Observational Aspect of Black Hole Dark Matter

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Abstract

Advances in high angular resolution astronomy make it conceivable that black hole dark matter could be detected via angular deviation effects.

Assuming the dark matter in the galaxy is made of solar mass black holes, there is a non-trivial probability that a line-of-sight through the galaxy, leads to μarcsec’s deviations, a value that has been discussed for various astronomical projects.

In cosmology the effects are magnified by an increased density at early times and an opening of angles due to redshift. If the dark matter is made of primordial black holes, present at the CMB, random deflections of the CMB photons lead to a limit on the angular resolution, approximately $3 \times 10^{-7} \sqrt{M/M_\odot} \text{ rad}$, with $M$ the mass of the black holes. Using the resolutions of $\sim 10^{-3} \text{ rad}$ demonstrated in observations of the “acoustic peaks” then implies the limit $(M/M_\odot) \lesssim 10^7$. While this large value seems uninteresting, improved resolutions would lead to significant limits or conceivably the discovery of primordial black holes.

1 Introduction

The problem of the nature of the dark matter is and remains one of the primary and most fascinating questions of contemporary science. While the search for elementary particle dark matter (“WIMPS”), particularly by means of the cryogenic technique\cite{1}, seems at present the most plausible path to its elucidation, some other possibilities have been discussed.

One of the most interesting of these possibilities is that the dark matter is made of black holes, primordial objects presumably originating from the very early universe. These have not been definitively excluded and it has been suggested that they could help in the understanding of structure formation in cosmology\cite{2}.

Discussions concerning this possibility have necessarily relied on indirect arguments. But it certainly would be more satisfying if there were direct observational evidence, for or against the black hole hypothesis. Here we would like
to consider how it may be possible to obtain such evidence by means of high angular resolution observations.

For a preliminary orientation we first consider the situation for the galaxy, and then turn to the CMB, where general relativistic effects lead to large enhancements.

2 Deflection and Distance

Our arguments are based on the deflection of light by massive bodies, as in the famous bending of light by the sun. If black holes make up the dark matter, the interstellar or intergalactic medium would be very “lumpy” on short distance scales, as opposed dark matter made of elementary particles.

The deflection angle $\delta \alpha$ of a ray passing a gravitating object of mass $M$ at impact parameter $b$, is

$$\delta \alpha = \frac{2 r_s}{b} \left( \frac{M}{M_\odot} \right) = \frac{6.3 \times 10^{-13}}{b} \left( \frac{M}{M_\odot} \right) \text{ly},$$

where we express $M$ in terms of the Schwarzschild radius $r_s = 2GM$ and normalize to the mass of the sun, $M_\odot$, where one has $r_s = 3.0 \text{km}$.

Thus for a deflection of $\delta \alpha$ or more there is a certain impact parameter $b$, to which we can associate a “cross section”

$$\pi b^2 = \pi \left( \frac{2r_s}{\delta \alpha} \right)^2 = 1.1 \times 10^2 \left( \frac{1}{\delta \alpha} \right)^2 \left( \frac{M}{M_\odot} \right)^2 \text{km}^2$$

$$= 1.3 \times 10^{-24} \left( \frac{1}{\delta \alpha} \right)^2 \left( \frac{M}{M_\odot} \right)^2 \text{ly}^2.$$ 

This is the cross section for having, in a single passage near a massive object, a deflection $\delta \alpha$ or more.

(Here and in the following we speak of the black holes as having a single, unique mass; if instead there is a spectrum of masses, $M$ should be understood as the average mass.)

If we consider $\delta \alpha$’s on the order of a $\mu$arcsec $= 4.8 \times 10^{-12} \text{rad}$, as in long baseline interferometry [3], this cross section can be $\sim \text{ly}^2$ for a solar mass object. Equivalently, passages at distances $\sim \text{ly}$ are potentially observable given such resolutions.

3 Probability of a $\delta \alpha$

One may turn Eq[2] into the probability for a deflection $\delta \alpha$ (or more) for a photon traveling a certain path by multiplying by the column density

$$\text{Prob}_{\delta \alpha} = \text{cross section} \times \text{column density} = \pi \left( \frac{2r_s}{\delta \alpha} \right)^2 \times \rho_2,$$
The column density \( \rho_2 \) is the two-dimensional density of the total number of black holes along the line-of-flight, projected on the perpendicular plane. (We are taking \( \text{Prob}_\delta \alpha \) to be small, a value approaching one implies multiple encounters.)

