Testing dark energy after pre-recombination early dark energy

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Abstract

In the studies on pre-recombination early dark energy (EDE), the evolution of Universe after recombination is usually regarded as ΛCDM-like, which corresponds that the equation of state of dark energy responsible for current accelerated expansion is $w = -1$. However, in realistic models, $w$ might be evolving. We consider the parametrizations of $w$ with respect to the redshift $z$ in Axion-like EDE and AdS-EDE models, respectively. We performed the Monte Carlo Markov chain analysis with recent cosmological data, and found that the bestfit $w(z)$ is compatible with $w_0 = -1, w_a = 0$ (the cosmological constant) and the evolution of $w$ is only marginally favored, which so has little effect on lifting the bestfit value of $H_0$.

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I. INTRODUCTION

Recently, local measurements of Hubble constant $H_0$ have showed $H_0 \sim 73 \text{km/s/Mpc}$ [1–5], which has $\gtrsim 4\sigma$ discrepancy to $H_0 \sim 67 \text{km/s/Mpc}$ suggested from the fit of cosmic microwave background (CMB) data [6] based on the $\Lambda$CDM model, which is so-called Hubble tension [7, 8], see also the most recent [9]. Currently, it is difficult to explain this conflict by systematic errors.

It has been widely thought that the Hubble tension might be a hint of new physics beyond $\Lambda$CDM, e.g.[10–14]. In early dark energy (EDE) scenario [15], see also [16–31], since the scalar field energy has a non-negligible fraction before recombination, the sound horizon

$$r_s^* = \int_{z_*}^{\infty} \frac{c_s}{H(z)} dz$$ (1)

is suppressed, where $z_*$ is the redshift at recombination, which naturally brings a higher $H_0$ (noting that CMB and BAO have fixed the angular scales

$$\theta_s^* = \frac{r_s^*}{D_A^*} \sim r_s^* H_0,$$ (2)

where $D_A^*$ is the angular diameter to last scattering surface), without spoiling fit to CMB and baryon acoustic oscillations (BAO) data, see also [32–35] for combined Planck+SPT dataset and [36, 37] for Planck+ACT dataset. In particular, an Anti-de Sitter (AdS) phase around recombination can allow $H_0 \simeq 73 \text{km/s/Mpc}$, so the corresponding AdS-EDE model [20, 21] can be $1\sigma$ consistent with local $H_0$ measurements. In Ref.[38], it has been found that the pre-recombination solutions of the Hubble tension implies a scale-invariant Harrison-Zeldovich spectrum of primordial scalar perturbation, i.e. $n_s = 1$ for $H_0 \sim 73 \text{km/s/Mpc}$.

The beyond-$\Lambda$CDM modifications after recombination have also been proposed e.g.[39–50], see also [51, 52] for recent reviews. The current accelerated expansion of our Universe suggests the existence of dark energy at present, with the equation of state (EOS) $w = p/\rho \simeq -1$. In the studies on pre-recombination EDE, the evolution of Universe after recombination is usually regarded as $\Lambda$CDM-like, which corresponds to $w = -1$. However, in realistic models, $w$ might be evolving, see e.g.[51] for a review, so having a consistency check for the $\Lambda$CDM model after recombination is significant.

In this paper, we will consider different parametrization of $w$ in Axion-like EDE and AdS-EDE models, respectively, and perform the Markov Chain Monte Carlo (MCMC) analysis
with PlanckCMB, BAO, Pantheon and $H_0$ dataset. This paper is organised as follows. In sect-II, we outline the parametrizations of $w$ of current dark energy. In sect-III, we perform the MCMC analysis and present our results. We conclude in sect-IV.

II. PARAMETRIZATIONS OF DARK ENERGY

In ΛCDM model, the dark energy activates as a cosmological constant with $w = -1$. However, in scalar field models of dark energy, e.g.[53–59], $w$ may be evolving or oscillating. Here, it is convenient to work with the parametrizations of $w(z)$.

The simplest parametrization is $w(z) = w_0 + w_1 z$, which is linear but diverge at high redshift, see also [61–63]. Thus the CPL parametrization [64, 65]

$$w(z) = w_0 + w_a \left( \frac{z}{1 + z} \right), \tag{3}$$

and the oscillating parametrization $w(z) = w_0 + w_1 \{1 - \cos[\ln(1 + z)]\}$ [66], see also [67, 69], have been widely applied. Both are well behaved at high $z$. The parametrizations degrading into linear at low $z$ also include logarithmic[70], etc [71–75], see also [76] for relevant comments.

III. MCMC RESULTS FOR EDE MODELS

In this section we will confront the Axion EDE and AdS-EDE models (with CPL and oscillating parametrizations, respectively) with recent cosmological data. We modified the Montepython-3.3 [77, 78] and CLASS [79, 80] codes to perform the MCMC analysis.

Here, we will set the SH0ES result [1] as the Gaussian prior. The datasets consist of Planck2018 high-l and low-l TTTEEE as well as Planck lensing likelihoods[81], the BOSS DR12 [82] with its full covariant matrix for BAO as well as the 6dFGS [83] and MGS of SDSS [84] for low-z BAO, the Pantheon data [85]. We consider chains to be converged when the Gelman-Rubin statistic[68] satisfies $R - 1 < 0.05$. 

A. Axion-like EDE model

In Axion-like EDE model [15], an oscillating axion field with the potential $V(\phi) \propto (1 - \cos[\phi/f])^n$, naturally arising in the string theory, is responsible for EDE. At the critical redshift, EDE starts to oscillate and dilutes away like a fluid with $w = (n - 1)/(n + 1)$. It is noted that $n = 3$ is better for a higher best-fit $H_0$ [15, 22].

