Characterization of a multimode coplanar waveguide parametric amplifier

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We demonstrate a Josephson parametric amplifier, based on a coplanar waveguide λ/4 resonator, connected to ground by means of a superconducting quantum interference device (SQUID). The fundamental resonant frequency is ∼1 GHz, and we use the higher modes for our measurements. We investigate three different pumping schemes, involving one mode or a pair of modes. The device can be current pumped by applying the pump tone to the input port of the resonator. An on-chip tuning line also allows for magnetic flux pumping. We show that we can approach quantum-limited noise performance while employing the single-mode flux-pumping scheme. We report a gain in excess of 20 dB and a saturation power of -133.5 dBm. The gain-bandwidth product depends on the employed pumping scheme and was shown to be as high as ~27.4 MHz.

I. INTRODUCTION

Due to the rapid advances in circuit quantum electrodynamics, there has been an increased interest in quantum-limited amplifiers. Amplifiers approaching this limit of minimally added noise have been developed in a number of different superconductor-based technologies, such as DC-SQUID amplifiers, traveling-wave parametric amplifiers, and resonator-based parametric amplifiers. In particular, systems based on Josephson junctions have been very successful and have found widespread use. For instance, they have been used for the generation and measurement of nonclassical states, quantum-limited measurement of nanomechanical oscillators, quantum feedback, and readout schemes for superconducting qubits.

Quantum-limited performance in Josephson parametric amplifiers has been reached in a number of configurations, all based on the modulation of the nonlinear inductance of a number of Josephson junctions. Often the Josephson junctions are configured in a SQUID geometry, or in an array of multiple SQUIDs. The junctions can be embedded in a resonant environment consisting of either a distributed circuit made up of one or multiple cavities, a lumped element circuit, or a combination of both. The Josephson inductance can be modulated in two different ways. The first option is by current pumping, where a strong tone at the signal input port modulates the phase over the junctions. The second way is to flux pump the Josephson inductance using an on-chip fast tuning line, to modulate the flux through the SQUID; this changes the critical current of the junctions, which, in turn, changes the Josephson inductance.

In this work, we present measurements of a Josephson parametric amplifier based on a superconducting coplanar waveguide (CPW) resonator. Usually the fundamental mode of the system is used for parametric amplification. Our device, however, is designed to have a relatively low fundamental frequency allowing us to measure parametric amplification using the higher modes, which we believe is novel. The motivation for this choice was to enable us to study the multimode pumping scheme described below. We operate the device both in the current-pumped and the flux-pumped scheme. In the latter case, we explore both the single-mode scheme, where the pump is resonant with twice the mode frequency, and the multimode scheme, where the pump is resonant with the sum of the resonant frequencies of two different modes. We make a comparison of these different modes of operation and study the gain, gain-bandwidth product, and saturation power. For the single-mode flux-pumping scheme and the single-mode current-pumping scheme, we also evaluate the noise performance of the device. We demonstrate that the amplifier reaches quantum-limited performance for the single-mode flux pumping case and that, within the parameter space investigated, flux-pumping performs better than current-pumping in our device.

The multimode flux-pumping scheme has similarities to previous work which uses a Josephson parametric converter. This device separates the signal and idler modes into two physically distinct cavities which are then connected by a more complicated network of Josephson junctions that requires multilayer fabrication with crossover wiring. We use a significantly less complex design, also allowing us to separate signal and idler, but rather than separating them over different resonators, we separate them over different modes of the same resonator. Compared to single-mode operation, this also gives us access to the full bandwidth of one of the modes, without having to worry about interference with the idler, as the idler will occur at a well-separated frequency.

