Review
Formulations, Solving Algorithms, Existing Problems and Future Challenges of Pre-Programmed PWM Techniques for High-Power AFE Converters: A Comprehensive Review

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Abstract: Considering the development of the hybrid wind and solar photovoltaic generation and smart grid, Active Front-End (AFE) converters for high-power applications are facing significant opportunities and challenges related to power quality and efficiency. The Pre-programmed Pulse-Width Modulation (PPWM) techniques can strictly control the harmonic spectrum of a specified voltage or current waveform generated by a high-power AFE converters, and have been extensively applied to reduce or even eliminate the harmonic distortion with low switching losses for high-power converters in order to deal with these issues aforementioned. For the PPWM techniques with low switching frequency, Selective Harmonic Elimination (SHE) and Selective Harmonic Mitigation (SHM) have been the prevailing solutions and gain widespread popularity, among which SHM can provide further control of the harmonic spectrum in cases of similar switching losses to SHE. Over the past several decades, the applications of SHE and SHM have been extended to high-power AFE converters. Thus, the aim of this study is to provide a comprehensive literature review regarding their various formulations, solving algorithms, and existing problems to high-power AFE converters. In addition, the suggestions for future applications of PPWM in high-power AFE converters are also discussed.

Keywords: pre-programmed pulse-width modulation (PPWM); selective harmonic elimination (SHE); selective harmonic mitigation (SHM); solving algorithm; high-power AFE converter

1. Introduction

High-power AFE converter is a very popular and significant topic in modern power electronics, especially considering the development of the hybrid wind and solar photovoltaic generation and smart grid [1–6]. Several specific industry applications of high-power AFE converters are shown in Figure 1, and the high-power AFE converter applications according to main industry branches are listed in Table 1 [7–12]. Many researchers and scientists are working on its related topology structures, controller designs and modulation techniques to improve the overall performance of the power system [13–37]. Among these research directions, modulation techniques are widely used to meet the rise in demand for high-power AFE converters, which require power losses have to be kept below acceptable limits of specific grid codes based on high-power AFE converters and the switching losses need to be reduced as much as possible. However, such a limited switching frequency (normally a few hundreds of hertz) will produce output voltage/current waveforms with high distortion. A suitable modulation technique should then be determined to reduce the harmonic distortion content in the power system [38–44].
Figure 1. Several specific industry applications of high-power AFE converters: (a) Photovoltaic application with six-pulse connection scheme; (b) Rolling mill application with twelve-pulse connection scheme; and (c) Wind power application with eighteen-pulse connection scheme.

It is now clear that the selection of the specific modulation technique affects the performance characteristics of high-power AFE converter in the power system to a great extent. Modulation techniques can be generally classified as carrier-based sinusoidal pulse-width modulation (CB-SPWM), space vector pulse-width modulation (SVPWM), and pre-programmed pulse-width modulation (PPWM) [45]. Additionally, switching losses and power quality will be the main constraints when modulation schemes are employed in high-power AFE converters.
Table 1. Industry applications of high-power AFE converters.

| Industry Branch | Specific Application                                                                 | Power Range (MW) | Rated Voltage (kV) |
|-----------------|--------------------------------------------------------------------------------------|------------------|-------------------|
| Power           | Power converters for solar panels/wind turbines, HVDC/FACTS links, coal mills.      | 1–40             | 2.3, 3.3, 4.0, 4.16, 4.2, 5.2, 6.6, 8.2, 10 |
|                 | Bucket wheel excavators, conveyor belts, ore mills.                                  | 2–15             | 2.3, 3.1, 3.3, 4.0, 4.1, 4.16 |
| Mining          | Sectional steel mill cold rolling mill, hot rolling mill drives, Booster-generators, Thrusters. | 2–25             | 2.4–13.8 |
| Metals          |                                                                                     | 2–20             | 2.3, 3.3, 4.16, 4.2, 6, 6.6, 6.9 |

What calls for special attention is that SPWM and SVPWM have several problems that are difficult to overcome when they are applied in the field of high-power applications, as the cooling system design limitations require restricted power losses. In addition, junction temperatures will also increase with the increase in switching losses, which will further lead to the lifetime reduction in the power devices. The switching frequency has to be low for these two modulation techniques to keep the switching losses below acceptable limits, but this presents high harmonic distortion around switching frequency, which leads to a bulky and expensive output filtering stage. Overall, high switching frequencies are inapplicable in high-power applications as they can cause more thermal losses, which will further damage the electrical switching equipment. To deal with the aforementioned problems, PPWM based on low switching frequency can be considered and applied, such as selective harmonic elimination (SHE), selective harmonic mitigation (SHM), hybrid SHE/SHM combinations, and PPWM with optimized switching patterns/sequences [46–54].

