Renormalization and two loop electroweak corrections to lepton anomalous dipole moments in the standard model and beyond (I): heavy fermion contributions

Tai-Fu Feng*, Xiu-Yi Yang

Department of Physics, Dalian University of Technology, Dalian, 116024, China

(Dated: January 12, 2009)

Abstract

Applying effective Lagrangian method and on-shell scheme, we analyze the electroweak corrections to anomalous dipole moments of lepton from some special two loop diagrams in which a closed heavy fermion loop is attached to the virtual gauge bosons or Higgs fields. As the masses of virtual fermions in inner loop are much heavier than the electroweak scale, we verify the final results satisfying the decoupling theorem explicitly if the interactions among Higgs and heavy fermions do not contain the nondecoupling couplings. At the decoupling limit, we also present the leading corrections to lepton anomalous dipole moments from those two loop diagrams in some popular extensions of the standard model, such as the fourth generation, supersymmetry, universal extra dimension, and the littlest Higgs with T-parity.

PACS numbers: 11.30.Er, 12.60.Jv, 14.80.Cp

Keywords: effective Lagrangian, anomalous dipole moments, two loop electroweak corrections

* email: fengtf@dlut.edu.cn
I. INTRODUCTION

At both aspects of experiment and theory, the magnetic dipole moments (MDMs) of leptons draw great attention of physicists because of their obvious importance. The anomalous dipole moments of lepton not only provide a potential window to detect new physics beyond the standard model (SM), but also can be used for testing loop effect in electroweak theories. The current experimental world average of the muon MDM is

\[ \sigma_{\mu}^{\text{exp}} = 11 659 208 \pm 6 \times 10^{-10}. \] (1)

Contributions to the MDM of muon are generally divided into three sectors: QED loops, hadronic contributions as well as electroweak corrections. The largest uncertainty of the SM prediction originates from the evaluation of hadronic vacuum polarization and light-by-light corrections. Depending on which evaluation of hadronic vacuum polarization is chosen, the differences between the SM predictions and experimental result lie in the range \(1.3\sigma \sim 3.8\sigma\) \cite{2,3}.

For the electroweak corrections, the standard one loop contribution amounts to \(19.5 \times 10^{-10}\), and the one loop corrections from new physics sector are generally suppressed by \(\Lambda_{\text{ew}}^2 / \Lambda_{\text{NP}}^2\). Here \(\Lambda_{\text{ew}}\) denotes the electroweak energy scale, and \(\Lambda_{\text{NP}}\) denotes the energy scale of new physics. Comparing with the analysis at one loop level, the two loop analysis is more complicated and less advanced. Utilizing the heavy mass expansion approximation (HME) together with the projection operator method, the authors of Ref.\cite{4} have evaluated the two loop standard electroweak corrections to muon MDM. Within the framework of CP conservation, Ref.\cite{5} presents the supersymmetric corrections from some special two-loop diagrams where a closed chargino (neutralino) or scalar fermion loop is inserted into those two-Higgs-doublet one-loop diagrams. Ref.\cite{6} discusses the contributions to muon MDM from the effective vertices \(H^\pm W^\mp \gamma, h_0(H_0)\gamma\gamma\) which are induced by the scalar quarks of the third generation. Furthermore, the contributions from two loop Bar-Zee-type diagrams to the electric dipole moments (EDMs) of light fermions are discussed extensively in literature \cite{7}.

In this paper, we calculate the corrections to the anomalous dipole moments of lepton from some special diagrams in which a closed heavy fermion loop is attached to the virtual
electroweak gauge or Higgs fields. The effective Lagrangian method can yield the one loop electroweak corrections to lepton MDMs and EDMs exactly in the SM and beyond, and has been adopted to calculate the two loop supersymmetric corrections for the branching ratio of $b \rightarrow s\gamma$ [8], neutron EDM [9] and lepton MDMs and EDMs [10, 11]. In concrete calculation, we assume that all external leptons and photon are off-shell, then expand the amplitude of corresponding triangle diagrams according to the external momenta of leptons and photon. Using loop momentum translating invariance, we formulate the sum of amplitude from those triangle diagrams corresponding to same self energy in the form which explicitly satisfies the Ward identity required by the QED gauge symmetry, then get all dimension 6 operators together with their coefficients. After the equations of motion are applied to external leptons, higher dimensional operators, such as dimension 8 operators, also contribute to the lepton MDMs and EDMs in principle. However, the contributions of dimension 8 operators contain the additional suppression factor $m_l^2/\Lambda_{\text{ew}}^2$ comparing with that of dimension 6 operators, where $m_l$ is the mass of lepton. Setting $\Lambda_{\text{ew}} \sim 100\text{GeV}$, one obtains easily that this suppression factor is about $10^{-6}$ for the muon lepton. Under current experimental precision, it implies that the contributions of all higher dimension operators ($D \geq 8$) can be neglected safely.

We adopt the naive dimensional regularization with the anticommuting $\gamma_5$ scheme, where there is no distinction between the first 4 dimensions and the remaining $D - 4$ dimensions. Since the bare effective Lagrangian contains the ultraviolet divergence which is induced by divergent subdiagrams, we give the renormalized results in the on-mass-shell scheme [12]. Additional, we adopt the nonlinear $R_\xi$ gauge with $\xi = 1$ for simplification [13]. This special gauge-fixing term guarantees explicit electromagnetic gauge invariance throughout the calculation, not just at the end because the choice of gauge-fixing term eliminates the $\gamma W^{\pm} G^\mp$ vertex in the Lagrangian.

This paper is composed of the sections as follows. In section II we introduce the effective Lagrangian method and our notations. We will demonstrate how to obtain the identities among two loop integrals from the loop momentum translating invariance through an example, then obtain the corrections from the relevant diagrams to the lepton MDMs and EDMs. Section III is devoted to the analysis and discussion in some concrete electroweak...
models. In section IV we give our conclusion. Some tedious formulae are collected in the appendices.

II. THE CORRECTIONS FROM THE RELATING DIAGRAMS

The lepton MDMs and EDMs can actually be written as the operators

\[ \mathcal{L}_{MDM} = \frac{e}{4 m_i} a_i \bar{l} \sigma^{\mu \nu} l \, F_{\mu \nu}, \]

\[ \mathcal{L}_{EDM} = -\frac{i}{2} d_i \bar{l} \sigma^{\mu \nu} \gamma_5 l \, F_{\mu \nu}. \]  

Here, \( \sigma^{\mu \nu} = i [\gamma_{\mu}, \gamma_{\nu}] / 2 \), \( l \) denotes the lepton fermion which is on-shell, \( F_{\mu \nu} \) is the electromagnetic field strength, \( m_i \) is the lepton mass and \( e \) represents the electric charge, respectively.

It is convenient to get the corrections from loop diagrams to lepton MDMs and EDMs in terms of the effective Lagrangian method, if the loop diagrams contain the virtual fields which are much heavier than the external lepton, i.e. \( m_\nu \gg m_i \) with \( m_\nu \) denoting the mass scale of virtual fields. Since \( \not{p'} = \not{p} = m_i \ll m_\nu \) for on-shell leptons and \( \not{k} \rightarrow 0 \ll m_\nu \) for photon, we can expand the amplitude of corresponding triangle diagrams according to the external momenta of leptons and photon. The two loop diagrams also contain some virtual light freedoms generally, such as virtual neutrinos, charged leptons or photon, and it is unsuitable to expand the propagators of light freedoms in powers of external momenta obviously. In order to obtain the corrections to lepton MDM and EDM properly, we should firstly match the amplitude of two loop diagrams from full theory to that of corresponding diagrams from effective theory which is composed by the QED Lagrangian and some higher dimension operators of light freedoms, then extract the Wilson coefficients of those high dimension operators which are only depend on the masses of virtual heavy freedoms as well as the possible matching scales. Finally, we strictly analyze the amplitude of corresponding diagrams from effective theory to obtain the contributions from the virtual light freedoms to lepton MDMs and EDMs. As discussed in the section IV it is enough to retain only those dimension 6 operators in later calculations:

\[ \mathcal{O}_{1}^{\mp} = \frac{1}{(4 \pi)^2} \bar{l} (\not{\Phi})^3 \omega_{\pm} l, \]
\[ \mathcal{O}_1^\mp = \frac{e Q_i^\mp}{(4\pi)^2} \left( i\mathcal{D}_\mu \right) \gamma^\mu F \cdot \sigma \omega_{\mp} l, \]

\[ \mathcal{O}_2^\mp = \frac{e Q_i^\mp l}{(4\pi)^2} [l F \cdot \sigma \gamma^\mu (i\mathcal{D}_\mu l), \]

\[ \mathcal{O}_3^\mp = \frac{e Q_i^\mp l}{(4\pi)^2} \bar{l} F \cdot \sigma \gamma^\mu \omega_{\mp} l, \]

\[ \mathcal{O}_4^\mp = \frac{e Q_i^\mp l}{(4\pi)^2} \bar{l} (\partial^\mu F_{\mu\nu}) \gamma^\nu \omega_{\mp} l, \]

\[ \mathcal{O}_5^\mp = \frac{m_i}{(4\pi)^2} \bar{l} (\not\!p)^2 \omega_{\mp} l, \]

\[ \mathcal{O}_6^\mp = \frac{e Q_i^\mp m_i}{(4\pi)^2} \bar{l} F \cdot \sigma \omega_{\mp} l, \]

with \( \mathcal{D}_\mu = \partial_\mu + ie A_\mu \) and \( \omega_{\mp} = (1 \mp \gamma_5)/2 \).

Certainly, all dimension 6 operators in Eq. (3) induce the effective couplings among photons and leptons. The effective vertices with one external photon are written as

\[ \mathcal{O}_1^\mp = \frac{ie Q_i^\mp}{(4\pi)^2} \left\{ \left( (p + k)^2 + p^2 \right) \gamma_\rho + (\not\!p + \not\!k) \gamma_\rho \not\!p \right\} \omega_{\mp}, \]

\[ \mathcal{O}_2^\mp = \frac{ie Q_i^\mp}{(4\pi)^2} (\not\!p + \not\!k) [\gamma_\rho, \gamma_\rho] \omega_{\mp}, \]

\[ \mathcal{O}_3^\mp = \frac{ie Q_i^\mp}{(4\pi)^2} [\gamma_\rho, \gamma_\rho] \gamma_\rho \omega_{\mp}, \]

\[ \mathcal{O}_4^\mp = \frac{ie Q_i^\mp}{(4\pi)^2} (k^2 \gamma_\rho - \not\!k \gamma_\rho) \omega_{\mp}, \]

\[ \mathcal{O}_5^\mp = \frac{ie Q_i^\mp}{(4\pi)^2} \left\{ (\not\!p + \not\!k) \gamma_\rho + \gamma_\rho \not\!p \right\} m_i \omega_{\mp}, \]

\[ \mathcal{O}_6^\mp = \frac{ie Q_i^\mp}{(4\pi)^2} [\gamma_\rho, \gamma_\rho] m_i \omega_{\mp}. \] (4)

If the full theory is invariant under the combined transformation of charge conjugation, parity and time reversal (CPT), the induced effective theory preserves the symmetry after the heavy freedoms are integrated out. The fact implies the Wilson coefficients of the operators \( \mathcal{O}_{2,3,6}^\mp \) satisfying the relations

\[ C_2^\mp = C_3^{\mp*}, \quad C_6^+ = C_6^{-*}, \] (5)

where \( C_i^\mp \) (\( i = 1, 2, \ldots, 6 \)) represent the Wilson coefficients of the corresponding operators \( \mathcal{O}_i^\mp \) in the effective Lagrangian. After applying the equations of motion to external leptons,
we find that the concerned terms in the effective Lagrangian are transformed into

\[
C^+ O_1^+ + C^+_2 O_3^+ + C_6^+ O_6^+ + C_6^+ O_6^-
\]

\[
\Rightarrow (C^+ + C_{2}^{*-} + C_6^+) O_6^+ + (C_{2}^{+*} + C^- + C_6^{+*}) O_6^-
\]

\[
= \frac{e Q_f m_i}{(4\pi)^2} \left\{ \Re \left( C^+_2 + C_2^{*-} + C_6^+ \right) \bar{l} \sigma^\mu\nu l + i \Im \left( C^+_2 + C_2^{*-} + C_6^+ \right) \bar{l} \sigma^\mu\nu 5 \right\} F_{\mu\nu} .
\]  

(6)

Here, \( \Re(\cdots) \) denotes the operation to take the real part of a complex number, and \( \Im(\cdots) \) denotes the operation to take the imaginary part of a complex number. Applying Eq.(2) and Eq.(6), we finally get

\[
a_l = \frac{4 Q_f m_i^2}{(4\pi)^2} \Re \left( C^+_2 + C_2^{*-} + C_6^+ \right) ,
\]

\[
d_l = -\frac{2 e Q_f m_i}{(4\pi)^2} \Im \left( C^+_2 + C_2^{*-} + C_6^+ \right) .
\]  

(7)

In other words, the MDM of lepton is proportional to real part of the effective coupling \( C^+_2 + C_2^{*-} + C_6^+ \), as well as the EDM of lepton is proportional to imaginary part of the effective coupling \( C^+_2 + C_2^{*-} + C_6^+ \).

After expanding the amplitude of corresponding triangle diagrams, we extract the Wilson coefficients of operators in Eq.(4) which are formulated in the linear combinations of one and two loop vacuum integrals in momentum space, then obtain the MDMs and EDMs of leptons. Taking those diagrams in which a closed heavy fermion loop is inserted into the propagator of charged gauge boson as an example, we show in detail how to obtain the MDMs and EDMs of leptons through the effective Lagrangian method.

A. The corrections from the diagrams where a closed heavy fermion loop is inserted into the self energy of \( W^\pm \) gauge boson

In order to get the amplitude of the diagrams in Fig.1, one can write the renormalizable interaction among the charged electroweak gauge boson \( W^\pm \) and the heavy fermions \( F_{\alpha,\beta} \) in a more universal form as

\[
\mathcal{L}_{WFF} = \frac{e}{s_w} W^{-\mu} \bar{F}_\alpha \gamma^\mu (\zeta^L_{\alpha\beta} \omega_- + \zeta^R_{\alpha\beta} \omega_+) F_\beta + h.c. ,
\]  

(8)
FIG. 1: The relating two-loop diagrams in which a closed heavy fermion loop is attached to virtual $W^\pm$ bosons, where the diagrams (e,f,g) contribute the counter terms to cancel the ultraviolet divergence arisen by divergent subdiagrams in (a,b,c,d) respectively.

where the concrete expressions of $\zeta^{L,R}_{\alpha\beta}$ depend on the models employed in our calculation.

The conservation of electric charge requires $Q_\beta - Q_\alpha = 1$, where $Q_{\alpha,\beta}$ denote the electric charges of the heavy fermions $F_{\alpha,\beta}$ respectively.

Applying Eq.(8), we write firstly the amplitude of those two loop diagrams in Fig.1. For example, the amplitude of Fig.1(a) can be formulated as

$$iA^{1(a)}_{ww,\rho}(p, k) = \overline{\psi}_f \int \frac{d^Dq_1}{(2\pi)^D} \frac{d^Dq_2}{(2\pi)^D} \left(-ie\frac{\Lambda^\varepsilon_{\text{RE}}}{\sqrt{2s_w}}\gamma^\mu\omega_-\gamma^\nu\omega_-\gamma^\rho\psi_f \right)$$

$$\times \frac{-i}{(q_1 - p - k)^2 - m_w^2} \left\{ie \left[ -g_{\mu\sigma}(2p + k - 2q_1)\rho + 2(g_{\rho\mu}k_\sigma - g_{\rho\sigma}k_\mu) \right] \right\}$$
Eq. (9) as

\[
\times \frac{-i}{(q_1 - p)^2 - m_w^2} \frac{-i}{(q_1 - p)^2 - m_w^2} \text{Tr} \left[ \left( i \frac{e \Lambda_{\text{RE}}}{s_w} \right) \gamma^\sigma \{ \xi_{\alpha \beta}^{L} \omega_{-} + \xi_{\alpha \beta}^{R} \omega_{+} \} \right] - i (\bar{q}_2 - \bar{p} + m_{F_3}) \left( q_2 - p \right)^2 - m_{F_3}^2. \tag{9}
\]

Here \( \Lambda_{\text{RE}} \) denotes the renormalization scale that can take any value in the range from the electroweak scale \( \Lambda_{\text{ew}} \) to the new physics scale \( \Lambda_{\text{NP}} \) naturally, and we adopt the shortcut notations: \( c_w = \cos \theta_w, \ s_w = \sin \theta_w \), with \( \theta_w \) denoting the Weinberg angle. Additionally, \( p, k \) are the incoming momenta of lepton and photon fields, \( \rho \) is the Lorentz index of photon. Certainly, the amplitude does not depend on how to mark the momenta of virtual fields for the invariance of loop momentum translation. It can be checked easily that the sum of amplitude for diagrams in Fig.1 satisfies the Ward identity required by the QED gauge invariance

\[
k^\mu A_{ww,\rho}(p, k) = e [\Sigma_{ww}(p + k) - \Sigma_{ww}(p)], \tag{10}
\]

where \( A_{ww,\rho} \) denotes the sum of amplitudes for the diagrams (a), (b), (c) and (d) in Fig.1 as well as \( \Sigma_{ww} \) denotes the amplitude of corresponding self energy diagram, respectively.

