Convolutional Deblurring for Natural Imaging

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Abstract—In this paper, we propose a novel design of image deblurring in the form of one-shot convolution filtering that can directly convolve with naturally blurred images for restoration. The problem of optical blurring is a common disadvantage to many imaging applications that suffer from optical imperfections. Despite numerous deconvolution methods that blindly estimate blurring in either inclusive or exclusive forms, they are practically challenging due to high computational cost and limited image quality reconstruction. Both conditions of high accuracy and high speed are prerequisite for high-throughput imaging platforms in digital archiving. It becomes equally important as reconstruction accuracy that how quickly the implemented algorithms are capable of recovering the latent images? In such platforms, deblurring is required after image acquisition to feed into the communication pipeline to be either stored, previewed, or processed for high-level interpretation. Therefore, on-the-fly correction of such images are highly preferred to avoid possible time delays, mitigate computational expenses, and increase the image perception quality. We bridge this gap by synthesizing a deconvolution kernel as a linear combination of Finite Impulse Response (FIR) even derivative filters that can be directly convolved with input blurry images to boost the frequency falloff of the Point Spread Function (PSF) associated with the optical blur. We employ a Gaussian lowpass filter to decouple the image denoising problem for image edge deblurring. Furthermore, we propose a blind approach to estimate the PSF statistics for two different models of Gaussian and Laplacian kernels that are common in many imaging pipelines. Thorough experiments are designed to test and validate the efficiency of the proposed method using naturally blurred images across six imaging applications and seven state-of-the-art deconvolution methods.

Index Terms—Image deconvolution, point spread function (PSF), optical blur, generalized Gaussian, MaxPol derivatives

I. INTRODUCTION

Blurring in many imaging modalities is caused by inadequate optics configuration for image acquisition. In an imperfect optical system, a ray of light, after passing through the optical setup, does not converge to an end point and instead spreads over the image domain. This is known as the point spread function (PSF) which characterizes the response (aka impulse response) of the optics system [1], [2]. The corresponding observation model is usually expressed by a linear convolution

\[ f_B(x) = f_L(x) * h_{PSF}(x) + \eta(x), \]

where, \( f_B \) is the blurry observation, \( f_L \) is the latent sharp image, \( h_{PSF} \) is the associated PSF kernel, and \( \eta \) is the noise contamination artifact. The problem of deblurring (aka deconvolution) refers to the restoration of the latent image from its blurry observation, which is inherently an ill-posed problem. In the case where the PSF is given, the deconvolution problem becomes “non-blind”, and it is referred as “blind” when the PSF is unknown. In either case, this is known to be a long-lasting problem in low-level vision tasks to address different blurring deficiencies for deconvolution such as lens aberrations [1], [4], turbid medium [5]–[11], out-of-focus [12]–[14], and motion artifact [15]–[18].

The energy falloff of the high frequency band is a common result of PSF which suppresses sharp edges and leads to blurry observations. Aberrations (including turbid medium and out-of-focus) are mainly identified by a symmetric PSF which preserves the image geometry and is known to be a common problem in many imaging modalities. It is known that no matter how well the system is in-focus, including no motion artifacts, the aberrations are still barriers to generating high quality images. One way to improve the image perception quality is to deploy more sophisticated optical hardware such high numerical aperture lens [1]. However, this is not a cost effective approach for applications such as consumer cameras. A more viable approach would be to integrate fast deblurring algorithms such as unsharp masking techniques in order to maintain the real-time acquisition problem [19], [20]. Although the processing cost of such masks is no more than a one-shot filter convolution, they do not necessarily comply with the inverse response of the PSF for falloff correction, and hence produce sub-par image quality with over-sharpening artifacts.

In this paper, we focus on the proper correction of the falloff frequency of the PSF by casting the image deconvolution into a one-shot convolution filter problem. Our method is divided into two main steps. We first find that the a priori PSF model can be inferred by the scale-space analysis of the blurred image in the Fourier domain. Following the falloff assumption of the natural image frequency spectrum, we blindly estimate the statistics of the PSF for two different models of Gaussian and Laplacian kernels as variants of the generalized Gaussian distribution. In the second step, we provide a closed form solution of the inverse PSF for deblurring by fitting a series of polynomials in the frequency domain and cast its equivalent representation in the spatial domain using a linear combination of FIR derivative filters. By doing so, we avoid ringing artifacts on the restored image, while optimally preserving edge information on both fine and coarse resolutions. We show that the proposed deblurring method is capable of covering diverse PSF models that could be introduced by various imaging paradigms.

A. Related Works

An overview of existing image deconvolution methods is listed in Table I, divided into seven different categories, for image recovery. Here, we describe each category and analyze their strengths and weaknesses for practical considerations.
**TABLE I**

**LIST OF EXISTING IMAGE DECONVOLUTION METHODOLOGIES, LISTED IN SIX CATEGORIES: STATISTICAL PRIOR MODELING; TIKHONOV REGULARIZATION; TOTAL VARIATION REGULARIZED MINIMIZATION APPROACHES; SPARSE MODELING; ITERATIVE SHRINKAGE; INCLUDING COMBINED APPROACH OF DIFFERENT REGULARIZATION; AND RECENTLY WITH EMERGING CNN MODELS.**

