A General Method for Designing n-Qubits Entanglement Creating Circuits and its Application in Different 4-qubits Entanglement Creating Circuits

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Abstract. One of the most important aspects of quantum computation is quantum entanglement, it is an interesting feature of quantum physics, and has many important applications such as quantum teleportation, cryptography, quantum computation, and dense coding. It is an essential task in quantum computing to design circuits that create an entangled system, and many circuits were implied in the quantum computers that generate the desired entangled state of a system. In this paper, we suggest a general method to design different quantum circuits that produce an entangled system for any number of qubits in different manners. Also, we apply the suggested method in order to design different 4-qubits entanglement creating circuits. The results show that it is possible to design different entanglement creating circuits for any number of qubits by following certain method that will be discussed in the paper, and in the case of 4-qubits system we are going to look only at two circuits, the first creates a “GHZ” like state and the second creates a four-basis entanglement state.

1. Introduction
Quantum entanglement is an important aspect of quantum physics, it is a quantum phenomenon describing the non-classical correlations within different subsystems[1]. In an entangled system a state of a particle cannot be described independently of the state of the other particles [2]. It is one of the key aspects that separate quantum from classical systems and has been the subject of the correlations noted by EPR (Einstein, Podolsky and Rosen)[3]. And was used to exclude non-local hidden variable description in quantum mechanics by Bell [4], [5].

Entanglement has many applications in quantum physics and quantum computation such as quantum teleportation [6], cryptography [7], quantum computation [8] and dense coding [9].

We are able to describe any changes in quantum states using quantum computation language, “a quantum computer is built from a quantum circuit containing wires and elementary quantum gates to carry around and manipulate the quantum information”[10], and for a quantum computer one of the most essential circuits are the circuits that creates an entangled systems. Many algorithms and circuits were designed in order to create an entangled system. In this paper, we are suggesting a general method to design different quantum circuits that produce an entangled system for n-qubits in different manners, and using the mentioned method, we are designing different 4-qubits system entanglement creating circuits.

The earliest circuit that creates entangled system is the one that entangles a two qubits system, and consists of a Hadamard gate acting on the first qubit followed by a CNOT gate acting on the second qubit as a target and the first qubit as control and the output of this circuit is a bell state or bell pair and can be represented in Figure 1 and the truth table of this circuit is represented in Table 1 [10].
Table 1. Bell states circuit truth table

| Input | Output |
|-------|--------|
| |\(\Phi^+ = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle\)| |
| |\(\Phi^- = \frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|11\rangle\)| |
| |\(\Psi^+ = \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle\)| |
| |\(\Psi^- = \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle\)| |

Figure 1. Bell states creating circuit.

As for the 3-qubits entanglement creating-circuit, Mezher B. Saleh and Loay E. George presented a circuit which entangles a 3-qubits system and consists of an X gate acting on the first qubit followed by a CNOT gate that acts on the second qubit as target and the first as control and two Hadamard gates on the first and the third qubit [11].

Many multi-qubit entangled states have been presented experimentally, as an example entanglement has been demonstrated up to 16-qubit in a superconducting system [12], also a genuine multipartite entanglement is demonstrated up to 18-qubits photonic system [13]. 12-qubits photonic system[14][15] and in 12-qubits superconducting system [16, [17].

In 2019 Alba Cervera-Lierta, José Ignacio Latorre and Dardo Goyeneche designed a series of quantum circuits that generate absolute maximally entangled states (AME)[18]. And in the same year a 20-qubits entanglement circuit was demonstrated by Gary J. Mooney, Charles D. Hill & Lloyd C. L. Hollenberg, which features the entanglement of 20-qubits in a superconducting quantum computer [19].

Over the past years many different quantum circuits that generate entangled systems was presented and studied, yet there wasn’t a general method that can be applied in order to design a quantum entanglement creating-circuit for any number of qubits, also only one type of 4-qubits entanglement was studied and had a designed circuit for, thus, we suggest a general method to design any entanglement creating circuit for any number of qubits. We also present two different 4-qubits entanglement creating circuits by following the mentioned method.

