SU(8) Grand Unification from Composite Quarks and Leptons

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Abstract

Despite many interesting attempts in the past, both theoretical and experimental, a possible composite structure of quarks and leptons is still an open problem. Meanwhile, some observed replications in their spectra, such as an existence of several quark color triplets, as well as the quark and lepton weak doublets, and as many as three identical quark-lepton families could mean that there might exist the truly elementary fermions, preons, being actual carriers of all the fundamental quantum numbers involved and composing the observed quarks and leptons at larger distances. Generally, certain regularities in replications of particles may signal about their constituent structure. Indeed, just regularities in the spectroscopy of hadrons observed in the nineteen-sixties made it possible to discover the quark structure of hadrons in the framework of the so-called Eightfold Way. We show that an inspirational Eightfold Way idea looks much more relevant when applied to a next level of matter elementarity, namely, to elementary preons and composite quarks and leptons. As we find below, just the eight preon model and their local metaflavor symmetry $SU(8)_{MF}$ may under certain natural conditions determine the fundamental entities of physical world at small distances and its total internal symmetry. Being exact for preons, it gets broken for composites down to a conventional $SU(5)$ GUT with an extra local family symmetry $SU(3)_F$ and three standard families of quarks and leptons. Though a tiny confinement scale for universal preons composing both quarks and leptons makes it impossible to directly confirm their composite nature, a simultaneous emergence of several extra $SU(5) \times SU(3)_F$ multiplets of heavy composite fermions may help with a model verification.

Keywords: Beyond Standard Model, Composite models, Grand unified models
1 Introduction

As is well known, there is no a viable unification symmetry scheme for classification of all observed quarks and leptons \cite{1, 2}. Indeed, while the known Grand Unified Theories, such as the \( SU(5) \), \( SO(10) \), or \( E(6) \) GUTs, work well when applied in a supersymmetric context with all three quark-lepton families included \cite{2, 4}, the very families are taken mechanically or, at best, classified by their own (discrete, global or local) symmetry. Any attempt to incorporate these families into a simple grand unification symmetry group framework leads to high symmetries with enormously extended representations which also contain lots of exotic states that never been detected \cite{1}. In fact, such states unavoidably appear in any anomaly free combinations of representations of possible \( SU(N) \) GUTs with the \( N \) value to be high enough to contain all three quark-lepton families \((N \geq 8)\). For the \( SO(10) \) and \( E(6) \) symmetry extensions the situation looks even worse. Actually, for the \( SO(4n + 2) \) groups \((n \geq 3)\), which only have admissible complex representations, there are equal number \( 2^{2n-5} \) of the left-handed and right-handed spinor multiplets \( 16_{L,R} \) of the \( SO(10) \) for quark-lepton families that requires an introduction of some special survival mechanism in order to prevent them from pairing and gaining the grand unification scale order masses. Meanwhile, a major objection to the \( E(6) \) symmetry extensions to the \( E(7) \) and \( E(8) \) which could in principle include all three families of quarks and leptons is that these extensions have only the self-conjugate representations that requires some special (mostly unnatural) symmetry breaking pattern to provide the chiral character of the weak interactions in the low-energy limit.

This problem may still motivate us to seek a solution in some subparticle or preon model for quark and leptons, despite many interesting attempts made in the past (some significant references can be found in \cite{2, 4}). A required solution might show, after all, which grand unification symmetry, if any, could accommodate all the presumably composite quark and lepton species. Some observed replications in their spectra, such as an existence of several quark color triplets, as well as the quark and lepton weak doublets, and as many as three identical quark-lepton families could mean that there might exist the truly elementary fermions, preons, being actual carriers of all the fundamental quantum numbers involved and composing the observed quarks and leptons at larger distances. Generally, certain regularities in replications of particles may signal about their constituent structure. Indeed, just regularities in the spectroscopy of hadrons observed in the nineteen-sixties made it possible to discover the quark structure of hadrons in the framework of the so-called Eightfold Way. This term was coined by Murray Gell-Mann in 1961 to describe a classification scheme for hadrons, that he had devised, according to which the known baryons and mesons are grouped into the eight-member families of some global hadron flavor symmetry \( SU(3) \) \cite{5}. This had finally led to the hypothesis of quarks which form the fundamental triplet of this symmetry, and, consequently, to a compositeness of the observed baryons and mesons. We try to show here that the inspirational Eightfold Way idea looks, as strange as it may seem, much more relevant when it is applied to a next level of the matter elementarity, namely, to elementary preons and composite quarks and leptons. As we find below, just the eight preon model and their local metaflavor symmetry \( SU(8)_{MF} \) may under certain natural conditions determine the
fundamental entities of physical world and its total internal symmetry at low energies.

Some heuristic motivation for such a picture may be related to the eight fundamental quantum numbers (charges) presently observed. They correspond in fact to the two weak isospin orientations, three types of colors and three species of quark-lepton families. Their basic carriers could be eight fermion preon fields $\mathcal{P}_i$ ($i = 1, \ldots , 8$) being presumably the fundamental octet of some basic metaflavor symmetry $SU(8)_{MF}$. Accordingly, we will refer to these preons as a collection of ”isons” $\mathcal{W}_a$ ($a = 1, 2$), ”chromons” $\mathcal{C}_k$ ($k = 1, 2, 3$) and ”famons” $\mathcal{F}_p$ ($p = 1, 2, 3$). We find below that, apart from some heuristic argumentation, the local metaflavor $SU(8)_{MF}$ symmetry as a basic internal symmetry of the physical world at small distances is indeed advocated by preon model for composite quarks and leptons.

We start by considering the $L$-$R$ symmetric composite model for quarks and leptons where the $N$ left-handed and $N$ right-handed constituent preons possessing some vectorlike local metaflavor symmetry $SU(N)_{MF}$ are bound by the chiral $SO(n)_L \times SO(n)_R$ gauge metacolor forces. They generally possess an exact chiral global symmetry $SU(N)_L \times SU(N)_R$ as well, once their $SU(N)_{MF}$ gauge metaflavor interactions are switched off.

Though these interactions may in principle break the preon chiral symmetry, they are typically too weak at the metacolor confining distances to influence the bound state spectrum. This spectrum is proposed to only depend on whether the preon chiral symmetry $SU(N)_L \times SU(N)_R$ or some of its part is preserved at large distances. This in turn determines the local metaflavor symmetry that could be observed at large distances through the massless or low-lying fermion composites emerged. Any extra metaflavor which is not protected by the preserved chiral symmetry lead to the superheavy composites with the metacolor scale $\Lambda_{MC}$ order masses and, therefore, appear unobserved at a laboratory scale.

We find that just the eight $L$-preons and eight $R$-preons and their observed metaflavor symmetry $SU(8)_{MF}$ appear as a result of a solution to the ’t Hooft’s anomaly matching condition \cite{6} providing the global chiral $SU(8)_L \times SU(8)_R$ symmetry preservation and masslessness of emerged composite fermions. We show that this happens if: (1) this condition is satisfied individually for the $L$-preon and $R$-preon composites and (2) each of these two series of composites fill the single irreducible representations of the $SU(N)_L$ and $SU(N)_R$ symmetry groups, respectively, rather than a set of their representations. Through the constraints on the admissible chiral symmetry $SU(N)_L \times SU(N)_R$ providing the massless composite fermions at large distances, the anomaly matching condition puts in general a powerful constraint on the underlying local metaflavor symmetry $SU(N)_{MF}$ as well. However, such an emerged $L$-$R$ symmetric $SU(8)_{MF}$ metaflavor theory certainly appears pure vectorlike for the identical $L$-preon and $R$-preon composite multiplets involved.

This means that, while preons are left massless being protected by their own metacolors, the composites being metacolor singlets will pair up and acquire heavy Dirac masses. We next show how the spontaneous $L$-$R$ symmetry violation caused by the simultaneously emerged composite scalars reduces the initially vectorlike metaflavor $SU(8)_{MF}$ symmetry down to one of its chiral subgroups being of significant physical interest. Particularly, this violation implies that, while there still remains the starting chiral symmetry $SU(8)_L$ for the left-handed preons and their composites, for the right-handed states one may only
have the broken chiral symmetry \([SU(5) \times SU(3)]_R\). Therefore, whereas nothing really happens with the left-handed preon composites still completing the total multiplet of the \(SU(8)_L\), the right-handed preon composites will form only some particular submultiplets of the \([SU(5) \times SU(3)]_R\) symmetry. As a result, we eventually come from the \(L-R\) symmetric metaflavor \(SU(8)_{MF}\) theory down to the conventional \(SU(5)\) GUT extended by an extra local family symmetry \(SU(3)_F\) describing the three standard families of quarks and leptons. Though the tiny confinement scale for universal preons composing both quarks and leptons makes it impossible to directly confirm their composite nature, simultaneous emergence of several extra \(SU(5) \times SU(3)_F\) multiplets of heavy composite fermions may help with a model verification. Some of them through a natural see-saw mechanism provide the physical neutrino masses which, in contrast to conventional picture, appear to follow an inverted family hierarchy. Others mix with ordinary quark-lepton families in a way that there may arise a marked violation of unitarity in the CKM matrix for leptons depending on an interplay between the compositeness scale and scale of the family symmetry \(SU(3)_F\). All issues mentioned above are successively considered below in the subsequent sections 2-7, and in the final section 8 we present our conclusion. For simplicity, we work in an ordinary spacetime framework, though extension to the conventional \(N = 1\) supersymmetry with preons and composites treated as standard scalar superfields could generally be made. As we see below, such an extension looks appealing in many aspects.

Some early attempt to classify quark-lepton families in the framework of the \(SU(8)\) GUT with composite quarks and leptons had been made quite a long ago \[7\], though with some special requirements which presently seem not necessary or could be in principle derived rather than postulated. Since then also many other things became better understood, especially the fact that the chiral family symmetry \(SU(3)_F\), taken by its own, was turned out to be rather successful in description of quark-lepton families. Meanwhile, as mentioned above, there has not yet appeared any valuable grand unified symmetry scheme with simultaneous classification of all observed quarks and leptons (though many interesting case studies have been carried out \[8\]). All that motivates us to address this essential problem once again in the framework of a significantly modified preon model for composite quarks and leptons. We will largely follow the recent paper \[9\].

2 Preons - metaflavors and metacolors

We start formulating the key elements of the preon model considered here. They are as follows below.

1. We propose that at small distances there are \(2N\) elementary massless left-handed and right-handed preons, described by the independent Weil spinors \(P_{iL}\) and \(Q_{iR}\) \((i = 1, \ldots, N)\), which possess a common local metaflavor symmetry \(SU(N)_{MF}\) unifying all known physical charges, such as weak isospin, color, and family number. The preons,

\[\text{Note that, unlike a conventional terminology, we call a metaflavor symmetry a real local flavor symmetry of preons rather than a spectator metaflavor symmetry appearing as result of a technical "gauging" of their global chiral symmetry which we discuss below.}\]
both $P_L$ and $Q_{iR}$, transform under the fundamental representation of the $SU(N)_{MF}$ and their metaflavor theory presumably has an exact $L$-$R$ symmetry. Actually, the $SU(N)_{MF}$ appears at the outset as some vectorlike symmetry which then breaks down at large distances to some of its chiral subgroup. All observed quarks and leptons are proposed to consist of these universal preons some combinations of which are collected in composite quarks, while others in composite leptons.

(2) The preons also possess a local chiral metacolor symmetry $G_{MC} = G_{MC}^L \times G_{MC}^R$ which contains them in its basic representation. In contrast to their common metaflavors, the left-handed and right-handed preon multiplets being vectorlike under the $SU(N)_{MF}$ symmetry, are taken to be chiral under the metacolor symmetry $G_{MC}$. They appear with different metacolors, $P^a_L$ and $Q^{a'}_{iR}$, where $a$ and $a'$ are indices of the corresponding metacolor subgroups $G_{MC}^L (a = 1, \ldots, n)$ and $G_{MC}^R (a' = 1, \ldots, n)$, respectively. These chiral $G_{MC}^{L,R}$ metacolor forces bind preons into composites - quarks, leptons and other states. As a consequence, there are two types of composites at large distances being composed individually from the left-right and left-handed preons, respectively. They presumably have a similar radius of compositeness, $R_{MC} \sim 1/\Lambda_{MC}$, where $\Lambda_{MC}$ corresponds to the scale of the preon confinement for the asymptotically free (or infrared divergent) metacolor symmetries. Due to the proposed $L$-$R$ invariance, the metacolor symmetry groups $G_{MC}^L$ and $G_{MC}^R$ are taken identical with the similar scales for both of sets of preons. We choose for metacolor the orthogonal symmetry group taking it, thereby, in a chiral form

$$G_{MC} = SO(n)_{MC}^L \times SO(n)_{MC}^R \quad (1)$$

This choice has some advantages over the unitary metacolor symmetry commonly used. Indeed, the orthogonal metacolor symmetry $[7, 10]$ is generically anomaly-free for its basic vector representation in which preons are presumably located. Also, the orthogonal metacolor allows more possible composite representations of the metaflavor symmetry $SU(N)_{MF}$ including those which are described by tensors with mixed upper and lower metaflavor indices.

(3) Under the local symmetries involved, both the metaflavor $SU(N)_{MF}$ and metacolor $G_{MC}$, the assignment of preons $P^a_L$ and $Q^{a'}_{iR}$ looks as

$$P^a_L (N, n, 1), \; Q^{a'}_{iR} (N, 1, n) \quad (2)$$

In fact, they also possess an accompanying chiral global $SU(N)_L \times SU(N)_R$ symmetry$^2$ in the limit when their common gauge $SU(N)_{MF}$ metaflavor interactions are switched off. We omitted here the Abelian chiral $U(1)_{L,R}$ symmetries since the corresponding currents have Adler-Bell-Jackiw anomalies given by the triangle graphs with a pair of metaphions $^{11}$. Accordingly, the current divergences for massless preons $P^a_L$ and $Q^{a'}_{iR}$ are given by

$$\partial_\mu J^\mu_L = \frac{g_{MC}^2}{16\pi^2} F^{[a,b]}_{[\mu} F^{\mu \rho \sigma]}_{b]} \epsilon^{\rho \sigma} \; , \; \partial_\mu J^\mu_R = \frac{g_{MC}^2}{16\pi^2} F^{[a',b']}_{[\mu} F^{\mu \rho \sigma]}_{b']} \epsilon^{\rho \sigma} \quad (3)$$

$^2$Note that the $SU(N)_L \times SU(N)_R$ is a chiral symmetry of the independent left-handed ($P^a_L$) and right-handed ($Q^{a'}_{iR}$) Weil spinors rather than chiral symmetry related to $L$- and $R$-components of the same Dirac spinors, as is usually implied. Instead of the $L$-$R$ basis used here, one could equally work in the pure left-handed basis where one may properly have the preon assignment, $P^a_L (N, n, 1)$ and $Q^{a'}_{iR} (\pi, 1, n)$, possessing the chiral symmetry $SU(N)_L \times SU(N)_L$. 

