Calculation of View Factors in Electric Arc Furnace Process Modeling

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The determination of view factors (VF) is a central component of the heat transfer model in comprehensive electric arc furnace process models. Due to the complex nature of the process, simplifying assumptions are necessary to allow the efficient estimation of VF. These simplifications mainly deal with the geometrical representation of the furnace and the methods to calculate the resulting VF. Such assumptions have been applied in several studies; however, while the results of the complete process models have been validated extensively, the calculated VF have not been examined in detail. Herein, two methods of calculating VF based on a geometry similar to that used in the process models proposed by MacRosty et al., Logar et al., and Meier et al. are presented and validated by comparing the results obtained using a commercial computational fluid dynamics software. The number of assumptions and simplifications is reduced when compared with previously published results and both methods are proven to be accurate while remaining computationally efficient enough for the application in process models for online applications and process optimization, where fast execution is necessary.

1. Introduction

The electric arc furnace (EAF) process is the second most productive steel making and the most important scrap recycling process route worldwide.[1,2] Due to the high energy and resource consumption as well as the significant emissions of the process, there has been a constant effort to optimize the operation of furnaces to make them more efficient and productive. The conditions within the furnace, such as high temperatures and strong electromagnetic fields, make direct measurements of important process parameters difficult, if not impossible in many cases. Therefore, process models have been developed to give a better understanding of the phenomena inside the furnace that cannot be characterized through measurements. Such models exist both for specific parts of the furnace or stages of the process, such as the operation of burners, slag foaming, or postcombustion, and for the complete process. The heat transfer within the furnace is an important part of these models and has a significant influence on the energy losses to cooling water and off-gas. Because of the high temperatures involved, thermal radiation is a highly relevant heat transfer mechanism for the EAF process. To accurately model the radiative heat transfer, the geometrical configuration of the surfaces within the furnace, their temperature, and the influence of the gas phase including its dust content have to be considered. While tools such as computational fluid dynamics (CFD) allow the accurate calculation of the heat transfer in complex geometries including gas and dust interference, such models are not computationally efficient enough to be used in online applications or optimization studies for the EAF process.[3] For these purposes, simplified but sufficiently accurate mathematical models are needed.

Such process models that include a detailed treatment of the radiative heat exchange have been published by MacRosty et al.[4] Logar et al.[5] and, based on that, Meier et al.,[6] Saboohi et al.,[7] Fathi et al.,[8] and Opitz et al.[9] All of these models are based on similar assumptions regarding the geometrical conditions during melting, using a cone-frustum-shaped void in the scrap moving down and expanding outward as the electrode bores down toward the melt and the charge melts. Three phase furnaces are approximated with a single central electrode and electric arc. MacRosty et al. further simplified the calculation of view factors (VF) by omitting the electrode and using an empirical function to estimate the VF from the electric arc to the scrap. Logar et al. used analytical equations for the calculations of their VF and used what they termed “auxiliary surfaces” to aid with the characterization of some of the more complex geometrical relations. Meier et al. further improved the calculation of the radiative heat transfer by including the gas phase in the calculations and adjusting some of the VF. Most of the assumptions and equations used by Logar et al. are however retained in the model proposed by Meier et al. While both...
Fathi et al. and Sabooahi et al. give no detailed description of the equations used for the determination of VF, they reference Logar et al. Opitz et al. used a discretized cone-frustum geometry for their model and introduced several additional simplifications such as omitting the electrode or using a cylinder with a calculated mean diameter instead of an actual cone-frustum for their calculation of VF. All these approaches have been shown to give satisfactory results when used in process models, allowing reasonable estimations of the heat transfer within the furnace. However, no direct validation of the calculated VF has been published and the errors that these assumptions and simplifications introduce have never been estimated in the literature.

In this article, two different approaches to the calculation of the VF are presented based on the hollowed cylinder geometry introduced by Hernandez et al.\textsuperscript{[10]} The methods proposed here address the problem of estimating VF of complicated geometries where blockages in the line of sight take place without the need of further simplifications such as the ones used in previously published studies. The good accuracy of the two approaches is demonstrated in a series of parametric studies where, for various configurations of the hollowed cylinder geometry, the VF are computed using the proposed approaches and compared with the results obtained using a commercial CFD software, Ansys Fluent. More importantly, the computation methods introduced in this article are fast; therefore, they can be included into process models for online control or offline optimization.

