The Importance of Monitoring Interval for Rockfall Magnitude-Frequency Estimation

Supplementary Material

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Supplementary Material 1: Level of Detection through the monitoring period

The original Level of Detection was set to 0.03 m (Fig. S1) but doubled here as described in the Methods.

![Graph showing Level of Detection (LoD) for individual change detections through time across a stable surface. A background level of noise of 0.01 – 0.03 m is apparent. Dates on the x-axis represent the first day of each month.]

Figure S1: Level of Detection (LoD) for individual change detections though time across a stable surface. A background level of noise of 0.01 – 0.03 m is apparent. Dates on the x-axis represent the first day of each month.

Supplementary Material 2: Power law modeling of the probability distribution of rockfall volumes

The magnitude-frequency relationship can be reported using either the probability density function (PDF) or the complementary cumulative distribution function (CCDF) [Bennett et al., 2012]. Central to PDF construction are histograms representing the frequency distribution, \( f(V_R) \), plotted in log-log space. Estimates of slope that result from least-squares regression in this approach are, however, subject to considerable systematic error [see Clauset et al., 2009]. An alternative to the probability density function is the complementary cumulative distribution function, which is used here:

\[
F(V_R) = F(V_R \geq V_{min}) = \left( \frac{V_R}{V_{min}} \right)^{-\alpha}
\]

where \( F(V_R) \) is the probability of a randomly selected rockfall exceeding a given volume \( V_R \), and the slope is denoted by \( \alpha \). The value of \( \alpha \) is related to \( \beta \), the exponent of the PDF, by:

\[
\alpha = \beta - 1
\]

The approach follows that presented by Clauset et al. [2009] and begins by estimating \( V_{min} \) and \( \alpha \) for the power law using maximum likelihood estimation. Values of \( V_{min} \) are selected to ensure that
the probability distribution of the empirical data and the best-fit power law are as close as possible above this value. A goodness-of-fit indicator, known as the \( p \)-value, is created to test whether a power law is a plausible fit based on the parameters \( \alpha \) and \( V_{\text{min}} \). This value is based on the distance between the distribution of the empirical data and the model (Kolmogorov-Smirnov), which is then compared with the distance between the empirical data and synthetic datasets \( (n = 1000) \) created from the same model. The derived \( p \)-value is the fraction of synthetic distances larger than the empirical distance.

Table S1: Fits to the CCDF for each inventory of \( T_{\text{int}} \). The \( p \)-value and \( V_{\text{min}} \), based on the fit to a power law, are provided alongside the Bayesian Information Criterion (BIC) for power law, log-normal, and exponential distributions. BIC values introduce a penalty for increased numbers of parameters in a model that may result in overfitting: \( \text{BIC} = \ln(n)k - 2\ln(L) \), where \( n \) is the number of observations, \( k \) is the number of model parameters, and \( L \) is the maximised value of the model likelihood function. A lower BIC, found here for the power law, indicates a preferable fit.

| \( T_{\text{int}} \) | \( p \)-value | \( V_{\text{min}} \) (m²) | BIC: power law | BIC: log-normal | BIC: exponential |
|-------------------|-------------|----------------|----------------|----------------|-----------------|
| 1 h               | 0.149       | 0.008          | -36906         | -32035         | -26357          |
| 3 h               | 0.646       | 0.011          | -12086         | -10201         | -7769           |
| 6 h               | 0.494       | 0.008          | -13306         | -11407         | -8694           |
| 12 h              | 0.832       | 0.006          | -14876         | -12930         | -9740           |
| 24 h              | 0.884       | 0.016          | -3421          | -2768          | -1906           |
| 96 h              | 0.934       | 0.021          | -1935          | -1469          | -845            |
| 7 d               | 0.917       | 0.022          | -1724          | -1306          | -743            |
| 14 d              | 0.828       | 0.022          | -1731          | -1353          | -872            |
| 21 d              | 0.131       | 0.018          | -2077          | -1654          | -1052           |
| 30 d              | 0.468       | 0.015          | -2843          | -2343          | -1791           |

If \( p > 0.1 \), we fail to reject the null hypothesis that both datasets originate from the same distribution. This value alone, however, is insufficient to determine whether the data follow a power law, in particular given that log-normal distributions can provide similar fits to power laws over many orders of magnitude [Cirillo et al., 2013; Clauset et al., 2009]. To test this, we applied a two-sample Kolmogorov Smirnov test to compare the empirical data to log-normal, exponential, and power law distributions (Fig. 3, main manuscript) \( > V_{\text{min}} \). The results rejected the null hypothesis that there was log-normal and exponential modeled data were from the same distribution as the empirical data (5% significance level), whereas the power law modelled data came from the same distribution for all inventories for \( V_R > V_{\text{min}} \) (Table S1). More specifically, when rockfall magnitude-frequency was represented as a CCDF, modeling of the data using a log-normal distribution failed to replicate the empirical data across all orders of magnitude.

