Dynamic Characteristics Analysis of Bearings Based on Modification
Cycloidal Pinwheel Reducer

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Abstract. Taking the cycloid pin gear reducer as the research object, the bearing force of the
cycloid pin gear reducer shaft is calculated under different modification amount. Based on the Hertz
theory, the contact stress and deformation of the bearing under different modification amounts are
calculated. Based on the transmission principle of cycloid pin wheel reducer, the model of cycloid
wheel and bearing is established and the finite element analysis is carried out. The analysis showed
that the Hertz theoretical solution is consistent with the finite element analysis solution, which
showed the correctness of the finite element results. The stress and deformation of the bearing
obtained under different modification amounts of the cycloid wheel are not changed much. It can
indicate that the cycloid wheel modification has no significant influence on the bearing force.
Therefore, the research content of this paper has certain reference value for the design and
optimization of cycloid pinwheel reducer.

Introduction

As a key component of cycloid pinwheel reducer, the normal operation of this bearing directly
affects the performance of the whole machine. Under cyclic loads, the inner and outer raceways and
rolling bodies of the bearing will suffer different forms of fatigue damage under high stress.
Therefore, it is of great significance to study the stress and deformation of the bearing in the
working process. [1,2,3]

Xinglin Pei et al. [4] established the finite element model of deep groove ball bearing with APDL
parametric language and analyzed the bearing stress. Canjiang Yao et al. [5] combined with Hertz
theory and finite element method, analyzed the stress and deformation of rolling bearing in RV
reducer under different working conditions. In this paper, the load of bearing is calculated based on
the different amount of modification of cycloid wheel. The stress and deformation of bearing under
different amount of modification of cycloid wheel is calculated by Hertz theory and finite element
method.

The Action Force of Pin Gear on Cycloid Wheel

The basic parameters of cycloid wheel are as follows: Central circle radius of pin gear \( r_p = 82 \text{mm} \),
Cylindrical radius of pin-tooth sleeve \( r_p = 4 \text{mm} \), Eccentricity \( a = 1.5 \text{mm} \), The number of teeth of pin
gear \( z_p = 40 \), The number of teeth of cycloid gear \( z_c = 39 \). According to the number of cycloid gear and
pinwheel teeth, the range of \( K_1 \) and \( K_2 \) can be determined to be [0.65, 0.9] and [1.0, 1.6] [6].

\[
K_1 = \frac{ac}{r_p}
\]

Where, the range of \( r_p \) is [66.67, 92.3]. The amount of modification \( \Delta r_p \) of the shift modification
method is to change \( r_p \) into \( r_p + \Delta r_p \), and then substitute the known \( r_p \) value into the value range of
the above solution, the value range of \( \Delta r_p \) is [-15.55, 10.3].

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\[ K_z = \frac{r_c \sin 180^\circ}{r_{rp}} \]  

(2)

Where, the range of \( r_p / r_{rp} \) is [12.8, 20.5], and the range of \( r_{rp} \) is [4.5, 5.2] by bringing the calculated range of \( r_p \) into the above range. The amount of modification \( \Delta r_{rp} \) of the equidistant modification method is to increase \( r_{rp} \) to \( r_{rp} + \Delta r_{rp} \), and then substitute the known \( r_{rp} = 4 \) into the above range to get the value range of \( \Delta r_{rp} \) is [0.5, 1.2] [7].

The initial clearance of the \( i \) pair of gear teeth along the normal direction of the point to be meshed is as follows [6].

\[ \Delta(\varphi)_i = \Delta r_{rp} \left(1 - \frac{\sin \varphi_i}{\sqrt{1 + K_i^2 - 2K_i \cos \varphi_i}}\right) - \Delta r_{rp} \left(\frac{1 - K_i \cos \varphi_i - \sqrt{1 - K_i^2 \sin \varphi_i}}{\sqrt{1 + K_i^2 - 2K_i \cos \varphi_i}}\right) \]  

(3)

Where, \( \varphi_i \) is the angle of the \( i \)th pin tooth relative to the center \( O_p \) of the pin tooth sleeve.

