Back analysis of mechanical parameters of rock masses based on multi-point time-dependent monitoring data

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Abstract. The physical and mechanical parameters of rock masses are important factors that affect the numerical simulation and safety evaluation of geotechnical engineering. Due to the discontinuity and heterogeneity of rock mass, the parameters obtained by laboratory test and field test can not represent the actual situation. With the development of computer technology, the parameter identification method based on machine learning provides a new way to solve the above problems. Aiming at the rheological model based on thermodynamics with internal state variables, a back analysis method of mechanical parameters based on IAGA-BP algorithm is proposed. The sensitivity of deformation parameters, damage effect parameters and viscoplastic parameters involved in the model are analyzed using range analysis method. And the parameters which have great influence on the deformation of surrounding rock are selected to reduce the dimension of output layer. The displacement corresponding to the representative time point are selected as the input layer neuron. A numerical model is established to finely simulate the driving process of TBM, which is used to generate training samples and test samples. BP, GA-BP, PSO-BP, APSO-BP and SVM are also used to inverse the parameters of surrounding rock. Two evaluation metrics are used to evaluate the performance of above algorithms, including prediction accuracy and stability. The method is applied to inverse parameters of surrounding rock of a tunnel. The results show that the time-dependent deformation curve of surrounding rock obtained by BP neural network optimized by improved adaptive genetic algorithm (IAGA-BP) fits well with the monitoring curve (the average prediction error is 3.8%, and the prediction stability is 16.3%), which is much better than other algorithms.

1. Introduction

Numerical simulation is a common method to evaluate the safety of rock masses in engineering. How to invert the mechanical parameters of rock masses according to the displacement time-history curve of multiple monitoring points is one of the key problems. In recent years, the emerging machine learning algorithms, which can well represent the nonlinear mechanical behavior of complex rock masses, provides a wide application prospect to invert the parameters of surrounding rock.

Taking a single-track railway tunnel in alpine area as the engineering background, Zhao et al. [1] constructed BP neural network, and inverted the mechanical parameters of surrounding rock by using the convergence displacement of vault and arch waist, so as to provide reference for later construction and lining optimization design. SunY et al. [2] combined BP neural network with the elitist non-dominated sorting genetic algorithm (NSGA-II) and applied it to the prediction of slope excavation on the right bank of a hydropower station, showing a good agreement between the predicted trend and the measured results. In order to overcome the disadvantage that the PSO algorithm is easy to fall into the local minimum in the later stage, Tian [3] embedded the simplex method into the PSO algorithm and
applied it to the inversion of elastic modulus of concrete dams. Literature [4,5] introduced fast strategy, direct search strategy, active learning strategy and neighborhood extremum strategy to optimize the PSO algorithm. Literature [6,7] applied support vector machine (SVM) to the parameter inversion of deep foundation pit and concrete gravity dam, which proves its advantage in small samples. At present, the majority of literatures either use the displacement convergence of multiple monitoring points for inversion without considering the time-dependent deformation, or only use the aging deformation of a single monitoring point for inversion. The parameter inversion considering multi-point time-dependent monitoring data still needs further study.

Aiming at the rheological model based on thermodynamics with internal state variables, a back analysis method of rock masses mechanical parameters based on IAGA-BP is proposed in this paper. The range analysis method is used to analyze the sensitivity of deformation parameters, damage effect parameters and viscoplastic parameters. And the parameters which have great influence on surrounding rock deformation are selected to reduce the dimension of output layer. Based on the numerical simulation of TBM tunneling process, learning samples and test samples are constructed. BP, GA-BP, PSO-BP (including its improved type) and SVM are also used to inverse the surrounding rock parameters. The prediction accuracy and stability of the above algorithms for test samples are analyzed and compared.

2. Inversion method based on IAGA-BP algorithm

The flow chart of inversion method based on IAGA-BP (including other comparison algorithms) is shown in Figure 1. The process consists of three parts: algorithm, numerical simulation and field monitoring. Numerical simulation is used to construct training samples and test samples. The data obtained from the field monitoring is used to construct the objective function to evaluate the similarity between the samples and the actual situation. When the objective function converges, the algorithm will output the inversion results.

