Cosmological perturbations from an inhomogeneous phase transition

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Abstract
A mechanism for generating metric perturbations in inflationary models is considered. Long-wavelength inhomogeneities of light scalar fields in a decoupled sector may give rise to superhorizon fluctuations of couplings and masses in the low-energy effective action. Cosmological phase transitions may then occur that are not simultaneous in space, but occur with time lags in different Hubble patches that arise from the long-wavelength inhomogeneities. Here an interesting model in which cosmological perturbations may be created at the electroweak phase transition is considered. The results show that phase transitions may be a generic source of non-Gaussianity.

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1. Introduction
The energy density during inflation is dominated by the inflaton potential energy. At the end of inflation, the energy stored in the inflaton potential is converted into particles, which decay and reheat the universe by thermalization to start the standard hot big bang phase.

In this paper, we consider a phase transition from a phase, \( A \), to another phase, \( B \), which are distinguished by the scaling of the energy density of a component \( \rho_i \). Namely, we consider a scaling of the energy density \( \rho_i \propto a^{-n_A} \) in phase \( A \) and \( \rho_i \propto a^{-n_B} \) in phase \( B \), where \( n_A \neq n_B \) causes generation of the density perturbations when the phase transition is inhomogeneous in space. The mechanism is very general and can be applied to many other models in which the transition between phases of different scaling is inhomogeneous in space, even if the transition is not a ‘phases transition’ in the strict meaning.

Before discussing an inhomogeneous phase transition, we review the mechanism of inhomogeneous reheating [1] to illustrate the basis of inhomogeneous scenarios. In [1], it has been argued that in realistic models of inflation the coupling of the inflaton to matter can be determined by the vacuum expectation values of fields in the underlying theory. If those fields
Due to the $\Gamma_\psi$ inhomogeneity, the decay of the $\psi$ particles does not occur simultaneously in space, which leads to a fluctuation of $t_{\text{dec}}$ and $\rho_{\text{dec}}/\rho_{\text{rh}}$. Thus, the evolution of the energy density is different in different patches, which results in density fluctuations.

(in the string theory they would be moduli fields from the compactified space) are light during inflation, they will fluctuate leading to density perturbations through the inhomogeneities of the coupling constants.

If the density perturbations created during inflation are negligible and the universe after inflation is filled with particles $\psi$ of mass $M_\psi$ and decay rate $\Gamma_\psi < H_I$, where $H_I$ is the Hubble parameter during inflation, spatial inhomogeneities in $\Gamma_\psi$ may lead to density perturbations when the particles decay into radiation. In deriving the magnitude of the density perturbations arising from the inhomogeneity, it is useful to compare the energy density in a region to the virtual hidden radiation $\rho_{\text{rh}}$, which scales as

$$\rho_{\text{rh}} \propto a^{-4},$$

and calculate the density perturbations on a uniform $\rho_{\text{rh}}$ surface. Here, we assume that there is no energy transition between the radiation density $\rho_{\text{rh}}$ and other components of the universe. Assuming that the domination by $\psi$ particles starts at $a_{\text{dom}} \equiv a(t_{\text{dom}})$ when $\rho_{\text{dom}} \equiv \rho(t_{\text{dom}}) \simeq \rho_\psi(t_{\text{dom}}) \simeq M_\psi^4$, the energy of the $\psi$ particles scales as matter in the domination interval, $t_{\text{dom}} < t < t_{\text{dec}}$:

$$\rho_\psi \simeq \rho \propto a^{-3},$$

with decay time $t \equiv t_{\text{dec}}$ defined by

$$\rho_{\text{dec}} \equiv \rho(t_{\text{dec}}) \simeq \Gamma_\psi^2 M_\psi^2.$$

Outside the domination interval, we assume that the energy density scales as radiation. Figure 1 shows a schematic representation of the inhomogeneous boundary that creates density fluctuations. Note that in this model the delay of the $\psi$ decay causes a delay in the evolution of the energy density. The calculation of the density perturbation is straightforward. Considering the $\psi$ domination interval, we find

$$\left( \frac{a_{\text{dec}}}{a_{\text{dom}}} \right)^3 = \frac{\rho_{\text{dom}}}{\rho_{\text{dec}}} = \frac{\rho_{\text{dom}}}{\Gamma_\psi^2 M_\psi^2}.$$

1 The ‘virtual hidden radiation’ is introduced just to keep track of the unperturbed spatially flat hypersurfaces.

2 Here we assume that the mass of the $\psi$ particles is a constant. Unlike the original argument in [1], we consider a uniform $\rho_{\text{dom}}$ and $\delta M_\psi = 0$ to simplify the argument.
where $\rho_{\text{dom}}$ and $M_p$ are uniform in space, while $\Gamma_\psi$ is inhomogeneous. Using $\rho_{\text{ch}}'$ in equation (1.1), we can find the energy density after the decay:

$$\rho \propto \frac{a_{\text{dec}}}{a_{\text{dom}}} \rho_{\text{ch}}' = \frac{\rho_{\text{dom}}}{\Gamma_\psi M_p^2} \Gamma_\psi^{2/3} \rho_{\text{ch}}' \rho_{\text{rh}}.$$  (1.5)

where the ratio $\rho / \rho_{\text{ch}}'$ is a time-independent constant after the decay. The density perturbation on a uniform $\rho_{\text{ch}}'$ surface is thus given by

$$\frac{\delta \rho}{\rho} = -\frac{2}{3} \frac{\delta \Gamma_\psi}{\Gamma_\psi},$$  (1.6)

which reproduces the limit $\Gamma_\psi / H_I \rightarrow 0$ in [2].

