Unbalance and resonance elimination with active bearings on a Jeffcott Rotor

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**Abstract**

In this contribution we have proven theoretically and practically that active bearings are able to eliminate both bearing forces and the resonance of a Jeffcott Rotor system. Active bearings can displace a rotor such that its center of mass always stays in the rotational center. The proposed collocated controller is able to keep this state at any rotational speed, leading to an elimination of bearing forces and resonances. We analytically demonstrated that the closed-loop system is *always* stable, even without knowledge of the rotor’s properties. The generalization of the proposed control approach for force-free operation either using displacement or force actuators enables its use for all kinds of active bearings. Moreover, the control approach allows a real time estimation of the rotor’s eccentricity. The low parameter count and the unproblematic stability behavior qualify the controller for many applications.

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1. Introduction

In the late 19th century, engineers believed a rotating machine could not be operated above its critical speed. Laval was the first to show practically that a rotor can operate beyond this limitation. Although Föppl and Jeffcott published correct explanations for the phenomenon, the Jeffcott Rotor prevailed in the engineering literature [1].

The center of mass never coincides with the center of rotation on real rotors. The resulting eccentricities cause rotor vibrations leading to rotor deflections and bearing forces. Both quantities get especially large when the rotor operates close to a critical speed. They are usually unwanted because they cause material stresses, noise and excessive bearing wear. The standard method to reduce vibrations is balancing, a process in which the rotor’s masses are redistributed to keep the rotor’s eccentricity reasonably small [2]. Even for balanced rotors, it is usually avoided to permanently operate a rotating machine close to a critical speed.

Active bearings based on electromagnetic actuators have been under heavy development since the 1980s and found applications in industry where conventional bearings might not be feasible. Although stiffness and damping can be adjusted, it is mandatory that the bearing forces caused by unbalance do not exceed the maximum actuator forces to maintain stability. The freedom of implementing different control schemes encouraged scientists to investigate methods to tackle the negative effects of unbalances. A large number of different algorithms emerged, which can be subcategorized in two

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One approach reduces the rotor displacements, which is indicated when high radial runout precision is required. On the downside, large actuator forces might be necessary to pull the rotor in the desired position [4,5]. In many applications, a perfect radial runout is unnecessary as long as the deflections do not exceed boundaries, for example in turbomachinery. In this case, it is desired to reduce the unbalance-induced bearing forces for a smoother machine operation and less actuator load. These methods reside under a variety of different names, such as Adaptive Autocentering Control [6], Adaptive Forced Balancing [7], Periodic Learning Control [8], Automatic Inertial Autocentering [9], Unbalance Compensation [10], Adaptive Feedforward Compensation [11] and Unbalance Force Rejection Control, a method described in an ISO-Standard [12]. Of all the approaches, two main concepts prevailed. The notch filter approach [13] suppresses the synchronous bearing forces and allows the rotor to rotate around its principal axis of inertia. As a shortcoming, it cannot stabilize the system at critical speeds. In contrast, the adaptive feedforward compensation injects a synchronous harmonic signal into the control system to compensate the unbalance forces [14]. The method can pass critical speeds when the rotational speed is changing slowly, however, stability is not necessarily guaranteed. Additionally, the exact nature of the injected signal and its physical interpretation remain unclear.

Inspired by the offered possibilities of active bearings, researchers started to look for alternatives based on other actuator principles. Palazzolo used piezoelectric actuators in combination with conventional ball bearings to reduce rotor vibrations [15–17]. After the feasibility of his approach was demonstrated, research focused on unbalance compensation. Lindenborn and Hasch were able to reduce bearing forces or rotor displacements in several orders of magnitude using adaptive feedforward compensation [18,19].

A large number of publications deal with unbalance problems and thereby reflect its general importance. Even though researchers tried to unify the approaches by introducing generalized notch filters [20], the connection between filtering and adaptive feedforward control is still obscure. Further the behavior of unbalance compensation in critical speeds is focus of discussion. While some experiments show that unbalance compensation does not work in the mechanical resonance, others show that both forces and displacements can be reduced simultaneously [3]. The practical usefulness of the unbalance compensation methods is further limited as most of them need accurate models and careful system design to work stably and reliably, impeding their field application.

For many years structural engineers treated almost exclusively rotordynamic problems and developed powerful tools to explain the motion of rotors [21]. In fact, the famous Hurwitz criterion for stability analysis originated from a rotordynamic problem [22,23]. On the other hand, the development of active bearings was mainly an achievement of control and electrical engineers. Consequently, control strategies were mainly described using transfer functions and block diagrams, methods which originated in signal theory. Despite the invaluable contribution of these methods for simple mechanical systems, they fail to adequately address rotordynamic questions such as equilibrium conditions, mode shapes or whirl directions. Structural engineers might be able to answer some of these questions but may get overwhelmed by complex concepts like feedforward compensation.

We think that a generalizing approach needs to consider control, electrical and structural engineering simultaneously.

