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Wess-Zumino term in tachyon effective action

Kazumi Okuyama

Enrico Fermi Institute, University of Chicago
5640 S. Ellis Ave., Chicago IL 60637, USA
E-mail: kazumi@theory.uchicago.edu

ABSTRACT: We show that the source of RR field computed from the boundary state describing the decay of a non-BPS brane is reproduced by a particular form of the Wess-Zumino term in the tachyon effective action. We also obtain a simple expression of the S-charge associated with rolling tachyons.

KEYWORDS: Tachyon Condensation, D-branes
1. Introduction

Decay of an unstable brane is a very interesting process which might help us to understand some properties of string theory in time-dependent backgrounds [1]–[15]. The dynamics of tachyon field $T$ on such a brane can be described by an effective field theory when the value of $T$ and its derivatives satisfy some conditions [15]. The proposed effective action of real tachyon $T$ on non-BPS $D_p$-brane in type-II string theory is given by [16, 17, 18, 19, 20, 4]

$$S_{\text{DBI}} = \int d^{p+1}\!x L = -M_p \int d^{p+1}\!x V(T) \sqrt{1 + \partial_\mu T \partial^\mu T}$$ (1.1)

where $M_p$ is the tension of the non-BPS brane. In this paper we use the unit $\alpha' = 1$. In (1.1), we suppressed the dependence on the gauge field and the scalar fields other than the tachyon. It was pointed out in [15] that if we take the potential $V(T)$ to be

$$V(T) = \frac{1}{\cosh T},$$ (1.2)

then the stress tensor of rolling tachyon computed from the boundary state is correctly reproduced by the effective action (1.1). This potential (1.2) was also discussed recently in [21, 22].

The tachyon effective action has another important term, i.e. the Wess-Zumino term (or the Chern-Simons term) describing the coupling of open string tachyon to the bulk RR field:

$$S_{\text{WZ}} = \int W(T) dT \wedge C_{\text{RR}}$$ (1.3)

where $W(T)$ is an even function of $T$ which vanishes as $T \to \pm \infty$. In order for a kink of $T$ connecting the two vacua at $T = \pm \infty$ to carry the correct RR charge and the tension of BPS $D(p-1)$ brane, $W(T)$ and $V(T)$ should satisfy the condition [22,1]

$$\int_{-\infty}^{\infty} W(T) dT = \frac{M_p}{M_{p-1}^{\text{BPS}}} \int_{-\infty}^{\infty} V(T) dT = 1.$$ (1.4)

\footnote{Our definition of $W(T)$ is different from that in [22] by the factor of $M_{p-1}^{\text{BPS}}$.}
Here $M_{p-1}^{BPS}$ is the tension of BPS $D(p-1)$-brane whose ratio to the tension $M_p$ of non-BPS $Dp$-brane is given by [2] 
\[
\frac{M_p}{M_{p-1}^{BPS}} = \frac{1}{\sqrt{2\pi}}.
\] (1.5)
Note that the last equality in (1.4) is satisfied for $V(T) = \frac{1}{\cosh\left(\frac{T}{\sqrt{2}}\right)}$.

In this short note, we will show that the coupling of the open string tachyon to the RR field computed from the boundary state is reproduced by setting 
\[
W(T) = \frac{1}{\sqrt{2\pi}} V(T) = \frac{1}{\sqrt{2\pi}} \cosh\left(\frac{T}{\sqrt{2}}\right).
\] (1.6)
This $W(T)$ obviously satisfies the condition (1.4).

In section 2, we consider some properties of the equation of motion obtained from the action (1.1) with $V(T)$ given by (1.2). In section 3, we show that the source of RR field computed from the boundary state is reproduced by (1.6). We also comment on the S-charge associated with the rolling tachyon. We conclude with a few comments in section 4.

