Two pentaquarks $P_c^+$ were discovered by LHCb collaboration as peaks in the proton-$J/\psi$ invariant mass. We perform the lattice QCD study of the scattering between $J/\psi$ meson and nucleon in the channels with $J^P = \frac{3}{2}^+, \frac{1}{2}^-, \frac{5}{2}^+, \frac{5}{2}^-$, where $P_c^+$ was discovered. This is the first lattice simulation that reaches the energies $4.3 - 4.5$ GeV where pentaquarks reside. The higher partial waves $L > 0$ are also explored for the first time. In this study we consider the single-channel approximation for scattering of $NJ/\psi$. Energies and eigenstates are extracted for the $NJ/\psi$ system at the zero total momentum for all six irreducible representations of the lattice irreducible representation. No significant energy shifts are observed. The number of eigenstates agrees with the number expected from non-interacting limit for scattering. This could possibly indicate that the $P_c$ resonances seen in experiment are a consequence of a coupling of the $NJ/\psi$ channel with other two-hadron channels.
1. Introduction

Two peaks in a proton-$J/\psi$ invariant mass were discovered in 2015 by LHCb [2]. This discovery was later confirmed by a model independent study in 2016 by the same collaboration [3]. Two resonances were observed, the broader with width $\Gamma = 205$ MeV and mass $M \approx 4380$ MeV and the narrower with $\Gamma = 40$ MeV and mass $M \approx 4450$ MeV. These resonances were later identified as hidden charm pentaquarks $P_c$ with minimal flavor structure $uudc\bar{c}$. LHCb found the best fit for spin-parity assignments $(J^P_c, J^P_{1/2}) = (3^-, 3^-)$, while acceptable solutions are also found for additional cases with the opposite parity, either $(3^+, 5^-)$ or $(\frac{5}{2}^+, \frac{3}{2}^-)$. $P_c$ resonances can strongly decay to a nucleon and a charmonium as well as to a charmed baryon and charmed meson.

At present there is no knowledge on $P_c$ resonances based on the first-principle lattice QCD. The lattice simulations of systems with flavor $\bar{c}cuud$ have never reached energies where the pentaquarks reside. Previous dynamical [4] and quenched [4] studies [5] presented results for $NJ/\psi$ and $N\eta_c$ potentials and phase shifts in s-wave using HALQCD method in one-channel approximation. These were extracted up to the energies 0.2 GeV above threshold. An attractive interaction was found in all channels explored, but not attractive enough to form bound states or resonances. The hadroquarkonium picture was considered in [6], where the static $\bar{c}c$ potential $V(r)$ was extracted for $m_c \rightarrow \infty$ as function of distance $r$ in the presence of the nucleon. The potential is found shifted down only by a few MeV due to the presence of the nucleon.

This is the first lattice simulation of $NJ/\psi$ scattering that reaches the energies 4.3 – 4.5 GeV where pentaquarks reside. We explore partial wave $L = 0$ and for the first time also $L > 0$. The aim is to explore the fate of pentaquark in one-channel approximation, where $NJ/\psi$ is decoupled from other two-hadron channels. We therefore perform a simulation of $NJ/\psi$ scattering in one-channel approximation in order to find whether $P_c$ features in the spectrum in this case. The detailed presentation of the study is given in [1], together with analogous simulation of the $N\eta_c$ channel.

The energy spectrum of the $NJ/\psi$ system in the non-interacting limit is an important reference case for scattering studies. The momenta $p = n\frac{2\pi}{L}$ of each hadron are discrete due to periodic boundary conditions of fermions in space on the lattice. The non-interacting energies of the nucleon-meson system are

$$E_{n,i} = E_N(p) + E_V(-p), \quad p = n\frac{2\pi}{L}, \quad \Delta E = E - E_{n,i}. \quad (1.1)$$

with $n \in \mathbb{N}^3$. The $E_{H=N,V}(p)$ are single hadron energies measured for different momenta on our lattice, they satisfy $E_{H=N,V}(p) = \sqrt{m_H^2 + p^2}$ in the continuum. Non-interacting energies $E_{n,i}$ are shown in Figure 1, together with experimental masses of $P_c$ resonances. In order to capture the region of both resonance, $NJ/\psi$ channel is explored up to $p^2 \leq 2$. The aim is to determine eigen-energies of the $NJ/\psi$ system. Energies $E_n$ are compared to non-interacting energies $E_{n,i}$ in search for the energy shift $\Delta E$. Significant non-zero energy shift $\Delta E \neq 0$ or an additional eigenstate could indicate the presence of a resonance state in the system [7].

