LOCALITY OF THE STRANGE SEA IN THE NUCLEON

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Abstract

We introduce the concept of “locality” for the strange sea in the nucleon, which measures proximity of the strange and anti-strange quarks in the momentum and coordinate spaces. The CCFR data for the strange and anti-strange distributions imply a “local” strange sea in the momentum space, which is unexpected in QCD and is at variance with the simple meson-cloud model where the strangeness is generated from the virtual transition of the nucleon to a hyperon plus a kaon. We present a simple model to interpret the CCFR data and to correlate momentum and coordinate space locality, yielding an upper bound of 0.005 fm$^2$ on the strange radius. We also discuss significances of locality for other charge-conjugation-odd observables.

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One of the fundamental properties of quantum field theory is the existence of the Dirac sea—infinite many fermions filling the negative energy states predicted by the Dirac equation. The excitation of the sea in the form of electron-positron pairs is partly responsible for the famous Lamb shift in the hydrogen spectra. Although novel, the sea effects are essentially perturbative in Quantum Electrodynamics because the fine structure constant is small. In Quantum Chromodynamics (QCD), however, the quark sea plays a much more important role in shaping the hadron structures due to the strong coupling. For instance, the recent measurements of the $g_1$ sum rule indicate that the quark sea is strongly polarized in a polarized nucleon. Furthermore, the data from deep-inelastic scattering show that the nucleon structure functions are dominated by the sea quarks at small Feynman $x$.

Despite its importance, the detailed structure of the quark sea in the nucleon, is largely unknown. Because of this, it is important to measure as many as possible observables that are directly linked to the sea degrees of freedom. Recently there have been many discussions in the literature about the “strange content” of the nucleon. These discussions are mainly motivated by the observation that strange quarks in the nucleon, if there are any, must be generated from the excitation of the Dirac sea. One hopes that once enough information is collected about the sea, a consistent picture may emerge to account for all properties measured.

In this paper, we discuss one particular aspect of the quark sea: the difference between quark and antiquark distributions in momentum and coordinate spaces. For up and down flavors, there is no physical way to separate the valence and sea quark contributions to any physical observables, other than defining the sea-quark distribution to be that of the anti-quarks. On the other hand, for the strange flavor, both quark and anti-quark distributions can be measured separately. Apart from the constraint that the total number of strange quarks must be equal to that of strange antiquarks, the two distributions do not have to be the same. The difference provides a unique window to the structure of the sea.

Depending upon interactions between quarks, there exist two extreme limits. In the first limit, quarks and antiquarks in the sea have exactly the same spatial or momentum wavefunctions. This can happen if they are tightly bound in pairs, or if they move independently but undergo similar interactions with other spectators. [We defer a discussion about the spin part of the wavefunctions to the latter part of the paper.] In the second limit, the quark and antiquark wavefunctions are qualitatively different from each other. This happens when quarks and antiquarks in the sea experience different interactions and move independently inside the nucleon. To differentiate the two limits, we introduce the concept of “locality of the sea”, which measures the average local similarity of quark and antiquark wavefunctions in coordinate or momentum spaces.

We are going to discuss two observables that have information one locality of the sea. If $s(\vec{r})$ and $\bar{s}(\vec{r})$ are spatial wavefunctions for the $s$ and $\bar{s}$ quarks in the nucleon, the strangeness radius,

$$\langle r^2 \rangle_s = \int d\vec{r} r^2 |s(\vec{r})|^2 - |\bar{s}(\vec{r})|^2 ,$$

is a measure of locality in the coordinate space. On the other hand, if $s(x)$ and $\bar{s}(x)$ are the strange quark and antiquark distributions, where $x$ is the momentum fraction of the nucleon carried by quarks in the infinite momentum frame, the positive-definite moment,
\[ L_s = \int_0^1 |s(x) - \bar{s}(x)| \, dx, \]  

is a measure of locality in momentum space. Clearly, a large \( \langle r^2 \rangle_s \) or \( L_s \) corresponds to small locality and a small \( \langle r^2 \rangle_s \) or \( L_s \) to large locality.

The remainder of the paper is organized as follows. We first comment on general expectations about the sea quark interactions in QCD. We then discuss the meson-cloud model \cite{4} for the strange sea, which seems to reflect the above expectations \cite{5}. We compare the model predictions with the strange quark and antiquark distributions extracted recently in neutrino deep-inelastic scattering by the CCFR collaboration \cite{6}. The qualitative difference motivated us to consider a new model for the strange sea, in which the strange and anti-strange quarks are perturbatively generated from the gluon splitting \textit{a la} Altarelli and Parisi \cite{7}. The interactions of \( s \) and \( \bar{s} \) with other quarks present in the nucleon are modelled by effective masses. After that, we attempt to correlate locality in momentum and coordinate spaces, giving a prediction for the strange radius. Finally, we comment on the effects of locality in other observables.