According the external momenta of leptons and photon, we expand the amplitude in Eq. (9) as

\[
i A_{1(a)}^{1}\rho(p, k) = -i e^5 \frac{\Lambda_{\text{RE}}^4}{2 s_w^4} \int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{q_1^2 (q_1^2 - m_w^2)^3 ((q_2 - q_1)^2 - m_{F_3}^2)}{(q_2 - m_{F_3}^2)}
\]

\[
\times \left\{ 1 + \frac{2 q_1 \cdot (3 p + k)}{q_1^2 - m_w^2} + \frac{2 q_1 \cdot p}{q_2^2 - m_{F_3}^2} - \frac{2 p^2 + (p + k)^2}{q_1^2 - m_w^2} - \frac{p^2}{q_2^2 - m_{F_3}^2} + \frac{4 (q_1 \cdot (p + k))^2 + 8 (q_1 \cdot p) (q_1 \cdot (p + k)) + 12 (q_1 \cdot p)^2}{q_1^2 - m_w^2} \right\} \frac{1}{(q_2 - m_{F_3}^2)^2}
\]

\[
+ 2 (g_{\rho \sigma} k_\sigma - g_{\rho \sigma} k_\rho) \text{Tr} \left[ \gamma^\sigma \left\{ \xi_{\alpha \beta}^{L} \omega_{-} + \xi_{\alpha \beta}^{R} \omega_{+} \right\} (\bar{q}_2 - \bar{p} + m_{F_3}) \right]
\]

\[
\times \gamma^\nu \left\{ \xi_{\alpha \beta}^{L} \omega_{-} + \xi_{\alpha \beta}^{R} \omega_{+} \right\} (\bar{q}_2 - \bar{p} + m_{F_3}). \tag{11}
\]

since we only consider the corrections to lepton MDM and EDM from dimension 6 operators.
Because the denominators of all terms are invariant under the reversal $q_1 \rightarrow -q_1, q_2 \rightarrow -q_2$, those terms in odd powers of loop momenta can be abandoned, and the terms in even powers of loop momenta can be simplified by

$$
\int \frac{d^D q_1 \, d^D q_2 \, q_{1\mu} q_{1\nu} q_{1\rho} q_{1\sigma} q_{1\alpha} q_{1\beta}}{(2\pi)^D (2\pi)^D ((q_2 - q_1)^2 - m_0^2)(q_1^2 - m_1^2)(q_2^2 - m_2^2)} \quad \int \frac{d^D q_1 \, d^D q_2 \, (q_1)^2 \, (q_2)^2 q_1 \cdot q_2}{D(D + 2)(D + 4)} \int \frac{d^D q_1 \, d^D q_2 \, q_{1\mu} q_{1\nu} q_{1\rho} q_{1\sigma} q_{1\alpha} q_{1\beta}}{(2\pi)^D (2\pi)^D ((q_2 - q_1)^2 - m_0^2)(q_1^2 - m_1^2)(q_2^2 - m_2^2)}
$$

$$
\int \frac{d^D q_1 \, d^D q_2 \, q_{1\mu} q_{1\nu} q_{1\rho} q_{1\sigma} q_{1\alpha} q_{1\beta}}{(2\pi)^D (2\pi)^D ((q_2 - q_1)^2 - m_0^2)(q_1^2 - m_1^2)(q_2^2 - m_2^2)} S_{\mu\nu\rho\sigma\alpha\beta} = -\frac{q_1^2 (q_1 \cdot q_2)^2 - (q_2^2) q_2^2}{D(D - 1)(D + 2)} T_{\mu\nu\rho\sigma\alpha\beta} - \frac{q_2^2 (q_1 \cdot q_2)^2 - (q_1^2) q_2^2}{D(D - 1)(D + 2)} T_{\mu\nu\rho\sigma\alpha\beta}
$$

$$
\int \frac{d^D q_1 \, d^D q_2 \, q_{1\mu} q_{1\nu} q_{1\rho} q_{1\sigma} q_{1\alpha} q_{1\beta}}{(2\pi)^D (2\pi)^D ((q_2 - q_1)^2 - m_0^2)(q_1^2 - m_1^2)(q_2^2 - m_2^2)} \times \left[ \frac{Dq_1^2 (q_1 \cdot q_2)^2 - (q_1^2) q_2^2}{D(D - 1)(D + 2)} S_{\mu\nu\rho\sigma\alpha\beta} + \frac{(q_1 \cdot q_2)^3 - q_2 q_1 \cdot q_2}{D(D - 1)(D + 2)} (g_{\mu\alpha} (g_{\nu\beta} g_{\rho\delta}) + g_{\mu\delta} (g_{\nu\alpha} g_{\rho\beta} + g_{\nu\beta} g_{\rho\alpha})) \right],
$$

and those similar formulae presented in Eq.(5) of Ref.[8], where the tensors are defined as

$$
T_{\mu\nu\rho\sigma\alpha\beta} = g_{\mu\nu} g_{\rho\sigma} + g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho},
$$

$$
S_{\mu\nu\rho\sigma\alpha\beta} = g_{\mu\nu} T_{\rho\sigma\alpha\beta} + g_{\mu\rho} T_{\nu\sigma\alpha\beta} + g_{\mu\sigma} T_{\nu\rho\alpha\beta} + g_{\mu\alpha} T_{\nu\rho\sigma\beta} + g_{\mu\beta} T_{\nu\rho\sigma\alpha}.
$$

Summing over those indices which appear both as superscripts and subscripts, we derive all possible dimension 6 operators in the momentum space together with their coefficients which are expressed in the linear combinations of one and two loop vacuum integrals. In a similar way, one obtains the amplitude of other diagrams. Before integrating with the loop momenta, we apply the loop momentum translating invariance to formulate the sum of those amplitude in explicitly QED gauge invariant form, then extract the Wilson coefficients of those dimension 6 operators listed in Eq.[3]. Actually, we can easily verify the equation

$$
\int \int \frac{d^D q_1 \, d^D q_2 \, q_{1\mu}}{(2\pi)^D (2\pi)^D (q_1^2 - m_1^2)(q_2^2 - m_2^2)((q_2 - q_1)^2 - m_0^2)} \equiv 0.
$$
Performing an infinitesimal translation \( q_1 \to q_1, q_2 \to q_2 - a \) with \( a_\rho \to 0 (\rho = 0, 1, \cdots, D) \),
one can write the left-handed side of above equation as
\[
\int \int \frac{d^Dq_1 d^Dq_2}{(2\pi)^D (2\pi)^D (q_1^2 - m_1^2)(q_2^2 - m_2^2)((q_2 - q_1)^2 - m_0^2)} q_{1\mu} \cdot q_{2\mu}
= \int \int \frac{d^Dq_1 d^Dq_2}{(2\pi)^D (2\pi)^D (q_1^2 - m_1^2)(q_2^2 - m_2^2)((q_2 - q_1)^2 - m_0^2)} q_{1\mu} \cdot q_{2\mu}
\times \left\{ 1 + \frac{2q_2 \cdot a}{q_2^2 - m_2^2} + \frac{2(q_2 - q_1) \cdot a}{(q_2 - q_1)^2 - m_0^2} + \cdots \right\}
\tag{15}
\]
This result implies
\[
\int \int \frac{d^Dq_1 d^Dq_2}{(2\pi)^D (2\pi)^D (q_1^2 - m_1^2)(q_2^2 - m_2^2)((q_2 - q_1)^2 - m_0^2)} q_{1\cdot q_2}
= \int \int \frac{d^Dq_1 d^Dq_2}{(2\pi)^D (2\pi)^D (q_1^2 - m_1^2)(q_2^2 - m_2^2)((q_2 - q_1)^2 - m_0^2)} q_{1\cdot q_2}
\tag{16}
\]
In a similar way, other identities listed in Ref. [8] can be derived. Using the expression of
two loop vacuum integral [14]
\[
\Lambda_{RE}^4 \int \int \frac{d^Dq_1 d^Dq_2}{(2\pi)^D (2\pi)^D (q_1^2 - m_1^2)(q_2^2 - m_2^2)((q_2 - q_1)^2 - m_0^2)} \frac{1}{D_{x_0, x, y, z}}
= \frac{\Lambda^2}{2(4\pi)^4} \Gamma^2(1 + \epsilon) \left( 4\pi x_R \right)^2 x^2 \left\{ -\frac{1}{\epsilon^2} (x_0 + x_1 + x_2)
+ \frac{1}{\epsilon^2} (2(x_0 \ln x_0 + x_1 \ln x_1 + x_2 \ln x_2) - x_0 - x_1 - x_2)
- 2(x_0 + x_1 + x_2) + 2(x_0 \ln x_0 + x_1 \ln x_1 + x_2 \ln x_2)
- x_0 \ln^2 x_0 - x_1 \ln^2 x_1 - x_2 \ln^2 x_2 - \Phi(x_0, x_1, x_2) \right\}
\tag{17}
\]
and
\[
\Phi(x, y, z) = (x + y - z) \ln x \ln y + (x - y + z) \ln x \ln z
+ (-x + y + z) \ln y \ln z + \text{sign}(\lambda^2) \sqrt{\lambda^2} |\Psi(x, y, z)|,
\]
\[
\frac{\partial \Phi}{\partial x}(x, y, z) = \ln x \ln y + \ln x \ln z - \ln y \ln z + 2 \ln x + \frac{x - y - z}{\sqrt{\lambda^2}} |\Psi(x, y, z)|,
\tag{18}
\]
one obtains easily
\[
\frac{\Lambda_{RE}^4}{\Lambda^2} \left\{ \int \int \frac{d^Dq_1 d^Dq_2}{(2\pi)^D (2\pi)^D (q_1^2 - m_1^2)(q_2^2 - m_2^2)((q_2 - q_1)^2 - m_0^2)} q_{1\cdot q_2} \right\}
\]
formulate the sum of the amplitude for Fig.1(a,b,c,d) satisfying QED gauge invariance and the relevant terms in the effective Lagrangian are formulated as:

\[
\mathcal{L}_{\text{Eff}}^{\text{tot}} = -\frac{(4\pi)^2 e^4}{s^4 Q_f} \cdot \Lambda_{\text{Re}}^4 \int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{1}{q_1^1 q_2^1 (q_1^2 - m_w^2)^2 (q_2^2 - m_{\tau}^2 - m_{\tau}^2)(q_2^2 - m_0^2)}
\]

which is equivalent to the identity Eq.(16). Here, \( \epsilon = 2 - D/2 \) with \( D \) denoting the dimension of space-time, \( \Lambda \) is an energy scale to define \( x_i = m_i^2/\Lambda^2 \), and \( x_R = \Lambda_{\text{Re}}^2/\Lambda^2 \). Additionally, \( \lambda^2 = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz \), and the concrete expression of \( \Psi(x,y,z) \) can be found in the appendix. Actually, the equation Eq.(19) provides a crosscheck of Eq.(17) and Eq.(18) rather than a verification of Eq.(16). In the limit \( z \ll x, y \), we can expand \( \Phi(x,y,z) \) according \( z \) as

\[
\Phi(x,y,z) = \varphi_0(x,y) + z\varphi_1(x,y) + \frac{z^2}{2!} \varphi_2(x,y) + \frac{z^3}{3!} \varphi_3(x,y) + 2z\left(\ln z - 1\right)\pi_1(x,y) + 2z^2 \left(\frac{\ln z}{2!} - \frac{3}{4}\right)\pi_2(x,y) + 2z^3 \left(\frac{\ln z}{3!} - \frac{11}{36}\right)\pi_3(x,y) + \cdots
\]

with

\[
\begin{align*}
\pi_1(x,y) &= 1 + \varrho_{1,1}(x,y), \\
\pi_2(x,y) &= -\frac{x + y}{(x-y)^2} - \frac{2xy}{(x-y)^3} \ln \frac{y}{x}, \\
\pi_3(x,y) &= -\frac{1}{(x-y)^2} - \frac{12xy}{(x-y)^4} - \frac{6xy(x+y)}{(x-y)^5} \ln \frac{y}{x},
\end{align*}
\]

and the concrete expressions of function \( \varphi_i(x,y) \) \( (i = 0, 1, 2, 3) \) can be found in appendix.

After applying those identities derived from loop momentum translating invariance, we formulate the sum of the amplitude for Fig.1(a,b,c,d) satisfying QED gauge invariance and CPT symmetry explicitly, and extract the Wilson coefficients of those operators in Eq.(3).

Since only the operators \( \mathcal{O}_{x}^{\pm}_{\alpha,\beta,\gamma} \) actually contribute to the MDMs and EDMs of leptons when the equations of motion are applied to the incoming and outgoing leptons separately, the relevant terms in the effective Lagrangian are formulated as:

\[
\mathcal{L}_{\text{eff}}^{\text{tot}} = -\frac{(4\pi)^2 e^4}{s^4 Q_f} \cdot \Lambda_{\text{Re}}^4 \int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{1}{q_1^1 q_2^1 (q_1^2 - m_w^2)^2 (q_2^2 - m_{\tau}^2 - m_{\tau}^2)(q_2^2 - m_0^2)}
\]
where $Q_f = -1$ represents the charge of leptons, and the expressions of form factors $\mathcal{N}_{\text{ww}}^i (i = 1, 2, 3, 4)$ are presented in appendix.

Integrating over loop momenta, one gets the following terms in the effective Lagrangian:

\[
\mathcal{L}_{\text{eff}}^{\text{ww}} = \frac{\sqrt{2} G_F \alpha_e x_w (4\pi)^2}{\pi^2 m_{\nu}^2} \frac{\Gamma^2 (1 + \varepsilon)}{(1 - \varepsilon)^2} \left\{ \right.
\]

\[
\begin{align*}
&\times \left[ - \frac{5}{24} \frac{x_{F\alpha} + x_{F\beta}}{x_w^2} \ln x_w + T_1 (x_{F\alpha}, x_{F\beta}) \right] (O_2^+ + O_3^-) \\
&+ \left( \zeta^{L*_{\alpha\beta}}_{\alpha\beta} - \zeta^{R*_{\alpha\beta}}_{\alpha\beta} \right) T_2 (x_{F\alpha}, x_{F\beta}) \left( O_2^- + O_3^+ \right) \\
&+ \left( \zeta^{L*_{\alpha\beta}}_{\alpha\beta} + \zeta^{R*_{\alpha\beta}}_{\alpha\beta} \right) \left( O_2^- - O_3^- \right) \\
&+ \left[ \right. \\
&\left. \frac{19}{72} x_w^2 - \frac{5}{12} \ln x_w + T_3 (x_{F\alpha}, x_{F\beta}) \left( O_2^- - O_3^+ \right) \\
&\left. + \left( \zeta^{L*_{\alpha\beta}}_{\alpha\beta} - \zeta^{R*_{\alpha\beta}}_{\alpha\beta} \right) \left( O_2^- - O_3^- \right) \right] \left( O_2^- - O_3^- \right) \right\} + \cdots ,
\end{align*}
\]

where $G_F = 1.16639 \times 10^{-5}$ GeV$^{-2}$ is the 4-fermion coupling, and $\alpha_e = e^2 / 4\pi$. Note that the above result does not depend on the concrete choice of energy scale $\Lambda$, and the concrete expressions of $T_i (x, y, z)$, $\phi_i (x, y)$ ($i, j = 1, 2 \cdots$) can be found in appendix.

The charged gauge boson self energy composed of a closed heavy fermion loop induces the ultraviolet divergence in the Wilson coefficients of effective Lagrangian, the unrenormalized $W^\pm$ self energy is generally expressed as

\[
\Sigma_{\mu\nu}^w (p, \Lambda_{\text{RE}}) = \Lambda^2 A_{w0}^w g_{\mu\nu} + \left( A_{1w}^w + \frac{p^2}{\Lambda^2} A_{2w}^w + \cdots \right) (p^2 g_{\mu\nu} - p_{\mu} p_{\nu}) \\
+ \left( B_{1w}^w + \frac{p^2}{\Lambda^2} B_{2w}^w + \cdots \right) p_{\mu} p_{\nu} ,
\]

where the form factors $A_{0,1,2}^w$ and $B_{1,2}^w$ only depend on the virtual field masses and renormalization scale. Here, we omit those terms which are strongly suppressed at the limit of
heavy virtual fermion masses. The corresponding counter terms are given as

$$\Sigma_{\mu\nu}^w(p, \Lambda_{RE}) = -\left[\delta m_w^2(\Lambda_{RE}) + m_w^2 \delta Z_w(\Lambda_{RE})\right]g_{\mu\nu} - \delta Z_w(\Lambda_{RE})\left[p^2g_{\mu\nu} - p_\mu p_\nu\right].$$  \hfill (25)

The renormalized self energy is given by

$$\hat{\Sigma}_{\mu\nu}^w(p, \Lambda_{RE}) = \Sigma_{\mu\nu}^w(p, \Lambda_{RE}) + \Sigma_{\mu\nu}^{wC}(p, \Lambda_{RE}).$$  \hfill (26)

For on-shell external gauge boson $W^\pm$, we have

$$\hat{\Sigma}_{\mu\nu}^w(p, m_w^2)e^\nu(p)\bigg|_{p^2=m_w^2} = 0,$$

$$\lim_{p^2\rightarrow m_w^2} \frac{1}{p^2 - m_w^2} \hat{\Sigma}_{\mu\nu}^w(p, m_w^2)e^\nu(p) = \epsilon_\mu(p),$$  \hfill (27)

where $\epsilon(p)$ is the polarization vector of $W^\pm$ gauge boson. Inserting Eq. (24) and Eq. (25) into Eq. (27), we derive the counter terms for the $W^\pm$ self energy in on-shell scheme as

$$\delta Z_{w}^{os}(m_w) = A_1^w + \frac{m_w^2}{\Lambda^2} A_2^w = A_1^w + x_w A_2^w,$$

$$\delta m_{w}^{2,os}(m_w) = A_0^w \Lambda^2 - m_w^2 \delta Z_{w}^{os}. $$  \hfill (28)

We should derive the counter term for the vertex $\gamma W^+W^-$ here since the corresponding coupling is not zero at tree level. In the nonlinear $R_\xi$ gauge with $\xi = 1$, the counter term for the vertex $\gamma W^+W^-$ is

$$i\delta C_{\gamma W^+W^-} = ie \cdot \delta Z_w(\Lambda_{RE})\left[g_{\mu\nu}(k_1 - k_2)_\rho + g_{\nu\rho}(k_2 - k_3)_\mu + g_{\rho\mu}(k_3 - k_1)_\nu\right],$$  \hfill (29)

where $k_i$ ($i = 1, 2, 3$) denote the incoming momenta of $W^\pm$ and photon, and $\mu, \nu, \rho$ denote the corresponding Lorentz indices respectively.

