Robust Optimization of a Permanent-Magnet Synchronous Machine Considering Uncertain Driving Cycles

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This article focuses on the robust optimization of a permanent-magnet (PM) synchronous machine while considering a driving cycle. The robustification is obtained by considering geometrical uncertainties caused by manufacturing inaccuracies, uncertainties linked to different driving styles, and uncertainties related to ambient parameters such as traffic and weather conditions. The optimization goal is to minimize the PM’s volume while maintaining the machine performance, i.e., the energy efficiency over the driving cycle and the maximal torque. The magnetic behavior of the machine is described by a partial differential equation (PDE) and is simulated by the finite-element method, employing an affine decomposition to avoid the reassembling of the system of equations due to the changing geometry. Sequential quadratic programming is used for the optimization, and stochastic collocation is applied to compute the moments of stochastic quantities. The robustness of the optimized configurations is validated by a Monte Carlo sampling. It is found that the uncertainties have a significant influence on the optimal PM configuration.

Index Terms—Optimization, permanent-magnet (PM) machines, robustness, uncertainty.

I. INTRODUCTION

PERMANENT-magnet (PM) synchronous machines (PMSMs) with buried magnets are widespread in electromobility due to their high efficiency and high power density. This allows a small machine for a fixed output power [1], [2]. PMs consist of rare-earth elements, presenting a limited and expensive resource, whose extraction and recycling are environmentally polluting [3]. Therefore, a reduction in the PM volume is desired while maintaining the performance of the machine. In practice, the machine’s performance is linked to the driving conditions. Hence, the efficiency over the full driving cycle has to be considered, which represents an energy efficiency due to the additional time component. This quantity can be calculated by a weighting based on, e.g., energy consumption [4] or operation point statistics [5].

The machine performance suffers from uncertainties occurring in the PM characteristics due to manufacturing inaccuracies [6] and in the driving cycle because of changing traffic, weather situations, and driving style. Thus, the optimal PM configuration has to be robust, i.e., even under these slight variations, the machine should still perform at least as specified.

To evaluate the performance of the machine, the finite-element method (FEM) is used. Ultimately, this results in a nonlinear robust optimization problem constrained by a partial differential equation (PDE). This overall problem has extremely high computational demands, because the PDE has to be solved once anew for every optimization step and, even more problematic, every stochastic quantity adds a dimension to the problem. Hence, the choice of the optimization method and the procedure to calculate the stochastic quantities are crucial. For this reason, metaheuristic optimization methods—as they are employed [7]—are unsuitable for this problem, as they converge slowly and, thus, need many optimization steps. In contrast, gradient-based methods typically converge fast, provided that the gradients of the objective and constraint function are given. Although this can be cumbersome for complicated functions, such preliminary computations are rewarded by a low iteration count of the embracing optimization procedure [8]. The large number of uncertain parameters is efficiently treated by stochastic collocation on Clenshaw–Curtis sparse grids [9], [10]. The performance is further improved using an affine decomposition, i.e., the geometry is divided into larger patches that can be deformed without reassembly [11].

This article is structured as follows. First, the concept of driving cycles and how to compute the performance of a machine with respect to a driving cycle as well as the modeling of uncertainties is explained in Section II. Then, the PMSM model and its computation are introduced in Section III. Section IV presents the robust optimization problem and method. It also discusses how to compute the stochastic quantities. Finally, the results and conclusions of this article are discussed in Section V.

II. DRIVING CYCLE

The urban driving cycle (UDC) is part of the New European Driving Cycle specified in [12]. It is defined by its velocity–time profile \( v(t) \) shown in Fig. 1(a). From this, the torque–speed profile \( (M_m, \omega_m) \) of Fig. 1(b) is obtained by

\[
\omega_m(t) = \frac{30v(t)}{\pi R_{wh}}, \quad M_m(t) = R_{wh} m_s a(t) + F_r + F_d(t) \frac{N_{drive}}{N} \tag{1}
\]

Manuscript received August 5, 2019; revised October 25, 2019; accepted November 3, 2019. Date of current version January 20, 2020. Corresponding author: Z. Bontinck (e-mail: zegerbontinck@gmail.com).

