CMB: A Look Inside the Inflaton

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Abstract

We show that if the field seeding the formation of the cosmic structures is a dynamically arising bosonic condensate, the features we observe in the CMB might be interpreted as a manifestation of its compositeness.

1 Introduction

Inflation has become a standard ingredient for the description of the very early Universe [1]. In fact, it solves some of the problems of the standard Big-Bang scenario and also makes predictions about cosmic microwave background radiation (CMB) anisotropies which are being measured with higher and higher precision.

The period of accelerated expansion is usually assumed to be driven by a real scalar field, the Inflaton, whose quantum fluctuations become the seeds for the formation of structures and whose signatures are encoded in the almost scale invariant power spectrum of the CMB. In fact Inflationary models [2] not only explain the large-scale homogeneity and isotropy of the universe, but also provide a natural mechanism to generate the observed magnitude of inhomogeneity. During the period of inflation, quantum fluctuations are generated within the Hubble horizon, and then stretch outside it to become classical. In the subsequent deceleration phase, these frozen fluctuations re-enter the horizon, and seed the matter and radiation density fluctuations observed in the universe.

With the development of cosmological observations, such as WMAP [3] and Sloan Digital Sky Survey [4] the $\Lambda$CDM model has shown to be an excellent fit to the WMAP three-year data. A nearly scale-invariant, adiabatic primordial power spectrum generated during inflation can be taken as the seed for the anisotropy of CMB. Even though a red power spectrum ($n_s < 1$) for the curvature perturbations is certainly a good fit to the WMAP data, a running spectral index with $\alpha = dn_s/d \log k$ slightly improves the fit [3]. The running is allowed not only by taking into account the WMAP data, but also in combination with other CMB and/or large scale structure information, such as 2dFGRS [5] and the Sloan Digital Sky Survey [4]. Further analysis of a possible running spectral index is discussed in [6] and theoretical explorations of WMAP results are carried out recently in [7]. The WMAP data also implies that a red power spectrum ($n_s < 1$)
at $k = 0.05\text{Mpc}^{-1}$ is favored and the running of the spectral index is not required at more than the 95% confidence level.

The CMB data might lead one to the attempt of reconstructing to high precision the inflaton potential, as done in refs. [3]. Further, it has been recently suggested that inflation might provide a window towards trans-Planckian physics [9, 10]. The reason for this is that inflation magnifies all quantum fluctuations and, therefore, red-shifts originally trans-Planckian frequencies down to the range of low energy physics. In this framework on might think that even non-commutativity arising at the highest energies (see [4]) might play a role in determining the CMB features.

In this article we would like to suggest a further possible interpretation of the CMB features, by assuming that the inflaton might not be a fundamental field but a dynamically arising bosonic condensate, or at least that this condensate could be at the origin of the CMB anisotropies. In refs. [12, 13] it is actually shown that a four-fermion interaction arises in General Relativity when fermions are covariantly coupled, and that at early times and high densities the correlation between pairs of fermions is enhanced. This enhancement leads to a BCS-like condensation of the fermions and opens a gap, indicating the dynamical formation of a bosonic condensate. If this is the case, it is possible that the features of the present day CMB are a sign of the compositeness of the inflaton (or curvaton) field. The correlation functions of a scalar field usually employed in the derivation the CMB power spectrum should then be substituted by the correlation functions for a composite scalar field operator in the form, for example, $\bar{\psi}\psi$ or $i\bar{\psi}\gamma_5\psi$. The leading behaviour of these correlation functions would be, as usual, those of a fundamental scalar field in a de Sitter spacetime, leading to a scale invariant spectrum. On the other hand the deviations from a scale invariant power spectrum, described by the spectral index $n_s$ and its running $\alpha$, might be the sign of the compositeness of the seeding field. By assuming a logarithmic correction to the two-point function (this is our ansatz, derived from the literature on composite scalar field correlation functions in flat spacetime [14]) we were able to derive values for spectral index and its running compatible with the experimental data.

