Constraining the host galaxy halos of massive black holes from LISA event rates

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Abstract. The coalescence of massive black hole binaries (with masses $10^{4}$–$10^{7} M_{\odot}$) leads to gravitational wave emission that is detectable out to high redshifts ($z \sim 20$) with the forthcoming LISA observatory. We combine the theoretically derived merger rates for dark matter haloes at various redshifts, with an empirically motivated prescription that connects the mass of a dark matter halo and that of its central black hole. Using the expected constraints on the (chirp or reduced) masses of binary black holes, their mass ratios and redshift uncertainties, we forecast the measurement precision on the occupation fraction, normalization and slope of the black hole mass-halo mass relation at various redshifts, assuming a five-year LISA survey for three different confidence scenarios. We use the expected sizes of the LISA localization ellipses on the sky to estimate the number of electromagnetic counterparts to the gravitational wave sources which are detectable by future wide-field optical surveys, such as LSST.

Keywords: gravitational waves / sources, massive black holes

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1 Introduction

Observations have now established that supermassive black holes inhabit the centres of most galaxies out to high redshifts [e.g., 1–5]. In the standard hierarchical structure formation scenario, the assembly of galaxies takes place via the repeated coalescences of their host dark matter haloes. Thus, massive black hole binaries, formed from the merger of galaxies each containing a massive black hole, are expected to be ubiquitous throughout cosmic time [6–14]. Direct observational evidence [15] for supermassive black hole binaries has now been found both locally [e.g., 16] and at intermediate redshifts, $z \approx 0.2$ [17]. The coalescence of binaries in the mass range $(10^4$–$10^7 M_\odot)$ leads to the emission of gravitational radiation at mHz frequencies, which is detectable by the Laser Interferometry Space Antenna (LISA) observatory [18] and the proposed TianQin space-borne detector [19–21]. Such coalescences are expected to occur more frequently at high redshifts, since the mergers of dark matter haloes are expected to be higher at early times [22]. LISA will be able to detect binary black hole mergers out to redshifts $z > 20$ with signal-to-noise ratios (SNRs) $\gtrsim 10$, and SNRs approaching $\sim 1000$ at low redshifts. The number of such events per year is estimated to be of the order of a few to a few thousand, though there is a large variation in the predictions of different models [e.g., 9, 23–28].

Gravitational wave measurements from massive black hole binary mergers with the LISA observatory will allow to infer several properties, such as the binary members’ masses (or, equivalently, the chirp mass and reduced mass of the system), spin vectors, rough sky location of the merger, and the source luminosity distance (which can be converted into a redshift for an assumed cosmology) at high precision [e.g., 29–34]. These measurements promise exciting new information constraining the seeds for the first supermassive black holes, their dynamical evolution and their relation to the observed luminosity function of quasars [10, 23, 35–37]. Subsets of the measured parameters are often highly correlated with each other, thus making it difficult to isolate a source from the entire population of coalescence events. It was, however, shown in ref. [34] that including precessional effects due to the interaction of one black hole’s spin with the gravitomagnetic fields from the other hole’s spin, breaks the degeneracies among several parameters, thus greatly improving the
accuracy in their measurement. In particular, masses are measurable by LISA to accuracies of $10^{-4} - 10^{-5}$, and luminosity distances to $0.2\% - 0.7\%$ at $z \sim 1$. Including the information in the spin precession also leads to an improvement in the localization of the sources on the sky to error ellipses with major axes of several tens of arcminutes, and minor axes a factor 2–4 times smaller. If an electromagnetic counterpart to the gravitational wave emission is found, the LISA sources are also expected to act as ‘standard sirens’, enabling a measurement of the expansion history of the universe [e.g., 38] and uncovering valuable tests of General Relativity [e.g., 30, 31, 39–41].

