Evaluating Douglas-fir and western hemlock volume growth in response to thinning and fertilisation

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Abstract

Background: Thinning and fertilisation are two silvicultural tools which can modify the growth of a stand. Thinning re-allocates the resources on a site to increase the growth of the trees remaining after the thinning but does not necessarily increase total stand volume as compared to an unthinned stand. Fertilisation is intended to increase the growth of all trees in a stand resulting in more volume. Understanding the response of fertilisation and thinning treatments is critical to making good silviculture prescriptions. To assist with making these prescriptions, yield models for Douglas-fir (Pseudotsuga menziesii (Mirb.) Franco var. menziesii) and western hemlock (Tsuga heterophylla (Raf.) Sarg.) on coastal sites of British Columbia, Canada were developed. Douglas-fir and western hemlock are the two most important species on these sites.

Methods: The data for the modelling came from a large fertilisation and thinning trial (EP703). The model for total volume is based on the Chapman-Richards function. One parameter of the model was expressed as a function of a previous measurement and the other two parameters, which are in turn expressed as a linear function of site index and thinning and/or fertilisation intensity.

Results: Independent models were fitted for both species in the study using maximum likelihood estimation. The models were programmed into a spreadsheet to evaluate the behaviour of the models and examine selected responses.

Conclusion: These growth and yield models for Douglas-fir and western hemlock allow forest practitioners to evaluate the outcomes of proposed silviculture prescriptions.

Keywords: Coastal British Columbia; Douglas-fir; Fertilisation response; Modelling; Simulation; Thinning response; Western hemlock

Background

Thinning and fertilisation are two silvicultural tools available to modify the growth of trees. Thinning is aimed at modifying the growth of a stand by removing trees to adjust the stand density (Smith 1986). While thinning reduces the total volume in a stand, it has the effect of re-allocating the growth resources such as light, moisture, and nutrients to the remaining trees. The intended result is fewer, but larger and more valuable trees. Since thinning reduces stand volume, increases in
and Swanzy 1986). The time taken to regain crown closure will depend on the thinning intensity, that is, the amount of volume removed from the stand, and the productivity of the site. The thinned trees can either be utilized (a commercial thinning) or left at the site (a pre-commercial thinning).

Fertilisation can be done anytime during the rotation and a response should be expected. The greatest response may be realized in younger stands when demands for nutrients are high (Chappell et al. 1992; Miller 1981) and also in older stands if N deficiency exists (Miller 1981).

Douglas-fir (Pseudotsuga menziesii (Mirb.) Franco var. menziesii) and western hemlock (Tsuga heterophylla (Raf.) Sarg.) are the two most common and commercially important species in coastal areas of British Columbia, Canada. In 2011, the total volume of timber harvested on the coast of British Columbia was 19.1 million m³ (approximately 28% of the total British Columbia harvest), of which 38.2% was hemlock and 32.6% was Douglas-fir (Ministry of Forests, Lands and Natural Resource Operations 2012).

In order to prepare efficient thinning and fertilisation prescriptions for these species, the effect of the silviculture prescriptions on the yield needs to be known. A growth and yield model that accounts for thinning and fertilisation and their interaction is useful in this regard.

The purpose of this study was to develop empirical growth and yield models that simulate the response (volume growth) to thinning and fertilisation treatments of Douglas-fir and western hemlock stands. The models can then be used to evaluate different thinning and fertilisation strategies to gain a better understanding of responses of these species to these treatments. The literature reports many different, and sometimes conflicting, responses to thinning and fertilisation. The models that are developed and described here will allow the responses of Douglas-fir and western hemlock on coastal areas of British Columbia to be explored. Western hemlock and Douglas-fir growth after thinning and/or fertilisation can be assessed and better understood with the models.

The modelling approach of Nishizono (2010) was closely followed. This seemed to be an expedient method of analysis since the Nishizono (2010) model fit the data well. It is also possible to implement the model in a spreadsheet, making it available from the published report without requiring specialized software. The liberty of making some modifications to Nishizono’s work to get a better fitting model was taken. The model was also modified to incorporate both thinning and fertilisation and their interaction. This increased the utility of the model and allowed it to explicitly simulate thinning and fertilisation treatments as interacting responses.

