Measured Stiffness Performance Evaluation Index of Industrial Robot in Side Milling

Jiabin Sun\textsuperscript{a} and Weimin Zhang\textsuperscript{b}

School of Mechanical Engineer, Tongji University, Shanghai 201804, China

\textsuperscript{a}13tjsunjb@tongji.edu.cn, \textsuperscript{b}weimin_zhang@hotmail.com

\textbf{Abstract.} Industrial robot is a potential alternative of machining tool in some certain process due to its flexibility and large workspace. The traditional stiffness model of robot cannot explain the situation that stiffness exits difference in opposite directions. Aiming at optimizing the robot side milling process, a measured stiffness performance evaluation index is proposed in this paper. It is deduced and calculated through actual measured data, which overcomes the shortcoming of other indexes deduced from traditional stiffness model. Experimental case indicates that the new index proposed in useful in feed direction and tool-workpiece relative position optimization.

\textbf{Keywords:} Robot milling, stiffness, performance evaluation index.

1. Introduction

Industrial robot is applied in some certain machining process due to its flexibility and large workspace. But the low stiffness caused by open-loop articulated serial structure directly affects the machining performance. At present, in the field of robot machining, in order to simplify the influencing factors, the most commonly used model is still the traditional stiffness model proposed by Salisbury [1], and Abele’s compliance model [2] is widely adopted because inverse of Jacobian matrix calculation is removed. Although the influence of external forces on the stiffness matrix is considered in the Conservative Congruence Transformation stiffness model proposed by Chen and Kao [3], the supplementary stiffness matrix is normally ignored due to the complicated calculation.

The researches on stiffness/compliance performance index are increasing, and the corresponding indexes are taking more factors into consideration, improving the practicability in different cutting scenarios. In 2015, Guo et al. [4] proposed the compliance performance index of robot based on the determinant of compliance matrix. In 2017, Lin et al. [5,6] proposed the deformation evaluation index (DEI) on the basis of compliance performance index. Because DEI mainly considers axial force, comprehensive stiffness performance index (CSPI) and normal stiffness performance index (NSPI) were proposed by Chen et al. [7] in 2019, so that the index can consider the external force direction more comprehensively. In the same year, Xiong et al. [8] selected the sum of squares of deformation of several key points as the evaluation index of deformation, and proposed a new deformation index. This index considered the complete compliance matrix, adopted the actual displacement of key points, followed the worst-case design method, and was independent of the coordinate system. Above indexes can be used for robot machining performance optimization, but in the certain case where only the direction of external force changes, they are not adequate to explain the stiffness difference.
2. Stiffness performance evaluation index

2.1. Stiffness performance index analysis

According to [4], all the compliance performance indexes generates an ellipsoid whose semi-axes are equal to the eigenvalues of translation compliance matrix \( C_{df} \) as shown in Figure 1.

Thus, the compliance performance of robot at a certain configuration can be represented by the volume of the ellipsoid, then, the overall stiffness performance index can be noted as

\[
    k_s = \frac{1}{\sqrt{\det(C_{df})}}
\]

Fig. 1 Compliance index ellipsoid

For different configurations of a robot, the configuration with greater \( k_s \) should possess better overall stiffness performance. Therefore, \( k_s \) can be used for configuration optimization in robot machining. According to Figure 1 and Equation (1), the stiffness performance should be same in one direction. However, in the practical case, stiffness performance changes when only force direction reverses. An auxiliary device is design to exert external force as seen in Figure 2. The device is consisted of a three-dimensional dynamometer, a spherical joint with link and a tension meter. The dynamometer is fixed on the end flange and the spherical joint is fixed in the center of the dynamometer. Considering that the robot and the dynamometer is a unit, the external force is approximately exerted to the center point by the tension meter which is bonded with the link.

