Two Component Dark Matters
in
$S_4 \times Z_2$ Flavor Symmetric Extra U(1) Model

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Abstract

We study cosmic-ray anomaly observed by PAMELA based on $E_6$ inspired extra U(1) model with $S_4 \times Z_2$ flavor symmetry. In our model, the lightest flavon has very long lifetime of $O(10^{18})$ second which is longer than the age of the universe, but not long enough to explain the PAMELA result $\sim O(10^{26})$ sec. Such a situation could be avoidable by considering that the flavon is not the dominant component of dark matters and the dominant one is the lightest neutralino. With appropriate parameter set, density parameter of dark matter and over-abundance of positron flux in cosmic-ray are realized at the same time. There is interesting correlation between spectrum of positron flux and $V_{MNS}$. No excess of anti-proton in cosmic-ray suggests that sfermions are heavier than 4 TeV and the masses of the light Higgs bosons are degenerated.
1 Introduction

Standard Model (SM) is successful theory of gauge interactions, however Higgs sector is not examined well. Therefore mass matrices of leptons and quarks are not well understood. Many unsolved puzzles of SM are left in these sectors; that is, e.g., why is the structure of mixing matrix of leptons (Maki-Nakagawa-Sakata matrix, $V_{MNS}$) very different from that of the mixing matrix of quarks (Cabibbo-Kobayashi-Maskawa matrix, $V_{CKM}$), especially why is the mixing angle $\theta_{23}$ maximal? Why is neutrino mass far smaller than those of other fermions? Why do generations exist?

We also find a problem in cosmology. In modern cosmology, the existence of the dark matter is clear. Recent cosmic-ray observation of PAMELA suggests that the dark matter decays mainly into leptons with very long lifetime [1][2][3]. Such a particle is not included in SM.

Separately from these puzzles, there is hierarchy problem why electroweak scale is much smaller than Planck scale. One of the solutions is to introduce supersymmetry (SUSY) [5]. However minimal supersymmetric standard model (MSSM) does not satisfy the solution, because we must fine-tune $\mu$-parameter in superpotential of MSSM, which is much smaller than Planck scale in order to realize appropriate electroweak symmetry breaking. This is called $\mu$-problem.

Another problem of MSSM is proton stability. The R-parity forbids baryon number violating trilinear terms in superpotential, however does not forbid quartic terms like $E^c U^c U^c D^c, LQQQ$. Such interactions reduce the lifetime of proton to unacceptable level [6]. Therefore the R-parity does not help the explanation of proton stability. The problem of proton lifetime of supersymmetric model is one of the most essential point in understanding generation structure.

With the motivation to solve flavor puzzles and hierarchy problem, we introduce new three symmetries. At first, we introduce non-Abelian discrete flavor symmetry $S_4 \times Z_2$, in order to explain that the mixing angle $\theta_{23}$ is maximal [7][8][9][10][11]. Because $V_{CKM}$ and $V_{MNS}$ are very different, it is expected that the representations of quarks and leptons are also different. Next, we introduce $U(1)_X$ gauge symmetry which forbids $\mu$-term [12]. Then, several new superfields must be introduced due to gauge anomaly cancellation condition; those are extra Higgs ($H^U, H^D$), singlet Higgs $S$ and exotic quarks ($g, g^c$). The extra Higgs bosons couple only to leptons, which induce the difference between $V_{CKM}$ and $V_{MNS}$. Moreover, the existence of exotic quarks is important to understand the meaning of generations. Finally we introduce $U(1)_Z$ gauge symmetry. Due to the anomaly cancellation condition, right-handed neutrino (RHN) superfield $N^c$ is introduced, then the smallness of neutrino mass is realized by seesaw mechanism. The two new $U(1)$ gauge symmetries and standard model gauge symmetry $G_{SM} = SU(3)_c \times SU(2)_W \times U(1)_Y$ can be embedded in $E_6$ as $G_{SM} \times U(1)_X \times U(1)_Z \subset E_6$, then MSSM and new superfields consist 27 of $E_6$ representation. With appropriate assignment of superfields under the flavor symmetry, the stability of proton is realized, which plays the most important role in the flavor symmetry. Thus we can understand that the generation structure is the new system to stabilize proton [8].

The new symmetries which are introduced above may also solve dark matter problems. As three $U(1)$ gauge symmetries include R-parity, lightest supersymmetric particle (LSP) is a candidate for dark matter. The positron flux observed by PAMELA is produced by the field which induces RHN mass and decays into leptons [13]. In this paper, we show our model is consistent with experimental results of dark matter. At first, we define our model in section 2. The estimations of relic abundance of dark matter and positron flux are given in section 3. Finally, we give conclusion of our analysis in section 4.

2 $S_4 \times Z_2$ flavor symmetric extra $U(1)$ model

2.1 Gauge symmetry

We extend the gauge symmetry from $G_{SM}$ to $G_2 = G_{SM} \times U(1)_X \times U(1)_Z \subset E_6$, and add new superfields $S, g, g^c, N^c$ which are embedded in 27 representation of $E_6$ with quark, lepton superfields $Q, U^c, D^c, L, E^c$ and Higgs superfields $H^U, H^D$. In order to break $U(1)_Z$ gauge symmetry, we introduce $G_{SM}$ singlet $\Phi$ and $\Phi^c$. The gauge representations of these superfields are given in Table 1. After the gauge symmetry breaking, as the R-parity symmetry

$$R = \exp \left[ \frac{i\pi}{20}(3x - 8y + 15z) \right]$$

(1)
remains unbroken, LSP is the candidate for dark matter. The invariant superpotential under the gauge symmetry \( G_2 \) is given by

\[
W = Y^U H^U QU^e + Y^D H^D QD^e + Y^E H^D LE^c \\
+ \lambda S H^U H^D + k S g g^e \\
+ Y N H^U LN^c + Y M \Phi N^c N^c \\
+ M_\Phi \Phi^c + y_1 QQg + y_2 g^2 U^c D^c + y_3 g E^c U^e + y_{4g} g^e LQ + y_{5g} D^c N^c,
\]

where first line consists of trilinear terms in MSSM. Second line generates effective \( \mu \) term \( \lambda \langle S \rangle H^U H^D \) by radiative symmetry breaking of \( U(1)_X \). Third line generates RHN mass term \( Y N \langle \Phi \rangle N^c N^c \) by radiative symmetry breaking of \( U(1)_Z \) and gives small neutrino mass by seesaw mechanism. Fourth line consists of unwanted terms which cause the problems such that the mass term \( M_\Phi \Phi^c \) prevents \( \Phi, \Phi^c \) from developing vacuum expectation values (VEVs) and the trilinear terms of exotic quarks destabilize proton. Note that Higgs superfields are extended to three generations. Generally, extra Higgs doublets cause the problem of flavor changing neutral currents (FCNCs).

### 2.2 Flavor symmetry

\[
\begin{array}{cccccccccc}
Q_1 & Q_2 & Q_3 & U^c_1 & U^c_2 & U^c_3 & D^c_1 & D^c_2 & D^c_3 \\
S_4 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
Z_2 & + & + & + & + & + & + & + & + \\
E^c_1 & E^c_2 & E^c_3 & L_1 & L_2 & N^c_1 & N^c_2 & H^D & H^L \\
S_4 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 1 \\
Z_2 & + & - & + & - & + & - & - & - \\
H^U_1 & H^U_2 & S_1 & S_3 & g_a & g_a^c & \Phi_1 & \Phi_3 & \Phi_a \\
S_4 & 2 & 1 & 2 & 1 & 3 & 3 & 2 & 1 \\
Z_2 & - & + & - & + & + & - & + & + \\
\end{array}
\]

Table 2: \( S_4 \times Z_2 \) assignment of superfields (Where the index \( i \) of the \( S_4 \) doublets runs \( i = 1, 2 \), and the index \( a \) of the \( S_4 \) triplets runs \( a = 1, 2, 3 \)).

In order to explain maximal mixing angle \( \theta_{23} \), we introduce \( S_4 \times Z_2 \) flavor symmetry. This symmetry solves the problems of superpotential defined in Eq.(2) at the same time. If we assign \( g, g^c, \Phi^c \) to \( S_4 \)-triplets and the other superfields to singlets or doublets, then \( M_\Phi, g_1, \ldots, g_5 \) are eliminated. As a result, superpotential is given by

\[
W = Y^U H^U QU^c + Y^D H^D QD^e + Y^E H^D LE^c \\
+ \lambda S H^U H^D + k S g g^e
\]
we call them flavons. which gives an appropriate neutrino mass. As the VEVs of Φ and Φ° have very large VEVs along the D-flat direction of ⟨Φ⟩ = ⟨Φ°⟩, where

\[ V = ⟨Φ⟩ = ⟨Φ°⟩ \sim \left( \frac{M_{\text{N}} m_{\text{SUSY}}}{a} \right)^{\frac{3}{4}} \sim 10^{11} \text{GeV} \frac{1}{\sqrt{a}} \left( \frac{m_{\text{N}} m_{\text{SUSY}}}{10 \text{TeV}} \right)^{\frac{3}{4}} . \]  

From the constraints on the lifetimes of proton and exotic quarks (see appendix C), the condition

\[ \frac{V^2}{M_p^2} \sim 10^{-12} \]  

must be satisfied [14]. From this condition, \( V \sim 10^{12} \text{GeV} \) is required. This value is realized by potential minimum condition as Eq.(4), when we take \( a \sim 10^{-2} \). The prediction for RHN mass is given by

\[ M_R \sim 10^{12} \text{GeV}, \]  

which gives an appropriate neutrino mass. As the VEVs of Φ and Φ° break not only U(1)_Z but also S_4 × S_2, we call them flavons.

### 2.3 Maki-Nakagawa-Sakata matrix \( V_{MNS} \)

The maximal mixing angle \( \theta_{23} \) of \( V_{MNS} \) is realized by the assignments that \( H^U, H^D, L, N^c \) are \( 2 + 1 \) of \( S_4 \) and \( E^c \) is \( 1 + 1' \) [7]. In order to reduce the number of parameters, we assign \( S \) and \( Φ \) to \( 2 + 1 \). If we assign \( Q, U^c \) and \( D^c \) to \( S_4 \)-singlets, then quarks do not couple to \( S_4 \) doublet Higgs and FCNC is suppressed. The flavor representations are given in Table 2. The leading order superpotential is given by

\[
W_{S_4 \times Z_2} = W_L + W_Q + W_H + W_g + W_Φ,
\]

\[
W_L = Y^N_2 \left[ H^U_2 L_i N^c_i + H^D_2 (L_1 N^c_1 - L_2 N^c_2) \right] + Y^N_3 H^U_3 L_i N^c_3 + Y^N_4 L_3 (H^U_1 N^c_1 + H^U_2 N^c_2) + Y^D_2 E^c_2 L_1 + Y^D_3 F^c_3 L_3 + Y^D_4 E^c_4 (H^D_1 L_2 - H^D_2 L_1) + \frac{1}{2} Y^M_1 Φ_3 (N^c_1 + N^c_2) + \frac{1}{2} Y^M_2 Φ_3 N^c_3,
\]

\[
W_Q = Y^U_2 H^U_2 Q_i U^c_i + Y^D_3 H^D_3 Q_i D^c_i \quad (i, j = 1, 2, 3),
\]

\[
W_H = λ_1 S_1 (H^U_1 H^D_1 + H^U_2 H^D_2) + λ_2 S_4 H^U_2 H^D_1,
\]

\[
W_g = k S_3 (g_1 g_1' + g_2 g_2' + g_3 g_3'),
\]

\[
W_Φ = \frac{a_1}{2 M_p} Φ_3^2 (Φ_1^2 + (Φ_2^2)^2 + (Φ_3^2)^2) + \frac{a_2}{2 M_p} (Φ_1^2 + Φ_2^2) [Φ_3^2 + (Φ_2^2)^2 + (Φ_3^2)^2] + \frac{a_3}{2 M_p} \left\{ 2 \sqrt{3} Φ_1 Φ_2 [(Φ_3^2)^2 - (Φ_2^2)^2] + [Φ_1^2 - Φ_2^2] [(Φ_3^2)^2 + (Φ_2^2)^2 - 2(Φ_1^2)^2] \right\}.
\]  

