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Dynamic Quality of an Aerostatic Thrust Bearing with a Microgroove and Support Center on Elastic Suspension

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Abstract: The disadvantage of aerostatic bearings is their low dynamic quality. The negative impact on the dynamic characteristics of the bearing is exerted by the volume of air contained in the bearing gap, pockets, and microgrooves located at the outlet of the feeding diaphragms. Reducing the volume of air in the flow path is a resource for increasing the dynamic quality of the aerostatic bearing. This article presents an improved design of an axial aerostatic bearing with simple diaphragms, an annular microgroove, and an elastic suspension of the movable center of the supporting disk. A mathematical model is presented and a methodology for calculating the static characteristics of a bearing and dynamic quality indicators is described. The calculations were carried out using dimensionless quantities, which made it possible to reduce the number of variable parameters. A new method for solving linearized and Laplace-transformed boundary value problems for transformants of air pressure dynamic functions in the bearing layer was applied, which made it possible to obtain a numerical solution of problems sufficient for practice accuracy. The optimization of the criteria for the dynamic quality of the bearing was carried out. It is shown that the use of an elastic suspension of the support center improves its dynamic characteristics by reducing the volume of compressed air in the bearing layer and choosing the optimal volume of the microgroove.

Keywords: aerostatic thrust bearing; dynamic quality; degree of stability; oscillation index; transient process; stability of a dynamic system

1. Introduction

To produce the load capacity in aerostatic bearings, flow rate limiters are usually used, which automatically regulate the pressure in the bearing air layer [1–6]. In practice, diaphragms are used as flow limiters, in which the limitation of the flow is created due to the air pressure, and capillaries, in which the pressure is regulated by the forces of viscous friction in the air lubricant. Diaphragms are used more often, among which nozzles with annular and simple diaphragms are distinguished [7,8].

The vulnerability of aerostatic bearings to instability is known, and oftentimes, this circumstance is decisive when choosing flow limiters. Bearings with annular diaphragms are always stable; however, compared to similar bearings with simple diaphragms, they have 1.5 times higher compliance [9]. The load capacity of bearings with annular diaphragms is also lower due to the discreteness of the feeders, while the use of simple diaphragms united by microgrooves helps to eliminate the noted drawback. At the same time, the presence of air pockets at the outlet of simple diaphragms and microgrooves is the reason for the loss of stability of the bearings and the extremely low quality of their dynamics [9,10]. The volume of air contained in the bearing layer also has a negative impact on the dynamics of the unit. Thus, a decrease in the volume of air in its flow path is a resource for increasing the dynamic quality of the aerostatic bearing.
Figure 1 shows a diagram of an improved circular aerostatic bearing with the shaft, 1 and base, 2, which is connected to a supporting disk, 3, of radius $r_0$. The bearing is powered from a source of compressed air through a hole, 8, of diameter $d$, from which air enters the microgroove, 7, made on the disk, 3, along the circumference of the radius $r_1$. The bearing center, 5, is supported by an elastic suspension, 4, in the form of a thin ring of thickness $\delta$. This ring, under the action of the pressure difference $p_s - p_k > 0$ in the hollow layer, 6, and the bearing layer, provides the necessary deformation $h_d$ of the material of the disk, 3, and the displacement of the central disk, 5, relative to the shaft, 1, by the value $h_s$, where $p_s$ is the injection pressure and $p_k$ is the pressure at the outlet of the diaphragms, 8.

![Figure 1. Calculation scheme of the bearing.](image)

Compared to a conventional support ($h_d = 0$), the volume of air contained in the bearing layer will be less due to deformation of $h_d$, where the gap $h_s$ on the central circle will always be less than the gap $h$ on the outer ring. In addition, due to deformation of the ring, 4, the vibration pattern of the bearing layer will change, which can also contribute to an increase in the dynamic quality of the bearing.

This paper considers a mathematical model of the stationary state of the bearing and calculates its optimal modes. On this basis, non-stationary mathematical modeling of the effectiveness of the proposed method for improving the dynamic characteristics and the calculation and study of the root stability criteria of the bearing were carried out. An example of the design calculation of the bearing is given.

2. Static Model of Bearing Movement

The static model includes the air mass flow balance equation:

$$q_k - q_h + q_{hs} = 0,$$  \hspace{1cm} (1)

as well as two equations of power balance of the movable elements of the support:

$$h_d = k_c (w_p - w_{hs}),$$  \hspace{1cm} (2)

$$w = f,$$  \hspace{1cm} (3)

$$w = w_h + w_{hs},$$  \hspace{1cm} (4)

where $q_k$, $q_h$, and $q_s$ are the mass flow rates of air through the gaps $h$, $h_s$, and the hole, 8, respectively, $w_h$, $w_{hs}$, and $w_p$ are the power reactions of compressed air in these gaps and the cavity, 6, and $w$ is the load capacity of the support [11].
The study of static characteristics was carried out in a dimensionless form. The scales of values are taken as: \( p_a \)—for air pressures, \( r_0 \)—for radial dimensions, \( h_0 \)—for the thickness of throttling slotted gaps, \( \frac{\pi h_0^2}{\rho T} \)—for mass air flow rates, and \( \pi r_0^2 p_a \)—for axial forces, where \( h_0 \) corresponds to the gap \( h \) in the bearing that perceives the design load \( f_0 \), \( p_a \) is the ambient pressure, \( \mu \) is the air viscosity, \( \Re \) is the universal gas constant, and \( T \) is the absolute gas temperature. Furthermore, dimensionless quantities are designated by capital Latin letters.

