Transport in an interacting wire connected to measuring leads and proximity effects

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We investigate transport through a finite interacting wire connected to noninteracting leads. The conductance of the pure wire is not renormalized by the interactions for any spatial variation of the interaction parameters $u, K$, and not even for Coulomb interactions restricted to the wire. We rigorously relate the conductance to the transmission, that turns out to be perfect. If $K$ varies abruptly at the contacts, an electron incident on the wire is reflected into a series of partial spatially separated charges which sum up to unity. For attractive interactions, the reflection at the contact is similar to Andreev reflection on a gapless superconductor. This process affects the density-density or pairing correlation functions: they are enhanced on the bulk of the wire as in an infinite Luttinger liquid, then extend to the external noninteracting leads in a way reminiscent of the proximity effect. The effect of impurities is governed by the wire parameter but is affected by the leads close to the contacts. Our results give a possible explanation to recent experiments on quantum wires by Tarucha et al. [1].

I. INTRODUCTION

Quantum wires open a new perspective to test the predictions of the well–developed theory of electron–electron interactions in one dimension, generically described as a Luttinger liquid. One of the theoretical predictions [2,3] is the renormalization of the conductance by the interactions, $g = Ke^2/h$, where $K$ is a key parameter depending on the interactions, with $K = 1$ for a noninteracting system. Recently, Tarucha et al. [1] studied relatively clean and long quantum wires: at high enough temperatures, when the impurities do not affect the transport, the conductance is quantized in units of $e^2/h$. Upon lowering the temperature, the impurities become effective, and the observed decrease in the conductance fits the power law behavior predicted by the Luttinger theory. Tarucha and al. extract a parameter $K \approx 0.7$, which contradicts $g = e^2/h \neq 0.7e^2/h$.

The model we propose provides a possible explanation for this paradox [4]. In contrast to previous transport results that overlook boundary effects in the wire, we take into account the measuring leads, supposed to be one-dimensional. The global system thus formed is treated as a Luttinger liquid with inhomogeneous parameters, $u(x), K(x)$ with $K$ set to unity and $u$ to the Fermi velocity $v_F$ on the external leads. The Hamiltonian is:

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1The model has been proposed simultaneously by D. Maslov and M. Stone [5], whose contribution is included in this volume.
\[ H = \int \frac{dx}{2\pi} \left[ \frac{u}{K} (\partial_x \Phi)^2 + uK (\partial_x \Theta)^2 \right] \]  

where the boson field \( \Phi \) is related to the particle density by \( \rho - \rho_0 = -\partial_x \Phi / \pi \), and \( \partial_x \Theta / \pi \) is the field canonically conjugate to \( \Phi \). The system is now similar to an elastic string with inhomogeneous sound velocity \( u(x) \) and compressibility \( u(x)/K(x) \). We restrict ourselves to spinless electrons for simplicity.

II. CONDUCTANCE OF THE PURE WIRE

In order to compute the conductance, we simulate the external reservoirs by the potential they impose on the asymptotic regions of the external leads. In the stationary regime, the current through the system depends only on these asymptotic values. There are many ways to compute the conductance, but the most straightforward and physically appealing one is through its relation to the transmission. There is no general Landauer formula for our interacting wire, where even the description in terms of quasiparticles fails; for the present model however we could establish it rigorously. \[4\]

Next we have to find the transmission of an incident electron on the interacting wire. For this purpose, we derive the equation of motion for the particle density, with the initial condition given by the injected charge of the incident electron. The total transmission turns out to be perfect! Thus the conductance is given by:

\[ g = \frac{e^2}{h}. \]  

It is worth noting that the perfect transmission allows to use the underlying argument of Landauer’s formula, where the reservoirs are simulated by the flux they inject. Our proof applies to any variation of the interaction parameters on the wire, but not only short-range interactions: it holds for any Coulomb interactions perfectly screened on the measuring leads, such as

\[ U(x, y) = \frac{f(x)f(y)}{|x - y|} \]

with \( f \) vanishing outside a finite region around the wire.

