Hybrid Numerical Methodology for Efficient Design and Optimization of Transmit-Array Antennas, X-Band Application

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ABSTRACT This paper presents a complete hybrid numerical methodology for the efficient design and optimization of large-scale Transmit-Array (TA) antennas for modern telecommunication applications. There are four main components to the proposed work and methodology: 1) the implementation through Python scripts of a hybrid scheme based on the Friis analytical formula for linking gain and phase of the primary source and elementary cells of the studied transmit-array; 2) the implementation of a Particle Swarm Optimizer (PSO) for efficient characterization of the optimal phase distribution on the in-plane lens maximizing the gain of the antenna and minimizing the side-lobe levels for multiple fed TA antennas; 3) the implementation of a full-wave Finite element and Interconnecting domain decomposition (FETI) for the final analysis of the TA radiating performance; 4) the design, optimization, fabrication and proof of concept of an X band transmit-array including the focal source. This work presents the main functionalities of the hybrid Python/CST tool associated with phase compensation PSO, FETI implementation for transmit-arrays and as an application of this numerical strategy, a new compact unit-cell operating in the X-band (thickness of 3.2 mm) able to easily generate Phase Rotations (PR) necessary for TAs with phase compensation on the aperture. The proposed unit-cell is a completely symmetric design including a metallic via interconnecting two identical square patches (including a circular hole in the center and a microstrip line) by crossing through a ground plane. A particle swarm optimization (PSO) routine is proposed as a way to quickly optimize the phase distribution of the transmit-array unit-cells. The optimization routine is tested through multiple sources and focal ratios, demonstrating a reduction of over 50% of the volume occupied by the antenna, while keeping a high gain (19.5 dBi) and overall good performance.

INDEX TERMS Transmit-array, X-band, linear polarization, phase compensation, phase rotation, finite element methods, finite element tearing and interconnecting, FETI, domain decomposition methods, FACTOPO, CST Microwave Studio.

I. INTRODUCTION

Transmit-array (TA) is a popular cost-effective solution for high-gain antennas at millimeter wavelengths, particularly for telecommunications or space applications. A transmit-array consists in the illumination of radiating elements distributed over two arrays and fed by a focal source (Figure 1) as proposed for example in the X-band [1], [2].
the Move ground terminals in the Ka-band [5], [6]. Recently, several configurations of transmit-arrays based on metasurfaces have been designed [7], [8], [9], [10]. These research works have demonstrated that metasurfaces can be used to significantly reduce the size of TAs and to allow dual-band operation and multi-functions. As shown in Figure 1, the focal source is illuminating the TA with a spherical wave front and a plane wave front is created on the other side in the $\theta_0$ direction.

The first contribution of the paper is a hybrid numerical methodology motivated by the fact that TAs are difficult to integrate in the final systems due to a focal ratio ($F/D$ see Figure 1) that is normally in the range of 0.5 to 1 [1]. A high ratio is required for optimal performance, but drastically increases the volume occupied by the antenna. Recent realizations have been able to reduce this focal ratio in the Ka band by applying an intermediate flat plate polarizer lens antenna [5], which acts with a patch antenna to illuminate the TA with a virtual focus further away from the TA. The work done here was able to attain a focal ratio of 0.55, which is a good improvement but requires the use of a second lens between the patch source and the main lens, thus increasing costs and the complexity of the design. Other works have tried to counter this problem by employing multiple feeds closer to the TA, which act as one equivalent source with a longer virtual focus [11]. Though this work managed to effectively reduce the focal length by a factor of 50%, the use of four horn antennas as a feed would create difficulties in realizing a feeding network for them, as well as the non-negligible weight and volume of these antennas. This work also fails to detail how the phase distribution is optimized for the multiple feed case. The electromagnetic simulation of such large TA antennas is a challenging problem to be solved through Full Wave simulations, since the problem is not periodic and there are no symmetry planes to reduce the size of the problem. In addition, the small size of the unit-cells compared to the wavelength of the incident wave requires a highly detailed mesh for good accuracy. Although it is possible to obtain the antenna performance characteristics through commercially available software such as ANSYS-HFSS [12] and CST Microwave Studio [13], the memory resources and especially the time required for the simulation are highly prohibitive to the design phase of these antennas. For instance in [14], using FEKO’s Method of Moments (MoM), a full wave simulation of a $30\times30$ reflect-array (analogous problem to a TA) in the Ka band takes about 27 hours of CPU time and 29.56 GB of memory. The work presented in this paper aims to propose a design and simulation methodology for large TA antennas that greatly reduces the full wave simulation efforts, which are however essential for the final analysis [15], thus reducing the time and memory consumption in the design phase of these antennas. Furthermore, this work also proposes the use of optimization techniques to achieve very low profiles for the final antenna, reducing the total occupied volume and allowing space constrained applications. A particle swarm optimization (PSO) routine is then proposed as a way to quickly optimize the phase distribution of the transmit-array unit-cells. For a multiple feed configuration, the simple algorithm allows the control of Side Lobe Levels (SLL) and axial gain.