We present the counter term diagrams to cancel the ultraviolet divergence contained by the bare effective Lagrangian in Fig. (e,f,g), and we can verify that the sum of corresponding amplitude satisfies the Ward identity required by the QED gauge invariance obviously.

Accordingly, the effective Lagrangian from the counter term diagrams is written as

$$\delta L_{\text{ew}}^C = \frac{e^2}{2s^2 \Lambda^2 Q_f} (4\pi x_w)^\varepsilon \frac{\Gamma(1 + \varepsilon)}{(1 - \varepsilon)} \left\{ A_0^w \left[ \frac{5}{12x_w^2} + \frac{19\varepsilon}{72x_w^2} - \frac{5\varepsilon}{12x_w^2} \ln x_w \right] + \frac{5\varepsilon}{12x_w} (A_1^w + x_w A_2^w) \right\}(O_2^- + O_3^-).$$
Adding the counter terms to bare Lagrangian Eq. (23), we cancel the ultraviolet divergence there. However, the diagrams in Fig. 1 include the virtual neutrino which belongs to light freedoms contained by the effective Lagrangian. It is unreasonable obviously in the above analysis that the propagators of virtual neutrino are expanded according to the external momenta. In order to obtain the corrections to lepton MDMs and EDMs from light freedoms properly, we match the sum of amplitude from full theory to that from effective theory [15] at first:

\[
\sum f_{\alpha,i}(m_{Vh}, m_{Vl}) \mathcal{O}_{i}^{\pm} = \sum f_{h,i}(m_{Vh}, \Lambda_{MA}) + f_{l,i}(m_{Vl}, \Lambda_{MA}) \mathcal{O}_{i}^{\pm},
\]

(31)

where \(\Lambda_{MA}\) represents the matching scale, and \(m_{Vh}, m_{Vl}\) denote the masses of virtual heavy and light freedoms respectively. The left-handed side of above equation denotes the amplitude from Fig. 1 derived through the above steps, the first term of right-handed side is the corrections to effective Lagrangian from heavy freedoms only, and the second term of right-handed side is the corrections obtained unsuitably to effective Lagrangian from light freedoms. Shrinking those heavy freedoms of Fig. 1 in a point, one obtains the corresponding Feynman diagrams of effective theory in Fig. 2. Expanding the amplitude of diagram for effective theory in powers of external momenta, we can derive the corrections

![Diagram](image-url)
\[ \sum_i f_{i,M}^\mp(m_{v_i}, \Lambda_{\text{MA}}) O_{i,M}^{\mp} \] which are originated from light freedoms purely. Inserting the concrete expressions of \( f_{i,M}^\mp(m_{v_i}, \Lambda_{\text{MA}}) \) into Eq.(31), one gets the corrections \( f_{i,M}^\mp(m_{v_i}, \Lambda_{\text{MA}}) \) which are originated from heavy freedoms only. Finally, we analyze the amplitude of Fig.2 strictly to get the corrections from virtual light freedoms to lepton MDMs and EDMs. Because the effective coupling among leptons and photon in Fig.2 is induced by the dimension 8 operators at least, the corrections from Fig.2 to lepton MDMs and EDMs contain the additional suppression factor \( m_l^2/\Lambda_{\text{EW}}^2 \) comparing with that only from the heavy freedoms. Under our approximation, the resulted lepton MDMs and EDMs are respectively formulated as

\begin{align*}
a_{i,F}^{\text{ww}} &= \frac{G_F m^2}{2\sqrt{2} \pi^2 s_w} x_w \left\{ \left( |\zeta^L_{\alpha\beta}|^2 + |\zeta^R_{\alpha\beta}|^2 \right) T_1(x_w, x_{F\alpha}, x_{F\beta}) ight. \\
&\quad + \left( |\zeta^L_{\alpha\beta}|^2 - |\zeta^R_{\alpha\beta}|^2 \right) T_2(x_w, x_{F\alpha}, x_{F\beta}) \\
&\quad + 2(x_{F\alpha} x_{F\beta})^{1/2} B(\zeta^R_{\alpha\beta} \zeta^L_{\alpha\beta}) T_3(x_w, x_{F\alpha}, x_{F\beta}) \right\}, \\
d_{i,F}^{\text{ww}} &= -\frac{G_F m c m}{2\sqrt{2} \pi^2 s_w} x_w (x_{F\alpha} x_{F\beta})^{1/2} B(\zeta^R_{\alpha\beta} \zeta^L_{\alpha\beta}) T_4(x_w, x_{F\alpha}, x_{F\beta}),
\end{align*}

which only depend on the masses of virtual fields. It should be clarified that the corrections to lepton EDMs from the diagrams (a,b,c,d) in Fig.1 do not depend on the concrete renormalization scheme adopted here since the relevant terms from bare Lagrangian do not contain the ultraviolet divergence. Using the expansion of \( \Phi(x,y,z) \) in Eq.(20), we get the asymptotic expressions of \( T_i(x, z, u), (i = 1, \cdots, 4) \) at the limit \( u \gg x \) as

\begin{align*}
T_1(x, z, u) &\approx -\frac{3 - 3Q\beta}{8x} + \frac{3 - 2Q\beta}{8x} \ln u - \frac{3 - 2Q\beta}{8x} \pi_1(z, u) \\
&\quad + \frac{(3 - 2Q\beta)z + 6u \partial \pi_1}{8x}(z, u) - \frac{(3 - 2Q\beta)z u + (3 + 2Q\beta)u^2 \partial^2 \pi_1}{8x}(z, u) \\
&\quad + \frac{1}{12x} u^2 (z - u) \frac{\partial^2 \pi_1}{\partial u^2}(z, u) - \frac{2(4 - Q\beta)z - (1 - 2Q\beta)u}{16x} \pi_2(z, u) \\
&\quad + \frac{7}{16x} (z - u) \frac{\partial \pi_2}{\partial u}(z, u) + \frac{z - u}{24} \pi_3(z, u) + \cdots, \\
T_2(x, z, u) &\approx -\frac{\ln u}{8x} + \frac{1}{8x} \pi_1(z, u) - \frac{(5 - Q\beta)u \partial \pi_1}{4x}(z, u) + \frac{u(z - u) \partial^2 \pi_1}{8x}(z, u) + \cdots, \\
T_3(x, z, u) &\approx \frac{1 - 3Q\beta}{24xu} + \frac{1 + Q\beta \partial \pi_1}{2x}(z, u) + \frac{1 - Q\beta \partial \pi_1}{4x}(z, u) - \frac{u(z - u) \partial^2 \pi_1}{16x}(z, u) \\
&\quad - \frac{Q\beta z - (3 + Q\beta)u \partial \pi_1}{8x}(z, u) - \frac{(1 - Q\beta)z - u}{8x} \frac{\partial^2 \pi_1}{\partial z \partial u}(z, u) \\
&\quad + \frac{3}{4x} \pi_3(z, u) - \frac{1 - Q\beta}{16x} (z - u) \left[ \frac{\partial \pi_2}{\partial u} + \frac{\partial \pi_2}{\partial z} \right](z, u) + \cdots,
\end{align*}
This implies that the leading contributions contained in the asymptotic form of Eq. 32 under the assumption $m_F = m_{F_\alpha} = m_{F_\beta} \gg m_w$ can be written as:

$$d_{iww}^{\alpha \beta} \approx \frac{G_{\alpha \beta} e m_l (2 + Q_\beta)}{16 \sqrt{2 \pi^3 s_w^3}} \Im (\zeta_{R_{\alpha \beta}}^* \zeta_{L_{\alpha \beta}}) + \cdots,$$

$$d_{iww}^{\alpha \beta} \approx -\frac{G_{\alpha \beta} e m_l (2 + Q_\beta)}{16 \sqrt{2 \pi^3 s_w^3}} \Re (\zeta_{R_{\alpha \beta}}^* \zeta_{L_{\alpha \beta}}) + \cdots,$$

where ellipses represent those relatively unimportant corrections.

Comparing the result in Eq. (32), the contributions from the corresponding diagrams contain the additional suppressed factor $m_l^2 / \Lambda_{EW}^2$ when both of virtual charged gauge bosons in Fig. 1(a,b,c,d) are replaced with the charged Goldstone $G^\pm$. However, we should consider the corrections from those two loop diagrams in which one of virtual charged gauge bosons is replaced with the charged Goldstone $G^\pm$ since it represents the longitudinal component of charged gauge boson in nonlinear $R_\xi$ gauge. For many extensions of the SM contain the charged Higgs, we also generalize the result directly to the diagrams in which a closed heavy loop is attached to the virtual $H^\pm$ and $W^\pm$ fields simultaneously.

B. The corrections from the diagrams where a closed heavy fermion loop is attached to the virtual $W^\pm, G^\pm (H^\pm)$ bosons

Similarly, the renormalizable interaction among the electroweak charged Goldstone/Higgs $G^\pm (H^\pm)$ and the heavy fermions $F_{\alpha \beta}$ can be expressed in a more universal form as

$$\mathcal{L}_{s_{\pm FF}} = \frac{e}{s_w} \left[ G^- F_{\alpha \beta} (G_{\alpha \beta}^{cL} \omega_- + G_{\alpha \beta}^{cR} \omega_+) F_\beta + H^- F_{\alpha \beta} (H_{\alpha \beta}^{cL} \omega_- + H_{\alpha \beta}^{cR} \omega_+) F_\beta \right] + h.c.,$$

where the concrete expressions of $G_{\alpha \beta}^{cL,R}, H_{\alpha \beta}^{cL,R}$ depend on the models employed in our calculation, the conservation of electric charge requires $Q_\beta - Q_\alpha = 1$. Generally, the couplings
FIG. 3: The two-loop diagrams in which a closed heavy fermion loop is attached to the virtual $W^\pm$, $G^\pm$ or $H^\pm$ bosons. In concrete calculation, the contributions from those mirror diagrams should be included also.

among the charged Goldstone/Higgs and leptons are written as

$$\mathcal{L}_{S^\pm l\nu} = \frac{e m_i}{\sqrt{2} m_w s_w} \left[ G^- \bar{l}_i \nu_i + B_c H^- \bar{l}_i \nu_i \right] + h.c. ,$$

where the parameter $B_c$ also depends on the concrete models adopted in our analysis. In full theory, the couplings in Eq. (35) induce the corrections to lepton MDMs and EDMs through the diagrams in Fig 3 and the corresponding diagram of effective theory is same as that presented in Fig.2.

After the steps taken in $WW$ sector, the corresponding corrections from the diagrams in Fig 3 to the lepton MDMs and EDMs are formulated as

$$d^{WG}_1 = \frac{G_F e_i m_i^2}{32 \sqrt{2} \pi^3 s_w^2} \left\{ \left( \frac{x_{F_3}}{x_w} \right)^{1/2} F_1 (x_w, x_{F_3}, x_{F_3}, x_{F_3}) \Re \left( G^{cL}_{\beta_\alpha \beta_\alpha} + G^{cR}_{\beta_\alpha \beta_\alpha} \right) \right\} ,$$

$$d^{WG}_4 = \frac{G_F e_i m_i^2}{64 \sqrt{2} \pi^3 s_w^2} \left\{ \left( \frac{x_{F_3}}{x_w} \right)^{1/2} F_1 (x_w, x_{F_3}, x_{F_3}, x_{F_3}) \Im \left( G^{cL}_{\beta_\alpha \beta_\alpha} + G^{cR}_{\beta_\alpha \beta_\alpha} \right) \right\} ,$$

where $s_w$ is the weak mixing angle.
\begin{align*}
&+ \left( \frac{F_{g_0}}{x_w} \right)^{1/2} F_2(x_w, x_w, F_{g_0}, F_{g_3}) G_{g_{\alpha\beta}}^L \left( G_{g_{\alpha\beta}}^R + G_{g_{\alpha\beta}}^L \right) \\
&+ \left( \frac{F_{g_3}}{x_w} \right)^{1/2} F_3(x_w, x_w, F_{g_0}, F_{g_3}) G_{g_{\alpha\beta}}^L \left( G_{g_{\alpha\beta}}^R - G_{g_{\alpha\beta}}^L \right) \\
&+ \left( \frac{F_{g_3}}{x_w} \right)^{1/2} F_4(x_w, x_w, F_{g_0}, F_{g_3}) G_{g_{\alpha\beta}}^L \left( G_{g_{\alpha\beta}}^R - G_{g_{\alpha\beta}}^L \right)
\end{align*}

and
\begin{align*}
d_{WH}^1 &= \frac{G_f \alpha^2 m_B^2}{32 \sqrt{2} \pi^2} \left( \left( \frac{F_{g_3}}{x_w} \right)^{1/2} F_1(x_w, x_w, F_{g_0}, F_{g_3}) G_{g_{\alpha\beta}}^R \left( G_{g_{\alpha\beta}}^R + G_{g_{\alpha\beta}}^L \right) \\
&+ \left( \frac{F_{g_3}}{x_w} \right)^{1/2} F_2(x_w, x_w, F_{g_0}, F_{g_3}) G_{g_{\alpha\beta}}^R \left( G_{g_{\alpha\beta}}^R + G_{g_{\alpha\beta}}^L \right) \\
&+ \left( \frac{F_{g_3}}{x_w} \right)^{1/2} F_3(x_w, x_w, F_{g_0}, F_{g_3}) G_{g_{\alpha\beta}}^R \left( G_{g_{\alpha\beta}}^R - G_{g_{\alpha\beta}}^L \right) \\
&+ \left( \frac{F_{g_3}}{x_w} \right)^{1/2} F_4(x_w, x_w, F_{g_0}, F_{g_3}) G_{g_{\alpha\beta}}^R \left( G_{g_{\alpha\beta}}^R - G_{g_{\alpha\beta}}^L \right) \right),
\end{align*}

The expressions of form factors $F_i(x, y, z, u)$ ($i = 1, \cdots, 4$) can be found in appendix.

Using the asymptotic formulae of form factors $F_i$ ($i = 1, \cdots, 4$) under the condition $z, u \gg x, y,$
\begin{align*}
F_1(x, y, z, u) &\simeq \frac{-3(2 - 3Q_\beta)z^3 - 32z^2u - (20 - 9Q_\beta)zu^2}{3(z - u)^4} \ln \frac{z}{u} \\
&\quad + \frac{(11 - 54Q_\beta)z^2 - (151 - 54Q_\beta)zu + 2u^2}{9(z - u)^3} \\
&\quad - \left[ \frac{5}{\partial u} + u \frac{\partial^2}{\partial u^2} \right] \left[ 2 \varphi_{11}(x, y) \pi_1(z, u) + \varphi_1(z, u) \right] \\
&\quad + \left[ (Q_\beta - 6) + \left( (4 - Q_\beta)z + (6 + Q_\beta)u \right) \frac{\partial}{\partial u} \\
&\quad - u(z - u) \frac{\partial^2}{\partial u^2} \right] \left[ \varphi_{11}(x, y) \pi_2(z, u) + \frac{1}{2} \varphi_2(z, u) \right] \\
&\quad - \frac{10(z - u)}{3} \left[ \varphi_{11}(x, y) \pi_3(z, u) \right. \\
&\quad \left. - \frac{1}{2} \varphi_3(z, u) \right] + \cdots,
\end{align*}

\begin{align*}
F_2(x, y, z, u) &\simeq \frac{-3z^3 - (35 - 3Q_\beta)z^2u - (29 - 12Q_\beta)zu^2 + 15(1 - Q_\beta)u^3}{3(z - u)^4} \ln \frac{z}{u}
\end{align*}
we simplify the expressions of Eq. (37) in the limit \( m = m_{c,R} = m_{c,L} \gg m_w \) as:

\[
\alpha_{t}^{WG} = \frac{G_F \alpha_s^s m_w^2}{32 \sqrt{2 \pi^3} \Sigma_w m_F} \left\{ \left\lfloor \frac{9}{4} - \frac{11}{18} Q_\beta + \frac{Q_\beta^2}{3} \ln \frac{m_F^2}{m_w^2} \right\rfloor \Re \left( G_{\beta \alpha \xi_{\alpha \beta}} + G_{\beta \alpha \xi_{\alpha \beta}} \right) + \frac{13 - 8 Q_\beta}{9} + \frac{2 - 4 Q_\beta}{3} \ln \frac{m_F^2}{m_w^2} \Re \left( G_{\beta \alpha \xi_{\alpha \beta}} + G_{\beta \alpha \xi_{\alpha \beta}} \right) \right\} ,
\]

\[
\alpha_{t}^{WG} = \frac{G_F \alpha_s^s m_w}{32 \sqrt{2 \pi^3} \Sigma_w m_F} \left\{ \left\lfloor \frac{9}{4} - \frac{11}{18} Q_\beta + \frac{Q_\beta^2}{3} \ln \frac{m_F^2}{m_w^2} \right\rfloor \Re \left( G_{\beta \alpha \xi_{\alpha \beta}} + G_{\beta \alpha \xi_{\alpha \beta}} \right) + \frac{13 - 8 Q_\beta}{9} + \frac{2 - 4 Q_\beta}{3} \ln \frac{m_F^2}{m_w^2} \Re \left( G_{\beta \alpha \xi_{\alpha \beta}} + G_{\beta \alpha \xi_{\alpha \beta}} \right) \right\} ,
\]
The results indicate that the corrections to $a_1$, $d_l$ from the diagrams in Fig.4 are suppressed in the limit $m_F = m_{F_a} = m_{F_3} \gg m_w$ unless the couplings $\mathcal{H}_{\beta\alpha}^{c,L,R}$ violate the decoupling theorem.

