On Holographic Realization of Logarithmic GCA

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Abstract

We study 2-dimensional Logarithmic Galilean Conformal Algebra (LGCA) by making use of a contraction of Topologically Massive Gravity at critical point. We observe that using a naive contraction at the critical point fails to give a well defined theory, though contracting the theory while we are approaching the critical point leads to a well behaved expression for two point functions of the energy-momentum tensors of LGCA.
1 Introduction

Beside the Schrodinger algebra [1–3] which is the most celebrated non-relativistic conformal algebra, another non-relativistic algebra named GCA [4] has recently received a lot of attention [5,6]. GCA is the only known non-relativistic conformal symmetry which scales space and time isotropically

$$x \to \lambda x, \quad t \to \lambda t.$$  \hspace{1cm} (1.1)

Usually non-relativistic conformal symmetries scale space and time anisotropically

$$x \to \lambda x, \quad t \to \lambda^\theta t.$$  \hspace{1cm} (1.2)

$\theta$ is called the anisotropy index. Looking for the most general non-relativistic conformal symmetries in $d \neq 2$ one ends up with the class of l-Galilei algebras [7]. Each element of the class is identified with a half-integer number $l$ which is the inverse of $\theta$. Being isotropic in scaling space and time is not GCA’s only special feature. GCA is as well unique by being obtained from contracting relativistic conformal symmetry in spatial dimensions.

$$x \to \frac{x}{c}, \quad t \to t, \quad c \to \infty.$$  \hspace{1cm} (1.3)

From a physical point of view it means that we investigate the behavior of system for low speeds or maybe low energies. GCA can be extended to an infinite dimensional algebra which is named full GCA. In 1 + 1- dimensions full GCA can be obtained from contraction of conformal symmetry which is two Virasoros [8]. This special feature of full GCA in 1 + 1-dimensions helps to know its quantum behavior from contracting $CFT_2$ [9,10].

Beside regular conformal field theories which are unitary, there is another class of conformal models which are not unitary and named logarithmic conformal field theories (LCFT). LCFT’s arise when the action of scaling operator on scaling fields are not diagonal but rather Jordan [11]

$$L_0\phi = h\phi, \quad L_0\psi = h\psi + \kappa\phi.$$  \hspace{1cm} (1.4)

Although being non-unitary, LCFT’s have a large variety of applications in field theories and statistical physics [2]. As regular $CFT_2$, LCFT’s can be contracted and yield logarithmic GCA (LGCA) [10].

One of the suggested realizations of $CFT_2$ in the context of holography is Topologically Massive Gravity (TMG) [16,17] away from the critical point. Recently this correspondence has been looked from the contraction point of view to suggest duality between contracted

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1 For two spatial dimensions a larger class has been claimed for the sake of respecting infinite dimensional symmetries of $CFT_2$ in 2 spatial dimensions [8]

2 For example see [12,15] and references therein.
TMG and GCA [18]. It is shown that TMG at the critical point might be dual to LCFT [19, 20]. In this paper we search the possibility of correspondence between LGCA on the boundary and contracted TMG at the critical point in the bulk. In section 2 we review GCA and its contraction from CFT$_2$ and as well its logarithmic representation. In section 3 we review Topologically Massive Gravity in critical point. In section 4 we utilize contraction approach to observe holographic realization of LGCA.

2 GCA and Contraction

Up to our knowledge the oldest reference to GCA is [4]. It was investigated later and called alt in [21]. Its exotic central charge has been of interest [22–24]. Later it got of interest in the context of holography [5, 9]. Full GCA which is infinite extension of GCA is represented by the following operators [25]

\[
T^n = -(n+1)t^n x_i \partial_i - t^{n+1} \partial_t, \\
M^m_i = t^{n+1} \partial_i, \\
J^n_{ij} = -t^n (x_i \partial_j - x_j \partial_i). \tag{2.1}
\]

This infinite algebra commutes as

\[
[T^m, T^n] = (m-n)T^{m+n}, \quad [T^m, J^n_a] = -nJ^{m+n}_a, \\
[J^m_a, J^n_b] = f_{abc}J^{m+n}_c, \quad [T^m, M^n_i] = (m-n)M^{m+n}_i, \\
[M^m_i, M^n_j] = 0, \quad [M^m_i, J^n_{jk}] = (M^{m+n}_j \delta_{ik} - M^{m+n}_k \delta_{ij}). \tag{2.2}
\]

