INTERLAYER SYNCHRONIZATION IN TIME-VARYING KURAMOTO-SAKAGUCHI NETWORKS

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We investigate the abrupt transition from low interlayer synchrony to high interlayer synchrony in a system of two identical layers of non-locally coupled Kuramoto-Sakaguchi oscillators using time-switching of the interlayer topology, while keeping the intralayer topology constant. As a measure of interlayer synchronization we introduce the interlayer order parameter, and determine the interlayer order parameter first in the static case as a function of the number of interlayer links observing a monotonically smooth transition to the fully synchronized state in the case that all nodes in layer 1 are connected to their corresponding nodes in layer 2. In the second step, we analyze the synchronization behavior of the multiplex network for a time-varying interlayer topology, observing abrupt transitions from the state of low synchrony to the state of high synchrony for a significantly lower number of interlayer connections. These abrupt transitions, even if not to full synchrony, appear only because of the periodic change of the interlayer topology, fostering the transition to occur for a significantly lower number of interlayer connections. We interpret this phenomenon as the shrinking, and ultimately, the nearly disappearance of the basin of attraction of a desynchronized network state. A comprehensive statistical analysis of this phenomenon is needed to interpret the results properly.

One of the first encouragements for analyzing networks composed of Kuramoto-Sakaguchi oscillators originates from neuroscience. The synaptic coupling can vary depending on the action on the neurons changing the coupling between the neurons. Consequently, the analysis of the impact of varying coupling ranges on the spiking behavior and its timings in neural networks was carried out in [9, 10]. Apart from neuroscience, many models of robust networks are studied in chemical, biological, and social networks, and many different systems. This study analyzes the effect of a time-varying interlayer coupling on the synchronization of a two-layer network composed of two weakly coupled rings of identical Kuramoto-Sakaguchi phase oscillators. We limit our investigation to one particular coupling composition, a non-locally coupled ring of Kuramoto-Sakaguchi oscillators resulting the two layers to be in the chimera state. Then we couple the two layers one-to-one weakly in order not to disturb the intralayer dynamics, stastically as well as time-varyingly.

Synchronization is an extraordinary characteristic of the dynamics of the networks of connected oscillators. Different synchronization regions are known, like a group of synchronization where the network is divided into two clusters of synchronous components or partial synchronization dynamics, which is chimera patterns where all networks are divided into coexisting regions of synchronized and desynchronized patterns. Chimera patterns were also investigated in various topological network. Those properties are studied in a variety of networks experimentally as well as theoretically. Especially, the important properties of a network of neurons have been investigated. Lately, related models have been considered for an all-to-all coupling topology, called the global coupling topology. Adaptive Kuramoto-Sakaguchi type networks show distinct complex topological responses on globally coupled networks. Critical, stable multi-frequency groups appear when the oscillators are divided into strongly coupled oscillators with a similar average frequency. This characteristics does not appear in the classical Kuramoto or Kuramoto-Sakaguchi phase oscillator dynamics. Furthermore, the groups demonstrate how to control a hierarchical composition; for instance, their dimensions are substantially different. This development is also described for adaptive neural networks of Morris-Lecar, Hindmarsh-Rose, and Hodgkin-Huxley models with either spike-timing-dependent plasticity rule. In networks modeling the brain, hierarchy and modularity have recently been introduced.

