On Neutrino Absorption Tomography of the Earth

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Abstract

We study the passage of UHE neutrinos through the Earth in order to perform an absorption tomography of its inner structure. The aim of this work is to study the extraction methods of the Earth’s density, in this conditions, we do not need to implement a realistic Monte Carlo simulation, as we are only interested in comparing the goodness of a standard method [1] with the one we propose. The Earth’s density is reconstructed using the 2-d Radon transform and we compare the density obtained considering neutral current regeneration through the complete transport equation, with the one obtained making use of the effective cross section approximation (standard method). We see that the effective cross section leads in general to inaccurate results, especially for flat initial neutrino fluxes, while the full transport equation method works regardless of the initial flux. Finally, an error propagation analysis made for different uncertainties in the surviving neutrino flux shows that the recovered density presents a percentage uncertainty less than two times the uncertainty in the flux.

I. INTRODUCTION

Neutrino absorption tomography has been considered as an alternative way to obtain information about the interior of the Earth that would be independent of any geophysical model (see [1] and references therein). In view of this, it is useful to understand the role of neutral current regeneration of neutrinos in their passage through the Earth. To that end, in the present work we perform the absorption tomography following two different approaches to take account of the neutral current regeneration effect in the density extraction method. The first approach, which we call the standard method or the effective cross section approach [2], is largely used in the literature consists on implementing an approximate solution to the transport equation by defining an effective cross section. The other method makes use of the complete integro-differential transport equation itself and not of its solution, and this is the approach we introduce in this work.

In the next section we start by describing both the standard and the transport equation approaches, and apply them for some initial neutrino fluxes. We use theorized neutrino fluxes in the range $10^4$GeV to $10^8$GeV coming from cosmic sources such as Active Galactic Nuclei (AGN) and Gamma Ray Bursts (GRB) as well as neutrinos generated by cosmic ray interactions in the atmosphere (ATM). All these fluxes are assumed to be isotropic, and the Earth density is considered spherically symmetric so that the angular data of the surviving flux can be used to recover the density be means of the 2-d Radon transform which is a standard technique, but as we have not found it explained in any article, we decided to include it in the appendix. In order to have a surviving neutrino flux after traversing the Earth, we consider the numerical solution to the complete transport equation for neutrino propagation. This topic is presented in section 2 and the reconstruction procedures are discussed in section 3, where we compare the results of our approach with the ones of the $\sigma_{eff}$-approach. For the sake of comparing the two density extraction methods, it suffices to consider the numerical solution for the surviving flux, since a realistic Monte Carlo study would be consistent with the numerical results for the surviving flux. Finally, in section 4 the error propagation is performed when recovering the Earth’s density following the transport equation approach, that is making no approximation related with the NC regeneration.
II. NEUTRINO PROPAGATION THROUGH THE EARTH. TRANSPORT EQUATION AND EFFECTIVE CROSS SECTION

As neutrinos travel through the Earth they may suffer charged current (CC) and neutral current (NC) interactions with the nucleons in their path. Neutrino oscillation within the Earth can be neglected for energies higher than 1000GeV \cite{3}, and since the CC and NC cross sections increase with the neutrino energy, we choose our energy range from 10^4GeV to 10^8GeV where the Earth is neither totally transparent nor opaque to neutrinos.

The change in a neutrino flux \( \phi_\nu(E) = \frac{dN}{dEdt} \) as it traverses the Earth can be divided into two effects: absorption and regeneration \cite{4}. Absorption is a decrease in the neutrino flux due to CC or NC interactions. When neutrinos pass through an amount of matter \( d\tau = n(r)dz \) in a distance \( dz \), where \( n(r) \) is the Earth’s number density, the change in the flux \( \phi_\nu(E, \tau) \) due only to absorption is proportional to \( \phi_\nu(E, \tau) \) by the total cross section \( \sigma_{tot}(E) = \sigma_{cc}(E) + \sigma_{nc}(E) \) which represents a probability of CC or NC interaction:

\[
\frac{d\phi_\nu(E, \tau)}{d\tau} = -\sigma_{tot}(E)\phi_\nu(E, \tau) \tag{1}
\]

Here \( \tau(z) \) is the amount of material found until a depth \( z \) that is,

\[
\tau(z) = \int_0^z d\tau'(n(z')) \tag{2}
\]

where the number density is the Avogadro’s constant times the density, \( n(z') = N_A \rho(z') \).