In this publication, we found a generalizing explanation for the elimination of unbalance forces on Jeffcott Rotors with active bearings. Our argumentation is based on a careful revision of the underlying kinematics and focuses on statically determinate rotors. Bearings – independent of the technology – keep the rotor in a defined position, maintaining a force equilibrium. In statically determinate systems, bearing reaction forces depend on the rotor load only. Current literature often reveals that active bearings exert controllable forces on a rotor. This statement contradicts the conditions of static stability. We point out that active bearings can displace a rotor without altering the force equilibrium and introduce a bearing displacement vector. The advantage of this approach is the consistent derivation of the equations of motion and its compatibility with structural mechanics. Our control design eliminates bearing reaction forces caused by unbalances. It drives the bearing displacements to keep the center of mass always in the rotational center. Some parts of the controller are defined in rotating coordinates, leading to a comprehensive derivation of the adaptive controller structure. The analysis of the closed-loop system reveals that the controller is able to keep the center of mass in the rotational center even at the speed of the mechanical resonance and that a force-free condition can be achieved at any rotational speed. Thus we prove that active bearings can eliminate the unbalance-induced rotor resonance. Remarkably, this controller has the property of unconditional stability, regardless of the rotor’s mass and stiffness. This property even holds for rotational speeds that match the resonance frequency. We generalize the control approach for different actuators, which we divide into two groups: Displacement and Force actuators. Displacement actuators have an inherent mechanical stiffness and are capable to support a rotor without additional control efforts. Force actuators, on the other hand, have no inherent mechanical stiffness and need an additional control system to support the rotor. In latter case, the controller must both support the rotor and eliminate the unbalance-induced bearing forces at the same time. We derive control laws for both actuator types, establishing a link between different active bearing principles and show that unbalance elimination is possible for any type. Inspired by the work of Herzog et al. [20], we take advantage of the fact that some controller parts are initially defined in rotating coordinates. The result is not only a simplified implementation of the controller but also allows further system insight: The controller’s state directly represents the eccentricity. Despite its simplicity, the controller directly enables access to the rotor’s balancing condition in physical meaningful units. The favorable stability behavior and the fact that no specific rotor knowledge is required make the controller suitable for industrial applications. The low count of adjustable parameters enables engineers to adjust the controller in the field without the need for an extensive rotordynamic simulation.

We stress that the obtained results are valid for Jeffcott Rotors only and do not consider effects such as gyroscopy, but we are currently working on the generalization for more complex rotor systems.
2. Mechanical model

2.1. The classical Jeffcott Rotor and its solutions

This chapter recapitulates the results that Jeffcott found in his pioneering work [1]. Despite its simplicity, it is able to explain the most important phenomena in rotordynamics. The governing equations get especially short and comprehensive when the considered rotor system is symmetric and complex notation is used. The real part of the complex coordinate denotes the horizontal axis of the cartesian coordinate system, while the imaginary part denotes the vertical one. According to Fig. 1, the Jeffcott Rotor consists of an elastic shaft with the stiffness $k$ which is supported by fixed supports at its ends. A disc with the mass $m$ is attached to the shaft, the complex coordinate $q_w$ denotes the intersection between shaft and disc. The disc’s center of mass $q_s$ is usually not coincident with the elastic center $q_w$, their distance is called eccentricity $\epsilon$:

$$q_s = q_w + \epsilon$$  \hspace{1cm} (1)

When the elastic center $q_w$ is displaced, it exerts a force $F_p$ on the disc, trying to restore the initial, undeflected position:

$$F_p = kq_w \quad k > 0$$  \hspace{1cm} (2)

Throughout the paper, the equations of motion are given with respect to the center of mass $q_s$ without loss of generality.

Newton’s second law of motion $\ddot{q} = -mF_s$ and Eqs. (1) and (2) yield

$$mq_s + kq_s = ke$$  \hspace{1cm} (3)

For description of the eccentricity a rotor-fixed hence positive rotating coordinate system is introduced. Coordinates which are defined in this system are identified by the superscript $\ast$ throughout this publication. When the shaft turns with the constant rotational speed $\Omega$, the angle between rotor-fixed and inertial coordinates $\varphi = \Omega t$ changes with time $t$. In rotor-fixed coordinates the eccentricity is constant:

$$\epsilon^\ast = \text{const.}$$  \hspace{1cm} (4)

A coordinate transformation to inertial coordinates yields

$$\epsilon = \epsilon^\ast e^{i\Omega t}.$$  \hspace{1cm} (5)

Eq. (3) gets more comprehensive when $m^{-1}k$ is replaced with the square of the natural frequency $\omega_0^2$.

$$\ddot{q}_s + \omega_0^2 q_s = \omega_0^2 e^{i\Omega t}$$  \hspace{1cm} (6)

The unbalance response of the rotor corresponds to the particular solution of Eq. (6). According to Euler’s method of solving linear homogeneous ordinary differential equations with constant coefficients, using the exponential function $q_s = q_s^\ast e^{i\Omega t}$ and its second derivative $\ddot{q}_s = -q_s^\ast \Omega^2 e^{i\Omega t}$ lead to the solutions for both shaft and mass displacements:

$$q_s = \left(\omega_0^2 - \Omega^2\right)^{-1} \omega_0^2 e^{i\Omega t}$$  \hspace{1cm} (7)

$$q_w = \left(\omega_0^2 - \Omega^2\right)^{-1} \Omega^2 e^{i\Omega t}$$  \hspace{1cm} (8)

Eq. (2) states that bearing forces $F_p$ are directly proportional to the shaft displacements $q_w$. For low rotational speeds $\Omega \ll \omega_0$, the center of mass moves around the elastic center, and the bearing forces remain small. The rotor’s deflections and forces get bigger when the rotational speed $\Omega$ approaches the natural frequency $\omega_0$. The solutions of Eqs. (7) and (8) are singular when the rotor reaches its critical speed at $\Omega = \omega_0$. Without damping the displacements and forces tend towards infinity with increasing time, and operating a rotor at its critical speed is usually not recommended. Jeffcott [1] discovered that the rotor deflections get smaller when the rotor enters supercritical range, $\Omega > \omega_0$. For $\Omega \gg \omega_0$, the center of mass approaches the...
rotational center, and both forces and deflections remain small. Fig. 5 visualizes the theoretical behavior of a rotor with respect to different rotational speeds $\Omega$.

When Eq. (8) is transformed to rotating coordinates, $q_{W}^{\Omega}$ is constant for each rotational speed: In rotating coordinates, the rotor deflects statically. The homogeneous solution represents the free oscillation of the system with the frequency $\omega_0$. It can be determined when the right-hand side of Eq. (6) is set to zero:

$$q_0 = q_0^+ e^{i\omega_0 t} + q_0^- e^{-i\omega_0 t}$$  \hspace{1cm} (9)

The solution is a superposition of two pointers rotating in opposite directions. The forward whirling part $q_0^+ e^{i\omega_0 t}$ rotates in positive direction, while the backward whirling part $q_0^- e^{-i\omega_0 t}$ rotates in the negative one. The constants $q_0^+$ and $q_0^-$ depend on the initial conditions. The homogeneous solution is symmetric and independent of the unbalance excitation.