We perform the MCMC analysis on the parameters set $\{\omega_b, \omega_{cdm}, H_0, \ln(10^{10}A_s), n_s, \tau_{reio}, \log_{10}a_c, f_{ede}, \phi_i, w_0, w_a\}$, where $\phi_i$ is the initial value of EDE field, $a_c$ is when the field starts rolling and $f_{ede}$ is the energy fraction of EDE at $a_c$. We also set $n = 3$ [15, 22]. In Table I, we present the MCMC results for the CPL and oscillating parameterizations of $w(z)$, respectively. In Fig.1, we show the 1σ and 2σ marginalized posterior distributions of model parameters.

| Parameters | ΛCDM $\pm$0.015 | Axi+CPL $\pm$0.018 | Axi+OSC $\pm$0.0256 |
|-----------|-----------------|-------------------|---------------------|
| $100\omega_b$ | 2.247(2.224)$\pm$0.014 | 2.247(2.289)$\pm$0.0188 | 2.256(2.277)$\pm$0.0256 |
| $\omega_{cdm}$ | 0.1182(0.1183)$+0.0008_{-0.0013}$ | 0.1243(0.1305)$+0.0023_{-0.0052}$ | 0.1261(0.1307)$+0.0032_{-0.0048}$ |
| $H_0$ | 68.16(68.23)$+0.56_{-0.4}$ | 70.06(72.05)$+0.85_{-1.34}$ | 70.09(71.90)$+1.18_{-0.94}$ |
| $\ln(10^{10}A_s)$ | 3.049(3.054)$+0.013_{-0.015}$ | 3.044(3.050)$+0.017_{-0.016}$ | 3.044(3.031)$+0.0154_{-0.0153}$ |
| $n_s$ | 0.9688(0.9696)$+0.0039_{-0.0042}$ | 0.9717(0.9853)$+0.0069_{-0.0104}$ | 0.9749(0.9850)$+0.0081_{-0.0110}$ |
| $\tau_{reio}$ | 0.0604(0.0636)$+0.0066_{-0.0075}$ | 0.0525(0.0505)$+0.0080_{-0.0081}$ | 0.0516(0.0448)$+0.0072_{-0.0076}$ |
| $f_{ede}$ | - | 0.045(0.052)$+0.0713_{-0.0370}$ | 0.047(0.054)$+0.0794_{-0.0229}$ |
| $\log_{10}a_c$ | - | $-3.652(-3.839)^{+0.2439_{-0.2057}}$ | $-3.664(-3.864)^{+0.2386_{-0.2112}}$ |
| $100\theta_s$ | 1.0422(1.0421)$+0.0005_{-0.0004}$ | 1.0416(1.0415)$+0.0003_{-0.0003}$ | 1.0415(1.0415)$+0.0003_{-0.0003}$ |
| $\sigma_8$ | 0.8078(0.81)$+0.0054_{-0.0066}$ | 0.8472(0.8597)$+0.0105_{-0.0236}$ | 0.8506(0.8538)$+0.0115_{-0.0119}$ |
| $w_0$ | -1 | $-1.021(-0.970)^{+0.0755_{-0.0883}}$ | $-1.008(-1.007)^{+0.0458_{-0.0449}}$ |
| $w_a$ | 0 | $-0.130(-0.293)^{+0.347_{-0.280}}$ | $-0.469(-0.444)^{+0.567_{-0.322}}$ |

TABLE I: Mean values (best-fits) of parameters in Axion-like EDE model for CPL and oscillating parameterizations, respectively, fitted to Planck2018+BAO+Pantheon+$H_0$ dataset.

In Fig.1, we see that the evolving $w$ has little effect on $H_0$ and $n_s$, compared with the model with $w = -1$ in Ref.[15]. However, the EDE parameters are constrained more tightly.

\footnote{We follow the name in Ref.[37].}
FIG. 1: Marginalized 1σ and 2σ posterior distributions of \( \{H_0, n_s, f_{ede}, w_0, w_a\} \) in Axion-like EDE model. The red lines are the results for the CPL parameterization and the blue ones are for the oscillating parameterization. The shadow regions of \( H_0 \) represent 1σ and 2σ values of local measurement [1]. The intersection point of dash lines in \( w_0-w_a \) plot corresponds to the ΛCDM model, with the shadow region corresponding to the Quintessence dark energy in the whole history.

by the oscillating parametrization. The result on \( w(z) \) is compatible with the cosmological constant \((w_0 = -1, w_a = 0)\), and only marginally favors the evolution of \( w \). Here and in AdS-EDE model (see Table-II), the amplitude \( \sigma_8 \) of matter density fluctuation at low redshift is larger than local measurements [86, 87], which, however, may be pulled lower by new physics beyond cold dark matter [88–90]².

We also follow the Ref.[18] and plot the difference \( \Delta C_l = C_{l,\text{model}} - C_{l,\text{ref}} \) in units of the cosmic variance per multipole

\[
\sigma_{CV} = \begin{cases} \\
\sqrt{2/(2l+1)}C_l^{TT}, & TT \\
\sqrt{1/(2l+1)}\sqrt{C_l^{TT}C_l^{EE} + (C_l^{TE})^2}, & TE \\
\sqrt{2/(2l+1)}C_l^{EE}, & EE \\
\end{cases}
\]

for both parametrizations in CMB TT, EE and TE spectrum in Fig.2, where \( C_{l,\text{ref}} \) is that for the ΛCDM model. Compared with the results in Ref.[15], the residual oscillations caused by

² Here, we will not involve it.
EDE at small scales are slightly amplified under both the CPL and oscillating parametrizations. The residuals of TT, EE and TE spectrum are within the cosmological variance at observable scales ($l \lesssim 2000$).

FIG. 2: $\Delta C_l / \sigma_{CV}$ for both parametrizations in Axion-like EDE model fitted to Planck2018+BAO+Pantheon+$H_0$ datasets. The reference model is the $\Lambda$CDM model. The left panel is that for the TT spectrum, the right one is for the EE and the bottom one is for the TE spectrum.