### 2 Cosmological Fermionic Condensate

In this subsection we outline the essential steps taken in [12] to describe the cosmological condensation. Let us start by considering purely gravitational dynamics as given by the Holst action

\[ S[e,A] = \frac{1}{16\pi G} \left( \int d^4x \, e_{\mu}^{\nu} e_{\rho}^{\nu} F_{\rho\mu}^b - \frac{1}{\gamma} \int d^4x \, e_{\mu}^{\nu} e_{\rho}^{\nu} \tilde{F}_{\rho\mu}^a \right), \]

which is a functional of the tetrad field $e_{\mu}^{a}$ [15]. Here $a = 0, 1, 2, 3$ is the internal Lorentz index, $\mu = 0, 1, 2, 3$ the coordinate index, and $e = \det e_{\mu}^a$. $F_{\mu\nu}^a$ is the curvature of the connection $A_{\mu\nu}^a$ defined as

\[ A_{\mu\nu}^a = \epsilon^c_{\mu}(e_{\nu}^{\mu} - \Gamma_{\mu\nu}^\alpha e_{\nu}^\alpha), \]

and $\Gamma_{\mu\nu}^\alpha$ are the Christoffel symbols and finally $\tilde{F}_{\mu\nu}^a = \frac{1}{2} \epsilon_{abc} F_{\mu\nu}^{cd} F_{\rho\sigma}^{cd}$ is the (internal) dual field strength.

The first term in (11) yields the tetrad formulation (Palatini action) of the Einstein-Hilbert action, the latter emerging when inserting a solution to the associated equation of motion ($A_{\mu\nu}^a$ being a torsion-free spin connection $\omega_{\mu\nu}^c[e]$) into (11) and using $g_{\mu\nu} = e_{\mu}^a e_{\nu}^a$. The second term is identically zero due to the Bianchi identity for the Riemann tensor. It follows that, regardless
of the value of $\gamma$, the action (1) is classically equivalent to the familiar Einstein-Hilbert action \[17\]. What holds true for pure gravity is no longer valid if minimally coupled chiral fermions are introduced. The equation of motion for the tetrad $e^\mu_a$ subject to a fermionic source is solved in terms of a connection $A^\mu_{cd}$, having two contributions, a torsion-free spin connection as in the purely gravitational case and a torsion term related to the axial fermion current. Upon substituting $A^\mu_{cd}$ back into the action, a four-fermion interaction of the following form emerges \[17\]:

$$S_{\text{int}} = \frac{K}{2} \int d^4x \left( \bar{\psi} \gamma_5 \gamma^a \psi \right) \left( \bar{\psi} \gamma_5 \gamma^a \psi \right) ; \quad K = -3\pi G \frac{\gamma^2}{\gamma^2 + 1} = -\frac{9}{8M_p^2} \frac{\gamma^2}{\gamma^2 + 1}. \quad (3)$$

Thus the Immirzi parameter acquires physical relevance through the presence of massless fermions, even though gravity is still treated classically.

The action describing the fermions reads

$$S_{\text{ferm}} = \int d^4x e \left[ \bar{\psi} i e^\mu_a \gamma^a D^\mu [e] \psi + \frac{K}{2} \left( \bar{\psi} \gamma_5 \gamma^a \psi \right) \left( \bar{\psi} \gamma_5 \gamma^a \psi \right) \right], \quad (4)$$

where $D^\mu [e]$ is the covariant derivative with respect to the connection $A$. Let us consider the system of interacting fermions in a de Sitter spacetime in FRW coordinates

$$ds^2 = dt^2 - a^2(t) d\vec{x} \cdot d\vec{x}, \quad (5)$$

where $a = a_0 e^{Ht}$ is the scale factor. In this case the vierbein reads

$$e_{\mu a} = \delta_{\mu 0} \delta_{a 0} - a(t) \delta_{\mu i} \delta_{a i}. \quad (6)$$

The consideration of a de Sitter spacetime is justified in an epoch where the energy density belonging to fluctuating degrees of freedom is sufficiently diluted as compared to the energy density of condensed degrees of freedom.

