Experimental Study of Thermal Convection in a Cylindrical Layer with a Longitudinal Partition at Modulated Rotation

A A Vjatkin1, V G Kozlov1, R R Sabirov2

1Laboratory of Vibration Hydromechanics, Perm State Humanitarian Pedagogical University, Perm, 614015, Russia
2 Perm National Research Polytechnic University, Perm, 614990, Russia

E-mail: *vijatkin_aa@pspu.ru

Abstract. Thermal convection of a liquid in a simply connected horizontal cylindrical layer rotating at a variable velocity is investigated experimentally. The inner boundary of the layer has a higher temperature. The cavity rotates rapidly and the liquid is stably stratified in a centrifugal force field. The parameters of the experiment correspond to the conditions when inertial waves are not excited (the modulation frequency is more than twice the cavity rotation frequency). The excitation thresholds of thermal convection and the structure of convective flows are studied depending on the rotation velocity and the parameters of the rotational vibrations of the cavity. It is found that the thermal convection is excited in a threshold way and its excitation is not associated with the manifestation of the mechanism of pendulum vibrational convection. A system of toroidal rolls with a spatial period commensurate with the layer thickness is observed in the cavity.

1. Introduction
Thermal convection in rotating systems is an actual and important problem [1-3], which attracts the attention of researchers from different fields. At uniform rotation, a non-isothermal fluid can be in a state of mechanical equilibrium in a centrifugal force field [4]. In an axisymmetric formulation this state corresponds to a temperature distribution with a temperature gradient directed to the axis of rotation. Otherwise, an intense centrifugal thermal convection develops in the rotating cavity [5-7], and the action of the inertial forces determines a number of specific features of this convection [8].

It is known that fluid vibrations can be a source of steady streaming [9,10]. In the case of a non-isothermal fluid, the averaged flows excited by vibrations (oscillating force fields) are called "Vibrational thermal convection" [11]. The effects of the vibrational thermal convection are able to manifest themselves in a great number of technological processes and natural phenomena which makes the study of this convection mechanism interesting for both applied and fundamental science [12-15]. The conditions of reduced gravity are especially favorable for the development this type of convection as on the board of spacecraft performing the high frequency rotational oscillations in a near-earth orbit the vibrational mechanism of convection may easily become the predominant one.

Rotation changes vibrational thermal convection qualitatively. For example, rotational vibrations of a cavity with a non-isothermal fluid trigger a very specific mechanism of "Pendulum vibrational convection" [16], which comes to the fore under the microgravity conditions. On the other hand, even at a uniform rotation of a cavity with a non-isothermal fluid in a static force field directed normal to the axis of rotation an averaged convection is excited in the fluid [17]. In this case, in the cavity reference frame, the static external field is converted into an oscillating (rotating) force field, which causes oscillations of the liquid relative to the cavity, and, as a consequence, excites the averaged (vibrational) thermal convection. Such vibrational convection in rotating cavities of different configuration has been studied by a team of authors in recent years.
All the factors called above make it actual to study the influence of the rotation velocity modulation on thermal convection in rotating cavities. Vibrational thermal convection in the case of pendulum (rotational) oscillations of the cavity is an object of practical interest. The excitation of the pendulum vibration convection in a plane layer and in a cylindrical layer with a longitudinal partition is associated with the presence of a rotational component of the cavity oscillations. A study [16] has shown that this mechanism for convection excitation is highly effective.

The proposed work is devoted to the experimental study of thermal convection in a rotating horizontal cylindrical layer with a longitudinal partition performing high-frequency rotational vibrations.

2. Experimental facility and procedure
The cuvette is a system of coaxial cylinders: an aluminum heat exchanger 1 (figure 1) and Plexiglas transparent pipes 2 and 3. The outer cylindrical surface of the heat exchanger 1 with a radius \( r = 30 \text{ mm} \) forms the inner boundary of the working layer. To ensure a high contrast when observing the convective structures in the layer and to protect against corrosion, a black self-adhesive film with a thickness of 50 \( \mu \text{m} \) is applied to the aluminum surface. A uniform heating of the surface is carried out by an electric heater located in the inner cavity of the heat exchanger.

