Dynamics of the Parker–Jeans Instability of Gaseous Disks Including the Effect of Cosmic Rays

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Abstract

The effect of cosmic rays on the Parker–Jeans instability in magnetized self-gravitating gaseous disks is studied using three-dimensional magnetohydrodynamic simulations with cosmic rays taken as a massless fluid with notable pressure. Cosmic-ray diffusion along the magnetic field is taken into account in the simulation. The initial configuration of the disk is a magnetized cold gas slab sandwiched between hot coronae (on top and bottom). We show that cosmic rays play an important role in the formation of filaments or clumps in some parameter regimes. In a certain range of the thickness of the gas slab, the cosmic-ray diffusion coefficient plays a decisive role in determining whether the filaments lie along or perpendicular to the magnetic field. We also consider the effect of rotation on the system.

Unified Astronomy Thesaurus concepts: Magnetohydrodynamical simulations (1966); Interstellar clouds (834); Cosmic rays (329); Magnetic fields (994); Collapsing clouds (267)

1. Introduction

In our Galaxy, the interstellar medium (ISM) comprises different components, such as different phases of gas, magnetic fields, and cosmic rays. All these components have similar energy density (e.g., Parker 1969; Ginzburg & Ptuskin 1976; Ferrière 2001; Cox 2005). It is known that cosmic rays are dynamically important in the structure and evolution of ISM, yet not many studies have been devoted to the influence of cosmic rays on the ISM. Nevertheless, in the past couple of decades efforts have been made on the influence of cosmic rays on instabilities (say, Parker instability, magneto-rotational instability) (e.g., Parker 1966; Kuznetsov & Ptuskin 1983; Hanasz 1997; Hanasz & Lesch 1997; Ryu et al. 2003; Kuwabara et al. 2004; Kuwabara & Ko 2006, 2015; Hanasz et al. 2009; Ko & Lo 2009; Lo et al. 2011; Heintz & Zweibel 2018), and on cosmic ray-modified structures and outflows (e.g., Ghosh & Ptuskin 1983; Ko & Webb 1987; Breitschwerdt et al. 1991, 1993; Ko 1991a, 1991b; Ko et al. 1991; Everett et al. 2008; Yang et al. 2012; Girichidis et al. 2016; Recchia et al. 2016; Pfrommer et al. 2017; Wiener et al. 2017; Farber et al. 2018). Generally speaking, cosmic rays often enhance instabilities. Depending on the context, they can assist or hinder outflows or winds. The diffusion of cosmic rays can affect the growth rate of instability. For instance, the growth rate of the Parker instability increases if the diffusion coefficient of cosmic rays is larger (Kuwabara et al. 2004). Moreover, we note that cosmic-ray diffusion may have some subtle effect on the dynamics of the system. The present work will illustrate one example.

An important subject in molecular cloud (MC) and star-formation research is the relation between magnetic fields and molecular clouds (e.g., see the review by Crutcher 2012). The orientation between magnetic fields and cloud filaments or cores reveals the dynamics of cloud collapse. Tassis et al. (2009) derived the intrinsic shapes and magnetic-field orientations of 24 MCs by statistical analysis using dust emission and polarization data from the Hertz polarimeter. They showed that the best-fitting intrinsic magnetic-field orientation is close to the direction of the minor axis of the oblate disks. Li et al. (2013) made use of near-infrared dust extinction maps and optical stellar polarimetry to compare the orientations between 13 filamentary clouds in the Gould Belt and their local intercloud magnetic fields. They obtained a bimodal distribution in which the clouds tend to be either parallel or perpendicular to the mean direction of the magnetic field. Soler et al. (2013) studied the relative orientation of the magnetic field with respect to the density structures by synthetic observations of the simulated turbulent molecular clouds. They adopted the Histogram of Relative Orientations (HRO) method, which utilizes the gradient to characterize the directionality of column density structures on multiple scales. They concluded that in most cases the orientation of the magnetic field is parallel to the density structure. However, in strongly magnetized cases, the orientation changes from parallel to perpendicular and the density is higher than a critical density. Planck Collaboration Int. (2016) evaluated the relative orientation of the magnetic field inferred from the Planck polarization observations at 353 GHz with respect to the gas column density structures for 10 nearby Gould Belt MCs by means of HRO. They found that the relative orientation changes from parallel to perpendicular with increasing column density.