| Author         | Year | Blind (B)/Scalable (N) | Statistical priors | Total variation | Sparse modeling | Iterative shrinkage | Combined regulation | Deep learning design |
|----------------|------|------------------------|--------------------|-----------------|-----------------|---------------------|---------------------|----------------------|
| Tao [21]       | 2018 | N                      |                    |                 |                 |                     |                     |                      |
| Wang [22]      | 2018 | N                      |                    |                 |                 |                     |                     |                      |
| Schuler [23], [24] | 2016 | B                      |                    |                 |                 |                     |                     |                      |
| Sun [18]       | 2015 | N                      |                    |                 |                 |                     |                     |                      |
| Hardis [25]    | 2015 | B                      |                    |                 |                 |                     |                     |                      |
| Xu [26]        | 2014 | N                      |                    |                 |                 |                     |                     |                      |
| Zhang [27]     | 2017 | N                      |                    |                 |                 |                     |                     |                      |
| Chan [28]      | 2017 | N                      |                    |                 |                 |                     |                     |                      |
| Romano [29]    | 2017 | N                      |                    |                 |                 |                     |                     |                      |
| Danielyan [30] | 2012 | N                      |                    |                 |                 |                     |                     |                      |
| Zoran [31]     | 2011 | N                      |                    |                 |                 |                     |                     |                      |
| Anwar [32]     | 2018 | B                      |                    |                 |                 |                     |                     |                      |
| Li [33]        | 2018 | B                      |                    |                 |                 |                     |                     |                      |
| Simoens [34]  | 2016 | NB                     |                    |                 |                 |                     |                     |                      |
| Kim [35]       | 2015 | B                      |                    |                 |                 |                     |                     |                      |
| Liu [36]       | 2014 | B                      |                    |                 |                 |                     |                     |                      |
| Mosleh [37]    | 2014 | N                      |                    |                 |                 |                     |                     |                      |
| Pan [38]       | 2013 | B                      |                    |                 |                 |                     |                     |                      |
| Kim [39]       | 2013 | B                      |                    |                 |                 |                     |                     |                      |
| Shen [40]      | 2012 | B                      |                    |                 |                 |                     |                     |                      |
| Sroubek [41]  | 2012 | B                      |                    |                 |                 |                     |                     |                      |
| Dong [42]      | 2011 | NB                     |                    |                 |                 |                     |                     |                      |
| Zhang [43], [44] | 2011 | B                      |                    |                 |                 |                     |                     |                      |
| Bai [45], [46] | 2018 | B                      |                    |                 |                 |                     |                     |                      |
| Lou [47]       | 2015 | N                      |                    |                 |                 |                     |                     |                      |
| Zhang [48]     | 2014 | N                      |                    |                 |                 |                     |                     |                      |
| Xu [49]        | 2012 | B                      |                    |                 |                 |                     |                     |                      |
| Chan [50]      | 2011 | N                      |                    |                 |                 |                     |                     |                      |
| Atanasov [51]  | 2010 | N                      |                    |                 |                 |                     |                     |                      |
| Li [52]        | 2018 | B                      |                    |                 |                 |                     |                     |                      |
| Bertorini [53] | 2010 | N                      |                    |                 |                 |                     |                     |                      |
| Che [54]       | 2009 | B                      |                    |                 |                 |                     |                     |                      |
| Wiener [55], [56] | 1949 | N                      |                    |                 |                 |                     |                     |                      |
| Xiao [57]      | 2016 | B                      |                    |                 |                 |                     |                     |                      |
| Zuo [58]       | 2013 | N                      |                    |                 |                 |                     |                     |                      |
| Krishnan [59] | 2011 | B                      |                    |                 |                 |                     |                     |                      |
| Dabov [60]     | 2008 | N                      |                    |                 |                 |                     |                     |                      |
| Neelamani [61] | 2004 | N                      |                    |                 |                 |                     |                     |                      |
| Whyte [62]     | 2014 | N                      |                    |                 |                 |                     |                     |                      |
| Schmidt [63]  | 2013 | N                      |                    |                 |                 |                     |                     |                      |
| Dong [64]      | 2013 | B                      |                    |                 |                 |                     |                     |                      |
| Sun [65]       | 2013 | B                      |                    |                 |                 |                     |                     |                      |
| Bishop [66]    | 2012 | B                      |                    |                 |                 |                     |                     |                      |
| Whyte [67]     | 2012 | B                      |                    |                 |                 |                     |                     |                      |
| Amizic [68]    | 2012 | B                      |                    |                 |                 |                     |                     |                      |
| Levin [69]     | 2011 | B                      |                    |                 |                 |                     |                     |                      |
| Krishnan [70] | 2009 | N                      |                    |                 |                 |                     |                     |                      |
| Bertero [71]  | 2009 | N                      |                    |                 |                 |                     |                     |                      |
| Bonettii [72]  | 2008 | N                      |                    |                 |                 |                     |                     |                      |
| Shan [73]      | 2008 | B/N                    |                    |                 |                 |                     |                     |                      |
| Yuan [74]      | 2007 | B                      |                    |                 |                 |                     |                     |                      |
| Fergus [75]    | 2006 | B                      |                    |                 |                 |                     |                     |                      |
| Biggs [76]     | 1997 | N                      |                    |                 |                 |                     |                     |                      |
| RL [77]        | 1974 | N                      |                    |                 |                 |                     |                     |                      |
1) Statistical priors: The idea is to formulate the occurrence of the underlying image as a conditional probability of a given blurry observation by maximum-a-posteriori (MAP) estimation. The early development of this method was proposed by Richardson-Lucy (RL) [73], [74] by recasting the solution in an iterative algorithm starting from an initial guess. The accelerated RL algorithm was proposed later by Biggs [72] using an adaptive line searching technique. We refer the reader to the comprehensive surveys in [75], [76] for the early development of these methods. With the emergence of digital consumer electronic cameras in the early 2000s, more practical deconvolution methods were released using the blind approach [4], [8], [13], [62], [63], [65]–[68], [70], [77]. With growing numbers of numerical solvers for alternating direction methods of multipliers (ADMM) (a variant of the splitting variable technique), the regulatory formulations were updated accordingly using different prior models [17], [64], [69], [70].

