CHEMICAL ENGINEERING | RESEARCH ARTICLE

Prediction of bed pressure drop, fluctuation and expansion ratios for three-phase fluidization of ternary mixtures of dolomite in a conical conduit

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Abstract: Hydrodynamics of conical fluidized bed differ from that of columnar beds by the fact that a velocity gradient exists along the axial direction of the bed. The gas–liquid–solid fluidized bed has emerged in recent years as one of the most promising devices for three-phase operations. Such a device is of considerable industrial importance as evident from its wide applications in chemical, refining, petrochemical, biochemical processing, pharmaceutical, and food industries. To explore this, experiments have been carried out to find the bed pressure drop, bed fluctuation, and expansion ratios for ternary mixtures of dolomite in a three-phase conical fluidized bed. The effect of superficial liquid and gas velocity, initial static bed height, average particle size, and cone angle on the above-mentioned three responses have been studied. Mathematical models have been developed for the responses using both dimensional and statistical analyses. The calculated values of the responses from the developed models have shown a very good agreement with the experimental ones.

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PUBLIC INTEREST STATEMENT

Experiments have been carried out to find the bed pressure drop, bed fluctuation, and expansion ratios for ternary mixture of dolomite in a three-phase conical fluidized bed. The effect of superficial liquid and gas velocity, initial static bed height, average particle size, and cone angle on the three responses have been studied. Mathematical models have been developed for the responses using both dimensional and statistical analyses. The calculated values of the responses from the developed models have shown a very good agreement with the experimental ones.
1. Introduction

With the development of fluidized bed coal combustion and the recent interest in the use of such beds for waste utilization and dry solids separation, potential applications of multi-component fluidized beds are on the rise. It is because of the fact that fluidized particles of uniform size at the beginning, may change due to attrition, coalescence, and chemical reaction, thereby affecting the quality of fluidization by high elutriation loss, de-fluidization, segregation, and inhomogeneous residence time in the bed leading to non-uniform products of wide particle size distribution. Therefore, Sau, Mohanty, and Biswal (2008) stated that proper characterization of bed dynamics for binary and multi-component mixtures in gas–solid systems is an important prerequisite for their effective utilization, where the combination of particle size, density, and shape influences the fluidization behavior.

Biswal, Sahu, and Roy (1982), Biswal, Samal, and Roy (1984), Biswal, Bhowmik and Roy (1984, 1985) developed theoretical models for minimum fluidization velocity and bed pressure drop for spherical particles for gas–solid systems in conical vessels. Due to angled walls, random and unrestricted particle movement occurs in a tapered surface with reduced back mixing. They, therefore, proposed a modified equation for the calculation of the maximum pressure drop. Later, Peng and Fan (1997) made an in-depth study of hydrodynamic characteristics of solid–liquid fluidization in a tapered bed and derived theoretical models for the prediction of minimum fluidization velocity and maximum pressure drop, based on the dynamic balance of forces exerted on the particle. However, the experiments were carried out for spherical particles only. Jing, Hu, Wang, and Jin (2000) and Shan et al. (2001) proposed models for $\Delta P_{mf}$ and $U_{mf}$ for gas–solid conical fluidized beds for spherical coarse and fine particles based on Peng and Fan (1997) models, but neglected pressure drop due to the kinetic change in the bed.

A correlation for fluctuation ratio in conical vessels for regular particles has been developed by Biswal, Samal, et al. (1984) using dimensional analysis approach based on four dimensionless groups neglecting the effect of density of gas and solid particles. They have developed a correlation for the bed fluctuation ratio for irregular particles in conical vessel. Singh, Roy, and Suryanarayana (1991) have developed correlations for bed fluctuation ratio for binary homogeneous and heterogeneous mixtures of spherical and non spherical particles in conical conduits. Singh, Suryanarayana, and Roy (1999) have also developed correlations for bed expansion ratio for cylindrical and non-cylindrical beds. Dora, Panda, Mohanty, and Roy (2013) have studied the bed expansion and fluctuation ratios in a gas–solid conical fluidized bed for homogeneous ternary mixture of irregular particles.

Current literature deals with the development of mathematical models for fluctuation and expansion ratios for binary mixtures in a conical bed with different cone angles. Practically no work has been carried out for three-phase fluidization in a conical bed. The objective of the present work is to study the hydrodynamic characteristics of ternary mixture in a three-phase conical fluidized bed with different cone angles viz. 4.61°, 5.13°, 7.47°, and 11.2° (incorporating the similar values of an earlier study (Dora et al., 2013) and to develop mathematical model for the determination of fluctuation and expansion ratios by dimensional analysis as well as statistical analysis.