The bulk of cosmic rays in ISM are low-energy (below a few hundred MeV). As they travel through ISM, they lose energy via ionization (and through damping of waves they excited). An increase in ionization rate can heat up gas and hinder diffusion of magnetic fields, thus affecting star-formation processes (e.g., Fatuzzo et al. 2006; Yusef-Zadeh et al. 2007; Glassgold et al. 2012; Bertram et al. 2015). We are interested in the dynamical influence of cosmic rays on star formation, in particular the formation and development of clouds.

Chou et al. (2000) studied the dynamics of the Parker–Jeans instability with a linear-stability analysis and magnetohydrodynamic (MHD) simulation. They showed the process of interstellar gas aggregation to molecular clouds. Kuwabara &
Ko (2006) added cosmic rays into the system and showed, by linear-stability analysis, that the self-gravitating gaseous disks is less unstable if cosmic-ray pressure is larger, and more unstable if the cosmic-ray diffusion coefficient is larger. However, the nonlinear development of the system has not been investigated. In view of recent progress in numerical techniques in MHD simulation with cosmic rays, we would like to revisit the problem of Parker–Jeans instability of a disk until the nonlinear stage. In the case of no cosmic-ray diffusion, the set of MHD equations with cosmic rays can be written in a fully conservation form (and cosmic rays can be expressed as a polytropic gas, $P_c \propto \rho^\gamma$; Kudoh & Hanawa 2016). The set of equations can then be simulated more precisely, e.g., in dealing with shock problems. The treatment of the cosmic-ray diffusion, which is the parabolic term in the cosmic-ray energy equation, has more restrictive time-step constraints than that in the system without cosmic-ray diffusion for explicit methods. Implicit methods can overcome such restrictions, but they involve inverting a large matrix, which is computationally expensive. Super-time-stepping methods (e.g., Alexiades et al. 1996) is a tradeoff between explicit and implicit methods in this regard. These methods can be viewed as an explicit Runge–Kutta method with several internal stages using the recursion relations associated with Chebyshev Polynomials. Meyer et al. (2012) presented a better stability super-time-stepping method, which is a multi-stage Runge–Kutta method based on the recursion sequence for Legendre polynomials instead of Chebyshev Polynomials. Usually, super-time-stepping methods are used in solving the heat conduction problem. On the other hand, it is possible to use such methods in solving the cosmic-ray diffusion problem. In this paper, we applied this method to solve the anisotropic cosmic-ray diffusion problem, and study the effect of cosmic rays on the Parker–Jeans instability through MHD simulations. A linear-stability analysis is performed for comparison.

This paper is organized as follows. In Section 2 we present the governing equations of the self-gravitating disk and the initial equilibrium model, the two-temperature layered disk. In Section 3 the results of MHD simulations are presented. Section 4 provides a summary and discussion.

2. Models

2.1. Two-fluid Self-gravitating Disk

We adopt a two-fluid MHD system. One fluid is the common magnetized thermal plasma and the other one is cosmic rays. Cosmic rays are considered a massless fluid with notable energy density (or pressure). Cosmic rays are coupled to the plasma via magnetic fluctuations, resulting in cosmic-ray advection and diffusion in the plasma. The energy exchange between the plasma and cosmic rays is facilitated by the work done by the cosmic-ray pressure gradient. In a rotating frame, the system is governed by the total mass, momentum, and energy equation of the system:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0, \tag{1}
\]

\[
\frac{\partial}{\partial t}(\rho V) + \nabla \cdot \left[ \rho V V + \left( P_g + P_e + \frac{B^2}{2\mu_0} \right) I - \frac{B B}{\mu_0} \right] = -\mathbf{\nabla} \cdot \left( \mathbf{g}_{\text{ext}} + 2\Omega \times V + \Omega \times (\Omega \times \mathbf{r}) \right), \tag{2}
\]

\[
\frac{\partial}{\partial t} \left( E + E_c + \frac{B^2}{2\mu_0} \right) + \nabla \cdot \left[ \left( E + E_c + P_g + P_e \right) V - \frac{(V \times B) \times B}{\mu_0} \right] = \nabla \cdot \left( \kappa_b \mathbf{b} \cdot \nabla E_c \right) - \rho V \cdot \left[ \nabla \psi - \mathbf{g}_{\text{ext}} + \Omega \times (\Omega \times \mathbf{r}) \right], \tag{3}
\]

supplemented by the cosmic-ray energy equation, the induction equation for magnetic fields and the Poisson equation for self-gravity:

\[
\frac{\partial E_c}{\partial t} + \nabla \cdot [(E_c + P_e) V] = V \cdot \nabla P_e + \nabla \cdot \left( \kappa_b \mathbf{b} \cdot \nabla E_c \right), \tag{4}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (V \times \mathbf{B}), \tag{5}
\]

\[
\nabla^2 \psi = 4\pi G\rho, \tag{6}
\]

where $E = E_k + E_g = \rho V^2/2 + P_g/\gamma_g - 1$ is the sum of kinetic and thermal energy density of the plasma; $\rho$, $V$, $P_g$, and $\gamma_g$ are the plasma density, flow velocity, thermal pressure, and polytropic index; $E_c = P_e/\gamma_e - 1$, $P_e$, and $\gamma_e$ are the energy density, pressure, and polytropic index for cosmic rays; $\psi$ and $\mathbf{g}_{\text{ext}}$ are the gravitational potential for self-gravity and the external gravitational acceleration; $\Omega$ is the rotational angular frequency; $\mathbf{B}$, $\mathbf{b} = \mathbf{B}/|\mathbf{B}|$ are the magnetic field and the unit vector in the direction of the magnetic field; $\kappa_b$ is the cosmic-ray diffusion coefficient along the magnetic field; and $\mathbf{I}$ is the unit tensor.

2.2. Equilibrium Model

We aim to study a local slab portion of a rotating, self-gravitating disk. We adopt a local Cartesian coordinate system $(x, y, z)$ such that $e_x = e_o$, $e_y = -e_r$, and $e_z = e_z$, where $(r, \phi, z)$ is the cylindrical coordinate system of the disk. We set up a simple hydrostatic equilibrium model as the initial background configuration for the simulation (see Chou et al. 2000). Assume the centrifugal force is balanced by the gravitational force in the horizontal direction, and all other quantities depend on $z$ only. In addition, assume the magnetic field is lying horizontally (and there is no cross field-line diffusion of cosmic rays). Then with $V = 0$, Equations (1), (3), (4), and (5) are satisfied automatically. There are only two equations left: the Poisson equation (Equation (6)),

\[
\frac{d^2 \psi}{dz^2} = 4\pi G\rho, \tag{7}
\]

and the momentum equation (Equation (2)), which becomes the magnetohydrostatic equation ($P_B = B^2/2\mu_0$),

\[
\frac{1}{\rho} \frac{dP_i}{dz} + \frac{d\psi}{dz} = \frac{1}{\rho} \frac{d}{dz} (P_g + P_B + P_e) + \frac{d\psi}{dz} = g_{\text{ext}} = 0, \tag{8}
\]

where $P_i$ is the total pressure and $g_{\text{ext}}$ is the external gravity due to other sources, for example, the stellar disk in the case of the Galactic disk. Eliminating $\psi$ from Equations (7) and (8), we
where \( h_i = \int dR/\rho \) can be called the total enthalpy. If the equation of state \( P_i = R(\rho) \) is given, then Equation (9) can be solved. We note that \( P_g, P_h, \) and \( P_e \) are not constrained by their energy equations (as they are satisfied automatically). Thus, for simplicity, we take \( P_h = \alpha P_g \) and \( P_e = \beta P_g \), and adopt a polytropic equation of state for the gas \( P_g \propto \rho^{\gamma_g} \). We then have \( h_i = C_z^2(1 + \alpha + \beta) / (\gamma_g - 1) \), where \( C_z^2 = \gamma_g P_g / \rho \) is the gas sound speed.

Furthermore, assume that the equilibrium gas layer is sandwiched between high-temperature gas layers given by

\[
T(z) = 0.5 \times \left[ T_{\text{cor}} + T_0 + (T_{\text{cor}} - T_0) \times \tanh \left( \frac{|z| - z_{\text{cor}}}{\Delta z} \right) \right],
\]

where \( z_{\text{cor}} \) and \( \Delta z \) are the half-thickness of the cold gaseous disk and the width of transition region between the cold gas and hot gas layer, \( T_0 \) and \( T_{\text{cor}} \) are the temperatures of the cold gas layer and hot gas sandwiching the cold gas layer, respectively. In Kuwabara & Ko (2006), \( T_{\text{cor}} \) was set as infinity for the linear-stability analysis. The initial equilibrium condition is obtained by solving Equation (9) numerically using the Runge–Kutta method.

