The stransverse mass, $M_{T2}$, in special cases

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Abstract: This document describes some special cases in which the stransverse mass, $M_{T2}$, may be calculated by non-iterative algorithms. The most notable special case is that in which the visible particles and the hypothesised invisible particles are massless – a situation relevant to its current usage in the Large Hadron Collider as a discovery variable, and a situation for which no analytic answer was previously known. We also derive an expression for $M_{T2}$ in another set of new (though arguably less interesting) special cases in which the missing transverse momentum must point parallel or anti parallel to the visible momentum sum. In addition, we find new derivations for already known $M_{T2}$ solutions in a manner that maintains manifest contralinear boost invariance throughout, providing new insights into old results. Along the way, we stumble across some unexpected results and make conjectures relating to geometric forms of $M_{eff}$ and $H_T$ and their relationship to $M_{T2}$.

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1 Introduction

The purpose of this note is to show how \( M_{T^2} \) [also known as the “transverse” mass] may be calculated by non-iterative algorithms in a small number of special cases, some of which are relevant to current usage patterns at the Large Hadron Collider.

There are at most ten people who will find this document interesting, and all of them would skip past any “motivating introductory waffle” if it were to be supplied. The remainder of the earth’s population will, quite rightly, find their local telephone directory a much better bed-time read, no matter how much effort I put in to motivating it. Insomniacs, journal referees, or those who are shocked to the core by the idea that a note may fail to motivate itself anywhere other than in the abstract or conclusions, may find in the Appendix some additional discussion of currency and relevance. However here in the introduction it would seem to make better sense to get straight down to business.

In Section 2 we describe our notation. In Section 3 we find the first non-iterative expression for \( M_{T^2} \) valid for the case where all particles are massless. At the end of that section we interpret this result and speculate on what it might be telling us. In Section 4 we start all over again, finding non-iterative expressions for \( M_{T^2} \) which are valid when the missing transverse momentum points in special directions. Many of the sub cases in Section 4 are new, and even where they are not, insight is provided by the new derivations.

On the applicability of the results herein.

It was noted in [2, 3] that the input momenta supplied to \( M_{T^2} \) may be grouped into two types – those that lead to “balanced” solutions, and those that lead to “unbalanced” solutions. See [3] for details of how these are defined. Both the general solution for \( M_{T^2} \) for “unbalanced” inputs, and the test that may be applied to inputs to determine whether or not they are “unbalanced”, are simple and have been known for some time. They may be found in [3]. In contrast, it is not expected that a general non-iterative algorithm for computing \( M_{T^2} \) for inputs that lead to “balanced” configurations exists. In-
indeed, from that statement derives part of the interest in the special cases considered herein for which non-iterative algorithms can be found.

Consequently, this note is only interested in determining and recording non-iterative solutions to MT2 for the case of inputs leading to “balanced” configurations. The unbalanced solution is already known in all generality. It will be assumed that all results in this note are taken to be “preceded” by a test for “unbalancedness” following [3], and that the results herein are only to be applied if that test fails.

On a separate matter we\(^1\) note that

\[
M_{T2}(a^\mu, b^\nu, \vec{\phi}) = M'_{CT2}(a^\mu, b^\nu, -\vec{\phi}) \tag{1.1}
\]

and so the results herein may trivially be turned into non-iterative expressions for \(M'_{CT2}\) [4] by changing the sign of the missing transverse momentum.

2 Notation

The transverse mass, \(M_{T2}\), [1] is\(^2\) the maximal lower bound on the mass of each member of a pair of identical parent particles which, if pair-produced at a hadron collider, could have each undergone a two-body decay into (i) a visible particle (or collection of particles) and (ii) an invisible object of hypothesised mass \(\chi\). The decay products of each parent are referred to as coming from different “sides” of the event. The momenta of the visible decay products of sides 1 and 2 will be referred to as \(a^\mu\) and \(b^\nu\) respectively. The momenta of the invisible daughters of sides 1 and 2 (or more usually the hypothesised momenta that they might take in some part of a calculation, since the true momenta are unknown) are referred to as \(p^\mu\) and \(q^\nu\) respectively. A consequence of the definition of \(M_{T2}\) (as a hadron collider variable) is that it is insensitive to the \(z\) components of the each of the visible momenta \(a^\mu\) and \(b^\nu\). It is only sensitive to the masses and transverse momenta of those objects. Accordingly, when performing our calculations we work exclusively in a Minkowski space of dimension 3=1+2 with signature \((+,-,-)\) rather than the usual 4=1+3 dimensions with signature \((+,-,-,-)\).\(^3\) Accordingly we view \(a^\mu\) as containing three components: \(a^\mu = (e_T, p_x, p_y) = (e_T, \mathbf{p})\) where \(\mathbf{p} = (p_x, p_y)\) and where \(p_x\) and \(p_y\) are the transverse components of \(a\)'s momentum, and \(e_T\) is defined by 

\[
e_T = \sqrt{m^2 + p_x^2 + p_y^2}
\]

in which \(m\) is the the “actual” (1+3 dimensional) mass of the particle. We denote the missing momentum 2-vector (another input to \(M_{T2}\)) as \(\mathbf{p} = (p_x, p_y)\). We denote by \(\chi\) the hypothesised mass of the species of invisible particle that was generated in the decay of each of the parents whose mass \(M_{T2}\) seeks to bound.

We use the usual Einstein summation convention and index raising and lowering notation to allow us to construct objects which are scalars with respect to “intrinsic” Lorentz transformations and rotations within our reduced \((+,-,-)\) transverse space. For example, if \(a^\mu = (e_T, \mathbf{p})\) then \(a_\mu = (e_T, -\mathbf{p})\) and \(a^\mu a_\mu = e_T^2 - |\mathbf{p}|^2\), and \(a^\mu b_\mu\) is a scalar in our reduced space.

Additionally, we will find it convenient to introduce an additional (non-standard) “bar notation” which is closely related to index raising and lowering. The “bar” creates a new Lorentz vector from an existing one, by reversal of the direction of its spatial component. Thus if \(a^\mu = (e_T, \mathbf{p})\) then \(\bar{a}^\mu = (e_T, -\mathbf{p})\). At first sight this may seem to be a backward step. Have we not already introduced index lowering? Surely \(\bar{a}^\mu = a_\mu\), what do we need the bar’s for? Hopefully the utility will become apparent in use. In short it is because “scalars” are not the only quantities we are interested in in the transverse plane, and we want the nature of our non-scalars to be evident without recourse to an abundance of indices. When constructing simple expressions for \(M_{T2}\) in special cases it is found often to be expedient to construct quantities which are not representations of the Lorentz group. Specifically we want to be able to construct quantities from pairs of Lorentz vectors which are not Lorentz scalars. One example is the quantity: \(E_T^2 E_T^2 + a_T \cdot b_T\). Although this

\(^1\)This was pointed out to me in the first instance by Chris Young.

