Fourier transforms Investigation of Global Solar Radiation at True Noon: in the Desert climatology

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Abstract: The aim of this work has been to obtain a typical annual time a function by the application of a calculation procedure based on a Fourier analysis to global solar radiation data of nine years at Helwan Egypt from June 1991 to August 1999. This function allows us to estimate the most probable values of the horizontal daily global solar radiation for every day of the year at Helwan Egypt. It is confirmed that the harmonic model presents an important percentage with 98% confidence bounds. The applied method permits its application to the new measurements of solar radiation that are being carried out, This will be necessary because of the number of years used in this study.

Key Words: Statistical Analysis - Global Solar Radiation - Fourier transforms

INTRODUCTION

The Solar radiation data needed for effective research into solar energy utilization. The physical and statistical routes to solar metrology related to each other. The parameters, which govern a physical model of the sky, take; hour-by-hour or day by day, values which fluctuate according to the fluctuating changes in the meteorological and environmental situations. These parameters include the sunshine hours, the relative humidity, cloud cover, temperature, and water vapor pressure. Nowadays, data are allowable for both these quantities at many locations in the world.

Several empirical formulae have been developed to calculate the solar radiation using various parameters[1-5]. These parameters include the sunshine hours, the relative humidity and sunshine hours[6-9], and the sunshine duration, relative humidity, maximum temperature, water vapor pressure, mean sea level and the ratio of water vapor pressure to the mean sea level pressure[10].

For a region with a climate subject to short-term meteorological variations, such as that of the UK, hourly averages provide a poor basis for modeling the short-term variation associated with solar irradiance. Many studies in the environment have investigated horizontal global solar irradiance at locations between latitude 40° N and 40° S. More recently, work has been carried out at higher latitudes such as: Behr, investigations were based at Hamburg, Germany (55°N)[12]; Badescu, considered global irradiation at two locations in Romania (44° N and 47° N)[13]; Barr et al., used climatic data from Alberta, Saskatchewan and Manitoba in Canada (49–55°N) to estimate global irradiation. Previous work has normally been based on horizontal global irradiation[14]. However, for a country at high latitude, such as the UK, it generally acknowledged that irradiation for both horizontal and vertical surfaces make a useful contribution to the assessment of overall sunlight levels for PV systems. In his work on tilted south-oriented surfaces in Germany, Behr recognized this and considered global irradiation on both the horizontal plane and a south-facing vertical surface[12].

This work tried to describe the state of global solar radiation in Helwan, Egypt and, to assess the relative effectiveness of true noon instantaneous irradiance observations at this site in terms of modeling long-term. Although considerable insight may be gained from direct graphical comparison of observations, this is subjective. Statistical methods are adopted to provide a more objective approach for deciding on appropriate averaging times, and assessing the loss of information attached to longer averaging times. Finally, to investigate the suite of empirical models used in the estimation of the global solar radiation at Helwan, Egypt.

MATERIALS AND METHODS

In a country that is predominantly deserted, the Nile River provides the lifeblood for Egypt's population. With 96% of Egyptians living astride the river, environmental issues are a central component of Egyptian life. Population growth, modernization, and increased economic development have brought environmental problems to the forefront, especially air pollution. In Cairo, emissions from vehicles and lead smelters, together with sand blowing in from the adjacent Western Desert, have created high levels of particulate matter in the atmosphere and a deadly combination for public health in the densely-populated capital. A very
common weather condition is characterized by calm or light air, increased humidity during the cold season. The measurements were taken on the terrace of research laboratory in the National Research Institute of Astronomy and Geophysics (NRIAG) in Helwan, Egypt (29.87 °N, 31.33 °E Elevation 124 m). The daily average of temperature ranges between 35.2 °C in July and 15.8 °C in January. The relative humidity fluctuates between 52.7% in June and 57.9% in December, and visibility about 5 Km as shown in Table 1. This restricted visibility is the result of the presence of solid particles in the atmosphere some of which act as condensation nuclei. During such a weather condition, the reduction of solar radiation is sometimes due to the increased quantity of water vapor, sometimes to the presence of increased quantities of aerosol particles and sometimes to the presence of both these influences. The deposition of dust and particulates matter, however, remains a major problem facing the PV power systems. Kipp & Zonen CM11 pyranometer placed on the horizontal plane to measure global irradiance and the direct solar irradiance full tracker. The data acquisition system consisted of a programmable devices data to treatment with transfer of data to a portable PC. The data were collected in binary form and subsequently converted to ASCII data collection was carried out on a continuous basis from June 1991 to August 1999.

