Algebraic Thinking in Solving Linier Program at High School Level: Female Student’s Field Independent Cognitive Style

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Abstract. The aim of this study was to describe algebraic thinking of high school female student’s field independent cognitive style in solving linier program problem by revealing deeply the female students’ responses. Subjects in this study were 7 female students having field independent cognitive style in class 11. The type of this research was descriptive qualitative. The method of data collection used was observation, documentation, and interview. Data analysis technique was by reduction, presentation, and conclusion. The results of this study showed that the female students with field independent cognitive style in solving the linier program problem had the ability to represent algebraic ideas from the narrative question that had been read by manipulating symbols and variables presented in tabular form, creating and building mathematical models in two variables linear inequality system which represented algebraic ideas, and interpreting the solutions as variables obtained from the point of intersection in the solution area to obtain maximum benefit.

1. Introduction

Algebra is one part of mathematics which is an extension of arithmetic [1][2] since algebra is represented by the language of symbols of the unknown variable [3][4]. Algebra can also be used to solve a problem in everyday life, so it makes algebra important and must be learned for all students [5][6].

Algebra standard for the school level corresponds to [5], that by learning algebra from kindergarten to high school level students are expected to be able to understand patterns, relationships and functions; students are able to present and analyze mathematical situations and structures that use algebraic symbols; students are able to use mathematical models to present and understand quantitative relationships; and students are able to analyze changes in various situations. Similarly in Indonesia, the standard of mathematical basic competences that must be achieved by high school students based on the curriculum of 2013 are using the concept of algebra to apply the exponent and logarithm rules that match the characteristics of the problems to be solved; using algebraic concepts to analyze absolute values in equations and inequalities; using algebraic concept to predict the sequences patterns and arithmetic and geometry series or other sequences through observation and reasoning; using the concept of algebra to describe various forms of expression which can be converted to quadratic equations; using algebraic concept to formulate the rules and the natures of algebraic function limits through observation of examples; using algebraic concept in solving problems in everyday life. One of the algebraic materials that must be studied by high school students is linier program material. Linier program is an optimization problem solving program, which maximizes or
minimizes a linear function that depends on linear constraints. This constraint can be an inequality or a linear equation. The problem of linear program in school is presented in narrative question form so that from the narrative question the students are expected to solve the problem by representing in the form of a linear function which contains symbols or variables that its value based on algebraic agreement is unknown. In that process, students do algebra thinking activities.

Some experts define algebraic thinking. According to [7] algebraic reasoning involves representing, generalizing, and formalizing patterns and regularity in all aspects of mathematics. In line with [8], algebraic thinking is using mathematical symbols and tools to analyze different situations by; representing that information mathematically in words, diagrams, tables, graphs, and equations; interpreting and applying mathematical findings, such as solving for unknowns, testing conjectures, and identifying functional relationships to the same situations and the new related situation. Meanwhile, he described three activities of algebraic thinking: generational activities, transformational activities, and global meta-level activities. The same thing [10], defines algebraic thinking as the ability of generalization, abstraction, analytic thinking, dynamic thinking, modeling, and organization. But according to [11], in his article entitled "Just What is Algebraic Thinking" that algebraic thinking has two main components, namely the ability to think mathematically and algebraic basic idea of. The ability to think mathematically is the ability to solve problems, the ability of representation, and the ability of quantitative reasoning while the algebraic basic idea is algebra as arithmetic generalization, algebra as the mathematics language, and algebra as a tool for a function and mathematical modeling.

To describe students’ algebraic thinking, it needs to be viewed from many dimensions because as human beings, students are unique human beings, in which one student is different from other students in many dimensions. Basically each individual is different one another. The dimensions of individual differences include intelligence, logical thinking ability, creativity, cognitive style, personality, values, attitudes, and interests. Cognitive style is different from intelligence and other ability dimensions. [12] divides the two cognitive styles namely field independent and field dependent. He distinguishes individuals who have field independent and field dependent cognitive style through the ability to answer Group Embedded Figure Test (GEFT) test within the specified time as well as certain criteria. Another dimension of individual differences is gender.

