A Comparison of p-p, p-Pb, Pb-Pb Collisions in the Thermal Model: Multiplicity Dependence of Thermal Parameters.

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An analysis is made of the particle composition (hadrochemistry) of the final state in proton-proton (p-p), proton-lead (p-Pb) and lead-lead (Pb-Pb) collisions as a function of the charged particle multiplicity $dN_{ch}/d\eta$. The thermal model is used to determine the chemical freeze-out temperature as well as the radius and strangeness saturation factor $\gamma_s$. Three different ensembles are used in the analysis namely, the grand canonical ensemble, the canonical ensemble with exact strangeness conservation and the canonical ensemble with exact baryon number, strangeness and electric charge conservation. It is shown that for high multiplicities (at least 20 charged hadrons in the mid-rapidity interval considered) the three ensembles lead to the same results.

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I. INTRODUCTION

In high energy collisions applications of the thermal-statistical model in the form of the hadron resonance gas model have been successful (see e.g. [1, 2] for two recent publications) in describing the composition of the final state e.g. the yields of pions, kaons, protons and other hadrons. In these descriptions use is made of the grand canonical ensemble and the canonical ensemble with exact strangeness conservation. In this paper we consider in addition the use of the canonical ensemble with exact baryon, strangeness and charge conservation. We also make a systematic analysis of the dependence on the charged particle multiplicity \( dN_{ch}/d\eta \) for the first time.

The identifying feature of the thermal model is that all hadronic resonances listed in [3] are assumed to be in thermal and chemical equilibrium. This assumption drastically reduces the number of free parameters as this stage is determined by just a few thermodynamic variables namely, the chemical freeze-out temperature \( T_{ch} \), the various chemical potentials \( \mu \) determined by the conserved quantum numbers and by the volume \( V \) of the system. It has been shown that this description is also the correct one [4–6] for a scaling expansion as first discussed by Bjorken [7]. After integration over \( p_T \) these authors have shown that:

\[
\frac{dN_i/dy}{dN_j/dy} = \frac{N_0^i}{N_0^j}
\]

where \( N_0^i \) \((N_0^j)\) is the particle yield of hadron \( i \) \((j)\) as calculated in a fireball at rest, while \( dN_i/dy \) is the yield of hadron \( i \) on the rapidity plateau. Hence, in the Bjorken model with longitudinal scaling and radial expansion the effects of hydrodynamic flow cancel out in ratios.

The yields produced in heavy-ion collisions have been the subject of intense discussions over the past few years and several proposals have been made in view of the fact that the number of pions is underestimated while the number of protons is overestimated. Several proposals to improve on this have been made recently:

- Incomplete hadron spectrum [8],
- chemical non-equilibrium at freeze-out [9][11],
- modification of hadron abundances in the hadronic phase [12][14],
- separate freeze-out for strange and non-strange hadrons [15][18],
- excluded volume interactions [19],
- energy dependent Breit-Wigner widths [20],
- use the phase shift analysis to take into account repulsive and attractive interactions [21][22],
- use the K-matrix formalism to take interactions into account [23].

These proposals improve the agreement with the observed yields and furthermore, some of them change the chemical freeze-out temperature, \( T_{ch} \) in only a minimal way like those presented recently in [20][22]. In the present analysis we therefore kept to the basic structure of the thermal model with a single freeze-out temperature and focus on the resulting thermal parameters \( T_{ch}, \gamma_s \) and the radius. All our calculations were done using the latest version of THERMUS [24][25].

Our results show some interesting new features:

- the grand canonical ensemble, the ensemble with strict strangeness conservation and the one with strict baryon number, strangeness and charge conservation agree very well for the particle composition in Pb-Pb collisions, they also agree well for p-Pb collisions but marked differences for p-p collisions are present. These differences disappear as the multiplicity of charged particles increases in the final state. Thus, p-p collisions with high multiplicities agree with what is seen in large systems like p-Pb and Pb-Pb collisions. Quantitatively this agreement starts when there are at least 20 charged hadrons in the mid-rapidity interval being considered. It also throws doubt on the applicability of the thermal model as applied to p-p collisions with low multiplicity.

- The convergence of the results in the three ensembles lends support to the idea that one reaches a thermodynamic limit where the results are independent of the ensemble being used.
II. ENSEMBLES CONSIDERED IN THE THERMAL MODEL

We compare in great detail three different ensembles based on the thermal model.