We give the parameter set to realize the maximal mixing angle of MNS matrix. We define non-negative VEVs as

\[
⟨ H^U_1 ⟩ = ⟨ H^U_2 ⟩ = \frac{1}{\sqrt{2}} v_u, \quad ⟨ H^U_3 ⟩ = v'_u, \quad ⟨ H^D_1 ⟩ = ⟨ H^D_2 ⟩ = \frac{1}{\sqrt{2}} v_d, \quad ⟨ H^D_3 ⟩ = v'_d,
\]

\[
⟨ S_1 ⟩ = ⟨ S_2 ⟩ = \frac{1}{\sqrt{2}} v_s, \quad ⟨ S_3 ⟩ = v'_s,
\]
\[ \langle \Phi_1 \rangle = v_4, \quad \langle \Phi_2 \rangle = \sqrt{v_1^2 + v_2^2 + v_3^2 - v_4^2}, \quad \langle \Phi_3 \rangle = V, \]
\[ \langle \Phi_1^c \rangle = \sqrt{\frac{V^2}{3} + v_1^2}, \quad \langle \Phi_2^c \rangle = \sqrt{\frac{V^2}{3} + v_2^2}, \quad \langle \Phi_3^c \rangle = \sqrt{\frac{V^2}{3} + v_3^2} \quad (v_{1,2,3,4} \ll V). \]

In \( W_L \), without loss of generality, we can define \( Y_{e,4}^N, Y_{l,2,3}^N, Y_{l,1}^N \) to be real and define the phase of \( Y_{3}^N \) as \( Y_{3}^N = |Y_{3}^N|e^{i\delta} \). We define mass parameters as

\[ M_1 = Y_{1}^MV, \quad M_3 = Y_{3}^MV, \]
\[ m_{\nu}^r = Y_{N}^N v_u, \quad m_{\nu}^l = Y_{3}^N v_u, \quad m_{\nu}^4 = Y_{4}^N v_u, \]
\[ m_{l}^1 = Y_{1}^Ev_d, \quad m_{l}^2 = Y_{2}^Ev_d, \quad m_{l}^3 = Y_{3}^Ev_d. \]

Using these parameters, the mass matrices of charged leptons and neutrinos are given by

\[ M_l = \frac{1}{\sqrt{2}} \begin{pmatrix} m_{l}^1 & 0 & -m_{l}^3 \\ 0 & \sqrt{2}m_{l}^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad M_D = \frac{1}{\sqrt{2}} \begin{pmatrix} m_{\nu}^r & m_{\nu}^l & 0 \\ m_{\nu}^l & -m_{\nu}^l & 0 \\ m_{\nu}^3 & m_{\nu}^4 & \sqrt{2}e^{i\delta}m_{\nu}^3 \end{pmatrix}, \]
\[ M_R = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_1 & 0 \\ 0 & 0 & M_3 \end{pmatrix}. \]

The following neutrino mass matrix is generated through the seesaw mechanism

\[ M_{\nu} = M_D M_R^{-1} M_D^T = \begin{pmatrix} \rho_2^2 & 0 & \rho_2 \rho_4 \\ 0 & \rho_2^2 & \rho_2 \rho_4 \\ \rho_2 \rho_4 & \rho_2 \rho_4 & \rho_2^2 + e^{2i\delta} \rho_3^2 \end{pmatrix}, \]

where

\[ \rho_2 = \frac{m_{\nu}^r}{\sqrt{M_1}}, \quad \rho_4 = \frac{m_{\nu}^l}{\sqrt{M_1}}, \quad \rho_3 = \frac{m_{\nu}^3}{\sqrt{M_3}}. \]

The mass eigenvalues and diagonalization matrix of charged leptons are given by

\[ V_l^T M_l V_l = \text{diag}(m_{\nu}^r, m_{\nu}^l, m_{\nu}^3) = ((m_{l}^1)^2, (m_{l}^2)^2, (m_{l}^3)^2), \]
\[ V_l = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 1 \\ 0 & 1 & 1 \\ -\sqrt{2} & 0 & 0 \end{pmatrix}, \]

and those of neutrinos are given by

\[ V_{\nu}^T M_{\nu} V_{\nu} = \text{diag}(e^{i(\phi_1 - \phi)}, e^{i(\phi_2 + \phi)} m_{\nu1}, m_{\nu2}, m_{\nu3}), \]
\[ V_{\nu} = \begin{pmatrix} -\sin \theta_\nu & e^{i\phi} \cos \theta_\nu & 0 \\ 0 & e^{i\phi} \cos \theta_\nu & 0 \\ e^{-i\phi} \cos \theta_\nu & \sin \theta_\nu & 0 \end{pmatrix}. \]

From Eq.(19) and Eq.(21), we obtain the MNS matrix as follows

\[ V_{MNS} = V_l^T V_{\nu} P_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} -\sqrt{2} e^{-i\phi} \cos \theta_\nu & -\sqrt{2} \sin \theta_\nu & 0 \\ -\sin \theta_\nu & -e^{i\phi} \cos \theta_\nu & 1 \\ -\sin \theta_\nu & e^{i\phi} \cos \theta_\nu & 1 \end{pmatrix} P_\nu, \]

where

\[ P_\nu = \text{diag}(e^{-i(\phi_1 - \phi)/2}, e^{-i(\phi_2 + \phi)/2}, 1). \]

Here it is worth mentioning that the lower bound of \( 0.04 \angle \theta_{13} \) was shown by the recent experiment reported by T2K [15] at 90% C.L., which could give a severe test to our model near future.

From the experimental bound [18], we impose the condition

\[ \tan \theta_\nu = \frac{1}{\sqrt{2}}, \quad m_{\nu_2}^2 - m_{\nu_1}^2 = 8.0 \times 10^{-5} (eV^2), \quad m_{\nu_2}^2 - m_{\nu_3}^2 = 2.5 \times 10^{-3} (eV^2), \]

\[ (v_{1,2,3,4} \ll V). \]
on the parameters, then the phase $\phi$ is given by

$$r \cos \phi = 0.361, \quad r = \frac{\rho_2}{\rho_4}.$$ (25)

Fixing the VEVs as

$$v_u = 10, \quad v'_u = 155.3, \quad v_d = 2.0, \quad v'_d = 77.8 \quad \text{(GeV)},$$ (26)

and the charged lepton masses as $[8][19]$

$$m^l_1 = 1.75 \text{GeV}, \quad m^l_2 = 487 \text{keV}, \quad m^l_3 = 103 \text{MeV},$$ (27)

Yukawa coupling constants are given by

$$Y_1^E = 0.875, \quad Y_3^E = 5.15 \times 10^{-2}, \quad Y_2^E = 6.25 \times 10^{-6}.$$ (28)

For the RHN mass parameters, we assume

$$V = 10^{12} \text{GeV}, \quad Y_1^M = Y_3^M = 1.$$ (29)

In order to investigate model dependence, we give two sample parameter sets A and B which are defined as follows

A : $r = 0.361$

$$\phi = \phi_1 = \phi_2 = 0.0^\circ, \quad \delta = 90.0^\circ,$$

$$\rho_2^2 = 1.80 \times 10^{-2} \text{eV}, \quad \rho_3^2 = 14.47 \times 10^{-2} \text{eV}, \quad \rho_4^2 = 13.78 \times 10^{-2} \text{eV},$$

$$m_{\nu_1} = 5.24 \times 10^{-2} \text{eV}, \quad m_{\nu_3} = 5.31 \times 10^{-2} \text{eV}, \quad m_{\nu_4} = 1.80 \times 10^{-2} \text{eV},$$

$$m_{\nu_2}^\nu = 4.24 \text{GeV}, \quad m_3^\nu = 12.0 \text{GeV}, \quad m_4^\nu = 11.7 \text{GeV},$$

$$Y_2^N = 0.424, \quad Y_3^N = 0.077, \quad Y_4^N = 1.17.$$ (30)

B : $r = 1.000$

$$\phi = 68.84^\circ, \quad \phi_1 = 41.55^\circ, \quad \phi_2 = 41.15^\circ, \quad \delta = 89.805^\circ,$$

$$\rho_2^2 = 5.03 \times 10^{-2} \text{eV}, \quad \rho_3^2 = 10.14 \times 10^{-2} \text{eV}, \quad \rho_4^2 = 5.03 \times 10^{-2} \text{eV},$$

$$m_{\nu_1} = 7.08 \times 10^{-2} \text{eV}, \quad m_{\nu_3} = 7.14 \times 10^{-2} \text{eV}, \quad m_{\nu_4} = 5.03 \times 10^{-2} \text{eV},$$

$$m_{\nu_2}^\nu = 7.09 \text{GeV}, \quad m_3^\nu = 10.02 \text{GeV}, \quad m_4^\nu = 7.09 \text{GeV},$$

$$Y_2^N = 0.709, \quad Y_3^N = 0.065, \quad Y_4^N = 0.709.$$ (31)

Generally, multi-Higgs model causes FCNC problems, however our assignments do not cause such problems. In the lepton sector, the interactions between charged leptons and Higgs bosons are given by

$$\mathcal{L}_l = Y_1^E l e^c c \left[ l_\mu \left( \frac{H_D^l - H^l_1}{\sqrt{2}} \right) + l_\tau \left( \frac{H_D^l + H^l_1}{\sqrt{2}} \right) \right] - Y_2^E H_3^D e^c l_\ell,$$

$$+ Y_3^E \mu^c \left[ l_\mu \left( \frac{H_D^l + H^l_2}{\sqrt{2}} \right) + l_\tau \left( \frac{H_D^l - H^l_2}{\sqrt{2}} \right) \right],$$ (32)

which do not contribute to $\tau \rightarrow e + \gamma, \mu \rightarrow e + \gamma$ processes. Because $\mathcal{L}_l$ has accidental $S_2$ symmetry such as

$$(H_1^l, H_2^l) \rightarrow (H_2^l, H_1^l), \quad (l_\mu, \mu^c) \rightarrow (-l_\mu, -\mu^c),$$ (33)

$\tau \rightarrow \mu + \gamma$ process is also not induced. Note that this $S_2$ symmetry is not the symmetry of whole theory, as the symmetry is violated in neutrino Yukawa couplings and flavon superpotential $W_\Phi$.

In the quark sector, as the quarks couple only to $H_D^l, H_D^l$, Higgs mediated FCNCs are not induced. In the basis that quark mass matrices are diagonal, the superpotential is written as

$$W_Q = Y_t H_1^D (Q_3) (U_3')' + Y_t H_3^D (Q_2) (U_2')' + Y_t H_4^D (Q_1) (U_1')'$$

$$+ Y_b H_1^D (Q_3) (D_3')' + Y_b H_3^D (Q_2) (D_2')' + Y_b H_4^D (Q_1) (D_1')'.$$ (34)

From here, we fix top, bottom and charm masses as $[8][19]$

$$Y_t v_u' = 172.5, \quad Y_b v_d' = 2.89, \quad Y_c v_u' = 0.624 \quad \text{(GeV)},$$ (35)
and assume exotic quark mass as
\[ kv'_s = 2000 \text{ (GeV)}, \tag{36} \]
in order to forbid the decay of lightest flavon into exotic quark pair. Then we fix the values of Yukawa coupling constants as
\[ Y_t = 1.12, \quad Y_b = 0.0371, \quad Y_c = 0.00405, \quad k = 1.0, \quad v'_t = 2000\text{GeV}, \quad v_s = 200\text{GeV}. \tag{37} \]

