Cumulative sums for edge determination of a single object in PET and SPECT images

Nicholas E Protonotarios¹², George M Spyrou³ and George A Kastis¹

¹ Research Center of Mathematics, Academy of Athens, Athens 11527, Greece
² Department of Mathematics, National Technical University of Athens, Athens 15780, Greece
³ Center of Systems Biology, Biomedical Research Foundation of the Academy of Athens, Athens 11527, Greece
E-mail: protost@hotmail.com

Abstract. The issue of edge determination of a single object in reconstructed nuclear medicine images has been examined thoroughly in the past, nevertheless most of the investigation has focused on the concepts of either numerical sinogram differentiation or segmentation. This work aims to develop an automated method for determining the contour of a single convex object in PET and SPECT reconstructed images, which can be used for computing body edges for attenuation correction, as well as for eliminating streak artifacts outside the specific object. This was accomplished by implementing a modified cumulative sums (CUSUM) scheme in the sinogram. Our method can automatically detect the object’s boundary in the reconstructed image. This approach has been tested in simulated as well as real phantoms and it performed efficiently for all convex objects. We were able to detect the contour of a single object in the image space, which in turn enabled us to eliminate streak artifacts outside and thus to obtain body edges necessary for attenuation correction.

1. Introduction

Emission computed tomography (ECT) and its clinical applications can be employed to various medical fields such as cardiology, oncology and neurology. The two predominant nuclear imaging ECT modalities are positron emission tomography (PET) and single photon emission computed tomography (SPECT); they may be applied in either clinical or preclinical cases. PET is based on the detection of annihilation photons produced by decaying radiopharmaceuticals (such as fludeoxyglucose ¹⁸F) injected into the patient and SPECT is based on the radiation of single photons detected by agent-labeled tracers (such as technetium ⁹⁹Tc).

Tomography is based on image reconstruction, specifically on the reconstruction from projection data such as sinograms. There exist two types of reconstruction algorithms, namely analytic and iterative. The main analytic reconstruction algorithm is filtered back-projection (FBP). FBP is fast and simple, however it usually generates “streak artifacts” in the reconstructions.

For the past three decades, edge determination through the sinogram of a single object for attenuation purposes in ECT has received considerable attention in the literature. Several methods have been suggested, [1–4]. These methods are applied to single objects being imaged and the first two of these methods emphasize on the skull.
In the current work, we developed an edge-determination method for nuclear medicine reconstructed images. This method can be used in order to eliminate FBP streak artifacts outside the single object under investigation, as well as to obtain object edges for attenuation correction. Our approach applies a simplified cumulative sums (CUSUM) scheme in the sinogram, as originally proposed by E. S. Page [5].

2. Materials and methods

2.1. The Radon transform and its attenuated generalization

Let \( g(x, y) \) denote the distribution of radioactivity. It is safe to assume that the function \( g \) is adequately smooth with compact support. If we introduce a set of so-called local coordinates, then every point \((x_1, x_2)\) on a line can be expressed in terms of the line’s (i) angle \( \theta \) with the \( x_1 \)-axis, (ii) distance from the origin \( \rho \), as well as (iii) local arc length \( \tau \):

\[ x_1 = \tau \cos \theta - \rho \sin \theta \quad \text{and} \quad x_2 = \tau \sin \theta + \rho \cos \theta. \quad (1) \]

We shall denote the Radon transform of \( g \) with \( \hat{g} \), defined as follows:

\[ \hat{g}(\rho, \theta) = \int_{-\infty}^{\infty} g(\tau \cos \theta - \rho \sin \theta, \tau \sin \theta + \rho \cos \theta) d\tau, \quad (2) \]

where

\[ \rho = x_2 \cos \theta - x_1 \sin \theta \quad \text{and} \quad \tau = x_1 \cos \theta + x_2 \sin \theta. \quad (3) \]

The sinogram of \( g(x_1, x_2) \) is none other than its Radon transform, \( \hat{g}(\rho, \theta) \), in matrix format and its elements \( \hat{g}_{ij} \) are given by

\[ \hat{g}_{ij} = \hat{g}(\rho_i, \theta_j), \quad i = 1, \ldots, P, \quad j = 1, \ldots, \Theta. \quad (4) \]

Similarly, the attenuated Radon transform of \( g \) with linear attenuation coefficient \( \mu(x_1, x_2) \), denoted by \( \hat{g}_\mu \), is defined as its weighted line integral, [6]:

\[ \hat{g}_\mu(\rho, \theta) = \int_{-\infty}^{\infty} e^{-\int_0^\infty \mu(x, \rho, \theta) ds} g(\tau \cos \theta - \rho \sin \theta, \tau \sin \theta + \rho \cos \theta) d\tau, \quad (5) \]

The attenuated sinogram of \( g(x_1, x_2) \) with attenuation \( \mu(x_1, x_2) \) is its attenuated Radon transform, \( \hat{g}_\mu(\rho, \theta) \), in matrix format and its elements \( \{\hat{g}_\mu\}_{ij} \) are given by

\[ \{\hat{g}_\mu\}_{ij} = \hat{g}_\mu(\rho_i, \theta_j), \quad i = 1, \ldots, P, \quad j = 1, \ldots, \Theta. \quad (6) \]

Henceforth, we drop the subscript and we will denote either the sinogram or the attenuated sinogram simply with \( \hat{g}_{ij} \).