Because the models are relatively simple, the responses can be evaluated analytically to some degree by examining the parameter estimates and can be more fully explored graphically.

Methods

Data

The data for this study come from Experimental Project 703 (EP703). This project is a large thinning and fertilisation trial established on the coast of British Columbia to examine the growth response of Douglas-fir and western hemlock to thinning and fertilisation (Darling and Omule 1989; Stone 1994). This study was initiated in 1971 and eventually encompassed 940 plots at 85 installations. Only plots that were pure Douglas-fir or pure western hemlock were analyzed, leaving 486 Douglas-fir plots and 222 western hemlock plots available for analysis. The plots were located in the Coastal Western Hemlock biogeoclimatic zone, zm1 (very dry maritime), zm2 (very dry maritime), vs1 (very wet maritime), ds1 (dry submaritime), and dm (dry maritime) subzone variants (Green and Klinka 1994). Installation locations and more specific plot information can be found in Stone (1994). Table 1 presents summary information about the plots at their first and last measurement.

The thinning treatments ranged from controls (0% removal) up to 50% removal. All but two of the thinned plots (one for each species) were thinned to maintain the thinning ratio (mean diameter of removed stems over the mean diameter of stems before thinning) at 0.8 – 0.9 (Darling and Omule 1989; Stone 1994). The other two thinned plots were thinned to 746 stems per hectare (Stone 1994). Fertilisation was applied at rates of 225, 450, 675, or 900 kg/ha with prilled urea (46% nitrogen, Darling and Omule 1989). The application was done manually except for 43 Douglas-fir plots which were fertilised aerially.

The total age (yrs), total standing volume (m³/ha), and volume (m³/ha) removed through thinning were available for each species in the plot at each measurement. Only trees that had a diameter at breast height of 5.0 cm or greater were measured. Since only pure Douglas-fir and western hemlock stands were of interest, a plot was classified as being pure Douglas-fir (pure western hemlock) if it contained 80% Douglas-fir (western hemlock) by volume, averaged over all measurements. However, regardless of the species composition, the total volume of all species in the plot was analyzed. The site index (m at breast height age 50) was estimated from the height and age of the target species using the Bruce (1981) or Wiley (1978) height-age models for Douglas-fir and western hemlock, respectively.
The modelling generally follows the procedure taken by Nishizono (2010). The base model is the Chapman-Richards function (Richards 1959):

\[ V_i = b_0 \left( 1 - e^{-b_1 x_i} \right)^{\frac{1}{b_2}} \]  

(1)

where \( V_i \) is standing total live volume (m\(^3\)/ha) at age \( t_i \) yrs, \( e \) is the base for natural logarithms, \( b_0, b_1, \) and \( b_2 \) are model parameters, and \( i \) indexes the measurement for a plot. By algebraically manipulating Equation 1 (Bailey and Clutter 1974), parameter \( b_1 \) was expressed as a function of a known volume \( V_{i-1} \) (m\(^3\)/ha) at age \( t_{i-1} \) yrs and parameters \( b_0 \) and \( b_2 \). In this analysis, \( V_{i-1} \) and \( t_{i-1} \) are the volume and age at the previous measurement.

\[ b_1 = \frac{\ln \left( 1 - \left( \frac{V_{i-1}}{b_0} \right)^{1-b_2} \right)}{t_{i-1}} \]  

(2)

where \( \ln \) is the natural logarithm. Substituting Equation 2 into Equation 1 for \( b_1 \) and adding a random error term \( (\varepsilon_i) \) yields Equation 3:

\[ V_i = b_0 \left\{ 1 - \left[ 1 - \left( \frac{V_{i-1}}{b_0} \right)^{1-b_2} \right]^{\frac{1}{b_2}} \right\}^{\frac{1}{b_2}} + \varepsilon_i \]  

(3)

