Scaling law of average failure rate and steady-state rate in rocks

SHENGWANG HAO,1,2 CHAO LIU,1 YINGCHONG WANG,1 and FUQING CHANG1

Abstract—The evolution properties in the steady stage of a rock specimen are reflective of the damage or weakening growth within and thus are used to determine whether an unstable transition occurs. In this paper, we report the experimental results for rock (granite and marble) specimens tested at room temperature and room humidity under three typical loading modes: quasi-static monotonic loading, brittle creep, and brittle creep relaxation. Deformed rock specimens in current experiments exhibit an apparent steady stage characterized by a nearly constant evolution rate, which dominates the lifetime of the rock specimens. The average failure rate presents a common power–law relationship with the evolution rate in the steady stage, although the exponent is different for different loading modes. The results indicate that a lower ratio of the slope of the secondary stage with respect to the average rate of the entire lifetime implies a more brittle failure.

Key words: Steady stage, time-to-failure, failure mode, rock.

1. Introduction

When a rock specimen is loaded quasi-statically in the laboratory, the accelerating deformation that leads to eventual failure always follows an apparent, constant strain rate stage. This stage is called the steady, or secondary stage, and can be observed in three types of experiments: monotonic loading by controlling the crosshead of the testing machine moving at a constant velocity (Hao et al. 2013), brittle creep testing (Brantut et al. 2014), and creep relaxation testing (Hao et al. 2014). Creep failure in rocks is typically classified within three temporal stages: primary creep, secondary creep, and accelerating tertiary creep (Scholz 1968; Okubo et al. 1991; Lockner 1993). The primary and tertiary stages receive the most attention because researchers believe they are more closely associated with failure. At the primary creep stage, the strain rate \( \dot{\varepsilon}(t) \) decays as a power law \( \dot{\varepsilon}(t) \sim t^{-p_1} \), called the Andrade’s law (Andrade 1910), with time following the application of the stress. The exponent \( p_1 \approx 2/3 \) (Andrade 1910). The power law decaying behavior for the rate of damage events (Amitrano and Helmstetter 2006) in the primary stage is similar to the modified Omori’s law (Omori 1894; Utsu 1961) \( n(t) \sim (c + t)^{-p_n} \), which describes the rate \( n(t) \) of earthquakes decaying with time after the main shock. The exponent \( p_n \) could differ from 1, though it is usually almost equal to 1. These relations could suggest a clue for understanding and predicting the delayed failure triggered by the main shock or other causes.

The tertiary creep stage represents rapid, unstable growth, and thus it should provide insight into the failure process. Power–law creep acceleration behavior during tertiary creep was revealed by researchers (Voight 1988, 1989; Guarino et al. 2002; Nechad et al. 2005). Voight’s relation (Voight 1988, 1989) \( \dot{\Omega} = \theta (\dot{\Omega}) - \Lambda = 0 \) has been widely accepted as the predominant method for describing the behavior of a material in the terminal stage of failure, where \( \Lambda \) and \( \theta \) are the constants of experience and \( \Omega \) is a measurable quantity such as strain. The dot refers to differentiation with respect to time. Similar power–law accelerations were also observed for natural structures, such as landslides (Saito and Uezawa 1961; Saito 1969; Petley et al. 2002), volcanoes (Voight 1988), or cliff collapses (Amitrano et al. 2005). Kilburn (2012) proposed a model to extend analyses to deformation under increasing stress and suggested an alternative relation between fracturing and stress. Hao et al. (2013) compressed granites and marbles in the laboratory by controlling the crosshead of the testing machine moving at a constant velocity.
They defined a response function as the change of the sample’s deformation with respect to the displacement of crosshead. The results showed that following a pseudo-steady stage, the response function increased rapidly as a power law relationship with displacement. Hao et al. (2016) presented a systematical analysis of this critical accelerating behavior and suggested a new relation \( \dot{\Omega}^{-1} \ddot{\Omega} \sim (t_f - t) \) to predict failure. where \( t_f \) represents the failure time. Efforts on describing the primary and tertiary stages improved our knowledge on the mechanism of failure and its prediction. The results also imply that the secondary stage as a stable stage between the primary and tertiary stages should reflect the specific properties of a specimen and its lifetime. Especially, the secondary stage reflects how a sample evolves from the decelerating stage to the accelerating stage.

