The Superconducting Phase Diagram of High-Tc Cuprates

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We address some basic issues of the superconducting part of the phase diagrams of the high-Tc cuprates, which so far have not been properly described. We derive, in particular, analytic expressions for \( T_c(x) \), the critical temperature as a function of doping for such materials. These exhibit the well-known dome-shaped superconducting phases, which are in excellent agreement with the experimental data for materials such as LSCO, Bi2201 and Hg1201. Our starting point is a BCS-like theory, defined in the oxygen lattice of the \( CuO_2 \) planes. Cooper pair formation occurs between holes belonging to the two different sublattices thereof, which are formed, respectively by ions with orbitals \( p_x \) and \( p_y \). This naturally leads to a d-wave SC order parameter. Our results imply the optimal doping occurs when such lattice is half-filled. The growth of the optimal temperature as well as the trend of the SC and AF domes to superimpose, which is verified in multi-layered cuprates, when the number, \( N \), of such planes is increased, can be simply understood as an enhancement of the relevant coupling parameter, namely, \( g \rightarrow Ng \). Our findings allow one to devise means and methods for increasing the critical SC temperature in cuprates.

Introduction.

Understanding the mechanism of high-Tc superconductivity in the cuprate materials is, at the same time, one of the most fascinating and challenging problems in physics. Thirty years after the experimental discovery of superconductivity in such materials [1], we have still several fundamental phenomenological issues of the high-Tc cuprates that cannot be properly accounted for by an underlying theory. To mention just a few of these issues, let us recall that so far we do not know the specific analytic expression for the curves representing the critical transition temperature as a function of doping, namely, \( T_c(x) \), which form the characteristic SC domes in all high-Tc materials. We also still do not have an explanation for the fact that the optimal transition critical temperature increases as a function of the number of adjacent \( CuO_2 \) planes, up to a point and then stabilizes, as one can observe, for instance, in the, \( Bi, Hg \) and \( Tl \) families of cuprates [2, 3]. Furthermore, we cannot explain why, when we consider multi-layered cuprates with a number, \( N \), of adjacent \( CuO_2 \) planes, the antiferromagnetic (AF) and superconducting (SC) domes found in the phase diagram of such materials come closer to each other and eventually superimpose, as we increase \( N \).

In this work, we start from a BCS-like theory, defined on the oxygen lattice of the \( CuO_2 \) planes of the cuprates. This describes the dynamics of the holes doped into the oxygen ions. Cooper pairs are formed by combining holes belonging to the two different sublattices of the oxygen square lattice, which contain respectively, \( p_x \) and \( p_y \) orbitals. This naturally leads to a d-wave SC order parameter.

The doping mechanism is explicitly taken into account by the introduction of a constraint on the fermion number. After integrating out the fermion degrees of freedom, we obtain an effective action in terms of the superconducting order parameter, \( \Delta_0 \) and the temperature. Then, minimizing the effective potential, which corresponds, to this action we are able to derive an expression relating \( \Delta_0, T \) and the doping parameter \( x \). From this, by making \( \Delta_0 = 0 \), we capture the threshold for the SC transition and thereby arrive at an analytic expression for the critical SC temperature as a function of doping, namely \( T_c(x) \). This reproduces the familiar SC domes found in the cuprates and is in excellent agreement with the experimental data for materials such as LSCO (\( La_{2-x}Sr_xCuO_4 \)), Bi2201 (\( BiSr_{2-x}La_xCuO_6 \)) and Hg1201 (\( HgBa_2CuO_{4+x} \)).

Our results clearly indicate that optimal doping occurs when the amount of stoichiometric doping, is such that the oxygen ions of the \( CuO_2 \) planes are half-filled. This observation suggests that an interplay between the two half-filled lattices, namely, copper and oxygen plays an important part in the SC mechanism in the cuprates. The increase of the optimal temperature, as well as the tendency of the SC and AF domes to superimpose as we increase the number of adjacent planes, in multi-layered cuprates, can be simply understood within our approach, as the effective coupling parameter \( g \) is enhanced by the number, \( N \), of such planes: \( g \rightarrow Ng \).

