Abstract

A class of grand unified theories with symmetry breaking scale of order $10^{16}\text{GeV}$ have a Higgs particle with mass in the $\text{TeV}$ scale. The cosmology of such theories is very different from usual. We study the cosmic strings obtained in such theories. These strings are much fatter than usual and their mass per unit length is reduced, resulting in a significant reduction in their cosmological effects. We also study the temperature evolution of such models.
1 Introduction

In recent years there has been much interest in unified fields theories, in particular, those involving supersymmetry. In a wide class of supersymmetric grand unified theories some of the phase transitions occur in the $\text{TeV}$ range, despite the gauge symmetry breaking scale being of order $10^{16}\text{GeV}$. This occurs in a rather generic class of supersymmetric theories in which the scalar field potential has ‘flat directions’. Such theories can arise in superstring theories [1]. The associated scalar fields also have mass in the $\text{TeV}$ range, usually arising from very small Higgs self-couplings. However, the gauge bosons have mass of order the grand unified scale.

Supersymmetry is looking increasingly likely to be involved in some underlying theory that unifies the interactions, whether it is a superstring theory or a supersymmetric grand unified theory. In such theories, the so-called flat directions are also a common feature, arising in a class of theories where the gauge symmetry is broken with a so-called F-term. Such models could solve the cosmological moduli problem. Whilst the physical content of such theories is understood, it is only recently that the cosmology has started to be investigated [2, 3, 4].

In a very innovative paper [2] Lyth and Stewart considered the cosmology of models with low mass Higgs particles. They showed that the evolution of the universe was radically different from usual. In addition to the usual inflationary period their model had a period of thermal inflation followed by a ‘cold big bang’ at $\text{TeV}$ scales. This was succeeded by Higgs particle decay and a ‘hot big bang’ at around $10\text{MeV}$ just prior to nucleosynthesis.

In this paper we investigate the cosmology of such theories, considerably extending previous work. We consider topological defects formed in such theories, in particular cosmic strings. In these theories the string profiles are considerably modified by the relatively small Higgs mass. We display the string profiles in section 2 for both the Higgs and gauge fields and calculate the string width and mass per unit length. We show that there is a modification factor depending on the logarithm of the ratio of the scalar and gauge particle masses. This modification results in the strings being ‘fat’, with decreased mass per unit length. In section 3 we discuss string dynamics, showing that they are produced after the period of friction domination, but otherwise evolve as normal strings. The resulting cosmology of such strings is also considered. We show that, if the gauge symmetry breaking scale is around $10^{16}\text{GeV}$, then the strings are less cosmologically significant than usual. We also discuss microphysical affects of such strings. In section
4 we comment on the temperature-time relation in our model and compare it with the temperature evolution discussed previously. By considering the decay of the Higgs particles in a half-life model, we show that the dramatic results of [2] are modified considerably.

We summarize our results in section 5.

2 The Structure of Low Higgs Mass Strings

In order to consider cosmic strings [3] in grand unified theories with low mass Higgs particles we can model the relevant features of the Higgs potential after supersymmetry breaking by the usual Mexican hat with a very small Higgs self coupling. We thus consider an effective Abelian Higgs model with Lagrangian,

\[ \mathcal{L} = (D^\mu \phi)^\dagger D_\mu \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \lambda (\phi^\dagger \phi - \eta^2/2)^2. \]

It should be emphasized that this potential is only appropriate for tree level calculations. Loop corrections within the Abelian Higgs model would usually necessitate a degree of fine tuning to protect the very small Higgs self coupling we are considering. However, in the context of the underlying supersymmetric theory this fine tuning is not a problem. We can consistently work with the Abelian Higgs model at the tree level with whatever couplings we choose.

We construct a Nielsen-Olesen vortex solution in the usual manner by setting

\[ \phi = \phi(r)e^{i\theta} \]

and considering only \( A_\theta \) non-vanishing. The energy per unit length of string is then given by

\[ \epsilon = 2\pi \int_0^\infty rdr [\phi_r^2 + \phi^2 (\frac{1}{r} - eA_\theta)^2 + \frac{1}{2} (A_{\theta,r} + A_\theta/r)^2 + \lambda (\phi^2 - \eta^2/2)^2] \]

It is convenient to introduce the scaled variables:

\[ \phi = \frac{\eta}{\sqrt{2}} \tilde{\phi}, \quad A_\theta = \eta \tilde{A}, \quad r = \frac{x}{\eta e}. \]