![Figure 1. Cavity scheme.](image)

The outer boundary of the working layer is formed by the transparent Plexiglas tube 2 (figure 1) with an inner radius \( R = 37 \text{ mm} \) and a wall thickness 3 mm. A partition is installed along the entire layer, eliminating the liquid seepage. The formed simply connected layer has the following geometric parameters: a thickness \( h = R - r = 7 \text{ mm} \), a relative radius \( \rho = r/R = 0.81 \), an average radius \( R_0 = (R + r)/2 = 33.5 \text{ mm} \), a length \( L = 183 \text{ mm} \). The outer surface of the cylinder 2 is washed by a thermostated water that circulates in the gap between the pipes 2 and 3 cooling the outer cylindrical boundary of the working layer. The coolant temperature in all series of experiments is 20 °C. At the ends, the cavity is hermetically sealed with flanges that set the coaxial arrangement of the cylinders 1-3.

A temperature measurement is carried out using resistance thermometers and a measuring device «Termodat». The sensors are made of copper wire with a diameter 0.02 mm. The measure temperatures are \( T_{in} \) at the inner heat exchanger (figure 1), \( T_{ex} \) at the outer boundary of the working layer, and \( T_{sh} \) the one of the coolant. The sensor that measures the temperature \( T_{ex} \) evenly laid on a self-adhesive film with dimensions \( L \times 2h \) and glued to the outer boundary of the layer. This approach makes it possible to obtain an integral temperature characteristic regardless of the structure of convective flows. The measuring device «Termodat» rotates with the cuvette. The power supply of the device, data transmission and connection of the electric heater to the DC power supply are carried out using a multichannel electrical collector.

The cuvette is driven by a stepper motor FL86STH156, which is controlled by a driver SMD-78. The motor step control system make it possible to set the modulated rotation according to the law \( \Omega = \Omega_{rot}(1 + \varepsilon \sin(\Omega_{ab} t)) \), where \( \Omega_{rot} \) is the average angular velocity of the cavity rotation, \( \Omega_{ab} = f_{ab}/2\pi \).
– a cyclic frequency of the angular oscillations, \( \varepsilon = \varphi_0 \Omega_{\text{lib}} / \Omega_{\text{rot}} \) – a libration amplitude. The paper considers the case \( \Omega_{\text{lib}} \geq 2 \Omega_{\text{rot}} \).

The experiment begins with filling the working layer with distilled water. Any gas and solid inclusions are absent. The coolant is supplied to the outer shell. The heat power on the electric heater is set using a DC source Mastech HY3020 and equals 32 W. In all series of experiments, the heater power stay constant regardless of the convection mode. Thus, the temperatures of the boundaries of the layer are set. The cuvette is driven into a relatively rapid rotation with a velocity \( f_{\text{rot}} = \Omega_{\text{rot}} / 2 \pi = 2 \) or 3 rps. After establishing a time-independent temperature distribution in the working layer (not less than 60 min.) the modulation of the rotation velocity is set. The cyclic frequency of the angular oscillations \( \Omega_{\text{lib}} \) is constant in a series of experiments. Then the amplitude of the librations \( \varepsilon \) is increased stepwise. At each step the temperature measurements are performed after reaching the stationary convection mode. The temperature difference of the layer boundaries \( \Theta = T_{\text{in}} - T_{\text{ex}} \) and the temperature drop on the wall of the Plexiglas pipe \( \Delta T = T_{\text{ex}} - T_{\text{sh}} \), are determined. The experiments are carried out at different values of linear frequency of modulation \( f_{\text{lib}} \) and velocity of the cavity rotation \( f_{\text{rot}} \). To visualize the convective currents an aluminum powder with a small amount of surfactant is used.