The Mechanism of Doping

An outstanding feature of all High-Tc cuprates is the presence of one or more \( CuO_2 \) planes, intercepting the primitive unit cell of such compounds. The \( CuO_2 \) planes have a lattice structure in which \( Cu^{++} \) ions occupy the sites and oxygen ions the links of a square lattice, in
which the copper ions are located a distance of \( a' = 3.8\AA \) apart. These are in a 3d\(^9\) electronic configuration, which results in one localized spin 1/2 per site, interacting with the neighbors through the super-exchange mechanism, which is ultimately responsible for the antiferromagnetic properties observed in the high-T\(_c\) cuprates. The system is a Mott-Hubbard insulator, from the point of view of the copper ions. From the point of view of the oxygen ions, however, the picture is different. Indeed, the oxygen ions are themselves, placed on the sites of a square lattice, with a lattice parameter \( a = \sqrt{2a'/2} = 1.9\sqrt{2} \\AA \) and present an electronic configuration that changes as a consequence of doping. In the case of the pure parent compounds the oxygen ions are doubly charged, namely: \( O^{-\times 2} \). In this case, such ions are in a 2p\(^6\) configuration, which corresponds to a completely filled p-band. Such a filled up band must be associated with a crystal structure with two electrons per site. By considering just the \( O^{-\times 2} \) square lattice, then one realizes these two electrons are precisely the ones belonging to the \( p_x \) and \( p_y \)-orbitals of the \( O^{-\times 2} \) ions, which form two square sublattices. We show below that the d-wave character of the SC order parameter allows one to infer that Cooper pairs form between electrons belonging to the \( p_x \) and \( p_y \)-orbitals of oxygen.

Consider now the process of doping, which we illustrate in the case of strontium doped \( La_2CuO_4 \). As we dope the system by replacing a fraction \( x \) of the \( La \) atoms with \( Sr \), which is an electron receptor, it follows that one of the two electrons of the oxygen ion \( p_x \) or \( p_y \)-orbitals is sucked by the \( Sr \) atom, thereby creating a hole in the relevant p-orbital. As we increase the strontium doped fraction \( x \), we will increase the number of holes in the oxygen lattice. This process will go on until we reach a situation where the oxygen lattice contains exactly one electron (or one hole) per site, namely becomes half-filled. Let us call \( x_0 \) the amount of strontium doping for which the oxygen lattice becomes half-filled. It follows that for an arbitrary amount of doping, \( x \), the net charge (in e-charge units) per oxygen ion can be written as

\[
N_e - N_h = 2 \left( 1 - \frac{x}{x_0} \right),
\]

where \( N_e \) and \( N_h \) are, respectively, the number of electrons and holes per oxygen ion. Notice that \( N_e - N_h = 2 \) for the undoped case \( (N_e = 2, N_h = 0) \), and, therefore, \( x = 0 \). In the half-filled case, when \( N_e = 0 \) and \( N_h = 0 \), conversely, we have \( x = x_0 \).

The Model.

We assume that the underlying pairing mechanism responsible for the superconductivity in cuprates, whatever it may be, generates an effective interaction hamiltonian on the oxygen square lattice, which determines the dynamics of the electrons and holes in the oxygen ions of the \( CuO_2 \) planes of the cuprates. This consists of a specific BCS interaction, which involves pairs of holes in which each component of the pair belongs to different oxygen sublattice, namely, to the \( p_x \) and \( p_y \) sublattices. The kinematics is determined by the usual tight-binding term, so the Hamiltonian is

\[
H = -t \sum_{R,d_i} \psi_{B\sigma}^\dagger(R + d_i) \psi_{A\sigma}(R) + hc
\]

\[
+ \sum_{R,d_i} \Delta(d_i) \left[ \psi_{B\sigma}^\dagger(R + d_i) \psi_{A\sigma}(R) - \psi_{B\sigma}(R + d_i) \psi_{A\sigma}^\dagger(R) \right]
\]

\[
+ \frac{1}{g} \sum_{R,d_i} \Delta^\dagger(R) \Delta(R), \tag{2}
\]

where \( t \) is the usual hopping parameter, \( g \), the interaction coupling parameter and \( \Delta^\dagger \) is the Cooper pair creation operator, the vacuum expectation value of which is a SC order parameter.

In the above expression, \( R \) denotes the sites of a square lattice and \( d_i \), \( i = 1, ..., 4 \), its nearest neighbors. \( \psi_{A,B\sigma}^\dagger \) is the creation operator of a hole with spin \( \sigma = \uparrow, \downarrow \) in sublattice \( A,B \). Such sublattices are formed, respectively, by the \( p_x \) and \( p_y \)-orbitals of oxygen ions. Such BCS interaction leads to Cooper pair formation, with a d-wave order parameter, as we shall see. The XY-asymmetry can be naturally ascribed to the different signs of overlapping \( p_x \) and \( p_y \)-orbitals along the oxygen lattice \( x \) and \( y \) directions (see Fig. 1), which imposes \( \Delta(d_x) = -\Delta(d_y) \).

