Modeling the filtration of liquid with ultra disperse solid particles in porous media

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Abstract. This paper considers an approach for modeling the filtration of a viscous liquid, containing ultra disperse solid particles, through the porous sample. The mechanism of microparticle retention during filtration of the suspension at supported pressure drop is discussed and the dependence of reduction of average permeability on particles size is documented.

1. Introduction
The depletion of traditional oil fields has led to the search for the methods of oil recovery from complex and unconventional reservoirs, which primarily include the rocks of the Bazhenov formation. The collector of this formation contains clay rocks and during filtration of the reservoir liquid, micro- and nanoparticles detach from the rock, causing pores plugging. Other important problems associated with the retention of the particle are mudcake formation during well drilling and migration of the proppant particles used for the hydraulic fracturing. The main problems which are relevant to fundamental understanding of deep bed filtration are the nature and conditions leading to the retention of particles throughout a porous sample, the change of pore structure due to deposition, and its effect on formation performance.

Typically, the following three mechanisms for retaining microparticles in the pores are considered: attachment, mechanical filtration, and sticking of particles in narrow pore throats (straining) [1]. The filtration of water, containing silicon oxide nanoparticles in the microspheres packed bed, and sedimentary rocks were studied in [2, 3] and [4] accordingly. It was shown that surface-modified nanoparticles could be transported through the sedimentary rock and the retention mechanism is reversible adsorption on the pore surface. The influence of various parameters, such as the particle size, pore size, ionic stress and filtration rate on permeability decrease during mechanical filtration was studied in [5, 6]. It was shown that the ratio of the particle size to the grain size defines considerably the pore plugging for this case. Another important parameter is the microparticle content in a fluid [7]. However, there is little available literature on the modelling of the pore plugging when the parameters of mechanical retention of the particle are discussed using experimental data.

The presence of several mechanisms of the pore plugging complicates significantly the formulating an adequate model of the damage of porous media during the filtration of liquid containing solid particles. The aim of this study is defining peculiarities of the retention of ultra disperse solid particles with the size comparable to the size of pore throats at supported pressure drop, when mechanical retention becomes preferable.
2. Model formulation

The filtration of reservoir fluid occurs under the action of external as well as reservoir pressure. Typically, reservoir fluid contains solid micro- and nanoparticles that are present in injected fluid or arise during the mobilization of natural minerals contained in the formation. Due to the retention of the solid particles in the channels, an irreversible decrease in the reservoir permeability occurs in the case of mechanical filtration due to blocking the pore throats by solid material. The degree of damage of the formation for this case is determined by many parameters such as density and apparent viscosity of liquid, filtration time, porosity, permeability, physicochemical properties of the rock, etc.

Mathematical model of the capture of microparticles, based on their mechanical retention in pore throats, is formulated in a framework of the linear theory of suspension filtration [8] without accounting the mobilization of retained particles by the flow. It allows us to use the retention coefficients $K$ to characterize the damage of the reservoir during filtration of the suspension in a porous medium. This model assumes that the retention rate is proportional to the filtration velocity and particle concentration in the flow. The governing system of equations includes the mass conservation equation for solid particles equation (1), the kinetic equation for particles accumulation showing the rate of porosity reduction during the capture of solid particles equation (2) and the continuity equation (3).

\[
\frac{\partial (m c)}{\partial t} + \frac{\partial c}{\partial x} = \frac{\partial m}{\partial t} \tag{1}
\]

\[
\frac{\partial \sigma}{\partial t} = -K \nu c \tag{2}
\]

\[
\frac{\partial \nu}{\partial x} = 0 \tag{3}
\]

Here $m$ and $m_0$ are the current and initial porosities, $\sigma = m - m_0$ is the porosity variation, $K$ is the filtration coefficient, $\nu$ is Darcy velocity, $k$ and $k_0$ are the current and initial permeability of the porous medium, $c$ is the volume concentration of particles. It is assumed that at a supported pressure drop on the sample, the blockage of the pore channels causes a decrease in the velocity of suspension and detachment of the particles by the flow is unfavorable.

