Measuring Chirally Odd Wave Functions with Helicity Flip Form Factors

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Abstract

We consider the role of chirally odd wave functions in hard exclusive reactions. Such wave functions have the quarks oriented in the opposite helicity configuration from those assumed in the short-distance limit and are generally associated with non-zero orbital angular momentum. Calculations in the impulse approximation allow for non-zero helicity flip amplitudes while the conventional factorization prescription for exclusive processes does not. By introducing a new approach, we show how helicity flip form factors are nevertheless calculable in QCD.

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Factorization and Hadron Helicity Flip

The theory of elastic form factors at large momentum transfer in QCD is well developed. Nevertheless, conventional theory cannot be compared with data for form factors involving hadronic helicity flip. This shortcoming is due to a theoretical choice of “factorization” scheme, and is not a property of the basic impulse approximations made in QCD. The main difference between helicity flip and non-flip quantities is that new wave functions appear in combinations that are not reducible to the quantities measured in non-flip reactions. This is a positive development: it means that the internal structure of the quarks in the hadron can be probed in a new way.

In the conventional approach[1], the factorization of a typical form factor $F(Q^2)$ reduces it to a convolution over the momentum fractions $x_{i,j}$ of the participating partons:

$$F(Q^2) = \int \prod_{ij} dx_i dx_j \phi_j^*(x_j, Q^2) H(x_i, x_j, Q^2) \phi_I(x_i, Q^2)$$  \hspace{1cm} (1)

Here, $H$ is a hard scattering kernel, representing the part of the amplitude that is perturbatively calculable; the $\phi$’s are objects called distribution amplitudes, which contain the non-perturbative information about the hadrons. This kind of factorization prescribes a dynamical symmetry which is manifested[1] in the hadronic helity conservation rule $\lambda_A + \lambda_B = \lambda_C + \lambda_D$ for reactions of the form $A + B \rightarrow C + D$. This rule is as general as the factorization. The key step is the relation of the distribution amplitudes $\phi$ to the wave functions $\psi$. A useful relation is obtained[2] in coordinate space, letting $b$ be the transverse separation between a pair of quarks:

$$\phi(x, Q^2) = 2\pi \int_0^\infty db Q J_1(Qb) \tilde{\psi}_{m=0}(x, b)$$  \hspace{1cm} (2)

Here we have expanded the b-space wave function in orbital angular momentum eigenstates with the z-axis along the direction of the momentum; $J_1$ is a Bessel function. The wave function participating in the model is selected by the hard scattering formalism to be carrying $m = 0$. This wave function is called the “short-distance” one. By angular momentum conservation the quark helicities for this case add up to be the hadron helicity. The nearly perfect chiral symmetry of perturbative QCD predicts that quark spins do not flip in the hard scattering, so that in the model the sum of the hadron helicities cannot change.

The success of the helicity conservation rule in comparison with data is uneven. There is a consistent pattern of its violation in hadron-hadron reactions. For a long time this has
been thought to be a problem for perturbative QCD. Recently[2,3], it has been recognized
that certain independent scattering processes[4] disobey the presumed factorization (1), even
at large $Q^2$. It has been proposed[2] that these processes are likely to be the explanation for
helicity violation in that case.

In certain photon initiated reactions the non-short distance aspects of independent
scattering seem not to be a problem. Thus (1) should apply at leading order to form factors,
and helicity non-conservation should be power suppressed compared to helicity conservation.
But being power suppressed does not say that the form factors are zero; why does the
factorization prescription say that they are zero?

A new approach[5] outlined here allows us to predict helicity flip form factors, or, more
objectively, to interpret them as measurements of hadron internal structure. The cases in
which we can do this are those in which the helicity flip term is the first, leading order
term which is linearly independent and separable by its Lorentz structure. For definiteness
we illustrate our study of the proton magnetic form factor $F_2(Q^2)$. We use the impulse
approximation and assume that we have perfect chiral symmetry in the hard collision. Thus
we have to understand how the quark, whose helicity is not flipped, can end up in a proton
whose helicity is flipped.

The photon causes the scattering of a quark out of one proton and into the next with
momentum transfer $Q^\mu$. From crossing symmetry this can be related to the antiquark-proton
scattering with ($t$-channel) momentum transfer $Q$. To study this we introduce a new object:
the off-diagonal, or transition amplitude we denote by $T$:

\[ T^{ij} = \int d^4x \, e^{ikx} < p + Q, s | T(\psi_i(0)\bar{\psi}_j(x)) | p, s > \] (3)

Here $i, j$ are the Dirac indices of the quark field, which tell how its spin is oriented. In the
diagonal case (same initial and final states, $Q = 0$) the imaginary part of $T$ from the cut
between the quarks is the parton distribution. By definition $T$ has an inclusive character: $T$
automatically sums over all but one Fock state components, no matter what their momenta,
spin, color, isospin, etc.

The electromagnetic current is chirally invariant and thus only the parts of $T$ satisfying
$\{T, \gamma^5\} = 0$ can be probed with a photon. This is what we mean by “chirally even”. The
opposite possibility is to be chirally odd, or $[T, \gamma^5] = 0$. The same selection rules occur in
deeply inelastic scattering, where certain inclusive parton distributions - the unpolarized and
the longitudinally polarized ones - are chirally even and “measurable”, while the chirally odd transverse polarization distribution is “unmeasurable” in that experiment[6]. The transverse polarization is a leading order distribution which can be measured with an anti-quark probe in Drell-Yan lepton pair production.

