At present and future hadron colliders, the precision physics program started in the past will be continued. In particular, a precise determination of the $W$ boson mass will be carried out. This requires the calculation of the radiative corrections and their implementation in Monte Carlo event generators for data analysis. In this talk, the status of the calculation of the order $\alpha$ electroweak radiative corrections is reviewed and a study of the impact of higher order QED corrections on the $W$ boson mass is presented.

1 Introduction

In addition to the program of discovery physics, experiments at the high-energy hadron colliders Tevatron RunII and the LHC are expected to continue the program of precision physics successfully carried out at LEP, SLC and the Tevatron itself. In particular, for precision tests of the Standard Model a very precise determination of the $W$ boson mass $M_W$ is important because this, together with an improved measurement of the top-quark mass, will put more severe indirect bounds on the mass of the Higgs boson.

The precision expected at the Tevatron is about 30 MeV per experiment per channel at Run IIa and 16 MeV at Run IIb, the latter being the same precision aimed at at the LHC [1]. Therefore, precise calculations and event
generators for the Drell-Yan process \( \overline{p}p \rightarrow W \rightarrow l\nu \) are strongly required.

From the experimental point of view, the \( W \) mass is extracted from the kinematics of the \( W \) boson decay \( W \rightarrow l\nu \), giving rise to a Jacobian peak in the distribution of the lepton transverse momentum \( p_T(l) \). However, the preferred quantity to determine \( M_W \) is the transverse mass spectrum \( M_T \) [1], which is less sensitive than the \( p_T(l) \) distribution to the \( W \) transverse motion.

Having in mind the precision anticipated at the Tevatron and the LHC for \( M_W \), accurate theoretical predictions including QCD and electroweak (EW) radiative corrections (RC) are necessary to precisely extract \( M_W \) from the data. In the following, we concentrate on EW RC.

2 Order \( \alpha \) electroweak radiative corrections

In the past years, the calculation of the full set of the \( \mathcal{O}(\alpha) \) EW RC to the single-\( W \) production in hadronic collisions was carried out. The first calculations were performed in the resonant \( W \) approximation (pole approximation) in Refs. [2, 3] and then the complete \( \mathcal{O}(\alpha) \) corrections were calculated in Ref. [4]. It is worth noticing that the non-resonant contributions are important in the tail of the \( M_T \) distribution, far from the \( M_W \) peak, because large Sudakov-like logarithms arise. Since the \( W \) width can be measured in the \( M_T \) tail, the full calculation is then mandatory.

In the present experimental analyses, the corrections of Ref. [4] are included. It comes out that EW corrections shift the \( W \) mass (in the measurements at the Tevatron Run Ib) by an amount of \(-65 \pm 20 \) MeV and \(-168 \pm 10 \) MeV for electron and muon channels, respectively [1]. It is known that these shifts are mainly due to final-state photonic corrections because of the presence of large collinear logarithms. In the presence of realistic selection criteria, the correction due to final-state photon radiation is of several per cent on the \( M_T \) spectrum in the peak region \( M_T \approx M_W \). This poses the question of the impact of higher-order QED corrections due to the multiple emission of (real and virtual) photons. These higher-order contributions are not presently included in data analysis at the Tevatron but they are estimated to introduce a systematic uncertainty of 20 MeV in the \( W \rightarrow e\nu_e \) decay channel and 10 MeV in the \( W \rightarrow \mu\nu_\mu \) decay [1]. This source of systematic uncertainty is not negligible in view of the foreseen experimental precisions and it can be reduced by means of improved theoretical calculations. Recent work in this direction includes the calculation of multi-photon corrections to leptonic \( W \) decays in the framework of the YFS approach implemented in the Monte Carlo (MC) generator \textsc{winhac} [5].
3 Higher-order QED corrections

In the approach here presented the real plus virtual corrections due to multi-photon radiation are computed in the leading-log approximation using the QED structure-function approach. The corrections are calculated by solving the QED DGLAP equation by means of the QED Parton Shower (PS) algorithm developed in Ref. [6]. Only radiation from the final-state leptons is presently included in our approach, because it is known that quark-mass singularities, originating from initial-state photon radiation, can be reabsorbed into a redefinition of the Parton Distribution Functions [7]. After this mass-factorization procedure, initial-state-radiation has only a small and uniform impact on the $M_T$ spectrum, while final-state radiation significantly distorts the shape of the $M_T$ distribution, affecting in turn the $M_W$ extraction.

The formulation is implemented into a MC generator, HORACE [8], which incorporates lepton identification criteria and detector resolution effects, in order to perform simulations for the hadronic process $p\bar{p} \rightarrow W \rightarrow l\nu$ ($l = e, \mu$) as realistic as possible. HORACE calculates QED corrections to all orders and at order $\alpha$, to disentangle the effect of higher-order contributions and to compare with the available $O(\alpha)$ programs. A first comparison was performed with WGRAD [3], showing good agreement. A more detailed analysis, comparing HORACE and WINHAC, is in progress [9].

To evaluate the shift on the fitted $W$ mass induced by higher-order QED corrections we used HORACE and performed $\chi^2$ fits to MC pseudo-data for the $M_T$ spectrum, simulating acceptance cuts, lepton identification criteria and (simplified) detector resolution effects. The center of mass energy considered in our study is $\sqrt{s} = 2$ TeV, corresponding to the Tevatron, but we checked that the conclusions of our analysis do not significantly change at the LHC energy. The steps of the fitting procedure are described in detail in Ref. [8]. The results of our analysis show that the mass shift due to higher-order effects is about 10 MeV for the $W \rightarrow \mu\nu$ channel and a few MeV for the $W \rightarrow e\nu$ channel, as a consequence of the different identification requirements for electrons and muons. Therefore, in view of the expected precision of 15-30 MeV for $M_W$ at the Tevatron Run II and at the LHC, it will be important to take multi-photon effects into account when extracting $M_W$ from the data.

4 Conclusions

In view of improved precision measurements of the $W$ mass at present and future hadron colliders, accurate theoretical predictions including EW ra-
diative corrections are strongly required for data analysis.

In recent years, the calculation of the full set of the $O(\alpha)$ EW RC was carried out \[3\,4\]. They are currently included in the experimental analyses and induce a shift on the extracted $W$ mass of the order of 100 MeV. The shift is mainly due to the photonic contributions.

A theoretical systematic uncertainty which can limit the precision of the future measurement is the effect of multi-photon emission. We studied the problem within a QED PS approach, by means of the event generator \textsc{HORACE}. We found that the shift due to these corrections is about 10 MeV in the $W \rightarrow \mu\nu$ channel and a few MeV in the $W \rightarrow e\nu$ channel. Therefore, they are important in view of the aimed precision of the order of 15-30 MeV.

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