In this paper, we combine the theoretically derived merger rates for dark matter haloes at various redshifts, with empirical expressions connecting the mass of a dark matter halo and that of its central black hole [42]. Introducing an occupation fraction parameter, $f_{bh}$ [43], that describes the fraction of host haloes that are expected to harbor central supermassive black holes, allows for an analytical computation of the merger rate of binary black holes as an explicit function of their masses, redshift and the parameters governing the black hole mass-halo mass relation. Using this merger rate in conjunction with the expected masses, ratios and redshift uncertainties [34] including the effects of precession, we forecast constraints on the three parameters: (i) $f_{bh}$, the occupation fraction of the black holes, (ii) $\gamma$, the power-law slope of the BH mass-halo mass relation when expressed in terms of the halo circular velocity $v_c$, and (iii) $\epsilon_0$, the amplitude of this relation, from a five-year LISA survey for three different confidence scenarios. Finally, we use the expected 3D error ellipsoid of localization of the merger with a LISA survey to place constraints on the expected number of electromagnetic counterparts to the gravitational wave sources, adopting LSST on the Vera Rubin Observatory as an example of a future wide-field optical survey for this purpose.

The paper is organized as follows. In section 2, we discuss the theoretical formalism involved in computing the merger rates of binary black holes from that of their host dark matter haloes. We then, in section 3, use the parameter constraints available in the literature [34] to place constraints on the free parameters of interest, given the expected level of uncertainty in the measurement of the remaining parameters. This requires a modification of the standard Fisher matrix procedure, which we describe in section 3.1, before computing the constraints in section 3.2 for three different confidence scenarios of LISA detection rates in section 3.3 and section 3.4. We constrain the expected number of electromagnetic counterpart galaxies to the binary black hole merger detectable in a future wide field survey like LSST in section 4. Finally, we summarize our conclusions and discuss future prospects in section 5.

2 Merger rates of massive black holes

The number of gravitational wave sources detectable by LISA is a convolution of: (i) the merger rate of the galaxies that contain black holes in the relevant mass range, and (ii) the occupation fraction of these galaxies, i.e. the fraction containing a black hole at their center. Since galaxies are known to reside in dark matter haloes described by the hierarchical scenario of structure formation, the merger of haloes is related to the coalescence rate of binary black holes.

We begin with the formalism for the merger rate of dark matter haloes per unit redshift ($z$) and halo mass fraction ($\xi$), as formulated by [44]:

$$dn_{halo} / dzd\xi = A \left( \frac{M}{10^{12} M_{\odot}} \right)^\alpha \xi^\beta \exp \left[ \left( \frac{\xi}{\xi_0} \right)^\gamma \right] (1 + z)\eta$$ (2.1)
where $M$ is the mass of the primary halo, $\xi$ is the mass ratio of the two merging haloes, and the parameters have the values $\alpha = 0.133$, $\beta = -1.995$, $\gamma_1 = 0.263$, $\eta = 0.0993$, $A = 0.0104$ and $\bar{\xi} = 9.72 \times 10^{-3}$.

Combining the above rate with the abundance of haloes with masses between $M$ and $M + dM$, we can convert it into a merger rate per unit logarithmic halo mass, as:

$$
\frac{dn_{\text{halo}}}{d\log_{10} M d\log_{10} \xi} = A \left( \frac{M}{10^{12} M_\odot} \right)^{\alpha} \xi^\beta \exp \left[ \left( \frac{\xi}{\bar{\xi}} \right)^{\gamma_1} \right] (1 + z)^\eta \frac{dn_{\text{halo}}}{d\log_{10} M},
$$

(2.2)

where $dn_{\text{halo}}/d\log_{10} M$ is the halo mass function, for which we adopt the Sheth Tormen form [45].

To predict the expected rate of binary black hole mergers, we combine the halo merger rate with the empirical relation connecting black hole and host halo mass [e.g., 42]:

$$
M_{\text{BH}} = M\epsilon_0 \left( \frac{M}{10^{12} M_\odot} \right)^{\gamma/3 - 1} \frac{(\Delta_v \Omega_m h^2)}{18\pi^2} \frac{\gamma/6}{(1 + z)^{\gamma/2}},
$$

