Strong phase shifts and color-suppressed tree amplitudes in $B \to DK^{(*)}$ and $B \to D\pi, D\rho$ decays

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Abstract

We analyze the decay processes $B \to DK, DK^*, D\pi,$ and $D\rho$ in a model-independent way. Using the quark diagram approach, we determine the magnitudes of the relevant amplitudes and the relative strong phase shifts. In order to find the most likely values of the magnitudes and the relative strong phases of the amplitudes in a statistically reliable way, we use the $\chi^2$ minimization technique. We find that the strong phase difference between the color-allowed and the color-suppressed tree amplitude can be large and is non-zero at 1σ level with the present data. The color-suppressed tree contributions are found to be sizably enhanced. We also examine the validity of factorization and estimate the breaking effects of flavor SU(3) symmetry in $B \to DK, D\pi$ and in $B \to DK^*, D\rho$.

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I. INTRODUCTION

A tremendous amount of experimental data on $B$ meson decays are being collected from $B$ factory experiments, such as Belle and BaBar. Experimentally plenty of two-body hadronic $B$ decays have been observed and a lot of theoretical works on these decay processes have been done. In particular, the first observation of the color-suppressed decay processes $\bar{B}^0 \to D^0\bar{K}^0$ and $\bar{B}^0 \to D^0\bar{K}^*$ by the Belle Collaboration [1] has drawn special attentions, since it allows one to do a complete isospin analysis of the $B \to DK^{(*)}$ modes together with the previously observed charged modes of the $B \to DK^{(*)}$ type.

Two-body hadronic $B$ meson decays to $DK^{(*)}$ and $D\pi$ final states have been of great interest. In these decay modes, there is no contribution from penguin diagrams so that theoretical uncertainties involved in the relevant QCD dynamics become much less. Thus, these modes serve as a good testing ground for various theoretical issues in hadronic $B$ decays, such as factorization hypothesis and final-state interactions. These processes are also expected to be useful for a determination of the CP violating phases, e.g., $\phi_3$ [2, 3, 4, 5].

It has been expected that in a heavy quark limit, certain two-body charmed $B$ decays, such as $B^0 \to D^{(*)}\pi^-$ [referred to as the class-1 (color-allowed) topology], can be explained well with the factorization hypothesis implying small final-state interactions. It has been confirmed in the QCD factorization approach [6]. However, in a recent work based on the perturbative QCD (PQCD) approach [7], it was pointed out that in order to explain $\bar{B}^0 \to D^0\pi^*$ [referred to as the class-2 (color-suppressed) topology] as well as $B^- \to D^0\pi^-$ [referred to as the class-3 (involving both color-allowed and color-suppressed) topology], there must exist a sizable relative strong phase $\delta_{12}$ between the class-1 and the class-2 amplitudes: e.g., $\delta_{12} = 59^0$. This relative strong phase arises from QCD dynamics through short-distance strong interactions and differs from the final-state strong phases through long-distance rescattering interactions.

Motivated by experimental measurements of the branching ratios (BRs) for $B \to D\pi$ and $B \to DK$ decays, some phenomenological studies have been performed to determine the possible final-state rescattering strong phases in these processes [8, 9, 10, 11, 12]. Especially, in Refs. [8, 12] the $B \to D\pi$ and $B \to DK^{(*)}$ modes were studied through the isospin analysis. However, in those works, the possibility of a sizable relative strong phase between the color-allowed and the color-suppressed tree amplitudes was completely ignored. On the
other hand, in Ref. [10], the $B \rightarrow D\pi$ ($D\rho$) and $B \rightarrow DK^{(*)}$ modes were analyzed in the topological quark diagram approach and in that analysis flavor SU(3) symmetry was assumed to combine the relevant amplitudes with each other.

In this work, we re-analyze the $B \rightarrow DK$ and $B \rightarrow D\pi$ modes as well as $B \rightarrow DK^*$ and $B \rightarrow D\rho$ in the quark diagram approach, focusing on the following interesting issues. (i) We estimate, in a model-independent way, the magnitude of the relative strong phases, taking into account the possibility of a sizable relative strong phase between the color-allowed and the color-suppressed tree amplitudes. This approach is different from that by Xing [8, 12], where the strong phase difference between the color-allowed and the color-suppressed tree amplitude was assumed to be zero. (ii) We first study the $B \rightarrow DK^{(*)}$ and $B \rightarrow D\pi$ ($D\rho$) independently, without using the flavor SU(3) symmetry, in order to avoid the possibly large effect of SU(3) breaking. (In fact, we shall see later that the SU(3) breaking effect can be sizable.) (iii) To determine the most likely values of the magnitudes of the relative strong phase shifts in a statistically reliable way, we do the $\chi^2$ analysis (with the flavor SU(3) and its breaking effect together) and explicitly show that the relative final-state strong phases in $B \rightarrow DK$ and $B \rightarrow D\pi$ are non-zero at 1σ level. (iv) We examine the validity of factorization approximation in these heavy → heavy type decay modes, and estimate the flavor SU(3) symmetry breaking effects in a model-independent way.

The paper is organized as follows. The decay modes $B \rightarrow DK$ and $B \rightarrow DK^*$ are studied in Sec. II and the modes $B \rightarrow D\pi$ and $B \rightarrow D\rho$ are analyzed in Sec. III. In Sec. IV, the $\chi^2$ analysis using $B \rightarrow DK$, $D\pi$ and $B \rightarrow DK^*$, $D\rho$ is presented. The breaking effects of the flavor SU(3) symmetry are estimated in Sec. V. We conclude the analysis in Sec. VI.