Applying a Fierz transformation to the current-current interaction in (3) yields

$$\left( \bar{\psi} \gamma_5 \gamma^a \psi \right) \left( \bar{\psi} \gamma_5 \gamma^a \psi \right) \rightarrow \frac{1}{N} \left( \bar{\psi} \psi \right)^2 + \frac{1}{N} \left( \bar{\psi} i \gamma_5 \psi \right)^2 + \ldots , \quad (7)$$

where the dots refer to flavor-nonsinglet contributions and products of vector and axial-vector currents, which do not lead to vacuum condensates. Allowing for a bare cosmological constant $\Lambda_0$, and denoting the bare reduced Planck mass by $M_0$, the complete action then takes the form

$$S = \int d^4x e \left\{ M_0^2 H^2 - \Lambda_0 + \bar{\psi} i e^\mu_a \gamma^a D^\mu [e] \psi + \frac{K}{2N} \left[ \left( \bar{\psi} \psi \right)^2 + \left( \bar{\psi} i \gamma_5 \psi \right)^2 + \ldots \right] \right\}, \quad (8)$$

where only the scalar flavor-singlet bilinears are shown explicitly. For condensation to take place $K$ needs to be positive, implying an imaginary $\gamma$ with $|\gamma| < 1$. In fact, only if $K > 0$ an attractive force occurs between fermions in the scalar channel \[18\]. In this case bound states form: The operator $\bar{\psi} \psi$ corresponds to a scalar field $\sigma$, while $\bar{\psi} i \gamma_5 \psi$ corresponds to a pseudoscalar field $\pi$. In addition, $(N^2 - 1)$ flavor-nonsinglet scalar and pseudoscalar fields $\sigma^k \sim (\bar{\psi} k^\mu \psi)$ and $\pi^k \sim (\bar{\psi} k^\mu i \gamma_5 \psi)$ appear. In the usual treatment of the Nambu–Jona-Lasinio model in Minkowski space \[18\] the dynamical breaking of chiral symmetry occurs for sufficiently large $K$. As a
result, the scalar fields $\sigma$ and $\sigma^k$ become massive, while the pseudoscalar fields $\pi$ and $\pi^k$ remain massless and represent Goldstone bosons (see, e.g., [19] for a review).

In a de Sitter background dynamical chiral symmetry breaking occurs for all values $K > 0$. Only the isosinglet fields are considered here. In fact, only the isosinglet scalar field $\sigma$ can acquire a nonzero vacuum expectation value and thus is relevant for de Sitter cosmology in the early Universe. In a de Sitter spacetime we expect that the particles associated with the remaining low-lying flavor-nonsinglet fields are sufficiently diluted to provide for the self-consistency of the de Sitter geometry.

### 3 Two Point Function in de Sitter Space-Time

Let us consider a scalar field $\phi$. We usually assume that it can be split into a classical background piece and a piece due to fluctuations according to

$$\phi(\tau, x) = \phi^{(0)}(\tau) + \delta\phi(\tau, x). \quad (9)$$

From now on we forget about the homogeneous part and assume that scalar field fluctuations are at the origin of the CMB we observe today. For convenience we have introduced the conformal time $\tau$, such that the metric is given by

$$ds^2 = a(\tau)^2 (dt^2 - dx^2). \quad (10)$$

In these coordinates the Klein-Gordon equation, ignoring the potential piece, becomes

$$\delta\phi''_k + \frac{2a'}{a} \delta\phi'_k + k^2 \delta \phi_k = 0, \quad (11)$$

where we have Fourier transformed in space and introduced the comoving momentum $k$. The conventions are such that

$$\delta\phi(x) = \frac{1}{(2\pi)^{3/2}} \int \delta\phi_k e^{ik\cdot x} d^3k. \quad (12)$$

