Non-adiabatic quench dynamics near anisotropic quantum critical point

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Abstract. We study non-equilibrium dynamics induced by a quench of a parameter near a quantum phase transition in the Kitaev model. On the basis of an exact solution for the time-dependent Schrödinger equation, we show that the density of excitations and residual energy after a quench scale with a quench rate \( \tau^{-1} \) as \( \tau^{-3/4} \) and \( \tau^{-5/4} \) respectively, when a parameter is quenched from a gapped phase toward the quantum critical point. These unusual scaling laws originate from an anisotropic nature of the quantum critical point in the Kitaev model. Generalizing these results, we provide novel scaling laws for a generic anisotropic quantum critical point in \( d \)-dimensional systems.

1. Introduction
Non-equilibrium dynamics of quantum systems near quantum critical points has been a subject of intense study in recent years [1, 2]. During such dynamics, a quantum system passes from one gapped phase to another via time evolution of a Hamiltonian parameter \( \lambda \) with a rate \( \tau^{-1} \) and an exponent \( \alpha = \lambda(t) = \lambda_0 |t/\tau|^\alpha \text{Sgn}(t) \), where \( \text{Sgn}(x) = 1(-1) \) for \( x > ( < ) 0 \) through an intermediate quantum critical point at \( \lambda = 0 \). At the critical point, the energy gap vanishes as \( \Delta(k) \sim |k|^z \) where \( z \) is the dynamical critical exponent. Thus the dynamics becomes non-adiabatic around a region near this point and the system fails to remain at the instantaneous ground state leading to formation of defects [3, 4, 5, 6, 7, 8, 9]. The density of these defects \( n \) and the residual energy produced in the process \( Q \) scale with universal exponents: \( n \sim \tau^{d-\nu d_\alpha/(d+z)\alpha_\alpha/(z\alpha+1)} \) and \( Q \sim \tau^{-(d+z)\nu\alpha/(z\alpha+1)} \), where \( \nu \) is the correlation length exponent and \( d \) is the system dimension [5, 7]. It is well-known that scaling laws do not change if the dynamics terminate at the critical point [8]. All of the above-mentioned studies apply to isotropic critical points where the scaling of the energy gap with the momentum is described by a single exponent \( z \). Recently, the anisotropic Dirac model with an anisotropic critical point is studied and it was shown that one needs multiple exponents to describe the scaling of the energy gap [10]. However such studies have not been carried out in the context of the Kitaev model and generic expressions for the scaling laws for \( n \) and \( Q \) for such critical points in arbitrary dimensions have not been provided.
Figure 1. Schematic representation of the Kitaev model on a honeycomb lattice. The bonds $J_1, J_2$ and $J_3$ shows nearest neighbor couplings between $x, y$ and $z$ components of the spins respectively. $\vec{n}$ represents the position vector of the midpoint of each vertical bond (unit cell). The vectors $\vec{M}_1$ and $\vec{M}_2$ are spanning vectors of the lattice. In the Fermionic representation of the model, the Majorana Fermions $a_{\vec{n}}$ and $b_{\vec{n}}$ sit at the bottom and top sites respectively of the vertical bond with center coordinate $\vec{n}$ as shown.

In this work, we study the aspect of non-equilibrium slow dynamics in the vicinity of anisotropic critical points with specific focus on the 2D Kitaev model. We derive a generic model-independent expression for the scaling of $n$ and $Q$ for such dynamics which takes a $d$-dimensional system from a gapped phase to the vicinity of an anisotropic critical point. We consider a scenario where the energy gap $\Delta_k$ vanishes as $k_z$ for $m$ momentum components $(i = 1, \cdots, m)$ and as $k_z'$ for the rest $d - m$ components $(i = m + 1, \cdots, d)$ with $z' \geq z$ at the critical point and show that the time-evolution of the Hamiltonian parameter $\lambda(t)$, which brings the system at the critical point at $t = 0$, leads to novel scaling laws for $n$ and $Q$:

