Top mode standard model and extra dimensions\textsuperscript{a}

MICHIO HASHIMOTO\textsuperscript{bc}
Department of Physics, Pusan National University, Pusan 609-735, Korea
E-mail: michioh@charm.phys.pusan.ac.jp

MASAHARU TANABASHI
Department of Physics, Tohoku University, Sendai 980-8578, Japan
E-mail: tanabash@tuhep.phys.tohoku.ac.jp

KOICHI YAMAWAKI
Department of Physics, Nagoya University, Nagoya 464-8602, Japan
E-mail: yamawaki@eken.phys.nagoya-u.ac.jp

ABSTRACT

We perform the most attractive channel (MAC) analysis in the top mode standard model with TeV-scale extra dimensions, where the standard model gauge bosons and the third generation of quarks and leptons are put in $D(= 6, 8, \cdots)$ dimensions. In order to make the scenario viable, only the attractive force of the top condensate should exceed the critical coupling, while other channels such as the bottom and tau condensates should not. It turns out that the top condensate can be the MAC for $D = 8$, whereas the tau condensation is favored for $D = 6$. On the basis of the renormalization group equations for the top Yukawa and Higgs quartic couplings, we predict masses of the top quark and the Higgs boson for $D = 8$ as $m_t = 172 - 175$ GeV and $m_H = 176 - 188$ GeV, respectively.

1. Introduction

What is the physics behind the electroweak symmetry breaking (EWSB)? Why are the masses of W, Z, and the top quark exceptionally large compared with those of other particles in the standard model (SM)? In the framework of the top quark condensate \textsuperscript{123}, which is often called the “Top Mode Standard Model” (TMSM), the chiral condensation of the top quark $\langle \bar{t}_L t_R \rangle \neq 0$ triggers the EWSB and then the top acquires its large mass of the order of the EWSB scale. The Higgs boson emerges as the scalar bound state of $tt$.

Along with the TeV-scale extra dimension scenario \textsuperscript{45}, the TMSM has been reconsidered by several authors. \textsuperscript{6789101112} In particular, Arkani-Hamed, Cheng, Dobrescu and Hall (ACDH) proposed a model where the SM gauge bosons and the third generation quarks/leptons live in the $D(= 6, 8, \cdots)$-dimensional bulk. \textsuperscript{8} The full bulk gauge dynamics was analyzed in Refs. \textsuperscript{910}, based on the ladder Schwinger-Dyson (SD) equation.

We here study the phenomenological implications. (For details see Ref. \textsuperscript{11}.) In order for only the top quark to acquire the dynamical mass of the order of the EWSB scale, the binding strength only for the top quark should exceed the critical one $\kappa_D^{\text{crit}}$. We thus analyze binding strengths of top, bottom and tau condensates, $\kappa_{t,b,\tau}$, by using

\textsuperscript{a}Talk given by M.H. at The 12th International Conference on Supersymmetry and Unification of Fundamental Interactions (SUSY 2004), June 17-23, 2004, Tsukuba, Japan

\textsuperscript{b}The present address is Department of Applied Mathematics, Western Science Centre, The University of Western Ontario, London, ON, Canada, N6A 5B7.

\textsuperscript{c}The new e-mail address: mhashimo@uwo.ca
2. tMAC analysis

Let us consider a simple version of the TMSM with extra dimensions where the SM gauge group and the third generation of quarks and leptons are put in the bulk \(D = 6, 8, \cdots\), while the first and second generations live on the 3-brane (four dimensional Minkowski space-time). The extra \(\delta = (D - 4)\) spatial dimensions are compactified at a TeV-scale \(R^{-1}\). In order to obtain a four dimensional chiral theory and to forbid massless gauge scalars, we compactify extra dimensions on the orbifold \(T^\delta/Z_2^\delta\). [9,11]

