CDCC studies on clustering physics

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Abstract. The continuum-discretized coupled-channels method (CDCC) has successfully been applied to studies on various reactions involving unstable nuclei. The four-body CDCC method is a new method to calculate three-body breakup continuum in a projectile, which gives an accurate analysis for four-body breakup systems. The eikonal reaction theory is a new approach to calculate inclusive breakup cross sections such as neutron removal cross sections. Moreover, CDCC is a useful tool for evaluation of nuclear data, which are important for nuclear engineering.

1. Introduction
Study on unstable nuclei is one of the main subject in nuclear physics. Breakup reactions involving such nuclei have played a key role in investigating the exotic properties. The observables such as breakup and neutron removal cross sections reflect information of the ground and resonance continuum states. Furthermore breakup reactions are also useful for searching the cluster structure of nuclei. Therefore in order to understand the exotic properties from the observables, an accurate analysis for describing the breakup mechanism is highly desirable.

The continuum-discretized coupled-channels method (CDCC) [1, 2, 3] is one of the most reliable methods for treating breakup processes in exclusive reactions such as elastic scattering, elastic-breakup reactions, and transfer reactions. CDCC has been proposed at first to solve three-body scattering problems, in which a projectile breaks up into two constituents. Recently, there are two important extensions for CDCC. One is a new method to calculate three-body breakup continuum in a projectile, which gives an accurate analysis for four-body breakup systems. This new method is called four-body CDCC [4, 5, 6, 7]. The other is a new approach to calculate inclusive breakup cross sections such as neutron removal cross sections. In the new approach called the eikonal reaction theory (ERT) [8], both nuclear and Coulomb breakup processes are consistently treated by CDCC without making the adiabatic approximation.

In SOTANCP3, we reviewed recent studies of CDCC. One is analyses of ⁶He breakup reactions and another is an application of CDCC to nuclear engineering. These works have been already reported in the CDCC review paper [3].

2. Analysis of ⁶He breakup reactions
⁶He is known as a two-neutron halo nucleus, and the structure can be described by an n + n + ⁴He three-body model. In the scattering of ⁶He, the projectile easily breaks up into three constitutes. This reaction is treated as a four-body system with three-body projectile and a
target nucleus. To describe the four-body breakup system, we adopt the four-body CDCC method.

For the $^6$He breakup reaction, we set a four-body model Hamiltonian as

$$H = T_R + U_{nT} + U_{xT} + U_{aT} + h_P,$$

where $T_R$ is a kinetic energy for the relative coordinate $R$ between the center-of-mass of $^6$He and a target, $U_{xT}$ is an optical potential between constituents $x$ in $^6$He and a target, and $h_P$ is an internal model Hamiltonian of $^6$He. Here, $U_{aT}$ includes the Coulomb interaction between $\alpha$ and a target.

In the four-body CDCC method, the total wave function is expanded in terms of bound and discretized continuum states of $^6$He,

$$\psi_{\text{CDCC}}^{(+)}(\xi, R) = \Phi_0(\xi)\chi_0^{(+)}(R) + \sum_{\nu} \Phi_\nu(\xi)\chi_\nu^{(+)}(R).$$

Here $\Phi_0$ and $\Phi_\nu$ represent bound and discretized continuum wave functions with the internal coordinate $\xi$, which is represented by a set of Jacobi coordinates, $\xi = (r, y)$. Relative wave functions between $^6$He and target, $\chi_0^{(+)}$ and $\chi_\nu^{(+)}$, are solved with the outgoing boundary condition. Momenta conjugate to coordinates $R$ and $(y, r)$ are defined by $P$ and $(p, k)$, respectively. To obtain the discretized continuum wave functions, the pseudo-state discretization method \cite{9, 10, 11, 12} is applied. In the pseudo-state discretization, the internal Hamiltonian of $^6$He is diagonalized with the $L^2$-type basis functions. In the present analysis, we adopt the Gaussian basis functions \cite{13}. The details of the four-body CDCC method are shown in Ref. \cite{6}.

The four-body $T$-matrix element to a breakup state with $(p, k)$ can be described by

$$T_\varepsilon(p, k, P) = \langle \psi_\varepsilon^{(-)}(p, k)\chi_\varepsilon^{(-)}(P)|U|\Psi^{(+)}\rangle_{\xi R}$$

with

$$U = U_{nT} + U_{xT} + U_{aT} - V_{PT}^{\text{Coul}}$$

where $V_{PT}^{\text{Coul}}$ is the Coulomb interactions between $^4$He and a target with the arguments are replaced by $R$. The final-state wave functions, $|\psi_\varepsilon^{(-)}(p, k)\rangle$ and $|\chi_\varepsilon^{(-)}(P)\rangle$, with the incoming boundary condition are defined by

