Dynamics and Vibrations of Rotors of Electrical Drives with Additively Manufactured Materials at Cryogenic Temperatures

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Abstract. The use of additive manufacturing and cryogenic cooling enables engineers to build electric machines with higher power densities than previously seen. Both the new manufacturing technologies and very low temperatures have a significant impact on the mechanical properties of materials. In the present work, a rotordynamics analysis of a permanent magnet synchronous motor concept from the AdHyBau project, funded by the German Federal Ministry for Economic Affairs and Energy, is conducted with an emphasis on the influence of additive manufacturing and cryogenic temperatures on vibrations and natural frequencies.

1. Introduction
The development of lightweight and efficient electric drives is one of the big challenges of the modern transport sector. In the AdHyBau project, novel methods of additive manufacturing and fiber-composite-metal hybrid materials are being developed. Together with cryogenic cooling, these new methods will allow for a significantly higher power density in electric drives compared to conventional designs. Lightweight design and cryogenic operating temperatures pose a number of challenges during the design process. Regarding the lightweight mechanical structure, not only does it have to withstand static loads, but it must also exhibit good dynamic behaviour under the influence of several excitations. In the present work, electromagnetic forces on the rotor of a permanent magnet synchronous motor (PMSM) are being investigated as the primary sources of nonsynchronous vibrations in the rotor of an electric drive. To characterize these forces, a finite element model of the investigated machine is created in the simulation environment COMSOL Multiphysics. The Maxwell Stress Tensor is used to derive the radial force on the rotor from the magnetic flux density. Then, a rotordynamics model is set up, into which the magnetic forces can be introduced in a simplified form. First, the rotordynamics analysis investigates the position of natural frequencies and the associated mode shapes. Next, the magnetic excitations are considered in the model and resulting deflections are calculated and presented in response plots. Furthermore, the impact of different parameters on natural frequencies and deflections is examined. Those are the operating temperature, a variation of the mechanical structure and the shaft length. This will allow engineers to estimate a range of parameters within which the machine design can vary. Lastly, torsional vibrations of the rotor during run-up from rest are investigated, again considering different operating temperatures as well as different rise times to design the entire system in which the machine operates accordingly.
2. Theoretical Foundations

2.1. Magnetic Forces on the Rotor of the Electric Machine

Corresponding with a cylindrical coordinate system, the magnetic forces on the rotor may be divided into radial, tangential and axial forces. The axial component can be neglected under the assumption of perfect symmetry in that direction. In the present work, the radial forces are being calculated from the magnetic flux density $B$ using the Maxwell Stress Tensor, which is defined as [1]

$$
\sigma_{ij} = \epsilon_0 E_i E_j + \frac{B_i B_j}{\mu_0} - \frac{1}{2} (\epsilon_0 E_i^2 + \frac{B_i^2}{\mu_0}) \delta_{ij}
$$

with the magnetic flux density $B$ and the electric field $E$. In the case at hand, the electric field may be neglected. Further reducing the tensor to the radial component of $B$ in the airgap yields

$$
\sigma_r = -\frac{B_r^2}{2\mu_0}
$$

Fluctuations of this force may be caused by the interaction of the rotor magnetic field with the stator teeth, resulting in the frequency [2]

$$
f_r = 2f_{el} \frac{s}{p}
$$

where $s$ is the number of stator teeth and $p$ is the number of pole pairs.

The tangential component of the force on the rotor can be expressed in terms of oscillations of the torque. The torque is calculated as the derivation of the magnetic energy over the rotational angle of the rotor. It is written as [2]

$$
T = -\frac{\partial W}{\partial \theta} = -\frac{gL}{2\mu_0} \frac{\partial}{\partial \theta} \int_0^{2\pi} B(\alpha, t)^2 d\alpha
$$

In most cases, the greatest cause of torque ripple is the cogging torque which is caused by oscillations of the magnetic reluctance between the permanent magnets and the slots of the stator. Its frequency depends on the rotational speed $U$, the number of pole pairs $p$ and the number of stator teeth $s$ [3]:

$$
f_{tr} = U \left( \frac{2s}{p} \right) p
$$

2.2. Fundamentals of Rotordynamics

In the following remarks, the mathematical model of a rotor is first described in its simplest form, being able to perform deviations in the cartesian $x$ and $y$ directions only. Introducing the complex deviation $r = x + jy$, the equation of motion takes the following form [4]:

$$
mi\ddot{r} + d_{ext} \dot{r} + \sum F_i \leftrightarrow \ddot{r} + 2\zeta \omega \dot{r} + \omega^2 r = \frac{1}{m} \sum F_i
$$

