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Feynman Amplitude Approach to study the Passage of a Jet in a Medium

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Abstract. In the Feynman amplitude approach for coherent collisions of a jet with medium partons, the Bose-Einstein symmetry with respect to the interchange of the exchanged bosons leads to a destructive interference of the amplitudes in most regions of the phase space but a constructive interference in some other regions. As a consequence, there is a collective longitudinal momentum transfer to the scatterers along the jet direction, each scatterer carrying a substantial fraction of the incident jet longitudinal momentum. The manifestation of the Bose-Einstein interference may have been observed in angular correlations of hadrons associated with a high-$p_T$ trigger in high-energy collisions at RHIC and LHC.

1. Introduction
To study the process of multiple collisions of a jet with medium partons, it is important to distinguish coherent and incoherent collisions [1]. The scattering process between the jet and a medium parton can be described by the exchange of a gluon. Such an exchange takes time. One can infer the gluon exchange time, which can also be called the longitudinal coherence time, from the longitudinal momentum transfer in a single two-body collision. In the rest-frame of the medium, the longitudinal momentum transfer $q_z$ in each two-body scattering is related to the transverse momentum transfer $q_T$ and the incident jet momentum $p_{jet}$ by

$$q_z = q_T^2 / 2p_{jet}.$$ (1)

The corresponding longitudinal coherence time is

$$\Delta t_{coh} = \hbar / q_z c = 2\hbar p_{jet} / q_T^2 c.$$ (2)

If $\Delta t_{coh}$ is much greater than the mean-free time $\lambda / c$ between collisions, the exchange of one gluon is finished before another collision begins and the jet-(medium partons) collisions are incoherent. If $\Delta t_{coh} \ll \lambda / c$, the exchange of one gluon is not completed before another collision begins and these jet-(medium partons) collisions are coherent.

For a mini-jet of $p_{jet} \sim 10$ GeV and a typical transverse momentum transfer of $q_T \sim 0.4$ GeV at RHIC and LHC, we estimate that the longitudinal coherence time $\Delta t_{coh}$ is of order 25 fm/c, which is much greater than the mean-free time for multiple collisions. Jet-(medium parton) multiple collisions at RHIC and LHC are coherent collisions.

Conventional investigations on coherent collisions of a fast particle with medium scatterers use the potential model [2]-[10] in which the scatterers are represented by static potentials and the
longitudinal recoils of the scatterers are considered as dependent variables that are appendages to the deflected motion of the incident particle. However, the coherent collisions of $p$ with $n$ medium partons in the reaction $p + a_1 + a_2 + \ldots + a_n \rightarrow p' + a'_1 + a'_2 + \ldots + a'_n$ is a $(1 + n)$-body collision with $3(n+1)$ degrees of freedom, subject to an over-all four-momentum conservation. A general treatment of coherent collisions necessitates the use of the both the transverse and longitudinal recoil momenta of the scatterers as independent dynamical variables, which is not allowed in the potential model. This leads us to forgo the potential model and to turn to the Feynman amplitude approach for the general treatment of coherent collisions.

The Feynman amplitude gives the probability amplitude as a function of the initial and final particle momenta of the interacting particles. For a set of given initial momenta, a set of medium final recoil momenta can be reached by different paths in different Feynman diagrams. There are thus uncertainties in the specifications of the vertices along $\Delta z_{\text{coh}}$ at which various virtual bosons are emitted. By Bose-Einstein (BE) symmetry, the total Feynman amplitude is the sum of all amplitudes with all interchanges of the virtual boson vertices. The summation of these amplitudes and the accompanying interference constitute the Bose-Einstein interference in the passage of the jet in the dense medium.

2. Feynman Amplitude Approach and Bose-Einstein Interference

![Feynman diagrams](image)

**Figure 1.** Feynman diagrams for the collision of a fast fermion $p$ (a) with two medium fermions $a_1$, and $a_2$, and (b) with $n$ medium fermions $a_1$, $a_2$, $\ldots$, and $a_n$.