(2.3)

which is consistent with observations [e.g., 46] in the local universe and assumes a power-law scaling of the black hole mass with virial velocity, $M_{\text{BH}} \propto v_c^{\gamma}$. The above relation is also consistent with the observed black hole-bulge mass relation [ref. 3, section 6.10 and eq. 10] coupled with the empirically derived stellar mass-halo mass relation [e.g., 47]. With this, eq. (2.2) can be recast as:

$$
\frac{dn_{\text{BH}}}{d\log_{10} M_{\text{BH}} dz dq} = f_{\text{bh}}^2 A_1 \left( \frac{M_{\text{BH}}}{10^{12} M_\odot} K(z, \gamma, \epsilon_0) \right)^{3\alpha/\gamma} \times q^{3/\gamma - 1 + 3\beta/\gamma} (1 + z)^\eta \exp \left[ \left( \frac{q}{\bar{q}} \right)^{3\gamma_1/\gamma} \right] \frac{dn_{\text{halo}}}{d\log_{10} M},
$$

(2.4)

where $q$ is the black hole mass ratio, related to $\xi$ as $q = \xi^{\gamma/3}$, $\bar{q}(\gamma) = \bar{\xi}^{\gamma/3}$ with $\bar{\xi} = 9.72 \times 10^{-3}$, $A_1 = (3/\gamma)^2 A$ and in which we have used the fact that $d\log_{10} M_{\text{BH}} = (\gamma/3)d\log_{10} M$. We have also introduced the occupation fraction $f_{\text{bh}}$, which measures the likelihood of merging haloes to contain black holes (which may in general, be a function of the halo mass, but is assumed to be constant here for simplicity, since the precise connection of the occupation fraction to host halo properties is currently unknown [43]). This then relates $q$ to $\xi$ as $q = \xi^{\gamma/3}$. The function $K(z, \gamma, \epsilon_0)$ is defined as:

$$
K(z, \gamma, \epsilon_0) = \epsilon_0 \left( \frac{\Delta_v \Omega_m h^2}{18\pi^2} \right)^{\gamma/6} (1 + z)^{\gamma/2}
$$

(2.5)

The merger rate of binary black holes is thus characterized by the three free parameters $f_{\text{bh}}, \gamma, \epsilon_0$ (with the other two, $\eta$ and $\alpha$ being inherited from the underlying halo merger rate).

### 3 Expected constraints

The future LISA observatory will be able to detect several mergers of massive binary black holes at high redshifts through their gravitational wave emission in the milli-Hertz (mHz) frequency range. In this section, we illustrate the constraints that forthcoming LISA detections can be used to place on the properties of the black hole mass-halo mass relation and occupation fraction at high redshifts, described through the parameters $f_{\text{bh}}, \gamma$ and $\epsilon_0$. 
3.1 Setting up the problem

To begin with, we use eq. (2.4) to define the observed rate of GW events per comoving volume per unit time [e.g., ref. 48], as:

\[ R(M_{\text{BH}}, q, z; \epsilon_0, f_{\text{bh}}, \gamma) = \frac{dn_{\text{BH}}}{d\log_{10} M_{\text{BH}} dq dz dt (1+z)} = H(z) f_{\text{bh}}^2 A_1(\gamma) \left( \frac{M_{\text{BH}}}{10^{12} M_\odot K(z, \gamma, \epsilon_0)} \right)^{3\alpha/\gamma} \times \frac{d_\nu}{d\log_{10} M} \left( q \right)^{3\gamma_1/\gamma} \exp \left[ \left( \frac{q}{\bar{q}(\gamma)} \right)^{3\gamma_1/\gamma} \right] \]  

(3.1)

in which \( H(z) \) is the Hubble parameter at redshift \( z \), and we have made the parameter dependences explicit: \( A_1(\gamma) = \left( \frac{3}{\gamma} \right)^2 A \) with \( A = 0.0104, \bar{q}(\gamma) = \bar{\xi}/3 \) with \( \bar{\xi} = 9.72 \times 10^{-3} \), and other constants have the values defined previously in eqs. (2.1) and (2.4). Note that the observed rate differs from the intrinsic rate by the redshift dilation factor of \((1+z)\).