This equation’s characteristic polynomial can be solved by the eigenvalues

$$
\lambda_{1,2} = -\frac{d_{ext}}{2m} \pm j \sqrt{\frac{k}{m} + \frac{d_{ext}^2}{4m^2}}
$$

The damped eigenfrequencies are expressed by the imaginary term. A critical damping of $d_{crit} = 2\sqrt{k/m}$ can also be extracted from the equation, although it seldomly becomes relevant in rotors [5]. Usually, the external damping is small, and the natural frequency is close to the that of an undamped system.

Beside the external damping, internal damping may also be present. It results from energy being dissipated by mechanisms of deformation such as micro-slip in press fits and becomes especially relevant in cases of small external damping [6]. After expressing the equation of motion in corotating coordinates and adding the internal damping, the equation takes the form
\[ m \ddot{r} + d_{\text{ext}} \dot{r} + d_i (\dot{r} - j \Omega r) + k r = \sum_i F_i \quad (8) \]

There is now a skew symmetric share of the stiffness matrix, which can lead to instability for certain values of \( d_i \) in proportion to \( d_{\text{ext}} \). Stable operations above the natural frequency may now be impossible. The range of stability can be increased by increasing the value of \( d_{\text{ext}} \).

Another phenomenon to be considered is the gyroscopic effect, that can gain relevance in rotors with large moments of inertia around the axes that are perpendicular to the axis of rotation. Using a complex rotational deflection \( \chi \), the equilibrium of moments can be expressed as [6]

\[ J_p \ddot{\chi} - \Omega J_{ax} \dot{\chi} + k_{x r} r + k_{r z} \chi = \sum_i M_i \quad (9) \]

Again, natural frequencies now depend directly on the rotational speed. The proportion of the two moments of inertia can also be relevant for stability.

Torsional vibration, although outside the reign of classical rotordynamics, may also be a cause for concern in many rotors. In most cases, the torsional motion can be treated as a one-degree-of-freedom system [7]. Its damping is usually very small. The natural frequency can therefore be approximated by the well-known expression \( \omega = \left( \frac{k_{\text{tor}}}{J_p} \right)^{1/2} \) [7].

3. State of the Art

Two publications about the development of electric drives with the consideration of cryogenic temperatures have been released in the past. Filipenko et al. [8] describe an electric motor with superconductive stator coils and cryogenic cooling. The magnetic flux density and the torque ripple are calculated using a finite element analysis. The necessity of qualifying materials for a cryogenic environment is mentioned. Haran et al. [9] describe the increase of the power density by incorporating fiber-composite materials. The increase of the magnetic potential difference allows for the armature mass to be reduced and its structural role to be taken by non-magnetic, more lightweight materials.

4. Methods

| Table 1. Properties of the electric machine. |
|---------------------------------------------|
| Quantity                  | Value    |
| rotor outer radius        | 240 mm   |
| length (yoke and PMs)     | 38 mm    |
| PM material               | NdFeB    |
| rotational speed          | 2500 rpm |
| operational torque        | 1900 Nm  |

4.1. Electromagnetic Simulation

To calculate the radial forces on the rotor as well as the torque ripple, a two-dimensional representation of the rotor is being set up in the simulation software COMSOL Multiphysics. The geometry of the armature and the magnets does not change along the rotational axis. As all electromagnetic properties are symmetrical along that axis, it is feasible to simulate the geometry as two-dimensional. Along the perimeter of the machine, the motor is sector-symmetrical with as many sectors as pole pairs. One of these sectors is depicted in Figure 1. The symmetry condition is defined using the magnetic vector potential \( A \). The vector potential at one edge of the sector must be the same as the one on the opposite side to ensure sector symmetry. On the inside and outside of the active parts, no mechanical structure is represented in the electromagnetics model. Instead, it is assumed that the surroundings consist of air with a permeability close to that of the vacuum.
For the finite element analysis, the geometry must be divided into discrete elements. The process of generating these elements is referred to as meshing. The density of the mesh determines the accuracy of the solution. In the present investigation, field properties in the air gap are of the highest interest. Therefore, the mesh in that area must be particularly fine. To guarantee a sufficient resolution, two boundary layers have been created along the dividing line between the corotating and the static part of the geometry.

