Abstract. I first discuss the NLO QCD calculation for the dissociation cross section of $J/\psi$ into open charm, which shows a large second order correction near the threshold. The large correction softens when effective thermal masses in the order of the binding energy are introduced for the quarks and gluons. Hence, with the thermal masses, the method is reliably applied to calculate the dissociation cross section of $J/\psi$, which according to recent lattice results seems to remain bound even up to $1.6 \times T_c$. The dominant contribution to the dissociation comes from the thermal gluons, while the effects due to thermal quarks are suppressed due to suppressed phase space. The relevance of this result is discussed in the context of RHIC and signature of QGP.

1. Quarkonium hadron interaction in QCD

Ever since $J/\psi$ suppression was suggested as a signature of quark gluon plasma formation in the early stages of a relativistic heavy-ion collision[1], much work have been performed in estimating the quarkonium hadron interaction in QCD. This is so because $J/\psi$ suppression is also expected to occur in the hadronic environment due to the absorption by the nucleons and by the comoving hadrons. Moreover, recent quenched lattice calculations show that the $J/\psi$ remains bound even up to $1.6 \times T_c$[2, 3, 4, 5]. Therefore, calculating the dissociation cross section of bound $J/\psi$ by thermal quarks and gluons became a necessity because such information are crucial in estimating the hitherto neglected effects of the dynamical quarks on the bound state from lattice calculation, and in quantitatively obtaining the survival probability of the bound state in the QGP state. To address all of these questions in QCD, the basic ingredients are the dissociation cross sections of quarkonium by partons. The hadronic cross sections are then obtained by folding the elementary partonic cross section by the parton distribution amplitudes inside the hadrons, while the thermal cross section are obtained by folding it with the thermal distributions inside the QGP.

Many model calculations have been performed to calculate the dissociation cross section by hadrons. As can be seen from fig.1 for the dissociation cross section of $J/\psi$ by pion, the calculations from each model vary greatly in their energy dependence and magnitude near the threshold. To understand the discrepancies and to obtain a consistent result, it is crucial to probe each model calculations further so as to determine the uncertainty and the valid energy range of each model calculations. The LO QCD approach was first performed by Peskin more than 25 years ago[6, 7]. The NLO calculation was performed only recently by making use of the Bethe-Salpeter amplitude and QCD factorization theorem[8]. As I will discuss in the next section, the second order correction becomes very large near threshold. This suggests that the QCD results are not reliable near threshold, and hence partly explains the differences with other model calculations in this energy region. The large second order correction in QCD can be traced down to a logarithmic behavior of the energy of the outgoing partons scaled with the binding energy. Therefore, for the charmonium case, where the binding energy is less than 1 GeV,
2. Quarkonium hadron interaction in QCD

2.1. Heavy quark propagation

Let us begin with some introduction on the propagation of heavy quarks in the QCD vacuum. The propagation of a heavy quark can be approximated by a perturbative quark propagation with a perturbative gluon insertion, which probes the non perturbative gluon field configuration in the QCD vacuum. Hence, the full heavy quark propagator is,

$$iS^A(q) = iS(q) + iS(q)(-igA)iS(q) \cdots ,$$

where, $iS(q) = i/(q - m)$ and $m$ is the heavy quark mass. The description in Eq.(1) is valid even for $q \to 0$, because $m \gg A \sim \Lambda_{QCD}$, where in the end only gauge invariant combination of the gauge field $A$ will remain after taking the vacuum expectation value.

2.2. System with two heavy quarks

The propagation of a system composed of a heavy quark and an antiquark can also be approximated by combined perturbative heavy quark propagator with gluon insertions. However, since there a two heavy quarks involved, based on the operator product expansion, the propagation can be typically written in the following form.

$$\Pi(q) = \cdots + \int_0^1 dx \frac{F(q^2, x)}{(4m^2 - q^2 - (2x - 1)^2q^2)^n} \langle G^n \rangle \cdots ,$$

where, $F(q^2, x)$ is a function depending on the structure of the two quark system and $\langle G^n \rangle \sim A_{QCD}^{2n}$ denotes the typical gauge invariant expectation value of gluonic operator of dimension $2n$. 

