**H**\(_\infty\) state estimation for memristive neural networks with randomly occurring DoS attacks

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**ABSTRACT**

This study deals with the problem of the \(H_{\infty}\) state estimation for discrete-time memristive neural networks with time-varying delays, where the output is subject to randomly occurring denial-of-service attacks. The average dwell time is used to describe the attack rules, which makes the randomly occurring denial-of-service attack more universal. The main purpose of the addressed issue is to contribute with a state estimation method, so that the dynamics of the error system is exponentially mean-square stable and satisfies a prescribed \(H_{\infty}\) disturbance attenuation level. Sufficient conditions for the solvability of such a problem are established by employing the Lyapunov function and stochastic analysis techniques. Estimator gain is described explicitly in terms of certain linear matrix inequalities. Finally, the effectiveness of the proposed state estimation scheme is proved by a numerical example.

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**1. Introduction**

As we all know, with the rapid development of automation and information technology in today’s society, artificial intelligence (AI) has become a research hot-spot in various fields, and artificial neural networks (ANNs), as the one of the most important part of artificial intelligence, have also been widely concerned (Fu & Chai, 2007; Rego & Araújo, 2021; Song et al., 2007). ANNs are inspired by biological nervous system, which provides a new solution for modelling, control and optimization of nonlinear systems. However, considering that the research objects are more and more complex and the calculation scale is more and more large, the traditional resistors can no longer meet the practical needs because of their inherent physical constraints. Therefore, a device with smaller physical size and higher integration is urgently needed.

In 1971, the existence of a two-port component is described in Chua (1971) and named memristor. In 2008, HP Lab completed the physical realization of memristor for the first time. Since then, memristor has attracted the attention of many scholars because of its memory function, extremely low power consumption, nano-scale size and many other advantages. The memory characteristics and learning function of memristors make the neural networks (NNs), composed of memristors, more similar to human brain, which brings new dawn for the development of NNs. Recurrent neural networks (RNNs) using memristors instead of resistors can achieve higher integration, the so-called memristive neural networks (MNNs) are widely valued in information processing, pattern recognition, brain-like research and other fields, and a number of numerous research on MNNs have emerged (Cao et al., 2020; Guo et al., 2015; Liu et al., 2018; Strukov et al., 2008; Wang et al., 2017; Yang et al., 2015; Zhang & Shen, 2013). However, it should be pointed out that a large amount of existing literatures are about continuous-time MNNs (CMNNs). Although the study of discrete-time MNNs (DMNNs) has more practical significance in engineering practice, DMNNs are still in a relatively neglected state compared with the CMNNs because of the failure to find a appropriate method to deal with state-related parameters.

The research of memristor neural network mainly includes two aspects: finding more economical materials and analysing the dynamics of systems. At present, there have been a large number of research on the dynamic behaviour analysis of CMNNs, but the related results with discrete-time are much less (Lv et al., 2021; Rakkiyappan et al., 2015; Xiao & Zeng, 2018; Xin et al., 2019; Zhu et al., 2018). It is worth mentioning that the
problem of state estimation has always been a hot topic in the study of complex networks. In practice, it is impossible or even unrealistic to know the state of neurons in the system directly. Therefore, how to estimate the state of internal neurons effectively based on the available measurement information has important research significance. And there have been many achievements on the state estimation of NNs (Huang et al., 2008; Ji et al., 2018; Liu et al., 2020; Rakkiyappan et al., 2015; Shen et al., 2011; Wang & Xue, 2018; Wang et al., 2005; Wang et al., 2012; Zou et al., 2015). Wang et al. (2005) studied the state estimation problem of neural networks with time-varying delays. The robust state estimation problem for a class of uncertain NNs with time-varying delay is studied in Huang et al. (2008). The $H_{\infty}$ state estimation problem of a class of DMNNs under random mixed delay and fading measurements is studied in Liu et al. (2020).

Nowadays, with the development of network communication technology, the activities and communication of the whole society depend on electronic technology. It is well known that many parts of communication network (such as sensors, controllers, etc.) are connected through shared media (Chen et al., 2013; Clark et al., 2012; Guan et al., 2016). However, due to the existence of public parts, attackers can enter the network through these vulnerable units to maliciously destroy the transmission data, resulting in the interruption of system stability. With the advance of communication technology, the ability of attackers to launch network attacks is also improving, and the forms of attacks are more complex and serious. In the past few years, the network security of complex networks has become a research focus. In the network without security protection device, attackers will maliciously intercept, tamper with and destroy the communication data in the transmission process, so as to destroy the stability of plants or move plants along the targets of the attackers (Ding et al., 2017; Ge et al., 2019; Liu et al., 2021; Yu & Yuan, 2020; Zhan et al., 2013; Zhao et al., 2020; Zhang et al., 2020). Generally, cyber-attacks are divided into three types (Ding et al., 2018): denial of service (DoS) attacks (Cao et al., 2021; Zhang et al., 2019), deception attacks (Teixeira et al., 2015; Yuan et al., 2021) and replay attacks (Huang et al., 2020; Zhu & Martinez, 2014).

DoS attacks are one of the most common and threatening attack types. For the networks with high real-time requirements, DoS attacks hinder the timeliness of data transmission, which not only reduces system performance, but even damages the stability of the entire system. There have been a lot of research work on DoS attacks. In Befekadu et al. (2015), the author uses Markov model to control random DoS attacks, while in Amin et al. (2009), the author uses Bernoulli distribution to describe them. In DoS attacks, an attacker can temporarily interrupt the connection between the systems and the internet, affecting the transmission of data packets. In addition to being described using Bernoulli or Markov models, such attacks have appropriate frequency and duration (Ding et al., 2021; Song et al., 2020). The duration and frequency of DoS attacks are clearly described in Preis and Tesi (2015), and the stability of closed-loop system is studied on this basis.

