Neutrino Oscillations Induced by Gravitational Recoil Effects

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Abstract

Quantum gravitational fluctuations of the space-time background, described by virtual $D$ branes, may induce the neutrino oscillations if a tiny violation of the Lorentz invariance (or a violation of the equivalence principle) is imposed. In this framework, the oscillation length of massless neutrinos turns out to be proportional to $E^{-2}M$, where $E$ is the neutrino energy and $M$ is the mass scale characterizing the topological fluctuations in the vacuum. Such a functional dependence on the energy is the same obtained in the framework of loop quantum gravity.

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The attempts to build the quantum theory of gravity have clearly shown that spacetime must have a non trivial topology at the Plank scale. The suggestion that spacetime could have a foam-like structure was, for the first time, advanced by Wheeler [1]. In the last years, the study of quantum fluctuations of the spacetime background has received a growing interest and today it represents a very active research area in physics [2, 3, 4, 5, 6, 7].

In the presence of Plank-size topological fluctuations, the quantum gravitational vacuum might behave as a non trivial medium. Such a behaviour has been proven in the framework of string theory [8] and of the canonical approach to quantum gravity [9] (see also [10]). In particular, we point out the underlying idea of Ref. [8]: quantum gravitational fluctuations in the vacuum must be modified by the passage of an energetic particle and the recoil will be reflected in back reaction effects on the propagating particle (see Ref. [11]).

From an experimental setting, the present status eludes any possibility to probe effects occurring at Plank energy. Nevertheless, it has been recently suggested that γ-ray bursts might be a possible candidate to test the theories of quantum gravity [12]. The argumentations of the authors are based on the peculiar physical properties of γ-ray bursts, i.e. their origin at cosmological distance and their high energy, which might make them sensitive to a dispersion scale comparable with the Plank scale [12].

In this note we investigate the possibility that the foamy structure of the gravitational background, described by the Ellis-Mavromatos-Nanopoulos-Volkov (EMNV) model [13], may induce the mixing of neutrinos if a tiny Lorentz invariance violation is imposed [14]. As a result, we find that the inverse of the neutrino oscillation length does depend on the square of the neutrino energy. This is the same functional dependence derived in Ref. [15] by using the loop quantum gravity approach (for a review, see [16]).

The EMNV model envisages virtual D branes as responsible of the foam structure of the space-time. The basic idea of this model is the recoil effect of a D brane struck by a boson [7, 11] or a fermion [13] particle, which would induce an energy dependence of the background metric through off-diagonal terms given by $G_{0i} \sim u_i$. Here $u_i$ is the average recoil velocity of the generic D brane [13], and it is of the order $u_i \sim E/M \ll 1$, where $E$ is the energy of the particle scattering off the D brane, and $M$ has the dimension of a mass which characterizes the quantum fluctuations scale. As a consequence of the off-diagonal term in the metric tensor ($G_{0i}$), the Lorentz invariance is broken.

Without pretending to be exhaustive, we just recall the main features of the EMNV model necessary for our considerations (for details, see Ref. [13] and references therein).

The momentum conservation during the recoil process implies an energy dependence of the metric tensor whose asymptotic (in the time) components are (for a D-dimensional spacetime)

$$G_{ij} = \delta_{ij}, \quad G_{00} = -1, \quad G_{0i} \sim u_i, \quad i, j = 1, \ldots, D - 1,$$

resulting constant in the spacetime coordinates. The metric (1) induces a variation of the light velocity given by $\delta c/c \sim -E/M$. 

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In view of the application to neutrino oscillations, we are interested to the recoil effects on fermions. A massless fermion is described by the covariant Dirac equation in curved spacetime

\[
[i\gamma^\mu(\nabla_\mu - \Gamma_\mu)]\psi = 0. \quad (2)
\]

The general relativity matrices \(\gamma^\mu\) are related to the Lorentz matrices \(\gamma^m\) by means of the vierbein fields \(e^m_\mu\) \((\gamma^\mu = e^m_\mu \gamma^m)\), with components given by

\[
e^m_\mu = \begin{pmatrix}
-1 & 0 & 0 & -u_1 \\
0 & -1 & 0 & -u_2 \\
0 & 0 & -1 & -u_3 \\
0 & 0 & 0 & 1
\end{pmatrix}.
\]