In terms of these variables the energy per unit length reduces to,

\[ \epsilon = \pi \eta^2 \int_0^\infty x dx [\tilde{\phi}_{,x}^2 + \tilde{\phi}^2 (\frac{1}{x} - \tilde{A})^2 + (\tilde{A}_{,x} + \tilde{A}/x)^2 + B(\tilde{\phi}^2 - 1)^2]. \]
where \( B = \lambda/2e \)

In order to obtain a model for the string we take the following forms for the string profiles:

\[
\tilde{\phi} = \begin{cases} 
\beta x^{\gamma} & 0 \leq x \leq \beta^{-1/\gamma} \\
1 & x > \beta^{-1/\gamma}
\end{cases}
\]

\[
\tilde{A} = \begin{cases} 
\frac{x}{X} & x \leq X \\
\frac{1}{x} & x > X
\end{cases}
\]

These approximations allow the gauge and scalar cores to have different radii and allow the scalar field to either delay its approach to its asymptotic form (large \( \gamma \)) or move to it more rapidly (small \( \gamma \)).

With these forms for the profile functions the energy per unit length becomes,

\[
\epsilon = \pi \eta^2 \left[ \frac{\gamma + \beta^2 X^{2\gamma}}{2} \left( \frac{1}{\gamma + 1} + \frac{1}{2\gamma + 4} \right) + \frac{2}{X^2} + B \beta^{-2/\gamma} \left( \frac{1}{2} - \frac{1}{\gamma + 1} + \frac{1}{4\gamma + 2} \right) \right].
\]

We now wish to vary the parameters so as to minimize the string energy. We first gain a rough idea of how the parameters vary with \( B \) and then proceed to a more careful minimization.

For small values of \( B \), that is small Higgs self coupling, we expect the energy to be small. This requires \( \gamma \) to be small, \( X \) to be large and \( \beta^2/\gamma \) to be small in order to suppress the individual terms in the energy. Assuming that each term in the energy decreases as the inverse of some scale \( l \) as \( B \) becomes small, we have

\[
\gamma \sim \frac{1}{l}, \quad X \sim \sqrt{l}, \quad \beta \sim \frac{1}{l}
\]

and

\[
B \sim l^{-1+2l}
\]

As an approximation we consider

\[
l = \frac{\log(1/B)}{2 \log \log(1/B)}.
\]

With this value of \( l \), \( B \beta^{-2/\gamma} \) indeed decreases more rapidly than \( 1/l \), thus our model for the string profile functions gives an energy per unit length that decreases at least as rapidly as

\[
\frac{2 \log \log(1/B)}{\log(1/B)}.
\]
We need to be slightly more careful about the minimization in order to determine how the core sizes scale with \( B \). We introduce some shorthand,

\[
Q = \frac{1}{2\gamma} - \frac{1}{\gamma + 1} + \frac{1}{2\gamma + 4}, \quad P = \frac{1}{2} - \frac{1}{\gamma + 1} + \frac{1}{4\gamma + 2},
\]

then vary the energy with respect to each of the parameters. Varying first with respect to \( X \) we find

\[
X = \left( \frac{2}{\beta^2 \gamma Q} \right)^{\frac{1}{1+2\gamma}}.
\]

Eliminating \( X \) and varying with respect to \( \beta \), we find that the value of \( \beta \) is given by

\[
\beta^2 = \left[ \frac{BP}{2} \left( \frac{Q\gamma}{2} \right) - \frac{1}{\gamma} \right]^{\frac{1+\gamma}{1+2\gamma}}.
\]

We saw above that \( \gamma \) is small for small values of \( B \), so we expand our expression for the energy about \( \gamma = 0 \) and keep only the leading terms;

\[
\epsilon \simeq \pi \eta^2 \left[ \frac{\gamma}{2} + \frac{B\gamma}{2\gamma} \right].
\]

Minimizing with respect to \( \gamma \) yields the constraint,

\[
\frac{K^2}{\log^2 B} + (K - 1)e^K = 0,
\]

where \( K = \gamma \log B \). There is a positive solution, \( K = 1 + O(1/\log^2 B) \), but this gives an unphysical negative value to \( \gamma \). For large \( \log^2 B \), the negative solution is

\[
K \simeq -\log (\log^2 B).
\]

Finally we have

\[
\gamma \simeq \frac{\log (\log^2 (1/B))}{\log (1/B)} = \frac{2 \log \log (1/B)}{\log (1/B)}.
\]

While this coincides with the form for \( \gamma \) we obtained by the naive argument, the forms of \( X \) and \( \beta \) are different. Working back through the constraints we find

\[
\beta \simeq B^{\frac{2}{\gamma}} = \frac{1}{\log (1/B)}, \quad X \simeq 2 \log (1/B).
\]
Fig. 1 shows a comparison of the actual Higgs and gauge field profiles of a low Higgs mass string with the model profiles for $B = 10^{-4}$.