Figure 1. $\sigma_{J/\psi + \pi}$ cross section from meson exchange model [9], quark exchange model [10] and perturbative QCD [7] calculations.
Table 1. Physical processes involving two heavy quarks where $4m^2 - q^2 \gg \Lambda^2_{QCD}$ and where a perturbative treatment such as Eq.(2) are possible.

| $q^2$                  | Process                                      | Expansion parameter       |
|------------------------|----------------------------------------------|----------------------------|
| $0$                    | Photo production of open charm               | $\frac{\Lambda^2_{QCD}}{4m}$ |
| $-Q^2 < 0$             | QCD sum rules for heavy quark system         | $\frac{\Lambda^2_{QCD}}{4m^2 + Q^2}$ |
| $m^2_{J/\psi} > 0$    | Dissociation cross section of bound states   | $\frac{\Lambda^2_{QCD}}{4m^2 - m^2_{J/\psi}}$ |

Figure 2. The Bethe-Salpeter equation for quarkonium.

The integration variable $x$ can be thought of as the momentum fraction carried by one of the heavy quark. Here, one notes that such perturbative expansion is valid when $4m^2 - q^2 \gg \Lambda^2_{QCD}$. There are several cases where this condition is satisfied and perturbative QCD treatments are possible. Those cases are summarized in Table 1.

In the last line of Table 1, $4m^2 - m^2_{J/\psi} \approx (2m + m_{J/\psi})\epsilon_0$, where $\epsilon_0$ is the binding energy of the $J/\psi$. In QCD if $m \rightarrow \infty$, the bound state becomes Coulombic and $\epsilon_0 = m [N_c g^2 / (16\pi)]^2 \gg \Lambda_{QCD}$ in the large $N_c$ limit. Therefore, the expansion parameter becomes small and the dissociation cross can be calculated using perturbative QCD.

2.3. Historical perspective on LO calculation

The LO calculation for the dissociation of quarkonium by hadrons have been performed by Peskin[6] and applied to the charmonium system by Bhanot and Peskin[7] more than 25 years ago. The result have been rederived by Kharzeev and Satz[11] and by Arleo et al.[12], and used in the analysis of $J/\psi$ suppression in heavy ion collision. All the derivation were based on calculating the real part of the correlation function using operator product expansion. Therefore, the derivation was rather time consuming and difficult to apply for a NLO calculation. Recently, we have derived anew the leading order pQCD result using the QCD factorization theorem [13]. Here, we will sketch the derivation. As in the original derivation by Peskin, one should note that the time scale of interaction to form the bound state, should be much smaller than that between the dipole and the hadrons. In momentum scale, it means that the bound state are produced by momentum scales of order $p \sim O(mg^2)$, while the typical scale for the interaction between the bound state and external hadron should be of order $\epsilon_0 \sim O(mg^4)$, which will be the separation scale in the OPE. Since the momentum scale of the bound state are larger than the separation scale, the bound state should be treated perturbatively, as in the Wilson coefficient. Hence, the bound state will be introduced as a vertex using the Bethe-Salpeter equation.

The Bethe-Salpeter equation, represented diagrammatically in Fig. 2, can be written as

$$\Gamma_\mu(p_1, -p_2) = -ig^2 C_F \int \frac{d^4K}{(2\pi)^4} \gamma^\alpha i \Delta(K + p_1 + p_2) \Gamma_\mu(K + p_1 + p_2, K)$$
Figure 3. Leading order diagrams for $\Phi + g \to Q + \bar{Q}$. The two lower diagrams are suppressed in the large $N_c$ limit.

