Application of TOPSIS Method for the Optimization of Physical and Mechanical Characteristics of Graphite and Alumina Reinforced ZA - 27 Alloy Hybrid Composites

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Abstract: The Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) method utilizes to choose the best one from several attributes, combinations and objectives. In this paper calculation steps of TOPSIS method and selection of the best combination of weight % of hybrid reinforcement, the optimization of Gr and Al2O3 reinforced ZA – 27 alloy hybrid composite on the basis of its physical and mechanical characteristics has been presented.

Keywords: TOPSIS, Reinforcement, Hybrid composite, Mechanical property

I. INTRODUCTION

The physical properties of final composite can be increased by adding the reinforcement material into the matrix material. Mostly researcher’s use two type of reinforcement material, the first one is synthetic fiber and second is natural fiber. We can further increase the properties of composite by mixing of secondary reinforcement. The hybrid composite is prepared by mixing of at least two reinforcement material into the matrix material [1]. The classification of matrix material can be as: Metals, Polymers, Ceramics, Carbon and Graphite. Some examples of metal matrix material are Aluminium, Copper, Titanium and ZA-27 [2]. Void is a physical property which remain unfilled during the preparation of composite, it effect the mechanical property. Number of voids reduce the longitudinal compressive strength, interlaminar shear strength and transverse tensile strength [3]. Hardness, compressive strength and charpy impact strength are some mechanical properties of material, where hardness resist the plastic deformation, wear, penetration and scratching [4], while compressive strength resist the direct pressure of applied compression force [5] and charpy impact strength resist the impact from a swinging pendulum, this test is carried to evaluate the toughness of any material [6].

Alternatives are the options from which we select the best one after evaluating and the selection of these are impacted by the criteria or attributes. For selecting the best alternative from some available alternatives TOPSIS can be one of the excellent decision making method. The fundamental idea of the (TOPSIS) technique is that the best chosen alternative not just has the lowest distance from the optimum solution but also has the largest distance from the worst solution [7-8]. The TOPSIS technique was first presented by Hwang and Yoon in 1981, with the fundamental thought originating from the compromise idea of the alternative solution selected had the nearest distance to optimum solution and having the farthest distance from the worst solution [9]. J. Papathanasiou et. al. [8] summarized the particularized steps involved in the TOPSIS method as follows:

1) Step 1: Formulation of Decision Matrix

\[ X_{ij} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \] ………… (1)

Where m represents the alternative and n is attributes, \( X_{ij} \) is specified with the inserting of every alternative and attributes.

2) Step 2: Normalize the decision matrix

\[ r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n}x_{ij}^2}} \] …………… (2)

\( i = 1, 2, 3, \ldots m \) and \( j = 1, 2, 3, \ldots n \)
3) **Step 3:** Calculation of weighted normalized decision matrix

\[
y_{ij} = w_i y_{ij} \quad i = 1, 2, 3 \ldots m \text{ and } j = 1, 2, 3 \ldots n
\]

…………. (3)

4) **Step 4:** Calculation of positive and negative ideal solution

On the basis of normalized weighted rank \((y_{ij})\) the ideal \(A^+\) and ideal \(A^-\) solution can be find out as follows:

\[
A^+ = (y_{i1}^+, y_{i2}^+, y_{i3}^+, \ldots, y_{in}^+) \\
A^- = (y_{i1}^-, y_{i2}^-, y_{i3}^-, \ldots, y_{in}^-)
\]

…………. (4)

…………. (5)

\[
y_j^+ = \begin{cases} 
\max_{i} y_{ij} & \text{if } j, \text{ benefit attribute} \\
\min_{i} y_{ij} & \text{if } j, \text{ cost attribute}
\end{cases}
\]

\[
y_j^- = \begin{cases} 
\max_{i} y_{ij} & \text{if } j, \text{ benefit attribute} \\
\min_{i} y_{ij} & \text{if } j, \text{ cost attribute}
\end{cases}
\]

5) **Step 5:** Calculation of distance with ideal solution

With a positive ideal solution distance is an alternative \(A_i\) is supposed as follows:

\[
D_i^+ = \sqrt{\sum_{j=1}^{n} (y_{ij}^+ - y_j^+)} \quad i = 1, 2, 3 \ldots m
\]

…………. (6)

With a negative ideal solution distance is an alternative \(A_i\) is supposed as follows:

\[
D_i^- = \sqrt{\sum_{j=1}^{n} (y_{ij}^- - y_j^-)} \quad i = 1, 2, 3 \ldots m
\]

…………. (7)

6) **Step 6:** Calculation of the preference value

For every alternative \((V_i)\) the preference value has been given as:

\[
V_i = \frac{D_i^-}{D_i^- + D_i^+} \quad i = 1, 2, 3 \ldots m
\]

…………. (8)

The greater value of \(V_i\) indicates that alternative \(A_i\) is preferred, at the end of calculation.

