A new distance measurement and its application in K-Means Algorithm

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Abstract

K-Means clustering algorithm is one of the most commonly used clustering algorithms because of its simplicity and efficiency. K-Means clustering algorithm based on Euclidean distance only pays attention to the linear distance between samples, but ignores the overall distribution structure of the dataset (i.e. the fluid structure of dataset). Since it is difficult to describe the internal structure of two data points by Euclidean distance in high-dimensional data space, we propose a new distance measurement, namely, view-distance, and apply it to the K-Means algorithm. On the classical manifold learning datasets, S-curve and Swiss roll datasets, not only this new distance can cluster the data according to the structure of the data itself, but also the boundaries between categories are neat dividing lines. Moreover, we also tested the classification accuracy and clustering effect of the K-Means algorithm based on view-distance on some real-world datasets. The experimental results show that, on most datasets, the K-Means algorithm based on view-distance has a certain degree of improvement in classification accuracy and clustering effect.

1 Introduction

Clustering is a data analysis problem in which a given dataset is grouped into several “classes” or “clusters” according to the similarity or distance of their features. Clustering is a great tool with a wide range of applications in data analysis, customer segmentation, recommender systems, search engines, image segmentation, semi-supervised learning, dimensionality reduction, preprocessing for other machine learning tasks, and more. The K-Means clustering algorithm was proposed by MacQueen [1] in 1967. It is one of the most commonly used clustering algorithms because of its simplicity and efficiency. Charles Elkan [2] proposed an important improvement to the K-Means algorithm in 2003. The improved algorithm greatly speeds up the algorithm by avoiding many unnecessary distance calculations. A smarter initialization step was introduced in 2006 by David Arthur and Sergei Vassilvitskii [3], proposing another important improvement to the K-Means algorithm, K-Means++, which makes the K-Means algorithm convergence to a suboptimal solution is less likely. Another important variant of the K-Means algorithm was proposed by David Sculley [4] in 2010. The algorithm is able to move the center point slightly using mini-batch K-Means in each iteration, rather than using the full dataset in each iteration, speeding up the algorithm by a factor of 3 to 4, and can be used for out-of-memory clustering large datasets. Feiping Nie et al [5] proposed K-Multiple-Means clustering algorithm by grouping data points of sub-clustering means into special k clusters. The Algorithm formalizes the multi-mean clustering problem into a single optimization problem, and uses an alternating optimization strategy to update the sub-cluster mean and the partition of the cluster. Other commonly used clustering algorithms [6] include hierarchical clustering [7], Gaussian mixture clustering [8], density-based clustering algorithm with noise DBSCAN [9], grid clustering STING algorithm [10], FCM fuzzy clustering algorithm [11], Mean Shift [12], Spectral Clustering Normalized Cuts [13], an algorithm for simultaneous clustering of samples and attributes Co-Clustering [14], deep clustering network DCN [15], multi-manifold clustering framework based on deep learning DMC [16], etc.

In the clustering algorithm, the sample set is often regarded as a point in the vector space, and
the distance between the samples is used to represent the similarity between the samples, so as to perform clustering, classification and other related tasks on the samples. Since similarity directly affects the result of clustering, its selection is the fundamental problem of clustering algorithm. Commonly used distances are Euclidean distance, Manhattan distance, Chebyshev distance, Minkowski distance, Mahalanobis distance, angle cosine, correlation coefficient, etc. Among them, Euclidean distance is used by most clustering algorithm because of its simple and small amount of calculation. However, in high-dimensional data space, it is far from enough to describe the distance between two data points with Euclidean distance, and it is difficult to describe the internal structure of the data. Van de Velden et al [17] used specific distance measures for different mixed data. Teng Qiu et al [18] improved clustering robustness and accuracy via distance ensemble and kernelization. Qi Li et al [19] proposed HIBOG method which can improve the clustering accuracy by ameliorating datasets with gravitation.