Dimensionless flow through the hole, \( 8 \), is determined by the expression [12]:

\[
Q_k = A_k \Pi(P_s, P_k),
\]

where \( \Pi \) is the Prandtl outflow function, and the aperture similarity criterion is:

\[
A_k = \frac{3\mu \Gamma d^2 \sqrt{\Re T}}{p_a h_0^3},
\]

where \( \Gamma = 2\sqrt{\left(\frac{2}{\gamma}\right)^{\frac{\gamma+1}{\gamma-1}}} \) and \( \gamma \) is an adiabatic air expansion index [9].

The pressure distribution function in the bearing layer [2] is:

\[
P(R) = \begin{cases} 
  P_k, & R \leq R_1, \\
  \sqrt{(P_k^2 - 1) \frac{\ln R}{\ln R_1} + 1}, & R_1 < R \leq 1.
\end{cases}
\]

Taking into account Equation (7), the formula for the flow rate in the bearing layer can be written in the form:

\[
Q_h = A_h H^3 (P_k^2 - 1),
\]

where \( A_h = -\frac{1}{\ln R_1} \).

Dimensionless force reactions were found by the following formulas [2,12]:

\[
W_{hs} = 2 \int_{0}^{R_1} R(P - 1) dR = R_1^2 (P_k - 1),
\]

\[
W_h = 2 \int_{R_1}^{1} R(P - 1) dR = 2J + R_1^2 - 1,
\]

\[
J = \int_{R_1}^{1} \sqrt{(P_k^2 - 1) \frac{\ln R}{\ln R_1} + 1} dR,
\]

\[
W_p = 2 \int_{0}^{R_1} R(P - 1) dR = R_1^2 (P_k - 1).
\]

In a dimensionless form, the model will include equations similar to (1)–(4):

\[
Q_k - Q_h + Q_{hs} = 0,
\]

\[
H_d = K_e (W_p - W_{hs}),
\]

\[
H_s = H - H_d,
\]

\[
W = F,
\]

as well as, obviously, \( H_s = H - H_d \).

When calculating the static characteristics of the bearing, dimensionless quantities were used as input parameters: radius \( R_1 \), injection pressure \( P_s \), deformation \( H_{d0} \) of the
ring, 4, at the design point $H = 1$, and normalized coefficient $\chi$ of the relative resistance of the diaphragms, 8:

$$\chi = \frac{P^2_k - 1}{P_2^2 - 1} \in [0, 1],$$  \hspace{1cm} (17)

which was used to calculate the pressures and the diaphragm similarity criterion.

Using Equation (17), we obtain the formula for calculating the pressure in the microgroove, 7:

$$P_k = \sqrt{1 + \chi (P_2^2 - 1)}.$$

According to (5)–(17), it is possible to find dimensionless force reactions, deformation $H_d$, coefficient $A_h$, flow rate $Q_h$ (flow rate $Q_{hs} = 0$), and a similarity criterion for the diaphragms:

$$A_k = \frac{Q_h}{\Pi(P_s, P_k)},$$  \hspace{1cm} (18)

By sequentially setting pressure values $P_k \in [1, P_s]$ in small steps, the corresponding force reactions, deformation $H_d$, gap $H = \frac{Q_h}{A_h(P_k^2 - 1)}$, and gap $H_s$ can be calculated. In the calculations, integral (12), which has no analytical quadrature, is calculated by the Simpson method [13].

Having written down the equations of the model in small $\Delta$ increments, we found the dimensionless bearing compliance to be:

$$K = -\frac{\Delta H}{\Delta F} = \frac{A_4 - A_5}{A_3 (A_1 + A_2)},$$  \hspace{1cm} (19)

where:

$$A_1 = \frac{2P_k}{\ln R_1} \int_{R_1}^1 \frac{R \ln R dR}{\sqrt{(P_k^2 - 1) \ln R_1 + 1}},$$  

$$A_2 = R_1^2 A_3 = 3 A_h H^2 \left( P_k^2 - 1 \right),$$  

$$A_4 = 2 A_h H^3 P_k,$$

$$A_5 = A_k \frac{\partial \Pi(P_s, P_k)}{\partial P_k} = A_k \left\{ \begin{array}{ll} 0, & P_k / P_s \leq 0.5, \\
0.5 P_1 - P_k, & \frac{P_k}{P_s} > 0.5. \end{array} \right. $$

3. Static Characteristics of the Bearing

Static compliance $K$ of the bearing does not depend on the elasticity coefficient of the ring, 4, deformation $H_d$, and gap $H_s$. The graphs in Figures 2 and 3 show the influence of the parameters $R_1$, $P_s$, and $\chi$ on this characteristic at the design point $H = 1$.

As can be seen from Figure 2, the curves of the dependence of the bearing compliance $K$ on the adjustment factor $\chi$ have an extreme character. The optimal value of the parameter of the relative resistance of the diaphragms is in the range $\chi = 0.45–0.46$.

Of interest is the nature of the change in the deformation $H_d$ and the gap $H_s$ from the load $F$. Figure 3 shows the dependences for four functions $H(F)$, $K(F)$, $H_d(F)$, and $H_s(F)$. The first of them is a load characteristic, the second represents the dependence of the compliance $K$ on the load $F$, and the last two show the nature of the dependence of the deformation $H_d$ and the gap $H_s$. The curves correspond to the calculated deformation $H_{d0} = 0.5$. Formula (27) makes it possible to determine the coefficient of elasticity $K_e$ of the ring, 4, at which the deformation $H_{d0}$ is ensured. At $R_1 = 0.75$ and $P_s = 4$, the coefficient $K_e = 0.731$. 


