Collective modes in free plasmas subjected to a radiation field

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Abstract. In this study we report the effects of an electromagnetic field on the physical properties of free plasmas. The calculations are carried out in the semi-classical approximation, i.e., the electromagnetic field is treated classically and the electrons from a quantum mechanical viewpoint. The results show that the collective modes are damped away more smoothly and in a smaller frequency range than those reported by previous studies. An exponential-like decay of the frequencies is readily observable from the plot of the plasmon frequency as a function of the external field amplitude. We successfully recreate the results of previous studies. We also obtain that the single photon processes has a pronounced effect on the decrease of the frequency range of modulation.

1. Introduction

The studies concerning the interaction of electromagnetic fields with electron plasmas using the Schrodinger equation to describe the electron states [2, 3, 4] has been done for a long time. In particular, photon-plasma interactions has been discussed for many authors. See [5] and references therein. Here we extend the work developed in ref. [2] where there is no photon process contributing to the collective modes in the plasma, in what they call a weak electron-radiation coupling, and also in references [3, 4] where two limiting cases are discussed, i.e., the strong-field limit such that only multiphoton processes are significant whereas the weak-field regime such that only single-photon processes are significant, respectively.

For this study, the laser beam is treated as a classical plane electromagnetic wave in the dipole approximation. We consider the laser linearly polarized along the z-axes, with the electric field along the x-axes. The electron states are described by the solution of the Schrodinger wave equation for an electron in the laser field. In this report we take into account a finite number of photons interacting with the electron plasma. This simple extension gives rise to a different dispersion relation of the electron waves. We see that an asymptotic value for the plasmon frequency exists as we increase the radiation wave number, and that this frequency never goes to zero, given a non-zero natural plasma frequency. The collective modes of the plasma are obtained numerically from the zeros of the dielectric function.

³ The term free plasma is used as unmagnetized plasma
2. Electron states
For a free plasma under the presence of an electromagnetic wave, the Schrödinger equation for the electrons takes the form:

\[ H \psi(r, t) = i\hbar \frac{\partial \psi(r, t)}{\partial t}, \]

where \( H \) is the hamiltonian of the system given by

\[ H = \frac{1}{2m_e} (p - eA(t))^2, \]

\( p \) is the momentum of the electron, \( A(t) \) is the vector potential of the radiation

\[ A(t) = \left( \frac{E}{\omega} \sin(\omega t) \right) \hat{x}, \]

and \( \omega \) is the frequency of the radiation. To solve this equation, given a time-dependent potential, we use a unitary transformation \([6, 7]\) of the form

\[ \psi(r, t) = U \Phi(r, t), \]

\( \Phi(r, t) \) being the solution of the Schrödinger equation for a free electron and \( U \) the unitary operator:

\[ U = \exp \left( \frac{i}{\hbar} \alpha(t) \cdot r \right) \exp \left( \frac{i}{\hbar} \beta(t) \cdot p \right) \exp \left( \frac{i}{\hbar} \eta(t) \right), \]

where \( \alpha(t), \beta(t) \) and \( \eta(t) \) are, respectively, the momentum translation generator, the spatial translation generator and a phase factor. Substituting this into equation (1) we obtain

\[ \frac{\partial \psi(r, t)}{\partial t} = \frac{i}{\hbar} \left( \frac{d\alpha}{dt} \cdot r + \frac{d\beta}{dt} \cdot p + \frac{d\eta}{dt} \right) \psi(r, t) + U \frac{\partial \Phi(r, t)}{\partial t}. \]

Multiplying by \( U^+ \) and after doing some manipulation:

\[ H_0 = \frac{p^2}{2m_e} + \frac{1}{2m_e} (\alpha(t) - eA(t))^2 + \frac{p}{m_e} (\alpha(t) - eA(t)) + \frac{d\alpha}{dt} \cdot r + \frac{d\beta}{dt} \cdot p - \frac{d\alpha}{dt} \beta + \frac{d\beta}{dt} \alpha + \frac{d\eta}{dt}, \]

where \( H_0 \) is the hamiltonian for the free electron. We proceed to solve the equation for \( \alpha, \beta \) and \( \eta \) (linear terms in \( r \) and \( p \) and independent terms are set equal to zero) and find:

\[ \psi(r, t) = \exp \left( \frac{i}{\hbar} F(t) \right) \exp (i \gamma_0 k \cdot r) \exp (ik \cdot r) \exp \left( -\frac{i}{\hbar} \varepsilon_k t \right), \]

where we have simplified using \( F(t) = -2\gamma_1 \omega t + \gamma_1 \sin(2\omega t) \), \( \gamma_0 = \frac{eE}{m_e\omega^2} \), \( \gamma_1 = \frac{e^2E^2}{8m_e\omega^4} \) and \( \varepsilon_k \) is the energy of the free electron with wave number \( k \).