Digital Object Identifier 10.1109/TMAG.2019.2952218

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The efficiency is the torque–speed trajectory in time using the Gaussian quadrature. This way, the energy and, thus, the efficiency over the UDC can be calculated by integrating over this trajectory in time with histogram heat map.

with \(a(t) = \ddot{v}(t)\) being the acceleration in \(m/s^2\), \(F_{rr} = C_{rr} m_g g\) and \(F_d(t) = (1/2)C_d \delta_{\text{front}} \beta^2(t)\) the rolling and drag force in N, \(R_{\text{wh}}\) is the wheel radius in m, \(N_{\text{drive}}\) is the number of machines on the axes, \(C_{rr}\) is the rolling resistance coefficient, \(m_g\) is the mass of the vehicle in kg, \(g\) is the gravitational acceleration in \(m/s^2\), \(C_d\) is the drag coefficient, \(\theta\) is the mass density of air, and \(S_{\text{front}}\) is the frontal area of the vehicle.

The torque–speed representation lacks the information of how long a specific operation point \((M_m, \omega_m)\) is operated, which is needed to compute the energy balance over the UDC. Former works partitioned the torque–speed plane and performed some sort of weighting [4], [5]. For instance, the torque–speed plane can be tiled into an \(8 \times 8\) grid and some sort of weighting is performed [4], [5]. For instance, the torque–speed plane can be tiled into an \(8 \times 8\) grid. Then, it is measured how many points of the torque–speed curve fall in each square while sampling the curve equidistantly. As a result, a histogram is obtained, which is visualized as a heat map on the grid [see Fig. 1(c)]. In contrast, this article considers the time information by including the time axis as the third component, leading to the UDC trajectory in Fig. 1(c). In this way, the energy and, thus, the efficiency over the UDC can be calculated by integrating over this torque–speed trajectory in time using the Gaussian quadrature. The efficiency is

\[
E(p) = \frac{\int_{t_0}^{t_{\text{end}}} \omega_m(t) M_m(t) \, dt}{\int_{t_0}^{t_{\text{end}}} (\omega_m(t) M_m(t) + m R_{\text{st}} I^2(t, p)) \, dt}
\]

where \(p\) collects the parameters of the PM geometry, \(m\) is the number of phases, \(R_{\text{st}}\) is the stator resistance in \(\Omega\), and \(I\) is the stator current in A given by the nonlinear relation

\[
\frac{M_m(t)}{N_{\text{pp}} m} = I \sin(\beta) (\Phi_0(p) + (L_d(p) - L_q(p)) I \cos(\beta))
\]

with \(N_{\text{pp}}\) as pole pair number, \(\Phi_0\) as magnetic flux at no-load in Wb, \(L_d\) as direct-axis inductance in H, \(L_q\) as quadrature-axis inductance in H, and \(\beta = \beta_{\text{opt}}(t, p)\) as the optimal current phase angle leading to the highest possible torque output [13]. Therefore, the maximal torque and (2) depend on the PM geometry and the driving cycle, and are affected by their uncertainties.

To model uncertainties in the driving cycle, the curve \(v(t)\) is interpreted as piecewise linear splines with 16 control points. Three scenarios arise: in scenario A, the control points \(v\) are deviated vertically in velocity [see Fig. 2(a)] while assuming them to be uniformly distributed

\[
v(\theta) \in \mathcal{U}((1 - \delta_v) \bar{v}, (1 + \delta_v) \bar{v})
\]

with \(\mathcal{U}(a, b)\) standing for the uniform distribution in the interval \([a, b]\) and \(\bar{v}\) as the quantity representing the stochastic nature, \(\delta_v\) as the maximal relative deviation, and \(\bar{v}\) as the control point’s mean values. This corresponds, e.g., to drivers ignoring the speed limits.