As an illustration of the Bose-Einstein interference of Feynman amplitudes, we consider first the example of the collision of a fermion $p$ with two fermion scatterers $a_1$ and $a_2$, in $p + a_1 + a_2 \rightarrow p' + a'_1 + a'_2$ in the Abelian gauge theory as shown in Fig. (1a). For simplicity, we assume the rest masses of the fermions to be the same and consider the high-energy limit in which $\{p_0, |p|, p'_0, |p'|\} \gg \{|a_i|, q_0, |q_i|\} \gg m$, for $i = 1, 2$. We assume conservation of helicity in high-energy collisions. The total amplitude $M$ is the symmetrized sum of the two diagrams in Fig. (1a). In the high-energy limit the total amplitude $M$ is

$$M(q_1, q_2) \sim g^4 \frac{2p \cdot \tilde{a}_1 2p \cdot \tilde{a}_2}{2m^3} \left( \frac{1}{2p \cdot q_1 - i\epsilon} + \frac{1}{2p \cdot q_2 - i\epsilon} \right) \frac{1}{q_2^2} \frac{1}{q_1^2},$$

where

$$\tilde{a}_i = \sqrt{\frac{a_{i0} + m a_i}{a_{i0} + m \frac{1}{2}}} + \sqrt{\frac{a'_{i0} + m a_i}{a'_{i0} + m \frac{1}{2}}}.$$  \hspace{1cm} (4)

The fermion $p'$ is outside the interaction region and is on the mass shell after the collision, leading to the constraint

$$(p - q_1 - q_2)^2 - m^2 \sim -2p \cdot q_1 - 2p \cdot q_2 \sim 0.$$  \hspace{1cm} (5)
Because of this mass-shell constraint (5), the two amplitudes in Eq. (3) are correlated with each other. The real parts of the amplitudes in Eq. (3) destructively cancel each other, leaving imaginary sharp distributions at $2p \cdot q_1$ and $2p \cdot q_2$,

$$M(q_1, q_2) \sim \frac{g^4}{2m} \frac{2p \cdot \tilde{a}_1}{m_1 q_1^2} \frac{2p \cdot \tilde{a}_2}{m_2 q_2^2} \left\{i\pi \Delta(2p \cdot q_1) + i\pi \Delta(2p \cdot q_2)\right\},$$  

(6)

where

$$\Delta(2p \cdot q_1) = \frac{1}{\pi} \left(\frac{\epsilon}{(2p \cdot q_1)^2 + \epsilon^2}\right),$$

which approaches the Dirac delta function $\delta(2p \cdot q_1)$ in the limit $\epsilon \to 0$.

Generalizing to the case of the coherent collision of a fast fermion with $n$ fermion scatterers in the process $p + a_1 + \ldots + a_n \to p' + a'_1 + \ldots + a'_n$, the total Feynman amplitude as given in Fig. (1b) is

$$M(q_1, q_2, \ldots, q_n) = \frac{g^{2n}}{2m} \left\{\prod_{i=1}^{n} \frac{2p \cdot \tilde{a}_i}{m_i q_i^2}\right\} \left\{\prod_{j=1}^{n-1} \frac{1}{\sum_{i=1}^{j} 2p \cdot q_i - i\epsilon} + \text{symmetric permutations}\right\} + \text{symmetric permutations},$$  

(7)

Using earlier results of Cheng and Wu [12] for the above sum of Feynman amplitudes with symmetric permutations,

$$\left\{\prod_{j=1}^{n-1} \frac{1}{\sum_{i=1}^{j} 2p \cdot q_i - i\epsilon} + \text{symmetric permutations}\right\} \Delta(\sum_{i=1}^{n} 2p \cdot q_i) = (2\pi i)^{n-1} \prod_{i=1}^{n} \Delta(2p \cdot q_i),$$