![Figure 1. The flow chart of inversion method based on IAGA-BP](image-url)
2.1. BP neural network

BP neural network is a kind of forward neural network, which consists of two parts: the forward transmission of information and the back propagation of error. If there is an error between the predicted value and the expected value, the gradient descent method is used to back propagate the error and iteratively modify the weight and threshold until the predetermined goal is achieved. The nonlinear mapping relationship between creep deformation and parameters of surrounding rock is complex. In order to improve the inversion accuracy, double hidden layer neural network is adopted, as shown in Figure 2. Because the number of hidden layer neurons has a great influence on the prediction accuracy, we combine the empirical formula and trial & error method to determine the optimal number of hidden layer neurons. When the number of neurons in hidden layer 1 is 16 and the number of neurons in hidden layer 2 is 15, the average prediction error is the lowest.

![Figure 2. Topological structure of double hidden layer BP neural network](image)

2.2. Improved adaptive genetic algorithm

Standard genetic algorithm has good global optimization performance, but it is also easy to premature convergence[8]. In order to solve this problem, Srinivas M et al. [9] proposed an improved genetic algorithm (AGA) in which the crossover probability and mutation probability change adaptively with the adaptive value. However, the algorithm overprotects the better individuals, which makes the excellent individuals evolve slowly. Furthermore, it is also easy to fall into local convergence. Chen et al. [10] further improved it and proposed an improved adaptive genetic algorithm (IAGA). The average value of the true fitness of a certain generation (the reciprocal of the absolute error of the training sample) is expressed by $EX$. The discrete degree of the true fitness is expressed by $DX$.

$$EX = \frac{f_{avg}}{N} = f_1 + f_2 + f_3 + \cdots + f_N$$

(1)

$$DX = \frac{f_1^2 + f_2^2 + f_3^2 + \cdots + f_N^2}{N} - f_{avg}^2$$

(2)

Define similarity coefficient $\varphi$ [11]:

$$\varphi = \frac{EX + 1}{\sqrt{DX}}$$

(3)

where $N$ is the number of individuals and $f_i (i = 1, 2, \cdots, N)$ is the true fitness value of the individual. On the basis of adaptive genetic algorithm, an improved crossover and mutation operator combined with similarity coefficient $\varphi$ is given as follows:

$$P_e = \begin{cases} 
\frac{k_1 - k_2}{1 + e^{-\varphi}} \times \frac{f_{max} - f'}{f_{max} - f_{avg}} + k_3 & f' \geq f_{avg} \\
k_4 & f' < f_{avg}
\end{cases}$$

(4)
\[
P_m = \begin{cases} 
  k_5 \frac{k_5}{k_5} \frac{f_{\text{max}} - f}{f_{\text{max}} - f_{\text{avg}}} + k_7 & \text{if } f \geq f_{\text{avg}} \\
  k_8 & \text{if } f < f_{\text{avg}}
\end{cases}
\]

where \( P_c \) is crossover probability and \( P_m \) is mutation probability. \( f' \) is the larger value of true fitness of cross individuals and \( f \) is the true fitness value of variation individual. \( f_{\text{avg}} \) is the average true fitness of the population and \( f_{\text{max}} \) is the maximum true fitness of the population. \( k_i \) \((i=1,2,\cdots,8)\) are adjustable parameters.

3. Constitutive model and parameter selection

3.1. The rheological model based on thermodynamics with internal state variables

Due to the high in-situ stress and complex structure of deep rock mass, it is easy to produce large extrusion deformation after face excavation, showing a strong time effect. In order to reasonably simulate the aging deformation of surrounding rock during TBM excavation, the paper adopts the rheological model based on thermodynamics with internal state variables [12].