II. $B \rightarrow DK$ AND $B \rightarrow DK^*$ DECAY MODES

First, let us consider the decay processes $B \rightarrow DK$. The decay amplitudes for two-body hadronic $B$ decays can be represented in terms of the basis of topological quark diagram contributions [13], such as $T$ (color-allowed tree amplitude), $C$ (color-suppressed tree amplitude), $E$ (exchange amplitude), and so on. The relevant decay amplitudes for $B \rightarrow DK$ can be written as

$$A_{DK}^{0-} \equiv \mathcal{A}(B^- \rightarrow D^0 K^-) = T_{DK} + C_{DK},$$
$$A_{DK}^{+-} \equiv \mathcal{A}(\bar{B}^0 \rightarrow D^+ K^-) = T_{DK},$$
\[
A_{00}^{DK} \equiv A(\bar{B}^0 \to D^0 \bar{K}^0) = C^{DK},
\]

where the topological amplitudes \(T^{DK}\) and \(C^{DK}\) are defined as

\[
X^{DK} \equiv |X^{DK}| e^{i\delta_X} \equiv |V_{cb} V_{us}^*| a_X e^{i\delta_X}, \quad (X = T, C)
\]

with the real amplitudes \(a_{T(C)}\) and the strong phases \(\delta_{T(C)}\). Note that no weak phase appears in the above amplitudes due to the Cabibbo-Kobayashi-Maskawa (CKM) factor \(V_{cb} V_{us}^*\).

From (1), the magnitudes \(|T^{DK}|\) and \(|C^{DK}|\) and strong phase difference \((\delta_T - \delta_C)^{DK}\) of the topological amplitudes can be determined in a model-independent way:

\[
|T^{DK}| = |A_{+0}^{DK}| = m_B \sqrt{\frac{8\pi}{p_{DK} \tau_0}} B^{DK}_{+0},
\]

\[
|C^{DK}| = |A_{00}^{DK}| = m_B \sqrt{\frac{8\pi}{p_{DK} \tau_0}} B^{DK}_{00},
\]

\[
\cos(\delta_T - \delta_C)^{DK} = \frac{|A_{+0}^{DK}|^2 - |A_{-0}^{DK}|^2 - |A_{00}^{DK}|^2}{2|A_{+0}^{DK}| \cdot |A_{00}^{DK}|} = \frac{(\tau_0/\tau_-) B^{DK}_{+0} - B^{DK}_{00} - B^{DK}_{-0}}{2\sqrt{B^{DK}_{+0} B^{DK}_{-0}}},
\]

where \(\tau_- (\tau_0)\) is the life time of \(B^- (\bar{B}^0)\). The magnitude of the momentum \(p_{DK}\) of the \(D(K)\) meson in the center of mass frame is given by

\[
p_{DK} = \frac{1}{2m_B} \sqrt{[m_B^2 - (m_D + m_K)^2][m_B^2 - (m_D - m_K)^2]}.
\]

Notice that \((\delta_T - \delta_C)^{DK}\) is the relative strong phase of the color-suppressed tree amplitude to the color-allowed tree amplitude.

Since the same relations (1) also hold for the corresponding \(B \to DK^*\) modes, the above result in (3) can be used for the relevant modes \(B^- \to D^0 K^{*-}\), \(\bar{B}^0 \to D^+ K^{*-}\) and \(\bar{B}^0 \to D^0 K^{*0}\) by simply replacing \(K\) by \(K^*\).

The experimental results on the BRs of \(B \to DK\) and \(DK^*\) as well as \(B \to D\pi\) and \(D\rho\) are shown in Table I. Using the measured BRs for \(B \to DK\) decays, we calculate the magnitudes of the color-allowed and the color-suppressed tree amplitudes and present the results in Table II. In Fig. 1, we show \(\cos(\delta_T - \delta_C)^{DK}\) versus \(|C^{DK}/T^{DK}|\). Due to the large uncertainty in the present data, it is still possible that the phase difference \((\delta_T - \delta_C)^{DK}\) vanishes. But, for the central values of the experimental data,

\[
(\delta_T - \delta_C)^{DK} = 63.0^\circ, \quad \text{or} \quad 297.0^\circ,
\]

\[
\frac{|C^{DK}|}{|T^{DK}|} = 0.50.
\]
TABLE I: The BRs of $B \to DK$, $DK^*(892)$, $D\pi$, and $D\rho$ modes in units of $10^{-4}$. 

| Mode       | Experimental value | Mode       | Experimental value |
|------------|--------------------|------------|--------------------|
| $B^- \to D^0K^-$ | $3.7 \pm 0.6$     | $B^- \to D^0K^*$ | $6.1 \pm 2.3$     |
| $\bar{B}^0 \to D^+K^-$ | $2.0 \pm 0.6$     | $\bar{B}^0 \to D^+K^*$ | $3.7 \pm 1.8$     |
| $\bar{B}^0 \to D^0\bar{K}^0$ | $0.50^{+0.13}_{-0.12} \pm 0.06$ | $\bar{B}^0 \to D^0\bar{K}^*$ | $0.48^{+0.11}_{-0.10} \pm 0.05$ |
| $B^- \to D^0\pi^-$ | $49.7 \pm 3.8$     | $B^- \to D^0\rho^-$ | $134 \pm 18$     |
| $\bar{B}^0 \to D^+\pi^-$ | $26.8 \pm 2.9$     | $\bar{B}^0 \to D^+\rho^-$ | $78 \pm 14$     |
| $\bar{B}^0 \to D^0\pi^0$ | $2.92 \pm 0.45$    | $\bar{B}^0 \to D^0\rho^0$ | $2.9 \pm 1.0 \pm 0.4$ |

TABLE II: The numerical results for $|T|$, $|C|$, $|C/T|$, and $\cos(\delta_T - \delta_C)$. The results shown in the last two columns are obtained from the $\chi^2$ fit for $(\chi^2_{min} + 1)$. 

