The Principle of Equivalence as a Guide towards Matrix Theory Compactifications.

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Abstract

The principle of equivalence is translated into the language of the world-volume field theories that define matrix and string theories. This idea leads to explore possible matrix descriptions of M-theory compactifications. An interesting case is the relationship between $D = 6 \mathcal{N} = 1 U(N)$ SYM and Matrix Theory on $K3$.

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1 Introduction.

In the last few years, important changes in the picture we have of the physics beyond the Planck scale have occurred. The publication of several articles that proposed and gave evidences for new dualities has helped to fill even more the space of theories M-theory has become. It was the case of Matrix Theory [1, 2, 3] and the AdS/CFT conjecture [4].

The guide for all has been the use of different quantum field theories to describe systems that include gravity. Based on D-brane world-volume physics, different limits have been found in which particular configurations of D-branes in certain backgrounds were able to account for all the degrees of freedom of the system.

What is most surprising of all this is that a wide variety of field theories seems to contain, at least in some limit, such an abstract symmetry as Einstein’s principle of equivalence. The purpose of this article is to show where it is hidden among the properties of the field theories and to find out that, in fact, it holds for all scales.

Once that is done, one can write down some minimum properties that allow to construct a theory of quantum gravity and take advantage of them in order to find some examples apart from the ones that are already known.

Led by this, I show the good features of one interesting case: $D = 6$ $\mathcal{N} = 1$ $U(N)$ SYM. I discuss its possibilities and how it could match into the moduli space of M-theory.

2 Quantum field theories and the principle of equivalence.

All solutions that have been tried until today to the problem of gravity quantization consist of a world-volume interpretation of one (Matrix Model) or two (String Theories) dimensional quantum field theories. It is an inter-
esting task to find out which are the common characteristics that allow all these theories to describe gravity and see how those properties constrain the construction of other possible theories.

One may think, to begin with, that the only requirements for the theories are to be quantum and to include in some way the principle of equivalence. This is achieved if the theory holds the following conditions:

- To be a quantum field theory. In principle, the number of dimensions is not important.
- To be renormalizable, because we want the theory to be consistent. I shall write more about this topic later.
- To have a sufficiently complex vacuum so as to be described by some continuous set of parameters that define a manifold of the kind of \( \mathbb{R}^d \) or any other that could be interpreted as a target space. If there are fermionic (Grassmann) parameters, they should be able to be arranged into spinorial representations of the rotation group of the bosonic space. This is in order to have a target super-space formulation. It is essential that all the parameters that define the vacuum have a (super)geometrical interpretation.
- This way, the theory can describe the dynamics of one single object. If we want a theory with a full second quantization, we had better find a family of quantum field theories whose moduli spaces are \( (\mathbb{R}^d)^N \), for every possible \( N \). In any other case, the world-volume will be unique, and multiple objects may only be included in a more indirect way. In string theories, they appear as different boundaries (vertex operators) of the world-sheet.
- The fields whose vacuum expectation values define the target manifold must possess a ‘flavour’ symmetry with a group \( SO(d) \) or \( SO(d - 1, 1) \).

These conditions are the world-volume implementation of the principle of equivalence. This is understood noting that, up to first order in perturbation theory (that means: in a small enough neighbourhood of the observer where
it is not much affected by interactions), all physical magnitudes depend exclusively on the positions of the objects (vacuum expectation values of scalars), because they are the only quantum numbers that define the vacuum. In particular, we can consider a probe -an observer- surrounded by some nearby objects; as they stand approximately in the same position, all of them suffer the same acceleration produced by the interaction with the rest of the universe. In this situation, the observer defines an inertial frame that does not feel at all the action of gravity over a sufficiently small volume around him. Moreover, the ‘laws of nature’ that the observer measures respect an $SO(d-1,1)$ symmetry, that is, the kinetics is that of special relativity. If the symmetry group is $SO(d)$, instead, we can either keep the theory as Galilean or try to interpret it as a light-cone formulation of a $d+1$-dimensional theory.

If we act like this we can recognize all the interactions as gravitational and therefore we can always find a geometrical interpretation of the involved forces. This holds independently of how small we take the typical scale to be.

