Encoding and readout strategies in magnetic resonance elastography

Christian Guenthner | Sebastian Kozerke

1 | INTRODUCTION

Magnetic resonance elastography (MRE) is a non-invasive imaging modality for the in vivo determination of biomechanical properties of tissue by measuring the propagation of externally induced shear waves. To this end, a modified phase-contrast MRI sequence is used to sensitize the image...
The acquired phase of an isochromat moving along a trajectory \( \vec{r}(t) \) while experiencing a gradient field \( \vec{G}(t) \) is given by\(^{47,48}\)

\[
\phi_\text{i}(t) = \gamma f t_0 \int_{t_0}^{t} \vec{r}(\tau) \cdot \vec{G}(\tau) \, d\tau,
\]

where \( t_0 \) is a reference time point for motion encoding and \( \gamma \) is the gyromagnetic ratio in rad/s/T, which is related to the more commonly used \( \gamma = \gamma / 2\pi \) in units of Hz/T, and \( \vec{r} \cdot \vec{G} \) is the matrix nomenclature of the dot product. A visualization of the harmonic isochromat movement and an exemplary gradient waveform is shown in Figure 1.

In the case of MRE, the motion of the isochromat can be described by a superposition of harmonic waves

\[
\vec{r}(t) = \sum_{f} \vec{r}_f \sin(2\pi f t + \theta_f)
\]

with frequencies \( f \), phases \( \theta_f \), and amplitudes \( \vec{r}_f \). Here, \( \theta_f \) and \( \vec{r}_f \) are the local phase offset and amplitude of the isochromat oscillation, and thus are in general spatially dependent. Additionally, an arbitrary number of harmonic waves of equal frequency \( f \) with corresponding displacements \( \vec{r}_f \) and phases \( \theta_f \) can coexist, which shall in the following be implied by the summation \( \sum_{\theta} \) and stated by \( \sum_{m} \) when summation over the waves \( m \) is explicitly needed. For the description of the MEG, we will assume the general properties

\[
\vec{G}(t) = \vec{G} \left( \frac{t}{T} \right),
\]

\[
\vec{G}_f(t \leq 0) = \vec{G}_f(t \geq N_0 T) = 0, \text{ and}
\]

\[
\forall n \in \{1, \ldots, N_0\} \setminus \{0\}; \quad \vec{G}_f(t) = \vec{G}_f(t + (n - 1) T).
\]

The period of the MEG is given by \( T_0^{-1} = T \), and the total duration is \( N_0 T = N_0 f_0^{-1} \), where \( N_0 \) is the number of MEG repetitions. Insertion of the isochromat motion into the phase equation and evaluation thereof after the full application of the MEG, i.e. at \( t = N_0 T \), leads to the total accumulated phase

\[
\phi_\text{i} = \gamma \int_{0}^{T_0} \vec{r}(\tau) \cdot \vec{G}(\tau) d\tau \sin(2\pi f \tau + \theta_f).
\]

Substitution of \( \sigma = (t - t_0) / T \) and \( d\tau = T d\sigma \) results in

\[
\phi_\text{i} = \gamma \int_{0}^{N_0} \vec{r}(\sigma T) \sin(2\pi f \sigma + 2\pi f t_0 + \theta_f) d\sigma.
\]

FIGURE 1  Visualization of the MEG and isochromat motion in their respective time frames.
where the encoding fraction $q_f = f / f_g = T_f$ is used. Next, with the application of the addition theorem for the sine-function\(^9\)

\[
\sin(a + b) = \begin{pmatrix} \sin(a) \\ \cos(a) \end{pmatrix}^T \begin{pmatrix} \cos(b) \\ \sin(b) \end{pmatrix}.
\]

(9)

the encoding equation can be derived

\[
\Phi_{t_o} = \sum_{t} r_f \left[ \int g(t) \cos(2\pi q_f t) \right] \begin{pmatrix} \cos(2\pi f t_0 + \theta_i) \\ \sin(2\pi f t_0 + \theta_i) \end{pmatrix}
\]

(10)

Here, $\tilde{E}_f$ is the encoding matrix, which fully describes the encoding of arbitrary waves using a given MEG.

**N.B.:** The encoding efficiency in MRE is proportional to $f^{-1}$ given that the encoding fraction is held constant. Thus, doubling the MRE frequency to reduce the total encoding time by a factor of two also decreases the accumulated phase by a factor of two.

### 2.1 Maximal phase

The maximal acquired phase can be determined from the encoding equation (Equation 10) by first realizing that for each $f$ the vector expression to the right is a two-dimensional unit vector rotating as a function of time $t_0$ and phase $\theta_i$. This vector is multiplied by the vector $r_f \tilde{E}_f$, which encodes the oscillation magnitude. Hence, the dot-product is maximized by the Euclidean norm $\| r_f \tilde{E}_f \|_2$. Thus, for a superposition of multiple oscillation frequencies $f$, the maximally acquired phase is given by

\[
\Phi_{t_o} \leq \sum_{t} \| r_f \tilde{E}_f \|_2.
\]

(11)

### 2.2 Unidirectional encoding: encoding with constant MEG direction

Until now, no assumptions have been made regarding the exact form of the MEGs except for their periodicity in the case of $N_0 > 1$. In unidirectional encoding MRE, MEGs that do not change orientation over time are used, and they can thus be written as

\[
\tilde{G}(t) = G \tilde{d} g(t),
\]

(12)

where $G$ is the *maximal gradient strength per channel*, $g(t)$ is the time-dependent gradient strength with $|g(t)| \leq 1$ and $\tilde{d}$ denotes the direction of the gradient field that fulfills $\forall i: |d_i| \leq 1$, i.e. the temporal gradient function $\tilde{G}(t)$ never exceeds the maximal gradient strength $G$ along any primary coordinate axis. Inserting this into Equation 10 leads to the encoding equation for unidirectional encoding

\[
\Phi_{t_o, d} = G \tilde{d} \sum_{t} r_f \begin{pmatrix} \int g(t) \cos(2\pi q_f t) \cos(2\pi f t_0 + \theta_i) \\ \int g(t) \sin(2\pi q_f t) \sin(2\pi f t_0 + \theta_i) \end{pmatrix}.
\]

(13)

Here, the encoding efficiency per gradient strength $\tilde{e}_f^T$ is introduced, which will be used in the further discussion of encoding schemes and the comparison thereof. In the following, $\Phi_{t_o, d}$ will be written to specify that the phase was measured along a constant encoding direction $\tilde{d}$.

The dot product in Equation 13 can also be rewritten using the geometric definition of the inner product $\vec{x} \cdot \vec{y} = \| \vec{x} \|_2 \| \vec{y} \|_2 \cos(\angle(\vec{x}, \vec{y}))$ of two vectors, where $\angle(\vec{x}, \vec{y})$ denotes the angle of the vector $\vec{x}$ with respect to the $x$-axis. With this identity, Equation 13 can be rewritten in the following form

\[
\Phi_{t_o, d} = G \tilde{d} \sum_{t} \| \tilde{e}_f \|_2 \cos(2\pi f t_0 + \theta_i - \angle(\tilde{e}_f))
\]

(14)
\[
\psi_f = \frac{\pi}{2} - \angle (\mathbf{e}_f).
\] (15)

In addition to the phase offset \(\psi_f\), Equation 14 shows that the magnitude of the encoding efficiency

\[
\epsilon_f = \| \mathbf{e}_f \|_2
\] (16)

solely holds the information on encoding efficiency, whereas its vector components are only required to extract the respective phase offset between the encoded signal and the wave.

### 2.3 Mono-frequency MRE

In mono-frequency MRE, the shear wave field is assumed to fluctuate with a single frequency component \(f\). Thus, the encoding equation, Equation 13, is further simplified to

\[
\phi_{t_0} = \sum_m G_{d_T, r} \epsilon_{Tf} \cos 2\pi ft_0 + \theta_m \sin 2\pi ft_0 + \theta_m.
\] (17)

where the index \(m\) is used to identify different harmonic contributions to the wave field, which add up forming motion on an arbitrary ellipse. The maximally encoded phase as stated in Equation 11 is also simplified and reads

\[
\phi_{t_0} \leq \sum_m \| \mathbf{r}_m \|_2 \| \mathbf{e}_f \|_2 = \sum_m Gr_m \epsilon_f.
\] (18)

### 2.4 Fourier space interpretation of encoding efficiencies

In this section, an alternative interpretation of encoding efficiencies and their evaluation is presented and some fundamental properties shall be derived. Again, the encoding efficiency is given by the integral

\[
\mathbf{E}_f = \frac{\gamma}{T} \int_0^T \mathbf{G}(\sigma) \left( \begin{array}{c} \sin(2\pi qf\sigma) \\
\cos(2\pi qf\sigma) \end{array} \right) \mathbf{e}_f^T.
\]

Introduction of a “window function”

\[
w(\sigma) = H(\sigma)H(N_0 - \sigma),
\] (19)

where \(H(\sigma)\) denotes the Heaviside step function, allows us to rewrite the encoding efficiency in the following form

\[
\mathbf{E}_f = \frac{\gamma}{T} \int_{-\infty}^{\infty} d\sigma w(\sigma) \mathbf{G}(\sigma) \left( \begin{array}{c} \sin(2\pi qf\sigma) \\
\cos(2\pi qf\sigma) \end{array} \right) \mathbf{e}_f^T.
\] (20)

This resembles the Fourier transform of \(\mathbf{G}(\sigma)\), with the definition

\[
\mathcal{F}\{f(x)|y\} = \int_{-\infty}^{\infty} dx e^{2\pi ixf(y)} \text{ and } \mathcal{F}^{-1}\{f(y)|x\} = \int_{-\infty}^{\infty} dy e^{-2\pi iyf(y)}.
\] (21)

Thus, the encoding efficiency can be restated as

\[
\mathbf{E}_f = \frac{\gamma}{T} \left( \text{Im} \left( \mathcal{F} \left[ w \mathbf{G} \right] (q_t) \right) \right) \text{Re} \left( \mathcal{F} \left[ w \mathbf{G} \right] (q_t) \right).
\] (22)

Using the convolution theorem,\(^{49}\) we finally arrive at

\[
\mathbf{E}_f = \frac{\gamma}{T} \left( \text{Im} \left( \mathcal{F}[w] \mathcal{F} \left[ \mathbf{G} \right] \right) \right) \text{Re} \left( \mathcal{F}[w] \mathcal{F} \left[ \mathbf{G} \right] \right),
\] (23)

where \(\ast\) denotes the convolution operator. Since the window function \(w\) is analytically known, its Fourier transform \(\hat{w}\) is easily found and given by a sinc function

\[
\hat{w}(q_t) = N_0 \frac{\sin(nN_0q_t)}{nN_0q_t} e^{inN_0q_t}.
\] (24)
Hence, the encoding efficiency is mainly given by the frequency spectrum of the MEG, which is additionally filtered by the windowing function. The magnitude of the encoding efficiency is then given by

$$\bar{E}_f = \frac{\gamma_q}{T} |F[w] \cdot F[\bar{G}]|.$$  \hfill (25)

**N.B.:** Increasing the MEG repetitions $N_q$ leads to better spectral sensitivity, since the half-width of $\tilde{w}(q_0)$ is reduced. Additionally, roots exist in the window function for $N_0 q_0 \notin \mathbb{N} \wedge N_0 q_0 \neq 1$, which can lead to roots in the encoding efficiency (see also Section 3.2).

In addition, since the sinc function is a representation of the Dirac delta function $\delta(q_0)$, i.e.

$$\lim_{N_0 \to \infty} N_0 \sin(nN_0 q_0) / nq_0 N_0 = \delta(q_0),$$  \hfill (26)

encoding efficiencies can be seen to converge against the spectrum of the MEG for large $N_0$

$$\lim_{N_0 \to \infty} \bar{E}_f = \frac{\gamma_q}{T} \left( \left| \text{Im} \left( F[\bar{G}] \right) \right| \cdot \text{Re} \left( F[\bar{G}] \right) \right).$$  \hfill (27)

### 2.5 Noise in MRE

For a full description of the encoding process in MRE, a consideration of how noise in the complex MRI signal impacts the acquired motion phase and reconstructed displacement field is given. The phase-to-noise ratio (PNR) per encoding direction and sampling point is given by

$$\text{PNR} = \frac{\phi_0}{\pi \text{SNR}},$$  \hfill (28)

where $\phi$ denotes the accumulated motion phase and $\text{SNR} > 2$ is assumed.\(^{38,50,51}\) Hence, PNR can be increased by either increasing the signal-to-noise ratio (SNR) or by increasing the encoded motion phase $\phi = G_0 \left\| \tilde{w} \right\|_2$. If a phase correction scheme is employed, e.g. unbalanced four-point, six-point, or tetrahedral encoding (see also Section 4) the recovered motion phases after decoding have 50% lower PNR than each individual phase image in the case of unbalanced four-point encoding and two times higher PNR in case of six-point and tetrahedral encoding. Influencing factors on SNR are reviewed in Section 6.