As can be seen from Figure 3, at low loads of $F$ when the pressure drop across the moving center is the greatest, the deformation is highest, as a result of which gap $H_s$ turns out to be much less than gap $H$. In comparison with the usual support due to the deformation of the suspension, 4, a significantly smaller volume of the bearing layer is provided, which should have a beneficial effect on the dynamic quality of the structure.

Deformation of $H_d$ in the region of low and moderate loads contributes to an effective decrease in the volume of the bearing layer. Consequently, the effect created by deformation can affect almost the entire range of loads.

The dependence of the volume of the bearing layer $V$ on the load $F$ at different values of deformation $H_{d0}$ is clearly shown in Figure 4.

Curve $H_{d0} = 0$ corresponds to the dependence $V(F)$ of a conventional bearing, and the remaining curves for which $H_{d0} > 0$ correspond to a bearing with a deformable suspension, 4. It is seen that deformation contributes to a significant decrease in the volume $V$ of the bearing layer. Thus, at load $F = 1$ and deformation $H_{d0} = 0.4$ at the design point, the volume decreases by almost one and a half times, and at $H_{d0} = 0.6$, it decreases more than twice.
The conclusion about the effectiveness of the proposed improvement can be made based on the study of non-stationary characteristics of the bearing.

**Figure 4.** Graph of the dependence of the bearing layer \( V \) volume on the load \( F \) for different values of deformation \( H_{i0} \).

### 4. Dynamic Model of Bearing Movement

In the unsteady mode, pressure \( p_k(t) \), gaps \( h(t) \) and \( h_0(t) \), deformation \( h_d(t) \), flow rates \( q_k(t) \) and \( q_h(t) \), and forces \( w_h(t) \), \( w_{hs}(t) \), \( w_p(t) \), and \( w(t) \) become functions of the current time \( t \). Moreover, new non-stationary functions appear, such as flow:

\[
q_v = \frac{v_k}{H_{0}T} \frac{dp_k}{dt},
\]

due to the compressibility of air in the microgroove 7, and the force of inertia:

\[
w_i = m_s \frac{d^2h}{dt^2},\]

of the shaft, 1, mass \( m_s \).

Taking this into account, the model of the bearing dynamics will take the form:

\[
q_k - q_{hs} + q_h - q_v = 0, \tag{22}
\]

\[
h_d = k_e (w_p - w_h), \tag{23}
\]

\[
w = w_h + w_{hs}, \tag{24}
\]

\[
w - w_i = f. \tag{25}
\]

The dynamic processes caused by small perturbations \( \Delta F(\tau) \) relative to the equilibrium state of the external dimensionless static load \( F \), where \( \tau \) is the dimensionless current time, are investigated. Linearization is applied to model (22)–(25), which is based on the assumption that in the investigated dynamic process, the variables change so that their deviations from steady-state values remain small at all times. The integral Laplace transform with respect to the current time was applied to the linearized model, as a result of which transformants of the deviations of the input load function \( \overline{\Delta F}(s) \) and output functions were obtained, where \( s \) is the variable of the Laplace transform [14]. The transformed linearized dimensionless mathematical model of the dynamics of small vibrations of the bearing is described by the equations:

\[
\overline{\Delta Q}_k - \overline{\Delta Q}_h + \overline{\Delta Q}_{hs} - \overline{\Delta Q}_v = 0, \tag{26}
\]
\[ \Delta H_d = K_c (\Delta W_p - \Delta W_{bs}), \]  
(27)  
\[ \Delta H - \Delta H_s - \Delta H_d = 0, \]  
(28)  
\[ \Delta W = \Delta W_b + \Delta W_{bs}, \]  
(29)  
\[ \Delta W - \Delta W_i = \Delta P. \]  
(30)

The function of dimensionless pressure in the bearing layer in the general case satisfies the boundary value problem for the Reynolds equation [2]:

\[
\begin{cases}
\frac{\partial}{\partial \tau} \left( RH^3 \frac{\partial^2 P}{\partial R^2} \right) = 2\sigma R \frac{\partial (P\lambda)}{\partial R}, \\
\frac{\partial P}{\partial \tau}(0, \tau) = 0, P(R_1, \tau) = P_0, P(1, \tau) = 1,
\end{cases}
\]
(31)

where \( \sigma = \frac{12 \mu R^2}{h_0 \rho U_0} \) is the number of squeezing and \( t_0 \) is the scale of the current time [9].

The transformed linearized problem corresponding to Equation (31) has the form:

\[
\begin{cases}
\frac{d}{dR} \left[ R \frac{d(\Delta P)}{dR} \right] = \frac{\rho_s}{\mu} R (H \Delta P + P \Delta H), \\
\Delta P(0, s) = 0, \Delta P(R_1, s) = \Delta P_a(s), \Delta P(1, s) = 0,
\end{cases}
\]
(32)

where \( \Delta P(R, s), \Delta H(s), \Delta P_a(s) \) are the Laplace transformants of the corresponding function deviations and \( H, P(R) \) are static values of the gap and pressure distribution functions.

