Characterizing Bell state analyzer using weak coherent pulses

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Received: 1 September 2020 / Accepted: 31 March 2021 / Published online: 19 April 2021
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Abstract
Bell state analyzer (BSA) is one of the most crucial apparatuses in photonic quantum information processing. While linear optics provide a practical way to implement BSA, it provides unavoidable errors when inputs are not ideal single-photon states. Here, we propose a simple method to deduce the BSA for single-photon inputs using weak coherent pulses. By applying the method to Reference-Frame-Independent Measurement-Device-Independent Quantum Key Distribution, we experimentally verify the feasibility and effectiveness of the method.

Keywords
Linear optical quantum circuits · Bell state analyzer · Photon number distribution · Photonic qubits

1 Introduction

Entanglement is at the heart of quantum information processing [1]. Bell state analyzer (BSA), an experimental apparatus which performs projective measurement onto maximally entangled two-qubit states, plays central roles in many photonic quantum information processing such as fundamental quantum physics [2,3], quantum key distribution [4,5], quantum teleportation [6–8], and quantum computation [9,10].

Figure 1a presents the conceptual diagram of an ideal BSA. When a two-qubit state $\rho_{ab}$ is given at the input modes $a$ and $b$, it returns one of the four Bell states with the probability of $p_i = \langle \Psi_i | \rho_{ab} | \Psi_i \rangle$. Here, $| \Psi_i \rangle$ where $i \in \{1, 2, 3, 4\}$ denotes one of four

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Fig. 1  

(a) Conceptual diagram of an ideal Bell state analyzer. Any two-qubit input state $\rho_{ab}$ returns one of the four Bell states, $|\psi^{\pm}\rangle_{ab}$ and $|\phi^{\pm}\rangle_{ab}$. If the input is not a two-qubit state, it returns a null outcome. 

(b) Polarization qubit Bell state analyzer using linear optics. BS : beamsplitter, PBS : polarizing beamsplitter, D : single-photon detector. Only two Bell states out of four can be determined using this scheme, so the intrinsic success probability is $p = 1/2$. It can provide certain outcomes even if the input is not a two-qubit state.

Bell states. Note that the overall BSA success probability $\sum p_i = 1$ for any two-qubit input states. If the input state is not prepared in the form of $\rho_{ab}$, e.g., mode $a$ has two particles, while mode $b$ is vacuum, it provides a null outcome.

In a photonic qubit system, the ideal implementation of BSA is not straightforward since it requires photon–photon interaction [1]. Instead, the linear optical BSA based on two-photon interference and post-selection has been widely applied for a various quantum information processing due to the simplicity and robustness of the implementation [11,12]. Figure 1b shows a typical linear optical BSA setup in polarization qubits.

The linear optical BSA, however, has a few drawbacks. With the scheme of Fig. 1 (b), $|\psi^{\pm}\rangle_{ab} = \frac{1}{\sqrt{2}} (|01\rangle \pm |10\rangle) = \frac{1}{\sqrt{2}} (|HV\rangle \pm |VH\rangle)$ input state is detected by $D_{12}$ or $D_{34}$ ($D_{14}$ or $D_{23}$) where $D_{ij}$ denotes the coincidence detection between $D_i$ and $D_j$. Here, $|H\rangle$ and $|V\rangle$ denote horizontal and vertical polarization states, respectively. The other two Bell states of $|\phi^{\pm}\rangle_{ab} = \frac{1}{\sqrt{2}} (|00\rangle \pm |11\rangle)$ cannot be detected by this scheme, so the intrinsic success probability is $p = 1/2$. One can think of other configurations for BSA; however, it is known that the success probability of the linear optical BSA can be at most $p = 1/2$ [13]. Besides, the linear optical BSA works properly only when both input modes $a$ and $b$ are strictly occupied by single-photon states. In other words, it can provide incorrect outcomes when the input state is not prepared in $\rho_{ab}$. For example, if input mode $a$ has two photons and $b$ is vacuum, the linear optical BSA can tell one of the Bell state outcomes, whereas the ideal one returns a null outcome. Therefore, the linear optical BSA should be carefully utilized with ideal single-photon inputs, or it can provide unwanted errors.

