Skyrmions in magnetic materials

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ABSTRACT
Skyrmions are vortex-like textures of magnetic moments found in some magnetically ordered materials. Here we describe the origin of the skyrmion and discuss the experimental realization of skyrmions in magnetic materials, the dynamic properties of the skyrmion and the potentially useful interaction of skyrmions with electrons. Our discussion is based on the physics of fields and allows us to touch on notions from field theory and topology that illustrate how skyrmions can be understood as part of a family of defects in an ordered field, and hint at other topological objects that await our discovery.

KEYWORDS
skyrmion; topological physics; magnetic materials.

1. Introduction

If you throw a stone into a still pond then the wave you set up on the surface of the water dies away, both at a distance far from the initial disturbance and after a relatively short interval of time. However, some waves in Nature are not like this. They are, instead, localised in space and trapped in existence for long periods, their removal costing a great deal of energy. There are many examples of these long-lived, wave-like lumps and this article concerns a family of them known as skyrmions.

Waves and lumps are excitations in fields. A field φ(x) is a machine that inputs a point x in spacetime and outputs the amplitude φ of the field at that point. From the point of view of the theory of fields, every particle in the Universe is an excitation in a quantum mechanical field. Each electron is an excitation in an electron field, each quark an excitation in a quark field, and so on. In the 1970s many high-energy physicists came to regard the lumps that occur in particle fields as sharing a similar status to the fundamental particles. At the same time, condensed matter physicists started treating lumps as an inevitable feature of ordered states of matter, representing a defect or imperfection in an ordered state. Skyrmions are one sort of lump that can occur in a field whose amplitude resembles a directed arrow at each point in space. At the outset we will characterise lumps as extended shapes in fields, held together by their internal interactions. An example of a skyrmion is shown in Figure 1, where the cones show the direction of the field amplitude at each point. Far from the skyrmion’s centre the arrows all point in the same direction (upwards in the figure). The skyrmion itself
Figure 1. Top: an example of a skyrmion. Bottom: the phase diagram of MnSi, with a small region of applied field and temperature, known as the $A$-phase, where a skyrmion lattice is observed.

resembles a vortex-like shape, with the field at the centre pointing downwards.

A key feature of the fields that we use to construct lumps like the skyrmion is that they must vary smoothly in space. Changing a field as a function of spatial coordinates results in an energy cost. However, the fields are also subject to potential-energy costs owing to their very presence in space and time and also to their interactions with themselves \[2\]. In magnets, these interactions can be traced back to the properties of atoms in solids and can be understood using arguments based on symmetry \[3\]. As usual in physics, the accountancy of minimising the total energy is the arbiter of whether a configuration such as a skyrmion can be realised or not. In fact, the crucial details of the energetics in this context can be described in terms of the overall shape of the field in space, not just on the precise details of the field configuration. The general study of such shapes in mathematics is known as topology, and so lumps like the skyrmion have been given the more respectable name of topological objects, topological defects or topological solitons \[4\]. The topologist doesn’t care about the exact distances and angles between points on the surface of an object (the study of these is known as geometry). In fact, the topologist imagines all objects as made from a mouldable clay that can be stretched and shrunk, subject to the rules that holes cannot be punched in the clay, and any existing holes cannot be healed up. Shapes that can be transformed into each other following the rules of this game are said to be homeomorphic to each other. There is a homeomorphism between a coffee cup and a ring donut (that is, a torus) because in preserving the loop that forms the cup’s handle, the cup can be moulded into a donut shape. Topologically then, the coffee cup and donut are identical \[2\].

In order to discover the skyrmion and the topological objects to which it is related, we will look at several systems distinguished by their dimensionality $d$. Specifically, we will discuss the one-dimensional ($d = 1$) line, the two-dimensional ($d = 2$) plane and the three-dimensional ($d = 3$) space of everyday existence. Low dimensionality (that is $d = 2$ and $d = 1$ behaviour) can be achieved experimentally in various materials in which the structure of the lattice confines the dominant interactions into one or two dimensions. If, for example, atoms interact principally along a line with much weaker interactions in the other two dimensions then, within a certain realm of applicability, the solid is effectively one-dimensional.

This might all seem rather abstract, but it is the ability to realise skyrmions experimentally that has led to a great explosion of interest in them in the last decade \[5–8\]. Although skyrmions turned out not to be useful models of elementary particles, they have since the late 2000s been observed, produced and studied experimentally via their occurrence as excitations in the microscopic fields of magnetic materials. Their
topological properties turn out to endow them with an unusual set of electromagnetic interactions. It has been argued that these properties are potentially useful, and the control of skyrmions could be exploited in future electronic devices. We will examine the possibility of control of skyrmions at the end of the paper. For now, we turn to a description of what a skyrmion is, and how they come to exist. We begin with a little history.

2. A brief history of the magnetic skyrmion

In 1961, the nuclear physicist Tony Skyrme proposed that topological objects could serve as a model of the nucleons that constitute nuclear matter [4,9]. Skyrme discussed these excitations using the (jokingly named) sine-Gordon model from field theory: a model that had originally been formulated in the completely different context of geometry, where it is related to the description of a space with negative curvature [1]. In Skyrme’s model, nucleons are taken to be topological lump-like excitations in a pion field (i.e. a field whose particle-like excitations are pions). With the emergence of the quark model in 1964, from independent work by Murray Gell-Mann and George Zweig, it was realised that neither nucleons nor pions were elementary particles, with both having quark constituents. Although skyrmions turned out not to model elementary particles they are still being studied today as an approximate description of atomic nuclei, with the original Skyrme model being developed to improve the match to low-energy properties of nuclei.

However, as an excitation in a field, the skyrmion can also be realised in a number of different contexts. The methods of field theory that had been important in the development of high energy physics were also applied across solid state (and later condensed matter) physics, following the work of Lev Landau and collaborators in the 1940s-60s [10]. Since then, skyrmions have regularly been suggested as possible excitations of a number of condensed matter systems such as $^3$He films, quantum Hall fluids, superconductors and liquid crystals [6].

In magnetic systems several interactions that can stabilise skyrmions have been identified and lead to the prediction of skyrmions with sizes ranging across a number of different length scales. These include (i) the dipolar interaction in thin magnetic films, which leads to magnetic bubbles: textures of characteristic size 100 nm–1 $\mu$m which can be related to skyrmions. There are also (ii) frustrated and (iii) multiple-spin exchange interactions which can result in skyrmions whose size is comparable to a lattice spacing. The main subject of this article is skyrmions’ proposed existence and subsequent observation in magnetic materials with chiral crystal symmetry, where skyrmions have a characteristic size of 5 – 100 nm. As we shall see, in magnets, skyrmions are related to helically twisted structures of magnetic moments. These helical arrangements can be understood using a model of the energetics of the magnet (the Bak-Jensen model, formulated in 1980 [11]) that invokes the Dzyaloshinskii-Moriya interaction, which is a hybrid effect that combines magnetic exchange and spin orbit interactions, to explain the origin of the twisting magnetic structure [3,12]. The key link to skyrmions was made by A.N. Bogdanov and collaborators in the late 1980s, who predicted the formation of a regularly repeating lattice of skyrmions in noncentrosymmetric (or chiral) magnetic materials [13] resembling the vortex lattice in type II superconductors.

