Doubly heavy baryons $\Omega_{QQ'}$ vs. $\Xi_{QQ'}$ in sum rules of NRQCD

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In the framework of two-point sum rules of NRQCD, the masses and couplings of doubly heavy baryons to the corresponding quark currents are evaluated with account of coulomb-like corrections in the system of doubly heavy diquark as well as the contribution of nonperturbative terms determined by the quark, gluon, mixed condensates and the product of gluon and quark condensates. The introduction of nonzero light quark mass destroys the factorization of baryon and diquark correlators even at the perturbative level and provides the better convergency of sum rules. We estimate the difference $M_\Omega - M_\Xi = 100 \pm 10$ MeV. The ratio of baryonic constants $|Z_\Omega|^2/|Z_\Xi|^2$ is equal to $1.3 \pm 0.2$ indicating the violation of SU(3) flavor symmetry for the doubly heavy baryons.

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I. INTRODUCTION

Testing the QCD dynamics of heavy quarks in various conditions provides us with a qualitative and quantitative knowledge that allows us to distinguish fine complex effects caused by the electroweak nature of CP-violation or physics beyond the Standard Model. The list of hadrons containing the heavy quarks as available to the experimental observations and measurements, was recently enriched by a new member, the long-lived $B_c$-meson, in addition to the heavy quarkonia $\bar{b}b$ and $\bar{c}c$ as well as the mesons and baryons with a single heavy quark. This success of CDF Collaboration in the first observation of $B_c$-meson was based on the progress of experimental technique in the reconstruction of rare processes with heavy quarks by use of vertex detectors. This experience supports a hope to observe other rare long-lived doubly heavy hadrons, i.e. the baryons containing two heavy quarks. As expected they have production rate and lifetime similar to the $B_c$-meson ones.

In the present paper we investigate the two-point sum rules of NRQCD. The light quark-doubly heavy diquark structure of baryon leads to the definite expressions for baryonic

*We omit the indices $QQ'$, when they actually do not lead to misidentification with usual strange baryons.
currents written in terms of nonrelativistic heavy quark fields. To relate the nonrelativistic heavy quark correlators to the full QCD ones we need to take into account the hard gluon corrections by means of solving the renormalization group equation known up to the two-loop accuracy.

The convergency of sum rules results are essentially improved by account for a nonzero light quark mass. As was mentioned in [6] the sum rules stability can be achieved by destroying the baryon-diquark factorization in the correlators. The convergency was obtained due to taking into account the nonperturbative interactions caused by higher dimension operators in contrast to ref. [7], where a signigicant instability of results were observed in full QCD sum rules with no product of quark and gluon condensates. We show that for the strange $\Omega_{QQ'}$ baryons this factorization is broken already in the perturbative limit which allows us to introduce a new criterion for the determination of baryon masses since we observe the stability of sum rules for the masses obtained from both correlators standing in front of two independent Lorentz structures for the spinor field of $\Omega_{QQ'}$.

Moreover our choice of baryonic current is convenient to take into account the $\alpha_s/v$ coulomb-like corrections [8] inside the doubly heavy diquark.

In Section II we describe the scheme of calculation. There we define the currents and represent the spectral densities in the NRQCD sum rules for various operators included into the consideration. The numerical estimates in comparison with the values obtained in potential models are given in Section III. The results are summarized in Conclusion.

II. NRQCD SUM RULES FOR DOUBLY HEAVY BARYONS

A. Description of the method

In order to determine the masses and coupling constants of baryons in sum rules we consider the two point correlators of interpolating baryon currents. The quantum numbers of doubly heavy diquark in the ground states are given by its spin and parity, so that $j_d^P = 1^+$ or $j_d^P = 0^+$ (if the identical heavy quarks form the diquark then the scalar state $j_d^P = 0^+$ is forbidden). Adding the light quark to form the baryon, we obtain the pair of degenerate states $j^P = 1_2^+$ and $j^P = 3_2^+$ for the baryons $\Xi_{cc}^o$, $\Xi_{bc}^o$, $\Xi_{bb}^o$ and $\Xi_{cc}^*$, $\Xi_{bc}^*$, $\Xi_{bb}^*$ with the vector diquark, and $j^P = 1_2^+$ for the $\Xi_{bc}^o$ baryons with the scalar diquark. Unlike the case of baryons with a single heavy quark [9], there is the only independent current component for each ground state. We find

$$J_{\Xi_{QQ'}^o} = [Q^T C \gamma_5 Q^j] q^k \epsilon_{ijk},$$

$$J_{\Xi_{QQ'}^o} = [Q^T C \gamma^m Q^j] \cdot \gamma_m \gamma_5 q^k \epsilon_{ijk},$$

$$J_{\Xi_{QQ'}^m} = [Q^T C \gamma^m \gamma_5 Q^j] q^k \epsilon_{ijk} + \frac{1}{3} \gamma^m [Q^T C \gamma^m Q^j] \cdot \gamma_m q^k \epsilon_{ijk},$$

\(^1\)The superscript $\diamond$ denotes various electric charges depending on the flavor of the light quark.
where $J_{R_{QQ}}$ satisfies the spin-3/2 condition $\gamma_n J_{R_{QQ}} = 0$. The flavor matrix $\tau$ is antisymmetric for $\Xi_{bc}'$ and symmetric for $\Xi_{QQ}$ and $\Xi_{QQ}'$. Here $T$ means transposition, $C$ is the charge conjugation matrix.