\(^2\)See proof in [5].

\(^3\)Accordingly, unless explicitly stated otherwise, any references to “Lorentz vectors” or “boosts”, etc, must be assumed to be in the reduced transverse space with signature \((+,-,-)\).
quantity (which is, for historical reasons known as \( A_T \) and is closely related to \( M_{CT} \) [6]) is manifestly not invariant under Lorentz transformations, it is nonetheless invariant under simultaneously applied boosts of equal magnitude but opposite direction of the constituent vectors \( a^\mu \) and \( b^\mu \). The usefulness of such a quantity was first noted in [6], wherein such transformations were named “contralinear boosts”, and we can thus consider \( A_T \) to be a “contralinear boost invariant scalar” or “contra-scalar” for short. At first sight, we do not appear to need to introduce any special notation to describe quantities like \( A_T \). For example, the Einstein summation convention itself allows us to write \( A_T = a^\mu a^\mu \) or equivalently as \( A_T = a_\mu a_\mu \). However, as we will need to work with both scalars and contra-scalars, sometimes in the same expression at the time of writing this section, it is straightforward to see that this “trivial zero” of \( a_\mu a_\mu \) occurs when \( \hat{a} \cdot \hat{b} = 0 \) and \( \hat{a}_{\mu} \hat{a}_{\mu} = \chi^2 = 0 \).

### 3 \( M_{T2} \) in the fully massless case \((a^2 = b^2 = \chi^2 = 0)\).

In this section it is our intention to write down a non-iterative expression for \( M_{T2} \) in the “fully massless case” i.e. in the case in which the input particles and the invisible particles are taken to be massless \( a^2 = b^2 = \chi^2 = 0 \). To the best of our knowledge this solution has not been reported elsewhere. This “fully massless case” is the situation in which \( M_{T2} \) is most frequently used, including cases such as the dijet configuration used in LHC supersymmetry searches [7–10].

We begin by noting that if the vector \( \hat{p} \) happens to lie “between” \( \hat{a} \) and \( \hat{b} \) (i.e. if \( \hat{p} \) lies inside the smaller of the two sectors of the transverse plane bounded by \( \hat{a} \) and \( \hat{b} \)) then \( M_{T2} \) in the fully massless case must be identically zero. We call this a “trivial zero” of \( M_{T2} \) in the fully massless case. One may prove that such a trivial zero exists because the constraint \( \hat{p} + \hat{q} = \hat{p} \) can be solved by taking \( \hat{p} \propto \hat{a} \) and \( \hat{q} \propto \hat{b} \). This is a partition of the missing transverse momentum that assigns transverse masses of zero to both sides of the event. In the notation that will be introduced later in this section, it is straightforward to see that this “trivial zero” of \( M_{T2} \) in the fully massless case occurs when \( (\hat{a} \cdot \hat{p})(\hat{a} \cdot \hat{b}) \geq 0 \) and \( (\hat{b} \cdot \hat{p})(\hat{a} \cdot \hat{b}) \leq 0 \).

We now exclude this trivial zero, and instead consider what happens when \( \hat{p} \) does not lie between \( \hat{a} \) and \( \hat{b} \). Here we already know (see e.g. equation (50) of [2]) that \( \hat{p} \) the splitting hypothesis which leads to the minimal “balanced” configuration satisfies the following relationship in terms of “transverse velocities” \( \mathbf{v} = \hat{p}/c_T \)

\[
(v_{p} - v_{a}) (v_{q} - v_{b}) \leq 0\]  

\[
(\mathbf{v}_p - \mathbf{v}_a) \times (\mathbf{v}_q - \mathbf{v}_b) = 0 \quad (3.1)
\]

\footnote{Note that the letter \( A \) in \( A_T \) has no connection to the letter \( a \) in \( E^a \) or \( \alpha_T \).}

\footnote{The fully massless case would not be appropriate where the visible momenta on each “side” are compound objects with significant masses – such as when \( M_{T2} \) is used on dileptonic \( t\bar{t} \) events to measure the top mass [11, 12].}
where the proportional symbol means parallel, and (ii) that when both visible particles are massless, the splitting hypothesis which leads to the \textit{minimal} \( M_{T2} \) solution is a “balanced” configuration. In the massless case, the transverse velocities are all represented by 2-D unit vectors since \( e_T = |\mathbf{p}| \).

We will therefore re-write equation (3.1) as

\[
(\mathbf{p} - \mathbf{a}) \propto (\mathbf{q} - \mathbf{b}). \tag{3.2}
\]

A direct consequence of equation (3.2) is that the angle \textit{between} \( q \) and \( p \) at the \( M_{T2} \) solution is fixed to the same value as the angular separation between \( a \) and \( b \).\(^6\) One possible such arrangement is therefore \( \mathbf{p} = \mathbf{b} \) and \( \mathbf{q} = \mathbf{a} \). The general configuration allowed by equation (3.2) can thus be parametrised by rotating this particular solution by an arbitrary angle \( \theta \). In other words, we can parametrise \( \mathbf{p} \) and \( \mathbf{q} \) in the following way:

\[
\mathbf{p} = \cos \theta \mathbf{b} + \sin \theta \mathbf{b}' \tag{3.3}
\]

\[
\mathbf{q} = \cos \theta \mathbf{a} + \sin \theta \mathbf{a}' \tag{3.4}
\]

where we intend the two vector \( \mathbf{a}' \) to be obtained from the two vector \( \mathbf{a} \) by a rotation by +90 degrees in the transverse plane, and likewise \( \mathbf{b}' \) to be obtained from \( \mathbf{b} \) by the same rotation. In effect, all that remains to do is to find \( \theta \), \(|\mathbf{p}|\) and \(|\mathbf{q}|\) by imposing the remaining constraints, namely (i) that the configuration be “balanced”, i.e.

