Two-loop R-parity violating Renormalisation Group Equations for non-standard soft
supersymmetry breaking in the context of the MSSM

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Abstract

We present the two-loop $\beta$-functions for non-standard soft supersymmetry-
breaking couplings including non-standard R-parity violating soft terms
in the Minimal Supersymmetric Standard Model

1 Introduction

It is known the MSSM is a gauge supersymmetric extension of the standard model,
with the addition of a number of dimension 2 and dimension 3 supersymmetry-breaking
mass and interaction terms. The model contains many new couplings, the soft SUSY
breaking interactions, which are arbitrary in the low energy effective theory, and the
reason for softly breaking terms are to avoid unwanted quadratic divergences. In order
to reduce the huge soft susy parameters very specific scenarios have been made about
physics at energies much higher than accessible energy such as mSUGRA and CMSSM.
They require an $N = 1$ supergravity theory at the Planck scale with a superpotential
of a very special and poorly motivated form. To obtain experimental predictions,
one should extrapolate the values of these couplings to the weak scale by using the
renormalisation group equations.

However a class of low-energy supersymmetric models can be constructed by imposing
some quite mild assumptions about the structure of the low-energy theory. The high-
energy origin of the models is irrelevant to this discussion. It is required that the fol-
lowing hold at the weak scale. First, the theory has minimal particle content consistent
with explaining the observed particles and being supersymmetric. This is the strongest
of assumptions and is a reasonable starting point. Second, the theory must have no
quadratic divergences. The absence of quadratic divergences is a major motivation for low-energy supersymmetry, and one should only allow all supersymmetry-breaking operators they do not cause quadratic divergences. To avoid quadratic divergences, one needs only to prohibit all dimension-four supersymmetry breaking interactions. To see this, notice that supersymmetry breaking is now accompanied by a mass parameter \( m \), so that a quadratic divergence in an operator would have a coefficient proportional to \( \Lambda^2 m \), where \( \Lambda \) is a cut-off scale. Only dimension-one operators could have such a coefficient. In theories with no scalars which are singlets under all symmetries of the theory, such an operator cannot occur.

An advantage of the model which is based on the above assumptions is that the parameter \( B \) is no longer dependent on the \( \mu \)-parameter. We know in the CMSSM the \( B \) parameter is set proportional to \( \mu \), and a nonzero \( B \) is required for suitable electroweak symmetry breaking. Therefore, \( \mu \) is required to be roughly of order \( 10^2 \) or \( 10^3 \)GeV, in order to allow a Higgs vev of order 174GeV. It is hard to understand why this parameter should be so small, and the same size as terms in the supersymmetry-breaking potential. However it is possible to make a model (based on the above assumption), and in this model the parameter \( \mu \) is no longer required in the supersymmetric potential because the general breaking potential permits a \( B \) term which is in principle unrelated to the parameter \( \mu \), so it is possible to set \( \mu \) to zero Ref\[2\].

It is well known that the MSSM is not, in fact, the most general renormalisable field theory consistent with the requirements of gauge invariance and naturalness; the unbroken theory is augmented by a discrete symmetry (R-parity) to forbid a set of baryon-number and lepton-number violating interactions, and the supersymmetry-breaking sector omits both R-parity violating soft terms and a set of non-standard (NS) soft breaking terms. There is a large literature on the effect of R-parity violation; a recent analysis (with standard soft-breaking terms) and references appears in Refs.\[3\] \[4\] \[5\] \[6\]; for earlier relevant work see in particular Ref\[7\]. The need to consider NS terms in a model independent analysis was stressed in Ref.\[8\]; for a discussion of the NS terms both in general and in the MSSM context see Refs.\[9\] \[10\] \[11\]; and for model-building applications see for example Refs.\[12\] \[13\]. For application of NS R-parity violating terms to leptogenesis, see Ref.\[14\].

In a previous paper Ref.\[11\] we gave the one-loop \( \beta \)-functions for non-standard param-
eters in the MSSM context. In this paper we extend the results to two loop corrections in the MSSM context.

2 The new soft breaking terms

The minimal supersymmetric standard model (MSSM) consists of a supersymmetric extension of the standard model, with the addition of a number of dimension 2 and dimension 3 supersymmetry-breaking mass and interaction terms. It became accepted when it was established that such a configuration is a natural consequence of supergravity when supersymmetry is broken in a hidden sector. However in Ref [9] it has been shown there are the new non-standard soft supersymmetry breaking terms which have no quadratic divergences. These terms are allowed according to the more general philosophy explained in the previous section. Now we review their results for a general $N = 1$ theory. We know the typical Lagrangian for the MSSM consists of two parts

$$\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{SOFT},$$

where $\mathcal{L}_{SUSY}$ is the Lagrangian for the supersymmetric gauge theory, containing the gauge multiplet ($A_\mu$, $\lambda$ that is the gaugino) and a matter multiplet (the spin-zero field $\phi_i$ and the spin-$\frac{1}{2}$ fields $\psi_i$). We assume a superpotential of the form

$$W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k.$$  

A renormalisable superpotential also contains quadratic and linear terms, but we assume there are no gauge singlet fields so there is no linear term, also we assume that an explicit quadratic term is not needed because such a term will be included as a special case from the new soft breaking terms Ref [9] [11]. A general soft breaking Lagrangian that prevents quadratic divergences is given by:

$$\mathcal{L}_{SOFT} = (m^2)^i_j \phi_i \phi_j + \left( \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j + \frac{1}{2} M \lambda \lambda + h.c. \right) + \mathcal{L}_{SOFT}^{new},$$

where $\mathcal{L}_{SOFT}^{new}$ introduces further possible dimension 3 terms in the case of a wide range of theories which preserve naturalness. It is given by:

$$\mathcal{L}_{SOFT}^{new} = \frac{1}{4} m^i_j \phi_i \phi_j \phi_k + \frac{1}{2} m^{ij} \psi_i \psi_j + m^a \psi_i \lambda_a + h.c.$$
The $m_A$ term is only possible in the presence of adjoint matter fields \[15\]. It is not an aspect of the MSSM, but often encountered in GUTs. Matter fermion mass terms ($m_F$ terms) and $r_{ij}^{jk}$ terms can still guarantee absence of quadratic divergences if they satisfy \[?\].

$$r_{ij}^{ij} - (m_F)_{ik}Y_{ijk} = 0$$  \hspace{1cm} (5)

A solution to Eq (5.5) comes from modifying the superpotential $W$ as follows:

$$W \rightarrow W + \frac{1}{2}(m_F)_{ij}\psi^i\psi^j$$  \hspace{1cm} (6)

since this new superpotential gives the appropriate fermion mass and interaction.

There are other possibilities, such as,

$$r_{ij}^{ij} = 0 \quad (but \quad r_{ij}^{jk} \neq 0)$$  \hspace{1cm} (7)

Of course if there is no gauge singlet chiral superfields then $r_{ij}^{ij} = (m_F)_{ik}Y_{ijk} = 0$ holds automatically. Therefore, in most cases in the MSSM we can keep these new soft breaking terms.

### 2.1 Non-standard terms in the MSSM

Now we present the case of the MSSM in the most general possible softly-broken version of the MSSM incorporating both R-parity violating (RPV) and non-standard (NS) terms. We include all possible soft supersymmetry breaking terms consistent with gauge invariance, which split the masses and couplings of particles and superpartners, but which do not remove the supersymmetric protection against large radiative corrections to scalar masses. It is interesting that, as we shall see, with the generalisation to the RPV case the connection between the NS terms and cubic scalar interactions involving supersymmetric mass terms is not universal.

We review the MSSM Lagrangian including all terms which avoid quadratic divergences. The superpotential is defined by

$$W = W_R + W_{NR},$$  \hspace{1cm} (8)

where $W_R$ and $W_{NR}$ have been given by Eq (4.2) and (4.3). However, we classify these parts again, and omit possible mass terms $H_1H_2$ and $LH_2$ because they are the
consequent terms of non-standard soft terms. In this new structure the \( W_R \) and \( W_{NR} \) terms are given by

\[
W_R = Y_u Q H_2 + Y_d Q H_1 + Y_e L H_1 \\
W_{NR} = \frac{1}{2}(\Lambda_E) E^c LL + \frac{1}{2}(\Lambda_U) U^c U^c + (\Lambda_D) D^c LQ.
\]

where generation \((i, j, \cdots)\), \(SU_2(a, b, \cdots)\), and \(SU_3(\alpha, \beta, \cdots)\) indices are contracted in “natural” fashion from left to right, thus for example

\[
\Lambda_D d c L Q \equiv \epsilon_{abc}(\Lambda_D)^{ijk}(d^c)_{ia} L^a Q^{bc}.
\]

We show complex conjugation by lowering the indices, thus \( (Y_u)_{ij} = (Y_u^*)_{ij} \).