The operator \(\nabla_\mu\) is the usual covariant derivative, \(\nabla_\mu = \partial_\mu + \Gamma^\nu_{\mu\nu}\), and \(\Gamma_\mu\) are the spin connections defined as

\[
\Gamma_\mu = \frac{1}{8} \left[\gamma^m, \gamma^n\right]e^e_m e^n_{\nu\mu}\]

(semicolon represents the covariant derivative). In terms of the metric (1), the Dirac equation (2) becomes

\[
[\gamma^m \partial_m - \gamma^0 (u_i \nabla_i)]\psi = 0. \quad (3)
\]

The action of the operator \(\gamma^\mu \partial_\mu\) allows to derive the dispersion relation [13]

\[
E^2 = p^2 - 2 (u_i p_i) E, \quad (4)
\]

where, as already noted, \(|\vec{u}| \sim E/M \ll 1\) and \(p\) is the momentum of the massless fermion. As argued in [13], if \(\gamma\)-rays bursts may emit pulses of neutrinos with \(E \sim 10^{10}\)GeV, the effects of the second term in (4) could be tested provided \(M \sim 10^{27}\)GeV.

Let us now assume that the \(u\)-term in (4) violates the Lorentz invariance. As argued by Coleman and Glashow [14] (see also [17]), this would imply that different species of neutrino massless may have different maximal attainable speeds that does not coincide with the light velocity \(c\). In such a case, neutrino oscillations can occur if the neutrino flavor eigenstates \(\nu_f\), \(f = e, \mu\), are linear superposition of the velocity eigenstates \(\nu_a\), \(a = 1, 2\), at infinite momentum. The relation between velocity and flavor eigenstates is

\[
\nu_\mu = \cos \theta \nu_1 + \sin \theta \nu_2, \\
\nu_e = \cos \theta \nu_2 - \sin \theta \nu_1,
\]

where \(\theta\) is the mixing angle.

From Eq. (4) it follows that the velocity \(v_a\) of the neutrinos eigenstates \(\nu_a\) can be written as (we consider one-dimensional motion)

\[
v_a \sim 1 + u_a = 1 + \alpha_a \frac{E}{M}, \quad a = 1, 2, \quad (5)
\]

where the notation \(u_a = \alpha_a E/M\) has been introduced. The parameter \(\alpha_a\) characterizes the maximal attainable velocity of neutrinos \(\nu_a\).
During their evolution, neutrinos propagate as a linear combination of velocity eigenstates whose energies are $E_a$, $a = 1, 2$. Taking into account Eq. (5), one infers that energy difference is

$$E_1 - E_2 = \delta v E \sim \delta \alpha \frac{E^2}{M}. \quad (6)$$

In Eq. (6), $\delta v = v_1 - v_2$ and $\delta \alpha = \alpha_1 - \alpha_2$. Thus, $\delta \alpha$ measures the degree of violation of the Lorentz invariance.

The probability that the neutrino preserves its flavor is therefore

$$P = \sin^2 \theta \sin^2 \left( \frac{\delta \alpha E}{2M} L \right), \quad (7)$$

being $L$ the cosmological distance traveled by neutrinos between the emission and detection. From (7) one infers the inverse of the oscillation length

$$\lambda_{\text{osc}}^{-1} \sim \delta \alpha \frac{E^2}{M}. \quad (8)$$

Bounds on $\delta \alpha$ can be estimated by using the relation $\lambda_{\text{osc}} \leq L$. For example, the values for the energy $E \sim 10^{10}$GeV and the mass scale $M \sim 10^{27}$GeV, the same of Ref. [13], and $L \sim 2.7 \times 10^{22}$km yield $\delta \alpha \geq 10^{-34}$.

The energy dependence of the oscillation length of neutrinos, Eq. (8), is the same of Ref. [13]. There the propagation of fermion fields (neutrinos) has been studied in the framework of the loop quantum gravity [16]. In that case, the oscillation length of neutrinos is given by $\lambda_{\text{osc}}^{-1} = \Delta \rho_1 E^2/M_P$, where the mass scale is characterized by the Plank mass $M_P$, and $\Delta \rho_1$ measures a violation of the equivalence principle [15].

It is worth noting that the possibility to generate neutrino oscillations (even for massless neutrinos) via the violation of the equivalence principle as in Refs. [18, 19], could also apply in the EMNV model. Results are the same discussed in this note (one obtains the oscillation length given by Eq. (8)) and do coincide with ones of Ref. [15].

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