Integrated Thermomechanical Characterization of Metals Using the Virtual Fields Method

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Abstract. In this paper, the possibility of characterizing the thermomechanical behavior of metals using the virtual fields method (VFM) and suitable specimens with heterogeneous strain and temperature fields was demonstrated using simulated experiments. The used geometry is a double-notched tensile test with a Gaussian distribution of temperature over the surface. The chosen constitutive model is the Johnson-Cook hardening law coupled with the Hill48 anisotropic yield criterion. First the VFM strategy and the simulated experiments are described. Then the results are presented showing three case studies, (i) only the effect of the temperature is identified, (ii) the whole set of constitutive parameters is identified at the same time, (iii) a two-step identification is performed. The potentiality of the method as well as the main problems are discussed extensively.

Introduction

Inverse methods are nowadays widely used to identify the constitutive parameters of materials from complex experiments, which involves heterogeneous states of stress/strain. The advantage of using heterogeneous stress states is that different loading conditions can be tested at the same time, collecting a richer information that can be used to identify advanced material models with a reduced number of experimental tests [1]. In recent years, remarkable advances were obtained in plasticity, where, for instance, the Virtual Fields Method (VFM) [2, 3] was used to identify advanced anisotropic plasticity models [4, 5]. The application of VFM to thermomechanical models, however, has not been investigated yet and only very recently a work was published on this topic [6].

Theory

The non-linear VFM for thermomechanical models is similar to the method used for plasticity, with the difference that the hardening model is a function of the equivalent plastic strain and the temperature. The method relies on the principle of virtual work and can be summarized as the minimization of the following cost function $\Psi$:

$$\Psi(\xi) = \int_{V} \sigma(\varepsilon_{DIC}, T_{IR}, \xi) : \varepsilon^{*} dV - \int_{\partial V} \mathbf{t} \cdot \mathbf{u}^{*} dS$$

where $\xi$ are the constitutive parameters that have to be identified, $\sigma$ is the Cauchy stress tensor, $\mathbf{t}$ are the external forces and $\mathbf{u}^{*}$ and $\varepsilon^{*}$ are the virtual displacement field and the corresponding virtual strain field, which can be arbitrarily chosen. The Cauchy stress can be reconstructed from the strain field using a constitutive model and a suitable reconstruction algorithm, such as the radial return in elasto-plasticity. In this case, the algorithm proposed by Rossi et al. was employed [7]. In the presented application, since a thermomechanical model is involved, the temperature field is also required to obtain the stress field.

From the experiments, the external forces are measured through the load cell, the strain field, i.e. $\varepsilon_{DIC}$, can be obtained through digital image correlation (DIC), and the temperature field, i.e. $T_{IR}$,
can be measured using an infrared (IR) camera. Therefore, the only unknowns are the constitutive parameters $\xi$ that are identified using a minimization algorithm.

The virtual fields $u^*$ and $\varepsilon^*$ can be generated using different methods, for instance they can be user-defined (UDVFs) or automatically generated using one of the approaches described in [3]; in this paper, sensitivity based virtual fields (SBVFs) were applied [8, 9].

Simulated experiments were used to assess the effectiveness of the method and the possibility of identifying the constitutive parameters using a reduced number of tests.

**Simulated Experiments**

The strain and temperature maps were obtained from Finite Element (FE) models. Although simulated experiments requires the simulation of the whole measurement chain [10, 11], for this preliminary study, the value of the strain (and temperature) obtained in each node of the FE model was simply reshaped over a regular grid, which reproduces a typical spatial resolution of a DIC measurement.

The selected geometry is a double-notched specimen, as shown in Fig. 1. Although the selection of the optimal specimen shape is largely debated in the inverse identification community, the double-notched specimen has proved to provide a dense set of heterogeneous data to feed the inverse identification of anisotropic plasticity models [12, 13]. In order to include information about the material anisotropy, we studied specimens obtained at three material texture orientations from the Rolling Direction (RD), namely $0^\circ$, $45^\circ$, $90^\circ$, all of them with a thickness of 1.2 mm.