To write the complete transport equation for neutrinos of our energy range, we have to add to (1) the effect of regeneration, which accounts for the possibility that neutrinos of energies \( E' > E \) may end up with energy \( E_\nu \) due to NC interactions with the nucleons. Unlike absorption, regeneration represents an increase in the flux at an energy \( E \) caused by NC interactions at energies \( E' > E \). Now, this change due to regeneration is proportional to each flux \( \phi_\nu(E', \tau) \) by the probability of such interactions which is given by \( \frac{d\sigma_{nc}(E', E)}{dE} \phi_\nu(E', \tau) \) so that we can write the transport equation as follows

\[
\frac{d\phi_\nu(E, \tau)}{d\tau} = -\sigma_{tot}(E)\phi_\nu(E, \tau) + \int_E^\infty dE' \frac{d\sigma_{nc}(E', E)}{dE} \phi_\nu(E', \tau). \tag{3}
\]

to be solved with the initial condition \( \phi_\nu(E, 0) = \Phi_\nu(E) \), where \( \Phi_\nu(E) \) is the initial flux.

The other approach consists on defining an effective cross section \( \sigma_{eff}(E, \Phi_\nu) \) in an attempt to incorporate both absorption and regeneration all together. Dividing the transport equation (3) by \( \phi_\nu(E, \tau) \) we obtain

\[
\frac{d\ln \phi_\nu(E, \tau)}{d\tau} = -\sigma_{tot}(E) + \int_{E_\nu}^\infty dE' \frac{d\sigma_{nc}(E', E)}{dE} \frac{\phi_\nu(E', \tau)}{\phi_\nu(E, \tau)}. \tag{4}
\]

In this last expression, the quotient of the fluxes \( \frac{\phi_\nu(E', \tau)}{\phi_\nu(E, \tau)} \) appears integrated on \( E' \), and these fluxes must be evaluated at a depth in the Earth corresponding to \( \tau \) nucleons traversed. The effective cross section approach consists in replacing the above quotient by the initial flux ratio \( \frac{\Phi_\nu(E')}{\Phi_\nu(E)} \)

\[
\frac{\phi_\nu(E', \tau)}{\phi_\nu(E, \tau)} \rightarrow \frac{\Phi_\nu(E')}{\Phi_\nu(E)}. \tag{5}
\]

and now we identify the effective cross section as

\[
\sigma_{eff}(E, \Phi_\nu) = -\sigma_{tot}(E) + \int_{E_\nu}^\infty dE' \frac{d\sigma_{nc}(E', E)}{dE} \frac{\Phi_\nu(E')}{\Phi_\nu(E)}. \tag{6}
\]

The effective cross section results then from the approximation (5) to use the initial fluxes (which are supposed as known) rather than the fluxes \( \phi_\nu(E, \tau) \) at different depths through the Earth which can be obtained using the original
transport equation (3). When using the effective cross section, the resulting flux for an amount of material $\tau$ traversed is

$$\phi_\nu(E, \tau) = \Phi_\nu(E)e^{-\sigma_{eff}(E)\tau}$$  (7)

We can compare this prediction for the surviving flux with the one obtained by solving the transport equation numerically. To do this, we use the different initial diffuse fluxes for: ATM neutrinos derived by Volkova [5], neutrinos coming from AGN according to Stecker-Salomon [6] and to Protheroe [7], and neutrinos from GRB derived by Waxman [8] (see figure 1).

