Abstract. A microscopic four-body description of near-threshold coherent photoproduction of the $\eta$ meson on the $(3N)$-nuclei is given. The photoproduction cross-section is calculated using the Finite Rank Approximation (FRA) of the nuclear Hamiltonian. The results indicate that the final state interaction of the $\eta$ meson with the residual nucleus plays an important role in the photoproduction process. Sensitivity of the results to the choice of the $\eta N T$-matrix is investigated. The importance of obeying the condition of $\eta N$ unitarity is demonstrated.

The high level of reliability of the modern few-body theory provides the means for making conclusions about underlying two-body interactions from experimental data on relevant few-body processes. The purpose of this report is to present the results of few-body calculations concerning the coherent $\eta$-photoproduction on the tritium and $^3$He targets,

$$^3\text{H}(\gamma, \eta)^3\text{H} \quad \text{and} \quad ^3\text{He}(\gamma, \eta)^3\text{He},$$

from which some conclusions about the $T$-matrices describing the elastic $\eta N$ scattering and the photoproduction process $N(\gamma, \eta)N$ could be made. To the best of our knowledge, no experimental data on such coherent reactions has been published yet.

To describe the few-body dynamics of this reaction, we employ the method based on the Finite-Rank Approximation (FRA) \cite{1} of the nuclear Hamiltonian $H_A$. This approximation consists in neglecting the continuous spectrum in the spectral expansion

$$H_A = E_0|\psi_0\rangle\langle\psi_0| + \text{continuum}$$

of this Hamiltonian. Physically, this means that we exclude the processes of (virtual) excitations of the nucleus during its interaction with the $\eta$ meson. It is clear that the stronger the nucleus is bound, the smaller is the contribution from such processes to the elastic $\eta A$ scattering. By comparing with the results of the exact Faddeev calculations, it was shown\cite{2} that even for $\eta d$ scattering (with weakest nuclear binding) the FRA method works reasonably well, which implies that we can obtain sufficiently accurate results applying this method to $\eta^3\text{H}$ and $\eta^3\text{He}$ scattering.

To include a photon into the FRA formalism, we follow the same procedure as in Ref.\cite{3} where the coherent $\eta$-photoproduction on deuteron was treated in the framework of the exact AGS equations and the photon was introduced by considering the $\eta N$ and
γN states as two different channels of the same system. This implies that the $T$-operator
describing the $\eta N$ interaction, should be replaced by $2 \times 2$ matrix, viz.

$$t_{\eta N} \rightarrow \begin{pmatrix} t_{\eta \gamma} & t_{\eta N} \\ t_{\eta N} & t_{N} \end{pmatrix},$$

(3)

where $t_{\eta \gamma}$ describes the Compton scattering, $t_{\eta N}$ the photoproduction process, and $t_{N}$ the
elastic $\eta N$ scattering. All calculations was performed in the first order on electromagnetic interaction.

The problem of constructing $\eta N$ potential or directly the corresponding $T$-matrix $t_{\eta N}$
has no unique solution since the only experimental information we have consists of the
two complex numbers, namely, position of the $S_{11}$-resonance pole $E_0 - i\Gamma/2$ and the $\eta N$
scattering length $a_{\eta N}$. In the present work, we use three different versions of $t_{\eta N}$ all of
which have the same separable form

$$t_{\eta N}(k', k; z) = g(k') \tau(z) g(k),$$

(4)

with the same formfactors $g(k) = (k^2 + \alpha^2)^{-1}$ (where $\alpha = 3.316 \text{ fm}^{-1}$, see Ref. [4])
but with different versions of the propagator $\tau(z)$. The version I is motivated by the
dominance of the $S_{11}$ resonance at the near-threshold energies and has simple Breit-Wigner form

$$\tau(z) = \frac{\lambda}{z - E_0 + i\Gamma/2},$$

(5)

which guaranties that the $t_{\eta N}$ has the resonance pole at $z = E_0 - i\Gamma/2$ (with $E_0 =$
1535 MeV $- (m_N + m_\eta)$ and $\Gamma = 150 \text{ MeV}$ see Ref. [5]). In this version of $\tau(z)$ the
strength parameter $\lambda$ is chosen to reproduce the $\eta N$ scattering length $a_{\eta N} = (0.55 +$
i0.3) fm.

