On Low-Energy Effective Action in $\mathcal{N} = 2$ Super Yang-Mills Theories on Non-Abelian Background

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Abstract

We compute the non-holomorphic corrections to low-energy effective action (higher derivative terms) in $\mathcal{N} = 2$, $SU(2)$ SYM theory coupled to hypermultiplets on a non-abelian background for a class of gauge fixing conditions. A general procedure for calculating the gauge parameters depending contributions to one-loop superfield effective action is developed. The one-loop non-holomorphic effective potential is exactly found in terms of the Euler dilogarithm function for specific choice of gauge parameters.
1 Introduction

Low-energy effective action of $\mathcal{N} = 2$ supersymmetric Yang-Mills theories is defined, in purely gauge superfield sector, by two effective potentials. The leading correction is given by holomorphic potential $\mathcal{F}(\mathcal{W})$ and the next-to-leading correction is written in terms of non-holomorphic potential $\mathcal{H}(\mathcal{W}, \bar{\mathcal{W}})$ where $\mathcal{W}$ and $\bar{\mathcal{W}}$ are $\mathcal{N} = 2$ superfield strengths (see e.g. the review [1]).

$\mathcal{N} = 2$ supersymmetry strongly restricts the form of holomorphic potential what was demonstrated by Seiberg and Witten for $SU(2)$ SYM model in Coulomb branch of inequivalent vacua in which the low energy theory has unbroken $U(1)$ gauge factors [2]. Extension of this result for various gauge groups and coupling to matter was given in ref. [8] (see also the review [9]). General form of holomorphic potential for arbitrary $\mathcal{N} = 2$ model is now well established.

Computation of non-holomorphic potential is more delicate and a general form of $\mathcal{H}(\mathcal{W}, \bar{\mathcal{W}})$ is still unknown although some contributions to $\mathcal{H}$ were obtained for special cases. In $\mathcal{N} = 2$ superconformal invariant models and $\mathcal{N} = 4$ SYM theory the non-holomorphic potential has been found in Coulomb phase [5] – [9]. Here all beta functions vanish and the evolutions under the renormalization group is trivial. This effective potential is turned out to be exact solution for $\mathcal{N} = 4$ SYM theory, its explicit form is given only by one-loop contribution, any higher-loop or instanton corrections are absent [6], [9] – [11]. However all above results correspond to abelian background $\mathcal{W}$ and $\bar{\mathcal{W}}$ for the theory, living on a point of general position of the moduli space, where one has the symmetry-breaking pattern: $SU(N) \to U(1)^{N-1}$ and all physical quantities vary smoothly over the moduli spaces. The moduli space becomes an orbifold, so that it is flat ”almost” everywhere else and has infinite curvature at the origin. The singularities of the moduli space are associated with the presence of new massless particles in the spectrum [13]. Besides, there exists a curve of marginal stability where otherwise stable BPS states become degenerate and can decay into a very few strong-coupling states [14]. In addition, all such points have an enhanced non-abelian symmetry which forms some non-abelian background. In this region, unstability of the low-energy approximation is expected to be broken down when the derivatives of scalar fields and $U(1)$-field strength become large. In particular, an analysis of such regions is important for understanding a quantum corrected form of BPS solution in strong coupling region [15]. As to non-abelian background, the non-holomorphic potential was found only for very special cases in refs. [8] – [12].

One of the basic approaches to evaluating the effective action is a derivative expansion. This approach allows to get the effective action in form of a series in derivatives of its functional arguments. Within $\mathcal{N} = 1$ supersymmetric derivative expansion, the leading contributions to effective action are formed by so called Kählerian and chiral superfield effective potentials. The Kählerian effective potential has a structure analogous to conventional effective potential, its form has been recently investigated for various $\mathcal{N} = 1$ supersymmetric models. Supercovariant derivatives depending corrections to Kählerian effective potential can be found using the methods developed in refs. [16] – [19]. We point out that the Kählerian effective potential naturally arises in $\mathcal{N} = 2$ SYM models if ones formulate these models in terms of $\mathcal{N} = 1$ superfields [21] and, as a result, it allows to construct the potentials $\mathcal{F}(\mathcal{W})$ and $\mathcal{H}(\mathcal{W}, \bar{\mathcal{W}})$ on its ground.

The powerful approach to evaluating the effective action in $\mathcal{N} = 2$ SUSY models can be
developed within harmonic superspace \cite{20} since this superspace provides a formulation of $\mathcal{N} = 2$ supersymmetric theories in terms of unconstrained $\mathcal{N} = 2$ superfields and, therefore, preserves a manifest off-shell $\mathcal{N} = 2$ supersymmetry. Structure of effective action of $\mathcal{N} = 2$ and $\mathcal{N} = 4$ SYM models in harmonic superspace has been studied in refs \cite{8, 11, 22, 23}.

Another line of current study of the effective action in extended SUSY theories is associated with realization of these theories on the world volume of branes. Such a realization provides a dual description of low-energy field dynamics in terms of D-brane theory. Webs of intersecting branes as a tool for studying the gauge theories with reduced number of supersymmetries have been introduced in ref. \cite{24}. The fivebrane construction has been successfully applied to the computation of holomorphic (or rather BPS) quantities of the four dimensional supersymmetric gauge theory (see refs. \cite{25, 26}). The fivebrane configurations corresponding to these $\mathcal{N} = 1$ supersymmetric gauge theories encode the information about the $\mathcal{N} = 1$ moduli spaces of vacua. The non-holomorphic quantities such as higher derivative terms in $\mathcal{N} = 2$ theories and the Kählerian potential of $\mathcal{N} = 1$ supersymmetric gauge theories have a special interest since they are not protected by supersymmetry. It was shown that the Kählerian potential on the Coulomb branch of $\mathcal{N} = 2$ theories is correctly reproduced from the classical dynamics of M-theory fivebrane. As to the non-holomorphic contributions to low-energy effective action, such as the higher derivative terms, a correspondence between string/brane approach and four-dimensional $\mathcal{N} = 2$ supersymmetric Yang-Mills theories beyond two-derivative level, is not completely established (see e.g. ref. \cite{25, 26, 27}). Moreover, in an arbitrary $\mathcal{N} = 2$ SYM model coupled to hypermultiplets, a general solution to the function $H$ and its derivative depending corrections in the points of moduli space corresponding to non-abelian background is still far to be found.

In this paper we discuss some aspects of structure of non-holomorphic effective potential for nonabelian background in order to pay attention on a problem of its gauge dependence. We show that for unbroken $SU(2)$ gauge group the one-loop non-holomorphic potential can be exactly calculated for a wide class of gauge fixing conditions. It is generally known that the contributions to the effective action, that contain factors of the classical equations of motion, aren’t uniquely defined. They are often ignored. An example of this ambiguity is a dependence of the effective action on the choice of gauge conditions in a gauge theory. This fact is related to the parameterization non-invariance of the conventional effective action (see e.g. ref. \cite{29}) and leads to a number of different effective actions corresponding to the one classical action.

We present an extended supersymmetrical $R_\xi$-gauge for SYM models within background field method. The choice of a gauge fixing term in spontaneous broken non-abelian gauge theories has a basic technical importance. It is known that the use of the $R_\xi$-gauge became a major step in the proof that Yang-Mills models are unitary, on-shell gauge independent and renormalizable quantum field theories. One of our motivation is to provide a useful "laboratory" for studying a full structure of low-energy EA in hypermultiplet model coupled non-abelian $\mathcal{N} = 2$ vector multiplet.

The structure of the paper is as follows:

In the second section we introduce general notations and remind the known procedure of reduction $\mathcal{N} = 2$ superfields and action to $\mathcal{N} = 1$ superspace. Section three presents the background field quantization method for the model under consideration. The extension of $R_\xi$-gauge fixing for the model is also introduced in this section. In the forth section we
study the gauge-dependence of $\mathcal{N} = 1$ superfield the Kähler potential and consider the problem of reconstruction $\mathcal{H}$. In short summary we discuss the results obtained.

## 2 $\mathcal{N} = 2$ SYM Theory in $\mathcal{N} = 1$ Superspace

A most simple and well developed description of four-dimensional supersymmetric field theories is formulation in terms of $\mathcal{N} = 1$ superspace. Although the $\mathcal{N} = 2$ supersymmetric models can be constructed in harmonic superspace $[20]$ preserving manifest $\mathcal{N} = 2$ supersymmetry, the $\mathcal{N} = 1$ formulation is still very useful and fruitful for study of the various quantum aspects in the $\mathcal{N} = 2$ supersymmetric models.