Commonly, the connecting structure of real-world systems is time-dependent, e.g., the brain neuronal plasticity, network of scientific collaborations, or mutation processes in evolutionary biology. To incorporate this feature into a description, many theoretical studies on synchronization have considered the effects of networks with time-varying topologies. Studies show that such settings have massive impact on the transition to synchronization. In fact, the synchronizability of networks is enhanced for different implementations of the topological time dependencies, i.e., the transitions to synchronization generally occur for lower values of the coupling parameters. The example are provided by, for instance, evolution along commutative graphs, adaptation of weights on-off coupling, different rewiring frequencies, and random connection. More recently, and of particular interest for this study, the enhancement of synchronizability due to time-varying topology has also been shown in networks in which the interacting structure is subdivided into different layers, i.e., multilayer networks. For instance, in a network of Hindmarsh-Rose neurons in which the coupling structure of both electrical and chemical synapses is time-varying, it has been found that the coupling strength required for the achievement of complete synchrony is significantly lower for increasing the rewiring frequency. Moreover, the enhancement effect has been also observed in a network with multiple layers in which only the structure of specific layers is time-varying. Furthermore, it has been shown that diverse partial synchronization patterns can be generated by multi-
plexing of adaptive networks. Recently, it has been found that noisy modulation of inter-layer coupling strength, called multiplexing noise, induces the inter-layer synchronization of spatio–temporal patterns. Despite such a significant understanding of the role played by time-dependent topologies in facilitating synchronization, some important issues are still await unresolved. In particular, the influence of the topological time-dependencies in the characteristics of the transitions to synchronization is mostly unknown.

In section I we review shortly the Kuramoto-Sakaguchi Network model, outlining the differences between the original Kuramoto model and the extension given by Kuramoto and Sakaguchi. In the following section II we define the network of two layers composed of identical Kuramoto-Sakaguchi oscillators in the chimera regime. Section III reviews first the global and local order parameters introduced by Kuramoto as a measure to find the regions of coherence and decoherence in a single layer network. If the system reaches full synchrony, the local order parameters for both layers coincide perfectly. As a second measure of interlayer synchronization, we introduced the interlayer order parameter indicating numerically the level synchrony between the two layers. In section IV, we analyze and discuss the behavior of the Kuramoto-Sakaguchi network, first in absence of interlayer links, showing that both layers are in the chimera state, then by increasing the number of interlayer links reaching full synchrony only in the case when all nodes in layer 1 are connected to their corresponding nodes in layer 2. Finally, we employ the idea of a time-varying interlayer topology, where the interlayer links are removed after a certain time period and the same number of interlayer links is instantaneously randomly connected again. The analysis of the interlayer order parameter as a function of the number of interlayer links shows sharp transitions from low synchrony to high synchrony. The high synchrony case, i.e. not having reached full synchrony, shows that the regions of coherence and incoherence coincide, while the local order parameters for the coherent regions are identical equal to 1, the local order parameters for the incoherent regions do not match perfectly. We finally close with the conclusion of the results.

I. THE KURAMOTO-SAKAGUCHI NETWORK MODEL

To get a deeper understanding on the interlayer synchronization phenomenon, we decided to extend the explosive synchronization model to a network consisting of two weakly coupled, identical Kuramoto-Sakaguchi layers. Again, we employ non-local coupling to ensure the establishment of chimera states. In contrast to the FHN oscillator, non-local coupling is not sufficient to create Chimera states. Therefore, the so-called Kuramoto-Sakaguchi oscillator is used. Kuramoto and Battogtokh introduced global order parameter in its simplest form in [12]. For a single layer, the Kuramoto system can be written as

\[ \dot{\theta}_i(t) = \omega_i + \frac{K}{N} \sum_{j} \sin(\theta_i - \theta_j). \]

For this model, it is not possible to generate chimera states with just employing non-local coupling. The extension of the Kuramoto model by the so-called phase-lag parameter \(\alpha\) opened the door to observe chimera states even for phase oscillators like the Kuramoto oscillator.

\[ \dot{\theta}_i(t) = \omega_i + \frac{K}{N} \sum_{j} \sin(\theta_i - \theta_j + \alpha). \]

Clearly, the Kuramoto model is the special case of the Kuramoto-Sakaguchi model for \(\alpha = 0\). Kuramoto and Battogtokh showed the existence of coherent and incoherent domains for the Kuramoto-Sakaguchi model without naming it Chimera state. The terminology ‘Chimera state’ was created by Abrams and Strogatz in [13]. Later, Panaggio and Abrams gave a summary of the phenomenon of Chimera states in [14].