It should be noted that damage, or weakening growth, in the secondary stage determines the unstable transition to the accelerating tertiary stage, and thus its evolution properties should contain information linked to the time to failure. In the laboratory tests, the secondary stage always dominates the lifetime of rock samples. Likewise in earthquake cycles, it is stated that throughout the interseismic period, the secondary stage is the dominant process (Perfettini and Avouac 2004). The slow speed of pre-earthquake deformation and strain accumulation (Chen et al. 2000; Shen et al. 2005; Meade 2007; Zhang 2013) in the Longmen Shan fault zone, which hosted the 2008 Wenchuan Mw 7.9 earthquake, China, led to an incorrect assessment of this hazard event. Therefore, revealing the relationship between the secondary stage and ultimate failure is critical to predicting failure and understanding the underlying mechanisms for it.

The strain rate in secondary stage creep is strongly dependent on the applied stress (Amiratiano and Helmstetter 2006). It was shown that the time to failure for a rock decreases with increasing mean stress (Scholz 1968; Kranz et al. 1982; Boukharov et al. 1995; Baud and Meredith 1997; Lockner 1998), and that the average time to failure and the applied stress exhibits an exponential relationship (Das and Scholz 1981). The experiments (Hao et al. 2013, 2014) indicated that the monotonic and creep-relaxation experiments also showed a pseudo-steady stage similar to the creep experiments. The dependence of the secondary creep rate and time to failure on the applied stress suggests that there should be a possible relationship between the secondary creep stage and the time to failure (Hao et al. 2014).

In this paper, we aim to establish an empirical relationship between the eventual failure and the evolution properties in the secondary stage in a rock sample under quasi-static tests. Monotonic (quasi-static constant displacement rate), brittle creep, and brittle creep relaxation experiments are the three typical laboratory experiments performed to investigate the dependency of the time-to-failure and failure modes on the secondary stage property.

2. Experimental methodology and material

Granite and marble, of the two major types of rocks found in the earth’s crust, are tested in this experiment. Some of the physical properties of these two types of rocks are listed in Table 1. The rocks were sampled from a depth of \( \sim 10 \) m from Beijing, China, and cut into prismatic blocks (40 mm in height and 16 mm \( \times \) 20 mm in cross-section). The surfaces of the specimens were cut for parallelism and perpendicularity between the faces, with particular emphasis on ensuring that the two ends were exactly perpendicular to the longitudinal axis of the sample. The specimens were intact, but had many randomly distributed intrinsic and natural microfractures. Figure 1 shows two microscopic images of crack pattern measured by SEM (scanning electron microscope) for two investigated rocks under loading to give an insight into their microstructures and fracture behaviors.

We performed three types of quasi-static experiments that were designed to simulate the three typical types of fault loading occurring in the upper crust. Figure 2 illustrates the experimental set-up and loading processes: monotonic loading (type 1), brittle creep (Amiratiano and Helmstetter 2006; Heap et al. 2011; Brantut et al. 2013, 2014) (type 2), and brittle creep relaxation (Hao et al. 2013) (type 3). In laboratory tests, the loading system consists of a load apparatus and a deformed sample. The load apparatus is always modeled by an analogous “elastic spring”
The displacement, $U$, of the crosshead of the testing machine includes the deformations of the loading apparatus, and that, $u$, of the rock samples. In the first type of experiment (type 1), a rock specimen is compressed by monotonically and quasi-statically moving the crosshead of the testing machine at a constant rate (i.e. $U$ is in linear relation with time). Brittle creep experiments (Amitrano and Helmstetter 2006; Heap et al. 2011; Brantut et al. 2013, 2014) were the second type of experiment (type 2) performed. In these experiments, a rock specimen was first loaded to a prescribed initial stress, which was then maintained at a constant value to observe the evolution of the deformation. The evolution of the deformation relates to the responses under applied and invariant stresses, (Benioff 1951; Scholz 1968; Singh 1975; Lockner 1993; Du and McMeeking 1995; Lienkaemper et al. 1997; Heap et al. 2011) and could be possibly analogous to some processes of fault weakening that may lead to a seismic rupture. Brittle creep relaxation (Hao et al. 2014) was the third type of experiment (type 3) performed. In this experiment, a rock specimen was first loaded to an initial state by imposing an initial displacement to the crosshead of the testing machine. Then, the displacement was held at a constant value. The rock specimen then undergoes a combination process in which it deforms but the stress relaxes because the crosshead of the testing machine is held constant. This experiment simulated the process in which the

Water absorption is the mass’s ratio of water absorbed in the atmosphere by unit volume rock with respect to the mass of the dry rock.
elastic energy release of the surroundings drives the damage propagation, or weakening of a fault zone after a main earthquake (Hao et al. 2014).

