Mathematical support for the formation a compromise tax on the profit of the region engineering enterprises

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Abstract. The task of managing the regional budget through the local income tax of business entities is very difficult. When solving it, it is necessary to take into account the interests of the authorities, businessmen and the population. The article is devoted to the search for such methods for calculating income tax rates that implement the necessary compromise of interests. The two-level algorithm proposed in the article takes into account the indicator of the integration of economic entities into the economy of the territory. First, the analysis and formation of tax rates for clusters of economic entities is carried out - taking into account the profits of entities and the number of employees in them. This is a top-level task. The group of socially significant economic entities for the region is studied at the lower level, its own optimization models are proposed for it. The results of model testing of models and algorithms are presented. It is shown that for machine-building enterprises a reasonable rate of profit tax to the local budget is 15.5%.

1. Introduction

At present, the leading role in the economic development of any state is assigned to domestic producers of goods and services. The results of each engineering enterprise, its profits, have a significant impact not only on its own state and development prospects, but also on the economic security of the region as a whole. At the same time, on the one hand, it is subject to taxation and a source of replenishment of the regional budget.

In this case, the regional authorities are interested in receiving the largest possible tax deductions. On the other hand, it is in the interests of both the regional authorities and the management of enterprises that the greatest net profit that can be invested in the development of production is obtained. The described contradiction is objective in nature, and the coordination of interests of the region and enterprises based on the search for a compromise value of the income tax rate in the regional budget is a task relevant to any territorial entity.

The problem of tax regulation of enterprises has repeatedly been the subject of discussion in many studies. Among them, we will mention the works in which general issues of fiscal policy are touched upon and the experience of various countries [1-11] is analyzed, as well as problems of optimizing the tax system and its coordination with the payment of transfers, prices and wages [12-15]. The tasks related to the formation of tax regulation mechanisms, including the use of mathematical tools, are widely covered in studies [2, 16-18].
The study is aimed at creating such methods of forming the income tax rate in the local budget, which will ensure harmonization between budget execution and the net profit of machine-building enterprises. At the same time, the methods actively use those statistical data that are available to the authorities of the territory.

2. Materials and methods

This study is based on the formal presentation of the economic system of a region as a combination of the following interrelated elements: regional authorities (regional administration); economic system of the region.

The economic system of the region is considered as a set of interacting enterprises engaged in economic activities in the region. We believe that regional enterprises can be combined into groups (classes) so that for enterprises of one group the income tax rate in the regional budget should be the same and possibly different from the other. These classes can be industries or economic activities. We assume that the enterprises of the region are divided into \( M \) -groups.

Consider one of the enterprise groups. For convenience, the group index will be omitted. The corporate income tax rate \( \alpha \) consists of two components:

\[
\alpha = \alpha^{\text{fed}} + \alpha^{\text{reg}},
\]

where \( \alpha^{\text{fed}} \) – the rate of profit tax in the Federal budget, uniform for all enterprises of the region; \( \alpha^{\text{reg}} \) – the tax rate in the regional budget (regional component) for this group of enterprises, which is set at the regional level and can take values from a specified interval \( \alpha \leq \alpha^{\text{reg}} \leq \bar{\alpha} \).

The strategic decision for the territorial administration bodies at the initial moment of time \( t = 0 \) is to make a decision on the territorial share of the profit tax rate for the class of economic entities. The term of such a decision is \( T \) years.

Let \( P_t \) there be the profit of the class of economic entities before deducting taxes at the moment of time \( t \ (t = 1, \ldots, T) \). Then \( R_t = \alpha^{\text{reg}} \cdot P_t \) is the size of the territorial tax that will go to the budget. At the same time, \( Z_t = \alpha \cdot P_t \) is a profit for business entities. Suppose that the leaders of the class of business entities have convinced the territorial management system that their minimum profit after taxes is equal to \( Z_t \), \( t = 1, \ldots, T \). The territorial management system has decided that \( R_t \) is a minimum share of taxes in the budget.

In other words, the received share of taxes in the budget is a compromise between the leadership of the territory and the class of economic entities. In this case, the following conditions must be met:

\[
R_t \geq \underline{R}, \quad Z_t \geq \underline{Z}, \quad t = 1, \ldots, T.
\]

The complexity of the formation of a compromise tax rate on profits is due to the fact that at the time of making decisions \( t = 0 \) he profits of class enterprises in each year of the period \( 1, \ldots, T \) are an unknown quantity. We offer to analyze the impact of tax policy on the amount of profits of a group of enterprises and tax revenues to the regional budget based on a mathematical model.