2) Tikhonov regularization: When the data fidelity is regularized in $\ell_2$-norm space to minimize a cost function, it becomes a variant of the Tikhonov regularization problem. The solution to this problem is given by quadratic minimization that can be accelerated by fast Fourier transform (FFT) and so reduce the computational complexity by the order of $O(n \log n)$. The early application of this regularization has been deployed in the classical Wiener deconvolution to regulate the image spectrum in the Fourier domain using a non-linearly weighted inverse blur response [55], [56]. More recent methods employ this framework for fast reconstruction [16], [53], [54]. Despite their efficiency, the sharpness of the reconstructed image edges are hampered by the Gibbs phenomenon, also known as ringing artifacts.

3) Iterative shrinkage: This is a variant of sparse reconstruction which recast the regularization problem in an iterative procedure where dominant feature coefficients are preserved during each iteration. Different regularizers can be found for image deconvolution in [57]–[61].

4) Variational regularization: Known as the total variation (TV) method, in which the priors for either the blur kernel or latent image are regulated by TV-norm [48]–[52], this norm preserves sharp edges while preventing Gibbs oscillations for recovery. As a common disadvantage, these methods suffer from visual blocking artifacts known as the “staircasing” artifacts.

5) Combined regularization: Combined approaches refer to deploying more than one regularization prior for recovery. This becomes more useful when both blur and image priors (in the blind case) could be fit in one regularization framework to address more complex formulations. The common practice to solve such regularized problem is to use split variable techniques to recast the algorithm in parallel and independently update each sub-modular task [54], [56]–[58], [40]–[45].

6) Deep Convolutional Neural Network (CNN): This is a variant of deep learning methods in which a convolutional network is trained to encode image features in multiple layers of decomposition. Each layer contains a set of convolutional filters to decompose the input feature map and then followed by an activation function e.g. ReLU, to identify features of interest to pass onto the next layer. The cascade arrangement of such layers provides an efficient way to decompose complex image structures for encoding/decoding [18], [21], [22], [24]–[26]. The common practice for developing these networks is to guide the training process by feeding in a pre-deconvolved image using Wiener/Tikhonov regularization to boost the performance results. A common disadvantage is the requirement of a train image set for training these networks. Since the latent image is not available, the train set is synthetically generated by a pre-defined blur kernel to obtain their blurry observation for training. Such an assumption however does not necessarily conform with the reality of blur observed in natural imaging applications.

7) Decoupled Methods: The problem of image deconvolution in the literature is jointly combined with denoising and deblurring, where prior assumptions are considered to regulate both inverse problems in one recovery framework. Recent developments decouple (separate) these into two sub-modular tasks, where the solution is usually cast by split variable minimization techniques for reconstruction. In fact, one can separately integrate a denoiser as a plug-in solution to address the denoising step [27], [28], [30]–[32], [78].

B. Remaining Challenges and Contributions

Despite vigorous research efforts, maintaining both precision and speed are still the main drawbacks of existing algorithms. High speed recovery simply means a “non-iterative” approach (or at least very few procedural algorithm) for practical implementation. When it comes to such demand, limited solutions exist such as Wiener [55], [56], Tikhonov [16], [53], Richardson-Lucy (RL) [72], and diagonalizing [35] based algorithms which are accelerated by fast Fourier transform (FFT). Despite their fast speed, they are prone to errors such as ringing artifacts and/or the loss of fine image detail edges. By contrast, the existing approaches with sophisticated deblurring involve heavy computational tasks where the problem is usually recast as an iterative minimization framework. Although recent techniques adopt CNN models to formulate the problem in feed-forward fashion and accelerate the recovery process using GPUs, existing limitations in blur modeling of natural images are still the main downside of such algorithms. In addition, the majority of deconvolution methods involve complicated parameter tuning procedures which limit their generalization ability.

The contributions of this paper in addressing the above challenges are as follows

- By adopting the decoupled approach, we observe that the problems of image deblurring and denoising should remain separate for reconstruction. We design a closed-form solution of the deblurring kernel as a linear combination of high-order FIR even derivative filters. For the numerical implementation, we employ the MaxPol library [79], [80] and call our deblurring method as the “1Shot-MaxPol” available to download from [1]. We consider the Gaussian lowpass filter as the denoising prior for the estimation of sharp deblurring edges.

[1] https://github.com/mahdihosseini/1Shot-MaxPol
We adopt the generalized Gaussian distribution to model PSF blur and analyze its feasibility range for recovery using the proposed deblurring method. We consider two variants of Gaussian and Laplacian models for blind estimation of blur statistics.

An adaptive tuning parameter is introduced based on the relative image entropy calculation to control the strength of deblurring.

Thorough experiments are conducted on 2054 natural images across diverse wavelength imaging bands. We blindly estimate the parameters for two model PSFs and feed them into seven state-of-the-art non-blind deblurring methods and compare their performances with our 1Shot-MaxPol deblurring. Empirical results indicate the superior performance of symmetric blur models, only consider symmetric kernels of even order of derivatives for approximation. The coefficients \( \alpha_n \) for \( n \in \{1, \ldots, n\} \) are undetermined. The inverse kernel \( D(x) \) in (3) is directly convolved with blurriness observation \( f_b \) for recovering

\[
f_R(x) = f_B(x) * [\delta(x) + D(x)] ,
\]

where, \( f_R(x) \approx f_l(x) \) is the recovered signal—approximant to the latent signal, and \( \delta(x) \) is the delta Kronecker function. Following the model design in (3), we define a dual approximation in the spatial and frequency domains, where the unknown coefficients \( \alpha_n \) are obtained in the frequency domain and then the equivalent representation in the spatial domain.