2. Discussion and results

In order to design any entanglement creating-quantum circuit, we suggest a general method that can be used for n-qubits. Each circuit should consist of (n-1) CNOT gates assembled where the first CNOT works on the second qubit as a target and on the first qubit as control, and the second CNOT works on the third qubit as a target and the second qubit (after the first CNOT operates on it) as control and so on. The circuit should also contain at least 1 Hadamard gate, and in order to produce an entangled state with m number of basis states we use \((m/2)\) Hadamard gates, the first acts on the 1st qubit and the second on the 2nd qubit, etc.
2.1 Four qubits entanglement creating circuits design

In order to design an entanglement creating circuit for a 4-qubits system we must know that there are different ways to entangle any quantum system, the states that represent the entanglement of a four qubits system differs in two ways. Firstly, the output states of the circuit differ according to the input states, and since we have 4 qubits it gives $2^4$ possible output states.

Secondly, the entangled 4 qubit states differ also in the number of the basis states of each entangled state, it can have two-basis entangled states (GHZ-like state), four-basis and even eight-basis, thus, by following the mentioned method we are considering two quantum circuits that generate entangled states of four qubits where each one gives a different number of basis states of the entangled state, the two circuits will consist of a set of three CNOT gates and will only differ in the number of the Hadamard gates in order to generate the desired state. We are going to present the mathematical procedure only for the input state $|0000\rangle$ and for the other input states, we only present the computational results that have been computed using MATLAB.

2.2.1 GHZ-like entanglement creating circuit of 4-qubits

The first circuit is the one that creates a GHZ-like entangled states, which means that the output state has two basis states. The circuit consists of one Hadamard gate acting on the first qubit followed by a three CNOT-gates as in Figure 2.

![Figure 2. GHZ like entanglement creating circuit of 4-qubits.](image)

Taking the case of the input $|0000\rangle$, the mathematical procedure of this circuit is as follows:

1. The Hadamard-gate acts on the first qubit:
   
   $H - gate|0\rangle = \frac{1}{\sqrt{2}} \left(\begin{array}{c}
   1 \\
   1 \\
   -1 \\
   -1
   \end{array}\right)|0\rangle = \frac{1}{\sqrt{2}} \left(\begin{array}{c}
   1 \\
   0
   \end{array}\right)$
   
   $H - gate|0\rangle = \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$  \hspace{1cm} (1)

   Taking tensor product between state 1 with the second qubit:
   
   $\left(\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle\right) \otimes |0\rangle = \frac{1}{\sqrt{2}} |00\rangle + \frac{1}{\sqrt{2}} |10\rangle$  \hspace{1cm} (2)

2. The first CNOT gate acts on the second qubit as a target, and the first qubit as control:
   
   $CNOT_1 |00\rangle = \left(\begin{array}{cccc}
   1 & 0 & 0 & 0 \\
   0 & 1 & 0 & 0 \\
   0 & 0 & 0 & 1 \\
   0 & 0 & 1 & 0
   \end{array}\right) |0\rangle = \left(\begin{array}{c}
   1 \\
   0 \\
   0 \\
   0
   \end{array}\right) = |00\rangle$
   
   $CNOT_1 |10\rangle = \left(\begin{array}{cccc}
   0 & 0 & 1 & 0 \\
   0 & 0 & 0 & 1 \\
   1 & 0 & 0 & 0 \\
   0 & 0 & 0 & 1
   \end{array}\right) |0\rangle = \left(\begin{array}{c}
   0 \\
   0 \\
   0 \\
   1
   \end{array}\right) = |11\rangle$

   $CNOT_1 \left(\frac{1}{\sqrt{2}} |00\rangle + \frac{1}{\sqrt{2}} |10\rangle\right) = \frac{1}{\sqrt{2}} |00\rangle + \frac{1}{\sqrt{2}} |11\rangle$  \hspace{1cm} (3)

3. Taking tensor product between state 3. and the 3\textsuperscript{rd} qubit:
(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle) \otimes |0\rangle = \frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|110\rangle \quad (4)

4. The second CNOT gate acts on the 3rd qubit as target and the second qubit as control:

\[
CNOT_2|00\rangle = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
1 \\
0 \\
0 \\
0
\end{pmatrix} = \begin{pmatrix}
1 \\
0 \\
0 \\
0
\end{pmatrix} = |00\rangle
\]

\[
CNOT_2|10\rangle = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
0 \\
1 \\
0 \\
0
\end{pmatrix} = \begin{pmatrix}
0 \\
0 \\
0 \\
1
\end{pmatrix} = |11\rangle
\]

\[
CNOT_2\left(\frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|110\rangle\right) = \frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|111\rangle \quad (5)
\]

5. Taking tensor product between state 5. and the 4th qubit:

\[
\left(\frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|111\rangle\right) \otimes |0\rangle = \frac{1}{\sqrt{2}}|0000\rangle + \frac{1}{\sqrt{2}}|1110\rangle \quad (6)
\]