Supplementary Material 3: Extracting elementary planar objects from 3D point cloud

In order to assess the influence of structural controls on the size distribution of rockfalls, the open source CloudCompare plug-in, Facets, was used to derive the horizontal and vertical width of visually persistent exposed planar discontinuity surfaces (facets) across the rock surface [Dewez et al., 2016]. The term ‘facets’ is used herein to describe the extracted discontinuities, while
acknowledging that the joints that separate contiguous discontinuities may be too small for identification from the point cloud data alone.

The examined rock face, East Cliff, comprises a number of sub-horizontal sandstone beds, with varying siltstone interbedding. As a result, no single co-planarity criterion could be used to extract facets across the entire cliff face. An initial approach was to divide the cliff face into exposures of separate lithologies. This division was prepared by collecting a series of orthoimages from the foreshore and merging these images into a single transect up the cliff. These images were draped over the full 3D model of the cliff face, described in Section 2 (main manuscript), enabling the different lithologies to be mapped (Fig. S1-S2).

Each lithology was assigned a bed number from 1-18 running from the top of the cliff to the bottom. Bed depths were measured using ten evenly spaced transects across the cliff face. The mean, median, and standard deviation of facet width and height is recorded for each bed. The resulting facet dimensions are listed in Table S2. Our comparison between the geometry of these facets and the recorded rockfalls draws upon the rockfall major axis. We consider this axis, equivalent to the largest one-dimensional measure of each rockfall, as most appropriate for interpreting the control of rock mass persistence over rockfall geometry.

Figure S2: Lithological units identified across the rock face. These are numbered from 1 at the top of the cliff (glacial till) to the buttress at the base of the cliff. The properties of these units are described in Table S2.

Figure S3: Detailed view of the generated facets compared against the slope model and high-resolution image, (a) Zoomed view of facets at the cliff top draped over a slope model. Colours are assigned randomly to aid distinction between individual facets, (b) slope model and image.
Table S2: Geometric properties of each lithology and its facets. Bed depth is presented as the mean of ten vertical profiles drawn at fixed intervals across each bed. The area of each bed is estimated without the areas of occlusion of the lighthouse scans. The average statistics of facet width and facet height, both of which are measured in 3D, show that the largest facets occur in the widely jointed sandstone beds (2 and 3) at the top of the cliff. The smallest are found in bed 7. These are contrasted against rockfall dimensions in order to examine the relationship between structural properties of the rock mass and its failure patterns.

| Lithology            | Bed number | Bed depth (m) | Area (m²) | Facet width (m) | Facet height (m) | Facet density (m²) |
|----------------------|------------|---------------|-----------|-----------------|------------------|-------------------|
|                      |            |               |           | Mean | Median | σ  | Mean | Median | σ  |               |
| Glacial Till         | 1          | 4.357         | 684.664   |      |        |    |      |        |    |               |
| Very widely jt Sst   | 2          | 2.173         | 164.853   | 0.534 | 0.377  | 0.475 | 0.515 | 0.369  | 0.500 | 3.82          |
| Widely jt Sst        | 3          | 3.033         | 491.510   | 0.527 | 0.373  | 0.475 | 0.496 | 0.377  | 0.436 | 3.80          |
| Intbd Sst/Sltst      | 4          | 0.756         | 64.714    |      |        |    |      |        |    |               |
| Intbd Sst/Sltst      | 5          | 2.257         | 437.038   |      |        |    |      |        |    |               |
| Closely jt Sst       | 6          | 1.013         | 226.737   |      |        |    |      |        |    |               |
| Closely jt Sst       | 7          | 2.310         | 477.830   | 0.202 | 0.134  | 0.236 | 0.191 | 0.134  | 0.210 | 5.13          |
| Intbd Sst/Sltst/CarbMud | 8        | 3.231         | 659.308   |      |        |    |      |        |    |               |
| Closely jt Sst       | 9          | 1.198         | 277.192   | 0.290 | 0.200  | 0.248 | 0.277 | 0.203  | 0.231 | 3.68          |
| Intbd Sst/Sltst/CarbMud | 10     | 4.770         | 984.195   |      |        |    |      |        |    |               |
| Intbd Sst/Sltst/CarbMud | 11   | 5.918         | 1.131     |      |        |    |      |        |    |               |
| Closely jt Sst       | 12         | 2.622         | 319.005   | 0.299 | 0.203  | 0.272 | 0.300 | 0.210  | 0.276 | 1.53          |
| Intbd Sst/Sltst/CarbMud | 13    | 2.730         | 205.831   |      |        |    |      |        |    |               |
| Closely jt Sst       | 14         | 1.201         | 69.501    | 0.399 | 0.309  | 0.261 | 0.391 | 0.310  | 0.235 | 1.59          |
| Intbd Sst/Sltst/CarbMud | 15   | 3.946         | 143.814   |      |        |    |      |        |    |               |
| Sid Sst              | 16         | 0.970         | 197.751   | 0.380 | 0.272  | 0.338 | 0.366 | 0.256  | 0.315 | 3.01          |
| Calc Sh              | 17         | 4.460         | 0.438     |      |        |    |      |        |    |               |
| Buttress             | 18         | Variable      | 3.057     |      |        |    |      |        |    |               |
Supplementary Material 4: Coefficient of determination for the fitting decay model to change in volume-frequency power law scaling coefficient

