Viable textures for the fermion sector

A. E. Cárcamo Hernández
Universidad Técnica Federico Santa María and Centro Científico-Tecnológico de Valparaíso
Casilla 110-V, Valparaíso, Chile

I. de Medeiros Varzielas
Department of Physics, University of Basel,
Klingelbergstr. 82, CH-4056 Basel, Switzerland and
School of Physics and Astronomy,
University of Southampton,
Southampton, SO17 1BJ, U.K.

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We consider a modification of the Fukuyama-Nishiura texture and compare it to the precision quark flavour data, finding that it fits the data very well. We then propose different viable textures for quarks, where only the Cabibbo mixing arises from the down sector, and extend to the charged leptons while constructing a complementary neutrino structure that leads to viable lepton masses and mixing.

I. INTRODUCTION

The flavour puzzle is not understood in the context of the Standard Model (SM), which does not specify the Yukawa structures and has no justification for the number of generations. As such, extensions addressing the fermion masses and mixing are particularly appealing.

In building models that address the flavour problem, it is important to know structures that lead to the observed fermion flavour data, and in this work we introduce two proposals.

We start by revisiting the Fukuyama-Nishiura (FN) texture [1, 2], which is no longer phenomenologically viable as shown in [3]. A simple modification is to modify the texture slightly by enabling a non-zero 11 entry, which we show is already a viable texture.

We then introduce other quark textures where the Cabibbo angle comes from the down quarks whereas the other mixing angles come from the up sector, which also successfully describes the quark masses and mixing. We extend this to the lepton sector, with charged leptons sharing the texture of the down quarks and the neutrinos significantly contributing to a viable PMNS mixing matrix.

Other works in the literature considering textures include [4, 5], and some recent works such as [6–8].

II. TEXTURES FOR THE QUARK SECTOR

A. Fukuyama-Nishiura texture and its modification

Proposed for leptons in [1] and also used for quarks in [2], the FN texture consists in mass matrices of the form:

\[
\begin{pmatrix}
0 & A_f & A_f \\
A_f & B_f & C_f \\
A_f & C_f & B_f
\end{pmatrix}
\]

In [3] it was shown that this texture doesn’t quite work currently, because it fails to reproduce the observed value of the CP violating phase \(\delta\). A simple modification of the texture is to consider

\[
\tilde{M}_f = \begin{pmatrix}
D_f & A_f & A_f \\
A_f & B_f & C_f \\
A_f & C_f & B_f
\end{pmatrix}
\]

with the complex phases included in the following way:

\[
M_f = P_f \tilde{M}_f P_f^\dagger, \quad P_f = \begin{pmatrix}
1 & 0 & 0 \\
0 & e^{-i\gamma_f} & 0 \\
0 & 0 & e^{-i\gamma_f}
\end{pmatrix}
\]

where \(A_f, B_f, C_f\) and \(D_f\) are real parameters. \(\tilde{M}_f\) is diagonalized by an orthogonal matrix \(R_f\):

\[
R_f^T \tilde{M}_f R_f = \text{diag} (-m_1, m_2, m_3)
\]
B. Mixing inspired textures

We consider now the following Mixing Inspired (MI) textures, where we consider non-zero off-diagonal entries leading to the Cabibbo angle in the down sector, and for

$$V = O^T_U P_U D O_D = \begin{pmatrix}
    c_{UCD} + \frac{1}{2} s_{UCD} (e^{i\sigma} + e^{i\tau}) & c_{UD} + \frac{1}{2} s_{UD} (e^{i\sigma} + e^{i\tau}) & \frac{1}{2} s_{U} (e^{i\sigma} - e^{i\tau}) \\
    s_{UCD} - \frac{1}{2} i c_{UCD} (e^{i\sigma} + e^{i\tau}) & s_{UD} - \frac{1}{2} i c_{UD} (e^{i\sigma} + e^{i\tau}) & \frac{1}{2} i c_{U} (e^{i\sigma} - e^{i\tau}) \\
    \frac{1}{2} s_{D} (e^{i\sigma} - e^{i\tau}) & \frac{1}{2} s_{D} (e^{i\sigma} + e^{i\tau}) & \frac{1}{2} (e^{i\sigma} + e^{i\tau})
\end{pmatrix}$$

Using the values of the quark masses at the $M_Z$ scale from Table I shown in Table II and varying the parameters $D_u, \sigma$ and $\tau$ we fitted the magnitudes of the CKM matrix elements, the CP violating phase and the Jarlskog invariant $J$ to the experimental values shown in Table II. The experimental values of CKM magnitudes and Jarlskog invariant are taken from [10].

