Nonperturbative contribution to the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi and Gribov-Levin-Ryskin equation

Bo He*

CCAST (World Laboratory), P.O.Box 8730
Beijing 100080, P.R. China

Department of Applied Physics, Shanghai Jiaotong University
Shanghai 200030, P.R. China

Abstract

By studying the nonperturbative contribution to the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi and Gribov-Levin-Ryskin equation, it is found that (i) the nonperturbative contribution suppresses the evolution rate at the low $Q^2$, small-x region; (ii) the nonperturbative contribution weakens the shadowing effect. The method in this paper suggests a smooth transition from the low $Q^2$ ("soft" ) region, where nonperturbative contribution dominates, to the large $Q^2$ ("hard" ) region, where the perturbative contribution dominates and the nonperturbative contribution can be neglected.

PACS numbers:12.38.Aw, 13.60.Hb

*Permanent Address: Department of Applied Physics, Shanghai Jiaotong University, Minhang Campus, Shanghai 200240, P.R. China. E-mail: hebobo@public6.sta.net.cn
The properties of parton distribution at small-x region (x is the value of the Bjorken variable) have recently been the important subject [1-5]. Recent measurements of the structure functions for the deep-inelastic ep scattering at HERA discovered their dramatic rise as x decreases from \(10^{-2}\) to \(10^{-4}\) [6,7]. The predictions of Glück, Reya and Vogt (GRV) [8] by using the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation [9] at very low \(Q^2\) (\(Q^2\) is the negative of the square of the four-momentum transferred by the lepton to the nucleon) are in broad agreement with this result. However, the GRV model fails in low \(Q^2\) quantitatively, that is to say, the evolution rate is faster than that of the experiments. It then becomes a challenging problem how to determine the structure function at the low \(Q^2\) region. Another important question is whether the shadowing effect, which the Gribov-Levin-Ryskin (GLR) equation [10] describes, can be observed by the current experiments at HERA.

The purpose of this letter is to study the DGLAP and GLR equation in low \(Q^2\) by considering the nonperturbative contribution. It will be showed that (i) the nonperturbative contribution suppresses the evolution rate at the low \(Q^2\), small-x region; (ii) the nonperturbative contribution weakens the shadowing effect. So the nonperturbative contribution is very important in the low \(Q^2\) region.

The DGLAP equation for the gluon distribution at small-x region in the DLLA is given by

\[
\frac{\partial^2 \frac{xg(x, Q^2)}{\partial \ln(1/x) \partial \ln(Q^2)}}{\partial \ln(1/x) \partial \ln(Q^2)} = \frac{3\alpha_s(Q^2)}{\pi} xg(x, Q^2)
\]

(1)

By considering the shadowing effect, the DGLAP equation can be modified in the form (called GLR equation):

\[
\frac{\partial^2 \frac{xg(x, Q^2)}{\partial \ln(1/x) \partial \ln(Q^2)}}{\partial \ln(1/x) \partial \ln(Q^2)} = \frac{3\alpha_s(Q^2)}{\pi} xg(x, Q^2) - \frac{81\alpha_s^2(Q^2)}{16R^2Q^2} [xg(x, Q^2)]^2
\]

(2)

1 Ref. [7] shows that the value of \(F_2\) given by GRV model is lower than that of the experiments at \(Q^2 = 0.4\text{GeV}^2\) while it is higher at \(Q^2 = 6.5\text{GeV}^2\).
In the previous studies people used the perturbative QCD effective coupling (the leading order coupling) $\alpha_s(Q^2)$ to study the equation, where

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f) \log(Q^2/\Lambda_{QCD}^2)}$$  \hfill (3)

By using the formula (3), the Eq.(2) can be cast in the form:

$$\partial_y \partial_t G(y, t) = cG(y, t) - \lambda \exp[-t - \exp(t)]G^2(y, t)$$  \hfill (4)

where $y = \ln(1/x)$, $t = \ln[\ln(Q^2/\Lambda_{QCD}^2)]$, $G(y, t) = xg(x, Q^2)$, $c = 12/(11 - 2n_f/3)$ with $n_f$ the number of quark flavors and $\lambda = 9\pi^2c^2/16\Lambda_{QCD}^2$. The equation (4) has been studied by many authors [11,12]. Eskola, Qiu and Wang [13] applied the eq.(4) to study the shadowing in heavy nuclei.