| Mode   | $|T| \ (10^{-7})$ | $|C| \ (10^{-7})$ | $|C/T|$ | $\cos(\delta_T - \delta_C)$ |
|--------|-------------------|-------------------|--------|-----------------------------|
| $B \to DK$ | $1.35 \sim 1.85$ | $0.68 \sim 0.92$ | $0.42 \sim 0.56$ | $0.03 \sim 0.73$ |
| $B \to D\pi$ | $4.4 \sim 6.8$  | $1.7 \sim 3.8$  | $0.51 \sim 0.69$ | $0.02 \sim 0.73$ |
| $B \to DK^*$ | $1.60 \sim 2.73$ | $0.68 \sim 0.90$ | $0.31 \sim 0.40$ | $0.1 \sim 1.0$ |
| $B \to D\rho$ | $7.9 \sim 11.6$ | $1.3 \sim 4.1$ | $0.32 \sim 0.41$ | $0.1 \sim 1.0$ |

Further, the $1\sigma$ region (whose boundary is shown as the ellipse in Fig. 1) obtained from the $\chi^2$ analysis (See Sec. IV for more detailed discussion) indicates that 

$$0.03 \leq \cos(\delta_T - \delta_C)^{DK} \leq 0.73 \ , \ 0.42 \leq \frac{|C^{DK}|}{|T^{DK}|} \leq 0.56 \ ,$$

(6)

where the possibility that $(\delta_T - \delta_C)^{DK} = 0$ is excluded. We also note that the best fit values (shown as the black dot in Fig. 1) with $\chi^2_{min}$/d.o.f. = 0.19/1 are

$$ (\delta_T - \delta_C)^{DK} = 71.3^0 , \quad \frac{|C^{DK}|}{|T^{DK}|} = 0.49 \ ,$$

(7)

which are in good agreement with those obtained for the central values of the data in Eq. (5).

The strong phase difference is quite sizable. It is also interesting to note that the contribution from the color-suppressed tree diagram could be larger than the previously estimated one, e.g., $|C^{DK}/T^{DK}| \approx 0.2$ given in Ref. [12]. In other works, the large color-suppressed tree contribution is favored by the present experimental data. 

For $B \to DK^*$ modes, we present $\cos(\delta_T - \delta_C)^{DK^*}$ versus $|C^{DK^*}/T^{DK^*}|$ in Fig. 2, and
FIG. 1: For $B \to DK$ decays, $\cos(\delta_T - \delta_C)^DK$ versus $|C^{DK}/T^{DK}|$. The mark “x” in the center denotes the result obtained from the central values of the experimental data. [The black dot is obtained from the $\chi^2$ fit with $\chi^2_{min}/d.o.f. = 0.19/1$ (See Sec. IV). The ellipse corresponds to the ($\chi^2_{min} + 1$) case.]

show the magnitudes of $T^{DK^*}$ and $C^{DK^*}$ in Table II. For the central values of the data,

$$(\delta_T - \delta_C)^{DK^*} = 57.3^\circ, \text{ or } 302.7^\circ,$$

$$(C^{DK^*}) = 0.36.$$  

As in the case of $B \to DK$ decays, we obtain a similar result for $B \to DK^*$ decays: the phase difference is sizable and the large color-suppressed tree contribution is favored.
Now let us examine the validity of the factorization approximation in $B \to D K$ decays. In the naive factorization approximation, the topological amplitudes $T^{DK}$ and $C^{DK}$ are given by

$$T^{DK} = \frac{G_F}{\sqrt{2}} (V_{cb} V_{us}^*) a_1^{\text{eff}} \langle K^- | \bar{s} \gamma^\mu(1 - \gamma_5) u | 0 \rangle \langle D^+ | \bar{c} \gamma^\mu(1 - \gamma_5) b | B^0 \rangle$$

$$= i \frac{G_F}{\sqrt{2}} (V_{cb} V_{us}^*) a_1^{\text{eff}} (m_B^2 - m_D^2) f_K F_0^{B \to D} (m_K^2) ,$$

$$C^{DK} = \frac{G_F}{\sqrt{2}} (V_{cb} V_{us}^*) a_2^{\text{eff}} \langle D^0 | \bar{c} \gamma^\mu(1 - \gamma_5) u | 0 \rangle \langle K^0 | \bar{s} \gamma^\mu(1 - \gamma_5) b | B^0 \rangle$$

$$= i \frac{G_F}{\sqrt{2}} (V_{cb} V_{us}^*) a_2^{\text{eff}} (m_B^2 - m_D^2) f_D F_0^{B \to K} (m_D^2) ,$$

where $V_{cb}$ and $V_{us}$ are the relevant CKM matrix elements, and $a_1^{\text{eff}}$ and $a_2^{\text{eff}}$ are the effective Wilson coefficients. $f_K(D)$ and $F_0^{B \to K(D)}(m_{D(K)}^2)$ denote the decay constant of a $K(D)$ meson and the hadronic form factor for the $B \to K(D)$ transition at $q^2 \equiv (p_B - p_{K(D)})^2 = m_{D(K)}^2$, respectively. We obtain

$$\left| \frac{a_2^{\text{eff}}}{a_1^{\text{eff}}} \right| = \frac{1}{r^{DK}} \left| C^{DK} \right| = \frac{1}{r^{DK}} \sqrt{\frac{B^{DK}_{00}}{B^{DK}_{--}}} ,$$

where

$$r^{DK} \equiv \frac{(m_B^2 - m_D^2) f_D F_0^{B \to K} (m_D^2)}{(m_B^2 - m_K^2) f_K F_0^{B \to D} (m_K^2)} .$$

For the central values of the data, $r^{DK} = 0.81$ and $\sqrt{\frac{B^{DK}_{00}}{B^{DK}_{--}}} = 0.50$, which lead to $\left| \frac{a_2^{\text{eff}}}{a_1^{\text{eff}}} \right| = 0.62$. For the 1σ range of the experimental values of the BRs, we find

$$0.46 \leq \left| \frac{a_2^{\text{eff}}}{a_1^{\text{eff}}} \right| \leq 0.84 .$$