A. Dual frequency approximation

The Fourier transform function of the inverse kernel (3) is

\[
D(\omega) = \sum_{n=1}^{N} \alpha_n \hat{d}_{2n}(\omega) = \sum_{n=1}^{N} (-1)^n \alpha_n \omega^{2n} ,
\]

where the frequency polynomials \( \omega^{2n} \) are the dual representations of the derivative kernels in the spatial domain. Note that the frequency response of all even order derivatives are real and hence the amplitude response of the linear summation is equal to its frequency response. Substituting the response (2) in the Fourier transform of the deconvolution problem in (4) gives

\[
f(\omega) \approx \left[ \hat{f}(\omega) \hat{h}(\omega) + \hat{\eta}(\omega) \right] \left( 1 + \hat{D}(\omega) \right) .
\]

The deconvolution method in (6) impacts both blur \( \hat{f}(\omega) \hat{h}(\omega) \) and noise \( \hat{\eta}(\omega) \) modules. The design of the inverse kernel \( \hat{D}(\omega) \) corrects the blur observation up to a certain cutoff frequency \( \omega \leq \omega_c \) and leaves the rest of the band \( \omega > \omega_c \) untouched, as that is mainly related to noise artifacts:

\[
\hat{D}(\omega) = \begin{cases} 
\hat{h}^{-1}(\omega) - 1 & 0 \leq \omega \leq \omega_c \\
0 & \omega_c < \omega \leq \pi
\end{cases}
\]

Here, the deconvolution kernel only enhances the informative image band and mainly suppresses through the rest of the high frequency spectrum for reconstruction. The solution to the design model (7) is inferred by fitting the frequency polynomials in (3) as a regression model to the inverse spectrum of blur function \( \hat{h}^{-1}(\omega) - 1 \). We obtain the unknown coefficients \( \alpha_n \) by solving a linear least square problem in (8). We consider the frequency band from the inverse spectrum that is below a certain threshold \( \hat{h}^{-1}(\omega) - 1 \leq T \) to avoid numerical instabilities for approximation. The merit of our design in (7) is the problems of denoising and deblurring decoupling, enabling them to be individually addressed for recovery. One can separately apply a denoiser as a plug-in tool if the input image is perturbed with noise.
B. Dual spatial approximation

For discrete approximation of the derivative filters in (5) we adopt the MaxPol solution introduced in [79], [80] to regulate the frequency response of the corresponding filter. In particular, MaxPol provides a closed-form solution to the FIR derivative kernels that can be regulated in terms of different parameter designs such as arbitrary order of differentiation $n$, wide cutoff frequency $\omega_c$ for lowpass filter design, and polynomial accuracy $\ell$ for high frequency resolution accuracy. MaxPol kernels are highly balanced in terms of accuracy and noise control, which offers a unique framework for discrete signal approximation, as opposed to their counterpart solutions such as Gaussian and Savitzky-Golay filters. For more information, we refer the reader to [79], [80] and the references therein. Figure 1 demonstrates an example of approximating an inverse deblurring kernel with two different cutoff parameters.

![Fig. 1. Inverse deconvolution $D(x)$ kernel design. The blurring kernel here is a generalized Gaussian form with parameters $\alpha = 2$ and $\beta = 1.5$. This type of kernel is usually common in many real imaging applications such as optical aberration and turbulent medium blur. Derivatives up to 14th order are used, i.e. $N = 7$, to design $D(x)$ with two different cutoffs $\omega_c = \{7\pi/8, \pi\}$.](image1)

C. Denoising as a decoupled module

A simple yet efficient method of denoising is introduced to avoid computational complexity and maintain the accurate recovery. The whole idea of inverse deconvolution kernel design in Section III was to balance the amplitude falloff of high frequency components caused by the PSF kernel. Ideally speaking, if no denoising/cutoff is considered, all of the frequency domain will be deconvolved according to the inverse kernel response. However, such full correction should be avoided due to noise contamination in real applications. Once the image is deconvolved by an inverse filter, we apply (convolve) similar symmetric blur kernels for denoising with less blur scale then that considered for deconvolution. This guarantees that the falloff of the high frequency amplitude will be balanced between noise cancellation and amplifying meaningful edge information. See Figure 2 for an example. We suggest using generalized Gaussian kernel for such denoising where the associated FIR kernel has no vanishing moment and hence does not cause edge hallucinations. For more information on the generalized Gaussian, please refer to Section III. Nevertheless one can deploy more sophisticated denoising methods such as pre-trained CNN model in [27] or the overcomplete dictionary design in [31] as a decoupled module.

D. Two dimensional deblurring framework

While our design in the previous section is applied in one dimension, for imaging applications this should be extended to two dimensions (2D). Let $f(x, y) \in \mathbb{R}^{N_1 \times N_2}$ represent the image in the 2D domain, with $N_1$ and $N_2$ being the number of discrete pixels along the vertical and horizontal axes, respectively. Assuming the blur operator is independently applied in both dimensions (separable mode), the linear model in (1) is revised to

$$f_B(x, y) = f_L(x, y) * h_{PSF}(x) * h_{PSF}(y) + \eta(x, y).$$  (8)

The corresponding deblurring kernels in both directions are designed by means of the approximation method in the previous section and applied to the blurry image for reconstruction

$$f_R(x, y) = f_B(x, y) * [\delta(x) + D(x)] * [\delta(y) + D(y)]$$  (9)

The prior assumption on the energy level of the blurring kernel is usually unknown for natural imaging problems. For proper adjustment, we define a tuning parameter $\gamma \in [0, 1]$ to control the significance of the deconvolution level

$$f_R(x, y) = f_B(x, y) + \gamma \nabla D f_B(x, y),$$  (10)

where $\nabla D f_B(x, y) = f_B(x, y) * [D_x + D_y + D_{xy}]$ gives the reconstructed image edges and $D_{xy} = D_x * D_y$ is the crossed deconvolution operator independently applied to the horizontal and vertical axes. Therefore, all of the convolution operations in (10) are conducted in one dimension with a total computational complexity of $O(4L)$ when $L$ is the tap-length of the FIR deconvolution operators.

