New measures to test modified gravity cosmologies

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Abstract. The observed accelerated expansion of the Universe may be explained by dark energy or the breakdown of general relativity (GR) on cosmological scales. When the latter case, a modified gravity scenario, is considered, it is often assumed that the background evolution is the same as the ΛCDM model but the density perturbation evolves differently. In this paper, we investigate more general classes of modified gravity, where both the background and perturbation evolutions are deviated from those in the ΛCDM model. We introduce two phase diagrams, $\alpha-f\sigma_8$ and $H-f\sigma_8$ diagrams; $H$ is the expansion rate, $f\sigma_8$ is a combination of the growth rate of the Universe and the normalization of the density fluctuation which is directly constrained by redshift-space distortions, and $\alpha$ is a parameter which characterizes the deviation of gravity from GR and can be probed by gravitational lensing. We consider several specific examples of Horndeski’s theory, which is a general scalar-tensor theory, and demonstrate how deviations from the ΛCDM model appears in the $\alpha-f\sigma_8$ and $H-f\sigma_8$ diagrams. The predicted deviations will be useful for future large-scale structure observations to exclude some of the modified gravity models.

Keywords: dark energy theory, modified gravity, gravitational lensing, redshift surveys

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1 Introduction

The accelerated expansion of the current Universe has been clarified by the observations of type Ia supernovae in the late 1990s [1, 2] and supported by observations of cosmic microwave background radiation (CMB) [3–5] and baryon acoustic oscillations (BAO) [6–12]. Broadly speaking, there are two approaches to explain the accelerating Universe; (i) introducing an additional energy component, called dark energy, to the total energy budget and (ii) modifying the action of gravity from the one predicted by Einstein’s theory of relativity. In dark energy models, one adopts the Einstein-Hilbert action and introduces a fluid matter with the equation-of-state parameter $w < -1/3$, where $w = -1$ corresponds to a cosmological constant. The simplest dark energy model is ΛCDM, and more generally, the $w$CDM or quintessence model has been considered [13–16]. On the other hand, the Einstein-Hilbert action itself is modified in the models of modified gravity theories. The models include $F(R)$ gravity [17–22], massive gravity [23–25], and Horndeski’s theory [26].

The differences of the models appear both in the background equations and in the linear perturbation equations. While ΛCDM is consistent with most of the observations, a tension between the values of the Hubble constant, $H_0$, constrained from early universe and late universe started to be recognized after the Planck mission reported the first results [4]. The discrepancy in the $H_0$ values between local observarions and CMB observations was first reported using Cepheid variables [27–29], and it has also been seen in the observations of strong lensing time delay and so on [30–35]. Moreover, there is another tension in a parameter, $f\sigma_8$, where $f$ is the growth rate of the universe and $\sigma_8$ is the normalization of the density fluctuation amplitude. This parameter can be directly constrained through peculiar velocities of galaxies in galaxy redshift surveys, known as redshift-space distortions.
(RSD) [36, 37], and is used to distinguish modified gravity models [38–42]. However, the $f\sigma_8$ values observed so far at $z < 1$ are systematically lower than the prediction of the best-fitting ΛCDM model from the Planck result [43–48], as pointed out by, e.g., refs. [49, 50]. Therefore, the importance of reconsidering the dynamics of the Universe in modified gravity models is increasing.

Modified gravity theories, in general, have too many degrees of freedom to be completely analyzed. Consequently, the observables of perturbation quantities (e.g. growth rate of the matter density perturbation) have been investigated only for simple models or for phenomenologically parametrized models. Moreover, the background evolution in modified gravity models is often assumed to be the same as that in the ΛCDM model. In this paper we take account of both the variations of background dynamics and those of perturbation quantities. For this purpose, we introduce new phase diagrams; the $\alpha-f\sigma_8$ and $H-f\sigma_8$ diagrams. Similar diagrams can be found in refs. [51–55]. Here $\alpha$ describes the effect on gravitational lensing and is defined in eq. (2.4) below in terms of the deflection potential $\Psi_{\text{defl}}$, and $\alpha = 0$ in the case of general relativity (e.g., [56–61]). Thus, these diagrams will enable us to investigate how each of the two key observations of large-scale structures, RSD and gravitational lensing, can constrain a given modified gravity model.

In this paper, we consider Horndeski’s theory, which is a general scalar-tensor theory. We do not adopt phenomenological parameterizations which have been commonly used in the literature. We focus several specific examples of Horndeski’s gravity and demonstrate how deviations from the ΛCDM model appear in the $\alpha-f\sigma_8$ and $H-f\sigma_8$ diagrams.

The contents of the paper are as follows. We briefly overview how models of dark energy and modified gravity behave on the $\alpha-f\sigma_8$ and $H-f\sigma_8$ diagrams in section 2. For this purpose we present the former and latter diagrams for the simple representative cases of dark energy and modified gravity, the $w$CDM and $F(R)$ gravity models, respectively. Then we describe Horndeski’s theory in section 3, and considering its typical examples, we present their $\alpha-f\sigma_8$ and $H-f\sigma_8$ diagrams in section 4. Our concluding remarks are given in section 5. A more detailed description of Horndeski’s theory is presented in appendix A. A short note on the possibility of having negative $\alpha$ is given in appendix B.

Throughout this paper, we adopt Natural units, $\hbar = c = k_B = 1$, and the gravitational constant $8\pi G$ is denoted by $\kappa^2 \equiv 8\pi/M_{\text{Pl}}^2$ with the Planck mass of $M_{\text{Pl}} = G^{-1/2} = 1.2 \times 10^{19}$ GeV.

