Generalizing the Sokolov-Ternov effect in intense laser fields

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Abstract

A consistent description of the radiative polarization for relativistic electrons in intense laser fields is derived by generalizing the Sokolov-Ternov effect in arbitrary field structure. The new form together with the spin-dependent radiation-reaction force provide a complete set of dynamical equations for electron momentum and spin in strong fields. When applied to varying intense fields, e.g. the laser fields, the generalized Sokolov-Ternov effect allows electrons to gain or lose polarization in any directions other than along the magnetic field in the rest frame of the electron. The self-consistent theory shows that highly relativistic electrons can obtain notable polarization in collision with an ultra-intense circularly polarized laser beam.

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**Introduction**

A charged particle with non-zero spin interacts with the external field via the Lorentz equation and the Stern-Gerlach force [1], where the particle can be deflected by the gradient of the magnetic field. For an ultra-relativistic electron in strong fields, when the field strength in its rest frame approaches the Schwinger limit [2], $\gamma$-photon emission [3] and its dependence on the electron’s spin state [4] as well as the consequent spin-dependent radiation-reaction (RR) [5] come into play and alter the electron dynamics. On the other hand, the spin vector evolves according to the Thomas-Bargmann-Michel-Telegdi (T-BMT) equation [6–8]. In the recent years, spin-relevant dynamics becomes a rising interest in the strong field regime [4,5,9–14] while the radiative polarization effect in varying fields, e.g. strong laser fields, is still not fully understood.

The spontaneous polarization of relativistic electrons in the static magnetic field due to the asymmetric spin-flip rates during photon emission is well known as the Sokolov-Ternov (S-T) effect [15]. By modeling the process with spin-flips of an ensemble of electrons in the spin-up/-down states in the static magnetic field, the S-T effect successfully interprets the polarization evolution in storage rings. This effect has been well confirmed and utilized to generate polarized electrons in numerous accelerator facilities [16–20]. The S-T effect was transplanted to a rotating standing wave recently [10], where the magnetic field axis is constant and polarization can build up co-axial to the magnetic field within a few laser periods. For more general consideration, the spin-flip probability rate in arbitrary field was calculated under the locally constant field approximation [4]. Alternatively, a so-called “quantum jump” scenario was adopted to the polarization process [12], by which the spin vector of the test particle falls onto the magnetic axis in the rest frame of the electron after a $\gamma$-photon emission. Both approaches converge when the initial spin orientations are parallel to the magnetic field in the rest frame of the electrons $\mathbf{B}_{\text{rest}}$. However, significant disparity arises for initial spins
perpendicular to \( \mathbf{B}_{\text{rest}} \). In the “spin-flip” approach, spins remain perpendicular during photon emission and no polarization is built up along the \( \mathbf{B}_{\text{rest}} \). The “quantum jump” picture forces the spin to project onto the \( \mathbf{B}_{\text{rest}} \), and the electron loses its initial polarization immediately after one single emission event.

In this article, we generalize the S-T effect in varying strong fields such that electron beams can gain polarization in a certain direction but do not lose polarization in other directions. We construct a self-consistent description of the spin dynamics in the strong field, based on the spin-dependent probability rates given by the quantum-electrodynamics (QED) calculations. The new approach is further applied to the collision process between circularly polarized (CP) lasers and un-polarized electrons. We show that notable longitudinal polarization is built up, which is not revealed in the previous studies. The generalized S-T model can be implemented into the particle-in-cell simulation to account for the polarization effect in laser-plasma interactions.

**Theoretical models**

**a) The generalized Sokolov-Ternov effect**

In the Sokolov-Ternov effect, electrons get polarized along the magnetic field through asymmetric spin-flip process where the electrons tend to be anti-parallel to the magnetic field. The polarization process is dominated by the equation

\[
\frac{d}{dt} N^\uparrow = \frac{dP_{\uparrow\downarrow}}{dt} N^\downarrow - \frac{dP_{\downarrow\uparrow}}{dt} N^\uparrow
\]  

(1)

where \( N^\uparrow, N^\downarrow \) denote the number of electrons in the spin-up/-down state along the chosen axis, e.g. the magnetic field direction in the S-T effect, and \( P \) denotes the probability rate of the transition between the spin-up/-down states. The polarization along the chosen axis is defined by \( \langle \mathbf{s} \rangle_{\text{axis}} = Pol = \frac{N^\uparrow - N^\downarrow}{N^\uparrow + N^\downarrow} \) and we have the equation of polarization
\[
\frac{d}{dt} Pol + (A + B) Pol = A - B
\]  

