Ghost imaging for an occluded object

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Abstract
Imaging for an occluded object is usually a significant problem. In this letter, we introduce an imaging scheme based on computational ghost imaging, which can obtain the image of a target object behind an obstacle. According to our theory, the distance between the object and the obstacle and the wavelength of the light source will affect the quality of the reconstructed image. In addition, if the bucket detector is placed far away from the obstacle, a tiny point-like detector without collecting lens can be applied to realize the imaging. The theoretical results above have been verified with our numerical simulations.

Keywords: computational ghost imaging, hidden target, diffraction

(Some figures may appear in colour only in the online journal)

1. Introduction
Ghost imaging is a novel imaging technique based on the intensity fluctuation correlations of light, and it was first proposed with entangled photons [1, 2]. Later, it was found that ghost imaging could also be realized using a classical thermal source [3], and there were many discussions about thermal ghost imaging [4–17].

In 2008, Shapiro proposed computational ghost imaging [18], and it was verified by experiments in 2009 [19]. Different from the conventional ghost imaging scheme, the computational ghost imaging technique applies a programmable light source, and the experimental setup can be simplified. Ghost imaging displays great potential in special situations, such as high lateral resolution imaging [20] and resistance of atmospheric turbulence [21, 22].

In addition to the above features, our recent work shows that ghost imaging may have even more advantages than conventional imaging techniques. Imaging for an occluded object is a significant problem, but in this letter we proved that, under appropriate conditions, one can obtain the image of an occluded object by applying the ghost imaging technique, even if the object is blocked by an unknown obstacle. This unique feature of the ghost imaging technique may be helpful in detecting hidden targets.

2. Theory
The schematic diagram of the computational ghost imaging technique for an occluded object is shown in figure 1. We see from the bucket detector that the target object is blocked by the obstacle. But when we use the computational ghost imaging technique, we can obtain the object’s image.

For simplification, we consider the one-dimensional case. Let \( \vec{C} \) and \( \vec{D} \) represent the transmission functions of the target object and the obstacle, where \( \vec{C} = [c_1, c_2, \cdots, c_N]^T \) and \( \vec{D} = [d_1, d_2, \cdots, d_N]^T \). The light emitted by the programmable light source illuminates the object, and its intensity distribution on the object plane can also be represented by a \( 1 \times N \) vector:

\[
\vec{S}(t) = \begin{bmatrix} s_1(t) & s_2(t) & \cdots & s_N(t) \end{bmatrix}^T.
\] (1)

The modulated light illuminates and passes through the target object, reaching the obstacle plane after \( z \) distance of propagating. First, for simplification, we assume that we can collect
all the transmitted light by using a bucket detector, and the bucket signal can be written as:

\[ B(t) = \sum_{m=1}^{N} \sum_{n=1}^{N} A_{mn} c_n s_n(t) \]  

(2)

where \( \{c_n\} \) and \( \{s_n(t)\} \) represent the elements of \( \mathcal{C} \) and \( \mathcal{D} \), \( \{A_{mn}\} \) represents the elements of the propagating matrix \( \mathbf{A} \). The second-order correlation function \([23]\) of this system is:

\[ G^{(2)}(n') = \langle s_{n'}(t) B(t) \rangle. \]  

(3)

Here, \( \langle \ldots \rangle \) represents the ensemble average. In this system, we assume that \( \{s_n(t)\} \) are independent and identically distributed. If we take enough measurements, we have \([23]\):

\[ \langle s_{n'}(t) s_{n}(t) \rangle_t = \delta(n', n) l + (\delta)^2, \]  

(4)

where \( \langle \delta \rangle \) is the average intensity of the light source, and \( l \) is the variance of the intensity. Therefore, the non-normalized second-order correlation function of the target object can be expressed by:

\[ G^{(2)}(n') = l c_{n'} \sum_{m=1}^{N} A_{mn} + O. \]  

(5)

where \( O = \langle \delta \rangle^2 \sum_{m=1}^{N} A_{mn} \) is a background term which is unrelated to \( n' \). In other words, it does not contain the spatial information of the object. So, we focus on the term \( l c_{n'} \sum_{m=1}^{N} A_{mn} \). Obviously, \( G^{(2)} \) is in proportion to \( c_{n'} \sum_{m=1}^{N} A_{mn} \). In order to find out the relationship between \( G^{(2)} \) and the transmission function of the target object, we need to investigate the form of the propagating matrix \( \mathbf{A} \).