\[
2 (|a| |\mathbf{p}| - a \cdot \mathbf{p}) = 2 (|b| |\mathbf{q}| - b \cdot \mathbf{q}) \tag{3.5}
\]

and (ii) that the momentum splitting condition is satisfied:

\[
\mathbf{p} + \mathbf{q} = \mathbf{p}. \tag{3.6}
\]

Substitution of the parametrisation of (3.3) and (3.4) into (3.5) leads to the constraint:

\[
|a| |\mathbf{p}| \left( 1 - \cos \theta (\hat{a} \cdot \mathbf{b}) - \sin \theta (\hat{a} \cdot \mathbf{b}') \right) = |b| |\mathbf{q}| \left( 1 - \cos \theta (\hat{a} \cdot \mathbf{b}) + \sin \theta (\hat{a} \cdot \mathbf{b}') \right) \tag{3.7}
\]

while substitution into the splitting condition of (3.6) and taking the dot-product with \( \mathbf{p}' \) (a unit-two vector obtained by rotating \( \mathbf{p} \) by +90 degrees in the transverse plane) leads to the constraint:

\[
+ |\mathbf{p}| \left( \cos \theta (\mathbf{b} \cdot \mathbf{p}') + \sin \theta (\mathbf{b} \cdot \mathbf{p}) \right) = - |\mathbf{q}| \left( \cos \theta (\hat{a} \cdot \mathbf{p}') + \sin \theta (\hat{a} \cdot \mathbf{p}) \right). \tag{3.8}
\]

All dependence on \(|\mathbf{p}|\) and \(|\mathbf{q}|\) may then be eliminated by taking the quotient of the last two constraints (3.7) and (3.8), which results in a single constraint of the form

\[
K_{\alpha \alpha} \sin^2 \theta + K_{\alpha \beta} \cos^2 \theta + K_{\alpha \gamma} \cos \theta \sin \theta + K_{\beta \gamma} \sin \theta + K_{\beta \beta} \cos \theta + K_1 = 0. \tag{3.9}
\]

Expressions for the coefficients \( K_{\alpha \alpha}, K_{\alpha \beta}, K_{\alpha \gamma}, K_{\beta \gamma} \) and \( K_1 \) are listed later in equations (3.23) to (3.28). We note that the left hand side of equation (3.9) viewed as a function of \( \theta \), (i) is bounded, (ii) is real, (iii) is continuous with period \( 2\pi \), (iv) is not constant (except in degenerate cases which we will not consider), (v) has no Fourier components with period smaller than \( \pi \), and therefore (again excluding degenerate cases which an implementation would need to deal with) has either two real roots or four real roots.

One method whereby \( \theta \) may be determined from equation (3.9) is to replace \( \cos \theta \) with \( \pm \sqrt{1 - \sin^2 \theta} \) before then taking an appropriate square in order to remove the \( \sqrt{\cdots} \) resulting in a quartic polynomial in \( \sin \theta \) of the form shown later in equation (3.17) where, for simplicity, \( \sin \theta \) has been

\(^6\)Note the ordering is \( \theta_{qp} = -\theta_{pq} = \theta_{ab} = -\theta_{ba} \).
abbreviated as $s$. One must remember that in promoting equation (3.9) to a quartic we have introduced spurious solutions — effectively those that have the “wrong” sign for $\cos \theta$, i.e. a sign which is incompatible with equation (3.9). Nevertheless, given any solution $s = s_0$ of equation (3.17) one can determine which sign of $\cos \theta$ is appropriate by returning to the un-squared form and checking consistency. For completeness, the exact nature of the test required to determine the sign of $\cos \theta$ is listed later in equations (3.13) to (3.16).

The two (or four) real roots $s \in \{s_1, s_2\}$ (or $s \in \{s_1, s_2, s_3, s_4\}$) of (3.17) may be obtained analytically and non-iteratively by many methods (such as that of Ferrari) and thence candidate values for $\sin \theta$ and $\cos \theta$ may be found by the methods already described. It remains only (i) to determine the magnitudes $|p|$ and $|q|$ in terms of these candidates, (ii) to dismiss any solutions yielding unphysical answers (such as complex $\theta$ or negative $|p|$ or negative $|q|$) and (iii) to determine which of the remaining solutions (if more than one) leads to the smallest value of either side of equation (3.5), this then being the desired result, namely $M_{T2}$. Steps (i) and (ii) may be achieved by noting that equation (3.8) uniquely fixes the ratio $\rho = \frac{|p|}{|q|}$ in terms of known quantities (see equation (3.12)), while the absolute value of $|p|$ and $|q|$ is fixed by taking the scalar product of equation (3.6) with $\hat{p}$ resulting in $|p| = |p|K\rho$ and $|q| = |p|K$ for $K$ as defined in (3.11).

The outcome is the following result for $M_{T2}$.

$$
(M_{T2}(a^b, b^c, \hat{p}))^2\big|_{m_a=m_s=x=0} = \begin{cases} 0 & \text{if } (\hat{a} \hat{a}') (\hat{a} \hat{b}') \geq 0 \text{ and } (\hat{b} \hat{b}') (\hat{a} \hat{b}') \leq 0 \\ 2 |a||p|K\rho (1 - \cos \theta(\hat{a} \hat{b}) - \sin \theta(\hat{a} \hat{b}')) & \text{otherwise}, \end{cases}
$$

where\[^7\]

$$
K = \left[ \rho \left( \cos \theta (b \hat{p}) - \sin \theta (b \hat{b}') \right) + \cos \theta (a \hat{p}) - \sin \theta (a \hat{b}') \right]^{-1},
$$

$$
\rho = \frac{\cos \theta (a \hat{b}') + \sin \theta (a \hat{b})}{\cos \theta (b \hat{b}') + \sin \theta (b \hat{b})},
$$

in which $\sin \theta$ and $\cos \theta$ are defined by

$$
\sin \theta = s,
$$

$$
\cos \theta = \begin{cases} +\sqrt{1-s^2} & \text{if } L_1 - L_2 = 0 \\ -\sqrt{1-s^2} & \text{if } L_1 + L_2 = 0 \end{cases},
$$

where

$$
L_1 = K_1 + K_{cc} + s(K_s + (-K_{cc} + K_{ss})s),
$$

$$
L_2 = -(K_c + K_{cs})\sqrt{1-s^2},
$$

and in which $s$ is the appropriate “real, $K > 0$” root\[^8\] of the equation

$$
As^4 + Bs^3 + Cs^2 + Ds + E = 0
$$

\[^7\]Note that in the second line of (3.10) one could use $2 |b||p|K(1 - \cos \theta(\hat{a} \hat{b}) + \sin \theta(\hat{a} \hat{b}'))$ in place of $2 |a||p|K\rho (1 - \cos \theta(\hat{a} \hat{b}) - \sin \theta(\hat{a} \hat{b}'))$.

