Primary Signal Phase Suppression Technique Based on Mutual Information Improved Variational Mode Decomposition

Jiaying Yue *, Jianpeng Lu, Sheng Wang, Nae Zheng, Long Zhang and Yi Han
People’s Liberation Army Strategic Support Force Information Engineering University, Zhengzhou 450001, China
*Correspondence: 18539931910@163.com

ABSTRACT

Specific emitter identification (SEI) identifies targets mainly by unintentional modulation of the signal. However, due to the high energy of the primary signal, once the primary signal changes, the recognition becomes less effective or even impossible using a feature database that is not updated. In this paper, we propose to use a mutual information improved variable mode decomposition (VMD) algorithm to suppress the primary signal phase of the transmitter. Furthermore, we simulate the feature extraction of the unintentional phase modulation of the transmitter signal and use support vector machine (SVM) for individual identification. The simulation results show that the algorithm improves the recognition rate by about 6% (0 dB) compared to the retained primary signal. The results demonstrate that our proposed phase suppression technique improves the adaptability and accuracy of individual identification of transmitters.

Keywords. specific emitter identification, variational mode decomposition, mutual information, primary signal suppression, feature extraction.

1. INTRODUCTION

Specific Emitter Identification (SEI) refers to the acquisition of fingerprint signals that can reflect the target’s identity through unintentional signals caused by the nonideality of emitter components [1-2]. However, unintentional modulation can be easily absorbed by intentional modulation, which makes feature extraction difficult. Traditional SEI technology typically extracts features from transient parts of a whole signal, such as the front and back edges [3-4] or steady-state parts of a whole signal [5-7]. It seems that the influence of primary signal does not attack much attention by re-searchers. As a result, there are relatively less studies on primary signal suppression. However, due to the high energy of the primary signal and its contribution to the individual characteristics, the change of the primary signal will lead to the change of the individual characteristics. At this point, if we continue to use the feature database before the change of the primary signal for recognition, the recognition effect will dramatically reduce or even failure in recognition. Therefore, if we can suppress the primary signal of the emitter, we can better extract the individual characteristics and thus improve the adaptability and accuracy of the SEI algorithm. In terms of the mechanism of unintentional modulation and individual feature extraction, suppressing the primary signal is feasible and has significant values in both theoretical research and future applications.

Many primary signal suppression techniques have been proposed. Wu et al. [8] used synchronous compression wavelet transform to suppress the primary signal and highlight the characteristics of unintentional modulation. Compared with the primary signal of the source, the recognition rate of the fractal box dimension feature was improved by approximately 10%. Lin et al. [9] performed Variational Mode Decomposition (VMD) on the radar pulse data and identifies the characteristics of unintentional modulation through the multi-domain features of each Intrinsic Mode Function (IMF). The VMD technique was initially proposed by Dragomiretskiy et al.[10]. It is similar to the Empirical Mode Decomposition (EMD) algorithm, but overcomes the disadvantages of the EMD, including 1) adding support for mathematical theory, 2) allowing error feedback in recursive filtering, and 3) dropping the predefined restrictions that define the IMF. As we known, the EMD algorithm has been widely used in the field of SEI. While VMD has more applications in fault diagnosis and other fields[11-13], the application of VMD to the problem of SEI has not been intensively investigated, with a few works have explored such application. Aghnaiya et al. [14-15] used VMD to decompose Bluetooth transient signals to obtain high-order moments as fingerprint feature for identifying devices, and proved that the performance of fingerprint features based on VMD algorithm is better than that of time-frequency distribution features based on HHT. Satija et al. [16] proposed VMD-SF and VMD-EM2 algorithms and evaluated the performance of SEI in single-hop and relaying scenarios. The VMD-SF outperforms the VMD-EM2 and traditional EMD-EM2 algorithm in terms of recognition rate and time complexity. However, at low signal-to-noise ratios (SNR) (e.g., 0 dB), the recognition rate is below 50%. Baldini et al.
applied VMD to wireless device recognition and evaluated the performance in presence of Additive White Gaussian Noise (AWGN) and fading effects channel (both Rician and Rayleigh). The performance of VMD still outperforms the other representations (i.e., time domain, frequency domain and EMD) at medium/high values of SNR, but at low values of SNR it is worse. He et al. [18] extracted the skewness and kurtosis characteristics of the signal through VMD technology without compensating for the distortion of the receiver, and diversity benefits can be obtained by using multiple distorted receivers. In recent years, new extensions of the VMD algorithm have also been proposed. For example, in 2021, Rehman et al. [19] proposed a Multi-variate Variational Mode Decomposition, but its application in the SEI field has not yet been investigated. Throughout the literature in recent years [14-17], the VMD algorithm has outperformed the traditional HHT and EMD algorithms in terms of recognition rate. However, due to the definition of the decomposition algorithm, it performs generally at low signal-to-noise ratios.

In the decomposition algorithm, such as the EMD algorithms and Intrinsic Time-scale Decomposition (ITD) algorithm, VMD requires the selection of components. In the work [20], the authors decomposed the original signal by the ITD algorithm. The appropriate IMF is selected based on the correlation coefficient, and then the permutation entropy, approximate entropy and sample entropy of each IMF are extracted to form the feature vector. Liu et al. [21] proposed the VMD based on mutual information optimization algorithm, and they applied it to noise reduction of pipeline leakage vibration signal. By selecting appropriate modes to reconstruct the signal, the localization accuracy can be effectively improved. In addition to the signal characteristics, the noise and false interference components in the signal also affect the accuracy of the time-domain correlation coefficient between the IMF components and the original signals. Therefore, mutual information entropy is mainly used to measure the correlation between two events, which is less affected by external interference factors [22].

Inspired by the application of VMD in fault diagnosis, which is similar to the SEI technology, whose features are also derived from unintentional modulation, we adopt the VMD based on mutual information optimization algorithm to suppress the Primary Signal Phase (PSP) of the emitter. This helps us to solve the problem that the high energy of the primary signal affects the individual characteristics and thus reduces the individual recognition effect. We simulated the feature extraction of Unintentional Phase Modulation (UPM) and evaluated the individual recognition rate using linear support vector machine. Simulation results show that the recognition rate of UPM is improved by about 6% (0dB) during the primary signal suppression process compared with retaining the primary signal. This result verifies the positive effect of the primary signal suppression process on the individual recognition technology of emitters.

2. METHODS

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2.1. Signal Fingerprint Model

For emitters, the power amplifier is the main source of fingerprint generation [23]. Moreover, the Taylor series model has been widely used due to its simplicity. For example, in the work [24], the simulation signal was generated by the Taylor series model. In this section, the nonlinearity of power amplifier is considered as the mechanism for simulation signal fingerprint. Since the Taylor series can better describe the input/output of memoryless nonlinear function, the emitted distortion signal \( z_m(t) \) can be modelled as

\[
  z_m(t) = \sum_{l=1}^{L} b_{m,l} \left( z_m(t)^l \right)
\]

where \( L \) is the order and \( b_{m,l} \) is the coefficient of Taylor polynomial.

2.2. Variational Mode Decomposition

The goal of VMD is to decompose the real-valued input signal into a discrete number of sub-signals, \( u_k \ (k = 1, 2, 3, \ldots, K) \), which has specific sparsity while reproducing the input. The VMD assumes that the signal consists of a finite number of IMFs with limited bandwidth in the spectral domain and each mode is most compact around a centre pulsation \( \omega_k \ (k = 1, 2, 3, \ldots, K) \). Under the constraint that the sum of the IMF components is equal to the input signal, the minimum
sum of the estimated band-widths of each eigenmode function component is sought. At the core of it are multiple adaptive Wiener filter banks, and the overall framework can be considered as a variational problem.

The decomposition process of the signal \( s(t) \) is described below, more details can be found in [10].

**a) Hilbert transform.** In order to obtain a unilateral frequency spectrum, the analytic signal of each mode is calculated by using the Hilbert transform.

\[
(\delta(t) + \frac{j}{\pi t}) \otimes u_k(t)
\]

where \( u_k \) \((k = 1, 2, 3,..., K)\) and \( \omega_k \) \((k = 1, 2, 3,..., K)\) denote the shorthand notations for the set of all modes and their centre frequencies. \( \otimes \) represent the convolution operation.

**b) Heterodyne.** The spectrum of each mode is transferred to the baseband and tuned to its respective estimated centre frequency by multiplying it with an exponential function.

\[
[(\delta(t) + \frac{j}{\pi t}) \otimes u_k(t)] e^{-j\omega_k t}
\]

**c) Wiener filtering.** The bandwidth is estimated by the H^{1} Gaussian smoothness of the demodulated signal, i.e., the square of the L2-norm of the gradient. Therefore, for a given signal \( s(t) \), the minimum of the sum of the estimated bandwidth of each mode component under the constraint that the sum of the order mode components is equal to the input signal is solved as a constrained variational problem.

c-1) Variational problem is denoted as:

\[
\min_{\{u_k\},\{\omega_k\}} \left\{ \sum_k \left\| \frac{\partial}{\partial \omega} \left[ (\delta(t) + \frac{j}{\pi t}) \otimes u_k(t) \right] e^{-j\omega t} \right\|_2^2 \right\}
\]

\[
s.t. \sum_k u_k = s(t)
\]

where \( \partial \) is the gradient of \( t \), \( \delta(t) \) is impulse function, and \( \sum_k = \sum_{k=1}^{K} \) can be understood as the sum of all modes.

c-2) To solve the optimal variational model, a quadratic penalty term \( \alpha \) and a Lagrange multiplier \( \lambda \) are introduced to remove the constraint of the problem. The augmented Lagrangian expression \( \mathcal{L} \) is as follows:

\[
\mathcal{L}(\{u_k\},\{\omega_k\},\lambda) = \alpha \sum_k \left\| \frac{\partial}{\partial \omega} \left[ (\delta(t) + \frac{j}{\pi t}) \otimes u_k(t) \right] e^{-j\omega t} \right\|_2^2 + \left\| s(t) - \sum_k u_k(t) \right\|_2^2 + \lambda \left\langle s(t) - \sum_k u_k(t) \right\rangle
\]

\[
(5)
\]

c-3) The saddle point of the Lagrange function is obtained by using Alternating Direction Method of Multiplication (ADMM) operator to update alternately and iteratively, which is the optimal solution of the variational model (see work [10] for details).

The solution \( u_k \) can be written as:

\[
\dot{u}_{k}^{n+1}(\omega) = \frac{\dot{s}(\omega) - \sum_{i \neq k} \dot{u}_i(\omega) + \dot{\lambda}(\omega)}{2 + 4\alpha(\omega - \omega_k)^2}
\]

\[
(6)
\]

The solution \( \omega_k^{n+1} \) is as follows:
The pseudocode for VMD is summarized in Table 1.

### Table 1. Pseudo code for VMD

| Algorithm 1 Pseudo code for VMD |
|----------------------------------|
| 1: Initialize $\hat{s}_1^n, \alpha^n, \lambda^n$, $n$ set to 0 |
| 2: Update mode $\hat{u}_k^n(\omega) = \frac{\hat{s}(\omega) - \sum_{i \neq k} \hat{u}_i^n(\omega) + \lambda^n(\omega)}{1 + 2\alpha(\omega - \omega_k)^2}$ |
| 3: Update Centre frequency $\omega_k^n : \omega_k^n = \frac{\int_0^\infty \omega |\hat{u}_k^n(\omega)|^2 d\omega}{\int_0^\infty |\hat{u}_k^n(\omega)|^2 d\omega}$ |
| 4: Update Lagrange multiplier $\lambda^{n+1}(\omega)$ until convergence $\sum_i \|\hat{u}_k^n - \hat{u}_i^n\|_2 < \epsilon$ |

2.3. Mutual Information

Shannon Entropy represents a mathematical measure of uncertainty. Mutual information [21] measures the information of one random variable that contains another random variable, i.e., it indicates the correlation between the two variables. If the correlation is higher, their mutual information will be larger. Consider two random signals $X$ and $Y$ with a joint distribution of $p(x,y)$ and a marginal probability density function of $p(x), p(y)$, the mutual information is defined as:

$$I(X,Y) = \sum_{x \in X} \sum_{y \in Y} p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$$

Because each IMF contains both noise signal and useful signal, and mutual information can measure the correlation between these two signals, mutual information is introduced here as a standard to measure the proportion of useful signals in IMFs.

2.4. Primary Signal Phase Suppression based on Improved VMD

SEI mainly focuses on the recognition of the unintentional modulation features of signals. Studies [8-9] have concluded that the primary signal has high energy and great impact on individual recognition. If the primary signal can be suppressed and the energy entropy of unintentional modulation features can be enhanced, the recognition will be more accurate.

In general, non-ideal emitter hardware will inevitably affect the ideal emitter signal. For example, subtle variations in the amplitude, frequency, and phase of the signal, which are considered as unintentional modulation of the signal by the emitter hardware. Unlike the intentional modulation of the carrier in a certain way during the design of the emitter signal waveform, the unintentional modulation carries the individual information. Tang [25] analysed the effect of hardware on the signal in terms of oscillator phase noise and power amplifier nonlinear distortion. It was demonstrated that the phase noise caused the sideband component of frequency spectrum. The unintentional modulation of both the oscillator and the power amplifier can be regarded as the influence of the additive phase noise. The emitter signal containing the phase noise and the channel white Gaussian noise can be expressed as...
\[ s(t) = x(t) + p(t) + n(t) \]  
(9)

where \( x(t) \) is the ideal emitter signal, \( p(t) \) is the phase noise, which carries the individual information, \( n(t) \) is the channel noise.

Theoretically, once the primary signal is suppressed, what remains is the sum of unintentional modulation and channel noise. To improve the robustness of the algorithm, the wavelet compression is introduced to filter the Gaussian noise.

The VMD algorithm can decompose the amplitude waves with different frequency domains into different mode functions. The simple synthesis of the mode functions decomposed by VMD cannot filter out the PSP, so it is necessary to filter the mode functions after the VMD decomposition of the emitter signals.

However, there is no observation method in the original algorithm for the mode function with strong energy phase. Thus, mutual information optimization was introduced. The phase components of the primary signal are filtered by mutual information, and then the suppressed signal phase is obtained by subtracting the reconstructed primary signal from the original signal. The optimization steps are as follows:

1. The phase of the received signal is extracted and the noise is removed using wavelet compression after unwrapping;
2. Different mode functions are obtained by VMD decomposition;
3. The mutual information between each mode function and the original phase is calculated;
4. The mode function with large mutual information is selected, and the PSP is re-constructed;
5. The PSP is subtracted from the original phase to obtain the noisy UPM;
6. Wavelet compression is used to obtain a more accurate UPM estimate.

3. EXPERIMENTS

The general process of SEI consists of five parts: signal front-end reception, signal preprocessing, feature extraction, classification, and database update. Here, we focus on the feature extraction and the classification part.

3.1. Feature Extraction

Our extracted UPM is based on the suppression of the PSP. The process of PSP suppression is completed by processing mutual information following the VMD de-composition. In the experimental simulation, we first perform the phase reconstruction of the primary signal, and then the original signal phase is subtracted from the reconstructed primary signal to obtain the UPM.

The chirp signal under the additive white Gaussian noise channel was selected in the experiment. The amplitude of the chirp signal was 1V, the starting frequency was 0Hz, the termination frequency was 200Hz, the signal time width was 1s, and the sampling frequency was 4410Hz. Then, the unintentional modulation characteristic of the signal is simulated by polynomial amplifier distortion according to the work of Liu et al. [23]. The number of radiation source categories \( K \) is set to 5. The power amplifier coefficients of five emitters are set as \( b_1 = [1, 0.5, 0.3], b_2 = [1, 0.08, 0.6], b_3 = [1, 0.01, 0.01], b_4 = [1, 0.01, 0.8], b_5 = [1, 0.5, 0.3] \).

At the same time, wavelet compression is used to denoise the original phase and UPM to obtain a more accurate phase estimate. In this section of the experiment, the Root Mean Square Error (RMSE) was used to measure the suppression effect on PSP. The experimental results include all the results of 200 times Monte Carlo.

\[
RMSE = \sqrt{\frac{1}{N} \sum_{n=1}^{N} (x_{\text{rec}}(n) - x_{\text{ideal}}(n))^2}
\]

3.1.1 Parameter selection of VMD algorithm If the value \( K \) is too small, aliasing may occur. Since the phase in the frequency domain is narrow and our goal is only to find the phase of the primary signal, it is not appropriate to have too large a \( K \) value; over-decomposition will occur and increase the decomposition time. Therefore, it is necessary to determine the appropriate number of decomposition layers. The decomposition performance at decomposition order of 2-6 was tested at 15dB, respectively. The decomposition performance of each order is measured under 200 Monte Carlo simulations.
Figure 1 shows the RMSE of estimation in three phases: the PSP, the original phase and the UPM. The RMSE decreases when the number of decomposition layers increases and stabilizes when the order is 3. Considering the decomposition performance and complexity, the third-order VMD de-composition is used in the subsequent experiments.

![Figure 1. VMD decomposition performance.](image)

**3.1.2 Parameter selection of VMD algorithm**

In order to select the components to be reconstructed properly, the mutual information entropy of each mode and the obtained signal is first calculated and compared. The higher the entropy, the higher the correlation and the more the primary signal contained. By default, the energy of the primary signal in the acquired signal is higher than that of the noise signal. The energy of the intentional modulation is higher than that of the unintentional modulation of the emitter signal.

From Figure 2, we observe that the mutual information value of IMF1 and IMF2 is significantly larger than that of IMF3. In this paper, the RMSE of IMF1, which is the primary signal, and IMF1 and IMF2, which form a new signal after reconstruction, were tested respectively. The results are shown in Table 2. We can see that taking two components is better than taking one component as the primary signal. Therefore, the IMF1 and IMF2 are retained and reconstructed to form a new phase as the phase of the primary signal, i.e., intentional phase modulation. The RMSE of the phase between the primary signal and the ideal Linear Frequency Modulation (LFM) signal is 0.0813.

![Figure 2. Information entropy of each IMF component.](image)

### Table 2. RMSE of the primary signal for different IMF components.

|       | IMF 1+IMF 2   | IMF 1   |
|-------|---------------|---------|
| PSP   | 0.0813        | 3.3837  |
| UPM   | 0.0813        | 3.3837  |
| original phase | 3.3804   | 0.0813  |

**3.1.3 Analysis of the results after primary signal suppression**

As shown in Equation (9), the ideal primary signal suppression is the synthesis of phase noise and white noise. Noise reduction is performed using wavelet compression technique before VMD decomposition and after primary signal suppression.
The experimental results of primary signal suppression are shown in Figure 3. Figure 3(a) shows the phase comparison between primary signal suppression based VMD and ideal suppression. From the overall signal perspective, the RMSE of PSP suppression based VMD and the ideal UPM is 0.0828. Compared with the ideal suppression signal, the edge effect on both sides leads to poor results, which is shown in Figure 3(b). The RMSE of this segment is 0.0385, which shows that the primary signal is well suppressed in the middle segment of the visible signal.

![Figure 3. Comparison diagram between the phase obtained after suppression and the ideal UPM. (a) The whole comparison diagram; (b) Enlarged comparison diagram.](image)

The received signal is the original signal. To improve the signal-to-noise ratio after decomposition, the wavelet compression transform is used for noise reduction. After VMD suppression, the reconstructed phase may still be a noisy phase. To obtain a more accurate phase estimate, the wavelet compression transform is used again to reduce the noise. First, we need to determine the appropriate wavelet order. When the SNR is 15dB, the denoising performance of the wavelet of orders 1-6 is tested separately. Figure 4 shows the denoising performance of the original signal. When the order is 2, the RMSE is the smallest, so order 2 is adopted in the subsequent experiments. Figure 4(b) shows the noise reduction effect of the UPM. The RMSE decreases with the increase of order, and stabilizes when the order is 4.

![Figure 4. Noise reduction performance under different wavelet order.](image)

After the wavelet parameters and other experimental parameters are selected, the SNR was set to 0~35dB, and 200 Monte Carlo simulations were carried out at 5dB intervals to calculate the RMSE at different SNRs. The statistical results are shown in Figure 5. We observe that the RMSE of PSP is larger than the UPM when the SNR is less than 5dB. Therefore, 3dB is subdivided between 0dB and 5dB. The experiment again confirmed that the original signal and UPM still maintain good estimation performance when the SNR is less than 5dB, despite the poor PSP. It also demonstrate that our proposed algorithm is robust.

![Figure 5. Statistical results of RMSE at different SNRs.](image)
3.2. Feature Extraction

After extracting fingerprint features, classifiers are used to recognize the emitter signals. Support vector machine (SVM) is a common classifier that is based on statistical learning theory. It has shown excellent performance in solving the recognition problems with small samples and high dimensionality. Therefore, we choose SVM as a classifier to perform recognition by 5-fold cross-validation.

Figure 6 shows the experimental results. We find that when the SNR is equal to 5dB, the recognition rate reaches 80%, and the recognition rate becomes 99.8% when the SNR is equal to 15dB. We also performed two comparative experiments, one with the primary signal suppression phase without wavelet compression to denoise (curve: ‘Original phase’), while the other is the original phase, as well as Wavelet transform, without primary signal suppression (curve: ‘UPM without noise reduction’). By comparing the recognition results of different SNR for multiple times, the effectiveness of the proposed algorithm has been demonstrated. In particular, the recognition rate is 6% higher than that of retaining the PSP and 10% higher than that of without wavelet compression when the SNR is equal to 0 dB~ 5dB.

This experiment targets the fingerprint features of phase extraction, so it is easier to distinguish the difference in emitters caused by phase noise. However, due to the decomposition algorithm, the signal changes greatly in the case of low SNR and the primary signal extraction effect will be affected. We use wavelet transform for noise reduction, a certain effect has been achieved from the experimental results, but the parameter selection of wavelet transform requires manual experience, so the next step should focus on the improvement of recognition methods under low signal-to-noise ratio.

4. CONCLUSIONS

In this paper, our goal is to extract the UPM feature and suppress the effect of the PSP on recognition task. We proposed to use VMD to decompose the signal phase. The PSP is reconstructed based on the mutual information by measuring the correlation between each IMF and the original phase. The estimation of the noisy UPM is obtained by phase subtraction. Wavelet compression is introduced and then a more accurate UPM is obtained by suppressing the PSP using an improved variable mode decomposition algorithm based on mutual information. The process of feature extraction and classification
of the LFM signal is simulated, the SVM classifier was used to identify different SNR ranges (0dB ~ 35dB). The results show that a higher classification performance (0dB ~ 5dB) can be obtained at a relatively low SNR compared to the preserved PSP. It can be concluded that the proposed method achieves a satisfactory classification performance at low SNR conditions.

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