Unconventional superconducting phases in a correlated two-dimensional Fermi gas of nonstandard quasiparticles: a simple model

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Abstract
We discuss a detailed phase diagram and other microscopic characteristics on the applied magnetic field–temperature (H–T) plane for a simple model of a correlated fluid represented by a two-dimensional (2D) gas of heavy quasiparticles with masses dependent on the spin direction and the effective field generated by the electron correlations. The consecutive transitions between the Bardeen–Cooper–Schrieffer (BCS) and the Fulde–Ferrell–Larkin–Ovchinnikov (FFLO) phases are either continuous or discontinuous, depending on the values of H and T. In the latter case, weak metamagnetic transitions occur at the BCS–FFLO boundary. We single out two different FFLO phases, as well as a re-entrant behaviour of one of them at high fields. The results are compared with those for ordinary Landau quasiparticles in order to demonstrate the robustness of the FFLO states against the BCS state for the case with spin-dependent masses (SDM). We believe that the mechanism of FFLO stabilization by SDM is generic: other high-field low-temperature (HFLT) superconducting phases should benefit from SDM as well.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Unconventional superconductivity in heavy-fermion and organic–metal systems is studied almost as frequently as high-temperature superconductivity and comprises a number of heavy-fermion and organic–metallic systems [1]. Among the states observed and discussed intensively recently is the superconductivity in the systems without space [2], and time [3] inversion symmetry, the Fulde–Ferrell–Larkin–Ovchinnikov (FFLO) state [4] and the states in which magnetic order, usually antiferromagnetic (AF), coexists with the FFLO [5, 6] or the Bardeen–Cooper–Schrieffer (BCS) type of state. Also, the FFLO states are discussed recently in the context of cold atomic fermionic gases [7] and quark-gluon plasma [8]. One of the basic motivations for these studies is the circumstance that the normal state can be represented by a Fermi fluid, albeit almost localized, so the nature of a paired state can be rationalized to a greater detail. Also, the intriguing feature of those superconductors is a cooperation rather than competition with magnetism [9]. The field-induced magnetism can be incorporated into the FFLO state, since there are substantial portions of the Brillouin-zone volume over which the quasiparticle excitations are gapless.

It is the latter topic (the FFLO appearance) which is the principal subject of this paper starting from a two-dimensional (2D) d-wave superconductor composed of unconventional (correlated) quasiparticles. Namely, we represent the heavy-fermion liquid by a gas of quasiparticles with the spin-direction-dependent effective masses (SDM), which were indeed observed in CeCoIn₅ and other systems [10] (in that
case $m^* \equiv m_\sigma, \sigma = \pm 1$ being the particle spin quantum number). Another non-trivial feature of our approach is the inclusion of the effective field $h_{\text{cor}}$ acting upon the magnetic moments in the spin-polarized state [11]. Both characteristics are generated by the electron correlations treated in the mean-field-type schemes [11]). The experimental motivation for our study is the observation of both SDM [10] and FFLO (or FFLO mixed with magnetism) [5] in the same heavy-fermion system CeCoIn$_5$ and the question we tackle is whether these two phenomena are interconnected. To address it we consider the simplest situation of an electron gas with the FFLO state in the simplest form (FF type with $\Delta(\mathbf{r}) = \Delta_0 e^{iQ \cdot \mathbf{r}}$). We also consider a $d$-wave form of the superconducting gap, i.e. $\Delta_{kQ} = \Delta_0 (\cos k_x - \cos k_y)$, where $Q$ is the Cooper-pair momentum ($Q \neq 0$ in the FFLO state). Such a form of the gap reflects the principal feature of quasi-two-dimensional superconductivity in strongly correlated electrons [3]. We show that the phase diagram with these high-field low-temperature (HFLT) $d$-wave superconducting phases in 2D differs remarkably from its 3D correspondent with the s-wave symmetry [15]).

Namely, several FFLO states appear in the present situation even when we disregard the possibility of their coexistence with antiferromagnetism [5, 6]. We also show that a weak metamagnetic transition accompanies the BCS $\rightarrow$ FFLO discontinuous transition.

