Photonic simulation of topological superconductor edge state and zero-energy mode at a vortex

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Photonic simulations of quantum Hall edge states and topological insulators have inspired considerable interest in recent years. Interestingly, there are theoretical predictions for another type of topological states in topological superconductors, but debates over their experimental observations still remain. Here we investigate the photonic analogue of the $p_x + ip_y$ model of topological superconductor. Two essential characteristics of topological superconductor, particle-hole symmetry and $p_x + ip_y$ pairing potentials, are well emulated in photonic systems. Its topological features are presented by chiral edge state and zero-energy mode at a vortex. This work may fertilize the study of photonic topological states, and open up the possibility for emulating wave behaviors in superconductors.

Topological state is a fundamentally new quantum state which is topologically distinct from all conventional states of matter. The first example is the quantum Hall (QH) state which has a full gap in the bulk and gapless chiral modes at the edge1–2. In the QH system, an external magnetic field is applied, explicitly breaking time-reversal (TR) symmetry. Beside this, another topological class of materials have been demonstrated invariant under TR, in the presence of strong spin-orbit coupling, which is named as topological insulator or quantum spin Hall (QSH) effect3–7. The QSH state in two dimension (2D) can be roughly understood as two copies of the QH state, where states with opposite spin propagate in opposite directions7.

In 2008 Haldane and Raghu demonstrated the possibility of achieving photonic analogue of the QH edge states of electrons, based on the fact that the edge states are a property of a one-particle eigenstates problem, which can be mapped into the Maxwell normal-mode problem8. Such photonic edge states were soon realized in microwave experiments in photonic crystals made with magneto-optic media9,10. Since then, various configurations have been proposed to achieve photonic topological states, such as aperiodic coupled-resonator system11 and dynamically modulated photonic resonator lattice exhibiting an effective magnetic field12. These topologically protected edge states ensure the absence of backscattering of waves by disorder, which suggest potential applications ranging from on-chip isolation10 to delay line11. Meanwhile photonic simulations of topological insulator without the breaking of TR symmetry have been designed in photonic metacrystal made with bi-anisotropic media13 and a lattice of optical ring resonators14. More recently, photonic Floquet topological insulator has been experimentally realized in a honeycomb lattice composed of coupled helical waveguides15, and synthetic magnetic fields for photons have been achieved by using linear silicon photonics16.

Interestingly, there is another type of topological states in chiral topological superconductor (TSC) which has a pairing gap in the bulk and gapless bound state at the edge6,7. Besides the edge state, the existence of zero-energy mode at a vortex core has been theoretically predicted17,18, which attracts great attention in the recent decade19. The simplest model for the TR breaking TSC is proposed in $p_x + ip_y$ pairing states17, whose Hamiltonian is given as

$$H_p = \begin{bmatrix} \frac{\hbar^2}{2m} - U' & \Delta(p_x + ip_y) \\ \Delta^*(p_x - ip_y) & \frac{\hbar^2}{2m} - U' \end{bmatrix},$$

where $\hat{p}$ is the momentum, $U'$ is the chemical potential, and the off-diagonal elements are pairing potentials. In the weak pairing phase with $U' > 0$, chiral edge states appear2. Although many attempts have been made to observe such edge states experimentally, there are still debates on them20–23.
In this work, we bridge the fields of TSC and photonics. A direct mapping is established between the $p_x + ip_y$ Hamiltonian of TSC and the effective Hamiltonian of a suitably designed electromagnetic medium. Photonic simulations of TSC edge state and zero-energy mode at a vortex are investigated. This study extends the essential physics of the $p_x + ip_y$ model to photonic systems, which may provide a clean and convenient platform to study topological states of TSC, due to the flexibility of material design and less difficulties in experimental observations. On the other hand, this idea may fertilize the study of photonic topological states, and inspire researchers to explore more fruitful topological effects.

**Results**

**Photonic analogue of the $p_x + ip_y$ Hamiltonian.** The main challenges of achieving photonic analogue of the $p_x + ip_y$ Hamiltonian lie in two aspects: (i) one of the critical properties of TSC is particle-hole symmetry, of which the photonic counterpart may seem unimaginable at first sight; (ii) pairing potential of the $p_x + ip_y$ pairing state is required to be a linear function of momentum (see the off-diagonal elements of the Hamiltonian), whereas such kind of interaction is not common in electromagnetic media. Here we propose a two-step approach to give a solution.