Based on the description, the researcher is interested in describing algebraic thinking of female students’ field-independent cognitive style in high school level in solving linear program problem. The researcher describes that algebraic thinking of female students’ field-independent cognitive style in solving linear program problem has 5 indicators that involve reading, writing, manipulating numbers and symbols using algebraic agreements; variable interpretation and variable expression; representing ideas for using expressions containing variables such as equations, inequalities, and functions; using the equivalent symbolic representation to manipulate formulas, expressions of equations, inequalities or functions; interpreting the solution. These five indicators are adapted from [11].

2. Method
The type of this research was descriptive qualitative. The data collection method used observation, documentation, and interview. Participants in this research were 216 female students of 11th grade high school students from State Senior High School 2 Mataram and State Senior High School 5 Mataram. This research instruments consisted of main instrument namely the researcher and supporting instrument consisting of 4 kinds of instrument namely GEFT Test, Mathematics ability problem, Linier Program Problem, and Interview Guidance. First 216 female students were given GEFT Test and Mathematics Ability Problem. Of the 216 female students, there were 149 students who had field independent cognitive style. Based on parameters that had been determined by the researcher, the results of GEFT test and the results of the mathematical ability of 149 students, there were 7 students selected to be given linear program problem and interviewed. The linier program Problem was “the flower shop ‘PUJI’ sells two kinds of hand bouquet that are cheap but interesting. The first series
consists of 4 roses, 2 dahlia flowers, and 3 tulips while the second series consists of 2 roses, 3 dahlia flowers, and 1 tulip flower. The seller only has stocks of 48 roses, 36 dahlia flowers, and 33 tulips. If the profit in the first series is Rp.15.000, - and that in the second series is Rp.12.000, - then do the following calculation.

- Determine all the possible inequalities models to express the need of roses, dahlias and tulips that are used to make two kinds of flower series, based on the data.
- From the result of the inequality system (a), show the graph and the shaded area of the solution in a system of plane Cartesian coordinate.
- Through the interpretation of the graph, what is the maximum profit that is gained by the seller? And how many series of the first flowers and the second flower that need to be made so that the seller can get the maximum profit?

3. Results and Discussion

Data collection on the research subject in working linier program problems and interview was conducted on March 13th, 2017. Based on the answers of seven male students of field independent cognitive-style, the algebraic thinking in solving linier program problem was described. Descriptions of their algebraic thinking were described as follows. Based on the first indicator, they could think about the problems they had read and then they could explain the general purpose of the narrative question and the questions given. They also found material related to the issues already read. Next they wrote out the essential elements for expressing algebra on the problems already read into a table. Two different tables are presented by them (see Figure 1 and see Figure 2). This was in accordance with [3], that algebraic is a language. Based on both tables, they could manipulate the existing elements in the problem into table [11].

![Figure 1. The first table form which is generated based on the first indicator of algebraic thinking](image1.png)

![Figure 2. The second table form which is generated based on the first and second indicators of algebraic thinking](image2.png)

In the second indicator, they can interpret the meaning and difference between the constants and the variables in the mathematical model that was already made (see Figure 2) and they could determine the constants for variables x and y and could interpret the function of constraint and purpose function [13]. The following is the interview with a research subject.

...
Researcher : Do you understand what you wrote in this table?
Subject : Yes, I do.
Researcher : Do you also understand these symbols? (while pointing at the table)
Subject : Yes, maam.
Researcher : Please explain it to me!
Subject : Yes, maam (while nodding). Because in the problem there are two flowers series then I suppose that the first series is x and the second series is y.
Researcher : Why are they supposed x and y?
Subject : Because those are the names of variables.
Researcher : May they be changed with other letters?
Subject : Yes, they can.
Researcher : How about this one (pointing at table 2 in column 2)?
Subject : Ohhh ..... it is coefficient.
Researcher : What is the meaning of 4x and 2y?
Subject : 4 shows the number of roses for the first series and 2 shows the number of roses for the second series.

Female students with field independent cognitive style could link the information available to the narrative question by writing and manipulating symbols such as variables and constants as representational capabilities [1][14][15][16].

Next to the third indicator, they could build a two-variable linier inequality system to create two types of flower series of three types of flowers. They could formulate and create all functional constraints in the form of linier inequalities and purpose functions in the form of equations for flower series (see Figure 3).