From point of view of $\mathcal{N} = 1$ supersymmetry a field content of pure $\mathcal{N} = 2$ SYM model is given by vector multiplet superfield $V$ and chiral superfield $\Phi$ and a field content of hypermultiplet is given by two chiral superfields $Q_+, \bar{Q}_-$. This allows to write an action $S$ of the $\mathcal{N} = 2$ SYM model coupled to hypermultiplet matter in $\mathcal{N} = 1$ superspace as follows

$$S = S_{\text{SYM}} + S_{\text{Hyper}}$$

$$S_{\text{SYM}} = \frac{1}{T(R)g^2} \text{tr} \left[ d^6z \left( \frac{1}{2} W^a W_a + d^8z \Phi e^{\Phi} e^{-V} \right) \right],$$

$$S_{\text{Hyper}} = \int d^8z (Q_+ e^V Q_+ + Q_- e^{-V} Q_-) + i \int d^6z Q_+ \Phi Q_+ + i \int d^6z \bar{Q}_- \bar{\Phi} Q_-,$$

where the superfields $V = V^A T^A$ and $\Phi = \Phi^A T^A$ form the $\mathcal{N} = 2$ gauge multiplet with component fields $(A_\mu, \lambda_{\pm}, \phi)$ belonging to the adjoint representation of gauge group $G$ and the superfields $Q_{\pm}$ form the hypermultiplet with component fields $(\psi_+, H_\pm, \psi_-)$ belonging to some representation $\mathcal{R}$ of $G$. We use the conventions of ref. $[30]$. It should be noted that the used gauge coupling constant $g$ is $\sqrt{2}$ times the usual $g$. The $T^A$ are the generators of a gauge group with $[T^A, T^B] = i f^{ABC} T^C$. These generators satisfy the normalizing conditions $\text{tr}(T^A T^B) = T(R) \delta_{AB}$, $(T^A)_{ij}(T^A)_{jk} = C(R) \delta_{ik}$ and $f^{ABC} f^{BCD} = C_2(G) \delta^{AB}$. The term $Q_- \Phi Q_+$ in the Lagrangian (3) means $Q_- i(T^A)_{ij} Q_+ j \Phi^A$.

The classical actions $S_{\text{SYM}}$ and $S_{\text{Hyper}}$ are gauge invariant and manifestly $\mathcal{N} = 1$ supersymmetric by the construction. However the full action $S$ is also invariant under the hidden $\mathcal{N} = 2$ supersymmetry transformations, which can be written in terms of covariant chiral superfields $\Phi_c = e^\Omega \Phi e^{-\Omega}$, $Q_{+c} = e^{\Omega} Q_+$ etc.

$$\delta \Phi_c = e^\Omega W_a,$$

$$\delta W_a = -\epsilon_a \nabla^2 \Phi_c + i \epsilon_a \nabla_{aa} \Phi_c,$$

$$\delta \bar{W}_a = -\bar{\epsilon}_a \nabla^2 \bar{\Phi}_c + i \epsilon_a \nabla_{aa} \bar{\Phi}_c.$$

$$\delta \bar{Q}_{+c} = \bar{Q}_{+c} (\Delta_1 \Omega) - \nabla^2 (Q_- c \chi),$$

$$\delta Q_{-c} = -(\Delta_1 \Omega) Q_{-c} + \nabla^2 (\chi Q_+ c),$$

$$\delta Q_{+c} = - (\Delta_2 \Omega) Q_{+c} + \nabla^2 (\bar{\chi} Q_- c),$$

$$\delta Q_{-c} = Q_{-c} (\Delta_2 \Omega) - \nabla^2 (\bar{\chi} Q_+ c),$$

$$\Delta_1 \Omega = e^{-\Omega} \delta e^\Omega = i \chi \Phi_c,$$

$$\Delta_2 \Omega = e^{\Omega} \delta e^{-\Omega} = i \bar{\Phi}_c \bar{\chi},$$

$$\chi = \lambda(\theta) + \bar{\lambda}(\bar{\theta}).$$

Here $\Omega$ is a complex superfield determining the gauge superfield $V$ in the form $e^V = e^\Omega e^\Omega$, $\lambda$ and $\bar{\lambda}$ are chiral and antichiral space-time independent superfield parameters with the
expansion $\lambda = \gamma + \frac{1}{2}\theta^\alpha \epsilon_\alpha + \theta^2(\beta_1 + i\beta_2)$, where the $\beta_1$, $\beta_2$ parameterize the $SU(2)/U(1)$ group, $\epsilon_\alpha$ are the anticommuting parameters presenting in the eqs (3) and $\gamma$ parameterizes the central charge transformations. Hypermultiplet action and corresponding $\mathcal{N} = 2$ supersymmetry transformations in terms of $\mathcal{N} = 1$ superspace were considered in refs. [30] and [31]. Invariance of the actions $S_{\text{SYM}}$ and $S_{\text{Hyper}}$ under the transformations (4, 5) can be checked straightforwardly. One points out also that both $\mathcal{N} = 2$ super Yang-Mills model and hypermultiplet model are the superconformal invariants [32]. Further we will use only the covariant chiral superfields and subscript $c$ will be omitted.

Low-energy effective action of the model under consideration is described by holomorphic scale dependent effective potential $\mathcal{F}(\mathcal{W})$ and non-holomorphic scale independent real effective potential $\mathcal{H}(\mathcal{W}, \bar{\mathcal{W}})$ where $\mathcal{W}$ is $\mathcal{N} = 2$ superfield strength. The corresponding contributions to effective action can be expressed in terms of $\mathcal{N} = 1$ superfields. The holomorphic part $\Gamma_F$ of low-energy effective action is written in $\mathcal{N} = 1$ form as follows [21]

$$\Gamma_F = \int d^4x d^2\theta \frac{1}{2} \mathcal{F}_{AB}(\Phi) W^{\alpha A} W^{\bar{B}}_{\alpha} + \int d^4x d^2\theta \mathcal{F}_A(\Phi) \bar{\Phi}^A + \text{h.c.}$$ (6)

We use the standard notation $\mathcal{F}_A = \frac{\partial}{\partial \Phi^A} \mathcal{F}, \mathcal{F}_{AB} = \frac{\partial}{\partial \Phi^A} \frac{\partial}{\partial \Phi^B} \mathcal{F}, \mathcal{H}_A = \frac{\partial}{\partial \Phi^A} \mathcal{H}, \mathcal{H}_{AB} = \frac{\partial}{\partial \Phi^A} \frac{\partial}{\partial \Phi^B} \mathcal{H}$ etc. The non-holomorphic contribution $\Gamma_H$ can be given in $\mathcal{N} = 1$ form using the metric, connection and curvature of natural Kähler geometry since the $\mathcal{H}$ is associated with a Kähler potential on a complex manifold defined modulo the real part of a holomorphic function

$$\Gamma_H = \int d^4x d^2\theta (g_{AB}[-\frac{1}{2} \nabla^{\alpha A} \phi^A \nabla_{\alpha B} \bar{\phi}^B + i \bar{W}^{B\dot{a}}(\nabla^\alpha W^A_{\alpha} + \Gamma^A_{CD} \nabla^\alpha \Phi^C W^D_{\alpha}) -$$

$$- (f^{ACD} \bar{W}^{B\dot{a}} \Phi^C \nabla_\alpha \Phi^D + f^{BCD} W^{A\alpha} \bar{\Phi}^C \nabla_\alpha \Phi^D) + (\nabla^2 \Phi^B + \frac{1}{2} \Gamma^{B\dot{a}}_{CD} W^{\alpha C} \bar{W}^{\dot{a} D}) \times$$

$$\times (\nabla^2 \Phi^A + \frac{1}{2} \Gamma^A_{EF} W^{E\alpha} W^F_{\alpha})] + \frac{1}{4} R_{ABCD}(W^{A\alpha} W^C_{\alpha} W^{B\dot{a}} \bar{W}^{\dot{a} D}) +$$

$$+ i \mu^A(\frac{1}{2} \nabla^\alpha W^A_{\alpha} + f^{ABC} \Phi^B \bar{\Phi}^C )],$$ (7)

where the last term in (7) written in terms (see ref. [33]) of the moment map (or Killing potential) $i \mu^A(\Phi, \bar{\Phi}) = f^{ABC} \mathcal{H} c^A \Phi^B$ and $g_{AB} = \mathcal{H}_{AB}, \Gamma^A_{BC} = g^{AD} \mathcal{H}_{BCD}, R_{ABCD} = \mathcal{H}_{ACBD} - g_{EF} \Gamma^E_{AC} \Gamma^F_{BD}$. The momentum map explains the nature of the auxiliary fields and it is used to write down the scalar potential of $\mathcal{N} = 2$ theories. It should be noted that on $\mathcal{N} = 1$ language the representation (7) for the non-holomorphic potential $\mathcal{H}(\mathcal{W}, \bar{\mathcal{W}})$ is essentially as expansion over derivatives $W, \Phi$ and an arbitrary number of external $\Phi$ superfields. Expression (7) can be also found directly from eq. (A-3). Coefficients of the expansion (7) containing derivatives $\mathcal{H}$ can be written by means of natural for Kähler geometry unit vectors $e^A = \Phi^A/\sqrt{\delta^2}, e^A = \bar{\Phi}^A/\sqrt{\delta^2}$, projectors $\Pi^{AB} = \delta^{AB} - e^A e^B$, $\Pi^{AB} = \delta^{AB} - e^A e^B$ and their derivatives. Being expressed in terms of component fields, the contribution to effective action $\Gamma_H$ contains at most four space-time derivatives. Obtaining the non-holomorphic contribution to the effective action in a form of an integral over $\mathcal{N} = 1$ superspace is based on decomposition of the non-abelian superfield strengths $\mathcal{W}, \bar{\mathcal{W}}$ in terms of $\mathcal{N} = 1$ superfields, which is given in the Appendix.