Based on the above considerations, this paper aims to study the $H_{\infty}$ state estimation problem of DMNNs with time delay under DoS attack. And Bernoulli model is established to describe the randomness of DoS attacks. In addition, considering the duration and frequency of attacks, the switching between attacks and non-attacks is described by the switching theory. By using stochastic analysis technique and linear matrix inequalities, the sufficient conditions satisfying $H_{\infty}$ performance are obtained, and the explicit expression of estimator gain is given. The main contributions of this paper as follows: (1) The $H_{\infty}$ state estimation problem of DMNNs with time-varying delays and randomly occurring DoS attacks is studied; (2) According to the theory of switched systems, depending on the frequency and duration of DoS attacks, the sufficient conditions that the error system can satisfy the exponentially mean-square stability and $H_{\infty}$ performance are obtained; (3) The estimator gain is obtained through a set of linear matrix inequalities. Finally, a simulation example is given to illustrate the effectiveness of the proposed estimator mechanism.

The remainder of this article is organized as follows, with the problem description and related preparation described in Section 2. The main content is in Section 3, including the sufficient conditions for the error system to satisfy the mean square exponential stability, the sufficient conditions for the error system to satisfy the $H_{\infty}$ performance index and the expression of the estimator gain. A simulation example is given in Section 4 to prove the effectiveness of the proposed method. Finally, the final conclusion is given in Section 5.

**Notation:** Unless otherwise specified, the notation used is fairly standard. $\mathbb{R}^n$ and $\mathbb{R}^{n \times m}$ denote the $n$-dimensional Euclidean space and the set of all $n \times m$ real matrices, respectively. $I_2([0, +\infty); \mathbb{R}^n)$ is the space of square-summable $n$-dimensional vector functions over $[0, +\infty)$. The notation $X \succeq Y$ (respectively, $X > Y$) where $X$ and $Y$ are symmetric matrices, means that $X - Y$ is positive semidefinite (respectively, positive definite). $I_2$ is used to represent the identity matrix with matching dimensions. The notation $P > 0$ means that $P$ is positive definite matrix. $A^T$ represents the transpose of $A$, $\text{diag} \{\cdots\}$ stands for a block-diagonal matrix. $E[\cdot]$ represents the expectation. The symbol $*$ indicates the symmetric term of a matrix.
2. Problem formulation

Consider DMNNs with time-varying delays in following form:
\[
\begin{align*}
v(s + 1) &= D(v(s))v(s) + A(v(s))f(v(s)) \\
&\quad + B(v(s))g(v(s - \tau(s))) + Lw(s) \\
y(s) &= Cv(s) + Gw(s) \\
z(s) &= Mv(s)
\end{align*}
\]  
(1)

where \( v(s) = [v_1(s) \ v_2(s) \ \cdots \ v_n(s)]^T \) and \( y(s) = [y_1(s) \ y_2(s) \ \cdots \ y_l(s)]^T \) donate the neuron state and the measurement output, respectively; \( z(s) = [z_1(s) \ z_2(s) \ \cdots \ z_r(s)]^T \) is the network output; \( D(v(s)) = \text{diag}\{d_1(v_1(s)), d_2(v_2(s)), \ldots, d_n(v_n(s))\} \) represents the self-feedback matrix; \( A(v(s)) = (a_{ij}(v(s))_{n \times n} \) and \( B(v(s)) = (b_{ij}(v(s))_{n \times n} \) are the network output weight matrix, the delay-dependent connection weight matrix, respectively; \( \tau(s) \) is the time-varying transmission delays and satisfies
\[
\tau_m \leq \tau(s) \leq \tau_M
\]  
(2)

where \( \tau_m \) and \( \tau_M \) are the upper and lower bounds of the time-varying delays, respectively, and they are the positive integers. The initial condition of (1) has the form \( v(s) = \zeta(s) = [\zeta_1(s) \ \zeta_2(s) \ \cdots \ \zeta_n(s)]^T \) for \( s \in [-\tau_M, 0] \).

According to the similar analysis (Liu et al., 2018), the state \( v(s) \) can be rewritten as:
\[
v(s + 1) = (\bar{D} + \Delta D(s))v(s) + (\bar{A} + \Delta A(s))f(v(s)) \\
&\quad + (\bar{B} + \Delta B(s))g(v(s - \tau(s))) + Lw(s)
\]  
(3)

where \( \bar{D} = \text{diag}\{d_1, d_2, \ldots, d_n\}, \bar{A} = (a_{ij})_{n \times n}, \bar{B} = (b_{ij})_{n \times n}, \Delta D(s) = H_F(s)N_D, \Delta A(s) = H_F(s)N_A, \Delta B(s) = H_F(s)N_B \), here \( d_i, a_{ij}, b_{ij} \) are known scalars, \( H, N_D, N_A \) and \( N_B \) are known real constant matrices, \( F_i(s) (i = 1, 2, 3) \) satisfies that \( F_i(s)F_i(s)^T \leq I \).

Furthermore, the data may be attacked during transmission, and the attack model can be describe as follows:
\[
\tilde{y}(s) = (1 - i(s))y(s - 1) + i(s)\hat{y}(s)
\]  
(4)

where \( \hat{y}(s) \) is the information received by the estimator actually. The stochastic variable \( i(s) \) is a stochastic variable and satisfying Bernoulli distribute white sequence, the form as follows:
\[
\text{Prob}[i(s) = 1] = \bar{\iota} \quad \text{Prob}[i(s) = 0] = 1 - \bar{\iota}
\]  
(5)

where \( \bar{\iota} \in [0, 1] \) are known constants.

Remark 2.1: Cyber-attacks are generally divided into DoS attacks, the deception attacks and reply attacks. The adversary prevents the estimation from receiving sensor information to carry out DoS attacks. In this paper, the randomly occurring behaviours of the DoS attacks are described by Bernoulli distribute white sequences with known conditional probability, and the estimator receives the normal measurements when \( i(s) = 1 \). In this article, we consider that the state of the system is switched between the occurrence of an DoS attacks and the occurrence of no DoS attacks. Denote the time instant, at which DoS attacks are launched by malicious adversaries, as \( t^k, k \in \mathbb{N} \).