B. AdS-EDE model

In AdS-EDE model [20], we consider the potential as $V(\phi) = V_0 \left( \frac{\phi}{M_p} \right)^4 - V_{ads}$, which is glued to $V(\phi) = 0$ at $\phi = \left( \frac{V_{ads}}{V_0} \right)^{1/4} M_p$ by interpolation, where $V_{ads}$ is the depth of AdS well. The existence of AdS phase enables the density $\rho_{ede}$ of field dilutes away faster, and so allows a larger EDE fraction but without spoiling fit to CMB and BAO data, which makes
AdS-EDE possible have a higher $H_0$.

We perform the MCMC analysis on the parameters set \{\(\omega_b, \omega_{cdm}, H_0, \ln(10^{10}A_s), n_s, \tau_{reio}, \ln(1 + z_c), f_{ede}, \alpha_{ads}, w_0, w_a\)\}, where \(z_c\) is the redshift when the field starts rolling, \(f_{ede}\) is the energy fraction of EDE at \(z_c\), and \(\alpha_{ads}\) corresponds to \(V_{ads} = \alpha_{ads}(\rho_m(z_c) + \rho_r(z_c))\), which will be fixed to \(3.79 \times 10^{-4}\), see [20]. In Table-II, we present the MCMC results of AdS-EDE model for the CPL and oscillating parameterizations, respectively. In Fig.3, we show the $1\sigma$ and $2\sigma$ marginalized posterior distributions of model parameters.

| Parameters | $\Lambda$CDM | AdS+CPL | AdS+OSC |
|------------|--------------|---------|----------|
| \(100\omega_b\) | (2.247, 2.224)$^{+0.015}_{-0.014}$ | (2.327, 2.320)$^{+0.0197}_{-0.0179}$ | (2.326, 2.324)$^{+0.0184}_{-0.0192}$ |
| \(\omega_{cdm}\) | 0.1182(0.1183)$^{+0.0008}_{-0.0013}$ | 0.1353(0.1338)$^{+0.00184}_{-0.00210}$ | 0.1354(0.1354)$^{+0.00185}_{-0.00203}$ |
| \(H_0\) | 68.16(68.23)$^{+0.56}_{-0.4}$ | 72.77(72.30)$^{+0.659}_{-0.805}$ | 72.73(72.51)$^{+0.778}_{-0.774}$ |
| \(\ln(10^{10}A_s)\) | 3.049(3.054)$^{+0.013}_{-0.016}$ | 3.071(3.067)$^{+0.0158}_{-0.0151}$ | 3.069(3.068)$^{+0.0152}_{-0.0144}$ |
| \(n_s\) | 0.9688(0.9696)$^{+0.0039}_{-0.0042}$ | 0.9938(0.9951)$^{+0.00488}_{-0.00468}$ | 0.9934(0.9948)$^{+0.00431}_{-0.00489}$ |
| \(\tau_{reio}\) | 0.0604(0.0636)$^{+0.0066}_{-0.0075}$ | 0.0530(0.0537)$^{+0.00807}_{-0.00815}$ | 0.0521(0.0523)$^{+0.00772}_{-0.00753}$ |
| \(f_{ede}\) | - | 0.1137(0.1081)$^{+0.00415}_{-0.00883}$ | 0.1135(0.1094)$^{+0.00772}_{-0.00753}$ |
| \(\ln(1 + z_c)\) | - | 8.168(8.169)$^{+0.0647}_{-0.0848}$ | 8.177(8.087)$^{+0.0759}_{-0.0821}$ |
| \(100\theta_s\) | (1.0421, 1.0421)$^{+0.0005}_{-0.0004}$ | 1.0410(1.0410)$^{+0.000304}_{-0.000308}$ | 1.0410(1.0411)$^{+0.000302}_{-0.000308}$ |
| \(\sigma_8\) | 0.8078(0.81)$^{+0.0054}_{-0.0066}$ | 0.8633(0.8636)$^{+0.0112}_{-0.0106}$ | 0.8651(0.8577)$^{+0.0112}_{-0.0118}$ |
| \(w_0\) | -1 | $-0.987(-0.989)^{+0.0734}_{-0.0753}$ | $-0.983(-0.960)^{+0.0463}_{-0.0477}$ |
| \(w_a\) | 0 | $-0.122(-0.154)^{+0.209}_{-0.248}$ | $-0.427(-0.562)^{+0.567}_{-0.344}$ |

| TABLE II: Mean values (best-fits) of parameters in AdS-EDE model for CPL and oscillating parameterizations, respectively, fitted to Planck2018+BAO+Pantheon+$H_0$ dataset. Actually, we can have the bestfit $H_0 \gtrsim 72$km/s/Mpc for AdS-EDE model without $H_0$ prior, see Appendix for the MCMC results based on Planck2018+BAO+Pantheon dataset. |

In Table-II and Fig.3, we see again that the evolving $w$ has little effect on $H_0$ and $n_s$, compared with AdS-EDE model with $w = -1$ in Ref.[20]. It is also clear that the parameterizations of dark energy hardly affect the EDE parameters. The result on $w(z)$ is still compatible with the cosmological constant ($w_0 = -1, w_a = 0$), and only marginally favors the evolution of $w$. The difference $\Delta C_l/\sigma_{CV}$ is plotted in Fig.4. Compared with the
results in Ref. [20], the parameterizations of dark energy not only maintain the character of oscillating in both TT and EE spectrum, but also strengthen the going-upwards of amplitude with $l$ in TT spectrum and the bump at $l \sim 200$ in EE spectrum. The residuals of the TT spectrum are within the cosmological variance for large scales ($l \lesssim 1500$) and become comparable to the $\sigma_{CV}$ as $l$ grows. However, the residuals of the EE and TE spectrum are larger than the $\sigma_{CV}$ in the $l \gtrsim 1300$ multipoles for CPL and $l \gtrsim 850$ for oscillating one, which may be detected.