In this paper we analyze a general form of the one-loop functionals $\Gamma_F$ and $\Gamma_H$ in the model under consideration using functional methods in $\mathcal{N} = 1$ superspace and revise the contributions to the effective action which determine a functional dependence of $\mathcal{F}$ and $\mathcal{H}$.
on the $\mathcal{N} = 2$ vector multiplet. Eqs (6,7) play a very important role in such an approach since they ensure a bridge between $\mathcal{N} = 1$ and $\mathcal{N} = 2$ descriptions and allow to restore manifestly $\mathcal{N} = 2$ supersymmetric functionals on the base of their $\mathcal{N} = 1$ projections.

Calculations of the low-energy effective action are based on the following reasoning: We compute one-loop contributions to the effective potential $K(\Phi, \bar{\Phi})$ induced by both $\mathcal{N} = 2$ vector multiplet and hypermultiplet in a wide class of gauge-fixing conditions. This effective potential depends on $\mathcal{N} = 1$ chiral superfields which are a part of the $\mathcal{N} = 2$ vector multiplet. The functionals (6,7) also contain terms depending only on $\Phi, \bar{\Phi}$ and this allows to restore such terms on the base of the given effective potential $K(\Phi, \bar{\Phi})$. It is known that this effective potential can not be written in the form $\bar{\Phi}F'(\Phi)$, which saturates the R-anomaly. The additional scale independent terms in the effective potential $K$ originate from a real function $H$. This function $H$ can be determined from comparison of the last term in (7) and effective potential $K$. The other terms in (7) arise from a momentum expansion in $\mathcal{N} = 1$ superspace and related by extra hidden $\mathcal{N} = 1$ supersymmetry to one another. Therefore, they can be exactly found on the base of the terms depending only on $\Phi, \bar{\Phi}$.

3 Background Field Quantization

3.1 Quantum-background splitting

To construct the effective action in the $\mathcal{N} = 1$ SYM theory with matter multiplets we use the background field method which is a powerful and convenient tool for studying the structure of a quantum gauge theory (see refs. [30, 33]). This method begins with the so-called background-quantum splitting of the initial gauge and matter superfields into two parts — into background superfields and the quantum superfields, according to the following transformation $e^{V} \rightarrow e^{\Omega}e^{V}e^{\check{\Omega}}$ and $\Phi \rightarrow \Phi + \phi$ in the actions (2, 3). As a result, these actions will be written as the functionals of the background superfields $\Omega, \check{\Omega}, \Phi, \check{\Phi}$ and quantum ones $V, \phi, \check{\phi}$.

To quantize the theory we impose the gauge-fixing conditions only on the quantum fields, introduce the corresponding ghosts and consider the background fields as the functional arguments of the effective action. Using the proper gauge fixing functions one can construct the effective action which will be invariant under the initial classical gauge transformations. Due to this property the effective action depends only on background strengths $W_\alpha$ and $\bar{W}_{\dot{\alpha}}$, covariantly-chiral superfields $\Phi$ and $\check{\Phi}$ and their covariant derivatives.

The gauge transformations of the quantum superfields $\phi$ and $V$ are written as follows

$$
\phi' = e^{i\Lambda}(\Phi + \phi)e^{-i\Lambda} - \Phi, \quad \bar{\phi}' = e^{i\check{\Lambda}}(\check{\Phi} + \bar{\phi})e^{-i\check{\Lambda}} - \check{\Phi},
$$

$$
\delta V = i(\check{\Lambda} - \Lambda) - \frac{i}{2}[V, \check{\Lambda} + \Lambda] + O(V^2),
$$

(8)

Namely these transformations must be fixed by proper gauge conditions imposed on the quantum superfields.

3.2 Gauge-fixing procedure

The basic step of the background field method is use of the gauge fixing conditions which are covariant under the background gauge transformations. We choose the proper gauge-
fixing conditions for the quantum superfields $V$ and $\phi$ in the form

$$ F^A = \nabla^2 V^A + i\lambda \frac{1}{\square_+} \nabla^2 \phi ^B \Phi ^C f^{ABC}, \quad F^A = \nabla^2 V^A - i\bar{\lambda} \frac{1}{\square_-} \nabla^2 \bar{\phi} ^B \bar{\Phi} ^C f^{ABC}, $$

(9)

where $\lambda, \bar{\lambda}$ are the arbitrary numerical parameters and standard notations $\square_{\pm}$ for Laplace-like operators in the superspace are used. In space of chiral and antichiral superfields these operators act accordingly

$$ \nabla^2 \nabla^2 = \square_+ = \square - i\tilde{W} \tilde{\nabla} - \frac{i}{2}(\nabla \tilde{W}), \quad \nabla^2 \nabla^2 = \square_- = \square - iW^a \nabla_a - \frac{i}{2}(\nabla W). $$

(10)

It is evident that the gauge fixing functions (9) are covariant under background superfield transformations. The gauge fixing functions (9) can be considered as a superfield form of so-called $R_{\xi}$-gauges which are used often in spontaneously broken gauge theories. Extension of $R_{\xi}$-gauge fixing conditions to $\mathcal{N} = 1$ superfield theories has been given in ref. [34].

Gauge fixing action corresponding to the functions (9) is constructed in the standard form

$$ S_{GF} = -\frac{1}{\alpha g^2} \int d^8z \left( F^A \bar{F}^A + b^A \bar{b}^A \right) $$

(11)

and depends on extra parameter $\alpha$. Invariance of this action under the background gauge transformations is evident

Action of the Faddeev-Popov ghosts $S_{FP}$ for the gauge fixing functions (9) has the form

$$ S_{FP} = \text{tr} \int d^8z \left( (\bar{c}' c - c' \bar{c}) - \left( c'[\Phi, \frac{\lambda}{\square_+} [\bar{c}, \Phi]] + c' \left[ \frac{\bar{\lambda}}{\square_-} [c, \Phi], \Phi \right] \right) \right). $$

(12)

The theory under consideration demands, besides Faddeev-Popov ghosts, the extra, so called Nielsen-Kallosh ghosts $b$ and $\bar{b}$. The full ghost action be

$$ S_{GH} = S_{FP} + \text{tr} \int d^8z \bar{b} b $$

(13)

The actions (11, 13) can be rewritten in more convenient form if we introduce the following $\Phi$-dependent denotations:

$$ X^{AB} = f^{ABC} \Phi ^C, \quad \bar{X}^{AB} = f^{ABC} \bar{\Phi} ^C. $$

(14)

Using these denotations, integrating by part and dropping the irrelevant for one-loop calculation terms one gets the expression for the gauge fixing action

$$ S_{GF} = -\frac{1}{\alpha g^2} \int d^4\theta \left( \nabla^2 V^A \bar{\nabla}^2 V^A + i\lambda V^A \bar{X}^{AB} \phi ^B - i\bar{\lambda} V^A X^{AB} \bar{\phi} ^B + \lambda \bar{\lambda} \nabla^2 \phi ^B \nabla^2 \bar{\phi} ^B \left( \frac{1}{\square_-} \bar{X}^{BA} \nabla^2 \frac{1}{\square_+} X^{AE} \right) \right) $$

(15)

We point out, because the parameter $\Lambda$ of the quantum field transformations (8) is chiral the ghosts $c, c'$ and $b$ are covariant chiral superfields. $\nabla_\alpha c = \nabla_\alpha c' = \nabla_\alpha b = 0$.

