The Application of Analytic Hierarchy Process and Fuzzy Mathematics Analysis in Risk Management

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Abstract—In the safety management system, risk management is the core and it is the key to complete risk management for scientific risk evaluation. However, in the previous risk evaluation, the safety sheet inspection technique is often used. Also, workers’ experience is referred to roughly estimate the risk degree and formulate management measures. This causes some bad influence on the safety management work. As a result, in order to safeguard work safety, a relatively scientific method is often used to analyze the project safety status, avoid from impacts of subjective factors, and conduct the quantitative expression on the safety status. This is the key and difficult point for the current risk management. In this paper, the author combines analytic hierarchy process (AHP) with fuzzy mathematics to construct the risk management analysis model and provides safeguard for the project safety.

Keywords—analytic hierarchy process; fuzzy mathematics; risk management; analytic model

I. INTRODUCTION

In theory, risk management is the management process that how a project or an enterprise minimize bad influence of risks in the risky environment [1]. However, in practice, inspection of project construction or enterprise management on safety loopholes combines with work experience to judge the safety status after scoring the safety checklist. Such a method is simple and easy to operate, but there are more impacts of subjective factors. Also, it doesn’t have the reliable scientific basis. In order to accurately evaluate the influencing factors of major risks and make workers scientifically and accurately look for high-risk loopholes, it is necessary to analyze and evaluate the suitable mathematical method in the analysis and evaluation process and construct the reasonable applicable model. The author thinks that AHP and fuzzy mathematics are relatively ideal evaluation methods.

AHP is a systematic and hierarchical analysis method with the qualitative and quantitative combination. It is an effective method to transform the semi-qualitative and semi-quantitative issue into the quantitative issue. AHP compares with multiple correlation factors layer by layer, so as to provide the quantitative basis for analyze, decide, predict or control development of things [2]. Based on the weight of correction factors on risks of material misstatement obtained by AHP, the fuzzy comprehensive evaluation model is constructed to obtain membership of material risks and determine the risk degree of material risks. According to the risk degree, the further work procedure is designed and implemented. Also, resources are reasonably allocated to decrease risks to the acceptable level [3].

In the risk management, personnel may use their experience to comprehensively analyze each factor of affecting risks in the risk prediction, so as to probably estimate the risk size. In order to avoid from limitations of human factors in the model application, the author thinks it is necessary to start from the systematic viewpoints and use the theories and methods of stereovision to judge influencing factors. To be specific, the influencing factors of risks are evaluated from the following two levels. One is to evaluate from the micro-level. The other one is to evaluate from the macro-level. As evaluating the micro-level, factors to be considered include: the selection and application of company policies, corporate target strategies and relevant operation risks, performance and evaluation, project properties, and internal project control. In macro-level, the factors to be considered get involved in industrial status, legal environment and supervision environment, as well as other external factors. To sum up, according to the principles of AHP, the hierarchal structure model of risk evaluation is constructed as follows.

Among which, the goal layer includes the risk evaluation layer (O). The criterion layers contain the micro-level (C₁) and macro-level (C₂). The index layers get involved in 8 indexes, such as selection and application of company polices, company performance and evaluation.

II. AHP

The integers between 1 and 8 in AHP are used as the scales to construct the judgment matrix. The multiple correlation factors are compared layer by layer to finally obtain the total ranking of each factor.

Step 1: According to total goals of problems, the systematic analytical hierarchy process is constructed

Step 2: In factors of affecting auditing judgment, risk preference of executors is the primary factor of influence. In order to overcome the impacts of subjective factors, AHP is used. Then, evaluation experts make comparisons on evaluation indexes and objects. AHP scoring principle is applied to construct the judgment matrix. It is targeted at each factor for comparison. For the ratio between two arbitrarily
factors \( c_i \) and \( c_j \) on the goals in \( a_{ij} \) table, the ratio scale in Table 1 is used to measure \( a_{ij} \), so as to obtain the comparative matrix \( A=(a_{ij}) \) between any two means. Obviously, there is:

\[
a_{ij} > 0, \quad a_{ij} = \frac{1}{a_{ji}}, \quad a_{ii} = 1
\]

### TABLE I. RATIO VALUE

| Scale \( a_{ij} \) | Meaning |
|------------------|---------|
| 1                | \( c_i \) and \( c_j \) have the same influence |
| 3                | The influence of \( c_i \) is slightly stronger than \( c_j \) |
| 5                | The influence of \( c_i \) is stronger than \( c_j \). |
| 7                | The influence of \( c_i \) is more obvious than \( c_j \). |
| 9                | The influence of \( c_i \) is more absolute than \( c_j \). |
| 2, 4, 6, 8       | The influence between \( c_i \) and \( c_j \) is between two adjacent levels. |
| 1/2, ..., 1/9    | The influence between \( c_i \) and \( c_j \) is the mutually inverse number of the above \( c_{ij} \). |

(1) The consistent index:

\[
BW = \lambda_{max}(W)
\]

Among which, \( \lambda_{max} \) represents the maximum eigenvalue of judgment matrix. \( n \) is the factor number of the judgment matrix.