In momentum space, we have the corresponding Hamil-
Hamiltonian
\[ H = \sum_{k,\sigma} \epsilon(k) \left[ \psi_{B\sigma}^\dagger(-k)\psi_{A\sigma}(k) + h.c. \right] + \sum_k \Delta(k) \left[ \psi_{B1}^\dagger(-k)\psi_{A1}(k) - \psi_{B1}^\dagger(k)\psi_{A1}^\dagger(-k) \right] + \frac{1}{g} \sum_k \Delta^\dagger(-k)\Delta(k), \]

where \( \epsilon(k) = 2t[\cos k_x a + \cos k_y a] \) is the usual tight-binding energy. When we start to dope and, consequently introduce holes in the system, it is natural to expect hole pockets to form around the points where \( \epsilon(k) = 0 \). Thus, we naturally expand about any one of the four equivalent points: \( \pm\frac{k_x}{2}, \pm\frac{k_y}{2} \) and thereby obtain \( \epsilon(k) \approx V_{eff}[k_x + k_y] \), where the characteristic velocity is given by \( V_{eff} = 2ta \). Using the xy-asymmetry of \( \Delta(d_1) \), which is caused by the asymmetric overlap of \( p_x \) and \( p_y \) orbitals described above, we obtain accordingly
\[ \langle \Delta(k) \rangle = \sigma_0[\cos k_x a - \cos k_y a], \]

which has d-wave symmetry.

An example of how this special BCS interaction can be derived from a system of interacting localized and itinerant spins can be found in [4, 5]. A similar hamiltonian is described in [6].

The cuprates can be classified according to the number of adjacent \( CuO_2 \) planes contained in their primitive unit cell. The index \( a \) indicates to which of the adjacent \( CuO_2 \) planes the electrons and holes belong. This index runs from 1 to \( N \), where \( N = 1, 2, 3, \ldots \), according to the number of planes the specific cuprate compound possesses. In this approach, we shall neglect interplane interactions.

Since \( \psi_{\sigma,a}^\dagger\psi_{\sigma,a} = \frac{N_a - N}{A} \), it follows that
\[ \sum_{a=1}^N \psi_{\sigma,a}^\dagger\psi_{\sigma,a} = 2A \left[ N - \frac{x}{x_0} \right] \equiv N\delta, \]

where \( A \) is the unit cell area of the oxygen lattice: \( A = 2 (1.9)^2 \AA^2 \).

In order to enforce the constraint (6), we add to the Lagrangean a term \( \lambda \left( \psi_{\sigma,a}^\dagger\psi_{\sigma,a} - N\delta \right) \), where \( \lambda \) is a Lagrange multiplier field upon which we must integrate. Conveniently integrating on the the complex field, \( \Delta \), which acts as a Hubbard-Stratonovitch field, the Hamiltonian pass to display a quartic interaction in the fermions, with coupling \( g \).

Conversely including the doping constraint and performing the quadratic functional integral over the fermion fields we obtain the effective action \( S[\sigma, \lambda] \),
\[ S[\Delta, \lambda] = \int d^3x \left[ -\frac{\Delta^\dagger\Delta}{g} - N\lambda\delta \right] + Ntr \ln A \]

where \( A = i\partial_0 - \mathcal{H} + \lambda I \otimes (-I). \) (Here \( I \) is the 2 \( \times \) 2 identity matrix and \( \mathcal{H} = \Psi_1^\dagger H \Psi_0 \)).

Minimizing this, namely
\[ \left[ \frac{\delta S}{\delta \Delta} \right]_{\Delta=\Delta_0, \lambda=\lambda_0} = 0; \quad \left[ \frac{\delta S}{\delta \lambda} \right]_{\Delta=\Delta_0, \lambda=\lambda_0} = 0. \]

we determine \( \Delta_0, \lambda_0 \).

The Critical Temperature: \( T_c(x) \)

Our main goal is to derive an expression for the critical temperature for the SC transition as a function of doping, \( T_c(x) \). For this purpose, we first note that the two equations in (8) imply relations among \( \Delta_0, \lambda_0, x \) and \( T \).