By using the “truncated Kaluza-Klein (KK)” effective theory \[5\], we calculate the RGEs for the four dimensional gauge couplings \(g_i (i = 3, 2, Y)\), (for details see Ref. [11])

\[
(4\pi)^2 \frac{d g_i}{d \mu} = b_i g_i^3 + b_i^{KK}(\mu) g_i^3, \quad (\mu \geq R^{-1})
\]

with \(b_3 = -7, b_2 = -\frac{19}{6}\) and \(b_Y = \frac{41}{6}\), where the RGE coefficients \(b_i^{KK}(\mu)\) are given by

\[
b_3^{KK}(\mu) = -11 N^g_{KK}(\mu) + \frac{\delta}{2} N^{gs}_{KK}(\mu) + \frac{8}{3} N^f_{KK}(\mu), \quad \text{for } SU(3)_c, \tag{2}
\]

\[
b_2^{KK}(\mu) = -\frac{22}{3} N^g_{KK}(\mu) + \delta N^{gs}_{KK}(\mu) + \frac{8}{3} N^f_{KK}(\mu) + \frac{1}{6} N^h_{KK}(\mu), \quad \text{for } SU(2)_W, \tag{3}
\]

\[
b_Y^{KK}(\mu) = \frac{40}{9} N^f_{KK}(\mu) + \frac{1}{6} N^h_{KK}(\mu), \quad \text{for } U(1)_Y. \tag{4}
\]

\(N^k_{KK}(\mu), k = g, gs, f, h\) denote the total numbers of KK modes below \(\mu\) for gauge bosons, gauge scalars, Dirac (4-component) fermions, and composite Higgs bosons, respectively.

We define the dimensionless bulk gauge couplings \(\hat{g}_i\) as \[9,10,11\]

\[
\hat{g}_i^2(\mu) = \frac{(2\pi R \mu)^\delta}{2^{\delta/2}} g_i^2(\mu), \quad (i = 3, 2, Y). \tag{5}
\]
Combining Eq. (5) with Eq. (1), we find the RGEs for \( \hat{g}_i \),

\[
\mu \frac{d}{d\mu} \hat{g}_i = \frac{\delta}{2} \hat{g}_i + \frac{\hat{g}_3^3}{(4\pi)^2} \frac{2^{\delta/2}}{(2\pi R \mu)^\delta} \left[ b_i + b_{KK}^i(\mu) \right].
\]  

(6)

We now analyze the energy scale \( \Lambda_{tM} \) where the top condensate is the most attractive channel (MAC) and only in the \( \bar{t}t \)-channel the binding strength exceeds the critical value \( \kappa_{\text{crit}}^D \), ("topped MAC" or "tMAC" scale),

\[
\kappa_t(\Lambda_{tM}) > \kappa_{\text{crit}}^D > \kappa_b(\Lambda_{tM}), \kappa_\tau(\Lambda_{tM}), \cdots.
\]  

(7)

The binding strengths \( \kappa_{t,b,\tau} \) are given by

\[
\kappa_t(\mu) = \frac{4}{3} \hat{g}_3^2(\mu)\Omega_{\text{NDA}} + \frac{1}{9} \hat{g}_Y^2(\mu)\Omega_{\text{NDA}}, \quad \text{for } \bar{t}t,
\]  

(8)

\[
\kappa_b(\mu) = \frac{4}{3} \hat{g}_3^2(\mu)\Omega_{\text{NDA}} - \frac{1}{18} \hat{g}_Y^2(\mu)\Omega_{\text{NDA}}, \quad \text{for } \bar{b}b,
\]  

(9)

\[
\kappa_\tau(\mu) = \frac{1}{2} \hat{g}_Y^2(\mu)\Omega_{\text{NDA}}, \quad \text{for } \bar{\tau}\tau.
\]  

(10)

We estimate \( \kappa_{\text{crit}}^D \) through the ladder SD equation. [9][10] The lowest possible values are

\[
\kappa_6^{\text{crit}} \simeq 0.122, \quad \kappa_8^{\text{crit}} \simeq 0.146.
\]  

(11)

We show the results of the tMAC analysis in Fig. 1. For \( D = 6 \), we find that the tMAC scale is squeezed out and the tau condensation is favored. For \( D = 8 \), on the other hand, it turns out that the tMAC scale \( \Lambda_{tM} \) satisfying Eq. (7) does exist,

\[
\Lambda_{tM} R = 3.5 - 3.6, \quad \text{for } R^{-1} = 1 - 100 \text{ TeV}.
\]  