II. THE KURAMOTO-SAKAGUCHI MODEL IN DOUBLE LAYER MULTIPLEX NETWORKS

We consider a network constructed of two identical layers of non-locally coupled Kuramoto-Sakaguchi oscillators. The interlayer coupling is weak and one-to-one, i.e. the node \(i\) in layer 1 is, if connected, only connected to its corresponding node \(i\) in layer 2. These kind of networks as aforementioned are called multiplex networks. The interlayer coupling is selected as weak in order not to influence the intralayer dynamics of the individual layers.

The consider the following system.

\[ \dot{\theta}_i^{(1)}(t) = \omega_i^{(1)} + \sigma_1 \sum_{j=1-R_1}^{i+R_1} \sin(\theta_i^{(1)} - \theta_j^{(1)} + \alpha^{(1)}) + \sigma_{12} \sin(\theta_i^{(1)} - \theta_i^{(2)}) \]
\[ + \sigma_{12} \sin(\theta_i^{(1)} - \theta_i^{(2)}) \]
\[ = \omega_i^{(2)} + \sigma_2 \sum_{j=1-R_2}^{i+R_2} \sin(\theta_i^{(2)} - \theta_j^{(2)} + \alpha^{(2)}) + \sigma_{21} \sin(\theta_i^{(2)} - \theta_i^{(1)}), \]

where \(\omega_i^{(j)}\) is called the natural frequency of the node \(i\) in layer \(j\).

III. ORDER PARAMETER FOR PHASE OSCILLATORS

A. Global and Local Order parameter

As a measure of the simultaneous coexistence of coherence and incoherence in a network, namely the Chimera states, Kuramoto and Battogtokh introduce global order parameter \(Z(t)\) in [12].

\[ Z(t) = \frac{1}{N} \left| \sum_{k=1}^{N} e^{i\theta_k(t)} \right|, \]
where \( \theta_k(t) \) is the phase of the \( k^{th} \)-oscillator at the instant \( t \). There are two limiting cases for the global order parameter \( Z(t) \). If \( Z(t) = 1 \) the network is coherent, whereas if \( Z(t) = 0 \), the network is incoherent. So, if \( 0 < Z(t) < 1 \) the network has domains of coherence and decoherence, consequently the global order parameter can be used to identify the existence of Chimera states.

Alongside with the global order parameter, the local order parameter \( Z_k(t) \) can be used as a measure to determine the regions of coherence and decoherence in the network.

\[
Z_k(t) = \frac{1}{2R+1} \left| \sum_{i=k-R}^{k+R} e^{i \theta_k(t)} \right|, \quad (5)
\]

where \( R \leq N/2 \).

### B. Interlayer synchronization order parameter

Synchronization in multilayer networks has only been studied based on the intralayer local order parameter \( Z_k(t) \) as defined in (5). This intralayer order parameter helps us to identify the regions of coherence and decoherence. On the other hand, a measure of interlayer synchronization has not been introduced yet in the literature. Therefore, to fill this gap we introduce the time-averaged interlayer order parameter as following

\[
Z = \frac{1}{N} \left| \left\langle \sum_{i=1}^{N} e^{i \theta^{(1)}_i(t) - \theta^{(2)}_i(t)} \right\rangle_{t} \right| = \frac{1}{NT} \int_{0}^{T} \left| \sum_{i=1}^{N} e^{i \theta^{(1)}(t) - \theta^{(2)}(t)} \right| dt. \quad (6)
\]

Using this definition we will determine the interlayer synchronization behavior of the two layer multiplex system of identical Kuramoto-Sakaguchi oscillators as given in equation (3). If the phase difference between node \( i \) in layer 1 and the corresponding node \( i \) in layer 2 vanishes, \( e^{i \theta^{(1)}_i(t) - \theta^{(2)}_i(t)} = 1 \), so if all nodes are synchronized, i.e. their phase difference of the corresponding nodes vanishes, the time averaged interlayer order parameter as defined in equation (3) becomes \( Z = 1 \).