Assuming that the concentration of particles is small, we can neglect the first term in the right-hand side of equation (1) as follows

\[
\frac{\partial \sigma}{\partial t} + \frac{\partial c}{\partial x} = 0 \tag{4}
\]

Substituting equation (2) into equation (4) and assuming that the filtration coefficient does not vary along the sample, one can obtain the equation for the concentration of the particles

\[
c(x) = c_0 \exp(-Kx) \tag{5}
\]

Substituting equation (5) into equation (2), and integrating over time, one can obtain the relation for determining the variation of the porosity as a function of time and distance from the inlet

\[
m(y,t) = m_0 - Kc_0 \exp(-Kx) \int_0^t \nu(t) dt \tag{6}
\]
The equation (6) can be used during processing of the experimental data for determining the filtration coefficient if the porosity is measured. If permeability is measured during the tests, then the model of the connection between porosity and permeability is needed. The following models can be used:

- model of a uniform coating of the parallel pores while the tortuosity variation does not take into account during plugging

\[ k = k_0 \times (m/m_0)^3 \]  

(7)

- “hydraulic radius” model that takes into account conservation of the granule size and specific surface area of porous media

\[ k = k_0 \times (m/m_0)^{3/2} / \left( (1 - m)/(1 - m_0) \right)^{3/2} \]  

(8)

The equation (8) assumes that specific surface area changes slowly in time and relative volume of the retained microparticles is small.

Taking into account equation (7), one obtained the equation for determining the variation of permeability along the length of the sample and in time during the pore plugging

\[ k(x, t) = k_0 (1 - Kc_0 \exp(-Kx) \int_0^t v(t) dt / m_0)^3 \]  

(9)

If the measure of the permeability variation occurs at the variable in time pressure drop, the Darcy equation for determination of the filtration velocity is necessary

\[ v(t) = -\frac{k(x)}{\mu} \frac{\partial p}{\partial x}, \quad v(t) = \frac{\Delta p(t)}{\int_0^L \frac{1}{k(x, t)} dx} \]  

(10)

Combining equation (8) and equation (9) gives the integral equation for determining the variation of the permeability in time

\[ k(x, t) = k_0 (1 - Kc_0 \exp(-Kx) \left( \int_0^\xi \frac{\Delta p(\zeta)}{\mu} \frac{d\zeta}{k(\eta, \zeta)} \right) / m_0)^3 \]  

(11)

This equation can be used for determination of the filtration coefficient using the experimental data.

3. Pore plugging peculiarities for the mechanical filtration

The proposed model was used for the determination of the dependence of filtration coefficient on the size of the particles using the experimental data [6]. In this paper, the filtration of a water-glycerin solution with aluminosilicate microparticles in a porous sample formed by the glass-sphere filling with an average size of 1.13 mm was considered. The particle-to-filling granule diameter ratio was varied from 0.046 to 0.109 and the volume particle concentration was varied from 0.001 to 0.02. For supported pressure drop, the filtration velocity was measured during the tests and dependence of the mean permeability on filtrate volume was obtained. Experimental data for the reduction of
Figure 1. Reduction of dimensionless average permeability depending on pumping volume of filtrate divided to pore volume during filtration of water glycerin solution containing microparticles with $d_p = 92.7 \mu m$ and $c_0=0.0003$, solid line shows the result of calculation for $K= 1.2$.

Dimensionless average permeability depending on pumping volume of filtrate divided on pore volume during filtration of water glycerin solution containing microparticles with $d_p = 92.7 \mu m$ and $c_0=0.0003$ are presented in figure 1. The solid line here shows the results of calculations of the reduction of permeability according to equation (11) if the filtration coefficient equals 1.2. As it is seen, the calculation results are in reasonable agreement with experimental data and proposed model can be used for prediction of the formation damage during filtration of reservoir fluid containing the solid microparticles.

Using experimental data [6] for other diameters of the particles, the dependence of filtration coefficient on the normalized diameter of solid particles was obtained. These data are presented in

Figure 2. Dependence of the filtration coefficient from the normalized diameter of solid particles.
figure 2 for filtration of microparticles with the average diameter $d_p$ of 77, 92.7 and 123 $\mu$m in water-glycerin solution with the viscosity of 20 cP in packed bed formed by spheres with diameter $D_g = 1.13$ mm. It was obtained that the filtration coefficient grows with increasing the particle-to-filling granule diameter ratio that is typical of mechanical filtration.

**Conclusions**

In this paper, the retention of ultra disperse solid particles with the size comparable to the size of pore throats during mechanical filtration is considered using a mathematical model of the capture of microparticles when filtration occurs under the supported pressure drop. Using analytical analysis, the equation for determination of the filtration coefficient using experimental data was obtained. This retention coefficient characterizes the damage of the reservoir during the filtration of suspension in a porous medium. It was shown that the filtration coefficient grows with increasing particle-to-filling granule diameter ratio. The approach proposed will promote understanding of the complex phenomena essential for the formulation of the comprehensive deep bed filtration theory that can be used for prediction of filtration in unconventional reservoirs including Bazhenov suite presented by kerogen–clayey–carbonate–siliceous rocks.

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