We have also introduced the notation $t$ for derivatives with respect to conformal time. If we then introduce the rescaled field $\mu = a\delta\phi$, the equation becomes

$$\mu''_k + \left(k^2 - \frac{a''}{a}\right) \mu_k = 0. \quad (13)$$

To proceed, we assume that the scale factor depend on conformal time as

$$a \sim \tau^{1/2 - \nu}, \quad (14)$$

where $\nu$ is a constant. An important example is $a \sim e^{Ht}$ with $H = \text{const.}$, where the change of coordinates gives

$$\frac{d\tau}{dt} = \frac{1}{a(t)} = e^{-Ht} \implies a(\tau) = -\frac{1}{H\tau}, \quad (15)$$

and we find that $\nu = \frac{3}{2}$. Note that the physical range of $\tau$ is $-\infty < \tau < 0$. The equation for the fluctuations, with $a$ of the form above, becomes

$$\mu''_k + \left(k^2 - \frac{1}{\tau^2} \left(\nu^2 - \frac{1}{4}\right)\right) \mu_k = 0. \quad (16)$$
This is a well known equation which is solved by Hankel functions. The general solution is given by

\[ f_k(\tau) = \frac{1}{\sqrt{2k}} \left( C_1(k) H_v^{(1)}(-k\tau) + C_2(k) H_v^{(2)}(-k\tau) \right), \tag{17} \]

where \( C_1(k) \) and \( C_2(k) \) are to be determined by initial conditions.

When quantizing this system one needs to introduce oscillators \( a_k(\tau) \) and \( a_{-k}^\dagger(\tau) \) such that

\[ \mu_k(\tau) = \frac{1}{\sqrt{2k}} \left( a_k(\tau) + a_{-k}^\dagger(\tau) \right) \tag{18} \]

\[ \pi_k(\tau) = \frac{1}{\tau} \mu_k(\tau) = -i \sqrt{\frac{k}{2}} \left( a_k(\tau) - a_{-k}^\dagger(\tau) \right), \tag{19} \]

obey standard commutation relations. These oscillators are time dependent, and can be expressed in terms of oscillators at a specific moment in time using the Bogolubov transformations

\[ a_k(\tau) = u_k(\tau) a_k(\tau_0) + v_k(\tau) a_{-k}(\tau_0) \tag{20} \]

\[ a_{-k}(\tau) = u_k^*(\tau) a_{-k}^\dagger(\tau_0) + v_k^*(\tau) a_k(\tau_0), \tag{21} \]

where

\[ |u_k(\tau)|^2 - |v_k(\tau)|^2 = 1. \tag{22} \]

The latter equation makes sure that the canonical commutation relations are obeyed at all times if they are obeyed at \( \tau_0 \). We can now write down the quantum field

\[ \mu_k(\tau) = f_k(\tau) a_k(\tau_0) + f_k^*(\tau) a_{-k}(\tau_0), \tag{23} \]

where

\[ f_k(\tau) = \frac{1}{\sqrt{2k}} \left( u_k(\tau) + v_k^*(\tau) \right) \tag{24} \]

is given by (17).

The usual procedure for fixing for the initial conditions is to consider the infinite past and choose a state annihilated by the annihilation operator, i.e.

\[ a_k(\tau_0) |0, \tau_0\rangle = 0, \tag{25} \]

for \( \tau_0 \to -\infty \). From (18) we conclude that

\[ \pi_k(\tau_0) = -ik\mu_k(\tau_0), \tag{26} \]

for \( \tau_0 \to -\infty \). Since the Hankel functions asymptotically behave as

\[ H_v^{(1)}(-k\tau) \sim \sqrt{\frac{2}{-k\tau \pi}} e^{-ik\tau} \]
\[ H_v^{(2)}(-k\tau) \sim H_v^{(1)*}(-k\tau), \tag{27} \]