Another way to generate cosmological perturbations from an inhomogeneous boundary is to consider an inhomogeneous end for the inflationary phase [3–6]. For inflationary expansion, the equation for the number of e-foldings is

$$N \equiv \ln \frac{a(t_e)}{a(t_N)},$$  (1.7)

where $t_N$ is the time when the long-wavelength inhomogeneity exits the horizon, and $t_e$ is the time when inflation ends. We define $\phi_N \equiv \phi(t_N)$ and $\phi_e \equiv \phi(t_e)$ for the inflaton field $\phi$.

Using $\rho_{\text{ch}}'$ and repeating the calculation given above, in place of equations (1.4) and (1.5), we obtain

$$ \left( \frac{a(t_e)}{a(t_N)} \right)^{1/3} \rho_{\text{ch}}' = e^{4N} \rho_{\text{ch}}'. $$  (1.8)

If we assume instant decay and instant thermalization after inflation, the energy density of the universe after inflation scales as radiation, and $\rho / \rho_{\text{ch}}'$ is a time-independent constant after inflation. Therefore, the density perturbation on a uniform $\rho_{\text{ch}}'$ surface, which is caused by the inhomogeneities in $N$, is given by

$$\frac{\delta \rho}{\rho} = 4\delta N,$$  (1.9)

and recovers the conventional $\delta N$ formula $\zeta = \delta N$. More specifically, we can calculate $\delta N$ from the $\phi_e$ inhomogeneity in the inflationary scenario using a very simple equation, $\delta N_{\phi_e} \simeq (\partial N/\partial \phi_e) \delta \phi_e$. In most inflationary scenarios, $N$ is given explicitly by $\phi_N$ and $\phi_e$.

Considering these two scenarios discussed above, the curvature perturbations created by the inhomogeneous boundaries are natural consequences of the inhomogeneities arising from long-wavelength fluctuations of light fields. In this paper, we consider inhomogeneous phase transitions in which the critical temperature is not homogeneous in space. If the potential energy dominates during a short interval, the phase may be dubbed mini-inflation. Following the uniform $\rho_{\text{ch}}$ calculation discussed above, we can calculate the density perturbations created at the phase boundary. To calculate the density perturbations we assume: (1) the beginning of the phase occurs simultaneously in space, but the end is inhomogeneous, (2) the transition occurs instantly just after the interval, and (3) all the energy stored in the potential is translated into radiation. Complementary scenarios for more generic situations require numerical study and are highly model dependent; thus they will be considered in future works. However,

3 See the appendix for the definition of the curvature perturbation $\zeta$ and the $\delta N$ formula that relates $\zeta$ to $\delta N$. 
we consider a particularly attractive model, featuring the possibility of inhomogeneous phase transitions at the electroweak (or more generically, unification) scale that may lead to the creation of a significant level of non-Gaussianity.

2. The model

2.1. A simple model for a second-order phase transition

To illustrate some typical features of finite temperature effects, we consider a real scalar field and a potential:

\[
L = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi),
\]

\[
V(\phi) = V_0 - \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{4} \lambda \phi^4,
\]

where \( V_0 \) is tuned so that the cosmological constant vanishes at the true minimum. The phenomenon of high-temperature symmetry restoration can be understood by the finite-temperature effective potential given by [7]

\[
V_T(\phi_c) = V(\phi_c) + \frac{T^4}{2\pi^2} \int_0^\infty dx \ln \left[ 1 - \exp \left( -\sqrt{x^2 + \frac{m_\phi^2 + 3\lambda \phi^2}{\mu^2}} \right) \right],
\]

where \( V(\phi_c) \) is the one-loop potential for zero temperature with the classical field \( \phi_c \):

\[
V(\phi_c) = -\frac{1}{2} m_\phi^2 \phi_c^2 + \frac{1}{4} \lambda \phi_c^4 + \frac{1}{64\pi^2} (m_\phi^2 + 3\lambda \phi_c^2)^2 \ln \left( \frac{m_\phi^2 + 3\lambda \phi_c^2}{\mu^2} \right),
\]

where \( \mu \) is a renormalization mass scale. At high temperatures, \( V_T \) can be expanded as

\[
V_T \simeq V(\phi_c) + \frac{1}{6} \lambda T^2 \phi_c^2 + O(T^4),
\]

which suggests that the temperature-corrected effective mass at \( \phi_c = 0 \) changes sign at a critical temperature:

\[
T_c \simeq \frac{2m_\phi}{\sqrt{\lambda}}.
\]