In the experiments, the rock specimens were compressed uniaxially along the 40-mm axis at room temperature, ranging from 10 to 30 °C, and room humidity, with average relative humidity ~62%. Uniaxial compression was achieved using a screw-driven crosshead, which is a universal electromechanical testing machine equipped with a load cell with an offset load of 1 kN. The deformation, \( u \), of a specimen was measured using 1 μm resolution extensometers located on the sides of the specimen. The displacement, \( U \), of the crosshead was continuously measured using a linear variable differential transformer with a resolution 1 μm.

In the type 1 experiment, the rock specimens were compressed by monotonically increasing the crosshead displacement at a rate of 0.02 mm/min (leading to a strain rate of approximately \( 8.3 \times 10^{-6} \) s\(^{-1} \)) until failure (i.e., no-hold step). The experimental protocols for type 1, type 2, and type 3 are illustrated in Figs. 3, 4 and 5. In brittle creep tests (type 2), the rock specimen was first loaded to an initial stress, (AB portion in Fig. 4) which was held at a constant value (AB portion in Fig. 4) while the deformation was measured. In the brittle creep relaxation experiment (type 3), the rock specimen was first rapidly loaded to the initial deformation state (OA part in Fig. 5) with a crosshead speed of 1.5 mm/min for the subsequent relaxation test. The crosshead was then held at this constant position (AB portion in Fig. 5) and the deformation and stress of the sample were measured as it relaxed. Hao et al. (2014) have detailed this loading process.
In this paper, the symbol $t_f$ denotes the failure time and $t_0$ represents the start time of the creep phase, or creep-relaxation phase, in brittle creep or creep-relaxation experiments. Thus, $(t_f - t_0)$ is the creep time or creep-relaxation time in these two experiments. The experimental parameters of all specimens tested in three types of experiments are listed in Tables 2, 3 and 4. It should be mentioned that there was an event of fracture that occurred during the testing of specimens SC-G-80-2 and SC-G-85-3. An audible sound was emitted by the fracture but the specimen did not fail completely. This small event induced a small jump in the curves (see Fig. 7a, b). Thus, the steady stage has a more direct relation with this small event than the macroscopic failure, and then we select the $u_f$ and $t_f$ corresponding to this small event to calculate the value of the average creep rate $\mu = \frac{u_f - u_0}{t_f - t_0}$.

We calculated the rate of deformation (or stress) $du/dU$, $d\sigma/dt$ or $d\varepsilon/dt$ by using the finite difference method. This constant rate stage is defined as the secondary stage in the present paper, and correspondingly, the value of the rate plateau is determined as the slope, $\dot{\varepsilon}S$, of the steady stage in the deformation (or stress) curves.

3. Results

3.1. Stages of evolution to failure

Let us have a close examination of the evolution of the response variables, such as stress or deformation, in the three experiments. Figure 3 shows a typical result for rock specimens tested in the type 1 experiment. The solid blue line plots the axial deformation ($u$) against time, and the solid red squares plot the stress–time curve. It can be seen that at the early stage, the stress–time relation is slightly convex upwards (also see Rudnicki and Rice 1975; Jeager et al. 2007; Hao et al. 2007, 2013) and the deformation growth is characterized by an initial convex upward phase of decreasing strain rate. Later, an almost linear stress–time relation, as well as a nearly linear $u \sim t$ relation, follows. Finally, the deformation, $u$,
of the sample increases rapidly to catastrophic failure, which is associated with a jump of stress (Fig. 3). It has been demonstrated that the catastrophic failure is induced by the elastic energy release from the loading apparatus, which occurs at some point in the strain softening section after peak stress (Salamon 1970; Hundson et al. 1972; Labuz and Biolzi 1991; Bai et al. 2005; Jeager et al. 2007; Hao et al. 2007, 2010, 2013). Figure 6 plots the deformation–displacement curves for specimens. It shows that specimens exhibit the typical three-staged behavior of primary, secondary, and terminal accelerated evolution to failure. In the primary stage, the deformation–displacement curve is concave downward and the slope of the deformation–displacement decreases with increasing displacement. Later, an almost linear deformation–displacement relation, i.e. a pseudo-steady stage follows. In the tertiary stage, the curve is concaved upwards and the deformation increases rapidly.

In brittle creep tests, rock specimens were loaded with different, constant applied stresses. Figure 4 shows a typical result of the type 2 experiment to illustrate the complete process of brittle creep testing. It can be seen that at the creep phase (AB portion) after imposing the initial stress (OA portion), the stress was held well at a constant value. Figure 7a–d shows the curves of strain against time for all samples during the creep phase, and the applied stresses are indicated in the corresponding figures. It can be seen that the deformation vs. time curves show typical brittle creep behavior characterized by the three-stage behavior as seen in previous studies (Amitrano and Helmstetter 2006; Heap et al. 2011; Brantut et al. 2013, 2014). Each primary creep stage is characterized by an initially high strain rate that decreased over time to reach an almost constant secondary stage strain rate, which is often interpreted as steady creep. Finally, the samples entered a tertiary phase characterized by an accelerated increase in strain. This
eventually resulted in macroscopic failure of the rock specimen.

