Permeability estimation of jointed rock mass using fractal network model

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Abstract Estimation of permeability in jointed rock mass is important to provide input parameters for numerical models simulating transient water flow. The influence of fractal distribution of rock joints on the permeability behavior of water transform is presented by using fractal geometry and Darcy’s law. The self-similar property of fracture network is applied to quantitatively describe the fractal distribution of rock joints. Based on the parallel theory of network and Darcy’s law, the effective permeability of jointed rock mass is proposed.

1. Introduction
Hydrogeological problems involving fractured rock are challenging to solve because contrasts in permeability of the fractures and surrounding rock matrix are extreme and localized, making flow strongly dependent on the interconnections between conductive fractures. The microstructural properties of fractured rock influence a variety of transport process either of industrial interest or of environmental significance. The parameters for depicting fractal properties of fractured rock include the fracture area fractal dimension, tortuosity fractal dimension, fracture density parameter, length of the fracture. Compared with conventional model of parameter estimation, the fractal model for permeability of fractured rock does not contain any empirical constants. Recent geological investigations have confirmed that subsurface fracture networks can commonly be described by fractal geometry. In the field of water flow problem, several reports are available that have characterized subsurface fractures and fault systems by means of fractal geometry.

It has long been recognized that improved numerical modeling techniques are required to more accurately simulate the large-scale deformation around underground excavations, which can define the potential flow paths of strata gases. However, to effectively model the deformation requires a determination of rock mass properties of the surrounding strata. Flow and transport in systems with fractal and self-affine surfaces and boundaries are relevant to a wide variety of scientific and industrial problems. For example, natural porous media and rock contain a wide variety of pores and fractures with broad distributions of sizes and shapes. There is now experimental evidence that the internal surface of the pores and the fractures is very rough, and that the roughness obeys fractal statistics. Conventionally a fracture surface is modeled as perfectly smooth, and thus flow in a single fracture was modeled as flow between parallel flat plates.

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Research methods of the permeability included experimental measurement, analytical solution and numerical simulation. Traditionally, determination of the permeability tensor is best accomplished through experimental measurements since many and various measurement techniques all yield consistent results. However, these measurements often require a large number of carefully controlled experiments and, in general, have no predictive capability (i.e., each new material must be handled on a case-by-case basis). As a consequence, analytical (for simple cases) or numerical predictions of the permeability have for the past few years been the goal of many researchers in the field of porous media flow. Sun(2007) presented the fractal dimension of the fractal model of dropwise condensation and its experimental study[1]. El Naschie(2007) proposed the deterministic quantum mechanics versus classical mechanical indeterminism[2]. Gao(2007) investigated the golden mean and fractal dimension of goose down[3]. El Naschie 2006) developed an eleven dimensional E-infinity fractal spacetime theory[4]. Giordano(2006) studied numerical analysis of hypersingular integral equations in the E-infinite Cantorian spacetime[5]. The fractal network model has been widely applied in natural fracture network simulation, thermal and hydrodynamic analysis of a fractal micro-channel network, flow and transport in hierarchically fractured rock and characterization of the pore structure of reservoir rocks[6-12]. The paper aim is to propose an estimation procedure for water permeability in fractured rock masses and to investigate the influence of fractal parameters of fracture network to equivalent permeability.

2. Fractal Distribution Model of Rock Joints

Quantitatively understanding of spatial clustering behavior of fracture patterns is fundamental to study fractured hydrology and the mechanical properties of discontinuous rocks. The major characteristic of fractal sets is their scaling properties related to self-similarity. For monofractals, scaling can be described by only one exponent (fractal dimension). Fractal geometry is one of the new concepts in the field of theoretical geometry that have been developed recently. The shapes that can be described by fractal geometry have two important properties: they are self-similar, and are characterized by their fractal dimensions. The investigation showed that the relationship between the fracture aperture w and the number of fractures M whose apertures are equal to or larger than w can be described by follows

\[ M = Cw^{-D} \] (1)

Where C is a constant and D is fractal dimension. The aperture of fracture changes versus hierarchical level based on fractal principle.

\[ w_{i+1} = \lambda w_i \] (2)

Where \( \lambda \) is the scale factor (0<\( \lambda \)<1). The typical fractured rock network is shown in Fig. 1.