6. The fourth CNOT gate acts on the 4th qubit as target bit and the 3rd qubit as control bit:

\[
CNOT_3|00\rangle = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
1 \\
0 \\
0 \\
0
\end{pmatrix} = \begin{pmatrix}
1 \\
0 \\
0 \\
0
\end{pmatrix} = |00\rangle
\]

\[
CNOT_3|10\rangle = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
0 \\
1 \\
0 \\
0
\end{pmatrix} = \begin{pmatrix}
0 \\
0 \\
0 \\
1
\end{pmatrix} = |11\rangle
\]

\[
CNOT_3\left(\frac{1}{\sqrt{2}}|0000\rangle + \frac{1}{\sqrt{2}}|1110\rangle\right) = \frac{1}{\sqrt{2}}|0000\rangle + \frac{1}{\sqrt{2}}|1111\rangle \quad (7)
\]

The output state is a maximally entangled state with equal amplitudes \(\frac{1}{\sqrt{2}}\), it is a GHZ-like entangled state. One of the properties of this type of entangled states is that we can identify the outcome of the measurement for the entire system by only measuring one qubit. As for the other possible input states, the computational results are shown in Table 2.

Table 2. computational result for the first circuit.

| Input | Output |
|-------|--------|
| 1     | \(\frac{1}{\sqrt{2}}|0000\rangle + \frac{1}{\sqrt{2}}|1111\rangle\) |
| 2     | \(\frac{1}{\sqrt{2}}|0011\rangle + \frac{1}{\sqrt{2}}|1100\rangle\) |
| 3     | \(\frac{1}{\sqrt{2}}|0111\rangle + \frac{1}{\sqrt{2}}|1010\rangle\) |
| 4     | \(\frac{1}{\sqrt{2}}|1000\rangle + \frac{1}{\sqrt{2}}|0111\rangle\) |
| 5     | \(\frac{1}{\sqrt{2}}|0001\rangle + \frac{1}{\sqrt{2}}|1110\rangle\) |
### 2.1.2. Four-basis entangled state creating circuit of 4-qubits

The second circuit that entangles the 4-qubits system, it generates an entangled state that has four-basis states with equal amplitudes, and consists of two Hadamard gates acting on the first and the second qubit and three CNOT-gates as in Figure 4.

![Figure 4](image-url)

**Figure 4.** four-basis states entanglement creating circuit of 4-qubits.

Taking the input of |0000⟩, the mathematical procedure of this circuit is as follows:

1. The second Hadamard-gate acts on the second qubit:

$$H_2|0⟩ = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 1 \\ -1 \end{array} \right) |0⟩ = \frac{1}{\sqrt{2}} |1⟩$$

$$H_2|0⟩ = \frac{1}{\sqrt{2}} |0⟩ + \frac{1}{\sqrt{2}} |1⟩$$

2. The first CNOT gate acts on state (8) as target and the first qubit as control, and since the first qubit is |0⟩ This means that state (8) remains unchanged after the first CNOT-Gate and equals to $\frac{1}{\sqrt{2}} |0⟩ + \frac{1}{\sqrt{2}} |1⟩$

|   |   |   |   |
|---|---|---|---|
| 6 | |0101⟩ | $\frac{1}{\sqrt{2}} |0110⟩ + \frac{1}{\sqrt{2}} |1001⟩$ |
| 7 | 0110⟩ | $\frac{1}{\sqrt{2}} |0010⟩ + \frac{1}{\sqrt{2}} |1101⟩$ |
| 8 | 0111⟩ | $\frac{1}{\sqrt{2}} |0101⟩ + \frac{1}{\sqrt{2}} |1010⟩$ |
| 9 | 1000⟩ | $\frac{1}{\sqrt{2}} |0000⟩ - \frac{1}{\sqrt{2}} |1111⟩$ |
| 10 | 1001⟩ | $\frac{1}{\sqrt{2}} |0111⟩ + \frac{1}{\sqrt{2}} |1000⟩$ |
| 11 | 1010⟩ | $\frac{1}{\sqrt{2}} |0011⟩ + \frac{1}{\sqrt{2}} |1100⟩$ |
| 12 | 1011⟩ | $\frac{1}{\sqrt{2}} |0100⟩ - \frac{1}{\sqrt{2}} |1011⟩$ |
| 13 | 1100⟩ | $\frac{1}{\sqrt{2}} |0001⟩ + \frac{1}{\sqrt{2}} |1110⟩$ |
| 14 | 1101⟩ | $\frac{1}{\sqrt{2}} |0110⟩ - \frac{1}{\sqrt{2}} |1001⟩$ |
| 15 | 1110⟩ | $\frac{1}{\sqrt{2}} |0010⟩ - \frac{1}{\sqrt{2}} |1101⟩$ |
| 16 | 1111⟩ | $\frac{1}{\sqrt{2}} |0101⟩ + \frac{1}{\sqrt{2}} |1010⟩$ |
3. Taking tensor product between state (1) and the 3rd qubit:
\[
\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \otimes |0\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|10\rangle
\]
(9)

