Arbitrary quantum state transfer under three parties participation

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Arbitrary quantum state transfer (AQST) is discussed in a system that atoms are trapped in three separate cavities which are connected via optical fibers. Through three parties cooperation, the AQST can be selectively implemented deterministically. The target state can be transferred to any of the parties with 100 percent fidelity and \( \frac{1}{2} \) success probability.

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Very recently, much attention has been paid to the study of the possibility of quantum information processing realized via optical fibers \(^{[1,2]}\). Generating an entangled state of distant qubits turns out to be a basic aim of quantum computation. It has been pointed out that implementing quantum entangling gate that works for spatially separated local processors which are connected by quantum channels is crucial in distributed quantum computation. Many schemes have been put forward to prepare engineering entanglement of atoms trapped in separate optical cavities by creating direct or indirect interaction between them \(^{[3-10]}\). Some of the schemes involve direct connection of separate cavities via optical fibers, others apply detection of the photons leaking from the cavities. All the implemented quantum gates work in a probabilistic way. To improve the corresponding success probability and fidelity, one must construct precisely controlled coherent evolutions of the global system and weaken the affect of photon detection inefficiency. In the scheme considered by Serafini et al \(^{[5]}\), the only required local control is synchronized switching on and off of the atom-field interaction in the distant cavities. In the scheme proposed by Mancini and Bose \(^{[11]}\), a direct interaction between two atoms trapped in distant cavities is engineered, the only required control for implementing quantum entangling gate is turning off the interaction between atoms and the locally applied laser fields. In the present letter, we propose an alternative scheme with particular focus on the establishment of three-qubit entanglement, which is suitable and effective for the generation of three-atom W-type state and two-atom Bell-state. To generate three-atom W-type state, the only control required is synchronized turning off the locally applied laser fields. While, To generate two-atom Bell-state, an additional quantum measurement performed on one of the atoms is needed. We demonstrate that the scheme works in a high success probability, and the atomic spontaneous emission does not affect the fidelity.

The schematic setup of the system is shown in Fig. 1. Three two-level atoms 1, 2 and 3 locate in separate optical cavities \( C_1 \), \( C_2 \) and \( C_3 \) respectively. The cavities are assumed to be single-sided. Three off-resonant driving external fields \( \epsilon_1 \), \( \epsilon_2 \) and \( \epsilon_3 \) are added on \( C_1 \), \( C_2 \) and \( C_3 \) respectively. In each cavity, a local weak laser field is applied to resonantly interact to the atom. Two neighboring cavities are connected via optical fiber. The global system is located in vacuum. Using the input-output theory, taking the adiabatic approximation \(^{[12]}\) and applying the methods developed in Refs. \(^{[11]}\) and \(^{[13]}\), we obtain the effective Hamiltonian of the global system as

\[
H_{\text{eff}} = J_{12} \sigma_1^- \sigma_2^- + J_{23} \sigma_2^- \sigma_3^- + J_{31} \sigma_3^- \sigma_1^- + \Gamma \sum_i (\sigma_i^- + \sigma_i^+) \tag{1}
\]

