possible $N=70$ subshell effect on structural evolution of cadmium isotope chain

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With effective charge of neutron ($e_{n} = 0.8$) and proton excitation of $(g_{9/2})^{-2}$, we self-consistently describe the electromagnetic feature of 11/2$^{-}$ isomers in $^{111–129}$Cd, including their electromagnetic moments and $7/2_2^+ \rightarrow 11/2_1^+$ E2 transitions. The resultant wave-functions present a sudden phase change at $N = 70$, which might be interpreted as the attractive-repulsive transition of the $Q_1^+ \cdot Q_1^+$ $pn$ interaction. This attractive-repulsive transition also explains the structural evolution of even-mass $^{110–128}$Cd. The two-body interaction elements of the surface-δ $Q_1^+ \cdot Q_1^+$ interaction demonstrates the difference between valence neutron orbits of $^{A<118}$Cd and that of $^{A>118}$Cd, which could be a typical effect of the potential $N = 70$ subshell.

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I. INTRODUCTION

Nuclear properties are generally affected by the nuclear shell structure. For example, nuclei with magic numbers of protons and/or neutrons (2, 8, 20, 28, 82, 126) are spherically shaped and are more tightly bound than their neighboring near-magic isotopes. As changing excitation energy, rotational frequency, or isospin, new shell/subshell gaps may emerge, and lead back to some nuclear structural evolution. Therefore, the prediction and observation of new shell gaps in atomic nuclei is of continuing great interest. Recently, the $N = 70$ subshell is proposed by exploring the impact of tensor terms on single-particle spectra [1]. However, experimental evidence on the existence of this subshell is rare. As far as we know, the only experimental investigation on this subshell focuses on Zr isotopes, and its conclusion tends to be negative [2]. It’s interesting to search potential $N = 70$ subshell effect in other isotope chains near Zr.

The behavior of the cadmium isotopes around $N = 70$ is most impressive. As shown in fig. 1 the $N = 70$ $^{118}$Cd reaches the lowest of $E(2+)$, and largest $E(4+)/E(2+)$ ratio, $B(E2, 2_1^+ \rightarrow 0_1^+)$, $\beta/\beta_{sp}$, which implies the largest deformation. In the $A \sim 100$ region, the nuclear deformation is usually driven by the proton-neutron ($pn$) interaction [3,4]. To understand whether and how the $N = 70$ subshell is possibly related to the Cd structural evolution, it’s desirable to investigate the $pn$ interaction along the Cd isotope chain in a microscopic point view. The even-mass Cd seems a typical collectquadrupole oscillator with $E(4+)/E(2+) \sim 2$ and strong $2_1^+ \rightarrow 0_1^+$ transition as shown in fig. 1(b, c). Their wave-functions can be too complicated to extract the $pn$ interaction. Therefore, we turn to discuss the possible $N = 70$ subshell on odd-mass Cd isotopes, whose simple wave-functions of single-particle (s.p.) states can be a sensitive probe to the $pn$ interaction.

Recently, the experimental observation of systematic 11/2$^{-}$ isomers in $^{111–129}$Cd was completed [11]. The linear behavior of the 11/2$^{-}$ quadrupole moments (denoted by $Q$) implies a seniority neutron configuration with a unpaired $h_{11/2}$ neutron. Corresponding neutron effective charge ($e_{n} \sim 2.5e$) and variable 11/2$^{-}$ magnetic moments (denoted by $\mu$) lead to the excitation of two proton holes in the $Z = 50$ core. Given the $pf$ shell closure, the pro-

FIG. 1: (color online) Structural evolution of even-mass $^{110–128}$Cd. Panels (a), (b), (c) and (d) presents their $E(2+)$, $E(4+)/E(2+)$, $B(E2, 2_1^+ \rightarrow 0_1^+)$ and $\beta/\beta_{sp}$, respectively. The experimental data of Panels (a), (b) and (c) is from ref. [6,7], except that $B(E2, 2_1^+ \rightarrow 0_1^+)$ of $^{120}$Cd is from ref. [8]. Adopted $\beta/\beta_{sp}$ in Panel (d) is from [6]. To complete $B(E2, 2_1^+ \rightarrow 0_1^+)$s and $\beta/\beta_{sp}$s in $^{110–128}$Cd, fitting curves of these two values with empirical systematic [10] are presented as “empirical fit” solid lines in Panels (c) and (d). The $N = 70$ position is highlighted by vertical blue lines.