2. Materials and methods

The experimental setup consists of a single-stage air compressor of sufficient capacity, an accumulator for storage of air at constant pressure (20 psig), a water tank, and a liquid pump (0.5 HP) as shown in Figure 1. Two rotameters, one for water (0–10 LPM) and the other for air (0–50 LPM) were used to measure the water and air flow rates, respectively. A 40 mesh screen at the bottom served
as the support as well as the distributor. The inside hollow space of the distributor was filled with glass beads of 1.5 cm outer diameter for uniform water and air distribution. The conical conduits with different cone angles are made up of Perspex sheets to allow visual observation. Detailed dimensions for the conical conduits used are given in Table 1. Two pressure tapings were provided for noting the bed pressure drop. A gate valve of 15 mm inner diameter was provided in the line to control the water flow to the bed. Two sets of manometer with carbon tetrachloride (for low pressure range) and mercury (for high pressure range) as manometric liquids were used to record the bed pressure drop. A high-speed digital camera has been used for verification of the maximum and minimum heights of the bed during fluidization.

Three closely sized samples of dolomite (as presented in Table 2) were used for the investigation. For ternary mixtures, fairly good mixing has been achieved by coning and quartering method as done in experimental practice and classification has been avoided since the ratio of the particle sizes of two successive fractions in the mixture was kept below 1.3. The scope of the experiments is presented in Table 2. A weighed quantity of the mixture was poured into the conical column. Prior to recording any data, the charge was vigorously fluidized with water at a velocity where no entrainment was observed. After a certain time, the water flow was suddenly stopped to obtain a mixed packed bed. The velocity of the water was then increased slowly allowing sufficient time to reach a steady state. Then air was passed through the bed as a dispersed phase. The mass velocity of the air ($G_f$) was increased slowly allowing sufficient time to reach a steady state. The rotameter and manometer readings were noted for each increment in flow rate from which the values of the

![Figure 1. Schematic diagram of experimental setup.](image)

### Table 1. Dimensions of conical conduits

| Dimension               | Tapered angle (in degree) |
|-------------------------|---------------------------|
|                         | 4.61                      | 5.13 | 7.47 | 9.52 | 11.2 |
| Bottom diameter (m)     | 0.048                     | 0.050| 0.042| 0.050| 0.045|
| Top diameter (m)        | 0.132                     | 0.135| 0.174| 0.212| 0.245|
| Height of the column (m)| 0.520                     | 0.470| 0.504| 0.483| 0.510|
superficial gas velocity and pressure drop were calculated. The above procedure was repeated for different values of initial static bed height \((H_s)\), average particle size \((D_{pavg})\), and cone angle \((\alpha)\).

In this work, both dimensional analysis and statistical analysis approaches have been used for the prediction of mathematical model for responses like bed pressure drop, fluctuation, and expansion ratios with minimum liquid fluidization mass velocity \((G_{mf})\) and bottom diameter \((D_O)\) as independent variables. In dimensional analysis, the following mathematical model has been used for different responses (Dora et al., 2013).

\[
\text{Response} = K \left( \frac{G_f}{G_{mf}} \right)^{a_1} \left( \frac{H_s}{D_O} \right)^{a_2} \left( \frac{D_{pavg}}{D_O} \right)^{a_3} (\tan \alpha)^{a_4} \tag{1}
\]

where \(a_1, a_2, a_3, \) and \(a_4\) are exponents and \(K\) is the coefficient.

For statistical analysis, central composite design (CCD) has been used to develop correlations with four independent dimensionless variables viz. \(\left( \frac{G_f}{G_{mf}} \right), \left( \frac{H_s}{D_O} \right), \left( \frac{D_{pavg}}{D_O} \right), \) and \((\tan \alpha)\) for the three dependent variables in dimensionless form such as bed pressure drop \(\Delta P\), bed fluctuation ratio \((f)\), and bed expansion ratio \((R)\).

The response has been used to develop an empirical model by statistical analysis that correlates the response of fluidized bed with process variables using a second-degree polynomial equation as given by Equation (2).

\[
Y = b'_0 + \sum_{i=1}^{n} b_i X_i + \sum_{i=1}^{n} b_i^2 X_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} X_i X_j \tag{2}
\]

where \(Y\) is the predicted response, \(b'_0\) the constant coefficient, \(b_i\) the linear coefficients, \(b_{ij}\) the interaction coefficients, \(b_{iij}\) the quadratic coefficients, and \(X_i, X_j\) are the coded values of process variables. The number of tests required \((N)\) for the CCD includes the standard \(2^n\) factorial with its origin at the center, \(2^n\) points fixed axially at a distance, say \(\alpha_s\), from the center to generate the quadratic terms, and replicate tests at the center; where \(n\) is the number of variables. The axial points are chosen such that they allow rotatability (Box & Hunter, 1957) which ensures that the variance in the model prediction is constant at all points equidistant from the design center. Replicates of the test at the center are very important as they provide an independent estimate of the experimental error. For four variables, the recommended number of tests at the center is six (Box & Hunter, 1961).
Once the desired range of values of the variables are defined, they are coded to lie at ±1 for the factorial points, 0 for the center points and ±α for the axial points. For statistical analysis, a statistical software package Design-Expert-8, Stat-Ease, Inc., Minneapolis, USA, has been used for regression analysis of the fluidized bed responses.

