The Horava-Lifshitz Type Quantum Field Theory and the Hierarchy Problem

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Abstract

We study the Lifshitz type extension of the standard model (SM) at the UV, with dynamical critical exponent \( z = 3 \). One loop radiative corrections to the Higgs mass in such a model are calculated. Our result shows that, the Hierarchy problem, which has initiated many excellent extension of the minimal SM, may be weakened in the \( z = 3 \) Lifshitz type quantum field theory.
I. INTRODUCTION

Our current description of the basic interactions in nature, based on the standard model (SM) of particle physics and general relativity, is in spectacular agreement with all known experiments. However, it is almost certainly fundamentally incomplete. The extreme fine-tuning needed to keep the Higgs mass small compared to the Planck scale (i.e., the Hierarchy problem) has motivated many extension of the minimal SM. All of these contain new physics, beyond the SM, which might be tested at the Large Hadron Collider (LHC). The most widely explored of these new physics is supersymmetry (SUSY).

Recently, a new quantum field theory of gravity with “dynamical critical exponent” \( z \) equal to 3 in the ultra-violet (UV) was introduced in \([1]\), which is called Horava-Lifshitz gravity. Although having no complete diffeomorphism invariance of the General Relativity but only the subset (a form of local Galilean invariance), this theory is power-counting renormalizable \([2]\) in the 3+1 spacetime and can be quantized using stochastic quantization method \([3]\). The general relativity and Lorentz symmetry in local frame can be recovered in the infrared limit. It may provide a ghost free UV complete theory of non-relativistic gravity around the flat space. Moreover, it was found that the evolution of the universe in the Horava-Lifshitz gravity might be singularity free \([4–7]\). As its characteristic, the theory exhibits scaling properties which are anisotropic between space and time

\[
x \rightarrow b x , \quad t \rightarrow b^z t .
\]  

(1)

Measuring dimensions of operators in the unit of spatial momentum, one gets \([x] = -1\) and \([t] = -z\). The system doesn’t possess the Lorentz invariance for nontrivial exponent \( z (z \neq 1) \) but possesses spatial rotational and translational invariance. The prototype of such a quantum field theory with \( z \neq 1 \) is the theory of a Lifshitz scalar in D+1 dimensions, first proposed as a description of tricritical phenomena. The action of the Lifshitz scalar can be written as

\[
S = \int dt d^D x \left\{ (\partial_t \phi)^2 - \lambda (\Delta \phi)^2 \right\},
\]

(2)

where \( \Delta \) is the spatial Laplacian. This action describes a free-field fixed point with critical exponent \( z = 2 \). Such fixed points with anisotropic scale invariance are called Lifshitz points. Properties of Lifshitz type field theory have been investigated in \([8]\). The construction of
gauge theories with Lifshitz fixed points in D+1 dimensions have been discussed in \([9–11]\). Such a theory flows naturally to the relativistic value \(z = 1\) at long distance, and therefore the Lorentz symmetry shall appear as an emergent symmetry.

In this paper we assume that the SM has a Lorentz non-invariant UV-completion and realize it at a Lifshitz fixed point with critical exponent \(z = 3\). A similar extension of the SM in Lorentz violating approach is discussed in \([12, 13]\). We focus on the one-loop radiative corrections to the Higgs mass. Our results show that the Hierarchy problem can be weakened in such a theory.

The outline of the paper is as follows: In section II, we will construct the SM at the Lifshitz fixed point. In section III, we will calculate the one-loop radiative corrections to the Higgs mass. Some conclusions are drawn in Section IV.

II. THE MODEL

In this section, we will construct the SM at the Lifshitz fixed point and derive propagators for scalar, vector and spinor fields, which shall be used for the loop calculations in the next section. The interactions for the spinor and Higgs fields, with dynamical critical exponent \(z\), can be written as

\[
S_F = \int dt d^3 \vec{x} \bar{\psi} i \left\{ \gamma^0 D_t - \alpha \gamma^i D_i (\vec{D})^{z-1} \right\} \psi , \quad (3)
\]

\[
S_H = \int dt d^3 \vec{x} \left\{ (D_t H)^2 - \beta [D_i (\vec{D})^{z-1} H]^2 - \frac{1}{2} \mu^2 (H^\dagger H) - \sum_{n=2}^{c} \frac{1}{2^n} \lambda_n (H^\dagger H)^n \right\} , \quad (4)
\]

