Leptonic Decays of Mesons in a Poincare-Covariant of a Quark Model

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Abstract
Using a relativistic constituent quark model based on point form of relativistic quantum mechanics with given set of degrees of freedom we investigate electroweak decays of the pion and $\rho$ mesons. All free parameters of quark model (the mass $u(d)$-quark and parameters which determines the confinement scale) are fixed by using relevant experimental data.
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1 Introduction

The researches of electroweak decays of hadrons always were of an important of an information about interaction of quarks. Today, electroweak decays of hadrons, which contain heavy quarks enable to measure parameters of a Standard Model (SM), and also serve for searches of effects of new physics i.e. physics outside of SM. In particular, the hadronic decays allow to define the elements of a matrix a Cabibbo-Kobayashi-Maskawa, angles of mixing. The leptonic decays of pseudoscalar mesons in models with two charged Higgs bosons become sensing to masses of these bosons. Such researches require the registration of a structure of hadrons, in particularly, of mesons. Now for exposition of relativistic bound systems there is a set of modes. Within the framework of relativistic Hamiltonian dynamics (see, for example, [2]) has shown that there is no unequivocal (unambiguous) separation of generators on a dynamic set (generators containing interaction $U$) and a kinematics set. The kinematics set can be connected to some subgroup of group the Poincare usually called as group of stability or the kinematics subgroup. In common, this subgroup contacts to a three-dimensional hypersurface, which remains invariant under an operation a Poincare of transformations from the given subgroup. For example, the instant form a RQM can be connected to a hypersurface $t = 0$, and form of dynamics on light front with a plane $t + Z = 0$.

The operators of three-dimensional momentum of a system and angular momentum do not contain in an instant form of dynamics interaction i.e. $\tilde{P} = P$ and $\tilde{J} = J$ ($\tilde{J} = (\tilde{M}^{23}, \tilde{M}^{31}, \tilde{M}^{12})$, while a mass operator $M$ (or Hamiltonian $\hat{P}_0 = \sqrt{M^2 + \hat{P}^2}$) and operator of a boost $\tilde{N} = (M^{01}, M^{02}, M^{03})$ include addenda with interaction. The description in a point form implies that the operators $\tilde{M}^\mu$ are same as for noninteracting particles i.e. $\tilde{M}^\mu = M^\mu$, and the terms with interaction are contained only in an operator 4 - momentum $\hat{P}$. In dynamics on light front we shall enter + and - components 4- vectors by a usual way:

\[ P^+ = (p^0 + p^\perp)/\sqrt{2}, \quad p^- = (p^0 - p^\perp)/\sqrt{2}. \]

We require that in the front form the operators $\tilde{P}^+, \tilde{P}_0, \tilde{M}^{12}, \tilde{M}^{+\perp}, \tilde{M}^{+\perp}$ ($j = 1, 2$) are the same as the corresponding free operators, and interaction terms may be present in the operators $\tilde{M}^{-\perp}$ and $\tilde{P}^-$. As the RQM 4-momentum of a bound system $P$ and total 4-momentum of particles $P_{12} = p_1 + p_2$, component this system (system of free quarks follows from explained above, in all forms of dynamics, for example) do not coincide i.e.

\[ P \neq P_{12} = p_1 + p_2. \]
The momentum \( p_i \) of particles of a relativistic system (quarks in a meson) can be transformed in full \( \vec{P}_{12} \) and relative momentum \( \vec{k} \) for selection of motion of a center mass system:

\[
\vec{k} = \vec{p}_1 + \frac{\vec{P}_{12}^2}{M_0} \left( \frac{\vec{P}_{12} \vec{p}_1}{\omega_{M_0} \vec{P}_{12}} + \omega_{m_1} \vec{p}_1 \right), \tag{2}
\]

Where

\[
M_0 = \omega_{m_1} (\vec{k}) + \omega_{m_2} (\vec{k}), \quad \omega_{m_1} (\vec{p}_1) = \sqrt{\vec{p}_1^2 + m_1^2}. \tag{3}
\]

The momentum \( Q \), \( \vec{q} \) system and derived base of an one-particle noninteracting system \( \vec{V} \) satisfy to a condition of a normalization:

\[
\sum_{ls} \int_0^\infty dk^2 k^2 N_c |\Psi^{J \mu}(kls)|^2 = 1. \tag{7}
\]

Here colour degrees of freedom of quarks we take into account by introduction of number of colours \( N_c \).

In such approach state vector of a meson is determined as a direct product of state vectors of free quarks with wave function \( \Psi^{J \mu}(kls) \)

\[
\left| \vec{V}, \mu, [JM] \right> = \sum_{ls} \int d^3k \frac{\omega_{m_1}(\vec{p}_1)\omega_{m_2}(\vec{p}_2)}{\omega_{m_1}(\vec{k})\omega_{m_2}(\vec{k})} (\sqrt{2}M_0^{3/2}) \Psi^{J \mu}(kls) \tag{8}
\]

Where \( \Psi^{J \mu}(kls; M) \sum_{m_\lambda \nu_{l_1}l_2} \langle s_1 \nu_{l_1}, s_2 \nu_{l_2} | \hat{S}\lambda \rangle | lm, s\lambda | J\mu \)

\( *Y_{lm}(\theta, \phi) D_{\lambda_1^\nu_1}^{1/2} (\vec{\tau} \vec{p}_1, P) D_{\lambda_2^\nu_2}^{1/2} (\vec{\tau} \vec{p}_2, P) * \left| p_1, \lambda_1 \right| \left| p_2, \lambda_2 \right> \).

3 The Basic requirements for an operator of a current

The operators of a current \( \vec{J}(x) \) bound system are necessary for an evaluation of constants of decays, charge form factors and other properties of relativistic particles. As \( \vec{J}(x) \) is four-vector, its property under an operation of transformations the Poincare same as well as at an operator four - momentum \( \vec{P}_\mu \). It is reduced in that commutation relation between \( \vec{J}(x) \) and generators of group with the Poincare \( \hat{M}^{\sigma\rho} \), \( \hat{P}_\mu \) are identical to commutation relations between 4 - momentum and generators:

\[
\left[ M^{\sigma\rho}, \hat{J}^\mu(x) \right] = i \left[ (\eta^{\sigma\rho} \hat{J}^\rho(x) - \eta^{\rho\sigma} \hat{J}^\sigma(x)) - i x^\rho \frac{\partial \hat{J}^\mu(x)}{\partial x^\sigma} - x^\sigma \frac{\partial \hat{J}^\mu(x)}{\partial x^\rho}, \right] \tag{9}
\]

\[
\left[ \hat{P}_\mu, \hat{J}_\nu(x) \right] = -i \frac{\partial \hat{J}_\nu(x)}{\partial x^\mu}. \tag{10}
\]
The translation invariance for an operator of a current reduces in the equation of an aspect:

\[ \hat{J}_\mu (x) = \exp \left( i \hat{P} x \right) \hat{J}_\mu (0) \exp \left( -i \hat{P} x \right). \]  

(11)

This equation makes possible to reduce a problem of searching \( \hat{J}_\mu (x) \) to a problem of searching \( \hat{J}_\mu (0) \). The requirement of Lorentz invariance for \( \hat{J}_\mu (0) \) reduces in a relation

\[ [\hat{M}^{\rho\sigma}, \hat{J}_\mu (0)] = i \left( g^{\mu\sigma} \hat{J}_\rho (0) - g^{\rho\sigma} \hat{J}_\mu (0) \right). \]  

(12)

If the theory is invariant in relation to spatial inversion and reflection of time, and the operators \( \hat{U}_P, \hat{U}_T \) are operators of these transformations, the operator of a current should satisfy to following conditions:

\[
\hat{U}_P (\hat{j}_0 (x^0, \vec{x}), \hat{J}(x^0, \vec{x})) \hat{U}_P^{-1} = \\
= (\hat{j}_0 (x^0, -\vec{x}), -\hat{J}(x^0, -\vec{x})), \\
\hat{U}_T (\hat{j}_0 (x^0, \vec{x}), \hat{J}(x^0, \vec{x})) \hat{U}_T^{-1} = \\
= (\hat{j}_0 (-x^0, \vec{x}), -\hat{J}(-x^0, \vec{x})) \]  

(13)

In addition to these equations the law can be used conservation of a current (equation of a continuity)

\[
\partial \hat{j}_\mu (x) / \partial x^\mu = 0. \]  

(14)

At last, the operator \( \hat{J}_\mu (x) \) in a RQM should satisfy to conditions of a so-called cluster separability (2) for multiparticle systems.

At evaluations many authors suppose, that the mathematical expressions for an operator of a current of a bound system and noninteracting system are equal. This condition (so-called relativistic impulse approximation)

\[ \hat{J}_\mu (0) = J_\mu (0) \]  

(15)

the Poincare - invariant of a relativistic quantum mechanics (3) can be realized without any assumptions only in point form of RQM. This result follows from a relation (14). In an instant form of dynamics and in dynamics on light front the relativistic impulse approximation (13) automatically is reduced in violations of commutation relations of Poincare group the for an current operator.

4 Leptonic constants of decays of mesons in a formalism a RQM

Constant \( f_p \) of a leptonic decay \( P(Q\bar{q}) \to l + \nu_l \) for a pseudoscalar meson \( P(Q\bar{q}) \) after deleting the element of a matrix \( V_{Q\bar{q}} \), are usually determined by a following relation:

\[
\langle 0 | \hat{j}_\mu (0) | \vec{P}, M_P, in \rangle = i \left( 1/2 \pi \right)^{3/2} \frac{1}{\sqrt{2 \omega M_P (\vec{P})}} P^\mu f_p, \]  

(16)

Where \( \hat{j}_\mu (0) \), and state vector of a meson with mass \( M_P \) undertake in Representation of the Heisenberg. State vector in this expression has Normalization:

\[ \langle \vec{P}, M | \vec{P}', M \rangle = \delta (\vec{P} - \vec{P}'). \]

The extraction of a constant of a decay from the matrix element of a current (10) is an independent problem and consequently we stay on it more in detail. For it we shall copy expression (10) as follows:

\[ \hat{j}_\mu = \\
\equiv \langle 0 | T \{ J_\mu^H (0) \} \hat{1} | \vec{P}, M_P, in \rangle \]  

(17)

where \( F(M) = 1/M^2, H_{l\nu}^h (x) = H_0^h (x) + H_{l\nu}^h (x) \) - total Hamiltonian interactions of quarks in a meson, and state vector of a meson with a velocity \( V \) is normalized with as follows:

\[
\langle \vec{V}, M | \vec{V}, M \rangle = 2 V_0 \delta (\vec{V} - \vec{V}^*), V_0 = \omega_M (\vec{P}) / M. \]  

(18)

Further in expression (13) inserting a full set of state vectors of two-particle system, forming base of an irreducible representation of Poincare group for free particles and using a Lippmann-Schwinger equation

\[ |j_{\nu}, in \rangle = \left( 1 + \lim_{\varepsilon \to +0} \frac{1}{M - M_0 + i \varepsilon} U \right) |p \rangle, \]  

(19)

( for a simplicity we have noted it in a system of rest of a meson, as all calculations we shall carry out in this frame of reference), which links state vector with representation of the Heisenberg and in representation interactions (state vector with an index \( V \), we come to the formula for the matrix element of a current:

\[ \hat{j}_\mu^P = \\
= \langle \nu | j_{\mu} (0) | \vec{P}, M, \nu \rangle \]  

(20)
with wave function for a pseudoscalar meson $\psi^P(k)$ (see (24)). Thus we also took into account, that owing to the Lippmann-Schwinger equation the relation is fulfilled:

$$\hat{M} \left| \vec{V}_{12}, k \right>_V = M_0 \left| \vec{V}_{12}, k \right>_V.$$  

(21)

Electroweak current of a free two-quark system in the formula (22) is determined by a form factor $G(k)$ i.e.

$$\langle 0 | T \{ J^H_{\mu}(0) \} \exp[i \int H^b_{int}(x) dx] | \vec{V}_{12}(k) \rangle_V (1/M_0^2) =$$

$$= i (1/2\pi)^{3/2} G_0(k)V_{12\mu}.$$  

(22)

An explicit of the matrix element (22) follows from method parametrization of the matrix elements of local operators. As in a point form of a vector of 4-velocities method parametrization of the matrix elements of local operators. In expression (24) operator of a current of a meson $\hat{J}_\mu^H(0)$ and Hamiltonian strong interaction $H^b_{int}(0)$ in a point form of dynamics the RQM without any assumptions can be defined through operators of free quark fields. Thus, as we marked early, the Poincare - invariance of a model in an outcome of this approximation is not breaker, in difference from an instant form of dynamics and dynamics on light front. Let’s mark also, that the wave function $\psi^P(k)$ is Lorentz-invariant, as in the point form of dynamics the operator of a boost does not contain interaction. In an instant form a RQM this property of wave functions has not a place.

The expression for Clebsch-Gordan coefficients of Poincare group is known (look, for example, [3]), and matrix element of a current (24) in quark base in case of a leptonic decay by an operator of a current $\hat{J}_\mu^H(0) = \hat{A}(0) \gamma^\mu (1 - \gamma_5) \hat{u}(0)$ is given by standard expression:

$$\langle 0 | \hat{J}_\mu^H(0) | p_1, \lambda_1, p_2, \lambda_2 \rangle_V =$$

$$= 1/(2\pi)^3 \sqrt{\pi} \omega_m / 2 \pi \omega_M (\vec{p}^2) \varepsilon^\mu_\rho (p).$$  

(25)

The similar scheme can be realized not only for decays of pseudoscalar mesons, but also for decays of vector mesons, and also for other reaction including of mesons. After an integration in a system of rest of a meson, we obtain following expression for leptonic constant of connection of a pion $f_\pi$ in the supposition of equality masses $u$ and $d$ of quarks ($m_u = m_d = m$) and ignoring QCD corrections:

$$F_\pi = N_\pi \frac{m}{\omega_m} \frac{k}{\omega_M} (\vec{p}) \psi^P(k)dk$$  

(26)

Let’s remark, that as the calculations were carried out in a system of rest of a meson, where the point form a RQM coincides an instant form, the outcome (26) coincides an outcome for this constants obtained in [4] within the framework of an instant form of dynamics .

