Formation of sharp eccentric rings in debris disks with gas but without planets

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‘Debris disks’ around young stars (analogues of the Kuiper Belt in our Solar System) show a variety of non-trivial structures attributed to planetary perturbations and used to constrain the properties of those planets1–5. However, these analyses have largely ignored the fact that some debris disks are found to contain small quantities of gas6–9, a component that all such disks should contain at some level6–11. Several debris disks have been measured with a dust-to-gas ratio of about unity4–9, at which the effect of hydrodynamics on the structure of the disk cannot be ignored2,12,13. Here we report linear and nonlinear modelling that shows that dust–gas interactions can produce some of the key patterns attributed to planets. We find a robust clumping instability that organizes the dust into narrow, eccentric rings, similar to the Fomalhaut debris disk14. The conclusion that such disks might contain planets is not necessarily required to explain these systems.

Disks around young stars seem to pass through an evolutionary phase when the disk is optically thin and the dust–to-gas ratio $\varepsilon$ ranges from 0.1 to 10. The nearby stars β Pictoris6,7,15–17, HD32297 (ref. 7), 49 Ceti (ref. 4) and HD 21997 (ref. 9) all host dust disks resembling ordinary debris disks and also have stable circumstellar gas detected in molecular CO, Na I or other metal lines; the inferred mass of gas ranges from lunar masses to a few Earth masses (Supplementary Information). The gas in these disks is thought to be produced by planetesimals or dust grains themselves, by means of sublimation, photodesorption10 or collisions11, processes that should occur in every debris disk at some level.

Structures may form in these disks by a recently proposed instability12,13. Gas drag causes dust in a disk to concentrate at pressure maxima18; however, when the disk is optically thin to starlight, the gas is most probably primarily heated by the dust, by photoelectric heating. In this circumstance, a concentration of dust that heats the gas creates a local pressure maximum that in turn can cause the dust to concentrate more. The result of this photoelectric instability could be that the dust clumps into rings or spiral patterns or other structures that could be detected by coronographic imaging or other methods.

Indeed, images of debris disks and transitional disks show a range of asymmetries and other structures that call for explanation. Traditionally, explanations for these structures rely on planetary perturbers—a tantalizing possibility. However, so far it has been difficult to prove that these patterns are clearly associated with exoplanets19,20.

Previous investigations of hydrodynamical instabilities in debris disks neglected a crucial aspect of the dynamics: the momentum equations for the dust and gas. Equilibrium terminal velocities are assumed between time steps in the numerical solution, and the dust distribution is updated accordingly. The continuity equation for the gas is not solved; that is, the gas distribution is assumed to be time-independent, despite heating, cooling, and drag forces. Moreover, previous investigations considered only one-dimensional models, which can only investigate azimuthally symmetrical ring-like patterns. This limitation also left open the possibility that, in higher dimensions, the power in the instability might collect in higher azimuthal wavenumbers, generating only unobservable clumps.

We present simulations of the fully compressible problem, solving for the continuity, Navier–Stokes and energy equations for the gas, and the momentum equation for the dust. Gas and dust interact dynamically through a drag force, and thermally through photoelectric heating. These are parametrized by a dynamical coupling time $\tau_d$ and a thermal coupling time $\tau_T$ (Supplementary Information). The simulations are performed with the Pencil Code21–24, which solves the hydrodynamics on a grid. Two numerical models are presented: a three-dimensional box embedded in the disk that co-rotates with the flow at a fixed distance from the star; and a two-dimensional global model of the disk in the inertial frame. In the former the dust is treated as a fluid, with a separate continuity equation. In the latter the dust is represented by discrete particles with position and velocities that are independent of the grid.

We perform a stability analysis of the linearized system of equations that should help interpret the results of the simulations (Supplementary Information). We plot in Fig. 1a–c the three solutions that show linear growth, as functions of $\varepsilon$ and $n = kH$, where $k$ is the radial wavenumber and $H$ is the gas scale height ($H = c_s/\sqrt{\gamma \Omega_K}$, where $c_s$ is the sound speed, $\Omega_K$ the Keplerian rotation frequency and $\gamma$ the adiabatic index). The friction time $\tau_f$ is assumed to be equal to $1/\Omega_K$. The left and middle panels show the growth and damping rates. The right panels show the oscillation frequencies. There is no linear instability for $\varepsilon \gtrsim 1$ or $n \lesssim 1$. At low dust load and high wavenumber the three growing modes appear. The growing modes shown in Fig. 1a have zero oscillation frequency, characterizing a true instability. The two other growing solutions (Fig. 1b, c) are overstabilities, given the associated non-zero oscillation frequencies. The pattern of larger growth rates at large $n$ and low $\varepsilon$ invites us to take $\zeta = \varepsilon n^2$ as characteristic variable and to explore the behaviour of $\zeta \ll 1$. The solutions in this approximation are plotted in Fig. 1f, g. The instability (red) has a growth rate of roughly 0.26$\Omega_K$ for all $\zeta$. The overstability (yellow) reaches an asymptotic growth rate of $0.2\Omega_K/2$, at ever-growing oscillation frequencies. Damped oscillations (blue) occur at a frequency close to the epicyclic frequency.