In a more general situation, one may introduce couplings to the fields in the background thermal bath. If the couplings of \( \phi \) to the fields in the background thermal bath are more significant than the self-coupling, a typical form of the potential with a thermal correction term is given by

\[
V = V_0 + (g^2 T^2 - \frac{1}{2} m_\phi^2) \phi^2 + \cdots,
\]

where \( g \) denotes the effective coupling of \( \phi \) to the fields in the thermal bath. In this case, the critical temperature is given by

\[
T_c \simeq \frac{m_\phi}{2g}.
\]

In this section, we consider the latter case where \( T_c \) is given by \( T_c \simeq \frac{m_\phi}{2g} \).

The phase transition is second order in the model discussed above. We consider two distinct cases:

(i) The energy density of the universe is dominated by the potential energy \( V_0 \) during the interval \( T_{dom} > T > T_c \). The universe is then dominated by radiation, due to instant decay. We assume that all the energy stored in the potential is converted into radiation just after the phase transition (i.e., we assume \( T_c = T_{dec} \). See also figure 2). Note that such a
Initially, the universe is dominated by radiation. The potential energy then starts to dominate at $T = T_{\text{dom}}$. The domination by the potential ends at $T = T_c$, where a phase transition occurs. Radiation domination starts after the phase transition.

The dominance of the vacuum potential cannot be attained in the simple $\phi^4$ model according to the Lagrangian (2.1). Additional degrees of freedom need to be present such that the effective potential takes the form (2.6). At the same time, $\lambda$ must be chosen sufficiently small to ensure a large value for $V_0$. This gives the condition $V_0 \sim m_\phi^4/4\lambda > \rho(T_c)$, which leads to $\lambda < O(0.1) \times g^4$.

The energy density of the universe is still dominated by radiation at $T = T_c$. After the phase transition at $T = T_c$, all the potential energy is converted into non-relativistic particles $\psi$ that scale as matter. The interval of the radiation domination may end at $T = T_{\text{dom}}$ when $\rho_\psi/\rho_{\text{rad}} \simeq 1$, or more generically the $\psi$ particles may decay into radiation at $T = T_{\text{dec}}$ before the domination. In this scenario, the inhomogeneous phase transition causes the inhomogeneities of the matter density. See also [13] in which the inhomogeneities of the curvatons are generated by inhomogeneous preheating.

In the former case, calculating the density perturbation is straightforward. We assume that the interval of domination by the potential energy starts at $T = T_{\text{dom}} \equiv T(t_{\text{dom}})$. Considering $\rho_{\text{ch}} \propto a^{-4}$ as before, after the phase transition at $T = T_c \equiv T(t_c)$ we find that

$$
\rho_\phi(t) \propto \left(\frac{T_{\text{dom}}}{T_c}\right)^4 \rho_{\text{ch}}.
$$

Just after the phase transition, the potential energy is converted into radiation. The energy density perturbation on a uniform $\rho_{\text{ch}}$ surface is thus given by

$$
\frac{\delta \rho}{\rho} = -4 \frac{\delta T_c}{T_c} = -4 \frac{\delta m_\phi}{m_\phi} + 4 \frac{\delta g}{g},
$$

where the curvature perturbation is given by

$$
\zeta = \frac{1}{4} \frac{\delta \rho}{\rho} = -\frac{\delta T_c}{T_c} = -\frac{\delta m_\phi}{m_\phi} + \frac{\delta g}{g}.
$$

This result can be obtained alternatively from the $\delta N$ formula $\zeta = \delta N$. For inflationary expansion during the $V_0$-dominated interval [8], the number of e-foldings is given by

$$
N = \ln \left(\frac{T_{\text{dom}}}{T_c}\right).
$$
which leads to \( \zeta = -\delta N = -\delta T_c/T_c \). In order to calculate the pure contribution from the inhomogeneous phase transition, we assume that all the energy stored in the potential is converted into radiation just after inflation. To understand the light-field potential, we consider a specific choice for the \( \sigma \)-dependent mass:

\[
m^2_\sigma(\sigma) = m^2_0 \left( 1 + \alpha \frac{\sigma^2}{\Lambda^2} \right),
\]

