Neveu-Schwarz Five-Branes at Orbifold Singularities and Holography

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We consider Type IIB Neveu-Schwarz five-branes transverse to $C^2/Z_n$ orbifolds and conjecture that string theory on the near horizon geometry is dual to the decoupled theory on the branes. We analyze the conformal field theory describing the near horizon region and the world volume non-critical string theory. The modular invariance consistency condition of string theory is exactly reproduced as the gauge anomaly cancellation condition in the little string theories. We comment on aspects of the holographic nature of this duality.
1. Introduction

Recent developments in string theory have exhibited deep connections between new consistent theories without gravity in various dimensions and M-theory compactifications. A beautiful example of this duality appears in the Matrix model description \([1]\) of M-theory backgrounds. In \([2]\), Maldacena has proposed a very interesting correspondence between superconformal field theories and M-theory backgrounds involving Anti-de Sitter spaces\(^1\). These dualities can be motivated by taking the decoupling limit of the field theories living on branes from bulk dynamics. From the point of view of the low energy supergravity approximation, the decoupling limit restricts the brane solution to the near horizon region. In the recent paper \([3]\), similar ideas have been applied to six dimensional non-critical string theories. Neveu-Schwarz (NS) five-branes were studied from two dual points of view, as decoupled theories and as supergravity solutions. The new duality proposed in \([3]\) relates the ultraviolet behavior of the theory on the branes to the string theory background defined by the near horizon region. This duality is very interesting since it can be tested in the world-sheet sigma model framework, and not only in a low energy supergravity description as in the AdS/CFT correspondence. It is also remarkable that the duality involves the “little string theories” which are not local quantum field theories \([4]\). Moreover, the duality provides a further example of holography \([5,6,7]\), since the little string theories can be thought of as living on the boundary of space-time.

In this paper we extend the above conjecture to Type IIB NS five-branes placed at a \(C^2/Z_n\) singularity\(^2\). The conjecture states that string theory on the near horizon geometry of NS five-branes at the orbifold singularity is dual to the decoupled theory on the NS five-branes at the singularity. The space-time theory has an exact conformal field theory description which can be described in detail. We check the validity of the conjecture by analyzing the consistency constraints on both sides of the duality. On the space-time side, consistency is determined by one-loop modular invariance of the partition function describing a string propagating in the near horizon of the branes. On the little string theory side, consistency is determined by gauge anomaly cancellation. We show that consistency on both sides of the duality is obtained at precisely the same value of the five-brane charge. This gives strong evidence for the finite \(N\) conjecture.

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1. This correspondence was made more precise in \([3,4]\).

2. The physics of branes at this singularity has also been considered in a similar context in the AdS/CFT correspondence \([10,11,12,13]\).
In section 2 we briefly review the conformal field theory description of the near horizon geometry of NS five-branes in flat space. We then construct the conformal field theory describing the near horizon geometry of the branes at the $C^2/Z_n$ singularity. The construction is an orbifold of the flat space theory which is based on the A series of the affine $\widehat{SU}(2)$ modular invariant partition functions. We find the necessary and sufficient conditions for modular invariance of the orbifold partition function. The details of the computation have been included in the Appendix for convenience. The case $n = 2$ presents some peculiar aspects which are considered in detail. In particular, we show that D little string theories corresponding to the D-series of affine $\widehat{SU}(2)$ modular invariant partition functions can be recovered using orbifold techniques.

In section 3 we study the $\mathcal{N} = (0, 1)$ little string theories realized on Type IIB NS five-branes at the singularity. We construct the gauge theory sector of the theory by analyzing the S-dual picture. We show that the consistency requirement for these theories both from the point of view of gauge anomaly cancellation and tadpole cancellation maps exactly to the modular invariance consistency requirement found in section 2. We match part of the spectrum of operators in short supersymmetry multiplets in the gauge theory to chiral primaries of the corresponding orbifold conformal field theory. Section 4 contains conclusions.

Note added for version 2: The conformal field theory describing the near horizon geometry was incorrect in the first version of this paper. In a subsequent paper by Kutasov, Larsen and Leigh [14] the correct, GSO projected, conformal field theory was described. The results and conclusions of our paper are unmodified from the first version and the operator matching is generalized to arbitrary $n$ by using the formalism of [14].

2. Neveu-Schwarz Five-Branes at Orbifold Singularities.

The supergravity solution for $N$ NS five-branes in flat space-time in the near horizon limit is given by

$$ds^2 = dx_6^2 + d\phi^2 + Nl_s^2 d\Omega_3^2$$

$$g_s(\phi) = e^{-\sqrt{\frac{4}{Nl_s}}}$$

$$H = -Nl_s^2 \epsilon_3$$

where $N$ is the five-brane charge and $l_s$ is the string length. String theory on this background with geometry $R^{5,1} \times R \times S^3$ and with a linearly varying dilaton has an exact
conformal field theory description \[13\]. The $R^{5,1}$ geometry is described by six free bosons and fermions and has $\mathcal{N} = (1, 1)$ world-sheet supersymmetry. The internal $R \times S^3$ manifold with the H-flux and the linear dilaton is described by an $\mathcal{N} = (4, 4)$ superconformal field theory consisting of two sectors. The radial linear dilaton corresponds to a free scalar $\phi$ with a background charge

$$Q = \sqrt{2 \over N}.$$  \hspace{1cm} (2.2)

The three sphere $S^3$ carrying H-flux is described by an $SU(2)$ WZW model at level\[2\] $k = N - 2$. The world-sheet theory is supersymmetrized by adding four free fermions $\psi^0_L(z), \psi^i_L(z), \psi^0_R(\bar{z}), \psi^i_R(\bar{z})$.

