observations that refer to low energy, approximately local, physics). The phrase “mass splittings in supermultiplets” refers precisely to properties of the (approximate) SuperPoincare generators defined by such observers. The DS Hamiltonian goes over in the limit to the Poincare Hamiltonian of the asymptotically flat observer. We use it, because our considerations will depend on the curvature of DS space.

Our second assumption is that the parameters in the local effective Lagrangian actually get contributions from “Feynman diagrams with virtual black holes in them”. There is not even a semi-rigorous justification for this assumption, and the following hand waving will have to suffice: Consider Feynman diagrams contributing to the masses of some of the particles in the theory. As we allow the momenta in internal loops to grow larger than the Planck scale, we encounter subgraphs which look like super-Planckian scattering amplitudes, amplitudes in which all kinematical invariants are larger than the Planck scale. According to classical general relativity, we expect such collisions to result in black hole production. I claim that the quantum mechanical interpretation of this is that there is no suppression of the probability of producing virtual black holes.

The reader may be disturbed by the feeling that such large energy and momentum transfer processes should be cut off in M-theory. My response is that the black holes themselves provide the cutoff. For example, probability one black hole production followed by Hawking evaporation, gives exponentially suppressed inclusive cross sections for finite numbers of particles with energy and momentum transfer much larger than the Planck scale.

Given our two assumptions we expect the SUSY breaking mass terms to be given by a thermodynamic average

\[
\frac{\int dM e^{S(M)-\beta M} \Delta m(M)}{\int dM e^{S(M)-\beta M}}
\]  

(4.2)

Here \(S(M)\) is the black hole entropy and \(\beta\) is the inverse Hawking temperature of DS space. \(\Delta m(M)\) is the contribution to SUSY breaking from virtual black holes of mass \(M\). We will restrict attention to four dimensions, since this is the only place where

\[4\]In weakly coupled string theory we find suppression of hard processes at a much lower scale. This however is only valid in an intermediate regime [14].
low energy SUGRA can have DS solutions. In that case \( S(M) = 4\pi M^2 = \pi R_S^2 \), while \( \beta = 2\pi R_D \). The integral is actually cut off when the Schwarzschild radius \( R_S = R_D \). It is easy to see that the integral is dominated by its upper endpoint (unless \( \Delta m \) falls extremely rapidly with black hole mass).

Our claim then is the SUSY breaking induced by DS space can be approximated by that due to virtual black holes of a size near the upper cutoff for Schwarzschild-DeSitter black holes. I hope to report on an estimate of this effect in the near future.

5. The fate of observers in an AsDs universe

There is a line in an old country and Western song that goes "DeSitter space is a lonely place . . ." . Indeed, once the cosmological constant takes over the expansion rate, everything that is not gravitationally bound to us soon passes outside our horizon. Worse, after baryons decay, gravitationally bound systems will cease to exist if they have not collapsed into black holes. And when quantum mechanics is taken into account even this ultimate refuge is lost to us, since the black holes decay. Eventually, the universe becomes full of elementary systems, each in its ground state in its own horizon volume (we are for the moment ignoring the Hawking radiation of DeSitter space).

Physics as we know it, which describes local interactions between systems which can communicate with each other, becomes increasingly irrelevant in such a universe, though the time scale for this to happen is enormously long. Thus, the usual apparatus of physics describes an epiphenomenon in an AsDS universe. One of the technical problems related to this observation is how one describes the physical answers that are relevant to us as exact, gauge invariant, mathematical quantities in such a theory.

In asymptotically flat space, the holographic principle tells us that we can calculate the S-matrix. So far we have found no other sensible physical quantities in Asymptotically Flat M-theory. But there is no S-matrix in AsDS spaces. One must really search for more local quantities, but it seems that any such search may have only an approximate nature. For example, one might imagine showing that the low energy effective Lagrangian description had the status of the first term in an asymptotic expansion of something. But what might that something be? If we extrapolate to high enough energy we are always required to ask questions about all of the degrees of freedom and their dependence on the global geometry of AsDS space. There is no exact quantum number that takes the place of energy. If we are willing to take the attitude that at sufficiently high energy we can neglect SUSY breaking, we can use the flat space, SUSY vacuum which best approximates our AsDS universe to calculate scattering amplitudes above the Planck scale. But we must recognize that at sufficiently high energies these amplitudes describe processes involving black holes larger than the DS radius. These
have nothing to do with anything in the real world, if the universe is AsDS. It is also far from obvious to me that one could find a systematic incorporation of the the SUSY violating corrections to these amplitudes into a more exact description of the world. In our view, SUSY violation is a consequence of the AsDS geometry of the universe, and might be incompatible with a description of the world in terms of scattering amplitudes. The phrase *SUSY violating scattering amplitude*, might be an oxymoron that made sense only at energies below the Planck scale.