The constant of a decay $f_V$ is determined from the matrix element of current

$$\langle 0 | J^\alpha_H(0) | \vec{P}, M_V, J = 1, \mu \rangle =$$

$$= (1/2\pi)^{3/2} f_V M_V \sqrt{2\omega_M} \varepsilon^\mu_\rho (p),$$  

(27)

Where $\varepsilon^\mu_\rho (p)$ - polarization vector meson with mass $M_V$. In (27) the meson current can be selected by an electromagnetic current of quarks

$$J^H_\alpha(0) = e_u \bar{u} \gamma_\alpha u + e_d \bar{d} \gamma_\alpha d$$  

(28)

for a decay $\rho^0 = (u\bar{u} - d\bar{d}) / \sqrt{2}$ - meson. Following similarly to case of a decay of a pseudoscalar meson we shall receive:

$$F_\rho = \frac{N_\rho}{3\sqrt{2\pi}} \frac{k}{\omega_M (\vec{k})} \left[ 2 + \frac{m}{\omega_m (\vec{k})} \right] \psi^P(k)$$  

(29)

This expression is calculated for a longitudinally polarized condition $\rho^0$ of a meson in a neglect by spin effects.
in quark-antiquark interaction. In common case, wave functions in the formulas for $f_\pi$ and $f_\rho$ should differ.

The equation of motion for bound $Q\bar{q}$ systems in a RQM is the relativistic equation with an effective potential $U$. However deriving of wave functions $\psi(k)$ and spectrum of masses from this equation is a challenge, which requires a separate research. In the given work we use simple model wave function, which depends on a scale parameter $1/\beta$:

$$\Psi_P(k) \equiv \Psi_P(k, \beta) = 2/\left(\sqrt{N_c} \beta^{3/2} \pi^{1/4}\right)\exp\left(-\frac{k^2}{2\beta^2}\right).$$  

Thus, in the given approach for pseudoscalar and vector mesons we have two free parameters is a mass $u(d)$ of a quark $M$ and parameter $\beta$. Expressions for leptonic constants with wave function (30) can be obtained in an analytical form with the help of special functions. So for a pion we have:

$$f_\pi = \frac{\sqrt{3}m}{\pi^{5/4} \Gamma\left(-\frac{1}{4}\right)} * \left(2^{3/4} \Gamma\left(-\frac{1}{4}\right) \Gamma\left(\frac{3}{4}\right) \right) _1F_1\left(\frac{1}{4}, \frac{1}{4}, \frac{m^2}{2\beta^2}\right) - \frac{2m^{3/2}}{\beta^{3/2}} \sqrt{\pi} \Gamma\left(-\frac{3}{4}\right) _1F_1\left(\frac{3}{2}, \frac{7}{4}, \frac{m^2}{2\beta^2}\right),$$  

where $1F_1(a; b; z)$ - degenerate hypergeometric function, and $\Gamma(z)$ - gamma-function. Similar, but more bulky The relation is received and for magnitude $f_\rho$. Parameters a Poincare - covariant of a quark model of electroweak decays it is possible to fix using, appropriate experimental data. In our calculations we used the following values for leptonic constants of decays: $f_\pi = 130.7 \pm 0.46$ MeV and $f_\rho = 152.9 \pm 4.8$ MeV ( constant of a decay $\rho^0$ of a meson in a pair $e^+e^-$ ) [11]. In the total we shall receive admissible with a point of view of an existing experimental data the following solved areas for mass of a quark $m$ and parameter $\beta$ of wave function (30), which satisfactorily describe both experiments, researched by us;:

$$m = 247 \pm 9 \text{ MeV}, \quad \beta = 323 \pm 8 \text{ MeV}.$$  

The data intervals for mass of easy quarks and for a parameter $\beta$ will be agreed restrictions obtained within the framework of the front form a RQM (see [4]) and a model, based on a solution of the equation of the Salpeter (see [11]). Using obtained parameters, we have a possibility to calculate other, independent from considered by us, interaction of mesons with light quarks.

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