We demonstrate a proof of concept in Figure 3 to deconvolve a starfish image under severe blurring conditions. The visual appearance of the blurred image makes it almost impossible to detect fine edges compared to its reference frame. The result of deblurring is also shown in the same figure where the recovery is one-to-one match compared to its reference frame. This particular experiment validates a perfect recovery under the linear assumptions in (10) for deblurring. In Figure 4 we also demonstrate the spectrum responses of blurred and deconvolved images compared to their reference for all three color channels. As shown, the deblurring model is capable of recovering...
(b) Blurred by 8  
(e) Green channel  
(c) Deconvolved by 10  

(d) $f_B \ast D_x$  
(e) $f_B \ast D_y$  
(f) $f_B \ast D_{xy}$  

Fig. 3. Deconvolving a blurred image using the inverse kernel $D$ with fullband design $\omega_c = \pi$ and significance level $\gamma = 1$. The purpose of this experiment is to perform a sanity check for full image recovery assuming that no noise is added, i.e. $\eta = 0$. The objective quality of blurred image is SSIM= 0.7247, PSNR=24.09, where the reconstructed image is a one-to-one match i.e. SSIM = 1, PSNR = $\infty$. The same blur model as that shown in Figure 1 is chosen.

a perfect spectrum range of different frequency bands. To calculate the spectrum response of each image channel, we integrate the amplitude spectrum of Fourier transform of the image in a subband frequency along a circular ring, shown in Figure 6(b).

Fig. 4. Amplitude spectrum of original, blurred, and deconvolved images shown for all three color channels. The spectrum of the deblurred image is very close to the original spectrum.

E. Adaptive level tuning

Here we define an adaptive measure to tune the deblurring significance level $\gamma$ by calculating the relative ratio of two image entropy

$$\gamma \triangleq \frac{E(f_B)}{E(\nabla D f_B) + T} \quad (11)$$

where $E(I) = - \sum p(k) \log p(k)$ stands for the entropy measurement of the input image with $p(k)$ being the histogram counts for gray level $k$. The threshold level ‘$T$’ is defined here to avoid the singularity that could be caused by sparse deblurring edges. The entropy calculates the histogram dispersion (aka average rate) of the image. The dispersion ratio in (11) defines the relative measure for proper adjustment of the blur image with respect to its deblurring edges.

III. BLUR MODELING AND ESTIMATION

In this section, we model the PSF blur kernel in natural imaging applications by the generalized Gaussian (GG) distribution. We study two model shapes of Gaussian and Laplacian distributions as particular cases of GG for blind estimation.

A. Modeling blur by Generalized Gaussian (GG)

The generalized Gaussian (GG) distribution was introduced by Subbotin [82] to revise the power law of Gauss’s distribution into more generalized sense

$$h_{GG}(x) = \frac{1}{2\Gamma(1 + 1/\beta)A(\beta, \sigma)} \exp \left| \frac{x}{A(\beta, \sigma)} \right|^\beta, \quad (12)$$

where $\beta$ defines the shape of the distribution function, $A(\beta, \sigma) = (\sigma^2 \Gamma(1/\beta)/\Gamma(3/\beta))^{1/2}$ is the scaling parameter, and $\Gamma(\cdot)$ is the Gamma function $\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt, \forall z > 0$. For instance, the standard Gaussian distribution, i.e. second order model, is determined by $\beta = 2$ and $A(2, \sigma)$. For more information on the distribution and how it is used in different engineering applications, please refer to [83] and the references therein. Figure 5 demonstrates examples of GG blur for a variety of selected shapes and scales. The shape and the scale of the distribution control the decay rate and energy concentration of the distribution, respectively. The amplitude spectra of the kernels are also shown in the same figure (second row). The spectrum response of the blur kernels are inversely related to their scales, where low scales maintain wider frequencies for transformation.

The GG model is used in several imaging applications to model static blur in natural imaging, such as atmospheric turbulence and optical aberrations [5], [13], [14], [68], [84], [85]. A common approach is to employ such kernels in a
non-blind fashion for image deconvolution. The shape and the scale are the two different characteristics that fit different blur applications. For instance, in weather-conditioned environments, the shape of atmospheric turbulence in haze imaging is close to \( \beta \approx 1.5 \) [84]. One of our goals in this section is to study the range of feasibility for designing deconvolution kernels, introduced in Section II, using different GG types. Figure 6(a) demonstrates the error of fitting GG blur with different contour levels identifying the error between the approximated inverse kernel and the ideal inverse response i.e. \( ||\hat{h}(\omega)||^{-1} - \hat{D}(\omega)||/||\hat{D}(\omega)||^{-1}|| \). Examples of inverse responses are shown in Figure 5 (third row). The shade of grays in Figure 6(a) show that a wide range of selected kernels with different shapes and scales can be associated to approximate their inverse response using MaxPol kernels.

\[
R(r) = \frac{\int_{0}^{2\pi} \hat{f}_B(r, \theta) d\theta}{\int_{0}^{2\pi} f_B(r/s, \theta) d\theta} \approx \frac{\int_{0}^{2\pi} \hat{h}(r, \theta) d\theta + cr}{s \int_{0}^{2\pi} h(r/s, \theta) d\theta + cr} \tag{15}
\]

The right-hand-side (RHS) of equation (15) is used to fit a certain blur model as a prior knowledge of the equation, where the data fidelity term is provided by approximating the ratio \( R(r) \) using two radial spectra of different image scales. To calculate the radial spectrum, we define a radial ring with fixed bin size \( \Delta r \) shown in Figure 6(b) and integrate the spectrum along the selected ring. The bin size determines the quantized level of the radial spectrum.