The PNR has a direct impact on the noise of the reconstructed displacement field, which is additionally dependent on the number of sampled mechanical wave offsets $N_p$. To quantify displacement noise, the displacement-to-noise ratio (DNR) was introduced\(^{52}\); this is given by

$$\text{DNR} = \frac{\left\| \tilde{w} \right\|_2}{\sigma_r} \propto \sqrt{\frac{N_p}{2}} G_0 / \text{SNR}.$$  \hfill (29)

In conventional mono-frequency MRE with more than four mechanical wave offsets, the standard deviation of the displacement noise, $\sigma_r$, can be estimated from the misfit of the fitted sinusoid and the measured data averaged over a region of interest.\(^{52,53}\)

Finally, an SNR measure based on Cauchy’s symmetric strain tensor $\tilde{\varepsilon}$ was proposed.\(^{53}\) The tensor is defined by local spatial derivatives of the time-harmonic displacement field

$$\tilde{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 \frac{du}{dx} & \frac{du}{dy} & \frac{du}{dz} \\ \frac{du}{dy} & 2 \frac{dv}{dy} & \frac{dv}{dz} \\ \frac{du}{dz} & \frac{dv}{dz} & 2 \frac{dw}{dz} \end{bmatrix},$$  \hfill (30)

where $(u, v, w)$ are the time-dependent motion field components in $(x, y, z)$ directions and $(x, y, z)$ denote the orthogonal coordinate axes. The octahedral shear strain (OSS) is given by

$$\text{OSS} = \frac{2}{3} \sqrt{(\varepsilon_{xx} - \varepsilon_{yy})^2 + (\varepsilon_{yy} - \varepsilon_{zz})^2 + (\varepsilon_{zz} - \varepsilon_{xx})^2 + 6(\varepsilon_{xy}^2 + \varepsilon_{yz}^2 + \varepsilon_{zx}^2)}.$$  \hfill (31)

The proposed OSS-SNR is defined by temporal and volumetric averaging of the OSS evaluated with the measured motion field and then divided by the same expression evaluated with motion noise in place of the actual motion field. Since the shear modulus is directly dependent on strain, OSS-SNR has been found to give more reliable SNR estimates than DNR and was claimed to better indicate reconstruction accuracy in cases where rigid body motion is present.\(^{53}\) However, OSS-SNR requires the calculation of spatial derivatives of the motion field and hence also depends on processing parameters, whereas DNR is parameterless and is a direct measure for the goodness of fit of the harmonic model.
2.6 Basic sequence types

All of the encoding schemes described in this review can be implemented in different basic MRI sequences. The first MRE sequences used a spoiled gradient recalled echo (GRE) sequence, where a steady-state longitudinal magnetization is reached through short repetition times and spoiling of remaining transverse magnetization before each successive RF pulse. Repetition times are chosen to be integer multiples of the periodicity of the wave to achieve a fixed phase relationship between the MRE sequence and the shear wave. A graphical depiction of a GRE sequence is given in Figure 2. MEGs (dashed blue) are introduced directly after the slice-selective RF excitation pulse, followed by phase encoding, readout, and phase-encoding-rewinder gradients applied together with spoilers in both measurement and slice directions. Low flip angles in conjunction with spoiling of residual transverse magnetization and fast repetition times lead to typically low SNR performance. Additionally, the echo signal is limited by $T_2^*$ decay, hence a decrease in motion sensitizing gradient frequency $f_g$ leads to an SNR increase proportional to $\exp\left(-f_g^{-1}/T_2^*\right)$. Hence, GRE-based MRE sequences are typically used in conjunction with high MRE frequencies and fractional encoding to reduce MEG durations (see also Sections 3.1 and 3.2).

To overcome SNR limitations, transverse magnetization can be reused within the next repetition cycle through balancing of all applied gradients. This sequence is generally known as balanced steady-state free precession (bSSFP) and is graphically shown in Figure 2. Here, the spoiling gradients in the measurement and readout directions of the spoiled GRE sequence are replaced by the respective balancing gradients. The incorporation of MEGs leads to wave-dependent steady-state magnitude and phase signals with very high sensitivities. However, the interplay of off-resonance effects and MEGs causes non-linear steady-state signal amplitude and phase behavior and leads to off-resonance-dependent encoding efficiencies. Additionally, bSSFP sequences are very sensitive to field inhomogeneities and are thus best used with short repetition times to reduce banding artifacts. Since the inclusion of MEGs in the bSSFP sequence prolongs the repetition time, fractional-wave encoding was developed to further reduce repetition times for low wave frequencies. Despite high SNR and PNR as well as very fast acquisition times, bSSFP has not found widespread use in MRE, which is due to the complex signal model and off-resonance effects.

Besides GRE-based sequences, spoiled SE-MRE is prominently used because of its $T_2$-decay echo intensity (rather than $T_2^*$) and the possibility of adding MEGs to either side of the echo pulse. An SE-based fractional encoding scheme with standard Cartesian readout is depicted in Figure 2. Here, the slice selective RF pulse is directly followed by phase encoding and measurement pre-phaser and a first bipolar MEG. The 180° echo pulse is also applied slice selectively and excess transverse magnetization is dephased after the echo pulse. The second bipolar MEG is applied such that

FIGURE 2 Comparisons of GRE-, SE- and bSSFP-based MRE sequence diagrams with standard Cartesian readout for the special case of fractional encoding
the encoded phase adds to the previously encoded phase by synchronizing it to the same wave cycle and taking the phase reversal due to the 180° 
echo pulse into account. The signal readout is centered around the echo time and followed by phase-rewinder and spoiling gradients in both slice 
and measurement directions. Due to $T_2$-decay-limited echo intensity, SE sequences are usually used with long echo times and incorporate multiple 
MEGs to enhance motion sensitivity. Because of specific absorption rate (SAR) limitations in conjunction with 180° echo pulses, SE-MRE is limited 
with regards to achievable repetition times and is hence often used with 90° flip angles, long repetition times, and fast-readout techniques such as 
EPI and spiral imaging, as reviewed later (see also Sections 3.1 and 3.2).

3 | “TEMPORAL” ENCODING IN MRE

For the successful reconstruction of stiffness maps, the shear wave needs to be probed with different offset times $t_0$ within the periodicity of the 
wave. Depending on the application, between 4 and 360 wave images have been reported; however, typically eight images are sufficient for recon-
struction of stiffness maps (see Table 1 and Table 2).

Apart from temporal wave resolution, the duration, shape, and direction of MEGs can be chosen to optimize encoding efficiency and sequence 
timing. In the following, an overview will be given of existing encoding schemes, their efficiency, and respective timing considerations.

3.1 | Full-wave encoding

In full-wave encoding the gradient frequency $f_g$ is chosen to be equal to the MRE shear wave frequency $f$, and thus can be considered to be a special 
case of single-frequency encoding. An exemplary GRE-based full-wave-encoding MRE sequence is displayed in Figure 3. The encoding efficiency is 
obtained by choosing $q = 1$ in Equation 13

$$\bar{e}_f^T = \gamma N_0 \int_0^T \cos(2\pi \sigma) \left( \begin{array}{c} \sin(2\pi \sigma) \\ \cos(2\pi \sigma) \end{array} \right)^T.$$  \hspace{1cm} (32)

Making use of the periodicity of $g(\sigma)$, the encoding efficiency can be further simplified to

$$\bar{e}_f^T = \gamma N_0 \int_0^T \cos(2\pi \sigma) \left( \begin{array}{c} \sin(2\pi \sigma) \\ \cos(2\pi \sigma) \end{array} \right)^T.$$  \hspace{1cm} (33)

This equation resembles the two first-order coefficients of the Fourier series expansion of $g(\sigma)$.

**N.B.:** In full-wave encoding, the lowest order Fourier coefficients ($n = 1$), i.e. the gradient components that are oscillating with the same frequency as the isochromat 

motion, contribute to wave encoding.

**Evaluation of the above equation for the case of a sinusoidal gradient leads to**

$$\bar{e}_f^T = \gamma N_0 \int_0^T \sin(2\pi \sigma) \left( \begin{array}{c} \sin(2\pi \sigma) \\ \cos(2\pi \sigma) \end{array} \right)^T = \gamma N_0 \left( \begin{array}{c} 1 \\ 0 \end{array} \right)^T.$$  \hspace{1cm} (34)

and the phase-equation of full-wave encoding \(^{58}\)

$$\phi_{t_0, G} = \sum_m \int_{t_m} \frac{d}{2f} \gamma N_0 \cos(2\pi ft_0 + \theta_{t,m}).$$  \hspace{1cm} (35)

In the case of a bipolar gradient with infinite slew rate, the encoding efficiency is given by

$$\bar{e}_f^T = \gamma N_0 \left( \frac{1}{f} \int_0^{1/2} \cos(2\pi \sigma) \left( \begin{array}{c} \sin(2\pi \sigma) \\ \cos(2\pi \sigma) \end{array} \right)^T - \frac{1}{f} \int_0^{1/2} \cos(2\pi \sigma) \left( \begin{array}{c} \sin(2\pi \sigma) \\ \cos(2\pi \sigma) \end{array} \right)^T \frac{2\pi N_0}{\pi ft} \left( \begin{array}{c} 1 \\ 0 \end{array} \right)^T \right. \hspace{1cm} (36)

and the signal phase is thus

$$\phi_{t_0, G} = \sum_m \int_{t_m} \frac{d}{2f} \gamma N_0 \cos(2\pi ft_0 + \theta_{t,m}).$$  \hspace{1cm} (36)

Accordingly, the encoding efficiency of a bipolar gradient with infinite slew is $4/\pi = 1.27$ times larger than the encoding efficiency of a sinu-
soideal gradient with equal maximal gradient strength $G$. It can be shown that finite-slew trapezoidal gradients with dwell-time $t_{\text{dwell}}$ and a single 
repetition $N_0 = 1$ have the same encoding efficiency as sinusoidal gradients of equal maximal gradient strength, given that $t_{\text{dwell}} = \alpha/(4f)$ with
| Leading Author | Year | Sequence | Dimension | Readout method | MRE/MEG frequency (Hz) | MEG shape | N0 | MEG gradient (mT/m) | MEG enc. directions | Wave images | Applied region | Matrix | FOV (mm) | Slice (mm) | Resolution (mm) | Flip angle | TR/TE (ms) |
|---------------|------|----------|-----------|---------------|-----------------------|-----------|----|-------------------|-------------------|-------------|---------------|---------|-----------|-------------|----------------|------------|------------|
| Muthupillai²   | 1995 | GRE 2D   | Cartesian | 200, 400      | T                     | 20        | n/a| 1 (S)             | 2-point          | 4           | porcine kidney | n/a     | n/a       | 10          | n/a            | n/a        | 40–300/20–60 |
| Muthupillai²⁸  | 1996 | GRE 2D   | Cartesian | 100–1100      | n/a                   | 2–30      | n/a| 1 (n/a)           | 2-point          | n/a         | phantom/calf  | 128 × 256| n/a       | n/a         | n/a            | n/a        | 10–60      |
| Muthupillai²⁹  | 1997 | GRE M2D  | Cartesian | 300           | n/a                   | 6         | n/a| 3 (XYZ)           | 6-point          | 1           | phantom       | 256 × 128×32| 256 × 256 | 64          | 2           | 1 × 2 × 2  | 30         |
| Sinkus²⁷       | 2000 | SE M2D   | Cartesian | 400           | S                     | 2 sp      | 23 | 4 (MPSR)          | 4-unb.           | 8           | phantom       | 128 × 128×n/a| 140 × 140 | n/a         | 3           | 1.1 × 1.1 × 3 | n/a        |
| Sinkus²⁰       | 2000 | SE M2D   | Cartesian | 60            | S                     | 2 sp      | 23 | 4 (MPSR)          | 4-unb.           | 8           | breast        | 128 × 128×n/a| 140 × 140 | n/a         | 3           | 1.1 × 1.1 × 3 | n/a        |
| Knuse²⁶        | 2000 | GRE 2D   | Cartesian | 75–300        | T                     | n/a       | 1 (n/a) | 2-point          | 8                | porcine misc | 64 × n/a   | 8–14 × 8–14 | 5         | n/a       | 30–45       | 100–170/20    |
| Weaver²¹       | 2000 | GRE 2D   | Cartesian | 100           | S                     | 1         | n/a | 3 (MPS)           | 3-noref          | 8           | phantom       | 256 × 128×12 | 120 × 120 | 36          | 3           | 2.1 × 4.2 × 3 | n/a        |
| Bieri²⁶        | 2006 | bSSFP 2D | Cartesian | 143           | FC                    | 1         | n/a | 1 (M)             | 2-point          | 1           | biceps        | 256 × 256  | 200 × 200 | 7           | 0.78 × .78   | 30         |
| Maderwald²²    | 2006 | GRE 2D   | EPI        | 142           | T                     | 3         | 25 | 1 (n/a)           | 2-point          | 8           | biceps        | 256 × 256  | 200 × 200 | n/a         | 0.78 × .78   | 30         |
| Maderwald²³    | 2006 | GRE 2D   | EPI        | 200           | T                     | 3         | 10 | 3 (MPSR)          | 6-point          | 8           | phantom       | 256 × 256  | 180 × 180 | n/a         | 0.7 × .7     | 30         |
| Glaser²⁴       | 2006 | SE 2D    | Cartesian | 200           | T                     | 2         | n/a | 1 (Z)             | 2-point          | 8           | breast        | 256 × 64   | 200 × 200 | 6           | 0.78 × 3.125 | n/a        |
| Glaser²⁵       | 2006 | SE-beam 1D| Cartesian | 200           | T                     | 2         | n/a | 1 (Z)             | 2-point          | 8           | breast        | 256       | 200       | n/a         | 0.78         | 90         |
| Glaser²⁶       | 2006 | GRE 2D   | Cartesian | 300           | T                     | 1         | n/a | 1 (Z)             | 2-point          | 8           | phantom       | 256 × 64   | 200 × 200 | 5           | 0.78 × 3.125 | 30         |
| Glaser²⁷       | 2006 | GRE-beam 1D| Cartesian | 300           | T                     | 1         | n/a | 1 (Z)             | 2-point          | 8           | phantom       | 256       | 200       | n/a         | 0.78         | 30         |
| Knuse²⁸        | 2008 | GRE 2D   | Cartesian | 100           | T                     | 1         | n/a | 1 (n/a)           | 2-point          | 8           | phantom       | 256 × 64   | 200 × 200 | 5           | 0.8 × 3      | 40         |
| Sack²⁹        | 2008 | SE 2D    | EPI        | 25, 50        | S                     | 1, 2      | 35 | 1 (S)             | 2-point          | 10          | brain         | 128 × 128  | 192 × 192 | 6           | 1.5 × 1.5    | n/a        |
| Sack³⁰        | 2009 | SE 2D    | EPI        | 32.35         | S                     | 1         | 25/35 | 3 (MPS)          | 6-point          | 10          | phantom       | n/a       | n/a       | n/a         | n/a            | n/a        |
| Riek³¹        | 2011 | GRE 2D   | Cartesian | 100–800       | S                     | 1–8      | 285 | 1 (S)             | 2-point          | 16          | phantom       | 128 × 128  | 40 × 40    | 2           | 0.3125 × 3.125 | n/a        |
| Klatt*³²       | 2013 | SE M2D   | Cartesian | 5000          | S                     | 1        | 600   | 3 (MPS)           | 6-point          | 8           | phantom       | 256 × 128×16| 200 × 100 | 8           | 0.78 × 0.78  | n/a        |
| Johnson³³      | 2013 | SE M2D   | spiral     | 50            | T                     | 2 sp     | n/a | 3 (XYZ)           | 6-point          | 8           | brain         | 128 × 128×20| 256 × 256×40| 2           | 2 × 2 × 2    | n/a        |
| Yin²⁹         | 2013 | SE M2D   | EPI        | 60            | FC                    | 2 sp     | n/a | 3 (XYZ)           | 6-point          | 4           | porcine misc. | 80 × 80×40  | 260 × 240 | 240          | 3.25 × 3.25  | 6          |
| Yin³⁰         | 2014 | SE 2D    | Cartesian | 500           | U                     | 2.46 sp  | 0–726.5 | 1 (S)             | 2-point          | 4           | phantom       | 128 × 40   | 40 × 40    | 2           | 0.3125 × 3.125 | 90         |
| Johnson³¹      | 2014 | SE M3D   | spiral     | 50            | FC                    | 2 sp     | n/a | 3 (XYZ)           | 6-point          | 4           | brain         | 120 × 120×120| 240 × 240×200| 2           | 2 × 2 × 2    | n/a        |
| Low³²         | 2015 | SE M2D   | EPI        | 60/90         | FC                    | n/a      | n/a | 3 (XYZ)           | 6-point          | 3           | kidney        | 72 × 72×20  | 360 × 360 | 370          | 3.5           | 5 × 5 × 3.5  | n/a        |
| Klatt*³³      | 2015 | SE M2D   | spiral     | 50            | FC                    | 2 sp     | 18   | 3 (XYZ)           | 6-point          | 8           | brain         | 128 × 128×40| 240 × 240×20 | 2           | 1.875 × 1.875 | n/a        |
| Klatt³⁴       | 2015 | SE M2D   | spiral     | 50            | FC                    | 2 sp     | 18   | 3 (XYZ)           | 6-point          | 8           | brain         | 128 × 128×40| 240 × 240×20 | 2           | 1.875 × 1.875 | n/a        |