$$x \Delta(K) \gamma_\alpha V(K + p_2),$$  \hfill (3)

where $C_F = (N_c^2 - 1)/N_c$, $i\Delta(K)$ is the quark propagator, and $iV(K)$ the gluon propagator. Changing variables to $q = p_1 + p_2$, $p \equiv (p_1 - p_2)/2$ and working in the rest frame of the bound state $\Phi$, the BS amplitude reduces to the Schrödinger equation with a relative bound state wave function $\psi(|\vec{p}|)$. Then the BS vertex can be written as,

$$\Gamma_\mu \left( \frac{q}{2} + p, -\frac{q}{2} + p \right) = \left( \epsilon_0 + \frac{\vec{p}\cdot\vec{p}}{m^2} \right) \psi(|\vec{p}|) \sqrt{\frac{m\Phi}{N_c}} \frac{1 + \gamma_0}{2} \gamma_\mu \frac{1 - \gamma_0}{2}.$$  \hfill (4)

where $\epsilon_0 = 2m - m_\Phi$. Using this BS vertex, one can obtain the LO matrix element for the LO graph given in Fig.3 by observing the following counting scheme.

$$|\vec{p}_1| \sim |\vec{p}_2| \sim O(mg^2), \quad k^0 = |\vec{k}| \sim O(mg^4).$$  \hfill (5)

These are obtained from noting that the binding energy $\epsilon_0 = m(N_c g^2/16\pi)^2 \sim O(mg^4)$ and that the three momentum of the heavy quarks are of $O(mg^2)$, and also using the non-relativistic energy conservation $m_\Phi + k_0 = 2m + |\vec{p}_1|^2/2m + |\vec{p}_2|^2/2m$.

3. Next to leading order result

The NLO result has been obtained in Ref.\[8\]. At this order, there are contributions with three body final states. These are $\Phi + q(g) \to Q + \bar{Q} + q(g)$. The counting scheme is similar to that of the LO with the incoming and outgoing parton momentum all of $O(mg^4)$. That is,

$$|\vec{p}_1| \sim |\vec{p}_2| \sim O(mg^2), \quad k^0 = |\vec{k}_1| \sim k^0_2 = |\vec{k}_2| \sim O(mg^4),$$  \hfill (6)

where $k_1$ is the incoming quark (or gluon) momentum, and $k_2$ is the outgoing quark (or gluon) momentum. The full calculation have been performed recently by T. Song and S. H. Lee\[8\]. Here, I will only highlight some findings.

3.1. Divergences

There are divergences that appear in the intermediate stages of the calculation. As an example, Fig.4 is the lowest order diagram that involves an incoming quark. If the quarks and gluons are massless, the cross section coming from Fig.4 has collinear divergence when the outgoing
energies of the Coulomb bound states. Specifically, from the relation $E = mc^2$, the binding energy is found to be $750$ MeV.

Here we show the result applied to the upsilon dissociation cross section. The two independent parameters of the theory is determined by fitting the physical masses of $\uppsilon(1S)$ and $\uppsi(2S)$ to the energies of the Coulomb bound states. Specifically, from the relation $m_{\uppsi(2S)} - m_{\uppsi(1S)} = \frac{3}{\epsilon_0}$, the binding energy is found to be $750$ MeV. Also, the bottom quark mass is found to be $5.1$ GeV from equating it to $(m_{\uppsi(1S)} + \epsilon_0)/2$. The coupling constant $g$ is then found to be $2.53$ from $g^2 = \frac{16\pi}{N_c \sqrt{\epsilon_0} / m}$.

Fig.5 is the plot for the elementary total cross section of $\Phi + q \rightarrow Q + Q + q$ and $\Phi + g \rightarrow Q + Q + g$ as a function of the incoming energy. In the right figure, the different lines represent different parts of the diagrams explained in [8]. In both figures, there are some energy regions where the cross sections are negative. The negative values come from mass factorization, because finite parts of the cross sections have been subtracted out and placed in the definition of the distribution function. Therefore, the cross section becomes physical only after folding the elementary cross sections with the parton distribution functions (PDF), and adding those to the LO contribution. Nonetheless, it is clear that there are large fluctuation near the threshold.
Figure 6. The left figure is the $\Upsilon(1S)+$nucleon total cross section to LO (dashed line) and to NLO (solid line). The right figure is the corresponding ratio between NLO and LO results.

and that the contributions from the gluon induced reactions are much larger than that from the quark.