The quadratic part of the full ghost action (13) which is relevant for one-loop calculations can be also given in terms of the denotions (14)

$$ S_{\text{ghost}} = \int d^8z \left( \bar{c}' A c^A - c' A \bar{c}^A + \bar{c}^B \frac{\lambda}{\square_-} \bar{X}^{BE} X^{EA} c^A + c' B \frac{\bar{\lambda}}{\square_-} \bar{X}^{BE} X^{EA} c^A + b^A \bar{b}^A \right). $$

(16)
To carry out the loop calculations, we expand the total action \( S = S_{\text{SYM}} + S_{\text{Hyper}} + S_{\text{GH}} + S_{\text{GF}} \) in power series in quantum fields. Only quadratic terms in this expansion are relevant in one-loop approximation. The corresponding quadratic part of total action can be written as follows

\[
S_2 = S_{\text{gauge}} + S_{\text{chiral}} + S_{\text{mix}} + S_{\text{ghost}}.
\]  

The contributions from the quantum field \( V \) is given by

\[
S_{\text{gauge}} = -\frac{1}{2g^2 T(R)} \int d^4 \theta V^A \left( \Box - iW^\alpha \nabla_\alpha - i\tilde{W}^\dagger \tilde{\nabla}_\dagger - (1 - \alpha^{-1})\{\nabla^2, \tilde{\nabla}^2\} - M \right)^{AB} V^B,
\]

where \( \Box = \frac{1}{2} \nabla^\alpha \nabla_\alpha \) is the background covariant d’Alambertian and \( W_\alpha, \tilde{W}_\dagger \) are the background field strengths. The mass matrix \( M^{BA} = -\frac{1}{2} \tilde{X}^{B \bar{E} X \bar{E} A} \) in the action \( S_{\text{gauge}} \) arises from term \( \frac{1}{2} \tilde{\Phi}[V, [V, \Phi]] \) in the action \( S_{\text{SYM}} \) (see (2)). The action \( S_{\text{mix}} \) contains terms mixing the quantum gauge and the chiral superfields:

\[
S_{\text{mix}} = \int d^4 \theta (\tilde{\phi}[V, \Phi] + [\tilde{\Phi}, V]\phi).
\]

The action \( S_{\text{chiral}} \) is quadratic in the quantum chiral superfield \( \phi \) and \( Q \)

\[
S_{\text{chiral}} = \text{tr} \left( \int d^4 \theta \bar{\phi} \phi + \int d^4 \theta (\bar{Q} + Q_+ + Q_- Q_-) + i \int d^2 \theta Q_- \Phi Q_+ + i \int d^2 \theta Q_+ \Phi Q_- \right).
\]

All one-loop contributions to effective action are given in terms of the functional trace \( \text{Tr} \ln(\hat{H}) \), where the operator \( \hat{H} \) is the matrix of the second variational derivatives of the action \( S_2 \) in all quantum fields. The one-loop effective action in the model under consideration reads

\[
\Gamma[V, \Phi] = \frac{i}{2} \text{Tr} \ln \hat{H}_{\text{SYM}} + i \text{Tr} \ln \hat{H}_{\text{Hyper}} - \frac{i}{2} \text{Tr} \ln \hat{H}_{\text{ghost}},
\]

with

\[
\hat{H}_{\text{SYM}} = \begin{pmatrix}
(\bar{O}_V)^{BA} & i\gamma X^{BA} \nabla^2 & -i\gamma \tilde{X}^{BA} \tilde{\nabla}^2 \\
-i\gamma \nabla^2 X^{BA} & (1 + \hat{R})^{BA} \nabla^2 \tilde{\nabla}^2 & 0 \\
-i\gamma \nabla^2 X^{BA} & 0 & (1 + \hat{R})^{BA} \tilde{\nabla}^2 \tilde{\nabla}^2 
\end{pmatrix}
\]

\[
\hat{H}_{\text{Hyper}} = \begin{pmatrix}
\delta^i_{\bar{J}} \nabla^2 \nabla^2 & i\bar{\Phi}^i \nabla^2 \\
i\bar{\Phi}^i \nabla^2 & \delta^i_{\bar{J}} \tilde{\nabla}^2 \tilde{\nabla}^2 
\end{pmatrix}
\]

\[
\hat{H}_{\text{ghost}} = \begin{pmatrix}
0 & \nabla^2(1 + G^T) \nabla^2 & 0 & 0 \\
0 & 0 & 0 & \nabla^2(1 + \tilde{G}) \nabla^2 \\
0 & 0 & \nabla^2(1 + \tilde{G}^T) \nabla^2 & 0
\end{pmatrix}
\]

where we use the following notation \( \hat{R}^{BA} = \sum_{\alpha} \tilde{X}^{B \bar{E} X \bar{E} A} \frac{1}{\alpha}, \)

\( R^{BA} = \frac{\lambda}{\alpha} \tilde{X}^{B \bar{E} X \bar{E} A} \quad \tilde{G}^{AB} = \sum_{\alpha} \tilde{X}^{A \bar{E} X \bar{E} B} \) and \( O_V = -\Box + iW^\alpha \nabla_\alpha + i\tilde{W}^\dagger \tilde{\nabla}_\dagger + (1 - \alpha^{-1})\{\nabla^2, \tilde{\nabla}^2\} + M \). Constants \( \gamma \) and \( \tilde{\gamma} \) are defined as \( \tilde{\gamma} = (1 - \tilde{\lambda}/\alpha), \gamma = (1 - \lambda/\alpha) \). The operator \( \hat{H}_{\text{SYM}} \) contains the contributions from \( \mathcal{N} = 1 \) vector and chiral multiplets forming \( \mathcal{N} = 2 \) gauge multiplet. One can see, that choice \( \lambda = \tilde{\lambda} = \alpha \) greatly simplifies all calculation because it diagonalizes the matrix \( \hat{H}_{\text{SYM}} \) and decouples the contributions.
from $\mathcal{N} = 1$ vector and chiral multiplets. However we will keep the gauge parameters $\lambda$ and $\alpha$ arbitrary and investigate an dependence of effective action on these parameters.

Since the EA is expressed in form of functional determinant of the differential operator $\hat{H}$, its calculation can be carried out on the base of Fock-Schwinger proper-time technique appropriately formulated in superspace (see aspects of such a formulation in refs. [18, 13, 33]).

Exact calculations of the functional traces defining the one-loop effective action is possible only for very specific backgrounds when eigenvalues and eigenfunctions of the operators under consideration are known, that is rather exception then a rule. Further we will use a derivative expansion of the effective action based on an symbol operator technique adapted to $\mathcal{N} = 1$ supersymmetric field models (see [19] for details). Our purpose is the calculations of the leading and subleading low-energy contributions to the one-loop effective action.

4 $\mathcal{N} = 1$ Kähler and Non-holomorphic $\mathcal{N} = 2$ Potentials

4.1 $\mathcal{N} = 1$ Kähler potential

In this section we study the form of the non-abelian low-energy effective action $\Gamma = \int d^8z K$ and its gauge dependence. We compute the one-loop contributions to Kähler effective potential $K$ induced by both $\mathcal{N} = 2$ vector multiplet and hypermultiplet. It is known that in the non-abelian case the Kähler potential cannot be written in the form $Im(\bar{\Phi}F'(\Phi))$ consistent with the rigid version of special geometry (see e.g. refs. [5, 17]). The additional terms originate from a real function $\mathcal{H}_0 (W, \bar{W})$ of the $\mathcal{N} = 2$ YM superfield strength $W$. The results obtained in the present paper are more general in compare with ones obtained in refs. [5, 12, 17] since we have used here the more general and complicated gauges.

We study the one-loop effective action for $SU(2)$-gauge model described by (2), (3) and (11) with $R_{\xi}$-gauge fixing (9) and Faddeev-Popov (16) terms in the case when the gauge vector field $V$ is purely quantum. We find Kähler potential in $\mathcal{N} = 1$ superspace, and then, the holomorphic $\mathcal{F}$ and nonholomorphic $\mathcal{H}$ potentials in $\mathcal{N} = 2$ superspace. To calculate these potentials we consider the diagrams with external $\Phi$, $\bar{\Phi}$ lines corresponding only to the constant field background. Such a choice of background superfields leads to a number of technical simplifications due to the absence of the background gauge field, which allows us replace all background covariant derivatives by flat ones (i.e. $\nabla \rightarrow D, \bar{\nabla} \rightarrow \bar{D}$). This provides a possibility to use the superspace projectors $P_1 = \frac{1}{6} \bar{D}^2 D^2, \ P_2 = \frac{1}{6} D^2 \bar{D}^2, \ P_T = -\frac{1}{6} D \bar{D} D\bar{D}$ and $\Pi_0 = P_1 + P_2$ and simplify the evaluations of the functional determinants (21).

Eq (24) allows us to write the ghost contribution as follows

$$\text{Tr} \ln (H_{\text{ghost}}) = \text{Tr} \left( \ln (1 + G) + \ln (1 + \bar{G}) \right) \Pi_0. \tag{25}$$

Notation $\text{Tr}(\cdots)$ means $\text{tr} \int d^8z (\cdots)$ as usual. Matrices $R$ and $G$ from the (22) and (24) are expressed in terms of $X$ and $M$. Using the identities $\text{tr} \ln O = \ln \det O$ and
\[ O = \frac{1}{N!} \epsilon^{a_1 \ldots a_N} \epsilon_{c_1 \ldots c_N} O^c_a O^d_b \ldots \] one can obtain (25)

\[ \text{Tr} \ln(H_{\text{ghost}}) = 2 \int d^8 z \left( \ln \left( 1 - \frac{\lambda}{\square} \Phi \Phi \right) + \ln \left( 1 - \frac{\bar{\lambda}}{\bar{\square}} \bar{\Phi} \bar{\Phi} \right) \right) \Pi_0, \quad (26) \]

where \( \bar{\Phi} \Phi \) means the scalar product in isospin space.