It can be observed that full GCA in 1 + 1-dimensions can be obtained directly from contracting conformal symmetry in 2-dimensions [8]. In fact in two dimensions

\[
z = x + t, \quad \bar{z} = x - t. \tag{2.3}
\]

conformal symmetry is two Virasoro algebra

\[
L_n = -z^{n+1} \partial_z, \quad \bar{L}_n = -\bar{z}^{n+1} \partial_{\bar{z}}. \tag{2.4}
\]

which commute as

\[
[L_m, L_n] = (m-n)L_{m+n} + \frac{c_L}{12} m(m^2-1)\delta_{m+n,0}, \\
[\bar{L}_m, \bar{L}_n] = (m-n)\bar{L}_{m+n} + \frac{c_R}{12} m(m^2-1)\delta_{m+n,0}. \tag{2.5}
\]

In the contraction limit we observe
\[ L_n = -(t + \frac{x}{c})^{n+1}(\partial_t - c\partial_x) = -t^{n+1}(-c\partial_x + \partial_t + (n + 1)\frac{x}{t}\partial_t + O(\frac{1}{c})), \]
\[ \mathcal{T}_n = -(t + \frac{x}{c})^{n+1}(-\partial_t - c\partial_x) = -t^{n+1}(c\partial_x + \partial_t + (n + 1)\frac{x}{t}\partial_t + O(\frac{1}{c})). \] (2.6)

Redefining operators

\[ T_n = L_n + \mathcal{T}_n, \quad M_n = \frac{L_n - \mathcal{T}_n}{c}. \] (2.7)

we end up with full GCA. Since we know the field theory of conformal symmetry in 2 dimensions via quantizing symmetry in (2.5) we can follow contraction and obtain quantized GCA in 1 + 1-dimensions [9]. Considering the states of \( \text{CFT}_2 \) which are characterized by their holomorphic and antiholomorphic scaling weights \( |h, \bar{h}\rangle \) in which

\[ L_0 |h, \bar{h}\rangle = h |h, \bar{h}\rangle, \quad \mathcal{T}_0 |h, \bar{h}\rangle = \bar{h} |h, \bar{h}\rangle. \] (2.8)

one can obtain states of \( \text{GCA}_2 \)

\[ T_0 |h, \bar{h}\rangle = (L_0 + \mathcal{T}_0) |h, \bar{h}\rangle = (h + \bar{h}) |h, \bar{h}\rangle, \]
\[ M_0 |h, \bar{h}\rangle = \frac{L_0 - \mathcal{T}_0}{c} |h, \bar{h}\rangle = \frac{h - \bar{h}}{c} |h, \bar{h}\rangle. \] (2.9)

As we observe, scaling states of \( \text{CFT}_2 \) are scaling states of GCA too. They are identified by their scaling weight and rapidity

\[ T_0 |\mathcal{H}, \xi\rangle = \mathcal{H} |\mathcal{H}, \xi\rangle, \quad M_0 |\mathcal{H}, \xi\rangle = \xi |\mathcal{H}, \xi\rangle. \] (2.10)

in which

\[ \mathcal{H} = h + \bar{h}, \quad \xi = \frac{1}{c}(h - \bar{h}). \] (2.11)

We have then

\[ h = \frac{1}{2}(\mathcal{H} + c\xi), \quad \bar{h} = \frac{1}{2}(\mathcal{H} - c\xi). \] (2.12)

Looking at commutation relations of \( \text{CFT}_2 \) generators (2.5) one can find new GCA central charges

\[ C_1 = c_R + c_L, \quad C_2 = \frac{c_R - c_L}{c}. \] (2.13)
in which
\[
[T_m, T_n] = (m - n)T_{m+n} + \frac{C_1}{12} m(m^2 - 1)\delta_{m+n,0},
\]
\[
[T_m, M_n] = (m - n)M_{m+n} + \frac{C_2}{12} m(m^2 - 1)\delta_{m+n,0},
\]
\[
[M_m, M_n] = 0.
\]
(2.14)

As it can be observed, to have finite and nonzero $H, \xi$, one need to have large $h, h$ and $c_R, c_L$ in opposite signs.