where \(\psi\) represents the SU(2)\(_L\) doublet or singlet, \(\gamma^i (i = 0 \sim 3)\) is the gamma matrix, \(D_\mu (\partial_\mu + ig_1 \tau^i W^i_\mu + ig_2 Y B_\mu )\) is the covariant derivative and \(c\) is the biggest integer satisfying the inequality \(c(3 - z) \leq 3 + z\). The engineering dimensions of \(\psi\) and \(H\), as well as coupling constants \(\alpha\) and \(\beta\), are given by

\[
[\psi] = \frac{3}{2}, \quad [H] = \frac{3 - z}{2}, \quad [\alpha] = [\beta] = 0 . \quad (5)
\]

The system has a free-field fixed point with \(z = 3\) for any spatial dimensions. In this case, \(c\) in Eq. (4) goes to infinity. The Yukawa interactions between the Higgs and spinor fields are

\[
S_Y = \int dt d^3 \vec{x} \left[ Y_e \bar{e}_L H E_R + Y_u \bar{q}_L H u_R + Y_d \bar{q}_L H d_R + \text{h.c.} \right] . \quad (6)
\]
Here \([Y_\alpha] = 3/2(z - 1)\). In Ref. [14], there is a novel approach responsible for the origin of fermion mass in a Lifshitz type extension of the SM including an extra scalar field. In this paper, we simply assume that Yukawa interactions are responsible for the origin of fermion masses after the electroweak symmetry spontaneously broken. For \(z = 3\), there are two other renormalizable terms:

\[
\kappa_g f_L^T \epsilon H f_R^T H f_R \epsilon H f_R^T H f_R,
\]

here \(f\) stands for quarks or leptons and the first term can be used to generate neutrino tiny Majorana masses without introducing right-handed Majorana neutrinos.

Now let’s construct the gauge field theory with arbitrary dynamical critical exponent \(z\) in 3+1 dimensions. We take \(A_0^a, A_i^a\) as the time and spatial components of gauge fields, separately. Our theory should be invariant under the following gauge transformation:

\[
A_\mu^a \to A_\mu^a + \frac{1}{g} \partial_\mu \epsilon^a + f^{abc} A_\mu^b \epsilon^c = A_\mu^a + \frac{1}{g} D_\mu \epsilon^a. \tag{7}
\]

Gauge invariant Lagrangian will be constructed from the field strengths, which are constructed from the commutations of covariant derivatives as \([D_i, D_j] = igE_i\) and \([D_i, D_j] = igF_{ij}\). The Lagrangian should contain a kinetic term which is quadratic in first time derivatives. Following the strategy proposed in Ref. [10], we obtain the following gauge invariant interactions

\[
S_{YM} = \int dt d^3 \vec{x} \left[ \frac{1}{2} Tr(E_i E_i) - \frac{1}{2\delta} Tr \left( \prod_{j}^{z-1} D_j F_k \right)^2 \right], \quad (z > 1) \tag{8}
\]

where \(Tr\) represents the trace for the gauge generators and \(\delta\) denotes dimensionless coupling constant. The engineering dimensions of the gauge field components and coupling constants at the corresponding fixed point \(z\) are

\[
[A_i] = \frac{z + 1}{2}, \quad [A_i] = \frac{3 - z}{2}, \quad [\gamma_i] = \frac{z - 1}{2}. \tag{9}
\]

The equation of motion for the gauge fields can be obtained from Eq. (8) by varying \(A_0\):

\[
\partial_i (\partial_i A_i) - D_i D_i A_0 = 0. \quad \text{We choose the following natural gauge-fixing condition:}
\]

\[
A_0 = 0, \quad \text{and} \quad \partial_i A_i = 0. \tag{10}
\]

According to the equation of motion, once we adopt the gauge-fixing condition in Eq. (10) at \(t = t_0\), this condition will continue to hold for all \(t\). Then we can derive the propagator for gauge fields using functional method

\[
\langle A_i^a A_j^b \rangle \propto \frac{-ig_{ij} \delta_{ab}}{k_0^2 - \delta k^6}. \tag{11}
\]
We can also derive the propagators for spinor and Higgs fields from Eq. (3) and (4), using the same method:

\[
\langle \psi_k \psi_l^\dagger \rangle \propto \frac{i}{\gamma^0 k_0 - \alpha \gamma^i k_i \delta_{kl}} \delta_{gf}, \quad \langle HH^\dagger \rangle \propto \frac{i}{k_0^2 - \beta k^5} .
\] (12)

