Octant sensitivity for large $\theta_{13}$ in atmospheric and long baseline neutrino experiments

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**ABSTRACT:** One of the unknown parameters in neutrino oscillation studies is the octant of the atmospheric neutrino mixing angle $\theta_{23}$. In this paper, we discuss the possibility of determining the octant of $\theta_{23}$ in the long baseline experiments T2K and NO$\nu$A in conjunction with future atmospheric neutrino detectors, in the light of non-zero value of $\theta_{13}$ measured by reactor experiments. We consider two detector technologies for atmospheric neutrinos – magnetized iron calorimeter and non-magnetized Liquid Argon Time Projection Chamber. We present the octant sensitivity for T2K/NO$\nu$A and atmospheric neutrino experiments separately as well as the combined sensitivity. For the long baseline experiments, a precise measurement of $\theta_{13}$, which can exclude degenerate solutions in the wrong octant, increases the sensitivity drastically. For $\theta_{23} = 39^\circ$ and $\sin^2 2\theta_{13} = 0.1$, at least $\sim 2\sigma$ sensitivity can be achieved by T2K + NO$\nu$A for all values of $\delta_{CP}$ for both normal and inverted hierarchy. For atmospheric neutrinos, the moderately large value of $\theta_{13}$ measured in the reactor experiments is conducive to octant sensitivity because of enhanced matter effects. A magnetized iron detector can give a 2$\sigma$ octant sensitivity for 500 kT yr exposure for $\theta_{23} = 39^\circ$, $\delta_{CP} = 0$ and normal hierarchy. This increases to 3$\sigma$ for both hierarchies by combining with T2K and NO$\nu$A. This is due to a preference of different $\theta_{23}$ values at the minimum $\chi^2$ by T2K/NO$\nu$A and atmospheric neutrino experiments. A Liquid Argon type detector for atmospheric neutrinos with the same exposure can give higher octant sensitivity, due to the interplay of muon and electron contributions and superior resolutions. We obtain a $\sim 3\sigma$ sensitivity for $\theta_{23} = 39^\circ$ for normal hierarchy. This increases to $\gtrsim 4\sigma$ for all values of $\delta_{CP}$ if combined with T2K/NO$\nu$A. For inverted hierarchy the combined sensitivity is around 3$\sigma$.

**KEYWORDS:** Neutrino Physics.
1. Introduction

The measurement of a non-zero value of the mixing angle $\theta_{13}$ by the reactor experiments Double-Chooz [1], Daya-Bay [2] and RENO [3] heralds a major breakthrough in the advancement of neutrino physics. The best-fit value of $\sin^2 \theta_{13}$ from latest global analysis of solar, atmospheric, reactor and accelerator data is $0.023 \pm 0.0023$ [4], which signifies non-zero $\theta_{13}$ at 10$\sigma$ level. Other global analyses also give similar results [5, 6]. This confirms the earlier observation of non-zero $\theta_{13}$ in T2K [7] and MINOS [8] experiments as well the indication of non-zero best-fit value of $\theta_{13}$ from previous analyses [9, 10]. Among the other oscillation parameters the solar parameters $\Delta m^2_{21}$ and $\theta_{12}$ are already well measured from solar neutrino and KamLAND experiments [11, 12]. The solar matter effect also dictates $\Delta m^2_{21} > 0$. The most stringent constraint on mass squared difference $\Delta m^2_{31}$ governing the atmospheric neutrino oscillations comes from the data from the MINOS experiment [13]. However the ordering of the third mass eigenstate with respect to the other two i.e. the sign of $\Delta m^2_{31}$ is not yet known. There are two possible arrangements of the neutrino mass states: (i) $m_1 < m_2 < m_3$ corresponding to Normal Hierarchy (NH) and (ii) $m_3 < m_1 < m_2$ corresponding to Inverted Hierarchy (IH). The mixing angle $\sin^2 \theta_{23}$ is mainly determined by the SuperKamiokande (SK) atmospheric neutrino data. However the octant in which...
Table 1: The best-fit values and 3σ ranges of neutrino oscillation parameters from Ref. [5]. The first and second rows in each case denote the values for a normal and inverted neutrino mass hierarchy respectively.

| Parameter                  | Best Fit | 3σ Range     |
|---------------------------|----------|---------------|
| $\Delta m^2_{21}$ [10^{-5} eV^2] | 7.54     | 6.99–8.18     |
| $|\Delta m^2_{31}|$ [10^{-3} eV^2] | 2.43     | 2.19 – 2.62   |
| $\sin^2 \theta_{12}$     | 0.307    | 0.26–0.36     |
| $\sin^2 \theta_{23}$     | 0.386    | 0.33–0.64     |
| $\sin^2 \theta_{13}$     | 0.0241   | 0.017–0.031   |
| $\delta$                  | 1.08\pi  | 0 – 2\pi      |

The first and second rows in each case denote the values for a normal and inverted neutrino mass hierarchy respectively.

this mixing angle lies is not yet decisively determined by the data. A full three-flavour fit to the SK data gives the best-fit for NH in the lower octant (LO) and IH in the higher octant (HO) keeping $\theta_{13}$ as a free parameter in the analysis [14]. Three-flavour global analysis of all available neutrino data give the best-fit $\theta_{23}$ in the lower octant. The best-fit values and 3σ ranges of oscillation parameters from global analysis in Ref. [5] is summarized in Table 1. At present there is no significant constraint on the CP phase $\delta_{CP}$ and the whole range from 0 – 2\pi is allowed at the 2σ level.

There are two aspects in the measurement of $\theta_{13}$ which is expected to play an important role in resolving the outstanding issues in neutrino oscillation physics. Firstly if $\theta_{13}$ is non-zero and relatively large — (i) CP violation in the lepton sector can be probed (ii) the earth matter effect on the propagation of neutrinos can be sizable. The latter facilitates the determination of mass hierarchy and octant in experiments in which neutrinos travel through an appreciable path length. The second aspect is the precision measurement of $\theta_{13}$: which helps in increasing the sensitivity in the determination of hierarchy, octant and $\delta_{CP}$.

The octant degeneracy means impossibility of distinguishing between $\theta_{23}$ and $\pi/2 – \theta_{23}$. This is generic and robust for vacuum oscillation probabilities that are functions of $\sin^2 2\theta_{23}$ e.g. the two flavour muon survival probability in vacuum [15]. If on the other hand the leading term in the probability are functions of $\sin^2 \theta_{23}$ (e.g. $P_{\mu e}$) then the inherent octant degeneracy is not there but lack of knowledge of other parameters like $\theta_{13}$ and $\delta_{CP}$ can give rise to octant degeneracy [16,17]. These issues may affect the octant sensitivity of the long baseline experiments T2K and NOνA where the matter effect is not very significant and in particular resonant matter effects do not get a chance to develop. Although conventionally the octant degeneracy refers to the indistinguishability of $\theta_{23}$ and $\pi/2 – \theta_{23}$, in view of the present uncertainty in the measurement of $\theta_{23}$ the scope of this can be generalized to any
value of $\theta_{23}$ in the wrong octant within its allowed range. In this paper by octant sensitivity we refer to this “generalized” definition. If in addition the hierarchy is unknown then there can also be the wrong-octant–wrong-hierarchy solutions and one needs to marginalize over the wrong hierarchy as well.

Atmospheric neutrinos pass through long distances in matter and they span a wide range in energy and can encounter resonant matter effects. In this case the octant sensitivity in $P_{\mu\mu}$ ensues from the term $\sin^4\theta_{13}\sin^22\theta_{13}^m$ \cite{18}. $P_{\mu\nu}$ in matter contains $\sin^2\theta_{13}\sin^22\theta_{13}^m$. Since at resonance $\sin^22\theta_{13}^m \approx 1$, the octant degeneracy can be removed. In this case also one can probe the effect of $\delta_{CP}$ uncertainty on the lifting of this degeneracy.

Atmospheric neutrinos provide fluxes of both neutrinos and antineutrinos as well as neutrinos of both electron and muon flavour. On one hand it provides the advantage of observing both electron and muon events. However on the other hand a particular type of event gets contributions from both disappearance and appearance probabilities. This can be a problem if the matter effects for these two channels go in opposite directions. Thus it is necessary to carefully study the various contributions and ascertain what may be the best possibility to decode the imprint of matter effects in atmospheric neutrino propagation. Three major types of detector technologies are under consideration at the present moment as future detector of atmospheric neutrinos.

(i) Water Cerenkov detectors: Such type of detectors have already been shown to be a successful option for atmospheric neutrino detection by the SuperKamiokande collaboration. This is sensitive to both electron and muon events and the energy threshold can be relatively low. Megaton detectors of this kind under consideration for future are HK, MEMPHYS \cite{19,20}. These cannot be magnetized and hence provide no charge identification capability. Multi-megaton detectors with ice also fall in this category. An example of such a detector is PINGU \cite{21,22}, which is a proposed upgrade of the DeepCore section of the IceCube detector for ultra-high energy neutrinos \cite{23} with a lower energy threshold for atmospheric neutrino detection.

(ii) Magnetized Iron Detectors: Such a detector for atmospheric neutrinos were proposed by the MONOLITH \cite{24} collaboration and is now actively pursued by the INO collaboration \cite{25}. This has a relatively high threshold and is mainly sensitive to muon neutrinos. These type of detectors offer the possibility of magnetization, thus making it possible to distinguish between muon and antimumon events, which enhances the sensitivity.

(iii) Liquid Argon Time Projection Chamber (LArTPC): Examples of such detectors are ICARUS and ArgoNeuT \cite{26,27}. The hallmark of these detectors are their superior particle identification capability and excellent energy and angular resolution. They are sensitive to both electron and muon events with good energy and direction reconstruction capacity for both type of events \cite{28}. Possibility of magnetization is also being discussed \cite{29,30}.

In this paper we study in detail the possibility of removal of octant degeneracy in view of the precise measurement of a relatively large value of $\theta_{13}$. There have been many earlier studies dealing with this subject both in the context of long baseline and atmospheric neutrinos. The importance of combining accelerator and reactor experiments and the role of a precise measurement of $\theta_{13}$ in abating the octant degeneracy have been considered in Refs. \cite{31–33}. The possibility of resolving the octant degeneracy using long
baseline experiments has also been explored in Refs. [34–39]. Recently octant sensitivity in the T2K/NOνA experiments has been investigated including the recent results on the measurement of non-zero $\theta_{13}$ by reactor experiments [40]. The octant sensitivity for atmospheric neutrinos in the context of magnetized iron calorimeter detectors was considered in Refs. [18,41,42], for water Cerenkov detectors in Ref. [19] and for LArTPC in Ref. [43].

We examine the octant sensitivity in the long baseline experiments T2K and NOνA and in the atmospheric neutrino experiments as well as the combined sensitivity of these experiments. In particular we address whether degeneracy due to $\theta_{13}$ can still affect octant determination at the current level of precision of this parameter. For fixed values of $\theta_{13}$ the effect of lack of knowledge of $\delta_{CP}$ on the octant determination capability of these experiments is also studied. We take into account the uncertainty of $\theta_{23}$ in the wrong octant and discuss how much this can influence the octant sensitivity. We present results for the two cases of known and unknown hierarchy.

For the study of atmospheric neutrinos we consider magnetized iron calorimeter detectors with charge sensitivity which is sensitive to the muon neutrinos. We also consider a non-magnetized LArTPC detector which can detect both electron and muon neutrinos. In particular we discuss the interplay between the muon and electron type events in the overall octant sensitivity. For our atmospheric analysis we assume a prior knowledge of hierarchy. Lastly we do a combined analysis of T2K, NOνA and atmospheric neutrinos and discuss the synergistic aspects between long baseline and atmospheric neutrino experiments.

The plan of the paper is as follows. In Section 2 we discuss the octant degeneracy at the level of oscillation and survival probabilities, for baselines corresponding to both atmospheric neutrinos and those relevant to NOνA and T2K. Section 3 discusses the analysis procedure and results. First we discuss the octant sensitivity in NOνA and T2K. Next we describe the results obtained for octant sensitivity using atmospheric neutrino detectors. Finally we present combined octant sensitivity of long baseline and atmospheric neutrino experiments. We end by summarizing the results.

2. Analysis of octant degeneracy

The ambiguity in the determination of the octant of $\theta_{23}$ may appear in the oscillation and survival probabilities as

(a) the intrinsic octant degeneracy, in which the probability is a function of $\sin^2 2\theta_{23}$ and hence the measurement cannot distinguish between $\theta_{23}$ and $\pi/2 - \theta_{23}$,

$$P(\theta_{23}^{tr}) = P(\pi/2 - \theta_{23}^{tr})$$ (2.1)

(b) the degeneracy of the octant with other neutrino parameters, which confuses octant determination due to the uncertainty in these parameters. In particular, this degeneracy arises in probabilities that are functions of $\sin^2 \theta_{23}$ or $\cos^2 \theta_{23}$. For such cases $P(\theta_{23}^{tr}) \neq P(\pi/2 - \theta_{23}^{tr})$. However for different values of the parameters $\theta_{13}$ and $\delta_{CP}$ the probability functions become identical for values of $\theta_{23}$ in opposite octants for different values of these parameters, i.e.

$$P(\theta_{23}^{tr}, \theta_{13}, \delta_{CP}) = P(\pi/2 - \theta_{23}^{tr}, \theta_{13}', \delta_{CP}')$$ (2.2)
where $\theta_{23}^{tr}$ denotes the true value of the mixing angle and the primed and unprimed values of $\theta_{13}$ and $\delta_{CP}$ lie within the current allowed ranges of these parameters. In the case of $\delta_{CP}$, this covers the entire range from 0 to $2\pi$, while for $\theta_{13}$ the current $3\sigma$ range is given by $\sin^2 2\theta_{13} = 0.07 - 0.13$. From the above equation it is evident that even if $\theta_{13}$ is determined very precisely, this degeneracy can still remain due to complete uncertainty in the CP phase. In fact, the scope of this degeneracy can be enlarged to define this as

$$P(\theta_{23}^{tr}, \theta_{13}, \delta_{CP}) = P(\theta_{23}^{wrong}, \theta_{13}', \delta_{CP}')$$

where $\theta_{23}^{wrong}$ denote values of the mixing angle in the opposite octant.

The features of the octant degeneracy and the potential for its resolution in different neutrino energy and baseline ranges can be understood from the expressions for the oscillation and survival probabilities relevant to specific ranges. We discuss below the probability expressions in the context of the fixed baseline experiments NO$\nu$A/T2K and for atmospheric neutrino experiments.

### 2.1 Neutrino Propagation in Matter

Neutrinos travelling through earth encounter a potential due to matter given as,

$$A = 2\sqrt{2} G_F n_e E = 2 \times 0.76 \times 10^{-4} \times Y_e \left[ \frac{\rho}{\text{g/cc}} \right] \left[ \frac{E}{\text{GeV}} \right] \text{eV}^2$$

(2.4)

where $G_F$ is the Fermi coupling constant and $n_e$ is the electron number density in matter, given by $n_E = N_A Y_e \rho$ ($N_A$ = Avogadro’s number, $Y_e$ = electron fraction $\sim 0.5$, $\rho$ = earth matter density.

The mass squared difference $(\Delta m_{31}^2)^m$ and mixing angle $\sin^2 2\theta_{13}^m$ in matter are related to their vacuum values by

$$\frac{(\Delta m_{31}^2)^m}{(\Delta m_{31}^2)^0} = \frac{\sqrt{(\Delta m_{31}^2 \cos 2\theta_{13} - A)^2 + (\Delta m_{31}^2 \sin 2\theta_{13})^2}}{\Delta m_{31}^2 \sin 2\theta_{13}}$$

$$\sin 2\theta_{13}^m = \frac{\Delta m_{31}^2 \sin 2\theta_{13}}{\sqrt{(\Delta m_{31}^2 \cos 2\theta_{13} - A)^2 + (\Delta m_{31}^2 \sin 2\theta_{13})^2}}$$

(2.5)

The MSW matter resonance [44–46] occurs and the mixing angle $\theta_{13}^m$ becomes maximal at neutrino energies and baselines for which the terms $\Delta m_{31}^2 \cos 2\theta_{13}$ and $A$ in the denominator of eq.(2.5) become equal. Hence the matter resonance energy $E_{res}$ is given by

$$E_{res} = \frac{|\Delta m_{31}^2| \cos 2\theta_{13}}{2 \times 0.76 \times 10^{-4} \times Y_e \times \rho}$$

(2.6)

Since the corresponding expression for antineutrinos is obtained by making the replacement $A \to -A$, it may be observed that the matter resonance occurs for the normal neutrino mass hierarchy (i.e. $\Delta m_{31}^2 > 0$) for neutrinos and for the inverted mass hierarchy (i.e. $\Delta m_{31}^2 < 0$) for antineutrinos.
2.2 Octant ambiguity in $P_{\mu e}$ and $P_{\mu\mu}$

For NO$\nu$A/T2K, the baselines are 812 and 295 Km respectively and the peak energies of the beams are in the range 0.5-2 GeV. For these values of baselines, the earth matter density is in the range $2.3 - 2.5$ g/cc, and the corresponding matter resonance energies are above 10 GeV. Hence the neutrino energies of both experiments lie well below matter resonance, and the oscillation probabilities will only display small sub-leading matter effects. The expressions of the relevant probabilities $P_{\mu\mu}$ and $P_{\mu e}$ in vacuum are given by the following expressions obtained in the one-mass scale dominant (OMSD) approximation,

$$P_{\mu\mu}^\nu = 1 - \sin^2 2\theta_{23} \sin^2 \left[ 1.27 \Delta m^2_{31} \frac{L}{E} \right] + 4 \sin^2 \theta_{13} \sin^2 \theta_{23} \cos 2\theta_{23} \sin^2 \left[ 1.27 \Delta m^2_{31} \frac{L}{E} \right]$$  \hspace{1cm} (2.7)

$$P_{\mu e}^\nu = \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \left[ 1.27 \Delta m^2_{31} \frac{L}{E} \right]$$  \hspace{1cm} (2.8)

The above probability expressions have sub-leading corrections corresponding to small matter effect terms and the solar mass-squared difference [47–51]. We observe the following salient features from these expressions:

(a) The disappearance channel $P_{\mu\mu}$ has in its leading order a dependence on $\sin^2 2\theta_{23}$, and hence is dominated by the intrinsic octant degeneracy. There is a small $\theta_{13}$-dependent correction in the measurement of $\theta_{23}$ which gives a minor ($\sim 1\%$) resolution of the degeneracy if $\theta_{13}$ is known precisely.

(b) The appearance channel $P_{\mu e}$ has the combination of parameters $\sin^2 \theta_{23} \sin^2 2\theta_{13}$, and hence does not suffer from the intrinsic octant degeneracy. However, the degeneracy of the octant with the parameter $\theta_{13}$ comes into play, since the above combination may be invariant for opposite octants for different values of $\theta_{13}$, and hence this degeneracy cannot get lifted with a measurement from such experiments alone [32]. This channel can be also affected by the large uncertainty in $\delta_{CP}$ when sub-leading corrections are included.

For atmospheric neutrinos, the relevant baselines and energies are in the range 1000 - 12500 Km and 1 - 10 GeV respectively. A large region in this L and E space exhibits strong resonant earth matter effects, since the earth densities in this baseline range (3 - 8 g/cc) correspond to resonance energies $E_{\text{res}} = 4 - 9$ GeV. Hence the relevant probability expressions $P_{\mu e}^m$, $P_{ee}^m$ and $P_{\mu\mu}^m$ can be written, in the OMSD approximation and with full matter effects, as [48,52]

$$P_{\mu e}^m = \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \left[ 1.27 \left( \Delta m^2_{31} \right)^m \frac{L}{E} \right]$$  \hspace{1cm} (2.9)
\[
P_{\mu\mu}^m = 1 - \cos^2 \theta_{13}^m \sin^2 2\theta_{23} \sin^2 \left[ 1.27 \frac{(\Delta m_{21}^2 + A + (\Delta m_{31}^2)^m)}{2} \right] \frac{L}{E} \\
- \sin^2 \theta_{13}^m \sin^2 2\theta_{23} \sin^2 \left[ 1.27 \frac{(\Delta m_{21}^2 + A - (\Delta m_{31}^2)^m)}{2} \right] \frac{L}{E} \\
- \sin^4 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 \left[ 1.27 (\Delta m_{31}^2)^m \right] \frac{L}{E} 
\]

(2.10)

\[
P_{ee}^m = 1 - \sin^2 2\theta_{13}^m \sin^2 \left[ 1.27 (\Delta m_{31}^2)^m \right] \frac{L}{E} 
\]

(2.11)

In this case, the following features are observed:

(a) The oscillation probability in matter \(P_{\mu\mu}^m\) is still guided to leading order by a dependence on \(\sin^2 \theta_{23}\). But strong resonant earth matter effects help in resolving the degeneracy since the mixing angle \(\theta_{13}\) in matter gets amplified to maximal values (close to 45°) near resonance. The combination \(\sin^2 \theta_{23} \sin^2 2\theta_{13}^m\) no longer remains invariant over opposite octants, since \(\sin^2 2\theta_{13}^m\) becomes close to 1 in both octants irrespective of the vacuum value of \(\theta_{13}\). This breaks the degeneracy of the octant with \(\theta_{13}\).

(b) The muon survival probability in matter \(P_{\mu\mu}^m\) has leading terms proportional to \(\sin^2 2\theta_{23}\), as in the vacuum case, which could give rise to the intrinsic octant degeneracy. But the strong octant-sensitive behaviour of the term \(\sin^4 \theta_{23} \sin^2 2\theta_{13}^m\) near resonance can override the degeneracy present in the \(\sin^2 2\theta_{23}\)-dependent terms.

(c) The electron survival probability \(P_{ee}^m\) is independent of \(\theta_{23}\) and hence does not contribute to the octant sensitivity.

(d) Since the \(P_{\mu e}^m\) channel is simply the CP conjugate of \(P_{\mu e}^m\), the probability level discussion in this section is applicable to \(P_{\mu e}^m\) also.

In the following discussion, we address the octant degeneracy due to \(\theta_{23}\) in the wrong octant, \(\theta_{13}\) and unknown values of \(\delta_{CP}\), at a probability level. The probability figures \([0-1]\) are drawn by solving the full three flavour propagation equation of the neutrinos in matter using PREM density profile [53]. In all these figures the left panels are for the NO\(\nu A\) peak energy and baseline (2 GeV, 812 Km), while the right panels are for a typical atmospheric neutrino energy and baseline (6 GeV, 5000 Km). The top row denotes the appearance channel \(P_{\mu e}\), while the bottom row denotes the disappearance channel \(P_{\mu\mu}\).

Figure \([0-1]\) depicts the probabilities \(P_{\mu e}\) and \(P_{\mu\mu}\) as a function of \(\sin^2 2\theta_{13}\) for \(\theta_{23}^{tr} = 39^\circ\) and \(\theta_{23}^{wrong} = 51^\circ\). The bands show the probability range in each case when \(\delta_{CP}\) is varied over its full range (0 to 2\(\pi\)). For a given fixed value of \(\sin^2 2\theta_{13}\), the distinction between \(\theta_{23}\) in the two octants can be gauged from the separation of the two bands along the relevant vertical line. The left panels show that for \(P_{\mu e}\), the \(\delta_{CP}\) bands overlap and the two hierarchies cannot be distinguished till nearly \(\sin^2 2\theta_{13} = 0.1\). The inset in the upper left panel shows the region of separation of the bands near \(\sin^2 2\theta_{13} = 0.1\) in detail. Hence the knowledge of the parameter \(\theta_{13}\) up to its current level of precision becomes crucial, since a \(\theta_{13}\) range including lower values would wash out the octant sensitivity derivable from such experiments due to the combined degeneracy with \(\theta_{13}\) and \(\delta_{CP}\). For \(P_{\mu\mu}\), the intrinsic degeneracy predominates and the effect of \(\delta_{CP}\) variation is insignificant.
Figure 1: Behaviour of the muon survival and oscillation probabilities as a function of $\sin^2 2\theta_{13}$ showing the $\theta_{23}$ octant degeneracy and its breaking. The left panels are for the NOvA peak energy and baseline (2 GeV, 812 Km), while the right panels are for a typical atmospheric neutrino energy and baseline (6 GeV, 5000 Km). The top row denotes the appearance channels $P_{\mu\mu}$ and $P_{e\mu}$, while the bottom row denotes the disappearance channel $P_{\mu\mu}$. The values of oscillation parameters chosen are $\theta^\text{true}_{23} = 39^\circ$, $\theta^\text{wrong}_{23} = 51^\circ$. The bands denote a variation over the full range of the phase $\delta_{CP}$. The inset shows the region of separation of the bands near $\sin^2 2\theta_{13} = 0.1$.

For the 5000 km baseline, due to the resonant matter effects breaking the octant degeneracy at the leading order, both $P_{\mu\mu}$ and $P_{\mu\mu}$ show a wider separation between the opposite-octant bands, even for small values of $\theta_{13}$. This is due to the $\sin^2 \theta_{23} \sin^2 2\theta^m_{13} (\sin^4 \theta_{23} \sin^2 \theta^m_{13})$ term in $P_{\mu\mu}$ ($P_{\mu\mu}$). The $\delta_{CP}$ bands in the right-hand panels are much wider because of the enhancement of the subleading terms due to matter effects. However, the enhancement is more for the leading order term which alleviates the degeneracy with $\delta_{CP}$.

Figure 2 shows the probabilities $P_{\mu\mu}$ and $P_{\mu\mu}$ as a function of $\sin^2 2\theta_{13}$. We have held
Figure 2: Same as Figure 1 with a fixed true and test $\delta_{CP} = 0$ and true $\theta_{23}^{tr} = 39^\circ$, with the band denoting a variation over the full allowed range of $\theta_{23}^{wrong} = 45^\circ$ to $54^\circ$ in the wrong octant.

fixed, true and test $\delta_{CP} = 0$ and true $\theta_{23}^{tr} = 39^\circ$, with the band denoting a variation over the full allowed range of $\theta_{23}^{wrong} = 45^\circ$ to $54^\circ$ in the wrong octant. Thus this figure reveals the effect of the uncertainty in the measurement of $\theta_{23}$ in the determination of octant, for a given value of $\sin^2 2\theta_{13}$. For the NO$\nu$A baseline, the survival probability $P_{\mu\mu}$ shows an overlap of the test $\theta_{23}$ band with the true curve, while in the probability $P_{\mu e}$ there is a small separation. Figures 1 and 2 thus indicate that for the NO$\nu$A baseline, the octant sensitivity from $P_{\mu e}$ is more affected by the uncertainty in $\delta_{CP}$ and less by the test $\theta_{23}$ variation, while for $P_{\mu\mu}$ the opposite is true. The plots for the atmospheric neutrino baseline show a clear breaking of the octant degeneracy in both $P_{\mu e}$ and $P_{\mu\mu}$ even for small values of $\theta_{13}$, indicating that the octant sensitivity from the atmospheric neutrino signal is stable against the variation of both $\delta_{CP}$ and the test value of $\theta_{23}$, even for small values of $\theta_{13}$. 
Having discussed octant sensitivity in the context of fixed $\theta_{13}$, we now come to the question of uncertainty in this parameter. The current reactor measurements of $\sin^2 2\theta_{13}$ have measured this parameter with a precision of 0.01 and hence the effect of $\theta_{13}$ uncertainty on octant degeneracy is largely reduced. In Figure 3, the energy spectra of the probabilities $P_{\mu e}$ and $P_{\mu\mu}$ are plotted. The figure shows the variation in the probabilities in each case when $\sin^2 2\theta_{13}$ is varied over three values – $\sin^2 2\theta_{13} = 0.07$, 0.1, and 0.13 covering the current allowed range, and for two illustrative values of $\theta_{23} – 39^\circ$ and $51^\circ$ in opposite octants. The value of $\delta_{CP}$ is fixed to be 0. The figure indicates that the separation of the LO and HO curves and hence the octant sensitivity depends on the value of $\sin^2 2\theta_{13}$ in opposite ways depending on whether the true $\theta_{23}$ value lies in the higher or lower octant.
Figure 4: Energy spectra of the probabilities $P_{\mu e}$ and $P_{\mu\mu}$ for the NO$\nu$A baseline (left panels) and for a sample atmospheric neutrino baseline 5000 Km (right panels). The figure shows the variation in the probabilities in each case when $\theta_{23}^{\text{wrong}}$ is varied over the entire allowed range in the wrong octant, as well as varying the test $\delta_{CP}$, fixing the true values $\theta_{23}^{tr} = 39^o$ and $\delta_{CP}^{tr} = 0$ in the solid curve. The spread due to variation of both test parameters is denoted by the blue band. The black band shows the variation with test $\delta_{CP}$ for a fixed $\theta_{23}^{tr} = 51^o$.

In $P_{\mu e}$, for $\theta_{23}^{tr} = 39^o$, lower values of true $\sin^2 \theta_{13}$ can give a higher octant sensitivity since they are more separated from the band of variation of the probability over the whole range of test $\sin^2 2\theta_{13}$, and as true $\sin^2 2\theta_{13}$ increases the degeneracy with the wrong $\theta_{23}$ band becomes more prominent. For $\theta_{23}^{tr} = 51^o$, the opposite is true, i.e. higher values of $\sin^2 2\theta_{13}^{tr}$ have a better separation with the wrong $\theta_{23}$ band. For example, for the NO$\nu$A baseline, if $\theta_{23}^{tr} = 39^o$, the $P_{\mu e}$ curve for $\sin^2 2\theta_{13}^{tr} = 0.07$ is well-separated from the band for $\theta_{23}^{wrong} = 51$, while the curve for $\sin^2 2\theta_{13}^{tr} = 0.1$ is seen to be just separated from it, and the $\sin^2 2\theta_{13}^{tr} = 0.13$ curve lies entirely within the band of test $\sin^2 2\theta_{13}$ variation with
the wrong octant. But if $\theta_{23}^{tr} = 51^\circ$, the $\sin^2 2\theta_{13}^{tr} = 0.07$ curve suffers from the degeneracy while the curves for $\sin^2 2\theta_{13}^{tr} = 0.1$ and upwards lie clearly outside the $\theta_{23}^{wrong}$ band. For $P_{\mu\mu}$, the effect of $\theta_{13}$ on the separation between the opposite octant bands is less, since the behaviour is governed by the intrinsic octant degeneracy.

For the 5000 km baseline, due to strong matter effects, the separation of the true and wrong $\theta_{23}$ bands is much better, and only the highest (lowest) values of $\sin^2 2\theta_{13}^{tr}$ suffer from a degeneracy with the wrong octant for $P_{\mu e}$ if the true octant is lower (higher). For $P_{\mu\mu}$, the behaviour is reversed.

Figure 4 again depicts the energy spectra of the probabilities $P_{\mu e}$ and $P_{\mu\mu}$. This figure shows the variation in the probabilities in each case when $\theta_{23}^{wrong}$ is varied over the entire allowed range in the wrong octant, as well as varying the test $\delta_{CP}$, fixing $\theta_{23}^{tr} = 39^\circ$ and $\delta_{CP}^{tr} = 0$. The solid black band denotes the variation with test $\delta_{CP}$ for a fixed $\theta_{23}^{wrong} = 51^\circ$.

For the NO$\nu$A baseline, the test probability bands show an almost complete overlap with the $\theta_{23}^{tr} = 39^\circ, \delta_{CP}^{tr} = 0$ curve. However, the octant sensitivity may still be retained due to spectral information. So there always exist specific energy regions and bins from which the octant sensitivity can be derived.

At 5000 km, the minimum separation between octants does not occur at or near $\theta_{23}^{wrong} = 90^\circ - \theta_{23}^{tr} = 51^\circ$ as can be seen from the solid black shaded region. The edge of the striped blue band corresponding to some other value of $\theta_{23}$ is closest to the ‘true’ curve and well-separated from it. This shows that there is octant sensitivity for this baseline even after including the uncertainty in $\delta_{CP}$ and $\theta_{23}$. The $\theta_{23}^{wrong}$ for which the minimum separation is likely to occur will be further discussed in the context of Figure 9.

### 3. Analysis and Results

In this section we present the results of our analysis for T2K, NO$\nu$A and atmospheric experiments. We also give results for octant sensitivity when the results from both type of experiments are combined.

In Refs. [54, 55], it has been shown that the atmospheric parameters $\Delta m^2_{atm}$ and $\theta_{\mu\mu}$ measured in MINOS are related to the oscillation parameters in nature, $\Delta m^2_{31}$ and $\theta_{23}$ using the following non-trivial transformations:

$$\sin \theta_{23} = \frac{\sin \theta_{\mu\mu}}{\cos \theta_{13}},$$

$$\Delta m^2_{31} = \Delta m^2_{atm} + (\cos^2 \theta_{12} - \cos \delta \sin \theta_{13} \sin 2\theta_{12} \tan \theta_{23}) \Delta m^2_{21}. \quad (3.2)$$

These transformations become significant in light of the moderately large measured value of $\theta_{13}$. Therefore, in order to avoid getting an erroneous estimate of octant sensitivity, we take these ‘corrected’ definitions into account. Thus, in calculating oscillation probabilities, we use the corrected parameters $\Delta m^2_{31}$ and $\theta_{23}$, after allocating the measured values to $\Delta m^2_{atm}$ and $\theta_{\mu\mu}$.

Our analysis procedure consists of simulating the experimental data for some specific values of the known oscillation parameters known as the ‘true values’. The experimental
data is generated for a fixed hierarchy and for the following fixed values of the parameters:

\[
\begin{align*}
(\Delta m_{21}^2)^{tr} &= 7.6 \times 10^{-5} \text{ eV}^2 \\
(\sin^2 \theta_{12})^{tr} &= 0.31 \\
(\sin^2 2\theta_{13})^{tr} &= 0.1 \\
(\Delta m_{atm}^2)^{tr} &= 2.4 \times 10^{-3} \text{ eV}^2,
\end{align*}
\]  

(3.3)

and specific values of \(\theta_{\mu\mu}^{tr}\) and \(\delta_{CP}^{tr}\). In the theoretical predictions which are fitted to the simulated experimental data, the ‘test’ parameters are marginalized over the following ranges:

\[
\begin{align*}
\delta_{CP} &\in [0, 2\pi) \\
\theta_{\mu\mu} &\in \begin{cases} 
(35^\circ, 45^\circ) &\text{(true higher octant)} \\
(45^\circ, 55^\circ) &\text{(true lower octant)} 
\end{cases} \\
\sin^2 2\theta_{13} &\in (0.07, 0.13).
\end{align*}
\]  

(3.4)

\(\Delta m_{21}^2\) and \(\sin^2 \theta_{12}\) are fixed to their true values since their effect is negligible. Also, after verifying that the effect of a marginalization over \(\Delta m_{31}^2\) is minimal, we have fixed \(\Delta m_{atm}^2\) to its true value for computational convenience.

In our calculation, the hierarchy is assumed to be known in all the atmospheric neutrino experiments, since the time-scale involved would ensure that the hierarchy is determined before any significant octant sensitivity is achievable. For NO\(\nu\)A/T2K, results are given both with and without prior knowledge of hierarchy, i.e. marginalizing over the test hierarchy.

Priors are taken in terms of the measured quantities \(\sin^2 2\theta_{13}\) and \(\sin^2 2\theta_{\mu\mu}\) as follows:

\[
\chi^2_{\text{prior}} = \left( \frac{\sin^2 2\theta_{\mu\mu}^{true} - \sin^2 2\theta_{\mu\mu}}{\sigma(\sin^2 2\theta_{\mu\mu})} \right)^2 + \left( \frac{\sin^2 2\theta_{13}^{true} - \sin^2 2\theta_{13}}{\sigma(\sin^2 2\theta_{13})} \right)^2
\]  

(3.5)

with the 1\(\sigma\) error ranges as \(\sigma_{\sin^2 2\theta_{\mu\mu}} = 5\%\) and \(\sigma_{\sin^2 2\theta_{13}} = 0.01\) unless otherwise stated. The latter is the error on \(\theta_{13}\) quoted recently by Double Chooz, Daya Bay and RENO [1–3].

3.1 NO\(\nu\)A and T2K

We simulate the current generation of long baseline experiments NO\(\nu\)A and T2K, using the GLoBES package [56,57] and its associated data files [58,59]. For T2K, we assume a 3 year run with neutrinos alone, running with beam power of 0.77 MW throughout. (We choose this low running time for T2K to compensate for the fact that their beam power will be increased to its proposed value over a period of a few years [60].) The energy resolutions and backgrounds are taken from Refs. [61–65]. The detector mass is taken to be 22.5 kT. For NO\(\nu\)A, we consider the set-up as described in Refs. [66,67], which is re-optimized for the moderately large measured value of \(\theta_{13}\). The 14 kT detector receives a neutrino and antineutrino beam for 3 years each, from the NuMI beam.
Figure 5: Marginalized octant sensitivity from a combination of NOνA and T2K, for the case of normal and inverted mass hierarchy. The test hierarchy has been fixed to be the same as the true hierarchy in this case. In this figure priors have been added.

Figure 6: Marginalized octant sensitivity from a combination of NOνA and T2K, for the case of normal and inverted mass hierarchy. The test hierarchy has been left free in this case. In this figure priors have been added.

Figures 5 and 6 depict the octant sensitivity from a combination of NOνA and T2K, marginalized over the test parameters and with added priors. The $\chi^2$ is defined as

$$\chi^2_{\text{tot}}^{\text{prior}} = \min(\chi^2_{\text{NOνA}} + \chi^2_{\text{T2K}} + \chi^2_{\text{prior}})$$

Here (and elsewhere), 'min' denotes a marginalization over the test parameters as outlined above. In Figure 5, the neutrino mass hierarchy is assumed to be known in each case, and in Figure 6 the mass hierarchy is taken to be unknown and therefore marginalized over.
Table 2: Marginalized octant sensitivity from NOνA, T2K and a combination of the two experiments for \(\delta_{CP} = 0\) and \(\sin^2 2\theta_{13} = 0\) without added priors (first three columns) and with added priors (fourth and fifth columns). 'prior' denotes the present prior of \(\sigma_{\sin^2 2\theta_{13}} = 0.01\) and 'prior_n' denotes a projected prior of \(\sigma_{\sin^2 2\theta_{13}} = 0.005\). Here normal hierarchy (inverted hierarchy) is assumed.

| \(\theta_{tr}^{23}\) | \(\chi^2 (\text{NO}\nu\text{A})\) | \(\chi^2 (\text{T2K})\) | \(\chi^2 (\text{NO}\nu\text{A} + \text{T2K})\) | \(\chi^2 (\text{NO}\nu\text{A} + \text{T2K} + \text{prior})\) | \(\chi^2 (\text{NO}\nu\text{A} + \text{T2K} + \text{prior}_n)\) |
|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 36             | 1.5 (4.1)       | 0.0 (0.8)       | 1.7 (5.8)       | 9.6 (14.8)      | 17.5 (26.7)     |
| 39             | 0.2 (0.4)       | 0.0 (0.1)       | 0.3 (0.6)       | 3.9 (7.3)       | 6.3 (12.0)      |
| 41             | 0.1 (0.1)       | 0.0 (0.0)       | 0.1 (0.1)       | 1.9 (3.6)       | 2.4 (5.4)       |
| 43             | 0.1 (0.1)       | 0.0 (0.0)       | 0.1 (0.1)       | 1.0 (0.8)       | 1.3 (1.1)       |
| 47             | 0.0 (0.1)       | 0.0 (0.0)       | 0.1 (0.1)       | 0.8 (1.0)       | 1.0 (1.3)       |
| 49             | 0.2 (0.3)       | 0.0 (0.0)       | 0.3 (0.5)       | 3.8 (2.3)       | 5.4 (3.1)       |
| 51             | 2.3 (1.1)       | 0.2 (0.1)       | 2.9 (1.5)       | 8.5 (6.0)       | 13.0 (8.2)      |
| 54             | 11.0 (6.8)      | 2.8 (0.5)       | 14.7 (9.4)      | 21.5 (16.1)     | 32.4 (22.6)     |

These plots are done for four specific true values of the CP phase, \(\delta_{CP} = 0, \pi/2, \pi\) and \(-\pi/2\).

Table 2 lists the values of the octant sensitivity from NO\nu\text{A} and T2K individually and from the combination for a specific value \(\delta_{CP} = 0\), and a set of true \(\theta_{23}\) values from both octants, without and with added priors, to depict the relative contributions from the two experiments. Figure 7 shows how the octant \(\chi^2\) for \(\theta_{tr}^{23} = 36^\circ\) varies with the test values of \(\theta_{23}\) and \(\delta_{CP}\) for the appearance and disappearance channels of NO\nu\text{A} and for T2K. In the figure, \(\theta_{13}\) and \(\delta_{CP}\) are marginalized in the left panel and \(\theta_{13}\) and \(\theta_{23}\) are marginalized in the right panel. Normal hierarchy is assumed in the figure. The following features are observed in the sensitivity behaviour of the two experiments and the appearance/disappearance channels in each case:

1. Figure 7 (left panel) shows that the \(\chi^2\) minima for the NO\nu\text{A} disappearance channel occur near \(\theta_{23}^{test} = \pi/2 - \theta_{tr}^{23}\), because of the predominant dependence on \(\sin^2 2\theta_{23}\), while for the appearance channel the minima occur near \(\theta_{23}^{test} = 45^\circ\), because of the \(\sin^2 \theta_{23}\) dependence. In the combination of NO\nu\text{A} appearance and disappearance channels (red solid curve in the figure), the \(\chi^2\) minima are near \(\theta_{23}^{test} = \pi/2 - \theta_{tr}^{23}\), following the behaviour of the disappearance channel \(\chi^2\), but the values are enhanced due to the contribution from the appearance \(\chi^2\) values at that point.

2. The \(\chi^2\) values are asymmetric across the lower and higher octants, as can be seen in Table 2. For example, \(\chi^2_{min}\) is about 1.5 for NO\nu\text{A} (appearance + disappearance) at \(\theta_{23}^{tr} = 36^\circ\) with \(\sin^2 2\theta_{13} = 0.1, \delta_{CP}^{tr} = 0\). For \(\theta_{23}^{tr} = 54^\circ\), \(\chi^2_{min}\) goes up to about 11 for NO\nu\text{A} (app + disapp) for the same value of \(\delta_{CP}^{tr}\).

3. The \(\chi^2\) values for NO\nu\text{A} and T2K strongly depends on the true value of \(\delta_{CP}\). We find for \(\delta_{CP}^{tr} = \pi/2\), \(\chi^2_{min}\) for NO\nu\text{A} (app + disapp) is 5.6 for \(\theta_{23}^{tr} = 36^\circ\) and 7.5
Figure 7: Octant sensitivity from NOνA appearance and disappearance channels and their combination and from T2K as a function of test $\theta_{23}$ (left panel) and test $\delta_{CP}$ (right panel). $\theta_{13}$ and $\delta_{CP}$ are marginalized in the left panel and $\theta_{13}$ and $\theta_{23}$ are marginalized in the right panel.

for $\theta^{tr}_{23} = 54^\circ$. This point will be illustrated in more detail later in the context of Figures [12 and 13].

4. From Figure 7 (right panel), we see that the $\chi^2$ for the disappearance channel is weakly dependent on the test value of $\delta_{CP}$, and remains small for all values of test $\delta_{CP}$. On the other hand, the $\chi^2$ for the appearance channel has a strong dependence on test $\delta_{CP}$, which is consistent with the behaviour of the respective probabilities seen in Section 2.2. In terms of test $\delta_{CP}$, the minima of the combination occur at test $\delta_{CP}$ values close to those for the appearance channel, which is the principal contributor to the $\delta_{CP}$ dependence of the sensitivity. The increased values of the combined NOνA $\chi^2$ minima at each test $\delta_{CP}$, can be attributed to the tension between the two channels with respect to $\theta^{test}_{23}$. Thus marginalization over $\delta_{CP}$ reduces the sensitivity to some extent because of the effect of $\delta_{CP}$ uncertainty in the appearance channel.

5. The $\chi^2$ minima for both appearance and disappearance channels are very low for all values of $\theta^{tr}_{23}$, indicating negligible octant sensitivity from the channels separately. This is apparent from the left panel of Figure 7, where the minimum values of $\chi^2$ for individual channels are quite small when marginalized over the entire $\theta_{23}$ range.

6. There is no major tension between NOνA appearance + disappearance and T2K appearance + disappearance. The $\chi^2$ minima occur at the same values of test $\sin^2 2\theta_{13}$ and test $\theta_{23}$, and for somewhat displaced values of test $\delta_{CP}$. This is because both T2K and NOνA roughly follow the octant behaviour dictated by the vacuum probabilities with subleading contributions of the $\delta_{CP}$-dependent terms. The slightly increased matter effect contribution for NOνA does not seem to have a significant effect. The
combined NOνA+T2K $\chi^2$ values are nearly equal to the sum of the corresponding individual NOνA and T2K values, as seen in Table 2. For example, $\chi^2_{min}$ for T2K appearance + disappearance is about 3 for $\theta_{23}^{tr} = 54^o$. The corresponding NOνA value is about 11. For NOνA+T2K, $\chi^2_{min}$ is about 15 for the same $\theta_{23}^{tr}$.

7. The addition of priors to NOνA+T2K enhances the sensitivity drastically, making it (for example) $(\chi^2_{min})_{\text{prior}} = 21$ for $\theta_{23}^{tr} = 54^o$ with the present value of the $\theta_{13}$ prior. This underscores the importance of precision measurement of $\theta_{13}$ on octant sensitivity. With more precise measurements of $\sin^2 2\theta_{13}$ expected from the reactor experiments, the octant sensitivity is expected to get better. If we take a stronger projected prior of $\sigma_{\sin^2 2\theta_{13}} = 0.005$ instead of the present value of $\sigma_{\sin^2 2\theta_{13}} = 0.01$, the results improve further, as seen in column 5 of the table, where prior$_n$ denotes the projected prior. Here, for example, the $\chi^2$ goes up from 3.9 to 6.3 for $\theta_{23}^{tr} = 39^o$, $\delta_{CP}^{tr} = 0$ and normal hierarchy. In the limit of infinite precision, i.e. if $\theta_{13}$ is held fixed, the $\chi^2$ goes up further to 7.7 for the same case.

8. We see from the Figures 5 and 6 that the $\chi^2$ is asymmetric between the lower and higher octants. The nature of this asymmetry depends on the true $\delta_{CP}$ value. For instance for $\delta_{CP}^{tr} = 0$, the $\chi^2$ values on the LO side are much lower than on the HO side, but this is reversed for other $\delta_{CP}^{tr}$ values. Note that the addition of priors makes the asymmetry less pronounced since the prior contributions for the lower and higher octants are roughly symmetrical. This can be seen by comparing the with and without prior values of $\chi^2$ in Table 4.

### 3.2 Atmospheric neutrinos

For our study of octant sensitivity in atmospheric neutrino experiments, we look at the following set-ups:

1. A large magnetized iron detector with an exposure of 500 kT yr, capable of detecting muon events with charge identification, using the following neutrino energy and angular resolution: $\sigma_{E_\nu} = 0.1\sqrt{E_\nu}$, $\sigma_{\theta_\nu} = 10^o$.

   Note that such a detector will be constructed by the India-based Neutrino Observatory (INO) collaboration [25]. The energy and angular resolutions of muons are available from INO simulation code [68, 69]. But the work on reconstruction of neutrino energy and angle requiring the resolutions for both muons and hadrons are in progress. In our neutrino analysis therefore we use fixed resolutions in terms of neutrino energy and angle as described above. Determination of the octant sensitivity of INO using the resolutions obtained from INO simulations is in progress.

2. A LArTPC with an exposure of 500 kT yr (unless otherwise stated) capable of detecting muon and electron events. No charge identification is assumed here. The angular resolutions are taken to be [43]:

   *In this table and some subsequent figures and tables, the $\chi^2$ values without priors are given only for comparison of the relative weightage of the contributions from different channels and different experiments.
\[ \sigma_{\theta_{13}} = 2.8^\circ, \]
\[ \sigma_{\theta_{23}} = 3.2^\circ. \] 

(3.7)

For the neutrino energy resolutions, we use the estimated value \( \sigma_{E_{\nu}} = 0.1\sqrt{E_{\nu}} \). A LArTPC has a very good angular resolution since ionization tracks can be transported undistorted over distances of several metres in highly purified LAr, allowing for excellent direction reconstruction by recording several projective views of the same event using wire planes with different orientations [28].

3.3 Octant sensitivity using atmospheric muon events in a magnetized iron detector

\[ \chi^2_{\text{tot prior}} = \min(\chi^2_{\text{Atm}} + \chi^2_{\text{prior}}), \]  

(3.8)

where

\[ \chi^2_{\text{Atm}} = \chi^2_{\mu} + \chi^2_{\bar{\mu}} \]

For the atmospheric analysis, the flux and detector systematic uncertainties are included using the method of \( \chi^2 \) pulls as outlined in [52, 70–72].

It is well known that the possibility of magnetization gives the iron detectors an excellent sensitivity to hierarchy [52, 68, 73–76]. Therefore it is plausible to assume that hierarchy would be determined before octant in these detectors. Thus in our analysis of atmospheric neutrinos, we assume the neutrino mass hierarchy fixed to be either normal or inverted,
and do not marginalize over the hierarchy. This is tantamount to the assumption that the wrong-octant-wrong hierarchy solutions are excluded. In Figure 8, we present the octant sensitivity of a magnetized iron calorimeter detector with an exposure of 500 kT yr. The second column of Table 4 displays the $\chi^2$ values for different values of $\theta_{23}$ in the LO. The $\chi^2$ in this case is symmetric about $\pi/4$, and therefore the values in the higher octant are similar. Assuming an inverted hierarchy gives worse results because in this case, matter resonance and hence octant sensitivity occurs in the antineutrino component of the event spectrum, for which the flux and detection cross-sections are lower. A $2\sigma$ sensitivity is obtained at $\theta_{23} = 39^\circ$ if NH is the true hierarchy.

3.4 Octant sensitivity using atmospheric events in a LArTPC

A LArTPC is sensitive to both muon and electron type events and one can study the interplay of both type of events in octant sensitivity using atmospheric neutrinos. We give the results separately for the atmospheric muon and electron signals as well as for the combined analysis.

In the following analysis we assume that the hierarchy will be determined by the combination of reactor, long baseline and INO experiments [68] before a LArTPC can give an octant measurement. Therefore we do not marginalize over hierarchy in the wrong octant.

The muon and electron event spectra are related to the oscillation probabilities as follows:

$$N_\mu \sim (\phi_\mu P_\mu \mu + \phi_e P_\mu e) \times (\text{CC cross - section, exposure, efficiency})$$

$$N_e \sim (\phi_\mu P_\mu e + \phi_e P_e e) \times (\text{CC cross - section, exposure, efficiency})$$

(3.9)

The antineutrino event rates are given by the same expressions with the flux and probabilities replaced by their antineutrino counterparts.

The $\chi^2$ in this case is given by

$$\chi^2_{\text{tot}}^{\text{prior}} = \min(\chi^2_{\text{Atm}} + \chi^2_{\text{prior}}),$$

where,

$$\chi^2_{\text{Atm}} = \chi^2_{\mu + \bar{\mu}} + \chi^2_{e + \bar{e}}$$

The atmospheric muon flux is approximately twice that of the electron flux. Hence the behaviour of the muon events is dictated by the muon survival probability and to a lesser extent by the $P_{e\mu}$ oscillation probability. The sensitivity derivable from muon events is somewhat compromised by the fact that $P_{e\mu}$ and $P_{\mu\mu}$ shift in opposite directions for a transition from one octant to another, as observed in Figure 3. On the other hand, for electron events the only source of the octant sensitivity is from $P_{\mu e}$, since the electron survival probability is independent of $\theta_{23}$. Therefore both muon and electron events individually give comparable values of octant sensitivity. The sensitivity from muon events is more strongly dependent on the value of $\theta_{\text{tr}}$, and is therefore higher than the sensitivity from electron events for values further away from maximal, getting closer to the latter as
\[ \theta_{23}^{tr} \] approaches 45°. This can be seen from Table 3 where we list the values of octant sensitivity from atmospheric muon and electron events in a LArTPC with an exposure of 500 kT yr, marginalized over the allowed ranges of the oscillation parameters. The neutrino mass hierarchy is fixed to be either normal or inverted. The results presented are for true values of \( \theta_{23} \) lying in the lower octant. The behaviour for \( \theta_{23}^{tr} \) in the upper octant is similar and symmetrical about \( \theta_{23}^{tr} = 45° \).

Figure 9 shows the behaviour of the octant sensitivity with the atmospheric neutrino signal in a LArTPC for a specific true value \( (\theta_{23})^{tr} = 39° \), as a function of the test values of \( \theta_{23} \), for both muon (left panel) and electron (right panel) events. For the latter the only octant-sensitive contribution is from \( P_{\mu e} \), which does not suffer from the intrinsic octant degeneracy at the leading order and the octant sensitivity is due to the \( \sin^2 \theta_{23} \times \sin^2 \theta_{13} \) term. Since \( \sin^2 \theta_{13}^{\nu} \) becomes close to 1 near matter resonance, the octant sensitivity increases proportionally with the test value of \( \sin^2 \theta_{23} \). Hence the minimum sensitivity is seen in the region near \( \theta_{23}^{test} \sim 45° \) in the wrong octant. The solid red curve in the left panel is for \( \sin^2 \theta_{13} = 0 \) for which the contribution comes from \( P_{\mu \mu} \) only. The curve exhibits no octant sensitivity due to the intrinsic degeneracy coming from the \( \sin^2 \theta_{23} \) term in \( P_{\mu \mu} \). This degeneracy is broken for higher values of \( \sin^2 \theta_{13} \) where both \( P_{\mu \mu} \) and \( P_{e \mu} \) can contribute. As a combined effect of all these factors the octant sensitivity does not have a clean proportionality with \( \theta_{23}^{test} \), but can have a minimum anywhere within the range \( 45° \ < \theta_{23}^{test} \ < \ (90° \ - \ \theta_{23}^{tr}) \) depending on the relative weightage of the two terms for specific energies, baselines and values of the neutrino parameters. This is reflected by the blue dot-dashed curve in the left panel.

We now study whether combining muon and electron events will give an enhanced octant sensitivity compared to that from the muon and electron event spectra individually. The last column of Table 3 lists the \( \chi^2_{\mu + e} \) values for NH (and IH). These values are obtained by marginalizing over all test parameters, but without adding priors. In Figure 10 the

| \( \theta_{23}^{tr} \) | \( \chi^2_{\mu} \) \( \text{NH(IH)} \) | \( \chi^2_{e} \) \( \text{NH(IH)} \) | \( \chi^2_{\mu + e} \) \( \text{NH(IH)} \) |
|-----------------|-----------------|-----------------|-----------------|
| 36              | 8.5 (1.5)       | 3.5 (1.4)       | 15.4 (5.8)      |
| 37              | 6.7 (1.3)       | 2.8 (1.1)       | 11.5 (4.8)      |
| 38              | 4.6 (1.0)       | 2.5 (0.8)       | 9.0 (3.7)       |
| 39              | 3.0 (0.7)       | 1.7 (0.6)       | 7.1 (2.8)       |
| 40              | 1.9 (0.6)       | 1.4 (0.5)       | 4.8 (1.7)       |
| 41              | 1.0 (0.3)       | 1.0 (0.3)       | 3.0 (0.9)       |
| 42              | 0.6 (0.2)       | 0.6 (0.2)       | 1.9 (0.5)       |
| 43              | 0.3 (0.1)       | 0.4 (0.1)       | 1.0 (0.3)       |
| 44              | 0.1 (0.1)       | 0.2 (0.1)       | 0.4 (0.2)       |

Table 3: Marginalized octant sensitivity from atmospheric muon, electron and muon + electron events with a LArTPC (500 kT yr exposure), for the case of normal (inverted) mass hierarchy, with \( \sin^2 2\theta_{13}^{\nu} = 0.1 \) and \( \delta_{CP}^{tr} = 0 \). Priors have not been included here.
Figure 9: Fixed-parameter octant sensitivity from atmospheric neutrinos with a LArTPC (500 kT yr exposure) as a function of test $\theta_{23}$ for $(\sin^2 2\theta_{13})_{tr} = 0.1$ and $(\theta_{23})_{tr} = 39^\circ$. The left panel is for muon events and the right panel is for electron events.

sensitivity is presented with the inclusion of priors. The following points may be noted from the listed results:

1. The values of the octant sensitivity that can be derived from the atmospheric muon and electron events separately are relatively low. A combined analysis of muon and electron events gives improved sensitivities which are better than the sum of the marginalized $\chi^2$ sensitivities from the two kinds of signals. This is because the muon and electron event spectra behave differently as functions of $\theta_{23}$ and hence their $\chi^2$ minima occur at different parameter values. Utilizing this fact, a marginalization over the sum of $\chi^2$ values gives an improved result compared to the individual contributions. The enhancement in the sensitivity due to this synergy can be anywhere between 20% and 50%, depending on the true value of $\theta_{23}$, as can be seen in Table 3.

2. The addition of prior information further improves the sensitivity by about 20 - 40%. The most significant contribution comes from the reduced error range of the parameter $\theta_{13}$ from recent reactor results. For an assumed normal hierarchy, a 3$\sigma$ signal of the octant may be achieved for $\theta_{23}^{tr} = 39^\circ (51^\circ)$ for a true lower octant (higher octant) with this exposure, as seen in Figure 10.

3. Assuming an inverted hierarchy gives worse results because in this case, matter resonance and hence octant sensitivity occurs in the antineutrino component of the event spectrum, for which the detection cross-sections are lower.

In Figure 11, the marginalized octant sensitivity from a LArTPC is plotted as a function of detector exposure for two true values of $\theta_{23}$. The figure shows that it is possible to
Figure 10: Marginalized octant sensitivity from both muon and electron events with a LArTPC (500 kT yr exposure), for the case of normal and inverted mass hierarchy. In this figure priors have been added.

Figure 11: Octant sensitivity for a non-magnetized LArTPC as a function of exposure. $\chi^2$ has been plotted for normal hierarchy for two values of $\theta_{23}$(true) – 36° and 39°.

achieve a 2$\sigma$ sensitivity for $\theta_{23}^{tr} = 39^\circ$ and a 3$\sigma$ sensitivity for $\theta_{23}^{tr} = 36^\circ$ from a LArTPC alone for an exposure as low as 120 kT yr, in the case of a normal hierarchy.

3.5 Effect of magnetization

Figure [10] shows the values of octant sensitivity derivable from a LArTPC with and without magnetization. The magnetization offers the possibility of charge identification. In the case of a LArTPC with magnetization, charge identification details are incorporated as given in
Refs. [29, 30, 43], assuming a 100% charge identification capability for muon events and a 20% charge identification capability in the energy range 1-5 GeV (none for higher energies) for electron events.

The octant sensitivity without charge identification is seen to be about 80% (60%) of that with charge identification capability with a similar exposure, in the case of a normal (inverted) mass hierarchy. This difference in behaviour is explained as follows.

If the true hierarchy is normal, neutrino events have octant sensitivity due to resonant matter effects, while antineutrino events do not. Therefore the antineutrino events for both octants are almost the same, say $k$. In the absence of charge-identification, we add these events, and

$$\chi^2 = \frac{([N^{HO} + k] - [N^{LO} + k])^2}{N^{HO} + k}.$$  

On the other hand, if we do have charge-identification, then the chisq is simply

$$\chi^2 = \frac{([N^{HO}] - [N^{LO}])^2}{N^{HO}} + \frac{([k] - [k])^2}{k}.$$  

In both cases, the numerator is same, but the denominator is more in the former case. Therefore charge-identification gives us higher sensitivity.

Note that when the hierarchy is normal, $k$ comes from the antineutrino events. Because of the flux and cross-section being small, $k$ is a relatively small number. But if the hierarchy is inverted, then $k$ will be because of neutrino events, whose higher flux and cross-section will make the denominator quite large. Therefore, the reduction in sensitivity due to loss of charge-identification will be more apparent for the inverted hierarchy. Charge identification capability has been shown to play a very crucial role in determination of mass hierarchy in such detectors [43]. However this does not seem to play such an important role for octant determination if we already assume a prior knowledge of the hierarchy. For a LArTPC without charge identification, if we assume that the hierarchy is not known then the octant sensitivity is unaffected for $|\theta_{23}^\text{atm} - 45^\circ| < 7^\circ$, i.e. for $39^\circ \leq \theta_{23}^\text{atm} \leq 51^\circ$, but for smaller (larger) values of $\theta_{23}^\text{atm}$ in the lower (higher) octant, there is a drop of 20-30% in the sensitivity. For a magnetized iron detector, we have checked that the effect of marginalizing over the hierarchy is negligible since the wrong hierarchy is excluded with a reliable confidence level for the exposure considered (500 kT yr). The same is true for a LArTPC with charge identification capability.

When we compare the octant sensitivity from a magnetized iron detector with the LArTPC results without charge identification, the iron detector sensitivities are about 40% of those with a LArTPC if the hierarchy is normal and about 50% of the LArTPC sensitivities if the hierarchy is inverted. The sensitivities from a LArTPC are higher than those from a magnetized iron detector. This is because of the possibility of realizing very high angular resolutions in the former type of detector and the significant contributions from muon as well as electron events.

3.6 Effect of $\delta_{CP}$

The dependence of the octant sensitivity on the CP phase $\delta_{CP}$ is different for atmospheric neutrino experiments and the long baseline experiments considered. Because of the strong
earth matter effects over a large range of atmospheric neutrino baselines, the behaviour of the corresponding oscillation probabilities is governed by the enhanced resonant features and their dependence on $\delta_{\text{CP}}$ is suppressed. On the other hand, for NO$\nu$A/T2K baselines, since the matter effects are smaller, $\delta_{\text{CP}}$ plays a greater role in the probability behaviour and hence in the octant sensitivity, because of the degeneracy of the octant with $\theta_{13}$ and $\delta_{\text{CP}}$ as explained previously.

![Figure 12: Marginalized octant sensitivity with priors as a function of true $\delta_{\text{CP}}$ from NO$\nu$A, T2K, NO$\nu$A + T2K and for an atmospheric neutrino experiment with a LArTPC for the case of normal and inverted mass hierarchy, for 2 values of $\theta_{23}^{\text{tr}}$ in the lower and higher octants.](image)

These features are reflected in Figure 12, in which the marginalized octant sensitivity for a LArTPC with atmospheric neutrinos, NO$\nu$A, T2K and their combinations are plotted as a function of the true $\delta_{\text{CP}}$ for $\theta_{23}^{\text{tr}} = 39^\circ$ (LO) and $51^\circ$ (HO) for both normal and inverted mass hierarchies. The $\delta_{\text{CP}}$ behaviour of the NO$\nu$A and T2K sensitivities are seen to be...
Figure 13: Wrong octant exclusion for NOνA+T2K in the true θ_{23} - true δ_{CP} plane. The left(right) panel is for NH(IH) as the true hierarchy. The brown(dense-dotted)/red(sparse-dotted)/gold(dashed)/green(solid) contours denote 1/2/3/4σ exclusion, respectively.

similar, and they follow an opposite behaviour with respect to δ_{CP} for θ_{23}^{tr} lying in the lower and higher octants, i.e. for θ_{23}^{tr} = 39° they are higher in the range 0° < δ_{CP}^{tr} < 180° and for θ_{23}^{tr} = 51° they are higher in the range −180° < δ_{CP}^{tr} < 0° [40].

For θ_{23}^{tr} = 39° (LO) and NH (the top left panel) the contribution from a LArTPC is higher than that from combined NOνA+T2K excepting for a narrow range δ_{CP}^{tr} = 90° to 135°. For θ_{23}^{tr} = 51° (HO) and NH (bottom left panel) the same trend is observed, except that in this case the combined NOνA+T2K has a higher sensitivity for the range δ_{CP}^{tr} = −15° to −90°.

For the top (bottom) right panels which are for θ_{23}^{tr} = 39° (51°) and IH, the NOνA + T2K as well as the standalone NOνA contribution to the sensitivity are higher than the atmospheric contribution over almost all values of true δ_{CP}. This is because the sensitivity from atmospheric neutrinos is less than half for an inverted hierarchy compared to normal hierarchy, while for NOνA and T2K the values are similar.

The T2K sensitivity is lower than NOνA in all cases, and lower than the atmospheric contribution for most values of true δ_{CP}, besides certain small ranges in the inverted hierarchy case where it is higher. The combined sensitivity from atmospheric neutrinos and NOνA + T2K follows the true δ_{CP} dependence of the NOνA + T2K sensitivity.

Finally, in Figure 13 we show the wrong octant exclusion sensitivity for NOνA+T2K in the true θ_{23} - true δ_{CP} plane. Contours are shown in this plane for 1, 2, 3 and 4σ octant exclusion. In the part of the parameter space that is enclosed within a certain contour, it is possible to exclude the wrong octant with the corresponding confidence level. As expected, the ability of the experiments to exclude the wrong octant is small around true θ_{23} ≃ 45°. As θ_{23} becomes more non-maximal, the exclusion ability becomes better. The effect of δ_{CP} on octant sensitivity for any given true value of θ_{23} can also be read from these plots. Since
the results using atmospheric neutrinos are almost independent of $\delta_{\text{CP}}$, the behaviour of the $\chi^2$ is similar to the plots in Figures 8 and 10 drawn for $\delta_{\text{CP}} = 0$.

3.7 Octant sensitivity from combined analysis of atmospheric electron and muon events with NO$\nu$A and T2K

Figure 14: Marginalized octant sensitivity from a combination of the atmospheric muon neutrino signal in a magnetized iron calorimeter detector (500 kT yr) + NO$\nu$A + T2K, for the case of normal and inverted mass hierarchy. In this figure priors have been added.

Figure 15: Marginalized octant sensitivity from a combination of the atmospheric muon + electron neutrino signal in a LArTPC (500 kT yr, without charge ID) + NO$\nu$A + T2K, for the case of normal and inverted mass hierarchy. In this figure priors have been added.
In this section we present the octant sensitivity from a combined analysis of simulated long baseline and atmospheric data. We add $\chi^2_{\text{prior}}$ to the combination to take into account the future precision measurements on $\theta_{13}$ and $\theta_{\mu\mu}$ and minimize the $(\chi^2_{\text{tot}})_{\text{prior}}$ given as,

$$(\chi^2_{\text{tot}})_{\text{prior}} = \min(\chi^2_{\text{NO}\nu A} + \chi^2_{\text{T2K}} + \chi^2_{\text{Atm}} + \chi^2_{\text{prior}})$$  \ (3.11)$$

Figure 14 shows the combined octant sensitivity using the atmospheric muon neutrino signal in a magnetized iron detector with NO$\nu$A and T2K. In Figure 15 the octant sensitivity computed from a combination of the atmospheric electron and muon neutrino signal in a LArTPC without charge identification capability with NO$\nu$A and T2K is plotted. In Table 4 we list the marginalized octant sensitivity with priors for a LArTPC (500 kT yr exposure), a magnetized iron detector (500 kT yr exposure), and a combination of each of them with NO$\nu$A + T2K, for the case of both NH and IH. All of the above results are for $\delta_{\text{CP}}^{\text{tr}} = 0$. For a magnetized iron detector, the combination with NO$\nu$A + T2K gives a 3$\sigma$ sensitivity at $\theta_{23} = 39^\circ$ for NH. A non-magnetized LArTPC + NO$\nu$A + T2K gives a 4$\sigma$ sensitivity in the same case. There is a tension between the behaviour of the NO$\nu$A/T2K octant sensitivity and the octant sensitivity from an atmospheric neutrino experiment as a function of test $\theta_{23}$, which can be seen by comparing Figures 7 and 9. For NO$\nu$A/T2K, the $\chi^2$ minima occur at or close to $\theta_{23}^{\text{test}} = \pi/2 - \theta_{23}^{\text{tr}}$, while for an atmospheric experiment the muon events may have $\chi^2$ minima anywhere between $\theta_{23}^{\text{test}} = 45^\circ$ and $\pi/2 - \theta_{23}^{\text{tr}}$ and the electron events have minima close to $\theta_{23}^{\text{test}} = 45^\circ$. This synergy leads to an enhancement of the octant sensitivity when the NO$\nu$A + T2K and atmospheric neutrino experiments are combined.

The combined $\chi^2$ values depend strongly on the true value of $\delta_{\text{CP}}$ following the behaviour of the NO$\nu$A/T2K $\chi^2$, as can be seen from the green (dot-dashed) curve in Figure 12. The LArTPC+NO$\nu$A+T2K combination is seen to reach a sensitivity of $> 4\sigma$ for

| $\theta_{23}^{\text{tr}}$ | LArTPC | Mag.Iron | LArTPC+NO$\nu$A+T2K | Mag.Iron+NO$\nu$A+T2K |
|--------------------------|--------|----------|-----------------------|------------------------|
| 36                       | 19.8 (11.5) | 8.5 (3.9) | 29.9 (21.9) | 21.1 (19.9) |
| 37                       | 15.6 (9.5) | 6.8 (3.2) | 24.6 (18.0) | 17.0 (16.3) |
| 38                       | 12.4 (7.0) | 5.3 (2.5) | 19.7 (14.3) | 13.2 (12.9) |
| 39                       | 8.9 (5.5) | 3.9 (1.9) | 15.5 (10.9) | 9.1 (9.8) |
| 40                       | 5.9 (3.5) | 2.7 (1.3) | 11.6 (8.0) | 7.1 (7.1) |
| 41                       | 4.0 (2.5) | 1.5 (0.9) | 7.7 (5.3) | 4.9 (4.8) |
| 42                       | 2.5 (1.6) | 0.6 (0.5) | 4.6 (3.0) | 3.2 (2.6) |
| 43                       | 1.5 (0.8) | 0.2 (0.2) | 1.6 (1.1) | 1.4 (1.0) |
| 44                       | 0.6 (0.6) | 0.1 (0.1) | 0.7 (0.5) | 0.2 (0.2) |

Table 4: Marginalized octant sensitivity with a LArTPC (500 kT yr exposure), a magnetized iron detector (500 kT yr exposure), and a combination of either of them with NO$\nu$A + T2K, for the case of normal (inverted) mass hierarchy, with $\sin^2 2\theta_{13}^{\text{tr}} = 0.1$ and $\delta_{\text{CP}}^{\text{tr}} = 0$. Priors have been included in this table.
Table 5: Marginalized octant sensitivity with a a combination of a LArTPC (500 kT yr exposure) and a magnetized iron detector (500 kT yr exposure), as well as a combination of both of them with NOνA + T2K, for the case of normal (inverted) mass hierarchy, with sin$^2 2\theta_{23} = 0.1$ and $\delta_{CP}^{tr} = 0$. Priors have been included in this table.

| $\theta_{23}^{tr}$ | Mag.Iron+ LArTPC | Mag.Iron+LArTPC+NOνA+T2K |
|------------------|------------------|--------------------------|
| 36               | 29.1 (15.6)      | 41.2 (27.0)              |
| 37               | 22.8 (12.9)      | 33.9 (22.3)              |
| 38               | 18.5 (9.7)       | 27.1 (17.7)              |
| 39               | 13.8 (7.6)       | 21.0 (13.5)              |
| 40               | 9.1 (5.0)        | 15.4 (9.8)               |
| 41               | 6.4 (3.5)        | 10.1 (6.5)               |
| 42               | 3.1 (2.1)        | 5.9 (3.7)                |
| 43               | 1.7 (1.0)        | 1.9 (1.4)                |
| 44               | 0.7 (0.7)        | 0.9 (0.7)                |

In this paper we have studied the possibility of determining the octant of the atmospheric mixing angle $\theta_{23}$ in the long baseline experiments T2K and NOνA as well as by atmospheric...
neutrino experiments. While the octant degeneracy conventionally refers to the indistinguishability between $\theta_{23}$ and $\pi/2 - \theta_{23}$, this can be generalized to include the whole range of allowed value of $\theta_{23}$ in the wrong octant, and we consider this generalized definition in our analysis.

We present a probability level discussion on the effect of uncertainty in $\theta_{13}$, $\delta_{CP}$ and values of $\theta_{23}$ in the wrong octant for baselines relevant to long baseline and atmospheric experiments. Below we summarize the salient features that emerge from our study at the probability level:

- For baselines where matter effects are small, the appearance channel probability $P_{\mu e}$ displays a degeneracy of the $\theta_{23}$ octant with the values of $\theta_{13}$ and $\delta_{CP}$ due to its dependence on the combination $\sin^2 \theta_{23} \sin^2 2\theta_{13}$ at leading order as well as its subleading $\delta_{CP}$ dependence. The disappearance channel probability $P_{\mu\mu}$ suffers from an intrinsic octant degeneracy between $\theta_{23}$ and $\pi/2 - \theta_{23}$ due to being a function of $\sin^2 2\theta_{23}$ at leading order.

- For NO$\nu$A/T2K baselines, there is a strong effect of the uncertainty in $\delta_{CP}$ in the appearance channel, and of the $\theta_{23}$ uncertainty in the disappearance channel. For instance, for the NO$\nu$A baseline the degeneracy between $\theta_{23} = 39^o$ and $51^o$ in $P_{\mu e}$ is lifted only for $\sin^2 2\theta_{13} \gtrsim 0.12$ if a variation over the entire range of $\delta_{CP}$ is taken into account.

- We find that after including the improved precision in $\theta_{13}$ from the recent reactor results, the octant degeneracy with respect to $\theta_{13}$ is largely reduced. Some amount of degeneracy with respect to $\theta_{13}$ still remains for higher (lower) values of $\theta_{13}$ in the allowed range for a true lower (higher) octant. Also, in general, information from the energy spectrum helps in reducing the degeneracy since for two different energies the degeneracy exists for different sets of parameter values.

- For a sample baseline of 5000 km traversed by the atmospheric neutrinos strong resonant matter effects help in lifting both the intrinsic octant degeneracy in $P_{\mu\mu}$ and the octant degeneracy with $\theta_{13}$ and $\delta_{CP}$ in $P_{\mu e}$. This is because the term $\sin^2 2\theta_{13}^m$ becomes close to 1 at or near matter resonance, and this makes the leading-order term proportional to $\sin^2 \theta_{23} (\sin^4 \theta_{23})$ in $P_{\mu e}$ ($P_{\mu\mu}$) predominate and give distinct values in the two octants irrespective of the vacuum value of $\theta_{13}$.

We perform a $\chi^2$ analysis of the octant sensitivity for T2K/NO$\nu$A and atmospheric neutrinos as well as a combined study. For atmospheric neutrinos we consider two types of detectors – magnetized iron calorimeter detectors capable of detecting muons and identifying their charge and a LArTPC (non-magnetized) which can detect both muons and electrons with superior energy and angular resolutions. For NO$\nu$A/T2K we present the results for both cases of known and unknown hierarchy while the atmospheric results are assuming hierarchy to be known. The main results are summarized below:

- The $\chi^2$ minima for the NO$\nu$A/T2K disappearance channel occur near $\theta_{23}^{best} = \pi/2 - \theta_{23}^{tr}$, because of the predominant dependence on $\sin^2 2\theta_{23}$, while for the appearance
channel the minima occur near $\theta_{23}^{test} = 45^o$, because of the $\sin^2 \theta_{23}$ dependence. This leads to a tension between the $\chi^2$ behaviour of the two spectra as a function of $\theta_{23}^{test}$, so that the combination of appearance and disappearance channels enhances octant sensitivity, but even then the $\chi^2$ is not very high. Combining the NO$\nu$A and T2K data leads to a higher octant sensitivity due to the addition of sensitivities from the two experiments. What plays a major role in enhancing the octant sensitivity in these experiments is the addition of priors, especially on $\theta_{13}$, which helps in ruling out the degenerate solutions in the wrong octant. After adding priors one can achieve a $\sim 2\sigma$ sensitivity at $\theta_{23} = 39^o$ for $\sin^2 2\theta_{13} = 0.1$ and $\delta_{CP} = 0$ for both normal and inverted hierarchies.

- A magnetized iron calorimeter gives a $2\sigma$ sensitivity to the octant for $\theta_{23} = 39^o$ and $\sin^2 2\theta_{13} = 0.1$ for a true normal hierarchy. A non-magnetized LArTPC can give a $3\sigma$ sensitivity for the same parameter values. The enhanced sensitivity of the LArTPC is due to the contribution of the electron events as well as superior energy and angular resolutions compared to an iron calorimeter detector. In a LArTPC, combining the muon and electron events gives improved sensitivities due to the different behaviour of the muon and electron event spectra with respect to $\theta_{23}$. If a LArTPC can have charge identification capability there will be a 20-40% increase in sensitivity, the enhancement being more in the case of an inverted hierarchy than for a normal hierarchy. Since we assume hierarchy to be already known, the magnetization of LArTPC (which is technically challenging) does not play a significant role. A marginalization over the hierarchy does not affect the results for a magnetized iron detector, since it already excludes the wrong hierarchy with a good confidence level for the same exposure. For a LArTPC, the results are unchanged by a hierarchy marginalization over the range $39^o \leq \theta_{23} \leq 51^o$, but suffer a drop of 20-30% for values of $\theta_{23}$ above and below this.

- Combining long baseline and atmospheric results can give a significant enhancement in the octant sensitivity because of the tension in the behaviour of the NO$\nu$A/T2K and atmospheric $\chi^2$ as functions of test $\theta_{23}$, arising due to the different energy and baseline ranges involved and the fact that strong matter effects dictate the probability behaviour for atmospheric baselines. Since the $\chi^2$ minima occur at different parameter values, there is a synergy between the experiments which gives an increased octant sensitivity. A combination of NO$\nu$A, T2K and LArTPC (500 kT yr) can give a nearly $3\sigma$ sensitivity for $\theta_{23} = 41^o$, a $4\sigma$ sensitivity for $\theta_{23} = 39^o$ and a $5\sigma$ sensitivity for $\theta_{23} = 37^o$ for $\sin^2 2\theta_{13} = 0.1$ and $\delta_{CP} = 0$ in the case of a normal hierarchy. The corresponding sensitivities for inverted hierarchy are somewhat less. A magnetized iron calorimeter combined with NO$\nu$A and T2K can give $\sim 3\sigma$ sensitivity for $\theta_{23} = 39^o$ for both NH and IH. Finally, a combination of a magnetized iron detector with a LArTPC (500 kT yr) and NO$\nu$A+ T2K gives a further improved sensitivity of $4.5\sigma$ for $\theta_{23} = 39^o$ and nearly $6\sigma$ for $\theta_{23} = 37^o$ for NH, and nearly $5\sigma$ for $\theta_{23} = 37^o$ for IH. These results hold good with a marginalization over the hierarchy.
• The octant sensitivity from an atmospheric neutrino experiment is nearly independent of the true value of $\delta_{CP}$, while the NO$\nu$A and T2K sensitivities are seen to be strongly $\delta_{CP}$-dependent and follow a definite shape as a function of $\delta_{CP}$. The shape gets flipped for a true lower or higher octant. For a normal mass hierarchy, the contribution of NO$\nu$A + T2K to the octant sensitivity is lower than that from a LArTPC measurement for almost all values of $\delta_{CP}$, while for an inverted hierarchy it is higher, due to the worse sensitivity from an atmospheric experiment in the inverted hierarchy case.

In conclusion, we find that the improved precision of $\theta_{13}$ and the different dependence on $\theta_{23}$ in the disappearance and appearance channels and in vacuum and matter probabilities leads to an enhanced octant sensitivity when long baseline and atmospheric neutrino experiments are combined.

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