All of this suggests that there is a somewhat more local description of holographic physics than any which exists at present. I presented a preliminary sketch of what such a formalism might look like at the Millenium conference in January. It involves a collection of Hilbert spaces $\mathcal{H}_i$, each of which is supposed to represent those states observable in the causal past of a finite number of points, in a cosmological spacetime which begins at a Big Bang singularity. More precisely, using the Bekenstein-Hawking-Bousso relation between areas and entropy, and a causal structure which is defined by mappings of the algebra of operators in one space into a subalgebra with (in general) nontrivial commutant in another, I proposed to reconstruct a spacetime directly from quantum mechanics. \(^5\)

In this formalism, the experience of a more or less localized observer is encoded in a sequence of Hilbert spaces of (exponentially) increasing dimension. Each space in the sequence is mapped into a tensor factor of the one succeeding it. In order to have unitary evolution, the full state in the successor Hilbert space must be determined by partial mappings from many different predecessor states. In general it is not required that the entire process, including an infinite sequence of steps, can be incorporated in a single Hilbert space of finite dimension. One consistent rule which allows this is that the Hilbert spaces in any sequence converge after a finite number of steps to a space of some fixed dimension, the same for every sequence. The inclusion maps become unitary mappings of this space into itself.

I would like to identify such a situation with an AsDS space, in the limit that the number of dimensions of the asymptotic Hilbert space is very large. Appropriately smooth unitary mappings between different sequences would represent the different ways in which the spacetime could be represented as the static patch of a given observer, each of whom perceives all of the things outside her horizon as a thermal gas.

In this view of the universe, the local degrees of freedom whose investigation is the province of experimental physics should be viewed as being ”on temporary loan” from the ”thermal DeSitter library”. As the DeSitter era unfolds, more and more of

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\(^5\)This is not the place for a detailed exposition of these ideas, which I hope to present at a later time.[12]
the observer’s degrees of freedom are ”returned to the shelf”: they get swept outside his horizon, and become part of the thermal background. It is interesting that the total number of borrowed degrees of freedom that we need to describe what we see is, even if we include the entropy in hypothetical black holes in the center of each galaxy, smaller by a factor of $10^{30}$ than the Bekenstein Hawking entropy corresponding to the cosmological constant. Thus, from a sufficiently cosmic viewpoint, the entire organized part of the universe may be just a small coherent fluctuation in a random system with an enormous number (nearly a googleplexus) of degrees of freedom. It may be that in the far future, after the universe has degenerated into a collection of frozen elementary systems, each in its own horizon volume, a new fluctuation in the Hawking radiation can form, and the whole process will begin again.

Let me conclude this section by repeating that the most important technical problem posed by this view of the AsDS universe is to realize the physical measurements we make in terms of exact mathematical statements about the finite dimensional Hilbert space associated with the spacetime.

6. Metaphysics

One of the most disturbing aspects of the proposal in this paper is that the theory of the universe involves a fixed integer $N$, the total number of quantum states in the universe. I believe that a discussion of the meaning of this number will depend on the distinction between equations of motion and boundary conditions in physics. It has long been apparent, that even if we find the ultimate physical laws encoded in a set of equations of motion, we will still have to deal with the question of what determines the boundary conditions. In cosmology, this question has traditionally been split into two parts: ”Do the spatial sections of a Friedmann-Robertson-Walker cosmology have a boundary (and what are the boundary conditions there)?”, and ”What are the initial conditions?”.

Einstein preferred closed cosmologies because he believed this eliminated the first of these questions. Various authors [13] have tried to address the second.