II. **OPTIMIZATION OF PHYSICAL AND MECHANICAL CHARACTERISTICS OF GRAPHITE AND ALUMINA REINFORCED ZA – 27 ALLOY HYBRID COMPOSITES BY TOPSIS METHOD**

On the basis of beneficiary and non – beneficiary attribute for the formulation of decision matrix, the selection criterion of the attributes has been provided in table I.

| S. No. | Attributes               | Selection criterion of attributes                  |
|-------|-------------------------|----------------------------------------------------|
| 1     | Void contents (VC)      | Non – beneficiary attribute (Lower the better)      |
| 2     | Hardness (H)            | Beneficiary attribute (Higher the better)           |
| 3     | Compressive strength (CS)| Beneficiary attribute (Higher the better)           |
| 4     | Charpy Impact strength (CIS)| Beneficiary attribute (Higher the better)       |
1) Step 1: Formulation of Decision Matrix According to the equation number 1 the decision matrix is formed, each row presents alternatives and each column denotes attributes. Four hybrid reinforcement wt.% alternatives and four attributes are consisting by the decision matrix, are describe in table II. For the formulation of decision matrix, experimental results of attributes are used and shown in table 2. Here all are beneficiary attributes except than void contents (VC).

TABLE II

| Composites       | VC   | H    | CS    | CIS   |
|------------------|------|------|-------|-------|
| ZA–27 + 0% GAl   | 1.63 | 61.3 | 282   | 36    |
| ZA–27 + 3.5% GAl | 1.46 | 67.28| 297   | 39    |
| ZA–27 + 7% GAl   | 0.64 | 72.1 | 314   | 43    |
| ZA–27 + 10.5% GAl| 1.95 | 59.33| 293   | 33    |

2) Step 2: The first element ($P_{11}$) of the normalized decision matrix according to equation 2, is calculated as 0.54331 ($P_{11} = 1.63 / \sqrt{(1.63^2 + 1.46^2 + 0.64^2 + 1.95^2)}$). Similarly, the other elements of the matrix are calculated, after that the normalized decision matrix R is set up like:

$$R = \begin{bmatrix}
0.54331 & 0.47011 & 0.47519 & 0.47454 \\
0.48665 & 0.51597 & 0.50046 & 0.51409 \\
0.21332 & 0.55293 & 0.52911 & 0.56682 \\
0.64997 & 0.45500 & 0.49372 & 0.43500
\end{bmatrix}$$

3) Step 3: Entropy method has been applied to calculate the weight of the attributes.

$$P_{ij} = \frac{x_{ij}}{\sum_{j=1}^{4} x_{ij}} \quad \text{........... (9)}$$

To calculate weight the first element ($P_{11}$) of the normalized matrix according to equation 9, is calculated as 0.28697 ($P_{11} = 1.63 / (1.63 + 1.46 + 0.64 + 1.95)$). Similarly, the other elements of the matrix are calculated, after that matrix P is set up like:

$$P = \begin{bmatrix}
0.028697 & 0.23576 & 0.23777 & 0.23840 \\
0.25704 & 0.25875 & 0.25042 & 0.25827 \\
0.11267 & 0.27729 & 0.26475 & 0.28476 \\
0.34330 & 0.22818 & 0.24704 & 0.21854
\end{bmatrix}$$

Entropy for $E_{1}$ attribute is calculated by;

$$E_j = -K \sum_{j=1}^{4} P_{ij} \ln P_{ij} \quad \text{and} \quad K = \frac{1}{\ln m} \quad \text{where m = number of alternative} \quad \text{--------- (10)}$$

The entropy and weight for each attributes are shown in table III;