Figure 5 illustrates a typical brittle creep relaxation result to demonstrate the complete process. As shown, the displacement was held constant (AB portion) well after the crosshead had reached the initial position (OA portion) (Fig. 5). The evolution curves of the deformation and stress at the constant displacement (AB) portion shown in the inset indicate that stress relaxation accompanied the increase in the deformation of the specimen. Figure 8 shows the results at the creep relaxation phase. It is clear that a typical stress relaxation process can be described as the rapid initial relaxation of stress, followed by a pseudo-steady stage in which the stress decreases at constant rate (Hao et al. 2014). This constant rate of stress relief terminates abruptly by an acceleration of stress loss and deformation that often culminates in catastrophic failure (Fig. 5).

In the brittle creep and creep relaxation experiments, some specimens immediately failed upon application of the initial stress or displacement. These experiments were discarded as they produced no data (the typical curves in type 2 and type 3 experiments are shown in Figs. 15 and 16 in Appendix, respectively). Some specimens did not fail during the available time window of 4 days. These experiments were also discarded (the exemplary curves in type 2 and type 3 experiments are shown in Figs. 15 and 16, respectively).

### 3.2. Scaling law of average failure rate and steady-state rate

It can be seen that the monotonic and creep-relaxation experiments showed a pseudo-steady stage similar with the creep experiments. To further characterize the three stages of the evolution of the
properties of rock failure, the first derivatives of the deformation with respect to the displacement \( \left( \frac{du}{dU} \right) \) for the monotonic experiments were plotted and two such plots are shown in Fig. 9 as examples. For the brittle creep experiments, the first derivatives of the deformation–time curves (i.e., the strain rate against time) are calculated and shown in Fig. 10. Figure 11 presents the curves of the strain and stress rates against time for the creep relaxation tests. It can be seen that the responses showed a common three-stage behavior for all three experiments. All of these plots demonstrated that there are long segments with an almost constant slope, as indicated by the horizontal sections of the curves. It is clear that the secondary stage dominated the lifetime of the specimens in all experiments under the three loading modes. The evolution characteristics of the secondary stage determine the transition from the steady state to the unstable state in the tertiary stage and thus determine whether macroscopic failure will occur.

For the brittle creep relaxation experiments and creep experiments, the resultant creep deformation is given by \( u_f - u_0 \), and hence, \( \mu = \frac{u_f - u_0}{b} \) represents the average creep deformation rate. Similarly, for the monotonic loading experiments, the average deformation rate is \( \mu = \frac{b}{u_f} \). Here, \( \mu_f \) is the resultant deformation and \( \mu_f \) is the resultant loading displacement (or a proxy measure of lifetime because \( U \) has a linear relationship with time). For a brittle rock material, the initiation, propagation, interaction, and coalescence of cracks are the main mechanism of deformation. Therefore, the deformation can be a proxy measurement of the change in damage during sample deformation. The ratio \( \lambda_s / \mu \) represents proportion of damage rate in the secondary state. A larger value of \( \lambda_s / \mu \) implies a smaller proportion of duration in the secondary stage, and consequently, a larger part of the damage is developed in the tertiary stage.

Therefore, as shown in Figs. 6, 7 and 8, a steep slope of the steady stage implies a short lifetime. The double-logarithm plots of the lifetime and \( \lambda_s \), are shown in Figs. 12, 13 and 14. The linear relationship in the double-logarithm plots indicates that \( \mu \) follows a power law

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\mu = A\lambda_s^z
\]

with the secondary creep rate \( \lambda_s \).

