Non-forward Balitsky-Kovchegov equation and Vector Mesons

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Considering the Balitsky-Kovchegov QCD evolution equation in full momentum space, we derive the travelling wave solutions expressing the nonlinear saturation constraints on the dipole scattering amplitude at non-zero momentum transfer. A phenomenological application to elastic vector meson production shows the compatibility of data with the QCD prediction: an enhanced saturation scale at intermediate momentum transfer.

1 Motivation

The saturation of parton densities at high energy has been mainly studied for the forward dipole-target scattering amplitude $T(r, q = 0, Y)$, where $r, q, Y$ are, respectively, the dipole size, the momentum transfer and the total rapidity of the process. For instance, the corresponding QCD Balitsky-Kovchegov (BK) equation \cite{BK} has been shown to provide a theoretical insight on the “geometric scaling” properties \cite{geometric} of the related $\gamma^*\text{-proton}$ cross-sections. Indeed, it can be related to the existence of a scaling for $T(r, q = 0, Y) \sim T(r^2 Q^2(Y))$ where the saturation scale is $Q^2(Y) \sim \exp{cY}$ and the constant $c$ can be interpreted as the critical speed of “travelling wave” solutions of the nonlinear BK equation \cite{travelling}. Our theoretical and phenomenological subjects are the extension of these properties to the non-forward amplitude $T(r, q \neq 0, Y)$, which is phenomenologically relevant for the elastic production of vector mesons in deep inelastic scattering.

2 BK equation in full momentum space

In order to study the properties of $T(r, q \neq 0, Y)$, one has first to deal with both conceptual and technical difficulties. It is known that the BK formalism has been originally derived in impact parameter $b$ but then its validity especially at large $b$ is questionable, since it leads to non physical power-law tails. Hence we start with the formulation of the BK equation in momentum $q$, which is more local but has a non-trivial nonlinear form \cite{local}. In fact, despite this problem, the general method of travelling wave solutions can be extended in the non-forward domain \cite{extended}. It consists in 3 steps: first, one solves the equation restricted to its linear
part which is related to the non-forward Balitsky Fadin Kuraev Lipatov (BFKL) equation \cite{7} for the dipole-dipole amplitude via factorisation and whose solution takes the form of a linear superposition of waves. Second, one finds that the nonlinearities act by selecting the travelling wave with critical speed $c$, in a way which, interestingly, is independent of the specific structure of the nonlinear damping terms. Third, one obtains after enough rapidity evolution, a solution which appears independent from initial conditions ($T_0 \sim r^{2\gamma_c}$), provided these are sharper than the critical travelling wave front profile $T \sim r^{2\gamma_c}$, with $\gamma_0 > \gamma_c$. Interestingly enough, QCD color transparency satisfies this criterium. Applying these general results on the non-forward case one finds the following QCD predictions, depending on the relative magnitude of three scales involved in the process, namely $q$, $k_T^{-1}$ (the target size) and $k_P^{-1} \equiv r$ (the projectile i.e. dipole size).

- Near-Forward region $q \ll k_T \ll k_P$: $Q_s^2(Y) \sim k_T^2 \exp cY$
- Intermediate transfer region $k_T \ll q \ll k_P$: $Q_s^2(Y) \sim q^2 \exp cY$
- High transfer region $q \ll k_T \ll k_P$: No saturation.

Our main prediction is thus the validity of the forward travelling wave solution extended in the non-forward intermediate-transfer domain but with an enhanced saturation scale by the ratio $q^2/k_T^2$, where $k_T$ is a typically small, nonperturbative scale. Hence we are led to predict geometric scaling properties with a purely perturbative initial saturation scale given by the transverse momentum. This saturation scale enhancement prediction is confirmed by numerical simulations of the BK solutions as shown in Fig.1.