In general, the propagating waves in 2D can be classified into transverse electric (TE) mode and transverse magnetic (TM) mode. Thanks to electromagnetic duality of Maxwell’s equations, equations for TE mode can be converted into that for TM mode by making transformations of $E_z \rightarrow H_{xy}$, $H_x \rightarrow -E_y$, $H_y \rightarrow -E_x$, $\varepsilon \rightarrow \mu$, and $\mu \rightarrow \varepsilon$. We suggest using band structures of TE and TM modes to mimic particle and hole states, respectively. However, electromagnetic duality is not enough for realizing photonic “particle-hole symmetry”, since particle-hole transformation also changes the sign of the kinetic energy. We design photonic band structures with positive- and negative-refractive-index bands by using metamaterials, which are also known as right-handed structures with positive- and negative-refractive-index bands by using metamaterials, which are also known as right-handed structures with positive- and negative-refractive-index bands by using metamaterials, which are also known as right-handed structures with positive- and negative-refractive-index bands by using metamaterials, which are also known as right-handed structures with positive- and negative-refractive-index bands by using metamaterials, which are also known as right-handed structures with positive- and negative-refractive-index bands by using metamaterials, which are also known as right-handed structures with positive- and negative-refractive-index bands by using metamaterials, which are also known as right-handed structures with positive- and negative-refractive-index bands by using metamaterials, which are also known as right-handed structures with positive- and negative-refractive-index bands by using metamaterials, which are also known as right-handed structures with positive- and negative-refractive-index bands by using metamaterials, which are also known as right-handed structures with positive- and negative-refractive-index bands by using metamaterials, which are also known as right-handed structures with positive- and negative-refractive-index bands by using metamaterials, which are also known as right-handed structures with positive- and negative-refractive-index bands by using metamaterials, which are also known as right-handed structures with positive- and negative-refractive-index bands by using metamaterials, which are also known as right-handed structures with positive- and negative-refractive-index bands by using metamaterials, which are also known as right-handed structures with positive- and negative-refractive-index bands by using metamaterials.

Figure 1 | Photonic simulation of TSC band structures. (a) A combination of electromagnetic duality and handedness inversion of photons mimics particle-hole transformation for electrons. For example, if a right-handed TE mode represent “particle”, a transformed left-handed TM mode can be considered as “hole”. (b,c) Band diagrams for particle-hole symmetric systems (b) without and (c) with $p_x + ip_y$ pairing potential. The $p_x + ip_y$ pairing potential leads to a gap with topological phase.

![Figure 1](link-to-figure)
By tuning \( m \) and \( U \), one can obtain a two-band diagram, as shown in figure 1b (colored blue).

Taking advantage of the electromagnetic duality of Maxwell’s equations, we can readily obtain the corresponding equations for TM mode from equation (2), which plays the role of “hole” state. Then we introduce handedness inversion, which flips the sign of permittivity and permeability. The material parameters are required to meet the conditions of

\[
\frac{\partial \mu_z}{\partial z} = -\frac{\partial \mu_z}{\partial z} \quad \text{and} \quad \frac{\partial \varepsilon_z}{\partial z} = -\frac{\partial \varepsilon_z}{\partial z}.
\]

Consequently, the effective Hamiltonian of the TM mode is obtained, \(-\mathbf{h}''(\mathbf{k}) + U\). The band diagram is plotted in figure 1b (colored pink). By tuning \( U \) to meet the condition \( U > m \), the right-handed TE band and the left-handed TM band cross at a finite frequency, as shown in figure 1b. The crossing frequency is \( \omega_0 \), which can be considered as photonic counterpart of zero energy in superconductor.

Next step is to emulate \( p_x + ip_y \) pairing potential, which is responsible for a gap opening at \( \omega_0 \). At first sight it might be very difficult to obtain photonic interaction that is a linear function of momentum. Here we introduce an alternative solution, which is named as odd-parity pairing4,30. In this picture, one can use constant interactions instead, if the interactions are applied between a wave function with odd parity and another with even parity. Note that for a single-orbital superconductor, odd-parity pairing is equivalent to \( p \)-wave pairing7,89.