Based on the actually received signal \( \hat{y}(s) \), the following state estimator is proposed to estimate the neuron state \( v(s) \):
\[
\begin{align*}
\hat{v}(s + 1) &= \bar{D}\hat{v}(s) + \bar{A}\hat{f}(v(s)) + \bar{B}\hat{g}(v(s - \tau(s))) \\
&\quad + K(\tilde{y}(s) - \bar{C}\hat{v}(s))
\end{align*}
\]  
(6)

\[
\tilde{z}(s) = M\hat{v}(s)
\]  
(6)

where \( \hat{v}(s) \in \mathbb{R}^n \) is the estimate of the state \( v(s) \), \( \tilde{z}(s) \in \mathbb{R}^r \) represents the estimate of the output \( z(s) \), and \( K \) is the estimator gain to be designed.

The estimation error can be described as:
\[
e(s + 1) = (\bar{D} - KC)e(s) + \Delta D(s)v(s) + (1 - \bar{\iota})KC\bar{v}(s) \\
&\quad - (i(s) - \bar{\iota})KC\bar{v}(s) + \bar{A}\hat{f}(v(s)) \\
&\quad + \Delta A(s)f(v(s)) + \bar{B}\hat{g}(v(s - \tau(s))) \\
&\quad + \Delta B(s)g(v(s - \tau(s))) - (1 - \bar{\iota})Ky(s - 1) \\
&\quad + (i(s) - \bar{\iota})K\hat{y}(s - 1) - (i(s) - \bar{\iota})K\tilde{w}(s) \\
&\quad + (L - \bar{\iota}K)\bar{w}(s)
\]  
(7)

where \( e(s) \triangleq v(s) - \hat{v}(s), \bar{f}(v(s)) \triangleq f(v(s)) - f(\bar{v}(s)), \bar{g}(v(s - \tau(s))) \triangleq g(v(s - \tau(s))) - g(\hat{v}(s - \tau(s))) \), \( \tilde{y}(s) \triangleq y(s) - \hat{y}(s) \) and \( \bar{z}(s) \triangleq z(s) - \bar{z}(s) \) is the output estimation error. And based on setting \( \eta(s) = [\bar{v}(s) \ \bar{e}(s)^T]^T \), we have the following function:
\[
\eta(s + 1) = W_1\eta(s) + (i(s) - \bar{\iota})W_2\eta(s) + W_3\bar{f}(s) \\
&\quad + W_4\bar{g}(s - \tau(s)) + W_5\bar{w}(s) \\
&\quad + (i(s) - \bar{\iota})W_6\bar{w}(s)
\]  
(8)

\[
\bar{z}(s) = M\eta(s)
\]  
(8)
where
\[
\begin{align*}
\tilde{f}(s) &= \begin{bmatrix} f^T(v(s)) & \bar{f}^T(v(s)) \end{bmatrix}^T, \\
\tilde{g}(s - \tau(s)) &= \begin{bmatrix} g^T(v(s - \tau(s))) & \bar{g}^T(v(s - \tau(s))) \end{bmatrix}^T,
\end{align*}
\]
\[
W_1 = \tilde{W}_1 + \Delta \mathcal{D}(s),
\]
\[
\tilde{W}_1 = \begin{bmatrix} \tilde{\bar{D}} & 0 & 0 \\
(1 - \bar{\delta})\bar{K} & \bar{\bar{D}} - \bar{\bar{K}}C & - (1 - \bar{\delta})K \\
\end{bmatrix},
\]
\[
\Delta \mathcal{D}(s) = \begin{bmatrix} \Delta \bar{D}(s) & 0 & 0 \\
\Delta \bar{D}(s) & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix},
\]
\[
W_2 = \begin{bmatrix} 0 & 0 \bar{0}^T \\
-K\bar{C} & \bar{K} & \bar{0} \\
0 & 0 & 0 \\
\end{bmatrix},
\]
\[
W_3 = \bar{A} + \Delta \bar{A}(s),
\]
\[
W_4 = \bar{\bar{B}} + \Delta \bar{B}(s),
\]
\[
\bar{\bar{A}} = \begin{bmatrix} \bar{A} & 0 & 0 \\
0 & \bar{A} & 0 \\
0 & 0 & 0 \\
\end{bmatrix},
\]
\[
\Delta \bar{A}(s) = \begin{bmatrix} \Delta \bar{A}(s) & 0 & 0 \\
\Delta \bar{A}(s) & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix},
\]
\[
\bar{\bar{B}} = \begin{bmatrix} \bar{B} & 0 & 0 \\
0 & \bar{B} & 0 \\
0 & 0 & 0 \\
\end{bmatrix},
\]
\[
\Delta \bar{B}(s) = \begin{bmatrix} \Delta \bar{B}(s) & 0 & 0 \\
\Delta \bar{B}(s) & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix},
\]
\[
W_5 = \begin{bmatrix} L \\
L - \bar{K}\bar{G} \\
\end{bmatrix},
\]
\[
W_6 = \begin{bmatrix} 0 & -\bar{K} \bar{G} \\
0 & 0 \\
\end{bmatrix},
\]
\[
\mathcal{M} = \begin{bmatrix} 0 & M & 0 \\
\end{bmatrix}.
\]

Here, we first introduce the following assumption and definition, and then go further.

**Assumption 2.1:** The neuron activation functions $f(\cdot)$ and $g(\cdot)$ satisfy $f(0) = g(0) = 0$ and
\[
\begin{align*}
[f(v) - f(y) - \Lambda_1(v - y)]^T[f(v) - f(y)] \\
- \Lambda_2(v - y) & \leq 0, \quad v, y \in \mathbb{R}^n \quad (v \neq y) \\
[g(v) - g(y) - \Upsilon_1(v - y)]^T[g(v) - g(y)] \\
- \Upsilon_2(v - y) & \leq 0, \quad v, y \in \mathbb{R}^n \quad (v \neq y)
\end{align*}
\]
where $\Lambda_1, \Lambda_2, \Upsilon_1$ and $\Upsilon_2$ are constant matrices.