(2)

where \( A = \frac{dP^I}{dt} \), \( B = \frac{dP^I}{dt} \) are defined for convenience. The solution for the equation is

\[
Pol(t) = \frac{A-B}{A+B} \left[ 1 - \exp \left( -\frac{t}{\tau} \right) \right] + Pol_0 \exp \left( -\frac{t}{\tau} \right)
\]

(3)

where \( \tau = \frac{1}{A+B} \) is the polarization time and \( Pol_0 \) is the initial polarization along the chosen axis.

In general, the above solution applies to any polarization axis other than the axis of magnetic field.

The spin-flip probability rate in arbitrary field due to photon emission is given in Ref. [4] via the spin density matrix method

\[
\frac{dn_{\text{flip}}}{d\psi d\delta} = -\frac{\alpha}{2b} \frac{\delta^2}{1 - \delta} \left\{ \frac{Ai'(z)}{z} - s_\zeta \frac{Ai(z)}{\sqrt{z}} - s_\kappa^2 \left( Ai_1(z) + \frac{Ai'(z)}{z} \right) \right\}
\]

(4)

where \( \psi \) is the phase, \( \delta \) the energy fraction of the emitted photon to the electron energy, \( \alpha \) is the fine structure constant, \( b = (\hbar k^\mu \cdot p_\mu)/m^2 c^2 \) is the energy parameter, \( z = \left[ \frac{\delta}{(1-\delta)\lambda_e} \right]^{2/3} \), \( \lambda_e = e\hbar/m^3 c^4 |F^{\mu\nu} \cdot p_{\mu\nu}| \) is the nonlinear quantum parameter and \( g = 1 + \frac{\delta^2}{2(1-\delta)} \). Here \( Ai \) & \( Ai' \) are the Airy function and its first order derivative, \( Ai_1(y) = \int_y^\infty Ai(x)dx \) and \( s_\zeta \) & \( s_\kappa \) are the components of spin vector on \( B_{\text{rest}} \) and \( k_{\text{rest}} \) (the wave vector of the field in the rest frame). The spin-flip probability was derived based on the locally constant field approximation (LCFA) that treats the external field as constant during the QED-photon emission process. It is applicable when \( a_0^3/\lambda_e \gg 1 \) [21–23] where \( a_0 = \frac{eE_0}{\omega mc^3} \).

Following Eq. (4), we define the spin-flip rates along the magnetic field \( \zeta \), electric field \( \eta \) and the wave vector \( \kappa \) in their pure states (|s| = 1) in the rest frame

\[
A_\zeta = \frac{1}{\sqrt{2}} \int \frac{\alpha}{2b} \frac{\delta^2}{1 - \delta} \left\{ \frac{Ai'(z)}{z} - s_\zeta \frac{Ai(z)}{\sqrt{z}} - s_\kappa^2 \left( Ai_1(z) + \frac{Ai'(z)}{z} \right) \right\} d\delta \cdot \frac{d\psi}{dt}
\]

(5a)

\[
B_\zeta = \frac{1}{\sqrt{2}} \int \frac{\alpha}{2b} \frac{\delta^2}{1 - \delta} \left\{ \frac{Ai'(z)}{z} + s_\zeta \frac{Ai(z)}{\sqrt{z}} - s_\kappa^2 \left( Ai_1(z) + \frac{Ai'(z)}{z} \right) \right\} d\delta \cdot \frac{d\psi}{dt}
\]

(5b)

\[
A_\eta = B_\eta = \frac{1}{\sqrt{2}} \int \frac{\alpha}{2b} \frac{\delta^2}{1 - \delta} \left\{ \frac{Ai'(z)}{z} \right\} d\delta \cdot \frac{d\psi}{dt}
\]

(5c)

\[
A_\kappa = B_\kappa = \frac{1}{\sqrt{2}} \int \frac{\alpha}{2b} \frac{\delta^2}{1 - \delta} \left\{ \frac{Ai'(z)}{z} - \left( Ai_1(z) + \frac{Ai'(z)}{z} \right) \right\} d\delta \cdot \frac{d\psi}{dt}
\]

