Comparison of two double-moment microphysics schemes in aspects of warm-rain droplet spectra and raindrop budget

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ABSTRACT
Simulation results of the WDM6 scheme and the Thompson scheme, both of which are commonly-used double-moment bulk microphysics schemes, are compared within the Weather Research and Forecasting model. The purpose of the comparison is to study the difference in the aspects of the warm-rain hydrometeor number concentrations, the droplet size distributions, and the budgets of the rain mixing ratio and number concentration. It is found that the WDM6 scheme overestimates the ratio and the amount of large precipitation, and underestimates those of small precipitation, compared to the Thompson scheme. The cloud number concentration (CNC) predicted in the WDM6 scheme is one to three orders of magnitude smaller than that of the Thompson scheme, which is set to the specific value of CNC. The cloud droplet spectra of the WDM6 scheme are broader. The WDM6 scheme produces a larger rain number concentration and smaller mass mean diameter of raindrops under the influence of both warm and cold rain processes—specifically, autoconversion and melting of snow and graupel. The WDM6 scheme produces a larger autoconversion rate and smaller total melting rate of snow and graupel than the Thompson scheme in the rain mixing ratio budget. The sign of the difference in the rain-cloud collection varies with region, and the rain-cloud collection process together with evaporation of rain have a major influence on the sign of the surface precipitation difference between the two schemes.

1. Introduction
Microphysics parameterization determines the evolution of hydrometeors, affects latent heat release, and interacts with dynamics. Thus, microphysics parameterization plays a central role in the prediction of the occurrence, development, and dissipation of weather systems. Double-moment schemes (Reisner, Rasmussen, and Bruintjes 1998; Coherent and Pinto 2000; Seifert and Beheng 2001; Morrison, Curry, and Khvorostyanov 2005; Thompson et al. 2008), which typically predict the number concentration in addition to the mixing ratio of hydrometeors, offer a more flexible treatment of droplet size distributions (DSDs), and outperform their single-moment counterparts (Morrison, Thompson, and Tatarskii 2009; Lim and Hong 2010; Jung, Xue, and Tong 2012). Double-moment schemes show advantage in the capability of investigating interactions of cloud and aerosols, e.g. in the Weather Research and Forecasting (WRF) model (Xie et al. 2013) and the Community Atmosphere model, version 5 (Xie et al. 2017).

Sensitivity tests of the simulation of microphysics schemes have been conducted for various weather systems. Different treatments of graupel and hail were found by Morrison and Milbrandt (2011) to be responsible for the differences in aspects of storm dynamics, reflectivity structure, surface precipitation, and cold pool between two microphysics schemes. Adams-Selin, van Den Heever, and Johnson (2013) tested the sensitivity of a bow-echo simulation to eight microphysics schemes in aspects of rainfall,
radar reflectivity, and hydrometeor mixing ratio, among others, with a focus on graupel parameter characteristics. The sensitivity of the simulation of tropical cyclone size to microphysics schemes was tested by Chan and Chan (2016). Simulations of different microphysics schemes were compared with remote-sensing observations with respect to the aspects of reflectivity, differential reflectivity, and microwave brightness temperature, among others, to analyze the characteristics of droplet spectra without explaining the cause of droplet spectra differences (Gao et al. 2011; Jankov et al. 2011; Molthan and Colle 2012; Johnson et al. 2016). However, few of these studies concentrated on droplet number concentrations, spectra and identifying microphysical causes of differences between microphysics schemes.

The objective of this paper is to explore the differences in droplet spectra and the microphysical causes of the differences between two commonly used double-moment bulk schemes in the WRF model (Skamarock and Klemp 2008) through a budget analysis.

### 2. Model configuration

During 19–21 July 2016, continuous heavy rain that mainly affected the provinces of Beijing, Tianjin, Hebei, Henan, Shandong, Shanxi, and Liaoning occurred in North China. The average accumulated precipitation from 1700 UTC 18 July to 0000 UTC 21 July in Beijing was 212.6 mm.