When the load is transferred, the torque applied to the cycloid wheel is \( T_c \), the contact deformation between the cycloid wheel and the pinwheel is \( w \) under the action of \( T_c \), and the bending deformation of the pin is \( f \), then the total deformation is equal to \( \delta = w + f \).

\[ \delta_i = \frac{l_i}{r_c} \delta_{\text{max}} \left(\frac{\sin \varphi_i}{\sqrt{1 + K_i^2 - 2K_i \cos \varphi_i}}\right) \delta_{\text{max}} \]  

(4)

Where, \( \delta_{\text{max}} \) is the maximum total deformation, \( \delta_{\text{max}} = w_{\text{max}} + f_{\text{max}} \). \( l_i \) is the distance from the common normal of the contact point between the \( i \)th pin tooth and the cycloid gear tooth to the midpoint of the cycloid gear; \( r_c \) is the nodal radius of the cycloid wheel.

\[ l_i = \frac{\sin \varphi_i}{\sqrt{1 + K_i^2 - 2K_i \cos \varphi_i}} r_c \]  

(5)

\[ W_{\text{max}} = \frac{2(1 - \mu^2)}{E} \times F_{\text{max}} \left(\frac{2}{3} + \ln \frac{16r_{rp} |\rho|}{c^2}\right) \]  

(6)

Where, \( F_{\text{max}} \) is the load of the most stressed tooth in the meshing tooth, \( b_c \) is the tooth width of the cycloid gear, \( \mu \) is the Poisson's ratio of the material, \( E \) is the elastic modulus of the material, \( \rho \) is the curvature radius of the tooth profile of the cycloid wheel at \( \varphi_0 = \arccos K_i \), \( c \) is the axial clearance between the cycloid gear and the pin gear housing.

\[ c = 4.99 \times 10^{-3} \sqrt{\frac{2(1 - \mu^2)}{E} \times F_{\text{max}} \times \frac{2|\rho|}{b_c} \frac{r_{rp} |\rho|}{|\rho| + r_{rp}} \left(\frac{1}{K_i} + 1 + 1 + K_i^2 |\rho| - (1 + z_p K_i^2) (|\rho| + r_{rp}) \right)^{\frac{1}{2}}} \]  

The bending deformation of the pin under the action of force \( F_{\text{max}} \) at the point of force action is as follows:

\[ f_{\text{max}} = \frac{F_{\text{max}} L^3}{48 E J} \times \frac{31}{64} \]  

(7)

Where, \( J \) is cross sectional moment of inertia and \( L \) is the distance between the two supporting points of the pin. When \( \delta_i \) is greater than \( \Delta(\varphi)_i \), the cycloid gear teeth and the pin teeth mesh with each other, otherwise they do not mesh.

The initial pin teeth number is \( m = 5 \), and the final pin teeth number is \( n = 8 \). The initial meshing angle is \( 26^\circ \), and the load of the most stressed tooth in the meshing tooth \( F_{\text{max}} \) is as follows:
\[ F_{\text{max}} = \frac{T_c}{\sum_{i=m}^{n} \left(\frac{1}{r_c} - \frac{\Delta(\theta)}{\delta_{\text{max}}})I_i\right) / \sum_{i=m}^{n} \left(\frac{1}{r_c} - \frac{\Delta(\theta)}{\delta_{\text{max}}})I_i\right)} = \frac{0.55T}{\sum_{i=m}^{n} \left(\frac{1}{r_c} - \frac{\Delta(\theta)}{\delta_{\text{max}}})I_i\right)} \]  \tag{8}