The complete soft-breaking terms are given by

\[
L_1 = \sum_{\phi} m^2_{\phi} \phi^c \phi + \left[ m^2_{H_1 H_2} + \sum_{i=1}^3 \frac{1}{2} M_i \phi_i \phi_i + h.c. \right] + \left[ h_u Q H_2 + h_d Q H_1 + h_e L H_1 + h.c. \right], \\
L_2 = m^2_L \tilde{L} + m^2_R \tilde{L} H_2 + \frac{1}{2} h_L \tilde{E}^c \tilde{L} \tilde{L} + \frac{1}{2} h_U \tilde{H}_1 \tilde{E}^c \tilde{D}^c + h.c., \\
L_3 = m_q \psi H_2 + R_5 H_2 \tilde{L} \tilde{E}^c + R_7 H_2 Q \tilde{D}^c + R_9 H_2 \tilde{Q} \tilde{E}^c + h.c., \\
L_4 = m_t \psi L H_2 + R_6 H_2 \tilde{L} \tilde{E}^c + R_8 H_2 Q \tilde{D}^c + R_4 \tilde{Q} \tilde{D}^c + h.c.
\]

where \( L_1 \) corresponds to SRPC(Standard R-parity Conserving), \( L_2 \) indicates SRPV(Standard R-parity Violating), \( L_3 \) shows NSRPC(Non-Standard R-parity Conserving) and \( L_4 \) corresponds to NSRPV(Non-Standard R-parity Violating) terms. These terms come from “pure” trilinears \( (\phi \phi \phi) \), “mixed” trilinears \( (\phi \phi^* \phi) \), scalar \( (\phi^* \phi) \) masses, a \( \phi \phi \) Higgs-boson mass-mixing, and \( \psi \psi \) terms in Eq (5.4).

We have separated the soft terms into Eqs. (5.38) and (5.39) because each group of couplings are not involved in the \( \beta \)-functions of the other group of couplings. In the Lagrangian we have two interaction terms \( (R_{3,4}) \) which cannot be generated by supersymmetric mass terms. These terms violate baryon and lepton numbers \( L, B \), so the effects of these two terms may be comparable to \( \Lambda_D, \Lambda_E \) and \( \Lambda_U \).

The full one-loop \( \beta \)-functions including NS soft terms and RPV terms have been presented in [11]. The full three loop \( \beta \)-functions including soft standard \( \beta \)-functions.
The two loop gauge $\beta$ functions and anomalous dimensions in the R-parity violating (RPV) case have been calculated in \cite{17}. In particular, Ref \cite{6} contained a complete set of one-loop $\beta$-functions for RPV parameters. The full two loop $\beta$-functions for the RPV couplings have been presented in \cite{16,18}. In this paper, we shall write down explicitly all possible R-parity violating terms in the framework of the Minimal Supersymmetric Standard Model, assuming the most general breaking of the MSSM incorporating both R-parity violating (RPV) and non-standard (NS) terms, and then compute results of two loop $\beta$-functions for all non-standard (NS) parameters of the MSSM in the most general possible softly-broken version of the MSSM. Here we give the full two-loop $\beta$-functions including NS soft terms and RPV terms.

The two loop-$\beta$-functions for $R_{1,9}$ are given as follows:

\begin{align*}
(16\pi^2)^2(\beta_{R_{1,9}}^{(2)})_{ij}^k &= -2(Y_d)^{il}(Y_e)_{kn}C_{e^l}(R_3)^l_{jm} - 2(A_D)^{im}(\Lambda_E)_{mpk}C_{e^l}(R_3)^l_{jmn} \\
&+ 2(A_D)^{ip}(\Lambda_D)_{pkm}C_{Q}(R_1)^{mj} - 2(Y_u)^{ip}(\Lambda_D)_{pkm}C_{Q}(R_9)^{mj} \\
&+ 2(Y_u)^{jp}(\Lambda_D)_{mpk}C_{e^l}(R_7)^{lm} - 4(A_u)^{jp}(\Lambda_D)_{pkm}C_{Q}(R_4)^{lm} \\
&+ (A_D)^{np}(\Lambda_E)_{mpk}(R_3)^{jm}C_{e^l} - 2(Y_d)^{jm}(Y_e)_{kn}(R_3)^{jm}C_{e^l} \\
&+ 2(Y_u)^{jp}(A_D)_{mpk}(R_1)^{jm}C_{e^l} - 2(Y_d)^{jp}(A_D)_{pkm}(R_9)^{jm}C_{H_1} \\
&- 2(Y_u)^{jp}(A_D)_{mpk}(R_7)^{lm}C_{H_2} - 2(A_u)^{jp}(A_D)_{pkm}(R_4)^{jm}C_{e^l} \\
&+ 4(Y_u)^{jm}(Y_e)_{kn}(\Lambda_D)_{mpq}(Y_e)^{pn}(R_9)^{qj} + 4(Y_d)^{jm}(Y_e)_{kn}(\Lambda_D)_{mpq}(\Lambda_E)^{nrp}(R_1)^{qj} \\
&+ 4(Y_d)^{jm}(Y_e)_{kn}(Y_e)_{pn}(R_9)^{qj} - 2(A_D)^{jm}(\Lambda_D)_{ikn}(\Lambda_D)_{mpq}(\Lambda_E)^{nrp}(R_1)^{qj} \\
&- 2(Y_d)^{jm}(\Lambda_D)_{ikn}(Y_e)_{qp}(R_1)^{qj} + 4(A_D)^{jm}(\Lambda_E)_{nk}(\Lambda_D)_{mpq}(\Lambda_E)^{nrp}(R_1)^{qj} \\
&- 4(A_D)^{jm}(\Lambda_E)_{nk}(Y_e)_{qm}(Y_e)^{rn}(R_1)^{qj} + 2(A_D)^{jm}(\Lambda_D)_{ikn}(\Lambda_D)_{mpq}(Y_d)^{np}(R_9)^{qj} \\
&+ 2(A_D)^{jm}(\Lambda_D)_{ikn}(Y_e)_{np}(R_3)^{qj} + 2(\Lambda_D)^{jm}(\Lambda_D)_{ikn}(\Lambda_E)_{pqm}(\Lambda_D)^{rnp}(R_3)^{qj} \\
&- 2(Y_d)^{jm}(\Lambda_D)_{ikn}(Y_e)_{np}(\Lambda_D)^{rnp}(R_3)^{qj} + 2(Y_d)^{jm}(\Lambda_D)_{ikn}(Y_e)_{np}(\Lambda_D)^{rnp}(R_3)^{qj} \\
&- 4(Y_u)^{jm}(\Lambda_D)_{ikn}(\Lambda_D)_{mpq}(\Lambda_D)^{rnp}(R_4)^{qj} - 4(\Lambda_U)^{jm}(\Lambda_D)_{ikn}(\Lambda_D)_{mpq}(\Lambda_D)^{rnp}(R_4)^{qj} \\
&- 4(\Lambda_U)^{jm}(\Lambda_D)_{ikn}(\Lambda_D)_{mpq}(\Lambda_D)^{rnp}(R_4)^{qj} - 4(\Lambda_U)^{jm}(\Lambda_D)_{ikn}(\Lambda_D)_{mpq}(\Lambda_D)^{rnp}(R_4)^{qj} \\
&- 4(\Lambda_U)^{jm}(\Lambda_D)_{ikn}(\Lambda_D)_{mpq}(\Lambda_D)^{rnp}(R_4)^{qj} - 2|C_{ac}|^2(R_1)^{qj} \\
&- 4(\Lambda_U)^{jm}(\Lambda_D)_{ikn}(\Lambda_D)_{mpq}(\Lambda_D)^{rnp}(R_4)^{qj} - 2|C_{ac}|^2(R_1)^{qj} \\
&- 4(\Lambda_U)^{jm}(\Lambda_D)_{ikn}(\Lambda_D)_{mpq}(\Lambda_D)^{rnp}(R_4)^{qj} - 2|C_{ac}|^2(R_1)^{qj}
\end{align*}
\[-2(R_4)_{m}^{j} (A_D)_{nkl} (\Lambda_U)_{imn} C_{uw^c} + 2(R_7)_{m}^{j} (A_D)_{ikn} (Y_u)_{n}^{j} C_{uw^c} \]
\[-2(R_3)_{m}^{j} (A_E)_{lnk} (A_D)_{mn} C_{Q} - 2(R_3)_{m}^{j} (Y_e)_{kl} (Y_d)_{mn} C_{Q} \]
\[-2(R_9)_{m}^{j} (A_D)_{nkl} (Y_d)_{imn} C_{Q} + 2(R_4)_{m}^{j} (A_D)_{nkl} (A_D)_{mn} C_{Q} \]
\[+ 2(R_4)_{m}^{j} (A_D)_{nkl} (A_D)_{imn} C_{L} + 2(R_4)_{m}^{j} (A_D)_{ikn} (Y_u)_{j}^{m} C_{L} \]
\[+ 2(R_1)_{m}^{j} (A_D)_{nkl} (A_D)_{mn} C_{L} - 2(R_9)_{m}^{j} (A_D)_{nkl} (Y_u)_{j}^{m} C_{L} \]
\[-(R_3)_{m}^{j} (A_E)_{ikn} (A_D)_{mn} C_{L} - 2(R_3)_{m}^{j} (Y_e)_{kl} (Y_d)_{mn} C_{L} \]
\[-12(Y_u)_{n}^{j} (Y_u)_{mn} (A_U)_{npq} (A_D)_{rkp} (R_4)^{mp} + 4(Y_u)_{n}^{j} (Y_u)_{mn} (Y_a)_{p}^{n} (A_D)_{rkp} (R_7)^{mp} \]
\[+ 12(Y_u)_{n}^{j} (Y_u)_{mn} (A_D)_{npq} (A_D)_{rkp} (R_1)^{pm} - 12(Y_u)_{n}^{j} (Y_u)_{mn} (Y_a)_{p}^{n} (A_D)_{rkp} (R_9)^{pm} \]
\[-12(Y_u)_{n}^{j} (Y_u)_{mn} (A_D)_{npq} (A_D)_{rkp} (R_3)^{mp} - 12(Y_u)_{n}^{j} (Y_u)_{mn} (Y_a)_{p}^{n} (A_D)_{rkp} (R_3)^{mp} \]
\[+(A_D)_{m}^{j} (A_D)_{ikn} (Y_u)_{n}^{j} (A_D)_{qmr} (R_7)^{rq} - (Y_d)_{m}^{j} (A_D)_{ikn} (Y_u)_{n}^{j} (Y_d)_{rq} (R_7)^{rq} \]
\[-(A_D)_{m}^{j} (A_D)_{ikn} (Y_u)_{n}^{j} (A_D)_{rmp} (R_7)^{mr} - (Y_d)_{m}^{j} (A_D)_{ikn} (Y_u)_{n}^{j} (Y_d)_{mr} (R_7)^{mr} \]
\[-(A_D)_{m}^{j} (A_D)_{ikn} (Y_u)_{n}^{j} (Y_e)_{m}^{q} (R_2)^{q} - (Y_d)_{m}^{j} (A_D)_{ikn} (A_D)_{np} (Y_u)_{qr} (R_1)^{qr} \]
\[-3(Y_u)_{n}^{j} (A_D)_{nkl} (Y_u)_{m}^{n} (Y_u)_{qr} (R_5)^{qr} - 2(Y_u)_{n}^{j} (Y_e)_{np} (Y_u)_{ji} (A_D)_{ik} (R_7)^{nr} \]
\[-4(Y_u)_{n}^{j} (A_U)_{pnm} (A_U)_{nl}^{m} (A_D)_{ikr} (R_7)^{nr} + 4(A_D)_{n}^{j} (A_D)_{np} (A_U)_{jm} (A_D)_{ikr} (R_4)^{nr} \]
\[+ 4(Y_u)_{n}^{j} (Y_d)_{nm} (A_U)_{jl}^{m} (A_D)_{ikr} (R_4)_{q}^{mr} + 2(A_D)_{n}^{j} (A_D)_{np} (A_U)_{jm} (A_D)_{ikr} (R_1)^{rn} \]
\[-4(Y_u)_{n}^{j} (A_U)_{pnm} (A_U)_{jm} (A_D)_{ikr} (R_5)^{rn} + 4(Y_a)_{n}^{j} (A_D)_{npn} (A_D)_{ml} (A_E)_{rkl} (R_5)_{q}^{nr} \]
\[-4(Y_u)_{n}^{j} (A_D)_{pnm} (A_D)_{jm} (A_D)_{ikr} (R_2)^{r} - 4(Y_u)_{n}^{j} (Y_d)_{p}^{m} (Y_a)_{lr} (R_7)^{r} \]
\[+ 4(Y_a)_{n}^{j} (A_D)_{mnp} (Y_a)_{lc} (Y_a)_{kr} (R_5)^{nr} - 4(A_D)_{n}^{j} (A_D)_{pm} (A_D)_{jm} (A_D)_{ikr} (R_4)^{nr} \]
\[+ 4(A_D)_{m}^{j} (A_D)_{pnm} (Y_a)_{lc} (A_D)_{kr} (R_4)_{q}^{nr} - 18(Y_u)_{n}^{j} (Y_u)_{lr} (R_1)_{k}^{im} C_{L} \]
\[-(A_D)_{n}^{j} (A_E)_{nkl} (R_4)_{m}^{i} C_{uw^c} - 2(A_D)_{n}^{j} (A_D)_{ikn} (R_1)_{i}^{k} C_{uw^c} \]
\[-2(Y_u)_{n}^{j} (A_D)_{ikn} (R_9)_{m}^{i} C_{uw^c} - 4(A_U)_{n}^{j} (A_D)_{ikn} (R_4)_{m}^{i} C_{Q} \]
\[+ 2(Y_u)_{n}^{j} (A_D)_{ikn} (R_7)^{i} C_{Q} + 4(R_4)_{n}^{j} C_{Q} (A_U)_{j}^{p} (A_D)_{qki} \]
\[-2(R_7)_{n}^{j} C_{Q} (Y_a)_{g}^{j} (A_D)_{ik} (Y_u)_{jq} - 2(R_9)_{n}^{j} C_{uw^c} (A_D)_{jm} (A_D)_{qkl} \]
\[+ 2(R_9)_{n}^{j} C_{uw^c} (Y_a)_{g}^{j} (A_D)_{qkl} + 2(R_1)_{n}^{j} C_{uw^c} (A_D)_{j}^{q} (A_E)_{lk} \]
\[+ 2(R_3)_{n}^{j} C_{uw^c} (Y_a)_{g}^{j} (A_D)_{j}^{q} (A_E)_{lk} + 4(R_1)_{n}^{j} C_{uw^c} \]
\[+ 4(R_1)_{n}^{j} (C_Q)^{2} - 2(R_1)_{n}^{j} (A_U)_{lmn} (A_U)_{j}^{p} (A_D)_{q}^{n} \]
\[-2(R_1)_{n}^{j} (Y_u)_{nl} (Y_u)_{j}^{p} (Y_a)_{q}^{n} - 2(R_1)_{n}^{j} (Y_u)_{nl} (Y_a)_{j}^{p} (Y_a)_{q}^{n} \]
\[+(R_1)_{n}^{j} (A_D)_{nm} (A_D)_{jm} (A_D)_{j}^{q} (A_D)_{l}^{p} \]
\[-(R_1)_{n}^{j} (A_D)_{nm} (A_D)_{jm} (A_D)_{l}^{p} - (R_1)_{n}^{j} (A_D)_{nm} (A_D)_{j}^{q} (A_D)_{l}^{p} \]
\[-(R_1)_{n}^{j} (Y_u)_{lk} (Y_u)_{jm} (Y_u)_{im} (Y_u)_{jm} \]
\[-(R_1)_{ij}^{ij}(Y_d)_{im}(Y_d)_{jm}^m(\gamma H_1) - (R_1)_{ij}^{ij}(Y_d)_{in}(Y_d)_{nj}^n(\gamma D)^{ij}_{p}\]
\[+(R_1)_{ij}^{ij}(\Lambda_D)_{mn}(Y_d)_{jm}^{m} (\gamma H_{1,L}) + (R_1)_{ij}^{ij}(Y_d)_{in}(\Lambda_D)^{nn}(\gamma H_{1,L})^p\]
\[-6(Y_u)_{im} (Y_u)_{jn} (R_1)_{ij}^{ij} (\gamma Q)_{p}^{j} - 6(Y_u)_{ml} (Y_u)_{nl} (R_1)_{ij}^{ij} (\gamma Q)_{p}^{j}\]
\[-2(R_1)_{ij}^{ij} C_{w\nu}(\gamma Q)^{j}_{p} - 2(R_1)_{ij}^{ij} C_{Q\gamma}(\gamma Q)^{j}_{p} - 2(R_1)_{ij}^{ij} C_{LJ}(\gamma Q)^{j}_{p}\]
\[+2(\Lambda_D)_{mn}(\gamma D)^{ij}_{p} (R_1)_{ij}^{ij} + 2(Y_u)_{ij}(\Lambda_D)_{mn} (\gamma D)^{ij}_{p} (R_0)^{ij}\]
\[-2(\Lambda_D)_{ij}^{ij}(\gamma D)^{ij}_{p} (R_1)_{ij}^{ij} - 2(Y_u)_{ij}(\gamma D)^{ij}_{p} (R_2)^{ij}\]
\[+4(\Lambda_D)_{mn}(\gamma D)^{ij}_{p} (R_1)_{ij}^{ij} + 2(Y_u)_{ij}(\gamma D)^{ij}_{p} (R_1)^{ij}\]
\[+2(Y_u)_{ij}(\gamma D)^{ij}_{p} (R_3)^{ij} + 2(Y_u)_{ij}(\gamma D)^{ij}_{p} (R_2)^{ij}\]
\[-3(R_1)_{ij}^{ij}(\gamma D)^{mn}(\gamma D)^{ij}_{p} + 3(R_1)_{ij}^{ij}(\gamma D)^{nm}(\gamma D)^{ij}_{p}\]
\[+(R_1)_{ij}^{ij}(\gamma D)^{ij}_{p} + 3(Y_u)_{ij}(\gamma D)^{ij}_{p} (R_n)^{ij}\]
\[\frac{9}{100}g_i + \frac{18}{4}g_i^2 + 2(R_1)_{ij}^{ij}\frac{132}{75}g_i^3 - 4g_i^4\]
\[+2(R_1)_{ij}^{ij}\frac{33}{300}g_i - \frac{3}{4}g_i^2 - 4g_i^3\]  