Since the 1960s, there has been a number of clear trends in the development of PPWM, aimed at solving scientific and technical problems in the fields of electromagnetic compatibility (EMC) and energy efficiency and increasing the output power quality of high-power AFE converters. PPWM was originally studied for traditional two- and three-level converters. Since then, after years of research, it has expanded to a variety of multilevel/hybrid AFE converters in numerous applications. At the beginning, it was found that adding some switching angles to the square voltage waveform could suppress low-order harmonics. Then, as the research further develops, the harmonic contents of a PWM waveform can be mathematically expressed by using Fourier series, which include a group of nonlinear and transcendental equations (mainly based on switching angles) [55–57]. Finally, a superior harmonic distortion performance can be realized when these PPWM methods apply a given set of switching angles that is obtained from particular mathematical calculations. The several distinct characteristics of PPWM are as follows [58–76]:

1. Working at low switching frequency, which is beneficial to reduce switching losses and improve the reliability and efficiency of the high-power AFE converter;
2. Allowing overmodulation, which can achieve high voltage gain as it causes an increased utilization of the DC bus;
3. Reducing or even removing additional filtering components/systems, and it can further reduce total cost of power systems;
4. For SHE, the specific low-order harmonics can be strictly eliminated while keeping the fundamental harmonic at a pre-determined value, which can avoid harmonic interference and resonance phenomena;
5. For SHM, its idea is to keep the harmonic content below the limits imposed by the particular applied grid codes, while also considering the resulting voltage/current total harmonic distortion (THD) from the perspective of power quality.
6. For hybrid SHE/SHM combinations, their performance indices can be optimized based on a given power quality aspect to a certain degree.

Despite the advantages of PPWM as mentioned above, there are some difficulties or challenges in applying this kind of approach. The first aspect is that the biggest challenge in the implementation of PPWM is to solve the sets of trigonometric equations with multiple switching angles through Fourier series analysis based on the output voltage/current waveforms. However, the sets of trigonometric equations, in nature, are highly nonlinear and transcendental. The existence of this situation means that no solutions, unique solutions, or multiple solutions may obtained over different modulation indices. Another aspect of these difficulties is that the switching angle sets are normally calculated offline as it takes a lot of computational work, then these results will be pre-stored in the look-up table (LUT); this is the reason why this approach is called pre-programmed. Finally, during the high-power AFE converter operation, these calculated switching angle sets are used. It should be noted that the offline nature of PPWM indicates the limitations of this approach.

Thus, the aim of this study is to provide an analytical literature review of PPWM development for high-power AFE converters and to make it serve as a useful and comprehensive material for understanding PPWM techniques, such as their features, advantages, and disadvantages. The state of the art and prominent issues in the PPWM methods are discussed. In addition, several well-established solving algorithms for calculating nonlinear and transcendental equations are reported, and illustrative simulation results are also provided.

This article is structured as follows. In Section 2, an overview of common PPWM output waveforms is described and their corresponding mathematical expressions are also proposed. Section 3 is devoted to the explanations of solving algorithms for achieving the solutions to the nonlinear and transcendental set of PPWM equations. Several illustrative examples based on solution trajectories, PPWM waveforms, and FFT spectra are provided to prove the feasibility of some commonly applied algorithms in Section 4. Section 5 deals with the existing problems of PPWM nowadays, and some suggestions for the future applications are also proposed. Finally, conclusions of the work are presented in Section 6.

2. PPWM Formulations

In the scientific and technical literature, the formulations of PPWM are on the basis of the decomposition of voltage/current PWM waveforms by using Fourier expansion series. It should be pointed out that PPWM formulations normally depend on the characteristics of a given waveform, such as unipolar, bipolar, stepped, multilevel, symmetrical, and asymmetrical waveforms [77–82]. All these features play the same important role in the analysis and determination of the form and complexity of a given solution space.