where \( \sigma \) is the light field, and \( \Lambda \) is the cut-off scale of the effective action. Note that a conventional interaction, \( \sim \alpha m^2_0 \sigma^2 \phi^2 / \Lambda^2 \), in the effective low-energy action may induce the \( \sigma \)-dependent mass. In this case, the thermal correction to the mass of the light field is \( m^2_\sigma(T) \approx \frac{\alpha m^2_0}{\Lambda^2} T^2 \), which is supposed to be smaller than the Hubble parameter, \( H^2 \approx \text{Max} \{ \rho^{\text{rad}}, V_0 \}/3 M^2_p \), as in the inhomogeneous reheating scenario discussed in [1]. If there is no significant potential other than the finite-temperature effective potential \( V(\phi) \), we find an effectively flat \( \sigma \)-potential during the symmetry restoration phase. Since the interaction depends on the values of the fields \( \sigma \) and \( \phi \), the background field trajectories after the phase transition may be sensitive to the initial conditions and the non-perturbative effects of the decay process, which means that the general evaluation of the cosmological parameters after the phase transition typically requires numerical calculations [9]. However, the numerical study related to such a non-perturbative process after the phase transition is highly model dependent and out of the scope of this paper. We thus assume that all the energy stored in the potential is instantly converted into radiation just after the phase transition, in order to single out the contribution from the inhomogeneous phase transition. In addition to the complexities of the decay process, the domain walls related to discrete symmetry breaking may cause a problem. However, cosmological domain walls can be made unstable and safe if a bias between the two vacua is induced by an effective interaction term that breaks the \( Z_2 \) symmetry. Note that for supergravity, domain walls caused by \( R \)-symmetry are safe since the supergravity interaction creates the required bias [10]. Therefore, for simplicity and to allow the calculation of the model-independent contribution from the inhomogeneous phase transition, we ignore the domain-wall problem in this paper, expecting that the walls decay instantly into radiation due to the bias between the two vacua.

The latter scenario is less trivial. Let us consider the case in which the energy density of the universe is dominated by radiation at \( T = T_c \) and the non-relativistic \( \psi \) particles decay into radiation at \( T = T_{\text{dec}} < T_c \), as is shown in figure 3. We assume that all the potential energy is translated into \( \psi \) particles at \( T = T_c \). We introduce the ratio \( r \equiv \rho_\psi / \rho \) and consider the case in which \( \psi \) does not dominate the universe (i.e., \( r(t_{\text{dec}}) < 1 \)). Assuming that the symmetry restoration phase starts at some uniform temperature \( T = T_R \), and introducing \( \rho_{\text{r}}^{\text{ih}} \) as before, at \( t = t_{\text{dec}} \) we find

\[
\rho_\psi(t) \propto \left( \frac{T_R}{T_c} \right)^4 \left( \frac{T_c}{T_{\text{dec}}} \right) \rho_{\text{r}}^{\text{ih}},
\]

where \( \rho_{\text{r}}^{\text{ih}} \) scales like radiation. The decay temperature, \( T_{\text{dec}} \equiv T(t_{\text{dec}}) \), is determined by

\[
\rho(t_{\text{dec}}) \approx \Gamma_\phi M^2_p,
\]

where we assume \( \delta \Gamma_\phi = 0 \). Therefore, the density perturbation is given by

\[
\frac{\delta \rho}{\rho} = -3 r \frac{\delta T_c}{T_c}.
\]
Initially, the universe is dominated by radiation. The potential energy is converted into non-relativistic matter at $T = T_c$. Then the matter decays into radiation at $T = T_{\text{dec}}$. In the figure, we show a case in which the non-relativistic matter dominates before the decay, but it is possible to consider a case in which the decay into radiation occurs before domination, as is discussed in the text.

which leads to the density perturbation given by

$$\frac{\delta \rho}{\rho} = -4r \frac{\delta T_c}{T_c}. \quad (2.17)$$

### 2.2. Non-thermal trapping

In the simple second-order example, we consider effective couplings that depend on light fields. Long-wavelength inhomogeneities of the light fields may lead to an inhomogeneous critical temperature $\delta T_c \neq 0$. Here, we consider another example, in which long-wavelength inhomogeneities of the number density of some particles cause an inhomogeneous end of the symmetry restoration phase.

During preheating, some of the kinetic energy of the inflaton is converted into excitations of the preheat field $\chi$. If $\chi$ couples to a field $\phi_a$ with a potential,

$$V(\phi_a, \chi) \sim V_0 - \frac{1}{2} m^2 \phi_a^2 + \frac{\phi_a^4}{\Lambda_{n-4}^2} - \frac{g^2}{2} \phi_a^2 \chi^2. \quad (2.18)$$

where the inflaton terms are omitted, the effective potential caused by the high density of the preheat field is given by [5, 11]

$$V_{\text{eff}}(\phi_a) \simeq V_0 - \frac{1}{2} m^2 \phi_a^2 + \frac{\phi_a^4}{\Lambda_{n-4}^2} + g |\phi_a| n_{\chi}. \quad (2.19)$$

For $\phi_a > 0$, the effective potential gives

$$V_{\text{eff}}(\phi_a) \simeq V_0 - \frac{1}{2} m^2 \left( \phi_a^2 - \frac{g n_{\chi}}{m^2} \right)^2 + \frac{g^2 n_{\chi}^2}{2m^2}. \quad (2.20)$$

