Comment on “The role of wetting heterogeneities in the meandering instability of a partial wetting rivulet” by Couvreur S. and Daerr A.

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Rivulets [1] and their meandering on a partially wetting surface [2] present an interesting problem, as complex behavior arises from a deceptively simple setup. Recently Couvreur and Daerr [3] suggested that meandering is caused by an instability developing as the flow rate $Q$ increases to a critical value $Q_c$, with stationary (pinned) meandering being the final state of the flow. We tried to verify this assertion experimentally, but instead produced results contradicting the claim of ref. [3]. The likely reason behind the discrepancy is the persistence of flow rate perturbations. Moreover, the theory presented in this paper cannot reproduce the states as considered and disagrees with other theories [4–6].

First, we tried reproducing the critical flow rate precipitating meandering as reported [3]. We were unable to do so with two carefully constructed experimental arrangements (one at the University of New Mexico, another at the University of North Carolina), both using the same substrate (glass), same fluid (water), and same flow parameters as the experiments of ref. [3], with the fluid supply following the design described in our previous work [5]. The stationary pattern that emerged was a non-meandering, straight flow over the span exceeding 2 m. This applies to flow rates 0.2–8 ml/s, while the range of flow rate of ref. [3] was 0.2–1.8 ml/s. The likely cause of this difference is the “constant level tank” Couvreur and Daerr employ: a constant (on average) level of fluid in the tank by itself does not guarantee that the instantaneous flow rate is constant (only the average), and the flow meandering is keenly sensitive to even modest flow rate $Q$ perturbations, as discussed in [5,7,8].

In a tank with a source of velocity fluctuations near the bottom (e.g., a pump), these fluctuations rapidly decay away from the source (consider exponential decay in Stokes’ second problem). Thus, the top (far) boundary well may appear unperturbed, while the discharge rate from the bottom of the tank is affected.

Any $Q$ variation (e.g., $Q$ increase) can temporarily destabilize a rivulet and mislead an observer into believing it has precipitated meandering. We have recorded [9] transient meandering in response to $Q$ increase or decrease between constant levels (how slow a rate change should be not to trigger meandering would be an interesting subject for further study). A stationary flow can be driven to meander with a short sequence of rate fluctuations (fig. 1) retaining average $Q$ and tank fluid level. In all these cases, we see almost immediate transition to meandering. However, after $Q$ becomes constant, the straight flow typically resumes, often in the matter of minutes, although sometimes it takes longer. Moreover, a “pinned” meandering pattern can be destroyed by a $Q$ increase, once again followed by formation of a straight rivulet [9].

The theory presented in ref. [3] may also contain unclear expositions, errors, or inconsistencies. It would help to specify that the axis of the independent coordinate $X$ ($x$ in dimensionless form) is normal to the direction of the rivulet, rather than pointing downstream. Setting $X$ to be the downstream direction and $h$ to be the deviation from the centerline (a common notation in the field) leads to apparent inconsistencies. The most important of these are as follows. First, in the balance of forces, terms that are normal to each other would be equated. Second, eq. (3) for
Comment

Fig. 1: (Colour on-line) (a) Straight rivulet at \( Q = 5.7 \text{ ml/s} \) (exists above meandering thresholds \( Q_{c} \), predicted in ref. [3], requires no surface “preparing” to develop). (b)–(e) Image sequence showing destruction and re-emergence of the straight rivulet following a sequence of three 1 s flow rate pulses with 1.5 s intervals (first pulse at \( t = 0 \)). Vertical image extent is 2 m.

\[ h'''(X) \] would contain stream curvature on the right-hand side, which cannot be taken as constant (as it is taken in [3]). Third, as a simple calculation shows, there would be no consistent limit for \( h(X) \) far downstream, as all solutions to eq. (3) would develop a singularity corresponding to eq. (3) following a sequence of three 1 s flow rate pulses with 1.5 s intervals (first pulse at \( t = 0 \)). Vertical image extent is 2 m.