In extracting sea distributions from hard processes, one usually makes the assumption that the sea quark distribution is equal to the sea antiquark distribution. This, however, is unfounded in QCD. Although charge-conjugation symmetry says that the quark distribution in a nucleon is equal to the antiquark distribution in an antinucleon, there is no symmetry in QCD lagrangian that allows to relate quark and antiquark distributions of the nucleon in the sea. At the level of Feynman diagrams, one can find a corresponding interaction of the sea antiquarks for every interaction of the sea quarks by changing the direction of quark propagation. However, the strengths of the interactions are different due to different color factors associated with directions of the quark lines. An example is shown in Fig. 1, where two diagrams are different only in the fermion-number flow of the loop. It is easy to see that the color factors from the two diagrams are quite different (signs are opposite). In fact, in the large \( N_c \) limit, the quark interaction is suppressed relative to the antiquark one by a factor \( 1/N_c^2 \). When similar diagrams with different topology are taken into account, it turns out that the quark and antiquark interactions have the same order in \( 1/N_c \). However, there is no constraint in QCD that they must have the same numerical coefficients.

If quarks and antiquarks in the sea indeed have significantly different interactions, the meson cloud model is a good representative of this. According to the model, the strange sea is generated from the dissociation of the nucleon into hyperons (\( \Sigma \) and \( \Lambda \)) plus kaons. Thus, the strange quark, mainly interacting with one up and one down quarks in \( \Lambda \), gives rise to the \( s(x) \) distribution. On the other hand, the strange antiquark, mainly interacting with the down quark in \( K^0 \), yields the \( \bar{s}(x) \) distribution. With a reasonable choice of parameters (a bag radius of 0.6 fm, for instance), Signal and Thomas predicted the distributions shown in Fig. 2. Not surprisingly, the two distributions have quite different shapes, as they remarked. The \( \bar{s} \) distribution is relatively soft, like a typical sea distribution. However, the \( s \) distribution is much harder, resembling the valence distributions in the nucleon. [A hard distribution means the large quark density at large \( x \), like valence distributions, and a soft distribution means the otherwise.] The model predicts that the momentum fraction of the nucleon carried by the strange quarks is almost twice as much as that carried by the strange antiquarks.

If the small locality of the strange sea is confirmed by data, it might be one of the
strongest evidences for the meson-cloud model. However, the recent CCFR data from neutrino deep-inelastic scattering show that the strange quark and antiquark distributions are almost identical within the experimental errors, with \( s(x) \) being slightly harder than \( \bar{s}(x) \): 
\[
\frac{s(x)}{\bar{s}(x)} \sim (1 - x)^{-0.46 \pm 0.87}.
\]
The data, analyzed to the next-to-leading order in the \( \overline{\text{MS}} \) scheme, is also shown in Fig. 2 \[2\]. In a recent paper by Koepf et al., the meson-cloud model was used to fit the \( \bar{s}(x) \) distribution by adjusting the cut-off in the kaon-baryon form factor \[3\], shown in the dashed curve in Fig. 2. Although the model can fit the data points for \( x > 0.3 \), it underpredicts the data at smaller \( x \). In the same spirit, one might try to fit the \( s(x) \) distribution by adjusting the bag radius used in Signal and Thomas’s calculation. Failed to find a dynamical reason, we introduce a model which can accommodate varying degree of locality and use it to fit the strange distributions. Our intention is to correlate locality in momentum and coordinate spaces.

Thus the meson-cloud model for the strange sea has a limited value. However, it is difficult to understand from the view of QCD why the strange distributions have such high degree of locality. Failed to find a dynamical reason, we introduce a model which can accommodate varying degree of locality and use it to fit the strange distributions. Our intention is to correlate locality in momentum and coordinate spaces.

We assume the strange pairs at scale \( \mu \) are produced perturbatively from the gluon distribution \( G(x) \) at the same scale. To regulate the infrared divergences in the perturbative calculation, we introduce effective masses for the strange quarks and antiquarks. Physically, the effective masses account for the mean-field interactions between the sea quarks under consideration and others present in the nucleon. The different interactions experienced by the strange quark and antiquarks are reflected by the difference in the effective masses. [Note that strictly speaking, different masses for quarks and antiquarks not only violate CPT in field theory, but also render the color current non-conserved. However, at the level of a model, we ignore these problems.] Intuitively, when a quark has a large effective mass, the corresponding distribution is hard.