### IV. RESULTS AND DISCUSSION

As aforementioned, the two layers are set to be identical. Therefore, the intralayer coupling strengths \( \sigma_1 \) and \( \sigma_2 \) of the links in layer 1 and layer 2, respectively, have to be equal, i.e. \( \sigma_1 = \sigma_2 \). Moreover, the coupling ranges \( r_1 = R_1/N \) and \( r_2 = R_2/N \) together with the coupling radii \( R_1 \) and \( R_2 \) have to be equal to ensure the the two layers to be identical. As the mean frequency is conserved in the Kuramoto model, and has no effect other than a constant phase shift, for simplicity we will select them uniformly as \( \omega^{(1)}_0 = 0 \). Additionally, the phase-lag parameter \( \alpha^{(1)} \) in layer 1 and layer 2 \( \alpha^{(2)} \), i.e. \( \alpha^{(1)} = \alpha^{(2)} = \pi/2 - 0.1 = 1.47 \). Finally, layer 1 and layer 2 will be coupled symmetrically and weakly to each other, i.e. \( \sigma_{12} = \sigma_{21} \ll \sigma_1 = \sigma_2 \).

Let us first summarize all parameters used in the following table:

| Parameter | Description of the Parameter |
|-----------|-----------------------------|
| \( N = 300 \) | Number of nodes per layer |
| \( \omega^{(1)}_0 = \omega^{(2)}_0 = 0 \) | natural frequency |
| \( \sigma_1 = \sigma_2 = 0.1 \) | intralayer coupling strength |
| \( r_1 = r_2 = 0.35 \) | intralayer coupling range |
| \( R_1 = R_1 \cdot N = 105 \) | intralayer coupling radius for layer 1 |
| \( R_2 = R_2 \cdot N = 105 \) | intralayer coupling radius for layer 2 |
| \( \sigma_{12} = \sigma_{21} = 0.01 \) | intralayer coupling strength |
| \( \alpha^{(1)} = \alpha^{(2)} = 1.47 \) | phase-lag parameter |
| \( N_{12} \) | Number of interlayer links |

TABLE I. List of all parameters used.

Proceeding to the analysis of the time-varying interlayer topology of the double layer multiplex Kuramoto-Sakaguchi network, we will use the parameters stated in table I following the procedure outlined in [33] for the FHN oscillator. First, we show using the spacetime plots and the local order parameters that both layers are in the chimera state for random initial conditions in the case that there is no interlayer connection.

![Spacetime plots](image1)

FIG. 1. For the case of no interlayer connection, i.e. \( N_{12} = 0 \), the spacetime plots for layer 1 (a) and layer 2 (b), as well as the local order parameters for layer 1 (c) and layer 2 (d) have been depicted.

For random initial conditions and no interlayer link, figure I shows that for the selected parameters stated in table I layer 1 and layer 2 are in the Chimera state. The spacetime plots in figure I(a) for layer 1 and figure I(b) for layer 2 together with the local order parameter at \( t = 3000 \) in figure I(c) and (d) show that both layers are in the chimera regime and that the size of the domains of coherence and decoherence are the same. So, we can state once again, that the size of the domains of coherence and decoherence are independent of the initial conditions.
In order to check if the proposed network (3) reaches full synchrony, we connect all nodes in layer 1 to their corresponding nodes in layer 2, i.e. the number of interlayer links $N_{12} = N - 300$.