3. Results and discussion

3.1. Bed pressure drop

Figure 2(a) shows the variation in bed pressure drop with superficial water velocity for initial static bed height of 0.15 m, mixture of 20:50:30 and cone angle of 7.47° for different superficial air velocities. It has been observed that the bed pressure drop decreases with the increase in superficial air velocity. This is due to the fact that increase in air velocity tends to increase the gas hold-up and which in turn decreases the density of the bed material (Dora, Mohanty, & Roy, 2012). Figure 2(b) shows the variation in bed pressure drop with respect to superficial water velocity for different mixture compositions at constant air velocity of 0.21 m/s, cone angle of 7.47°, and initial static bed height of 0.15 m. It is observed from Figure 2(b) that with the increase in average particle size the bed pressure drop decreases. The decrease may be due to increase in void space with increase in particle size. Similarly, Figure 2(c) shows the variation in bed pressure drop with superficial water velocity for a mixture of 20:50:30, air velocity of 0.21 m/s, and cone angle of 7.47° for different values of initial static bed heights. It is observed that with the increase in initial static bed height, the bed pressure drop increases. This is due to the fact that the pressure drop required for counterbalancing the weight of the bed increases with the increase in initial static bed height. The variation in bed pressure drop with superficial water velocity for different cone angles at an initial static bed height of 0.15 m, superficial air velocity of 0.21 m/s, and for a mixture of 20:50:30 is presented in Figure 2(d). It is clear that with increase in cone angle, pressure drop also increases. This is due to the fact that for a given initial static bed height, with increase in cone angle, the weight of the bed increases, and therefore the pressure drop required to counterbalance the weight of the bed also increases.
3.2. Bed fluctuation ratio

The bed fluctuation ratio ($r$) is defined as the ratio of the highest to the lowest bed height of the fluidized bed in expansion, i.e., $r = h_h / h_1$. The variation in bed fluctuation ratio with superficial water velocity for initial static bed height of 0.15 m with 20:50:30 mixture and cone angle of 7.47° for different values of superficial air velocity is shown in Figure 3(a). It has been observed that the bed fluctuation increases with increase in air velocity. Figure 3(b) shows the variation in bed fluctuation ratio with
superficial water velocity for different mixtures at constant air velocity of 0.21 m/s, initial static bed height of 0.15 m, and cone angle of 7.47°. It is observed that with decrease in particle diameter (i.e. increase in percentage of fines), the bed fluctuation ratio ($r$) increases. The fines entrained by the bubbles compared to course particles are carried to greater height in the bed, resulting in the increase in fluctuation ratio. The variation in bed fluctuation ratio with superficial water velocity for

Figure 2(d). Variation in bed pressure drop with superficial water velocity for cone angles at $H_s = 0.15$ m, $D_{avg} = 0.00127$ m, and $V_g = 0.21$ m/s.

Figure 3(a). Variation in bed fluctuation ratio with superficial water velocity for different superficial air velocity at $H_s = 0.15$ m, $D_{avg} = 0.00127$ m, and cone angle = 7.47°.
mixture of 20:50:30 at air velocity of 0.21 m/s and cone angle of 7.47° for different initial static bed heights is shown in Figure 3(c). Similarly, Figure 3(d) presents the variation in bed fluctuation ratio with superficial water velocity for different cone angles for a mixture of 20:50:30 at air velocity of 0.21 m/s and initial static bed height of 0.15 m. With the increase in bed height and cone angle, bed...
fluctuation ratio is found to decrease as observed from Figures 3(c) and 3(d). Whether it is an increase in bed height or an increase in cone angle, the vertical lift of the particles in the upper segment of the bed for both the above cases is reduced to a considerable extent, thereby reducing the fluctuation of the top of the expanded bed surface and consequently the fluctuation ratio.

Figure 3(d). Variation in bed fluctuation ratio with superficial water velocity for cone angles at $D_{pavg} = 0.00127$ m, $V_g = 0.21$ m/s, and $H_s = 0.15$ m.