It is known [see, for example, ref. 34] that LISA measurements from \( 10^4 \) binary black hole mergers with masses \( 10^5 - 3 \times 10^6 M_\odot \) over \( z \sim 1-5 \) can constrain the individual (chirp or reduced) black hole masses \( M_{\text{BH}} \), redshifts \( z \) and mass ratios \( q \), with a relative precision ranging from 0.1%–10% in various scenarios. Our objective is to use this information, together with the rate equation above, to measure how well a given detection scenario can constrain the three free parameters \( \epsilon_0, f_{\text{bh}} \) and \( \gamma \). In its most generic form, the above problem can be expressed as a constraint equation:

\[ f(K_i; U_j) = 0, \]  

(3.2)

on a function \( f \) of (i) ‘known’ variables \( K_i, i = 1 \) to \( n \), all of which are determinable to a specified degree of accuracy, i.e. \( \Delta K_i/K_i \) is known for all \( i \), and (ii) ‘unknown’ variables \( U_j, j = 1 \) to \( m \), which are the parameters we wish to estimate the errors on. In our present case, \( f \) is the difference between the event rate \( \mathcal{R} \) and its fiducial value, the \( K_i \)’s are the set \{\( M_{\text{BH}}, q, z \)\}, the \( U_j \)’s are the set \{\( \epsilon_0, f_{\text{bh}}, \gamma \)\}, and \( i = j = 3 \). Given \( f \) and \( \Delta K_i/K_i \) for all \( i \), we need to estimate \( \Delta U_j/U_j \) for all \( j \).

3.2 Parameter constraints

Towards handling the above (non-standard) situation, we refine the standard Fisher matrix formalism\(^1\) by summing the Fisher components at all known incidences of the ‘known’ parameters. We begin by using eq. (3.1) to evaluate the observed event rate \( \mathcal{R} \) in bins of \( \log M_{\text{BH}}, q \) and \( z \), spaced over the relevant ranges in each of the known parameters \( K_i \), where constraints are available [49] as follows:

\[ \log_{10}(M_{\text{BH}}/M_\odot) = \{5, 5.5, 6, 6.5\} \]
\[ q = \{0.1, 0.3, 1.0\} \]
\[ z = \{1, 3, 5\} \]

\(^1\)Note that the approach described here is also useful in the generic scenario when: (i) one does not have a clearly defined likelihood function for the parameter constraints, and (ii) one is dealing with correlated parameters of which a subset are unknown, with the known ones being characterized by the probability distribution of their errors.
This allows us to express the per-bin variation of $R$ in each $\{\log_{10} M_{\text{BH}}, q, z\}$ bin as:

$$\Delta R_{\text{bin}} = \frac{\partial R}{\partial \log_{10} M_{\text{BH}}} \Delta \log_{10} M_{\text{BH}} + \frac{\partial R}{\partial z} \Delta z + \frac{\partial R}{\partial q} \Delta q,$$  \hspace{1cm} (3.3)

where the quantity on the r.h.s. is evaluated in each bin. This step allows us to use the information available in the distributions of $\Delta K_i/K_i$, because we are counting all the incidences. This is equivalent to simulating the sample of individual values. The resultant standard deviation on $R$ in each bin then becomes

$$\sigma_{R_{\text{bin}}} = \Delta R_{\text{bin}} / \sqrt{N_{\text{events}}/N_{\text{bins}}},$$

where $N_{\text{events}}$ is the number of events and $N_{\text{bins}}$ is the number of bins. In practice, we may replace the $\Delta R_{\text{bin}}$ computed from eq. (3.3) by an average value representing an assumed confidence scenario of uncertainties in the ‘known’ parameters.

