Created-by-current states in long Josephson junctions

T. L. Boyadjiev\textsuperscript{1,2}, O. Yu. Andreeva\textsuperscript{3}, E. G. Semerdjieva\textsuperscript{1,4} and Yu. M. Shukrinov\textsuperscript{1,5(a)}

\textsuperscript{1} JINR - Dubna, Moscow Region, 141980, Russia
\textsuperscript{2} Sofia University - Sofia, 1164, Bulgaria, EU
\textsuperscript{3} Tumen Thermal Networks OAO TGK 10 - Tobolsk, 626150, Russia
\textsuperscript{4} University of Plovdiv - Plovdiv, 4000, Bulgaria, EU
\textsuperscript{5} Physical Technical Institute - Dushanbe, 734063, Tajikistan

received 25 September 2007; accepted in final form 27 June 2008
published online 5 August 2008

PACS 74.50.+r – Tunneling phenomena; point contacts, weak links, Josephson effects
PACS 05.45.-a – Nonlinear dynamics and chaos

Abstract – Critical curves “critical current-external magnetic field” of long Josephson junctions with inhomogeneity and variable width are studied. We demonstrate the existence of regions of magnetic field where some fluxon states are stable only if the external current through the junction is different from zero. Position and size of such regions depend on the length of the junction, its geometry, parameters of inhomogeneity and form of the junction. The noncentral (left and right) pure fluxon states appear in the inhomogeneous Josephson junction with the increase in the junction length. We demonstrate new bifurcation points with change in width of the inhomogeneity and amplitude of the Josephson current through the inhomogeneity.

Introduction. – Starting from the classical paper of Owen and Scawlapino [1], the long Josephson junctions (LJJ) are attracting attention of both experimentalists and theoreticians. They are good candidates for a wide range of applications such as superconducting quantum interference devices, Josephson voltage standards, logic elements, and Josephson flux-flow oscillators because of their nonlinear behavior and quantum effects [2–4]. On the other hand, the nonlinearity makes the LJJ a very complex system, and some aspects of its properties have not been investigated up to now. Particularly, the experimental methods to determine the vortex structure under the critical curve are not yet well developed. But there are different kinds of stable states in the LJJ, whose bifurcation curves are close to the critical curve of the junction and manifest themselves in the experiments [2,5]. Particularly, some “metastable” states (experimental points under the overlap curve) were observed in ref. [6]. Static bound states of fluxons in LJJ with artificial inhomogeneities were experimentally found in ref. [5]. Experimental and theoretical investigation of such states is an interesting and actual problem.

An important step of the sample characterization in the experiment with LJJ’s is the measurement of its static properties, \textit{i.e.} the dependence of the critical current \(I_c\) on the external magnetic field \(H\). This dependence provides a technique to evaluate the several important parameters of LJJ from experiment, particularly, the critical current density, the magnetic flux penetration field, the effective magnetic thickness [7]. Since the solution of the corresponding boundary value problem (BVP) cannot be performed analytically in the general case, the most straightforward way to study the dependence \(I_c(H)\) is its numerical simulation.

In our previous study we investigated the stability of the vortices in LJJ with inhomogeneities (LJJI) [8,9] and the vortex structure in exponentially shaped Josephson junctions (ESJJ) [10–12]. The obtained bifurcation curves of the mixed fluxon-antifluxon states in LJJI [9] and pure fluxon states in ESJJ show the intervals of magnetic field, where these states are stable only if the current through the junction is not equal to zero. These peculiarities were not mention at that time. We call these states the CbC-states (created-by-current), and the corresponding regions of the external magnetic field CbC-regions.