$$
n \sim \tau^{-(m+(d-m)z/z'|\nu\alpha}/(z\nu\alpha+1)},
$$

$$
Q \sim \tau^{-(m+z)+(d-m)z'/\nu\alpha}/(z\nu\alpha+1)}.
$$

Our results reproduce their well-known counterparts for the isotropic case ($z = z'$) as special cases. We also show, by exact analytical solution for linear time evolution ($\alpha = 1$), that the two-dimensional (2D) Kitaev model provides an explicit realization of the scaling laws mentioned above with $d = z' = 2$ and $m = \nu = z = 1$ leading to $n \sim \tau^{-3/4}$ and $Q \sim \tau^{-5/4}$. We also corroborate the scaling laws mentioned above by numerical studies of the Kitaev model for arbitrary power-law time evolution.

2. Anisotropic critical points

We begin with the study of slow dynamics in the Kitaev model [12, 13, 14, 15]. The Hamiltonian for this model, schematically represented in Fig. 1, is given by

$$
H_K = \sum_{j+l = \text{even}} (J_1 \tau_{j,l}^x \tau_{j+1,l}^x + J_2 \tau_{j-1,l}^y \tau_{j,l}^y + J_3 \tau_{j,l}^z \tau_{j+1,l}^z),
$$

where $\tau_{j,l} = (\tau_{j,l}^x, \tau_{j,l}^y, \tau_{j,l}^z)$ denote Pauli matrices at the site $(j, l)$ of the honeycomb lattice. $J_1$, $J_2$, and $J_3$ represent nearest-neighbor couplings between $x, y$ and $z$ components of the spins respectively. It is well-known that $H_K$ can be represented in terms of Fermionic fields by a straightforward Majorana transformation: $a_{jl} = (\prod_{k=-\infty}^{j-1} \tau_{ik}^x \prod_{i=-\infty}^{j-1} \tau_{il}^z) \tau_{jl}^y$ for even $j+l$.
and $b_{jl} = (\prod_{i=1}^{j-1} \prod_{l=1}^{l-1} \tau^{z}_{ik} \prod_{i=1}^{j-1} \prod_{l=1}^{l-1} \tau^{z}_{jl}) \tau^{z}_{jl}$ for odd $j + l$ [13, 14, 15]. This leads to the Fermionic Hamiltonian

$$H_F = i \sum_{\tilde{n}} \left( J_1 b_{\tilde{n}1-\tilde{M}_1} + J_2 b_{\tilde{n}1+\tilde{M}_2} + J_3 D_{\tilde{n}} b_{\tilde{n}a_{\tilde{n}}} \right),$$

where $\tilde{n} = \sqrt{3} n_1 + \left( \frac{\sqrt{3}}{2} i + \frac{\sqrt{3}}{2} j \right) n_2$ denote the midpoints of the vertical bonds. Here $n_1, n_2$ run over all integers so that the vectors $\tilde{n}$ form a triangular lattice whose vertices lie at the centers of the vertical bonds of the underlying honeycomb lattice. The Majorana Fermions $a_{\tilde{n}}$ and $b_{\tilde{n}}$ sit at the bottom and top sites respectively of the bond labeled $\tilde{n}$. The vectors $\tilde{M}_1 = \sqrt{3} i - \frac{1}{2} j$ and $\tilde{M}_2 = \sqrt{3} i + \frac{1}{2} j$ are spanning vectors for the lattice, and $D_{\tilde{n}}$ can take the values $\pm 1$ independently for each $\tilde{n}$. The crucial point that makes the solution of Kitaev model feasible is that $D_{\tilde{n}}$ commutes with $H_F$, so that all the eigenstates of $H_F$ can be labeled by specific values of $D_{\tilde{n}}$. It is well-known that the ground state of the model corresponds to $D_{\tilde{n}} = 1$. Hence the off-diagonal terms $\alpha_{\tilde{n}}$ remain time independent, and the quench problem reduces to a Landau-Zener problem for each $\tilde{k}$.