The contribution of the hypermultiplet to the effective action for any representation of gauge group is given by

\[ \text{Tr} \ln(H_{\text{Hyper}}) = \frac{1}{2} \text{Tr} \ln \left( 1 + \left( \begin{array}{cc} \frac{1}{\square} \Phi \Phi P_2 & 0 \\ 0 & \frac{1}{\bar{\square}} \bar{\Phi} \bar{\Phi} P_1 \end{array} \right) \right) = \frac{1}{2} \text{Tr} \int d^8 z \ln \left( 1 + \frac{\Phi \Phi}{\square} \right) \Pi_0 \quad (27) \]

where the trace is taken over the representation of the hypermultiplet. The eigenvalues of the matrix \( (\Phi \Phi)^{i \bar{j}} \) containing in the definition of hypermutiplet contribution \( \text{Tr} \ln H_{\text{Hyper}} \) in the fundamental representation are \( ((\Phi \Phi) \pm \sqrt{(\Phi \Phi)^2 - \Phi^2 \bar{\Phi}^2})/4 \). For adjoint representation we have

\[ \text{Tr} \ln(H_{\text{adj}}^{\text{Hyper}}) = \frac{1}{2} \text{Tr} \ln \left( 1 - \frac{X \bar{X}}{\square} \right)^B A \Pi_0 = \int d^8 z \ln(1 - \frac{(\Phi \Phi)}{\square}) \Pi_0. \quad (28) \]

The other contributions in (21) are given by

\[ \text{Tr} \ln H_{\text{SYM}} = \text{Tr} \ln \left( 1 - \frac{M}{\square} \right) P_T + \]

\[ + \ln \left( 1 - \frac{1}{\square} (\Phi \Phi)(\lambda + \bar{\lambda}) + \lambda \bar{\lambda} \left( \frac{\Phi \Phi}{\square} \right)^2 \right) \Pi_0 + \]

\[ + \ln \left( 1 - \frac{1}{\square} (\Phi \Phi)(\lambda + \bar{\lambda}) + \lambda \bar{\lambda} \left( \frac{\Phi \Phi}{\square} \right)^2 - \frac{1}{2} \lambda \bar{\lambda} \left( \Phi \Phi - \Phi^2 \bar{\Phi}^2 \right) + \]

\[ + \frac{\alpha}{\square^2} \left( (\Phi \Phi)^2 - \Phi^2 \bar{\Phi}^2 \right) \left( \frac{\lambda \bar{\lambda}}{4} (\Phi \Phi) + \frac{\lambda + \bar{\lambda}}{2} - \frac{\alpha}{4} \right) \Pi_0 \right]. \]

The computations for the first term in (29) lead to

\[ \text{Tr} \ln(1 - \frac{\Phi \Phi}{\square}) P_T = \ln \left( 1 - \frac{\Phi \Phi}{\square} \right) P_T + \ln \left( 1 - \frac{\Phi \Phi}{\square} + \frac{(\Phi \Phi)^2 - \Phi^2 \bar{\Phi}^2}{4 \square^2} \right) P_T \quad (30) \]

Taking into account the results above one can obtain the Kähler potential in the model under consideration. For actual computation we use the technique which was described in detail in ref. [19]. According to this technique in the case under consideration it is sufficient to fulfil the following replacements

\[ \square \to - k^2, \quad \int d^4 k \to i \int_0^\infty \frac{k^2 dk^2}{(4\pi)^2}, \quad \Pi_0 \to - \frac{2}{k^2}, \quad P_T \to \frac{2}{k^2}, \quad (31) \]

and integrate over \( k^2 \). The momentum integral is divergent and needs regularization. All cut-off dependence contributes only to renormalization of the initial action. The typical integrals are given by following expression

\[ \int_0^{\Lambda^2} dk^2 \ln \left( 1 + \frac{A}{k^2} \right) = -A \ln \frac{A}{e \Lambda^2} \quad (32) \]
where the notation $t$ is given by this expression. Dependence of the one-loop effective action on all gauge parameters is which automatically vanishes for abelian background fields $\Phi$. This is the main result of equation (35) was found in ref. [17]. Result for Landau-DeWitt gauge is obtained at $\alpha = 0, \lambda = \bar{\lambda} = 1$. Note that (35) in the gauge $\alpha = \lambda = \bar{\lambda} = 1$, which can be naturally called as Fermi-DeWitt, two last terms in the first line (34) are exactly cancelled by (35) while the first term (34) is being doubled.

Using the integral (32) in (35) we obtain

$$K_{GD} = e_1 \ln(-e_1) + e_2 \ln(-e_2) + e_2 \ln(-e_2),$$

where $e$'s are the roots of the polynomial, which appears in process of integration of (35)

$$e_1 = -\frac{2}{3} + \frac{1}{6} R_1, \quad e_2 = -\frac{2}{3} - \frac{1}{12} R_1 + \frac{i}{12} R_2,$$

Finding the roots of the polynomial and using eq. (36) we get the final result for (35):

$$K_{GD}(s^2, \gamma) = \Phi \bar{\Phi} K_{GD} = \lambda \frac{\Phi \bar{\Phi}}{(4\pi)^2} \left( -\frac{2}{3} \ln(s^2) + \frac{R_1}{12} \ln(e_1^2) + \frac{i}{12} R_2 \ln(e_2) \right),$$

where we have used the notation $s^2 = 1 - \frac{\Phi^2 \bar{\Phi}^2}{(\Phi \bar{\Phi})^2} < 0$.

2) The effective action $\Gamma_{SYM} = \Gamma_V + \Gamma_{GD}$ induced by $N = 2$ vector multiplet contains vector loop contribution

$$K_V = -\frac{1}{(4\pi)^2} \int dk^2 \ln(1 + \frac{(\Phi \bar{\Phi})}{k^2}) + \ln(1 + \frac{(\Phi \bar{\Phi}) - \sqrt{\bar{\Phi}^2 \Phi^2}}{2k^2}) + \ln(1 + \frac{(\Phi \bar{\Phi}) + \sqrt{\bar{\Phi}^2 \Phi^2}}{2k^2}) =$$

$$= \frac{1}{(4\pi)^2} \left( \frac{\Phi \bar{\Phi}}{e^2 \Lambda^4} \ln(1 + \frac{\Phi \bar{\Phi}}{e^2 \Lambda^4}) + \frac{\Phi \bar{\Phi}}{\sqrt{\Phi^2 \bar{\Phi}^2}} \ln(t + \sqrt{\Phi^2 \bar{\Phi}^2} \left[ \frac{t + \frac{1}{2}}{t - \frac{1}{2}} \ln \frac{t + 1}{2} + \frac{1}{2} \ln \frac{t - 1}{2} \right] \right),$$

where the notation $t = \frac{\Phi \bar{\Phi}}{\sqrt{\Phi^2 \bar{\Phi}^2}}$ was introduced, plus

3) The gauge dependent contribution

$$K_{GD} = \frac{1}{(4\pi)^2} \int dk^2 \ln \left( 1 + \frac{(\Phi \bar{\Phi})}{k^2} + \frac{(\Phi \bar{\Phi}) - \sqrt{\bar{\Phi}^2 \Phi^2}}{2k^2} \right) \left[ -\frac{\lambda \bar{\lambda}}{2} + \alpha \left( \frac{\lambda \bar{\lambda}}{4k^2} + \frac{\lambda + \bar{\lambda}}{2} - \frac{\alpha}{4} \right) \right] =$$

$$= \frac{1}{(4\pi)^2} \int dk^2 \ln \left( \frac{k^2 - e_1}{k^2} \cdot \frac{k^2 - e_2}{k^2} \cdot \frac{k^2 - e_3}{k^2} \right),$$

The final result is a sum of three terms:

1) The hypermultiplet contribution to effective action

$$K_{Hyper}^{fund} = -\frac{1}{(4\pi)^2} \int dk^2 \ln(1 + \frac{(\Phi \bar{\Phi}) + \sqrt{(\bar{\Phi} \Phi)^2 - \Phi^2 \bar{\Phi}^2}}{4k^2}) =$$

$$= -\frac{1}{(4\pi)^2} \int dk^2 \ln(1 + \frac{(\Phi \bar{\Phi}) - \sqrt{(\bar{\Phi} \Phi)^2 - \Phi^2 \bar{\Phi}^2}}{4k^2}) =$$

$$= -\frac{1}{(8\pi)^2} (\Phi \bar{\Phi}) \left( \ln \frac{\Phi^2 \bar{\Phi}^2}{16e^2 \Lambda^4} + s \ln \frac{1 + s}{1 - s} \right),$$

(33)
\[
R_1 = \Delta_1 + \Delta_2, \quad R_2 = \Delta_1 - \Delta_2, \quad \Delta_{1,2} = \left(-b \pm \sqrt{-a^2 + b^2}\right)^{\frac{1}{2}},
\]
\[
a = 4 + 3s^2(\gamma^2 - 4\gamma + 2), \quad b = -8 + 9s^2(2\gamma^2 - 5\gamma + 4), \quad \gamma = \frac{a}{s},
\]
and we have assumed \(\lambda = \bar{\lambda}\). This form of gauge-dependent part of Kähler potential allows to investigate a gauge dependence of non-holomorphic effective potential \(\mathcal{H}\).

