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An inverse vibration-based approach towards modelling and damage identification in nonlinearly vibrating structures. Application for delamination detection in a composite beam.

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Abstract This study explores the possibilities for inverse analysis and modelling from data of a nonlinearly vibrating structure. We are suggesting a statistical approach based on singular spectrum analysis (SSA). The method is based on a free decay response, when the structure is given an initial disturbance and is left to vibrate on its own. The measured vibration response is decomposed into new variables, the principal components, which are used to uncover oscillatory patterns in the structural response. In this study an application of the methodology for the purposes of delamination detection in a composite beam is explored.

1. Introduction.
The free decay response of a linearly vibrating structure is a decaying signal which in the frequency domain is a function of the natural frequencies of the structure and will have peaks at these natural frequencies. Most real structures have nonlinearities induced by their material, the geometry of the structure, as well as by its supports and joints. In some occasions it is possible to approximate a nonlinear vibratory behaviour with a linear one. But on a number of cases the presence of nonlinearities should be taken into account. Structures made of composite materials are one such example. It is known that composite materials and structures made of composites demonstrate quite complex vibratory behaviour which on most occasions cannot be approximated as linear. The free decay response of composite structures is quite difficult or on some occasions even impossible to analyse in the frequency domain especially because of the nonlinearities present in the structure which result in nonlinear vibration response signal. An example of such a free decay signal is given in Figure 1 where the signal is given in the time-domain and the frequency domain. It can be seen that the signal is nonlinear and it is difficult to draw conclusions for the structure’s behaviour from these signal representations. The presence of noise and other uncertainties coming from the environmental conditions should not be forgotten because they also contribute to the nonlinear and non-stationary nature of the obtained signal.

There are different attempts to treat the problem for nonlinear structural vibration analysis and identification ranging from completely deterministic approaches to approaches based on probability and statistical methods. Deterministic modelling methods are based on deterministic analytical or numerical models for the nonlinear and non-stationary behaviour of certain structures, vis. composite shells, beams and or plates [1,2]. It should be noted that such methods are on most occasions oriented and developed for a certain structural and material type and in general they cannot be easily or
automatically applied to other structures or materials. Inverse identification methods for such vibratory problems can be deterministic but in the majority of cases they are based on statistical time series models like e.g. ARMA and NARMA ones. A more detailed review and classification of such methods can be found in e.g. [3].

Another big class of modelling approaches attempt to model the structural response time series using nonlinear time series analysis methods. Although these methods are generally oriented towards modelling deterministic systems most of the methods operate on a statistical basis making statistical assumptions. Probabilistically based methods are generally based on certain hypotheses and when applied for identification purposes treat the parameters being identified as random variables.

Figure 1. Free decay response measured for a 1m long, 20mm wide and 6mm thick carbon fibre beam

The method suggested here uses a time series approach where the time lagged vibration response measurements are considered as measured variables and are analysed using principal components analysis. This is used in order to reduce the number of time domain components of the measured signal and also to uncover oscillation patterns in it. This is certainly not the first paper to explore a statistical approach and not the first paper to use PCA for the purposes of damage assessment. But there are rather few applications of statistical methods and nearly no papers for the application of PCA for delamination detection in composite structures. In this study the time domain measured vibration response is treated as a random time dependent multidimensional variable which is analysed using singular spectrum analysis (SSA) which is an alternative of PCA. The main idea of PCA is to reduce the number of the measured time or frequency domain components in the measured discrete vibration response, retaining most of their variance. It has been previously applied in the time domain as well as over frequency domain data by e.g. [4] and the author of the present study [5]. As a result a vibrating system is represented by a new but much smaller set of variables vis. the several principal components. The new principal components can be associated to certain important properties of the structure.
including the presence of damage as well as its location in the structure and its size/extent. SSA is an alternative of PCA which is developed for time dependent variables [6]. It has the same properties as PCA but in addition it is applied in order to uncover oscillation patterns in the measured time series. This study suggests to use SSA rather than conventional modal analysis in order to analyse the oscillation patterns in the vibration response of a nonlinearly vibrating structure. The oscillations in the vibration response signal of a linearly vibrating structure can be associated with its modal behaviour and they correspond to its natural frequencies and are characterised by its mode shapes. Modal analysis cannot be applied to structures with nonlinearities. Also experimentally oscillations and harmonics are quite difficult or even impossible to uncover form the measured time or frequency domain response. For structures made of composites it is known that they usually demonstrate complex vibratory behaviour which might exhibit double or very close resonances [7]. Our suggestion is to use SSA in place of modal analysis to uncover the oscillations/periodicity in the vibration response of nonlinearly vibrating structures. To our knowledge this is the first study to suggest this idea. The advantage of SSA is that it can uncover oscillations at any frequencies rather than at a certain set of frequencies.

