What is the Evidence for the Color Glass Condensate? *

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Abstract

I introduce the concept of the Color Glass Condensate. I review data from HERA and RHIC which suggest that such a universal form of matter has been found.

1 What is the Color Glass Condensate?

The ideas for the Color Glass Condensate originate in the result for the HERA data on the gluon distribution function shown in Fig. 1(a) [1] The gluon density is rising rapidly as a function of decreasing x. This was expected in a variety of theoretical works,[2]-[4] and has the implication that the real physical transverse density of gluons must increase.[2]-[3],[5]. This follows because total cross sections rise slowly at high energies but the number of gluons is rising rapidly. This is shown in Fig. 1(b). This led to the conjecture that the density of

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Figure 1: (a) The HERA data for the gluon distribution function as a function of $x$ for various values of $Q^2$. (b) A physical picture of the low $x$ gluon density inside a hadron as a function of energy.

Gluons should become limited, that is, there is gluon saturation. [2]-[3], [5]

The low $x$ gluons therefore are closely packed together. The strong interaction strength must become weak, $\alpha_S \ll 1$. Weakly coupled systems should be possible to understand from first principles in QCD. [5]

This weakly coupled system is called a Color Glass Condensate for reasons we now enumerate:[6]

- **Color** The gluons which make up this matter are colored.
- **Glass** The gluons at small $x$ are generated from gluons at larger values of $x$. In the infinite momentum frame, these larger momentum gluons travel very fast and their natural time scales are Lorentz time dilated. This time dilated scale is transferred to the low $x$ degrees of freedom which therefore evolve very slowly compared to natural time scales. This is the property of a glass.
- **Condensate** The phase space density

$$\rho = \frac{1}{\pi R^2} \frac{dN}{dyd^2p_T} \quad (1)$$
is generated by a trade off between a negative mass-squared term linear in the density which generates the instability, $-\rho$ and an interaction term $\alpha_S \rho^2$ which stabilizes the system at a phase space density $\rho \sim 1/\alpha_S$. Because $\alpha_S << 1$, this means that the quantum mechanical states of the system associated with the condensate are multiply occupied. They are highly coherent, and share some properties of Bose condensates. The gluon occupation factor is very high, of order $1/\alpha_S$, but it is only slowly (logarithmically) increasing when further increasing the energy, or decreasing the transverse momentum. This provides saturation and cures the infrared problem of the traditional BFKL approach.[7]

Implicit in this definition is a concept of fast gluons which act as sources for the colored fields at small x. These degrees of freedom are treated differently than the fast gluons which are taken to be sources. The slow ones are fields. There is an arbitrary $X_0$ which separates these degrees of freedom. This arbitrariness is cured by a renormalization group equation which requires that physics be independent of $X_0$. In fact this equation determines much of the structure of the resulting theory as its solution flows to a universal fixed point.[6]-[9]

There is evidence which supports this picture. One piece is the observation of limiting fragmentation. This phenomena is that if particles collide at some fixed center of mass energy and the distribution of particles are measured as a function of their longitudinal momentum from the longitudinal momentum of one of the colliding particles, then these distributions do not change as one goes to higher energy, except for the new degrees of freedom that appear. This is true near zero longitudinal momentum in the center of mass frame because new degrees of freedom appear as the center of mass energy is increased. In the analogy with the CGC, the degrees of freedom, save the new ones added in at low longitudinal momentum, are the sources. The fields correspond to the new degrees of freedom. The sources are fixed in accord with limiting fragmentation. One generates an effective theory for the low longitudinal momentum degrees of freedom as fixed sources above some cutoff, and the fields generated by these sources below the cutoff. A recent measurement of limiting fragmentation comes from the Phobos experiment at RHIC shown in Fig. 2 [10]

Of course the perfect scaling of the limiting fragmentation curves is only an approximation. As shown by Jalilian-Marian, the limiting
Figure 2: Limiting fragmentation and the RHIC data.