### 4.2 \(\mathcal{N} = 2\) non-holomorphic potential

In previous subsection we have found the one-loop Kähler effective potential \(K(\Phi, \bar{\Phi})\) induced by both \(\mathcal{N} = 2\) vector multiplets and hypermultiplets. As it has been mentioned in refs. [16, 17], the Kähler potential in the nonabelian case determines not only by the holomorphic function \(F\). The additional terms originate from a real function \(\mathcal{H}(W, \bar{W})\) of the \(\mathcal{N} = 2\) Yang-Mills superfield strength \(W\), which is integrated over full \(\mathcal{N} = 2\) superspace. We can derive the one-loop contribution to \(F\) and \(\mathcal{H}\) comparing the last term in decomposition (7) with Kähler potential. It leads to

\[
K(\Phi, \bar{\Phi}) = \bar{\Phi}^A F_A + \Phi^2 (\bar{\Phi}^A H_A) - (\Phi \bar{\Phi})(\Phi^A H_A).
\]

(39)

where

\[
\Phi^A H_A = 0, \quad \bar{\Phi}^A H_A = -\frac{2\Phi^2}{(\Phi \bar{\Phi})} s^2 \frac{\partial \mathcal{H}}{\partial s^2}.
\]

It is well known that \(\beta\)-function and axial anomaly exactly arise from holomorphic potential \(F\). This fact gives us a unique recept for extracting contributions from Kähler potential, which can be associated with holomorphic and non-holomorphic potentials respectively.

Using the expressions (33) and (34) and the reconstruction formula (39), ones find, in accordance with ref. [16], the contributions to holomorphic potential \(\mathcal{F}(W)\) and to non-holomorphic potential \(\mathcal{H}(W, \bar{W})\) depending on the \(\mathcal{N} = 2\) superfield strength \(W\):

\[
\mathcal{F}_{\text{Hyper}}^{\text{fund}} = \frac{-1}{(8\pi)^2} W^2 \ln \frac{W^2}{e^2 \Lambda^2}
\]

(40)

\[
\mathcal{F}_{\text{Vector}} = \frac{1}{(16\pi)^2} \ln \frac{1+s}{1-s}
\]

(41)

\[
\mathcal{H}_{\text{Hyper}} = \frac{1}{2} \left(\int_0^t du \ln \frac{u}{u-1} + 2 \ln \frac{t+1}{2} \ln \frac{t-1}{2}\right).
\]

(42)

\[
\mathcal{H}_{\text{Vector}} = \frac{-1}{(8\pi)^2} \left(-\text{Li}_2(1-t^2) + 2 \ln \frac{t+1}{2} \ln \frac{t-1}{2}\right).
\]

(43)

It is interesting to point out that eq. (43) can be exactly rewritten in terms of Euler dilogarithm function \(\text{Li}_2(t)\) (see, e.g. ref. [35])

\[
\mathcal{H}_{\text{Vector}} = \frac{-1}{(8\pi)^2} \left(-\text{Li}_2(1-t^2) + 2 \ln \frac{t+1}{2} \ln \frac{t-1}{2}\right)
\]

(44)

Our further aim is to obtain off-shell gauge-dependent contribution to \(\mathcal{H}\) from the gauge-dependent part of the full Kähler potential. In this case eq. (33) is written in the form

\[
-2s^2(1-s^2) \frac{d\mathcal{H}_{\text{GD}}}{ds^2} = K_{\text{GD}}(s^2),
\]

(45)
where $\tilde{K}_{GD}$ was introduced in eq. (38), $s^2 = 1 - 1/t^2$ and $t = \frac{W^2}{\sqrt{W^4+W^2}}$, $t \in [0, 1]$. It has already been noticed that $K_{GD} = 0$ at $s^2 \to 0$ and therefore $H_{GD}$ vanishes on-shell.

We see the holomorphic potential $F$ is gauge independent. All dependence on gauge-fixing parameters is concentrated in the term $H_{GD}$ of non-holomorphic potential $H$. Our next aim is to obtain the $H_{GD}$. The form (38) of Kähler potential is not very convenient for this aim and further analysis because of its complicated structure, though it reproduces all known results as partial cases. Therefore we reformulate (38) to more simple and suitable form using the special algebraic methods. Let us present (38) as a formal power series. Eq. (36) is nothing but a determination of a symmetrical function via the polynomial roots. According to the fundamental theorem in theory of symmetrical functions (see e.g. ref. [36]) “any entire rational symmetrical function can be uniquely rewritten as a entire rational function of elementary symmetrical functions” (i.e. coefficients of the polynomial).

To represent (36) as an entire rational function we expand the logarithms into a formal power series

$$K_{GD} = -\sum_{n=1}^{\infty} \frac{1}{n} S_n,$$

where the power symmetrical functions of the roots $e_1, e_2, e_3$ of the form

$$S_n = e_1(1+e_1)^n + e_2(1+e_2)^n + e_3(1+e_3)^n$$

has been used. Using classical recursion Newton’s formulae we can uniquely express $S_n$ in terms of elementary symmetrical functions.

It is well known that the roots $e_i$ of an algebraic equation are always satisfy the Vieta relations. For the roots (37) of the polynomial, which appears from the numerator in the logarithm of (35) we have

$$-e_1 e_2 e_3 = g_3, \quad e_1 e_2 + e_2 e_3 + e_1 e_3 = g_2, \quad e_1 + e_2 + e_3 = -2,$$

where elementary symmetrical functions are given from (37, 48) as $g_2 = 1 + s^2(-\frac{1}{2} + \gamma(1 - \frac{s^2}{4})), \quad g_3 = s^2 \frac{\gamma}{4}$.

Multiplying eq. (47) by $e_1 + e_2 + e_3$ and using identities (48) we obtain the recursion relation

$$S_{n+1} - S_n - (1-g_2)S_{n-1} + (1-g_2+g_3)S_{n-2} = 0.$$  

Using this relation one can evaluate any $S_n$ step by step. Writing out the few first symmetrical functions

$$S_1 = 2(1 - g_2), \quad S_2 = -2(1 - g_2) - 3g_3, \quad S_3 = -2(1 - g_2)^2 - g_3,$$

$$S_4 = -6(1 - g_2)^2 - 6(1 - g_2)g_3 - g_3, \ldots$$

one can see that $S_n \sim s^2$ for any $n$. It allows to simplify integration in eq. (45). In addition, we note that each $S_n$ includes $g_3$ linearly.

Moreover, the known Waring formulae (see. e.g. [36]) allow to express $S_n$ for any $n$ directly in terms of $g_2, g_3$. In order to get all $S_n$, it is very useful to introduce a generating function defined by a formal power series

$$G(\tau) = \sum_{k=1}^{\infty} \tau^{k-1} S_k,$$
then any \( S_n \) can be found with help of differentiations of the generating function \( G \) with respect to \( \tau \). It also allows us to express a general term of the sequence \( S_n \) in terms of symmetrical functions \( g_2 \) and \( g_3 \) instead of the roots \( e_i \). Since the functions \( g_2, g_3 \) are known from the integral (33), we can avoid finding the roots \( e_i \) for analysis \( H_{GD} \) at all.