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ton excitation of Cd isotopes can be attributed to five \((g_9/2)^{-2}\) configurations. Thus, possible configurations of \(11/2^+\) isomers are limited, which simplifies our numerical investigation. On the other hand, the degree of freedom from the proton excitation provides the opportunity to understand the evolution of \(pn\) interaction along the Cd isotope chain.

II. WAVE-FUNCTIONS

Our calculation adopts a \(\chi^2\)-fitting process to \(11/2^-\) \(Q\) values for obtaining the wave-function of \(11/2^-\) isomer. The numerical process can be described as follows. First of all, we calculate the \(Q/\mu\) matrix elements for the seniority neutron configuration and the \((\pi g_9/2)^{-2}\) proton excitation. Secondly, we perform a serial of trial calculations to optimize \(e_\nu\) and model space. Finally, the best-fit wave-functions of \(11/2^-\) isomers are obtained by the fitting process to experimental \(Qs\) under constraints of experimental \(\mu s\) from Table I of ref. 11.

The calculation of neutron \(Q/\mu\) matrix elements follows ref. 11. The neutron configuration of \(Cd\) \(11/2^-\) isomers supposedly corresponds to the \(\nu h_{11/2}\) s.p. excitation, i.e. \(\nu h_{11/2}S\nu(111)^{-2}/2\), where \(S\nu\) is the neutron pair with spin zero, and \(A\) is the mass. For such neutron configuration, the occupation of the \(\nu h_{11/2}\) orbit is estimated to be \(n = 1 + \frac{8}{9}(A - 111)\), assuming \(S\nu\)s occupy \(s^1/2, d^3/2, d^5/2, h^11/2\) orbits averagely. Thus, \(\langle \nu h_{11/2}S\nu(111)^{-2}/2 | Q | \nu h_{11/2}S\nu(111)^{-2}/2 \rangle = \frac{8}{9}(A - 111)\), where \(Q\) is the \(Q\) operator, and \(\langle \nu h_{11/2} | Q | \nu h_{11/2} \rangle = -0.269b\) as in ref. 11. Correspondingly, \(\langle \nu h_{11/2}S\nu(111)^{-2}/2 | Q | \nu h_{11/2}S\nu(111)^{-2}/2 \rangle = -1.339\mu N\), where \(\mu\) is the proton operator, and the neutron spin Lande factor \(g_\nu = -3.827 \times 0.7\).

According anti-symmetry of fermion, \((g_9/2)^{-2}\) proton configurations have only five spins: \(I = 0h, I = 2h, I = 4h, I = 6h\) and \(I = 8h\) (denoted by \(S\nu, D\nu, G\nu, I\pi, K\pi\), respectively). Their \(Q/\mu\) matrix elements can be calculated according to ref. 12. Assuming a solid \(N = 64\) subshell \(2, 13, 17\), the \(0^+_1, 2^+_1, 4^+_1, 6^+_1\) and \(8^+_1\) states of “semi-magic” \(112\)Cd correspond to \(S\nu, D\nu, G\nu, I\pi, K\pi\) configurations. The effect charge of proton is adjusted to achieve an agreement with experimental \(Q\) of the \(112\)Cd \(2^+_1\) state. There are strong E2 transitions of \(2^+_1 \rightarrow 0^+_1\) and \(4^+_1 \rightarrow 2^+_1\) in \(112\)Cd, which can not be purely explained by proton s.p. motion on \(g_{9/2}\) orbit. The neutron collective excitation across the \(N = 64\) subshell is still non-negligible in Cd isotopes. To consider this point, the magnitudes of \(\langle S\nu | Q | D\nu \rangle\) and \(\langle D\nu | Q | G\nu \rangle\) are extracted from B(E2, \(2^+_1 \rightarrow 0^+_1\)) and B(E2, \(4^+_1 \rightarrow 2^+_1\)) for \(112\)Cd, while their signs still follow the \((g_9/2)^{-2}\) calculation. For proton \(\mu\) matrix elements, the proton Lande factors \(g_\mu = 1\) and \(g_\mu s = 5.586 \times 0.7\) are adopted as usual. Adopted proton electromagnetic matrix elements are listed in Table I.

