EXTREME DOMAIN WALL–BLACK HOLE COMPLEMENTARITY IN $N = 1$ SUPERGRAVITY WITH A GENERAL DILATON COUPLING

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Abstract

We study supersymmetric (extreme) domain walls in four-dimensional ($4d$) $N = 1$ supergravity theories with a general dilaton coupling $\alpha > 0$. Type I walls, which are static, planar (say, in $(x, y)$ plane) configurations, interpolate between Minkowski space-time and a vacuum with a varying dilaton field. We classify their global space-time with respect to the value of the coupling $\alpha$. $N = 1$ supergravity with $\alpha = 1$, an effective theory from superstrings, provides a dividing line between the theories with $\alpha > 1$, where there is a naked (planar) singularity on one side of the wall, and the theories with $\alpha < 1$, where the singularity of the of the wall is covered by the horizon. The global space-time (in $(t, z)$ direction) of the extreme walls with the coupling $\alpha$ is the same as the global space-time (in $(t, r)$ direction) of the extreme magnetically charged black holes with the coupling $1/\alpha$. 

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Over the last few years topological defects in general, and black holes, in particular, have been studied extensively in theories in which an additional scalar field, the dilaton, couples to such defects. The dilaton adds new features to the nature of such configurations, in particular in the region of the space-time where it blows-up. A special class of such configurations correspond to the extreme configurations, which can be shown to be supersymmetric configurations whose mass saturates the corresponding Bogomol’nyi bound. They are thus of special interest since they correspond to configurations with the minimal

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1For a review see Ref. and references therein.
energy in its class, and can be viewed as genuine solitons.

In this paper we study extreme domain wall configurations interpolating between isolated supersymmetric vacua of the four-dimensional (4d) $N = 1$ supergravity theory with a general dilaton coupling specified by parameter $\alpha > 0$. In particular we classify their space-time structure with respect to the coupling $\alpha$. The dilaton field, a scalar component of a linear multiplet, acts as an additional matter source, adding new features to configurations in this type of supergravity theories. We shall concentrate on extreme configurations, i.e., those are the ones which preserve part “$N = \frac{1}{2}$” of the supersymmetry and they saturate the corresponding Bogomol’nyi bound for the energy density of the wall. Specifically, we address extreme Type I dilatonic domain walls, which turn out to be static, planar configurations, interpolating between Minkowski space-time and a vacuum with a varying dilaton field. We shall compare their global space structure with the one of the corresponding extreme charged dilatonic black holes \[2\ldots6\ldots1\].

We study such configurations within 4d $N = 1$ supergravity theory with a linear supermultiplet whose scalar component corresponds to the dilaton field. Within the Kähler superspace formalism\[2\] the dilaton field $\phi$ is represented as a real part of the scalar component $S \equiv e^{-2\phi/\sqrt{\alpha}} + ia$ of the chiral superfield $\mathcal{S}$. $\mathcal{S}$ has the following restricted form of the Kähler potential

$$K(S, \mathcal{S}) = -\alpha \ln(S + \mathcal{S}). \quad (1)$$

and no superpotential ($W(S) = 0$). The field $\phi$ is defined in $S$ so that its kinetic energy is normalized. Note, that in 4d $N = 1$ supergravity the value of $\alpha$ in \[1\] is an arbitrary positive constant with $\alpha = 1$ corresponding to an effective theory of superstring vacua. $\mathcal{S}$ also couples linearly to the kinetic energy of the Yang-Mills superfield $W_{\mu a}$, i.e., yielding a term in the Lagrangian proportional to

$$\mathcal{S} \text{ tr } (W_{\mu a} W^{\mu a}). \quad (2)$$

In addition, the theory contains chiral superfields, $\mathcal{T}_i$, with Kähler potential $K_M(\mathcal{T}_i, \overline{\mathcal{T}}_i)$ and nonzero superpotential $W_M(\mathcal{T}_i)$.