- Grand canonical ensemble (GCE), the conservation of quantum numbers is implemented using chemical potentials. The quantum numbers are conserved on the average. The partition function depends on thermodynamic quantities and the Hamiltonian describing the system of $N$ hadrons:

$$Z_{GCE} = \text{Tr} \left[ e^{-\frac{H - \mu_N}{T}} \right]$$

which, in the framework of the thermal model considered here, leads to

$$\ln Z_{GCE}(T,\mu,V) = \sum_i g_i V \int \frac{d^3p}{(2\pi)^3} \exp \left( -\frac{E_i - \mu_i}{T} \right)$$

in the Boltzmann approximation, $g_i$ is the degeneracy factor of hadron $i$, $V$ is the volume of the system, $\mu_i$ is the chemical potential associated with the hadron. The yield is given by:

$$N_{i}^{GCE} = V \int \frac{d^3p}{(2\pi)^3} \exp \left( -\frac{E_i}{T} \right),$$

where we have put the chemical potentials equal to zero, as relevant for the beam energies at the Large Hadron Collider considered here. The decays of resonances have to be added to the final yield

$$N_{i}^{GCE}(\text{total}) = N_{i}^{GCE} + \sum_j B(r(j \rightarrow i)) N_{i}^{GCE}.$$  

- Canonical ensemble with exact implementation of strangeness conservation, we will refer to this as the strangeness canonical ensemble (SCE). There are chemical potentials for baryon number $B$ and charge $Q$ but not for strangeness:

$$Z_{SCE} = \text{Tr} \left[ e^{-\frac{H - \mu_N}{T} \delta(S,\sum_i S_i)} \right]$$

The delta function imposes exact strangeness conservation, requiring overall strangeness to be fixed to the value $S$, in this paper we will only consider the case where overall strangeness is zero, $S = 0$. This change leads to [26]:

$$Z_{SCE} = \frac{1}{(2\pi)^2} \int_0^{2\pi} d\phi e^{-iS\phi} Z_{GCE}(T,\mu_B,\lambda_S)$$

where the fugacity factor is replaced by

$$\lambda_S = e^{i\phi}$$

$$N_{i}^{SCE} = V \frac{Z_i^C}{Z_{S=0}} \sum_{k,p=-\infty}^{\infty} \frac{a_3^p a_2^k a_1^{-2k-3p-s}}{a_3^p a_2^k a_1^{-2k-3p-s}} I_k(x_2) I_p(x_3) I_{-2k-3p-s}(x_1),$$

where $Z_{S=0}^C$ is the canonical partition function

$$Z_{S=0}^C = e^{S_0} \sum_{k,p=-\infty}^{\infty} \frac{a_3^p a_2^k a_1^{-2k-3p}}{a_3^p a_2^k a_1^{-2k-3p}} I_k(x_2) I_p(x_3) I_{-2k-3p}(x_1),$$

where $Z_k^i$ is the one-particle partition function calculated for $\mu_S = 0$ in the Boltzmann approximation. The arguments of the Bessel functions $I_*(x)$ and the parameters $a_1$ are introduced as,

$$a_s = \sqrt{S_s/S_{-s}}, \quad x_s = 2V \sqrt{S_s S_{-s}},$$

where $S_s$ is the sum of all $Z_k^i(\mu_S = 0)$ for particle species $k$ carrying strangeness $s$. As previously, the decays of resonances have to be added to the final yield

$$N_{i}^{SCE}(\text{total}) = N_{i}^{SCE} + \sum_j B(r(j \rightarrow i)) N_{i}^{SCE}.$$
Canonical ensemble with exact implementation of \( B, S \) and \( Q \) conservation, we will refer to this as the full canonical ensemble (FCE). In this ensemble there are no chemical potentials. The partition function is given by:

\[
Z_{FCE} = \text{Tr} \left[ e^{-\frac{(H-H_s^\mu)}{T}} \delta(B_i, \sum_j B_j) \delta(Q_i, \sum_j Q_j) \delta(S_i, \sum_j S_j) \right]
\]

(12)

\[
Z_{FCE} = \frac{1}{(2\pi)^{3/2}} \int_0^{2\pi} d\alpha e^{-iB\alpha} \int_0^{2\pi} d\psi e^{-iQ\psi} \int_0^{2\pi} d\phi e^{-iS\phi} Z_{GCE}(T, \lambda_B, \lambda_Q, \lambda_S)
\]

(13)

where the fugacity factors have been replaced by

\[
\lambda_B = e^{i\alpha}, \quad \lambda_Q = e^{i\psi}, \quad \lambda_S = e^{i\phi}.
\]

(14)

As before, the decays of resonances have to be added to the final yield

\[
N_{i,FCE}^{(\text{total})} = N_{i,FCE}^{(\text{total})} + \sum_j \text{Br}(j \to i) N_{i,FCE}^{(\text{final})}.
\]

(15)

In this case the analytic expression becomes very lengthy and we refrain from writing it down here, it is implemented in the THERMUS program [24].