![Fig. 1 Typical fractured rock network](image)

To generate a fractal fracture network, the hierarchical fracture networks are generated as lattice structure. A square template consists of two fractures of aperture \( w_1 \). The new fractures of aperture \( w_2 \)
are produced to form the second level of fractal network. The new fractures of aperture \( w_i \) may be continued until the required fractal fracture network at the specified hierarchical level is obtained as shown in Fig.2. The index \( i \) shows the level of hierarchical fracture networks. Suppose that \( N_0 \) is the number of original fractures for the square; and \( N_s \) is the number of new addition fractures between two higher level fractures. Fig. 2 depicts the pictures of fracture distribution after adding new fractures for two and three times.

![Fig. 2: Addition of new fractures for hierarchical level \( i=3 \).](image)

3. Influence of Fractal Distribution of Rock Joints on the Permeability Behavior

The fractured rock masses having various fracture apertures can be considered as a series of tortuous fracture networks with variable apertures. As shown in Fig.2, the length of tortuous fracture is calculated as follows

\[
L_i = \mu L
\]

Where \( \mu \) is the tortuous factor of fracture, \( L_i \) is the length of tortuous fracture. The equivalent permeability of fracture network system including \( N \) tortuous fractures for water flow problem is expressed as follows

\[
k_0 = \frac{Nw_0^2}{12\mu L}
\]

Where \( k_0 \) is the equivalent permeability, \( w_0 \) is the aperture of fracture, and \( L \) is the side length of square, as shown in Fig.2. Based on the parallel principles of flow networks, the equivalent permeability of hierarchical fracture networks for very hierarchical level is expressed as follows

\[
k_1 = N_0 \frac{w_1^2}{12\mu L} = N_0^0\lambda^0 \frac{w_1^2}{12\mu L}
\]

\[
k_2 = N_0N_s \frac{w_2^2}{12\mu L} = N_0N_s^2\lambda^2 \frac{w_1^2}{12\mu L}
\]

\[
k_3 = N_0N_s \frac{w_3^2}{12\mu L} = N_0N_s^4\lambda^4 \frac{w_1^2}{12\mu L}
\]

\[
k_i = N_0N_s \frac{w_i^2}{12\mu L} = N_0N_s^{i-1}\lambda^{2i-2} \frac{w_1^2}{12\mu L}
\]

Applying the summation formula for a geometric series yields
where \( k_e \) is the equivalent permeability of fracture networks in rock masses system. The dimensionless equivalent permeability \( K_e \) can be expressed as follows:

\[
K_e = \frac{k_e}{N_0} = \frac{w_i^2}{12\mu L} \frac{1-(N_0\lambda^2)^m}{\mu(1-N_0\lambda^2)}
\]

(10)

Fig. 3 Influence of scaling factor \( \lambda \) to permeability

Fig. 3, 4 and 5 show the influence of fractal parameters on equivalent permeability. The investigation show that the equivalent permeability of fractal network system in rock masses will increase as increasing scaling factor \( \lambda \) and number of divide \( N_s \).

4. Conclusion
Fractures exist over a wide range of scales, from microfractures to largest faults. Their patterns display a self-similar geometry, at least at statistical sense, that repeat over various scales. The physical properties of hierarchical structures have been paid much attention in recent years since these structures are believed to arise in various contests. The microstructural properties of fractured rock masses can affect a variety of transport process either of industrial interest, or of environmental significance. A fractal permeability model for fractured rock masses was developed based on fractal geometry and water seepage principles in fracture networks. The influences of fractural parameters of fracture network on permeability were investigated. The mechanism of water transform in jointed rock mass is governed by the characteristics of the fractal distribution of discontinuities.
Fig. 5 Influence of tortuous factor $T$ to permeability

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