In this letter we study the CbC-states of mixed fluxon states \(\Phi^{-1}\Phi^1\), \(\Phi^1\Phi^{-1}\) and pure fluxon states \(\Phi^n\). We demonstrate new bifurcation points with change in width of the inhomogeneity and amplitude of the Josephson current through the inhomogeneity. The influence of the model parameters on CbC-regions in the LJJI and ESJJ is investigated.
Method of calculation. — First we start with the LJJ case. We consider the Josephson junctions in the overlap geometry [2]. In order to study a linear stability of the static distribution \(\varphi(x)\) of the magnetic flux \(\phi(t, x)\), we write the perturbed expression for \(\phi(t, x)\) as \(\phi(t, x) = \varphi(x) + \xi(t)\psi(x)\), where \(\varepsilon\) is a parameter [13,14]. Then in a first approximation with respect to \(\varepsilon\), we get the following boundary value problem (BVP) for \(\varphi(x)\):

\[-\varphi_{xx} + j_C(x)\sin\varphi + \gamma = 0, \quad \varphi(-l) = \varphi(l) = h_c\]  

(1)

and the corresponding Sturm-Liouville problem (SLP):

\[-\psi_{xx} + \sigma(x)\psi_x + q(x)\psi = \lambda\psi, \quad \psi_x(-l) = \psi_x(l) = 0.\]  

(2)

Here all the quantities are in dimensionless form [3], \(h_c\) is the external magnetic field, \(\gamma\) is the external current, and \(\lambda\) is an eigenvalue of the SLP, \(2l\) is a length of the junction. The function \(j_C(x)\) represents the Josephson current amplitude, so \(|j_C(x)| \leq 1\). Because of the nonlinearity the BVP, eq. (1), has more then one solution for given parameters (counted set in case \(h_c = 0, \gamma = 0, l \to \infty\)) [15].

The potential \(q(x) = J_D(x)\cos\varphi(x)\) is defined by the concrete static solution \(\varphi(x)\), which is asymptotically stable with respect to small perturbations, if \(\lim_{t \to \infty}|\xi(t)| \to 0\). As the junction’s length \(l < \infty\) in the case under consideration and the potential \(|q(x)| \leq 1\), the SLP, eq. (2), is regular, so there exists a discrete low-bounded set of real eigenvalues \(\lambda_n, n = 0, 1, 2, \ldots\) and the minimal one \(\lambda_0 \geq -1\) [16].

The corresponding eigenfunctions satisfy the norm condition \(\int_{-l}^{l} \psi^2(x) dx = 1\). In case \(\lambda_0 < 0\) the static solution \(\varphi(x)\) is unstable, i.e. \(|\xi(t)| \to \infty\), when \(t \to \infty\). Solutions of the BVP, eq. (1), eq. (2), describe the different fluxon states in LJJ. A numerical simulation simplifies the study and makes it possible to estimate the range of variation of the parameters in which one can expect stability or instability of the magnetic flux distributions in the Josephson junction.

The models of LJJ depend on \(m\) parameters, such as junction’s length \(2l\), external magnetic field \(h_c\), external current \(\gamma\) and so on. Let us denote the vector of all the parameters by \(p\). In all cases we presume that the possible solutions of non-linear BVP eq. (1) continuously depend on the set \(p\), i.e. \(\varphi = \varphi(x, p)\). It follows that the potential of SLP eq. (2) generated by the solution \(\varphi(x, p)\), depends on the model parameters \(p\) as well, i.e., \(q = q(x, p)\). Finally, the corresponding eigenvalues and eigenfunctions of eq. (2) are continuous functions of \(p\), i.e. \(\lambda_n = \lambda_n(p), \psi_n = \psi_n(x, p)\).

Hence, the static solution \(\varphi(x, p)\) is asymptotically (exponentially) stable with respect to small space-time perturbations in some bounded subset of the parameter region, if the minimal eigenvalue satisfies \(\lambda_0(p) > 0\) [14]. Every point on the hypersurface \(\lambda_0(p) = 0\) is the bifurcation point for the solution under consideration. The values of the parameters which satisfy \(\lambda_0(p) = 0\), are called the bifurcation (critical) values for this solution. The cross-section of this hypersurface by a hyperplane, which correspond to fixed values of \(m - 2\) parameters, determine the bifurcation curve for the remaining two parameters. From the experimental point of view, the most important are the bifurcation curves of the kind “critical current-magnetic field”:

\[\lambda_0(\gamma, h_c) = 0.\]  

(3)

The numerical algorithm for the determination of the bifurcation curves is proposed in [17] and was successfully applied to various physical problems [9].

