Thermal Effects and Flat Direction Baryogenesis

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Abstract

In this paper we provide a detailed numerical study of the influence of thermal effects on the original picture of the Affleck-Dine (AD) baryogenesis. These effects are found to modify the results greatly in some cases. We estimate the baryon/entropy ratio and provide numerical results on the typical behaviour of the charge as a function of the strength of the potential and other parameters.
1. Introduction

Affleck-Dine baryogenesis from flat directions [1, 2] is a natural mechanism to explain the baryon/entropy ratio. The current known value of $\Omega_b h^2$ is $0.019 \pm 0.002$ from nucleosynthesis and $0.031 \pm 0.005$ from BOOMERANG data [3]. This corresponds to $n_B/n_{\gamma} \approx 5.1 \times 10^{-9}$ and $n_B/n_{\gamma} \approx 8.3 \times 10^{-9}$ respectively. The ingredients needed in general to generate the baryon asymmetry are CP violating A-terms, terms which lift flat directions, and supersymmetry breaking terms in the early Universe which give rise to negative mass squared of order of $H^2$, where $H$ is the Hubble constant. At the level of renormalizable interactions there are several flat directions which can be found in MSSM, i.e., directions where $F$ and $D$ terms in the potential vanish. A simple example is the $H_u L$ flat direction. These flat directions can be lifted by nonrenormalizable terms which we might imagine to be associated with the Planck scale, i.e., suppressed by some power of $M_\ast$. $^1$ In that situation as a result of the balance between soft induced SUSY breaking potential and higher order nonrenormalizable terms the field can acquire a large expectation value which evolves as some power of $t$. Eventually, the negative mass squared term, which decreases as $1/t^2$, becomes comparable with $m_{3/2}^2$ term and the AD-field starts to oscillate. This is the moment when baryon charge, which is generated by the torque due to the different phases and time dependence of A-terms, “freezes” at some value. Terms which lift the flat direction in general have the form $\frac{\phi^{2n+4}}{M^{2n}}$. We will principally consider $n = 1, 2, 3$ and refer to these as the $n = 1, 2, 3$ cases. $^2$ One then can do a simple estimate to obtain the ratio $n_B/n_{\gamma}$. For example, in the $n = 1$ case, $n_B/n_{\gamma}$ is of order $\left(\frac{m_{3/2}}{M_\ast}\right)^{1/2}\sin(\delta) \sim 10^{-9}\sin(\delta)$ or so. $^2$ The resulting baryon/entropy ratio is a bit lower because one has to take into account that, for example, in the case of the $H_u L$ direction lepton number is produced first, which later has to be converted to baryon number. Also, the AD condensate interacts with thermalized inflaton decay products and some part of it can be evaporated before the charge is produced. There is a related issue of Q-ball formation and the corresponding evaporation rate, which is different from that of homogeneous condensate. There have been a number of papers on the subject in which authors considered thermal effects relative to AD baryogenesis as well as Q-ball formation. In these paper we will consider AD baryogenesis in the absence of Q-balls. Taking into account the formation of Q-balls will alter the whole picture. We refer the reader to [4] as for this subject.

Recently it was noticed [5] that due to the evolution of the AD condensate in the background of inflaton decay products there is an interesting effect that takes place in addition to the physics of the original scenario. It was observed that superpotential interactions couple the flat directions to other fields. These fields acquire masses induced by the flat-direction vev but they may be sufficiently small so that fields come to thermal

$^1$By $M_\ast$ we denote reduced Planck mass, $M_\ast = 10^{18}$GeV.

$^2$By $\delta$ we denote CP-violating phase.
equilibrium with the inflaton decay products. In such cases the flat direction starts to oscillate at earlier time than usually estimated because it acquires thermal mass \( y^2 T^2 \) which decreases with time as \( 1/t \). Since \(-H^2\) falls more rapidly with time, the difference \( y^2 T^2 - H^2 \) eventually becomes positive. That normally happens much earlier than \( t \sim m_{3/2}^{-1} \).

It was also argued that the main source for generating baryon asymmetry in that case are \( A \)-terms which are proportional to \( T \). In [6] it was pointed out that such \( A \)-terms are suppressed, in general, by symmetries so that there is no temperature enhancement of the \( A \)-term. However, it was noted that there is an additional source of \( A \)-terms that can be efficient in the \( n = 2, 3 \) cases. It was also shown that there is an additional thermal contribution to the potential of the form \( T^4 \log(\phi^2) \). In the \( n = 2, 3 \) cases, this defines the time when the condensate starts to oscillate, rather than the \( y^2 T^2 \) contribution found in [5].