Despite these imperfections, the linear optical BSA has been widely applied with non-ideal single-photon inputs. For instance, Measurement-Device-Independent Quantum Key Distribution (MDI-QKD), where an untrusted third party performs Bell state measurement (BSM) onto optical pulses from two distant communication parties, is usually implemented with weak coherent pulses (WCP) [14–17]. The use of non-ideal optical pulses causes nonzero intrinsic quantum bit error rate (QBER) in certain bases. Note that in Reference-Frame-Independent MDI-QKD (RFI-MDI-QKD), a more advanced QKD protocol which simultaneously provides high level of security.
and implementation practicality, these nonzero QBERs suggest complicated parameter estimation associated with the security [18–20].

In this paper, based on the recently proposed technique to characterize linear optical networks using non-ideal input states [21–24], we propose a method to characterize a linear optical BSA using WCP. In particular, we present a method to deduce the BSA result for ideal single-photon input states using the experimental data with WCP. In order to present the effectiveness of our method, we apply this method to MDI-QKD and RFI-MDI-QKD and show that the security-associated parameter estimation can be more simple and experiment friendly.

2 Theory

2.1 Characterizing linear optical quantum circuits

Let us see how one can characterize general linear optical quantum circuits for ideal single-photon inputs using the experimental data with WCP. It will be applied to the linear optical BSA in the following section. Assuming that we input WCP with the same average photon number at the input modes \( a \) and \( b \), \( \mu_a = \mu_b = \mu \), the coincidence counts between \( D_i \) and \( D_j \) are presented as

\[
N(D_{ij})^{\mu,\mu} = \kappa_i \kappa_j \mu^2 e^{-2\mu} P(D_{ij}|1_P, 1_Q) \\
+ \kappa_i \kappa_j \frac{\mu^2}{2} e^{-\mu} \{ P(D_{ij}|2_P, 0) + P(D_{ij}|0, 2_Q) \} \\
+ O(\mu^r, \mu^s, \mu^t, \mu^u),
\]

where \( P(D_{ij}|m_P, n_Q) \) is the conditional probability of coincidence detection \( D_{ij} \) when the input mode \( a \) (\( b \)) is occupied by \( m \) (\( n \)) photons with \( P \) (\( Q \)) polarization state. Here, \( \kappa_i \) denotes the detection efficiency of \( D_i \). The last \( O(\mu^r, \mu^s, \mu^t, \mu^u) \) term presents the higher-order contribution where the integer powers of \( r, s, t, \) and \( u \) denote the number of photons following Poissonian distribution of each input mode.

By blocking one of the input modes, one can obtain the coincidence counts as

\[
N(D_{ij})^{\mu,0} = \kappa_i \kappa_j \frac{\mu^2}{2} e^{-\mu} P(D_{ij}|2_P, 0) + O(\mu^r), \\
N(D_{ij})^{0,\mu} = \kappa_i \kappa_j \frac{\mu^2}{2} e^{-\mu} P(D_{ij}|0, 2_Q) + O(\mu^s). 
\]

Note that Eqs. (1) and (2) ignore the cases when total number of input photons is smaller than two since they do not provide a coincidence detection.

From Eqs. (1) and (2), we can isolate \( P(D_{ij}|1_P, 1_Q) \) from all other \( P(D_{ij}|m_P, n_Q) \) where \( m_P \neq 1 \) and \( n_Q \neq 1 \) as

\[
P(D_{ij}|1_P, 1_Q) = \frac{N(D_{ij})^{\mu,\mu} - N(D_{ij})^{\mu,0} - N(D_{ij})^{0,\mu} - O(\mu^r, \mu^u)}{\kappa_i \kappa_j \mu^2 e^{-2\mu}},
\]

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where $\mathcal{O} (\mu^t \mu^u) = \mathcal{O} (\mu^r, \mu^s, \mu^t \mu^u) - \mathcal{O} (\mu^r) - \mathcal{O} (\mu^s) \geq 0$. By dropping the $\mathcal{O} (\mu^t \mu^u)$ term in Eq. (3), one can obtain the upper bound of $P (D_{ij} | 1_P, 1_Q)$ as

$$P_{ub} (D_{ij} | 1_P, 1_Q) = \frac{N (D_{ij})^{\mu, \mu} - N (D_{ij})^{\mu, 0} - N (D_{ij})^{0, \mu}}{\kappa_i \kappa_j \mu^2 e^{-2\mu}}. \quad (4)$$