\[^8\]The quartic polynomial in $s$ has four roots, of which at least two are real and at most two form a complex conjugate pair. If there are only two real roots, one will lead to $K > 0$ and the other to $K < 0$. If there is more than one real root having $K > 0$, the one leading to the smallest value of $M_{T2}$ should be chosen. Degenerate cases are not discussed.
in which

\[ A = K_{cs}^2 + (K_{ss} - K_{cc})^2 \]  
\[ B = 2(K_{cs}K_c + K_s(K_{ss} - K_{cc})) \]  
\[ C = K_s^2 - K_{cs}^2 + K_c^2 + 2(K_{ss} - K_{cc})(K_1 + K_{cc}) \]  
\[ D = 2(-K_{cs}K_c + K_s(K_1 + K_{cc})) \]  
\[ E = (K_1 - K_c + K_{cc})(K_1 + K_c + K_{cc}) \]

wherein

\[ K_{ss} = - (\delta \hat{p})(\hat{a} \hat{b}') \]  
\[ K_{cc} = - (\sigma \hat{p}')(\hat{a} \hat{b}) \]  
\[ K_s = (\sigma \hat{p}) \]  
\[ K_c = (\sigma \hat{p}') \]  
\[ K_{cs} = -(\sigma \hat{p})(\hat{a} \hat{b}) - (\delta \hat{p}')(\hat{a} \hat{b}') \]  
\[ K_1 = 0 \]

in which we have introduced two new two-vectors \( \sigma \) and \( \delta \) according to

\[ \sigma = a + b, \quad \text{and} \]
\[ \delta = a - b. \]

Throughout the above we have adopted a notation in which unit-vectors carry a “hat”, while a “prime” (as in \( b' \)) indicates rotation of the two-vector through 90 degrees in the transverse plane. A consequence of this is that

\[ (v \cdot w) \equiv |v| |w| \cos \theta_{vw} \]  
\[ (v \cdot w') \equiv |v| |w| \sin \theta_{vw} \]

for arbitrary two vectors \( v \) and \( w \).

3.1 Remarks on the fully massless case (\( a^2 = b^2 = \chi^2 = 0 \)).

We note that for real constants \( \lambda \) and \( \mu \) satisfying \( \lambda \mu \geq 0 \), the solution in the above special case has the following property:

\[ M_{T2}^2 (\lambda a^\mu, \lambda b^\mu, \lambda \hat{p}) \big|_{m_a=m_b=\chi=0} = \lambda \mu M_{T2}^2 (a^\mu, b^\mu, \hat{p}) \big|_{m_a=m_b=\chi=0} \]

or equivalently

\[ M_{T2}^2 (a^\mu, b^\mu, \hat{p}) \big|_{m_a=m_b=\chi=0} = \sqrt{|a| |b| |\hat{p}|} M_{T2}^2 \left( \sqrt{|a|} \hat{a}, \sqrt{|b|} \hat{b}, \hat{p} \right) \big|_{m_a=m_b=\chi=0} \]

which could be interpreted as saying that the non-trivial dependence of \( M_{T2} \) on its inputs in this special case is confined to three dimensionless parameters, of which two are relative angles of the visible and missing transverse momenta, and one is the ratio of the momenta of the two visible particles.

\(^{9}\)The property described may easily be proved without using the \( M_{T2} \) solution. It is sufficient to note that the balanced condition (that the transverse masses on each sides are equal for the splitting that achieves the minimal transverse mass for either side) is not disturbed by scaling the missing momenta or by scaling both visible momenta equally.
Another interpretation of this result, is that $M_{T^2}$ in the fully massless case can be decomposed into a “magnitude” part

$$\rho_{T^2} = \sqrt{\sqrt{|a| |b| |p|}}$$

and an “angular” part

$$\theta_{T^2} = M_{T^2} \left( \sqrt{\frac{|a|}{|b|}} \hat{a}, \sqrt{\frac{|b|}{|a|}} \hat{b}, \hat{p} \right)$$

such that $M_{T^2} = \rho_{T^2} \theta_{T^2}$. Interestingly, $\theta_{T^2}$ seems to have only very mild dependence on $|b|/|a|$ and so is, to a relatively good approximation, just a universal function of $\theta_{ab}$ and $\theta_{bp}$ which is small for back to back events and large for pencil-like collimated events, multiplied by a normalisation function $f(|b|/|a|)$ whose maximum occurs when $|b|/|a| = 1$ and which is only slowly varying with $|b|/|a|$.

Accordingly, much of the “signal” structure of $M_{T^2}$ in the massless case comes from the $\rho_{T^2}$ part. Finally, we remark in passing, that $\rho_{T^2}$ is, in effect, a geometric mean of the magnitudes of the input momenta. Contrast this with the effective mass $M_{\text{eff}}$ [13] (sometimes also referred to as $H_T$ or similar) which is proportional to the algebraic mean of the input momenta. We therefore learn that there is a sense in which $M_{T^2}$ is acting a bit like $M_{\text{eff}}$ or $H_T$ but in “log space” rather than in “linear space”. One might conjecture whether there is anything to learn from that in the wider context ... for example, why is so much attention paid to linear sums? It is an interesting open question as to whether it would be useful to construct geometric versions of the effective mass or $H_T$ such as

$$M_{\text{geom}} = \left( |p| \prod_{i=1}^n |a_i| \right)^{1/(n+1)}$$

(assuming one invisible) or

$$M_{\text{geom}} = \left( |p|^2 \prod_{i=1}^n |a_i| \right)^{1/(n+2)}$$

(assuming two invisibles) or

$$H_T^{\text{geom}} = \left( \prod_{i=1}^n |a_i| \right)^{1/n}$$

or variants thereof in which the mean was taken over the number of “parents” (1 or 2) rather than the number of constituents $n$.

4 $M_{T^2}$ in the “$\hat{p} = Q(a + b)$” case.