Similarly to what Nishizono (2010) did, the remaining two parameters, \( b_0 \) and \( b_2 \), were expressed as functions of site index, thinning regime, and fertilisation regime:

\[ b_0 = b_{00} + b_{01} \times SI + b_{02} \times T + b_{03} \times F + b_{04} \times T \times F + b_{05} \times SI \times T + b_{06} \times SI \times F \]

\[ b_2 = b_{20} + b_{21} \times SI + b_{22} \times T + b_{23} \times F + b_{24} \times T \times F + b_{25} \times SI \times T + b_{26} \times SI \times F \]  

(4)

where \( SI = \) site index (m at breast height age 50), \( b_{ij}, i = 1, 2, j = 0, 1, \ldots , 6 \) are model parameters, \( T \) is thinning intensity, \( F \) is fertilisation intensity, and \( \varepsilon \) is a random error term with the usual assumption that the \( \varepsilon \) are identically and independently distributed as \( \mathcal{N}(0, \sigma^2) \). Thinning intensity (T) is the volume (m\(^3\)/ha) removed divided by the time (years) since the thinning. Fertilisation intensity (F) is the amount of fertiliser applied (kg/ha) divided by the time (years) since fertilisation.

Table 1 Summary information for the first and last measurements of the study plots

|                | Douglas-fir | Western hemlock |
|----------------|------------|----------------|
|                | First      | Last           | First    | Last         |
| Total age (yrs)| 38         | 64             | 47       | 72           |
| (18 – 76)     | (39 – 100) | (29 – 71)      | (55 – 93)|             |
| Ave. DBH (cm) | 16.4       | 24.2           | 22.0     | 32.7         |
| (4.5 – 46.2)  | (11.4 – 49.7)| (8.2 – 45.5) | (10.0 – 57.3)| |
| Ave. height (m)| 16.0      | 23.3           | 24.2     | 34.1         |
| (3.9 – 38.3)  | (11.3 – 41.2)| (11.0 – 41.1)| (12.4 – 46.6)| |
| SPH           | 1651       | 1223           | 1753     | 925          |
| (240 – 4840)  | (340 – 3580)| (243 – 7660) | (229 – 2400)| |
| Volume (m\(^3\)/ha)| 277.2    | 660.8          | 607.2    | 1118.9       |
| (42.1 – 1087.0)| (241.7 – 1618.4)| (172.2 – 1515.3)| (546.5 – 1869.8)| |
| Age at thinning (yrs) | 37         | 46             |
| (15 – 73)     | (28 – 68)             |             |
| Pre-thin volume (m\(^3\)/ha)| 312.8    | 369.8          |
| (62.5 – 1126.4)|           | (2480.0 – 1433.6) | |
| Volume removed (m\(^3\)/ha)| 85.6    | 107.2          |
| (13.1 – 369.6)|           | (78.9 – 623.3) | |
| Site index (m) | 26         | 28             |
| (14 – 42)     | (20 – 36)             |             |
| No. measurements | 7.3         | 7.1            |
| (4 – 9)       | (3 – 8)              |             |

DBH is diameter at breast height. SPH is stems per hectare. The average for the variable is presented with values in parenthesis indicating the range for the variable. The thinning statistics are for the thinned plots only.
Three significant modifications to Nishizono’s (2010) model and analysis were made: modifying the definition of thinning intensity, adding fertilisation, and analyzing the models as marginal or population average models (Diggle et al. 2002). These modifications are discussed more extensively in the Discussion section.

Mortality is not explicitly modelled. The dependent variable, standing live volume (V m³/ha), is net of mortality. Therefore, the model implicitly incorporates endemic mortality. If catastrophic mortality events such as massive windthrow or insect attack occur, then the models cannot be expected to give reasonable volume predictions. Since only endemic mortality was included in the model, data for which there was evidence of catastrophic volume loss was removed from the analysis data set.