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| \(D_e\) | \(|Q|\langle Q | D_e \rangle \times e_\pi\) | \(-4.75e \times r_0^2\) |
| \(G_e\) | \(|Q|\langle Q | G_e \rangle \times e_\pi\) | \(-2.13e \times r_0^2\) |
| \(I_\pi\) | \(|Q|\langle Q | I_\pi \rangle \times e_\pi\) | \(+1.207e \times r_0^2\) |
| \(K_\pi\) | \(|Q|\langle Q | K_\pi \rangle \times e_\pi\) | \(+5.71e \times r_0^2\) |
| \(S_\pi\) | \(|Q|\langle Q | S_\pi \rangle \times e_\pi\) | \(-14.265e \times r_0^2\) |
| \(D_e\) | \(|Q|\langle Q | G_e \rangle \times e_\pi\) | \(-12.164e \times r_0^2\) |
| \(D_e\) | \(|\pi|\langle |\pi | D_e \rangle \times e_\pi\) | \(+3.698\mu N\) |
| \(G_e\) | \(|\pi|\langle |\pi | G_e \rangle \times e_\pi\) | \(+6.75\mu N\) |
| \(I_\pi\) | \(|\pi|\langle |\pi | I_\pi \rangle \times e_\pi\) | \(+9.783\mu N\) |
| \(K_\pi\) | \(|\pi|\langle |\pi | K_\pi \rangle \times e_\pi\) | \(+12.809\mu N\) |
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The neutron \(\nu h_{11/2}S\nu(111)^{-2}/2\) and proton \((\pi g_9/2)^{-2}\) configurations are coupled into five \(I^2 = 11/2^-\) bases for the quantum description for \(11/2^-\) isomers of \(111-129\) Cd. Previous calculations \(13\) demonstrated that the \(S\nu\) and \(D\nu\) configurations can be predominate in the Cd low-lying states. Therefore, we first perform \(\chi^2\)-fitting to the experimental \(Qs\) of \(111-129\)Cd with a trial wave-function as \(\nu h_{11/2}S\nu(111)^{-2}/2 \otimes (C_S S\pi + C_D D\pi)\)\(^{-1/2}\) where \(C_S\) and \(C_D\) are fitting variables, and the experimental \(\mu s\) are taken as the fitting constraints. This fitting is denoted by “\(SD\) fitting” in this work for convenience. We also note that \(e_\nu\) strongly affects the goodness of our \(\chi^2\)-fitting, and thus the root mean square (r.m.s.) of the fitting results is plotted as a function of \(e_\nu\) in fig. 2. The fitting with \(S\nu\) and \(D\nu\) provides \(-0.16e\) error, i.e. the same order of \(Qs\), which is unsatisfactory. Additional proton configuration is required. Therefore, we preform three other fittings with trial wave-functions as \(\nu h_{11/2}S\nu(111)^{-2}/2 \otimes (C_S S\pi + C_D D\pi + C_G G\pi)\)\(^{-1/2}\), \(\nu h_{11/2}S\nu(111)^{-2}/2 \otimes (C_S S\pi + C_D D\pi + C_I I\pi)\)\(^{-1/2}\), and \(\nu h_{11/2}S\nu(111)^{-2}/2 \otimes (C_S S\pi + C_D D\pi + C_K K\pi)\)\(^{-1/2}\) (denoted by “\(SD, SDI, and SDK\)”, respectively), where \(C_G, C_I\) and \(C_K\) are additional fitting variables. Their r.m.s.-\(e_\nu\) relations are also plotted in fig. 2. The \(SD\) and \(SDK\) fittings provide r.m.s.\(-0.05eb\), which is improved, however, still unsatisfactory. On the other hand, the \(SD\) fitting reaches an almost accurate agreement with experiments (0.001eb r.m.s.) at \(e_\nu = 0.8e\). Therefore, we conclude that the \(SD\) wave-function is capable of self-consistent description on \(Q\) and \(\mu\) values of the \(Cd\) \(11/2^-\) isomers.

