On the Fuzzy Decomposition of Fuzzy Codewords with the Aid of Relative Weight

A. Neeraja, B. Amudhambigai

Department of Mathematics Sri Sarada College for Women, Salem, Tamil Nadu, India
E-mail: neeru572010@gmail.com, rbamudha@yahoo.co.in

Abstract. Accurate transmission of a message is always of prime importance in day-to-day life. Messages of paramount importance require greater caution while transmission, so that it is not disrupted by any means. But due to several problems there may be some sort of misinterpretation on the received message. Based on the above mentioned factors, in this paper, a new idea of the Unique Decipherability of Fuzzy Codewords, their decomposition and the ambiguity of fuzzy codewords based on their relative weight are proposed, which handles several special issues in Information Theory.

1. Introduction

The process of deciphering the correct message from a wrong one based on the parameters where it went wrong has always been the prime notion of studying coding theory. The notion of coding theory was first initially proposed by Shannon [4], in which he proposed the transmission of additional bits in a much smaller way. After Shannon, several studies [2,3,6,7] were carried out on studying the theories and varieties of codes, their decipherability. This was further developed by Hamming [9] who developed the ingenious method of detection of errors, and also several profound theories on correcting those errors. This theory of error correction comprises the study using Maximum Likelihood Decoding [5]. The notion of fuzzy sets was initially proposed by Zadeh [10]. The prime focus of fuzzy logic lies between assigning degree of association in the closed interval between 0 and 1, for all the members in a given collection. A new perspective of coding theory based on fuzzy logic was studied in [1]. Following the idea, the notions of Unique Decipherability of Fuzzy Code, it’s fuzzy concatenative independence, fuzzy coding decomposition of fuzzy codewords are studied here along with their properties. The advantage of this method lies in the fact that every disrupted message has a unique membership value within the interval [0,1], thus helping us in identifying the proximity of the misinterpreted words. This way of partition is utilised in the process of Maximum Likelihood Decoding of Fuzzy Codewords studied in [8].

We have organized this paper as described below: Section - 1 consists of Introduction and Preliminaries. In Section - 2, the properties of Unique Decipherability of a fuzzy code, its fuzzy concatenative independence, fuzzy segregation and the notion of fuzzy coding partition of fuzzy codes are studied along with their properties.
Preliminaries

The basic preliminaries required for the study of this paper are given below:

**Definition 1.1.** [1] A *Binary Code* is a sequence of Zeroes and Ones, and they are known as codewords.

**Definition 1.2.** [1] Let \((F_n^q)\) be the collection of ordered n-tuples \(a = a_1, a_2, \ldots, a_n\) where each \(a_i \in F_q\). The elements belonging to \(F_n^q\) are called *vectors or words*.

**Definition 1.3.** [1] The weight of a vector \(x\) in \(F_n^2\), denoted by \(w(x)\), is defined as the number of ones (1s) in \(x\).

**Definition 1.4.** [1] Suppose \(x\) is a codeword of \(C\). If \(w_1, w_2, \ldots, w_k\) are defined as the positions of ones (1s) in \(x\), then \(w_1 + w_2 + \ldots + w_k\) are called the *relative weight* of codeword \(x\).

Since 11...1 is a codeword of \(C\), then its relative weight is

\[
1 + 2 + \ldots + n = \frac{n(n+1)}{2}
\]

Thus this weight is called a *Maximum Relative Weight* of codeword \(x\) of \(C\).

**Definition 1.5.** [1] The *Relative Weight* of a codeword \(x\) in \((F_2)^n\) is denoted by \(J(x)\) and it is defined by

\[
J(x) = \frac{w(x)}{\text{Maximum Relative Weight}}
\]

**Definition 1.6.** [1] Let \(C\) be a Code. The function \(J: C \rightarrow [0, 1]\) is defined as a *Fuzzy Code* if it satisfies:

(i) \(J(x + y) \geq \min\{J(x), J(y)\}\)

(ii) \(J(-x) = J(x)\)

(iii) \(J(xy) \leq \max\{J(x), J(y)\}\), for every \(x, y \in C\).

Here, the members of the fuzzy code \(J(C)\) are called as fuzzy codewords.

