The evidence of cosmic acceleration and observational constraints

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Abstract. Directly comparing the 6 expansion rates measured by type Ia supernovae data and the lower bound on the expansion rate set by the strong energy conditions or the null hypothesis that there never exists cosmic acceleration, we see 3σ direct evidence of cosmic acceleration and the Rh = ct model is strongly excluded by the type Ia supernovae data. We also use Gaussian process method to reconstruct the expansion rate and the deceleration parameter from the 31 cosmic chronometers data and the 6 data points on the expansion rate measured from type Ia supernovae data, the direct evidence of cosmic acceleration is more than 3σ and we find that the transition redshift \( z_t = 0.60^{+0.21}_{-0.12} \) at which the expansion of the Universe underwent the transition from acceleration to deceleration. The Hubble constant inferred from the cosmic chronometers data with the Gaussian process method is \( H_0 = 67.46 \pm 4.75 \) Km/s/Mpc. By fitting two different two-parameter models to the observational data, we find that the constraints on the model parameters from either the full distance modulus data by the Pantheon compilation or the compressed expansion rate data are very similar, and the derived Hubble constants are consistent with the Planck 2018 result. Our results confirm that the 6 compressed expansion rate data can replace the full 1048 distance modulus data from the Pantheon compilation. We derive the transition redshift \( z_t = 0.61^{+0.24}_{-0.16} \) by fitting a simple \( q(z) \) model to the combination of cosmic chronometers data and the Pantheon compilation, the result is consistent with that obtained from the reconstruction with Gaussian process.

Keywords: cosmic acceleration, cosmological parameters, Gaussian process method

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1 Introduction

The observations of type Ia supernovae (SNe Ia) [1, 2] suggest that the Universe is currently undergoing accelerated expansion. This raises a vital question about the mechanism of this accelerated expansion: what is the cause and nature of the accelerated expansion? Many efforts have been made to understand this question and two approaches were usually used. One approach is to modify general relativity at the cosmological scale, such as the Dvali-Gabadadze-Porrati model [3], f(R) gravity [4–6], and dRGT ghost-free massive gravity [7, 8]. The other approach is to introduce an exotic matter component dubbed as dark energy which has negative pressure and contributes about 70% of the matter content of the Universe to drive the cosmic acceleration. Although the cosmological constant named as ΛCDM model is the simplest candidate for dark energy and is consistent with current observations, it also faces problems such as fine tuning and coincidence problems. Furthermore, there exist many orders of magnitude discrepancy between the theoretical estimation and astronomical observations for the cosmological constant [9]. Therefore, dynamical dark energy models such as the quintessence model [10–14] are usually considered. For a recent review of dark energy, please see Ref. [15–18].

The Hubble constant $H_0 = 67.27 \pm 0.60$ km/s/Mpc inferred from Planck 2018 measurement on the cosmic microwave background anisotropy (CMBR) with the assumption of ΛCDM model [19] is in 4.4σ tension with the local measurement $H_0 = 74.03 \pm 1.42$ km/s/Mpc by the Hubble Space Telescope (HST) observations of 70 long-period Cepheids in the large Magellanic Cloud [20]. Combining the distance measurement from gravitational wave and the identification of local host galaxy from the electromagnetic counterpart, gravitational wave becomes a standard siren and can be used to measure the Hubble constant [21]. The detection of the first gravitational
wave event GW170817 and its electromagnetic counterpart GRB170817A from a binary neutron star merger measures \(H_0\) as \(H_0 = 70.0^{+12.9}_{-8.8}\) km/s/Mpc \([22]\). This value is consistent with both local and high redshift measurements due to the large error bar. By reconstructing the observational data of the Hubble parameter \(H(z)\) from cosmic chronometers (CCH) and baryon acoustic oscillation (BAO) with Gaussian process (GP) method, it was found that \(H_0 \sim 67 \pm 4\) km/s/Mpc \([23]\). Using the Gaussian kernel in the GP method, the reconstruction of CCH data and SNe Ia data from the Pantheon compilation \([24]\) and the HST CANDELS and CLASH Multi-Cycle Treasury (MCT) programs \([25]\) (Patheon+MCT) gives \(H_0 = 67.06 \pm 1.68\) km/s/Mpc \([26]\). Applying the GP method with the Matérn kernel to the combination of CCH, BAO and SNe Ia data, it was found that \(H_0 = 68.52^{+0.94+2.51_{(sys)}}_{-0.94}\) km/s/Mpc \([27]\). The results with different kernels are consistent with each other. These values prefer the lower value determined from Planck 2018 data and is in tension with the local measurement from distance ladder.

The evidence for cosmic acceleration and the measurement on the Hubble constant from Planck 2018 data were obtained by fitting the observational data, so they depend on the models used in the fitting. The zero acceleration model (eternal coasting \([28]\) or \(R_{\text{m}} = ct\) model \([29]\)) is also consistent with some observational data \([30–35]\). To be model independent, many parametric and none-parametric model independent (in the sense that it does not use a particular cosmological model) methods were proposed to study the evolution of the deceleration parameter \(q(z)\), the geometry of the Universe and the property of dark energy.

By comparing the bound set by the null hypothesis that the Universe never experiences an accelerated expansion with the observational data, the energy conditions may be used to provide direct and model independent evidence of cosmic acceleration \([36–43]\). Although this kinematic method does not assume any gravitational theory and matter content, it cannot provide us with any detailed information about the cosmic acceleration, like the transition redshift at which the expansion of the Universe underwent the transition from accelerated expansion to decelerated expansion. \([40, 41]\). As emphasized in \([40, 41]\), great caution is needed to correctly interpret the result from falsifying the null hypothesis. The violation of the bound set by the null hypothesis provides direct evidence that cosmic acceleration once occurred, and the fulfillment of the bound doesn’t mean no cosmic acceleration at all, which is the reason why no evidence of acceleration was found in accelerating cosmologies in Ref. \([43]\). In this paper, we analyze the direct evidence of cosmic acceleration using the CCH data and the expansion rate data \(E(z)\) from the Pantheon+MCT SNe Ia compilation \([25]\).