(Continues)
**TABLE 1** (Continued)

| Leading Author | Year | Sequence | Dimension | Readout method | MRE/MEG frequency (Hz) | MEG shape | MEG gradient (mT/m) | MEG enc. directions | Wave images | Applied region | Matrix | FOV (mm) | Slice (mm) | Resolution (mm) | Flip angle | T<sub>R</sub>/T<sub>E</sub> (ms) |
|----------------|------|----------|-----------|----------------|------------------------|-----------|---------------------|---------------------|-------------|---------------|--------|----------|-----------|----------------|-------------|-----------------|
| Numano         | 2015 | GRE      | 2D        | Cartesian      | 100                    | T         | n/a                 | 1 (M)               | 8           | phantom      | 256 × 256 | 180 × 180 | 5         | 0.7 × 0.7      | 20          | 40/2.3          |
| Numano         | 2015 | GRE      | 2D        | Cartesian      | 50                     | T         | n/a                 | 1 (M)               | 8           | psoas muscle | 256 × 256 | 300 × 300 | 10        | 1.17 × 1.17    | 20          | 40/2.3          |
| Anderson       | 2016 | SE       | M3D       | spiral         | 50                     | FC        | n/a                 | 3 (XYZ)             | 4           | brain         | 120 × 120 × 80 | 240 × 240 × 120 | 2         | 2 × 2 × 2      | n/a         | 1800/73         |
| Yin            | 2017 | SE       | M2D       | Cartesian      | 800                    | U         | 6 sp                | 1 (S)               | 4           | brain mouse   | 128 × 128 × 10 | 24 × 24 × 10    | 1         | 0.19 × 0.19 × 1 | 90          | 1000/20.6       |
| Yin++          | 2017 | SE       | M2D       | Cartesian      | 800                    | U         | 6 sp                | 6 (vd)              | 4           | brain mouse   | 128 × 128 × 8  | 24 × 24 × 8     | 1         | 0.19 × 0.19 × 1 | 90          | 1000/20.6       |

*Sample interval modulation. 1Simultaneous MRE and DWI. 2Simultaneous MRE and DTI. n.a., not applicable; n/a: not available.
Sequence: BEAM: pencil-beam excitation.
Dimension: 1D/2D, 1D/2D acquisition; M2D, stacked 2D acquisition; M3D, multislab acquisition; 3D, full 3D acquisition.
N0: sp, number of MEGs is split between pre- and post-echo-pulse in SE-MRE. MEG shape: T, bipolar trapezoidal; S, sinusoidal; FC, flow-compensated; U, unipolar.
Encoding directions: M, measurement; P, phase; S, slice; R, reference; X, Y, Z, specific mentioning of main gradient axes; vd, variable directions.
Encoding scheme: 2-point, two-point difference; 4-unb., unbalanced four-point; 3-noref, three point encoding without reference phase correction.
| Leading author | Year | Sequence | Dimension | Readout method | MRE frequency (Hz) | MEG frequency (Hz) | MEG shape | NO | MEG gradient (mT/m) | MEG directions | MEG enc. scheme | Wave images | Applied region | Matrix | FOV (mm) | Slice (mm) | Resolution (mm) | Flip angle | TR/TE (ms) |
|----------------|------|----------|-----------|---------------|-------------------|-------------------|-----------|----|-------------------|----------------|----------------|-------------|---------------|--------|-----------|-----------|----------------|----------|-------------|
| Sack           | 2004 | GRE-Beam | 1D        | Cartesian     | 150               | 150-750           | S         | 2-20| 40                | 1 (M)          | 2-point        | n/a         | phantom       | 256    | 240       | n.a.      | 0.93           | 2000/    |
| Klatt          | 2006 | bSSFP   | 2D        | Cartesian     | 51                | 150               | T         | 1   | 35                | 1 (S)          | 2-point        | 20          | liver         | 128 × 128 | 300 × 300  | 10        | 2.34 × 2.34 | 20       | 9.8/       |
| Klatt          | 2006 | bSSFP   | M2D       | Cartesian     | 51                | 150               | T         | 1   | 35                | 1 (S)          | 2-point        | 20          | liver         | 64 × 64 × 11 | 300 × 300 × 110 | 10        | 4.69 × 4.69 × 10 | 20       | 9.8/       |
| Rump           | 2007 | bSSFP   | 2D        | Cartesian     | 48.5              | 500               | T         | 4   | 30                | 1 (S)          | 2-point        | 16          | heart         | 128 × 64 | 400 × 400  | 5         | 3.13 × 6.25 | 50       | 5.2/       |
| Klatt*         | 2007 | SE       | 2D        | EPI           | 25-62.5           | 60                | S         | 4   | 35                | 1 (S)          | 2-point        | 40          | brain         | 128 × 128 | 200 × 200  | 6         | 1.56 × 1.56 | 90       | 3000/149   |
| Klatt*         | 2007 | SE       | 2D        | EPI           | 25-62.5           | 50                | S         | 1   | 20                | 1 (S)          | 2-point        | 40          | liver         | 64 × 128 | 300 × 300  | 10        | 4.69 × 2.34 | 90       | 500/64     |
| Asbach*        | 2008 | SE       | 2D        | EPI           | 25-62.5           | 50                | S         | 1   | 20                | 1 (S)          | 2-point        | 40          | liver         | 64 × 128 | 300 × 300  | 10        | 4.69 × 2.34 | 90       | 500/64     |
| Sack*          | 2009 | GRE      | 2D        | Cartesian     | 24.3              | 500               | T         | 1   | 25-35             | 3 (MPSR)       | 4-urb.        | heart       | 128 × 128 | 320 × 320  | 5         | 2.5 × 2.5   | 15       | 5.16/      |
| Nedredal       | 2011 | SE       | M2D       | EPI           | 60                | 80                | FC        | 1   | n/a               | 1 (S)          | 2-point        | 4           | liver         | n/a × n/a × 40 | n/a × n/a × 120 | 3         | 3 × 3 × 3   | 90       | 1634/48    |
| Romano*        | 2012 | SE       | M2D       | EPI           | 50                | 60                | FC        | 3   | n/a               | 3 (MPS)        | n/a           | 8           | brain        | 128 × 112 | 256 × 224 × 60 | 2         | 2 × 2 × 2   | n/a      |
| Garteiser*     | 2013 | GRE      | M2D       | Cartesian     | 28-84             | 120               | T         | 1   | 21                | 3 (MPSR)       | 4-urb.        | 8           | liver        | 80 × 80 | 320 × 320 × 36 | 4         | 4 × 4 × 4   | 20       | 9/21       |
| Garteiser*     | 2013 | GRE      | M2D       | Cartesian     | 28-84             | 160               | T         | 1   | 30                | 3 (MPSR)       | 4-urb.        | 8           | liver        | 80 × 80 | 320 × 320 × 32 | 4         | 4 × 4 × 4   | 20       | 6/91       |
| Sahebjavaher*  | 2013 | SE       | M2D       | EPI           | 50                | 100               | S         | 2sp | 60                | 3 (MPS)        | 3-noref       | n/a         | prostate     | 64 × 64 | 192 × 192 × 60 | 3         | 3 × 3 × 3   | n/a      | 2700/44    |
| Sahebjavaher*  | 2014 | GRE      | M2D       | Cartesian     | 70                | 119               | T         | 1   | 48                | 3 (XYZR)       | 4-urb.        | 8           | prostate     | 128 × 128 | 256 × 256 × 48 | 2         | 2 × 2 × 2   | 25       | 344/9.2    |
| Hirsch*        | 2014 | SE       | M2D       | EPI           | 30-60             | 50                | FC        | 1   | 30                | 3 (XYZ)        | 3-noref       | 8           | liver        | 128 × 104 × 10 | 350 × 284 × 50 | 5         | 2.7 × 2.7   | 90       | 182/54     |
| Braun*         | 2014 | SE       | M2D       | EPI           | 40-60             | 36-71             | FC        | 1,2 | 50                | 3 (XYZ)        | 3-noref       | 8           | brain        | 200 × 186 × 20 | 200 × 186 × 20 | 1         | 1 × 1 × 1   | 90       | 5640/76    |
| Corbin*        | 2016 | GRE      | 2D        | Cartesian     | 100               | 160               | T         | 1   | 20                | 1 (S)          | 2-point       | 3           | liver (swine)| 128 × 128 | 300 × 300  | 10        | 2.34 × 2.34 | 15       | 10.7/9     |

*Multiphase MRE; n.a., not applicable; n/a, not available.

Sequence: BEAM: pencil beam excitation.

Dimension: 1D/2D, 1D/2D acquisition; M2D, stacked 2D acquisition; M3D, multislab acquisition; 3D, full 3D acquisition.

NO: sp, number of MEGs is split between pre- and post-echo-pulse in SE-MRE. MEG shape: T, bipolar trapezoidal; S, sinusoidal; FC, flow-compensated.

Encoding directions: M, measurement; P, phase; S, slice; R, reference; X, Y, Z, specific mentioning of main gradient axes.

Encoding scheme: 2-point, two-point difference; 4-urb., unbalanced four-point; 3-noref, three point encoding without reference phase correction.
\( \alpha \approx 0.748 \) being the approximate solution of \( \alpha^2 = 8|\sin(\pi\alpha/2)| \). In other words, both MEG waveforms have the same encoding efficiency given that the dwell-time is approximately 75% of the maximum dwell-time \( 1/(4f) \).

In Table 1, studies using full-wave encoding are listed and compared regarding key MRE imaging parameters. For high wave frequencies \( (f \lesssim 100\text{Hz}) \), both SE and GRE sequences have been used in conjunction with full-wave encoding. However, since the signal of SE sequences are dependent on \( T_2 \) decay rather than the \( T_2^* \) decay of GRE sequences, SE is preferred over GRE-based sequences for low encoding frequencies \( (f \lesssim 100\text{Hz}) \) for SNR reasons. Additionally, SE sequences can benefit from inherently longer echo times by applying multiple MEGs \( (N_0 > 1) \) even at low wave frequencies\(^{63,65}\) as well as motion sensitization before and after the echo pulse.\(^{29,57,68,70,73}\) However, long echo times prolong the acquisition of full volumetric three-dimensional MRE datasets, leading to increased acquisition times for SE-based sequences. Thus, accelerated readout with EPI\(^{28,29,35,65}\) or spiral\(^{68,70,73}\) trajectories as well as parallel imaging\(^{28,29,35}\) are increasingly used in SE-based full-wave encoded MRE (see also Section 6).

### 3.2 Fractional-wave encoding

To overcome the echo-time limitations of full-wave encoding, fractional-wave encoding employs MEGs of usually higher frequency than the underlying wave frequency (see Figure 3). In this way, only a fraction of the wave is encoded at the expense of a reduced encoding efficiency, but leading to shorter motion-encoding times.\(^{38,75}\) The encoded phase is in general described by Equation 13, and in the case of a single MRE frequency component it is given by Equation 17.

For the case of sinusoidal MEGs, the encoding efficiency can be shown to be given by

\[
\epsilon_f = \frac{y}{2nf(q_f^2 - 1)} \left( \sin\left(2\pi n_0 q_f \right) \right)^T.
\]

Equation 37 illustrates, that in fractional encoding, the phase relationship between encoded signal and wave is now dependent on the encoding fraction \( q_f \). The norm of the encoding efficiency is then given by\(^{38}\)

\[
\epsilon_f = \frac{y}{fn} \left| \frac{q_f \sin(nq_f N_0)}{q_f^2 - 1} \right|.
\]

**N.B.:** The encoding efficiency is zero whenever

\[
q_f \neq 1; \epsilon_f = 0 \iff N_0 q_f \in \mathbb{N}.
\]

This condition is referred to as the filter condition of MRE for sinusoidal gradients.\(^{38,62}\) It has been successfully used to selectively measure non-linear effects of shear wave propagation\(^{75}\) or to optimize encoding in multi-frequency MRE (see Section 3.6).\(^{83}\)

---

**FIGURE 3** Sequence diagram comparing GRE full-wave and fractional-wave MRE. For both schemes, \( T_R \) is an integer multiple of the wave periodicity.
In Figure 4, the gradient shape and respective encoding efficiencies \( \epsilon_f \) for constant MEG frequency are plotted for sinusoidal, rectangular, and trapezoidal MEGs. Encoding efficiency can be seen to be highly dependent on the gradient shape, especially in the high-q region.

In Table 2, publications using fractional-wave encoding are referenced and compared regarding key MRE imaging parameters. Since encoding is sped up by reducing the duration of the MEG, fractional encoding is mainly used for low wave frequencies (\( f \leq 100\text{Hz} \)). Additionally, due to the reduced repetition time, speed-up through fast readouts such as EPI and spiral imaging is not as prominently used as in full-wave encoding.

3.3 | Multishot acquisition

With the development of fractional encoding, the duration of a single shot, i.e. excitation, wave encoding, and readout, can become shorter than half a wave period. Hence, multiple shots can be fit into a single wave period, further accelerating the readout.\(^{38,79}\) Rump et al. fitted four shots of fractional bSSFP-MRE acquisitions into a single wave period in order to sample the wave at zero, 90°, 180°, and 270° mechanical phase offsets.\(^{38}\) These were then combined into two phase-difference acquisitions to correct for phase offsets and to double motion sensitivity.