The second contribution of the paper presents, as an application of the hybrid methodology, a novel X-band unit-cell architecture that improves the bandwidth while reducing the footprint. One particularity of the new unit-cell is that it transforms a vertical polarization into a horizontal polarization. It therefore behaves like a polarizer. The design offers a simple and good optimization of the impedance matching level based on its line theory model (introduced in the section V-A). Another particularity of this unit-cell is a lower profile ($0.12 \lambda$) compared to thick multi-layer structures such as Frequency Selective Surfaces (FSS). Compared to the state of the art, the new proposed unit-cell design (Figure 2) is perfectly symmetric [1] allowing i) reduction of the profile of the cell with a neglected mutual coupling between the adjacent cells and ii) easy phase shifting control [2] over a 360°-range by simply rotating the T-line (see Figure 2). It also enables the bandwidth to be increased by combining two coupling techniques between the Rx and Tx patch antennas, i.e. the coupling i) through the metallic via and directly ii) through the hole (slot) in the ground plane. One of the originalities of the proposed design is the internal feeding of the patch antenna compared to a standard patch excitation technique and the good mastery and comprehension of the impedance matching as explained in section V. The different impedances can be given by the EM simulation softwares ($Z_{ant}$,$Z_{line}$ and $Z_{via}$). This unit-cell can be modelled as illustrated in Figure 2. The PSO optimization routine is tested through multiple sources and focal ratios, demonstrating a reduction of over 50% of the volume occupied by the antenna, while keeping a high gain ($\geq 19.5$) dBi and overall good performance.

The first part of this paper will describe the hybrid analytical tool and the PSO optimizer for rapid design and optimization of TAs including the search for the optimal source. In the second part, we will discuss the FETI-2LM full-wave tool used for the analysis of the final design of the antenna. Next, we will describe in the third part the design of a new unit-cell operating in linear polarization and the
simulated and measured performance of a 10λx10λ TA made up of 400 unit-cells populating the aperture through a 1-bit phase compensation and fed by an ATM 90-441-6(13971-D) horn source. Finally, the simulated performance of the TA fed by a microstrip patch array for demonstrating the reduction of over 50% of the volume occupied by the antenna will be described.

II. HYBRID ANALYTICAL/FULL-WAVE TOOL

In order to allow fast simulation of TA antennas, thus making parametric studies possible, a hybrid tool was developed to perform accurate simulations in under 0.5 seconds. This hybrid tool was proposed at first by [1] and developed as a MATLAB routine, and in our work is retrofitted with an object-oriented strategy coded in Python to efficiently design transmit-arrays and obtain their performance during parametric studies. The methodology used in this tool is based on the Friis formula for linking gain and phase of the primary source and the elementary cells characterized initially with full wave simulation tools.

In the simulations, the feed is considered to be well suited at the frequency of interest, so no mismatch is considered at this point. The power injected (and therefore radiated) by the feed is considered as $P_1$. Considering the free space propagation, the available power can be calculated at each of the cells using the Friis equation, Figure 3 summarizes the power budget. The phase center of the feed is considered to be at a point $(x_s, y_s, -f)$ and each cell has coordinates $(x_m, y_m, 0)$, thus the power link between these two objects can be calculated as:

$$ P_3^m = \left( \frac{\lambda}{4\pi f_{rm}} \right)^2 G_{SP}(\theta_{SP}^m, \phi_{SP}^m) G_{AE}(\theta_{AE}^m, \phi_{AE}^m) P_1 $$

(1)

Where $P_3^m$ is the available power on each of the m-unit cells, $G_{SP}$ and $G_{Rx}$ are, respectively, the source and unit cells Rx Gain, $r_m$ the distance between the feed and the m-th cell and the couples $(\theta_{SP}^m, \phi_{SP}^m), (\theta_{AE}^m, \phi_{AE}^m)$ are the incidence angles described in Figure 3. As the position of both feed and unit cells are known, the incidence angle at each lens can be calculated easily.

As the incident power $P_3^m$ is known through the Friis equation, the incident wave $a_1^m$ can be easily calculated as:

$$ a_1^m = \sqrt{P_3^m} e^{-j2\pi f_{rm} x} e^{j\phi_{SP}^m} $$

(4)

Where $\phi_{SP}^m$ is the phase of the incident field from the feed at the m-th cell. The total incident power is therefore calculated as:

$$ P_3 = \sum |a_1|^2 $$

(5)

The computation of the transmitted waves is immediate through the use of the S matrix. Here, the incident wave $a_2$
is taken as zero, since the TA is treated in the transmission mode. Thus:

\[ b^m_2 = |S_{21}(\omega)|\alpha^m_1 e^{j\phi^m_{\text{trans}}} \quad (6) \]