Now, we investigate the propagation progress, and we consider the dispersed case. Let \( u_0(n, t') \) be the instantaneous field distribution of the source on the target object plane at time \( t' \). For simplification, the dimension and the pixel size at the target object and obstacle planes are equal. For pixels with a transverse size of \( \Delta z \), after \( z \) distance of traveling, the field distribution on the obstacle plane can be written as \([24]\):

\[ u(m, t') = \frac{e^{ikz(\Delta z)^2}}{i(\Delta z)^2} \sum_{n=1}^{N} u_0(n, t') e^{ikz[(m-n)^2-4]} \]  

(6)

where \( N \) is the number of pixels, and \( e^{ikz[(m-n)^2-4]} \) is usually denoted by \( h_{\lambda,\lambda}(m-n) \), which is called the point spread function. We can obtain the intensity distribution on the obstacle plane:

\[ I_{obs}(m, t) = \int_{t}^{t+\Delta t} [h^*(m-1)t') + h^*(m-n)u_0(n,t') + \ldots + h^*(m-n)u_0(n,t') + \ldots + h^*(m-n)u_0(n,t') + \ldots + h(m-n)u_0(n,t') + \ldots + h(m-n)u_0(n,t') + \ldots + h(m-n)u_0(n,t')] dt'. \]  

(7)

where \( \Delta t \) is a short period of time. Here, we assume that the light source is incoherent. In this case, we have:

\[ I_{obs}(m, t) = \int_{t}^{t+\Delta t} u^*(m, t') u(m, t') dt' = \int_{t}^{t+\Delta t} [h^*(m-1)t') + h^*(m-n)u_0(n,t') + \ldots + h^*(m-n)u_0(n,t') + \ldots + h(m-n)u_0(n,t') + \ldots + h(m-n)u_0(n,t') + \ldots + h(m-n)u_0(n,t')] dt'. \]  

(8)

We can simplify equation (7) into:

\[ I_{obs}(m, t) = \sum_{n'=1}^{N} \sum_{n=1}^{N} h^*(m-n')h(m-n)I_{obj}(n, t). \]  

(9)

where \( I_{obj}(n, t) \) is the intensity distribution through the target object plane. Equation (9) can also be written in matrix form:

\[ I_{obs} = \begin{bmatrix} A_{11} & A_{12} & \ldots & A_{1n} & \ldots & A_{1N} \\ A_{21} & \vdots & \ddots & \vdots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ A_{m1} & A_{m2} & \ldots & A_{mn} & \ldots & A_{mN} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ A_{N1} & A_{N2} & \ldots & A_{NN} \end{bmatrix} I_{obj}, \]  

(10)

where

\[ A_{mn} = |h(m-n)|^2 = \frac{(\Delta x)^2}{\lambda^2} \sum_{n'=1}^{N} e^{-\frac{4\pi(z-n^2)}{\lambda^2}} \]  

(11)

is called the intensity point spread function. Figure 2 gives the intensity point spread function curve of light sources with several typical wavelengths at different distances of propagating.

Figure 2 shows that the intensity point spread function is influenced by the distance of propagating \( z \) and the wavelength of the light source \( \lambda \). When \( \lambda \) and/or \( z \) is big enough, the intensity point spread function approaches a constant which is unrelated to the spatial coordinates.

The physics behind this progress is as follows. The illuminating light carries the information of the object, propagates a distance of \( z \), and reaches the obstacle plane. Due to the propagation of the light, the information of the object spreads around on the obstacle plane. Every single point on the obstacle plane produces an Airy pattern on the obstacle plane, and the Airy patterns overlap with each other. As a result, every single pixel on the obstacle plane contains the information from multiple points on the object plane. As the distance \( z \) or wavelength \( \lambda \) increases, the area of every Airy pattern increases. When the magnitudes of \( z \) and/or \( \lambda \) are great enough, we can assume
that every pixel on the obstacle plane contains the information from all of the points on the object plane. Therefore effective information about the object can always reach the bucket detector via the outside of the obstacle’s border.

The diffraction on the obstacle plane is actually a similar process: after a distance of traveling, the transmitted light reaches the bucket detector plane. Note that, like the situation we discussed above, if this distance is far enough, every pixel on the bucket detector plane contains the information from all of the points on the obstacle plane. Therefore, in this case, it is not necessary to collect all of the transmitted light. Instead, in ideal conditions, even a tiny point-like detector can finish the task. With the assumption of a long distance between the bucket detector and the obstacle, the intensity on the bucket detector plane is almost evenly distributed. The bucket signal can be written as:

\[ B'(t) = \alpha \sum_{m=1}^{N} d_m \sum_{n=1}^{N} A_{mn} c_n s_n(t), \]  

where \( \alpha \in (0, 1) \) is a constant which depends on the size of the bucket detector. Based on the discussions above, we can now
explain why the ghost imaging technique can realize the imaging of an occluded object. From equation (5) we know that $\vec{G}^{(2)}$ is in proportion to $c_{n'} \sum_{m=1}^{N} d_{m} A_{mn}$. Both the information of the target object and the obstacle are contained in $\vec{G}^{(2)}$. While the magnitude of $\lambda z$ is big enough, the elements in the propagating matrix $\vec{A}$ approach a constant which is unrelated to the spatial coordinates. The second-order correlation function of the target object is:

$$\vec{G}^{(2)}(n') \approx l o d A_{n'n'} + \alpha O,$$  \hspace{1cm}  (13)

where $\vec{d} = \sum_{n=1}^{N} d_{n}$, $\vec{A} = \frac{1}{N} \sum_{m=1}^{N} \sum_{n=1}^{N} A_{mn}$. In this case, the spatial information of the obstacle is eliminated. $G^{(2)}$ is now in proportion to the target object’s transmission function, and the image of the object can be obtained correctly. Notice that one can obtain the image of the target object in this case, even if the shape of the obstacle is unknown. The reason is that, different from the conventional imaging technique, ghost imaging is a kind of computational imaging scheme which is based on the intensity fluctuation correlations, and the imaging quality is only sensitive to the fluctuation of the total (or average) intensity of the transmitted light. When the distance between the object and the obstacle is far enough, the obstacle has very limited effects on the fluctuation of the total intensity. The result in this case is that, even under the effect of an obstacle, the ghost imaging scheme will not fail and we can still obtain the image of the target object.