Bifurcation curves in LJJ with inhomogeneity. — The static configurations of finite-length junction in the presence of an external magnetic field were studied by many authors [1,8,9,14,18–21]. Particularly, Pagano et al. [18] demonstrated the bifurcation curves of the fluxon states for arbitrary junction lengths in the three different regimes typically observed: small, intermediate, and long junction. We consider here the LJJ whose barrier layer contains one resistive rectangular inhomogeneity, characterized by its width \(\Delta\), the position of the center of the inhomogeneity \(\zeta\) and the portion of the Josephson current \(k_J\) through it. The existence of the inhomogeneity leads to the local change of the Josephson current through the junction, which can be modeled by \(j_C(x) = 1 + k_J\) inside of the inhomogeneity and \(j_C(x) = 1\) outside of it. At \(k_J > 0\) the value of the current through the inhomogeneity exceeds the value of the current in other parts of the junction and such inhomogeneity are considered as a shunt [14]. At \(k_J \in [-1, 0)\) the value of the current through the inhomogeneity is less than in other parts of the junction and such inhomogeneity represents a microresistor. Changing the thickness of the barrier layer inside of the inhomogeneity, we can model the transformation from the “shunt” to the “microresistor” inhomogeneity. The value \(k_J = 0\) corresponds to a homogeneous junction. Results of the numerical solution of the non-linear eigenvalue problem [9] eq. (1), eq. (2) are presented in fig. 1, where the curve “critical current-external magnetic field” of the junction with \(2l = 7\) and the inhomogeneity with width \(\Delta = 0.7\) in the center of the junction is shown. This...
Table 1: Physical parameters for bifurcation states $M$, $\Phi^1$, $\Phi^{-1}\Phi^1$ and $\Phi^1\Phi^{-1}$.

| Type        | $\gamma_{cr}$ | $N$     | $\Delta\varphi/2\pi$ | $\varphi(0)/\pi$ |
|-------------|---------------|---------|------------------------|------------------|
| $\Phi^1$    | 0.228         | 1.317   | 1.69                   | 1.438            |
| $M$         | 0.065         | 0.073   | 0.847                  | 0.048            |
| $\Phi^{-1}\Phi^1$ | 0.034     | 1.654   | 2.28                   | 1.58             |
| $\Phi^{-1}\Phi^{-1}$ | 0.017   | 1.43    | 2.19                   | 1.329            |
| $\Phi^1\Phi^{-1}$ | -0.017   | 2.57    | 2.19                   | 2.671            |
| $\Phi^{-1}\Phi^{-1}M$ | -0.034   | 2.346   | 2.28                   | 2.42             |
| $\Phi^1$    | -0.065        | 0.073   | 0.847                  | -0.048           |
| $\Phi^1$    | -0.228        | 0.683   | 1.69                   | 0.562            |

curve is “constructed” as envelope of bifurcation curves, eq. (3), which belong to different possible bound states [9]. The inhomogeneity leads to the appearance on the critical curves of the fragments of bifurcation curves of the stable mixed states like $\Phi^n\Phi^{-n}$ and $\Phi^{-n}\Phi^n$, $n = 1, 2, \ldots,$ which are not stable in the homogeneous case. In one’s turn this leads to the non-monotonic decrease of the maxima of the critical curve, when the magnetic field increases. As we can see in fig. 1, in the investigated region of magnetic field the critical curve is an envelope of the critical curves for the Meissner state $M$, mixed distributions $\Phi^1\Phi^{-1}$, $\Phi^{-1}\Phi^1$ and pure fluxon states $\Phi^1$, $\Phi^3$, $\Phi^5$, $\Phi^7$. All “even” pure states like $\Phi^2$, $\Phi^4$, etc., are unstable, so their bifurcation curves, eq. (3), do not share the critical curve of the junction as whole.