The original K-Means clustering algorithm based on Euclidean distance directly calculates the straight-line distance in high-dimensional space, and it is difficult to cluster the dataset according to the fluid structure of data. Inspired by the geodesic distance in three-dimensional space, we define a new distance metric formula named view-distance and apply it to the K-Means algorithm. The experimental results show that, compared with the Euclidean distance, the K-Means algorithm based on view-distance can cluster the S-curve and Swiss roll datasets according to the fluid structure of data. In addition, the classification accuracy and clustering effect on most real-world datasets are also improved to a certain level. In section 2, we first give the definition of view-distance and prove the properties of corresponding distance matrix. Then we propose the similarity discrimination gain, introduce the calculation formula and show the contour map of the view-distance function in three and multidimensional space. In section 3, we apply view-distance to K-Means algorithm and test the performance of new algorithm for clustering ability according to the fluid structure of data on synthetic datasets. Finally, we test the classification ability and clustering effect of the new algorithm on real-world datasets.

2 A new distance measurement

2.1 Definition of view-distance

Definition 1. Given a sample set \( X \subset \mathbb{R}^m \), \( x = (x_1, x_2, ..., x_m)^T \in X \) and \( y = (y_1, y_2, ..., y_m)^T \in X \), the distance between \( x \) and \( y \) is defined by

\[
d_v = (m - 2)! \sum_{1 \leq i < j \leq m} d_E \left((x_i, x_j), (y_i, y_j)\right),
\]

where \( d_E \left((x_i, x_j), (y_i, y_j)\right) \) represents the Euclidean distance between the vector \((x_i, x_j)\) and \((y_i, y_j)\), \( m \geq 2 \), abbreviated as view-distance.

Since the Euclidean distance satisfies the axioms of nonnegativity, symmetry and trigonometric inequality, it follows from formula (1) that the view-distance also satisfies three axioms.

Remark 1. In this definition, we consider not only the Euclidean distances between two samples, but also the projection distances on different hyperplanes, see Figure 1 or Figure 5.

Remark 2. According to the formula (1), in high-dimensional sample space, since the view-distance between every two sample points will have constant \((m - 2)!\), so the coefficient \((m - 2)!\) has no effect on the clustering of the data. The actual calculation formula is

\[
d_v = \sum_{1 \leq i < j \leq m} d_E \left((x_i, x_j), (y_i, y_j)\right).
\]

In the sense of new distance metric, the distance between two points in a three-dimensional space is the sum of their projected Euclidean distances on three two-dimensional planes, and the distance
between two points in a multidimensional space is the sum of the Euclidean distances of their projections on \( m!/2 \) two-dimensional planes. A four-dimensional space is composed of 4 three-dimensional spaces, or 12 two-dimensional spaces; a five-dimensional space is composed of 5 four-dimensional spaces, or 20 three-dimensional spaces, or 60 two-dimensional spaces. Therefore, in some practical applications, we may select some hyperplane projections according to the fluid structure of data to obtain a simpler distance formula.

Figure 1: View-distance and Euclidean distance in three-dimensional space. \( xy, xz, yz \) represent the Euclidean distance of two points projected on three two-dimensional planes, and \( E \) represents the Euclidean distance between two points.

**Lemma 1** \([20]\). Let \( E = (e_{ij})_{n \times n} \) be the matrix where \( e_{ij} \) corresponding to the Euclidean distance, then \( E \) is a distance matrix if and only if the following three conditions hold:

1. \( E \) is a nonnegative symmetric matrix;
2. \( \text{diag}(E) = 0 \), i.e., the diagonal elements of matrix \( E \) are all 0;
3. \( (I - es^T)E(I - es^T) \) is a positive semidefinite matrix, where \( e = (1, 1, ..., 1)^T \), \( s = (1, 0, ..., 0)^T \).

**Theorem 1.** Let \( V = (v_{ij})_{n \times n} \) be the matrix where \( v_{ij} \) corresponding to the view-distance, then \( V \) is also a distance matrix.

