Reconstruction of CMB Temperature Anisotropies with Primordial CMB Induced Polarization in Galaxy Clusters

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ABSTRACT

Scattering of cosmic microwave background (CMB) radiation in galaxy clusters induces polarization signals determined by the quadrupole anisotropy in the photon distribution at the location of clusters. This "remote quadrupole" derived from the measurements of the induced polarization in galaxy clusters provides an opportunity of reconstruction of local CMB temperature anisotropies. In this Letter we develop an algorithm of the reconstruction through the estimation of the underlying primordial gravitational potential, which is the origin of the CMB temperature and polarization fluctuations and CMB induced polarization in galaxy clusters. We found a nice reconstruction for the quadrupole and octopole components of the CMB temperature anisotropies with the assistance of the CMB induced polarization signals. The reconstruction can be an important consistency test on the puzzles of CMB anomaly, especially for the low quadrupole and axis of evil problems reported in WMAP and Planck data.

Key words: (cosmology:) cosmic microwave background – cosmology: theory

1 INTRODUCTION

Large-scale anomalies have been reported in Cosmic Microwave Background (CMB) temperature map with several independent observations. A low quadrupole of CMB temperature anisotropies was first found in COBE data (Hinshaw et al. 1996) then confirmed by WMAP (WMAP Collaboration et al. 2003). The so-called axis of evil, which is an unusual alignment of the preferred axes of the quadrupole and octopole are found by several authors (de Oliveira-Costa et al. 2004; Land and Magueijo 2005; Samal et al. 2008). Other anomalies include power asymmetry in north/south hemisphere (Eriksen et al. 2004; Hansen et al. 2009) and an anomalous cold spot (Cruz et al. 2006). If the anomalies are not caused by foreground residuals or systematic effects, we are facing a challenge of understanding of fundamental physics and the nature of the cosmos.

The polarization of the CMB is expected to provide valuable information on the nature of CMB anomalies. It is generated through Thomson scattering of temperature anisotropies on the last scattering surface (Hu and White 1997). If the anomalies in the CMB temperature are primordial, the polarization should thus exhibit similar peculiarities (see Vielva et al. 2011; Frommert and Ensslin 2010, for examples related to the Cold Spot, axis of evil, respectively).

One of the question here is that: Is it possible to reconstruct CMB temperature map for the low multipole from other independent observations? If the answer is yes, then the reconstructed temperature map may be used to distinguish whether the anomalies are primordial or systematic. CMB polarization is one of the candidates. Due to the poor correlation coefficients (less than 0.5 for multipole l = 2 and l = 3), however, the reconstruction from only CMB polarization can not achieve our goal. The scattering of CMB photons in clusters of galaxies may shed light on this attempt.

The scattering of CMB photons in clusters of galaxies induces a polarization signal, which is determined by the quadrupole anisotropy in the photon distribution at the cluster location (Sazonov and Sunyaev 1999). Therefore, the remote quadrupole in distant clusters, in principle, is investigable by the measurements of this induced polarization signal. The measurements of these signals were originally proposed to suppress the cosmic variance
uncertainty (Kamionkowski and Loeb 1997; Bunn 2006; Portsmouth 2004). Moreover, the magnitude of these signals gives some clues about the evolution of the CMB quadrupole to probe the dark energy. Of particular interest here is that it probes three-dimensional information of potential fluctuations around our last scattering surface (Seto and Sasaki 2000). Even though this induced polarization in galaxy clusters has not been observed, its detection contains rich information for study of cosmology.

In this Letter we propose an independent observation for the study of the CMB anomalies. We explore a practically useful cosmological probe from the measurements of remote quadrupole: the reconstruction of CMB temperature anisotropies for low multipole. Provided by the strong correlation of the remote quadrupole in low redshifts with the local CMB (Hall and Challinor 2014), the reconstructed CMB temperature anisotropies are accurate enough for distinguishing the sources of low quadrupole and axis of evil problems.

