KKW Analysis for the Dyadosphere of a Charged Black Hole

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Abstract

The Keski-Vakkuri, Kraus and Wilczek (KKW) analysis is used to compute the temperature and entropy in the dyadosphere of a charged black hole solution. For our purpose we choose the dyadosphere region of the Reissner-Nordström black hole solution.

Our results show that the expressions of the temperature and entropy in the dyadosphere of this charged black hole are not the Hawking temperature and the Bekenstein-Hawking entropy, respectively.

Keywords: KKW analysis, dyadosphere of a charged black hole

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1 INTRODUCTION

One of the most interesting problem of relativity is the evaluation of the temperature and entropy of black holes and this issue still attracts considerable attention in the literature. The important method of Keski-Vakkuri, Kraus and Wilczek (KKW) [1] has been used to compute the temperature and entropy of Schwarzschild-type black hole solution [2] and, after this, has been applied to other black hole space-times [3]-[5]. It is important to point out that in the (KKW) analysis, the total Arnowitt-Desser-Misner mass [6] is

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fixed but the mass of the Schwarzschild black hole decreases due to the emitted radiation. For solving the problem of singularities and to avoid them at the horizon the Painlevé [7] coordinate transformation is used and this enable us the study of the across-horizon physics such as the black hole radiation. The black hole temperature depends not only on the characteristics of the black hole but, also, on the energy of the emitted shell of energy. Furthermore, the black hole entropy is not given by Bekenstein and Hawking formula for the specific black hole.

Also, the (KKW) generalized analysis can be applied successfully for evaluating the temperature and entropy of a black hole solution which is not of Schwarzschild-type. The generalized (KKW) analysis case was studied by Vagenas [8] and he gave the formulas for the temperature and entropy of a black hole solution described by a metric which satisfies the condition $A(r) \cdot B^{-1}(r) \neq 1$. About the Hawking radiation, we are allowed to conclude that is viewed as a tunneling process which emanates from the non-Schwarzschild-type black hole solution.

Vagenas [8] used a more general coordinate transformation in order to apply the (KKW) analysis to non-Schwarzschild-type black holes. The two conditions: 1) the regularity at the horizon which ensures that we can study the across-horizon physics and 2) the stationarity of the non-static metric which implies that the time direction is a Killing vector, were fulfilled in order to generalize the (KKW) analysis. This generalized analysis furnishes us the exact expressions of the temperature and entropy of the non-Schwarzschild-type black holes which are not the Hawking temperature $T_H$ and the Bekenstein-Hawking entropy $S_{BH}$. Other interesting results on black hole thermodynamics were obtained [9]. Furthermore, we use the (KKW) generalized analysis in order to compute the temperature and entropy of a magnetic stringy black hole solution [10].

In this paper we use the Keski-Vakkuri, Kraus and Wilczek (KKW) analysis to compute the temperature and entropy in the dyadosphere of a charged black hole solution. We choose the dyadosphere region [11] of the Reissner-Nordström black hole solution.
2 KKW ANALYSIS FOR THE DYADOSPHERE OF A CHARGED BLACK HOLE

Ruffini and collaborators [12]-[13] demonstrated that the event horizon of a charged black hole is surrounded by a special region called the dyadosphere where the electromagnetic field exceeds the Euler-Heisenberg critical value for electron-positron pair production. They studied certain properties of the dyadosphere corresponding to the Reissner-Nordström space-time [12]-[13]. In addition, the new concept of dyadosphere was introduced by Ruffini [12] to explain gamma ray bursts.

De Lorenci, Figueiredo, Fliche and Novello [11] computed the correction for the Reissner-Nordström metric from the first contribution of the Euler-Heisenberg Lagrangian and got the metric

\[ ds^2 = -(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\sigma Q^4}{5 r^6}) dt^2 + 
(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\sigma Q^4}{5 r^6})^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2. \]

This metric is of the type

\[ ds^2 = -A(r) dt^2 + A(r)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2, \]

with \( A(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\sigma Q^4}{5 r^6} \).