The mathematical expression of the output voltage PWM waveform generated by a high-power AFE converter $u(\omega t)$ is based on the Fourier expansion series, which is written as:

$$u(\omega t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(n\omega t) + b_n \sin(n\omega t))$$

(1)

where $\omega$ is the radian frequency of the output voltage, $n$ is the harmonic order, $a_0$, $a_n$, and $b_n$ are the Fourier series coefficients, among which $a_0$ determines the amplitude of the DC component, and $a_n$ and $b_n$ determine the amplitude of the $n$th harmonic order (here, taking the three-level waveform with equal voltage levels in the range of $2\pi$ as an example):

$$a_0 = \frac{1}{\pi} \int_{0}^{2\pi} u(\omega t) d(\omega t),$$

$$a_n = \frac{1}{\pi} \int_{0}^{2\pi} u(\omega t) \cos(n\omega t) d(\omega t),$$

$$b_n = \frac{1}{\pi} \int_{0}^{2\pi} u(\omega t) \sin(n\omega t) d(\omega t).$$

(2)
The Fourier expansion series, as is clear from Equation (2), includes the calculation of a set of switching angles $\alpha (\alpha = \omega t)$, and these switching angles are used to determine the particular harmonics to be controlled (usually to be eliminated or mitigated).

Figure 2 shows the classification of common PPWM types and formulations. The problem of SHE can be formulated as a minimization function seeking complete elimination of the selected harmonics. As for SHM, its problem can sometimes be expressed as a minimization function that tries to find a local or global minimum value of the selected harmonic rather than its complete elimination, and sometimes it can be formulated as a minimization function that provides the solutions to make each selected harmonic value as well as the voltage/current THD lower than the limit of the grid codes applied [83–87]. In some specific applications, such as the high-power four-leg inverter, a combination of SHE and SHM can be used [88–90].

![Diagram](https://via.placeholder.com/150)

**Figure 2.** Classification of common PPWM types and formulations.

When considering applying the above PPWM approaches, the main factor that needs to be properly controlled to a required value is the fundamental frequency. The tradeoff between harmonic contents and feasibility analysis of solutions should be seriously considered for all multilevel waveforms, which means that the applied PPWM method should be conducive to the convergence of solutions and the higher continuity of solutions.

Previously, the PPWM formulations can be simply and clearly determined by the quantity of switching transitions within one cycle for two-level output voltage waveforms. However, two factors should be considered for multilevel PPWM formulations: the quantity of switching transitions within one cycle and the distribution of these switching transitions at different levels of output voltage waveforms.

The latter has the influence on convergence and continuity of solutions and can also affect the complexity of PPWM formulations. Therefore, equations of a single form that can be universally used for PPWM waveforms are significant to handle this kind of technique. Next, the brief descriptions of several common PPWM formulations are given below.

### 2.1. PPWM Formulation with Quarter-Wave Symmetry

The simplest PPWM formulation for multilevel waveforms is quarter-wave symmetry, as shown in Figure 3.
Figure 3. Three-level waveform with quarter-wave symmetry.

It requires the least number of equations and can remarkably simplify the calculation process of solving the PPWM problem due to the absence of the dc component, even harmonics and the sine coefficients of odd harmonics. Therefore, the output voltage can be reduced to

\[
u(\omega t) = \sum_{n=1}^{\infty} \left( \frac{4}{n\pi} \sum_{k=1}^{N} (-1)^{k+1} \cos(n\alpha_k) \right) \sin(n\omega t) \tag{3}\]

where \(N\) is the number of switching, \(k\) is the sequence number of the switching angle from 1 to \(N\). Additionally, the following limit must be satisfied for the switching angles within a quarter of the period:

\[0 < \alpha_1 < \alpha_2 < \ldots < \alpha_N < \pi/2 \tag{4}\]

2.2. PPWM Formulation with Half-Wave Symmetry

Compared to quarter-wave symmetric multilevel waveforms, PPWM formulation with half-wave symmetry can expand the number of feasible solutions and also has the potential to better improve harmonic performance, as shown in Figure 4.

Figure 4. Three-level waveform with half-wave symmetry.

The dc component and the even harmonics in half-wave symmetry are eliminated, and therefore only the sine and cosine components of the odd harmonics need to be controlled. Then, the expression of multilevel waveform with half-wave symmetry is written in the following form:

\[
u(\omega t) = \sum_{n=1}^{\infty} \left( \frac{2}{n\pi} \sum_{k=1}^{2N} (-1)^{k+1} \sin(n\alpha_k) \right) \cos(n\omega t) + \frac{2}{n\pi} \sum_{k=1}^{2N} (-1)^{k+1} \cos(n\alpha_k) \sin(n\omega t) \tag{5}\]
Additionally, the following requirement must be satisfied for the switching angles within a half of the period:

\[ 0 < \alpha_1 < \alpha_2 < \ldots < \alpha_N < \ldots < \alpha_{2N} < \pi \] (6)

2.3. PPWM Formulation with Asymmetry

Complete cancellation of the symmetry requirements causes the increased complexity of the asymmetrical PPWM formulation, such as computational burden, difficulty to achieve valid solutions and suboptimal harmonics. Therefore, this kind of PPWM formulation remains the least attractive choice, as shown in Figure 5.