**Assumption 2.2:** Denote the number of on/off DoS attacks occurring on the interval $[s_0, s_1]$ as $N(s_0, s_1)$, there exist the chatter bound $N_0 \geq 0$ and average dwell-time $\tau_a$ satisfy:
\[
N(s_0, s_1) \leq N_0 + \frac{(s_1 - s_0)}{\tau_a}. \tag{11}
\]

**Remark 2.2:** In this article, the system is considered as a switching system involving that the attack is active or silent. In order to make sure stability, the frequency and average dwell time of DoS attack are constrained as shown in Assumption 2.2. Denote the time instant, at which DoS attacks are launched by malicious adversaries, as $s_k^k k \in \mathbb{N}$. A basic problem is to determine a minimum $\tau_a$ so that the system can meet the prescribed performance index.

Our main idea in this article is to design an appropriate estimator (6) such as:

1. System (8) with $w(s) = 0$ is exponentially mean-square stable.
2. Under zero initial conditions, for given disturbance attention level $\gamma > 0$ and all nonzero $w(s)$
\[
\sum_{s=0}^{\infty} \mathbb{E}\|\tilde{z}(s)\|^2 \leq \gamma^2 \sum_{s=0}^{\infty} \mathbb{E}\|w(s)\|^2. \tag{12}
\]

3. **Main results**

In this section, the $H_\infty$ state estimation of the augmented system (8) is addressed by utilizing the Lyapunov stability analysis approach. To start with, we give the following useful lemmas that will be needed for the subsequent derivation in this paper.

**Lemma 3.1 (Schur complement):** Given constant matrices $\Psi_1, \Psi_2, \Psi_3$ where $\Psi_1 = \Psi_1^T$ and $\Psi_2 > 0$, then
\[
\Psi_1 + \Psi_3^T \Psi_2^{-1} \Psi_3 < 0
\]
if and only if
\[
\begin{bmatrix} \Psi_1 & \Psi_3^T \\
\Psi_3 & -\Psi_2 \\
\end{bmatrix} < 0.
\]

**Lemma 3.2 (S-procedure):** Let $R = R^T, U$ and $E$ be real matrices with appropriate dimensions, and $F^T F \leq I$. Then, the inequality $R + UFE + (UFE)^T < 0$ holds if and only if there exists a scalar $\vartheta > 0$ such that $R + \vartheta UU^T + \mu^{-1} E^T E < 0$ or, equivalently,
\[
\begin{bmatrix} R & U & \vartheta E^T \\
* & -\vartheta I & 0 \\
* & * & -\vartheta I \\
\end{bmatrix} < 0.
\]
Lemma 3.3: Given scalars $0 < \varepsilon < 1$, $\sigma > 1$, if exists a Lyapunov function $V_{i(s)}(s) = \eta^T(s)P_{i(s)}\eta(s)$ such that the following

$$V_i(s + 1) - V_i(s) \leq -\varepsilon V_i(s),$$

$$V_i(s_i^k) \leq \sigma V_i(s_i^k)$$

hold, where $i = s_i^k$, $j = s_i^{k-1}$. Then for any switching signal, the system (8) is exponentially mean-square stable with the average dwell-time

$$\tau_a \geq \tau_a^* = -\frac{\ln \sigma}{\ln(1 - \varepsilon)}.$$  

Sufficient conditions for the augmented system (8) to satisfy $H_\infty$ performance constraint (12) are established in the following results.

Theorem 3.1: Let state estimation gain $K$ be given. The system (8) with $w(s) = 0$ is exponentially mean-square stable if there exist matrices $P_{i(s)} = \text{diag}(P_{i(s),1}, P_{i(s),2}, P_{i(s),3}) > 0$, $Q > 0$, and scalars $\lambda_1 > 0$, $\lambda_2 > 0$, $0 < \varepsilon < 1$, $\sigma > 1$ such that:

$$\begin{aligned}
\tilde{\Pi}_{i(s)} = & \begin{bmatrix}
\tilde{\Theta}_{11} & 0 & \tilde{\Theta}_{13} & \tilde{\Theta}_{14} \\
* & \tilde{\Theta}_{22} & 0 & \tilde{\Theta}_{24} \\
* & * & \tilde{\Theta}_{33} & \tilde{\Theta}_{34} \\
* & * & * & \tilde{\Theta}_{44}
\end{bmatrix} < 0, \\
\end{aligned}$$  

and the average dwell-time $\tau_a$ satisfy

$$\tau_a \geq \tau_a^* = -\frac{\ln \sigma}{\ln(1 - \varepsilon)}.$$  

Proof: Select the following Lyapunov functional candidate:

$$V_{i(s)}(s) = V_{i(s),1}(s) + V_2(s) + V_3(s)$$  

where

$$V_{i(s),1}(s) = \eta^T(s)P_{i(s)}\eta(s),$$

$$V_2(s) = \sum_{i=s-\tau(s)}^{s-1} \eta^T(i)Q\eta(i),$$

$$V_3(s) = \sum_{i=s-\tau(m+1)}^{s-1} \sum_{i=j}^{s-1} \eta^T(i)Q\eta(i).$$

For the convenience of analysis, $s_i^k$, $k \in \mathbb{N}$ is used to represent the switching time sequence of switching signals $i(s)$ for $s \in [s_i^k, s_i^{k+1}]$. Then, under $w(s) = 0$, letting $\mathbb{E}[\Delta V(s)] = \mathbb{E}[V(s + 1) - V(s)]$, along the trajectory of system (8), we have