where \( \sigma_i^- \) and \( \sigma_i^+ \) (\( \sigma_i^+ \)) \( i = 1, 2, 3 \), are spin and spin raising (lowering) operators of atom \( i \), \( \Gamma \) represents the local laser field added on the atom. To keep the validity of adiabatic
approximation, we assume $\Gamma \ll J_{12}(J_{23}, J_{31})$. And
\begin{align*}
J_{12} &= 2|\chi|^2Im\left\{a_1a_2^* (M_1 + i\kappa e^{i\phi_{01} + i\phi_{13}})/(M_1^2 - W_1^2)\right\}, \\
J_{23} &= 2|\chi|^2Im\left\{a_2a_3^* (M_2 + i\kappa e^{i\phi_{01} + i\phi_{13}})/(M_2^2 - W_2^2)\right\}, \\
J_{31} &= 2|\chi|^2Im\left\{a_3a_1^* (M_3 + i\kappa e^{i\phi_{01} + i\phi_{13}})/(M_3^2 - W_3^2)\right\},
\end{align*}
where $\kappa$ is the cavity leaking rate, $\chi = \frac{2}{\sqrt{N}}$, $g$ is the coupling strength between atom and cavity field, $\Delta$ is the detuning. In deducing Eq. (1), the condition $\Delta = \kappa \gg g$ is assumed, $M = i\Delta + \kappa$, $W = \kappa^2 e^{i(\phi_{01} + \phi_{13})}$. The phase factors $\phi_{01}, \phi_{32}$, and $\phi_{13}$ are the phases delay caused by the photon transmission along the optical fibers. And
\begin{align*}
\alpha_1 &= \frac{M^2e_1 + \kappa^2 e^{i(\phi_{02} + \phi_{13})}e_2 + Mke^{i\phi_{13}}e_3}{M^3 - W_1^3}, \\
\alpha_2 &= \frac{M^2e_2 + \kappa^2 e^{i(\phi_{01} + \phi_{13})}e_3 + Mke^{i\phi_{21}}e_1}{M^3 - W_2^3}, \\
\alpha_3 &= \frac{M^2e_3 + \kappa^2 e^{i(\phi_{01} + \phi_{13})}e_1 + Mke^{i\phi_{32}}e_2}{M^3 - W_3^3},
\end{align*}
We assume that $e_1 = e_2 = e_3 = e_0$, $\phi_{21} = \phi_{32} = \phi_{13} = \phi_0$. This leads to
\begin{align}
\alpha_1 = \alpha_2 = \alpha_3 = \alpha_0, \\
J_{12} = J_{23} = J_{31} = J_0.
\end{align}
The Hamiltonian in Eq. (1) is now written as
\begin{equation}
H_{eff} = H_{zz} + H_z, \tag{5}
\end{equation}
where
\begin{equation}
H_{zz} = J_0(\sigma_1^z\sigma_2^z + \sigma_2^z\sigma_3^z + \sigma_3^z\sigma_1^z), \quad H_z = \sum_i \Gamma_i (\sigma_i^- + \sigma_i^+). \tag{6}
\end{equation}
Eq. (5) represents the Hamiltonian of an Ising ring model. The entanglement of the ground state of the above Hamiltonian has already been discussed. Here, we study the entanglement of the evolved system state governed by the Hamiltonian. Under the condition $\Gamma_i \ll J_0$, the secular part of the effective Hamiltonian can be obtained through the transformation $UH_{zz}U^{-1}$, $U = e^{-iH_{zz}t}$, as
\begin{equation}
\hat{H} = \sum_{ijk} \Gamma_i \sigma_i^z(1 - \frac{1}{2}\sigma_j^z\sigma_k^z). \tag{7}
\end{equation}
where the subscripts $ijk$ are permutations of 1, 2, 3.

The straight forward interpretation of this Hamiltonian is: the spin of an atom in the Ising ring flips if and only if its two neighbors have opposite spins.

For the initial states that one or two of the atoms are excited, the system state is restricted within the subspace spanned by the following basis vectors
\begin{align}
|\phi_1\rangle &= |ggg\rangle, |\phi_2\rangle = |egg\rangle, |\phi_3\rangle = |eeg\rangle, \\
|\phi_4\rangle &= |gee\rangle, |\phi_5\rangle = |gge\rangle, |\phi_6\rangle = |ege\rangle. \tag{8}
\end{align}
We firstly consider a case where $\Gamma_1 = \Gamma_3 = 0$. The Hamiltonian in Eq. (7) is now written as
\begin{equation}
\hat{H} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}. \tag{9}
\end{equation}
The eigenvalues of the Hamiltonian can be obtained as $E_{1,2} = \pm \sqrt{2} \Gamma_2, E_{3,4,5,6} = 0$, and the corresponding eigenvectors are
\begin{equation}
|\psi_i\rangle = \sum_j S_{ij}|\phi_j\rangle \tag{10}
\end{equation}
where
\begin{equation}
S = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 1 \\
\end{pmatrix}. \tag{11}
\end{equation}
For initial system state $|\Psi(0)\rangle = \sum_i c_i(0)|\psi_i\rangle$, the evolving system state can be written as $|\Psi(t)\rangle = \sum_i c_i(t)|\psi_i\rangle$, where the coefficients $c_i(t)$ are given by
\begin{equation}
c_i(t) = \sum_j [S^{-1}]_{ij}[S c(0)]_je^{-iE_jt}, \tag{12}
\end{equation}
where $c(0) = [c_1(0), c_2(0), c_3(0), c_4(0), c_5(0), c_6(0)]^T$, and $S$ is the $6 \times 6$ unitary transformation matrix between eigenvectors and basis vectors.