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**Table 2**

Experimental conditions and results for all type 1 experiments

| Sample number | \( \sigma_{\text{max}} \) (MPa) | \( \sigma_f \) (MPa) | \( u_f \) (mm) | \( U_f \) (mm) | Sample number | \( \sigma_{\text{max}} \) (MPa) | \( \sigma_f \) (MPa) | \( u_f \) (mm) | \( U_f \) (mm) |
|---------------|-------------------------------|-------------------|----------------|----------------|---------------|-------------------------------|-------------------|----------------|----------------|
| T1-M-1        | 189.2                         | 189.0             | 0.221          | 0.550          | T1-G-1        | 77.76                        | 77.89             | 0.271           | 0.596          |
| T1-M-2        | 183.7                         | 182.6             | 0.262          | 0.573          | T1-G-2        | 58.45                        | 58.73             | 0.258           | 0.548          |
| T1-M-3        | 180.9                         | 179.3             | 0.272          | 0.570          | T1-G-3        | 69.79                        | 69.97             | 0.243           | 0.585          |
| T1-M-4        | 182.5                         | 180.1             | 0.272          | 0.560          | T1-G-4        | 59.85                        | 60.10             | 0.252           | 0.563          |
| T1-M-5        | 110.4                         | 108.6             | 0.220          | 0.397          | T1-G-5        | 39.179                       | 40.146            | 0.328           | 0.423          |
| T1-M-6        | 127.6                         | 126.4             | 0.205          | 0.397          | T1-G-6        | 61                           | 61.40             | 0.304           | 0.610          |
| T1-M-7        | 125.6                         | 124.4             | 0.241          | 0.396          | T1-G-7        | 61.88                        | 61.98             | 0.228           | 0.542          |
| T1-M-8        | 113.3                         | 112.4             | 0.260          | 0.407          | T1-G-8        | 42.46                        | 42.63             | 0.216           | 0.431          |
| T1-M-9        | 133.5                         | 132.5             | 0.211          | 0.398          | T1-G-9        | 49.05                        | 49.46             | 0.311           | 0.526          |
| T1-M-10       | 111.5                         | 110.1             | 0.236          | 0.375          | T1-G-10       | 41.19                        | 41.76             | 0.303           | 0.502          |
| T1-M-11       | 122.7                         | 121.4             | 0.233          | 0.384          | T1-G-11       | 46.61                        | 47.77             | 0.292           | 0.509          |
| T1-M-12       | 174.5                         | 173.2             | 0.335          | 0.542          | T1-G-12       | 62.96                        | 63.36             | 0.213           | 0.515          |
| T1-M-13       | 121.0                         | 119.9             | 0.216          | 0.386          | T1-G-13       | 68.54                        | 68.7              | 0.206           | 0.559          |
| T1-M-14       | 196.1                         | 196.1             | 0.240          | 0.582          | T1-G-14       | 71.69                        | 71.96             | 0.233           | 0.581          |
| T1-M-15       | 174.4                         | 174.4             | 0.186          | 0.481          | T1-G-15       | 74.51                        | 74.91             | 0.277           | 0.583          |
| T1-M-16       | 172.6                         | 172.5             | 0.183          | 0.466          |               |                              |                   |                |                |
| T1-M-17       | 88.8                          | 87.9              | 0.289          | 0.395          |               |                              |                   |                |                |
| T1-M-18       | 91.3                          | 90.8              | 0.287          | 0.395          |               |                              |                   |                |                |
| T1-M-19       | 182.4                         | 181.3             | 0.298          | 0.584          |               |                              |                   |                |                |

Type 1 experiments, \( M \) marble, \( G \) granite. Load rate: \( dU/dt = 0.02 \) mm/min

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Table 3
Experimental conditions and results for all type 2 experiments

| Sample number | \( \sigma_{\text{max}} \) (MPa) | \( u_0 \) (mm) | \( u_f \) (mm) | \( t_f - t_f \) (s) |
|---------------|-------------------------------|----------------|----------------|------------------|
| T2-G-80-1     | 181.2                         | 0.208          | 0.262          | 570              |
| T2-G-80-2     | 181.2                         | 0.216          | 0.267          | 6355             |
| T2-G-80-3     | 181.2                         | 0.208          | 0.232          | 5775             |
| T2-G-85-1     | 190.6                         | 0.227          | 0.265          | 133              |
| T2-G-85-2     | 190.6                         | 0.226          | 0.255          | 397              |
| T2-G-85-3     | 190.6                         | 0.216          | 0.227          | 2051             |
| T2-G-90-1     | 203.1                         | 0.247          | 0.271          | 56               |
| T2-G-90-2     | 203.1                         | 0.222          | 0.248          | 19               |
| T2-G-90-3     | 203.1                         | 0.237          | 0.269          | 115              |
| T2-G-90-4     | 203.1                         | 0.239          | 0.267          | 122              |
| T2-G-90-5     | 203.1                         | 0.228          | 0.252          | 290              |
| T2-G-90-6     | 203.1                         | 0.231          | 0.262          | 539              |
| T2-G-90-7     | 203.1                         | 0.232          | 0.254          | 830              |
| T2-G-90-8     | 203.1                         | 0.227          | 0.252          | 9929             |
| T2-G-90-9     | 203.1                         | 0.227          | 0.245          | 1317             |
| T2-G-90-10    | 203.1                         | 0.232          | 0.250          | 378              |
| T2-G-90-11    | 203.1                         | 0.224          | 0.257          | 4569             |
| T2-G-90-12    | 203.1                         | 0.232          | 0.253          | 481              |
| T2-G-95-1     | 212.5                         | 0.293          | 0.321          | 23               |
| T2-G-95-2     | 212.5                         | 0.289          | 0.310          | 20               |
| T2-G-95-3     | 212.5                         | 0.251          | 0.276          | 32               |
| T2-G-95-4     | 212.5                         | 0.250          | 0.279          | 153              |
| T2-G-95-5     | 212.5                         | 0.244          | 0.266          | 230              |
| T2-G-95-6     | 212.5                         | 0.236          | 0.269          | 559              |