$$\begin{aligned}
\mathbb{E}[\Delta V_{i(s),1}(s)] &= \mathbb{E}[V_{i(s+1),1}(s + 1) - V_{i(s),1}(s)] \\
&= \mathbb{E}\left\{\eta^T(s + 1)P_{i(s+1)}\eta(s + 1) - \eta^T(s)P_{i(s)}\eta(s)\right\} \\
&= \mathbb{E}\left\{\left(W_1\eta(s) + W_3\tilde{f}(s) + W_4\tilde{g}(s - \tau(s))\right)^T P_{i(s+1)} \right. \\
&\quad \times \left(W_1\eta(s) + W_3\tilde{f}(s) + W_4\tilde{g}(s - \tau(s))\right) - \eta^T(s)P_{i(s)}\eta(s)\right\} \\
&= \mathbb{E}\left\{\eta^T(s)\left(W_1^TP_{i(s+1)}W_1 - P_{i(s)}\right)\eta(s) \\
&\quad + \tilde{f}^T(s)W_3^TP_{i(s+1)}\tilde{f}(s) \\
&\quad + \tilde{g}^T(s - \tau(s))W_4^TP_{i(s+1)}\tilde{g}(s - \tau(s)) + 2\eta^T(s)W_1^TP_{i(s+1)}W_4\tilde{g}(s - \tau(s)) \\
&\quad + 2\eta^T(s)W_3^TP_{i(s+1)}W_4\tilde{f}(s) \\
&\quad + 2\eta^T(s)W_3^TP_{i(s+1)}W_4\tilde{g}(s - \tau(s))\right\}. \\
\end{aligned}$$

Similarly, it follows that

$$\begin{aligned}
\mathbb{E}[\Delta V_2(s)] &= \mathbb{E}[V_2(s + 1) - V_2(s)] \\
&= \mathbb{E}\left\{\eta^T(s)Q\eta(s) - \eta^T(s - \tau(s))Q\eta(s - \tau(s)) \\
&\quad + \sum_{i=s+1-\tau(s+1)}^{s-1} \eta^T(i)Q\eta(i) - \sum_{i=s+1-\tau(s)}^{s-1} \eta^T(i)Q\eta(i)\right\}.
\end{aligned}$$
functions which corresponds to Lemma 3.3. Therefore, it can be accomplished.

\[ \mathbb{E}\{\Delta V_3(s)\} \leq \mathbb{E}\left\{ \eta^T(s)Q\eta(s) - \eta^T(s - \tau(s))Q\eta(s - \tau(s)) \right\} + \sum_{i=s+1}^{s-t_m} \eta^T(i)Q\eta(i) \] (20)

and

\[ \mathbb{E}\{\Delta V_{i(s)}(s)\} \leq \mathbb{E}\left\{ \eta^T(s)\Pi_{i}(s)\eta(s) - \sum_{i=s+1}^{s-t_m} \eta^T(i)\eta(i) \right\}. \] (21)

Subsequently, according to (15), (19)–(21), we can easily access to that

\[ \mathbb{E}\{\Delta V_{i(s)}(s)\} \leq \mathbb{E}\left\{ \zeta^T(s)\Pi_{i}(s)\zeta(s) \right\} \] (22)

where

\[
\zeta(s) = \begin{bmatrix} \eta^T(s) & \eta^T(s - \tau(s)) & \bar{f}^T(s) & \bar{g}^T(s - \tau(s)) \end{bmatrix}^T, \\
\Pi_{i}(s) = \begin{bmatrix} \Theta_{11} & 0 & \Theta_{13} & \Theta_{14} \\ * & \Theta_{22} & 0 & 0 \\ * & * & \Theta_{33} & \Theta_{34} \\ * & * & * & \Theta_{44} \end{bmatrix}.
\]

Next, we deduce from Assumption 2.1 that the nonlinear functions \( \bar{f}(s) \) and \( \bar{g}(s - \tau(s)) \) satisfy

\[
\left[ \bar{f}(s) - (\mathcal{J} \otimes \Lambda_1)\eta(s) \right]^T \left[ \bar{f}(s) - (\mathcal{J} \otimes \Lambda_2)\eta(s) \right] \leq 0,
\]
\[
\left[ \bar{g}(s - \tau(s)) - (\mathcal{J} \otimes \Upsilon_1)\eta(s - \tau(s)) \right]^T \left[ \bar{g}(s - \tau(s)) \right] \leq 0.
\]

It is evident from (22) and (23) that

\[
\mathbb{E}\{\Delta V_{i(s)}(s)\} \leq \mathbb{E}\left\{ \zeta^T(s)\Pi_{i}(s)\zeta(s) - \lambda_1\bar{f}^T(s) - (\mathcal{J} \otimes \Lambda_1)\eta(s) \right\} + \sum_{i=s+1}^{s-t_m} \eta^T(i)\Pi_{i}(s)\eta(i) \geq 0.
\]

\[
\mathbb{E}\{\Delta V_{i(s)}(s)\} < -\varepsilon\mathbb{E}\{V_{i(s)}(s)\}
\]

can obviously be obtained, which corresponds to Lemma 3.3. Therefore, it can be concluded that the system (8) with \( w(s) = 0 \) is exponentially mean-square stable, which ends the proof of Theorem 3.1.

**Theorem 3.2:** Let \( K \) and disturbance attention level \( \gamma > 0 \) be given. Under the zero initial condition, the system (8) is exponentially mean-square stable with \( w(s) = 0 \) and satisfies \( H_{\infty} \) performance constraint (12) for all nonzero \( w(s) \) if there exist matrices \( P_{i(s)} = \text{diag}\{P_{i(s,1)}, P_{i(s,2)}, P_{i(s,3)}\} > 0, Q > 0, \) and scalars \( \lambda_1 > 0, \lambda_2 > 0, 0 < \varepsilon < 1, \sigma > 1 \) such that:

\[
\Pi_{i}(s) = \begin{bmatrix} \Theta_{11} & 0 & \Theta_{13} & \Theta_{14} & \Theta_{15} \\ * & \Theta_{22} & 0 & 0 & 0 \\ * & * & \Theta_{33} & \Theta_{34} & \Theta_{35} \\ * & * & * & \Theta_{44} & \Theta_{45} \\ * & * & * & * & \Theta_{55} \end{bmatrix} < 0,
\]

\[
P_{i(s)} - \sigma P_{i(s-1)} < 0, (s) \neq (s - 1)
\]

and the average dwell-time \( \tau_d \) satisfy

\[
\tau_d \geq \tau_d^* = -\frac{\ln \sigma}{\ln(1 - \varepsilon)}
\]

where

\[
\Theta_{11} = \tilde{\Theta} + \mathcal{M}^T\mathcal{M}, \quad \Theta_{15} = W^TP_{i(s+1)}W_5,
\]
\[
\Theta_{35} = W_5^TP_{i(s+1)}W_5, \quad \Theta_{45} = W_5^TP_{i(s+1)}W_5,
\]
\[
\Theta_{55} = W_5^TP_{i(s+1)}W_5 - \gamma^2I
\]

and other parameters are defined as Theorem 3.1.