2. Decomposition of a Fuzzy Codeword

This section is devoted to study the notion of Fuzzy Segregation of a fuzzy code, Unique Decipherability of Fuzzy Code, Fuzzy Concatenative Independence of Fuzzy Code, the fuzzy coding partition and the notion of total ambiguity of fuzzy code are defined and some of their properties are studied. Throughout this section \(C\) denotes a code comprising codewords of varying lengths.

**Definition 2.1.** Let the collection of codewords, \(\{C_1, C_2, \ldots, C_n\}\) be the *segregation of \(C\)*, such that each collection \(C_i\) contains codewords of the same length. Let \(J(C_i) = \{J(C_{i1}), J(C_{i2}), \ldots, J(C_{in})\}\) be the respective relative weights associated with \(C\). The collection \(J(C) = \{J(C_1), J(C_2), \ldots, J(C_n)\}\) is called the *Fuzzy Segregation* associated with \(C\) if each fuzzy codeword \(J(C_{ik})(k=1,2,\ldots,n) \in J(C_i)\) has the same maximum relative weight.

**Definition 2.2.** Let the collection of codewords, \(\{C_1, C_2, \ldots, C_n\}\) be the *segregation of \(C\)*, such that \(J(C)\) is the fuzzy segregation associated with \(C\). Then the fuzzy decomposition of an arbitrary fuzzy codeword \(J(C_{it})\) is given by \(J(C_{it}) = J(C_{ij})\) where \(i = 1,2,\ldots,n\) and \(1 \leq j \leq n\).

**Definition 2.3.** Let the collection of codewords, \(\{C_1, C_2, \ldots, C_n\}\) be the *segregation of \(C\)*, such that \(J(C)\) is the fuzzy segregation associated with \(C\). Then the fuzzy decomposition of an arbitrary fuzzy codeword \(J(C_{it})\) is given by \(J(C_{it}) = J(C_{ij})\) where \(i = 1,2,\ldots,n\) and \(t = 1,2,\ldots,m\). (i.e.) If

\[
J[C_{i1} \oplus C_{j2} \oplus ... \oplus C_{in}] = J[C_{j1} \oplus C_{j2} \oplus ... \oplus C_{jm}] \text{ (for } i \neq j \text{ and } i,j = 1,2,\ldots,n)\]
implies $C_{i1} = C_{j1}, C_{i2} = C_{j2}, ..., C_{in} = C_{jm}$. A fuzzy code that is not a UDFC is an ambiguous fuzzy code.

**Definition 2.4.** Let the collection of codewords, $\{C_1, C_2,...,C_n\}$ be the segregation of $\mathcal{C}$, such that $J(\mathcal{C})$ is the fuzzy segregation associated with $\mathcal{C}$. Let $\mathcal{D} = \{D_{11}, D_{12}, ..., D_{1m}\}$ (m $\neq$ n), be another collection of codewords different from $\mathcal{C}$ such that $J(D_{ik}) = J(C_{ij})$ for any arbitrary fuzzy codeword $J(C_{ij}) \in J(\mathcal{C})$ (i, j, k=1,2,...,n and i $\neq$ j) The collection $J(\mathcal{C})$ is said to be **Fuzzy Concatenatively Independent** if the following conditions holds:

(i) $J(\oplus(D_{ij})) \leq \forall J(C_{ik})$ for i = 1,2,...,n, j = 1,2,...,m and k = 1,2,..., n, j $\neq$ k.

(ii) Each fuzzy codeword in $J(\mathcal{C})$ has a different relative weight.

**Definition 2.5.** Let the collection of codewords, $\{C_1, C_2,...,C_n\}$ be the segregation of $\mathcal{C}$, such that $J(\mathcal{C})$ is the fuzzy segregation associated with $\mathcal{C}$ and it is fuzzy concatenatively independent.

Any arbitrary Fuzzy Codeword $J(C_i)$ is said to have a $\mathcal{C}_p$ - **Fuzzy Decomposition**, $J(C_p) = J[C_{i1} \oplus C_{i2} \oplus ... \oplus C_{in}]$ (i = 1, 2,...,n) if the following conditions are satisfied:

(i) $J(C_i) \leq \forall J(C_{ij})$, i = 1,2,...,n, where $J(C_{ij}) \in J(\mathcal{C}_1)$, $J(C_{ij}) \in J(\mathcal{C}_2)$ and so on.