2. Geometry of the EAF

Modern EAFs have a complex inside geometry usually including an oriel for eccentric bottom tapping and installations such as burners, lances, and injectors. To provide a single generalized geometrical configuration to represent different furnaces and allows the efficient calculation of the relevant radiative heat fluxes, a simplified geometry is used for process models. The most relevant assumption in this and earlier works is that the EAF can be represented by an axially symmetrical furnace with a single central electrode and arc. This simplification allows for a significantly easier calculation of VF while retaining most relevant features of the furnace. Furthermore, the assumption that all surfaces can be treated as gray or even black bodies is made to further simplify the calculation of the radiative heat transfer.

2.1. Cone-Frustum Geometry

The existing process models mentioned in Section 1 utilize a cone-frustum-shaped hole inside the scrap heap to represent the volume of metal that has already been molten during the meltdown stage of a batch. While the exact representation may differ in some detail among the various works, the general concept and the definition of the different surfaces remain similar to what is shown in Figure 1.

During meltdown, the cone-frustum-shaped void moves downward into the scrap and expands outward, progressively exposing both the melt surface and the water-cooled wall. This allows the determination of the sizes and relative position of all surfaces based on inputs such as bath height, remaining scrap volume, and the size of the electric arc for any point of time during the simulation.

2.2. Hollowed Cylinder Geometry

While the cone-frustum approach has been applied successfully to several EAF process models, Hernandez et al. have argued that a hole with vertical walls may represent better the shape of the metallic scrap during the bore-down and meltdown phases of the EAF process.\textsuperscript{[10]} Furthermore, while the cone-frustum shape used by Logar et al. and others used a fixed angle of the cone during the simulation, the height of the remaining scrap and the diameter of the hole can be adjusted more freely when using this adjusted geometry. A comparison of the previously used cone-frustum and the proposed vertical bore geometry is shown in Figure 2.

In principle, this geometry is comparable with a cone-frustum with a $90^\circ$ angle, however with an independently adjustable height and diameter. Both the cone-frustum and this adjusted geometry allow the different surfaces inside the furnace to be separated into simple shapes such as rings, disks, cylinder surfaces, and so on, mostly oriented parallel or at a right angle to each other, facilitating the estimation of VF. Overall, this adjusted geometry was found to better match the progression of meltdown observed during experiments.\textsuperscript{[10]} allows more freedom when adjusting the relationship between the radius and height of the cylinder, and is easier to describe mathematically when determining the VF. The cone-frustum geometry was found to overestimate the amount of scrap melting within the early stages of meltdown and combined with complete process models the adjusted geometry allows a better fit to measured data such as the electrode position and the temperatures of cooling water for the roof and wall panels. Furthermore, even if a cone-frustum geometry with a steep angle would give a somewhat more accurate description of the actual geometry, the calculation would become significantly more complex and more assumptions or simplifications would be necessary, likely negating any benefit of the more accurate geometrical description.
3. Calculation of VF

The VF from surface $A_1$ to $A_2$, denoted as $VF_{A_1\rightarrow A_2}$, is defined as the fraction of the total diffuse energy emitted by surface $A_1$ that is directly emitted toward and intercepted by $A_2$. Using the definitions given in Figure 3a and assuming gray or black bodies the VF between two surfaces can be obtained through the relationship given in Equation (1).

$$VF_{dA_1\rightarrow dA_2} = \int_{\phi} \int_{\psi} \frac{\sin(\psi) \cos(\phi)}{\pi} d\psi d\phi$$  \hspace{1cm} (2)

Depending on the shape of the surfaces involved and their relative position to each other, the determination of the function describing the angles and distances needed for the calculation of the view factors can become quite complex. For numerous common configurations of surfaces or bodies, analytical expressions have been derived and are publicly available. For other configurations where analytical expressions are not available, numerical methods have to be applied, making the calculation of these VF potentially more computationally demanding and time-consuming.

