Mathematical model for spreading of COVID-19 virus with the Mittag–Leffler kernel

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Abstract
In the Nidovirales order of the Coronaviridae family, where the coronavirus (crown-like spikes on the surface of the virus) causing severe infections like acute lung injury and acute respiratory distress syndrome. The contagion of this virus categorized as severed, which even causes severe damages to human life to harmless such as a common cold.

In this manuscript, we discussed the SARS-CoV-2 virus into a system of equations to examine the existence and uniqueness results with the Atangana–Baleanu derivative by using a fixed-point method. Later, we designed a system where we generate numerical results to predict the outcome of virus spreadings all over India.

KEYWORDS
AB-derivative, coronavirus, fixed-point techniques, fractional calculus, mathematical models

1 | INTRODUCTION

The on-going pandemic of COVID-19, viral pneumonia break out in late 2019 in Wuhan, China, which has its spread worldwide across 210 nations named as “SARS-CoV-2.” It is raging around the world with an immense toll in terms of human, economic, and social impact. Within a short span, it raises an alert in every country all over the world like a pandemic disease, which urges every nation to forecast their precautionary actions to control and contain the wild spread of the virus as the severity of the disease will harm human life badly. Since the novel coronavirus is new to the world to forecast some impact of the pandemic situation and to build a mitigation plan the similarity effects of Severe Acute Respiratory Syndrome (SARS) and Middle East Respiratory Syndrome epidemics in 2003 and
2009 were used for study and analysis. From the study of the initial spread of COVID-19, many of the mathematical models were came into the act from contributors across the world to determine the severity of the spread. Whenever a contagious disease extends its tributary, it follows certain patterns of spread which widely help us to identify and monitor the dynamics of the disease outbreaks. The method we used to estimate the spread of the disease is a factor that drives us to finalize the measures to get rid of infectious diseases. The outbreak of the disease within the country or state for time is usually nonlinear, which propels us to design the system where we can study those dynamic nonlinear phenomena. By this system, we can able to define the transmission of such contagious disease, which helps us to interpret the remedial measures to stop or contain the spread of contagious disease.

In recent years, fractional differentiation has been drawing increasing attention in the study of social and physical behaviors where scaling power law of fractional order appears universal as an empirical description of such complex phenomena. The fractional-order models are more adequated than the previously used integer-order models because fractional-order derivatives and integrals describe the memory and hereditary properties of different phenomena [1].

Koca [2] investigated the Ebola virus spreading within a particular place of the population by $AB$-derivative. Dokuyucu and Dutta [3] discussed the model for Ebola virus with the Caputo derivative without a singular kernel in the fractional order. Dong et al. [4] derived a model for the granular SEIR epidemic with fractional order. Babaei et al. [5] investigated the impact of therapy treatments on the control of HIV/AIDS spread in prisons, and there is no cure, or some treatment methods are available. Baleanu et al. [6] studied a Rubella disease model with Caputo–Fabrizio derivative. Danane et al. [7] discussed Hepatitis B virus infection with antibody immune response. Bekiros and Kouloumpou [8] described the infectious disease dynamics with the SBDiEM model. Moore et al. [9] studied the Caputo–Fabrizio derivative to the treatment compartment for the HIV/AIDS. Xiao and Chen [10] analyzed a predator–prey model with the disease. Ucar [11] discussed the existence and uniqueness result for a smoking model with nonsingular derivatives. Agarwal and Singh [12] modeled the transmission dynamics of the Nipah virus of fractional order. Khan et al. [13] studied the Caputo–Fabrizio derivative for modeling the dynamics of hepatitis. Veeresha et al. [14] analyzed the numerical solution of Schistosomiasis disease in biological phenomena. Khan et al. [15] used the nonsingular Mittag–Leffler Law for the HIV-TB co-infection model. For more references to the $AB$-derivative, the reader can refer References [16–26].

Atangana [27] constructed a model for the effect of lockdown by proposing new fractal fractional differential and integral operators. Khan and Atangana [28] discussed the novel coronavirus (2019-nCov) with $AB$-derivative. Shaikh et al. [29, 30] described and analyzed the pandemic spreading of COVID-19, the outbreak in India with fractional derivative. Arino and Portet [31] described the spreading of COVID-19 in the population by the SLIAR epidemic model. Abdo et al. [32] studied the existence and stability of the novel coronavirus (COVID-19) model. Fanelli and Piazza [33] analyzed of the spreading of COVID-19 in China, Italy, and France. Memon et al. [34] discussed the epidemiological system using real incidence data from Pakistan. Valentim et al. [35] described a fractional calculus to improve tumor growth models. Luo et al. [36] investigated an epidemic model for pulse vaccination strategy. Gao et al. [37] analyzed the pulse vaccination and saturation incidence of a delayed epidemic model. Veeresha et al. [38, 39] discussed the q-homotopy analysis transform method and fractional natural decomposition method in COVID-19 model, and refer References 40–56 for models.