The rest of the paper is organised as follows. The next paragraph introduces SSA as and the way it is applied to vibrating structures. 3 considers the case study of a vibrating composite beam and the problem considered. 4 is dedicated to the analysis of the obtained results.

2. The suggested SSA method and its application to vibrating structures

The vibration response of structure is measured by measuring its acceleration, velocity or displacement. These quantities are time series. In this investigation the measured accelerations are used. So one will be in possession of time series which on most occasions consists of equidistant time measurements, i.e.

$$x_i = x(t_i) = x(t_0 + (i-1)\Delta t)$$

where $\Delta t$ is the sampling interval.

The idea of SSA is simple: a principal components analysis is applied to variables analysed being lagged versions of a single time series variable. More specifically our $p$ variables are given by equation (2) where $i=1,2,...,p$. It is also assumed that we have $m$ realisations $x_j = (x_{j1}, x_{j2}, \ldots, x_{jp})$ of the $p$-dimensional variable $x$. So one is in possession of a data matrix $X$, which elements are $X_{ij}, i=1,2,...,p$ and $j=1,2,...,m$ and it has the vectors $x_j$ as its rows. The new variables, the principle components, $z_k$ are linear functions of $x_i$ found using the covariance matrix of $X$, $\Sigma$ according following the next relations

$$z_k = A^\prime z_k$$

$$\Sigma - \lambda^2 I = 0$$

where $I$ is the identity matrix, $\lambda$ are the eigen-values of $\Sigma$ and $A$ is the matrix of its eigen-vectors. Usually the first several PC’s, corresponding to the first largest eigen values, contain most of the variance of the original variables and thus the variables $x_{ij}, j=1,2,...,p$ can be represented by a much smaller number of new variables $z_k, k=1,2,...,q, q<<p$ which are independent.

If a time series has an oscillatory component, SSA will have an associated pair of eigenvectors with identical eigen-values. We are suggesting to use SSA in order to uncover periodicity in the measured vibration response. These periodic components will correspond to the natural frequencies of the structure under investigation. The corresponding eigen-vectors can be also regarded as characteristics of the vibrating system as the new principal components. Another important issue about PCA and SSA is that for the case of two or more categories of variables the distance between the new PC’s from the same category tends to decrease while in the same time the distance between PC’s from different categories increases.

3. Damage assessment using SSA
Damage assessment using vibration measurements is based on the fact that changes introduced in the structure result in change in its vibration response. It is generally argued that damage causes stiffness reduction in the structure. It is known that some damage scenarios change the natural frequencies of the structure under investigation. In this study we consider free decay response and thus the oscillatory patterns are associated with the natural frequencies of the structure. As a result of the SSA analysis one obtains the corresponding loading vectors for the PC’s (the eigen vectors) and also the eigen values. Thus if one does an analogy to modal analysis SSA gives the possibility to easily estimate the eigen vectors as the eigen values from equation (2) which characterise the time series in question. Our argument is that these characteristics will change as a result of damage since the structural properties are changed and the response signal will be changed. And we prove this in the following example.