![Marginalized 1σ and 2σ posterior distributions of $\{H_0, n_s, f_{ede}, w_0, w_a\}$ in AdS-EDE model. The red lines are the results for the CPL parameterization and the blue ones are for the oscillating parameterization. The shadow regions are the same as in Fig.2.](image)

**FIG. 3:** Marginalized 1σ and 2σ posterior distributions of $\{H_0, n_s, f_{ede}, w_0, w_a\}$ in AdS-EDE model. The red lines are the results for the CPL parameterization and the blue ones are for the oscillating parameterization. The shadow regions are the same as in Fig.2.

C. Discussion

We present the $H_0 - r^*_s$ contours for the CPL and oscillating parametrizations, respectively, with colored scatters as $w_0$ in Fig.5. We see that $w_0$ at 1σ contour is closed to -1, which is consistent with the cosmological constant. As expected, we have $H_0 \simeq 73$ and $w_0 \simeq -1$ for AdS-EDE model.

We list the $\chi^2$ of all datasets for different models in Table-III, respectively. We find that all best-fit models are improved over the best-fit $\Lambda$CDM model by $\Delta \chi^2 \sim -20$. We
FIG. 4: $\Delta C_l/\sigma_{C_V}$ for both parametrizations in AdS-EDE model fitted to Planck2018+BAO+Pantheon+$H_0$ datasets. The reference model is the $\Lambda$CDM model. The left panel is that for the TT spectrum, the right one is for the EE and the bottom one is for the TE spectrum.

see that both parameterizations reduce the $\chi^2$ of Axion EDE model markedly, but slightly reduce that of the AdS-EDE model. This suggests that with the evolving $w$ of current dark energy, Axion model seems to be favored over the AdS model. However, here since the AdS parameter $\alpha_{ads}$ (relevant with the depth of AdS well) is fixed as $3.79 \times 10^{-4}$, releasing $\alpha_{ads}$ might gives better fit for the AdS-EDE model. It is also noted that although the $\chi^2_{CMB}$ in Axion EDE model is reduced, its fit to BAO dataset is worsened seriously.

To test whether the smoothing effect of lensing to the CMB power spectrum is consistent with that measured by the lensing reconstruction, the lensing potential is often scaled by a consistency parameter $A_L$, theoretically $A_L = 1$ [91]. It has been pointed out that Planck data seems favor a closed universe [92], while flat universe suggests $A_L = 1.180 \pm 0.065$
FIG. 5: $H_0 - r_s^*$ contours for the CPL and oscillating parametrizations with colored scatters by $w_0$. The red contour is that for Axion EDE model and the blue one is for AdS-EDE model. The shadow region corresponds to 1 $\sigma$ value of local $H_0$ measurement.

| Dataset            | $\Lambda$CDM | AdS+CPL   | AdS+OSC   | Axi+CPL | Axi+OSC |
|--------------------|---------------|-----------|-----------|---------|---------|
| Planck high-l TT,TE,EE | 2347.5        | 2346.52   | 2344.81   | 2338.89 | 2336.30 |
| Planck low-l EE    | 398.2         | 396.04    | 395.77    | 395.73  | 396.29  |
| Planck low-l TT    | 23.9          | 20.63     | 20.81     | 20.94   | 20.78   |
| Planck lensing     | 9.1           | 10.07     | 10.81     | 10.33   | 9.36    |
| BAO BOSS DR12      | 1.8           | 3.51      | 3.51      | 4.61    | 5.08    |
| BAO smallz 2014    | 2.2           | 1.61      | 2.00      | 2.88    | 3.38    |
| Pantheon           | 1026.9        | 1028.24   | 1027.49   | 1027.12 | 1027.52 |
| HST                | 15.4          | 0.21      | 0.29      | 0.89    | 0.95    |
| **Total**          | **3825**      | **3806.83**| **3805.50**| **3803.40**| **3799.67**|

TABLE III: $\chi^2$ of all datasets for different models.

(Planck TT,TE,EE+lowE) [6], which is called the lensing anomaly. The oscillating parameterization of dark energy might help to explain this problem.

We set $A_L$ as a MCMC parameter, and show the posterior distribution of parameters set $\{H_0, n_s, w_0, w_a, A_L\}$ in Axion-like EDE model with the oscillating parametrization in Fig.6. We see that the bestfit of $H_0$ is consistent with that in sect-III.A, and $A_L = 1.0421^{+0.036}_{-0.050}$.
but with a slightly smaller $\Omega_m$, see Fig. 7. Thus it is possible to seek for certain oscillating parametrizations to alleviate the lensing anomaly.

FIG. 6: Marginalized 1σ and 2σ posterior distributions of $\{H_0, n_s, w_0, w_a, A_L\}$ in Axion-like EDE model with the oscillating parametrization. It is obvious that $A_L$ is very closed to 1, which alleviates the lensing anomaly.

FIG. 7: $A_L$-$\Omega_m$ contour in Axion-like EDE model with the oscillating parametrization. The dash line corresponds to $A_L=1$. 
IV. CONCLUSIONS

In the studies on pre-recombination EDE, the evolution of Universe after recombination is usually regarded as ΛCDM-like, which corresponds to $w = -1$. However, in realistic models, $w$ might be evolving or oscillating. Here, we investigate the effects of different parametrization of $w$ in Axion-like EDE and AdS-EDE models, respectively.

We performed the MCMC analysis with recent cosmological data, and found that bestfit $w(z)$ is compatible with $w_0 = -1, w_a = 0$ (the cosmological constant), and the evolution of $w$ is only marginally favored. Particularly, our study confirmed again that AdS-EDE model (here the bestfit $H_0 \simeq 72.7 \text{km/s/Mpc}$) is more efficient in solving the Hubble tension. The parametrizations of $w$ i.e. the evolution of current dark energy, in EDE models has little effect on the bestfit values of $H_0$ and $n_s$, thus the shift of primordial scalar spectral index scales still as $\delta n_s \simeq 0.4 \frac{\delta H_0}{H_0}$ [38], which suggests a scale-invariant Harrison-Zeldovich spectrum ($n_s = 1$) for $H_0 \sim 73 \text{km/s/Mpc}$, see also Table-I,II.