While the numerator of Eq. (4) can be directly obtained from the experimental data, the denominator requires precise calibration of the detection efficiencies and the average photon number. In order to avoid the calibration problem, we investigate the sum of single-photon counts when one of the modes is blocked as

$$N (D_i) = \kappa_i \mu e^{-\mu} \{ P (D_i | 1_P, 0) + P (D_i | 0, 1_Q) \} + \mathcal{O} (\mu^r) + \mathcal{O} (\mu^s), \quad (5)$$

where $P (D_i | 1_P, 0)$ and $P (D_i | 0, 1_Q)$ are determined by the input polarization states. By dropping the last two terms of Eq. (5), one can obtain the lower bound of $\kappa_i \mu e^{-\mu}$ as

$$\left( \kappa_i \mu e^{-\mu} \right)_{lb} = \frac{N (D_i)}{\{ P (D_i | 1_P, 0) + P (D_i | 0, 1_Q) \}}. \quad (6)$$

Since Eq. (6) provides the lower bound of denominator of Eq. (4), applying Eq. (6) to Eq. (4) still provides the upper bound of $P (D_{ij} | 1_P, P_{ub}, 1_Q)$.

### 2.2 Application to BSA and RFI-MDI-QKD

Let us first apply the above linear optical quantum circuit characterization scheme to linear optical BSA in MDI-QKD. In MDI-QKD, two communication parties, Alice and Bob, transmit optical pulses to a third party who performs BSM [4,5]. Note that the MDI-QKD is usually implemented using WCP with decoy states [14–17]. Let us first discuss the case when both communication parties transmit the states in $Z$-basis, i.e., either $|0\rangle = |H\rangle$ or $|1\rangle = |V\rangle$. Since a PBS transmits (reflects) the horizontal (vertical) polarization state, the horizontal and vertical polarization states are detected at $\{D_1, D_3\}$ and $\{D_2, D_4\}$, respectively. Therefore, for the horizontal polarization single input, one can find $P (D_{13} | 2_H, 0) = P (D_{13} | 0, 2_H) = 1/2$, and all other $P (D_{ij} | 2_H, 0) = P (D_{ij} | 0, 2_H) = 0$. Likewise, the vertical single input gives $P (D_{24} | 2_V, 0) = P (D_{24} | 0, 2_V) = 1/2$, while all other $P (D_{ij} | 2_V, 0) = P (D_{ij} | 0, 2_V) = 0$. Therefore, except for $D_{13}$ and $D_{24}$, Eq. (1) in $Z$-basis becomes simplified as

$$N (D_{ij})^{\mu, \mu} = \kappa_i \kappa_j \mu^2 e^{-2\mu} P_{ub} (D_{ij} | 1_H, 1_H). \quad (7)$$

Considering $D_{13}$ and $D_{24}$ does not account for the BSA result, one can find that, in $Z$-basis, the linear optical BSA results with WCP are identical to those with ideal single-photon inputs. This result coincides with that the QBER in $Z$-basis (when both Alice and Bob choose $Z$-basis) using WPC can be $Q_{ZZ}^{\mu, \mu} = 0$ and, thus, is used for secret key distribution [5].
In $X$- and $Y$-basis, the difference between WPC and ideal single-photon states becomes visible. Since $P(D_{ij}|2_P, 0)$ and $P(D_{ij}|0, 2_Q)$ contribute to the BSM results, the QBERs in $X$- and $Y$-bases with the WCP cannot be lower than $Q^{\mu,\mu}_{XX}$, $Q^{\mu,\mu}_{YY} \geq 0.25$, whereas those with single-photon states can be $Q^{1,1}_{XX}$, $Q^{1,1}_{YY} = 0$. For the input polarization states in these bases, $|P\rangle = \frac{1}{\sqrt{2}} (|H\rangle + e^{i\theta_p} |V\rangle)$, the single-count probability becomes $P(D_i|1_P, 0) = P(D_i|0, 1_Q) = 1/4$ for all $D_i$. Therefore, from Eqs. (4) and (6), we find

$$P_{ub}(D_{ij}|1_P, 1_Q) = \frac{N(D_{ij})^{\mu,\mu} - N(D_{ij})^{\mu,0} - N(D_{ij})^{0,\mu}}{\frac{1}{4} N(D_i) N(D_j)}.$$  