In this section we consider results for $M_{T^2}$ that are valid in the regime in which the missing momentum is proportional to (though not necessarily in the same direction as) the sum of the visible momenta from each side of the event. In this section, masses are general and need not be zero. In other words, we concern ourselves here with the case $\hat{p} = Q(a + b)$ for some real constant $Q$ satisfying $-\infty < Q < +\infty$. The results we will find are

- the general solution for $Q = -1$,
- the general solution for $Q = 0$ and
- the general solution for any $Q$ but with the requirement that $m_a = m_b$ (“$m$”).

Before establishing our new results, we first comment on what is already known in these regimes.

\[ \text{Note that where the number of ingredients } n \text{ for } M_{\text{eff}} \text{ and } H_T \text{ can vary between events, the constant of proportionality, } n, \text{ will also vary between events.} \]
4.1 Previous results

The “$Q = -1$ case” corresponds to an absence of Upstream Transverse Momentum (UTM). It has already been shown in [3] and [14] that in this case

$$M_{T2}^{bal}(a^\mu, b^\mu, \mathbf{p} = -(a + b))^2 = \chi^2 + A_T + \sqrt{(A_T^2 - m_a^2 m_b^2) \left( 1 + \frac{4\chi^2}{2AT - m_a^2 - m_b^2} \right)}.$$  (4.1)

One thing this shows is that the dynamic dependence of $M_{T2}$ on its inputs (in that special case) is contained entirely within the contralinear boost invariant quantity $A_T$. The existing proofs provide no clear reason as to where that invariance comes from. Herein we will re-prove that result using a method that maintains manifest contralinear boost invariance at all times, and in doing so (1) we will gain some insight as to where the invariance comes from, and (2) we will be lead to make further generalisations of the result to the case $Q \neq -1$.

The $Q = 0$ case (i.e. the case in which $p_T = 0$) received a small amount of attention in [2]. Specifically, it was recorded therein that

$$M_{T2}^{bal}(a^\mu, b^\mu, \mathbf{p} = 0) |_{m_a = m_b = m} = \chi^2 + m^2 + \chi \sqrt{2(A_T + m^2)}$$

Herein we go beyond that result by generalising it to the case that $m_a \neq m_b$. Furthermore we gain the result by a method maintaining manifest contralinear boost invariance throughout.

The case where both $Q = +1$ and $m_a = m_b = m$ received, perhaps inadvertently, some attention in [4]. Specifically [4] gave an expression for $M_{CT2}$ (note, not $M_{T2}$) in the case where the visible particles are massless ($m_a = m_b = 0$) and there is no UTM (i.e. $\mathbf{p} = -(a + b)$). No explicit claims relating $M_{CT2}$ solutions to $M_{T2}$ solutions are made in [4], however using (1.1) we can see that the result of [4] corresponds to an expression for $M_{T2}$ in which the visible particles are still massless ($m_a = m_b = 0$) but in which there is a large amount of UTM, since $\mathbf{p} = +(a + b)$. Indeed, the observation that the $M_{CT2}$ result of [4] corresponded to an $M_{T2}$ result was the trigger for writing this paper. We will re-prove that result, but our result will then go beyond it as it will neither require $Q = +1$, nor require the visible particles to me massless.

4.2 The new results

We shall prove the following results by methods that maintain manifest contralinear boost invariance at all times:

$$M_{T2}^{bal}(a^\mu, b^\mu, \mathbf{p} = 0)^2 =$$

$$\equiv \chi^2 + \frac{m_a^2 + m_b^2}{2} + \frac{(m_a^2 - m_b^2)^2}{2(2AT - m_a^2 - m_b^2)} + \sqrt{(A_T^2 - m_a^2 m_b^2) \left( \frac{(m_a^2 - m_b^2)^2}{2AT - m_a^2 - m_b^2} + \frac{4\chi^2}{2AT - m_a^2 - m_b^2} \right)}$$

$$\equiv \chi^2 + A_T - \frac{2(A_T - m_a^2)(A_T - m_b^2)}{2AT - m_a^2 - m_b^2} + \sqrt{(A_T^2 - m_a^2 m_b^2) \left( 1 + \frac{(A_T - m_a^2)(A_T - m_b^2)}{(2AT - m_a^2 - m_b^2)^2} \right)}.$$  (4.2)

\footnote{The dependence of (4.1) on $A_T$ was not evident in the labyrinthine result first published in [3]. The exclusive dependence of the result on $A_T$ was first noted by [14], ostensibly by simplification of the result of [3]. The contralinear boost invariance of the result only becomes manifest in the final step of that simplification, and thus provides little insight as to where it comes from.}
and

$$M_{T2}^{\text{bal}}(a^\mu, b^\nu, p = +Q(a + b))^2 =$$

$$= \begin{cases} 
    M_{T2}^2(a^\mu, b^\nu, p = 0) & (\text{see equation } (4.2)) \\
    \chi^2 + A_T + \sqrt{(A_T^2 - m_a^2 m_b^2)} \left( 1 + \frac{\chi^2}{2A_T - m_a^2 - m_b^2} \right) & \text{if } Q = 0 \\
    \chi^2 + (1 + Q)m^2 - A_T Q + \sqrt{(A_T + m_a^2)(Q^2 - A_T - m_b^2) + 2\chi^2} & \text{if } m_a = m_b = m \\
    \chi^2 + (1 + Q)m^2 - A_T Q + \sqrt{(A_T - m_a^2 m_b^2) \left( Q^2 + \frac{4\chi^2}{2A_T - m_a^2 - m_b^2} \right)} & \text{otherwise, } 4.4 \\
    \chi^2 + (1 + Q)rac{m_a^2 + m_b^2}{2} - A_T Q + \sqrt{(A_T^2 - m_a^2 m_b^2) \left( Q^2 + \frac{4\chi^2}{2A_T - m_a^2 - m_b^2} \right)} & \text{if } Q = -1 \text{ or } m_a = m_b = m \text{ otherwise.} 
\end{cases}$$

We note that the RHS of (4.3) is always less than the RHS of (4.1).