Here, we investigate the mathematical model for spreading of COVID-19 virus in the world with Atangana–Baleanu fractional derivative. Consider the following systems:
In last section, we illustrate the numerical solutions for Equations (1.1) by using $AB$-derivative in graphical method.

\[ \left\{ \begin{align*}
(0)^{ABC}D_t^\zeta(S_p(t)) &= \Delta_b - \lambda_d S_p - \frac{a_S(I_e + \beta_A A_p)}{N} - \gamma_Q S_p Q_p \\
(0)^{ABC}D_t^\zeta(E_p(t)) &= \frac{a_S(I_e + \beta_A A_p)}{N} + \gamma_Q S_p Q_p - (1 - \phi_A) \delta_{RI} E_p - \phi_A \mu_{EI} E_p - \lambda_d E_p \\
(0)^{ABC}D_t^\zeta(I_p(t)) &= (1 - \phi_A) \delta_{RI} E_p - (\sigma_{RI} + \lambda_d) I_p \\
(0)^{ABC}D_t^\zeta(A_p(t)) &= \phi_A \mu_{EI} E_p - (\rho_{RA} + \lambda_d) A_p \\
(0)^{ABC}D_t^\zeta(R_p(t)) &= \sigma_{RI} I_p + \rho_{RA} A_p - \lambda_d R_p \\
(0)^{ABC}D_t^\zeta(Q_p(t)) &= \kappa_{1Q} I_p + \nu_{AQ} A_p - \eta_Q Q_p \\
\end{align*} \right. \]

where $0 < \zeta < 1$, $N$ represents the total population of people, $S_p(t)$ represents susceptible people, $E_p(t)$ represents exposed people, $I_p(t)$ represents infected people, $A_p(t)$ represents asymptotically infected people, $R_p(t)$ represents recovered or removed people, $Q_p(t)$ represents people in the reservoir or market (people affected directly by seafood), $\Delta_b$ and $\lambda_d$ represent birth and natural death rate, $\alpha_1$ is disease transmission coefficient, $\beta_A$ is transmission multiple (asymptomatically infected), $\gamma_Q$ is disease transmission coefficient from seafood places to susceptible people, $\phi_A$ is proposition of asymptomatic infection, $\sigma_{RI}$ is removal rate, $\rho_{RA}$ is recovery rate, $\eta_Q$ is removing rate of viruses from the seafood market, $\mu_{EI}$ is transmission rate of becomes infected, $\delta_{RI}$ is transmission rate after completing the incubation period, $\nu_{AQ}$ is asymptotically infected directly contributing the virus from market, and $\kappa_{1Q}$ is infected symptoms of the virus from market.

In Section 2, we described about essential results and propositions, which will be important for main problems. In Section 3, we derived the solution for the system of Equation (1.1). In Section 4, we discussed the existence results. Uniqueness results for the above system of equation derived in Section 5. In last section, we illustrate the numerical solutions for Equations (1.1) by using $AB$-derivative in graphical method.

### 2 | PRELIMINARIES

- The left R–L integral is [1, 57]

\[ (0)^{\zeta}I^x(t) = \frac{1}{\Gamma(\zeta)} \int_0^t (t-s)^{\zeta-1}x(s)ds, \]

where $\zeta > 0$.

- The left R–L derivative is [1, 57]

\[ (0)^{\zeta}D^x(t) = \frac{d}{dt} \left( \frac{1}{\Gamma(1-\zeta)} \int_0^t (t-s)^{-\zeta}x(s)ds \right), \]

where $0 < \zeta < 1$.

- The Caputo derivative is [1, 57]

\[ (0)^{\zeta}D^x(t) = \frac{1}{\Gamma(1-\zeta)} \int_0^t (t-s)^{-\zeta}x'(s)ds, \]

where $0 < \zeta < 1$.

- The Caputo $AB$-derivative is [58]
\( (\frac{A^B}{0} D^\xi x)(t) = \frac{B(\xi)}{1 - \xi} \int_0^t x'(s) E_\xi \left[ -\frac{\xi}{1 - \xi} (t - s)^\xi \right] ds, \)

where \( \zeta \in [0, 1], \ x' \in H'(a, b), \ a \leq b, \) and \( B(\zeta) \) is a normalizing positive function satisfying \( B(0) = B(1) = 1. \)

- The associative fractional integral of (2.1) is

\[
(\frac{A^B}{0} I^\xi x)(t) = \frac{1 - \xi}{B(\xi)} x(t) + \frac{\xi}{B(\xi)} (\frac{A^B}{0} I^\xi x)(t), \tag{2.2}
\]

where \( \frac{A^B}{0} I^\xi \) is the left R–L integral given in (2.1).

**Proposition 2.1** For \( 0 < \zeta < 1, \) we conclude that [59, 60]

\[
(\frac{A^B}{0} I^\xi (\frac{A^B}{0} D^\xi x))(t) = x(t) - x(0) E_\xi (\lambda t^\xi) - \frac{\xi}{1 - \xi} x(0) E_{\xi, \zeta + 1}(\lambda t^\xi) = x(t) - x(0).
\]

### 3 | SOLUTION PART

By using \( AB \)-fractional integral on both side of (1.1)

\[
\begin{align*}
\frac{A^B}{0} I^\xi (\frac{A^B}{0} D^\xi f)(S_p(t)) &= \frac{A^B}{0} I^\xi \left[ \Delta_b - \lambda_d S_p - \frac{a_s S_p (I_p + \beta I_s A_p)}{N} - \gamma Q S_p Q_p \right] \\
\frac{A^B}{0} I^\xi (\frac{A^B}{0} D^\xi f)(E_p(t)) &= \frac{A^B}{0} I^\xi \left[ \frac{a_s S_p (I_p + \beta I_s A_p)}{N} + \gamma Q S_p Q_p - (1 - \phi_a) \delta R I_p - \phi A E I_p - \lambda_d E_p \right] \\
\frac{A^B}{0} I^\xi (\frac{A^B}{0} D^\xi f)(I_p(t)) &= \frac{A^B}{0} I^\xi [(1 - \phi_a) \delta R I_p - (\sigma_R I + \lambda_d) I_p] \\
\frac{A^B}{0} I^\xi (\frac{A^B}{0} D^\xi f)(A_p(t)) &= \frac{A^B}{0} I^\xi [\phi A E I_p - (\rho_R A + \lambda_d) \Delta_p] \\
\frac{A^B}{0} I^\xi (\frac{A^B}{0} D^\xi f)(R_p(t)) &= \frac{A^B}{0} I^\xi [\sigma_R I_p + \rho_R A \Delta_p - \lambda_d R_p] \\
\frac{A^B}{0} I^\xi (\frac{A^B}{0} D^\xi f)(Q_p(t)) &= \frac{A^B}{0} I^\xi [\kappa I_q I_p + \nu A Q p - \eta Q Q p].
\end{align*}
\]