### 2.4 Higgs sector

Higgs potential is given as follows,
\[
V = V_F + V_D + V_A + V_{m^2}, \tag{38}
\]
\[
V_F = |\lambda_1 S_3 H_1^D + \lambda_5 S_1 H_3^D|^2 + |\lambda_1 S_3 H_2^D + \lambda_5 S_2 H_3^D|^2
+ |\lambda_2 S_3 H_3^D + \lambda_4 (S_1 H_1^D + S_2 H_2^D)|^2
+ |\lambda_1 S_3 H_1^U + \lambda_5 S_1 H_3^U|^2 + |\lambda_1 S_3 H_2^U + \lambda_5 S_2 H_3^U|^2
+ |\lambda_3 S_3 H_1^U + \lambda_5 (S_1 H_2^U + S_2 H_3^U)|^2
+ |\lambda_3 H_1^U H_1^D + \lambda_5 H_1^U H_3^D| + |\lambda_4 H_1^U H_2^D + \lambda_5 H_2^U H_3^D|^2 + |\lambda_1 H_1^U H_1^D + H_2^U H_2^D|^2
\]
where we can define \( \lambda_{1,3,4,5} \) to be real without loss of generality, and we assume all the soft SUSY breaking parameters are real to avoid complex VEVs.

\[
V_D = \frac{1}{2} g^2_Y \left( \frac{1}{2} |H_a^U|^2 - \frac{1}{2} |H_a^D|^2 \right) + \frac{1}{2} g^2_Z \sum_A ((H_a^U)^\dagger T^A H_a^U + (H_a^D)^\dagger T^A H_a^D)^2
+ \frac{1}{2} g^2_Z (-2 |H_a^U|^2 - 3 |H_a^D|^2 + 5 |S_a|^2)^2, \tag{40}
\]

\[
V_A = -\lambda_1 A_1 S_3 (H_1^U H_1^D + H_2^U H_2^D) - \lambda_3 A_3 S_3 H_3^D
- \lambda_4 A_4 H_3^U (S_1 H_1^D + S_2 H_2^D) - \lambda_5 A_5 (S_1 H_1^U + S_2 H_2^U) H_3^D + \text{h.c.,} \tag{41}
\]

\[
V_{m^2} = -m^2_{11}(H_1^U)^2 + m^2_{22}(H_2^U)^2 + m^2_{33}(H_3^U)^2 + m^2_{33}(H_3^D)^2 + m^2_{33}(H_3^D)^2 + m^2_{33}(H_3^D)^2
- m^2_{33}(S_3)^2 + m^2_{33}(S_3)^2
- m^2_{33}(S_3)^2 + m^2_{33}(S_3)^2 + m^2_{33}(S_3)^2 + m^2_{33}(S_3)^2 + \text{h.c.}, \tag{42}
\]

where we can define \( \lambda_{1,3,4,5} \) to be real without loss of generality, and we assume all the soft SUSY breaking parameters are real to avoid complex VEVs.

If \( S_1 \times Z_2 \) breaking terms; \( m^2_{BU}, m^2_{BD}, m^2_{BS} \), violate accidental \( O(2) \) symmetry of Higgs potential and fix the VEV directions (Eq.(13)) to realize \( \theta_{23} = 45^\circ \). This potential has \( S_2 \) symmetry such as
\[
H_1^U \leftrightarrow H_2^U, \quad H_1^D \leftrightarrow H_2^D, \quad S_1 \leftrightarrow S_2. \tag{43}
\]

Minimizing this potential, we get mass matrices of Higgs bosons. The results are given in Appendix A. In the same manner, we add soft \( S_1 \times Z_2 \) breaking terms in flavon sector to avoid domain wall problem [20].

### 3 Dark Matter

Here we show that our model is consistent with cosmic-ray observation of PAMELA. Decaying dark matter scenarios with Non-Abelian discrete flavor symmetries have been done by Ref. [16].

#### 3.1 LF decay width

We assume that the candidate for decaying dark matter is the lightest flavon (LF). In the six flavon superfields; \( \Phi_a, \Phi^c_a \ (a = 1, 2, 3) \), only one linear combination is super-heavy and the other five superfields have TeV scale masses. As LF cannot decay into other flavons, it has very long lifetime. Due to the non-renormalizable

\[ \text{[We would like to thank Referee for the suggestion.] \footnote{1}} \]
interactions with light particles, LF becomes unstable dark matter. Among the interactions, the source term of RHN mass

$$W_{\text{eff}} = \frac{(Y^N H^U L)^2}{2Y^M (V + \Phi_3)} \sim \frac{(Y^N H^U L)^2}{2M_R} \left(1 - \frac{\Phi_3}{V}\right) = \frac{1}{2} m_\nu \left(\frac{H^U L}{v}\right)^2 \left(1 - \frac{\Phi_3}{V}\right)$$ (44)

is the unique interaction to emit leptons without emitting quarks [21], where $v$ is VEV of $H^U$. We estimate the positron flux using this interaction. Due to the factor $1/v$, the Higgs which develops the smallest VEV gives the largest contribution to LF decay. Therefore we can neglect the contribution from $H^U_3$, because $v_u \ll v'$ as one can see from Eq.(26). This effect is important to suppress weak boson emission. Due to the enhancement factor $m_{LF}/v_u$, LF decay width is dominated by 4-body decay as follows

$$\Gamma(LF \to \nu + \nu) \ll \Gamma(LF \to \nu + l + H^+) \sim \Gamma(LF \to \nu + l + W^+) \ll \Gamma(LF \to l + H^+ + H^+).$$ (45)

From the spectrum of positron flux observed by PAMELA, we assume

$$m_{LF} = 4\text{TeV}.$$ (46)

If we assume all sfermions which couple to LF are heavier than 4TeV, the other interactions do not contribute to LF decay. The interactions which contribute to LF decay is given as follows

$$\mathcal{L}_{2\nu} = \frac{1}{2} C_{\nu L} \phi_{\nu L} \left\{(H_1^U)^0 (H_1^U)^0 + \left(H_2^U\right)^0 (H_1^U)^0 (\nu_e \nu_e + r^2 \nu_\mu \nu_\mu + r^2 \nu_\tau \nu_\tau) + 2\sqrt{2} r(\nu_\mu - \nu_\tau) (H_2^U)^0 (H_1^U)^0 \nu_e - \sqrt{2} r(\nu_\mu + \nu_\tau) [(H_1^U)^0 (H_2^U)^0 \nu_e + (H_2^U)^0 (H_1^U)^0 \nu_e]\right\},$$ (47)

$$\mathcal{L}_{1\nu} = -\frac{1}{2} C_{\nu R} \phi_{\nu R} \left\{2 \left[(H_1^U)^0 (H_1^U)^+ + (H_2^U)^0 (H_2^U)^+\right] (e\nu_e + r^2 \nu_\mu \nu_\mu + r^2 \nu_\tau \nu_\tau) + 2\sqrt{2} r[(H_1^U)^0 (H_2^U)^+ + (H_2^U)^0 (H_1^U)^+ \nu_e + \nu_e (\nu_\mu + \nu_\tau)] - \sqrt{2} r[(H_1^U)^0 (H_2^U)^+ - (H_2^U)^0 (H_1^U)^+ \nu_e + \nu_e (\nu_\mu + \nu_\tau)]\right\},$$ (48)

$$\mathcal{L}_{2l} = \frac{1}{2} C_{l L} \phi_{l L} \left\{(H_1^U)^+ (H_1^U)^+ + (H_2^U)^+ (H_2^U)^+ \nu_e + \nu_e (\nu_\mu + \nu_\tau)\right\}$$

$$C_{\nu L} = \frac{\epsilon \phi_{\nu L}}{\sqrt{2} v_u^2},$$ (50)

where $\epsilon$ is the flavon mixing parameter which is defined by

$$\Phi_3 = \frac{\epsilon \phi_{\nu L}}{\sqrt{2}},$$ (51)

where $\phi_{\nu L}$ is LF field.

Using Eq.(47)-(49), the LF decay widths are given as follows

$$\Gamma_{2\nu} = \Gamma_{2\nu} = (6 + 10r^2 + 12r^4) \Gamma_0,$$ (52)

$$\Gamma_{1\nu} = \Gamma_{1\nu} = \Gamma_e + \Gamma_\mu + \Gamma_\tau = (2 + 8r^2 + 8r^4) \Gamma_0,$$

$$\Gamma_{2l} = \Gamma_{2l} = \Gamma_2 e + \Gamma_2 \mu + \Gamma_2 \tau + \Gamma_{e\mu} + \Gamma_{e\tau} = (8 + 12r^2 + 16r^4) \Gamma_0,$$

$$\Gamma_{1\text{lepton}} = \Gamma_{1\nu} + \Gamma_{1l} = (10 + 20r^2 + 24r^4) \Gamma_0 = \Gamma_{\text{anti-lepton}},$$

$$\Gamma_{\text{total}} = 2(\Gamma_{2\nu} + \Gamma_{1\nu} + \Gamma_{2l}) = 2(16 + 30r^2 + 36r^4) \Gamma_0,$$

$$\Gamma_0 = \frac{m_{\nu L}^2}{16\pi} \left(\frac{\epsilon \phi_{\nu L}^2}{32\pi^2 v_u^2 V}\right)^2 (0.1111),$$ (56)
where we classify the final states only by charged lepton flavor $\epsilon, \mu, \tau$. The rates of lepton flavor emitted by LF decay are given by

\[
p_c = \frac{9 + 8r^2}{9 + 16r^2 + 20r^4}, \quad p_\mu = p_\tau = \frac{4r^2 + 10r^4}{9 + 16r^2 + 20r^4}.
\]

Anti-lepton flux depends on Majorana phase $\phi$ through Eq.(25), such as $e$-dominant for $r < 1$ and $(\mu, \tau)$-dominant for $r > 1$ (see Fig. 1). For each parameter set, LF lifetime is estimated as follows

\[
A : \Gamma_{\text{total}}^{-1} = 3.72 \times 10^{11} \epsilon^{-2} \text{sec}, \quad \Gamma_{\text{anti-lepton}}^{-1} = 1.17 \times 10^{12} \epsilon^{-2} \text{sec},
\]

\[
B : \Gamma_{\text{total}}^{-1} = 7.01 \times 10^{11} \epsilon^{-2} \text{sec}, \quad \Gamma_{\text{anti-lepton}}^{-1} = 2.13 \times 10^{12} \epsilon^{-2} \text{sec}.
\]

Hereafter we assume $\epsilon < 10^{-3}$ to avoid extinction of LF.

3.2 Relic Abundance of LF

At early stage of the universe, flavon multiplets are produced through $U(1)_Z$ gauge interaction \cite{13}. Since we assume that reheating temperature is low enough to avoid gravitino over-production as $T_{RH} < 10^7$ GeV \cite{22}, this interaction is never thermal equilibrium. Therefore we assume non-thermal production of flavons and boundary condition $n_{LF}(T_{RH}) = 0$.

For the chiral multiplets $(\psi_L, \Psi)$, $U(1)_Z$ gauge interaction is given by

\[
\mathcal{L}_{U(1)_Z} = ig_z A^\mu \sum_i z_i \left[ \bar{\psi}_{i,L} \gamma_\mu \psi_{i,L} + \Psi_i \partial_\mu \Psi_i^\dagger - \Psi_i^\dagger \partial_\mu \Psi_i \right],
\]

from which we calculate production cross sections of flavon multiplets $(\phi, \Phi)$. From Eq.(13), the $U(1)_Z$ gauge boson mass is nearly equal to $16g_z V$.