In this paper we further investigate this scenario. We consider different types of thermal effects which are relevant for different choices of \( n \). We analyse the parameter space in greater detail, i.e., the dependence of the resulting baryon/entropy ratio on the parameters of the lagrangian. These parameters are the Yukawa constant, gauge coupling constant, the relative phase between \( A \)-terms, and the coefficients in front of the \( A \)-terms.

The \( n = 1 \) case is found not to generate sufficiently large \( n_B/n_\gamma \), but the \( n = 2, 3 \) cases can rather easily generate the needed number for a wide range of parameters. Throughout we assume for simplicity that the ratio of the inflaton mass to the reduced Planck mass is of order of \( 10^{-5} \), taking, in general, the mass of inflaton \( m_I \) to be \( 10^{13} \text{GeV} \) and reduced Planck mass \( M_* \) to be \( 10^{18} \text{GeV} \). These parameters appear in the estimate of the reheating temperature \( T_R \) as well as in the estimate of the baryon number violating terms. Therefore, the choice of these parameters is important for the estimate of \( n_B/n_\gamma \). We discuss this in detail in Section 4.

This paper is organized as follows: in Section 2 we discuss the origin of two thermal effects as well as their relevance in the \( n = 1, 2, 3 \) cases. In Section 3 we introduce additional \( A \)-terms which will be the sources for generating the baryon asymmetry. In Section 4 we provide the estimates and numerical results for the baryon/entropy ratio. In Section 5 we give some details of the numerical simulations.

\(^3\)y is the coupling constant and T is the temperature.
2. Thermal effects

As was argued in [5] and [6] there are two types of thermal-induced contributions to the potential. The first is due to the following mechanism. Consider some field $\chi$ which couples to the flat direction via the superpotential $W \sim y\chi\chi\phi$ and interacts with the dilute plasma produced by the inflaton decay. Because of $\chi$’s coupling to the flat direction, $\chi$ acquires mass $m_\chi = y\phi$. If this mass is less than the temperature of the thermal plasma this field will come to thermal equilibrium, giving an effective temperature-dependent mass to the flat direction. Let us to do some estimates to see when this effect might be important. Consider the case $n = 1$. Since

$$H_{th} < \frac{T_R^2}{y^4 M_*},$$

where $H_{th}$ is the value of Hubble constant when thermalization of $\chi$ occurs. Taking $M_*$ to be $10^{18}$GeV and $T_R = 10^{10}$GeV one obtains:

$$H_{th} < \left(\frac{0.01}{y}\right)4 \times 10^{10}\text{GeV.}$$ (2)

On the other hand, if $\phi$ gets a thermal mass, oscillations start when $yT \sim H$, which gives the value of the Hubble constant $H_o$ at this moment

$$H_o \sim y^{4/3} T_R^{2/3} M_*^{1/3} = \left(\frac{y}{0.01}\right)^{4/3} 10^{10}\text{Gev.}$$ (3)

In order for this thermal term to be relevant oscillations have to start not later than $\chi$ becomes thermalized, so that we have $H_o/H_{th} \leq 1$, which means $y \leq 0.01$. Provided $y$ is sufficiently small, there is a $y^2T^2|\phi|^2$-contribution to the potential, which affects the time when oscillation of $\phi$ begins. If $y$ is bigger than 0.01, then oscillations start when $y^3T^2$ term is generated, which means that $H_o$ is defined by $H_{th}$. Estimating $H_{th}$ for the $n = 2$ case gives:

$$n = 2 : H_{th} < \frac{T_R^6}{y^{12} M_*^5} = \left(\frac{0.01}{y}\right)^{12} 10^{-6}\text{GeV,}$$ (4)

which is too late for thermalization process to have any effect on $V(\phi)$ unless $y \leq 0.001$. For $n = 3$ from the condition $y\phi < T$ one obtains $\left(\frac{y}{0.01}\right) < 10^{-2}$. We conclude that in the $n = 2, 3$ cases this effect does not take place unless $y$ is very small.

