Quantum numbers of the pentaquark states $P_c^+$ via symmetry analysis

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Abstract
We investigate the quantum numbers of the pentaquark states $P_c^+$, which are composed of 4 (three flavors) quarks and an antiquark, by analyzing their inherent nodal structure in this paper. Assuming that the four quarks form a tetrahedron or a square, and the antiquark is at the ground state, we determine the nodeless structure of the states with orbital angular momentum $L \leq 3$, and in turn, the accessible low-lying states. Since the inherent nodal structure depends only on the inherent geometric symmetry, we propose the quantum numbers $J^P$ of the low-lying pentaquark states $P_c^+$ may be $\frac{3}{2}^-$, $\frac{5}{2}^-$, $\frac{3}{2}^+$ and $\frac{5}{2}^+$, independent of dynamical models.

Keywords: pentaquark states, exotic hadrons, color confinement, symmetry analysis

1. Introduction
Searching for the multiquark states has attracted great attention for more than four decades [1–7]. Analyzing the $J/\psi p$ resonances in experiments at the LHCb indicates that there exist pentaquark states including heavy quarks (c and $\bar{c}$) [8]. One of the two resonances has a mass of $4380 \pm 8 \pm 29$ MeV and a width of $205 \pm 18 \pm 86$ MeV, and another has a mass of $4498.8 \pm 1.7 \pm 2.5$ MeV and a width of $39 \pm 5 \pm 19$ MeV. They are proposed to hold opposite parity, with one state having spin 3/2 and the other 5/2. Even though quite a lot of theoretical investigations have been accomplished consequently (see for example, [9–29], for reviews, see [3, 4]), the concrete spin and parity have not yet been well-assigned to the two states. Recently the pentaquark states have been updated as $4312$ MeV with a width about $9.8$ MeV, $4440$ MeV with a width about $20.6$ MeV and $4457$ MeV with a width about $20.6$ MeV [30], and the quantum numbers of the states have not yet been determined either [31], although many theoretical works have tried to shed light on this. For instance, by considering the states with molecular structure, the quantum numbers are proposed as $J^P = \frac{1}{2}^- \cdot \frac{1}{2}^- \cdot \frac{3}{2}^-$ (or $\frac{5}{2}^-$) (see for example [32–39]), or $\frac{1}{2}^- \cdot \frac{3}{2}^-$, and excited states with $J^P = \frac{1}{2}^+$ [40], which are not consistent with those proposed in reference [41]. Furthermore, stringently speaking, pentaquark states refer to the color singlets composed of 4 quarks and 1 antiquark. In such a compact pentaquark scenario, their quantum numbers are proposed as $J^P = \frac{3}{2}^- \cdot \frac{1}{2}^- \cdot \frac{3}{2}^-$ via considering the uncertainty and the rearrangement decay properties (see, e.g., reference [42]). Since fundamental calculation for the pentaquark states via QCD is not available at the present stage, we investigate the quantum numbers by analyzing the symmetry, more explicitly the intrinsic nodal structure, of the color singlets including 4 three flavor quarks and 1 antiquark in this paper.

The remainder of this paper is organized as follows: in section 2, we give a classification of the wave functions of the pentaquark states according to the symmetries of the quarks. Then in section 3 we assume that the 4 quarks form a tetrahedron or a square and analyze the nodeless structure of the states with orbital angular momentum $L \leq 3$, in turn, proposing the quantum numbers of the pentaquark states. Finally, we give our conclusion and some remarks in the last section.

2. Structure of the wave functions
It has been known that the wave functions of few-body systems can usually be written as a coupling of the orbital part
and an internal part, and the inherent nodal surface (INS) analysis approach is quite powerful to assign the quantum numbers to a system (see, e.g., references [43–46]). One can definitely take the INS analysis approach to investigate the pentaquark states \( (q^4q) \) systems at the quark model level since they are typically few-body systems. Since quarks and antiquarks can never identified with each other, there does not exist antisymmetric restrictions on the wave function when interchanging a quark with the antiquark. However, if we consider only the 4 quarks, it should be antisymmetric, i.e., it has the symmetry \([14] \), if we ignore the current quark mass difference. Considering the system of the 4 quarks involve three flavors (e.g., \( uudc, uuds, uudb \), and so on), one knows well that the system possesses the internal symmetry \( SU_{CSF}(18) \supset SU_C(3) \otimes SU_F(3) \otimes SU_S(2) \). Meanwhile the orbital part holds the symmetry of the permutation group \( S_4 \).