As all produced flavon multiplets decay into LF finally, LF number density is given by

\[
n_{LF} = 5N_{LF}(n_\phi + n_\Phi),
\]

where 5 is the number of light flavon superfields, $n_\phi, n_\Phi$ are number density of one flavon multiplet and $N_{LF} \sim O(1)$ is LF production rate which means how many LFs are produced per one degree of freedom of flavon multiplets. The Boltzmann equation for $n_{LF}$ is given by

\[
\dot{n}_{LF} + 3Hn_{LF} = 2480N_{LF}CT^8,
\]

\[
C = \frac{21}{(2\pi)^3} \left( \frac{1}{32V^2} \right)^2,
\]

from which we get

\[
\Omega_{LF} h^2 = \frac{m_{LF}s_0 h^2}{\rho_c} \left[ \frac{15 \times 2480 \times 21 m_P N_{LF}}{2\pi^2(341.25) \times 30.67(2\pi)^5} \left( \frac{1}{32V^2} \right)^2 T_{RH}^3 \right],
\]

\[
= 5.06 \times 10^{-9} N_{LF} \left( \frac{T_{RH}}{10^5 \text{GeV}} \right)^3,
\]

Figure 1:
where [18]

\[
H = 1.66 \sqrt{g_*} \frac{T^2}{m_P},
\]

\[
g_* = 341.25,
\]

\[
m_P = 1.22 \times 10^{19}\text{GeV},
\]

\[
s_0 = 2890/\text{cm}^3,
\]

\[
\rho_c = 1.05 \times 10^4 \text{eV/cm}^3.
\]

For \( T_{RH} < 10^7\text{GeV} \), LF does not dominate dark matter \( (\Omega_{LF} h^2 \ll \Omega_{DM} h^2 = 0.11) \), thus other dark matter should be considered as we will discuss later. Such multi-component dark matter is discussed in [26]. Although the number density is very low, the short lifetime of LF enables us to explain cosmic-ray observation. The effective lifetime of LF is defined as

\[
\tau_{\text{eff}} = \Gamma^{-1}_{\text{anti-lepton}} \left( \frac{\Omega_{DM}}{\Omega_{LF}} \right),
\]

and the following values are obtained for each parameter set

\[
A : \tau_{\text{eff}} = 6.0 \times 10^{25}\text{sec}, \quad \epsilon^2 N_{LF} \left( \frac{T_{RH}}{10^5\text{GeV}} \right)^3 = 4.2 \times 10^{-7},
\]

\[
B : \tau_{\text{eff}} = 7.0 \times 10^{25}\text{sec}, \quad \epsilon^2 N_{LF} \left( \frac{T_{RH}}{10^5\text{GeV}} \right)^3 = 6.6 \times 10^{-7}.
\]

Eq.(67) and Eq.(68) are satisfied for example, if we put \( N_{LF} \sim 1, T_{RH} \sim 10^5\text{GeV}, \epsilon \sim 10^{-3} \).

The positron flux from the decay of LF is calculated as

\[
\Phi(E_{e^+}) = \frac{v_{e^+}}{4\pi} \frac{1}{m_{LF}\tau_{\text{eff}}} \int dE' G_{e^+}(E_{e^+}, E') \sum_{\ell=e^+,\mu^+,\tau^\pm} p_\ell \frac{dN_\ell e^+}{dE'},
\]

where \( v_{e^+} \) is the velocity of the positron, \( G_{e^+} \) is the Green’s function which is expressed in [3], \( p_\ell \) is expressed in Eq.(57) and \( dN_\ell e^+ / dE' \) is the fragmentation function produced from the decay of \( \ell \) to \( e^+ \). The fragmentation function is calculated by using the event generator pythia [27] and the result is shown in Fig.2. We can evaluate the positron flux from the decay of LF by using the fragmentation function. The results for each parameter set A and B are shown in Fig.3.

**Figure 2:** The fragmentation function calculated by pythia for parameter set A (left) and B (right).

From the gamma-ray observations [4], the constraint for \( \tau \)-flux is given by

\[
(\tau_{\tau})_{\text{eff}} = \frac{\Gamma_{\text{lepton}}}{\Gamma_{\tau}} \tau_{\text{eff}} = \frac{10 + 20r^2 + 24r^4}{8r^2 + 20r^4} \tau_{\text{eff}} \geq 2.1 \times 10^{26}\text{sec},
\]

(70)
As the weak boson emission is suppressed by factor \((\frac{1}{\sqrt{2}})^2\), small \(\tau\) model is favored. This is the new information about neutrino sector extracted by cosmic-ray observations.

### 3.3 Higgs decay width

No excess of anti-proton flux in cosmic-ray constrains the species of the particles emitted by LF decay [23]. As the weak boson \(Z,W^\pm\) and the chargino decay mainly into quarks, LF should not decay into these particles so much. The weak boson emission is suppressed by factor \((v_u/m_{LF})^2\) and the chargino emission channel is kinematically closed for heavy sfermion scenario. In order to forbid the weak boson and the chargino emission from Higgs boson decay, we assume light Higgs scenario.

In the Higgs potential Eqs.\((38)-(42)\) and mass terms of the neutralinos and the charginos

\[
\mathcal{L} \supset -i\sqrt{2}(H_u^\dagger)^I[\bar{g}_2\lambda_2^A T_2^A + 3g_y\lambda_Y - 2g_x\lambda_X]h^U_a
\]
\[- i\sqrt{2}(H_d^D)^I[\bar{g}_2\lambda_2^A T_2^A - 3g_y\lambda_Y - 3g_x\lambda_X]h^D_a - i\sqrt{2}(S_a)^I[5g_x\lambda_X]s_a
\]
\[- \frac{1}{2}M_2\lambda_2^A \lambda_2^A - \frac{1}{2}M_Y \lambda_Y \lambda_Y - \frac{1}{2}M_X \lambda_X \lambda_X + (W_H)_{\theta^2} + h.c. \quad (A = 1, 2, 3),
\]

we assume the parameters as follows

\[
g_2 = 0.652, \quad g_y = \frac{1}{6}g_Y \quad (g_Y = 0.357), \quad g_x = \frac{1}{2\sqrt{6}}g_Y = 0.073,
\]
\[
\lambda_1 = 0.065, \quad \lambda_3 = 0.4, \quad \lambda_4 = 0.398, \quad \lambda_5 = 0.75,
\]
\[
M_X = M_2 = 200, \quad M_Y = 180 \quad (\text{GeV}),
\]
\[
A_1 = 0.0, \quad A_3 = A_4 = A_5 = 1.0 \quad (\text{TeV}),
\]
\[
m_{B_H}^2 = 0.0408, \quad m_{B_D}^2 = m_{B_S}^2 = 0.02 \quad (\text{TeV}^2).
\]

Mass matrices of neutralinos and charginos are given in appendix A and the values of mass eigenvalues and mixing matrices are given in appendix B.

We consider only the mass eigenstates which dominate \(H_{1,2}^\dagger\) such as

\[
\phi_1^*(91.50), \quad \phi_2^*(121.96), \quad \phi_3^*(152.48), \quad \rho_1^*(112.12), \quad \rho_0^*(145.53),
\]
\[(H^+_3)^*(90.70), \quad (H^+_3)^*(130.02) \quad (\text{GeV}),
\]

where \(\phi_1^*, \rho_1^*, (H^+_3)^*\)' are \(S_2\)-odd and the others are even. As \(S_2\) forbids interaction \(\phi_1^* ZZ\) and \(\phi_1^*\) is not emitted through \(Z^* \rightarrow Z + \phi_1^*\), LEP bound \(m_H \geq 114.4\text{GeV}\) is not imposed on \(\phi_1^*\). As the masses of these Higgs bosons are well degenerated, they do not emit weak bosons or charginos.

Figure 3: The positron flux calculated for parameter set A (left) and B (right).

which is estimated for each parameter set as follows

\[
\begin{align*}
A : \quad & \quad (\tau_\tau)_{\text{eff}} = 5.6 \times 10^{26}\text{sec}, \\
B : \quad & \quad (\tau_\tau)_{\text{eff}} = 1.4 \times 10^{26}\text{sec}.
\end{align*}
\]

As the parameter set B is severe to satisfy Eq.\((70)\), small \(\tau\) model is favored. This is the new information about neutrino sector extracted by cosmic-ray observations.
The neutralinos into which these Higgs bosons can decay are two singlino dominant neutralinos

\[ \eta'_1 (41.92), \quad \eta'_4 (44.55) \text{ (GeV)}, \]

where \( \eta'_1 \) is \( S_2 \)-odd and LSP. \( S_2 \)-even neutralino \( \eta'_4 \) can decay into \( \eta'_1 \) through \( \eta'_4 \rightarrow \eta'_1 + \mu + \tau \) without emitting quarks. As \( S_2 \)-odd Higgs boson can not decay into quarks, we consider only the decay of \( S_2 \)-even Higgs bosons.

The decay widths of \( \phi'_4, \phi'_5, \rho'_6 \) due to the Yukawa interactions

\[ \mathcal{L} \supset Y_c (H^U_3)^0 cc + Y_b (H^D_3)^0 bb + Y'_1 [(H^D_1)^0 \eta'_1 e_1 + (H^D_2)^0 \eta'_2 e_2] + h.c., \]

are given as follows

\[ \Gamma(\phi'_4 \rightarrow c + \bar{c}) = 3.63 \times 10^{-6} \Gamma_{2d}, \]
\[ \Gamma(\phi'_4 \rightarrow b + \bar{b}) = 1.54 \times 10^{-4} \Gamma_{2d}, \]
\[ \Gamma(\phi'_4 \rightarrow \tau + \bar{\tau}) = 1.10 \times 10^{-6} \Gamma_{2d}, \]
\[ \Gamma(\phi'_5 \rightarrow c + \bar{c}) = 1.61 \times 10^{-5} \Gamma_{2d}, \]
\[ \Gamma(\phi'_5 \rightarrow b + \bar{b}) = 2.68 \times 10^{-4} \Gamma_{2d}, \]
\[ \Gamma(\phi'_5 \rightarrow \tau + \bar{\tau}) = 4.90 \times 10^{-5} \Gamma_{2d}, \]
\[ \Gamma(\rho'_6 \rightarrow c + \bar{c}) = 1.91 \times 10^{-7} \Gamma_{2d}, \]
\[ \Gamma(\rho'_6 \rightarrow b + \bar{b}) = 4.96 \times 10^{-8} \Gamma_{2d}, \]
\[ \Gamma(\rho'_6 \rightarrow \tau + \bar{\tau}) = 6.89 \times 10^{-8} \Gamma_{2d}, \]

where \( \Gamma_{2d} \) is 2-body decay width of scalar. The interactions with the neutralinos \( \eta'_1, \eta'_4, \)

\[ \mathcal{L} \supset -\lambda_1 \left\{ S_3 [(h^U_1)^0 (h^D_1)^0] + (h^U_2)^0 (h^D_2)^0 \right\} + s_3 [(h^U_1)^0 (h^D_1)^0] + (h^U_2)^0 (h^D_2)^0 \]
\[ + s_3 [(h^U_1)^0 (h^D_1)^0] + (h^U_2)^0 (h^D_2)^0 \right\} \]
\[ - \lambda_3 [S_3 (h^U_3)^0 (h^D_3)^0] + s_3 (h^U_3)^0 (h^D_3)^0] + s_3 (h^U_3)^0 (h^D_3)^0] \]
\[ - \lambda_4 \left\{ (h^U_1)^0 [s_1 (h^U_1)^0] + s_2 (h^D_2)^0] + (h^U_1)^0 [s_1 (h^D_1)^0] + s_2 (h^D_2)^0] \]
\[ + (h^D_2)^0 [s_1 (h^U_1)^0] + s_2 (h^U_1)^0] \]
\[ - i \sqrt{2} \sum_i [(h^U_i)^0] \left\{ \frac{1}{2} g_2 \lambda_2^3 + \frac{1}{2} g_Y \lambda_Y - 2 g_x \lambda_X \right\} (h^U_i)^0 \]
\[ - i \sqrt{2} \sum_i [(h^D_i)^0] \left\{ \frac{1}{2} g_2 \lambda_2^3 - \frac{1}{2} g_Y \lambda_Y - 3 g_x \lambda_X \right\} (h^D_i)^0 \]
\[ - i \sqrt{2} \sum_i S^i \left[ 5 g_x \lambda_X \right] s_i + h.c. \]

\[ = -0.0620 \phi'_4 + 0.127 \phi'_5 + 0.0299i \rho'_6 \eta'_1 \eta'_4 \]
\[ -0.0716 \phi'_4 + 0.117 \phi'_5 + 0.0142i \rho'_6 \eta'_1 \eta'_4 + h.c. \]

give

\[ \Gamma(\phi'_4 \rightarrow \eta + \eta) = 208 \times 10^{-4} \Gamma_{2d}, \]
\[ \Gamma(\phi'_5 \rightarrow \eta + \eta) = 864 \times 10^{-4} \Gamma_{2d}, \]
\[ \Gamma(\rho'_6 \rightarrow \eta + \eta) = 3179 \times 10^{-6} \Gamma_{2d}, \]

which dominate the decay widths of \( \phi'_4, \phi'_5, \rho'_6 \).

