A simple semi-Markov model of functioning of agricultural cleaning and transport system

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Abstract. The harvesting process in agriculture with transport provision is considered as the work of a two-element technical system consisting of a harvesting unit and a vehicle defined by the harvest-transport system (TCB). The functioning of such system is a causal change of its states. Transitions of the studied system from one state to another occur at random points in time due to the occurrence of failures of its elements for various reasons (technical, technological, operational, organizational, etc.). The distribution of the waiting time for the transition of a system from one state to another can be either exponential (that is, there is a pure Markov process with discrete states and continuous time), and any other (except exponential). The simplest semi-Markov model has been developed for the stationary mode of the system functioning, which leads to the final values of the probabilities of its states. A systematic, probabilistic-statistical approach and a graph-analytical apparatus of the theory of semi-Markov processes were used. The obtained semi-Markov model of TCB operation was tested on potato harvesting using the KPK-2-01 potato harvester. The values of the probability-time characteristics of the model confirmed the correctness of the proposed method.

1. Introduction
The harvesting process in agriculture (harvesting grain and vegetable crops, forage harvesting, etc.) in most cases is carried out by a harvest-transport system (TCB) consisting of a harvest unit and an accompanying vehicle [1]. The performance of such system is affected by random factors due to organizational, operational, technical, technological, climatic, human and other reasons. In this regard, the system may be in a healthy or inoperable state due to the "failure" of one of its components. Practice shows that the technical-technological group of events has the greatest weight in which the TCB cannot perform the functions assigned to it. The time spent by the system in any state is a continuous random value (NSV), obeying some distribution law with a known distribution function or probability density. Therefore, to analyze the functioning of such systems, it is advisable to apply the probabilistic-statistical approach [2], in particular the use of the theory of Markov and semi-Markov processes.

A characteristic feature of the Markov process with continuous time is that at any given moment in time the further course of such process is determined only by its state at that moment and does not depend on the nature of the process flow in the preceding periods (the “no after-effect” property). In this case, the distribution of the transition time from one state of the system to another is exponential with the parameter $\lambda$. In the case of Markov processes, the Kolmogorov differential equations and systems
of algebraic equations for the limit probabilities of the states of the system are valid [3]. Most of the real processes are characterized by the presence of aftereffect, which makes it impossible to use Markov models. For system analysis of the dynamics of a wide class of recoverable systems and queuing systems, semi-Markov models come to the aid of researchers, in which the distribution of the system's residence time in any state represents an arbitrary distribution function that differs from the distribution function of the exponential distribution law. In the educational and scientific literature [4, 5, 6], the term “nested Markov process” (VMP) is also used. The meaning of this term is that the Markov process of transition between system states occurs within another, non-Markov process, in which another process is embedded. Therefore, the development and substantiation of the mathematical apparatus of semi-Markov models for evaluating the functioning of complex systems, including agricultural harvest-transport systems, is highly relevant.

2. Material and methods
The considered harvest-transport system can simultaneously be in one and only in one state: \( S_i = S_1, S_2, \ldots, S_n \). Let us suppose that at the initial moment of time \( t_0 \) the harvest-transport system is in one of the possible states \( S_i \) and spends there a random time \( T_{ik} \), and then, with probability \( p_{ik} \), it passes into the \( S_2 \) state and so on (Fig. 1). If the length of each step representing the time the system is in a state of a random variable with an arbitrary distribution function, then we will get a semi-Markov process.

Thus, we can distinguish two main properties of the semi-Markov process [6]:

- transitions between states are Markov as for a Markov chain;
- the distribution of time spent in any of the states is arbitrary with time function (except exponential), including that it can be a constant.

Since the use of semi-Markov models is difficult in the case of a large number of system states, we first consider three states: \( S_0 \) - fault-free operation of the FB, \( S_1 \) - inoperable state of the system due to a failure of the cleaning unit due to technical and technological reason, \( S_2 \) - inoperable state of the system due to for vehicle failure or waiting. The state graph of the system under consideration is shown in Fig. 2. The probabilities of the transition from state \( i \) to state \( j \) at the time of the jump characterize the
“Markov” part of the semi-Markov process in question, the meaning of which is that after the nth step, the TCF will be in the \( S_j \) state if before it was in the \( S_i \) state.