When $n_{\chi}$ is very large, the field $\phi_a$ is trapped by a strong attraction from the origin. During the interval of the trapping, the potential barrier decreases, since $n_{\chi}$ scales as $n_{\chi} \propto a^{-3}$, and ultimately tunneling occurs below the critical number density [5, 11] given by

$$n_{\chi} \lesssim n_c \equiv \frac{m^3}{g}. \quad (2.21)$$
If the energy density is dominated by the potential energy $V_0$, the trapping leads to an inflationary expansion. The number of e-foldings elapsed during this interval is

$$N = \frac{1}{3} \ln \left( \frac{n_x(t_i)}{n_x(t_e)} \right),$$

(2.22)

where $t_i$ and $t_e$ are the time when the domination by the potential energy starts and when the inflationary expansion ends. In this model, inhomogeneities in the initial number density, $n_x(t_i)$, can be created by inhomogeneous preheating [12, 13]. The inhomogeneities in the preheating arise from the long-wavelength fluctuations of the multi-field trajectory for the symmetry-breaking potential. This is the origin of $\delta N$ discussed in [5]. In addition to the inhomogeneities in $\delta n_x(t_i)$, we may consider inhomogeneities in $n_c$, which are non-zero if $m$ and $g$ are modulated at the end of the trapping phase. Using the $\delta N$ formalism, for $\delta n_x(t_e) \simeq \delta n_c$ we find

$$\delta N = -\frac{1}{3} \frac{\delta n_c}{n_c} \simeq -\frac{\delta m}{m} + \frac{1}{3} \frac{\delta g}{g}.$$

(2.23)

2.3. Electroweak phase transition

The main obstacle in building a model of an inhomogeneous electroweak phase transition is that the long-wavelength inhomogeneities of the light field must survive until the electroweak phase transition, when the Hubble parameter is much lower than the gravitino mass. In supergravity models inspired by string theory, there are many light fields (moduli) in the effective action, but typically the mass of the moduli fields is expected to be of the same order as the gravitino mass, where the gravitino mass is generically given by $m_{3/2} \sim \Lambda_{\text{SUSY}}^2 / M_p$, where $\Lambda_{\text{SUSY}} > \text{TeV}$ is the supersymmetry breaking scale [14]. The mass of the moduli $m_{3/2}$ is clearly larger than $H_{\text{EW}} \equiv T_{\text{EW}}^2 / M_p$, where $T_{\text{EW}}$ is the critical temperature for the electroweak phase transition. Therefore, if an inhomogeneous phase transition occurs at the electroweak phase transition, the inhomogeneities of the effective action must be inherited from the fluctuations of the light fields whose potential is protected by symmetry before the electroweak phase transition, while the moduli potential must be lifted after the phase transition. The above condition for the inhomogeneous electroweak phase transition might seem very severe, but in string theory there is at least one specific example that may induce an inhomogeneous phase transition at the electroweak scale. We consider intersecting D-brane models, which are an interesting possibility for string model building, allowing us to devise models that are sensibly close to the minimal supersymmetric standard model (MSSM) in terms of particles and gauge groups [15]. A remarkable feature of this scenario is that the flavor structure of the Yukawa couplings may arise from the matter fields located at different intersections, with the resulting Yukawa couplings expressed by the classical instanton action of the minimal world-sheet area:

$$Y \propto \exp \left( -\frac{A}{2\pi \alpha'} \right),$$

(2.24)

where $A$ is the minimal world-sheet area of the intersection. If the model is constructed from D6-branes in type IIA string theory wrapping orientifolds of $R_4 \times T^2 \times T^2 \times T^2$ [15], there will be shift symmetries that correspond to the brane motion in the internal space. If the shift symmetries are not broken in the effective action, the minimal world-sheet area remains

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4 Inhomogeneous preheating accompanied by instant decay may directly lead to the creation of curvature perturbation [12]. In this section, we consider a preheating field that does not lead to instant decay [11, 13].
as an arbitrary parameter. Considering moduli fields $\sigma_i, (i = 1, 2, 3)$ for the three branes constituting a triangle in the internal space, we find

$$\delta A(\sigma_i) \simeq \sum_i \frac{\partial A}{\partial \sigma_i} \delta \sigma_i + \sum_{ij} \frac{\partial^2 A}{\partial \sigma_i \partial \sigma_j} \delta \sigma_i \delta \sigma_j. \quad (2.25)$$

It would be better for our purpose to consider a simple form of $A(\sigma)$ and consider the inhomogeneity $\delta \sigma$ to obtain

$$\delta A(\sigma) \simeq A' \delta \sigma + A''(\delta \sigma)^2 \equiv \alpha_1 \frac{\delta \sigma}{\Lambda_A} + \alpha_2 \left( \frac{\delta \sigma}{\Lambda_A} \right)^2. \quad (2.26)$$