Figure 4(a). Variation in bed expansion ratio with superficial water velocity for different superficial air velocity at $H_s = 0.15$ m, $D_{pavg} = 0.00127$ m, and cone angle = $7.47^\circ$.
3.3. Bed expansion ratio
The variation in bed expansion ratio ($R$) with superficial water velocity for the column with cone angle of 7.47° and initial static bed height of 0.15 m with a mixture of 20:50:30 for different values air velocities is shown in Figure 4(a). It is clear from that with the increase in air velocity the bed expansion ratio increases. Figure 4(b) shows the variation in bed expansion ratio with respect to superficial water velocity for different mixtures at constant air velocity of 0.21 m/s, initial static bed height of 0.15 m, and cone angle of 7.47°. It is clear from Figure 4(b) that with an increase in particle size (i.e. decrease in percentage of fines in the mixture), the expansion ratio decreases because of reduction in the amount of fine particles in the mixture, which are generally lifted to a relatively greater height. Figure 4(c) shows the variation in bed expansion ratio with superficial water velocity for the column with cone angle of 7.47°, air velocity of 0.21 m/s, and mixture of 20:50:30 for different values of initial static bed height. Similarly, Figure 4(d) presents the variation in bed expansion ratio with superficial water velocity for columns with different cone angles for a mixture of 20:50:30 at air velocity of 0.21 m/s, and an initial static bed height of 0.15 m. It is clear from Figures 4(c) and 4(d) that with an increase in the initial static bed height and the column cone angle, the expansion ratio decreases. The reason is same as that has been explained in Section 3.2 above for the reduction in the bed fluctuation ratio in these two cases.

3.4. Development of correlations for pressure drop, fluctuation, and expansion ratio by dimensional analysis
The dimensional analysis for the prediction of bed pressure drop, fluctuation, and expansion ratios for a conical fluidized bed have been found to depend upon four dimensionless groups. The values of the dimensionless groups along with their corresponding responses are presented in Table 3. The number of runs as shown in Table 3 is 20. The individual exponents are obtained by plotting a power trend for each response with respect to that of each independent variable keeping the others constant at their respective “zero” levels. After obtaining the exponents, the coefficient is obtained as suggested by Davis (1978). The developed correlations for the three responses are as follows:

$$\frac{\Delta P_f}{\Delta P_{mf}} = 6 \times 10^{-12} \left( \frac{G_f}{G_{mf}} \right)^{-0.216} \left( \frac{H_s}{D_O} \right)^{0.142} \left( \frac{D_{pavg}}{D_O} \right)^{-7.06} (\tan \alpha)^{0.085}$$  \hspace{1cm} (4)

$$r = 8 \times 10^6 \left( \frac{G_f}{G_{mf}} \right)^{0.109} \left( \frac{H_s}{D_O} \right)^{-0.084} \left( \frac{D_{pavg}}{D_O} \right)^{6} (\tan \alpha)^{-0.058}$$  \hspace{1cm} (5)

$$R = 7 \times 10^6 \left( \frac{G_f}{G_{mf}} \right)^{0.093} \left( \frac{H_s}{D_O} \right)^{-0.675} \left( \frac{D_{pavg}}{D_O} \right)^{3.84} (\tan \alpha)^{0.2}$$  \hspace{1cm} (6)

The term $\Delta P_{mf}$ used in the Equations (4 and 8) is calculated from Equation (7) which is correlated using author’s data.

$$\Delta P_{mf} = 3 \times 10^{-6} \left( \frac{H_s}{D_O} \right)^{0.84} \left( \frac{D_{pavg}}{D_O} \right)^{-3.08} (\tan \alpha)^{0.013}$$  \hspace{1cm} (7)

The negative values of the exponent show the antagonistic effect and the positive values of exponent show the synergistic effect of the independent variables on the responses. Figures 5–7 show the comparison of the calculated values of responses with the experimental ones for bed pressure drop ratio, fluctuation, and expansion ratios, respectively. From Figure 5, the value of the coefficient of correlation is found to be 0.97, where the developed correlation for bed pressure drop ratio shows a very good agreement with the experimental data. Similarly, from Figures 6 and 7 the coefficients of correlations are found to be 0.82 and 0.97 for bed fluctuation and expansion ratios, respectively.
3.5. Development of regression model equations using response surface methodology based central composite design

On the basis of experimental data the models have been developed by employing response surface methodology (RSM)-based central composite design (CCD). Analysis of variance (ANOVA) has been used to estimate the statistical parameter. Table 4 represents the complete experimental range and level of variables where Table 5 shows design of experiment together with the experimental result for the three responses. The final empirical models in terms of coded factor (excluding the
Figure 4(d). Variation in bed expansion ratio with superficial water velocity for cone angles at $D_{\text{pavg}} = 0.00127$ m, $V_g = 0.21$ m/s, and $H_s = 0.15$ m