The helicity-flip form factors can be investigated in terms of $T$, but one cannot go further to the level of factorization given in Eq.(1) and still save the calculation. The problem of the helicity flip form factor is the same problem as finding a helicity flip amplitude in antiquark-proton scattering.

As mentioned earlier, it has been shown in $pp \rightarrow pp$ scattering that the independent scattering kinematics allows all possible orbital angular momenta to participate in the scattering process. When this occurs the hadron can flip its helicity. Is there a similar process in quark-proton scattering which can do the same? An example independent scattering contribution is shown in figure 1.

We believe that these types of processes are the unique configurations which can do the job, except for “endpoint” contributions (which would totally destroy the power counting and are apparently Sudakov suppressed). In general we can decompose the Dirac structure of $T$:

$$T^{ij} = u_a(p,s)\bar{u}_b(p',s')t^{ij}_{ab}$$

$$t^{ij}_{ab} = \Gamma^{ij} \cdot V_{ab} \cdot U$$

where $\Gamma$ are matrices spanning the Dirac space (the set $1, \gamma^5, \gamma^\mu$, etc.), $V$ and $U$ are tensors contracted with the $\Gamma$’s and functions of the invariants $Q^2, Q \cdot k$, etc. We have suppressed the flavor dependence on the struck quark. The part of the tensor $t^{ij}_{ab}$ for the helicity flip of interest must be be chirally even in indices $i, j$, since there is no quark helicity flip, and odd in indices $a, b$ for the proton helicity flip. We can make a list of the invariant amplitudes forming $t^{ij}_{ab}$:

$$t^{ij}_{ab} = t_1(\gamma^\mu)^{ij}(\gamma_\mu)_{ab} + t_2(\gamma^\mu)^{ij}(i\sigma_{\mu\nu}(p-p')^\nu)_{ab} + t_3(i\sigma_{\lambda\nu}(p-p')^\nu)^{ij}(i\sigma^{\lambda\nu}(p-p')_\nu)_{ab} + ... \quad (4)$$

In (4) we indicate that there are many possible orthogonal Dirac projections, which we do not bother to write down. What we want is the measurable ones, which we now show are the first two. Putting together the factors, the form factor are calculated:

$$\bar{u}(p', s')(F_1\gamma^\mu + \frac{iF_2}{2m}\sigma^\mu\nu q_\nu)u(p, s) = \frac{1}{4} \int d^4k T r[\gamma^\mu T]$$

$$= \bar{u}(p', s')\gamma^\mu(ps) \int d^4kt_1 + \bar{u}(p', s')i\sigma^\mu\nu q_\nu(ps) \int d^4kt_2 \quad (5)$$
The relation (5) is perfectly general, as can be checked by inserting the definition (3) and obtaining the matrix element of the electromagnetic current operator $<p's' | J^\mu_{cm} | ps>$. We can read off the form factors:

$$F_1 = \int d^4k t_1; \quad F_2 = \int d^4k t_2$$

Our physical picture is that quark hadron scattering measures non-zero orbital angular momentum in the wave function; with the insertion of a hard probe which “pinches the ends” of the scattering shut, we obtain the form factors.

Now consider power counting of the process. From Fig. (1), we have two hard gluons, two quark wave functions contracted into distribution amplitudes, and two wave functions for the relative orbital angular momentum of the quarks. Working in transverse separation space we will have a Fourier transform $\exp(ib \cdot Q)$ so that for large $Q$, each power of $b$ scales like $1/Q$. A wave function carrying orbital angular momentum $m$ goes like $b^m$ as $b \to 0$. We need at least one power of $b$ for a unit of orbital angular momentum, and then we need two because the integration interval is symmetric. One then anticipates the terms contributing to the form factor $F_2(Q^2)$ as:

$$F_2(Q^2) = (1/Q^2)\alpha_s^2 \prod_{1}^2 \int dx_i d^2b \sum_{j,k} \phi_i^* \tilde{\psi}_j(b)e^{iQ \cdot b} \phi_j \tilde{\psi}_k(b)$$

where $j$ and $k$ are the indices in an orbital angular momentum expansion, and we let the $x$-dependence be implicit. By power counting then, $F_2$ goes like $1/Q^6$, and is calculable if all the wave functions are known. It follows that a measurement of $F_2$ measures the orbital angular momentum content of the quarks in the proton. Of course, a series of definitions is needed to say exactly which wave functions are measured. This is a new result - previously $F_2$ was merely argued to be “higher twist” and the helicity flip was attributed to quark mass terms. Recent SLAC data[7] shows that $F_2$ conforms to the power counting above and is not anomalously small in magnitude. This allows us to conclude that the projections onto $m = 1$ wave functions of quarks in the proton are about the same size as the $m = 0$ parts. This is consistent with the picture that helicity flip in the $pp \to pp$ hard scattering rate is not suppressed at large $Q^2$, and occurs due to the intrusion of quark orbital angular momentum permitted by independent scattering.

The spin structure of hadrons is growing more and more interesting. The idea that helicity flip form factors measure non-zero orbital angular momentum can be tested in color transparency: quasiexclusive electroproduction knockout of protons (or self-analyzing deltas).
from a nuclear target, with measured final state polarization. Non-zero $m$ should preferentially be filtered away at large $A$. We expect more theoretical work in this area, and exciting interaction with experiment.

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