The WDM6 scheme (Lim and Hong 2010) and the Thompson scheme (Thompson et al. 2008) are used in this study. These schemes were chosen because they have been widely used and tested (e.g. Loughner et al. 2011; Bove et al. 2014; Clark et al. 2014; Varble et al. 2014), and the main point of interest of the study is their different treatment of cloud number concentration (CNC). Both schemes predict the mixing ratio of cloud, rain, ice, snow, graupel, and the number concentration of rain. Additionally, the WDM6 scheme predicts the number concentration of cloud condensation nuclei (CCN) and clouds, while the Thompson scheme predicts the number concentration of ice and sets the CNC as a constant. The gamma distribution,

\[ n(D) = N_0 D^\alpha \exp(-\lambda D), \quad (1) \]

and the four-parameter normalized gamma function,

\[ n(D) = N_0 \Gamma(\alpha) D^{\alpha - 1} \exp(-\lambda D) \Gamma(\alpha), \]

are adopted for rain DSD in the Thompson scheme and the WDM6 scheme, respectively. Here, \( \lambda \) is the slope parameter, whereas \( \alpha \) and \( \nu \) are the dispersion parameters. Besides, \( N, N_0, D, H, \) and \( D \) represent the droplet number concentration, intercept parameter, and diameter of droplets, respectively. Additional details of parameter definitions of the schemes are presented in Tables 1 and 2. The simulations performed using the WRF model (version 3.6) cover the 18-h period from 0000 to 1800 UTC 20 July, during which time precipitation was mainly located in Beijing. The model setup is shown in Table 3.

### 3. Model simulation results

#### 3.1 Precipitation

The simulated 0000–1800 UTC accumulated precipitation is presented in Figure 1(a,b). Generally, simulation using either scheme agrees with the observations (Figure 1(d)) in location. Both simulations have a rainfall center of over 300 mm in central Beijing and the simulated accumulated precipitation of both schemes is larger than the observations. The

### Table 1. Parameter definitions in the Thompson microphysics scheme. The fall speed relation for the Thompson scheme is \( V(D) = \gamma D^2 \exp(-\beta D) \), where \( \gamma = (\rho_0/\rho_m)^{0.5} \) and \( \rho_0 \) is the standard air density at sea level. The mass relation is \( m = a D^b \).

| Species       | \( N_0 \) | \( a \) | \( b \) | \( c \) | \( d \) |
|--------------|---------|--------|--------|--------|--------|
| Cloud        | 3 \times 10^3 | 5 | 1000 | \( \pi \rho_\nu/6 \) | 3 | 0.316946 \times 10^3 | 0 | 0 |
| Rain         | Prognostic | 0 | 1000 | \( \pi \rho_\nu/6 \) | 3 | 4854 | 1 | 195 |
| Ice          | Prognostic | 0.6357 | 890 | \( \pi \rho/6 \) | 2 | 1847.5 | 1 | 0 |
| Graupel      | \( 10^{-5} - 3 \times 10^{-6} \) | 0 | 500 | \( \pi \rho_\nu/6 \) | 3 | 442 | 0.89 | 0 |
| Snow         | Temperature dependent | 0 | 100 | \( \pi w/6 \) | 0.069 | 2 | 40 | 0.55 | 100 |

### Table 2. Parameter definitions in the WDM6 microphysics scheme. The fall speed relation for the WDM6 scheme is \( V(D) = \gamma D^2 \). The particle size distribution of the WDM6 scheme is \( n(D) = N \frac{\alpha}{\Gamma(\alpha)} D^{\alpha - 1} \exp[-\lambda D] \). The mass relation is \( m = a D^b \).