The matrix structure of correlator for two baryonic currents with the spin of 1/2 has the form

$$\Pi(w) = i \int d^4x e^{ipx} \langle 0| T\{J(x), \bar{J}(0)\}|0\rangle = \gamma F_1(w) + F_2(w), \quad (2)$$

where $w$ is defined by $p^2 = (M + w)^2$, $M = m_Q + m_{Q'} + m_s$, $m_{Q,Q'}$ are the heavy quark masses and $m_s$ is the strange quark mass. The appropriate definitions of scalar formfactors for the 3/2-spin baryon are given by the following:

$$\Pi_{\mu\nu}(w) = i \int d^4x e^{ipx} \langle 0| T\{J_\mu(x), \bar{J}_\nu(0)\}|0\rangle = -g_{\mu\nu}\gamma F_1(w) + \tilde{F}_2(w) + \ldots, \quad (3)$$

where we do not concern for distinct Lorentz structures. The scalar correlators $F$ can be evaluated in a deep euclidean region by employing the Operator Product Expansion (OPE) in the framework of NRQCD,

$$F_{1,2}(w) = \sum_d C_d^{(1,2)}(w) O_d, \quad (4)$$

where $O_d$ denotes the local operator with a given dimension $d$: $O_0 = \hat{1}$, $O_3 = \langle \bar{q}q \rangle$, $O_4 = \langle \alpha_s G^2 \rangle$, ..., and the functions $C_d(w)$ are the corresponding Wilson coefficients of OPE. For the contribution of quark condensate operator we explore the following OPE up to the terms of the fourth order in $x$ (the derivation is presented in Appendix):

$$\langle 0| T\bar{s}_a i \gamma_i \bar{s}_b(0)|0\rangle = -\frac{1}{12}\delta^{ab}\delta_{ij}\langle \bar{s}s \rangle \cdot \left[1 + \frac{x^2(m_0^2 - 2m_s^2)}{16} + \frac{x^4(\frac{\alpha_s}{\pi} G^2) - \frac{3}{2} m_s^2(m_0^2 - m_s^2))}{288}\right] \cdot \left[1 + \frac{x^2}{24^2} \left(\frac{3m_0^2}{4} - m_s^2\right)\right]. \quad (5)$$

Note that at $m_s \neq 0$ the expansion of quark condensate gives contributions in both correlators in contrast with the sum rules for $\Xi_{QQ}'$. Putting $m_s = 0$ and neglecting the higher condensates, the authors found the factorization of diquark correlator in $F_2$ and full baryonic correlator in $F_1$. This fact was the physical reason for the divergency of SR method.

We write down the Wilson coefficient in front of unity and quark-gluon operators by making use of the dispersion relation over $w$,

$$C_d(w) = \frac{1}{\pi} \int_0^\infty \frac{\rho_d(\omega)d\omega}{\omega - w}, \quad (6)$$

where $\rho$ denotes the imaginary part in the physical region of NRQCD.

To relate the NRQCD correlators to the real hadrons, we use the dispersion representation for the two point function with the physical spectral density given by the appropriate
resonance and continuum part. The coupling constants of baryons are defined by the following expressions:

\[
\langle 0 | J(x) \Xi(\Omega)_{\bar{Q}Q}(p) \rangle = i Z_{\Xi(\Omega)_{\bar{Q}Q}} u(v, M_{\Xi(\Omega)}) e^{i p x},
\]

\[
\langle 0 | J^m(x) \Xi(\Omega)_{\bar{Q}Q}(p, \lambda) \rangle = i Z_{\Xi(\Omega)_{\bar{Q}Q}} u^m(v, M_{\Xi(\Omega)}) e^{i p x},
\]

where the spinor field with the four-velocity \(v\) and mass \(M\) (the mass of baryon) satisfies the equation \(\dot{u}(v, M) = u(v, M)\), and \(u^m(v, M)\) denotes the transversal spinor.

Then we use the nonrelativistic expressions for the physical spectral functions

\[
\rho_{\text{phys}}^{1, 2}(\omega) = \frac{M}{2M_{\text{phys}}} |Z|^2 \delta(\bar{\Lambda} - \omega),
\]

where we have performed the substitution \(\delta(p^2 - M^2) \rightarrow \frac{1}{2M^2} \delta(\bar{\Lambda} - \omega)\), here \(\bar{\Lambda}\) is the binding energy of baryon and \(M = M + \bar{\Lambda}\). The nonrelativistic dispersion relation for the hadronic part of sum rules has the form

\[
\int \frac{\rho_{\text{phys}}^{1, 2} d\omega}{\omega - w} = \frac{1}{2M_{\text{phys}}} \frac{|Z|^2}{\bar{\Lambda} - w}.
\]

We suppose that the continuum densities starting from the threshold \(\omega_{\text{cont}}\), is modelled by the NRQCD expressions. Then, in the sum rules equalizing the correlators calculated in NRQCD and given by the physical states, the integration above \(\omega_{\text{cont}}\) cancel each other in two sides of sum rules. Further, we write down the correlators in the deep underthreshold point of \(w = -M + t\) with \(t \rightarrow 0\).