Thus, the mass per unit length for these type of cosmic strings is

$$\mu = \frac{\eta^2}{\log (B^{-1})}$$

Hence, for symmetry breaking scale $\eta$ the mass per unit length is reduced by the logarithmic factor. A similar result was obtained numerically in ref [3], though the full profile functions were not obtained. For small $\lambda$ this reduction factor can be an order of magnitude, hence affecting the cosmological predictions in this model. Similarly, the gauge and scalar cores are vastly different. The scalar core size is set by the inverse Higgs’ mass, whilst the gauge core is increased by $\log B^{-1}$. Hence, in these low mass Higgs models the string width is considerably fatter than in the usual case. That these strings are much fatter was recognised in ref [4], though the reduction in $\mu$ was not realised.

3 The Evolution and Cosmology of Low Higgs Mass Strings

In the standard string model there are two distinct periods of string evolution, depending on the significance of the plasma. The length scale above which friction is dominant is set by [5]

$$l_f = \frac{\mu}{\sigma \rho}$$

where $\mu$ is the mass per unit length of string, $\rho$ is the energy density of scatterers and $\sigma$ is the scattering cross-section per unit length. While the mass per unit length of these strings is suppressed by a logarithmic factor, the scattering cross-section is unchanged. The dominant process is Aharonov-Bohm scattering, with the cross-section being determined by the momentum of the scattering particles, $\sigma \propto T^{-1}$.

In standard string models friction domination ends well before the electroweak transition. The analysis is similar in this modified picture, with a logarithmic correction appearing due to the lower mass per unit length. The main difference is that strings are formed much later, at around the time of the electroweak transition. Thus strings form well after the period
Figure 1: Comparison of the actual Higgs and gauge field profiles (solid lines) of a low Higgs mass string with the model profiles (dashed lines) in this case $B = 10^{-4}$. 
of possible friction domination. Otherwise their evolution is simply that of normal string with a suitably reduced mass per unit length.

Whilst the usual Aharonov-Bohm scattering cross-section is the same for our fat strings as for the normal GUT strings the same is not true for the inelastic cross-section. For example, the baryon violating catalysis cross-section will depend on the string radius in realistic GUT models, unlike the case in toy $U(1)$ models. This dependence arises from the group generators in the string gauge field and it also depends on the scattering particle. The full details are shown in [7]. Consequently, the catalysis cross-section will be bigger for fat strings than the corresponding usual GUT strings. This could result in some erasing of a primordial baryon asymmetry.

The cosmological effects of cosmic strings are determined by the parameter $G\mu$. The anisotropy in the microwave background radiation, the density fluctuations and the gravitational lensing are all proportional to $G\mu$. With the logarithmic reduction factor, as given in the previous section, the cosmological effects of these fat strings will be just those of ordinary strings formed at a slightly lower energy scale. This reduction in the gravitational effects of the strings prevents them from playing a major role in structure formation, unless the unification temperature is slightly raised by a corresponding amount. Similarly, the power spectrum is likely to be the same shape as for the usual GUT scale strings. These strings are likely to evade constraints arising from microwave background and large scale structure measurements. As these strings are appearing in an inflationary model, this lack of string density perturbations is not a problem, perturbations arising during the inflationary era can seed large scale structure formation. There has been recent interest in mixed models where the density perturbations are produced by both cosmic strings and inflationary perturbations. If the cosmic strings are of the fat type discussed here then their contribution to density perturbations will be reduced unless unification is raised to compensate for the logarithmic factor.

The decay of cosmic string loops can result in the production of cosmic rays and also produce a baryon asymmetry. In [8,9] it was shown that the decay of GUT scale string loops could account for the resulting baryon asymmetry, particularly in models with a low freeze out temperature. The corresponding case for our fat strings is rather more complicated. This is because the analysis in [8] was performed at the Ginsberg temperature where thermal fluctuations can no longer restore the symmetry. However, since the fat strings under consideration are produced in first order phase transitions, it seems more appropriate to consider the transition temperature itself. In
which case, we have a massive enhancement factor coming from the increase in the string width. The results of ref [8] are enhanced by a factor of $\lambda^{-1}$, in which case this model can easily account for the observed baryon asymmetry.