Maximum likelihood estimation (procedure NLMIXED in SAS, SAS Institute Inc. 2004) was used to fit the model to the data. The variance was modelled as a power of the time since the last measurement (Pinheiro and Bates 2000), i.e., Var(ε) = σ²(t – t−1)². Serial correlation should not be an issue since volume is being projected for only one measurement interval. For each measurement in each plot, the standing volume (Vi m³/ha) was the dependent variable and the total age (ti years) and the standing volume (Vi−1) and total age (ti−1) from the previous measurement were independent variables in the fitting process, as well as the other variables in Equation 4. Consequently, the volume from the first observation for each plot was not used as a dependent variable since there was not a previous measurement to provide a value for Vi−1 and ti−1. For all plots except one Douglas-fir plot, the thinning was done immediately after the plot was established. Therefore, the post-thinning volume (as opposed to the pre-thinned volume) was considered to be the observation for that year except for the one plot where the thinning was not done at plot establishment. For this plot, the pre-thinned volume was used as Vi for that year. The post-thinning volume was obtained by subtracting the volume removed in the thinning from the pre-thinned volume and it was used as Vi−1 for the next observation. Each species was analyzed independently from the other. Normal quantile-quantile (QQ) plots of the standardized residuals were made to check for normality and plots of the standardized residuals versus the fitted values were made to check that the residuals were homoscedastic.

Results

Modelling

The results of the analysis of Equation 3 with parameters given in Equation 4 are presented in Table 3. The power parameter for the weights (δ) was 0.8162 for Douglas-fir and 0.6806 for western hemlock. Parameter σ in the variance model was 5.602 for Douglas-fir and 9.153 for western hemlock, although these values are hard to interpret on their own since the weights also have to be considered to fully understand the error structure. The QQ plots showed slight non-normality in the standardized residuals for Douglas-fir but the western hemlock standardized residuals were normal. There was no evidence

| Table 2 Input parameters used in the simulation runs |
|---------------------------------------------------|
| **Input**                                         |
| Parameter                                         | Values |
| Site index (m) (Douglas-fir)                       | 21  29  37 |
| Site index (m) (Western hemlock)                   | 25  29  33 |
| Thinning age (yr)                                 | 30   |
| Thinning level (% volume removed)                  | 0  20  35  50 |
| Fertilisation level (kg/ha)                        | 450 |
| Fertilisation age (yr)                             | 30  50 |
| Start age (yr)                                    | 20   |
| Start volume (m³/ha) (Douglas-fir)                | 30 @ site index 21  80 @ site index 29  130 @ site index 37 |
| Start volume (m³/ha) (Western hemlock)            | 20 @ site index 25  55 @ site index 29  90 @ site index 33 |
| Finish age (yr)                                   | 100  |
of heteroscedasticity for both species, indicating that the variance structure was adequately modelled.

Simulation
The results of the simulations for selected scenarios are discussed in the following section.

Discussion
This research results in models that predict stand volume after thinning and fertilisation for Douglas-fir and western hemlock on the coast of British Columbia. The predictions are made from stand volume at a known age, target age of the predicted volume, site index, and thinning and fertilisation intensity. Thinning intensity is defined as volume (m$^3$/ha) removed in the thinning divided by the time since thinning. Fertilisation intensity is similarly defined as the amount of urea fertiliser (kg/ha) applied divided by the number of years since the application. Therefore, the effect of both thinning and fertilisation on volume growth decreases over time. The models that were developed are dynamic in the sense of Diéguez-Aranda et al. (2006). That is, model predictions are based on a known volume/age point ($V_{i-1}, t_{i-1}$ in Equation 3), which is then projected forwards or backwards in time. The models are path invariant which means that the projections from age $t_{i-1}$ to age $t_1$ can be made in a single time step or multiple time steps and the same predicted volume will be obtained (Diéguez-Aranda et al. 2006).

This work closely follows that of Nishizono (2010), the major difference being the definitions of thinning intensity and the addition of fertilisation. Analyzing the models using a thinning intensity defined as the volume removed in a previous thinning, as was used by Nishizono (2010) resulted in a poorer fit. The definition of thinning intensity that was implemented (volume removed divided by time since thinning) results in the effect of the thinning in the model dwindling over time. The stands in the data set were only thinned once; hence only one thinning was modelled. It is not intuitive how to modify the implemented thinning intensity definition for multiple entries. Two options are to either accumulate the thinning intensities or to calculate the thinning intensity as if a previous thinning had not been done. The two methods would give similar results if the thinnings are far apart in time.

