Fifth Memoir

or

Letter on the resistance of air to the movement of pendulums.

Denis Diderot

If the place where Newton calculates the resistance caused by air to the movement of the pendulum embarasses you, do not let your self-steem be afflicted by it. As the greatest geometers will tell you, in the depth and laconicity of the Principia one encounters everywhere motives to console a man of penetrating mind who has had some difficulty in understanding them; and you will see shortly that there is another reason that seems even better to me – that the hypothesis this author started with might not be exact. Something surprises me however: that you were advised to seek me in order to free you from your embarassment. It is true that I studied Newton with the purpose of elucidating him. I should even tell you that this work was pursued, if not successfully, at least with great vivacity. But I did not think of it any longer since the Reverend Fathers Le Sueur and Jacquier made their commentaries public, and I did not feel tempted to ever reconsider it. There was, in my work, a few things you would not find in the work of these great geometers and a great many things in theirs you most surely would not find in mine. So what do you ask of me? Even though mathematical matters were once much familiar to me, to ask me now about Newton is to talk of a dream of the year past. However, to persevere in the habit of pleasing you I will leaf through my abandoned drafts, I will consult the sagacity of my friends and tell you what I can learn from them, telling you also, with Horace:

‘Si quid novisti rectius istis, candidus imperti; si non, hiutere mecum.’

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Proposition I.

Problem.

Let a pendulum \( M \), which describes an arc \( BA \) in air, be attached to a fixed point \( G \) through the straight line \( GM \). One asks for its velocity at any given point \( M \), given that it was let go from point \( B \) (see fig. 2).

Let \( GM = a \), \( NA = b \), \( AP = x \). The weight is \( p \). The resistance that air causes on \( M \) when it has a velocity \( g \) is equal to \( f \). The velocity of the pendulum at point \( M \) is equal to equal to \( v \).

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†Born: 5 October 1713 in Langres, France. Died: July 31 1784 in Paris, France.

1Horace, Epistolae, Liber I, ep. VI, vers. 67, 68 Edition. [Transl.: If you can better these, please tell me. If not, follow them with me.]
If one assumes, as all physicists do, that the resistance in air and in other fluids is proportional to the square of the velocity, the resistance at point $M$ is equal to $f v^2 / g^2$, and this resistance, acting along [the arc] $Mm$ will act so as to decrease the velocity. Moreover, one can easily see that the weight $p$ acting along $MQ$ can be decomposed into two other forces: one, acting along $MR$, is compensated by the resistance of the string or straight line $GM$, while the other acts along $Mm$, perpendicularly to $GM$, and is equal to

$$\frac{p \times MP}{GM} = \frac{p \sqrt{2ax - x^2}}{a}.$$

So, the total accelerating force at point $M$ that causes the body to move along $Mm$ is equal to

$$\frac{p \sqrt{2ax - x^2}}{a} - \frac{fv^2}{g^2}.$$

But the time it takes to traverse $Mm$ is equal to $Mm/v$ and the element or increase in velocity is equal
to the accelerating force multiplied by time. Thus
\[
\left( \frac{p \sqrt{2ax - x^2}}{a} - \frac{f v^2}{g^2} \right) \times \frac{Mm}{v} = dv.
\]

In this equation I substitute the little arc \( Mm \) by its value \( -\frac{a \, dx}{\sqrt{2ax - x^2}} \), with a minus sign, because as the pendulum descends the velocity increases while \( x \) becomes smaller. I have
\[
p \, dx + \frac{f v^2 \times ax}{g^2 \sqrt{2ax - x^2}} = v \, dv.
\]
whose integral is
\[
\frac{v^2}{2} = pb - px + \int \frac{f v^2 \times ax}{g^2 \sqrt{2ax - x^2}} \, dx.
\]
I added the constant \( pb \) since \( v = 0 \) when \( x = b \), that is when the pendulum is at point \( B \) it falls due to its own weight.

First, one will notice in this equation that if a pendulum falls in vacuum or in a non-resistant medium, it will have a velocity [given by] \( v^2 = pb - px \). However, as the resistance of air is much smaller than the weight \( p \), the real value of \( v^2 \) will differ little from \( 2pb - 2px \) and one may then replace \( f v^2 \) by \( f(2pb - 2px) \), and this will cause but a very small error.