4. The structure and the experiment performed
The structure in consideration is a composite beam made of carbon fibre with the following dimensions: length 500 mm, width 50 mm and thickness 6mm. They are made of ten layers. The beam is clamped at both ends. The experiment performed is the so-called ‘bump test’ where the beam is given a bump by hammer and is then left to vibrate on its own. The acceleration in the middle of the beam is measured. The time domain signals are used to perform the SSA analysis while the frequency domain signals are used to estimate the natural frequencies of the structure. The tests are repeated twenty times and thus twenty different signal realisations are recorded. Each signal is measured for one second and is sampled at 500 Hz so that we can uncover the first several natural frequencies of the beam.

For the SSA one is in possession of \( n' = 20 \) signals \( x_i \), each one made of \( p=1000 \) points. Thus our data matrix \( S(n',p) \) is made of 20 rows and 1000 columns.

We introduce delamination across the whole width of the beam between two layers. Different delamination sizes are introduced by changing the length of the delamination. We introduce three different delamination sizes, namely small delamination-10mm, medium delamination-20mm and large delamination -30mm. Delamination is also introduced in three different locations with respect to the beam length, namely 10 mm from the left end of the beam, in the middle of the beam and 10 mm from the right end of the beam. These are referred to as left, centre and right delamination, respectively. We also introduce three different positions of delamination across the beam thickness, between the 1st and 2nd layers, between the 5th and 6th layers and between the 9th and the 10th layers, which are referred to as upper, middle and lower delaminations respectively.

5. Delamination detection and estimation-Analysis of results.

5.1. Delamination detection using the natural frequencies.
We first look at the frequency domain vibration response signals for the intact beam and try to estimate its natural frequencies of vibration. One such signal is given on Figure 1. We were able to get the first three natural frequencies from all the recorded signals. The fourth and the fifth natural frequencies were obtained from 15 tests and in five tests we were not able to obtain them. Secondly it should be noted that the obtained estimates of the natural frequencies possess quite some variance which is between 5% and nearly 8%. Furthermore the changes of the natural frequencies due to delamination turned out to be in the range of the errors of the estimates. These excluded the natural frequencies as possible features for delamination detection.

5.2. Delamination detection using singular spectrum analysis
Next we consider the results from the SSA. Let us first look for oscillation patterns in the free decay behaviour. Table 1 gives the first twelve PC’s with the corresponding individual and cumulative variances and the eigen-values/frequencies for the undamaged beam. It can be seen that there are five pairs of eigen-vectors with equal or very close eigen-values in the first twelve PC’s. The first two PC’s have equal variances of 18.2% and altogether they are responsible for 36.4% of the total...
variance. A similar pattern can be observed for the third and the fourth PC’s and thus can be assumed to define an oscillation pattern in the beam behaviour. The sixth and the seventh PC’s also define oscillation and the same is true for the 9-th and the 10-th and the 11-th and the 12-th PC’s which suggest two more oscillation patterns in the beam behaviour. The fifth and the eighth PC’s do not seem to define oscillation patterns.

| PC No | Individual variance | Cumulative variance | Eigen value/frequency | Oscillation pattern |
|-------|---------------------|---------------------|-----------------------|---------------------|
| 1     | 18.2                | 18.2                | 39.5829(6.3 Hz)       | +                   |
| 2     | 18.2                | 36.4                | 39.5829(6.3 Hz)       | +                   |
| 3     | 12                  | 48.4                | 194.773(31 Hz)        | +                   |
| 4     | 12.2                | 60.6                | 197.9145(31.5 Hz)     | +                   |
| 5     | 10                  | 70.6                | -                     |                     |
| 6     | 4.1                 | 74.7                | 571.753(91 Hz)        | +                   |
| 7     | 3.9                 | 78.6                | 571.753(91 Hz)        | +                   |
| 8     | 3                   | 81.6                | -                     |                     |
| 9     | 2.6                 | 84.2                | 753.96(120 Hz)        | +                   |
| 10    | 2.5                 | 86.7                | 757.1015(120.5 Hz)    | +                   |
| 11    | 2                   | 88.7                | 980.148(156 Hz)       | +                   |
| 12    | 2                   | 90.7                | 977.0065(155.5 Hz)    | +                   |