2.2. The Jeffcott Rotor with active bearings

The purpose of a rotor bearing is to keep the rotor in the desired position – this is true for both passive and active bearings. The Jeffcott Rotor is a statically determinate system. Bearing forces are reaction forces – they do not depend on the bearing itself, but on external loads applied to the shaft. They are governed by the equilibrium conditions, regardless of the bearing type. Imagine a non-rotating horizontal Jeffcott Rotor under gravitational load $F_G$ acting on the center of mass. In static conditions the bearings must develop a reaction force $F_L$ of exactly the same magnitude, $F_G = F_L$, regardless whether the bearing is active or passive, the bearing principle or the control strategy. Failing to do so would result in static instability.

When the bearing force is entirely determined by external loads and consequently cannot be changed without losing stability, it is clear that active bearings cannot control forces in statically determined systems. An active bearing enables the displacement of the shaft in the complex plane. In the presented case, the active bearings can statically move the shaft to any desired position in the complex plane without bearing forces. Fig. 2 shows the mechanical model of the actively supported Jeffcott Rotor. The introduced actuator displacements are represented by a red arrow. The active bearing force $F$ depends on the shaft deflection only and is proportional to the distance between elastic center $q_W$ and actuator displacement $a$:

$$F = k (q_W - a)$$  \hspace{1cm} (10)

The equations of motion for rotors with active bearings can finally be found with Newton’s second law $m\ddot{q}_S = -F$ and Eqs. (1) and (10):

$$m\ddot{q}_S + kq_S = k\dot{a} + k\varepsilon$$  \hspace{1cm} (11)

Special attention should be paid to the right-hand side of the equation. Structural engineers call equations, where the stiffness $k$ appears on both sides of the equation a support or footpoint excitation. The mathematical expression $\dot{a}$ represents a bearing displacement, and although the unit of the right-hand side expression $ka$ is a force, substituting it with an ‘actuator force’ $F_A$ will lead to ambiguous if not incorrect interpretations of the system behavior. The only valid definition for the occurring bearing force is given in Eq. (10). Different right-hand side formulations represent different physical setups. Let us neglect the eccentricity for a moment ($\varepsilon = 0$) to further explain the difference. An equation $m\ddot{q}_S + k\dot{q}_S = ka$ represents a system with actuators placed at shaft’s end. When the actuator is moving statically, the shaft does not bend. In contrast, an equation of the form $m\ddot{q}_S + k\dot{q}_S = F_A$ represents a system where the shaft’s end is attached to a fixed support similar to Fig. 1 and the actuator force $F_A$ is acting at the disc. In this case, an actuator force indeed causes a bending of the shaft. We consider this difference as one of the key elements in this work.

Eq. (11) is certainly not new and authors of standard literature for active bearings found similar expressions. The actuator displacement $\dot{a}$ on the right hand side is usually called ‘reference input’ and is considered to be a feature to compensate...
static rotor displacements caused by external influences [3, 25]. As a static displacement compensation does not contribute to the dynamic system behavior, they are usually omitted in the subsequent dynamic system analysis. Our approach differs from the standard way and uses the actuator displacements \( a \) as a dynamic input, not a static one. The system is controlled by dynamically displacing bearing point \( a \), leading to a disc displacement.

3. Control strategy

The control objective is to eliminate unbalance-induced bearing forces. These forces vanish when masses’ acceleration is zero and the mass is kept at the rotational center, \( q_s = 0 \). According to Eq. (1), the elastic center then rotates around the rotational center, \( q_W = - \varepsilon \). Additionally, the elastic constraints of Eq. (10) must be satisfied. The undeflected shaft exerts forces neither at the mass nor at the bearing, which is the case when both actuator displacement and elastic center are coincident, \( a = q_W \). Fig. 3 illustrates the kinematics of the force-free condition. Adjusting the actuator displacement \( a \) in the described way might seem like an easy task, but it is challenging due to the following reasons:

- The eccentricity \( \varepsilon \) is not constant in inertial coordinates, it depends on the rotational speed \( \Omega \), see Eq. (5).
- The eccentricity \( \varepsilon^+ \) is constant in the positive rotating coordinate system, but it is usually unknown. The problem gets more complicated as the eccentricity \( \varepsilon^+ \) is a complex quantity which has two degrees of freedom in the complex plane.
- The solution for a linear differential equation is the superposition of both homogeneous and particular parts. Even if it is possible to displace the actuators in a way that above conditions are met, only the particular solution is affected. The homogeneous solution from Eq. (9) is still performing undamped oscillations which generate bearing forces. For the controller design also the homogeneous solution has to be addressed.

3.1. Control principle

We stated that the rotor operates force-free when the actuator displacement is geometrically exactly opposite to the eccentricity, \( a = - \varepsilon \) as it permits the center of mass to stay in the rotational center. In the positive rotating coordinate system, both eccentricity and actuator displacement must be constant in force-free condition. We therefore design a controller with a compensating displacement element \( q_F^+ \) which is defined in the positive rotating coordinate system. The controlled system should finally result in force-free condition, hence the compensating element \( q_F^+ \) must converge to the negative eccentricity \( -\varepsilon^+ \). A necessary condition for convergence is stability. Speaking in general terms, a system is stable when its overall energy balance is negative, and the overall controller behavior should be dissipative. As minimal requirement, the compensating element \( q_F^+ \) should not excite the rotor. According to the principle of work, no energy will be transferred to the mechanical system when the velocity of the compensating element \( \dot{q}_F^+ \) is geometrically perpendicular to the bearing force \( F^+ \). Fig. 4a illustrates the geometric properties of the controller element while the governing control law is given in Eq. (12). The complex unit \( i = \sqrt{-1} \) represents the property of perpendicularity. The real parameter \( \tilde{c}_e \) is the adaption speed of the controller element:

\[
\dot{q}_F^+ = - \tilde{c}_e F^+
\]