| Species       | \( N_0 \) | \( a \) | \( v \) | \( p \) | \( a \) | \( b \) | \( c \) | \( d \) |
|--------------|---------|--------|--------|--------|--------|--------|--------|--------|
| Cloud        | Prognostic | 3 | 1 | 1000 | \( \pi \rho_\nu/6 \) | 3 | 841.9 | 0.8 |
| Rain         | Prognostic | 1 | 2 | 1000 | \( \pi \rho_\nu/6 \) | 3 | 1.49 \times 10^4 | 1.31 |
| Ice          | - | - | - | - | 0.2899 | 2 | 1.49 \times 10^4 | 1.31 |
| Graupel      | \( 4 \times 10^{-6} \) | 1 | 1 | 500 | \( \pi \rho_\nu/6 \) | 3 | 330 | 0.8 |
| Snow         | Temperature dependent | 1 | 1 | 100 | \( \pi \rho/6 \) | 3 | 11.72 | 0.41 |

### Table 3. Model configuration.

| Outer domain | Inner domain |
|--------------|--------------|
| Grids        | 151 \times 130, 9 km, 40 layers | 196 \times 199, 3 km, 40 layers |
| Cumulus      | Kain–Fritsch | None |
| Radiation    | RRTM longwave, Dudhia shortwave |
| Land surface | NOAH-MP |
| Surface layer| Revised MM5 |
| PBL          | YSU |
| Microphysics | WDM6 or Thompson |

\[ n(D) = N \frac{\alpha}{\Gamma(\alpha)} D^{\alpha - 1} \exp[-\lambda D] \Gamma(\alpha), \quad (2) \]
probability distribution of hourly precipitation of the Thompson scheme is closer to the observation than that of the WDM6 scheme. The WDM6 scheme underestimates the ratio of precipitation smaller than 5 mm h$^{-1}$ and overestimates the ratio of larger precipitation at the same time (Figure 1(f)). As shown in Table 4, the average precipitation...
is larger when the WDM6 scheme produces more precipitation. Meanwhile, the WDM6 scheme overestimates larger precipitation and underestimates smaller precipitation compared to the observation. Two typical domains (marked as domain A and B in Figure 1(c)) around Beijing are chosen to explore the differences and the corresponding microphysical causes in hydrometeor characteristics between the two schemes. The simulated radar reflectivity of both schemes compares well with the observation in domain A. However, the WDM6 scheme shows a delay of precipitation in domain B, as presented in Figure 2. In domain A, the WDM6 scheme generally produces more precipitation than the Thompson scheme, while the opposite happens in domain B. Domain A is mainly dominated by updrafts, with a maximum average vertical velocity exceeding 0.5 m s⁻¹. Downdrafts are dominant at a lower level in domain B in both schemes, as shown in Figure 3.

### 3.2 Hydrometeors

Time–height series of the warm-rain hydrometeor number concentrations of the two schemes are presented in Figure (4a–f). The treatment of CNC is quite different between the two schemes. In the Thompson scheme, the CNC is set at a constant of $3 \times 10^8$ m⁻³,

| Region          | WDM6 | Thompson | Observation |
|-----------------|------|----------|-------------|
| WDM6 > Thompson| 64.83| 51.35    | 56.37       |
| WDM6 ≤ Thompson| 29.61| 37.66    | 37.86       |

Figure 2. Time–height series of median observed Beijing S-band radar (39.81°N, 116.47°E) and simulated radar reflectivity (units: dBZ): (a) observations in domain A; (b) observations in domain B; (c) WDM6 scheme simulation in domain A; (d) WDM6 scheme simulation in domain B; (e) Thompson scheme simulation in domain A; (f) Thompson scheme simulation in domain B.
as recommended for a continental system. However, in the WDM6 scheme, the CNC is predicted with an initial CCN number concentration (CNNC) of $10^{9}$ m$^{-3}$ (Yum and Hudson 2002). We also tested the WDM6 scheme with an initial CNNC of $10^{10}$ m$^{-3}$, since the CNNC is about $10^{9}$–$10^{10}$ m$^{-3}$ at a supersaturation of 0.4%–0.6% according to observations in North China (Shi and Duan 2007; Deng et al. 2011; Li et al. 2015). The results, which are not presented here, showed that the simulation is more sensitive in the choice of microphysics scheme than the initial CNNC. The CNC in the WDM6 scheme is within the range of $10^{5}$–$10^{7}$ m$^{-3}$, which is one to three orders of magnitude smaller than the typical CNC magnitude of $10^{8}$ m$^{-3}$ (Huang and Xu 1999), except during 0000–0100 UTC. The rain number concentration (RNC) in the WDM6 scheme, with two maxima at 2 km and 5 km, is over 10 times higher than that in the Thompson scheme.