### 4.2.1 Proof for the case when $p = 0$.

A consequence of $p = 0$ is that the invisible daughter particles (being the only sources of missing transverse momentum) must be back-to-back in the lab frame. We can enforce the conditions (a) that the invisible daughter hypotheses be back-to-back, and (b) that they share a common mass, by writing $q = \bar{p}$. To calculate the value of $M_{T2}$ in the “balanced” case it is therefore sufficient to perform an Euler-Lagrange minimisation using the Lagrangian

$$\mathcal{L} = \frac{1}{2} ((a + p)^2 + (\bar{b} + p)^2) + \lambda \left( p^2 - \chi^2 \right) + \mu \left( (a + p)^2 - (\bar{b} + p)^2 \right)$$

in which $\lambda$ and $\mu$ are Lagrange multipliers, the former enforcing the constraint that the invisible daughters have mass $\chi$, and the latter enforcing the constraint which gives us the balanced case. The resultant Euler-Lagrange equation for $p$ (i.e. $\partial \mathcal{L}/\partial p = 0$) then reduces to

$$p = Aa + B\bar{b}$$

(4.7)

for unknown constants $A$ and $B$ (functions of the Lagrange multipliers). We can determine $A$ and $B$ by substituting them back into the two constraints, making $A$ and $B$ the solution of the simultaneous equations

$$(Aa + B\bar{b})^2 = \chi^2$$

(4.8)

$$(A + 1)a + B\bar{b})^2 = (Aa + (B + 1)\bar{b})^2$$

(4.9)

which reduce to

$$A^2 m_a^2 + B^2 m_b^2 + 2ABA_T = \chi^2,$$

(4.10)

$$(2A + 1)m_a^2 - (2B + 1)m_b^2 = 2(A - B)A_T,$$

(4.11)

where we have once again defined $A_T = (a, \bar{b})$. It now only remains to solve these two simultaneous equations in order to determine $A$ and $B$ in terms of $A_T$, $\chi$, $m_a$ and $m_b$, and then to substitute the values so determined into equation (4.7) to determine $p$, before finally to substituting this value of $p$ into $(a + p)^2$ (or $(\bar{b} + p)^2$ since it will be the same) in order to determine $M_{T2}^2$ for this balanced case. This leads to the result shown earlier in equations (4.2) and (4.3) and concludes the proof.
4.2.2 Proof for the case when p ≠ 0.

We begin by defining two new transverse Lorentz vectors k and r according to \( k = Qa - p \) and \( r = Qb - q \). Next we demonstrate that if \( k = \bar{r} \) then (i) the missing momentum condition \( \mathbf{p} = \mathbf{p} + \mathbf{q} \), and (ii) the condition for the critical \( M_{T2} \) splitting hypothesis to be balanced, are both satisfied (provided that \( m_a = m_b \) or \( Q = -1 \)). Let us consider (i) first. If \( k = \bar{r} \) then \( Qa - p = Qb - q \) which implies \( Q(a - b) = p - q \) which, taking the transverse components, implies \( Q(a + b) = p + q \) as required. Now we must prove (ii). \( k = \bar{r} \) implies \( k^2 = r^2 \) which implies \( (Qa - p)^2 = (Qb - q)^2 \) which implies \( Q^2m_a^2 - 2Q(a.p) + \chi^2 = Q^2m_b^2 - 2Q(b.q) + \chi^2 \) which (if \( Q \neq 0 \))12 implies \( 2(a.p) - 2(b.q) = Q(m_a^2 - m_b^2) \) which implies \( (a + p)^2 - (b + q)^2 = m_a^2 - m_b^2 + Q(m_a^2 - mb^2) = (1 + Q)(m_a^2 - m_b^2) \). This allows us to see, as required, that \( k = \bar{r} \) implies that the “balanced” condition is satisfied if \( Q = -1 \) or \( m_a = m_b \).

We are now in a position to claim that \( M_{T2} \) for the case under consideration will be given by the solution to the Euler Lagrange problem with free parameters \( k, \lambda, \mu \) with Lagrangian

\[
\mathcal{L}(k, \lambda, \mu) = (a + p)^2 + \frac{\lambda}{2}(p^2 - \chi^2) + \frac{\mu}{2}(q^2 - \chi^2) \tag{4.12}
\]

\[
= ((1 + Q)a - k)^2 + \frac{\lambda}{2}((Qa - k)^2 - \chi^2) + \frac{\mu}{2}((Qb - k)^2 - \chi^2) \tag{4.13}
\]

which gives us again a (different) Euler-Lagrange equation for \( k \) of the form

\[
k = Aa + Bb
\]

for some, as yet undetermined, constants \( A \) and \( B \) which may be found by solving the remaining constraint equations associated with \( \lambda \) and \( \mu \) namely:

\[
((A - Q)a + Bb)^2 = \chi^2 \tag{4.14}
\]

\[
((Aa + (B - Q)b)^2 = \chi^2 \tag{4.15}
\]

or equivalently

\[
(A - Q)^2m_a^2 + B^2m_b^2 + 2(A - Q)BA_T = \chi^2 \tag{4.16}
\]

\[
A^2m_a^2 + (B - Q)^2m_b^2 + 2A(B - Q)A_T = \chi^2. \tag{4.17}
\]

Taking the difference we discover

\[
2AQ(A_T - m_a^2) + Q^2m_a^2 = 2BQ(A_T - m_b^2) + Q^2m_b^2
\]

which (if \( Q \neq 0 \) as before) allows us to eliminate either \( A \) or \( B \) from the preceding equations, leaving at worst a quadratic expression for whichever quantity remains. With \( A \) and \( B \) now determined in terms of \( Q \), \( m_a^2 \), \( m_b^2 \) and \( A_T \) it only remains to find the balanced \( M_{T2} \) solution by substituting into the expression \( M^2 = \frac{1}{2}((a + p)^2 + (b + q)^2) = \frac{1}{2}((a + QA - k)^2 + (b + Qb - k)^2) \) for \( k \) as defined in equation (4.2.2). This results in the single expression:

\[
M^2 = \chi^2 + (1 + Q)m_a^2 + m_b^2 - A_TQ + \sqrt{(A_T^2 - m_a^2m_b^2) \left( Q^2 + \frac{4\chi^2}{2A_T - m_a^2 - m_b^2} \right)} \tag{4.18}
\]

which we recall is is only “meaningful” if either \( Q = -1 \) or \( m_a = m_b \). Specialising the above expression for \( M^2 \) for both of those cases leads to the right hand sides of (4.4) and (4.5) and thus concludes the proof.

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12We have already considered the \( Q = 0 \) case separately and with greater generality (see for example equation (4.2)) valid for \( m_a \neq m_b \) and therefore the invalidity of the proof in the case \( Q = 0 \) need not concern us here. However, for completeness we note that the limit \( Q \to 0 \) of the solution we are about to obtain is well defined and is the same as that of (4.2), at least in the case \( m_a = m_b \) under consideration, and therefore the answer need not carry \( Q \neq 0 \) qualifiers.