Where, \( T \) is the torque acting on the output axis. \( F_{\text{max}0} \) is given as the initial value of \( F_{\text{max}} \), and \( \delta_0 \) is obtained. Then calculate \( F_{\text{max}1} \) according to the obtained \( \delta_0 \), and compare the values of \( F_{\text{max}0} \) and \( F_{\text{max}1} \). If it conforms to \( |F_{\text{max}k} \) - \( F_{\text{max}k-1}|| \leq 0.1\% \), take \( F_{\text{max}} = (F_{\text{max}0} + F_{\text{max}1}) / 2 \). If it does not meet the requirements, \( F_{\text{max}1} \) will be taken into solution \( \delta_0 \), iterative solution until it meets the conditions, and the stress value of each tooth will be calculated.

**Force of Pillar Pin on Cycloid Wheel**

There is a certain distance between the pin sleeve and the pin hole along the common normal direction. This distance is called the initial clearance, which is expressed by \( \Delta W_i \).

\[ \Delta W_i = \frac{\Delta}{2} (1 - \sin \alpha_i) \]  \tag{9}

Where, \( \alpha_i \) is the angle of the pin center relative to \( O_p \).

When the pin sleeve contacts the pin hole and transmits the torque, the pin will have a small deformation, which is represented by \( \epsilon_i \).

\[ \epsilon_i = \epsilon_{\text{max}} \sin \alpha_i \]  \tag{10}

Where, \( \epsilon_{\text{max}} \) is \( \epsilon_{\text{max}} = W_{w_{\text{max}}} + f_{w_{\text{max}}} \), \( W_{w_{\text{max}}} \) is the contact deformation of the pin sleeve, \( f_{w_{\text{max}}} \) is the bending deformation of the pin.

\[ W_{w_{\text{max}}} = \frac{2(1 - \mu^2)}{E} \frac{Q_{\text{max}}}{\pi b_c} \left[ \frac{2}{3} + \ln \left( \frac{16r_n r_c}{c^3} \right) \right] \]  \tag{11}

Where, \( r_n \) is the radius of pinhole of theoretical column on cycloid wheel and \( r_m \) is the radius of pin sleeve. And

\[ c = 9.98 \times 10^{-3} \sqrt{\frac{(1 - \mu^2)}{E} \frac{Q_{\text{max}}}{b_c} \left( \frac{r_n r_c}{a} \right)} \]

\[ f_{w_{\text{max}}} = \frac{Q_{\text{max}} L_c^2}{3EJ_s} \]

Because \( \Delta W_i \) is the clearance between the pin sleeve and the hole, \( \epsilon_i \) is the deformation, only the pin whose \( \epsilon_i \) is greater than \( \Delta W_i \) is the pin that participates in the transmission of torque.

According to the moment balance conditions, it can be obtained that:

\[ T_c = \sum_{i=m}^{n} Q_i l_i \]  \tag{12}

Where, \( l_i \) is the force arm from the center of the pin to \( O_p \). \( Q_i \) is the force between the pin sleeve and the pin hole of cycloid wheel. \( Q_i = ((\epsilon_i - \Delta W_i)/\epsilon_{\text{max}})Q_{\text{max}} \).

\[ Q_{\text{max}} = \frac{0.55T}{R_c \sum_{i=m}^{n} \left( \sin \alpha_i - \frac{\Delta W_i}{\epsilon_{\text{max}}} \right) \sin \alpha_i} \]  \tag{13}

Where, \( R_c \) is the radius of the center circle of the pin.

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Take the initial value $Q_0$, according to the above method, the calculation is iterated until it conforms to $|Q_{\text{max}k} - Q_{\text{max}k-1}| \leq 0.1\% Q_{\text{max}k}$, output $Q_{\text{max}}$ and calculate the force $Q_i$ of each meshing tooth. $Q_0 = 4T_c/R_wz_w$.

**Acting Force of Rotary Arm Bearing**

The force from the bearing to the cycloid wheel is balanced with the resultant force from the pin teeth and the pillar pin to the cycloid wheel, as shown in Fig. 1.