The tree-level low-energy action of 4d $N = 1$ supergravity theory with a linear supermultiplet is therefore specified by a separable Kähler potential $K = K_M(\mathcal{T}_i, \overline{\mathcal{T}}_i) + K(S, \mathcal{S})$, superpotential $W = W_M(\mathcal{T}_i)$, and the gauge coupling function $f_{ab} = \delta_{ab} S$.

In the following we shall study the extreme domain walls and compare their global space-time to the one of the charged black holes within the above class of supergravity theories. For the sake of simplicity we shall assume that walls are formed due to isolated minima of one matter field $T$ (a scalar component of a chiral superfield), only. In addition, we address Abelian charged black holes with one $U(1)$ gauge group factor (Kac-Moody level $k = 1$),

\[2\] The Lagrangian with the chiral superfield $\mathcal{S}$ is at the classical level equivalent to the one with the linear supermultiplet $L$, where $\mathcal{S}$ and $L$ are related through the duality transformation. See, e.g., Ref. [7] and references therein.
only. We also set the axion field \( a \), the imaginary part of \( S \), to zero. The bosonic part of the tree level Lagrangian is then of the form:

\[
\mathcal{L} = \sqrt{-g} \left[ -\frac{1}{2} R + K_T \nabla \phi \nabla \phi - 2^{-\alpha} e^{2\sqrt{\alpha}\phi} \bar{V} - 2^{-\frac{2\phi}{\sqrt{\alpha}}} F_{\mu \nu} F^{\mu \nu} \right].
\]  

(3)

where

\[
\bar{V} = e^{K_T M} \left[ K_T T | D_T W_M |^2 - (3 - \alpha) |W_M|^2 \right]
\]

(4)
is the part of the potential, which depends only on the matter field \( T \). Here \( K_T T \equiv (\partial_T \partial_T K_M)^{-1} \) and \( D_T W_M \equiv e^{-K_M} \partial_T (e^{K_M} W_M) \). We use the metric convention (+−−−) and set the gravitational constant \( \kappa \equiv 8 \pi G = 1 \).

The extreme wall solutions correspond to the the case when gauge fields are turned off, i.e., \( F_{\mu \nu} = 0 \), and the matter potential \( \bar{V} \) (Eq. (3)) has two isolated supersymmetric (\( D_T W_M (T) = 0 \)) minima with \( W_M (T) = 0 \) and \( W_M (T) \neq 0 \), respectively. The corresponding wall solution (Type I) is a static, planar configuration (say, in the \((x, y)\) plane located at \( z \sim 0 \)). It interpolates between, say, \( z > 0 \), the supersymmetric vacuum with \( W_M (T) = 0 \), and \( z < 0 \), the supersymmetric vacuum with \( W_M (T) \neq 0 \). The metric \( \text{Ansatz} \) for planar, static domain wall solutions

\[
ds^2 = A(z) (dt^2 - dz^2 - dx^2 - dy^2)
\]

(5)
is conformally flat. The scalar fields \( T(z) \), and \( \phi(z) \) depend on \( z \), only. Using a technique of the generalized Israel-Nester-Witten form, similar to the one developed for the study of supergravity walls without the dilaton field in Ref. [9], one can show that the field equations, the self-dual Bogomol’nyi equations, are of the form:

\[
0 = \text{Im}(\partial_z T \frac{D_T W_M}{W_M})
\]

\[
\partial_z T = -(2^{-\alpha} A e^{2\sqrt{\alpha}\phi})^{1/2} e^{K_M/2} |W_M| \frac{K_T T}{W_M} \frac{D_T W_M}{W_M}
\]

\[
\partial_z \ln A = 2(2^{-\alpha} A e^{2\sqrt{\alpha}\phi})^{1/2} e^{K_M/2} |W_M|
\]

\[
\partial_z \phi = -\sqrt{\alpha}(2^{-\alpha} A e^{2\sqrt{\alpha}\phi})^{1/2} e^{K_M/2} |W_M|.
\]

(6)
The energy density of the wall, \( \sigma \), saturates the Bogomol’nyi bound. For a thin wall with the boundary conditions \( A(0) = 1 \) and \( \phi(0) = \phi_0 \), \( \sigma \) is of the form:

\[
\sigma = \sigma_{\text{ext}} \equiv 2^{1-\frac{\phi}{2\sqrt{\alpha}\phi_0}} e^{\frac{K_M}{2}} |W_M|_{z=0^{-}}
\]

(7)

Here the subscript \( 0^{-} \) refers to the side of the wall with \( W_M (T) \neq 0 \).