The generating function \( G \) satisfies an algebraic equation which can be derived by multiplying the recursion relation by \( \tau^k \) and summing over powers \( k \). The solution to this equation is

\[
G(\tau) = \frac{2(1 - g_2) - 4(1 - g_2)\tau - 3g_3\tau + 2(1 - g_2 + g_3)\tau^2}{1 - \tau - (1 - g_2)\tau^2 + (1 - g_2 + g_3)\tau^3}.
\]  

As a result we obtain an expansion of \( K_{GD} \) in terms of elementary symmetrical functions \( g_2, g_3 \):

\[
K_{GD} = -\sum_{n=1}^{\infty} \frac{1}{n!} \left( \frac{d}{d\tau} \right)^{n-1} G(\tau)|_{\tau=0}.
\]  

Now, it is useful to introduce the new parameters \( g = -1/2 + (\gamma/2 - 1)^2, \ g_3 = \gamma/4, \ p = g + g_3, \ u = 1 - s^2 \). Using the binomial formula for derivatives of the generating function (52) in (53), we rewrite the equation (45) in the following form

\[
-2u \frac{dH_{GD}}{du} = \sum_{k=0}^{\infty} \frac{1}{(k + 3)!} (4g - g_3(k + 1)(k + 5)) \left( \frac{d}{d\tau} \right)^k Y|_{\tau=0},
\]  

where \( Y^{-1} = 1 - \tau - g\tau^2 + p\tau^3 + u(g\tau^2 - p\tau^3) \). It is useful to extract, in the right hand side of eq. (54), the powers of \( u \) and rewrite this relation in form of double sum

\[
-2u \frac{dH_{GD}}{du} = \sum_{l=0}^{\infty} u^l \sum_{k=0}^{\infty} \frac{k!}{(k + 3)!} (4g - g_3(k + 1)(k + 5)) \left[ \frac{1}{k!} \left( \frac{d}{d\tau} \right)^k Q_l \right]|_{\tau=0},
\]  

and

\[
Q_l = \frac{(-g\tau^2 + p\tau^3)^l}{(1 - \tau - g\tau^2 + p\tau^3)^{l+1}}.
\]

Last expression allows to find \( H_{GD} \) as a series with a coefficient, at each given power of \( u \), depending on elementary symmetrical functions. Hence, we finally can rewrite (45) in terms of elementary symmetrical functions. We see that the right hand side (53) can be written via rational functions for any given choice of gauge parameters. For some partial choice of gauge parameters, arbitrary term of series can be found exactly. Therefore, directly finding several first derivations \( \left( \frac{d}{d\tau} \right)^k Q_l \) at \( \tau = 0 \) (for example by Mathematica software) in right side (53) one can restore the general term of the series (55) and then directly fulfil summation over powers \( u \). As a result, the gauge-dependent part of effective action can always be written in any given gauge as a series over powers of elementary symmetrical functions. The actual summation of such a series can be realized for any specific choices of gauge parameters. We point out that the procedure described above can be used, in principle, for evaluating a functional determinant for an arbitrary higher order non-minimal operator in the low-energy approximation. Actually, the only we need is a system of roots of a polinomial corresponding to the operator in the momentum representation.
For example, taking only linear in $\alpha$ and $\lambda$ terms in (55) we obtain the few first terms in the expansion of $H_{GD}$ for arbitrary $\alpha$, $\lambda$:

$$H_{GD}(\lambda, \alpha) = \frac{1}{(4\pi)^2} \left[ \lambda \left( \sqrt{2} \ln(1 + \sqrt{2}) - \ln(2) \right) + \frac{\alpha}{2} (3 - \ln(\varepsilon)) + \cdots \right] \ln(t) + O(t), \quad (56)$$

where $\varepsilon$ is IR cut-off parameter. The occurrence of IR-divergence could be seen directly from (46) and (50). As it has been mentioned above $S_n \sim g_3$ and therefore the corresponding series is divergent. But, for $\alpha = 0$ the IR-divergence does not appear (i.e. in the Landau-DeWitt gauge: $\alpha = 0, \lambda = 1$).

Let’s consider the Landau-DeWitt gauge in more detail. At such a choice $Y^{(k)}$ in (54) becomes enough simple

$$Y^{(k)} = k! \left( \frac{1}{1 - a^2} - \frac{(-a)^{k+1}}{2(1 + a)} - \frac{a^{k+1}}{2(1 - a)} \right), \quad a^2 = \frac{s^2}{2} \quad (57)$$

and the general term in right side (54) can be exactly found. For example

$$\sum_{k=0}^{\infty} \frac{k! a^k}{(k + 3)!} = \frac{1}{6} {}_2F_1(1, 1; 4; a) = \frac{1}{4a^3} \left[ a(3a - 2) - 2(1 - a)^2 \ln(1 - a) \right], \quad (58)$$

and

$$\sum_{k=0}^{\infty} \frac{k!}{(k + 3)!} = \frac{1}{6} {}_2F_1(1, 1; 4; 1) = \frac{1}{4}, \quad (59)$$

where ${}_2F_1$ is the Gauss hypergeometric function (see, e.g. [35]).

Finally, the expression (54) becomes

$$(1 - 2a^2) \frac{dH_{GD}}{da} = \frac{1-a}{a} \ln(1-a) + \frac{1+a}{a} \ln(1+a) \quad (60)$$

and we obtain $H_{GD}$ by integration

$$2(4\pi)^2 H_{GD} = \ln(2) \ln(1 - s^2) + \frac{1}{\sqrt{2}} \ln \left( \frac{\sqrt{2} - 1}{\sqrt{2} + 1} \right) \ln(1 - s^2) - \text{Li}_2 \left( \frac{s^2}{2} \right) +$$

$$+ \frac{\sqrt{2} - 1}{\sqrt{2}} \left[ \text{Li}_2 \left( \frac{s - 1}{\sqrt{2} - 1} \right) + \text{Li}_2 \left( -\frac{s + 1}{\sqrt{2} - 1} \right) \right] +$$

$$+ \frac{\sqrt{2} + 1}{\sqrt{2}} \left[ \text{Li}_2 \left( \frac{s + 1}{\sqrt{2} + 1} \right) + \text{Li}_2 \left( \frac{1-s}{\sqrt{2} + 1} \right) \right]. \quad (61)$$

It should be noted that for the considered gauge, $H_{GD}$ can be also found by direct integration from (33) with the same result (61). That proves correctness of the described above method. We remind that $s^2 = 1 - \frac{W^2 W^2}{(WW)^2} \leq 0$. Of course, on-shell, where $W$ and $\bar{W}$ are abelian, we have $s = 0$ and therefore $H_{GD}$ vanishes as the other contributions to $H$.

We emphasize that expressions (42, 43) and (61) are exact results within one-loop approximation. Of course, they can be expanded in series in two limit cases: $t \to 1$ and $t \to 0$. At $t \to 1$ (almost abelian case) we have

$$H_{Vector} \sim -\frac{1}{(8\pi)^2} \left( (t - 1) \ln \left( \frac{t - 1}{2} \right) + \cdots \right),$$

$$H_{Hyper} \sim \frac{1}{(8\pi)^2} \left( 2(t - 1) - \frac{1}{3} (t - 1)^2 + \cdots \right). \quad (62)$$
To analyze the analytical properties of $\mathcal{H}_{GD}$ near of this point it is useful to present solution to eq. (60) by the series

$$
2(4\pi)^2\mathcal{H}_{GD} = -\frac{1}{2} \ln(1 - s^2) + \sum_{k=1}^{\infty} \left( \frac{s^2}{2} \right)^{k+1} \frac{2F_1(1, k + 1; k + 2; s^2)}{(2k + 1)(k + 1)^2}.
$$

(63)

This representation is defined when $|s^2| < 1$ and provides the obvious power expansion. Moreover, we can analytical extend $\mathcal{H}_{GD}$ in the complex plane with the cut $[1, \infty]$ in order to analyze its behavior near of all branching points. For example, from the known identity

$$
2F_1(1, k + 1; k + 2; s^2) = (k + 1)\Phi(s^2, 1, k + 1),
$$

where $\Phi$ is so called the Lerch transcendental function (see, e.g. [35]). Taking into account that $\lim_{s^2 \to 1} \Phi(s^2, 1, k + 1)/\ln(1 - s^2) = -1$ we get at $s^2 \to 1$ (or $t^2 \to -\infty$) the logarithmical branch point

$$
\mathcal{H}_{GD} \sim \ln(1 - s^2)(\ln(2) + \frac{1}{\sqrt{2}} \ln(\frac{\sqrt{2} - 1}{\sqrt{2} + 1})) + \ldots
$$

(64)

At small $t$, large $-s^2$ (equivalently, large ”mass”) we have an asymptotic Schwinger-DeWitt perturbation series in inverse powers of ”mass” $\sqrt{W^2\bar{W}^2}$ and a logarithmic branch point, that can be seen from another representation of eq.(63)

$$
2(4\pi)^2\mathcal{H}_{GD} = -\frac{1}{2} \ln(t^2) + \sum_{k=1}^{\infty} \left( \frac{t^2 - 1}{2} \right)^{k+1} \frac{2F_1(k + 1, k + 1; k + 2; 1 - t^2)}{(2k + 1)(k + 1)^2}.
$$

(65)

Summation of the asymptotic series at $t \sim 0$ leads to

$$
\mathcal{H}_{GD} \sim -\frac{1}{(8\pi)^2} (t^2 \ln(t^2) + \sqrt{2}\pi t + \ldots)
$$

(66)

In this region on the isolated branch of the multifunction we also have

$$
\mathcal{H}_{\text{Vector}} \sim \frac{-1}{(8\pi)^2} \left( 2i\pi t - t^2 \ln(t^2) + \ldots \right),
$$

$$
\mathcal{H}_{\text{Hyper}} \sim \frac{1}{(8\pi)^2} \left( \pi t - t^2 + \ldots \right).
$$

(67)

Let us remember that the appearance of the imaginary part related to the second term in eq. (63), which (as it has been mentioned earlier) is missing in the Fermi-DeWitt gauge. Such a behavior is not unusual and it looks like quite analogous to a well-known exactly solvable model in effective field theory — namely the Euler-Heisenberg effective action. It is pointing out some more property of the Euler-Heisenberg effective action at small mass (strong external field), it possesses by logarithmic branch point as well as $\mathcal{H}_{GD}$, $\mathcal{H}_{\text{Vector}}$ while at large mass (weak external field) there exists an asymptotic series expansion in inverse powers of mass (see e.g. [37]).