(8)

Equation (8) implies that the upper bound of coincidence probability $P_{ub}(D_{ij})$, i.e., the BSM results, for ideal single-photon input states can be deduced from coincidence and single counts using WCP inputs. Considering the upper bound of the coincidence probability $P_{ub}(D_{ij})$ involving the high-order photon number contributions, we can infer the upper bounds of QBERs for ideal single-photon inputs of $Q^{1,1}_{RS}$ in $R$- and $S$-bases for Alice and Bob where $R, S \in \{X, Y\}$. Note also that all the experimental data $N(D_{ij})$, $N(D_i)$, and $N(D_j)$ in Eq. (8) can be obtained from the ordinary decoy-based MDI-QKD experiment.

Applying the above method to RFI-MDI-QKD using WCP finds more interesting results in the security analysis. In RFI-MDI-QKD, the security analysis requires the QBER in $Z$-basis, $Q^{1,1}_{ZZ}$ and the $C$ parameter given as [18]

$$C = \langle X_AX_B \rangle^2 + \langle XX \rangle^2 + \langle YY \rangle^2 + \langle XY \rangle^2 + \langle YX \rangle^2$$

$$= \left(1 - 2Q^{1,1}_{XX}\right)^2 + \left(1 - 2Q^{1,1}_{YY}\right)^2 + \left(1 - 2Q^{1,1}_{XY}\right)^2.$$  

(9)

It is notable that both $Q^{1,1}_{ZZ}$ and $C$ are independent of the reference frame rotation, and thus, it does not require pre-shared reference frames between Alice and Bob. Note that $Q^{1,1}_{ZZ} = 0$ and $C = 2$ for RFI-MDI-QKD with ideal single-photon input states. While
\[ Q_{ZZ}^{1,1} = Q_{ZZ}^{\mu,\mu} \] can be directly obtained using WCP, calculating \( C \) requires \( Q_{RS}^{1,1} \). Our method provides a simple way to obtain the upper bound of \( Q_{RS}^{1,1} \) using WCP, and thus, with Eq. (9), we can obtain the lower bound of \( C \). Note that obtaining the lower bound of \( C \) is significant in the security analysis in QKD since it is essential to assume the worst case scenario for the QKD security analysis. Comparing to the conventional way to calculate \( C \) in RFI-MDI-QKD which requires accurate calibration of the average photon numbers of signal and decoy states, QBERs with different average photon number inputs, gains for different average photon number inputs, etc. (see Eqs. (6) and (7) of Ref. [18]), our method provides much more simple and robust mean to estimate \( C \). Note that the recently developed RFI-MDI-QKD using fewer quantum states can also be implemented with this method in order to simplify the experimental implementation of RFI-MDI-QKD [25].

### 3 Experiment

In order to compare the deduced BSM results using WCP and ideal single-photon inputs, we performed the experiments using attenuated laser pulses and photon pairs from spontaneous parametric down conversion (SPDC). The single-photon pairs at 1556 nm are generated by type II SPDC using 10-mm periodically polled KTP crystal pumped by femtosecond laser pulses. The WCP are obtained by attenuating femtosecond laser pulses. In order to make the spectral bandwidths identical, both light sources are filtered by the same interference filters with 3 nm bandwidth. Then, the optical pulses are sent to Alice and Bob who correspond to the transmitters of MDI-QKD, see Fig. 2.

In order to erase the first-order interference of WCP, Alice and Bob employ mirrors attached with piezoelectric translators (PZT) [26,27]. The PZTs are independently modulated during the experiment. The polarization states of \(|\varphi\rangle_A\) and \(|\varphi\rangle_B\) are encoded using half- and quarter-waveplates (H, Q), and then, the optical pulses are sent to Charlie who performs BSM using a linear optical BSA. The 15 dB neutral density filters (ND) at the optical paths simulate the quantum channel loss in the QKD communication. In order to obtain \( P(D_{ij}|1_P, 1_Q) \) using WCP, the BSM was performed when both Alice and Bob send optical pulses, \( N(D_{ij})^{\mu,\mu} \), and only one of Alice and Bob sends an optical pulse, \( N(D_{ij})^{\mu,0} \), \( N(D_{ij})^{0,\mu} \), and \( N(D_{ij})^{0,0} \). Note that, in the MDI-QKD scenario, the later corresponds to the case when one of the transmitters sends signal state, while the other sends vacuum decoy state.