For comparison, in the PQCD approach [14], it is estimated that

$$\left| \frac{a_2^{\text{eff}}}{a_1^{\text{eff}}} \right| \sim 0.38 .$$

In QCD factorization, the effective Wilson coefficient $a_1^{\text{eff}}$ for $B \to D K$ modes is the same as that for $B \to D \pi$ modes, to a good approximation [6]. The $a_1^{\text{eff}}$ for $B \to D \pi$ is presented in next section. It is known [6] that in this approach $|a_2^{\text{eff}}|$ can not be reliably calculated, because the mechanism of color transparency is not operative for the class-2 decays, such as $\bar{B}^0 \to D^0 \pi^0$ and $\bar{B}^0 \to D^0 K^0$, where the emission particle is a heavy charm meson. In next section, an illustrative value of $|a_2^{\text{eff}}/a_1^{\text{eff}}|$ is shown.
Similarly, for $B \to DK^*$ decays, the amplitudes $T^{DK^*}$ and $C^{DK^*}$ can be written as

\[
T^{DK^*} = \frac{G_F}{\sqrt{2}} (V_{cb} V_{us}^*) a_1^{eff} \langle K^*|\bar{s}\gamma_\mu(1-\gamma_5)u|0\rangle \langle D^+|\bar{c}\gamma_\mu(1-\gamma_5)b|\bar{B}^0\rangle , \\
C^{DK^*} = \frac{G_F}{\sqrt{2}} (V_{cb} V_{us}^*) a_2^{eff} \langle D^0|\bar{c}\gamma_\mu(1-\gamma_5)u|0\rangle \langle K^*|\bar{s}\gamma_\mu(1-\gamma_5)b|\bar{B}^0\rangle .
\]

We use the following parametrization [15]:

\[
\langle 0|V_\mu|K^*\rangle = f_{K^*} m_{K^*} \epsilon_{K^*} \mu , \\
\langle K^*|\bar{s}\gamma_\mu(1-\gamma_5)b|\bar{B}^0\rangle = \epsilon_{\mu\nu\rho\sigma} \epsilon_{K^*}^\nu p_B^\rho \bar{p}_K^\sigma \frac{2V^{B\to K^*}(p_D^2)}{m_B + m_{K^*}} \\
- i\epsilon_{K^*}(m_B + m_{K^*})A_1^{B\to K^*}(p_D^2) \\
+ i(p_B + p_{K^*})\mu \epsilon_{K^*} \cdot p_B \frac{A_2^{B\to K^*}(p_D^2)}{m_B + m_{K^*}} \\
+ i p_D \mu \epsilon_{K^*} \cdot p_B \frac{2m_{K^*}}{p_D^2} \left(A_3^{B\to K^*}(p_D^2) - A_0^{B\to K^*}(p_D^2)\right) ,
\]

where $f_{K^*}$ and $\epsilon_{K^*}$ denote the decay constant and the polarization vector of the $K^*$ meson, respectively. $V^{B\to K^*}$ and $A_i^{B\to K^*}$ ($i = 0, 1, 2, 3$) are the form factors for the $B \to K^*$ transition and given by the QCD sum rules on the light-cone [15]. With these form factors, we obtain

\[
\frac{|a_2^{eff}|}{|a_1^{eff}|} = \frac{1}{r^{DK^*}} \left|\frac{C^{DK^*}}{T^{DK^*}}\right| = \frac{1}{r^{DK^*}} \sqrt{\frac{B_{00}^{DK^*}}{B_{-+}^{DK^*}}} .
\]

In the $B$ rest frame, $r^{DK^*}$ is given by $r^{DK^*} = \frac{4}{3}$, where

\[
a = \left\{-(m_B + m_{K^*})A_1^{B\to K^*}(m_D^2) + (m_B - m_{K^*})A_2^{B\to K^*}(m_D^2) + 2m_{K^*}(A_3^{B\to K^*}(m_D^2) - A_0^{B\to K^*}(m_D^2))\right\} (p_B \cdot \epsilon_{K^*}^\nu) , \\
b = \sqrt{\frac{B_{00}^{DK^*}}{B_{-+}^{DK^*}}} f_{K^*} \xi_D \lambda^{1/2}(m_B^2, m_D^2, m_{K^*}^2) .
\]

Here $\xi_D$ is the Isgur-Wise function and $\lambda(m_B^2, m_D^2, m_{K^*}^2) = m_B^3 + m_B^2 + m_{K^*}^2 - 2m_B^2m_D^2 - 2m_B^2m_{K^*}^2 - 2m_D^2m_{K^*}^2$. Using the central values of the data, we obtain $r^{DK^*} = 0.82$ and $\sqrt{\frac{B_{00}^{DK^*}}{B_{-+}^{DK^*}}} = 0.36$, which give $|a_2^{eff}/a_1^{eff}| = 0.44$. For the 1σ range of the experimental data, the allowed value of the ratio $|a_2^{eff}/a_1^{eff}|$ is in between 0.31 and 0.69.