By using proposition, we get

\[
\begin{align*}
S_p(t) - S_p(0) &= \frac{A^B}{0} I^\xi \left[ \Delta_b - \lambda_d S_p - \frac{a_s S_p (I_p + \beta I_s A_p)}{N} - \gamma Q S_p Q_p \right] \\
E_p(t) - E_p(0) &= \frac{A^B}{0} I^\xi \left[ \frac{a_s S_p (I_p + \beta I_s A_p)}{N} + \gamma Q S_p Q_p - (1 - \phi_a) \delta R I_p - \phi A E I_p - \lambda_d E_p \right] \\
I_p(t) - I_p(0) &= \frac{A^B}{0} I^\xi [(1 - \phi_a) \delta R I_p - (\sigma_R I + \lambda_d) I_p] \\
A_p(t) - A_p(0) &= \frac{A^B}{0} I^\xi [\phi A E I_p - (\rho_R A + \lambda_d) \Delta_p] \\
R_p(t) - R_p(0) &= \frac{A^B}{0} I^\xi [\sigma_R I_p + \rho_R A \Delta_p - \lambda_d R_p] \\
Q_p(t) - Q_p(0) &= \frac{A^B}{0} I^\xi [\kappa I_q I_p + \nu A Q p - \eta Q Q p].
\end{align*}
\]

Substitute the initial conditions to the above system of equation becomes

\[
\begin{align*}
S_p(t) - S_p(0) &= \frac{A^B}{0} I^\xi \left[ \Delta_b - \lambda_d S_p - \frac{a_s S_p (I_p + \beta I_s A_p)}{N} - \gamma Q S_p Q_p \right] \\
E_p(t) - E_0 &= \frac{A^B}{0} I^\xi \left[ \frac{a_s S_p (I_p + \beta I_s A_p)}{N} + \gamma Q S_p Q_p - (1 - \phi_a) \delta R I_p - \phi A E I_p - \lambda_d E_p \right] \\
I_p(t) - I_0 &= \frac{A^B}{0} I^\xi [(1 - \phi_a) \delta R I_p - (\sigma_R I + \lambda_d) I_p] \\
A_p(t) - A_0 &= \frac{A^B}{0} I^\xi [\phi A E I_p - (\rho_R A + \lambda_d) \Delta_p] \\
R_p(t) - R_0 &= \frac{A^B}{0} I^\xi [\sigma_R I_p + \rho_R A \Delta_p - \lambda_d R_p] \\
Q_p(t) - Q_0 &= \frac{A^B}{0} I^\xi [\kappa I_q I_p + \nu A Q p - \eta Q Q p].
\end{align*}
\]

(3.1)
Using (2.2) in (3.2), we get

\[
Z_1(t, S_p) = \Delta_b - \lambda_d S_p - \frac{\alpha_1 S_p(I_p + \beta_A A_p)}{N} - \gamma Q S_p Q_p
\]

\[
Z_2(t, E_p) = \frac{\alpha_1 S_p(I_p + \beta_A A_p)}{N} + \gamma Q S_p Q_p - (1 - \phi_A) \delta_R E_p - \phi_A \mu_E E_p - \lambda d E_p
\]

\[
Z_3(t, I_p) = (1 - \phi_A) \delta_R I_p - (\sigma_R + \lambda_d) I_p
\]

\[
Z_4(t, A_p) = \phi_A \mu_E E_p - (\rho_R + \lambda_d) A_p
\]

\[
Z_5(t, R_p) = \sigma_R I_p + \rho_R A_p - \lambda d R_p
\]

\[
Z_6(t, Q_p) = \kappa I Q I_p + \nu A Q A_p - \eta Q Q_p.
\]

By using the above consideration in system of Equation (3.1), we get

\[
S_p(t) - S_0 = \frac{AB}{0} \int Z_1(t, S_p) \text{d}t
\]

\[
E_p(t) - E_0 = \frac{AB}{0} \int Z_2(t, E_p) \text{d}t
\]

\[
I_p(t) - I_0 = \frac{AB}{0} \int Z_3(t, I_p) \text{d}t
\]

\[
A_p(t) - A_0 = \frac{AB}{0} \int Z_4(t, A_p) \text{d}t
\]

\[
R_p(t) - R_0 = \frac{AB}{0} \int Z_5(t, R_p) \text{d}t
\]

\[
Q_p(t) - Q_0 = \frac{AB}{0} \int Z_6(t, Q_p) \text{d}t.
\]