The $r^2$ values attributed to the rockfall volume frequency distributions (Fig. 3a; main manuscript) based on each value of $T_{int}$ are presented in Fig. S3. The decay in exponent, $\beta$, with $T_{int}$ is presented for a range of LoDs used in rockfall detection (Fig. S4). These present minimal variation across the range of the $T_{int}$, given that the LoD defined in Williams et al. (2018) and in Fig. S1, has been increased for the purpose of inventory comparison (see Methods).

Figure S4: $r^2$ values for the fitting decay model to change in volume-frequency power law scaling coefficient.

Figure S5: Decay in the exponent of the rockfall volume frequency distribution with $T_{int}$, where the Level of Detection (LoD) used to identify rockfalls is varied.
Supplementary Material 5: Increase in recorded rockfall area within developing failure scar, prior to the largest 30 rockfall events

Figure S6: Rockfall activity prior to the largest 30 rockfall events, plotted against $T_{\text{int}}$. For each rockfall, the total area of measured rockfalls within its scar is recorded for each inventory ($T_{\text{int}}$). The area recorded for each $T_{\text{int}}$ is normalized by the greatest area recorded (most commonly $T_{\text{int}} = 1$ h; see example inset). Below $T_{\text{int}} \sim 4$ d – 7 d, more frequent monitoring yields an increase in observable pre-failure activity ($A_{\text{norm}}$), indicating that the 30 individual rockfall events are likely to have failed within this timescale.

Supplementary Material 6: Results of fitting decay model to change in volume-frequency power law scaling coefficient

Three-parameter asymptotic regression, $\beta_{T_{\text{int}}} = \beta_0 + C_1 \cdot C_2 \log_{10} T_{\text{int}}$

Coefficients (with 95% confidence bounds):

$\beta_0 = 1.961 (1.928, 1.993)$ | $C_1 = 0.4271 (0.3496, 0.5046)$ | $C_2 = 0.0925 (-0.0014, 0.1865)$

Goodness of fit:

$SSE = 0.006581 (-)$ | $R^2 = 0.9604$ | $\text{Adjusted } R^2 = 0.9491$ | $\text{RMSE} = 0.03066 (-)$ | $p\text{ value} = 0.0325$

Supplementary Material 7: Results of fitting decay model (Equation 3, main manuscript) to change in difference between rockfall geometry and discontinuity-defined blocks

Three-parameter asymptotic regression, $\delta \bar{a} = \bar{a}_0 + C_1 \cdot C_2 \log_{10} T_{\text{int}}$

Coefficients (with 95% confidence bounds):

$\bar{a}_0 = -0.8797\text{ m} (-0.9269, -0.8326)$ | $C_1 = 1 (0.7621, 0.7855)$ | $C_2 = 0.8345 (0.8023, 0.8667)$

Goodness of fit:

$SSE = 0.006783\text{ m}^2$ | $R^2 = 0.9586$ | $\text{Adjusted } R^2 = 0.9534$ | $\text{RMSE} = 0.02612\text{ m}$ | $p\text{ value} = 0.0308$
**Supplementary Material 8**: Distribution of facet and rockfall \((T_{int} = 1 \text{ h} \text{ and } 30 \text{ d})\) major axes within each bed.

Figure S7: Difference in major axis lengths across beds with exposed joint structure. The distribution of facet and rockfall dimensions is most similar when rockfalls are monitored at \(T_{int} = 1 \text{ h}\). At \(T_{int} = 30 \text{ d}\), rockfalls are on average larger than the facets in the beds from which they originate.
References used within the supplementary material:

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