From the above results, we see that if one assumes the naive factorization in $B \to DK$ and $B \to DK^*$ decays, the favored value of the ratio $|a_2^{eff}/a_1^{eff}|$ is much larger than the usual estimate $|a_2^{eff}/a_1^{eff}| \sim 0.25$ [12]. This can be possibly understood if the magnitude of the color-suppressed tree amplitude $C^{DK^*}$ is effectively enhanced due to non-factorizable contributions as in the PQCD approach [7] or final-state interactions [16].
III. $B \to D\pi$ AND $D\rho$ DECAY MODES

Let us turn to $B \to D\pi$ decays. The decay amplitudes can be represented in terms of the topological amplitudes $T^{D\pi}$, $C^{D\pi}$ and $E^{D\pi}$:

$$A_{0+}^{D\pi} \equiv A(B^- \to D^0\pi^-) = T^{D\pi} + C^{D\pi},$$
$$A_{+-}^{D\pi} \equiv A(B^0 \to D^+\pi^-) = T^{D\pi} + E^{D\pi},$$
$$\sqrt{2}A_{00}^{D\pi} \equiv \sqrt{2}A(B^0 \to D^0\pi^0) = -C^{D\pi} + E^{D\pi}, \quad (18)$$

where the topological amplitudes $T^{D\pi}$, $C^{D\pi}$ and $E^{D\pi}$ are defined as

$$X^{D\pi} \equiv |X^{D\pi}|e^{i\delta_X'} \equiv |V_{cb}V_{us}^*|a_X'e^{i\delta_X'}, \quad (X = T, C, E) \quad (19)$$

with the real amplitude $a_{T(C,E)}'$ and the strong phases $\delta_{T(C,E)}'$. The above amplitudes involve no weak phase because of the CKM factor $V_{cb}V_{us}^*$. From the measured BR for $B \to D\pi$ which involves only the $W$-exchange diagram, we obtain $|E^{D\pi}| = (0.71 \pm 0.10) \times 10^{-7}$ GeV. To be even more conservative, considering the SU(3) breaking effect, we allow that $|E^{D\pi}|$ lies within the $2\sigma$ range, which leads to $|E^{D\pi}| = (0.71 \pm 0.20) \times 10^{-7}$ GeV. Further, we allow that $(\delta_T' - \delta_E')^{D\pi}$ can vary from 0 to $2\pi$.

The amplitudes and the phase differences can be written as

$$|T^{D\pi}| = \sqrt{\frac{8\pi m_B^2 B^{D\pi}}{p_{D\pi} m_{D\pi}^2}} - |E^{D\pi}|^2 \sin^2(\delta_T' - \delta_E')^{D\pi} - |E^{D\pi}| \cos(\delta_T' - \delta_E')^{D\pi},$$
$$|C^{D\pi}| = \sqrt{\frac{16\pi m_B^2 B^{D\pi}}{p_{D\pi} m_{D\pi}^2}} - |E^{D\pi}|^2 \sin^2(\delta_C' - \delta_E')^{D\pi} + |E^{D\pi}| \cos(\delta_C' - \delta_E')^{D\pi},$$

$$\cos(\delta_T' - \delta_C')^{D\pi} = \frac{1}{2|T^{D\pi}||C^{D\pi}|} \left[ \frac{8\pi m_B^2 B^{D\pi}}{p_{D\pi} m_{D\pi}^2} - |T^{D\pi}|^2 - |C^{D\pi}|^2 \right],$$
$$\cos(\delta_C' - \delta_E')^{D\pi} = -\frac{1}{2|C^{D\pi}||E^{D\pi}|} \left[ \frac{16\pi m_B^2 B^{D\pi}}{p_{D\pi} m_{D\pi}^2} - |C^{D\pi}|^2 - |E^{D\pi}|^2 \right]. \quad (20)$$
The above relations hold for the corresponding $B \to D\rho$ decay modes as well and can be used for the relevant $B \to D\rho$ decays by simply replacing $\pi$ by $\rho$.

Similarly to the case of $B \to DK$ decays, using the experimental result for $B \to D\pi$, we compute the magnitudes of the relevant amplitudes and the phase differences. Our numerical result is shown in Fig. 3 as a graph of $\cos(\delta_T' - \delta_C')_{D\pi}$ versus $|C_{D\pi}/T_{D\pi}|$. The magnitudes of $T_{D\pi}$ and $C_{D\pi}$ are shown in Table II. The best fit values (shown as the black dot in Fig. 3) with $\chi^2_{\text{min}}/\text{d.o.f.} = 0.19/1$ are (See Sec. IV)

\[ |T_{D\pi}| = 5.85 \times 10^{-7}\text{GeV}, \quad |C_{D\pi}| = 3.56 \times 10^{-7}\text{GeV}, \]
\[ |E_{D\pi}| = 0.86 \times 10^{-7}\text{GeV}, \quad (\delta_T' - \delta_C')_{D\pi} = 71.3^\circ. \] (21)

For $B \to D\rho$ decays, we also obtain similar results. In this case, since only the upper bound for the BR of $\bar{B}^0 \to D_s^+K^{*-}$ (which involves only the annihilation contribution) is known at present, we use $|E_{D\rho}| = (0.71 \pm 0.20) \times 10^{-7}$ GeV as in the case of $B \to D\pi$. As we shall see in Sec. IV this treatment turns out to be reasonable. We present the graph of $\cos(\delta_T' - \delta_C')_{D\rho}$ versus $|C_{D\rho}/T_{D\rho}|$ in Fig. 4. The best fit values (shown as the black dot in Fig. 4) with $\chi^2_{\text{min}}/\text{d.o.f.} = 0.17/1$ are (See Sec. IV)

\[ |T_{D\rho}| = 9.57 \times 10^{-7}\text{GeV}, \quad |C_{D\rho}| = 3.55 \times 10^{-7}\text{GeV}, \]
\[ |E_{D\rho}| = 0.75 \times 10^{-7}\text{GeV}, \quad (\delta_T - \delta_C) = 32.4^\circ. \] (22)

Let us turn to examine the validity of the factorization in $B \to D\pi$ and $B \to D\rho$ decays.
FIG. 4: For $B \to D\rho$ decays, $\cos(\delta_T' - \delta_C')^{D\rho}$ versus $|C^{D\rho}/T^{D\rho}|$. [The black dot is obtained from
the $\chi^2$ fit with $\chi^2_{\text{min}}/\text{d.o.f.} = 0.17/1$ (See Sec. IV). The half ellipse corresponds to the $(\chi^2_{\text{min}} + 1)$
case.]