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The parametric amplifier consists of a λ/4 coplanar waveguide resonator terminated to ground by means of a SQUID. We fabricate the circuit using electron-beam lithography and standard two-angle evaporation of aluminum. We installed the amplifier in a reflection measurement setup (see Fig. 1) and cooled it down in a dilution cryostat with a base temperature below 20 mK. We used the higher modes ($m = 2, 3, 4$) for our measurements, whose resonant frequencies are then found at $f_m(0) \approx (2m + 1) \times f_0(0)$ and can be tuned down to $f_m(\Phi_0/2) \approx (2m) \times f_0(0)$ (see Fig. 2). Simultaneous numerical fitting of the different modes to the characteristic equation, eq. (1), results in a SQUID inductance participation ratio $\gamma_1 = L_{sq}/L_r \approx 1.6\%$ for a SQUID inductance of $L_{sq} \approx 200\mu$H. The flux-dependent mode frequencies $f_m(\Phi_{DC})$ for this sample, as well as the fitting results, are shown in Fig. 2. We also measured a second sample, where the resonator design was left unchanged but the SQUID inductance was increased. This resulted in an increased SQUID inductance participation ratio $\gamma_2 = L_{sq}/L_r \approx 5.1\%$. The SQUID capacitance participation ratio $C_{sq}/C_r$ is much smaller than $\gamma_{1,2}$, and therefore the last term in eq. (1) can be neglected for both samples.

The resonator is coupled to the 50 Ω measurement line by means of an interdigitated coupling capacitor, $C_c$ (see Fig. 1). The device is strongly overcoupled, meaning that the internal losses are significantly smaller than the external ones. The resulting loaded quality factor ($Q$) depends on the mode number and scales approximately as $Q_m \approx Q_0/(2m + 1)$, where $Q_0$ is the loaded quality factor of the fundamental mode. In these samples, $Q_m$ is about 500 for mode 2, 380 for mode 3, and 225 for mode 4.
then phase sensitive and can in principle operate without adding noise. For the nondegenerate case, however, the idler and signal differ in frequency and the amplifier gain is phase insensitive. The minimal added noise in this case is the standard quantum limit of half a photon.

When the signal and idler fall within the linewidth of a single mode of the resonator \( f_s \approx f_i \approx f_m \) we can apply an AC flux-pumping tone resonant to twice the mode frequency \( f_{\text{p,flux}} \approx 2 \times f_m \). This will be referred to as the single-mode flux-pumping scheme (see Fig. 3a).

Another option is that the signal and idler fall within the linewidth of two different modes \( f_s \approx f_m \) and \( f_i \approx f_n \). We can then apply an AC flux-pumping tone which is resonant with the sum of two different modes \( f_{\text{p,flux}} = f_m + f_n \) and this is called the multimode flux-pumping scheme (see Fig. 3b).

In the current-pumping scheme, two pump photons are combined to provide energy transfer into the signal and idler modes (see Fig. 3c). The pump frequency is then close to the signal frequency \( f_p \approx f_s, f_i \). The amplifier is operated in this regime by applying a strong microwave tone, resonant with the mode of interest, directly to the input line of the parametric amplifier \( f_{\text{p,input}} = f_m \approx f_s \). However, this tone needs to be filtered out at the output of the measurement setup to avoid interference with the measurements. This case will hereafter be referred to as the single-mode current-pumping scheme.

**A. Single-mode flux pumping**

The first pumping scheme we analyze is the single-mode flux-pumping scheme. The frequency of the mode is determined by the applied DC flux, which in our measurements was \( \Phi_{\text{DC}} \sim 0.44 \Phi_0 \). The pump frequency was chosen to be twice the resonant frequency of mode 2: \( f_p \approx 2 \times f_2 (0.44 \Phi_0) \approx 2 \times 4.446 \text{ GHz} \). The exact resonant frequency of the mode involved will however also depend on the applied pump amplitude due to the pump-induced frequency shift\(^{21,22}\).

To map out the region in which we see gain, the pump was scanned both in power and in frequency. A small signal tone was applied with an offset of 100 kHz compared to the pump frequency, \( f_s = f_p/2 + 100 \text{ kHz} \), to operate the device in the nondegenerate mode. By measuring and comparing the reflected signal power both with the pump on and off, we were able to calculate the gain (see Fig. 3a).

We also measured the increase of the noise floor at the signal frequency with the pump tone on, as compared to when the pump was off. The maximum gain obtained at this DC flux bias point was 25 dB (for a pump power \( P_p = -48 \text{ dBm} \) at the parametric amplifier and a pump frequency \( f_p = 8.888 \text{ GHz} \)). At these pump conditions, the noise floor had also risen by 6.5 dB, limiting the improvement in signal-to-noise ratio (SNR) to 18.5 dB. By choosing slightly different pump parameters we reached a maximal SNR improvement of 20 dB.