2. Model: unconventional gas of quasiparticles with real-space pairing

The principal features of our approach have been defined earlier [14] (cf particularly sections V–VII). We start from the effective quasiparticle picture which is common to both narrow band and hybridized correlated-electron systems [11, 12, 16, 17]. The explicit form of quasiparticle energies in the gas of correlated quasiparticles is

$$\xi_{k\sigma} = \frac{\hbar^2 k^2}{2m_\sigma} - \sigma (h + h_{\text{cor}}) - \mu,$$

where $\mu$ is the chemical potential, $h = g_\mu B H_s$ and the mass enhancement factor in the large-$U$ limit [17] in the simplest situation is

$$\frac{m_\sigma}{m_B} = \frac{1 - n_\sigma}{1 - n} = 1 - n/2 - \sigma \frac{m}{2(1 - n)},$$

with $m \equiv n_\uparrow - n_\downarrow$ being the spin polarization and $n = n_\uparrow + n_\downarrow$ the band filling. Also $\sigma = \pm 1$ is the spin quantum number, $m_B$ is the band mass, $\Delta m \equiv m_2 - m_1$ is the mass difference and $m_\text{av} \equiv (m_1 + m_2)/2$ is the average mass. As one can see, the spin–dependent mass enhancement is particularly strong for an almost half-filled case when $1 - n \equiv \delta \ll 1$, i.e. for the quasiparticles close to the Mott–Hubbard localization. Here the superconducting phases in this 2D $d$-wave superconductor are discussed in detail and compared briefly with the previous results [14, 15]. In connection with this one should note that the concept of SDM has also been used in the context of the coexistence of ferromagnetism and superconductivity [18].

Even though our considerations represent a model situation, we assume the following values of the parameters, emulating the heavy-fermion systems: the filling $n = 0.97$, the elementary square-cell area $S = (4.62 \, \text{Å})^2$, the starting $(H_s = 0)$ quasiparticle mass $m_\sigma = 100m_\text{av}$ (data for CeCoIn$_5$ [10]), and the pairing potential cutoff and magnitude $h_{\text{cor}} = 17 \, \text{K}$ and $V_0 = 90 \, \text{K}$, respectively. The characteristic energy scale associated with spin fluctuations in CeCoIn$_5$ is $T_{dd} = 10 \, \text{K}$ [19a, 19b]—a value comparable to our $h_{\text{cor}}$. For those parameters, the chemical potential was equal to $\mu \approx 126 \, \text{K}$. This means that $V_0 \lesssim \mu$ and the (weak-coupling) BCS approximation can be regarded only as a proper solution on a quantitative level. Additionally, the chemical potential is readjusted in the superconducting state so that $n$ is constant. The pairing potential has the separable $d$-wave form

$$V_{k,k'} = -V_0 (\cos k_x - \cos k_y) (\cos k'_x - \cos k'_y),$$

which differs slightly from that used in [15d]. For thus-defined quasiparticles with energy $\xi_{k\sigma}$, we derive their correspondents $E_{k\sigma}$ in the superconducting states [14, 20]:

$$E_{k\sigma} = E_k + \sigma \xi_{k\sigma}^{(s)}.$$

$$E_k = \sqrt{\xi_{k\sigma}^{(s)2} + \Delta^2_{kQ}},$$

$$\xi_{k\sigma}^{(s)} = \frac{1}{2} (\xi_{k+Q/2\uparrow}^{\sigma} + \xi_{k-Q/2\downarrow}^{\sigma}),$$

$$\xi_{k\sigma}^{(a)} = \frac{1}{2} (\xi_{k+Q/2\uparrow}^{\sigma} - \xi_{k-Q/2\downarrow}^{\sigma}),$$

as well as the free-energy functional $\mathcal{F}$ and the system of four self-consistent integral equations for the field $h_{\text{cor}}$, magnetization $\vec{m}$, gap magnitude $\Delta_Q$ and chemical potential $\mu$. Explicitly, starting from the free-energy functional $\mathcal{F}$, we obtain the corresponding integral equations of the following form:

$$\mathcal{F} = -k_B T \sum_{k\sigma} \ln(1 + e^{-\beta E_{k\sigma}}) + \sum_{k} (\xi_{k\sigma}^{(s)} - E_k)$$

$$+ N \Delta^2_{Q} \frac{V_0}{V} + \mu N + \frac{N}{n} \vec{m} h_{\text{cor}},$$

$$h_{\text{cor}} = -\frac{n}{N} \sum_{k\sigma} f(E_{k\sigma}) \frac{\partial E_{k\sigma}}{\partial \vec{m}} + \frac{n}{N} \sum_{k} \sum_{\sigma} \frac{\partial \xi_{k\sigma}^{(s)}}{\partial \vec{m}} \left(1 - \frac{\xi_{k\sigma}^{(s)}}{E_k}\right),$$