There is a subtlety regarding the inclusion of fermions. They induce supersymmetry in the target space. In general, if the supermultiplet is large enough, this could add interactions mediated by vector or scalar fields with spin one that may include gauge groups. The argument above must, therefore, be generalized because interactions may depend not only on the bosonic coordinates, but also on the fermionic ones. That means that the interactions will be supergravitational. In this case the principle of equivalence is enhanced so that the local symmetry observed in the inertial frame is not only the one spanned by the Poincaré group but the whole supersymmetry algebra.

Somehow, this conditions represent the minimum properties a world-volume theory must have in order for the target theory to include gravity.
3 Known examples: String Theories and the Matrix Model.

Let us first see what is the most simple example of the philosophy outlined in the previous section: String Theories. Polyakov and Green-Schwarz actions are just the actions of two-dimensional free scalar fields. The vacuum of the theory is parameterized by the vacuum expectation values of the positions of the strings, that, indeed, define a Minkowskian space with \( SO(d - 1, 1) \) symmetry. The theory does not have a complete second quantization because it is not a family, but just a single field theory. The only way in which several strings can be included is by performing a number of punctures in the single world-sheet when scattering amplitudes are calculated. So multiple strings are obtained taking advantage of the fact that the world-sheet has two dimensions and so an arbitrary number of boundaries (holes) can be put on it.

Let us now move on to the Matrix Model. Exactly the same arguments can be made to see that all conditions are held \(^2\). This ensures that the principle of equivalence is an exact symmetry for all scales, which is not a trivial result. There are certain subtleties related in part to the light-cone interpretation that I want to clarify. In principle, the conditions described in the first section guarantee that we are describing objects of some class moving in a gravitational background, but that does not mean that the objects naturally come out of the gravity theory. To make it clearer: it is not necessary that the graviton should be one of the objects that the theory can explicitly describe.

This can be better understood with an example. Take the Dirac-Born-

\(^2\)There are two papers [16] that explicitly say that finite-N Matrix Model breaks the principle of equivalence even for long distances. This was deduced assuming the existence of backgrounds with vacuum expectation values for off-diagonal terms. Those are just quantum fluctuations. The statements I make above only refer to stable vacuum configurations of the theory.
Infeld gauge theory that describes the short-distance dynamics of general D-$p$-branes with positive Ramond-Ramond charge. Regardless of whether or not there is any realistic physical situation in which the only significant degrees of freedom are those of the D-$p$-branes with positive charge, the action includes, among others, the gravitational interaction. What I am trying to say is that the conditions in the second section do not impose the action to be realistic nor to completely determine a whole theory, just to describe gravity in a quantum manner.

As it is clear that gravitons are well defined asymptotic states in nature, we can only interpret D-brane actions, and some others that I shall later consider, as partial descriptions or formulations adapted to particular physical configurations where, for example, graviton states are irrelevant.

We begin now a search, among some simple classes of theories, for some examples, apart from those already known, that could describe interesting physics.

4 Bosonic theories.

One should now see how other gauge theories can be interpreted as gravitational ones. I shall begin with bosonic Yang-Mills theories in different dimensions. It is necessary to dimensionally reduce the action, at least in some directions because the vacuum expectation values of gauge fields are pure gauge and do not span any moduli space. The dimensional reduction gives us a number of scalars that determine the dimension of the configuration space.

The rôle of the gauge group is to define the vacuum and to produce a particular potential. The $U(N)$ set of groups is particularly well adapted to include a many-particle configuration space into its vacuum degeneracy, so we shall study it first.

When one makes a field theory calculation, similar to that in [6] the
long distance potential that is obtained is $V \propto r^{1+p}$ independently of the original dimensions of the YM. $p$ is the number of spatial dimensions that have not been reduced. This potentials only coincide with a Newton long-distance limit in two dimensions and therefore, in the general case they would represent gravitational fields that do not vanish in the infinite. They are not of any physical interest.

We only have left the possibility to interpret that the objects we are describing are not point particles, not even branes, but extended objects like bodies in three dimensions with a particular mass distribution. Coordinates should, in this case, parameterize the center of mass of each object. If we choose the correct distribution, we can classically obtain virtually any potential that grows with the distance faster than the Newtonian. Except, maybe, for very particular situations, this interpretation is hardly realistic.