## 2 Dark energy and modified gravity

In models in which dark energy is given by a matter field, commonly its energy momentum tensor is decoupled from the other matter components. In this case, the main effect of dark energy is to modify the background evolution of the Universe, and hence its effect to the growth of linear perturbations is indirect. On the other hand, in modified gravity theories of dark energy, its effective energy momentum tensor is likely to be coupled to the matter components. As a result, not only the background evolution of the Universe but also the linear perturbation equations may significantly deviate from those in the ΛCDM model.

In the following, we assume the metric,

$$ ds^2 = -(1 + 2\Psi(t, x))dt^2 + a^2(t)(1 + 2\Phi(t, x))\delta_{ij}dx^idx^j, \quad (2.1) $$
and adopt notations in Fourier space as

\[ \Psi(t, k) + (1 + \alpha(t, k))\Phi(t, k) = 0, \tag{2.2} \]
\[ \delta \rho(t, k)/\rho(t) = \delta(t, k) = \delta(k)D(t, k), \tag{2.3} \]

where \( k \) is the wave number, \( D(t, k) \) is the growing mode of the matter density perturbation, and \( \delta(k) \) describes the scale dependence of \( \delta(t, k) \) at initial time \( t = t_i \) under the normalization of \( D \) as \( D(t_i, k) = 1 \).

In eq. (2.2), we introduced a quantity \( \alpha \), which vanishes in Einstein gravity (i.e., \( \Psi + \Phi = 0 \)). The relation of \( \alpha \) with the well-known gravitational slip parameter, \( \eta \equiv -\Phi/\Psi \) [39], is \( \eta = 1/(1 + \alpha) \). Thus \( \alpha \) characterizes how much a given gravity model deviates from General Relativity (GR). Because the strength of gravitational lensing is determined by the deflection potential [62],

\[ \Psi_{\text{defl}} \equiv \Psi - \Phi = \frac{2 + \alpha}{1 + \alpha} \Psi = \left( 2 - \frac{\alpha}{1 + \alpha} \right) \Psi, \tag{2.4} \]

we see that the gravitational lensing effect is reduced if \( \alpha > 0 \) or \( \alpha < -2 \) and enhanced if \( -2 < \alpha < 0 \). We note that there is a subtlety in this interpretation. It will be shown in section 4 that, for a class of modified gravity models considered in this paper, the lensing effect is unaffected if we compare it for the same mass distribution.

To see this, let us first introduce \( G_{\text{eff}} \) which relates the Newton potential to the matter density perturbation in modified gravity:

\[ \Delta \Psi = 4\pi G_{\text{eff}} \delta \rho a^2. \tag{2.5} \]

Let us also introduce \( G_{\text{light}} \) that would relate the deflection potential \( \Psi_{\text{defl}} \) to \( \delta \rho \),

\[ \Delta \Psi_{\text{defl}} = 4\pi G_{\text{light}} \delta \rho a^2. \tag{2.6} \]

We have \( G_{\text{eff}} = G \) and \( G_{\text{light}} = 2G \) in GR. On the other hand, for modified gravity, the \( \alpha \)-dependence of \( G_{\text{eff}} \) is found in section 4 as

\[ G_{\text{eff}} = \frac{2 (1 + \alpha)}{2 + \alpha} G. \tag{2.7} \]

From eqs. (2.4) and (2.5), we find

\[ \Delta \Psi_{\text{defl}} = \frac{2 + \alpha}{1 + \alpha} \Delta \Psi = \frac{2 + \alpha}{1 + \alpha} \cdot 4\pi G_{\text{eff}} \delta \rho a^2. \tag{2.8} \]

Hence we obtain

\[ G_{\text{light}} = \frac{2 + \alpha}{1 + \alpha} G_{\text{eff}}. \tag{2.9} \]

Inserting eq. (2.7) into the above \( G_{\text{eff}} \) gives \( G_{\text{light}} = 2G \), that is, the resulting \( \Psi_{\text{defl}} \) will be the same as that in GR, independent of \( \alpha \).

However, as the mass distribution is measured by its gravitational effect that includes the modification in the effective gravitational constant, what can be compared is the lensing effect for the same \( \Psi \). This means our original interpretation is correct from an observational point of view. More precisely speaking, it is the gravitationally inferred mass distribution that is overestimated if \( \alpha > 0 \) or \( \alpha < -2 \) and otherwise if \( -2 < \alpha < 0 \), while the gravitational lensing effect remains the same.
To quantify the growth of linear perturbations, one usually considers the combination, \( f_\sigma \), inferred from galaxy surveys, where \( f = d \ln D / d \ln a \) and \( \sigma \) is the normalization parameter of the density perturbation spectrum. In the following, to test modified gravity models, we evaluate the ratio of \( f_\sigma \) in a given model to that in the \( \Lambda \)CDM model, \( f_\sigma / f_{\sigma,\Lambda CD M} \). Note that the value of \( f \) should approach unity in any viable model of gravity in the high-redshift limit. We also assume that the value of \( \sigma \) coincides with that of the \( \Lambda \)CDM model in the high redshift limit. Thus, this ratio becomes unity at high redshifts. Since \( f_\sigma \) depends not only on the modifications of the linear perturbation equations but also on the background evolution of the Universe, below we will study correlations between \( f_\sigma \) and \( \alpha \) and between \( f_\sigma \) and the Hubble expansion rate \( H \) in various models of gravity. The evolution equation of the quasi-static mode of the matter density contrast \( \delta = \delta \rho_m / \rho_m \) is expressed as [61, 64]

\[
\ddot{\delta} + 2H \dot{\delta} - 4\pi G_{\text{eff}} \rho_m \delta = 0.
\] (2.10)

The parameter \( f_\sigma \) will be evaluated by using eq. (2.10) with appropriate initial conditions.

