Comment on

The two-phase issue in the $O(n)$ non-linear
$\sigma-$model: a Monte-Carlo study

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In their recent paper [1] Alles et al present new numerical evidence favoring the absence of a massless phase in the $O(N)$ nonlinear $\Sigma$ models with $N > 2$. While we have nothing to say about their numerics, we would like to repeat our objections to some of the statements made by Alles et al:

1) ‘The results for $O(8)$ support the asymptotic freedom scenario.’

As we have stressed repeatedly in the past [4], Kupianen [3] proved rigorously that the $1/N$ expansion produces the correct asymptotic expansion at fixed $\beta = \beta/N$ and the only issue is whether the expansion is uniform in $\beta$. Since it is known rigorously that the spherical model has $\eta = 0$, if a nonuniformity in $\beta$ does in fact exist, to see any deviations from $\eta = 0$ one would have to probe larger and larger values of $\beta$ as one increases $N$. Therefore if one wishes to determine the universality class of the $O(N)$ models with $N > 2$ (as revealed by the value of $\eta$), one should investigate $O(3)$ not $O(8)$, where the true asymptotic value may emerge only at huge correlation length.
2) ‘Assuming finite-size scaling (FSS), it has been shown that $O(3)$ presents asymptotic scaling starting from $\xi = 10^5$.‘

As we stated in our Comments to the Kim and Caracciolo et al papers [4], contrary to their claims, these authors have not established the existence of asymptotic scaling in $O(3)$, but in fact implicitly preassumed it. Indeed, it is again a rigorous fact that perturbation theory (PT) in $1/\beta$ at fixed lattice size $L$ gives the correct asymptotic expansion and the only open issue is whether this expansion is uniform in $L$. In their FSS investigations, Kim and Caracciolo et al assumed that there exists a $\beta$ independent $L_{\text{min}}$ on which one can apply FSS. The existence of such an $L_{\text{min}}$ is equivalent to the assumption that the model is asymptotically free, since this is a true property of PT and the latter is surely valid at fixed $L = L_{\text{min}}$. As we showed in a recent paper [5], the corrections to FSS, although quite small are there and thus the claims of Kim and Caracciolo et al are questionable.

3) ‘The $O(3)$ model with Symanzik action does not show KT behavior.’

The Symanzik action was invented precisely to improve the agreement of lattice PT with continuum PT and the latter does produce $\eta = 0$. The price is the introduction of an anti-ferromagnetic coupling, whose effect could be a lack of monotonicity of certain thermodynamic variables. Consequently it is hard to know what the small variation observed by Alles et al in $R_{KT}$ is supposed to indicate. On the other hand, the constancy of $R_{KT}$ with precisely $\eta = 1/4$ and the clear drop of $R_{PT}$ found by us for the model with standard action seems at the very least intriguing, if in fact the asymptotic freedom scenario is the correct one. To allow the reader to form a better impression whether for the Symanzik action the KT or the PT scenario looks more likely, Alles et al should display (on the same scale) both $R_{KT}$ and $R_{PT}$.

References

[1] B.Allés, A.Buonanno and G.Cella, *The two-phase issue on the O(N) nonlinear $\sigma-$model: a Monte Carlo Study*, [hep-lat/9608002](http://arxiv.org/abs/hep-lat/9608002).

[2] A.Patrascioiu and E.Seiler, *Nucl.Phys.* B **443** (1995) 596.

[3] A.Kupiainen, *Comm.Math,Phys.* 73 (1980) 273.

[4] A.Patrascioiu and E.Seiler, *Phys.Rev.Lett.* 73 (1994) 3325; *Phys.Rev.Lett.* 76 (1996) 1178.

[5] A.Patrascioiu and E.Seiler, *Can Numerics on Small Lattices Reveal the True Continuum Limit?*, to appear in *Phys.Lett. B*. 