Now, let us focus on the \(SD\) fitting results with \(e_\nu = 0.8e\). Depending on inputed initial \(C_S, C_D\) and \(C_G\) variables, this fitting process can reach two intrinsically different final wave-functions for single Cd isotope, which both accurately reproduce experimental \(Q\) value. This double-valued behavior provides the \(SD\) wave-function more opportunity to reach a reasonable fitting, which is why the \(SD\) fitting is superior to \(SDI\) and \(SDK\) ones.

TABLE I: Adopted proton electromagnetic matrix elements in this work (see text for detailed calculation description). \(e_\pi\) is the proton effective charge, and \(r_0^2 \sim 4^{1/2} \text{ fm}^2\).

We also find that the most obvious difference between the two SDG final wave-functions is their phase of $C_G$ related to $C_S$, according to which we classify the SDG resultant wave-functions as two types denoted by $SDG^+$ and $SDG^-$. $C_S$ and $C_S$ of $SDG^+$ wave-functions have the same phase; those of $SDG^-$ ones have different phases. We present the $SDG^+$ and $SDG^-$ wave-functions separately in fig. 3.

In fig. 3 the $Q$ values calculated with $SDG^+$ and $SDG^-$ wave-functions are compared with corresponding experimental $Q$s from ref. [11]. The agreement between calculation and experiments directly demonstrates the reliability of our calculation.

III. ATTRACTION-REPULSION TRANSITION BASED ON THE $N = 70$ SUBSHELL

The $SDG$ fitting provides two wave-functions with different phases. It’s essential to decide which phase the physical reality favors. Strong E2 transitions from $7/2^+_1$ states to $11/2^-$ isomers in $^{113-119}\text{Cd}$ may partially solve this problem. We will use both $SDG^+$ and $SDG^-$ wave-functions to calculate these E2 transition rates, and the comparison between experiments and calculated results tells which type of wave-function is superior.

Before the B(E2) calculation, the main configuration of the $7/2^+_1$ state should be clarified. We note that the strong E2 transition of $7/2^+_1 \rightarrow 11/2^+_1$ could empirically imply the similarity between the $11/2^-$ isomer and $7/2^+_1$ state. Thus, the $7/2^+_1$ wave-function could be a proton-neutron recoupling of the $11/2^-\pi^+$ wave-function, i.e. $[v_{h_{11/2}}A_{-11/2}^{\pi^+} \otimes D_{\pi}]l^\pi = 7/2^-$ or $[v_{h_{11/2}}A_{-11/2}^{\pi^+} \otimes G_{\pi}]l^\pi = 7/2^-$. On the other hand, $7/2^-$ excitation energies related to $11/2^-$ isomer, i.e. $E(7/2^-) - E(11/2^-)$, seem to have similar trend to $E(2^-) + E(4^+)$ of corresponding even-even core as shown in fig. 3. Pearson’s correlation coefficients [19] of $E(7/2^-) - E(11/2^-)$ related to $E(2^-) + E(4^+)$ are calculated to be 0.955 and 0.975, respectively, which mathematically suggests their highly possible positive correlations. $2^-_1$ and $4_1^+$ states of even-mass $^{110-130}\text{Cd}$ isotopes correspond to a quadrupole oscillator as shown in fig. 3(b, c). Thus, the $7/2^+_1$ state could also be attributed to a quadrupole excitation based on the $11/2^-$ isomer. Fig. 4 already suggests the main configuration of $11/2^-$ isomer as $[v_{h_{11/2}}A_{-11/2}^{\pi^+} \otimes S_{\pi}]l^\pi = 11/2^-$. Thus, one can rule out $[v_{h_{11/2}}A_{-11/2}^{\pi^+} \otimes G_{\pi}]l^\pi = 7/2^-$ for the main component of $7/2^+_1$ state, and reasonably assume it as $[v_{h_{11/2}}A_{-11/2}^{\pi^+} \otimes D_{\pi}]l^\pi = 7/2^-$, where $D_{\pi}$ replaces $S_{\pi}$.
to be responsible for the quadrupole excitation.