Before a time-dependent study of the electromagnetic field during operations can be performed, a stationary study must take place to find the ideal initial rotation of the rotor relative to the stator for maximum torque. After this has been completed, a transient study takes place, calculating the radial force on the rotor using the Maxwell Stress Tensor and the torque fluctuations. Both properties are being investigated globally and, in the case of the radial force, at certain points at the outer diameter of the rotor to extract one-dimensional data that can then be transformed to the frequency domain using the Fast Fourier Transformation.

4.2. Rotordynamic Simulation

For the rotordynamic calculations, both a 1D beam model and 3D representations of the rotor, one of which is shown in Figure 2, have been created. All connections between different parts of the geometry are considered to be fixed. The shaft and the support structure for the armature may be connected by multi-material deposition, while the connection between the support structure and the armature may be realized with a press fit in the axial direction. Internal damping, although it may be caused by micro-slip in the press fit, can therefore not be considered in the present investigation. Two different support structures have been investigated. Both are manufactured from a high-performance aluminium-scandium alloy. One of the structures has been designed as an extremely lightweight structure with the benefits of additive manufacturing in mind, while the other one is a simplified version that can easily be achieved with conventional manufacturing methods.

To fully characterize the dynamical behaviour of the rotor, different types of studies must be conducted. First, eigenfrequency analyses are being performed to describe the natural frequencies depending on the rotational speed. If natural frequencies are found to be close to either the rotational speed or one of the excitation frequencies, frequency dependent studies are performed additionally where the deformation response to the excitations can be modelled. All the mentioned studies are conducted for different sets of parameters. The parameters are the operating temperature which affects

**Figure 1.** Geometry of the electromagnetic simulation.

**Figure 2.** Geometry of the 3D model.
the modulus of elasticity, the different mechanical structures, and the length of the shaft between the bearings. Several simplifications have been made. External and internal damping have not been considered. This is a reasonable simplification as damping is usually very small in electric drives [5]. Furthermore, the bearings are assumed as perfectly stiff. The magnetic excitations are included in the rotordynamics model in their simplest form without higher harmonics.

5. Results

![Figure 3](image1.png)

**Figure 3.** Radial flux density at different points in the air gap.

![Figure 4](image2.png)

**Figure 4.** Frequency spectrum of the pressure measured above a PM.

5.1. Electromagnetic Simulation

The results of the calculation show that the sign of the radial component of the magnetic flux density in the airgap follows the magnetization of the permanent magnets. The magnetic field of the three permanent magnets per sector accounts for the bigger proportion of the flux density, which is clearly visible in the changes of $B$ in Figure 3, where the top line depicts $B$ in the airgap over the center of a PM. Analyzing $B$ for certain points in the airgap in corotating coordinates shows that although the amount of $B$ at those points depends mostly on the proximity to a permanent magnet, there is also a component that fluctuates while the rotor rotates and is therefore caused by the interaction with the stator geometry. From the values of the radial component of $B$, the pressure on the rotor can be calculated using the Maxwell Stress Tensor. The Fast Fourier Transformation of the time-dependent data shows the frequency spectrum of the radial forces (Figure 4). The dominating frequency is $f = f_{el} \cdot 3 = 3250$ Hz. Harmonics occur but have a much smaller amount than the fundamental oscillation.
The torque over the time has been evaluated globally for one machine segment. To obtain the overall torque, the data must be multiplied by the number of segments, which is the number of pole pairs. The result of this is shown in Figure 5. It can be used to verify the model by examining the amount of the torque and comparing it to analytical approximations. The results show that the torque fluctuates with $f_{tr} = f_{el} \times 6 = 6500\text{ Hz}$. This oscillation is most likely being generated by the reluctance between the PMs of the rotor and the teeth of the stator, as described above. The Fast Fourier Transformation shows that harmonics only occur with very small amounts.

![Figure 5](image_url) Torque over the time of one electric period.