Before presenting detailed studies on modified gravity models, let us first consider the simplest dark energy model, i.e. the \( \Lambda \)CDM model. Note that \( \alpha = 0 \) because there is no deviation from GR in this model. The \( H - f_\sigma \) diagram is depicted in figure 1. The initial conditions are set at \( z = 10 \) where they coincide with those of the \( \Lambda \)CDM model. The figure shows that the deviation in \( f_\sigma \) is almost proportional to that in the Hubble rate at each redshift, with negative proportionality coefficients. Thus \( f_\sigma \) in \( \Lambda \)CDM models becomes smaller than that in the \( \Lambda \)CDM model for models with a larger Hubble rate \( H \) which corresponds to \( w < -1 \). We note that the linear perturbation equations in the \( \Lambda \)CDM model are unchanged from those of the \( \Lambda \)CDM model. Therefore, the smaller value of \( f_\sigma \) due to a larger \( H \) may be regarded as a purely background effect. We should note that if we apply a different boundary condition, e.g., \( H_{\Lambda CD M} = H_{\Lambda \Lambda CD M} \) and \( \Omega_m = 0.31 \) at \( z = 0 \), the quantitative results will be different though the qualitative tendency will remain the same.

As we mentioned in the above, the behavior of the matter density perturbation is not independent from the background evolution of the Universe in general. In some theories, e.g. \( F(R) \) gravity theories, even if the difference between the model and GR at the background level is negligibly small, there can be \( O(1) \) difference at perturbation level due to the scale dependence of the matter density perturbation that enhances the deviation from GR on small scales. As a characteristic example for such a case, let us consider \( F(R) \) gravity with

\[
F(R) = R - 2\Lambda + |F_{R0}| \frac{R_0^2}{R},
\] (2.11)

where \( R_0 \) and \( \Lambda \) are the current values of the Ricci scalar and the cosmological constant, respectively. For the parameter \( F_{R0} \) in the range \( 10^{-6} < |F_{R0}| < 10^{-4} \ll \Lambda / R_0 \), this model is known to reproduce the background evolution of the \( \Lambda \)CDM model almost exactly [63]. In figure 2, we plot the predicted values of \( \alpha \) and \( f_\sigma \) for several redshifts. In general, the function \( \delta(t, k) \) is not separable, in the other words, the \( k \)-dependence of \( D(t, k) \) in (2.3) cannot be ignored in \( F(R) \) gravity theories. Here, for definiteness, we plot \( f_\sigma \), i.e. \( f_\sigma \) at \( k = (8h^{-1}\text{Mpc})^{-1} \). It shows that the density perturbation in this model always evolves faster than that in the case of the \( \Lambda \)CDM model, and \( \alpha \) is always positive.
3 More general modified gravity models

In the previous section, we studied two examples: $w$CDM model and $F(R)$ gravity theory. In the $w$CDM case, the linear perturbation equations are the same as those in the ΛCDM model, while the background evolution of the Universe is different. On the other hand, in the $F(R)$ gravity case, the background evolution of the Universe is the same as the ΛCDM model, while the linear perturbation equations are different. In more general modified gravity theories, both of them will be different from the ΛCDM model. Here we consider Horndeski’s theory. It is a general scalar-tensor theory which includes $F(R)$ gravity as a special case [64–66].

The action in Horndeski’s theory is given by [26, 67, 68]

$$S_H = \sum_{i=2}^{5} \int d^4 x \sqrt{-g} \mathcal{L}_i,$$

where

$$\mathcal{L}_2 = K(\phi, X),$$

$$\mathcal{L}_3 = -G_3(\phi, X) \Box \phi,$$

$$\mathcal{L}_4 = G_4(\phi, X) R + G_{4X} \left[ (\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right],$$

$$\mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - \frac{G_{5X}}{6} \left[ (\Box \phi)^3 - 3(\Box \phi)(\nabla_\mu \nabla_\nu \phi)^2 + 2(\nabla_\mu \nabla_\nu \phi)^3 \right].$$

Here, $K$, $G_3$, $G_4$, and $G_5$ are generic functions of $\phi$ and $X = -\partial_\mu \phi \partial^\mu \phi / 2$, and the subscript $X$ means a derivative with respect to $X$. The total action is the sum of $S_H$ and the matter.
action $S_{\text{matter}}$ which contains baryons and cold dark matter. More details are referred to appendix A.

The recent observations of gravitational wave event GW170817 [69] and its electromagnetic counterparts [70–72] showed that the propagation speed of gravitational waves should satisfy

$$|c_T^2 - 1| \lesssim 10^{-15},$$

in the relatively recent Universe. This bound implies the sound speed of the tensor mode, given by

$$c_T^2 = \frac{G_4 - XG_5\phi - XG_5\phi}{G_4 - 2XG_4\phi - X(G_5\phi H - G_5\phi)},$$

should be almost unity, where and below the subscript $\phi$ means a derivative with respect to $\phi$. If the terms $XG_5\phi$, $XG_4\phi$, $XG_5\phi H$, $XG_5\phi$ are satisfied for most healthy theories of gravity (see figure 3). Thus, it is reasonable to assume that the terms proportional to $G_3\phi$, $G_5\phi$, and $G_5\phi$ are not relevant for the current accelerated expansion of the Universe. Hence we set $G_4\phi = G_5 = 0$ in the following.