2. Some properties of $S_{DBI}$

To see the dynamics of tachyon field described by the action (1.1) with (1.2), it is convenient to perform a field redefinition from $T$ to $T$ as [2] 
\[
\sinh\frac{T}{\sqrt{2}} = \frac{T}{\sqrt{2}}.
\] (2.1)

In terms of this variable $T$, the lagrangian becomes 
\[
\frac{\mathcal{L}}{M_p} = -\frac{1}{1 + \frac{T^2}{2}} \sqrt{1 + \frac{T^2}{2} + \partial_\mu T \partial^\mu T},
\] (2.2)
and the equation of motion derived from this lagrangian is 
\[
\left(\partial_\mu \partial^\mu T + \frac{1}{2} T^2\right)
\left(1 + \frac{T^2}{2}\right) + \partial_\mu T \partial^\mu T \partial_\nu T - \partial_\mu T \partial^\nu T \partial_\nu T = 0.
\] (2.3)

When the tachyon $T$ depends only on time $t$, the second and the third terms in (2.3) cancel, and the equation of motion reduces to 
\[
-\partial_t^2 T + \frac{1}{2} T = 0.
\] (2.4)

Therefore, the general solution of homogeneous tachyon is given by
\[
T = T_+ e^{\frac{\sqrt{2}}{2} t} + T_- e^{-\frac{\sqrt{2}}{2} t},
\] (2.5)
with some constant coefficient $T_\pm$.

\footnote{From the late time behavior of the source for the RR field in the boundary state, Sen showed that $W(T)$ goes like $e^{-\frac{\sqrt{2}}{2} T}$ for large $T$ [1], [7] has this property.}

\footnote{The converse statement was proved in [2]. They showed that the effective action is fixed to have the form (1.2) by requiring: (1) $T = e^{\frac{\sqrt{2}}{2} t}$ is a solution of the equation of motion at each order in the power series expansion in $T$. (2) The on-shell lagrangian agrees with the disk partition function.}
An interesting fact is that the form of stress tensor as a function of time is reproduced from this effective action \([15]\). For instance, for the tachyon configuration describing the “full brane”

\[
\frac{T}{\sqrt{2}} = \cosh \frac{t}{\sqrt{2}},
\]

the on-shell lagrangian is evaluated as

\[
\frac{\mathcal{L}}{M_p} = \frac{1}{1 + e^{2(t-t_0)}} + \frac{1}{1 + e^{2(t+t_0)}} - 1.
\]

This expression agrees with the disk partition function \(Z(x^0)\) in the presence of boundary tachyon operator \(T_{WS} = \lambda \cosh(X^0/\sqrt{2})\), if we identify the coupling \(\lambda\) and the parameter \(t_0\) in (2.6) as

\[
\sin \pi \lambda = e^{-\frac{1}{\sqrt{2}}t_0}.
\]

In general, the worldsheet tachyon operator \(T_{WS}\) in the \((-1)\) picture is different from the tachyon field \(T\) in the effective action \((2.2)\). In particular, the periodicity of \(\lambda\) cannot be reproduced by any first derivative effective actions \([15]\). Moreover, the relative coefficient of \(T_{00}\) and \(T_{ij}\) computed from \((1.1)\) does not agree with the boundary state result for the full brane. As pointed out in \([8]\), if a tachyon configuration has a turning point \(\partial_0 T = 0\), the effective action \((1.1)\) cannot reproduce the stress tensor computed from the boundary state.

However, after taking the “half S-brane” limit,

\[
t \to t + a, \quad t_0 \to t_0 + a, \quad a \to \infty \quad \text{with} \quad t, t_0 \quad \text{fixed},
\]

we can actually identify \(T_{WS}\) with \(T\). In this limit, the tachyon configuration and the on-shell lagrangian become

\[
\frac{T}{\sqrt{2}} = e^{\frac{1}{\sqrt{2}}(t-t_0)}, \quad -\frac{\mathcal{L}}{M_p} = \frac{1}{1 + e^{2(t-t_0)}},
\]

and the stress tensor agrees with the worldsheet computation for the half S-brane with \(T_{WS} = \lambda e^{X^0/\sqrt{2}}\) \(([3], [4])\). It was argued in \([22]\) that the action \((1.1)\) is reliable only for the fluctuation of tachyon field around a single exponential \(e^{\pm t/\sqrt{2}}\) (i.e. \(T_+ = 0\) or \(T_- = 0\) in \((2.5)\)).

