THE NEUTRON STAR MASS–RADIUS RELATION AND THE EQUATION OF STATE OF DENSE MATTER

ANDREW W. STEINER, JAMES M. LATTIMER, AND EDWARD F. BROWN

1 Joint Institute for Nuclear Astrophysics, National Superconducting Cyclotron Laboratory, and the Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA; steiner3@umich.edu, efbrown@msu.edu
2 Institute for Nuclear Theory, University of Washington, Seattle, WA 98195, USA
3 Department of Physics & Astronomy, Stony Brook University, Stony Brook, NY 11794, USA; james.lattimer@stonybrook.edu

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ABSTRACT

The equation of state (EOS) of dense matter has been a long-sought goal of nuclear physics. EOSs generate unique mass versus radius ($M - R$) relations for neutron stars, the ultra-dense remnants of stellar evolution. In this work, we determine the neutron star mass–radius relation and, based on recent observations of both transiently accreting and bursting sources, we show that the radius of a 1.4 solar mass neutron star lies between 10.4 and 12.9 km, independent of assumptions about the composition of the core. We show, for the first time, that these constraints remain valid upon removal from our sample of the most extreme transient sources or of the entire set of bursting sources; our constraints also apply even if deconfined quark matter exists in the neutron star core. Our results significantly constrain the dense matter EOS and are furthermore consistent with constraints from both heavy-ion collisions and theoretical studies of neutron matter. We predict a relatively weak dependence of the symmetry energy on the density and a value for the neutron skin thickness of lead which is less than 0.20 fm, results that are testable in forthcoming experiments.

Key words: dense matter -- stars: neutron -- X-rays: binaries -- X-rays: bursts

Online-only material: color figures

1. INTRODUCTION

The masses of several neutron stars have been precisely measured using pulsar timing (Lorimer 2008); simultaneous mass and radius measurements, however, are considerably less certain. The leading candidates for such measurements are bursting neutron stars that show photospheric radius expansion (PRE; van Paradijs 1979; for a review, see Lewin et al. 1993) and transiently accreting neutron stars in quiescence (Rutledge et al. 1999).

Observations have already begun to determine the $M - R$ relation for neutron stars and to place strong constraints on the equation of state (EOS) of dense matter. Previously derived constraints (Read et al. 2009; Özel et al. 2010; Steiner et al. 2010; Steiner & Gandolfi 2012) have several limitations, including the use of fixed parameterizations for the EOS. Those works neither showed their results to be independent of their parameterizations, including the possibility of deconfined quark matter, nor did they address the full set of sources analyzed here. Özel et al. (2010) considered only PRE sources, but these may be subject to considerable systematic errors (Steiner et al. 2010; Boutloukos et al. 2010; Suleimanov et al. 2011). Steiner et al. (2010) considered both sources but used a smaller data set. Hebeler et al. (2010) and Steiner & Gandolfi (2012) took advantage of modern predictions for pure neutron matter near the saturation density to constrain the $M - R$ relation, but ignored systematic uncertainties associated with the observations. These systematic uncertainties could invalidate M and R results for all the PRE burst sources and/or some quiescent low-mass X-ray binary (qLMXB) sources, in which case the constraints on the EOS obtained by these works might be significantly altered. In this Letter, we address these limitations and attempt to make a more complete accounting of the systematic uncertainties due to modifications in either the EOS model or assumptions about the data set. Our constraints on, for example, the radius for a 1.4 solar mass neutron star are in some cases less stringent than those obtained previously, but are more robust because of the more careful accounting of the uncertainties described above.

2. EQUATION-OF-STATE PARAMETERIZATIONS AND DATA SET

At the lowest energy densities ($\leq 15$ MeV fm$^{-3}$ or mass densities $\lesssim 3 \times 10^{13}$ g cm$^{-3}$) the pressure–density relation is well understood (Haensel et al. 2007). Between 15 and 200–300 MeV fm$^{-3}$, the EOS is well described by four parameters: the incompressibility, the skewness, the magnitude of the symmetry energy ($S_v$), and the parameter describing the density derivative of the symmetry energy ($L$), all evaluated at the nuclear saturation density (approximately 150 MeV fm$^{-3}$). These parameters are constrained to varying degrees by experimental data (Lattimer & Lim 2012), including nuclear masses (Kortelainen et al. 2010), neutron skin thicknesses (Chen et al. 2010), giant dipole resonances and dipole polarizabilities (Trippa et al. 2008; Tamii et al. 2011; Piekarewicz et al. 2012), and heavy-ion collisions (Tsang et al. 2009). High-density matter is constrained by (1) causality (the speed of sound must not exceed the speed of light), (2) hydrodynamical stability, and (3) having a sufficient maximum mass (it must be greater than the largest well-determined neutron star mass, $1.97 \pm 0.04 M_\odot$ for PSR J1614–2230; Demorest et al. 2010). In addition, we impose the constraint that implied neutron star masses cannot be less than what is thought to be achievable in supernova, about $0.8 M_\odot$.