The insert to the fig. 1 shows the details of formation of the critical curve in the marked region. In all our figures in this paper the solid lines correspond to the positive direction of the current, the dashed lines correspond to the negative direction of the current. The critical curve of the junction consists of pieces of the bifurcation curves for the $\Phi^1$, $\Phi^1\Phi^{-1}$ and $\Phi^3$ states in this region.

The bifurcation curves for mixed $\Phi^1\Phi^{-1}$ and $\Phi^{-1}\Phi^1$ states demonstrate an interesting peculiarity. As we can see, in the intervals of magnetic field $(h_1, h_2)$ and $(h_3, h_4)$ these Cbc-states are stable only if the current through the junction is not equal to zero. A value of the current, which is needed for the creation of stable mixed distributions $\Phi^1\Phi^{-1}$ and $\Phi^{-1}\Phi^1$ in Cbc-regions, depends on the value of the external magnetic field. Table 1 presents the values of the physical parameters for the bifurcation states $\Phi^{-1}\Phi^1$ and $\Phi^1\Phi^{-1}$ at $h = 1.9$ in comparison with $M$ and $\Phi^1$.

Here the average value of the magnetic flux (number of fluxons) is defined as a functional $N[\varphi] = \frac{1}{2\pi} \int_{-1}^{1} \varphi(x) dx$ and the full magnetic flux through the junction is $\Delta\varphi = \varphi(l) - \varphi(-l)$ [9]. For non-bifurcation pure fluxon states at $\gamma = 0$ the number of vortices equals the integer number: $N[\Phi^n\Phi^{-n}] = n$, while for non-bifurcation mixed fluxon states at $\gamma = 0$ we obtain $N[\Phi^n\Phi^{-n}] + N[\Phi^{-n}\Phi^n] = 2n$ [9]. For non-bifurcation the stable Meissner solution $N[M] = 0$. As we can see from the table, the found bifurcation values satisfy the following relations. For $\Phi^1$ states half of the sum of $N$ for positive and negative currents equals $n$. For Cbc-states we should take the sum of $N[\Phi^n\Phi^{-n}]$ and $N[\Phi^{-n}\Phi^n]$ for opposite current directions to have the integer number $2n$. The magnetic flux in the center of junction $\varphi(0)/2\pi$ fulfills a similar relationship.

The distribution of the internal magnetic field $\varphi(x)$ along the junction for bifurcation states $M$, $\Phi^1$, and $\Phi^{-1}\Phi^1$ in LJJ with length $2l = 7$, width of inhomogeneity $\Delta = 0.7$ at $h_c = 1.9$. The inhomogeneity in LJJ attracts the bound states and it leads to the deformation of the distributions of the magnetic field for these states. In fig. 3 we present the bifurcation curves for vortices in LJJ with length $2l = 12$ and the same width of inhomogeneity $\Delta = 0.7$ in the center of junction and the zero amplitude of Josephson current through inhomogeneity $j_c = 0$. It clearly shows the tendency of changing of the different bifurcation curves with the increase in magnetic field. The main features of this critical curve coincide with the results for LJJ with $2l = 7$. Particularly, we observe here the mixed states as well, and, as a result, the non-monotonic decrease of the maxima of the junction’s critical curve with magnetic field.