In order to find the critical temperature \( T_c(x) \), we impose the condition \( \sigma_0 = 0 \), (or \( \Delta_0 = 0 \)), which expresses the fact that the system is on the verge of the SC phase. Then, for \( \sigma_0 = 0 \) the conditions expressed in (8) lead to expressions for \( \lambda_0(x) \) and \( T_c(x) \), namely,
\[ \lambda_0(x) = \left( \frac{\delta}{\delta x} \right) \frac{Ng g_c}{Ng - g_c} = 2\gamma(x_0 - x), \]

and
\[ T_c(x) = \frac{\ln 2}{\ln 2 + \ln \cosh \left( \frac{\lambda_0(x)}{2T_c(x)} \right)} \]

In the above expressions, \( \gamma(N) = \frac{Ng g_c}{Ng - g_c} \), where \( g_c = \alpha / \Lambda \), is a threshold coupling parameter below which there is no SC phase. Here, \( \alpha = 2\pi V_{eff}^2 \) and \( \Lambda = 0.018 eV \approx h V_{eff}/a = \sqrt{2\pi\alpha,a} \) is the energy cutoff, which naturally appears on a lattice [7].

In the previous equations, \( T_{max} \), given by
\[ T_{max} = \frac{\alpha}{2\ln 2} \frac{\xi(N)}{g_c} \quad \xi(N) = \frac{Ng - g_c}{Ng} \]

is the transition temperature at optimal doping and occurs at \( x = x_0 \) namely \( T_{max} = T_c(x_0) \). This means optimal doping occurs at half-filling for the oxygen lattice. Notice that \( \gamma(N) = \frac{Ng}{Ng - g_c} = \frac{g_c}{2Ng \xi(N)} \). We see that \( T_{max} \) depends linearly on the couplings: \( Ng - g_c \), a behavior completely different from the one exhibited by the transition temperature in conventional SC, where there is an
exponential dependence. This kind of behavior has been extensively studied before [5, 7].

We describe now the results of our theory applied to three single plane cuprates ($N = 1$), namely LSCO, Bi2201 and Hg1201.

Results

1. LSCO

Starting from (10) we can write (see Supplementary Material)

$$T_c(x) = \frac{\ln 2 \, T_{\text{max}}}{\ln 2 + \frac{2|x_0 - x|}{T_c(x)} + \frac{1}{2} \left\{ \exp \left[ -\frac{2\gamma|x_0 - x|}{T_c(x)} \right] - 1 \right\}} \tag{12}$$

The solution $T_c(x)$ of this implicit equation for the critical temperature of the SC transition is depicted in Fig. 2.

![Fig. 2](image1.png)

FIG. 2: Solution of Eq. (12) for the SC dome of LSCO. Experimental data from [9–12]

For obtaining this result, we used $\gamma = 0.020 \text{ eV}$, $x_0 = 0.16$ and $T_{\text{max}} = 0.0031 \text{ eV}$ (Notice that $T(K) = 11604 T(eV)$). Observe also, that with the choice of $\gamma$, we adjust only one parameter of our analytic expression for $T_c$. Entering the experimental values of $T_{\text{max}}$ and $x_0$ and using the value of $\Lambda$, we find $g = 0.40226 \text{ eV}^{-1}$, $g_c = 0.30766 \text{ eV}^{-1}$ and $\xi(1) = 0.23517$.

Taking the limit $T_c \to 0$ in (12), we find the two quantum critical points where the SC dome starts at $T = 0$. These are given by

$$x_{SC}^+ = x_0 \pm \frac{T_{\text{max}}}{\gamma} \ln 2 = x_0 \pm \alpha A x_0 \frac{\xi^2(N)}{g_c^2}. \tag{13}$$

Inserting the above numerical values, for $N = 1$, we find: $x_{SC}^+ = 0.053$ and $x_{SC}^- = 0.2669$.

It is instructive to compare our result with the empirical curve, obtained by fitting the data for the LSCO dome, namely, the parabola [2, 3, 17]

$$T_c(x) = T_{\text{max}} \left[ 1 - 2.616(x_0 - x)^2 \right], \tag{14}$$

In the next figure we superimpose it with our solution of (12)

![Fig. 3](image2.png)

FIG. 3: The empirical parabolic fit for the SC dome of LSCO, dotted line, superimposed with our solution for Eq. (12), solid line. Experimental data from [9–12]