Thus one has
\[
\frac{v^2}{2} = 2pb - 2px + 2 \int \frac{f(2pb - 2px) \times ax}{g^2 \sqrt{2ax - x^2}} \, dx.
\]
for the approximate value of \( v^2 \). It is now a question of finding the integral of the term under the sign \( \int \), and the difficulty reduces to integrating \( \frac{ba \, dx}{\sqrt{2ax - x^2}} \). It should be remarked that this integral is such that it must be 0 when \( x = b \). Or, the integral of the first term \( \int \frac{ba \, dx}{\sqrt{2ax - x^2}} \) is \( b \times (arc \, AM - arc \, AB) \). To this I added the constant \( -b \times arc \, AB \) so that \( \int \frac{ba \, dx}{\sqrt{2ax - x^2}} \) is equal to 0 when \( x \) is equal to \( b \); thus one has
\[
\int \frac{ba \, dx}{\sqrt{2ax - x^2}} = -b \times arc \, BM.
\]
Now, to find the integral \( \int \frac{-a \, dx}{\sqrt{2ax - x^2}} \), I write it as
\[
\int \frac{-a \, x \, dx}{\sqrt{2ax - x^2}} = \int \frac{a^2 \, dx - ax \, dx}{\sqrt{2ax - x^2}} - \int \frac{a^2 \, dx}{\sqrt{2ax - x^2}}.
\]
where the integral is \( a\sqrt{2ax - x^2} - a \times AM = a \times (MP - AM) \), to which the constant \( -a(BN - AB) \) must be added for the same reason explained above; one will have then
\[
\int \frac{-a \, x \, dx}{\sqrt{2ax - x^2}} = -a \times (BO - BM).
\]
Hence
\[
v^2 = 2pb - 2px - \frac{2f \times 2pb \times BM}{g^2} - \frac{2f \times 2pa \times (BO - BM)}{g^2}.
\]

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2 In the original there is a mass \( m \) missing on the right hand side of the equation. So, for the following equations to hold, one has to consider \( m = 1 \). [N. of T.]

3 In the original, a factor \( g^2 \) is missing in the second term on the right hand side. [N. of T.]
Corollary I.

When the pendulum arrives at $A$, one has

$$v^2 = 2p_b - \frac{2f \times 2p_b \times BA}{g^2} - \frac{2f \times 2p_a \times (BN - BA)}{g^2}.$$

Corollary II.

Thus from (fig. 3), if one takes $An = b - \frac{2f \times BA}{g^2} - \frac{2f \times (BN - BA)}{g^2}$ one has $v^2 = 2p \times An$, that is, the velocity at $A$ is the same as the one the pendulum would have if it had fallen in vacuum from $b$ to $A$.

Corollary III.

If arc $AB$ has but a few degrees, $BN$ will be almost equal to $BA$; in this case one may assume $v^2 = 2p - \frac{2f \times 2p_b \times BA}{g^2}$.

Proposition II.

Problem.

Suppose (fig. 4) that a pendulum $A$, placed in the vertical $GA$, receives an impulse or velocity $h$ along the horizontal $AR$. One asks for its velocity at any given point $M$.

Solution

Using the same naming of variables as before, the retarding force will be

$$\frac{p \sqrt{2ax - x^2}}{a} + \frac{fv^2}{g^2},$$

as the resistance now helps the weight, continuously diminishing the velocity of the pendulum. Thus one will have

$$-dv = \left(\frac{adx}{v \sqrt{2ax - x^2}}\right) \times \left(\frac{p \sqrt{2ax - x^2}}{a} + \frac{fv^2}{g^2}\right).$$

I write $-dv$ since, as $x$ increases, $v$ decreases, so that

$$-vdv = pdx + \frac{fv^2 \times adx}{g^2 \sqrt{2ax - x^2}}.$$

and adding the constants

$$\frac{h^2 - v^2}{2} = px + \int \frac{fv^2 \times adx}{g^2 \sqrt{2ax - x^2}}.$$