As regular CFT the logarithmic CFT can be contracted and yield LGCA. If we consider LCFT scaling states
\[
L_0|h, 0\rangle = h|h, 0\rangle,
\]
\[
L_0|h, 1\rangle = h|h, 1\rangle + \kappa|h, 0\rangle.
\]
(2.15)

and set $\kappa = 1$ then under contraction procedure we find \[10\]
\[
T_0|h, \overline{h}, 0\rangle = (h + \overline{h})|h, \overline{h}, 0\rangle = H|H, \xi, 0\rangle,
\]
\[
M_0|h, \overline{h}, 0\rangle = \frac{h - \overline{h}}{c}|h, \overline{h}, 0\rangle = \xi|H, \xi, 0\rangle,
\]
\[
T_0|h, \overline{h}, 1\rangle = (h + \overline{h})|h, \overline{h}, 1\rangle + |h, \overline{h}, 0\rangle = H|H, \xi, 1\rangle + |H, \xi, 0\rangle,
\]
\[
M_0|h, \overline{h}, 1\rangle = \frac{h - \overline{h}}{c}|h, \overline{h}, 1\rangle + \frac{1}{c}|h, \overline{h}, 0\rangle = \xi|H, \xi, 1\rangle.
\]
(2.16)

So we end up with Jordan scaling and diagonal rapidity. However considering a complex $\kappa$ we observe that Jordan form for rapidity arises as well \[26\]. Since LCFT can be realized in the bulk at the critical point of TMG, we now ask if we can follow contraction and realize LGCA as well.

3 Topologically Massive Gravity at the critical point

Three dimensional gravity with or without cosmological constant term has no propagating modes, but when one adds higher derivative terms to the action nontrivial degrees of freedom arise. One of the best known models in this regard is Topologically Massive Gravity \[16\][17] in which the higher derivative term is Chern-Simons term. In other words
\[
I = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left[ R + \frac{2}{l^2} + \frac{1}{\mu} \mathcal{L}_{CS} \right]
\]
(3.1)

where
\[ \mathcal{L}_{CS} = \frac{1}{2} \epsilon^{\lambda \mu \nu} \Gamma_{\lambda \sigma}^\alpha \left[ \partial_\mu \Gamma^\sigma_{\alpha \nu} + \frac{2}{3} \Gamma^\sigma_{\mu \tau} \Gamma^\tau_{\nu \alpha} \right] \]  

(3.2)

In general adding higher derivative term causes the instability due to the presence of ghost-like modes. Nevertheless one can show [27] that TMG at the critical value for coupling of Chern-Simons term to Einstein-Hilbert term is stable above the AdS_3 vacuum with Brown-Henneaux condition. With this boundary condition the left-moving degrees of freedom are pure gauge and related symmetry charges are zero.

The equations of motion of this action is

\[ R_{\mu \nu} - \frac{1}{2} R g_{\mu \nu} + \frac{1}{\ell^2} g_{\mu \nu} + \frac{1}{\mu} C_{\mu \nu} = 0 \]  

(3.3)

where Cotton tensor is

\[ C_{\mu \nu} = \epsilon^{\alpha \beta \mu} \nabla_\alpha (R_{\beta \nu} - \frac{1}{4} R g_{\beta \nu}) \]  

(3.4)

TMG on an asymptotically locally AdS_3 geometry may have a dual CFT with these central charges

\[ c_R = \frac{3\ell}{2G_N} \left( \frac{\mu \ell + 1}{\mu \ell} \right), \quad c_L = \frac{3\ell}{2G_N} \left( \frac{\mu \ell - 1}{\mu \ell} \right). \]  

(3.5)

So one can take the non-relativistic limit from both sides of this duality and check if this correspondence is still valid in this limit. Recently it is shown [18] that in the non-relativistic limit, the form of two-point correlation functions in both sides are equal and have GCA structure away from the critical point. However equation of motion (3.3), at the critical point has a non-trivial solution [28] that do not obey Brown-Henneaux conditions which may be interpreted as the left-moving excitations. The general solution of linearized equation of motion at the critical point in the finite neighborhood of the conformal boundary at \( \rho = 0 \) has the form of

\[ ds^2 = \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g_{ij} dx^i dx^j, \]  

(3.6)

in which

\[ g_{ij} = b_{(0)ij} \log(\rho) + g_{(0)ij} + \left( b_{(2)ij} \log(\rho) + g_{(2)ij} \right) \rho + \cdots \]  

(3.7)

When \( b_{(0)ij} \) is turned on, this solution has not structure of AlAdS space-time. But with machinery of AdS/CFT [29], it is shown in [19] that TMG at the critical point might be dual