The other contribution to the potential comes from the modification of the coupling constant when some sfermions which are coupled to the flat direction gain masses. The effective potential for $\phi$ (see [6]) is then

$$V_{eff} \sim \alpha^2 T^4 \log(|\phi|^2),$$ (5)

$^{4}T_d$ is the inflaton decay rate.
where $\alpha$ is the gauge coupling. This contribution causes $\phi$ to oscillate when $H^2 \sim \frac{\partial V_{\text{eff}}}{\partial |\phi|^2}$. For $n = 1$, $\phi$ starts to oscillate at $H_o \sim \alpha T_R = (\alpha_{0.01})10^8\text{GeV}$. For $n = 2$: $H_o \sim \alpha_{0.01}^{6/5}T_R(\frac{T_R}{M_*})^{1/5} = (\alpha_{0.01})^{6/5}10^6\text{GeV}$, while for $n = 3$: $H_o \sim \alpha_{0.01}^{4/3}T_R(\frac{T_R}{M_*})^{1/3} = (\alpha_{0.01})^{4/3}10^5\text{GeV}$. If we now look at the ratio of $H_{\text{osc}}$’s due to both thermal contributions in the $n = 1$ case, we see that

$$\frac{H_{\text{osc}}^{(1)}}{H_{\text{osc}}^{(2)}} \sim \frac{y^{4/3}M_*}{\alpha_T} = \frac{y^{4/3}}{\alpha}10^3;$$

where by $H_{\text{osc}}^{(1)}$ and $H_{\text{osc}}^{(2)}$ we denote the values of the Hubble constant when the AD field starts to oscillate if there is only $\alpha^2 T^4 \log(|\phi|^2)$ or $y^2 T^2$ thermal contribution to the potential, respectively.

That estimate shows that in the $n = 1$ case the $y^2 T^2$ term dominates over the $\alpha^2 T^4 \log(|\phi|^2)$-potential unless the ratio $\frac{y^{4/3}}{\alpha} \leq 10^{-3}$. Therefore, we will analyze the $n = 1$ case with the $y^2 T^2$ term only, while for $n = 2, 3$ we will consider only the logarithmic thermal term.

3. $A$-terms

To build up a baryon or lepton asymmetry one needs to have corresponding $U(1)$ violating $A$-terms. The misalignment between their phases then exerts a torque, making $\phi$ to roll down towards one of the discrete minima along the angular direction and settle there. In the original scenario [2] there were two $A$-terms: $Am_3/2\phi^{n+3}/M_*^n$, which is the usual supersymmetry breaking $A$-term, and $aH\phi^{n+3}/M_*^n$, which is induced because of the finite energy density in the Universe during inflation.\(^5\) At later times ($H < m_{3/2}$), because of its dependence on $H$, the second term is no longer important, but at the times $H \sim m_{3/2}$, when the flat direction starts to oscillate, this term is of comparable size with the first one and a sufficient amount of charge is produced.

As we learned in the previous section, $\phi$-oscillations generically start much earlier than $H \sim m_{3/2}$. One might then consider some other sources of $A$-terms, which can be relevant in this case. These additional $A$-terms may arise in the following way: consider the superpotential

$$\delta W = \int d^4 \theta f(I) \frac{\phi^{n+3}}{M_*^n};$$

where $f(I)$ is some holomorphic function of the inflaton field $I$. Taking the first two

\(^5\) $A$ and $a$ are some complex constants.
terms in the polynomial expansion of $f(I)$ in powers of $I/M_*$ leads to the superpotential

$$\delta W = \frac{1}{M_*} (a I + b \frac{I^2}{M_*}) \phi^{n+3} M_*^n,$$

(8)

where, in general, $a$ and $b$ are complex constants which do not need to have the same phase. $I$ decreases as $I_0(t_0/t)$, so that the second term is somewhat suppressed with respect to the first, but at the time when oscillations of the $\phi$-field start this is not necessarily a large suppression and one might expect to generate a reasonable baryon number. We will investigate this particular case in greater detail in the next section.