Even after this simplification, the theory still remains quite complicated. For example, the effective Newton constant that can be defined on sufficiently small scales $k^2 \gg a^2H^2$ takes the form [61],

$$G_{\text{eff}} = \frac{1}{16\pi G_4} \left( \frac{A}{B} + O(a^2H^2/k^2) \right);$$

$$A \equiv G_4C_{\text{kin}} + 4G_4\phi,$$\n
$$B \equiv G_4C_{\text{kin}} - \frac{1}{4}\phi^4G_3^2 - \phi^2G_3\phiG_4\phi + 3G_4^2,$$\n
where we have introduced an effective coefficient of the kinetic term $C_{\text{kin}}$

$$C_{\text{kin}} = K_X - 2G_3\phi + \phi(2G_3\phi + \phi^2G_3\phi) + \phi^2G_3\phi + 4H\phi G_3\phi.$$\n
As for $\alpha$ which describes the modification of the lensing effect, it is expressed using eq. (3.11) as [61]

$$\alpha = \frac{G_4\phi(2G_4\phi + \phi^2G_3\phi)}{G_4C_{\text{kin}} + G_4\phi(-\phi^2G_3\phi + 2G_4\phi) + O(a^2H^2/k^2)}.$$\n
The above equation shows that $\alpha$ is non-vanishing only if $G_4$ is non-trivial, i.e. is $\phi$-dependent. It also shows that $\alpha < 0$ is realized only if $G_3 = 0$ and $G_4 > 0$ which are satisfied for most healthy theories of gravity (see figure 3). Thus $\alpha = 0$ if $G_4\phi = 0$, $\alpha > 0$ if $G_4\phi \neq 0$ and $G_3 = 0$, while $\alpha$ can be positive or negative if $G_3 \neq 0$ and $G_4\phi \neq 0$.

In passing we note that $f(R)$ gravity theories correspond to the case [64–66]:

$$K = \frac{M_{\text{pl}}^2}{16\pi}(F - RF_{,R}), \quad G_3 = G_5 = 0, \quad G_4 = \frac{1}{2\sqrt{8\pi}}M_{\text{pl}}\phi, \quad \phi = \frac{1}{\sqrt{8\pi}}M_{\text{pl}}F_{,R}.$$\n
Because $G_3 = 0$, $\alpha$ is always positive in $f(R)$ gravity with $K_X > 0$. We should note that $O(a^2H^2/k^2)$ terms in eqs. (3.8) and (3.12) are to be taken into account in $F(R)$ gravity models [64]. The case $K_X \leq 0$ with $G_3 = 0$ or $K_X - 2G_3\phi \leq 0$ may be also acceptable if instabilities are absent. Such a case is discussed in appendix B, but we will not consider it in the main text for simplicity.
Figure 3. Classification of Horndeski’s models in terms of $\alpha$ and $f$ under the assumptions $K_X > 0$ and $G_4 > 0$. Nonzero $\alpha$ is only realized when $G_{4\phi} \neq 0$, and $\alpha < 0$ is realized only if $G_{4\phi} \neq 0$ and $G_{3X} \neq 0$.

4 Specific examples

Let us now consider a few specific examples of Horndeski’s gravity that show small but observationally interesting deviations from the ΛCDM model. To compare with the ΛCDM model, we will assume the same amount of matter as that in the ΛCDM model, i.e. $\Omega_m, 0 h^2$ is fixed, and fix the ratio $f\sigma_8,_{\Lambda\text{CDM}}/f\sigma_8 = 1$ at sufficiently high redshifts, $z \geq 10$. In what follows, to evaluate the effects of functions $K(\phi, X)$, $G_3(\phi, X)$, and $G_4(\phi)$, we assume their forms as

$$K(\phi, X) = X + K_2X^2 - (V_0 + V_1\phi + m^2\phi^2),$$

$$G_3(\phi) = g\phi,$$

$$G_4(\phi) = \frac{1}{2\kappa^2} \exp\left[\lambda \frac{\phi}{M_{pl}}\right],$$

where $K_2, V_0, V_1, m^2, g$ and $\lambda$ are constant parameters.

In this above class of models, the expressions for $\alpha$ and $G_{\text{eff}}$ are greatly simplified as

$$\alpha = \frac{2G_4^2}{G_4C_{\text{kin}} + 2G_4^2},$$

$$G_{\text{eff}} = \frac{G_4C_{\text{kin}} + 4G_4^2}{16\pi G_4(G_4C_{\text{kin}} + 3G_4^2)}.$$  

where $C_{\text{kin}}$ defined in eq. (3.11) is also simplified as

$$C_{\text{kin}} = K_X - 2G_{3\phi}.$$  

As $G_{4\phi}$ should be small enough to guarantee the proximity to Newton gravity in the large scale structure formation, the above two equations imply that $\alpha$ is small and positive if $C_{\text{kin}} = O(1)$ and $> 0$, and $G_{\text{eff}} \approx G$.

Now we are in a position to evaluate the effect of $\alpha$ on gravitational lensing for the same mass density distribution. Since the resulting gravitational potential is proportional to the effective gravitational constant $G_{\text{eff}}$, the resulting gravitational lensing effect is proportional
\[ y = 1.00 - 1.53(x-1) + 1.40(x-1)^2 \]

\[ y = 1.00 - 1.50(x-1) + 1.46(x-1)^2 \]

\[ y = 1.00 - 1.47(x-1) + 1.54(x-1)^2 \]

**Figure 4.** $H - f\sigma_8$ diagram in the case $K = X - (V_0 + V_1\phi)$, $G_3 = 0$ and $G_4 = \text{const.}$ for various of $V_1$. The initial conditions are set as $\phi = 0.5 M_{\text{pl}}$ and $\dot{\phi} = -2V_1/(9H)$ at $z = 10$. The best-fitting quadratic curves are shown for the redshifts 1.0 (dotted), 0.5 (dashed) and 0.1 (solid).