It can be shown \[13,16\], that this theory has an extended $\mathcal{N} = (4, 4)$ superconformal symmetry containing an $\hat{SU}(2)_1 \times \hat{SU}(2)_{k+1} \times \hat{U}(1)$ current algebra. The standard $\mathcal{N} = (4, 4)$ superconformal algebra is the subalgebra generated by the following supersymmetry currents

$$G^0_L = J^0_L \psi^0_L + \sqrt{2 \over N} \left[ J^i_L \psi^i_L + \psi^1_L \psi^2_L \psi^3_L - \partial \psi^0_L \right]$$
$$G^i_L = J^0_L \psi^i_L + \sqrt{2 \over N} \left[ -J^i_L \psi^0_L + \epsilon^{ijk} J^j_L \psi^k_L - {1 \over 2} \epsilon^{ijk} \psi^0_L \psi^3_L \psi^k_L - \partial \psi^i_L \right]$$  \hspace{1cm} (2.3)

where $J^i_{L,R}$ are the bosonic $\hat{SU}(2)_k$ currents

$$J_L(z) = {k \over 2} g \partial g^{-1}, \quad J_R(\bar{z}) = {k \over 2} g^{-1} \bar{\partial} g,$$  \hspace{1cm} (2.4)

and $J^0_{L,R}$ are the $U(1)$ currents associated with the $\phi$ coordinate. The expression for the right moving supercurrents is similar. Note that in addition to the superconformal symmetry, a complete description of the string background involves a modular invariant conformal field theory. It has been shown in \[17\] that the conformal field theory describing the near horizon geometry of NS five-branes in flat space corresponds to the modular invariant WZW model in the A series. In \[3\], string theory on this background was conjectured to be dual to the decoupled theory on NS five-branes. This is a holographic description since the little string theories live on the boundary $\phi \to \infty$ of space-time.

Here, we consider the conformal field theory description of a string propagating on the near horizon geometry of NS five-branes transverse to $\mathbb{C}^2/\mathbb{Z}_n$ orbifold singularities. For

\[3\] This is the value of the level after decoupling the fermions by a chiral rotation.
technical reasons that will become clear later, we restrict to \( n \) prime. The orbifold group acts by

\[
z_1 \rightarrow e^{\frac{2\pi i}{n}} z_1, \quad z_2 \rightarrow e^{-\frac{2\pi i}{n}} z_2
\]  

(2.5)

The near horizon geometry for NS-branes embedded in this orbifold can be easily obtained from (2.1) by acting on the solution by (2.5). This only acts on the \( S^3 \) geometry which now becomes a Lens space \( S^3/Z_n \). In order to construct the conformal field theory describing the propagation of a string in this background we must identify the geometric \( Z_n \) orbifold action in the original conformal field theory. This can be easily derived by parameterizing the \( S^3 \) by the Euler coordinates \( 0 \leq \theta < \pi, 0 \leq \phi < 2\pi, 0 \leq \psi < 4\pi \) such that

\[
z_1 = re^{i\frac{\phi + \psi}{2}} \cos \frac{\theta}{2}, \quad z_2 = re^{i\frac{\phi - \psi}{2}} \sin \frac{\theta}{2}.
\]  

(2.6)

Identifying \( S^3 \) with the \( SU(2) \) group manifold, an arbitrary group element can be written

\[
g = e^{-i\frac{\psi}{2} \sigma_3} e^{-i\frac{\theta}{2} \sigma_2} e^{-i\frac{\phi}{2} \sigma_3}.
\]  

(2.7)

Then the action (2.5) translates into

\[
r \rightarrow r, \quad \theta \rightarrow \theta
\]

(2.8)

\[
\phi \rightarrow \phi, \quad \psi \rightarrow \psi + \frac{4\pi}{n}.
\]

It follows that \( Z_n \) is embedded in \( SU(2) \) along the Cartan generator and it acts on the group manifold by left multiplication

\[
g(z, \bar{z}) \rightarrow h^{-1} g(z, \bar{z}), \quad h = e^{\frac{2\pi i}{n} \sigma_3}.
\]  

(2.9)

Note that this results in a left/right asymmetric action on the bosonic currents (2.4)

\[
J_L(z) \rightarrow h^{-1} J_L(z) h, \quad J_R(\bar{z}) \rightarrow J_R(\bar{z}).
\]  

(2.10)

The radial dilaton field \( \phi \) is obviously \( Z_n \) invariant. Note that if \( n = 2 \), \( Z_2 \) is isomorphic to the center of \( SU(2) \) and both right and left moving currents are left invariant.

The orbifold action on fermions can be derived starting from the world-sheet realization of the \( SO(4) \simeq SU(2)_L \times SU(2)_R \) R-symmetry of the space-time theory \([17,3]\). As detailed above, the model has an \( SU(2)_L \times SU(2)_R \) symmetry generated by the bosonic
left and right moving currents. At the same time, the fermions $\psi^i_{L,R}$ transform in the adjoint of another level two $SU(2)_{L,R}$ symmetry generated by

$$A^i_L = \frac{1}{2} \epsilon_{ijk} \psi_j^L \psi_k^L, \quad A^i_R = \frac{1}{2} \epsilon_{ijk} \psi_j^R \psi_k^R.$$  \hspace{1cm} (2.11)

The $SO(4)$ symmetry is generated by the total currents

$$\tilde{J}_{L,R} = J_{L,R} + A_{L,R}.$$  \hspace{1cm} (2.12)