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where $F_{\mu\nu}^{[a,b]}$ and $F'_{\mu\nu}^{[a',b']}$ are the metagluon field strengths being in adjoint representations of the $SO(n)^{L,R}_{MC}$ groups, respectively, while $g_{MC}$ and $g'_{MC}$ are the appropriate gauge coupling constants.\footnote{Note that all fields and other quantities related to the "right" metacolor group $SO(n)^{R}_{MC}$ are denoted everywhere by the same letters as those of the "left" metacolor group $SO(n)^{L}_{MC}$ but taken with a prime symbol. The metagluon indices are given by two different sets of the bold Latin letters: $a, b, c, d, ...$ for the $SO(n)^{L}_{MC}$ and $a', b', c', d', ...$ for the $SO(n)^{R}_{MC}$. The metagluon indices are given by ordinary lowercase Latin letters, while lowercase Greek letters stand, as usual, for conventional spacetime indices.} Thus, the $U(1)_{L,R}$ symmetries which might be associated with chiral preon number symmetries in the classical Lagrangian, appear broken by the quantum corrections.\footnote{Nevertheless, one could presumably still "feel" these symmetries at the small distances, $r \ll R_{MC}$, where the corrections \footnote{The point is, however, the chiral discrete subgroups $Z_{N}^{L,R}$ of the $U(1)_{L,R}$ symmetries are still preserved. Under $Z_{N}^{L,R}$ the preons transform as $P_{iL}^{a} \rightarrow e^{i2\pi q_{L}^{(i)}/N}P_{iL}^{a}$, $Q_{iR}^{a'} \rightarrow e^{i2\pi q_{R}^{(i)}/N}Q_{iR}^{a'}$ (4) where the discrete charges $q_{L,R}^{(i)}$ are integers being defined modulo $N$ only. They are taken to be $q_{L,R}^{(i)} = 1$ for all $N$ preon species being in the fundamental multiplet of the metaflavor $SU(N)_{MF}$. We call them the chiral discrete preon numbers. The conditions for the $Z_{N}^{L,R}$ symmetries to be anomaly-free in the triangles $Z_{N}^{L,R} - SO(n)^{L,R}_{MC} - SO(n)^{L,R}_{MC}$ are given, respectively, by the simple equations $\sum_{i}^{N} q_{L,R}^{(i)} = 0 \mod N$ (5) which are automatically satisfied. Note that chiral discrete symmetries $Z_{N}^{L}$ and $Z_{N}^{R}$ can be equally considered as the centers of the groups $SU(N)_{L}$ and $SU(N)_{R}$, respectively. Therefore, the total chiral symmetry of all the preons involved is in fact the symmetry $K(N) = SU(N)_{L} \times SU(N)_{R}$ (6) which also includes its center identified as $C(N, N) = Z_{N}^{L} \times Z_{N}^{R}$ (7) We will refer to the latter, in what follows, as the chiral discrete preon number symmetry.} The point is, however, the chiral discrete subgroups $Z_{N}^{L,R}$ of the $U(1)_{L,R}$ symmetries are still preserved. Under $Z_{N}^{L,R}$ the preons transform as:

$$P_{iL}^{a} \rightarrow e^{i2\pi q_{L}^{(i)}/N}P_{iL}^{a} , \quad Q_{iR}^{a'} \rightarrow e^{i2\pi q_{R}^{(i)}/N}Q_{iR}^{a'}$$

(4) where the discrete charges $q_{L,R}^{(i)}$ are integers being defined modulo $N$ only. They are taken to be $q_{L,R}^{(i)} = 1$ for all $N$ preon species being in the fundamental multiplet of the metaflavor $SU(N)_{MF}$. We call them the chiral discrete preon numbers. The conditions for the $Z_{N}^{L,R}$ symmetries to be anomaly-free in the triangles $Z_{N}^{L,R} - SO(n)^{L,R}_{MC} - SO(n)^{L,R}_{MC}$ are given, respectively, by the simple equations $\sum_{i}^{N} q_{L,R}^{(i)} = 0 \mod N$ (5)

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(4) Obviously, the preon condensate $\langle \bar{P}_{L}Q_{R} \rangle$ which could cause the metacolor scale $\Lambda_{MC}$ order masses for composites is principally impossible in the left-right metacolor model.\footnote{Note that all fields and other quantities related to the "right" metacolor group $SO(n)^{R}_{MC}$ are denoted everywhere by the same letters as those of the "left" metacolor group $SO(n)^{L}_{MC}$ but taken with a prime symbol. The metagluon indices are given by two different sets of the bold Latin letters: $a, b, c, d, ...$ for the $SO(n)^{L}_{MC}$ and $a', b', c', d', ...$ for the $SO(n)^{R}_{MC}$. The metagluon indices are given by ordinary lowercase Latin letters, while lowercase Greek letters stand, as usual, for conventional spacetime indices.} This is in sharp contrast to an ordinary QCD case where the left-handed and right-handed quarks forms the $\langle \bar{q}_{L}q_{R} \rangle$ condensate thus leading to the color scale $\Lambda_{C}$ order masses for composite mesons and baryons. The fact that there is no the $\langle \bar{P}_{L}Q_{R} \rangle$ type condensate may be generally considered as a necessary but not yet a sufficient condition for masslessness of composites. The genuine massless fermion composites are presumably

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only those which preserve chiral symmetry of preons (6) at large distances that is controlled by the ’t Hooft’s anomaly matching (AM) condition [6]. We also consider here the composite scalar fields which could break the $SU(N)_{MF}$ metaflavor theory down to the Standard Model (SM). Generally, they being no protected by any symmetry will become very heavy (with masses of the order of the scale $\Lambda_{MC}$) and decouple from a low-lying particle spectrum. However, candidate(s) for the composite SM Higgs boson could also be expected under some special dynamical requirements.

(5) To summarize, we introduced $2N$ left-handed and right-handed preons, $P_{a R}^i$ and $Q_{a R}^i$ ($i = 1, \ldots, N; a = 1, \ldots, n; a' = 1, \ldots, n$), possessing some local metaflavor symmetry $SU(N)_{MF}$ and chiral metacolor symmetry $SO(n)_{MC}^L \times SO(n)_{MC}^R$ with numbers of metaflavors ($N$) and metacolors ($n$) not yet determined. We argued that these preons also possess the chiral symmetry $K(N)$ (6) in the limit when their gauge $SU(N)_{MF}$ metaflavor interactions are switched off. Though these interactions break the chiral symmetry $K(N)$, we propose for what follows that they are typically too weak at the metacolor confining distances to influence the bound state spectrum. On the other hand, just a preservation of the chiral symmetry $K(N)$ determines the particular metaflavor symmetry $SU(N)_{MF}$ which could be observed at large distances through the massless composites emerged. Indeed, any extra $\tilde{N}$ metaflavors (extra $\tilde{N}$ pairs of $P$ and $Q$ preons), which are not provided by a preserved chiral symmetry, will form superheavy composites with the metacolor scale $\Lambda_{MC}$ order masses and, therefore, appear unobserved at a laboratory scale. If that would be the case, one might have the chiral symmetry $K(N + \tilde{N})$ for preons, while the lower symmetry $K(N)$ for composites. However, for simplicity’s sake, we accept below that just the preserved chiral symmetry $K(N)$ eventually determines an observed metaflavor symmetry $SU(N)_{MF}$ so as to exclude any extra preons in the theory. Note that, apart from preons being the actual carriers of both the metaflavors and metacolors, there may also exist a number of elementary chiral fermions only possessing the metacolors. We will refer to them as ”sterilons” which can be used for construction of composites as well. A rather natural case could be sterilons being the gaugino (metagluino) multiplets, $S_{L}^{[a,b]}$ and $S_{R}^{[a',b']}$ of the $SO(n)_{MC}^L$ and $SO(n)_{MC}^R$, respectively, if one would properly supersymmetrize the metacolor theory. Accordingly, their masslessness might be protected by gauge invariance rather than the abovementioned chiral symmetry $K(N)$ (6) being solely related to the massless preons $P_{a R}^i$ and $Q_{a R}^i$, respectively.

3 AM conditions for N metaflavors

3.1 Preamble

Usually, for tackling mathematical problems related to conservation of the chiral symmetry $SU(N)_{L} \times SU(N)_{R}$, one turns it into the would-be local symmetry group with some spectator gauge fields and fermions [6] being in fact mathematical tools only. This trick allows to properly analyze the corresponding gauge anomaly cancellation thus checking the chiral symmetry preservation for massless preons and composites at both small and large distances. Though this symmetry is usually referred to as a metaflavor symmetry,
we will call it the spectator gauge symmetry. An actual metaflavor theory in our model is
an input local vectorlike $SU(N)_{MF}$ symmetry unifying $N$ left-handed and $N$ right-handed
preons, while their chiral symmetry $SU(N)_L \times SU(N)_R$ is in fact global. It is important
to see that, whereas in the $SU(N)_{MF}$ metaflavor theory gauge anomalies of preons and
composites are automatically cancelled out between left-handed and right-handed states
involved, in the spectator gauge $SU(N)_L \times SU(N)_R$ theory all anomalies have to be can-
celled by special multiplets of the metacolorless spectator fermions introduced individually
for the $SU(N)_L$ and $SU(N)_R$ sectors of the theory. As was mentioned above, though the
$SU(N)_{MF}$ metaflavor interactions may in principle break the preon chiral symmetry (6),
they are typically too weak to influence the bound state spectrum.

In the proposed preon model the AM condition (6) states in general that the chiral $SU(N)^3_L$ and $SU(N)^3_R$ triangle anomalies related to $N$ left-handed and $N$ right-handed preons have to match those for massless composite fermions being produced by the $SO(n)^3_{MC}$ and $SO(n)^3_{MC}$ metacolor forces, respectively. Actually, fermions composed from the left-handed preons and those composed from the right-handed ones have to inde-
pendently satisfy their own AM conditions. In contrast, in the local $SU(N)_{MF}$ metaflavor theory being as yet vectorlike, the $SU(N)^3_{MF}$ metaflavor triangle anomalies of the $L$-preons
and $R$-preons, as well as anomalies of their left-handed and right-handed composites, will
automatically compensate each other for any number $N$ of the starting preon species.
However, the AM condition through the constraints on the admissible chiral symmetry
$SU(N)_L \times SU(N)_R$ providing the masslessness of composite fermions at large distances,
may put in general a powerful constraint on this number, and thereby on the underlying
local metaflavor symmetry $SU(N)_{MF}$ itself as a potential local symmetry of massless (or
light) composites. This actually depends on the extent to which the accompanying global
chiral symmetry (6) of preons remains at large distances.

In one way or another, the AM condition

$$\sum_r i_r a(r) = na(N)$$

for preons (the right side) and composite fermions (the left side) should be satisfied. For
the sake of brevity, the equation (8) is simultaneously written for both the left-handed and
right-handed preons and their composites. Here $a(N)$ and $a(r)$ are the group coefficients
of triangle anomalies related to the groups $SU(N)_L$ or $SU(N)_R$ in (6) whose coefficients
are calculated in an ordinary way,

$$a(r)d^{ABC} = Tr\{(T^A T^B )T^C\}_r$$

where $T^A$ ($A, B, C = 1, \ldots, N^2 - 1$) are the $SU(N)_{L,R}$ generators taken in the corre-
sponding representation $r$. The $a(N)$ corresponds to a fundamental representation and is
trivially equal to $\pm 1$ (for left-handed and right-handed preons, respectively), while $a(r)$ is

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5This is somewhat similar to an interrelation between local and global symmetries in the conventional
quark model with the lightest u and d quarks where the actual local flavor theory is determined by
the unified electroweak symmetry $SU(2)_W \times U(1)_Y$ rather than their chiral global symmetry $SU(2)_L \times
SU(2)_R \times U(1)_{L+R}$.  

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related to a representation \( r \) for massless composite fermions. The value of the factors \( i_r \) give a number of times the representation \( r \) appears in a spectrum of composite fermions and is taken positive for the left-handed states and negative for the right-handed ones. The anomaly coefficients for composites \( a(r) \) contain an explicit dependence on the number of preons \( N \), due to which one could try to find this number from the AM condition taken separately for the \( L- \) and \( R- \) preons and their composites. In general, there are too many solutions to the condition \( (8) \) for any value of \( N \). Nevertheless, for some special, though natural, requirements an actual solution may only appear for \( N = 8 \), as we will see below.

Apart from the AM conditions \( (8) \), there could be in general another kind of constraint on composite models \( [6] \). This constraint requires the anomaly matching condition to work, even if any number of the initially introduced \( N \) preons acquire an infinite masses and consequently decouple from an entire theory. Indeed, this extra constraint makes the AM conditions to be independent of a number of metaflavors \( N \) and, as a result, classification of composite fermions becomes quite arbitrary. The point is, however, that in our preon model with a chiral metacolor symmetry \( SO(n)^{LMC}_L \times SO(n)^{RMC}_R \) the left-handed and right-handed preons might only form the Majorana masses which are in fact forbidden by the input local metaflavor symmetry \( SU(N)_{MF} \) of the theory. Apart from that, such a constraint may appear generally irrelevant since, as can be seriously argued \( [2] \), the nonperturbative effects may not be analytic in the preon mass so that the theories for massless and massive preons are turned out to be quite different. One way or the other, we will not consider this extra constraint in our composite model.