To simplify the calculation, we work in a special coordinate system. We assume that the nucleon is moving in the \( z \) direction, \( P = (P^0, 0, 0, P^3) \). Choose two light-cone null vectors \( p = P^+ / \sqrt{2}(1, 0, 0, 1) \) and \( n = 1 / (\sqrt{2}P^+)(1, 0, 0, -1) \) with \( p^2 = 0, n^2 = 0, p \cdot n = 1 \), where \( P^+ = 1 / \sqrt{2}(P^0 + P^3) \) is a component of the nucleon momentum in light-cone coordinates. The nucleon moment is now \( P = p + \frac{M^2}{2} n \). The quark distribution in the nucleon can be calculated with \[4\]
\[
f(x) = \frac{1}{2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P|\bar{\psi}(0)\gamma_\mu\psi(\lambda n)|P\rangle,
\]
where \( |P\rangle \) is the nucleon state normalized as \( \langle P|P'\rangle = 2P^0(2\pi)^3\delta^3(\vec{P} - \vec{P}') \).

Using the above equation, we calculate the strange distribution in a gluon,
\[
f_s(x, \mu^2) = \frac{\alpha_s}{4\pi} \int_0^{\mu^2} dk_+^2 \langle k^2_+ + \bar{m}_s^2 \rangle \frac{x^2 + (x - 1)^2 + 2x(1 - x)m_s\bar{m}_s + (1 - x)^2(m_s^2 - \bar{m}_s^2)}{[(k^2_+ + \bar{m}_s^2) + (1 - x)(m_s^2 - \bar{m}_s^2)]^2}
\]
where \( m_s \) and \( \bar{m}_s \) are the effective masses for the strange quarks and antiquarks, respectively, and \( \mu \) is the transverse momentum cut-off defining the renormalization scale of the
distribution. Since we are going to compare our result with the experimental data at 1 GeV$^2$, we take $\mu^2 = 1$ GeV$^2$ accordingly. We choose the strong-coupling constant, $\alpha_s$, to be 0.5, which roughly corresponds to the $\overline{\text{MS}}$ coupling at the cut-off scale. The strange antiquark distribution in a gluon has the similar expression as in Eq. (4), except $x$ and $1 - x$ need to be interchanged.

Convoluting $f_s(x, \mu^2)$ with the gluon distribution $G(x, \mu^2)$, we predict the strange distribution,

$$xs(x, m_s, \bar{m}_s) = \int_x^1 \frac{dy}{y} x f_s \left( \frac{x}{y}, m_s, \bar{m}_s \right) G(y, \mu^2 = 1 \text{GeV}^2),$$

(5)

Using the CTEQ3 gluon distribution [10] and fitting Eq. (5) to the CCFR data [6], we get $m_s = 260 \pm 70$ MeV, and $\bar{m}_s = 220 \pm 70$ MeV, with large $m_s$ correlated with small $\bar{m}_s$ and vice versa. The quality of the fit with $m_s = 260$ MeV and $\bar{m}_s = 220$ MeV is shown in Fig. 3. Given the simplicity of the model, the agreement is remarkably good. The effective masses $m_s$ and $\bar{m}_s$ are equal within the experimental errors. The difference in the central values, if significant at all, reflects weakly the points of the meson-cloud model.

Closely related to the momentum-space locality is the strange radius of the nucleon introduced by Jaffe in Ref. [11]. Experimentally, the strange radius can be extracted from the strange contribution to the elastic form factor of the nucleon. Several experiments at CEBAF have been planned to measure the form factor through parity-violating electron scattering [3]. Theoretically, the strange radius measures the net strangeness (strange minus antistrange quark) distribution in the nucleon (see Eq. (1)). There exist several conflicting predictions of the strange radius in the literature. The dispersion analysis in Ref. [11], assuming the second isoscalar vector-meson pole is dominated by $\phi$, gave $\langle r^2 \rangle_s = 0.13$ fm$^2$. In coordinate space, a positive strange radius means that the strange quarks dominate over anti-strange quarks at large radius. On the other hand, the SU(3) Skyrme model predicted a strange radius in the range of $-0.11$ to $-0.21$ fm$^2$ [12]. The sign here is consistent with the meson-cloud model. Finally, the kaon-loop calculations yielded a strange radius of order $-0.01$ fm$^2$ [13].