In scenario B, the control points are shifted horizontally in time [see Fig. 2(b)] by choosing the variations as

\[
\delta t_i(\theta) = \begin{cases} 
\alpha \tau(\theta)(t_i - t_i-1), & \text{for } \tau(\theta) < 0 \\
\alpha \tau(\theta)(t_i+1 - t_i), & \text{otherwise}
\end{cases}
\]

with \(\tau(\theta) \in \mathcal{U}(-1, 1)\) and a factor \(\alpha \in (0, 1)\) controlling the strength of the deviation. A combination of both scenarios is also possible (scenario A + B). These three scenarios can mimic changing traffic situations or different driving habits.

Instead of manipulating the velocity–time profile itself, one can also vary the rolling resistance coefficient \(C_{rr}\) and the drag coefficient \(C_d\), which are used to compute \(M_{m}(t)\) from \(v(t)\) (scenario C). In this way, uncertain weather and road conditions can be simulated, as \(C_{rr}\) can increase by a factor up to \(\delta_{rr} = 1.3\) on a wet road [14], and \(C_d\) by a factor up to \(\delta_d = 1.2\) due to raindrops contaminating the vehicle surface [15]. It is assumed that these mechanical coefficients are uniformly distributed

\[
C_{rr}(\theta) \in \mathcal{U}(C_{rr,\text{dry}}, \delta_{rr} C_{rr,\text{dry}})
\]

\[
C_d(\theta) \in \mathcal{U}(C_{d,\text{dry}}, \delta_d C_{d,\text{dry}})
\]

where \(C_{rr,\text{dry}}\) and \(C_{d,\text{dry}}\) represent the coefficients of dry weather conditions.
Please note that one can conveniently incorporate arbitrarily complicated driving cycles due to the usage of splines. If measured data are available, then one may use a Karhunen–Loève expansion to obtain a mean cycle and perturbations [16].

III. COMPUTATION OF PMSMs

To save the computational effort, one typically considers a 2-D cross section of one single pole of the PMSM (see Fig. 3). The PM (marked in green) is characterized by its width \( p_1 \), its height \( p_2 \), and its distance to the rotor surface \( p_3 \), which are collected in a vector \( p \). The manufacturing imperfections are modeled by a uniform distribution

\[
p(\theta) \in \mathcal{U}(\bar{p} - \delta_p, \bar{p} + \delta_p)
\]

where \( \bar{p} \) contains the mean values of the parameters in mm and \( \delta_p \) is the maximal deviation in mm.

The magnetic behavior of the PMSM is described by

\[
\nabla \times (\nabla \times \vec{A}(p)) = \vec{J}_{sc} - \nabla \times \vec{H}_{pm}(p) \quad \text{on} \quad \Omega_d(p)
\]

with \( \nabla \) the relutivity in m/H, \( \vec{A} \) the magnetic vector potential (MVP) in Wb/m, \( \vec{J}_{sc} \) the source current density in A/m\(^2\), and \( \vec{H}_{pm} \) the PM’s source field in A/m. The MVP is discretized by

\[
\vec{A}(p) \approx \sum_{j=1}^{N_{FE}} u_j(p)N_j(x,y)\vec{e}_z,
\]

where \( N_{FE} \) is the number of degrees of freedom, \( u_j \) are the unknown coefficients, \( N_j(x,y) \) are the 2-D Cartesian nodal shape functions, \( \ell_z \) is the length of the machine, and \( \vec{e}_z \) is the Cartesian unit vector in the z-direction. On the stator and shaft boundaries \( \Gamma_{st} \) and \( \Gamma_{sh} \), homogeneous Dirichlet boundary conditions (BCs) are imposed, while antiperiodic BCs are set on the left and right sides of the pole, \( \Gamma_{L} \) and \( \Gamma_{R} \) (see Fig. 3). The FEM and the Ritz–Galerkin ansatz lead to the system of equations