We have shown that $Z_n$ is embedded in the bosonic $SU(2)_L$ along the Cartan generator. In order to maintain the geometric interpretation, it follows that the orbifold group should be similarly embedded in the fermionic $SU(2)_L$. Therefore, $Z_n$ acts trivially on the right moving fermions. The correct action on the left moving fermions can be fixed by requiring that the world-sheet theory possess at least the $\mathcal{N} = (1, 1)$ supersymmetry of a Type II string. Therefore, we must choose the action on the fermions such that $G^0_L$ is left invariant under the orbifold action. Defining the currents $J^\pm_L = J^1_L \pm iJ^2_L$, the formula (2.10) yields

$$J^+_L \rightarrow e^{\frac{4\pi i}{n}} J^+_L,$$

$$J^-_L \rightarrow e^{-\frac{4\pi i}{n}} J^-_L,$$

$$J^3_L \rightarrow J^3_L.$$  \hspace{1cm} (2.13)

Invariance of $G^0_L$ in (2.3) fixes the action on the world-sheet fermions to be

$$\psi^+_L \rightarrow e^{\frac{4\pi i}{n}} \psi^+_L,$$

$$\psi^-_L \rightarrow e^{-\frac{4\pi i}{n}} \psi^-_L,$$

$$\psi^3_L \rightarrow \psi^3_L,$$  \hspace{1cm} (2.14)

where $\psi^\pm_L = \psi^1_L \pm i\psi^2_L$. When this action is combined with (2.13), it follows that the relevant theories are in fact asymmetric orbifolds \cite{18}. This fact is counterintuitive since the orbifold projection follows from a well defined geometric action.

The superconformal symmetry of the orbifold theory is determined by the currents (2.3) left invariant by the $Z_n$ action. In the right moving sector, all four spin $3/2$ currents are preserved since the action on them is trivial. In the left moving sector, it is easy to check that only $G^0$ and $G^3$ are left invariant if $n \geq 3$. This follows from the embedding of the $Z_n$ twist (2.13), (2.14) along the Cartan generator. Therefore the resulting theory has $\mathcal{N} = (2, 4)$ superconformal symmetry. In the case $n = 2$ all supercurrents are left invariant.
Note that these models constitute exceptions from the standard rule correlating spacetime and world-sheet supersymmetry [19].

2.1. The Orbifold Theory

In order to define consistent string backgrounds, the orbifold conformal field theories we construct must have modular invariant partition functions. Since the orbifold group acts asymmetrically, it is expected that significant restrictions on the allowed theories [18] will be imposed. As showed in the following, the modular invariance constraint restricts the value of the five-brane charge \( N = k + 2 \), to be an integer multiple of \( n \). Here \( k \) is the level of the bosonic WZW model as defined below (2.2).

In this section we analyze these constraints and compute the orbifold partition functions for \( n \geq 3 \) and prime. The case \( n = 2 \) presents certain peculiar aspects and will be studied separately. The method applied in the following is a generalization of the bosonic orbifolds discussed in [20,21,22].

The orbifold partition function can be constructed in the usual way

\[
Z_{\text{orb}} = \frac{1}{n} \sum_{p, q=0}^{n-1} (\alpha^p, \alpha^q), \tag{2.15}
\]

where \( \alpha \) is the \( n^{th} \) root of unity. The term \((\alpha^p, \alpha^q)\) corresponds to the partition function over states in the Hilbert space twisted by \( \alpha^p \) and with the operator \( \alpha^q \) inserted in the trace. In this notation the untwisted partition function \( Z_1 \) is given by

\[
Z_1 = \frac{1}{n} \sum_{q=0}^{n-1} (1, \alpha^q). \tag{2.16}
\]

Using the action of the modular group on \((\alpha^p, \alpha^q)\), the full partition function for \( n \) prime can be conveniently rewritten as

\[
Z_{\text{orb}} = \left(1 + \sum_{i=1}^{n} T^i S\right) Z_1 - Z, \tag{2.17}
\]

where \( T \) and \( S \) are the generators of the modular group \( SL(2, \mathbb{Z}) \) and \( Z \) is the partition function of the original model.

\(^4\) This is explained by the fact that the present models do not satisfy the hypotheses of the theorem. We thank T. Banks for explanations on these issues.
Modular invariance imposes the following constraint\footnote{For \( n \) not prime extra terms appear in the partition function (2.17) which have to be separately modular invariant, imposing extra conditions on the untwisted partition function. This is only a technical detail; we do not expect new physical phenomena if \( n \) is not prime.}

\[ T^n S Z_1 = S Z_1. \] (2.18)

The GSO projected partition function describing a string propagating in the background defined by (2.1) factorizes into bosonic and fermionic parts. Taking into account the fact that the fermions are effectively free, we have

\[ Z = Z_X^4 Z_A^A Z_W^W Z_\phi |Z_F|^2. \] (2.19)

\( Z_X \) and \( Z_\phi \) denote the partition function of a free scalar and that of the Liouville field respectively. \( Z_A^A Z_W^W \) is the A-modular invariant \( \widehat{SU}(2)_k \) partition function obtained by summing diagonally the \( \widehat{SU}(2)_k \) characters over the highest weight integrable representations

\[ Z_A^A Z_W^W = \sum_{j=0}^{k/2} |\chi^j|^2, \] (2.20)

and

\[ Z_F = \frac{1}{2} \left( \frac{\theta_3(q)}{\eta(q)} \right)^4 - \frac{1}{2} \left( \frac{\theta_4(q)}{\eta(q)} \right)^4 - \frac{1}{2} \left( \frac{\theta_2(q)}{\eta(q)} \right)^4 \] (2.21)

is the partition function of the free fermions. Note that we have divided by the infinite volume corresponding of the six non-compact bosons.