The CCFR data seem to imply that the strange radius is small. To directly reach this conclusion, one must find a model to correlate the strange distributions with the elastic form factor. [It is not impossible to find a wavefunction from which the momentum space locality implies nothing for the coordinate space locality. However, such a wavefunction seems unnatural.] One such model is the particle-plus-core model suggested by Gunion, Brodsky and Blankenbecler [14] many years ago. For our purpose, the simplest version without spin degrees of freedom is qualitatively sufficient. Using $\psi_s(k_\perp, x)$ to denote the wavefunction for the strange quarks, the strange quark distribution is,

$$s(x) = \frac{1}{2x(1-x)} \int \frac{d^2k_\perp}{(2\pi)^3} |\psi_s(k_\perp, x)|^2.$$

(6)

On the other hand, the strange form factor is,

$$F_s(q^2) = \int_0^1 \frac{dx}{2x(1-x)} \int \frac{d^2k_\perp}{(2\pi)^3} \left[ \psi_s(k_\perp, x) \psi_s(k_\perp + (1-x)q_\perp, x) - \psi_s(k_\perp, x) \psi_s(k_\perp + (1-x)q_\perp, x) \right].$$

(7)
Thus one can read the wavefunction from an expression for $s(x)$ and calculate the corresponding elastic form factor. Almost by definition, the small momentum-space locality is directly translated to the small coordinate-space locality. Without an explicit calculation, we can estimate the strange radius through a dimensional analysis,

$$|\langle r^2 \rangle_s| \sim \frac{(m_s - \bar{m}_s)}{\mu^3} \leq 0.005 \text{ fm}^2.$$  \hspace{1cm} (8)

Except for an uncertain sign, the bound is smaller than any of the theoretical prediction mentioned above.

To provide support for the expectation that the momentum space locality may mean coordinate space locality, we consider the neutron charge distribution. The valence quark distributions measured in hard processes directly provide the charge distribution in momentum space. In Fig. 4, we have shown the positive charge distribution from the up quark and the negative charge distribution from the down quarks as fitted in CTEQ3 \cite{10}. The total charge distribution, shown in the solid line, is considerably smaller than individual charge distributions, indicating strong locality of charges in momentum space. From this, one would expect that the charge distribution in coordinate space is quite local. This is supported by the small charge radius of the neutron $-0.12 \text{ fm}^2$. \[\text{[Notice, however, that the sign correlation in momentum and coordinate spaces is counter-intuitive.]}\]

One might generalize the notion of “local similarity” of the sea to the spin degrees of freedom. For instance, if quarks and antiquarks tend to have the same spin wavefunctions, the strange anomalous magnetic moment might be small. This spin locality is not expected in QCD even though the spatial degrees of freedom show a high degree of locality. A heuristic example is the neutron anomalous magnetic moment, which is large despite the charge distribution is very much local. However, QCD dynamics may surprise us again. The issue can be resolved by measuring the helicity distributions of the strange quarks ($\Delta s(x)$) and antiquarks ($\Delta \bar{s}(x)$) as well as the strange anomalous magnetic moment $\mu_s$.

Generalizing the concept of locality to all charge-conjugation-odd observables is tempting but subtle. The charge-conjugation operation involves not only the interchange of $s$ and $\bar{s}$ quarks, but also the change of the sign for the gluon fields. The nucleon is definitely very different if all gluon fields are charge-conjugated. However, for observables and/or models without the explicit gluon fields, one can discuss the generalized locality through the observation that all charge-conjugation-odd observables vanish in a state with definite charge-conjugation properties. In fact, if one defines a charge-conjugation operation as merely the interchange of $s$ and $\bar{s}$ quarks and if the sea has definite charge-conjugation symmetry, the matrix elements of C-odd operators with strange fields only, such as $\langle PS|\bar{s}\sigma^{\mu\nu}s|PS\rangle$ and $\langle P'|\bar{s}\gamma^\mu s|P\rangle$, vanish. In a phenomenological model proposed by Jaffe and Lipkin to account for the spin properties of the nucleon \cite{15}, the sea quarks form $0^{++}$ and $1^{++}$ pairs. Here the sea is charge-conjugation even.