The boundary value problem (32) for the segment \( R \in [R_a, R_b] \) can be represented as:

\[
\begin{cases}
R \frac{d^2(\Delta P)}{dR^2} + \frac{d(\Delta P)}{dR} = \frac{\rho_s}{\mu} R (H \Delta P + P \Delta H), \\
\alpha_a \Delta P(R_a, s) + (1 - \alpha_a) \frac{d}{dR} (R_a, s) = \beta_a \Delta P_a(s), \\
\alpha_b \Delta P(R_b, s) + (1 - \alpha_b) \frac{d}{dR} (R_b, s) = \beta_b \Delta P_b(s),
\end{cases}
\]
(33)

where \( \Delta P(R, s), \Delta H(s), \Delta P_a(s), \Delta P_b(s) \) are Laplace deviations of the corresponding functions and \( H \) and \( P(R) \) are the static gap and pressure function, respectively.

Using the superposition method, we represent the required function in the form:

\[ \Delta P(R, s) = U_a(R, s) \Delta P_a + U_b(R, s) \Delta P_b + U_t(R, s) \Delta H. \]  
(34)

Substituting Equation (34) into Equation (33) and performing the separation of variables, we obtain problems for determining functions \( U_a, U_b, U_t \), which can be written in the following general form:

\[
\begin{cases}
R \frac{d^2(PLI)}{dR^2} + \frac{d(PLI)}{dR} = \frac{\rho_s}{\mu} R (HLI + \lambda P), \\
\alpha_t U(R_t, s) + (1 - \alpha_t) \frac{d}{dR} (R_t, s) = \beta_t, \\
\alpha_b U(R_b, s) + (1 - \alpha_b) \frac{d}{dR} (R_b, s) = \beta_b.
\end{cases}
\]
(35)

For \( \lambda = 1, \alpha_a = 1, \alpha_b = 0, \beta_a = 0, \) and \( \beta_b = 0, \) we obtain a boundary value problem for the function \( U_t \) for any model, and for \( \lambda = 0 \) we obtain problems for the functions \( U_a \) or \( U_b \).

The algebraic finite difference method [15] was applied to the solution of problem (35). For this, the segment \([R_a, R_b]\) was divided into an even number \( n \) of segments, and a system of algebraic equations is written for the internal nodes of the grid:

\[
R_i \frac{P_{i+1} U_{i+1} - 2 P_i U_i + P_{i-1} U_{i-1}}{g^2} + \frac{P_{i+1} U_{i+1} - P_{i-1} U_{i-1}}{2g} = \frac{\rho_s R_i}{H^3} (HLU_i + \lambda P_i),
\]
(36)

where \( g = (R_b - R_a)/n \) is the grid step, \( U_i \) are the values of the function at the grid nodes, and \( i = 1, 2, \ldots, n-1. \)
System (36) is supplemented with the boundary conditions:

\[
\begin{align*}
\alpha_a U_0 + \frac{1 - \alpha_a}{2g} (-3U_0 + 4U_1 - U_2) &= (1 - \lambda)\beta_a, \\
\alpha_0 U_n + \frac{1 - \alpha_0}{2g} (3U_n - 4U_{n-1} + U_{n-2}) &= (1 - \lambda)\beta_b.
\end{align*}
\]  

(37)

When deriving Equation (37), derivatives of the second order of accuracy \(O(g^2)\) at the end of segment [16] were used.

Systems (36) and (37) were solved by the sweep method [17]. For this, the recurrent formula is applied:

\[U_{i-1} = x_i U_i + y_i.\]  (38)

The first two equations from Equations (36) and (37) have the form:

\[
\begin{align*}
a_1 U_0 + a_2 U_1 - a_3 U_2 &= a_0, \\
b_1 U_0 - b_2 U_1 + b_3 U_2 &= b_0,
\end{align*}
\]  

(39)

where:

\[
a_1 = \alpha_a = \frac{3(1 - \alpha_a)}{2g}, a_2 = \frac{2(1 - \alpha_a)}{g}, a_3 = \frac{1 - \alpha_a}{2g}, a_0 = (1 - \lambda)\beta_a,
\]

\[
b_1 = \frac{P_0}{g} \left( \frac{R_1}{g} - \frac{1}{2} \right), b_2 = R_1 \left( \frac{2P_1}{g^2} + \frac{\alpha s}{H^2} \right), b_3 = \frac{P_2}{g} \left( \frac{R_1}{g} + \frac{1}{2} \right), b_0 = \frac{R_1 P_3 \lambda \alpha s}{H^3}.
\]

Using Equations (38) and (39), we found the initial sweep coefficients:

\[
x_1 = -\frac{c_2}{c_1}, y_1 = \frac{c_0}{c_1}, \]  (40)

where \(c_1 = b_1 + \frac{b_2 a_1}{a_3}, c_2 = \frac{b_2 a_2}{a_3} - b_2, c_0 = b_0 - \frac{b_2 a_0}{a_3}\).