The orbifold group acts nontrivially only in the sector consisting of \((J^i, \psi^i)\) which describes a supersymmetrized \( \mathcal{N} = (1, 1) \) WZW model. As explained in [14], the orbifold theory is best described by rewriting the left moving sector of this theory in terms of an \( \mathcal{N} = 2 \) minimal model corresponding to the coset space \( SU(2)/U(1) \) and a free superfield \((\xi, \psi^3)\) corresponding to the \( U(1) \) current \( J_3^+ \). The current algebra can be written as

\[ J_3^+ = \frac{ik}{2} \partial \xi \]
\[ J_3^- = e^{i\xi} \psi^\para_+ \]
\[ J_3^- = e^{-i\xi} \psi^\para_- \] (2.22)
The scalar $\xi(z)$ is a compact scalar of unit radius and $\psi^\pm_{para}$ are chiral parafermion fields which parametrize the bosonic coset and the supercurrents of the minimal model. In this realization of the WZW model, the partition function (2.19) is

$$Z = \frac{1}{2} \sum_{s=0}^{3} \rho_s \sum_{j=0}^{k/2} \sum_{2m=-k-1}^{k+2} \chi_{jms}^{N=2}(q) L_{m}^{(k+2)}(q) \left( \frac{\Theta_s}{\eta} \right)^{3} \tilde{\chi}_{j}^{k}(\bar{q}) Z^4_{X} Z_{\phi} Z_{F}. \quad (2.23)$$

$\chi_{jms}^{N=2}(q)$ are minimal model characters. The index $s$ labels the different spin structures on the torus, $(\Theta_0, \Theta_1, \Theta_2, \Theta_3)$ are the standard theta functions $(\theta_2, \theta_3, \theta_1, \theta_4)$, and $(\rho_s) = (-1, 1, 0, -1)$. The compact scalar characters are labeled by $L_{m}^{(k+2)}$.

The $Z_n$ group acts by shifting the $\xi$ coordinate (2.22)

$$\xi \rightarrow \xi + \frac{4\pi}{n}. \quad (2.24)$$

This follows because the $Z_n$ twist is embedded along the Cartan generator parametrized by $\xi$ and therefore does not act on the minimal model characters. The untwisted partition function can be computed by inserting the shift operator along the $\xi$ coordinate. Since the momentum of the compact boson is $m$, one must insert the projector $\frac{1}{n} \sum_{l=0}^{n-1} e^{\frac{2\pi i m l}{n}}$ in the partition function (2.23). The result is

$$Z_1 = \frac{1}{2} \sum_{s=0}^{3} \rho_s \sum_{j=0}^{k/2} \sum_{2m=-k-1; 2m\equiv 0(n)}^{k+2} \chi_{jms}^{N=2}(q) L_{m}^{(k+2)}(q) \left( \frac{\Theta_s}{\eta} \right)^{3} \tilde{\chi}_{j}^{k}(\bar{q}) Z^4_{X} Z_{\phi} Z_{F}. \quad (2.25)$$

The orbifold partition function can be obtained from formula (2.17). This requires knowing the modular properties of the various characters. The computation outlined in the appendix leads to the final formula

$$Z_{orb} = \frac{1}{2} \sum_{s=0}^{3} \rho_s \sum_{j=0}^{k/2} \sum_{2m'=-k-1}^{k+2} \chi_{jms}^{N=2}(q) L_{m'}^{(k+2)}(q) \left( \frac{\Theta_s}{\eta} \right)^{3} Z^4_{X} Z_{\phi} Z_{F} + \frac{1}{2} \sum_{s=0}^{3} \rho_s \sum_{j=0}^{k/2} \sum_{2m=-k-1; 2m\equiv 0(n)}^{k+2} \chi_{jms}^{N=2}(q) L_{m}^{(k+2)}(q) \left( \frac{\Theta_s}{\eta} \right)^{3} Z^4_{X} Z_{\phi} Z_{F}. \quad (2.26)$$

The explicit computation shows that the orbifold theory is modular invariant if and only if

$$k + 2 \equiv 0 \ (\text{mod} \ n). \quad (2.27)$$
Therefore \( p = \frac{k+2}{n} \) is an integer number. The sum over \( m', m'' \) in the first term of (2.20) is restricted to

\[
m' - m'' \equiv 0 \pmod{p}, \quad m' + m'' \equiv 0 \pmod{n}, \quad m' \neq m''
\]

and represents the contribution of the twisted sectors. The second term is identical to (2.25) and represents the contribution of the untwisted sector.

The physical meaning of the modular invariance constraint (2.27) is that the resulting conformal field theory is consistent if and only if the five-brane charge is an integer multiple of \( n \). As we will see in section three, this consistency condition is exactly reproduced in the holographic dual theory on NS five-branes at the orbifold through gauge anomaly cancellation.

2.2. D-series and \( C^2/Z_2 \) orbifolds

As mentioned before, the case \( n = 2 \) exhibits some peculiar aspects that deserve further comments. First, note that in this case, \( Z_2 \) is isomorphic to the center of \( SU(2) \). Therefore, it acts trivially on the conserved Kac-Moody currents. Since it also acts trivially on fermions, it follows that the orbifold action leaves the eight supercurrents (2.3) invariant. However, \( Z_2 \) acts nontrivially on the Ramond sector spin fields which transform in the doublet of \( SU(2) \).

Note that for \( n = 2 \) the orbifold partition function can be written in the form

\[
Z_{\text{orb}} = \frac{1}{2} \sum_{s=0}^{4} \rho(s) \sum_{j=0}^{k/2} \sum_{2m=-k-1,2m=0(2)} \left[ \chi_{jms}^{N=2} + \chi_{(\frac{k}{2}-j)ms}^{N=2} \right] \hat{x}_j^k L_m^{(k+2)} \left( \frac{\Theta_s}{\eta} \right)^3 Z_X Z_\phi \bar{Z}_F \tag{2.29}
\]

where the non-diagonal terms of the form \( \chi_{(\frac{k}{2}-j)ms}^{N=2} \hat{x}_j^k \) correspond to the twisted sectors. Modular invariance constrains the level \( k \) to be even.