(2) The random consistent index \( RI \) is shown as follows:

### TABLE II. THE RANDOM CONSISTENT INDEX

| \( n \) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|--------|---|---|---|---|---|---|---|---|---|----|----|----|----|
| \( RI \) | 0 | 0 | 0.58 | 0.90 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 | 1.49 | 1.51 | 1.54 | 1.56 |

(3) The consistent ratio index:

\[
CR = \frac{\overline{C}}{\overline{M}}
\]

CR<0.1, it shows factors’ judgment thinking is consistent, thus \( W_0 \) can be used as the weight vector.

Step 3: Confirm the integrated weight. For each judgment matrix, the normalization processing of vectors is conducted to obtain the weight vector \( W \).

Step 4: Conduct the consistent inspection. In general, actual judgment matrix may not be consistent. In other words, judgment tendency of experts is inconsistent. As a result, it is necessary to do the consistent inspection, so as to ensure that the weight has the higher reliability. The following indexes are mainly inspected as follows:

(3) The consistent ratio index:

\[
CR = \frac{\overline{C}}{\overline{M}}
\]

CR<0.1, it shows factors’ judgment thinking is consistent, thus \( W_0 \) can be used as the weight vector.

Step 5: Total ranking of layer: The single ranking results of all elements in the same level are used to calculate the weight of all factors by aiming at the last level. According to the weight, the same method of AHP is used to obtain the total ranking results of hierarchy.

### III. FUZZY MATHEMATICS

Step 1: Confirm the factor set. Considering the relationship between risk factors, it is divided into micro-level factors and macro-level factors in line with different properties of factors(primary factors). Micro-level factors are affected by 5 secondary factors, while macro-level factors are affected by 3 secondary factors, as shown in Figure 1, so as to determine the factor set of two primary factors:

- Micro-level: \( A = \{1, 2, 3, 4, 5\} \)
- Macro-level: \( B = \{1, 2, 3, 4, 5\} \)

Step 2: Determine the judgment set. The judgment set {maximum influence, higher influence, high influence, general influence, slight influence and no influence} of 6 elements can be given to each secondary factor. According to the judgment set composed of secondary factors, the judgment set {maximum risk, higher risk, high risk, general risk, slight risk, and no risk} of 6 elements is given to primary factors.

Step 3: Confirm the fuzzy judgment matrix

For the construction of secondary factors’ judgment matrix, experts respectively evaluate and mark the factors of affecting risk degree.

\[
r_{ij} = \frac{\text{Number of people rated as the } j \text{th}}{\text{Total number of judges}} (i = 1, 2, A; m; j = 1, 2, A; n)
\]

The judgment results are used to construct the single judgment matrix for each factor set. Among which, the \( i \)th secondary factor set is judged as \( (r_{1i}, r_{2i}, \ldots, r_{ni}), (i = 1, 2, A, m) \), thus the fuzzy evaluation matrix of the corresponding secondary factor set is:

\[
R = \left[\begin{array}{ccc}
  r_{11} & r_{12} & r_{1n} \\
  r_{21} & r_{22} & r_{2n} \\
  & M & O \\
  r_{m1} & r_{m1} & r_{mn}
\end{array}\right]
\]
In this way, the fuzzy evaluation matrix of two secondary factors respectively is $R_1$ and $R_2$.

Step 4: The judgment set of secondary factors is constructed. To begin with, weight of each factor obtained by AHP conducts the normalization processing to obtain the corresponding weight vector:

$$W = (w_1, w_2, w_3, w_4, w_5, w_6)$$  \hspace{1cm} (7)

Also, judgment matrix of each factor set conducts the normalization processing to obtain:

Next, $i^{th}$ secondary factor set conducts the primary comprehensive judgment. The model $M(\Lambda, \nu)$ is used to calculate the judgment set of the $i^{th}$ factor set as:

$$\Phi = W_i R_i^*$$  \hspace{1cm} (8)

Step 5: Construct the secondary comprehensive judgment set of primary factors.

Construct the judgment set matrix of two primary factors as the judgment matrix of primary factors, namely:

$$R = (V11, V12) T = \begin{bmatrix} V_{11} & V_{12} & V_{13} & V_{14} & V_{15} & V_{16} \\ V_{21} & V_{22} & V_{23} & V_{24} & V_{25} & V_{26} \end{bmatrix}$$  \hspace{1cm} (9)

Secondary factors are used for the secondary comprehensive judgment. The model $M(\Lambda, \nu)$ is used to calculate the judgment set of the $i^{th}$ factor set:

$$U = W_i R = (U_1, U_2, U_3, U_4, U_5, U_6)$$  \hspace{1cm} (10)

Step 6: Use the maximum membership principle to solve the membership degree of the target layer. Each vector in the result matrix is analyzed. According to the maximum membership principle, the risk degree of the corresponding comment set for the numerical maximum is the degree of risks. Through the ranking of risk factors, the maximum risk factors of affecting the project are determined. On the basis of the overall risk evaluation, the prevention method for project risks is formulated.

IV. SUMMARY

The core of the safety management system is the risk management. By analyzing loopholes, it is necessary to control in advance and prevent from unsafe events. Through the combined application of AHP and fuzzy mathematical analysis, project risks conduct the scientific evaluation. Also, scientific means are used for management. This shows the important significance on safety level. In this paper, the author constructs the AHP and fuzzy mathematics model to provide the available model for risk management and offers the effective reference to safety management.

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