(12)

3. Prediction of \( m_t \) and \( m_H \)

We predict the top quark mass \( m_t \) and the Higgs boson mass \( m_H \) in a way used by ACDH [3]. This is similar to the approach of Bardeen, Hill and Lindner [3]: The masses \( m_t \)
and $m_H$ are predicted by using the RGEs of the top Yukawa and Higgs quartic couplings ($y$ and $\lambda$) with compositeness conditions,

$$y(\mu) \to \infty, \quad \frac{\lambda(\mu)}{y(\mu)^4} \to 0, \quad (\mu \to \Lambda). \quad (13)$$

While the compositeness scale $\Lambda$ in Ref. [8] was treated as a free parameter to be adjusted for reproducing the experimental value of $m_t$, we identify $\Lambda$ with the tMAC scale $\Lambda_{tM}$ and hence $\Lambda$ is no longer an adjustable parameter but constrained as Eq. (12). Thus we can test our model by comparing the predicted $m_t$ with the experimental value.

We show the results of $m_t$ and $m_H$ for $D = 8$, $R^{-1} = 10$ TeV in Fig. 2 where the tMAC scale $\Lambda_{tM}$ is shown by the shaded region. We then predict $m_t$ and $m_H$ as

$$m_t = 172 - 175 \text{ GeV}, \quad m_H = 176 - 188 \text{ GeV}, \quad \text{for} \quad R^{-1} = 1-100 \text{ TeV}. \quad (14)$$

The predicted value of $m_t$ is acceptable. The prediction for the mass of the composite Higgs boson shown in Eq. (14) can be tested in collider experiments such as LHC.

4. Summary and discussions

We have performed the tMAC analysis. We found that the region of the tMAC scale is squeezed out for $D = 6$, while it does exist for $D = 8$, $\Lambda_{tM} = (3.5-3.6)R^{-1}$. The prediction of the top quark mass for $D = 8$ is successful and the (composite) Higgs boson with the characteristic mass shown in Eq. (14) should be discovered at LHC.

For a viable model with $D = 6$ the gauged Nambu-Jona-Lasinio (GNJL) model in the bulk may be helpful. [12]

5. References

[1] V. A. Miransky, M. Tanabashi, and K. Yamawaki, Phys. Lett. B 221, 177 (1989); Mod. Phys. Lett. A 4, 1043 (1989).
[2] Y. Nambu, Fermi Inst. Rep. 89-08, 1989; in Proc. of 1989 Workshop on Dynamical Symmetry Breaking, ed. T. Muta and K. Yamawaki (Nagoya U., Nagoya, 1990).
[3] W. A. Bardeen, C. T. Hill and M. Lindner, Phys. Rev. D41, 1647 (1990).
[4] I. Antoniadis, Phys. Lett. B246, 377 (1990).
[5] K. R. Dienes, E. Dudas and T. Gherghetta, Phys. Lett. B436, 55 (1998); Nucl. Phys. B537, 47 (1999).
[6] B. A. Dobrescu, Phys. Lett. B461, 99 (1999); [hep-ph/9903407]
[7] H. C. Cheng, B. A. Dobrescu, and C. T. Hill, Nucl. Phys. B589, 249 (2000).
[8] N. Arkani-Hamed, H. C. Cheng, B. A. Dobrescu and L. J. Hall, Phys. Rev. D62, 096006 (2000).
[9] M. Hashimoto, M. Tanabashi and K. Yamawaki, Phys. Rev. D64, 056003 (2001).
[10] V. Gusynin, M. Hashimoto, M. Tanabashi and K. Yamawaki, Phys. Rev. D65, 116008 (2002).
[11] M. Hashimoto, M. Tanabashi, and K. Yamawaki, Phys. Rev. D69, 076004 (2004).
[12] V. P. Gusynin, M. Hashimoto, M. Tanabashi and K. Yamawaki, [hep-ph/0406194]