The “eXpresso” sequence, a fractional GRE-based MRE sequence, follows a different approach, where multiple slices are acquired within a single wave cycle.\(^{79}\) This leads to a mechanical phase offset between the slices, which can be corrected for in an additional processing step before the stiffness reconstruction is performed. In Figure 5, the “eXpresso” sequence is depicted for the acquisition of three slices during one wave period. The second and third slices will be acquired with a mechanical phase offset of 120° and 240°, respectively.

3.4 | Sample interval modulation.

Sample interval modulation (SLIM) is capable of measuring all three motion components of a single-frequency shear wave simultaneously.\(^{67,71}\) To this end, the wave’s motion components are encoded with different apparent frequencies by time-shifting the MEGs with respect to one another. In Equation 14, we have stated the MRE phase of a general shear wave field measured with a unidirectional MEG as a function of the time-offset \( t_0 \) between the onset of the MEGs and an arbitrary reference point of the wave field. In conventional mono-frequency MRE, the offset time \( t_0 \) is chosen to sample the wave at \( N_p \) equidistant time points \( t_i = f^{-1} i/N_p (i \in \{1...N_p\}) \) within one period \( f^{-1} \) of the wave. The acquired phase series within a voxel is thus given by

\[
\Phi_{p,d} = \sum_m \mathbf{G}_{d} \cdot \mathbf{r}_{f,m} \cdot \left[ \epsilon_{f,m} \right] \sin \left( \frac{2\pi}{N_p} i + \theta_{f,m} + \frac{N}{2} - \angle \left( \epsilon_{f,m} \right) \right),
\]

(40)

FIGURE 4  Encoding efficiency (left) and gradient amplitude plots (right) for sinusoidal (dashed gray), infinite-slew (dot-dashed gray) and a slew-rate-limited trapezoidal (black) bipolar MEG. The roots of the encoding efficiency, i.e. the filter condition, are dependent on the underlying gradient shape and gradient repetition and can be “tuned” to selectively suppress motion components.

FIGURE 5  Multishot fractional GRE-MRE sequence (“eXpresso-sequence”) acquiring one k-space line for three slices within one mechanical wave cycle. The different starting times of the slice acquisitions during one vibration cycle result in a mechanical phase offset between the slices, which needs to be corrected for in a post-processing step.
Hence, the phase series represents exactly one phase cycle as the underlying wave. Application of a discrete Fourier transform

\[ \mathcal{F}\left[\phi_{j} \right] = N_p \sum_{n} \phi_{n} e^{-i2\pi j \epsilon_{n}} \]  

(41)

yields one non-zero element at \( j = 1 \) and its complex conjugate partner at \( j = N_p \). However, if the shear wave is probed with the same number of sampling points, but with an integer multiple \( k \) of the offset time, the phase will be given by

\[ \phi_{k \epsilon_d} = \sum_{m} \Gamma_{d} \sin \left(2\pi \frac{k}{N_p} + \theta_{f,m} + \frac{\pi}{2} - \angle \left( \epsilon_{f} \right) \right) \]  

(42)

resulting in a non-zero element at \( j = k \) in the aforementioned discrete Fourier transformation. Thus, time-shifting the MEGs such that the three orthogonal motion components are probed with different sampling intervals leads to three distinct peaks in the frequency spectrum of the phase, each corresponding to the respective encoded motion amplitude as well as respective mechanical phase.\(^{67,71}\)

In Figure 6, the first, third, and fifth sampling patterns of a SLIM SE-MRE acquisition scheme are shown. The MEGs in read, phase, and slice directions are time-shifted to project the motion direction onto different frequency components of the signal phase. In addition to temporal shifting, the periodicity of the shear wave field can be used to shorten the offset times as depicted in the fifth pattern. In Figure 7, the real part of the complex wave images corresponding to the three displacement components in \( x, y, \) and \( z \) directions as well as real and imaginary components of the shear stiffness are compared for conventional full-wave encoded MRE and SLIM-accelerated MRE acquisitions.\(^{71}\)

In principle, SLIM MRE is capable of acquiring the same motion field with the same DNR as conventional uni-directional MRE with eight mechanical-phase offsets in approximately one-third of the scan time. However, echo times must be sufficiently large to allow for the temporal shifting of the MEGs, leading to either prolonged echo times or lower encoding efficiency, the induced wave field must be free of harmonics, and additional wave-dependent phase accrual caused by imaging gradients is not corrected for (see also Section 4). A similar encoding concept was derived by Nir et al. using optimization-based design.\(^{84}\)
Simultaneous multi-frequency MRE

In simultaneous multi-frequency MRE, shear waves are introduced comprising multiple frequency components. These are simultaneously measured using the broad-band sensitivity of MEGs (see Equation 38 for sinusoidal MEGs and Figure 4). Following Equation 13, the MRE phase signal is then given by a superposition of the shear components oscillating at their respective frequencies. Application of a Fourier transformation leads to distinct peaks, allowing for separation of the respective shear amplitudes.\(^{77}\) The frequency resolution of simultaneous multi-frequency MRE is given by

\[
\delta f = (\Delta t N_p)^{-1},
\]

where \(\Delta t\) is the sampling interval and \(N_p\) is the number of sampling points (\(N_p = 40, \Delta t = 2\) ms in Reference 77). Due to the high number of temporal sampling points, multi-frequency MRE is typically performed in single-slice, unidirectional MRE experiments.\(^{77,85-87}\) In Figure 8, a set of multi-frequency brain images is shown for 25, 37.5, 50, and 62.5 Hz with motion sensitization in the through-plane direction. The difference in wavelength can be clearly observed in the displacement images in the top row and corresponding shear wave speeds in the bottom row.

In comparison to the sequential measurement at different frequencies, simultaneous multi-frequency MRE comes with two drawbacks that both lead to lowered SNR. First, the maximally applicable vibration strength per frequency component is reduced, as the transducer’s total output power is limited and has to be distributed to the different frequencies. Second, since the broadband sensitivity of the MEGs is used for encoding, a global optimum yielding sufficient sensitivity to all applied frequencies has to be found, which is in contrast to sequential measurements, where it is possible to fine-tune each sequence to best encode the applied frequency.\(^{88}\)

Selective spectral displacement projection

Selective spectral displacement projection (SDP) uses the filter condition (Equation 39) to simultaneously measure the displacement of three different frequencies polarized along three different mutually orthogonal directions. The filter condition specifies the roots of the encoding efficiency of sinusoidal MEGs and is given by \(N_q = N_0 f/g \in \mathbb{N} \wedge q \neq 1\). Thus, if a combination of shear wave frequency \(f\) and MEG properties \(N_0, fg\) fulfills the above condition, the wave is not encoded. By choosing orthogonal MEGs that fulfill the above condition for two of the three shear frequencies at the same time, the oscillation of only one frequency component is encoded in each direction.\(^{83}\) In contrast to simultaneous multi-frequency MRE, the different frequency components are probed with equal encoding efficiency, which is especially important in high-frequency MRE as found in phantom and pre-clinical animal studies. Here, \(N_0\) is typically much larger than 1, leading to highly suppressed encoding efficiency for \(f \neq g\) (see also inset in Figure 9).

In Figure 9, an exemplary GRE-based SDP-MRE sequence is shown. Here, the MEG in the read direction oscillates at 5 kHz with 15 cycles, the MEG in the phase direction oscillates at 6 kHz and 18 cycles, and the MEG in the slice direction oscillates at 7 kHz and 21 cycles. The inset in the top-right corner depicts the dependency of the encoding efficiency for the three MEGs on the underlying MRE frequency. The red circles mark the roots of the encoding efficiencies of two MEGs coinciding with the peak of the third MEG.
PHASE CORRECTION SCHEMES

The phase of an MRE measurement \( \phi \) in a single voxel can be approximated by the accumulated phase caused by the MEGs \( \phi_{b_0, d} \) a phase term \( \phi_{b_0} \) collating field inhomogeneity and local susceptibility changes, and motion-encoded phase from imaging gradients \( \phi_{t, \text{Imaging}} \) such as the slice or phase-encoding gradient (Figure 10, red-shaded gradients). Neglecting geometric distortions, which may additionally lead to subtle spatially varying bias in the MRE signal, the phase in a single measurement can be approximated by

\[
\phi = \phi_{b_0, d} + \phi_{b_0} + \phi_{t, \text{Imaging}} (\pm \eta).
\]  

(44)

Additionally, a random variable \( \eta \) can be included to account for phase noise. Three orthogonal motion directions should be encoded, which needs according to Equation 13 at least three measurements with encoding gradients that together are a basis of \( \mathbb{R}^3 \). Additionally, the phase offset \( \phi_{b_0} + \phi_{b_0, \text{Imaging}} \) needs to be corrected for, and accordingly a minimum of four measurements is required if a single time point is considered.

To date, three-directional MRE is performed with three different encoding schemes. Four-point unbalanced encoding consists of the measurement with three orthogonal encoding directions as well as a reference scan with disabled MEGs to measure the sum \( \phi_{b_0} + \phi_{b_0, \text{Imaging}} \) separately for each mechanical phase offset \( t_0 \) (Figure 10, solid MEGs and reference scan). This reference phase is then subtracted from each encoded direction.\(^{24,36,57,79}\)

The second scheme, which is commonly referred as “tetrahedral encoding” in the MRE literature, is a variant of the four-point unbalanced encoding that is sensitive to motion along the four diagonals of a regular cube. Here, a reference scan with simultaneous motion encoding along all three encoding directions is acquired, and in the subsequent three measurements one gradient channel is inverted at a time. Accordingly, the motion phase in tetrahedral encoding is decoded by subtraction of the reference from each encoded direction, comparable to unbalanced four-point encoding.\(^{89}\) The scheme however has higher encoding efficiency, as the reference is acquired with inverted motion sensitization.

The third scheme is six-point encoding, where three orthogonal directions are measured with non-inverted and inverted gradients (Figure 10, solid and dashed MEGs, no reference scan). Subtraction of inverted and non-inverted scans leads to doubled motion sensitivity and the cancellation
of the spurious phase $\phi_b + \phi_{b,\text{imaging}}$, since the sign of the latter is not inverted with the MEGs. This scheme is found in many three-directional MRE acquisitions* as well as in uni-directional MRE, where only one wave component is encoded.\(^1\)

In addition to correcting for phase accrual induced by the imaging gradients, the extraction of the displacement fields may require phase unwrapping prior to spectral analysis. The discussion and comparison of these techniques is beyond the scope of this review. Barnhill et al. developed and compared unwrapping algorithms for unwrapping 4D MRE datasets.\(^9\) For completeness, some temporal and spatio-temporal unwrapping prior to spectral analysis. The discussion and comparison of these techniques is beyond the scope of this review. Barnhill et al. developed and compared unwrapping algorithms for unwrapping 4D MRE datasets.\(^9\) For completeness, some temporal and spatio-temporal unwrapping methods that have been proposed for use in MRE will be listed here: Goldstein's two- and three-dimensional unwrapping,\(^61,91\) Flynn's two-dimensional phase unwrapping,\(^80,92-94\) gradient-based unwrapping,\(^28,95,96\) unwrapping based on minimum cost flow analysis,\(^52,97,98\) 3D quality guided unwrapping,\(^68,99,100\) and 3D quality guided flood-fill unwrapping.\(^19,101\)

5 | MEG DESIGN

Apart from "temporal" encoding and phase-correction schemes, the encoding efficiency of the MEG is a key property that can be tuned. Generally, encoding gradients in phase contrast MRI can be classified according to their gradient moments.

Since a major field of MRE employs fractional encoding, we first will take a look at the low-$q$ regime, thus MEG frequencies much higher than the underlying shear wave frequency $f$. A Taylor series expansion about $q = 0$ of the general encoding efficiency (Equation 13) is given up to third order by

$$\varepsilon_f^T = y_q N_q \int_0 T dQ(\sigma) \left( \begin{pmatrix} 2n\sigma q_f \\ \frac{1}{1-2i(\sigma\sigma)^2} q_f^2 \end{pmatrix} + O(q_f^4) \right) = y_q \int_0 T M_n q_f \left( M_0 - 2n^2 M_2 q_f^2 \right) + O(q_f^4).$$

(45)

Here, we use the "reduced" gradient moments $\tilde{M}_n$ of the temporal gradient strength function $g(\sigma)$

$$\tilde{M}_n = y_q \int_0 N_q dQ(\sigma) \sigma^n = G^{-1} \left( \begin{pmatrix} 1 \\ \vdots \\ n+1 \end{pmatrix} \right) M_n, \text{ with } M_n = \int_0 T dt G(t) t^n.$$

which is connected to the "physical" gradient moments $M_{n,T}$ by normalizing them to unit length and unit gradient strength. Equation 45 reveals that the encoding efficiency in the low-$q$ regime is a polynomial in $q_f$, where the $n$th polynomial coefficient is proportional to the $(n+1)$th-gradient moment.

Since the applied MEGs are typically zeroth-moment compensated ($M_0 = 0$) so that they do not interfere with phase encoding, the low-$q$ regime is only controlled by the first and second moments of the gradient waveform. In the following section, we want to discuss bipolar ($M_1 \neq 0$) and flow-compensated MEGs ($M_1 = 0$) and summarize their encoding properties.

5.1 | Bipolar MEGs

Bipolar MEGs feature positive and negative gradient lobes with equal area, such that no phase dependency on the position of the isochromat is introduced. They feature a high first moment $M_1$ and show a $q_f^2$ dependency in the low-$q$ regime, where the encoding efficiency is given by

$$\varepsilon_f^T = 2nM_1 q_f \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right) = 2nM_1 q_f / C_3.$$  

(46)

In Table 3, the encoding efficiencies of different representations of bipolar MEGs are summarized together with their respective moments and references to publications using these MEGs. The analytical encoding efficiency of sinusoidal gradients has already been discussed in Section 3.2 and compared with different gradient waveforms in Figure 4 (left).