Fragmentation curves are given by the total quark, antiquark and gluon distribution functions of the fast particle measured at a momentum scale $Q^2_{sat}$ appropriate for the particle that it collides with.[11] The saturation momentum $Q_{sat}$ will play a crucial role in our later discussion. It is a momentum scale which is determined by the density of gluons in the CGC

$$\frac{1}{\pi R^2} \frac{dN}{dy} \sim \frac{1}{\alpha_s} Q^2_{sat}$$

(2)

The saturation momenta turns out to depend on the total beam energy because the longitudinal momentum scale of the target particle at fixed $x$ of the projectile will depend upon the beam energy. It is nevertheless remarkable how small these violations appear to be.

The CGC may be defined mathematically by a path integral:

$$Z = \int_{X_0} [dA][dj] exp (iS[A,j] - \chi[j])$$

(3)

What this means is that there is an effective theory defined below some cutoff in $x$ at $X_0$, and that this effective theory is a gluon field in the presence of an external source $j$. This source arises from the
quarks and gluons with \( x \geq X_0 \), and is a variable of integration. The fluctuations in \( j \) are controlled by the weight function \( \chi[j] \). It is \( \chi[j] \) which satisfies renormalization group equations which make the theory independent of \( X_0 \).[8]-[14],[6] The equation for \( \chi \) is called the JIMWLK equation. This equation reduces in appropriate limits to the BFKL and DGLAP evolution equations.[4], [15] The theory above is mathematically very similar to that of spin glasses.

There are a variety of kinematic regions where one can find solutions of the renormalization group equations which have different properties. There is a region where the gluon density is very high, and the physics is controlled by the CGC. This is when typical momenta are less than a saturation momenta which depends on \( x \),

\[
Q^2 \leq Q^2_{sat}(x)
\]  

The dependence of \( x \) has been evaluated by several authors, [2],[16]-[18], and in the energy range appropriate for current experiments has been determined by Triantafyllopoulos to be

\[
Q^2_{sat} \sim (x_0/x)^\lambda \text{GeV}^2
\]

where with about 15\% uncertainty \( \lambda = 0.3 \). The value of \( x_0 \) is not determined from the renormalization group equations and must be found from experiment.

There is also a region of very high \( Q^2 \) at fixed \( x \), where the density of gluons is small and perturbative QCD is reliable. It turns out there is a third region intermediate between high density and low where there are universal solutions to the renormalization group equations and scaling in terms of \( Q^2_{sat} \).[17] In this region and in the region of the CGC, distribution functions are universal functions of only \( Q^2/Q^2_{sat}(x) \). The extended scaling region is when

\[
Q^2_{sat} \leq Q^2 \leq Q^4_{sat}/\Lambda^2_{QCD}
\]

These various regions are shown in Fig. 3 in the plane of \( ln(1/x) \) and \( ln(Q^2) \)

### 2 Why is the CGC Important?

The Color Glass Condensate is a new universal type of matter:
Figure 3: The various regions where there are a CGC, extended scaling, and low density of glue.

- **Matter:** The separation between the gluons in the CGC is small compared to the size of the system. Due to Lorentz time dilation, the lifetime of this matter is long compared to natural time scales.

- **New:** This matter can only be probed at high energy, and it may be produced in ultrarelativistic nuclear collisions.

- **Universal:** The CGC is universal independent of the type of hadron which generated it. Universality of this matter implies it is of fundamental interest.

**The Color Glass Condensate is a theory of:**

- The origin of glue and sea quarks in hadrons.
- The origin and nature of cross sections and particle production.
- The distribution of valence quantum numbers at small x.
- The initial conditions for the matter which evolves into the Quark Gluon Plasma at RHIC.

### 3 What does a high energy hadron look like?

In Fig. 4, a hadron is shown in its rest frame and as viewed from...
the infinite momentum frame. The picture does not look Lorentz
invariant. This is because the hadron is made of many different Fock
space components, some containing valence quarks and a few gluons
and sea quarks and some containing valence quarks plus many sea
quarks and gluons. At low energies, one is sensitive to matrix elements
which involve the valence quarks and a few gluons and sea quarks.
For high energy collisions, typical matrix elements involve the valence
quarks plus many sea quarks and gluons. At high energies, the hadron
appears as if it is a gluon wall.