Table 3. Design of experiments for dimensional analysis

| Run | $\frac{g_s}{g_{ref}}$ | $H_s \times 10^2$ | $\frac{D_{\text{pavg}}}{D_o}$ | $\tan \alpha$ | $\frac{\Delta P_s}{\Delta P_{ref}}$ | $r$ | $R$  |
|-----|----------------------|-------------------|-----------------------------|---------------|------------------------------------|-----|-----|
| 1   | 2.38                 | 3.02              | 2.63                        | 0.131         | 0.280                              | 1.45| 3.90|
| 2   | 2.98                 | 3.02              | 2.63                        | 0.131         | 0.292                              | 1.44| 3.60|
| 3   | 3.57                 | 3.02              | 2.63                        | 0.131         | 0.300                              | 1.43| 2.89|
| 4   | 4.17                 | 3.02              | 2.63                        | 0.131         | 0.312                              | 1.40| 2.61|
| 5   | 4.76                 | 3.02              | 2.63                        | 0.131         | 0.320                              | 1.35| 2.46|
| 6   | 3.57                 | 2.92              | 2.63                        | 0.131         | 0.436                              | 1.02| 2.56|
| 7   | 3.57                 | 2.97              | 2.63                        | 0.131         | 0.367                              | 1.23| 2.90|
| 8   | 3.57                 | 3.02              | 2.63                        | 0.131         | 0.314                              | 1.34| 3.13|
| 9   | 3.57                 | 3.09              | 2.63                        | 0.131         | 0.242                              | 1.45| 3.38|
| 10  | 3.57                 | 3.14              | 2.63                        | 0.131         | 0.226                              | 1.48| 3.44|
| 11  | 3.57                 | 3.02              | 0.25                        | 0.131         | 0.351                              | 1.07| 2.54|
| 12  | 3.57                 | 3.02              | 1.44                        | 0.131         | 0.282                              | 1.28| 2.88|
| 13  | 3.57                 | 3.02              | 2.63                        | 0.131         | 0.230                              | 1.39| 3.11|
| 14  | 3.57                 | 3.02              | 3.81                        | 0.131         | 0.157                              | 1.50| 3.36|
| 15  | 3.57                 | 3.02              | 5.00                        | 0.131         | 0.141                              | 1.53| 3.42|
| 16  | 3.57                 | 3.02              | 2.63                        | 0.080         | 0.323                              | 1.31| 3.74|
| 17  | 3.57                 | 3.02              | 2.63                        | 0.089         | 0.336                              | 1.30| 4.08|
| 18  | 3.57                 | 3.02              | 2.63                        | 0.131         | 0.344                              | 1.29| 4.31|
| 19  | 3.57                 | 3.02              | 2.63                        | 0.167         | 0.356                              | 1.26| 4.56|
| 20  | 3.57                 | 3.02              | 2.63                        | 0.198         | 0.364                              | 1.21| 4.62|
Insignificant terms) for bed pressure drop $\left( \frac{\Delta P_f}{\Delta P_{mf}} \right)$, bed fluctuation ratio ($r$), and bed expansion ratio ($R$) have been presented in Equations (8), (9), and (10), respectively, as under:

$$
\frac{\Delta P_f}{\Delta P_{mf}} = 0.203 + 0.0068A - 0.026B - 0.028C + 0.0039D - 0.0007AC - 0.0016A^2 + 0.012C^2 \tag{8}
$$

$$
r = 1.424 - 0.02A + 0.112B + 0.085C - 0.02D + 0.006BC - 0.001CD - 0.025C^2 + 0.004D^2 \tag{9}
$$

$$
R = 3.252 - 0.38A + 0.22B + 0.17C + 0.142D - 0.025AB - 0.016AC + 0.009BD + 0.06A^2 - 0.051C^2 \tag{10}
$$

ANOVA has been used to analyze the experimental factors on bed pressure drop, bed fluctuation, and expansion ratios and to estimate the statistical parameters. The ANOVA for the responses, $\left( \frac{\Delta P_f}{\Delta P_{mf}} \right)$, $r$ and $R$ are represented in Tables 6-8. The $F$-values for the models as depicted in Equations (8–10) are found to be 22.17, 94.72, and 232.33, respectively, which implies that the models are significant. The model $F$-values show that 99.99% of noise has been avoided. Value of “prob. $> F$” less than 0.0500 indicate model terms are significant. Thus, for $\left( \frac{\Delta P_f}{\Delta P_{mf}} \right)$, the model terms A, B, C, D, AC, $A^2$, and $C^2$ are significant. Similarly, for bed fluctuation and expansion ratio the model terms A, B, C, D, BC, CD, $C^2$, and $D^2$ are significant and for bed expansion ratio the model terms A, B, C, D, AB, AC, BD, $A^2$, and $C^2$ are significant.

