Double point contact in Quantum Hall Line Junctions

Eun-Ah Kim and Eduardo Fradkin
Department of Physics, University of Illinois at Urbana-Champaign,
1110 W. Green St., Urbana, IL 61801-3080, USA
(Dated: March 22, 2022)

We show that multiple point contacts on a barrier separating two laterally coupled quantum Hall fluids induce Aharonov-Bohm (AB) oscillations in the tunneling conductance. These quantum coherence effects provide new evidence for the Luttinger liquid behavior of the edge states of quantum Hall fluids. For a two point contact, we identify coherent and incoherent regimes determined by the relative magnitude of their separation and the temperature. We analyze both regimes in the strong and weak tunneling amplitude limits as well as their temperature dependence. We find that the tunneling conductance should exhibit AB oscillations in the coherent regime, both at strong and weak tunneling amplitude with the same period but with different functional form.

Two-dimensional electron fluids in large magnetic fields offer an ideal setting to study non-trivial quantum coherence effects in strongly interacting macroscopic systems. It is well known that the excitations supported by the edges of quantum Hall fluids provide an ideal window to study this new physics. Already there are a number of very interesting experiments which have uncovered the non-trivial Luttinger liquid behavior of these edge states.

Recently Kang et al. used a new experimental setup in which two quantum Hall fluids are laterally coupled along an atomically precise barrier. In these experiments Kang and coworkers found that for filling factors \( \nu \gtrsim 1 \), and for some range of filling factors, a pronounced zero bias conductance (ZBC) peak appears in the tunneling conductance of the device. (The same effect reappears for \( \nu \gtrsim 2 \).) Two alternative mechanisms have been proposed to explain these experiments: a) Landau level mixing induced by the barrier potential, and b) tunneling at isolated quantum point contacts.

In Ref. we showed that the salient features of the experiment of Kang et al. can be successfully explained by modeling the system as a pair of (coupled) chiral Luttinger liquids (the edge states on each side of the barrier) in the presence of a single point contact (PC). In particular we showed that inter-edge Coulomb interaction yields an effective reduced Luttinger parameter \( K < 1 \) and that for \( \nu \gtrsim 1 \), the system crosses over to the strong tunneling amplitude regime, leading to the appearance of zero-bias peak in the tunneling conductance with a peak value at \( T = 0 \) of \( G_0 = Ke^2/h \). This crossover is controlled by the energy scales of this system: the bias voltage \( V \), the crossover scale \( T_K \), and the temperature \( T \) (and the possibility by a small spin polarization for \( \nu \sim 2 \)). We also predicted an increase in the height of the zero-bias conductance (ZBC) peak for \( T \lesssim T_K \).

However, if the barrier contains more than one tunneling center (as it surely does) a number of interesting quantum coherence effects must take place, and it is of interest to investigate quantum coherence effects of multiple impurities and their competition with thermal fluctuations. An example of effects of this sort has been considered some time ago by Chamon et al., who proposed a quasi-particle interference experiment based on a two-tunneling center device in the fractional quantum Hall regime, as a way to measure directly the fractional statistics of Laughlin quasi-particles. However, in the FQH regime, only the case of weak tunneling centers needs to be considered since in this regime tunneling at a point contact is an irrelevant perturbation. Instead, for \( \nu \gtrsim 1 \), the system is in the strong tunneling limit for \( T \lesssim T_K \), which is not accessible by perturbation theory. In this regime, tunneling processes become dominant, and an instanton expansion is required to describe the physics. This problem is closely related to that of scattering centers in quantum wires, first discussed by Kane and Fisher. Our analysis closely follows their approach.

In this paper, we analyze the quantum Hall Line junction with two PC’s both in the strong and weak tunneling amplitude limits. We show that the two-PC system may be in a coherent or in an incoherent regime depending on the distance \( a \) separating the tunneling centers. In the coherent regime the system exhibits Aharonov-Bohm (AB) oscillations in the form of a series of resonant tunneling processes in the strong tunneling limit. Instead, the AB effect in the weak tunneling limit has a simple sinusoidal form. In contrast, in the incoherent regime, the strong and weak tunneling limits are related by duality. Naturally, a realistic barrier must contain more than two PC’s, which will result in a more complex structure of AB oscillations than what we find for just two PC’s. Nevertheless it is also natural to expect that as the temperature is lowered this pattern will reveal itself step by step with the strongest PC’s giving rise to the most prominent features of the interference pattern. The observation of this AB interference pattern will provide a way to sort out whether the ZBP observed by Kang et al. is due to Landau level mixing or to PC tunneling, since the former mechanism predicts a smooth non-periodic dependence of the tunneling conductance on the magnetic field.