2. Single hadron operators

In order to determine non-interacting energies of the $NJ/\psi$ system, the energies of nucleon

\footnote{Due to simplicity, $p$ will be used instead of $\mathbf{p}$ from here on.}
and meson are computed separately. We used 3 standard nucleon interpolators (Eq. 2.1) and 2 standard mesonic interpolators (Eq. 2.2) for each value of relative momenta \( p \).

\[
N(\vec{p}, t) = \sum_\epsilon \epsilon_{\alpha\beta} P^\dagger \Gamma^1 u(\vec{x}, t) (u^T(\vec{x}, t) \Gamma^2 d(\vec{x}, t)) e^{i\vec{p} \cdot \vec{x}} \ , \quad (\Gamma^1, \Gamma^2) : (1, C\gamma_5), (\gamma_5, C), (1, r_5 C \gamma_5) \\
V(\vec{p}, t) = \sum_\epsilon \epsilon(\vec{x}, t) \Gamma(\vec{p}, t) e^{i\vec{p} \cdot \vec{x}} \quad \Gamma : \gamma_5, \gamma_5, i = x, y, z
\]  \hspace{1cm} (2.1)

\[
O_{\Gamma_{1,2}}^{[J_{L,m}]} = \sum_{m_1, m_2} C_{L_1 m_1, S_{L_2} m_2} \sum_{R \in O} \gamma_s R_1 (R p) N_{m_1} (R p) M_{m_2} (-R p) . \hspace{1cm} (3.1)
\]

These are subduced to the chosen irrep \( \Gamma \) (Eq. 3.2) using subduction coefficients \( \gamma_{\Gamma_{1,2}}^{J_{L,m}} \) from [12].

\[
O_{[p, \Gamma_{1,2}]}^{[J_{L,S}]} = \sum_{m_1} \gamma_{[p, \Gamma_{1,2}]}^{J_{L,S}} O_{[p, \Gamma_{1,2}]}^{[J_{L,S}]} \ . \hspace{1cm} (3.2)
\]

On the lattice, the operators \( O_{[p, \Gamma_{1,2}]}^{[J_{L,S}]} \) form a reducible representation with respect to the lattice group \( O_h \). One has to employ operators which transform according to irreducible representations \( \Gamma^p \). Those are listed in Table 1, where several \( J^p \) contribute to a given irrep \( \Gamma^p \).

| irrep \( \Gamma^p \) | \( J^p \) |
|------------------|-------|
| \( G^\pm_1 \)    | \( \frac{3}{2} \pm \frac{7}{2} \) |
| \( G^\pm_2 \)    | \( \frac{5}{2} \pm \frac{7}{2} \) |
| \( H^\pm \)      | \( \frac{3}{2} \pm \frac{5}{2} \pm \frac{7}{2} \) |

Table 1: Irreducible representations \( \Gamma^p \) of the discrete lattice group \( O_h \), together with a list of \( J^p \) that a certain irrep contains.

\( P_e \) states with \( J = 3/2^\pm \) or \( 5/2^\pm \) could be seen in irreps \( G^\pm_2 \) or \( H^\pm \). A simple example of the \( NJ/\psi \) operator at \( p = 0 \) that transform according to the \( H^\pm \) irrep is

\[
O_{[J_{L,S}=-1]}^{[J_{L,S}=(\frac{1}{2}, \frac{3}{2}, \frac{5}{2})]} (0) = N_i (0) (V_+(0) - iV_s (0)) . \hspace{1cm} (3.3)
\]
at the sink and its creation operator $\bar{c}$ lead to multiple linear-dependent operators. For each irrep one can find linearly independent basis of operator-types. The employed operator-types can be found in appendix of [1]. All remaining operators can be written as a linear combination of these.