![Figure 3](image-url)

**Figure 3. Photo results for the third and the fourth indicator of algebraic thinking**

The fourth indicator, they could find all the intersection points of the linier inequalities pair. They could find and complete the completion set of the linier inequality system that had been created by elimination. They substituted on a linier inequality system by using basic operations (addition, subtraction, multiplication, division) between linier inequalities to obtain a cutoff point in the Cartesian plane graph of two pairs of known linier inequalities. They attempted to describe the results of algebraic analysis in the form of a graphic interpretation in the Cartesian coordinate system (see Figure 3). They were able to determine the shaded area of the linier inequality system as the solution area of the linier inequality system (see Figure 3).

The last indicator, they could interpret three cutoff points in the solution area on the graph that had been made then they tested the three points by substituting in the equation of the objective function that had been made to obtain the maximum profit (see Figure 4). The cutoff point that generated the maximum profit was called the maximum point and they could interpret the numbers at the maximum point obtained as the value of the questioned variable as in the following interview.

Researcher : What is the meaning of maximum profit Rp.207.000, - at the point (9,6)?
Subject: So the seller gains maximum profit as much as Rp. 207,000, and ... hmm ... (while looking at the table that had been created). Well ... the meaning of point 9 is the number of the first flower series because it was exemplified before by x and the second flower series was exemplified by y, so the number of both flower series that can be made by the seller are 6.

Figure 4. Photo results for the fifth indicators of algebraic thinking

4. Conclusion
The research result states that algebra thinking from 7 high school female students in field independent cognitive style can solve liiner program problem. They have an ability to represent algebraic ideas from the narrative questions that have been read by manipulating symbols and variables presented in two different table forms. From these two tables they could create and construct a mathematical model in a two-variable liiner inequality system based on algebraic agreement as a constraint function and objective function. They could interpret meaning and difference between constants and variables on constraints function and objective functions that have been made. They were able to complete a two-variable liiner inequality system to search for all possible cutoff points to illustrate the results of the algebra analysis of the completion area into a graph of the Cartesian coordinate system. Then they interpret and test the cutoff points of maximum profit after being substituted on the objective function, and they are able to conclude the maximum point as the value of the questioned variable.

References
[1] Kaput J J 2008 What is Algebra?What is Algebraic Reasonning? (New York: Lawrence Erlbaum Associates)
[2] Vance J H 1998 Number Operations from An Algebraic Perspective Teaching Children Mathematics 4 pp 282-285.
[3] Usiskin Z 1997 Doing Algebra in Grades K-4 Teaching Children Mathematics 3 pp 346-356
[4] Blair L 2003 It’s Elementary: Introducing Algebraic Thinking Before High School SEDL Letter 15(1) pp 16-20
[5] National Council of Teachers of Mathematics 2000 Principles and Standards for School Mathematics (Reston, VA: NCTM)
[6] Chambers D 1994 The Right Algebra for All Educational Leadership 51 p 85
[7] Van de Walle J A 2004 Elementary and Middle School Mathematics: Teaching Developmentally 5th ed (New York: Longman)
[8] Herbert K & Brown R H 1997 Patterns as Tools for Algebraic Reasoning Teaching Children Mathematics 3
[9] Kieran Carolyn 2004 Algebraic Thinking in the Early Grades: What Is It? The Mathematics
Educator Vol. 8(1) pp139-151

[10] Lew Hee Chan 2004 Developing Algebraic Thinking In The Earlier Grades: A Case Study of The South Korean Elementary School Mathematics Curriculum The Mathematics Educator 8

[11] Kriegler S 2008 Just what is algebraic thinking UCLA: Department of Mathematis. Available online also at: http://www.math.ucla.edu/~kriegler/pub/algebrat.html [accessed in Manila, Philippines: December 5, 2015].

[12] Witkin H A, Moore C A, Goodenough DR, and Cox P W 1975 Field-dependent and field-independent cognitive styles and their educational implications ETS Research Report Series 2 pp 1-64

[13] Battista M & Van Auken Borrow C 1998 Using Spreadsheets to Promote Algebraic Thinking Teaching Children Mathematics 4 pp 470-478.

[14] Wagner S 1983 What are These Things Called Variables? Mathematics Teacher 76 pp 474-479

[15] Skemp R R 1987 The Psychology of Learning Mathematics (Hillsdale NJ: Lawrence Erlbaum Associates)

[16] Schoenfeld A H & Arcavi A 1988 On The Meaning of Variables Mathematics Teacher 81 pp 420-427.