We can now use the Fisher formalism to forecast the expected uncertainties on the ‘unknowns’ $\{\log \epsilon_0, \gamma, f_{\text{bh}}\}$, for varying numbers of LISA events in a 5-year survey. The fiducial values of these parameters, around which the errors are computed, are taken to be $U_{i,\text{fid}} = \{-5.02, 4.53, 0.56\}$ which are consistent with observations in nearby galaxies [42, 46]. We calculate the $(i, j)$th element of the Fisher matrix $\mathcal{F}$, by summing over its contributions from each bin:

$$\mathcal{F}_{ij} = \sum_{\text{bin}} \frac{1}{(\sigma_{R_{\text{bin}}})^2} \frac{\partial R_{\text{bin}}}{\partial U_i} \frac{\partial R_{\text{bin}}}{\partial U_j},$$  \hspace{1cm} (3.4)

where $U_i = \{\epsilon_0, f_{\text{bh}}, \gamma\}$, and the derivatives $\partial R_{\text{bin}}/\partial U_i$ are computed in each bin $i$.\(^3\)

### 3.3 Confidence scenarios

The number of LISA detections predicted to take place every year is fairly uncertain [e.g., 50, 51]. Here, we consider three different confidence scenarios for the constraints on the parameters: (i) Optimistic, (ii) Pessimistic and (iii) Intermediate, which correspond to $(\Delta R/R) = 0.001, 0.1$ and 0.01 respectively for $10^4$ events, consistently with the expectations of refs. [49, 52]. Within each scenario, we further consider three different numbers of LISA events: 500, 1000 and 2000 respectively observed over a 5-year period (corresponding to 100, 200 and 400 events per year) to compute $\sigma_{R_{\text{bin}}}$, and thus the elements of the Fisher matrix in eq. (3.4), as detailed in the previous subsection.

From the Fisher matrix $\mathcal{F}$, we can now compute the standard deviations on each parameter, $p_i = \{K_i, U_i\}$, when the others are marginalized over, using the expression, $\sigma(p_i) = \sqrt{(\mathcal{F}^{-1})_{ii}}$. For the three ‘unknown’ parameters $\{\log \epsilon_0, \gamma, f_{\text{bh}}\}$, the values of $\sigma(U_i)/U_{i,\text{fid}}$, with $U_{i,\text{fid}} = \{-5.02, 4.53, 0.56\}$ are listed in table 1. The corresponding 1-$\sigma$ and 2-$\sigma$ contours for the two extreme situations (pessimistic, with lowest number of events, and optimistic, with the highest number of events) are plotted in figure 1.

### 3.4 Middle scenario

We focus on an intermediate, or middle scenario to explore the constraints achievable for individual redshifts. In this scenario, the relative error is taken to be between the two cases considered above, at $\Delta R/R = 0.01$ (for $10^4$ events), and we fix the number of detections to 1000 (i.e. 200 per year observed over 5 years). Within this scenario, we consider two individual redshifts, one at $z \sim 3$ (which lies within the range in which constraints are available) and

\(^2\)For ease of computation, we use $\log \epsilon_0$ instead of $\epsilon_0$ in the numerical results.

\(^3\)Note that this calculation makes the implicit assumptions that the bins are independent since we neglect cross-correlations between the bins.
Figure 1. Extreme ends of the forecasting ability on the unknown parameters \(\{\log \epsilon_0, \gamma, f_{bh}\}\) with LISA. Panel (a): 1-\(\sigma\) and 2-\(\sigma\) confidence contours assuming the optimistic scenario for \(\Delta R\) and 400 LISA detections per year. Panel (b): pessimistic scenario for \(\Delta R\) and 100 LISA detections per year.
Table 1. Expected relative errors, $\sigma(U_i)/U_i$, on the ‘unknown’ parameters \{$\log \epsilon_0, \gamma, f_{bh}$\} around their fiducial values $U_{i,fid} = \{-5.02, 4.53, 0.56\}$, for the pessimistic and optimistic scenarios considered in the main text. In each case, the forecasted parameter constraints assume a 5-year LISA survey with 100, 200 and 400 events per year.

| No. of events | $\log \epsilon_0$ | $\gamma$ | $f_{bh}$ | No. of events | $\log \epsilon_0$ | $\gamma$ | $f_{bh}$ |
|---------------|----------------|---------|-------|---------------|----------------|---------|-------|
| 500           | 0.017          | 0.022   | 0.046 | 500           | 0.0017         | 0.0022  | 0.0046 |
| 1000          | 0.015          | 0.019   | 0.038 | 1000          | 0.0015         | 0.0019  | 0.0038 |
| 2000          | 0.012          | 0.016   | 0.032 | 2000          | 0.001          | 0.002   | 0.003  |