4. $n_B/n_\gamma$: estimates and numerical results

Before we move to the discussion of numerical results let us provide some crude estimates for the baryon number. In the $n = 1$ case we assume that $y \leq 0.01$ and $y^2 T^2$ dominates over the logarithmic term. Then, $H_o \sim y^{4/3} T^2 (\frac{M_*}{T})^{1/3}$. We take the equation

$$\frac{dn_B}{dt} = |V_B| \sin(\delta),$$

(9)

where $\delta$ is some effective phase which comes from the phase difference of two $A$-terms, $V_B$ is baryon number violating part of the potential. For the $A$-terms which come from superpotential $\delta W = \frac{1}{M_*} (a I + b \frac{I^2}{M_*}) \phi^{n+3} M_*^n \delta$ would be the phase of $b$ after we redefine the phase of $\phi$ to make the $aI$ term real.\(^6\)

Taking the initial amplitude of $I$ to be of order of $M_*$ and replacing $\frac{dn_B}{dt}$ by the product $n_B H_o$ one obtains

$$n_B t^2 = \frac{b H_o \phi^4}{M_* M_*^2 H_o^2} = by^{4/3} T^{2/3} M_*^{1/3} \approx b (\frac{y}{0.01})^{4/3} 10^{11} \text{GeV}.$$

(10)

We compute $n_B t^2$ instead of $n_B$ because it is more convenient, since after the oscillations begin the baryon density decreases as $1/t^2$. For the same reason it is more convenient to compute $n_\gamma t^2 = T_R^{3/2} t_d^2$ where $t_d = (\Gamma_d)^{-1}$, and $\Gamma_d \approx m_I^3/M_*^2$ is the inflaton decay rate. Since $T = (H \Gamma_d M_*^2)^{1/4}$, the reheating temperature is given by $T_R = (\Gamma_d M_*)^{1/2}$. Then, $T_R^{3/2} = M_*^{3/2} t_d^{-1/2}$. Taking $m_I \approx 10^{13} \text{GeV}$ one obtains $\Gamma_d \approx 10^3 \text{GeV}$, $T_R \approx 10^{10} \text{GeV}$ and $n_\gamma t^2 \approx 10^{25} \text{GeV}$. The value of $T_R$ is somewhat large from the perspective of the gravitino problem. The baryon/entropy ratio is

$$n = 1 : \frac{n_B}{n_\gamma} \approx b (\frac{y}{0.01})^{4/3} 10^{-14} \sin(\delta_b),$$

(11)

\(^6\)The phase of $I$ is assumed to be a constant, so that one can absorb its phase in $b$ and consider the inflaton to be real.
which turns out to be too small unless \( y \sim 1 \). Actual numerical study shows that for a wide range (see Fig. 1) of \( y \) the resulting baryon/entropy ratio is somewhat larger:

\[
\frac{n_B}{n_\gamma} \approx (b \sin(\delta_b))(10^{-14} - 10^{-13}),
\]

and almost independent of \( y \). There is some uncertainty which depends on the choice of \( M_* \) as well as \( m_I \). However, in order for \( n = 1 \) case to be viable we would need to consider a much smaller ratio of \( M_*/m_I \) than \( 10^5 \).

One can also consider \( \Gamma_d = \epsilon m_\gamma^3/M_*^2 \), where \( \epsilon \) is some new parameter, i.e., \( \Gamma_d = m_\gamma^3/\Lambda^2 \), where \( \Lambda \) is some new scale different from \( M_* \). Then \( T_R \) is lower than before by a factor of \( \epsilon^{1/2} \). For \( \epsilon \sim 10^{-4} \), \( T_R \sim 10^6 \)GeV, which is an upper bound on \( T_R \) to avoid gravitino problem. If one combines the result of Eq. (10) and the value for \( T_R^3 t_d^2 \) in terms of \( \epsilon \), the inflaton mass, and \( M_* \), one gets that \( n_B/n_\gamma = \gamma y^{4/3} e^{5/6}(m_I/M_*)^{5/2} \). This is an even smaller number than at \( T_R = 10^{10} \)GeV. So if one tries to fit the value of \( T_R \) in a consistent way to avoid gravitino problem one obtains that the \( n = 1 \) case seems to be even less acceptable. In the literature sometimes the value of \( T_R \) is considered as a free parameter. In that case the value of the ratio \( n_B/n_\gamma \sim T_R^{-7/2} \) is very sensitive to the value of the reheating temperature and our estimate for the resulting baryon/entropy ratio changes by several orders of magnitude. For example, if one takes \( T_R = 10^6 \)GeV again and keeps \( \Gamma_d \approx 10^3 \)GeV one will obtain \( n_B/n_\gamma \approx 10^{-9} \). One can also treat \( t_d \) as a free parameter as well. This, in fact, would significantly relax constraints on \( n_B t^2 \) and would soften our conclusions. There was also the discussion in the literature [7], that the gravitino problem can actually be avoided even at temperatures higher than \( 10^{10} \)GeV.

