Quark-mass variation effect on big bang nucleosynthesis

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Abstract. We calculate the effect of variation in the light-current quark mass, $m_q$, on standard big bang nucleosynthesis. A change in $m_q$ at the time of nucleosynthesis affects nuclear reaction rates, and hence primordial abundances, via changes in the binding energies of light nuclei. It is found that a relative variation of $\delta m_q/m_q = 0.016 \pm 0.005$ provides better agreement between observed primordial abundances and those predicted by theory. This is largely due to resolution of the existing discrepancies for $^7$Li. However this method ignores possible changes in the position of resonances in nuclear reactions. The predicted $^7$Li abundance has a strong dependence on the cross-section of the resonant reactions $^3$He ($d, p$) $^4$He and $t$ ($d, n$) $^4$He. We show that changes in $m_q$ at the time of BBN could shift the position of these resonances away from the Gamow window and lead to an increased production of $^7$Li, exacerbating the lithium problem.

1. Introduction

Measurements of the primordial baryon-to-photon ratio $\eta$ from the cosmic microwave background from WMAP [1], coupled with precise measurements of the neutron half-life [2], have made big bang nucleosynthesis (BBN) an essentially parameter-free theory [2, 3, 4]. In this paradigm excellent agreement has been obtained between predicted and observed abundances of deuterium and $^4$He (see, e.g. the Particle Data Group review [2] and references therein). However there is some disagreement for $^7$Li, the only other element for which the abundance has been measured to an accuracy at which fruitful comparison with theory can be made. While the “lithium problem” has been known for some time, it has been exacerbated by recent measurements of the $^3$He($\alpha, \gamma$) $^7$Be reaction [5]. Standard BBN theory with $\eta$ provided by WMAP5 overproduces $^7$Li by a factor of 2.4 – 4.3 (around 4 – 5$\sigma$) [4].

One possible solution to the lithium problem is that the physical constants of the early Universe may have been slightly different. Recently, Ref. [6] examined the response of BBN to variation of several physical parameters, including binding energies, in a linear approximation. These were coupled with calculated dependences of binding energies on $m_q$ in [7], which found that the $^7$Li
abundance discrepancy could be resolved by a variation in light-quark mass of $\delta m_q/m_q = 0.013 \pm 0.002$. Crucially, the $^4$He and d abundances were found to be relatively insensitive to $m_q$ and so the existing agreement between theory and observation in these elements was maintained. In this work we re-examine the dependence of light element production on variation of the dimensionless parameter $X_q = m_q/A_{QCD}$ where $m_q$ is the light-quark mass and $A_{QCD}$ is the pole in the running strong coupling constant. We follow [7] and assume that $A_{QCD}$ is constant, calculating the dependence on the small parameter $m_q$. This is not an approximation. Rather it only means that we measure all dimensions ($m_q$, cross sections, etc) in units of $A_{QCD}$. Therefore $\delta m_q/m_q$ should be understood as $\delta X_q/X_q$. We take into account several effects that were not previously considered, most importantly the nonlinear dependence on $m_q$ and variation of resonance positions.

2. Variation of binding energies

The energy released in each reaction, $Q$, is determined by the nuclear binding energies. As noted in [6], the $Q$-values affect the forward (exothermic) reaction rates via phase space and radiative emission factors. For radiative capture reactions at low energy $E$ the $Q$-dependence is

$$\sigma(E) \sim E^2 \sim (Q + E)^3.$$  \hspace{1cm} (1)

For low-energy reactions with two nucleons in the exit channel the dependence is proportional to the outgoing channel velocity, $v \sim (Q+E)^{3/2}$. When the outgoing particles are charged, the Gamow factor of the exit channel can also contribute:

$$\sigma(E) \sim (Q + E)^{1/2} \exp\left(-\sqrt{E_g/Q}(Q + E)\right).$$  \hspace{1cm} (2)

The Gamow factor appears because of the Coulomb barrier to the reaction; $E_g = 2\pi^2 Z_1^2 Z_2^2 \alpha^2 \mu e^2$ where $\alpha$ is the fine-structure constant, $Z_1$ and $Z_2$ are the charge numbers of the products, and $\mu$ is the reduced mass of the products. At BBN temperatures we can usually assume that $E \ll Q$. Expanding in $Q$,

$$\sigma = \sigma_0\left[1 + 1/2(1 + \sqrt{E_g/Q})\delta Q/Q + \ldots\right],$$  \hspace{1cm} (3)

and we see that the Gamow term in (2) is generally small (it was neglected in [6]). However it can be important for some reactions, for example in $^7$Be (n, p) $^7$Li, $\sqrt{E_g/Q} = 2.17$, i.e. it triples the effect of $\delta Q$ on the reaction rate.