5.2 | Flow-compensated MEGs

Flow-compensation, or first-order moment nulling, is a phase-contrast technique to suppress phase artifacts occurring from motion with constant velocity along the encoding direction originating from blood flow, breathing, or generally intra-voxel motion. Many publications using flow-compensated gradients are based on the original description of Muthupillai,\(^58\) where the encoding efficiency of sinusoidal flow-compensated gradients was calculated. Such a gradient is depicted in solid black in Figure 11 (right). Cosinusoidal gradients can be seen as the theoretical equivalent to sinusoidal gradients, but with infinite slew rate at the beginning and end of the gradient. Their encoding efficiency is plotted against the encoding fraction in Figure 11 (left) and compared with sinusoidal MEGs. Its analytical encoding efficiency is stated in Table 3.

*References 29,35,36,62,68,70,73,74,83.

\(^1\)References 1,27,38,48,60,62-66,69,74-78,81.
Table of encoding efficiencies, slew rates, and first- and second-order moments for different MEG shapes. Bipolar (infinite slew) and cosinusoidal gradient shapes are included as theoretical models and hence have not been used for encoding in MRE. The filter condition for each waveform can be readily derived by calculating the roots of the encoding efficiency. Trapezoidal flow-compensated MEGs are prominently used; however, complexity and degrees of freedom render an analytical solution of the general encoding efficiency and second moment infeasible. Hence, only the special case \( q = 1 \) is given.

| Name                  | Shape                          | Parameter | Encoding efficiency [rad/T] | Max. slew | First moment \( (M_1) \) | Second moment \( (M_2) \) | References          |
|-----------------------|--------------------------------|-----------|----------------------------|-----------|---------------------------|------------------------|----------------------|
| Sinusoidal            | \( q, N_0 \)                   | \( \epsilon_f = \frac{\gamma |q \sin(qN_0)|}{\pi f} \) | \( \frac{2\pi f}{q} \) | \( -\gamma G N_0 q^2 \frac{2\pi f}{q} \) | \( -\gamma G N_0^2 q^3 \frac{2\pi f}{q} \) | 4, 6, 11–14, 24, 27, 28, 32 |
| Bipolar (infinite slew)| \( q \)                        | \( \epsilon_f = \frac{2\gamma \sin^2 \left( \frac{\pi q}{2} \right)}{\pi f} \) | \( \infty \) | \( -\gamma G q^2 \frac{2\pi f}{q} \) | \( -\gamma G q^3 \frac{2\pi f}{q} \) | n.a.                  |
| Trapezoidal           | \( q, t_{\text{dwell}} \)      | \( \epsilon_f = \frac{2\gamma \sin^2 \left( \frac{\pi q}{2} \right) \sin \left( \frac{q - 2\beta}{2} \right)}{\pi^2 \beta} \) | \( \frac{G}{t_{\text{dwell}}} \) | \( -\gamma G \frac{q^2}{4f^2} (q-2\beta) \) | \( -\gamma G \frac{q^3}{4f^2} (q-2\beta) \) | 1, 5, 7, 10, 11, 15, 25, 26, 31, 33, 35, 36, 39 |
| Cosinusoidal          | \( q, N_0 \)                   | \( \epsilon_f = \frac{\gamma |q \sin(qN_0)|}{\pi f} \) | \( \infty \) | 0 | \( \gamma G N_0 \frac{q^3}{2\pi^2 f^2} \) | n.a.                  |
| Trapezoidal flow compensated | \( t_{\text{dwell}}, q = 1 \) | \( \epsilon_f = \frac{\gamma |\cos(2\pi f) + \sin \left( \frac{\beta}{2} \right) N \beta + \sin \left( \frac{\beta}{2} \right) N \beta|}{2\pi^2 \beta} \) | \( \frac{G}{t_{\text{dwell}}} \) | 0 | \( \gamma G \frac{1}{16f} (\beta^2 - 1) (3\beta - 1) \) | 16, 18–21, 23, 29, 30, 34 |

\( f \), wave frequency; \( q = f_{\text{fp}} \) encoding fraction; \( \gamma \), gyromagnetic ratio; \( N_0 \), number of gradient repetitions; \( t_{\text{dwell}} \), gradient dwell time; \( \text{SR} = G/t_{\text{dwell}} \), slew rate; \( G \), maximal gradient strength; \( \beta = ft_{\text{dwell}} \), ratio of gradient dwell-time and wave period \( f^{-1} \).

n.a., not applicable.
According to Equation 45, the encoding efficiency of flow-compensated gradients in the low-\(q\) regime is given by
\[
\epsilon_f = \frac{2\pi^2 M_2 q^3 f_f f_0}{C_0^2 / C_1 + O q^5 f_f / C_0 / C_1} \approx \frac{2\pi^2 M_2 q^3 f_f f_0}{C_0^2 / C_1}.
\] (47)

Hence, the efficiency of flow-compensated gradients for small \(q\) is proportional to \(q^3\), compared with \(q^2\) for bipolar gradients, which is crucial for fractional encoding schemes. Hence, flow-compensated gradients are mostly employed in full-wave encoding \(^{28,29,35,70,73}\) and in fractional encoding with \(q > 1/2\).\(^ {17,27,28,80}\)

6 | ADVANCED READOUT AND SUB-NYQUIST TECHNIQUES

In the previous sections, we have reviewed different MRE sequences, “temporal” encoding schemes, and phase correction as well as encoding efficiencies. In the following, the emphasis will lie on efficient readout.

6.1 | Alternative \(k\)-space coverage

Besides sequential readout of single Cartesian \(k\)-space lines, which has been assumed and depicted in sequence graphs so far, alternative ways of traversing \(k\)-space are readily being employed in MRE. The most often used is echo-planar imaging (EPI), which acquires multiple parallel \(k\)-space lines sequentially within a single repetition of the sequence.\(^ {102}\) A SE-based full-wave-encoding MRE sequence with EPI readout and an echo-train length of 5 is depicted in Figure 12 and Figure 13. EPI is prominently used with SE-MRE sequences in particular because of the inherently longer repetition times of SE-based sequences compared with GRE.\(^ {28,29,35,65}\)

EPI sequences with long echo trains are subject to a low-/high-pass filter effect as \(k\)-space lines acquired before and after the echo time are damped because of \(T_2^*\)-related signal decay, resulting in blurring. Additionally, the switching of strong gradients as they are used with MRE results in eddy currents induced in the conducting parts of the MRI scanner. The resulting spurious magnetic fields lead to geometric distortions, such as shifting and shearing of the image. First-order eddy currents cause delays of the gradient waveform and the readout, leading to a line-by-line misalignment of acquired echoes. This misalignment leads to Nyquist ghosting in the phase-encoding direction. In addition, the

**FIGURE 11**  Encoding efficiency for constant MEG frequency and gradient amplitude plots for sinusoidal, non-flow-compensated (dashed gray) and cosinusoidal, flow-compensated (black) MEG shapes. N.B.: Encoding efficiency for flow-compensated gradients is highly suppressed for encoding fractions smaller than 1. The dot-dashed lines show the low-\(q\) regime expansion for cosinusoidal and sinusoidal gradients.

**FIGURE 12**  Spoiled SE-based MRE sequence with echo planar imaging readout, an echo-train length of 5, and full-wave encoding with four repetitions of sinusoidal MEGs. The inset shows the five acquired \(k\)-space lines and the traversal between lines due to the blip-gradients in the phase-encoding direction.
acquisition of multiple echoes increases the readout duration, which in turns leads to pronounced artifacts due to local static magnetic field offsets.103

Another efficient way of traversing $k$-space is the spiral readout,104 which is depicted for the case of constant velocity and constant density in Figure 13. In spiral acquisition the readout is started in the $k$-space center and has the potential benefit of traversing the whole $k$-space within a single shot. In multi-shot spiral acquisitions, shot-by-shot differences caused by eddy currents and motion are distributed equally over the whole $k$ space, leading to blurring instead of Nyquist ghosting as in EPI. Spirals feature very effective $k$-space sampling since the corners are not acquired, reduced motion sensitivity because of constantly changing gradient directions, partial flow compensation during the readout leading to reduced flow-induced artifacts, efficient use of gradient power, and oversampling of the $k$-space center, which can be used for self-navigation.105-107 In addition, spiral trajectories can be specifically tuned to the needs and conditions of the readout, since readout duration, number of interleaves, sampling density, and velocity can be independently adapted within the limits of the gradient system through the use of variable density spirals.108

The advantages of spirals come at the cost of a complex reconstruction process of the images, which typically employs regridding and subsequent Fourier transformation.109,110 In addition, local field inhomogeneities may cause artifacts in the final image due to the long readout times. These artifacts can be resolved through non-linear iterative reconstruction such as multi-frequency interpolation, although this renders the reconstruction process even more elaborate and time consuming.111 Furthermore, eddy-current-induced $B_0$ and trajectory changes, which lead to blurring, can only be accounted for if a complete model of the gradient response is known, e.g. by measuring with a field camera.112,113 In addition, concomitant gradient fields, i.e. higher-order magnetic fields that accompany gradient fields according to Maxwell’s equations, cause spatially dependent blurring of spiral scans.114 Since these effects are non-linear in space, additional correction methods need to be applied114, or full inversion of the encoding operator, i.e. phase conjugate reconstruction, needs to be performed, which is currently only feasible in the case of low-resolution imaging.115

Spiral imaging has been successfully performed in SE-based full-wave MRE, with as few as six spiral interleaves to cover the whole $k$-space of a $128 \times 128$ pixel image (240 x 240 mm$^2$ field of view, FOV).68,70,71,73

6.2 | Reduced FOV techniques
The FOV in MRI is given by the extent of the object and must be chosen larger in order to avoid fold-over artifacts. In SE-MRI acquisitions, the FOV in the phase-encoding direction can be reduced by applying RF excitation and echo pulses slice-selectively, while the slice-selection gradient of the echo pulse is chosen orthogonal to the excitation slice-selection gradient. In this way, only the spins residing in the intersecting volume produce a signal leading to a reduction of the effective FOV. In this way, the number of phase-encoding steps can be reduced to increase acquisition speed (reduced FOV, rFOV-MRE).63

Disabling the phase-encoding gradient leads to a projection of the FOV onto the frequency-encoding direction and thus the acquisition of a 1D image (beam SE-MRE). Since GRE-based MRE acquisitions only have one RF pulse, the excitation and echo formation with orthogonal slice-selection gradients is not possible. However, 2D-spatially selective RF pulses can be used instead. Here, a combination of a modulated RF pulse and a suitable gradient waveform is used.116-118 With rFOV MRE and beam MRE, the acquisition of full MRE datasets can be restricted to a small region of interest, thus enabling the fast acquisition of full wave fields with high resolution.63,119,120

6.3 | Multi-slice and 3D readout
The acquisition of volumetric wave displacements is crucial to successful inversion of the wave equation and extraction of shear-stiffness maps.121 Until now, we have assumed that 2D slices are acquired by employing slice-selective RF excitation pulses. In sequential multi-slice imaging (M2D),
these 2D slices are consecutively acquired and stacked, forming a volumetric data set. In order to increase the steady-state signal, stacked 2D images are usually acquired in an interleaved fashion (MS) such that the effective repetition-time per slice is increased (see Figure 14).

For spoiled GRE-based sequences, the transverse magnetization is given by

\[ \mathbf{S} = \mathbf{S}_0 \frac{1 - e^{-T_{1p}/T_1^*}}{1 - e^{-2T_{1p}/T_1^*}} e^{T_{2R} T_1^*}, \]

assuming that the flip angle is chosen to be the Ernst angle, \( \alpha = \cos^{-1}\left(\frac{1}{e^{T_{1p}/T_1^*}}\right) \). The signal increase \( h_{\text{GRE}}(n) \) through extending the effective repetition time to \( nT_R \) is then given for spoiled GRE by

\[ h_{\text{GRE}}(n) = \frac{1 - e^{-nT_{1p}/T_1^*}}{1 - e^{-2nT_{1p}/T_1^*}} \sqrt{1 - e^{-2nT_{1p}/T_1^*}}. \]

For spoiled SE sequences with a 90° excitation pulse and 180° echo pulse, the SNR increase is given by

\[ h_{\text{SE}}(n) = \frac{1 - 2e^{-nT_{1p}/T_1^*}}{1 - 2e^{-2nT_{1p}/T_1^*}} + e^{-nT_{1p}/T_1^*} T_{2R} = \frac{1 - e^{-nT_{1p}/T_1^*}}{1 - e^{-2nT_{1p}/T_1^*}} \]

The SNR of interleaved acquisition (MS) relative to non-interleaved stacked images (M2D) is then given by

\[ \text{SNR}_{\text{MS}} = h(N_{\text{slices}}) \text{SNR}_{\text{M2D}}. \]

True 3D readout, where the z-dimension is also encoded using a phase-encoding gradient (see Figure 14), leads to even higher SNR increase in spoiled GRE sequences. In Figure 15, SNRs of GRE and SE sequences are compared for sequential and interleaved stacked 3D acquisitions. Interleaved acquisition always leads to an increase in SNR. The SNR of a volumetric readout is given by

\[ \text{SNR}_{\text{3D}} = \sqrt{N_{\text{slices}}} \text{SNR}_{\text{M2D}} = \sqrt{N_{\text{slices}}} \text{SNR}_{\text{MS}}. \]

where the reduction in effective repetition time compared with MS might mitigate the \( \sqrt{N_{\text{slices}}} \) SNR boost in SE sequences. In Figure 15, the SNR of a true 3D acquisition (\( N_{\text{slabs}} = 1 \)) is compared with interleaved stacked images for SE and GRE sequences. GRE always benefits in terms of SNR from true 3D readout; however, SNR increase might only be a couple of percent for short repetition times.122 SE-based sequences might lose SNR with short repetition times or gain SNR for long repetition times.

For SE sequences, a compromise between stacked 2D slices and true 3D encoding is given by the multi-slab technique (M3D). Here, multiple parallel “partial slabs” (\( N_{\text{slabs}} \)) of the full volume are selectively excited, and multiple slices are then encoded within these volumes using phase encoding, hence combining the SNR increase of 3D readout, while limiting the loss in steady-state magnetization (see Figure 14). In spoiled GRE sequences, the mutli-slab acquisition technique is not favorable in terms of SNR, as shown in Figure 15. The SNR of multi-slab 3D readout is given by

\[ \text{SNR}_{\text{M3D}} = \sqrt{N_{\text{slabs}}} \text{SNR}_{\text{M2D}}. \]

**FIGURE 14** Graphical depiction of the different volumetric readout schemes, where the red lines denote acquired \( k \)-lines and the arrows show the acquisition order. From left to right, sequential readout of stacked images (M2D), interleaved acquisition of multiple slices (MS) to increase the actual \( T_R \) per slice, true 3D readout through volume excitation and phase encoding in phase and slice directions, and multi-slab 3D readout (M3D) mixing sequential and 3D readout by sequentially exciting volumetric slabs and phase encoding in the slice direction within the slabs. The latter is performed to balance the long effective \( T_R \) of MS and the SNR increase of 3D readout.