\footnote{Note that if \( b_{(0)ij} = 0 \) this solution obeys Brown-Henneaux boundary condition and AdS vacuum is the case for which \( b_{(0)ij} = 0 \) and \( g_{(0)ij} = \delta_{ij} \)}
to logarithmic conformal field theory treating $b_{(0)ij}$ perturbatively. Two point correlation functions will be:

$$<T_{zz}(z,\bar{z})T_{zz}(0)> = \frac{3l/2G_N}{z^4}, \quad <t_{z\bar{z}}(z,\bar{z})t_{z\bar{z}}(0)> = \frac{-3l/2G_N}{z^4} \quad (3.8)$$

Starting from the above equation and performing contraction we obtain:

$$<T_{zz}T_{zz}> = \frac{3l}{2G_N} \frac{1}{(t-\frac{z}{c})^4} = \frac{3l}{2G_N} t^{-4} + \frac{6l}{G_N} \frac{x}{ct} + ..., \quad (3.9)$$

It can be easily seen that by no means one can redefine the parameters to keep spatial dimension in the contraction limit. It is because in (3.8) factors of logarithmic term and $z^{-4}$ term are coupled to each other. In other words at the first sight it seems that since TMG at the critical point corresponds to a special subclass of LCFT’s, its contraction fails to give an interesting result. However in [30] a different approach to LCFT was introduced. In this approach LCFT is obtained as vanishing limit of left central charge. In [19] the authors utilized this insight to evaluate TMG/LCFT correspondence. While taking the contraction limit of the final result (3.8) fails to keep spatial dimension, taking both limits simultaneously and carefully one can help spatial dimension survive.

4 Contraction of TMG/LCFT

4.1 Contraction of CFT at $c_L \to 0$

In the approach presented in [30] it is supposed that as $c_L$ goes to zero, beside energy-momentum tensor there is another operator $X$ with conformal dimension of $(2+\eta(c_L),\eta(c_L))$ which in the limit approaches to $(2,0)$. The non-vanishing two point-functions are

$$<T_{zz}T_{zz}> = \frac{c_R}{z^4}, \quad <T_{zz}T_{zz}> = \frac{c_L}{z^4}, \quad <XX> = \frac{1}{c_L z^{4+2\eta(c_L)}}$$

Defining $t_{zz} = -\frac{1}{c_L} X - \frac{1}{c_L} T_{zz}$ and letting $c_L \to 0$, one approach LCFT two-point functions

$$<T_{zz}(z)t_{zz}(0,0) >= \frac{b}{2z^4}, \quad <t_{z\bar{z}}(z,\bar{z})t_{z\bar{z}}(0,0) >= \frac{-b \log(|z^2|)}{z^4} \quad (4.2)$$
Now we try to redefine energy-momentum tensors and try to consider both limits simultaneously. To do so we let

\[
T_1 = \frac{1}{\sqrt{c_L}} T_{zz} + \frac{\beta(c_L)}{c_L} X + \frac{\gamma(c_L)}{c_L} T,
\]

\[
T_2 = \frac{1}{c} \left( \frac{1}{\sqrt{c_L}} T_{zz} - \frac{\beta(c_L)}{c_L} X - \frac{\gamma(c_L)}{c_L} T \right),
\]

\[
T_3 = T_{zz} + \sqrt{c_L} T_{zz},
\]

and observe by taking limits of \( c_L \to 0 \) and \( c \to \infty \) simultaneously we obtain

\[
<T_1 T_1> = \frac{1}{c_L} <T_{zz} T_{zz}> + \frac{\beta^2}{c_L^2} <XX> + \frac{\gamma^2}{c_L^2} <T_{zz} T_{zz}>
\]

\[
= \frac{c_R}{c_L} t^{-4} (1 + 4 \frac{x}{ct} + ...) + \frac{\beta^2 \alpha t^{-4\eta}}{c_L^2} (1 - 4 \frac{x}{ct} + ...) + \frac{\gamma^2}{c_L} t^{-4} (1 - 4 \frac{x}{ct} + ...)
\]

\[
= t^{-4} \left( \frac{c_R + \beta^2 \alpha t^{-4\eta} + \gamma^2}{c_L} \right) + 4t^{-4} \left( \frac{c_R - \beta^2 \alpha t^{-4\eta} - \gamma^2}{c_L} \right) \frac{x}{ct}
\]