In this paper we stick to \( T_R \approx 10^{10} \)GeV according to the estimate which we get from the expression \( T = (H \Gamma_d M_*^2)^{1/4} \) for the temperature during the inflation at \( H \approx \Gamma_d \) as well as the estimate for \( \Gamma_d \) mentioned above. Therefore, for that choice of parameters, we can conclude that \( n = 1 \) case can hardly be acceptable. However, with all the remarks above, one can probably consider the \( n = 1 \) case to be a borderline case.

In the case of \( n = 2 \), a similar analysis gives \( H_\phi \sim \alpha^{6/5} T_R^{6/5} M_*^{-1/5} \) (now there is no \( y^2 T^2 \)-term and oscillations start due to the logarithmic contribution). Since \( \phi \) behaves as \((H M_*^2)^{1/3}\) one obtains the following estimate for \( n_B t^2 \):

\[
\begin{align*}
n = 2 : n_B t^2 &\approx b \alpha^{4/5} T_R^{4/5} M_*^{1/5} \approx b(\frac{\alpha}{0.1})^{4/5} 10^{11} \text{GeV},
\end{align*}
\]

and the corresponding baryon/entropy ratio is

\[
\begin{align*}
n = 2 : \frac{n_B}{n_\gamma} &\approx b(\frac{\alpha}{0.1})^{4/5} 10^{-13} \sin(\delta_b).
\end{align*}
\]

This is very different from the numerical result, which is

\[
\begin{align*}
\frac{n_B}{n_\gamma} &\approx (b \sin(\delta_b))(10^{-10} - 10^{-9}),
\end{align*}
\]
for wide range of $\alpha$ (see Figs.2, 3). We want to point out here that as in the case $n = 1$ the dependence of the result on the coupling constant is rather weak, as opposed to what one gets estimating $n_B/n_\gamma$ analytically.

Before we explain why the numerical result is so different from the naive estimate, we want to repeat the analysis above for the $n = 3$ case. In that case $\phi = (HM^3_\ast)^{1/4}$; the operator which creates the charge is $bH^\ast_0 M^1_\ast / \gamma$. Estimating $n_B t^2$ at $H = H_0$ one gets

$$n_B t^2 \approx bH^\ast_0 M^1_\ast / \gamma \sin(\delta_b).$$

(16)

Taking $T_R = 10^{10}$ GeV and $M_\ast = 10^{18}$ GeV one gets

$$n_B t^2 \approx b\alpha^{2/3} \sin(\delta_b)10^{12}$$

and, the corresponding baryon/entropy ratio

$$\frac{n_B}{n_\gamma} = \frac{n_B t^2}{T_R^3 t_d^2} \approx b\alpha^{4/5} \sin(\delta_b)10^{-13},$$

(18)

which is too small for all reasonable values of $b$, $y$ and $\sin(\delta_b)$. One can notice that this rough estimate predicts the decrease of baryon/entropy ratio as $y$ gets smaller as well as in the $n = 2$ case. Actual numerical study, as in case $n = 2$, gives a quite different value of that ratio. The baryon/entropy ratio for the wide range of $\alpha$ is

$$\frac{n_B}{n_\gamma} \approx b\sin(\delta_b)(10^{-8} - 10^{-9}),$$

(19)

which could be consistent with experimentally known value if $b \sim (0.1 - 0.01)$ and $\delta_b \sim 1$.