As can be seen, interactions given in Eqs. (3), (4) and (8) are incomplete. The full theory should contain all operators with dimension less than \( z + 3 \), which are not forbidden by symmetries. Therefore, following terms should be added to the Lagrangian: 

\[
-\alpha_1 \bar{\psi} i \gamma^i D_i \psi, \quad -\beta_1 |D_i H|^2, \quad -1/2 \text{Tr}(F_{ij})^2, \quad (-1/2 \delta_1) \text{Tr}[D_j F_{ik}]^2 .
\]

That is why we use the symbol “\( \propto \)” instead of “\( = \)” in Eqs. (11) and (12). However, when \( \Lambda > \Lambda_{IR} \), where \( \Lambda_{IR} \) stands for infrared cut off, below which the Lorentz symmetry is recovered, these terms will be the subdominant contribution to the propagators. As a result, we can safely use these propagators to preform one-loop calculations at \( \Lambda (> \Lambda_{IR}) \).

A distinctive feature of the Lifshitz type quantum field theory is that, the dispersion relation becomes \( E^2 - \sum \alpha_i k^{2i} = m^2 \) \([15]\), where \( \alpha_i \) are marginal coupling parameters. Then the speed of light can be written as

\[
c = \left(\sum_n n \alpha_n k^{2n-1}\right) \left(\sum_n \alpha_n k^{2n}\right)^{-\frac{1}{2}},
\] (13)

where \( k = |k| \). In our case, \( \alpha_1 = 1, \alpha_2 = \delta_1 \) and \( \alpha_3 = \delta \). One finds that, if \( z \geq 2 \) the speed of light goes to infinity in the UV. For \( z = 3 \), the discrepancy of the speed of light at the UV and IR can be used to explain the time delays in gamma-ray bursts \([11]\).

### III. HIERARCHY PROBLEM

For a long time, there were only two solutions to the Hierarchy Problem: SUSY and technicolor, and SUSY is heavily favored. In recent years, there are several other new ways to address the Hierarchy problem, including ADD models \([16]\), little Higgs models \([17]\), twin Higgs models \([18]\), folded SUSY \([19]\), Lee-Wick SM \([20]\), and so on. In this section, we will explore a new solution to the Hierarchy problem in the Lifshitz type quantum field theory. We assume \( z = 3 \) and then calculate one-loop radiative corrections to the Higgs mass in such a theory, using the propagators for scalar, spinor and vector fields presented in Eqs. (11) and (12). Relevant feynman diagrams are listed in Fig. (1).
Fig. 1 (a) comes from Higgs self interaction. Direct calculation to this diagram results in

$$\lambda \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - \beta k^6 - \mu^2} = \frac{\lambda}{24\pi^2\sqrt{\beta}} \ln \frac{\Lambda_{UV}}{\Lambda_{IR} + \mu^2},$$

where $\lambda$ is the four Higgs self-interaction coupling constant and $\Lambda_{UV}$ is ultraviolet cut off. We may find that there is only logarithmic divergence instead of quadratic divergence in Eq. (14). Fig. 1 (b) comes from Yukawa interactions. Direct calculation to this diagram results in

$$\int \frac{d^4k}{(2\pi)^4} (-1)^{Tr} \left[ Y^\alpha_{ik} \gamma^0 k_0 - \alpha \gamma^i k_i \frac{2\pi}{4} \right] \cdot Y^\alpha_{ik} \gamma^0 (k_0 - p_0) - \alpha \gamma^i (k_i - p_i) (k - p)^2$$

$$= T \int \frac{d^4k}{(2\pi)^4} \frac{k_0(k_0 - p_0) - \alpha^2 k \cdot (k - p) k^2(k - p)^2}{(k_0^2 - \alpha^2 k^6)[(k_0 - p_0)^2 - \alpha^2(k - p)^6]}$$

$$\approx \frac{T}{24\pi^2 \alpha} \ln \left( \frac{\Lambda_{UV}}{\Lambda_{IR}} \right),$$