Table 1. Variances in per cent and eigen-values for the first twelve PC’s

We now look at the corresponding eigen-values which define the frequencies of oscillation for the PC’s with close/equal variances (see Table 1). It should be noted that they are quite close to the natural frequencies of the beam. So as was expected the oscillation patterns found by SSA correspond to the natural frequencies of vibration of the beam. Thus obviously SSA was able to find these natural frequencies while in this particular case it was difficult to obtain them experimentally. SSA got these oscillation patterns quite easily and readily.

Table 2 gives the percentage changes for the first PC with delamination for the different delamination locations and sizes. It seem to contain information for the presence of delamination and its size. But it does not seem to change a lot as a result of the position of delamination. The second PC has very similar behaviour: again it changes with delamination but is not sensitive to its location. This behaviour is to be expected and it can be explained by the fact that the first natural frequencies are global structural characteristics. The percentage changes for the third PC are given in Table 3. This PC seems to contrast between central and side (left/right) delamination. It also changes as a result of the size of the delamination but these changes are smaller than the ones experienced by the first two PC’s. The fourth PC has similar behaviour as the behaviour of the third PC- it also contrasts between central and side delamination. The sixth and the seventh PC’s seem to contrast between the two positions of delamination according to the beam thickness—namely between upper/lower and middle delamination. Table 4 gives the percentage changes of the sixth PC with delamination. It can be perceived that the changes for delamination close to the surface of the beam—upper and/or lower delamination—are much smaller as compared to the changes that this PC experiences for middle delamination. The seventh PC has similar properties.

6. Some comments and conclusions
This work considers the use of singular spectrum analysis, which is an alternative of principal components analysis, for the purposes of description and analysis of the vibratory behaviour of a freely vibrating structure. In this paper the application of SSA is demonstrated for the purposes of
delamination assessment in a composite beam. It is shown that the PC’s contain a lot of information regarding the presence and the location of delamination in the beam. So it can be argued that the PC’s obtained can be used with much more success than the natural frequencies of the structure for the purposes of delamination assessment.

| Left/right(side) | small | medium | large |
|------------------|-------|--------|-------|
| Upper/lower | 6.0 | 8.6 | 12.0 |
| Middle | 6.6 | 8.9 | 12.0 |

| Centre | small | medium | large |
|--------|-------|--------|-------|
| Upper/lower | 6.1 | 10.0 | 14.0 |
| Middle | 6.7 | 10.8 | 14.9 |

Table 2. PC1 percentage change with delamination.

| Left/right(side) | small | medium | large |
|------------------|-------|--------|-------|
| Upper/lower | 4.1 | 6.0 | 10.0 |
| Middle | 4.7 | 6.8 | 11.3 |

| Centre | small | medium | large |
|--------|-------|--------|-------|
| Upper/lower | 7.2 | 9.1 | 13.0 |
| Middle | 7.8 | 9.8 | 14.1 |

Table 3. PC3 percentage change with delamination.

| Left/right(side) | small | medium | large |
|------------------|-------|--------|-------|
| Upper/lower | 5.1 | 6.0 | 9.0 |
| Middle | 8.0 | 10.6 | 15.6 |

| Centre | small | medium | large |
|--------|-------|--------|-------|
| Upper/lower | 5.6 | 6.8 | 11.0 |
| Middle | 8.9 | 11.0 | 16.1 |

Table 4. PC6 percentage change with delamination.

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