In the following MHD simulations, it is set as a finite value (Shibata et al. 1989). The scale height of the density is defined as \( H = C_{\text{ab}}/\sqrt{(1 + \alpha + \beta)/(2\pi G\rho_0\gamma_g)} \), where the subscript 0 denotes the value at the midplane. Quantities are normalized to the density, velocity, and length, \( \rho_0, C_{\text{ab}}, \) and \( H_0 = C_{\text{ab}}/\sqrt{2\pi G\rho_0\gamma_g} \), respectively. As fiducial values, we pick \( \rho_0 = 1.67 \times 10^{-19} \text{ kg m}^{-3}, \) \( C_{\text{ab}} = 5 \text{ km s}^{-1}, \) \( H_0 = 20 \text{ pc}, \gamma_g = 1.05, \gamma_c = 4/3, \) and the unit of time is \( H_0/C_{\text{ab}} \sim 4 \text{ Myr}. \)

The cosmic-ray diffusion coefficient is estimated to be \( 3 \times 10^{21} \text{ m}^2 \text{s}^{-1} \) (e.g., Berezinskii et al. 1990), and the normalized diffusion coefficient \( \kappa_i \) is 100. Here, we neglect the external gravity because it does not have a significant influence on the dynamics of the system when the ratio of the external gravity to the self-gravity is less than one (e.g., Chou et al. 2000).

3. Three-dimensional Cosmic-Ray MHD Simulation with Self-gravity

3.1. Numerical Procedure

We solve the MHD equations supplemented by the cosmic-ray energy equation and the Poisson equation for self-gravity by numerical simulation. We adopt a Harten–Lax–van Leer Discontinuities method (Miyoshi & Kasano 2005) for the advection part of the numerical solver. The self-gravity part (Poisson equation) is solved by the finite-difference method, and the large matrix inversion by the biconjugate gradients stabilized (BICGStab) method. We apply a super-time-stepping scheme called a second-order accurate s-stage Runge–Kutta Legendre scheme (RKL2; e.g., Meyer et al. 2012) to solve the cosmic-ray diffusion part in the cosmic-ray energy equation, which is the parabolic mathematically and is known to be computationally expensive. Usually, this part is solved by an implicit scheme to prevent the restrictive time-step constraints. However, we already have applied the implicit method to solve the Poisson equation for self-gravity, and it will be too computationally costly to again apply the implicit method for the cosmic-ray diffusion part. Thus, we select the super-time-stepping scheme as the computational cost of super-time-stepping is somewhere in between the implicit and explicit methods.

We calculate a slab portion of a rotating or non-rotating, self-gravitating disk in Cartesian coordinates. The models that we studied are listed in Table 1. In all models, the initial ratios of magnetic-field pressure to gas pressure \( \alpha \) and cosmic-ray pressure to gas pressure \( \beta \) are set to one. The initial magnetic field of all models is \( B(z) e_z \) (i.e., in the azimuthal direction \( e_z \) of the disk). The thickness of the slab is thin \( (z_{\text{cor}} = 0.9) \) for model 1, model 2, model 3, model 5, model 6, and \( (z_{\text{cor}} = 0.6) \) for model 7, and thick \( (z_{\text{cor}} = 3.0) \) for model 4 and model 8. The cosmic-ray diffusion coefficient is high \( (\kappa_i = 100.0) \) for model 1 and model 4, middle \( (\kappa_i = 10.0) \) for model 7, and low \( (\kappa_i = 1.65) \) for model 2, model 5 and model 8, and no-diffusion for model 3 and model 6. The rotation effect is applied only in model 5. The \( x \)-direction, \( y \)-direction, and \( z \)-direction correspond to the azimuthal direction, the inward radial direction, and the rotation axis of the disk. The size of the simulation box in the \( x \)- and \( y \)-directions for each model is decided from the wavelength of the maximum growth rate given by the linear analysis and is shown as \( \lambda_x \) max and \( \lambda_y \) max in Table 1. Figure 1 shows the initial gas pressure distribution for the thin slab case and the thick slab case. The number of grid points in each direction is...
We assume periodic boundaries for \( x = x_{\text{min}} \) and \( x = x_{\text{max}} \), \( y = y_{\text{min}} \) and \( y = y_{\text{max}} \), and a free boundary for \( z = z_{\text{min}} \) and \( z = z_{\text{max}} \).