Indeed, eq. (3) for the cross-sectional profile \( h(x) \) is \( h'''(x) = -\alpha h'(x) \), with \( \alpha \) being a constant depending on the parameters of the flow. In eq. (4) a cubic, area-preserving, polynomial expansion to this equation is assumed, stating \( h = \theta_{s}/2(1 - x^{2})(1 + Ax/3) \), where \( \theta_{s} \) is the (tangent of) contact angle of the equilibrium profile and \( A \) is the asymmetry parameter. This expansion, applied for \( -1 < x < 1 \), does not represent a true solution of the differential equation and introduces artificial constraints, such as an additional linear dependence between derivatives evaluated at \( x = \pm 1 \). However, the most serious criticism concerns the assumption of constancy of the cross-section, used as an additional condition to determine the form of polynomial (p. 2 of the paper). The constancy of area, i.e. \( \int_{-1}^{1} h(x)dx \), does not follow from the equation for \( h(x) \) or from any other physical principle. The quantity that is constant for all steady states is the fluid flux through a given cross-section and not the area. Since the theory is based on the \textit{deviations} from the straight rivulet, inaccurate description of variations of cross-sectional profile casts doubt on the validity of the whole theory.

In addition, the polynomial ansatz itself introduces errors, as shown in fig. 2, providing an exact solution of eq. (3) (solid line) and the corresponding polynomial (dashed line) satisfying exactly the same boundary conditions for \( h(-1) \) and \( h'(-1) \) with \( h'''(-1) \) for eq. (3) chosen so \( h(1) = 0 \), for a particular value of \( \theta_{s} \). Clearly, the area under the solid curve is greater than that under the dashed curve. The difference between the areas changes with the contact angles, tending to increase when one of

the contact angles is increasing, precisely where the theory is applied. This area mismatch is also present if one were to assign the same derivatives to the exact solution and polynomial ansatz at both ends.

Some of the difference in the interpretation of results may come from a different approach to time scales. The time necessary for a straight rivulet to establish in our experiment varies from minutes to hours. During the transition to straight steady state (which is sustainable indefinitely—for days), meandering patterns that appear may look stationary, but destabilize in a matter of minutes.

Finally, let us comment on the interpretation of theoretical results after eq. (8). The authors take the RMS of curvature and use it as a length parameter in the problem. Our previous measurements show that for meandering, all measurable quantities, properly averaged, satisfy a power law distribution [7]. In the light of this observation, any curvature RMS is likely to depend on the particular cutoff and numerical procedure and thus may not be suited for use as a robust length scale.

The problem of rivulet meandering in a variety of settings is interesting both as an example of a simple flow with surprisingly complex behavior, and because of its practical importance in a wide variety of areas. However, this very complexity requires that the problem is treated rigorously and with attention to detail—both in theoretical considerations and in experimental approaches.

REFERENCES

[1] Powell G. D. and Rothfeld L. B., \textit{AIChe J.}, \textbf{12} (1966) 972.
[2] Culkin J. B. and Davis S. H., \textit{AIChe J.}, \textbf{30} (1984) 263.
[3] Couvreur S. and Daerr A., \textit{EPL}, \textbf{99} (2012) 24004.
[4] Park C. W. and Homsy G. M., \textit{J. Fluid Mech.}, \textbf{139} (1984) 291.
[5] Birnir B., Mertens K., Putkaradze V. and Vorobieff P., \textit{J. Fluid Mech.}, \textbf{607} (2008) 401.
[6] Daerr A., Eggers J., Limat L. and Valade N., \textit{Phys. Rev. Lett.}, \textbf{106} (2011) 184501.
[7] Birnir B., Mertens K., Putkaradze V. and Vorobieff P., \textit{Phys. Rev. Lett.}, \textbf{101} (2008) 114501.
[8] Mertens K., Putkaradze V. and Vorobieff P., \textit{J. Fluid Mech.}, \textbf{531} (2005) 49.
[9] Vorobieff P., Fathi N., Putkaradze V. and Mertens K., \textit{Bull. Am. Phys. Soc.}, \textbf{57} (2012) 411; \url{http://www.me.unm.edu/~kalmoth/dfd12/video.html}. 54002-p2