There is a well known problem associated with Einstein’s suggestion, if one believes that quantum theory is the ultimate description of nature, and also believes in an ultraviolet cutoff. A closed universe with a UV cutoff must have a finite number of states. If we try to associate the cutoff with a cutoff of short distances, we immediately run into a problem. The volume of the universe changes with time, so the number of states allowed by a short distance cutoff would appear to change as well. This violates unitarity.

The advent of holographic cosmology [15][8] has resolved this conundrum. The obvious conjecture that follows from this work is that the number of states in a cos-
mology is the exponential of one fourth of the area in Planck units of a maximal set of holographic screens. I believe that ultimately this prescription will be turned around. Cosmology will be derived from quantum mechanics, with spacetime geometry being computed from the number of quantum states.

From this point of view, the natural distinction between cosmological boundary conditions will be in terms of the number of quantum states that they admit. We first have the possibility of a finite number, and then infinity. We expect that systems with a finite number of states can describe either AsDS universes or recollapsing universes. It is likely that the distinction between the two is simply whether we require an infinite or a finite number of steps in our choice of time evolution.

With an infinite number of states it is natural to look for some operator on the Hilbert space whose eigenspaces with finite eigenvalue are finite dimensional and then to make a finer classification in terms of the behavior of the density of states at large eigenvalue. Geometrically we would expect this to map into the problem of black hole entropy in cosmologies with no finite area cosmological horizon.

I think that, apart from the apparent observational evidence for a cosmological constant, our reaction to the choice between finite and infinite cosmologies (in the present sense) can at best be an emotional one. On the one hand, it is reasonable to think that nothing is actually infinite - that infinity or infinitesimal always refers to an idealization that makes problems more easy to treat mathematically (in the practical, rather than the rigorous sense of mathematics). Then one will be saddled with the annoying question of why a particular finite number is chosen. This may lead one prefer to accept infinity as a reality, though I would claim that the various choices among behaviors of the asymptotic spectrum of black holes will be equally annoying. One may find that insisting on a large asymptotic symmetry group somewhat restricts the possibilities, but the plethora of exactly stable Poincare and AdS vacua of M-theory makes this seem unlikely. The only theoretical basis for resolving this problem would seem to be to prove a theorem that every system with an infinite number of states which is asymptotically describable by a large smooth geometry, becomes supersymmetric in the asymptotic limit. As, I have noted, there is some meager evidence for this conjecture.

Given that N is finite, the question of how it is chosen might have two generic kinds of answer:

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6Provision must probably be made for duplication of information on different screens. An incomplete draft of a proposal for a completely quantum mechanical and holographic formalism for cosmology was outlined in my talk at Strings at the Millenium in CalTech. A somewhat more extended presentation of these ideas is in preparation[12].
• In the fullness of time we might show that $N$ had to satisfy some number theoretic property that is satisfied by $[0, 1, 2, 216, 2^{1620} + 23, 2^{10250} + 13365, \ldots]$. Or perhaps it is the unique solution to some number theory problem.

• There is some meta-dynamics which gives rise to quantum systems with different values of $N[16]$. Perhaps it is even some kind of deterministic dynamics and could alleviate our unease with the application of probabilistic ideas to the whole universe. In such a system we might find either a true dynamical explanation of the value of $N$, or the framework for an anthropic determination of this single parameter.

The point about these possible answers is that they have very little to do with physics in the universe we observe (hence the title of this section). Our best strategy is probably to ignore the question. The most useful attitude would appear to be to assume $N$ is a boundary condition and hope that many features of the dynamics have universal properties for large but finite $N$. Thus the characterization of the formula $M_{SUSY} \sim \Lambda^{1/8}$ as a formula for a critical exponent.

7. Some remarks on phenomenology

One of the most interesting features of the proposal in this paper is that it solves what I consider one of the primary phenomenological problems of M-theory, namely why we do not live in one of the many stable supersymmetric ground states of the theory. The answer is simply that we do not have enough states. Poincare invariant ground states have an infinite number of excitations, at least all of the scattering states of gravitons.