The low-density part of all the EOS parameterizations (except for strange quark stars) is described as in Steiner et al. (2010). All of the parameters in the low-density EOS, as well as the parameters in the high-density EOS models, are described with uniform prior distributions. Our fiducial EOS model (A) parameterizes the high-density EOS as a set of two piecewise
continuous power laws defining pressure $P = \varepsilon^{1+1/n}$ as a function of energy density $\varepsilon$. Model A has four high-density parameters: the transition energy density between the low-density EOS and the first polytrope, the transition energy density between the first and second polytrope, and the two polytropic indices, $n_1$ and $n_2$. We also employ an EOS model (B) which is similar to model A, except that the exponents in the two polytropes are parameterized with uniform priors in $\Gamma_i = 1 + 1/n_i$ instead of $n_i$. The upper and lower limits of $\Gamma_i$ correspond to the upper and lower limits of $n_i$ from model A and Steiner et al. (2010). A third EOS model (C) is piecwise linear EOS: the low-density EOS is used up to energy densities of 200 MeV fm$^{-3}$, and then consists of line segments that begin and end at four fixed energy densities: 400, 600, 1000, and 1400 MeV fm$^{-3}$. The linear relation between energy densities of 1000 and 1400 MeV fm$^{-3}$ is extrapolated to higher energy densities when necessary. The pressure on the energy density grid is allowed to be as large as 600 MeV.

A fourth EOS model (D) assumes that matter is a polytrope at intermediate densities and quark matter at high densities. The pressure of quark matter is described by Alford et al. (2005),

$$P = \frac{3a_1}{4\pi^2} \mu^4 - \frac{3a_2}{4\pi^2} \mu^2 - B,$$  

(1)

where $\mu$ is the quark chemical potential. The quantity $B$ is the bag constant and simulates confinement. The parameter $a_1$ describes corrections to the leading coefficient from a non-interacting Fermi gas (for which $a_1 = 1$). Corrections from perturbative quantum chromodynamics at high density suggest $0.6 < a_1 < 1$. The parameter $a_2$ approximately describes corrections from the finite strange quark mass $m_s$ and the quark superfluid gap $\Delta$ and is given by $a_2 = m_s^2 - 4\Delta^2$ (see also Steiner & Gandolfi 2012). In order to include the effects of a possible mixed phase in a model-independent way, a polytrope is added in between the low-density EOS and quark matter. In this model, there are five parameters in total: the transition energy density between low densities and the polytrope, the polytropic index, the transition energy density between the polytrope and quark matter (used to fix the value of $B$), $a_2$, and $a_3$. To describe bare strange quark stars, model E applies Equation (1) at all densities, with neither a low-density EOS nor an intermediate polytrope. Model E thus has three parameters: $a_2, a_3$, and $B$. All parameter ranges (i.e., the boundaries of the prior distributions) are large enough to ensure that the results do not change significantly when the range is increased. Our results are sensitive to the shape of these prior distributions, and our variation in EOS models effectively probes variations of these shapes.

Our baseline data include eight neutron stars: four which produced PRE X-ray bursts (4U 1608–522 (Güver et al. 2010a), KS 1731–260 (Özel et al. 2011), EXO 1745–248 (Özel et al. 2009), and 4U 1820–30 (Güver et al. 2010b)) and four qLMXBs in the globular clusters M13 (Webb & Barret 2007), $\omega$ Cen (Webb & Barret 2007), 47 Tuc (Heinke et al. 2006), and NGC 6397 (Guillot et al. 2011). There are a handful of other neutron star mass and radius constraints, such as those for RXJ 1856 (Pons et al. 2002), but all of these may be subject to even larger uncertainties than the observations we use here and would thus not strongly modify our final results. Our baseline model interprets the PRE burst sources with an extended photosphere, and assumes the same distribution of distances as in Steiner et al. (2010).

We do find that neutron star radii depend (albeit weakly) on the EOS model used to analyze the data, causing the lower and upper 95% confidence limits to vary by about 0.8 km. This effectively tests the robustness of our constraints under variations of the shape of the prior distributions. We also consider several modifications to the baseline data set. Suleimanov et al. (2011) have suggested that the X-ray spectra for PRE sources are affected by accretion and the eclipse of the neutron star by the disk. This affects the normalization at late times and we take this into account by increasing the color correction factor $f_c$, taking $1.45 < f_c < 1.8$ (modification I). Alternatively, Boutloukos et al. (2010) have suggested that the color correction factors are instead smaller as a result of magnetic confinement of the X-ray burst; this is considered in modification II in which we assume $1 < f_c < 1.35$. Some previous works assumed that the photosphere of PRE bursts is coincident with the neutron star surface (Özel et al. 2010), and we also consider this scenario (III). Finally, we test the sensitivity of our results to the removal of any one source or class of sources by removing X7 (IV) or M13 (V), by removing all PRE sources (VI), and by removing all the qLMXBs (VII).