(8)

we obtain the sum Feynman amplitude $M$, including all the symmetric permutations of the exchanged bosons [1]

$$M(q_1, q_2, \ldots, q_n) \Delta(\sum_{i=1}^{n} 2p \cdot q_i) = \frac{g^{2n}(2\pi i)^{n-1}}{2m^{2n-1}} \prod_{i=1}^{n} \frac{2p \cdot \tilde{a}_i}{q_i^2} \Delta(2p \cdot q_i).$$

(9)

This can be alternatively rewritten as

$$M(q_1, q_2, \ldots, q_n) = \frac{g^{2n}(2\pi i)^{n-1}}{2m^{2n-1}} \left(\prod_{i=1}^{n} \frac{2p \cdot \tilde{a}_i}{q_i^2}\right) \sum_{j=1}^{n} \left\{\prod_{i=1, i\neq j}^{n} \Delta(2p \cdot q_i)\right\}. $$

(10)

Equation (10) for the Feynman amplitude as a sum of the product of delta functions of $2p \cdot q_i$, is similar to previous Bose-Einstein interference results obtained for the emission of many photons or gluons in bremsstrahlung and for the sum of ladder and cross-ladder amplitudes in the collision of two fermions [11]-[16].

The above considerations for the Abelian theory can be extended to the non-Abelian theory [1]. For the coherent collision of an energetic parton with parton scatterers in non-Abelian cases, we find that the complete Bose-Einstein symmetry in the exchange of virtual gluons consists not only of space-time exchange symmetry but also color index exchange symmetry. Nevertheless, there is always a space-time symmetric and color-index symmetric component of the Feynman amplitude that behaves in the same way as the Feynman amplitude in the Abelian case. For this space-time symmetric and color-index symmetric component, the recoiling partons behave in the same way as in collisions in the Abelian case. As the amplitude of this component involves products of the singular delta functions, it is also the dominant component and provides the dominant contribution to the total Feynman amplitude.

For gluon scatterers, there is a minor modification in the longitudinal momentum distribution with $\tilde{a}_i$ of quark scatterers in Eq. (10) replaced by [1]

$$\bar{a}_i = \frac{a_i + a'_i}{2}.$$  

(11)
3. Consequences of the Bose-Einstein Interference

The total Feynman amplitude $M$ in (10) reveals that the BE interference of the Feynman amplitudes gives rise to delta-function type constraints $\Delta(2p \cdot q_i)$, which impose the conditions

$$q_{i0} - q_{iz} = \frac{p_T \cdot q_{iT}}{p_{jet}} \sim 0 \quad \text{or} \quad q_{i0} \sim q_{iz}. \quad (12)$$

The gluon propagator $1/q^2_i$ in Eq. (10) becomes

$$\frac{1}{q^2_i} = \frac{1}{q_{i0}^2 - q_{iz}^2 - |q_{iT}|^2} \sim \frac{1}{|q_{iT}|^2}. \quad (13)$$

We obtain the important result that as a consequence of the BE interference of the Feynman amplitudes, the scatterers tend to come out at small $q_{iT}$ peaking at zero, with $q_{i0} \sim q_{iz}$.

To proceed further, we need the distribution of the longitudinal momentum transfer $q_{iz}$ of the scatterers, which is determined by the Feynman amplitude $M$ and the phase-space factors via the differential cross section given by [1]

$$d^4\sigma = \frac{|M|^2(2\pi)^4\delta^4(p + \sum_{i=1}^n a_i - p' - \sum_{i=1}^n a'_i)}{\prod_{i=1}^n f_{p_i}\prod_{i=2}^n (m/p_{io}) \prod_{i=1}^n T_i} \left\{ \frac{d^4p'2mD_{p'}(p')}{(2\pi)^3} \right\} \left\{ \prod_{i=1}^n \frac{d^4a'_i2m_iD_i(a'_i)}{(2\pi)^3} \right\}, \quad (14)$$

where $f_{p_i}$ is the flux factor for the collision between $p_i$ and the scatterer $a_i$ of rest mass $m_i$,