Harmonic Analysis: The type of analysis any function, continuous within an interval, can be represented by an infinite series of sine and cosine functions. This series is called a Fourier series, and the method of finding the functions Fourier analysis. If only a finite number of points exist in the interval to be analyzed, a finite number of sine and cosine will be able to account for all the observations. The determination of a finite sum of sine and cosine terms is called harmonic analysis. This is the correct one for investigating the harmonics of an identifiable frequency under the assumption that the time series is genuinely periodic. Each harmonic may (but does not necessarily) have a distinct physical meaning. The misuse of the concept, that the time series repeats itself exactly every n observation, has been responsible for the acceptance of probably more spurious hypotheses than any other statistical or applied mathematical tool.. (It) breaks down completely when applied to a statistical function[5]. Recall that if \( f(x) \) is infinitely differentiable at \( x=a \), then the Taylor series of \( f \) is defined as:

\[
f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!(x-a)^n}
\]  

(1)

i.e., we have a “representation” of the function \( f \). We have known from experience that Taylor series representations are useful in many ways. Similarly, there exist other types of representations, which are of great utility. In this paper we are concerned, specifically, with the representation of periodic function by trigonometric series of the form:

\[
f(x) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n \pi x}{L} + b_n \sin \frac{n \pi x}{L} \right)
\]  

(2)

Harmonic analysis means the computation of the coefficients \( a_n \) and \( b_n \). Now, with the coefficients computed according to eq.2, the right-hand side of eq.1 is called the Fourier series of \( f \); the coefficients \( a_0 , a_1 , b_1 , a_2 , b_2 , ... \) and are called the Fourier coefficients of \( f \), and the individual terms in eq.1 are generally referred to as harmonic. Similarly, for the case of interest expressed by eq.2, we can say that if \( S(t), \tau(t), G(t) \) are represented by discrete Fourier series with equal period and \( N \) harmonics, then the corresponding complex Fourier coefficients are related by:

\[
S_n(t) = \sum_{m} G_m(t) \tau_{n-m}
\]  

(3)

Note that \( n \) harmonics (\( n+1 \) Fourier coefficients, and \( n \) complex conjugate terms) are used in the calculation. The Fourier coefficients \( G_n(t) \) and \( \tau_n(t) \) need only be evaluated once for specific historical weather data and for glazing assemblies, respectively. Thus, the coefficients \( S_n(t) \) can be calculated from \( G_n(t) \) and \( \tau_n(t) \) without the need to first determine \( G_n(t) \) and \( \tau_n(t) \). The discrete Fourier series representation of \( \tau(t) \) is

\[
\tau(t) = \sum_{n} \tau^n(a) / n!(x-a)^n
\]  

(4)

Note that the number of harmonics employed to represent \( \tau(t) \), \( G(t) \) can be different. The method followed, which is used in an independent form for the data of Helwan, Fourier analysis is applied to the set of annual series year after year. This permits the removal of the random phenomena and the obtaining of the representative equation of each year from the retained harmonics in function of the explained variance for each of them. In our case, in a generalized form, only the annual harmonic is retained. Carrying out the analysis year after year allows us to notice quickly if there exists a year with any differences in its behavior with regard to the rest of the years of the studied series and also use less computer time.

Statistical Calculation of Autoregressive Parameters: Suppose \( N \) observations \( x_1,x_2,x_3,....,x_N \) are taken sequentially, at equally spaced intervals of time
on a variable $X$. The observation taken at time $t$ is referred to as $x_t$ ($t=1,2,\ldots,N$), and the series of observations as the time series $X_t$. To investigate the dependence of $x_t$ on its previous values $x_{t-1},x_{t-2},\ldots$, the autocorrelation coefficients (CC) at lags $k$, $r(k)$, given by;

$$CC(k) = \frac{\sum_{t=1}^{N-k} (x_t - \bar{x})(x_{t+k} - \bar{x})}{\sum_{t=1}^{N} (x_t - \bar{x})^2}, \quad k = \pm 1,\pm 2,\ldots$$

where

$$\bar{x} = \frac{1}{N} \sum_{t=1}^{N} x_t$$

are calculated. Heuristically, $CC(k)$ may be regarded as the estimate CC between $x_t$ and $x_{t+k}$ ($t=1,2,3,\ldots,N-k$, $k=0,+1,+2$. Since autocorrelation was expected to be present, Autoregressive Integrated Moving Average models, which provide a wide range of stochastic processes for modeling series, were used\[16].