**Proof.** Obviously, the properties (1) (2) of Lemma 1 are easily derived from the definition of the distance matrix and view-distance. Next, we assume that \( x = (x_1, x_2, ..., x_m)^T \), \( y = (y_1, y_2, ..., y_m)^T \), and \( E_{ij} = (e'_{ij})_{m \times m} \) is an Euclidean distance matrix, where

\[
e'_{ij} = d_E((x_i, x_j), (y_i, y_j)).
\]

Then, it follows that

\[
v_{ij} = \sum_{1 \leq i < j \leq m} d_E((x_i, x_j), (y_i, y_j)),
\]

\[
V = \sum_{1 \leq i < j \leq m} E_{ij}.
\]

Since \( E_{ij} \) is an Euclidean distance matrix, derived from the Lemma 1, \( (I - es^T)E(I - se^T) \) is a positive semidefinite matrix, where \( e = (1, 1, ..., 1)^T \), \( s = (1, 0, ..., 0)^T \), so for every non-zero column vector \( u \in \mathbb{R}^m \), we have

\[
u^T(I - es^T)V(I - se^T)u = u^T(I - es^T) \sum_{1 \leq i < j \leq m} E_{ij}(I - se^T)u,
\]

\[
u^T(I - es^T)V(I - se^T)u = \sum_{1 \leq i < j \leq m} u^T(I - es^T)E_{ij}(I - se^T)u \geq 0,
\]
which indicates that \((I - es^T)V(I - se^T)\) is a positive semidefinite matrix, that is \(V\) is a distance matrix.

**Property 1.** Let \(V = (v_{ij})_{n \times n}\) be the view-distance matrix and \(E = (e_{ij})_{n \times n}\) be the Euclidean distance matrix. Let \(\lambda_{V_{\text{max}}}\) and \(\lambda_{E_{\text{max}}}\) be the largest eigenvalue of \(V\) and \(E\), \(\rho(V)\) and \(\rho(E)\) be the spectral radius of \(V\) and \(E\). The view-distance matrix has the following properties,

1. \(V\) has only one eigenvalue greater than zero,
2. \(\rho(V) = \lambda_{V_{\text{max}}}\), the spectral radius of \(V\) is equal to its largest eigenvalue,
3. \(\lambda_{V_{\text{max}}} \geq \lambda_{E_{\text{max}}}\), when \(n = 2\), \(\lambda_{V_{\text{max}}} = \lambda_{E_{\text{max}}}\), when \(n > 2\), \(\lambda_{V_{\text{max}}} > \lambda_{E_{\text{max}}}\).

**Proof.** The property (1) can be proved by the property of the Euclidean distance matrix \([21]\). Now let us prove the properties (2) and (3).

(2) Let \(\lambda_1, \lambda_2, ..., \lambda_{n-1}, \lambda_n = \lambda_{V_{\text{plus}}}\) be the eigenvalues of the view-distance matrix \(V\), where \(\lambda_{V_{\text{plus}}}\) is the positive eigenvalue of matrix \(V\). It follows from property (1) that

\[
\lambda_i < 0, i = 1, 2, ..., n - 1,
\]
\[
\sum_{i=1}^{n} \lambda_i = \text{Tr}(V) = 0.
\]

Therefore, we can obtain that

\[
\lambda_n = -\sum_{i=1}^{n-1} \lambda_i,
\]

which yields that

\[
\rho(V) = \max_{1 \leq i \leq n} |\lambda_i| = \lambda_{V_{\text{max}}} = \lambda_{V_{\text{plus}}}.
\]

(3) According to property (2), We only need to prove that the spectral radius of \(V\) is greater than the spectral radius of \(E\). Note that both the view-distance matrix \(V\) and the Euclidean distance matrix \(E\) are nonnegative matrices, and \(v_{ij} \geq e_{ij}\), it follows that \(V \geq E\). Then, according to nonnegative matrix theorem \([22]\), we have

\[
\rho(V) = \lambda_{V_{\text{max}}} \geq \lambda_{E_{\text{max}}} = \rho(E).
\]

Similar to the kernel function, the view-distance and Euclidean distance are regarded as the mapping function between sample points in the sample space, and the corresponding matrix is the distance matrix. According to property (3), when the dimension of the sample space is larger than 2, the spectral radius of the view-distance matrix is larger than the spectral radius of the Euclidean distance matrix, that is, the maximum stretching ability of the view-distance is stronger than the Euclidean distance.

### 2.2 Similarity discrimination gain

If we increase the sample space by one dimension, the effect on the similarity of differentiating sample points is defined as dimensional number similarity discrimination gain.