Figures 8–10 show the comparison of the calculated and actual data obtained for bed pressure drop, bed fluctuation, and expansion ratios, respectively. In Figure 8, the value of coefficient of correlation ($R^2$) has been found to be 0.95, which shows that the model developed agrees well with the experimental data. Similarly, Figures 9 and 10 show the comparison of experimental values with the calculated ones obtained from models shown in Equations (9 and 10) for bed fluctuation and expansion ratios, respectively. The $R^2$ values have been found to be 0.98 and 0.99 for the responses “$r$” and
“R,” respectively, which show that the models developed for both bed fluctuation and expansion ratios agree well with the experimental values. The adequate precision of 18.95, 37.01, and 59.92 obtained for bed pressure drop, fluctuation, and expansion ratios indicate adequate signals. Thus, the model evaluated can be used to navigate the design space. The fair correlation coefficients might have resulted by the insignificant terms in Table 5 and is most likely due to four different
### Table 4. Level of independent variables

| Variables | Symbol | $-\alpha$ | $-1$ | 0   | $+1$ | $+\alpha$ |
|-----------|--------|----------|------|-----|------|----------|
| $\alpha_{\text{in}}$ | A      | 0.25     | 1.44 | 2.63| 3.81 | 5.00     |
| $\alpha_{\text{out}}$ | B      | 2.38     | 2.98 | 3.57| 4.17 | 4.76     |
| $\alpha_{\text{error}} \times 10^2$ | C      | 2.92     | 2.97 | 3.02| 3.09 | 3.14     |
| $\tan \alpha$ | D      | 0.080    | 0.089| 0.131| 0.167| 0.198    |

### Table 5. Design of experiments for statistical analysis

| Run | $H_s$ | $D_{\text{avg}} \times 10^2$ | $G_f$ | $G_{mf}$ | $\tan \alpha$ | $P_f$ | $P_{mf}$ | $R$ | $R$ |
|-----|-------|-----------------------------|-------|---------|----------------|------|---------|-----|-----|
| 1   | 2.98  | 2.97                        | 1.44  | 0.089   | 0.249          | 1.27 | 3.10    |     |     |
| 2   | 4.17  | 2.97                        | 1.44  | 0.089   | 0.265          | 1.24 | 2.47    |     |     |
| 3   | 2.98  | 3.09                        | 1.44  | 0.089   | 0.194          | 1.49 | 3.55    |     |     |
| 4   | 4.17  | 3.09                        | 1.44  | 0.089   | 0.207          | 1.45 | 2.82    |     |     |
| 5   | 2.98  | 2.97                        | 3.81  | 0.089   | 0.202          | 1.41 | 3.39    |     |     |
| 6   | 4.17  | 2.97                        | 3.81  | 0.089   | 0.215          | 1.37 | 2.70    |     |     |
| 7   | 2.98  | 3.09                        | 3.81  | 0.089   | 0.157          | 1.66 | 3.89    |     |     |
| 8   | 4.17  | 3.09                        | 3.81  | 0.089   | 0.168          | 1.61 | 3.10    |     |     |
| 9   | 2.98  | 2.97                        | 1.44  | 0.167   | 0.258          | 1.24 | 3.37    |     |     |
| 10  | 4.17  | 2.97                        | 1.44  | 0.167   | 0.275          | 1.20 | 2.69    |     |     |
| 11  | 2.98  | 3.09                        | 1.44  | 0.167   | 0.201          | 1.45 | 3.86    |     |     |
| 12  | 4.17  | 3.09                        | 1.44  | 0.167   | 0.215          | 1.41 | 3.08    |     |     |
| 13  | 2.98  | 2.97                        | 3.81  | 0.167   | 0.209          | 1.37 | 3.70    |     |     |
| 14  | 4.17  | 2.97                        | 3.81  | 0.167   | 0.223          | 1.34 | 2.95    |     |     |
| 15  | 2.98  | 3.09                        | 3.81  | 0.167   | 0.163          | 1.61 | 4.23    |     |     |
| 16  | 4.17  | 3.09                        | 3.81  | 0.167   | 0.174          | 1.56 | 3.37    |     |     |
| 17  | 2.38  | 3.02                        | 2.63  | 0.131   | 0.188          | 1.47 | 4.28    |     |     |
| 18  | 4.76  | 3.02                        | 2.63  | 0.131   | 0.215          | 1.39 | 2.68    |     |     |
| 19  | 3.57  | 2.92                        | 2.63  | 0.131   | 0.263          | 1.21 | 2.83    |     |     |
| 20  | 3.57  | 3.14                        | 2.63  | 0.131   | 0.159          | 1.66 | 3.72    |     |     |
| 21  | 3.57  | 3.02                        | 0.25  | 0.131   | 0.337          | 1.10 | 2.61    |     |     |
| 22  | 3.57  | 3.02                        | 5.00  | 0.131   | 0.177          | 1.53 | 3.46    |     |     |
| 23  | 3.57  | 3.02                        | 2.63  | 0.080   | 0.194          | 1.48 | 2.91    |     |     |
| 24  | 3.57  | 3.02                        | 2.63  | 0.198   | 0.210          | 1.39 | 3.49    |     |     |
| 25  | 3.57  | 3.02                        | 2.63  | 0.131   | 0.204          | 1.42 | 3.25    |     |     |
| 26  | 3.57  | 3.02                        | 2.63  | 0.131   | 0.204          | 1.42 | 3.25    |     |     |
| 27  | 3.57  | 3.02                        | 2.63  | 0.131   | 0.204          | 1.42 | 3.25    |     |     |
| 28  | 3.57  | 3.02                        | 2.63  | 0.131   | 0.204          | 1.42 | 3.25    |     |     |
| 29  | 3.57  | 3.02                        | 2.63  | 0.131   | 0.204          | 1.42 | 3.25    |     |     |
| 30  | 3.57  | 3.02                        | 2.63  | 0.131   | 0.204          | 1.42 | 3.25    |     |     |
Table 6. ANOVA for $\Delta P_{\text{up}}$