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3 We are concerned with classical solutions of the low-energy effective action. Thus, we neither include higher derivative terms, nor possible mixed Kähler-Lorentz and mixed Kähler-Gauge anomalies (see, e.g., Ref. [8] and references therein). Eventually, such corrections should be included in the full treatment of the theory.
The first two equations in (6) describe the evolution of the matter field $T = T(z)$ with $z$. In (6) the third equation for $A(z)$ and the fourth equation for $\phi(z)$ imply:

$$
\tilde{A}(z) \equiv A(z)e^{2\phi(z)/\sqrt{\alpha}} = e^{2\phi_0/\sqrt{\alpha}}.
$$

(8)

The above equation is true everywhere in the domain wall background. It implies that there is a choice (8) of the frame where the metric is flat.

The explicit form of solutions in the thin wall approximation is of the form:

$$
A(z) = 1, \quad z > 0;
A(z) = \left[1 - \frac{1}{2}(\alpha - 1)\sigma_{ext}|z|\right]^{-2\alpha}, \quad \alpha \neq 1, \quad z < 0;
A(z) = e^{-\sigma_{ext}|z|}, \quad \alpha = 1, \quad z < 0,
$$

(9)

and the dilaton field satisfies Eq.(8). On one side of the wall ($z > 0$) the space-time is Minkowski; both, $A(z)$ and $\phi$, are constant. However, on the other side of the wall ($z < 0$), both, $A(z)$ and $\phi(z)$, change its value. There the curvature is of the form:

$$
R = \frac{3(2-\alpha)\sigma_{ext}^2}{2}\left[1 - \frac{1}{2}(\alpha - 1)\sigma_{ext}|z|\right]^{-\frac{2\alpha}{\alpha - 1}}, \quad \alpha \neq 1;
R = \frac{3}{2}\sigma_{ext}^2 e^{-\sigma_{ext}|z|}, \quad \alpha = 1.
$$

(10)

Such walls therefore act as “windows” from the Minkowski space-time into the new type of space-times with varying dilaton field. The nature of these space-times depends crucially on the value of parameter $\alpha$:

- $\alpha = 0$ corresponds to the case of ordinary supergravity walls [9,10], i.e., the dilaton field does not couple to the matter fields (which form the wall). For $z < 0$ the induced space-time side is anti-de Sitter. At ($t = \infty, z = -\infty$,) there is a Cauchy horizon with the zero surface gravity ($K \equiv \frac{1}{2}\partial_z[\ln A(z)]_{z=-\infty} = 0$) and thus zero temperature ($T \equiv K/(2\pi) = 0$) [11]. Geodesic extensions of the space-time were given in Refs. [11] and [12]. The most symmetric geodesic extension comprises of a system of an infinite lattice of semi-infinite Minkowski space-times separated by an anti-de Sitter core. The Carter-Penrose diagram in the ($z,t$) direction is given in Figure 1a.

- $\alpha < 1$ corresponds to the walls where curvature blows up at $z = -\infty$. On the other hand, null geodesics reach $z = -\infty$ in infinite affine time $\tau \equiv \int^{-\infty} dz$, i.e., $z = -\infty$ corresponds to the event horizon. Therefore the planar singularity at $z = -\infty$ is null, i.e., it is covered by the event horizon. The associated temperature $T = 0$. The Carter-Penrose diagram in the ($z,t$) direction is given in Figure 2a.

- $\alpha = 1$ corresponds to the case of stringy dilatonic wall [13] of 4$d$ tree-level effective string action. The wall interpolates between the constant dilaton vacuum (Minkowski space-time) and the linear dilaton vacuum, i.e., $\phi = \sigma_{ext}|z|/2$. Note, that now the frame (8) with the flat metric is the sigma model frame (string frame) of the string theory, i.e., strings do not see the wall. At $z = -\infty$, where the curvature (and the dilaton) blow up, the metric has an event horizon as well [13]. Again, the singularity
is null, i.e., it is covered by the event horizon, and the global space-time is the same as the one for the walls with $\alpha < 1$ (see Figure 2a). However, the temperature associated with the horizon is now finite, i.e., $T = \sigma_{ext}/(4\pi)$.

- $\alpha > 1$ corresponds to the case where the metric becomes singular at a finite coordinate distance $|z|_{sing} = 2\sigma_{ext}/(\alpha - 1)$ from the wall. For $\alpha \neq 2$, the curvature blows-up at $|z|_{sing}$. For $\alpha = 2$, $R = 0$ but $R_{\mu\nu}R^{\mu\nu} = \infty$ at the singularity. Null geodesics reach $|z|_{sing}$ is a finite time i.e., the affine time $\tau$ is finite. Thus, the planar singularity is naked. The Carter-Penrose diagram in the $(z,t)$ plane for this type of dilatonic domain walls is given in Figure 2a. In addition, the surface gravity $K$ blows-up at this point, and thus $T = \infty$.

Extreme stringy dilatonic walls ($\alpha = 1, T = \sigma_{ext}/(4\pi)$) therefore serve as a dividing line between extreme walls ($\alpha < 1, T = 0$) with the (planar) singularity covered by the horizon and the extreme walls ($\alpha > 1, T = \infty$) with the naked (planar) singularity.

We would now like to compare the above solutions to the ones of the extreme magnetically charged black holes with a general dilaton coupling \[3,5\]. They correspond to the spherically symmetric solutions of the Lagrangian (3) with the matter fields $T$ turned off, i.e., $V \equiv 0$, however, with non-zero gauge fields $F_{\mu\nu} \neq 0$. The (Einstein frame) metric is of the form \[3,5\]:

$$ds^2 = \lambda(r)dt^2 - \lambda(r)^{-1}dr^2 - R(r)d\Omega^2, \quad (11)$$

with:

$$\lambda(r) = \left(1 - \frac{r_0}{r}\right)^{\frac{2\alpha}{1+\alpha}}, \quad R(r) = r^2 \left(1 - \frac{r_0}{r}\right)^{\frac{2}{1+\alpha}}. \quad (12)$$

The dilaton field $\phi$ and the magnetic field are of the form:

$$e^{2\phi/\sqrt{\alpha}} = \left(1 - \frac{r_0}{r}\right)^{-\frac{2}{1+\alpha}}, \quad F_{\theta\phi} = P \sin \theta. \quad (13)$$

Here $P$ is the magnetic charge of the black hole, $r_0^2 = P^2(\frac{1+\alpha}{\alpha})$ and the mass $M$ of the black hole is

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\[4\] Such a class of domain walls were first found \[14\] for discrete values of parameter $\alpha = (2, 3, ...)$.

\[5\] Note that in general, Lagrangian (3) describes the bosonic part of 4d $N = 1$ supergravity theory. Charged black holes with $\alpha = 1$ (arising from the string theory) and $\alpha = 1/3$ (arising from the 5d Kaluza-Klein theory \[3\]), however, correspond to solutions of $N = 4$ and $N = 8$ supergravity Lagrangians, respectively. The Bogomol’nyi bounds for charged dilatonic black holes were derived for $\alpha = 1$ and $\alpha = 1/3$ in Refs. \[3\] and \[15\], respectively.

\[6\] The corresponding electrically charged black holes have the same Einstein frame metric, however, the dilaton solution is related to the corresponding magnetic one by the transformation $\phi \to -\phi$. 

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Interestingly, the global space-time structure (and the related thermal properties) of the extreme magnetically charged dilatonic black holes bear striking similarities to the one of the corresponding domain wall configurations, however, now the role of \( \alpha \) is inverted:

- \( \alpha = \infty \) corresponds to the case, when the dilaton field does not couple to the gauge fields. The solution therefore corresponds to the extreme Reissner-Nordström black hole, which has the time-like singularity at \( r = 0 \) and \( r = r_0 \) corresponds to the Cauchy horizon. Its global space-time structure (see Figure 1b) in the \((r, t)\) direction is the same as the one of the Type I supergravity walls in the \((z, t)\) direction. In the latter case, however, the time-like singularity is replaced by the wall. The corresponding temperature of the black hole \( T = 0 \).

- \( \infty > \alpha > 1 \) corresponds to solutions with the curvature singularity at \( r = r_0 \). Null radial geodesics reach \( r = r_0 \) in infinite time, i.e., affine time \( \tau \equiv \int_{r_0}^{r} dr/\lambda = \infty \). Therefore \( r = r_0 \) corresponds to the null singularity (see Figure 1b). The corresponding temperature \( T = 0 \).

- \( \alpha = 1 \) corresponds to the stringy extreme magnetically charged black hole with the null singularity at \( r = r_0 \) (see Figure 2b), however, the temperature \( T = M/(8\pi) \) is finite.

- \( \alpha < 1 \), corresponds to solutions, where the singularity at \( r = r_0 \) is reached by a null geodesics in a finite (affine) time. Thus, the singularity is naked (see Figure 3b) and the temperature \( T \) is infinite.

Thus, extreme magnetically charged stringy dilatonic black holes \((\alpha = 1, T = M/(8\pi))\) serve as a dividing line \([4]\) between extreme charged dilatonic black holes \((\alpha > 1, T = 0)\) with the singularity covered by the horizon and those \((\alpha < 1, T = \infty)\) with the naked singularity.

We would like to emphasize the complementarity between the extreme dilatonic domain walls and extreme magnetically charged dilatonic black holes. The global space-time in the \((t, z)\) slice for extreme walls with coupling \( \alpha \) is the same as the one in the \((t, r)\) slice for extreme magnetically charged black holes with coupling \( 1/\alpha \) (See Figures 1-3). Between the two solutions the role of \( \alpha \) is inverted, while the role of \( W_M(T) \) on one side of the wall and the magnetic charge \( P \) of the black hole are interchanged. The origin of such a complementarity can be traced to the form of the Lagrangian (3), where in the case of the walls the potential (the source for the wall) is modulated by the dilaton coupling of the type \( e^{2\sqrt{\alpha} \phi} \), while in the case of the black hole the kinetic energy of the gauge field (the source for the magnetically charged configuration) is modulated by a complementary dilaton coupling of the type \( e^{-2\phi/\sqrt{\alpha}} \).

One would further expect that a two \((t, z)\) dimensional effective action for the wall with the metric Ansatz (5), and a two \((t, r)\) dimensional effective action for the black hole with the metric Ansatz (10), bear similarities\([5]\). It, however, turns out that such a

\[ M^2 = P^2 \left( \frac{1 + \alpha}{\alpha} \right) \]  

\(^7\) I would like to thank D. Youm for collaboration on this point.
similarity between the two actions is not transparent. One can, however, show that near the singularity, the metric \( (t, z) \) of the wall with the coupling \( \alpha \) is the same as the metric \( (t, r) \) of the black hole with the coupling \( 1/\alpha \). Namely, in the region \( r - r_0 \equiv \rho \to 0^+ \), the the coordinates \((t, \rho)\) of the black hole with the coupling \( \alpha \) and the coordinates \((t, z)\) of the wall with the coupling \( \tilde{\alpha} \equiv 1/\alpha \) are related in the following way:

\[
[1 - \frac{1}{2}(\tilde{\alpha} - 1)\sigma_{\text{ext}}|z|]^{\frac{2}{2(\tilde{\alpha} - 1)}} = \left(\frac{\rho}{r_0}\right)^{2\alpha - 1}, \quad \alpha \neq 1
\]
\[
e^{-\sigma_{\text{ext}}|z|} = \frac{\rho}{r_0}, \quad \alpha = 1
\]

where \( r_0 = \frac{2}{(1 + \tilde{\alpha})\sigma_{\text{ext}}} \) and \( \tilde{\alpha} \equiv 1/\alpha \).

Near the singularity the dilaton blows up in both cases, however, unlike the corresponding two-dimensional metric slices, the coordinate dependence of the dilaton near the singularity is different in either case. This fact is also reflected in the different form of the corresponding two-dimensional effective actions.

We have studied extreme domain walls in \( N = 1 \) supergravity with a general dilaton coupling \( \alpha \). We found that such configurations are static, planar configurations interpolating between isolated supersymmetric vacua with the varying dilaton field. Type I walls interpolate between the Minkowski vacuum and the supersymmetric vacuum with the varying dilaton. For \( \alpha > 1 \) the walls have a (planar) naked singularity on one side of the wall, while for \( \alpha \leq 1 \), the singularity is covered by the horizon. The extreme magnetically charged black holes with the coupling \( \alpha \) have the same global space-time structure as the Type I wall with the coupling \( 1/\alpha \). Interestingly, only in the case with \( \alpha = 1 \), which corresponds to the tree-level low-energy theory of \( \mathcal{N} = 1 \) 4d superstring vacua, the above types of configurations do not have naked singularities.

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Figure Captions

Figure 1: In Figure 1a the Penrose-Carter diagram in the \((z, t)\) direction for the extreme Type I ordinary domain wall \((\alpha = 0)\) is presented. It corresponds to the most symmetric geodesic extension and comprises of a system of an infinite lattice of semi-infinite Minkowski space-times \((M)\) separated by an anti-de Sitter core. The compact null coordinates define the axes: \(u, v = 2 \tan^{-1}(t \mp z)\). The domain wall region is denoted with the thin lines. Cauchy horizons (dashed lines) are the nulls separating the anti-de Sitter patches. Figure 1b represents the Penrose-Carter diagram for the extreme magnetically charged Reissner-Nordstrom black hole \((\alpha = \infty)\) in the \((r, t)\) direction. The jagged line represents the time-like singularity and the dashed lines are the corresponding Cauchy horizons. Note a formal similarity between Figures 1a and 1b.

Figure 2: In Figure 2a the Penrose-Carter diagram in the \((z, t)\) plane for extreme Type I dilatonic domain wall with \(0 < \alpha \leq 1\) is presented. The compactified null coordinates are \(u, v = 2 \tan^{-1}(t \mp z)\). The wall (denoted by a thin line) separates the semi-infinite Minkowski space-time \((M)\) and the space-time with a varying dilaton field and the null singularity covered by the horizon (jagged line). Figure 1b corresponds to the Penrose-Carter diagram in the \((r, t)\) plane for the extreme magnetically charged black hole \(\alpha > 1\). The jagged line corresponds to the null singularity covered by the horizon.

Figure 3: In Figure 3a the Penrose-Carter diagram in the \((z, t)\) plane for extreme Type I dilatonic domain wall with \(\alpha > 1\) corresponds to the the wall (denoted by a thin line) separating the semi-infinite Minkowski space-time \((M)\) and the space-time with a varying dilaton field and the naked (planar) singularity (jagged line). The compactified null coordinates are \(u, v = 2 \tan^{-1}[t \mp (z + |z|_{\text{sing}})]\). Figure 2b represents the Penrose-Carter diagram for the extreme magnetically charged dilatonic black hole \((\alpha < 1)\) in the \((r, t)\) plane. The singularity (jagged line) is naked.
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