III. DYNAMIC PROCESS OF TRANSMISSION

It is not obvious to guess the perfect transmission when the interactions are abruptly switched on the wire. Let’s restrict ourselves to short-range interactions, with a parameter \( K \) on the wire, a situation we deal with in the rest of the paper. An incident electron starts to interact with the electrons of the wire, and is partially reflected at the contact with coefficient \( \gamma \)

\[ \gamma = \frac{1 - K}{1 + K} \]

This coefficient comes from two matching conditions of the boson field at the contacts, required by the equation of motion: the continuity of the current (the interactions conserve momentum) and of \((u/K)\rho\) \(^2\)
How can it be transmitted in the stationary regime? This is because the subsequent reflections are of hole (electron) type if the first one is of electron (hole) type, which is the case of repulsive (attractive) interaction: this is illustrated in fig. 1. The incident electron is transmitted into a series of spatially separated partial charges that sum up to unity for times long compared to the traversal time of the wire. The wire behaves as a Fabry-Perot resonator.

\[
\begin{array}{c}
1 \\
y \\
-a +a
\end{array}
\]

\[
t = 0
\]

\[
\begin{array}{c}
\gamma \\
1-\gamma
\end{array}
\]

\[
t_0
\]

\[
\begin{array}{c}
\gamma \\
-\gamma(1-\gamma) \\
1-\gamma
\end{array}
\]

\[
t + t_0
\]

\[
\begin{array}{c}
-\gamma(1-\gamma) \\
\gamma(1-\gamma)
\end{array}
\]

\[
t + 2t_0
\]

FIG. 1. The transmission process of an incident electron on the wire in the case where \( K > 1 \) and \( u = v_F \) for simplicity. We denote: \( t_y = (-a - y)/v_F \), and \( t_2 = 2a/u = 1/T_L \) the traversal time of the wire by the density oscillations. At \( t_y + (2n + 1)t_2 \) (resp. \( t_y + 2nt_2 \)), a charge \( \gamma^{2n}(1 - \gamma^2) \) (resp. \( -\gamma^{2n-1}(1 - \gamma^2) \)) comes out at \( a \) (resp. \(-a\)). The first reflected charge is of hole type, while the subsequent ones are of electron type. If \( K < 1 \), the signs of the reflected charges are exchanged.

IV. PROXIMITY EFFECT

Before considering the effect of impurities on the conductance, let’s ask the following question: we know that an infinite Luttinger liquid has a tendency towards superconducting order or the formation of a charge density wave, depending on whether the interactions are attractive or repulsive. This will make it a more or less good conductor. Now that we connect it to measuring leads, the conductance is always equal to the noninteracting value \( e^2/h \). Does the tendency to one or the other type of order persists? Yes, it does, and even more it extends in the noninteracting leads, recalling the proximity effect.

The previous dynamic process due to change in interactions affects the correlation functions: note that the latter depend on the temperature, while the transport and transmission did not. Let us for instance focus on the charge density correlation functions. In the high temperature limit, when the distance between the contacts is higher than the thermal coherence length \( L_T = u/T \), the multiple reflections are washed out, so that we recover the same behavior in the bulk as in an infinite Luttinger liquid with parameter \( K \). But there is still the first reflection on the contacts with coefficient \( \gamma \), that affects the charge density correlation in their vicinity up to distances of the order of \( L_T \): it reduces (enhances) them whenever the interactions on the wire are repulsive (attractive). This yields an effective local parameter \( K_a = 2K/(1 + K) = 1 - \gamma \), proportional to the transmission coefficient of an incident electron on the contact: \( K_a \) is greater (smaller) than \( K \) for repulsive (attractive) interactions. The external noninteracting leads moderate the effect of interactions near the contacts, but they also feel the effect of the
interactions within the wire: indeed, the charge density correlations are enhanced (reduced) in the leads if \( K < 1 \) \((K > 1)\), up to a distance given by \( L_T \).