2 SIMULATIONS OF CMB SKY AND CMB INDUCED POLARIZATION IN DISTANT GALAXY CLUSTERS

The CMB temperature anisotropies and E-mode polarization from a single plane wave can be written as (Ma and Bertschinger 1995)

$$\Delta T(\mathbf{k}, \eta, \hat{n}) = \sum_i (-i)^i (2l + 1) \Delta \theta^i(\mathbf{k}, \eta) P_l(\hat{k} \cdot \hat{n}),$$

where $P_l$ is Legendre polynomial, $\Delta \theta^i$ is the transfer function with $X = T, E$ for temperature anisotropies or E-mode polarization, respectively. To compute the spherical harmonics coefficients of the temperature anisotropy and E-mode polarization $a_{X,lm}$ and some derivations later, we need the additional theorem of the spin-weighted spherical harmonics (Ng and Liu 1999)

$$\sum_m s_1 Y_{lm}(\hat{n}') s_2 Y_{lm}(\hat{n}) = \sqrt{\frac{2l+1}{4\pi}} (-1)^{s_1-s_2} a_{lm}(\beta, \alpha) e^{-i\alpha s_1 \gamma},$$

where $\alpha$, $\beta$ and $\gamma$ are the Euler angles as being composed of a rotation $\alpha$ around $\hat{e}_x$, followed by $\beta$ around the new $\hat{e}_y$ and finally $\gamma$ around $\hat{e}_z$. Then the spherical harmonics coefficients are

$$a_{X,lm} = (-i)^l 4\pi \int d^4 k Y_{lm}^*(\hat{k}) \Delta X_l(\mathbf{k}, \eta).$$

On the other hand, the polarization effect in the distant clusters of galaxies arises from the presence of the quadrupole component of the CMB in the rest-frame of a cluster. For a cluster located in the $z$ direction, the primordial CMB quadrupole induced polarization is (Ramos, da Silva and Liu 2012)

$$(Q_T \pm iU_T) = \sqrt{\frac{2\pi}{15}} \int d^3 k e^{i k \cdot \hat{n}} \Delta T_2(\mathbf{k}, \eta) Y_{22}(\hat{n}).$$

where $\tau$ is the optical depth across the cluster, $Q_T$ and $U_T$ are Stokes parameters in the unit of brightness temperature. We adopt here, by convection, $Q_T < 0$ ($Q_T > 0$) for a N-S (E-W) polarization component and $U_T > 0$ ($U_T < 0$) for a NE-SW (NW-SE) component. We have assumed that free electrons in a cluster see the same CMB quadrupole because the primordial CMB temperature quadrupole has variations on much larger scales than the extent of individual clusters (Ramos, da Silva and Liu 2012). Using the addition theorem of spin-weighted spherical harmonics in Eq.(2), we rewrite Eq. (5) for a cluster located in any line of sight direction $\hat{n}$

$$(Q_T \pm iU_T) = \frac{2\sqrt{6\pi}}{5} \int d^3 k e^{i k \cdot \hat{n}} \Delta T_2(\mathbf{k}, \eta) \sum_m Y_{2m}(\mathbf{k}) \mp 2 Y_{2m}(\hat{n}).$$

All the CMB temperature anisotropy and its polarization and the polarization signal at distant clusters of galaxies can be computed directly from Eqs.(3) and (5), provided that the $\Delta T_l(\mathbf{k}, \eta)$ for each wave-mode is known. Since the evolution of $\Delta T_l(\mathbf{k}, \eta)$ is independent of the direction of $\mathbf{k}$, we may write $\Delta T_l(\mathbf{k}, \eta) = \Delta T_l(\hat{k}, \eta) \Psi(\mathbf{k})$, where $\Psi(\mathbf{k})$ is the primordial gravitational potential. This primordial gravitational potential $\Psi(\mathbf{k})$ that appears in Eq. (3) and (5) is the origin of CMB temperature and polarization fluctuations and primordial CMB induced polarization in galaxy clusters. Therefore, estimating the primordial gravitational potential in three-dimensional Fourier space is our first step for the reconstruction.

It is usual to assume that the two-point correlation function of the $\Psi(\mathbf{k})$ has the form

$$<\Psi^*(\mathbf{k})\Psi(\mathbf{k}')> = P_{\Psi}(k)\delta^3(\mathbf{k} - \mathbf{k}'),$$

and the power spectrum $P_{\Psi}(k)$ obeys a power-law

$$P_{\Psi}(k) = A k^{n_a - 4},$$

with $A$ being a normalization factor and $n_a$ a spectral index of the scalar perturbations.