The dc component, as well as both the odd and even harmonics, need to be controlled in the asymmetrical PPWM formulation. Therefore, the formula of asymmetrical multilevel waveform should be rewritten as:

\[
u(\omega t) = \frac{1}{4\pi} \left[ \sum_{k=1}^{4N} (-1)^{k+1} \alpha_k \right] + \sum_{n=1}^{\infty} \left( \frac{2}{\pi n} \left[ \sum_{k=1}^{4N} (-1)^{k+1} \cos(n\alpha_k) \right] \cos(n\omega t) \right)
\]

\[ + \frac{2}{\pi n} \left[ \sum_{k=1}^{4N} (-1)^{k+1} \sin(n\alpha_k) \right] \sin(n\omega t) \] (7)

Figure 5. Three-level waveform with asymmetry.

Additionally, the following constraint must be met for the switching angles within a whole period:

\[ 0 < \alpha_1 < \alpha_2 < \ldots < \alpha_N < \ldots < \alpha_{4N} < 2\pi \] (8)

For the majority of high-power AFE converter topologies, the PPWM formulations presented above are convincing based on the principle of equal output voltage levels in amplitude. Furthermore, with the extensive development and strong popularization of renewable energy technologies, such as photovoltaic applications, converter topologies based on cascaded H-bridge (CHB) normally work with unequal/variable output voltage levels in amplitude. The formulations of unequal/variable output voltage levels can be classified as [91,92]: (1) unequal and constant voltages; (2) unequal and variable voltages; and (3) a combination of constant and variable voltages.

3. Solving Algorithms for PPWM Techniques

Finding feasible solutions of the PPWM methods is a crucial task and the option of an appropriate solving algorithm considerably relies on the PPWM formulation of the output waveform discussed in Section 2. In the past few decades, scientists and research groups have made great efforts to develop and improve a large number of solving algorithms in order to find the unique optimal solution or multiple optimal solutions in the PPWM formulations, such as numerical methods, algebraic methods, and intelligent optimization methods [93–125]. This section specifically introduces these solving algorithms focusing on
their characteristics and the classification with commonly used methods is presented in Figure 6.

![Figure 6. Classification of PPWM solving algorithms and methods.](image)

### 3.1. Numerical Methods

There are normally two factors should be considered in solving the PPWM formulations with numerical methods:

- The iteration efficiency of algorithm;
- The determination of initial values for algorithm.

The former factor has been well improved with the vigorous development of the field of mathematics, therefore the latter factor has actually become the main issue that needs to be considered in solving the PPWM formulations. A satisfactory estimate of the initial values can ensure the convergence of the final solutions to a large extent. Although sometimes the initial values may be predictable for output waveforms with a small quantity of switching angles, they may not work well or they are difficult to predict for output waveforms with a large quantity of switching angles.

The Newton–Raphson method was intensively studied in the early stage to solve the PPWM equations, and several approaches that can be chosen to determine the initial values are as follows. The incremental value of modulation index $M$ is used to calculate the PPWM equations, which is based on the assumption that the equations change continuously with the modulation index, and the current solution will continue to be directly used as the initial value of the new equation with a slight increase in $M$ [126]. For a given modulation index $M$, some linear functions can be achieved to calculate the initial values for the PPWM equations with a small quantity of switching angles and then extended to the PPWM equations with a large number of switching angles [46]. Using predictive algorithms to find the initial values for the PPWM equations along the tangent direction of a feasible point on the solution trajectory with an incremented value of modulation index $M$, and then use the Newton–Raphson method to find the exact feasible solutions [127,128]. Method based on the equal area and barycenter superposition of the PPWM signal with sinusoidal reference signal is applied to determine the starting point of the solution trajectory [129–131]. The mirror surplus harmonic method can be combined in a phase-shift technique to provide the initial values while reducing the computational burden of harmonic suppression [132]. The conventional sine-triangle PWM can also be applied to generate the initial values. Using the orthonormal set of Walsh functions to find a more tractable PPWM equations instead of Fourier series representations [71,133,134]. Using the homotopy algorithm to relax the limits of defining initial values of PPWM equations due to its convergence domain is bigger than that of Newton–Raphson method [135,136].
3.2. Algebraic Methods

Algebraic methods can overcome the limits in numerical methods, which can achieve feasible solutions of PPWM equations without the need of initial values. These methods apply trigonometric identities to transform the nonlinear and transcendental equations of PPWM into a set of equivalent polynomial equations, and then use resultant theory to calculate the resultant system of polynomial equations and finally try to find all feasible solutions of the switching angles based on a given PPWM formulation.