To the deflection potential given by

\[ \Psi_{\text{defl}} = \frac{2 + \alpha}{1 + \alpha} \Psi = \frac{2 + \alpha G_{\text{eff}}}{1 + \alpha G} \Psi_{\text{GR}}, \]

where $\Psi_{\text{GR}}$ is the gravitational potential of the mass distribution if gravity were GR. Using eqs. (4.4) and (4.5), we find

\[ G_{\text{eff}} = \frac{1}{\kappa^2 G_4} \frac{1 + \alpha}{2 + \alpha} G = \frac{1 + \alpha}{2 + \alpha} \exp \left[ -\lambda \frac{\phi}{M_{\text{pl}}} \right] G, \]

which gives

\[ \Psi_{\text{defl}} = 2 \exp \left[ -\lambda \frac{\phi}{M_{\text{pl}}} \right] \Psi_{\text{GR}} = \exp \left[ -\lambda \frac{\phi}{M_{\text{pl}}} \right] \Psi_{\text{defl,GR}}, \]

where $\Psi_{\text{defl,GR}}$ is the deflection potential in the case of GR. Thus we see that the gravitational lensing effect is independent of $\alpha$ for the same mass density distribution. In particular, it remains essentially the same as that in GR if $\lambda \phi / M_{\text{pl}} \ll 1$, as we mentioned in section 2.

### 4.1 Linear potential, minimally coupled with gravity

First we consider the case $K = X - (V_0 + V_1\phi)$ with $G_3 = 0$ and $G_4 = \text{const.}$, which represents a quintessential field [14]. In this case, $\alpha$ always vanishes as seen from eq. (3.12). Figure 4 shows the correlations between $f\sigma_8$ and $H$ for various values of $V_1$ at several different redshifts. One sees that a higher Hubble rate is accompanied with a lower growth rate of the matter density perturbation, which is similar to the case of the $w$CDM model shown in figure 1. Note that $\alpha = 0$ also in the $w$CDM model. These similarities are due to the fact that both models have no non-minimal coupling with gravity.
Next we consider the effect of non-minimal coupling with gravity, namely, the case where $G_4 = \exp\left[\lambda \phi/M_{pl}\right]/(2\kappa^2)$ with $K = X - V_0$ and $G_3 = 0$ (e.g., see [73, 74]). Figure 5 shows the correlations between $f\sigma_8$ and $H$ and between $f\sigma_8$ and $\alpha$. The $H - f\sigma_8$ diagram on the left is similar to that of the minimally coupled linear potential case except for the inclinations of the fitted curves. On the other hand, the $\alpha - f\sigma_8$ diagram is quite different: $\alpha$ is always non-negative as in the case of $F(R)$ gravity, see figure 2 while it vanishes in the minimally coupled linear potential case. We also note that the ratio $f\sigma_8/f\sigma_{8,\Lambda CDM}$ can be greater or smaller than unity depending on the sign of $\lambda$ in contrast to the $F(R)$ gravity case where the ratio is always greater than unity. The similarity and difference from the $F(R)$ gravity case can be explained by the fact that $\alpha$ does not depend on the sign of $\lambda$ or $G_{4\phi}$, while $G_{eff}$ may increase or decrease depending on the sign of $\lambda$.

### 4.2 Flat potential, non-minimally coupled with gravity

This model is similar to a combination of the models considered in subsections 4.1 and 4.2. However, it differs from such a model in that there is an oscillation of the equation of state parameter induced by the oscillation of the scalar field, which gives rise to an oscillatory evolution in the Hubble rate as can be seen in the left panel of figure 6. We find $f\sigma_8$ in this model is always smaller than that in the $\Lambda$CDM model even at $z = 0.5$ at which $H/H_{\Lambda CDM} < 1$. This result may be understood if we note the fact that $f(z) = d\ln D/d\ln a$, which $\sigma_8(z)$ is proportional to, depends non-locally on time, or it is a hysteresis effect of the oscillatory evolution in the present case. We also note that $\alpha$ does not vary much within the range of parameters we adopted, with the values in the range $0.02 \sim 0.04$. 

$$K = X - (V_0 + m^2 \phi^2), \quad G_3 = 0, \quad G_4 = \frac{1}{2\kappa^2} e^{\lambda \phi/M_{pl}}. \tag{4.10}$$
Actually the small positiveness of $\alpha$ is a common feature in models that satisfy $K_X - 2G_{4\phi} = O(1)$ and $G_{4\phi} \neq 0$, provided that the background evolution does not deviate much from that in the $\Lambda$CDM model. This may be seen by comparing eqs. (4.4) and (4.5). If we require $G_{4\phi}$ to be small enough to guarantee the proximity to Newton gravity, namely, $G_{4\phi}^2/G_4 \ll 1$, these two equations implies that $\alpha$ is small and positive if $C_{\text{kin}} = O(1)$ and $G_{\text{eff}} \approx G$.

### 4.4 Non-canonical kinetic term

If we want to consider models with larger values of $\alpha$ and possibly substantial deviations of $G_{\text{eff}}$ from Newton gravity, we need to relax the assumption $C_{\text{kin}} = K_X - 2G_{4\phi} = O(1)$. From eq. (3.12), one notices that $\alpha = O(1)$ can be realized if $2G_{4\phi}^2 \gg C_{\text{kin}}G_4$. Since $K_X = 1$ and $G_3 = 0$ for a canonical scalar field, one has to resort to a non-canonical scalar field model to achieve it. For definiteness, we propose the following model,

$$K(\phi, X) = X + K_2X^2 - V_0, \quad G_3 = \frac{\phi}{2},$$

(4.11)

which gives $C_{\text{kin}} = 2K_2X$. For $K_2 = O(H_0^2M_{pl}^2)$, $C_{\text{kin}} = O(\phi^2/V_0) \ll 1$ for a slowly rolling scalar field. Thus this model can achieve $C_{\text{kin}} \ll 1$ and hence $\alpha = O(1)$. Here we focus on the case $K_2 > 0$ because the scalar field would become non-dynamical if $K_2 = 0$ and it would become a ghost if $K_2 < 0$.