To form a mathematical model, we will assume that the administration of the region knows the following aggregated statistical information on the performance of the enterprise group.: \( Z_0 \) – the net profit for the year \( t = 0 \); \( K_0 \) – the fixed assets at the beginning of the year \( t = 1 \); \( k \) – the capital productivity ratio as the ratio of fixed assets at the beginning of the year \( t = 0 \) to sales revenue; \( \delta \) – the share of net profit spent on reinvestment \( \left( \delta \in [0,1] \right) \); \( c \) – the unit cost of production; \( \beta \) – the output tax rate.
We introduce the following notation: $K_t$ – the fixed assets of the enterprise group at the end of the year $t$; $I_t$ – the volume of investments in the group of companies in the year $t$; $Y_t$ – the volume of products manufactured by the group. The aggregated discrete model for assessing the impact of the tax rate on the profit of a group of enterprises represents a system of relations:

$$K_t = K_{t-1} + \delta \cdot Z_{t-1} + I_t; \quad (1)$$
$$Y_t = k \cdot K_t; \quad P_t = (1-c) \cdot Y_t; \quad (2)$$
$$Z_t = \left(1 - (\alpha^{fed} + \alpha^{reg})\right) \cdot P_t - \beta \cdot Y_t, \quad t = 1,...,T. \quad (3)$$

Note that (1) - (3) may be inconvenient for carrying out practical calculations. For such cases, we give a differential analogue of the model, which we call the aggregated differential model for assessing the impact of the corporate income tax rate:

$$\frac{dK^m(t)}{dt} = \delta_m \cdot Z^m(t) + I^m(t); \quad (4)$$
$$Y^m(t) = k_m \cdot K^m(t); \quad \Pi^m(t) = (1-c) \cdot Y^m(t); \quad (5)$$
$$Z^m(t) = \left(1 - (\alpha^{fed} + \alpha^{reg}_m)\right) \cdot \Pi^m(t) - \beta \cdot Y^m(t). \quad (6)$$

Taking into account relations (5) - (6), equation (4) takes the following form:

$$\frac{dK^m(t)}{dt} = a \cdot K^m(t) + I(t), \text{ where } a = \left(1 - \alpha^{fed}_m - \alpha^{reg}_m\right) \cdot (1-c) \cdot k_m - \beta \cdot k_m. \quad (7)$$

Solution (7) depends on the type of function $I^m(t)$. In the case when investments are constant, i.e. $I^m(t) = I_0$, differential equation (7) has the following solution:

$$K^m(t) = e^{at} \left(K^m_0 + \frac{I_0}{a}\right) - \frac{I_0}{a}. \quad (8)$$

3. Results and discussion

Based on the described approach, an algorithm was created for forming a compromise tax rate for the profit of a group of regional enterprises, including the following sequence of steps:

Step 0. Setting the parameters and initial values of the model (1) - (3). Setting the allowable interval for changing the tax rate $\alpha \leq \alpha^{reg} \leq \bar{\alpha}$.

Setting values $R_t$ and $Z_t$, $t = 1,...,T$.

Step 1. Formation of the final set of values of the regional component of the income tax rate:

$$V = \left\{(\alpha^{reg})^1,...,(\alpha^{reg})^S\right\} \subseteq [\alpha^{reg}, \bar{\alpha^{reg}}]. \quad (9)$$

Step 2. For each bid set $V$ by the formulas (1) - (3) are calculated: $K_t, Y_t, \Pi_t, Z_t$.

Step 3. As a compromise, bets are selected to ensure the implementation of inequalities:

$$R_t = \alpha^{reg} \cdot P_t \geq R_t; \quad Z_t \geq Z_t; \quad t = 1,...,T. \quad (10)$$
As an example of the work of the algorithm, we present the calculations made on the basis of statistical data of a group of mechanical engineering enterprises of the Voronezh region. The parameters of the model (1) - (3) were determined on the basis of statistical data officially submitted by the Federal Statistics Service. The period of the year is chosen as the time \( T = 4 \) year. 2018 is adopted as the initial year (planning year).

The maximum value of the corporate income tax rate is 20%. The current income tax rate in the Federal Budget of the Russian Federation is 2% \( (\alpha_{\text{fed}} = 0.02) \), and in the regional budget it varies from 12.5 to 18%.

Table 1 shows the calculation results (in millions of rubles) for the two options for the rate of income tax in the regional budget: \( \alpha_{1}^{\text{reg}} = 0.155 \), \( \alpha_{2}^{\text{reg}} = 0.18 \).

### Table 1. Basic data for the algorithm.

| Year, t | \( \alpha_{1}^{\text{reg}} = 0.155 \) | \( \alpha_{2}^{\text{reg}} = 0.18 \) |
|---------|-------------------|-------------------|
|         | \( R_t \)      | \( Z_t \)      | \( R_t \)      | \( Z_t \)      |
| 1       | 3555.72         | 18441.90        | 4436.96        | 17946.36       |
| 2       | 6714.29         | 37024.74        | 7813.98        | 35988.10       |
| 3       | 10122.31        | 55689.08        | 11752.25       | 54125.73       |
| 4       | 13533.36        | 74455.47        | 15711.30       | 72359.76       |

Discussion of the calculation results with representatives of regional authorities and their comparison with the planned values of profits of enterprises of the mechanical engineering complex and revenues to the regional budget allowed determining that the rate of income tax in the regional budget of 15.5% is a compromise.

4. Conclusion

The task of managing the regional budget through the local income tax of business entities is very difficult. When solving it, it is necessary to take into account the interests of the authorities, businessmen and the population. The article is devoted to the search for such methods for calculating income tax rates that implement the necessary compromise of interests. The two-level algorithm proposed in the article takes into account the indicator of the integration of economic entities into the economy of the territory. First, the analysis and formation of tax rates for clusters of economic entities is carried out - taking into account the profits of entities and the number of employees in them. This is a top-level task. The group of socially significant economic entities for the region is studied at the lower level, its own optimization models are proposed for it. The results of model testing of models and algorithms are presented. It is shown that for machine-building enterprises a reasonable rate of profit tax to the local budget is 15.5%.

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