5 Conclusions

We have detailed non-iterative algorithms for calculating $M_{T2}$ valid in a number of new special cases. One of these is the “fully massless case” which is the scenario in which $M_{T2}$ is used most frequently at the LHC. The other cases (most but not all of which are new) apply when the transverse missing momentum is parallel or anti-parallel to the vector sum of the visible momenta. Furthermore, in the cases for which non-iterative solutions were already known, we have found new derivations which are manifestly contralinear boost invariance at all times, providing advances in insight over earlier derivations. Along the way, we have stumbled in Section 4.2.2 across a number of interesting conjectures into the nature of variables like $M_{AT}$ and $H_T$, and have also gained therein better insight into the nature of $M_{T2}$ as a geometric mean in the fully massless case.

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A Appendix

Many techniques have been proposed for measuring the masses of the new particles which it is hoped the Large Hadron Collider will produce (see [15] for a recent review). Some of these techniques use the kinematic variable known as $M_{T2}$ [1] which may be thought of either as a natural extension of the transverse mass $M_T$ [16? ] to events containing pairs of mother particles, each undergoing a decay into a mixture of visible and invisible daughter particles, or as an event-by-event bound on the kinematic properties of such events [5].

Much of the literature that has developed $M_{T2}$ methods [2, 3, 6, 11, 14, 17–34] is concerned with the kinematic properties of the variable, the properties of its endpoints, and how (or whether) one can use these to place constraints on, or perhaps even measure 13, the masses of new pair produced particles and/or their invisible daughters. An entirely different use for $M_{T2}$ has been highlighted [7, 8] by members of the large general-purpose LHC experiments – identifying properties of $M_{T2}$ that explain why it is useful as a “cut” or “discovery” variable. As a consequence of its sensitivity to the mass scale of the pair produced parents, and as a consequence of its definition as a kinematic bound, it is particularly good at suppressing the low multiplicity low-mass-scale standard model processes (principally QCD and pair production of top quarks) which can be backgrounds to new-physics signatures with few visible particles into the final state.14 Early indications of the performance of $M_{T2}$ in early ATLAS data were very encouraging [9], and indeed it was pleasing to see that the most stringent expected limits on di-squark production from the 2010 LHC data came came from the use, by ATLAS [10], of $M_{T2}$ in this way.

However, as the instantaneous luminosity increases, it becomes necessary either to pre-scale triggers15 or to increase the trigger thresholds (e.g. the minimum transverse jet momenta). In particular, the QCD dijet cross section is so large that long before design luminosity is reached, one will find it necessary to apply to single and di-jet triggers either very large pre-scales or very high jet $p_T$ thresholds to prevent QCD events saturating the trigger. All this is bad news for any new-physics searches that hope to look for signals containing only two jets in association with missing transverse momentum, such as su-

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13Thus far, $M_{T2}$ has only been used once in anger to measure the mass of a particle – the top-quark in CDF in the dilepton channel [12]. The results are promising, and we are told that the top quark mass measurement with $M_{T2}$ has “the smallest total systematic uncertainty” of any in that channel [12].

14Consider, for example, supersymmetry in the case that the only thing that can be produced are squark pairs, each decaying to a quark jet and an (invisible) neutralino.

15To “pre-scale” a trigger means to accept at random only a fixed and pre-determined fraction of the events that pass it.
persymmetric models in which all sparticles are heavy except the squarks and the neutralino LSP (lightest supersymmetric particle). Searches for such “low multiplicity” signals are compromised if the only triggers they can pass are single or dijet or missing transverse momentum triggers. Fortunately this is not the end of the story. Since the majority of the QCD events contain back-to-back jets, some experiments have implemented “ΔΦ” triggers – i.e. triggers which only accept events if the leading two jets (above some \( p_T \) threshold) have an angular separation in the transverse plane which is less than a pre-defined value (such as 0.9π). QCD events find it much harder to pass ΔΦ triggers than, say, di-squark susy events, and so such triggers can remain unpre-scaled for a greater length of time than the corresponding mono- and di-jet triggers. Though ΔΦ triggers are conceptually easy to understand and implement, \( M_{T2} \) is expected to discriminate QCD from susy much better than ΔΦ [35]. The lack of fast methods for evaluating \( M_{T2} \) has, however, presented a hurdle to the adoption of \( M_{T2} \) as a trigger.

As it is not possible to write down closed-form analytic expressions for \( M_{T2} \) in the general case, \( M_{T2} \) is usually evaluated using numerical libraries such as [36] and [37] which use iterative algorithms and are therefore too slow to use in LHC experiment triggers.\(^{16}\)

One of the motivations for this study is therefore the hope that methods of calculating \( M_{T2} \) quickly and reliably can be found, in the cases of interest to experiments, so that \( M_{T2} \) may be implemented as a trigger variable.

The other arguably more important motivation for this study is pure mathematical interest. Ref [5] uncovered very useful mathematical insights into the nature of \( M_{T2} \) which allowed the creation of what is, at present, the fastest and most accurate algorithm for the evaluation of \( M_{T2} \).\(^{18}\) It is not always possible to predict what fruit a mathematical investigation will bring. The buds ripening here are those interpretations of Section 3.1.

References

[1] C. G. Lester and D. J. Summers, *Measuring masses of semi-invisibly decaying particles pair produced at hadron colliders*, Phys. Lett. B463 (1999) 99–103, [hep-ph/9906349].

[2] A. Barr, C. Lester, and P. Stephens, \( m(T2) \) : *The Truth behind the glamour*, J. Phys. G29 (2003) 2343–2363, [hep-ph/0304226].

[3] C. Lester and A. Barr, \( M_{T2Gen} \) : *Mass scale measurements in pair-production at colliders*, JHEP 12 (2007) 102, [arXiv:0708.1028].

[4] W. S. Cho, J. E. Kim, and J.-H. Kim, *Shining on buried new particles*, arXiv:0912.2354.

[5] H.-C. Cheng and Z. Han, *Minimal kinematic constraints and \( M_{T2} \)*, JHEP 12 (2008) 063, [arXiv:0810.5178].

[6] D. R. Tovey, *On measuring the masses of pair-produced semi-invisibly decaying particles at hadron colliders*, JHEP 04 (2008) 034, [arXiv:0802.2879].