(5d)
The spin-flip rates given in Ref. [4] indicate that the electron can only get polarized along the direction \( \zeta \). Combining it with Eq. (3) one can find that the electrons can only get depolarized along other directions where we have \( A = B \). Eq. (3) and (5) together describe the evolution of polarization due to photon emission. In the classical limit that \( \chi_e \ll 1 \), our theory reproduces the well-known polarization limit in the S-T effect, i.e. \( (A - B)/(A + B) \), of -0.924 and the polarization time scale of S-T effect \( \tau_{S-T} = \frac{9\sqrt{3}}{15} \frac{h^2}{mme} \frac{\tau}{Y} \chi_e^{-3} \) [24] as shown in Fig. 1. The polarization time in the \( \kappa \) direction \( \tau^\kappa \) is also shown and one can find that \( \tau^\eta = \tau^\zeta \) as \( A^\eta + B^\eta = A^\zeta + B^\zeta \). Disparity emerges when \( \chi_e \) is approaching or going beyond unity, where the polarization time is larger, and the polarization limit declines.

![Fig. 1](image-url) (left axis) The polarization limit \( (A - B)/(A + B) \) (solid). (right axis) The polarization time \( \tau = 1/(A + B) \) divided by \( \gamma \) along the \( \zeta \) direction (dashed), the classical approximation of the S-T effect (dotted) and polarization time in the \( \kappa \) direction (dotted-dashed). The polarization time in the \( \eta \) direction equals that in the \( \zeta \) direction.

One should notice that in Eq. (4) we consider the spin vector \( s \), which is the quantum average of a certain spin state [25]. Following that, the polarization along each axis is derived, representing the average over all possible spin states. A key point in our model is that we deal with polarization instead of individual spins, therefore the results are insensitive to the choice of quantization axis or initial spin.
orientations as they are for previous approaches.

Apart from the radiative polarization effect, the spin vector, as well as the polarization, undergoes precession around an axis in the rest frame of the electron according to the T-BMT equation (see Ref. [26])

\[
\frac{d}{dt} \mathbf{s} = \frac{a_e}{mc} \mathbf{s} \times \left[ \left( a_e + \frac{1}{y} \right) \mathbf{B} - a_e \frac{y}{y+1} (\mathbf{B} \cdot \mathbf{B}) - \left( a_e + \frac{1}{y+1} \right) \mathbf{B} \times \mathbf{E} \right] \tag{6}
\]

where \( a_e \approx 1.16 \times 10^{-3} \) [27] is the anomalous magnetic moment of electron [28], \( \mathbf{B} \) is the magnetic field, \( \mathbf{E} \) is the electric field, \( \mathbf{B} = \mathbf{v}/c \) is the normalized velocity. Eq. (3) and (6) offer a complete description of the spin dynamics. In our case, we consider precession of the polarization vector by replacing the spin vector \( \mathbf{s} \) with \( \mathbf{Pol} \equiv \langle \mathbf{s} \rangle \).

b) Spin-dependent radiation-reaction

The dynamics becomes self-consistent when one includes the spin-dependent radiation-reaction. For the polarization \( \langle \mathbf{s} \rangle \), the radiation probability rate is [4]

\[
\frac{dP^s}{d\psi d\delta} = -\frac{a}{b} \left\{ A_1(z) + g \frac{2A_1(z)}{z} + \langle \mathbf{s} \rangle \delta \frac{A_1(z)}{\sqrt{2}} \right\} \tag{7}
\]

By averaging the radiated photon energy using Eq. (7),

\[
\frac{d\bar{\delta}^s}{d\psi} = \int \delta \frac{dP^s}{d\psi d\delta} d\delta \tag{8}
\]

RR force is thus the recoil of the averaged energy loss from photon emission by momentum conservation. As shown by Eq. (7), radiation energy has a negative dependence on the magnetic component thus the electrons in the anti-parallel state tends to radiate more energy and experience strong RR force.