Given the HEMT noise temperature, specified to be \( T_H = 5 \pm 1 \text{ K} \), and an estimate of the insertion loss of the components installed between the HEMT and the parametric amplifier of \( A = -3.5 \pm 1 \text{ dB} \), we can calculate the amount of noise added by the parametric amplifier.
(as referred to its input). We start by defining the SNR improvement ($\Delta SNR$) from the case were we have only the HEMT ($SNR_H$) to the case where we also turn the parametric amplifier on ($SNR_{H,J}$) as follows:

$$\Delta SNR = \frac{SNR_{H,J}}{SNR_H} = \frac{T_{N,\text{in}} + \frac{T_H}{A} - T_{J,\text{in}} + \frac{T_J}{A^2 G_J}}{T_{N,\text{in}} + T_J + \frac{1}{A^2 G_J}}.$$ (3)

Here $T_{N,\text{in}}$ is the noise temperature of the field in the input line, which is ideally close to vacuum noise $\hbar \omega/(2k_B)$. $T_J$ and $G_J$ are the noise temperature and the power gain of the parametric amplifier, respectively. We can now express $T_J$ as follows,

$$T_J = \frac{T_H}{A} \left( \frac{1}{\Delta SNR} - \frac{1}{G_J} \right) + T_{N,\text{in}} \left( \frac{1}{\Delta SNR} - 1 \right).$$ (4)

Converting the noise temperature of the parametric amplifier, $T_J$, to the number of noise photons added by the parametric amplifier is done by multiplying $T_J$ with $k_B/(\hbar \omega)$. We calculated the added number of noise photons with the aforementioned assumptions for all points where the gain was larger than 10 dB (see Fig. 3a-f). We can see that there is a region where the amplifier approaches the quantum limit, which for nondegenerate pumping is half a photon of added noise. The minimum value reached is $0.66^{+0.39}_{-0.28}$ added noise photons, which corresponds to a noise temperature of the parametric amplifier of $T_{J,\text{min}} = 141^{+83}_{-60}$ mK at a signal frequency of 4.4441 GHz.

The bandwidth of the amplifier was measured by fixing the pump at $f_p = 8.887$ GHz. We then swept the signal tone in a range of $f_p/2 \pm 16.5$ MHz, for different pump powers. The bandwidth is then extracted as the full width at half maximum of the gain peak (3 dB bandwidth, $BW$). The peak value of the power gain, $G_{\text{peak}}$, is also extracted. The gain-bandwidth product is then defined as $GBWP = \sqrt{G_{\text{peak}} \cdot BW}$. The gain, bandwidth and GBWP are shown, as a function of pump power, in Fig. 4b. The GBWP of the device was on average 8 MHz.

Lastly, we also measured the saturation power of the parametric amplifier, by fixing the pump frequency at $f_p = 8.887$ GHz and pump power $P_p = -49.5$ dBm, and the signal frequency at 4.444 GHz, resulting in a gain of 10.5 dB at small signal power. The gain was then measured as a function of the signal power and we found that the saturation power, defined as the signal power at which the gain is decreased by 1 dB, was -139.5 dBm.

B. Multimode flux pumping

We are not limited to flux pumping of a single mode, but we can also pump with a pump frequency $f_p = f_m + f_n$, with $m \neq n$. In this case, we expect that any signal tone with a frequency close to that of the modes which are being pumped will be amplified. In the available measurement bandwidth of our measurement setup,
and with the DC flux bias \( \Phi_{DC} = 0.45\Phi_0 \), we have access to three different modes, \( f_2 \), \( f_3 \), and \( f_4 \), with resonant frequencies 4.391 GHz, 6.186 GHz and 8.008 GHz, respectively (with the pump turned off).

The reflection coefficient of the input port of the parametric amplifier was measured while sweeping the signal tone in a frequency range around the pair of modes being pumped \( (f_m \) and \( f_n \)). The difference in the magnitude of the reflection coefficient for pump on and off was then recorded and the peak in the gain fitted with a Lorentzian to extract the peak value and the bandwidth. Maps of the gain peak and the bandwidth for all possible combinations of modes 2, 3, and 4 are shown in Fig. 5. We achieve positive gain for all possible combinations of modes. The maximum gain we can achieve depends on the chosen pair of modes, and can be as high as 20 dB, which is very similar to the gain observed in the single-mode flux-pumping scheme. There is clearly a symmetry in the structure of the gain regions between the modes involved. The frequency of the input tones at which the gain is maximal, for each pump frequency and pump power, adds up to the pump frequency as expected.