Using (2.2) in (3.2), we get

\[
S_p(t) - S_0 = \frac{1-\xi}{B(\xi)} \int Z_1(t, S_p) \text{d}t + \frac{\xi}{B(\xi)} \int Z_1(t, S_p) \text{d}t
\]

\[
E_p(t) - E_0 = \frac{1-\xi}{B(\xi)} \int Z_2(t, E_p) \text{d}t + \frac{\xi}{B(\xi)} \int Z_2(t, E_p) \text{d}t
\]

\[
I_p(t) - I_0 = \frac{1-\xi}{B(\xi)} \int Z_3(t, I_p) \text{d}t + \frac{\xi}{B(\xi)} \int Z_3(t, I_p) \text{d}t
\]

\[
A_p(t) - A_0 = \frac{1-\xi}{B(\xi)} \int Z_4(t, A_p) \text{d}t + \frac{\xi}{B(\xi)} \int Z_4(t, A_p) \text{d}t
\]

\[
R_p(t) - R_0 = \frac{1-\xi}{B(\xi)} \int Z_5(t, R_p) \text{d}t + \frac{\xi}{B(\xi)} \int Z_5(t, R_p) \text{d}t
\]

\[
Q_p(t) - Q_0 = \frac{1-\xi}{B(\xi)} \int Z_6(t, Q_p) \text{d}t + \frac{\xi}{B(\xi)} \int Z_6(t, Q_p) \text{d}t.
\]

The above system of Equations (3.3) is the solution of the systems of Equations (1.1).
4 | EXISTENCE SOLUTIONS

**Theorem 4.1** The kernels $Z_1, Z_2, Z_3, Z_4, Z_5,$ and $Z_6$ satisfy the Lipschitz condition and contraction if the following inequality holds:

$$0 \leq a_1 < 1$$
$$0 \leq a_2 < 1$$
$$0 \leq a_3 < 1$$
$$0 \leq a_4 < 1$$
$$0 \leq a_5 < 1$$
$$0 \leq a_6 < 1.$$

**Proof.** Consider $Z_1(t, S_p) = \Delta_b - \lambda_d S_p - \frac{a_3 S_p (I_p + \beta A_p)}{N} - \gamma_{QS} Q_p$.

Let $S_p$ and $S_{p1}$ be two function, so we have following:

$$\|Z_1(t, S_p) - Z_1(t, S_{p1})\| = \| \Delta_b - \lambda_d S_p - \frac{a_3 S_p (I_p + \beta A_p)}{N} - \gamma_{QS} Q_p \|
$$

$$\leq \| \Delta_b - \lambda_d S_{p1} - \frac{a_3 S_{p1} (I_p + \beta A_p)}{N} - \gamma_{QS} Q_{p1} \|
$$

$$\leq \| \Delta_b - \left[ \lambda_d - \frac{a_3 I_p}{N} - \frac{a_3 \beta A_p}{N} - \gamma_{QS} Q_p \right] S_p
$$

$$- \left[ \Delta_b - \left[ \lambda_d - \frac{a_3 I_{p1}}{N} - \frac{a_3 \beta A_{p1}}{N} - \gamma_{QS} Q_{p1} \right] S_{p1} \right] \|
$$

$$\leq \| \Delta_b - I S_p - \Delta_b + I S_{p1} \|
$$

$$\leq a_1 \| S_p(t) - S_{p1}(t) \|$$

where $I = \lambda_d - \frac{a_3 I_p}{N} - \frac{a_3 \beta A_p}{N} - \gamma_{QS} Q_p$.

Consider $u = \max_{t \in J} \| S_p(t) \|$, $v = \max_{t \in J} \| E_p(t) \|$, $w = \max_{t \in J} \| I_p(t) \|$, $x = \max_{t \in J} \| A_p(t) \|$, $y = \max_{t \in J} \| R_p(t) \|$, and $z = \max_{t \in J} \| Q_p(t) \|$. Hence, the Lipschitz condition is satisfied for $Z_1$ and if $0 \leq a_1 < 1$, then it is also a contraction for $Z_1$.

Similarly, the other kernels satisfy the Lipschitz condition as follows:

$$\|Z_2(t, E_p) - Z_2(t, E_{p1})\| \leq a_2 \|E_p(t) - E_{p1}(t)\|$$
$$\|Z_3(t, I_p) - Z_3(t, I_{p1})\| \leq a_3 \|I_p(t) - I_{p1}(t)\|$$
$$\|Z_4(t, A_p) - Z_4(t, A_{p1})\| \leq a_4 \|A_p(t) - A_{p1}(t)\|$$
$$\|Z_5(t, R_p) - Z_5(t, R_{p1})\| \leq a_5 \|R_p(t) - R_{p1}(t)\|$$
$$\|Z_6(t, Q_p) - Z_6(t, Q_{p1})\| \leq a_6 \|Q_p(t) - Q_{p1}(t)\|.$$

**Theorem 4.2** Let $\tilde{M} \subset H$ be bounded and $l, m, n, k, p, q > 0$ such that

$$\|S_p(t_2) - S_p(t_1)\| \leq C_1 \|t_2 - t_1\|$$
$$\|E_p(t_2) - E_p(t_1)\| \leq C_2 \|t_2 - t_1\|$$
\begin{align*}
\|I_p(t_2) - I_p(t_1)\| & \leq C_3\|t_2 - t_1\| \\
\|A_p(t_2) - A_p(t_1)\| & \leq C_4\|t_2 - t_1\| \\
\|R_p(t_2) - R_p(t_1)\| & \leq C_5\|t_2 - t_1\| \\
\|Q_p(t_2) - Q_p(t_1)\| & \leq C_6\|t_2 - t_1\| \\
\end{align*}

where \( S_p, E_p, T_p, A_p, R_p, Q_p \in \hat{M} \). Then \( T(M) \) is compact.