The left figure of Fig.6 shows the LO and the NLO total dissociation cross section, while the right figure shows their ratio. As can be seen in the figure, the perturbative QCD approach is acceptable only in some limited energy region, and has large higher order corrections in the threshold region.

4. Lessons to learn

4.1. Gluon vs quark induced dissociation

As can be seen from Fig.5, the contribution from gluons are an order of magnitude larger than that from the quarks. The reason for such difference is not restricted to the case of Upsilon system, but is a general result coming from the following facts.

- There are many more gluon induced diagrams.
- Even for the diagrams in Fig.4, the contribution is larger when the incoming and outgoing quarks are replaced by gluons.

4.2. Origin of large correction

The reason for large fluctuation of the cross section near threshold can be traced down to the logarithmic behavior of the energy of the outgoing massless parton. The explicit form for Fig.4 is given by,

$$\log \left[ \frac{2k_{2,0}}{\epsilon_0} \right]$$

(7)

where $k_{2,0}$ is the energy of the outgoing parton and $\epsilon_0$ is the binding energy of the bound state. This term varies from $-\infty$ to 0, as $k_{2,0}$ changes from 0 to $\epsilon_0/2$, which corresponds to the threshold region.

For the $J/\psi$, $\epsilon_0 \approx 700\text{MeV}$. Therefore, an effective thermal mass of few hundred MeV is large enough for the quarks and gluons to tame the fluctuation and to make the higher order corrections convergent near the threshold. Such thermal mass occur in the QGP state at temperatures higher than the critical temperature $T > T_c$. 
5. Dissociation of $J/\psi$ due to thermal gluons and quarks
The previous discussions justifies the use of perturbative QCD method to estimate the dissociation of $J/\psi$ by thermal quarks and gluons.

5.1. Gluon vs quark effects
As discussed in the previous section, a thermal mass for the quark and gluon of a few hundred MeV is enough to make the perturbative calculation meaningful. Moreover, from general grounds, the order $\alpha^2$ dissociation induced by quarks are much smaller than that from gluons. Hence the thermal width acquired by $J/\psi$ from quarks will be much smaller than that from the gluons.

![Figure 7](image)

**Figure 7.** Lowest order dissociation cross section of $J/\psi$ by gluons and quark.

Fig.7 shows the elementary dissociation cross section with an effective thermal mass of 200 MeV for the quarks and gluons. The Solid (dashed) line represents the lowest order dissociation due to a gluon (quark). For the gluon, the lowest order appears at order $\alpha$, while for the quarks, it does at order $\alpha^2$. The parameters have been fitted to the charmonium mass assuming a Coulomb bound state. Such procedure gives 800 MeV for the binding, but the value have been artificially changed to 400 MeV in producing the cross sections shown in the right figure of Fig.7.

Fig.8 represents the effective thermal cross section of gluons and quarks calculated from the following formula.

$$\langle \sigma \rangle = \frac{\int \sigma(p)p^2 f(p) dp}{\int p^2 f(p) dp}, \quad (8)$$

where $f(p)$ is the thermal distribution of the parton.

Finally, Fig.9 represents the effective thermal width calculated from the following simple formula.

$$\Gamma_{eff} = deg \cdot \int \sigma(p)f(p)p^2 dp, \quad (9)$$

where $deg$ is the degrees of freedom of the parton, and we have assumed the relative velocity to be $c$. We can conclude the following.

(i) The effects of quarks to the thermal width of the $J/\psi$ are negligible compared to that of gluons. Therefore, the conclusions about the fate of $J/\psi$ up to $2 \times T_c$ from quenched lattice result, will not be modified by the introduction of dynamical fermions.

(ii) The thermal width becomes in the order of 1 GeV at temperatures of above 600 MeV.
Acknowledgments
I would like to thank the organizers for inviting me to this exciting conference.

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Figure 8. Effective thermal cross sections.

Figure 9. Effective thermal width of $J/\psi$ at finite temperature.