Let \( [f]_{LO} \), \( [f]_{IC} \) and \( [f]_{FS} \) be the irreducible representation (IRREP) of the groups associated with the orbital, the color and the flavor-spin space, respectively, we should have

\[
[f] = [f]_{LO} \otimes [f]_{IC} \otimes [f]_{FS}.
\]

The lack of experimental observation of free 'color charge' indicates that all the hadronic states should be \( SU(3) \) color-singlets. For the \( qqqq \) system, the IRREP of the \( SU(3) \) color space should be \([222] \). Consequently, we have the IRREP of the color part of the 4-quark system as \([f]_{IC} = [211] \). Then \([f]_{LO} \otimes [f]_{FS} \) should be \([31] \), which is the conjugate of \([211] \). In turn, the configuration of \([f]_{LO} \) and \([f]_{FS} \) can be fixed with the reduction principle of the direct product of the IRREPs of the unitary group.

The obtained possible \([f]_{FS} \) and the corresponding \([f]_{LO} \) are listed in table 1. Corresponding to each IRREP \([f]_{FS} \), the flavor and spin decomposition of the three flavors \( q^4q \) state and the \( q^4\bar{q} \) state have been given in reference [47]. We recall them here in tables 2 and 3.

It is obvious that such an orbital and flavor-spin configuration space is very large. Therefore we should pick out the significant ones for the low-lying pentaquark states.

### 3. Inherent nodal structure analysis

From tables 1–3 we can notice that, given an IRREP of the orbital wave function, we can deduce the corresponding flavor and spin part of the wave function. Generally, the orbital wave function depends on the geometric configuration and the dynamics, or, in the present article, we should say models.

In this section, we analyze the structure of the wave functions of the pentaquark states independent of dynamical models.

The zero points for the wave function are known as nodes. The locus for the nodes is a surface, i.e. the nodal surface, in configuration space. For usual bound states of a quantum system, the fewer nodes the configuration contains, the lower energy the state has. The dynamical nodal surface depends on the dynamics of the system, while the INS relies only on the inherent geometric configuration of the system.

Let \( \mathcal{A} \) be a geometric configuration in the coordinate space of a quantum system, and let \( \hat{O} \) be an element of the operation on the wave function \( F(\mathcal{A}) \), we have

\[
\hat{O} F(\mathcal{A}) = F(\hat{O}\mathcal{A}).
\]

If the operation \( \hat{O} \) leaves the configuration \( \mathcal{A} \) invariant, i.e. \( \hat{O}\mathcal{A} = \mathcal{A} \), the above equation becomes

\[
\hat{O} F(\mathcal{A}) = F(\mathcal{A}).
\]

If there do not exist common non-zero solutions for the wave functions governed by equation (3), all the \( F(\mathcal{A}) \) must be zero at the configuration \( \mathcal{A} \), then a nodal surface appears.

The wave function of the 4 quarks with total angular momentum \( J \) and parity \( \pi \) can be written as

\[
\Psi = \sum_{L,S,\lambda} \Psi_{L,S,\lambda}^S.
\]
Table 3. Spin-flavor decomposition of the $q^4q$ states (taken from reference [47]). The subscripts stand for the dimensions of the IRREP.

| $\delta_{ij}$ | $SU_q(6)$ | $SU(3)$ | $SU(2)$ |
|---------------|------------|----------|----------|
| [4]           | $[51111]_{700}$ | $[51]_6 \otimes [5]_6$ | $[41]_4 \otimes [41]_4$ |
|               | $[51]_6 \otimes [41]_4$ | $[42]_7 \otimes [41]_4$ | $[42]_7 \otimes [32]_2$ |
|               | $[42]_7 \otimes [32]_2$ | $[33]_{10} \otimes [32]_2$ | $[41]_1 \otimes [5]_6$ |
|               | $[41]_1 \otimes [5]_6$ | $[41]_1 \otimes [41]_4$ | $[41]_1 \otimes [32]_2$ |
|               | $[41]_1 \otimes [32]_2$ | $[32]_1 \otimes [32]_2$ | $[32]_1 \otimes [32]_2$ |

| [4] + [31]    | $[411111]_{56}$ | $[41]_1 \otimes [41]_4$ | $[32]_1 \otimes [32]_2$ |

| [31]          | $[421111]_{1134}$ | $[51]_6 \otimes [41]_4$ | $[51]_6 \otimes [32]_2$ |
|               | $[42]_7 \otimes [41]_4$ | $[42]_7 \otimes [32]_2$ | $[33]_{10} \otimes [32]_2$ |
|               | $[33]_{10} \otimes [32]_2$ | $[41]_1 \otimes [5]_6$ | $[41]_1 \otimes [41]_4$ |
|               | $[41]_1 \otimes [41]_4$ | $[41]_1 \otimes [32]_2$ | $[32]_1 \otimes [32]_2$ |
|               | $[32]_1 \otimes [32]_2$ | $[32]_1 \otimes [32]_2$ | $[32]_1 \otimes [32]_2$ |

| [31] + [22] + [211] | $[321111]_{70}$ | $[41]_1 \otimes [32]_2$ | $[32]_1 \otimes [32]_2$ |
|                   | $[32]_1 \otimes [32]_2$ | $[32]_1 \otimes [32]_2$ | $[32]_1 \otimes [32]_2$ |
|                   | $[32]_1 \otimes [32]_2$ | $[32]_1 \otimes [32]_2$ | $[32]_1 \otimes [32]_2$ |

| [22]           | $[33111]_{360}$ | $[51]_6 \otimes [32]_2$ | $[42]_2 \otimes [41]_4$ |
|               | $[42]_2 \otimes [41]_4$ | $[42]_2 \otimes [32]_2$ | $[33]_{10} \otimes [5]_6$ |
|               | $[33]_{10} \otimes [5]_6$ | $[33]_{10} \otimes [41]_4$ | $[33]_{10} \otimes [32]_2$ |
|               | $[33]_{10} \otimes [32]_2$ | $[41]_1 \otimes [41]_4$ | $[41]_1 \otimes [32]_2$ |
|               | $[41]_1 \otimes [32]_2$ | $[32]_1 \otimes [32]_2$ | $[32]_1 \otimes [32]_2$ |
|               | $[32]_1 \otimes [32]_2$ | $[32]_1 \otimes [32]_2$ | $[32]_1 \otimes [32]_2$ |

| [211]          | $[32111]_{540}$ | $[42]_7 \otimes [41]_4$ | $[42]_7 \otimes [32]_2$ |
|               | $[42]_7 \otimes [32]_2$ | $[33]_{10} \otimes [41]_4$ | $[33]_{10} \otimes [32]_2$ |
|               | $[33]_{10} \otimes [32]_2$ | $[41]_1 \otimes [32]_2$ | $[41]_1 \otimes [32]_2$ |
|               | $[41]_1 \otimes [32]_2$ | $[32]_1 \otimes [32]_2$ | $[32]_1 \otimes [32]_2$ |
|               | $[32]_1 \otimes [32]_2$ | $[32]_1 \otimes [32]_2$ | $[32]_1 \otimes [32]_2$ |

Table 3. (Continued.)

| $\delta_{ij}$ | $SU_q(6)$ | $SU(3)$ | $SU(2)$ |
|---------------|------------|----------|----------|
| [211] + [1111] | $[2222111]_{20}$ | $[32]_1 \otimes [32]_2$ | $[32]_1 \otimes [32]_2$ |
|               | $[32]_1 \otimes [32]_2$ | $[41]_1 \otimes [32]_2$ | $[41]_1 \otimes [32]_2$ |

where $\Psi_{L,\lambda}^\pm = \sum_{i, M, M_i} C_{L, M, S_M}^i F_{L, \lambda}^\pm \chi_{S_M}^\lambda$, (5)

We assume that the 4 quarks form an equilateral tetrahedron (ETH) or a square, which is illustrated in figure 1. We determine all the accessible configurations with $L \leq 3$, extending the results of reference [48]. For the case of the ETH, we denote $O$ as the center of the 4 quarks, $i, j, k$ form the frame, and $p_{12} \perp \hat{k}, p_{34} \perp \hat{k}$. Referring to $R_{\hat{k}}$ as a rotation about the axis along the vector $\vec{\delta}$ by an angle $\delta$, $p_{ij}$ as an interchange of the particles $i$ and $j$, $p_{ijk}$ as permuting the 3 and 4 particles, and $P$ as a space inversion, the ETH is evidently invariant to the operations:

$$\hat{O}_1 = p_{12} P_{34} R_{\hat{k}}^e.$$  \hspace{1cm} (6)

$$\hat{O}_2 = p_{34} R_{\hat{P}}^e P.$$  \hspace{1cm} (7)

$$\hat{O}_3 = p_{1423} R_{\hat{k}}^e P.$$  \hspace{1cm} (8)

$$\hat{O}_4 = p_{2431} R_{\hat{k}}^e P.$$  \hspace{1cm} (9)

Inserting each of these operators into equation (3), we have a set of equations:

$$\hat{O}_i - E F_{L, \lambda}^\pm (A) = 0.$$

(10)
They are homogeneous linear algebraic equations. If they are solved easily. The obtained results are shown in table 6, where $A_\pi$ denotes the accessible configuration, however they are less constrained in the neighborhood of the ETH, since it holds the highest symmetry. To check such an idea, we have performed calculations where the shape in the left panel of figure 1 is invariant under the operations $\hat{O}_1$, $\hat{O}_2$ and $\hat{O}_3$, but not $\hat{O}_4$. The obtained results are listed as those marked with ETH3 in table 6. It shows apparently that all the configurations accessible to the ETH are definitely accessible to ETH3, and more nodeless configurations are allowed for ETH3.

For the case of the 4 quarks forming a square, the operations for the configuration to be invariant are

$$\hat{O}_1 = p_{12} p_{34} \hat{P},$$

$$\hat{O}_2 = R^j_k \hat{P},$$

$$\hat{O}_3 = p_{34} R^j_k,$$

$$\hat{O}_4 = p_{1342} R^j_k.$$  

Inserting them into equation (3), we get another set of equations as equation (10). The obtained accessible configuration of $(L\pi\lambda)$ and the IRREPs of the $S_4$ group of the wave

![Figure 1. Left panel: body frame illustration of the equilateral tetradedron (ETH) configuration. Right panel: body frame illustration of the square.](image)
functions are listed as those marked as ‘square’ in table 6. In the neighborhood of the square, for example, prolonged along the \( j \)-direction, it is still invariant under \( \hat{O}_1 \), \( \hat{O}_2 \) and \( \hat{O}_3 \), but not \( \hat{O}_0 \). The accessible configurations are also given in table 6 marked ‘square\(^3\)’. It is evident that the square-accessible component is inherently nodeless in the neighborhood of the square.

Combining tables 2 and 6 one can get the number of accessible (nodeless) states \( J^P_L^Z = L^Z \oplus S \) with orbital angular momentum \( L \leq 3 \). The obtained results are listed in table 7.

Considering the fact that, for usual bound states of the quantum system, the fewer nodes the configuration contains, the lower energy the state has, one can recognize that \( J^P \) of the lowest energy state involving 4 quarks with three flavors is \( J^P_L = 2^+ \) (the accessible number is 50). While that of the second and the third lowest states are \( 2^- \) and \( 3^- \) (the accessible numbers are 47 and 43, respectively). If the symmetry of the configurations are less constrained, such as those for ETH\(_3\) and square\(_3\), the result almost stays the same (the accessible numbers become 53, 47 and 46, respectively, but \( J^P_L \) of the third lowest changes to \( 1^- \)).

In our scheme regarding pentaquark states, we notice that the \( P^+_c \) states are composed of a 4-quark cluster involving a \( c \)-quark (\( uudc \)) and an antiquark \( \bar{c} \) at the ground state. The total spin and the parity of the \( P^+_c \) states should be \( J^P (P^+_c) = J^P_4 (uudc) \oplus \frac{1}{2} (\bar{c}) \in \{ \frac{3}{2}^+, \frac{5}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+ \} \). Since the \( c \)-quark is much heavier than the \( u \)– and \( d \)-quarks, we can assume that the interaction between the cluster and the \( \bar{c} \)-quark is similar to that between the \( c \)- and \( \bar{c} \)-quarks. In view of heavy quark interaction, the energy sequence (from lower to higher) of the doublet with mainly \( J^P_4 = 2^+ \), is \( \{ \frac{3}{2}^- \}, \{ \frac{5}{2}^- \} \), \( \{ \frac{3}{2}^+, \frac{5}{2}^+ \} \), respectively. Taking the fact that the spin–orbital splitting is proportional to the orbital angular momentum into consideration, one can assign the energy sequence of the \( J^P \) states to \( \{ \frac{3}{2}^- \}, \{ \frac{3}{2}^+, \frac{5}{2}^- \}, \frac{5}{2}^- \) .

It should also be mentioned that the pentaquark states do not appear as geometric tetrahedrons or squares. It is well known that a microscopic particle is in fact the quantization of a quantum field, or simply a quantum wave. The ETH configuration of the state will then appear as a spheroid, the deformed tetrahedron (for example that corresponding to the ETH\(_3\)) will appear as a pear, and the state with square configuration (Square\(_3\)) will display as an oblate ellipsoid (prolate ellipsoid). In this sense the pentaquark states may have finite quadruple moment or dipole moment. Then measuring the electromagnetic property of the pentaquark states could assign the configuration of the states.

### 4. Summary and remarks

In summary, by analyzing the symmetry, more concretely the inherent nodal structure, of the system including 4 quarks involving three flavors, we show dynamical model independently that the quantum numbers of the lowest-lying pentaquark states \( P^+_c \) could mainly be \( J^P = \frac{3}{2}^- \) with orbital angular momentum \( L = 2 \). The quantum numbers of the other two low-lying pentaquark states \( P^+_c \) could be \( J^P = \frac{3}{2}^+, \frac{5}{2}^- \). And
there may exist another $P_c$ pentaquark state with $J^P = \frac{5}{2}^-$, which may be observed by analyzing the structure of the peak around 4312 MeV. More experiments or analyzing the existing experiments with broader viewpoints (see, e.g., references [49, 50]) are then expected.

It is finally remarkable that our present results seem only valid for the pentaquark states without molecular structures, namely no color singlet 3-quark states and meson-like states. In fact we could not rule out the molecular structure, since we have not constrained the location of the antiquark in our analysis. If the antiquark locates at the center of the configurations that the 4 quarks formed or on the line perpendicular to the ‘plane’ formed by the 4 quarks (the balance of the interactions between the antiquark and each of the quark may result in such features), a molecular structure really may not be involved. If the antiquark does not locate at the center or the 4 quarks’ configuration is deformed, the antiquark may form a meson-like state with the quark closing to it, and the other 3 quarks may form a baryon. In this sense the molecular structure emerges. Anyway, our present result is a qualitative result based upon symmetry analysis. Taking the presently assigned configuration and the obtained results into dynamical model calculations can apparently help to reduce the load of numerical calculation tasks. In addition, the work analyzing the inherent nodal structure of the pentaquark states with molecular configurations will be given in a future article.

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Table 7. Obtained number of the accessible (nodeless) states $J^P$ with orbital angular momentum $L \leq 3$ of the ETH configuration and square configuration of the 4-quark system and the corresponding less restricted ones.

| Configuration | $0^+$ | $0^-$ | $1^+$ | $1^-$ | $2^+$ | $2^-$ | $3^+$ | $3^-$ | $4^+$ | $4^-$ | $5^+$ | $5^-$ |
|---------------|------|------|------|------|------|------|------|------|------|------|------|------|
| ETH square   | 7    | 10   | 19   | 22   | 25   | 28   | 15   | 26   | 4    | 15   | 0    | 4    |
| ETH          | 7    | 7    | 18   | 15   | 25   | 19   | 15   | 17   | 4    | 10   | 0    | 2    |
| ETH square1  | 9    | 10   | 21   | 22   | 26   | 28   | 15   | 26   | 4    | 15   | 0    | 4    |
| ETH square2  | 11   | 7    | 25   | 15   | 27   | 19   | 15   | 17   | 4    | 10   | 0    | 2    |
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