There are some applied approaches of PPWM technologies proposed by algebraic methods. The multiple-angle formulas and the variables substitution are employed to transform the PPWM equations to their corresponding polynomial equations, and then the polynomial equations are converted to their equivalent triangular equations by using the resultant elimination theory, and finally all valid solutions of switching angles can be obtained [137,138]. The Chebyshev function can also be applied to convert the trigonometric equations related to PPWM problems into their algebraic equivalents, thereby achieving less computing time and superior convergence ability [139]. The symmetric polynomial theory and the power sum can be introduced to reduce the degree of the resultant polynomial equations [75,140–142]. Another approach is the Wu-method, which can convert polynomial equations to a characteristic triangular set with the same zero set as the original polynomial equations under the help of symmetric polynomial theory [143]. One procedure of solving PPWM equations can be guaranteed by the extension theorem in the Groebner bases theory, and then all feasible solutions can be found [144].

The main limitation of these algebraic methods is that the order/degree of polynomials increases as the number of harmonics needed to be considered increases (or the number of switching angles increases), which increases the computational burden and thus causes low efficiency.

3.3. Intelligent Optimization Methods

Currently, the intelligent optimization methods are studied and admired by more and more researchers to solve the PPWM problems. Here, the PPWM problem can be reformulated as an optimization problem, which will be minimized by the fitness function and the constraint functions, and meanwhile modern stochastic search algorithms can be applied to find all feasible solutions. The reasons why intelligent optimization methods are popular can be defined as:

- They have lower requirement or are less dependent on the determination of initial values for PPWM formulations than numerical methods. This demand exists as high-power AFE converters are used in more and more applications and the methods with the need of initial values are not competent for this case;
- They are easy and clear for understanding and implementation due to the great development of the artificial intelligence techniques in the past few decades.

A differential evolution (DE) algorithm can be used to find the optimal switching angles for the PPWM technique by transforming the nonlinear and transcendental equations of PPWM to a constrained optimization problem [145,146]. Genetic algorithms (GAs) have been used to find feasible solutions to the PPWM problem in many research areas, such as selected harmonic elimination, reduction in the line current harmonic and determination of the optimal switching angles with balanced/unbalanced dc sources [147–149]. The particle swarm optimization (PSO) algorithm is another powerful optimization tool for different PPWM formulations, which can realize functions such as the elimination of harmonics and minimization of the voltage/current THD [150–152].

Although these intelligent optimization methods show their abilities and strong robustness to solve the PPWM equations without the need of initial values, they are sensitive to the setting parameters of the algorithm applied and the converge speeds are normally slower than the numerical methods and algebraic methods.

A key issue in the real-time implementation of numerical and intelligent optimization methods is that we cannot be sure whether there is a feasible solution or not. Additionally,
it is also not clear that the failure to achieve a feasible solution is caused by the selection of the setting parameters of the algorithm applied or there is indeed no such valid solution for the PPWM equations.

3.4. Other Improved Methods for Real-Time/Online Implementation

Most of the solving algorithms mentioned above are based on off-line calculation, which are no longer suitable for online/real-time implementation. The online/real-time implementation is a hard and challenging task to PPWM problems.

Nevertheless, some corresponding algorithms and methods have been already successfully proposed by some researchers. One approach is presented to greatly reduce the mathematical complexity of PPWM equations, which is able to realize the real-time implementation for PPWM by the microcontroller [153,154]. In this method, the nonlinear and transcendental PPWM equations are transformed to a set of algebraic equations by Chebyshev polynomials, and then continue to be converted to a single polynomial by generalized Newton’s identities. Finally, the values of the switching angles are represented by the roots of this polynomial, and they can be computed in real-time by direct substitution of the reference phase voltage angle. Introducing the curve-fitting mode with a piecewise linear representation to represent the nonlinear trajectory of the switching angles as straight line segments, it can then calculate switching angles online by digital signal processor (DSP) with ease [58,155]. A mathematical model based on Chebyshev polynomials is applied to generate switching angles in real time by field-programmable gate array (FPGA) [156]. The application of artificial neural network (ANN) in the real-time implementation of PPWM require the offline training of the neural network for the switching angles. After that, this approach can generate optimal switching angles for a certain range of modulation indices [157–159]. Another method, called model predictive control (MPC), can control the selected harmonic components in real time with low switching frequency [160,161]. The regular-sampled PWM techniques can be employed to realize the easy real-time implementation by the microprocessor, but it requires a complex nonlinear sampling process in order to generate optimized PWM [162–164].