Equation (36) is presented in the form:

\[a_i U_{i+1} - b_i U_i + c_i U_{i-1} = d_i,\]  (41)

where:

\[
a_i = \frac{P_i + 1}{g} \left( \frac{R_i}{g} + \frac{1}{2} \right), b_i = R_i \left( \frac{2P_i}{g^2} + \frac{\alpha s}{H^2} \right), c_i = \frac{P_{i-1}}{g} \left( \frac{R_i}{g} - \frac{1}{2} \right), d_i = \frac{\lambda \alpha s R_i \lambda s}{H^3}.
\]

Substituting Equation (41) into Equation (38), we found recursive formulas for the sweep coefficients:

\[
x_{i+1} = -\frac{a_i}{b_i - c_i x_i}, y_{i+1} = \frac{c_i y_i - d_i}{b_i - c_i x_i}. \]  (42)

The value of \(U_n\) required for the backward sweep is obtained from the second equation in Equation (37) and the last equation in Equation (36):

\[U_n = \frac{e_6 - e_5 y_n}{e_4 + e_5 x_n},\]  (43)

where:

\[
e_3 = \alpha_b + \frac{3(1 - \alpha_b)}{2g}, e_2 = \frac{2(1 - \alpha_b)}{g}, e_1 = \frac{1 - \alpha_b}{2g}, e_0 = (1 - \lambda)\beta_b,
\]

\[
e_4 = a_{n-1} - \frac{c_{n-1} e_3}{e_1}, e_5 = \frac{c_{n-1} e_2}{e_1} - b_{n-1}, e_6 = d_i - \frac{c_{n-1} e_0}{e_1}.
\]

The load capacity factors are expressed by the integral:

\[A_w = 2 \int_{R_1}^{R_2} RU dR.\]  (44)
In the general case, the formula for the flow rate transformant in the bearing layer has the form:

$$\Delta Q_h = -3H^2R\frac{d^2}{dR^2}H - 2H^3\frac{d(P\Delta P)}{dR} = A_{qh}\Delta H + A_{qa}\Delta P_a + A_{qb}\Delta P_b,$$

where:

$$A_{qh} = A_{q0} - 2H^3 R\frac{d(P\Delta P)}{dR}, \quad A_{qa} = -2H^3 R\frac{d(P\Delta P)}{dR}, \quad A_{qb} = -2H^3 R\frac{d(P\Delta P)}{dR}, \quad A_{q0} = \frac{3\lambda H^2 (P_2 - \beta^2)}{ln(R_a/R_b)}.$$

At the edges of the segment $[R_a, R_b]$:

$$B_a = -2H^3 R\frac{d(P\Delta P)}{dR} R = R_a = \frac{R_a H^3}{8} (P_2 U_2 - 4P_1 U_1 + 3P_3 U_0), \quad A_{q0} = (A_{q0} + B_a),$$

$$B_b = -2H^3 R\frac{d(P\Delta P)}{dR} R = R_b = \frac{R_b H^3}{8} (3P_4 U_n - 4P_{n-1} U_{n-1} + P_{n-2} U_{n-2}), \quad A_{q0} = (A_{q0} + B_b).$$

The transformant of the force reaction of the bearing layer thickness $H_b$ is determined by the Simpson formula [13]:

$$\Delta W_{hs} = 2 \int_0^{R_1} R\Delta P dR = A_{whs} \Delta H_s + A_{wks} \Delta P_k,$$

(45)

where $A_{whs} = 2 \int_0^{R_1} RU_d dR, A_{wks} = 2 \int_0^{R_1} RU_k dR$.

The transformed function of the flow rate in the bearing layer is:

$$\Delta Q_{hs} = A_{qhs} \Delta H_s + A_{qks} \Delta P_k.$$

(46)

The force reaction of the bearing layer in the gap of thickness $H$ is:

$$\Delta W_h = 2 \int_{R_1}^{R} R\Delta P dR = A_{wh} \Delta H + A_{wk} \Delta P_k,$$

(47)

where $A_{wh} = 2 \int_{R_1}^{R} RU_h dR, A_{wk} = 2 \int_{R_1}^{R} RU_k dR$.

The flow rate transformant at the inlet to the gap of thickness $H$ is:

$$\Delta Q_h = A_{qh} \Delta H + A_{qk} \Delta P_k.$$

(48)

The coefficients of Equations (44)–(48) were found as a result of a fourfold solution of problem (35) with the values of the parameters $\lambda, \alpha_a, \beta_a, \alpha_b, \beta_b$ given in Table 1.

| Gap | $\lambda$ | $\alpha_a$ | $\beta_a$ | $\alpha_b$ | $\beta_b$ | Coefficients |
|-----|-----------|-------------|-----------|-------------|-----------|--------------|
| $H_s$ | 1 | 1 | 0 | 1 | 0 | $A_{qhs}, A_{qks}$ |
|     | 0 | 0 | 0 | 1 | 1 | $A_{whs}, A_{wks}$ |
| $H$  | 1 | 1 | 0 | 1 | 0 | $A_{qhs}, A_{qks}$ |
|     | 0 | 1 | 1 | 1 | 0 | $A_{whs}, A_{wks}$ |

The transformation formula for the flow rate due to the air compressibility in the microgroove, 7, has the form:

$$\Delta Q_v = A_v s \Delta P_k,$$

(49)
where $A_v = \sigma V_k$, $V_k = \frac{V_{kh}}{\sigma_{kh}}$ is the dimensionless volume of the microgroove.

The flow function through the diaphragm, $\Delta Q_k$, is determined by the formula:

$$\Delta Q_k = A_k \Delta P_k.$$  

(50)

The formula for the dimensionless transformant of the inertia force has the form:

$$\Delta W_i = M_s^2 \Delta H,$$  

(51)

where $M_s = m_{sh} \pi r^2 t_{hp}$ is the dimensionless mass of the shaft.