For gain values larger than 5 dB, we also calculated the GBWP. The average GBWP for the different pairs of modes is shown in Table I. The GBWP is higher when the pair of pumped modes is higher in frequency, because the linewidths grow with increasing mode number.

The saturation power was also measured for this pumping scheme. We biased the pump to provide maximum gain and then swept the signal power. The saturation powers can be found in Table II.

### Table I. Multimode flux pumping. Average gain-bandwidth product GBWP\(_{\text{avg},m,n}\) for the different pairs of modes \( f_m \) and \( f_n \), when flux pumping at \( f_p = f_m + f_n \). The average was calculated over all points in Fig. 6 where the gain was larger than 5 dB, marked by the contours in Fig. 6.

| \( f_p \) [MHz] | GBWP\(_{\text{avg},m}\) [MHz] | GBWP\(_{\text{avg},n}\) [MHz] |
|-----------------|------------------------|------------------------|
| \( f_{m=2} + f_{n=3} \) | 4.6 | 10.7 |
| \( f_{m=2} + f_{n=4} \) | 13.1 | 15.3 |
| \( f_{m=3} + f_{n=4} \) | 26.5 | 27.4 |

### Table II. Multimode flux pumping. Saturation power \( P_{\text{Sat},m,n} \) for the different pairs of modes \( f_m \) and \( f_n \), when flux pumping at \( f_p = f_m + f_n \). The pump power was chosen to provide maximum gain in the different modes.

| \( f_p \) [GHz] | \( P_p \) [dBm] | \( P_{\text{Sat},m} \) [dBm] | \( P_{\text{Sat},n} \) [dBm] |
|-----------------|-----------------|-----------------|-----------------|
| \( f_{m=2} + f_{n=3} = 10.446 \) | -31 | -141.5 | -141.5 |
| \( f_{m=2} + f_{n=4} = 12.372 \) | -26 | -141.5 | -137.5 |
| \( f_{m=3} + f_{n=4} = 14.140 \) | -29 | -135.5 | -133.5 |

### C. Single-mode current pumping

The last pumping scheme which is discussed here is the current-pumping scheme. These measurements were performed on a similar device differing only in SQUID inductance participation ratio (\( \gamma_2 = 5.1\% \)). In the current-pumping scheme, a strong pump tone is applied to the input port of the parametric amplifier at the resonant frequency \( f_p = f_n \), thus performing four-wave mixing. The mode frequency, at the applied DC flux bias \( (\Phi_{DC} = 0.4\Phi_0) \), was \( f_2(0.4\Phi_0) = 4.223 \) GHz.

As before, we map out the region of gain by scanning the pump both in power and frequency (see Fig. 7). The frequency range of interest is now around the mode frequency. A small signal tone, offset from the pump by 100 kHz to avoid a phase-dependent gain, is combined at room temperature with the strong pump tone. After reflecting off the parametric amplifier, the pump tone is filtered out digitally at room temperature. Both the gain, the increase in noise floor and the SNR improvement are then extracted in a similar fashion as in the single-mode flux-pumping case and are presented as a function of pump parameters in Fig. 7.

The maximum gain is found to be 15 dB for a pump frequency of \( f_p = 4.213 \) GHz and a pump power of \( P_p = -124 \) dBm. However, there is an increase in the noise floor at these settings, limiting the SNR improvement to 8.5 dB. At optimal settings for this flux bias point, the maximum SNR improvement was 10.5 dB. In analogy to the single-mode flux-pumping scheme, we can calculate the number of added noise photons using eq. 4. In this case, the HEMT noise temperature \( T_{Ht} \) was estimated to...
FIG. 6. (color online) Multimode flux pumping. Power gain observed at the different mode frequencies $f_m$ and $f_n$, as a function of pump power and pump frequency. The device shows gain when the pump frequency matches the sum of two mode frequencies, $f_p = f_m + f_n$. All possible combinations of the three available modes show a region of gain in the pump parameter space and a symmetry between the two modes involved. Panels (a), (b) and (c) show the combinations of modes $f_2 + f_3$, $f_2 + f_4$ and $f_3 + f_4$, respectively, measured at the indicated mode number. The white dashed contours border the region over which the GBWP was averaged (gain larger than 5 dB) (see Table I).