The tension on the Hubble constant may not caused by the fitting model \([44]\). To check the tensions in data, null tests with the reconstruction of some smooth functions from observational data may be used \([45–54]\). The GP method is one of the most widely used model independent method to reconstruct a function and its derivatives from discrete data points without invoking any specific model. This method has been used in cosmology to reconstruct cosmological parameters and probe the property of cosmic acceleration \([23, 26, 27, 54–75]\). In this paper, we use the GP method to reconstruct the expansion rate \(E(z)\) and the deceleration parameter \(q(z)\) from the
CCH and Pantheon+MCT data. To probe the property of cosmic acceleration and dark energy with the combination of different observational data, we parameterize the deceleration parameter $q(z)$ with a simple two-parameter model $q(z) = 1/2 + (q_1 z + q_2)/(1 + z)^2$ [76, 77] and the equation of state parameter $w(z)$ with the SSLCPL model [78, 79] which approximates the dynamics of general thawing scalar fields over a large redshift range.

The paper is organized as follows. In section 2, we introduce the observational data used in this paper. The null hypothesis method and the direct evidences for cosmic acceleration from Pantheon+MCT data and CCH data are then discussed. In section 3, we introduce the GP method and use the method to reconstruct the expansion rate and deceleration parameter by combining the CCH and Pantheon+MCT SNe Ia data. Two particular parameterizations are used to fit the observational data in section 4. We conclude the paper with some discussions in section 5.

2 Observational data

The Hubble parameter directly probes the expansion history of the Universe by its definition $H = \dot{a}/a$, where $a$ denotes the cosmic scale factor and $\dot{a}$ is its rate of change with respect to the cosmic time $t$. Since the Hubble parameter is related the differential redshift time relation as

$$H(z) = -\frac{1}{(1 + z)} \frac{dz}{dt} \approx -\frac{1}{(1 + z)} \frac{\Delta z}{\Delta t}$$

and $dz$ is obtained from spectroscopic surveys, so a measurement of $dt$ gives the Hubble parameter which is independent of the cosmological model. Based on the spectroscopic differential evolution of passively evolving galaxies, CCH method obtains the expansion rate $dz/dt$ by taking a pair of massive and passively evolving galaxies at two different redshifts [80]. We show the 31 CCH data points of $H(z)$ compiled by [23, 26, 27, 52, 81, 82] in table 1. These data cover a redshift range up to $z \sim 2$ and are obtained without assuming any particular cosmological model. There exit systematic uncertainties associated with the stellar population synthesis models like BC03 [83] and MaStro [84], and a possible contamination due to young underlying stellar components in quiescent galaxies [82], so the data is model dependent in this sense. Here we use the measurements on $H(z)$ with the BC03 model. To keep the data to be model independent as minimally as possible, we don’t use the $H(z)$ data determined by BAO measurements in this paper.

The Pantheon sample [24] is the largest SNe Ia sample which include 1048 spectroscopically confirmed SNe Ia and the furthest SN reaches approximately redshift $z \sim 2.3$. It consists of 279 spectroscopically confirmed SNe Ia with redshift $0.03 < z < 0.68$ discovered by the Pan-STARRS1 Medium Deep Survey [92], samples of SNe Ia from the Harvard Smithsonian Center for Astrophysics SN surveys [93], the Carnegie SN Project [94], the Sloan digital sky survey [95] and the SN legacy survey [96], and high-z data with the redshift $z > 1.0$ from the Hubble space telescope cluster SN survey [97], GOODS [98] and CANDELS/CLASH survey [99, 100]. The calibration systematics is
Table 1. The 31 CCH data with the BC03 model. The unit for $H(z)$ is km/s/Mpc.

| $z$  | $H(z)$ | $\sigma_{H(z)}$ | Ref. | $z$  | $H(z)$ | $\sigma_{H(z)}$ | Ref. |
|------|--------|----------------|------|------|--------|----------------|------|
| 0.07 | 69.0   | 19.6           | [85] | 0.4783 | 80.9   | 9.0           | [86] |
| 0.09 | 69.0   | 12.0           | [87] | 0.48  | 97.0   | 62.0          | [88] |
| 0.12 | 68.6   | 26.2           | [85] | 0.593 | 104.0  | 13.0          | [89] |
| 0.17 | 83.0   | 8.0            | [87] | 0.68  | 92.0   | 8.0           | [89] |
| 0.179| 75.0   | 4.0            | [89] | 0.781 | 105.0  | 12.0          | [89] |
| 0.199| 75.0   | 5.0            | [89] | 0.875 | 125.0  | 17.0          | [89] |
| 0.2  | 72.9   | 29.6           | [85] | 0.88  | 90.0   | 40.0          | [88] |
| 0.27 | 77.0   | 14.0           | [87] | 0.9   | 117.0  | 23.0          | [87] |
| 0.28 | 88.8   | 36.6           | [85] | 1.037 | 154.0  | 20.0          | [89] |
| 0.352| 83.0   | 14.0           | [89] | 1.3   | 168.0  | 17.0          | [87] |
| 0.3802| 83.0  | 13.5           | [86] | 1.363 | 160.0  | 33.6          | [90] |
| 0.4  | 95.0   | 17.0           | [87] | 1.43  | 177.0  | 18.0          | [87] |
| 0.4004| 77.0  | 10.2           | [86] | 1.53  | 140.0  | 14.0          | [87] |
| 0.4247| 87.1  | 11.2           | [86] | 1.75  | 202.0  | 40.0          | [87] |
| 0.4497| 92.8  | 12.9           | [86] | 1.965 | 186.5  | 50.4          | [90] |
| 0.47 | 89.0   | 49.6           | [91] |

reduced substantially by cross-calibrating all of the SN samples. The distance modulus $\mu$ of SNeIa was derived from the observation of light curves through the SALT2 light-curve fitter

$$\mu_{\text{obs}} = m_B - M_B + \alpha \cdot X_1 - \beta \cdot C + \Delta_M + \Delta_B,$$