**Proof.** Let

\begin{align*}
W_1 &= \max_{0 \leq t \leq 0 \leq t \leq S_{p}} Z_1(t, S_{p}(t)), \\
W_2 &= \max_{0 \leq t \leq 0 \leq E_{p} < m} Z_2(t, E_{p}(t)), \\
W_3 &= \max_{0 \leq t \leq 0 \leq E_{p} < m} Z_3(t, I_{p}(t)), \\
W_4 &= \max_{0 \leq t \leq 0 \leq A_{p} < k} Z_4(t, A_{p}(t)), \\
W_5 &= \max_{0 \leq t \leq 0 \leq R_{p} < p} Z_5(t, R_{p}(t)), \\
W_6 &= \max_{0 \leq t \leq 0 \leq Q_{p} < q} Z_6(t, Q_{p}(t)),
\end{align*}

\( \exists \) a constants \( l, m, n, k, p, q > 0 \) such that \( \|S_{p}(t)\| < l, \|E_{p}(t)\| < m, \|I_{p}(t)\| < n, \|A_{p}(t)\| < k, \|R_{p}(t)\| < p, \|Q_{p}(t)\| < q \). For all \( S_{p}, E_{p}, I_{p}, A_{p}, R_{p}, Q_{p} \in \hat{M} \), we get

\begin{align*}
\|TS_{p}(t)\| &= \|1 - \frac{\zeta}{B(\zeta)} Z_1(t, S_{p}) + \frac{\zeta}{B(\zeta)} \int_{0}^{t} (t - s)^{(\zeta - 1)} Z_1(s, S_{p}) ds\| \\
&\leq \frac{1 - \zeta}{B(\zeta)} \|Z_1(t, S_{p})\| + \frac{\zeta}{B(\zeta)} \frac{1}{\Gamma(\zeta)} \int_{0}^{t} (t - s)^{(\zeta - 1)} Z_1(s, S_{p}) ds\| \\
&\leq \frac{1 - \zeta}{B(\zeta)} \|Z_1(t, S_{p})\| + \frac{\zeta}{B(\zeta)} \frac{(t - 0)^{\zeta}}{\zeta \Gamma(\zeta)} \|Z_1(s, S_{p}) ds\| \\
&\leq \frac{1 - \zeta}{B(\zeta)} \|Z_1(t, S_{p})\| + \left( \frac{(t - 0)^{\zeta}}{B(\zeta) \Gamma(\zeta)} \right) \|Z_1(s, S_{p}) ds\| \\
&\leq \left( \frac{1 - \zeta}{B(\zeta)} + \frac{(t - 0)^{\zeta}}{B(\zeta) \Gamma(\zeta)} \right) W_1.
\end{align*}

Similarly,

\begin{align*}
\|TE_{p}(t)\| &\leq \left( \frac{1 - \zeta}{B(\zeta)} + \frac{(t - 0)^{\zeta}}{B(\zeta) \Gamma(\zeta)} \right) W_2 \\
\|TI_{p}(t)\| &\leq \left( \frac{1 - \zeta}{B(\zeta)} + \frac{(t - 0)^{\zeta}}{B(\zeta) \Gamma(\zeta)} \right) W_3 \\
\|TA_{p}(t)\| &\leq \left( \frac{1 - \zeta}{B(\zeta)} + \frac{(t - 0)^{\zeta}}{B(\zeta) \Gamma(\zeta)} \right) W_4 \\
\|TR_{p}(t)\| &\leq \left( \frac{1 - \zeta}{B(\zeta)} + \frac{(t - 0)^{\zeta}}{B(\zeta) \Gamma(\zeta)} \right) W_5 \\
\|TQ_{p}(t)\| &\leq \left( \frac{1 - \zeta}{B(\zeta)} + \frac{(t - 0)^{\zeta}}{B(\zeta) \Gamma(\zeta)} \right) W_6.
\end{align*}
Consequently, $T(M)$ is bounded.

Next, to examine $t_1 < t_2$ and $S_p(t), E_p(t), I_p(t), A_p(t), R_p(t), Q_p(t) \in \tilde{M}$ and then for every $\varepsilon > 0$ if $\|t_2 - t_1\| \leq \delta$, we get

$$
\|TS_p(t_2) - TS_p(t_1)\| = \left\| \frac{1 - \zeta}{B(\zeta)} [Z_1(t_2, S_p(t_2)) - Z_1(t_1, S_p(t_1))] \\
+ \frac{\zeta}{B(\zeta)} \int_0^{t_2} (t_2 - s)^{(\zeta - 1)}Z_1(s, S_p(s))ds - \int_0^{t_1} (t_1 - s)^{(\zeta - 1)}Z_1(s, S_p(s))ds \right\|
\leq \frac{1 - \zeta}{B(\zeta)} \|Z_1(t_2, S_p(t_2)) - Z_1(t_1, S_p(t_1))\|
+ \frac{\zeta}{B(\zeta)} \int_0^{t_2} (t_2 - s)^{(\zeta - 1)}Z_1(s, S_p(s))ds - \int_0^{t_1} (t_1 - s)^{(\zeta - 1)}Z_1(s, S_p(s))ds \\
\leq \left( \frac{1 - \zeta}{B(\zeta)} + \frac{(t_2 - t_1)^\zeta}{B(\zeta)(\Gamma(\zeta))} \right) \|Z_1(t_2, S_p(t_2)) - Z_1(t_1, S_p(t_1))\|.
$$

(4.1)