2. Bi2201 and Hg1201

Now, for Bi2201 and Hg1201, we have different equations for $T_c(x)$ in the underdoped, $x < x_0$ and overdoped, $x > x_0$ regions. The reason is the expansions we have to make, starting from (10), are necessarily different for $x_0 - x > 0$ and $x_0 - x < 0$, thus leading to different equations in each region. Indeed, we obtain (see Supplementary Material)

$$T_c(x) = \frac{\ln 2 \, T_{\text{max}}}{\ln 2 + \frac{2|x_0 - x|}{T_c(x)} + \frac{1}{2} \left\{ \exp \left[ -\frac{2\gamma|x_0 - x|}{T_c(x)} \right] - 1 \right\}} \tag{15}$$

for $x < x_0$ and

$$T_c(x) = \frac{\ln 2 \, T_{\text{max}}}{\frac{\gamma|x_0 - x|}{T_c(x)} + \ln \left[ 1 + \exp \left[ -\frac{2\gamma|x_0 - x|}{T_c(x)} \right] \right]} \tag{16}$$

for $x > x_0$.

Let us consider Bi2201 first. The solution $T_c(x)$ of these implicit equations for the critical temperature of the SC transition of Bi2201 is depicted in Fig. 4.

For obtaining this result, we used $N = 1$, $\gamma = 0.0120 \text{ eV}$, $x_0 = 0.29$ and $T_{\text{max}} = 0.0030 \text{ eV}$, which
imply \( g = 0.40800 \, eV^{-1} \) and \( g_c = 0.31552 \, eV^{-1} \) and \( \xi(1) = 0.22666 \).

Taking the limit \( T_c \to 0 \) in (12), we find the two quantum critical points where the SC dome starts at \( T = 0 \). These are given by

\[
x_{SC}^- = x_0 - \frac{T_{max}}{\gamma} \ln 2 \quad \text{and} \quad x_{SC}^+ = x_0 + \frac{T_{max}}{3\gamma} \ln 2.
\] (17)

Inserting the above numerical values, we find: \( x_{SC}^- = 0.1175 \) and \( x_{SC}^+ = 0.3475 \).

Now consider Hg1201. Then, using Eqs. (15) and (16) with parameters \( \gamma = 0.0310 \, eV \), \( x_0 = 0.25 \) and \( T_{max} = 0.00835 \, eV \), we obtain the solution \( T_c(x) \) of this implicit equation for the critical temperature of the SC transition, which is depicted in Fig. 5. The above values imply \( g = 2.14781 \, eV^{-1} \) and \( g_c = 0.78612 \, eV^{-1} \) and \( \xi(1) = 0.63398 \).

Inserting the above numerical values in (17), we now find: \( x_{SC}^- = 0.064 \) and \( x_{SC}^+ = 0.3119 \).

**Increase of \( T_{max} \) with The Number of Adjacent Planes**

It is an evident experimental fact that the optimal transition temperature becomes higher as one increases the number of CuO\(_2\) planes per primitive unit cell. Bi2201 and Hg1201, for instance, are single-layered materials, which have multi-layered relatives with a higher optimal temperature.

The mercury family, for instance, consists of [15, 16, 20, 21]: Hg1201 (single-layered) (\( T_{max} = 97 \, K \)), Hg1212 (double-layered) (\( T_{max} = 125 \, K \)), Hg1223 (triple-layered) (\( T_{max} = 137 \, K \)), Hg1234 (four-layered)(\( T_{max} = 127 \, K \)) and Hg1245 (five-layered)(\( T_{max} = 120 \, K \)). It shows an increase of the higher optimal temperatures as the number of adjacent layers is increased from \( N = 1 \) to \( N = 3 \). Then for \( N = 4, 5 \), \( T_{max} \) stabilizes at a temperature approximately corresponding to \( N = 2 \).

For the bismuth family, conversely, we have Bi2201 (single-layered) (\( T_{max} = 34 \, K \)), Bi2212 (double-layered) (\( T_{max} = 92 \, K \)), Bi2223 (triple-layered) (\( T_{max} = 108 \, K \)).

From (11), we see that we may express the optimal temperature of a multi-layered cuprate with \( N \) adjacent CuO\(_2\) planes in terms of the same temperature of the single-layered one, as

\[
T_{max}(N) = \frac{\xi(N)}{\xi(1)} T_{max}(1)
\] (18)

Observing that \( \xi(N) \) is a monotonically increasing function of \( N \), the obvious effect of increasing the number of adjacent planes is to increase \( T_{max} \). This follows directly from the enhancement of the coupling parameter, namely: \( g \to Ng \).

