Comment on “Specific Heat and Shape Transitions in Light $sd$ Nuclei”

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Abstract

This comment re-examines the origin of structure seen in the computed specific heat of finite nuclei. In a recent paper, Civitarese and Schvellinger suggest that such structure is due to model-space truncation in the calculations. We reaffirm our conclusion that the structure is caused by a collective-to-non-collective phase transformation at low temperatures, signaled by a change in the nuclear level density below 10 MeV excitation energy.

21.60.Ev, 21.60.Fw, 21.10.Re, 27.30.+t
In a recent paper Civitarese and Schvellinger [1] have presented the results of calculations of the thermal properties of eleven even-even sd nuclei in the SU3 model of Elliott [2]. Within the framework of this model they calculated the specific heat and the ensemble average of the intrinsic quadrupole moment of these nuclei in the canonical ensemble. Since the latter quantity does not vanish as the temperature is raised, although a peak is observed in the specific heat, they claim to have presented evidence against the occurrence of phase transitions in light nuclei. Similar results have also been obtained by Dukelsky et. al. [3]. Moreover, Civitarese and Schvellinger suggest that their results are in contradiction to those of our earlier work [4]. This is not the case, and in the present comment we simply wish to set the record straight with regard to our arguments in favor of the existence of a low temperature collective-to-non-collective phase transition in nuclei [1–8].

In a series of papers we have attempted to compare finite-temperature Hartree-Fock (FTHF), or approximate canonical calculations, with canonical calculations which use the exact eigenstates of the same realistic effective Hamiltonian operator [4–10] in the same model space [4–7,11–14]. For this reason we have restricted our calculations to the sd shell, where exact shell-model calculations are possible. In the FTHF calculations in both $^{20}$Ne [6] and $^{24}$Mg [13] the ensemble average of the quadrupole moment vanishes, and this was taken to indicate the occurrence of a shape transition [15,16]. However, as we have previously pointed out [4–6,13], this behavior appears to an artifact of the finite-temperature mean-field calculation and depends on the volume of the system [17,18]. The canonical-ensemble average of the quadrupole moment [13] or the Hill-Wheeler deformation parameter $\beta$ [6] calculated with exact shell model eigenstates does not vanish, in agreement with results obtained in the SU3 model [11,12]. More recent calculations of the ensemble average of the quadrupole moment squared $Q^2$. $Q^2$ indicate that it is discontinuous in the FTHF approximation, while no discontinuity is observed in the canonical calculations [19]. Here it should also be noted that for light nuclei the thermal fluctuations in the quadrupole moment and $\beta$ are large and, when taken into account, wash out any differences between the predictions of the finite-temperature mean-field calculations and the exact canonical
calculations \cite{13,20}. For this reason we have expressed some concern about attempts to confirm experimentally the existence of such shape transitions \cite{8}.

The question now arises: Does a low-temperature phase transition (or, more correctly, a phase transformation) take place in nuclei? Clearly it cannot be a true phase transition because these take place only in infinite systems. However, if we use as operational definition of a phase transition in a finite system, the remnant of the true phase transition which would occur in the thermodynamic limit of the finite system \cite{21,22}, then we feel that there is evidence for a non-collective-to-collective phase transition in finite nuclei. One of the universal features of finite nuclei is an abrupt change in the density of states at excitation energies of 10 MeV or less \cite{23}. At lower excitation energies the spectrum of most nuclei is sparse and dominated by a relatively small number of collective states. With increasing excitation energy, the independent particle degrees of freedom dominate and the density of states grows exponentially. Moreover, since the nuclear force is short-ranged and saturates rather quickly one would expect that such a change in the many-body level density might also occur in nuclear matter. A recent semi-empirical determination of the properties of nuclear matter \cite{24} has indeed provided evidence for such a low-temperature phase transition, which is consistent with the predictions of realistic finite-temperature BCS calculations \cite{25}.

We have therefore suggested that the peak seen in the specific heat, $C$, in both FTHF calculations and canonical calculations for finite nuclei \cite{1,5} is indicative of this collective-to-non-collective phase transition. In both cases, it is useful to note \cite{26,27} that the specific heat is given by

$$C = \frac{\partial E}{\partial T} = T \frac{\partial S}{\partial T}$$

where $E$ is the ensemble average of the energy and $T$ is the temperature. The entropy is given by

$$S = \log \rho$$

where $\rho$ is the many-body density of states. Peaked structures in the specific heat, which we
have identified as signatures of the aforementioned phase transition, can be directly related to abrupt changes in the many-body level density [4,5], which are clearly due to dynamical effects [7]. Clearly one must be careful to ascertain that any peaked structure in $C$ does not arise from the finite size of the model space [3,28]. For this reason we have performed canonical calculations of $C$ using not only the eigenstates of an $sd$ shell effective interaction, but also the experimental nuclear structure information [4]. In $^{20}$Ne in both cases a peaked structure is observed in $C$, which agrees with the FTHF predictions [4]. If the continuum contribution is taken account [4,7,29], the peak in $C$ remains, although the behavior of $C$ changes at higher temperatures. Furthermore, we have also shown [5] that the FTHF does very good job of approximating the change in the many-body level density seen in the shell-model calculations. This change in the spectral distribution, or normalized level density, is also seen in the SU3 model calculations in Figure 2 of Ref [1].

In conclusion, we would like to point out that the results in Ref [1], in our opinion, do not disagree with our previous work but, on the contrary, seem to support our conclusions with respect to the existence of a low-temperature collective-to-non-collective phase transition in nuclei.

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