\[(16\pi^2)^2(\beta_R^{(2)})_{ik}^{ij} = \frac{6(Y_d)^{mn}(Y_u)_{jn}(Y_u)_{jn}^n C_{w\nu} (R_5)^{ij}}{-12(Y_u)^{im}(Y_u)_{jn} (\Lambda_D)_{mn} (\gamma D)^{ij}_{p} + 12(Y_u)^{mn}(Y_u)_{jn}(\gamma D)^{ij}_{p}\]
\[-12(Y_u)^{im}(Y_u)_{jn} (\Lambda_D)_{mn} (\gamma D)^{ij}_{p} - 4(R_2)^{ij}[C_{H_2}^2 - 6(R_2)^{ij}(Y_u)_{jm}(Y_u)_{jm}^m C_{H_1}\]
\[-2[C_{H_1}]^2(R_2)^{ij} - 2[C_{c\nu}]^2(R_2)^{ij} + 4(R_2)C_{H_2} C_{H_1}\]
\[-4(R_2)^{ij} C_{c\nu} C_{H_2} - 6(R_3)^{ij}(Y_u)_{jm}(Y_u)_{jm}^m C_{H_2}\]
\(-6(Y_e)^i(A_D)_{nm}(Y_u)^{(n)}(Y_u)_{rp}(R_\theta)^{mp} + 6(Y_e)^i(A_D)_{nm}(A_D)^{npq}(Y_u)_{rp}(R_\tau)^{mp} \)
\(-6(Y_e)^i(A_D)_{nm}(A_U)^{npq}(Y_u)_{rp}(R_4)^{mp} + 6(Y_e)^i(A_D)_{nm}(Y_u)^{(n)}(Y_u)_{rp}(R_\tau)^{mp} \)
\(-12(Y_e)^i(A_D)_{mn}(Y_d)^{(m)}(Y_u)_{ls}(R_\theta)^{nr} - 12(A_\Lambda)^{ipq}(A_D)_{mn}(Y_d)^{(m)}(Y_u)_{ls}(R_\tau)^{nr} \)
\(-18(Y_e)^i(A_D)_{ln}(R_\tau)^{ml}C_{H_2} - 18(Y_e)^i(A_\Lambda)_{ln}(R_\tau)^{ml}C_{H_2} \)
\(-6(Y_e)^i(Y_u)_{al}(R_2)^{j}C_{H_2} - 6(Y_d)^{lm}(Y_d)_{ln}(R_3)^{ml}C_{e} \)
\(+6(R_3)^iC_{e}(Y_d)^{(q)}(Y_u)_{ql} + 4(R_2)^i[C_{e}]^2 + 4(R_2)^i[C_{H_2}]^2 \)
\(-2(R_2)^i(A_\Lambda)_{lnn}(Y_e)^{(m)}(\gamma_L)^{\eta} - 2(R_2)^i(Y_e)^{(n)}(Y_e)^{(m)}(\gamma_L)^{\eta} \)
\(+3(R_2)^i(Y_e)^{(m)}(Y_e)^{(n)}(\gamma_Q)^{\eta} - (R_2)^i(Y_e)^{(n)}(Y_e)^{(m)}(\gamma_L)^{\eta} \)
\(+(R_3)^i(A_\Lambda)_{nmn}(Y_e)^{(m)}(\gamma_L)^{\eta} = (R_3)^i(Y_e)^{(n)}(Y_e)^{(m)}(\gamma_L)^{\eta} \)
\(+3(R_3)^i(A_D)_{nmn}(Y_e)^{(m)}(\gamma_D)^{\eta} + 3(R_3)^i(A_D)_{nmn}(Y_d)^{(m)}(\gamma_Q)^{\eta} \)
\(+2(R_2)^i(Y_e)^{(m)}(Y_e)^{(n)}(\gamma_LH_1)^{\eta} = (R_2)^i(Y_e)^{(n)}(Y_e)^{(m)}(\gamma_LH_1)^{\eta} \)
\(+6(A_D)_{nmn}(Y_e)^{(n)}(R_\tau)^{ml}(\gamma_Q)^{\eta} = 2(R_2)^i(Y_e)^{(n)}(R_2)^{p}(\gamma_D)^{\eta} \)
\(-2(Y_e)^{(n)}(Y_e)^{(m)}(\gamma_H_1) = 2(Y_e)^{(n)}(Y_e)^{(m)}(R_2)^{p}(\gamma_D)^{\eta} \)
\(-2(A_\Lambda)_{lnn}(Y_e)^{(m)}(R_\tau)^{ml}(\gamma_L)^{\eta} = 6(A_D)_{nmn}(Y_e)^{(n)}(R_\tau)^{ml}(\gamma_D)^{\eta} \)
\(-6(A_D)_{nmn}(Y_e)^{(n)}(R_\tau)^{ml}(\gamma_Q)^{\eta} = 2(R_2)^i(Y_e)^{(n)}(R_2)^{p}(\gamma_LH_1)^{\eta} \)
\(-2(Y_e)^{(n)}(Y_e)^{(m)}(R_\tau)^{ml}(\gamma_LH_1)^{\eta} = 2(R_2)^i(C_{e}(\gamma_L)^{\eta} - 2(R_2)^i[C_{H_1}(\gamma_H_1)] \)
\(-2(R_2)^i[C_{H_2}(\gamma_H_2)] = 6(Y_d)^{lm}(Y_u)^{(n)}(\gamma_Q)^{\eta} \)
\(-6(Y_d)^{lm}(Y_u)_{ln}(R_3)^{nl}(\gamma_Q)^{\eta} = 6(Y_d)^{lm}(Y_u)_{ln}(\gamma_Q)^{\eta}(R_3)^{ml} \)
\(-3(R_2)^i(Y_e)^{(n)}(Y_e)^{(m)}(\gamma_U)^{\eta} = 3(R_2)^i(Y_e)^{(n)}(Y_e)^{(m)}(\gamma_Q)^{\eta} \)
\(-2(R_2)^i[C_{H_2}(\gamma_H_2)] \)
\(= -4(Y_d)^{lp}(A_\Lambda)^{kpm}(C_{Q}(R_0)^{ni} + 4(A_\Lambda)^{lp}(A_D)^{kpm}(C_{Q}(R_1)^{ni}) \)
\(-4(Y_e)^{li}(Y_d)^{nk}(C_{Q}(R_1)^{ni} + 4(Y_u)^{pi}(A_D)^{kpm}(C_{L}(R_3)^{nj}) \)
\(-4(A_U)^{lp}(A_U)^{nph}(C_{e}(R_3)^{nj}) - 4(Y_e)^{li}(A_D)^{kpm}(R_0)^{ml}C_{H_1} \)
\(+4(A_\Lambda)^{npq}(A_D)^{kpm}(R_1)^{ml}C_{L} - 4(Y_e)^{np}(Y_d)^{(p)}(R_2)^{j}C_{H_2} + 4(Y_e)^{np}(A_D)^{kpm}(R_3)^{nj}C_{H_2} \)

\((16\pi^2)^2(\beta_{K_3}^{(2)})^{ij}_{K_3} = -4(Y_d)^{lp}(A_\Lambda)^{kpm}(C_{Q}(R_0)^{ni} + 4(A_\Lambda)^{lp}(A_D)^{kpm}(C_{Q}(R_1)^{ni}) \)

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\[+4(Y_e)^{mj}(Y_d)_{nk}(\Lambda_E)_{qmn}(\Lambda_D)^{prq}(R_{ij})^q = 4(Y_e)^{mj}(Y_d)_{nk}(\Lambda_E)_{pq}(R_{ij})^q - 4(Y_e)^{mj}(Y_d)_{nk}(\Lambda_E)_{mq}(Y_d)^{r_n}(R_{ij})^q
\]

\[4(Y_e)^{ij}(\Lambda_D)_{kln}(Y_e)_{pq}(\Lambda_D)^{prq}(R_{ij})^q + 4(Y_e)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)_{qmn}(\Lambda_D)^{rpq}(R_{ij})^q
\]

\[4(Y_e)^{jmil}(\Lambda_D)_{kln}(Y_e)_{mq}(Y_d)^{n_r}(R_{ij})^q + 4(Y_e)^{mj}(Y_d)_{nk}(\Lambda_D)^{prq}(R_{ij})^q
\]

\[4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)_{mq}(\Lambda_D)_{prq}(R_{ij})^q + 4(Y_e)^{mj}(Y_d)_{nk}(\Lambda_D)^{prq}(R_{ij})^q
\]

\[-4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)_{pq}(Y_d)^{mpr}(R_{ij})^q - 4(Y_e)^{ij}(\Lambda_D)_{kln}(Y_e)_{qp}(\Lambda_D)^{prq}(R_{ij})^q
\]

\[-4(Y_e)^{ij}(\Lambda_D)_{kln}(Y_e)_{qp}(Y_d)^{opr}(R_{ij})^q + 4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_D)_{pq}(Y_d)^{opr}(R_{ij})^q
\]

\[-8(\Lambda_U)^{jmil}(\Lambda_U)_{nk}(\Lambda_U)_{qpm}(\Lambda_U)^{nrp}(R_{ij})^q - 12(Y_e)^{ij}(\Lambda_D)_{kln}(Y_e)_{pq}(\Lambda_D)^{nrp}(R_{ij})^q
\]

\[-12(Y_e)^{ij}(Y_d)_{ik}(Y_e)_{pq}(\Lambda_D)^{nrp}(R_{ij})^q - 4(Y_e)^{mj}(Y_d)_{nk}(\Lambda_D)_{pq}(Y_d)^{opr}(R_{ij})^q
\]

\[-4(R_{ij})^{ij}(C_{dc})^2 - 2[C_{dc}]^2(R_{ij})^q + 2[C_{dc}]^2(R_{ij})^q
\]

\[-4(R_{ij})^{ij}C_{dc}C_{dc} - 4(R_{ij})^{ij}C_{dc}C_{dc}
\]

\[2(R_{ij})^{ij}(\Lambda_U)_{nk}(Y_e)^{m}C_{we} - 4(R_{ij})^{ij}(Y_d)_{nk}(Y_e)^{m}C_{we}
\]

\[4(R_{ij})^{ij}(\Lambda_U)_{nk}(\Lambda_U)^{nm}C_{we} - 4(R_{ij})^{ij}(\Lambda_D)_{nk}(Y_e)^{m}C_{we}
\]