where $m_B$ corresponds to the observed peak magnitude in rest-frame $B$ band, $X_1$ is the time stretching of the light curve, $C$ describes the supernova color at maximum brightness, $M_B$ is the absolute $B$-band magnitude of a fiducial SN Ia with $X_1 = 0$ and $C = 0$, $\Delta_M$ is a distance correction based on the host-galaxy mass of the SN and $\Delta_B$ is a distance correction based on predicted biases from simulations. The parameters $\alpha$ and $\beta$ characterize luminosity-stretch, and luminosity-color relations. Since the absolute magnitude of a SN Ia is degenerated with the Hubble constant, the corrected magnitudes $\mu + M_B$ are given for cosmological model fitting [24]. The nuisance parameters $\alpha$, $\beta$ and $H_0$ should be marginalized. The statistical uncertainty and systematic uncertainty are also given in Ref. [24]. The total uncertainty matrix of the distance modulus is given by

$$\Sigma_\mu = D_{\text{stat}} + C_{\text{sys}},$$

where the statistical matrix $D_{\text{stat}}$ has only a diagonal component and $C_{\text{sys}}$ is the systematic covariance. We take into account all the statistical uncertainties as described by their full covariance matrix.

Recently, Riess et al. combine the Pantheon sample with 15 SNe Ia at redshift $z > 1$ discovered in the CANDELS and CLASH Multi-Cycle Treasury (MCT) programs using WFC3 on the Hubble Space Telescope and compress the raw distance measurements to expansion rate $E(z)$ at six redshifts in the range $0.07 < z < 1.5$ by assuming
a flat universe with $\Omega_k = 0$ [25], the results and the correlation matrix of $E(z)$ are shown in table 2. Because of the assumption of a flat universe, the results of $E(z)$ are cosmological model dependent in this sense. The last point $E(z = 1.5)$ is not Gaussian, the symmetrization of the upper and lower bounds gives $E(1.5) = 2.924 \pm 0.675$ [27], or $E(1.5) = 2.67 \pm 0.675$ [74], and the Gaussian approximation is $E(1.5) = 2.78 \pm 0.59$ [82].

| $z$  | $E(z)$          | Correlation Matrix |
|------|-----------------|--------------------|
| 0.07 | 0.994 ± 0.023   | 1.00               |
| 0.2  | 1.113 ± 0.020   | 0.40 1.00          |
| 0.35 | 1.122 ± 0.037   | 0.52 -0.13 1.00    |
| 0.55 | 1.369 ± 0.063   | 0.35 0.35 -0.18 1.00 |
| 0.9  | 1.54 ± 0.12     | 0.02 -0.08 0.19 -0.41 1.00 |
| 1.5  | 2.69$^{+0.86}_{-0.52}$ | 0.00 -0.06 -0.05 0.16 -0.21 1.00 |

Table 2. Pantheon+MCT SN Ia measurements of $E(z)$ [25].

BAO is a powerful standard ruler to probe the angular diameter distance and the Hubble parameter evolution. The isotropic and anisotropic BAO measurements are summarized in tables 3 and 4, respectively [101]. The covariance matrix associated with the data in table 4 is

$$
C = \begin{pmatrix}
0.0150 & -0.0357 & 0.0071 & -0.0100 & 0.0032 & -0.0036 & 0 & 0 \\
-0.0357 & 0.5304 & -0.0160 & 0.1766 & -0.0083 & 0.0616 & 0 & 0 \\
0.0071 & -0.0160 & 0.0182 & -0.0323 & 0.0097 & -0.0131 & 0 & 0 \\
-0.0100 & 0.1766 & -0.0323 & 0.3267 & -0.0167 & 0.1450 & 0 & 0 \\
0.0032 & -0.0083 & 0.0097 & -0.0167 & 0.0243 & -0.0352 & 0 & 0 \\
-0.0036 & 0.0616 & -0.0131 & 0.1450 & -0.0352 & 0.2684 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.1358 & -0.0296 & 0.0492 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.0296 & 0.1358 & 0.0492
\end{pmatrix}
$$

Table 3. Isotropic BAO data.

| Data set   | Redshift | $D_V(z)/r_d$ | Ref.       |
|------------|----------|--------------|------------|
| 6dF        | z=0.106  | 2.98 ± 0.13  | [102]      |
| MGS        | z=0.15   | 4.47 ± 0.17  | [103]      |
| eBOSS quasars | z=1.52  | 26.1 ± 1.1   | [104]      |

In a spatially flat universe, the Hubble distance is

$$
D_H(z) = \frac{c}{H(z)},
$$

the angular diameter distance is

$$
D_A(z) = \frac{c}{1 + z} \int_0^z \frac{dx}{H(x)},
$$
Table 4. Anisotropic BAO data. In the third column, $A$ means $D_A(z)/r_d$ and $H$ means $D_H(z)/r_d$.  

| Data set      | Redshift | $D_{A/H}(z)/r_d$ | Ref. |
|---------------|----------|------------------|------|
| BOSS DR12     | z=0.38   | 7.42($A$)        | [105]|
| BOSS DR12     | z=0.38   | 24.97($H$)       | [105]|
| BOSS DR12     | z=0.51   | 8.85($A$)        | [105]|
| BOSS DR12     | z=0.51   | 22.31($H$)       | [105]|
| BOSS DR12     | z=0.61   | 9.69($A$)        | [105]|
| BOSS DR12     | z=0.61   | 20.49($H$)       | [105]|
| BOSS DR12     | z=2.4    | 10.76($A$)       | [106]|
| BOSS DR12     | z=2.4    | 8.94($H$)        | [106]|

The luminosity distance is

$$d_L(z) = c(1 + z) \int_0^z \frac{dx}{H(x)},$$

(2.6)

and the effective distance $D_V(z)$ is [107]

$$D_V(z) = \left[ \frac{d_L^2(z)}{(1+z)^2} \frac{cz}{H(z)} \right]^{1/3}.$$ 

(2.7)