The CCFR data provide a strong evidence that quarks and antiquarks in the strange sea encounter similar interactions. It is important to check independently that the CCFR method of extracting the strange distributions actually works. In deep-inelastic scattering, there is another way to measure the strange distribution through combining the structure function $F_2(x)$ measured from muon and neutrino scatterings \cite{16}. In a recent paper by
Barone et al. [17], they analyzed both methods by taking into account the quark mass and current non-conservation effects. They concluded that to the next-to-leading order, both extractions of the strange distributions are consistent. Hadron facilities provide new opportunities to access to the strange distributions in the nucleon. Intensive kaon beams, if available, can probe the strange sea directly through Drell-Yan processes [18]. The polarized RHIC has the capability of selecting hard, weak-interaction subprocess through single-spin asymmetry, and thus strange quarks, with their unique weak coupling, can be identified through their transition to charm quarks [19]. Clearly, the polarized and unpolarized strange distributions deserve to be studied more extensively in the future experiments.

To summarize, we introduced the concept of “locality” for the strange sea in the nucleon, which measures the local similarity of strange quark and antiquarks in momentum and coordinate spaces. We pointed out that the CCFR data on the strange distributions indicate that the strange sea is highly local, a picture unexpected in QCD and at variance with the meson-cloud model. We introduced a simple perturbative model for the strange sea, which fits the CCFR data very well. In our model, locality of the sea is reflected by the difference of the effective masses for the strange quarks and antiquarks. By correlating the strange distribution with the elastic form factor, we argue that the strange radius of the nucleon is smaller than 0.005 fm$^2$, a prediction to be verified soon by CEBAF experiments. Finally, we discussed the extension of the locality concept to other observables and commented on the experimental status of the strange distributions.

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REFERENCES

[1] J. Ashman et al., Phys. Lett. B206 (1988) 364; Nucl. Phys. B328 (1989) 1; B. Adeva et al., Phys. Lett. B302 (1993) 533; P. L. Anthony et al., Phys. Rev. Lett. 71 (1993) 959; K. Abe et al., Phys. Rev. Lett. 74 (1995) 346.
[2] T. Sloan, G. Smadja and R. Voss, Phys. Rep. 162 (1988) 45.
[3] M. Musolf, T. W. Donnelly, J. Dubach, S. J. Pollock, S. Kowalski, and E. J. Beise, Phys. Rep. 239 (1994) 1.
[4] J.D. Sullivan, Phys. Rev. D 5 (1972) 1732; A. W. Thomas, Phys. Lett. B126 (1983) 97.
[5] A.I. Signal and A.W. Thomas, Phys. Lett. B191 (1987) 205; A.W. Thomas, Prog. of Theoretical Phys. Suppl. No. 91 (1987) 204.
[6] A.O. Bazarko et al., Z. Phys. C 65 (1995) 189.
[7] G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298.
[8] W. Koepf, L. L. Frankfurt, and M. Strikman, TAUP-2273-95, hep-ph/9507218.
[9] R. L. Jaffe and X. Ji, Phys. Rev. Lett. 67 (1991) 552.
[10] H. L. Lai et. al., Phys. Rev. D51 (1995) 4763.
[11] R. L. Jaffe, Phys. Lett. B229 (1989) 275.
[12] N. W. Park, J. Schechter and H. Weigel, Phys. Rev. D43 (1991) 869.
[13] M. Musolf and M. Burkardt, Z. Phys. C 61 (1994) 433.
[14] J. F. Gunion, S. J. Brodsky, and R. Blankenbecler, Phys. Rev. D8 (1973) 287.
[15] R. L. Jaffe and H. J. Lipkin, Phys. Lett. B 266 (1991) 458.
[16] NMC, P. Amaudruz et al., Phys. Lett. B295 (1992) 159.
[17] V. Barone, M. Genovese, N. N. Nikolaev, E. Predazzi and B. G. Zakharov, DFTT 24/95, hep-ph/9505343.
[18] The NA3 Collaboration, J. Badier et al., Z. Phys. C 26 (1985) 489.
[19] The RHIC spin collaboration, A proposal to Relativistic Heavy-Ion Collider at Brookhaven, 1991.
FIGURES

FIG. 1. Feynman diagrams representing interactions of the sea quark and antiquark in a pair with valence quarks in the nucleon.

FIG. 2. The meson-cloud model predictions for the strange and anti-strange distributions in the nucleon and the CCFR data. The solid lines are from Ref. [5] and the dashed line from Ref. [8].

FIG. 3. Comparison of our model predictions ($m_s = 260$ MeV and $\bar{m}_s = 220$ MeV) with the CCFR data.

FIG. 4. The neutron charge distributions as functions of the Feynman $x$. The positive charge $\rho_+$ is from the valence $u$ quark and the negative charge $\rho_-$ from the valence $d$ quarks.
Figure 1
Fig. 3
Neutron Charge Distributions

Fig. 4