Figure 7 shows the $H - f\sigma_8$ and $\alpha - f\sigma_8$ diagrams in this case for various values of $K_2$ in units of $K_2/(H_0^2M_{pl}^2)$. As seen from the left panel, the background evolution of the Universe is almost same as that in the $\Lambda$CDM, with $f\sigma_8/f\sigma_{8,\Lambda\text{CDM}} > 1$. This behavior is similar to $F(R)$ gravity. The right figure shows that $\alpha = 1$ is certainly realized in this case. $\alpha = 1$ means that the gravitational lensing effect is enhanced by a factor 3/2. We note that, for a more general form of $C_{\text{kin}} = K_X - 2G_{4\phi}$, we may achieve any value of $\alpha$ in the range $0 < \alpha < 1$ while keeping the $\Lambda$CDM like background evolution intact. We also note that $G_{\text{eff}}$ would substantially deviate from the Newtonian $G$ if $G_4 \approx (2k^2)$ as seen from eq. (4.8).

The initial value of the scalar field is chosen rather arbitrarily, except for the model discussed in section 4.3, but in a way that it can exhibit characteristic, qualitative features of each model. For example, in the case of the model considered in section 4.1, the tendency shown in figure 4 remains the same for other choices of the initial conditions.
for section 4.3, the initial condition is chosen so that it reproduces the observationally constrained/indicated evolution of $w$. One may worry about the validity of the small scale approximation $k^2/(a^2 H^2) \gg 1$ because some of modified gravity theories have strong dependence on the wave number $k$. In fact, the matter density perturbation can strongly depend on the scale in Horndeski’s theory if there is a hierarchy between $|K(\phi X)|$, $M_{\text{pl}} H^2 G_3(\phi, X)$, and $H^2 G_4$ or if there is a hierarchy between their derivatives. In the case of $F(R)$ gravity, $|K_{,\phi\phi}| \sim 1/f_{RR} \gg H^2$ should be satisfied to accord with observations and these conditions cause the scale dependence of the matter density perturbation. The reason why the scale dependence appears is that there are two limits [76]: $k^2/(a^2 H^2) \gg 1$ and $1 \gg H^2 f_{RR}$. The Compton wavelength is determined by balancing these two conditions, then the condition $k^2/(a^2 H^2) \gg 1$ is superior to $1 \gg H^2 f_{RR}$ inside the Compton wavelength. The cases we considered in this section do not have such a hierarchy, therefore, the small scale approximation $k^2/(a^2 H^2) \gg 1$ is always valid if we focus on a scale which is deep inside the horizon.

5 Conclusion

We have investigated a class of dark energy models based on Horndeski’s theory of modified gravity which exhibit small but interesting deviations from the ΛCDM model not only in the background evolution but also in the linear perturbation level. The models include the $w$CDM, $F(R)$ gravity, and four kinds of Horndeski gravity models. To classify the properties of these models, we have introduced two diagrams; $H - f\sigma_8$ and $\alpha - f\sigma_8$ diagrams. We have found that these two diagrams provide a useful tool to distinguish the differences in the observational predictions of different models from each other.

Figure 8 shows the summary of our results. The right panel shows the $\alpha - f\sigma_8$ diagram, which exhibits deviations in the behavior of linear perturbations from GR, at $z = 0.5$ for various models with various parameters. The canonical scalar model with linear potential (denoted by $V_1$, discussed in section 4.1) does not have a direct coupling to gravity, hence $\alpha = 0$. The non-minimally coupled scalar model with flat potential (denoted by G4, discussed in section 4.2) gives non-vanishing $\alpha$ but the values are too small ($\sim O(0.01)$) to be seen in the figure, whereas the non-minimally coupled scalar model with quadratic potential (denoted...
Figure 8. Summary of the $H - f\sigma$ (left) and $\alpha - f\sigma_8$ (right) diagrams for various dark energy models at redshift $z = 0.5$. In the legend, $V_1$, $G4$, $G4m^2$, and $G4Kx$ represent the cases studied in subsections 4.1, 4.2, 4.3, and 4.4, respectively. The $wCDM$ model and $F(R)$ gravity model are also plotted, respectively, in the $H - f\sigma$ diagram and $\alpha - f\sigma_8$ diagram for comparison. The black cross expresses on the left panel 1$\sigma$ constraint from the observation by Alam et al. [77]. Note that the horizontal axis of the right panel mixes linear and logarithmic scales for clarity.

by $G4m^2$, discussed in section 4.3). As for the non-minimally coupled scalar model with non-canonical kinetic term (denoted by $G4Kx$, discussed in section 4.4), $\alpha = 1$, that is the dynamics of the linear perturbation is quite different from GR.

In all of the models studied in this paper, we have found $\alpha > 0$. As discussed in appendix B, there exist theoretically acceptable models with negative $\alpha$, but they all satisfy $\alpha < -2$. In fact, the condition $\alpha < -2$ is necessary to guarantee the positivity of $G_{\text{eff}}$ as can be seen from eq. (4.8). Thus either $\alpha > 0$ or $\alpha < -2$, our result implies that gravity becomes effectively stronger than GR in all the models. In other words, if we are to measure the mass density distribution, we would overestimate it if we use the gravitational dynamics, while we would obtain a correct estimate if we use gravitational lensing observations. Whether this has any substantial implications to observational data analysis is an issue to be studied.

The left panel shows the $H - f\sigma_8$ diagram at $z = 0.5$ for various models with various parameters. It shows the dependence of the growth rate of linear perturbations on the difference in the background evolution of the Universe. We see that in all models except for the non-minimally coupled scalar model with quadratic potential or non-canonical kinetic term, there is a tendency that larger $H$ gives smaller $f\sigma_8$ in comparison with $\Lambda$CDM. In the case of the quadratic potential model ($G4m^2$), because of the oscillatory feature in $H$, the fact that $f\sigma_8$ are smaller for smaller $H$ depends on the redshift, as well as on the choice of the model parameters. So it is difficult to discuss a general tendency in this model. In the case of the non-canonical kinetic term model ($G4Kx$), the dependence of $f\sigma_8$ on $H$ seems very small. This is explained by the fact that this model can mimic the $\Lambda$CDM model very well as long as the background evolution of the Universe is concerned.