For $B \to D\pi$ decays, neglecting the small $E^{D\pi}$,

$$\left| \frac{a_2^{\text{eff}}}{a_1^{\text{eff}}} \right| = \frac{1}{r^{D\pi}} \left| \frac{C^{D\pi}}{T^{D\pi}} \right| = \frac{1}{r^{D\pi}} \sqrt{\frac{2B_{00}^{D\pi}}{B_{-+}^{D\pi}}} = 0.54 \sim 0.70 \ ,$$

(23)

where

$$r^{D\pi} = \frac{1}{\sqrt{2}} \frac{(m_B^2 - m_\pi^2)f_D F_0^{B \to \pi}(m_D^2)}{f_\pi F_0^{B \to D}(m_\pi^2)} = 0.54 \ .$$

(24)

For comparison, in PQCD calculation [14], it is predicted that

$$\left| \frac{a_2^{\text{eff}}}{a_1^{\text{eff}}} \right| = 0.42 \sim 0.50 \ ,$$

(25)

when the contribution from the exchange diagrams is neglected, and

$$\left| \frac{a_2^{\text{eff}}}{a_1^{\text{eff}}} \right| = 0.37 \sim 0.45 \ ,$$

(26)

when the contribution from the exchange diagrams is included. In the QCD factorization
approach [6], $|a_1^{\text{eff}}|$ is estimated as $|a_1^{\text{eff}}| \approx 1.05$. But, as commented in the previous section,
in this approach $|a_2^{\text{eff}}|$ can not be reliably estimated. For an illustration, a rough estimation
shows

$$\left| \frac{a_2^{\text{eff}}}{a_1^{\text{eff}}} \right| \sim 0.24 \ .$$

(27)

For $B \to D\rho$ decays, neglecting the small $E^{D\rho}$,

$$\left| \frac{a_2^{\text{eff}}}{a_1^{\text{eff}}} \right| = \frac{1}{r^{D\rho}} \left| \frac{C^{D\rho}}{T^{D\rho}} \right| = \frac{1}{r^{D\rho}} \sqrt{\frac{2B_{00}^{D\rho}}{B_{-+}^{D\rho}}} = 0.24 \sim 0.42 \ .$$

(28)
In the $B$ rest frame, $r^{D\rho}$ is given by $r^{D\rho} = \frac{a'}{b'} = 0.60$, where

$$a' = \frac{1}{\sqrt{2}} \left\{ -(m_B + m_\rho)A_1^{B\rightarrow\rho}(m_D^2) + (m_B - m_\rho)A_2^{B\rightarrow\rho}(m_D^2) + 2m_\rho \left( A_3^{B\rightarrow\rho}(m_D^2) - A_0^{B\rightarrow\rho}(m_D^2) \right) \right\} (p_B \cdot \epsilon_\rho^*) ,$$

$$b' = \sqrt{\frac{m_B m_D}{2m_B m_D}} f_{D\rho} \xi_D \lambda^{1/2}(m_B^2, m_D^2, m_\rho^2) .$$

(29)

As in the cases of $B \rightarrow DK$ and $B \rightarrow DK^*$, the large values of $|a_{2eff}/a_{1eff}|$ are favored for $B \rightarrow D\pi$ and $B \rightarrow D\rho$ decays. It indicates that for $B \rightarrow D\pi$ and $B \rightarrow D\rho$ the color-suppressed tree contributions to $C^{D\pi}$ and $C^{D\rho}$ are effectively enhanced. The possible mechanism for this enhancement is either the short-distance non-factorizable contribution [7] or large final-state rescattering interactions [16], or both of them.

IV. THE $\chi^2$ ANALYSIS USING $B \rightarrow DK$, $D\pi$, AND $B \rightarrow DK^*$, $D\rho$

In order to find the most likely values of the magnitudes of the topological amplitudes and the strong phase shifts in $B \rightarrow DK$, $D\pi$, and $DK^*$, $D\rho$, we do the $\chi^2$ analysis using the BRs of these decay processes. First we assume the flavor SU(3) symmetry between the topological amplitudes for $B \rightarrow DK$ and $D\pi$ (similarly for $B \rightarrow DK^*$ and $D\rho$). Then we will take into account the SU(3) breaking effect.

A. The $B \rightarrow DK$ and $D\pi$ case

Assuming the flavor SU(3) symmetry, we have six observables (the measured BRs of $B \rightarrow DK$ and $D\pi$, as shown in Table I) and five parameters $[|T^{DK}|, |C^{DK}|, |E^{D\pi}|, (\delta_T - \delta_C), (\delta_T - \delta_E)]$ so that the degrees of freedom (d.o.f.) for the fit is 1. Without considering the SU(3) breaking effect, $T^{D\pi}$ and $C^{D\pi}$ are given by $T^{D\pi} = T^{DK} \left( \frac{V_{ud}}{V_{us}} \right)$ and $C^{D\pi} = C^{DK} \left( \frac{V_{ud}}{V_{us}} \right)$, respectively. In this case we find that $\chi^2_{min/d.o.f.} = 3.34/1$ indicating a poor fit. Taking into account the SU(3) breaking at first order, such as $T^{D\pi} = T^{DK} \left( \frac{V_{ud}f_\pi}{V_{us}f_K} \right)$ and $C^{D\pi} = C^{DK} \left( \frac{V_{ud}}{V_{us}} \right)$, we find the best fit with $\chi^2_{min/d.o.f.} = 0.19/1$. The corresponding parameter values are

$$|T^{DK}| = 1.64 \times 10^{-7}\text{GeV}, \quad |C^{DK}| = 0.81 \times 10^{-7}\text{GeV}, \quad |E^{D\pi}| = 0.86 \times 10^{-7}\text{GeV}, \quad (\delta_T - \delta_C) = 71.3^\circ, \quad (\delta_T - \delta_E) = 91.2^\circ .$$

(30)
The $\cos(\delta_T - \delta_C)$ versus $|C/T|$ obtained from the above ones for $\chi^2_{\text{min}}$ is depicted for $B \to DK$ in Fig. 2 and for $B \to D\pi$ in Fig. 4. The result for $(\chi^2_{\text{min}} + 1)$ is also shown as an ellipse in the same figures and their numerical values are shown in Table II.