We begin by describing our model (see Fig.1) which has two PC’s, one located at \( x = -a/2 \) and the other located at \( x = a/2 \). In the notation of Ref. the local...
The effective action for the tunneling center degrees of freedom is

$$\mathcal{H}_t = t_1 \psi^\dagger_1 \psi_1 \delta(x + a/2) + t_2 \psi^\dagger_+ \psi_- \delta(x - a/2) + \text{h.c.,}$$

(1)

where $t_1$ and $t_2$ are tunneling amplitudes, hereafter referred to as the “coupling constants”. The right and left moving chiral Fermi fields $\psi^\pm$ can be bosonized in terms of the right and left moving chiral bosons $\phi^\pm$, as

$$\psi^\pm(x) \propto \frac{1}{\sqrt{2\pi}} e^{\pm i\phi^\pm(x)} \pm i k_F x.$$  

Similarly, the normally ordered density operators are given by

$$J_\pm = \frac{1}{\sqrt{2\pi}} \partial_x \phi^\pm.$$  

Notice that the tunneling operators at $x = \pm a/2$ have a relative phase of $2k_F a$, which cannot be “gauged away” by shifting the bosonic field $\phi = \phi_+ + \phi_-$ by a constant. However, the Fermi momentum is directly connected to the position of the edge or the effective width $d$ of the barrier.

The potential $\varphi(x)$ can be identified with $\varphi(x) \propto -\frac{1}{2\pi} \omega_0 T \mp \frac{1}{2\pi} x$.

Thus, in general there exists a single value $\varphi_0 = \pm \frac{1}{2\pi} \omega_0 T$ for which the resonance was tuned by a factor $\frac{1}{2\pi} \omega_0 T$.

By comparing Eq. (1) and Eq. (2) we see that in the incoherent regime $S_{\text{eff}}$ reduces to

$$S_{\text{eff}} = \frac{1}{\beta} \sum_{\nu \neq 0} \frac{\left| \omega_{\nu n} \right|}{8\pi} \left[ X_{\nu n}^{(1)} \right]^2 + \frac{\left| \omega_{\nu n} \right|}{8\pi} \left[ X_{\nu n}^{(2)} \right]^2$$

(3)

where $\omega_n = 2\pi n T$ are the Matsubara frequencies, $X_{\nu n}^{(i)}$ are the Fourier components of $X_\nu(\tau)$, and $T$ is the temperature. Notice that $\frac{\sqrt{K}}{2\pi 2\sqrt{K}} X_1$ measures the charge transferred along the barrier and $\frac{\sqrt{K}}{2\pi 2\sqrt{K}} X_2$ measures the charge transferred to the island (the cross-hatched region in Fig. 1).

The term $e^{-\omega_0}$ in the denominator of Eq. (3) accounts for the coherence between two tunneling centers. As a result, at sufficiently high temperatures $Ta \gg 1$, the regime quantum coherence effects are washed away. Thus, in physical units, we can identify two extreme regimes in which the effective action simplifies: the coherent regime with $\hbar v/a \gg k_B T$ in which the PC’s are strongly coupled, and the incoherent regime with $\hbar v/a \ll k_B T$, in which the PC’s act independently. Thus, in the coherent regime, the effective action of Eq. (3) becomes

$$S_{\text{eff}} = \frac{1}{\beta} \sum_{\nu \neq 0} \frac{\left| \omega_{\nu n} \right|}{8\pi} \left[ X_{\nu n}^{(1)} \right]^2 + \frac{\left| \omega_{\nu n} \right|}{8\pi} \left[ X_{\nu n}^{(2)} \right]^2$$

(4)

where $V_f$ is the effective potential

$$V_f(X_1, X_2) = \frac{1}{4\pi \alpha} X_2(\tau)^2 + 2\Gamma \cos \left( \sqrt{\frac{K}{2}} X_1 \right) \cos \left( \sqrt{\frac{K}{2}} X_2 - \frac{\pi}{\Phi} \right)$$

(5)

In contrast, in the incoherent regime $S_{\text{eff}}$ reduces to

$$S_{\text{eff}} = \frac{1}{\beta} \sum_{\nu \neq 0} \frac{\left| \omega_{\nu n} \right|}{8\pi} \left[ X_{\nu n}^{(1)} \right]^2 + \frac{\left| \omega_{\nu n} \right|}{8\pi} \left[ X_{\nu n}^{(2)} \right]^2$$

(6)

By comparing Eq. (3) and Eq. (6) we see that in the incoherent regime the PC’s are effectively decoupled while in the coherent regime they are strongly coupled, and $X_2$ becomes massive, with a mass of order $1/a$. Also note that the strength of non-local interaction in the incoherent regime is exactly twice that of the coherent regime.