To summarize, the standard models we examine include the baseline case (A), three variations of the EOS (B–D), and six other modifications of the baseline model varying the included data or its interpretation (A I–A VI). In addition, we also examine several more extreme scenarios which are described below.

We use the Bayesian method of Steiner et al. (2010), using marginal estimation to determine the posterior probability densities of quantities of interest. The marginal estimation integrals are performed using Markov Chain Monte Carlo.

3. RESULTS AND DISCUSSION

In our baseline analysis, we find strong constraints on the $M$–$R$ curve and on the dense matter EOS: the radius of a 1.4 $M_\odot$ neutron star is between 11.2 and 12.3 km (95% confidence). The permissible radius range encompassing all variations of the EOS and interpretations of the astrophysical data, but not including the more extreme scenarios, is 10.4–12.9 km (95% confidence), only moderately larger than the baseline result. The 68% and 95% confidence ranges are displayed in the upper and middle portions of Table 1. We determine the $M$–$R$ relation for a range of neutron star masses (see Figure 1). We also determine the 68% and 95% confidence intervals of the EOS of dense matter (Figure 2). The estimated uncertainty of the pressure is approximately 30%–50% at all densities achievable in neutron star interiors. In addition, the posterior distributions of the central energy density of the maximum mass star imply that the highest central density is $\varepsilon \sim 1200$ MeV fm$^{-3}$ (Lattimer & Prakash 2005).

Producing significantly different neutron star radii requires extreme assumptions regarding the EOS and the data. We now consider more extreme scenarios, which are presented in the bottom portion of Table 1 (see also Figure 3). To achieve significantly smaller radii, we must assume both that the color correction factor is anomalously small ($\ll 1.3$) for all of the PRE sources and that the EOS has strong phase transitions (model C). In this case, we get radii as small as 9 km. Increasing the maximum mass constraint (modification VIII), as would be the case if the estimated most-likely mass of the pulsar B1957+20 is 2.4 $M_\odot$ (van Kerkwijk et al. 2011), slightly increases radii relative to the baseline case.

We obtain even larger radii, up to almost 14 km, if we add the long PRE burst source 4U 1724–307 and further assume, as suggested in Suleimanov et al. (2011; modification IX), that
Table 1
Limits for the Radius of a 1.4 Solar Mass Neutron Star for All of the Models Considered in This Work

| EOS Model | Data Modifications | \( R_{95\%} \) (km) | \( R_{68\%} \) (km) | \( R_{95\%} \) (km) | \( R_{68\%} \) (km) |
|-----------|--------------------|---------------------|-------------------|-------------------|-------------------|
| A (2 polytropes) | - - - | 11.18 | 11.49 | 12.07 | 12.33 |
| B (2 polytropes) | - - - | 11.23 | 11.53 | 12.17 | 12.45 |
| C (line segments) | - - - | 10.63 | 10.88 | 11.45 | 11.83 |
| D (hybrid w/quarks) | - - - | 11.44 | 11.69 | 12.27 | 12.54 |

| Variations in the EOS model |
|-----------------------------|
| EOS Model | Data Modifications | \( R_{95\%} \) (km) | \( R_{68\%} \) (km) |
| A (2 polytropes) | I (high \( f_C \)) | 11.82 | 12.07 |
| A | II (low \( f_C \)) | 10.42 | 10.58 |
| A | III (redshifted photosphere) | 10.74 | 10.93 |
| A | IV (without X7) | 10.87 | 11.19 |
| A | V (without M13) | 10.94 | 11.25 |
| A | VI (no PREs) | 11.23 | 11.56 |
| A | VII (no qLMXBs) | 11.17 | 11.96 |
| Global limits | - - - | 10.42 | 10.58 |

| Variations in the data interpretation |
|-----------------------------|
| EOS Model | Data Modifications | \( R_{95\%} \) (km) | \( R_{68\%} \) (km) |
| A | I (high \( f_C \)) | 11.82 | 12.07 |
| A | II (low \( f_C \)) | 10.42 | 10.58 |
| A | III (redshifted photosphere) | 10.74 | 10.93 |
| A | IV (without X7) | 10.87 | 11.19 |
| A | V (without M13) | 10.94 | 11.25 |
| A | VI (no PREs) | 11.23 | 11.56 |
| A | VII (no qLMXBs) | 11.17 | 11.96 |
| Global limits | - - - | 10.42 | 10.58 |