Similarly, there will be a large amount of cosmic rays produced by the decaying string loops, mainly in the form of $TeV$ Higgs particles. There may also be cosmic rays produced by the infinite string network [10]. Since the particles emitted by the network are going to be mainly $TeV$ scale Higgs particles and their decay products [11], they will evade the bounds of ref [10].

However, since the dominant microphysical effects occur shortly after string formation, these will be diluted by any subsequent reheating. If there is sufficient reheating both the baryon asymmetry produced and the cosmic ray flux will be diluted, and could be diluted below observational limits. We discuss this in the next section.

4 The Temperature-Time Relationship For Late Transitions

The extremely low Higgs mass in these models delays the breaking of the GUT symmetry until the temperature is of order the Higgs mass, $\sim 1 \text{ TeV}$. Even with the very flat Higgs potential corresponding to such low self couplings, the vacuum energy density is of order $10^{36} \text{GeV}^4$ and is much greater than the radiation energy density (of order $10^{12} \text{GeV}^4$ at symmetry breaking). Following symmetry breaking the vacuum energy density is converted into Higgs particles and the universe enters a phase of matter domination [2]. In [2] it is argued that this phase lasts for the lifetime of a typical Higgs particle, of order $10^{23} \text{GeV}^{-1}$, then the Higgs particles decay, leading to a reheating of the Universe. This reheating produces a standard radiation dominated epoch that encompasses the final period of nucleosynthesis.

While this approach provides a reliable estimate of the temperature at which radiation domination recommences, the temperature evolution during the Higgs dominated phase is rather different. It is more appropriate to think of the Higgs lifetime as a half-life and consider a continuous transfer of energy from the matter to radiation fields.

If $r_d$ is the probability of a Higgs particle decaying per unit time, the evolution of the radiation and matter energy densities are given by

$$ \frac{d}{dt} \rho_r = -4H\rho_r + \rho_m r_d, \quad \frac{d}{dt} \rho_m = -3H\rho_m - \rho_m r_d. $$
These expressions are valid for both a second order transition and for the first order transition we expect at weak coupling. Bubble collisions produce a sea of 'soft' quanta, with energy less than the corresponding instantaneous reheat temperature. Now, even instantaneous conversion of the Higgs' potential energy to radiation would only give a reheat temperature of around $10^9$ GeV, much less than the mass of the particles mediating Higgs decay. Thus the Higgs decay rate will be that of static Higgs particles.

We can understand how the energy densities evolve by considering some approximate solutions. In a flat, matter dominated Universe, the evolution of the matter energy density is given by,

$$\frac{d}{dt}\rho_m = -3\sqrt{\frac{8}{3}\pi G\rho_m\rho_m - \rho_m r_d}.$$ 

and we have $\rho_m = e^{-r_d t} x$ with

$$x_f^{1/2} = x_i^{1/2} - \frac{3}{r_d}\sqrt{\frac{8}{3}\pi G}\left[e^{-r_d t_f/2} - e^{-r_d t_i/2}\right].$$

To leading order in $r_d t$ this gives

$$\rho_m = \left[x_i^{1/2} - \frac{3}{2}\sqrt{\frac{8}{3}\pi G}\left(t_f - t_i\right)\right]^{-2}.$$ 

Assuming that $x_i$ is very large for $t_i$ close to zero, we have the standard form,

$$\rho_m = \left[\frac{3}{2}\sqrt{\frac{8}{3}\pi G}\left]\right]^{-2}.$$ 

As expected, for $r_d t << 1$ the decay of the Higgs particles has little effect on their energy density. However, significant energy is transferred into radiation. With $\rho_m$ varying as $t^{-2}$, we can solve for the radiation energy density,

$$\rho_r = at^{-8/3} + \frac{4}{15}\frac{r_d}{5}\frac{1}{3\pi G}\frac{1}{t}.$$ 

The particular integral varies as $t^{-1}$ and so quickly dominates the complementary function. Physically, the initial radiation is diluted and redshifted, quickly leaving only the radiation from the decaying Higgs particles. After a possible initial increase in the temperature, the temperature decreases as $t^{-1/4}$.

Assuming that these small $r_d t$ forms persist until matter-radiation equality, we have equality at $r_d t = 5/3$. Now, the Hubble parameter is given by
Figure 2: The Higgs (solid line) and radiation energy densities as functions of time

\( H = \frac{2}{3t} \), thus at this time the Higgs' half-life and the expansion time-scale are comparable and Higgs decay becomes important in determining \( \rho_m \). Following equality the remaining Higgs particles decay rapidly, but the energy liberated is only of order the radiation energy density and there is little further reheating.