**Proof:** Next, the \( H_{\infty} \) performance of system (8) is investigated. Consider the same Lyapunov functional as (15) is chosen and (25), we have

\[
\mathbb{E}\{V_{i(s+1)}(s + 1) - (1 - \varepsilon) V_{i(s)}(s)\} + \mathbb{E}\{z^T(s)z(s)\} - \gamma^2\mathbb{E}\{w^T(s)w(s)\}
\]

\[\leq \mathbb{E}\left\{ \eta^T(s) \left( W^TP_{i(s+1)}W_1 + (\tau_M - \tau_m + 1)Q - P_{i(s)} \right) + \mathcal{M}^T\mathcal{M}\eta(s) + \bar{f}^T(s)W_5^TP_{i(s+1)}W_3\bar{f}(s) + \bar{g}^T(s - \tau(s))W_5^TP_{i(s+1)}W_3\bar{g}(s - \tau(s)) + w^T(s)w_5^T(P_{i(s+1)} - \gamma^2I)w(s) + 2\eta^T(s)W_5^TP_{i(s+1)}W_5\bar{g}(s - \tau(s)) + 2\eta^T(s)W_5^TP_{i(s+1)}W_5\bar{g}(s - \tau(s)) + 2\eta^T(s)W_5^TP_{i(s+1)}W_5\bar{g}(s - \tau(s)) + 2\eta^T(s)W_5^TP_{i(s+1)}W_5\bar{g}(s - \tau(s)) + 2\eta^T(s)W_5^TP_{i(s+1)}W_5\bar{g}(s - \tau(s)) \right\} < 0.
\]

\[
\mathbb{E}\{\Delta V_{i(s+1)}(s\ +\ 1)\} < -\varepsilon\mathbb{E}\{V_{i(s)}(s)\}
\]
\[ + \bar{g}^T(s - \tau(s))W_k^sP_{\delta_s^k+1}W_k^s w(s) \]  
\[ \leq \mathbb{E} \left[ \phi(s) \Pi_{i(s)} \phi(s) \right] < 0 \]  
(28)

where
\[ \phi(s) = [\xi^T(s) \ w^T(s)]^T. \]

In what follows, let us investigate the impact of switches induced by DoS attacks. On the interval \([s_0, s_1]\), the switching instants are denoted as \(s_0 < s_1^0 < s_1^1 < \ldots < s_i^j < s < s_1\). Since there are only two states of the system, namely, normal transmission state and DoS attacked, the following proof is discussed in two cases: (1) donating intervals without switches as \([s_i^j, s_i^j+1]\); (2) an existing switch \(s = s_i^j - 1\).

**Case (1):** Intervals \([s_i^j, s_i^j+1]\) without switches, it follows directly from (28) that

\[ \mathbb{E}[V_{i(s)}(s)] = \mathbb{E}[V_{i(s)}(s)] \]
\[ \leq (1 - \varepsilon)\mathbb{E}[V_{i(s)}(s-1)] + \gamma^2 \mathbb{E}[w^T(s)w(s)] \]
\[ - \mathbb{E}[z^T(s)z(s)] \]
\[ \leq (1 - \varepsilon)^2 \mathbb{E}[V_{i(s)}(s-2)] \]
\[ + \sum_{i=s-1}^{s-2} (1 - \varepsilon)^{s-1-i} \gamma^2 \mathbb{E}[w^T(i)w(i)] \]
\[ - \sum_{i=s-1}^{s-2} (1 - \varepsilon)^{s-1-i} \mathbb{E}[z^T(i)z(i)] \]
\[ \leq \cdots \]
\[ \leq (1 - \varepsilon)^{s-s_0} \mathbb{E}[V_{i(s)}(s)] \]
\[ + \sum_{i=s_0}^{s_1} (1 - \varepsilon)^{s_1-j} \gamma^2 \mathbb{E}[w^T(j)w(j)] \]
\[ - \sum_{i=s_0}^{s_1} (1 - \varepsilon)^{s_1-j} \mathbb{E}[z^T(j)z(j)]. \]  
(29)

**Case (2):** Suppose there is a switch from instant \(s_i^j - 1\) to \(s_i^j\), we have

\[ \mathbb{E}[V_{i(s)}(s)] \leq (1 - \varepsilon)^{s-s_0} \mathbb{E}[V_{i(s)}(s)] \]
\[ + \sum_{i=s_0}^{s_1} (1 - \varepsilon)^{s_1-j} \gamma^2 \mathbb{E}[w^T(j)w(j)] \]
\[ - \sum_{i=s_0}^{s_1} (1 - \varepsilon)^{s_1-j} \mathbb{E}[z^T(j)z(j)] \]
\[ \leq \sigma(1 - \varepsilon)^{s-s_0} \mathbb{E}[V_{i(s)}(s)] \]
\[ + \sum_{i=s_0}^{s_1} (1 - \varepsilon)^{s_1-j} \gamma^2 \mathbb{E}[w^T(j)w(j)] \]
\[ - \sum_{i=s_0}^{s_1} (1 - \varepsilon)^{s_1-j} \mathbb{E}[z^T(j)z(j)]. \]  
(30)

where \(N(s_0, s_1)\) represents the number of on/off DoS attacks occurring on the interval \([s_0, s_1]\). And then

\[ \mathbb{E}[V_{i(s)}(s)] \leq \sum_{i=s_0}^{s_1} \sigma^N(s) (1 - \varepsilon)^{s_1-j} \gamma^2 \mathbb{E}[w^T(j)w(j)] \]
\[ - \sum_{i=s_0}^{s_1} \sigma^N(s) (1 - \varepsilon)^{s_1-j} \mathbb{E}[z^T(j)z(j)]. \]  
(31)

Invoking (27) and the definition of the average dwell-time,

\[ N(i, s) \leq N_0 + \frac{s - i}{\tau_a} \leq N_0 - (s - i) \frac{\ln(1 - \varepsilon)}{\ln \sigma} \]  
(32)

which implies that

\[ \sigma^N(s) \leq (1 - \rho)^{i-s} \sigma N_0. \]
Under zero initial condition and $\mathbb{E}[V(t(\infty))|\omega] \geq 0$, then we have
\[
\sum_{i=0}^{\infty} \mathbb{E}[\tilde{z}(i)\tilde{z}(i)] \leq \gamma^2 \sum_{i=0}^{\infty} \mathbb{E}[w^T(i)w(i)],
\]
(33)
the proof of Theorem 3.2 is completed.