### 3.2 Strengthened AM condition: SU(8) unification

To strengthen the AM condition one may think that it would be more appropriate to have all composite quarks and leptons in a single representation of the unified symmetry group \( SU(N)_{MF} \) rather than in some set of its representations. This, though would not largely influence the gauge sector of the unified theory, could make its Yukawa sector much less arbitrary. Apart from that, the composites belonging to different representations would have in general the different discrete \( Z^L_R \) preon numbers that could look rather unnatural. In this manner, the strengthened AM condition suggests that only some particular \( SU(N)_{MF} \) multiplets of the left-handed and right-handed composites being in the corresponding representations of chiral symmetry groups \( SU(N)_{L,R} \) in \( (6) \) may contribute. It is clear that these \( L- \) and \( R- \) multiplets have to be similar to remain the \( SU(N)_{MF} \) metaflavor theory anomaly free. Let us propose for the moment that we only have the minimal three-preon fermion composites. They are formed by the metacolor forces which correspond to the \( SO(3)^{LMC}_L \times SO(3)^{RMC}_R \) symmetry case in \( (11) \) with metagluons, \( A^{a}_\mu \) and \( A^{'a}_\mu \) transforming, like preons \( P_{aL}^\mu \) and \( Q_{aR}^\mu \) themselves, according to the vector representations of the \( SO(3)^{LMC}_L \) and \( SO(3)^{RMC}_R \), respectively. We will, therefore, require that only some single representation \( r_0 \) for massless three-preon states has to satisfy the AM condition that simply gives in \( (8) \)

\[
\begin{equation}
a(r_0) = 3
\end{equation}
\]

individually for \( L- \) preon and \( R- \) preon composites.
Now, calculating the anomaly coefficients for all possible three $L$-preon and three $R$-preon metacolorless composites one has, respectively,

\[
\begin{align*}
\Psi^{\{ijk\}}_{L L,} & , \Psi'^{\{ijk\}}_{R L}, \pm (N^2/2 + 9N/2 + 9), \\
\Psi^{\{ijk\}}_{L R,} & , \Psi'^{\{ijk\}}_{R R}, \pm (N^2/2 - 9N/2 + 9), \\
\Psi^{\{ijk\}}_{L L',} & , \Psi'^{\{ijk\}}_{R L'}, \pm (N^2/2 + 7N/2 - 1), \\
\Psi^{\{ijk\}}_{L R',} & , \Psi'^{\{ijk\}}_{R R'}, \pm (N^2/2 - 7N/2 - 1)
\end{align*}
\]

(11)

where all appropriate third-rank representations of the $SU(N)_{L,R}$ are listed (the traces have been subtracted from the tensors at the last two lines). According to the notation taken, we denote the $R$-preon composites by the same letters as the $L$-preon ones but taken with the prime symbol (anomaly coefficients for right-handed composites have to be taken with an opposite sign). In contrast to conventional QCD, for an orthogonal metacolor there could also exist the simplest composite states $\Psi^{i}_{L}$ and $\Psi'^{i}_{R}$ constructed out of single $P^a_{iL}$ and $Q^a_{iR}$ preons, whose metacolor charges are screened by the corresponding metagluon fields of $SO(3)_{MC}^{L}$ and $SO(3)_{MC}^{R}$, respectively. Though they could in principle contribute into the AM condition, we do not consider these fundamental $SU(8)_{MF}$ multiplets as appropriate composite candidates for physical quarks and leptons. In our left-right model the composite states $\Psi^{i}_{L}$ and $\Psi'^{i}_{R}$ generally pair up and receive heavy Dirac masses (as is argued later).

Putting now each of the above anomaly coefficients in (11) into the AM condition (10) one can readily find that there is a solution with an integer $N$ only for the last tensors $\Psi^{i}_{[jk]L}$ and $\Psi'^{i}_{[jk]R}$, and this is in fact the unique "eightfold" solution

\[
N^2/2 - 7N/2 - 1 = 3, \quad N = 8 .
\]

(12)

This means that among all possible chiral symmetries $K(N)$ in (6) only $SU(8)_{L} \times SU(8)_{R}$ could in principle provide the massless fermion composites at large distances that in turn identifies the metaflavor $SU(8)_{MF}$ symmetry as a possible unified symmetry of the left-handed and right-handed preons and their composites which presumably include, among many other states, the ordinary quarks and leptons.

Remarkably, the same solution $N = 8$ independently appears if one requires that the yet unknown metaflavor symmetry $SU(N)_{MF}$ has to possess the right $SU(5)$ GUT classification [15] for the observed quarks and leptons. Indeed, for an adequate $SU(5)$ assignment the "right" $SU(N)_{MF}$ representation has to contain the equal numbers of the $SU(5)$ anti-quintets $\bar{5}$ and decuplets $10$ to be in accordance with the Standard Model classification and an observation. Decomposing $SU(N)_{MF}$ into $SU(5) \times SU(N - 5)$ one can calculate these numbers for all its 3-index multiplets presented in (11). Actually, as can be easily seen, neither of them but the representations $\Psi^{i}_{[jk]L}$ and $\Psi'^{i}_{[jk]R}$ appear to contain both chiral anti-quintets and decuplets of the $SU(5)$ GUT. Moreover, an equality

\[6\] The anomaly coefficients for all possible third-rank tensors given above have been first calculated in [7, 10].
of their numbers in these representations determines the total symmetry group itself for which such an equality is solely possible. In fact, this equality when being required for the multiplets $\Psi_{[jk]}^L$ and $\Psi_{[jk]}^R$ reads as

$$\frac{(N - 5)(N - 6)}{2} = N - 5$$

(13)

with the number of anti-quintets on the left side and number of decuplets on the right one. As a result, we come once again to the eightfold $SU(8)_{MF}$ metaflavor symmetry ($N = 8$).

Let us note, however, that the $SO(3)_{LMC}^L \times SO(3)_{LMC}^R$ metacolors providing the three-preon structure of composite quarks and leptons may appear insufficient for the preon confinement, unless one invokes some special strong coupling regime [13]. Generally, for $N$ preons possessing the asymptotically free $SO(n)$ metacolor symmetry, one must require the inequality [14]

$$n > 2 + 2N/11$$

(14)
as a necessary condition for their confinement inside of quarks and leptons. One can readily see that for a commonly used minimal metacolor number value, $n = 3$, at most five preon species ($N = 5$) with an underlying metaflavor symmetry $SU(5)_{MF}$ is only admissible. This certainly seems to be not enough for all elementary quantum numbers (or metaflavors) presently observed including those for quark-lepton families. For the next odd number, $n = 5$, one may have up to sixteen ($N = 16$) admissible preon species that could be quite sufficient. We consider here just this case in the strengthened AM condition [10]. Thus, the metacolor forces now correspond to the $SO(5)_{LMC}^L \times SO(5)_{LMC}^R$ symmetry case with gauge metagluon fields $A^a_{[a,b]}$ and $A^b_{[a\prime,b\prime]}$ transforming according to adjoint representations of the $SO(5)_{LMC}^L$ and $SO(5)_{LMC}^R$ groups, respectively. Remarkably, due to an orthogonal nature of metacolor, that allows a metacolor charge to be screened by its own metagluons, hand in hand with the five-preon fermion composites, the three-preon composites [11], as well as the one-preon composites, may also appear in the bound spectrum in this case. We consider here just these minimal composite states in the new strengthened AM condition

$$a(r_0) = 5$$

(15)

individually for $L$-preon and $R$-preon composite states ignoring the five-preon states as somewhat exotic ones.

Checking all the representations [11] of the $SU(N)_{L,R}$ we find that the AM condition works again for the multiplets $\Psi_{[jk]}^L$ and $\Psi_{[jk]}^R$ provided that they are taken together with the fundamental multiplets $\Psi_{il}^{(1,2)}$ and $\Psi_{iR}^{(1,2)}$ corresponding to composites in which the preon metacolor is properly screened by a pair of metagluons[7].

$$\Psi_{[jk]}^L + \Psi_{il}^{(1)} + \Psi_{il}^{(2)}, \quad \Psi_{[jk]}^R + \Psi_{iR}^{(1)} + \Psi_{iR}^{(2)}.$$  

(16)

7 Or, alternatively, one can take $\Psi_{il}^{(1)}$ and $\Psi_{iR}^{(1)}$ as the single preon states screened by a pair of metagluons, while $\Psi_{il}^{(2)}$ and $\Psi_{iR}^{(2)}$ as the actual three-preon states given by the "trace" tensors obtained after taking traces out of the proper three-index tensors $\Psi_{[jk]}^L$ and $\Psi_{[jk]}^R$ in [10]. Their wave functions can be found in the section 4.2.
Again, using the AM condition (15) for the tensors (16) one eventually has the same equation (12) for the number of preon species $N$, thus coming once again to the eightfold solution $N = 8$. Generally, due to direct screening effects of the orthogonal metacolor the massless three-preon and one-preon composites mentioned above may appear for any odd number of metacolors rather than in the $n = 5$ metacolor case only. Indeed, one can easily check that for any higher odd $n$ value the AM condition also works for the three-preon tensors $\Psi^i_{[jk]L} (\Psi^{n}_{[jk]R})$ extended by some number $p$ of one-preon multiplets $\Psi^i_{iL} (\Psi^i_{iR})$

$$\Psi^i_{[jk]L} + p\Psi_{iL} , \Psi^{n}_{[jk]R} + p\Psi^i_{iR}$$

provided that they all are properly screened by the corresponding metagluons. Actually, the AM condition now leads to the equation generalizing the above anomaly matching condition (12)

$$N^2/2 - 7N/2 - 1 + p = n$$

One can see that there appear solutions in (18) only for $n - p = 3$ and, therefore, one has again solution for the eightfold chiral symmetry $SU(8)_L \times SU(8)_R$ and, thereby, the eightfold metaflavor symmetry $SU(8)_{MF}$. Indeed, the $n = 5$ metacolor case leading to the composite multiplets (16) is just one particular, though minimal confining model case which satisfies the generalized anomaly matching condition (18).

### 3.3 Minimality matching condition

We have seen above that for a minimal metacolor case, $n = 3$, the strengthened AM condition (10) determines the $SU(8)_{MF}$ metaflavor symmetry as the only possible unified symmetry of massless preons and their composites which are located in the representations $\Psi^i_{[jk]L}$ and $\Psi^{n}_{[jk]R}$. Nonetheless, for the higher numbers of metacolors some other candidates may also emerge. Thanks to the orthogonal metacolor screening effect, we may consider again only three-preon composite tensors ignoring the high-number preon states as some exotic ones. One can immediately check that there appears a new candidate even in the five-metacolor case. Indeed, the strengthened AM condition (15) for the third-rank antisymmetric composite multiplets in (11)

$$\Psi^i_{[jk]L} , \Psi^{n}_{[jk]R} ; N^2/2 - 9N/2 + 9 = 5$$

has a solution for an integer $N$ and this is again the eightfold solution, $N = 8$. Therefore, we have in fact two competing eightfold models conditioned by the survived $SU(8)_L \times SU(8)_R$ chiral symmetry at large distances. Meanwhile, only the model with composite multiplets $\Psi^i_{[jk]L}$ and $\Psi^{n}_{[jk]R}$ may give, as was clearly seen above, an adequate description of the quark-lepton families.

Some way to discriminate the model with composite multiplets (19), as well as models with other three-preon tensors in (11) would be an existence of an extra selection rule related to minimality of possible global preon numbers for composites, both left-handed and right-handed ones. Since, as we discussed in the section 2 above, the corresponding symmetries $U(1)_{L,R}$ are broken by anomalies, one may only turn to the discrete preon
numbers related to the center $Z^L_N \times Z^R_N$ of the chiral $SU(N)_L \times SU(N)_R$ symmetry involved. So, according to this selection rule only composites satisfying the discrete preon number matching condition

$$Z^{L,R}_N(\text{preons}) = Z^{L,R}_N(\text{composites})$$  \hspace{1cm} (20)$$

might presumably appear in physical spectrum. We cannot fundamentally argue why the extra selection rule (20) should work in general. However, it seems reasonable that the orthogonal metacolor allowing (in contrast to unitary metacolor symmetry case) too many possible composite configurations may somehow dynamically single out composites with a minimal discrete preon number.

If the minimality matching condition (20) works, then the composite multiplets in (16) and (17) having the minimal unit $Z^{L,R}_N$ charges (as those of the preons themselves) may emerge massless at large distances, while the composite third-rank multiplets (19) having the triple discrete charges should presumably acquire heavy masses. In general, only one more multiplet of massless composites with minimal $Z^{L,R}_N$ charges, namely $\Psi^i_{\{jk\}}L (\Psi^a_i)^R_{\{jk\}}$, could in principle appear in the collection (11). However, as one can readily check, the AM condition for this multiplet would require an enormously large number of metacolors $n$ even for a low number of metaflavor species $N$ that seems hardly admissible.

So, we will mainly consider in what follows the set of multiplets (16) satisfying the AM conditions in the $n = 5$ metacolor case that eventually leads to the $SU(8)_{MF}$ metaflavor symmetry which may contain three standard families of quarks and leptons. Generally, the whole class of the $n$ metacolor $SU(8)_{MF}$ theories emerging through the massless composite multiplets (17) appears the best possible choice, though one has still vectorlike theory with similar left-handed and right-handed composite multiplets. Remarkably, this class is also selected by the discrete preon number matching condition (20) discussed above. We find in the section 5 below that the proposed selection rule (20) becomes particularly important when the starting $L-R$ symmetry in the theory is spontaneously broken and, as a result, three chiral families of quarks and leptons emerge.

4 Composites - the L-R symmetry phase

4.1 Multiplets of preon and composites

According to the strengthened AM conditions discussed above only the chiral symmetry (6) of eight preon species

$$K(8) = SU(8)_L \times SU(8)_R$$  \hspace{1cm} (21)$$

with the center

$$C(8,8) = Z^L_8 \times Z^R_8$$  \hspace{1cm} (22)$$

may appear unbroken at both small and large distances. This in turn selects, among many other $SU(N)_{MF}$ alternatives, just the local metaflavor $SU(8)_{MF}$ theory as the only possible unified theory of massless preons and composites. So, we have at small distances the sixteen left-handed and right-handed preons given by the Weil fields

$$P^a_{iL}, \quad Q^a_{iR} \quad (i = 1, \ldots, 8; \quad a = 1, \ldots, 5; \quad a' = 1, \ldots, 5)$$  \hspace{1cm} (23)$$

13
belonging to the fundamental octets and quintets of the $SU(8)_{MF}$ symmetry and metacolor symmetry $SO(5)^L_{MC} \times SO(5)^R_{MC}$, respectively. At large distances, on the other hand, we have massless composites \((16)\) which are located in the left-handed and right-handed multiplets of the $SU(8)_{MF}$

\[216_{[jk]L}^i + 8_{L}^{(1)} + 8_{L}^{(2)}, \quad 216_{[jk]R}^i + 8_{R}^{(1)} + 8_{R}^{(2)},\]

where their dimensions are explicitly indicated. Recall that, while the three-preon composites $216_{L,R}$ are properly screened by single metagluons of the $SO(5)^{L,R}_{MC}$, the one-preon composites $8_{L,R}^{(1,2)}$ are screened by pairs of metagluons.