It should be mentioned that there was an event of fracture that occurred during the testing of specimens SC-G-80-2 and SC-G-85-3. We heard a sound emitted by the fracture but the specimen did not fail completely. This small event induced a small jump in the curves (see Fig. 7a, b). Thus, the stable stage has a more direct relation with this small event than the macroscopic failure, and then we select the \( u_f \) and \( t_f \) corresponding to this small event to calculate the value of the average creep rate \( \mu \).

Table 4
Experimental conditions and results for all type 3 experiments

| Sample number | \( U \) (mm) | \( \sigma_0 \) (MPa) | \( \sigma_f \) (MPa) | \( u_0 \) (MPa) | \( u_f \) (MPa) | \( t_f - t_f \) (s) |
|---------------|-------------|-----------------|-----------------|-------------|-------------|------------------|
| T3-G-1        | 0.770       | 238.4           | 227.9           | 0.241       | 0.231       | 347              |
| T3-G-2        | 0.750       | 222.2           | 237.1           | 0.214       | 0.232       | 797              |
| T3-G-3        | 0.760       | 2040            | 229.1           | 0.174       | 0.193       | 497              |
| T3-G-4        | 0.730       | 228.0           | 209.3           | 0.222       | 0.243       | 2471             |
| T3-G-5        | 0.710       | 2140            | 193.6           | 0.214       | 0.238       | 1345             |
| T3-G-6        | 0.682       | 169.7           | 120.9           | 0.107       | 0.182       | 125              |
| T3-G-7        | 0.753       | 214.7           | 183.1           | 0.169       | 0.219       | 285              |
| T3-G-8        | 0.753       | 225.4           | 198.3           | 0.147       | 0.189       | 208              |
| T3-G-9        | 0.741       | 203.1           | 170.0           | 0.220       | 0.281       | 593              |
| T3-G-10       | 0.721       | 202.7           | 149.8           | 0.233       | 0.324       | 926              |
| T3-M-1        | 0.760       | 225.8           | 214.8           | 0.226       | 0.237       | 593              |
| T3-M-2        | 0.783       | 186.0           | 157.8           | 0.160       | 0.239       | 83               |
| T3-M-3        | 0.756       | 191.9           | 152.9           | 0.543       | 0.382       | 129              |
| T3-M-4        | 0.756       | 166.9           | 127.2           | 0.342       | 0.456       | 64               |

It is noted that there was an event of fracture that occurred during the testing of specimens SC-G-80-2 and SC-G-85-3. We heard a sound emitted by the fracture but the specimen did not fail completely. This small event induced a small jump in the curves (see Fig. 7a, b). Thus, the stable stage has a more direct relation with this small event than the macroscopic failure, and then we select the \( u_f \) and \( t_f \) corresponding to this small event to calculate the value of the average creep rate \( \mu \).

\( T2 \) type 3 experiments, \( M \) marble, \( G \) granite
The power law exponent was approximately 0.70 [0.72 ± 0.04 for the marble specimens (Fig. 12a) and 0.70 ± 0.04 for the granite specimens (Fig. 12b)] in the monotonic loading experiments. The exponent \( \alpha \) is 0.92 ± 0.04 in the brittle creep experiments (Fig. 13), which is almost a linear relationship between \( \mu \) and \( \lambda_\mu \). On the other hand, the exponent \( \alpha \) is about 0.85 [0.84 ± 0.03 (Fig. 14a) for stress relaxation and 0.86 ± 0.03 (Fig. 14b) for deformation evolution] in the brittle creep relaxation experiments. Our results indicate that the power law relation (1) is fitted well for the three types of experiments.