Fig. 2 Force exerting device

Two robot configurations are selected as test sample, noted as P1 and P2. Firstly, incremental tensile forces are exerted through tension meter as direction of external force keeps invariant, corresponding displacement of robot end are measured by the method in [9].
Good linear relationship between force magnitude and displacement can be seen in Figure 3, which follows the conception of stiffness ellipsoid. Secondly, forces with invariant magnitude and opposite directions are exerted, that is to pull and push the tension meter with 150N force respectively while keeping the link still. Significant displacement difference appears when the direction of force turns opposite, this situation is illustrated in Figure 4. According to stiffness ellipsoid, the stiffness should be the same in a straight line, but practical case proves that stiffness is different in positive and negative direction. This is because that the stiffness ellipsoid is deduced based on the traditional stiffness model of robot,

\[ \mathbf{K} = \mathbf{J} \mathbf{K}_e \mathbf{J}^T \].

A section of the matrix concerned with external load is normally ignored, that is \( \mathbf{K}_e \) in the

\[ \mathbf{K} = \mathbf{J} \mathbf{K}_e (\mathbf{K}_e - \mathbf{K}) \mathbf{J}^T \].

Thus, stiffness ellipsoid is not always appropriate, especially in the situation that external forces direction matters.

### 2.2. Measured stiffness performance evaluation index

Robot milling is a complex process as common industrial robot may have six degrees of freedom (DOF). Straight line side milling is studied in this paper for deduce influencing factors, meaning that the end-effector moves similar to the 3-DOF milling center actuated by the robot. The feed direction and the relative location between tool and workpiece decide the cut-in mode (climb-cut or reverse-cut), which results in cutting forces with opposite directions. As the stiffness ellipsoid cannot illuminate the stiffness difference between opposite directions, a practical index is introduced in this paper for this certain robot milling case.

The external force transferred from the end effector to the end flange can be decomposed to three orthogonal directions in the end flange coordinate which is same to the base coordinate (Figure 5). Considering the external force as a vector, the direction changes when the vector is in different octant. Specially, the external force vectors in the 1st octant and the 8th octant have opposite directions, and the directions are all positive and all negative respectively. Thus, the external force in the 1st octant is noted as

\[ \mathbf{f}_1 = \begin{bmatrix} f_{x1} & f_{y1} & f_{z1} \end{bmatrix}^T \]

and corresponding displacement is

\[ \mathbf{d}_1 = \begin{bmatrix} d_{x1} & d_{y1} & d_{z1} \end{bmatrix}^T \].

For the 8th octant, the footer is changed to 8.
The relationship between external force and displacement can be express as

\[
\begin{bmatrix}
  f_x \\
  f_y \\
  f_z \\
\end{bmatrix} =
\begin{bmatrix}
  k_{11} & k_{12} & k_{13} \\
  k_{21} & k_{22} & k_{23} \\
  k_{31} & k_{32} & k_{33} \\
\end{bmatrix}
\begin{bmatrix}
  dx \\
  dy \\
  dz \\
\end{bmatrix} =
\begin{bmatrix}
  K_{df}^T \\
  K_{df}^T \\
  K_{df}^T \\
\end{bmatrix}
\begin{bmatrix}
  d \\
\end{bmatrix}
\]

(2)

The matrix \( K_{df} \) can be calculated with groups of measured force-displacement data, and the calculated matrix is noted as measured translation stiffness matrix \( K_{mdf} \). The measure method is as follows. The force-exerting link remains in the 1st octant of end flange coordinate, and the spherical joint is used to adjust the direction. Once the link moves to a new configuration, it is pulled and pushed through tension meter several times randomly (Figure 6), meanwhile, the external force and displacement are measured and transferred to end flange coordinate. \( K_{mdf} \) can be divided into

\[
\begin{bmatrix}
  f_{i1} \\
  f_{i2} \\
  f_{i3} \\
\end{bmatrix} =
\begin{bmatrix}
  k_{i1} \\
  k_{i2} \\
  k_{i3} \\
\end{bmatrix}
\begin{bmatrix}
  d_x \\
  d_y \\
  d_z \\
\end{bmatrix} =
\begin{bmatrix}
  K_{df}^T \\
  K_{df}^T \\
  K_{df}^T \\
\end{bmatrix}
\begin{bmatrix}
  d \\
\end{bmatrix}
\]

(3)

in which \( i=x, y, z \), and corresponding \( j=1, 2, 3 \) respectively.