![Figure 1. Bearing Force Diagram](image)

So the horizontal resultant force, the vertical direction of the cycloid wheel and the force of the bearing to the cycloid wheel as follows:

$$
\sum F_x = \sum_{i=1}^{n} F_i \cos \theta_i, \quad \sum F_y = \sum_{i=1}^{n} F_i \sin \theta_i, \quad F_i = \sqrt{(\sum F_{x,i})^2 + (\sum F_{y,i})^2}
$$

(14)

The different amount of modification are substituted into the solution are shown in Tab. 1.

| $\Delta r_p$/mm | $\Delta r_p$/mm | $\Sigma F_x$/N | $\Sigma F_y$/N | $\Sigma Q_i$/N | $F_i$/N |
|-----------------|-----------------|----------------|----------------|----------------|---------|
| 0.5             | -0.15           | 3094.8         | 83.7816        | 3361.48        | 4507.89 |
| 0.6             | -0.07           | 3155.8         | 83.4794        | 3361.04        | 4549.89 |
| 0.7             | 0.01            | 3145.2         | 88.3592        | 3360.50        | 4538.63 |
| 0.8             | 0.09            | 3161.0         | 97.0632        | 3359.40        | 4542.55 |
| 0.9             | 0.15            | 3167.0         | 93.1358        | 3360.38        | 4550.25 |
| 1.0             | 0.22            | 3156.8         | 95.7706        | 3360.38        | 4541.26 |
| 1.1             | 0.28            | 3158.9         | 91.5987        | 3360.82        | 4546.04 |
| 1.2             | 0.35            | 3156.7         | 93.6068        | 3360.72        | 4542.99 |

**Hertz Theory Calculation**

The problems of contact stress and deformation of rolling bearings is successfully solved by Hertz elastic contact theory. For the contact problem inside the rolling bearing, the contact surface is an ellipse and the surface pressure is semi-ellipsoidal. According to the point contact calculation formula [8], take the deep groove ball bearing 6206 as an example, the bearing parameters are shown in Tab. 2 and the calculation results are shown in Tab. 3.

**Finite Element Analysis**

According to the transmission principle of the cycloid pinwheel reducer, the transmission geometry model is established. The parameters of the finite element analysis are defined and the dynamic characteristics of the bearing are analyzed based on the actual working conditions of the cycloid pinwheel drive. The cycloid wheel and bearing material are defined as GCr15 with an elastic modulus of 2.07E+5 MPa, a Poisson's ratio of 0.3 and a density of 7850 kg/m^3. In the mesh generation, the mixed tetrahedral and hexahedral method is used as shown in Fig. 2. According to the definition of the contact surface and the target surface [8], the roller surface is defined as the contact surface, and the inner and outer races of the bearing are the target surface.
Table 2. Bearing basic parameters.

| Bearing material | Outer diameter | Inner diameter | Rolling element diameter | Width | Elastic Modulus | Poisson's ratio | Number of rolling elements |
|------------------|----------------|----------------|--------------------------|-------|----------------|-----------------|---------------------------|
| GCr15            | 62mm           | 30mm           | 9.525mm                  | 15mm. | 207GPa         | 0.3             | 8                         |

Table 3. Hertz theory calculation result.