But in contrast to the case $2l = 7$, the new “noncentral” fluxon states appear here in addition to the “central” ones pinned to the inhomogeneity. We found that in the case under consideration the noncentral bound states $\Phi^1$, 47008-p3
\( \Phi^1, \Phi^2 \) and \( \Phi^{-2} \) are stable only. The distribution of the internal magnetic field along the junction for \( \Phi^1 \) and \( \Phi^2 \) is shown in fig. 2(b). For Josephson junction with \( 2l = 12 \) the states \( \Phi^1 \) and \( \Phi^2 \) are not stable without electric current at any value of the external magnetic field. We have not found the stable \( \Phi^1\Phi^{-1} \) and \( \Phi^{-1}\Phi^1 \) states in this case also, so we consider that they are replaced with \( \Phi^2 \) and \( \Phi^{-2} \) when the length of LJJ is increased. Or we can say that for LJJ with \( 2l = 12 \) the width’s value \( \Delta = 0.7 \) is small enough to stabilize the \( \Phi^1\Phi^{-1} \) and \( \Phi^{-1}\Phi^1 \) states. We can see below in fig. 5(a) the transformation of the \( \Phi^2 \) state into \( \Phi^1\Phi^{-1} \) and \( \Phi^{-1}\Phi^1 \) with the increase in \( \Delta \). The \( \Phi^2 \) and \( \Phi^{-2} \) demonstrate the CbC-regions which look like ChC-regions for mixed states \( \Phi^1\Phi^{-1} \) and \( \Phi^{-1}\Phi^1 \) for small junction length. Figure 3 allows us to represent, from the unified point of view, the influence of the inhomogeneity on the bifurcation curves of the fluxon states in LJJ. It leads to: a) the mixed fluxon-antifluxon states \( \Phi^0\Phi^0 \); b) the non-monotonic decrease of the maxima of the junction’s critical curve with magnetic field; c) right and left fluxon states; d) a different position of the CbC-states of the different fluxon states.

The influence of the inhomogeneity width. — The influence of the inhomogeneity width on the CbC-regions for the mixed state \( \Phi^1\Phi^{-1} \) in the case of \( 2l = 7 \) is demonstrated by fig. 4. It shows the dependence of the minimal eigenvalue \( \lambda_0 \) on the external current \( \gamma \) for \( \Phi^1 \) and \( \Phi^2 \) for the LJJ with two values of the width of inhomogeneity \( \Delta = 0.5 \) and \( \Delta = 1 \) at \( h_e = 2.2 \). The zeroes of \( \lambda_0 \) determine the critical currents for the corresponding states. The two values of the current \( \gamma_{cr} \) determine the lower and upper critical currents for the CbC-states at fixed value of the external magnetic field \( h_e = 2.2 \). We can see that with increase in \( \Delta \) the interval of the CbC-region for \( \Phi^1\Phi^{-1} \) increases. The influence of the inhomogeneity \( \Delta \) on the bifurcation curve for \( \Phi^2 \) is demonstrated in fig. 5(a). We show here the bifurcation curves at \( \Delta = 0.3; 0.5; 1 \). This figure demonstrates the transformation \( \Phi^2 \rightarrow (\Phi^1\Phi^{-1}, \Phi^{-1}\Phi^1) \) with increase in \( \Delta \).

There is a bifurcation region in the interval \( 0.3 < \Delta < 0.5 \), where the state \( \Phi^2 \) is getting unstable, but \( \Phi^1\Phi^{-1} \) and \( \Phi^{-1}\Phi^1 \) are stable. The details of such transformation are shown in fig. 5(b). As we can see, the state \( \Phi^2 \) is still stable at \( \Delta = 0.45 \), but the corresponding region of the magnetic field decreases with \( \Delta \). This region disappears completely in the interval \( 0.45 < \Delta < 0.5 \).