![Figure 2. $N_{12} = 300$](image)

Figure 2 shows the spacetime plots for (a) layer 1 and (b) layer 2, as well as the local order parameter for (c) layer 1 and (d) layer 2 in the case $N_{12} = N = 300$. Evidently, the regions of coherence and decoherence in both layers now overlap if all nodes in layer 1 are connected to its corresponding node in layer 2. The reason for the selection of a weak coupling strength was to ensure that the interlayer coupling does not impact the intralayer dynamics of the individual layers notably. As one can see clearly in figure 2 the chimera patterns have not been influenced, i.e. they still appear, and the size of the domains of coherence and decoherence has not changed, or if only insignificantly. It is clear that the domains of coherence and decoherence appear now at the same nodes in the corresponding layers. In figure 2 (b) we show for the case of full synchrony, the combination of figure 2 (c) and (d) clearly indicating the overlap of the domains in both layers. In the next step, we increase gradually the number of interlayer links and determine the interlayer order parameters as a function of the number of interlayer links $N_{12}$.

![Figure 3. (a) Interlayer synchronization order parameter as a function of the number of interlayer links $N_{12}$ with not switching. (b) local order parameter for the case $N_{12} = N = 300$](image)

In figure 3 (a) we can see that the interlayer order parameter increases monotonically with increasing number of interlayer links $N_{12}$, reaching full synchrony, i.e. $Z = 1$, only for $N_{12} = N = 300$. So, we can conclude, full synchronisation can only be reached if all nodes in layer 1 are connected to the corresponding node in layer 2. In this case, as aforementioned the regions of coherence and decoherence coincide illustrated using the local order parameter $Z_0(t)$ exemplarily for $t = 3000$ as illustrated in figure 3 (b).

Finally, following the process we already used in 55, we want to investigate if it possible to reach synchrony for a significantly fewer number of interlayer links than $N_{12} = N = 300$. Analogously to the analysis of the network of the FHN oscillators, for a given number of interlayer links $N_{12}$ we start now removing the randomly distributed links after a given time, called the switching time $T_S$, and immediately connecting the same number of interlayer links randomly again. Statistically, the distribution of the connections after each time interval $T_S$ is most likely different to the distribution of connections before. So, we determine the interlayer order parameter for different switching times as a function of the number of interlayer links $N_{12}$.

![Figure 4. Interlayer synchronization order parameter as a function of the number of interlayer links $N_{12}$ for switching times (a) $T_S = 5$, (b) $T_S = 35$, (c) $T_S = 45$, (d) $T_S = 55$, (e) $T_S = 65$, and (f) $T_S = 95$.](image)

Figure 4 depicts the interlayer order parameter as a function of the number of interlayer links $N_{12}$ with switching times (a) $T_S = 5$, (b) $T_S = 35$, (c) $T_S = 45$, (d) $T_S = 55$, (e) $T_S = 65$, and (f) $T_S = 95$. We can observe, that for every switching time we can observe abrupt changes in the interlayer order parameter. Unfortunately, we could not find a combination of switching time $T_S$ and number of interlayer links where the system reaches full synchrony between the two layers. Interestingly, although we could not pin down the situation where the system makes a transition from the desynchronized to the fully synchronized state yet, the system shows for the every
switching time an abrupt change in the interlayer order parameter, leading to partial synchronization of the coherent regions of the two layers. The most significant transition between low interlayer order parameter (Z ≈ 0.3) to high interlayer order parameter (Z ≈ 0.7) could be observed for \( T_S = 5 \), indicating that the two layers are widely synchronized. Let us have a closer look at the case of switching time \( T_S = 5 \) and \( N_{12} = 70 \), where we could observe the biggest jump in the interlayer order parameter. Therefore, we will first show the spacetime plots with their corresponding local order parameters. In order to show, the effect we will combine the interlayer order parameters for both layers in one graph.

**FIG. 6.** For switching time \( T_S = 5 \) and \( N_{12} = 70 \) space time plot for layer 1 (a) and layer 2 (b) and local order parameter for layer 1 (c) and layer 2 (d)

In figure 5 we can observe, that the regions of coherence and decoherence in layer 1 and layer 2 are at the same positions, indicating that the weak interlayer coupling made the regions of coherence and decoherence in layer 1 and layer 2 coincide. A closer look at the local order parameter, by plotting the local order parameters for layer 1 and layer 2 in figure 6 shows, that the local order parameters for the coherent domains coincide, but the local order parameters for the incoherent domains do not overlap as perfect. But, nevertheless we can interpret this as a transition to a interlayer synchrony. Furthermore, it seems that the combination of switching and the change of the number of interlayer links impacts the domains of decoherence. A more careful statistical analysis has to be carried out to show how switching impacts the intralayer behavior of the individual layers.