Besides the initial flux, we also need to assume a density for the Earth to solve the transport equation or to use the $\sigma_{eff}$-approach, so we take this density to be given by the Preliminary Reference Earth Model (PREM) [9], which is plotted in figure 2. Neutrinos will then follow a path through the Earth towards the detector as illustrated in figure 3.

Solving the transport equation by Euler’s method and by the iterative method described in [10] give the same results which are also consistent with the ones in [3]. For illustration we show in figures 4 and 5 the normalized final flux or shadowing factor

$$S(E, \theta) = \frac{\phi_\nu(E, \tau)}{\Phi_\nu(E)}$$  (8)

as well as the shadowing factor $S_{eff}$ according to the $\sigma_{eff}$ approach

$$S_{eff}(E, \theta) = e^{\sigma_{eff}(E, \Phi_\nu)\tau}$$  (9)

for nadir angles $\theta = \{80^0, 40^0, 0^0\}$ from the downward normal to the surface where the detector is (figure 3).

We note that the two approaches agree when dealing with fast decreasing initial spectra such as ATM, while for flatter initial fluxes, the $\sigma_{eff}$ approach trends to overestimate the surviving flux, (and hence the shadowing factor) as the nadir angle decreases.
III. TOMOGRAPHY OF THE EARTH

In the appendix we see that the Radon transform of the number density is the amount of material traversed \( \tau \) appearing in the transport equation, and the expressions for the Radon transform and its inverse transform are obtained exploding the assumed spherical symmetry of the density. For reconstructing the number density we will then use (see the appendix)

\[
n(r) = -\frac{1}{\pi} \int_{\arcsin \frac{r}{R}}^{\pi/2} \frac{d\theta'}{\sqrt{R^2 \sin^2 \theta' - r^2}} \frac{d\tau(\theta')}{d\theta'}
\]  

(10)

This expression relates the number density of the Earth at a distance \( r \) from its center, with the different amounts of matter traversed at different nadir angles through the Earth. Then, measuring \( \tau(\theta') \) at all the possible nadir angles between 0 y \( \pi/2 \) we may obtain \( n(r) \).

To use equation (10), we must be able to infer \( \frac{d\tau}{d\theta} \) from the data of the surviving neutrino flux. We may then do this inference under the transport equation approach or under the \( \sigma_{eff} \)-approach and reconstruct a density in each case.

Leaving for the next section a treatment of the error involved, we now generate data for the surviving flux at the 91 nadir angles 90°, 89°, ..., 0° as we did in the previous section for (80°, 40°, 0°). These data are generated using the complete transport equation, assuming the density given by the PREM (figure 2), and now we consider two different initial spectra for the sum of ATM, AGN, and GRB neutrinos (figure 6).

Once we have the surviving neutrino flux, we have to obtain \( \frac{d\tau}{d\theta} \) in order to do the tomography of the Earth. The usual approach is to use the effective cross section to relate the surviving flux with \( \tau(\theta') \) through (7), but as we pointed out in the previous section, this approach is only satisfactory for very fast decreasing spectra. In this conditions, we expect that the reconstructed density may not be accurate in general.

As an alternative method, we propose to use the complete integro-differential transport equation (not its solution) without approximation to directly obtain \( \frac{d\tau(\theta)}{d\theta} \) from the surviving flux \( \phi_{\nu}(E, \theta) \), which can be done noting that

\[
\frac{d\phi_{\nu}(E, \theta)}{d\theta} = \frac{d\phi_{\nu}(E, \tau(\theta))}{d\tau} \cdot \frac{d\tau}{d\theta}.
\]

(11)

As the angular variation of the number of nucleons \( \tau \) is independent of the energy of the neutrino flux, if we integrate over the our neutrino energy range \( \Delta E = (10^4\text{GeV}, 10^7\text{GeV}) \) at both sides of (11) and use the transport equation (3), we find that

\[
\frac{d\tau(\theta)}{d\theta} = \frac{N(\theta)}{D(\theta)}
\]