1) The coherent regime. The potential $V_f(X_1, X_2)$ is periodic in $X_1$ with period $2\pi \sqrt{2/K}$. The mass term breaks this lattice translation symmetry in $X_2$ direction. Thus, in general there exists a single value $X_2 = X^0_2$ which minimizes the potential along the $X_2$ axis, unless the resonance condition $\Phi/\phi_0 = (\text{half-integer})$ is satisfied. However, when the flux satisfies $\Phi = (n + \frac{1}{2}) \phi_0$, the potential $V_f(X_1, X_2)$ acquires the additional symmetry $V_f(X_1, X_2) = V(X_1 + \pi \sqrt{2/K} - X_2)$. Thus, in this case there are two values of $X_2$ which minimize the potential. This effect is analogous to the resonance phenomena first pointed out by Kane and Fisher except for that in that case the resonance was tuned by a

![Diagram](image-url)
gate voltage. The resonance found here is the result of the Aharonov-Bohm effect which enables the transfer of half an electron when the flux penetrating the island is exactly half-integer flux quantum. We will use the instanton expansion \[ \text{to examine the coherent regime in the strong tunneling limit } \Gamma \gg 1/(aK), \] and perturbation theory in the weak tunneling limit \( \Gamma \ll 1/(aK) \). The weak tunneling limit, \( \Gamma \ll 1/(aK) \):

\[ G(t) = \frac{e^2}{h} G^\text{pert}(t) + \Delta G^\text{res}(t) \]

(a) weak tunneling

limit amplitude

(b) strong tunneling

limit amplitude

FIG. 2: (a) AB oscillations in the weak tunneling limit of the coherent regime; (b) AB effect in the strong tunneling limit of the coherent regime for \( K < 1/4 \).

Here the effective potential is dominated by the mass term which is minimized for \( X_2 = 0 \). Hence in this case \( X_2 \) can be integrated out resulting simply in a finite flux-independent renormalization of the tunneling amplitude: \( V_\Gamma \to 2\Gamma \cos(\pi\frac{K}{\phi_0}) \cos(\sqrt{\frac{K}{2}}X_1) \). Consequently, a lowest order perturbative calculation using the Keldysh formalism \[ \text{results an expression for } G_\Gamma(V = 0, T) \text{ in the coherent weak tunneling limit:} \]

\[ G_\Gamma(0, T) = \frac{e^2}{h} \frac{K}{2} \frac{\Gamma(1/2)\Gamma(K)}{\Gamma(1/2 + K)} \left( \frac{T}{T_K^\text{CW}} \right)^{2K-2} \times \frac{1}{2} \left( 1 + \cos(2\pi\Phi/\phi_0) \right) + \cdots, \]

where \( T_K^\text{CW} = \Lambda \left( \frac{2\pi}{\Phi} \right)^{1/(1-K)} \) is a crossover scale and \( \Lambda \) is a high-energy cutoff. Hence, in the weak tunneling limit of the coherent regime, there is an Aharonov-Bohm interference effect with the usual oscillatory form (albeit with a reduced amplitude), as well as an offset.

The strong tunneling limit: For \( \Gamma \gg 1/(aK) \), the system can be either off-resonance or on-resonance. When the system is off-resonance, \( V_\Gamma \) has a single minimum at \( X_2 = X_2^o \approx \sqrt{2K/(\kappa a - \pi n)} \), where \( n \) is an integer. In this case, there is an energy gap of order \( 1/aK \) to the states with other values of \( X_2 \), i.e. the island-charge fluctuation is effectively suppressed (Coulomb blockade). Thus, \( X_2 \) is frozen to a non-zero value and the problem reduces to a single point contact system. In this regime the saturation value of the tunneling conductance is necessarily equal to \( Ke^2/h \). We can calculate the leading corrections to this result using the instanton technique. In this case, the instanton is an electron tunneling process across the island between two disconnected pieces of the barrier. We will denote the instanton fugacity by \( \zeta \), and compute the lowest order correction to the tunneling conductance \( \Delta G^\text{inst} = G_\Gamma - Ke^2/h \) due to these tunneling processes. To the lowest order in \( \zeta \) we find

\[ \Delta G^\text{inst}_\Gamma = \frac{\pi}{4} \frac{\Gamma((1/2)\Gamma(1/K))}{\Gamma(1/2 + 1/4K)} \left( \frac{T}{T_K^\text{CSR}} \right)^{(1+g)/2} + \cdots, \]

where \( T_K^\text{CSR} = \Lambda \left( \frac{2\pi}{\Phi} \right)^{1/(1-K)} \) is a crossover scale determined by the instanton fugacity \( \zeta \) and \( \Lambda \).