**FIGURE 15** Relative SNR as a function of the number of acquired slices for GRE- and SE-based 3D scans. For GRE, acquisition with the Ernst angle is assumed, while SE is assumed to use 90° flip angles. Interleaved stacked images show higher SNR than sequential stacked images, due to elongation of the effective repetition time per slice. Full 3D acquisition (\( N_{\text{slabs}} = 1 \)) always shows higher SNR for GRE sequences; however, multislab (\( N_{\text{slabs}} > 1 \)) does not improve SNR. For SE sequences, SNR in volumetric acquisition qualitatively differ depending on the \( T_R/T_1 \) combination.
\[
\text{SNR}_{\text{M2D}} = \sqrt{\frac{N_{\text{slabs}}}{N_{\text{slabs}} h(N_{\text{slabs}})}} \text{SNR}_{\text{M2D}} = \sqrt{\frac{N_{\text{slabs}} h(N_{\text{slabs}})}{N_{\text{slabs}} h(N_{\text{slabs}})}} \text{SNR}_{\text{M3D}}.
\]

In Table 1, Table 2, and Table 4, publications using 3D readout are compared regarding key MRE imaging parameters. Most current volumetric MRE acquisitions are performed using sequential or interleaved stacked 2D scans.\textsuperscript{12} Multislab acquisition was recently developed and applied in conjunction with spiral MRE readout,\textsuperscript{70,73} whereas full 3D acquisition was also recently performed together with compressed sensing and dynamic nuclear polarization MRE.\textsuperscript{124}

### 6.4 Sub-Nyquist techniques

Besides speeding up acquisitions by effective coverage of the k-space, image acquisition can also be sped up through undersampling. Partial Fourier (PF) fully samples only a fraction of the k-space and completes the missing information by making use of Hermitian symmetry of the k-space for real-valued signals (see Figure 16).\textsuperscript{122} Since MRI signals are complex valued, a low-resolution phase image is usually estimated from the fully sampled center of k-space and then subtracted from the measured data, creating an approximately real-valued image with Hermitian symmetry (homodyne reconstruction).\textsuperscript{125} PF with homodyne reconstruction is well established for the acquisition of magnitude images; however, for the acquisition of phase-contrast images, Hermitian symmetry is no longer given. A low-resolution phase estimation might be sufficient to restore symmetry in special cases; however, in MRE, the localized strong phase variations are key to successful stiffness reconstruction, which cannot be recovered by low-resolution phase estimates. Hence, only zero-filling is a valid reconstruction method, but does not lead to higher resolution than centered acquisition of the same number of k-space lines.\textsuperscript{122}

Parallel imaging techniques, e.g. sensitivity encoding (SENSE) and GRAPPA, utilize the spatial sensitivity of multi-receive coils to unfold the aliasing artifacts that occur when undersampling is performed with regular patterns (see Figure 16). For SENSE, additional sensitivity maps need to be acquired in a separate scan. The undersampled data is then directly reconstructed by inversion of the encoding matrix that takes the point-spread function of the undersampling pattern and the sensitivities into account.\textsuperscript{126} GRAPPA uses additional auto calibration signals, which are fully sampled but low resolution. A model is built in the fully-sampled k-space region that allows the estimation of the missing information from the surrounding acquired k-lines. In a next step, this model is then applied to the missing k-space data, completing the images for each coil separately. These coil images are then reconstructed and coil-combined to form an unaliased final image.\textsuperscript{127}

Local noise enhancement in parallel imaging is typically characterized by the geometry factor. For SENSE this factor can be readily calculated and is given by

\[
g = \sqrt{\text{diag} \left( \hat{E}^H \hat{\Psi}^{-1} \hat{E} \right) \cdot \text{diag} \left( \left( \hat{E}^H \hat{\Psi}^{-1} \hat{E} \right)^{-1} \right)},
\]

where \(\hat{\Psi}\) is the noise covariance matrix of the coil elements and \(\hat{E}\) is the encoding matrix. The SNR in SENSE is then given by

\[
\text{SNR}_{\text{SENSE}} = \frac{\text{SNR}_0}{\sqrt{R g}}
\]

where \(\text{SNR}_0\) is the SNR of a fully sampled scan and \(R\) is the acceleration factor.\textsuperscript{122,126,128} In the case of GRAPPA, the calculation of the geometry factor is also possible.\textsuperscript{129} Parallel imaging acquisition is well established for phase-contrast imaging and readily used in accelerated MRE (see Table 4).

In compressed sensing (CS), the inherent sparsity of MRI images in the image itself or a sparse transformed domain thereof is used to reconstruct randomly undersampled data. The randomness of the sampling pattern leads to noise-like interference, which can be filtered in the sparse domain of the MRI images by non-linear thresholding.\textsuperscript{130,131} CS has recently been used in ultra-low field MRE in conjunction with dynamic nuclear polarization (DNP).\textsuperscript{124} Here, Cartesian 3D acquisition with phase encoding in both phase and slice encoding directions was paired with variable density Gaussian undersampling patterns.\textsuperscript{132} However, the influence of CS on phase contrast images is not yet fully understood and needs further scrutiny in the future.

In Figure 16, typical sampling patterns for the discussed sub-Nyquist methods are visualized and compared. In Table 4, references to MRE publications using sub-Nyquist techniques are presented and compared regarding key MRE imaging parameters.

### 7 UNCONVENTIONAL MRE ACQUISITION TECHNIQUES

In the following, "exotic" MRE sequences, which do not fall into the usually performed categories of MRE scans and techniques described above, are briefly reviewed.
### Table 4: Comparison of MRE publications using sub-Nyquist techniques for accelerated acquisition

| Leading author | Year | Sequence | Dimension | Readout method | Sub-Nyquist technique (factor) | MRE frequency (Hz) | MEG frequency (Hz) | MEG shape | N0 | MEG gradient (mT/m) | MEG enc. directions | MEG enc. scheme | Applied region | Flip angle | Matrix | FOV (mm) | Slice (mm) | Resolution (mm) | T₁/T₂ (ms) | Wave images |
|----------------|------|----------|-----------|----------------|-------------------------------|-------------------|-------------------|-----------|----|-------------------|-------------------|----------------|---------------|------------|-----------|-----------|----------|------------|------------|-----------|-------------|
| Sack²⁶        | 2009 | GRE      | 2D        | Cartesian      | G (2)                         | 24.3              | 500               | T         | 1  | 25, 35            | 3 (MPSR)         | 4-unb.         | heart         | 15         | 128 × 128 | 320 × 320 | 5          | 2.5 × 2.5  | 5.16/360   |
| Nedredal¹⁷     | 2011 | SE       | M2D       | EPI            | Pl (2)                        | 60                | 80                | FC        | 1  | n/a              | 1 (S)             | 2-point        | liver         | 90         | n/a × n/a | n/a × n/a | 3          | 3 × 3 × 3  | 1634/48    |
| Garteiser²⁹    | 2013 | GRE      | M2D       | Cartesian      | S (2)                         | 28–84             | 120               | T         | 1  | 21               | 3 (MPSR)         | 4-unb.         | liver         | 20         | 80 × 80   | 320 × 320 | 4          | 4 × 4 × 4  | 9.21/8     |
| Garteiser      | 2013 | GRE      | M2D       | Cartesian      | S (2)                         | 28–84             | 160               | T         | 1  | 30               | 3 (MPSR)         | 4-unb.         | liver         | 20         | 80 × 80   | 320 × 320 | 4          | 4 × 4 × 4  | 6.91/8     |
| Yin²⁹          | 2013 | SE       | M2D       | EPI            | Pl (3)                        | 60                | 60                | FC        | 2  | sp               | 3 (XYZ)          | 6-point        | porcine liver/spleen | n/a        | 80 × 80   | 260 × 260 | 6          | 325 × 3.25 | 1667/48    |
| Sahebjavaher²⁴ | 2014 | GRE      | M2D       | Cartesian      | S(2), PF(80%)                 | 70                | 119               | T         | 1  | 48               | 4 (XYZR)         | 4-unb.         | prostate      | 25         | 128 × 128 | 256 × 256 | 2          | 2 × 2 × 2  | 344/92     |
| Hirsch²⁸       | 2014 | SE       | M2D       | EPI            | G (2)                         | 30–60             | 50                | FC        | 1  | 30               | 3 (XYZ)          | 3-noref        | liver         | 90         | 128 × 104 | 350 × 284 | 5          | 2.7 × 2.7  | 182/54     |
| Braun³⁰        | 2014 | SE       | M2D       | EPI            | G (3)                         | 40–60             | 36–71             | FC        | 1, 2 | 50               | 3 (XYZ)          | 3-noref        | brain         | 90         | 200 × 186 | 200 × 186 | 1          | 1 × 1 × 1  | 5640/76    |
| Numano²²       | 2015 | GRE      | 2D        | Cartesian      | S (2)                         | 100               | 100               | T         | 1  | n/a              | 1 (M)            | n/a           | phantom      | 20         | 256 × 256 | 180 × 180 | 5          | .7 × .7     | 40/2.3     |
| Numano         | 2015 | GRE      | 2D        | Cartesian      | S (2)                         | 50                | 50                | T         | 1  | n/a              | 1 (M)            | n/a           | psoas major muscle | 20         | 256 × 256 | 300 × 300 | 10         | 1.17 × 1.17 | 40/2.3     |
| Low³⁵          | 2015 | SE       | M2D       | EPI            | Pl (2)                        | 60/90             | 60/90             | FC        | n/a | n/a              | 3 (XYZ)          | 6-point        | kidney        | n/a        | 72 × 72   | 360 × 360 | 3.5        | 5 × 5 × 3.5 | 1323–1584/37-40 |
| Anderson⁷³     | 2016 | SE       | M3D       | spiral         | S (3)                         | 50                | 50                | FC        | 2  | sp               | 3 (XYZ)          | 6-point        | brain         | n/a        | 120 × 120 | 240 × 240 | 2          | 2 × 2 × 2  | 1800/73    |
| Salameh¹²⁴     | 2016 | bSSFP    | 3D        | Cartesian      | CS                            | 103               | 206               | S         | 1  | 1                | 4 (MPSR)         | 4-unb.         | phantom      | 90         | 65 × 64   | 98 × 173  | 9          | 1.5 × 2.7  | 39/24      |

n.a., not applicable; n/a, not available.

**Dimension:** 1D/2D, 1D/2D acquisition; M2D, stacked 2D acquisition; M3D, multislab acquisition; 3D, full 3D acquisition.

**Sub-Nyquist technique:** PI, parallel imaging (type not specified); G, GRAPPA; S, SENSE; CS, compressed sensing.

**N0:** sp, number of MEGs is split between pre- and post-echo-pulse in SE-MRE. MEG shape: T, bipolar trapezoidal; S, sinusoidal; FC, flow-compensated.

**Encoding directions:** M, measurement; P, phase; S, slice; R, reference; X, Y, Z, specific mentioning of main gradient axes.

**Encoding scheme:** 2-point, two-point difference; 4-unb., unbalanced four-point; 3-noref, three point encoding without reference phase correction.
6.17 Calculated and if needed optimized to achieve a certain diffusion weighting without destroying the phase

repulsion algorithm\textsuperscript{135} and then clustered into four groups corresponding to four mechanical phase offsets. All diffusion directions are then

corresponding to DWI, MRE, and\textsuperscript{133} it makes use of the underlying Brownian motion of water self-diffusion by switching motion sensitizing gradients. These gradient fields cause intra-voxel dephasing of magnetization vectors in the presence of random motion of isochromats, leading to signal magnitude decay. The signal loss due to diffusion can be approximated by

\begin{align}
S &= S_0 e^{-bD},
\end{align}

where \(D\) is the local diffusion constant and \(b\) the "b-value" of the applied gradients \(G(t)\textsuperscript{122}\)

\begin{align}
b &= \gamma^2 \int_0^T (f_0(t)G(t))^2 dt.
\end{align}

Acquisition of a single diffusion-weighted image leads to local contrast changes due to the underlying diffusion mechanism. Acquisition of multiple images with different \(b\)-weightings allows for the quantification of the local diffusion coefficient. This coefficient is however also influenced by other physiological effects such as perfusion, and thus is usually referred to as the apparent diffusion coefficient (ADC)\textsuperscript{122}.

Since diffusion is not generally isotropic, anisotropic diffusion, e.g. along brain or muscle fibers, can be encountered. In the case of anisotropic diffusion, Equation 48 is generalized to

\begin{align}
S &= S_0 \exp \left(-\gamma^2 \int_0^T f_0(t)\left(\vec{\nabla} (f_0(t)\vec{G}(t))\right)^T \vec{D} \left(\vec{f}_0(t)\vec{G}(t)\right)\right).
\end{align}

where \(\vec{D}\) is the diffusion tensor and \(\vec{G}\) the gradient waveform used for diffusion weighting\textsuperscript{122}. Because of the symmetry properties of the diffusion tensor, a total of six non-parallel gradient directions suffices to encode the full diffusion tensor, while the acquisition of at least 30 directions has become the standard to estimate complex intra-voxel anisotropy\textsuperscript{134}.

Already in the early stages of MRE, reconstruction of the stiffness tensor was proposed\textsuperscript{57}. To help the reconstruction of anisotropic stiffness properties, the combination of diffusion tensor imaging (DTI) and elastography was used\textsuperscript{17,99}. However, in these studies, separate scans for DTI and MRE datasets were performed. Since MRE uses the phase-contrast principle with strong gradients for the displacement detection, diffusion weighting can be simultaneously encoded in the magnitude of the complex signal (diffusion MRE, dMRE), leading to co-registered datasets\textsuperscript{74}. The acquisition of diffusion-weighted images together with MRE is straightforward, since only the \(b\)-value of the applied MEGs needs to be calculated and if needed optimized to achieve a certain diffusion weighting without destroying the phase-locked nature of the MEGs in MRE\textsuperscript{69}.

A comparison of conventional DWI, MRE, and dMRE is shown in Figure 17.