In all three cases we have also taken into account the strangeness saturation factor \( \gamma_s \) [27] which enters as a multiplicative factor, raised to the power of the strangeness content, in the particle yields. Keeping this factor fixed at one does not change the fixed message, only the resulting value of \( \chi^2 \) is increased indicating a worsening of the fits.

These three ensembles are applied to p-p collisions at 7 TeV in the central region of rapidity [28], to p-Pb collisions at 5.02 TeV [29, 30] and to Pb-Pb collisions at 2.76 TeV [31–33] with particular focus on the dependence on the charged particle multiplicity. It is well known that in this kinematic region, one has particle - antiparticle symmetry and therefore there is no net baryon density and also no net strangeness. The different ensembles nevertheless give different results because of the way they are implemented. A clear size dependence is present in the results of the ensembles. In the thermodynamic limit they should become equivalent. Clearly there are other ensembles that could be investigated and also other sources of finite volume corrections. We hope to address these in a future publication.

A similar analysis was done in [34–36] for p-p collisions at 200 GeV but without the dependence on charged multiplicity.

For p-p collisions we have taken the five particle species listed in table I where we also compare the measured values with the model calculations. For p-Pb and Pb-Pb collisions we included the \( \Omega \) measurements in our analysis, so that six particle species were considered for p-Pb and Pb-Pb. We have checked explicitly that for the five bins in p-p collisions where the \( \Omega \) has also been measured, there is no difference in the outcome for the values of \( T_{ch}, \gamma_s \) and the radius.

As shown in [37, 38] the \( \phi \) meson is not described very well and has not been included.

### III. COMPARISON OF DIFFERENT ENSEMBLES.

In Fig. 1 we show the chemical freeze-out temperature as a function of the multiplicity of hadrons in the final state [28]. As explained in the previous section the freeze-out temperature has been calculated using three different ensembles. The highest values are obtained using the canonical ensemble with exact conservation of three quantum numbers, baryon number \( B \), strangeness \( S \) and charge \( Q \), all of them being set to zero as is appropriate for the central rapidity region in p-p collisions at 7 TeV. In this ensemble the temperature drops strongly from the lowest to the highest multiplicity.

| Particle Species | \( dN/dy \) (data) | \( dN/dy \) (model) | Canonical S | Canonical B, S, Q | Grand Canonical |
|------------------|------------------|------------------|-------------|------------------|---------------|
| \( \pi^\pm \)     | 7.88 ± 0.38      | 6.78             | 6.76        | 6.96             |
| \( K_S^0 \)      | 1.04 ± 0.05      | 1.16             | 1.16        | 1.15             |
| \( p, (\bar{p}) \) | 0.44 ± 0.03      | 0.50             | 0.50        | 0.50             |
| \( \Lambda \)    | 0.302 ± 0.020    | 0.279            | 0.262       | 0.246            |
| \( \Xi^- (\Xi^+) \) | 0.0358 ± 0.0023  | 0.035            | 0.035       | 0.036            |
FIG. 1. The chemical freeze-out temperature, $T_{\text{ch}}$, obtained for three different ensembles. The black points are obtained using the grand canonical ensemble, the blue points use exact strangeness conservation while the red points have built-in exact baryon number, strangeness and charge conservation. Circles are for p-p collisions at 7 TeV, squares are for p-Pb collisions at 5.02 TeV while triangles are for Pb-Pb collisions at 2.76 TeV.

The lowest values for $T_{\text{ch}}$ are obtained when using the grand canonical ensemble, in this case the conserved quantum numbers are again zero. The results are clearly different from those obtained in the previous ensemble, especially in the low multiplicity intervals. They gradually approach each other and they become equal at the highest multiplicities.

For comparison with the previous two cases we also calculated $T_{\text{ch}}$ using the canonical ensemble with only strangeness $S$ being exactly conserved using the method presented in [26]. In this case the results are close to those obtained in the grand canonical ensemble, with the values of $T_{\text{ch}}$ always slightly higher than in the grand canonical ensemble. Again for the highest multiplicity interval the results become equivalent. As can be seen in Fig. 1 even though all the ensembles produce different results, for high multiplicities the results converge to a common value close to 160 MeV.