Similarly for fertilisation, the variable $F$ (fertilisation intensity) is the amount of fertiliser applied (kg/ha) divided by the time (years) since fertilisation. This allows the growth rate to increase rapidly after fertilisation but over time the effect of fertilisation dissipates.

Unlike Nishizono (2010), who analyzed the thinning model as a nonlinear mixed-effects model, the model was fitted as a nonlinear population-average model. The response (volume) of the population average was of more concern than predicting the response of individuals, making the population-average model more appropriate (Diggle et al. 2002, p. 130). Furthermore, in application it is unlikely that there will be enough measurements to adequately calibrate a mixed-effects model. Unlike linear models, a nonlinear mixed-effects model with the random parameters set to their expected value of 0 does not give unbiased estimates of the population mean (Meng and Huang 2009, p. 245). Hence, in the situation where the model will not be calibrated for individuals, it is preferable to fit the model as a population-average model.

In the data analysis, the models project the volume for one measurement only resulting in short projection periods. Consequently, evaluating the residuals from the analysis does not give a good indication of model performance over a long period of time, as would be useful in a real application. A more pragmatic test was performed by projecting the volumes with the first post-treatment observation (or first observations for the control plots) and examining the residuals from

Table 3 Results of the analysis of model 3 for Douglas-fir and western hemlock

| Species          | Parameter | Estimate | Std. Err. |
|------------------|-----------|----------|-----------|
| Douglas-fir      | $b_{00}$  | −906.7   | 116.3     |
|                  | $b_{10}$  | 88.23    | 4.707     |
|                  | $b_{20}$  | −175.1   | 23.92     |
|                  | $b_{30}$  | 21.40    | 3.803     |
|                  | $b_{40}$  | 6.376    | 1.075     |
|                  | $b_{50}$  | 11.92    | 1.450     |
|                  | $b_{60}$  | −0.6717  | 0.1054    |
|                  | $b_{10}$  | 0.8412   | 0.02315   |
|                  | $b_{21}$  | −0.007438| 0.0008311 |
|                  | $b_{22}$  | −0.001296| 0.0001893 |
|                  | $b_{23}$  | 0.0004199| 0.00004177|
|                  | $b_{24}$  | −0.000007131| 0.000001324|

| Western hemlock  | $b_{00}$  | −895.1   | 189.6     |
|                  | $b_{10}$  | 92.84    | 7.214     |
|                  | $b_{20}$  | 148.9    | 14.14     |
|                  | $b_{30}$  | −0.1918  | 0.01657   |
|                  | $b_{40}$  | −0.9547  | 0.326     |
|                  | $b_{50}$  | 1.355    | 0.04396   |
|                  | $b_{60}$  | −0.02524 | 0.001689  |
|                  | $b_{23}$  | 0.0002371| 0.00006200|
|                  | $b_{25}$  | −0.00006663| 0.000007122|

Parameter estimates that were significantly different from 0 at $\alpha = 0.05$ are presented. Other parameters are not in the model and hence have a value of 0. Parameter estimates and their standard errors are shown to 4 significant digits.
these projections. This was done for the control plots and plots that only had one treatment in order to isolate the model behaviour by treatment type. The residual variance increased as the length of the projection period increased as is typical in yield projections. The control plots did not show any bias over the projection period, nor did the fertilised western hemlock plots. The thinned Douglas-fir plots and the fertilised Douglas-fir plots each showed a bias of about 1 m$^3$/ha/yr and the thinned western hemlock plots had a bias of less than 2 m$^3$/ha/yr. These biases are small but nevertheless could be an area of future refinement to the models. In practice, periodic stand volume measurements could be taken and used as input to the model to reduce the length of future stand volume projections.