$$\left[T_R + V_{PT}^{\text{Coul}}(R) - (E_{\text{tot}} - \varepsilon)\right]|\chi_\varepsilon^{(-)}(P)\rangle = 0,$$

$$[h_P - \varepsilon]|\psi_\varepsilon^{(-)}(p, k)\rangle = 0,$$

where $E_{\text{tot}} - \varepsilon = (hP)^2/(2\mu_R)$ and $\varepsilon = (hp)^2/(2\mu_\xi) + (hP)^2/(2\mu_\varepsilon)$ for reduced masses $\mu_R$ and $\mu_\xi$ of coordinates $R$ and $\xi$, respectively. Inserting the approximate complete set defined as

$$1 \approx |\Phi_0\rangle\langle\Phi_0| + \sum_{\nu} |\Phi_\nu\rangle\langle\Phi_\nu|,$$

we can find that the $T$-matrix element is well approximated by

$$T_\varepsilon(p, k, P) \approx \sum_{\nu \neq 0} \langle \psi_\varepsilon^{(-)}(p, k)|\Phi_\nu\rangle T_\varepsilon,$$

where $T_\nu$ is the $T$-matrix element to a $\nu$-th discrete breakup state $\Phi_\nu$ with an eigenenergy $\varepsilon_\nu$,

$$T_\nu = \langle \Phi_\nu|\chi_\varepsilon^{(-)}(P)|U|\Psi^{(+)}\rangle_{\text{CDCC}}.$$
Here Eq. (8) is derived by replacing $P$ by $P_n$ in $\chi^{(-)}(P)$. The $T_n$ are obtainable by CDCC, but it is quite hard to calculate the smoothing factor $\langle \psi^{(-)}(p, k) | \Phi_n \rangle$ directly with either numerical integration [7, 14, 15]. Hence, we propose a new way of obtaining the differential cross section with respect to $\varepsilon$ without calculating the smoothing factor [16].

Using the exact $T$ matrix, The differential cross section as a function of $\varepsilon$ is calculated by

$$
\frac{d^2 \sigma}{d\varepsilon d\Omega_P} = \int dp' dp'' \delta(\varepsilon - \varepsilon') |T^{(-)}(p', k', P')|^2 = \frac{1}{\pi} \text{Im} \mathcal{R}(\varepsilon). 
$$

Here $\mathcal{R}(\varepsilon)$ is the generalized response function defined as

$$
\mathcal{R}(\varepsilon) = \int d\varepsilon d\varepsilon' \langle \Psi^{(+)}(\varepsilon') | U | \chi^{(-)}(\varepsilon) \rangle \mathcal{R}(\varepsilon, \varepsilon, \varepsilon') \langle \chi^{(-)}(\varepsilon) | U | \Psi^{(+)}(\varepsilon') \rangle \mathcal{R}.
$$

with the three-body Green’s function of $^6$He

$$
G^{(-)}(\varepsilon, \varepsilon, \varepsilon') = \lim_{\eta \to +0} \langle \varepsilon | \frac{1}{\varepsilon - h_P - i\eta} | \varepsilon' \rangle.
$$

For the sides of $G^{(-)}$ in Eq. (11), we insert an approximate complete set represented by Eq. (7) and then $\mathcal{R}$ is calculated as

$$
\mathcal{R}(\varepsilon) = \sum_{\nu} \sum_{\nu'} \langle \Psi^{(+)} | U | \chi^{(-)} | \Phi_{\nu} \rangle \langle \Phi_{\nu} | G^{(-)}(\varepsilon, \varepsilon, \varepsilon') | \Phi_{\nu'} \rangle \langle \Phi_{\nu'} | \chi^{(-)} | U | \Psi^{(+)} \rangle,
$$

approximated by

$$
\mathcal{R}(\varepsilon, \varepsilon, \varepsilon') \approx \sum_{\nu} \sum_{\nu'} \mathcal{T}^{\nu}_{\nu'} \langle \Phi_{\nu} | G^{(-)}(\varepsilon, \varepsilon, \varepsilon') | \Phi_{\nu'} \rangle T_{\nu}.
$$

In Eq. (13), there is no the smoothing factor, as expected. Furthermore, the propagator $G^{(-)}$ operates only on spatially damping functions $\Phi_{\nu}$, so that the calculation of $\langle \Phi_{\nu} | G^{(-)} | \Phi_{\nu'} \rangle$ becomes feasible as show below.