Note that the strengthened AM conditions propose that the chiral symmetry subgroups $SU(8)_{L}$ and $SU(8)_{R}$ of the left-handed and right-handed preons in \((21)\) have to remain individually. At the same time, in the vectorlike metaflavor $SU(8)_{MF}$ theory these $L$- and $R$-preons act jointly, so that all triangle anomalies at both small and large distances always appear automatically compensated. Decomposing the $SU(8)_{MF}$ composite multiplets \((24)\) into the $SU(5) \times SU(3)$ components one has

\[216_{L,R} = [(5 + 10, \overline{3}) + (45, 1) + (20, 1)]_{L,R}, \quad 8_{L,R}^{(1,2)} = [(5, 1) + (1, 3)]_{L,R}^{(1,2)}\]

where the first term for the left-handed composites in $216_{L}$, $(5 + 10, \overline{3})_{L}$, could be associated with the standard $SU(5)$ GUT assignment for quarks and leptons \([15]\) extended by some family symmetry, which we will denote for what follows by $SU(3)_F$. The other submultiplets in \((25)\) become heavy, as we show later, and decouple from an observed low-lying particle spectrum. Below, we discuss in more detail some possible $L$-preon and $R$-preon composites, both fermions and scalars, being of an important interest.

### 4.2 Structure of composite fermions

The determination of an explicit form of wave functions for the composite states \((24)\) is a complicated dynamical problem related to the yet unknown dynamics of the preon confinement. We propose that some basic features of the left-handed composites built from the $P$-preons and their metagluons may be simply given by the gauge invariant expressions

\[
\Psi_{[jk]L}(x) \propto \epsilon_{abcde} \left( F^{[cde]}_{\mu \nu} \right)_{P_{kL}}^b \gamma^{\mu} \gamma^{\nu} \Psi_{[jL]}^a, \\
\Psi_{[jL]}(x) \propto \epsilon_{abcde} \left( F^{[cde]}_{\mu \nu} \right)_{P_{jL}}^b \left( F^b_{\mu \nu} \right)_{P_{cL}}^a
\]

for the three-preon and one-preon states, respectively ($F^{[cde]}_{\mu \nu}$ is the metagluon stress tensor related to the metacolor symmetry $SO(5)^{L}_{MC}$ with the antisymmetric fifth-rank tensors $\epsilon_{abcde}$ and vector representation indices $a, b, c, d$ and $e$). The preon covariant derivative $D^\nu P^b_{jL}$ in \((26)\) should contain terms corresponding to both metacolor and metaflavor local symmetries involved

\[
D^\nu P^b_{jL} = \left[ \delta^b_{c} \delta^d_{e} \partial^\nu + g_{MC}(I \cdot A^\nu)_{c}^{b} \delta^d_{e} + g_{MF} \delta^b_{c} (T \cdot B^\nu)_{jL} \right] P_{cL}^e
\]
where \( I \) and \( T \) stand for sets of the \( SO(5)_{MC}^R \) and \( SU(8)_{MF} \) generators, while \( \mathbf{A}^\nu \) and \( \mathbf{B}^\nu \) are the corresponding gauge field multiplets taken with their coupling constants. In the valent preon approximation, the left-handed preon currents \( (26) \) correspond to the zero mass bound states with a spin of \( 1/2 \) and a helicity \(-1/2\) being formed by two left-handed preons and one anti-preon plus a metagluon (or one left-handed preon plus a pair of metagluons) which are moving in a common direction.

In a similar way one can construct the right-handed preon composites which correspond to multiplets of states with a spin of \( 1/2 \) and helicity \(+1/2\) composed from right-handed \( Q \)-preons and metagluons of the metacolor symmetry \( SO(5)_{MC}^R \). This is simply achieved by making the proper replacements in \( (26) \) leading to the composite states

\[
\Psi_{iR}^\nu(x) \propto \epsilon_{a'b'c'd'e'} \left[ \left( \gamma_5 \mathbf{T}^i \right) \gamma^\mu D^\nu Q_{jR}^{b'} \right] Q_{kR}^{c'} F_{\mu\nu}^{d'e'} \\
\Psi_{iR}^\nu(x) \propto \epsilon_{a'b'c'd'e'} \left[ \left( \gamma_5 \mathbf{T}^i \right) \gamma^\mu D^\nu Q_{jR}^{b'} \right] Q_{kR}^{c'} F_{\mu\nu}^{d'e'}
\]

(28)

where the antisymmetric fifth-rank tensors \( \epsilon_{a'b'c'd'e'} \) and vector representation indices \( a', b', c' \) and \( d' \) now belong to the metacolor symmetry \( SO(5)_{MC}^R \) with a metagluon \( A^\mu_{\nu d'e'} \).

Accordingly, the covariant derivative \( D^\nu Q_{jR}^{b'} \) has a form

\[
D^\nu Q_{jR}^{b'} = \left[ \delta_{b'}^{d'} A_{\nu}^{\mu} \gamma_{\mu} + g'_{MC} (T \cdot A^\nu)^{\mu} \gamma_{\mu} + g_{MF} (T \cdot B^\nu)^{\mu} \gamma_{\mu} \right] Q_{kR}^{c'}
\]

(29)

where the \( SO(5)_{MC}^R \) set of generators \( I' \) and the corresponding gauge field multiplet \( \mathbf{A}^\nu \) with its coupling constant \( g'_{MC} \) are now presented.

In principle, apart from the composites \( (26) \) and \( (28) \), one might construct the composites with inverse chiralities, particularly, the right-handed multiplets \( \Psi_{[jk]R}^i \) and \( \Psi_{iR}^\nu \) composed from the left-handed preons \( P_{aL}^\alpha \) and the left-handed multiplets \( \Psi_{[jk]L}^i \) and \( \Psi_{iL}^\nu \) composed from the right-handed preons \( Q_{aR}^{\alpha} \). If so, they might pair up with the starting multiplets \( (26) \) and \( (28) \) and, thereby, cause the metacolor scale \( \Lambda_{MC} \) order masses individually for the \( P \)- and \( Q \)-preon composites. The point is, however, that due the chiral symmetry preservation, according to which the AM condition selects just the multiplets \( (26) \) and \( (28) \) as the generically massless ones, the inverse chirality multiplets mentioned above are not allowed to appear in the composite spectrum. In contrast, all other composite fermion multiplets listed above in \( (11) \) may possess both chiralities individually for the \( P \)- and \( Q \)-preon composites. As a result, they will pair up and acquire the metacolor scale order Dirac masses. In fact, all of them then decouples from the low-lying fermion spectrum and may only contribute in the coupling running at superhigh energies.

### 4.3 Composite scalars

Let us now turn to the heavy scalar composites which may cause spontaneous breaking of the \( L-R \) symmetry in the theory, as well as the metafort foam symmetry \( SU(8)_{MF} \) itself down to the Standard model and finally its breaking as well. We schematically present below some of them. First, there are the two-preon composites giving the massive effective scalar...
states which complete in general both two-index symmetric and antisymmetric multiplets of the $SU(8)_{MF}$

\[
\begin{align*}
\chi_{(ij)}(x) &\propto P^a_{[iL}C P^a_{jL]} , \quad \chi'_{(ij)}(x) \propto P^a_{[iL}C P^a_{jL]} \\
\chi^a_{(ij)}(x) &\propto Q^a_{[iR}C Q^a_{jR]} , \quad \chi'^a_{(ij)}(x) \propto Q^a_{[iR}C Q^a_{jR]}
\end{align*}
\]

(30)

where $C$ stands for the charge-conjugation matrix. They have the following $SU(5) \times SU(3)_{MF}$ decomposition

\[
\begin{align*}
28 &= (5,3) + (10,1) + (1,\overline{3}) , \\
36 &= (5,3) + (15,1) + (1,6)
\end{align*}
\]

(31)

according to which they might break the family symmetry $SU(3)_{MF}$ when developing vacuum expectation values (VEVs) on the components $(1,\overline{3})$ and $(1,6)$, respectively.

Next is the preon-antipreon composites belonging to the adjoint representations of the $SU(8)_{MF}$

\[
\phi_j^a(x) \propto T^{a}_{L} \gamma_\mu D^\mu P^a_{jL} , \quad \phi'_j^a(x) \propto \overline{T}^{a}_{R} \gamma_\mu D^\mu Q'^a_{jR}
\]

(32)

where $D^\mu$ and $D'^\mu$ are the covariant derivatives with respect to the metacolor and metaflavor symmetries given above in (27) and (29), respectively. The $SU(5) \times SU(3)_{MF}$ structure of these multiplets

\[
63 = (24,1) + (\overline{5},3) + (5,\overline{3}) + (1,8) + (1,1)
\]

(33)

shows that they can be used for the "diagonal" breaking all the symmetries involved - $SU(8)_{MF}$, $SU(5)$ and $SU(3)_{MF}$ - depending on which components their VEVs are developed.

Further, there may be scalars which are consisted of three preons combined with metaflavorless fermions $S^{[ab]}_L$ and $S'^{[a'b']}_R$

\[
\begin{align*}
\Phi_{[ijkl]}(x) &\propto \epsilon_{abcdef} \left( P^a_{[iL}C P^b_{jL]} \right) \left( P^c_{kL}C S^{[de]}_L \right) , \\
\Phi'_{[ijkl]}(x) &\propto \epsilon_{a'b'c'd'e'f'} \left( Q'^a_{[iR}C Q'^b_{jR]} \right) \left( Q'^c_{kR}C S'^{[d'e']}_R \right)
\end{align*}
\]

(34)

which belong to the third-rank antisymmetric representation of the $SU(8)_{MF}$. Thereby, apart from preons, being the actual carriers of metaflavors and metacolors, we have used a pair of elementary sterile fermion multiplets, $S^{[ab]}_L$ and $S'^{[a'b']}_R$ having only the $SO(5)^L_{MC}$ and $SO(5)^R_{MC}$, metacolors, respectively. We called these fermions "sterilons" above in the section 2 treating them as possible massless or light gauginos in the supersymmetrized metacolor theory.\footnote{Remarkably, as one can notice from equations (26), (28) and (34), our basic composites, both fermions $\Psi_{[ijL]}$ ($\Psi'_{[ijL]}$) and scalars $\Phi_{[ijkl]}$ ($\Phi'_{[ijkl]}$) contain the gauge superfield components of the would-be supersymmetric metacolor theory - gauge vector fields $A^a_{\mu} (A'^a_{\mu})$ and gauginos $S^{[ab]}_L (S'^{[a'b']}_R)$, respectively.}

In principle, one might consider some other sterilon containing scalar multiplets as well, particularly, the composite multiplets $\Phi_{[ijkl]}(x)$ and $\Phi'_{[ijkl]}(x)$ consisting of two preons and anti-preon, rather than three preons as in (34). However, it is clear that
such scalars, if existed, would be extremely unstable dissociating into our massless fermion multiplets $\Psi^i_{[jk]L}$ and $\Psi^i_{[jk]R}$ and sterilon (screened by the corresponding metagluons). The same could be said about the one-preon scalar multiplets $\Phi_i(x)$ and $\Phi'_i(x)$ containing sterilon together with metagluons - they will readily decay into the screened preons and sterilon. So, there are practically left only the third-rank scalars as some simple choice for the sterilon containing scalar multiplets. The $SU(5) \times SU(3)_{MF}$ decomposition of them gives

$$56 = (\overline{10}, 1) + (10, 3) + (5, \overline{3}) + (1, 1)$$

(35)

that means an immediate breaking of the starting symmetry $SU(8)_{MF}$ down to $SU(5) \times SU(3)_{MF}$ when their VEVs are developed on the component $(1, 1)$.

All the above basic scalars develop large VEVs which determine in general some effective mass scale in the theory. They all provide an appropriate breaking of the $SU(8)_{MF}$ GUT down to the Standard Model. Further breaking may be related to the exotic scalars consisting of four preons

$$\varphi_{[ijkl]}(x) \propto \begin{pmatrix} P_{[iL}^a C P_{jL}^a \end{pmatrix} \begin{pmatrix} P_{kL}^b C P_{lL}^b \end{pmatrix}$$

$$\varphi'_{[ijkl]}(x) \propto \begin{pmatrix} Q_{[iR}^a' C Q_{jR}^a' \end{pmatrix} \begin{pmatrix} Q_{kR}^b C Q_{lR}^b \end{pmatrix}$$

(36)

where together with pure antisymmetrical multiplets there could also be many other ones with a mixed symmetry. They all are similar to the exotic states of QCD, like the $(g\overline{q})(q\overline{q})$ states and others. Even though they may still be bound, it is conceivable that the significantly weaker four-preon attraction makes these very unstable scalars to develop a hierarchically small VEV just needed for a conventional breaking of the Standard Model.

The antisymmetrical fourth-rank multiplets are in fact the self-conjugated multiplets of the $SU(8)_{MF}$ with the $SU(5) \times SU(3)_{MF}$ structure

$$70 = (\overline{5}, 1) + (5, 1) + (10, \overline{3}) + (\overline{10}, 3)$$

(37)

containing the $SU(5)$ quintets which could break down the Standard Model itself, and also give masses to quarks and leptons. So, using all the effective scalar fields listed above one may finally come to the realistic breaking pattern of the starting metaflavor symmetry $SU(8)_{MF}$.

### 4.4 Gravitational conversion of vectorlike fermions

In conclusion, let us note that the whole $L$-$R$ symmetric $SU(8)_{MF}$ metaflavor theory so far considered certainly appears pure vectorlike for the identical $L$-preon and $R$-preon.

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9This implies in general some special fine tuning in total potentials of all the scalar fields involved to provide, apart the small VEVs of the Higgs multiplets, also the proper doublet-triplet splittings in them so as to have the light electroweak doublets and heavy color triplets which would mediate a proton decay. Interestingly, while the first problem is generically solved in supersymmetric extension of any $SU(N)$ grand unification, a solution to the second one in the framework of the so-called missing VEV conjecture naturally appears only for the supersymmetric $SU(8)$ GUT [17].
composite multiplets (24) whose masslessness is individually provided by their own chiral symmetry preservation. Nonetheless, while preons themselves are left massless being protected by their own metacolors, these left-handed and right-handed composites being metacolor singlets may acquire masses. Indeed, they will pair up with each other in (24) and, thereby, acquire nonzero Dirac masses. This presumably happens due to the gravitational transitions caused by virtual black holes acting as intermediate particles, just as they presumably do in gravitationally induced proton decay [18]. In fact, the above transitions between the similar $L$-preon and $R$-preon composite multiplets in (24) can readily proceed via virtual black holes in a way that all local symmetries involved are preserved. In fact, these transitions basically depend on the compositeness scale $\Lambda_{MC}$ and scale of quantum gravity determined, as usual, by the Planck mass $M_{Pl} \simeq 1.2 \cdot 10^{19} GeV$. As we find later, the scale $\Lambda_{MC}$ appears close to the Planck scale, due to which masses of all the composites involved are expected to be very heavy. This would make our $SU(8)_{MF}$ metaflavor theory meaningless unless the proposed $L$-$R$ symmetry is broken so as to allow some submultiplets in (24) to be left truly massless. One could hope that such breaking may eventually exclude the right-handed submultiplet $(5 + 10, \bar{3})_R$ in the composite spectrum (25), while leaving there its left-handed counterpart, $(\bar{5} + 10, 3)_L$, which can be then uniquely associated with the observed three families of ordinary quarks and leptons. We consider all that in detail in the next section.