Now we show how an arbitrary quantum state $a|e\rangle + \beta|g\rangle$ be transferred from a cavity to another. To do this, we assume Alice, Bob and Charlie hold atoms 1, 2 and 3 respectively, and atom 1 is supposed to be initially in state $a|e\rangle + \beta|g\rangle$, where $a$ and $\beta$ are complex numbers and fulfill normalization condition, atoms 2 and 3 are in ground state. For initial state $|\Psi(0)\rangle = |\phi_1\rangle$, one can get
\begin{align}
c_1(t) &= \cos^2 t, \\
c_2(t) &= -\sin\Gamma_2t, \\
c_3(t) &= c_4(t) = c_5(t) = c_6(t) = 0. \tag{13}
\end{align}
At $\Gamma_2 t = k\pi + \frac{\pi}{2}$, Alice, Bob and Charlie synchronously turn off driving fields $e_1, e_2, e_3$ and laser fields $L_2$, the state of the atoms evolves to $(a|e\rangle e_{12} + \beta|g\rangle e_{12}) \otimes |\psi_3\rangle$. Now, Alice turns on her local laser field $L_1$. Recall that atom 1 resonantly interacts with $L_1$ and decouples from cavity field $C_1$. Furthermore, since the driving fields are turned off, the Ising-type interaction between atoms is damaged. Then, the dynamics of atom 1 is only governed effectively by the Hamiltonian
\begin{equation}
H_1 = \Gamma_1(\sigma_1^- + \sigma_1^+) \tag{14}
\end{equation}
FIG. 2: Fidelity of transferring arbitrary quantum state from Alice to Bob with versus time.

One can get
\[
|\Psi(t)_{12} = |e_1(\alpha \cos \Gamma t |e_2 - i \beta \sin \Gamma t |g_2)
+ |g_1(-i \alpha \sin \Gamma t |e_2 + \beta \cos \Gamma t |g_2). \quad (15)
\]
At \(\tan \Gamma (t - t_1) = 1\), Alice performs measurement \(|e_1(e)\) on her atom, the atomic state of Bob then is
\[
|\Psi_2 = \alpha |e_2 - i \beta |g_2 \quad (16)
\]. By using a rotation
\[
H_{\text{rot}} = \begin{pmatrix}
1 & 0 \\
0 & e^{i \frac{\pi}{4}}
\end{pmatrix}. \quad (17)
\]

Bob can obtain a state \(|\Psi_2 = \alpha |e_2 + \beta |g_2\). Thus, Alice, Bob and Charlie cooperatively implement a perfect deterministic quantum state transfer.

Similarly, using this method, the arbitrary quantum state can also be transferred to Charlie.

We let \(\alpha = \cos(\theta)\) and \(\beta = \sin(\theta)\), and define the average success probability of the quantum state transfer as
\[
P = \frac{1}{2\pi} \int_0^{2\pi} P(\theta) d\theta \quad (18)
\]
the average fidelity of the quantum state transfer as
\[
F = \frac{1}{2\pi} \int_0^{2\pi} F(\theta) d\theta \quad (19)
\]
where \(P(\theta) = \cos^2 \theta \cos^2 \Gamma_1(t - t_1) + \sin^2 \theta \sin^2 \Gamma_1(t - t_1), F(\theta) = \frac{1}{\pi \alpha \beta} \cos^2 \theta \cos \Gamma_1(t - t_1) + \sin^2 \theta \sin \Gamma_1(t - t_1))\). We can easily see that \(P = \frac{1}{2}\).

In Fig. 3, we show the fidelity \(F\) with respect to time. At \(\Gamma_1(t - t_1) = k\pi + \frac{\pi}{2},\) the arbitrary quantum state \(\alpha |e_1 + \beta |g_2\) can be transferred from Alice to Bob with success probability \(\frac{1}{2}\) and fidelity 100 percent.

We have put forward a scheme to transfer arbitrary quantum state from atom to another. In this scheme, the transfer can be deterministically implemented by three parties’s cooperation. The average success probability can approach \(\frac{1}{2}\), while the average fidelity can approach 100 percent. The transfer will be terminated if one of the parties had a mishandling or the communication channel had been illegally observed anywhere. So, this scheme provides a relative more secure quantum communication than those only using two parties. Furthermore, we can see that, the transfer can be implemented selectively. The arbitrary quantum state can be transferred from Alice to Bob, or from Alice to Charlie, or from Bob to Charlie, and so on. From an extending point of view, this kind of system may act as a perform of quantum network.

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