(ii) If $J(C_{ij}) \leq \forall J(C_{il})$; i = 1,2,...,n, then $J(C_{ij}) \not< \forall J(C_{il})$; i = 1,2,...,n, 1 $\leq$ l $\leq$ n and so on.

**Definition 2.6.** Let $\mathcal{C}$ be a code comprising codewords of varying lengths and let $\mathcal{C} = \{C_1, C_2,...,C_n\}$ be the segregation of $\mathcal{C}$, such that $J(\mathcal{C})$ is the fuzzy segregation associated with $\mathcal{C}$. The collection $J(\mathcal{C})$ is said to be a **fuzzy coding decomposition** if $J(\mathcal{C})$ is fuzzy concatenatively independent and any arbitrary fuzzy codeword $J(C_i)$ has a unique $\mathcal{C}_p$ - Fuzzy Decomposition.

**Remark 2.7.** Let $\mathcal{C}$ be a code comprising codewords of varying lengths and let $\mathcal{C} = \{C_1, C_2,...,C_n\}$ be the segregation of $\mathcal{C}$, such that $J(\mathcal{C})$ is the fuzzy segregation associated with $\mathcal{C}$. Suppose that an arbitrary fuzzy codeword $J(C_i)$ has two fuzzy decompositions such that $J(C_i) = J(C_{il}) = J(C_{jm})$, where $J(C_{il}) = J[C_{i1} \oplus C_{i2} \oplus ... \oplus C_{is}]$ and $J(C_{jm}) = J[C_{j1} \oplus C_{j2} \oplus ... \oplus C_{jt}]$ (i, j=1,2,...,n, i $\neq$ j and s,t $\leq$ n). The fuzzy decomposition of $J(C_i)$ is called a **prime fuzzy decomposition** if for $l < s$ and $m < t$, $J(C_{il}) \neq J(C_{jm})$ for any $v = 1,2,...,l$ and $w = 1,2,...,m$.

**Remark 2.8.** An arbitrary fuzzy codeword $J(C_i)$, having two fuzzy decompositions such that $J(C_i) = J[C_{i1} \oplus C_{i2} \oplus ... \oplus C_{is}] = J[C_{j1} \oplus C_{j2} \oplus ... \oplus C_{jt}]$ (i $\neq$ j and s,t $\leq$ n) can be uniquely represented as a prime fuzzy factorization $J(C_i) = J[C_{i1} \oplus C_{i2} \oplus ... \oplus C_{ik_2-1}] = J[C_{j1} \oplus C_{j2} \oplus ... \oplus C_{jl_2-1}]

\vdots

J(C_n) = J[C_{kn} \oplus ... \oplus C_{is}] = J[C_{ln} \oplus ... \oplus C_{jt}]$.

**Remark 2.9.** Let $\mathcal{C}$ be a code comprising codewords of varying lengths and let $\mathcal{C} = \{C_1, C_2,...,C_n\}$ be the segregation of $\mathcal{C}$, such that $J(\mathcal{C})$ is the fuzzy segregation associated with $\mathcal{C}$. Then for a unique fuzzy decomposition of an arbitrary fuzzy codeword $J(C_i)$, there exists a unique $\mathcal{C}_p$ - Fuzzy Decomposition of $J(C_i)$. Thus for an arbitrary fuzzy codeword with two distinct $\mathcal{C}_p$ - Fuzzy Decompositions, we have two distinct fuzzy decompositions.