| Observable | Modified FN texture | Experimental Value |
|------------|---------------------|--------------------|
| $|V_{ud}|$ | 0.974 | 0.97427 ± 0.00015 |
| $|V_{us}|$ | 0.225 | 0.22534 ± 0.00065 |
| $|V_{ub}|$ | 0.00351 | 0.00351 ± 0.00015 ± 0.00014 |
| $|V_{cd}|$ | 0.225 | 0.22520 ± 0.00065 |
| $|V_{cb}|$ | 0.973 | 0.97344 ± 0.00016 |
| $|V_{ts}|$ | 0.0412 | 0.0412 ± 0.0001 ± 0.00025 |
| $|V_{tb}|$ | 0.00867 | 0.00867 ± 0.00029 ± 0.00031 |
| $|V_{td}|$ | 0.0404 | 0.0404 ± 0.0001 ± 0.00025 |
| $|V_{tb}|$ | 0.999 | 0.999146 ± 0.000021 ± 0.000046 |
| $J$ | $2.96 \times 10^{-5}$ | $(2.96 \pm 0.00020) \times 10^{-5}$ |
| $\delta$ | $69.2^\circ$ | $68^\circ$ |

Table II. Comparison of our fit for the modified FN texture to the experimental CKM magnitudes, CP violating phase and Jarlskog invariant.

Our results correspond to the values:

$$D_u = 3.13 \times 10^{-3} GeV, \quad \sigma = 87.9^\circ,$$
$$D_d = -2.35 \times 10^{-4} GeV, \quad \tau = 92.6^\circ.$$  

(8)

The obtained magnitudes of the CKM matrix elements, the CP violating phase and the Jarlskog invariant are in excellent agreement with the experimental data.
the ups leading to the remaining (small) mixing angles:

\[ M_U = \frac{v}{\sqrt{2}} \begin{pmatrix} c_1 \lambda^8 & 0 & a_1 \lambda^3 \\ 0 & b_1 \lambda^4 & a_2 \lambda^2 \\ 0 & 0 & a_3 \end{pmatrix}, \]

\[ M_D = \frac{v}{\sqrt{2}} \begin{pmatrix} c_1 \lambda^7 & f_1 \lambda^6 & 0 \\ 0 & f_2 \lambda^5 & 0 \\ 0 & 0 & g_1 \lambda^3 \end{pmatrix}, \quad (9) \]

where \( \lambda = 0.225 \) is one of the Wolfenstein parameters, \( v = 246 \) GeV the symmetry breaking scale and \( a_k \) \( (k = 1, 2, 3) \), \( b_1, c_1, g_1, f_1, f_2, e_1 \) are \( \mathcal{O}(1) \) parameters. From comparison with the Wolfenstein parameterisation, we conclude that most dimensionless parameters given in Eq. (9) can be real, excepting \( a_1 \).

Therefore, the up and down type quark masses are approximately given by:

\[ m_u \simeq c_1 \lambda^8 \sqrt{2} v, \quad m_c \simeq b_1 \lambda^4 \sqrt{2} v, \quad m_t \simeq a_1 \lambda^3 \sqrt{2} v, \]

\[ m_d \simeq \frac{c_1 \lambda^7}{\sqrt{2}} v, \quad m_s \simeq f_2 \lambda^5 \sqrt{2} v, \quad m_b \simeq g_1 \lambda^3 \sqrt{2} v \quad (10) \]

The Wolfenstein parameterisation \( \text{[11]} \) of the CKM matrix is:

\[ V_W \simeq \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A \lambda^3 (\rho - i \eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A \lambda^2 \\ A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 & 1 \end{pmatrix}, \quad (11) \]

with

\[ \lambda = 0.22535 \pm 0.00065, \quad A = 0.811^{+0.022}_{-0.012}, \quad (12) \]

\[ \bar{\lambda} = 0.313^{+0.026}_{-0.013}, \quad \bar{\eta} = 0.345^{+0.013}_{-0.014}, \quad (13) \]

\[ \bar{\lambda} \simeq \rho \left(1 - \frac{\lambda^2}{2}\right), \quad \bar{\eta} \simeq \eta \left(1 - \frac{\lambda^2}{2}\right). \quad (14) \]

From the comparison with \( \text{[11]} \), we find:

\[ a_3 \simeq 1, \quad a_2 \simeq A, \quad a_1 \simeq -A \sqrt{\rho^2 + \eta^2} e^{i \delta}, \quad (15) \]

\[ b_1 \simeq \sqrt{\frac{m_2}{\lambda^8 m_t^2}} \simeq 1.43, \quad c_1 \simeq \sqrt{\frac{m_2}{\lambda^8 m_t^2}} \simeq 1.27, \quad (16) \]

note that \( a_1 \) is required to be complex.