We also show $\Delta C_1 / \sigma_{CV}$ for both parametrizations in CMB TT, EE and TE spectrum, and found that compared with the results in Refs.[15, 20], the parameterization of $w(z)$ basically maintains the shape of spectrum, but slightly amplifies the residual oscillations caused by EDE at small scale, which might be detectable. In addition, we also found that the oscillating parametrization could alleviate the lensing anomaly. Thus it is interesting to test other parametrizations of current dark energy in EDE cosmologies.

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Appendix A: MCMC results in AdS-EDE model without $H_0$ prior

We show the MCMC results of AdS-EDE model for the CPL parametrization, based on Planck2018+BAO+Pantheon dataset without $H_0$ prior.

| Parameters | AdS+CPL |
|------------|---------|
| $100\omega_b$ | $2.331(2.328)^{+0.0194}_{-0.0210}$ |
| $\omega_{cdm}$ | $0.1345(0.1340)^{+0.00186}_{-0.00227}$ |
| $H_0$ | $72.44(72.36)^{+0.704}_{-0.925}$ |
| $\ln(10^{10}A_s)$ | $3.070(3.075)^{+0.0156}_{-0.0167}$ |
| $n_s$ | $0.9955(0.9935)^{+0.00462}_{-0.00488}$ |
| $\tau_{reio}$ | $0.0537(0.0553)^{+0.00838}_{-0.00876}$ |
| $f_{ede}$ | $0.1122(0.1059)^{+0.00446}_{-0.00909}$ |
| $\ln(1+z_c)$ | $8.1943(8.2317)^{+0.0844}_{-0.0943}$ |
| $100\theta_s$ | $1.0411(1.0410)^{+0.000328}_{-0.000349}$ |
| $\sigma_8$ | $0.8584(0.8649)^{+0.0119}_{-0.0123}$ |
| $w_0$ | $-0.969(-0.979)^{+0.056}_{-0.040}$ |
| $w_a$ | $-0.173(-0.146)^{+0.108}_{-0.174}$ |

TABLE IV: Mean values (best-fits) of parameters in AdS-EDE model for the CPL parameterizations, fitted to Planck2018+BAO+Pantheon dataset.

[1] A. G. Riess, S. Casertano, W. Yuan, L. M. Macri and D. Scolnic, Astrophys. J. 876, no.1, 85 (2019) doi:10.3847/1538-4357/ab1422 [arXiv:1903.07603 [astro-ph.CO]].

[2] G. C. F. Chen, C. D. Fassnacht, S. H. Suyu, C. E. Rusu, J. H. H. Chan, K. C. Wong, M. W. Auger, S. Hilbert, V. Bonvin and S. Birrer, et al. Mon. Not. Roy. Astron. Soc. 490, no.2, 1743-1773 (2019) doi:10.1093/mnras/stz2547 [arXiv:1907.02533 [astro-ph.CO]].

[3] K. C. Wong, S. H. Suyu, G. C. F. Chen, C. E. Rusu, M. Millon, D. Sluse, V. Bonvin, C. D. Fassnacht, S. Taubenberger and M. W. Auger, et al. Mon. Not. Roy. Astron. Soc. 498, no.1, 1420-1439 (2020) doi:10.1093/mnras/stz3094 [arXiv:1907.04869 [astro-ph.CO]].
[4] W. L. Freedman, B. F. Madore, D. Hatt, T. J. Hoyt, I. S. Jang, R. L. Beaton, C. R. Burns, M. G. Lee, A. J. Monson and J. R. Neeley, et al. doi:10.3847/1538-4357/ab2f73 [arXiv:1907.05922 [astro-ph.CO]].

[5] C. D. Huang, A. G. Riess, W. Yuan, L. M. Macri, N. L. Zakamska, S. Casertano, P. A. White-lock, S. L. Hoffmann, A. V. Filippenko and D. Scolnic, doi:10.3847/1538-4357/ab5ddd [arXiv:1908.10883 [astro-ph.CO]].

[6] N. Aghanim et al. [Planck], Astron. Astrophys. 641, A6 (2020) [erratum: Astron. Astrophys. 652, C4 (2021)] doi:10.1051/0004-6361/201833910 [arXiv:1807.06209 [astro-ph.CO]].

[7] A. G. Riess, Nature Rev. Phys. 2 (2019) no.1, 10-12 doi:10.1038/s42254-019-0137-0 [arXiv:2001.03624 [astro-ph.CO]].

[8] L. Verde, T. Treu and A. G. Riess, Nature Astron. 3, 891 doi:10.1038/s41550-019-0902-0 [arXiv:1907.10625 [astro-ph.CO]].

[9] A. G. Riess, W. Yuan, L. M. Macri, D. Scolnic, D. Brout, S. Casertano, D. O. Jones, Y. Murakami, L. Breuval and T. G. Brink, et al. [arXiv:2112.04510 [astro-ph.CO]].

[10] L. Knox and M. Millea, Phys. Rev. D 101, no.4, 043533 (2020) doi:10.1103/PhysRevD.101.043533 [arXiv:1908.03663 [astro-ph.CO]].

[11] R. G. Cai, Z. K. Guo, S. J. Wang, W. W. Yu and Y. Zhou, Phys. Rev. D 105 (2022) no.2, L021301 doi:10.1103/PhysRevD.105.L021301 [arXiv:2107.13286 [astro-ph.CO]].

[12] M. Z. Lyu, B. S. Haridasu, M. Viel and J. Q. Xia, Astrophys. J. 900 (2020) no.2, 160 doi:10.3847/1538-4357/aba756 [arXiv:2001.08713 [astro-ph.CO]].

[13] B. S. Haridasu, M. Viel and N. Vittorio, Phys. Rev. D 103 (2021) no.6, 063539 doi:10.1103/PhysRevD.103.063539 [arXiv:2012.10324 [astro-ph.CO]].