\[
K_v(p)u(p) = \vec{J}_{sc} + \vec{J}_{pm}(p)
\]

where \( K_v \) is the FEM system matrix, \( \vec{J}_{sc} \) and \( \vec{J}_{pm} \) form the right-hand side, and \( u \) collects the coefficients of the discretized MVP. From \( u \), the quantities of interest (QoI) can be calculated in post-processing. To simplify notation, we state

\[
\mathcal{E}(u(p)) = \mathcal{E}(p) \quad \text{and} \quad M_{max}(u(p)) = M_{max}(p).
\]

As the system matrix as well as the right-hand side depend on the PM geometry \( p \), they would have to be assembled anew in every optimization step, which is critical as the matrix assembly requires a high computational effort. To circumvent this, an affine decomposition is employed [11]. The computational domain is subdivided in a fixed region and a region with a changing geometry (see Fig. 3). The system matrix and right-hand side in (9) are decomposed into fixed submatrices and subvectors multiplied by coefficients that fully inherit the dependence on \( p \). In this way, the submatrices and subvectors need to be assembled only once beforehand, while the weighting coefficients are promptly computed by evaluating an analytical formula every time the geometry changes.

The FEM solution \( u \) is used to calculate the parameters \( L_{ds}, L_{q}, \) and \( \Phi_0 \) with help of the loading method introduced in [17]. Furthermore, the stator current \( I \) and ultimately the energy efficiency (2) and torque are calculated.

IV. ROBUST OPTIMIZATION

Due to the uncertainties in the driving cycle and the PM geometry, a robust minimal PM configuration is desired. To this end, the expectation value \( \mathbb{E}[\cdot] \) and the standard deviation \( \text{std}[\cdot] \) of the QoIs are needed and the standard deviation is weighted with a so-called risk-aversion parameter \( \lambda = \sqrt{\mathcal{E}} \), yielding the robust optimization problem [18]

\[
\begin{aligned}
\min_{p} J(p) &= \tilde{p}_1\tilde{p}_2 + \lambda \text{std} \{\tilde{p}_1, \tilde{p}_2\} \\
\text{s.t.} \quad \mathcal{E}(p) &+ \lambda \text{std} \{\mathcal{E}(p)\} \leq 0 \\
M_{max}(p) &- \mathbb{E}[M_{max}(p)] + \lambda \text{std} [M_{max}(p)] \leq 0 \\
G(p) &
\end{aligned}
\]

Here, \( \mathcal{E} \) and \( M_{max} \) are the desired energy efficiency and maximal mechanical torque, respectively, and \( G(p) \) is a function considering geometrical constraints.

In the process of stochastic collocation, the probabilistic integrals defining the expectation value and standard deviation of some QoI \( q \) are approximated by

\[
\mathbb{E}[q] \approx \sum_{k=1}^{N_c} \omega_k q(z_k), \quad \text{std}[q] \approx \left( \sum_{k=1}^{N_c} \omega_k(q(z_k) - \mathbb{E}[q])^2 \right)^{1/2}
\]

where \( N_c \) is the number of collocation points, \( \omega_k \) are the weights, and \( z_k \) are the collocation points. If the points are chosen based on a tensor product grid, the method suffers from the curse of dimensionality, i.e., the number of needed evaluations rises exponentially with the number of uncertain parameters [9], such that this method is only usable for scenario C at the very most (see the third column in Table I), as 250 evaluations require approximately 1 h of computation.

| Scenario | #Parameters | #Evaluations (full) | #Evaluations (sparse) |
|----------|-------------|---------------------|-----------------------|
| C        | 5           | 3725                | 241                   |
| B        | 7           | 78125               | 589                   |
| A        | 11          | 48,828,125          | 2069                  |
| A+B      | 15          | > 3 · 10^{10}       | 5021                  |
time on a standard computer. Instead, Smolyak sparse grids are employed, which represent the subsets of the full tensor grids [9]. Furthermore, Clenshaw–Curtis knots are used to lower the number of needed PDE evaluations further [10]. As a result, the curse of dimensionality is significantly weakened (see the fourth column in Table I).