To achieve the photonic “odd-parity pairing”, we introduce here the reflection operator \( M \) in 2D point groups4,30,33, Take for example the reflection \( M_x (x \rightarrow -x, y \rightarrow y) \) in the \( xy \) plane. Under this operation, \( k_x \) changes its sign (\( k_x \rightarrow -k_x \)). To keep Maxwell’s equations invariant under \( M \), we should have \( H_y \rightarrow -H_y \) and \( E_x \rightarrow -E_x \). Similar arguments can be applied to \( M_y \). In contrast, for any reflection \( M \) in the \( xy \) plane, \( E_z \) and \( H_z \) keep their signs. In this picture, \( E_z \) and \( H_z \) act as “odd-parity” wave functions, while \( E_x \) and \( H_x \) act as “even-parity” wave functions. Therefore, we introduce interactions between \( E_z \) and \( E_x \), and between \( H_z \) and \( H_x \), respectively. We also notice that the required pairing potentials are expected for not only opening a gap but also breaking TR symmetry. As a result, the “odd-parity pairing” interactions are designed by employing complex off-diagonal elements in permittivity and permeability tensors,

\[
\begin{pmatrix}
\varepsilon_+ & 0 & k_1 \\
0 & \varepsilon_+ & -ik_2 \\
k_1^* & ik_2^* & \varepsilon_z
\end{pmatrix}, \quad \mu =
\begin{pmatrix}
\mu_+ & 0 & k_3 \\
0 & \mu_+ & ik_4 \\
k_3^* & ik_4^* & \mu_z
\end{pmatrix}.
\]

Without loss of generality, we assume \( k_1 = k_2 = k_3 = k_4 = k \). In this case, the system has rotational symmetry in the \( k_x \) plane.

After the above two steps, we obtain the effective Hamiltonian for the TSC-like photonic medium,

\[
H_{\text{eff}} = \left( \mathbf{h}(\mathbf{k}) - U \right) - \Delta \left( \begin{array}{ll}
\frac{\partial}{\partial k_x} & \frac{\partial}{\partial k_y} \\
\frac{\partial}{\partial k_y} & -\frac{\partial}{\partial k_x} + U
\end{array} \right), \quad \Delta = \left( \begin{array}{ll}
0 & i \Delta \\
-i \Delta & 0
\end{array} \right),
\]

where \( \Delta = \omega_0 c/\varepsilon \). This Hamiltonian presents “particle-hole symmetry”,

\[
\Gamma H_{\text{eff}}(k)\Gamma^\dagger = -H_{\text{eff}}(-k), \quad \Gamma = \left( \begin{array}{ll}
0 & 1 \\
1 & 0
\end{array} \right).
\]

The pairing potential \( \Delta \) results in a TSC-like gap centered at \( \omega_0 \), as shown in figure 1c. To verify the topological feature of the TSC-like medium, we analytically calculate the winding number. It is known that topological nature of a system changes only when the gap closes. This property enables us to investigate topological feature in the extreme case that the gap has nearly zero width. Here we use a general method of approximation for this problem. First, consider the case of absent pairing potential, as shown in figure 1b. The eigen-functions can be obtained by diagonalizing the \( 6 \times 6 \) Hamiltonian described by equation (7). As all interesting physics are determined by the bands crossing zero energy, one may extract the \( 6 \times 2 \) eigen-functions for the corresponding two bands and project out the others. Next, we open an extremely small gap by introducing a very weak pairing potential. An appropriate approximation is used here: for sufficiently small variation of \( k \), treat this term as a perturbation, so as to get an expression for the Hamiltonian in terms of the gapless solutions. Consequently, we use the \( 6 \times 2 \) eigen-functions of the gapless solutions to extract the \( 2 \times 2 \) sub-matrix of the pairing case approximately,

