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The Influence of the Magnetic Field on the Properties of Neutron Star Matter

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In the mean field approximation of the relativistic $\sigma$-$\omega$-$\rho$ model, the magnetic fields are incorporated, and its influence on the properties of n-p-e neutron star matter are studied. When the strength of the magnetic field is weaker than $\sim 10^{18}$ G, the particle fractions and chemical potentials, matter energy density and pressure hardly change with the magnetic field; when the strength of the magnetic field is stronger than $\sim 10^{20}$ G, the above quantities change with the magnetic field evidently. Furthermore, the pressure is studied in both thermodynamics and hydrodynamics. The difference between these two ways exits in the high density region, that is, the thermal self-consistency may not be satisfied in this region if the magnetic field is considered.

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1. Introduction
Over the past several ten years, the observations have indicated that large magnetic fields exist at the surface of the neutron stars[1]. From the spindown rates of pulsars, the magnetic fields of 558 pulsars of the catalog by Taylor[2] lies between $B = 1.7 \times 10^8$ G (PSR B1957+20) and $B = 2.1 \times 10^{13}$ G (PSR B0154+61), with a typical value $B = 1.3 \times 10^{12}$ G, most of young pulsars having a surface field in the range $B \sim 0.1 - 2 \times 10^{13}$ G. Several proofs suggest that soft $\gamma$-ray repeaters(SGRS),

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and perhaps the so-called anomalous X-ray pulsars (AXPS), are neutron stars with magnetic fields \( \geq 10^{14} \text{G} \)—the so-called magnetars.\textsuperscript{3} Furthermore, the observed X-ray luminosities of the AXPs may require a field strength \( B \geq 10^{16} \text{G} \).\textsuperscript{4} The population statistics of SGRs suggest that magnetars may constitute a significant fraction (\( \geq 10\% \)) of the neutron star population.\textsuperscript{5} A considerable amount of studies have been devoted to the structure of the magnetic field outside the neutron star, in the so-called magnetosphere, in relation with the pulsar emission mechanism (for a review, see e.g.\textsuperscript{6}). The strength of the magnetic field in the interior of neutron stars remains unknown, its estimated values can be \( 10^{18} \text{G} \) for a star with \( R \approx 10 \text{km} \) and \( M \approx 1.4M_\odot \).\textsuperscript{7}

The effect of the neutron star strong interior magnetic fields has been studied recently.\textsuperscript{8,9} Broderick etc. found that the magnetic fields of strengths larger than \( 10^{16} \text{G} \) affect the equation of state (EOS) of dense matter directly through drastic changes in the composition of matter.\textsuperscript{10} The EOS is altered by both the Landau quantization of the charged particles (such as protons, electrons, etc.) and the interactions of the magnetic moments, including the anomalous magnetic moments of the neutral particles (such as the neutron, strangeness-bearing \( \Lambda \)-hyperon etc.), with the magnetic field.

We study the properties of n-p-e neutron star by incorporating the magnetic field with the relativistic \( \sigma-\omega-\rho \) model. In other works, the pressure is usually studied by thermodynamics. As we know, the pressure can be calculated by both thermodynamics and hydrodynamics, which are equivalent in nuclear matter when the magnetic field is absent. Considering the magnetic field in neutron stars, the difference between these two ways will exist as pointed out in this paper, especially in the high density region. The theoretical formalism is outlined in section 2; in section 3, the numerical results are presented; and finally, we give our summary in section 4.

