Scaling of particle production with number of participants in high-energy $A+A$ collisions in the parton-cascade model

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In view of the recent WA98 data of $\pi^0$ spectra from central $Pb + Pb$ collisions at the CERN SPS, we analyze the production of neutral pions for $A + A$ collisions across the periodic table at $\sqrt{s} = 17$ AGeV and 200 AGeV within the framework of the parton-cascade model for relativistic heavy ion collisions. The multiplicity of the pions (having $p_T > 0.5$ GeV) in the central rapidity region, is seen to scale as $\sim (N_{\text{part}})^\alpha$, where $N_{\text{part}}$ is the number of participating nucleons, which we have approximated as $2A$ for central collisions of identical nuclei. We argue that the deviation of $\alpha$ ($\simeq 1.2$) from unity may have its origin in the multiple scattering suffered by the partons. We also find that the constant of proportionality in the above scaling relation increases substantially in going from SPS to RHIC energies. This would imply that the (semi)hard partonic activity becomes a much cleaner signal above the soft particle production at the higher energy of RHIC, and thus much less dependent on the (lack of) understanding of the underlying soft physics background.

Recently, the WA98 Collaboration has published data for the production of neutral pions up to transverse momenta of $p_\perp \simeq 4$ GeV/c, in central $Pb + Pb$ collisions at 160 A GeV/c incident momentum, corresponding to $\sqrt{s} \simeq 17$ A GeV. Two most interesting features of these data emerge when compared to corresponding data from $pp$ collisions and collisions involving lighter nuclei: a) an approximate invariance of the spectral shapes, i.e., a near indepenence of the slope of the neutral pion $p_\perp$ spectra; b) a simple scaling of the $\pi^0$ with the number of participating nucleons, if the number of participants is large ($\gtrsim 30$). In the present work, we use the event generator VNI which embodies the physics of the parton-cascade model for ultra-relativistic heavy-ion collisions to analyze these observations. The model attempts to describe the nuclear dynamics on the microscopic level of particle transport and interactions, by evolving the multi-particle system in space-time from the instant of nuclear overlap all the way to the final-state hadron yield. For details we refer the interested reader to Refs. 2,3.

A simple consideration may illustrate the features of particle production within this
approach and its relevance to multiple parton scattering. Let $x$ denote the number of partons in each nucleon, and let each parton suffer $\nu$ collisions during the partonic stage. Assuming that each virtual parton radiates $r$ partons, we see that the number of produced partons will vary as

$$N_{\text{partons}} \propto \nu (1 + r) \times A,$$

and as we expect, $N_{\text{hadrons}} \propto N_{\text{partons}}$, one realizes immediately that if the partons interact only once, the multiplicity of the partons, and hence the multiplicity of hadrons, will scale as $A$. It is also clear that if every parton interacts with every other parton then $\nu \propto A$, and the number of materialized partons would scale as $A^2$. That can happen, if the system would live for an infinitely long time. However, this is not the case. Rather than that, in relativistic heavy ion collisions, the partonic matter will expand, dilute, and eventually convert into hadrons. Thus a given parton may undergo $\nu \sim R/\lambda$ interactions; where $R$ is the transverse size of the system and $\lambda$ is the mean free path of the parton. Noting that $R \sim A^{1/3}$, we immediately see that the number of materialized partons, and hence the number of produced particles would scale as $\approx A^{4/3}$. An experimental verification of this scaling behaviour could be a direct manifestation the formation of a dense partonic matter!

We shall demonstrate now that these simple considerations are indeed confirmed by a detailed simulation with the event generator VNI on the basis of the parton-cascade/cluster-hadronization model. We first consider the recently measured transverse momentum distribution of $\pi^0$-production in central collisions of $Pb + Pb$ at CERN SPS obtained by the WA98 collaboration [1]. To make contact with the experimental data, the simulations were done for the range of impact parameters $0 < b < 4.5$ fm, which corresponds to $10\%$ of minimum-bias cross-section. The result of our model calculation, shown as the solid histogram in Fig. 1a, is seen to be in decent agreement with the experimental measurements. The model results do not include the final-state interaction among produced hadrons yet, but it is likely [4] that the agreement will further improve once the effect of cascading hadrons is included. The dashed histogram in Fig. 1 gives the soft contribution while the solid curve gives a hydrodynamic prediction (without the contribution of resonance decays).