For the best fit, we find that $|C^{DK}/T^{DK}| = 0.49$, which indicates the relatively large color-suppressed tree contribution. The best fit value for $|E^{D\pi}|$ is in good agreement with $|E^{D\pi}| = (0.71 \pm 0.20) \times 10^{-7}$ GeV used in Sec. III. Our result indicates that the exchange contribution in $B \to D\pi$ decay can be sizably enhanced as well, which is contrary to the usual estimate in the QCD factorization. Notice that within the flavor SU(3) symmetry with a reasonable SU(3) breaking effect in the $B \to DK$ and $D\pi$ decays, the strong phase difference between the color-allowed and -suppressed decay amplitudes does not vanish at the level of one standard deviation.

**B. The $B \to DK^*$ and $D\rho$ case**

Similarly to the $B \to DK$ and $D\pi$ case, we have the six measured BRs of $B \to DK^*$ and $D\rho$ and the five parameters. Assuming the flavor SU(3) symmetry $[T^{D\rho} = T^{DK^*}(V_{ud}/V_{us})]$, we find that $\chi^2_{\text{min}}/\text{d.o.f.} = 0.10/1$. The corresponding parameters are

$$
|T^{DK^*}| = 2.24 \times 10^{-7}\text{GeV}, \quad |C^{DK^*}| = 0.81 \times 10^{-7}\text{GeV}, \quad |E^{D\rho}| = 0.86 \times 10^{-7}\text{GeV},
(\delta_T - \delta_C) = 39.7^\circ, \quad (\delta_T - \delta_E) = 65.3^\circ. \tag{31}
$$

Taking into account the SU(3) breaking effect $[T^{D\rho} = T^{DK^*}(V_{ud}/f_\rho V_{us}/f_K^*)$ and $C^{D\rho} = C^{DK^*}(V_{ud}/V_{us})]$, we find another good fit with $\chi^2_{\text{min}}/\text{d.o.f.} = 0.17/1$. The corresponding parameters in this case are

$$
|T^{DK^*}| = 2.23 \times 10^{-7}\text{GeV}, \quad |C^{DK^*}| = 0.81 \times 10^{-7}\text{GeV}, \quad |E^{D\rho}| = 0.75 \times 10^{-7}\text{GeV},
(\delta_T - \delta_C) = 32.4^\circ, \quad (\delta_T - \delta_E) = 34.8^\circ. \tag{32}
$$

The numerical values of $|C/T|$ and $\cos(\delta_T - \delta_C)$ for $(\chi^2_{\text{min}} + 1)$ are shown in Table II.

Unlike the $B \to DK$ and $D\pi$ case, good fits are obtained for both cases of the SU(3) symmetry and the broken SU(3) symmetry. This can be understood that in the $B \to DK^*$ and $D\rho$ case the flavor SU(3) breaking factor $f_\rho/f_{K^*}$ is almost equal to unity, while in the $B \to DK$ and $D\pi$ case the breaking factor $f_\pi/f_K$ is relatively large. Further, the parameters obtained in the case of the SU(3) symmetry are quite similar to those obtained in the broken...
SU(3) symmetry, except the parameter \((\delta_T - \delta_E)\) which shows a sizable difference in the two cases. The black dots in Figs. 3 (for \(B \to D K^*\)) and 4 (for \(B \to D \rho\)) show \(\cos(\delta_T - \delta_C)\) versus \(|C/T|\) for \(\chi_{\text{min}}^2/d.o.f. = 0.17/1\). Those values for \((\chi_{\text{min}}^2 + 1)\) are shown as a half ellipse in the same figures.

We notice that the relatively large color-suppressed tree contribution is favored in the \(B \to D K^*\) and \(D \rho\) case as well: \(|C/T| = 0.36\) for \(\chi_{\text{min}}^2/d.o.f. = 0.17/1\). The magnitude of \(E^{D\rho}\) is also consistent with the one used in Sec. III. It implies that the BR for \(\bar{B}^0 \to D_s^+ K^{*-}\) would be similar to that for \(\bar{B}^0 \to D_s^+ K^-\). This will be tested with future experimental results on the BRs for these decay modes.

V. FLAVOR SU(3) SYMMETRY BREAKING EFFECT

Let us estimate the flavor SU(3) symmetry breaking effect in \(B \to D K\) (\(D K^*\)) and \(B \to D \pi\) (\(D \rho\)) decays. If flavor SU(3) were exact, one would get for \(B \to D K\) and \(D \pi\),

\[
\frac{T^{DK}}{V_{cb}V_{us}^{*}} = \frac{T^{D\pi}}{V_{cb}V_{us}^{*}}, \quad \frac{C^{DK}}{V_{cb}V_{us}^{*}} = \frac{C^{D\pi}}{V_{cb}V_{us}^{*}},
\]

and for \(B \to D K^*\) and \(D \rho\),

\[
\frac{T^{DK*}}{V_{cb}V_{us}^{*}} = \frac{T^{D\rho}}{V_{cb}V_{us}^{*}}, \quad \frac{C^{DK*}}{V_{cb}V_{us}^{*}} = \frac{C^{D\rho}}{V_{cb}V_{us}^{*}}.
\]