where $T \equiv -Tr[Y_e e^\dagger + 3Y_u Y_u^\dagger + 3Y_d Y_d^\dagger]$. To calculate integral in Eq. (15), we have used the approximation $p \ll k$, which is good enough for $p \ll \Lambda_{IR}$. Fig. 1 (c) comes from gauge interactions of Higgs field, which gives

$$\left( \frac{3}{2} g_2^2 + \frac{1}{2} g_1^2 \right) p^4 \int d^4k \frac{i}{k_0^2 - \delta k^6} = \frac{p^4}{48\pi^2 \sqrt{\delta}} \left( 3g_2^2 + g_1^2 \right) \ln \left( \frac{\Lambda_{UV}}{\Lambda_{IR}} \right),$$

where $p$ is the spatial momentum of Higgs field and $g_1$, $g_2$ are gauge coupling constants corresponding to $U(1)_Y$ and $SU(2)_L$, respectively.

To sum up, traditional feynman diagrams listed in Fig. 1, that are quadratic divergent in the SM, become logarithmic divergent in the $z = 3$ Lifshitz type quantum field theory. Actually, in this theory there are two other feynman diagrams that may contribute to Higgs mass. We list them in Fig. 2.

Direct calculation to Fig. 2 (a) results in

$$Q \ p^2 \left[ \int \frac{d^4k}{k_0^2 - \delta k^6} \right] = Q \ p^2 \frac{1}{576\pi^4 \delta} \left[ \ln \left( \frac{\Lambda_{UV}}{\Lambda_{IR}} \right) \right]^2,$$
FIG. 2: Extra Feynman diagrams in the $z = 3$ Lifshitz type quantum field theory, that contribute to the Higgs mass.

where $Q \equiv 1/4(6g_2^4 + 3g_1^2g_2^2 + g_1^4)$. Direct calculation to Fig. 2 (b) gives

$$R \left[ \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - \delta k^0} \right]^3 = R \frac{1}{13824\pi^6\delta^{3/2}} \left[ \ln \left( \frac{\Lambda_{UV}}{\Lambda_{IR}} \right) \right]^3,$$

(18)

where $R \equiv 1/8(10g^6 + 6g_1^2g_4^4 + 3g_1^4g_2^2 + g_1^6)$. It is clear from Eqs. (17) and (18) that, Fig. 2 (a) and (b) can not lead to terrible quadratic divergence.

Let’s go to investigate the effect on Hierarchy problem from the interaction of Higgs doublet and four fermions. To guarantee the gauge invariance, such interaction can be written as $\zeta_{gf}^{kl} f_L^g f_R^f H f_L^k H f_R^l$. Then the Feynman diagram similar to Fig. 2 (a) with gauge field loop changed with fermion loop may contribute to the Higgs mass. Actually, fermions are massless above the electroweak scale and such fermionic blob like Feynman diagram does not work at all. We come to the same conclusion for the interaction, like $\kappa_{gf}^{CL} \varepsilon H f_L^T \varepsilon H$.

As can be seen in the upper calculations, we only considered the UV contribution to the Hierarchy problem. Actually, in the $\Lambda_{IR}$, Lorentz invariance recovers as accidental symmetry [1] and Lifshitz type quantum field theory goes back to the SM. Taking into account the contribution from IR, the total corrections to the Higgs mass should be

$$\delta m_H^2 \propto A \times \Lambda_{IR}^2 + B \times \ln \left( \frac{\Lambda_{UV}}{\Lambda_{IR}} \right) + \cdots,$$

(19)

where $A$ and $B$ stand for coefficients. There are constraints on the scale of Lorentz symmetry violation from HESS [21], MAGIC [22] and FERMI [23] experiments, which may be $\Lambda_{IR} \sim 10^{11}$ GeV [24]. Taking this result into Eq. (19), we find that the Hierarchy problem is still there. But in this case, $\delta m_H$ is proportional to $\Lambda_{IR}$ not to the Planck scale, such that the Hierarchy problem is weakened.
IV. SUMMARY

In this paper we have considered a Lifshitz type extension of the SM at the UV with dynamical critical exponent $z = 3$. We have written down the full interactions and derived the propagators for scalar, spinor and vector fields. Then we have focused on calculating one-loop radiative corrections to the Higgs mass. Our results show that the Hierarchy problem can be weakened in the $z = 3$ Lifshitz type quantum field theory. But still, there are many other problems for such a theory, which are important and interesting but beyond the scope of this paper. A detailed study to these topics will be shown in somewhere else.

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