**C. The corrections from the diagram where a closed heavy fermion loop is inserted into the self energy of $Z$ gauge boson**

In order to get the amplitude of the diagram in Fig.4, we write the renormalizable interaction among the electroweak neutral gauge boson $Z$ and the heavy fermions $F_{\alpha\beta}$ in a
more universal form as

$$\mathcal{L}_{ZFF} = \frac{e}{2s_w c_w} Z^\mu \bar{F}_\alpha \gamma_\mu (\xi^L_{\alpha\beta} \omega_- + \xi^R_{\alpha\beta} \omega_+) F_\beta ,$$

(40)

where the expressions of $\xi^L,R_{\alpha\beta}$ depend on the concrete models employed in our calculation, and the CPT symmetry requires $\xi^L_{\alpha\beta} = \xi^{L*}_{\beta\alpha}$, $\xi^R_{\alpha\beta} = \xi^{R*}_{\beta\alpha}$.

FIG. 5: The diagrams of effective theory correspond to those diagrams in Fig.4. Where the diagram (a) corresponds to Fig.4(a) and (b) in full theory, and the diagram (b) corresponds to Fig.4(c) and (d) in full theory respectively.

It is easy to check that the divergent amplitude of Fig.4(a) satisfies the Ward identity:

$$k^\rho A^{(a)}_{zz,\rho} (p, k) = e [\Sigma_{zz} (p + k) - \Sigma_{zz} (p)] ,$$

(41)
where $p$, $k$ are the incoming momenta of lepton and photon fields, $\rho$ is the Lorentz index of photon, $A^{(a)}_{zz,\rho}$ denotes the sum of amplitude for the diagrams in Fig. 4(a), and $\Sigma_{zz}$ denotes the amplitude of corresponding self energy diagram, respectively. After a tedious calculation, the bare Lagrangian from the diagram Fig. 4(a) can be written as

$$\mathcal{L}^{{\text{eff}}}_{2(a)} = -\frac{G_F \alpha_S}{4\sqrt{2}\pi^2} \left( \frac{1}{\epsilon} \right)^2 (4\pi x_R)^2 \left( \xi^{L}_{\alpha\beta} \xi^{L}_{\alpha\beta} + \xi^{R}_{\alpha\beta} \xi^{R}_{\alpha\beta} \right) \left[ \frac{1}{\epsilon} \frac{x_{F\alpha} + x_{F\beta}}{x_z^2} + \frac{2}{3x_z} + T_5(x_{F\alpha}, x_{F\beta}) \right]$$

$$\times \left[ (T_2 - Q_f s_w^2)(O_2 + O_3) + Q^2 s_w^4 (O_2^+ + O_3^+) \right]$$

$$+ \left( \xi^{R}_{\alpha\beta} \xi^{L}_{\alpha\beta} + \xi^{L}_{\alpha\beta} \xi^{R}_{\alpha\beta} \right) (x_{F\alpha} x_{F\beta})^{1/2} \left[ \frac{2}{3x_z} + \frac{2}{\epsilon} \frac{x_{F\alpha} + x_{F\beta}}{x_z^2} + \frac{2}{x_z^2} \ln x_z \right]$$

$$- \frac{11}{3x_z^2} \left[ (T_2 - Q_f s_w^2)(O_2 + O_3) + Q^2 s_w^4 (O_2^+ + O_3^+) \right]$$

$$+ 3Q_f \left( \xi^{L}_{\alpha\beta} \xi^{L}_{\alpha\beta} + \xi^{R}_{\alpha\beta} \xi^{R}_{\alpha\beta} \right) (x_{F\alpha} x_{F\beta})^{1/2} \left[ \frac{1}{\epsilon} \frac{x_{F\alpha} + x_{F\beta}}{x_z^2} \right.$$

$$\frac{\theta_{2,1}(x_{F\alpha}, x_{F\beta})}{x_z^2} - \frac{x_{F\alpha} + x_{F\beta}}{x_z^2} \left( 3 \frac{1}{x_z^2} \right) \left( O_6^+ + O_6^- \right)$$

$$- 6Q_f \left( \xi^{L}_{\alpha\beta} \xi^{R}_{\alpha\beta} + \xi^{R}_{\alpha\beta} \xi^{L}_{\alpha\beta} \right) (x_{F\alpha} x_{F\beta})^{1/2} \left[ \frac{1}{\epsilon} \frac{x_{F\alpha} + x_{F\beta}}{x_z^2} \right.$$

$$\frac{\theta_{2,1}(x_{F\alpha}, x_{F\beta})}{x_z^2} - \frac{x_{F\alpha} + x_{F\beta}}{x_z^2} \left( 1 + \frac{1}{x_z^2} \right) \left( O_6^+ + O_6^- \right) \right] \right) + \cdots \right), \quad (42)$$

where $T_2 = -1/2$ represents the isospin of charged leptons.

Generally, the unrenormalized self energy of the weak gauge boson $Z$ can be written as

$$\Sigma^2_{\mu\nu}(p, \Lambda_{\text{RE}}) = \Lambda^2 A_0^2 g_{\mu\nu} + \left( A_1^2 + \frac{p^2}{\Lambda^2} A_2^2 + \cdots \right) (p^2 g_{\mu\nu} - p_{\mu} p_{\nu})$$

$$+ \left( B_1^2 + \frac{p^2}{\Lambda^2} B_2^2 + \cdots \right) p_{\mu} p_{\nu}. \quad (43)$$

Here, we omit those terms which are strongly suppressed at the limit of the large virtual fermion masses, and those form factors are actually decided by the masses of virtual fields and the renormalization scale. Correspondingly, the counter terms are given as

$$\Sigma^2_{\mu\nu}(p, \Lambda_{\text{RE}}) = -\left[ \delta m^2_{\alpha} (\Lambda_{\text{RE}}) + m_{\alpha}^2 \delta Z_{\alpha} (\Lambda_{\text{RE}}) \right] g_{\mu\nu} - \delta Z_{\alpha} (\Lambda_{\text{RE}}) \left[ p^2 g_{\mu\nu} - p_{\mu} p_{\nu} \right]. \quad (44)$$

The renormalized self energy is expressed as

$$\hat{\Sigma}^2_{\mu\nu}(p, \Lambda_{\text{RE}}) = \Sigma^2_{\mu\nu}(p, \Lambda_{\text{RE}}) + \Sigma^2_{\mu\nu}(p, \Lambda_{\text{RE}}). \quad (45)$$
For on-shell external gauge boson \( Z \), we have

\[
\hat{\Sigma}_{\mu}^g(p, m_z)e^\nu(p)\Big|_{p^2 = m_z^2} = 0,
\]

\[
\lim_{p^2 - m_z^2 \to 0} \frac{1}{p^2 - m_z^2} \hat{\Sigma}_{\mu}^g(p, m_z)e^\nu(p) = \epsilon_\nu(p), \tag{46}
\]

where \( \epsilon(p) \) is the polarization vector of neutral gauge boson. From Eq. (46), we get the counter terms at electroweak scale in on-shell scheme:

\[
\delta Z^a_{\mu}(m_z) = A_1^z + \frac{m_z^2}{\Lambda^2} A_2^z = A_1^z + x_z A_2^z,
\]

\[
\delta m^2_{\mu}(m_z) = A_0^z \Lambda^2 - m_z^2 \delta Z^a_{\mu} \tag{47}
\]

Accordingly, the effective Lagrangian from the counter term diagrams is written as

\[
\delta \mathcal{L}^C_{zz} = i \frac{G_F \alpha_w x_w}{\sqrt{2} \sigma_w^2} \frac{\Gamma^2(1 + \epsilon)}{(1 - \epsilon)^2} \left\{ \left[ A_0^z \left( \frac{1}{3x_z^2} + \frac{11 \epsilon}{18 x_z^2} - \frac{\epsilon}{3} \ln x_z \right) + \frac{\epsilon}{3x_z^2} (A_1^z + x_z A_2^z) \right] \right\}
\]

\[
\left( (T_f^Z - Q_f s^2_w)^2 (O^-_2 + O^-_3) + Q^2_f s^4_w (O^+_2 + O^+_3) \right)
\]

\[
+ \frac{2}{x_z} \left[ \left( T_f^Z - Q_f s^2_w \right)^2 (O^-_2 + O^-_3) + Q^2_f s^4_w (O^+_2 + O^+_3) \right]
\]

\[
+ \left( \xi^L_{\alpha \gamma} + \xi^R_{\alpha \gamma} \right) (x_{F_{\alpha}} x_{F_{\beta}})^{1/2} \left[ - \frac{1}{3 \ln x_z} + \frac{2 \theta_{2,1} (x_{F_{\alpha}}, x_{F_{\beta}})}{x_z^2} + \frac{2}{x_z^2} \ln x_z \right]
\]

\[
- \frac{11}{3x_z^2} \left[ (T_f^Z - Q_f s^2_w)^2 (O^-_2 + O^-_3) + Q^2_f s^4_w (O^+_2 + O^+_3) \right]
\]

\[
+ 3Q_f \left( \xi^L_{\alpha \gamma} + \xi^R_{\alpha \gamma} \right) s^2_w (T_f^Z - Q_f s^2_w) \left[ - \frac{1}{3 \ln x_z} + \frac{2 \theta_{2,1} (x_{F_{\alpha}}, x_{F_{\beta}})}{x_z^2} + \frac{2}{x_z^2} \ln x_z \right]
\]

\[
- \frac{\theta_{2,1} (x_{F_{\alpha}}, x_{F_{\beta}})}{x_z^2} - \frac{x_{F_{\alpha}} + x_{F_{\beta}}}{x_z^2} \left( \frac{3}{2} + \ln x_z \right) (O^+_6 + O^-_6)
\]

\[
- 6Q_f \left( \xi^L_{\alpha \gamma} + \xi^R_{\alpha \gamma} \right) s^2_w (T_f^Z - Q_f s^2_w) (x_{F_{\alpha}} x_{F_{\beta}})^{1/2} \left[ \frac{1}{3 \ln x_z} + \frac{2 \theta_{2,1} (x_{F_{\alpha}}, x_{F_{\beta}})}{x_z^2} + \frac{2}{x_z^2} \ln x_z \right]
\]

\[
- \frac{\theta_{2,1} (x_{F_{\alpha}}, x_{F_{\beta}})}{x_z^2} - \frac{1 + \ln x_z}{x_z^2} (O^+_6 + O^-_6) \right\} + \cdots, \tag{48}
\]
As a result of the preparation mentioned above, we use the contributions from the counter
term diagram in Fig.4(a) b) to cancel the ultraviolet divergence in Eq. (42). After matching
the amplitude from Fig.4(a, b) to that from the diagram of effective theory (Fig.5(a)) and
analyzing the corrections from light freedoms properly, we get the theoretical prediction on
the lepton MDMs as
\[
a_{l(a+b)}^{zz} = -\frac{G_F \alpha \xi_{l} m_{l}^2}{24 \sqrt{2} \pi^3 s_w^2 c_w^4} \left| (\xi_{l}^L \right|^2 + \left| \xi_{l}^R \right|^2) \left[ (T_f^Z - Q_f s_w^2)^2 + Q_f^2 s_w^4 \right] T_5(x_z, x_{F_a}, x_{F_\beta}), \tag{49}
\]
In fact, the four-fermion interaction does not induce the corrections to lepton MDMs and
EDMs in Fig.5(a). Using the asymptotic expression of
\[
T_5(x_z, u, z) \simeq -\frac{1}{9x} - \frac{z + u}{6x} \pi_2(z, u) + \frac{(z - u)^2}{18x} \pi_3(z, u) + \cdots, \tag{50}
\]
one finally obtains the leading corrections contained in the asymptotic form of Eq.(49)
under the assumption \( m_F = m_{F_a} = m_{F_\beta} \gg m_w \):
\[
a_{l(a+b)}^{zz} \propto O(m_w^2/m_F^2) + \cdots, \tag{51}
\]
where ellipsis represents those relatively unimportant corrections.

In a similar way, we can verify the identity
\[
k^\rho \mathcal{A}^{(c+d)}_{zz, \rho}(p, k) = 0. \tag{52}
\]
\( \mathcal{A}^{(c+d)}_{zz, \rho} \) denotes the sum of amplitude for the diagrams in Fig.4(c, d). After the steps adopted
above, the corrections from Fig.4(c, d) to lepton MDMs and EDMs are formulated as
\[
a_{l(c+d)}^{zz} = -\frac{Q_f G_F \alpha \xi_{l} m_{l}^2}{64 \sqrt{2} \pi^3 s_w^2 c_w^4} \left( \left| (\xi_{l}^L \right|^2 + \left| \xi_{l}^R \right|^2) \left[ (T_f^Z - Q_f s_w^2)^2 + Q_f^2 s_w^4 \right] T_6(x_z, x_{F_a}, x_{F_\beta}) \right)
\]
\[
+ \left( \left| (\xi_{l}^L \right|^2 + \left| \xi_{l}^R \right|^2) \left[ (T_f^Z - Q_f s_w^2)^2 - Q_f^2 s_w^4 \right] T_7(x_z, x_{F_a}, x_{F_\beta}) \right)
\]
\[
- \text{Re}(\xi_{l\beta}^L, \xi_{l\beta}^R) \left[ (T_f^Z - Q_f s_w^2)^2 + Q_f^2 s_w^4 \right] (x_{F_a}, x_{F_\beta})^{1/2} T_8(x_z, x_{F_a}, x_{F_\beta}) \right)
\]
\[
- Q_f \left( \left| (\xi_{l}^L \right|^2 + \left| \xi_{l}^R \right|^2) s_w^2 (T_f^Z - Q_f s_w^2) T_9(x_z, x_{F_a}, x_{F_\beta}) \right),
\]
\[
a_{l(c+d)}^{zz} = \frac{Q_f G_F \alpha \xi_{l} \cdot m_{l}^2}{8 \sqrt{2} \pi^3 s_w^2 c_w^4} \left( \left| (\xi_{l}^L \right|^2 + \left| \xi_{l}^R \right|^2) \left[ (T_f^Z - Q_f s_w^2)^2 + Q_f^2 s_w^4 \right] \right)
\]
\[
\times \left( \frac{\partial^2}{\partial x_z \partial x_{F_\beta}} - \frac{\partial^2}{\partial x_z \partial x_{F_a}} \right) \left( \Phi(x_z, x_{F_a}, x_{F_\beta}) - \varphi_0(x_{F_a}, x_{F_\beta}) \right) \right)
\]
\[
- \frac{1}{16} \left[ (T_f^Z - Q_f s_w^2)^2 + Q_f^2 s_w^4 \right] T_0(x_z, x_{F_a}, x_{F_\beta}) \right). \tag{53}
\]
}\)
As a closed heavy fermion loop is inserted into neutral or charged gauge boson propagator, the counter terms to self energy diagrams of Z or W gauge bosons induce the renormalization for Weinberg angle

\[
\frac{\delta s}{s_w} = - \frac{c_w^2}{2s_w^2} \left[ \frac{\delta m_{\text{os}}^2}{m_w^2} - \frac{\delta m_{\text{os}}^2}{m_z^2} \right] \simeq \frac{c_w^2}{2s_w^2} \Delta \rho_F + \cdots ,
\]

where the dots indicate again nonleading contributions. Furthermore, \( \Delta \rho_F \) denotes the 1-loop corrections from the heavy fermions to the \( \rho \) parameter, which appears in the ratio of weak neutral to charged current amplitudes and comparisons of the Z and W\(^\pm \) masses. Using the above equation, one can express the corresponding counter terms for the vertices \( Z\bar{\ell}l \) as

\[
i\delta C_{Z\ell} = \frac{ie}{s_w c_w} \left[ \frac{\delta s}{2 s_w^2} + s_w \delta s_w \omega_+ \right].
\]

Inserting the counter terms into one loop standard model diagrams, we finally obtain

\[
\alpha^2 = - \frac{e^2 m_l^2}{3(4\pi)^2 s_w^2 s_w^2 R^2} \left( 1 \right. \left. - 2s_w^2 + 2s_w^4 \right) \frac{\delta s}{s_w} = - \frac{G_F m_l^2}{12\sqrt{2} \pi^2 s_w^2} \left( 1 \right. \left. - 2s_w^2 + 2s_w^4 \right) \Delta \rho_F + \cdots .
\]

In principle, we should consider the corrections from the counter terms for \( W^+ \bar{\nu}l \):

\[
i\delta C_{W^+\bar{\nu}l} = - \frac{ie}{\sqrt{2} s_w^2} \delta s_w \omega_- .
\]

However, the corrections can be absorbed in the one-loop result as we parametrize the final result in terms of \( G_F \) determined from the muon’s lifetime.