We do not need, however, to force the theories to describe multi-particle systems. Take, for instance, the $SU(2)$ theory. The vacuum defines a single particle geometric space. We can now interpret the theory as the description of the dynamics of one particle moving in a particular background which is responsible for the potential. The theories do not seem to be very useful although it should be pointed out that $V \propto r$ potentials appear over thin shells around spherically symmetric objects in any dimension. It is not clear, however, what kind of boundary conditions should be imposed.

Without any further investigation on this topic, we move on to some, more interesting theories.

## 5 Supersymmetric theories.

Supersymmetry can be added introducing fermions in the world-volume theory. The easiest way to do this is by imposing supersymmetry among the world-volume fields. This can only be done if the number of fermionic degrees of freedom that we get can be arranged into a spinor of the target space.
There, they will be interpreted as fermionic coordinates (supercharges). This happens in 10, 6 and 4 dimensions. There are probably more cases in higher dimensions, but with little physical interest.

For them to be fundamental theories -by that I mean: defined in a whole phase space, it is necessary that they should give appropriate long range forces, that is, Newton potentials. If the potential is stronger than that -it was the case in the bosonic theories- they cannot be used to describe open spaces, because the graviton fields do not vanish in the infinite. If the potential is weaker, it would at most describe a gravitational theory whose action does not contain the scalar $R$, but higher powers or derivatives of it. To my knowledge, these do not have any use in any realistic situation.

It is possible to make the calculation of the potential at tree level. In ten dimensions it was made in [3, 7], and the other cases are simple extensions. Again, we consider these three theories reduced to $0 + 1$ dimensions. The gauge group is $U(2)$ because we need the potential between two particles. The results of these computations are

\[
V_{D=10}(r) \propto \frac{v^4}{r^7}, \\
V_{D=6}(r) \propto \frac{v^2}{r^3}, \quad \text{and} \\
V_{D=4}(r) \propto \frac{v^2}{r^3}.
\] (1)

The former case can be recognized as the potential between D0-branes, and the resulting fundamental theory is, of course, Matrix Theory. It is also very interesting that the six-dimensional case also gives a Newton potential and, therefore, a long-distance metric that reproduces its effects can be written. It is

\[
ds^2 = \left(1 + \frac{gY_M}{2r^3}\right) (dr^2 + r^2d\Omega_4^2).
\] (2)
In our more or less systematic search for realistic gravitational theories, we have eventually found this good candidate. I shall concentrate on it from now on. It can be extended to a Born-Infeld-Dirac type action with an $SO(5,1)$ space-time symmetry and, therefore, relativistic in the target space. It is renormalizable and contains gravity. In principle, it describes one type of object (BPS saturated) moving in a supergravitational background. The Lagrangian does not explicitly include free graviton states so one could question if it is enough to define a whole theory, that is, if it contains all the information needed. The argument supporting this is that gravity is different from any other interaction in that it acts exactly the same over any physical object independently of any other quantum number but the stress-energy tensor. That means that any object can probe it and extract the gravitational Lagrangian because all behave the same. Analyzing the potential or the force between the particles in terms of their positions and velocities, it is possible to identify a complete action formed by an infinite series of terms in derivatives and powers of the curvature scalar $R$. This is always possible because what we have is a field of accelerations, rather than a strength field. The sixteen supercharges that we know the theory to have help us complete every term with the necessary fields. To make it short, what we should do is calculate scattering processes, then deduce the potential terms created by them and finally, identify the extensions of supergravity that reproduce them. I have already taken the first step when I have compared the supergravity ‘Newton’ potential to the SYM one-loop potential. This is exactly the philosophy followed by Matrix Theory. The main conjecture there is to assume that all the objects that appear in the theory are either gravitons or bound states of them. In particular, the ‘algorithm’ I have just described to obtain the supergravity series has been partially applied to the D0-brane action to get the $D = 10 \, N = 2$ supergravity action and some corrections [14].
fact, as Matrix Theory compactifications and simple D-brane interactions have shown [4, 5], any D-brane action can be used to calculate gravitational interactions if the momentum transfer is restricted to the directions which are orthogonal to the D-planes. The difference between using the complete DBI action and just the SYM term is that by taking the light-cone in the eleventh direction (using SYM), it is made sure -according to the conjecture- that all the objects are described by the same exact action. That is, in the fundamental string action, D-branes appear as non-perturbative, solitonic states; the D-brane action is related to it through $S$ and $T$ dualities so it contains the same information, but differently organized so that it is useful in different physical circumstances. The BFSS’s guess is that the Matrix Model Hamiltonian perturbatively describes all the objects of M-theory in the light-cone.