The cosmic shear power spectrum in weak lensing surveys and the redshift-space power spectrum in galaxy surveys directly constrain $\alpha$ and $f\sigma_8$, respectively. The latter observable can further probe $H$ using BAO as a standard ruler. The blue bars denote observational 1$\sigma$ error bars obtained by Alam et al. [77]. Since it is a 1$\sigma$ constraint, no reliable conclusion can be drawn, but it seems that when compared to the $\Lambda$CDM model, those models that yield slightly larger $H$ with slightly smaller $f\sigma_8$ are preferred. It is, however, important to
note that most of the current observational analyses have been performed assuming ΛCDM and GR, a so-called consistency test. In order to test a modified gravity model, one needs to use a theoretical template of the power spectrum with the given gravity model [47, 78, 80]. Ref. [79] has derived an analytic formula of the matter power spectrum in real space under Horndeski’s theory. Further theoretical efforts are required in order to constrain general modified gravity models on the \( H-f\sigma \) and \( \alpha-f\sigma \) diagrams proposed in this paper.

So far, observational constraints are not so severe yet to exclude any of these models. But eventually we will be able to exclude most of the models as observational accuracies improve. The \( H-f\sigma \) and \( \alpha-f\sigma \) diagrams we introduced, or their variants, may play an important role at such a stage.

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A Horndeski’s theory

Let us first recapitulate the action in Horndeski’s theory [26, 67, 68],

\[
S_H = \sum_{i=2}^{5} \int d^4x \sqrt{-g} \mathcal{L}_i, \tag{A.1}
\]

where

\[
\mathcal{L}_2 = K(\phi, X), \tag{A.2}
\]

\[
\mathcal{L}_3 = -G_3(\phi, X)\Box\phi, \tag{A.3}
\]

\[
\mathcal{L}_4 = G_4(\phi, X)R + G_4X \left[ (\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right], \tag{A.4}
\]

\[
\mathcal{L}_5 = G_5(\phi, X)G_\mu\nu\nabla_\mu \nabla_\nu \phi - \frac{G_5X}{6} \left[ (\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu \nabla_\nu \phi)^2 + 2(\nabla_\mu \nabla_\nu \phi)^3 \right]. \tag{A.5}
\]

Here, \( K, G_3, G_4, \) and \( G_5 \) are generic functions of \( \phi \) and \( X = -\partial_\mu \phi \partial^\mu \phi / 2 \), and the subscript \( X \) means a derivative with respect to \( X \). The total action is the sum of \( S_H \) and the matter action \( S_{\text{matter}} \).

The Friedman equations are given by [68]

\[
\rho_{\text{matter}} + \sum_{i=2}^{5} \mathcal{E}_i = 0, \tag{A.6}
\]

where

\[
\mathcal{E}_2 = 2XK_X - K, \tag{A.7}
\]

\[
\mathcal{E}_3 = 6X\phi HG_3X - 2XG_3\phi, \tag{A.8}
\]

\[
\mathcal{E}_4 = -6H^2G_4 + 24H^2X(G_4X + XG_{4XX}) - 12HX\dot{\phi}G_4X - 6H\dot{\phi}G_4\phi, \tag{A.9}
\]

\[
\mathcal{E}_5 = 2H^2\dot{\phi}(5G_{5X} + 2XG_{5XX}) - 6H^2X(3G_{5\phi} + 2XG_{5\phi X}), \tag{A.10}
\]
and
\[ p_{\text{matter}} + \sum_{i=2}^{5} P_i = 0, \]  
(A.11)

where

\[ P_2 = K, \]  
(A.12)
\[ P_3 = -2X\left(G_{3\phi} + \ddot{\phi}G_{3X}\right), \]  
(A.13)
\[ P_4 = 2(3H^2 + 2\dot{H})G_4 - 4H^2X\left(3 + \frac{\dot{X}}{HX} + 2\frac{\dot{H}}{H^2}\right)G_4X \]
\[ -8HXG_{4XX} + 2(\ddot{\phi} + 2H\dot{\phi})G_{4\phi} + 4XG_{4\phi\phi} + 4X(\ddot{\phi} - 2H\dot{\phi})G_{4\phi X}, \]  
(A.14)
\[ P_5 = -2X(2H^3\phi + 2H\dot{H}\phi + 3H^2\ddot{\phi})G_{5X} - 4H^2X^2\ddot{\phi}G_{5XX} \]
\[ + 4HX(\dot{X} - HX)G_{5\phi X} + 2H^2X\left(3 + 2\frac{\dot{X}}{HX} + 2\frac{\dot{H}}{H^2}\right)G_{5\phi} + 4HX\phi G_{5\phi X}. \]  
(A.15)

Here, \( H = \dot{a}/a \) is the Hubble rate function and the dot means derivative with respect to time and \( p_{\text{matter}} \) and \( \rho_{\text{matter}} \) are the matter energy density and the pressure, respectively. The equation of motion of the scalar field is given by varying the action with respect to \( \phi(t) \):

\[ \frac{1}{a^3} \frac{d}{dt}(a^3J) = P_\phi, \]  
(A.16)

where

\[ J = \dot{\phi}K_X + 6HXG_{3X} - 2\dot{\phi}G_{3\phi} + 6H^2\dot{\phi}(G_{4X} + 2XG_{4XX}) - 12HXG_{4\phi X} \]
\[ + 2H^3X(3G_{5X} + 2XG_{5XX}) - 6H^2\dot{\phi}(G_{5\phi} + XG_{5\phi X}), \]  
(A.17)
\[ P_\phi = K_\phi - 2X(G_{3\phi\phi} + \ddot{\phi}G_{3\phi X}) + 6(2H^2 + \dot{H})G_{4\phi} + 6H(\dot{X} + 2HX)G_{4\phi X} \]
\[ - 6H^2XG_{5\phi\phi} + 2H^3X\dot{\phi}G_{5\phi X}. \]  
(A.18)