we find that the vacuum choice correspond to the choice \( C_2(k) = 0 \) (and \( |C_1(k)| = 1 \)).
We have now fully determined the quantum fluctuations, and it is time to deduce what their effect will be on the CMBR. To do this, we compute the size of the fluctuations according to
\[
\langle \hat{\varphi}(\eta, \mathbf{x}) \hat{\varphi}(\eta, \mathbf{x}') \rangle = \int \frac{dk}{k} \frac{\sin k|\mathbf{x} - \mathbf{x}'|}{k|\mathbf{x} - \mathbf{x}'|} \frac{k^3|\varphi_k(\eta)|^2}{2\pi^2} = \int \frac{dk}{k} \frac{\sin k|\mathbf{x} - \mathbf{x}'|}{k|\mathbf{x} - \mathbf{x}'|} P_{\varphi}(k, \eta).
\] (26)

from which
\[
P(k) = \frac{4\pi k^3}{(2\pi)^3} \left| \delta \varphi_k \right|^2 = \frac{k^3}{2\pi^2 a^2} \left| \mu_k \right|^2 = \frac{k^3}{2\pi^2 a^2} \left| f_k \right|^2 = \frac{k^3}{2\pi^2 a^2} \frac{|-\tau|}{4} H^{(1)}(-k\tau)^2. \]

This we should evaluate at late times, that is, when \( \tau \to 0 \). In this limit the Hankel function behaves as
\[
H^{(1)}(-k\tau) \sim \sqrt{\frac{2}{\pi}} (-k\tau)^{-\nu},
\] (28)

and we find
\[
P \sim \frac{1}{4\pi^2} \frac{1}{a^2} (-\tau)^{1-2\nu} k^{3-2\nu} \sim \frac{1}{4\pi^2} H^2 k^{3-2\nu}.
\] (29)

Here we have used [14] to get rid off the \( \tau \) dependence. Furthermore, if \( \nu \sim 3/2 \) and we have a slow roll, \( H \) is nearly constant and can be used to set the scale of the fluctuations. In particular, we find the well known scale invariant spectrum if \( \nu = 3/2 \),
\[
P = \frac{1}{4\pi^2} H^2.
\] (30)

4 The Condensate Corrections

Let us assume that the previously described condensation mechanism is realized in the inflationary expansion of the Universe and that the dynamically generated composite scalar field is at the origin of the curvature perturbations. The composite nature of this field would then become apparent in its correlation functions. In particular the equal time 2 point correlation function discussed above \( \langle \hat{\varphi}(\eta, \mathbf{x}) \hat{\varphi}(\eta, \mathbf{x}') \rangle \) would be replaced by the 2-point function for \( \bar{\psi} \psi \) so that the power spectrum will be defined by
\[
\langle (\bar{\psi}\psi)(\eta, \mathbf{x})(\bar{\psi}\psi)(\eta, \mathbf{x}') \rangle \equiv \int \frac{dk}{k} \frac{\sin k|\mathbf{x} - \mathbf{x}'|}{k|\mathbf{x} - \mathbf{x}'|} P(k, \eta).
\] (31)

This would lead to corrections to the pure scalar field correlation function, as we expect the leading behavior not to be affected. We thus assume that the Fourier transform of the 2-point function will be modified as, factorizing the leading behavior,
\[
\tilde{\xi}(k) \simeq \frac{1}{k^3} \left( 1 + \delta \tilde{\xi} \right)
\] (32)

Given the relation
\[
P(k) = \frac{k^3}{2\pi} \tilde{\xi}(k) \simeq k^{n_s-1}
\] (33)

one obtains
\[
\log \left( 1 + \delta \tilde{\xi} \right) \simeq (n_s - 1) \log k
\] (34)
By referring to the literature (see [14]) we make the ansatz for the corrections to $\tilde{\xi}$

$$\delta\tilde{\xi} = \log \left[ 1 + A \log \left( \frac{B^2}{k^2} \right) \right]$$

(35)

where $A$ is the amplitude of the logarithmic correction and $B$ is the unit in which $k$ is measured. This leads to an expression for the spectral index

$$n_s \simeq 1 + \frac{\log \left[ 1 + A \log \left( \frac{B^2}{k^2} \right) \right]}{\log \left( \frac{k}{B} \right)}$$