The starting parameters for the site index, start and finish ages, and thinning levels in the simulations (Table 2, Figures 1, 2, 3 and 4) were chosen to cover the range of the data. The initial volumes were obtained from runs of the Tree And Stand Simulator (TASS) growth and yield model (Mitchell 1975; Mitchell and Cameron 1985). Although the 100-year end time for the simulations is longer than typical rotation ages for Douglas-fir and western hemlock in coastal British Columbia (Brown 2003; Spittlehouse 2003), it may be informative to investigate the response to thinning and/or fertilisation over this longer time horizon.

Interpreting the effect of parameters $b_0$ and $b_2$ on model behaviour is somewhat complicated by the formulation. This is an artefact of parameter $b_1$ in Equation 1 being a function of parameters $b_0$ and $b_2$. Parameter $b_0$ in Equations 1 and 3 is the asymptote; hence larger values of $b_0$ indicate a greater potential volume from the site. Parameter $b_2$ is indicative of the growth rate of the stand with higher values indicating faster growth.

However, a graphical approach has to be taken to fully assess the behaviour of the models because of the interaction between parameters $b_0$ and $b_2$ in Equations 3/4. Snowdon (2002) classifies growth responses to silvicultural treatments as being a type 1 or type 2 response. Type 1 responses are characterized by a temporary increase in growth rate that reduces the time needed to reach a given stage of stand development. Type 2 responses are characterized by a real and sustained change in stand productivity. The response to fertilisation can be either type 1 or type 2, depending on the amount of fertiliser and the nutrient deficiency of the site (Snowdon 2002). Thinning will most likely be a type 1 response since the volume of a thinned stand rarely exceeds the volume of an unthinned stand, although the response to thinning may be sustained for quite a while. Given the model formulation in Equations 3/4, the response to both fertilisation and thinning dwindles over time making the response a type 1 response. Simulations using average start values in Table 2 confirm that both thinning and fertilisation are type 1 responses for western hemlock and Douglas-fir. Initially, fertilisation increases growth and thinning decreases stand growth but over time the stand growth for both treatments converge to the growth of the untreated stand. Note, however, that an extreme
thinning will reduce the volume to a point where the growth of the treated stand diverges from the growth of the untreated stand over a typical rotation period.

The response of Douglas-fir and western hemlock to thinning and fertilisation can now be compared to published responses using these new models. This is done in part to assess the integrity of the model and to evaluate specific responses. The assessment is done analytically by examining the parameters of the model (Equation 4) but mostly by using graphical techniques.

More productive sites (higher site index) have increased growth and yield

Site index is a measure of site productivity; better sites produce a greater volume of timber and the trees on the site are faster growing (Carmean 1975). Parameter $b_0$ increases and parameter $b_2$ decreases with site index for both species. As gleaned from the value of parameter $b_0$, the potential yield (asymptote) increases as site index increases. A graphical analysis (Figure 1) shows that growth also increases as site index increases even though parameter $b_2$ decreases. The models behave as expected with respect to site index.

The response to thinning depends on the thinning intensity

On a per tree basis, a heavy thinning will reduce competition more than a lighter thinning and hence the growth response is expected to increase as the thinning intensity increases (Pukkala et al. 2002). However, on a stand level the response is less clear because there are fewer trees to contribute to stand growth. Cañellas et al. (2004) found that *Quercus pyrenaica* growth increased with increasing thinning intensity but declined with the heaviest thinning. D’Amato et al. (2011) did not find any stand-level growth response to thinning of *Picea glauca* stands. For *Pinus sylvestris* stands, stand volume increment decreased as thinning intensity increased and the difference increased over time (del Río et al. 2008). To evaluate the fitted models, volume was plotted against age at 4 levels of thinning that occurred at age 30 assuming a site of medium quality (site index = 29 m) for both species (Figure 2). In absolute terms, the volume in the thinned stands did not approach the volume in the unthinned stands. That is, the volume lost in the thinning
was not recovered through increased growth of the trees left on the site. However, on a relative basis, the thinned stands had proportionally more volume at the end of the 100 year simulation than was removed. That is, for Douglas fir, the volume lost when compared to the unthinned stands was 4%, 12%, and 23% for the 20%, 35%, and 50% removals, respectively. The volume lost for western hemlock was 9%, 19%, and 32% for the same levels of removal.