Note that for simplifying we have implied the functionality of \( \alpha, \beta, \gamma, \eta \) to \( c_L \). Now requiring

\[
\lim_{c_L \to 0} \left( \beta^2 \alpha \frac{t^{-4\eta}}{c_L^2} + \gamma^2 \right) = -c_R
\]

we obtain

\[
<T_1 T_1> = t^{-4} \lim_{c_L \to 0} \left[ (\gamma^2)' + (\beta^2 \frac{\alpha}{c_L^2})' \right] + 4t^{-4} \lim_{c_L \to 0, c \to \infty} \left( \frac{2c_R}{c L} \right) \frac{x}{ct}
\]

\[
+ t^{-4} \lim_{c_L \to 0} \left[ -2\eta \beta^2 \frac{\alpha}{c_L^2} \right] \ln t^2
\]

As well we observe

\[
<T_1 T_2> = \frac{1}{c} \left( \frac{1}{c_L} <T_{zz} T_{zz}> - \frac{\beta^2}{c_L^2} <XX> - \frac{\gamma^2}{c_L^2} <T_{zz} T_{zz}> \right)
\]

\[
= t^{-4} \left( \frac{c_R - \beta^2 \alpha t^{-4\eta} - \gamma^2}{c_L} \right) \frac{1}{c} + 4t^{-4} \left( \frac{c_R + \beta^2 \alpha t^{-4\eta} + \gamma^2}{c_L} \right) \frac{1}{c^2} \frac{x}{t}
\]

which at the limits results in

\footnote{Which "r" indicate derivative respect to \( c_L \).}
\[ <T_1T_2> = t^{-4} \lim_{c_L \to 0, c \to \infty} \left( \frac{2c_R}{cL} \right) \]  
(4.8)

For other two-point functions we obtain

\[ <T_1T_3> = t^{-4}(\lim_{c_L \to 0} \gamma + c_R), \quad <T_2T_3> = <T_3T_3> = 0. \]  
(4.9)

If we define

\[ \lim_{c_L \to 0} \left[ \left( \frac{\gamma^2}{\gamma^2} + \left( \frac{\beta^2 \alpha}{c_L^2} \right) \right) \right] = C_1, \quad \lim_{c_L \to 0, c \to \infty} \left( \frac{2c_R}{cL} \right) = C_2, \]

\[ \lim_{c_L \to 0} \left[ -2 \eta' \beta^2 \alpha \right] = C_3, \quad \lim_{c_L \to 0} \gamma + c_R = C_4. \]  
(4.10)

The two-point correlation functions of non-relativistic limit will be

\[ <T_1T_1> = C_1 t^{-4} + 4C_2 t^{-4} \frac{x}{t} + C_3 t^{-4} \ln t^2, \]
\[ <T_1T_2> = C_2 t^{-4}, \quad <T_1T_3> = C_4 t^{-4}, \]
\[ <T_2T_2> = <T_2T_3> = <T_3T_3> = 0. \]  
(4.11)

As it can be observed with coupling both limits we derived a much larger class of LGCA in which beside the time, spatial dependency appears in two-point functions. Now we try to consider taking parallel limits in gravity side to see if we can observe spatial dimension in two-point functions.

### 4.2 Topologically Massive Gravity at \( \mu l \to 1 \)

Note that with the above procedure we can determine the relative behavior of two limits; one is critical limit \( \mu l \to 1 \) and the other is contraction limit \( c \to \infty \). Let us review some important points of section (7) of [19]. To find two-point correlation functions with the AdS/CFT machinery, one needs to calculate the second expansion of action \( I_2 \) and linearized equations of motion around the groundstate, AdS vacuum, simultaneously. Linearized equations of motion give the most general asymptotic form of the solution:

\[ h_{ij} = h_{(-2\lambda)ij}\rho^{-\lambda} + h_{(0)ij} + h_{(2)ij}\rho + h_{(2-2\lambda)ij}\rho^{1-\lambda} + h_{(2+2\lambda)ij}\rho^{\lambda+1} + ... \]  
(4.12)

with

\[ \mu l = 2\lambda + 1. \]  
(4.13)

Equations of motion of TMG are third order in derivatives. This means that one needs three boundary condition to determine the solution. The first condition is regularity in
\[ \rho = \infty \text{ limit. So, we are left with two other conditions which are reflected in two boundary sources } h_{(2\lambda)i}, h_{(0)i}, \text{ for which we define the corresponding CFT operators } T_{ij} \text{ and } X_{ij}. \]