2.2. The CUSUM algorithm for edge determination

In medical applications such as PET and SPECT, both the sinogram and the attenuated sinogram have finite supports and hence we may assume that for every row of the sinogram there are two (left and right) “essentially zero-count” intervals. We also assume that these intervals’ lengths are equal to \( L \) pixels and we treat \( L \) as a user-defined parameter. Due to the assumed left-right symmetry and without loss of generality, we will focus on finding the abrupt changes at the left side of the sinogram in a row-by-row basis. We introduce the CUSUM (cumulative sums) statistic \( C^{(j)}_n \):

\[ C^{(j)}_0 = 0 \quad (7a) \]

\[ C^{(j)}_n = \max\{0, C^{(j)}_{n-1} + \hat{g}_{n,j} - (\mu^{(j)}_C + \lambda \sigma^{(j)}_C)\}, \quad n = 1, \ldots, P \quad (7b) \]
where \( \mu_{C}^{(j)} \) and \( \sigma_{C}^{(j)} \) are the mean and standard deviation of the “essentially zero-count” interval, i.e.:

\[
\mu_{C}^{(j)} = \frac{1}{L} \sum_{i=1}^{L} \hat{g}_{ij}, \quad \sigma_{C}^{(j)} = \sqrt{\frac{1}{L} \sum_{i=1}^{L} \left( \hat{g}_{ij} - \mu_{C}^{(j)} \right)^2},
\]

and \( \lambda \) is (a user-defined) positive, usually fixed at \( \lambda = 3 \), the value of which measures the size of the shift we aim to detect. It is obvious that, until it detects the abrupt change in question, the CUSUM statistic is non-decreasing, i.e. \( C_{k}^{(j)} \geq C_{l}^{(j)} \) for all \( k > l \).

In order to determine the left edge, we must find the index \( i_{L}^{(j)} \) of \( \rho \) such that the CUSUM statistic \( C_{i}^{(j)} \) stops being zero for all \( i \geq i_{L}^{(j)} \), i.e.

\[
i_{L}^{(j)} = \max_{1 \leq n \leq P} \ker C_{n}^{(j)}.
\]

(9)

Similar considerations are valid for the right index \( i_{R}^{(j)} \), only this time we need to find the minimum of the kernel of the right CUSUM statistic \( D_{n}^{(j)} \)

\[
i_{R}^{(j)} = \min_{1 \leq n \leq P} \ker D_{n}^{(j)}.
\]

(10)

Given the above indices, we denote the non-zero interval of the variable \( \rho \) in row \( j \) with \( A^{(j)} \):

\[
A^{(j)} = \left[ \rho_{i_{L}^{(j)}}, \rho_{i_{R}^{(j)}} \right].
\]

(11)

Given the information of \( A^{(j)} \) for all rows, we are able to construct both the sinogram mask as well as the image mask.

### 2.3. Edge determination through sinogram and image masking

It is now evident that the sinogram mask \( S \) will be created as follows

\[
S(\rho_{i}, \theta_{j}) = \begin{cases} 
1, & \text{if } \rho_{i} \in A^{(j)} \\
0, & \text{otherwise}
\end{cases}.
\]

(12)

The next step is to calculate the image mask, \( M \). If we consider one of the results of Kastis et al. [7], namely that a pixel which is outside the boundary spanned by an object and hence has a value of 0, can be singled out from the sinogram by first identifying the detector locations \( \rho_{i} = \rho_{i}(\theta_{j}) \) for all angles \( \theta_{j} \) that receive contribution from this pixel. If this is the case, then for every \( (x_{1}, x_{2}) \) if there is even one \( \theta_{j} \) such that \( \hat{g}(\rho_{i}, \theta_{j}) = 0 \) it follows that \( g(x_{1}, x_{2}) \) must be zero. This condition emerges from the nature of the sinogram, since it is a sum of positive terms (counts) and thus equals zero only when all terms are zero. In this manner, we are able to “reconstruct” the sinogram mask and calculate the resulting image mask,

\[
M(x_{1}^{(m)}, x_{2}^{(n)}) = \begin{cases} 
0, & \text{if } \exists \theta^{*} \text{ such that } S(\rho(\theta^{*}), \theta^{*}) = 0 \\
1, & \text{otherwise}
\end{cases}.
\]

(13)

The resulting cleared image will then just be the element-by-element multiplication of the original image with the above image mask. This way the edges of the object are determined.