Eventually, the optimization problem is solved with a standard gradient-based method for constrained nonlinear optimization problems (sequential quadratic programming). The gradients of the objective and constraint functions are determined manually and semi-analytically by symbolic computing.

V. RESULTS AND CONCLUSION

The robust optimization of a PMSM considering an uncertain driving cycle and a PM geometry is now applied to the PMSM model of Section III, where an initial PM configuration with a cross-sectional area $S_0 = 133 \text{ mm}^2$ is chosen, and the model is discretized with $N_{\text{FEQ}} = 8128$ nodes. The desired energy efficiency is set to the value of the initial configuration ($\mathcal{E}_d = 97.07\%$), and the desired maximal torque is adapted to the maximal occurring torque in the UDC ($M_{\text{max, d}} = 2.57 \text{ Nm}$). The deviation parameters are set to $\delta_y = 0.2$ and $\alpha = 0.78$ for the control point’s vertical [see (3)] and horizontal shifts [see (4)], respectively; $\delta_{\text{in}} = 1.3$ and $\delta_{\text{out}} = 1.2$ for the mechanical coefficients [see (5) and (6)]; $\delta_p = 0.2$ mm for the PM characteristics [see (7)]; and $\delta_{\text{wh}} = 120 \cdot 10^{-3} \text{ m}$, $N_{\text{drive}} = 2$, and $S_{\text{front}} = 1 \text{ m}^2$ for the vehicle. To validate the robustness of the found optimal PM configuration, Monte Carlo sampling is used, where the success rate (SR) is calculated as the ratio of the number of successful samples, i.e., the samples fulfilling the desired energy efficiency and maximal torque, and the total number of samples, which is here chosen as 10000.

The initial configuration, and the nominal and robust optima are listed in Table II. The robustness of every configuration has been validated not only for the scenario it has been optimized for but also for every scenario. The resulting SRs are shown in Table II. It is observed that the optimal PM configurations for the scenarios A, B, and A + B are very similar and robust with respect to the scenario it has been optimized for, while the initial configuration as well as the nominally optimized configuration are never robust. In addition, the optimal configurations for A, B, and A + B are for each other robust. Thus, it seems that it does not matter in which direction the control points of the UDC are deviated to model uncertain driving cycles. The largest PM configuration is returned for scenario C, for which its own robustness validation scores only an SR of 91.14\%, and for which the optimized configurations for the other scenarios perform very badly. To compare the initial PM configuration with the optimized one for scenario C, Fig. 4 shows the resulting UDCs in the current–speed plane, since the maximal torque depends on the PM configuration and the maximal current does not. It is observed that the optimized PMSM (white curve) needs less current to achieve the same speeds as the initial one (red curve). Moreover, the operation-pointwise efficiency $\epsilon_{\text{opt}} - \epsilon_0$ is positive or at least zero in the whole operating area, meaning that the performance is generally improved for every operating point. Especially, low-speed operating points benefit from the optimization.

Manipulating the mechanical coefficients shows the greatest impact on the UDC and, thus, on the performance, resulting into a bigger PM and a less robust configuration. Moreover, it has been indicated that a robust optimization is essential to obtain a robust PM configuration when uncertainties play a role. To perform such a robust optimization, the proposed method in this article has proved to be computationally efficient.

ACKNOWLEDGMENT

This work was supported in part by the German BMBF in the context of the SIMUROM under Project 05M2013, in the context of the PASIROM under Project 05M18RDA, in part by the Deutsche Forschungsgemeinschaft under Grant SCH01562/3-1, and in part by the Excellence Initiative of the German Federal and State Governments and the GSCE at TU Darmstadt under Grant GSC 233/2.

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