be $2 \pm 0.5$ K, and the insertion loss $A$ of the components installed between the HEMT and the parametric amplifier of $-2.35\pm 1$ dB. The number of added noise photons was calculated for all points where the gain was larger than 5 dB (see Fig. 7), and reaches a minimum of $2^{ \pm 0.8}$. The GBWP was also extracted by fixing the pump at $f_p = 4.214$ GHz and $P_p = -124.5$ dBm and was found to be limited to $\sim 1.5$ MHz. We also measured the saturation power. For this measurement the pump was set for maximum gain. The signal tone was then detuned from the pump with 100 kHz and swept in power. The power at which the gain decreased by 1 dB was -145.5 dBm.

IV. DISCUSSION AND SUMMARY

Having explored the different pumping schemes, we can now make a comparison. Within the investigated pump parameter space, we can see that single- and multimode flux pumping both perform better than single-mode current pumping. The maximally achieved power gain for both the single- and multimode flux-pumping cases reached up to 20 dB, while in the current-pumped scheme we could only reach about 15 dB.

When comparing the GBWP we see that also here flux pumping gives significantly better results, with an average of 8 MHz for single-mode flux pumping compared with only 1.5 MHz for current pumping. In the multimode flux-pumping scheme, we see that the GBWP increases with the mode number, with a maximum GBWP of 27.4 MHz. We also note that the GBWP is increasing faster than the linewidth of the modes involved.

For the single-mode flux pumping and the single-mode current pumping, we can also make a comparison of the noise performance of the parametric amplifier. We can see that the flux-pumping scheme provides better results, and that we approach the standard quantum limit of half a photon of added noise. In the current-pumped scheme, our parametric amplifier adds a minimum noise of 2 photons.

Finally we can also compare the saturation power, and again it seems that the flux-pumping scheme performs better than the current-pumping scheme. The best overall performance was reached for single-mode flux pumping, where the saturation power was -139.5 dBm. In the multimode flux pumping scheme we can, however, reach a saturation power of -133.5 dBm. Moreover, the saturation power is dependent on the mode number and is in general higher for modes with a higher frequency. In the current-pumping scheme, the saturation power was limited to -145.5 dBm.

In summary, we measured Josephson parametric amplifiers consisting of a CPW $\lambda/4$ resonator terminated to ground by means of a SQUID. The fundamental frequency of the resonator is close to 1 GHz, allowing us to easily access multiple modes of the resonator. We compared three different pumping schemes and determine that flux pumping the parametric amplifier performs better than current-pumping for the investigated parameter space. The amplifier approached quantum-limited noise performance for the single-mode flux pumping scheme. The multimode flux-pumping scheme, where we pump different pairs of modes of the same resonator, is a novel
The optimal point of operation for this flux value is at $f_p = 4.215$ GHz and $P_p = -124.5$ dBm and shows a SNR improvement of 10.5 dB compared to when the parametric amplifier is turned off. The region with maximal gain is bordered by a region with deamplification, which can be as large as -10 dB. For pump conditions where the gain was larger than 5 dB, we calculated an estimate of the number of added noise photons by the parametric amplifier. The parametric amplifier adds $2^{1+2.22} / 0.87$ photons of noise, in the region with gain larger than 10 dB.

FIG. 7. (color online) Single-mode current pumping. (a) Power gain and (b) increase in noise floor, (c) improvement in SNR, and (d) added noise observed at $f_p + 100$ kHz as a function of pump power and pump frequency. The maximum gain of 15 dB is found at $f_p = 4.213$ GHz and $P_p = -124$ dBm. At this point the noise floor has increased by 6.5 dB, giving a SNR improvement of 8.5 dB. The optimal point of operation for this flux value is at $f_p = 4.215$ GHz and $P_p = -124.5$ dBm and shows a SNR improvement of 10.5 dB compared to when the parametric amplifier is turned off. The region with maximal gain is bordered by a region with deamplification, which can be as large as -10 dB. For pump conditions where the gain was larger than 5 dB, we calculated an estimate of the number of added noise photons by the parametric amplifier. The parametric amplifier adds $2^{1+2.22} / 0.87$ photons of noise, in the region with gain larger than 10 dB.
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