Actually, fig. 5 shows typical features of the catastrophe theory [22] in the current vs. the magnetic field plane. This correspondence is stressed by fig. 5(c), where the dependence of the total energy \( F \) on \( \gamma \) is shown for stable distributions \( \Phi^2 \), \( \Phi^1\Phi^{-1} \) and unstable ones related to them at \( h_e = 1.85 \) (see fig. 5b). The cusps in the \( B_1 \) point is a bifurcation point for \( \Phi^2 \) with change in the external current. Correspondingly, the points \( B_2 \) and \( B_3 \) are the cusps of the bifurcation points for mixed distribution \( \Phi^{-1}\Phi^1 \). Note, that the interval along the \( \gamma \)-axis which corresponds to the points \( B_2 \) and \( B_3 \) has not the point \( \gamma = 0 \), i.e. the current in the \( B_2 \)-point is a “creation current”, and in the point \( B_3 \) an “annihilation current” of the distribution \( \Phi^{-1}\Phi^1 \). The states the two horns (in the interval of magnetic field \( h_1 < h < h_2 \) in the insert to fig. 1) reflect the fact that these states cannot be stable without current through the junction. A detailed study of the observed features in correspondence of the catastrophe theory will be performed somewhere else.

Shifting of the inhomogeneity. — Figure 6 shows the bifurcation curves for \( \Phi^1 \) in LJJ with \( 2l = 7, \Delta = 0.7 \) and \( k_J = -1 \) at different positions of the inhomogeneity \( \zeta = 0; 0.5; 1; 1.5; 2 \) and \( 2.5 \). We found that when the inhomogeneity is shifted from the center of the junction, the pure fluxon states have the CbC-regions as well. The case \( \zeta = 0 \) correspond to the position of the inhomogeneity in the center of the junction.

The influence of the current value through the inhomogeneity. — The decrease in the value of the parameter \( k_J \), which characterizes the amplitude of the Josephson current through the inhomogeneity, leads to the same transformation \( \Phi^2 \rightarrow (\Phi^1\Phi^{-1}, \Phi^{-1}\Phi^1) \) as
we have observed with the increase in \( \Delta \). We demonstrate it for LJJ with \( 2l = 7 \) in fig. 7(a), where the transformation of the dependence \( N(h_e) \) for \( \Phi^2 \) with the value of the amplitude of the Josephson current through the inhomogeneity is shown. As we can see from this figure, a bifurcation of the states exists in the interval \(-1 < k_J < -0.735 \). We found that a new bifurcation point exists at \( k_J \approx -0.77 \). The dependence \( N(h_e) \) in LJJ with \( 2l = 7, \Delta = 0.7, \zeta = 0 \) and \( k_J = -1 \) is shown in fig. 7(b). It clearly demonstrates the relations for \( N \) for the pure and mixed bifurcation states we mentioned above. Particularly, for the pure fluxon state \( \Phi^3 \) the sum of the \( N(h_e) \) for positive and negative current equals 3, and for the states \( \Phi^2 \Phi^{-2}, \Phi^{-2} \Phi^2 \) we get \( N[\Phi^2 \Phi^{-2}] + N[\Phi^{-2} \Phi^2] = 4 \).

**Exponentially shaped LJJ.** As a second example, we consider an exponentially shaped LJJ [10–12]. In ref. [12] we demonstrated that the ESJJ is equivalent to the Josephson junction with distributed inhomogeneity. So we may expect the \( \text{CbC} \)-regions in the critical curves of the ESJJ as well. The corresponding BVP for the ESJJ has the form

\[
-\varphi_{xx} + \sin \varphi + \sigma (\varphi_x h_e - \gamma) = 0, \quad \varphi_x(-l) = h_e, \varphi_x(l) = h_e,
\]

where \( 0 \leq \sigma \ll 1 \) is the form parameter.
Results of the numerical solution of this BVP combined with the corresponding SLP [10] are presented in fig. 8(a), where the bifurcation curves for Meissner solution and four first fluxon states are shown for \( l = 7 \) and \( \sigma = 0.07 \). As we can see, the bifurcation curves of pure fluxon states in ESJJ demonstrate the CbC-regions. The CbC-states are realized at positive current in low magnetic fields and negative current in high magnetic fields. The exponential shape of the junction leads to the appearance of the distributed “geometrical” current \( \sigma (\varphi_c - h_c) \), which modifies the bifurcation curves of the states in comparison with the rectangular homogeneous junction [12]. Bifurcation curves for \( \Phi^1 \) at \( l = 7 \) and different values of \( \sigma \) are shown in fig. 8(b). The increase in \( \sigma \) leads to the increase of the CbC-region. With the increase in \( \sigma \) a situation might be realized where the dependence of the upper critical current on the external magnetic field in the CbC-regions gets non-monotonic. We found that such non-monotonic behavior of the bifurcation curve \( \Phi^1 \) in low magnetic fields appears in ESJJ with the increase in junction’s length as well.