The complex scaling method (CSM) [17, 18, 19, 20] is now applied to evaluating $\langle \Phi_{\nu} | G^{(-)} | \Phi_{\nu'} \rangle$. The scaling transformation operator $C(\theta)$ and its inverse are defined by

$$
\langle \mathbf{r}, \mathbf{y} | C(\theta) | f \rangle = e^{i3\theta} f(\mathbf{r} e^{i\theta}, \mathbf{y} e^{i\theta}),
$$

$$
\langle f | C^{-1}(\theta) | \mathbf{r}, \mathbf{y} \rangle = \{e^{-i3\theta} f(\mathbf{r} e^{-i\theta}, \mathbf{y} e^{-i\theta})\}^*.
$$

Using the operators, one can get

$$
\langle \Phi_{\nu} | G^{(-)}(\theta) | \Phi_{\nu'} \rangle = \langle \Phi_{\nu} | C^{-1}(\theta) G^{(-)}(\theta) C(\theta) | \Phi_{\nu'} \rangle,
$$

where

$$
G^{(-)}(\theta) = \lim_{\eta \to +0} \frac{1}{\varepsilon - \varepsilon^\theta - i\eta}.
$$

with $h_P^\theta = C(\theta) h_P C^{-1}(\theta)$. When $-\pi < \theta < 0$, the scaled propagator $\langle \xi | G^{(-)}(\theta) | \xi' \rangle$ is a damping function of $\xi$ and $\xi'$. It should be noted that although the scaling angle in general calculations with CSM has been taken as positive, the angle in the present situation becomes negative since $G^{(-)}$ has the incoming boundary condition. Hence, it can be expanded with $L^2$-type basis functions with high accuracy:

$$
G^{(-)}(\theta) \approx \sum_i \frac{\phi_i^\theta}{\varepsilon - \varepsilon_i^\theta},
$$
where $\phi_i^\theta$ is an $i$-th eigenstate obtained by diagonalizing $h_P^\theta$ in a modelspace spanned by $L^2$-type basis functions:

$$
\langle \tilde{\phi}_i^\theta | h_P^\theta | \phi_i^\theta \rangle = \varepsilon_i^\theta \delta_{ii'}.
$$

(19)

In virtue of CSM, thus, we do not need to evaluate exact three-body continuum state $\psi_\xi^{(-)}(k, p)$ to obtain $\langle \Phi_\nu | G^{(-)} | \Phi_\nu' \rangle$.

Inserting Eq. (18) into Eq. (11) through Eq. (16) leads to a useful form of

$$
\frac{d^2 \sigma}{d\varepsilon d\Omega_P} \approx \frac{1}{\pi} \text{Im} \sum_i T_i^\theta \tilde{T}_i^\theta
$$

(20)

with

$$
\tilde{T}_i^\theta \equiv \sum_{\nu'} \langle \tilde{\phi}_i^\theta | C(\theta) | \Phi_\nu' \rangle T_{\nu'\nu},
$$

(21)

$$
T_i^\theta \equiv \sum_{\nu} T_{\nu}^* \langle \Phi_{\nu'} | C^{-1}(\theta) | \phi_i^\theta \rangle.
$$

(22)

The principal result of this paper is that $C(\theta)$ and $C^{-1}(\theta)$ operate only on the spatially damping function $\Phi_\nu$. This makes the calculation of $\langle \tilde{\phi}_i^\theta | C(\theta) | \Phi_\nu' \rangle$ and $\langle \Phi_\nu | C^{-1}(\theta) | \phi_i^\theta \rangle$ feasible and makes it possible the convergence of $d^2 \sigma / d\varepsilon d\Omega_P$ with respect to extending the modelspace, as shown below. In other words, $C(\theta)$ and $C^{-1}(\theta)$ are not allowed to act on a non-damping function such as the plane wave, since the scaled function diverges asymptotically in this case.

Figure 1. Comparison of the breakup cross section calculated by CDCC (solid line) with experimental data for (a) $^6\text{He} + ^{12}\text{C}$ scattering at 240 MeV/nucleon and (b) $^6\text{He} + ^{208}\text{Pb}$ scattering at 240 MeV/nucleon [16]. The dot-dashed, dotted, and dashed lines correspond to contributions of $0^+$, $1^-$, and $2^+$ breakup, respectively. The experimental data are taken from Ref. [21].

