Numerical Study on Dynamic Buckling of Composite Bar with One Edge Simply Supported and Opposite Edge Clamped

LIU Chao¹, YANG Qiang¹* and HAN Zhijun²

¹College of Mechanical and Vehicle Engineering of Taiyuan University of Technology, Taiyuan University of Technology, Taiyuan, Shanxi 030024, China
²College of Biomedical Engineering of Taiyuan University of Technology, Taiyuan University of Technology, Taiyuan, Shanxi 030024, China

* YANG Qiang’s e-mail: yangqiang62@126.com

Abstract. Considering the stress wave effect, the dynamic buckling problem of a composite rod with one side fixed on one side being axially impacted by a rigid mass is studied. Based on the Hamilton principle and the axial inertia and rotational inertia, the dynamic buckling control equation is derived. For the dimensionless, the central differential method is used to solve the governing equation. The buckling mode of the composite bar with different initial deflections under different impact mass, impact velocity, critical length and layup mode is obtained by B-R criterion. The results show that the impact velocity, impact mass, critical length and layup mode have an effect on the dynamic buckling of composite rods, and the initial deflection changes have a significant effect on the buckling mode.

1. Introduction

Composite bar has high value in engineering applications due to their high strength and high stiffness. The dynamic buckling problem of rigid mass block axial impact composite bar is widely used in engineering practice such as aviation and military. Bolotin V V [1] studied the dynamic buckling of composite plates with cracks and discussed the relationship between crack growth and buckling. Borchani W [2] used the analytical method to study the buckling problem of the double-sided elastic bar. Ghayesh M H [3] used the Galerkin method to study the nonlinear buckling and post-buckling problems of Timoshenko beams under axial loads. Mottram J T [4] compares the experimental value of the critical load when the I-beam is buckling with the theoretical value. Pouresmaeeli S [5] studied the buckling problem of functionally graded carbon nanobar and determined the critical buckling load by Galerkin method, and compared the results with the literature. Gunda J B [6] studied the buckling problem of composite materials by analytical method. In summary, the existing research on the buckling of composite bar with rigid masses is insufficient. Since the axial inertia is considered, there is no corresponding analytical solution, so the difference method becomes an important method to solve such problems.

In this paper, the buckling problem of a rigid mass with axially impacted composite bar with initial defects is studied. Considering the stress wave effect, axial inertia and rotational inertia, control equations of dynamic buckling are derived by Hamilton principle. The control equations are dimensionless which is solved by central difference method; the effects of stress wave effect, initial deflection, axial inertia and lay-up angles on dynamic buckling are discussed. The results show that the stress wave effect, initial deflection, axial inertia, and lay-up angles have significant effects.
2. The control equations of dynamic buckling

Figure (2-1) is the composite bar with the left end simply, the right end clamped, and the length L is struck by a mass of initial velocity \( V_0 \) and mass M. The stress wave propagates in the bar as shown in Figure (2-2).

When the stress wave propagates into the \( L_{cr} \) (critical length) within the bar, the buckling occurs suddenly and the bending of the bar conforms to the flat interface assumption. The bar moves only in the principal plane \((z, x)\). All stresses and displacements are independent of \( y \). The displacement in the \( y \) direction of any point in the bar is zero, and the displacements in the \( z \) and \( x \) directions are respectively \((u, w)\). By Hamilton’s principle The buckling control equation for composite bar is \([7]\):

\[
\begin{align*}
N’ - \rho \ddot{u} &= 0 \\
N’w + Nw'' + Q' - \rho \ddot{w} &= 0 \\
M' - Q - \frac{1}{12} \rho h w'' &= 0
\end{align*}
\]

Where

\[
N = A_{11} \left[ u' + \frac{1}{2} (w')^2 \right] + B_{11} \phi
\]

\[
M = B_{11} \left[ u' + \frac{1}{2} (w')^2 \right] + D_{11} \phi
\]

\[
(A_{11}, B_{11}, D_{11}) = \sum_{k=1}^{n} Q_{11} k (l, z, z^2) dl
\]

\( \bar{Q}_{11} \) is the \( k \)-th layer off-axis stiffness, \( k \) is the number of layups, and \( n \) is the total number of layups. Where: \( N \) is the axial force, \( M \) is the moment, \( \rho \) is the density, \( h \) is the bar thickness, \( \phi \) is the rotation angle, \( c \) is the stress wave velocity, \( A_{11} \) is the tensile stiffness, \( B_{11} \) is the coupling stiffness, and \( D_{11} \) is the bending stiffness.