**The effect of fertilisation on tree growth is relatively short-lived**

Nitrogen (N) is the most limiting nutrient in Douglas-fir and western hemlock sites (Bennett et al. 2003; Chappell et al. 1991). Consequently, fertilisation with N is more common than fertilising with other nutrients. The period of time for which tree growth is elevated after fertilisation is variable but relatively short, e.g. 5 – 10 years (Bennett et al. 2003; Binkley and Reid 1985), although exceptions can be found (Binkley and Reid 1985). When fertiliser is applied at the time of thinning the maximum response is observed about 5 years after treatment (Snowdon 2002). It has been hypothesised that the length of time that a fertilisation effect is observed depends on the amount of fertilisation applied in relation to the amount of the nutrient on site before fertilisation (Miller 1981). These hypotheses can be evaluated with the models. For each species, sites at three levels of site index were fertilised with 450 kg/ha of N fertiliser at age 50. This level of fertiliser is generally the maximum that is economically feasible (Barclay and Brix 1985). The percent change in growth as compared to an unfertilised site with the same site index was plotted (part a of Figure 3 is for Douglas-fir; part b is for western hemlock).

The response to fertilisation is much greater for Douglas-fir than it is for western hemlock, and the response lasts substantially longer for Douglas-fir. The response and the period of time over which the response is observed decreases as the site index increases. This is likely due to the better sites (as indicated by a higher site index) having a higher store of N on site at the time of the fertilisation, hence N is less limiting in growth. A similar response was observed by Heath and Chappell (1989) for Douglas-fir. A few points need to be made with this analysis:

1. Confidence intervals for the predicted percent change in volume growth are not available, so it cannot be determined when the change in volume growth is not significantly different from zero.
2. There is a significant interaction term between fertiliser amount and site index for Douglas-fir (but not western hemlock) and the coefficient for this term is negative (Table 3, parameter b_{06}). This results in a predicted loss of volume growth at high levels of fertilisation application and site index. This is more likely an artifact of the modelling than a real response since the predicted loss of growth is small and may not be significantly different from zero.
3. As with all models, users have to be wary when extrapolating these models. The response to fertilisation is extrapolated in terms of the length of the response.

**Interaction between fertilisation and thinning**

It may be of interest to determine whether the effects of thinning and fertilisation are additive or if there is an interaction between the two treatments (Miller et al. 1979; Zhang et al. 2005). This can be investigated with the models. The models for both Douglas-fir and western hemlock have significant interaction terms (parameters b_{04} and b_{24} for Douglas-fir, parameter b_{04} only for western hemlock, Table 3). In a much smaller experiment, McWilliams and Thérien (1996) did not find any interaction between thinning and fertilisation in Douglas-fir. Simulation was used to investigate how this interaction term affects volume. Three site index levels were assumed with corresponding volumes at age 20 (Table 2) for each species. Thinning and fertilisation treatments (35% removal with 450 kg/ha urea fertiliser) were applied at age 30. The same scenario was applied with the interaction term removed from the model. The difference in volume is then due to the thinning/fertilisation interaction. The volume-age trajectories are plotted in Figure 4, parts a (Douglas-fir) and b (western hemlock). The effect of the interaction between fertilisation and thinning is slight for Douglas-fir (approximately 20 – 25 m³/ha at age 100 for low and medium sites, practically no difference for high sites) and virtually non-existent for western hemlock. Interestingly, though, the very slight effect of the thinning × fertilisation interaction for western hemlock is negative which is due to the negative estimated value of parameter b_{04} in Equation 4. The lack of a significant interaction response for western hemlock (even though the parameter for this term is statistically significant) can be attributed to the small estimated value for parameter b_{04} in Equation 4. When this parameter is multiplied by the thinning and fertilisation effects (T and F, respectively, in Equation 4), the term does not have a large effect on the asymptote, particularly as the thinning and fertilisation effects diminish over time.