So, if $f = 0$ one gets $v^2 = h^2 - 2px$ and one can replace $v^2$ in the expression $\int \frac{fv^2 \times adx}{g^2 \sqrt{2ax - x^2}}$ by its approximate value $h^2 - 2px$ as in the preceding problem. This will give

$$v^2 = h^2 - 2px - 2 \int \frac{fh^2 \times adx}{g^2 \sqrt{2ax - x^2}} + 2 \int \frac{f \times 2paxdx}{g^2 \sqrt{2ax - x^2}}$$

$$= h^2 - 2px - \frac{2fh^2}{g^2} \times AM + \frac{2f \times 2pax}{g^2} \times (AM - MP).$$

\[4\text{There is a misprint in the original. There appears a } du \text{ instead of a } dv. \text{ [N. of T.]}\]
Let \( AN \) be the height the pendulum would have reached in vacuum. One has 
\[
v^2 = 2p \times PN - \frac{2f \times 2p \times AN \times AM}{g^2} + \frac{2f \times 2pa}{g^2} \times (-MP + AM)
\]
(1)

**Corollary I.**

Hence (fig. 5) when the body arrives at point \( c \) such that 
\[
Nn = \frac{2f \times AN \times Ac}{g^2} + \frac{2f \times a \times (nc - Ac)}{g^2} \]
the velocity \( v \) will be equal to 0.

**Corollary II.**

Since \( nc \) and \( Ac \) differ little from \( NC \) and \( AC \), it follows that to find point \( c \) where the body stops, or the height \( n \) it reaches, one has to take 
\[
Nn = \frac{2f \times AN \times AC}{g^2} + \frac{2fa \times (NC - AC)}{g^2}.
\]

**Corollary III.**

If arc \( AC \) comprises just a few degrees, \( AC \) will be nearly equal \( AN \) and one will have 
\[
Nn = \frac{2f \times AN \times AC}{g^2}
\]
approximately.

**Corollary IV.**

If a pendulum (fig. 6) descends from \( B \), its velocity at \( A \), which I called \( h \), will be equal (Corol. II, Prop. I) to that it would have if falling in vacuum from height 
\[
A_n = b - \frac{2fx \times BA}{g^2} - \frac{2fa \times (BN - BA)}{g^2}
\]
and it will ascend to height \( A_\nu \) (Corol. II, Prop. II) is equal to 
\[
A_n - \frac{2fx \times AN \times AC}{g^2} + \frac{2fa \times (NC - AC)}{g^2}.
\]
And since \( nc \) and \( Ac \) differ little from \( BN \) and \( BA \), we have 
\[
A_\nu = \frac{b - \frac{4fx \times BA}{g^2} + \frac{4fa \times (BN - BA)}{g^2}}{g^2}.
\]

**Corollary V.**

If arc \( AB \) comprises but a few degrees, we have 
\[
A_\nu = b - \frac{4fx \times BA}{g^2} = AN \times \left(1 - \frac{4fx \times BA}{g^2}\right).
\]
Or, under the same assumption, the arcs \( AC \) and \( Ak \) are to each other approximately as the roots of the abscissae \( AN \), \( A_\nu \). For, in the circle, the chords are to each other as the roots of the abscissae; or, the arcs can be replaced by the chords. Thus
\[
Ck = AC \times \frac{\sqrt{AN} - \sqrt{A_\nu}}{\sqrt{AN}}.
\]
or
\[
\sqrt{A_\nu} = \sqrt{AN \times \left(1 - \frac{4f \times BA}{g^2}\right)} = \sqrt{AN} \sqrt{\left(1 - \frac{4f \times BA}{g^2}\right)}.
\]
Since \( 4f \times BA/g^2 \) is very small compared to 1, one may replace \( \sqrt{1 - \frac{4f \times BA}{g^2}} \) by \( 1 - \frac{2f \times BA}{g^2} \), because they are nearly equal and one also knows that if \( \alpha \) is a small fraction, \( \sqrt{1 - \alpha} \) is approximately equal to \( 1 - \alpha/2 \). Thus
\[
Ck = AC \times \frac{2f \times BA}{g^2} = \frac{2f \times BA^2}{g^2}.
\]
(2)

Hence the difference \( Ck \) between the arc \( BA \) of descent and the arc \( Ak \) of the ascent is as the square of the arc \( AB \).

\[5\]There is a misprint here. The correct form should be \( \sqrt{AN \left(1 - \frac{4f \times BA}{g^2}\right)} \). [N. of T.]
Corollary VI.