First we need to substitute this asymptotic form to second expansion of action and determine diverging part of the action. Then after finding the proper covariant counterterms to vanish diverging part, we need to substitute again the asymptotic form in renormalized action \( I_{2,\lambda,tot} = I_2 + I_{ct} \). In this step one can determine the one-point functions of two corresponding CFT operators with functionally differentiation of renormalized action \( I_{2,\lambda,tot} \) with respect to the two sources \( h_{(2\lambda)i}, h_{(0)i} \).

\[
< X_{ij} > = - \frac{4\pi}{\sqrt{-g_{AdS}}} \frac{\delta I_{2,\lambda,tot}}{\delta h^{ij}_{(2\lambda)}}, \quad < T_{ij} > = \frac{4\pi}{\sqrt{-g_{AdS}}} \frac{\delta I_{2,\lambda,tot}}{\delta h^{ij}_{(0)}}.
\] (4.14)

To find the two-point functions one needs to functionally differentiate one-point functions with respect to the sources. However in one-point functions there exist some terms that their forms are not fully determined by asymptotic analysis. In this step of holographic renormalization one needs to solve the linearized equations of motion exactly to fully determine the dependency of these terms to the sources. After following this procedure, we end up with these two-point functions:\(^5\)

\[
< T_{zz}(z,\bar{z})T_{zz}(0) > = \frac{3l}{2G_N} \frac{\lambda+1}{2\lambda+1} \frac{1}{\bar{z}^4},
\]

\[
< T_{zz}(z,\bar{z})X_{zz}(0) > = - \frac{1}{2G_N} \frac{\lambda(\lambda+1)(2\lambda+3)}{2\lambda+1} \frac{1}{z^{2\lambda+4} \bar{z}^{2\lambda}}.
\] (4.15)

Defining

\[
T_1 = \frac{1}{\sqrt{\lambda}} T_{zz} - \frac{1}{\lambda} X, \quad T_2 = \frac{1}{c} \left( \frac{1}{\sqrt{\lambda}} T_{zz} + \frac{1}{\lambda} X \right), \quad T_3 = T_{zz} + \sqrt{\lambda} T_{zz}.
\] (4.16)

and looking over (4.15) and (4.13), we obtain

\[
< T_1 T_1 > = \frac{\mu l + 1}{2\mu l(\mu l - 1)} \left( \frac{3l}{G_N} \frac{1}{\bar{z}^4} - \frac{l}{G_N} (\mu l + 2) \frac{1}{z^{(\mu l - 1)} + 4z^{\mu l - 1}} \right)
= \frac{\mu l + 1}{2\mu l(\mu l - 1)} t^{-4} \left( \frac{3l}{G_N} [1 + 4 \frac{x}{ct} + ...] - \frac{l}{G_N} (\mu l + 2) [1 - 4 \frac{x}{ct} + ...] t^{-2(\mu l - 1)} \right)
= \frac{\mu l + 1}{2\mu l} t^{-4} \left( \frac{l}{G_N} \frac{3 - (\mu l + 2)t^{-2(\mu l - 1)}}{\mu l - 1} + 4 \frac{l}{G_N} \frac{3 + (\mu l + 2)t^{-2(\mu l - 1)}}{\mu l - 1} \frac{x}{ct} \right).
\] (4.17)

considering

\(^5\)One first uses lightcone coordinates \( v, u = t \pm x \) and replaces \( v \rightarrow z, u \rightarrow \bar{z} \), the complex boundary coordinates.
\[ \lim_{\mu l \to 1} \frac{3 - (\mu l + 2)t^{-2(\mu l - 1)}}{\mu l - 1} = 3 \ln t^2 - 1, \quad (4.18) \]

we end up

\[ \langle T_1 T_1 \rangle = t^{-4} \left( -\frac{l}{G_N} + 4 \frac{6l}{G_N} \frac{x}{c} + \frac{3l}{G_N} \ln t^2 \right). \quad (4.19) \]