The decay widths of \( (H^U_3)^0 \) due to Yukawa interactions

\[ \mathcal{L} \supset -Y_c (H^U_3)^0 se - Y'_1 [(H^D_1)^0 e^\nu_1 + (H^D_2)^0 e^\nu_2] + h.c. \]
are given by
\[
\Gamma((H_3^-)^\prime \to s + \bar{c}) = 1.95 \times 10^{-7} \Gamma_{2d},
\]
(91)
\[
\Gamma((H_3^+)^\prime \to \tau + \bar{\nu}_\tau) = 2.48 \times 10^{-6} \Gamma_{2d}.
\]
(92)

From these estimations, \((H_3^\pm)^\prime\) decay gives dominant contribution to anti-proton flux.

As one charged lepton emission from LF decay accommodates one charged Higgs emission at even rate of \((H_3^\pm)^\prime\) and \((H_3^\mp)^\prime\), the quark flux is estimated as
\[
\tau_{\text{quark}} = \left[ \frac{1}{2} \left( 0 + 0.195 \right) \right]^{-1} \tau_{\text{eff}} = 27.4 \tau_{\text{eff}}.
\]
(93)

For each parameter sets, we get
\[A: \tau_{\text{quark}} = 1.6 \times 10^{27} \text{sec},\]
(94)
\[B: \tau_{\text{quark}} = 1.9 \times 10^{27} \text{sec},\]
(95)
from which the spectrum of anti-proton flux is given in Fig. 4. There is no inconsistency in anti-proton flux.

![Graph showing antiproton flux](image)

**Figure 4:**

### 3.4 Relic Abundance of LSP

Finally we estimate the relic abundance of LSP. The interactions between \(\eta_1, \eta_4\) and \(Z\) are given by
\[
\mathcal{L} \supset \frac{1}{2} \bar{\psi}_1 \gamma^\mu (-\partial_\mu - i G_1 Z \gamma_5) \psi_1 + \frac{1}{2} \bar{\psi}_4 \gamma^\mu (-\partial_\mu - i G_4 Z \gamma_5) \psi_4
\]
\[
+ i G(f_L) \bar{\psi}_f \gamma^\mu Z \mu \bar{\psi}_f + i G(f_R) \bar{\psi}_f \gamma^\mu Z \mu \bar{\psi}_f,
\]
(96)
where
\[
G_1 = 0.00823, \quad G_4 = 0.0119,
\]
(97)
\[
G(\epsilon_L) = -\frac{g_Y^2}{\sqrt{g_Y^2 + g_Z^2}} = -0.172, \quad G(\epsilon_R) = \frac{-g_Y^2 + g_Z^2}{2 \sqrt{g_Y^2 + g_Z^2}} = 0.200,
\]
(98)
\[
G(\nu_L) = -\frac{\sqrt{g_Y^2 + g_Z^2}}{2} = -0.372,
\]
(99)
\[
G(u_L) = \frac{2g_Y^2}{3 \sqrt{g_Y^2 + g_Z^2}} = 0.114, \quad G(u_R) = \frac{g_Y^2 - 3g_Z^2}{6 \sqrt{g_Y^2 + g_Z^2}} = -0.257,
\]
(100)
\[
G(d_L) = -\frac{g_Y^2}{3 \sqrt{g_Y^2 + g_Z^2}} = -0.057, \quad G(d_R) = \frac{g_Y^2 + 3g_Z^2}{6 \sqrt{g_Y^2 + g_Z^2}} = 0.315.
\]
(101)

\[\text{(101)}\]
These interactions give dominant contribution to annihilation of $\eta_1', \eta_4'$. As $G_i \ll G(\nu_L)$, the contributions $Z \to \eta\eta$ to Z-decay width is negligible. Therefore LEP bound $m \geq 46\text{GeV}$ is not imposed on $\eta_1', \eta_4'$. The relic abundance of the neutralino is calculated by the formula

$$x_F = \ln \left( \frac{0.0955m_pm_i(a + 6b/x_F)}{(g_\ast x_F)^{\frac{1}{2}}} \right)$$  \hspace{1cm} (102)$$

$$\Omega h^2 = \frac{8.76 \times 10^{-11}g_\ast^{-\frac{1}{2}}x_F}{(a + 3b/x_F)\text{GeV}^2},$$  \hspace{1cm} (103)$$

where $m_1 = 41.92\text{GeV}, m_4 = 44.55\text{GeV},$

$$a_{f,i} = \frac{2c_f}{\pi} \left[ \frac{m_f(G_i/2)}{4m_i^2 - M_Z^2} (G(f_L) - G(f_R)) \right]^2 \left( 1 - \frac{m_i^2}{m_f^2} \right)^{\frac{1}{2}},$$  \hspace{1cm} (104)$$

$$b_{f,i} = \frac{1}{6} \left( -\frac{9}{2} + \frac{3}{4} \frac{m_f^2}{m_i^2 - m_f^2} \right) a_{f,i}$$

$$+ \frac{c_f}{3\pi} \left[ \frac{m_i(G_i/2)}{4m_i^2 - M_Z^2} \right]^2 \left[ G^2(f_L) + G^2(f_R) \right] \left( 4 + \frac{2m_i^2}{m_f^2} \right) \left( 1 - \frac{m_i^2}{m_f^2} \right)^{\frac{1}{2}},$$  \hspace{1cm} (105)$$

c_f$ is color factor such as $c_f = 1$ for $SU_c(3)$ singlet, $c_f = 3$ for triplet, and $m_Z = 91.2$ GeV. For the approximation $m_f/m_i = 0$, these coefficients are given by

$$a_1 = a_{b,1} + a_{r,1} = 5.01 \times 10^{-11}\text{GeV}^{-2},$$  \hspace{1cm} (106)$$

$$a_4 = a_{b,4} + a_{r,4} = 1.22 \times 10^{-9}\text{GeV}^{-2},$$  \hspace{1cm} (107)$$

$$b_1 = 1.53 \times 10^{-8}\text{GeV}^{-2},$$  \hspace{1cm} (108)$$

$$b_4 = 4.23 \times 10^{-7}\text{GeV}^{-2}. $$  \hspace{1cm} (109)$$

The relic abundance of $\eta_1'$ is given by

$$g_\ast = 75.75,$$

$$x_F = 22.32,$$

$$T_F = 1.88 \text{ GeV},$$

$$\Omega_1 h^2 = 0.106,$$  \hspace{1cm} (110)$$

and that of $\eta_4'$ is given by

$$g_\ast = 72.25,$$

$$x_F = 25.52,$$

$$T_F = 1.75 \text{ GeV},$$

$$\Omega_4 h^2 = 0.0052.$$  \hspace{1cm} (111)$$

As $\eta_4'$ is converted into $\eta_1'$, relic abundance of LSP is given by

$$(\Omega_{CDM} h^2) = \Omega_1 h^2 + \Omega_4 h^2 = 0.111,$$  \hspace{1cm} (112)$$

which realizes density parameter of dark matter.

## 4 Conclusion

We have considered dark matter based on $S_4 \times Z_2$ flavor symmetric extra U(1) model. The results are as follows. There exists appropriate parameter set to realize relic abundance of dark matter and positron flux observed by PAMELA at the same time. The dominant component of dark matter is LSP and the origin of positron flux is given by the decay of LF which generates the mass of RHN. There is deep connection between PAMELA observation and neutrino mass. The long life time of LF results in large RHN mass and
the spectrum of positron flux depend on Majorana phase $\phi$ in $V_{MNS}$. Therefore, cosmic-ray observation gives new information about the structure of neutrino mass matrix.

From the fact that there is no excess of anti-proton flux in cosmic-ray, we can guess about the particle spectrum. As sfermions can decay into quarks, weak boson and charginos, LF must not decay into those particles, which suggests sfermions are heavier than $4\text{TeV}$. Although this is also favorable from the viewpoint of the FCNC constraints, experimental verification becomes difficult. However experimental verification of our scenario is not impossible. From the fact that Higgs does not decay into weak boson or charginos, we can expect that Higgs boson is light and degenerated, therefore the examination of the mass spectrum of Higgs boson is possible.

A Mass matrices

Neutral CP even Higgs boson

\[ H_a^{\nu} \supset \frac{\phi_{\nu,a}}{\sqrt{2}} \quad H_a^{D} \supset \frac{\phi_{D,a}}{\sqrt{2}} \quad S_a \supset \frac{\phi_{S,a}}{\sqrt{2}} \quad (a = 1,2,3), \]

\[ -\mathcal{L} \supset \frac{1}{2} \phi_i M_{ij} \phi_j, \quad \phi_i = \left( \begin{array}{c} \phi_{\nu,a} \\ \phi_{D,a} \\ \phi_{S,a} \end{array} \right), \quad (i,j = 1,2,\cdots,9), \]

\[ M_{i,1}^2 = M_{i,2}^2 = \sqrt{2} m_{BD}(v_\nu/v_u) + \lambda_1 A_1 v_s(v_d/v_u) + \lambda_5 A_5 v_s(v_d/v_u) - \lambda_1^2 v_\nu^2/2 - \lambda_5^2 v_s^2/2 \]

\[ - \left[ (\lambda_1 \lambda_4 + \lambda_3 \lambda_5) v_s v_\nu' + (\lambda_4 \lambda_5 + \lambda_1 \lambda_3) v_d' v_u' \right] (v_u'/v_u) + \left[ \frac{1}{4} (g_Y^2 + g_2^2) + 4g_s^2 \right] v_u^2, \]

\[ M_{i,2}^2 = \lambda_3^2 v_\nu^2 + \lambda_4^2 v_s^2 + \frac{1}{2} \left( g_Y^2 + g_2^2 \right) v_u^2, \]

\[ M_{i,3}^2 = M_{i,3}^2 = -m_{BD}^2 + \left[ (\lambda_1 \lambda_4 + \lambda_3 \lambda_5) v_\nu' v_s + (\lambda_4 \lambda_5 + \lambda_1 \lambda_3) v_d' v_u' \right] / \sqrt{2} \]

\[ + \left[ \sqrt{2} \left( \frac{1}{4} (g_Y^2 + g_2^2) + 4g_s^2 \right) v_u v_u', \right. \]

\[ M_{i,4}^2 = \lambda_4^2 v_\nu^2 + \lambda_5^2 v_s^2 + \frac{1}{2} \left( g_Y^2 + g_2^2 \right) v_u^2, \]

\[ M_{i,5}^2 = m_{BD}^2 + \left[ (\lambda_1 \lambda_5 + \lambda_3 \lambda_4) v_\nu' v_s + (\lambda_4 \lambda_5 + \lambda_1 \lambda_3) v_d' v_u' \right] / \sqrt{2} \]

\[ + \left[ \sqrt{2} \left( \frac{1}{4} (g_Y^2 + g_2^2) + 4g_s^2 \right) v_u v_u', \right. \]

\[ M_{i,6}^2 = \lambda_5^2 v_\nu^2 + \lambda_4^2 v_s^2 + \frac{1}{2} \left( g_Y^2 + g_2^2 \right) v_u^2, \]

\[ M_{i,7}^2 = \lambda_5^2 v_\nu^2 + \lambda_4^2 v_s^2 + \frac{1}{2} \left( g_Y^2 + g_2^2 \right) v_u^2, \]

\[ M_{i,8}^2 = \lambda_5^2 v_\nu^2 + \lambda_4^2 v_s^2 + \frac{1}{2} \left( g_Y^2 + g_2^2 \right) v_u^2, \]

\[ M_{i,9}^2 = \lambda_5^2 v_\nu^2 + \lambda_4^2 v_s^2 + \frac{1}{2} \left( g_Y^2 + g_2^2 \right) v_u^2, \]

\[ M_{i,10}^2 = \lambda_5^2 v_\nu^2 + \lambda_4^2 v_s^2 + \frac{1}{2} \left( g_Y^2 + g_2^2 \right) v_u^2, \]

\[ M_{i,11}^2 = \lambda_5^2 v_\nu^2 + \lambda_4^2 v_s^2 + \frac{1}{2} \left( g_Y^2 + g_2^2 \right) v_u^2, \]