Table 2. Expected relative errors, $\sigma(U_i)/U_i$, on the ‘unknown’ parameters \{$\log \epsilon_0, \gamma, f_{bh}$\} around their fiducial values $U_{i,fid} = \{-5.02, 4.53, 0.56\}$, for the ‘middle scenario’ at two individual redshifts, $z \sim 3$ and $z \sim 8$. The forecasts assume a 5-year LISA survey with 200 events per year.

| Redshift | $\log \epsilon_0$ | $\gamma$ | $f_{bh}$ |
|----------|----------------|---------|-------|
| $z \sim 3$ | 0.006 | 0.007 | 0.016 |
| $z \sim 8$ | 0.005 | 0.009 | 0.014 |

Another at $z \sim 8$ (which lies outside this range), in order to illustrate the possible evolution of the constraining ability with respect to redshift.\(^4\) These are shown in figure 2 and table 2. Some of the constraints improve upon reaching higher redshifts, $z \gg 1$, assuming that the uncertainties on the known parameters continue to hold.

4 Localization and electromagnetic counterparts

Binary neutron star mergers detected in gravitational waves are expected to have an electromagnetic counterpart [e.g., 53–57], which allows for the identification of a host galaxy. It was shown that gravitational wave observations by LIGO-Virgo from the merger of two neutron stars have the potential to constrain the Hubble constant to within a few percent in five years, if a host galaxy is identified, either from a direct electromagnetic counterpart or from a statistical analysis of a catalogue of potential host galaxies [e.g., 38, 58–70].

Electromagnetic counterparts to stellar-origin binary black holes have also been predicted in the literature [e.g., 71, 72] via GRB afterglows. Another possible electromagnetic counterpart to the LIGO candidate event S190521g was reported recently by the Zwicky Transient Factory [ZTF; 73].

Here, we explore the possibility of identifying the potential host galaxies of LISA-detected binary supermassive black holes, using a catalogue of candidate electromagnetic counterparts detected by a future photometric survey, using LSST on the Vera Rubin Observatory\(^5\) as an example. We focus throughout on electromagnetic counterparts from the stellar light of the host galaxy (which is much longer lived than the transient counterparts arising from the emission from hot gas around the compact objects). If the electromagnetic

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\(^4\)The $z \sim 8$ case is to be considered very optimistic since the errors on the parameters are expected to be far worse by then [49].

\(^5\)https://www.lsst.org/.
Figure 2. Intermediate or middle scenario for constraints on the unknown parameters, now focusing on two fixed values of redshift. Panel (a): 1-σ and 2-σ confidence contours assuming the intermediate number, i.e 200 LISA detections per year, but focused on $z \sim 3$. Panel (b): same as left panel, but focused on $z \sim 8$. 
counterpart can be unambiguously identified, its sky position and redshift can be measured accurately. In the absence of a precise identification, we combine the expected uncertainties in the error ellipse parameters and $\Delta z/(1+z)$ (from ref. [34]) with the number of galaxies per unit sky area in the relevant range detectable by LSST, to estimate the total number of potential host galaxies needed to be searched for in order to identify the counterpart.

The properties of the detected sources are modelled using the LSST redshift selection function [74], which is modified from the form in the LSST Science Book [75] for galaxies having $i$-band magnitudes $20.5<i<25.5$:

$$\phi(z) \propto z^{1.28} \exp \left( -\frac{z}{0.41} \right)^{0.97}$$  \hspace{1cm} (4.1)

The corresponding galaxy surface number density is derived from the stellar mass corresponding to each black hole mass bin. This is calculated using the results of ref. [76] to assign stellar masses, $M_*$ to the black hole host dark matter haloes, which in turn are derived from the black hole mass-halo mass relation of ref. [42]. The derived stellar masses are converted into $i$-band magnitudes using the $M_*/L_i$-$(g-i)$ relation of ref. [77], assuming a typical $(g-i)=1.5$ for LSST-detected spiral galaxies [LSST Science Book, ref. 75, table 3.1]. The $K$-correction is added following the estimates$^6$ for $z \sim 0.5$, consistently with the SDSS findings from figure 6 of ref. [78], which also indicates evidence that the $K$-correction flattens at higher redshifts.