For $D_{\tilde{n}} = 1$, Eq. (2) can be diagonalized as

$$H_F = \sum_{\tilde{k}} \psi_{\tilde{k}}^{\dagger} H_{\tilde{k}} \psi_{\tilde{k}},$$

where $\psi_{\tilde{k}} = (a_{\tilde{k}}^{\dagger}, b_{\tilde{k}}^{\dagger})$ are Fourier transforms of $a_{\tilde{n}}^{\dagger}$ and $b_{\tilde{n}}^{\dagger}$, the sum over $\tilde{k}$ extends over half the Brillouin zone (BZ) of the triangular lattice formed by the vectors $\tilde{n}$, and $H_{\tilde{k}}$ can be expressed in terms of the Pauli matrices $\sigma^z$ in particle-hole space as

$$H_{\tilde{k}} = 2 \left\{ J_1 \sin(\tilde{k} \cdot \tilde{M}_1) - J_2 \sin(\tilde{k} \cdot \tilde{M}_2) \right\} \sigma^1 + 2 \left\{ J_3 + J_1 \cos(\tilde{k} \cdot \tilde{M}_1) + J_2 \cos(\tilde{k} \cdot \tilde{M}_2) \right\} \sigma^2.$$

The spectrum consists of two bands with energies $E^\pm_{\tilde{k}} = \pm E_{\tilde{k}}$ [15], where

$$E_{\tilde{k}} = 2 \left\{ \left\{ J_1 \sin(\tilde{k} \cdot \tilde{M}_1) - J_2 \sin(\tilde{k} \cdot \tilde{M}_2) \right\}^2 + \left\{ J_3 + J_1 \cos(\tilde{k} \cdot \tilde{M}_1) + J_2 \cos(\tilde{k} \cdot \tilde{M}_2) \right\}^2 \right\}^{1/2}.$$

For $|J_1 - J_2| \leq J_3 \leq J_1 + J_2$, the bands touch each other, and the energy gap $\Delta_{\tilde{k}} = E^+_{\tilde{k}} - E^-_{\tilde{k}}$ vanishes for special values of $\tilde{k}$ leading to a gapless phase. In particular we note that for $J_1 = J_2 = 1$ and $J_3 = 2$, the gap vanishes at $\tilde{k}_c = (2\pi/\sqrt{3}, 0)$ and around this point $\Delta_{\tilde{k}} \sim k_y$ and $\Delta_{\tilde{k}} \sim (k_x - 2\pi/\sqrt{3})^2$. Thus this critical point constitutes an example of an anisotropic critical point with $z = m = 1$ and $d = z' = 2$. We note that such an anisotropic scaling occurs for any non-zero value of $J_1$ and $J_2$ at $J_3 = (J_1 + J_2)$.

We now consider a dynamics in this model $J_3(t) = (J_1 + J_2 - J t/\tau)$ from $t = -\infty$ to $t = 0$ at a fixed rate $1/\tau$ which brings the system from a gapped phase to the anisotropic critical point at $\tilde{k}_c$. Although this quench problem can be solved for any $J_1$ and $J_2$, we shall fix $J_1 = J_2 = J$ for simplicity and scale all energies (times) by $J$ ($h/J$) in the subsequent analysis. This choice does not change the scaling properties which we seek. Also, to study the time evolution of the system, we note that after an unitary transformation $U = \exp(-i\sigma^1 \pi/4)$, we obtain $H_F = \sum_{\tilde{k}} \psi_{\tilde{k}}^{\dagger} H_{\tilde{k}}' \psi_{\tilde{k}}$, where $H_{\tilde{k}}' = U H_{\tilde{k}} U^\dagger$ is given by

$$H_{\tilde{k}}' = 2 \left\{ (g_{\tilde{k}} - t/\tau) \sigma^3 + \alpha_{\tilde{k}} \sigma^1 \right\},$$

where $\alpha_{\tilde{k}} = \sin(\tilde{k} \cdot \tilde{M}_1) - \sin(\tilde{k} \cdot \tilde{M}_2)$ and $g_{\tilde{k}} = 2 + \cos(\tilde{k} \cdot \tilde{M}_1) + \cos(\tilde{k} \cdot \tilde{M}_2)$. Hence the off-diagonal elements of $H_{\tilde{k}}'$ remain time independent, and the quench problem reduces to a Landau-Zener problem for each $\tilde{k}$. 

3
The state of the system after the quench at time $t = 0$ can be found by solving the Landau-Zener problem at each $\vec{k}$ with the initial condition $\psi_{\vec{k}}^0(t = -\infty) = |1\rangle = (0,1)^T$ for all $\vec{k}$. After some algebra, one obtains for a given $\vec{k}$ and at $t = 0$ [17]

$$|\psi_{\vec{k}}\rangle^d = e^{-\pi\alpha_{\vec{k}}^2/4} \left( e^{3i\pi/4} D_{\mu_{\vec{k}}} (\nu_{\vec{k}}) |1\rangle + \alpha_{\vec{k}} \sqrt{\tau} D_{\mu_{\vec{k}}+1} (\nu_{\vec{k}}) |0\rangle \right),$$

where $\nu_{\vec{k}} = 2i g_{\vec{k}} \sqrt{\tau} \exp(-i \pi/4)$, $\mu_{\vec{k}} = -i 2\alpha_{\vec{k}} \tau$ and $D_{\mu}$ are parabolic cylinder functions. The excited state at $t = 0$, solved by diagonalizing $H_{\vec{k}}^0(t = 0)$, yields, for a given $\vec{k}$, $|\psi_{\vec{k}}^+\rangle = [(E_{\vec{k}}^+ - 2g_{\vec{k}})|1\rangle + 2\alpha_{\vec{k}}|0\rangle]/D_{\vec{k}}$, where $D_{\vec{k}} = [(E_{\vec{k}}^+ - 2g_{\vec{k}})^2 + 4\alpha_{\vec{k}}^2]^{1/2}$. Thus the probability of defect formation, given by $p_{\vec{k}} = |\langle \psi_{\vec{k}}^+ | \psi_{\vec{k}} \rangle|^2$, can be obtained as

$$p_{\vec{k}} = \frac{4\alpha_{\vec{k}}^2 e^{-\pi\alpha_{\vec{k}}^2/4}}{D_{\vec{k}}^2} \left| \alpha_{\vec{k}} \sqrt{\tau} D_{\mu_{\vec{k}}-1} (\nu_{\vec{k}}) + \frac{E_{\vec{k}}^+ - 2g_{\vec{k}}}{2\alpha_{\vec{k}}} \times e^{-3i\pi/4} D_{\mu_{\vec{k}}} (\nu_{\vec{k}}) \right|^2.$$  

(8)

Since $\tau$ is large for slow dynamics, the contribution to the defect formation comes from a small region near the critical point where $\Delta_{\vec{k}}$ is sufficiently small for $\vec{k} \approx \vec{k}_c$. The density of defects can be thus estimated by expanding $p_{\vec{k}}$ about $\vec{k} = \vec{k}_c$: $n \approx \int d\delta k_x d\delta k_y p_{\vec{k} = \vec{k}_c+\delta k}$, where the limits of integration can now be safely extended to infinity. To compute this integral, we note that around $\vec{k} = \vec{k}_c$, $\alpha_{\delta k} \approx 3\delta k_y$ and $g_{\delta k} \approx 3(\delta k_x^2 + 3\delta k_y^2)/4$. Thus a redefinition of variables $\delta k_x \to \delta k'_x = \delta k_x \tau^{1/4}$ and $\delta k_y \to \delta k'_y = \delta k_y \tau^{1/2}$ allows us to extract the $\tau$ dependence of the defect density

$$n \approx \int d\delta k_x d\delta k_y p_{\delta k}, \sim \tau^{-3/4} \int d\delta k'_x d\delta k'_y p_{\delta k'}.$$  

(9)

A similar analysis can be carried out for computation of residual energy $Q = (2\pi)^{-2} \int d^2 k p_{\vec{k}} \Delta_{\vec{k}}$. Here we note that near the critical point $\vec{k} = \vec{k}_c$, $\Delta_{\vec{k}} \approx 4\sqrt{9\delta k_y^2 + 9(\delta k_x^2 + 3\delta k_y^2)^2}/16$ and thus scale as $\tau^{-1/2}$. Thus one obtains

$$Q \approx \int d\delta k_x d\delta k_y \Delta_{\delta k} p_{\delta k},\sim \tau^{-5/4} \int d\delta k'_x d\delta k'_y \Delta_{\delta k'} p_{\delta k'}.$$  

(10)

Equations (9) and (10) show that $n \sim \tau^{-3/4}$ and $Q \sim \tau^{-5/4}$ at the critical point. These scaling laws do not conform to the predictions of earlier works on defect production during passage through isotropic quantum critical points [5] or critical surfaces [15]; their origin lies in the anisotropic scaling of $\delta k_x$ and $\delta k_y$ with the quench time $\tau$.

To generalize these results for arbitrary $d$-dimensional anisotropic critical points, where the energy gap $\Delta_{\vec{k}} \sim k_i^z$ for $m$ directions and $\sim k_i^z$ for $d-m$ directions, we provide a simple phase space argument as first proposed in Ref. [4]. We consider a general power-law quench with $\lambda(t) = \lambda_0 |t/\tau|^{\alpha} \text{Sgn}(t)$ which starts at $t = -\infty$ and reaches the critical point at $t = 0$. We first note that the adiabaticity condition breaks down when the rate of change of the energy gap become equivalent to the square of the gap: $d\Delta_{\vec{k}}/dt \geq \Delta_{\vec{k}}^2$. Since $\Delta_{\vec{k}} \sim \lambda^{2\nu} |t/\tau|^{\nu\alpha}$, we find that the time spent by the system in the non-adiabatic regime is given by $t \sim \tau^{\nu\alpha}/(\nu\alpha+1)$. The scaling of the energy gap in this regime can thus be written as $\Delta_{\vec{k}} \sim \tau^{-\nu\alpha}/(\nu\alpha+1)$. The phase space for defect production is given by $\Omega_n \sim k_1 \cdots k_d$. Since $\Delta_{\vec{k}} \sim k_i^z$ for $i = 1, \cdots, m$ and $k_i^z$ for $i = m+1, \cdots, d$, we finally obtain

$$n \sim \tau^{-\left[\frac{m+(d-m)z}{z'}\right] \nu\alpha}/(\nu\alpha+1).$$  

(11)
Figure 2. Numerical results on the defect density $n$ and residual energy $Q$. The time-dependent Schrödinger equation is solved in the momentum space for systems with size up to $512 \times 512$ unit cells. The parameter $\alpha$ specifying the evolution of $J_3 = 2 - |t/\tau|^\alpha \text{Sgn}(t)$ is chosen as $\alpha = 1$, 3 and 5. The lines indicate the power laws expected from Eqs. (11) and (12) for $d = z' = 2$ and $z = \nu = m = 1$, $n \sim \tau^{-3\alpha/[2(\alpha+1)]}$ and $Q \sim \tau^{-5\alpha/[2(\alpha+1)]}$. The agreement between curves obtained numerically and the corresponding power laws is remarkable. In all plots, $t$ varies from an initial value $t_{in} = -3\tau$ to a final value $t_f = 0$.

A similar argument can also be presented for the residual energy. We note that for $z \leq z'$, the leading behavior of the energy gap near the quantum critical point, where the defects are produced, is $\Delta^2_k \sim k_i^2$ for $1 \leq i \leq m$. Thus the phase space for the residual energy production is $\Omega_Q \sim \Delta^2_k k_1 \cdots k_d$ leading to a scaling of $Q$ as

$$Q \sim \tau^{-(m+z)+(d-m)z/z'}|\alpha/(z\alpha+1)|.$$  

(12)

We note that the scaling laws, Eqs. (11) and (12), reproduce their isotropic counterparts for $z = z'$ leading to $n \sim \tau^{-3\alpha/[2\alpha+1]}$ and $Q \sim \tau^{-5\alpha/[2\alpha+1]}$ [7, 5, 8]. Also, the scaling of the Kitaev model for linear time evolution elaborated in this work is reproduced for $d = z' = 2$, and $z = \nu = \alpha = 1$ leading $n \sim \tau^{-3/4}$ and $Q \sim \tau^{-5/4}$. Moreover, we note that the scaling of defect density for a linear quench through a gapless surface can also be obtained from Eq. (11) by noting that for such quenches the energy gap depends only on the $m$ momenta components orthogonal to the $d-m$ dimensional gapless surface. This can be represented by putting $z' \to \infty$ (since $k_i \sim \Delta^{1/z'}_k$) leading to the scaling law $n \sim \tau^{-m\nu/(z\nu+1)}$ [15]. Thus Eqs. (11) and (12) reproduce all earlier results on defect production for slow dynamics across quantum critical lines and surfaces as special cases. Finally, we would like to point out that the maximum values of these exponents is 2 which can be obtained by similar considerations as in the cases of isotropic critical points [8].

To verify these scaling laws, we now study non-linear power-law dynamics in the Kitaev model numerically. To this end, we again restrict ourselves to $J_1 = J_2 = 1$ and evolve $J_3(t) = (2 - |t/\tau|^\alpha \text{Sgn}(t))$ for $-\infty \leq t \leq 0$ so that the anisotropic critical point is reached at $t = 0$. The corresponding time-dependent Hamiltonian is given by $H(\vec{k};t) = \sum \psi^\dagger_k \begin{bmatrix} g_k & |t/\tau|^\alpha \text{Sgn}(t) \sigma^3 + \alpha_k \sigma^1 \end{bmatrix} \psi_k$. We solve the time-dependent Schrödinger equation $i\partial_t \psi_k = H(\vec{k};t) \psi(\vec{k})$ numerically for each $k$, compute $p_k$, and use it to obtain the defect density $n = \int d^d k p_k$ and $Q = \int d^d k \Delta_k p_k$ numerically as a function of $\tau$ and $\alpha$. The plots of $n$ and $Q$ vs $\tau$ are shown in Fig. 2 for several representative values of $\alpha$. The lines in the figure indicate the power laws expected from Eqs. (11) and (12) ($n \sim \tau^{-3\alpha/[2(\alpha+1)]}$ and $Q \sim \tau^{-5\alpha/[2(\alpha+1)]}$) for $d = z' = 2$ and $z = \nu = m = 1$. The agreement between the numerical and theoretical results
3. Conclusion

In conclusion, we have shown that the Kitaev model constitutes an example of a two-dimensional model with an anisotropic critical point. We have also demonstrated that the presence of such an anisotropic critical point leads to novel scaling laws defect density and residual energy during slow power-law dynamics which takes the system from a gapped phase to the vicinity of such a critical point. We have generalized our results for such scaling laws for \(d\)-dimensional systems with such anisotropic critical point. We note that there has been suggestions of experimental realization of the Kitaev model using ultracold atomic system [19]. In the event of such a realization, the simplest experimental test of our theory would involve measurement of defect density \(n\) following a slow ramp. Such experiments has recently been performed for standard ultracold boson systems [20].

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