Our suggestion about the origin of SUSY breaking probably has more practical implications for SUSY phenomenology as well. For example suppose that the generation structure of the standard model is related to a discrete gauge symmetry that is spontaneously broken at an energy scale well below the Planck scale. We have attributed the dominant contribution to SUSY breaking to very high energy black hole states. These states will be insensitive to the low energy breaking of generation symmetry and might well produce flavor singlet squark mass matrices. Alternatively, the mere fact that SUSY breaking comes from a thermal average over a large number of states might produce flavor singlet mass matrices, without appeal to symmetries (Of course, we probably want to have flavor symmetries to explain the quark mass matrix.). One might imagine the possibility of deriving the minimal SUGRA spectrum, or some other simple pattern of SUSY breaking, from this scenario.
Another general conclusion would appear to be that the gravitino mass, as well as the masses of any moduli which originate from SUSY breaking, will be of order $\Lambda^{1/4}$. This causes well known cosmological difficulties, which must be solved.

Finally one may hope that the current approach to cosmology will eventually solve the vacuum selection problem of string/M-theory. In the limit of vanishing cosmological constant, our approach implies that the finite dimensional Hilbert space of an AsDS M-theoretic cosmology, approaches that of an asymptotically flat SUSY vacuum of M-theory. This is presumably the state which describes scattering of particles inside gravitationally bound clusters during the pre-asymptotic stage of the AsDS universe.

The question of which flat SUSY background we approach in the limit might depend on initial conditions - that is, in the small $\Lambda$ limit, the Hilbert space might break up into superselection sectors and different cosmological evolutions might end up in different sectors. On the other hand, one might hope for a more unique and universal answer. At any rate, the question is certainly tied up with that of initial conditions for cosmology.

Certain features of the desired background can be understood from general considerations. It must be supersymmetric, and its low energy effective Lagrangian must have a small deformation corresponding to a SUSY violating DS space. This makes it virtually certain that the SUSY background cannot have any moduli. Small deformations of a SUSY Lagrangian with moduli will generally give rise to cosmologies with varying moduli, rather than a DS space. In [17] I discussed a general analysis of inflationary cosmologies deriving from M-theory. Approximate moduli were argued to be good inflaton candidates, and the discrepancy between the inflation and SUSY breaking scales was attributed to the existence to a submanifold of approximate moduli space where SUSY and a discrete R symmetry were restored. Much of the postinflationary dynamics of the universe depended on the dimension of this submanifold. The present considerations suggest that one wants it to be a point, as has long been advocated by Dine [18]. This suggests that, in order to find the vacuum state of M-theory that describes the universe approximately, one must search for an isolated point in the approximate moduli space of an $N = 1$ compactification, which preserves SUSY and a discrete R-symmetry.

8. Conclusions

It should be obvious that the claims made here are somewhat tentative and unformed. One aspect of the subject that I find rather confusing is the relation of the fundamental theory to the low energy effective Lagrangian. Despite the UV/IR correspondence, I believe it is correct that physics below the Planck scale is governed by a locally supersymmetric effective Lagrangian. In [12] I have suggested that local SUSY is in
fact connected to the arbitrary choice of holographic screen, and should therefore be a fundamental symmetry, not to be broken. Since we expect the scale of SUSY breaking to be much smaller than the Planck scale there should be an effective Lagrangian description of low energy physics which is locally supersymmetric, which means that SUSY breaking appears spontaneously. The SUSY breaking scale and cosmological constant should simply be set by tuning parameters in this Lagrangian.

The confusing point is that in this description there appears to be a low energy origin for SUSY breaking. Some chiral field’s F term gets a nonzero expectation value. I suspect that the correct description will simply introduce SUSY breaking through a Volkov-Akulov [19] goldstino multiplet. The SUSY breaking scale and cosmological constant will be put in by hand. They are related by a formula of the form \( M_{SUSY} = K M_P (\Lambda/M_P^4)^{1/8} \). This formula can only be understood, and the constant \( K \) calculated, within the framework of the full theory. Similarly, the couplings of the Goldstino to other low energy fields, which determine the phenomenology of SUSY breaking, will depend on high energy physics. Only if the conjectures about relating high energy physics to black hole physics, which were adumbrated in section 4 are correct, will we be able to extract any details of the SUSY spectrum without a full understanding of the quantum mechanics of M-theory.

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