Where \( \phi^m_{\text{trans}} \) is the transmission phase of the \( S_{21} \) parameter at the \( m \)-th cell, calculated through equation 2. By neglecting the coupling between adjacent cells, the total power radiated by the Tx array is therefore:

\[ P_A = \sum |b^m_2|^2 \quad (7) \]

Since each transmitted wave is known, the total electric field radiated in the direction \((\theta, \phi)\) can be obtained as the superposition of the fields radiated by each unit-cell at the Rx side. The array theory enables calculation of the power density of the total radiated field at an observation point \((r, \theta, \phi)\) as:

\[ |AF| = |\sum b^m_2 e^{j2\pi(\cos(\phi - \phi^m_{\text{trans}})/\lambda)}| \]

\[ S(\theta, \phi) = \frac{D^m_{\text{trans}}(\theta, \phi)}{4\pi r^2} |AF(\theta, \phi)| \quad (9) \]

Where \( D^m_{\text{trans}} \) is the unit-cell directivity at the free space side (Rx side). The directivity of the TA can be obtained as the ratio of its power density and the total isotropic radiated power:

\[ D(\theta, \phi) = \frac{S(\theta, \phi)}{S(\theta, \phi)} = \frac{4\pi r^2}{P_A} S(\theta, \phi) \quad (10) \]

The TA gain is calculated in a similar way, but instead of the total radiated power, the total input power is used:

\[ G(\theta, \phi) = \frac{4\pi r^2}{P_1} S(\theta, \phi) = \frac{P_2}{P_1} D(\theta, \phi) = \eta_{\text{11}} D(\theta, \phi) \quad (11) \]

The hybrid tool makes it possible to carry out parametric studies very quickly by varying parameters such as the focal length and thus, to choose optimal values in the predimensioning phases. Therefore, a parametric study can be performed to determine the optimal focal length, taking into account several performance indicators, in particular the axis gain (Figure 11), the Side Lobe Level (SLL), the power efficiency and the aperture efficiency as shown in Table 2 for several focal ratios F/D.

### III. PSO OPTIMIZER

Considering for example the \( 20 \times 20 \) transmit-array analyzed in section V, the optimization should find a 400 dimensional vector that satisfies the constraints of gain and SLL. This high dimensional problem would require a high computational effort to find the global minimum, which makes the use of gradient associated methods difficult due to the need for estimating a large gradient vector or an approximation to the Hessian matrix. Also, the search space of the multiple feed TA might not be differentiable, thus prohibiting the use of these methods.

A way around this high dimensional problem is to consider only solutions in which the phase gradient is symmetric around the \( x \) and \( y \) axis. In doing so, the 400 dimensional problem becomes a 100 dimensional one. Even though this method reduces the computational efforts to find a solution, the gradient associated methods still took too much time to converge (if ever). An optimization technique that has been used in solving many EM problems, such as phased array synthesis [17]-[18], is the particle swarm optimization (PSO).

PSO is a stochastic population based optimization algorithm in which "particles", or candidate solution vectors, move around the optimization hyperspace to find the global maximum. A detailed explanation of the algorithm can be found in [19]. The principle of the algorithm is quite simple, given a starting point of a certain number of particles, the population of the "swarm", is initialized by randomly placing them around this initial guess. Each position is then scored by a cost function giving the "fitness" of these points. Each particle knows its personal best position \((p_{\text{best}})\) and the global best position ever passed by any of the particles \((g_{\text{best}})\). These values are updated after each iteration. To update the position of each particle, its velocity is defined as:

\[ v = \omega v_{n-1} + c_1 r_{\text{and}}(x - p_{\text{best}}) + c_2 r_{\text{and}}(x - g_{\text{best}}) \quad (12) \]

Where \( \omega, c_1, c_2 \) are coefficients controlling the impact of each term and \( r_{\text{and}}() \) is a random number between 0 and 1. Equation 12 establishes that the each particle’s velocity is composed of three components. \( \omega \) is called the inertia weight which keeps the particle "traveling" in the same direction it was going at the last iteration. \( c_1 \) is the personal knowledge, it attracts the particle towards its best personal position. Finally, \( c_2 \) is the social knowledge weight that attracts the particle towards the global best found by the swarm. After a given number of iterations, the particles will have globally searched the optimization hyperspace and they may converge on a maximum point.