However, when $\lambda z$ decreases, the curve of the intensity point spread function approaches the $\delta$ function. Thus, the non-opposite angle elements of the propagating matrix $\vec{A}$ approach zero. In this case, the second-order correlation function of the target object can be written as:

$$\vec{G}^{(2)}(n') \approx l o A_{n'n'} c_{n'} d_{n'} + \alpha O.$$  \hspace{1cm}  (14)

Obviously, $\vec{G}^{(2)}$ is in proportion to $c_{n'} d_{n'}$, the product of the transmission function of the target object and the obstacle. We will obtain a mixture image of the target object and the obstacle, and we cannot revive the image of the target object correctly.

Thus, to realize the imaging for an occluded object, the distance between the target object and the obstacle should be far enough. In addition, in order to obtain an image with higher quality, we can increase the wavelength of the illuminating light. Furthermore, if we place the bucket detector far away from the obstacle, it is possible to use a tiny detector to realize the imaging.

3. Numerical simulations

To verify our theoretical results, numerical simulations are carried out, and the robustness of this imaging system is also judged. A schematic diagram of our numerical simulation is shown in figure 1. We take 1200000 measurements for every simulation, and the field distribution of the light source is modulated into a Gaussian distribution. The distance between the spatial light modulator and the target object is taken to be 0.50 m. The size of the bucket detector is $0.08 \times 0.08$ mm, and it is placed 10.00 m away from the obstacle, in the center of the bucket detector plane (on the optic axis). As figure 3 shows, the target object is an opaque arrow, and the obstacle is a ‘ghost’-shaped opaque plate; both of them are placed in the center in their planes. The size of the target object is $1.20 \times 0.72$ mm, and the size of the obstacle is about $2.08 \times 2.08$ mm. Both the target object and the obstacle plane are pixelated into two $64 \times 64$ pixel images, with pixel width $\Delta x = 0.04$ mm.

We investigate the influence of the distance between the target object and the obstacle and the wavelength of the light source, respectively.

![Figure 6. The reconstructed images of the target object under different detection SNRs. The distance between the target object and the obstacle is 3.0 m, and the wavelength of the light source is 632.8 nm.](image-url)
3.1. The influence of the distance between the target object and the obstacle

In this part, we use a 632.8 nm laser as the light source. In order to study the influence of the target object on the imaging quality, we change the distance between the target object and the obstacle, and reconstruct the image of the target object by using the computational ghost imaging technique, respectively. The results of our numerical simulation are shown in figure 4. It is clear that, when the distance between the target object and the obstacle is far enough, it is possible to realize the imaging for an occluded object by applying the computational ghost imaging technique.

3.2. The influence of the wavelength of the light source

In this part, the distance between the target object and the obstacle is taken to be 3.0 m. In order to find out the influence of the wavelength on the imaging quality, we use light sources with different wavelengths to implement computational ghost imaging for the target object. The results of our numerical simulations are shown in figure 5. Clearly, we can get a clearer view of the target object by applying a light source with longer wavelength. However, the spatial resolution of the reconstructed image is decreased.

3.3. The robustness of this imaging system

Many computational imaging schemes fail with the effect of noise, thus it is necessary to judge the performance of our imaging scheme under the influence of background noise. We use the signal-to-noise ratio (SNR) to describe the effect of the background noise on the bucket signal, which is defined as:

\[ \text{SNR} = 10 \log_{10} \frac{B}{N_b}, \]  

(15)

where \( B \) is the average intensity of the bucket signal, \( N_b \) is the average intensity of the background noise, and the noise is Gaussian noise. The reconstructed images of the target object under different SNRs are shown in figure 6.

The results show that when the SNR of the bucket signal is 7.8985 dB, the image of the object can still be recognized. The imaging scheme fails when the SNR is lower than 5.3867 dB. Thus, this imaging scheme can partly resist the effect of noise.

4. Conclusions

In conclusion, we have proved that ghost imaging can realize imaging for an occluded object. According to the above discussions, we find that this unique feature is based on the fact that the ghost imaging technique is based on the intensity fluctuation correlations. Due to the diffraction of the light, a bucket detector (with limited size) can always capture useful fluctuation information which is caused by different illuminating patterns passing through the object. If the distance between the target object and the obstacle is far enough, the image of the target object can be reconstructed accurately. In addition, a better image of the target object can be obtained by using a light source with longer wavelength, but the resolution of the reconstructed image is decreased. In addition, it is possible to realize the imaging by using a tiny point-like bucket detector if the detector is placed far away from the obstacle. Numerical simulations have been carried out, and the results agree with our theoretical analysis.

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