[14] S. Vagnozzi, Phys. Rev. D 104 (2021) no.6, 063524 doi:10.1103/PhysRevD.104.063524 [arXiv:2105.10425 [astro-ph.CO]].

[15] V. Poulin, T. L. Smith, T. Karwal and M. Kamionkowski, Phys. Rev. Lett. 122, no.22, 221301 (2019) doi:10.1103/PhysRevLett.122.221301 [arXiv:1811.04083 [astro-ph.CO]].

[16] P. Agrawal, F. Y. Cyr-Racine, D. Pinner and L. Randall, [arXiv:1904.01016 [astro-ph.CO]].

[17] S. Alexander and E. McDonough, Phys. Lett. B 797, 134830 (2019) doi:10.1016/j.physletb.2019.134830 [arXiv:1904.08912 [astro-ph.CO]].

[18] M. X. Lin, G. Benevento, W. Hu and M. Raveri, Phys. Rev. D 100, no.6, 063542 (2019) doi:10.1103/PhysRevD.100.063542 [arXiv:1905.12618 [astro-ph.CO]].
[19] F. Niedermann and M. S. Sloth, Phys. Rev. D 103, no.4, L041303 (2021) doi:10.1103/PhysRevD.103.L041303 [arXiv:1910.10739 [astro-ph.CO]].

[20] G. Ye and Y. S. Piao, Phys. Rev. D 101, no.8, 083507 (2020) doi:10.1103/PhysRevD.101.083507 [arXiv:2001.02451 [astro-ph.CO]].

[21] G. Ye and Y. S. Piao, Phys. Rev. D 102 (2020) no.8, 083523 doi:10.1103/PhysRevD.102.083523 [arXiv:2008.10832 [astro-ph.CO]].

[22] T. L. Smith, V. Poulin and M. A. Amin, Phys. Rev. D 101, no.6, 063523 (2020) doi:10.1103/PhysRevD.101.063523 [arXiv:1908.06995 [astro-ph.CO]].

[23] J. Sakstein and M. Trodden, Phys. Rev. Lett. 124, no.16, 161301 (2020) doi:10.1103/PhysRevLett.124.161301 [arXiv:1911.11760 [astro-ph.CO]].

[24] T. Karwal, M. Raveri, B. Jain, J. Khoury and M. Trodden, [arXiv:2106.13290 [astro-ph.CO]].

[25] M. Braglia, W. T. Emond, F. Finelli, A. E. Gumrukcuoglu and K. Koyama, Phys. Rev. D 102 (2020) no.8, 083513 doi:10.1103/PhysRevD.102.083513 [arXiv:2005.14053 [astro-ph.CO]].

[26] M. X. Lin, W. Hu and M. Raveri, Phys. Rev. D 102 (2020), 123523 doi:10.1103/PhysRevD.102.123523 [arXiv:2009.08974 [astro-ph.CO]].

[27] S. Nojiri, S. D. Odintsov, D. Saez-Chillon Gomez and G. S. Sharov, Phys. Dark Univ. 32 (2021), 100837 doi:10.1016/j.dark.2021.100837 [arXiv:2103.05304 [gr-qc]].

[28] V. I. Sabla and R. R. Caldwell, Phys. Rev. D 103 (2021) no.10, 103506 doi:10.1103/PhysRevD.103.103506 [arXiv:2103.04999 [astro-ph.CO]].

[29] A. Moss, E. Copeland, S. Bamford and T. Clarke, [arXiv:2109.14848 [astro-ph.CO]].

[30] E. McDonough, M. X. Lin, J. C. Hill, W. Hu and S. Zhou, [arXiv:2112.09128 [astro-ph.CO]].

[31] A. Gómez-Valent, Z. Zheng, L. Amendola, V. Pettorino and C. Wetterich, Phys. Rev. D 104, no.8, 083536 (2021) doi:10.1103/PhysRevD.104.083536 [arXiv:2107.11065 [astro-ph.CO]].

[32] A. La Posta, T. Louis, X. Garrido and J. C. Hill, [arXiv:2112.10754 [astro-ph.CO]].

[33] A. Chudaykin, D. Gorbunov and N. Nedelko, JCAP 08 (2020), 013 doi:10.1088/1475-7516/2020/08/013 [arXiv:2004.13046 [astro-ph.CO]].

[34] A. Chudaykin, D. Gorbunov and N. Nedelko, Phys. Rev. D 103 (2021) no.4, 043529 doi:10.1103/PhysRevD.103.043529 [arXiv:2011.04682 [astro-ph.CO]].

[35] J. Q. Jiang and Y. S. Piao, Phys. Rev. D 104 (2021) no.10, 103524 doi:10.1103/PhysRevD.104.103524 [arXiv:2107.07128 [astro-ph.CO]].

[36] J. C. Hill, E. Calabrese, S. Aiola, N. Battaglia, B. Bolliet, S. K. Choi, M. J. Devlin, A. J. Duiv-
envoorden, J. Dunkley and S. Ferraro, et al. [arXiv:2109.04451 [astro-ph.CO]].

[37] V. Poulin, T. L. Smith and A. Bartlett, Phys. Rev. D 104 (2021) no.12, 123550
doi:10.1103/PhysRevD.104.123550 [arXiv:2109.06229 [astro-ph.CO]].

[38] G. Ye, B. Hu and Y. S. Piao, Phys. Rev. D 104 (2021) no.6, 063510
doi:10.1103/PhysRevD.104.063510 [arXiv:2103.09729 [astro-ph.CO]].

[39] E. Di Valentino, A. Melchiorri, E. V. Linder and J. Silk, Phys. Rev. D 96, no.2, 023523 (2017)
doi:10.1103/PhysRevD.96.023523 [arXiv:1704.00762 [astro-ph.CO]].

[40] S. Vagnozzi, Phys. Rev. D 102, no.2, 023518 (2020) doi:10.1103/PhysRevD.102.023518
[arXiv:1907.07569 [astro-ph.CO]].