The simplicity of this method allows for easy adaptation of the routine, therefore making it capable of being applied to multiple transmit-array configurations with very little effort. Furthermore, it does not require differentiability (nor the approximation of derivatives) and it has the natural capacity to avoid local minima by its random movements. Inspired by other array synthesis problems ([20], [18] and [17]), an optimization module was developed. The goal was to maximize the axis gain, and keep the SLL under a determined threshold level \( SLL_{\text{min}} \), thus the cost function was defined as:

\[ f(x) = -\text{Gain}(\theta = 0^\circ) + \Pi(SLL) \quad (13) \]
lems with billions of unknowns. They are of course very especially, the FEM method requires the solution of prob-
(FEM). For computationally challenging TA applications circuit engineering applications is the Finite Element Method electromagnetic simulations [22] of antennas or microwave successful frequency domain computational methods for formed with exact full wave methods. One of the most The final analysis of TA radiating performance can be per-
be optimized to maintain the gain, while reducing the SLL. is too elevated. The 1-bit phase distribution on the array will (left image in Figure 5), presents a high gain, but a SLL that be neglected and equation 2 is used. This new distribution (2). They comprise metallized layers of concentric square will be designed to provide a phase correction Φx,y each sub-domain. Each unit-cell at position (x,y), is a 100 dimensional vector that is mapped as a phase distribution on the array. For each position of the optimization hyperspace, the hybrid tool is used to obtain the performance of the TA. Based on the cost function, each position is then scored and used to update the particles’ velocities and position. To ensure better performance, the constriction factor proposed by [21] was applied, along with velocity clamping. Upper and lower bounds for the positions were also used to ensure consistency with a 1-bit quantification.

A 50-particles population was considered, with a total of 10,000 iterations. The algorithm presented an overall good speed of execution, being able to execute a full iteration, that is 50 executions, in under 9 seconds. For the final results, a similar penalty regarding a minimum acceptable gain was also included in the cost function to avoid gain reduction. The gamma parameter impacts the overall evolution of the optimization in the sense that it may lead to a solution that privileges the constraint function over the gain optimization. The best compromise found was to set γSLL = γgainmin = 3.

Rather than starting the optimization of the phase dist-
ribution from a random initial point, the first approach is to divide the transmit-array into 4 “independent” parts. The phase distribution is calculated for a single source feeding each quadrant. The influence of the other feeds will, at first, be neglected and equation 2 is used. This new distribution (left image in Figure 5), presents a high gain, but a SLL that is too elevated. The 1-bit phase distribution on the array will be optimized to maintain the gain, while reducing the SLL.

IV. FETI-2LM FINAL ANALYSIS
The final analysis of TA radiating performance can be performed with exact full wave methods. One of the most successful frequency domain computational methods for electromagnetic simulations [22] of antennas or microwave circuit engineering applications is the Finite Element Method (FEM). For computationally challenging TA applications especially, the FEM method requires the solution of problems with billions of unknowns. They are of course very difficult to solve with a single computational domain using commercial software such as CST Microwave Studio [13] or HFSS [12], especially when the problems of interest involve multi-scale geometries, with very small details compared to the wavelength. This is the case for X band or Ka-band transmit-array applications. The FETI-2LM method used to solve the Maxwell equations ([23], [24], [25]), splits the initial TA computational domain into non-overlapping sub-domains. The Finite Element solution of the local problems was first obtained with direct solvers [26] such as PARDISO. As proposed in a previous work [23], two Lagrange Multipliers are introduced to impose tangent field continuity at the interfaces of the sub-domains, and the reduced problem at the interfaces is solved with an iterative ORTHODIR method [27]. The realization of the complete meshing of the TA will require a considerable effort and would require considerable memory resources. This mesh is therefore not built into our simulation methodology.

Even with an automatic mesh splitter such as METIS [28], the TA being non-periodic, a single domain pre-treatment step would be long and difficult to implement. Our alternative strategy makes it possible to virtually build the mesh of the complete TA with only 3 types of sub-domain meshes (“lens unit cell”, “empty unit-cell” and “horn” described in Figure 6). As a consequence, our implementation of the FETI-2LM method only requires that the mesh has a reasonable size for each sub-domain. Each unit-cell at position (x,y) of the TA will be designed to provide a phase correction Φlens(x,y) (2). They comprise metallized layers of concentric square patches printed on substrate layers as described in Figure ?? . For example, considering a 1-bit phase compensation application, the in-plane TA is designed with a set of Nelem=2 elementary cells populating the TA.

In summary, the simulation strategy used is as follows:

- Step 1: Design and optimization of the Nelem=2 unit-cell geometry with CST Microwave Studio [13]

FIGURE 5: 1 bit phase gradient of the TA, initial guess (left) and final optimization result (right).

\[
\Pi(SLL) = \begin{cases} 
0 & \text{if } SLL \leq SLL_{\text{min}} \\
\gamma |SLL - SLL_{\text{min}}| & \text{else}
\end{cases}
\]

Where \(\Pi(SLL)\) is a penalty function only applied if the SLL is over the threshold level, with the penalty able to be controlled through the \(\gamma\) parameter. Each particle corresponds to a 100 dimensional vector that is mapped as a phase distribution on the array. For each position of the optimization hyperspace, the hybrid tool is used to obtain the performance of the TA. Based on the cost function, each position is then scored and used to update the particles’ velocities and position. To ensure better performance, the constriction factor proposed by [21] was applied, along with velocity clamping. Upper and lower bounds for the positions were also used to ensure consistency with a 1-bit quantification.

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FIGURE 6: Exploded image of the TA domain decompo-
sition (from bottom to top: horn (cyan), empty (purple and green), TA (red)
Step 2: Automatic mesh generation of the \( N_{\text{elem}}=2 \) elementary unit-cells with the GID pre-processor [29].

Step 3: Assignment of the \( N_{\text{elem}}=2 \) elementary mesh cells populating the lens to the sub-domains and calculation cores.

Step 4: FETI-2LM simulations using massively parallel clusters.

Step 5: Extraction of the radiation patterns of the TA.

V. APPLICATION TO THE DESIGN OF A LINEAR POLARIZATION TRANSMIT-ARRAY IN THE X BAND

For illustrating the efficiency of the hybrid numerical strategy proposed, we discuss the design and the optimization of a new low-profile X band unit-cell and the reduction of the volume occupied by the antenna (F/D) with a patch array as a focal source instead of an initial horn source.

A. PHASE ROTATION UNIT-CELL (DESIGN, MANUFACTURE AND MEASUREMENT)

The main idea of the new design consists in minimizing as much as possible the size of the unit-cell with the aim of increasing the number of cells populating a transmit-array and thus allowing its aperture efficiency to be increased. To achieve this, two patches and a metallic via have been placed in the center of the unit-cell (Figure 2). With this configuration, the patch antennas cannot be coupled because their impedances in the center are zero \((S_{11} = -1)\). To modify these impedances, the antennas have been loaded by a circular slot of radius \( R_{\text{cyl}} \) in the center of the unit-cell, and a microstrip line of width \( W_{\text{line}} \) (of impedance \( Z_{\text{line}} \)) has been used to connect the metallic via (of impedance \( Z_{\text{via}} \)) to the patch antenna (of impedance \( Z_{\text{ant}} \)). This new configuration (antenna with internal excitation) enables us to introduce a phase delay by rotating the transmission line (keeping the same \( Z_{\text{ant}} \)) and achievement of a perfect phase compensation (all the required phases on the array). The design challenge consists in matching the different impedances. The transmission line theory can be used (formula, lossless case):

\[
Z_{\text{via}} = Z_{\text{line}} \frac{Z_{\text{ant}} + j\tan\left(\frac{2\pi l}{\lambda_g}\right)Z_{\text{line}}}{Z_{\text{line}} + j\tan\left(\frac{2\pi l}{\lambda_g}\right)Z_{\text{ant}}} \tag{15}
\]

where \( l = R_{\text{cyl}} \) and \( \lambda_g \) is the guided wavelength. A simple stub of length \( L_{\text{step}} \) was added as an additional design parameter for optimization, see Figure 7. The proposed unit-cell that is intended to be used for the manufacture of a 1-bit transmit-array is presented in Figure 7 and Table 1. It contains 3 metallic parts and operates at 10 GHz in linear polarization. The in-plane dimensions of the cell are 15 mm x 15 mm \((\lambda/2 \times \lambda/2)\) and its thickness is 3.2 mm. The unit-cell is made up of two patches connected by a metallic via. Between these two patches is inserted an 18 \( \mu \)m thick copper ground plane surrounded by 2 substrates (Rogers RO4003 1.524 mm thick with \( \varepsilon = 3.55 \) relative permittivity and a dissipation factor tan \( \delta \) of 0.0027). The two substrates are glued with a RO4450F pre-preg of thickness 100 \( \mu \)m.

By rotating the upper patch, the unit-cell must allow Phase Rotations (PR) through phase-shifts on the transmission coefficient S21.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{figure7}
\caption{Unit-cell in sectional view}
\end{figure}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
L_{\text{patch}} & 6.5 mm \\
W_{\text{patch}} & 6.45 mm \\
R_{\text{cyl}} & 1.65 mm \\
W_{\text{line}} & 1.4 mm \\
R_{\text{via}} & 0.5 mm \\
L_{\text{step}} & 0.3 mm \\
L_{\text{cell}}=W_{\text{cell}} & 15 mm \\
\hline
\end{tabular}
\caption{Unit-cell dimensions}
\end{table}

Based on these considerations, the design optimization of this unit-cell was carried out using the commercial electromagnetic CST simulation software [13]. Periodic array boundary conditions are imposed during these simulations permitting the mutual coupling between adjacent cells to be taken into account. The frequency response of the optimized cell is shown in Figure 8 (red curve) and is characterized by a bandwidth of nearly 900 MHz around 10 GHz which corresponds to a bandwidth of almost 9% and an insertion loss limited to -0.3 dB (green curve).

The manufactured unit-cell measurements (Figure 9) were carried out using a WR-90 waveguide with suitable adaptor and a Wave Guide Measurement System (WGMS). The S11 measurements (Figure 8 dashed red curve) clearly show a -10 dB bandwidth of 870 MHz centered at 10 GHz which is reasonably similar to the simulated values.

Otherwise, the unit-cell has a simulated gain of 4.8 dBi which makes it a good candidate for transmit-array design. For this use, the cell is able to produce, by rotations of the upper patch, accurate phase-shifts on the lens transmit-array.
FIGURE 8: Simulated and measured unit-cell S-parameters

in-plane surface. As specified for example in [30], [31], the phase shift must be constant in frequency, in this case for two rotations (90° and 270°), of 180° over the entire bandwidth. Our unit-cell permits rotation of the upper patch through +/- 90° allowing a 180° phase shift. Figure 9 and Figure 10 show the two cells thus designed and the validation by the simulation of the phase shift of 180° on the useful frequency band (90° in red and 270° in blue).

FIGURE 9: Top and bottom manufactured U slot patches: 90° (first row) and 270° (second row) unit-cells

B. INITIAL TRANSMIT-ARRAY IMPLEMENTATION WITH SINGLE FEED CONFIGURATION

A 20x20 transmit-array (TA) operating in the X-band and centered at 10 GHz is now considered. It consists of 400 unit-cells of dimensions 15 mm x 15 mm populated with the two unit-cells previously described in section V. The in-plane dimension of the array is then 300 mm x 300 mm and the primary source chosen is a rectangular 15 dB ATM 90-441-6(13971-D) horn operating between 8.2 GHz and 12.4 GHz.

The design of the TA is optimized in terms of focal length (F) with the hybrid analytical/exact tool presented in section II, to efficiently design transmit-arrays and obtain their performance.

As presented previously in section II, it is first necessary, using commercial software such as CST Microwave Studio [13], to obtain the gain and phase diagrams of the primary source (making it possible to determine the distribution of the electric field illuminating the array on the primary source side) as well as the unit-cell (allowing evaluation of the radiation performance of the transmit-array). The principle of a transmit-array is to associate with each unit-cell a phase gradient which then allows, once the array has been configured, a zenithal beam angle $\theta_0$ to be obtained when the feed is at the central position (x=0, y=0). Each unit-cell of the TA has been designed to provide a phase correction $\Phi_{\text{Lens}}(x,y)$ at position $(x,y)$ and defined by equation (2). As a result and for the central position of the horn source, the phase at the output of the transmit-array is given by equation (3). However, the previous expression is a perfect compensation law and consequently there will be as many gradients as there are unit-cells, which greatly complicates the manufacturing. Thus, it is possible to consider a non-perfect phase quantization, with 2, 4 or 8 phase states. 1-bit phase compensation consists of two phase states: 90° and 270°, 2-bit compensation consists of four phase states, and so on. The greater the number of phase states, the greater the complexity of the array, the greater the precision of the phase gradient and therefore the precise angle $\theta_0$ of beamforming.

In our work, 1-bit phase compensation was chosen in order to simplify the design and manufacture of the transmit-array. The hybrid tool makes it possible to carry out parametric studies very quickly by varying parameters such as the focal length and thus, select optimal values in the design. Thus, a parametric study was performed to determine the optimal focal length. For this, several parameters have been taken into account, in particular the axis gain (Figure 11), the Side Lobe Level (SLL), the power efficiency and the aperture efficiency.

FIGURE 10: Phase-shift between the two cells
as shown in Table 2 for several focal ratio F/D.

![Figure 11: Gain as a function of F/D](image)

**TABLE 2: TA performance versus focal ratio F/D**

| Focal Ratio | 0.5 | 0.75 | 1.33 |
|-------------|-----|------|------|
| Gain max (θ = 0°) | 18.96 | 21.88 | 23.07 |
| Directivity max (θ = 0°) | 20.69 | 23.63 | 25.41 |
| SLL (dB) | -15.51 | -20.93 | -19.41 |
| Power Efficiency (%) | 66.56 | 66.76 | 58.3 |
| Aperture Efficiency (%) | 9.31 | 18.32 | 25.11 |

![Figure 12: 1-bit compensation cells distribution of the 400 unit-cells TA prototype (PR=90° in blue and PR=270° in red)](image)

As a result of various parametric investigations carried out with the hybrid tool, the optimal focal ratio resulting in higher gains for this TA is 0.75 corresponding to a distance of 225 mm separating the source and the array. After studying the manufacturing complexity with several compensations (1-bit, 2-bit, 3-bit and perfect compensation), the model chosen for this TA is the 1-bit compensation (Figure 12), providing a gain difference of 3.4 dB compared to the perfect compensation and requiring only two different cells for populating the lens array. The maximum axial gain calculated by the hybrid tool for a 1-bit compensation model is 21.88 dBi. In order to confirm the efficiency of the TA before manufacture, full wave simulations are carried out with the Finite Element Tearing and Interconnecting (FETI) method in the frequency domain developed in [32] and briefly presented in section IV [15]. During the FETI simulation, the 20x20 physical array is completed by 2 empty elementary cells of the same size all around the array in order to remove the absorbing boundary surface far away from the physical array. The total number of sub-domains is then equal to 1153 (24x24 lens array) + 24x24 (empty array to adjust the focal distance) + 1 (horn).