For \( \sigma = 0 \) we obtain the Reissner-Nordström solution. De Lorenci et. al. [11] showed that the correction term \( \frac{\sigma Q^4}{5 r^6} \) is of the same order of magnitude as the Reissner-Nordström charge term \( \frac{Q^2}{r^2} \). The metric describes a black hole with an event horizon at \( r_+ \).

The (KKW) methodology [1] in the case of a black hole background which belongs to the family of geometries of Schwarzschild-type requires that the metric should be regular at the event horizon. Also, the total Arnowitt-Desser-Misner mass \( M_{ADM} \) have to be well-defined so we get \( A(r) \to 1 \), as \( r \to \infty \). We have the Painlevé [7] coordinate transformation given by

\[ \sqrt{A(r)} dt = \sqrt{A(r)} d\tau - \sqrt{\frac{1 - A(r)}{A(r)}} dr, \]
where \( \tau \) is the new time coordinate. The metric given by (1) becomes

\[
d s^2 = -A(r) \, d\tau^2 + 2\sqrt{1 - A(r)} \, d\tau \, dr + dr^2 + r^2 \, d\theta^2 + r^2 \sin^2 \theta \, d\varphi^2.
\]  (4)

The radial null geodesics are

\[
\cdot r = \pm 1 - \sqrt{1 - A(r)}.
\]  (5)

In the equation above, the upper (lower) sign corresponds to the outgoing (ingoing) geodesics under the assumption that \( \tau \) increases towards future.

The total Arnowitt-Deser-Misner mass \( M_{ADM} \) is fixed and the mass \( M \) of the black hole fluctuates because a shell of energy \( \omega \) which constitutes of massless particle considering only the s-wave part of emission, is radiated by the black hole. Now, we are in the situation when the massless particles travel on the outgoing geodesics which are due to the varying mass \( M \) of the black hole. The metric becomes

\[
d s^2 = -A(r, M - \omega) \, d\tau^2 + 2\sqrt{1 - A(r, M - \omega)} \, d\tau \, dr + dr^2 + r^2 \, d\theta^2 + r^2 \sin^2 \theta \, d\varphi^2.
\]  (6)

We get for the outgoing radial null geodesics a new formula

\[
\cdot r = 1 - \sqrt{1 - A(r, M - \omega)}.
\]  (7)

We make the approximation

\[
\sqrt{1 - A'} \approx 1 - \frac{1}{2} A',
\]  (8)

where \( A' = A(r, M - \omega) \) and and thus the imaginary part of the action (see equation (19) in [8])

\[
\text{Im} I = \text{Im} \int_{r_+(M - \omega)}^{r_+} \int_{0}^{+\omega} \frac{d\omega'}{1 - \sqrt{1 - A'}} \, dr.
\]  (9)

We obtain for the metric given by (1)

\[
\text{Im} I = \frac{\pi}{2} [r_+^2(M) - r_+^2(M - \omega)].
\]  (10)
Now, we evaluate the temperature of the black hole (see equation (20) in [8]) and we obtain

\[ T_{bh}(M, \omega) = \frac{\omega}{\pi} \left[ r_+^2 (M) - r_+^2 (M - \omega) \right]^{-1}. \] (11)

The expression of the entropy is given by

\[ S_{bh} = S_{BH} - \pi \left[ r_+^2 (M) - r_+^2 (M - \omega) \right]. \] (12)

The Hawking temperature \( T_H \) in the dyadosphere of the charged black hole is defined as

\[ T_H = \frac{r_+ - r_-}{A_h} = \frac{r_+ - r_-}{4 \pi r_+^2}. \] (13)

The entropy of the black hole is different from the Bekenstein-Hawking entropy formula \( S_{BH} \) that is given by

\[ S_{BH} = \frac{A_h}{4} = \pi r_+^2. \] (14)

3 DISCUSSION

One of the most attractive methods used to evaluate the temperature and entropy of black holes is the (KKW) analysis. In some recent investigations about this important issue, the temperature and entropy of black holes, the importance of the (KKW) analysis is emphasized [14]. An interesting study about the Hawking radiation and the temperature and entropy of black holes was made by M. Angheben et. al. [15], and A. J. M. Medved and E. C. Vagenas [16]. In these works, the authors also pointed out some interesting results obtained with the (KKW) analysis.