| Source | Sum of squares | df | Mean square | F value | p-value | prob. > F | Remarks |
|--------|----------------|----|-------------|---------|---------|-----------|---------|
| Model  | 0.041338       | 14 | 0.002953    | 22.1716 | <0.0001 | Significant |
| A      | 0.001127       | 1  | 0.001127    | 8.464783| 0.0108  | Significant |
| B      | 0.016269       | 1  | 0.016269    | 122.1613| <0.0001 | Significant |
| C      | 0.018772       | 1  | 0.018772    | 140.957 | <0.0001 | Significant |
| D      | 0.000364       | 1  | 0.000364    | 2.736673| 0.0188  | Significant |
| AB     | 1.17E-05       | 1  | 1.17E-05    | 0.087717| 0.7712  |           |
| AC     | 8.24E-06       | 1  | 8.24E-06    | 0.061879| 0.0369  | Significant |
| AD     | 2.58E-07       | 1  | 2.58E-07    | 0.001937| 0.9655  |           |
| BC     | 0.000119       | 1  | 0.000119    | 0.892444| 0.3598  |           |
| BD     | 3.72E-06       | 1  | 3.72E-06    | 0.027935| 0.8695  |           |
| CD     | 2.62E-06       | 1  | 2.62E-06    | 0.019707| 0.8902  |           |
| A^2    | 7.53E-05       | 1  | 7.53E-05    | 0.565063| 0.0439  | Significant |
| B^2    | 1.1E-05        | 1  | 1.1E-05     | 0.082479| 0.7779  |           |
| C^2    | 0.004092       | 1  | 0.004092    | 30.7263 | <0.0001 | Significant |
| D^2    | 6.88E-05       | 1  | 6.88E-05    | 0.516917| 0.4832  |           |
| Residual| 0.001998       | 15 | 0.000133    |         |         |           |
| Lack of fit| 0.001998   | 10 | 0.0002      |         |         |           |
| Pure error| 0             | 5  | 0           |         |         |           |
| Cor total| 0.043336     | 29 |             |         |         |           |

Table 7. ANOVA for bed fluctuation ratio ($r$)

| Source | Sum of squares | df | Mean square | F value | p-value | prob. > F | Remarks |
|--------|----------------|----|-------------|---------|---------|-----------|---------|
| Model  | 0.521339       | 14 | 0.037239    | 94.72371| <0.0001 | Significant |
| A      | 0.009906       | 1  | 0.009906    | 25.19707| 0.0002  | Significant |
| B      | 0.302247       | 1  | 0.302247    | 768.827 | <0.0001 | Significant |
| C      | 0.176156       | 1  | 0.176156    | 448.0882| <0.0001 | Significant |
| D      | 0.010108       | 1  | 0.010108    | 25.71098| 0.0001  | Significant |
| AB     | 4.01E-05       | 1  | 4.01E-05    | 0.101934| 0.7539  |           |
| AC     | 1.81E-05       | 1  | 1.81E-05    | 0.046144| 0.8328  |           |
| AD     | 1.29E-06       | 1  | 1.29E-06    | 0.003288| 0.9550  |           |
| BC     | 0.000565       | 1  | 0.000565    | 1.437841| 0.0491  | Significant |
| BD     | 4.03E-05       | 1  | 4.03E-05    | 0.102444| 0.7533  |           |
| CD     | 1.82E-05       | 1  | 1.82E-05    | 0.046375| 0.0324  | Significant |
| A^2    | 0.000396       | 1  | 0.000396    | 1.007668| 0.3314  |           |
| B^2    | 0.000784       | 1  | 0.000784    | 1.994946| 0.1783  |           |
| C^2    | 0.017746       | 1  | 0.017746    | 45.14168| <0.0001 | Significant |
| D^2    | 0.000525       | 1  | 0.000525    | 1.335683| 0.0259  | Significant |
| Residual| 0.005897       | 15 | 0.000393    |         |         |           |
| Lack of fit| 0.005897     | 10 | 0.00059     |         |         |           |
| Pure error| 0             | 5  | 0           |         |         |           |
| Cor total| 0.527236      | 29 |             |         |         |           |
Table 8. ANOVA for bed expansion ratio (R)