We can compare the entropy generated in this approximation with that obtained by assuming that all of the Higgs' decay at \( t = 1/r_d \). In both cases we have a phase of matter domination with \( R \propto t^{2/3} \) which ends at \( t \sim 1/r_d \). This is followed by a phase of radiation domination with an initial radiation energy density,

\[ \rho_r \sim \frac{r_d^2}{\frac{2}{3} \pi G} \cdot \]

The entropy generation and temperature at equality are similar in both approximations, but the temperature evolutions are very different as is the amount of entropy produced subsequent to any given intermediate temperature.

The forms of \( \rho_r \) and \( \rho_m \) for \( r_d = 10^{-22} GeV \) and the initial conditions given above are shown in fig2. It can be seen that there is a small amount of prompt reheating, but then the universe cools monotonically with matter domination ending just before \( \rho_r \) drops below about \( 10^{-12} GeV^{-4} \) and nucleosynthesis begins.

The temperature evolutions for continuous and instantaneous Higgs decay are compared in fig3. The most significant differences between the two
arise for temperatures above $T_{\text{reheat}}$, these temperatures are only attained once and occur before much of the entropy generation in the instantaneous model. In the continuous decay model these temperatures are reached later and, particularly in the case of temperatures close to $T_{\text{reheat}}$, there is much less entropy generation at subsequent epochs.

In the instantaneous picture, the temperature increases by a factor of around $10^{10}$ at reheating. Thus the ratio of the relic density to the entropy density, $n/s$, for any relic produced at temperatures above the reheat temperature is reduced by a factor of around $10^{30}$ at reheating. In our continuous case, the scale factor has the usual matter dominated form, $R \propto t^{2/3}$, while the temperature has the somewhat unusual form, $T \propto t^{-1/4}$. The relic density to entropy density ratio then has the form,

$$\frac{n}{s} \propto \frac{R^{-3}}{T^{3}} \propto t^{-1.25}$$

For a relic formed at a temperature $T_x$ at time $t_x$, we have a dilution factor, 

$$d.f. = \left(\frac{t_{\text{reheat}}}{t_x}\right)^{1.25} = \left(\frac{T_x}{T_{\text{reheat}}}\right)^{5}$$

For processes occurring at around the TeV scale, the dilution factor is around $10^{30}$ as in the instantaneous case. However, if the process occurs at a lower temperature, the dilution factor is reduced.
In the baryogenesis scenario of [8], the baryon asymmetry was sufficient to account for that required by nucleosynthesis. Our mechanism discussed in the previous section enhances this by the factor $\lambda^{-1}$. However, the dilution factor we have found is considerably greater than this. Consequently, the resulting baryon asymmetry will be diluted below that required by nucleosynthesis. A very late baryogenesis mechanism is needed in this class of models in order to avoid dilution. Similarly, emitted cosmic rays are likely to be significantly diluted in this class of models.

5 Conclusions

We have considered the properties of cosmic strings in theories where the Higgs self-coupling is small. Such theories arise in supersymmetric, grand unified theories with flat directions. We have shown the resulting strings are much fatter than usual, with vastly different gauge and scalar field core sizes. The gauge core is increased by a logarithmic factor, whilst the scalar core is the inverse Higgs mass. Similarly, the mass per unit length of the string is reduced by the logarithmic suppression factor relative to usual cosmic string models. This alters the cosmology of such strings. The gravitational properties are reduced by this factor. Consequently, if such strings are to play a role in large scale structure then the unification temperature needs to be increased by a corresponding amount. This makes such models less attractive from a particle physics viewpoint, though they could still arise from superstring theories. Similarly, the increase in string width changes their microphysical properties. It results in an increased baryon catalysis cross-section and a vast increase in the number of particles produced by loop decay. Whilst this latter effect could produce the observed baryon asymmetry and result in massive production of cosmic rays from such strings, it is likely that such observable effects will be diluted by the subsequent reheating.

The evolution of the universe is greatly modified in this class of theories, with the universe undergoing a late reheating. Using a realistic model for the decay of the $TeV$ scale Higgs particle, we have shown how the evolution is modified. There is prompt reheating following the phase transition, the universe then undergoes a period of matter domination as it cools monotonically until radiation domination begins just before nucleosynthesis. This scenario is rather different from that proposed in [2] where all Higgs particles were taken to decay at the typical lifetime of order $10^{23}GeV^{-1}$. In
particular, relics produced at temperatures above the reheat temperature suffer significantly less dilution if the Higgs decay is continuous than they would if the decay was instantaneous.

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