In Fig. 2 we plot our results for the $p_{\perp}$ spectra of $\pi^0$'s for a number of central $AA$ collisions at $\sqrt{s} = 17$ A GeV for various $A + A$ systems from $A = 16$ to $A = 197$. One observes that they are almost identical in shape with a universal slope for $p_{\perp} \lesssim 1.5$ GeV/c. On the other hand, the deviations appearing at larger $p_{\perp}$ for heavier systems are indicative of enhanced multiple scattering there. Similar results (not displayed here) were obtained at RHIC energies. In order to verify this scaling more closely, we have calculated, as a function of the nuclear mass number $A$, the production of $\pi^0$'s in the central rapidity region ($-0.5 < y < 0.5$) having transverse momenta $p_{\perp} \geq 0.5$ GeV/c. The latter choice minimizes the influence of pions having their origin in decay of resonances. This kinematic window was motivated [1] by the WA98 collaboration in their measurement of the $\pi^0$ yield. Fig. 3 displays the simulation results for central $A + A$ collisions across the periodic table, at CERN SPS center-of-mass energy $\sqrt{s} = 17$ A GeV, while Fig. 4 shows the same for RHIC energy $\sqrt{s} = 200$ A GeV. The solid lines are fits to the model results, represented by the symbols, and scale as
Figure 1. Transverse mass spectra of neutral pions in central collisions of 158 AGeV Pb + Pb collisions. The solid histogram represents our result from the parton-cascade/cluster-hadronization model. The dashed histogram gives the contribution of the soft-part.

Figure 2. Transverse momentum spectra of neutral pions in central collisions of identical nuclei at $E_{cm} = 17$ A GeV. The symbols are our results for various systems, from $O + O$ up to $Au + Au$. All results are normalized to the case $A = 16$.

\[ N_{\pi^0} \propto (N_{\text{part}})^{\alpha}, \quad (2) \]

where $N_{\text{part}} = 2A$ is the number of participating nucleons, and $\alpha$ being extracted as:

\[ \alpha \approx \begin{cases} 1.16 & \text{at } \sqrt{s} = 17 \text{ A GeV} \\ 1.23 & \text{at } \sqrt{s} = 200 \text{ A GeV} \end{cases} \quad (3) \]

It is interesting to note that $\alpha \approx 1.2$ is in excellent agreement with the corrected WA98 results \[.\]

For comparison, the dashed lines correspond to a linear scaling $\sim (N_{\text{part}})^{1.0}$, whereas the dashed-dotted lines indicate a hypothetical scaling with the nuclear overlap factor $T_{AA}(b = 0) \sim (N_{\text{part}})^{1.42}$. A linear scaling would reflect a single-collision situation, and a scaling with $T_{AA}$ would indicate a Glauber-type multiple-collision scenario, in which nucleons suffer several collisions along their incident straight-line trajectory, without deflection and without energy loss. Comparing the three curves, we can conclude that our simulation results rise significantly slower than with $T_{AA}$, because firstly, the particles change direction through the collisions, and secondly, they are subject to a collision time of the order of the inverse momentum transfer, during which they cannot rescatter. On the other hand, our calculated $\pi^0$ yields grow much faster than linear with $A$, due to multiple scatterings.
In summary, we have demonstrated here that the observed scaling of the number of produced particles with the number of participants, in heavy-ion $A + A$ collisions, as well as the approximate shape-independence of the transverse momentum spectra, are satisfactorily reproduced by the parton-cascade / cluster-hadronization model. We must add that a more accurate description of the $p_T$ spectra at larger $p_T$ will need a readjustment of the hadronization model.

Dr. Klaus Geiger, a dear colleague, a very good friend, and a brilliant physicist was killed in an air-crash in September 1998. This is one of the last pieces of work which we completed together [7].

**REFERENCES**

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6. The WA98 group reported at the QM'99 meeting (see talk by T. Peitzmann) that the value of $\alpha$ reported by them in Ref. [1] $\approx 1.3$ was now corrected to $\approx 1.15–1.20$. This is in much better agreement with our prediction which were made before the revised data were reported.
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