\[
H_{\text{eff}} = \left( \begin{array}{cc}
\sqrt{m^2 + k_x^2 + k_y^2} - U & -i \Delta (k_x + ik_y) \\
-i \Delta (k_x - ik_y) & \sqrt{m^2 + k_x^2 + k_y^2}
\end{array} \right)
\]

\[
\equiv d_x \sigma_x + d_y \sigma_y + j_z \sigma_z,
\]

where \( \sigma_{xy} \) are the Pauli matrices. It can be directly mapped into the \( p_x + ip_y \) TSC Hamiltonian described by equation (1). From equation (9), one can calculate the winding number analytically,

\[
N_w = \frac{1}{4\pi} \int dk_x \int dk_y \frac{\partial \mathbf{d}}{\partial k_x} \times \frac{\partial \mathbf{d}}{\partial k_y}.
\]

In the present model, it gives \( N_w = \theta |U| - |m| = 1 \), providing a direct evidence for the topological phase.

**Simulation of TSC edge state.** The non-zero winding number promises a chiral edge state localized at the interface between the TSC-like medium and another opaque medium. Let the interface be at \( x = 0 \), and the opaque medium be located in the region \( x < 0 \). The dispersion relation of the edge state is illustrated by the red line in figure 2a, where the shaded areas indicate the projected band diagram of infinite TSC-like medium. The material parameters of the TSC-like medium are taken as \( \varepsilon_\perp = 0.8, \varepsilon_\parallel = 1.1, \mu_\perp = 0.1, \mu_\parallel = 0.1 \), which lead to \( \delta \varepsilon_\perp = -\delta \mu_\perp = -0.2, \delta \varepsilon_\parallel = -\delta \mu_\parallel = 0.1 \). \( \omega_0 = 89 \), and \( \kappa = 0.04 \) (with the unit of GHz). These parameters ensure the particle-hole-like symmetry of the effective Hamiltonian. From figure 2a, one can see a single branch of edge state with positive group velocity, which suggests broken TR symmetry and is expected to support one-way propagation. Note that another branch with negative group velocity can be realized by changing the permittivity and permeability tensors described by equation (6) into their conjugations, which makes the winding number flip its sign. To demonstrate the edge state intuitively, we also perform numerical calculations in real space by employing finite-difference frequency-domain (FDFD) method. As shown in figure 2b, the edge state distributes almost uniformly along the interface between two media and can pass through sharp bends at the corner without scattering.

One may question that the material parameters described by equation (6) with certain symmetries might be very difficult to be realized. Fortunately, it is known that topological nature of a system would not change if the gap doesn’t close, and thus the edge state is topologically protected and insensitive to small perturbations. Taking advantage of this property, the strict requirements for material parameters with four pairs of off-diagonal elements could be reduced. Here, we remove two of them by letting \( \kappa_2 = \kappa_3 = 0 \) in equation (6), and perturb the symmetric Drude parameters of permittivity and per-
meability, for example, \( \varepsilon_{\perp} = 2 - 140^2/\omega^2, \varepsilon_{\parallel} = 2 - 121^2/\omega^2, \mu_{\perp} = 1 - 77^2/\omega^2, \mu_{\parallel} = 1 - 94^2/\omega^2, k_1 = 0.08, \text{and } k_2 = 0.04 \) (with the unit of GHz). In this case, the perfect form of the TSC-like Hamiltonian described by equation (7) no longer remains, and the band structure adiabatically deforms from the symmetric one without gap closing. Compared with figures 2a and 2c, the bulk gap in figure 2c is narrower, and the edge state in figure 2d shows broken rotational symmetry in the \( k_{\parallel} \) plane (the wavelength along the \( y \) direction is shorter than that along the \( x \) direction). Nevertheless, the edge state still exists. It provides an evidence for the topologically protected edge state against material-parameter perturbations, which may facilitate experimental realizations.