Next let us briefly discuss the inhomogeneous tachyon configuration. It is easy to see that the configuration with a single exponential

\[
T = e^{ik_\mu X^\mu}
\]

solves the equation of motion \((2.3)\) provided \(k_\mu\) is on-shell:

\[
k_\mu k^\mu = \frac{1}{2}.
\]

However, the sum of on-shell exponentials, e.g.

\[
T = \frac{1}{2} e^{\frac{1}{\sqrt{2}}(t+ix)} + \frac{1}{2} e^{\frac{1}{\sqrt{2}}(t-ix)} = e^{\frac{1}{\sqrt{2}}t} \cos \left( \frac{1}{2} x \right)
\]

(2.13)
is not a solution of (2.3) because of the non-linearity of the equation of motion. It is interesting that this effective action (2.2) knows that the boundary interaction $e^{X_0/2} \cos(X/2)$ is not exactly marginal in the superstring case [5], although the single exponential $e^{(X_0 \pm iX)/2}$ is exactly marginal. It seems difficult to find an exact inhomogeneous solution of (2.2). Instead, we can solve (2.2) numerically, or solve it perturbatively along the line of [5]. It was observed that, during the inhomogeneous decay of an unstable brane, defect branes are formed at a finite time [3, 9] and the slope of the kink becomes infinitely steep [26]. It would be interesting to see whether the tension of the defect brane calculated by this effective action agrees with the tension of the BPS brane.

3. Wess-Zumino term and S-charge

In this section we will show that the source of RR field read off from the boundary state is reproduced by the Wess-Zumino term (1.3) with the choice

$$W(T) = \frac{1}{\sqrt{2\pi}} V(T).$$

The source of RR field generated by the rolling tachyon $T_{WS} = \lambda \cosh X_0 / \sqrt{2}$ is [4]

$$j(t) = \frac{\sin \pi \lambda}{\sqrt{2\pi}} \left[ \frac{e^{\sqrt{2}t}}{1 + \sin^2 \pi \lambda e^{\sqrt{2}t}} - \frac{e^{-\sqrt{2}t}}{1 + \sin^2 \pi \lambda e^{-\sqrt{2}t}} \right].$$

This source and the bulk RR $p$-form field couple via the Wess-Zumino term on the world-volume of non-BPS $Dp$-brane

$$S_{WZ} = \int_{R^{1,p}} J \wedge C_{RR} = \int dt d^p \xi j(t) C_{1,...,p}$$

where $J = j(t) dt$.

Using the parameter $t_0$ (2.8), the source $j(t)$ is rewritten as

$$j(t) = \frac{1}{\sqrt{2\pi}} \left[ \frac{e^{\sqrt{2}t - t_0}}{1 + e^{\sqrt{2}(t-t_0)}} - \frac{e^{-\sqrt{2}t - t_0}}{1 + e^{-\sqrt{2}(t-t_0)}} \right] = \frac{1}{\sqrt{2\pi}} \sinh \frac{t_0}{\sqrt{2}} \frac{\sinh \frac{t}{\sqrt{2}}}{\cosh^2 \frac{t}{\sqrt{2}} + \sinh^2 \frac{t_0}{\sqrt{2}}}.$$

Note that the normalization of $j(t)$ was determined by requiring that the S-brane with $T_{WS} = \lambda \sinh X_0 / \sqrt{2}$ should carry $\pm 1$ of unit RR charge [1]. $j(t)$ has poles at $t = \pm t_0 + \pi i (2n+1)/\sqrt{2}$ and the residues are

$$2\pi i \text{Res } j(t) \bigg|_{t = \pm t_0 + \frac{\pi i (2n+1)}{\sqrt{2}}} = \pm (-1)^n \quad (n \in \mathbb{Z}).$$

The alternating sign of residues corresponds to the fact that, when $t_0 = 0$ (or $\lambda = \frac{1}{2}$), the Wick rotated configuration describes an array of branes and anti-branes.

From (3.3), (2.6) and (2.1), one can easily see that the RR source $J$ is written as

$$J = \frac{1}{\sqrt{2\pi}} \frac{dT}{1 + \frac{1}{2} T^2} = \frac{1}{\sqrt{2\pi}} V(T) dT.$$
This relation holds also for the half S-brane. (The RR source of the half S-brane is obtained by taking the limit \((2.9)\) in \((3.1)\).) Therefore, we conclude that the function \(W(T)\) is given by \(\frac{1}{p^2} \approx V(T)\), at least for the on-shell tachyon. Strictly speaking, the effective action \(S_{DBI}(1.1)\) is reliable only for the half S-branes. However, as we saw above, the disk partition function \(Z = \langle B|0_{NS}\rangle\) and the RR source \(j = \langle B|C_{RR}\rangle\) are correctly reproduced by the effective action \((1.1), (1.3)\) even for the full brane. It would be interesting to understand the physical meaning of this fact.