The next two subsections study two different cases of generalized Gaussian for blur scale estimation: Gaussian (\( \beta = 2 \)) and Laplacian (\( \beta = 1 \)).

1) Gaussian blur kernel: The 2D Gaussian blur kernel is defined by \( h(x, y) = \frac{1}{2\pi\alpha^2} \exp(-x^2 + y^2)/2\alpha^2 \) and its 2D Fourier transform is \( \hat{h}(\omega_x, \omega_y) = \exp(-\alpha^2/2(\omega_x^2 + \omega_y^2)) \). Converting the domain from Cartesian to polar we obtain a rotation invariant kernel \( \hat{h}(r) = \exp(-\alpha^2 r^2/2) \) and substituting this into ratio spectrum (15) simplifies to

\[
R(r) \approx \frac{\exp(-\alpha^2 r^2/2 + c'r)}{s \exp(-\alpha^2 r^2/2s^2 + c'r)} \tag{16}
\]

The discrete measurements of the ratio \( R(r) \) in (16) is also calculated by the ratio between the radial spectrum of the original image and its downsampled image with scale \( s > 1 \).

2) Laplacian blur kernel: The 2D Laplacian blur kernel in separable mode is defined by \( h(x, y) = 1/2\alpha^2 \exp(-\sqrt{2}/\alpha(|x| + |y|)) \) and its 2D Fourier transform is \( \hat{h}(\omega_x, \omega_y) = 4/(2 + \alpha^2 \omega_x^2 + \alpha^2 \omega_y^2) \) [89]. Converting the domain from Cartesian to polar yields \( \hat{h}(r, \theta) = 4/(4 + 2\alpha^2 r^2 + \alpha^4 \cos^2\theta \sin^2\theta) \) while unlike the Gaussian in previous section is a rotation-dependent spectrum. Therefore, we need to calculate the radial spectrum of the blur image defined in (14) for this particular case. First, we revise the integral in (14) by

\[
\int_{0}^{2\pi} \hat{f}_B(r/s, \theta) d\theta \approx \int_{0}^{2\pi} \frac{A}{B + \sin^2\theta} d\theta + c \tag{17}
\]

where

\[
A = 16s^5/\alpha^4 r^5 \quad \text{and} \quad B = (16s^4 + 8\alpha^2 r^2 s^2)/\alpha^4 r^4. \tag{18}
\]

The integral in (17) is definite and it can be identified as a line integral by change of variable \( z = \exp i\theta \) and substituting this into the Euler formula, we have \( \sin \theta = (z - z^{-1})/2i \). By plugging this into the integrand in (17), we have

\[
\int_{0}^{2\pi} \frac{A}{B + \sin^2\theta} d\theta = \int_{C} \frac{-4Azdz}{i[z^4 - (4B + 2)z^2 + 1]} \tag{19}
\]
where the integral is applied around a closed unit circle. By taking another change of variable $u = z^2$, the integral in (19) simplifies to

$$\int_0^{2\pi} \frac{A}{B + \sin^2\theta} d\theta = 4iA \int_C \frac{du}{u^2 - (4B + 2)u + 1}. \quad (20)$$

We find the poles inside the unit circle from the denominator and apply the Residue theorem to calculate the integral value. The roots of the denominator are

$$\begin{cases} r_1 = 2B + 1 + 2\sqrt{B^2 + 1} \\ r_2 = 2B + 1 - 2\sqrt{B^2 + 1} \end{cases} \quad (21)$$

where, the root $r_1$ is outside the unit circle and does not apply. The second root $r_2 < 1$ for any $B$ and hence the residue of the integrand (20) at $r_2$ can be computed by

$$\text{Res}_{r_2} f(u) = \lim_{u \to r_2} \frac{u - r_2}{(u - r_2)(u - r_1)} = -\frac{1}{4\sqrt{B^2 + 1}} \quad (22)$$

Substituting (22) to the integral in (20) gives

$$4iA \int_C = 4iA \left[ 2\pi i \sum \text{Res} \right] = \frac{2\pi A}{\sqrt{B^2 + 1}} \quad (23)$$

Finally, the radial spectrum in (17) is obtained by

$$\int_0^{2\pi} \tilde{f}_B(r/s, \theta) d\theta \approx \frac{2\pi A}{\sqrt{B^2 + 1}} + c \quad (24)$$

The radial spectrum in (24) can now be used to approximate the ratio spectrum $R(r)$ defined in (15).

C. Synthetic validation

In both blur models (Gaussian/Laplacian), the unknown parameters scale $\alpha$ and noise level $\sigma'$ are obtained by solving a non-linear least square problem from [81]. Figure 7 demonstrates an example for blur parameter estimation. It is worth noting that we employ a good in-focus image for experimenting and synthetically blur it to estimate its scale for validation. If the original image is naturally blurred, the final blur will be the combination of natural and synthetic blur. The original image is shown in Figure 7(a) and is blurred with a Gaussian kernel of scale $\alpha = 1$ and perturbed by AWGN noise with standard deviation $\sigma = 1/255$ in unit scale shown in Figure 7(b). The blurred image is downsampled by a factor of $s = 2$, as shown in Figure 7(c). We employ MaxPol kernel of 0-order derivative with tap-length $l = 16$ and cutoff parameter $P = 24$ for downsampling to preserve most of the frequency spectrum. The radial spectra of both images is calculated and shown in Figure 7(d)-7(f). The ratio spectrum is fitted to the model in (16) and both unknown parameters $\alpha$ and $\sigma'$ are approximated. It worth noting the ratio spectrum defined for both Gaussian and Laplacian yield valid estimations through blur assessment. These models will be used in the experiment section to truly validate the blur levels of natural images and design their corresponding inverse kernels for deblurring.