These improved methods for real-time/online implementation of PPWM normally require the use of microprocessor techniques, which need large memory capabilities for the LUTs. Sometimes, the accuracy of the calculated switching angles could be reduced in order to approximate the valid solutions of PPWM, and this will impose restrictions on the use of PPWM.

4. Illustrative Examples Based on Solution Trajectories, PPWM Waveforms, and FFT Spectra

The different definitions of the PPWM problems mainly depend on the number of switching angles and voltage levels, resulting in an infinite number of possible results. In this section, solution trajectories, PPWM waveforms, and FFT spectra of the typical three-level waveforms with quarter-wave symmetry are presented in terms of the relationship between switching angles and modulation indices. All feasible solutions of the switching angles for PPWM are achieved in the modulation range from 0.7 to 1.15 (step size is 0.01), and the line voltage spectra on the converter side are obtained when the modulation index \( M \) is 0.8. In addition, the general structure of the solving process of PPWM is proposed in Figure 7.
4.1. Results of SHE Implementation

In SHE, the magnitude of fundamental component and harmonics that can be eliminated are expressed as:

\[
\begin{align*}
H_1 &= \frac{4}{\pi} \sum_{k=1}^{N} (-1)^{k+1} \cos(\alpha_k) = M \\
H_n &= \frac{4}{\pi} \sum_{k=1}^{N} (-1)^{k+1} \cos(n\alpha_k) = 0, \quad \text{where } n = 5, 7, 11, \ldots
\end{align*}
\]  

(9)

Meanwhile, Equation (9) also needs to satisfy the constraint below:

\[0 < \alpha_1 < \alpha_2 < \ldots < \alpha_N < \pi/2\]  

(10)

Then, the objective function in SHE is as follows:

\[E = (H_1 - M)^2 + \ldots + H_n^2 \rightarrow \min\]  

(11)

Figures 8–10 illustrate several SHE solutions by built-in function \texttt{fsolve} in MATLAB (MathWorks, 2017b, Natick, MA, USA) for six-pulse, twelve-pulse, and eighteen-pulse connection schemes [102], respectively. It is important to note that this method has the characteristics of fast convergence speed and high precision when a proper initial value is given. Additionally, it is clear from these figures that the selected harmonics are all effectively removed from the spectra, which prove the feasibility of this method. In addition, the solution trajectories tend to be linear over the modulation range from 0.7 to 1.15, which can facilitate real-time/online implementation.

In order to find more feasible solutions or multiple solutions, the PSO algorithm is applied and its results [150] are compared with the solution in Figure 8a obtained by \texttt{fsolve} function. The result of the first case (Case 1) obtained by the PSO algorithm is the same as the solution in Figure 8a, but there are two other different results (Case 2 and Case 3), as shown in Figure 11.
Figure 8. Results of solution trajectories, PPWM waveforms and FFT spectra for six-pulse connection scheme based on `fsolve` function: (a) SHE problem with five switching angles; (b) SHE problem with seven switching angles; (c) SHE problem with fifteen switching angles; and (d) SHE problem with seventeen switching angles.
Figure 9. Results of solution trajectories, PPWM waveforms and FFT spectra for twelve-pulse connection scheme based on \textit{fsolve} function: (a) SHE problem with five switching angles; (b) SHE problem with seven switching angles.

Figure 10. Results of solution trajectories, PPWM waveforms and FFT spectra for eighteen-pulse connection scheme based on \textit{fsolve} function: (a) SHE problem with five switching angles; (b) SHE problem with seven switching angles.
As can be seen from Figure 11, all groups of solutions can effectively eliminate the selected harmonics, but their solution trajectories, PPWM waveforms, and FFT spectra differ. Sometimes, the same solution will be obtained using the PSO algorithm, considering the modulation range between 1.01 and 1.1 in Figure 11. In this case, it is necessary to run the PSO algorithm several times until other solutions are obtained. However, it is unknown whether there are multiple solutions, a unique solution, or even no solution under one modulation index.

To compare the differences of these three groups of solutions more deeply, the comparison of AFE-side line voltage THD for these three different cases is provided, as shown in Figure 12.

**Figure 11.** Results of solution trajectories, PPWM waveforms and FFT spectra for six-pulse connection scheme based on PSO algorithm: (a) SHE problem with five switching angles—Case 2; (b) SHE problem with seven switching angles—Case 3.

**Figure 12.** Comparison of AFE-side line voltage THD for these three different cases.
Additionally, it can be seen from Figure 12 that there is a significant difference in AFE-side line voltage THD, which can provide another new idea for how to better reduce the overall THD of the power system. For instance, the combined solution with the lowest THD among these three groups of solutions can be applied instead of using only single group of solutions.

4.2. Results of SHM Implementation

In SHM, the objective function becomes:

\[ E = \sqrt{H_5^2 + H_7^2 + \ldots + H_{49}^2} \rightarrow \min \] (12)

Additionally, it should meet the following constraints based on the applied grid codes. Here, grid code EN 50160 and quality CIGRE WG36-05 standard [165,166] are employed for demonstration.

\[ \begin{align*}
|H_1 - M| & \leq L_1 \\
|H_n| & \leq L_n, \text{ where } n = 5, 7, 11, \ldots, 49.
\end{align*} \] (13)

where \( L_1 \) is the limitation that determines the value of the fundamental component depending on the modulation index \( M \), which should ideally be close to 0; \( L_n \) is the allowed limits for each \( n \)th harmonic component.

Figure 13 shows two groups of SHM solutions based on 13 and 15 switching angles [167] by built-in function \textit{fmincon} in MATLAB (MathWorks, 2017b, Natick, MA, USA), respectively.

**Figure 13.** Results of solution trajectories, PPWM waveforms and FFT spectra based on \textit{fmincon} function: (a) SHM problem with thirteen switching angles; (b) SHM problem with fifteen switching angles.
In Figure 8c, the magnitudes of the 47th and 49th harmonics are large in the SHE solution with fifteen switching angles, which do not meet the requirements of the grid codes. Even though this problem can be solved by increasing the number of switching angles, as shown in Figure 8d, it will bring more switching losses.

As mentioned in Section 1, SHM is proposed to make the output results of the power system conform to the requirements specified by the grid codes. Compared with the solutions in Figure 8c,d, both of the two cases in Figure 13 satisfy the limits of the grid codes without increasing the switching loss, and can even reduce the switching loss. In addition, it is important to note that the switching loss of the first case with thirteen switching angles is less than the second case with fifteen switching angles, but the THD (considered up to harmonic order 49th) of the second case with fifteen switching angles is better than the first case with thirteen switching angles. Therefore, there is a trade-off between switching losses and THD.

5. Existing Problems and Future Challenges

As discussed in Sections 2 and 3, the numerical methods have the features of fast convergence speed and high precision when a proper initial value is given, but there is no general calculation method for the determination of initial values for PPWM. This problem can be solved by algebraic methods and intelligent optimization methods, as their implementations do not require the determination of initial values.

However, the computation burden of algebraic methods is very heavy, making them difficult to realize in real time or online. For this reason, the intelligent optimization methods with real-time/online implementation for PPWM will become the focus and hotspot of future research. In the research process, the following issues need to be noted and to be studied more deeply:

- The convergence speed of intelligent optimization methods is not as fast as that of numerical methods, which should be further improved;
- The precision of the solution by intelligent optimization methods with the minimization of the objective function is limited by the development of microprocessors. Sometimes, the solutions with low accuracy can be applied for some engineering applications;
- For intelligent optimization methods, it is unknown whether there is no solution, a unique solution, or even multiple solutions, or if there is a unique solution, or even no solution, under one condition;
- Realization with dynamic response, etc.

In addition, a review of the scientific literature shows that the main factors affecting the output voltage/current spectrum on the high-power AFE converter side using PPWM are the accuracy of synchronization with the grid, the frequency of the voltage/current control loop, the sampling frequency of microcontroller (DSP/FPGA), the passive filter, the voltage distortion and impedance of the network, the voltage distortion and impedance in the circuit network, the response speed of the control system, the balance of the DC-link voltage, and the applied PPWM switching pattern/sequence of the semiconductor module in the high-power AFE converter [168].