It is interesting to compare this example to the \( Z_2(-1)^{F_L} \) orbifold of the same model, where \((-1)^{F_L}\) represents space-time fermion number in the left moving sector. This orbifold conformal field theory describes string theory in the near horizon region of Type IIB NS five-branes at an orbifold five-plane. The effect of this extra twist is to undo the \( Z_2 \) projection in the left moving Ramond sector so that the untwisted partition function reads

\[
Z_1 = \sum_{j=0, \ 2j \in \mathbb{Z}} \chi_j^k \tilde{x}_j^k Z_X^4 Z_\phi |Z_F|^2. \tag{2.30}
\]
The orbifold partition function is then

\[ Z_{\text{orb}} = Z_{DWZW}^{D} Z_{\phi}^{4} |Z_{F}|^{2} \]  

(2.31)

where \( Z_{DWZW}^{D} \) is the D-modular invariant partition function of \( \widehat{SU(2)}_{k} \), as also found in [20,21,22]. Therefore, by using orbifold techniques we have shown that the near horizon geometry of the \( D_{N} \) little string theory is given by the D-modular invariant \( \widehat{SU(2)}_{k} \) partition function. Here \( k + 2 = 2N - 2 \), which is the total five-brane charge. In [5] this conformal field theory was conjectured to be dual to the six dimensional \( \mathcal{N} = (1,1) \) \( D_{N} \) little string theory.

In order to understand better the difference between the two models, let us consider the effect of gauging \((-1)^{F_L} \) on the space-time supercharges. The corresponding world-sheet currents are:

\[ Q_{L}^{A\alpha} = S_{L}^{\alpha} \Sigma_{L}^{A}, \quad Q_{R}^{B\bar{\alpha}} = S_{R}^{\bar{\alpha}} \Sigma_{R}^{B} \]  

(2.32)

where \( L, R \) refer to world-sheet chirality, \( \alpha, \bar{\alpha} \) are indices of the \( 4, \bar{4} \) representations of \( \text{Spin}(1,5) \) and \( A, B \) are doublet indices of the \( SU(2)_{L} \times SU(2)_{R} \) R-symmetry. The fields \( \Sigma_{L,R}^{A,B} \) are spin fields in the Ramond sector of the internal superconformal field theory. Although these formulas are identical in the two cases, gauging \((-1)^{F_L} \) leaves them invariant in agreement with [23]. At the same time, gauging \( Z_{2} \) projects the left moving supercharges out. Similar differences will appear in all sectors of the theory characterized by non-zero space-time fermion number.

3. Little String Theories at Orbifold Singularities

In this section, we analyze the world volume theories of Type IIB NS five-branes localized at \( C^2/Z_{n} \) orbifold singularities. These theories have been first considered by Intriligator in [24]. As explained there, the bulk modes decouple in the limit \( g_{s} \to 0, \) with \( M_{s} \) fixed, leading to new \( \mathcal{N} = (0,1) \) non-critical string theories. At low energies, these are expected to flow to an interacting \( \mathcal{N} = (0,1) \) superconformal fixed point. The infrared degrees of freedom consist of two sectors. The first sector corresponds to the degrees of freedom

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6 In order to keep the notation simple, the ghost factors of world-sheet operators will not be explicitly written throughout the paper.

7 This limit corresponds in the M-theory picture to keeping the tension of the strings living in the five-brane constant.
freedom localized at the singularity and it can be described along the moduli space by \( n \mathcal{N} = (0,1) \) tensor multiplets and \( n \mathcal{N} = (0,1) \) hypermultiplets. Note that in the absence of the five-branes, the latter would reduce to the infrared degrees of freedom of the \( A_n \), \( \mathcal{N} = (0,2) \) non-critical string theory. The second sector corresponds to the five-brane degrees of freedom and is described by certain gauge theories.

The gauge theory sector is derived based on the S-dual picture of IIB Dirichlet five-branes transverse to an orbifold singularity \([25,26,27]\). The field content is determined by choosing a representation of \( Z_n \) on the Chan-Paton factors. If we choose a general representation \( R = \bigoplus_{i=0}^{n-1} v_i R_i \), where \( R_i \) are the one dimensional representations of \( Z_n \) and \( v_i \) their multiplicities, then the projection yields a gauge theory with gauge group

\[
\prod_{i=0}^{n-1} U(v_i), \tag{3.1}
\]

and \( n \) hypermultiplets in the bifundamental representation \((v_i, \overline{v}_{i+1})\). The allowed representations are constrained by gauge anomaly cancellation. Cancellation of the \( \text{Tr} F^4 \) term in the anomaly polynomial restricts the allowed representations to the regular one, where \( v_i = p \) for all \( i \). Then, the anomalies can be canceled by a six-dimensional Green-Schwarz mechanism which relies essentially on the twisted sector tensor multiplets associated to the orbifold singularity. Finally, the \( U(1) \) factors in the quiver theory are anomalous due to the presence of bifundamental matter \([25,29]\). Therefore they become massive by pairing with the hypermultiplets, except for the diagonal \( U(1) \) which decouples since it has no charged matter. We are finally left with a \( \prod_{i=0}^{n-1} SU(p) \) quiver gauge theory with bifundamental matter. The restriction to the regular representation can be alternatively viewed as a tadpole cancellation condition \([27]\). As noted in \([24]\), the effective gauge coupling depends linearly on the Coulomb branch coordinates spanned by the scalar components of the tensor multiplets. The absence of Landau poles indicates the existence of a non-trivial fixed point at the origin.