We apply the new smoothing approach with CSM to analyses of $^6\text{He}$ breakup cross sections on $^{12}\text{C}$ and $^{208}\text{Pb}$ at 240 MeV/nucleon [21]. Figure 1 shows the calculated breakup cross sections compared with the experimental data. These data have already been analyzed by four-body
distorted wave Born approximation (DWBA) [22] and the eikonal approximation [23]. For $U_{nT}$ and $U_{\alpha T}$ in the reaction on $^{208}$Pb, we take the same potentials used in Ref. [23]. Meanwhile $U_{nT}$ in the case of $^{12}$C is taken from the global nucleon-nucleus potential [24] and the optical potential for $\alpha^{12}$C system is evaluated from the $^{12}$C + $^{12}$C potential at 200 MeV/nucleon [25] by changing the radius parameter from $^{12}$C to $\alpha$. In $^6$He + $^{12}$C scattering at 240 MeV/nucleon, nuclear breakup is dominant, while Coulomb breakup to 1$^-$ continuum is dominant for $^6$He + $^{208}$Pb scattering. For $^{12}$C target, we can see the clear peak around $\varepsilon = 1$ MeV corresponding the $2^+$-resonance of $^6$He. The present theoretical result is consistent with the experimental data except for the peak of the $2^+$-resonance. Similar overestimations are also seen in the results of four-body DWBA. We expect that the overestimation can be partly solved by taking into account of the resolution of the experimental data. For $^{208}$Pb target, the present method underestimates the experimental data at $\varepsilon \gtrsim 2$ MeV. A possible origin of the underestimation are that the inelastic breakup reactions are not included in the present calculation. As mentioned in Ref. [22], the inelastic breakup effect is not negligible, and the elastic breakup cross section calculated with four-body DWBA also underestimates the data. Furthermore, for the present reaction systems, we calculated one- and two-neutron stripping cross sections with ERT. The results will be reported in the forthcoming paper [26].

3. Application of CDCC to nuclear engineering

![Angular distribution of the elastic differential cross section of $n + ^6$Li scattering for incident energies between 7.47 and 24.0 MeV [27]. Experimental data are taken from Refs. [31, 32, 33].](image)

Since CDCC is a fully quantum-mechanical method, it is applicable to reactions at low incident energies, which are important for nuclear engineering. In this field, Lithium isotopes are important for not only tritium breeding material in DT fusion reactors but also a candidate for target material in the intense neutron source of IFMIF. Recently, accurate nuclear data of nucleon induced reactions on $^{6,7}$Li are highly required for incident energies up to 150 MeV.
In Ref. [27], we performed a microscopic analysis of \( n + ^6\text{Li} \) scattering by the three-body CDCC method, in which \(^6\text{Li}\) is described as a \( d + \alpha \) system. As the interaction between \( n \) and \(^6\text{Li}\), we applied a folding model with the JLM effective interaction [30], where the normalization of the imaginary part, \( \lambda_w \), is optimized to reproduce the elastic cross sections. Details of the calculation are shown in Ref. [27]. Figure 2 shows the differential elastic cross sections of \( n + ^6\text{Li} \) for incident energies between 7.47 and 24.0 MeV. One sees that the results of the CDCC calculation (the solid lines) are in good agreement with the experimental data. The dashed lines represent the results of a single-channel calculation without couplings to the breakup states. Thus breakup effects are significant to reproduce the angular distributions of the elastic scattering. For all incident energies, we take \( \lambda_w = 0.1 \) to reproduce the data.

![Graph showing differential elastic cross sections](image)

**Figure 3.** The neutron spectra calculated by CDCC with the JLM interaction comparing with measured data at selected angular points in the laboratory system [27]. The experimental data are taken from Ref. [31].

In Fig. 3, the calculated neutron spectra are compared with the experimental data at selected scattering angles in the laboratory frame and incident energies. Components of the breakup states of \(^1\text{S}, ^1\text{D}, ^2\text{D}, \) and \(^3\text{D}\) are represented by the dash-dotted, dashed, dotted, and thin solid lines, respectively. One sees that the CDCC calculation with the finite resolution of the experimental apparatus [31] gives a good agreement with experimental data in the high neutron energy region. On the other hand, in the low neutron energy region, which corresponds to highly exited states of \(^6\text{Li}\), the calculated cross section considerably undershoot the experimental data. This indicates that experimental data contain contribution from the \((n, 2n)\) process due to a four-body breakup reaction \(^6\text{Li}(n, nnp)\alpha\), as indicated in Ref. [31]. Estimate of these effects with four-body CDCC will be very important.

### 4. Summary

CDCC is one of the most reliable methods for studying on unstable nuclei via the breakup reaction. Recently, we propose the four-body CDCC method that can describe three-body breakup processes of the projectile in the scattering. For analyses of \(^6\text{He}\) reactions, the four-body CDCC well reproduces the breakup cross sections. Thus the four-body CDCC is indispensable for investigating of properties of unstable nuclei.

CDCC with the JLM interaction is expected to be a powerful framework for the data evaluation of the \(^6\text{Li}(n, n')\) reactions. Once the JLM parameter is determined by an analysis of elastic scattering, evaluation of the inelastic cross sections and neutron breakup spectra can
be done with no free adjustable parameters. This is a very important feature of the present framework that enables a quantitative calculation of the cross sections for nuclear engineering studies.

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