Figure 3 shows the BSM results with (a) single-photon inputs from SPDC, (b) WCP with \( \mu = 0.25 \), and (c) deduced single-photon inputs using WCP. Here, \( \mu \) is determined at the output of Alice and Bob, and thus, the effective average photon number at Charlie becomes \( \mu \sim 0.008 \) after 15 dB of quantum channel loss. It shows that the BSM results for \( ZZ \)-basis inputs are all similar among different optical inputs. On the other hand, the BSM results for other bases present the difference. The BSM results with WCP are clearly different from those with SPDC single-photon inputs. However, one can obtain very similar results with the SPDC single-photon inputs by deducing single-photon inputs using WCP. This can be quantified by QBERs in
Fig. 3 Bell state measurement results with (a) single-photon inputs from SPDC, (b) WCP with $\mu = 0.25$, and (c) deduced single-photon inputs using WCP. Here, $\mu$ is determined at the outputs of Alice and Bob, and thus, the effective average photon number at Charlie becomes $\mu \sim 0.008$ after 15 dB of quantum channel loss.

MDI-QKD scenario, see Table 1. For ZZ-basis, QBERs with all inputs are close to $Q_{ZZ} = 0$. For XX- and YY-bases, however, QBER with WCP is much higher than those with SPDC inputs. Note that the intrinsic QBER limit with WCP in these bases is $Q_{XX} = Q_{YY} = 0.25$. By applying our method to deduce the single-photon inputs, QBERs become as low as those with SPDC inputs. Note, however, lowering QBERs does not mean that we can utilize these bases for secret key generation. It happens as a result of statistical treatment and does not effective on the individual events.

In order to investigate the effectiveness of our method in RFI-MDI-QKD, we have performed the protocol with respect to reference frame rotation $\beta$ at Bob’s channel. As shown in Fig. 2, the reference frame rotation was implemented by rotating a half-waveplate (H$_\beta$) which is located between two quarter-waveplates at 45° [28]. Figure 4 shows QBERs for (a) WCP and (b) deduced single-photon inputs using WCP with respect to $\beta$. The $\beta$-independent $Q_{ZZ} < 0.02$ is not presented. By deducing the

| Inputs      | ZZ-basis | XX-basis | YY-basis |
|-------------|----------|----------|----------|
| $Q^{\text{SPDC}}$ | 0.01 ± 0.001 | 0.034 ± 0.002 | 0.038 ± 0.002 |
| $Q^{\mu,\mu}$ | 0.018 ± 0.001 | 0.269 ± 0.004 | 0.265 ± 0.004 |
| $Q^{1,1}$ | 0.014 ± 0.002 | 0.037 ± 0.013 | 0.030 ± 0.013 |
single-photon inputs, the visibility of the sinusoidal oscillation increases from $V_{\mu,\mu} = 0.47 \pm 0.002$ to $V_{1,1} = 0.94 \pm 0.01$. Figure 4c presents estimated $C$ for various inputs with respect to $\beta$. While all $C$ are invariant under the reference frame rotation, it clearly shows $C_{1,1}$ for deduced single-photon inputs using WCP is similar to $C_{\text{SPDC}}$ for single-photon inputs using SPDC. The estimated $C$ is $C_{\mu,\mu} = 0.48 \pm 0.002$, $C_{1,1} = 1.75 \pm 0.014$, and $C_{\text{SPDC}} = 1.77 \pm 0.004$, respectively. Different mean photon numbers for WCP do not provide much difference in calculating $C_{\mu,\mu}$ and $C_{1,1}$, see Fig. 4 (d). These results clearly show that $C$ can be obtained with our method which does not require precise calibration of QBERs and gains of optical pulses with different average photon numbers. Considering the quantum channel loss, the effective average photon numbers at Charlie becomes $\mu \in [0.0032, 0.016]$ corresponding to the output mean photon numbers at Alice and Bob, $\mu \in [0.1, 0.5]$. With this tiny change in the effective average photon numbers at Charlie, $C_{\mu,\mu}$ and $C_{1,1}$ are almost invariant under the $\mu$ changes at Alice and Bob. However, a smaller number of coincidence counts provide larger experimental uncertainties.

