Learning trajectory for teaching number patterns using RME approach in junior high schools

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Abstract: This research aimed at developing a learning trajectory based on realistic mathematics education (RME) approach, for teaching number patterns to the students at grade eight junior high schools. The research employed design research approach proposed by Gravemeijer and Cobb. The activities of the research consisted of a cyclic process of preparing for the experiment, conducting the experiment, and retrospective analysis. The learning trajectory for teaching number patterns was designed during preparing for the experiment phase, and then it was validated by three math educators. It was tried out in a small group, followed by try out in a regular classroom during conducting the experiment phase. Retrospective analysis was conducted after each try out to improve the learning trajectory. 32 grade eight students of a junior high school in Padang were involved as the subjects of the research. Data were collected through observations, analyzing students’ works, videotaping, and test. Collected data were analyzed descriptively. Through this research we developed a RME-based learning trajectory for teaching number patterns. The learning trajectory reached the criteria of content and construct validity; reflected the state of the art RME, internally consistent, and potential to develop students’ reasoning. It also satisfied the criteria of practicality in which the learning trajectory worked as intended in the classrooms. The results of students’ works showed that the learning trajectory could also improve the students’ reasoning in learning number patterns.

Keywords: learning trajectory, RME, number patterns

1. Introduction
There are a lot of patterns in this world, such as in the solar system, calendar, music, maps, light, and waves. Patterns also can be found easily in our surrounding such as on the floor, in the supermarket, or on the street. Kayaspor [1] mentioned that everything in our life has only mathematical patterns. Therefore, patterns become an important topic in mathematics. Learning about patterns can help students developing their reasoning [2], especially inductive reasoning [3]. Patterns also can guide students to make a generalization [4], as it very crucial and useful in solving mathematical problems [5,6]. Meanwhile, [7] said that the generalization is a heartbeat of mathematics.

There are two main types of patterns in math namely number patterns and shape patterns. Both types of patterns can be found in school mathematics curriculum. In the recent Indonesian curriculum for junior high schools, capable of using number patterns in making prediction, generalization, and in solving mathematical problems become an important goal to achieve by the students. Although number patterns are frequently found in our daily life and become an integral part in math curriculum, the results from several researches showed that most students found difficulties in learning number
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patterns [8-12]. The difficulties faced by the students were especially in understanding the problems, finding the patterns, and making generalization.

The students found difficulties in learning number patterns because they did not learn the topic in a meaningful way. The learning process tended to more focus on finding the formula. Therefore, the students lack experiences in exploring the number patterns. As a matter of fact, explorations are very important activities in learning number patterns [13, 14]. In addition, examples of number patterns presented by math teachers were mostly focused on numbers. Meanwhile, many literatures (see 6,15,16] showed that number patterns can be presented in form of dots, arrangement of toothpicks, triangle or rectangle representations, spider web, or Lego, so that the students have opportunity to explore number patterns from various perspectives.

Considering the difficulties faced by the students and referring to the results found by researchers mentioned above, in this research we developed a learning trajectory (LT) for teaching number patterns using realistic mathematics education (RME) approach. A LT learning trajectory is the sequences of activities or tasks to guide students to achieve a specific instructional goal [17]. At the beginning, a LT is designed in form of a hypothetical learning trajectory (HLT). A HLT consists of the learning goal, the learning activities, and the hypothetical learning processes—a prediction and anticipation of how the students’ thinking and understanding will evolve in the context of the learning activities [17,18].

In designing the LT for teaching number patterns, we referred to several researchers that already developed LT for teaching other topics in mathematics [see 18-26]. Most of these researches showed that the LT’s could help the students to improve their conceptual understanding and their mathematical thinking abilities. We also used RME approach as theoretical basis for designing the LT because of several reasons. Firstly, in RME students start learning a mathematical concept by exploring a contextual problem [27, 28]. It would give a benefit to the students if they have opportunity to learn number patterns by exploring the contextual problems which are related to their daily live. Secondly, the activities of learning mathematics in RME are sequenced in such a way so that students experience the processes of horizontal and vertical mathematization [29]. It means that the students will have opportunity to explore number patterns by using their informal knowledge which guides them to reinvent a formal formula of a number pattern. The process of mathematization can be seen in Figure 1.

![Figure 1. Processes of horizontal and vertical mathematization (Source: Gravemeijer [29])](image)

Thirdly, in the processes of mathematization, the students will also experience the characteristic of RME for instructional design, namely didactical phenomenology, guided reinvention, emerging models [29,30] and characteristic of RME for teaching and learning such as students free productions and students contributions [31,32]. The results of various researches revealed that RME approach could stimulate the students’ thinking and their mathematical abilities [see 18-26]. In this research, we
investigate the impact of RME approach on the students’ reasoning ability. The research question that we formulated in this research was: what are the characteristics of the learning trajectory for teaching number patterns using RME approach which are valid, practical, and effective to stimulate students’ reasoning ability?

2. Research Method
This research used design research approach proposed by Gravemeijer and Cobb [33] which characterized by a cyclic process of preparing for the experiment, conducting the experiment, and retrospective analysis. The cyclic process is described in Figure 2.

![Figure 2. The cyclic process of design research (Source: Gravemeijer & Cobb [33])](image)

The activities conducted in each phase of the research, focus of the activities, and data collection techniques are presented in Table 1.

| No. | Research phase       | Research activities                                                                 | Data collection          |
|-----|----------------------|-------------------------------------------------------------------------------------|--------------------------|
| 1.  | Preparing for the    | - Need analysis                                                                     | - Checklist              |
|     | experiment           | - Analyzing the curriculum and essential concepts                                  | - Document analysis      |
|     |                      | - Analyzing students’ characteristics                                               | - Observations           |
|     |                      | - Analyzing relevant literatures                                                    | - Interviews             |
|     |                      | - Designing the HLT                                                                 | - Expert judgment        |
|     |                      | - Validating the HLT to three math educators                                        |                          |
|     |                      | - Revising the HLT                                                                 |                          |
| 2.  | Conducting the       | - Small group try out: investigating the practicality of the HLT                    | - Observations           |
|     | experiment           | - Classroom try out: investigating the practicality and the effectiveness of the     | - Analyzing students’    |
|     |                      | HLT on students’ reasoning ability                                                 | works                    |
|     |                      |                                                                                        | - Video taping           |
|     |                      |                                                                                        | - Test                   |
| 3.  | Retrospective analysis| - Discussing and reflecting on the results of conducting the experiment phase to    | - Document analysis      |
|     |                      | revise and improve the HLT to get the learning trajectory for teaching number        |                          |
|     |                      | patterns which are valid, practical, and effective to improve students reasoning    |                          |
|     |                      | ability                                                                            |                          |
The subjects of the research, when the HLT was tried out in a small group, were six grade eight students from a junior high school in Padang, West Sumatra Indonesia. Meanwhile, the try out in the classroom involved 32 eight grade students from the same school.

3. Results and Discussions
After conducting need analysis, analyzing curriculum, essential concepts, students’ characteristics, and reviewing literatures on how to teach number patterns, it was designed the HLT for teaching number patterns using RME approaches. There were seven activities involved in the HLT and in each activity the students solve one or two contextual problems. The goals of the activities were to facilitate the students to:

a. Recognizing and finding pattern using context of light decorations
b. Finding patterns of odd and even numbers using context of decorations on the wall
c. Finding patterns of triangle, square, and rectangle numbers using context of decorations made from bottle caps
d. Finding the $n^{th}$ term of an arithmetic sequence using context of the chairs in a theatre
e. Finding the sum of $n$-terms of an arithmetic sequence using context of the chairs in a theatre
f. Finding the $n^{th}$ term of a geometric sequence using context of “like” on Instagram
g. Finding the sum of $n$-terms of a geometric sequence using context of “like” on Instagram

The HLT was validated by three experts in mathematics education who familiar with RME approach. After discussion and revision processes with the experts, the HLT reached the criteria of validity (content and construct validity, see [34]). Some criteria approved by the experts were: 1) the contextual problems provided for the students are potential to help them reinventing the concepts about number patterns; 2) learning activities provided for the students can guide them to experiencing the process of horizontal and vertical mathematization; 3) The HLT reflected the characteristics for instructional design as well as for teaching and learning [see 27,29,31]; 4) The HLT potential to improve students’ reasoning ability in learning number patterns.

The results of small group try out indicated that the HLT for teaching number patterns using RME approach could work as intended. The students could solve the contextual problem in providing time and they learn the topic without many difficulties. The predictions and anticipations prepared by the researches could help the students to achieve the goals of instructions. The same situations were found during classroom try out. It means that the HLT satisfied the criteria of practicality [34].

Finally, the HLT for teaching number patterns using RME approach could stimulate students’ reasoning ability. The following parts will present some activities in the HLT and their impacts on the students’ reasoning ability.

In the first activity, the students were given the next problem:

*Rani saw lights decoration on a garden city. The lights were changed from green, yellow, and red in every second. Rani curious, if she saw the green lights on the first second, what color she might be seen on the 15th second? Can you help Rani to answer her curiosity?*

At the beginning most students solve the problems without structure (just by writing H = green, K = yellow, and M = red repeatedly, without aware of the time). After the teacher challenged them to present the answer so that it more understandable, some students wrote the answer in a table as can be seen in an example below.
From the student’s answer we can see that the contextual problem could stimulate the students to recognize that the problem was about finding a pattern and later on it help them to discover the pattern. To bring the students thinking to the higher level, the teacher raised probing question that was prepared in the HLT: “after observing the pattern in the table, can you find the colour of lights that will be on at the 15th second without completing the table?” A student came with the following answer.

The student divided 15 by 3 because there were three colours. As the reminder of the division equal zero, he came to the conclusion that the colour of light that will on at the 15th minute might be the third colour which was M (red). From the explanation above we can see that the probing questions raised by the teachers could stimulate the students’ thinking and reasoning. This finding inline to the results reveal by [35] about the potential of teachers’ probing questions when teaching mathematics using RME approach.

When finding the pattern of odd numbers using the context of decoration on the wall, the students also showed a good understanding of the problem and they could come with brilliant mathematical idea as can be seen in the figure below.

Tata wanted to put paper decoration on the wall of her bedroom. She made the paper decoration using isosceles triangles and arranged them as the following.

As Tata loved number 12, she made the decoration until 12 layers. Can you predict how many triangles Tata needed for the 12th layer, and the total number of triangles she needed for the decoration? Explain your answer!

One of the students’ answers:
When the students were asked to find the general pattern of odd numbers, they could come with the correct answer that can be seen in following figure.

In general, the LT designed in this research could help the students to reinvent the concepts about number patterns such as the patterns of odd and even numbers, triangle, square, and rectangle numbers, arithmetic sequences, and geometric sequences. At the end of the lesson we gave a reasoning test to 32 students. The indicators for the reasoning test were making a conjecture, finding a pattern, giving a reason and proof, and making a conclusion. The result of the test showed that 25 of 32 students (78.1%) got the score more than 75 (ideal score = 100). The LT also could improve students’ reasoning ability as most of the students could give reasons for most of their answers in solving the contextual problems. These results strengthen the findings from the previous researches that the LT and RME approach could improve students’ understanding and students’ reasoning in learning mathematics [see 18, 21, 25, 36, 37]

4. Conclusion

Through this research we developed a learning trajectory for teaching number patterns using RME approach in junior high schools. There were seven activities developed in the HLT and in each activity the students solve one or two contextual problems. The learning trajectory reached the criteria of content and constructs validity with characteristics; it reflected the state of the art RME (including the characteristic of RME for instructional design and for teaching and learning); it internally consistent and potential to develop students’ reasoning. The learning trajectory also satisfied the criteria of practicality in which the learning trajectory worked as intended in small group try out and in the classrooms. The learning trajectory also effective in improving the students’ reasoning in learning number patterns. As a learning trajectory is very helpful in building students’ understanding when learning a mathematics topic, it is highly recommended to develop learning trajectories for other mathematics topics using the local contexts.

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