The thermodynamic force conjugate with viscoplastic internal variable \( \lambda(\lambda_1, \lambda_2) \) and \( \chi \) is:

\[
f^{vp}_1 = \sqrt{J_2}
\]

\[
f^{vp}_2 = (1 + b \chi)(a I_1 + \sqrt{J_2})
\]

\[
f_s = b \lambda_2 (a I_1 + \sqrt{J_2})
\]

where \( a \) and \( b \) are material parameters (the same below). \( J_2 \) is the second invariant of stress bias. \( I_1 \) is the first invariant of stress tensor.

The viscoplastic strain rate equation is:

\[
\varepsilon^{vp}_m = a[(1+b \chi) \dot{\lambda}_2 + b \dot{\lambda}_2 \dot{\chi}]
\]

\[
\varepsilon^{vp}_{ij} = \left[ \dot{\lambda}_1 + (1+b \chi) \dot{\lambda}_2 + b \dot{\lambda}_2 \dot{\chi} \right] \frac{s_{ij}}{2 \sqrt{J_2}}
\]

where \( \varepsilon^{vp}_m \) is viscoplastic volumetric strain. \( \varepsilon^{vp}_{ij} \) is viscoplastic deviatoric strain.

The evolution equation of internal variables is:

\[
\dot{\lambda}_1 = \frac{1}{\eta_{p1}} \left( f^{vp}_{1} - h \lambda_1 \right)
\]

\[
\dot{\lambda}_1 = \kappa_{p2} \left( f^{vp}_{2} - \frac{R}{R} \right)^p
\]

\[
\dot{\chi} = \kappa_{p3} \exp(m \chi) \left( \frac{f_s}{R} \right)^2 \text{sign}(\dot{\lambda}_1)
\]

\[
\langle x \rangle = \begin{cases} 
  x & x > 0 \\
  0 & x \leq 0
\end{cases}
\]

where \( \eta_{p1}, \kappa_{p2} \) and \( \kappa_{p3} \) are viscosity coefficients. \( p, h \) and \( R \) are material constant. \( m \) is the equation parameter, which has no physical meaning and only makes the calculation stable. \( \text{sign}(x) \) is the sign function.
3.2. Parameter selection
The rheological model based on thermodynamics with internal state variables proposed by Zhang [12] involves as many as 11 parameters. It is of great significance to screen the parameters of the model because too many parameters will reduce the accuracy of inversion.

The deformation modulus $E$ is usually measured by indoor uniaxial compression tests. However, the mechanical properties of rock masses in the construction site are often different from those tested in the laboratory. Therefore, the deformation modulus $E$ should be regarded as one of the mechanical parameters to be inverted. Poisson's ratio can be regarded as a known number in the process of inversion because the variation range of it is small. Due to the short duration of monitoring and measurement in the construction site, it can be considered that the surrounding rock has not yet entered the stage of accelerated creep, that is, the damage effect of rock masses does not need to be considered. Three viscoplastic parameters, i.e., $h$, $\eta_{p1}$, $R$, are taken as one of the parameters to be inversed using the range analysis method, as shown in Table 1.

To sum up, the physical and mechanical parameters of rock masses to be inversed are $E$, $h$, $\eta_{p1}$, $R$.

| Factor | $\eta_{p1}$ (GPa·s) | $\sigma'$ (MPa) | $R$ (MPa) | $h$ (GPa) | $\kappa_{p2}$ (s$^{-1}$) | $p$ |
|-------|------------------|----------------|---------|--------|-------------------|----|
| $K_{ij}$ | 198.02 | 188.48 | 198.96 | 199.21 | 186.03 | 188.33 |
| $K_{3j}$ | 194.14 | 189.17 | 191.57 | 193.59 | 188.44 | 188.69 |
| $K_{4j}$ | 187.87 | 188.58 | 186.71 | 188.72 | 188.63 | 188.87 |
| $K_{5j}$ | 183.08 | 188.70 | 183.47 | 182.89 | 189.84 | 188.46 |
| $R_{ij}$ | 179.42 | 187.60 | 181.82 | 178.12 | 189.59 | 188.18 |

Factor order: $h > \eta_{p1} > R > \kappa_{p2} > \sigma' > p$

4. Engineering example and result analysis

4.1. Project overview and monitoring layout
A highway tunnel in southwest China is mainly constructed by double shield TBM, with the total length about 4.78km and the diameter of 9.13m. The average buried depth of the tunnel is more than 300m and the maximum principal stress is more than 30Mpa, which is a typical high crustal stress tunnel. During the shutdown and maintenance of TBM, an advanced horizontal drill is used to drill a small angle hole (inclination angle $14^\circ$) along the tunnel axis. 8 groups of height difference displacement sensors are put into the hole and grouted[13]. The distribution of the height difference displacement sensors is shown in Figure 3.