We implemented our algorithm in GNU Octave, which is a freely redistributable high-level interpreted language and software.
2.4. Simulation study
We performed a simulation study by employing the simulated sinogram of a simulated image-quality (IQ) phantom under Poisson noise of 10% of the total counts. Software for Tomographic Image Reconstruction (STIR) [8] was used to simulate the GE Discovery ST PET/computer tomography (CT) scanner. This specific scanner consists of 24 detector rings with diameter of 886.2 mm each. Every detector ring consists of 70 blocks, each of which consisting of an array of $6 \times 6$ crystals, i.e. a total of 420 crystals per ring. The sinograms’ dimensions are $P = 221$ detectors and $\Theta = 210$ angles, corresponding to a detector bin size of 3.195 mm, see [9]. We used an $128 \times 128$ square image grid. We also set the parameter values $\lambda = 3$ and $L = 60$ pixels. The reconstruction was performed in STIR by employing 2D FBP with a ramp filter and a cutoff at Nyquist frequency.

3. Results
The algorithm’s execution time was measured less than 5 seconds. The study was executed on a TOSHIBA laptop with Intel® Core™ i7-4710 HQ Processor, running on a 64-bit Windows® 10 environment.

The initial sinogram, the sinogram edge determination and the sinogram mask are shown in Fig. 1. The reconstructed image, image mask and masked image are shown in Fig. 2. This specific digital phantom simulates the human torso. It is clear from Fig. 2c that the object is preserved intact and that FBP artifacts outside the object have been removed. Our algorithm has significantly eliminated the streak artifacts outside the object. The cleared image consists of 5471 nonzero pixels instead of the uncleared image, which consists of $128^2 = 16,384$ pixels, i.e. a percentage of less than 12% of the original number of nonzero pixels of the uncleared image.

4. Discussion and conclusions
Our main goal is to automatically determine the edges of single-object nuclear medicine images. In order to accomplish this goal, we implemented a CUSUM-based algorithm and tested it on an image quality simulated phantom. We performed sinogram edge determination and hence created a sinogram mask. Then we mapped the masked sinogram onto the image space and therefore we performed image masking and finally we determined the edges of the object under investigation.

The determination of the boundaries of a single object from its sinogram projections is essential for the determination of the attenuation correction map, if unavailable. Our method
is accurate and efficient and can be proven to be extremely useful, since it can be employed for other applications as well, such as computing body contours for attenuation correction in PET/SPECT systems that do not have a CT. Furthermore, in reconstruction techniques other than FBP (where the reconstruction is performed on a pixel-by-pixel basis), our method can substantially reduce reconstruction time, since it essentially cancels out the masked, zero-count pixels.

A concise method for the determination of the edges of single-object ECT images has been developed and tested. With our CUSUM-based method, we are able to systematically detect the boundaries of an object via its sinogram. Edge determination is essential for eliminating the streak artifacts outside the object under investigation, as well as for performing attenuation correction.

Acknowledgments
This work was partially supported by the research programme “Inverse Problems and Medical Imaging” (200/842) of the Research Committee of the Academy of Athens. The authors would like to thank Dr. Tsiamyrtzis of Athens University of Economics and Business (AUEB) for his brilliant suggestions.

References
[1] M. Bergström, J. Litton, L. Eriksson, C. Bohm, and G. Blomqvist. Determination of object contour from projections for attenuation correction in cranial positron emission tomography. Journal of Computer Assisted Tomography, 6(2):365–372, 1982.
[2] M. Hosoba, H. Wani, H. Toyama, H. Murata, and E. Tanaka. Automated body contour detection in SPECT: effects on quantitative studies. Journal of Nuclear Medicine, 27(7):1184–1191, July 1986.
[3] J. Case, M. King, D.-S. Luo, E. J. Soares, M. S. Z. Rabin, et al. Determination of concave body outlines from spect projection data. In IEEE Nuclear Science Symposium and Medical Imaging Conference Record, volume 2, pages 944–948. 1, 1995.
[4] L. R. Barnden, J. Dickson, and B. F. Hutton. Detection and validation of the body edge in low count emission tomography images. Computer Methods and Programs in Biomedicine, 84(23):153 – 161, 2006. Medical Image Segmentation Special Issue.
[5] E. S. Page. Continuous inspection schemes. Biometrika, 41(1/2):100–115, 1954.
[6] A. S. Fokas, A. Iserles, and V. Marinakis. Reconstruction algorithm for single photon emission computed tomography and its numerical implementation. Journal of The Royal Society Interface, 3(6):45–54, 2006.
[7] G. A. Kastis, D. Kyriakopoulou, A. Gaitanis, Y. Fernández, B. F. Hutton, and A. S. Fokas. Evaluation of the spline reconstruction technique for PET. Medical physics, 41(4):042501, 2014.
[8] K. Thielemans, C. Tsoumpas, S. Mustafovic, T. Beisel, P. Aguiar, N. Dikaios, and M. W. Jacobson. Stir: software for tomographic image reconstruction release 2. *Physics in Medicine and Biology*, 57(4):867, 2012.

[9] V. Bettinardi, M. Danna, A. Savi, M. Lecchi, I. Castiglioni, M. Gilardi, H. Bammer, G. Lucignani, and F. Fazio. Performance evaluation of the new whole-body PET/CT scanner: Discovery st. *European Journal of Nuclear Medicine and Molecular Imaging*, 31(6):867–881, 2004.