Now the sum rules in the scheme of moments with respect to \(t\) can be written down as follows:

\[
\frac{1}{\pi} \int_{0}^{\omega_{\text{cont}}} \frac{\rho_{1, 2} d\omega}{(\omega + M)^n} = \frac{M}{2M} \frac{|Z|^2}{M^n},
\]

where \(\rho_j\) contains the contributions given by various operators in OPE for the corresponding scalar functions \(F_j\). Introducing the following notation for the \(n\)-th moment of two point correlation function:

\[
\mathcal{M}_n = \frac{1}{\pi} \int_{0}^{\omega_{\text{cont}}} \frac{\rho(\omega) d\omega}{(\omega + M)^{n+1}},
\]

we can obtain the estimates of baryon mass \(M\), for example, as the following:

\[
M[n] = \frac{\mathcal{M}_n}{\mathcal{M}_{n+1}},
\]

and the coupling is determined by the expression

\[
|Z[n]|^2 = \frac{2M}{M} \mathcal{M}_n M^{n+1},
\]

where we see the dependence of sum rule results on the scheme parameter.
B. Calculating the spectral densities

In this subsection we present analytical expression for the perturbative spectral functions in the NRQCD approximation. The evaluation of spectral densities involves the standard use of Cutkosky rules [10], with some modification motivated by NRQCD

heavy quark : \( \frac{1}{p_0 - (m + \frac{p^2}{2m})} \rightarrow 2\pi i \cdot \delta(p_0 - (m + \frac{p^2}{2m})) \),

light quark : \( \frac{1}{p^2 - m^2} \rightarrow 2\pi i \cdot \delta(p^2 - m^2) \).

We derive the spin symmetry relations for all the spectral densities due to the fact that in the leading order of the heavy quark effective theory the spins of heavy quarks are decoupled, so

\[ \rho_{1,0}^{\ast} = 3\rho_{1,0}^{\prime}, \quad \rho_{2,0}^{\ast} = 3\rho_{2,0}^{\prime}, \]

and we have the following relation for the baryon couplings in NRQCD:

\[ |Z_1|^2 = 3|Z_0|^2 = 3|Z_0|^2. \]

Using the smallness of the strange quark mass we use the following expansions in \( m_s \) for the perturbative spectral densities standing in front of unity operator (\( m_{QQ'} = m_Qm_{Q'}/(m_Q + m_{Q'}) \) is the reduced diquark mass, \( \mathcal{M}_{diq} = m_Q + m_{Q'} \)):

\[ \rho_{1,0}^{\ast} (\omega) = \frac{\sqrt{2}(m_{QQ'}\omega)^{3/2}}{15015\pi^2(\mathcal{M}_{diq} + \omega)^3} (\eta_{1,0}(\omega) + m_s\eta_{1,1}(\omega) + m_s^2\eta_{1,2}(\omega)), \]

where we have found

\[ \eta_{1,0}(\omega) = 16\omega^2(429\mathcal{M}_{diq}^2 + 715\mathcal{M}_{diq}\omega + 403\mathcal{M}_{diq}\omega^2 + 77\omega^3), \]
\[ \eta_{1,1}(\omega) = 104\omega(231\mathcal{M}_{diq}^3 + 297\mathcal{M}_{diq}\omega + 121\mathcal{M}_{diq}\omega^2 + 15\omega^3), \]
\[ \eta_{1,2}(\omega) = \frac{10}{(\mathcal{M}_{diq} + \omega)^2} (3003\mathcal{M}_{diq}^5 + 9009\mathcal{M}_{diq}^4\omega + 9438\mathcal{M}_{diq}^3\omega^2 + 4290\mathcal{M}_{diq}^2\omega^3 + 871\mathcal{M}_{diq}\omega^4 + 77\omega^5). \]

The first term of this expansion reproduces the result obtained in [6]. A new feature is the appearance of nonzero perturbative \( \rho_{1,0}^{\ast} \) density which is proportional to \( m_s \),

\[ \rho_{2,0}^{\ast} (\omega) = \frac{2\sqrt{2}\omega(m_{QQ'}\omega)^{3/2}m_s}{105\pi^2(\mathcal{M}_{diq} + \omega)^2} (\eta_{2,0} + m_s\eta_{2,1} + m_s^2\eta_{2,2}), \]

and
\[ \eta_{2,0} = 42\omega (M_{\text{diqu}}^2 + 48M_{\text{diqu}}\omega + 14\omega^2), \]
\[ \eta_{2,1} = 3(35M_{\text{diqu}}^2 + 28M_{\text{diqu}}\omega + 5\omega^2), \]
\[ \eta_{2,2} = \frac{1}{(M_{\text{diqu}} + \omega)^2}(105M_{\text{diqu}}^3 + 315M_{\text{diqu}}^2\omega + 279M_{\text{diqu}}\omega^2 + 77\omega^3). \]  

(19)

The account for the coulomb-like interaction leads to the finite renormalization of the diquark spectral densities before the integration over the diquark invariant mass by the introduction of Sommerfeld factor \( C \), so that

\[ \rho_{\text{diquark}}^C = \rho_{\text{diquark}}^{\text{bare}} \cdot C \]  

(20)

with

\[ C = \frac{2\pi\alpha_s}{3v_{QQ'}} \left[ 1 - \exp \left( -\frac{2\pi\alpha_s}{3v_{QQ'}} \right) \right]^{-1}, \]  

(21)

where \( v_{12} \) denotes the relative velocity of heavy quarks inside the diquark, and we have taken into account the color anti-triplet structure of diquark. The relative velocity is given by the following expression:

\[ v_{QQ'} = \sqrt{1 - \frac{4m_Qm_{Q'}}{Q^2 - (m_Q - m_{Q'})^2}}, \]  

(22)

where \( Q^2 \) is the square of heavy diquark four-momentum. In NRQCD we take the limit of low velocities, so that denoting the diquark invariant mass squared by \( Q^2 = (M_{\text{diqu}} + \epsilon)^2 \), we find

\[ C = \frac{2\pi\alpha_s}{3v_{QQ'}}, \quad v^2_{QQ'} = \frac{\epsilon}{2m_{QQ'}}, \]

at \( \epsilon \ll m_{QQ'} \).