$$f_{p_i} = \frac{4\sqrt{(p_i \cdot a_i)^2 - (m m_{iT})^2}}{(2m)(2m_i)}. \quad (15)$$

and $m_{iT} = \sqrt{m_i^2 + a_{iT}^2}$. The final fast particle $p'$ resides outside the medium and its state can be described as being on the mass shell,

$$\frac{D_{p'}(p')}{2p'_0} \sim \frac{\delta(p'_0 - \sqrt{p'^2 + m^2})}{2p'_0} = \delta(p'^2 - m^2). \quad (16)$$

The states of the medium scatterer after the collision, $a'_i$, can be described by

$$D_i(a'_i) = \frac{\Gamma_i/2\pi}{[a'_{i0} - \sqrt{(a'_i - q_i A)^2 + (m_i + S)^2 + g A_0]^2 + \Gamma_i/4]} \quad \text{for} \quad i = 2, \ldots, n,$$

where $A = \{A, A_0\}$ and $S$ are the vector and scalar mean fields experienced by the medium scatterer $a'_i$ after the collision, respectively. As the mean fields and scatterer widths increase with medium density and are presumably quite large and dominant for a dense medium, we shall approximately represent $D_i(a'_i)$ as an average constant that is only a weak function of $a'_i$. Other descriptions of $D_i(a'_i)/2a_{i0}$ for the states of the scatterers are also possible but may not be as general; they can be the subjects for future investigations.

In the high-energy limit, the constraint $\Delta(2p \cdot q_i)$ is the same as the constraint $\Delta((p - q_i) - m^2)$ which can be interpreted as an intermediate state $p_{i+1}=(p - q_i)$ appearing as a quasi-particle on the mass shell as a result of the interference of many amplitudes. The quantity $T_i$ with $\{i = 2, \ldots, n\}$ in Eq. (14) is the mean lifetime of the intermediate quasi-particle state $p_i$ of the incident fast particle, prior to its exchanging a virtual boson with $a_i$.

We change variables from $a'_i$ to $q_i = a'_i - a_i$, integrate over $d^3p'$, and we obtain

$$d^0\sigma = \frac{1}{4} \left( \frac{\alpha^2}{mp_{jet}} \right)^n \delta(p'_0 + \sum q_{i0} - p_0) \left\{ \prod_{i=1}^n \frac{8D_i(2p' \cdot \tilde{a}_i)^2 dq_{i0} dq_{iT}}{m2a_{i0} \cdot |q_{iT}|^4} \right\}, \quad (17)$$
We introduce the fractional longitudinal momentum kick

\[ x_i = \frac{q_i}{p_{jet}}, \quad dq_i = p_{jet} dx_i, \tag{18} \]

we obtain then approximately the differential cross section

\[ d^n \sigma \sim C f(x_1, x_2, x_3, \ldots, x_n) \delta(1 - x_1 - x_2 - x_3 - \ldots - \frac{p_{jet}'}{p_{jet}}) \prod dx_i dq_i T \delta(q_{iT}^1)^4 \delta(q_{iT}^2)^4 \ldots \delta(q_{iT}^n)^4, \tag{19} \]

where \( C \) is a weak function of \( x_i \) and \( q_{iT} \), and \([1]\)

\[ f(x_1, x_2, x_3, \ldots, x_n) \sim \begin{cases} 1 & \text{for quark scatterers,} \\ \frac{1}{x_1 x_2 x_3 \ldots x_n} & \text{for gluon scatterers.} \end{cases} \tag{20} \]

The average longitudinal momentum fraction is approximately

\[ \langle x_i \rangle \sim \begin{cases} \frac{1}{2n} & \text{for quark scatterers,} \\ 1 - \frac{m_g T}{p_{jet}} / \left( n \cosh^{-1} \left( \frac{p_{jet}}{nm_g T} \right) \right) & \text{for gluon scatterers.} \end{cases} \tag{21} \]