The sound horizon at the drag redshift $z_d$ is

$$r_d = \frac{c}{\sqrt{3}} \int_{z_d}^{\infty} \frac{dz}{\sqrt{1 + \left(3\Omega_b/4\Omega_r\right)/(1+z)H(z)},}$$

(2.8)

and the drag redshift $z_d$ is fitted as [108]

$$z_d = \frac{1291(\Omega_m h^2)^{0.251}}{1 + 0.659(\Omega_m h^2)^{0.828}}[1 + b_1(\Omega_b h^2)^{b_2}],$$

(2.9)

where

$$b_1 = 0.313(\Omega_b h^2)^{-0.419}[1 + 0.607(\Omega_m h^2)^{0.674}],$$

(2.10)

and

$$b_2 = 0.238(\Omega_m h^2)^{0.223}.$$\hspace{1cm} (2.11)

In this work we take $\Omega_b h^2 = 0.02236$ and $\Omega_m h^2 = 2.469 \times 10^{-5}$ [19], where the dimensionless parameter $h = H_0/(100 \text{ km/s/Mpc})$. To use the data, we calculate

$$\chi^2_{BAO} = \chi^2_{iso} + \chi^2_{aniso},$$

(2.12)

$$\chi^2_{iso} = \sum_i \left( \frac{v_i - d_{iso}^i}{\sigma_i} \right)^2,$$

(2.13)

$$\chi^2_{aniso} = (w - d_{aniso}^T C^{-1} (w - d_{aniso}^T),$$

(2.14)

where the vectors $d_{iso}$ and $d_{aniso}$ are the isotropic and anisotropic data from tables 3 and 4, respectively, and $v$ and $w$ are the predictions for these vectors in a given cosmological model.
2.1 Null hypothesis

The strong energy conditions $\rho + 3p \geq 0$ and $\rho + p \geq 0$ tell us that

\[ q(t) \equiv -\frac{\ddot{a}}{aH^2} \geq 0, \quad (2.15) \]
\[ \dot{H} - \frac{k}{a^2} \leq 0. \quad (2.16) \]

The condition (2.15) means no acceleration and the condition (2.16) means no super-acceleration. The model with $q = 0$ is also called eternal coasting [28] or $Rh = ct$ model [29]. Integrating equation (2.16) yields

\[ H(z) \geq H_0 \sqrt{1 - \Omega_k + \Omega_k(1 + z)^2}, \quad (2.17) \]

where the Hubble constant $H_0$ denotes the current value of the Hubble parameter $H(z)$ and $\Omega_k = -k/(a_0 H_0^2)$. For a spatially flat universe, $\Omega_k = 0$, the above condition becomes $H(z) \geq H_0$.

By using the redshift $z$, the deceleration parameter $q(t)$ is related with the Hubble parameter $H(t)$ as

\[ \ln \frac{H(z)}{H_0} = \int_0^z \frac{1 + q(z')}{1 + z'} dz', \quad (2.18) \]

Substituting equation (2.15) into equation (2.18), we get

\[ H(z) \geq H_0(1 + z). \quad (2.19) \]

Therefore, if the universe has never experienced an accelerated expansion, then equation (2.19) is always satisfied. Because of the integration effect, even if the condition (2.19) is satisfied at some redshifts, it does not mean that the universe has never experienced an accelerated expansion [40, 41]. However, if the condition (2.19) is violated at some redshifts, we conclude that the universe once experienced accelerated expansion. Based on the strong energy condition (2.19), the CCH data on $H(z)$ and the SNe Ia data on $E(z)$ can be used to see whether the Universe ever experienced accelerated expansion or not, i.e., we can compare equation (2.19) with observational data to show direct evidence of cosmic acceleration in a model independent way.

For $z \geq 0$, we have

\[ H_0(1 + z) \geq H_0 \sqrt{1 - \Omega_k + \Omega_k(1 + z)^2}. \quad (2.20) \]

Therefore, once the energy condition $\rho + 3p \geq 0$ is satisfied, then the condition $\rho + p \geq 0$ is also satisfied. On the other hand, if the universe experiences super-accelerated expansion, it must also experience accelerated expansion, so we can compare the conditions (2.17) and (2.19) with observational data to show direct evidence of both accelerated and super-accelerated expansion. Although we derive the conditions (2.17) and (2.19) from the strong energy conditions in the standard cosmological framework, these bounds actually are independent of the strong energy conditions and can be applied to more general cases because they just depend on the conditions (2.15) and (2.16). In other words, the lower bound (2.19) for decelerated expansion is independent of gravitational theory and it just assumes Friedmann-Robertson-Walker (FRW) metric.
2.2 Direct evidence for cosmic acceleration

Now, we use the Pantheon+MCT SNe Ia measurements on $E(z)$ and CCH data of $H(z)$ to show the direct evidence for cosmic acceleration. The advantage of $E(z)$ data is that it is independent of the Hubble constant and the drawback is that it assume $\Omega_k = 0$, so it is model dependent in this sense. For the compressed SNe Ia data at six redshifts, we compare $E(z)$ data with $1+z$ and 1 to show the evidence of accelerated expansion or super-accelerated expansion, respectively. We plot the $E(z)$ data from table 2 along with the null hypotheses (2.17) and (2.19) in figure 1. From figure 1, we see that all the low redshift points ($z < 1$) violate the lower bound (2.19) even at $3\sigma$ level, so we have $3\sigma$ evidence for cosmic acceleration, but we don’t see strong evidence for decelerated expansion due to the lack and poor quality of data at high redshifts. The first point also provides weak evidence of super-accelerated expansion. Therefore, the Pantheon+MCT SNe Ia data is strongly against the $Rh = ct$ model. Note that this does not mean that the expansion of the Universe is always accelerating up to the redshift $z \sim 1$ or there is no decelerated expansion at all. The $E(z)$ graph just provides us with the evidence for cosmic acceleration and it does not give us any information on the property of cosmic acceleration. This evidence is independent of any gravitational theory and it just assumes the flat FRW metric. Fitting $Rh = ct$ model to Pantheon+MCT data, we get $\chi^2 = 85.29$. For $\Lambda$CDM model, the best fit is $\Omega_{m0} = 0.265 \pm 0.029$ and $\chi^2 = 7.69$, so $Rh = ct$ model is strongly disfavoured by the Pantheon+MCT data.