Whereas the inviscid solution has growth even for very small wavelengths, viscosity will cap power at this regime, leading to a finite fastest-growing mode (Supplementary Information), which we reproduce numerically (Fig. 1h). Although there is no linear growth for $\varepsilon \lesssim 1$, we show that there exists nonlinear growth for $\varepsilon = 1$. We show in Fig. 1i the time evolution of the maximum dust surface density $\Sigma_d$ (normalized by its initial value, $\Sigma_0$). A qualitative change in the behaviour of the system (a bifurcation) occurs when the noise amplitude of the initial velocity ($u_{rms}$) is raised far enough, as expected from nonlinear instabilities25,26. We emphasize this result because, depending on the abundance of H$_2$, the range of $\varepsilon$ in debris disks spans both the linear and nonlinear regimes. The parameter space of $\tau_T$ and $\tau_f$ is explored in one-dimensional models in Supplementary Information, showing robustness.

In Fig. 2 we show the linear development and saturation of the photoelectric instability in a vertically stratified local box of size...
In a small region (high dust-to-gas ratio and high frequency), the system can be characterized by overstability. Conversely, solutions represent the linear regime, where a perturbation is damped, imaginary components indicate oscillations, and the real part characterizes the growth rate.

Solutions are for axisymmetric perturbations $\psi = \psi_0 e^{\theta t + \text{ikx}}$, where $\psi_0$ is a small amplitude, $x$ is the radial coordinate in the local Cartesian co-rotating frame, $k$ is the radial wavenumber, $t$ is time, and $\theta$ is the complex frequency. Positive real $s$ means that a perturbation grows, negative $s$ indicates decay, and imaginary $s$ represents oscillations.

Solutions are for axisymmetric perturbations ($\theta = 0$) and non-axisymmetric modes ($\theta \neq 0$). The overall agreement is excellent. The growth rates are only very slightly underestimated. Nonlinear growth occurs when the amplitude of the initial perturbation exceeds a critical value, which marks the onset of nonlinear instability.

Nonlinear growth. Although there is no linear instability for $e = 1$, growth occurs when the amplitude of the initial perturbation exceeds a critical value, which marks the onset of nonlinear instability. Growth rates are typically lower in the nonlinear regime compared to the linear regime, and the growth rate increases with increasing dust-to-gas ratio.

The simulation shows that stratification does not quench the instability. The flow develops into a dynamic system of narrow rings. Whereas some of the rings break into arcs, some maintain axisymmetry for the whole timespan of the simulation. The system oscillates between stable and unstable states, with the frequency of these oscillations depending on the dust-to-gas ratio and the orbital period.

We consider now a two-dimensional global model. The resulting flow, in the $r-\phi$ plane ($r$ is radius and $\phi$ is azimuth), is shown in Fig. 3a–c. At selected snapshots, the flow develops into a dynamic system of narrow rings. The flow is characterized by oscillations in the radial and azimuthal directions, with the oscillation frequency depending on the dust-to-gas ratio and the orbital period.

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non-axisymmetric instability is observed, and the dust forms stripes. c, Time evolution of the vertically and azimuthally averaged density, showing the formation of well-defined rings. d, Time evolution of the maximum dust density. The instability saturates at about 70 orbits in this case. The slowdown compared with the growth rate $d_2/2$ predicted in Fig. 1 is because of the use of viscosity, and the background pressure needed for the stratification. The dimensionless parameter $\beta = \gamma (c_s/c_v)^3$ measures the strength of this term. e, Maximum growth rate, showing that linear instability exists as long as $\beta < 1$. The maximum growth rate decreases smoothly from $d_2/2$ for $\beta = 0$, to zero for $\beta = 1$. f, The structure formed in the dust density at $t = 50$ (about eight orbits) for different values of $\beta$. At moderate values, growth still occurs at a significant fraction of the dynamical time. The run shown in a-d used $\beta = 0.5$.

A development of the model is that some of the rings start to oscillate, seeming eccentric. These oscillations are epicycles in the orbital plane, with a period equaling the Keplerian, corresponding to the free (Supplementary Information). We also check that when the conditions for the streaming instability are considered, the photoelectric instability dominates (Supplementary Information).
oscillations in the right-hand side of Fig. 1a–c. We check (Supplementary Information) that they correspond to eigenvectors for which \( u = \gamma \); that is, gas and dust velocities coinciding. For this mode, the drag force and back-reaction are cancelled. So, for maintaining the eccentricity, this mode is being selected from among the other modes in the spectrum. This is naturally expected when the dust-to-gas ratio is very high. For \( \epsilon \ll 1 \), the gas is strongly coupled to the dust, canceling the gas–dust drift velocity in the same way that \( \gamma \ll 1 \) does in the opposite way, by strongly coupling the dust to the gas. In this configuration, the freely oscillating epicyclic modes can be selected.

We plot in Fig. 3e one of the oscillating rings, showing that its shape is better fitted by an ellipse (red dotted line) than by a circle (black dotted line). The eccentricity is 0.03, which is close to the eccentricity found\(^{29}\) for the ring around HD 61005 (\( \epsilon = 0.045 \pm 0.015 \)). We also notice that some of the clumps in Fig. 3 should become very bright in reflected light, because they have dust enhancements of an order of magnitude. In conclusion, the proposed photoelectric instability provides simple and plausible explanations for rings in debris disks, their eccentricities, and bright moving sources in reflected light.

Recent work\(^{29}\) suggests that the ring around Fomalhaut is confined by a pair of shepherding terrestrial-mass planets, below the current detection limits. Detection of gas around the ring would be a way to distinguish that situation from the one we propose. At present, only upper limits on the amount of gas in the Fomalhaut system exist\(^{10}\); however, they are relatively insensitive because they probe CO emission, and CO could easily be dissociated around this early A-type star.

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**Supplementary Information** is available in the online version of the paper.

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