Equation (8) is the wave function for an electron in an electromagnetic field given by (3).

We can see that the wave function in (8) forms an orthonormal set. So, it is possible to expand the wave function of an electron in a local potential using (8) as basis:

\[ \Psi_k(r, t) = \sum_k a_k(t) \psi(r, t). \]
Assuming a weak local potential of the form
\[
\varphi(r, t) = \int dq \int d\Omega \exp(iq \cdot r) \exp(-i\Omega t) \varphi(q, \Omega) + c.c. \tag{10}
\]
we determine the coefficients \(a_k(t)\) using perturbation theory [8]:
\[
a_{k+q}(t) = e \cdot \exp(-i\gamma_0 q_x t) \sum_{m, \Omega} i^m J_m(q_x \gamma_0) \varphi(q, \Omega) \times \\
\frac{\exp \left( \frac{\hbar}{\epsilon_k+q - \epsilon_k - \hbar \Omega - m \hbar \omega - i\zeta} \right) (\zeta \to 0^+) \cdot e^{-i\Omega t}}{\epsilon_k+q - \epsilon_k - \hbar \Omega - m \hbar \omega - i\zeta} \right), \tag{11}
\]
where we have used the identity
\[
\exp(i\alpha \cos x) = \sum_m i^m J_m(\alpha) \exp(i m x). \tag{12}
\]
Finally, we can write the wave function as
\[
\Psi_k(r, t) = \exp \left( \frac{i}{\hbar} F(t) \right) \exp(i\gamma_0 k_x (1 - \cos(\omega t))) \exp \left( -\frac{i}{\hbar} \epsilon_k t \right) \exp(i k \cdot r) \times \\
\left\{ 1 + e^{i\gamma_0 q_x \cos(\omega t)} \cdot \sum_{q, \Omega} i^m J_m(q_x \gamma_0) \cdot \frac{\varphi(q, \Omega) \exp(-i(\Omega + m \omega t))}{\epsilon_k+q - \epsilon_k - \hbar \Omega - m \hbar \omega - i\eta} \exp(iq \cdot r) \right\} \tag{13}
\]
This last wave function corresponds to the electrons of the plasma under the incidence of the radiation given by (3).

3. Dielectric function

Now that we know the states of the electrons via (13), we can obtain the fluctuation of the charge density
\[
\rho_k(r, t) = -e |\Psi_k(r, t)|^2 - \rho^{(0)}_k(r, t), \tag{14}
\]
\(\rho^{(0)}_k(r, t)\) being the charge density in the absence of a weak local potential, i.e., the charge density given by the \(\varphi(r, t)\) distribution. Neglecting terms of higher orders in \(\varphi\):
\[
\rho_k(r, t) = -e^2 \sum_{q, \Omega} \exp(iq \cdot r) \varphi(q, \Omega) \exp(-i\gamma_0 q_x \cos(\omega t)) \times \\
\left\{ \frac{\exp(-i(\Omega + m \omega t))}{\epsilon_k+q - \epsilon_k - \hbar \Omega - m \hbar \omega - i\eta} + \frac{\exp(i(\Omega + m \omega t))}{\epsilon_k+q - \epsilon_k + \hbar \Omega + m \hbar \omega + i\eta} \right\}. \tag{15}
\]
Assuming a maxwellian distribution \(f_k\), we have the total fluctuation as
\[
\rho(r, t) = \sum_k f_k \rho_k(r, t). \tag{16}
\]
Using, once more, relation (12), we obtain
\[
\rho(r, t) = -e^2 \sum_{q, \Omega} \exp(iq \cdot r) \varphi(q, \Omega) \exp(-i\Omega t) \times \\
\sum_{m, m'} i^{m-m'} J_m(q_x \gamma_0) J'_m(q_x \gamma_0) \exp(-i(m - m') \omega t) \cdot \Pi(q, \Omega + m \omega), \tag{17}
\]
where $\Pi(q, \Omega) = \sum_k \frac{f_{k+q} - f_k}{\varepsilon_{k+q} - \varepsilon_k - m\Omega - m}$ is the electronic polarizability and $m$ corresponds to the number of photons involved in the process$^4$. We take the real part of (17) to calculate the fluctuation. This fluctuation induces a potential in the medium given by Poison equation

$$\nabla^2 \varphi_{\text{ind}}(r, t) = -4\pi \rho(r, t).$$

(18)