Substituting Equations (44)–(51) into Equations (26)–(41), we obtained a system of linear equations with input $\Delta F$ and output $\Delta H$, $\Delta H_s$, $\Delta H_d$, $\Delta P_k$ functions:

$$A \begin{bmatrix} \Delta V \\ \Delta H \\ \Delta H_s \\ \Delta H_d \\ \Delta P_k \end{bmatrix} = E \begin{bmatrix} \Delta F \\ 0 \\ 0 \end{bmatrix},$$  

(52)

where $A$ is a complex matrix:

$$[V] = \begin{bmatrix} \Delta H \\ \Delta H_s \\ \Delta H_d \\ \Delta P_k \end{bmatrix}, [E] = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$  

The dynamic compliance of the bearing is determined by the transfer function:

$$K(s) = \frac{\Delta H}{\Delta F} = -|A_H| |A|,$$  

(53)

where $|A|$ is the determinant of the matrix $A$ of system (52) and $A_H$ is the matrix formed from matrix $A$ by replacing the first column with $[E]$ in accordance with Cramer’s rule.

5. Characteristic Polynomial and Criteria for the Dynamic Quality of the Bearing

The bearing dynamics model is a non-linear system with distributed parameters. After linearization, it becomes linear, but it still remains a system with distributed parameters, since transfer function (53) can be obtained by numerical methods based on the solution of the above-mentioned differential equations.

To assess the dynamic quality of the bearing, transfer function (53) was approximately represented in the form:

$$K(s) = \frac{b_0 + b_1 s + b_2 s^2 + \ldots + b_{n-1} s^{n-1} + b_n}{1 + a_1 s + a_2 s^2 + \ldots + a_m s^m},$$  

(54)

where $m < n$, $m > 0$, and $n > 0$, $n$ and $m$ are natural numbers, and $a_i$ and $b_i$ are real numbers.

The number $m$ is constant for the transfer function $K(s)$ and is determined by its smallest natural value $m$, for which:

$$\lim_{s \to \infty} s^m K(s) \rightarrow \frac{b_{n-m}}{a_n} \neq 0.$$  

Numerical experiments have shown that for a given transfer function, $m = 2$. This corresponds to the difference in the orders of the polynomials of the transfer functions of the aerostatic and hydrostatic bearings, the models of which take into account the effect of the shaft mass inertia on their dynamics [11,18]. The unknown transfer function coefficients were found using the iterative method described in [19].

To assess the dynamic quality of linear systems, the following root criteria were used: the degree of stability $\eta$ and damping of oscillations over the period $\xi$. The degree of stability $\eta$ characterizes the speed of the system, while the criterion $\xi$ is suitable for assessing the stability margin of the system [18–20].
6. Dynamic Characteristics of the Bearing

The calculation of the criteria for the dynamic quality of the bearing was carried out at a unit dimensionless mass $M_s = 1$, varying the parameters $P_s$, $R_1$, $\chi$, $H_{d0}$, $\sigma$, and $V_k$. The graphs in Figures 5–7 show curves for $P_s = 4$, $R_1 = 0.75$ and $\chi = 0.45$.

![Figure 5](image1)

**Figure 5.** Graph of the dependence of the stability degree $\eta$ on the volume $V_k$ for different values of the parameter $H_{d0}$ at $\sigma = 25$.

![Figure 6](image2)

**Figure 6.** Graph of the dependence of the attenuation for the period $\xi$ on the volume $V_k$ for different values of the parameter $H_{d0}$ at $\sigma = 25$.

Among the parameters that do not affect the static characteristics but affect the dynamic characteristics of the bearing are the deformation parameter $H_{d0}$, the compression number $\sigma$, and the volume of the microgroove $V_k$. The influence of these parameters is of particular interest, since they are a resource for optimizing the dynamic characteristics of the bearing.
Figures 5 and 6 show the dependences of the stability degree $\eta$ and damping for the period $\xi$ on the volume $V_k$ at a fixed $\sigma = 25$ and different values of deformation $H_{d0}$ for the regime of the design point.

The graphs show that an ordinary support with a rigid center 5 ($H_{d0} = 0$) is stable only for a small $V_k$. However, with an increase in $H_{d0}$, when the rigid center, 5, acquires the ability to perform additional displacement, the support becomes stable even at the volume of the microgroove, which is larger by one order of magnitude or more than the volume of the bearing layer. Moreover, even a small displacement has a significant effect on the stability of the support: for example, an unstable conventional support already becomes stable at $H_{d0} = 0.1$. A further increase in $H_{d0}$ contributes to an increase in the response rate, which, depending on the values of other parameters, can fluctuate within a significant range of $H_{d0} = 0.25$–0.5. For the graph in Figure 5, the maximum response rate falls on $H_{d0} = 0.32$. A further increase in $H_{d0}$ leads to a decrease in the response speed, while the support remains stable.

The graph in Figure 6 shows the dependences of the criterion $\xi$ on the transient characteristic’s oscillation. This makes it possible to assess the stability margin of the bearing. It can be seen that at $H_{d0} > 0.1$ near the extremum points, there is a rapid increase in the values of the criterion $\xi$, which indicates a decrease in the oscillation of the system and, consequently, an increase in its damping. Already at $H_{d0} > 0.2$ for $V_k > V_{k,\text{opt}}$, where $V_{k,\text{opt}}$ is the volume corresponding to the maximum value of the stability degree $\eta$, the criterion $\xi > 90\%$, which characterizes the bearing as a well–damped dynamic system [14,20,21].