[41] L. Visinelli, S. Vagnozzi and U. Danielsson, Symmetry 11, no.8, 1035 (2019)
doi:10.3390/sym11081035 [arXiv:1907.07953 [astro-ph.CO]].

[42] E. Di Valentino, A. Melchiorri, O. Mena and S. Vagnozzi, Phys. Dark Univ. 30, 100666 (2020)
doi:10.1016/j.dark.2020.100666 [arXiv:1908.04281 [astro-ph.CO]].

[43] Ö. Akarsu, J. D. Barrow, L. A. Escamilla and J. A. Vazquez, Phys. Rev. D 101, no.6, 063528 (2020)
doi:10.1103/PhysRevD.101.063528 [arXiv:1912.08751 [astro-ph.CO]].

[44] Ö. Akarsu, S. Kumar, E. Özüeker and J. A. Vazquez, Phys. Rev. D 104 (2021) no.12, 123512
doi:10.1103/PhysRevD.104.123512 [arXiv:2108.09239 [astro-ph.CO]].

[45] W. Yang, E. Di Valentino, S. Pan, Y. Wu and J. Lu, Mon. Not. Roy. Astron. Soc. 501 (2021)
no.4, 5845-5858 doi:10.1093/mnras/staa3914 [arXiv:2101.02168 [astro-ph.CO]].

[46] W. Yang, E. Di Valentino, S. Pan and O. Mena, Phys. Dark Univ. 31 (2021), 100762
doi:10.1016/j.dark.2020.100762 [arXiv:2007.02927 [astro-ph.CO]].

[47] W. Liu, L. A. Anchordoqui, E. Di Valentino, S. Pan, Y. Wu and W. Yang, [arXiv:2108.04188
[astro-ph.CO]].

[48] G. Alestas, D. Camarena, E. Di Valentino, L. Kazantzidis, V. Marra, S. Nesseris and
L. Perivolaropoulos, [arXiv:2110.04336 [astro-ph.CO]].

[49] L. Perivolaropoulos and F. Skara, Phys. Rev. D 104 (2021) no.12, 123511
doi:10.1103/PhysRevD.104.123511 [arXiv:2109.04406 [astro-ph.CO]].

[50] S. Bag, V. Sahni, A. Shafieloo and Y. Shtanov, Astrophys. J. 923 (2021) no.2, 212
doi:10.3847/1538-4357/ac307e [arXiv:2107.03271 [astro-ph.CO]].

[51] E. Di Valentino, O. Mena, S. Pan, L. Visinelli, W. Yang, A. Melchiorri, D. F. Mota, A. G. Riess
and J. Silk, Class. Quant. Grav. 38 (2021) no.15, 153001 doi:10.1088/1361-6382/ac086d
[arXiv:2103.01183 [astro-ph.CO]].

[52] L. Perivolaropoulos and F. Skara, [arXiv:2105.05208 [astro-ph.CO]].

[53] B. Ratra and P. J. E. Peebles, Phys. Rev. D 37 (1988), 3406 doi:10.1103/PhysRevD.37.3406

[54] I. Zlatev, L. M. Wang and P. J. Steinhardt, Phys. Rev. Lett. 82 (1999), 896-899

[55] R. R. Caldwell, Phys. Lett. B 545 (2002), 23-29 doi:10.1016/S0370-2693(02)02589-3 [arXiv:astro-ph/9908168 [astro-ph]].

[56] B. Feng, X. L. Wang and X. M. Zhang, Phys. Lett. B 607 (2005), 35-41 doi:10.1016/j.physletb.2004.12.071 [arXiv:astro-ph/0404224 [astro-ph]].

[57] Z. K. Guo, Y. S. Piao, X. M. Zhang and Y. Z. Zhang, Phys. Lett. B 608 (2005), 177-182 doi:10.1016/j.physletb.2005.01.017 [arXiv:astro-ph/0410654 [astro-ph]].

[58] H. Wei, R. G. Cai and D. F. Zeng, Class. Quant. Grav. 22 (2005), 3189-3202 doi:10.1088/0264-9381/22/16/005 [arXiv:hep-th/0501160 [hep-th]].

[59] H. Wei and R. G. Cai, Phys. Rev. D 72 (2005), 123507 doi:10.1103/PhysRevD.72.123507 [arXiv:astro-ph/0509328 [astro-ph]]. doi:10.1103/PhysRevLett.82.896 [arXiv:astro-ph/9807002 [astro-ph]].

[60] L. Hart and J. Chluba, Mon. Not. Roy. Astron. Soc. 493, no.3, 3255-3263 (2020) doi:10.1093/mnras/staa412 [arXiv:1912.03986 [astro-ph.CO]].

[61] A. R. Cooray and D. Huterer, Astrophys. J. Lett. 513, L95-L98 (1999) doi:10.1086/311927 [arXiv:astro-ph/9901097 [astro-ph]].

[62] P. Astier, Phys. Lett. B 500, 8-15 (2001) doi:10.1016/S0370-2693(01)00072-7 [arXiv:astro-ph/0008306 [astro-ph]].

[63] J. Weller and A. Albrecht, Phys. Rev. D 65, 103512 (2002) doi:10.1103/PhysRevD.65.103512 [arXiv:astro-ph/0106079 [astro-ph]].

[64] M. Chevallier and D. Polarski, Int. J. Mod. Phys. D 10, 213-224 (2001) doi:10.1142/S0218271801000822 [arXiv:gr-qc/0009008 [gr-qc]].

[65] E. V. Linder, Phys. Rev. Lett. 90, 091301 (2003) doi:10.1103/PhysRevLett.90.091301 [arXiv:astro-ph/0208512 [astro-ph]].

[66] B. Feng, M. Li, Y. S. Piao and X. Zhang, Phys. Lett. B 634, 101-105 (2006) doi:10.1016/j.physletb.2006.01.066 [arXiv:astro-ph/0407432 [astro-ph]].