(12)

where

\[
N(\theta) = \int_{\Delta E} dE \frac{d\phi_{\nu}(E, \theta)}{d\theta},
\]

\[
D(\theta) = \int_{\Delta E} dE \left( \int_{-\infty}^{\infty} dE' \frac{d\sigma_{\nu c}(E', E)}{dE'} \phi_{\nu}(E', \theta) - \sigma_{tot}(E)\phi_{\nu}(E, \theta) \right).
\]

Inserting \( \frac{d\tau(\theta)}{d\theta} \) given by (12) into (10) yields the density reconstructed under the transport equation approach.

The densities recovered are plotted in figure 7, together with the original one given by the PREM. We would like to point out again that we reconstruct the earth density using the differential transport equation. We do not need to use the corresponding solution. As the differential transport equation is locally valid we have considered it and the corresponding measurable surviving flux on the earth surface.

We clearly observe that the recovered density using the effective cross section approach is higher that the density that actually caused the attenuation, while with the transport equation approach the reconstructed density acceptably
reproduces the original one. We note that the effective cross section leads to worse results the flatter is the initial flux, as we saw in the previous section. This is due to the fact that NC regeneration is more important when there are “many” neutrinos of higher energy than a given one, that then may happen to regenerate to that lower energy. With a fast-decreasing spectra such as ATM alone, regeneration is not very important, and hence the $\sigma_{eff}$-approach also works for recovering the original density. However, in our range of energy where absorption tomography is in principle possible, other sources of neutrinos also are expected to contribute (AGN, GRB, etc.) and dominate in a total neutrino flux in that range. This justifies the use of the complete transport equation approach rather than the standard one.

IV. ERROR PROPAGATION

In this section we present the error propagation analysis considering only the uncertainty in the surviving neutrino flux. We have not taken into account theoretical uncertainties in the cross section such as the error due to extrapolating to high energies, or the uncertainty accounting for possible new physics. The method we suggest for performing a neutrino absorption tomography clearly depends on the surviving neutrino flux at each nadir angle. IceCube [11] in the southern ice seems to be the most promising neutrino detector, with 1km$^3$ of detection volume with photomultipliers placed regularly to catch the Cherenkov light emitted when CC produced muons travel in the ice. The angular resolution is expected to be of about 1° (or better), which is the angular resolution we implemented for doing the density extraction.

As the uncertainty in a total measured neutrino flux depends among other things on the surviving flux being measured, we do not know what it will be for certain. Still, to test the recovery procedure, we assume three different uncertainties in the detected flux which are in the order of the expected ones in [11,12], and propagate each uncertainty to obtain the corresponding one to the recovered density. To do this, we implement the transport equation approach, in which $d\frac{\tau(\theta)}{d\theta}$ is given by (12). So for an uncertainty in the flux $\Delta \phi_{\nu}(E, \theta)$ we find that the uncertainties in $N(\theta)$ and $D(\theta)$ are given by

$$\Delta N(\theta) = \frac{d}{d\theta} \int_{\Delta E} dE \Delta \phi_{\nu}(E, \theta)$$

(13)

$$\Delta D(\theta) = \sqrt{(\Delta d_1(\theta))^2 + (\Delta d_2(\theta))^2},$$

(14)

where

$$\Delta d_1(\theta) = \int_{\Delta E} dE \sigma_{tot}(E) \Delta \phi_{\nu}(E, \tau)$$

$$\Delta d_2(\theta) = \int_{\Delta E} dE \int_{E}^{\infty} dE' \frac{d\sigma_{nc}(E', E)}{dE} \Delta \phi_{\nu}(E', \tau).$$