Based on the above theorems, the desired estimator is designed as follows.

**Theorem 3.3:** Let the disturbance attention level $\gamma > 0$ be given. Under the zero initial condition, the system (8) is exponentially mean-square stable and satisfies $H_\infty$ performance constraint (12) for all nonzero $w(t)$ if there exist matrices $P(t) = \text{diag}[P_1(t), P_2(t), \ldots, P_N(t)] > 0, Q > 0, Y > 0$ and positive scalars $\lambda_1 > 0, \lambda_2 > 0, 0 < \varepsilon < 1, \sigma > 1, \theta$ such that
\[
\Xi(t) = \begin{bmatrix} \Omega & \hat{H} & \theta \hat{N}^T \\ \ast & -\theta I & 0 \\ \ast & \ast & -\theta I \end{bmatrix} < 0,
\]
(34)

\[
P(t) - \sigma P(t-1) < 0, \forall t \neq t(s-1)
\]
(35)

where
\[
\Xi(t) = \begin{bmatrix} \Omega_{11} & 0 & \Omega_{13} \\ 0 & \Omega_{22} & 0 \\ \ast & \ast & \Omega_{33} \end{bmatrix},
\]
\[
\Phi_2 = \begin{bmatrix} W_{1} & 0 & \Phi_2 \end{bmatrix},
\]
\[
\Phi_1 = \begin{bmatrix} \Psi_1 & 0 & P(t) \end{bmatrix},
\]
\[
\Psi_1 = \begin{bmatrix} P(t) & 0 & 0 \\ \ast & \ast & \ast \end{bmatrix},
\]
\[
\Psi_2 = \begin{bmatrix} P(t) & 0 & 0 \\ \ast & \ast & \ast \end{bmatrix},
\]
\[
\Omega_{11} = \hat{\Omega}_{11} - W_{1}P(t)W_{1}, \quad \Omega_{13} = \lambda_1 \hat{\Omega}_{13},
\]
\[
\Omega_{33} = -\lambda_2 I, \quad \Omega_{44} = -\lambda_2 I, \quad \Omega_{55} = -\gamma^2 I,
\]
\[
\hat{H} = \begin{bmatrix} \hat{H}^T P(t) & 0 & 0 & 0 & 0 \\ \hat{H}^T P(t) & 0 & 0 & 0 & 0 \\ \hat{H}^T P(t) & 0 & 0 & 0 & 0 \end{bmatrix},
\]
\[
\hat{N} = \begin{bmatrix} N_A & 0 & 0 \\ 0 & N_A & 0 \\ 0 & 0 & N_B \end{bmatrix},
\]
\[
\hat{H} = \begin{bmatrix} H & 0 & 0 \\ H & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \hat{N}_D = \begin{bmatrix} N_D & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},
\]
\[
\hat{H} = \begin{bmatrix} H & 0 & 0 \\ H & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \hat{N}_D = \begin{bmatrix} N_D & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},
\]
and other parameters are defined as Theorem 3.1. If the foregoing conditions hold, a desired estimator gain is given by
\[
K_1(t) = P(t)^{-1}Y.
\]
(36)

**Proof:** Donate
\[
\Phi_1 = [P(t+1)W_1 0 P(t+1)W_3 P(t+1)W_4 P(t+1)W_5].
\]
According to (36), we have that from (34)
\[
\Xi(t) = \begin{bmatrix} \hat{\Omega} & \hat{H} & \theta \hat{N}^T \\ \ast & -\theta I & 0 \\ \ast & \ast & -\theta I \end{bmatrix} < 0
\]
(37)
where
\[
\hat{\Omega} = \begin{bmatrix} -P(t+1) & \Phi_1 \end{bmatrix},
\]
\[
\Phi_1 = \begin{bmatrix} P(t+1) \hat{W}_1 & P(t+1) \hat{A} & P(t+1) \hat{B} & \hat{\Psi}_2 \end{bmatrix}.
\]
Then, by using S-procedure, the (37) can be described as follows
\[
\hat{\Omega} + (\hat{H}^T \Phi(s) \hat{N}) + (\hat{H}^T \Phi(s) \hat{N})^T < 0
\]
(38)
where $\hat{\Phi}(s) \triangleq \text{diag}[\hat{F}_1(s), \hat{F}_1(s), \hat{F}_1(s), \hat{F}_2(s), \hat{F}_2(s), \hat{F}_2(s), \hat{F}_3(s), \hat{F}_3(s), \hat{F}_3(s)]$. Rearrange (38)
\[
\begin{bmatrix} -P(t+1) & \Phi_1 \end{bmatrix} < 0.
\]
(39)
Pre-multiplying and post-multiplying (39) by diag($P(t+1)^{-1}$, I, I, I, I, I, I, I) and its transpose, one can find that condition (34) guarantees the (25) by Lemma 3.1. The augmented system (8) is exponentially mean-square stable and satisfies $H_\infty$ performance constraint, and the appropriate estimator gain can be obtained.