The equation is transferred to the inertial coordinate system. To do so, the relation from fixed to rotating coordinates \( \dot{q}^+_R = q^+_e e^{-i \omega t} \) is derived with the product rule, yielding \( \dot{q}^+_e = (\dot{q}_f - \tilde{i} \omega q_f) e^{-i \omega t} \). The transformation for the actuator force \( F^+ = F e^{-i \omega t} \) leads to the controller element formulation in inertial coordinates

\[
\dot{q}_f = \tilde{i} \omega q_f - \tilde{c}_e F
\]

The controller from Eq. (13) does not destabilize the system, but cannot stabilize it either. Therefore, a dissipative element is
Eq. (14) is a mathematical equivalent to a mechanical spring-damper system. The parameter $c_D$ represents the inverse damping coefficient and the parameter $k_D$ serves as a return spring. The damping element of Fig. 4c is the only element where the directions of force and velocity are the same:

\[ \dot{a} = -c_D F - k_D a \]

With the superposition of the adaption and damping elements from Eqs. (13) and (14), the system can be stabilized. However, the controller parameters have to be chosen carefully, since stability depends on both mechanical rotor properties and controller parameters. The problem of conditional stability for a certain parameter field is present in many unbalance compensation algorithms [6,14,26].

The stability of a system is independent of the unbalance excitation and depends on the eigenvalues of the homogeneous solution only. Recalling the homogeneous solution of the Jeffcott Rotor from Eq. (9), it is interesting that the free whirling of the rotor is symmetric. One part of the solution is defined in the positive rotating coordinate system, while the other part of the solution is defined in a negative rotating coordinate system.

The control element from Eq. (12) compensates only the positive part of the homogeneous solution and does not affect the negative one. By introducing a compensating element that is defined in the negative rotating coordinate system, the proposed controller can be symmetrized so that it compensates also the backward whirling. The definition of this compensating element is given in Eq. (15), the superscript $-c$ denotes the negative rotating coordinate system. Fig. 4b illustrates the backward compensating element:

\[ \dot{a^-} = +i c^- F^- \]

Again, this element has to be transferred from the negative rotating coordinate system to the inertial one:

\[ \dot{a}_g = -i \Omega a_g + i c^- F \]

The proposed controller finally consists of three governing elements: The forward compensation mechanism from Eq. (12), the backward compensation mechanism from Eq. (15) and the damping element from Eq. (14). The final actuator displacement is the superposition of each displacement:
The controller equations can be conveniently summarized in state-space:

\[
\begin{bmatrix}
\dot{\mathbf{q}}_S \\
\dot{\mathbf{q}}_\Omega \\
\dot{\mathbf{a}}_F \\
\dot{\mathbf{a}}_B \\
\dot{\mathbf{a}}_D
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
-\mathbf{c}_D k & 0 & 0 & 0 & 0 \\
\mathbf{i} \Omega & \mathbf{D} & \mathbf{F} & \mathbf{C} & \mathbf{I} \\
0 & \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} \\
\mathbf{a} & \mathbf{c}_g & \mathbf{c}_k & \mathbf{c}_p & \mathbf{c}_e
\end{bmatrix}
\begin{bmatrix}
\mathbf{q}_S \\
\mathbf{q}_\Omega \\
\mathbf{a}_F \\
\mathbf{a}_B \\
\mathbf{a}_D
\end{bmatrix}
\]

The controller uses the bearing force \( F \) as a control input while it controls the bearing displacement \( a \). Both force and displacement are bound to the same degree of freedom. Such a system is called \textit{collocated} and is usually well suited for control [27]. The system matrix \( \mathbf{A}_g \) depends on the rotational speed \( \Omega \). Standard controllers have fixed parameters, while the input/output behavior of this controller changes with the rotational speed. We derived the \textit{adaptive} structure of the controller with a coordinate transformation from the rotating coordinate systems (Eqs. (12) and (15)) to the inertial one (Eqs. (13) and (16)). Knowing the rotor’s exact rotational speed is necessary for the controller to work properly.

### 3.2. The closed-loop solution

The subsequent analysis of the control system will be performed in state-space. The representation of the complete system in state space is possible by combining the controller from Eq. (18) with the state-space representation of the mechanical system from Eq. (11) and the bearing force from Eq. (10):

\[
\dot{\mathbf{x}} = \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{e} e^{i \Omega t}
\]

(19)

We focus on the response of the controlled system due to unbalance excitation. Although the state-space representation of Eq. (19) is more complex than the differential Eq. (6) of the Jeffcott Rotor, its particular solution can be found in a similar manner by substituting the vector of exponential functions \( \mathbf{x} = \mathbf{x} e^{i \Omega t} \) and its derivative \( \dot{\mathbf{x}} = i \Omega \mathbf{x} e^{i \Omega t} \) in Eq. (19), where \( \mathbf{I} \) represents the identity matrix:

\[
\mathbf{x}^* = (i \Omega \mathbf{I} - \mathbf{A})^{-1} \mathbf{B} \mathbf{e}^+
\]

(20)

The particular solution only exists if \( (i \Omega \mathbf{I} - \mathbf{A}) \) is nonsingular:

\[
\text{det}(i \Omega \mathbf{I} - \mathbf{A}) = -2 \mathbf{c}_E \mathbf{c}_O^2 (i \Omega + \mathbf{c}_p k_D)
\]

(21)

Assuming the rotor is rotating \( \Omega \neq 0 \) and all other parameters are real and nonzero, the solution exists. This result is somehow surprising as the uncontrolled Jeffcott Rotor from Eq. (7) indeed has a singularity at \( \Omega = \Omega_0 \), where the rotor gets excited at its resonance frequency. The controlled system has no singularity and hence no \textit{resonance} in its operational range.

The direct inversion of \( (i \Omega \mathbf{I} - \mathbf{A})^{-1} \) is a cumbersome task. In Section 3, we already defined how the kinematics of the force-free particular solution should look like. If the \textit{guessed} solution for \( \mathbf{x}^* \) solves Eq. (20), a direct inversion can be avoided.