The corrected spectral densities are equal to

\[ \rho_1^C(\omega) = \frac{m_{QQ'}^2\alpha_s\omega(2M_{\text{diqu}} + \omega)}{6\pi^2(M_{\text{diqu}} + \omega)^3}(\eta_{1,0}^C + m_s\eta_{1,1}^C + m_s^2\eta_{1,2}^C), \]  

(23)

where

\[ \eta_{1,0}^C = (2M_{\text{diqu}} + \omega)^2\omega^2, \]
\[ \eta_{1,1}^C = \frac{3(2M_{\text{diqu}} + \omega)\omega}{(M_{\text{diqu}} + \omega)}(4M_{\text{diqu}}^3 + 6M_{\text{diqu}}^2\omega + 4M_{\text{diqu}}\omega^2 + \omega^3), \]
\[ \eta_{1,2}^C = \frac{1}{(M_{\text{diqu}} + \omega)^2}(12M_{\text{diqu}}^4 + 24M_{\text{diqu}}^3\omega + 32M_{\text{diqu}}^2\omega^2 + 20M_{\text{diqu}}\omega^3 + 5\omega^4). \]  

(24)

We see that the first term again reproduces the result of \( \mathbb{4} \). For the \( \rho_2^{CQ_{QQ'}} \) we find

\[ \rho_2^C = \frac{m_s m_{QQ'}^2(2M_{\text{diqu}} + \omega)\omega\alpha_s}{2\pi(M_{\text{diqu}} + \omega)^2}(\eta_{2,0}^C + m_s\eta_{2,1}^C + m_s^2\eta_{2,2}^C), \]  

(25)
\[ \eta_{C,0} = (2M_{diq} + \omega)\omega, \]
\[ \eta_{C,1} = \frac{2}{M_{diq} + \omega}(2M_{diq}^2 + 2M_{diq}\omega + \omega^2), \]
\[ \eta_{C,2} = \frac{2}{(M_{diq} + \omega)^2}(2M_{diq}^2 + 2M_{diq}\omega + \omega^2). \]  

(26)

The use of these expansions numerically leads to very small deviations from the exact integral representations of spectral densities (about 0.5%), but they are more convenient in calculations.

The contribution to the moments of the spectral densities determined by the light quark condensate can be calculated by the exploration of (5)

\[ M^{(1)}_{qq}(n) = -\frac{(n+1)!}{n!}P_1M_{diq}^{(n+1)} + \frac{(n+3)!}{n!}P_3M_{diq}^{(n+3)} \]
\[ M^{(2)}_{qq}(n) = P_0M_{diq}^{(n)} - \frac{(n+2)!}{n!}P_2M_{diq}^{(n+2)} + \frac{(n+4)!}{n!}P_4M_{diq}^{(n+4)}, \]  

(27)

where we have introduced the coefficients of expansion in \( x \) by \( P_i \) (see (5) and Appendix). The n-th moment of two-point correlator function of diquark is denoted by \( M_{diq}^{(n)} \). Then the diquark spectral density takes the following form:

\[ \rho_{diq} = \frac{12\sqrt{2}m_{QQ'}^{3/2}\sqrt{\omega}}{\pi}, \]  

(28)

which has to be multiplied by the Sommerfeld factor \( C \), wherein the variable \( \epsilon \) is substituted by \( \omega \), since in this case there is no integration over the quark-diquark invariant mass. This corrected density is

\[ \rho_{diq}^C = \frac{48\pi\alpha_s m_{QQ'}^2}{3}, \]  

(29)

and it is independent of \( \omega \).

The corrections due to the gluon condensate are given by the density

\[ \rho_{C}^{G^2}(\omega) = \frac{(m_Q^2 + m_{Q'}^2 + 11m_Qm_{Q'})m_{QQ'}^{5/2}\sqrt{\omega}}{21 \cdot 2^{10}\sqrt{2\pi}m_{Q}^2m_{Q'}^2(M_{diq} + \omega)^2}(\eta_{G,0}^G + m_s\eta_{G,1}^G + m_s^2\eta_{G,2}^G), \]  

(30)

with

\[ \eta_{G,0}^G = 84M_{diq}^3 + 196M_{diq}^2\omega + 133M_{diq}\omega^2 + 11\omega^3, \]
\[ \eta_{G,1}^G = -\frac{2(210M_{diq}^3 + 70M_{diq}^2\omega + 21M_{diq}\omega^2 + 3\omega^3)}{M_{diq} + \omega}, \]
\[ \eta_{G,2}^G = \frac{2(210M_{diq}^3 + 70M_{diq}^2\omega + 21M_{diq}\omega^2 + 3\omega^3)}{(M_{diq} + \omega)^2}, \]  

(31)

where we again make the expansion in \( m_s \). In the case of nonzero quark mass we get the nonzero density proportional to \( m_s \).
\[ \rho_2^{G^2}(\omega) = \frac{m_s(m_Q^2 + m_{Q'}^2 + 11m_Qm_{Q'})\sqrt{\omega}m_{Q'}^{5/2}}{3 \cdot 2^9 \sqrt{2\pi} m_Q^2 m_{Q'}^2 (M_{\text{diquark}} + \omega)} (\eta_{2,0}^{G^2} + m_s \eta_{2,1}^{G^2}), \]

with

\[
\eta_{2,0}^{G^2} = -(9M_{\text{diquark}} + \omega), \\
\eta_{2,1}^{G^2} = \frac{9M_{\text{diquark}} + \omega}{M_{\text{diquark}} + \omega}. \tag{33}
\]

For the product of condensates \( \langle \bar{q}q \rangle \langle \alpha_s G^2 \rangle \), wherein the gluon fields are connected to the heavy quarks, it is convenient to compute the contribution to the two-point correlation function itself. We have found

\[ F_2^{\bar{q}qG^2}(\omega) = -\frac{m_{QQ'}^{5/2}(m_Q^2 + m_{Q'}^2 + 11m_Qm_{Q'})}{2^9 \sqrt{2\pi} m_Qm_{Q'}(-\omega)^{5/2}}, \tag{34} \]

and we put \( F_1^{\bar{q}qG^2}(\omega) = 0 \), since we have restricted the dimension of condensate operators, while the corresponding term in \( F_1 \) would appear in the fifth order in expansion \( \langle \alpha_s \rangle \). The result is given in the form, which allows the analytical continuation over \( \omega = -M + w \).