Let us consider a possible mechanism for generating an effective potential related to $A$. Assuming that the Yukawa couplings are generated by the mechanism and considering the standard one-loop correction to the Higgs field potential from the top fermion loop, we obtain

$$\Delta m^2_H \sim -\frac{3}{4\pi^2} Y^2_t m^2_{\text{top}} \ln \left( \frac{\mu}{m_{\text{top}}} \right), \quad (2.27)$$

where $m_{\text{top}}$ denotes the scalar top mass [17]. Here we consider $Y_t \propto \exp(-A/(2\pi\alpha'))$. In the MSSM, the one-loop correction from the top Yukawa coupling destabilizes the Higgs potential and causes electroweak symmetry breaking. From the one-loop correction term in equation (2.27), we find that the world-sheet area can be stabilized after the electroweak symmetry breaking [16]. In this case, the free motion of the D6-branes in the internal space is protected by the shift symmetry before the electroweak symmetry breaking. However, after the electroweak symmetry breaking, which is induced by the loop correction in the MSSM electroweak symmetry-breaking scenario, the shift symmetries are partly broken, and the minimal world-sheet area is stabilized in the low-energy effective action. Although the scenario depends greatly on the specific details of the intersecting brane models, a generic implication of the scenario is that the inhomogeneous phase transition occurs whenever the shift symmetries are not explicitly broken before the phase transition. A similar mechanism may work at the GUT phase transition, and the inhomogeneous phase transition may lead to a cosmological signature of the intersecting brane models.

3. Conclusions and discussions

We have studied a mechanism for generating primordial density perturbations in inflationary models. We considered long-wavelength inhomogeneities of light scalar fields that cause superhorizon fluctuations of couplings and masses in the effective low-energy action. Since the effective couplings and masses are not homogeneous in space, cosmological phase transitions may occur that are not simultaneous in space. It is possible to create the primordial curvature perturbation from the mechanism, but more generally, the scenario of an inhomogeneous phase transition allows for non-Gaussianity to occur in the spectrum after inflation [18, 19]. It is useful to specify the level of non-Gaussianity by the nonlinear parameter $f_{\text{NL}}$, which is usually defined by the Bardeen potential $\Phi$:

$$\Phi = \Phi_{\text{Gaussian}} + f_{\text{NL}} \Phi_{\text{Gaussian}}^2. \quad (3.1)$$

Using the Bardeen potential, the curvature perturbation $\zeta$ is given by

$$\Phi = \frac{1}{2} \zeta. \quad (3.2)$$
When we consider ‘additional’ non-Gaussianity created at the inhomogeneous phase transition, the first-order perturbation is generated dominantly by the usual inflaton perturbation. Therefore, the ‘additional’ second-order perturbation is not correlated to the first-order perturbation. In this case, the nonlinear parameter is estimated as [20]

$$6 \frac{f_{\text{NL}}}{5} \simeq \frac{1}{N_\phi^2} \left[ N_\sigma^2 N_{\sigma \sigma} + N_{\sigma \sigma} P_\sigma \log(k_b L) \right],$$  \hspace{1cm} (3.3)

where $\zeta$ can be expanded by the $\delta N$ formalism as

$$\zeta \simeq N_\phi \delta \phi + N_\sigma \delta \sigma + \frac{1}{2} N_{\phi \phi} \delta \phi^2 + \frac{1}{2} N_{\sigma \sigma} \delta \sigma^2 + \cdots,$$  \hspace{1cm} (3.4)

and we assume that the perturbation can be separated as

$$\zeta = \zeta^{(\phi)} + \zeta^{(\sigma)}.$$  \hspace{1cm} (3.5)

Here $k_b \equiv \min(k_i)$ ($i = 1, 2, 3$) is the minimum wavevector of the bispectrum, and $L$ is the size of a box in which the perturbation is defined. A useful simplification is [21]

$$f_{\text{NL}} \simeq \left( \frac{1}{1300} \frac{N_{\sigma \sigma}}{N_\phi^2} \right)^3.$$  \hspace{1cm} (3.6)

The scenario of adding non-Gaussianity from the inhomogeneous phase transition is interesting, since for the effective low-energy action, higher-dimensional couplings may naturally appear with light fields in a decoupled sector.

Consider a simple example discussed in section 2.1 with $\delta m_\phi \neq 0$ and $\delta g = \delta \lambda = 0$. Considering the initial value for the light field $\sigma$ in equation (2.12), a modest assumption would be $\sigma \simeq 0$. From equation (2.10), the curvature perturbation created from the inhomogeneous phase transition is purely second order and given by

$$\zeta^{(\sigma)} \simeq -\frac{\delta m_\phi}{m_\phi} \simeq -\alpha \frac{\delta \sigma^2}{2\Lambda^2} = -\frac{\alpha H_I^2}{2\Lambda^2(2\pi)^2},$$  \hspace{1cm} (3.7)

where $H_I$ is the Hubble parameter when the long-wavelength inhomogeneity of the light field $\sigma$ exits the horizon during inflation. Thus we find from the $\delta N$ formula;

$$N_{\sigma \sigma} = -\frac{\alpha}{\Lambda^2}.$$  \hspace{1cm} (3.8)