3. Experimental results
In the case of relatively rapid rotation and temperature gradient directed towards the rotation axis the action of the centrifugal force of inertia set the state of mechanical quasi-equilibrium in the liquid layer; the Nusselt number \( Nu = 1 \).

![Figure 2](image1.png)  
*Figure 2. Dependence of the Nusselt number on the amplitude of the modulation of the rotational velocity.*

![Figure 3](image2.png)  
*Figure 3. Threshold curves for different velocities of rotation.*

The rotational vibrations with small amplitude are unable to break the quasi-equilibrium state of the liquid layer. The horizontal parts of the curves in figure 2 show that the heat transport through the layer stays unchanged and equal to the molecular one in the area of small amplitude. When the amplitude of the rotation velocity modulation reaches the critical value the heat transport through the layer begins to increase. In figure 2 there is a break on the heat transport curves which indicates the change in the flow regime. The heat transport through the layer increases due to the appearance of the system of the toroidal vortices periodic along the axis of rotation (figure 4). The size of the vortices is commensurate with the layer thickness. It is important to note that the found convective flow is significantly different from that observed in [16]. With a further increase in the amplitude of angular oscillations, the intensity of convective flows increases monotonically.
The threshold value of the amplitude $\varepsilon^*$ of the modulation of the rotation velocity changes non-monotonically with increasing libration frequency (figure 3). With increase in $f_{\text{lb}}$ the threshold value of $\varepsilon^*$ first decreases but then begins to increase. A non-monotonic view of the dependencies in figure 3 may indicate different physical mechanisms of convection excitation on the left and right parts of the V-shaped curves.

4. Discussion

The equations of pendulum thermal vibrational convection are obtained in [16] and considered the convection in a flat layer and a coaxial gap performing high-frequency translational-rotational oscillations is also considered here. When it comes to a cylindrical layer with a partition these equations are also can be used to describe the case of rotational oscillation on the background of the uniform rotation (modulated rotation). In contrast to [16] the liquid will also be affected by the centrifugal force of inertia playing the role of the static force field.

Thus, the averaged thermal convection in a simply connected rotating horizontal cylindrical layer performing high-frequency rotational oscillations of small amplitude is determined by the centrifugal Rayleigh number $R_{\text{CaR}}$, the vibrational parameter $R_v$ and the parameter $R_k$:

$$R_{\text{Ca}} = \frac{\Omega^2 R \beta \vartheta h^3}{v \chi}, \quad R_v = \frac{(\epsilon R \Omega \delta \vartheta h^3)^2}{2v \chi}, \quad R_k = \frac{((\epsilon \Omega)^2 R \beta \delta \vartheta h^3)^2}{2v \chi}$$

The dimensionless frequency in all experiments corresponds to the condition of high frequencies, $\omega = \Omega_n h^3 / \nu > 10^4$.

Let us consider the threshold curves (Figure 5) in the space of various control parameters. In Figure 5 a dashed line shows the excitation threshold of pendulum vibrational convection, obtained in [16]. The points on the graph are grouped according to the series of experiments carried out at different velocities of rotation. The range of parameters corresponds to large negative values of the Rayleigh centrifugal number – the fluid is in a state of stable stratification. According to the theory, in a thin cylindrical layer with a partition, the rotational vibrations lead to a renormalization of the static force field. The parameters of the performed experiment correspond to the condition $R_{\text{Ca}} / R_v \approx 100$. This explains the absence of the slope of the threshold curves corresponding to a certain rotation velocity.

The theoretical excitation threshold of the vibrational pendulum convection (figure 5) is located significantly higher than the experimental threshold curves. This indicates that the observed effect of excitation of intense convection at relatively weak rotational vibrations is associated with the manifestation of another mechanism.
Figure 5. Threshold points on the plane of parameters defining the pendulum vibrational thermal convection. Points 1 and 2 correspond to the designations in figure 3.

Figure 6. Thresholds of the averaged convection excited in a rotating horizontal cylindrical layer by the action of an external gravitational force field on the plane of governing parameters.