![Figure 1](image.png)

**Figure 1.** The Pantheon+MCT SNe Ia measurements on $E(z)$ with $1\sigma$, $2\sigma$ and $3\sigma$ errors. The dashed line corresponds to the $Rh = ct$ model with $q(z) = 0$, the dotted line denotes $E(z) = 1$ which represents the model with $\dot{H} = 0$ in a spatially flat universe, and the solid line shows the best fit $\Lambda$CDM model.

Now we compare the CCH measurements on the Hubble parameter $H(z)$ with the null hypothesis (2.17) and (2.19) and the result is shown in figure 2. Since the Hubble constant $H_0$ appears in equations (2.17) and (2.19), and the latest result $H_0 = 67.27 \pm 0.60$ km/s/Mpc from Planck 2018 [19] is in tension with the local measurement.
$H_0 = 74.03 \pm 1.42 \text{ km/s/Mpc}$ from HST [20] at $4.4\sigma$ level, so the direct evidence of cosmic acceleration from CCH data is affected by the uncertainty in $H_0$ and this method depends on the cosmological model as the way the Hubble constant depends on. In figure 2, we take both the values $H_0 = 67.27 \pm 0.60 \text{ km/s/Mpc}$ and $H_0 = 74.03 \pm 1.42 \text{ km/s/Mpc}$. For $H_0 = 74.03 \text{ km/s/Mpc}$, some of the low redshift ($z < 1$) CCH data violate the lower bound of the null hypothesis (2.19), and the three points at $z = 0.68$, $z = 0.781$ and $z = 1.53$ violate the lower bound even at $2\sigma$ confidence level. So we expect that the Universe once experienced accelerated expansion, but this does not mean that the accelerated expansion happened up to $z = 1.53$. For $H_0 = 67.27 \text{ km/s/Mpc}$, the data points that violate the lower bound (2.19) become less and only two points at $z = 0.68$ and $z = 1.53$ violate the lower bound at $2\sigma$ confidence level. Therefore, the evidence strongly depends on the value of $H_0$. For both choices of $H_0$, we see $2\sigma$ evidence of cosmic acceleration and no strong evidence of super-accelerated expansion, note that this result applies to any value of $\Omega_k$. We may argue that those points which violate the lower bound are outliers, so we fit the $Rh = ct$ model to the CCH data and we get the best fit $H_0 = 62.34 \pm 1.43 \text{ km/s/Mpc}$ and $\chi^2 = 16.62$. For the $\Lambda$CDM model, the best fit is $\Omega_{m0} = 0.32 \pm 0.06$, $H_0 = 68.11 \pm 3.09 \text{ km/s/Mpc}$ and $\chi^2 = 14.50$. In terms of $\chi^2$ statistics, it seems that both $Rh = ct$ and $\Lambda$CDM model fit CCH data well. To avoid the possible outlier problem, in the next section we reconstruct the data using Gauss process method.

![Figure 2. The CCH data with 1σ and 2σ uncertainties. The solid lines corresponds to $q(z) = 0$ with $H_0 = 67.27 \text{ km/s/Mpc}$ and the blue shaded area corresponds to $H_0 = 67.27 \pm 0.60 \text{ km/s/Mpc}$. The dotted line denotes $H(z) = H_0 = 67.27 \text{ km/s/Mpc}$. The dashed line corresponds to $q(z) = 0$ with $H_0 = 74.03 \text{ km/s/Mpc}$ and the blue shaded area corresponds to $H_0 = 74.03 \pm 1.42 \text{ km/s/Mpc}$.](image)

## 3 Reconstruction of observational data with Gaussian Process

Because of insufficient and low quality of observational data, we use the Gaussian Process (GP) method to find a smooth function $f(x)$ that best represents a set of
observational data points \( f(x_i) \pm \sigma_i \). The GP method assume that the value of the function \( f(x) \) at any point \( x \) follows a Gaussian distribution. At each \( z_i \), the value of \( f(z_i) \) is drawn from a Gaussian distribution with mean \( u(z_i) \) and variance \( k(z_i, z_i) \). Besides, \( f(z_i) \) and \( f(z_j) \) are correlated by the covariance function (or kernel function) \( k(z_i, z_j) \).

A GP is written as

\[
    f(x) \sim \text{GP}(\mu(x), k(x, x')), \tag{3.1}
\]

so the kernel function plays a crucial role in the GP method and must be selected beforehand. In this sense, GP is model dependent although it is independent of cosmological models. There are three widely used kernel functions with two degrees of freedom. The Gaussian/squared-exponential kernel

\[
    k(x_i, x_j) = \sigma_f^2 \exp \left( -\frac{(x_i - x_j)^2}{2l_f^2} \right), \tag{3.2}
\]

where \( \sigma_f \) and \( l_f \) are hyperparameters. The Cauchy kernel

\[
    k(x_i, x_j) = \frac{\sigma_f^2 l_f}{(x_i - x_j)^2 + l_f^2}. \tag{3.3}
\]

The Matérn kernel

\[
    k(x_i, x_j) = \sigma_f^2 \frac{2^{1-\nu}}{\Gamma(\nu)} \left( \frac{\sqrt{2\nu(x_i - x_j)^2}}{l_f} \right) ^\nu K_\nu \left( \frac{\sqrt{2\nu(x_i - x_j)^2}}{l_f} \right), \tag{3.4}
\]

where \( K_\nu \) is the modified Bessel function with \( \nu \) being positive. Here we choose the Gaussian kernel. The hyperparameters are determined from the observed data by minimizing the log likelihood function

\[
    \ln \mathcal{L} = \ln p(y|X, \sigma_f, l) \\
    = -\frac{1}{2} (y - \mu)^T [K(X, X) + C]^{-1} (y - \mu) \\
    - \frac{1}{2} \ln |K(X, X) + C| - \frac{n}{2} \ln(2\pi), \tag{3.5}
\]

where \( X = [x_1, x_2, ..., x_n]^T \) are the inputs, \( K(X, X) \) is the covariance matrix with components \( k(x_i, x_j) \), \( y \) is the vector of observed data and \( C \) is the covariance matrix of the observed data.