Similarly,

$$
\begin{align*}
\|TE_p(t_2) - TE_p(t_1)\| & \leq \left\{ \frac{1 - \zeta}{B(\zeta)} + \frac{(t_2 - t_1)^\zeta}{B(\zeta)(\Gamma(\zeta))} \right\} \|Z_2(t_2, E_p(t_2)) - Z_2(t_1, E_p(t_1))\| \\
\|TI_p(t_2) - TI_p(t_1)\| & \leq \left\{ \frac{1 - \zeta}{B(\zeta)} + \frac{(t_2 - t_1)^\zeta}{B(\zeta)(\Gamma(\zeta))} \right\} \|Z_3(t_2, I_p(t_2)) - Z_3(t_1, I_p(t_1))\| \\
\|TA_p(t_2) - TA_p(t_1)\| & \leq \left\{ \frac{1 - \zeta}{B(\zeta)} + \frac{(t_2 - t_1)^\zeta}{B(\zeta)(\Gamma(\zeta))} \right\} \|Z_4(t_2, A_p(t_2)) - Z_4(t_1, A_p(t_1))\| \\
\|TR_p(t_2) - TR_p(t_1)\| & \leq \left\{ \frac{1 - \zeta}{B(\zeta)} + \frac{(t_2 - t_1)^\zeta}{B(\zeta)(\Gamma(\zeta))} \right\} \|Z_5(t_2, R_p(t_2)) - Z_5(t_1, R_p(t_1))\| \\
\|TQ_p(t_2) - TQ_p(t_1)\| & \leq \left\{ \frac{1 - \zeta}{B(\zeta)} + \frac{(t_2 - t_1)^\zeta}{B(\zeta)(\Gamma(\zeta))} \right\} \|Z_6(t_2, Q_p(t_2)) - Z_6(t_1, Q_p(t_1))\|.
\end{align*}
$$

(4.2)

Consider

$$
\|Z_3(t_2, I_p(t_2)) - Z_3(t_1, I_p(t_1))\| = \|[(1 - \phi_A)\delta_{R^E}E_p - (\sigma_{R^I} + \lambda_A)I_p(t_2)] \\
- [(1 - \phi_A)\delta_{R^E}E_p - (\sigma_{R^I} + \lambda_A)I_p(t_1)]\| \\
\leq (\sigma_{R^I} + \lambda_A)\|I_p(t_2) - I_p(t_1)\| \\
\leq C_3\|t_2 - t_1\|.
$$

(4.3)

Similarly,

$$
\begin{align*}
\|Z_1(t_2, S_p(t_2)) - Z_1(t_1, S_p(t_1))\| & \leq C_1\|t_2 - t_1\| \\
\|Z_2(t_2, E_p(t_2)) - Z_2(t_1, E_p(t_1))\| & \leq C_2\|t_2 - t_1\| \\
\|Z_4(t_2, A_p(t_2)) - Z_4(t_1, A_p(t_1))\| & \leq C_4\|t_2 - t_1\| \\
\|Z_5(t_2, R_p(t_2)) - Z_5(t_1, R_p(t_1))\| & \leq C_5\|t_2 - t_1\| \\
\|Z_6(t_2, Q_p(t_2)) - Z_6(t_1, Q_p(t_1))\| & \leq C_6\|t_2 - t_1\|.
\end{align*}
$$

(4.4)
Now, substitute the systems (4.3) and (4.4) in Equations (4.1) and (4.2), we get

\[
\begin{align*}
||T_{S_p}(t_2) - T_{S_p}(t_1)|| & \leq \left( \frac{1 - B(\xi)}{B(\xi)} + \frac{(t_2 - t_1)\xi}{B(\xi)\Gamma(\xi)} \right) C_1 ||t_2 - t_1|| \\
||T_{E_p}(t_2) - T_{E_p}(t_1)|| & \leq \left( \frac{1 - B(\xi)}{B(\xi)} + \frac{(t_2 - t_1)\xi}{B(\xi)\Gamma(\xi)} \right) C_2 ||t_2 - t_1|| \\
||T_{I_p}(t_2) - T_{I_p}(t_1)|| & \leq \left( \frac{1 - B(\xi)}{B(\xi)} + \frac{(t_2 - t_1)\xi}{B(\xi)\Gamma(\xi)} \right) C_3 ||t_2 - t_1|| \\
||T_{A_p}(t_2) - T_{A_p}(t_1)|| & \leq \left( \frac{1 - B(\xi)}{B(\xi)} + \frac{(t_2 - t_1)\xi}{B(\xi)\Gamma(\xi)} \right) C_4 ||t_2 - t_1|| \\
||T_{R_p}(t_2) - T_{R_p}(t_1)|| & \leq \left( \frac{1 - B(\xi)}{B(\xi)} + \frac{(t_2 - t_1)\xi}{B(\xi)\Gamma(\xi)} \right) C_5 ||t_2 - t_1|| \\
||T_{Q_p}(t_2) - T_{Q_p}(t_1)|| & \leq \left( \frac{1 - B(\xi)}{B(\xi)} + \frac{(t_2 - t_1)\xi}{B(\xi)\Gamma(\xi)} \right) C_6 ||t_2 - t_1|| \\
\end{align*}
\]

that is, \( ||T_{S_p}(t_2) - T_{S_p}(t_1)|| \to 0 \) as \( t_2 \to t_1 \). Similarly, for all \( T(\tilde{M}) \to 0 \) as \( t_2 \to t_1 \). Thus \( T(\tilde{M}) \) is equicontinuous.

According to Arezla–Azcoli theorem, “Let \( \Omega \) be compact Hausdroff Metric space. Then \( \tilde{M} \subseteq C(\Omega) \) is equicontinuous if \( \tilde{M} \) is uniformly bounded and uniformly continuous,” \( T(\tilde{M}) \) is compact.