These results mesh very nicely with the expectations that follow by generalizing the conjecture in \([3]\). In our context, the duality states that string theory on the near horizon region of NS five-branes at a \( C^2/Z_n \) orbifold singularity is dual to the decoupled theory of NS five-branes at the singularity. Consistency of these two theories is achieved at

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8 Recently, interesting gauge theories have been constructed in the context of orbifolds with discrete torsion by choosing projective representations of the discrete group \([28]\).
exactly the same values of the five-brane charge, that is, a multiple of $n$. The matching of consistency conditions provides strong evidence for the finite $N$ duality. In brief, modular invariance of the conformal field theory describing space-time is mapped to gauge anomaly cancellation in the little string theory.

3.1. Operator Matching and Holography

The conformal field theory approach is expected to shed some light on the holographic nature of the $\mathcal{N} = (0,1)$ string-theories. In [5], the six dimensional $\mathcal{N} = (0,2)$ and $\mathcal{N} = (1,1)$ non-critical string theories have been conjectured to be dual to full string theories in linear dilaton backgrounds. Here we propose that this new duality can be extended to the $\mathcal{N} = (0,1)$ theories discussed in this section.

In order to support this proposal, the main idea is to compare the spectrum of chiral primaries of the WZW orbifolds to operators of $\mathcal{N} = (0,1)$ theories in short representations of the supersymmetry algebra. As described in the previous subsection, the low energy gauge theory on the branes is the quiver $SU(p)^n$ theory which can be identified to the $Z_n$ projection of a $\mathcal{N} = (1,1) U(N)$ gauge theory ($N = pn$). The R-symmetry of the latter is $Spin(4) \simeq SU(2)_L \times SU(2)_R$ which is identified with the $SU(2)_L \times SU(2)_R$ level $N$ current algebra in the conformal field theory. The $Z_n$ projection is embedded in both cases in $SU(2)_L$ so that the R-symmetry of the quiver gauge theory is $SU(2)_R$.

Let $X^m$, $m = 1 \ldots 4$, denote the scalar components of the $\mathcal{N} = (1,1)$ vector multiplet. In the absence of the $Z_2$ projection, the theory contains short supersymmetry multiplets

$$\text{Tr} \left( X^{i_1} X^{i_2} \ldots X^{i_{2j+2}} \right), \quad 0 \leq 2j \leq N - 2,$$

which transform in the symmetric traceless $(j + 1, j + 1)$ representation of $SU(2)_L \times SU(2)_R$. The first descendant fields in the multiplet are generated by the action of the supercharges $Q_L, Q_R$. On group theory grounds, the action of $Q_L$ on (3.2) results in two irreducible $SU(2)_L$ representations of spins $j + \frac{1}{2}$ and $j + \frac{3}{2}$. However, the higher spin operators are absent since the operators (3.2) are chiral with respect to the $\mathcal{N} = (1,1)$ supersymmetry algebra. Therefore, the descendant contains the $(j + \frac{1}{2}, j)$ component. Note that the latter is still a chiral representation with respect to $Q_R$, therefore it defines a

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9 We thank N. Seiberg for very helpful explanations on various aspects of the following discussion.
$Q_R$-short multiplet. A similar discussion holds for $Q_R$-descendants, inverting the roles of $Q_L$ and $Q_R$.

\textit{i) Untwisted sector.} Recall that in the covering $\mathcal{N} = (4,4)$ theory the chiral primary operators in the NS-NS sector are of the form\textsuperscript{10} [31, 33]

$$\psi V_j \bar{\psi} \bar{V}_j e^{\beta_j \phi}, \quad \beta_j = j \sqrt{\frac{2}{N}}, \quad (3.3)$$

Since the fermions transform in the spin $j = 1$ representations of $SU(2)_{L,R}$, the operators in [31] decompose in $j-1, j, j+1$ irreducible blocks. As explained in [3], the BRST chiral primaries correspond to the $j+1$ representation (the others being either unphysical or descendants). Furthermore, they can be holographically identified with the six dimensional short multiplets (3.2). The orbifold projection leaves invariant the operators with $2(j+1) \equiv 0 \pmod{n}$.

Another set of operators can be derived from the action of left moving $\mathcal{N} = (4,4)$ spectral flow \textsuperscript{31} on (3.3). The resulting R-NS operators are of the form\textsuperscript{11}

$$\Sigma V_j \bar{\psi} \bar{V}_j e^{\beta_j \phi} \quad (3.4)$$

where $\Sigma$ is an $Spin(4)$ spin-field of definite chirality obtained by bosonising the four free fermions. Since the spin field $\Sigma$ transforms in the $j = \frac{1}{2}$ representation of $SU(2)_L$, the operators (3.4) decompose in irreducible blocks of spin $j \pm \frac{1}{2}$. The physical operators are selected by imposing BRST invariance. The relevant piece of the BRST operator is (up to ghost factors) determined by the gauged superconformal generator

$$G^0_L = J^0_L \psi^0_L + \sqrt{\frac{2}{N}} [J^1_L \psi^1_L + \psi^2_L \psi^3_L - \partial \psi^0_L]. \quad (3.5)$$

A direct computation shows that the physical operators correspond to the spin $j + \frac{1}{2}$ representation in (3.4) while the spin $j - \frac{1}{2}$ representation is not BRST invariant. The orbifold projection leaves invariant the operators corresponding to $2j + 1 \equiv 0 \pmod{n}$.

The operator identification for the orbifold theories emerges from the above elements.

In the six dimensional noncritical string theory, the $Z_n$ invariant operators are of the form (3.2) with $2j + 2 \equiv 0 \pmod{n}$ and first order $Q_L$-descendants with $2j + 1 \equiv 0 \pmod{n}$. As noted in the discussion following (3.2) both classes define short $\mathcal{N} = (0,1)$ multiplets.