Equation (21) indicates that the ratio of the average longitudinal momentum transfer \( q_i \) to the incident jet momentum \( p_{jet} \) is approximately inversely proportional to \( n \). The longitudinal momentum transfer \( q_i \) is much greater than \( |q_{iT}| \), as the transverse momentum transfer peaks at small values of \( |q_{iT}| \). As a consequence, the angular direction of the momentum transfer is predominantly along the jet direction. Thus, in a coherent collision there is a collective quantum many-body effect arising from Bose-Einstein interference such that each scatterer receives a substantial momentum transfer, predominantly along the jet direction, with a magnitude that is approximately inversely proportional to the number medium scatterers in the collision, \( n \).

4. Signatures of Bose-Einstein Interference

The results in the above sections provide information on the signatures for the occurrence of the Bose-Einstein interference in the coherent collision of a jet with medium partons \([1]\):

(i) The medium scatterers recoil collectively.

(ii) Each scatterer acquires a longitudinal momentum kick \( q_z \) predominantly along the incident jet direction that is approximately inversely proportional to the number of scatterers \( n \).

(iii) The Bose-Einstein interference is a quantum many-body effect. It occurs only in the multiple collision of the fast jet with two or more scatterers. Therefore there is a threshold, corresponding to the requirement of two or more scatterers in the multiple collision, \( n \geq 2 \).

To inquire whether Bose-Einstein interference may correspond to any observable physical phenomenon, it is necessary to separate the incident jet from the scatterers. Such a separation is kinematically possible in \( \Delta \phi - \Delta \eta \) angular correlation measurements of produced pairs with a high-\( p_T \) trigger \([17]-[44]\). In Figs. 2, we show the angular correlations with a high-\( p_T \) trigger for the most central collisions: (a) PHOBOS data of Au-Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV at RHIC \([39]\), (b) CMS data of pp collisions at \( \sqrt{s_{NN}} = 7 \) TeV at LHC \([40]\), and (c) CMS data of PbPb collisions at \( \sqrt{s_{NN}} = 2.76 \) TeV at LHC \([41]\). Particles in the “jet” part of the correlation measurement with \( \Delta \eta \sim 0 \) and \( \Delta \phi \sim 0 \) in a narrow cone can be identified as arising predominantly from the fragmentation of a jet. As the trigger particle has a \( p_T \) of order a
Figure 2. $\Delta \phi - \Delta \eta$ correlation of produced hadrons in the most central collisions with a high $p_T$ trigger: (a) PHOBOS data of AuAu collisions at $\sqrt{s_{NN}} = 200$ GeV at RHIC [39], (b) CMS data of $pp$ collisions at $\sqrt{s_{NN}} = 7$ TeV at LHC [40], and (c) CMS data of PbPb collisions at $\sqrt{s_{NN}} = 2.76$ TeV at LHC [41].
by comparing jet fragment yields in AuAu and pp collisions, then \( q_z \sim 0.8 - 1.0 \text{ GeV} \) and \( \langle n \rangle \sim 6 \) for the most central AuAu collisions at \( \sqrt{s_{NN}} = 200 \text{ GeV} \). In another momentum kick model analysis for the highest multiplicity pp collisions at \( \sqrt{s_{NN}} = 7 \text{ TeV} \) at the LHC, the momentum kick model gives \( q_z \sim 2 \text{ GeV} \) and \( f_R \langle n \rangle \sim 1.5 \) corresponding approximately to \( \langle n \rangle \sim 2.4 \). The experimental data give an average longitudinal momentum transfer \( q_z \) that is approximately inverse proportional to the number of scatterers \( \langle n \rangle \), in approximate agreement with the second signature of the Bose-Einstein interference.