The arising question is whether the observed convective structures are related to the averaged convection generated by an external force field rotating in the cavity reference frame. A review of experimental and theoretical studies of this vibrational convection mechanism can be found in [17]. At rotation around a horizontal axis, in addition to the classical centrifugal Rayleigh number, the averaged convection of a non-isothermal fluid is determined by a modified vibration parameter 

\[ R_{vg} = \left( \frac{g \beta \Theta h}{2\nu \Omega r_{at}^2} \right) \cdot \left( R_{tr} + R_{t} \right) \]

It should be noted that this mechanism generates the averaged convection even in a stably stratified fluid, at negative values of the centrifugal Rayleigh number. The results of studying such convection in a rotating horizontal cylindrical liquid layer heated from the inside [17] are presented with a dashed curve in figure 6. Points 3 show the threshold values of the averaged convection excitation in a rotating layer with a partition in the absence of librations. The observations indicate that the excitation thresholds of the convection in a layer with a partition are in a qualitative agreement with those in a cylindrical layer without a partition [17]. At the same time, in the librating layer with a partition the critical values of the vibration parameter \( R_{vg} \) (points 1 and 2), turn out to be several orders of magnitude lower.

The performed analysis indicates that the observed effect of the excitation of intense convection of a non-isothermal liquid at a relatively weak libration forcing is not associated with the action of the listed mechanisms.

Let us dwell on the discussion of the nature of the threshold increase in heat transfer and analyze the oscillating motion of a fluid in an unevenly rotating cylindrical layer with a partition and boundaries of different temperatures.

A theoretical model [16] does not take the rotation into account, and the theoretical threshold curve in figure 5 is valid for two-dimensional perturbations in the form of rolls extended along the axis of rotation. In the experiments under consideration, the convective structures have the form of toroidal vortices, which can explain the discrepancy between the experimental results and theoretical predictions (figure 5) and the results of experiments with the pendulum vibrations in the absence of rotation.

As it has already been noted, the pendulum vibrational convection arising during combined (translational-rotational) oscillations is determined by the parameter \( R_{tr} \). The mechanism of convection is based on the interaction of the “isothermal” pulsating velocity field associated with the rotational...
component of the oscillations with the temperature field. The rotational vibration with an amplitude $\varphi_0$ excite the shear oscillations of the liquid in a thin cylindrical layer ($h \ll R_0$) with an angular amplitude $2\varphi_0$. The "isothermal" tangential oscillations of the liquid near the cavity wall occur with a significant amplitude $\varphi_0h$. The theoretical description in [16] is given in the approximation of high vibration frequencies, when the thickness of the Stokes layer relative to the layer thickness is considered negligible. The processes occurring in the viscous boundary layers are not considered in the theory.

The excitation of the three-dimensional convective vortices (figure 4) can be caused by the loss of stability of the viscous boundary layers. In the case of rotational vibrations against the background of uniform rotation, the boundary layers experience quasi-stationary instability, and three-dimensional structures (such as Görtler vortices) develop only on a fraction of the period when the instantaneous velocity profile in the boundary layer becomes unstable.

5. Conclusion

The excitation thresholds of averaged thermal convection in a rotating horizontal simply connected cylindrical. The excitation thresholds of thermal convection in a rotating horizontal cylindrical layer with a longitudinal partition under rotational vibrations were investigated experimentally. A layer of a fluid stably stratified in the field of centrifugal forces was considered (the inner boundary has a higher temperature). It was found that with an increase in the amplitude of oscillations, thermal convection is excited in a threshold manner in the form of a system of toroidal three-dimensional vortices, the size of which is commensurate with the layer thickness. The high efficiency of rotational vibrations was noted – the convection occurs at a relatively weak vibration action. The discovered effect is not described by the available theoretical models of vibrational thermal convection and requires further theoretical and experimental studies.

Acknowledgments

This work was supported by the Russian Science Foundation (project № 18-71-10053).

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