| More extreme scenarios |
|-----------------------------|
| EOS Model | Data Modifications | \( R_{95\%} \) (km) | \( R_{68\%} \) (km) |
| C (line segments) | II (low \( f_C \)) | 9.17 | 9.34 |
| A (2 polytropes) | VIII (\( M_{\text{max}} > 2.4 \)) | 12.14 | 12.29 |
| E (bare quark star) | - - - | 10.19 | 10.64 |

| Scenario motivated by Suleimanov et al. (2011) |
|-----------------------------|
| EOS Model | Data Modifications | \( R_{95\%} \) (km) | \( R_{68\%} \) (km) |
| A (2 polytropes) | IX (see the text) | 12.35 | 12.83 |

Note. Model A and the assumption \( 1.33 < f_C < 1.47 \) for the PRE sources are assumed unless specified otherwise.

Figure 1. Comparison of the predicted \( M-R \) relation with the observations. The shaded regions outline the 68% and 95% confidences for the \( M-R \) relation; these include variations in the EOS model and the modifications to the data set (see Table 1) but not the more extreme scenarios. The lines give the 95% confidence regions for the eight neutron stars in our data set. (A color version of this figure is available in the online journal.)

Figure 2. Predicted pressure as a function of baryon density of neutron star matter as obtained from astrophysical observations. The region labeled “NS 68%” gives the 68% confidence limits and the region labeled “NS 95%” gives the 95% confidence limits. Results for neutron star matter from effective field theory (Hebeler et al. 2010; see inset), from quantum Monte Carlo (Gandolfi et al. 2012), and from constraints inferred from heavy-ion collisions (Danielewicz et al. 2002) are also shown for comparison. (A color version of this figure is available in the online journal.)

While we are able to significantly constrain the \( P-\varepsilon \) relation, determination of the composition of neutron star cores is not yet possible. To probe the core composition, we consider EOS model E, which describes the entire star by the high-density quark matter EOS used in model D, i.e., a self-bound strange quark star. In the mass range 1.4–2 solar masses, the radii are not significantly different from our baseline model so that there is
no strong preference for either strange quark or hadronic stars; however, model E predicts radii significantly less than 10 km for low masses ($<1.2 \, M_{\odot}$).

Our neglect of rotation is unlikely to affect our conclusions. Rotation increases the radius at the equator and decreases the radius at the poles, and this could be relevant for the interpretation of some PRE X-ray bursts: the rotation rate of 4U 1608−522 is 619 Hz. However, for EOSs that are likely to reproduce the observational data, this rotation rate increases the radius by less than 10% (Weber 1999). This introduces an uncertainty smaller than that due to variations in $f_c$, which we have already taken into account. The rotation rates for the qLMXBs in our sample are unknown. Assuming that they are similar to other qLMXBs, however, means that the effect of rotation is smaller than that of their distance uncertainties.

The relationship between pressure and energy density (Figure 2) that we determine from our baseline analysis from observations is consistent with effective field theory (Hebeler et al. 2010) and quantum Monte Carlo (Gandolfi et al. 2012; Steiner & Gandolfi 2012) calculations of low-density neutron matter. Note that these neutron matter result are incompatible with the Suleimanov et al. (2010) interpretation of 4U 1724−307 (Suleimanov et al. 2011) which suggested exclusion of short PRE bursts and qLMXBs M13 and 41.1–83.4 MeV to 95% confidence. The allowed values of $L$ are substantially larger for model C because this parameterization more effectively decouples the low- and high-density behaviors of the EOS.

Our preferred range for $L$ is similar to that obtained from other experimental and observational studies (Tamii et al. 2011; Tsang et al. 2012; Steiner & Gandolfi 2012; Lattimer & Lim 2012) and experimental studies (e.g., Tsang et al. 2012; Tamii et al. 2011). Our results suggest that the neutron skin thickness of $^{208}$Pb (Typel & Brown 2000; Steiner et al. 2005) is less than about 0.20 fm. This result is independent of the EOS models (which include possible phase transitions) and data modifications described above. It is compatible with experiment (Horowitz et al. 2001) and also with measurements of the dipole polarizability of $^{208}$Pb (Reinhard & Nazarewicz 2010).

While we have endeavored to take into account some systematic uncertainties in our analysis, we cannot rule out corrections due to the small number of sources and to possible drastic modifications of the current understanding of low-mass X-ray binaries. Nevertheless, it is encouraging that these astrophysical considerations agree not only with nuclear physics experiments but also with theoretical studies of neutron matter at low densities and heavy-ion experiments at higher densities.
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