### RESULTS AND DISCUSSION

Scaled Hourly Global Irradiation: Table 1, shows; monthly mean hours of global solar radiation (G) W/m², bright sunshine (S) Hour, maximum temperature (T) °C, water vapor pressure (V) (hpa), mean sea level MSL (hpa), and relative humidity (RH) % at Helwan station.

| Month | G (W/m²) | S(h) | T (°C) | V(hpa) | MSL(hpa) | RH % |
|-------|----------|------|--------|--------|----------|------|
| Jan.  | 533      | 7.6  | 20.1   | 10.8   | 1019     | 64.7 |
| Feb.  | 680      | 7.5  | 18.6   | 8.8    | 1022     | 59   |
| Mar.  | 814      | 8.4  | 20.8   | 10     | 1015     | 56.7 |
| Apr.  | 856      | 9.5  | 25.6   | 10.5   | 1014     | 48   |
| May   | 895      | 11   | 32.6   | 14.2   | 1013     | 48.3 |
| Jun.  | 898      | 11.9 | 34.5   | 18.6   | 1009     | 52.7 |
| Jul.  | 889      | 11.6 | 35     | 22.3   | 1009     | 59   |
| Aug.  | 875      | 11   | 33     | 22.5   | 1011     | 62.3 |
| Sep.  | 813      | 10   | 32     | 20     | 1013     | 59.7 |
| Oct.  | 667      | 9.1  | 29.3   | 17.2   | 1014     | 60   |
| Nov.  | 570      | 8.3  | 25.1   | 14.2   | 1016     | 62.7 |
| Dec.  | 479      | 7.3  | 20.2   | 10.7   | 1019     | 62.3 |

Fig. 1 shows clearness index of mean global solar radiation at true noon at clear sky day at Helwan region which given the high clearness index at summer season (June, July, August) around 70-80% in the clear sky days and gives the minimum in the winter season around 40-50%. Can be polynomial fitting and given the Coefficient of determination, RMSE =69% and describe by the equation;

$$K_I = 0.39 + 0.13S/S_0 - 0.01M^2$$

Fig. 2 shows, corresponding between Clearness index at true noon and S/S0, which are given the linear fitting eqn;

$$K_I = 0.86 - 0.15S/S_0$$

Where RMSE = 0.044. Fig. 3 shows the annual variation of global solar radiation at true noon at Helwan, Egypt. Three dimension shows may benefit description data as the maximum is lie in ± 4 hours before and after around true noon around 750-900 W/m². Fig. 4 shows annual monthly variation of global solar radiation at true noon at Helwan, region, Egypt. The general trend describes the depression in the line fitting by 26% in the last eight years and can be polynomial fitting and given the Coefficient of determination, RMSE =79%, and describe by the equation;

$$G = 753.8 - 0.26M$$

General trend is shows; decrease of variation in the hours of measurements afternoon from the measurement before noon. This due to the east direction of Helwan region is clean and impurity from the industrial factories or any source of air pollutants, while west direction of Helwan having many sources of air pollutants. Generally, the maximum intensity of global solar radiation lay in true noon. It is indicate to the sinusoidal shapes treatment and can be describing by Predication of global solar radiation equation which possible to describe by; RMSE: 38.78 (with 98% confidence bounds):

$$G(M) = \sum_{i=1}^{n} [a_0 + a_n\cos(Mw) + b_n\sin(Mw)]$$

Where; descriptions of parameters are in eqn 10 cleared in Table 2. Fig. 5 shows annual hourly Variation of air temperature °C at Helwan, Egypt. It is clear that the mean maximum of Temperature after true noon ranged from 34-36 °C until reach to the sunset in summer season and the minimum at the winter season.