**Remark 3.1:** This article aims to study the $H_\infty$ state estimation problem of discrete-time delayed MNNs subject to DoS attacks. For DMNNs with time-varying delays, considering the DoS attack with a certain success probability, the frequency of DoS is described according to the switching system theory. In a word, a new unified framework is established to account for the $H_\infty$ state estimation of the DMNNs with time-varying delays and randomly occurring DoS attacks.
4. Illustrative example

In this section, a numerical example is given to illustrate the effectiveness of the proposed method. Consider DMNNs (1) with parameters as

\[d_1(v_1(\cdot)) = \begin{cases} 0.975, & |v_1(\cdot)| > 0.1, \\ -0.175, & |v_1(\cdot)| \leq 0.1, \end{cases}\]

\[d_2(v_2(\cdot)) = \begin{cases} 0.370, & |v_2(\cdot)| > 0.1, \\ 0.230, & |v_2(\cdot)| \leq 0.1, \end{cases}\]

\[a_{11}(v_1(\cdot)) = \begin{cases} 0.268, & |v_1(\cdot)| > 0.1, \\ 0.732, & |v_1(\cdot)| \leq 0.1, \end{cases}\]

\[a_{12}(v_1(\cdot)) = \begin{cases} 0.150, & |v_1(\cdot)| > 0.1, \\ -0.350, & |v_1(\cdot)| \leq 0.1, \end{cases}\]

\[a_{21}(v_2(\cdot)) = \begin{cases} 0.360, & |v_2(\cdot)| > 0.1, \\ 0.040, & |v_2(\cdot)| \leq 0.1, \end{cases}\]

\[a_{22}(v_2(\cdot)) = \begin{cases} 0.270, & |v_2(\cdot)| > 0.1, \\ 0.130, & |v_2(\cdot)| \leq 0.1, \end{cases}\]

\[b_{11}(v_1(\cdot)) = \begin{cases} 0.350, & |v_1(\cdot)| > 0.1, \\ 0.350, & |v_1(\cdot)| \leq 0.1, \end{cases}\]

\[b_{12}(v_1(\cdot)) = \begin{cases} 0.220, & |v_1(\cdot)| > 0.1, \\ 0.280, & |v_1(\cdot)| \leq 0.1, \end{cases}\]

\[b_{21}(v_2(\cdot)) = \begin{cases} 0.015, & |v_2(\cdot)| > 0.1, \\ 0.095, & |v_2(\cdot)| \leq 0.1, \end{cases}\]

\[b_{22}(v_2(\cdot)) = \begin{cases} -0.650, & |v_2(\cdot)| > 0.1, \\ 0.250, & |v_2(\cdot)| \leq 0.1, \end{cases}\]

and other parameters are given as

\[C = \begin{bmatrix} 0.400 & 0.200 \end{bmatrix}, \quad L = \begin{bmatrix} 0.250 \\ 0.150 \end{bmatrix},\]

\[G = 0.050, \quad M = \begin{bmatrix} -0.800 & 0.500 \end{bmatrix}.\]

The activation functions \(f(v(s))\) and \(g(v(s))\) are chosen as

\[f(v(s)) = \begin{bmatrix} -\text{tanh}(0.1v_1(s)) \\ \text{tanh}(0.3v_2(s)) \end{bmatrix},\]

\[g(v(s)) = \begin{bmatrix} -\text{tanh}(0.15v_1(s)) \\ \text{tanh}(0.3v_2(s)) \end{bmatrix},\]

which satisfy the Assumption 2.1 with

\[\Lambda_1 = \begin{bmatrix} -0.6 & 0 \\ 0 & -0.4 \end{bmatrix}, \quad \Lambda_2 = \begin{bmatrix} -0.3 & 0 \\ 0 & 0.6 \end{bmatrix},\]

\[\Upsilon_1 = \begin{bmatrix} 0 & 0 \\ 0 & -0.4 \end{bmatrix}, \quad \Upsilon_2 = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.2 \end{bmatrix}.\]

In the example, the parameter of DoS attacks are taken as \(\tau = 0.9\), the disturbance attenuation level \(\gamma = 0.9\). The discrete delays are set as \(\tau(s) = 3 + \cos(s\pi)\), then we have \(\tau_m = 1\) and \(\tau_M = 2\).

Solving the Theorem 3.3, we can obtain the feasible solution:

\[\gamma = \begin{bmatrix} 0.4508 \\ -2.3442 \end{bmatrix}.\]

Case (a): \(\iota(s) = 1\), there is no DoS attack

\[P_{1,2} = \begin{bmatrix} 3.5712 & -7.0349 \\ -7.0349 & 22.9131 \end{bmatrix}, \quad K_1 = \begin{bmatrix} -0.1906 \\ -0.1608 \end{bmatrix}.\]

Case (b): \(\iota(s) = 0\), there exist DoS attack

\[P_{0,2} = \begin{bmatrix} 3.9142 & -6.5245 \\ -6.5245 & 31.8767 \end{bmatrix}, \quad K_0 = \begin{bmatrix} 0.5906 \\ 0.1256 \end{bmatrix}.\]

In the simulation, the exogenous disturbances are chosen as \(w(s) = \cos(s - 1)/(10s + 1)\). The initial values of the neuron state are random values and obey the uniform distribution on \([-1, 1]\). Figures 1–4 show simulation results. Figure 1 plots the ideal measurements output and the received signals. Figure 2 describes \(z(s)\) and its estimation \(\hat{z}(s)\). Figure 3 depicts the estimation errors. And Figure 4 shows the instants of the DoS attacks. It can be indicated that, the proposed estimator could provide satisfactory state estimations for DMNNs under the DoS attacks.

5. Conclusions

In this paper, the \(H_\infty\) state estimation problem of a class of DMNNs with time-varying delay subject to DoS attacks is studied. Using the attacks’ frequency and durations to describe DoS attacks, the modelled network can
be essentially described as a switched system that handles between DoS attacks and no DoS attacks. By using Lyapunov stability theorem, the system satisfies the mean-square exponential stability and $H_{\infty}$ performance index, and a set of linear matrix inequalities are used to express the sufficient conditions for the system to satisfy the target performance. Finally, a simulation example is given to verify the effectiveness of the results. In the future other types of cyber-attacks can be considered instead of DoS attacks. What needs to be made clear is that how to consider cyber-attacks and network security issues in complex networks and reduce the impact of attacks deserves further study.

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