**Simulation of zero-energy mode at a vortex.** Another prominent feature of \( p_x + ip_y \) superconductor is zero-energy mode at a vortex.\(^{17,34-37} \) It is a solution of the Bogoliubov-de Gennes (BdG) equation in polar coordinates with \( \Delta(r,\theta) = \Delta_0(r)\exp(\pm i\theta), \) characterized by a bound state at zero energy at a vortex core which can be reasonably considered as a small circular edge with vanishing density at the center\(^8 \). Zero-energy mode is another important topological excitation, but earlier works in photonics only concentrated on edge states. Here we introduce azimuth-dependent off-diagonal elements in permittivity and permeability tensors with cylindrical symmetry, for example, \( \kappa_1 = \kappa_2 = \kappa \exp(-i\theta) \) and \( \kappa_3 = \kappa_4 = \kappa \exp(+i\theta) \). This design mimics a counter-clockwise vortex with a singularity at the center. FDFD method is employed to numerically calculate the field distributions in a finite medium at zero energy (\( \omega_0 \)). The results are shown in figure 3. Two types of bound states are clearly seen: one appears at the edge and the other locates at the center “vortex core”. We have also confirmed that the bound state at the vortex only exists at \( \omega_0 \), which is indeed a “zero-energy” mode. It is known that the zero-energy modes at the vortex and at the edge can be described as the Jackiw-Rebbi solution in the 1D Dirac equation at the domain wall\(^{38,39} \). Here we reproduce them in photonic structures. Further calculations illustrate that the mode around the center rotates counter-clockwise around the vortex core, while the other rotates clockwise along the edge. The electric and magnetic fields also show different patterns between these two modes: in one mode \( E_z \) and \( H_z \) oscillate in phase, while in the other \( E_z \) and \( H_z \) oscillate out of phase.

**Discussion**

It should be pointed out that the effective TSC Hamiltonian proposed in this work is based on the presence of complex off-diagonal elements in permittivity and permeability tensors, which can be brought by gyrotrropic responses of materials. We note that in the past decade, the study of metamaterials has made remarkable progress in tailoring \( \varepsilon \) and \( \mu \) of an effective medium, offering flexible ways for manipulating electromagnetic wave propagation, such as cloaking\(^{40,41} \), optical analogue of quantum interference\(^{42} \) and Fano resonance\(^{43} \), and also mimicking black hole\(^{44} \). Most recently, the increased interests in achieving topological phases and edge states in photonic systems suggest that the presence of gyrotrropic and chiral responses in materials may lead to fruitful results\(^{45,46} \), and some of them have already been realized in experiments\(^{47,48,49} \). For more complicated cases, such as the vortex model in this work, position-dependent gyrotrropic parameters are taken into account. It may inspire researchers to tailor gyrotrropic responses of materials as well as \( \varepsilon \) and \( \mu \) in the future.

In summary, we have established a direct mapping between the \( p_x + ip_y \) model of topological superconductor and suitably designed electromagnetic medium. Two essential points of this mapping, particle-hole symmetry and \( p_x + ip_y \) pairing, have been investigated in detail. A combination of electromagnetic duality and handedness inversion of photons was proposed to mimic particle-hole trans-
formation for electrons. The analogous \( p_x + i p_y \) pairing potentials are achieved by introducing complex off-diagonal elements in permittivity and permeability tensors. Finally, we obtained a \( 2 \times 2 \) effective Hamiltonian that has the identical mathematical structure of the TSC Hamiltonian, and mimicked TSC edge state and zero-energy mode at a vortex. This study bridges the fields of superconductor and photonics, providing a good platform for simulating wave behaviors in superconductors physics at room temperature. The \( p_x + i p_y \) model for TSC is the first example, and other interesting effects such as Andreev reflection will be further investigated.

**Methods**

Band dispersions of both bulk media and edge states were obtained by analytically solving Maxwell’s equations with effective permittivity and permeability. The effective Hamiltonian of the proposed photonic structures and its mapping into the \( p_x + i p_y \) model of topological superconductor were carried out by theoretical deduction under appropriate approximations. Winding number was calculated analytically from the effective Hamiltonian. Numerical simulations were performed by using FDFD method.

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**Author contributions**

W.T., L.C. and H.Q.L. conceived the idea. W.T. designed the photonic model and carried out electromagnetic calculation. L.C. carried out calculation for effective Hamiltonian and winding number. X.J. carried out FDFD simulation. L.C. and H.Q.L. provided theoretical analysis and interpretation. All authors co-wrote the paper.

**Additional information**

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