Non local screening in a vortex line liquid

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We show that the recent experiments\cite{1} reporting the onset of the non-local conductivity in the vortex state of YBaCuO single crystals indicate the presence of a new liquid phase of vortices. This phase is intermediate between the normal metal and the Abrikosov lattice. We use the mapping of the vortex problem to the problem of bose liquid to determine theoretically the properties of the proposed vortex liquid phase and compare them with the data.

In a recent Letter H. Safar et al reported\cite{1} the results of the transport measurements in the mixed state of YBaCuO single crystals with the field applied along $c$-direction of the crystal performed in a transformer geometry shown in Fig 1. Two sets of measurements were done. In the first one the current was injected through the pair of contacts (1,4) and the potential was measured between contacts (2,3) or (6,7). In the second set the current was injected through contacts (1,5) and the potential was measured between contacts (2,6), (3,7) or (4,8). In the first setup the current flows predominantly along $ab$ plane of the crystal whereas in the second setup the current flows mostly in $c$ direction.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig1.png}
\caption{Geometry of dc transformer experiment. Heavy dots denote contacts.}
\end{figure}

It was found\cite{1} that in the first setup the potential drop between contacts (6,7) is smaller than the drop between (2,3) at high temperatures, but becomes undistinguishable from it below some critical temperature $T_{th}$. If interpreted in terms of the apparent resistivity ratio $\rho_c/\rho_{ab}$ such observation implies that this ratio tends to zero at $T \rightarrow T_{th}(H)$. This temperature $T_{th}(H)$ is significantly higher than the transition temperature $T_g(H)$ defined by $\rho_{ab}(T_g) = 0$. However, the experiments of the second setup showed that the ratio of the potentials $V_{45}/V_{26}$ becomes much larger when $T \rightarrow T_{th}(H)$ indicating that apparent ratio $\rho_c/\rho_{ab}$ extracted from these measurements tends to infinity. Finally, an independent set of measurements indicate that resistivity in $c$-direction extracted from measurements with the uniform current $\rho_c(T_{th}) = 0$\cite{1}.

Safar et al interpreted their results as an evidence for non local transport in the mixed state; in this note we shall show that this non local transport may be a signature of a phase transition into the novel vortex liquid state characterized by a non-zero phase rigidity in the direction of the magnetic field\cite{1}. We shall argue that $T_{th}$ should be identified with the transition temperature into the intermediate vortex liquid state.

We begin with the estimates of the parameters of the vortex system in the regime of the experiments\cite{1}. We use the value of the penetration length $\lambda = 1400\mu m$ and anisotropy ratio $M/m = 50$ to estimate a vortex entanglement length (cf.\cite{1}) in $c$-direction: $L = 1.5(1 - t)/B \mu m$ where $t = T/T_c$ and $B$ is measured in Tesla which gives $L \sim 0.03 \mu m$ in the conditions of\cite{1}. Thus, the vortices are entangled on a much shorter scales than the sample thickness (0.03 mm). This estimate rules out the simple explanation of the experiment\cite{1} that each individual vortex remains straight on the scales of the sample thickness. The absence of current dissipation in $c$-direction shows that although vortices wander at short scales they remain relatively straight at large scales due to collective effects. The dissipation of the uniform in-plane current shows that the vortex lattice is not formed.

In order to describe the liquid of vortices state quantitatively we mapped the vortex problem to the model of strongly interacting two dimensional bosons\cite{1}. We shown that the problem of Gibbs equilibrium state of the system of infinitely long vortices at temperature $T$ is formally equivalent to the quantum ground state problem of two-dimensional interacting Bose liquid (with the "2D Plank constant" $\hbar_{2D} = T$) at zero temperature. The direction of the vortex space along the magnetic field ($\hat{z} \parallel c$) plays the role of imaginary time of the 2D quantum theory. The interaction between bosons in the quantum model has two parts: static 2D Coulomb part and dynamic transverse current-current interaction mediated...
by the fluctuating 2D "magnetic" field (cf. [3]). This mapping identifies the Abrikosov lattice and the crystal of bosons, the normal metal and the Bose superfluid liquid. In [3] we also found a new vortex liquid phase between Abrikosov lattice and normal metal which is mapped into the "normal" (i.e. non-superfluid) liquid of bosons. Here we shall use this mapping to find the response of the vortex phase in the conditions of experiment [1]. In [3] we derived the duality relations between the response functions in the vortex system and the response function for the boson system and found that the new intermediate phase has non-zero superfluid density in c-direction: \( j_z = -n_s c A_z \) but has a finite resistivity to a uniform current in ab-directions. The phase transition line normal metal-new phase was predicted in Ref. [4]. Fig. 1a is surprisingly close to \( T_{lb}(H) \) line reported in [7]. Below we show that in this state the non-uniform in-plane current which satisfies \( \int j_{ab} dz = 0 \) is non-dissipative. This resolves the apparent contradiction of observations because in the set up yielding \( \rho_{ab}/\rho_c \to 0 \) the in-plane current obeys \( \int j_{ab} dz = 0 \).

We prove the existence of a non-dissipative current \( j_{ab}(q_{z} \neq 0) \) repeating the derivation of response functions (cf. Section VI.A.1 of [3]) for the in-plane currents.

\[
D(q) = \langle A_\alpha(q) A_\alpha(-q) \rangle = \frac{4\pi T}{q^2 + \Sigma^2(q)}; \\
\Pi(q) = \frac{q^2}{A^2 + q^2 \Sigma^2(q)} \tag{1}
\]

where \( \Pi(q) \) is the longitudinal response of the dual boson liquid and \( g^2 = \frac{\phi^2}{4\pi \lambda} \) is its interaction constant i.e. the analog of the electric charge \( e_{2D} \). Here and below we explicitly consider only isotropic superconductors; we shall restore anisotropy factor \( m/M \) only at the very end using general scaling arguments [4].

The intermediate phase corresponds to the normal liquid of bosons. Since \( q_z \) play the role of the frequency in the boson model \( \Pi(q) \) is strongly dependent on the ratio \( q_z/q_{\perp} \). In the limit \( q_{\perp} \to 0 \) the longitudinal response of the boson system is described by the finite effective 'conductivity' \( \sigma : g^2 \Pi(q_z) = \sigma |q_z| \). To estimate the effective conductivity we note that normal liquid is realized in the regime of the strong interaction when the dimensionless interaction parameters are the order of unity [4]. In this regime the only combination of parameters with dimensionality of conductivity is the "quantum" conductivity \( \sigma_Q = e_{2D}^2/\hbar v_{2D} \). Identifying \( g \to e_{2D}, T \to h_{2D} \), we estimate \( \sigma \sim \frac{q_{\perp}^2}{2\pi T} \). Using this expression for \( \Pi(q_z) \) we get the correlator of the in-plane electromagnetic vector potential:

\[
D(q_z) = \frac{4\pi T}{\frac{q_z^2}{2\pi} + \Lambda^{-1}} \tag{2}
\]

where \( \Lambda = \lambda^2 \sigma = \frac{q_{\perp}^2}{8\pi^2 T} \).

Eq. (2) shows that the in-plane magnetic field decays at large distances \( z \gtrsim \Lambda \) as \( B(z) \sim B_0 \frac{1}{z} \) in this state, being screened on a scale of \( \Lambda \) by a non-dissipative current \( j \sim \frac{1}{\Lambda^2} \) along the boundary. To see this, consider the effect of the external current flowing in the \( x \) direction on the edge of the sample \( j_z(z) = J_x(z) \delta(z) \). According to Eq. (2) it induces magnetic field \( B_y(q_z) = q_z \cdot D(q_z) \cdot J_x(z) \sim 1/(q_z + \text{sgn}(q_z) \Lambda^{-1}) \). After integration over \( q_z \) one gets the induced field \( B_y(\mathbf{q}) \sim B_0 \frac{1}{q_z} \) and the shielding current \( j_z(\mathbf{q}) \propto dB_y/dz \). Note that in the superfluid phase of bose liquid one would get \( \sigma \sim 1/q_z \) and \( \Pi(q_z) \sim \text{const} \), so the electromagnetic correlation function would acquire its usual for normal metal form \( D(q_z) \propto q_z^{-2} \). The universal length scale \( \Lambda \) is about \( 400 \mu \text{m} \) at \( T = 90 \text{ K} \) which is larger than the sample thickness \( d \) ( note that the rescaling [4] taking into account anisotropy does not affect the relation between \( \Lambda \) and \( d \), since both quantities scale in the same way). The existence of non-dissipative in-plane current at \( q_z \neq 0 \) in the vortex liquid implies that \( \rho_{xx}(q_z \neq 0) = 0 \) at \( T < T_{lb}(H) \). We interpret observations [4] as a signature of vanishing of \( \rho_{xx}(q_z \neq 0) \) at \( T > T_{lb}(H) \), which supports a phenomenological description proposed in Ref. [3].

The Eq. (3) for \( D(q_z) \) was derived in the limit of in-plane homogeneous current, i.e. \( q_{\perp} \to 0 \). Using the conventional diffusion form for the density-density correlator

\[
\Pi(q) \sim \frac{\sigma q_z^2}{|q_z| + D q_{\perp}^2} \tag{4}
\]

we see that Eq. (4) remains valid if \( q_{\perp}^2 \ll q_z/D \). Here \( D \) has the meaning of "diffusion coefficient" associated with "conductivity" \( \sigma \). Thus, \( \sigma \approx e_{2D}^2 D \left( \frac{m_{2D}}{2\pi\lambda^2} \right) \) where the last factor in parenthesis is the density of states for 2D particles with mass \( m_{2D} = (\Phi_0/4\pi\lambda)^2 \). The "quantum-limit" expression for \( \sigma \), which we used above means that the mean free path of bosons is of the order of their separation, i.e. \( D \approx h_{2D}/m_{2D} \).

Now we estimate the relative size of wavevectors \( q_z \) and \( q_{\perp}^2 \) for the conditions of the experiment [1]. After the rescaling [4] takes into account with the mass anisotropy factor \( m/M = 0.02 \), the relevant wavevectors become \( q_z \approx \sqrt{m_{2D}/d}, q_{\perp} \approx 1/d_{\text{plane}} \), with the sample thickness \( d = (3-6) \times 10^{-3} \text{ cm} \) and the relevant lateral dimension \( d_{\text{plane}} \approx 0.05 \text{ cm} \). With the above estimates for \( D \), the condition \( q_{\perp}^2 \ll q_z/D \) is satisfied for any reasonable sample size.

We expect also that the results of the dc transformer measurements in thick samples \( d \gtrsim \Lambda \) should be qualitatively different from the samples reported in [1,2] due to the screening effects. Finally, we note that the results obtained in twinned samples for the tilted field will be severely modified by the interaction between vortices and twins because in these conditions each vortex line
intersects the twin boundary which affects strongly the motion and equilibrium positions of vortex lines.

In conclusion, the observations strongly support the existence of a new vortex line liquid state sandwiched between the normal state and the vortex glass.

We are grateful to H. Safar for useful discussions. We acknowledge the support of the innovation partnership grant of the State of NJ and of the grant M6M000 from International Science Foundation.

[1] H. Safar, P. L. Gammel, D. A. Huse, S. N. Majumdar, L. F. Schneemeyer, D. J. Bishop, D. Lopez, G. Nieva and F. de la Cruz, Phys. Rev. Lett. 72, 1272 (1994).
[2] F. de la Cruz, D. Lopez and G. Nieva (unpublished).
[3] M. V. Feigelman, V. B. Geshkenbein and V. M. Vinokur, JETP Letters, 52, 546 (1990).
[4] M. V. Feigelman, V. B. Geshkenbein, L. B. Ioffe and A. I. Larkin, Phys. Rev. B 48, 16641 (1993).
[5] D. R. Nelson Phys. Rev. Lett. 60, 1973 (1988).
[6] G. Blatter, V. B. Geshkenbein, A. I. Larkin, Phys. Rev. Lett. 68, 875 (1992); cf. also detailed discussion in G. Blatter et al, Rev. Mod. Phys. (1995).
[7] D. A. Huse and S. N. Majumdar, Phys. Rev. Lett. 71, 2473 (1993).