In the six-dimensional case that we are studying, we do not have so many dualities to guarantee the completeness of the theory but the fact that we can describe at least some quantum objects interacting through gravity up to any energy or length scale is very encouraging.

One may think that the lack of so much supersymmetry as in the Matrix Model, and therefore, the lack of some non-renormalization theorems may cause some problems in the relation between the SYM and the supergravity. The answer in no. The only requirement is that the theory be renormalizable because the comparison is made just by calculating physical magnitudes. In particular, the potential is obtained studying scattering processes. The fact that the coefficients of the potential are or not corrected by higher loop terms is not important because we are not comparing the theories loop by loop, but their physical effects as a whole. That is, the whole SYM series is equivalent to the whole -infinite- supergravity series.

This is all we can say about this theory, considered independently of M and string theories. It seems to be complete, in the sense that it possesses all the information of a six-dimensional extended supergravity theory, valid
up to all scales. However, the description of many degrees of freedom (including gravitons) gets very complex at small distances because one has to construct the supergravity series that gives more and more complicated interactions among the gravitons as one considers smaller distances. In fact, the mechanism that would regularize the theory at high energies -parallel to the appearance of very degenerated massive states in string theory- is the presence of massive bound states of gravitons. This is the same guess as in Matrix Theory, where the bound states include, among others, membranes and, therefore, strings in ten dimensions. This makes it very difficult to know which are the relevant degrees of freedom at high energies or find, for instance, a partition function for the complete theory.

T-duality shows that whenever a small volume limit is taken in any string or matrix theory, states with discrete momentum different from zero are, in effect, decoupled as in any other field theory, but there always exists a set of other quantum numbers (string windings in the simplest cases) that gather into a continuous spectrum that can be interpreted, thanks to T-duality, as the opening of new dimensions in the dual theory. This does not seem to be the case: there is not any obvious Kaluza-Klein spectrum and the continuous spectrum is just six-dimensional. This gives us three possibilities: the theory might be disconnected from M-theory compactifications; it could be connected, but with a K-K spectrum that is hidden as some bound states of gravitons; finally the theory may need to be completed with more fields (or simply more information). The difficulties in calculating bound states in supergravity series seems to make it impossible to decide which is true, but I shall give some hints.

7 A possible relation to the Matrix Model.

Looking at its form, very similar to the usual Matrix Model, one is pushed to try to locate this six dimensional model in a ‘nearby place’ in the moduli
space of M-theory. I shall now study this possibility.

The Lagrangian describes BPS states whose supermultiplet is generated by 8 supercharges. This means that the effective theory should have 16 of them, and should not be chiral. In six dimensions, there are several theories whose low energy supergravities share those characteristics, but all of them are related by different dualities. The most interesting to us for its simple relation to M-theory is type IIA string theory compactified on a $K3$ manifold. The $K3$ may be taken to any singular limit with -for example- orbifold singularities. This changes the degeneracy of the effective model as well as the gauge symmetry. The guides that will lead us to find out the particular situation that may be described by $D = 6 \mathcal{N} = 1 \ U(N)$ SYM are its spectrum of objects and their behaviour.

The only objects that are explicitly present are heavy point-like BPS states. Just like Matrix Theory, it contains information about other massless objects like six-dimensional gravitons but they are only included in the interactions.