It is useful to rewrite \(J\) as

\[
J = \frac{1}{\pi} \tan^{-1}\left(\frac{T}{\sqrt{2}}\right). \tag{3.6}
\]

From this expression, it is obvious that the integral of \(J\) is 1 when integrated over the kink configuration of \(T\) connecting the two vacua \(T = \pm \infty\). Therefore, \(J = W(T)dT\) satisfies the condition \((1.4)\).

The equation of motion of the bulk RR \(p\)-form field with the source term \((3.2)\) is given by

\[
(-\partial^2_t + \partial^2_{\mathbf{x}}) C_{1,\ldots,p} = -j(t) \delta^{d-p}(\mathbf{x}), \tag{3.7}
\]

where \(\mathbf{x}\) is the transverse coordinates of the non-BPS \(Dp\)-brane. The S-charge carried by the rolling tachyon is defined by \([1]\)

\[
Q_s(t) = \int d^{d-p}x F_{0,\ldots,p} = \int d^{d-p}x \partial_t C_{1,\ldots,p}. \tag{3.8}
\]

From this definition of S-charge, one can show that \(Q_s(t)\) is related to \(j(t)\) in a simple way:

\[
\frac{d}{dt} Q_s(t) = j(t). \tag{3.9}
\]

Using \((3.6)\), we can integrate the relation \((3.9)\) to obtain

\[
Q_s(t) = \frac{1}{\pi} \tan^{-1}\left(\frac{T(t)}{\sqrt{2}}\right), \tag{3.10}
\]

up to a constant shift.

We can check the general formula \((3.9)\) by the explicit computation as follows. For the full brane, the RR \(p\)-form generated by the source \((3.3)\) is

\[
C_{1,\ldots,p} = \frac{1}{\sqrt{2\pi} 2^{\nu} \Gamma(\nu + 1) \text{vol}(S^{8-p})} \sum_{n=0}^{\infty} (-1)^n \left[ e^{\frac{2n+1}{\sqrt{2}}(t-t_0)} - e^{-\frac{2n+1}{\sqrt{2}}(t+t_0)} \right] \times
\]

\[
\times \left(\frac{2n+1}{\sqrt{2}r}\right)^\nu K_{\nu} \left(\frac{2n+1}{\sqrt{2}r}\right), \tag{3.11}
\]

where \(\nu = \frac{d-p}{2} - 1\), \(\text{vol}(S^{d-1}) = \frac{2\pi^d/2}{\Gamma(d/2)}\), and \(K_{\nu}(z)\) is the modified Bessel function. \(r = \sqrt{\mathbf{x}^2}\) is the radial coordinate of the transverse directions. Plugging this expression into the definition \((3.8)\) of \(Q_s(t)\), the S-charge of the full brane is obtained as

\[
Q_{\text{full}}(t) = Q_+(t) + Q_-(t) \tag{3.12}
\]
where $Q_{\pm}(t)$ are the S-charge of the half S-branes:

$$Q_+(t) = \frac{1}{\pi} \tan^{-1} \left( e^{\frac{1}{\sqrt{2}}(t-t_0)} \right), \quad Q_-(t) = \frac{1}{\pi} \tan^{-1} \left( e^{-\frac{1}{\sqrt{2}}(t+t_0)} \right).$$

(3.13)

It is easy to see that this expression satisfies (3.9). As a function of time, $Q_{\text{full}}(t)$ has a kink at $t = t_0$ and an anti-kink at $t = -t_0$, and approaches $\frac{1}{2}$ at large $|t|$. (More precisely, $Q_{\text{full}}(t)$ has a “half” (anti)kink at $t = \pm t_0$. See the discussion below.) Note that the S-charge is independent of time when $\lambda = \frac{1}{2}$, or $t_0 = 0$ [11]

$$Q_{\text{full}}(t) \bigg|_{t_0=0} = \frac{1}{2}.$$  

(3.14)

This is consistent with the relation (3.9) since there is no source of RR field when $\lambda = \frac{1}{2}$.