To estimate the SU(3) breaking effect, let us take the central values of the data as a typical example. We find for \(B \to D K\) and \(B \to D \pi\),

\[
\frac{T^{DK}/(V_{cb}V_{us}^{*})}{T^{D\pi}/(V_{cb}V_{us}^{*})} = 1.21, \quad \frac{C^{DK}/(V_{cb}V_{us}^{*})}{C^{D\pi}/(V_{cb}V_{us}^{*})} = 1.29,
\]

and for \(B \to D K^*\) and \(B \to D \rho\),

\[
\frac{T^{DK*}/(V_{cb}V_{us}^{*})}{T^{D\rho}/(V_{cb}V_{us}^{*})} = 0.96, \quad \frac{C^{DK*}/(V_{cb}V_{us}^{*})}{C^{D\rho}/(V_{cb}V_{us}^{*})} = 1.27.
\]

The above result shows that the SU(3) breaking effect can be sizable: i.e., about \((20 - 30)\%\) at the amplitude level, except the color-allowed tree amplitudes for \(B \to D K^*\) and \(D \rho\). Our result for the color-allowed tree amplitudes in \([8]\) and \([9]\) agrees with that of Ref. \([8]\). But, the result for the color-suppressed tree amplitudes does not agree with the estimate in the naive factorization shown in \([8]\) and shows about two or three times larger breaking effect. It again indicates that the color-suppressed tree amplitudes can not be reasonably estimated by the naive factorization, because they can be effectively enhanced by non-factorizable effect and final-state interactions, as discussed before.
VI. CONCLUSION

We studied $B \to D K$, $D K^*$ and $B \to D \pi$, $D \rho$ decay processes in a model-independent way. Using the quark diagram decomposition of the decay amplitudes and the present experimental result on the relevant BRs, we determined the magnitudes and the relative strong phase shifts of the relevant amplitudes.

First we analyzed the $B \to D K^{(*)}$ and $B \to D \pi \ (D \rho)$ modes separately from each other so that the flavor SU(3) symmetry is not needed to combine the relevant amplitudes in $B \to D K^{(*)}$ and $B \to D \pi \ (D \rho)$ with each other. As shown in Sec. V, the SU(3) breaking effect can be sizable in these modes. Further, in order to determine the most likely values for the relative strong phases and the magnitudes of the amplitudes in a statistically reliable way, we used the $\chi^2$ minimization technique. In this case, we used the flavor SU(3) symmetry, but took its breaking effect into account as well.

Our results show that the strong phase differences between the color-allowed and the color-suppressed tree amplitudes can be large: for instance, for the $B \to D K \ (D \pi)$ mode, the best fit value for $(\delta_T - \delta_C)$ is $71.3^\circ$. It should be emphasized that $(\delta_T - \delta_C)$ is non-zero at 1σ level (Figs. 1 and 3). This result is obtained from the statistical approach and clearly different from those of the previous works, where $(\delta_T - \delta_C)$ was assumed to be 0° [8, 12], or the vanishing $(\delta_T - \delta_C)$ could not be excluded with the present data [10].

Another interesting result is that in $B \to D K^{(*)}, D \pi, D \rho$ decays, the color-suppressed tree contributions are effectively enhanced, which is inconsistent with the naive expectation in the factorization approximation. For example, the best fit value is $|C/T| = 0.49$ for $B \to D K$, and $|C/T| = 0.60$ for $B \to D \pi$. These ratios are quite larger than previously estimated ones as in [8, 12], but are consistent with the recent results as in [7, 16, 17].

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REFERENCES

[1] P. Krokovny et al. [Belle Collaboration], Phys. Rev. Lett. 90, 141802 (2003) arXiv:hep-ex/0212066.

[2] D. Atwood, I. Dunietz and A. Soni, Phys. Rev. Lett. 78, 3257 (1997) arXiv:hep-ph/9612433; Phys. Rev. D 63, 036005 (2001) arXiv:hep-ph/0008090.

[3] M. Gronau and J. L. Rosner, Phys. Lett. B 439, 171 (1998) arXiv:hep-ph/9807447.

[4] J. H. Jang and P. Ko, Phys. Rev. D 58, 111302 (1998) arXiv:hep-ph/9807496.

[5] C. S. Kim and S. Oh, Eur. Phys. J. C 21, 495 (2001) arXiv:hep-ph/0009082.

[6] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 591, 313 (2000) arXiv:hep-ph/0006124.

[7] Y. Y. Keum, T. Kurimoto, H. N. Li, C. D. Lu and A. I. Sanda, Phys. Rev. D 69, 094018 (2004) arXiv:hep-ph/0305335.

[8] Z. z. Xing, High Energy Phys. Nucl. Phys. 26, 100 (2002) arXiv:hep-ph/0107257.

[9] M. Neubert and A. A. Petrov, Phys. Lett. B 519, 50 (2001) arXiv:hep-ph/0108103.

[10] C. W. Chiang and J. L. Rosner, Phys. Rev. D 67, 074013 (2003) arXiv:hep-ph/0212274.

[11] H. Y. Cheng, Phys. Rev. D 65, 094012 (2002) arXiv:hep-ph/0108096.

[12] Z. z. Xing, Eur. Phys. J. C 28, 63 (2003) arXiv:hep-ph/0301024.

[13] M. Gronau, O. F. Hernandez, D. London and J. L. Rosner, Phys. Rev. D 50, 4529 (1994) arXiv:hep-ph/9404283; ibid. 52, 6356 (1995) arXiv:hep-ph/9504326; ibid. 52, 6374 (1995) arXiv:hep-ph/9504327.

[14] Y. Y. Keum, 3rd Workshop On The Unitarity Triangle: CKM 2005, 15-18 March 2005, San Diego, California.

[15] P. Ball, eConf C0304052, WG101 (2003) arXiv:hep-ph/0306251.
[16] H. Y. Cheng, C. K. Chua and A. Soni, arXiv:hep-ph/0409317.

[17] S. Mantry, D. Pirjol and I. W. Stewart, Phys. Rev. D 68, 114009 (2003) arXiv:hep-ph/0306254.