Calculated B(E2, 7/2\textsuperscript{+} → 11/2\textsuperscript{−})s with above assumed initial [\nu h_{11/2} S_{\nu}^{(A-111)/2} \otimes D_{\nu}]^{I^+ = 7/2^{-}} configuration and final SDG wave-functions from fig. \textbf{3} are compared with available experimental data in fig. \textbf{6}. Our calculations reasonably reproduces the experimental B(E2) values. On the other hand, test calculations for simplicity) only provide \sim 1W.u. B(E2)s, which also rules out such configuration for 7/2\textsuperscript{−} state in a posteriori point of view. According to fig. \textbf{6} currently available experimental data favors SDG\textsuperscript{+} wave-functions for N < 70 and SDG\textsuperscript{−} ones for N > 70. A sudden phase change of C\textsubscript{G} is identified at N = 70.

In the framework of the perturbation theory, the phase of C\textsubscript{G} in SDG wave-function corresponds to the sign of Hamiltonian matrix element, \langle \nu h_{11/2} S_{\nu}^{(A-111)/2} \otimes S_{\pi} | H | \nu h_{11/2} S_{\nu}^{(A-111)/2} \otimes G_{\nu} \rangle. This element is assigned as a particle-hole type pn interaction by parity and angular momentum conservation. This pn interaction is mathematically written as Q\textsubscript{\pi} \cdot Q\textsubscript{\nu}, where Q\textsuperscript{\pi} can be an arbitrary particle-hole operator with I^\pi = 4^+. The sign of some interaction represents its repulsiveness or attractiveness. Thus, a positive C\textsubscript{G} demonstrates an attractive Q\textsubscript{\pi} \cdot Q\textsubscript{\nu} interaction, and vice versa. The sudden change of C\textsubscript{G} (or the E2 transition in fig. \textbf{4}) at N = 70 is interpreted as a transition from attractive Q\textsubscript{\pi} \cdot Q\textsubscript{\nu} to repulsive one.

In the perturbation theory, the magnitude of C\textsubscript{G} also represents the strength of Q\textsubscript{\pi} \cdot Q\textsubscript{\nu} interaction. As shown in fig. \textbf{3} the magnitude of C\textsubscript{G} and the strength of Q\textsubscript{\pi} \cdot Q\textsubscript{\nu} interaction both tend to increase with respect to the mass number regardless of the C\textsubscript{G} phase. We note that the total pn interaction of observable \textsuperscript{110-129}Cd isotopes is presumably attractive. As increasing the mass number, the attractive Q\textsubscript{\pi} \cdot Q\textsubscript{\nu} interaction increases the attractiveness of the total pn interaction before N = 70; correspondingly, the repulsive Q\textsubscript{\pi} \cdot Q\textsubscript{\nu} interaction after N = 70 reduces it. As a result, the Cd isotope chain reaches the largest deformation at N = 70 as shown in fig. \textbf{1}. In a word, the behavior of the Q\textsubscript{\pi} \cdot Q\textsubscript{\nu} interaction drives the structural evolution of even-mass \textsuperscript{110-129}Cd.