Figure 2. State graph and transitions of agricultural harvest - transportation system

The matrix of transition probabilities is:

\[
P = \begin{pmatrix}
S_0 & S_1 \\
S_0 & 0 & p_{01} & p_{02} \\
S_1 & p_{10} & 0 & 0 \\
S_2 & p_{20} & 0 & 0 \\
\end{pmatrix}
\]

(1)

The sum of the probabilities of each row of the matrix (1) must be equal to one, i.e. the following condition is met:

\[
\sum_{i=0}^{2} p_{ij} = 1.
\]

(2)

In practice, the \( p_{ij} \) they are calculated as a fraction of the time when technical and technological failures occur in the total set of different types of failures based on the physical nature of the agricultural harvesting process using statistical data [7]. Each non-zero element \( p_{ij} \) of the matrix (1) of the transition probabilities is assigned a random variable \( T_{ij} \) - the time the system is in the state \( S_i \) (provided that the next state of the system is state \( S_j(t) \)). This value corresponds to the conditional distribution function \( Q_{ij}(t) \) (the probability that the time spent in state \( i \) does not exceed \( t \)) or the conditional density of distribution \( f_{ij}(t) = Q'_{ij}(t) \). Let us set the matrix (3) \( Q(t) = \{Q_{ij}(t)\} \) of the conditional distribution functions of the system's residence time in the \( i \)-th state before transition to the \( j \)-th state. The analytical form of the distribution functions \( Q_{ij}(t) \) can be determined statistically by advancing and testing statistical hypotheses about the types of distribution laws:

\[
Q(t) = \begin{pmatrix}
S_0 & S_1 \\
S_0 & 0 & Q_{01} & Q_{02} \\
S_1 & Q_{10} & 0 & 0 \\
S_2 & Q_{20} & 0 & 0 \\
\end{pmatrix}
\]

(3)

Theoretical studies and analysis of the functioning of some technical systems [6,7,8,9] show that when choosing independent distribution functions \( Q_{ij}(t) \), functions of the form are most often used:
\[ Q_{ij} = 1 - \exp(-\lambda_i t), \]
\[ Q_{ij} = 1 - (1 + q_i t) \cdot \exp(-q_i t), \quad (4) \]
\[ Q_j = \begin{cases} 0, t < 0 \\ \frac{t}{T_j}, t < T_i \\ 1, t \geq T_i, \end{cases} \]

where \( \lambda_i \) - an exponential distribution parameter; \( q_i \) - the parameter of the Erlang law; \( t, T_i \) - temporal characteristics of the system states.

According to the theorems of multiplication and addition of probabilities, the unconditional distribution function and the probability density of the total waiting time of a system in the \( S_i \) state will be:

\[ F_i(t) = P(t_i < t) = \sum_{j=1}^{n} p_{ij} \cdot Q_{ij}(t), \quad (5) \]
\[ f_i(t) = \sum_{j=1}^{n} p_{ij} \cdot Q'_{ij}(t). \quad (6) \]

The limiting values of the transition probabilities \( p_{ij}(t) \) of a system of an embedded Markov process for a time not exceeding \( t \) are determined by the formula [6]:

\[ P_{ij} = \lim_{t \to \infty} p_{ij} = \int_0^t \prod_{k=0}^{i-1} \left[ 1 - Q_{ik}(t) \right] dQ_{ij}(t) = \int_0^t \prod_{k=0}^{i-1} \left[ 1 - Q_{ik}(t) \right] Q'_{ij}(t) dt. \quad (7) \]

The mathematical expectation of the time of the TCF in the \( i \)-th state requires taking into account the unconditional distribution function or independent functions \( Q_{ij}(t) \) of the distribution of the time the system is in state \( i \) before going to the \( j \)-th state and is determined by [6]:

\[ \overline{t}_i = \int_0^t [1 - F_i(t)] dt = \int_0^t \prod_{j=1}^{n} [1 - Q_{ij}(t)] dt. \quad (8) \]

3. Research results and discussion

An important characteristic of the semi-Markov process is the limiting values of the probabilities \( \pi_i \) of the states of the system of the nested Markov process, which are determined from a system of algebraic equations, which is compiled from the matrix equation:

\[ \pi = \pi^T \cdot P, \quad (9) \]

where \( \pi \) - the matrix-column, whose elements are the probabilities of the stationary (established) mode of the system; \( \pi \) is the matrix-row transposed by it and \( P \) is the matrix of the limiting transitional probabilities of the TCB. According to the matrix multiplication rule, the matrix equation (9), supplemented by the normalization condition, is written as a system:

\[ \begin{cases} \pi_i = \sum_{j=1}^{n} \pi_j P_{ji}, \\ \sum_{i=1}^{n} \pi_i = 1. \end{cases} \quad (10) \]
This system of linear algebraic equations is linearly dependent. Excluding one of the equations (it follows as a consequence of the others) and, using the method of successive substitutions, we obtain the values of the final probabilities $\pi_i$ of the embedded $\pi_i$ Markov process.