5 Composites - partially broken L-R symmetry

5.1 Strategy

We propose a spontaneous breaking of the chiral symmetry $K(8)$ (21) in the right-handed preon sector to induce a basic $L$-$R$ asymmetry in the emerged $SU(8)_{MF}$ metaflavor theory being as yet vectorlike. We will analyze this breaking first in the framework of the spectator gauge $SU(8)_L \times SU(8)_R$ model switching off the metaflavor interactions and then consider its physical consequences in our $SU(8)_{MF}$ theory. Actually, this breaking may appear due to a possible condensation of massive composite scalars which unavoidably appear in the theory together with composite fermions. We will see that just due to compositeness of the scalars involved, while such breaking does not change the chiral symmetry of the preons themselves, it may crucially change the chiral symmetry of their composites. Some of these scalars (30), (32), (34) and (36) composed, respectively, from the left-handed ($P$) and right-handed ($Q$) preons we considered in the previous section. The preon metacolor interactions may generally produce at large distances (beyond a confinement area) some effective potential due to which these scalars develop VEVs violating global and local symmetries involved. We propose that the third-rank scalars (31), $\Phi_{[ijk]}$, $\Phi_{[ijk]}'$ containing the sterilons $S_L^{[ab]}$ and $S_R'^{[a'b']}$ may develop the largest VEVs in the theory that is a somewhat natural breaking pattern in our $SO(5)_{MC}^L \times SO(5)_{MC}^R$ metacolor theory.

If from those two scalars only the $Q$-preon scalar multiplet $\Phi_{[ijk]}'$ develops VEV one comes, as we show below, to the spontaneous violation of the $K(8)$ symmetry (21) including
its center (22). This violation will presumably have a form

\[
K(8) \rightarrow [SU(8)]_L \times [SU(5) \times SU(3)]_R ,
\]
\[
C(8, 8) \rightarrow Z_8^L \times (Z_5 \times Z_3)^R
\]

that breaks the chiral symmetry of right-handed \(Q\)-preon composites, while the chiral symmetry of the left-handed \(P\)-preon composites is left intact. This in turn means that the underlying \(L-R\) symmetry becomes spontaneously broken in the theory at large distances.

### 5.2 How may it work

The breaking pattern (38) may be caused by some typical \(L-R\) symmetric polynomial potential for scalars \(\Phi_{ijk}\) and \(\Phi'_{ijk}\)

\[
U = -M_U^2(\Phi^2 + \Phi'^2) + h_1(\Phi^2\Phi^2 + \Phi'^2\Phi'^2)^2 + h_2(\Phi^4 + \Phi'^4) + h_3(\Phi^2\Phi'^2) + \cdots ,
\]

(with the notations \(\Phi^2 = Tr(\Phi^+\Phi)\), \(\Phi'^2 = Tr(\Phi'^+\Phi')\), \(\Phi^4 = Tr(\Phi^+\Phi^+\Phi)\) and \(\Phi'^4 = Tr(\Phi'^+\Phi'^+\Phi')\) used) which presumably are induced by the multi-preon interactions at large distances. Likewise, these interactions may also produce the Yukawa preon-scalar potential of the type

\[
L_Y = \frac{1}{M_Y^3} \left\{ \epsilon_{abcde} \left( P_{iL}^a C P_{jL}^b \right) \left( P_{kL}^c C S^{[de]}_L \right) \right\} \Phi_{ijk} \]
\[
+ \left\{ \epsilon_{a'b'c'd'e'} \left( Q_{iR}^{a'} C P_{jR}^{b'} \right) \left( Q_{kR}^{c'} C S^{[d'e']}_R \right) \right\} \Phi'_{ijk} \} + \cdots
\]

being initially \(L-R\) symmetric as well. These potentials including in principle all possible high dimension couplings (denoted above by dots) are evidently non-renormalizable and can be only considered as an effective approach being valid at sufficiently low energies.\(^{10}\) All dimensionful couplings with the mass parameters \(M_U, M_Y\) and others are proportional to appropriate powers of some UV cutoff which in our case can be ultimately related to the preon confinement energy scale \(\Lambda_{MC}^\ast\).

As in the known left-right models [2], in the potential (39) for a natural range of the parameters, particularly, for \(h_3 > 2(h_1 + h_2)\) and properly chosen the higher dimension coupling constants, the scalars \(\Phi_{ijk}\) and \(\Phi'_{ijk}\) given in (34) and 35) may develop the totally asymmetric VEV configuration

\[
\langle \Phi_{ijk} \rangle = 0 , \quad \langle \Phi'_{ijk} \rangle = \delta_p^i \delta_q^j \delta_r^k \epsilon^{pqr} M_{LR} \quad (p, q, r = 1, 2, 3)
\]

This VEV containing the antisymmetric third-rank tensor \(\epsilon^{pqr}\) of the \(SU(3)\) group certainly leads to the chiral symmetry breaking for the right-handed composites, \(SU(8)_R \rightarrow [SU(5) \times SU(3)]_R\), leaving intact the \(SU(8)_L\) symmetry for the left-handed ones. In terms

\(^{10}\)These multi-preon interaction model looks somewhat similar to the well-known multi-fermion interaction schemes used in the other contexts for chiral symmetry breaking [19] or spontaneous Lorentz violation [20].
of the spectator gauge chiral symmetry, this means that all non-diagonal gauge bosons related to the broken generators of the coset \( SU(8)_R / [SU(5) \times SU(3)]_R \) acquire at large distances the scale \( M_{LR} \) order masses. That is, these bosons having been strictly transversal at small distances acquire longitudinal parts containing the composite Goldstone modes beyond the preon confinement area. This means that, though the massless right-handed preons still possess the \( SU(8)_R \) symmetry, the masslessness of their composites at large distances is now solely controlled by its remained \([SU(5) \times SU(3)]_R\) part. Indeed, in contrast to the conserved gauge currents of the preons, the gauge currents of their composites related to the broken generators are no longer conserved even in the classical Lagrangian. The more so, the triangle anomalies of composites related to generators of the spectator gauge \( SU(8)_R \) symmetry, other than those of its \([SU(5) \times SU(3)]_R\) subgroup, are no longer matched with anomalies of preons. In other words, the spectator fermions which cancel anomalies of the right-handed preons cannot cancel all anomalies of their composites. In fact, anomalies related to the triangle graphs of composites with massive spectator gauge bosons having longitudinal parts cannot be cancelled. This causes a proper reduction of a possible chiral symmetry for right-handed preons and their massless composites, just as is shown above in (38), both spontaneously and through the anomalies. Remarkably, the latter is crucially related to the compositenes of the scalars \( \Phi_{[ijk]} \) and \( \Phi'_{[ijk]} \) which are solely emerged at large distances. For elementary scalars existing at all distances one would only have a spontaneous symmetry breaking of the chiral symmetry that could not change the anomaly matching condition for right-handed preons and their composites.

A similar symmetry breaking effect related to the anomalies can be clearly seen as well by considering the new four-preon interaction

\[
\frac{M_{LR}}{M_Y^2} \epsilon^{pqr} \epsilon_{ab'c'd'e'} \left( Q_{R}^{a'} C Q_{R}^{b'} \right) \left( Q_{R}^{c'} C S_{R}^{d'e'} \right) \tag{42}
\]

which follows from the Yukawa potential terms in (40) once the asymmetric VEV (41) is developed (\( Q_{pR}^{a'} \) are the right-handed family preon species, \( p = 1, 2, 3 \)). One can readily see that the interaction (42) possessing only \([SU(5) \times SU(3)]_R\) symmetry will necessarily modify the AM condition for right-handed states at large distances. Indeed, this four-preon coupling induces radiative corrections to the triangle graphs with circulation of composites containing the family preons \( Q_{pR}^{a'} \). As a result, anomalies related to the graphs with the non-diagonal spectator gauge bosons are not longer matched with those for the right-handed preon themselves. There is left only the \([SU(5) \times SU(3)]_R\) symmetry correspondence between short and large distances.

5.3 Metaflavor theory with broken L-R symmetry

Let us now turn to the metaflavor interactions having been so far switched off and see what happens in our \( SU(8)_{MF} \) metaflavor gauge theory. One can readily see that the asymmetric VEV configuration (41) related to the third-rank scalars \( \Phi_{[ijk]} \) and \( \Phi'_{[ijk]} \) may lead to the natural spontaneous violation of the starting L-R symmetry in the \( SU(8)_{MF} \) theory. First of all, this symmetry itself becomes, as usual, spontaneously broken at the scale \( M_{LR} \) down to the \( SU(5) \times SU(3) \) symmetry group which we may identify as
the conventional $SU(5)$ grand unification times the $SU(3)_F$ family symmetry. The point is, however, that the non-diagonal gauge bosons receiving masses due to this symmetry breaking are no longer elementary at large distances. Indeed, they acquire the longitudinal parts containing the composite Goldstone modes which consist of the right-handed preons, as follows from the VEV given above (11). Thereby, these modes will only couple to the right-handed preon composites leaving the left-handed ones basically intact. As a result, while the $SU(8)_{MF}^3$ triangle anomalies related to $P$- and $Q$-preons automatically cancel each other, the same anomalies related to the emerged left-handed and right-handed composites are no longer cancelled. This means that the gauge $SU(8)_{MF}$ metaflavor symmetry, similar to the chiral $SU(8)_R$ discussed above, not only spontaneously breaks in an ordinary way, but also breaks by anomalies. Thereby, while one has the $SU(8)_{MF}$ symmetry at small distances, there only exist the $SU(5) \times SU(3)_F$ metaflavor symmetry beyond the preon confinement area. This symmetry will then spontaneously breaks down to the Standard Model by the other composite scalars (30), (32) and (36) introduced above, as is discussed in the next section. We propose that, unlike the third-rank scalars $\Phi_{[ijk]}$ and $\Phi'_{[ijk]}$ considered here, these scalars do not break the $L$-$R$ symmetry. In fact, this may readily appear, if in their potentials of the type used above (39) the analogous parameters are arranged in an opposite way. As a result, all such $P$-preon and $Q$-preon scalars will develop the similar VEVs preserving the $L$-$R$ symmetry.

5.4 Spectrum of survived composites

Finally, let us consider the whole spectrum of composites to which the spontaneous $L$-$R$ symmetry violation may lead in the survived $SU(5) \times SU(3)_F$ metaflavor theory. As we have seen, while there still remains the starting chiral symmetry $SU(8)_L$ for the left-handed preons and their composites, for the right-handed preon composite states we only have the broken symmetry given in (38). Therefore, whereas nothing changes for the $L$-preon composites, the $R$-preons will only compose some particular submultiplets in (25) according to the matching condition for the broken chiral symmetry (38). In general, these submultiplets may not include the three right-handed quark-lepton families $(5 + 10, 3)_R$.

We might simply postulate it in the $L$-$R$ symmetry broken phase as some possible ansatz being allowed by the different chiral symmetries of the $L$-preon and $R$-preon composites. Nonetheless, one can try to derive it using the proposed discrete preon number matching condition (20) at least as some heuristic argument. Note that after the $L$-$R$ symmetry breaking (11), together with the chiral symmetry $K(8)$ its discrete center group, though being remained for preons, is properly reduced for composites, just as is shown in (38). Particularly, the discrete $Z_8^R$ symmetry of the right-handed composites comes to

$$Z_8^R \rightarrow Z_5^R \times Z_3^R$$ (43)

while the $Z_{8L}$ symmetry of the left-handed composites is left intact. Furthermore, in accordance with the matching condition for the broken chiral symmetry $[SU(5) \times SU(3)]_R$ for

\footnote{Note that the effective chiral symmetry for the left-handed composites (emerging in an absence of their metaflavor interactions) is in fact higher than their actual metaflavor symmetry $SU(5) \times SU(3)_F$.}
R-preons and their composites, $Z_5^R$ and $Z_3^R$ can also be considered as discrete symmetries of the right-handed quintet preons $Q_{sR}$ ($s = 1, \ldots, 5$) of $SU(5)_R$ and triplet preons $Q_{aR}$ ($a = 1, 2, 3$) of $SU(3)_R$, respectively, which are thereby separated. Namely, the $Q$-preon discrete symmetry in the broken $L$-$R$ symmetry phase is viewed from the large distances as the product (43) rather than the universal $Z_8^R$ symmetry for all eight preons, as was in its unbroken phase (4). Now, if we require the discrete preon charge matching (20) for preons and composites, the states collected in the submultiplet $(5 + 10, 3)_R$ will never appear in physical spectrum since they possess both of discrete $Z_5^R$ and $Z_3^R$ charges, while the preons $Q_{sR}$ and $Q_{aR}$ have, by definition, only one of them. Indeed, if the $Z_5^R$ and $Z_3^R$ transformations are defined for $Q$-preons as

$$Q_{sR} \to e^{i2\pi q_{(5)}^R/5}Q_{sR}, \quad Q_{aR} \to e^{i2\pi q_{(3)}^R/3}Q_{aR} \tag{44}$$

then the $Z_5^R \times Z_3^R$ charges for them are $\left(q_{(5)}^R, 0\right)$ and $\left(0, q_{(3)}^R\right)$, while for their composite submultiplets $(5, 3)_R$ and $(10, 3)_R$ in (25) they come to $\left(-q_{(5)}^R, 2q_{(3)}^R\right)$ and $\left(2q_{(5)}^R, -q_{(3)}^R\right)$, respectively. At the same time, all other composite right-handed submultiplets in (24) readily match the $Z_5^R \times Z_3^R$ charges for preons (normally, they may be taken to be unit charges, $q_{(5)}^R = q_{(3)}^R = 1$).