### C. Matching with full QCD

To connect the NRQCD sum rules to the baryonic couplings in full QCD we have to use the relation

\[ J^{QCD} = \mathcal{K}_J(\alpha_s, \mu_{\text{soft}}, \mu_{\text{hard}}) \cdot J^{NRQCD}, \]

where the coefficient \( \mathcal{K}_J(\alpha_s, \mu_{\text{soft}}, \mu_{\text{hard}}) \) depends on the soft normalization scale \( \mu_{\text{soft}} \). The \( \mathcal{K} \)-factor obeys the matching condition at the hard scale \( \mu_{\text{hard}} = M_{\text{diquark}} \) and is determined by the anomalous dimensions of effective baryonic currents which are independent of the diquark spin in the leading order. They are known up to the two loop accuracy \( \mathbb{F} \). In our consideration we use the one-loop accuracy, since the subleading corrections in the first \( \alpha_s \) order are not available yet. So,

\[
\gamma = \frac{d \ln \mathcal{K}_J(\alpha_s, \mu_{\text{soft}}, \mu_{\text{hard}})}{d \ln(\mu)} = \sum_{m=1}^{\infty} \left( \frac{\alpha_s}{4\pi} \right)^m \gamma^{(m)},
\]

\[
\gamma^{(1)} = \left( -2C_B(3a - 3) + 3C_F(a - 2) \right), \tag{35}
\]

where \( C_F = (N_c^2 - 1)/2N_c, \ C_B = (N_c + 1)/2N_c \) for \( N_c = 3 \) and \( a \) is the gauge parameter. In Feynman gauge \( a = 1 \), and we get \( \gamma^{(1)} = -4 \). So, in the leading logarithmic approximation and to the one-loop accuracy we find

\[
\mathcal{K}_J(\alpha_s, \mu_{\text{soft}}, \mu_{\text{hard}}) = \left( \frac{\alpha_s(\mu_{\text{hard}})}{\alpha_s(\mu_{\text{soft}})} \right)^{\gamma^{(1)}_{2\beta_0}}, \tag{36}
\]

where \( \beta_0 = 11 - 2/3N_F = 9 \). Further, we determine the soft normalization scale for the NRQCD estimates by the average momentum transfer inside the doubly heavy diquark, so
that $\mu_{\text{soft}}^2 = 2m_{QQ}T_{\text{diq}}$, where $T_{\text{diq}}$ is the kinetic energy in the system of two heavy quarks, which is phenomenologically independent of the heavy quark flavours and approximately equal to 0.2 GeV \([12]\). Then, the coefficients $K_J$ are equal to

$$K_{\Omega_{cc}} \approx 1.95, \quad K_{\Omega_{bc}} \approx 1.52, \quad K_{\Omega_{bb}} \approx 1.30,$$

with the characteristic uncertainty about 10% basically due to the variation of hard and soft scale points $\mu_{\text{hard}, \text{soft}}$. Note that the values of $K_J$ do not change the estimates of baryon masses, but they are essential in the evaluation of baryon couplings.

**III. NUMERICAL RESULTS**

Evaluating the two-point sum rules, we explore the scheme of moments. We point out the well-known fact that an essential part of uncertainties is caused by the variation of heavy quark masses. Indeed, the results of sum rules for the systems containing two heavy quarks strongly depend on the choice of masses, and this fact allows us to pin down the values of masses with a high precision up to 20 MeV \([13]\), so that $m_b = 4.60 \pm 0.02$ GeV, $m_c = 1.40 \pm 0.03$ GeV, which are extracted from the two-point sum rules for the families of $\Upsilon$ and $\psi$. To get these values we have use the correlators evaluated up to the same accuracy in $\alpha_s$, i.e. we have put the quark loop with the appropriate Sommerfeld factor. Then, the stability criterion for the leptonic constant of heavy quarkonium strictly fixes the heavy quark masses, which are close to the results of \([14]\). The same sum rules are also explored to estimate the couplings determining the coulomb-like interactions inside the heavy quarkonia

$$\alpha_s(bb) = 0.37, \quad \alpha_s(cc) = 0.60,$$

since they fix the absolute normalization of corresponding leptonic constants known experimentally.

Since the squared size of diquark is two times larger than that of the meson the effective coulomb constants have to be rescaled according to the equation of evolution in QCD. We use the one-loop evolution equation

$$\alpha_s(QQ') = \frac{\alpha_s(QQ')}{1 + \frac{9}{4\pi}\alpha_s(QQ') \ln 2}.$$

So,

$$\alpha_s(bb) = 0.45, \quad \alpha_s(bc) = 0.58, \quad \alpha_s(cc) = 0.85.$$

The values of condensates are taken in the ranges $\langle \bar{q}q \rangle = -(0.26 \div 0.27 \text{ GeV})^3$, $m_0^2 = 0.75 \div 0.85 \text{ GeV}^2$, $\langle \pi^2 \rangle = (1.7 \div 1.8) \cdot 10^{-2} \text{ GeV}^4$.