The magnitude increaseasment of the Q\textsubscript{\pi} \cdot Q\textsubscript{\nu} interaction with respect to neutron number is reasonable according to the N\textsubscript{p}N\textsubscript{n} scheme of the pn interaction \textsuperscript{20}. However, its sudden attractive-repulsive change at N = 70 required a neutron-number sensitive explanation. We schematically calculate the two-body interaction elements to illustrate the attractiveness/repulsiveness of the Q\textsubscript{\pi} \cdot Q\textsubscript{\nu} interaction between a \textsubscript{g}\textsubscript{9/2} proton and a neutron at d\textsubscript{3}/2, d\textsubscript{5}/2 or h\textsubscript{11/2} (s\textsubscript{1/2} orbit is not involved in the Q\textsubscript{\nu} operator). Given the short-range property of nuclear force, we expend a surface-\delta \textsuperscript{21,22} pn interaction into particle-hole formalism, i.e. a serial of Q\textsubscript{\pi} \cdot Q\textsubscript{\nu} terms, where the Q\textsubscript{\pi} \cdot Q\textsubscript{\nu} term is extracted specifically. Two-body interaction elements of this particle-hole term are calculated with formalism from ref. \textsuperscript{12}, and listed in Table \textbf{11}. The most of two-body interaction after d\textsubscript{5}/2 and d\textsubscript{5}/2 orbits are attractive; on the contrary, those for h\textsubscript{11/2} orbit are mostly repulsive. Thus, the attractiveness/repulsiveness of Q\textsubscript{\pi} \cdot Q\textsubscript{\nu} interaction implies the valence neutron orbit. As state above, the Q\textsubscript{\pi} \cdot Q\textsubscript{\nu} interaction for A < \textsuperscript{118}Cd is attractive, and corresponding valence neutrons are active at d\textsubscript{5}/2/d\textsubscript{5}/2 orbits; while valence neutrons of A \geq \textsuperscript{118}Cd occupy the h\textsubscript{11/2} orbit with a repulsive Q\textsubscript{\pi} \cdot Q\textsubscript{\nu} interaction. This change of valence neutron orbit across N = 70 could be attributed to a typical subshell effect, as demonstrated in fig. \textbf{11} taking \textsuperscript{117,119}Cd as example. Therefore, we conclude that the structural evolution of Cd isotopes.
TABLE II: Two-body interaction elements of the surface-δ $Q^+_\delta \cdot Q^+_\delta$ component below and above the possible $N = 70$ gap. This calculation is based on Eq. (9.15) of ref. [23] with $V(\text{eff})=1$, $\alpha = 0$ and $F^0 = 1$. These three parameters are arbitrarily determined by definition of the surface-δ interaction [21][22].

| $N$ | $(g_{9/2}d_{3/2})^4|V_{pn}|(g_{9/2}d_{3/2})^4$ | $(g_{9/2}d_{3/2})^3|V_{pn}|(g_{9/2}d_{3/2})^5$ | $(g_{9/2}d_{3/2})^4|V_{pn}|(g_{9/2}d_{3/2})^5$ | $(g_{9/2}d_{3/2})^6|V_{pn}|(g_{9/2}d_{3/2})^5$ | $(g_{9/2}d_{3/2})^2|V_{pn}|(g_{9/2}d_{3/2})^2$ | $(g_{9/2}d_{3/2})^3|V_{pn}|(g_{9/2}d_{3/2})^4$ | $(g_{9/2}d_{3/2})^5|V_{pn}|(g_{9/2}d_{3/2})^5$ | $(g_{9/2}d_{3/2})^6|V_{pn}|(g_{9/2}d_{3/2})^5$ |
|------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|--------------------------------------------------|
| $<70$ | $-0.340$ | $-0.387$ | $-0.225$ | $-0.064$ | $0.446$ | $0.439$ | $-0.102$ | $-0.522$ | $-0.351$ |
| $>70$ | $-0.340$ | $-0.387$ | $-0.225$ | $-0.064$ | $0.446$ | $0.439$ | $-0.102$ | $-0.522$ | $-0.351$ |

may be a fingerprint of the potential $N = 70$ subshell.

IV. SUMMARY

To summary, we extract the wave-function of the $11/2^-$ isomers in odd-mass $^{111-129}$Cd according to their electromagnetic feather, including electromagnetic moments and B(E2) values. According the perturbation theory, resultant wave-functions present an attractive-repulsive transition of the $pn$ interaction across $N = 70$, which also lead to the structural evolution of even-mass $^{110-128}$Cd. Considering the short-range property of the nuclear force, this attractive-repulsive transition may be further attributed to the possible $N = 70$ subshell.

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