## 5 | Uniqueness Solutions

In an earlier section, we expressed the existence solution for SARS-CoV-2 spreading predict with \( AB \)-derivative by using a fixed-point method. Now, we proceed uniqueness results of the system (3.4) with initial conditions.

\[
\begin{align*}
||T(S_{p_1}(t)) - T(S_{p_2}(t))|| & = \left| \frac{1 - B(\xi)}{B(\xi)} \right| Z_1(t, S_{p_1}(t)) - Z_1(t, S_{p_2}(t)) \\
& + \left| \frac{B(\xi)}{B(\xi)\Gamma(\xi)} \right| Z_1(t, S_{p_1}(t)) - Z_1(t, S_{p_2}(t))|| \\
& \leq \left( \frac{1 - B(\xi)}{B(\xi)} \right) ||Z_1(t, S_{p_1}(t)) - Z_1(t, S_{p_2}(t))|| \\
& + \left( \frac{B(\xi)}{B(\xi)\Gamma(\xi)} \right) \left( \frac{1 - \xi}{\xi} \right) ||Z_1(t, S_{p_1}(t)) - Z_1(t, S_{p_2}(t))|| \frac{(t - 0)\xi}{\xi}\Gamma(\xi) \\
& \leq \left( \frac{1 - B(\xi)}{B(\xi)} + \frac{(t - 0)\xi}{B(\xi)\Gamma(\xi)} \right) a_1 ||S_{p_1}(t) - S_{p_2}(t)||. \\
\end{align*}
\]

Similarly,

\[
\begin{align*}
||T(E_{p_1}(t)) - T(E_{p_2}(t))|| & \leq \left( \frac{1 - B(\xi)}{B(\xi)} + \frac{(t - 0)\xi}{B(\xi)\Gamma(\xi)} \right) a_2 ||E_{p_1}(t) - E_{p_2}(t)|| \\
||T(I_{p_1}(t)) - T(I_{p_2}(t))|| & \leq \left( \frac{1 - B(\xi)}{B(\xi)} + \frac{(t - 0)\xi}{B(\xi)\Gamma(\xi)} \right) a_3 ||I_{p_1}(t) - I_{p_2}(t)|| \\
||T(A_{p_1}(t)) - T(A_{p_2}(t))|| & \leq \left( \frac{1 - B(\xi)}{B(\xi)} + \frac{(t - 0)\xi}{B(\xi)\Gamma(\xi)} \right) a_4 ||A_{p_1}(t) - A_{p_2}(t)||
\end{align*}
\]
FIGURE 1 Numerical results of the susceptible people $S_p(t)$ with respect to time $t$ when $n$ varies from 1 to 3

\[
\|T(R_{p_1}(t)) - T(R_{p_2}(t))\| \leq \left( \frac{1 - \zeta}{B(\zeta)} + \frac{(t - 0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) a_5\|R_{p_1}(t) - R_{p_2}(t)\| \\
\|T(Q_{p_1}(t)) - T(Q_{p_2}(t))\| \leq \left( \frac{1 - \zeta}{B(\zeta)} + \frac{(t - 0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) a_6\|Q_{p_1}(t) - Q_{p_2}(t)\|.
\]

Therefore, if the following conditions holds:

\[
\begin{align*}
\left( \frac{1 - \zeta}{B(\zeta)} + \frac{(t - 0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) a_1 & < 1 \\
\left( \frac{1 - \zeta}{B(\zeta)} + \frac{(t - 0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) a_2 & < 1 \\
\left( \frac{1 - \zeta}{B(\zeta)} + \frac{(t - 0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) a_3 & < 1 \\
\left( \frac{1 - \zeta}{B(\zeta)} + \frac{(t - 0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) a_4 & < 1 \\
\left( \frac{1 - \zeta}{B(\zeta)} + \frac{(t - 0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) a_5 & < 1 \\
\left( \frac{1 - \zeta}{B(\zeta)} + \frac{(t - 0)^\zeta}{B(\zeta)\Gamma(\zeta)} \right) a_6 & < 1.
\end{align*}
\]

\[
\therefore T \text{ is a contraction. These certain that the model has a unique solution.}
\]

6 | NUMERICAL RESULTS

Here, we discussed the model of SARS-CoV-2 with $AB$-derivative. Consider the parametric values from the literature \[29\] in Table 1. Assume 35% of total initial population $N = S_0 + E_0 + I_0 + A_0 + R_0 + Q_0 = 481, 747, 192$, where $E_0 = 1, 724, 266; I_0 = 745; A_0 = 413; R_0 = 66$, susceptible case can be discovered as $S_0 = N - (E_0 + I_0 + A_0) - R_0$ and $Q_0 = 10,00,000$. The birth rate
Δ_b = \lambda_d \times N \text{ and natural death rate } \lambda_d = \frac{1}{69.50} \text{ (life expectancy in India is 69.50 per year 2019) are derived in the absence of infection.}

Consider the fractional order \( \zeta = 0.5 \) and using the above parametric values, \( n \) varies from 1 to 3 in Equation (3.3). The range of \( n \) values varies according to the time and the above approximate values to identify the SARS-CoV-2 viruses growth in people simultaneously in Figures 1–6.

\[
S_{p_n}(t) = 1 + 3465807.135 - 0.0072 \times S_{p_{n-1}}(t) - 2.5947e - 10 \times S_{p_{n-1}}(t) \times I_{p_{n-1}}(t)
- 1.5423e - 10 \times S_{p_{n-1}}(t) \times A_{p_{n-1}}(t) - 6.1550e - 09 \times S_{p_{n-1}}(t) \times Q_{p_{n-1}}(t)
+ \frac{1955408.38556}{0.5} \times t^{0.5} - \frac{0.00406224}{0.5} \times t^{0.5} \times S_{p_{n-1}}(t)
\]
FIGURE 4  Numerical results of the asymptotically infected people $A_p(t)$ with respect to time $t$ when $n$ varies from 1 to 3