IV. Experiments

We evaluate the proposed 1shot-MaxPol deblurring by conducting experiments in terms of reconstruction accuracy, computational complexity, and scalability for two different Gaussian and Laplacian blur models. We evaluate the reconstruction accuracy by adopting a no-reference sharpness quality assessment (NR-FQA) metric using the maximum local variation (MLV) method introduced in [90] where high values indicate better focus resolution and low values indicate the opposite. The processing speed of this metric is quite fast and can be used to evaluate large image databases such as the one introduced in Section IV-A. For comparison, we select eight non-blind image deconvolution methods, including Krishnan [59], EPLL [32], Chan-DeconvTV [51], IDD-BM3D [31], MLP [23], Simoes [35], Chan-PlugPlay [28], and IRCNN [27]. For more information on the procedural steps and functionality of these methods please refer to the Section “Comparison Methods” in the supplementary document of this paper. The associated PSF used in all non-blind deconvolution methods is provided by the blind estimation approach proposed in Section III for each test image. Using the same PSF for all methods provides a benchmark for a fair comparison of reconstruction quality. We estimate both Gaussian and Laplacian blur models with two different scales of $s = \{2, 4\}$.

A. Selected Natural Image Database

Here we describe the selected natural image databases for deblurring. We have collected 2054 images across different imaging modalities with optical realm covering between 350nm to 850nm spectral wavelength such as visible (RGB), hyperspectral (multi-channel), and near-infrared (NIR) (single channel). The optics involved in such modalities cause blurring.
due to lens imperfection and turbid medium. The overview of the selected natural databases is listed in Table II

| Database          | Camera            | Format | #Bit | Patch Size | # |
|-------------------|-------------------|--------|------|------------|---|
| LROC [91]         | Narrow-Angle      | TIF    | 8-Gray | 512 × 512   | 937 |
| McMaster [92]     | Kodak Film        | TIF    | 8-RGB | 500 × 500   | 18  |
| Hyperspec. [93]   | Hamamatsu         | TIF    | 12-RGB | 1024 × 1344 | 63  |
| RGB-Scene [92]    | Nikon D90         | TIF    | 8-RGB | 682 × 1024  | 477 |
| NIR-Scene [94]    | Canon T1i         | TIF    | 8-NIR | 682 × 1024  | 477 |
| Haze [11], [95]   | N/A               | TIF    | 8-RGB | ~ 500 × 600 | 82  |

1) Lunar Reconnaissance Orbiter Camera (LROC) Images: The LROC images are high resolution photos captured by two narrow angle cameras (NAC) mounted on the Lunar Reconnaissance Orbiter (LRO) satellite launched by NASA on 18 June 2009 [91], [96]–[98]. The mission objective of LROC is to map the surface of Moon to identify future landing sites as well as the scientific exploration of key targets. The high resolution images are released by Arizona State University every three months to the NASA Planetary Data System (PDS) publicly available to download from [http://www4.comp.polyu.edu.hk/~cslzhang/CDM_Dataset.htm](http://www4.comp.polyu.edu.hk/~cslzhang/CDM_Dataset.htm). The NAC camera projects a 700-mm focal-length telescope imaging onto linear array CCD camera with spectral response between 400–760nm. The camera provides a near diffraction-limited performance with 0.5° pixel resolution over a combined 5- km swath at a nominal 50-km lunar altitude. The NAC images are sampled at 12 bits and converted to 8 bits, then a lossless compression is applied prior to downlink. The modular transfer function (MTF) of the camera (aka PSF) yields a wide span over the Nyquist band [91]. Such a wide span guarantees that the majority of image frequency information is preserved, and we show in our experiments that these image can be well recovered. We have cropped 937 gray channel image patches of 512 × 512 pixel resolution from six different Lunar craters: Hell Q, Luminous Piazzaro, Jackson, Tycho, Burg, and Rozhdestvenskiy W.

2) Hyperspectral Imaging Camera: Hyperspectral radiance images are captured in 33 different wavelength filters sampled within [400, 720] nm [93]. The image are acquired with a monochrome camera with CCD arrays of 1024 × 1344 pixels. The database is from 30 natural scenes including rural and urban photos captured in the Minho region of Portugal. The optical lens aberration integrated in camera has a line spread function close to a Gaussian function. We have rendered an RGB image from all 33 Hyperspectral frames. Note that we have applied image deblurring on gamma corrected RGB images as suggested by the authors in [93].

3) McMaster Images: The McMaster database is constructed with true RGB color from Kodak films [92]. The database contains 18 image patches of size 500 × 500 pixels and are cropped from eight image scenes. The images have high saturation with abrupt color transitions and sharp structures [92].

4) NIR-RGB Scene Dataset: This dataset is introduced in [94], where the images are captured by Nikon D90 and Canon T1i cameras using both visible and near infrared (NIR) filters with 750nm cutoff between the two filters. Images are processed after image acquisition using white balance correction. Both NIR and RGB images are registered using SIFT features and the final images are re-sampled. Note that such re-sampling deteriorates the high frequency spectrum which can negatively impact the quality of image deblurring.

5) Haze Images: Haze and foggy images refer to outdoor imaging in bad weather conditions. Turbid media such as atmospheric particles, smoke, and water droplets absorb the scattered light and prevent it from reaching the camera sensor. Therefore, the acquired image is unclear and needs to be de-hazed [11], [95]. We obtain 41 haze images used in the literature and de-haze them using two methods described in [11], [95], i.e. 82 de-hazed images in total. We show here that by adding the deconvolution module, the recovered images become more clearer.