**Proposition 2.10.** Let $\mathcal{C}$ be a code comprising codewords of varying lengths and let $\mathcal{C} = \{C_1, C_2,...,C_n\}$ be the segregation of $\mathcal{C}$, such that $J(\mathcal{C})$ is the fuzzy segregation associated with $\mathcal{C}$. The fuzzy segregation $J(\mathcal{C})$ associated with $\mathcal{C}$ is a fuzzy coding decomposition iff for every prime fuzzy decomposition of an arbitrary fuzzy codeword $J(C_p) = J[C_{i1} \oplus C_{i2} \oplus ... \oplus C_{is}] = J[C_{j1} \oplus C_{j2} \oplus ... \oplus C_{jt}]$ an integer n exists, such that, for $l \leq s$ and $m \leq t$, $J(C_{il})$, $J(C_{jm})$ $\in J(\mathcal{C}_n)$, i, j = 1,2,...,n and i $\neq$ j, s,t $\leq$ n.
Proof. Let us assume that \( J(\mathcal{C}) \) is a fuzzy coding decomposition and let \( J(C_p) = J[C_{i1} \oplus C_{i2} \oplus \ldots \oplus C_{in}] = J[C_{j1} \oplus C_{j2} \oplus \ldots \oplus C_{jn}] \) be the two prime fuzzy decompositions of an arbitrary fuzzy codeword \( J(C_p) \) and let \( J(C_{p1} \oplus C_{p2} \oplus \ldots \oplus C_{pm}) \) be its unique \( C_p \) - Fuzzy Decomposition. To prove our hypothesis we have to show that \( n = 1 \). On the contrary assume that \( n > 1 \). Since, \( J(\mathcal{C}) \) is a fuzzy coding decomposition it has a unique \( C_p \) - Fuzzy Decomposition and hence for \( l < s \) and \( m < t \), we have \( J(C_p) = J[C_{i1} \oplus C_{i2} \oplus \ldots \oplus C_{il}] \neq J[C_{j1} \oplus C_{j2} \oplus \ldots \oplus C_{jm}] \). Let \( C_{i1}, C_{i2}, \ldots, C_{in}, C_{j1}, C_{j2}, \ldots, C_{jn} \) be the two prime fuzzy decompositions of an arbitrary fuzzy codeword \( J(C_p) \) and let \( J(C_{p1} \oplus C_{p2} \oplus \ldots \oplus C_{pm}) \) be its unique \( C_p \) - Fuzzy Decomposition.

To show that \( J(\mathcal{C}) \) is a fuzzy coding decomposition initially prove that the members of \( J(\mathcal{C}) \) are fuzzy concatenatively independent. Let us assume the contrary that they are not fuzzy concatenatively independent. Then by Definition 2.4, for any two fuzzy codewords \( J(C_{il}) \in J(\mathcal{C}_{i}) \) and \( J(C_{jm}) \in J(\mathcal{C}_{j}) \), where \( 1 \leq l \leq n \) and \( 1 \leq m \leq n \) and \( i, j = 1,2,\ldots,n \), \( i \neq j \). \( J(C_{il}) = J(C_{jm}) \). Then \( J(C_p) = J[C_{i1} \oplus C_{i2} \oplus \ldots \oplus C_{in}] = J[C_{j1} \oplus C_{j2} \oplus \ldots \oplus C_{jn}] \), where \( J(C_{il}) \in J(\mathcal{C}_i) \) \( (m = 1,2,\ldots,s) \) and \( J(C_{jm}) \in J(\mathcal{C}_j) \) \( (n = 1,2,\ldots,t) \). From Remark 2.8, \( J(C_p) \) has a prime fuzzy decomposition such that \( J(C_{il}) = J[C_{i1} \oplus C_{i2} \oplus \ldots \oplus C_{il-1}] = J[C_{j1} \oplus C_{j2} \oplus \ldots \oplus C_{jl-1}] \) \( (g \leq s \) and \( h \leq t) \), which is a contradiction as \( C_{il} \in \mathcal{C}_i \) \( (m = 1,2,\ldots,s) \) and \( C_{jl} \in \mathcal{C}_j \) \( (n = 1,2,\ldots,t) \) whenever \( i \neq j \).

To complete the proof we now show that, any arbitrary fuzzy codeword has a unique \( J(C_p) \) fuzzy decomposition. Suppose on the contrary that there are two distinct \( J(C_p) \) fuzzy decompositions, then by Remark 2.9, we have two distinct fuzzy decompositions for \( J(C_p) \), such that \( J(C_p) = J[C_{i1} \oplus C_{i2} \oplus \ldots \oplus C_{is}] = J[C_{j1} \oplus C_{j2} \oplus \ldots \oplus C_{js}] \). Since \( i \neq j \), we have by Remark 2.8, a prime fuzzy decomposition such that

\[
J(C_1) = J[C_{i1} \oplus C_{i2} \oplus \ldots \oplus C_{ik2-1}] = J[C_{j1} \oplus C_{j2} \oplus \ldots \oplus C_{jk2-1}]
\]

\[
J(C_2) = J[C_{k2} \oplus \ldots \oplus C_{k3-1}] = J[C_{l2} \oplus \ldots \oplus C_{l3-1}]
\]

\[
\vdots
\]

\[
J(C_n) = J[C_{kn} \oplus \ldots \oplus C_s] = J[C_{ln} \oplus \ldots \oplus C_t].
\]