The naive expectation fails because the approximation of $dn_B/dt$ by $n_B H_0$ is too crude with respect to actual integration. One can understand why the amplitude of $n_B t^2$ grows as $y$ gets smaller, when looking at the behavior of $n_B t^2$ at some values of $y$ as a function of time. From numerical results it is clear that the later the oscillations of $\phi$ start the more oscillations $n_B t^2$ undergoes before it “freezes”. With each oscillation, the operator which is responsible for the charge production contributes more and more to the amplitude of $n_B t^2$. That might explain the unexpected behavior of charge in the $n = 2, 3$ cases. One can see that the behavior of the charge for $n = 1$ with the $y^2 T^2$-term is quite different from its behavior for $n = 2, 3$ with the logarithmic term. Namely, for $n = 2, 3$, before the value of a charge “freezes”, it experiences many more oscillations than in the $n = 1$ case, gaining larger amplitude with each of them. One can see this from numerical results in Fig.1-6, which show the evolution of the charge with time and with the strength of the potential (Yukawa coupling in the case $n = 1$, and gauge coupling in the $n = 2, 3$ cases). The first three plots demonstrate the expected behavior of the charge, i.e., it
oscillates at the time $t < t_o$ and freezes afterwards. The difference between $n = 1$ and $n = 2, 3$ is that for $n = 1$ the charge typically oscillates one or two times, while in the $n = 2, 3$ cases it oscillates much more. Correspondingly, for $n = 1$, the numerical result is close to the naive estimate, but for $n = 2, 3$ numerical value is much larger.

The Figs.1-3 further distinguish these cases. One can see that changing the Yukawa coupling in the phenomenologically interesting region does not produce major changes in the amplitude of $n_B t^2$ in the $n = 1$ case. However, if one changes the value of the gauge coupling in the $n = 2, 3$ cases one can see that $n_B t^2$ oscillates with $\log(\alpha)$ and its amplitude grows with the decreasing of $\alpha$, which is against the naive analytical estimate. We believe that this happens due to the logarithmic nature of the thermal potential in these cases.

Dependence of numerical result of the baryon/entropy ratio on $b$ and $\delta_b$ doesn’t bring any surprises with respect to the crude estimate above. In Fig.7 one can see that dependence on $\delta_b$ is $\sin(\delta_b)$, and from Fig.8 the dependence on $b$ is a linear function. Thus for any $b$ and $\delta_b$ one knows how to reproduce the baryon/entropy ratio from the results in the equations (12, 15, 19).

5. Details of the numerical simulations

The equation for the evolution of the $\phi$ is

$$\ddot{\phi} + 3H \dot{\phi} + \frac{\partial V}{\partial \phi^*} = 0. \tag{20}$$

In general $V$ has the form $V = V_0(|\phi|^2) + V_B(\phi) + V_B(\phi^*)$, where $V_0$ determines the evolution of $|\phi|$ and $V_B$ is the part of the potential which is responsible for the creating of the baryon charge. It also has a minor effect on evolution of $|\phi|$, which can be neglected in the cases we consider. The potentials we wish to study have the form:

$$V_0 = (-H^2 + m_{3/2}^2)|\phi|^2 + V_{th}(T) + |\frac{\partial W}{\partial \phi}|^2, \tag{21}$$

$$V_B = aH |\phi|^4 + b H^2 |\phi|^4 + Am_{3/2}W$$

As we discussed before, the terms which are proportional to $m_{3/2}$ are not important. Dropping them, for $n = 1$ we have:

$$V = (-H^2 + g^2 T^2)|\phi|^2 + \lambda^2 |\phi|^6 M_*^2 + (aH |\phi|^4 M_* + b H^2 |\phi|^4 M_* + h.c.), \tag{22}$$

for $n = 2$:

$$V = -H^2 |\phi|^2 + \alpha^2 T^4 \log(|\phi|^2) + \lambda^2 |\phi|^8 M_*^2 + (aH |\phi|^5 M_* + b H^2 |\phi|^5 M_*^2 + h.c.), \tag{23}$$
and for $n = 3$:

$$V = -H^2|\phi|^2 + \alpha^2 T^4 \log(|\phi|^2) + \lambda^2 \frac{|\phi|^{10}}{M_*^6} + (aH \frac{\phi^6}{M_*^3} + bH^2 \frac{\phi^6}{M_* M_*^3} + \text{h.c.}).$$  \hspace{1cm} (24)$$

We took into account that different thermal effects are relevant for $n = 1$ and $n = 2, 3$. Introducing radial and angular parts of $\phi$ by $\phi = Re^{i\Omega}$ one gets corresponding equations for $R$ and $\Omega$:

$n = 1$:

$$\ddot{R} + 3H \dot{R} + (-H^2 - \dot{\Omega}^2 + y^2 T^2 + \frac{\lambda^2 R^4}{M_*^2}) R = 0,$$

$$\ddot{\Omega} + (3H + \frac{2\dot{R}}{R}) \dot{\Omega} = |a| \frac{R^2 H}{M_*} \sin(4\Omega) + |b| \frac{R^2 H^2}{M_*} \sin(4\Omega - \delta_b);$$

$n = 2$:

$$\ddot{R} + 3H \dot{R} + (-H^2 - \dot{\Omega}^2 + \alpha^2 T^4 / R^2 + \frac{\lambda^2 R^6}{M_*^2}) R = 0,$$

$$\ddot{\Omega} + (3H + \frac{2\dot{R}}{R}) \dot{\Omega} = |a| \frac{R^2 H}{M_*} \sin(5\Omega) + |b| \frac{R^2 H^2}{M_*} \sin(5\Omega - \delta_b);$$

$n = 3$:

$$\ddot{R} + 3H \dot{R} + (-H^2 - \dot{\Omega}^2 + \alpha^2 T^4 / R^2 + \frac{\lambda^2 R^6}{M_*^2}) R = 0,$$

$$\ddot{\Omega} + (3H + \frac{2\dot{R}}{R}) \dot{\Omega} = |a| \frac{R^2 H}{M_*} \sin(6\Omega) + |b| \frac{R^2 H^2}{M_*} \sin(6\Omega - \delta_b).$$

We are dropping the contribution from $V_B$ at $t < t_o$ to the equations for $R$ for the following reasons. Let us to compare the behavior of $R$ induced by the $\frac{\lambda^2 R^4}{M_*^2}$ and the $|a| \frac{R^2 H}{M_*} \cos(4\Omega)$ terms separately ($n = 1$ case). We know that the first term causes $R$ to behave as $(H M_*)^{1/2}$. The second term causes $R$ to behave in the same way, provided $\Omega$ is small, so that $\cos(4\Omega) \approx 1$. The smallness of $\Omega$ is guaranteed by its equation of evolution and, in fact, is well seen in numerical results. Therefore, qualitatively it does not bring any changes to the evolution of $R$.\footnote{In fact, one can also neglect $\dot{\Omega}^2$ term in the equation for $R$ since this term is small compared to the others for any $H > H_o$. It becomes, however, important after the oscillations start. At this moment baryon charge is already produced and fixed. Later evolution of the baryon number is not interesting because of that and one does not need to run simulations further.} The other term, which is proportional to $|b|$, is suppressed by $(m_I/M_*)$ and falls down more rapidly with time with respect to the $A$-term. Hence, neglecting these terms in the equation for $R$ makes sense, since that does not change the evolution of $R$ and simplifies the numerical and analytical analysis of the equations.

For numerical simulations it is more convenient to introduce a dimensionless field $r = R t$ and scale everything by $m_I$, which we take to be $10^{13}\text{GeV}$. Then, the equations...
above with all the simplifications which were discussed before take the form:

\( n = 1: \)

\[
\ddot{r} + \left( -H^2 + y^2 T^2 + \epsilon^2 \frac{\lambda r^4}{t^2} \right) r = 0,
\]

\[
\dddot{\Omega} + \frac{2 \dot{r}}{r} \dot{\Omega} = |a| \epsilon^2 \frac{r^2 H}{t^2} \sin(4\Omega) + |b| \epsilon^2 \frac{r^2 H^2}{t^2} \sin(4\Omega - \delta_b);
\]

\( n = 2: \)

\[
\ddot{r} + \left( -H^2 + \alpha^2 t^2 T^4/r^2 + \epsilon^4 \frac{\lambda r^6}{t^6} \right) r = 0,
\]

\[
\dddot{\Omega} + \frac{2 \dot{r}}{r} \dot{\Omega} = |a| \epsilon^4 \frac{r^4 H}{t^4} \sin(5\Omega) + |b| \epsilon^4 \frac{r^4 H^2}{t^4} \sin(5\Omega - \delta_b);
\]

\( n = 3: \)

\[
\ddot{r} + \left( -H^2 + \alpha^2 t^2 T^4/r^2 + \epsilon^6 \frac{\lambda r^8}{t^8} \right) r = 0,
\]

\[
\dddot{\Omega} + \frac{2 \dot{r}}{r} \dot{\Omega} = |a| \epsilon^6 \frac{r^6 H}{t^6} \sin(6\Omega) + |b| \epsilon^6 \frac{r^6 H^2}{t^6} \sin(6\Omega - \delta_b),
\]

where \( \epsilon = M_1/M_* \approx 10^{-5} \). Solving these equations numerically and noting that \( n_B t^2 = r^2 \dot{\Omega} \), we can plot the behavior of \( n_B t^2 \) as a function of time as well as find its asymptotic value as a function of different parameters.