3.1. Monte Carlo Method for the Calculation of VF

The Monte Carlo (MC) statistical approach is a powerful tool that is being widely used by researchers to calculate VF of radiative heat exchanges in complex geometries. This technique works in the following manner: an emitting surface $A_1$ in an enclosure with $m$ surfaces is set to emit an arbitrary number of energy packages (bundles) $N$. Each bundle is emitted from a random position in the surface $A_1$ in a random 3D direction. The task of the algorithm is to identify which of the other $m-1$ surfaces intercept the bundle. In a spherical coordinate system, the components in the azimuthal ($\phi$) and the zenith ($\theta$) directions follow a predetermined probability density function of emission. After all bundles have been emitted, the VF between the emitter surface $A_1$ and a receiver surface $A_2$ (see Figure 4) is computed as the total number of intercepts in $A_2$ ($N_{A_2}$) divided by the total number of emitted bundles $N$ as given in Equation (3).

$$VF_{A_1\rightarrow A_2} = \frac{N_{A_2}}{N}$$  \hspace{1cm} (3)

One of the disadvantages of the MC approach is that due to its statistical nature, the energy balances will not sum up properly. Nonetheless, for a large number of emitted bundles (larger than 100 000) authors have suggested that the errors in the computation of the VF can be contained to less than 3%.

3.1.1. MC Method for Exchanges with Blockages in the Line of Sight

One of the major advantages of the MC approach to calculating VF is that with minor modifications, the core algorithm described in Section 3.1 can also compute VF with blockages in their lines of sight. The philosophy of the modified MC algorithm is simple and can be better explained with an example. For reasons of space, we only present the procedure for calculating the VF from the melt to the other surfaces in the EAF enclosure. In an earlier work, the MC algorithm utilized for estimating the VF from the arc to the other surfaces was presented. The MC algorithm starts by emitting a bundle from a random location in the melt in a random direction ($\phi, \psi$) (see Figure 3). Exchanges between the melt and other surfaces are prone to
blockages in their lines of sight, and, according to their relative position to each other, blockages can occur only in one possible order. By analyzing the geometry of the exchanges, one can determine that, on the one hand, bundles arriving at the arc cannot suffer any blockages, whereas, on the other hand, the probability that a bundle arrives at the wall of the furnace is the smallest. This is because any emission toward the wall can be intersected by all other surfaces (the arc, the electrode, and the scrap pile). The order of possible blockages for any emitted bundle can be established as follows: 1) If a bundle is emitted in the direction of the arc, no other surface can impede this exchange. Therefore, this is the first surface at which the interception of the bundle must be evaluated. 2) If a bundle is emitted in the direction of the horizontal electrode, only the arc can block the exchange. Therefore, the interception at the horizontal electrode must be computed after having evaluated a possible interception by the arc. 3) When a bundle is emitted in the direction of the vertical electrode, no other surface can interfere in the exchange. However, if a bundle is emitted from a radial position larger than the radius of the electrode in the melt, an interception on the vertical surface occurs only for azimuthal angles for which a landing in the arc or the horizontal surface of the electrode does not take place. On the contrary, if the location of the emission is under the electrode, then bundle can never reach the vertical surface of the electrode. These considerations can be embedded in a single logic evaluation, which evaluates the interception of the bundle by the vertical surface of the electrode only if no interception in the arc and the horizontal electrode is possible. 4) The interception of a bundle on the vertical surface of the melt scrap can only be hindered by the arc, the horizontal, and the vertical surfaces of the electrode. Therefore, an interception in these surfaces must be evaluated beforehand. 5) A bundle emitted in the direction of the wall or the roof of the furnace can be blocked by the previous four surfaces. On the contrary, while the roof of the furnace will always exchange heat with the pool of liquid after the bore-down period, the wall does not. From this condition, one can conclude that it is logical to evaluate first the interception at the roof.

After having established the order, as earlier, the algorithm moves from surface to surface, evaluating the interception of each emitted bundle. If an interception occurs at the evaluated surface, the bundle is counted and the process is repeated. If, on the contrary, the bundle does not land at the evaluated surface, the algorithm moves to the subsequent one until all the surfaces have been evaluated. This hierarchical evaluation of the receiving surface is performed for all the emitted bundles. The VF from the melt to the horizontal electrode, the solid metal, and the roof can be calculated using Equation (3). In the algorithm, the VF from the melt to the wall is estimated using the summation rule at the wall surface, and by applying the inverse rule to the VF from the arc to the melt, which was calculated beforehand, using the MC algorithm at the arc surface. This avoids the discrepancies that occur by computing the same VF using the MC method at two different surfaces. The logic diagram of the program for the calculations of the VF from the melt is shown in Figure 4.