**Definition 2.** Given a sample point \(x, x = (x_1, x_2, ..., x_m)^T \in \mathbb{R}^m\) and \(x' = (x_1, x_2, ..., x_{m+1})^T \in \mathbb{R}^{m+1}\), the dimensional number similarity discrimination gain of \(x\) is defined by

\[
\alpha = \frac{\|x'\|}{\|x\|} - 1,
\]

where \(\|\cdot\|\) represents a norm related to the defined distance.
If we increase the difference in a certain dimension, the effect on distinguishing the similarity of two points is defined as certain dimension similarity discrimination gain.

**Definition 3.** Given a sample point \( x, x = (x_1, x_2, ..., x_{m-1}, 0)^T \in \mathbb{R}^m \) and \( x' = (x_1, x_2, ..., x_{m-1}, x_m)^T \in \mathbb{R}^m \), the certain dimension similarity discrimination gain of \( x \) is defined by

\[
\beta = \frac{\|x'\|}{\|x\|} - 1,
\]

where \( \|\cdot\| \) represents a norm related to the defined distance.

**Definition 4.** Given a vector \( x = (x_1, x_2, ..., x_m)^T \in \mathbb{R}^m \), \( v \)-norm is defined by

\[
\|x\|_v = \sum_{1 \leq i < j \leq m} \sqrt{x_i^2 + x_j^2}.
\]

Since the 2-norm satisfies the axioms of nonnegativity, homogeneity and trigonometric inequality, it follows from formula (3) that the \( v \)-norm also satisfies three axioms.

The calculation formula of dimensional number similarity difference gain corresponding to the \( v \)-norm is

\[
\alpha_v = \sum_{1 \leq i < j \leq m} \sqrt{x_i^2 + x_j^2}.
\]

The calculation formula of certain dimension similarity difference gain corresponding to the \( v \)-norm is

\[
\beta_v = \sum_{1 \leq i < j \leq m} \sqrt{x_i^2 + x_j^2} - 1.
\]

**Theorem 2.** Let \( \alpha_2 \) and \( \alpha_v \) be the dimensional number similarity difference gain defined by 2-norm and \( v \)-norm, respectively, then, when \( m = 2 \),

\[
\alpha_v \geq \alpha_2.
\]

**Proof.** According to the definition of dimensional number similarity difference gain

\[
\alpha_2 = \sqrt{x_1^2 + x_2^2 + x_3^2} - 1,
\]

\[
\alpha_v = \sqrt{x_1^2 + x_2^2} + \sqrt{x_1^2 + x_3^2} + \sqrt{x_2^2 + x_3^2} - 1.
\]

Then, derived from the inequality

\[
\sqrt{x_1^2 + x_2^2} + \sqrt{x_1^2 + x_3^2} + \sqrt{x_2^2 + x_3^2} \geq \sqrt{x_1^2 + x_2^2 + x_3^2},
\]

that is

\[
\alpha_v \geq \alpha_2.
\]

Theorem 2 shows that if we add a new dimension to a two-dimensional sample space, using the view-distance to distinguish the similarity of the sample points is better than the Euclidean distance.

In three-dimensional space, the Euclidean distance between each point \( x = (x, y, z)^T \) and the origin point is defined by

\[
d_E = \sqrt{x^2 + y^2 + z^2},
\]

and the view-distance is

\[
d_v = \sqrt{x^2 + y^2 + \sqrt{x^2 + z^2} + \sqrt{y^2 + z^2}}.
\]
In four-dimensional space, the Euclidean distance between every point $x = (x_1, x_2, x_3, x_4)^T$ and the origin point is defined by

$$d_E = \sqrt{x_1^2 + x_2^2 + x_3^2 + x_4^2},$$

and the view-distance is

$$d_V = \sqrt{x_1^2 + x_2^2} + \sqrt{x_1^2 + x_3^2} + \sqrt{x_1^2 + x_4^2} + \sqrt{x_2^2 + x_3^2} + \sqrt{x_2^2 + x_4^2} + \sqrt{x_3^2 + x_4^2}.$$

The three-dimensional of Euclidean distance and view-distance contour maps are shown in Figure 2. The contour maps of view-distance in different dimensions are shown in Figure 3. This is clear to see that as the space dimension increases, the view-distance between two points is gradually much larger than the Euclidean distance.

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**Figure 2:** The contour maps of view-distance and Euclidean distance when $z$ axis coordinate is $0$, $0.5$ and $1$. With the increase of $z$ axis coordinate, the view-distance and the Euclidean distance increase, but the difference between the view-distance and the Euclidean distance also increases.