The contributions from the corresponding diagrams contain the additional suppressed factor \( m_l^2/\Lambda_{\text{ew}}^2 \) when both of virtual neutral gauge bosons in Fig[4] are replaced with the neutral Goldstone \( G^0 \). Nevertheless, we should consider the corrections from those two loop diagrams in which one of virtual neutral gauge bosons is replaced with the neutral Goldstone \( G^0 \) since it represents the longitudinal component of charged gauge boson in nonlinear \( R_\xi \) gauge. For many electroweak theories contain the neutral CP-even and CP-odd Higgs, we also generalize the result directly to the diagrams in which a closed heavy loop is attached on the virtual neutral gauge boson Z and neutral scalars \( h^0, A^0 \).
D. The corrections from the diagrams where a closed heavy fermion loop is attached to the virtual $Z, G^0$ or $h^0$ bosons

Similarly, the renormalizable interaction among the electroweak neutral Goldstone/Higgs $G^0 (h^0, A^0)$ and the heavy fermions $F_{\alpha, \beta}$ can be expressed in a more universal form as

\[
\mathcal{L}_{s0\mathcal{E}F} = \frac{e}{s_w} \left[ -i G^0 \bar{F}_\alpha (G^{N}_{\alpha \beta} \omega_- - G^{N\dagger}_{\alpha \beta} \omega_+) F_\beta + h^0 \bar{F}_\alpha (\mathcal{H}^E_{\alpha \beta} \omega_- + \mathcal{H}^{E\dagger}_{\alpha \beta} \omega_+) F_\beta \\
- i A^0 \bar{F}_\alpha (\mathcal{H}^O_{\alpha \beta} \omega_- - \mathcal{H}^{O\dagger}_{\alpha \beta} \omega_+) F_\beta \right] + h.c., \quad (58)
\]

where the concrete expressions of $G^N_{\alpha \beta}, \mathcal{H}^E_{\alpha \beta}$ and $\mathcal{H}^O_{\alpha \beta}$ depend on the models employed in our calculation, and the conservation of electric charge requires $Q_\beta = Q_\alpha$. Generally, the couplings among the neutral Goldstone/Higgs and leptons are written as

\[
\mathcal{L}_{s_{0\mathcal{E}}} = \frac{e m_e}{2 m_{w^*} s_w} \left[ - i G^0 \bar{l} \gamma_5 l + B_\mathcal{E} h^0 \bar{1} l - i B_\mathcal{O} A^0 \bar{1} \gamma_5 l \right], \quad (59)
\]

and the parameters $B_\mathcal{E}, B_\mathcal{O}$ depend on the concrete models adopted in our analysis. In full theory, the couplings in Eq. (58) induce the corrections to lepton MDMs and EDMs from the diagrams in Fig. 6. Accordingly, the corresponding diagram of effective theory is presented in Fig. 5(b). In principle, we should also consider the corrections from those two loop diagrams in which one of virtual $Z$ gauge bosons is replaced with the neutral Goldstone/Higgs $G^0 (h^0, A^0)$ in Fig. 4(a). However, the sum of amplitude from those two loop diagrams does not contribute to the lepton MDMs and EDMs when one of virtual neutral gauge bosons in Fig. 4(a) is replaced with the neutral Goldstone/Higgs $G^0 (h^0, A^0)$.
After the steps taken in $WW$ sector, we formulate the corresponding corrections to the lepton MDMs and EDMs from the diagrams in Fig.6 as

\[
aZ^{h^0}_l = - \frac{G_F \alpha m^2_{\mu} B_\mu}{64 \pi^3 s^2_w c^2_w} (T_f^Z - 2 Q_f s^2_w) (x_w x_F^3)^{1/2} \left[ 2 \ln x_F - \ln x_F^3 \right] \theta_{0,1} (x_z, x_h)
\]

\[
aZ^{h^0}_l = \frac{G_F \alpha m^2_{\mu} B_\mu}{64 \pi^3 s^2_w c^2_w} \left[ (T_f^Z - 2 Q_f s^2_w) (x_w x_F^3)^{1/2} \left[ 2 \ln x_F - \ln x_F^3 \right] \theta_{0,1} (x_z, x_h)
\]

\[
aZ^{h^0}_l = \frac{G_F m^2_{\mu} B_\mu}{128 \pi^3 s^2_w c^2_w} \left[ (T_f^Z - 2 Q_f s^2_w) (x_w x_F^3)^{1/2} \left[ 2 \ln x_F - \ln x_F^3 \right] \theta_{0,1} (x_z, x_h)
\]

\[
aZ^{A^0}_l = - \frac{G_F \alpha m^2_{\mu} B_\mu}{64 \pi^3 s^2_w c^2_w} (T_f^Z - 2 Q_f s^2_w) (x_w x_F^3)^{1/2} \left[ 2 \ln x_F - \ln x_F^3 \right] \theta_{0,1} (x_z, x_A)
\]

\[
aZ^{A^0}_l = \frac{G_F m^2_{\mu} B_\mu}{128 \pi^3 s^2_w c^2_w} \left[ (T_f^Z - 2 Q_f s^2_w) (x_w x_F^3)^{1/2} \left[ 2 \ln x_F - \ln x_F^3 \right] \theta_{0,1} (x_z, x_A)
\]

\[
aZ^{A^0}_l = \frac{G_F \alpha m^2_{\mu} B_\mu}{128 \pi^3 s^2_w c^2_w} \left[ (T_f^Z - 2 Q_f s^2_w) (x_w x_F^3)^{1/2} \left[ 2 \ln x_F - \ln x_F^3 \right] \theta_{0,1} (x_z, x_A)
\]

The expressions of form factors $F_i(x, y, z, u)$ ($i = 5, 6$) can be found in appendix. In the heavy mass limit $m_F = m_{e} = m_{f} \gg m_{h}, m_{A}, m_{w}$, we have

\[
aZ^{h^0}_l = - \frac{G_F \alpha m^2_{\mu} m_w B_\mu}{64 \pi^3 s^2_w c^2_w m_F} \left[ \theta_{1,1} (m_z^2, m^2_w) - \ln m_F^2 - 1 \right]
\]

\[
aZ^{A^0}_l = \frac{G_F \alpha m^2_{\mu} m_w B_\mu}{32 \pi^3 s^2_w c^2_w m_F} \left[ \theta_{1,1} (m_z^2, m^2_w) - \ln m_F^2 - 1 \right]
\]

\[
aZ^{G}_l = \frac{G_F \alpha m^2_{\mu} m_w}{32 \pi^3 s^2_w c^2_w m_F} \ln \left( \frac{m^2_{e}}{m^2_w} \right) (T_f^Z - 2 Q_f s^2_w) \Delta \left( \theta_{1,1} (m_z^2, m^2_w) - \ln m_F^2 - 1 \right)
\]

\[
dZ^{h^0}_l = - \frac{G_F \alpha m^2_{\mu} B_\mu}{64 \pi^3 s^2_w c^2_w m_F} \left[ \theta_{1,1} (m_z^2, m^2_w) - \ln m_F^2 - 1 \right]
\]

\[
dZ^{h^0}_l = \frac{G_F \alpha m^2_{\mu} B_\mu}{128 \pi^3 s^2_w c^2_w m_F} \left[ \theta_{1,1} (m_z^2, m^2_w) - \ln m_F^2 - 1 \right]
\]

\[
dZ^{h^0}_l = \frac{G_F \alpha m^2_{\mu} B_\mu}{128 \pi^3 s^2_w c^2_w m_F} \left[ \theta_{1,1} (m_z^2, m^2_w) - \ln m_F^2 - 1 \right]
\]
\[ d_{l}^{ZA0} = \frac{G_{F} \alpha_{e} e_{m_{w}} B_{O}}{128 \pi^{3} s_{w}^{2} c_{w}^{2} m_{F}} \left[ \theta_{1,1} \left( m_{z}^{2}, m_{A}^{2} \right) - \ln \frac{m_{F}^{2}}{m_{Z}^{2}} - 1 \right] \]
\[ \times \left( T_{f}^{Z} - 2 Q_{f} s_{w}^{2} \right) \Im \left( H_{\beta \alpha}^{0} \xi_{L}^{\beta \alpha} + H_{\beta \alpha}^{0 \dagger} \xi_{R}^{\beta \alpha} \right), \]
\[ d_{l}^{ZG} = \frac{G_{F} \alpha_{e} e_{m_{w}}}{128 \pi^{3} s_{w}^{2} c_{w}^{2} m_{F}} \ln \left( \frac{m_{F}^{2}}{m_{Z}^{2}} \right) \left( T_{f}^{Z} - 2 Q_{f} s_{w}^{2} \right) \Im \left( G_{F}^{N} \xi_{L}^{N} + G_{F}^{N \dagger} \xi_{R}^{N} \right). \] (61)

E. The corrections from the diagrams where a closed heavy fermion loop is attached to the virtual \( \gamma \) and \( Z \) bosons

When a closed fermion loop is attached to the virtual \( \gamma \) and \( Z \) gauge bosons (Fig. 7), the corresponding diagrams of effective theory are presented in Fig. 8. Taking the steps above, one can get the tedious correction to the effective Lagrangian. If we ignore the terms which are proportional to the suppression factor \( 1 - 4 s_{w}^{2} \), the correction from this sector to the lepton MDMs from this sector is drastically simplified as

\[ a_{l}^{\gamma Z} = \frac{G_{F} \alpha_{e} Q_{f} Q_{a} m_{w}^{2}}{32 \sqrt{2} \pi^{3} s_{w}^{2}} \left( \xi_{L}^{\alpha} - \xi_{R}^{\alpha} \right) x_{w} T_{7}(x_{a}, x_{F_{a}}), \] (62)

and the correction to the lepton EDMs is zero. In the limit \( m_{F} = m_{F_{a}} \gg m_{a} \), we approximate the correction to the lepton MDMs from this sector as

\[ a_{l}^{\gamma Z} = \frac{G_{F} \alpha_{e} Q_{f} Q_{a} m_{w}^{2} m_{z}^{2}}{32 \sqrt{2} \pi^{3} s_{w}^{2} m_{F}^{2}} \left( \xi_{L}^{\alpha} - \xi_{R}^{\alpha} \right) \left[ 35 + \ln \frac{m_{F}^{2}}{m_{z}^{2}} \right]. \] (63)
FIG. 8: The diagrams of effective theory corresponds to those diagrams in Fig. 7. Where the

diagram (a) corresponds to Fig. 7(a) in full theory, and the diagram (b) corresponds to Fig. 7(b) in

full theory respectively.

F. The corrections from the diagrams where a closed heavy fermion loop is at-
tached to the virtual $\gamma$, $G^0 (h^0, A^0)$ bosons

As a closed fermion loop is attached to the virtual neutral Higgs and photon fields, a
real photon can be emitted from either the virtual lepton or the virtual charginos in the self
energy diagram. When a real photon is emitted from the virtual charginos, the corresponding
"triangle" diagrams belong to the typical two-loop Bar-Zee-type diagrams [16]. Within the
framework of CP violating MSSM, the contributions from two-loop Bar-Zee-type diagrams
to the EDMs of those light fermions are discussed extensively in literature [7]. When a
real photon is attached to the internal standard fermion, the correction from corresponding
triangle diagram to the effective Lagrangian is zero because of the Furry theorem, this point
is also verified through a strict analysis. Replacing the virtual neutral gauge boson with
photon in Fig. 6, one obtains the relevant diagrams in full theory. Meanwhile, the diagram
of effective theory is same as that presented in Fig. 8(a). After the steps adopted above, the

corresponding corrections to lepton MDMs and EDMs from this sector are expressed as

$$
\begin{align*}
    a_1^{\gamma h^0} &= \frac{G_F \alpha_e Q_f Q_\alpha m_i^2 B_0}{8 \pi^3} \Re(\mathcal{H}_{\alpha \alpha}^{\varepsilon})(x_w x_{F_\alpha})^{1/2} T_{11}\left(x_h, x_{F_\alpha}, x_{F_\alpha}\right), \\
    d_1^{\gamma h^0} &= -\frac{G_F \alpha_e Q_f Q_\alpha m_i^2 B_0}{8 \pi^3} \Im(\mathcal{H}_{\alpha \alpha}^{\varepsilon})(x_w x_{F_\alpha})^{1/2} T_{12}\left(x_A, x_{F_\alpha}, x_{F_\alpha}\right), \\
    a_1^{\gamma A^0} &= -\frac{G_F \alpha_e Q_f Q_\alpha m_i^2 B_0}{16 \pi^3} \Re(\mathcal{H}_{\alpha \alpha}^{O})(x_w x_{F_\alpha})^{1/2} T_{11}\left(x_A, x_{F_\alpha}, x_{F_\alpha}\right), \\
    d_1^{\gamma A^0} &= -\frac{G_F \alpha_e Q_f Q_\alpha m_i^2 B_0}{16 \pi^3} \Im(\mathcal{H}_{\alpha \alpha}^{O})(x_w x_{F_\alpha})^{1/2} T_{12}\left(x_A, x_{F_\alpha}, x_{F_\alpha}\right),
\end{align*}
$$

(64)
Note here that the corrections from this sector to the MDM of lepton depend on real parts of the effective couplings $H_{\alpha\alpha}$, and the corrections from this sector to the EDM of lepton depend on imaginary parts of the effective couplings $H_{\alpha\alpha}$. In the limit $m_F = m_{F\alpha} \gg m_h, m_A$, the above expressions are simplified as

\[
a_{\gamma h}^l = \frac{G_F^\alpha e Q_f Q_{\alpha} m_{w}^2 m_{w}}{8\pi^3 m_F} \Re(H_{\alpha\alpha}^E) \left[ 1 + \ln \frac{m_F^2}{m_h^2} \right],
\]

\[
a_{\gamma A}^l = \frac{G_F^\alpha e Q_f Q_{\alpha} m_{w}^2 m_{w}}{16\pi^3 m_F} \Im(H_{\alpha\alpha}^O) \left[ 1 + \ln \frac{m_F^2}{m_A^2} \right],
\]

\[
d_{\gamma h}^l = \frac{G_F^\alpha e Q_f Q_{\alpha} m_{w}^2 m_{w}}{8\pi^3 m_F} \Im(H_{\alpha\alpha}^E) \left[ 1 + \ln \frac{m_F^2}{m_h^2} \right],
\]

\[
d_{\gamma A}^l = -\frac{G_F^\alpha e Q_f Q_{\alpha} m_{w}^2 m_{w}}{16\pi^3 m_F} \Re(H_{\alpha\alpha}^O) \left[ 1 + \ln \frac{m_F^2}{m_A^2} \right].
\]

Similarly, the corrections to the lepton MDMs and EDMs from the $\gamma G$ sector are:

\[
a_{\gamma G}^l = \frac{G_F^\alpha e Q_f Q_{\alpha} m_{l}^2 m_{l}}{8\pi^3 m_F} \Re(G_{\alpha\alpha}^N) \left( x_w x_{F\alpha} \right)^{1/2} T_{12}(x_z, x_{F\alpha}, x_{F\alpha}),
\]

\[
d_{\gamma G}^l = \frac{G_F^\alpha e Q_f Q_{\alpha} m_{l}^2 m_{l}}{16\pi^3 m_F} \Im(G_{\alpha\alpha}^N) \left( x_w x_{F\alpha} \right)^{1/2} T_{11}(x_z, x_{F\alpha}, x_{F\alpha}).
\]

The corrections from this sector to the MDM of lepton are proportional to real parts of the effective couplings $G_{\alpha\alpha}^N$, and the corrections from this sector to the EDM of lepton are proportional to imaginary parts of the effective couplings $G_{\alpha\alpha}^N$, separately. In the limit $m_F = m_{F\alpha} \gg m_z$, we have

\[
a_{\gamma G}^l = \frac{G_F^\alpha e Q_f Q_{\alpha} m_{l}^2 m_{l}}{8\pi^3 m_F} \Re(G_{\alpha\alpha}^N) \left[ 1 + \ln \frac{m_F^2}{m_z^2} \right],
\]

\[
d_{\gamma G}^l = \frac{G_F^\alpha e Q_f Q_{\alpha} m_{l}^2 m_{l}}{16\pi^3 m_F} \Im(G_{\alpha\alpha}^N) \left[ 1 + \ln \frac{m_F^2}{m_z^2} \right],
\]

which are suppressed by the masses of heavy fermions.