\[ M_{i,12}^2 = \lambda_5^2 v_\nu^2 + \lambda_4^2 v_s^2 + \frac{1}{2} \left( g_Y^2 + g_2^2 \right) v_u^2, \]
\[ M_{1,4}^- = M_{1,4}^0 = -\lambda_1 A_1 v'_s + 3\lambda_2^2 v_u v_d / 2 + (\lambda_4 \lambda_5 + \lambda_1 \lambda_3) v'_u v'_d + \left[ -\frac{1}{4}(g_Y^2 + g_2^2) + 6g_2^2 \right] v_u v_d, \]
\[ M_{1,5}^- = M_{1,5}^0 = \lambda_1^2 v_u v_d / 2 + \left[ -\frac{1}{4}(g_Y^2 + g_2^2) + 6g_2^2 \right] v_u v_d, \]
\[ M_{1,6}^- = M_{1,6}^0 = -\lambda_5 A_5 v_s / \sqrt{2} + \sqrt{2}\lambda_5^2 v_u v'_d + (\lambda_4 \lambda_5 + \lambda_1 \lambda_3) v'_u v_d / \sqrt{2} \]
\[ + \sqrt{2}\left[ -\frac{1}{4}(g_Y^2 + g_2^2) + 6g_2^2 \right] v_u v'_d, \]
\[ M_{1,7}^- = M_{1,7}^0 = -\lambda_4 A_4 v_u / \sqrt{2} + \sqrt{2}\lambda_4^2 v'_u v_d + (\lambda_4 \lambda_5 + \lambda_1 \lambda_3) v_u v'_d / \sqrt{2} \]
\[ + \sqrt{2}\left[ -\frac{1}{4}(g_Y^2 + g_2^2) + 6g_2^2 \right] v'_u v_d, \]
\[ M_{1,8}^- = M_{1,8}^0 = -\lambda_3 A_3 v'_u + 2\lambda_3^2 v'_u v'_d + (\lambda_1 \lambda_3 + \lambda_4 \lambda_5) v_u v_d + 2 \left[ -\frac{1}{4}(g_Y^2 + g_2^2) + 6g_2^2 \right] v'_u v'_d, \]
\[ M_{1,9}^- = M_{1,9}^0 = -\lambda_2 A_2 v'_u + 3\lambda_2^2 v_u v_s / 2 + (\lambda_1 \lambda_4 + \lambda_3 \lambda_5) v'_u v'_s + [-10g_2^2] v_u v_s, \]
\[ M_{2,7}^- = M_{2,7}^0 = \lambda_3^2 v_u v_s / 2 + [-10g_2^2] v_u v_s, \]
\[ M_{2,8}^- = M_{2,8}^0 = -\lambda_4 A_4 v'_u / \sqrt{2} + \sqrt{2}\lambda_4^2 v'_u v_s + (\lambda_1 \lambda_4 + \lambda_3 \lambda_5) v_u v'_s / \sqrt{2} + \sqrt{2}\left[ -10g_2^2 \right] v'_u v'_s, \]
\[ M_{2,9}^- = M_{2,9}^0 = -\lambda_5 A_5 v'_u / \sqrt{2} + \sqrt{2}\lambda_5^2 v'_u v_s + (\lambda_1 \lambda_5 + \lambda_3 \lambda_4) v_u v'_s / \sqrt{2} + \sqrt{2}\left[ -10g_2^2 \right] v'_u v'_s, \]
\[ M_{3,7}^- = M_{3,7}^0 = -\lambda_4 A_4 v'_u + 3\lambda_4^2 v'_u v'_s / 2 + (\lambda_1 \lambda_3 + \lambda_4 \lambda_5) v'_u v'_s + [-15g_2^2] v_d v_s, \]
\[ M_{3,8}^- = M_{3,8}^0 = \lambda_2^2 v_d v_s / 2 + [-15g_2^2] v_d v_s, \]
\[ M_{3,9}^- = M_{3,9}^0 = -\lambda_5 A_5 v'_u / \sqrt{2} + \sqrt{2}\lambda_5^2 v'_u v'_s / 2 + (\lambda_1 \lambda_5 + \lambda_3 \lambda_4) v_d v'_s / \sqrt{2} + \sqrt{2}\left[ -15g_2^2 \right] v'_u v'_s, \]
\[ M_{4,7}^- = M_{4,7}^0 = -\lambda_4 A_4 v'_u + 3\lambda_4^2 v'_u v'_s / 2 + (\lambda_1 \lambda_3 + \lambda_4 \lambda_5) v'_u v'_s + [-15g_2^2] v'_u v'_s, \]
\[ M_{4,8}^- = M_{4,8}^0 = \lambda_2^2 v_d v_s / 2 + [-15g_2^2] v_d v_s, \]
\[ M_{4,9}^- = M_{4,9}^0 = -\lambda_5 A_5 v'_u / \sqrt{2} + \sqrt{2}\lambda_5^2 v'_u v'_s / 2 + (\lambda_1 \lambda_5 + \lambda_3 \lambda_4) v_d v'_s / \sqrt{2} + \sqrt{2}\left[ -15g_2^2 \right] v'_u v'_s, \]
\[ M_{5,6}^- = M_{5,6}^0 = -\lambda_3 A_3 v'_u + 2\lambda_3^2 v'_u v'_d + (\lambda_1 \lambda_5 + \lambda_3 \lambda_4) v_d v_s + 2 \left[ -15g_2^2 \right] v'_u v'_s. \]

\textbf{Neutral CP odd Higgs boson}

\[ H_u^U \supset \frac{i\rho_{u,a}}{\sqrt{2}}, \quad H_d^D \supset \frac{i\rho_{d,a}}{\sqrt{2}}, \quad S_a \supset \frac{i\rho_{s,a}}{\sqrt{2}} \quad (a = 1, 2, 3), \]
\[ -\mathcal{L} \supset \frac{1}{2} m_i M_{ij} \rho_j, \quad \rho_i = \left( \begin{array}{c} \rho_{u,a} \\ \rho_{d,a} \\ \rho_{s,a} \end{array} \right), \quad (i, j = 1, 2, \ldots, 9), \]

\[ M_{1,1}^2 = M_{2,2}^2 = \sqrt{2} m_{3BU}^2 (v'_u / v_u) + \lambda_1 A_1 v'_s (v_d / v_u) + \lambda_5 A_5 v_s (v'_d / v_u) - \lambda_1^2 v_s^2 / 2 - \lambda_5^2 v_s^2 / 2 \]
\[ - [(\lambda_1 \lambda_3 + \lambda_3 \lambda_5) v_u v'_s + (\lambda_1 \lambda_4 + \lambda_1 \lambda_5) v'_u v'_d] (v'_u / v_u), \]
\[ M_{1,2}^2 = \lambda_2^2 v_u^2 / 2 + \lambda_4^2 v_d^2 / 2, \]
\[ M_{1,3}^2 = M_{2,3}^2 = -m_{3BU}^2 + [(\lambda_1 \lambda_4 + \lambda_4 \lambda_5) v'_u v'_s + (\lambda_1 \lambda_5 + \lambda_3 \lambda_4) v_d v'_d] / \sqrt{2}, \]
\[ M_{3,3}^2 = \sqrt{2} m_{3BU}^2 (v_u / v'_u) + \lambda_3 A_3 (v'_u / v'_d) + \lambda_4 A_4 v_s (v'_u / v'_d) \]
\[ - [(\lambda_1 \lambda_4 + \lambda_3 \lambda_5) v'_u v'_s + (\lambda_4 \lambda_5 + \lambda_1 \lambda_3) v'_u v'_d] (v'_u / v'_d), \]
\[ M_{4,4}^2 = M_{5,5}^2 = \sqrt{2} m_{3BU}^2 (v'_u / v_u) + \lambda_1 A_1 v'_s (v_d / v_u) + \lambda_4 A_4 v_s (v'_d / v_u) - \lambda_1^2 v'_s^2 / 2 - \lambda_4^2 v'_s^2 / 2 \]
\[ - [(\lambda_1 \lambda_3 + \lambda_3 \lambda_5) v'_u v'_s + (\lambda_1 \lambda_4 + \lambda_1 \lambda_5) v'_u v'_d] (v'_u / v_u), \]
\[ M_{4,5}^2 = \lambda_2^2 v_s^2 / 2 + \lambda_4^2 v_d^2 / 2, \]
\[ M_{4,6}^2 = M_{5,6}^2 = -m_{3BU}^2 + [(\lambda_1 \lambda_5 + \lambda_3 \lambda_4) v'_u v'_s + (\lambda_1 \lambda_5 + \lambda_3 \lambda_4) v_d v'_d] / \sqrt{2}, \]
\[ M_{5,7}^2 = M_{6,7}^2 = \sqrt{2} m_{3BU}^2 (v'_u / v'_d) + \lambda_3 A_3 (v'_u / v'_d) + \lambda_5 A_5 v_s (v'_u / v'_d) \]
\[ - [(\lambda_1 \lambda_3 + \lambda_3 \lambda_5) v'_u v'_s + (\lambda_1 \lambda_4 + \lambda_1 \lambda_5) v'_u v'_d] (v'_u / v_u), \]
\[ M_{6,8}^2 = M_{7,8}^2 = \sqrt{2} m_{3BU}^2 (v'_u / v_u) + \lambda_1 A_1 v'_s (v_d / v_u) + \lambda_4 A_4 v_s (v'_d / v_u) - \lambda_1^2 v'_s^2 / 2 - \lambda_4^2 v'_s^2 / 2 \]
\[ - [(\lambda_1 \lambda_3 + \lambda_3 \lambda_5) v'_u v'_s + (\lambda_1 \lambda_4 + \lambda_1 \lambda_5) v'_u v'_d] (v'_u / v_u), \]
\[ M_{7,7}^2 = \lambda_2^2 v_s^2 / 2 + \lambda_4^2 v_d^2 / 2, \]

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\[ M_{7,9}^2 = -m_{BS}^2 + [(\lambda_1 \lambda_5 + \lambda_3 \lambda_4) v_d v_u + (\lambda_1 \lambda_4 + \lambda_3 \lambda_5) v_u v_d] / \sqrt{2}, \]
\[ M_{9,9}^2 = \sqrt{2} m_{BS}^2 (v_d / v_u) + \lambda_1 A_1 (v_u v_d / v_u') + \lambda_3 A_3 (v_u' / v_u') \]
\[ - [(\lambda_1 \lambda_5 + \lambda_3 \lambda_4) v_d v_u + (\lambda_1 \lambda_4 + \lambda_3 \lambda_5) v_u v_d] / \sqrt{2}, \]
\[ M_{7,4}^2 = M_{9,5}^2 = \lambda_1 A_1 v_d' + (\lambda_4 \lambda_5 - \lambda_1 \lambda_3) v_d v_u' - \lambda_1^2 v_u v_d / 2, \]
\[ M_{1,5}^2 = M_{2,4}^2 = \lambda_1^2 v_u v_d / 2, \]
\[ M_{1,6}^2 = M_{2,6}^2 = \lambda_5 A_5 v_d / \sqrt{2} + (\lambda_1 \lambda_3 - \lambda_4 \lambda_5) v_u' v_d / \sqrt{2}, \]
\[ M_{3,4}^2 = M_{5,5}^2 = \lambda_4 A_4 v_u / \sqrt{2} + (\lambda_1 \lambda_3 - \lambda_4 \lambda_5) v_u' v_d / \sqrt{2}, \]
\[ M_{2,6}^2 = \lambda_3 A_3 v_u' + (\lambda_4 \lambda_5 - \lambda_1 \lambda_3) v_u v_d, \]
\[ M_{7,7}^2 = M_{9,7}^2 = \lambda_5 A_5 v_d' + (\lambda_4 \lambda_5 - \lambda_3 \lambda_4) v_d' v_u' - \lambda_5^2 v_d v_u / 2, \]
\[ M_{1,7}^2 = M_{2,7}^2 = \lambda_5^2 v_d v_u / 2, \]
\[ M_{1,8}^2 = M_{2,8}^2 = \lambda_4 A_4 v_u + (\lambda_4 \lambda_5 - \lambda_3 \lambda_4) v_u' v_d' - \lambda_4^2 v_u v_d / 2, \]
\[ M_{4,8}^2 = M_{5,8}^2 = \lambda_3 A_3 v_u' + (\lambda_4 \lambda_5 - \lambda_3 \lambda_4) v_u v_d, \]
\[ M_{6,7}^2 = M_{7,8}^2 = \lambda_5 A_5 v_u / \sqrt{2} + (\lambda_4 \lambda_5 - \lambda_1 \lambda_3) v_d v_u'/ \sqrt{2}, \]
\[ M_{6,9}^2 = \lambda_3 A_3 v_u' + (\lambda_4 \lambda_5 + \lambda_3 \lambda_4) v_u' v_d. \]