One way or another, the simplest combination of the right-handed submultiplets in (25) which may simultaneously satisfy the AM conditions (15) implied for the $[SU(5) \times SU(3)]_R$ symmetry, as well as the above discrete preon number matching condition (20) is in fact given by the collection

$$[(45, 1)_R + (5, 8 + 1)_R + (1, 3)_R]_{216} + 2[(5, 1)_R + (1, 3)_R]_8 + 2(1, 3)_R \tag{45}$$

The first and second terms (45) contain submultiplets following from the multiplets $216_R$ and $8_R$ in (25), respectively, while the submultiplet $(1, 3)_R$ has to appear two times more in order to appropriately restore the anomaly coefficient balance for the $R$-preon composites. The massless states (45), though they are individually protected by preservation of the chiral symmetry $[SU(5) \times SU(3)]_R$, will generally pair up with the similar left-handed submultiplets in $216_L + S_L^{(1)} + S_L^{(2)}$ (25)

$$\left[(5 + 10, 3)_L + (45, 1)_L + (5, 8 + 1)_L + (24, 3)_L + (1, 3)_L + (1, \overline{6})_L\right]_{216} + 2[(5, 1)_L + (1, 3)_L]_8 \tag{46}$$

due to the quantum gravitational conversion discussed in the section 4.4. As a result, due to the expected closeness of the compositeness scale $\Lambda_{MC}$ to the Planck scale $M_P$, the Dirac masses of all such composites appear very heavy. Eventually, only some special submultiplets in the starting left-handed and right-handed composite multiplets (25) including those which contain ordinary quarks and leptons may emerge at present laboratory energies.
6 Some immediate physical consequences

6.1 Quarks, leptons and beyond

We have seen in the section 5.2 above that the \( L-R \) symmetry breaking in the right-handed preon sector initiated by the composite scalar condensation \((41)\) makes the starting metaflavor symmetry \( SU(8)_{MF} \) to be reduced at large distances to the product of the standard \( SU(5) \) GUT and family symmetry \( SU(3)_{F} \)

\[
SU(8)_{MF} \rightarrow SU(5) \times SU(3)_{F}
\]

This \( SU(5) \times SU(3)_{F} \) symmetry of composites appearing in the \( L-R \) symmetry broken phase is in essence the chiral remnant of the initial vectorlike \( SU(8)_{MF} \) symmetry of preons that only exists at small distances and breaks by anomalies at the large ones. Accordingly, the massless composite fermions survived during the gravitational conversion of similar \( L \)-preon and \( R \)-preon composites in \((45, 46)\) and decoupling them from a low-energy spectrum are given now by the collection of the \( SU(5) \times SU(3)_{F} \) multiplets

\[
(5 + 10, 3)_{L} + (24, 3)_{L} + (1, 6)_{L} + 2(1, 3)_{R}
\]

which automatically appear free from both the \( SU(5) \) and \( SU(3)_{F} \) anomalies.

The first term in \((48)\) contains just three conventional families of quarks and leptons which are presented below

\[
\begin{bmatrix}
(u \\
\tilde{d})_{L}
\end{bmatrix}
\begin{bmatrix}
(c \\
\tilde{s})_{L}
\end{bmatrix}
\begin{bmatrix}
t \\
\tilde{b}
\end{bmatrix}_{L} \leftrightarrow \mathcal{C}_{l} \mathcal{W}_{a} \overline{F}^{p}
\]

\[
\begin{bmatrix}
\tilde{u}_{L} & \tilde{c}_{L} & \tilde{t}_{L}
\end{bmatrix} \leftrightarrow \epsilon^{lmn} \mathcal{C}_{m} \mathcal{C}_{n} \overline{F}^{p}
\]

\[
\begin{bmatrix}
\tilde{d}_{L} & \tilde{s}_{L} & \tilde{b}_{L}
\end{bmatrix} \leftrightarrow \epsilon^{pqr} \mathcal{F}_{p} \mathcal{F}_{q} 
\]

On the right side one can see their preon content in terms of the left-handed Weil spinor fields \( \mathcal{C}_{l} \), \( \mathcal{W}_{a} \) and \( \mathcal{F}_{p} \) with a proper implication of the antisymmetric tensors \( \epsilon^{lmn} \), \( \epsilon^{pqr} \), \( \epsilon^{ab} \) of the groups \( SU(3)_{C} \), \( SU(3)_{F} \) and \( SU(2)_{W} \), respectively. Here, \( \mathcal{C}_{l} \) \((l, m, n = 1, 2, 3)\) stand for the “chromons” being the basic triplet of the color \( SU(3)_{C} \), then \( \mathcal{W}_{a} \) \((a, b = 1, 2)\) denotes the “isons” forming the elementary doublet of the weak isotopic spin \( SU(2)_{W} \) and, finally, the family number carriers are given by the “famons” \( \mathcal{F}_{p} \) \((p, q, r = 1, 2, 3)\) which belong to the fundamental triplet of the family symmetry \( SU(3)_{F} \). Note that in accordance with a usual assignment of quarks and leptons to the left-handed multiplets
in the \( SU(5) \) GUT, the right-handed quark and lepton states are taken in \( (5) \) in terms of their anti-states taken in the left-handed basis. Also, the lepton doublets are presented in terms of the anti-doublets, as they are usually treated in the anti-quintet \( \overline{5}_L \) of the \( SU(5) \). From the table \( (49) \) one can readily find the electric charges of chromons, isons and famons, respectively,

\[
Q_C = -1/3; \quad Q_{W_1} = 1, \quad Q_{W_2} = 0; \quad Q_F = 0. \tag{50}
\]

It is important to note that the compositeness scale \( \Lambda_{MC} \) for universal preons composing both quarks and leptons appears too high to directly observe their composite nature \[21\]. Indeed, one can readily see from the quark-lepton preon structure shown above in \( (49) \) that the quark pair \( u + d \) contains the same preons as the antiquark-antilepton pair \( \overline{u} + e^+ \). This eventually would lead to the process

\[
u + d \rightarrow \pi^0 + e^+ \tag{51}
\]

and consequently to the proton decay \( p \rightarrow \pi^0 + e^+ \) just due to a simple rearrangement of preons inside the proton. To prevent this, the compositeness scale \( \Lambda_{MC} \) has to be of the order of the scale of the \( SU(5) \) GUT or even larger, \( \Lambda_{MC} \gtrsim M_{GUT} \approx 2 \cdot 10^{16} \text{ GeV} \). This scale is also in accordance with the energy scale at the end of inflation in a conventional cosmological scenario \[22\] due to which any potentially stable heavy composite will be safely inflated out (like the magnetic monopoles involved in the theory). On the other hand, most of such heavy composites can decay into ordinary quarks and leptons by the virtual black hole mechanism described in the section 4.4 provided that all accompanying local symmetries are preserved.

### 6.2 Quark-lepton masses and mixings

The quarks and leptons presented above in the submultiplet \( (5+10,3)_L \) receive their masses from the \( SU(8) \) invariant Yukawa couplings with composite scalars introduced in the section 4.3. As was already mentioned above, in contrast to the third-rank scalars \( (34) \) causing the basic \( L-R \) symmetry breaking in the theory, the other composite scalars \( (30), (32) \) and \( (36) \) develop presumably pure \( L-R \) symmetrical VEVs. Actually, in the \( SU(8)_{MF} \) theory there is some doubling of the identical scalar multiplets which are composed individually from the left-handed and right-handed preons. We call them the \( L \) - and \( R \)-scalars to show that they selectively interact with the left-handed and right-handed composite fermions, respectively. As matter of fact, in the Yukawa couplings of the left-handed composite fermions presented in \( (48) \) the \( L \)-scalars may only contribute. Particularly, for the submultiplet \( (5+10,3)_L \) one has, as in a conventional \( SU(5) \) GUT, the two independent \( SU(8)_{MF} \) invariant couplings

\[
\begin{align*}
\frac{1}{M} \left[ \Psi_{\{jk\}L} C \Psi_{\{mp\}L} \right] \phi^{[jkmpl]}(a_u \chi_{[u]} + b_u \chi_{[i]}) \\
\frac{1}{M} \left[ \Psi_{\{jk\}L} C \Psi_{\{in\}L} \right] \phi^{[jkmpl]}(a_d \chi_{[p]} + b_d \chi_{[p]})
\end{align*} \tag{52}
\]
with different index contraction for the up quarks, and down quarks and leptons, respectively. The mass $\mathcal{M}$ stands for some effective scale in the theory that is related to the metacolor scale $\Lambda_{MC}$, while $a_{u,d}$ and $b_{u,d}$ are some dimensionless constants of the order of 1. Note that these couplings contain two types of scalars with the appropriate $SU(5) \times SU(3)_F$ components: the exotic four-preon $\varphi$ multiplet given in (56) and (57) containing the $SU(5)$ quintets $(5,1)_u$ and $(\bar{5},1)_d$ to break the Standard Model at the electroweak scale $M_{SM,15}$ and the basic two-preon $\chi_{[ij]}$ and $\chi_{(ij)}$ multiplets in (30) and (31) with components $(1,\bar{3})$ and $(1,6)$ to properly break the $SU(3)_F$ family symmetry at the large scale $M_F$. We call them the "vertical" and "horizontal" scalars, respectively. Taken together, they presumably determine masses and mixings of all quarks and leptons. Otherwise, we would have to introduce the multi-preon scalar composites with both vertical and horizontal indices that seems to be too complicated. After the $SU(8)_{MF}$ symmetry breaking (47) the Yukawa couplings acquire the transparent form (all metaflavor indices are omitted)

$$
\begin{align*}
\left[(10,\bar{3})_L C(10,\bar{3})_L\right] (5,1)_u [a_u (1,\bar{3}) + b_u (1,6)]/\mathcal{M} ,
\left[(\bar{5},\bar{3})_L C(10,\bar{3})_L\right] (\bar{5},1)_d [a_d (1,\bar{3}) + b_d (1,6)]/\mathcal{M}
\end{align*}
$$

(53)

where we only include those components of the vertical scalar $\varphi$ and horizontal scalars $\chi_{[ij]}$ and $\chi_{(ij)}$ which develop the VEVs. Just the horizontal scalar VEVs determine through the Yukawa couplings (53) the mass matrices for quarks and leptons

$$
\begin{align*}
\hat{m}^u_{pq} &= \langle 5,1 \rangle_u [a_u \langle 1,\bar{3} \rangle_{[pq]} + b_u \langle 1,6 \rangle_{[pq]}]/\mathcal{M} ,
\hat{m}^d_{pq} &= \langle 5,1 \rangle_d [a_d \langle 1,\bar{3} \rangle_{[pq]} + b_d \langle 1,6 \rangle_{[pq]}]/\mathcal{M}
\end{align*}
$$

(54)

where the angle brackets denote the corresponding VEVs, while $p,q = 1,2,3$ stand for family indices. The matrices $\hat{m}^u_{pq}$ and $\hat{m}^d_{pq}$ are defined at the grand unified scale and have to be then extrapolated down to an actual mass range for quarks and leptons. Depending on which components the above symmetrical and asymmetrical VEVs are developed, one comes to different texture zero types for all matrices involved. The strong hierarchies of the quark-lepton masses and mixings may be now explained by somewhat softer hierarchies between the breaking directions of the $SU(3)_F$ family symmetry whose

\[13\] Therefore, in contrast to a conventional $SU(5)$ GUT our model contain the two different scalar quintets stemming from the same self-conjugated multiplet $\varphi^{[ijkl]}$ of the $SU(8)_{MF}$. We denoted these quintets as $(5,1)_u$ and $(\bar{5},1)_d$ to stress that they give masses to the up and down quarks, respectively (see below).

\[14\] In this case, there also would appear the stronger gauge hierarchy problem to arrange the VEVs of all these scalars in an appropriate way.

\[15\] Note that hand in hand with a good relation between masses of the $b$-quark and $\tau$-lepton there also appear in the matrix $\hat{m}^d$, as usual in the $SU(5)$ GUT, the bad relations for the lighter down quarks and leptons. However, this readily can be alleviated in our model by introducing in (62) an extra higher dimension Yukawa coupling which includes the adjoint composite scalar $\phi'_{ij}$ as well

$$
\frac{1}{\Lambda^2} \left[ \Psi_{[ijkl]}^i L C \Psi_{[nm]}^j L \right] \phi'_{ij} \varphi^{[ijkl]} (a'_{ijk} \chi_{[ij]} + b'_{ik} \chi_{[ik]})
$$

This will decouple masses of the first two quark-lepton families from each other leaving intact masses of the third one.
scale $M_F$ is imposed to be close to the effective scale $M$ (see some significant references in the section 7).

### 6.3 Neutrino masses

While quarks and charged leptons acquire masses in a way described above, the neutrinos are still left massless. However, in contrast to the standard $SU(5)$ GUT our model contains some generic candidates for singlet heavy neutrinos to eventually generate masses of the physical ones. Indeed, the initially massless extra submultiplets $(24,3)_L$, $(1,\overline{6})_L$ and $(1,3)^{(1,2)}_R$ in the survived collection become then heavy acquiring the family scale order masses once the $SU(3)_F$ symmetry breaks down. In order to sufficiently suppress all flavor-changing transitions, which would be mediated by the family gauge bosons, this scale $M_F$ has to be at least of the order $10^5 \text{ GeV}$, though in our model, due to the VEVs developed by composite scalars, it may be as large as the scale $M_{GUT}$ and even larger. The above submultiplets receive the heavy Majorana masses from the $SU(8)_{MF}$ invariant Yukawa couplings

\[
\left[\Psi_{[jk]}^i L C \Psi_{[im]}^j L \right] \left( a_{ex} \chi^{[km]} + b_{ex} \chi^{\{km\}} \right) \quad (55)
\]

for the left-handed composites and in a similar way for the right-handed ones (the coupling constants $a_{ex}$ and $b_{ex}$ are dimensionless and of the order of 1). In the decomposed $SU(5) \times SU(3)_F$ invariant form these couplings look as

\[
[(24,3)_LC(24,3)_L + (1,\overline{6})_LC(1,\overline{6})_L][a_{ex}(1,3) + b_{ex}(1,\overline{6})] \quad (56)
\]

when either the $SU(5)$ indices or the $SU(3)_F$ indices are only contracted. As can be readily seen by comparing the Yukawa couplings (55) and (56) with (52) and (53), all extra submultiplets in (48), acquire eventually the mass-matrices being basically similar in form to those for quarks and leptons. Accordingly, some significant mass hierarchies (by two or three orders of magnitude) between extra state families are also expected.