$$\vec{m} = \frac{n}{N} \sum_{k\sigma} \frac{\sigma f(E_{k\sigma})}{E_k},$$

$$\Delta_Q = \frac{V_0}{N} \sum_{k} (\cos k_x - \cos k_y)^2 \frac{1 - f(E_{k\uparrow}) - f(E_{k\downarrow})}{2E_k} \Delta_Q,$$

$$n = n_\uparrow + n_\downarrow = \frac{1}{N} \sum_{k\sigma} \left[ \nu_k^2 f(E_{k\sigma}) + \nu_k^2 [1 - f(E_{k\sigma - \sigma})] \right].$$
Those quantities determine the physical free energy in different (BCS, FFLO, NS) states which are compared to obtain the phase diagram and other microscopic characteristics, as we discuss below. Note that we limit ourselves to a single $Q$ (Fulde–Ferrell-type) solution [14, 20] as we intend to describe superconductivity with SDM in the simplest case and thus test the importance of the quasiparticles’ mass spin-direction dependence. In that situation, the whole problem comprises a simultaneous solution of those four integral equations for $\mu$, $\bar{m}$, $\Delta Q$ and $h_{\text{cor}}$ for fixed $Q$ followed by subsequent minimization of the thus-obtained physical free energy $\mathcal{F}$ with respect to $Q$.

Note also that for the case of quasiparticles with spin-independent masses (SIM) we use the dispersion relation

$$\xi_{k\sigma}^{(\text{SIM})} = \frac{\hbar^2 k^2}{2m_{av}} - \sigma \hbar - \mu.$$ 

Then, the free-energy functional has the same form as in (8) but with the $\frac{\hbar^2}{2m_{av}}$ term absent (because $h_{\text{cor}} = 0$ in that case) and the system of self-consistent equations for $\bar{m}$, $\Delta Q$ and $\mu$ is given by (10)–(12). In this system of equations $E_{k\sigma}^{(\text{SIM})}$ and $E_k$ are calculated using (4)–(7) with the dispersion $\xi_{k\sigma}^{(\text{SIM})}$ instead of (1).

3. Results: BCS versus FFLO states

The overall phase diagram in the 2D case on the applied magnetic field ($H_a$)–temperature ($T$) plane is exhibited in figure 1 for the cases with spin-dependent (SDM) ((a), (b)) and spin-independent (SIM) (c) effective masses. The FFLO phase is robust only in the former case, as for the $s$-wave solution for the three-dimensional gas [14], although the difference is greater in the 3D case. The specific difference is that, in the present case, two distinct phase-boundary lines appear inside the FFLO state, as detailed in figure 1(b): the topmost and the bottommost parts (red colour) have the Cooper-pair momentum $Q$ oriented along the $k_x$ (or $k_y$) direction, whereas the middle phase (blue colour) has $Q$ along the diagonal ($k_x = k_y$). Also, superconductivity of FFLO type exists up to the field of 35 T in the SDM case, i.e. to a field about four times larger than that for the SIM case. Hence, the former system indeed belongs to the class of high-field low-temperature superconductors.

To visualize the detailed nature of the transition to the FFLO phase we have plotted in figure 2 profiles of the gap $\Delta Q$ along the diagonal ($k_x = k_y$). Figure 1 illustrates a re-entrant high-field behaviour for the FFLO1 phase (cf also figure 1(c) for the SIM case). Note also that the FFLO states exist far beyond the second critical field $H_{c2}$ for the BCS state, marked by the dashed line.

Note that, for SDM with increasing temperature, the transition from BCS to FFLO state occurs at higher fields, in qualitative agreement with experimental results [5]. The different FFLO phases are exhibited in (b). The red region corresponds to the Cooper-pair momentum $Q$ in the $k_y$ direction ($\theta_0 = 0$), whereas the blue one to the momentum along the diagonal ($k_x = k_y$, $\theta_0 = \pi/4$). Note that this anisotropy results solely from the $d$-wave gap symmetry, as the unpaired gas is isotropic. The dashed line marks the BCS critical field $H_{c1}$; the dotted–dashed line marks $H_{c2}$ for the solution with $\theta_0 = 0$.