TABLE III

| Attributes | $E_j$ | $DP_j = 1 - E_j$ | $W_j = \frac{DP_j}{\sum_{i=1}^{4} DP_i}$ |
|------------|-------|-----------------|----------------------------------|
| VC         | 0.95245 | 0.04755 | 0.88169 |
| H          | 0.99777 | 0.00223 | 0.04134 |
| CS         | 0.99938 | 0.00062 | 0.01149 |
| CIS        | 0.99647 | 0.00353 | 0.06545 |
The first element \((y_{11})\) of the weighted normalized matrix is found by multiplying the assigned weight to the first attribute and the first element of normalized matrix \((y_{11} = w_1 \times n_{11} = 0.88169 \times 0.54331 = 0.47903)\). Similarly, the other elements of the matrix are calculated and then weighted normalized matrix \(y\) is set up like:

\[
\begin{bmatrix}
0.47903 & 0.01943 & 0.00545 & 0.03105 \\
0.42907 & 0.02133 & 0.00575 & 0.03364 \\
0.18808 & 0.02285 & 0.00607 & 0.03709 \\
0.57307 & 0.01880 & 0.00567 & 0.02847
\end{bmatrix}
\]

4) **Step 4:** According to equation 4 and 5, the optimal solution \(A^+\) is determined by taking the maximum values because these are elements of optimal solution and the non-optimal solution \(A^-\) is determined by taking the minimum values.

\[
A^+ = [0.18808, 0.02285, 0.00607, 0.03709]
\]

\[
A^- = [0.57307, 0.01880, 0.00545, 0.02847]
\]

5) **Step 5:** According to equation 6, the value of first element \((0.29103)\) of the distance from optimal solution matrix \((D^+)\) is calculated by \(((0.47903-0.18808)^2 + (0.01943-0.02285)^2 + (0.00545-0.00607)^2 + (0.03105-0.03709)^2)^{1/2}\). Similar calculation procedure has been followed for calculating the other values as;

\[
D^+ = [0.29103, 0.24101, 0, 0.38510]
\]

Similarly, by using Equation 7, the value of first element \((0.09407)\) of the distance from non-optimal solution matrix \((D^-)\) is calculated by \(((0.47903-0.57307)^2 + (0.01943-0.01880)^2 + (0.00545-0.00545)^2 + (0.03105-0.02847)^2)^{1/2}\). Same calculation procedure has been followed for calculating the other values as;

\[
D^- = [0.09407, 0.14411, 0.38510, 0.00022]
\]

6) **Step 6:** With the use of equation 8, the value of first element \((0.244)\) nearest to the optimal solution matrix \((V_i)\) is calculated by \((0.09407/(0.29103 + 0.09407))\). Similar calculation procedure has been followed for calculating the other values as;

\[
V_i = [0.244, 0.374, 1, 0.005]
\]

7) **Step 7:** In this step the computation of preference order for graphite and alumina reinforced ZA – 27 alloy hybrid composites has been done by taking the greater value as a priority, shown in table IV.

| Hybrid reinforcement wt.% | Preference values \((V_i)\) | Rank |
|---------------------------|---------------------------|------|
| 0% GAl                    | 0.244                     | 3    |
| 3.5% GAl                  | 0.374                     | 2    |
| 7% GAl                    | 1                         | 1    |
| 10.5% GAl                 | 0.005                      | 4    |

8) **Step 8:** This step discusses the arrangement of graphite and alumina reinforced ZA – 27 alloy hybrid composite alternatives. The preference values of these composite have been arranged by giving the priority to greater values. The sequence of ranks is given below, show that 7wt.% of hybrid reinforcement have optimum result for physical and mechanical properties of graphite and alumina reinforced ZA – 27 alloy hybrid composites i.e. 7wt.% GAl > 3.5wt.% GAl > 0wt.% Gal > 10.5wt.% GAl.

### III. CONCLUSION

For multiple objectives TOPSIS is a good decision making method, with simple principle it can evaluate different targets at same time. This paper present the optimum result for the physical and mechanical characteristics of the different wt.% of Gr and Al2O3 reinforced ZA – 27 alloy hybrid composite. It is found that 7 wt.% of graphite and alumina reinforced hybrid composite have best result of physical and mechanical characteristics and the order of optimizations is; 7wt.% GAl > 3.5wt.% GAl > 0wt.% Gal > 10.5wt.% GAl.
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