**Figure 3:** Three-dimensional, four-dimensional and five-dimensional view-distance contour maps when $x_3, x_4, x_5$ take different values. Columns 1-4 show that the view-distance increases gradually with each dimension added, even if the dimension increases by 0. Lines 1-3 show that as the values $x_3, x_4$ and $x_5$ increase, the higher the dimension, the faster the view-distance increases.
3 Application on K-Means Algorithm

3.1 Dataset introduction

(1) Synthetic dataset [23]: The s-curve dataset and swiss roll datasets are commonly used artificial datasets for manifold learning and clustering algorithms, and are named for its shapes similar to s and swiss roll.

(2) Real-world dataset [24]: flower classification dataset iris, breast tissue dataset breast, measure the geometric properties of grains belonging to three different wheat varieties dataset seeds, contain six types of glass and their equivalent according to their oxide content dataset glass, chemical analysis of three different varietals from the same region of Italy dataset wine, describe the values of four categorical attributes for each of the 2201 people on board the Titanic when it struck an iceberg and sank dataset titanic, predict the cellular localization of proteins dataset yeast, describe the features of the nucleus present in the image dataset wdbc.

Table 1: Descriptions of the test datasets (1-2: synthetic datasets; 3-10: real-world ones)

| Index | Dataset   | Number | Dimension | Category | Distribution | Size   |
|-------|-----------|--------|-----------|----------|--------------|--------|
| 1     | s-curve   | 1500   | 3         |          |              | 4500   |
| 2     | swiss roll| 1500   | 3         |          |              | 4500   |
| 3     | iris      | 150    | 4         | 3        | 50/50/50     | 600    |
| 4     | breast    | 106    | 9         | 6        | 22/21/14/15/16/18 | 954 |
| 5     | seeds     | 210    | 7         | 3        | 70/70/70     | 1470   |
| 6     | glass     | 214    | 9         | 6        | 70/76/17/13/9/29 | 1926  |
| 7     | wine      | 178    | 13        | 3        | 59/71/48     | 2314   |
| 8     | titanic   | 2201   | 3         | 2        | 1490/711     | 6603   |
| 9     | yeast     | 1484   | 8         | 10       | 5/20/30/35/44/51/163/244/429/463 | 11872 |
| 10    | wdbc      | 569    | 30        | 2        | 212/357      | 17070  |

3.2 Synthetic datasets

We first test the performance of original K-Means algorithm for clustering ability according to the fluid structure of data on the s-curve and swiss roll datasets and use DBSCAN clustering algorithm.
and hierarchical clustering algorithm on the s-curve and swiss roll datasets at the same time.

The experimental results show that the original K-Means algorithm has a general effect and can partially divide the data along the fluid structure of data. The hierarchical clustering algorithm works better, it can divide the data along the fluid structure of data, but the boundary of the categories is not a neat dividing line. The DBSCAN clustering algorithm does not perform well on the datasets.

The swiss roll dataset has a fluid structure, so it is more reasonable to calculate the similarity between sample points by geodesic distance than by Euclidean distance. But the calculation of geodesic distance is very complex and difficult to be applied in high-dimensional sample space. The view-distance is simple to calculate and closer to geodesic distance in high-dimensional space. In the swiss roll dataset, the plane effect of the three distances is shown in Figure 5.

![Figure 5: (A) shows the effect map of geodesic distance, view-distance and Euclidean distance on swiss roll dataset. The yellow line represents the geodesic distance between two points, the blue line represents the Euclidean distance between two points, and the black line represents the view-distance between two points. (B) shows the contour plot of view-distance and Euclidean distance when z axis coordinate is 0.](image)

Then we apply view-distance to the K-Means algorithm, and test the performance of new algorithm for clustering ability according to the fluid structure of data on the s-curve and swiss roll datasets. The experimental results show that the K-Means algorithm based on view-distance has achieved good results in s-curve and swiss roll datasets, the dataset can be clustered according to the fluid structure of data, and the boundary between categories is a neat dividing line. The effect is shown in the following Figure 6.