![Figure 13: Measurements of the X-band TA prototype](image)

During the FETI simulation, the domain decomposition solver takes into account the position of each cell on the array and its orientation, given that we have considered a 1-bit compensation. Figure 12 shows the two types of cells considered, PR=90° in blue and PR=270° in red. Figure 14 shows the simulated E plane gain diagrams obtained with the Python hybrid tool and the FACTOPO full wave domain decomposition tool.

![Figure 14: Measured and simulated gain pattern of the transmit-array for F/D=0.75 (225 mm), H-plane (φ = 90°)](image)
FIGURE 15: Measured and simulated (full-wave) cross-polarization gains

FIGURE 16: Full wave and hybrid simulated gain pattern of the transmit-array for F/D=0.75 (225 mm), E-plane ($\phi=0^\circ$)

FIGURE 17: Measured and simulated (full-wave) gain versus frequency for F/D=0.75 (225 mm)

shows that the full-wave axis gain (20.81 dBi on the red curve) is 1.07 dB less than the hybrid tool simulated gain (21.88 dBi on the red curve). This difference may be due to the fact that the hybrid-tool simulation does not take into account all electromagnetic phenomena such as diffractions by the edges of the array as well as the coupling effects between the source and the array. Both simulations are close to the measured gain pattern (21.01 dBi on the black curve) which is affected by a 1.5° shift, no doubt due to focal source misalignment. Additionally, both measurements and simulations showed a high cross-polarization (around 10 dBi) (Figure 15). Indeed, the large slot defined on the ground plane of the unit-cell design enables a non negligible amount of direct coupling between the incident wave and the upper patch. In Figure 17, showing the evolution of the simulated and measured axial gain as a function of the frequency, we observe that the gain is greater than 20 dBi in the frequency band [9.5 - 10.4 GHz] which is consistent with the 900 MHz bandwidth observed on the $S_{11}$ measurements of the unit-cell (Figure 8). The experimental set-up allows for lens in-plane translation in front of the stationary primary feed. The evolution of the measured realized gain versus the beam tilt angle is presented in Figure 18. Measurements show that a beam scanning from -22.5° to +25° is achieved with 3 dB scan loss.

Table 3 shows the elapsed time of the different phases of the simulation methodology. The simulation time using CST, Hybrid tool and PSO routine are obtained on a standard PC with 1 Intel(R) Xeon(R) CPU E5-1603 @2.8GHz processor. The FACTOPO simulation used 1153 Intel Xeon 2680v4 core processors. The simulation times demonstrate clearly the efficiency of the hybrid code for fast design of transmit-arrays.

C. TRANSMIT-ARRAY IMPLEMENTATION WITH MULTIPLE FEED CONFIGURATION

In order to achieve high gain with reduced focal ratios, a multiple feed TA is presented and designed with four sources
placed at the center of each quadrant, as Figure 19 shows. The source separation is chosen to be \(0.5D\), since it has been shown that the maximal gain for multiple feed TA is obtained around this value [33]. By approaching the four sources to the unit-cells, the illumination of the array becomes more uniform and thus higher directivities and gains can be obtained, since the four feeds will act as one equivalent source with a larger focal length. To avoid the use of bulkier sources like horn antennas, the optimization studies are carried out using standard microstrip patch antennas.

The objective of this work is not to achieve a large bandwidth and therefore the multiple feed source is designed to operate at 10 GHz following the steps described in [34] using the same substrate as that used for the unit-cell. Afterwards it is simulated using CST Microwave Studio and the results show a maximal gain of 6 dBi. To properly place the sources, a \(2 \times 2\) array is designed using coaxial feeding and microstrip lines. The use of patches makes the feeding of the source much easier. A single 50 \(\Omega\) coaxial cable is required to feed the structure and the power division is realized through power dividers with the microstrip lines, thus making the design simpler, cheaper and lighter.

| Simulation                  | Elapsed Time     |
|-----------------------------|------------------|
| Feed Source (CST)           | 7 min 30 s       |
| Unit-Cell (CST)             | 3 min 30s        |
| Hybrid Code (Single Execution) | 0.2 s          |
| FACTOPO (per frequency)     | 53 min           |
| PSO Routine (per iteration of 50 Particles) | 9 s |

TABLE 3: Elapsed time consumed by each part of the hybrid numerical methodology

To analyse correctly the Figure 21, note that it presents the radiation pattern of four patch antennas taking into account the large distance between them (about \(5 \lambda\)). So, it is not the radiation pattern of an array antenna composed by four elements.