The \( \rho_2 \) can in turn be expressed in terms of the presumably approximately known dark matter mass column density and the unknown black hole mass \( M \) as

\[
\rho_2 = \frac{\mu}{M}
\]

where the dark matter mass column density is expressed as \( \mu/\text{unit area} \), with \( \mu \) a mass.

Since \( r_s^2 \sim M^2 \) one sees that Eq 3 is proportional to \( M \) and vanishes as \( M \to 0 \), as expected when the medium becomes less “lumpy” and approaches a smooth continuum.

4 Galaxy

We first apply the above estimates to the case of the galaxy. In his original discussion of ‘Machos’, Paczynski [4] briefly mentioned this possibility of the direct observation of angular deviations, but dismissed it as immeasurably small. However, since that time there have been great improvements in angular resolutions; also a brief discussion for the galaxy will allow us to establish the ideas without the complications of general relativity.

While an accurate calculation of \( \rho_2 \) requires an integration of the dark matter density along the flight path, we can crudely estimate it for a source in the Milky Way (or for rays traveling through a nearby galaxy) by taking the presumed dark matter density near the earth of \( 0.4 \text{GeV/cm}^3 = 7.1 \times 10^{-28} \text{kg/cm}^3 = 6.0 \times 10^{20} \text{kg/ly}^3 \) and a typical galactic travel distance of \( 10^4 \) light years. One thus obtains

\[
\rho_2 \approx \frac{1}{M} \left( 6.0 \times 10^{20} \text{kg/ly}^3 \right) \times 10^4 \text{ly} = 3.0 \times \frac{(M_{\odot}/M)}{\text{ly}^2} \text{ galaxy}
\]

Combining Eq 2 and Eq 3 one has the probability of a deflection of \( \delta \alpha \) or more in a typical passage through the galaxy

\[
\text{Prob}_{\delta \alpha} \approx 3.7 \times 10^{-24} (1/\delta \alpha)^2 = 1.6 \times 10^{-1} \times (1/\delta \alpha')^2 (M/M_{\odot}) \text{ galaxy}.
\]

In the first writing \( \delta \alpha \) is in radians, while in the second \( \delta \alpha' \) is in \( \mu \text{arcsec} \). Thus if the dark matter is made up of solar mass (or more) black holes, a ray crossing the galaxy has a substantial chance of a several \( \mu \text{arcsec} \) deflection. This estimate is of course quite approximate, and will depend, for example, on whether the flight path is through regions of high or low dark matter density.

The next question is how this very small angular deflection might be detected, given that there are generally larger effects from the motion of the source and the earth. To obtain an observable signal, one might look for a motion of \( \delta \alpha \). The black hole, like other objects in the galaxy, will be moving with some
velocity. This velocity will lead to a change in \( b \) in Eq \( \text{1} \) and so a change in \( \delta \alpha \). Since we are considering passages at distances \( b \) on the order of \( a \) ly or less and the typical velocity of objects in the galaxy \( v \approx 2 \times 10^{-3} \) ly in one year, it is possible that a significant change in \( b \) can occur over a timescale of years. The presence of the black holes would be thus signaled as a small ‘noise’, with a timescale on the order of years, on the more smoothly varying angular positions of the sources.

A shorter timescale for the ‘noise’ could be obtained by raising the threshold of detection for \( \delta \alpha \), implying smaller \( b \), but then a reduced probability \( P_{\text{rob}} \). This behavior with respect to threshold would be useful in establishing the reality of a possible positive signal.

5 CMB As Source

We now turn to observations on the CMB. Instead of considering a particular source, we consider the typical deflection of a photon and argue that due to the randomness of this deflection smaller angular features will be washed out, i.e. the presence of black holes implies a limit on the angular resolution possible.