**G. The corrections from the diagrams where a closed heavy fermion loop is attached to the virtual $\gamma$ bosons**

At the two loop level, there are QED diagrams involving a photon vacuum polarization subdiagrams. For leptons or quarks, these contributions are of course known. If heavy
fermions contribute, the two loop QED contributions are modified by the photon vacuum polarization (Fig.9). Adopting the same steps in WW sector, we formulate the corrections from Fig.9 to lepton MDM and EDM respectively as:

\[ a_l^{\gamma\gamma} = \frac{\sqrt{2} G_F \alpha_e Q_l^2}{45 \pi^3} \frac{m_l^2}{m_w^2} m_{\mu}^2, \]
\[ d_l^{\gamma\gamma} = 0, \]  
(68)

which coincide with the well known results in Ref. [24]. For masses \( m_{\mu} \geq 100\text{GeV} \), these corrections from this sector to the muon MDM \( a_\mu \) are below \( 10^{-13} \) and hence negligible.

FIG. 9: The two-loop diagrams in which a closed heavy fermion loop is attached to the virtual \( \gamma \) bosons, where subdiagram (a) represents the two loop diagram in full theory, (b) represents the corresponding counter diagram, and (c) is the corresponding diagram of effective theory.

III. THE LEADING TERMS IN TWO LOOP CORRECTIONS TO \( a_l \) IN CONCRETE ELECTROWEAK THEORIES

In this section, we will present the leading terms from two loop corrections to \( a_l \) within some electroweak models discussed extensively in literature.
A. \textbf{the SM}

Within the framework of SM, only top quark belongs to the fermion which mass is heavier than that of weak gauge bosons. The couplings in Eq. (8) and Eq. (40) are respectively expressed as

\[
\xi^L_{tt} = -1 + \frac{4}{3}s_w^2, \quad \xi^R_{tt} = -\frac{4}{3}s_w^2, \\
\xi^L_{tdi} = -\frac{V_{tdi}}{\sqrt{2}}, \quad \xi^R_{tdi} = 0.
\]

(69)

Using the unitary property of Cabibbo-Kabayashi-Maskawa (CKM) matrix $V$ and $m_z = m_w/c_w$, one formulates the leading corrections from top quark to lepton MDMs as

\[
a^{SM}_{2L} = \frac{G_F\alpha_m^2}{8\sqrt{2}\pi^3 s^2_w} \left\{ 3 \left( 1 - \frac{104}{9} s^2_w - \frac{16}{9} s^2_w \ln \frac{m^2_t}{m^2_h} \right) \right. \\
- \frac{G_F m^2_t}{12\sqrt{2}\pi^2 s^2_w} \left( 1 - 2s^2_w + 2s^4_w \right) \Delta \rho_{SM} \right\}.
\]

(70)

In order to produce the terms $\propto m^2_t$, we take the limit $s^2_w \to 1/4$ used in Ref[4], and approximate the last term as $-5G_F m^2_t \Delta \rho_{SM}/24\sqrt{2}\pi^2$. Using the leading contributions $\Delta \rho_{SM} \approx 3e^2 m^2_t/4(4\pi)^2 s^2_w m^2_w + \cdots$ in the limit $m_t \gg m_w$, we recover the terms $\propto m^2_t$ in Ref[4] perfectly.

B. \textbf{the Extension of SM with the Fourth Generation}

Besides top quark, the extension of SM with the fourth generation also includes additional three heavy fermions: $t', b', \tau'$. Correspondingly, the couplings in Eq. (8) and Eq. (40) are separately written as

\[
\xi^L_{t't'} = -1 + \frac{4}{3}s_w^2, \quad \xi^R_{t't'} = -\frac{4}{3}s_w^2, \\
\xi^L_{b'b'} = -1 + \frac{2}{3}s_w^2, \quad \xi^R_{b'b'} = \frac{2}{3}s_w^2, \\
\xi^L_{\tau\tau'} = 1 - 2s_w^2, \quad \xi^R_{\tau\tau'} = -2s_w^2, \\
\xi^L_{\nu_{t't'}}, \quad \xi^R_{\nu_{t't'}} = 0, \\
\xi^L_{\nu_{b'b'}}, \quad \xi^R_{\nu_{b'b'}} = 0.
\]

(71)
Assuming \( m'_{t} = m'_{\nu} = m'_{\tau} = m'_{F} \gg m_{w} \) and applying the unitary property of \( 4 \times 4 \) CKM matrix \( V \), we finally obtain

\[
a_{2L}^{4G} = a_{2L}^{SM} + \frac{G_{\mu} \alpha_{e} m_{F}^{2}}{48 \sqrt{2} \pi^{3} s_{w}^{2}} \left[ 31 + 12 |V_{\nu\nu}|^{2} \right] - \frac{G_{\mu} m_{F}^{2}}{12 \sqrt{2} \pi^{2} s_{w}^{2}} (1 - 2 s_{w}^{2} + 2 s_{w}^{4}) \Delta \rho_{4G} .
\]

(72)

Here, the 1-loop corrections to \( \rho \)-parameter from the heavy fermions of 4th generation can be written as

\[
\Delta \rho_{4G} = \frac{\alpha_{e}}{16 \pi s_{w}^{2}} \frac{m_{F}^{2}}{m_{w}^{2}} \left[ 19 + 12 \ln \left( \frac{m_{F}^{2}}{m_{w}^{2}} \right) \right] .
\]

(73)

In this model, the dominant contributions from Higgs sector are originated from the two-loop \( \gamma h \) diagrams. Under the assumption \( m'_{t} = m'_{\nu} = m'_{\tau} = m'_{F} \), those contributions are zero since the anomalous cancelation. There is no correction from those heavy fermions to lepton EDMs also in the extension of SM with the fourth generation.

**C. the minimal supersymmetric extension of SM**

In this extension of SM, the additional possible heavy fermions include two charginos \( \chi_{1,2}^{\pm} \) and four neutralinos \( \chi_{i}^{0} (i = 1, \ldots, 4) \) [17]. The couplings among weak gauge bosons and heavy fermions are given as

\[
\begin{align*}
\xi_{\chi_{3}^{\pm} \chi_{3}^{\pm}}^{L} &= 2 \delta_{\alpha \beta} \cos 2 \theta_{w} + (U_{L}^{\dagger})_{\alpha 1} (U_{L})_{1 \beta} , \\
\xi_{\chi_{3}^{\pm} \chi_{3}^{\pm}}^{R} &= 2 \delta_{\alpha \beta} \cos 2 \theta_{w} + (U_{R}^{\dagger})_{\alpha 1} (U_{R})_{1 \beta} , \quad (\alpha, \beta = 1, 2) , \\
\xi_{\chi_{0}^{i} \chi_{0}^{j}}^{L} &= N_{\alpha 4}^{t} N_{4 \beta} , \\
\xi_{\chi_{0}^{i} \chi_{0}^{j}}^{R} &= N_{\beta 3}^{t} N_{3 \alpha} , \quad (\alpha, \beta = 1, \ldots, 4) , \\
\zeta_{\chi_{3}^{\pm} \chi_{0}^{i}}^{L} &= N_{\alpha 2}^{t} (U_{L})_{1 \beta} - \frac{1}{\sqrt{2}} N_{\alpha 4}^{t} (U_{L})_{2 \beta} , \\
\zeta_{\chi_{3}^{\pm} \chi_{0}^{i}}^{R} &= N_{2 \alpha} (U_{R}^{\dagger})_{1 \beta} + \frac{1}{\sqrt{2}} N_{3 \alpha} (U_{R}^{\dagger})_{3 \beta} , \quad (\alpha = 1, \ldots, 4; \beta = 1, 2) .
\end{align*}
\]

(74)

Here, the \( 4 \times 4 \) matrix \( N \) denotes the mixing matrix of neutralinos, two \( 2 \times 2 \) matrices \( U_{L}, U_{R} \) denote the left- and right-handed mixing matrices of charginos, respectively. In the limit of
heavy masses, the mass spectra of charginos and neutralinos are respectively approached as

\[ m_{\chi^\pm} \approx \text{diag}(|m_2|, |\mu_H|), \]

\[ m_{\chi^0} \approx \text{diag}(|m_1|, |m_2|, |\mu_H|, |\mu_H|). \]  

(75)

Here, \( \mu_H \) represents the \( \mu \) parameter in superpotential, and \( m_2, m_1 \) denote the soft breaking masses of \( SU(2) \times U(1) \) gauginos, respectively.

Applying Eq.(74) and Eq.(75), we get the two loop corrections to lepton MDMs in the heavy mass limit \(|m_1| = |m_2| = |\mu_H| = \mu_F \gg \mu_w\) as

\[ a_{2L}^{MSSM} = a_{2L}^{SM} + \frac{41G_F m_1^2}{96 \sqrt{2} \pi^2 s_w^2} - \frac{G_F m_1^2}{12 \sqrt{2} \pi^2 s_w^2} (1 - 2 s_w^2 + 2 s_w^4) \Delta \rho_{SU SY}, \]  

(76)

the 1-loop corrections to \( \rho \)-parameter from the heavy supersymmetric fermions can be written as \[18\]

\[ \Delta \rho_{SU SY} = \frac{\alpha_e}{16 \pi s_w^2} \left[ -6 + \frac{(1 - 2 s_\beta^2)^2}{c_w^2} + \frac{8 c_w^2 c^2 - 2 s_w^2 + 2 s_w^4}{c_w^2 - 1/(2 c^2)} \ln(2 c_w^2)}, \]

\[ + \frac{8 s_\beta^2 c_w^2 + 4 s_w^2 + 2 c_w^4}{c_w^2 - 1/(2 s_\beta^2)} \ln(2 s_\beta^2) \right]. \]  

(77)

The abbreviation symbols \( c_\beta = \cos \beta, \ s_\beta = \sin \beta \) with \( \tan \beta = v_2/v_1 \) denoting the ratio between the absolute values of two vacuum expectations: \( v_1, v_2 \). As \( 1 \leq \tan \beta \leq 60 \) and \( \mu_F \sim 1 \text{TeV} \), the contributions from this sector to muon MDMs is well below \( 10^{-11} \) which can be ignored safely.

At large \( \tan \beta \), the dominant two-loop supersymmetric corrections to lepton anomalous dipole moments are originated from those Bar-Zee type diagrams which are analyzed extensively. To obtain the corrections from those sectors, we formulate the relevant couplings as

\[ \mathcal{H}^{c,L}_{\chi^\pm \chi^0} = -c_\beta \left[ \frac{1}{\sqrt{2}} (U^\dagger_R)_{2\beta} \left( \frac{s_w}{c_w} N_{1\alpha} + N_{2\alpha} \right) + (U^\dagger_R)_{1\beta} N_{3\alpha} \right], \]

\[ \mathcal{H}^{c,R}_{\chi^\pm \chi^0} = s_\beta \left[ \frac{1}{\sqrt{2}} (U^\dagger_L)_{2\beta} \left( \frac{s_w}{c_w} N_{1\alpha} + N_{2\alpha} \right) - (U^\dagger_L)_{1\beta} N_{3\alpha} \right], \]

\[ \mathcal{H}^c_{\chi^\pm \chi^0} (h^0_k) = \mathcal{Z}^c_{1k} (U_L)_{2\alpha} (U_R)_{1\beta} + \mathcal{Z}^c_{2k} (U_L)_{1\alpha} (U_R)_{2\beta}, \ (k = 1, 2), \]

\[ \mathcal{H}^O_{\chi^\pm \chi^0} (h^0_k) = -s_\beta (U_L)_{2\alpha} (U_R)_{1\beta}, \]  

(78)
with $Z^e$ is the mixing matrix of two CP even Higgs. Assuming $|\mu_H| = |m_2| = |m_1| = m_F$ and $\theta_1 = \theta_2 = \theta_\mu$, we expand the effective couplings in Eq.39 and Eq.65 in powers of $m_w/m_F$ and get

\begin{align}
\sum_{\alpha,\beta} \mathcal{H}^{eL}_{x_\alpha x_\beta} \zeta^{L}_{x_\alpha x_\beta} &= \mathcal{O}\left(\frac{m^2}{m_F^2}\right), \\
\sum_{\alpha,\beta} \mathcal{H}^{eL}_{x_\alpha x_\beta} \zeta^{R}_{x_\alpha x_\beta} &= \mathcal{O}\left(\frac{m^2}{m_F^2}\right), \\
\sum_{\alpha,\beta} \mathcal{H}^{eR}_{x_\alpha x_\beta} \zeta^{L}_{x_\alpha x_\beta} &= \mathcal{O}\left(\frac{m^2}{m_F^2}\right), \\
\sum_{\alpha,\beta} \mathcal{H}^{eR}_{x_\alpha x_\beta} \zeta^{R}_{x_\alpha x_\beta} &= \frac{m_w}{m_F} e^{i\theta_\mu}, \\
\sum_{\alpha} \mathcal{H}^{e0}_{x_\alpha x_\alpha} (l^0_k) &\propto \frac{m_w}{m_F} e^{i\theta_\mu}, \\
\sum_{\alpha} \mathcal{H}^{e0}_{x_\alpha x_\alpha} &\propto \frac{m_w}{m_F} e^{i\theta_\mu},
\end{align}

(79)

where $\theta_{1,2} = \arg(m_{1,2})$, $\theta_\mu = \arg(\mu_H)$ are the corresponding CP violating phases. Applying the above equations and $\mathcal{B}_E = \mathcal{B}_O = \mathcal{B}_C = \tan \beta$, we find

\begin{align}
a_{2L} &= \frac{G_F \alpha_e m_e^2 m_w^2 \tan \beta}{8\sqrt{2}\pi^3 s^2 w m_F^2} \left[A + B \ln \frac{m^2}{m_F^2}\right] \cos \theta_\mu, \\
d_{2L} &= \frac{G_F \alpha_e e m_e^2 m_w^2 \tan \beta}{16\sqrt{2}\pi^3 s^2 w m_F^2} \left[C + D \ln \frac{m^2}{m_F^2}\right] \sin \theta_\mu.
\end{align}

(80)

Here, the form factors $A$, $B$, $C$, $D$ depend on the masses of higgs and the mixing matrix of neutral CP-even higgs.

The two loop corrections to lepton MDMs and EDMs are proportional to $1/m_F^2$ in large $\tan \beta$ limit, which are consistent with the result presented in Ref.[5] qualitatively. Using HME approximation and projection operator method, Ref.[5] approaches the corrections to the muon MDM from two loop diagrams in which a closed chargino/neutralino loop is inserted into those two Higgs doublet one loop diagrams as

\begin{align}
a_{2L}^{\text{MSSM}} ([5]) &= 11 \times 10^{-10} \left(\frac{\tan \beta}{50}\right) \left(\frac{100 \text{GeV}}{M_{\text{SUSY}}}\right)^2 \text{sign}(\mu_H),
\end{align}

(81)

under the assumption $\mu_H = m_2 = (3c_w^2/5s_w^2)m_1 = m_A = M_{\text{SUSY}}$ and large $\tan \beta$ limit, where $m_A$ denotes the mass of neutral CP-odd Higgs.
FIG. 10: The supersymmetric corrections to the muon MDM \(a_\mu\) and varies with the supersymmetric scale \(M_{\text{SUSY}}\) when \(\mu_{H} = m_{2} = (3c_{w}^{2}/5s_{w}^{2})m_{1} = m_{A} = M_{\text{SUSY}}\) and \(\tan \beta = 5, 50\). Here the solid line stands for the two-loop contributions from neutralino/chargino sector with \(\tan \beta = 50\), the dash line stands for the results of Eq.(81) with \(\tan \beta = 50\); the dot line stands for the two-loop contributions from neutralino/chargino sector with \(\tan \beta = 5\), the dash-dot line stands for the results of Eq.(81) with \(\tan \beta = 5\).