There is also a notable difference in the warm-rain DSD between the two schemes. The DSDs are explicitly predicted in bin schemes, whereas they are calculated by the number concentration, mixing ratio, and the predefined DSD forms in bulk schemes. The warm-rain DSDs of 0300 UTC and 0600 UTC at 2 km and 5 km are taken as examples and shown in Figure 5. It is found that the WDM6 scheme produces wider cloud spectra, with more cloud droplets of 30 μm in diameter or larger.

Meanwhile, the raindrop spectra are narrower in the WDM6 scheme, and numerous small raindrops with diameters of approximately 0.1 mm are responsible for the larger RNC. It has also been found, in other research, that the WDM6 scheme tends to produce a large RNC and a high proportion of small raindrops (Gao et al. 2011; Adams-Selin, van Den Heever, and Johnson 2013; Morrison et al. 2015; Johnson et al. 2016). The mean mass diameter of raindrops ($D_{mr}$) influences the terminal velocity, radar reflectivity, and microphysical processes of evaporation, collection, etc. Time–height series of the median $D_{mr}$ are presented in Figure 4(g–j). As can be seen, $D_{mr}$ increases while descending to the ground in both schemes during most simulation periods, which demonstrates the ability of reflecting ‘size-sorting’ in double-moment microphysics schemes. Generally, the $D_{mr}$ of the WDM6 scheme is smaller and varies more widely with altitude.

### 3.3 Budget analysis of raindrops

Vertical profiles of horizontally averaged and temporally averaged budget terms of RNC and rain mixing ratio between 0100 and 1400 UTC in domain A and B are presented in Figure 6. The period 0100–1400 UTC is chosen because the rainfall rate is quite different in the two schemes in both domains during this period.
There is a more significant difference in the budget terms of RNC than that of rain mixing ratio between the two schemes. The main sources of RNC include autoconversion (Autoconversion in Figure 6) and melting of snow and graupel (Melting-snow and Melting-graup in Figure 6; hereafter, Melt_sg), and all of these terms are larger in the WDM6 scheme. Autoconversion is the process of
collision and coalescence of cloud droplets to form raindrops. The autoconversion rate of the RNC in the WDM6 scheme is one to three orders of magnitude larger than in the Thompson scheme. The parameterization of autoconversion, which is positively associated with the power of cloud droplet diameter, follows Cohard and Pinty (2000) in both MP schemes. Thus, the larger autoconversion rate is a result of the wider cloud spectra produced by the WDM6 scheme. Melt_sg is responsible for the maximum RNC at approximately 5 km (Figure 4(c, d)) in the WDM6 scheme. The rain mixing ratio produced by Melt_sg is similar in magnitude, and thus the difference in the RNC source of Melt_sg is caused by the different treatment of snow and graupel number concentrations between the two schemes. Self-collection/breakup (Self-Collection in Figure 6), which is the process of raindrops collecting other raindrops or breaking up after collision, is the main sink of the RNC and forms a negative feedback effect on the larger RNC.

The rain mixing ratio is mainly increased by rain-collecting cloud droplets (Accretion-Cloud in Figure 6) and Melt_sg, and decreased by falling to the ground (Sedimentation in Figure 6). In domain A, rain-cloud collection is comparable with Melt_sg, while Melt_sg is the dominant source of the rain mixing ratio in domain B in both schemes. Evaporation of raindrops is vigorous in domain B, especially in the WDM6 scheme. Autoconversion is an important source of the RNC, while it contributes little to the rain mixing ratio, indicating that the raindrop size produced by autoconversion is much smaller than that produced by Melt_sg. The term Accretion-cmix in the WDM6 scheme represents the process of clouds collected by snow and graupel in a warm area converting to rain, which could be considered another form of Melt_sg.

Figure 5. Average droplet spectra of warm-rain hydrometeors of 0300 UTC and 0600 UTC at 2 km and 5 km, with the results of the WDM6 scheme and the Thompson scheme presented by the solid line and dashed line, respectively: (a) cloud spectrum in domain A; (b) cloud spectrum in domain B; (c) rain spectrum in domain A; (d) rain spectrum in domain B.
Figure 6. Average profiles of the budget terms of the (a–d) RNC (units: m$^{-3}$ s$^{-1}$) and (e–j) rain mixing ratio (units: 10$^{-6}$ kg kg$^{-1}$ s$^{-1}$). The first and third rows represent the results in domain A. The second and fourth rows represent the results in domain B. The first, second, and third columns represent the results of the WDM6 scheme, the Thompson scheme, and the difference between them (WDM6 minus Thompson), respectively.
The differences in the main budget terms of the rain mixing ratio between the two schemes (WDM6 minus Thompson) are presented in Figure 6(g,j). The melting of snow and graupel are treated as one term because they are similar, and the Accretion-c&mix term of the WDM6 scheme is also regarded as the melting of snow and graupel. In the Thompson scheme, Melt_sg is larger in the rain mixing ratio budget, while it is smaller in RNC budget in both domain A and B, indicating the $D_{mr}$ of raindrops produced by Melt_sg is larger in the Thompson scheme than that of the WDM6 scheme. As discussed previously, the autoconversion process produces a large number of small raindrops. Vigorous autoconversion in the WDM6 scheme is another reason for the smaller $D_{mr}$. Rain-cloud collection does not change the RNC, which leads to an increase in the $D_{mr}$. Rain-cloud collection is found to be larger in the WDM6 scheme in domain A, making $D_{mr}$ in the WDM6 scheme comparable with that in the Thompson scheme. In domain B, evaporation is larger due to the larger total surface area of numerous smaller raindrops in the WDM6 scheme. Besides, rain-cloud collection is smaller, and thus the $D_{mr}$ is much smaller in the WDM6 scheme. As a result, precipitation is smaller in the WDM6 scheme in domain B. The difference in rain-cloud collection could be an explanation of the overestimation of large precipitation and underestimation of small precipitation.

4. Summary

In this study, the WDM6 scheme and the Thompson scheme, both of which are double-moment schemes for raindrops, are compared in a simulation of an unusually heavy rainfall event in North China on 20 July 2016. Although the simulation basically agrees with observations in terms of rainfall location and accumulated precipitation when either of the two schemes is used, there are notable differences in the aspects of surface precipitation, warm-rain DSDs and droplet number concentration, among others. The major conclusions can be summarized as follows:

The probability distribution of the hourly precipitation of the Thompson scheme is closer to the observations than that of the WDM6 scheme. The WDM6 scheme overestimates (underestimates) the ratio and amount of large (small) precipitation.

The CNC is predicted in the WDM6 scheme but is set as constant in the Thompson scheme. The WDM6 scheme produces a much smaller CNC and wider cloud droplet spectra compared to the Thompson scheme.

The WDM6 scheme produces over 10 times more raindrops than the Thompson scheme, as a result of both a larger rate of autoconversion and melting of snow and graupel. The raindrop spectra are narrower, and numerous raindrops with diameters of 0.1 mm are responsible for the larger RNC in the WDM6 scheme.

For the rain mixing ratio budget terms, the WDM6 scheme produces a larger autoconversion rate and smaller total melting rate of snow and graupel than the Thompson scheme. The rain-cloud collection rate is larger in the WDM6 scheme in the area dominated by updrafts. As a result, a large number of small raindrops develop by collecting cloud droplets, leading to more precipitation in the WDM6 scheme. Meanwhile, in the area where convection is not vigorous, the precipitation of the WDM6 scheme is less than that of the Thompson scheme. The reason could be the smaller rain-cloud collection term and the larger evaporation rate owing to the larger total surface area of numerous small raindrops.

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