Noting that the uncertainty on the $M_{BH} - M_*$ is of the order of 0.29 dex [3] and that on the $(g-i) - L_i/M_*$ is 0.1 dex, the combined uncertainty on the $i$-magnitudes is $\sim 0.30$ dex. The error on the $M_{BH}$ values from LISA is very small, of the order of sub-percent [34]. Using this range in the $i$-magnitudes as the $i$-bin widths, we can now estimate the number of galaxies per square arcmin within each $i$- and redshift bin, using the formula $N(< i) = 46 \times 10^{0.31(i-25)}$ arcmin$^{-2}$ from the LSST Science Book [ref. 75, eq. 3.7] multiplied by the redshift selection function of LSST above.

From the results of binary black hole merger analyses (e.g., ref. [34], see table IV), the median values of the major and minor ellipse axes ($2a$ and $2b$) for sources with black hole masses in the range $\{10^5,10^7\}M_\odot$ are expected to be in the range 13 arcmin to about 81 arcmin. Assuming a redshift localization of $\Delta z/z = 0.01$ around $z \sim 1$ (of the order of the uncertainty in $\Delta d_L/d_L$)$^7$, we find the expected mean number of LSST sources in each error ellipsoid as a function of black hole mass, as shown in figure 3. The plot shows that LSST is expected to detect of order $\sim 100$–200 galaxies with the black hole masses above $10^6.5M_\odot$.

Electromagnetic counterparts of supermassive binary black hole mergers in gas disks have been well studied in the literature [e.g., 81–83]. It is possible that galaxies that are intrinsically fainter than the LSST limit above will enter the regime of detectability due to the bright flare caused by the binary black hole merger, which is dependent on the gas content and other properties of the host [83, 84], including the gas density profile and feedback effects. This in turn, opens up the possibility of subsequent follow-up searches for the host galaxy with other instruments.

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$^6$http://kcor.sai.msu.ru/.

$^7$The relation between luminosity distance and redshift is nonlinear and dependent on the assumed cosmology — which can, in turn, be constrained, if an electromagnetic counterpart is found. For simplicity in the present analysis, we assume the same order of magnitude of the estimated relative errors in $\Delta d_L/d_L$ and $\Delta z/z$, noting that for a standard $\Lambda$CDM cosmology consistent with the latest constraints [79, 80], $\Delta d_L/d_L \sim 0.01$ corresponds to $\Delta z/z \sim 0.0081$ at $z \sim 1$. 

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5 Discussion

In this paper, we have computed the rate of massive binary black hole mergers (of masses $10^4$–$10^7 M_\odot$) out to high redshifts ($z \gtrsim 5$) and connected it up to the number of events detectable by the forthcoming LISA observatory. Our theoretical framework assumes black holes to be associated with host dark matter haloes, with the black hole mass scaling as a power law with the halo circular velocity, as needed to match the observed luminosity function of quasars [22, 42]. The parameters (normalization, $\epsilon_0$ and slope, $\gamma$) of this empirically motivated relation [46], which matches the latest constraints in the local universe [3] are assumed to hold to high redshifts as well [85].

The mergers of massive black holes are assumed to follow those of their underlying host dark matter haloes. Halo mergers [44] are assumed to lead to black hole coalescence without delay. We also neglected the halo merger timescale, assuming it to be much shorter than the Hubble time. This is a valid assumption if the black hole binaries do not have extreme mass ratios ($q \equiv M_1/M_2 < 20$), [e.g., 9, 11, 12], we additionally expect extreme mass ratio inspirals (EMRIs) to be depleted because of the long dynamical friction times for small sub-haloes in big haloes [86, 87] and thus restrict to $q > 0.1$ in the calculations. We introduce an occupation fraction parameter, $f_{bh}$, which measures the probability that the dark matter halo hosts a seed black hole. Theoretical models based on merger trees have shown that values of $f_{bh} \gtrsim 0.1$ can accurately reproduce the evolution of the quasar luminosity function at redshifts $0 < z < 6$, as well as the mass function of remnant supermassive black holes at $z = 0$ [43] including the effects of recoils from gravitational wave emission [88] and triple systems [12].