Now we go down in temperature, until \( L_T \gg L \): there is thermal coherence all over the wire, and the multiple reflections affect the correlation function, whose dependence on the temperature is now determined by the external leads. Since the latter are noninteracting, there is no dependence on temperature. But there is a dependence on the wire length, that a transmitted charge from the leads have crossed many times! For points either very close to the contacts or far from them by a distance of the order of \( L \), i.e. well on the bulk, the local correlation function can be obtained by the substitution \( T \to T_L = u/L \) in the high temperature result, thus is governed respectively by \( K_a \) and \( K \). For other points, this substitution no more works. We summarize the results in a simplified form in the table I. The charge density correlations are still enhanced (reduced) compared to their noninteracting value (reached far away in the external leads) if \( K < 1 \) \((K > 1)\): this enhancement (reduction) extends in the external leads up to a distance \( \sim L \).

The same reasoning holds for the pairing correlation functions: they are enhanced (reduced) whenever the charge density correlation are reduced (enhanced), so that it is easy to translate the previous statements. The local parameter at the contact is obtained by taking the inverse of \( K \) in \( K_a \), thus it is the average of \( K \) with the lead parameter: \((1 + K)/2\). Whenever the interactions are attractive, \( K > 1 \), the tendency towards superconducting order extends in the noninteracting leads up to the shorter length scale. For instance, at an external frequency \( \omega \ll T_L \), and at \( T \ll T_L \), the longer is the interacting wire, the most enhanced and the most extended in the leads are the pairing correlations. Now let’s come back to the reflection of an incident electron on the contact, that affected the correlation functions as already explained: when \( K > 1 \), we have \( \gamma < 0 \), thus a partial hole is reflected back: this is the analogous of an Andreev reflection at the interface between a normal metal and a gapless superconductor: the electron energy is obviously greater than the vanishing gap, thus the reflection is partial. In our case, we get exactly one hole reflected in the limit \( K \gg 1 \).

| \( \omega \) | \( T_L \) | \( T_L \) | \( T_a \) | \( T_x \) | \( \omega \) |
|---|---|---|---|---|---|
| \( \omega < T_L \) | \( T_L^{K_a-1} T_x^{K-K_a} \) | \( T_L^{K_a-1} T_x^{K-K_a} \) | \( \omega^{K_a-1} T_x^{K-K_a} \) | \( \omega^{K_a-1} \) |

**TABLE I.** The local CDW correlation function for different points \( x \) on the interacting wire, divided by its value in a noninteracting wire. All the energies have to be divided by the bandwidth \( \Lambda \). \( T_x = u/(a - |x|) \) is the inverse of the time taken by a plasmon to go from \( x \) to the closest contact. The behavior on the external lead can be deduced simply by replacing \( K \) by unity, and \( T_x \) by \( v_F/(|x| - a) \). \( \omega \) is an energy variable that can be either a frequency, the inverse of time, or the temperature. When \( T_x \sim T_L \) (resp. \( T_x \sim \Lambda \)), the behavior is governed only by \( K \) \((K_a = 2K/(1 + K))\). The pairing correlation function can be inferred by replacing \( K \) by \( 1/K \), thus \( K_a \) by \( K_a = 2/(1 + K) \). On the external leads, we let \( K \to 1 \), but keep \( K_a \).

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\[^3\]In Ref. [4], we defined a local conductance at the contact, given by \( g_+ = g_0 K_a \). It verifies \( g_0 K_1 \leq g_+ \leq 2 g_0 K_1 \) recalling similar inequalities at a N-S interface.
V. BACKSCATTERING POTENTIAL

The conductance of the pure wire did not get affected by the reflections at the contacts due to change in interactions: the interactions as well as those reflections conserve the total momentum of the electrons. Obviously, this is no more the case in the presence of a backscattering potential. Does the Luttinger liquid have a chance to show up in the dirty wire? The effect of backscattering is determined by the charge density correlations: since we’ve already seen their sensitivity to interactions, we can already guess the answer: yes, the interactions will affect the conductance when electrons are backscattered. We refer to the previous discussion of the inhomogeneity of the charge density correlation function, showing how the reduction in the conductance depends on the impurity location. The table 1 gives the correction to the conductance in the presence of a barrier at \( x \), by letting \( \omega = T \).

We performed a renormalization procedure explicitly at finite temperature for a barrier placed in our finite wire [6]; we verify explicitly that the interactions are not renormalized by the barrier, so that the perturbative computation of the conductance is sufficient.