![Figure 6: Left figure shows the effect of K-Means clustering based on view-distance on s-curve dataset(3 categories). Right figure shows the effect of K-Means clustering based on view-distance on swiss roll dataset(8 categories). The dataset can be clustered according to the fluid structure of data, and the boundary between categories is a neat dividing line.](image)

### 3.3 Real-world datasets

#### 3.3.1 Classification accuracy

We apply the view-distance to K-Means algorithm and KNN algorithm, and test their classification accuracy on the real-world datasets. The experimental results show that the K-Means and KNN
algorithm based on view-distance has higher classification accuracy than the original K-Means and KNN algorithm based on Euclidean distance in most datasets. The classification accuracy effect is shown in following Table 2.

### Table 2: Classification accuracy table.

| Dataset | K-Means | view-K-Means | KNN | view-KNN |
|---------|---------|--------------|-----|----------|
| iris    | 96.67%  | 96.67%       | 98% | 98%      |
| breast  | 52.83%  | 52.83%       | 81.13% | 85.85% |
| seeds   | 90.95%  | 91.90%       | 88.57% | 93.81% |
| glass   | 54.21%  | 55.14%       | 78.97% | 83.18% |
| wine    | 70.22%  | 70.79%       | 97.75% | 97.75% |
| titanic | 78.33%  | 78.33%       | 57.20% | 77.92% |
| yeast   | \1\3   | \            | 68.53% | 68.73% |
| wdbc    | 88.75%  | 90.16%       | 94.2% | 95.08%  |

#### 3.3.2 Clustering effect

Common evaluation indicators of clustering effect include Adjusted Rand index [26, 27], homogenization score [28, 29], mutual information [30], v measure [30], FMI [31], etc. We calculate several common evaluation index values of the original K-Means algorithm and the K-Means algorithm based on view-distance to evaluate its clustering effect. At the same time, we calculate the performance of AffinityPropagation clustering algorithm [32], Mean Shift clustering algorithm using flat kernel, Birch clustering algorithm [33], and hierarchical clustering algorithm. The experimental results show that the clustering indexes of the new algorithm are better than those of the original K-Means Algorithm based on Euclidean distance in most datasets. Compared with other clustering algorithms, new algorithm also has some advantages in most clustering indexes. The clustering effects of different datasets are are shown in the following Table 3.

### Table 3: The clustering effect table. (ARI, homo, AMI, v_measure, FMI represent the clustering indexes in turn: adjusted rand index, homogenization score, mutual information, v measure, FMI.)

| Dataset | Metric | AP | MeanShift | BIRCH | Agg | K-Means | ours |
|---------|--------|----|-----------|-------|-----|---------|------|
| iris    | ARI    | 0.4344 | 0.5681 | \1\4 | 0.8577 | 0.9039 | 0.9039 |
|         | homo   | 0.9146 | 0.5794 | \   | 0.8640 | 0.8983 | 0.8983 |
|         | AMI    | 0.6309 | 0.7316 | \   | 0.8625 | 0.7315 | 0.8984 |
|         | v_measure | 0.6426 | 0.7337 | \   | 0.8642 | 0.7337 | 0.8997 |
|         | FMI    | 0.5988 | 0.7715 | \   | 0.9233 | 0.7715 | 0.9356 |
| breast  | ARI    | 0.2382 | 0.2179 | 0.1163 | 0.2399 | 0.3454 | 0.3894 |
|         | homo   | 0.5315 | 0.3021 | 0.2030 | 0.4515 | 0.5345 | 0.5596 |
|         | AMI    | 0.4395 | 0.4217 | 0.2839 | 0.4754 | 0.5789 | 0.5936 |
|         | v_measure | 0.5103 | 0.4416 | 0.3101 | 0.5187 | 0.6084 | 0.6259 |
|         | FMI    | 0.3670 | 0.5002 | 0.4373 | 0.4411 | 0.5298 | 0.5436 |
| seeds   | ARI    | 0.2262 | 0.3071 | \   | 0.3770 | 0.6898 | 0.7109 |
|         | homo   | \6739 | 0.2344 | \   | 0.3829 | 0.5962 | 0.6188 |
|         | AMI    | 0.4215 | 0.2983 | \   | 0.4133 | 0.6410 | 0.6664 |
|         | v_measure | 0.4618 | 0.3080 | \   | 0.4267 | 0.6492 | 0.6740 |
|         | FMI    | 0.3946 | 0.5856 | \   | 0.5837 | 0.7877 | 0.8024 |