To compare our result with that presented in Ref.[5] numerically, we take the same assumption \(\mu_{H} = m_{2} = (3c_{w}^{2}/5s_{w}^{2})m_{1} = m_{A} = M_{\text{SUSY}}\) on the parameter space of supersymmetry. In addition, the existence of a CP-even SM like Higgs with mass above 115 GeV sets a strong constraint on the parameter space of the employed model. To address this problem, we include all loop corrected effects in the Higgs potential [25], and choose the Yukawa couplings of the 3rd generation sfermions as \(A_{t} = A_{b} = A_{\tau} = M_{\text{SUSY}}\). Considering those points above, we present the numerical results in Fig.10. For large \(\tan \beta\) case, our numerical results agree with the approximation presented in Eq.(81) very well. It implies that the equation Eq.(81) fits the exact result perfectly for large \(\tan \beta\) and \(\mu_{H} = m_{2} = (3c_{w}^{2}/5s_{w}^{2})m_{1} = m_{A} = M_{\text{SUSY}}\).
D. the littlest Higgs with T-parity

In the framework of the littlest Higgs with T-parity, all the SM particles are even, as well as the corresponding mirror fields are odd under the discrete T-transformation \[19\]. In order to avoid dangerous contributions to the Higgs mass from one-loop quadratic divergences, we introduce additionally one T-even top quark \(T^+\) together with its mirror partner, the T-odd top quark \(T^-\) besides the SM fermions \(f^i\) and their mirror partners \(f_H^i\). The couplings among the SM gauge bosons and heavy fermions in Eq.(40) and Eq.(8) are respectively given as \[20\]

\[
\begin{align*}
\xi^L_{T^+_+} &= -\eta^2_L \frac{v^2}{f^2} + \frac{4}{3} s^2_w, & \xi^R_{T^+_+} &= \frac{4}{3} s^2_w, \\
\xi^L_{T^+_t} &= -\eta_L \frac{v}{f}, & \xi^R_{T^+_t} &= 0, \\
\xi^L_{T^-_-} &= \xi^R_{T^-_-} = \frac{4}{3} s^2_w, \\
\xi^L_{u_H^i u_H^j} &= \xi^R_{u_H^i u_H^j} = -1 + \frac{4}{3} s^2_w, \\
\xi^L_{d_H^i d_H^j} &= \xi^R_{d_H^i d_H^j} = 1 - \frac{2}{3} s^2_w, \\
\xi^L_{\nu_H^i \nu_H^j} &= \xi^R_{\nu_H^i \nu_H^j} = -1, \\
\xi^L_{e_H^i e_H^j} &= \xi^R_{e_H^i e_H^j} = 1 - 2 s^2_w, \\
\zeta^L_{T^+_b} &= \frac{V_{tb}}{\sqrt{2}} \eta_L \frac{v}{f}, & \zeta^R_{T^+_b} &= 0, \\
\zeta^L_{u_H^i u_H^j} &= \zeta^R_{u_H^i u_H^j} = \frac{\delta_{ij}}{\sqrt{2}}, \\
\zeta^L_{\nu_H^i \nu_H^j} &= \zeta^R_{\nu_H^i \nu_H^j} = \frac{\delta_{ij}}{\sqrt{2}}.
\end{align*}
\]  

(82)

Here, \(f\) is the breaking scale of a large \(SU(5)/SO(5)\) symmetry, and

\[
\eta_L = \frac{\lambda_1^2}{\lambda_1^1 + \lambda_2^2},
\]  

(83)

with \(\lambda_{1,2}\) represent the Yukawa couplings of top quark sector. Additionally, the relations among the masses of heavy fermions are presented as

\[
m_{T^+_+} = \frac{f}{v} \frac{m_t}{\sqrt{\eta_L (1 - \eta_L)}},
\]
Applying the equations above, we give the leading corrections from heavy fermions to lepton MDMs in the limit \( f = m_{\ell_i} = m_{\ell_{\ell_i}} = m_F \gg m_w \) as

\[
a_{2L}^{LHT} = a_{2L}^{SM} + \frac{5G_F \alpha_e m_{\ell_i}^2}{6\sqrt{2}\pi^3 s_w^2} - \frac{G_F m_{\ell_i}^2}{12\sqrt{2}\pi^2 s_w^2} \left( 1 - 2s_w^2 + 2s_w^4 \right) \Delta \rho_{LHT},
\]

the 1-loop corrections to \( \rho \)-parameter from heavy fermions can be written as [21]

\[
\Delta \rho_{LHT} = \frac{\eta_L}{2f} \left[ -\frac{1}{2} + \ln \frac{m_{\ell_i}^2}{m_w^2} \right] \Delta \rho_{SM}.
\]

E. the universal extra dimension

If all particles of the SM are zero modes of corresponding 5-dimension bulk fields [22], the KK excitations of fermion acquire the masses

\[
m_{f_{n(i)}} = \sqrt{m_{f_i}^2 + \frac{n^2}{R^2}} , \quad (n = 1, 2, \cdots),
\]

where \( m_{f_i} \) denotes the mass of corresponding SM field, and \( R \) is the compactification radius. To fit the present experimental data, we choose \( 1/R \geq 200 \text{ GeV} \). Furthermore, we formulate the couplings among the zero modes of weak gauge bosons and the KK excitations of fermions as [23]
\[ \zeta^L_{i(n)_{d_j}} = - s_{i(n)} \frac{V_{ij}}{\sqrt{2}} , \quad \zeta^R_{i(n)_{d_j}} = 0 , \]
\[ \zeta^L_{i(n)_{\nu_j}} = \delta_{ij} \frac{\sqrt{2}}{\sqrt{2}}, \quad \zeta^R_{i(n)_{\nu_j}} = 0 , \]  
\( (88) \)

with

\[ \tan 2\alpha_{i(n)} = \frac{m_{i(n)}}{n/R} , \quad (f_i = u_i, d_i, \nu_i, e_i, \quad i = 1, 2, 3) , \]
\[ c_{i(n)} = \cos \alpha_{i(n)} , \quad s_{i(n)} = \sin \alpha_{i(n)} . \]  
\( (89) \)

Using Eq.\( (87) \), Eq.\( (88) \) and Eq.\( (89) \), we formulate the leading contributions from the KK excitations of fermions as

\[ a^{UED}_{2L} = a^{SM}_{2L} - \frac{G_F \alpha_e m^2_i}{2\sqrt{2\pi^3 s_w^2}} . \]  
\( (90) \)

IV. CONCLUSION

In this work, we have investigated the electroweak corrections to the lepton MDMs and EDMs from some two loop diagrams in which a closed heavy fermion loop is inserted into those two Higgs doublet diagrams. Adopting on-shell scheme, we subtract the ultraviolet divergence caused by the subdiagrams and get the theoretical predictions on lepton MDMs. As the masses of virtual fermions in inner loop are much heavier than the electroweak scale, we verify the final results satisfying the decoupling theorem explicitly if the interactions among Higgs and heavy fermions do not contain the nondecoupling couplings. Our results are universal for all extensions of the SM where the interactions among the electroweak gauge bosons and heavy fermions are renormalizable. As application of our analysis, we present the leading corrections to lepton MDMs in some popular extensions of the SM, such as the fourth generation, supersymmetry, universal extra dimension, and the littlest higgs with T-parity.
Acknowledgments

The work has been supported by the National Natural Science Foundation of China (NNSFC) with Grant No. 10675027.

APPENDIX A: THE FORM FACTORS

\[ N_{ww}^{(1)} = -\frac{24}{(q_1^2 - m_w^2)^3} \left[ -\frac{(q_1^2)^2 q_1 \cdot q_2 - (q_2^2)^2 q_2^2}{D + 2} + 4 \cdot \frac{q_1^2 q_1 \cdot q_2^2 - (q_2^2)^2 q_2^2}{D(D + 2)} \right] \\
+ \frac{6}{D + 2} \frac{q_1^2(q_2^2)^2 - q_1 \cdot q_2(q_2^2)^2}{(q_2^2 - m_{F_B}^2)^3} + \frac{18}{D + 2} \frac{(q_1^2)^2 q_2^2 - q_1^2 q_1 \cdot q_2 q_2^2}{(q_2^2 - m_w^2)^2(q_2^2 - m_{F_B}^2)} \\
+ \frac{12}{D + 2} \frac{q_2^2 q_1 \cdot q_2 q_2^2 - q_1^2(q_2^2)^2}{(q_2^2 - m_w^2)(q_2^2 - m_{F_B}^2)^2} - \frac{2}{(q_2^2 - m_{F_B}^2)^2} \left[ (1 - \frac{Q_\beta}{D}) q_1^2 q_2^2 \right] \\
- \frac{1}{(q_1^2 - m_w^2)(q_2^2 - m_{F_B}^2)} \left[ \left( \frac{3 - \frac{2}{D} Q_\beta}{q_1^2 q_2^2} \right) q_1 \cdot q_2 \right] \\
+ \left( 1 + \frac{2}{D} \right) q_1 \cdot q_2 \frac{q_2^2}{q_2^2} \left[ \left( \frac{D}{4} - \frac{Q_\beta}{2} \right) q_1^2 - \left( \frac{D}{4} + \frac{1}{2} + \frac{1}{D} \right) (1 + \frac{1}{D} Q_\beta) q_1 \cdot q_2 \right] \\
- \frac{1}{q_1^2 - m_w^2} \left[ \left( \frac{D}{4} + 2 - (1 + \frac{2}{D}) Q_\beta \right) q_1^2 - \left( \frac{D}{2} + 1 + \frac{2}{D} - \frac{4}{D} Q_\beta \right) q_1 \cdot q_2 \right], \]

\[ N_{ww}^{(2)} = \frac{2}{D} \frac{Q_\beta q_1^2 q_2^2 - q_1 \cdot q_2 q_2^2}{(q_2^2 - m_{F_B}^2)^2} - \frac{1}{q_2^2 - m_{F_B}^2} \left[ \frac{Q_\beta}{2} q_1^2 - \left( \frac{1}{2} + \frac{1}{D} - \frac{1}{D} Q_\beta \right) q_1 \cdot q_2 \right] \\
+ \frac{2}{D} \frac{Q_\beta q_1^2 q_1 \cdot q_2 - q_1^2 q_2^2}{(q_1^2 - m_w^2)(q_2^2 - m_{F_B}^2)} + \frac{q_2^2 q_1 \cdot q_2}{q_1^2 - m_w^2}, \]

\[ N_{ww}^{(3)} = \frac{6(D - 2)}{D(D + 2)} \frac{q_1 \cdot q_2 q_2^2}{(q_2^2 - m_{F_B}^2)^3} \left( 1 - \frac{2}{D} \frac{Q_\beta}{q_2^2 - m_{F_B}^2} \right) \frac{q_1 \cdot q_2}{(q_2^2 - m_{F_B}^2)^2}. \]
\[
\begin{align*}
&\frac{24(D - 2)}{D(D + 2)} \frac{(q_1^2)^2}{(q_1^2 - m_w^2)^3} - \frac{1}{D} (Q_\beta - 1) \frac{q_1 \cdot q_2}{(q_2^2 - m_{F_\beta}^2)((q_2 - q_1)^2 - m_{V_\alpha}^2)} \\
&+ \frac{18(D - 2)}{D(D + 2)} \frac{q_1^2 q_1 \cdot q_2}{(q_1^2 - m_w^2)^2(q_2^2 - m_{F_\beta}^2)} + \frac{12(D - 2)}{D(D + 2)} \frac{q_1^2 q_2^2}{(q_1^2 - m_w^2)(q_2^2 - m_{F_\beta}^2)^2} \\
&- 3\left(1 - \frac{6}{D}\right) \frac{q_1^2}{(q_1^2 - m_w^2)^2} - \frac{1}{(q_1^2 - m_w^2)(q_2^2 - m_{F_\beta}^2)} \left[\left(\frac{3D - 6}{D + 2} - \frac{2}{D} Q_\beta\right) q_1^2ight] \\
&- \left(\frac{D^2 + 20}{D(D + 2)} - \frac{2}{D} Q_\beta\right) q_1 \cdot q_2 - \frac{2}{D} \frac{(Q_\beta - 1) q_1 \cdot q_2}{(q_1^2 - m_w^2)((q_2 - q_1)^2 - m_{V_\alpha}^2)} ; \\
\mathcal{N}_{ww}^{(4)} &= \frac{2}{D} Q_\beta \left[\frac{q_1^2}{(q_1^2 - m_w^2)(q_2^2 - m_{\bar{\chi}_\beta}^2)} + \frac{q_1 \cdot q_2}{(q_2^2 - m_{\bar{\chi}_\beta}^2)^2}\right] \\
&+ \frac{2}{D} (Q_\beta - 1) \left[\frac{q_1^2}{(q_1^2 - m_w^2)((q_2 - q_1)^2 - m_{\bar{\chi}_\alpha}^2)} + \frac{q_1 \cdot q_2}{2(q_2^2 - m_{\bar{\chi}_\beta}^2)((q_2 - q_1)^2 - m_{\bar{\chi}_\alpha}^2)}\right]. \\
\end{align*}
\]

**APPENDIX B: THE FUNCTIONS**

The definition of $\Psi(x, y, z)$ is written as:

- $\lambda^2 > 0$, $\sqrt{y} + \sqrt{z} < \sqrt{x}$:
  \[
  \Psi(x, y, z) = 2 \ln\left(\frac{x + y - z - \lambda}{2x}\right) \ln\left(\frac{x - y + z - \lambda}{2x}\right) - \ln\frac{y}{x} \ln\frac{z}{x} \\
  - 2L_{i_2}\left(\frac{x + y - z - \lambda}{2x}\right) - 2L_{i_2}\left(\frac{x - y + z - \lambda}{2x}\right) + \frac{\pi^2}{3} ,
  \]
  where $L_{i_2}(x)$ is the spence function;

- $\lambda^2 > 0$, $\sqrt{x} + \sqrt{z} < \sqrt{y}$:
  \[
  \Psi(x, y, z) = \text{Eq.}\,(B1)\,(x \leftrightarrow y) ;
  \]

- $\lambda^2 > 0$, $\sqrt{x} + \sqrt{y} < \sqrt{z}$:
  \[
  \Psi(x, y, z) = \text{Eq.}\,(B1)\,(x \leftrightarrow z) ;
  \]
\( \lambda^2 < 0: \)

\[
\Psi(x, y, z) = 2 \{ Cl_2\left(2 \arccos\left(\frac{-x + y + z}{2\sqrt{yz}}\right)\right) + Cl_2\left(2 \arccos\left(\frac{x - y + z}{2\sqrt{xz}}\right)\right) \\
+ Cl_2\left(2 \arccos\left(\frac{x + y - z}{2\sqrt{xy}}\right)\right) \},
\]

where \( Cl_2(x) \) denotes the Clausen function.

The expressions of \( \varphi_0(x, y), \varphi_1(x, y), \varphi_2(x, y) \) and \( \varphi_3(x, y) \) are given as

\[
\varphi_0(x, y) = \begin{cases} 
(x + y) \ln x \ln y + (x - y) \Theta(x, y), & x > y; \\
2x \ln^2 x, & x = y; \\
(x + y) \ln x \ln y + (y - x) \Theta(y, x), & x < y. 
\end{cases}
\]

(B5)

\[
\varphi_1(x, y) = \begin{cases} 
-\ln x \ln y - \frac{x + y}{x - y} \Theta(x, y), & x > y; \\
4 - 2 \ln x - \ln^2 x, & x = y; \\
-\ln x \ln y - \frac{x + y}{y - x} \Theta(y, x), & x < y. 
\end{cases}
\]

(B6)

\[
\varphi_2(x, y) = \begin{cases} 
\frac{(2x^2 + 6xy) \ln x - (6xy + 2y^2) \ln y}{(x-y)^3} - \frac{4xy}{(x-y)^3} \Theta(x, y), & x > y; \\
-\frac{5x + 2}{3x} \ln x, & x = y; \\
\frac{(2x^2 + 6xy) \ln x - (6xy + 2y^2) \ln y}{(x-y)^3} - \frac{4xy}{(y-x)^3} \Theta(y, x), & x < y. 
\end{cases}
\]

(B7)

\[
\varphi_3(x, y) = \begin{cases} 
-\frac{12xy(x+y)}{(x-y)^5} \Theta(x, y) - \frac{2(x^2 + 6xy + y^2)}{(x-y)^4}, & x > y; \\
-\frac{53x^2 + 1}{150x^2} \ln x, & x = y; \\
-\frac{12xy(x+y)}{(y-x)^5} \Theta(y, x) - \frac{2(x^2 + 6xy + y^2)}{(y-x)^4}, & x < y; \\
+ \frac{2(x^3 + 20xy^2 + 11x^2y) \ln x - 2(y^3 + 20xy^2 + 11x^2y) \ln y}{(x-y)^5}, & x < y. 
\end{cases}
\]

(B8)

with

\[
\Theta(x, y) = \ln x \ln \frac{y}{x} - 2 \ln(x - y) \ln \frac{y}{x} - 2\text{Li}_2\left(\frac{y}{x}\right) + \frac{\pi^2}{3}.
\]