The stress wave is transmitted to the \( L_{cr} \) position, and the composite bar undergoes dynamic buckling. Neglecting the shear deformation and the rotational inertia, the dynamic buckling control equation of the composite bar as follows:

\[
A_{11} (u' + w'w') = \rho \ddot{u}
\]

\[
D_{11} w'''' + A_{11} \left[ u' + \frac{1}{2} (w')^2 \right] \cdot w'' + A_{11} (u''\cdot w') \cdot w'' = \rho \ddot{w}
\]

Take the following dimensionless quantities

\[
c = \left( \frac{A_{11}}{\rho h} \right)^{\frac{1}{2}}, \lambda = \left( \frac{D_{11}}{A_{11}} \right)^{\frac{1}{2}}, \bar{w} = \frac{w}{\lambda}, \bar{u} = \frac{u}{\lambda}, \bar{x} = \frac{x}{\lambda}, \ddot{t} = \frac{t}{t_{cr}}
\]

The dimensionless form of the control equations are calculated by bringing formula (9) into
formula (7), (8):
\[ w'' + \left( \frac{1}{2} (w')^2 \right) \cdot w' + \left( \frac{u'' + w'}{\lambda} \right) \cdot w' = \frac{f}{\lambda^2} \cdot w \]  
(10)

Equations (10) and (11) satisfy boundary conditions (12) and initial conditions (13)
\[
\begin{align*}
\bar{u}(0, \bar{r}) &= u''(0, \bar{r}) = 0, \bar{u}(\bar{r}, \bar{r}) = \bar{u}'(\bar{r}, \bar{r}) = 0 \\
\bar{w}(0, \bar{r}) &= w''(0, \bar{r}) = 0, \bar{w}(\bar{r}, \bar{r}) = w''(\bar{r}, \bar{r}) = 0 \\
\bar{w} = \bar{w}_0, \bar{w} = \bar{w}_0, \bar{u} = \bar{u}_0, \bar{u} = \bar{u}_0 (0 \leq \bar{x} \leq 1)
\end{align*}
\]
(12)

\[
\begin{align*}
\bar{w} &= \bar{w}_0, \bar{w} = \bar{w}_0, \bar{u} = \bar{u}_0, \bar{u} = \bar{u}_0 (0 \leq \bar{x} \leq L / L_c)
\end{align*}
\]
(13)

Where \( t \) is time, \( t_c \) is critical time, and \( L_c \) is critical length. The axial initial displacement is calculated by formula (14)
\[
\bar{u}_0 = -\frac{Mv_0 t_c}{\rho Ah^2} \exp \left( \frac{Mx \cdot L_c}{M} \right) + \frac{M^2 v_0 t_c}{\rho Ah^2} \]
(14)

Medium \( A \) is the cross-sectional area of the bar. Cite w-direction initial displacement[7] as formula (15):
\[
\bar{w}_0 = c_0 \left[ \sin(n_2 \pi) - \frac{n_1}{n_2} \sin(n_2 \pi) \right]
\]
(15)

3. Difference format for control equations

Bring central differential and backward differential formats into control equations, the difference format of the control equations are obtained as follows.
\[
\bar{u}_{j+1} = \Delta T^2 \cdot \left( \bar{u}' + \bar{w}' \cdot \bar{w}' \right) + 2 \cdot \bar{u}_j - \bar{u}_{j-1}
\]
(16)

\[
\bar{w}_{j+1} = \Delta T^2 \cdot \frac{\lambda^2}{L_c^2} \left[ \bar{w}' + \left( \bar{u}' + \bar{w}' \cdot \bar{w}' \right) \cdot \bar{w}' + \left( \bar{u}' + \frac{1}{2} (\bar{w}')^2 \right) \cdot \bar{w}' \right] + 2 \bar{w}_j + \bar{w}_{j-1}
\]
(17)

4. Numerical Simulation

In this paper, dynamic buckling of carbon//epoxy bar is discussed and material parameters are as shown in Table 1[7]. The influence of different initial imperfections, impact mass and lay-up angles on dynamic buckling of composite bar is discussed by programming formula (10), (11) with Matlab. Take displacement step \( \Delta x = 0.06 \), and time step \( \Delta t = 0.06 \). Then, obtained result is converted to the actual displacement by \( w = \frac{w}{x L_c} \).

Table 1 properties for Carbon//epoxy resin composite bar

| \( E_1 \) (Gpa) | \( E_2 = E_3 \) (Gpa) | \( \mu_{23} \) | \( \mu_{12} = \mu_{13} \) (Gpa) | \( G_{12} = G_{13} \) (Gpa) | \( G_{23} \) (Gpa) | density \( (kg/m^3) \) | Bar length \( (m) \) | Cross-sectional area \( (m^2) \) |
|-----------------|-----------------|--------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 139             | 9.4             | 0.33         | 0.31            | 4.5             | 2.98            | 1583            | 1               | 0.02×0.011      |

Figure 4-1 (a), (b) indicates that dynamic buckling mode diagrams of a composite bar with different initial imperfections when impact mass, impact speed and the lay-up angles are constant. In Figure (a) \( n_1 = 1, n_2 = 2 \), and in Figure (b) \( n_1 = 2, n_2 = 3 \). \( L_c \) in Figure (a) is 0.25, 0.42m and 0.65m, and Figure (b) is same as Figure (a).
Figure 4-1 Different initial imperfections of buckling mode on bar.