In addition, there is a further point to observations on the CMB. So far our discussion would apply to any compact gravitating object— the rays would have to pass improbably close to the object to see a behavior truly characteristic of black holes. And many such objects have been discussed, such as “Machos”, “Brown Dwarfs”, and so forth. However, at least with the present understanding of cosmology, such compact objects would be formed after the formation of structure, and would not be present at the CMB. On the other hand, primordial black holes would presumably arise from very early times \([2]\). Therefore, a detection at the CMB would strongly support the case that the objects are indeed primordial.

For the CMB case, general relativistic effects, arising from an opening of the deflection angle for the observer, and the high density of dark matter at earlier times, lead to a large quantitative enhancement relative to Eq \( \text{6} \).

5.1 Opening of Angles

The deflection angle produced at some high redshift is magnified when observed “now”, due to the redshift of general relativity. We work in the simplest FRW model for cosmology, where one has the scale factor \( a(t) = (t/t_{\text{now}})^{2/3} \) after the formation of the CMB, with the value \( t_{\text{now}} = 2.9 \times 10^{17} \) s.

In the local frame of the black hole, the photon before deflection has the 4-vector \( k \) and after deflection \( k' \). The scalar 4-product is \( kk' = \omega^2 (1 - \cos \delta \alpha) \approx \frac{1}{2} \omega^2 (\delta \alpha)^2 \). We take the two photons to have the same frequency and to differ by a small angle, as for photons deflected by a massive black hole. A scalar product is conserved under parallel transport, and since the photons come to us by free fall or parallel transport, one has from a deflection \( \delta \alpha \) at cosmological time \( t \) the relation to the \( \delta \alpha \) at present, ‘now’, \( (\omega_t \delta \alpha_t)^2 = (\omega_{\text{now}} \delta \alpha)^2 \).
Therefore the deviation angles are in the ratio of the frequencies and are so increased by the redshift.

\[ \delta \alpha = \frac{1}{a(t)} \times \delta \alpha_t \]  

(7)

where \( a(t) \) is the scale factor at the cosmological time \( t \) of the scattering.

This means that a deflection \( \delta \alpha \) ‘now’ originates from a smaller deflection at high redshift, and since these will occur at larger impact parameter, they have a higher probability than would be the case without this angular effect. (This logic can also be used to show that a nonrelativistic transverse motion has only a small effect on \( \delta \alpha \); also applicable for the galaxy case.)

### 5.2 Increase of Density

A second effect is the increased density of the dark matter at early times, which is expected to vary as \( 1/a^3 \), such that number density at time \( t \) is \( \rho/a^3 \), where \( \rho \) is the present number density. The contribution to the column density from a cosmic time interval \( dt \) is then

\[ d\rho_2 = \frac{\rho}{a^3} dt = \frac{\mu}{M} \frac{1}{a^3} dt , \]  

(8)

where \( \mu \) is the cosmological dark matter mass density at present.

### 5.3 Probability Integral

Taking these two effects into account

\[ d\text{Prob}_{\delta \alpha} = \pi b^2(t) d\rho_2 = \pi (\frac{2r_s}{a(t)\delta \alpha})^2 d\rho_2 , \]  

(9)

using Eq(7) and Eq(1) to give \( b(t) = 2r_s/(a(t)\delta \alpha) \). Combining with Eq(8)

\[ \int d\text{Prob}_{\delta \alpha} = \frac{\mu}{M} \pi (\frac{2r_s}{\delta \alpha})^2 \int \frac{1}{a^3} dt , \]  

(10)

The factor \( 1/a^5 \) will lead to large enhancement over the simple dimensional factors:

\[ \int_{\text{cmb}}^{\text{now}} \frac{dt}{a^5} = \frac{t_{\text{now}}}{t_{\text{cmb}}} \frac{3}{7} (x^{7/3} - 1) = 7.2 \times 10^9 \times t_{\text{now}} \]  

(11)

with \( x = t_{\text{now}}/t_{\text{cmb}} = 2.4 \times 10^4 \), integrating over the time since \( t_{\text{cmb}} = 3.8 \times 10^5 \) y.