However, it should be noted that in the large $Q^2$ the formula (3) derived from perturbative QCD is a good approximation while in the low $Q^2$ nonperturbative contribution should be included. To avoid the ghost-pole problem in the behavior of a running coupling, Shirkov and Solovtsov [14] obtained the QCD running coupling in the leading order as:

$$\alpha_{an}(Q^2) = \frac{12\pi}{(33 - 2n_f) \log(Q^2/\Lambda_{QCD}^2) + \log(1 - Q^2/\Lambda_{QCD}^2)}$$  \hfill (5)

The second term on the right-hand side of Eq. (5) is clearly the nonperturbative contribution. It is noted that $\alpha_{an}(Q^2 = 0GeV^2) = \frac{12\pi}{(33 - 2n_f) \log(1)}$ depends only on group factors and does not depend on the $\Lambda_{QCD}$. It can be found that in the large $Q^2$ the running coupling $\alpha_{an}(Q^2)$ is dominated by perturbative contribution and the nonperturbative contribution can be neglected while in the low $Q^2$ the nonperturbative contribution is very notable.

By applying the formula (5) to DGLAP equation\(^2\), the Eq. (1) can be written as:

$$\partial_y \partial_t G(y, t) = cG(y, t)[1 + \frac{\exp(t)}{1 - \exp(e^t)}]$$  \hfill (6)

\(^2\)The reason that the formula (5) replaces formula(3) in DGLAP equation will be given in the ending of the text.
Adopting the semi-classical approximation \[4,12\], which amounts to keeping only the first order derivatives of the function \( z(y, t) = \log[ G(y, t) ] \), Eq. (6) is rewritten as:

\[
\partial_y z(y, t) \partial_t z(y, t) = c [1 + \frac{\exp(t)}{1 - \exp(e^t)}] \tag{7}
\]

Eq. (7) can be solved by using the method of characteristic \[4,12\]. Let

\[
p = \partial_t z(y, t), \quad q = \partial_y z(y, t) \tag{8}
\]

Eq. (7) can be written as the following general form:

\[
F(p, q, t, y, z) = 0 \tag{9}
\]

The characteristic differential equations of Eq. (9) have the following form:

\[
\frac{dt(\tau)}{d\tau} = F_p, \quad \frac{dy(\tau)}{d\tau} = F_q, \quad \frac{dz(\tau)}{d\tau} = pF_p + qF_q,
\]

\[
\frac{dp(\tau)}{d\tau} = -(F_t + pF_z), \quad \frac{dq(\tau)}{d\tau} = -(F_y + qF_z), \tag{10}
\]

where \( \tau \) is the "inner" time.

\( F_p, F_q, F_t, F_y, F_z \) is

\[
F_p = q, \quad F_q = p, \quad F_y = 0, \quad F_z = 0.
\]

\[
F_t = -ce^t[1 - \exp(e^t) + \exp(e^t + t)]/[1 - \exp(e^t)]^2 \tag{11}
\]

Same as Ref. \[12\], the initial conditions to solve Eqs. (10) are:

\[
t_0 = \log[\log(Q_0^2/\Lambda_{QCD}^2)] = \log[\log(4GeV^2/\Lambda_{QCD}^2)],
\]

\[
y_0 = \log(1/x_0) = \log(100), p_0 = c/\delta_{bare}, z_0 = \log(3.38),
\]

\[
q_0 = c[1 + \frac{e^{t_0}}{1 - \exp(e^{t_0})}]/p_0, \quad \Lambda_{QCD} = 0.2GeV, \delta_{bare} = 0.5. \tag{12}
\]

For clear comparison between the results of Eqs. (10) with those of DGLAP equation not including the nonperturbative contribution, we solve the Eqs. (10) numerically by...
adopting the Runge-Kutta methods [12]. Fig. 1 shows the evolution path \((y,t)\) corresponding to the dashed line by solving the Eqs. (10) compared with the evolution path \((y,t)\) corresponding to the solid line by solving the characteristic differential equations of DGLAP not including nonperturbative contribution.