The third signature for the occurrence of the Bose-Einstein interference is the presence of a threshold at \( n = 2 \). The collective recoil of the scatterers along the jet direction occurs only when the number of scatterers exceed two. This signature will indicate a sudden onset of the ridge yield as a function of an increase in the number of kicked medium scatterers, \( n \). The number of kicked medium scatterers \( n \) increases with centrality, as can be represented either by an increase in the number of participants \( N_{\text{part}} \), or by an increase in the multiplicity number \( N_{\text{mult}} \). We need a description of the number of medium parton scatterers \( n \), the participant number \( N_{\text{part}} \), or the multiplicity number \( N_{\text{mult}} \), as a function of centrality.

Previously, we have worked out the number of medium parton scatterers, the number of participants, and parton energy loss, as a function of centrality for nucleus-nucleus collisions using a geometrical model of jet-(medium parton) collisions [48]. The basic assumption is a jet-medium parton cross section, the systematics of the density of the hot medium, and the behavior of the fragmentation function as a function of energy loss. The available data of \( R_{AA} \) as a function of centrality and the number of medium scatterers at the most central collision as inferred from the correlation data provide the needed reference data to determine the centrality dependency of the number of medium scatterers. In the momentum kick model, as it is assumed that the associated particle yield is proportional to the number of scatterers, the momentum kick model can predict the centrality dependency of the associated ridge yield.

We show in Fig. 3 and the experimental ridge yield per high-\( p_T \) trigger as a function of \( N_{\text{part}} \) for AuAu and CuCu collisions at \( \sqrt{s} = 200 \) and 62 GeV at RHIC [20, 24, 39]. We also show in Fig. 3 the theoretical yields obtained in the momentum kick model [48], where the ridge yield at the most central collision at \( N_{\text{part}} = 320 \) for AuAu collision at \( \sqrt{s_{NN}} = 200 \text{ GeV} \) was calibrated as \( n = 6 \) [48]. With such a calibration, the threshold values in \( N_{\text{part}} \) at which \( n = 2 \) can be located. Theoretical ridge yields from the momentum kick model analysis, modified to include

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**Figure 3.** The ridge yield per high-\( p_T \) trigger as a function of the participant number \( N_{\text{part}} \) for nucleus-nucleus collisions at \( \sqrt{s_{NN}} = 200 \) and 62 GeV [31, 39]. The curves are the momentum kick model results of [48] modified to include the Bose-Einstein interference threshold effect of \( n \geq 2 \).
the Bose-Einstein interference threshold effect of \( n \geq 2 \), are shown as the solid curves for AuAu collisions, and as dashed curves for CuCu collisions in Fig. 3. The theoretical thresholds in Fig. 3 will be smoothed out by the uncertainties in the estimates of the number of scatterers and the fluctuations of the number of scatterers as a function of \( N_{\text{part}} \). Although the experimental data appear to be consistent with theory and with the presence of thresholds, the large error bars in the STAR data in Figs. 3(a) and 3(b) preclude a definitive conclusion. On the other hand, the PHOBOS measurement as shown in Fig. 3(c) indicates more clearly a threshold at \( N_{\text{part}} \sim 70 \), in agreement with the predicted \( n \gtrsim 2 \) threshold for the occurrence of the collective medium parton recoil that is a signature for the occurrence of the BE interference.

For the ridge yield as a function of multiplicity in \( pp \) collisions, it is necessary to determine the centrality dependence of both the ridge yield and the charge multiplicity. In the momentum kick model, the ridge yield is proportional to the number of kicked medium partons. The (average) number of kicked medium partons per jet, \( f_R\langle n(b) \rangle \), can be evaluated as a function of the impact parameter \( b \) for \( AA \) collisions [48, 49]. We apply a similar analysis to \( pp \) collisions by treating the colliding protons as extended droplets as in the Chou-Yang model [54]. For the multiplicity number of produced particles as a function of centrality, we evaluate the participant droplet elements as a function of centrality and assume that the multiplicity is proportional to the participant droplet elements. By combining both number of kicked medium partons and the multiplicity number as a function of centrality, we obtain the momentum kick model results of the associated ridge yield as a function of multiplicity shown in Fig. 4. The data points are from the CMS Collaboration for \( pp \) collisions at \( \sqrt{s_{NN}} = 7 \) TeV. The dashed curves are the momentum kick model results without a threshold, and the solid curves are the results of the momentum kick model with the assumption of a threshold at \( n = 2 \). Results in Fig. 4 indicate the possible presence of a ridge threshold at \( n = 2 \), in suggestive support of the third signature for the occurrence of the Bose-Einstein interference.