### 3.2. Numerical Results

In this subsection, we show the results of a CR-MHD simulation on the formation of self-gravitating clouds by imposing random perturbation to the initial equilibrium state described in Section 2.2. The imposed perturbation is a velocity perturbation in the horizontal plane \( \delta V_x, \delta V_y \) whose amplitude is distributed randomly between \(-0.05 \leq \delta V_x, \delta V_y \leq 0.05\) (Chou et al. 2000). Figures 2–9 show the results of model 1 to model 8 consecutively, in which the time for model 1 is \( t = 17.5 \), model 2 is \( t = 23.5 \), model 3 is \( t = 23.5 \), model 4 is \( t = 11.0 \), model 5 is \( t = 32.0 \), model 6 is \( t = 19.0 \), model 7 is \( t = 11.0 \), and model 8 is \( t = 94.0 \) Myr.
$t = 22.0$, and model 8 is $t = 15.5$, where the unit of time is about 4 Myr. The left panel in each figure shows the normalized density distribution and the magnetic-field lines. The isosurface shows the normalized density at value equals to 1.7, and the lines show the magnetic-field lines. The right panel in each figure shows the normalized cosmic ray and thermal gas pressure distribution on the plane $z = 0.0$. Cosmic-ray pressure is on a color scale, and thermal gas pressure is in contours (the range of contours is $1.0 \leq P_g \leq 3.0$ with an interval of 0.5).

Initially, the cold gas is distributed uniformly in the $x$- and $y$-directions. As time proceeds, Parker–Jeans instability causes the gas to coalesce, but it develops into different structures in different models. In model 1, model 4, model 5, model 6, and model 8, filamentary structures are formed with their long-axis perpendicular to the magnetic field. In model 2, the filaments break up into clumps. In model 3 (the one without cosmic-ray
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FIGURE 8. Same as Figure 2, except for model 7 (α = 1.0, β = 1.0, κ_j = 10.0, \(z_{\text{cor}} = 0.6\), \(\Omega = 0.0\)) at \(t \sim 88\) Myr.

FIGURE 9. Same as Figure 2, except for model 8 (α = 1.0, β = 1.0, κ_j = 1.65, \(z_{\text{cor}} = 3.0\), \(\Omega = 0.0\)) at \(t \sim 62\) Myr.

diffusion), the filamentary structures with a long-axis parallel to the magnetic field are formed. These results show that the cold gas coalesces or collapses to form filamentary clouds. Depending on the cosmic-ray diffusion coefficient and rotation, the filaments may align with or be perpendicular to the magnetic field. When the diffusion coefficient is large/small (model 1/model 3), long filaments form perpendicular/parallel to the magnetic field, and if the diffusion coefficient is somewhere in between (model 2), the filaments may turn into clumps with weak directionality. When compared with thin slab cases (model 1, model 2, model 3, model 6, or model 7) and thick slab cases with a small cosmic-ray diffusion coefficient (model 8), the deformation of magnetic field lines is larger in the thick slab case with a large cosmic-ray diffusion coefficient (model 4) and in the case with rotation (model 5). Moreover, in the thick slab case the range of thermal pressure variation between the midplane (\(z = 0\)) and the half-thickness (\(z = z_{\text{cor}}\)) is wider than that for the thin slab cases. The effect of magnetic buoyancy is larger in the thick slab case with a large cosmic-ray diffusion coefficient.

The distribution of the cosmic-ray pressure matches with thermal gas pressure in model 2, model 3, model 5, model 7, and model 8, while they are almost uncorrelated in model 1 and model 4. The cosmic-ray diffusion coefficient of model 1 and model 4 is large, so the cosmic-ray pressure is nearly uniform. As a result, the contribution of cosmic rays to the cold gas coalescent is weak. On the other hand, when the diffusion coefficient is small (model 2, model 3, model 5, model 7, and model 8), the cosmic-ray pressure gradient is more significant and the distribution of gas is strongly affected (Kuwabara et al. 2004).