Also, by the previous part of the proposition, the fuzzy code \( J(\mathcal{C}) \) is fuzzy concatenatively independent and hence, \( J(C_{il}) \neq J(C_{jl}) \) \( (i, j = 1,2,\ldots,n \text{ and } i \neq j) \). Let \( N' \) denote the number of vectors in an arbitrary fuzzy codeword and suppose that \( N'[C_{ij}] < N'[C_{ik}] \), then it is clear that \( N'[C_{ij}] \geq 2 \). Since \( J(\mathcal{C}) \) has a \( C_p \) fuzzy decomposition, for \( C_j \in \mathcal{C} \) if \( J(C_{ij}) \in J(\mathcal{C}_j) \) and \( J(C_{ij}) \notin J(C_{j2}) \).

Thus an arbitrary fuzzy codeword in a prime fuzzy decomposition \( J(C_h) \) \( (1 \leq h \leq n) \) cannot occur in both decompositions of \( J(C_{ij}) \) and \( J(C_{ik}) \). Hence, we have \( N'[C_{ij}] \leq N'[C_{ij}] \implies N'[C_{ij}] \leq 2 \). Since \( J(\mathcal{C}) \) is expressed both in terms of \( J(C_{ij}) \) and \( J(C_{ik}) \). But since they are fuzzy concatenatively independent, we have \( J(C_1) \leq \{J(C_{ij})\} \) \( (i=1,2,\ldots,n) \).

Let \( J(C_{j1}) = J[C_{i1} \oplus C_{i2} \oplus \ldots \oplus C_{im}], 1 \leq m < n \). By the hypothesis, \( n > 1 \) and \( N'[C_{ij}] \geq 2 \), therefore a prime fuzzy factor \( J(C_{m1}) \neq J(C_{1}) \) or \( J(C_{m1}) \neq J(C_{11}) \), which implies, \( J(C_{m1}) \leq J(C_1) \) and \( J(C_{m1}) \leq J(C_{11}) \). But \( J(C_1) \leq J(C_{ij}) \) and \( J(C_{11}) \leq J(C_{ij}) \) and also \( J(C_{m1}) \) has a prime fuzzy decomposition, which is a contradiction. Thus the fuzzy segregation \( J(\mathcal{C}) \) has a unique \( J(C_p) \) fuzzy decomposition and hence \( J(\mathcal{C}) \) is a fuzzy coding decomposition. □

**Definition 2.11.** Let the collection of codewords, \( \{\mathcal{C}_1, \mathcal{C}_2,\ldots,\mathcal{C}_n\} \) be the *segregation of* \( \mathcal{C} \), such that \( J(\mathcal{C}) \) is the fuzzy segregation associated with \( \mathcal{C} \). The fuzzy segregation \( J(\mathcal{C}) \) is a trivial fuzzy segregation if \( J(\mathcal{C}) \) has only one fuzzy segregation.

**Definition 2.12.** The fuzzy segregation \( J(\mathcal{C}) \) is called a discrete fuzzy segregation if each fuzzy segregation in \( J(\mathcal{C}) \) has only one fuzzy codeword.
Remark 2.13. A fuzzy segregation \( J(\mathcal{C}) \) is a UD\( F\mathcal{C} \) iff the discrete fuzzy segregation of \( J(\mathcal{C}) \) is a fuzzy coding decomposition. Since each discrete fuzzy segregation has only one fuzzy codeword, the fuzzy decomposition is unique and hence this is true.

Definition 2.14. Let the collection of codewords, \( \{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_n\} \) be the segregation of \( \mathcal{C} \), such that \( J(\mathcal{C}) \) is the fuzzy segregation associated with \( \mathcal{C} \). A fuzzy cross section \( J(\mathcal{C}_i) \) of \( J(\mathcal{C}) \) is a collection containing exactly one fuzzy codeword from each fuzzy segregation of \( J(\mathcal{C}) \).