We showed that the gauge-dependent part of off-shell effective action can be found with an arbitrary level of accuracy and at any choice of gauge fixing parameters. The form of the non-holomorphic effective potential has an essential arbitrariness due to its explicit gauge dependence. In particular, this fact leads to the ambiguous definition of
$R_{ABCD}(W^A\bar{W}^\alpha W^C\bar{W}^\beta \bar{W}^D)_{\alpha\beta}$ term from eq. (7), which should reproduce the leading term in the expansion of the non-abelian analog of the Born-Infeld action (see, e.g. [28]). The structure of the tensor $R_{ABCD}$ is enough cumbersome. Besides this we point out that the symmetrized trace $(F^+)^2(F^-)^2/\phi^2\bar{\phi}^2$, defying the full set of $F^4$-terms in effective action also contains the various contractions $\phi^A, \bar{\phi}^A$ with $F^A$. Existence a large class of gauge theory operators, which correspond to supergravity modes and contain nontrivial extra factors (depending on $\phi^A, \bar{\phi}^A$), in non-abelian Born-Infeld action was discussed in refs. [28, 38].

To conclude this subsection we note that unlike the abelian case, $\mathcal{N} = 2$-supersymmetry itself can not uniquely fix a form of next-to-leading term in the effective action because of its explicit gauge dependence.

5 Summary

We have studied the non-holomorphic potential depending on non-abelian strengths $W$ and $\bar{W}$ in $\mathcal{N} = 2$ supersymmetric theory of $SU(2)$ gauge multiplet coupled to hypermultiplet. The theory under consideration was realized in terms of $\mathcal{N} = 1$ superfields and described by $\mathcal{N} = 1$ gauge superfield interacting with three chiral superfields in some representations of gauge group. We quantized the theory withing background field method using three-parametric $\mathcal{N} = 1$ supersymmetric $R_\xi$-type gauge and constructed the corresponding quadratic action describing the one-loop effective action.

We have calculated the Kählerian effective potential depending on $\mathcal{N} = 1$ chiral superfield projection of $\mathcal{N} = 2$ superfield strength taking the values in Lie algebra of $SU(2)$ group and containing all three gauge parameters. Using the special methods of the polynomial algebra we developed a general recurrent procedure of obtaining manifestly $\mathcal{N} = 2$ supersymmetric non-holomorphic potential for a class of gauge parameters under consideration. This potential reproduces all previous results on non-holomorphic potential in non-abelian background as partial case. The procedure we have developed to compute the gauge dependent contribution to effective one-loop effective action is quite generic and, in principle, it can be considered as a new method of calculating an one-loop effective action for arbitrary (non-minimal, higher order) differential operators.

The special case of supersymmetric Landau-DeWitt gauge was investigated in more details. It is turned out to be that the non-holomorphic potential is exactly found for this case in terms of Euler dilogarithm function. We have also studied the various limiting situations of the non-holomorphic potential, in particular ”near on-shell” limit and ”large mass” $\sqrt{W^2\bar{W}^2}$ limit. It is interesting to point out that Euler dilogarithms occur in many problems associated with quantum $\mathcal{N} = 2$ supersymmetric field models (see e.g. ref. [39]).

6 Acknowledgments

The authors are grateful to S.J. Gates for the useful comments. The work was supported in part by INTAS grant, INTAS-00-00254. I.L.Buchbinder is also grateful to RFBR grant, project No 02-02-04002 and to DFG grant, project No 436 RUS 113/669 for partial support. The work of N.G.Pletnev and A.T.Banin was supported in part by RFBR grant, project No 00-02-17884.
Appendixes

A \ N = 1 \text{ structure of } \mathcal{N} = 2 \text{ superfield strengths}

The $\mathcal{N} = 2$ SYM theory is usually formulated in the ordinary $\mathcal{N} = 2$ superspace by imposing certain constraints on the gauge and super covariant derivative $\nabla_{\alpha a}$ and $\nabla^a_{\alpha}$

$$
\{\nabla_{\alpha a}, \nabla_{\beta b}\} = i C_{\alpha \beta} C_{\alpha \beta} \mathcal{W}, \quad \{\nabla^a_{\alpha}, \nabla^b_{\beta}\} = i C^{a b} C_{\alpha \beta} \mathcal{W}, \quad \{\nabla_{\alpha a}, \nabla^b_{\beta}\} = i \delta^b_\alpha \nabla_{\alpha \beta}, \quad \text{(A. 1)}
$$

where $\mathcal{W}, \overline{\mathcal{W}}$ are chiral (antichiral) scalar superfield strengths, respectively (see ref. [30]). The $\mathcal{N} = 2$ chiral superfield $\overline{\nabla} \alpha \mathcal{W} = 0$ is reducible, unlike its $\mathcal{N} = 1$ counterpart. To achieve an irreducible superfield we may additionally impose important the constrains of reality condition

$$
\nabla^\alpha a \nabla_{\alpha b} \mathcal{W} = C_{a c b d} \overline{\nabla} \alpha i a \nabla_{\alpha c} \overline{\mathcal{W}}. \quad \text{(A. 2)}
$$

The rigid $SU(2)_R$ indices are raised or lowered with the help of the antisymmetric invariant tensor $C_{a b}$, with $C_{1 2} = C^{1 2} = 1$. It is customary to represent the solution to these constraints in the form of an $\mathcal{N} = 1$ superfield expansion. We defines $\mathcal{N} = 1$ superfield components both in the adjoint representation nonabelian gauge group as $\mathcal{W} = \phi$, $\nabla_{\alpha a} \mathcal{W} = -W_a$ etc. The bar denotes setting $\bar{\theta}^2 = \eta^a = 0$, $\bar{\eta}^2 = \bar{\eta} = 0$. As a result of reducing $\mathcal{N} = 2$ superfield to $\mathcal{N} = 1$ form we find:

$$
\mathcal{W} = \Phi^A - \eta^a W_A - \eta^2 \nabla^2 \Phi + i \eta^a \bar{\eta}^a \nabla_{\alpha a} \Phi + \eta^2 \bar{\eta}^2 (i \nabla^a W_a + f^{ABC} \bar{\Phi} B \nabla_{\alpha} \bar{\Phi} C) + \eta^2 \eta^a f^{ABC} \Phi B \nabla_{\alpha} \Phi C + \eta^2 \eta^2 (\nabla A + \frac{1}{2} f^{ABC} \bar{\Phi} B \nabla_{\alpha} \bar{\Phi} C - f^{ABC} (\nabla^a \Phi B) W_a - \frac{1}{2} f^{ABC} \Phi B \Phi D E)
$$

$$
\tilde{\mathcal{W}} = \tilde{\Phi}^A - \bar{\eta}^a \tilde{W}_A - \bar{\eta}^2 \nabla^2 \tilde{\Phi} - i \bar{\eta}^a \bar{\eta}^a \nabla_{\alpha a} \tilde{\Phi} + \bar{\eta}^2 \bar{\eta}^2 (i \nabla^a \tilde{W}_a + f^{ABC} \tilde{\Phi} B \nabla_{\alpha} \tilde{\Phi} C) + \bar{\eta}^2 \bar{\eta}^a f^{ABC} \tilde{\Phi} B \nabla_{\alpha} \tilde{\Phi} C + \bar{\eta}^2 \bar{\eta}^2 (\nabla A + \frac{1}{2} f^{ABC} \tilde{\Phi} B \nabla_{\alpha} \tilde{\Phi} C - f^{BCA} (\nabla^a \tilde{\Phi} B) \tilde{W}_a - \frac{1}{2} f^{ABC} \tilde{\Phi} B \tilde{\Phi} D E).
$$

Component expansion $\mathcal{N} = 1$ superfields is well known. Namely this eq. (A. 3) has been used to derive $\mathcal{N} = 1$ form of low-energy effective action (7). Another way of derivating was given in ref. [3].

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