For our case, the system (10) takes the form:

\[
\begin{align*}
\pi_0 &= P_{10}\pi_1 + P_{20}\pi_2, \\
\pi_1 &= P_{01}\pi_0, \\
\pi_2 &= P_{02}\pi_0, \\
\pi_0 + \pi_1 + \pi_2 &= 1.
\end{align*}
\]

\begin{align*}
\pi_0 &= \frac{P_{20}}{1 + P_{20} + P_{20}P_{01} - P_{10}P_{01}}, \\
\pi_1 &= \frac{P_{20}P_{01}}{1 + P_{20} + P_{20}P_{01} - P_{10}P_{01}}, \\
\pi_2 &= \frac{1 - P_{10}P_{01}}{1 + P_{20} + P_{20}P_{01} - P_{10}P_{01}}.
\end{align*}

(11)

For the semi-Markov process as a whole, the limiting (final) probabilities $P_i$ of the states of the harvest-transport system are determined from the expression:

\[P_i = \pi_i \cdot \tilde{t}_i \left( \sum_{j=1}^{n} \pi_j \cdot \tilde{t}_j \right)^{-1},\]

(12)

where $\tilde{t}_i$ - the average time the system is in the $i$-th state. Considering that, $P_{10} = P_{20} = 1$, formulas (11) will be simplified:

\[\begin{align*}
\pi_0 &= 0.5, \\
\pi_1 &= 0.5P_{01}, \\
\pi_2 &= 0.5(1 - P_{01}).
\end{align*}\]

(13)

To find numerical values of the quantities $\pi_i$, it is necessary to find $P_{01}$. Let the independent distribution function $Q_{01}$ of the TCF residence time in the operational state $S_0$ before the transition the inoperative state $S_1$ due to the failure of the cleaning unit be given by:

\[Q_{01} = 1 - (1 + q_1 t) \exp(-q_1 t),\]

where $q_1$ - the parameter of the Erlang law. Considering that:

\[Q_{01}' = \left[1 - (1 + q_1 t) \exp(-q_1 t)\right]' = q_1^2 t \cdot \exp(-q_1 t),\]

let us apply formula (7), integrate in parts twice and calculate a definite integral using the Newton-Leibniz formula. Omitting detailed calculations, the limiting transient probability $P_{01}$ will be:

\[P_{01} = \int_0^{t_a} \left[1 - (1 + (q_1 t) \exp(-q_1 t))\right] q_1^2 t \cdot \exp(-q_1 t) dt = q_1^2 \int_0^{t_a} (t + q_1 t^2) \cdot \exp(-q_1 t) dt = \frac{3}{4} \exp(-2q_1 t_a) \left( \frac{1}{2} q_1^2 t_a^2 + q_1 t_a + \frac{3}{4} \right),\]

where $t_a$ - the average recovery time of a combine harvester. To calculate the numerical value of $P_{01}$, we use the statistical data obtained on potato harvesting using the harvest-transport system in the form of a KPK-2-01 combine and the MTZ-80 tractor [10], i.e. $P_{01} = 0.75$. Then the values of the final probabilities of the embedded Markov process are determined by formulas (13):

\[
\begin{align*}
\pi_0 &= 0.5, \\
\pi_1 &= 0.375, \\
\pi_2 &= 0.125.
\end{align*}\]
Finally, the limiting (final) probabilities \( P_i \) of the states of the harvest-transport system in the stationary mode of operation on potato harvest will be equal to:

\[
P_0 = \frac{\pi_0 \cdot \tilde{t}_0}{\pi_0 \cdot \tilde{t}_0 + \pi_1 \cdot \tilde{t}_1 + \pi_2 \cdot \tilde{t}_2} = 0.614, \quad P_1 = \frac{\pi_1 \cdot \tilde{t}_1}{\pi_0 \cdot \tilde{t}_0 + \pi_1 \cdot \tilde{t}_1 + \pi_2 \cdot \tilde{t}_2} = 0.289, \quad P_2 = \frac{\pi_2 \cdot \tilde{t}_2}{\pi_0 \cdot \tilde{t}_0 + \pi_1 \cdot \tilde{t}_1 + \pi_2 \cdot \tilde{t}_2} = 0.095.
\]

The results indicate that the developed semi-Markov model (11), (12) can be used to find probability-time indicators of harvesting processes in agriculture. The values of the final probabilities, calculated from the statistical data of combine harvesting potatoes with KPK-2-01, correspond to reality and satisfy the normalization condition \( P_0 + P_1 + P_2 \approx 1 \).

4. Conclusion

It is advisable to describe any agricultural harvesting process with transport supply using the theory of semi-Markov processes due to the presence of aftereffect, when the state of the harvest-transport system depends on the nature of the process flow in previous periods. The approach used in constructing the model of functioning of the TCF in the stationary mode has wide universality. By changing the state graph of the system and using the known laws of the distribution of time intervals in the threads to restore its working capacity, it is possible to obtain more complex models. The values of the output probabilistic-time indicators of the models will allow a more accurate analysis of the established mode of agricultural harvesting, the development of appropriate measures to improve them, as well as to calculate the reliability parameters of harvest-transport systems.

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