FIGURE 5  Numerical results of the recovered or removed people $R_p(t)$ with respect to time $t$ when $n$ varies from 1 to 3

\[
E_p(t) = 1 + 2.5947e - 10 \times S_{p_{n-1}}(t) \times I_{p_{n-1}}(t) + 1.5423e - 10 \times S_{p_{n-1}}(t) \times A_{p_{n-1}}(t) \\
+ 6.1550e - 09 \times S_{p_{n-1}}(t) \times Q_{p_{n-1}}(t) - 0.0096 \times E_{p_{n-1}}(t) \\
+ 1.46329e - 10 \times I_{p_{n-1}}^0.5 \times S_{p_{n-1}}(t) \times I_{p_{n-1}}(t) + 8.7017e - 11 \times I_{p_{n-1}}^0.5 \times S_{p_{n-1}}(t) \times A_{p_{n-1}}(t)
\]
**TABLE 1** Approximate values of parameter implemented in the SARS-CoV-2 (COVID-19 model) (1.1)

| S. No. | Notation with values | Parametric representation |
|--------|----------------------|--------------------------|
| 1.     | Δ₃ = 6.9316 × 10⁶   | Birth rate               |
| 2.     | α₄ = 2.5 × 10⁻¹     | Contiguous rate          |
| 3.     | λ₃ = 1.44 × 10⁻²    | Natural death rate       |
| 4.     | β₃ = 5.944 × 10⁻¹   | Transmission multiple rate (asymptomatically infected) |
| 5.     | δ₃ = 4.7876 × 10⁻³  | After incubation period Rₚ from Aₚ |
| 6.     | μ₃ = 5 × 10⁻²       | Incubation period of infected Iₚ from Eₚ |
| 7.     | λ₄ = 1.243 × 10⁻²   | Proposition of asymptomatic infected Aₚ |
| 8.     | λ₅ = 1.231 × 10⁻⁸   | Transmission coefficient from market to Sₚ |
| 9.     | σ₄ = 9.871 × 10⁻²   | Removal rate from Iₚ |
| 10.    | ρ₆ = 8.543 × 10⁻²   | Recovery rate from Aₚ |
| 11.    | 𝜅₆ = 3.98 × 10⁻⁴   | Infected Iₚ from Qₙ |
| 12.    | 𝜅₆ = 1 × 10⁻³      | Asymptomatic infected Aₚ from Qₙ |
| 13.    | 𝜅₆ = 1 × 10⁻²      | Removing virus rate from Qₙ |

\[
Lₚₙ(t) = 1 + 0.0024 \times Eₚₙ₋₁(t) - 0.0566 \times Iₚₙ₋₁(t) + \frac{0.0013}{0.5} \times t^{0.5} \times Eₚₙ₋₁(t) \\
\quad - \frac{0.0319}{0.5} \times t^{0.5} \times Aₚₙ₋₁(t),
\]

\[
Aₚₙ(t) = 1 + 3.1075e - 04 \times Eₚₙ₋₁(t) - 0.4344 \times Aₚₙ₋₁(t) + \frac{1.7533e - 04}{0.5} \times t^{0.5} \times Eₚₙ₋₁(t) \\
\quad - \frac{0.2451}{0.5} \times t^{0.5} \times Aₚₙ₋₁(t),
\]

\[
q₈n(t) = \frac{3.4727e - 09}{0.5} \times t^{0.5} \times Sₚₙ₋₁(t) \times Qₚₙ₋₁(t) - \frac{0.0056}{0.5} \times t^{0.5} \times Eₚₙ₋₁(t),
\]
\[ R_{pn}(t) = 1 + 0.0494 \times I_{pn-1}(t) + 0.4272 \times A_{pn-1}(t) - 0.0072 \times R_{pn-1}(t) + \frac{0.0278}{0.5} \times t_0.5 \times I_{pn-1}(t) \\
+ \frac{0.241}{0.5} \times t_0.5 \times A_{pn-1}(t) - \frac{0.0041}{0.5} \times t_0.5 \times R_{pn-1}(t), \]

\[ Q_{pn}(t) = 1 + 1.9900e - 04 \times I_{pn-1}(t) + 5.000e - 04 \times A_{pn-1}(t) - 0.005 \times Q_{pn-1}(t) \\
+ \frac{1.1228e - 04}{0.5} \times t_0.5 \times I_{pn-1}(t) + \frac{2.8210e - 04}{0.5} \times t_0.5 \times A_{pn-1}(t) - \frac{0.0028}{0.5} \times t_0.5 \times Q_{pn-1}(t). \]

\section*{7 Conclusion}

In this article, we studied a SARS-CoV-2 nonlinear infection-spreading model among various phenomena concerning time by using the Atangana–Baleanu derivative. We derived the solutions, existence results, and unique solutions for the system of equations by the fixed-point method. By using the Mittag–Leffler Kernel with an approximate value, the numerical results will be calculated and plotted in each stage of people affected or recovered from SARS-Cov-2 in graphically. We obtained the approximate value of those stages to predict the outcome of SARS-CoV-2 spreading in the country or state or province. In future, by utilizing the AB-derivative we can predict the infections of pandemic disease like SARS-CoV-2 (COVID-19).

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\section*{Conflict of Interest}

The authors declare that they have no competing interests.

\section*{Authors Contributions}

All the authors contributed equally and they read and approved the final manuscript for publication.

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