\textsuperscript{10} The operators are written in $(-1)$ picture, but the ghost part is omitted for simplicity.

\textsuperscript{11} These operators are written in $(-1/2)$ picture, again omitting the ghost factors.
The former are naturally identified with invariant NS-NS chiral primaries while the latter can be related to invariant R-NS chiral primaries. This gives further support of the finite \(N\) conjecture.

\textit{ii) Twisted sectors.} Here the operator content can be deduced from the partition function (2.26). Recall that the minimal model primary operators are of the form \([32]\). Restricting to the NS sector, the theory also contains descendants of the form \(V_{j+1}^j\) which generate the submodule \(\mathcal{H}_{j+1,s=2}^{j+1}\) \([32]\). As observed in \([3]\), in the context of \(\mathcal{N} = (1, 1)\) theories, it is not clear if the left-right asymmetric operators can be identified in the six dimensional theory. Therefore, in the following we will focus on left-right symmetric twisted chiral primaries. Taking into account the restrictions (2.28), and \([14]\), these are given by

\[
V_{j+1}^j e^{-ij\xi} (\bar{V})_{j+1}^j e^{i\beta_j\phi}\tag{3.6}
\]

with

\[
2(j + 1) \equiv 0 \pmod{p}, \quad 2(j + 1) \leq k + 1. \tag{3.7}
\]

Therefore we obtain \(n - 1\) twisted chiral primaries with \(SU(2)_R\) isospin given by

\[
2j + 2 = p, 2p, \ldots, (n - 1)p. \tag{3.8}
\]

The corresponding six dimensional operators can be found by analogy with \([5]\). Note that for the D-series studied there, there is a single twisted symmetric operator which has been identified with the Pfaffian of the low energy gauge theory. In the present case, the quiver gauge theory contains dibaryon operators \([33]\) which can be thought as generalizing the Pfaffian operator. For concreteness, let \(i^a, \bar{i}^a\) denote indices in the \(p, \bar{p}\) representation of the \(a\)-th \(SU(p)\) factor of the gauge group, \(a = 0, \ldots, n - 1\). Let \(X_{a,a+1}, \bar{X}_{a+1,a}\) denote the bifundamental hypermultiplets corresponding to the edges of the quiver diagram. Note that \((Z_{a,a+1}^A) \equiv \left(X_{a,a+1}, -\bar{X}_{a+1,a}\right)\) transforms as a doublet of \(SU(2)_R\). For each pair \((a, a + 1)\) we can construct the operator

\[
U_{a,a+1} = \epsilon_{i_1}^{a_1} \ldots \epsilon_{i_p}^{a_p} \epsilon_{\bar{i}_1}^{a+1_1} \ldots \epsilon_{\bar{i}_p}^{a+1_p} Z_{a,a+1}^{A_{a,a+1}} \ldots Z_{a,a+1}^{A_{a,a+1}}. \tag{3.9}
\]

The indices \(A_{a,a+1}^k\) are symmetrized so that the operators (3.9) transform in the spin \(\frac{p}{2}\) representation of \(SU(2)_R\). Then, we propose to identify the symmetric twisted operators

\[\text{Note that here the } N = 2 \text{ minimal characters are labelled according to } [32], \text{ hence they are related to those occurring in (2.23) by linear transformations.}\]
with the operators

\[ S(U_{01}U_{12} \ldots U_{a,a+1}) \]  

(3.10)

where \( S \) denotes the projection operator on the top \( SU(2)_R \) isospin.

4. Conclusions

To summarize, the main result of the present paper is an extension of the conjecture in \[ \text{Ref.} \] to five-branes transverse to an \( C^2/Z_n \) orbifold. This provides new holographic examples of string theory backgrounds relating the \( R^{5,1} \times R \times S^3/Z_n \) geometry with a radial linear dilaton to \( \mathcal{N} = (0,1) \) six dimensional non-critical string theories. These theories flow at low energies to quiver gauge theories coupled to \( n \), \( \mathcal{N} = (0,1) \) tensor multiplets representing the degrees of freedom localized at the singularity. The infrared dynamics is governed by an interacting superconformal fixed point.

In spite of the reduced supersymmetry, the conjectured duality can be tested by analyzing the conformal field theory orbifold in parallel with the brane theories. Along these lines, we have shown that the modular invariance constraints of the orbifold theory are mapped to anomaly cancellation conditions in the decoupled \( \mathcal{N} = (0,1) \) theory. We have also matched the left-right symmetric chiral primary operators of the worldsheet superconformal algebra with short multiplets of the six dimensional \( \mathcal{N} = (0,1) \) supersymmetry algebra. The twisted sector multiplets enter in a novel way in the operator matching. The main puzzle present in the operator analysis is the role of the tensor multiplet operators. Since they are neutral under the \( SU(2)_R \) R-symmetry, it is unclear whether we can identify them with states in the spectrum of the orbifold conformal field theory.