| Source | Sum of squares | df | Mean square | F value | p-value prob. > F | Remarks |
|--------|----------------|----|-------------|---------|------------------|---------|
| Model  | 6.033103       | 14 | 0.430936    | 232.3384| <0.0001          | Significant |
| A      | 3.456181       | 1  | 3.456181    | 1863.395| <0.0001          | Significant |
| B      | 1.171439       | 1  | 1.171439    | 631.5797| <0.0001          | Significant |
| C      | 0.694681       | 1  | 0.694681    | 374.5363| <0.0001          | Significant |
| D      | 0.483748       | 1  | 0.483748    | 260.8122| <0.0001          | Significant |
| AB     | 0.009999       | 1  | 0.009999    | 5.3909  | 0.0347           | Significant |
| AC     | 0.004565       | 1  | 0.004565    | 2.461187| 0.0175           | Significant |
| AD     | 0.004048       | 1  | 0.004048    | 2.182244| 0.1603           |          |
| BC     | 0.001634       | 1  | 0.001634    | 0.881237| 0.3627           |          |
| BD     | 0.001449       | 1  | 0.001449    | 0.78136 | 0.3073           | Significant |
| CD     | 0.000662       | 1  | 0.000662    | 0.356726| 0.5592           |          |
| A²     | 0.097375       | 1  | 0.097375    | 52.49967| <0.0001          | Significant |
| B²     | 0.002153       | 1  | 0.002153    | 1.160727| 0.2983           |          |
| C²     | 0.073656       | 1  | 0.073656    | 39.71154| <0.0001          | Significant |
| D²     | 0.002413       | 1  | 0.002413    | 1.301148| 0.2719           |          |
| Residual| 0.027822      | 15 | 0.001855    |         |                  |          |
| Lack of fit| 0.027822  | 10 | 0.002782  |         |                  |          |
| Pure error | 0            | 5  | 0          |         |                  |          |
| Cor total  | 6.060925    | 29 |            |         |                  |          |

Figure 8. Comparison of the values of $\frac{\Delta P_f}{\Delta P_{mf}}$ calculated from Equation (8) with the experimental ones.
variables selected in wide ranges with a limited number of experiments, as well as the nonlinear influence of the investigated parameters on process response.

4. Conclusion
In this study, the hydrodynamic behavior of three-phase fluidized bed for homogenous ternary mixtures in conical conduits has been carried out. In three-phase fluidization, the hydrodynamic variables studied are bed pressure drop, bed fluctuation, and bed expansion ratios. As pressure drop in a fluidization system is an important parameter for the design and fabrication of the fluidized bed reactor, its value should be as low as possible from economic point of view. In this study by the use of a secondary fluidizing medium (air) beyond minimum fluidization the bed pressure drop can be significantly reduced. Simultaneously, the study of both
dimensional analysis and RSM-based CCD and quadratic programming are used to model the influence of four process parameters on the three responses. Mathematical correlations are derived for the three responses using sets of experimental data and ANOVA. Predicted values obtained using the model equations have been found to be in very good agreement with the experimental values. Hence, the developed correlations can be used for the design of three-phase conical fluidized bed systems with homogeneous ternary mixtures of irregular particles of bed materials within the ranges of the operating parameters investigated.

Nomenclature

- \( D_o \) bottom diameter of tapered bed (m)
- \( D_{pav} \) average diameter of ternary mixture (m)
- \( H_s \) initial static bed height (m)
- \( \Lambda \) cone angle (in degree)
- \( \Delta P_{mf} \) pressure drop at minimum fluidization velocity (N/m²)
- \( \Delta P_f \) pressure drop during fluidization (N/m²)
- \( G_f \) flow rate of fluid at fluidization condition (kg/h m²)
- \( G_{mf} \) flow rate of fluid at minimum fluidization condition (kg/h m²)
- \( r \) bed fluctuation ratio
- \( R \) bed expansion ratio
- \( N \) number of runs
- \( n \) number of independent variables
- \( h_1 \) highest bed height (m)
- \( h_2 \) lowest bed height (m)
- \( V_g \) superficial gas velocity (m/s)

Funding
The authors received no direct funding for this research.

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Citation information
Cite this article as: Prediction of bed pressure drop, fluctuation and expansion ratios for three-phase fluidization of ternary mixtures of dolomite in a conical conduit, R.K. Padhi, D.T.K. Dora, Y.K. Mohanty, G.K. Roy & B. Sarangi, Cogent Engineering (2016), 3: 1181821.

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