Table 2: Describe of the different parameters for eqn 10.

| P Parameter | Value | Range |
|-------------|-------|-------|
| $w$         | 0.52  | 0.523 | 0.53 |
| $a_0$       | 0.748 | 0.727 | 0.799 |
| $a_1$       | 142   | 113.8 | 170.7 |
| $a_2$       | -25   | 22.2  | -15.02 |
| $a_3$       | -25   | 20.6  | -4.24  |
| $a_4$       | 10.1  | 33.2  | 5.08   |
| $a_5$       | -16.5 | 42.3  | -19.11 |

From To From To
Fig. 6 shows annual hourly Variation of relative humidity % at Helwan, Egypt. It is clear that the mean maximum after true noon ranged from 70-80 % in the winter season and minimum at the summer season from 40-50 %. Figs. 7-10 shows, G, D, T, and RH appreciable periodicity accompanied by intervals of peak height variation \( F(x) \). The distinguished of these variables are formed the long-period variables by an often more complex light curve and by smaller amplitude. The name "Irregular Variables" given to that solar radiation whose intensity is too independent of time for it to be possible to define a cycle around true noon approximately at 12' hour the Fourier transform, however, shows a strong peak at True noon) with Confidence level around

Fig. 1: Annual variation of Clearness index at true noon at clear sky day.

Fig. 2: Corresponding between Clearness index at true noon and S/S0.

Fig. 3: Annual hourly Variation of global solar Radiation at Helwan, Egypt.

Fig. 4: Annual Monthly Variation of global solar radiation at true noon at Helwan, Egypt.

Fig. 5: Annual hourly Variation of air temperature °C at Helwan, Egypt.
99% for G, D, T, RH as corresponding 11.68, 11.59, 11.67, and 11.34. Similarly, the $F(x)$ ranged as 37.9, 3.53, 51.63, 18.05. Inspecting the results, it is apparent that the models agree quite well with the monitored data.

One of the puzzles regarding the occurrence of G, which came to radiation early, was what appeared to be a seasonal effect; it was observed that fewer events occurred during the radiation winter than at of the times of year. There is no reason to suppose that the sun became less active in December and January, but taking a Kt as a reference as shown in Fig. 10. Table 3 shows, statistical behavior for; G, D, T, RH. That indicate to that the skew ness decrease as even in winter, the G frequency distributions are negative skewed (i.e. with a left tail), (i.e. less peaked that the Fourier) and tend to be bimodal.

Table 3: Statistical behavior for; G, D, T, RH

|   | SE  | VA  | AD  | SD  | CV  | SK  | KU  |
|---|-----|-----|-----|-----|-----|-----|-----|
| G | 0.37| 140.53 | 9.61 | 11.85 | 1.55 | 1.38 | 0.49 |
| D | 0.04| 1.36  | 1.00 | 1.16  | 0.94 | 0.83 | -0.82|
| T | 0.48| 239.54 | 12.33| 15.47 | 1.54 | 1.48 | 0.86 |
| RH| 0.18| 33.59 | 4.75 | 5.79  | 1.46 | 1.34 | 0.28 |

**CONCLUSION**

A simple yet an effective stochastic simulation model of global solar radiation at true noon are present in this paper. The introduction of the time dependent frequency distribution concept leads to a reliable simulator. The main conclusions come out from the present work are:

1. This study confirms that the permissible extrapolation of solar irradiation values from measurements site are dependent upon the characteristics of the region’s solar climate and therefore will vary in different parts of the country.
2. The efforts of the researchers in finding statistical interrelationships between some commonly available meteorological parameters and the simultaneous corresponding value of the main solar irradiation quantities have been examined and compared which are given high correlation coefficient which allows depending on it.
3. Developed to frequency distribution; that is to a description statistical analysis, at Helwan, Egypt, of the main quantities of interest such as global solar radiation at true noon and to the statistical modeling of the observed empirical frequency distribution are listed.
4. At test works carried out to investigate the frequency distribution of many solar irradiation related quantities (G, D, T, RH) measured on various time intervals.
5. Correlations, with regional regression coefficients are established to estimate model for Helwan region.
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