![Figure 3. Layout of height difference displacement sensors](image-url)
4.2. Numerical model
According to the tunnel design information, the numerical model is established to finely simulate the TBM tunneling process and the support effect of shield and segment. The total number of grid elements is 376000 and the number of nodes is 387941.

![Numerical model of the tunnel](image)

**Figure 4.** Numerical model of the tunnel

4.3. Results
Considering the inversion accuracy and efficiency, we constructed 111 samples by orthogonal test, 10% of which were used for verification and 5% for testing. Inversion results show that the average prediction error of IAGA-BP is 3.8% and the stability is 16.3%, as shown in Figure 5–8. Compared with other algorithms, the prediction error and stability are greatly improved, as shown in Table 2. However, there are still errors between the inversion curve and the monitoring curve, which are speculated to be caused by the following three reasons:

1. There are errors in the monitoring data of the construction site. The height difference displacement sensors take the farthest end as the displacement reference point, while the displacement reference point itself has deformation.

2. Rock mass is discontinuous, heterogeneous and anisotropic, but it may not be considered so comprehensively in the process of calculation. The properties of the surrounding rock in the first half of the tunnel test section are good, while that in the latter half are broken. There is a sudden change in lithology, while the numerical model assumes that the surrounding rock is homogeneous.

3. IAGA-BP based inversion method has its own prediction error. Furthermore, due to the limitation of calculation time cost, the number of samples is small, which limits the prediction accuracy.

![Fitness evolution curve (IAGA-BP)](image)

**Figure 5.** Fitness evolution curve (IAGA-BP)

![Test sample error (IAGA-BP)](image)

**Figure 6.** Test sample error (IAGA-BP)
Figure 7. Percentage of prediction error (IAGA-BP)

Figure 8. Comparison of predicted value and expected value (IAGA-BP)

Table 2. Inversion accuracy and stability of different algorithms (test samples)

| Evaluation         | Algorithm | IAGA-BP | BP  | GA-BP | PSO-BP | APSO-BP | SVM |
|--------------------|-----------|---------|-----|-------|--------|---------|-----|
| Average prediction | 3.8       | 24.4    | 5.5 | 6.5   | 5.9    | 10.0    |     |
| Stability (%)      | 16.3      | 267.8   | 93.9| 74.2  | 48.7   | 17.0    |     |

Figure 9. Comparison between inversion deformation curve and monitoring curve

5. Conclusion

In this paper, an inverse analysis method of rock mass mechanical parameters based on IAGA-BP algorithm is proposed. Then, the parameters which have great influence on surrounding rock deformation are selected. Relying on a tunnel project in southwest China, the TBM tunneling process is
simulated in detail, and the learning samples and test samples are constructed. The IAGA-BP inversion results are analyzed and compared with BP, GA-BP, PSO-BP, APSO-BP, SVM and other algorithms. The conclusion of the study are as follows:

1. The deformation of surrounding rock in transitional creep stage and steady creep stage is mainly sensitive to deformation parameters $E$ and viscoplastic parameters $h$, $\eta_p$ and $R$. In order to reduce the difficulty of inversion and improve the accuracy of inversion, only the above four parameters can be inverted.

2. The prediction accuracy and stability of the inversion method based on IAGA-BP are verified, with the average prediction error $3.8\%$ and the prediction stability $16.3\%$, which is much better than other algorithms. The time-dependent deformation curve of surrounding rock obtained by inversion fits well with the monitoring curve, which proves that the method is feasible.

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