Let’s make more comments about the barrier at the contact: this is indeed a simple way of modeling a mismatch at the opening of a quantum wire into the two-dimensional gas. The interactions are expected to be repulsive, thus \( K_a < 1 \), and the conductance can be notably decreased in the limit of a long wire. But when we get long wires, we can’t avoid impurities in their bulk: according to our results, the latter dominate the scattering at the contacts because \( K < K_a < 1 \). For the contact to dominate, the interactions have to be attractive, \( K > 1 \) in which case \( 1 < K_a < K \): in this case, the external leads reduce the local attraction, thus enhance the effect of the backscattering near the contact.

Let’s now consider an extended disorder on the wire, with Gaussian distribution. There are many powers that emerge in the conductance. Let’s write the dominant ones, without giving the explicit coefficients:

\[
g = \frac{e^2}{\hbar} \left[ 1 - \frac{L}{L_e} \omega^{2(K-1)} - \frac{\alpha}{L_e} \omega^{2(K_a-1)} \right]
\]

with \( \omega = \max(T,T_L)/\Lambda \), \( \Lambda \) being the bandwidth, and \( \alpha \sim u/\Lambda \). The contribution from the impurities near the contacts dominates for \( K > K_c = (3 + \sqrt{17})/4 \), at \( T < T_L \).

If we restore the spin of electrons, the pure wire conductance is just multiplied by 2 since the transport depends only on the charge degrees of freedom. Conceptually, the extension of the backscattering effects is easy, but a more richer behavior emerges. This will be the subject of a separate publication.

We note finally the coherence of our results with the experiment by Tarucha and al [1]. The ballistic conductance equals \( 2e^2/h \), and a decrease with temperature with a power law saturating at \( T < T_L \) is observed. But the exponent in this power law yields the wire parameter only if there is at least one impurity in the bulk of the wire. We cannot really decide about the impurity distribution in the measured wires. If the bulk of a wire is clean, the reduction comes exclusively from the backscattering at the contacts, which would yield a parameter \( K = 0.5 \), different from the value \( K = 0.7 \) one infer in the presence of impurities on the bulk. The observed decrease of the conductance with the wire length is not a strong argument for an extended disorder: a barrier yields also a correction saturating at \( L^{K(x)-1} \), with \( K(x) = K \) on the bulk and \( K_a \) near the contacts.
VI. SUMMARY

To summarize, the conductance of an interacting one-dimensional wire connected to perfect one-dimensional measuring leads is equal to $2e^2/h$, for any range of the interactions on the wire, and as long as they conserve the momentum of the electrons. But the Luttinger liquid behavior gets revealed in the presence of backscattering potential whose effect depends in a nontrivial way on its distribution through the wire.

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Transport dans un fil quantique connecté à des fils de mesure

Lorsqu’on connecte un liquide de Luttinger à des fils de mesure parfaits, nous montrons que sa conductance n’est plus renormalisée par les interactions, contrairement au résultat jusqu’ici admis. Nous démontrons ce résultat non seulement pour un profil arbitraire des paramètres sur le fil central, mais aussi en présence d’interactions Coulombiennes écrantées sur les fils de mesure. Le moyen le plus direct de le montrer est d’établir rigoureusement que la conductance est donnée par le coefficient de transmission d’un électron à travers le fil, et que la transmission est parfaite dans le régime stationnaire. Ainsi, pour des interactions à courte portée branchées discontinûment sur le fil, un électron incident est transmis en une série de charges partielles spatialement séparées, dont la somme vaut l’unité.

Si les interactions sont attractives, la réflexion d’un électron sur le contact est l’analogue d’une réflexion d’Andreev. Les fonctions de corrélation correspondant à la tendance à l’ordre supraconducteur sont renforcées en s’étendant dans les fils externes, rappelant ainsi l’effet de proximité.

En présence d’impuretés, la conductance devient sensible aux interactions, mais dépend à la fois de l’emplacement des impuretés et des paramètres du fil mesuré ainsi que celui des fils de mesure. Nos résultats donnent une explication possible au paradoxe soulevé par les expériences récentes de Tarucha et al sur des fils quantiques.