The absolute error in $d\frac{\tau(\theta)}{d\theta}$ is then

$$\Delta \left[ d\frac{\tau(\theta)}{d\theta} \right] = \sqrt{\left( \frac{\Delta N(\theta)}{D(\theta)} \right)^2 + \left( \frac{N(\theta)}{D(\theta)} \Delta D(\theta) \right)^2},$$

(15)

and by means of (10), we obtain the uncertainty in the number density recovered:

$$\Delta n(r) = \frac{1}{\pi} \int_{\arcsin \frac{r}{R}}^{\pi/2} \frac{d\theta'}{\sqrt{R^2 \sin^2 \theta' - r^2}} \left( \Delta \left[ d\frac{\tau(\theta')}{d\theta'} \right] \right),$$

(16)

that yields the corresponding uncertainty in the density through $\Delta \rho(r) = \Delta n(r)/N_A$, which we illustrate by error bars in the figures 8 and 9 for the three uncertainties of 15%, 10%, and 5% in the surviving neutrino flux. We show the results with the error bars considering only the initial flux produced by AGN (Stecker-Salomon), GRB and
ATM neutrinos added together, as they are substantially the same the other initial spectra considered before (AGN (Protheroe) + GRB + ATM) under the transport equation approach.

We observe that the error propagated to the recovered density is about $1.6 - 1.7$ times the error in the flux detected.

V. FINAL REMARKS

We have presented in this work a simple method to extract the Earth’s density based on the complete transport equation, without using any approximation as the use of the effective cross section would imply. We have compared the results obtained by our method with the ones obtained by the $\sigma_{eff}$-method. We can conclude that our method allows an accurate reconstruction for each initial flux we have considered, while the standard approach trends to overestimate the density accounting for its inaccuracy in dealing with the NC regeneration.

To estimate the uncertainties in the reconstructed density, we have propagated the relative uncertainty in the surviving neutrino flux and found that it is increased about 60% - 70% by the procedure.

APPENDIX A: RADON TRANSFORM AND THE EARTH’S DENSITY

The 2-d Radon transform [13] of a continuous function $n(x, y)$ can be defined as

$$ R_n(p, \vec{\xi}) = \int_{\mathbb{R}^2} dx \ dy \ n(x, y) \ \delta(p - \vec{x} \cdot \vec{\xi}) $$

(A1)

$R_n(p, \vec{\xi})$ (see figure 10) is thus $n(x, y)$ integrated along the straight at a distance $p$ from the origin and normal to $\vec{\xi} = (\cos \theta, \sin \theta)$ which is an unitary vector defined by $\vec{\xi} = (\cos \theta, \sin \theta)$, where the angle $\theta$ is measured from the x-axis (Fig.3).

If $R_n(p, \vec{\xi})$ is continuous, then $n(x, y)$ can be recovered using the inverse Radon transform. The latter can be obtained relating the 2-d Radon transform with the 2-d Fourier transform $F_n(\lambda \vec{\xi})$

$$ F_n(\lambda \vec{\xi}) = \int_{\mathbb{R}^2} dx \ dy \ e^{i\lambda \vec{x} \cdot \vec{\xi}} n(\vec{x}), $$

(A2)

where $\vec{x} = (x, y)$.

Since $\int_{-\infty}^{\infty} dp \ \delta(p - \vec{x} \cdot \vec{\xi}) = 1$,

$$ F_n(\lambda \vec{\xi}) = \int_{-\infty}^{\infty} dp \delta(p - \vec{x} \cdot \vec{\xi}) \int_{\mathbb{R}^2} dx \ dy \ e^{i\lambda \vec{x} \cdot \vec{\xi}} n(\vec{x}) $$

(A3)

and as the Dirac’s delta $\delta(p - \vec{x} \cdot \vec{\xi})$ allows us to write $p$ instead of $\vec{x} \cdot \vec{\xi}$, we obtain

$$ F_n(\lambda \vec{\xi}) = \int_{-\infty}^{\infty} dp \ e^{i\lambda p} \int_{\mathbb{R}^2} dx \ dy \ \delta(p - \vec{x} \cdot \vec{\xi}) \ n(\vec{x}). $$