Equations (A.6), (A.11), and (A.16) control the background evolution of the Universe. In the same manner as the quintessence model, eqs. (A.11) and (A.16) are equivalent when eq. (A.6) holds. Equations (A.6) and (A.11) can be rewritten in the well-known form

\[ 3H^2 = \kappa^2(\rho_{\text{matter}} + \rho_\phi), \]  
(A.19)
\[ -3H^2 - 2\dot{H} = \kappa^2(p_{\text{matter}} + p_\phi), \]  
(A.20)

where we defined \( \rho_\phi \) and \( p_\phi \) as

\[ \rho_\phi \equiv \sum_{i=2}^{5} \mathcal{E}_i + \frac{3H^2}{\kappa^2}, \quad p_\phi \equiv \sum_{i=2}^{5} P_i - \frac{1}{\kappa^2}(3H^2 + 2\dot{H}). \]  
(A.21)

These equations define the effective energy density and effective pressure, respectively. As for the matter part, we may set \( p_{\text{matter}} = 0 \) as the pressures of both baryons and cold dark matter are negligible.
B Possibility of $\alpha < 0$

As mentioned in section 4, a negative $\alpha$ can be realized in the case $G_{4\phi} \neq 0$ and $G_{3X} \neq 0$. To see this, we recapitulate the expression for $\alpha$,

$$\alpha = \frac{G_{4\phi} (2G_{4\phi} + \phi^2 G_{3X})}{G_4 \left[ K_X - 2G_{3\phi} + \phi(2G_{3X} + \dot{\phi}^2 G_{3XX}) + \dot{\phi}^2 G_{3\phi X} \right]}.$$

(B.1)

Hence we have $\alpha < 0$ if we consider a model with $G_{4\phi} (2G_{4\phi} + \phi^2 G_{3X}) < 0$, which may be realized by making $\dot{\phi}^2 G_{3X}$ the same order of $G_{4\phi}$. A simplest model would be to set $G_{3} = -\chi/m^{3}$ and $K_{X} = 1$ with $m^{3} \lesssim H_{0}^{2} M_{\text{pl}}$. But we have not checked if this model could give an observationally viable model or not.

Another possibility is to consider $C_{\text{kin}} = K_{X} - 2G_{3\phi} \ll 1$. In this case it seems there are many ways to realize a negative $\alpha$. Here let us consider the case $C_{\text{kin}} = K_{X} - 2G_{3\phi} < 0$. In this case, we should take care of no ghost and no gradient instability conditions. No ghost condition and the condition $c_{s}^{2} \geq 0$ are expressed as [75, 81]

$$K_{X} + \dot{\phi}^{2} K_{XX} - 2G_{3\phi} - \dot{\phi}^{2} G_{3\phi X} + 3H \dot{\phi}(2G_{3X} + \dot{\phi}^{2} G_{3XX}) + \frac{3}{4} \left( \frac{2G_{4\phi} - \dot{\phi}^{2} G_{3X}}{G_{4}} \right)^{2} > 0,$$

(B.2)

$$G_{4} C_{\text{kin}} + 4G_{4\phi} - \frac{1}{4} (2G_{4\phi} - \dot{\phi}^{2} G_{3X})^{2} \geq 0,$$

(B.3)

where $G_{4} > 0$ is assumed to prevent the instability in the tensor perturbation.

A simple example to realize a negative $\alpha$ without instabilities is the case $G_{3} = 0$, $K(\phi, X) = -c X$, $c > 0$, and $G_{4} = e^{\lambda \phi / M_{\text{pl}}} / (2\alpha^{2})$. In this case, the stability conditions (B.2) and (B.3) are expressed by a single equation;

$$-c G_{4} + 3G_{4\phi}^{2} > 0,$$

(B.4)

which is re-expressed as

$$8\pi c < \frac{3}{2} \lambda^{2} \exp[\lambda \phi / M_{\text{pl}}].$$

(B.5)

The expressions for $\alpha$ and $G_{\text{eff}}$ are given by

$$\alpha = \frac{2G_{4\phi}^{2}}{-c G_{4\phi} + 2G_{4\phi}^{2}} = \frac{\lambda^{2} e^{\lambda \phi / M_{\text{pl}}}}{-8\pi c + \lambda^{2} e^{\lambda \phi / M_{\text{pl}}}}$$

$$G_{\text{eff}} = \frac{-c G_{4} + 4G_{4\phi}^{2}}{16\pi G_{4}(-c G_{4} + 3G_{4\phi}^{2})} = \exp[-\lambda \phi / M_{\text{pl}}] \frac{16\pi c - 4\lambda^{2} e^{\lambda \phi / M_{\text{pl}}}}{16\pi c - 3\lambda^{2} e^{\lambda \phi / M_{\text{pl}}} G} .$$

(B.6)

We note that this gives the same expression for $G_{\text{eff}}$ as the one obtained for the models discussed in section 4 when expressed in terms of $\alpha$,

$$G_{\text{eff}} = \frac{2 + 2\alpha}{2 + \alpha} \exp[-\lambda \phi / M_{\text{pl}}] G .$$

(B.7)

The stability condition (B.5) implies that $\alpha$ may take the value in the range, $-\infty < \alpha < -2$ and $0 \leq \alpha < \infty$. The stability also guarantees the positivity of $G_{\text{eff}}$. A negative $\alpha$ can be realized for $\lambda^{2} e^{\lambda \phi / M_{\text{pl}}} < 8\pi c < 3\lambda^{2} e^{\lambda \phi / M_{\text{pl}}} / 2$. Both for negative and positive values of $\alpha$, the effective gravitational force becomes twice as strong as GR in the limit of large $|\alpha|$.
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