To predict the function values \( f_* = [f_{*1}, f_{*2}, ..., f_{*m}]^T \) at the test locations \( X_* = [x_{n+1}, x_{n+2}, ..., x_{n+m}]^T \), the predictive normal distribution is

\[
    p(f_*|X, y, X_*) = \mathcal{N}(\hat{\mu}, \hat{\Sigma}), \tag{3.6}
\]

\[
    \hat{\mu} = K(X_*, X)^T (K(X, X) + C)^{-1} (y - \mu(X)) + \mu(X_*), \tag{3.7}
\]

\[
    \hat{\Sigma} = K(X_*, X_*) - K(X_*, X)^T (K(X, X) + C)^{-1} K(X_*, X). \tag{3.8}
\]
The public available python package GaPP [61] is used to do the GP reconstruction. We reconstruct \(E(z)\) by combining the CCH data on the Hubble parameter and the Pantheon+MCT data on \(E(z)\) to show direct evidence of cosmic acceleration. We first determine the Hubble constant \(H_0\) from the reconstructed \(H(z)\) function at \(z = 0\) by using the 31 CCH data, then we multiply the 6 \(E(z)\) data by this \(H_0\) and add these \(H(z)\) points to CCH data. We reconstruct \(H(z)\) again with these 37 \(H(z)\) data and obtain the Hubble constant \(H_0\) from this reconstruction, then we multiply the 6 \(E(z)\) data by this new value of \(H_0\) and update the \(H(z)\) data with these new 6 points, the process is repeated until a convergent value of \(H_0\) is obtained. We find that \(H_0 = 64.03 \pm 2.95 \text{ km/s/Mpc}\) which is lower than that in [26]. The reason is that when we multiply smaller \(H_0\) to \(E(z)\), then we get smaller \(H(z)\). So if we add these \(H(z)\) to the CCH data, we expect to get smaller value of \(H_0\) in the next step. To avoid this problem, we just use the inferred value of \(H_0 = 67.46 \pm 4.75 \text{ km/s/Mpc}\) from the first GP reconstruction of the CCH data and divide the CCH data by this \(H_0\) to obtain \(E(z)\) from the CCH data, then we add these \(E(z)\) data to the Pantheon+MCT data to reconstruct \(E(z)\) function. The result is shown in figure 3. Because the reconstructed results start from \(z = 0\) and \(E(z = 0) = 1\) by definition, the convergence at \(E(z = 0) = 1\) decrease the constraint ability of this method near \(z = 0\) (the other reason is the Hubble law at low redshift). From figure 3, we see 3\(\sigma\) evidence of accelerated expansion in the redshift ranges \(0.1 \lesssim z \lesssim 1\), but no significant evidence for decelerated expansion. Again the results don’t mean that in the redshift ranges \(0.1 \lesssim z \lesssim 1\) the universe always experienced accelerated expansion.

![Figure 3. GP reconstruction of \(E(Z)\) from CCH+Patheon+MCT data. The blue solid line is the mean of the reconstruction and the shaded areas are 1\(\sigma\), 2\(\sigma\) and 3\(\sigma\) errors. The dashed line corresponds to \(q(z) = 0\) \((E(z) = 1 + z)\) and the dotted line corresponds to \(E(z) = 1\).](image)

In order to get detailed information about the acceleration and the transition redshift, we reconstruct the deceleration parameter \(q(z)\) from the reconstructed \(E(z)\) and \(E'(z)\) by using the relation \(q(z) = E'(z)(1 + z)/E(z) - 1\), and the result is shown in figure 4. We see that accelerated expansion happened until \(z \lesssim 0.3\) at the 2\(\sigma\) level. The
mean of reconstruction suggests that the transition from deceleration to acceleration happened at $z_t = 0.60^{+0.21}_{-0.12}$.

Figure 4. GP reconstruction of the deceleration parameter $q(z)$ from CCH+Pantheon+MCT. The blue solid line is the mean of the reconstruction and the shaded areas are 1σ and 2σ errors. The dashed line corresponds to $\Lambda CDM$ model and the dotted line corresponds to $q(z) = 0$.

4 Observational constraints on acceleration and dark energy

In this section, we first consider a simple parametrization of the deceleration parameter $q(z)$ [76]

$$q(z) = 1 + q_1 z + q_2 (1 + z)^2.$$  \hspace{1cm} (4.1)

In this model, $q_0 = 1/2 + q_2$. Substituting equation (4.1) into equation (2.18), we get

$$H(z) = H_0 (1 + z)^2 \exp \left[ \frac{q_2}{2} + \frac{q_1 z^2 - q_2}{2(1 + z)^2} \right].$$  \hspace{1cm} (4.2)

To fit the model to CCH data, we calculate

$$\chi^2_H = \sum_i \frac{[H_{\text{obs}}(z_i) - H_{\text{th}}(z_i)]^2}{\sigma_i^2}. \hspace{1cm} (4.3)$$

For the SNe Ia data, we consider the distance modulus measurements from Pantheon compilation and the expansion rate measurements from Pantheon+MCT separately. For the Pantheon+MCT data, we calculate

$$\chi^2_{SN} = \sum_{ij} [E_{\text{obs}}(z_i) - E_{\text{th}}(z_i)] C^{-1}_{E}(z_i, z_j) [E_{\text{obs}}(z_j) - E_{\text{th}}(z_j)]. \hspace{1cm} (4.4)$$
For the distance modulus data, we combine equations (4.2) and (2.6) to get the distance modulus
\[ \mu_{th} = 5 \log_{10} \left[ d_L(z)/\text{Mpc} \right] + 25, \]
then we calculate
\[ \chi^2_{SN} = \sum - \sum^{-1} \Delta \mu, \]  
where \( \Delta \mu = \mu_{obs} - \mu_{th} \) and \( \Sigma_{\mu} \) is the total covariance matrix. Finally, the total chi-square is given by
\[ \chi^2 = \chi^2_H + \chi^2_{SN}. \]  
Because there is no matter density \( \Omega_{m0} \) in this model, so we don’t use the BAO data for this model.