\[4(R_{ij})^{ij}(\Lambda_D)_{nk}(\Lambda_E)^{nm}C_{we} - 4(R_{ij})^{ij}(\Lambda_D)_{nk}(Y_e)^{m}C_{we}
\]

\[4(\Lambda_E)^{jmil}(\Lambda_D)_{nk}(\Lambda_D)^{nrp}(R_{ij})^q - 2(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(Y_e)^{m}(\Lambda_E)_{rqp}(R_{ij})^q
\]

\[2(Y_e)^{ij}(\Lambda_D)_{kln}(Y_e)^{m}(\Lambda_E)_{rqp}(R_{ij})^q + 4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_D)^{nrp}(R_{ij})^q
\]

\[-6(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)^{nm}C_{we} - 4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(Y_e)^{m}C_{we}
\]

\[-4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)^{nm}C_{we} - 4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(Y_e)^{m}C_{we}
\]

\[4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)_{pq}(Y_d)^{opr}(R_{ij})^q - 4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_D)^{nrp}(R_{ij})^q
\]

\[4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)_{pq}(Y_d)^{opr}(R_{ij})^q - 4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_D)^{nrp}(R_{ij})^q
\]

\[4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)_{pq}(Y_d)^{opr}(R_{ij})^q - 4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_D)^{nrp}(R_{ij})^q
\]

\[4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)_{pq}(Y_d)^{opr}(R_{ij})^q - 4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_D)^{nrp}(R_{ij})^q
\]

\[4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)_{pq}(Y_d)^{opr}(R_{ij})^q - 4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_D)^{nrp}(R_{ij})^q
\]

\[4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)_{pq}(Y_d)^{opr}(R_{ij})^q - 4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_D)^{nrp}(R_{ij})^q
\]

\[4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)_{pq}(Y_d)^{opr}(R_{ij})^q - 4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_D)^{nrp}(R_{ij})^q
\]

\[4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)_{pq}(Y_d)^{opr}(R_{ij})^q - 4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_D)^{nrp}(R_{ij})^q
\]

\[4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)_{pq}(Y_d)^{opr}(R_{ij})^q - 4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_D)^{nrp}(R_{ij})^q
\]

\[4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)_{pq}(Y_d)^{opr}(R_{ij})^q - 4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_D)^{nrp}(R_{ij})^q
\]

\[4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)_{pq}(Y_d)^{opr}(R_{ij})^q - 4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_D)^{nrp}(R_{ij})^q
\]

\[4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)_{pq}(Y_d)^{opr}(R_{ij})^q - 4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_D)^{nrp}(R_{ij})^q
\]

\[4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)_{pq}(Y_d)^{opr}(R_{ij})^q - 4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_D)^{nrp}(R_{ij})^q
\]

\[4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)_{pq}(Y_d)^{opr}(R_{ij})^q - 4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_D)^{nrp}(R_{ij})^q
\]

\[4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)_{pq}(Y_d)^{opr}(R_{ij})^q - 4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_D)^{nrp}(R_{ij})^q
\]

\[4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_E)_{pq}(Y_d)^{opr}(R_{ij})^q - 4(\Lambda_E)^{jmil}(\Lambda_D)_{kln}(\Lambda_D)^{nrp}(R_{ij})^q
\]
\[-2(R_3)^i j C_d (\gamma D)^i_k + 4(\Lambda_E)^j m l (\Lambda_D)_{k p n} (\gamma L)^n_p (R_3)^{p i}\]
\[-4(Y_e)^m j (Y_d)_{p k} (\gamma H^1) (R_1)^{p m} - 4(Y_e)^j i (\Lambda_D)_{k n p} (\gamma L)^n_i (R_0)^{p i}\]
\[-4(\Lambda_U)^{i m l} (\Lambda_U)_{p m k} (\gamma D)^i_p (R_3)^{p m} - 4(Y_e)^j i (Y_d)_{n k} (\gamma Q)^n_i (R_2)^j\]
\[+4(\Lambda_E)^j m l (\Lambda_D)_{k l n} (R_1)^{p i} (\gamma L)^p_m - 4(Y_e)^m j (Y_d)_{n k} (R_1)^{p i} (\gamma L)^p_m\]
\[-4(Y_e)^j i (\Lambda_D)_{k l n} (R_3)^{m i} (\gamma H^1) - 4(\Lambda_U)^{i m l} (\Lambda_U)_{n l k} (R_3)^{p i} (\gamma D)^p_m\]
\[+4(\Lambda_E)^j m l (\Lambda_D)_{k l n} (R_0)^{m i} (\gamma L H^1)_m - 4(Y_e)^j i (\Lambda_D)_{k l n} (R_1)^{m i} (\gamma L H^1)_m\]
\[-4(Y_e)^j i (Y_d)_{i k} (R_2)^j (\gamma H^2) + 4(Y_e)^j i (\Lambda_D)_{k n l} (R_2)^j (\gamma H^2)\]
\[+4(\Lambda_E)^j m l (\Lambda_D)_{k l n} (\gamma Q)^n_p (R_1)^{p m} - 4(Y_e)^m j (Y_d)_{n k} (\gamma Q)^n_p (R_1)^{p m}\]
\[-4(Y_e)^j i (\Lambda_D)_{k l n} (\gamma Q)^n_p (R_0)^{p m} + 4(Y_e)^m j (\Lambda_D)_{k l n} (\gamma Q)^n_p (R_1)^{p m}\]
\[+4(\Lambda_U)^{i m l} (\Lambda_U)_{n l k} (\gamma U)^p_i (R_3)^{p m} - 4(Y_e)^j i (Y_d)_{i k} (\gamma H^1) (R_2)^j\]
\[+4(Y_e)^j i (\Lambda_D)_{k n l} (\gamma L)^n_i (R_5)^{p j} + 4(Y_e)^j i (\Lambda_D)_{k n l} (\gamma L H^1)^n_i (R_2)^j\]
\[+4(Y_e)^j i (Y_d)_{i k} (\gamma L H^1) p (R_3)^{p j} - 2(R_3)^i j (\Lambda_D)^{i m n} (\Lambda_D)_{k p m} (\gamma Q)^n\]
\[+2(R_3)^j i (\Lambda_D)^{i m n} (\Lambda_D)_{k p m} (\gamma L)^n_i - 2(R_3)^j i (Y_d)_{i k} (\gamma Q)^n_i (\gamma L)^n_i\]
\[-2(R_3)^i j (Y_d)_{i k} C_Q (R_3)^{i n} - 2(R_3)^j i (\Lambda_D)_{m p k} (\gamma D)^{i n}_m + 2(R_3)^i j (Y_e)^m l (\Lambda_D)^{i m n} (\gamma L H^1)^{i n}\]
\[+2(R_3)^j i (\Lambda_D)^{i m n} (\Lambda_D)_{m p k} (\gamma D)^{i n}_m + 2(R_3)^j i (Y_e)^m l (\Lambda_D)^{i m n} (\gamma L H^1)^{i n}\]
\[+2(R_3)^j i (\Lambda_D)^{i m n} (Y_d)_{m k} (\gamma L H^1)^n_m + 2(R_3)^j i \left[\frac{99}{25} g_4^i - 4 g_4^i\right]\]
\[-2(R_3)^i k \left[\frac{33}{75} g_4^k - 4 g_3^k\right] + 2(R_3)^j i \left[\frac{132}{75} g_4^k - 4 g_3^k\right]\]

\[(16\pi^2)\left(\beta^{(2)}_{R_4}\right)_k = -2(Y_e)^j p (\Lambda_U)_{n p k} C_d (R_2)^{i n} + 2(Y_d)^j p (\Lambda_U)_{n p k} C_d (R_2)^{i n}\]
\[-2(\Lambda_D)^{j p l} (\Lambda_D)_{n p k} C_d (R_1)^{i n} - 2(\Lambda_D)^{j p l} (\Lambda_D)_{k p m} C_Q (R_4)^{i n}\]
\[-2(Y_d)^j i (Y_d)_{n k} C_Q (R_3)^{i n} - 2(Y_u)^j p (\Lambda_U)_{p m k} (R_7)^{i n} C_{H^2}\]
\[+2(Y_d)^j p (\Lambda_U)_{m p k} (R_0)^{i m} C_{H^1} - 2(\Lambda_D)^{j l i} (\Lambda_U)_{n p k} (R_1)^{i m} C_L\]
\[+2(\Lambda_D)^{n p j} (\Lambda_D)_{k p m} (R_4)^{i m} C_{d c} + 2(Y_d)^j n (Y_d)_{n k} (R_4)^{i m} C_{d c}\]
\[+4(Y_d)^j l (\Lambda_D)_{n l k} (Y_d)_{j p} (\Lambda_U)^{n r p} (R_4)^{i q} + 4(\Lambda_D)^{i m j} (\Lambda_U)_{n l k} (\Lambda_D)_{p m q} (\Lambda_U)^{n r p} (R_4)^{i q}\]
\[-4(Y_e)^j l (\Lambda_U)_{l n k} (Y_u)_{j p} (\Lambda_U)^{n r p} (R_4)^{i q} - 2(\Lambda_D)^{j m l} (\Lambda_D)_{k l n} (\Lambda_D)_{m p q} (\Lambda_D)^{r p n} (R_4)^{i q}\]
\[-2(\Lambda_D)^{m l j} (\Lambda_D)_{k l n} (Y_d)_{j q m} (Y_d)^{n r} (R_4)^{i q} - 2(Y_e)^j m (Y_d)_{n k} (\Lambda_D)^{r p n} (R_4)^{i q}\]
\[-2(Y_d)^j m (Y_d)_{n k} (Y_d)_{q m} (Y_d)^{n r} (R_4)^{i q} - 2(Y_e)^j l (\Lambda_U)_{l n k} (Y_u)_{j p} (Y_d)^{n r p} (R_4)^{i q}\]
\[+2(Y_e)^j l (\Lambda_U)_{l n k} (Y_u)_{j p} (\Lambda_D)^{n r p} (R_1)^{i q} - 2(\Lambda_D)^{j m l} (\Lambda_D)_{l n k} (\Lambda_D)^{r p n} (R_0)^{i q}\]
\[-2(R_4)_{ij}^i C_Q(\gamma Q)^j - 4(R_4)_{ij}^i C_{d_c}(\gamma D)^j_k + 2(A_D)^{ml}(A_D)_{knp}(\gamma L)^n_{(R_4)^p}m
\]
+2(Y_4)^{lm}(Y_d)_{pk}(\gamma H_i)(R_4)^p_k - (A_D)^{ml}(Y_d)_{pk}(\gamma H_i)(R_4)^p_k
\]
\[-2(Y_4)^{lm}(A_D)_{knp}(\gamma L H_i)^n_{(R_4)^p}m - 2(Y_4)^{ji}(A_U)_{npk}(\gamma U)^i_{(R_7)^p}
\]
\[-2(A_D)^{lmj}(A_U)_{pkn}(\gamma D)^j_{(R_1)^p}k - 2(Y_4)^{ji}(A_U)_{pkn}(\gamma D)^j_{(R_1)^p}k
\]
\[-2(A_D)^{mlj}(A_D)_{kln}(\gamma D)^l_{(R_4)^p}n - 2(Y_4)^{jm}(Y_d)_{nkl}(\gamma D)^m_{(R_4)^p}l
\]
\[-2(Y_4)^{ji}(A_U)_{lkn}(\gamma R)^i_{(R_7)^p}n - 2(A_D)^{imj}(A_U)_{lkn}(\gamma R)^i_{(R_7)^p}n
\]
\[-2(A_D)^{imj}(A_U)_{lkn}(\gamma L H_i)^n_{(R_4)^p}n + 2(Y_4)^{ji}(A_U)_{nlk}(R_i)^n_{(R_7)^p}
\]
\[-2(Y_4)^{ji}(A_U)_{nlk}(R_i)^n_{(R_7)^p}n - 2(A_D)^{mlj}(A_D)_{klm}(\gamma U)^i_{(R_7)^p}
\]
\[-2(Y_4)^{ji}(A_U)_{nlk}(\gamma R)^i_{(R_7)^p}l - 2(A_D)^{ij}(A_D)_{nlk}(\gamma R)^i_{(R_7)^p}l
\]
\[-2(Y_4)^{ji}(A_U)_{nlk}(\gamma U)^i_{(R_7)^p}n - 2(Y_4)^{jl}(Y_d)_{nlk}(\gamma U)^l_{(R_7)^p}
\]
\[-2(Y_4)^{ji}(A_U)_{nlk}(\gamma L H_i)^n_{(R_4)^p}l + 2(A_D)^{imj}(A_U)_{nlk}(\gamma L H_i)^n_{(R_4)^p}l
\]
\[\frac{(16\pi^2)^2 (\beta_{R_5}^{(2)})_{ij}^{(1)}}{6} = +6(A_D)^{lp}(Y_u)_{pm} C_{uv}(R_3)^i_{m} + 6(A_D)^{np}(Y_u)_{pm}(R_3)^i_{n} C_{dc}
\]
\[-12(A_D)^{ml}(Y_u)_{ln}(A_D)_{npk}(Y_q)_{m}^n (R_3)^p_{l} - 12(A_D)^{ml}(Y_u)_{ln}(Y_d)_{pm}(Y_u)_{pm}(R_2)^i_{j}
\]
\[12(A_D)^{ml}(Y_u)_{ln}(A_U)^{npk}(Y_q)_{m}^n (R_3)^p_{l} - 4(R_5)^{ij}[C_{H_2}]^2
\]
\[-2[C_{H_2}]^2 (R_3)^i_{j} - 2[C_{H_2}]^2 (R_3)^i_{j} - 4(R_5)^{ij}C_{uv} C_{H_2}
\]
\[6(R_3)_{il}^j(Y_u)_{nl}(A_U)_{lm}(C_{H_2} + 6(R_3)_{il}^j(Y_u)_{nl}(A_D)_{lm} C_{H_2}
\]
\[+12(Y_u)^{ji}(Y_u)_{mn}(A_D)^{nq}(Y_u)_{pr}(R_4)^q_{p} + 6(Y_u)^{ji}(Y_u)_{mn}(A_D)^{nq}(Y_u)_{pr}(R_1)^{mp}_{q}
\]
\[-6(Y_u)^{ji}(Y_u)_{mn}(Y_u)_{np}(Y_q)_{m}^n (R_3)^p_{l} - 12(A_D)^{ji}Y_{mn}(A_D)_{npk}(Y_u)_{pm}(R_2)^i_{j}
\]
\[12(Y_u)^{ji}(Y_u)_{mn}(A_D)^{nq}(Y_u)_{pr}(R_3)^p_{l} + 12(Y_u)^{ji}(Y_u)_{mn}(A_D)^{nl}(Y_u)_{npk}(Y_4)^i_{q}
\]
\[+6(A_D)^{mi}(Y_u)_{ln}(R_3)^i_{m} C_{dc} - 6(R_3)^{ij}C_{uv}(A_D)^{miq}(Y_u)_{q}
\]
\[+4(R_5)^{ij}[C_{H_2}]^2 - 2(R_5)^{ij}(A_U)_{lm}(A_D)^{npm}(\gamma L)^n_{p}
\]
\[-2(R_5)^{ij}(A_U)_{lm}(Y_u)^{mj}(\gamma L H_i)^n_{p} - 2(R_5)^{ij}(Y_u)_{nl}(Y_e)^{pj}(\gamma L)^n_{p}
\]
\[-(R_5)^{ij}(A_D)_{nml}(\gamma L)^{pmi}(\gamma L)^n_{p} - (R_5)^{ij}(Y_e)_{ln}(Y_e)^{pj}(\gamma L)^n_{p}
\]
\begin{align}
(16\pi^2)^2(\beta_{R^2}^{(2)})^{ij} &= +2(Y_u)^{ip}(Y_u)_{np}C_Q(R_5)^{nj} + 4(\Lambda_U)^{pj}(Y_u)_{np}C_Q(R_5)^{in} \\
&+ 2(\Lambda_D)^{jp}(Y_u)_{pm}C_{\varphi_c}(R_1)^{in} - 2(Y_d)^{pi}(Y_u)_{pm}C_{\varphi_c}(R_9)^{in} \\
&+ 2(Y_u)^{ip}(Y_u)_{mp}(R_T)^{mj}C_{H_2} + 4(\Lambda_D)^{pjn}(Y_u)_{mp}(R_3)^{in}C_{d_c} \\
&+ 2(\Lambda_D)^{jnp}(Y_u)_{pm}(R_1)^{in}C_L - 2(Y_d)^{pi}(Y_u)_{pm}(R_9)^{in}C_{H_1} \\
&- 2(Y_u)^{il}(Y_u)_{nt}(Y_u)_{qr}(R_T)^{qj} + 4(\Lambda_U)^{mj}(Y_u)_{nt}C_{\varphi qos}(\Lambda_D)^{prn}(R_1)^{qg} \\
&- 4(\Lambda_U)^{jmn}(Y_u)_{nt}(\Lambda_U)^{qmn}(Y_d)^{np}(R_3)^{qg} - 4(\Lambda_D)^{jmn}(Y_u)_{tm}(\Lambda_U)^{nmq}(\Lambda_G)^{rup}(R_4)^{qg} \\
&- 2(C_Q)^{j}(R_T)^{ij} - 2(C_{d_c})^2(R_T)^{ij} - 4(R_T)^{ij}C_{d_c}C_{H_2} \\
&- 4(R_T)^{ij}C_QC_{H_2} - 4(R_4)^{il}(Y_u)_{nt}(\Lambda_U)^{nmj}C_{d_c} \\
&- 2(R_9)^{il}(Y_u)_{nt}(Y_d)^{nj}C_{d_c} - 2(R_1)^{il}(Y_u)_{nt}(\Lambda_D)^{jmn}C_{d_c} \\
&+ 2(R_4)^{il}(Y_u)_{nt}(Y_d)^{nj}C_{d_c} + 2(R_4)^{il}(Y_u)_{nt}(\Lambda_D)^{jmn}C_{d_c} \\
&- 2(R_T)^{ij}(Y_u)_{nt}(Y_d)^{nj}C_{H_2} - 3(Y_u)^{il}(Y_u)_{nt}(Y_d)^{nj}(Y_u)_{qr}(R_9)^{qg} \\
&+ 3(Y_u)^{il}(Y_u)_{nt}(\Lambda_D)^{jmn}(Y_u)_{qr}(R_1)^{qj} + 3(\Lambda_D)^{jml}(Y_u)_{nt}(Y_u)^{mn}(\Lambda_D)^{jnm}(R_T)^{qg} \\
&+ 3(Y_d)^{ij}(Y_u)_{nt}(Y_u)^{nj}(R_T)^{qj} - (\Lambda_D)^{jml}(Y_u)_{nt}(R_3)^{qg}(R_3)^{qg} \\
&- (Y_d)^{ij}(Y_u)_{nt}(Y_u)^{nj}(Y_u)_{qr}(R_3)^{qg} - (\Lambda_D)^{jml}(Y_u)_{nt}(Y_u)^{nj}(Y_u)_{mr}(R_3)^{qg} \\
&+ 4(\Lambda_D)^{qpr}(\Lambda_E)^{jnp}(\Lambda_U)^{imj}(Y_u)_{nt}(R_1)^{qg} - 4(Y_d)^{ip}(\Lambda_U)^{jmn}(\Lambda_U)^{im}(Y_u)_{nt}(R_1)^{qg} \\
&- 2(\Lambda_D)^{qpr}(\Lambda_E)^{jnp}(\Lambda_D)^{jml}(Y_u)_{nt}(R_3)^{qg} - 2(\Lambda_D)^{qpr}(Y_d)_{nt}(Y_u)^{nj}(R_3)^{qg}. 
\end{align}
\[\begin{align*}
-2(Y_d)^{ij}(Y_e)_{mn}(A_D)^{imj}(Y_u)_{nr}(R_3)^{ln} &+ 4(A_D)^{jpp}(A_D)_{npn}(A_U)^{lmj}(Y_e)_{ul}(R_4)^{nr} \\
+ 4(Y_d)^{ij}(Y_d)_{np}(A_D)^{imj}(Y_u)_{nl}(R_3)^{nr} &- 4(A_D)^{jpp}(Y_d)_{pn}(Y_u)_{id}(Y_d)_{rl}(R_3)^{nr} \\
+ 2(Y_d)^{ij}(Y_d)_{pl}(Y_u)_{il}(R_9)^{pr} &+ 2(Y_u)_{il}(R_7)^{ij}C_{dc} \\
+ 4(A_U)^{imj}(Y_u)_{nl}(R_3)^{ln}C_Q &+ 2(A_D)^{jml}(Y_u)_{nq}(R_1)^{im}C_Q \\
-2(Y_d)^{ij}(Y_u)_{ln}(R_9)^{in}C_Q &- 2(R_1)^{iln}C_Q(A_D)^{jmq}(Y_u)_{qf} \\
+ 2(R_9)^{il}C_Q(Y_d)^{qi}(Y_u)_{qf} &+ 4(R_4)^{il}C_Q(A_U)^{pmj}(Y_u)_{qf} \\
+ 2(R_7)^{ij}C_{dc}(Y_u)^{iq}(Y_u)_{ln} &+ 4(R_7)^{ij}[C_{dc}]^2 + 4(Y_u)^{ij}[C_Q]^2 \\
-2(R_7)^{ij}(Y_d)_{nl}(Y_d)^{im}(\gamma_{H_2}) &+ 2(R_7)^{ij}(Y_d)_{nl}(A_D)^{jpm}(\gamma_{LH_1})_p \\
-2(R_7)^{ij}(A_D)^{nml}(A_D)^{jpm}(\gamma_{L})_p &- 2(R_7)^{ij}(A_U)_{nmj}(A_U)^{jpmj}(\gamma_D)_p \\
-2(R_7)^{ij}(A_U)^{nml}(A_U)^{jpmj}(\gamma_U)_p &- (R_7)^{ij}(Y_u)_{nl}(Y_u)^{im}(\gamma_{H_2}) \\
-(R_7)^{ij}(Y_u)_{ln}(Y_u)^{ip}(\gamma_U)_p &- 2(Y_u)_{il}C_Q(\gamma_D)_l \\
-2(Y_u)^{ij}C_Q(\gamma_Q)_l &- 2(R_7)^{ij}[C_{H_2} \gamma_{H_2}] \\
+ 2(Y_u)^{ij}(Y_d)_{pl}(\gamma_U)_p^2(R_7)^{pj} &+ 2(A_D)^{jml}(Y_u)_{np}(\gamma_Q)_p^2(R_4)^{lp} \\
-2(Y_d)^{ij}(Y_u)_{np}(\gamma_Q)_p^2(R_4)^{lp} &+ 4(A_U)^{lmj}(Y_u)_{pl}(\gamma_U)_p^2(R_4)^{lp} \\
+ 2(Y_u)^{ij}(Y_u)_{nl}(R_7)^{ij}(\gamma_{H_2}) &+ 4(A_U)^{lmj}(Y_u)_{nl}(R_4)^{pl}(\gamma_D)_p \\
+ 2(A_D)^{mlj}(Y_u)_{nq}(R_1)^{in}(\gamma_{LH_1})_m &+ 2(A_D)^{jml}(Y_u)_{ln}(R_9)^{in}(\gamma_{LH_1})_m \\
-2(Y_d)^{ij}(Y_u)_{ln}(R_1)^{in}(\gamma_{LH_1})_m &- 2(Y_d)^{ij}(Y_u)_{ln}(R_9)^{in}(\gamma_{H_2}) \\
+ 2(Y_u)^{ij}(Y_u)_{nl}(\gamma_{H_2})_p^2(R_7)^{pj} &+ 2(A_D)^{jml}(Y_u)_{ln}(\gamma_U)_p^2(R_1)^{ln} \\
-2(Y_d)^{ij}(Y_u)_{ln}(\gamma_U)_p^2(R_1)^{ln} &+ 4(A_U)^{lmj}(Y_u)_{ln}(\gamma_U)_p^2(R_1)^{ln} \\
-3(R_7)^{ij}(Y_u)^{mn}(Y_u)_{np}(\gamma_U)_p^2 &- 3(R_7)^{ij}(Y_u)^{mn}(Y_u)_{np}(\gamma_Q)_p^2 \\
-2(R_7)^{ij}[\frac{33}{100}g_1^4 + \frac{3}{4}g_2^2] &+ 2(R_7)^{ij}[\frac{99}{25}g_1^4] \\
+ 2(R_7)^{ij}[\frac{33}{100}g_1^4 + \frac{3}{4}g_2^2] \\
\end{align*}\]
Two-loop $\beta$-functions for the various $\phi\phi^*$ mass terms are given by:

\begin{align}
(16\pi^2)^j \beta^{[2]}_{mQ_j} &= (16\pi^2)^k \beta^{[2]}_{MSSM} - 2(Y_u)_{jk} (Y_u)^{pq} (R_1)^{ik} (R_1)_{pq} - 2(Y_u)_{jk} (Y_u)^{pq} (R_0)^{ik} (R_0)_{pq} \\
-2(Y_a)_{jk} (Y_a)^{pq} (Y_a)^{ik} (R_1)_{pq} - 2(Y_a)_{jk} (Y_a)^{pq} (R_0)^{ik} (R_0)_{pq} \\
-(R_a)^{ij} (Y_a)^{im} (Y_a)^{j} (Y_a)_{pq} - (R_a)^{ij} (Y_a)^{im} (Y_a)_{pq} \\
+(R_a)^{ij} (Y_a)^{im} (Y_a)^{j} (Y_a)_{pq} - (R_a)^{ij} (Y_a)^{im} (Y_a)_{pq} \\
-(R_a)^{ij} (Y_a)^{im} (Y_a)^{j} (Y_a)_{pq} - (R_a)^{ij} (Y_a)^{im} (Y_a)_{pq} \\
+(R_a)^{ij} (Y_a)^{im} (Y_a)^{j} (Y_a)_{pq} - (R_a)^{ij} (Y_a)^{im} (Y_a)_{pq} \\
-6(Y_u)_{ij} (Y_u)^{im} (R_1)^{jp} (Y_a)_{pq} - 2(R_a)^{ij} C_{\phi\phi^*} (Y_a)_{pq} \\
(22)
\end{align}
\[-2(Y_d)^{i k}(\Lambda_D)_{k m p}(R_9)^{j m}(R_1)^{m n} + 2(Y_d)^{i k}(Y_d)_{p k}(R_9)^{j m}(R_0)_{j n}\]
\[-2C_{H_2}(R_7)^{i k}(R_7)_{j k} + 4C_{d_4}(R_4)^{i k}(R_1)^{i j}_{k j}\]
\[-2C_{H_1}(R_9)^{i k}(R_9)_{j k} + 2C_L(R_1)^{i k}_{j k}\]
\[-2C_{c_3}(R_1)^{i k}_{j k} - 2C_{c_3}(R_9)^{i k}(R_9)_{j k}\]
\[-2C_{d_4}(R_7)^{i k}(R_7)_{j k} - 4C_Q(R_9)^{i k}(R_4)_{j k}\]
\[-2(R_4)^{i k}(R_4)_{j m}(\gamma_Q)^{m n} - (R_9)^{i k}(R_9)_{j m}(\gamma_U)^{m n}\]
\[-(R_7)^{i k}(R_7)_{j m}(\gamma_D)^{m n} - (R_1)^{i k}_{j m}(\gamma_U)^{m n}\]
\[-2(R_4)^{i k}(R_4)_{j m}(\gamma_Q)^{m n} - (R_7)^{i k}(R_7)_{j k}(\gamma_H_2)\]
\[-(R_9)^{i k}(R_9)_{j k}(\gamma_H_1) - (R_1)^{i k}_{j k}(\gamma_L)^{i m}\]

\[(23)\]