Hence (fig. 7) if one knows the arc $BAC$ that a pendulum describes when falling from point $B$, one can easily find the arc $bAk$ that it will describe when falling from point $b$: it suffices to find $Ak$, which one gets by making $(BA - AC) : (bA - Ak) = BA^2 : bA^2$.

Corollary VII.

Hence it follows that (fig. 6) if a pendulum describes the arc $BA$ in air, one may find its velocity at point $A$ by dividing the line $N\nu$ in two equal parts at point $n$. Then this velocity (Corol. III, Prop. I) is very close to that one would get when falling in vacuum from the height $b - \frac{2f \times BA}{g^2} = b - \frac{N\nu}{2}$.

Corollary VIII.

One has $AC^2 : Ac^2 = An : An$. That is $AC^2 : (AC^2 - 2Cc \times AC) = AN : (AN - Nn)$. Therefore $Nn = \frac{2Cc \times AC \times AN}{AC^2} = \frac{2Cc \times AN}{AC^2}$. For the same reason we have $N\nu = \frac{2Ck \times AN}{AC^2}$. Thus $Ck : Cc = N\nu : Nn$. Therefore $c$ is the middle point of the arcs $Ck$. Thus, instead of dividing $N\nu$ in two equal parts, we can divide $Ck$ in two equal parts to obtain the arc $Ac$ that body $A$, on ascension, would have traversed in vacuum.

Corollary IX.

If pendulum $A$ is a small sphere, the resistance $f$, all other things being equal, is inversely proportional to the diameter of this sphere and its density. But the resistance caused by air on two spheres of different diameters goes as the surface or the square of the diameter, and this resistance has to be divided by the mass, which is the density multiplied by the third power of the diameter. From this it follows that the arc $Ck$, all other things being equal, is like $AB^2$ divided by the product of the diameter of the sphere and its density.

It is up to you, M***, to see if I can now make use of the propositions, since one wants to determine the changes in the movement caused by the resistance of air in pendulums used to study the collision of bodies. You will notice, without difficulty, that corollaries VI, VII and VIII will give you the velocity that two pendulums would have or receive in the lowest point where they supposedly collide.

Mr Newton, as you may well know, did not believe in neglecting this resistance. He talked about the collision of bodies in the first book of his *Principia*, and seems to have made $Ck$ proportional not to the square of the arc traversed, as we found it to be, or as you would suppose, since this was the place in his work that kept you from advancing, but to the arc solely: and this is what left for me to show you. To this effect, let me transcribe his text, to which I will add the comments that I find in the papers that the Reverend Fathers Jacquier and Le Sueur condemned to oblivion, preventing from their excellent Commentaries, that which I meditated upon.

**Newton’s Text**

Let, says Newton (*Princip. Mathem. pag. 50, see fig. 8*) two spherical bodies $A$ and $B$ be suspended from the points $C$ and $D$ through two equal and parallel lines $AC$ and $BD$ such that these lengths describe...
two semicircles $EAF, GBH$, divided in two equal parts by the radii $CA, CB$. Move body $A$ to any $R$ on the arc $EAF$. Remove $B$ and let $A$ fall: if, after one oscillation, it returns to point $V$, then $RV$ will express the retardation caused by the resistance of air. Take $ST$ equal to the fourth part of $RV$ and place it in the middle such that $RS$ equals $TV$, that is $RS$ is to $ST$ as $3$ is to $2$: $RS$ will express very closely the retardation after the descent from $S$ to $A$. Replace the body you have removed. Let $A$ fall from $S$. Its velocity at the reflection point $A$ will be, without appreciable error, the same velocity it would have if it had fallen from point $T$. Therefore its velocity will be given by the chord $TA$, as all notable geometers know that the velocity of a pendulum at its lowest point of the arc it describes is like the chord of this arc. If the body $A$, after the collision, returns to point $S$ and the body $B$ to point $K$, remove $B$ and find the point $u$ from where $A$, after falling, would return to $r$ such that $st$ is the fourth part of $ru$ and $sr$ is equal to $tu$. The chord $tA$ will express the velocity $A$ will have at point $A$ after its reflection, since $t$ is the real and correct place to which $A$ would return in the absence of air resistance. The place $K$ to which body $B$ returns should be corrected using the same method, finding the point $l$ it would reach in vacuum. This is how one does the experiments as if in vacuum. Finally one has, so to say, to multiply body $A$ by the chord $TA$ which expresses the velocity, to obtain its movement at point $A$ immediately before the collision, and by the chord $tA$ to have it right after the collision. One has to search, using the same method, the quantities of movement before and after collision of two bodies which were let go at the same time from two different points, and find, by comparing its movements, the effect of the collision. This is how I performed my experiments with pendulums 10 feet long, with equal as well as with unequal bodies, which I let fall from afar through distances, for example, of 8, 12 and 16 feet. I found, without having erred in the measured quantities by three inches, that the changes caused by direct collision in the direction contrary to the movement of the bodies were equal and consequently, action is always equal to reaction, etc.’