5.2. Rotordynamic Simulation

![Figure 6](image_url) Campbell plot for the lowest frequency range.

The eigenfrequency analysis with the simplified geometry for the lower frequency range shows that the lowest eigenfrequency lies well above the operating rotational speed of 2500 rpm or 43.7 1/s, as can be seen in the Campbell plot in Figure 6. The lowest natural frequencies are those of the torsional and axial mode. The torsional mode could potentially be excited by the torque ripple during run-up, when the torque ripple matches the torsional natural frequency.

Looking at the Campbell plots for the frequencies around the magnetic excitation frequencies (Figures 7 and 8), we find a large number and density of natural frequencies, many of which lie close to the excitations at operating state. When examining the mode shapes, the conclusion can be drawn that the higher natural frequencies almost exclusively concern deformations of small parts of the geometry like membrane-shaped parts of the disk or stiffening fins. Still, response analyses in the frequency domain are necessary to determine if dangerous deformations can occur. The response plots confirm that no significant deflections are to be expected at the excitation frequencies.
The different operating temperatures of 77 K, 300 K and 400 K are considered as an influencing variable to the modulus of elasticity of the structural materials of the shaft and the support structure, titanium and an aluminium-scandium alloy. The data for this is taken from previous studies as well as from [10] and [11].

Table 2. Young’s modulus of structural materials for different temperatures.

| Temperature | Titanium Ti6Al4V | aluminium-scandium alloy |
|-------------|------------------|--------------------------|
| 77 K        | 132 GPa          | 76 GPa                   |
| 300 K       | 116 GPa          | 69 GPa                   |
| 400 K       | 105 GPa          | 63.5 GPa                 |

The results show that lowering the temperature leads to an increase of all natural frequencies. This can be seen in Figures 9 and 10 for the example of the torsional eigenfrequency. This result can be explained by the proportionality of the modulus of elasticity to the stiffness, while the mass and the moments of inertia remain constant.
Evaluating the natural frequencies and deflections for the two different structural designs, the conclusion can be drawn that an additively manufactured lightweight structure can have the same dynamic properties as a much heavier conventional part. The lightweight structure at hand has been designed with a focus on static loads and seeks to have the same behaviour under static loads than a conventional structure. In this case, similar static behaviour appears to imply very similar dynamic behaviour as well.

The shaft length (or distance between bearings) in directly related to the eigenfrequencies as the stiffness of the shaft length is one of the most fundamental variables of its stiffness. We have seen from the abovementioned eigenfrequency analysis that bending modes do not occur close to the rotational speed. For a shaft length of $l = 2000$ mm, the first bending mode matches the frequency of the rotational speed at operating state. The frequency of the torsional mode also decreases with an increase in shaft length due to a reduction of torsional stiffness. As bending modes could not be detected in earlier studies with much smaller shaft length, this can also be used to validate the methods of the analysis itself by proving that bending modes can in fact be detected in the system.

Transient studies of run-up are of interest because only during run-up does the frequency of the torque ripple (cogging torque) momentarily match the natural frequency of the torsional mode. The results show that a significant torsional deflection occurs when the frequency of the cogging torque and the torsional eigenfrequency coincide, which can be recognized in Figure 12. For different operating temperatures, the earlier results are validated as the deflection during run-up becomes smaller for lower temperatures because of the increased stiffness and occurs later due to the increased value of the natural frequencies. Different rise times directly influence the amount of deflection as well.
6. Conclusions

The present work has shown that nonsynchronous magnetic forces do not excite significant oscillations in the investigated machine during the operating state. However, during run-up, torsional vibrations may occur due to cogging torque. Cryogenic operating temperatures may lead to stiffer mechanical parts and therefore higher eigenfrequencies, making the system more robust if fatigue is not considered. The evaluation of two different mechanical designs has shown that additively manufactured lightweight parts can have a similar dynamic behaviour to heavier, more conventional mechanical structures. Several tasks remain to be resolved in the future. As soon as the state of the AdHyBau project allows for it, a bearing design needs to be conducted and the bearings must be considered in the dynamic evaluation of the rotor. Furthermore, the results of the present research may be validated by incorporating a set of boundary conditions such as external and internal damping. Regarding the vibrations during run-up, a fatigue strength verification needs to be performed to draw the correct conclusions from the present findings.

7. References

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