For the acquisition of simultaneous DTI and MRE, 12 gradient directions are chosen, uniformly distributed in 3D, according to an electrostatic repulsion algorithm\textsuperscript{135} and then clustered into four groups corresponding to four mechanical phase offsets. All diffusion directions are then acquired using the same gradient waveform, but with varying mechanical phase offsets. In this way, a full three-directional MRE dataset as well as the full diffusion tensor can be encoded\textsuperscript{74}. Drawbacks of the dMRE and DTI-MRE are local noise enhancement due to strong diffusion weighting (\(PNR = \text{SNR} = e^{-2bD}\)) as well as intra-voxel phase dispersion. The latter is a result of phase variations within a voxel in an MRE experiment leading to
signal loss and hence the possibility of biasing dMRE and DTI-MRE results. However, efforts to increase spatial resolution in MRE will work in favor of dMRE and DTI-MRE as IVPD-related signal loss is reduced.

8 | SUMMARY

MRE encoding and readout strategies have evolved significantly over the last two decades. In this review an attempt has been made to present wave-encoding and readout approaches in a unified mathematical way to allow for a comparison of the wave-encoding and SNR efficiency of the various methods described in the literature. Beyond established encoding schemes including full- and fractional-wave encoding, emerging techniques such as SLIM and multi-shot encoding have been included.

Synchronization of the wave-encoding gradients with the frequency of the mechanical wave to be imaged sets bounds on the readout duration. In this context, properties of fast readout techniques have been summarized. For the acquisition of high-resolution, volumetric, three-directional MRE datasets, highly accelerated MRE readouts are necessary, which can be achieved through the synergistic combination of existing fast readout, parallel-imaging, and sub-Nyquist methods, which have been summarized too.

The generic treatment of encoding gradient design and efficiency as well as SNR efficiency of the sequence as described in the present work allows optimization of standard and advanced MRE sequences within the gradient performance limits of a given MR system. Accordingly, efforts thereof may prove important to advance the robustness and, ultimately, the application of MRE.

While the focus of this review has been on sequence design and properties, MR system imperfections, the impact of involuntary subject motion and other image artifacts have not been treated. It is noted that the requirement of strong encoding gradients in MRE may accentuate imperfections of the gradient chain including eddy currents and temperature-induced drifts, which in turn can compromise the accuracy and precision of MRE encoding and readout. Further advances in MR system performance control and correction as well as motion compensation techniques are expected to address these issues. In this relation, no specific mention of magnetic field strength has been made either, since most work in MRE has been conducted on clinical MR systems operating at 1.5 and 3.0 T. The SNR benefit of higher static fields is intrinsically linked to increased field inhomogeneities, which limit longer and hence more time-efficient readouts. Future work is required to demonstrate potential benefits of using static fields beyond 3.0 T for MRE.

ACKNOWLEDGEMENT

This project has received funding from the European Union’s Horizon 2020 Research and Innovation Programme under Grant Agreement 668039.

ORCID

Christian Guenthner @ http://orcid.org/0000-0001-8707-7016
Sebastian Kozerke @ http://orcid.org/0000-0003-3725-8884

REFERENCES

1. Muthupillai R, Lomas D, Rossman P, Greenleaf J, Manduca A, Ehman R. Magnetic resonance elastography by direct visualization of propagating acoustic strain waves. Science (80-. ). 1995;269(5232):1854-1857. https://doi.org/10.1126/science.7569924

2. Yin M, Talwalkar JA, Glaser KJ, et al. Assessment of Hepatic Fibrosis With Magnetic Resonance Elastography. Clin Gastroenterol Hepatol. 2007;5(10):1207-1213.e2. https://doi.org/10.1016/j.cgh.2007.06.012

3. Rouvière O, Yin M, Dresner MA, et al. MR Elastography of the Liver: Preliminary Results. Radiology. 2006;240(2):440-448. https://doi.org/10.1148/radiol.2402050606
16. Murphy MC, Huston J, Jack CR, et al. Decreased brain stiffness in Alzheimer's disease determined by magnetic resonance elastography. Eur Radiol. 2008;18(11):2535-2541. https://doi.org/10.1002/erm2.1051-5

17. Singh S, Venkatesh SK, Wang Z, et al. Diagnostic Performance of Magnetic Resonance Elastography in Staging Liver Fibrosis: A Systematic Review and Meta-analysis of Individual Participant Data. Clin Gastroenterol Hepatol. 2015;13(3):440-451.e6. https://doi.org/10.1016/j.cgh.2014.09.046

18. Singh S, Venkatesh SK, Loomba R, et al. Magnetic resonance elastography for staging liver fibrosis in non-alcoholic fatty liver disease: a diagnostic accuracy systematic review and individual participant data pooled analysis. Eur Radiol. 2016;26(5):1431-1440. https://doi.org/10.1007/s00330-015-3949-z

19. Low G. General review of magnetic resonance elastography. J Radiol. 2016;8(1):59. https://doi.org/10.4329/wjr.v8i1.59

20. Talwalkar JA. Elastography for Detecting Hepatic Fibrosis: Options and Considerations. Gastroenterology. 2008;135(1):299-302. https://doi.org/10.1053/j.gastro.2008.05.038

21. Talwalkar JA, Yin Y, Han JC, Xi ZN, Shen H, Gao PY. Magnetic Resonance Elastography of Brain Tumors: Preliminary Results. Acta Radiol. 2011;52(3):327-330. https://doi.org/10.1080/02841850.2010.480044

22. Sahebjavaher RS, Baghani A, Honarvar M, Sinkus R, Salcudean SE. Transperineal prostate MR elastography: Initial in vivo results. Magn Reson Med. 2013;69(2):411-420. https://doi.org/10.1002/mrm.24268

23. Arani A, Da Rosa M, Ramsay E, Plewes DB, Haider MA, Chopra R. Incorporating endorectal MR elastography into multi-frequency Dual Parameter Reconstruction. NMR Biomed. 2013;26(5):1431-1440. https://doi.org/10.1002/nbm.3118

24. Talwalkar JA, Yin Y, Xi ZN, Shen H, Gao PY. Magnetic Resonance Elastography of Brain Tumors: Preliminary Results. Acta Radiol. 2007;48(1):112-115. https://doi.org/10.1080/02841850.2006.400012

25. Talwalkar JA, Yin M, Venkatesh S, et al. Feasibility of In Vivo MR Elastographic Splenic Stiffness Measurements in the Assessment of Portal Hypertension. J Magn Reson Imaging. 2013;46(5):863-870. https://doi.org/10.1016/j.jmri.2012.12.024
35. Low G, Owen NE, Joubert I, et al. Reliability of magnetic resonance elastography using multislice two-dimensional spin-echo echo-planar imaging (SE-EPI) and three-dimensional inversion reconstruction for assessing renal stiffness. J Magn Reson Imaging. 2015;42(3):844-850. https://doi.org/10.1002/jmri.24826

36. Sack I, Rump J, Elgeti T, Samani A, Braun J. MR elastography of the human heart: Noninvasive assessment of myocardial elasticity changes by shear wave amplitude variations. Magn Reson Med. 2009;61(3):668-677. https://doi.org/10.1002/mrm.21878

37. Rump J, Klatt D, Braun J, et al. Fast planar steady-state free precession MR elastography on human liver. Proc Int Soc Magn Reson Med. 2006;14:399.

38. Rump J, Klatt D, Braun J, Warmuth C, Sack I. Fractional encoding of harmonic motions in MR elastography. Magn Reson Med. 2007;57(2):388-395. https://doi.org/10.1002/mrm.21152

39. Robert B, Sinkus R, Gennisson JL, Fink M. Application of DENSE-MR-elastography to the human heart. Magn Reson Med. 2009;62(5):1155-1163. https://doi.org/10.1002/mrm.22124

40. Kollipaka A, McGee KP, Araoz PA, et al. MR elastography as a method for the assessment of myocardial stiffness: Comparison with an established pressure-volume model in a left ventricular model of the heart. Magn Reson Med. 2012;67(1):120-127. https://doi.org/10.1002/mrm.22361

41. Kollipaka A, Aggarwal SR, McGee KP, et al. Magnetic resonance elastography as a method to estimate myocardial contractility. J Magn Reson Imaging. 2012;36(1):120-127. https://doi.org/10.1002/mrm.22191

42. Manduca A, Ophir J, Dersner MA, et al. Magnetic resonance elastography: Non-invasive mapping of tissue elasticity. Med Image Anal. 2001;5(4):237-254. https://doi.org/10.1016/S1361-8415(00)00039-6

43. Conturo TE, Smith GD. Signal-to-noise in phase angle reconstruction: Dynamic range extension using phase reference offsets. Magn Reson Med. 1990;15(3):420-437. https://doi.org/10.1002/jmri.1880010620

44. Bernstein MA, Ikezaki Y. Comparison of phase difference and complex difference processing in phase-contrast MR angiography. J Magn Reson Imaging. 1991;1(6):725-732. https://doi.org/10.1002/jmri.18901.1.6.1664.000038

45. Venkatesh SK, Yin M, Ehman RL. Magnetic resonance elastography of liver: Technique, analysis, and clinical applications. J Magn Reson Imaging. 2013;37(3):544-555. https://doi.org/10.1002/jmri.23731

46. Pepin KM, Ehman RL, McGee KP. Magnetic resonance elastography (MRE) in cancer: Technique, analysis, and applications. Prog Nucl Magn Reson Spectrosc. 2015;90-91:32-48. https://doi.org/10.1016/j.pmrms.2015.06.001

47. Moran PR. A flow velocity zeugmatographic interlace for NMR imaging in humans. Magn Reson Imaging. 1982;1(4):197-203. https://doi.org/10.1016/0730-725X(82)90170-9

48. Muthupillai R, Ehman RL. Magnetic resonance elastography. Nat Med. 1996;2(5):601-603. https://doi.org/10.1038/nm0596-601

49. Bronstein IN, Semendyayev KA, Musiol G, Muehlig H. Handbook of Mathematics. Berlin, Heidelberg: Springer Berlin Heidelberg; 2007. https://doi.org/10.1007/978-3-540-27212-2

50. Bernstein MA, Ikezaki Y. Comparison of phase difference and complex difference processing in phase-contrast MR angiography. J Magn Reson Imaging. 1991;1(6):725-729. https://doi.org/10.1002/jmri.1880010620

51. Conturo TE, Smith GD. Signal-to-noise in phase angle reconstruction: Dynamic range extension using phase reference offsets. Magn Reson Med. 1990;15(3):420-437. https://doi.org/10.1002/jmri.18901.1.6.1664.000038

52. Sack I, Rump J, Elgeti T, Samani A, Braun J, Warmuth C, Sack I. Fractional encoding of harmonic motions in MR elastography. Magn Reson Med. 2007;57(2):388-395. https://doi.org/10.1002/mrm.21152

53. Oppelt A, Graumann R, Barfuss H, Fischer H, Hartl W, Schajow W, FISP – a new fast MRI sequence. Electromedica. 1986;54:15-18. http://mri-q.com/uploads/3/7/7/4/3274160/oppe A.t.pdf

54. Bieri O, Maderwald S, Ladd ME, Scheffler K. Balanced alternating steady-state elastography. Magn Reson Med. 2006;55(2):233-241. https://doi.org/10.1002/mrm.20812

55. Muthupillai R, Rossman PJ, Lomas DJ, Greenleaf JF, Riederer SJ, Ehman RL. Magnetic resonance imaging of transverse acoustic strain waves. Magn Reson Med. 2006;55(6):1381-1389. https://doi.org/10.1002/mrm.20913

56. Wurtz H, Muthupillai R, Rossman PJ, Lomas DJ, Greenleaf JF, Riederer SJ, Ehman RL. Magnetic resonance imaging of transverse acoustic strain waves. Magn Reson Med. 2006;55(6):1381-1389. https://doi.org/10.1002/mrm.20913

57. Kruse SA, Rose GH, Glaser KJ, et al. Magnetic resonance elastography of the brain. Neuroimage. 2008;39(1):231-237. https://doi.org/10.1016/j.neuroimage.2007.08.030

58. Sack I, Beierbach B, Hamhaber U, Klatt D, Braun J. Non-invasive measurement of brain viscoelasticity using magnetic resonance elastography. NMR Biomed. 2008;21(3):265-271. https://doi.org/10.1002/nbm.1189
66. Riek K, Klatt D, Nuzha H, et al. Wide-range dynamic magnetic resonance elastography. J Biomech. 2011;44(7):1380-1386. https://doi.org/10.1016/j.jbiomech.2010.12.031

67. Klatt D, Yasar TK, Royston TJ, Magin RL. Sample interval modulation for the simultaneous acquisition of displacement vector data in magnetic resonance elastography: theory and application. Phys Med Biol. 2013;58(24):8663-8675. https://doi.org/10.1088/0031-9155/58/24/8663

68. Johnson CL, McGarry MDJ, Van Houten EEW, et al. Magnetic resonance elastography of the brain using multishot spiral readouts with self-navigated motion correction. Magn Reson Med. 2013;70(2):404-412. https://doi.org/10.1002/mrm.24473

69. Yin Z, Magin RL, Klatt D. Simultaneous MR elastography and diffusion acquisitions: Diffusion-MRE (dMRE). Magn Reson Med. 2014;71(5):1682-1688. https://doi.org/10.1002/mrm.25180

70. Johnson CL, Holtrop JL, McGarry MDJ, et al. 3D multislab, multishot acquisition for fast, whole-brain MR elastography with high signal-to-noise efficiency. Magn Reson Med. 2014;71(2):477-485. https://doi.org/10.1002/mrm.25065

71. Klatt D, Johnson CL, Magin RL. Simultaneous, multidirectional acquisition of displacement fields in magnetic resonance elastography of the in vivo human brain. J Magn Reson Imaging. 2015;42(2):297-304. https://doi.org/10.1002/jmri.24806

72. Numano T, Mizuhara K, Hata J, Washio T, Homma K. A simple method for MR elastography: A gradient-echo type multi-echo sequence. Magn Reson Imaging. 2015;33(1):31-37. https://doi.org/10.1016/j.mri.2014.10.002

73. Anderson AT, Van Houten EEW, McGarry MDJ, et al. Observation of directional-dependent mechanical properties in the human brain with multi-excitation MR elastography. J Mech Behav Biomed Mater. 2016;59:538-546. https://doi.org/10.1016/j.jmbbm.2016.03.005

74. Yin Z, Kearney SP, Magin RL, Klatt D. Concurrent 3D acquisition of diffusion tensor imaging and magnetic resonance elastography displacement data (DTI-MRE): Theory and in vivo application. Magn Reson Med. 2017;77(1):273-284. https://doi.org/10.1002/mrm.26121

75. Sack I, Mcgowan CK, Samani A, Luginbuhl C, Oakden W, Plewes DB. Observation of nonlinear shear wave propagation using magnetic resonance elastography. Magn Reson Med. 2004;52(4):842-850. https://doi.org/10.1002/mrm.20238