\[(16\pi^2)^2 (\beta_{m_{\text{GUT}}}^{[2]})^m_j = (16\pi^2)^2 (\beta_{m_{\text{GUT}}}^{[2]\text{MSSM}}) - 2(Y_u)_{k j}(Y_u)^{q p}(R_9)^{k i}(R_9)^{q p} - 2(Y_u)_{k j}^p(Y_u)^{q p}(R_1)^{k i}_{p q}
+ 8(\Lambda_U)^{i k}(\Lambda_D)_{k m p}(R_4)^{j m}(R_1)^{m n} - 8(\Lambda_U)^{i k}(\Lambda_D)_{k m p}(R_4)^{j m}(R_9)_{n j}
- 4(Y_u)^{k i}(Y_u)_{d k}(R_9)_{n j} - 2(Y_u)^{k i}(Y_u)^{m n}(R_9)_{k j}
- 2C_{d_4}(R_7)^{i k}_{j k} - 4C_L(R_1)^{i k}_{k j}\]

\[(24)\]

\[(16\pi^2)^2 (\beta_{m_{\text{GUT}}}^{[2]})^m_j = (16\pi^2)^2 (\beta_{m_{\text{GUT}}}^{[2]\text{MSSM}}) - 8(\Lambda_U)^{i j k}(Y_u)_{a j}(R_4)^{n m}(R_5)^{m n}(R_1)^{m n}(R_2)^{m n}(R_9)_{j m}
+ 4(\Lambda_D)^{i k l}(Y_u)^{m n}(R_3)^{m n}(R_9)^{m k} - 6(\Lambda_D)^{j m k}(\Lambda_D)_{n m p}(R_7)^{k i}(R_5)^{q p}
- 6(Y_d)_{k j}(Y_d)^{q p}(R_7)^{i k}(R_7)^{q p} - 8(\Lambda_U)^{i j k}(Y_u)_{p k}(R_4)^{j m}(R_7)^{m n}(R_7)^{m n}\]

\[(25)\]
\[
(16\pi^2)^2 \beta_{m_z^2}^{(2)} = \left(16\pi^2\right)^2 \beta_{m_z^2}^{(2)MSSM} + 12(\Lambda_D)^{k\ell}(\Lambda_U)^{\nu\mu}(R_1)^{\nu\mu}(R_4)^{\nu\mu} + 6(\Lambda_D)^{k\ell}(\Lambda_U)^{\nu\mu}(R_4)^{\nu\mu} -6(\Lambda_D)^{k\ell}(\Lambda_U)^{\nu\mu}(R_4)^{\nu\mu} -3(\Lambda_E)^{k\ell}(\Lambda_U)^{\nu\mu}(R_4)^{\nu\mu} + 12C_{\ell}(R_1)^{k\ell}(R_4)^{k\ell} + 3(\Lambda_E)^{k\ell}(\Lambda_U)^{\nu\mu}(R_4)^{\nu\mu} -3(\Lambda_E)^{k\ell}(\Lambda_U)^{\nu\mu}(R_4)^{\nu\mu} -3(\Lambda_E)^{k\ell}(\Lambda_U)^{\nu\mu}(R_4)^{\nu\mu} -3(\Lambda_E)^{k\ell}(\Lambda_U)^{\nu\mu}(R_4)^{\nu\mu} -3(\Lambda_E)^{k\ell}(\Lambda_U)^{\nu\mu}(R_4)^{\nu\mu}
\]

(26)

\[
(16\pi^2)^2 \beta_{m_\tau^2}^{(2)} = \left(16\pi^2\right)^2 \beta_{m_\tau^2}^{(2)MSSM} -6(\gamma_L)^{k\ell}(\gamma_L)^{\nu\mu}(R_1)^{\nu\mu}(R_4)^{\nu\mu} -6(\gamma_L)^{k\ell}(\gamma_L)^{\nu\mu}(R_4)^{\nu\mu} + 6C_{\ell}(R_1)^{k\ell}(R_4)^{k\ell} + 3(\Lambda_E)^{k\ell}(\Lambda_U)^{\nu\mu}(R_4)^{\nu\mu} -3(\Lambda_E)^{k\ell}(\Lambda_U)^{\nu\mu}(R_4)^{\nu\mu} -3(\Lambda_E)^{k\ell}(\Lambda_U)^{\nu\mu}(R_4)^{\nu\mu} -3(\Lambda_E)^{k\ell}(\Lambda_U)^{\nu\mu}(R_4)^{\nu\mu} -3(\Lambda_E)^{k\ell}(\Lambda_U)^{\nu\mu}(R_4)^{\nu\mu}
\]

(27)

\[
(16\pi^2)^2 \beta_{m_\mu^2}^{(2)} = \left(16\pi^2\right)^2 \beta_{m_\mu^2}^{(2)MSSM} -6(\gamma_L)^{k\ell}(\gamma_L)^{\nu\mu}(R_1)^{\nu\mu}(R_4)^{\nu\mu} -6(\gamma_L)^{k\ell}(\gamma_L)^{\nu\mu}(R_4)^{\nu\mu} + 6C_{\ell}(R_1)^{k\ell}(R_4)^{k\ell} + 3(\Lambda_E)^{k\ell}(\Lambda_U)^{\nu\mu}(R_4)^{\nu\mu} -3(\Lambda_E)^{k\ell}(\Lambda_U)^{\nu\mu}(R_4)^{\nu\mu} -3(\Lambda_E)^{k\ell}(\Lambda_U)^{\nu\mu}(R_4)^{\nu\mu} -3(\Lambda_E)^{k\ell}(\Lambda_U)^{\nu\mu}(R_4)^{\nu\mu} -3(\Lambda_E)^{k\ell}(\Lambda_U)^{\nu\mu}(R_4)^{\nu\mu}
\]

(28)
where

\[+12C_{H_2}(R_T)^{kl}(R_T)_{kl} + 4C_{H_2}(R_2)^{kl}(R_2)_{kl}\]

\[+4C_{H_2}(R_2)^{kl}(R_2)_{kl} - (R_2)^{ij}(R_2)^{mn}(\gamma_4)_{ij}^m\]

\[-3(R_T)^{ik}(R_T)_{kl}(\gamma_4)_{lm}^m - (R_2)^{ik}(R_2)_{kl}(\gamma_4)_{lm}^m\]

\[-3(R_T)^{ik}(R_T)_m(\gamma_4)_{kl}^m - (R_2)^{ik}(R_2)_m(\gamma_4)_{kl}^m\]

\[-(R_2)^{ik}(R_2)_m(\gamma_4)_{kl}^m\]

(29)

\[(16\pi^2)^2(\beta_{m_R}^{(2)})^i = (16\pi^2)^2(\beta_{m_R}^{(2)\text{RPV}}) + 6(\Lambda_D)^{ik}(\Lambda_D)_{ik}(R_9)^{nm}(R_1)_{km}^{pl} - 6(\Lambda_D)^{ik}(Y_4)_{ik}(R_9)^{nm}(R_3)_{km}\]

\[-3(Y_5)_{ik}(\Lambda_D)^{ik} (R_9)^{mn}(R_9)_{km}^{kl}\]

\[+12C_L(R_9)^{kl}(R_1)_l^{kl} - 2C_{H_2}(R_2)^{kl}(R_2)_k\]

\[-2C_{ce}(R_5)^{il}(R_2)^{kl}(R_2)_{km}(\gamma_4)_{il}^m\]

\[-3(R_9)^{ik}(R_1)_m(\gamma_4)_{kl}^m - (R_5)^{ik}(R_2)_m(\gamma_4)_{kl}^m\]

\[-(R_5)^{ik}(R_2)_m(\gamma_4)_{kl}^m\]

(30)

where

\[C_Q = \frac{1}{2} g_3^2 + \frac{1}{2} g_2^2 + \frac{1}{2} g_1^2, \quad C_{uc} = \frac{1}{2} g_3^2 + \frac{1}{2} g_2^2, \quad C_{dc} = \frac{1}{2} g_3^2 + \frac{1}{2} g_2^2, \quad C_{ce} = \frac{1}{2} g_3^2 + \frac{1}{2} g_2^2, \quad C_{he} = \frac{1}{2} g_3^2 + \frac{1}{2} g_2^2.\]

(31)

We have the following results for the \(\phi\phi\)-type terms:

\[(16\pi^2)^2(\beta_{m_R}^{(2)})^i = (16\pi^2)^2(\beta_{m_R}^{(2)\text{RSM}}) + (16\pi^2)^2(\beta_{m_R}^{(2)\text{MSSM}})\]

(32)

\[(16\pi^2)^2(\beta_{m_R}^{(2)})^i = (16\pi^2)^2(\beta_{m_R}^{(2)\text{RPV}}) + (16\pi^2)^2(\beta_{m_R}^{(2)\text{MSSM}})\]

(33)

In our calculation we use the results of Ref [19] which gives two-loop \(\beta\)-functions for a general gauge susy theory for standard and non-standard soft susy braking terms, and also we assume \(m_F\) is zero in Eq. (4) [19] therefor \(m_4\) and \(m_r\) are zero in Eqs. (14) and (15)).

To check our results, first we have calculated one loop RGEs for both standard and non-standard terms, and have compared with ref [11]. Our results are the same as theirs. Moreover, to test the method we have obtained the two loop standard soft braking \(\beta\)-functions which are consistent with results in ref [20].

20
3 Conclusion

In this paper we have expanded the study of the RG evolution of non-standard soft terms up two loop, and presented the two-loop renormalisation of the R-parity violating extension of the MSSM with the most general possible set of soft breaking terms consistent with naturalness.

Typically, we expect effects of the two loop \( \beta \)-functions make a difference of several percent in compare with effects of one loop on the standard running analysis such as Higgs physics and the scalar quark sector of the MSSM; however it is quite difficult to make consequential estimates of the size of the two-loop corrections without committing to a specific model. Moreover it is desirable from the point of view of consistency to use the full set of \( \beta \)-functions.

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