Charged Higgs boson

\[ H_u^a \supset H_u^+, \quad H_d^a \supset H_u^{a+3} \quad (a = 1, 2, 3), \]
\[ - L \supset H_u^a M_{ij} H_d^j, \quad H_u^+ = \begin{pmatrix} H_u^+ \\ (H_d^a)^{-1} \end{pmatrix} \quad (i, j = 1, 2, \cdots, 6), \]

\[ M_{1,1}^2 = \sqrt{2} m_{BD}^2 (v_u / v_d) + \lambda_1 A_1 v_d (v_u / v_d) + \lambda_5 A_5 v_u (v_d' / v_u) - \lambda_1^2 v_d^2 - \lambda_5^2 v_u^2 / 2 + (v_d')^2 \]
\[ - [(\lambda_1 \lambda_4 + \lambda_3 \lambda_5) v_d v_u + (\lambda_4 \lambda_5 + \lambda_3 \lambda_4) v_d' v_u] (v_u' / v_d), \]
\[ - \frac{1}{4} \lambda_2^2 (v_d + (v_u')^2 - v_u^2 + (v_d')^2) + \frac{1}{4} g_2^2 v_u^2, \]
\[ M_{1,2}^2 = \lambda_5^2 v_d / 2 + \frac{1}{4} g_2^2 v_u^2, \]
\[ M_{1,3}^2 = M_{2,3}^2 = -m_{BD}^2 + (\lambda_1 \lambda_4 + \lambda_3 \lambda_5) v_d v_u / \sqrt{2} + \frac{1}{2 \sqrt{2}} g_2^2 v_d^2 v_u, \]
\[ M_{3,3}^2 = \sqrt{2} m_{BD}^2 (v_u / v_d) + \lambda_5 A_3 v_u (v_d' / v_u') + \lambda_4 A_4 v_u (v_d' / v_u) - \lambda_4^2 v_u^2 - \lambda_5^2 (v_u')^2 \]
\[ - [(\lambda_1 \lambda_4 + \lambda_3 \lambda_5) v_d v_u + (\lambda_4 \lambda_5 + \lambda_3 \lambda_4) v_d' v_u] (v_u' / v_u) \]
\[ - \frac{1}{4} \lambda_2^2 (v_u + (v_u')^2 - v_d^2 + (v_u')^2) + \frac{1}{4} g_2^2 (v_u')^2, \]
\[ M_{4,4}^2 = M_{5,5}^2 = \sqrt{2} m_{BD}^2 (v_d / v_u) + \lambda_1 A_1 v_u (v_d / v_u) + \lambda_4 A_4 v_u (v_d / v_u) \]
\[ - \lambda_4^2 v_u^2 - \lambda_1^2 (v_u'^2 / 2 + (v_u')^2) - [(\lambda_1 \lambda_5 + \lambda_3 \lambda_4) v_d v_u' + (\lambda_4 \lambda_5 + \lambda_3 \lambda_4) v_u v_d'] (v_d' / v_d) \]
\[ + \frac{1}{4} \lambda_2^2 (v_u' + (v_u')^2 - v_d^2 + (v_u')^2) + \frac{1}{4} g_2^2 v_d^2, \]
\[ M_{4,5}^2 = \lambda_4^2 v_u'^2 / 2 + \frac{1}{4} g_2^2 v_d^2, \]
\[ M_{4,6}^2 = M_{5,6}^2 = -m_{BD}^2 + (\lambda_1 \lambda_5 + \lambda_3 \lambda_4) v_d v_u / \sqrt{2} + \frac{1}{2 \sqrt{2}} g_2^2 v_d v_u, \]
\[ M_{6,6}^2 = \sqrt{2} m_{BD}^2 (v_u / v_d) + \lambda_5 A_3 v_u (v_d' / v_u') + \lambda_5 A_5 v_u (v_d' / v_u') - \lambda_5^2 v_u^2 - \lambda_5^2 (v_u')^2 \]
\[ - [(\lambda_1 \lambda_5 + \lambda_3 \lambda_4) v_d v_u' + (\lambda_4 \lambda_5 + \lambda_3 \lambda_4) v_u v_d'] (v_u' / v_d). \]
\[
\begin{align*}
M_{1,4}^2 &= M_{2,5}^2 = \lambda_1 A_1 v'_s - \lambda_1 [\lambda_1 v_u v_d + \lambda_3 v_u' v_d] + \frac{1}{4} g_2^2 v_u v_d, \\
M_{1,5}^2 &= M_{2,4}^2 = \frac{1}{4} g_2^2 v_u v_d, \\
M_{1,6}^2 &= M_{2,6}^2 = \frac{5}{2} A_5 s / \sqrt{2} - \frac{5}{2} [\lambda_1 v'_u v_d + \lambda_5 v_u v_d] / \sqrt{2} + \frac{1}{2 \sqrt{2}} g_2^2 v_u v_d, \\
M_{3,4}^2 &= M_{2,5}^5 = \lambda_4 A_4 v_s / \sqrt{2} - \lambda_4 [\lambda_1 v_u v_d + \lambda_5 v_u v_d] / \sqrt{2} + \frac{1}{2 \sqrt{2}} g_2^2 v_u v_d, \\
M_{3,6}^2 &= \lambda_3 A_3 v'_s - \lambda_3 [\lambda_1 v_u v_d + \lambda_3 v_u' v_d] + \frac{1}{2} g_2^2 v_u' v_d. 
\end{align*}
\] (115)

**1-loop corrections to Higgs mass**

In order to satisfy the experimental bound for the lightest neutral CP even Higgs boson mass, the contributions from 1-loop corrections are important [24][25]. We add 1-loop contributions

\[
\Delta V = \frac{1}{64 \pi^2} \text{Str} \left[ M^4 \left( \frac{\ln M^2}{\Lambda^2} - \frac{3}{2} \right) \right]
\] (116)

to Higgs potential. The dominant contributions are given by trilinear terms \(Y_i H_i^U Q_3 U_3^c\) and \(k S_3 (g_1 g'_1 + g_2 g'_2 + g_3 g'_3)\). From the mass terms of squark and scalar g-quark

\[
- \mathcal{L} \supset \left[ m_{Q_3}^2 + (Y_i (H_i^U)^0)^2 \right] |U_3|^2 + \left[ m_{U_3}^2 + (Y_i (H_i^D)^0)^2 \right] |U_3|^2 + \left[ m_g^2 + (k v'_s)^2 \right] (|g'_1|^2 + |g'_2|^2 + |g'_3|^2) + \left[ m_g^2 + (k v'_s)^2 \right] (|g'_1|^2 + |g'_2|^2 + |g'_3|^2) + \left[ k A V v'_s + k \lambda_3 (H_3^U)^0 (H_3^D)^0 \right] (g_1 g'_1 + g_2 g'_2 + g_3 g'_3) + h.c.
\]

\[
= \left( U_3^*, U_3 \right) \begin{bmatrix}
- m_{Q_3}^2 + (Y_i v'_u)^2 & Y_i A v'_u + Y_i A v'_s v_d & Y_i A v'_u + Y_i A v'_s v_d \\
Y_i A v'_u + Y_i A v'_s v_d & m_{U_3}^2 + (Y_i v'_u)^2 & m_{U_3}^2 + (Y_i v'_u)^2 \\
Y_i A v'_u + Y_i A v'_s v_d & m_{U_3}^2 + (Y_i v'_u)^2 & m_{U_3}^2 + (Y_i v'_u)^2
\end{bmatrix} \left( \begin{bmatrix}
U_3 \\
(U_3^*)^* \\
\end{bmatrix}\right)
+ \sum_i \left( g_i + g_i^* \right) \begin{bmatrix}
- m_g^2 + (k v'_s)^2 & k A k v'_s + k \lambda_3 v'_u v_d & m_g^2 + (k v'_s)^2 \\
k A k v'_s + k \lambda_3 v'_u v_d & m_g^2 + (k v'_s)^2 & m_g^2 + (k v'_s)^2
\end{bmatrix} \left( \begin{bmatrix}
1 \\
1 \\
\end{bmatrix}\right) \right)
\] (117)

Mass eigenvalues are given by

\[
M_{1,5}^2, M_{1,6}^2 = \frac{1}{2} \left[ m_{Q_3}^2 + m_{U_3}^2 + 2(Y_i v'_u)^2 \pm \sqrt{(m_{Q_3}^2 - m_{U_3}^2)^2 + 4Y_i^2(A v'_u + \lambda_3 v'_u v_d)^2} \right],
\]

\[
M_{3,4}^2, M_{3,6}^2 = \frac{1}{2} \left[ m_g^2 + m_g^2 + 2(k v'_s)^2 \pm \sqrt{(m_g^2 - m_g^2)^2 + 4k^2(A k v'_s + \lambda_3 v'_u v_d)^2} \right].
\] (118)

For simplicity, we assume

\[
m_{Q_3}^2 = m_{U_3}^2 = m_g^2 = m_{g'}^2 = m_Q^2 = 16 \text{TeV}^2, \quad A_t = A_k = 0.0 \text{TeV},
\]

then Eq.(118) is rewritten by

\[
M_{1,5}^2, M_{1,6}^2 = m_{Q_3}^2 + (Y_i v'_u)^2 \pm Y_i \lambda_3 v'_u v_d,
\]

\[
M_{3,4}^2, M_{3,6}^2 = m_g^2 + (k v'_s)^2 \pm k \lambda_3 v'_u v_d.
\] (120)

As potential minimum condition is modified as

\[
\frac{\partial (V + \Delta V)}{\partial X} = 0, \quad X = (v'_u, v'_d),
\] (121)

and we must add the terms

\[
\Delta M_{3,3}^2 = \frac{1}{2} \frac{\partial^2 V}{\partial (v'_u)^2} - \frac{1}{2} \frac{\partial V}{\partial (v'_u)} \frac{\partial^2 V}{\partial (v'_u)^2},
\]

\[
\Delta M_{6,6}^2 = \frac{1}{2} \frac{\partial^2 V}{\partial (v'_d)^2} - \frac{1}{2} \frac{\partial V}{\partial (v'_d)} \frac{\partial^2 V}{\partial (v'_d)^2},
\]

\[
\Delta M_{3,6}^2 = \frac{1}{2} \frac{\partial^2 V}{\partial (v'_u) \partial (v'_d)},
\] (122)
to the neutral CP-even Higgs boson mass. We fix renormalization point as
\[ \Lambda = 4 \text{TeV}. \]

Chargino
\[
\mathcal{L} \supset \chi^+_i M_{ij} \chi^+_j + h.c.,
\]
\[
h^U_a = \begin{pmatrix} h^U_{ua} \\ h^U_{da} \end{pmatrix}_a, \quad h^D_a = \begin{pmatrix} h^D_{da} \\ -i h^D_{da} \end{pmatrix}_a \quad (a = 1, 2, 3),
\]
\[
\chi^+_i = \begin{pmatrix} (h^U_{ua})_a \\ -i (h^D_{da})_a \end{pmatrix}, \quad \chi^-_i = \begin{pmatrix} (h^D_{da})_a \\ -i (h^D_{da})_a \end{pmatrix}, \quad w^\pm = \frac{\lambda^3_{13} \mp i \lambda^3_{12}}{\sqrt{2}} \quad (i, j = 1, 2, 3, 4),
\]
\[
M = \begin{pmatrix}
\lambda_1 v_s'/\sqrt{2} & 0 & \lambda_5 v_s' \sqrt{2} & g_2 v_u' \sqrt{2} \\
0 & \lambda_1 v_s'/\sqrt{2} & \lambda_5 v_s' \sqrt{2} & g_2 v_u' \sqrt{2} \\
g_2 v_d' \sqrt{2} & \lambda_4 v_s' \sqrt{2} & \lambda_3 v_s' & g_2 v_d' \\
g_2 v_d' \sqrt{2} & \lambda_4 v_s' \sqrt{2} & \lambda_3 v_s' & M_2
\end{pmatrix}.
\]