Clarifications

Here is Newton’s text and now the clarifications that I promised to give you. If a body falls from $R$ to $A$ (fig. 9) in a non resistant medium, its veloicy is, as we know, equal to that it would acquire if it had fallen from point $T$. Therefore its velocity will be given by the chord $TA$, as all notable geometers know that the velocity of a pendulum at its lowest point of the arc it describes is like the chord of this arc. If the body $A$, after the collision, returns to point $S$ and the body $B$ to point $K$, remove $B$ and find the point $u$ from where $A$, after falling, would return to $r$ such that $st$ is the fourth part of $ru$ and $sr$ is equal to $tu$. The chord $tA$ will express the velocity $A$ will have at point $A$ after its reflection, since $t$ is the real and correct place to which $A$ would return in the absence of air resistance. The place $K$ to which body $B$ returns should be corrected using the same method, finding the point $l$ it would reach in vacuum. This is how one does the experiments as if in vacuum. Finally one has, so to say, to multiply body $A$ by the chord $TA$ which expresses the velocity, to obtain its movement at point $A$ immediately before the collision, and by the chord $tA$ to have it right after the collision. One has to search, using the same method, the quantities of movement before and after collision of two bodies which were let go at the same time from two different points, and find, by comparing its movements, the effect of the collision. This is how I performed my experiments with pendulums 10 feet long, with equal as well as with unequal bodies, which I let fall from afar through distances, for example, of 8, 12 and 16 feet. I found, without having erred in the measured quantities by three inches, that the changes caused by direct collision in the direction contrary to the movement of the bodies were equal and consequently, action is always equal to reaction, etc.’

T. Exponatur igitur haec velocitas per chordam arcus $TA$: nam velocitatem Penduli in puncto inmo esse ut chorda arcus quem cadendo descriptis, Propositio est Geometris notissima. Post reexionem perveniat corpus $A$ ad locum $s$, et corpus $B$ ad locum $K$. Tollatur corpus $B$ et inveniatur locus $v$, a quo si corpus $A$ demittatur, et post unam oscillationem redeat ad locum $r$, sit $st$ pars quarta ipsius $rv$ sita in medio, ita videlicet ut $rs$ et $tv$ aequentur: et per chordam arcus $TA$ exponatur velocitas quam corpus $A$ proxime post reexionem habuit in loco $A$. Nam $t$ erit locus ille verus et correctus, ad quem corpus $A$, sublata aeris resistentia, ascendere debuisset. Simili methodo corrigendus erit locus $k$, ad quem corpus $B$ ascendiit, et inveniendus locus $l$, a quo si corpus illud ascendere debuisset in vacuo. Hoc pacto experiri licet omnia perinde ac si in vacuo constituti essemus. Tandem ducendum erit corpus $A$ in chordam arcus $TA$, quae velocitatem ejus exhibet, ut habeatur motus ejus in loco $A$ proxime ante reexionem; deinde in chordam arcus $IA$ ut habeatur motus ejus in loco $A$ proxime post reexionem. Et simili methodo uti corpora duo simul demittuntur de locis diversis, inveniendi sunt motus utriusq; tam ante, quam post reexionem; et tum demum conferendi sunt motus inter se et colligendi effectus reexionis. Hoc modo in Pendulis pedum decem rem tentando, idque in corporibus tam inaequalibus quam aequalibus, et faciendo ut corpora de intervallis amplitissimis, puta pedum octo, duodecim vel sexdecim concurrerent, reperi semper sine errore trium digitorum in mensuris, ubi corpora sibi mutuo occurrebant, aequales esse mutationes motuum corporibus in partes contrarias illatae, atque ideo actionem et reactionem semper esse aequales, etc. [Diderot].
non-resistant medium. And, instead of ascending to $Ay = An$, the resistance of the medium does not let it ascend beyond $V$.