It is worth mentioning that the simulations realized with the hybrid code include (in this case) 4 objects of the "primary source" or "feed" type. In other words, each source is treated as an independent antenna. Since the spacing between the elements is large (\(5\lambda_0\)), the inter-elements coupling is very reduced. In addition, for each patch, the TA lens is placed in the far field region, but regarding the whole patch array the lenses will not be in this region since its dimensions are much bigger than the ones from a single patch. Therefore, the use of the radiation pattern of the whole patch array of the TA gain and therefore taken into account in the hybrid simulation. The gain measurements closely matched the simulations, with a peak gain of 9 dBi as seen in Figure 21.
will lead to false results from the hybrid code, justifying the modeling of the patch antennas as point sources.

In section V we designed an X-band $20 \times 20$ TA fed by a 16 dBi gain horn antenna with a focal ratio of 0.75. This optimization algorithm is used to achieve a reduction of at least 50% of the focal ratio by using multiple feeds. Figure 5 shows the comparison between the starting point of the optimization and the final solution for the proposed reduced ratio (0.375) found by the optimization routine. The radiation pattern of the multi-feed case is compared to that of a single patch (see Table 4) placed at a focal ratio of 0.75. As the $2 \times 2$ patch array acts as one single source placed further away from the TA lens, this "single patch" simulation gives the minimum achievable SLL for the same value of gain (19.5 dB). Finally, the multiple feed is also compared to the single horn feed TA presented earlier in figure 22.

| Source                  | $2 \times 2$ Patch Array |
|-------------------------|--------------------------|
| Directivity (dBi)       | 26.74                     |
| Gain (dBi)              | 19.49                     |
| SLL (dB)                | -16.98                    |
| Aperture Efficiency (%) | 37.5                      |

TABLE 4: Comparison of TA performances with a single or multiple feed

Table 4 shows that the optimization was able to achieve an SLL of -14.31 dB, which represents an increase of only 2.67 dB when compared to the single patch feed. As shown, the optimization of the phase distribution enabled a reduced focal length without a major degradation of the radiation characteristics of the TA. In addition, the antenna efficiency increased, a high gain of 19.55 dBi was maintained and the total volume of the antenna was halved. This passage from a focal length of 225 mm to 112.5 mm allows for much easier integration in constrained spaces. Figure 26 shows the convergence of the PSO algorithm towards the optimal gain of the TA as a function of the number of cycles.

When compared to the single horn feed (see Figure 22), in the case of a large focal length (black and red curves), the multiple feed presents smaller gain, higher SLLs and smaller aperture efficiency. Nevertheless, in the case of a reduced focal ratio ($F = 112.5$ mm) it has much better performance than the horn (dark blue curve). In this case, the multiple feed presents a well-defined main beam and a higher axial gain, justifying its use as a solution for focal length reduction.

In Figure 23, we observe a good match of the simulated and measured radiation pattern of the multiple feed TA, with side lobes well placed. As the manufactured source presented 1 dB of loss in the transmission lines, this impact was considered for the final simulation which led to a simulated realized gain of 18.55 dBi. The gain measurements showed a difference of -1.6 dB when compared to the simulations. Multiple tests did not show evidence of misalignment between the antennas and this reduction was therefore attributed to edge effects on the feeding patches, which could not be modeled in the hybrid code, and calibration problems with the anechoic chamber.

Since patch gains are low, the focal length reduction can be pushed further without loss of performance. Focal ratios of 0.312 and 0.25 were also studied, with both configurations having high gains and SLLs under the required threshold level of -14 dB as Figure 24 shows. The reduced focal ratio also reduces spillover losses and therefore increases the antenna power efficiency by up to 38%. The phase distributions showed in Figure 25 are remarkably close using the solution found for a ratio of 0.312 as a starting point for the 0.25 case, the optimization converged with less than 200 iterations.

VI. CONCLUSION

This paper presents a coherent hybrid numerical strategy for the design, optimization and final analysis simulations of TAs. As a demonstration of the numerical efficiency of the proposed methodology, the paper discusses the design, simulation and measurements of a new low profile and wide-
equal to 0.75, the full wave simulated gain is 20.81 dBi and the measured gain is 21.01 dBi, demonstrating that this transmit-array concept is very promising and theoretically well understood.

With the objective of reducing the overall volume of the antenna, a particle swarm optimization routine was tested through multiple simulations using different focal ratios and sources. The results show good performance, with gains above 19 dBi and side lobe levels under -14 dB. A volume reduction of more than 50% was achieved, allowing ratios as low as 0.25, with a gain reduction of only 2 dBi. The method is therefore an easy tool for the rapid design of space constrained transmit-array antennas, with performance that is easily adjustable through the cost functions. More advanced PSO techniques, such as cooperative swarms [19], will be investigated in future works to increase the algorithm performance.

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