The main source of uncertainties in the ratios of the baryonic couplings is the ratio of the condensates of the strange quark and light quark. We use $\langle ss \rangle/\langle \bar{q}q \rangle = 0.8 \pm 0.2$ that corresponds to the variations of the sum $(m_u + m_d)[1 \text{ GeV}] = 12 \div 14 \text{ MeV} \([15]\)$.

So, we have described the set of parameters entering the scheme of calculations. In Figs. \([1-3]\) we present the results of the two-point sum rules for the masses of $\Xi_{bc}$ and $\Omega_{bc}$ (the figures for the other baryons are similar). For the $\Omega_{bc}$-baryons one can observe the stability of mass
with respect to the changing of the moment numbers in both correlators. We suppose it is connected with the destroying of diquark-Ω baryon factorization in the perturbative limit in contrast to the Ξ-baryons. The stability regions for $F_1$ and $F_2$ are not coincide because the contributions of higher dimension operators become valuable at the different numbers of moments. However, the quantity $1/2(M_1[n] + M_2[n])$ has the larger stability region, and we explore this fact to determine the Ω baryons masses as well as that of Ξ baryons. The theoretical uncertainties in the Ω baryons masses are mainly determined by the difference between the values of baryon masses at the regions of stability.

Then, we investigate the difference between the masses $1/2((M_{1,\Omega}+M_{2,\Omega})-(M_{1,\Xi}+M_{2,\Xi}))$ shown in Fig. 4. In our scheme of baryon masses determination this quantity has the meaning of the difference between the Ω and Ξ baryon masses. It has the large region of stability and is determined with a good precision. We obtain

$$\Delta M = M_{\Omega bc} - M_{\Xi bc} = M_{\Omega cc} - M_{\Xi cc} = M_{\Omega bc} - M_{\Xi bc} = 100 \pm 10 \text{ MeV}.$$  

The uncertainty in the Ξ-baryons masses are determined through the uncertainty in the Ω-baryons masses and that of in $\Delta M$. So, for the masses we find the following results:

$$
\begin{align*}
M_{\Omega bc} &= 6.89 \pm 0.05 \text{ GeV}, & M_{\Xi bc} &= 6.79 \pm 0.06 \text{ GeV}, \\
M_{\Omega tb} &= 10.09 \pm 0.05 \text{ GeV}, & M_{\Xi tb} &= 10.00 \pm 0.06 \text{ GeV}, \\
M_{\Omega cc} &= 3.65 \pm 0.05 \text{ GeV}, & M_{\Xi cc} &= 3.55 \pm 0.06 \text{ GeV}.
\end{align*}
$$  

The obtained values are in agreement with the calculations in the framework of nonrelativistic potential models $[10,17]$, though the models based on the calculation of three body systems with the pair interactions $[17]$ give slightly higher values of masses. In $[10]$ the other method of baryon mass determination was used, since the quantities $M_{1,\Xi}$ and $M_{2,\Xi}$ separately have no good stability in the sum rules. So, the difference of $M_1 - M_2$ close to zero was stable. The use of $\frac{1}{2}(M_1 + M_2)$ stability criterion results in the Ξ-QQ masses coinciding with those of $[10]$ up to 10 MeV. Figs. 4, 5 show the dependence of baryon couplings calculated in the moment scheme of NRQCD sum rules. Numerically, we find

$$
\begin{align*}
|Z_{\Omega cc}|^2 &= (10.0 \pm 1.2) \cdot 10^{-3} \text{ GeV}^6, & |Z_{\Xi cc}|^2 &= (7.2 \pm 0.8) \cdot 10^{-3} \text{ GeV}^6, \\
|Z_{\Omega bc}|^2 &= (15.6 \pm 1.6) \cdot 10^{-3} \text{ GeV}^6, & |Z_{\Xi bc}|^2 &= (11.6 \pm 1.0) \cdot 10^{-3} \text{ GeV}^6, \\
|Z_{\Omega tb}|^2 &= (6.0 \pm 0.8) \cdot 10^{-2} \text{ GeV}^6, & |Z_{\Xi tb}|^2 &= (4.2 \pm 0.6) \cdot 10^{-2} \text{ GeV}^6.
\end{align*}
$$  

In Fig. 6 we present the sum rules results for the ratio of baryonic constants $|Z_{\Omega bc}|^2/|Z_{\Xi bc}|^2$. We have also found

$$|Z_{\Omega bc}|^2/|Z_{\Xi bc}|^2 = |Z_{\Omega cc}|^2/|Z_{\Xi cc}|^2 = |Z_{\Omega tb}|^2/|Z_{\Xi tb}|^2 = 1.3 \pm 0.2.$$  

The uncertainty of this result as was mentioned above is mainly connected with the pourly known ratio of $\langle \bar{s}s \rangle / \langle \bar{q}q \rangle = 0.8 \pm 0.2$.

For the sake of comparison, we derive the relation between the baryon coupling and the wave function of doubly heavy baryon evaluated in the framework of potential model, where we have used the approximation of quark-diquark factorization. So, we find

$$|Z^{PM}| = 2\sqrt{3} |\Psi_{d}(0) \cdot \Psi_{t,s}(0)|,$$  

(42)
where $\Psi_d(0)$ and $\Psi_{l,s}(0)$ denote the wave functions at the origin for the doubly heavy diquark and light (strange) quark-diquark systems, respectively. In the approximation used, the values of $\Psi(0)$ were calculated in [18] in the potential by Buchmüller–Tye [18], so that

\[
\sqrt{4\pi} |\Psi_d(0)| = 0.53 \text{ GeV}^{3/2}, \\
\sqrt{4\pi} |\Psi_s(0)| = 0.64 \text{ GeV}^{3/2}, \\
\sqrt{4\pi} |\Psi_{cc}(0)| = 0.53 \text{ GeV}^{3/2}, \\
\sqrt{4\pi} |\Psi_{bc}(0)| = 0.73 \text{ GeV}^{3/2}, \\
\sqrt{4\pi} |\Psi_{bb}(0)| = 1.35 \text{ GeV}^{3/2}.
\]