As well we observe

\[ \langle T_1 T_2 \rangle = \frac{\mu l + 1}{2\mu l(\mu l - 1)c} t^{-4} \left( \frac{3l}{G_N} [1 + 4 \frac{x}{ct} + ...] + \frac{l}{G_N} (\mu l + 2) [1 - 4 \frac{x}{ct} + ...] t^{-2(\mu l - 1)} \right) \]
\[ = \frac{\mu l + 1}{2\mu l} t^{-4} \left( \frac{l}{G_N} 3 + (\mu l + 2)t^{-2(\mu l - 1)} \frac{1}{\mu l - 1} \frac{1}{c} + 4 \frac{l}{G_N} 3 - (\mu l + 2)t^{-2(\mu l - 1)} \frac{x}{c^2t} \right) \quad (4.20) \]

and

\[ \langle T_1 T_3 \rangle = \frac{3l}{G_N}, \quad \langle T_2 T_2 \rangle = \langle T_2 T_3 \rangle = \langle T_3 T_3 \rangle = 0. \quad (4.21) \]

Comparing with the forms of two-point functions of LGCA introduced in previous section

\[ \langle T_1 T_1 \rangle = C_1 t^{-4} + 4C_2 t^{-4} \frac{x}{t} + C_3 t^{-4} \ln t^2, \]
\[ \langle T_1 T_2 \rangle = C_2 t^{-4}, \quad \langle T_1 T_3 \rangle = C_4 t^{-4}, \quad (4.22) \]
\[ \langle T_2 T_2 \rangle = \langle T_2 T_3 \rangle = \langle T_3 T_3 \rangle = 0 \]

We conclude TMG at the critical point can be dual to LGCA with the following central charges

\[ C_1 = -\frac{l}{G_N}, \quad C_2 = \frac{6l}{G_N}, \quad C_3 = \frac{3l}{G_N}, \quad C_4 = \frac{3l}{G_N}. \quad (4.23) \]

5 Conclusions and Outlook

In this paper we explored some aspects of TMG/LGCA holography. We observe that for this duality to exist at least at the level of two-point correlation functions, we should carefully take both limits. One limit is the contraction limit in which the speed of light \( c \) goes to infinite and the other is the "chiral" limit \( \mu l \to 1 \) that corresponds to vanishing left central
charge. Taking both limits separately, contracted two-point correlation functions depends only on time. However considering both limits simultaneously; $\mu l - 1$ coupled to $\frac{1}{c}$; we observe existence of spatial dimension in final results. It means that when contracting a model one needs to be careful to consider all possible limits to span the space of all variable of the contracted model more thoroughly.

We should notify that when an algebra is contracted there is no necessity that contracting its representation as well yields thorough representation of the contracted model. We can utilize this apparatus if we have a good knowledge about the contracted model. For $CFT_2/GCA_2$ since previously many aspects of its representation and as well its two point functions were known, this approach had something strong to say. Beside two-point functions many more reasons were presented in [9] to support this approach. Otherwise, contraction approach may span only a subclass of representations of the contracted algebra and just give some insights about the contracted model. So we should mention that though our final contraction yields spatial term for two-point functions but a concrete statement is subject to more thorough investigation.

Another three dimensional gravity with higher derivative terms has been introduced in [33]. The corresponding action is given by

$$S = \frac{1}{16\pi G_N} \int_R d^3x \sqrt{-G} \left[ R - 2\lambda \right] - \frac{1}{m^2} \frac{1}{16\pi G_N} \int_R d^3x \sqrt{-G} \left[ R^\mu\nu R_{\mu\nu} - \frac{3}{8} R^2 \right]. \quad (5.1)$$

This model is known as New Massive Gravity (NMG). It is believed that NMG model on an asymptotically locally $AdS_3$ geometry may have a dual CFT whose central charges are given by [34, 35] (see also [36, 37])

$$c_L = c_R = \frac{3l}{2G_N} \left( 1 - \frac{1}{2m^2l^2} \right). \quad (5.2)$$

At the critical value, $m^2l^2 = \frac{1}{2}$, where the central charges are zero it has been shown [38] that the model admits a new vacuum solution which is not asymptotically locally $AdS_3$. Near the boundary, form of this solution is similar to TMG’s solution in critical point [3.6], (3.7), however, relations between the coefficients are different due to the dynamical differences of two theories. It is shown that NMG at the critical point [39, 40] may be dual to LCFT. In contrast to TMG, this model has parity symmetry, so the left and right central charges are equal. As a result this model may not be applicable for studying NMG/GCA holography. However the thorough examination is left: If we can by the contraction procedure followed in this paper introduce proper NMG/LGCA holography or not.

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