![Fig. 1: Geometry and FE meshing of the notched specimen, including the ROI whose full-field data were directly involved in the non-linear VFM identification. All the dimensions are in mm.](image)

The experiments were simulated by using the commercial software Abaqus/Standard®, employing CPS4R 4-nodes elements and assuming the plane stress mechanical state under tensile deformation. In this study, we considered a constitutive model that reproduces the thermomechanical properties of an advanced high-strength steel, namely the TRIP780, using a Johnson-Cook (J-C) hardening law. The strain-rate sensitivity of the material was not considered in this analysis, therefore the corresponding term in the J-C model was removed, leading to the following equation:
\[ \sigma(\varepsilon_p) = (A + B\varepsilon_p^n)(1 - \tilde{T}^m) , \quad (2) \]

where \( A, B \) and \( n \) are the material coefficients of the hardening law, while \( m \) and \( \tilde{T}^m \) account for the thermal response. In particular, \( \tilde{T}^m \) assumes different values depending from the material point’s temperature with respect to a reference test temperature \( T_t \) and the material melting temperature \( T_m \):

\[ \tilde{T} = \begin{cases} 
0 & T < T_t \\
\frac{T - T_t}{T_m - T_t} & T_t < T < T_m \\
1 & T > T_m 
\end{cases} \quad (3) \]

Besides, the material plastic anisotropy was modelled through the Hill48 model based on the width-to-thickness strain ratio – also known as Lankford coefficient – from uniaxial tensile tests on three material orientations (typically 0°, 45° and 90° with respect to the RD). All the material parameters are listed in Table 1.

Table 1: Reference material coefficients used for the FE analysis.

| J-C model | Hill48 model |
|-----------|--------------|
| A (MPa)   | B (MPa)      | n  | m  | \( T_t \) (K) | \( T_m \) (K) | \( R_0 \)  | \( R_{45} \) | \( R_{90} \) |
| 494       | 1429         | 0.79 | 0.86 | 293.15   | 1673       | 0.896       | 0.925       | 1.062       |

The distribution of temperature was imposed on each nodal location by using a Gaussian law, where the maximum temperature is positioned on the upper part of the notches. In this way it was possible to create an heterogeneous distribution spanning from 533 K to 773.15 K, as depicted in Fig. 2. In real experiments, heterogeneous temperature distributions can be generated using appropriate heating sources, e.g. resistance or induction heating, where only a portion of the specimen is directly heated. The idea is that a single test will reproduce multiple thermal conditions that can be used to identify the constitutive model.

Fig. 2: Heterogeneous temperature distribution on the notched specimen gauge area. The colorbar displays the temperature values in Kelvin.

The coupled Johnson-Cook and Hill48 constitutive model was implemented in the FE solver by means of an UMAT user subroutine. Fig. 3 reports an example of stress and equivalent plastic strain (PEEQ) computed at the end of the FE simulation on the specimen with material orientation at 0° from the RD. It is worth noticing that the alteration of material properties due to temperature distribution produces an asymmetric structure in the mechanical fields, with the plastic deformation that does not localize as usual in the center of the notches but is shifted towards the zone with higher temperature.
Fig. 3: Equivalent plastic strain (PEEQ) and Cauchy stress components generated in the gauge area of the specimen at 0° at the last step of the FE analysis.

The stress distribution is also heterogeneous, as illustrated Fig. 3, shear stress ($\sigma_{XY}$) and transverse stress ($\sigma_{XY}$) are activated in different zones of the specimen.

**Results**

The non linear VFM was used to identify the parameters with different strategies. Among the 9 parameters listed in Table 1, only 7 are unknowns since the test temperature $T_t$ and the melting temperature of the material $T_m$ are acknowledged. The outcomes are summarized in Table 2.