However, when the magnetic field is tuned to a resonance, the projection of \( V_\Gamma \) has two degenerate minima at \( X_2 = \pm \sqrt{2K/\Phi} \), where \( g \) represents the renormalization of the Luttinger parameter (or “compactification radius”) of \( X_2 \); at the strong tunneling fixed point \( g_0 = 1 \). Hence the island effectively behaves like a two-level system and instantons connecting degenerate minima \( (X_1 = m\pi\sqrt{2K}/X_2^o) \) and \( (X_1 = (m \pm 1/2)\pi\sqrt{2K}, X_2 = -X_2^o) \) with integer \( m \) correspond to electrons hopping on and off the island with the hopping amplitude \( \zeta' \). In terms of these instantons, with fugacity \( \zeta' \), the partition function is \[ \text{(14)} \]

\[ Z = \sum_{n} \int_{0}^{\beta} d\tau_{2n} \cdots \int_{0}^{T} d\tau_{1} e^{-\sum_{i<j} V_{ij}}, \]

\[ V_{ij} = \frac{1}{2K} \left( q_i^0 q_j^0 + g q_i^0 q_j^0 \right) \ln |\Lambda(\tau_i - \tau_j)| \]

As long as \( \zeta'/\Lambda \ll 1 \), we can use this partition function to calculate semi-classically the lowest order correction to \( \Delta G^\text{res}_\Gamma \approx G^\text{res}_\Gamma - Ke^2/h \):
Phase transition or crossover? Having understood the strong and the weak tunneling limits in the coherent regime, we now inquire if these separated by a phase transition or by a smooth cross-over. The RG flow for this problem, originally derived in Ref. [13], is:

\[
\frac{\delta \zeta'}{\delta t} = \frac{1}{4K}((4K - 1) - g)\zeta' \quad (12a)
\]

\[
\frac{\delta K}{\delta t} = -\frac{8}{\Lambda^2} e^2 g. \quad (12b)
\]

This resulting flow, shown qualitatively in Fig. 3 has a fixed point at \((g, \zeta') = (4K - 1, 0)\) which depends on \(K\), and on the initial value of \(g\), which is \(g_0 = 1\). The asymptotic behavior of the system depends on both the value of \(K\) and the initial value of \(\zeta'\). Thus, for \(1/4 \leq K < 1/2\), there is a quantum phase transition at \(T = 0\) between a phase in which the tunneling conductance \(G_t\) saturates to \(Ke^2/h\) as \(T \rightarrow 0\), and a phase in which \(G_t\) vanishes as \(T \rightarrow 0\). Instead, for \(K < 1/4\), there is a crossover as the system flows to a line of strong tunneling fixed points \(g \rightarrow g_s, \zeta' \rightarrow 0\), each of which yielding a different scaling law for Eq. (13). For \(T > 0\), in all cases, there will be a crossover between strong and weak tunneling fixed points as finite temperature will eventually stop the flow. The data of Ref. [8] suggests that Coulomb interactions reduce the Luttinger parameter to a small value \(K \sim 0.2\).

2) The incoherent regime: In this regime, \(\exp(-\beta a) \rightarrow 0\), thermal fluctuations overwhelm coherence effects and the two PC’s behave as if they were decoupled from each other. To the lowest order in \(1/\Lambda\), the zero bias tunneling conductance in the weak tunneling limit \(T \gg \Lambda\) calculated perturbatively is

\[
G_t = \frac{e^2}{h} - \frac{2}{K} \frac{\Gamma(1/2)\Gamma(1/K)}{\Gamma(1/1 + 1/2)} \left( T_0^{K_S} \right)^{2/K-2} + \ldots (13)
\]

with \(T_0^{K_S} = \frac{\Lambda}{\zeta} \frac{1}{e^2 K}\). Similarly, a semi-classical calculation in the strong tunneling limit \(T \ll \Lambda\) leads to