(A4)

That is, the 2-d Fourier transform of $n(\vec{x})$ is equal to the 2-d Fourier transform of the 2-d Radon transform of $n(\vec{x})$, which is known as the Fourier Slice Theorem:

$$ F_n(\lambda \vec{\xi}) = \int_{-\infty}^{\infty} dp \ e^{i\lambda p} \ R_n(p, \vec{\xi}). $$

(A5)
Then, by means of the inverse Fourier transform we recover \( n(\vec{x}) \),

\[
n(\vec{x}) = \frac{1}{(2\pi)^2} \int d^2(\lambda\vec{\xi}) \ e^{-i\lambda\vec{x} \cdot \vec{\xi}} F_n(\lambda\vec{\xi}) \quad (A6)
\]

Noting that the unitary vector \( \vec{\xi} = (\cos \phi, \sin \phi) \), we use the polar coordinates \((\lambda, \phi)\) and (A6) becomes

\[
n(\vec{x}) = \frac{1}{(2\pi)^2} \int_0^\infty d\lambda \lambda \int_0^{2\pi} d\phi \ e^{-i\lambda\vec{x} \cdot \vec{\xi}} F_n(\lambda\vec{\xi}) \quad (A7)
\]

Expressing \( F_n(\lambda\vec{\xi}) \) according to (A5), defining \( p' := p - \vec{x} \cdot \vec{\xi} \), and \( \mathcal{R}_n(p', \vec{\xi}) \) as

\[
\mathcal{R}_n(p', \vec{\xi}) := \frac{1}{2\pi} \int_0^{2\pi} d\phi \ R_n(\lambda\vec{\xi}) \quad (A8)
\]

we can write \( n(\vec{x}) \) as

\[
n(\vec{x}) = \frac{1}{2\pi} \int_0^\infty d\lambda \lambda \int_{-\infty}^{\infty} dp \ e^{i\lambda p} \mathcal{R}_n(p, \vec{\xi}) \quad (A9)
\]

Expressing the above equation in terms of the function \( \text{sign}(\lambda) \), performing integration by parts in \( p \), and using the Fourier transform of \( \text{sign}(\lambda) \), we obtain

\[
n(\vec{x}) = -\frac{1}{2\pi} \int_{-\infty}^{\infty} dp \frac{\mathcal{R}_n'(p, \vec{\xi})}{p} \quad (A10)
\]

which expresses \( n(\vec{x}) \) in terms the \( p \) derivative of its Radon transform in a simplified fashion.

In the following, we use the above results to relate the Earth’s density with its Radon transform in a simple expression.

We assume an spherically symmetric number density for the Earth which we denote by \( n(r) \) and begin by noting that the amount of nucleons \( \tau(z) \) found along a path of depth \( z = 2R \cos \theta \) in the Earth is nothing but the Radon transform of the number density \( n(r) \) of the Earth:

\[
\tau(\theta) = \int_0^z dz' n(z') = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \ n(\vec{x}) \ \delta(p - \vec{\xi} \cdot \vec{x}) \quad (A11)
\]

where now \( p = R \sin \theta \) and \( R \) is the Earth’s radius. In the polar coordinates \((r, \phi)\) with the origin in the Earth’s center, \( \vec{\xi} \cdot \vec{x} = r \cos(\phi - \theta) \), then we may write

\[
\tau(\theta) = \int_0^{2\pi} d\phi \int_0^r dr \ n(r) \ \delta(p - r \cos(\phi - \theta)). \quad (A12)
\]