To compute the simulation data, we first generate the three-dimensional primordial gravitational potential $\Psi(\mathbf{k})$ by drawing a random number from a gaussian distribution with variance $P_{\Psi}(k)$ for each wave-mode. The time evolution of the transfer function $\Delta \theta^i(\mathbf{k}, \eta)$ is calculated numerically by the CMBFast (Seljak and Zaldarriaga 1996) Boltzmann code, assuming the cosmological parameters from the Planck 2013 results (Planck Collaboration et al. 2014). Different skies can be generated by changing the seed of our random number generator routine. Given the simulated primordial gravitational potential, we generate CMB data $a_{T,lm}$ and $a_{E,lm}$ using Eq.(3) and polarization data in clusters $Q_T$ and $U_T$ using Eq. (5). The observed CMB induced polarization in clusters is linear proportional to the cluster’s optical depth, which may be extracted from X-ray surface brightness observations if the temperature profile is known. Here we simply assume $\tau = 1$. The position of the galaxy clusters is assumed to be randomly distributed in the universe from $z = 0$ to 4. We perform the integration in Eqs.(3) and (5) by summing the contribution from each Fourier mode in spherical coordinates. In each radial direction of $\mathbf{k}$, we have sampled uniformly 240 modes in logarithm space from $k = 7 \times 10^{-6}$ to $1.4 \times 10^{-1}$ h/Mpc. The angular directions of ($\theta_k, \phi_k$) are obtained by the Healpix scheme (Gorski et al. 2005) at resolution-5 map.
3 RECONSTRUCTION OF CMB TEMPERATURE ANISOTROPIES

We present in Fig. (1) a typical sky realization of the CMB temperature anisotropies ($l \leq 16$) and its quadrupole and octopole components. In order to reconstruct this CMB temperature map with CMB induced polarization in galaxy clusters from the same realization, we first estimate the primordial gravitational potential using Bayes' theorem

$$P_r(\Psi|d) \propto P_r(d|\Psi)P_r(\Psi),$$

where $P_r(\Psi)$ is the prior probability, $P_r(d|\Psi)$ is the likelihood of obtaining the data $d = \{Q_T, U_T\}$ given a realization of primordial gravitational potential $\Psi$. The estimation of $\Psi$ is obtained by minimizing the function

$$f = \sum_{j=1}^{n_c} \frac{(\hat{Q}_{T,j} - Q_{T,j})^2}{2\sigma^2_{Q_T}} + \sum_{j=1}^{n_c} \frac{(\hat{U}_{T,j} - U_{T,j})^2}{2\sigma^2_{U_T}} + \sum_{k=1}^{n_k} \frac{\Psi_k^2}{2P_{\Psi}},$$

where $\sigma_{Q_T}$ and $\sigma_{U_T}$ are instrument noise of observation, $n_k$ is the number of Fourier modes in the fitting, $n_c$ is the number of observed clusters of galaxies, $\Psi_k$ is the discretized $\Psi(k)$, $\hat{Q}_{T,j}$ and $\hat{U}_{T,j}$ are the reconstructed Stokes parameters in the $j$th galaxy cluster with the estimated $\Psi$. The last term in the right side is the logarithm of the prior.

There must be many possible sets of $\Psi$ corresponding to one observation within the desired uncertainty since the number of modes $n_k$ is much bigger than the number of data $n_c$. Therefore, the reconstructed $\Psi$ differ with the input, while the reconstructed Stokes parameters in the galaxy clusters do not change too much. However, reducing $n_k$ to avoid the overfitting produces another problem. It generates the spurious correlation between the induced polarization in clusters and the CMB temperature anisotropy and $E$-mode polarization once we reduce the number of modes $n_k$ in ra-
In Fig. (2) we present the reconstructed temperature map \((l \leq 10)\) and its quadrupole and octopole components with the estimated \(\Psi\), which is obtained by fitting to the simulated data from 300 galaxy clusters whose redshifts \(z < 1.0\) in our catalog. It is obvious that the reconstructed temperature map loses power on small angular scales. However, with the assistance of strong correlation of remote quadrupole with local CMB, the quadrupole component is almost completely reconstructed and the octopole component is also similar to the simulated one. It shows the feasibility of the reconstruction with our algorithm and the results should be useful for the study of anomalies in CMB maps. For the higher multipole, the reconstruction becomes worse due to the more deviation of the transfer functions from that of remote quadrupole with increasing \(l\).