We are left with the task of evaluating the dimensional factor

\[ \frac{\mu}{M} \pi (\frac{2r_s}{\delta \alpha})^2 t_{\text{now}} \]  

(12)

The “cross section” was evaluated in Eq(2)

\[ \pi (\frac{2r_s}{\delta \alpha})^2 = 1.3 \times 10^{-24} \left( \frac{1}{\delta \alpha} \right)^2 \left( \frac{M/M_\odot}{2} \right)^2 \text{ly}^2 \]  

(13)
The remaining factor $\frac{\mu}{M}$ of $t_{\text{now}}$ can be interpreted as the column density for a distance $t_{\text{now}}$ ($\sim$ Hubble distance) at the present dark matter density, without the general relativistic effects. To evaluate this we take the present dark matter mass density $\mu$ at 1/4 the critical value \cite{6}:

$$\mu = 1.0 \times 10^{-9} M_\odot/\text{ly}^3,$$  \hspace{1cm} (14)

which together with $t_{\text{now}} = 9.2 \times 10^9\text{ yr}$ yields

$$\frac{\mu}{M} t_{\text{now}} = 9.2 \times (M_\odot/M)/\text{ly}^2$$ \hspace{1cm} (15)

Interestingly, this is about the same as the estimate for the galaxy in Eq \cite{5} a factor $10^6$ in the densities has been cancelled by a similar factor in the distances. Thus the main difference vis-a-vis the galactic effects arises from Eq \cite{11}.

To finally estimate the integrated Eq \cite{10} for photons from the CMB we put together Eq \cite{11}, Eq \cite{13} and Eq \cite{15} to find

$$\text{Prob}_{\delta \alpha} = 8.6 \times 10^{-14} \left( \frac{1}{\delta \alpha} \right)^2 (M/M_\odot) = 3.7 \times 10^9 \left( \frac{1}{\delta \alpha} \right)^2 (M/M_\odot) \text{ CMB}$$ \hspace{1cm} (16)

where again $\delta \alpha$ is in radians while $\delta \alpha'$ is in $\mu$arcsec.

\section{5.4 Discussion}

To convert these results into a possibly observable effect, consider the question of the angular resolution possible in observations on the CMB. When $\text{Prob}_{\delta \alpha}$ is of order one, a photon from a point on the CMB, will undergo a deflection $\delta \alpha$ with high probability. Since these deflections are random in direction, the angular position of points will be spread by $\delta \alpha$. This implies a limit on the angular resolution, set by the requirement that $\text{Prob}_{\delta \alpha}$ be less than one. Setting $\text{Prob}_{\delta \alpha} \approx 1$ in Eq \cite{16} gives

$$\delta \alpha_{\text{lim}} \approx 2.9 \times 10^{-7} \sqrt{(M/M_\odot)} \quad \delta \alpha'_{\text{lim}} \approx 6.1 \times 10^4 \sqrt{(M/M_\odot)},$$ \hspace{1cm} (17)

as an approximate limit for the best obtainable resolution in observations on the CMB, in radians or $\mu$arcsec respectively.

To see how this might work in practice, we take the observation of the “acoustic peaks” in the temperature fluctuations of the CMB. (We stress that here we are not interested in the temperature fluctuations themselves but simply in the demonstration of an angular resolution.) These features have been observed out to $l \sim 1000$, implying angular resolutions of about $\sim 10^{-3} \text{ rad}$. Using Eq \cite{17} this implies an approximate upper limit on the mass of possible black holes

$$(M/M_\odot) \lesssim 10^7$$ \hspace{1cm} (18)

This does not seem a very stringent limit, the mass $M \sim 10^7 M_\odot$ is larger than that already known for black holes in galaxies \cite{7}.
However, the argument does suggest that higher resolution observations on the CMB could lead to significant restrictions or perhaps even positive evidence for dark matter black holes. The latter is necessarily more difficult since it would be necessary to eliminate other possible angular averaging effects, both instrumental and natural. In this connection it should be noted that Eq(17) is frequency independent, reflecting the achromatic behavior of light in gravitational fields, a feature that would not be expected for most background effects.

Finally, to carry out such studies it would be necessary to identify small angle features on the CMB, analogous to the “acoustic peaks”, which could be used to demonstrate the desired high angular resolution. A way around this problem could be establishing coherence of the radiation field over long baselines [8].

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