Figure 7 shows the dependence of the stability degree $\eta$ on the parameter $\sigma$, which also only affects the dynamic properties of the bearing. It can be seen that at the design point, the bearing can be both stable ($\eta > 0$) and unstable ($\eta < 0$). An increase in the volume $V_k$ of the microgroove promotes an expansion of the stability area and an increase in the performance of the bearing. The dependences $\eta (\sigma)$ are also extreme. This means that each value of the volume $V_k$ corresponds to the optimal value of the number $\sigma$ from the point of view of speed. With an increase in $V_k$, the speed peak shifts towards a lower $\sigma$, which corresponds to large values of the dimensional gap $h_0$. In this case, the peaks themselves describe an extreme curve, which indicates that there is also an optimal value $V_{k}$ in terms of speed. Together with the previously made conclusions, this indicates that for each set of values of the parameters $P_s$, $R_1$, and $\chi$, there is an optimal set of values for the parameters $H_{d0}$, $\sigma$, and $V_k$, which only affect the dynamics of the bearing. However, optimization of the design point mode does not guarantee that the bearing will have optimal dynamic
characteristics over the entire range of load variation $F$. In this range, with large volumes of microgrooves, such a bearing may even be unstable. As the analysis of the calculated data has shown, the instability region can occupy up to half of this range, which falls on small and moderate loads.

From the graph in Figure 8, which shows the curves of the dependence of the stability degree $\eta$ on the load $F$, it can be seen that at large volumes $V_k$ and small deformations $H_{d0}$ in the region of small and medium loads, the bearing is unstable. Stability in this range is provided only for $H_{d0} > 0.4$.

Analysis of the graph in Figure 8 shows that the best in terms of performance is the curve corresponding to $H_{d0} = 0.5$. For this mode, stability takes place in the entire range of loads at $\sigma = 25, V_k = 16$.

7. An Example of a Bearing Design Calculation

To conclude the study, we give an example of calculating the dimensional characteristics of the bearing, corresponding to $\sigma = 25, V_k = 16$.

Let us take the bearing radius $r_0 = 40.10^{-3}$ m and the ambient pressure $p_a = 0.1 \cdot 10^6$ Pa. The maximum dimensionless bearing load $F_{\text{max}} = 2.42$, and the maximum load $f_{\text{max}} = \pi r_0^2 p_a F_{\text{max}} = 1.2 \cdot 10^3$ N. The dimensionless design load at $\chi = 0.45$ is $F = 1.42$, and the dimensionless design load is $f = \pi r_0^2 p_a F = 0.71 \cdot 10^3$ N.

Using the expression for the dimensionless mass of the shaft at $M_s = 1$, we find the formula for calculating the scale of the real time:

$$t_0 = \frac{1}{r_0} \sqrt{\frac{m_s h_0}{\pi p_a}}. \tag{55}$$

Substituting Equation (55) into the expression for the compression number $\sigma$ of problem (32), we obtain the formula for calculating the gap at the design point:

$$h_0 = r_0 \frac{\pi v_0}{m_s p_a} \left( \frac{12 \mu}{\sigma} \right)^{\frac{1}{2}}. \tag{56}$$

Taking the mass of the shaft $m_s = 5$ kg and the viscosity of the air $\mu = 17.2 \cdot 10^{-6}$ Pa.s, using Equation (56), we find the gap $h_0 = 18 \cdot 10^{-6}$ m. Using Equation (55), we find the
scale of the real time $t_0 = 0.42 \cdot 10^{-3}$ s. The well-known formula [14] for determining the duration $t_p$ of the decay of the transient response at $\eta = 0.07$ gives $t_{pp} = \frac{3h}{\Pi} = 0.02$ s.

At temperature $T = 293$ K, a gas constant $R = 287.14$ m$^2$/s$^2$ Kz and an adiabatic air expansion index $\gamma = 1.4$, we calculate $\Gamma = 2\sqrt{\gamma} \left( \frac{2}{\gamma} \right) \frac{\chi^{\frac{1}{1-\gamma}}}{\chi^{\frac{1}{1-\gamma}}} = 1.7$. Dimensionless pressure in the microgroove $P_k = \sqrt{1 + \chi (P_s^2 - 1)} = 2.78$. From the condition of equality of the flow rates $Q_k$ and $Q_\varphi$, we find the criterion for the similarity of the diaphragm $A_k = \frac{A_k (P_s^2 - 1)}{12 \Pi \varphi_k} = 12.8$. Let us take the number of diaphragms $n_k = 3$. Using Equation (15), we calculate the diameter of simple diaphragms $d = h_k \sqrt{\frac{A_k \rho h_0}{3 \mu \Pi \gamma \Phi}} = 0.4 \cdot 10^{-3}$ m.

Using Formula (14), we determine the dimensionless design lubricant flow rate $Q = 23$. The dimensional mass flow rate $q = \frac{\pi h_0^2 P_s^2}{12 \mu \Pi \gamma} Q = 2.4 \cdot 10^{-3}$ kg/s.

The structural volume at design point $q = \frac{\pi h_0^2 P_s^2}{12 \mu \Pi \gamma} Q = 2.4 \cdot 10^{-3}$ kg/s.

The cross-sectional area of the microgroove is $s_k = \frac{v_k}{2 \pi \gamma \Phi k} = -2.44 \cdot 10^{-6}$ m$^2$, while its depth is $l_k = \sqrt{s_k} = 1.2 \cdot 10^{-3}$ m. For dimensionless $K_e = 0.731$, we obtain the dimensional coefficient of elasticity $k_e = \frac{k_e h_0}{\pi \varphi_k P_s} = 4.67 \cdot 10^{-6}$ m.

8. Conclusions

This paper proposes an improved technical solution for an axial aerostatic bearing with an elastic suspension of the supporting disk, simple diaphragms, and an annular microgroove. The results of mathematical modeling and theoretical research of stationary and non-stationary operation modes of support are presented, and the possibility of improving its dynamic characteristics by reducing the volume of the central part of the bearing layer is shown. It has been established that the optimal choice of the compression number, the volume of the microgroove, and the coefficient of elasticity of the suspension, which ensures the displacement of the support center, provides the bearing with high performance and a high margin of stability. It is shown that the use of a moving center makes it possible to reduce the volume of the bearing layer by a factor of two or more and, due to this, to increase the speed of the bearing by 3–4 times. With optimal adjustment of the bearing parameters, the oscillation of the transient processes decreases until their transition to a qualitatively new state—they become almost aperiodic, which indicates that the bearing acquires the properties of a well-damped dynamic system. An example of the design calculation of the structure is given.

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**References**

1. Schenk, C.; Buschmann, S.; Risse, S.; Eberhardt, R.; Tünnermann, A. Comparison between flat aerostatic gas-bearing pads with orifice and porous feedings at high-vacuum conditions. *Precis. Eng.* 2008, 32, 319–328. [CrossRef]
2. Constantinescu, V.N. *Gas Lubrication*; The American Society of Mechanical Engineers: New York, NY, USA, 1969; p. 621.
3. Bhat, N.; Kumar, S.; Tan, W.; Narasimhan, R.; Low, T.C. Performance of inherently compensated flat pad aerostatic bearings subject to dynamic perturbation forces. *Precis. Eng.* 2012, 36, 399–407. [CrossRef]
4. Mizumoto, H.; Arii, S.; Kami, Y.; Goto, K.; Yamamoto, T.; Kawamoto, M. Active inherent restrictor for air-bearing spindles. *Precis. Eng.* **1996**, *19*, 141–147. [CrossRef]

5. Araki, R.; Takita, A.; Nachaisit, P.; Shu, D.-W.; Fujii, Y. Frictional Characteristics of a Small Aerostatic Linear Bearing. *Lubricants* **2015**, *3*, 132–141. [CrossRef]

6. Maamari, N.; Krebs, A.; Weikert, S.; Wegener, K. Centrally fed orifice based active aerostatic bearing with quasi-infinite static stiffness and high servo compliance. *Tribol. Int.* **2019**, *129*, 297–313. [CrossRef]

7. Al-Bender, F. *Air Bearings: Theory, Design and Applications*; John Wiley & Sons: Hoboken, NJ, USA, 2021; p. 592.

8. Rowe, W.B. *Hydrostatic, Aerostatic and Hybrid Bearing Design*; Elsevier: Amsterdam, The Netherlands, 2012.

9. Pinegin, S.; Tabachnikov, Y.; Sipenkov, I. *Static and Dynamic Characteristics of Gas-Static Supports*; Nauka: Moscow, Russia, 1982; p. 265.

10. Zhang, J.; Zou, D.; Ta, N.; Rao, Z. Numerical research of pressure depression in aerostatic thrust bearing with inherent orifice. *Tribol. Int.* **2018**, *123*, 385–396. [CrossRef]

11. Kodnyanko, V.A.; Shatokhin, S.N. Theoretical study on dynamics quality of aerostatic thrust bearing with external combined throttling. *FME Trans.* **2020**, *48*, 342–350. [CrossRef]

12. Dasgupta, S.; Papadimitru, H.; Vazirani, U. *Algorithms*; Publishing Group URSS: Moscow, Russia, 2019; p. 320.

13. Dwight, H. *Tables of Integrals and Other Mathematical Data*; The Macmillan Company: New York, NY, USA, 1961.

14. Besekersky, V.; Popov, E. *Theory of Automatic Control Systems*; Profession: Saint Petersburg, Russia, 2003; p. 752.

15. Yudin, D. *Computational Methods of Decision Theory*; Lan’: Saint Petersburg, Russia, 2014; p. 320.

16. Maltsev, I. *Fundamentals of Linear Algebra*, 2nd ed.; Lan’: Saint Petersburg, Russia, 2016; p. 480.

17. Demidovich, B.; Maron, I.; Shuvalova, E. *Numerical Methods of Analysis. Approximation of Functions, Differential and Integral Equations*, 5th ed.; Lan’: Moscow, Russia, 2010; p. 400.

18. Kodnyanko, V.; Kurzakov, A. Quality of Dynamics of Gas-static Thrust Bearing with Movable Carrying Circle on Elastic Suspension. *Tribol. Ind.* **2019**, *41*, 237–241. [CrossRef]

19. Kodnyanko, V.; Shatokhin, S.; Kurzakov, A.; Pikalov, Y. Mathematical Modeling on Statics and Dynamics of Aerostatic Thrust Bearing with External Combined Throttling and Elastic Orifice Fluid Flow Regulation. *Lubricants* **2020**, *8*, 57. [CrossRef]

20. Shatokhin, S.; Kodnyanko, V. Load and flow rate characteristics of an axial pressurized gas bearing with an active compensation of gas flow. *Soviet Mach. Sci.* **2020**, *6*, 110–115.

21. Kodnyanko, V.; Shatokhin, S.; Kurzakov, A.; Pikalov, Y. Theoretical analysis of compliance and dynamics quality of a lightly loaded aerostatic journal bearing with elastic orifices. *Precis. Eng.* **2021**, *68*, 72–81. [CrossRef]