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1 A noncentrosymmetric crystal is one where points at coordinates ($x$, $y$, $z$) are not equivalent to points at coordinates ($-x$, $-y$, $-z$). This is the case for a crystal that is chiral, by which we mean the arrangement of atoms has a particular handedness.
Figure 2. (a) A disordered magnetic state. This configuration has magnetisation \( M = 0 \) and so does (b), which is produced by turning all of the spins through 180°. (c) An ordered magnetic phase with \( M = M_0 \). Turning the spins through 180° results in (d) which has \( M = -M_0 \).

The experimental observation of magnetic skyrmions was relatively recent. In the noncentrosymmetric magnet MnSi there is a small region of applied magnetic field and temperature that corresponds to a region of the phase diagram known as the anomalous phase or \( A \)-phase. In 2009, small angle neutron scattering (SANS) measurements allowed the identification of this phase as hosting a lattice formed from skyrmions \[14\]. This was closely followed by observations of a skyrmion lattice in (Fe,Co)Si using microscopy techniques \[15\]. Since then, magnetic skyrmions have been observed in a number of magnetic materials (examples include FeGe, Cu$_2$OSeO$_3$ and GaV$_4$S$_8$) \[16\] and contexts (thin magnetic layers, near interfaces in heterostructured magnets, in patterned nanostructures) \[17\]. There have also been observations of more complicated magnetic structures related to skyrmions and recent results suggesting skyrmion phases might exist in larger classes of material where they are stabilised by frustrated spin interactions \[18\].

3. Topological defects in ordered magnets

In this section we examine the physics of skyrmions, with the goal of showing what a skyrmion is and how it arises. We will do this by building a small family of topological objects that can exist as so-called defects in ordered magnets \[17\].

A simple cartoon picture of a disordered magnet is shown in Figure 2(a). The arrows represent spins (or magnetic moments) on atoms, arranged on a lattice. The system can be understood via its symmetry: the magnetisation \( M \) (that is, the average magnetic moment) is zero, since as many arrows point up as down. If we turn each of the moments through 180° then we obtain the situation shown in Figure 2(b), which also has \( M = 0 \). We therefore cannot tell from the magnetisation that we have transformed the system. This inability to tell that a change has been made is known as a symmetry. In contrast to magnetic disorder, an ordered magnet is shown in Figure 2(c) and (d).
This has magnetisation $M_0 \neq 0$ and has lost its previous symmetry in that we can now tell from the magnetisation if a rotation is made to the arrows. We can see this by again turning the arrows through 180°, which reverses the magnetisation $M_0 \rightarrow -M_0$. We say that the symmetry has been broken on magnetic ordering. In Nature, this sort of ordering is observed to take place via a magnetic phase transition at a temperature $T_c$.

A simple mathematical description of a magnetically ordered state is provided by the Landau model \cite{2101718}, which describes the magnet via a free energy

$$F(T, M) = F_0 + a_0 (T - T_c) M^2 + \lambda M^4,$$  \hspace{1cm} (1)\hspace{1cm}

where $T$ is temperature and $a_0$ and $\lambda$ are positive constants. The inclusion of only even powers of $M$ in this function ensures that the function $F$ is symmetrical with respect to the reversal of the magnetisation. This free energy is plotted for $T > T_c$ and $T < T_c$ in Figure 3 where we see that the number and position of its minima change significantly with $T$, depending on whether we are above or below the critical temperature $T_c$. This is so important because the equilibrium state of the system is found by minimising this free energy function. From Figure 3(a) we can see that it is minimised by $M = 0$ for $T > T_c$, which corresponds to a magnetically disordered state. For $T < T_c$ the free energy has two minima, found at $M = \pm M_0$, where $M_0 \neq 0$, corresponding to the system aligning spins up ($M = +M_0$) or down ($M = -M_0$).

It is worth noting for later that the term $\lambda M^4$ is essential here for forming the two minima for $T < T_c$ and, as a result, for stabilising magnetic order. For this reason it is sometimes called the stability term.

This picture of an ordered system was originally formulated by Lev Landau \cite{10} and its use in modern Condensed Matter Physics was further popularised in the 1970s.
and 80s by Philip Anderson [19], who teaches us to look for a number of key features when a symmetry is broken. The important one for our purposes is the notion of a defect. In the ordered state described above, the average magnetisation field $M(x)$ is a constant, independent of position $x$. (This assumes we average the magnetisation over blocks of the material such that the graininess of individual atoms is unresolvable.) Constant $M(x)$ corresponds to the whole system sitting in one of the potential minima in Figure 3(b). We can, however, ask an interesting question: what if half of the magnet (the left half, say) fell into the minimum on the left (with $M = -M_0$) while the right half fell into the minimum on the right (with $M = M_0$)? We would then have the state of affairs shown in Figure 3(c), where the field changes its value as a function of position $x$. The magnetisation field must vary smoothly and so it passes from spin down ($-M_0$) on the left to spin up ($M_0$) on the right, making the characteristic shape shown, which is known as a kink or domain wall. The kink is very stable since to remove it would cost us a semi-infinite amount of energy. This is because it would require our lifting each moment (on the left, say) over the hump in the potential shown in Figure 3(b), in order to make the field uniform. The kink does, however, lie a finite energy above the ground state, which is a uniform field configuration. This is because it costs energy to vary the field as a function of spatial coordinate (a phenomenon known as rigidity, which is another property that Anderson tells us to expect at all symmetry-breaking phase transitions). The result of the energetic considerations is a kink of finite size, as shown in Figure 3(c).

The energetics of a defect can be evaluated in an upgrade of our free energy that will be useful to us later. We use a magnetisation field $\phi(x)$, which is a measure of the magnetisation evaluated at a point. (It is conventional here to switch notation to describe the spatially variation in terms of a field $\phi(x)$ rather than in terms of the magnetisation $M$ we had before.) The free energy is $F = \int d^d x f(\phi)$, where $d$ is the dimensionality of the space and the free energy density $f(\phi)$ is given by

$$f(\phi) = c(\nabla \phi) \cdot (\nabla \phi) + a_0(T - T_c)(\phi \cdot \phi) + \lambda (\phi \cdot \phi)^2,$$

where $c$ is another positive constant. Notice how the first term tells us that it costs energy for the field to vary in space. The other terms resemble the ones we considered before in equation (1).

The example discussed above of the domain wall is essentially a one-dimensional one (with the field $\phi(x)$ varying in its size as we follow it along $x$ from left to right). In different numbers of dimensions we can produce a family of excitations all related to the kink. Here it will be important to distinguish between the dimensionality $d$ of the space (or atomic lattice in a solid) and the dimensionality $D$ of the field $\phi$. The kink exists for $d = 1$ and $D = 1$, which is to say we need only consider a 1$d$ line of spins where each spin is 1$D$ in that it can point either up or down.

Next consider the case of $d = 2$ and $D = 2$, that is, two-dimensional space and a field that can point in any direction in a 2$D$ plane. The analogue of the double-well potential in Figure 3(b) resembles the bottom of a punted wine bottle or a Mexican hat, as shown in Figure 4(a). We again ask what sort of excitation can live in this potential. Like the kink, the answer involves the field breaking symmetry by sitting in a different minimum of the potential in different regions of space, subject to the

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2 There is a slight disconnect here between the discrete world of spins and the continuous one of fields. Generally we imagine spins as each having a fixed magnitude or length, which rotate or flip, but don’t change their length in space. Recall, however, that we average over many spins to get the field $\phi(x)$ and so $|\phi(x)|$ can take any value, reflecting the number of up and down spins in the volume over which we average.
Figure 4. (a) The potential energy for $d = D = 2$. (b) The vortex excitation. (c) The hedgehog or monopole excitation for $d = D = 3$. (Adapted from Ref [2].)

all-important constraint that the field vary smoothly from place to place. The defect in this case is known as a vortex, an example of which is shown in Figure 4(b). The vortex has a core at its centre and has a field that swirls around the core.

An important point about the vortex is that there are lots of very similar structures we can make, for example, by globally rotating all of the arrows by some fixed angle. In fact, from the point of view of topology each of these excitations is equivalent. The quantity that defines the topological properties of the vortex is its integer winding number $w$. This quantity counts the number of times the arrows rotate through $2\pi$ radians as we follow a circle around the vortex core. The diagram shows a $w = 1$ vortex, since the arrows make a complete rotation as we follow a circle around the vortex core. It is possible to make vortices with $w = 2$. In contrast to a $w = 1$ object, a $w = -1$ object, known as an antivortex, doesn’t have the arrows pointing in the opposite direction, but rather has arrows that wrap in the opposite direction as the circle is traversed around the core.

In the three dimensional case of $D = 3, d = 3$ we have a configuration called a hedgehog (or monopole) shown in Figure 4(c). Here the winding number is given by considering the 3D field $\phi(x^1, x^2)$, where $x^1$ and $x^2$ are coordinates allowing us to locate points on a closed surface (conventionally we choose angles $x^1 = \theta$ and $x^2 = \varphi$, for example), and we evaluate the integral

$$w = \frac{1}{4\pi} \int dx^1 dx^2 \hat{\phi} \cdot \left( \frac{\partial \phi}{\partial x^1} \times \frac{\partial \phi}{\partial x^2} \right),$$

where $\hat{\phi} = \phi/|\phi|$ is the normalised (unit) field and where the surface over which we

\[^3\text{There is an important subtlety here related to the topology: the field must go to zero at the vortex core because the direction of } \phi \text{ is not defined at that point. This property of going through zero also occurs for the kink (as it goes from } -M_0 \text{ to } M_0 \text{) and will occur at the centre of the hedgehog described below. The existence of these zeros is described mathematically by the so-called Poincaré-Hopf theorem.}\]
integrate surrounds the core of the hedgehog. The integrand in this expression gives an element of the solid angle swept out by the vectors $\phi$. By comparing the integral of this quantity with $4\pi$ we can therefore compute how many times these vectors wrap around a sphere. In the same way that we can globally rotate the $D = 2$ arrows of the vortex without changing $w$, a combed hedgehog, with all of its arrows rotated globally by the same amount, also has the same winding number as the conventional hedgehog [see Figure 5, top].

The vortex and hedgehog introduce a new feature compared to the domain wall: they cost an infinite amount of energy! This can be understood by inspection of the vortex. It is swirly at large distances from the core, so that the fields never becomes uniform. The first term in equation 2 then keeps costing energy causing a volume integral over the free energy density to diverge. This energetic cost is a consequence of Derrick’s theorem and is important in judging whether each of these objects can hope to exist. That is, if an object costs an infinite amount of energy to create, it is not going to be realised in a system (at least without some other physical property being introduced) [24]. Specifically, Derrick investigated static field configurations as they are scaled up and down in their spatial size. If a field configuration is stable, then there is a point where the energy is stationary with respect to such a scaling. If the field configuration has no such stationary point then Derrick’s theorem says it cannot exist [4]. The instability of the vortex is a consequence of the lack of any stationary point in its energy as its characteristic size $L$ increases, since the energetic cost of the configuration increases monotonically with $L$. In addition to Derrick’s theorem is also the problem of the core of the vortex or monopole: this costs a lot of energy as there is a lot of swirliness near the core.

So far, each of our three examples have involved $d = D$. However, this does not have to be the case. Consider the case of $D = 3$, where field arrows can point in any Cartesian direction. This time we will constrain them to be of fixed length so, if placed at the centre of a sphere of fixed radius, their tips meet some point on the surface of the sphere. (The surface of the spherical ball of unit radius is called $S^2$ by mathematicians, so we sometimes say that the field lives in the space $S^2$). For the case above where $D = d = 3$, we had the hedgehog. We will now squash the $d = 3$
hedgehog flat, so it can be accommodated into space with \( d = 2 \). This squashing is carried out mathematically using the stereographic projection as shown in Figure 5 [20]. This is a mathematical method that projects the south pole to the origin of a plane, the equator to a unit circle, the southern hemisphere to the region inside the circle, and the northern hemisphere to the region outside the circle, with the northern hemisphere sent off to very large distances.

The shape that results from enacting this projection on the hedgehog is our first example of a skyrmion. The skyrmion formed from all arrows being radial is known as a hedgehog skyrmion or, more conventionally, a Néel skyrmion. This reflects the fact that if one considers a one-dimensional cut through the diameter of a skyrmion, the spins form a Néel domain wall configuration, where the spin rotates through \( 2\pi \) along the direction of the wall. This can be contrasted with the state of affairs resulting from the projection of a combed hedgehog. This is known as a Bloch skyrmion since a corresponding one-dimensional cut through the diameter of this object gives a Bloch-type magnetic domain wall where spins rotate perpendicular to the direction of the wall. (Note that these Bloch and Néel skyrmions are topologically equivalent, with both having a winding number \( w = 1 \).) The method of making cuts to reveal walls, allows us to understand another skyrmion configuration: this one has \( w = -1 \) and is known as an antiskyrmion [22]. This case is more complicated in that cuts through the diameter of the skyrmion differ from one another. There are two cuts through this object (at angles \( 0 \) and \( \pi/2 \)) that give inequivalent Néel walls and two (at angles \( \pi/4 \) and \( 3\pi/4 \)) that give inequivalent Bloch walls (shown in Figure 6 of Ref. [23]).

As with vortices, we can also, in principle, make skyrmions with larger winding numbers. Although the winding numbers are useful invariants, it is more useful to use the skyrmion charge \( Q_s \), [6] which is the same as the winding number up to a sign. The sign is supplied by the polarity of the skyrmion (i.e. the difference between spin directions at the centre of the skyrmion and far from the skyrmion).

Mathematically [5], we can build a class of skyrmion textures in a \( d = 2 \) plane using a \( D = 3 \) field with components \( \phi(r) = [\cos \Phi(\varphi) \sin \Theta(r), \sin \Phi(\varphi) \sin \Theta(r), \cos \Theta(r)] \), where \( r = (r \cos \varphi, r \sin \varphi) \) labels the spatial coordinates in the 2-dimensional plane. Plugging this into the expression for the winding number in equation [3], we find \( w = \frac{1}{4\pi} [\cos \Theta(r)]^\varphi_{r=0}^{\varphi=2\pi} \{\Phi(\varphi)\}_{\varphi=0}^{\varphi=2\pi} \). If the spins point downwards at \( r = 0 \) and up at \( r = \infty \), then the first square-bracketed term gives 2 and we define \( 2\pi m = [\Phi(\varphi)]_{\varphi=0}^{\varphi=2\pi} \), where \( m \) is an integer known as the vorticity. We then have a winding number \( w = m \) and we can define \( \Phi(\varphi) = m\varphi + \gamma \), where \( \gamma \) is a global phase that changes the direction of the spins with \( \gamma = 0, \pi \) corresponding to Néel skyrmions, while \( \gamma = \pm \pi/2 \) corresponds to Bloch skyrmions.

Another way to build a skyrmion is to consider a vortex (with \( d = 2 \) and \( D = 2 \)) and then give the arrows the freedom to move in a third dimension by setting \( D = 3 \). The spins at large distances from the core will remain in the plane, but since the vortex core costs a great deal of energy, the spins in the centre will develop a component in the third direction, curling downwards (or upwards) as we move inwards towards the core. This texture is called a meron. We can think of the structure in terms of part of a hedgehog on a sphere, whose projection into \( d = 2 \) results in the meron [Figure 6] [24]. The lower hemisphere of the combed hedgehog gives the meron when projected

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4 The existence of antiskyrmions provides one means of creating skyrmions: a skyrmion-antiskyrmion pair could be spontaneously created from the \( w = 0 \) ground state. Since the pair has a total winding number \( w = 0 \) (i.e. the sum of the winding numbers of a \( w = 1 \) skyrmion and a \( w = -1 \) antiskyrmion) this is allowed by topology.

5 The projection in Figure 6 is not strictly the same as the ordinary stereographic projection in Figure 5.
Figure 6. A meron (left) results from the projection of the lower hemisphere of a hedgehog; an antimeron results from the projection of the upper half. The skyrmion can be viewed as a sum of a meron and antimeron. (Based on a figure from Ref [24]).

into the \( d = 2 \) plane. From this point of view, a meron is half a skyrmion. (It does not have an integer winding number, since the spins don’t wrap completely around the sphere.) The upper hemisphere of the combed hedgehog is can then be used to generate an antimeron and the skyrmion can be pictured as a meron-antimeron pair, formed from glueing a meron and antimeron together on the sphere and then carrying out the stereographic projection.

4. A brief introduction to topology and field theory

Skyrmions are often described in terms of their topological properties and so here we shall very briefly touch on some of the mathematics underlying the description of the physics in terms of topology and the consequences for the theory of fields \[221\].

Spaces in which topology is important are given names: the real line is denoted \( \mathbb{R} \), the two-dimensional plane is called \( \mathbb{R}^2 \) and the \( n \)-dimensional vector space is \( \mathbb{R}^n \). One can define a product space so \( \mathbb{R} \times \mathbb{R} = \mathbb{R}^2 \). Similarly \( \mathbb{R} \times \mathbb{R} \times \cdots \times \mathbb{R} = \mathbb{R}^n \). A segment of a \( \mathbb{R} \) joined end to end gives a circle, denoted \( S^1 \). As mentioned above, the sphere is called \( S^2 \), which is a two-dimensional space, because by ‘sphere’ we mean the surface of a ball, not its interior. The sphere cannot be embedded in \( \mathbb{R}^2 \), which is to say that you cannot fully represent a sphere on a piece of paper without cutting it in some way. (However \( S^2 \) can, of course, be embedded in \( \mathbb{R}^3 \): there are balls (e.g. tennis balls) in three-dimensional space.] \[2\]

Once we have a space, we can define a path through the space. If we join the ends of the path then it becomes a loop. Topologically, some paths can be continuously deformed into each other and we find that in a particular space the set of possible loops can be divided up into a number of classes. These classes can then be mapped onto a mathematic group (called the fundamental group, given the symbol \( \pi_1 \)). For example, in \( \mathbb{R}^n \) all loops are contractible (i.e. they can be continuously shrunk to a point) and so there is only one class of loop and we say that \( \pi_1 \) is a trivial group consisting of the identity element. It is more interesting if the space has a hole in it, so that loops can be divided into different classes, each one characterised by the integer number of times the loop winds round the hole. This integer is the winding number that we met in the last section. As a result, \( \pi_1 = \mathbb{Z} \), where \( \mathbb{Z} \) denotes the set

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\[^6\text{Nothing crucial will hinge on the details presented in this section, so it can be skipped on a first reading.}\]
of integers. Another very similar example is \( \pi_1(S^1) = \mathbb{Z} \) (in words: loops wrap around a circle an integer number of times). [2]

In field theory, the partition function for a system is a useful quantity that enables the average thermodynamic properties of a complicated system to be calculated. The partition function can be written as a functional integral \([21]\) 

\[
Z = \sum_w \int D\phi \, e^{-S[\phi]},
\]

where \( S \) is the action for the field theory, which is closely related to the free energy we met in the previous section. Often the action falls apart into two pieces \( S = S_{\text{top}} + S_0 \), where \( S_{\text{top}} \) is a topological part of the action. This topological action often takes the form \( S_{\text{top}} = i\theta w \), where \( w \) is the winding number (again) and \( i\theta \) is the topological action of a field configuration with \( w = 1 \). In that case we have

\[
Z = \sum_w e^{-i\theta w} \int D\phi \, e^{-S_0[\phi]}.
\]

This means that if we find a theory that predicts an action \( S_{\text{top}} = \frac{i\theta}{4\pi} \int d^2x \phi \cdot \left( \frac{\partial \phi}{\partial x_1} \times \frac{\partial \phi}{\partial x_2} \right) \), we expect skyrmion textures. This indeed turns out to be the case for some theories.

In the previous sections we argued that topological objects are stable owing to the very large energy cost that would be incurred in removing them from existence. This is sometimes elevated to the idea of topological protection whereby it not possible to change the winding number of a topological object without introducing a singularity in a field (something forbidden by the mathematics). It should be remembered however that this is an idealisation (often based on infinitely large systems, or systems of reduced dimensionality) and in physical realisation there are often many ways of adding, removing or altering defects, as we shall see shortly.

This is all rather austere and involved. However, we can justify the existence of skyrmions in magnetic materials using simpler insights from the free energy, as we did above. We turn to this next.

5. Chiral interactions in noncentrosymmetric magnets

Magnetic materials are built from magnetic moments, or spins. This discrete structure is immediately somewhat different to our picture of continuous and smooth fields in the previous sections. When the variation of the directions of magnetic spins takes place over distances that are long compared to a length \( a \), the separation of atoms, then the continuum picture of fields is likely to be a good approximation; when this is not the case then the field picture is expected to break down. In this latter case, changing the system in a way that does not preserve the topology might still cost appreciable amounts of energy, but will not be forbidden.

At the energy scales encountered in solids, a magnetic spin \( \mathbf{S} \) can be thought of as a quantum top: a localised arrow representing angular momentum. Spins interact in various ways, the most common being the exchange interaction between two spins, which can be written \( \hat{H} = -J\mathbf{S}_1 \cdot \mathbf{S}_2 \), where \( J \) is an exchange constant. Other means of interaction are possible such as those based on spin orbit (SO) coupling [3]. In an ion, SO coupling induces orbital moments that lead to single-ion anisotropy: a contribution to the energy arising from the direction of the electronic spin \( \mathbf{S} \) with respect to the system’s crystal axes. When ions interact via an exchange interaction in the presence of SO coupling, processes are allowed that combine these interactions. These
interactions seem complicated at first sight, but simply rely on considering the possible combinations, in series, of SO interactions and exchange interactions. For example, starting with both ions in their ground states, one possible process involves the SO interaction lifting one ion out of its ground state and then the exchange interaction returning it to the ground state. Another involves the exchange interaction lifting an ion from its ground state and then the SO returning it. Taken together, the effect of these processes is to lead to an effective interaction between ions that reflects the effect of both of these interactions into an anisotropic exchange coupling known as the Dzyaloshinsky-Moriya (DM) interaction [3,12]. For a discrete system of spins the Hamiltonian for the DM interaction takes the form

$$H_{\text{DM}} = D \cdot (S_1 \times S_2) ,$$

where the so-called DM vector $D$ depends on the details of the induced orbital moments.

In many solids, the DM vector $D$ will be non-zero, although it is strongly constrained via Neumann’s principle (that is, that the Hamiltonian shares at least the symmetry of the underlying crystal system). As a result, if the two ions have a centre of inversion midway between them, such that the symmetry operation swaps $S_1 \leftrightarrow -S_2$, this implies $H_{\text{DM}} = -H_{\text{DM}}$ and so $H_{\text{DM}} = 0$, implying that $D$ must vanish. The role of $D$ in stabilising skyrmions, and its vanishing when there is a centre of symmetry, explains why most putative cases of skyrmions in bulk materials have been in noncentrosymmetric systems. It is worth remembering though that $D$ is often nonzero at interfaces between materials, leading to skyrmion textures that can be stabilised in multilayers. For a pair of ions, it is usually possible to strongly constrain the direction of $D$ through symmetry arguments. When dealing with the continuum version of the interaction appropriate for a description in terms of fields, the same considerations apply but may be generalised through the use of a phenomenological approach based on Lifshitz invariants (LIs) [10]. These are antisymmetric functions of the form $\left( m_i \frac{\partial m_j}{\partial x} - m_j \frac{\partial m_i}{\partial x} \right)$ which occur in the free energy of some systems. The DM interaction results in a macroscopic contribution to the free energy written $F_{\text{DM}} = g \int \phi \cdot (\nabla \times \phi)$. We can then upgrade the free energy from the previous sections, so we have [6,14]

$$F = \int d^d x \left[ c (\nabla \phi)^2 + a (\phi \cdot \cdot \phi) + g \phi \cdot (\nabla \times \phi) - \phi \cdot H + \lambda (\phi \cdot \phi)^2 \right] ,$$

where $H$ is an applied magnetic field which gives the additional contribution to the energy of $-\phi \cdot H$ known as the Zeeman term, that simply reflects the fact that the spins would like to point along the direction of the field.

In order to see the sorts of spin fields that can exist in systems described by this free energy, it is helpful to switch to Fourier space, describing the fields in terms of the sum of plane waves with wavevectors $q$. We take the solid to be a finite sized box of volume $V$, where the boundary conditions imply that the waves are quantised. We then define a (discrete) Fourier transform $\phi(x) = \frac{1}{\sqrt{V}} \sum_q \phi_q e^{iq \cdot x}$, where $\phi_q^\dagger = \phi_{-q}$.

Here $q = 0$ corresponds to a constant (ferromagnetic) spin texture. A plane wave of spin density would have a single nonzero $q$. In general, many magnetic structures observed in Nature can be represented by smooth fields $\phi(x)$ built from relatively few distinct $q$'s (usually $\leq 4$). Expressed in terms of the Fourier variables, the free energy
in equation 6 is written

\[
F = \sum_q \left[ (cq^2 + a)(\phi_q \cdot \phi_{-q}) + igq \cdot (\phi_q \times \phi_{-q}) - \phi_q \cdot H \right] + \lambda \sum_{q_1,q_2,q_3,q_4} \left[ (\phi_{q_1} \cdot \phi_{q_2}) \left( \phi_{q_3} \cdot \phi_{q_4} \right) \delta^{(3)}(q_1 + q_2 + q_3 + q_4) \right],
\tag{7}
\]

which is our key equation and will tell us how skyrmions can come into being. The second (DM) term gives us a preference for non-collinear magnetic states (i.e. it makes a negative contribution to the free energy for some swirling spin structures). In fact, in the absence of the Zeeman term, the ground state of this model is a helical spin structure (right handed if \( g > 0 \)) with a wavevector of magnitude \( q = g/c \).

The DM interaction therefore gives non-collinear magnetic structures as solutions, but it is not clear yet that it gives rise to skyrmions. As we saw before, swirling textures such as skyrmion solutions can be killed by Derrick’s theorem. For \( d = 2 \), the positive DM term \( g \int d^2 x \phi \cdot (\nabla \times \phi) \) scales as \( L \), the characteristic length of the region over which the field changes, which is cause for concern, since it implies that a skyrmion should have \( L = \infty \) if the integral is negative (which it will be if the DM term favours swirling spin structures). Skyrmions therefore seem to want to unwind to save energy, with Derrick’s theorem telling us that unless the energy has a stationary point with respect to variations in \( L \), then the skyrmion cannot exist. However the Zeeman term \( -\int d^2 x \phi \cdot H \) in equation 7 comes to our rescue since it scales as \( L^2 \), making a positive contribution since the spins forming a skyrmion are not lying along the field direction. We then have an energy going as \( E = -a_1 L + a_2 L^2 \), with \( a_1 \) and \( a_2 \) positive constants. This has a stable minimum (when \( L = a_1/2a_2 \)), giving a prediction, via Derrick’s theorem, of swirling spin structures with a finite size. One moral here is that a magnetic field is needed to stabilise the skyrmion.

Two-dimensional skyrmions are therefore possible, at least energetically, and it turns out that skyrmion structures that can exist stably in our model free energy can indeed be written down. In addition to the argument above for single skyrmions, we can investigate the skyrmion lattice phase, since this is the one that was observed, in MnSi for example [14]. Let’s consider the quartic term in equation 7 with prefactor \( \lambda \), since this provides the stability of the broken symmetry magnetic structure in a material. Since the Zeeman term is also necessary to realise skyrmions, we will try a solution with a ferromagnetic component \( q_1 = 0 \) to reflect the Zeeman term’s preference for aligned spins. With this included, the stability term becomes

\[
[\phi(x) \cdot \phi(x)]^2 \rightarrow \sum_{q_2,q_3,q_4} \left( \phi_{q_2} \cdot \phi_{q_3} \right) \left( \phi_{q_4} \cdot \phi_{q_5} \right) \delta^{(3)}(q_2 + q_3 + q_4).
\tag{8}
\]

For simplicity we constrain the \( q \)'s to have equal magnitudes and end up with a contribution to the energy proportional to \( \delta^{(3)}(q_1 + q_2 + q_3) \), implying that we seek \( q \)-vectors that form an equilateral triangle. Just such an energetic contribution is seen elsewhere in physics in the energetics of the liquid-solid transition [14,18]. In that case there is a similar term in the free energy that leads to a first order phase transition between a liquid and a regular repeating crystal lattice. In our case this gives rise to the skyrmion lattice (SkL), which is a triangular arrangement of skyrmions in a two-dimensional plane with the arrangement copied in the layers of atoms in the third dimension, such that a tube-like structure is set up in three dimensions. This is rather
like the vortex lattice in type II superconductors and is shown in Figure 7. It can be demonstrated via simulations \cite{14} that thermal fluctuations make the skyrmion lattice phase the equilibrium state of our model in a small pocket of the phase diagram close to the critical temperature $T_c$.

6. Realising and detecting magnetic skyrmions

The SkL phase was initially found experimentally in a relatively small number of materials in a rather restricted set of circumstances: typically a narrow region of temperature and applied magnetic field, usually within a few Kelvin below the magnetic ordering temperature. The skyrmion phase is usually surrounded in the temperature-applied field phase diagram by other, non-collinear, helimagnetically ordered phases.

It seemed from the first results on bulk systems that skyrmions were not particularly stable, owing to this very small extent of the SkL phase in the phase diagram. However, the stability of the skyrmion state seems to depend essentially on the dimensionality of the system and it was found that in the two-dimensional limit (corresponding to materials with a thickness of only one skyrmion) skyrmions are observed over a far larger region of temperature and field, with this region shrinking dramatically as the sample size is increased towards the three-dimensional, bulk limit. In fact, things are not as limited as they seem in the bulk owing to another feature of skyrmions: they can be made to exist outside of their equilibrium phase. As a result of fast cooling, it is possible to temporarily trap skyrmions in existence in a metastable state \cite{25}. This opens up the possibility of creating, observing and controlling individual skyrmions outside of the confines of the often very restricted SkL region of the phase diagram.

A periodic structure like a lattice can be detected using diffraction techniques and SkLs were originally measured using small angle neutron scattering (SANS). Although diffraction identifies periodic structures in Fourier space, real space imaging is also possible using Transmission Electron Microscopy (TEM) magnetic imaging techniques (known collectively as Lorentz microscopy). These are based on the fact that the Lorentz force from internal magnetic fields deflects electrons as they pass through a
material, altering the phase of the electronic wavefunction. The resulting images that are generated can be used to measure the magnetisation of individual skyrmions on an absolute scale. Other experimental techniques that have been employed including x-ray methods to measure the winding number of the skyrmion tubes, x-ray holography and scanning transmission x-ray microscopy to image skyrmions and muon-spin relaxation to probe the static and dynamic field distribution of the skyrmion lattice. Magnetometry is routinely used to measure skyrmions using AC magnetic susceptibility. Susceptibility measures the response in the magnetisation $M$ of a material to a driving field $H$ given by $M = \chi H$. In AC measurements the idea is to apply a small, alternating component of the applied field and measure the response $\chi'$ occurring in phase and the response $\chi''$ occurring out of phase. It is often found that the in-phase susceptibility $\chi'$ has a temperature- and field-dependent magnitude that correlates closely with the skyrmion lattice phase, while changes in $\chi''$ also marks out the skyrmion lattice phase boundaries [5].

The first discoveries of magnetic skyrmions were made in several materials with the noncentrosymmetric “B20” crystal system (with space group $P2_13$). Although it was pessimistically suggested at the time that this might be the only set of materials that could host them, skyrmions have since been found in bulk crystals of many other systems. As described in Section 2, the initial observation of the SkL was made in the $A$-phase of MnSi in 2009 using SANS, when Mühlbauer and co-workers [14] observed a magnetic pattern with intensity maxima that had a six-fold rotational symmetry. Since then SANS has been used widely to identify the SkL in bulk systems. After the discovery of the SkL in MnSi, the same group then found an $A$-phase with similar properties in the doped semiconductor Fe$_{0.5}$Co$_{0.2}$Si [27]. Further evidence for this interpretation was provided by a real-space observation of a two-dimensional skyrmion lattice using Lorentz transmission electron microscopy on a thin film of Fe$_{0.5}$Co$_{0.5}$Si [15]. Skyrmion lattices have since been found in many more materials [8], including the multiferroic insulator Cu$_2$OSeO$_3$ [28] and can form at room temperature in Co-Zn-Mn metallic alloys (which is a material with a $\beta$-Mn structure and was the first non-B20 skyrmion-hosting material to be identified). There are now extensive reports of a large range of different skyrmion textures, including Néel skyrmions (in GaV$_4$S$_8$) [29], and antiskyrmions (in Mn$_{1.4}$Pt$_{0.9}$Pd$_{0.1}$Sn) [22].

In parallel with these discoveries are a large class of skyrmion systems where the skyrmions are stabilised near an interface in multilayer materials. The mechanism is again based on the DM interaction, although here it is the DM interaction that arises at an interface between two materials. This is realised by a thin (< 1 nm) ferromagnetic layer of material coupled to a material such as Pt or Ir, which has a large SO interaction. The resulting DM interaction, along with dipolar coupling, stabilises skyrmions (often Néel type) with characteristic size of several hundred nanometres [7,8].

Furthermore, although we have presented one means of realising skyrmions in the previous section: using the DM interaction, we should not discount the other proposals for ways that skyrmions could be stabilised in magnets. For example, the possibility of using frustration has attracted interest recently with several candidate skyrmion-hosting materials identified such as centrosymmetric Gd$_2$PdSi$_3$ [16]. (Here, however, the small size of the skyrmions is such that we might expect the field description to break down.)

Another very important experimental technique that has been used in this context is the Hall effect: that is, through a transverse voltage observed when current is passed along a material in a perpendicular magnetic field. In skyrmion-hosting materials we
often measure very large Hall effects. These Hall effects contain contributions that cannot be explained by the classical Hall effect (owing to the interaction of the conduction electrons and the external field) nor the anomalous Hall effect (due to the interaction of conduction electrons with the magnetisation of a ferromagnetic conductor). This extra contribution is known as a topological Hall effect and results from considering the dynamics of skyrmions and their interactions with electrons, which is the subject to which we now turn.

7. Skyrmion dynamics

Skyrmions and skyrmion lattices are not just static structures, but are subject to a rich range of behaviour as a function of time. While it is often useful to imagine the skyrmion as a rigid object whose transport involves a change in the coordinates of its centre, it should be borne in mind that magnetic skyrmion dynamics always involve the collective evolution of a system’s magnetic spins as a function of time. The dynamics of magnetic spins can be understood using the Landau-Lifshitz-Gilbert equation of motion, which is a first-order differential equation given (in the absence of damping) by

\[ \dot{n} = -\alpha n \times h, \]

where \( n \) gives the moment direction, \( \alpha \) is the gyromagnetic ratio, and \( h \) is an effective magnetic field that combines the externally applied field, and internal interactions in the material \( [10] \). (This equation of motion for the field \( n(r, t) \) is designed to look like that for a single moment that we briefly discuss below.)

Interestingly for our purposes, the dynamics of magnetic systems depend on a geometrical notion: the Berry phase \( [21] \). The Hamiltonian of a system relies on several parameters, one example being a magnetic field. We can imagine taking a spin-\( S \) particle in its ground state, with its spin aligned along the direction of an applied magnetic field vector and then slowly rotating the magnetic field. If we are slow enough then, during this process, the Hamiltonian describing the system changes, but the spin never leaves the instantaneous ground state. (Classically we imagine the spin staying aligned along the field direction as we rotate the field.) During the process of rotating the field, we move it in such a way that we rotate the tip of the field vector around a closed loop on a sphere, so that the field direction finishes exactly where it started. We might expect that the spin wavefunction will have acquired a phase factor reflecting (i) the time that has elapsed during the process and (ii) the energy of the particle in its instantaneous ground state, and this does indeed turn out to be the case. However, it also turns out that the wavefunction acquires an additional phase \( \gamma \), that depends only on the geometry of the situation (that is, the details of the path we chose to rotate the field around; not the speed the field was rotated around the loop). This additional phase factor is known as a Berry phase. For a time-dependent magnetic field of constant magnitude and a spin \( S \mathbf{n} \), the phase \( \gamma \) is given by an integral with a familiar integrand: the one we encountered in equation \( [3] \). Specifically, the phase is given by

\[ \gamma = \frac{S}{2} \int \! dx^1 dx^2 \mathbf{n} \cdot \left( \frac{\partial \mathbf{n}}{\partial x^1} \times \frac{\partial \mathbf{n}}{\partial x^2} \right), \tag{9} \]

where we integrate over the area enclosed on the surface of a unit sphere by the journey of the tip of the magnetic field vector \( [21] \).

\[ \text{It is worth noting that there is not a one-to-one correspondence between observation of the topological Hall effect and the presence of skyrmions.} \]
It is interesting to note that, when considered in terms of fields, the Berry phase $\gamma$ leads to a contribution to the equation of motion for spins that, when considered along with the Zeeman energy (proportional to $-S \cdot H$), gives rise to the well-known equation of motion for spins $\dot{S} \propto S \times H$. In other words, the familiar first-order equation of motion describing the quantum analogue of the classical precession of a spin resulting from the torque $S \times H$, can also be explained as resulting from this subtle geometrical phase [21]. Perhaps unsurprisingly for objects built from large numbers of spins, the Berry phase will also contribute to give us the equation of motion for skyrmions, as we now describe.

In considering their dynamics, we describe skyrmions in terms of a skyrmion coordinate $R(t) = (X(t), Y(t))$, which tells us the position of a skyrmion in the $X$-$Y$ plane. We can consider the motion of spin vectors as a function of space and time $n(r, t)$, but if the only source of motion is the translation of a rigid skyrmion then we can write $n(r - R(t))$, and dynamics then involve the spin configuration changing because of displacements of the skyrmion position. If we take the Berry phase into account along with the coupling of spins to local magnetic fields one can derive a Landau-Lifshitz Gilbert equation specifically for skyrmions, given by [6]

$$M_s \ddot{R} = -\frac{\partial V}{\partial R} + G \left( \hat{z} \times \dot{R} \right).$$

(10)

Here $M_s$ is an effective mass of a skyrmion, reflecting the energetic cost of its having a velocity $\dot{R}$ and $V$ is a potential reflecting the energy cost of the skyrmion sitting in local magnetic fields. The parameter $G$ is known as the gyrotropic constant and is given by $G = 2hSQ_s/a^2$ where $S$ is the spin, $Q_s$ is the skyrmion charge and $a$ is the underlying lattice spacing. In fact, the equation of motion in equation (10) looks rather like the Lorentz force law in electromagnetism $F = q(E + v \times B)$ for the forces on an electric charge $q$ in $E$- and $B$-fields. In this picture $\partial V/\partial R$ plays the role of the $E$-field, while $-G\hat{z}$ becomes the $B$-field, constrained to point along the $z$-direction (i.e. out of the plane of the skyrmion). When the mass vanishes the two terms must cancel and we have $G\dot{R} = -\hat{z} \times \left( \partial V/\partial R \right)$, which implies that massless skyrmions experience a drifting motion, moving perpendicular to the directions of “$E$” and “$B$”.

A first example of skyrmion dynamics that makes use of these ideas is the motion of a single skyrmion, confined to a circular disk [6]. In this case, the potential $V$ has the form of a restoring force, deflecting the skyrmion from the edges of the disk. The resulting motion is described by two gyration modes which involve the skyrmion coordinate undergoing cyclotron-like, circular motion, such that the centre of the skyrmion orbits the centre of the disk. The two modes represent clockwise and anticlockwise rotations, with frequencies that differ by a factor $G/M_s$ and thus reflect the topological charge $Q_s$ of the skyrmion. A second example is the dynamics of the entire lattice of rigid skyrmions. In this case there are two modes of excitation: an acoustic one and an optical one. These again represent uniform clockwise and anticlockwise rotations, but this time they are rotations of the skyrmion lattice. The acoustic mode is a Goldstone mode [2,17] that reflects fact that the lattice of skyrmions breaks translational symmetry (just like the atoms in a crystal lattice, whose Goldstone mode is an acoustic phonon). Although by analogy with phonons we might expect a linear dispersion for skyrmions, the acoustic mode actually has a dispersion $\omega \propto k^2$, just like the spin wave excitations of a ferromagnet. This state of affairs is actually due the curious, quantum mechanical feature of skyrmion dynamics: the coordinates of the skyrmion, have a non zero commutator that obeys $[X, Y] = ia^2/(4\pi SQ_s)$ [6].
So far we have only looked at the dynamics of rigid skyrmions that occur due to the movement of the skyrmion coordinate. Another class of skyrmion dynamics involves the modes that describe the periodic deformation of the skyrmion’s own magnetic structure \[6\]. These deformations can be parametrised by considering a contour around the core of the skyrmion, connecting points with a constant spin direction \(n\), with radius \(C(t, \phi)\), where \(\phi\) is the angle in the plane and \(t\) is time. For the unexcited skyrmion this contour will usually be a circle with radius \(C_s\), while it will have some more complicated behaviour as a function of time and the angle \(\phi\) when excited. The mode structure of the skyrmion can then be written

\[
C(t, \phi) = C_s + C_0(t) + C_1(t)e^{i\phi} + C_{-1}(t)e^{-i\phi} + \ldots
\]

(11)

Here the lowest order dynamic contribution \(C_0(t)\) is a breathing mode: the skyrmion expands and contracts as a function of time. The next-lowest energy excitations reflect small displacements of the skyrmion’s centre from the origin, to a position \(\mathbf{R} = (X,Y)\) as we had before. As might be expected, the modes \(C_1\) and \(C_{-1}\) again reflect the cyclotron orbits of the centre of the skyrmion. These modes are shown in Figure 6 of Ref. \[23\]. An advantage of this approach of permitting the skyrmion to deform is that it provides an expression for the skyrmion’s effective mass, which is given by

\[
M_s = G^2 R_s^2 / J,
\]

where \(R_s\) is the linear size of the skyrmion, \(J\) is the magnetic exchange constant and \(G\) is the gyrotropic constant. Key here is that the mass scales with the area \(R_s^2\) of the skyrmion: large skyrmions are more massive than small ones. Experimentally, microwave resonance experiments have been used to excite the breathing mode and the lowest order rotating modes of the skyrmions in the gigahertz regime \[26\]. Using this technique, microwaves are absorbed at the characteristic resonant frequencies of the magnetic structure, allowing a spectroscopic insight into the presence of skyrmions and their dynamics.

8. Spin transfer torque and emergent electrodynamics

The large contributions skyrmions give to the Hall effect that we met earlier reflects some notable features of their electrodynamics that we discuss here. Hund’s rule coupling is the name we give to the quantum mechanical effect where a single electron has its spin biased in the direction of a local moment \[17\]. (It is familiar from atomic physics, where it partially explains the spin ground state of a magnetic ion and arises from electrostatic considerations and the Pauli exclusion principle.) Skyrmions interact with electrons via Hund’s rule coupling, which aligns the electronic spin along the direction of the local magnetic moment of the skyrmion \(n\). Since electrons move very fast (that is, much faster than skyrmion spins are moving), the electron has enough time to completely orient its spin in the direction of \(n\). (As described above, this can lead to a geometric Berry phase.) Rotating electron spins requires that the skyrmion supplies a torque, and so classical mechanics tells us that there must be an equal and opposite torque acting on the spins in the skyrmion. This leads to an effect known as the spin transfer torque that allows skyrmions to be moved by electric currents \[5,6\]. An important example is a current flowing through a skyrmion which, in the absence of dissipation, gives rise to a drift of the skyrmion in the direction of the current.

We saw above [equation (10)] how the forces that cause the motion of rigid skyrmions bear a resemblance to those that result from Lorentz’s force law in electromagnetism. The rather different case of the motion of an electron that passes through a skyrmion
also enjoys a link to electromagnetism (although it is worth keeping these two cases separate). In the current case, where the electron’s motion is determined by Hund’s rule coupling, the electron acts just as if it is passing through a electromagnetic field. It is not simply that the field straightforwardly reflects the local electromagnetic forces from each of the magnetic ions on the electron (this would need to take Pauli exclusion into account for example); it is as if there is a new electromagnetic field present, determined by the topology of the skyrmion. This has been called an emergent electromagnetic field: all of the microscopic, quantum mechanical Hund’s rule couplings act in a complicated manner and what emerges is a rather simple effective force field that has the mathematical structure of a classical electromagnetic field [5][31].

In conventional electrodynamics, the electromagnetic field is described in terms of a gauge field [2] with components (in natural units, where the speed of light is set to unity)
\[ A^\mu = (V, A^x, A^y, A^z), \]
that causes a force on electric charges moving at a velocity \( v \) with components \( qF^\mu\nu v^\mu \), where \( F^\mu\nu \) are the components of the Faraday tensor, a \( 4 \times 4 \) matrix built from the components \( A^\mu \) using a four-dimensional cross product, written as [2]
\[
F^\mu\nu = \left( \frac{\partial A^\nu}{\partial x^\mu} - \frac{\partial A^\mu}{\partial x^\nu} \right). \tag{12}
\]

For skyrmions, the twists and turns of the skyrmion spin texture can be thought of as giving rise to an emergent electromagnetic gauge field \( a \), with components \( a = (U, a^x, a^y, a^z) \), that couples to the electrons in exactly the manner that the real electromagnetic gauge field \( A^\mu \) does. In the skyrmion case, the electrons couple to the components \( f^\mu\nu \) of an emergent Faraday tensor, which is determined by the spin configuration of the skyrmion via [6]
\[
f^\mu\nu = -\frac{\hbar}{q_e} \left( \frac{\partial a^\nu}{\partial x^\mu} - \frac{\partial a^\mu}{\partial x^\nu} \right) = -\frac{\hbar}{2q_e} \mathbf{n} \cdot \left( \frac{\partial \mathbf{n}}{\partial x^\mu} - \frac{\partial \mathbf{n}}{\partial x^\nu} \right), \tag{13}\]

where \( q_e \) is the electronic charge. Perhaps we should not be surprised to see the now-familiar integrand from equation [3] providing the components of the effective Faraday tensor.

This abstract picture can be made more vivid by picking out the emergent electric \( e \) and magnetic \( b \) fields from \( f^\mu\nu \). Specifically we have [6]
\[
b = -\frac{\hbar}{q_e} (\nabla \times \mathbf{a}), \quad e = -\frac{\hbar}{q_e} (\nabla U - \frac{\partial a}{\partial t}). \tag{14}\]

In words, the first expression says that the skyrmion spin texture is the source of emergent magnetic field; the second equations says that the moving skyrmion is the source of electric field.

We motivated our discussion of skyrmion dynamics by an attempt to understand the large topological contribution to the Hall effect seen in materials when skyrmions are present [6]. The size of this contribution follows from the magnetic flux \( \Phi \) experienced by an electron from the presence of the skyrmion. This is given by \( \Phi = -\hbar Q_s / q_e \), which turns out to be a very large field for a typical skyrmion: typically tens of tesla. Following Ref [6], let’s examine an example of the motion of a skyrmion and an electron (Figure [5]). If we apply an external \( B \)-field along the \( z \)-direction then an electron with charge \( q_e \) traveling along the \( x \)-direction with velocity \( u \) experiences a Lorentz force \( q_e u \times B \), deflecting the electron in a direction perpendicular to \( u \) and \( B \), in this case
along +y. If the applied $B$-field stabilises a skyrmion, then the emergent magnetic field $b$ from a skyrmion is typically much larger than the external magnetic field $B$, in the opposite direction (owing to the minus sign in $\Phi = -hQ_s/q_e$), and therefore deflects the electron exactly in the opposite direction: along $-y$. The electron therefore experiences a large Hall effect due to the presence of the skyrmion. However, the motion of electrons also causes the skyrmion to drift in the direction of the electron flow with velocity $v$. Since the skyrmion is now at motion, this causes an emergent electric field $e = b \times v$ directed along $-y$ that (owing to the electron’s negative charge) produces a force on the electron along $+y$. This lessens the amount of deflection along $-y$ compared to the case of a static skyrmion. The moving skyrmion therefore causes a smaller Hall effect compared to a static one.

9. Conclusions

One reason why skyrmions have received so much attention is the possibility of using them in spintronics applications. Based on the discussion in the previous section, there is a prospect of using skyrmions to provide a very efficient coupling of an electric or spin current to a magnetic structure. Since skyrmions carry an invariant skyrmion charge $Q_s$ (or winding number $w$) it has been suggested that this could be used to encode information. Compared to magnetic domain walls used in many magnetic memory applications, there is experimental evidence that skyrmions could be transported at surprisingly low energy cost. Great excitement followed from the observation that an spin transfer torque allowed an electrical current density of just $2.2 \times 10^6$ Am$^{-2}$ to cause motion of the skyrmion lattice [32]. Although, happily, this is very small indeed compared to the $10^{11}$ Am$^{-2}$ required to move ferromagnetic domain walls by spin transfer torque, if high speed manipulation of the skyrmions is attempted then the current densities become similar. This would seem to imply that although skyrmions might constitute a new, low-energy avenue for magnetic data storage, logical bit-wise operations or spintronic technologies, there is still much work to be done in learning how to best exploit skyrmions for technology. We would, for example, prefer to use small (< 10 nm) skyrmions to maximise the density of data and to realise them at room temperature and without the need for an applied magnetic field. As a route towards their exploitation, there has been much discussion of the properties of skyrmions confined in various geometries. Finite-sized systems allow different pathways to creating and annihilating skyrmions, owing to the lowered energy barriers at boundaries.
A much-discussed example of a skyrmion-based data storage device is the racetrack model [8]. Here the skyrmions resemble the beads on an abacus whose presence or absence provide the series of 1s and 0s required to store digital information. Such a device would need to comprise (i) a writing module based on localised skyrmion creation; (ii) some means of moving skyrmions; and (iii) a means of reliably detecting them. We are some way from having this level of control, but it does not seem unreasonable. Other ideas to utilise skyrmions include proposals for creating logic gates and even for probabilistic computing devices [7,8]. With the current excitement in the field, it is likely there will be many other suggestions for applications as magnetic skyrmions are investigated more widely.

From the point of view of realising topological objects, perhaps predictably, the skyrmion doesn’t exhaust the possibilities of finding complex, particle-like excitations in condensed matter systems and beyond. An example of a more complicated object is a $d = 3$ topological excitation, known as hopfion [33], shown in Figure 11 of Ref. [23]. If we take a finite length of skyrmion tube, each cross-section contains a $d = 2$ skyrmion. If we twist one end of the tube through $2\pi$ and join it to the other end, we form a closed loop. This loop has the topology of a hopfion. In the $d = 2$ skyrmion, one spin in the texture points towards each direction once. If the magnetisation points downwards at the centre of the skyrmion then, in the hopfion we have constructed from the skyrmion tube, the magnetisation points downwards along a circle in space. The same is true for any other spin direction, which also forms a loop. The defining property of the hopfion is that each of these loops formed by following a particular spin direction links another one only once. We say that the hopfion linking number $Q$ is one. As always, this does not exhaust the possibilities and further multihopfions can be constructed with $Q > 1$.

We have seen how topological defects such as the skyrmion (or hopfion) are members of a family of stable field excitations. These can be treated on the same sort of basis as “fundamental” particles with their own set of properties and interactions. While the existence of skyrmions in magnetic materials is now well established, their existence as excitations in other matter and gauge fields is not so firmly founded. However, it might well be that defects formed in the fields of the early Universe and have resulted in detectable features that we will encounter in the future. Cosmological speculation aside, the idea of encoding information in domain wall defects is a very important technological one and the potential for skyrmions to be used low-energy-cost data devices makes it likely that they will continue to be widely discussed in the coming years in one form or another. Whatever one’s field of interest within physics, the story of the magnetic skyrmion suggests the usefulness and unifying potential of notions from topology and field theory and hints that this is an area to watch in the future.

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