Remarkably, the three SM singlet states in the submultiplet $(24,3)_L$ appearing in its $SU(3)_C \times SU(2)_W \times SU(3)_F$ decomposition as $(1,1,3)_L$ could be considered as candidates for the heavy right-handed neutrinos $N_R^p$ taken as the left-handed anti-neutrinos $\bar{N}_L^p$ in the considered basis ($p$ stands for the family symmetry index, $p = 1,2,3$). As mentioned above, they develop masses

\[
m_{N_R^p} = f^p M_F, \quad f^p = (0.001 - 1) \quad (57)
\]

including a possible mass hierarchy between heavy neutrinos of different families. The massless physical neutrinos contained in the submultiplet $(5,\overline{3})_L$ in (48) may mix with them, thus acquiring the tiny Majorana masses through the see-saw mechanism in their common $6 \times 6$ mass-matrix. Actually, this mixing may appear from the high-dimensional $SU(8)_{MF}$ Yukawa couplings

\[
\frac{1}{M^3} \left[\Psi_{[jk]}^i L C \Psi_{[im]}^j L \right] \varphi_{[lpqr]} \left( \chi^{[jp]} \chi^{[kq]} \chi^{[mr]} + \ldots \right) \quad (58)
\]
with the above vertical and horizontal scalars $\phi$ and $\chi$ developing VEVs on the $SU(5) \times SU(3)_F$ components $(5,1)_d$ and $(1,3)$ plus $(1,\bar{6})$, respectively (the dimensionful coupling constant is properly determined here by the effective scale $\mathcal{M}$, while dots stand for other terms with anti-symmetrical and symmetrical horizontal scalars with all possible index contractions). This coupling after electroweak and family symmetry breakings at the corresponding scales $M_{SM}$ and $M_F$ leads to the mixing terms for the submultiplets $(\bar{5},3)_L$ and $(24,3)_L$ being of the order

$$(M_F/\mathcal{M})^3 M_{SM}$$

(59)

that in turn induces masses for physical neutrinos. These masses according to (59) and (57) come to

$$m_{\nu[]} \sim (M_F/\mathcal{M})^6 M^2_{SM}/f_F M_F$$

(60)

which appear to be of the right order provided that the scale of family symmetry $M_F$ is of the order of the scale $\mathcal{M}$ that, as we have seen above, is also required for masses of quarks and leptons. Apart from that, the scale $M_F$ has to be close to the scale of the $SU(5)$ GUT given above to be in agreement with observed limitations on mass spectrum of physical neutrinos [22]. One can see that since masses of heavy neutrinos $N_p$ follow in (55) the mass hierarchy of quark-lepton families, masses of physical neutrinos have to obey the inverted hierarchy so that the electron neutrino appears to be the heaviest one with mass value up to $1eV$ or even larger.

### 6.4 Heavy states

The compositeness scale $\Lambda_{MC}$ may on its own cause a limit on on the composite fermion masses appearing as a result of the quantum gravitational transitions between similar states in the right-handed multiplets (45) and left-handed multiplets (46)

$$[(45,1)_{L,R} + (5,8 + 1)_{L,R} + (1,3)_{L,R}]_{216} + 2[(5,1)_{L,R} + (1,3)_{L,R}]_8$$

(61)

into each other, as was argued in the section 4.4. From dimensional arguments related to a general structure of the three-preon composites proposed above in (26) and (28), masses of these vectorlike multiplets could be of the order

$$M_V \sim (\Lambda_{MC}/M_{Pl})^{9} \Lambda_{MC}$$

(62)

that corresponds to the high-dimension interaction between left-handed and right-handed composites of the type (the metacolor indices are omitted)

$$\frac{1}{M^9_{Pl}} \left[(\bar{P}^j_R \gamma_\mu P^j_R) \bar{F}_{F^L} F_{L}^\mu \nu \right] \left[(\bar{Q}^i_L \gamma^\rho Q^j_L) Q_{kL} F_{F^\nu}^i \right] + h.c.$$  

(63)

Similarly, the screened one-preon states

$$(5,1)^{(1,2)}_{L,R} + (1,3)^{(1,2)}_{L,R}$$

(64)
in (61) acquire masses when being pairing with each other through the analogous high-

dimension interactions of the $P$- and $Q$-preons including their metagluons

$$
\frac{1}{M_{Pl}} \left[ (F_{\mu\nu} F^{\mu\nu}) T^f_L \right] \left[ (F'_{\rho\sigma} F'^{\rho\sigma}) Q_{iR} \right] + h.c. \quad (65)
$$

appearing from their structure given in (26) and (28). Again, from the dimensional argu-

ments one may conclude that these masses has a natural order

$$
(\Lambda_{MC}/M_{Pl})^7 \Lambda_{MC} \quad (66)
$$

that may be significantly larger than masses (62) of the 3-preon states. These gravitation-

ally induced masses are indeed very sensitive to the confinem ent scale $\Lambda_{MC}$. In order for

the vectorlike multiplets (61) and (64) to be enough heavy, say with masses $10^5 \text{GeV}$ and

higher, the scale $\Lambda_{MC}$ has to be close to the Planck scale $M_{Pl}$, namely $\Lambda_{MC} \gtrsim 5 \cdot 10^{17} \text{GeV}$, as is readily seen from (62).

Note that some of heavy vectorlike submultiplets, particular

ly, $(5,8)_{L,R}$ in (61) may mix with the physical submultiplet $(\overline{5}, 3)_L$ in (48) which contains the lepton doublet and down antiquarks. This mixing appears through the above Yukawa couplings (55) with simultaneous contractions the $SU(5)$ and $SU(3)_F$ indices, that leads instead of (56) to the couplings

$$
[(5, 8)_L C(\overline{5}, 3)_L][a_{ex}(1, 3) + b_{ex}(1, \overline{6})] \quad (67)
$$

These couplings after the $SU(3)_F$ symmetry breaking cause the family scale $M_F$ order mixing masses in the corresponding triangular mass matrices (describing either left-right or right-left fermion component transitions but not both simultaneously). Though these matrices cannot significantly disturb the masses of all the states involved, they may induce a large admixture of the heavy states in the left-handed leptons in $(\overline{5}, 3)_L$ that in turn could strongly influence their CKM matrix. To properly suppress this mixing one has to generally propose that the non-diagonal mass term of the heavy multiplets $(5,8)_{L,R}$ in (61) is at least less than its diagonal mass $M_V$ (62) induced by gravity, namely,

$$
M_F \lesssim (\Lambda_{MC}/M_{Pl})^9 \Lambda_{MC} \quad (68)
$$

For the naturally high family scale $M_F$ being at least of the order of the $SU(5)$ GUT scale, as is required from the masses of neutrinos discussed above, the metacolor scale $\Lambda_{MC}$ in (68) has to appear very close to the Planck scale $M_{Pl}$. Particularly, for $M_F \sim M_{GUT}$ and $\Lambda_{MC} \sim M_{Pl}$ one has for an angle of the above extra mixing

$$
\tan \theta \sim M_F/M_V \sim (M_F/M_{Pl})(M_{Pl}/\Lambda_{MC})^{10} \quad (69)
$$

a reasonably small value $\theta \sim M_{GUT}/M_{Pl} \sim 10^{-3}$. Generally, depending on actual values of the scales $\Lambda_{MC}$ and $M_F$, there could be expected some marked violation of unitarity in the CKM matrix for leptons which may be of a special interest for observations. In contrast, for quarks, due to mixing of only right-handed down quarks in (67), the CKM matrix is left practically intact.

\textsuperscript{16}For simplicity, we do not consider analogous mixings of the physical multiplet $(\overline{5}, 3)_L$ with the three-preon multiplets $(5,1)_{L,R}$ and screened one-preon preon multiplets $(5,1)_{L,R}^{1,2}$ in (61).
6.5 Basic scenarios

Actually, there are two phases in the model: the $L$-$R$ symmetry phase with the initial vectorlike $SU(8)_{MF}$ symmetry of the left-handed and right-handed preons and the broken $L$-$R$ symmetry phase with the $SU(5) \times SU(3)_F$ symmetry of the chiral composites having principally different multiplet spectra for the right handed and left-handed states given above in (45, 46). Accordingly, the massless composite fermions survived after the gravitational conversion of similar $L$-preon and $R$-preon composites are finally collected in the $SU(5) \times SU(3)_F$ multiplets (15). In contrast to preons, there is no the $SU(8)_{MF}$ unification for composites, because it is broken by anomalies at the large distances, as we discussed in the section 5.

According to all the symmetry breakings involved, our preon model contains in fact a few basic mass scales determined by the generic compositeness scale $\Lambda_{MC}$. They are: the $L$-$R$ symmetry breaking scale $M_{LR}$ where the starting metaflavor symmetry $SU(8)_{MF}$ of preons breaks down to the $SU(5) \times SU(3)_F$ symmetry for composites; then the grand unification scale $M_{GUT}$ and the family scale $M_F$ where the $SU(5)$ and $SU(3)_F$ get broken, respectively. These breakings are provided by the basic scalar composites (30), (32) and (34) developing the large VEVs in the theory that eventually leads to the Standard Model. Further breaking may be only related to the exotic scalars consisting of four preons (36) which, as we proposed in the section 4.3, develop the VEV of the electroweak scale order $M_{SM}$. The model predicts three types of the composite fermion states which are: (1) the three families of ordinary quarks and leptons $(\bar{5} + 10, 3)_L$ in (45) with masses at the electroweak scale $M_{SM}$, (2) the chiral multiplets $(24, 3)_L + (1, 6)_L + 2(1, 3)_R$ with the Majorana type masses at the family scale $M_F$ and (3) the vectorlike multiplets (61) with the gavitationally induced Dirac masses (62) and (66). The latter gives one more high scale $M_V$ in the theory which may be located in a wide range of masses up to the Planck scale $M_{Pl}$. Further details depend on a taken scenario with a particular interplay between the high mass scales mentioned above.

The most natural scenario seems to be the case with the Planck scale as the effective theory scale, $\mathcal{M} \sim M_{Pl}$, when all the high scales are of the same order determined by the generic scale $\Lambda_{MC}$ which itself is taken near the Planck scale

$$M_{LR} \sim M_{GUT} \sim M_F \sim M_V \sim \Lambda_{MC} \sim M_{Pl}$$  \hspace{1cm} (70)

As a result, the starting $SU(8)_{MF}$ symmetry breaks at once to the Standard Model at the Planck scale. This means that only ordinary quarks and leptons will contribute to gauge coupling unification in the $SU(5)$ GUT whose scale $M_{GUT}$ has to be increased now to the Planck mass $M_{Pl}$. This, as usual, can be done by an embedding the whole model into $N = 1$ supergravity [2] with a proper extension of particle spectra involved. Due to the highest scales used, this scenario at low energies is not much different from a conventional supergravity grand unification. Actually, it additionally predicts only a strong mixing of lepton doublets with heavy states given in (69) with near maximal angle ($\theta \sim \pi/4$) and, thereby, some significant violation of unitarity in the lepton CKM matrix. Neither physically interesting neutrino masses, nor any extra composites at intermediate scales are provided.
The next may be the case with the $SU(5)$ GUT scale as an effective theory scale, $M \sim M_{\text{GUT}}$, where the $M_{\text{GUT}}$ is considered as a conventional supersymmetric $SU(5)$ GUT scale. This is somewhat lower than the compositeness scale $\Lambda_{\text{MC}}$ which cannot be less than $5 \cdot 10^{17}$ GeV, as was stated above. At the same time, the $L$-$R$ symmetry breaking scale $M_{LR}$ (which at the same time is the scale of the $SU(8)_{\text{MF}}$ as well) should always be near the scale $\Lambda_{\text{MC}}$ to cause the principally different composite spectra, \((45)\) and \((46)\), for the left-handed and right-handed preons, respectively. If otherwise $M_{LR} \ll \Lambda_{\text{MC}}$, only the similar multiplets \(216_i^{jkL} + 8^{(1)}_i L + 8^{(2)}_i L\) \((24)\) of the yet unbroken $SU(8)_{\text{MF}}$ metaflavor symmetry would appear in a composite spectrum. They all then pair up with each other and, thereby, acquire heavy Dirac masses that would make the whole model to be meaningless. So, we propose that only the $L$-$R$ symmetry breaking scale $M_{LR}$ is of the Planck scale order, others are less

\[ M_{LR} \sim \Lambda_{\text{MC}} \sim M_{\text{Pl}}, \quad M_{\text{GUT}} \sim M_F \sim M_V \]  

(71)

that implies some two-three order of magnitude hierarchy between these two groups of scales. In this scenario, one may have the right order masses for physical neutrinos, as one can see from (60). Also there is a significant mixing of lepton doublets with heavy states mentioned above in (69) provided that masses $M_V$ of the vectorlike multiplets is also of the order of $M_{\text{GUT}}$. For that, the compositeness scale has to be $\Lambda_{\text{MC}} \approx 6 \cdot 10^{18}$ GeV, as follows from (62). Thus, the whole picture may look as follows. There are only $L$-preons and $R$-preons at the Planck scale with the metaflavor symmetry $SU(8)_{\text{MF}}$ which runs over a short distance down to the compositeness scale $\Lambda_{\text{MC}}$ located a bit lower. Then at the scale $M_{LR} \sim \Lambda_{\text{MC}}$ this symmetry breaks to the $SU(5) \times SU(3)_{\text{F}}$ in the $L$-$R$ asymmetric way so that there appears two different left-handed and right-handed composite spectra, \((45)\) and \((46)\), respectively. The similar vectorlike submultiplets in them get the gravitationally induced Dirac masses \((62)\) and \((66)\), while chiral multiplets receive Majorana masses from the family symmetry breaking in \((55)\) and \((56)\). They all decouple at the scale $M_{\text{GUT}}$ where the $SU(5)$ and $SU(3)_{\text{F}}$ break, and below this scale there are only left the three standard families of massless quarks and leptons. They in turn get their masses at the scale $M_{\text{SM}}$ where the Standard Model breaks down.