The controller was designed to keep the mass in the rotational center, so both position \( \mathbf{q}_S \) and velocity \( \mathbf{q}_\Omega \) of the mass must be zero. The actuator displacement \( \mathbf{a} \) should match the negative eccentricity. The total actuator displacement \( \mathbf{a} \) is a superposition of elements, however only the forward compensating element that is defined in the same coordinate system as the eccentricity can contribute to the unbalance response. The forward compensating element \( a_F \) must therefore match the negative eccentricity \( -e \) while the other elements \( a_B \) and \( a_D \) are assumed to be zero. The preliminary, guessed solution is defined as

\[
\mathbf{x}^* = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T
\]

(22)

Eq. (22) is the correct particular solution when it satisfies Eq. (20):
Eq. (23) reveals that the preliminary solution $x^+$ solves the differential equation and is therefore the unbalance response of the controlled rotor. The controller was designed in a way that the mass stays always in the center $q_S=0$ while the shaft center rotates driven by the actuator around the midpoint, $q_W=a=-\varepsilon$. According to Eq. (10), the bearing force $F$ is always zero since $q_W$ and $a$ are coincident. Fig. 5 compares the analytical solutions for the controlled and uncontrolled system.

We want to highlight some properties of the solution. Eqs. (21) and (22) reveal that there is no resonant effect even at the natural frequency $\omega_0$ of the rotor. Furthermore, the controller parameters $\tilde{c}_C$, $c_D$ and $k_D$ have no influence on the particular solution. Even the rotor mass $m$ or the system stiffness $k$ have no effect on the steady-state solution. The proposed controller in Eq. (18) does not depend on magnitude or position of the eccentricity $\varepsilon$, as long as the bearing force $F$ can be measured. However, it is important to know the rotational speed $\Omega$ of the rotor at all times.

The designed controller is able to fully eliminate both the bearing force induced by unbalance and the resonance at any given rotational speed $\Omega \neq 0$, even at the rotor’s natural frequency $\omega_0$.

3.3. Proof of stability

When the controlled system is stable, the homogeneous solution vanishes with proceeding time, afterwards only the particular solution contributes to the total system response. The homogeneous system $x = Ax$ is stable if the real parts of its
eigenvalues are negative. We check the stability of the system using the Routh–Hurwitz stability criterion [23]. A necessary condition for stability is that all coefficients of the characteristic polynomial are positive:

$$\det(\lambda I - A) = a_5 \lambda^5 + a_4 \lambda^4 + a_3 \lambda^3 + a_2 \lambda^2 + a_1 \lambda + a_0$$

$$a_5 = 1$$
$$a_4 = c_0 \omega^2 + k_0$$
$$a_3 = \omega^2 + 2ck_c \omega + m^{-1}k$$
$$a_2 = c_0 \omega^2 + 2ck_c k_0 \omega + c_0 k_0 m^{-1}k$$
$$a_1 = m^{-1}k \omega^2$$
$$a_0 = c_0 k_0 m^{-1}k \omega^2$$

(24)

The controller parameters are $c_0$ and $k_0$ as well as the mechanical parameters $m$ and $k$ are all assumed to be positive. However it turns out that some coefficients of the characteristic polynomial have odd exponents of $\omega$, leading to possible instability for negative rotational speeds $-\omega$. We introduce a factor $c_C$ so that all exponents of $\omega$ are even only:

$$\omega \tilde{=} c_C \omega$$

(25)

The necessary stability criterion is met when $c_C$ is assumed to be positive. A sufficient stability criterion is the positiveness of all 5 Hurwitz determinants $\Delta$:

$$\Delta_1 = c_0 \omega + k_0$$
$$\Delta_2 = c_0 k^2 (2c_C \omega^2 + m^{-1})$$
$$\Delta_3 = c_0 k^2 (2c_C (k + k_0) + 4c_C^2 k_0 \omega^2 + 4c_C k_0 m^{-1} \omega^2 + k_0 m^{-1} \omega^2)$$
$$\Delta_4 = 2c_C c_0 k m^{-1} \omega^2$$
$$\Delta_5 = 2c_C c_0 k k_0 m^{-2} \omega^2$$

All Hurwitz determinants are positive as only even exponents of $\omega$ appear. We have finally proven that the controlled system is always stable. This result is remarkable as typical implementations of unbalance compensation usually guarantee stability for certain fields of controller parameters only.

We just demonstrated that stability is always given, even at the operating point of the mechanical resonance $\omega_0$. Disturbances, for example shock excitations, cause the rotor to perform additional oscillations. It is guaranteed that these oscillations exponentially decrease and the mass returns to the rotational center and the bearing forces vanish. In practice, the rotor’s eccentricity might suddenly change through the loss of rotor parts. Directly after this event, transient bearing forces will occur until the homogeneous solution vanishes and the rotor returns to its force-free steady-state.

4. Controller implementation

In the previous chapters we assumed actuators and bearings to be massless and perfectly stiff. We will drop the second assumption and introduce a bearing or actuator stiffness $k_L$ for non-perfect conditions. The actuator displacement $a$ and the shaft end are no longer coincident and the shaft end gets a new massless coordinate $q_L$. It is not coincident with the bearing displacement $a$ unless the bearing forces are zero. We further reassign the former shaft stiffness $k$ to be $k_R$ in the following sections. The system stiffness $k$ is now reassigned to be the overall system stiffness. The unbalance response from Eq. (22) is independent of the overall stiffness $k$ so the bearing or actuator stiffness $k_L$ has no influence on the force-free condition of the system in steady-state. Fig. 6 shows the modified system:

$$k = (k_R + k_i)^{-1}k_kk_i$$

(26)

For controller implementation, it is inevitable to discuss the physical realization of the active bearing. As discussed before, we can reduce the large number of actuator principles to two general properties: Displacement controlled actuators and force controlled actuators. An ideal displacement controlled actuator has an infinite inherent stiffness. Consequently, control inputs are translated into actuator displacements, independently of the external loads applied. The stiffness property of the actuator qualify it as shaft support, even without a control mechanism. Piezoelectric actuators are an idealized example of this category. A force controlled actuator on the other hand has no inherent stiffness. The control input is translated into an actuator force, independent of the actuator displacement. An example of this category are voice coils and electromagnets. Force actuators cannot work as support without a stabilizing control mechanism. Depending on the actuator type, we will derive different controller formulations.
4.1. Displacement actuators

According to the assumptions made in Section 3, the physical realization of an active bearing of this kind consists of force sensors, displacement actuators and a mechanical bearing which are all connected in series. The system setup is perpendicular, representing both real and imaginary axes. Sample implementations can be found in [28,29].