| $F_r/N$ | $\sigma_1/MPa$ | $a_1/mm$ | $b_1/mm$ | $\delta_1/mm$ | $\sigma_2/MPa$ | $a_2/mm$ | $b_2/mm$ | $\delta_2/mm$ |
|---------|----------------|----------|----------|----------------|----------------|----------|----------|----------------|
| 4507.89 | 2920           | 2.132    | 0.192    | 0.0187         | 2558           | 1.839    | 0.254    | 0.0193        |
| 4549.89 | 2929           | 2.139    | 0.193    | 0.0189         | 2569           | 1.845    | 0.255    | 0.0194        |
| 4538.63 | 2926           | 2.137    | 0.193    | 0.0188         | 2564           | 1.843    | 0.255    | 0.0194        |
| 4542.55 | 2927           | 2.138    | 0.193    | 0.0188         | 2565           | 1.844    | 0.255    | 0.0194        |
| 4550.25 | 2930           | 2.139    | 0.193    | 0.0189         | 2570           | 1.845    | 0.256    | 0.0195        |
| 4541.26 | 2926           | 2.137    | 0.193    | 0.0188         | 2565           | 1.844    | 0.255    | 0.0194        |
| 4546.04 | 2928           | 2.138    | 0.193    | 0.0188         | 2567           | 1.845    | 0.255    | 0.0194        |
| 4542.99 | 2927           | 2.138    | 0.193    | 0.0188         | 2565           | 1.844    | 0.255    | 0.0194        |

Where, $\sigma_1$ is inner ring and rolling element contact stress. $a_1$ is Contact ellipse long half shaft. $b_1$ is Contact ellipse short half shaft. $\delta_1$ is Contact deformation. $\sigma_2$ is Outer ring and rolling element contact stress. $a_2$ is Contact ellipse long half shaft. $b_2$ is Contact ellipse short half shaft. $\delta_2$ is Contact deformation.

A torque is applied to the bearing according to the actual working condition of the bearing. A constraint is applied on the end face of the crankshaft in order to limit the simulated bearing moving axially. Because the assembly is rotating with the crankshaft rotating, the torque $B$ is applied at a position where the bearing is not mounted on the other side of the crankshaft. As shown in Fig. 3.

**Figure 2. Schematic diagram of meshing.**  **Figure 3. Schematic diagram of applying load constraints.**

**Result Analysis**

The magnitude and distribution of contact stress and deformation of bearing inner and outer rings and rollers are obtained under different modification of cycloid wheel. As shown in Fig. 4 and 5. The results showed that the maximum contact stress of the rolling element lied at the contact point between the rolling element and the inner ring. The dangerous position of bearing fatigue damage is the contact position between rolling element and raceway of bearing inner ring. The contact stress decreases from contact point to both sides. The contact area is elliptical which is consistent with Hertz theory.

**Figure 4. Contact stress diagram.**  **Figure 5. Diagram of rolling element deformation.**

The solution is consistent by contrast with finite element solution and Hertz theory, which showed the correctness of the finite element analysis shown in Fig. 6-9. The contact stress and deformation have little change under different modification amount, which indicates that the modification of cycloid wheel has little influence on the contact stress and deformation of bearing.
When $\triangle r_p$ is 0.5mm and $\triangle r_p$ is -0.15mm, the contact stress and deformation of inner and outer rings are the smallest, so the modification amount is more appropriate.

![Figure 6. Contact Stress Diagram of Inner Ring and Roller.](image)

![Figure 7. Contact Stress Diagram of Outer Ring and Roller.](image)

![Figure 8. Contact Deformation Diagram of Inner Ring and Roller.](image)

![Figure 9. Contact Deformation Diagram of Outer Ring and Roller.](image)

**Conclusion**

(1) The bearing forces of cycloid pinwheel reducer are obtained based on the different modifications of cycloid wheel. The analysis results are consistent by using the two method of finite element and Hertz theory, which shows that the finite element method has certain accuracy. And the bearing stress and deformation area can clearly and intuitively observed. As can compensate for the shortcomings of the traditional theoretical calculation. It also provided some references for optimum design and reliability calculation of bearings.

(2) From the analysis, it can be seen that the rolling element and the bearing raceway contact in a small area. Even if the load is not very large, the bearing will also be subject to great contact stress. When the load is too large, plastic deformation will easily occur leading to fatigue damage in the contact area. This analysis has a certain reference value for the study of bearing life. The change of cycloid wheel modification on bearing force can be understood more deeply and can also provide some reference for choosing appropriate the modification amount of cycloid wheel.

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