Fitting the model to CCH+Pantheon and CCH+Pantheon+MCT, we obtain the constraints on the model parameters \( q_1 \) and \( q_2 \) along with the Hubble constant and the results are shown in figure 5. The 1σ constraints on the model parameters are shown in table 5. From table 5 and figure 5, we see that the constraints on the model parameters \( H_0, q_1 \) and \( q_2 \) are very similar with either CCH+Pantheon or CCH+Pantheon+MCT data and the results are consistent, so we can replace the full distance modulus data by the compressed expansion rate data. The Hubble constant is consistent with the Planck 2018 result.

For comparison, we also fit the curved ΛCDM model and \( Rh = ct \) model to the CCH+Pantheon data. Because the Pantheon+MCT data assume a spatially flat universe, so we don’t fit the curved ΛCDM model and \( Rh = ct \) model to this data. For the curved ΛCDM model, we get \( H_0 = 69.4 \pm 2.0 \) km/s/Mpc, \( \Omega_{m0} = 0.33 \pm 0.06, \) \( \Omega_{k0} = -0.08 \pm 0.16 \) and \( \chi^2 = 1050.37, \) this result is also shown in table 6. For the \( Rh = ct \) model, we get \( H_0 = 62.34 \pm 1.43 \) km/s/Mpc and \( \chi^2 = 1140.65. \) Both the simple \( q(z) \) and the curved ΛCDM models fit the data well and the Hubble constant is consistent with Planck 2018 result. In terms of Akaike Information Criterion (AIC) which is defined as \( \chi^2 + 2n, \) where \( n \) is the number of parameters in the model, we get AIC=1056.37 for the curved ΛCDM model and AIC=1142.65 for the \( Rh = ct \) model. So comparing with the simple \( q(z) \) model and the curved ΛCDM model, the \( Rh = ct \) model is strongly disfavored by the CCH+Pantheon data.

Table 5. The 1σ constraints on the model parameters for the simple \( q(z) \) model. QDa denotes the data sets CCH+Pantheon and QDb denotes the data sets CCH+Pantheon+MCT.

| Data sets | \( H_0 \) (km/s/Mpc) | \( q_1 \) | \( q_2 \) | \( \chi^2 \) | AIC  |
|-----------|---------------------|---------|---------|--------|------|
| QDa       | 69.14 ± 1.86        | −0.19 ± 0.43 | −1.18 ± 0.10 | 1050.77 | 1056.77 |
| QDb       | 69.38 ± 1.87        | −0.21 ± 0.44 | −1.18 ± 0.11 | 20.86   | 26.86 |

By using the observational constraints, we reconstruct \( q(z) \) and the result is shown in figure 6. Figure 6 shows 3σ evidence for cosmic acceleration in the redshift range 0 ≤ \( z \) ≤ 0.25 and 2σ evidence for cosmic deceleration in the past with \( z \gtrsim 1. \) The transition redshift when the Universe underwent the transition from deceleration to acceleration is \( z_t = 0.61_{-0.16}^{+0.24} \) at the 1σ confidence level. This result is consistent with that in the last section obtained from GP reconstruction and ΛCDM model.
Figure 5. The $1\sigma$ and $2\sigma$ contour plots for the simple $q(z)$ model.

Figure 6. The reconstruction of the deceleration parameter by using the constraints from CCH+Pantheon data for the simple $q(z)$ model. The solid line is drawn by using the best fit parameters. The shaded areas are the $1\sigma$, $2\sigma$ and $3\sigma$ uncertainties. The red dotted line denotes the best fit $\Lambda$CDM model.
Table 6. The 1σ constraints on the model parameters for the ΛCDM model. QDa denotes the data sets CCH+Pantheon and SDa denotes the data sets CCH+BAO+Pantheon.

| Data sets | $H_0$ (km/s/Mpc) | $\Omega_{m0}$ | $\Omega_{k0}$ | $\chi^2$ | AIC |
|-----------|-----------------|--------------|-------------|---------|-----|
| QDa       | 69.4 ± 2.0      | 0.33 ± 0.06  | −0.08 ± 0.16| 1050.37 | 1056.67 |
| SDa       | 68.9 ± 1.8      | 0.325 ± 0.014| −0.09 ± 0.05| 1065.63 | 1071.63 |

To use the BAO data and constrain the matter energy density, now we consider the SSLCPL model [78, 79]. This model approximates the dynamics of general thawing scalar fields over a large redshift range. It has the same form as the commonly used Chevallier-Polarski-Linder (CPL) model which parameterizes the equation of state as [109, 110]

$$w(z) = w_0 + w_a \frac{z}{1 + z}.$$  \hspace{1cm} (4.7)

For the SSLCPL model, the parameter $w_a$ is not an independent parameter, i.e.,

$$w_a = 6(1 + w_0) \frac{(\Omega_{\phi0}^{-1} - 1)[\sqrt{\Omega_{\phi0}} - \tanh^{-1}(\sqrt{\Omega_{\phi0}})]}{\Omega_{\phi0}^{-1/3} - (\Omega_{\phi0}^{-1} - 1) \tanh^{-1}(\sqrt{\Omega_{\phi0}})}.$$  \hspace{1cm} (4.8)

where $\Omega_{\phi0}$ is the dark energy density normalized by the current critical energy density. The SSLCPL model has only one free parameter $w_0$ and it reduces to the ΛCDM model when the parameter $w_0 = -1$. With this explicit degeneracy relation between $w_0$ and $w_a$, we expect to get tighter constraints on $\Omega_{\phi0}$ and $w_0$ for the SSLCPL model than the CPL model does. For a spatially flat universe, the Friedmann equation for the CPL parametrization becomes

$$\frac{H^2}{H_0^2} = \Omega_{r0} (1 + z)^4 + \Omega_{m0} (1 + z)^3 + (1 - \Omega_{r0} - \Omega_{m0}) (1 + z)^3 (1 + w_0 + w_a) \exp \left( -\frac{3w_a z}{1 + z} \right),$$  \hspace{1cm} (4.9)

where $\Omega_{r0}$ is radiation density parameter, $\Omega_{m0}$ is matter density parameter and 1 − $\Omega_{r0} - \Omega_{m0}$ = $\Omega_{\phi0}$ is the dark energy density parameter. To fit the flat SSLCPL model to the observational data and obtain the best fit parameters, we minimize

$$\chi^2 = \chi^2_H + \chi^2_{SN} + \chi^2_{BAO}.$$  \hspace{1cm} (4.10)