Table 2: Summary of the identification outcomes using different approaches: single parameter identification, the simultaneous inverse calibration of all the constitutive coefficients and the two-step identification procedure.

| Used test | VFs      | A     | B     | n     | m     | $R_0$ | $R_{45}$ | $R_{00}$ |
|-----------|----------|-------|-------|-------|-------|-------|----------|----------|
| 1st case study |          |       |       |       |       |       |          |          |
| 0°        | UDVFs    | -     | -     | -     | 0.8773| -     | -        | -        |
| err [%]   | -        | -     | -     | 2.0   | -     | -     | -        | -        |
| 2nd case study |          |       |       |       |       |       |          |          |
| 0°+45°+90° | UDVFs    | 326.58| 966.32| 0.7745| 8.5427| 0.8526| 0.8546   | 1.0042   |
| err [%]   | 33.8     | 32.3  | 1.9   | 893.3 | 4.8   | 7.6   | 5.4      |
| 0°+45°+90° | SBVFs    | 468.29| 1406.18| 0.8493| 1.1198| 0.7662| 0.8944   | 0.8965   |
| err [%]   | 5.2      | 1.5   | 7.5   | 30.2  | 14.4  | 3.3   | 15.0     |
| 3rd case study |          |       |       |       |       |       |          |          |
| 0° (293.15 K) | UDVFs    | 484.042| 1371.54| 0.7575| -     | -     | -        | -        |
| err [%]   | 2.0      | 4.0   | 4.1   | -     | -     | -     | -        | -        |
| 0°+45°+90° | UDVFs    | -     | -     | -     | 0.9024| 0.8598| 0.8612   | 1.0148   |
| err [%]   | -        | -     | -     | 4.9   | 4.0   | 6.8   | 4.4      |

As first case study, a single test was used to identify the exponent $m$, which is the term that drives the thermal effect in the J-C law. This example simulate a case where the material behavior at room...
temperature, i.e. the first terms of the J-C law and the \( R \)-values, is already known and only the effect of the temperature must be evaluated. In this case, a single test is sufficient to correctly identify the parameter, demonstrating that this approach can be used to use a single test to identify the material behavior at different temperatures, while multiple experiments at different temperatures would be required in the standard calibration.

In the second case study, instead, three tests with a temperature gradient and different orientations were used to identify simultaneously the whole set of parameters of the constitutive model. Both UDVFs (using the virtual fields defined in [12]) and SBVFs were used. In this case, the identification problem was more difficult to solve and a large error is observed. In general, the sensitivity based VFs produced a lower error, especially in the identification of the J-C parameters, while a slightly worst performance is obtained for the Hill48 parameters. For both methods, the most difficult parameter to identify is the exponent \( m \). From this analysis it turns out that the selected three experiments are not sufficient to identify the whole set of parameters at the same time. However, the identification error on each parameter listed in Table 1, often, it is not a proper criterion to assess the accuracy of the identification. In fact, in case of non-uniqueness of the solution, the identified parameters could give a satisfactory description of the material behavior in the investigated range of temperature and equivalent plastic strain even if they are different from the reference ones. An in depth investigation of the non-uniqueness issue is beyond the scope of the present paper and will be tackled in future studies.

The third case study shows a different identification strategy. The problem is split into two steps, first parameters \( A \), \( B \) and \( n \) were identified using a single test at room temperature; then the exponent \( m \) and the anisotropic parameters \( R_0 \), \( R_{45} \) and \( R_{90} \) were calibrated using the three tests with the temperature gradient. Only UDVFs were used in the VFM algorithm, however, all parameters were identified with a low error level, below 5%.

**Conclusion**

In this paper, simulated experiments were used to demonstrate the possibility of using VFM to identify the constitutive parameters of a thermomechanical plasticity model, which also involves anisotropy. Double-notched specimens were used to this purpose, with the same geometry already developed for other applications. Different case studies were taken into consideration. The main outcomes are:

- the accuracy of the thermomechanical VFM was assessed through virtual experiments;
- a single specimen with a heterogeneous temperature fields can be used to calibrate the thermal exponent of a J-C model;
- the simultaneous identification of the whole set of parameters from the experiments is difficult and a high level of error is observed even using simulated data;
- the sensitivity based VFs allow to consistently reduce the overall error although some parameters are still not identified properly;
- a two-step strategy can be used to improve the identification, in this case, using four experiments it was possible to identify properly the whole set of constitutive parameters.

The results discussed here are still a preliminary study of the thermomechanical VFM, the proposed geometry and temperature distribution can be improved and, in the simulated experiments, it is necessary to introduce the effect of the measurement system to have a realistic estimate of the error. Finally the non-uniqueness issue must be investigate to evaluate the actual accuracy of the calibrated parameters, even if they look rather different from the reference ones.
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