Rotation is considered in model 5. Using the linear-stability analysis method in Kuwabara & Ko (2006), we work out how the maximum linear growth rate of model 5 depends on a different angular velocity \(\Omega\) (i.e., parameters other than \(\Omega\) are \(\alpha = 1.0, \beta = 1.0, \kappa_j = 1.65, \) and \(z_{\text{cor}} = 0.9\)). Figure 10 shows the dependence of the maximum growth rate on \(\Omega\). In the figure, \(\sigma_{\text{max}}\) is the maximum growth rate in the \(x\)-direction, i.e., the maximum growth rate of perturbations, which does not depend on \(y\); and a similar definition applies to \(\sigma_{\text{max}}\). \(\sigma_{\text{max}}\) decreases as \(\Omega\) increases and becomes zero for \(\Omega \geq 0.27\). Hence, for a large enough \(\Omega\), perturbations in the \(x\)-direction outgrow those in the \(y\)-direction. We set \(\Omega = 0.3\) in model 5 such that \(\sigma_{\text{max}} = 0.0\). Figure 6 indicates that the gas forms long filaments perpendicular to the magnetic field, i.e., perturbation variations grow predominantly in the \(x\)-direction. This is consistent with the prediction of the linear-stability analysis.

Figure 11 shows the time evolution of the perturbed gas density at the position where the density reaches its maximum value at the end of the simulation. The growth is fastest in model 4, the thick slab case with a large cosmic-ray diffusion.
coefficient. For thin slab cases, the growth of model 1 is faster than that in model 2, which in turn is faster than that in model 3. We deduce that the smaller the cosmic-ray diffusion coefficient, the smaller the growth rate (Kuwabara et al. 2004). The smallest growth rate is in model 5, in which the instability is suppressed by the effect of rotation. All models evolve linearly at first and shift to a nonlinear stage later, and the gas cloud collapses (density tends to large values) eventually.

4. Summary and Discussion

We succeeded in analyzing the evolution of a self-gravitating two-temperature layered gas slab using an MHD simulation and cosmic rays. The gas slab is susceptible to Parker and Jeans instabilities. Cosmic rays play an interesting dynamical role, in particular, when diffusion of cosmic rays is taken into account. We only considered diffusion along the magnetic field.

Generally speaking, the cold gas slab develops into filamentary structures, but the direction of the filament with respect to the magnetic field depends on the cosmic-ray diffusion coefficient. When there is a large diffusion coefficient the filaments form preferentially perpendicular to the magnetic field, while when there is a small diffusion coefficient the filaments prefers to lie along the magnetic field. For an intermediate diffusion coefficient, clumps may form instead of filaments, and it will be impractical to describe alignment. These results agree well with the linear-stability analysis by Kuwabara & Ko (2006). We show the results of our linear analysis in Figures 12 and 13. The figures plot the maximum growth rates ($\sigma_x, \sigma_y$) against the thickness of the gas slab $z_{cor}$ (for the definition of $\sigma_x$ and $\sigma_y$, see Section 3.2 or Figure 10). Note that $\sigma_{max}$ does not depend on $\kappa$. If $\sigma_x > \sigma_y$, then the instability variations grow faster in the $x$-direction (the direction of the magnetic field), and thus the cold gas prefers to coalesce into filaments perpendicular to the magnetic field. In contrast, if $\sigma_x < \sigma_y$, the filaments form along the magnetic field. In Figure 12, we observe that the thickness of the slab in the range $0.65 \leq z_{cor} \leq 1.1$ has an interesting feature: $\sigma_x$ can be larger or smaller than $\sigma_y$ depending on the cosmic-ray diffusion coefficient. We call this the “interchange zone.” Therefore, the direction of the filaments formed from the Parker–Jeans instability depends on the diffusion coefficient. As can be read from Figure 12, in the linear stage of model 1 ($\kappa_{||} = 100.0$), $\sigma_x > \sigma_y$, which

![Figure 10](image1.png)

**Figure 10.** Dependence of the maximum growth rate on angular velocity for $\alpha = 1.0$, $\beta = 1.0$, $\kappa_{||} = 1.65$ and $z_{cor} = 0.9$. $\sigma_x$ ($\sigma_y$) is the maximum growth rate of perturbations, which does not depend on $y$ ($x$).

![Figure 11](image2.png)

**Figure 11.** Each curve show the time evolution of the perturbed gas density at the position where the density reaches its maximum value at the end of the simulation.

![Figure 12](image3.png)

**Figure 12.** Dependence of the maximum growth rates $\sigma_x$ and $\sigma_y$ on the half-thickness of the slab $z_{cor}$ for the case $\alpha = 1.0$, $\beta = 1.0$, and $\Omega = 0.0$. Note that $\sigma_y$ is independent of $\kappa$. The $\sigma_{max}$ in each model is plotted for the convenience of comparing the simulation results.