In the static limit of potential models, these parameters result in the estimates

\[
|Z_{\Omega_{cc}}^{PM}|^2 = 8.8 \cdot 10^{-3} \text{ GeV}^6, \\
|Z_{\Xi_{cc}}^{PM}|^2 = 6.0 \cdot 10^{-3} \text{ GeV}^6, \\
|Z_{\Omega_{bc}}^{PM}|^2 = 1.6 \cdot 10^{-2} \text{ GeV}^6, \\
|Z_{\Xi_{bc}}^{PM}|^2 = 1.1 \cdot 10^{-2} \text{ GeV}^6, \\
|Z_{\Omega_{bb}}^{PM}|^2 = 5.6 \cdot 10^{-2} \text{ GeV}^6, \\
|Z_{\Xi_{bb}}^{PM}|^2 = 3.9 \cdot 10^{-2} \text{ GeV}^6.
\] (43)

The estimates in the potential model (43) are close to the values obtained in the sum rules of NRQCD [11]. We also see that the SU(3)-flavor splitting for the baryonic constants $|Z_\Omega|^2/|Z_\Xi|^2$ is determined by the ratio $|\Psi_s(0)|^2/|\Psi_l(0)|^2 = 1.45$ which is in agreement with the sum rules result. The values obtained in the NRQCD sum rules have to be multiplied by the Wilson coefficients coming from the expansion of full QCD operators in terms of NRQCD fields, as they have been estimated by use of corresponding anomalous dimensions. This procedure results in the final estimates

\[
|Z_{\Omega_{cc}}| = (38 \pm 5) \cdot 10^{-3} \text{ GeV}^6, \\
|Z_{\Xi_{cc}}| = (27 \pm 3) \cdot 10^{-3} \text{ GeV}^6, \\
|Z_{\Omega_{bc}}| = (36 \pm 4) \cdot 10^{-3} \text{ GeV}^6, \\
|Z_{\Xi_{bc}}| = (27 \pm 3) \cdot 10^{-3} \text{ GeV}^6, \\
|Z_{\Omega_{bb}}| = (10 \pm 1) \cdot 10^{-2} \text{ GeV}^6, \\
|Z_{\Xi_{bb}}| = (70 \pm 8) \cdot 10^{-3} \text{ GeV}^6.
\] (44)

IV. CONCLUSION

In this paper the NRQCD sum rules applied to the doubly heavy baryons have been considered. The nonrelativistic approximation for the heavy quark fields allows us to fix the structure of baryonic currents (the light quark-doubly heavy diquark) and to take into account the coulomb-like interactions inside the doubly heavy diquark. The presence of both the nonzero mass of light quark and the contribution of nonperturbative terms of the quark, gluon, mixed condensates and the product of condensates destroys the factorization of the correlators. This fact provides the convergency of sum rules for each correlator and allows us to obtain the reliable results for the masses and baryonic constants, which agree with the estimates in the framework of potential models. We also have calculated the mass splitting of $\Omega$ and $\Xi$ doubly heavy baryons and the ratio of baryonic constants $|Z_\Omega|^2/|Z_\Xi|^2$.

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V. APPENDIX

Here the derivation of expansion (5) is briefly presented. The calculations are done in the technique of fixed point gauge [19], so we write down the expansion of quark field:

\[ q(x) = q(0) + x^\alpha D_\alpha q(0) + \frac{1}{2} x^\alpha x^\beta D_\alpha D_\beta q(0) + ... , \]

and in the evaluation of \( \langle 0 | Tq^a_i(x)\bar{q}^b_j(0) | 0 \rangle \), where \( i \) and \( j \) are the spinor indices, \( a, b \) are the color indices, we have to know how to get the vacuum average of type \( \langle 0 | D_\alpha ... D_\omega q(0) \bar{q}(0) | 0 \rangle \).

The main formulae are the followings:

- the definitions of condensates
  \[ \langle q^a_i(0)\bar{q}^b_j(0) \rangle_0 = -\frac{1}{12} \delta^{ab} \delta_{ij} \langle \bar{q}q \rangle , \]
  \[ \langle G^{\alpha\beta}_{a\beta} G^{\alpha'\beta'}_{a'} \rangle = \frac{\delta^{aa'}}{96} (g_{\alpha\alpha'} g_{\beta\beta'} - g_{\alpha\beta} g_{\alpha'\beta'}) \langle G^2 \rangle , \]
  \[ \langle \bar{q} i g G^{\alpha\beta}_{a\beta} t^a \sigma_{\alpha\beta} q \rangle_0 = m_0^2 \langle \bar{q}q \rangle , \]

- the commutator of covariant derivatives
  \[ [D_\alpha, D_\beta] = -ig G^{a}_{\alpha\beta} t^a , \]

and the equation of motion for the spinor field

\[ \bar{D}q = -im_q q. \]

Form the last two equations we derive the so-called quadratic Dirac equation,

\[ D^2 q = -m_q^2 q + \frac{\sigma_{\alpha\beta}}{2} ig G^{a}_{\alpha\beta} t^a q. \]

Now it is an easy challenge to obtain the first term in expansion (5).