[67] R. Lazkoz, V. Salzano and I. Sendra, Phys. Lett. B 694, 198-208 (2011) doi:10.1016/j.physletb.2010.10.002 [arXiv:1003.6084 [astro-ph.CO]].
[68] A. Gelman and D. B. Rubin, Statist. Sci. 7, 457-472 (1992) doi:10.1214/ss/1177011136
[69] S. Pan, E. N. Saridakis and W. Yang, Phys. Rev. D 98, no.6, 063510 (2018) doi:10.1103/PhysRevD.98.063510 [arXiv:1712.05746 [astro-ph.CO]].
[70] G. Efstathiou, Mon. Not. Roy. Astron. Soc. 310, 842-850 (1999) doi:10.1046/j.1365-8711.1999.02997.x [arXiv:astro-ph/9904356 [astro-ph]].
[71] E. M. Barboza, Jr. and J. S. Alcaniz, Phys. Lett. B 666, 415-419 (2008) doi:10.1016/j.physletb.2008.08.012 [arXiv:0805.1713 [astro-ph]].
[72] J. Z. Ma and X. Zhang, Phys. Lett. B 699, 233-238 (2011) doi:10.1016/j.physletb.2011.04.013 [arXiv:1102.2671 [astro-ph.CO]].
[73] C. J. Feng, X. Y. Shen, P. Li and X. Z. Li, JCAP 09, 023 (2012) doi:10.1088/1475-7516/2012/09/023 [arXiv:1206.0063 [astro-ph.CO]].
[74] G. Pantazis, S. Nesseris and L. Perivolaropoulos, Phys. Rev. D 93, no.10, 103503 (2016) doi:10.1103/PhysRevD.93.103503 [arXiv:1603.02164 [astro-ph.CO]].
[75] W. Yang, S. Pan and A. Paliathanasis, Mon. Not. Roy. Astron. Soc. 475, no.2, 2605-2613 (2018) doi:10.1093/mnras/sty019 [arXiv:1708.01717 [gr-qc]].
[76] E. Ó. Colgáin, M. M. Sheikh-Jabbari and L. Yin, Phys. Rev. D 104, no.2, 023510 (2021) doi:10.1103/PhysRevD.104.023510 [arXiv:2104.01930 [astro-ph.CO]].
[77] B. Audren, J. Lesgourgues, K. Benabed and S. Prunet, JCAP 02, 001 (2013) doi:10.1088/1475-7516/2013/02/001 [arXiv:1210.7183 [astro-ph.CO]].
[78] T. Brinckmann and J. Lesgourgues, Phys. Dark Univ. 24, 100260 (2019) doi:10.1016/j.dark.2018.100260 [arXiv:1804.07261 [astro-ph.CO]].
[79] J. Lesgourgues, [arXiv:1104.2932 [astro-ph.IM]].
[80] D. Blas, J. Lesgourgues and T. Tram, JCAP 07, 034 (2011) doi:10.1088/1475-7516/2011/07/034 [arXiv:1104.2933 [astro-ph.CO]].
[81] N. Aghanim et al. [Planck], Astron. Astrophys. 641, A5 (2020) doi:10.1051/0004-6361/201936386 [arXiv:1907.12875 [astro-ph.CO]].
[82] S. Alam et al. [BOSS], Mon. Not. Roy. Astron. Soc. 470 (2017) no.3, 2617-2652 doi:10.1093/mnras/stx721 [arXiv:1607.03155 [astro-ph.CO]].
[83] F. Beutler, C. Blake, M. Colless, D. H. Jones, L. Staveley-Smith, L. Campbell, Q. Parker, W. Saunders and F. Watson, Mon. Not. Roy. Astron. Soc. 416, 3017-3032 (2011) doi:10.1111/j.1365-2966.2011.19250.x [arXiv:1106.3366 [astro-ph.CO]].
[84] A. J. Ross, L. Samushia, C. Howlett, W. J. Percival, A. Burden and M. Manera, Mon. Not. Roy. Astron. Soc. 449, no.1, 835-847 (2015) doi:10.1093/mnras/stv154 [arXiv:1409.3242 [astro-ph.CO]].

[85] D. M. Scolnic et al. [Pan-STARRS1], Astrophys. J. 859, no.2, 101 (2018) doi:10.3847/1538-4357/aab9bb [arXiv:1710.00845 [astro-ph.CO]].

[86] H. Hildebrandt, F. Köhlinger, J. L. van den Busch, B. Joachimi, C. Heymans, A. Kannawadi, A. H. Wright, M. Asgari, C. Blake and H. Hoekstra, et al. Astron. Astrophys. 633 (2020), A69 doi:10.1051/0004-6361/201834878 [arXiv:1812.06076 [astro-ph.CO]].

[87] C. Heymans, T. Tröster, M. Asgari, C. Blake, H. Hildebrandt, B. Joachimi, K. Kuĳken, C. A. Lin, A. G. Sánchez and J. L. van den Busch, et al. Astron. Astrophys. 646 (2021), A140 doi:10.1051/0004-6361/202039063 [arXiv:2007.15632 [astro-ph.CO]].

[88] I. J. Allali, M. P. Hertzberg and F. Rompineve, Phys. Rev. D 104 (2021) no.8, L081303 doi:10.1103/PhysRevD.104.L081303 [arXiv:2104.12798 [astro-ph.CO]].

[89] G. Ye, J. Zhang and Y. S. Piao, [arXiv:2107.13391 [astro-ph.CO]].

[90] S. J. Clark, K. Vattis, J. Fan and S. M. Koushiappas, [arXiv:2110.09562 [astro-ph.CO]].

[91] E. Calabrese, A. Slosar, A. Melchiorri, G. F. Smoot and O. Zahn, Phys. Rev. D 77, 123531 (2008) doi:10.1103/PhysRevD.77.123531 [arXiv:0803.2309 [astro-ph]].

[92] E. Di Valentino, A. Melchiorri and J. Silk, Nature Astron. 4, no.2, 196-203 (2019) doi:10.1038/s41550-019-006-9 [arXiv:1911.02087 [astro-ph.CO]].