The interception of a bundle by a surface can be determined using the surface interception technique. An illustrative example of this methodology is shown in Figure 5, where the emission of a bundle that lands on the furnace wall is analyzed.

In Figure 5, the bundle is emitted with a zenith angle (\(\phi\)) that is larger than the angle (\(\theta_1\)), which determines a landing in the roof of the furnace. A landing in either the solid metal, the electrode horizontal, or the arc is also not feasible because the angle of emission 90° – \(\phi\) is larger than the surface intersection angle for these three surfaces (\(\theta_1, \theta_2, \theta_3\)) (see Figure 5a). Furthermore, the polar angle of emission (\(\psi\)) is larger than the blocking angle created by the electrode (\(\delta\)). This ensures that the bundle is not blocked by the vertical surface of the electrode (see Figure 5b).

At all times, the intersection angles in Figure 5 can be determined using geometrical information of the furnace, the electrode, the arc, and the hollowed cylinder (radiiuses and heights).

3.2. Approximation using Analytical Expressions

As an alternative to the numerical MC method, a different approach relying on analytical expressions and a few simplifications was derived for the calculation of VF where an analytical description of the exact configuration is not available.

3.2.1. Use of Rotational Symmetry

An alternative method for the calculation of VF involving surfaces of bodies located along the axis of the rotational
symmetry can be derived using projections of the receiving surfaces on the hemisphere of the radiative field (see Figure 5) and some simplifying assumptions. The method is illustrated using the calculation of VF from the vertical surface of the electrode and the geometry shown in Figure 6a as an example.

The surfaces of the roof \(A_1\), wall \(A_2\), scrap \(A_3\), and melt \(A_4\) irradiated by a differential surface element \(dA_0\) on the electrode can be projected onto a hemisphere above the surface element. The projected surfaces \(a_1\) through \(a_4\) are defined by the angles \(\alpha_1\) through \(\alpha_4\). These angles \(\alpha_1, \ldots, \alpha_4\), however, will vary throughout the circumference as the distance between the surfaces changes depending on the direction of emission. As shown in Figure 6b, the distance travelled by the emission in the radial direction \(S_{1a}\) is shorter than that in the direction perpendicular to the radius \(S_{1b}\).

As long as \(r_1\) remains significantly larger than \(r_0\), the relative difference between \(S_{1a}\) and \(S_{1b}\), however, remains small and the difference in the angles depending on the direction of emission can be ignored. With this simplification, the projected surfaces and the resulting solid angles can be described easily. Using the
coordinate system shown in Figure 7 and by converting Equation (2) accordingly, Equation (4) is derived for the calculation of \(VF\) for the electrode and arc. The error resulting from this assumption, especially for cases where the difference between \(S_{1a}\) and \(S_{1b}\) may become relevant, is evaluated in Section 4.

\[
VF_{dA_{1}-dA_{2}} = \int_{\alpha}^{\alpha+d\alpha} \int_{\beta}^{\beta+d\beta} \frac{\sin^2(\alpha) \sin(\beta)}{\pi} d\alpha d\beta
\]  

(4)

Assuming \(\alpha\) to be independent of \(\beta\) and integrating leads to Equation (5) for the \(VF\) for a surface element \(dA_0\) on the electrode or arc to the surface \(A_i\) with the angles \(\alpha_i\) and \(\alpha_{i-1}\) characterizing the boundaries of \(A_i\), as shown in Figure 8.

\[
VF_{dA_0-A_i} = \alpha_i - \sin(\alpha_i) \cos(\alpha_i) - \alpha_{i-1} + \sin(\alpha_{i-1}) \cos(\alpha_{i-1})
\]  

(5)

The overall \(VF\) is then obtained by integrating numerically over the total area of the emitting surface. Due to the rotational symmetry, discretization and numerical integration are only necessary in the vertical direction of the electrode and arc as the \(VF\) from a horizontal differential strip on the surface are equal to those of a differential element at the same height.

Figure 5. Surface intersection angles of the MC algorithm: a) side view and b) top view of the furnace.

Figure 6. a) Definitions of angles and surfaces for elements on the axis of symmetry; b) varying distances throughout the circumference.