We fit the parameters \( e_1, f_1, f_2 \) and \( g_1 \) in Eq. (9) to reproduce the down type quark masses and quark mixing parameters. The results for the CKM matrix elements, the Jarlskog invariant \( J \) and the CP violating phase \( \delta \) in Tables \( \text{[III]} \) and \( \text{[IV]} \) correspond to the best fit values:

\[ e_1 \simeq 0.6, \quad f_1 \simeq 0.59, \quad f_2 \simeq 0.57, \quad g_1 \simeq 1.42. \quad (17) \]

\begin{tabular}{|c|c|c|c|}
\hline
Observable & MI textures & Experimental Value \\
\hline
\( m_u (\text{MeV}) \) & 641 & 635 \pm 86 \\
\( m_d (\text{MeV}) \) & 59.2 & 57.7^{+16.8}_{-15.7} \\
\( m_s (\text{GeV}) \) & 2.82 & 2.82^{+0.09}_{-0.04} \\
\( m_c (\text{MeV}) \) & 0.487 & 0.487 \\
\( m_t (\text{MeV}) \) & 102.8 & 102.8 \pm 0.0003 \\
\( m_b (\text{MeV}) \) & 1.75 & 1.75 \pm 0.0003 \\
\hline
\end{tabular}

Table III. MI textures and experimental values of charged fermion masses.

\begin{tabular}{|c|c|c|c|}
\hline
Obs. & Wolfenstein & MI textures & Experimental \\
\hline
\( V_{ud} \) & 0.9746 & 0.9743 & 0.97427 \pm 0.00015 \\
\( V_{cb} \) & 0.22535 & 0.2253 & 0.22534 \pm 0.00065 \\
\( V_{td} \) & 0.00868 & 0.00351 & 0.00351^{+0.0015}_{-0.0014} \\
\( V_{ud} \) & 0.22535 & 0.22501 & 0.22520 \pm 0.00065 \\
\( V_{cd} \) & 0.9746 & 0.97349 & 0.97344 \pm 0.00016 \\
\( V_{td} \) & 0.0412 & 0.0411 & 0.0412^{+0.0014}_{-0.0005} \\
\( V_{tb} \) & 0.00342 & 0.0110 & 0.00807^{+0.00029}_{-0.00031} \\
\( V_{ub} \) & 0.0412 & 0.0398 & 0.0404^{+0.0005}_{-0.0005} \\
\( V_{tb} \) & 1 & 0.999147 & 0.999146^{+0.000021}_{-0.000066} \\
\hline
\( J \) & 2.90 \times 10^{-5} & 2.94 \times 10^{-5} & (2.96^{+0.20}_{-0.16}) \times 10^{-5} \\
\hline
\( \delta \) & 69.9^\circ & 68^\circ & 68^\circ \\
\hline
\end{tabular}

Table IV. Wolfenstein, MI textures and experimental values of CKM parameters.

The experimental values of the CKM magnitudes and the Jarlskog invariant are taken from Ref. \( \text{[10]} \).

As can be seen, the quark masses and the CKM matrix obtained from these textures are in excellent agreement with the experimental data. The agreement with the experimental data is as good as in the models of Refs. \( \text{[6]} \) and better than, for example, those in Refs. \( \text{[7]} \),

## III. LEPTON SECTOR

We extend the texture we used for the down quarks to the charged leptons:

\[ M_l = \frac{v}{\sqrt{2}} \begin{pmatrix} x_1 \lambda^8 & y_1 \lambda^6 & 0 \\ 0 & y_2 \lambda^5 & 0 \\ 0 & 0 & z_3 \lambda^3 \end{pmatrix}. \quad (18) \]

Therefore, \( M_l M_l^T \) can be approximately diagonalized by a rotation matrix \( R_l \) according to:
\[ R_l^T M_l R_l \approx \begin{pmatrix} m_e^2 & 0 & 0 \\ 0 & m_\mu^2 & 0 \\ 0 & 0 & m_\tau^2 \end{pmatrix} \]

\[ R_l \approx \begin{pmatrix} \cos \gamma & \sin \gamma & 0 \\ -\sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (19) \]

The charged lepton masses are approximately:

\[ m_e \approx x_1 \lambda^8 \sqrt{2}, \quad m_\mu \approx y_2 \lambda^5 \sqrt{2}, \quad m_\tau \approx z_3 \lambda^3 \sqrt{2}. \quad (20) \]