The mechanical setup is collocated because displacements and forces are set and measured at the same node [27] such that the controller can be implemented according to Eq. (18). The complex-valued inputs and outputs represent the respective axes in the Cartesian coordinate system. Put in other words, the controller of Eq. (18) controls both axes. For a practical implementation, a conversion from the Cartesian coordinate system to the complex coordinate system is performed by multiplying one axis with the complex operator $i$. One main control feature is the distinction between the rotor’s forward and backward whirl, which can only be used when the controller has access to the force measurements of both axes. A decoupled implementation of Eq. (18) for each axis will probably not work as expected.

Many rotor systems do not run at exactly one speed, but operate at multiple speeds to cover different operating points. Since the controller’s system matrix is speed-dependent, a specific controller would be needed for every operating point nullifying the advantages of the original plain approach. We found a way to implement an adaptive controller without adding additional complexity to the system. During the definition of the controller in Section 3 two controller elements were defined in rotating coordinates. The original definition of the compensating elements of Eqs. (12) and (15) is independent of the rotational speed $\Omega$. By reformulating the controller in different coordinate systems, the parameter dependency can be removed. We define a modified state-space vector $\tilde{x}_R$ such that the controller elements can be represented in their original coordinate system:

$$\tilde{x}_R = \begin{bmatrix} a_t^+ & a_t^- & a_d \\ a_t^+ & a_t^- & 0 \\ a_t^+ & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \begin{bmatrix} a_t^+ & a_t^- & a_d \\ a_t^+ & a_t^- & 0 \\ a_t^+ & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i\Omega & 0 & 0 \\ 0 & -i\Omega & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_R \\ T \end{bmatrix} + A \begin{bmatrix} \dot{x}_R \\ T \end{bmatrix}$$  

The transformed coordinates of Eq. (28) are inserted in the state-space Eqs. (18):

$$\tilde{x}_R = T^{-1} \begin{bmatrix} A_R - \hat{A} \end{bmatrix} \begin{bmatrix} x_R \\ T \end{bmatrix} + T^{-1} \begin{bmatrix} B_R \end{bmatrix} F = \begin{bmatrix} A_R - \hat{A} \end{bmatrix} \begin{bmatrix} \tilde{x}_R \\ T \end{bmatrix} + T^{-1} \begin{bmatrix} B_R \end{bmatrix} F \tag{29}$$

The system matrix $(A_R - \hat{A})$ is a sparse, diagonal matrix with only one constant element and finally independent of the rotational speed $\Omega$. The measured forces are multiplied with the input matrix $B_R$ and transformed with the inverse transformation matrix $T^{-1}$. Since the system matrix is decoupled, each element of the state vector $\tilde{x}_R$ can be calculated separately. The control states in their respective coordinate systems are transformed to the inertial coordinate system using the transformation matrix $T$, to calculate the final actuator displacement $a$:

$$a = C_R \begin{bmatrix} T \end{bmatrix} \begin{bmatrix} x_R \\ T \end{bmatrix} \tag{30}$$

The input–output behavior of the transformed controller from Eqs. (29) and (30) is identical to the behavior of the parameter-dependent differential equation from Eq. (18), with the advantage that the controller’s system matrix is speed-
independent. The adaptive structure is achieved by simple multiplications with the time and speed-dependent transformation matrices $T$ and $T^{-1}$. We point out that some parts of the controller structure are similar to the one presented by Herzog et al. [20].

Despite the implementation advantages, we want to highlight some additional benefits that might not be obvious at first sight. We found in Eq. (22) that the forward compensating element equals the negative eccentricity $a_F = -\varepsilon$. In steady-state, the controller directly stores the rotor’s eccentricity as a displacement variable. When the absolute position $\varphi$ of the rotating coordinate system is known, the variable argument $\varOmega = \varphi$ can be substituted in the transformation matrix $T$ and the controller stores both magnitude and phase of the eccentricity $e^+$. Similarly, the controller element $a_B$ compensates and stores backward whirl excitations that may result from noncompliant rotors. Sub- or super-harmonic excitations as well as other disturbances will excite the damping element $a_D$. Latter controller elements represent deviations from the ideal machine operation and may be used for a simple machine health monitoring.

Alternatively, the dissipating element $a_D$ could be implemented as a mechanical spring-damper element, see Fig. 4d. The actuator then only controls the compensating elements $a_F$ and $a_B$ and is connected in series. A mechanical spring-damper element always keeps the rotor well-damped and is beneficial in case of failure of the active system. Stability is guaranteed since the matrix structure of the overall system remains unchanged.

We also assumed that the actuator is attached to the inertial coordinate system. In steady-state, the actuator performs circular motions with the radius $|\varepsilon|$. The actuator needs therefore a higher bandwidth than the maximum rotational frequency, which rules out slower actuator principles. However, in many applications the rotor’s unbalance $e^+$ changes very slowly with time. Accordingly, only small actuator bandwidths are needed when the displacement actuator is installed on the rotor. Adapting the controller for a rotating actuator is simple, only the output Eq. (30) has to be transformed to the positive rotating coordinate system. If the shaft forces are also measured on the rotor, the input force equations can be transformed to rotating coordinates, $F = F^e e^{\varOmega t}$. Since these modifications do not alter the structure of the closed-loop system, stability is still guaranteed.