Even for the initial condition $\sigma \simeq 0$, the nonlinear parameter for the inhomogeneous phase transition is significant. Considering the usual normalization for the first-order perturbation, we find [8]

$$|N_\phi \delta \phi| \simeq 5 \times 10^{-5}.$$  \hspace{1cm} (3.9)

The nonlinear parameter is thus given by

$$f_{\text{NL}} \simeq \left( 10^6 \times \frac{H_I^2}{\Lambda^2} \right)^3.$$  \hspace{1cm} (3.10)

Considering the modest bound for the nonlinear parameter $|f_{\text{NL}}| < 100$, the above result puts a significant upper bound on the inflationary scale or on the effective couplings that contain decoupled light fields.

For the electroweak phase transition, we find for the simple case ($A = A(\sigma)$):

$$\zeta^{(\sigma)} = -\frac{\delta (\Delta m_H)}{\Delta m_H} \simeq \frac{\delta A}{2\pi \alpha'} \simeq \frac{\alpha_1}{2\pi \alpha'} \frac{\delta \sigma}{\Lambda_A} + \frac{\alpha_2}{2\pi \alpha'} \left( \frac{\delta \sigma}{\Lambda_A} \right)^2.$$  \hspace{1cm} (3.11)

With regard to the non-Gaussianity, we find from the above equation that $\alpha$ in the standard
Calculation is simply replaced by $-\alpha_2/(2\pi\alpha')$ for the electroweak phase transition with the effective scale $\Lambda_A = \Lambda$.

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Appendix. $\delta N$ formalism for the curvature perturbation

Here we consider two different definitions for the curvature perturbations [22]. The comoving curvature perturbation ($\zeta$) can be related to the curvature perturbation on uniform-density hypersurfaces ($\zeta$) by studying the evolution at large scales. The gauge-invariant combinations for the curvature perturbations can be constructed as follows:

$$
\zeta = -\psi - H \frac{\delta \rho}{\rho}, \quad R = \psi - H \frac{\delta q}{\rho + p},
$$

(A.1)

where $\delta q$ is the momentum perturbation that is expressed as $\delta q = -\dot{\phi}\delta \phi$ for the inflaton $\phi$ with a standard kinetic term. Linear scalar perturbations of a Friedman–Robertson–Walker (FRW) background were considered:

$$
\text{d}s^2 = -(1 + 2\Lambda) \text{d}t^2 + 2a^2(t)\nabla_i B \text{d}x^i \text{d}t + a^2(t)[(1 - 2\psi)\gamma_{ij} + 2\nabla_i \nabla_j E] \text{d}x^i \text{d}x^j.
$$

(A.2)

Here $\rho$ and $p$ denote the energy density and the pressure. Spatially flat hypersurfaces and uniform-density hypersurfaces are defined by $\psi = 0$ and $\delta \rho = 0$, respectively.

Besides the curvature perturbations defined above, it is useful to define the perturbed expansion rate with respect to the coordinate time. The perturbed expansion rate is expressed as

$$
\delta \tilde{\theta} \equiv -3\dot{\psi} + \nabla^2 \sigma,
$$

(A.3)

where the scalar describing the shear is

$$
\sigma = \dot{E} - B.
$$

(A.4)

Choosing the gauge whose slicing is flat at $t_{\text{ini}}$ and uniform density at $t$, the $\delta N$ formula is given by

$$
\zeta = \frac{1}{3} \int_{t_{\text{ini}}}^{t} \delta \tilde{\theta} \, \text{d}t = \delta N.
$$

(A.5)

The $\delta N$ formula is sometimes expressed by

$$
\zeta = \delta N = -H \frac{\delta \rho}{\rho} \bigg|_{\psi = 0},
$$

(A.6)

where $\delta N$ is the perturbed expansion to uniform-density hypersurfaces with respect to spatially flat hypersurfaces, and $\delta \rho$ must be evaluated on spatially flat hypersurfaces.