![Figure 6](image_url)

Deformation curves in monotonic tests. The curves exhibit the apparent tri-stage behaviors. T1 type 1 experiments, G granite, M marble

![Figure 7](image_url)

Creep curves for different applied stress levels. a Load level: 80% peak stress; b load level: 85% peak stress; c load level: 90% peak stress; d load level: 95% peak stress. The curves exhibit the apparent tri-stage behaviors. The specimens present different failure times and strain under the same stress level. T2 type 2 experiments, G granite, M marble
The subcritical growth of micro cracks, i.e. crack growth, can occur when the stress intensity factor $K$ is lower than its critical value (also called as fracture toughness), $K_c$, is suggested to be the main mechanism responsible for brittle creep of rocks (Scholz 1972; Atkinson and Meredith 1987; Lockner 1993; Amitrano and Helmstetter 2006). Subcritical crack growth can be caused by several competing mechanisms, including: stress corrosion, diffusion, dissolution, ion exchange and microplasticity (Atkinson 1984; Atkinson and Meredith 1987). The theory of stress corrosion postulates that the reaction between strained bonds and the environmental agent produces a weakened (an activated) state that can then be broken at lower stresses than the unweakened bonds. The model of diffusion assumes that mass transport can be the dominant mechanism of subcritical crack growth. The dissolution model suggests that the growth rate of cracks is controlled by the silica dissolution rate. According to the theory of ion exchange, if the chemical environment contains species which can undergo ion exchange with species in the solid phase and if there is a gross mismatch in the size of these different species, then lattice strain can result from ion exchange which can facilitate
crack growth. The theory of micro-plasticity suggests that the dislocation activity allowing plastic flow may be a significant mechanism of crack propagation. It is assumed that stress corrosion is the main mechanism of subcritical growth in shallow crustal conditions (Atkinson 1984; Atkinson and Meredith 1987).

The crack velocity fits a power law with stress intensity factor (Charles 1958; Atkinson 1984; Atkinson and Meredith 1987; Amitrano and Helmstetter 2006)

$$\frac{V}{V_0} = \left( \frac{K}{K_c} \right)^q \exp\left( -\frac{H}{R T} \right) \quad K_0 < K < K_c$$

where $V$ is the crack growth velocity, $R$ is the gas constant, $T$ is the temperature, $H$ is the activation energy, and $K_0$ is the threshold value below which no crack propagation is observed. Based on Eq. (2), a power–law relationship of the time-to-failure with the applied stress (Charles 1958; Cruden 1974; Kranz 1980; Lockner 1993; Amitrano and Helmstetter 2006) has been derived analytically and observed experimentally

$$t_f = t_s \left( \frac{\sigma}{\sigma_s} \right)^{-p}$$

The constants $t_s$ and $\sigma_s$ depend on rock properties and ambient conditions (Scholz 1972). An exponential relation (Wiederhorn and Bolz 1970; Das and Scholz 1981; Amitrano and Helmstetter 2006)

$$t_f = t_s \exp\left( -b \frac{\sigma}{\sigma_s} \right)$$

has also been suggested. These two empirical relations (3) and (4) are equivalent in terms of correlation coefficient (Amitrano and Helmstetter 2006) and indicate the stress dependence of the time-to-failure.

A theoretical derivation presented by Main (2000) gives a relationship between the strain rate $\dot{\varepsilon}$ and stress

$$\dot{\varepsilon} = C\sigma^m,$$

where $C$ is a constant. Then, Eqs. (3) and (5) suggest a possible power law relation

![Figure 11](image1.png)

Stress rate–time curves in the creep-relaxation experiments. G granite

![Figure 12](image2.png)

Double-logarithm plots of $l$ and $k_s$ in the monotonic loading tests. a Marble, b granite. The data fit well a power law. The two types of rock exhibit few differences with the exponents: marble is 0.72 and granite is 0.70
between the strain rate and the time-to-failure similar with the relation (1). It should be noted that if $\kappa = (e_f - e_0)$, then expression (6) reproduces our empirical relation (1).

Under quasi-static loading experiments, the damage evolution processes in rocks could present some common properties. In three types of experiments, the first stage is either related to the elastic closing of the pre-existing cracks and pores, or to the condition where the crack-closure rate exceeds the crack-opening and propagation rates (Rudnicki and Rice 1975; Heap et al. 2009; Hao et al. 2013). This leads to an effect of strain hardening in the type 1 (Hao et al. 2013) and type 2 experiments (Lockner and Byerlee 1980; Lockner 1993). The secondary stage corresponds to the stage where damage to rock samples occurs randomly, and thus the spatial distribution of the strain field presents weak fluctuations (Hao et al. 2007, 2010). This occurs so that the macroscopic physical quantities (e.g., average strain or stress) that describe the global average mechanical responses evolve steadily. The diffused distribution of acoustic emission events has also been observed throughout secondary stage in the creep experiments and quasi-constant acoustic emission rate experiments (Lockner et al. 1991; Lockner 1993). The accelerating evolution in the tertiary stage always results from the localization and coalescence of cracks observed in monotonic loading experiments (Hao et al. 2007, 2010), creep experiments, and in quasi-constant acoustic emission rate experiments (Lockner et al. 1991).