In many cases, engineers will implement the control algorithm on a digital controller with the sampling time $t_S$. A simple method to derive a discrete controller is the forward finite difference approach, where the derivative $\ddot{x}$ of Eq. (29) is approximated using the current state vector $\ddot{x}$ and the state vector of the next timestep $\ddot{x}_{t+1}$. The calculated actuator displacement is given in Eq. (32):

$$\frac{\ddot{x}_{t+1} - \ddot{x}}{t_S} \approx (A_{x} - \dot{A})\ddot{x} + T^{-1}B_{x}F$$

$$\ddot{x}_{t+1} \approx \left((A_{x} - \dot{A})t_S + I\right)\ddot{x} + T^{-1}B_{x}F$$

$$a_{t+1} = C_{x}T\ddot{x}_{t+1}$$

One problem of explicit discretization methods is their conditional stability, and the system might get instable when the sampling time $t_S$ is large. On the positive side, the calculation of the state-space equations only requires little computational performance.

4.2. Force actuators

Force actuators have no inherent stiffness and are unable to support the rotor without an additional controller. Fig. 7a shows a floating rotor with an actuator force acting at the shaft end $q_L$. The origin of static instability is found in Eq. (26), since for zero actuator stiffness $k_L$ the overall system stiffness $k$ is zero and the system is unable to generate restoring forces.
As soon as the shaft displacement $q_L$ can be measured and used for control, a binding between shaft end $q_L$ and displacement vector $a$ can be established. The situation is illustrated in Fig. 7b:

$$F = k_L(q_L - a)$$  \hspace{1cm} (33)$$

The basic control law of Eq. (33) reproduces electronically an actuator stiffness $k_L$ that is needed to guarantee static stability of the controlled system. Put in other words, the controller of a force actuator needs to ensure the positiveness of the overall stiffness $k$. In all previous considerations, the actuator displacement $a$ was a real physical, measurable quantity. In this case, $a$ is a calculation quantity that may be best described as a virtual dynamic position setpoint. Inserting Eq. (33) into the controller Eq. (18) yields

$$\dot{x}_R = (A_R - B_R C_R k_L) x_R + B_R k_L q_L$$ \hspace{1cm} (34)$$

$$F = k_L(q_L - C_R x_R).$$ \hspace{1cm} (35)$$

The resulting controller accomplishes two tasks. It stabilizes the rotor statically $k > 0$ and displaces the rotor in a way that the mass stays in the rotational center at any given rotational speed $\Omega$. Because the control law leads to the same state-space model from Eq. (19) it is consequently stable for any rotational speed $\Omega$. For a simple implementation, we use the transformed vector $\tilde{x}_R$ which was already introduced in Eq. (27):

$$\dot{\tilde{x}}_R = T^{-1} \left( A_R - \tilde{A} - B_R C_R k_L \right) T \tilde{x}_R + T^{-1} B_R k_L q_L$$ \hspace{1cm} (36)$$

In contrast to its displacement counterpart, the system matrix $(A_R - \tilde{A} - B_R C_R k_L)$ is densely populated, making a decoupling of the different controller states difficult. It is however possible to derive a time-discrete version for sampled systems using the forward difference method:

$$\ddot{\tilde{x}}_{R+1} \approx T^{-1} \left( (A_R - \tilde{A} - B_R C_R k_L) I_T + I \right) T \tilde{x}_R + T^{-1} B_R k_L I_T q_L$$ \hspace{1cm} (37)$$

$$F_{R+1} = k_L(q_L - C_R T \tilde{x}_{R+1})$$ \hspace{1cm} (38)$$

The dense population of system matrix prevents the elimination of the transformation matrix $T$. We point out that the discretized version is neither an exact solution nor does it share the property of unlimited stability with the exact solution. The transformed state vector $\ddot{\tilde{x}}_{R+1}$ contains the same information about the rotor as we explained in the previous paragraph.

5. Experimental validation

We use a rotor test rig with an active bearing to demonstrate that the proposed control approach is able to eliminate both resonances and bearing forces under real-world conditions. The experimental validation is performed for displacement actuators only, as our test rig is equipped with piezoactuators. In contrast to the theory presented in the previous chapters, our test rig is only equipped with only one active bearing plane, while the other one is passive.

5.1. Experimental setup

Our test rig can be seen in Fig. 8. It consists of a rotor with a disc and a flexible shaft. The steel shaft’s diameter is 9 mm,
and the length from bearing to bearing spans 190 mm. The disc weighs $m = 2.5$ kg. The shaft is driven by an electric motor which transmits the torque through a flexible joint to allow radial shaft movements. The active bearing on the left-hand side uses two piezoelectric actuators to displace the bearing assembly. The actuators have a maximum stroke of $\delta_{\text{MAX}} = 60 \mu$m. Pre-tension springs are used to keep the actuator compressed under all operating conditions and to ensure the mechanical integrity of the bearing assembly.

Piezoelectric force sensors are attached between the actuators and the support frame. Due to their small size, they are barely visible in the test rig photograph. They are used as control inputs. The direct collocation of sensor and actuator is a necessary requirement for the presented control approach. Eddy-current sensors measure the disc’s position. They have a monitoring function only and are not used for control. Strain gages enable the accurate measurement of the actuator deflections.

We use a standard PC with a National Instruments PCI-6259 I/O card for control and data acquisition, running Simulink Real-Time. A sampling frequency of $f_s = 15$ kHz minimizes discretization problems and avoids actuator noise. Piezoelectric actuators have a hysteresis effect, which is particularly distinct in large signal domain. A subsidiary PI-control that uses the measured actuator displacements provided by the strain gauges effectively removes the hysteresis effect. We emphasize that this control loop is not necessary to demonstrate the working principle, it does however improve the test results by removing higher hysteresis-induced harmonics. In the presented theory, we assumed that the bearing forces are exactly known. On our test rig however, the bearing assembly is supported by both the actuators and the pre-tension springs. In contrast to the previous assumptions, the forces on the actuators and bearing differ. When the actuator displacement $a$ and the pre-tension spring stiffnesses $k_R$ is known, we can derive the bearing force $F$ from the measured actuator forces $F_A$ using the formula $F = F_A - k_R a$. After the conversion of the actuator forces $F_A$ into the bearing forces $F$, the signal is fed to the controller which is implemented using Eqs. (29) and (30). The controller outputs are then passed to the subsidiary piezoposition controller.