In the non-interacting limit, one expects several degenerate $N(p)V(-p)$ eigenstates for most of $J^P$ (or irreps) and relative momenta $p > 0$. In the continuum, different combinations of $(L,S)$ lead to a given $J^P$ ($|L - S| \leq J \leq |L + S|$) due to the non-zero spins of the scattering particles. The linearly independent combinations $(L,S)$ represent linearly independent eigenstates, so each of them should feature as an independent eigenstate in the spectrum. On the lattice, different spins $J^P$ can contribute to a given irrep $\Gamma^p$ as listed in Table 1. Linearly independent combinations corresponding to $(J^P,L,S)$, that subduce to given irrep $\Gamma^p$, now present linearly-independent eigenstates. The numbers of these states are summarized in Table 2 - those are the number of degenerate eigenstates in a given row of irrep in the non-interacting limit. The number of linearly independent operator-types is also equal to the number of degenerate eigenstates.

In the elastic approximation there is no contraction connecting $J/\psi$ and $N$ interpolators, as shown in Figure 2. Therefore a single-hadron correlation function can be simulated separately

| irrep | $N(p)J/\psi(-p)$ | irrep | $N(p)J/\psi(-p)$ |
|-------|-----------------|-------|-----------------|
|       | $p^2 = 0$ | $p^2 = 1$ | $p^2 = 2$ | $p^2 = 0$ | $p^2 = 1$ | $p^2 = 2$ |
| $G_1^+$ | 0 | 2 | 3 | $G_2^-$ | 0 | 1 | 3 |
| $G_1^-$ | 1 | 2 | 3 | $H^+$ | 0 | 3 | 6 |
| $G_2^-$ | 0 | 1 | 3 | $H^-$ | 1 | 3 | 6 |

Table 2: The number of the expected degenerate eigenstates for each row of irrep (in non-interacting limit). Each of those linearly independent eigenstates should appear in the spectrum. This number is equal to the number of the linearly-independent operator-types.

\[
\begin{align*}
C_{\Gamma^p}^{WN;\gamma} & = C_{\gamma \to x}^N C_{\gamma \to y}^V - i C_{\gamma \to x}^N C_{\gamma \to y}^V + i C_{\gamma \to x}^V C_{\gamma \to y}^V + C_{\gamma \to x}^V C_{\gamma \to y}^V, \\
C_{\Gamma^p}^{H;\gamma} & = \langle \Omega | H^{\alpha_{\Gamma^p}} | \Omega \rangle, \quad H = N, V.
\end{align*}
\]

Similarly, all two-hadron correlators in our study can be expressed in terms of $\langle 0|N_{m_i'}(p')\bar{N}_{m_i}(p)|0\rangle$, $\langle 0|V_i(p')V_i^\dagger(p)|0\rangle$ and $\langle 0|P(p')P^\dagger(p)|0\rangle$, which were pre-computed for all combinations of $p', p = 0, 1, 2, i, i' = x, y, z$ and $m_s, m_{s'} = 1/2, -1/2$. 

Figure 2: Wick contractions considered in our simulation for one-channel approximation.
4. Lattice setup

All simulations were performed on $N_f = 2$ ensemble with parameters listed in table 3, that was generated in context of the work [8, 9].

| $N^3 \times N_T$ | $\beta$ | $a[\text{fm}]$ | $L[\text{fm}]$ | #config | $m_{\pi}[\text{MeV}]$ |
|------------------|---------|----------------|-------------|---------|----------------|---|
| $16^3 \times 32$ | 7.1     | 0.1239(13)    | 1.98(2)    | 281     | 266(3)         |   |

Table 3: Parameters of the lattice ensemble.

Wilson-Clover action is used for light quarks while for charm quarks Fermi lab approach is employed. Full distillation was used for quark smearing. 48 eigenvectors were used for smearing of light quarks in nucleon, while charm quarks in charmonium were smeared with use of 96 eigenvectors.

5. Results

Resulting eigen-energies for single and two hadron system are presented. All results are obtained from the correlated one exponential fits and the errors are calculated using jack-knife method.