(36)

The behavior of the spectral index is shown in Fig. 1 for positive $A$ and in Fig. 2 for negative $A$. In the first case the spectral index is greater than 1 (blue spectrum) whereas in the second case it is lower than 1 (red spectrum). In both cases the running of the spectral index $\alpha$ can be as large $\alpha \simeq -0.1$ and is always negative. If one would like to accommodate a blue spectrum for small $k$ and a red spectrum for high $k$ one would be required to allow for a dependence (running) of $A$ on $k$ or allow for a more generic dependence of the 2-point function correction.
5 Conclusions

We have shown that by assuming the curvature perturbations at the origin of the CMB are due
to a dynamically arising bosonic condensate, the features observed in the power spectrum can
be interpreted as an expression of its composite nature. In particular we were able to derive
a red power spectrum compatible with the observed CMB data with a negative running which
can be of the order of magnitude $\alpha \simeq -0.05$.

References

[1] A.D. Linde, Particle physics and inflationary cosmology (Harwood, Chur, Switzerland,
1990). A.R. Liddle and D.H. Lyth, Cosmological inflation and large-scale structure (Cam-
bridge University Press, Cambridge, England, 2000).

[2] A.H. Guth, Phys.Rev.D 23(1981)347; A.D. Linde, Phys.Lett.B 108(1982)389;

[3] D.N. Spergel et al., astro-ph/0603449

[4] M. Tegmark et al., astro-ph/0608632

[5] S. Cole et al., Mon.Not.Roy.Astron.Soc. 362(2005)505, astro-ph/0501174

[6] B. Feng, J.Q. Xia and J. Yokoyama, astro-ph/0608365.

[7] Q.G. Huang, M. Li and J.H. She, hep-th/0604186
    J.M. Cline and L. Hoi, JCAP 0606(2006)007;
    G. Ballesteros, J.A. Casas and J.R. Espinosa, JCAP 0603(2006)001;
    J. Martin and C. Ringeval, JCAP 0608(2006)009;
    Q.G. Huang, Phys.Rev.D 74(2006)063513;
    Q.G. Huang and M. Li, Nucl.Phys.B 755(2006)286;
    H. Peiris and R. Easther, JCAP 0607(2006)002;
    R. Easther and H. Peiris, JCAP 0609(2006)010;
    S.M. Leach, A.R. Liddle, J. Martin and D.J. Schwarz, Phys.Rev.D 66(2002);
    G. L. Alberghi, R. Casadio and G. Venturi, Phys. Rev. D 60, 124018 (1999)
    S.M. Leach and A.R. Liddle, Mon.Not.Roy.Astron.Soc. 341(2003)1151;
    L. Abalidi and D.H. Lyth, JCAP 08(2006)013;
    A.R. Liddle and S.M. Leach, Phys.Rev.D 68(2003)103503.

[8] J. E. Lidsey, A. R. Liddle, E. W. Kolb, E. J. Copeland, T. Barreiro and M. Abney, Rev.
Mod. Phys. 69, 373 (1997);
    S. M. Leach and A. R. Liddle, Phys. Rev. D 68, 123508 (2003);
    C. Caprini, S. H. Hansen and M. Kunz, Mon. Not. Roy. Astron. Soc. 339, 212 (2003);
    S. Cole et al. [The 2dFGRS Collaboration], Mon. Not. Roy. Astron. Soc. 362, 505 (2005);
    B. Feng, J. Q. Xia and J. Yokoyama, JCAP 0705, 020 (2007);
    J. M. Cline and L. Hoi, JCAP 0606, 007 (2006);
    G. Ballesteros, J. A. Casas and J. R. Espinosa, JCAP 0603, 001 (2006);
    J. Martin and C. Ringeval, “Inflation after WMAP3: Confronting the slow-roll and exact
power spectra JCAP 0608, 009 (2006);
L. Alabidi and D. H. Lyth, JCAP 0608, 013 (2006);
G. L. Alberghi, R. Casadio and A. Tronconi, Mod. Phys. Lett. A 22, 339 (2007)
R. Easther and H. Peiris, JCAP 0609, 010 (2006);
D. J. H. Chung, G. Shiu and M. Trodden, Phys. Rev. D 68, 063501 (2003);
H. Peiris and R. Easther, JCAP 0607, 002 (2006);