We took advantage of the algorithm’s simplicity and adjusted the controller parameters manually. We excited the standstill rotor with a small hammer and adjusted the parameters $c_D$ and $k_D$ until the excitation response quickly vanishes. The unbalance compensation parameter $c_C$ can be adjusted easiest when the rotor is operated close to its resonance. We increased the value until the controller converged quickly.

In contrast to theory, we observed instability problems of the algorithm. Surprisingly, these problems occurred at rotational speeds that did not match the resonance frequency of the passive system. Our investigation revealed that the problem arises from an inaccurate estimation of the bearing forces. Small errors between the real spring stiffness $k_j$ and the estimated stiffness $\bar{k}_j$ lead to residual forces that cause instability. We assume that this problem only occurs outside the resonance because the ratio between bearing force and actuator displacement is very small, and consequently residual errors $k_j - \bar{k}_j \neq 0$ significantly contribute to the system response. To ensure stability, we augmented the controller’s system matrix by introducing additional elements $\delta$ in $A_\delta$ to prevent an integrator windup:

$$\dot{x}_R = \left( A_R - \bar{A} - A_\delta \right) x_R + \bar{T} B_R F$$

with $A_\delta = \begin{bmatrix} \delta & 0 & 0 \\ 0 & \delta & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(39)

The controller of Eq. (39) loses its theoretical capability of eliminating resonances, however it does prevent instability of the practical system when bearing forces $F$ are not accurately known. In practice we did not observe a performance degradation for small values of the parameter $\delta$.

5.2. Results and discussion

We demonstrate the theory with a rotor run-up from standstill to a maximum speed of 9000 rpm. Our theoretical consideration neglected the rotational acceleration, so we chose a small run-up acceleration of 25 rpm/s. We recorded both the bearing forces and the disc displacements for passive and controlled run-up. In passive run-up, the piezoelectric actuators were separated from the amplifiers. For evaluation, we transformed the data to complex coordinates and determined the magnitudes. The hull curves of both disc displacements and bearing forces are presented in Fig. 9.

The passive rotor has a distinct resonance at 75 Hz, where the disc displacements exceed 0.4 mm and the bearing forces reach 95 N. In controlled case, no clear resonance peak is visible neither in the displacements nor in the forces.

We highlight that the theoretical considerations only relied on some very basic assumptions, and numerous physical effects that occur in real-world rotors were not considered. In contrast to theory, we have a slightly different kinematic setup, our test rig’s mass distribution is continuous, the flexible joint prevents a perfect force-free static displacement of the rotor, the bearing forces are not perfectly known, the bearings have clearance, the rotor is not perfectly isotropic, and so on. Despite all these neglected factors, the theoretical results from Fig. 5 and the results from Fig. 9 match very well. We consider this as strong evidence that resonances can be eliminated not only theoretically, but also practically.
6. Conclusion

In this publication, we have proven theoretically and practically that active bearings can eliminate both resonances and bearing forces for a Jeffcott Rotor system. We derived a model-free controller and analytically confirmed that the closed-loop system is stable for any parameter or rotor configuration. We were able to derive different controllers for displacement as well as force actuators and further generalized the concepts of unbalance force elimination in one simple yet elegant theory.

The starting point for our investigation was the careful revision of the dynamic equations and the underlying rotor kinematics. In contrast to common literature for active bearings, we derived from the equations of motion that an active bearing controls displacements, not forces. Bearing forces need to satisfy the equilibrium conditions and cannot be arbitrarily changed without losing static stability. We reckon the imprecise use of the terms *bearing* and *actuator* may have contributed to this problem. We found that the bearing displacements are represented by a displacement vector which leads to consistent equations of motion.

On the basis of this revision, we developed a collocated controller that measures the bearing forces and controls the actuator displacements. The controller’s definition in different coordinate systems leads to a comprehensive explication of its speed-dependency. Because of the controller’s simplicity, we were able to derive an analytical solution for the controlled rotor. Surprisingly, the particular solution is independent of the rotor’s mass and stiffness and the eccentricity does not need to be known. It is moreover independent of the controller parameters and exists for any rotational speed $\Omega \neq 0$. The eccentric mass is unable to excite the controlled system, leading to an elimination of the unbalance-induced resonance. We demonstrated that bearing forces are zero at all operating points and displacements never exceed the magnitude of the eccentricity. The physical explication for this phenomenon is not damping, but the actuators’ possibility to change the system’s rotational axis.

A truly surprising property is the proven unlimited stability of the system, even without knowledge of the rotors mass, stiffness or eccentricity. The stability is even guaranteed when the rotor is operated at the exact speed of its mechanical resonance. The controller parameters such as the damping coefficient have no influence on the unbalance response, they just determine the speed of convergence.

We found a generalized theoretical viewpoint for different active bearing types. Therefore, we defined two different types of actuators. Displacement actuators are able to support the rotor due to their inherent stiffness while force actuators have no inherent stiffness and cannot support the rotor without additional control. The controllers for each actuator types are similar, except that the force controller provides additional static stability for the rotor. We demonstrated that our theory also covers rotor-fixed actuators. One major advantage of this concept is the reduced actuator bandwidth, which may be much smaller than the rotor’s rotational frequency.

Fig. 9. Test rig run-up from standstill to 9000 rpm with an acceleration of 25 rpm/s. The resonance is clearly visible in the uncontrolled case. In controlled case, both amplitude and force peaks are eliminated.
The introduction of rotating coordinates not only led to easier controller equations, but also provides additional system insight. The presented controllers are able to directly represent both magnitude and phase of the eccentricity in physical meaningful units.

We provided evidence for the practical feasibility of our approach using a test rig with piezoelectric actuators. In contrast to theory, we experienced some instability problems outside the resonance. The reason was found in the indirect and therefore inaccurate measurement of bearing forces. Although the presented theory did not cover most of the real-world effects, we verified that the test rig behavior matches the theory.

Currently, we are working on a generalization of the approach for arbitrary rotors.

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