It is notable that the uncertainties of $Q$ and $C$ for the deduced single-photon inputs are larger than those with WCP and SPDC. For instance, $C_{1,1}$ has larger uncertainty than $C_{\mu,\mu}$ and $C_{\text{SPDC}}$. It is because that the results with the deduced single-photon inputs are calculated from three sets of experimental data, i.e., $N(D_{ij})_{\mu,\mu}$, $N(D_{ij})_{\mu,0}$, and $N(D_{ij})_{0,\mu}$, and during the data analysis, the small errors in the experimental data.
propagate and cause increasing errors in the final parameter estimation. We note that this happens due to the finite number of data size.

4 Conclusion

To summarize, we have proposed and experimentally verified a method to deduce BSA for single-photon inputs using WCP. We have also applied the method to RFI-MDI-QKD and verified the effectiveness in estimating the security-associated parameter C. We note that applying our method to MDI- and RFI-MDI-QKD in more realistic experimental conditions including the finite key size analysis would be necessary for future work.

It is noteworthy that while we have focused on the BSA in this work, our method can be applied to any linear optical quantum circuits. Therefore, it can be applied to check the performance quality of linear optical quantum circuits when single-photon inputs are unavailable. We also remark it would be an interesting research direction to extend our methods to larger linear optical quantum circuits with a large number of inputs including GHZ state analyzer.

Acknowledgements This work was supported by the NRF programs (2019M3E4A1079777, 2019R1A2C2006381, 2019M3E4A107866011), the IITP programs (2020-0-00947, 2020-0-00972), and the KIST research program (2E30620).

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

References

1. Horodecki, R., Horodecki, P., Horodecki, M., Horodecki, K.: Quantum entanglement. Rev. Mod. Phys. 81, 865 (2008)
2. Branciard, C., Rosset, D., Liang, Y.-C., Gisin, N.: Measurement-device-independent entanglement witnesses for all entangled quantum states. Phys. Rev. Lett. 110, 060405 (2013)
3. Kim, Y.-S., Pramanik, T., Cho, Y.-W., Yang, M., Han, S.-W., Lee, S.-Y., Kang, M.-S., Moon, S.: Informationally symmetrical Bell state preparation and measurement. Opt. Express 26, 29539 (2018)
4. Braunstein, S.L., Pirandola, S.: Side-channel-free quantum key distribution. Phys. Rev. Lett. 108, 130502 (2012)
5. Lo, H.-K., Curty, M., Qi, B.: Measurement-device-independent quantum key distribution. Phys. Rev. Lett. 108, 130503 (2012)
6. Bennett, C.H., Brassard, G., Crépeau, C., Jozsa, R., Peres, A., Wooters, W.K.: Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Phys. Rev. Lett. 70, 1895–1899 (1993)
7. Bouwmeester, Pan, J.-W., Mattle, K., Eibl, M., Weinfurter, H., Zeilinger, A.: Experimental quantum teleportation. Nature 390, 575 (1997)
8. Gottesman, D., Chuang, I.L.: Demonstrating the viability of universal quantum computation using teleportation and single-qubit operations. Nature 402, 390 (1999)
9. Knill, E., Laflamme, R., Milburn, G.J.: A scheme for efficient quantum computation with linear optics. Nature 409, 46 (2001)
10. Gao, W.-B., Goebel, A.M., Lu, C.-Y., Dai, H.-N., Wagenknecht, C., Zhang, Q., Zhao, B., Peng, C.-Z., Chen, Z.-B., Chen, Y.-A., Pan, J.-W.: Teleportation-based realization of an optical quantum two-qubit entangling gate. Proc. Natl. Acad. Sci. USA 107, 20869 (2010)

11. Mattle, K., Weinfurter, H., Kwiat, P.G., Zeilinger, A.: Dense coding in experimental quantum communication. Phys. Rev. Lett. 76, 4656 (1996)

12. Ma, X.-S., Zotter, S., Kofler, J., Ursin, R., Jennewein, T., Brukner, Č, Zeilinger, A.: Experimental delayed-choice entanglement swapping. Nature Phys. 8, 479 (2012)