Figure 7. Coordinate system for arc and electrode.
This allows the use of an analytical expression describing the view factor from a differential element in a coaxial ring to another coaxial ring, separated by a coaxial cylinder.\textsuperscript{[19]} By numerically integrating the given expression for the VF from an infinitesimal element on the melt surface to this approximated roof surface, the VF from the melt to the roof can be estimated. The radii are determined for each surface element. Discretization and numerical integration are necessary for the radial direction only as the VF do not vary along the circumference for an axially symmetrical geometry.

Similarly, the VF from the melt to the wall surface is estimated using the expression for the VF from an infinitesimal element on the end plane of a cylinder to its lateral surface with an interfering coaxial cylinder.\textsuperscript{[20]} As with the radius for the roof, for the irradiated surface of the wall, the height of the surface is determined by blockage by the scrap and varies throughout the circumference. A constant averaged height is used to simplify the problem and make the analytical expression applicable, allowing the estimation of the VF using numerical integration along a single direction. The maximum and minimum heights can be calculated using Equation (8) and (9) and are limited to the value of $h_2$.

$$H_{\text{min}} = h_1 \frac{r_2 - R_i}{r_1 - R_i}$$  \hspace{1cm} (8)

$$H_{\text{max}} = h_1 \frac{\sqrt{r_2^2 - r_0^2} + \sqrt{r_1^2 - r_0^2}}{\sqrt{R_1^2 - r_0^2} + \sqrt{R_2^2 - r_0^2}} \frac{h_2}{R_i}$$ \hspace{1cm} (9)

These approximations and the constant angle used to determine the VF of the electrode and arc will introduce some errors into the calculation of the VF. These errors depend on the exact configuration and the full geometrical configuration of the EAF enclosure. In Section 4, the results for specific cases will be discussed and the errors will be quantified for different VF by comparison with the results from other methods of calculating VF.

3.3. Application of Methods to the Calculation of VF in the EAF Enclosure

All the VF needed for the computation of the radiative exchange in the EAF enclosure can be obtained either using the MC method, the approximations described earlier, or by using available analytical expressions. A short description of the analytical expressions utilized here is shown in Table 1.\textsuperscript{[12]}

In the following tables detailing the applied methods for the calculation of each considered VF, the use of the summation rule will be indicated by sum, with sum roof meaning that the VF in question can be determined by applying the summation rule to the VF already calculated for the roof. A zero indicates that there is no direct line of sight between the two surfaces involved under any circumstances and the VF can be set to zero. The reciprocity of VF allows all VF to be determined once half the matrix is known. Therefore, for any combination of two surfaces only one of the two VF involved is given in the matrix.
3.3.1. Overview of MC Approach

The coefficients of the VF matrix are estimated in three stages. First, the MC algorithm (Section 3.1) is run sequentially for the arc, the vertical electrode, and the melt surfaces. In these calculations, the summation rule is always enforced at the wall, and precomputed VF are included in subsequent calculations using the reciprocity rule. For example, the VF from the arc to the melt is estimated using the MC algorithm at the arc surface and then utilized in the computations of the VF from the melt and the vertical solid scrap surfaces using the reciprocity rule.

Second, the VF that can be calculated using analytical expressions are estimated. For the case of the VF that describes the exchanges of a surface with itself (vertical scrap and wall of the furnace) the mathematical relationship C-91 was used, assuming that the radius of the internal cylinder is that of the electric arc.

Finally, the remaining VF were estimating by applying the summation rules at different surfaces as needed. Both the use of precomputed VF within the individual MC at each surface and the enforcement of summation rules at the various surfaces avoid the incongruences that arise from running the MC algorithm to calculate the same VF at different surfaces, as well as the errors when closing the VF balances (summation rule).

The VF matrix computed using the MC algorithms is shown in Table 2.

3.3.2. Overview of Analytical Approach

Table 3 shows the methods used to determine the VF for the analytical approach and the approximations described in Section 3.2, with C-54* and C-56* indicating the method shown in Section 3.2.2 and RS the use of rotational symmetry as given in Section 3.2.1.