Proposition 2.15. Let the collection of codewords, \( \{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_n\} \) be the segregation of \( \mathcal{C} \), such that \( J(\mathcal{C}) \) is the fuzzy segregation associated with \( \mathcal{C} \), such that \( J(\mathcal{C}) \) is a fuzzy coding decomposition. Then any fuzzy cross section \( J(\mathcal{C}_i) \) of \( J(\mathcal{C}) \) is a UD\( F\mathcal{C} \).

Proof. Let \( J(\mathcal{C}_1) = \{J(C_{11}), J(C_{12}), \ldots, J(C_{1n})\}, J(\mathcal{C}_2) = \{J(C_{21}), J(C_{22}), \ldots, J(C_{2m})\} \) and so on and let \( J(\mathcal{C}_i) = \{J(C_{i1}), J(C_{i2}), \ldots\} \) be the fuzzy cross section of \( J(\mathcal{C}) \). Suppose that \( J(\mathcal{C}_i) \) is not a UD\( F\mathcal{C} \), then any arbitrary fuzzy codeword \( J(C_{ih}) \in J(\mathcal{C}_i) \) \( (i, s = 1,2,\ldots,n) \), will have two distinct fuzzy decompositions. (i.e.,\( J(C_{ih}) = J(C_{ij} \oplus C_{i2} \oplus \ldots \oplus C_{is}) = J(C_{1j} \oplus C_{12} \oplus \ldots \oplus C_{1t}) \), where \( J(C_{ij}), J(C_{jk}) \in J(\mathcal{C}_i) \) \((i,j=1,2,\ldots,n \text{ and } i \neq j \text{ and } 1 \leq h \leq s, 1 \leq k \leq t) \). Since they are distinct, we have \( J(C_{ih}) \neq J(C_{jk}) \).

Since \( J(\mathcal{C}_i) \) is a fuzzy cross section of \( J(\mathcal{C}) \), it is clear that \( J(C_{ih}), J(C_{jk}) \in J(\mathcal{C}) \). Here \( J(\mathcal{C}) \) is a fuzzy coding decomposition and suppose that \( J(\mathcal{C}_i) \) is not a fuzzy coding decomposition then \( J(\mathcal{C}_i) \) can have two distinct \( J(\mathcal{C}_i) \) - fuzzy decompositions and by our assumption \( J(C_{ih}) \neq J(C_{jk}) \) which implies that the fuzzy codewords are different, which is a contradiction to the fact that \( J(\mathcal{C}) \) is a fuzzy coding decomposition. Hence, \( J(\mathcal{C}_i) \) is a UD\( F\mathcal{C} \).

Definition 2.16. Let the collection of codewords, \( \{\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_n\} \) be the segregation of \( \mathcal{E} \), such that \( J(\mathcal{E}) \) is the fuzzy segregation associated with \( \mathcal{E} \). Then a fuzzy characteristic segregation \( J(\mathcal{E}_i) \) for \( J(\mathcal{E}) \) is a discrete fuzzy segregation, defined as

\[
\chi(\mathcal{E}(\mathcal{C}_{ij})) = \begin{cases} 
1_{\mathcal{C}} & \text{if } J(C_{ij}) \leq J(C_{11} \oplus \ldots \oplus C_{1n}) \\
0_{\mathcal{C}} & \text{otherwise}
\end{cases}
\]

If \( \mathcal{N} \) denotes the number of codewords in a collection, then a fuzzy code is said to be totally ambiguous if \( \mathcal{N}(\mathcal{C}) > 1 \) and \( J(\mathcal{E}_i) \) is a trivial fuzzy segregation.

Proposition 2.17. Let \( \mathcal{C} \) be a code comprising codewords of varying lengths and let \( \mathcal{C} = \{\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_n\} \) be the segregation of \( \mathcal{C} \), such that \( J(\mathcal{C}) \) is the fuzzy segregation associated with \( \mathcal{C} \). Suppose that \( J(\mathcal{C}_i) = \{J(\mathcal{C}_{i1}), J(\mathcal{C}_{i2}), \ldots, J(\mathcal{C}_{in})\} \) be the fuzzy characteristic segregation of \( J(\mathcal{C}) \). If \( \mathcal{N}(J(\mathcal{C}_{ji})) > 1 \) for some \( j \geq 1 \), then \( J(\mathcal{C}_{ji}) \) is a totally ambiguous fuzzy code.