6. Summary and Conclusions
A general treatment of coherent collisions necessitates the use of both the transverse and the longitudinal recoil momenta of the scatterers as independent dynamical variables. In the potential model [2, 3], however, the longitudinal momenta transfers cannot be independent...
dynamical variables. This leads us to forgo the potential model and to use the Feynman amplitude approach for coherent collisions.

In the Feynman amplitude approach for the coherent collisions of a fast particle on \( n \) scatterers, there are \( n! \) different orderings in the sequence of collisions along the fast particle path at which various virtual bosons are exchanged. By Bose-Einstein symmetry, the total Feynman amplitude is the sum of the \( n! \) amplitudes for all possible interchanges of the virtual bosons. The summation of these \( n! \) Feynman amplitudes and the accompanying interference constitute the Bose-Einstein interference in the passage of the fast particle in the dense medium.

Our interest in examining this problem has been stimulated by the phenomenological successes of the momentum kick model in the analysis of the angular correlations of hadrons produced in high-energy heavy-ion collisions [45]-[52]. We seek a theoretical foundation for the origin of the longitudinal momentum kick along the jet direction postulated in the model. We find that in the coherent collisions of an energetic fermion with \( n \) fermion scatterers at high energies in the Abelian theory, the symmetrization of the Feynman scattering amplitudes with respect to the interchange of the exchanged bosons leads to the Bose-Einstein interference, resulting in a sharp distributions at \( p \cdot q_i \sim 0 \). The Bose-Einstein symmetry constraints of \( p \cdot q_i \sim 0 \) limit the transverse momentum transfers of the scatterers to small values of \( q_{iT} \). The longitudinal momenta of the scatterers get their share of longitudinal momenta from the jet, resulting in the collective momentum transfer to the scatterers along the jet direction.

For coherent collisions of an energetic parton with parton scatterers in non-Abelian cases, we find that the complete Bose-Einstein symmetry in the exchange of virtual gluons consists not only of space-time exchange symmetry but also color index exchange symmetry. Nevertheless, there is always a space-time symmetric and color-index symmetric component of the Feynman amplitude that behaves in the same way as the Feynman amplitude in the Abelian case. Furthermore, as these amplitudes involve products of singular delta functions, they are also the dominant components. There is thus a finite probability for the parton scatterers to acquire a momentum transfer along the trigger jet direction, each carrying a significant fraction of the longitudinal momentum of the incident jet. The collective recoil has a threshold, requiring at least two scatterers for the occurrence of the quantum interference. Theoretical analysis of the magnitude of the Feynman diagram matrix elements and phase space factors indicates that the average magnitude of the momentum kick should be approximately inversely proportional to the number of scatterers.

In high-energy nuclear collisions at RHIC and LHC, an incident high-\( p_T \) jet and the scatterers can be kinematically separated by angular correlation measurements. A comparison of the jet and ridge components reveals the presence of a collective momentum transfer to the scatterers along the jet direction, and a phenomenological analysis using the momentum kick model supports the approximate inverse proportionality relation between the magnitude of the longitudinal momentum kick and the number of scatterers. The empirical ridge yield as a function of centrality suggests the possible presence of a threshold at \( n \sim 2 \), as required for the quantum Bose-Einstein interference. Thus, the manifestation of the Bose-Einstein interference effects may have been experimentally observed in the \( \Delta \phi - \Delta \eta \) angular correlation of hadrons associated with a high-\( p_T \) trigger in high-energy collisions at RHIC and LHC.

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