Of course the Color Glass Condensate description is Lorentz invari-
ant. In fact, there is a very subtle duality of description. If one views
the hadron in the infinite momentum frame, one is scattering from
high density of gluonic matter. In the rest frame of the hadron, where
the probe has very high energy, the Color Glass Condensate appears
through coherent multiple scattering of the probe valence quarks. One
can prove these descriptions are mathematically equivalent.[19]-[20]

The form of the fields associated with the CGC can also be easily
seen. In the infinite momentum frame, the entire hadron is Lorentz
contracted into a small region of \( x^- = t - z \). The glassy nature of
the fields makes them independent of the light cone time \( x^+ = t + z \).
Therefore the only large component of \( F^{\mu\nu} \) is \( F^{i+} \sim \partial/\partial x^- A^i \). A

Figure 4: A hadron is shown at rest and in the infinite momentum frame.
little algebra shows that
\[ \vec{E} \perp \vec{B} \perp \vec{z} \]  \hspace{1cm} (7)

In Fig. 5, these fields are shown. They correspond to the Lienard-Wiechart potential for a boosted electron, except that they are colored and they have a random polarization and color.

Figure 5: The distribution of colored fields within the CGC.

The density of gluons per unit area defines a momentum scale, the saturation momenta,

\[ \frac{1}{\pi R^2} \frac{dN}{dy} \sim \frac{1}{\alpha_S} Q_{sat}^2(x) \]  \hspace{1cm} (8)

We insert the extra factor of $1/\alpha_S$ because the fields are classical, and therefore their density should scale in this way. This form is guaranteed because the system is almost scale invariant. Small violations of scaling arise because $\alpha_S$ is measured at $Q_{sat}$. 
Experimental Evidence in Support of CGC

In this section, I discuss the accumulated evidence from HERA and RHIC, and elsewhere, in support of the hypothesis of a Color Glass Condensate.

4.1 Geometrical Scaling

Geometrical scaling is the observation\cite{21}-\cite{22} that the deep inelastic cross section for virtual photon scattering as a function of $Q^2$ and $x$ is really only a function of

$$
\sigma^{\gamma p} \sim F(Q^2/Q_{sat}^2)
$$

(9)

where the saturation momentum is taken to be

$$
Q_{sat}^2 \sim (x_0/x)^\lambda \ 1 \text{GeV}^2
$$

(10)

and $\lambda \sim 0.3$ and $x_0 \sim 10^{-4}$. This scaling works for $x \leq 10^{-2}$ and for the available data in $Q^2$. The data is shown in Fig. 6

It is straightforward to understand why this scaling works for the small $Q^2 \leq Q_{sat}^2$. This is the region of the CGC, and there is only one dimensionful scale which characterizes the system: the saturation momentum\cite{16}. The surprise is that there is an extended scaling window for $Q_{sat}^2 \leq Q^2 \leq Q_{sat}^4/\Lambda_{QCD}^2$\cite{17}. This can be proven analytically. As well, one now has reliable computation of the dependence on $x$ of the saturation momentum, that is, one knows the exponent $\lambda$ to about 15% accuracy, and it agrees with what is seen from the geometrical scaling curve\cite{18}. What is not determined from the theory of the CGC is the scale $x_0$, and this must be found by experiment. This comes from the boundary conditions for the renormalization group equations.

4.2 The Structure Function $F_2$

Using the dipole description of the virtual photon wavefunction, the structure function $F_2$ can be related to the gluon distribution function which arises from the CGC. One can compute and compare to data. There are 3 unknown parameters in this description: the hadron size,
the scale $x_0$ and the quark mass. In addition, the parameter $\lambda$ which controls the energy dependence of the saturation momentum is determined by experiment to better accuracy than it is currently known theoretically.[16]-[24] The results for the description of the data are remarkably good for $x \leq 10^{-2}$ and $Q^2 \leq 45 \text{ GeV}^2$, as shown below in Figs. 7-8[24]

One should note that this description includes both the high and low $Q^2$ data. Descriptions based on DGLAP evolution can describe the large $Q^2$ points. The CGC description is very economical in the number of parameters which are used.

### 4.3 Diffraction and Quasi-Elastic Processes

The CGC provides a description of the underlying structure of gluonic matter inside a hadron. As such, it should be sensitive to probes of the
transverse extent of this matter, which can be experimentally studied in diffraction and related quasi-elastic particle production.\cite{25}

For diffraction by a virtual photon, a good theoretical first principles computation is only available for small mass states which are produced by the virtual photon.\cite{26}-\cite{33}. In Fig. 9, a computation of the ratio of the diffractive to total deep inelastic cross section for various $Q^2$ and produced masses is shown. The agreement is good for small masses, and even reasonable for large masses.

There are additional computations of quasi-elastic $\rho$ meson production and $J/\Psi$ production\cite{32}-\cite{33}. These are shown in Figs. 10-11. They agree well up to an overall normalization uncertainty for the $\rho$
Figure 8: More of the CGC description of $F_2$.

meson associated with knowledge of the $\rho$ meson wavefunction. The various curves correspond in the figures correspond to different models for these wavefunctions.

4.4 Qualitative Understanding of Total Hadronic Cross Sections

The elastic and total cross section of $pp$ scattering as a function of energy is shown in Fig. 12. The cross section is slowly varying as a function of energy and is believed to grow as $ln^2(E)$. To understand this behaviour, imagine that we take some probe and try to penetrate
Figure 9: The ratio of diffractive to total cross sections for deep inelastic scattering.

the hadron. The cross section is defined by the impact parameter at which the hadron become opaque. We expect that at fixed $x$, the impact parameter distribution of matter inside a hadron fall off at large $b$ like $e^{-2m_x b}$. At fixed impact parameter, we expect that the number of gluons grows as $(x_0/x)^\lambda$. Setting

$$(x_0/x_{\text{min}})^\lambda \exp(-2m_x b) \sim 1$$

(11)

gives

$$\sigma \sim b^2 \sim \ln^2(1/x_{\text{min}}) \sim \ln^2(E)$$

(12)

Here $x_{\text{min}}$ is the minimal value of $x$ accessible for some energy $E$ and goes as $x_{\text{min}} \sim \Lambda_{QCD}/E$.

This simple physically motivated picture give the expected $\ln^2(E)$ growth of the total cross section. It has proven difficult to make these simple arguments rigorous.[34]-[35].
5 Heavy Ion Collisions

The collision of two ultrarelativistic heavy ions can be visualized as the scattering of two sheets of colored glass, as shown in Fig. 13. [36]-[39] At very early times after the collision the matter is at very high energy density and in the form of a CGC. As time goes on, the matter expands. As it expands the density of gluons decreases, and gluons begin to propagate with little interaction. At later times, the interaction strength increases and there is sufficient time for the matter to thermalize and form a Quark Gluon Plasma. This scenario is shown in Fig. 14, with realistic estimates for energy density and time scales appropriate for the RHIC heavy ion accelerator.

5.1 The Multiplicity

The CGC allows for a direct computation of the particle multiplicity in hadronic collisions. If one naively tries to compute jet production, the total multiplicity is infrared divergent. This follows because of the

Figure 10: The CGC description of quasi-elastic $\rho$ meson production.
1/$p_T^4$ nature of the perturbative formula for gluon production

$$\frac{1}{\pi R^2} \frac{dN}{dy d^2 p_T} \sim \frac{1}{\alpha_S} \frac{Q_s^{4}}{p_T^4}$$  \tag{13}

In the CGC, when $p_T \leq Q_{sat}$, this formula is cutoff. This means that the total gluon multiplicity goes as

$$\frac{1}{\pi R^2} \frac{dN}{dy} \sim \frac{1}{\alpha_S} Q_{sat}^{2}$$  \tag{14}

One can compute the proportionality constant and before the RHIC data appeared, predictions were made for the gluon multiplicity. In Fig. 15, the predictions for the first RHIC run are presented. The CGC was one of the few models which got the multiplicity correct.

Also, the dependence of the multiplicity on the number of participants can also be computed, realizing that the saturation momentum
should be (for not too small $x$) proportional $N_{part}^{1/3}$. This leads to

$$\frac{dN}{dy} \sim \frac{1}{\alpha_S}$$

so that we have a very slow logarithmic dependence on the number of participants. This was a prediction of the CGC and it agreed with experiment, as shown in Fig. 16.[40]-[43]

One can go even further and compute the dependence of the multiplicity on rapidity and centrality, and the transverse momentum distribution of produced hadrons by using CGC initial conditions matched together with a hydrodynamic calculation which evolves the matter thought the Quark Gluon Plasma.[44] The results describe the data remarkably.[45]-[46] This hydrodynamic simulation of Nara and Hi-
rano does remarkably well.

5.2 High $p_T$ Particles

The early results from RHIC on gold-gold collisions revealed that the high $p_T$ production cross sections were almost an order of magnitude below that expected for jet production arising from incoherent parton-parton scattering.[47] This could be either due to initial state shadowing of the gluon distribution inside the nuclei,[48] or to final state jet quenching.[49] For centrally produced jets, the $x$ of the parton which produces a 5-10 GeV particle is of order $10^{-1}$, and this is outside the region where on the basis of the HERA data one expects the effects of the CGC to be important. Nevertheless, nuclei might be different than protons, so it is not a priori impossible.

The crucial test of these two different mechanisms is the comparison of dA scattering to pp. If there is suppression of jet in dA collisions, then it is an initial state effect. The experiments were performed, and all there is little initial state effect for centrally produced jets.[50] The suppression of centrally produced jets in AA collisions at RHIC is indeed due to final state interactions, that is jet quenching.

This is not in contradiction with the existence of a CGC. The particles which control the multiplicity distribution in the central region are relatively soft, and arise from $x \sim 10^{-2}$. To probe such small $x$ degrees of freedom at high transverse momentum at RHIC requires
that one go to the forward region.[51]-[52]

If one uses naive Glauber theory to compute the effects of shadowing by multiple scattering, one expects that if one goes into the forward region of the deuteron, the probe propagates through more matter in the nucleus. This is because we probe all of the gluons with $x$ greater than the minimum $x$ of the nucleus which can be seen by the deuteron. Going more forward makes this minimum $x$ smaller. Now multiple scattering will produce more particles at some intermediate value of $p_T$. (At very high $p_T$, the effects of multiple scattering will disappear.) This is the source of the Cronin peak and it is expected to occur at $p_T$ of $2 - 4$ GeV. Clearly the height of this peak should increase as one goes more forward on the side of the deuteron, and should increase with the centrality of the collision.[53] A result of such a computation is shown in Fig. 18.

Classical rescattering effects are included in the computation of the properties of the CGC. There is another effect however and that is quantum evolution generated by the renormalization group equations. It was a surprise that when one computed the evolution of the gluon
distribution function including both effects, the quantum evolution dominated. This means that the height of the Cronin peak, and the overall magnitude of the gluon distribution decreased as one went from backwards to forward angles.[54]– [56] The results of one such computation are shown in Fig. 19 It was also a surprise how rapid the effect set in.

The Brahms experiment at RHIC recently presented data on the ratio of central to peripheral transverse momentum distributions.[57] The ratio $R_{CP}$ is defined in such a way that if the processes were due to incoherent production of jets, then $R_{CP} = 1$. A value less than one indicates suppression, and a value larger than one indicates a Cronin type enhancement. The results for a variety of forward angles for $R_{CP}$ as a function of $p_T$ is shown in Fig. 20 a. There is clearly a decrease in $R_{CP}$ as one goes to forward angles, in distinction from the predictions of classical multiple scattering. The effect is very rapid in rapidity, as was expected from computations of the CGC. In Fig. 20 b, the ratio $R_{CP}$ is shown as a function of $p_T$ for the forward pseudorapidity $\eta \sim 3$ for less central and more central events. The ratio decreases for more central collisions, against the expectation of classical multiple
scattering and consistent with the CGC hypothesis.

Similar results have been seen in the Star and Phobos experiment as shown in Fig.21 [58]-[59] Phenix has also shown very dramatically the dependence upon centrality of $R_{dAu}$ and pseudorapidity for hadron with $1\ GeV \leq p_T \leq 3\ GeV$,[60] as seen in Fig. 22 This data beautifully illustrates the Cronin enhancement on the gold side and the depletion on the deuteron side, and as well the dependence on centrality. It appears that classical multiple scattering dominates on the gold side, and quantum evolution on the deuteron side. By a happy coincidence, these effects nearly cancel in the mid-rapidity region, making RHIC a very good machine for studying QGP effects at midrapidity for hard probes. The $J/\Psi$ has also been measured in the Phenix experiment and shows similar dependence on centrality and pseudorapidity as does the hadron spectra[60].

Figure 17: Results of hydrodynamic simulation with a CGC initial condition for the multiplicity as a function of $y$ and centrality, and for the transverse momentum distribution at zero rapidity as a function of transverse momentum and centrality.
Figure 18: The expectations of classical multiple scattering for the $p_T$ distribution in dA collisions.

5.3 Alternative Explanations

Although classical multiple scattering cannot explain the effects seen at RHIC in the forward dA experiments, is it possible that some other theory of shadowing can do it? It has not been the expectation from a number of computations which build in shadowing.[61] When any of these models are corrected to include the effects of classical multiple scattering, they fail to describe the dA data.[62]-[63] Nevertheless, someone will surely find some parameterization of shadowing based on somewhat shadowy assumptions and modeling which when combined with classical multiple scattering will describe the data. The question which must be asked is how robust is the underlying model which predicts these distributions. The CGC provides a robust theoretical framework based on first principles from QCD in which such
shadowing and a variety of other effects are predicted. The CGC also explains a variety of other phenomena, as we have seen.

One of the tests of the CGC hypothesis will be the forward backward correlations for jets in \(dA\) collisions, where one of the jets is at forward pseudorapidity. If there is a CGC present, the \(p_T\) of the jet will be broadened on a scale of order of the saturation momentum. At present, this measurement has not been reported\[64\], nor have there been reliable theoretical computations.

6 Summary

My colleague Dima Kharzeev was quoted in the press as saying about the CGC and the recent RHIC results

- This is nothing short of a major discovery.
- it’s going to trigger a real revolution in nuclear physics

By now, it should be clear why I support his position. The measurements at RHIC which support the idea that there is ultradense
Figure 20: (a) The ratio $R_{CP}$ as a function of $p_T$ for various forward pseudo-rapidities. (b) The ratio $R_{CP}$ at a fixed forward pseudorapidity as a function of $p_T$ for less central and more central dA collisions.

matter, and that at early times this matter has the properties predicted for a Color Glass Condensate is without doubt a major discovery. I think however in order to make the case for the Color Glass Condensate compelling, one needs to supplement the RHIC data with electron scattering data from HERA. Additional tests will come from hard processes measured in LHC and potentially definitive tests from experiments at eRHIC.

A revolution involves at least the following:

- Revolutions involve major realignments of traditional relationships between large groups of people.
- Friends try to kill one another. Sometimes successfully.
- There are bad consequences if you try to make a revolution and fail.

The kind of debate over the CGC hypothesis is less whether the CGC can describe phenomena observed in HERA and RHIC, but whether or not there are alternative explanations. These alternative explanations often involve model computations, and do not try to unify the wide range of phenomena described in the talk. It is always very difficult to falsify a model because there is always some arbitrariness in any model. Also, models are a bit like a hydra: If you rule out one model two new ones will appear. On the other hand a theory, such as that of the CGC, must pass much more stringent tests, and
Figure 21: (a) The ratio $R_{CP}$ as a function of $p_T$ as measured in Star for dA collisions at various pseudorapidities. (b) The ratio $R_{dAu}$ as a function of pseudorapidity at fixed $p_T$ values and different centralities.

Figure 22: The pseudorapidity distribution of the ratio $R_{CP}$ for stopped particles with $1 \text{ GeV} \leq p_T \leq 3 \text{ GeV}$ at different centralities as measured in Phenix.

if it is wrong, it must be discarded. Historically, such debates are never resolved on the basis of whether a theory or a model with free parameters can best describe data. The conclusion will almost always be that some model can. The reason that theories become accepted is because they have a simple and unifying intellectual framework, that the arguments which motivate the theory are compelling, and that it describes, within the accuracy of various approximations, a wide variety of diverse phenomena.

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