[9] R. H. Brandenberger and J. Martin, Phys. Rev. D 71, 023504 (2005);
    J. Martin and R. Brandenberger, Phys. Rev. D 68, 063513 (2003);
    J. Martin and R. H. Brandenberger, Phys. Rev. D 63, 123501 (2001)
G. L. Alberghi, R. Casadio and A. Tronconi, Phys. Rev. D 74, 103501 (2006)

[10] R.H. Brandenberger and J. Martin, Phys. Rev. D 63, 123501 (2001)
    G.L. Alberghi, D. A. Lowe and M. Trodden, JHEP 9907, 020 (1999)
    A. Kempf and J.C. Niemeyer, Phys. Rev. D 64, 103501 (2001)
    R. Easther, B.R. Greene, W.H. Kinney, and G. Shiu, Phys. Rev. D 64, 103502 (2001);
    Phys. Rev. D 66, 023518 (2002)
    U.H. Danielsson, Phys. rev. D 66, 023511 (2002)

[11] R. Brandenberger and P. M. Ho, “Noncommutative spacetime, stringy spacetime uncertainty principle, and
    Phys. Rev. D 66, 023517 (2002) [AAPPs Bull. 12N1, 10 (2002)]
    S. Cremonini, Phys. Rev. D 68, 063514 (2003)
    S. Alexander, R. Brandenberger and J. Magueijo, Phys. Rev. D 67, 081301 (2003)
    G.L. Alberghi, K. Goldstein and D. A. Lowe, Phys. Lett. B 578, 247 (2004)
    G. L. Alberghi and A. Tronconi, Phys. Rev. D 73, 027702 (2006)
    Q. G. Huang and M. Li, JHEP 0306, 014 (2003)
    Q. G. Huang and M. Li, JCAP 0311, 001 (2003)
    Q. G. Huang and M. Li, Nucl. Phys. B 755, 286 (2006)
    F. Lizzi, G. Mangano, G. Miele and M. Peloso, JHEP 0206, 049 (2002)
    G. L. Alberghi, R. Casadio and A. Tronconi, Phys. Rev. D 74, 103501 (2006)
    K. Fang, B. Chen and W. Xue, Phys. Rev. D 77, 063523 (2008)

[12] F. Giacosa, R. Hofmann and M. Neubert, JHEP 0802, 077 (2008) R. Hofmann, Nucl. Phys.
    B 740, 195 (2006)

[13] S. Alexander and T. Biswas, arXiv:0807.4468 [hep-th].
    S. Alexander and D. Vaid, arXiv:hep-th/0702064
    S. H. S. Alexander and D. Vaid, arXiv:hep-th/0609066.

[14] M.E.Peskin and D.V.Shroeder An Introduction to Quantum Field Theory 1995, Addison-Wesley
    A. A. Andrianov, V. A. Andrianov and R. Rodenberg, JHEP 9906, 003 (1999)
    A. A. Andrianov, V. A. Andrianov, V. L. Yudichev and R. Rodenberg, Int. J. Mod. Phys.
    A 14, 323 (1999).

[15] S. Holst, Phys. Rev. D53, 5966 (1996).

[16] A. Ashtekar, Phys. Rev. Lett. 57, 2244 (1986); Phys. Rev. D36, 1587 (1987).
[17] A. Perez and C. Rovelli, Phys. Rev. D73, 044013 (2006)

[18] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); ibid. 124, 246 (1961).

[19] S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992).

[20] S. Hannestad, S.H. Hansen, F.L. Villante and A.J.S. Hamilton, Astropart.Phys.17(2002)375,
    S. Hannestad, S.H. Hansen and F.L. Villante, Astropart.Phys.16(2001)137,