Assuming that \( g \) and \( g_c \) are the same for all members of a family, we have for the bismuth family: \( \xi(1) = 0.22666 \), \( \xi(2) = 0.61333 \), \( \xi(3) = 0.74222 \), which according to (18) gives

\[
T_{max}(2) = 2.7058 T_{max}(1) = 91.99 \, K;
\]

\[
T_{max}(3) = 3.27446 T_{max}(1) = 111.33 \, K.
\]

These should be compared with the experimental values [2, 3]: 92 \, K and 108 \, K, respectively.

Accordingly, for the mercury family we have: \( \xi(1) = 0.63398 \), \( \xi(2) = 0.81699 \), \( \xi(3) = 0.87799 \), which implies

\[
T_{max}(2) = 1.28866 T_{max}(1) = 125.0 \, K;
\]

\[
T_{max}(3) = 1.38488 T_{max}(1) = 134.33 \, K.
\]

The last two values should be compared with the experimental values [2, 3]: 125 \, K and 137 \, K.

We see that our theoretical values for the optimal temperature of the members of the Bi and Hg families, are in excellent agreement with the experimental value for
Superposition of the SC and AF Domes

An evident effect in the phenomenology of multilayered cuprates is the observation that, as we increase the number of CuO$_2$ planes per primitive unit cell, the SC and AF domes come closer to each other and eventually superimpose [22]. From (17), we clearly see that the quantum critical point $x_{SC}^\pm$ decreases as we increase $N$, the number of adjacent planes, as a result of the enhancement of the effective coupling constant $g$. Hence it will eventually come inside the AF dome that will consequently be superimposed to the SC dome. Indeed, using (13), (17) and considering that $\xi(N)$ is a monotonically increasing function of $N$ we can understand why $x_{SC}^\pm(N)$ decreases as $N$ increases, therefore producing a superposition of the SC and AF domes. Furthermore, since the innermost planes are far from the charge reservoirs, they remain underdoped, and consequently in the AF phase while the outermost planes are efficiently doped and go into the SC phase.

Conclusions.

Our results indicate that the mechanism of Cooper pair formation in the cuprates, whatever it may be, produces a BCS-type effective theory for the holes doped into the system, whose hamiltonian is defined on the oxygen lattice of the CuO$_2$ planes. Cooper pairs are formed by holes belonging to the two $p_x$ and $p_y$ sublattices, naturally yielding a d-wave SC order parameter. Integrating over the fermions and minimizing the resulting effective action, we derive an implicit equation for the critical SC temperature as a function of doping: $T_c(x)$. The solution of such equation is, then compared with the experimental data for different compounds. Such a theory provides a description of the SC phase diagram of the cuprates that is in excellent agreement with experimental data for different materials. Optimal doping corresponds to a half-filled oxygen lattice. Such result highlights the interplay between a half-filled Mott-Hubbard insulator in the copper lattice and a band insulator, originally with two electrons per site, in the oxygen lattice, which eventually becomes half-filled through doping.

For single-layered cuprates, the maximal temperature, at optimal doping, has a clear linear dependence on the difference between the effective coupling parameter, $g$ and a threshold critical coupling $g_c$, below which the SC phase would disappear. This is in sharp contrast with conventional superconductivity, where $T_c$ presents no threshold and an exponential dependence on the coupling parameter. This feature is certainly responsible for explaining why the critical temperatures are higher in the cuprates. The increase of $T_{max}$ with the number of adjacent planes as well as the superposition of SC and AF domes in multi-layered cuprates can be understood as a consequence of the enhancement of the coupling $g \rightarrow Ng$ produced by the presence of these planes. Based on our results one can devise a way to increase $T_c$ in cuprates: this would be achieved by effectively doping the innermost planes in multilayered cuprates. For that purpose, one should have materials with a unit cell containing as much layers as possible but with no more than two layers separating the charge reservoir locations. Equivalently, one should have multilayered cuprates with a larger number of charge reservoirs intercalated with such layers, in the primitive unit cell.

Our results open a new avenue of investigation of the physical properties of High-Tc cuprates, with outstanding possibilities. Among these, how to describe the pseudogap transition within this framework, how to include the antiferromagnetic phase in the picture, how to describe the interplay of the AF and SC phases, how to describe the resistivity above $T_c$.

The crucial issue in high-Tc superconductivity, of course, remains the underlying mechanism of pair formation. The results reported here could be a concrete step forward towards the complete understanding of the nature of this mechanism.

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