3. The accuracy of the algorithm is less than 50% on this dataset.
4. The algorithm does not converge on this dataset.
| Dataset  | Metric | AP  | MeanShift | BIRCH | Agg  | K-Means | ours  |
|---------|--------|-----|-----------|-------|------|---------|-------|
| glass   | ARI    | 0.1903 | 0.2619 |       | 0.2303 | 0.2993 | **0.3114** |
|         | homo   | **0.5542** | 0.4058 |       | 0.3192 | 0.4043 | 0.4094 |
|         | AMI    | 0.3315 | 0.4120 |       | 0.3307 | 0.4642 | **0.4673** |
|         | v_measure | 0.4119 | 0.4678 |       | 0.3614 | 0.4860 | **0.4892** |
|         | FMI    | 0.3450 | 0.5579 |       | 0.4966 | 0.5733 | **0.5755** |
| wine    | ARI    | 0.1767 | 0.3198 |       | 0.3218 | 0.3470 | **0.3563** |
|         | homo   | **0.5100** | 0.3494 |       | 0.4434 | 0.3762 | 0.3961 |
|         | AMI    | 0.3113 | 0.3525 |       | **0.4553** | 0.3756 | 0.3995 |
|         | v_measure | 0.3338 | 0.3595 |       | **0.4613** | 0.3823 | 0.4060 |
|         | FMI    | 0.3633 | 0.5627 |       | 0.5717 | 0.5750 | **0.5852** |
| titanic | ARI    |       |       | 0.0971 | 0.1276 | 0.2744 | **0.2932** | **0.2932** |
|         | homo   |       |       | 0.1663 | 0.0804 | 0.1570 | **0.1715** | **0.1715** |
|         | AMI    |       |       | 0.0998 | 0.0774 | **0.5288** | 0.0774 | 0.0774 |
|         | v_measure |       |       | 0.1006 | 0.0777 | **0.5294** | 0.0777 | 0.0777 |
|         | FMI    |       |       | 0.4503 | 0.5997 | **0.8393** | 0.5997 | 0.5997 |
| yeast   | ARI    | 0.0265 | 0.0061 |       | 0.1189 | 0.1360 | **0.1738** |
|         | homo   | **0.4302** | 0.0224 |       | 0.2544 | 0.2679 | 0.2865 |
|         | AMI    | 0.1882 | 0.0338 |       | 0.2177 | 0.2089 | **0.2475** |
|         | v_measure | 0.2483 | 0.0424 |       | 0.2293 | 0.2164 | **0.2566** |
|         | FMI    | 0.1107 | **0.4693** |       | 0.2779 | 0.3703 | 0.3746 |
| wdbc    | ARI    | 0.0571 | 0.0841 |       | **0.6476** | 0.5951 | 0.6413 |
|         | homo   | **0.6902** | 0.0855 |       | 0.5195 | 0.4875 | 0.5202 |
|         | AMI    | 0.2272 | 0.1235 |       | 0.5288 | 0.5126 | **0.5337** |
|         | v_measure | 0.2384 | 0.1276 |       | 0.5294 | 0.5132 | **0.5344** |
|         | FMI    | 0.2531 | 0.7193 |       | **0.8393** | 0.8238 | 0.8390 |

## 4 Conclusion

In the sense of the new distance metric, the distance between two points in three-dimensional space is the sum of the Euclidean distances from their projections on three two-dimensional planes, which is closer to the geodesic distance to a certain degree. Therefore, the K-Means algorithm based on the view-distance can cluster the data according to the fluid structure of data while keeping simple and efficient, and the boundary between the categories is a neat dividing line. As the dimension of the sample space increases, the view-distance between two points will increase in $m(m - 1)/2$ (m is the characteristic dimension of the sample), gradually becoming much larger than the Euclidean distance, so the data separability is also enhanced. In most real-world datasets, the classification accuracy and clustering effects are improved to a certain degree. However, with the further increase of the dimension of the sample space, the calculation amount of the view-distance between two sample points will also increase greatly, so the view-distance is slower to compute in the higher dimensional sample space than the Euclidean distance.

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