SELF-ORGANIZED CRITICALITY IN GLUON SYSTEMS
AND ITS CONSEQUENCES

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It is pointed out, that color-singlet gluon clusters can be formed in hadrons as a consequence of self-organized criticality (SOC) in systems of interacting soft gluons, and that the properties of such spatiotemporal complexities can be probed experimentally by examining inelastic diffractive scattering. Theoretical arguments and experimental evidence supporting the proposed picture are presented. As a consequence of the space-time properties of the color-singlet gluon clusters due to SOC in gluon systems, a simple analytical formula for the differential cross section for inelastic diffractive hadron-hadron scattering can be derived. The obtained results are in good agreement with the existing data. Furthermore, it is shown that meson radii can be extracted from inelastic diffractive scattering experiments. The results obtained for pion and kaon charge radii are compared with those determined in meson form factor measurements.

This talk is a short summary of three papers written at FU Berlin in collaboration with C. Boros, T. Meng, R. Rittel, and Y. Zhang.

In 1987, Bak, Tang and Wiesenfeld observed that open, dynamical, complex systems far from equilibrium may evolve into critical states, where local perturbations can propagate like avalanches (called BTW-avalanches) over all length and time scales. In contrast to critical behavior in equilibrium thermodynamics, the evolution into this very delicate state needs no fine tuning of external parameters: The criticality is self-organized (SOC). The distributions of the size $S$ and the lifetime $T$ of such BTW-avalanches obey power laws:

$$D_S(S) \propto S^{-\mu} \quad \text{and} \quad D_T(T) \propto T^{-\nu}$$

known as the “fingerprints of SOC”. Since the first observation of SOC, there has been vast interest in this fast developing field. The reason is that SOC provides so far the only known mechanism to generate spatiotemporal complexity, which is ubiquitous in Nature. In the macroscopic world, many complex systems have been found showing this kind of behavior: sand- and rice-piles, the crust of earth exhibiting earthquakes of all sizes, the stock market and even the system of biological genera on earth. As particle physicists we ask the following questions:

- Does SOC also exists at the fundamental level of matter, in the world of
quarks and gluons?

- How to probe this experimentally? What are the measurable consequences?

At the beginning, we search for an appropriate candidate for a system to investigate and recall experimental results and theoretical facts that are of considerable importance: First, it has been observed\(^6\) that soft gluons dominate the small-\(x_B\) region of deep-inelastic electron-proton scattering (DIS) at HERA. Thus, there are many soft gluons in the proton. Second, gluons may \textit{directly} interact with each other through local gluon-gluon coupling prescribed by the QCD-Lagrangian. This distinguishes their behavior from that of other elementary particles such as photons. Third, due to emission and absorption of gluons, the number of gluons is not a conserved quantity. This shows, that systems of interacting soft gluons should be considered as \textit{open, dynamical, complex systems with many degrees of freedom}. This can be considered as the zeroth fingerprint for the existence of SOC in such systems.

What are the possible BTW-avalanches in the gluon system in the proton? Another experimental observation is very helpful in this connection: Large-rapidity gap events (LRG) have been observed\(^6\) in the small-\(x_B\) region of DIS, the same region where the soft gluons dominate. In such events, the virtual photons encounter colorless objects originating from the proton (called Pomeron in Regge phenomenology\(^6\)). Therefore, this type of events is very similar to inelastic diffractive hadron-hadron scattering in which such colorless objects also play the dominating role. The interaction of these color-singlets with the proton remnant should be of Van-der-Waals type. Thus, they can be easily knocked out by the projectile in reactions with relatively small momentum transfer. This makes it relatively easy to “examine” them without the proton remnant in inelastic diffractive scattering processes.

It has been suggested\(^1,2\) that the colorless objects are BTW-avalanches in form of color-singlet gluon clusters due to SOC in the system of interacting soft gluons in the proton. In order to test this proposal, a systematic analysis of the existing data has been performed\(^2\), where we made use of the fact, that the size of the cluster is the measurable quantity \(x_P\) and that the diffractive structure function \(F_2^{D(3)}\) divided by \(x_P\) can be interpreted as the size distribution of the clusters. The result of the analysis shows (see fig. \(^3\)), that the size and the lifetime distribution\(^7\) of the color-singlet gluon clusters indeed show power-law behavior Eq.(1) with exponents \(\mu \approx 2\) and \(\nu \approx 2\). Furthermore, this behavior is universal and can be seen in all inelastic diffractive scattering processes (for details see Ref.\(^2\)), as it is expected if SOC is the underlying production mechanism for the colorless objects.
Figure 1. $F^{D(3)}(\beta, Q^2, x_P)/x_P$ as a function of $x_P$ for a wide range of the kinematical variables $Q^2$ and $\beta$ in a double-logarithmic plot. This figure shows, that the power-law behavior with an exponent $\approx -2$ is independent of those variables. Data are taken from Ref.\cite{8,9}.