Improperly adjusting the parameters of the PI controllers in the voltage/current control loops in dq synchronous reference frame will lead to significant fluctuations in the modulation index M and in the phase shift angle between the converter side and grid side, which will cause the incorrect formation of the PPWM switching patterns/sequences for the high-power AFE converters. The same bad effect can also be caused by the unfiltered distortion in the feedback of the phase current and phase locked loop (PLL) control loops when the control system is synchronized with the voltage vector of the grid. The settings of the capacity of the cable connection and the parameters of the line filters can cause resonance effects near the switching frequency of the high-power AFE converter, leading to severe distortion of the feedback signals of the power system, and further causing the
operation failures of the high-power AFE converter or other devices at the point of common coupling (PCC) [169].

Although it is complicated to fine-tune control systems of the high-power AFE converters by using PPWM techniques and many other related issues, systems based on them continue to be actively used nowadays. Many manufacturers guarantee the EMC of their devices only relying on the indicators of THD and the individual harmonic components not exceeding the 50th harmonic, the reason is that the current EMC standards do not specify the above indicators within the range between 2.5 and 150 kHz [170]. The test results from several metallurgical enterprises in Russia also confirm the existence of unsolved scientific and technical problems to ensure the EMC of high-power AFE converters [171,172]. Therefore, there is a need for the development of new power quality standards to solve the problem of electromagnetic compatibility for high-power AFE converters connected with the grid.

Overall, all of the above factors are closely related to each other and require a deep analysis based on the problems in the fields of electromagnetic compatibility (EMC) and energy efficiency and increasing the output power quality of high-power AFE converters with PPWM.

6. Conclusions

This literature review provides a greater understanding of PPWM techniques as the attractive modulation techniques for AFE converters in high-power applications. The reasons PPWM techniques are widely used are that they can work with reduced/low switching frequency and can effectively control the harmonic spectrum and voltage/current THD. This article focuses on the PPWM formulations for different output multilevel waveforms, solving algorithms, existing problems, and suggestions of research trends in PPWM techniques. The main achievements and contributions of this review can be summarized as follows:

1. The PPWM formulations for different output multilevel waveforms and their respective characteristics play a significant role in determining the complexity of optimization problem in PPWM techniques and achieving feasible solutions of switching angles. The level of output voltages and the number of switching angles are other two elements that will affect the definition of PPWM equations;

2. Three common PPWM output waveforms are presented based on the principle of equal voltage levels in amplitude, such as quarter-wave symmetric, half-wave symmetric, and asymmetrical waveforms, and their corresponding mathematical expressions are also proposed, among which the PPWM formulation with quarter-wave symmetry offers the simplest form and can be easier implemented. In addition, there are also output waveforms with unequal or variable voltages in amplitude;

3. Determination of the solving algorithms to find the feasible solution of switching angles for PPWM techniques are the work that requires careful consideration. There are two factors that should be considered: (1) PPWM output waveforms/formulation and (2) Practical goals. A large number of solving algorithms/methods are discussed and classified into four different groups: (1) Numerical methods; (2) Algebraic methods; (3) Intelligent optimization methods; and (4) Other improved methods for real-time/online implementation, based on whether they require initial value determination, whether they can achieve multiple solutions, and they are capable of real-time/online operation, etc.;

4. Determination of the objective optimization function is another significant aspect of PPWM techniques. It can either aim to eliminate selected harmonics or relax the harmonic limits that consider minimizing voltage/current THD or complying with requirements of applied grid codes;

5. The existing problems of solving algorithms for PPWM techniques are brought up for discussion. The numerical methods struggle to calculate the initial values, even when they have rapid convergence speed and high precision if a suitable initial value is provided. The algebraic methods can achieve feasible solutions without
the need of initial values, but they have heavy computation burden, which is not suitable for real-time/online implementation. Therefore, the intelligent optimization methods with real-time/online implementation will be the focus and hotspot in this research direction;

6. Several main factors that will influence the output voltage/current spectrum on the high-power AFE converter side using PPWM techniques are listed. The situation of the EMC of many devices, designed nowadays by manufacturers only considering indicators of THD, and the individual harmonic components not exceeding the 50th harmonic, should be noted. For this reason, the more advanced and normalized power quality standards need to be specified.

In the end, PPWM techniques have great potential and hold tremendous promise in various industrial applications due to its characteristics of low switching frequency and controllability of the switching pattern/sequence, such as adjustable speed drives (ASDs), flexible AC transmission systems (FACTS), and renewable energy systems (RESs), although the dynamic response still needs to be seriously analyzed and evaluated in PPWM techniques when they are applied into practical applications.

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