An exception to the rule (1) is found in the reaction p (n, $\gamma$) d, an important reaction because d is a precursor to all further nucleosynthesis. This reaction is sensitive not only to $Q$ but also to the position of the virtual level with energy $\varepsilon = 0.07$ MeV. The sensitivity of this reaction to $Q$ was calculated in [8]

$$<\sigma \nu> \sim [1 + (5/2 + \sqrt{Q/\varepsilon})\delta Q/Q].$$  \hspace{1cm} (4)

Note that [6,7] did not take variation of the virtual level into account.

We denote the sensitivity of nuclear binding energies to the light-current quark mass $m_q$ by

$$K = (\delta E/E)/(\delta m_q/m_q).$$  \hspace{1cm} (5)

Values of $K$ for several light nuclei were presented in Refs. [7, 9]. We use the “best values” from these papers, given by the AV18+UIX nuclear Hamiltonians, with hadron mass variations calculated in terms of the $m_q$ using the Dyson-Schwinger equation calculation of [10]. From these one calculates the $m_q$-dependence of the $Q$ values, and therefore the reaction rates, and therefore the primordial abundances of light elements in BBN.

Comparing the observed and predicted abundances we obtain for $^4$He, d, and $^7$Li respectively, $\delta m_q/m_q = -0.002 \pm 0.037$, 0.012 $\pm$ 0.011, and 0.018 $\pm$ 0.006. The three data sets are therefore consistent, with weighted mean $\delta m_q/m_q = 0.016 \pm 0.005$.  \hspace{1cm} (6)

The $^4$He abundance has a low sensitivity to $m_q$. It is worth pointing out that the more tightly constrained fraction, $Y_p = 0.2477 \pm 0.0029$ [11] instead of conservative fraction $Y_p = 0.249 \pm 0.009$ [12].
used in our calculation is also consistent with the variation (6). Taking into account the nonlinear dependence of BBN abundances on \( m_q \) is important, particularly for \(^7\)Li. In fact, if we assume a linear response, as was done in [6, 7], we instead obtain \( \delta m_q/m_q = 0.014 \pm 0.002 \).

3. Resonances

Of the most important reactions in BBN, the mirror reactions \(^3\)He (d, p) \(^4\)He (reaction 1), and \( t \) (d, n) \(^4\)He (reaction 2) are the only reactions where the cross-section is dominated by a fairly narrow resonance. Therefore, one can hope for sensitivity of primordial abundances to the position of these resonances. Both of these reactions have the cross-sections with the general form

\[
\sigma(E) = \exp(-\sqrt{E_g/E}/E) P(E)/(E-E_r)^2 + \Gamma_r^2/4,
\]

where \( E_g \) is the Gamow energy of the reactants, \( E_r \) and \( \Gamma_r \) are resonance parameters, and \( P(E) \) is a polynomial chosen to fit the measured reaction cross-section. In this work we use the cross-section fits of Ref. [13], which give \( E_r^{(1)} = 0.183 \text{ MeV}, \Gamma_r^{(1)} = 0.256 \text{ MeV} \) and \( E_r^{(2)} = 0.0482 \text{ MeV}, \Gamma_r^{(2)} = 0.0806 \text{ MeV} \) for reactions 1 and 2, respectively.

Consider modification of the resonance positions, \( E_r \rightarrow E_r + \delta E_r \), due to a variation of the fundamental constant \( m_q \). Reaction 1 will be affected in the following way. The resonance is an excited state of \(^5\)Li; that is, a compound nucleus with three protons and two neutrons: we call this state \(^5\)Li*. Similarly there is a state \(^5\)He* for reaction 2. Then

\[
E_r^{(1)} = E_{Li^*} - E_{He} - E_d, \quad E_r^{(2)} = E_{He^*} - E_r - E_d,
\]

and so \( E_{Li^*} = -9.76 \text{ MeV} \) and \( E_{He^*} = -10.66 \text{ MeV} \). The change in the resonance position due to a variation in \( m_q \) is therefore

\[
\delta E_r^{(1)} = \delta E_{Li^*} - \delta E_{He} - \delta E_d = (K_{Li^*}E_{Li^*} - K_{He}E_{He} - K_d E_d)\delta m_q/m_q
\]

with the \( K \) defined by (5).