The above phase transitions can be connected with the magnetization changes. This is because the FFLO
Figure 2. (a) Gap parameter $\Delta_1$ (in units of K) and (b) Cooper-pair momentum $Q$ in units of the Fermi momentum difference $\Delta k_F \equiv \Delta k_F^\uparrow - \Delta k_F^\downarrow$, both for the SDM case on $H_a$-$T$ plane. Transitions between various phases are seen as a change of the magnitude of the gap: the lower-field transitions are first-order, whereas the transition to normal state is continuous.

Figure 3. Spin polarization $\bar{m} \equiv n_\uparrow - n_\downarrow$ as a function of applied field. Note the weak jumps corresponding to the discontinuous transitions at $T = 0.02$ and $0.50$ K for the SDM case (a) and much larger in the SIM case (b). For the SDM case all transitions at $T = 1$ K are continuous.

phase encompasses semi-macroscopic regions of $k$-space with gapless quasiparticle excitations in the superconducting phase. This means that the magnetization curve will show a nontypical behaviour, particularly in the vicinity of the transition to the FFLO state, as displayed in figure 3. Namely, the $\bar{m}(H_a)$ exhibits a weak metamagnetic behaviour accompanied by a weak jump at the two lower-field transition points. It is surprising at first glance that the corresponding jump is much larger in the SIM case. However, one must remember that in the SDM case the field $h_{cor}$ compensates largely the applied field (see figure 4 for details). The spin magnetization does not include the magnetic dipole moment which may arise from an inhomogeneous current-carrying state when $Q \neq 0$. Obviously, the FFLO state may coexist with AF (or SDW) or spin-flop phases, but these cases are not discussed here, as we would like to characterize in detail here the ‘pure’ FFLO state to single out its novel features in the SDM case.

To compare our results with those for the three-dimensional system and s-wave pairing symmetry we recall here the mechanism behind the FFLO stabilization by SDM presented in [14] (cf section VI there). Namely, SDM compensates the Zeeman effect influence by reducing the Fermi wavevector’s splitting. Therefore, a superconducting state with SDM has higher critical fields (here $h_{c2} = 10$ T for SIM and $h_{c2} = 36$ T for the SDM case, cf figure 1). The FFLO state benefits from SDM by a greater extent than BCS because spin polarization $\bar{m}$ in the latter is smaller (cf figure 3), and from (2) the mass difference $\Delta m \propto \bar{m}$. Therefore, in BCS the mass difference is smaller and the Fermi wavevector’s splitting larger than in the FFLO (the Zeeman term influence is compensated less effectively). Hence, at $T = 0$ the FFLO fills about $1/2$ of the phase diagram for SIM and about $2/3$ for SDM. On the other hand, as temperature $T$ increases, the spin polarization increases in the BCS state (see figure 3) allowing a larger mass difference $\Delta m$ and reducing the Fermi wavevector’s splitting, enhancing superconductivity. This is
why the transition line between BCS and FFLO is curved upwards in the SDM case. In the present situation, the BCS state can have a substantial spin polarization already at \( T = 0 \) (unlike in the 3D, s-wave case) and therefore the BCS state can benefit from SDM already at \( T = 0 \), and the FFLO state is not stabilized so spectacularly here, as it was in the 3D case (where in the BCS phase \( \bar{m} \approx 0 \) at \( T = 0 \)).

For the sake of completeness, we draw in figure 4 the effective field induced by the correlations. The jumps reflect the discontinuous transitions discussed above. The field \( h_{\text{cor}} \) (in units of \( h_{\text{cor}}/\hbar \) for the unpaired Fermi sea) increases both with increasing temperature and field. The mass difference \( \Delta m = m_1 - m_1 \) changes with the applied field, reflecting the change in \( \bar{m}(H) \); the relative difference \( \Delta m / m_{av} \) reaches about 10\% for an applied field of the order of 30 T.

4. Conclusions

In summary, we have singled out different FFLO states in a 2D gas of correlated quasiparticles with spin-dependent effective masses (SDM) and effective field induced by the electron correlations, as well as compared them briefly with those in the SIM case [15]. A number of FFLO phases appear and these phases are stable in unusually high fields only for the case with SDM, which were indeed discovered in CeCoIn\(_5\) and other systems [10]. It is suggested that these nonstandard properties of quasiparticle states should be their universal feature for all the systems close to the f- or d-electron Mott–Hubbard localization if the atomic disorder effects are very weak. Namely, other HFLT phases (including various FFLO phases mixed with antiferromagnetism) can be stabilized from having SDM as well, since they always have higher spin polarization than the uniform superconducting state, and then SDM compensate the Zeeman term influence more effectively [6].

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