We see again that the basin of attraction of the desynchronized state shrinks, but we could not yet find the condition where the basin of attraction of the desynchronized state vanishes. This is just an intermediate result of this phenomenon. As we have clear indicators that full synchrony exists for the case \( N_{12} = N \) switched or not switched, and we can observe certain abrupt transitions, further research will reveal the results for an abrupt transition from the desynchronized state to the fully synchronized state, where the basin of attraction of the desynchronized state vanishes, has to be carried out.

As already discussed above, although we could not yet observe the transition from a desynchronized state to full synchrony, nevertheless we can observe the intriguing phenomenon of abrupt transitions from states of lower synchrony to higher synchrony with the implementation of switching. We also illustrated for the case of the biggest jump in the interlayer order parameter, that the regions of coherence and decoherence overlap, while the incoherence seems to behave different in the different layers. We also can state, that the difference in the order parameter at the transitions seems to be depending on the switching time. Also two abrupt changes of the order parameter could be observed. With shorter switching times, the average time one node is connected to its corresponding node is higher. A comprehensive statistical analysis of this phenomenon is needed to interpret the results properly.

V. CONCLUSIONS

In analogy to the study of the FHN oscillator we extended our study to the Kuramoto-Sakaguchi oscillator to investigate if such a collective transition also appears for phase oscillators. As a measure of synchronization we introduced the interlayer order parameter as counter part to the synchronization error in the FHN case. The parameters for the Kuramoto-Sakaguchi network were selected in such a way that the individual layers show Chimera patterns under non-local intralayer coupling. Non-local intralayer coupling is not sufficient to impose Chimera behavior, therefore the phase-lag parameter had to be selected analogously to the coupling angle in the cross-coupling matrix of the FHN system as \( \alpha = \pi/2 – 0.1 \). With weak interlayer coupling strength, we studied the interlayer synchronization behavior of the Kuramoto-Sakaguchi first for static interlayer topology with varying number of interlayer links, observing a smooth transition from the desynchronized state to the fully synchronized state with increasing number of interlayer links. Full synchrony can only be reached in case of all nodes in layer 1 being connected to the corresponding node in layer 2. Then, the interlayer connection topology is changed for a fixed number of interlayer...
links at given time-intervals, namely after each time-interval $T_3$ all $N_2$ links are removed and connected randomly at different positions. Reconnection for individual links is not prohibited in two succeeding time-intervals. We observe for different switching times $T_3$ abrupt changes in the interlayer order parameter for varying number of interlayer links $N_2$. These sharp changes in the interlayer order parameter as a function of the number of interlayer links indicates, that under certain conditions the domains of coherence and incoherence overlap for both networks. The local order parameters in this case show that the coherent parts of the layers coincide, whereas the incoherent parts do not match perfectly resulting in an interlayer order parameter $Z < 1$. This phenomenon has been shown explicitly for the case $T_3 = 5$ and $N = 70$. The state of full synchrony, could not be established yet for a number of interlayer conditions $N_2 < N$ using time-varying interlayer connection switching. Further statistical analyses have to be conducted to finalize this work. The partial synchronization of the interlayer networks for the coherent domains indicates a collective behavior of the nodes in the coherent domain, whereas the incoherent domains seem not to synchronize perfectly. Further studies are needed to explain this behavior. Of course, also in this case we can observe that switching leads to a shrinking basin of attraction of the desynchronized states without vanishing up to our current knowledge.

ACKNOWLEDGMENTS

DATA AVAILABILITY

The data that support the findings of this study are available from the authors upon reasonable request.

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