Neutralino
\[
\mathcal{L} \supset \frac{1}{2} \eta_i M_{ij} \eta_j + h.c.,
\]
\[
\eta = \begin{pmatrix} (h^U_{ua})_a \\ s_a \\ i \lambda \end{pmatrix}_a, \quad \lambda = \begin{pmatrix} \lambda_Y \\ \lambda^3_2 \\ \lambda_X \end{pmatrix}_a \quad (a = 1, 2, 3; \quad i, j = 1, 2, \cdots, 12),
\]
\[
M = \begin{pmatrix}
0 & M_{ud} & M_{us} & M_{u\lambda} \\
M_{du} & 0 & M_{ds} & M_{d\lambda} \\
M_{su} & M_{sd} & 0 & M_{s\lambda} \\
M_{\lambda u} & M_{\lambda d} & M_{\lambda s} & M_{\lambda \lambda}
\end{pmatrix},
\]
\[
M_{ud} = \begin{pmatrix}
\lambda_1 v_s' & 0 & \lambda_5 v_s' \sqrt{2} \\
0 & \lambda_1 v_s' & \lambda_5 v_s' \sqrt{2} \\
\lambda_4 v_d' \sqrt{2} & \lambda_4 v_d' \sqrt{2} & \lambda_3 v_s' \\
0 & \lambda_4 v_d' \sqrt{2} & \lambda_3 v_s'
\end{pmatrix} = M^T_{du},
\]
\[
M_{us} = \begin{pmatrix}
\lambda_5 v_d' & 0 & \lambda_1 v_d' \sqrt{2} \\
0 & \lambda_5 v_d' & \lambda_1 v_d' \sqrt{2} \\
\lambda_4 v_d' \sqrt{2} & \lambda_4 v_d' \sqrt{2} & \lambda_3 v_d' \\
0 & \lambda_4 v_d' \sqrt{2} & \lambda_3 v_d'
\end{pmatrix} = M^T_{su},
\]
\[
M_{ds} = \begin{pmatrix}
\lambda_3 v_u' & 0 & \lambda_1 v_u' \sqrt{2} \\
0 & \lambda_3 v_u' & \lambda_1 v_u' \sqrt{2} \\
\lambda_5 v_u' \sqrt{2} & \lambda_5 v_u' \sqrt{2} & \lambda_3 v_u' \\
0 & \lambda_5 v_u' \sqrt{2} & \lambda_3 v_u'
\end{pmatrix} = M^T_{sd},
\]
\[
M_{u\lambda} = \begin{pmatrix}
g y v_u/2 & -g_2 v_u/2 & -2 g_2 v_u \\
g y v_u/2 & -g_2 v_u/2 & -2 g_2 v_u \\
g y v_u'/\sqrt{2} & -g_2 v_u'/\sqrt{2} & -2 g_2 v_u'/\sqrt{2} \\
-g y v_d/2 & g_2 v_d/2 & -3 g_2 v_d \\
g y v_d'/\sqrt{2} & g_2 v_d'/\sqrt{2} & -3 g_2 v_d'
\end{pmatrix} = M^T_{u\lambda},
\]
\[
M_{d\lambda} = \begin{pmatrix}
-g y v_d/2 & g_2 v_d/2 & -3 g_2 v_d \\
g y v_d/2 & g_2 v_d/2 & -3 g_2 v_d \\
-g y v_d'/\sqrt{2} & g_2 v_d'/\sqrt{2} & -3 g_2 v_d'
\end{pmatrix} = M^T_{d\lambda},
\]
\[
M_{s\lambda} = \begin{pmatrix}
0 & 0 & 5 g_2 v_s \\
0 & 0 & 5 g_2 v_s \\
0 & 0 & 5 \sqrt{2} g_2 v_s'
\end{pmatrix} = M^T_{s\lambda},
\]
\[
M_{\lambda \lambda} = \begin{pmatrix}
-M_Y & 0 & 0 \\
0 & -M_2 & 0 \\
0 & 0 & -M_X
\end{pmatrix}.
\]

B Mixing matrices and mass eigenvalues

Mass eigenvalues \((m_i : \text{GeV})\) and diagonalization matrix \(U = (u_1, u_2, \cdots)\) are given as follows.
Neutral CP-even Higgs
For Higgs bosons, the diagonalization matrices are defined as

\[(M^2)' = U^T M^2 U = \text{diag}(m_1^2, m_2^2, \cdots).\] (126)

| \(i\) | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \(u_i\) | -0.698 | -0.113 | -0.0006 | -0.613 | 0.329 | -0.115 | 0.0006 | 0.003 | -0.059 |
|      | 0.698 | 0.113 | 0.0006 | -0.613 | 0.329 | -0.115 | 0.0006 | 0.003 | -0.059 |
|      | 0    | 0    | 0    | -0.384 | -0.808 | -0.036 | 0.00009 | -0.009 | -0.445 |
|      | 0.0015 | -0.0005 | -0.0012 | 0.0008 | 0.006 | -0.706 | -0.0008 | -0.030 |
|      | -0.0015 | 0.005 | 0.707 | -0.0012 | -0.008 | 0.006 | -0.706 | -0.0008 | -0.030 |
|      | 0    | 0    | 0    | 0    | 0    | -0.273 | -0.360 | -0.031 | -0.033 | 0.054 | 0.889 |
|      | 0.113 | -0.698 | -0.0056 | 0.118 | -0.025 | -0.693 | 0.006 | 0.071 | -0.002 |
|      | -0.113 | 0.698 | 0.0056 | 0.118 | -0.025 | -0.693 | 0.006 | 0.071 | -0.002 |
|      | 0    | 0    | 0    | 0    | 0    | 0    | 0.014 | 0.101 | 0.00009 | 0.993 | -0.052 |
| \(m_i\) | 91.50 | 537 | 2005 | 121.96 | 152.96 | 539 | 2007 | 1018 | 1425 |

Neutral CP-odd Higgs

| \(i\) | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \(u_i\) | -0.703 | 0.073 | -0.040 | 0.004 | 0.0005 | 0.700 | 0.074 | -0.002 | -0.059 |
|      | 0.703 | -0.073 | -0.040 | 0.004 | -0.0005 | 0.700 | 0.074 | -0.002 | -0.059 |
|      | 0    | 0    | -0.889 | 0.086 | 0    | -0.088 | -0.009 | -0.029 | -0.044 |
|      | 0.0014 | 0.008 | 0.008 | -0.00008 | 0.707 | 0.0003 | 0.008 | -0.706 | 0.030 |
|      | -0.0014 | 0.008 | 0.008 | -0.00008 | -0.707 | 0.0003 | 0.008 | -0.706 | 0.030 |
|      | 0    | 0    | 0.448 | -0.004 | 0    | -0.049 | -0.005 | -0.033 | -0.892 |
|      | -0.073 | -0.703 | -0.006 | -0.070 | 0.008 | 0.074 | -0.700 | -0.008 | -0.002 |
|      | 0.073 | 0.703 | -0.006 | -0.070 | -0.008 | 0.074 | -0.700 | -0.008 | -0.002 |
|      | 0    | 0    | -0.079 | -0.991 | 0    | -0.012 | 0.099 | -0.001 | -0.034 |
| \(m_i\) | 112.12 | 532 | 0    | 0    | 0    | 2005 | 145.53 | 535 | 2008 | 1423 |

Charged Higgs

| \(i\) | 1   | 2   | 3   | 4   | 5   | 6   |
|-----|-----|-----|-----|-----|-----|-----|
| \(u_i\) | 0.707 | -0.000055 | -0.703 | 0.041 | -0.001 | 0.059 |
|      | -0.707 | 0.000055 | -0.703 | 0.041 | -0.001 | 0.059 |
|      | 0    | 0    | 0.089 | 0.893 | -0.029 | 0.441 |
|      | 0.000055 | 0.707 | -0.0018 | -0.008 | -0.706 | -0.030 |
|      | -0.000055 | -0.707 | -0.0018 | -0.008 | -0.706 | -0.030 |
|      | 0    | 0    | 0.049 | -0.447 | -0.032 | 0.893 |
| \(m_i\) | 90.70 | 2005 | 130.02 | 0    | 2007 | 1422 |

Neutralino
For neutralinos, the diagonalization matrix is defined as

\[U^T M U = \text{diag}(m_1, m_2, \cdots m_{12}).\] (129)
### Chargino
For the charginos, the diagonalization matrices are defined as

\[
\begin{align*}
\chi^- &= U_-(\chi^-)', \quad \chi^+ = U_+(\chi^+)', \\
U_+^T M U_- &= \text{diag}(m_1, m_2, m_3, m_4) = (130, 111, 193, 830), \\
U_+ &= \begin{pmatrix}
0.707 & 0.685 & 0.104 & 0.140 \\
-0.707 & 0.685 & 0.104 & 0.140 \\
0 & -0.209 & 0.066 & 0.976 \\
0 & 0.131 & -0.987 & 0.095
\end{pmatrix},
\quad U_- = \begin{pmatrix}
-0.707 & -0.696 & 0.085 & 0.088 \\
0.707 & -0.696 & 0.085 & 0.088 \\
0 & 0.140 & 0.128 & 0.982 \\
0 & -0.102 & -0.984 & 0.143
\end{pmatrix}.
\end{align*}
\]

These mass eigenvalues are consistent with the experimental mass bounds [18]

- Charged Higgs : \( m \geq 79.3 \text{ GeV} \),
- Neutral CP-even Higgs : \( m \geq 114.4 \text{ GeV} \),
- Neutral CP-odd Higgs : \( m \geq 93.4 \text{ GeV} \),
- Chargino : \( m \geq 94 \text{ GeV} \),
- Neutralino : \( m \geq 46 \text{ GeV} \).

Note that \( \rho'_3, \rho'_4, (H_4^\pm)' \) are Nambu-Goldstone boson which are eaten by gauge bosons.

### C The lifetimes of exotic quarks
As the R-parities of exotic quarks are odd, at least there must be one sfermion which is lighter than exotic quarks, to make them unstable. Now we assume the right handed slepton \( E^c_l \) is lighter than \( 2\text{ TeV} \) and the other sfermions are heavier than \( 4\text{ TeV} \). For simplicity, we assume there is no mixing between \( E^c_l \) and \( L_i \).
Through the non-renormalizable interaction
\[ \mathcal{L} \supset - \sum_i \frac{c_i V^2}{\sqrt{3} M_P} (\psi_{g,1} + \psi_{g,2} + \psi_{g,3}) E_i^c u_i^c + h.c., \] (132)
the exotic quarks \( \psi_{g,1-3} \) can decay into \( u_i^c \) and \( E_i^c \), where \( c_i \) are \( O(1) \) coefficients. The lifetimes are estimated as follows
\[ \Gamma(\psi^\dagger_{g,j} \rightarrow E_i^c + u_i^c) = \frac{2 T e V}{16 \pi} \left( \frac{c_i V^2}{\sqrt{3} M_P^2} \right)^2 = \frac{c_i^2}{1.7 [\text{sec}]}, \] (133)
from which we must put \( c_i \sim 4 \) in order to satisfy the cosmological constraint for exotic particle, \( \tau < 0.1 \text{sec} \).
The interaction Eq.(132) comes from
\[ W \supset \sum_i \frac{c_i}{M_P} \Phi_3(\Phi_1^c g_1 + \Phi_2^c g_2 + \Phi_3^c g_3) E_i^c U_i^c, \] (134)
which may contribute to LF decay through
\[ \mathcal{L} \supset - \sum_{i,j} \frac{c_i V}{M_P} \alpha_{ij} \phi_{LF} \psi_{g,j} E_i^c u_i^c + h.c., \] (135)
where \( \alpha_{ij} \) are given by linear combinations of the flavon mixing parameters. In this paper, we assume \( \alpha_{ij} \) are small enough and this interaction does not give sizable contribution to LF decay width.

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