In summary, we performed the numerical simulation of the critical curves of long Josephson junctions with inhomogeneity and variable width. In both cases we demonstrated the CbC-regions of the magnetic field, where fluxon states are stable only if the current through the junction is different from zero. We showed that the position and size of these regions depend on the length of the junction, its geometry, the parameters of inhomogeneity and the form of the junction. We consider that the development of the experimental methods for detection of ChC-states will open a perspective for their applications.

***

We thank I. V. Puzynin and N. M. Plakida for useful discussion and cooperation. This work is partially supported by Sofia University Scientific foundation under Grant No 135/2008 and RFBR grant 08-02-00520-a.

REFERENCES

[1] Owen C. S. and Scalapino D. J., Phys. Rev., 164 (1967) 538.
[2] Barone A. and Paterno J., Physics and Applications of the Josephson Effect (John Wiley and Sons) 1982.
[3] Licharev K. K., Dynamics of Josephson Junctions and Circuits (Gordon and Breach, New York) 1986.
[4] Petraglia A., Pedersen N. F., Christiansen P. L. and Ustinov A. V., Phys. Rev. B, 55 (1997) 8490.
[5] Vistavkin A. N., Drachevski Yu. F., Cosheletz V. P. and Serpuchenko I. L., Low Temp. Phys., 14 (1988) 646.
[6] Schmidtal K., Phys. Rev. B, 2 (1970) 2526.
[7] Goldobin E. and Ustinov A. V., Phys. Rev., 59 (1999) 11532.
[8] Filipov A. T., Gal’pern Yu. S., Boyadjiev T. L. and Puzynin I. V., Phys. Lett. A, 120 (1987) 47.
[9] Puzynin I. V. et al., Part. Nucl. (Dubna), 38 (2007) 70.
[10] Semerdjieva E. G., Boyadjiev T. L. and Shukrinov Yu. M., Low Temp. Phys., 30 (2004) 610.
[11] Shukrinov Yu. M., Semerdjieva E. G. and Boyadjiev T. L., J. Low Temp. Phys., 139 (2005) 299.
[12] Semerdjieva E. G., Boyadjiev T. L. and Shukrinov Yu. M., Low Temp. Phys., 31 (2005) 1110.
[13] Fogel M. B. et al., Phys. Rev. B, 15 (1977) 1578.
[14] Gal’pern Yu. S. and Filipov A. T., Sov. Phys. JETP, 59 (1984) 894.
[15] Iliev I. D., Khristov E. Kh. and Kirichev K. P., Spectral Methods in Soliton Equations (Longman Scientific & Technical, Wiley) 1994.
[16] Levitan B. M. and Sargsian I. S., Introduction to Spectral Theory, in Transl. Math. Monogr. (AMS, Providence, RI) 1975.
[17] Boyadjiev T. L. et al., Communications of JINR P11-88-409 (Dubna) 1988.
[18] Pagano S. et al., Phys. Rev. B, 43 (1991) 5364.
[19] Caputo J.-G., Flytzanis N., Gaididei Y., Stefanakis N. and Vavalis E., Supercond. Sci. Technol., 13 (2000) 423.
[20] Stefanakis N. and Flytzanis N., Supercond. Sci. Technol., 14 (2001) 16.
[21] Ustinov A. V., Long Josephson Junctions and Stacks, in Superconductivity in Networks and Mesoscopic Structures, edited by Giovannella C. and Lambert C. (AIP) 1998.
[22] Poston Tim and Stewart Ian, Catastrophe Theory and its Applications (Dover) 1996.