The S-charge $Q_s(t)$ at fixed $t$ is not a gauge invariant quantity. The physical S-charge is the difference of $Q_s(t)$ in the far future and the far past:

$$\Delta Q_s = Q_s(t = +\infty) - Q_s(t = -\infty).$$  

(3.15)

This is the definition of the integral of RR flux over the “transverse sphere” surrounding S-brane in lorentzian space [1]. For the half S-brane and the full brane, their physical S-charges are

$$\Delta Q_{\pm} = \pm \frac{1}{2}, \quad \Delta Q_{\text{full}} = 0.$$  

(3.16)

The S-charge $\Delta Q_{\pm} = \pm \frac{1}{2}$ of half S-brane reflects the fact that tachyon field describing the half S-brane runs from $T = 0$ to $T = \infty$ which is the half way between the two vacua $T = \pm \infty$.

The tachyon configuration of the form $\sinh t/\sqrt{2}$, which connects the two vacua $T = \pm \infty$, can be obtained from the $\cosh t/\sqrt{2}$ configuration (2.6) by the shift $t \rightarrow t + \pi i/\sqrt{2}$, $t_0 \rightarrow t_0 + \pi i/\sqrt{2}$ [3]. Under this shift of $t$ and $t_0$, $T$ in (2.6) becomes

$$T = \frac{\sinh \frac{t}{\sqrt{2}}}{\cosh \frac{t_0}{\sqrt{2}}}. $$  

(3.17)

The on-shell lagrangian (or the disk partition function) of this configuration is the same as the cosh case (2.7), but the S-charge becomes

$$Q_{\sinh}(t) = Q_+(t) - Q_-(t),$$  

(3.18)

and it satisfies $\Delta Q_{\sinh} = 1$ (we normalized $j(t)$ by requiring this relation [11]).

4. Discussion

In this paper, we showed that the source $j(t)$ of RR field computed from the boundary state is reproduced by the Wess-Zumino term of the form (1.6). We should comment on the relation between our $W(T)$ and the Wess-Zumino term obtained in BSFT [27, 28]. The function $W(T)$ in (4.4) is quite different from the BSFT result $W(T) \sim e^{-T^2/4}$. As
emphasized in [22], the two actions have different regimes of validity, i.e. our $W(T)$ is valid for nearly on-shell configurations while BSFT is valid far off mass shell. However, in view of the topological nature of WZ term in BSFT, namely the Chern character of superconnection, we expect our $W(T)$ is related to that in BSFT by a nontrivial field definition.

We have two pictures suggesting there are some branes at $t = \pm t_0$ (or its shift in the imaginary direction):

1. From the form of $Q_{\text{full}}(t)$, when $t_0$ is large the full brane can be well approximated by the following configuration: The unstable $Dp$-brane exists only in the region $|t| < t_0$ and ends on the half $Sp$-branes at $t = \pm t_0$, and there is no brane before $-t_0$ and after $t_0$. It was discussed in [29] that the non-BPS branes in euclidean space can end on “half D-branes” which carry the half unit of RR charge. This is based on the fact that the unstable branes can be regarded as sphalerons in string theory [30].

2. In [14, 11], it was observed that when $t_0 = 0$ the closed string state produced by the unstable $Dp$-brane is closely related to the array of BPS $D(p - 1)$-branes at $t = \pi i(2n + 1)/\sqrt{2}$. This comes from the fact that the disk partition function $Z(t)$ has poles at $t = \pi i(2n + 1)/\sqrt{2}$. When we turn on $t_0$, the poles of $Z(t)$ and $j(t)$ are shifted to $t = \pm t_0 + \pi i(2n + 1)/\sqrt{2}$. The closed string emission rate from the full brane is related to the cylinder amplitude between (anti)D-branes sitting at $t = \pm t_0 + \pi i(2n + 1)/\sqrt{2}$ [15].

It would be interesting to study the relation between these two pictures (see [11] for a discussion on the relation between S-branes and D-instantons).