All measured \( n \) groups of data (here \( n=16 \) for precision) and Equation 3 are substituted into Equation 2, then

\[
\begin{bmatrix}
  f_{i1} \\
  f_{i2} \\
  f_{i3} \\
\end{bmatrix} =
\begin{bmatrix}
  K_{mdf}^T \\
  K_{mdf}^T \\
  K_{mdf}^T \\
\end{bmatrix}
\begin{bmatrix}
  d_i \\
\end{bmatrix}
\]

(4)

\[
\begin{bmatrix}
  f_{s, real} \\
\end{bmatrix} =
\begin{bmatrix}
  K_{df}^T \\
  K_{df}^T \\
  K_{df}^T \\
\end{bmatrix}
\begin{bmatrix}
  d_{s, real} \\
\end{bmatrix}
\]

(5)

Thus, \( K_{mdf} \) can be calculated by

\[
\min e(K_{mdf}^T) = \| f_{s, real} - K_{mdf}^T d_{s, real} \|
\]

(6)

For one row of \( K_{mdf} \), the square of force magnitude and displacement ratio based on the conception of rayleigh quotient is noted as
\[ R_{\text{mod}} = \frac{|f|}{|d|} = \frac{d'(K_{\text{mod}}'K_{\text{mod}})d}{d'd'} \]  \hspace{1cm} (7)

Note \( Q_{\text{mod}}(d) = \sqrt{R_{\text{mod}}(d)} \), then

\[ |f| = Q_{\text{mod}}(d) \times |d| \]  \hspace{1cm} (8)

\( Q_{\text{mod}}(d) \) represents the requisite force magnitude to produce unit displacement, and it is a function of \( d \). \( K_{\text{mod}}'K_{\text{mod}} \) in \( Q_{\text{mod}}(d) \) is used to express the stiffness performance in \( i \) direction, and measured stiffness performance evaluation index is noted as

\[ k_i = \sqrt{K_{\text{mod}}'K_{\text{mod}}} \]  \hspace{1cm} (9)

A larger \( k_i \) means better stiffness performance in \( i \) direction.

3. Index application

Measured stiffness performance evaluation indexes of 1st and 8th octant are \( k_{i1} = [k_{i1} \ k_{j1} \ k_{z1}] \) and \( k_{i8} = [k_{i8} \ k_{j8} \ k_{z8}] \) respectively.

\( k_{i1} > k_{i8} \) means that the stiffness performance in the negative direction of coordinate axis is better than that of positive direction; on the contrary, \( k_{i1} < k_{i8} \). In a robot milling case (Figure 7), milling path is a straight and short line, where the robot configuration variation can be ignored. The cutting parameters and robot configuration are kept settled, only the position of the workpiece is changed from P1 to P2, so that the cutting force directions are opposite.

![Fig. 7 Robot milling case](image)

Here, along the direction of \( F_{x1} \), \( k_{i1} = 8.5 \times 10^6 \text{N/m} \) at P1; and along the direction of \( F_{x2} \), \( k_{i2} = 6.3 \times 10^6 \text{N/m} \) at P2. As the cutting parameters are the same, the cutting forces should be comparable. In fact, there exists a 25N difference as shown in Figure 8. Due to the worse stiffness performance in \( F_{x2} \) direction, the robot deforms greatly when it is subjected to cutting force. In order to reach the expected position, the force exerted by the robot on the workpiece is larger than that at P1. Therefore, the reaction force received by the robot is also larger when the workpiece is located in P2. In conclusion, P1 position is the better machining position.
4. Summary
Due to the problem that traditional stiffness model and corresponding index of robot cannot explain the stiffness difference between two opposite directions. Measured stiffness performance evaluation index is proposed in this paper. It is a practical index which is deduced and calculated based on measured force-displacement data, and can be used for tool-workpiece relative location and feed direction optimization. A robot milling case is applied to prove the practicability of this index.

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