Put this way, the arc $RV$ expresses the retardations produced by the resistance of air in all the retardation I mentioned. But these oscillations are each one smaller than the other. To have the retardation of any of them in particular, I have to divide the arc $RV$ into unequal parts; and as these oscillations are in a number of four, the retardation of the first oscillation is larger than the fourth part of $RV$; and its fourth part, larger than the retardation of the fourth oscillation. But there is a point $S$ such that a fall through arc $SA$ will have a retardation given exactly by the fourth part of $RV$.

Let us find this point $S$. To find it, let $RA = 1$; $RV = 4b$; $SA = x$. If we assume the retardations are proportional to the traversed arcs, one will have the retardation $Rr$ of the arc $RA$ traversed is equal to $\frac{b}{x}$. And $A\rho$, the second arc, is equal to $AR = RA - Rr = 1 - \frac{b}{x}$. The same way $\rho N$, the retardation of arc $A\rho$, is $(1 - \frac{b}{x}) \times \frac{b}{x} = \frac{b^2}{x^2}$. Hence $AN$, the third arc, is $A\rho - p\rho N = 1 - \frac{2b}{x} + \frac{b^2}{x^2}$. And the retardation $Nn$ of the arc $AN$ is \( (1 - \frac{2b}{x} + \frac{b^2}{x^2}) \times \frac{b}{x} = \frac{b^2}{x^2} + \frac{b^3}{x^3} \). Hence $Ay = An = AN - Nn$, the fourth arc, is equal to $1 - \frac{3b}{x} + \frac{3b^2}{x^2} + \frac{b^3}{x^3}$. Thus, the retardation $V y$ of the fourth arc is $\frac{b}{x} - \frac{3b^3}{x^3} + \frac{3b^5}{x^5} - \frac{b^4}{x^4}$. One thus has

$$Rr, \text{ retardation of the first arc, is equal to } \frac{b}{x}.$$ $$\rho N, \text{ retardation of the second, is equal to } \frac{b}{x} - \frac{b^2}{x^2}.$$ $$Nn, \text{ retardation of the third, is equal to } \frac{b}{x} - \frac{2b^2}{x^2} + \frac{b^3}{x^3}.$$ $$Vy, \text{ retardation of the fourth, is equal to } \frac{b}{x} - \frac{3b^3}{x^3} + \frac{3b^5}{x^5} - \frac{b^4}{x^4}.$$ And since $Rr + \rho N + Nn + Vy = 4b$, we have the equation \( \frac{2b}{x} - \frac{6b^2}{x^2} + \frac{4b^3}{x^3} - \frac{b^4}{x^4} = 4b \), or $x^4 - x^3 + \frac{3b^2}{x^2} - \frac{b^2}{x^2} + \frac{b}{x} = 0$, whose approximate solution will give us the value of $x$.

To find it, we drop the last two terms \(-\frac{b^2}{x^2} + \frac{b}{x}\) which are much smaller than the other terms since $b$ is very small. One is left with $x^4 - x^3 + \frac{3b^2}{2} = 0$, or $x^2 - x + \frac{3b}{2} = 0$, an equation whose root is $x = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{3b}{2}}$. However, $\sqrt{\frac{1}{4} - \frac{3b}{2}}$ is very close to $\frac{1}{2} - \frac{3b}{4}$ from which [we have that] $x$ is very close to $\frac{1}{2} + \frac{b}{2} - \frac{3b}{4} = 1 - \frac{3b}{2}$.