4. The 2nd CNOT-Gate acts on state (9):
\[
CNOT_2|00\rangle = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 \\
0 \\
0 \\
0
\end{pmatrix} = \begin{pmatrix}
1 \\
0 \\
0 \\
0
\end{pmatrix} = |00\rangle
\]
\[
CNOT_2|10\rangle = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
0 \\
1 \\
0 \\
0
\end{pmatrix} = \begin{pmatrix}
0 \\
1 \\
0 \\
0
\end{pmatrix} = |11\rangle
\]
\[
CNOT_2\left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|10\rangle\right) = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle
\]
(10)

5. Taking tensor product between state (10) and the 4th qubit:
\[
\left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle\right) \otimes |0\rangle = \frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|110\rangle
\]
(11)

6. The 3rd CNOT-Gate acts on state (11) where the 4th qubit is the target and the 3rd qubit is the control:
\[
CNOT_3|00\rangle = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 \\
0 \\
0 \\
0
\end{pmatrix} = \begin{pmatrix}
1 \\
0 \\
0 \\
0
\end{pmatrix} = |00\rangle
\]
\[
CNOT_3|10\rangle = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
0 \\
1 \\
0 \\
0
\end{pmatrix} = \begin{pmatrix}
0 \\
0 \\
1 \\
0
\end{pmatrix} = |11\rangle
\]
\[
CNOT_3\left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|10\rangle\right) = \frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|111\rangle
\]
(12)

7. The first Hadamard – Gate acts on the first qubit:
\[
H_1|0\rangle = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 1 \\ 1 \\ -1 \\ -1 \end{array}\right) \left(\begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \end{array}\right) = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \end{array}\right)
\]
\[
H_1|0\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle
\]
(13)

8. Taking tensor product between state (12) and state (13):
\[
\frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{2}(|0000\rangle + |1110\rangle + |0001\rangle + |1111\rangle)
\]
(14)

The output state is a four-basis entangled state with equal amplitudes (\(\frac{1}{\sqrt{2}}\)), the difference in the number of the basis states gives different properties than the GHZ-like state. The measurement outcome for this type of entanglement can be identified by measuring only 2 qubits. As for the other possible input states, the computational results are shown in Table 3.

**Table 3.** computational result for the second circuit.

| Input | Output |
|-------|--------|
| 1 | \(\frac{1}{2}(|0000\rangle + |1110\rangle + |0001\rangle + |1111\rangle)\) |
3. Conclusions
For any quantum system with any number of qubits (n-qubits quantum system), it is possible to entangle the system in different manners by using the proper circuit, in which we use a set of CNOT and Hadamard gates, where the number of CNOT gates should always be (n-1) and at least 1 Hadamard gate, and with the number of Hadamard gates we can decide the number of basis states that the entangled output state has. For m number of basis states that we want the entangled output state we want to use $\frac{m}{2}$ Hadamard gates.

As for a 4-qubits system as an application for the mentioned method we presented two different quantum circuits to generate an entangled state, the first generates a two-basis entangled state (GHZ-like state), the second is generating a four-basis entangled state. The circuits consist of a set of three CNOT gates and the number of the basis states can be determined by the number of the Hadamard gates used in the circuit, where we use $\frac{m}{2}$ Hadamard gates to generate the number of basis states we desire to have. Both the two mentioned circuits have 16 possible output states, the output states differ in the way the 4 qubits are entangled and is decided by the input states, those differences can be helpful in many quantum computation applications such as quantum teleportation and quantum information splitting by giving a verity of states as options to be used in those applications, the output states also differ in the signs of the state’s amplitudes, the variety of states’ signs can be helpful in
distinguishing entanglement states, also the different state’s signs give an increase in entropy which has a significant impact on quantum encryption and cryptography.

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