At this point it is rather easy to analyze the behavior of the charge. In fact, the equation for \( r \) is no longer dependent on \( \Omega \), so that the second equation describes the evolution of \( \Omega \) in the “background” of the radial component, which, in turn, is given by the first equation in each case. The evolution of \( r \) is easy to understand just looking at its equation. First, it falls down as some power of \( t \), which depends on \( n \), and then at \( H \sim H_o \) starts to oscillate. Numerical comparison of the full system of equations without all these simplifications which we made above shows that at \( t < t_o \) this approximation is reliable.

6. Conclusions

Clearly thermal effects are important in the evolution of AD condensate and modify the original mechanism of AD baryogenesis. We find that in the \( n = 1 \) case it is difficult to reproduce the known value of the baryon/entropy ratio even if taking into account some uncertainty due to the choice of the value of the reheating temperature, so that this case should probably be considered borderline at best. However, the \( n = 2, 3 \) cases give plausible results for a wide range of the parameters of the potential. We have shown that the naive estimate of \( n_B/n_\gamma \) fails by several orders of magnitude to reproduce the observed numerical value. This effect is most clearly seen in the \( n = 2, 3 \) cases, when the radial component of the AD field evolves in the logarithmic potential. This happens due to the behavior of the charge before it freezes, which might seem a bit surprising.
Instead of decreasing with $H_o$ it actually slightly grows. We have also shown numerically that dependence on a few other parameters of the potential such as the phase difference between the $A$ terms and the coefficient of the $A$ term $b$ is of the expected form.

**Acknowledgements.**

We would like to thank M. Dine and M. Graesser for useful conversations and for the discussion of the results of this paper.

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7. Figures and Captions

Fig.1: This graph illustrates the behavior of the asymptotic value of $n_B t^2$ as a function of $\ln(y)$ for $n = 1$. The choice of parameters here is $a = b = 1; \delta_b = \pi/3$.

Fig.2: This graph illustrates the behavior of the asymptotic value of $n_B t^2$ as a function of $\ln(\alpha)$ for $n = 2$. The choice of parameters here is $a = b = 1; \delta_b = \pi/3$.

Fig.3: This graph illustrates the behavior of the asymptotic value of $n_B t^2$ as a function of $\ln(\alpha)$ for $n = 3$. The choice of parameters here is $a = b = 1; \delta_b = \pi/3$.

Fig.4: This graph illustrates generic behavior of $n_B t^2$ as a function of time in the $n = 1$ case; cases a, b, c, d correspond to $y_a = 0.01, y_b = 0.005, y_c = 0.0025, y_d = 0.001$. The values of other parameters are taken to be $\delta_b = \pi/3, a = b = 1 = \lambda$.

Fig.5: This graph illustrates generic behavior of $n_B t^2$ as a function of time in the $n = 2$ case; cases a, b, c correspond to $\alpha_a = 0.1, \alpha_b = 0.07, \alpha_c = 0.03$. The values of other parameters are taken to be $\delta_b = \pi/3, a = b = 1 = \lambda$.

Fig.6: This graph illustrates the behavior of $n_B t^2$ for $n = 3$, case a corresponds to $\alpha = 0.1$, case b to $\alpha = 0.7$, and case c to $\alpha = 0.5$. The values of other parameters are taken to be $\delta_b = \pi/3, a = b = 1 = \lambda$.

Fig.7: This graph illustrates the behavior of the asymptotic value of $n_B t^2$ as a function of $\sin(\delta_b)$ for $n = 2$. The choice of parameters here is $a = b = 1; \alpha=0.05$.

Fig.8: This graph illustrates the behavior of charge as a function of $b$ for $n = 2$. The choice of parameters here is $a = 1, \delta_b = \pi/3, \alpha=0.03$. 
Figure 1:
Figure 2:
Figure 3:
Figure 4:
Figure 6:
Figure 7:
Figure 8: