Numerical simulation of the dynamics evolution of alluvial mining quarries

T P Lyubimova¹, A P Lepikhin², Ya N Parshakova¹

¹Institute of continuous media mechanics UB RAS, 614013, Academika Koroleva st, 1, Perm, Russian
²Mining Institute UB RAS, Sibirskaya, 78 –а, Perm, Russia

E-mail: lyubimovat@mail.ru

Abstract. Alluvial mining quarry (or placer mining) is one of the main techniques for extracting important building materials such as sand and gravels. Prediction of quarries detrimental effects on the hydraulic regimes of rivers, in particular on flow regimes, has been carried on in full details in 0, 1 and 2D problem formulations (in the latter case, a depth-averaging is applied). However, the prediction of the quarry behavior itself is unfeasible, though such information would be of paramount importance for estimating the adverse effect on the river bed. This work studies the dynamics evolution of alluvial mining quarries in the framework of two-dimensional formulation based on width-averaging. The Euler multiphase model, which allows simulating separately the behavior of several interacting phases, is implemented. The conducted numerical experiments show that the upstream part of the quarry is eroded more intensively than the downstream one, displacing the quarry up-stream. This effect was observed during numerous field case-studies.

1. Introduction

Exploitation of alluvial deposits of construction materials, principally sand and gravels, leads in general to the implantation of alluvial mining quarries. To minimize detrimental effects of such quarries, it is important to predict the impacts of extraction on the hydrological regime of water body. Traditionally, such predictions are made in the framework of hydrodynamic models using “zero”- one- or two-dimensional problem formulations. In [1] the results of calculations of the sand quarry behavior in the framework of a one-dimensional model are presented. The behavior of quarries of different sizes, differing both in length and width, is shown. The behavior of the quarry depending on the flow rate is investigated, the sanding of the quarry is found to be accelerated with the increase in flow rate, while the shape of the quarry varies with time in different ways for different flow rates. The paper [2] begins with an overview of the sand and gravel industry in general. It concludes with a discussion, by way of selected examples, of the broad range of potential impacts and describes some techniques that have been successfully used to prevent or limit those impacts. Where the river enters the pit immediately moves upstream, and the river bed starts to be scoured [3]. In the latter case, a depth-averaging is applied. A more effective way to obtain the desired estimates is to perform computational hydrodynamic
experiments in two-dimensional formulation based on a width-averaging. Such an approach is presented and implemented in this paper.

2. Mathematical model
The dynamics of alluvial mining quarry was modeled using the multiphase Euler model, which allows simulating separately the behavior of several interacting phases, which might be any of the three media — liquid, solid or gas phases in any combination.

The description of multiphase flows as interpenetrating continua includes the concept of phase volume fraction \( \alpha_q \). The phase volume is a space occupied by each phase. The laws of mass and momentum conservation are assumed to be satisfied for each phase independently. The volume of phase \( q \) is defined as \( V_q = \int_\Omega \alpha_q dV \), with \( \sum_{q=1}^n \alpha_q = 1 \).

The equation for the law of momentum conservation and continuity equation are solved independently for each phase.

The equation of continuity for phase \( q \) is written as

\[
\frac{\partial}{\partial t} \left( \alpha_q \rho_q \overline{v}_q \right) + \nabla \cdot \left( \alpha_q \rho_q \overline{v}_q \overline{v}_q \right) = \sum_{p=1}^n \left( \dot{m}_{pq} - \dot{m}_{qp} \right) ,
\]

where \( \overline{v}_q \) is the velocity of phase \( q \), \( \dot{m}_{pq} \) describes the mass transfer from phase \( p \) to phase \( q \) while \( \dot{m}_{qp} \) describes the mass transfer in the opposite direction.

The law of conservation of momentum for a liquid phase \( (q = l) \) is written as

\[
\frac{\partial}{\partial t} \left( \alpha_q \rho_q \overline{v}_q \right) + \nabla \cdot \left( \alpha_q \rho_q \overline{v}_q \overline{v}_q \right) = -\alpha_q \nabla p + \nabla \overline{\tau}_q + \alpha_q \rho_q g +
\]

\[
+ \sum_{p=1}^n \left( \overline{R}_{qp} + \dot{m}_{pq} \overline{v}_p - \dot{m}_{qp} \overline{v}_q \right) + \left( \overline{F}_{q} + \overline{F}_{lq,p} + \overline{F}_{v,m,q} \right)
\]

where \( \overline{\tau}_q \) is the shear stress tensor of the phase \( q \), \( p = s \) :

\[
\overline{\tau}_q = \alpha_q \mu_q \left( \nabla \overline{v}_q + \nabla \overline{v}_q^T \right) + \alpha_q \left( \lambda_q - \frac{2}{3} \mu_q \right) \nabla \cdot \overline{v}_q \overline{I}
\]

Here \( \mu_q \) and \( \lambda_q \) are the shear and bulk viscosities of the phase \( q \), \( \overline{F}_{q} \) is the external force, \( \overline{F}_{lq,p} \) is the buoyancy force, \( \overline{F}_{v,m,q} \) is the virtual mass force, \( \overline{R}_{qp} \) is the force of interaction between the phases, \( p \) is common pressure for all phases, \( \overline{v}_{pq} \) is the inter-phase velocity, which is defined as follows: if \( \dot{m}_{pq} > 0 \) (i.e. the mass of the phase \( q \) is transferred into the mass of phase \( p \)) then \( \overline{v}_{pq} = \overline{v}_p \) and vice versa. Additionally, if \( \dot{m}_{pq} > 0 \), then \( \overline{v}_{pq} = \overline{v}_q \) and if \( \dot{m}_{pq} < 0 \), then \( \overline{v}_{pq} = \overline{v}_p \).

The inter-phase force depends on the friction, pressure, particle packing and other factors. For this force the following conditions are verified: \( \overline{R}_{qp} = -\overline{R}_{pq} \), \( \overline{R}_{qq} = 0 \).

\[
\sum_{p=1}^n \overline{R}_{pq} = \sum_{p=1}^n K_{pq} \left( \overline{v}_p - \overline{v}_q \right)
\]

Here, \( K_{pq} \) is the coefficient of momentum exchange between the phases.

For a solid phase the law of momentum conservation has the following form:
\[
\frac{\partial}{\partial t} (\alpha_i \rho_i \tilde{v}_i) + \nabla \cdot (\alpha_i \rho_i \tilde{v}_i) = -\alpha_i \nabla p - \nabla \cdot \tau_s + \alpha_i \rho_i \tilde{g} + \\
+ \sum_{p=1}^{n} \left( \tilde{R}_li + \tilde{m}_li \tilde{v}_i - \tilde{m}_li \tilde{v}_li \right) + \left( \tilde{F}_s + \tilde{F}_{lif} + \tilde{F}_{vm} \right)
\]

Here \( p_s \) is the pressure of the solid phase.

The exchange coefficient between the liquid and the solid phases can be written as

\[
K_{sl} = \frac{\alpha_i \rho_i f}{\tau_s}
\]

where \( \tau_s \) is "the particle relaxation time" (or particle drag coefficient), which is defined as \( \tau_s = \rho_s d_i^2 / (18 \mu_l) \) and \( d_i \) is the diameter of particles in the solid phase, \( \rho_s \) the solid density, and \( \mu_l \) the dynamic viscosity (assuming a Stokes flow and a Reynolds number less than unit).

The coefficient \( f \) can be derived from several models. In this study, we use the Gidaspow model of closely packed layer of particles [5]. Particle packing depends on their quantity and particle diameter. The calculation of maximum packing for a liquid phase with solid inclusions is presented in [6].

The stress tensor includes the shear and bulk viscosities of the solid phase, which are related to the particle momentum due to particle motions and their collisions. The shear viscosity consists of the collision-induced part, the kinetic part and the part associated with additional friction:

\[
\mu_s = \mu_{s,col} + \mu_{s,kin} + \mu_{s,fr}
\]

To describe the turbulence pulsations of velocity, we use the disperse \( k - \epsilon \) model of turbulence. The random motion of the solid phase in the liquid phase turbulent flow is the major contribution to the energy change in the system. The contribution of inter-particle collisions is negligible.

The equations for the turbulent kinetic energy \( k \) and the rate of its dissipation \( \epsilon \) are given as

\[
\frac{\partial}{\partial t} (\alpha_i \rho_i k_i) + \nabla \cdot (\alpha_i \rho_i \tilde{U}_i k_i) = \nabla \cdot \left( \alpha_i \frac{\mu_i}{\sigma_u} \nabla k_i \right) + \alpha_i G_{k,i} - \alpha_i \rho_i e_i + \alpha_i \rho_i \Pi_{k,i}
\]

\[
\frac{\partial}{\partial t} (\alpha_i \rho_i e_i) + \nabla \cdot (\alpha_i \rho_i \tilde{U}_i e_i) = \nabla \cdot \left( \alpha_i \frac{\mu_i}{\sigma_u} \nabla e_i \right) + \alpha_i e_i \left( C_{1k} G_k - C_{2k} \rho_i e_i \right) + \alpha_i \rho_i \Pi_{e,i}
\]

Here \( G_{k,i} = \mu_i S^2 \) is the generation of the turbulent kinetic energy due to the mean velocity gradient, where \( S = \sqrt{2 S_y S_y} \) is the norm of the tensor of the mean strain rate, \( S_y = \frac{1}{2} \left( \frac{\partial v_j}{\partial x_j} + \frac{\partial v_i}{\partial x_j} \right) \); \( \Pi_{k,i} \) defines the influence of the solid phase on the liquid phase, \( C_{1k}, C_{2k}, \alpha_k, \sigma_k \) are constants.

The problem was solved using unsteady approach. The calculations were carried out in two-dimensional formulation based on the averaging in the horizontal direction perpendicular to the flow of the river. At initial time, the quarry was assumed to be rectangular (Fig.1) or with the inclined lateral walls (Fig.2).
Fig. 1. Computational domain at initial time with water (in blue), and sand (in red). (a) the entire computational domain, (b) the fragment of the grid covering the studied domain

The no-slip condition was imposed at the lower boundary of the studied domain, modeling the river bottom. The upper boundary of the studied domain, corresponding to the free liquid surface, was considered as nondeformable. It was supposed to obey the vanishing condition of the normal velocity component and tangential stresses. On the lateral boundaries of the domain, the condition of zero velocity derivative with respect to the normal was imposed.

A constant flow velocity $\bar{V}$ with one nonzero component was assumed at the input of the studied domain.

$$x = 0: \quad v_x = V, \quad v_y = 0$$

At the exit of the studied domain, the derivatives with respect to $x$ of all functions were assumed vanishing (equal to zero) as the boundary conditions.

As initial conditions, the velocity of the basic flow is imposed equal to the velocity at the input of the studied domain: $\bar{v}_0 = \bar{V}$.

3. Mathematical model

The calculations were performed in a 200x14 m domain, with horizontal dimension of 200 m and vertical extension of 14 m. The length of the quarry was 100 m and the depth was 6 m. The velocity of the base flow was taken to be equal to 0.3 m/s. The size of the particles forming the bottom and walls of the quarry were about $d = 150 \mu m$, the particle density was $\rho_s = 2.65 \times 10^3$ kg/m$^3$. The calculations were made up to 400,000 seconds (about 4.63 days).

The numerical simulations showed that a vortex appears near the bottom of the quarry (Fig. 2). During simulations, it appears that this vortex is moving in such a way that it causes the erosion of the front (up-stream) edge of the quarry more intensively than the downstream one.

For a sand shear viscosity of the order of $10^5$ kg/m s, the sanding of the quarry occurs slowly (Fig.3). The quarry erosion moves upstream and is accompanied by the accumulation of the sediments inside the quarry. With decreasing levels and increasing slopes one can observe the erosion of the river bottom, which propagates upstream (against the base stream). In the downstream part of the quarry, the
growth of the slopes of the water surface and flow velocities is observed, which, as a rule, increases the size of the bottom ridges. The depth of the riverbed penetration is proportional to the quarry length. The area occupied by the quarry is gradually covered by sediments intensively delivered from the first (upstream) region.

Fig. 3. Dynamics of the quarry erosion. (a) – 9,000 s (2.5 h), (b) – 400,000 s (4.63 days).

Figure 4 illustrates the regimes with uniform sanding of the quarry in the case where the flow velocity is higher than the non-eroding velocity. One can see the erosion of the bed bottom in the upstream region adjacent to the quarry and gradual sanding of the quarry itself.

Fig. 4. Dynamics of erosion of quarry (a) – 5,000 s (1.39 h), (b) – ~10^6 s (11.574 days).

With increasing distance from the quarry, the sediment flow rate gradually increases along the length extension of the river. For length quarries of 50, 100 and 200 m, the time of sanding of the quarry was found to be equal to 6, 13 and 27 months, respectively, for the average size of the river bed particles equal to 0.15 mm and an average velocity of the base flow equal to 0.3 m/s.

4. Conclusions

The calculations showed that, at the bottom of the quarry, the water stream motion generates a vortex in such direction that the upstream wall of the quarry is eroded more intensively than the downstream wall. Filling of the quarry with sediments occurs due to a subsequent downward displacement of the upstream wall of the quarry. The suspended sediments can partially settle out in the downstream part of the quarry, which leads to gradual sanding of the quarry accompanied by a change of the flow depth in the flow regions upstream and downstream from the quarry at a distance exceeding its length. In the
downstream region, the overall erosion of the river bed takes place. The zone of maximal erosion of the bottom is adjacent to the downstream wall of the quarry and spreads over this wall. This effect was recorded in multiple field observations.

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