We can now integrate in \( \phi \) making use of the Dirac’s delta property \( \delta(f(\phi)) = \sum_i \delta(\phi - \phi_i) \frac{1}{|f'(\phi_i)|} \) where \( \phi_i \) are the zeros of \( f(\phi) \), and as this Dirac’s delta in (A12) implies that

\[
\cos(\phi_i - \theta) = \frac{p}{r} \quad \text{then} \quad |p| < r,
\]

and expression (A12) becomes
\[
\tau(\theta) = \int_{|p|}^{\infty} \frac{2\pi n(r)}{\sqrt{r^2 - p^2}} \, dr.
\] (A13)

Then as \(n(r)\) vanishes for \(r > R\),
\[
\tau(\theta) = R_n(p) = \int_{R^2 \sin^2 \theta}^{R^2} \frac{n(r) \, d(r^2)}{\sqrt{r^2 - R^2 \sin^2 \theta}}.
\] (A14)

We can recover \(n(r)\) by means of the expression for the inverse Radon transform (A10):
\[
n(r) = -\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \frac{dp}{p} \frac{d}{dp} \int_{0}^{2\pi} d\phi \, R_n(p + \xi \cdot \hat{x})
\] (A15)

defining \(p' := p + \xi \cdot \hat{x}\), we can write
\[
n(r) = -\frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{0}^{2\pi} \frac{dp' \, d\phi}{p' - r \cos(\phi - \theta)} \frac{dR_n(p')}{dp'}.
\] (A16)

Now, the integral in \(\phi\) is defined for \(p' > r\) and \(p' < -r\), then by the Cauchy theorem it turns out that
\[
\int_{0}^{2\pi} \frac{d\phi}{p' - r \cos(\phi - \theta)} = \begin{cases} 
\frac{2\pi}{p'^2 - r^2} & \text{si } p' > r \\
-\frac{2\pi}{p'^2 - r^2} & \text{si } p' < -r 
\end{cases}
\]

Then, as it must be \(|p'| < R\) so that \(R_n(p')\) does not vanish, defining \(\theta'\) such that \(p' = R \sin \theta'\), and recalling that \(R_n(p') = \tau(\theta')\), we obtain that
\[
n(r) = -\frac{1}{\pi} \int_{\arcsin \frac{r}{R}}^{\pi/2} \frac{d\theta'}{\sqrt{R^2 \sin^2 \theta' - r^2}} \frac{d\tau(\theta')}{d\theta'},
\] (A17)

which allows us to express the Earth’s density in terms of the slope in the amount of nucleons \(\tau\).

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Figure Captions

**Figure 1:** Initial muon neutrino fluxes plotted separately: AGN(Protheroe) [7], AGN(Stecker-Salomon) [6], GRB [8] and ATM [5].

**Figure 2:** Earth density as predicted by the 'Preliminary Reference Earth Model' [9].

**Figure 3:** Neutrino path towards the detector.

**Figure 4:** Shadowing factors for ATM and GRB fluxes for nadir angles $80^0$, $40^0$, and $0^0$.

**Figure 5:** Shadowing factors for AGN (S-Salomon) and AGN (Protheroe) fluxes for nadir angles $80^0$, $40^0$, and $0^0$.

**Figure 6:** Total initial neutrino fluxes from AGN + GRB + ATM. AGN(P) stands for the flux by Protheroe, and AGN(S-S) is the neutrino flux predicted by Stecker and Salomon.

**Figure 7:** Recovered densities compared with the PREM density (solid line). Boxes and stars represent the densities obtained with the transport equation and with the $\sigma_{eff}$ methods respectively.

**Figure 8:** Densities recovered with error bars for uncertainties of 15% and 10% in the surviving flux.

**Figure 9:** Density recovered with error bars for an uncertainty of 5% in the surviving flux.

**Figure 10:** $R^2$ region where the Radon transform is defined. The unit vector $\xi$ and the number $p$ define the integration path.
Fig. 1
Fig. 2
Fig. 3
Fig. 6
Fig. 8

\[ \Delta \phi_v = 15\% \]

\[ \Delta \phi_v = 10\% \]
Fig. 9

\[ \Delta \phi_\gamma = 5\% \]