3 QCD Saturation Model for Exclusive VM production

The differential cross-section for exclusive vector meson (VM) production at HERA, see Fig.2, can be theoretically obtained from the non-forward dipole-proton amplitude and from $\Phi_{V, T, L}^\gamma$, the overlap functions between the (longitudinal and transverse) virtual photon and

Figure 2: $\rho$ (H1) and $\phi$ (ZEUS) differential cross-sections at $W = 75$ GeV
vector meson wave-functions [8]. For completion, we used two different VM wave-functions of the literature, without noticeable difference in our conclusions. One writes

\[
\frac{d\sigma_{T,L}^{\gamma^* p \rightarrow Vp}}{dq^2} = \frac{1}{16\pi} \left| \int d^2r \int_0^1 dz \, \Phi_{T,L}^V(z,r;Q^2,M_V^2) \, e^{-izq \cdot r} \, T(r,q,Y) \right|^2 ,
\]

Following theoretical prescriptions, we consider a forward dipole-proton amplitude \( \mathcal{N}_{11M} \) satisfactorily describing the total DIS cross-sections in a saturation model [9]. We just make the saturation scale varying with \( q^2 \), following the trend shown in Fig.1 and starting from the forward model one \( Q^2_s(Y) \), one writes

\[
T(r,q,Y) = 2\pi R_p^2 e^{-Bq^2} \mathcal{N}_{11M}(r^2 Q^2_s(Y,q)) ; \quad Q^2_s(q,Y) = Q^2_s(Y) (1 + c q^2) .
\]

The factor \( 2\pi R_p^2 e^{-Bq^2} \) comes from the non-perturbative proton form factor. For clarity of the analysis, we considered only \( B \) and \( c \) as free parameters of the non-forward parametrisations, the others being independently fixed by the forward analysis.

In Table 1 one displays the \( \chi^2/\text{points} \) obtained by a fit of \( \rho \) (47 data points) and \( \phi \) (34 points) total elastic production cross-sections and of \( \rho \) (50 data points) and \( \phi \) (70 points) differential cross-sections. The Table compares the saturation fits for fixed and \( q^2 \)-dependent scales, with a favour for the enhanced-scale model in the total. The model gives a comparable fit with a more conventional non-saturation model using a \( Q^2 \)-dependent slope \( B \propto M_V^2 + Q^2 \). Some of our results for the cross-sections are displayed in the figures. In Fig.2 one shows the results of the fit for \( \rho \)-production (H1) and \( \phi \)-production (ZEUS) differential cross-sections for a total \( \gamma^* - p \) energy \( W = 75 \text{GeV} \) and different \( Q^2 \) values. Let us finally present our predictions for the

| Cross-sections | \( q^2 \)-Sat. | fixed-Sat. |
|----------------|----------------|-----------|
| \( \rho \), \( \sigma_{el} \) | 1.156 | 1.732 |
| \( \rho \), \( \frac{d\sigma}{dt} \) | 1.382 | 1.489 |
| \( \phi \), \( \sigma_{el} \) | 1.322 | 2.247 |
| \( \phi \), \( \frac{d\sigma}{dt} \) | 1.076 | 0.931 |
| Total | 1.212 | 1.480 |

Table 1: Comparison of the \( \chi^2/\text{points} \)
DVCS cross-section, which is obtained without any free parameter from our analysis. In Fig. 3, they are compared with the available data and the agreement is good in the simple chosen parametrisation.

4 Conclusions

Let us summarize our new results

- **Saturation at non-zero transfer**: The Balitsky-Kovchegov QCD evolution equation involving full momentum transfer predicts (besides the known $q = 0$ case) saturation in the *intermediate* transfer range, namely for $Q_0 < q < Q$, where $Q_0$ (resp. $Q$) is the target (resp. projectile) typical scale.
  - **Characterisation of the universality class**: The universality class of the corresponding travelling-wave solutions is governed by a purely perturbative saturation scale $Q_s(Y) \equiv q^2 \Omega(Y)$, where $\Omega(Y) \sim e^{cY}$ is the same rapidity evolution factor as in the forward case. Consequently the *intermediate transfer* saturation scale gets enhanced by a factor $q^2/Q_0^2$.
  - **Phenomenology of Vector mesons**: The QCD predictions are applied in the experimentally accessible *intermediate transfer* range of vector meson production. The model uses an interpolation between the forward and non-forward saturation scale together with a parameter-frozen forward saturation model. It fits better the data on $\rho$ (H1) and $\phi$ (ZEUS) cross-sections than for a non-enhanced saturation.
  - **Prospects**: The next phenomenological prospect is to add charm to the discussion, both with the modification of the forward case by including the charm contribution [10] and by also considering the production of $\Psi$ mesons. On a theoretical ground, it would be interesting to go beyond the mean-field approximation of the BK equation.

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