In order to extend the MI textures to the lepton sector we also consider a neutrino structure, within a type I seesaw implementation, with a 6 × 6 neutrino mass matrix:

\[ M_\nu = \begin{pmatrix} 0_{3\times3} & M_\nu^D \\ (M_\nu^D)^T & M_R \end{pmatrix}, \quad (21) \]

\[ M_\nu^D = \begin{pmatrix} 0 & \lambda^{(v)}_{12} \sqrt{2} & \lambda^{(v)}_{13} \sqrt{2} \\ \lambda^{(v)}_{21} \sqrt{2} & 0 & \lambda^{(v)}_{23} \sqrt{2} \\ \lambda^{(v)}_{31} \sqrt{2} & \lambda^{(v)}_{32} \sqrt{2} & 0 \end{pmatrix}, \]

\[ M_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix} = \begin{pmatrix} w^{-1} & 0 & 0 \\ 0 & p^{-1} & 0 \\ 0 & 0 & q^{-1} \end{pmatrix}. \quad (22) \]

With \((M_R)_{ij} \gg v\), the light neutrino mass matrix is given by the type I seesaw formula:

\[ M_\nu = M_\nu^D M_R^{-1} (M_\nu^D)^T \begin{pmatrix} c^2p & 0 & cdp \\ 0 & a^2w & abw \\ cdp & abw & wb^2 + pd^2 \end{pmatrix}. \quad (23) \]

Varying the parameters \(A, B, C, D\) and \(\gamma\) we fitted \(\Delta m^2_{21}, \Delta m^2_{31}\) (note that we define \(\Delta m^2_{ij} = m_i^2 - m_j^2\)), \(\sin^2 \theta_{12}, \sin^2 \theta_{13}\) and \(\sin^2 \theta_{23}\) to the experimental values \[^{12}\] in Table \(V\) for the normal hierarchy neutrino mass spectrum. The best fit result is:

\[ \Delta m^2_{21} = 7.62 \times 10^{-5} eV^2, \quad \Delta m^2_{31} = 2.55 \times 10^{-3} eV^2, \]

\[ m_{\nu_1} = 0, \quad m_{\nu_2} \approx 9 meV, \quad m_{\nu_3} \approx 50 meV, \quad \gamma \approx -0.35 \pi, \]

\[ \sin^2 \theta_{12} = 0.32, \quad \sin^2 \theta_{13} = 0.0246, \quad \sin^2 \theta_{23} = 0.613, \]

\[ A \approx 3.35 \times 10^{-2} eV, \quad B \approx 2.17 \times 10^{-2} eV, \]

\[ C \approx 5.38 \times 10^{-3} eV, \quad D \approx 3.60 \times 10^{-3} eV. \quad (24) \]

Comparing Eq \((24)\) with Table \(V\) we see that the mass squared splittings \(\Delta m^2_{21}\) and \(\Delta m^2_{31}\) and mixing parameters \(\sin^2 \theta_{12}, \sin^2 \theta_{13}\) and \(\sin^2 \theta_{23}\) are in excellent agreement with the experimental data. Note that here we considered all leptonic parameters to be real for simplicity, but a non-vanishing CP phase in the PMNS mixing matrix can be generated by making e.g. \(y_1\) or \(y_2\) complex.

| Parameter | \(\Delta m^2_{21}(10^{-5} eV^2)\) | \(\Delta m^2_{31}(10^{-3} eV^2)\) | \(\sin^2 \theta_{12}\) | \(\sin^2 \theta_{13}\) | \(\sin^2 \theta_{23}\) |
|-----------|---------------------------------|---------------------------------|----------------|----------------|----------------|
| Best fit  | 7.62                           | 2.55                            | 0.320          | 0.613          | 0.0246         |
| 1σ range  | 7.43–7.81                      | 2.46–2.61                       | 0.303–0.336    | 0.573–0.635    | 0.0218–0.0275  |
| 2σ range  | 7.27–8.01                      | 2.38–2.68                       | 0.29–0.35      | 0.38–0.66      | 0.019–0.030    |
| 3σ range  | 7.12–8.20                      | 2.31–2.74                       | 0.27–0.37      | 0.36–0.68      |                |

Table V. Experimental ranges of leptonic mixing parameters from \[^{12}\] for the case of normal hierarchy.

### IV. CONCLUSIONS

In this paper we studied two sets of textures. We started with a simple modification of the Fukuyama-Nishiura texture and showed that the extra parameter makes it viable in the quark sector. We then considered mixing inspired textures based on the idea that the Cabibbo mixing originates from the down sector whereas the up sector contributes to the remaining mixing angles. From the mixing inspired textures we are able to obtain a very good fit to the quark masses and CKM mixing parameters. Finally we extended the mixing inspired texture to the leptons: by reusing the down quark texture in the charged lepton sector, and proposing a compatible texture for the neutrinos, such that we obtain the observed leptonic mass and mixing parameters.
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