(B9)
The functions adopted in the text are written as
\[ g_{i,j}(x, y, z) = \frac{x^i \ln^j x - y^i \ln^j y}{x - y}, \quad \Omega_i(x, y; u, v) = \frac{x^i \Phi(x, u, v) - y^i \Phi(y, u, v)}{x - y}, \]
\[ T_1(x, y, z) = \frac{(y - z)^2}{3x^3} (1 + g_{i,1}(y, z)) - \frac{1}{48x^2} \left[ 10g_{2,1}(y, z) + (31 - 12Q_\beta)y + (19 + 12Q_\beta)z - (22 + 12Q_\beta)y \ln y - (16 - 12Q_\beta)z \ln z \right] - \frac{1}{8x} \left[ 3(1 - Q_\beta) - 3 - 2Q_\beta \right] \ln z \]
\[ - \frac{1}{48} \left\{ \partial^4 \partial^4 \left[ (y - z)^2 \Phi - x^2 \Phi \right] (x, y, z) \right\} - \frac{1}{6} \frac{\partial^4 \partial^4}{\partial x^3 \partial z} \left[ (y - z) \Phi + xz \Phi \right] (x, y, z) \]
\[ + \frac{6}{48} \partial^4 \partial^4 \partial^4 \partial^4 \left[ (y - z) \Phi - xz \Phi \right] (x, y, z) \]
\[ - 2 \frac{\partial^4 \partial^4}{\partial x \partial z} \left[ z^2 (y - z) \frac{\Phi(x, y, z) - \varphi_0(y, z)}{x} + z^2 \Phi(x, y, z) \right] \]
\[ + 3 \frac{\partial^4 \partial^4}{\partial x \partial z} \left[ (y - z) \frac{\Phi(x, y, z) - \varphi_0(y, z)}{x} + \left( 4y - 4z + 3x \right) \Phi(x, y, z) \right] \]
\[ + 6 \frac{\partial^3 \partial^3}{\partial x \partial z} \left[ \frac{5}{2} (1 - Q_\beta) z (y - z) \frac{\Phi(x, y, z) - \varphi_0(y, z)}{x} + \frac{3}{2} \Phi(x, y, z) \right] \]
\[ - 3 \frac{\partial^3 \partial^3}{\partial x \partial z} \left[ 3z (y - z) \frac{\Phi(x, y, z) - \varphi_0(y, z)}{x} + \left( 6 - Q_\beta \right) y \right] \]
\[ + (11 - 3Q_\beta) z \Phi(x, y, z) - (6 - Q_\beta) x \Phi(x, y, z) \right\}, \]
\[ T_2(x, y, z) = - \frac{1}{16} \left\{ \frac{2 \ln z}{x} - \frac{4}{x^2} (y - z + y \ln y - z \ln z) \right\} \]
\[ + \partial^3 \partial^3 \left[ (1 - 2Q_\beta) z \Phi(x, y, z) - (y - z) \frac{\Phi(x, y, z) - \varphi_0(y, z)}{x} \right] \]
\[ - \partial^2 \partial^2 \partial^2 \partial^2 \left[ (3 - 5Q_\beta) \Phi(x, y, z) - (3 - Q_\beta) (y - z) \frac{\Phi(x, y, z) - \varphi_0(y, z)}{x} \right] \]
\[ - \partial^3 \partial^3 \partial^3 \partial^3 \left[ Q_\beta (x \Phi - (Q_\beta y + (2 - Q_\beta) z) \Phi \right] (x, y, z) \]
\[ - 2 \partial^2 \partial^2 \left[ \Phi(x, y, z) + (y - z) \frac{\Phi(x, y, z) - \varphi_0(y, z)}{x} \right] \right\}, \]
\[ T_3(x, y, z) = \frac{5}{12x^2} g_{1,1}(y, z) + \frac{7}{6x^2} + \frac{1 - 3Q_\beta}{24xz} - \frac{1 - Q_\beta}{8x^2} \ln y + \frac{4 - Q_\beta}{8x^2} \ln z \]
\[ - \frac{1}{48} \left\{ \frac{\partial^4 \partial^4}{\partial x \partial z} \left[ (y - z) \Omega_0 - z \Omega_1 \right] (x, y, z) \right\}. \]
\[-3(1 - Q_\beta) \frac{\partial^3}{\partial x \partial y \partial z} [(y - z) \frac{\Phi(x, y, z) - \varphi_0(y, z)}{x} - \Phi(x, y, z)]
+ 3(1 - Q_\beta) \frac{\partial^3}{\partial y \partial x} [(y - z) \frac{\Phi(x, y, z) - \varphi_0(y, z)}{x} - \Phi(x, y, z)]
- \frac{\partial^4}{\partial x^4} (x \Phi(x, y, z)) + 3 \frac{\partial^4}{\partial x^3 \partial z} [(y - z) \Phi - x \Phi(x, y, z)]
- 6 \frac{\partial^4}{\partial x^2 \partial y^2} (z \Phi(x, y, z)) - 6 \frac{\partial^3}{\partial x \partial y} (x, y, z)
+ 3 \frac{\partial^3}{\partial x^2 \partial z} [(3 - Q_\beta)(y - z) \frac{\Phi(x, y, z) - \varphi_0(y, z)}{x} + (1 - Q_\beta) \Phi(x, y, z)]
+ 3(1 - Q_\beta) \frac{\partial^3}{\partial x^2 \partial y} [(y - z) \frac{\Phi(x, y, z) - \varphi_0(y, z)}{x} - \Phi(x, y, z)] \},

T_4(x, y, z) = -\frac{1}{16} \left\{ Q_\beta \left[ \frac{2}{zx} - 2 \frac{\partial^3 \Phi}{\partial x^2 \partial y} (x, y, z)
+ \frac{\partial^3}{\partial x^2 \partial y} [(y - z) \frac{\Phi(x, y, z) - \varphi_0(y, z)}{x} - \Phi(x, y, z)] \right] - Q_\alpha \left[ 2 \frac{\partial^3 \Phi}{\partial x^2 \partial y} (x, y, z)
- \frac{\partial^3}{\partial x^2 \partial y} [(y - z) \frac{\Phi(x, y, z) - \varphi_0(y, z)}{x} - \Phi(x, y, z)] \right] \right\},

T_5(x, y, z) = \frac{2}{3x} + \left( -\frac{1}{3x^2} + \frac{4 \ln x}{3x^2} \right) (y + z)
+ \frac{7}{6x^2} + \frac{2}{3x^2} \ln x \right) (y \ln y + z \ln z)
+ \frac{2}{3x^3} - \frac{4}{3x^3} \ln x \right) (y - z)^2 (1 + \varrho_{1,1}(y, z))
+ \frac{23}{6x^2} (y + z) \varrho_{1,1}(y, z) - \frac{5 \varrho_{2,1}(y, z)}{x^2}
- \frac{1}{3x^2} \left( 1 - \frac{2(y + z)}{x} \right) (\Phi(x, y, z) - \varphi_0(y, z))
+ \frac{1}{3x} \left( \frac{y + z}{x} - \frac{2(y - z)^2}{x^2} \right) \varphi_1(y, z)
+ \frac{1}{3x} \left( 1 - \frac{3(y + z)}{x} + \frac{2(y - z)^2}{x^2} \right) \frac{\partial \Phi}{\partial x}(x, y, z)
- \frac{1}{3} \left( 1 - \frac{2(y + z)}{x} + \frac{(y - z)^2}{x^2} \right) \frac{\partial^2 \Phi}{\partial x^2}(x, y, z)
- \frac{(y - z)^2}{3x^2} \varphi_2(y, z),

T_6(x, y, z) = \frac{4}{x} \ln z - \frac{4}{x^2} \left( y - y \ln y - z + z \ln z \right) + \frac{\partial^3}{\partial x^2 \partial z} [(y - 3z - x) \Phi(x, y, z)]
- 2 \frac{\partial^3}{\partial x \partial z^2} \left[ \frac{yz - z^2}{x} \Phi(x, y, z) - \varphi_0(y, z) \right]
- \frac{\partial^2}{\partial x \partial z} \left[ \Phi(x, y, z) - \frac{5}{x} (y - z) (\Phi(x, y, z) - \varphi_0(y, z)) \right] \right\]
\[-2 \frac{\partial^2}{\partial x^2} \left( \frac{y-z}{x} \left( \Phi(x, y, z) - \varphi_0(y, z) \right) + 2\Phi(x, y, z) \right), \]

\[T_7(x, y, z) = -\frac{1}{x^2} \left( \varphi_0 - (y-z) \frac{\partial \varphi_0}{\partial z} \right)(y, z) + \left[ 2z \frac{\partial^3 \Phi}{\partial x \partial z^2} + \frac{\partial^2 \Phi}{\partial x^2} \right] \frac{y-z}{x^2} \frac{\partial \Phi}{\partial z} (x, y, z), \]

\[T_9(x, y, z) = -4 \left( \frac{\partial^3 \Phi}{\partial x^2 \partial z} + \frac{\partial^3 \Phi}{\partial x^2 \partial y} \right)(x, y, z) + \frac{4}{x^2} \left( \ln y - \ln z \right) + \frac{2}{x} \left( \frac{\partial^3 \Phi}{\partial x^2 \partial z} - \frac{\partial^3 \Phi}{\partial x \partial y \partial z} \right) \left[ \Phi(x, y, z) - \frac{y-z}{x} \left( \Phi(x, y, z) - \varphi_0(y, z) \right) \right], \]

\[T_{12}(x, y, z) = \frac{1}{x} \left\{ -4(2 + \ln y) \left( \ln x - 1 \right) - \frac{\partial}{\partial z} \left[ (1 + 2\frac{y-z}{x}) \Phi \right](x, y, z) \right. \]

\[
+ \frac{\partial}{\partial z} \left[ \left( 1 + 2\frac{y-z}{x} \right) \varphi_0 + 2(y-z)\varphi_1 \right](y, z) \right\}, \]

\[F_1(x, y, z, u) = 2 \left( (2 - Q_\beta) \ln u + 1 - 2Q_\beta \right) \varrho_{o,1}(x, y) - \frac{6(z-u)}{xy} \]

\[- \frac{6(z \ln z - u \ln u)}{xy} + \frac{Q_\beta xy + 2(x+y)(z-u)}{x^2 y^2} \varphi_0(z, u) \]

\[- \frac{Q_\beta z - (2 + Q_\beta) u \varphi_0(y, z, u)}{xy} - \frac{u(z-u) \varphi_0(z, u)}{xy} \Omega_0(z, u) \]

\[+ \left( Q_\beta - (Q_\beta z - (2 + Q_\beta) u) \frac{\partial}{\partial u} - u(z-u) \frac{\partial^2 \varphi_0}{\partial u^2} \right) \Omega_{-1}(x, y; z, u), \]

\[- \left. \left( \frac{\partial}{\partial u} + u \frac{\partial^2}{\partial u^2} \right) \Omega_0(x, y; z, u) - \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right)^2 \Omega_1(x, y; z, u) \right\} - \left( z - u \right) \Omega_0(x, y; z, u) \]

\[- \frac{\partial \Omega_0}{\partial u}(x, y; z, u) - 2(z-u) \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) \Omega_{-1}(x, y; z, u), \]

\[F_2(x, y, z, u) = 2 \left( \ln u - 1 - (1 - Q_\beta)(2 + \ln z) \right) \varrho_{o,1}(x, y) - \frac{6(z-u)}{xy} \]
\[
-6(z \ln z - u \ln u) - \frac{Q_5 xy}{x^2 y^2} (x + y)(z - u) \varphi_0(z, u) \\
+ \frac{z + u \partial \varphi_0}{xy} (z, u) + (1 - Q_\beta) \frac{z - u \partial \varphi_0}{xy} (z, u) - u(z - u) \frac{\partial^2 \varphi_0}{\partial u^2} (z, u) \\
+ \left( - Q_\beta + (z + u) \frac{\partial}{\partial u} + (1 - Q_\beta) (z - u) \frac{\partial}{\partial z} - u(z - u) \frac{\partial^2}{\partial u^2} \right) \Omega_{-1}(x, y; z, u) \\
+ \left( - \frac{\partial}{\partial u} - (1 - Q_\beta) \frac{\partial}{\partial z} + u \frac{\partial^2}{\partial u^2} \right) \Omega_0(x, y; z, u) \\
+ \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right)^2 \left[ \Omega_1(x, y; z, u) - (z - u) \Omega_0(x, y; z, u) \right] \\
+ \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) \left[ -2 \Omega_0(x, y; z, u) + 4u \frac{\partial \Omega_0}{\partial u} (x, y; z, u) \right] \\
-2(z - u) \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) \Omega_{-1}(x, y; z, u),
\]

\[
F_3(x, y, z, u) = -2(2 + \ln u) \varphi_0(x, y) + \frac{1}{xy} \varphi_0(z, u) - \frac{z - u \partial \varphi_0}{xy} (z, u) \\
+ (1 - 2Q_\beta) \frac{\partial}{\partial u} \Omega_0(x, y; z, u) + (1 - (z - u) \frac{\partial}{\partial u}) \Omega_{-1}(x, y; z, u),
\]

\[
F_4(x, y, z, u) = 2(2Q_\beta + \ln u - (1 - Q_\beta) \ln z) \varphi_0(x, y) \\
- \frac{Q_5}{xy} \varphi_0(z, u) + \frac{z - u}{xy} \left( \frac{\partial}{\partial u} + (1 - Q_\beta) \frac{\partial}{\partial z} \right) \varphi_0(z, u) \\
+ \left( - Q_\beta + (z - u) \frac{\partial}{\partial u} + (1 - Q_\beta)(z - u) \frac{\partial}{\partial z} \right) \Omega_{-1}(x, y; z, u) \\
- \left( \frac{\partial}{\partial u} + (1 - Q_\beta) \frac{\partial}{\partial z} \right) \Omega_0(x, y; z, u),
\]

\[
F_5(x, y, z, u) = \frac{1}{xy} \frac{\partial}{\partial u} \left[ (z - u) \varphi_0 \right] (z, u) + \frac{1}{x - y} \left( \frac{\partial}{\partial u} \left[ (1 + \frac{z - u}{x}) \Phi \right] (x, z, u) \\
- \frac{\partial}{\partial u} \left[ (1 + \frac{z - u}{y}) \Phi \right] (y, z, u) \right],
\]

\[
F_6(x, y, z, u) = -\frac{1}{xy} \frac{\partial}{\partial u} \left[ (z - u) \varphi_0 \right] (z, u) + \frac{1}{x - y} \left( \frac{\partial}{\partial u} \left[ (1 - \frac{z - u}{x}) \Phi \right] (x, z, u) \\
- \frac{\partial}{\partial u} \left[ (1 - \frac{z - u}{y}) \Phi \right] (y, z, u) \right).
\]

(B10)

[1] [The Mzon $g - 2$ Collaboration], Phys. Rev. Lett. 92(2004)161802.

[2] J. P. Miller, E. de Rafael and B. L. Roberts, Mzon ($g$-2): experiment and theory, Rep. Prog. Phys. 70(2007)795.
[3] F. Jegerlehner, Acta Phys. Polon. B 38(2007)3021.
[4] A. Czarnecki, B. Krazse and W. J. Marciano, Phys. Rev. D. 52(1995)2619; Phys. Rev. Lett. 76(1996)3267; T. Kukhto, E. Kuraev, A. Schiller and Z. Silagadye, Nucl. Phys. B. 371(1992)567.
[5] S. Heinemeyer, D. Stöckinger and G. Weiglein, Nucl. Phys. B. 690(2004)62; ibid. 699(2004)103.
[6] C. Chen, C. Geng, Phys. Lett. B. 511(2001)77.
[7] A. Pilaftsis, Phys. Rev. D. 58(1998)096010; Phys. Lett. B435(1998)88; A. Pilaftsis, C. E. M. Wagner, Nucl. Phys. B. 533(1999)3; M. Carena, J. Ellis, A. Pilaftsis, C. E. M. Wagner, ibid. 586(2000)92; ibid. 625(2002)345.
[8] Tai-Fu Feng, Phys. Rev. D 70(2004)096012.
[9] Tai-Fu Feng, Xue-Qian Li, Jukka Maalampi, Xinmin Zhang, Phys. Rev. D. 71(2005)056005.
[10] Tai-Fu Feng, Tao Huang, Xue-Qian Li, Xin-Min Zhang, Shu-Min Zhao, Phys.Rev.D. 68(2003)016004; Tai-Fu Feng, Xue-Qian Li, Lin Lin, Jukka Maalampi, and He-Shan Song, Phys. Rev. D 73(2006)116001.
[11] Tai-Fu Feng, Lin Sun, and Xiu-Yi Yang, Phys. Rev. D 77(2008)116008; Nucl. Phys. B. 800(2008)221.
[12] M. Bohm, H. Spiesberger, W. Hollik, Fortsch. Phys. 34(1986)687; A. Denner, ibid. 41(1993)307.
[13] L. F. Abbott, Nucl. Phys. B 185(1981)189; M. B. Gavela, G. Girardi, C. Malleville, and P. Sorba, Nucl. Phys. B 193(1981)257; N. G. Deshpande, M. Nayerimonfared, Nucl. Phys. B 213(1983)390.
[14] A. I. Davydychev and J. B. Tazsk, Nucl. Phys. B. 397(1993)123.
[15] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. 68(1996)1125; R. Grigjanis, P. J. O'Donnell, M. Sutherland and H. Navelet, Phys. Rep. 228(1993)93.
[16] S. M. Barr and A. Zee, Phys. Rev. Lett. 65(1990)21.
[17] M. J. Ramsey-Mzsolf, S. Su, Phys. Rept. 456(2008)1.
[18] C. S. Lim, T. Inami, and N. Sakai, Phys. Rev. D 29(1984)1486; M. Dress, K. Hagiwara, Phys. Rev. D 42(1990)1709.
[19] J. Hubisz and P. Meade, Phys. Rev. D. 71(2005)035016.

[20] M. Blanke, A. J. Buras, A. Poschenrieder, S. Reckziegl, C. Tarantino, S. Uhlig and A. Weiler, JHEP 01(2007)066.

[21] M. C. Chen, S. Dawson, Phys. Rev. D 70(2004)015003.

[22] H. Georgi, A. K. Grant and G. Hailz, Phys. Rev. D. 63(2001)064027; T. Appelquist, H.-C. Cheng and B. Dobrescu, Phys. Rev. D. 64(2001)035002.

[23] A. J. Buras, M. Spranger, and A. Weiler, Nucl. Phys. B. 660(2003)225.

[24] B. Laztrzp, E. De Rafael, Phys. Rev. 174(1968)1835; M. Samzel, G.-W. Li, Phys. Rev. D. 44(1991)3935, *ibid.* 48(1993)1879 (Erratum).

[25] S. Heinemeyer, W. Hollik and G. Weiglein, Ezr. Phys. J. C 9(1999)343; Comp. Phys. Comm. 124(2000)76.