76. Klatt D, Asbach P, Rump J, et al. In Vivo Determination of Hepatic Stiffness Using Steady-State Free Precession Magnetic Resonance Elastography. Invest Radiol. 2006;41(12):841-848. https://doi.org/10.1097/01.rdi.0000037046.16303.8f

77. Klatt D, Hamhaber U, Asbach P, Braun J, Sack I. Noninvasive assessment of the rheological behavior of human organs using multifrequency MR elastography: a study of brain and liver viscoelasticity. Phys Med Biol. 2007;52(24):7281-7294. https://doi.org/10.1088/0031-9155/52/24/006

78. Asbach P, Klatt D, Hamhaber U, et al. Assessment of liver viscoelasticity using multifrequency MR elastography. Magn Reson Med. 2008;60(2):373-379. https://doi.org/10.1002/mrm.21636

79. Garteiser P, Sahebjavaher RS, Ter Beek LC, et al. Rapid acquisition of multifrequency, multislice and multidirectional MR elastography data with a fractionally encoded gradient echo sequence. NMR Biomed. 2013;26(10):1326-1335. https://doi.org/10.1002/nbm.2998

80. Braun J, Guo J, Lützkendorf R, et al. High-resolution mechanical imaging of the human brain by three-dimensional multifrequency magnetic resonance elastography at 7T. Neuroimage. 2014;90:308-318. https://doi.org/10.1016/j.neuroimage.2013.12.032

81. Corbin N, Vappoo J, Breton E, et al. Interventional MR elastography for MRI-guided percutaneous procedures. Magn Reson Med. 2016;75(3):1110-1118. https://doi.org/10.1002/mrm.25694

82. Sack I, Gedat E, Bernarding J, Buntkowsky G, Braun J. Magnetic resonance elastography and diffusion-weighted imaging of the sol/gel phase transition in agarose. J Magn Reson. 2004;166(2):252-261. https://doi.org/10.1016/j.jmr.2003.10.020

83. Yasar TK, Klatt D, Magin RL, Royston TJ. Selective spectral displacement projection for multifrequency MRE. Phys Biol. 2013;58(16):5771-5781. https://doi.org/10.1088/0031-9155/58/16/5771

84. Nordal I, Vappoo J, Breton E, et al. Interventional MR elastography for MRI-guided percutaneous procedures. Magn Reson Med. 2016;75(3):1110-1118. https://doi.org/10.1002/mrm.25694

85. Sack I, Gedat E, Bernarding J, Buntkowsky G, Braun J. Magnetic resonance elastography and diffusion-weighted imaging of the sol/gel phase transition in agarose. J Magn Reson. 2004;166(2):252-261. https://doi.org/10.1016/j.jmr.2003.10.020

86. Yasar TK, Klatt D, Magin RL, Royston TJ. Selective spectral displacement projection for multifrequency MRE. Phys Biol. 2013;58(16):5771-5781. https://doi.org/10.1088/0031-9155/58/16/5771

87. Hirsch S, Braun J, Sack I. Magnetic Resonance Elastography: Physical Background And Medical Applications. Weinheim, Germany: Wiley-VCH Verlag GmbH & Co. KGaA; 2016. https://doi.org/10.1002/9783527696017.

88. Oliphant TE, Manduca A, Ehman RL, Greenleaf JF. Complex-Valued Stiffness Reconstruction for Magnetic Differential Equation. Magn Reson Med. 2001;45(September 2000):299-310.

89. Bamber J, Kennedy P, Johnson CL, Mada M, Roberts N. Real-time 4D phase unwrapping applied to magnetic resonance elastography. Magn Reson Med. 2015;73(6):2321-2331. https://doi.org/10.1002/mrm.25332

90. Goldstein RM, Zebker HA, Werner CL. Satellite radar interferometry: Two-dimensional phase unwrapping. Radio Sci. 1988;23(4):713-720. https://doi.org/10.1029/RS023i004p00713

91. Glaser KJ, Felmlee JP, Manduca A, Ehman RL. Shear stiffness estimation using intravoxel phase dispersion in magnetic resonance elastography. Magn Reson Med. 2003;50(6):1256-1265. https://doi.org/10.1002/mrm.10641

92. Hirsch S, Klatt D, Freimann F, Scheel M, Braun J, Sack I. In vivo measurement of volumetric strain in the human brain induced by arterial pulsation and harmonic waves. Magn Reson Med. 2013;70(3):671-682. https://doi.org/10.1002/mrm.24499

93. Yasar TK, Royston TJ, Magin RL. Wideband MR elastography for viscoelasticity model identification. Magn Reson Med. 2013;70(2):479-489. https://doi.org/10.1002/mrm.24495

94. Papazoglou S, Xu C, Hamhaber U, et al. Scatter-based magnetic resonance elastography. Phys Med Biol. 2009;54(7):2229-2241. https://doi.org/10.1088/0031-9155/54/7/025

95. Fehlner A, Papazoglou S, McGarry MD, et al. Cerebral multifrequency MR elastography by remote excitation of intracranial shear waves. NMR Biomed. 2015;28(11):1426-1432. https://doi.org/10.1002/nbm.3388
97. Costantini M. A novel phase unwrapping method based on network programming. IEEE Trans Geosci Remote Sens. 1998;36(3):813-821. https://doi.org/10.1109/36.763674
98. Qin EC, Sinkus R, Geng G, et al. Combining MR elastography and diffusion tensor imaging for the assessment of anisotropic mechanical properties: A phantom study. J Magn Reson Imaging. 2013;37(1):217-226. https://doi.org/10.1002/jmri.23797
99. Wang H, Weaver JB, Perreard II, Doyley MM, Paulsen KD. A three-dimensional quality-guided phase unwrapping method for MR elastography. Phys Med Biol. 2011;56(13):3935-3952. https://doi.org/10.1088/0031-9155/56/13/012
100. Petrov AY, Doherty PD, Sellier M, Chase JG. Multi-frequency inversion in Rayleigh damped Magnetic Resonance Elastography. Biomed Signal Process Control. 2014;13:270-281. https://doi.org/10.1016/j.bspc.2014.04.006
101. Spottiswoode BS, Zhong X, Hess AT, et al. Tracking myocardial motion from cine DENSE images using spatiotemporal phase unwrapping and temporal fitting. IEEE Trans Med Imaging. 2007;26(1):15-30. https://doi.org/10.1109/TMI.2006.884215
102. Mansfield P. Multi-planar image formation using NMR spin echoes. J Phys C Solid State Phys. 1977;10(3):L55-L58. https://doi.org/10.1088/0022-3719/10/3/004
103. Le Bihan D, Poupon C, Amadon A, Lethimonnier F. Artifacts and pitfalls in diffusion MRI. J Magn Reson Imaging. 2006;24(3):478-488. https://doi.org/10.1002/jmri.20683
104. Ahn CB, Kim JH, Cho ZH. High-Speed Spiral-Scan Echo Planar NMR Imaging-I. IEEE Trans Med Imaging. 1986;5(1):2-7. https://doi.org/10.1109/TMI.1986.4307732
105. Delattre BMA, Heidemann RM, Crowe LA, Vallée J-P, Hyacinthe J-N. Spiral demystified. Magn Reson Imaging. 2010;28(6):862-881. https://doi.org/10.1016/j.mri.2010.03.036
106. Liu C, Bamber R, Kim DH, Moseley ME. Self-navigated interleaved spiral (SNAILS): Application to high-resolution diffusion tensor imaging. Magn Reson Med. 2004;52(6):1388-1396. https://doi.org/10.1002/mrm.20288
107. Liao J-R, Pauly JM, Brosnan TJ, Pelc NJ. Reduction of motion artifacts in cine MRI using variable-density spiral trajectories. Magn Reson Med. 1997;37(4):569-575. https://doi.org/10.1002/mrm.1910370416
108. Kim D, Adalsteinsson E, Spielman DM. Simple analytic variable density spiral design. Magn Reson Med. 2003;50(1):214-219. https://doi.org/10.1002/mrm.10493
109. Jackson JI, Meyer CH, Nishimura DG, Macovski A. Selection of a convolution function for Fourier inversion using gridding. IEEE Trans Med Imaging. 1986;10(3):473-478.
110. O’Sullivan JD. A Fast Sinc Function Gridding Algorithm for Fourier Inversion in Computer Tomography. IEEE Trans Med Imaging. 1985;4(4):200-207. https://doi.org/10.1109/TMI.1985.4307723
111. Man L-C, Pauly JM, Macovski A. Multifrequency interpolation for fast off-resonance correction. Magn Reson Med. 1997;37(5):785-792. https://doi.org/10.1002/mrm.1910370523
112. Vannesjo SJ, Graedel NN, Kasper L, et al. Image reconstruction using a gradient impulse response model for trajectory prediction. Magn Reson Med. 2016;76(4):45-56. https://doi.org/10.1002/mrm.25041
113. King KF, Ganin A, Zhou XJ, Bernstein MA. Concomitant gradient field effects in spiral scans. Magn Reson Med. 1999;41(1):103-112. https://doi.org/10.1002/(SICI)1522-2594(199901)41:1<103::AID-MRM15>3.0.CO;2-M
114. Maeda A, Sano K, Yokoyama T. Reconstruction by Weighted Correlation for MRI with Time-Varying Gradients. IEEE Trans Med Imaging. 1988;7(1):26-31. https://doi.org/10.1109/42.39296
115. Pauly J, Nishimura D, Macovski A. A k-space analysis of small-tip-angle excitation. J Magn Reson. 1989;81(1):43-56. https://doi.org/10.1016/0278-0136(89)90265-5
116. Börnert P, Alfeld B. On spatially selective RF excitation and its analogy with spiral MR image acquisition. Magn Reson Mater Physics, Biol Med. 1998;7(3):166-178. https://doi.org/10.1016/S1352-8861(98)00077-5
117. Hardie C, Cline HE. Broadband nuclear magnetic resonance pulses with two-dimensional spatial selectivity. J Appl Phys. 1989;66(4):1513-1516. https://doi.org/10.1063/1.34411
118. Yuan L, Glaser KJ, Rouviere O, et al. Preliminary assessment of one-dimensional MR elastography for use in monitoring focused ultrasound therapy. Phys Med Biol. 2007;52(19):5909-5919. https://doi.org/10.1088/0031-9155/52/19/012
119. Bensamoun SF, Glaser KJ, Ringleb SJ, Chen Q, Ehman RL, An K-N. Rapid magnetic resonance elastography of muscle using one-dimensional projection. J Magn Reson Imaging. 2008;27(5):1083-1088. https://doi.org/10.1002/jmri.21307
120. Van Houten EEW, Miga MI, Weaver JB, Kennedy FE, Paulsen KD. Three-dimensional subzone-based reconstruction algorithm for MR elastography. Magn Reson Med. 2001;45(5):827-837. https://doi.org/10.1002/mrm.1112
121. Bernstein MA, King KF, Zhou XJ. Handbook of MRI Pulse Sequences. Amsterdam: Elsevier Academic Press; 2004.
122. Edelstein WA, Glover GH, Hardy CJ, Redington RW. The intrinsic signal-to-noise ratio in NMR Imaging. Magn Reson Med. 1986;3(4):604-618. https://doi.org/10.1002/1050-1252(1986)3:4<604::AID-MRM3>3.0.CO;2-T
123. Salameh N, Sarracanie M, Armstrong BD, Rosen MS, Comment A. Overhauser-enhanced magnetic resonance elastography. NMR Biomed. 2016;29(5):607-613. https://doi.org/10.1002/nbm.3499
124. McGibney G, Smith MR, Nichols ST, Crawley A. Quantitative evaluation of several partial fourier reconstruction algorithms used in mri. Magn Reson Med. 1993;30(1):51-59. https://doi.org/10.1002/mrm.1910300109
125. Prazekmann KP, Weiger M, Scheidegger MB, Boesiger P. SENSE: Sensitivity encoding for fast MRI. Magn Reson Med. 1999;42(5):952-962. https://doi.org/10.1002/(SICI)1522-2594(199911)42:5<952::AID-MRM15>3.0.CO;2-X
126. Griswold MA, Jakob PM, Heidemann RM, et al. Generalized Autocalibrating Partially Parallel Acquisitions (GRAPPA). Magn Reson Med. 2002;47(6):1202-1210. https://doi.org/10.1002/mrm.10171
128. Pruessmann KP, Weiger M, Börnert P, Boesiger P. Advances in sensitivity encoding with arbitrary k-space trajectories. Magn Reson Med. 2001;46(4):638-651. https://doi.org/10.1002/mrm.1241

129. Breuer FA, Kannengiesser SAR, Blaimer M, Seiberlich N, Jakob PM, Griswold MA. General formulation for quantitative G-factor calculation in GRAPPA reconstructions. Magn Reson Med. 2009;62(3):739-746. https://doi.org/10.1002/mrm.22066

130. Donoho DL. Compressed sensing. IEEE Trans Inf Theory. 2006;52(4):1289-1306. https://doi.org/10.1109/TIT.2006.871582

131. Lustig M, Donoho D, Pauly JM. Sparse MRI: The application of compressed sensing for rapid MR imaging. Magn Reson Med. 2007;58(6):1182-1195. https://doi.org/10.1002/mrm.21391

132. Sarracanie M, Armstrong BD, Stockmann J, Rosen MS. High speed 3D overhauser-enhanced MRI using combined b-SSFP and compressed sensing. Magn Reson Med. 2014;71(2):735-745. https://doi.org/10.1002/mrm.24705

133. Beauchamp NJ, Ulug AM, Passe TJ, van Zijl PC. MR diffusion imaging in stroke: Review and controversies. Radiographics. 1998;18(5):1269-1283. https://doi.org/10.1148/radiographics.18.5.9747619

134. Jones DK. The Effect of Gradient Sampling Schemes on Measures Derived from Diffusion Tensor MRI: A Monte Carlo Study. Magn Reson Med. 2004;51(4):807-815. https://doi.org/10.1002/mrm.20033

135. Jones DK, Horsfield MA, Simmons A. Optimal strategies for measuring diffusion in anisotropic systems by magnetic resonance imaging. Magn Reson Med. 1999;42(3):515-525. https://doi.org/10.1002/mrm.14

How to cite this article: Guenthner C, Kozerke S. Encoding and readout strategies in magnetic resonance elastography. NMR Biomedicine. 2018;31:e3919. https://doi.org/10.1002/nbm.3919