Like the DGLAP evolution equation, by using the formula (5), the GLR equation can be cast in the form:

\[
\frac{\partial_y \partial_t G(y,t)}{G(y,t)} = c G(y,t)[1 + \frac{\exp(t)}{1 - \exp(e^t)}] - \lambda \exp[-t - \exp(t)][1 + \frac{\exp(t)}{1 - \exp(e^t)}] G^2(y,t) \quad (13)
\]

The semi-classical approximation of the Eq. (13)

\[
\frac{\partial_y z(y,t) \partial_t z(y,t)}{z(y,t)} = c[1 + \frac{\exp(t)}{1 - \exp(e^t)}] - \lambda \exp[-t - \exp(t) + z][1 + \frac{\exp(t)}{1 - \exp(e^t)}]^2 \quad (14)
\]

By using the method of characteristic, the solution of Eq. (14) are showed in Fig. 2.

To conclude, recent experiments at HERA have supplied much information about the nucleon structure at both large \(Q^2\) and low \(Q^2\). In large \(Q^2\), the DGLAP equation derived from perturbative QCD can describe the behavior of parton distribution. The challenging problem is how to make a unified treatment on nucleon structure at both large \(Q^2\) and low \(Q^2\). This letter proposes a way to meet the requirement. It is well known that the parton distribution includes both perturbative QCD and nonperturbative QCD effects. The input distribution reflects the nonperturbative QCD, and the DGLAP equation itself is the perturbative QCD. So it seems that the DGLAP equation describing the perturbative QCD effect and the input distribution describing the nonperturbative QCD effect together can give the comprehensive description to the parton distribution. However, it can be seen clearly that the input distribution does not include all nonperturbative effects, that is to say, some nonperturbative effects are reflected through the running coupling. By considering the nonperturbative effects in running coupling, it is a natural way to apply the DGLAP equation in the low \(Q^2\) region. So the evolution equation itself includes both perturbative and nonperturbative effects. It should be noted that this way is a work ansatz, which can not be derived from the theory. In viewing the Fig. 1, it can be found that the nonperturbative contribution to DGLAP is very notable.
Although the predictions of GRV model by applying the DGLAP evolution equation at very low $Q^2$ are in broad agreement with HERA experiments, the evolution rate which results from the model is faster than that of the experiments. By considering the nonperturbative contribution to DGLAP, the discrepancy between GRV model and the experiments can be explained naturally. From analysing the DGLAP equation, it can be found that the running coupling determines the evolution rate, which becomes slow by considering the nonperturbative contribution, especially in very low $Q^2$ such as $Q^2 = 0.65 GeV^2$.

Recently, one of the important questions is whether the shadowing effect can be observed by the current experiments at HERA. Some people say “can” such as Shabelski and Treleani [15] while other people say “cannot” such as Golec-Biernat, Krasny and Riess [16]. Ayala, Gay Ducati and Levin [17] argue that the shadowing effect is large in the gluon distribution but small in $F_2(x, Q^2)$. Like the DGLAP equation, the GLR equation can be treated by the same method. In this paper, a firm conclusion about this question does not be made. Nevertheless, in viewing the Fig.2, it can be concluded that shadowing effect in the GLR equation, which is modified by the nonperturbative contribution, is not so notable as what has been studied in the case when gluons concentrate in "hot-spots" within proton ($R = 2 GeV^{-1}$). By analysing the value of $\alpha_s(Q^2)$ and $\alpha_{an}(Q^2)$, the simple explanation of this result is that the linear term in the GLR equation is proportional to $\alpha_s(Q^2)$ or $\alpha_{an}(Q^2)$ while the nonlinear term is proportional to $\alpha_s^2(Q^2)$ or $\alpha_{an}^2(Q^2)$, so the shadowing effect is weakened due to $\alpha_s(Q^2) > \alpha_{an}(Q^2)$, especially at low $Q^2$ region. The result means that the nonperturbative contribution weakens the shadowing effect. Some people [13] discussed the nuclear shadowing by applying the GLR evolution equation without considering the nonperturbative contribution. It might be interesting to restudy the nuclear shadowing by applying the GLR equation (14), which is modified by the nonperturbative contribution.