Given the expected uncertainties in the measurement of black hole masses and ratios at various redshifts (the ‘known’ parameters), one can use the LISA detection rate to place
constraints on the remaining ‘unknown’ parameters, viz. the occupation fraction \((f_{bh})\), normalization \((\epsilon_0)\) and slope \((\gamma)\) of the black hole-halo mass relation. To evaluate the prospects for this goal, we modified the standard Fisher matrix approach and accounted for the available constraints on the subset of ‘known’ parameters in three different confidence scenarios, each assumed to have 100, 200 or 400 event detections per year, for a survey of a 5 year duration. In so doing, we have found that the occupation fraction of black holes \((f_{bh})\) and the parameters governing the black hole mass to halo mass evolution \((\epsilon_0\) and \(\gamma)\) can be constrained to percent or sub-percent levels of accuracy around \(z \sim 1–5\), depending on the scenario under consideration. If the uncertainties on the measured source parameters are assumed to hold to higher redshifts \((z \sim 8)\), then the parameter constraints become tighter.

We have also explored the possibility of detecting the electromagnetic counterpart from the host galaxy stellar light associated with the massive or supermassive binary black hole merger, using future wide-field photometric surveys, such as LSST. We assumed that the electromagnetic follow up occurs in the post-merger period, once the black holes have reached coalescence. Given the expected range of sensitivity of LSST, and an assumed conversion between the black hole masses, their host galaxy masses and the corresponding \(i\)-band magnitudes, we expect roughly 100–200 electromagnetic counterparts to fall within the expected LSST sensitivity range for binaries with masses above \(M_{BH} \sim 10^{6.5} M_\odot\) at \(z \sim 1\). These figures are comparable to the estimates derived by ref. [84], who address a different problem: that of monitoring the LISA sources with LSST in the 2–3 weeks preceding the merger. Another difference is that the analysis of ref. [84] considers the gas accretion emission from the merger whereas we focus on stellar light. However, after accounting for a localization cut corresponding to the LISA error ellipse, and imposing photometric redshift and luminosity bounds (which makes their assumptions fairly comparable to those in our present analysis), their derived number of counterpart candidates reaches \(N_{\text{counterpart}} \sim 1–1000\), consistently with the present findings.

A caveat to the error estimates is the assumed Gaussian approximation to the likelihood function, which is the basis for the direct computation of the Fisher information matrix. This approximation is almost certain to underestimate the true uncertainty since it misses the possible long tail of the likelihood function, and is known to break down at low number counts [e.g., 34]. Thus, the results presented here should be taken as optimistic. There is also the possibility of multiple supermassive black hole binaries residing in the same host halo [e.g., 12, 89] with corresponding gravitational-wave emission signatures [e.g., 9, 90, 91]. The probability of multiple black holes in the nuclei of galaxies increases with increasing host halo mass and redshift. Simulations [12] show that only about 30\% of galaxies with haloes of masses \(10^{11} M_\odot\) at \(z \sim 6\) contain more than two supermassive black holes at redshifts \(2 < z < 6\), while lower mass galaxies rarely host more than two supermassive black holes at any point in their assembly history. Numerical simulations of triple black hole systems have been shown to produce distinct signatures in the gravitational wave spectrum [91], though their detectability with LISA relies on the development of adequate analysis techniques to extract the signal amidst the large confusion noise. The recoil associated with the gravitational wave emission [e.g., 88, 92] can lead to escaping supermassive black holes. About 10 percent [12] of such black holes are ejected at velocities \(> 2000\) km s\(^{-1}\) and expected to spend a few Gyr in the outskirts of the halo. It would be interesting to consider the above effects in simulation forecasts for LISA. A more detailed analysis would also include the timescales for mergers and address the dependence of the parameter constraints on the estimated time before the merger [e.g., 52], which we leave to future work.
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