Changes to the cross-section of reaction 1 affects the primordial abundances of \(^3\)He and \(^7\)Be, while changes in reaction 2 affect abundances of \( t \) and \(^7\)Li. Since \( t \) and \(^3\)He are not well constrained observationally, we choose to focus on \(^7\)Li. In figure 1 we present \(^7\)Li abundance against variation of light quark mass \( \delta m_q/m_q \) at \( \eta = 6.23 \times 10^{-10} \), the WMAP5 value. For such a value of \( \eta \), the majority of \(^7\)Li is created as \(^7\)Be (which \( \beta \)-captures to \(^7\)Li) via the reaction \(^3\)He (\(^4\)He, \( \gamma \)) \(^7\)Be.

We need to find \( K_{Li^*} \) (and similarly \( K_{He^*} \)). One assumption is that the mass-energy of the resonance varies with the mass-energy in the incoming channel [6]; in this case the resonance does not
shift. This assumption corresponds to \( K_{\text{Li}} = -1.54 \) and \( K_{\text{He}} = -1.44 \). It corresponds to the solid line in figure 1. A more reasonable guess is to assume that the variation of the resonant state \(^5\text{Li}\) will be approximately the same as that of the ground state \(^5\text{Li}\). This can be seen by considering the resonance and the ground state configurations as residing in the same potential. The sensitivity of the ground state \(^5\text{He}\) to \( m_q \) has been calculated \( K_{\text{He}} = -1.24 \) [9]; \( K_{\text{Li}} \) was not calculated explicitly, but its value will be very close to that of \(^5\text{He}\). Our assumption of equal variation of the ground and excited state then gives

\[
K_{\text{Li}} = -3.35 , \quad K_{\text{He}} = -3.19
\]  

(10)

This assumption corresponds to the dashed line in figure 1. The equal-variation assumption in the previous paragraph represents an upper limit on the relationship between the ground and excited state. In reality the potential-dependence of the states may be different, in which case the shift of the \(^5\text{Li}\) or \(^5\text{He}\) resonance may be smaller than the shift of the ground state. On the other hand a minimum value of \( K \) for the resonance states is that of the ground state, \( K_{\text{Li}} = K_{\text{He}} = -1.24 \). A reasonable, conservative, estimate is to take the average of these extremal values: \( K_{\text{Li}} = -2.29 \) and \( K_{\text{He}} = -2.21 \); this is the dot-dashed line in figure 1. Ultimately however, we require a nuclear calculation of sensitivity, of the kind presented in Refs. [6, 7]. The first calculation has been done in [14]. The result \( K_{\text{Li}} = -2.48 \) and \( K_{\text{He}} = -2.16 \) is remarkably close to our estimated values. The effect of \( \delta E^{(1)} \) on BBN can be understood in the following way. When the cross-section is convolved with a Maxwellian distribution, the exponential term gives rise to the “Gamow window” at energy \( E_g/E_g = (kT/2E_g)^{2/3} \). This reaction is most active at \( kT \approx 0.07 \) MeV, at which time the Gamow window is at \( E_g = 0.180 \) MeV. This is remarkably close to the resonance energy for this reaction \( E_r = 0.183 \) MeV. Therefore movement of the resonance position in either direction will reduce the cross-section for this reaction at the relevant temperatures. In turn this reduces the amount of \(^3\text{He}\) that is destroyed via reaction 1, leaving more to react with \(^4\text{He}\) to produce \(^7\text{Be}\). On the other hand the effect of this reaction on \( d \) and \( ^4\text{He} \) abundances is minimal. The effect of \( \delta E^{(2)} \) is very similar: it reduces the amount of \( t \) destroyed in reaction 2, leaving more tritium to react with \(^4\text{He}\) to produce \(^7\text{Li}\) directly. Despite this production channel being suppressed at high \( \eta \), the effect of \( \delta E^{(2)} \) is still important for \(^7\text{Li}\) production because the relative effect of the variation is larger \( \delta E^{(2)} / I^{(2)} > \delta E^{(1)} / I^{(1)} \). The trends seen in figure 1 are the same even at low \( \eta \) since both reaction pathways behave in much the same way to variation in \( m_q \).

From figure 1 we see that taking shifts in the resonance positions into account can destroy the agreement between theory and observation previously obtained by varying \( m_q \). In the case where the shifts in the ground and resonant states vary by the same amount (dashed line), the \(^7\text{Li}\) discrepancy actually gets worse with variation in light quark mass. On the other hand the milder “averaged \( K \)” response (dot-dashed line) still significantly challenges the conclusions of Section II.

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