Since the tensor \( x_\alpha ... x_\omega \) is the symmetric one, we may perform the symmetrization

\[ D_\alpha ... D_\omega \rightarrow \{ D_\alpha, ..., D_\omega \}_+ , \]

to find the n-th term of expansion for \( \langle \bar{q}(x)q(0) \rangle \), which equals

\[ \frac{1}{n!} x_\alpha ... x_\omega \langle \bar{q}(0) \rangle D_\alpha ... D_\omega q(0) \rangle = \frac{1}{n!} x_\alpha ... x_\omega \langle \bar{q}(0) \{ D_\alpha, ..., D_\omega \}_+ q(0) \rangle . \]

Note, the tensor \( \langle \bar{q}(0) \{ D_\alpha, ..., D_\omega \}_+ q(0) \rangle \) is also symmetric one.

The second term of expansion is derived from

\[ \langle \{ D_\alpha, D_\beta \}_+ q^i_\rho(0) \bar{q}^j_\eta(0) \rangle = -2! \ P_2 \cdot g_{\alpha\beta} \delta^{ij} \delta_{\rho\eta} \langle \bar{q}q \rangle , \]
and the coefficient \( P_2 \) is determined by contracting the indices \( \alpha, \beta \) and using the quadratic Dirac equation,

\[ P_2 = (m_0^2 - 2m_q^2)/192. \]
The third term can be derived from the following structure:

\[ \langle \{ D_\alpha, D_\beta, D_\delta \} + q_i^i(0) \bar{q}_i(0) \rangle = -3! \, \mathcal{P}_3 \cdot \delta^{ij} ( (\gamma_\alpha)_{\rho\eta} g_{\beta\delta} + (\gamma_\beta)_{\rho\eta} g_{\alpha\delta} + (\gamma_\delta)_{\rho\eta} g_{\alpha\beta} ) (\bar{q} q) \].

Then, contracting \( \alpha \) and \( \beta \) and using of the equation of motion, the quadratic Dirac equation and the commutation relation, we obtain

\[ \mathcal{P}_3 = -i \, m_q (3m_0^2/4 - m_q^2)/576. \]

This includes the evaluation of vacuum averages

\[ \langle D^2 D_\alpha q(0) \bar{q}(0) \rangle, \langle D_\beta D_\alpha D_\beta q(0) \bar{q}(0) \rangle \text{ and } \langle D_\alpha D^2 q(0) \bar{q}(0) \rangle. \]

Considering the structure

\[ \langle \{ D_\alpha, D_\beta, D_\delta, D_\xi \} + q_i^i(0) \bar{q}_i(0) \rangle = -4! \, \mathcal{P}_4 \cdot \delta^{ij} \delta_{\rho\eta} ( g_{\alpha\beta} g_{\delta\xi} + g_{\alpha\delta} g_{\beta\xi} + g_{\alpha\xi} g_{\delta\beta} ) (\bar{q} q) \]

contracted over any pair of indices, we derive

\[ \mathcal{P}_4 = (\pi^2 \langle \alpha_s/\pi G^2 \rangle + 3/2 \, m_q^2 (m_0^2 - m_q^2))/3456. \]

Here we evaluated the following types of vacuum expectations:

\[ \langle D^2 D_\alpha q(0) \bar{q}(0) \rangle, \langle D_\alpha D_\beta D_\alpha D_\beta q(0) \bar{q}(0) \rangle, \langle D_\alpha D^2 D_\alpha q(0) \bar{q}(0) \rangle. \]

Then the OPE for the quark condensate can be expressed in terms of \( \mathcal{P}_i \) by

\[ \langle q_i^i(x) \bar{q}_i^j(0) \rangle = -\delta^{ij} (\bar{q} q) \cdot (\mathcal{P}_0 \delta_{\rho\eta} + \mathcal{P}_1 x_\alpha \gamma_{\rho\eta} + \mathcal{P}_2 \delta_{\rho\eta} x^2 + \mathcal{P}_3 x_\alpha \gamma_{\rho\eta} x^2 + \mathcal{P}_4 \delta_{\rho\eta} x^4), \]

with \( \mathcal{P}_0 = 1/12, \) and \( \mathcal{P}_1 = -i m_q/48. \)

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FIG. 1. The $\Xi_{bc}$ (lower curve) and $\Omega_{bc}$ (upper curve) masses obtained in the NRQCD sum rules from the first correlator $F_1$. 

FIG. 2. The $\Xi_{bc}$ (lower curve) and $\Omega_{bc}$ (upper curve) masses obtained in the NRQCD two point sum rules from the second correlator $F_2$. 

$M_{\Xi,\Omega_{bc}}$, GeV

$M_{\Xi,\Omega_{bc}}$, GeV
\[ M_{\Xi_{bc}}, \text{GeV} \]

\[ \Delta M, \text{GeV} \]

FIG. 3. The \( \Xi_{bc} \) (lower curve) and \( \Omega_{bc} \) (upper curve) masses obtained by averaging the results from both correlators.

FIG. 4. The mass difference \( \Delta M = M_{\Omega_{bc}} - M_{\Xi_{bc}} \) obtained from the results shown in Fig 3.
FIG. 5. The couplings $|Z_{\Omega_{bc}}^{(1,2)}|^2$ calculated in NRQCD sum rules for the formfactors $F_1$ and $F_2$ (solid and dashed lines, correspondingly) in the scheme of moments for the spectral densities.

FIG. 6. The couplings $|Z_{\Xi_{bc}}^{NR}|^2$ calculated in NRQCD sum rules for the formfactors $F_1$ and $F_2$ (solid and dashed lines, correspondingly) in the scheme of moments for the spectral densities.
FIG. 7. The ratio $\frac{|Z_{0\bar{b}}|^2}{|Z_{\Xi_{bb}}|^2}$ calculated in NRQCD sum rules in the scheme of moments for the spectral densities at $\langle \bar{s}s \rangle / \langle \bar{q}q \rangle = 0.8$. 