13. Calsamiglia, J., Lütkenhaus, N.: Maximum efficiency of a linear-optical Bell-state analyzer. Appl. Phys. B: Lasers and Opt. 72, 67 (2001)

14. Choi, Y., Kwon, O., Woo, M., Oh, K., Han, S.-W., Kim, Y.-S., Moon, S.: Plug-and-play measurement-device-independent quantum key distribution. Phys. Rev. A 93, 032319 (2016)

15. Yin, H.-L., Chen, T.-Y., Yu, Z.-W., Liu, H., You, L.-X., Zhou, Y.-H., Chen, S.-J., Mao, Y., Huang, M.-Q., Zhang, W.-J., Chen, H., Li, M. J., Nolan, D., F. Z., Jiang, X., Wang, Z., Zhang, Q., Wang, X.-B., Pan, J.-W.: Measurement-device-independent quantum key distribution over a 404 km optical fiber. Phys. Rev. Lett. 117, 190501 (2016)

16. Park, C.H., Woo, M.K., Park, B.K., Lee, M.S., Kim, Y.-S., Cho, Y.-W., Kim, S., Han, S.-W., Moon, S.: Practical plug-and-play measurement-device-independent quantum key distribution with polarization division multiplexing. IEEE Access 6, 58587 (2018)

17. Liu, H., Wang, W., Wei, K., Fang, X.-T., Li, L., Liu, N.-L., Liang, H., Zhang, S.-J., Zhang, W., Li, H., You, L., Wang, Z., Lo, H.-K., Chen, T.-Y., Xu, F., Pan, J.-W.: Experimental demonstration of high-rate measurement-device-independent quantum key distribution over asymmetric channels. Phys. Rev. Lett. 122, 160501 (2019)

18. Wang, C., Song, X.-T., Yin, Z.-Q., Wang, S., Chen, W., Zhang, C.-M., Guo, G.-C., Han, Z.-F.: Phase-reference-free experiment of measurement-device-independent quantum key distribution. Phys. Rev. Lett. 115, 160502 (2015)

19. Wang, C., Yin, Z.-Q., Wang, S., Chen, W., Guo, G.-C., Han, Z.-F.: Measurement-device-independent quantum key distribution robust against environmental disturbances. Optica 4, 1016 (2017)

20. Liu, H., Wang, J., Ma, H., Sun, S.: Polarization-multiplexing-based measurement-device-independent quantum key distribution without phase reference calibration. Optica 5, 902 (2018)

21. Yuan, X., Zhang, Z., Lütkenhaus, Norbert, Ma, X.: Simulating single photons with realistic photon sources. Phys. Rev. A 94, 062305 (2016)

22. Navarrete, Á., Wang, W., Xu, F., Curty, M.: Characterizing multi-photon quantum interference with practical light sources and threshold single-photon detectors. New J. Phys. 20, 043018 (2018)

23. Aragoneses, A., Islam, N.T., Eggleston, M., Lezama, A., Kim, J., Gauthier, D.J.: Bounding the outcome of a two-photon interference measurement using weak coherent states. Opt. Lett. 43, 3806 (2018)

24. Zhang, Y.-Z., Wei, K., Xu, F.: Generalized Hong-Ou-Mandel quantum interference with phase-randomized weak coherent states. Phys. Rev. A 101, 033823 (2020)

25. Lee, D., Hong, S.-J., Cho, Y.-W., Lim, H.-T., Han, S.-W., Jung, H., Moon, S., Lee, K.J., Kim, Y.-S.: Reference-frame-independent measurement-device-independent quantum key distribution using fewer quantum states. Opt. Lett. 45, 2624 (2020)

26. Kim, Y.-S., Slattery, O., Kuo, P.S., Tang, X.: Conditions for two-photon interference with coherent pulses. Phys. Rev. A 87, 063843 (2013)

27. Kim, Y.-S., Slattery, O., Kuo, P.S., Tang, X.: Two-photon interference with continuous-wave multi-mode coherent light. Opt. Express 22, 3611 (2014)

28. Yoon, J., Pramanik, T., Park, B.-K., Cho, Y.-W., Lee, S.-Y., Kim, S., Han, S.-W., Moon, S., Kim, Y.-S.: Experimental comparison of various quantum key distribution protocols under reference frame rotation and fluctuation. Opt. Comm. 441, 64 (2019)

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