We forecast the observation strategy for the induced polarization in galaxy clusters. To quantify the goodness of the reconstruction, we defined the dimensionless correlation coefficient \(r_l = \sum_m a_{T,lm} \bar{a}_{T,lm} / (2l + 1) \sqrt{C_l C_l}\) and error of reconstruction \(E_l = \sum_m (a_{T,lm} - \bar{a}_{T,lm})^2 / (2l + 1)C_l\) with \(C_l\) the input temperature power spectrum from simulation. We make 100 realizations and select the observed galaxy clusters with redshifts \(z \leq z_h\). We show the average of \(r_l\) and \(E_l\) as function of \(n_c\) for \(l = 2, 3\) and 4 in Fig (3) with fixed \(z_h = 1\).

With tens of observed galaxy clusters the quadrupole component can be perfectly reconstructed and be useful for the investigation of low quadrupole problem. The reconstruction of higher multipole components improves with the increasing number of observed galaxy clusters. We also test the reconstruction by adding \(E\)-mode polarization data (dashed curves). With the additional data \(a_{E,lm}\) for \(l \leq 10\), the reconstruction of octopole and higher multipole anisotropies improves significantly through the \(TE\) correlation. The relation of the reconstruction and the depth of the observation is shown in Fig. (4). The remote quadrupole of observed in high redshift galaxy clusters probes the universe on scales smaller to the local CMB, so the reconstruction of octopole and higher multipole improves with increasing \(z_h\).

The procedure we have here is something of an idealization. We assume the optical depth through the galaxy clusters is precisely measured and the instrumental noise is significantly small \((\sigma_a = \sigma_T = \sigma_{lm} = 10^{-3} \mu K)\). We also ignore all the contaminations of the polarization signal, for example, the polarization induced by the transverse peculiar velocity of the galaxy clusters (Sazonov and Sunyaev 1999; Ramos, da Silva and Liu 2012) and the background polarization. The signal is probably detectable in the next-generation polarization experiments with broad frequency coverage such as PRISM (Prism Collaboration et al. 2013) but the separation of the quadrupole signal from the other contaminants would be an experimental challenge.

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\[\sum_{\nu} \frac{\omega}{\nu} = 1\text{,} \sum_{\nu} \omega = 1\text{,} \sum_{\nu} \omega^2 = 1\]
REFERENCES

Bunn E. F., Phys. Rev. D, 73, 123517
Cruz M. et al., 2006, MNRAS, 369, 57
de Oliveira-Costa A. et al., 2004, Phys. Rev. D, 69, 063516
Eriksen H. K. et al., 2004, ApJ, 605, 14
Frommert M. and Ensslin T. A., 2010 MNRAS, 403, 1739
Gorski K. M. et al., 2005, ApJ, 622, 759
Hall A. and Challinor A., 2014, Phys. Rev. D, 90, 063518
Hansen F. K. et al., 2009, ApJ, 704, 1448
Hinshaw G. et al., 1996, Astrophysical Journal Letters, 464, L25
Hu W. and White M., 1997, New Astron., 2, 323
Kamionkowski M. and Loeb A., 1997, Phys. Rev. D, 56, 4511
Land K. and Magueijo J., 2005, Phys. Rev. Lett., 95, 071301
Ma C. and Bertschinger E., 1995, ApJ, 455, 7
Ng K.-W. and Liu G.-C., 1999, Int. J. Mod. Phys. D8, 61
Planck Collaboration et. al., 2014, A& A, 571, 16
Portsmouth J., 2004, Phys. Rev. D, 70, 063504
PRISM Collaboration et al., ArXiv e-prints, 1306.2259 (2013)
Ramos E. P. R. G., Silva A. da and Liu G.-C., 2012, ApJ, 757, 44
Samal P. K. et al., 2008, MNRAS, 385,1718
Sazonov S. Y. and Sunyaev R. A., 1999, Mon. Not. R. Astron. Soc. 310, 765
Seljak U. and Zaldarriaga M., 1996, ApJ, 469, 437
Seto N. and Sasaki M., 2000, Phys. Rev. D, 62, 123004
Vielva P. et al., 2011, MNRAS, 410, 33
WMAP Collaboration et al., 2003, ApJS, 148, 175