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Appendix A. The Orbifold Partition Function

In this appendix we compute the orbifold partition function and derive the modular invariance condition \( k + 2 \equiv 0 \pmod{n} \). The starting point is the untwisted partition function \( Z_1 \):

\[
Z_1 = \frac{1}{2} \sum_{s=0}^{3} \rho_s \sum_{j=0}^{k/2} \sum_{2m = -k-1}^{2m \equiv 0(n)} \chi_{jms}^{N=2}(q) L_m^{(k+2)}(q) \left( \frac{\Theta_s}{\eta} \right)^3 \phi_j^{k}(q) Z_{\phi} \bar{Z}_F. \tag{A.1}
\]

The modular group acts on the characters in (A.1) according to

\[
\Theta_s \left( \frac{-1}{\tau} \right) = \sum_{s'=0}^{3} C_{ss'} \frac{\Theta_{s'}}{\eta}(\tau)
\]

\[
\Theta_s(\tau + 1) = \sum_{s'=0}^{3} \exp \left( \frac{\pi(1-s^2)}{4} \right) D_{ss'} \frac{\Theta_{s'}}{\eta}(\tau)
\]

\[
\chi_{jms}^{N=2} \left( \frac{-1}{\tau} \right) = \frac{1}{k+2} \sum_{s'=0}^{3} \sum_{j'=0}^{k/2} \sum_{m'=-k-1}^{k+2} \frac{\chi_{ss'} e^{\frac{\pi i m m'}{k+2}} \sin \left[ \frac{\pi(2j+1)(2j'+1)}{k+2} \right]}{k+2} \chi_{j'm's'}^{N=2}(\tau)
\]

\[
\chi_{jms}^{N=2}(\tau + 1) = \exp \left( 2\pi i \left[ \frac{j(j+1)}{k+2} - \frac{m^2}{k+2} + \frac{s^2}{8} \right] \right) \chi_{jms}^{N=2}(\tau)
\]

\[
L_m^{(k+2)} \left( \frac{-1}{\tau} \right) = \frac{1}{\sqrt{2(k+2)}} \sum_{m'=-k-1}^{k+2} \exp \left( \frac{2\pi i m m'}{k+2} \right) L_{m'}^{(k+2)}(\tau)
\]

\[
L_m^{(k+2)}(\tau + 1) = \exp \left( \frac{2\pi i m^2}{k+2} \right) L_m^{(k+2)}(\tau)
\]

\[
\chi_j^k \left( \frac{-1}{\tau} \right) = \sqrt{\frac{2}{k+2}} \sum_{j'=0}^{k/2} \sin \left[ \frac{\pi(2j+1)(2j'+1)}{k+2} \right] \chi_j^k(\tau)
\]

\[
\chi_j^k(\tau + 1) = \exp \left( 2\pi i \left[ \frac{j(j+1)}{k+2} \right] \right) \chi_j^k(\tau). \tag{A.2}
\]

The matrices \( C_{ss'} \) and \( D_{ss'} \) are given by

\[
C_{ss'} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad D_{ss'} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{A.3}
\]

We will first assume \( k + 2 = np \) with \( p \in \mathbb{Z} \) and prove that the resulting partition function is modular invariant. The details of the computation will also show that if \( k+2 = np + r, \ r = 1, \ldots, p - 1 \), modular invariance does not hold. The partition function can
be obtained by acting with the modular group on the untwisted partition function \(2.17\). In order to evaluate \(S \cdot Z_1\), we need the following orthogonality relations

\[
\sum_{j=0}^{k/2} \sin \left[ \frac{\pi (2j + 1)(2j' + 1)}{k + 2} \right] \sin \left[ \frac{\pi (2j + 1)(2j'' + 1)}{k + 2} \right] = \frac{k + 2}{2} \delta_{j,j''} \quad (A.4)
\]

\[
\sum_{m=-k-1}^{k+2} e^{\frac{4\pi i m m'}{k+2}} e^{-\frac{4\pi i m m''}{k+2}} = 2p \delta_{m'-m'' \equiv 0 (p)}.
\]

Then, a direct evaluation shows that

\[
S \cdot Z_1 = \frac{1}{2n} \sum_{s=0}^{3} \rho_s \sum_{j=0}^{k/2} \sum_{m',m''=-k-1}^{k+2} \chi_{j}^{N=2}(q) L_{m''}^{(k+2)}(q) \left( \frac{\Theta_s}{\eta} \right)^3 \chi_{j}^{k}(q) Z_{\phi} Z_{F} \quad (A.5)
\]

where the sum over \(m',m''\) is restricted to \(m' - m'' \equiv 0 (p)\). The \(T^l\) action on \(A.5\) induces a the following phase to each term in \(A.5\)

\[
\exp 2\pi i l \left[ \frac{m'^2 - m''^2}{k + 2} \right]. \quad (A.6)
\]

In particular, it is clear that \(T^n S Z_1 = S Z_1\), therefore modular invariance is satisfied. The sum over \(l = 0, \ldots n - 1\) can be performed by noting that

\[
\sum_{l=0}^{n-1} e^{2\pi i l \frac{m'^2 - m''^2}{k + 2}} = n \delta_{m'+m'' \equiv 0 (n)}. \quad (A.7)
\]

After subtracting the original partition function \(Z\), the final result is

\[
Z_{orb} = \frac{1}{2} \sum_{s=0}^{4} \rho_s \sum_{j=0}^{k/2} \sum_{m'=m''=-k-1}^{k+2} \chi_{j}^{N=2}(q) L_{m''}^{(k+2)}(q) \left( \frac{\Theta_s}{\eta} \right)^3 Z_{\phi} Z_{F} + \quad (A.8)
\]

\[
\frac{1}{2} \sum_{s=0}^{4} \rho_s \sum_{j=0}^{k/2} \sum_{m=-k-1,2m \equiv 0(n)}^{k+2} \chi_{j m s}^{N=2} L_{m}^{(k+2)} \chi_{j}^{k}(q) Z_{\phi} Z_{F}
\]

where \(m',m''\) are constrained by

\[
m' - m'' \equiv 0 (\text{mod } p), \quad m' + m'' \equiv 0 (\text{mod } n), \quad m' \neq m''. \quad (A.9)
\]

Finally, if \(k + 2\) is not a multiple of \(n\), an explicit evaluation of \(S Z_1\) shows that the relation \(T^n S Z_1 = S Z_1\) does not hold. Therefore the orbifold theory is not modular invariant in this case.
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