An alternative way (version II) of constructing the two-body $T$-matrix $t_{\eta N}$ is to solve
the corresponding Lippmann-Schwinger equation with an appropriate separable potential
having the same form-factors $g(k)$. However, a one-term separable $T$-matrix obtained
in this way, does not have a pole at $z = E_0 - i\Gamma/2$. To recover the resonance behaviour
in this case, we use the trick suggested in Ref. [6], namely, we use a potential with an
energy-dependent strength, which resulted in

$$\tau(z) = -\frac{4\pi \alpha^3}{\mu_{N}} \cdot \frac{\Lambda(\zeta - z) + C\zeta}{\zeta - z - [\Lambda(\zeta - z) + C\zeta]/(1 - i\sqrt{2\pi\mu_{\eta N}/\alpha})^2},$$

(6)

where the constants $\Lambda$, $C$, and $\zeta$ are chosen in such way that the corresponding scattering amplitude reproduces the same (as for version I) scattering length $a_{\eta N}$ and has a pole
at $z = E_0 - i\Gamma/2$. Moreover, it is consistent with the condition of the two-body unitarity
because it obeys the Lippmann-Schwinger equation.

The version III has the same functional form as version I, but different value of $\lambda$
which is now fixed using the condition of the $\eta N$ unitarity, namely, $(1 - 2\pi i t_{\eta N})(1 -$
$2\pi i t_{\eta N}^{-1}) = 1$. The resulting $t_{\eta N}$ gives $a_{\eta N} = (0.76 + i0.61) \text{ fm}.$

Therefore all the three versions of $t_{\eta N}$ have a pole at $z = E_0 - i\Gamma/2$, the first two of
them reproduce the same $a_{\eta N}$, the versions II and III are consistent with the unitarity
condition but give different $a_{\eta N}$. 


In constructing the photoabsorption (production) $T$-matrix $t^\eta(E)$, we use the corresponding on-shell $T$-matrix $t^\eta_{\text{on}}(E)$ of Ref. [7] and extend it off the energy shell,

$$t^\eta_{\text{off}}(k', k; E) = \frac{\kappa^2 + E^2}{\kappa^2 + k'^2} t^\eta_{\text{on}}(E) \frac{\alpha^2 + 2\mu_{\eta N}E}{\alpha^2 + k^2},$$

(7)

using Yamaguchi form–factors which disappear (go to unity) on the energy shell with $\kappa$ being a parameter. Varying $\kappa$ in our calculations, we found that the dependence of the photoproduction cross-sections on the choice of this parameter is rather weak, and we simply put $\kappa = \alpha$. It is known that $t^\eta$ is different for neutron and proton. In this work we assume that they have the same functional form (7) and differ by a constant factor, $t^\eta_n = At^\eta_p$. Multipole analysis [8] gives for this factor the estimate: $A = -0.84 \pm 0.15$.

To obtain the nuclear wave function $\psi_0$ (which is needed for the expansion (2) of $H_A$), we solve the few-body equations of the Integro-Differential Equation Approach (IDEA) [9] with the Malfliet–Tjon potential [10].

Figures 1 and 2 show the results of our calculations for the total (integrated over the directions of the outgoing meson) cross-section $\sigma$ of the coherent processes (1). The calculations were done for two nuclear targets, namely, $^3\text{H}$ and $^3\text{He}$. The curves corresponding to the three versions of $t^\eta$ are denoted respectively as (I), (II), and (III).

![Graph](image-url)

**FIGURE 1.** Total photoproduction cross-section on $^3\text{He}$ and $^3\text{H}$ nuclei for different $t^\eta$-matrices. Triangles are the results for $^3\text{He}$ taken from Ref.[11].

As is seen in Fig.1 (left plot), the two versions of $t^\eta$, (I) and (II), give significantly different results despite the fact that both of them reproduce the same $a_{\eta N}$ and the $S_{11}$ resonance. This indicates that the scattering of the $\eta$ meson on the nucleons (final state interaction) is very important in the description of the photoproduction process. This conclusion becomes even more substantiated when our curves are compared to the corresponding points (triangles) calculated for the $^3\text{He}$ target in Ref. [11] where the final state interaction was treated using an optical potential of the first order. It is well-known that the first-order optical theory is not adequate at the energies near resonances. This is the reason why the calculations of Ref. [11] underestimate $\sigma$ near the threshold.

Another conclusion, following from the fact that the curves (I) in Fig.1 (left plot) are significantly different from the corresponding curves (II), is that the two-body unitarity is important as well. To clarify this statement, we compare (see Fig.1, right plot) three curves corresponding to the three choices of $\tau(z)$ in (4). Surprisingly, the curves (II) and...
(III) almost coincide despite the fact that they correspond to different $\alpha_{\eta N}$ while both obey the two-body unitarity condition.

Fig. 2 shows the dependence of $\sigma$ on the choice of the parameter $A$ for the cases of $^3$He and $^3$H target (left and right plots respectively). An interesting observation here is that the cross-section for $\eta$ photoproduction is more sensitive to this parameter when tritium rather than on $^3$He target is used. This means that between these two nuclei, the tritium is a preferable candidate for a possible experimental determination of the ratio $A$.

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