The only way to simplify the spectrum of M-Theory so as to have just one kind of object seems to be taking the light-cone in the eleventh dimension, that is, to use the Matrix description. So we had better look for the Matrix model of type IIA string on a $K3$. This has already been studied [5, 13] and seen to be described by the theory of longitudinal M-5-branes wrapping $S^1 \times K3$ or equivalently IIA D4-D0-brane bound states just wrapping the $K3$. The particular field theory is not explicitly known, but some properties have been studied.

Indeed, $D = 6 \mathcal{N} = 1 \ U(N)$ SYM does have enough supercharges to generate the hypermultiplet of the five-brane. Moreover, wrapped five-branes behave like point-like objects in the open directions. The straightforward guess is that the diagonal elements of the YM scalar matrices should be interpreted as the transverse positions of the five-branes.

However, we only have one arbitrary parameter to fix -the coupling, so it
is clear that we cannot reproduce the moduli space of the $K3$. To solve this, we have to see what happens to the link between them: the supergravity. Six-dimensional type IIA supergravity is determined by the string coupling and the vacuum expectation values of the scalars that appear after the compactification. The latter can be arranged into a $24 \times 24$ matrix that can, at generic points, be diagonalized to give, basically, the 24 couplings of a gauge group that is broken to $U(1)^{24}$. Therefore, the moduli space is, generically, 25-dimensional. Our case is, however, simpler, because we are describing just one kind of object. It is a BPS state and so its gauge charge is the same as its mass. That is why, the interaction of our object is determined by one coupling. In general, in the Matrix models, the complexity of the moduli space is carried by the manifold in which the field theory of the model is defined over.

The only physical situation where the particular form of the manifold looses importance is when its size is reduced to zero. In the Matrix models, the field theories are defined over the T-dual manifold. When none of the manifolds -neither the original nor the dual- is large or small, the target moduli space is defined by the vacuum expectation values of the scalars and the Wilson lines of the gauge fields. If the T-dual volume is vanishingly small, the field theory is defined in a point and we can therefore expect a quantum mechanical formulation. Besides, when the volume is small, all the internal degrees of freedom can be integrated out so that the only physical magnitudes that survive -and therefore should appear in the action- should be the positions of the particles. This is a kind of ‘dimensional reduction’ of the Matrix model. This argument is also valid for large volumes because of the self-T-duality of type IIA superstring in six dimensions.

This tells us that the provisional conjecture that should be proposed is that the Matrix model that describes the $D = 6$ non-chiral $\mathcal{N} = 2$ string theories in the DKPS limit and with $V_{K3} \to 0$, is $D = 6$ $\mathcal{N} = 1$ $U(N)$ SYM reduced to $0 + 1$ dimensions. The relation between the Yang-Mills and the
IIA string coupling should be \( g \):

\[
g_{YM} = g_s^{-1/3} V_{11}^{-1}
\]  

(3)

This helps us know what kind of limit we should take the volume to. The relation between the original volume and the T-dual is

\[
\tilde{V} = \frac{1}{m_p^{10} R_5^5 V(K3 \times S^1)}
\]  

(4)

where the volumes are measured in units of the eleven-dimensional Planck length. In units of \( \alpha' \), the original volume is

\[
V_{\alpha'} = \frac{V(K3 \times S^1)}{R^5 m_p^{10}} g_s^{10/3}
\]  

(5)

What this is telling us is that the limit we are looking for is that with a T-dual small volume, which means that the original compact volume must be large compared to the Planck length, while remaining vanishing as \( g_s \to 0 \) in terms of \( \alpha' \). Besides, if we want the D4-brane description to be valid -weakly coupled-, we should take the original volume to infinity quicker than \( g_s^{-1/3} \). Its world-volume theory is not renormalizable, but that does not matter as long as all its fields are decoupled. I shall be more concrete in next section. This is the addition to the DKPS limit that seems to simplify the spectrum enough to allow us to use a theory with much fewer degrees of freedom.

8 Compactifications.

Apart from the small dual volume limit, there are other reasons why we have such a simple theory in six dimensions when the equivalent compactification on a torus is so difficult to define. Of course, we have presumably decoupled the gauge theory, which carries most of the complexity, and besides, the supersymmetry breaking reduces a lot the number of degrees of freedom.
In particular, the supermultiplets are short enough not to necessarily include gravitons. A similar argument was pointed out in [11] when discussing compactifications of Matrix Theory on Calabi-Yau manifolds.