Proof. Let us assume that \( \mathcal{N}(J(\mathcal{C}_{ji})) > 1 \) and suppose that \( J(\mathcal{C}_{ji}) \) is not a totally ambiguous fuzzy code, then it does not have a trivial fuzzy segregation. Hence \( J(\mathcal{C}_{ij}) = \{J(C_{11}), J(C_{12}), \ldots, J(C_{1n})\} \). Let this be the characteristic fuzzy segregation of \( J(\mathcal{C}_{ij}) \) and clearly \( \mathcal{N}(J(\mathcal{C}_{1i})) \geq 2 \). Hence, we have \( J(\mathcal{C}_i) = \{J(\mathcal{C}_{i1}), J(\mathcal{C}_{i2}), \ldots, J(C_{1n}), \ldots, J(C_{2n}), \ldots, J(C_{ni})\} \), which is not possible as \( J(\mathcal{C}_i) \) is the characteristic fuzzy segregation and hence it can contain only one codeword from each \( \mathcal{C}_{ji} \). Hence \( J(\mathcal{C}_{ji}) \) is a totally ambiguous fuzzy code.

Proposition 2.18. If all the proper subsets of \( \mathcal{C} \) are UD\( F\mathcal{C} \). Then either \( \mathcal{C} \) is UD\( F\mathcal{C} \) or it is a totally ambiguous fuzzy code.

Proof. Suppose that \( J(\mathcal{C}) \) is not a UD\( F\mathcal{C} \). Let \( J(\mathcal{C}_i) = \{J(\mathcal{C}_1), J(\mathcal{C}_2), \ldots, J(\mathcal{C}_n)\} \) be the characteristic fuzzy decomposition of \( J(\mathcal{C}) \). Let us assume the contrary, that \( J(\mathcal{C}) \) is not a totally ambiguous fuzzy code. Then \( \mathcal{N}(J(\mathcal{C}_{ji})) \geq 2 \) and there exists a fuzzy decomposition of any \( J(\mathcal{C}_i) \) in \( J(\mathcal{C}) \) as \( J(\mathcal{C}) \) is not a UD\( F\mathcal{C} \). Thus \( J(\mathcal{C}_i) \) is not a discrete fuzzy decomposition and hence by Proposition 2.17, \( J(\mathcal{C}_i) \) is a totally ambiguous fuzzy code, which contradicts our assumption. Hence \( J(\mathcal{C}) \) is a UD\( F\mathcal{C} \).
This notion of decipherability can be further employed in maximum likelihood decoding which plays a vital role in analysing the wrongly sent message which is studied in [8].

3. Conclusion

The method of relative weight is implemented in this paper to analyse the various notions of decipherability and ambiguity of fuzzy codewords. This helps in locating the exact place of disruption because of its uniquely decipherable and segregating property, thereby fixing the disruption faster. The scope for future study includes the implementation of this fuzzy segregation in the diction of words in certain regional languages.

Acknowledgement

The authors would like to express their sincere gratitude and thanks to the referees for their suggestions towards the betterment of this paper.

References

[1] Ayten Ozkan and E. Mehmet Ozkan, A Different Approach to Coding theory Pakistan Journal of Applied Sciences, (2002) 2(11):1032-1033.
[2] Byrne E., Ravagnani A., Partition-Balanced Families of Codes and Asymptotic Enumeration in Coding Theory Information Theory, (2018).
[3] Berstel J., Perrin D., The Theory of Codes, Academic Press, New York, (1985).
[4] Claude E. Shannon, A Mathematical Theory of Communication Bell System Technical Journal. (1948), 27:379–423, 623–656.
[5] Forney Jr. G.D., Convolutional codes II. Maximum-likelihood decoding Information and Control (1974), 25:3, 222-266.
[6] Guzman F.:, Decipherability of codes, Journal of Pure and Applied Algebra. 141 (1999) 13-35.
[7] Lempel A. On multiset decipherable codes, IEEE Trans. Inform. Theory, (1986), 32, 714-716.
[8] Neeraja A., Amudhambigai B., A Novel Way of Detecting and Correcting Transmitted Errors using Fuzzy Logic, (Communicated).
[9] Richard W. Hamming. Error Detecting and Error Correcting Codes, Bell System Technical Journal, (1950), 29:147–160.
[10] Zadeh, L.A., Fuzzy Sets Inform. Control, (1965), 8 .338-353.