2. Basic theory

The Lagrangian density is given by:

\[
\mathcal{L} = \bar{\psi} [i\gamma_\mu D^\mu - m + g_s \phi - \gamma_\mu g_\omega \omega^\mu - \frac{1}{2} g_\rho \gamma_\mu \vec{r} \cdot \vec{\rho}] \psi + \frac{1}{2} g^\mu \phi \partial_\mu \phi - \frac{1}{2} m_\phi^2 \phi^2 - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} - \frac{1}{4} \rho_{\mu\nu} \rho^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho} \cdot \vec{\rho} + \bar{\psi} [i\gamma_\mu D^\mu - m_e] \psi_e - U(\sigma),
\]

\[
U(\sigma) = \frac{1}{3} c \phi^3 + \frac{1}{4} d \phi^4
\]

\[
\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu
\]

\[
\rho_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu
\]
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\[ D^\mu = \partial^\mu + iqA^\mu \]
\[ A^0 = 0, \vec{A} = (0, xB_m, 0), \] (5)

\[ q \] is the charge of a particle, \[ A^\mu = (A^0, \vec{A}) \] is the four dimensional electromagnetic vector, we select it as Eq. (5) to obtain a magnetic field along \( z \) axis. We don’t consider the anomalous magnetic moments, so neutrons are not affected by magnetic field due to its neutrality. The charge of proton is \( e \), its move equation, the formalism of the field operator and eigenvalue are different from the case in which the magnetic field is ignored. And its move equation is:

\[ i\gamma_\mu \partial^\mu \psi_p - [m - g_s \phi + \gamma_\mu g_\omega \omega^\mu + \frac{1}{2} g_\rho \gamma_\mu \vec{\rho} \cdot \vec{\rho} + e\gamma_\mu A^\mu] \psi_p = 0, \] (6)

the general solution of Eq. (6) is:

\[ \psi_p \approx \exp(-i\varepsilon H t + ip_y y + ip_z z) f_{p_y, p_z}(x), \] (7)

where \( f_{p_y, p_z}(x) \) is the four-component solution, and Eq. (6) can be converted into:

\[ [-i\alpha_x \frac{\partial}{\partial x} + \alpha_y (p_y - exB_m) + \beta m^* + \alpha_z p_z + U_{0, p}^H] f_{p_y, p_z}(x) = \varepsilon^H f_{p_y, p_z}(x), \] (8)

\[ m^* = m - g_s v, \] (9)

\( v \) is the mean field of \( \sigma \) meson field,

\[ U_{0, N}^H = \frac{g_\sigma^2}{m_\omega^2} \rho_B + \frac{1}{4} \frac{g_\rho^2}{m_\rho^2} \rho_3 I_{3N}, \quad N = p, n, \] (10)

\( I_{3N} \) is the projection of nucleon isospin along \( z \) axis, \( I_{3p} = 1, I_{3n} = -1 \).

\[ \rho_B = \rho_p + \rho_n, \quad \rho_3 = \rho_p - \rho_n, \] (11)

\( \rho_B, \rho_n, \rho_p \) and the following \( \rho_e \) are the density of baryons, neutrons, protons and electrons respectively. By resolving the Eq. (8), \( \psi_p \) is obtained:

\[ \psi_p^{(\pm)}(x) = \frac{1}{\sqrt{2\varepsilon_{\nu}^H (\varepsilon_{\nu}^H + p_z)}} \begin{pmatrix} (\varepsilon_{\nu}^H + p_z) I_{\nu, p_y}(x) \\ -i\sqrt{2eB_m} \nu I_{\nu - 1, p_y}(x) \\ -m^* I_{\nu, p_y}(x) \\ 0 \end{pmatrix}, \] (12)
ψ_{p1}^{(-)}(x) = \frac{\exp(\pm iH t - ip_y y - ip_z z)}{\sqrt{2\varepsilon'_\nu(\varepsilon'_\nu - p_z)}} \begin{pmatrix} -m^* I_{\nu,-p_y}(x) & 0 \\ (-\varepsilon'_\nu + p_z) I_{\nu,-p_y}(x) & i\sqrt{2eB}\varepsilon'_\nu I_{\nu-1,-p_y}(x) \end{pmatrix}, \quad (13)

ψ_{p2}^{(+)}(x) = \frac{\exp(-iH t + ip_y y + ip_z z)}{\sqrt{2\varepsilon'_\nu(\varepsilon'_\nu + p_z)}} \begin{pmatrix} 0 & -m^* I_{\nu-1,p_y}(x) \\ i\sqrt{2eB}\varepsilon'_\nu I_{\nu,p_y}(x) & (\varepsilon'_\nu + p_z) I_{\nu-1,p_y}(x) \end{pmatrix}, \quad (14)

ψ_{p2}^{(-)}(x) = \frac{\exp(iH t - ip_y y + ip_z z)}{\sqrt{2\varepsilon'_\nu(\varepsilon'_\nu + p_z)}} \begin{pmatrix} i\sqrt{2eB}\varepsilon'_\nu I_{\nu,-p_y}(x) & -m^* I_{\nu-1,-p_y}(x) \\ (-\varepsilon'_\nu + p_z) I_{\nu-1,-p_y}(x) & 0 \end{pmatrix}, \quad (15)

\varepsilon'_\nu = \sqrt{p_z^2 + m^2 + 2eB} = \varepsilon_H - U_{0,p}, \quad (16)

I_{\nu,p_y}(x) \text{ is normalized as:}

\int dx I_{\nu,p_y}(x) I_{\nu',p_y}(x) = \delta_{\nu\nu'}, \quad \sum_{n=0}^\infty I_{\nu,p_y}(x) I_{\nu',p_y}(x') = \delta(x - x'). \quad (17)

\nu \text{ is the Landau quantum number, } \psi_p \text{ has the same form as } \psi_p \text{ with the proton quantities replaced by the electron ones. The proton propagator can be constructed by } \psi_p. \text{ Furthermore, by the known neutron propagator, we can present the energy density } \varepsilon \text{ and pressure } p \text{ in the mean field approximation:}

\varepsilon = \frac{\gamma_n}{(2\pi)^3} \int_{k_F} d^3k \sqrt{k^2 + m^2 + U_{0,n}\rho_n + eB_m}(\sum_{\nu=0}^{\nu_{\max}} g_{\nu} \int_{-p_{3F}}^{p_{3F}} dp_{3}\varepsilon'_\nu) + \frac{eB_m}{(2\pi)^2} \int_{-p_{3F}}^{p_{3F}} dp_{3} \varepsilon'_\nu \rho_{\nu} + \frac{eB_m}{(2\pi)^2} \int_{-p_{3F}}^{p_{3F}} dp_{3} \varepsilon'_\nu f_{\nu} + \frac{1}{2}m_{\nu}^2 v^2 - \frac{1}{2}m_{\nu}^2 V_0 v^2 - \frac{1}{2}m_{\nu}^2 V_0^2 + \frac{1}{3}cv^3 + \frac{1}{4}dv^4, \quad (18)

p = \frac{1}{3} \frac{\gamma_n}{(2\pi)^3} \int_{k_F} d^3k \frac{k^2}{\sqrt{k^2 + m^2}} + \frac{eB_m}{(2\pi)^2} \int_{-p_{3F}}^{p_{3F}} dp_{3} \sum_{\nu=0}^{\nu_{\max}} g_{\nu} \frac{p_{3}\nu^2 + \nu eB_m}{\varepsilon'_\nu} + \frac{1}{3} \frac{eB_m}{(2\pi)^2} \int_{-p_{3F}}^{p_{3F}} dp_{3} \sum_{\nu=0}^{\nu_{\max}} g_{\nu} \frac{p_{3}\nu^2 + \nu eB_m}{\varepsilon'_\nu} - \frac{1}{2}m_{\nu}^2 v^2 + \frac{1}{2}m_{\nu}^2 V_0 v^2 + \frac{1}{2}m_{\nu}^2 V_0^2 - \frac{1}{3}cv^3 - \frac{1}{4}dv^4, \quad (19)
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\[ \rho_p = \frac{eB_m}{2\pi^2} \sum_{\nu=0}^{\nu_{\text{max}}} p_3 F_{\nu}, \quad (20) \]

\[ \rho_e = \frac{eB_m}{2\pi^2} \sum_{\nu=0}^{\nu_{\text{max}}} p_3 F_{\nu}, \quad (21) \]

\[ \rho_n = \frac{3}{(2\pi)^3} \int_{k_F} d^3k, \quad (22) \]

chemical equilibrium and neutrality conditions in neutron stars give

\[ \mu_n = \mu_p + \mu_e, \quad \rho_p = \rho_e. \quad (23) \]

The Landau level degeneracy factor \( g_{\nu} \) is 1 for \( \nu = 0 \) and 2 for \( \nu > 0 \). \( \mu_n, \mu_p \) and \( \mu_e \) are the neutron, proton and electron chemical potential, \( V_0^{\omega}, V_0^{\rho} \) are the mean fields of the time-like component of the \( \omega \) meson field and the time-like isospin 3-component of \( \rho \) meson field respectively.

3. Numerical calculation

The values of the constants are adjusted to reproduce the

Fig. 1. The curve of energy density of n-p-e neutron star matter changing with baryon density in different magnetic field. The solid and dashed represent the cases in \( B_m \leq 10^{18} \text{G}, B_m = 10^{20} \text{G} \) respectively.

saturation properties of nuclear matter in the mean field approximation of the relativistic \( \sigma-\omega-\rho \) model, \( g_s = 7.17, g_\omega = 7.16, g_\rho = 8.55 \), \( c = 100.02 \text{MeV}, d = 372.26 \). Fig. 1 depicts the relation between the energy density of n-p-e neutron star matter and baryon density in different magnitude of uniform magnetic field. With the magnetic field increasing, the energy density doesn’t change if \( B_m \leq 10^{18} \text{G} \) and decreases rapidly if \( B_m \) accesses to \( 10^{20} \text{G} \). In Fig. 2 we plot the particle fraction, \( Y_i = \frac{\rho_i}{\rho_B}, i = n, p \), as a function of
Fig. 2. The fractions of neutrons (solid) and protons (dashed) of n-p-e neutron star matter change with baryon density in the magnetic field of $\leq 10^{16}$ G, $10^{18}$ G, $10^{20}$ G (neutron is from top to bottom and proton is adverse) respectively.

Fig. 3. The relation between the potential of neutrons (solid), electrons (dashed) and baryon density in the magnetic field of $< 10^{18}$ G, $10^{20}$ G (from top to bottom) respectively.

We study the pressure by Eq.(19) as the dashed in Fig. 4. For comparison, we also study it by thermodynamics, that is $p = \rho B^2 \frac{\partial}{\partial \rho B} \left( \frac{\varepsilon}{\rho B} \right)$. These two results, which are equivalent in nuclear matter, are different in the high density region, and more evident if $B_m > 10^{18}$ G. Its cause may be that more complex factors are not considered, for example, the particle magnetic momentum and anomalous magnetic momentum, the non uniformity of the magnetic field etc..

4. Summary
By appropriating the magnetic field in the relativistic $\sigma-\omega-\rho$ model, we have studied the properties of n-p-e neutron star matter. It is found that the properties are nearly...
Fig. 4. The pressure of n-p-e neutron star matter changes with baryon density in the magnetic field of \(10^{18} \text{G}, 10^{20} \text{G}\) (from top to bottom). The solid and the dashed represent the results by thermodynamics and hydrodynamics respectively.

invariant with the magnetic field when \(B_m \leq 10^{18} \text{G}\). To observe the effect of the magnetic field clearly, we assume the value of \(B_m\) to be \(10^{20} \text{G}\), regardless of its physical meaning. As we expected, the properties are altered evidently. In addition, the discrepancy exits between the pressures studied by two ways as mentioned above in the high density region. For simplicity, some complex factors which are presumed to be present in the interior of neutron star are omitted. This rude treatment may be just the cause of the discrepancy between pressures by thermodynamics and hydrodynamics.

Further, the quark phase has been studied at finite temperature or finite chemical potential in detail. It will be also valuable that the magnetic field in neutron stars is studied considering these factors.

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