And the last interesting scenario could be the case with the $SU(3)_{\text{F}}$ family symmetry scale taken as the effective scale in the theory, $M \sim M_F$, which could be as low as possible. As we mentioned above, in order not to come in conflict with possible flavor-changing processes this scale has to be at least of the order $10^5$ GeV. At the same time, to avoid an inadmissible mixing of lepton doublets with those in heavy vectorlike multiplets \((61)\) masses of the latters have also to be of the family scale order or less, $M_V \lesssim M_F$. So, one has for all scales involved

\[ M_{LR} \sim \Lambda_{\text{MC}} \sim M_{\text{Pl}}, \quad M_{\text{GUT}} \gtrsim M_F \gtrsim M_V \gtrsim 10^5 \text{GeV} \]  

(72)

that for the relatively low family scale $M_F$ implies strong hierarchy between compositeness scale $\Lambda_{\text{MC}}$ and an effective theory scale $M$. Meanwhile, the low gravitational conversion scale $M_V$ can be readily reached by only a slight lowering of the metacolor scale in (62) down to $\Lambda_{\text{MC}} \gtrsim 5 \cdot 10^{17}$ GeV. Eventually, one has, apart the three families of ordinary
quarks and leptons with masses at the electroweak scale $M_{SM}$, the heavy chiral multiplets $\{48\}$ and heavy vectorlike multiplets $\{61\}$ with the Majorana and Dirac masses (respectively) in the interval $(10^5 - 10^{16})$ GeV. Indeed, these extra heavy states might in principle affect the gauge coupling unification. The point is, however, that they all are presented by the unsplit multiplets of the $SU(5) \times SU(3)_F$ symmetry whose components have the same mass. Therefore, they may influence unification of the $SU(5)$ gauge coupling constants in the two-loop approximation only that can be largely ignored in both ordinary and supersymmetric theories.

Note that none of the above scenarios proposes the further unification of gauge coupling constants $g_5$ and $g_{3F}$ of the $SU(5)$ and $SU(3)_F$, respectively, at the $SU(8)$_MF unification scale. Indeed, as was mentioned above, the $SU(8)$_MF does not exist for composites due to anomalies appearing at the large distances. Thus, the $SU(8)_MF$ gauge constant $g_8$ is only applicable to gauge interactions of preons and may substantially differ from gauge coupling constants $g_5$ and $g_{3F}$ of composites. Nonetheless, some significant impact on the $g_8$ coupling constant running may appear from the threshold effects at the compositeness scale $\Lambda_{MC}$ where the preons themselves come into play. An appropriate running would make it possible for the $SU(8)_MF$ metaflavor theory of preons to be further unified with gravity at the Planck scale $M_{Pl}$. Anyway, a coupling unification in the preon model for quarks and leptons deserves a special consideration that we plan to address elsewhere.

Meanwhile, one may conclude that, together with an origin of the standard $SU(5)$ grand unification, the most important prediction of the left-right preon model considered here is, indeed, an existence of the local family symmetry $SU(3)_F$ for quark-lepton generations. In fact, all quarks and leptons with the both left-handed and right-handed components appear as antitriplets of the $SU(3)_F$, as is readily seen from their submultiplet $(\bar{5} + 10, 3)_L$ itself in $\{48\}$. This means that the $SU(3)_F$ is in essence a chiral symmetry, rather than a vectorlike symmetry such as, for example, the conventional color symmetry $SU(3)_C$. We briefly sketch the $SU(3)_F$ family symmetry and some of its basic applications below.

7 The family symmetry $SU(3)_F$

The flavor mixing of quarks and leptons is certainly one of the major problems that presently confronts particle physics. Many attempts have been made to interpret the pattern of this mixing in terms of various family symmetries - discrete or continuous, global or local. Among them, the chiral family symmetry $SU(3)_F$ introduced first in the preon model framework [7] and then developed by its own by many authors [22-26, 28-31] seems most promising.

Generically, the chiral family symmetry $SU(3)_F$ possesses the following four distinctive features:

(i) It provides a natural explanation of the number three of observed quark-lepton families correlated with three species of massless or light ($m_\nu < M_Z/2$) neutrinos contributing to the invisible $Z$ boson partial decay width;

(ii) Its local nature conforms with the other local symmetries of the Standard Model,
such as the weak isospin symmetry $SU(2)_W$ or color symmetry $SU(3)_C$, thus leading to the family-unified SM with a total symmetry $SM \times SU(3)_F$.

(iii) Its chiral nature, according to which both the left-handed and right-handed quarks and leptons are proposed to be fundamental antitriplets of the $SU(3)_F$, provides the hierarchical mass spectrum of quark-lepton families as a result of a spontaneous symmetry breaking at some high scale $M_F$ which could in principle located in the range from $10^5$ GeV (to properly suppress the flavor-changing processes) up to the grand unification scale $M_{GUT}$ and even higher. Actually, any family symmetry should be completely broken in order to conform with reality at lower energies. Meanwhile, this symmetry should be chiral, rather than vectorlike, since a vectorlike symmetry would not forbid the family invariant masses, thus leading in general to uniform rather than hierarchical mass spectra. Interestingly, both known examples of local vectorlike symmetries, electromagnetic $U(1)_{EM}$ and color $SU(3)_C$, appear to be exact symmetries, while all chiral symmetries including conventional grand unifications $SU(5), SO(10)$ and $E(6)$ appear broken;

(iv) Thereby, due to its chiral structure, the $SU(3)_F$ admits a natural unification with all known GUTs in a direct product form, both in an ordinary and supersymmetric framework, thus leading to the family-unified GUTs, $GUT \times SU(3)_F$, beyond the Standard Model.

So, if one takes these natural criteria seriously, any other candidate for flavor symmetry, except for the local chiral $SU(3)_F$ symmetry, can be excluded. Indeed, the $U(1)$ family symmetry does not satisfy the criterion (i) and is in fact applicable to any number of quark-lepton families. Also, the $SU(2)$ family symmetry can contain, besides two light families treated as its doublets, any number of additional (singlets or new doublets of $SU(2)$) families. All global non-Abelian symmetries are excluded by the criterion (ii), while the vectorlike symmetries are excluded by the last criteria (iii) and (iv).

Among applications of the $SU(3)_F$ symmetry, the most interesting ones related to description, both in the Standard Model and GUTs, of the quark and lepton masses and mixings [23] including the neutrino masses and oscillations [25]. Indeed, spontaneous breaking of this symmetry gives some guidance to the observed hierarchy between elements of the quark-lepton mass matrices and presence of texture zeros in them, that in the preon model framework we schematically discussed above in the section 6. Remarkably, the $SU(3)_F$ invariant Yukawa couplings like those presented in [52, 53] are always accompanied by an accidental global chiral $U(1)$ symmetry [26] which can be identified with the Peccei-Quinn symmetry [28], thus giving a solution to the strong $CP$ problem. For the relatively low family scale $M_F$, the $SU(3)_F$ gauge bosons will also enter into play so that there may become important many flavor-changing rare processes [27] including some of their astrophysical consequences [29]. In the framework of supersymmetric theories [30], both SM and GUT, the family $SU(3)_F$ symmetry hand in hand with hierarchical masses and mixings for quarks and leptons leads to an almost uniform mass spectrum for their superpartners with a high degree of flavor conservation, that makes this symmetry existence even more significant in the SUSY case. The special sector of applications is related to a new type of topological defects - flavored cosmic strings and monopoles appearing due to the spontaneous violation of the $SU(3)_F$ which may be considered as possible candidates for the cold dark matter in the Universe [31].
In this context, the question naturally arises whether the $SU(3)_F$ family symmetry has its origin in the preon model considered here or it rather appears as an independently postulated symmetry. This may depend in fact on the basic scenarios considered in the section 6.5. The most crucial difference between these two cases is related to an existence in the preon model of some heavy $SU(5) \times SU(3)_F$ multiplets located at scales from $O(100)\text{ TeV}$ up to the Planck mass. They are the chiral multiplets \textbf{(15)} with the family scale $M_F$ order masses and vectorlike multiplets \textbf{(61)} with the gravitationally induced masses being presumably of the same order. If they are relatively light, they may be directly observed and they can also influence the $SU(5)$ gauge coupling unification. If they are too heavy, they can still strongly affect the mass matrices for down quarks and leptons that eventually leads to a significant violation of unitarity in the lepton CKM matrix. Moreover, for the high scale family symmetry one has in the preon model some actual candidates for heavy neutrinos that provides, as was shown in the section 6.3, some natural see-saw mechanism for physical neutrinos.

8 Conclusion and outlook

We started by stating that there is still left a serious problem in particle physics related to classification of all observed quark-lepton families. This may motivate us to continue seeking a solution in some subparticle or preon models for quarks and leptons. In this connection, it is worth underlining at the outset that none of the presently popular $SU(5)$, $SO(10)$ and $E(6)$ GUTs satisfies a somewhat evident criterion of possible elementarity of quarks and leptons. It seems likely that, if they were elementary they all should be contained in a single fundamental representation of grand unified symmetry, rather than in a set of its representations. This, as well as replication of quantum numbers carried by quarks and leptons, could tell about their composite structure formed by preons which might be elementary carriers of those quantum numbers. We have shown above that in accordance with these heuristic arguments the preon model may under certain natural conditions determine a local metaflavor $SU(8)_MF$ symmetry as a basic internal symmetry of the physical world at small distances that is then passed on to large distances in the terms of the $SU(5) \times SU(3)_F$ symmetry for composite quarks and leptons.

Let us now recall all stages we passed to come to the main results presented here. We started with the $2N$ left-handed and right-handed preons, $P^{iL}_{ai}$ and $Q^{iR}_{a'i}$ ($i = 1, \ldots, N$; $a, a' = 1, \ldots, n$), possessing some local metaflavor symmetry $SU(N)_{MF}$ and chiral metacolor symmetry $SO(n)_{MC}^L \times SO(n)_{MC}^R$ with numbers of metaflavors ($N$) and metacolors ($n$) not yet determined. We argued that these preons also possess the chiral symmetry $K(N) \textbf{(18)}$ in the limit when their gauge $SU(N)_{MF}$ metaflavor interactions are switched off. Though these interactions break the $K(N)$ symmetry, they are typically too weak at the metacolor confining distances to influence the formation of composites. Requiring a preservation of an appropriate chiral symmetry in the bound state spectrum, provided that this spectrum is only given by some single representation of $K(N)$ rather than a set of its representations, we found that just the chiral symmetry $K(8) \textbf{(21)}$ is solely selected as a universal chiral symmetry at all distances for an asymptotically free $SO(5)_{MC}^L \times SO(5)_{MC}^R$ metacolor
theory. Simultaneously, this determines the effective local metaflavor symmetry $SU(8)_{MF}$ that could be in principle observed at large distances through the low-lying composites emerged. Indeed, any possible extra $N - 8$ metaflavors, which are not provided by a preserved chiral symmetry, could only emerge in superheavy composites with the metacolor scale $\Lambda_{MC}$ order masses, thus appearing unobserved at a laboratory scale. However, such $L-R$ symmetric $SU(8)_{MF}$ metaflavor theory certainly is pure vectorlike for the identical $L$-preon and $R$-preon composite multiplets involved that is inadmissible. Actually, this means that, while preons are left massless being protected by their own metacolors, the composites being metacolor singlets will pair up and acquire heavy Dirac masses. It is rather clear that such a theory is meaningless unless the $L-R$ symmetry is partially broken that seems to be a crucial point in our model. In this connection, some natural mechanism for spontaneous $L-R$ symmetry breaking caused by the simultaneously emerged composite scalars has been proposed. According to it, while nothing happens with the left-handed preon composites still completing the total multiplets of the $SU(8)_L$, the right-handed preon composites will only form some particular submultiplets of the $[SU(5) \times SU(3)]_R$ symmetry. As a result, the triangle anomalies corresponding to generators of the $SU(8)_R$, other than those of its $[SU(5) \times SU(3)]_R$ subgroup, are left uncompensated that causes the proper decreasing of the chiral symmetry $K(8)$ shown in (38). Meanwhile, the $L-R$ symmetric metaflavor $SU(8)_{MF}$ theory of preons breaks down to an effective chiral $SU(5) \times SU(3)_F$ theory for composites containing the conventional $SU(5)$ GUT times an extra local family symmetry $SU(3)_F$ describing the three standard quark-lepton families.

Though the tiny radius of compositeness for universal preons composing both quarks and leptons makes it impossible to immediately confirm their composite nature, a few special $SU(5) \times SU(3)_F$ multiplets of extra composite fermions, predicted by the theory at the energies from $O(100)$ $TeV$ up to the Planck scale, may appear of an actual experimental interest. Some of them can be directly observed, the others populate the $SU(5)$ GUT desert. Due to their mixing with ordinary quark-lepton families there may emerge a significant violation of unitarity in the lepton CKM matrix depending on the interplay between the compositeness scale and scale of the family symmetry $SU(3)_F$.

Let us also briefly comment on some ways along which these results can be extended. The first and immediate one could be a supersymmetric extension of the present model, though many other supersymmetric preon models have been proposed [2, 4]. We introduced above some metaflavorless preons called sterilons which would be in fact gaugino (metagluino) in the supersymmetrized metacolor theory. This can be only considered as the first step which may be then continued totally within a supersymmetric context. Generally, as a conventional SUSY still remains undiscovered, one could think that it may be generically related to preons rather than quarks and leptons in the Standard Model. And, as a possible result, one might only expect an existence of some effective SUSY at energies being much higher than the electroweak scale.

Another important question we have not yet considered here is an emergence of composite vector field in the theory. Generally, the vector composites have to be heavy with masses of the order of the compositeness scale $\Lambda_{MC}$ unless they appear as effective gauge fields. The point is, however, that some massless composite vector fields could nonetheless appear in a theory as the vector Goldstone bosons related to spontaneous violation of
Lorentz invariance. Such violation could in principle appear through the multi-preon interactions similar to those given in the section 5.2. Actually, one could start with a global (rather than local) metaflavor symmetry $SU(N)_{MF}$ which is then converted into the local one through the contact multi-preon interactions [20] or some nonlinear constraint put on the preon currents (see in this connection [33] and the later works [34]). If so, the quarks and leptons, on the one hand, and the mediator gauge fields (photons, weak bosons, gluons etc.), on the other, could be composed at the same order distances determined by the preon confinement scale $\Lambda_{MC}$. In other words, there may be a high energy limit to the division of matter beyond which one can not go. Indeed, a conventional division of matter from atoms to quarks is naturally related to the fact that matter is successively divided, whereas the mediator gauge fields are left intact. However, situation may be drastically changed if these spontaneously emerging gauge fields become composite as well. Note also that closeness of the compositeness scale $\Lambda_{MC}$ to the Planck scale $M_{Pl}$ in the present model seems to make it in principle possible for the $SU(8)_{MF}$ metaflavor theory to be further unified with gravity, particularly in the case when a graviton, like other gauge fields, is presumably composite. This and other things mentioned above seem to be especially interesting and worth further pursuing.

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