![Figure 13](image4.png)

**Figure 13.** Dependence of the maximum growth rates $\sigma_x$ and $\sigma_y$ on the half-thickness of the slab $z_{cor}$ for the case $\alpha = 1.0$, $\beta = 10.0$ and $\Omega = 0.0$. Note that $\sigma_y$ is independent of $\kappa$.
predicts that the filaments are perpendicular to the magnetic field. This is exactly how the nonlinear stage of model 1 behaves (see Figure 2). Similarly, in the linear stage of model 3 ($\kappa| = 0.0$), $\sigma_x < \sigma_y$, and Figure 4 shows exactly what is predicted: filaments form along the magnetic field. In model 2, $\sigma_x \approx \sigma_y$, and the gas coalesces to form clumps. Now, if we increase cosmic-ray pressure from $\beta = 1.0$ to 10.0, the size of the “interchange zone” increases; see Figure 13.

Tracing the history of the evolution of the gas density at the position where the density has its maximum value at the end of the simulation, we learn that the growth rate depends on the cosmic-ray diffusion coefficient, the thickness of the slab and rotation; see Figure 11. The growth rate increases as the diffusion coefficient increases or the thickness of the slab increases. However, rotation suppresses the density growth rate and the suppression is different in $\sigma_x$ and $\sigma_y$; see Figure 10. $\sigma_y$ is strongly suppressed for large $\Omega$.

The influence of cosmic rays on the formation filaments or clumps from cold gas slabs through Parker–Jeans instability can be summarized in two ways: their effects on Jeans instability and their effects on Parker instability. In addition to fluid with significant pressure, cosmic rays help counter the self-gravity of the gas, i.e., they reduce or suppress Jeans instability. However, if there is cosmic-ray diffusion along the magnetic field, then the effect of cosmic-ray pressure on supporting the gas along the field lines reduces, while the effect has its full strength across the field lines (e.g., Appendix A of Kuwabara & Ko 2006). Hence the large diffusion coefficient along the magnetic field exacerbates the tendency of cloud collapse along the field lines, and the filaments preferentially form perpendicular to the magnetic field (e.g., Figures 2 and 5).

As Parker instability develops, matter tends to slide down along magnetic-field lines to the foot points and coalesce to form clouds. However, in the case of small or zero diffusion coefficient, larger cosmic-ray gradient is established and impedes the matter motion toward the foot point (Kuwabara et al. 2004). This facilitates the formation of filaments along the magnetic field (e.g., Figure 4).

When the thickness of the gas slab is larger than the “interchange zone,” Jeans instability dominates and $\sigma_x$ is always larger than $\sigma_y$ (see Figure 12 or 13). The filaments form perpendicular to the magnetic field. On the other hand, when the thickness of the gas slab is smaller than the “interchange zone,” it is conducive to Parker instability and $\sigma_x$ is always smaller than $\sigma_y$ (see Figure 12 or 13). In this case, the filaments prefer to lie along the magnetic field.

Observations showed that the galactic magnetic fields are anchored at molecular clouds (e.g., Han & Zhang 2007; Li et al. 2009; Li & Henning 2011). The existence of such ordered magnetic fields implies that the morphology of the magnetic field tends to be preserved during the process of giant molecular cloud formation. Therefore, it is important to study the early stages of their formation, which this work has done while showing the effects of cosmic rays on this process. Recently, bimodal distributions of orientation between cloud sand magnetic fields were observed in Gould Belt molecular clouds (e.g., Li et al. 2013; Planck Collaboration Int., 2016). Soler et al. (2013) confirmed this with a synthetic observation of the simulated turbulent molecular clouds. In addition, Soler & Hennebelle (2017) showed that the direction change is an indication of compressive motions resulting from either gravitational collapse or converging flows. In such phenomena, the effect of cosmic-ray diffusion may play an important role, as shown in this work.

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References
Alexiades, V., Amiez, G., & Gremaud, P. A. 1996, CNME, 12, 31
Berezinskii, V., Bulanov, S., Dogiel, V., Ginzburg, V., & Ptuskin, V. 1990, in Astrophysics of Cosmic Rays, ed. V. L. Ginzburg (New York: North-Holland)
Bertram, E., Glover, S. C. O., Clark, P. C., & Klessen, R. S. 2015, MNRAS, 451, 3679
Breitschwerdt, D., McKenzie, J., & Volk, H. 1991, A&A, 245, 79
Breitschwerdt, D., McKenzie, J., & Volk, H. 1993, A&A, 269, 54
Choi, W., Matsumoto, R., Tajima, T., Umekawa, M., & Shibata, K. 2000, ApJ, 538, 710
Cox, D. P. 2005, ARA&A, 43, 337
Crutcher, R. M. 2012, ARA&A, 50, 29
Everett, J. E., Zweibel, E. G., Benjamin, R. A., et al. 2008, ApJ, 674, 258
Farber, R., Ruszkowski, M., Yang, H.-Y. K., & Zweibel, E. G. 2018, ApJ, 856, 112
Fatuzzo, M., Adams, F. C., & Melia, F. 2006, ApJ, 653, L49
Ferrière, K. M. 2001, RvMP, 73, 1031
Ghosh, A., & Ptuskin, V. S. 1983, Ap&SS, 92, 37
Ginzburg, V. L., & Ptuskin, V. S. 1976, RvMP, 48, 161
Girichidis, P., Naab, T., Walsh, S., et al. 2016, ApJ, 816, L19
Glassgold, A. E., Galli, D., & Padovani, M. 2012, ApJ, 756, 157
Han, J. L., & Zhang, J. S. 2007, A&A, 464, 609
Hanasz, M. 1997, A&A, 327, 813
Hanasz, M., & Lesch, H. 1997, A&A, 321, 1007
Hanasz, M., Wótański, D., & Kowalik, K. 2009, ApJ, 706, L15
Heintz, E., & Zweibel, E. G. 2018, ApJ, 860, 97
Ko, C.-M. 1991a, A&A, 242, 85
Ko, C.-M. 1991b, A&A, 251, 713
Ko, C.-M., Dougherty, M., & McKenzie, J. 1991, A&A, 241, 62
Ko, C.-M., & Lo, Y.-Y. 2009, ApJ, 691, 1587
Ko, C. M., & Webb, G. M. 1987, ApJ, 323, 657
Kadow, Y., & Hanawa, T. 2016, MNRAS, 462, 4517
Kuwabara, T., & Ko, C. M. 2006, ApJ, 636, 290
Kuwabara, T., & Ko, C. M. 2015, ApJ, 798, 79
Kuwabara, T., Nakamura, K., & Ko, C. M. 2004, ApJ, 607, 828
Kuznetsov, V. D., & Ptuskin, V. S. 1983, Ap&SS, 94, 5
Li, H., Dowell, C. D., Goodman, A., Hildebrand, R., & Novak, G. 2009, ApJ, 704, 891
Li, H., Fang, M., Henning, T., & Kainulainen, J. 2013, MNRAS, 436, 3707
Li, H., & Henning, T. 2011, Natur, 479, 499
Lo, Y. Y., Ko, C. M., & Wang, C. Y. 2011, CoPhC, 182, 177
Meyer, C. D., Balsara, D. S., & Aslam, T. D. 2012, MNRAS, 422, 2102
Miyoishi, T., & Kusano, K. 2005, JCoPh, 208, 315
Parker, E. 1966, ApJ, 145, 811
Parker, E. N. 1969, SSRv, 9, 651
Pfrommer, C., Pakmor, R., Schaal, K., Simpson, C. M., & Springel, V. 2017, MNRAS, 465, 4500
Planck Collaboration Int. 2016, A&A, 586, A138
Recchia, S., Blasi, P., & Morlino, G. 2016, MNRAS, 462, 4227
Ryu, D., Kim, J., Hong, S. S., & Jones, T. W. 2003, ApJ, 589, 338
Shibata, K., Tajima, T., Matsumoto, R., et al. 1989, ApJ, 338, 471
Soler, J. D., & Hennebelle, P. 2017, A&A, 607, A2
Soler, J. D., & Hennebelle, P. Martin, P. G., et al. 2013, ApJ, 774, 128
Tassis, K., Dowell, C. D., Hildebrand, R. H., Kirby, L., & Vaillancourt, J. E. 2009, MNRAS, 399, 1681
Wiener, J., Oh, S. P., & Zweibel, E. G. 2017, MNRAS, 467, 646
Yang, H. Y. K., Ruszkowski, M., Ricker, P. M., Zweibel, E., & Lee, D. 2012, ApJ, 761, 185
Yusef-Zadeh, F., Wardle, M., & Roy, S. 2007, ApJ, 665, L123