We would also like to mention that another spin-dependent force, the Stern-Gerlach force, is induced by the gradient of the magnetic field which can be triggered by the strong field of laser pulse. However,
this effect is weak in the considered regime [9] when compared to the spin-dependent RR effect [5]. Therefore, the Stern-Gerlach effect is not included in the following calculations.

c) Numerical methods

The theoretical model based on Eq. (3), (6) and (7) is a complete set of description for spin-dependent electron dynamics. The particle motion between each photon emission event is treated classically following the Lorentz equation \( \frac{dp}{dt} = -e(E + v \times B) \). The radiation-reaction effect is considered by adding a radiation-reaction force to the electron, which is achieved by losing momentum of \( \delta^s \cdot \sqrt{\gamma^2 - 1}mc^2 \cdot \frac{dp}{dt} \Delta t \) at each time step where \( \Delta t \) is the time step. Since we consider polarization, each test electron is assigned with a polarization property to represent the average of the spin vectors for an ensemble of electrons. The evolution of the polarization is calculated by Eq. (3), (5) and (6).

For comparison considerations we also perform QED Monte-Carlo (MC) simulations of RR [29] and radiative spin-dynamics [5] for the “spin-flip”, “quantum jump” and S-T effect. Here each test electron is assigned with a spin vector that evolves according to the T-BMT equation and the “spin-flip” process or the “quantum jump” process.

While our model takes QED average for the photon emission, for an electron beam with many particles, the average in each phase space cell can be a good representation of those treated stochastically.

Results

We then apply our approach to the collision between relativistic electrons and intense laser pulses to evaluate the polarization effect. First, we benchmark our results with the QED-MC simulation by
colliding a linearly polarized laser with electrons polarized along the magnetic field direction of the laser, e.g. y-axis, as shown in Fig. 2(a). The LP laser is approximated by

\[ E = \hat{x}E_x = \hat{x}E_0 \exp \left( -\frac{x^2 + y^2}{w_0^2} \right) \cos(\psi) \cos^2 \left( \frac{\psi}{2N} \right) \]

and \( B = \hat{y}E_x/c \), where \( E_0 \) is the electric amplitude corresponding to \( a_0 = 100 \), \( w_0 = 2\lambda \) is the beam waist, \( \psi \) is the laser phase and \( N = 20 \) is the full length of the pulse in the unit of wavelength (800nm). Radiation-reaction and spin precession are not considered in the benchmarking to avoid stochastic effects induced by MC calculation of RR which is important when \( \chi_e > 0.1 \) [30–32] so that we can explicitly compare the polarization effect. One can clearly see that the polarization evolution of our energy averaged modelling agrees with those of the MC results. The polarization decreases every half period and slightly increases every other half period because the magnetic field of laser is oscillating during the collision, which is consistent with the known S-T effect that electron spins tend to be aligned towards the negative direction of the magnetic field.

For further benchmarking, polarization evolution for the longitudinal polarized electrons are shown in Fig. 2(b). One can see that while previous models are inconsistent with each other, our model agrees with the spin-flip model in terms of \( Pol_x \) and the quantum jump model for \( Pol_y \). On the other hand, the spin-flip model is unable to gain any polarization in the y-direction and the quantum jump model predicts a fast depolarization of \( Pol_x \). The results for the unpolarized electrons are shown in Fig. 2(c) where the quantum jump model agrees well with the generalized S-T model in terms of \( Pol_y \) and all the models show vanishing polarization in \( Pol_x \) and \( Pol_z \) (not shown here). For initialization of the un-polarized electrons in the MC algorithm, unit spin vectors of the test electrons are uniformly scattered to the \( 4\pi \) spaces suggested in Ref. [12]. The results indicate that while previous models depend on specifically chosen initial conditions, e.g. spin orientations with respect to the chosen
quantization axis, the generalized S-T model considering polarization is consistent in handling any initial directions.

**Fig. 2** The polarization during the collision with a LP laser pulse. (a) $Po_{l_y}$ of the electrons initially polarized along the magnetic field of the LP laser (y-axis). The polarization $Po_{l_y}$ of the generalized S-T model (black solid) and the MC results of the S-T (black dashed) formula, spin-flip model (blue dotted) and quantum jump model (red dotted-dashed) coincide with each other. (b) $Po_{l_z}$ (left axis, starting from $Pol = 1$) and $Po_{l_y}$ (right axis, starting from $Pol = 0$) of the electrons initially polarized along the wave vector of the laser (z-axis). The generalized S-T model agrees with the spin-flip model in terms of $Po_{l_z}$ and with the quantum jump model in terms of $Po_{l_y}$. (c) $Po_{l_y}$ of the initially unpolarized electrons.