Figure 4-1 (a), (b) shows that with the propagation of stress waves, buckling is constantly occurring and expanding, that is, as the stress wave propagates, higher-order modes are excited and the number of buckling modes increases; Figure (b) With the propagation of stress waves, the amplitude of buckling peaks and troughs is much larger than that of Fig. (a), and the number of buckling modes is more than that of Fig. (a); the initial deflection has a significant effect on dynamic buckling; The stress wave effect has a significant effect on the dynamic buckling of composite rods.

Figure 4-2(a) is a graph showing the relationship between different $M$ and the corresponding $V_0$, and 4-2(b) expresses that dynamic buckling modes diagram of different impact mass when initial imperfections, impact speed and lay-up angles are constant.

Figure 4-3 remarks that the buckling mode of different lay-up angles when initial imperfections, impact speed and impact mass are constant.
Figure 4-3 Different lay-up angles of buckling mode on bar.

Figure 4-3(a) shows that the lay-up angles are 45/45/45/45, the buckling mode peak and trough value are the largest, the buckling mode is the largest, and the first half wavelength is the smallest. When the lay-up angles are 0/0/0/0, the buckling mode diagram has the smallest peak and trough value, and the first half wavelength is the largest. It is shown that for an same lay-up angle composite bar, the smaller the layer angle, the better the composite's ability to resist buckling deformation; the change in layer angle has a significant effect on dynamic buckling. When the lay-up angles are 0/45/0/45/0, the peak and valley values of the buckling mode diagram are less than the layer angle of 45/45/45/45/45, the number of modes is less, and the first half wavelength is more. Large, indicating that the cross-laying resistance to buckling deformation is better than non-cross-layering. When the lay-up angles are 0/90/0/90/0, the peak and trough values increase slightly compared to the 0/45/0/45/0 layer, and the first half wavelength decreases slightly, indicating that for the cross-laying The composite bar of the layer, the change of the layup has little effect on the dynamic buckling.

Figure 4-3(b) shows that the lay-up angles are 45/45/45/45/45, and the buckling mode diagram has the smallest peak and trough value, and the first half wavelength is the largest. It is indicated that the peak and trough values of the buckling mode diagram gradually increase with the increase of the lay-up angle of each layer, and the first half wavelength gradually decreases. When the lay-up angles are 45/45/60/60/45/45, the peak wave trough is larger than the lay-up angles of 60/60/45/45/60/60. It is indicated that for the composite bar, the change of the intermediate layer lay-up angle has a greater influence on the dynamic buckling than the change of the angle of the two-layer lay-up.

5. Conclusions
Considering the stress wave effect, taking into account the influence of axial inertia, the dynamic buckling control equation of the rigid mass impact composite bar is obtained by Hamilton principle. The control equations are dimensionless which is solved by central difference method. The differential equations of the control equations were programmed by Matlab. The effects of initial deflection, stress wave effect, impact quality and lay-up angles on the dynamic buckling of composite bar were discussed.

Initial deflection, stress wave effect, impact quality, and lay-up angles have significant effects on buckling of composite bar. The critical speed decreases with the increase of the impact mass. The impact mass in the sensitive area changes drastically with the change of the critical speed. In the non-sensitive area, this change is obviously flat.

Acknowledgments
Thanks to the support from the National Natural Science Foundation of China (11372209).

References
[1] Bolotin V V , Bolotin V V . Delaminations in composite structures: Its origin, buckling, growth and stability [J]. Composites Part B Engineering, 1996, 27(2):129-145.
[2] Borchani W, Lajnef N, Burgueño R. Energy method solution for the postbuckling response of an axially loaded bilaterally constrained beam [J]. Mechanics Research Communications, 2015, 70: S0093641315001652.

[3] Ghayesh M H, Farokhi H. Post-buckling dynamics of Timoshenko microbeams under axial loads [J]. International Journal of Dynamics and Control, 2015, 3(4): 403-415.

[4] Mottram J T. Lateral-torsional buckling of a pultruded I-beam [J]. Composites, 1992, 23(2): 81-92.

[5] Pouresmaeeli S, Fazelzadeh S A. Uncertain Buckling and Sensitivity Analysis of Functionally Graded Carbon Nanotube-Reinforced Composite Beam [J]. International Journal of Applied Mechanics, 2017, 09(05): 1550002-5125.

[6] Gunda J B, Venkateswara Rao G. Post-buckling and Large Amplitude Free Vibration Analysis of Composite Beams: Simple Intuitive Formulation [J]. Journal of The Institution of Engineers (India): Series C, 2016, 97(2): 175-183.

[7] L.F Wang, Z.J.Han, X.P.Yan, G YLu. The Influence of First Order Shear Deformation on Dynamic Buckling of Composite Bar [J]. Applied Mechanics and Materials. Vol 751, pp 195-199, Apr., 2015.