[7] A. J. Barr and C. Gwenlan, *The race for supersymmetry: using \( M_{T2} \) for discovery*, Phys. Rev. D80 (2009) 074007, [arXiv:0907.2713].

[8] A. J. Barr, C. Gwenlan, C. G. Lester, and C. J. S. Young, *A comment on 'Amplification of endpoint structure for new particle mass measurement at the LHC'*, arXiv:1006.2568.

[9] ATLAS Collaboration, *The ATLAS Collaboration, Early supersymmetry searches in channels with jets and missing transverse momentum with the ATLAS detector*, Tech.
[10] ATLAS Collaboration, Search for squarks and gluinos using final states with jets and missing transverse momentum with the ATLAS detector in sqrt(s) = 7 TeV proton-proton collisions, arXiv:1102.5290.

[11] W. S. Cho, K. Choi, Y. G. Kim, and C. B. Park, Measuring the top quark mass with mT2 at the LHC, Phys. Rev. D78 (2008) 034019, [arXiv:0804.2185].

[12] CDF Collaboration, T. Aaltonen et. al., Top Quark Mass Measurement using mT2 in the Dilepton Channel at CDF, Phys. Rev. D81 (2010) 031102, [arXiv:0911.2956].

[13] D. R. Tovey, Measuring the SUSY mass scale at the LHC, Phys. Lett. B498 (2001) 1–10, [hep-ph/0006276].

[14] W. S. Cho, K. Choi, Y. G. Kim, and C. B. Park, Measuring superparticle masses at hadron collider using the transverse mass kink, JHEP 02 (2008) 035, [arXiv:0711.4526].

[15] A. J. Barr and C. G. Lester, A Review of the Mass Measurement Techniques proposed for the Large Hadron Collider, J. Phys. G37 (2010) 123001, [arXiv:1004.2732].

[16] UA1 Collaboration, G. Arnison et. al., Experimental observation of isolated large transverse energy electrons with associated missing energy at s1/2 = 540 GeV, Phys. Lett. B122 (1983) 103–116.

[17] B. C. Allanach, C. G. Lester, M. A. Parker, and B. R. Webber, Measuring sparticle masses in non-universal string inspired models at the LHC, JHEP 09 (2000) 004, [hep-ph/0007009].

[18] A. J. Barr, C. G. Lester, M. A. Parker, B. C. Allanach, and P. Richardson, Discovering anomaly-mediated supersymmetry at the LHC, JHEP 03 (2003) 045, [hep-ph/0208214].

[19] W. S. Cho, K. Choi, Y. G. Kim, and C. B. Park, Gluino transverse mass, Phys. Rev. Lett. 100 (2008) 171801, [arXiv:0709.0288].

[20] B. Gripaios, Transverse observables and mass determination at hadron colliders, JHEP 02 (2008) 053, [arXiv:0709.2740].

[21] A. J. Barr, B. Gripaios, and C. G. Lester, Weighing WIMPs with kinks at colliders: Invisible particle mass measurements from endpoints, JHEP 02 (2008) 014, [arXiv:0711.4008].

[22] G. G. Ross and M. Serna, Mass determination of new states at hadron colliders, Phys. Lett. B665 (2008) 212–218, [arXiv:0712.0943].

[23] M. M. Nojiri, G. Polesello, and D. R. Tovey, A hybrid method for determining SUSY particle masses at the LHC with fully identified cascade decays, JHEP 05 (2008) 014, [arXiv:0712.2718].

[24] M. Serna, A short comparison between mT2 and mCT2, JHEP 06 (2008) 004, [arXiv:0804.3344].

[25] A. J. Barr, G. G. Ross, and M. Serna, The precision determination of invisible-particle masses at the LHC, Phys. Rev. D78 (2008) 056006, [arXiv:0806.3224].

[26] W. S. Cho, K. Choi, Y. G. Kim, and C. B. Park, M_{T2}-assisted on-shell reconstruction of missing momenta and its application to spin measurement at the LHC, Phys. Rev. D79 (2009) 031701, [arXiv:0810.4853].

[27] M. Burns, K. Kong, K. T. Matchev, and M. Park, Using subsystem mT2 for complete mass determinations in decay chains with missing energy at hadron colliders, JHEP 03 (2009) 143, [arXiv:0810.5576].

[28] A. J. Barr, A. Pinder, and M. Serna, Precision Determination of Invisible-Particle Masses at the CERN LHC: II, Phys. Rev. D79 (2009) 074005, [arXiv:0811.2138].

[29] A. J. Barr, B. Gripaios, and C. G. Lester, Measuring the Higgs boson mass in dileptonic W-boson decays at hadron colliders, JHEP 07 (2009) 072, [arXiv:0902.4864].

[30] A. J. Barr, B. Gripaios, and C. G. Lester, Transverse masses and kinematic constraints: from the boundary to the crease, JHEP 11 (2009) 096, [arXiv:0908.3779].

[31] G. Polesello and D. R. Tovey, Supersymmetric particle mass measurement with the boost-corrected contransverse mass, JHEP 03 (2010) 030, [arXiv:0910.0174].

[32] I.-W. Kim, Algebraic singularity method for mass measurement with missing energy, Phys. Rev. Lett. 104 (2010) 081601, [arXiv:0910.1149].

[33] P. Konar, K. Kong, K. T. Matchev, and
M. Park, *Superpartner mass measurements with 1D decomposed $M_{T2}$*, arXiv:0910.3679.

[34] P. Konar, K. Kong, K. T. Matchev, and M. Park, *Dark matter particle spectroscopy at the LHC: Generalizing $m_{T2}$ to asymmetric event topologies*, arXiv:0911.4126.

[35] L. Randall and D. Tucker-Smith, *Dijet Searches for Supersymmetry at the LHC*, *Phys. Rev. Lett.* 101 (2008) 221803, [arXiv:0806.1049](http://arxiv.org/abs/0806.1049).

[36] A. J. Barr and C. G. Lester, “Oxbridge transverse mass library.” [http://www.hep.phy.cam.ac.uk/~lester/mt2/index.html](http://www.hep.phy.cam.ac.uk/~lester/mt2/index.html).

[37] H.-C. Cheng and Z. Han, “UCD transverse mass library.” [http://particle.physics.ucdavis.edu/hefti/projects/doku.ph](http://particle.physics.ucdavis.edu/hefti/projects/doku.php).