The following physical picture for inelastic diffractive scattering processes emerges: The beam particle sees a cloud of color-singlet gluon clusters due to self-organized criticality (SOC) in the gluon system. The size and lifetime distribution of the clusters obey universal power laws. There is no typical size, no typical lifetime and no static structure for the clusters. The color-singlet gluon cluster is knocked out by the beam particle. Both break up producing the usually unidentified hadronic system $X$.

What are the measurable consequences? Since the space-time properties
Figure 2. The differential cross section \( \frac{d\sigma}{dt} \) for inelastic diffractive scattering off proton using different projectiles \((p, \pi^+, K^+, \gamma \text{ and } \gamma^*)\) as function of \( \sqrt{-t} \). The data are taken from Refs.[11-15]. The solid lines represent the \( \exp(-2a\sqrt{-t}) \)-dependence, where \( a = \sqrt{\frac{3}{5}}r_p \) with the proton radius \( r_p \). See text or Refs.[1,2] for details.

of the cluster cloud are known, we choose an optical geometrical approach, where the beam particle is considered as high frequency wave passing the target cloud of colorless gluon clusters. Taking into account 1. the properties of the gluon clusters due to SOC, 2. the confinement of single (colored) gluons, 3. causality and 4. the cluster distribution, it is possible to determine the differential cross section for inelastic diffractive scattering processes

\[
\frac{d\sigma}{dt} = C \cdot \exp(-2a\sqrt{-t}),
\]  

where \( t \) is the square of the four-momentum transfer and \( a = \sqrt{\frac{3}{5}}r_p \) is directly related to the proton radius and \( C \) is an unknown normalization constant. This behavior is an until now unknown regularity and in very good agreement with the experimental data for inelastic diffractive proton-proton and (antiproton-proton) scattering.

Here, we present a further test by comparing Eq.(2) with the experimental data for inelastic diffractive scattering off protons using different pro-
Since no integrated data seems to be available, we plot different boxes for different values of \( x_P \). Since the behavior in \( p_\perp \) should be independent of the value of \( x_P \) according to Eq.(3), the slope parameter \( a \) should be equal in all these boxes. This is confirmed by a fit to Eq.(3) which leads to values for the meson radii of \( r_{\pi^\pm} = 0.62 \pm 0.04 \) fm, and \( r_{K^\pm} = 0.53 \pm 0.03 \) fm. These values are in agreement with those obtained from form factor measurements (see Refs.[3,10] for details).

In summary: Systems of interacting soft gluons should be considered as open, dynamical, complex systems far from equilibrium. The fingerprints of SOC exist in inelastic diffractive scattering. The colorless objects which
play the dominating role in such reactions are BTW-avalanches in the form of color-singlet gluon clusters due to SOC in the gluon system in the proton. The differential cross section $d\sigma/dt$ can be calculated in an optical geometrical model taking the SOC properties of the gluon system into account and is given by Eq. (3). The slope $a$ in Eq. (3) is directly related to the radius of the target hadron, which remains intact in the reaction. There is good agreement with the experimental data.

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**References**

1. T. Meng, R. Rittel and Y. Zhang, *Phys. Rev. Lett.* **82**, 2044 (1999).
2. C. Boros, T. Meng, R. Rittel, K. Tabelow and Y. Zhang, *Phys. Rev. D* **61**, 094010 (2000).
3. T. Meng, R. Rittel, K. Tabelow and Y. Zhang, [hep-ph/9910331](https://arxiv.org/abs/hep-ph/9910331).
4. P. Bak, C. Tang and K. Wiesenfeld, *Phys. Rev. Lett.* **59**, 381 (1987); *Phys. Rev. A* **38**, 364 (1988).
5. See e.g. P. Bak in *How Nature Works*, (Springer-Verlag, New York, 1996).
6. See e.g. H. Abramowicz and A. Caldwell, *Rev. Mod. Phys.* **71**, 1275 (1999); A.M. Cooper-Sarkar, R.C.E. Devenish and A. De Roeck, *Int. J. Mod. Phys. A* **13**, 3385 (1998); and papers cited therein.
7. R. Rittel, PhD thesis, FU Berlin, October 2000.
8. H1 Collaboration, C. Adloff et al, *Z. Phys. C* **76**, 613 (1997).
9. ZEUS Collaboration, J. Breitweg et al, *Eur. Phys. J. C* **6**, 43 (1999).
10. K. Tabelow, PhD thesis, FU Berlin, in preparation.
11. M.G. Albrow et al, *Nucl. Phys. B* **108**, 1 (1976).
12. M. Adamus et al, *Z. Phys. C* **39**, 361 (1988).
13. F.C. Winkelmann et al, *Phys. Rev. Lett.* **32**, 121 (1974).
14. ZEUS Collaboration, J. Breitweg et al, *Eur. Phys. J. C* **2**, 237 (1998).
15. ZEUS Collaboration, J. Breitweg et al, *Eur. Phys. J. C* **1**, 81 (1998).