Among other things, this allows us to compactify further and see what happens in lower dimensions at least when half the supersymmetry is broken. We can choose the manifolds to be tori and take advantage of the T-duality of gauge theories shown by [9], which is general and independent of the dimension. This way we could define Matrix Theory on $S^1 \times K3 \times T^d$. As gauge theories are renormalizable up to three dimensions, we can compactify down to $D = 2 + 1$, where the description is in terms of $D = 3 + 1 \mathcal{N} = 2$ SYM.

The situation is particularly interesting when compactifying just one dimension. In that case, one can be more explicit. We take advantage of the following set of dualities. We begin with type IIA on $K3 \times S^1$ and make a matrix model of it. T-duality leads us to a IIB D5-brane wrapping $\tilde{K}3 \times \tilde{S}^1$. That is the same as another $\tilde{M}$-theory on $\tilde{T}^5/\mathbb{Z}_2 \times \tilde{S}^1$ where one of the directions of the five-torus is light-like. We can choose any of the eleven dimensions to define a type IIA coupling constant so we make a flip, similar to the one in [10], and choose the direction of $\tilde{S}^1$. That one is not light-like, so we get the non-perturbative definition of IIA string theory on $\tilde{T}^5/\mathbb{Z}_2$. We can get the Lagrangian of this theory in the limit we are interested in led by the calculation in [10].

Let us start with the Lagrangian of N D0-branes moving on $K3 \times S^1$. By T-duality, it becomes a six-dimensional SYM defined over a $\tilde{K}3 \times \tilde{S}^1$. This is true at least at low energies. If the theory is not renormalizable, it should be appropriately completed at high energies. If we take its volume to be vanishingly small, we can make a dimensional reduction of the theory. What we are left with are the zero-modes of the scalars and of the gauge fields. However, in the $K3$, all loops are contractible and so no Wilson line can be defined and the zero modes of the gauge fields are pure gauge except for the
component in the direction of the circle. In this moment we interchange the rôle of that circle and the one related to the string coupling. This puts the Lagrangian exactly in the form of an $\mathcal{N} = 1 D = 5 + 1$ SYM reduced to $D = 1 + 1$ and that is precisely what we expected. The coupling is

$$g_{YM} = m_s^{-4} V_{K3}^{-1} R_5^{-1} R_{11}$$

The interchange of the fifth and the eleventh dimensions gives an IIB-like S-duality related, like in the ten-dimensional case, to the modular transformations of the $5 - 11$ torus. The fact that the theory on the $K3$ decouples helps us deal with a field theory which could, in principle, be non-renormalizable. Being a well defined compactification of M-theory, we suppose that the world-volume six-dimensional field theory exists and that is enough for our purposes. On the other hand, the T-dual theory may be strongly coupled even when the original one is weakly coupled; however, after the swap of the two circles is performed, the string coupling is related to the size of the circle we choose and independent of the original coupling constant.

It is interesting to note that when one takes the coupling of the $D = 1 + 1$ field theory to zero, one gets something that could be interpreted to be a six-dimensional Green-Schwarz action in second quantization. One could be tempted to relate these to the little strings that move through the world-volume of five-branes. However, our strings interact through gravity and so they do not have any relationship.

If that were the whole story, then a lot of evidence would be in favour of this theory to be one more of the matrix models. However there are a couple of things that we have not taken into account. We have not solved the problem of the lack of continuous spectrum even when we have explicitly used T-duality so we must have forgotten something. It is the existence of open string instantons related to the axion $B^{\mu\nu}$'s topological configurations around the $K3$'s two-cycles. I shall be more explicit, the Fourier-Mukai
transform [13] that plays the rôle of the T-duality in the $K3$, changes the original D0-brane with a bound state of a D4-brane wrapping the $K3$ with another D0-brane. The T-duality along the direction of the second circle transforms the system into a bound state of a D5 and a D1-brane. The light degrees of freedom are not only the gauge ones, because the open strings with both ends attached to the D1-brane can have world-sheets which are surfaces that wrap non-trivially around the two-cycles of the $K3$. These states are instantons because they are localized solutions both in space and time. Their energy is proportional to the area of the two-cycles, which goes to zero when the volume of the $K3$ is taken to zero. These light states are the ones that provide the continuous spectrum that we missed. These states are also involved when one tries to see which is the effect of the T-duality in the S-dual theory, the heterotic string on a four-torus.

This means that the SYM quantum mechanics cannot, by itself, describe all the theory. We must add the instantons. The final conjecture should therefore be that: ‘M-theory on $S^1 \times K3$ in the light-cone is dual to the system formed by the $D = 6 \mathcal{N} = 1 U(N)$ SYMQM BPS states in interaction with the instantons’. The theory is not as simplified as we expected. The situation is somewhat similar to what happens in the matrix model for the six-torus. There, there are two objects that do not decouple: the D6-branes and the D0-branes [5, 11]. That case is, however, worse because the light D0-branes contain gravity multiplets. In our case, the instantons have the same states as open strings at tree level -that is precisely what they are- so their description is much simpler. In fact, the inclusion of instantons just means the inclusion of open strings attached to the 0-branes in such a way that their world-sheet is a sphere around a two-cycle of the $K3$. This makes their dynamics relatively easy to treat.
Degeneracy of the quantum states: gauge symmetry and bound states.

One characteristic I have not written about yet is the gauge group. In M-theory, it is given by the Kaluza-Klein interpretation of the higher dimensions. In the case we are dealing with, we can take advantage of the S-duality we have at hand between type IIA string theory on $K^3$ and the $E_8 \times E_8$ heterotic string on a four-torus. The gauge group at a generic point in the moduli space is $U(1)^{24}$, but it can be enhanced at singular points where some degrees of freedom become massless. From the type IIA point of view, that happens whenever the $K^3$ is deformed to develop a singularity and some of its two-cycles are reduced to zero size. In the perturbative theory, the D2-branes that wrap the degenerate two-cycles appear as massless point-particles in the six-dimensional theory and are the ones responsible for the symmetry enhancement.

It is straightforward to understand that from our non-perturbative point of view, the instantons, whose energies are proportional to the surface of the two-cycles, will take the rôle of the D2-branes. That is, the particular form of the $K^3$, including its singularities, is encoded in the dynamics of the instantons. They reproduce the whole moduli space and generate the different target gauge groups.

When the gauge group is non-abelian, the 0-branes acquire another index that works as a quantum number that does not appear in the Hamiltonian. The freedom to add numbers to the labels of the states always exists in quantum mechanics. In the same way that Pauli spin is added by hand, any other number could also be added. The justification must come from a more general theory. In our case, the theory in eleven dimensions.

There is an important feature that this theory should have in order to relate it to a matrix model of M-theory: bound states at threshold. They represent the Kaluza-Klein partons that come out from the compactified
eleventh dimension. The calculation of the Witten index of the SYM was performed in [12]. The result was that there were no such states. This is what we expected because part of the interaction that we have in our case was not considered. The possibility is not negligible that even the fields that we have decoupled in the small-volume limit may have some ‘topological’ effects in the Witten index calculation, which is basically a problem of field counting (I mean: one field with nearly infinite mass is still one field).

This does not prevent us from dealing with the bound states. We know from their existence, thanks to the dualities and the limits we have taken, and that will be enough for dynamical purposes. The only change is the same as in the gauge degrees of freedom: we should add another index that tells us that whenever two 0-branes coincide in the same point, the degeneracy is two (there is one free and one bound state).

10 Conclusions.

I have shown how the principle of equivalence is hidden in the world-volume interpretation of quantum fields. The properties of the fields in the parametric space of the trajectory have little in common with their characteristics in the target space. In particular, most symmetries of the target theory come from symmetries of the vacuum configurations of the world-volume fields. I have studied how this is achieved in Matrix and String theories.

This insight leads to uncover certain theories that can probably be useful to study Matrix Theory compactifications. $D = 6 \mathcal{N} = 1$ SYM has been studied with more depth and seems to be the correct theory that, together with the dynamics of some open string instantons, is able to describe M-theory on $S^1$(light-like)$\times K3$. 
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