In this paper, the DGLAP and GLR equation are solved by applying the method of characteristic. From the initial conditions (12), it can be found that the start point
is $Q^2 = 4GeV^2$, which is not very low, because in very low $Q^2$ the evolution equation might be too complicated to treat and the semi-classical approximation is not a good approximation. However, the conclusions shown in this paper can be deduced easily to the very low $Q^2$ region, where the nonperturbative contribution becomes more dominant because the difference between the formula (3) and formula (5) $\alpha_s(Q^2) - \alpha_{an}(Q^2)$ which is $(0.5 - 0.4) = 0.1$ at $Q^2 = 0.65GeV^2$ is much more notable than that $(0.3 - 0.29) = 0.01$ at $Q^2 = 4GeV^2$.

In summary, it is believed that QCD, which includes the perturbative part and the nonperturbative part, is a complete theory to describe all strong interaction experiments. Nevertheless, as the fundamental dynamical model, the DGLAP or GLR equation itself only reflects the perturbative effect. Unfortunately, almost all strong interaction experiments such as the deep-inelastic $ep$ scattering at HERA involve both perturbative effect and nonperturbative effect. The purpose of the paper is to develop a fundamental dynamical model which itself includes nonperturbative effect. By studying the model, we make the conclusions: (i) the nonperturbative contribution suppresses the evolution rate at the low $Q^2$, small-x region; (ii) the nonperturbative contribution weakens the shadowing effect. Those conclusions are helpful to explain the recent experiments in low $Q^2$ region.

If the results of this paper are compared with the recent HERA data in detail, the quark distribution must also be discussed in the low $Q^2$ region, but it should be noted that the recent HERA data is available at a few isolated values of averaged x and $Q^2$, especially in very low $Q^2$, so how to analyse the HERA experiments and compare the experiment data with the results of the model developed in this paper will be a challenging work. Therefore in this paper the quark distribution is not discussed. However, the method developed in this paper can be easily extended to the evolution equation for the quark distribution. Even the method can also be applied to discuss the parton distribution at large-x, low $Q^2$ region, because the DGLAP equation modified by the nonperturbative contribution can be applied to both the large-x region and small-x region (in small-x region, it is possible that the GLR equation describes the parton behavior, but it is difficult to check the GLR equation because the nonperturbative contribution weakens...
the shadowing effect showed in this paper).

The main purpose of this paper is to developed a method, which the evolution equation itself includes the nonperturbative contribution. The method has the important theoretical feature that it suggests a smooth transition from the low $Q^2$ ("soft") , where nonperturbative contribution dominates, to the large $Q^2$ ("hard") region, where the perturbative contribution dominates and the nonperturbative contribution can be neglected. Although the method is only a first step in considering the nonperturbative contribution to QCD dynamical equations, it can be checked not only by the HERA experiments but also by other strong interaction experiments because the method proposed in this paper is adaptable to wide region.

The author is grateful to C.Wang for his helpful discussions. This work was supported in part by the Foundation of Shanghai Jiaotong University.

References

[1] L.N.Lipatov, Phys. Rep. 286 (1997) 131
[2] B.Badelek, J.Kwiecinski, Rev. Mod. Phys. 68 (1996) 445
[3] B.Badelek, K.Charchula, M.Krawczyk, and J.Kwiecinski, Rev. Mod. Phys. 64(1992) 927
[4] L.V.Gribov, E.M.Levin and M.G.Ryskin, Phys. Rep. 100 (1983) 1
[5] E.M.Levin and M.G. Ryskin, Phys. Rep. 189 (1990) 269
[6] I.Ab et al., Nucl. Phys. B 407 (1993) 515; M.Derrick et al., Phys. Lett. B 316 (1993) 412;
[7] ZEUS Collaboration, M.Derrick et al., Phys. Lett. B 407 (1997) 432
Figure Captions

Fig. 1. The dashed curve corresponds to the results of Eqs. (10) and the solid curve corresponds to the results of the DGLAP equation without considering the nonperturbative contribution.

Fig. 2. As for Fig. 1, except for the GLR equation in the case $R = 2eV^2$. 
Fig. 1, Bo He, Nonperturbative contribution to...
Fig. 2, Bo He, Nonperturbative contribution to...