\[
G_t = \frac{e^2}{h} K \pi (2K) \frac{\Gamma}{\Lambda} \frac{\Gamma}{\Gamma(K) \Gamma(K + 1/2)} \left( T_0^{K_S} \right)^{(2K-2)} + \ldots (14)
\]

with \(T_0^{K_S} = \Lambda (\frac{1}{\zeta} \frac{1}{e^2 K})\). Eq. (13) and Eq. (14) show that, in the incoherent regime, weak and strong tunneling limits are exactly dual to each other. They also have the same scaling behavior as the single PC case studied in Ref. [11], which explains why the single PC picture works.

We close with a few comments on multi-impurity effects. Here we have discussed in detail coherence effects in a two PC system. Naturally, a realistic barrier (even an “atomically precise” one) will have a number of such defects. It is clear that a multi-point contact extension of our analysis will lead to a complex interference pattern due to the existence of many competing pathways. Also, one expects a broad distribution of defects, both in tunneling amplitudes and in relative separation. Thus, at a given temperature, the strongest effects will be due to the closest defects with the largest tunneling amplitudes. Thus, as \(T\) is lowered, coherent Aharonov-Bohm oscillations will become increasingly more complex. Conversely, as \(T\) is raised these effects are washed-out and the system will eventually reach the single impurity limit.

After this work was completed we became aware of the experiments of Kataoka et al. [20] on an anti-dot in a quantum Hall system. Although the main focus of that work is the behavior of the system near \(\nu = 2\) (which we will discuss elsewhere) we wish to note that the device of Ref. [20] is in the strong tunneling coherent regime discussed above. Further, the temperature dependence of the oscillations observed in Ref. [20] are remarkably similar to what we find for Aharonov-Bohm oscillations here near \(\nu = 1\). Also, Sim et al. [21] have recently developed a model for the anti-dot device.

We thank Prof. W. Kang for several useful and stimulating discussions. This work was supported in part by the National Science Foundation through the grant DMR01-32990.

[1] F. P. Milliken, C. P. Umbach, and R. A. Webb, Solid State Comm. 97, 309 (1996).
[2] A. M. Chang, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 77, 2538 (1996).
[3] M. Grayson, D. C. Tsui, L. N. Pfeiffer, K. W. West, and A. M. Chang, Phys. Rev. Lett. 80, 1062 (1998).
[4] L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne, Phys. Rev. Lett. 79, 2526 (1997).
[5] R. de Picciotto, M. Reznikov, M. Heiblum, V. Umansky, G. Bunin, and D. Mahalu, Nature 389, 162 (1997).
[6] X. G. Wen, Phys. Rev. B 41, 12838 (1990).
[7] C. L. Kane and M. P. A. Fisher, Phys. Rev. Lett. 68, 1220 (1992).
[8] W. Kang, H. L. Stormer, L. N. Pfeiffer, K. W. Baldwin, and K. W. West, Nature 403, 59 (2000).
[9] A. Mitra and S. M. Girvin, Phys. Rev. B 64, R41309 (2001).
[10] M. Kollar and S. Sachdev, Phys. Rev. B 65, R121304 (2002).
[11] E.-A. Kim and E. Fradkin, Phys. Rev. B 67, 045317 (2003).
[12] C. Chamon, D. E. Freed, S. A. Kivelson, S. L. Sondhi, and X. G. Wen, Phys. Rev. B 55, 2331 (1997).
[13] C. L. Kane and M. P. A. Fisher, Phys. Rev. B 46, 15233 (1992).
[14] See, for instance, Bosonization by Michael Stone, World Scientific (Singapore, 1994), and references therein.
[15] P. W. Anderson, G. Yuval, and D. R. Hamann, Phys. Rev. B 1, 4464 (1970).
[16] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys. 59, 1 (1987).
[17] A. Schmid, Phys. Rev. Lett. 51, 1506 (1983).
[18] L. P. Kadanoff and G. Baym, Quantum Statistical Mechanics (W. A. Benjamin, 1962).
[19] J. Rammer and H. Smith, Rev. Mod. Phys. 323, 58 (1986).
[20] M. Kataoka, C. J. B. Ford, M. Y. Simmons, and D. A. Richie, Phys. Rev. Lett. 89, 226803 (2002).
[21] H.-S. Sim, M. Kataoka, H. Yi, N. Y. H. M.-S. Choi, and S.-R. E. Yang, cond-matt/0305642.