In Fig. 2 we show results for the strangeness saturation factor $\gamma_s$ [27]. In this case we obtain again quite substantial differences in each one of the three ensembles considered. The highest values being found in the canonical ensemble with exact strangeness conservation. Note that the values of $\gamma_s$ become compatible with unity, i.e. with chemical
equilibrium for all light flavors.

In Fig. 3 the radius at chemical freeze-out obtained in the three ensembles is presented. As in the previous figures, the results become independent of the ensemble chosen for the highest multiplicities while showing clear differences for low multiplicities.

Our results show that there is a strong correlation between some of the parameters. The very high temperature obtained in the canonical BSQ ensemble (FCE) correlates with the small radius in the same ensemble. Particle yields increase with temperature but a small volume decreases them, hence the correlation between the parameters.

Table 2 shows the $\chi^2$ values obtained for the three ensembles considered in this paper. The values confirm the earlier statement that these values throw doubt on the applicability of the thermal model in p-p collisions. Fixing $\gamma_s = 1$ does not change the physics message but considerably worsens the resulting $\chi^2$ values.

The fits to the hadronic yields obtained in p-p collisions at 7 TeV in five different centrality bins are shown in Figs 4.
FIG. 3. The chemical freeze-out radius obtained for three different ensembles. The black points were obtained using the grand canonical ensemble, the blue points uses exact strangeness conservation while the red points have built-in exact baryon number, strangeness and charge conservation. Circles are for p-p collisions at 7 TeV, squares are for p-Pb collisions at 5.02 TeV while triangles are for Pb-Pb collisions at 2.76 TeV.

and 5. The upper panels show the yields while the lower panels show the ratios of the measured data divided by the fit values for the three different ensembles considered here. The three lines corresponding to the fits are often very close to each other and overlap, hence they are not always visible on the figures.

IV. DISCUSSION AND CONCLUSIONS

In this paper we have investigated three different ensembles to analyze the variation of particle yields with the multiplicity of charged particles produced in proton-proton collisions at the center-of-mass energy of $\sqrt{s} = 7$ TeV [28], p-Pb collisions at 5.02 TeV [29, 30] and Pb-Pb collisions at 2.76 TeV [31–33].

We have kept the basic structure of the thermal model as presented in [24] and focused on the resulting thermal
parameters $T_{ch}, \gamma_s$ and the radius and their dependence on the final state multiplicity. We note in this regards that recent improvements on the treatment of the particle yields do not lead to substantial changes of the chemical freeze-out temperature, $T_{ch}$ [20, 22]. Our results show two new interesting features:

- a comparison of the grand canonical ensemble, the ensemble with strict strangeness conservation and the one with strict baryon number, strangeness and charge conservation agree very well for large systems like p-Pb and Pb-Pb, but show marked differences for p-p collisions. These differences tend to disappear as the multiplicity of charged particles increases in the final state of p-p collisions. This supports the fact that p-p collisions with high multiplicities agree with what is seen in large systems like Pb-Pb. Quantitatively this starts happening when there are more than 20 charged hadrons in the mid-rapidity interval being considered. It also throws doubt on the applicability of the thermal model in low multiplicity p-p collisions.

- The convergence of the results in the three ensembles lends support to the notion a thermodynamic limit is reached where results are independent of the ensemble being used.

We believe that it is of interest to note that all three ensembles lead to the same results when the multiplicity of charged particles $dN_{ch}/d\eta$ exceeds 20 at mid-rapidity. This could be interpreted as reaching the thermodynamic limit since the three ensembles lead to the same results. It would be of interest to extend this analysis to higher beam energies and higher multiplicity intervals.

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FIG. 4. Hadronic yields in p-p collisions at 7 TeV in centrality bins 1, 2 and 3 corresponding to 0 - 4.7 % (left panel) and 4.7 - 14 % (middle panel) and 14 - 28 % (right panel) respectively. The lower panel shows the ratio of experimental data divided by the fit results. The black points were obtained using the grand canonical ensemble, the blue points uses exact strangeness conservation while the red ones have built-in exact baryon number, strangeness and charge conservation. In some cases the bars overlap. The data are taken from Ref. [28].

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FIG. 5. Hadronic yields in p-p collisions at 7 TeV in centrality bins 4 and 5 corresponding to 28 - 48 % (left panel) and 48 - 100 % (right panel) respectively. The lower panel shows the ratio of experimental data divided by the fit results. The black bars were obtained using the grand canonical ensemble, the blue ones uses exact strangeness conservation while the red ones have been obtained using exact baryon number, strangeness and charge conservation. The data are taken from Ref. [28].

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