### Table II

**2054 SELECTED NATURAL IMAGES FOR DEBLURRING ACROSS DIFFERENT MODALITIES CONTAINING OPTICAL BLUR**

| Database          | Camera            | Format | #Bit | Patch Size | # |
|-------------------|-------------------|--------|------|------------|---|
| LROC [91]         | Narrow-Angle      | TIF    | 8-Gray | 512 × 512   | 937 |
| McMaster [92]     | Kodak Film        | TIF    | 8-RGB | 500 × 500   | 18  |
| Hyperspec. [93]   | Hamamatsu         | TIF    | 12-RGB | 1024 × 1344 | 63  |
| RGB-Scene [92]    | Nikon D90         | TIF    | 8-RGB | 682 × 1024  | 477 |
| NIR-Scene [94]    | Canon T1i         | TIF    | 8-NIR | 682 × 1024  | 477 |
| Haze [11], [95]   | N/A               | TIF    | 8-RGB | ~ 500 × 600 | 82  |

### Table III

**AVERAGE PERFORMANCE ON ALL RECOVERED IMAGES BY MEANS OF DIFFERENT METHODS ACROSS DIFFERENT BLUR MODELS. THE AVERAGE NR-FQA SCORE FOR ORIGINAL DATABASES IS 0.5408.**

| Model               | Gaussian (β = 2) | Laplacian (β = 1) |
|---------------------|------------------|-------------------|
| Scale               |                   |                   |
| s = 2               | 0.9009            | 0.7415            | 0.7346          |
| s = 4               | 0.6832            | 0.6582            | 0.6539          |
| Krishnan [59], [69] | 0.8583            | 0.7464            | 0.7396          |
| Chan-PlugPlay [28]  | 0.7387            | 0.7359            |                 |
| MLP [23]            | 0.8867            | 0.7390            | 0.7233          |
| Chan-DecovTV [51]   | 0.9090            | 0.7461            | 0.7387          |
| IRCNN [27]          | 0.9114            | 0.7359            | 0.7264          |
| Simoes [35]         | 0.8856            | 0.7315            | 0.7264          |
| IDD-BM3D [31]       | 0.9264            | 0.7461            | 0.7387          |
| 1Shot-MaxPol        | 0.9283            | 0.8792            | 0.8561          |

### B. Performance Analysis

Figure 8 demonstrates the NR-FQA analysis for each deblurring method. The associated PSFs are blindly estimated for both Gaussian and Laplacian models with scale factor s = 2. The statistical distribution of the scores are shown in box-plots corresponding to 25 and 75 percentiles to exclude the outliers (gray circles) and the median scores are shown as red lines overlaid on the box-plot. The figure also includes the original image scores performed on six different categories of the databases explained in Section IV-A. The median NR-FQA for the original LROC, NIR, and Hyperspectral databases are relatively low (high blur) compared to the RGB-Scene, McMaster, and Haze databases. The overall performance of all methods is also demonstrated in Table III across different PSF models, where the top three performing methods are shown in bold numbers. Note that 1Shot-MaxPol and IDD-BM3D outrank the other methods for all four different blur models, where IRCNN and Chan-PlugPlay rank in third place for the Gaussian and Laplacian, respectively. The majority of methods provide reasonable performance on Gaussian models, but provide inferior performances to 1Shot-MaxPol using Laplacian blur. This is simply because the majority of the deblurring methods do not generalize for diverse blurring models for recovery, e.g. IRCNN is trained using only Gaussian blur. In
As one of the main objectives of this paper was to develop fast deblurring method (while maintaining a good performance accuracy), we are keen to analyze the computational complexity of different deblurring methods used in this paper for comparison. We design two sets of experiments to investigate this. First, we analyze the CPU time versus different image sizes for reconstruction. For the CPU time measure, all the experiments are done on a Windows station with an AMD FX-8370E 8-Core CPU 3.30 GHz. Figure 10(a) demonstrates this complexity, where 1Shot-MaxPol outranks the second and the third top methods i.e. Simoes and Krishnan, respectively. For instance, 1Shot-MaxPol is 3.43 and 8.03 times faster than the top second and the third methods on recovering an image tile of size $1024\times 1024$. We perform the second type of assessment by analyzing the computation speed versus average NR-FQA of different methods. This is shown in Figure 10(b) where a large y-axis value indicates a high accuracy and a small x-axis value indicates low time consumption. Thus, an ideal method should be located at the top-left corner of the plot. Despite the fact that both 1Shot-MaxPol and IDD-BM3D provide the highest accuracy reconstruction, it worth noting that 1Shot-MaxPol is 71 times faster than IDD-BM3D. This easily puts our proposed method as one of the leading algorithms for optical deblurring in digital archiving applications.

### V. Conclusion

In this paper we have introduced a novel deblurring method for natural images to correct optical blur (in symmetric form) such as aberrations, defocus, and turbid medium. The method is called “1Shot-MaxPol” and cast as a one-shot convolution filter for blur correction. The merit of our design is to decouple the problems of deblurring and denoising and address them individually. For the case of deblurring, we first blindly estimated the PSF statistics for blur modeling using the novel approach of scale-space analysis of the image blur in the Fourier domain. We then constructed an FIR filter kernel for
Fig. 9. Deblurring examples of LROC and McMaster images using different methods. For better visual comparison, please turn off image-smoothing option from Adobe Acrobat view software.

Fig. 10. Computational complexity analysis of all comparison methods: (a) progression of speed by increasing image size, and (b) NRF-QA comparison vs computation time using an image size of $512 \times 512$.

natural image deblurring by casting its dual representation in the Fourier domain for inverse approximation. For the case of denoising, we offered two optional designs for cutoff frequency regulation for inverse deblurring kernel design and Gaussian filter to mitigate the noise effect on deblurred edges. We have gathered 2054 natural images from six different databases available online that contained optical blur. Experiment results shows that our deblurring method significantly outperform the existing state-of-the-art methods in terms of no-reference focus quality assessment, visual perception error, and computational complexity.

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