5 In this appendix, ‘$\psi$’ is used for a metric perturbation.
References

[1] Dvali G, Gruzinov A and Zaldarriaga M 2004 Cosmological perturbations from inhomogeneous reheating, freezeout, and mass domination Phys. Rev. D 69 083505 (arXiv:astro-ph/0305548)
[2] Dvali G, Gruzinov A and Zaldarriaga M 2004 A new mechanism for generating density perturbations from inflaton Phys. Rev. D 69 023505 (arXiv:astro-ph/0303591)
[3] Bernardeau F, Kofman L and Uzan JP 2004 Modulated fluctuations from hybrid inflation Phys. Rev. D 70 083004 (arXiv:astro-ph/0403315)
[4] Lyth D H 2005 Generating the curvature perturbation at the end of inflation J. Cosmol. Astropart. Phys. JCAP11(2005)006 (arXiv:astro-ph/0510443)
[5] Matsuda T 2007 Cosmological perturbations from inhomogeneous preheating and multi-field trapping J. High Energy Phys. JHEP07(2007)035 (arXiv:0707.0543 [hep-th])
[6] Matsuda T 2006 Elliptic inflation: generating the curvature perturbation without slow-roll J. Cosmol. Astropart. Phys. JCAP09(2006)003 (arXiv:hep-ph/0606137)
[7] Lyth D H and Riotto A 2006 Generating the curvature perturbation at the end of inflation in string theory Phys. Rev. Lett. 97 121301 (arXiv:astro-ph/0607326)
[8] Matsuda T 2003 Non-tachyonic brane inflation Phys. Rev. D 67 083519 (arXiv:hep-th/0302035)
[9] Kolb E W and Turner M S 1990 The early universe Front. Phys. 69 1
[10] Matsuda T 2000 Weak scale inflation and unstable domain walls Phys. Lett. B 486 300 (arXiv:hep-th/0002194)
[11] Matsuda T 1998 On the cosmological domain wall problem in supersymmetric models Phys. Lett. B 436 264 (arXiv:hep-th/9804409)
[12] Matsuda T 2008 Non-standard kinetic term as a natural source of non-Gaussianity J. High Energy Phys. JHEP10(2008)089 (arXiv:0810.3291 [hep-ph])
[13] Matsuda T 2007 Hybrid curvaton from broken symmetry J. High Energy Phys. JHEP09(2007)027 (arXiv:0708.4098 [hep-ph])
[14] Matsuda T 2007 No curvaton or hybrid quintessential inflation J. Cosmol. Astropart. Phys. JCAP08(2007)003 (arXiv:0707.1948 [hep-ph])
[15] Matsuda T 2008 Curvatons and inhomogeneous scenarios with deviation from slow-roll J. Cosmol. Astropart. Phys. JCAP12(2008)001 (arXiv:0811.1318 [hep-ph])
[16] Matsuda T 2007 Brane inflation without slow-roll J. High Energy Phys. JHEP03(2007)096 (arXiv:astro-ph/0610402)
[17] Matsuda T 2009 Remote inflation: hybrid-like inflation without hybrid-type potential arXiv:0904.2821 [astro-ph.CO]
[18] Matsuda T 2009 Remote inflation as hybrid-like sneutrino/MSSM inflation arXiv:0905.4328 [hep-ph]
[19] Matsuda T 2008 Modulated inflation Phys. Lett. B 665 338 (arXiv:0801.2648 [hep-ph])
[20] Matsuda T 2008 Modulated inflation from kinetic term J. Cosmol. Astropart. Phys. JCAP05(2008)022 (arXiv:0804.3268 [hep-th])
[21] Matsuda T 2003 F-term, D-term and hybrid brane inflation J. Cosmol. Astropart. Phys. JCAP11(2003)003 (arXiv:hep-ph/0302078)
[22] Matsuda T 2005 Comment on the stability of the Yukawa couplings and the cosmological problems of intersecting brane models Gen. Relativ. Gravit. 37 1297 (arXiv:hep-ph/0309314)
[23] Matsuda T 2002 Activated sphalerons and large extra dimensions Phys. Rev. D 66 047301 (arXiv:hep-ph/0205331)
[24] Ibanez L E and Ross G G 1982 SU(2)-L X U(1) symmetry breaking as a radiative effect of supersymmetry breaking in guts Phys. Lett. B 110 215
[18] Bartolo N, Komatsu E, Matarrese S and Riotto A 2004 Non-Gaussianity from inflation: theory and observations

Phys. Rep. 402 103 (arXiv:astro-ph/0406398)

[19] Yadav A P S and Wandelt B D 2008 Evidence of primordial non-Gaussianity (fNL) in the Wilkinson microwave anisotropy probe 3-year data at 2.8σ
Phys. Rev. Lett. 100 181301 (arXiv:0712.1148)

[20] Lyth D H 2006 Non-Gaussianity and cosmic uncertainty in curvaton-type models J. Cosmol. Astropart. Phys. JCAP06(2006)015 (arXiv:astro-ph/0602285)

Suyama T and Takahashi F 2008 Non-Gaussianity from symmetry J. Cosmol. Astropart. Phys. JCAP08(2008)007 (arXiv:0804.0425 [astro-ph])

[21] Lyth D H and Rodriguez Y 2005 Non-Gaussianity from the second-order cosmological perturbation Phys. Rev. D 71 123508 (arXiv:astro-ph/0502578)

[22] Matsuda T 2009 Evolution of the curvature perturbations during warm inflation J. Cosmol. Astropart. Phys. JCAP06(2009)002 (arXiv:0905.0308 [astro-ph.CO])

Matsuda T 2009 Evolution of curvature perturbation in generalized gravity theories arXiv:0906.0643 [hep-th]