The results for fitting both the CCH+BAO+Pantheon data (we label this data sets as SDa) and CCH+BAO+Pantheon+MCT data (we label this data sets as SDb) are shown in figure 7 and table 7. From figure 7 and table 7, we see that the constraints on the model parameters from both SDa and SDb data are very similar and they are consistent, and both results are consistent with flat ΛCDM model. As shown in tables 6 and 7, both flat SSLCPL and curved ΛCDM model fit the observational data well.
Table 7. The 1σ constraints on the model parameters for the SSLCPL model. SDa denotes the data sets CCH+BAO+Pantheon and SDb denotes the data sets CCH+BAO+Pantheon+MCT.

| Data sets | $H_0$ (km/s/Mpc) | $\Omega_{\phi 0}$ | $w_0$ | $\chi^2$ | AIC |
|-----------|-----------------|-----------------|-------|---------|-----|
| SDa       | 66.79 ± 1.40    | 0.687 ± 0.013   | −1.03 ± 0.07 | 1068.48 | 1074.48 |
| SDb       | 66.90 ± 1.39    | 0.688 ± 0.013   | −1.04 ± 0.07 | 38.85   | 44.85 |

Figure 7. The 1σ and 2σ contour plots for the SSLCPL model.

5 Discussion

The null hypothesis that cosmic acceleration never happened gives the kinematic bound $E(z) \geq 1 + z$. In standard cosmology, the null hypothesis $q(z) \geq 0$ is equivalent to the strong energy condition $\rho + 3p \geq 0$. The six $E(z)$ data from Pantheon+MCT can be compared directly with the lower bound to give direct evidence of cosmic acceleration. The five low redshift data points with $z < 1$ lie outside the lower bound at the 3σ confidence level, and the only one high redshift data crosses the bound at the 1σ level. Therefore, we have 3σ direct evidence of cosmic acceleration. This direct evidence does
not assume any gravitational theory or cosmological model. The only caveat from this
direct evidence is that the $E(z)$ assumes a spatially flat universe. Although there is
no strong evidence for decelerated expansion, it does not mean the cosmic acceleration
started at least from $z \sim 0.9$ or there is no decelerated expansion in the redshift ranges
$0 < z < 0.9$ because of the integration effect of the deceleration parameter. Due to
the integration effect, even if the transition from cosmic acceleration to deceleration
happened at the redshift $z \sim 0.6$, the expansion rate remains outside the bound until
$z \geq 2$. The direct evidence does exclude the $Rh = ct$ at the 3σ confidence level.
Comparing the $Rh = ct$ model with ΛCDM model by the $\chi^2$ statistics, the $Rh = ct$
model is also strongly disfavoured by the $E(z)$ data. We also use the CCH data to
give direct evidence of cosmic acceleration. Due to large error bars in the data and the
uncertainties in the value of the Hubble constant, only several data points lie outside
the lower bound. Those data points may be outliers, so the evidence from CCH data
is not convincing.

The GP method was used to reconstruct the $E(z)$ and $q(z)$ functions from the
CCH and Pantheon+MCT data. The Hubble constant $H_0 = 67.46 \pm 4.75 \text{ km/s/Mpc}$
inferred from the reconstructed $H(z)$ by CCH data is consistent with the Planck 2018
result, but it has a little tension with the local measurement even though the error
bar is big. The reconstructed $E(z)$ shows more than 3σ direct evidence for cosmic
acceleration up to the redshift $z \sim 1$, and the reconstructed $q(z)$ function gives the
transition redshift $z_t = 0.60^{+0.21}_{-0.12}$ at which the expansion of the Universe underwent the
transition from acceleration to deceleration.

Fitting the simple two-parameter parametrization $q(z) = 1/2 + (q_1 z + q_2)/(1 + z)^2$
to CCH+Patheon and CCH+Patheon+MCT data we get consistent constraints on the
model parameters. The best fit Hubble constant is $H_0 = 69.14 \pm 1.86 \text{ km/s/Mpc}$. This
value is consistent with the Planck 2018 result and has a little tension with the local measurement. By using the fitted parameters from CCH+Patheon, we reconstruct
$q(z)$ and get the transition redshift $z_t = 0.61^{+0.24}_{-0.16}$ which is consistent with that from
GP method. We also fit the SSLCPL model to the combination of CCH, BAO and
SNe data, and we get consistent results with either Pantheon or Patheon+MCT
data. The Hubble constant $H_0 = 66.79 \pm 1.4 \text{ km/s/Mpc}$ from fitting the SSLCPL
model to the CCH+BAO+Pantheon data is consistent with the Planck 2018 result
and is in tension with the local measurement at 3.6σ confidence level. The addition
of the type Ia SNe data helps the CCH and BAO data to tighten the error bar on the
Hubble constant, but it does not affect the value of the Hubble constant because of
the arbitrary normalization of the luminosity distance.

In conclusion, the expansion rate measured from Pantheon+MCT gives more than
3σ direct evidence for cosmic acceleration. In fitting cosmological models, we can use
the six compressed data points on the expansion rate instead of the full Pantheon
compilation. The CCH and BAO data prefers lower value of the Hubble constant
which is consistent with Planck 2018 result.
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