However, the S-T effect as well as the spin-flip MC model cannot account for the polarization when the magnetic no longer stays co-axial to the polarization direction as the spin vector of the test particle only flips along itself. Our generalized S-T model is capable of handling such situations like in the laser fields where the direction of the magnetic field varies within the scale of laser periods. We evaluate the polarization during the collision between an un-polarized electron of $\gamma_0 = 1000$ and a circularly polarized (CP) laser pulse of $\alpha_0 = 100$. The CP pulse is approximated by the superposition of two LP pulses of

$$E = \hat{x}E_x + \hat{y}E_y = \frac{E_0}{\sqrt{2}} \exp \left[ -\frac{(x^2 + y^2)}{w_0^2} \right] \cos \left( \frac{\psi}{2N} \right) \left[ \hat{x} \sin(\psi) + \hat{y} \sin(\psi - \pi/2) \right]$$

and

$$B_x = -\hat{y}E_y/c, \quad B_y = \hat{x}E_x/c.$$ 

The evolution of the polarization components is shown in Fig. 3 where electron gains longitudinal polarization ($Po_{l_z}$) while the transverse polarization is merely
dragged along the magnetic field via the polarization effect. When T-BMT equation is not included as shown in Fig. 3(b), the electron gains a net polarization due to the non-oscillating magnetic field along z-axis in the rest frame (the black line in Fig. 3(c)) and the total polarization can therefore build up. However, when T-BMT is considered (Fig. 3(a)), the spin precession carries the polarization and redistributes the $Pol_z$ to be positive. Unfortunately, $B_{z}^{\text{rest}}$ will further depolarize the positive $Pol_z$ as they are parallel in this situation. For higher polarization in the collision with CP pulse, one can increase the field strength and the electron energy as the polarization time $\tau \sim \gamma \chi^{-3} \sim \gamma^{-2} a_0^{-3}$. However, $B_{z}^{\text{rest}}$ comes from the angle between the momentum of electron and the wave vector of the CP pulse. Simply increasing electron energy would decrease the angle and $B_{z}^{\text{rest}}$ may not be increased. The interplay between these processes will be further investigated in the upcoming work.

One should notice that at sufficiently high laser intensity and electron energies, the electron-positron pair production [33] becomes significant. However, pair production is well suppressed under the parameters we are interested in as shown in previous work [32].
Fig. 3 (a) The components of the polarization and the total polarization (dashed) during the collision. (b) The components of the polarization when T-BMT is not considered (c) The components of the magnetic field in the rest frame for x, y components (left axis) and z component (right axis).

**Discussion**

We compare the longitudinal polarization of the spin-flip and the quantum jump model with that of our generalized S-T model with the same configuration in Fig.3, which is shown in Fig. 4(a) with $a_0 = 100$ and $γ_0 = 10^3$. Due to the rotation symmetry of CP wave, $Pol_x$ is also shown in Fig. 4(b).
The three models show disparate results in both longitudinal and transverse polarization. The generalized S-T model predicts a longitudinal polarization between those from the spin-flip model and the quantum-jump model. On the other hand, the results of the quantum jump model agree with the conclusion of Ref. [12] that electrons cannot get polarized in the CP pulse. The transverse polarization oscillates with consistent period and gain no net value in the three cases. Again, our model shows an oscillation amplitude between the other two.

The precession affects the polarization of all these models in the similar way as that in Fig. 3, where $Pol_z$ is tilted towards positive direction. This result might be explained by the interplay between the radiative polarization and the precession effect, which will be further investigated in the upcoming work.

**Fig. 4** Comparison of the polarization effect in the collision between un-polarized electrons and CP laser among the spin-flip model, quantum jump and our generalized S-T model for $Pol_z$ (a) and $Pol_x$ (b).

**Conclusion**

In conclusion, we derive the Sokolov-Ternov-like polarization effect and generalize it to arbitrary directions other than along the magnetic field. The generalized Sokolov-Ternov model predicts
electron polarization by colliding with circularly polarized laser pulse. By considering the spin-dependent radiation-reaction, the spin precession and the spin polarization effect, we present a consistent description of the spin dynamics in the intense laser field.

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