For the VF from the vertical scrap surface to itself and the melt surface, it is assumed that the electrode reaches all the way down to the melt. In reality, different cases occur where part or all of the height within the hole formed by the scrap is actually occupied by the arc and not the electrode. The added complexity of

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**Table 1.** Indices and description of VF calculations.

| Index | Description |
|-------|-------------|
| C-53  | Annulus to coaxial annulus of different radius, both annuli have inner radius of blocking coaxial cylinder |
| C-54  | Coaxial annular rings separated by coaxial cylinder |
| C-55  | Annular ring between two concentric cylinders to inside of outer cylinder, inner radius of ring is equal to radius of inner cylinder |
| C-56  | Ring on annulus between coaxial cylinders to inner surface of outer cylinder |
| C-91  | Interior of finite length right circular coaxial cylinder to itself |

**Table 2.** Calculation of VF using MC method.

| Surface       | Roof | Wall | Scrap horizontal | Scrap vertical | Melt | Arc | Electrode vertical | Electrode horizontal |
|---------------|------|------|------------------|----------------|------|-----|-------------------|---------------------|
| Roof          | 0    | C-55 | C-55             | Sum roof       | MC   | MC  | MC                | 0                   |
| Wall          | –    | C-91 | Sum scrap horizontal | Sum scrap vertical | MC   | MC  | MC                | C-55                |
| Scrap horizontal | –    | –   | 0                | 0              | 0    | MC  | MC                | C-53                |
| Scrap vertical | –    | –   | –                | C-91           | 0    | MC  | MC                | Sum electrode horizontal |
| Melt          | –    | –   | –                | –              | 0    | MC  | MC                | MC                  |
| Arc           | –    | –   | –                | –              | –    | 0   | 0                 | MC                  |
| Electrode vertical | –    | –   | –                | –              | –    | –   | 0                 | 0                   |
| Electrode horizontal | –    | –   | –                | –              | –    | –   | 0                 | 0                   |

**Table 3.** Calculation of VF using analytical expressions and approximations.

| Surface       | Roof | Wall | Scrap horizontal | Scrap vertical | Melt | Arc | Electrode vertical | Electrode horizontal |
|---------------|------|------|------------------|----------------|------|-----|-------------------|---------------------|
| Roof          | 0    | C-55 | C-55             | Sum roof       | C-54*| RS  | RS                | 0                   |
| Wall          | –    | C-91 | C-55             | Sum wall       | C-56*| RS  | RS                | C-55                |
| Scrap horizontal | –    | –   | 0                | 0              | 0    | RS  | RS                | C-53                |
| Scrap vertical | –    | –   | –                | C-91           | C-55 | RS  | RS                | C-55                |
| Melt          | –    | –   | –                | –              | 0    | RS  | RS                | C-53                |
| Arc           | –    | –   | –                | –              | –    | 0   | 0                 | RS                  |
| Electrode vertical | –    | –   | –                | –              | –    | –   | 0                 | 0                   |
| Electrode horizontal | –    | –   | –                | –              | –    | –   | 0                 | 0                   |
these different cases would, however, make the use of a simple analytic expression impossible. For most cases, the error resulting from this approximation is small and can be determined using the sum of all VF for the melt and vertical scrap surfaces.

### 4. Results and Discussion

The accuracy of the methods proposed in Section 3 is studied in two study cases where the internal radius and the height of the pile of scrap are varied independently. The base geometry for these computations is shown in Figure 9. Here, the used dimensions were chosen so that the geometry roughly represents that of an industrial EAF and relevant phenomena such as the blocking of radiation between different surfaces and the arc can be observed. As only the relative sizes and positions of the surfaces to each other are relevant for the calculation of VF, the unit of measurement is not relevant. For both study cases, it is assumed that the arc is in contact with the melt surface.

In the first study case, from now on referred to as case 1, the height of the scrap, shown in Figure 9 as 0.8, was varied from 0.001 to 2.299, corresponding to a furnace with the remaining scrap just covering the bath surface and a furnace completely filled with scrap. All other values were kept constant. In the second study case, referred to as case 2, the radius of the hole in the scrap pile was varied from 0.2001 to 1.7499. This describes the hole increasing in diameter from barely larger than the electrode to just covering inside of the wall at the radius of the furnace vessel. Again, all other values were kept constant.

#### 4.1. Ansys Fluent Case for Validation

The surface-to-surface (S2S) radiation model in Ansys Fluent was used to obtain high-fidelity, but computationally expensive, numerical solutions for the VF in the furnace. These high-fidelity results can then be used to compute the errors of the less accurate but faster algorithms presented in Section 3.

The S2S model computes VF by calculating the VF from each volume element in the discretized domain to every other element.[21]

This study focuses on how the VF change as the geometry inside the furnace changes during the scrap bore-down phase. The VF0 are therefore computed for a range of incrementally changed geometries as described earlier. This means that each incremental geometry change has to be remeshed for the S2S computation. Mesh independence studies were, however, only done for a sample of two geometries. For the case where the height of the scrap inside the furnace is varied (case 1), the geometry with a scrap height of 1.472 along with the constant hole radius of 1.400 was chosen. For the case where the diameter of the hole in the scrap is increased (case 2), the geometry with a hole radius of 1.068 and the constant scrap height of 0.8 was considered.

The mesh independence study for both VF computations is shown in Figure 10, based on the VF from the arc to the electrode, bore (hole in the scrap), and furnace roof. The absolute error is defined as given in Equation (10).

$$\text{Absolute error} = |V_{i} - V_{\text{finest}}|$$  \hspace{1cm} (10)

Here, $V_{i}$ is the view computed using the $i$th mesh density and $V_{\text{finest}}$ is the VF computed using the finest mesh. The meshes automatically created by Mesher without any additional refinement compute the VF with an error $<0.75 \times 10^{-1}$ based on automatically created meshes using roughly 59 000 elements for case 1 and 90 000 elements for case 2. Considering the automatically generated and finer meshes, the largest percentage error relative to $V_{\text{finest}}$ is 0.58% for the VF from the arc to the electrode tip for case 2. It is therefore concluded that for the purpose of computing VF only, the mesh automatically generated by Mesher is sufficiently refined.

The fact that the error in Figure 10 does not decrease monotonically with increasing mesh density could be due to Mesher not evenly increasing the number of volume elements on both surfaces as the mesh is refined. Computing the VF between a coarsely meshed and a finely meshed surface will result in an error due to the coarsely meshed surface. Because the S2S model computes the VF from every volume element to every other volume element, this error occurs for every element of the finely meshed surface, possibly resulting in a larger error than in the previous case where both surfaces were coarsely meshed.

In the next refinement step where the mesh is further refined on the surface that was previously still coarsely meshed, the error then decreases as expected.

An analytical solution exists for the VF from the arc to the electrode tip[22] (C-77: outer surface of cylinder to annular disk at end of cylinder). This analytical solution is used to verify this one VF of all the computed geometry cases. The maximum absolute error for the VF from arc to the electrode tip in the S2S calculations is given in Equation (11).

$$\text{Absolute error}_{\text{electrode tip}} = \max |V_{S2S} - V_{\text{analytical}}|$$

$$= 7.12 \times 10^{-4}$$  \hspace{1cm} (11)

This corresponds to a percentage error relative to the analytical VF of 0.68%.
4.2. VF Results

Because of the difficulty in remeshing and exporting VF for each case, a smaller number of geometries was used for the 2S2 calculations when compared with the other methods. For VF with steep changes, the linear interpolation between these points will therefore be visible in some cases. Direct comparison of the results is possible at the points evaluated with the S2S approach and the errors will be determined based on these points and not the interpolation between them.

4.2.1. VF\text{arc-roof} and VF\text{arc-wall}

Figure 11a shows the VF from the arc to the roof and wall for case 1. The largest error is observed for the analytical method at a scrap height of 0 and amounts to 0.025 for the VF\text{arc-wall} compared with the S2S and MC results. At a height of roughly 0.3, the results of the analytical and MC methods converge and remain within 0.01 of each other. The deviation of the S2S results, visible at scrap heights between 0.5 and 0.75 for example, is due to the smaller number of sample points and the linear interpolation between the points. At points that the S2S results were exported for, the deviation of the MC method is within 0.01 of each other. The deviation among the three results for a radius of up to 0.6 again occurs due to the smaller number of sample points utilized in the S2S method. Meanwhile, for a radius greater than 0.6, the results of the three methods are in good agreement.

4.2.2. VF\text{electrode-roof} and VF\text{electrode-wall}

Figure 12a,b shows the results of the computations for the VF from the electrode to the roof and wall for cases 1 and 2. For both cases, the MC and S2S results show good agreement throughout the whole domain for both the VF. The error of the analytical approximation method increases as the height of the scrap decreases for the VF from the electrode to the wall, for the VF from the electrode to the roof, it remains almost constant through the whole range of the analysis. This is expected as the radius of the electrode is larger than that of the arc which leads to a greater error as stated in Section 3.2.1.

Overall, the difference to the S2S and MC results remains within 0.025 and the behavior of the VF can be reproduced for both cases, albeit with an offset. Even the case of almost identical radii of the emitting and blocking or receiving surfaces, encountered here when the radius of the hole inside the scrap is almost equal to that of the electrode, does not lead to any significant increase in the model error.

4.2.3. VF\text{melt-roof} and VF\text{melt-wall}

Figure 13a,b shows the VF from the melt to the roof and wall, allowing the error resulting from the simplifications described in Section 3.2.2 to be quantified. Both the MC and the analytical method show deviations of up to 0.03 from the VF calculated using S2S. For the MC method, the error is smaller and comparable with that of the previously discussed VF for the VF\text{melt-roof} and increases only for VF\text{melt-wall}, whereas the analytical approximation yields similar errors for both VF. The agreement of both methods with the S2S results remains within acceptable
margins, with the MC showing a slightly better accuracy when compared with the analytical approximation.

The study cases and VF discussed earlier include at least one application for each of the methods and simplifications used for the estimation of VF described in Section 3. The remaining VF needed to fully specify the heat exchange in the EAF system that are not shown here can be determined using the same methods, VF algebra or simple analytical expressions.\textsuperscript{[12]} The varying accuracy of the presented methods for different VF can be attributed to several factors. For the analytical approximation method, the accuracy is high when exact analytical descriptions can be used and reduced somewhat where the necessary simplifications and assumptions are relevant. The MC method, being a numerical method, has a more uniform accuracy and therefore is more accurate than the analytical approximation in some cases and less accurate in others. Further errors are introduced through the use of the summation rule and reciprocity in both methods, where negligible errors in the calculation of one VF can lead to significant errors for another, especially when the sizes of the surfaces involved are different by orders of magnitude.

Figure 11. VF of the arc for a) case 1 and b) case 2.
4.3. Execution Speed

The calculation of one set of VF for the geometry discussed here takes roughly 1 ms per configuration using the analytical method, 200 ms with the MC method, and several seconds when applying S2S (not considering the time necessary to create a mesh for each new configuration). For the purpose of online applications of the EAF process model, such as operators’ guidance or control, the analytical approximations can be run for every iteration, taking the simulation time of a batch with a tap-to-tap time of about 60 min to less than 60 s. Considering that the total execution time for one iteration of the model, including all other model aspects such as thermochemistry, heat transfer, and phase changes, is about 3.5 ms, the impact of the VF estimation on the overall runtime is significant. From this perspective, the analytical approximation clearly outperforms the MC method. One alternative of implementing the MC algorithm in a complete process model is by computing the VF every $n$th step of the simulation timeline rather than at every iteration of the integrator. Following this strategy, simulation times in the same range of those obtained with the analytical approximation can be obtained. For the case of a discretization of 1 min, the
Errors induced in the computation of the energy fluxes from the arc can be contained to less than 3% in comparison with performing the computation of the VF every second of the simulation timeline.

5. Conclusion

Two different methods for the estimation of VF in the EAF enclosure were derived and their results were validated using detailed S2S calculations. Both methods provide accurate estimations of VF, with the MC method being more accurate but slower than the approach using analytical expressions and simplifying assumptions. The maximum errors in the computations of the models presented here are contained to 0.025 for the MC and 0.03 for the analytical expressions, in respect to the values computed using Ansys Fluent with the S2S method.

The fast execution time of the analytical method of calculation of VF allows its integration in complete EAF process models without the need of further simplifications. The computational load of the MC method, on the contrary, requires a discretized
approach of calculation of VF, where its computation is not performed at every step of the integrator. Both methods, however, can be integrated into EAF process models for online applications, providing estimations of VF without the additional assumptions and simplifications used in previous works.

When used within such process models and compared with previously proposed methods based on a cone-frustum geometry, the methods were found to significantly accelerate the simulation and allow better fitting to measured data for cooling water temperatures and the position of the electrode, which can be used as an indicator of the progression of the meltdown.

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Conflict of Interest

The authors declare no conflict of interest.

Keywords

electric arc furnace, process models, view factors

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