**Comparison to published yield tables**

In this section, the models developed here (Equations 3/4 with appropriate parameters from Table 3) are compared to other published yield tables. For western hemlock, the comparison is made to yield tables published by Barnes (1962) and Taylor (1934). These tables are for
fully stocked stands with a minimum diameter limit of 1.5 cm, making them approximately the same as the EP703 hemlock plots. However, both of these yield tables are for mixed species stands, and no thinning or fertilisation information is available. Barnes (1962) published yield tables for Oregon/Washington, Alaska, and British Columbia; the British Columbia tables were used in this comparison. At the same starting conditions at age 20, both the Taylor and Barnes yield curves are approximately the same for site index 25 and 29 stands. The western hemlock model predicts greater yields than these two yield tables at site index 25 and 29, but at site index 29 the yields are similar below age 60. At site index 34, the yields from Equations 3/4 and from Barnes’ yield table are almost identical (Taylor did not publish yield tables for site indexes greater than 32).

The fitted Douglas-fir model was compared graphically to published yield tables calculated using the Douglas-fir simulator (DFSIM) (Curtis et al. 1982). The DFSIM is a whole-stand growth and yield model developed specifically for Douglas-fir in the Pacific Northwest region of North America (Curtis et al. 1981). It is able to simulate natural and planted stands with pre-commercial and commercial thinning, as well as fertilisation. Four stand types were compared (natural, planted, planted and commercially thinned, natural with pre-commercial thinning and fertilisation) at three levels of site index (25.9, 32.0, and 38.1). These correspond to Tables 1, 2, 7, and 11, parts A, B, and C in Curtis et al. (1982). The simulations started at age 30. The starting volumes for the fitted model were taken from the corresponding tables in Curtis et al. (1982). A utilisation limit of 4 cm (1.6 inches) was used in the comparison.
when more than one utilisation limit was published. The commercial thinning and fertilisation treatments were applied at the same rate and age as in the published tables (Curtis et al. 1982). The thinning removed approximately 30% of the volume and the fertilisation was done with 225 kg/ha of N. The pre-commercial thinning in the fourth scenario was not modelled because the thinning was done too early for the fitted model to have any validity.

Figure 5 parts a – d show the results of the comparison. For natural stands (part a), there is good correspondence between the two models for the first 20 to 30 years after which the fitted model estimates higher volumes than those estimated by the DFSIM. The deviations between the two models increase with increasing site index. Curtis et al. (1981) had very little data for total ages greater than 60 years even in untreated natural stands. The plantation yield curves (part b) show similar yield projections over the longer term, but again the DFSIM yields are extrapolations beyond about total age 45 (Curtis et al. 1981). Unlike the natural stand yield curves, the differences between the two models decreases as site index increases. Part c shows that use of the DFSIM and Equations 3/4 lead to dissimilar volume predictions for thinnings regimes. Both before and after thinning, volume growth rates are greater using the DFSIM model than for Equations 3/4 at all levels of site index, Figure 5c. As noted previously, a slight bias may be present in Equations 3/4 for thinned stands, but not to the extent that it would explain the differences in yield indicated in part c of Figure 5. The two models may be projecting different populations as there are no data for trees over age 45 in the DFSIM development data set. The fertilisation scenario (part d) shows good agreement between the two models, particularly at higher site indexes. The models developed here do not show as much fertiliser response as the DFSIM at the lower site index level.

Conclusion
The growth and yield model developed by Nishizono (2010), which is based on the Chapman-Richard’s model, was fitted to Douglas-fir and western hemlock data. The resulting models are able to predict responses to thinning and fertilisation. Several modifications were made to Nishizono’s model which improved the behaviour of the models. Various scenarios were devised to demonstrate what type of management questions could be answered.

Competing interests
The author declares that he has no competing interests.

Author’s contributions
GN was the primary author and undertook the data extraction and analysis.

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