To further demonstrate the relationship between failure and the secondary stage, let us focus on the brittle creep relaxation experiments. For a plastic material, stress relaxation is achieved through a process in which the initial imposed elastic strain is replaced over time by an inelastic strain. However, for heterogeneous brittle materials such as rocks, a
drop in the stress is induced by the development of
damage, which includes the initiation, propagation,
coalescence, and growth of cracks and defects.

It should be mentioned that the evolution of the
macroscopic response depends on the microphysical of
the rock specimen, which induces sample-specificities
of rock specimens. The evolution properties (e.g., the
slope of the secondary stage) of the response (strain
or stress) curves and the eventual failure are deter-
mined by the properties of damage evolution for a
specific specimen and the loading condition. In type 1
and type 3 experiments, the elasticity of the testing
machine (the stiffness of the loading apparatus is
\( \sim 130 \text{kN/mm} \)) played an extremely important role in
promoting failure. The condition of catastrophic rup-
ture in type 1 experiments is controlled by two main
factors (Bai et al. 2005; Jeager et al. 2007; Hao et al.
2007, 2010, 2013): the stiffness ratio between the load
apparatus and the rock sample, and the damage evolu-
tion properties of the rock. The condition of
catastrophic rupture in type 3 experiments is controlled
by three factors (Hao et al. 2013): the stiffness ratio
between the load apparatus and the rock sample, the
initial applied displacement, and the damage evolution
properties of rock. In the type 2 experiment, failure is
determined mainly by the initial stress level and the
specific damage evolution properties of a rock speci-
men. Therefore, it is difficult to decouple these
combined effects of loading level, testing machine, and
sample-specificities of rock specimens. Both the slope
of the secondary stage and failure time are commonly
dependent on these conditions. As a result, the vari-
ability of these factors do not change the scaling law
between the two main macroscopic responses (\( \lambda_s \) and
\( \mu \)), but lead to the differences in \( \lambda_s \), power exponent and
failure time among the specimens.

The ratio \( \lambda_s/\mu \) between the average deformation
rate \( \lambda_s \) of the secondary stage and the average rate \( \lambda_e \)
for whole lifetime reflects the contribution of the
secondary stage to the whole failure.

5. Conclusions

In the three types of our quasi-static experiments,
rock specimens presented an apparent long steady (or
pseudo-steady) stage followed by a rapid deformation
stage, leading to a macroscopic failure. The steady
stage dominates almost the entire lifetime of a rock
specimen. A steep slope of the steady stage implies a
short lifetime. These properties promise a unified
description of the relationship between the time-to-
failure and failure with the evolution properties in the
steady stage.

The present experimental results show that the
lifetime of rock specimens can be commonly
expressed as a power–law relationship with the slope
of the steady stage [cf. Eqs. (1) and (6)]. The power
law exponent is approximately 0.70 in the monotonic
loading experiments, 0.92 \( \pm \) 0.04 in the brittle creep
experiments, and approximately 0.85 in the brittle
creep relaxation experiments. Further investigations
are needed to determine the reason for the different
exponent values. Under the creep experiments, the
approximate linear relationship of the lifetime of rock
specimens with a steady stage slope provides a
potential method for predicting time-to-failure by
using a linear extrapolation.

\( \lambda_s/\mu \), which is the ratio of the evolution rate in
the steady stage with respect to the average rate of the
total lifetime, is proposed to describe the failure
mode. A larger value of \( \lambda_s/\mu \) means a smaller
damage proportion occurred in the steady stage,
and consequently, a larger portion of damage
occurred in the tertiary stage, corresponding to
more brittle failure.

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Appendix: results for specimens that failed
immediately and did not fail after a long time
loading

See Figs. 15 and 16.
Figure 15
Results of the brittle creep tests: one specimen failed immediately while trying to hold the stress and one did not fail after a long load time. 

a The results of a specimen that did not fail after a long load time. After approximately 12.16 h of testing, the data are too large to be stored.

b The curves of stress vs. time of a specimen that failed immediately while trying to hold the stress

Figure 16
Results in creep relaxation tests: one specimen failed immediately when trying to hold the displacement and one that did not fail after a long time loading. 

a The results of a specimen that did not fail after a long time loading. After about 5.1 h in the tests, the data is too large to be stored.

b The results of a specimen that failed immediately when try to hold the displacement

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