We used the (KKW) analysis introduced in [1] in order to evaluate the temperature and entropy in the dyadosphere of the Reissner-Nordström black hole solution. We conclude that the temperature and entropy in the dyadosphere region of the Reissner-Nordström black hole solution are different from the Hawking temperature \( T_H \) and the Bekenstein-Hawking entropy \( S_{BH} \), respectively. The temperature and entropy in the dyadosphere depend on the \( r_+ \) and \( r_- \) and on \( r_+ \) parameters, respectively. Since we don’t know the explicit expression of \( r_+ \) and \( r_- \), we are allowed only to assume that these
expressions depend on the emitted particle’s energy. Our results sustain the importance of the KKW analysis [1].

4 References

References

[1] P. Kraus and F. Wilczek, *Nucl. Phys.* **B433**, 403 (1995); P. Kraus and F. Wilczek, *Nucl. Phys.* **B437**, 231 (1995); E. Keski-Vakkuri and P. Kraus, *Phys. Rev.* **D54**, 7407 (1996).

[2] M. K. Parikh and F. Wilczek, *Phys. Rev. Lett.* **85**, 5042 (2000).

[3] S. Hemming and E. Keski-Vakkuri, *Phys. Rev.* **D64**, 044006 (2001); Y. Kwon, *Il Nuovo Cimento* **B115**, 469 (2000).

[4] E. C. Vagenas, *Phys. Lett.* **B503**, 399 (2001); E. C. Vagenas, *Mod. Phys. Lett.* **A17**, 609 (2002); E. C. Vagenas, *Phys. Lett.* **B533**, 302 (2002); M. R. Setare and E. C. Vagenas, *Phys. Lett.* **B584**, 127 (2004); M. R. Setare and E. C. Vagenas, [hep-th/0405186](https://arxiv.org/abs/hep-th/0405186).

[5] A. J. M. Medved, *Class. Quant. Grav.* **19**, 589 (2002); A. J. M. Medved, *Phys. Rev.* **D66**, 124009 (2002).

[6] R. Arnowitt, S. Deser and C. W. Misner, in *Gravitation: An Introduction to Current Research*, ed. by L. Witten (Wiley, New York, 1962).

[7] P. Painlevé, *C. R. Acad. Sci.* (Paris) **173**, 677 (1921).

[8] E. C. Vagenas, *Phys. Lett.* **B559**, 65 (2003).

[9] Michele Atzano, A. J. M. Medved and E. C. Vagenas, JHEP **0509**, 037, (2005); A. J. M. Medved and E. C. Vagenas, Mod.Phys.Lett. **A20**, 1723, (2005); A. J. M. Medved and E. C. Vagenas, Mod.Phys.Lett. **A20**, 2449, (2005); A. J. M. Medved and E. C. Vagenas, Phys.Rev. **D70**, 124021, (2004).

[10] I. Radinschi, Proceedings of 5-th Conference of Balkan Society of Geometers, August 29 - September 2, 2005 Mangalia, Romania, [gr-qc/0412111](https://arxiv.org/abs/gr-qc/0412111)
[11] V. A. De Lorenci, N. Figueiredo, H. H. Fliche and M. Novello, A&A \textbf{369}, 690 (2001).

[12] R. Ruffini, in \textit{XLIX Yamada Conference on Black Holes and High Energy Astrophysics}, edited by H. Salto, (Univ. Acad. Press., Tokyo, 1998)

[13] G. Preparata, R. Ruffini and S. -S. Xue, A&A \textbf{L87}, 338 (1998); G. Preparata, R. Ruffini and S. -S. Xue, J.Korean Phys.Soc. \textbf{42}, S99 (2003)

[14] M. R. Setare, hep-th/0504179

[15] M. Angheben, M. Nadalini, L. Vanzo and S. Zerbini, JHEP \textbf{0505}, 014 (2005)

[16] A. J. M. Medved and E. C. Vagenas, Mod.Phys.Lett. \textbf{A20}, 2449, (2005).