Using (17) and the Fourier transform of (18), we obtain

$$\varphi_{\text{ind}}(q, \Omega) = \frac{4\pi \epsilon_0^2}{q^2} \sum_m J_m^2(q \gamma_0) \Pi(q, \Omega + m\omega),$$

(19)

which is the induced part of the full local potential:

$$\varphi(q, \Omega) = \varphi_{\text{ext}}(q, \Omega) + \varphi_{\text{ind}}(q, \Omega) = \frac{\varphi_{\text{ext}}(q, \Omega)}{\epsilon(q, \Omega)},$$

(20)

$\epsilon(q, \Omega)$ being the dielectric function of the plasma. The roots of the real part of this dielectric function give us the frequency of the longitudinal waves (collective oscillations, i.e., plasmons) in the plasma. We, therefore, separate $\epsilon(q, \Omega)$ into a real and an imaginary part. We proceed to the classical limit by letting $\hbar \to 0$ and, after some algebraic work, we are left with:

$$\epsilon_{\text{real}}(q, \Omega) = 1 - \omega_p^2 \sum_m J_m^2(q \gamma_0) \frac{1}{\lambda^2} \left( 1 + \frac{2 \langle q \cdot v \rangle}{\lambda} + \frac{3 \langle (q \cdot v)^2 \rangle}{\lambda^2} \right) \exp \left( -\frac{\epsilon_\gamma}{k_B T} \right),$$

(21)

where $\lambda = m\omega + \Omega$, $\omega_p$ is the plasma natural frequency, $\epsilon_\gamma = 2\gamma_1 \omega$ is the energy of the electromagnetic radiation and the average $\langle g(q, v) \rangle$ is taken with respect to the function $f_k$.

4. Collective modes

The roots of (21) give us the frequencies of collective oscillations in the plasma as a function of their wave number ($\Omega(q)$). Here we encounter a numerical difficulty, as the summation over $m$ does not allow us to get an analytical result for this frequencies. However, if we fix a value for $q$, we obtain an expression depending only on $\Omega$, which is easily solvable using a bisection method in FORTRAN language$^9$. The sum over $m$ is truncated for a value of $m$ beyond which the terms become negligible. In this way, we take values of $q$ from zero to about 20000 $m^{-1}$, and for each $q$ we get a frequency value. So, we can plot the dispersion relation ($\Omega$ versus $q$) of this system for various values of the radiation frequency. In the same manner, we can fix values of $E$ and get the dependence of $\Omega$ with $E$. These plots are shown in figures (1) and (3). We see that, for any given value of $\omega$ (the frequency of the radiation), the asymptotic value for the plasmon frequency is the same, the difference is that for larger values of $\omega$ the plasmon frequency decays slower. From this we see that, for a radiation with large enough energy, the plasma remains unperturbed.

We can also vary the number of photons involved in the process and see how the curves react for a natural plasma frequency of $6.10^{11} s^{-1}$. This plots are shown in figures (2) and (4).

In figure (2) we note a clear restriction to the range of frequencies allowed to the plasmons as $m$ increases. So, as more photons participate in the process, the more energetic the plasmons are (given the same value of $q$). For all plots, the temperature of the plasma is $k_B T = 1, 6.10^{-19} J$.

$^4$ Notice how $m$ appears as the number of $\hbar \omega$ contributing to the polarizability.
Figure 1. The natural plasma frequency for this case is $6.10^{11}\text{s}^{-1}$ and $E=10\text{ V/m}$. Notice that, as we increase the radiation frequency, the dispersion decays slower. The asymptotic value for $\Omega$ is about $4.2.10^{11}\text{s}^{-1}$.

Figure 2. There is an asymptotic value for the plasmon frequency as we increase both $m$ and $q$. The curve for $m=0$ reproduces exactly that of ref. [2].

Figure 3. The natural plasma frequency for this case is, also, $6.10^{11}\text{s}^{-1}$ and the value of $q$ is generated randomly in the interval of figure (1). For the $E$ dependence we see an almost exponential like decay, which agrees with exponential term that appears in (21).

Figure 4. We see the tendency of the curve becoming an exponential-like decay as we increase the number of photons in the process.

5. Conclusion
The number of photons involved in the interaction between the plasma and the electromagnetic radiation is of extreme importance. We see that a single photon "elevates" the range of frequencies of the plasmons (see figure (2)), and, as we increase the number of photons, an asymptotic value appears for these frequencies.

The plasmon frequency decays very rapidly with increasing field amplitude ($E$) in an exponential-like curve (see figure (3)). As we increase the number of photons in the process, we no longer see an oscillation in the curve of $\Omega$ versus $E$ as reported by [2]. We are currently extending these results allowing nonlinearities, and putting forward a full numerical calculation of the dielectric constant.
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