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References

[1] M. Gutperle and A. Strominger, Spacelike branes, J. High Energy Phys. 04 (2002) 018 hep-th/0202210.

[2] A. Sen, Rolling tachyon, J. High Energy Phys. 04 (2002) 048 hep-th/0203211.

[3] A. Sen, Tachyon matter, J. High Energy Phys. 07 (2002) 065 hep-th/0203265.

[4] A. Sen, Field theory of tachyon matter, Mod. Phys. Lett. A 17 (2002) 1797 hep-th/0204143.

[5] A. Sen, Time evolution in open string theory, J. High Energy Phys. 10 (2002) 003 hep-th/0207105.

[6] A. Sen, Time and tachyon, hep-th/0209122.

[7] P. Mukhopadhyay and A. Sen, Decay of unstable D-branes with electric field, J. High Energy Phys. 11 (2002) 047 hep-th/0208142.
[8] A. Strominger, Open string creation by S-branes, hep-th/0209090.
[9] F. Larsen, A. Naqvi and S. Terashima, Rolling tachyons and decaying branes, J. High Energy Phys. 02 (2003) 039 hep-th/0212248.
[10] M. Gutperle and A. Strominger, Timelike boundary Liouville theory, hep-th/0301038.
[11] A. Maloney, A. Strominger and X. Yin, S-brane thermodynamics, hep-th/0302146.
[12] T. Okuda and S. Sugimoto, Coupling of rolling tachyon to closed strings, Nucl. Phys. B 647 (2002) 101 hep-th/0208196.
[13] S.-J. Rey and S. Sugimoto, Rolling tachyon with electric and magnetic fields: T-duality approach, Phys. Rev. D 67 (2003) 086008 hep-th/0301049.
[14] S.-J. Rey and S. Sugimoto, Rolling of modulated tachyon with gauge flux and emergent fundamental string, hep-th/0303133.
[15] N. Lambert, H. Liu and J. Maldacena, Closed strings from decaying D-branes, hep-th/0303139.
[16] A. Sen, Supersymmetric world-volume action for non-BPS D-branes, J. High Energy Phys. 10 (1999) 008 hep-th/9909062.
[17] M.R. Garousi, Tachyon couplings on non-BPS D-branes and dirac-Born-Infeld action, Nucl. Phys. B 584 (2000) 281 hep-th/0003122.
[18] E.A. Bergshoeff, M. de Roo, T.C. de Wit, E. Eyras and S. Panda, T-duality and actions for non-BPS D-branes, J. High Energy Phys. 05 (2000) 003 hep-th/0003221.
[19] J. Kluson, Proposal for non-BPS D-brane action, Phys. Rev. D 62 (2000) 126003 hep-th/0004106.
[20] M.R. Garousi, On-shell S-matrix and tachyonic effective actions, Nucl. Phys. B 647 (2002) 117 hep-th/0209063.
[21] F. Leblond and A.W. Peet, SD-brane gravity fields and rolling tachyons, hep-th/0303036.
[22] D. Kutasov and V. Niarchos, Tachyon effective actions in open string theory, hep-th/0304045.
[23] A. Sen, Dirac-Born-Infeld action on the tachyon kink and vortex, hep-th/0303057.
[24] A. Sen, Non-BPS states and branes in string theory, hep-th/9904207.
[25] N.D. Lambert and I. Sachs, Tachyon dynamics and the effective action approximation, Phys. Rev. D 67 (2003) 026005 hep-th/0208217.
[26] J.M. Cline and H. Firouzjahi, Real-time D-brane condensation, hep-th/0301101.
[27] T. Takayanagi, S. Terashima and T. Uesugi, Brane-antibrane action from boundary string field theory, J. High Energy Phys. 03 (2001) 019 hep-th/0012210.
[28] P. Kraus and F. Larsen, Boundary string field theory of the dd-bar system, Phys. Rev. D 63 (2001) 106002 hep-th/0012198.
[29] N. Drukker, D.J. Gross and N. Itzhaki, Sphalerons, merons and unstable branes in AdS, Phys. Rev. D 62 (2000) 086007 hep-th/0004131.
[30] J.A. Harvey, P. Horava and P. Kraus, D-sphalerons and the topology of string configuration space, J. High Energy Phys. 03 (2000) 021 hep-th/0004143.