**Remarks on this approximation**

**1st.** Notice that $-b^2x^2 + \frac{b^3}{2} < 0$, since $x > b$, from which it follows that $x^4 - x^3 + \frac{3b^2}{2} = 0$. Thus $x > \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{3b}{2}}$. But $\frac{1}{2} - \frac{3b}{2}$ is little larger than $\sqrt{\frac{1}{4} - \frac{3b}{2}}$, so when replacing $\sqrt{\frac{1}{4} - \frac{3b}{2}}$ by $\frac{1}{2} - \frac{3b}{2}$ one is giving back to $x$ a little of what one took from it. From which it follows that this approximation is as simple and accurate as we could wish, given the assumption that the retardations are as the arcs and not the square of the arcs.

**2nd.** The retardations $\frac{b}{x}$, $\frac{b}{x} - \frac{b^2}{x^2}$, etc. are in geometrical progression.

**3rd.** One may solve the equation \( \frac{4b}{x} - \frac{6b^2}{x^2} + \frac{4b^3}{x^3} - \frac{b^4}{x^4} = 4b \) exactly if one makes the approximation $1 - \frac{4b}{x} + \frac{6b^2}{x^2} - \frac{4b^3}{x^3} + \frac{b^4}{x^4} = 1 - 4b$. Thus $1 - \frac{b}{x} = \sqrt{1 - 4b}$ or $x = \frac{b}{1 - \sqrt{1 - 4b}}$. 

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4th. That, to find the place \(V\), we have \(st : tu = 2 : 3\) and that \(tu = sr\). From which it follows that \(su : sr = 5 : 3\). Let \(As = 1\), \(sr = x\). We have \(Au = 1 + \frac{5x}{4}\); \(Ar = 1 - x\). Or \(Au\) is to \(Ar\) almost as \(AV : AR\). So, if we make \(AV : AR = m : n\), we will have \(m : n = (1 = \frac{5x}{3}) : (1 - x)\). From this \(n + \frac{5nx}{3} = m - mx\) and \(\frac{m-n}{m+5n/3} = (3 \times \frac{m-n}{5m+5n}) \times \ As\) since we assumed \(As = 1\).

One may now determine the point \(V\) through experiments, letting a pendulum fall from point \(V\) until it returns to a point \(r\), where the distance \(sr\) [from \(r\)] to \(s\) is such that it is equal to \(su \times \frac{2}{3}\) or, one may simply take \(st = \frac{As}{AS} \times ST\).

Here is, to me very well clarified it seems, the entire passage of Newton on the retardation of pendulums caused by the resistance of air. From it, there seems to follow that this author assumed the retardations [proportional] to the arcs while, according to the preceding propositions, we found it to be as the squares of the arcs. You may object, undoubtedly, that Newton has the experiments on his side; and that with this hypothesis he found action to be always equal to reaction\[7\]; and that, for example, if body \(A\) with 9 degrees of movement, after colliding with \(B\) initially at rest, kept moving with 2 while \(B\) departed with 7; that if bodies collided coming from opposite directions, \(A\) with 12 degrees, \(B\) with 6 and that \(A\) reflected with 2 \(B\) reflected with 8, etc.

Being advised to never doubt the exactitude and good faith of Newton, I would nonetheless like to remind you that this did not prevent his experiments on colors from being repeated. Why would one not do this in this particular case, where the author started with an hypothesis which calculations clearly contradict and where it is even easier to make a mistake since the velocities are represented by quantities whose differences are very small, namely, the chords of the arcs traversed before and after retardations? If you think this is not enough, since it is Newton’s great name [after all], I am vexed; as for me, I cannot agree with him. I have for Newton all deference one may accord to the unique men of his kin and tend strongly to the belief that he has truth at his side. But, even so, it is better to be sure of if. I invite therefore all those who like the good physics to restart their experiments and tell us if the retardations are as those that Newton seems to have assumed, proportional to the arcs traversed; or those that the calculations give us, proportional to the square of these arcs.

\[7\]Ut si corpus \(A\) incidebat in corpus \(B\) quiescens cum novem partibus motus, et amissis septem partibus pergebat post reflexionem cum duabus; corpus \(B\) resiliebat cum partibus istis septem. Si corpora obviam ibant, \(A\) cum duodecim partibus et \(B\) cum sex, et redibat \(A\) cum duabus; redibat \(B\) cum octo, facta etc. [Diderot].