5.1 Individual energies of $N$ and $J/\psi$

The energies of nucleon and $J/\psi$ meson for various momenta $p^2 = 0, 1, 2$ are given in Table 4. Those are needed to determine (1.1).

| particle | $p^2$ | $E_n a$ | $\sigma_{E_n}a$ | fit range | $p^2$ | $E_n a$ | $\sigma_{E_n}a$ | fit range |
|----------|-------|---------|----------------|-----------|-------|---------|----------------|-----------|
| $N$      | 0     | 0.701   | 0.019          | [6, 9]    | 0     | 1.539   | 0.001          | [10, 14]  |
|          | 1     | 0.769   | 0.028          | [7, 10]   | 1     | 1.576   | 0.001          | [10, 14]  |
|          | 2     | 0.849   | 0.054          | [7, 9]    | 2     | 1.613   | 0.001          | [9, 12]   |

Table 4: Fitted energies for single hadrons.

5.2 $NJ/\psi$ channel

Eigen-energies of $NJ/\psi$ system were extracted from the correlation matrices using GEVP. This big correlation matrices give rather noisy eigenvalues, therefore we restricted our analysis to a smaller subset, where each operator-type is represented by two operators: both meson operators (2.2) and the first nucleon operator (2.1). Energies are obtained from the eigenvalues using the correlated one-exponential fits, while their errors are calculated using jack-knife method. All fits were performed for $t = [7, 10]$.

The observed spectrum is shown in Fig. 3. The energies are compatible with non-interacting ones (1.1) within our errors. We establish all almost-degenerate states expected in the non-interacting limit (Table 2). So our lattice results show no significant energy shift or any additional eigenstate. $P_c$ candidate channels ($J^P = \frac{1}{2}^+$ and $\frac{3}{2}^-$) are compared to the analytic prediction of eigen-energies in a scenario with Breit-Wigner-type $P_c(4450)$ or $P_c(4380)$ resonances, shown in Figure 4. In
1.1. Are compared to the analytic predictions of a scenario with non-interacting \( O_2 \). Dashed lines are non-interacting \( N J / \psi \) energies (Eq. 1.1). Green and turquoise dash-dotted lines are experimental values of \( M_P \). Center of rectangle is eigen-energy \( E_n \) and its height corresponds to \( 2 \sigma_E \). The number of expected states in non-interacting case is added to a figure on a upper left side at each data set.

![Figure 3](image-url)

**Figure 3:** Energy spectrum of \( N J / \psi \) system in all 6 irreps of \( O_2 \). Dashed lines are non-interacting \( N J / \psi \) energies (Eq. 1.1). Green and turquoise dash-dotted lines are experimental values of \( M_P \). Center of rectangle is eigen-energy \( E_n \) and its height corresponds to \( 2 \sigma_E \). The number of expected states in non-interacting case is added to a figure on a upper left side at each data set.

![Figure 4](image-url)

**Figure 4:** The energies of eigenstates in a scenario with a Breit-Wigner-type \( P_c \) (4450) or \( P_c \) (4380) resonances, assuming that it is coupled only to \( N J / \psi \) channel and decoupled from other two-hadron channels. This scenario renders an additional eigenstate near \( M_P \pm \Gamma_P \) with respect to the non-interacting case.

A scenario featuring \( P_c \) (Figure 4) would expect an additional eigenstate (with respect to the non-interacting case) at an energy close to \( M_P \). These additional eigenstates are not found in our study. We conclude that the scenario based on a Breit-Wigner-type \( P_c \) resonances, coupled solely to \( N J / \psi \), is not supported by our lattice data.

6. Conclusion

We perform a \( N_f = 2 \) lattice QCD simulation of \( N J / \psi \) scattering in the one-channel approximation, where \( N \) denotes a proton or a neutron. The resulting energies of eigenstates in Figure 3 are compared to the analytic predictions of a scenario with non-interacting \( N J / \psi \) system and a scenario featuring a \( P_c \) resonance coupled to a single channel. We find that the extracted lattice
spectra is consistent with the prediction of an almost non-interacting \(NJ/\psi\) system within errors of our calculation. The scenario based on a Breit-Wigner-type \(P_c\) resonance, coupled solely to \(NJ/\psi\), is not supported by our lattice data. This might suggest that the strong coupling between the \(NJ/\psi\) with other two-hadron channels might be responsible for the existence of the \(P_c\) resonances in the experiment. Future lattice simulations of coupled-channel scattering are needed to investigate this hypothesis.

More details on this study can be found in [1], where also \(N\eta_c\) scattering in one-channel approximation is considered.

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