B → K*ℓ+ℓ−(ρ ℓν_ℓ) helicity analysis in the LEET

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Abstract

We calculate the independent helicity amplitudes in the decays B → K*ℓ+ℓ− and B → ρℓν_ℓ in the so-called Large-Energy-Effective-Theory (LEET). Taking into account the dominant O(α_s) and SU(3) symmetry-breaking effects, we calculate various single (and total) distributions in these decays making use of the presently available data and decay form factors calculated in the QCD sum rule approach. Differential decay rates in the dilepton invariant mass and the Forward-Backward asymmetry in B → K*ℓ+ℓ− are worked out. Measurements of the ratios R_i(s) = dΓ_H_i(s)/dΓ_H(s)(B → K*ℓ+ℓ−)/dΓ_H(s)(B → ρℓν_ℓ), involving the helicity amplitudes H_i(s), i = 0,+1,−1, as precision tests of the standard model in semileptonic rare B-decays are emphasized. We argue that R_0(s) and R_−(s) can be used to determine the CKM ratio |V_{ub}|/|V_{cs}| and search for new physics, where the later is illustrated by supersymmetry.

1 Introduction

Rare B decays involving flavour-changing-neutral-current (FCNC) transitions, such as b → sγ and b → sℓ+ℓ−, have received a lot of theoretical interest, especially after the first measurements reported by the CLEO collaboration of the B → X_sγ decay. The current world average based on the improved measurements by the CLEO, ALEPH and BELLE collaborations, B(B → X_sγ) = (3.22 ± 0.40) × 10^−4, is in good agreement with the estimates of the standard model (SM), which we shall take as B(B → X_sγ) = (3.50 ± 0.50) × 10^−4, reflecting the parametric uncertainties dominated by the scheme-dependence of the quark masses. The decay B → X_sγ also provides useful constraints on the parameters of the supersymmetric theories, which in the context of the minimal supersymmetric standard model (MSSM) have been recently updated in [1].

Exclusive decays involving the b → sγ transition are best exemplified by the decay B → K*γ, which have been measured with a typical accuracy of ±10%, the current branching ratios being B(B^± → K*±γ) = (3.82±0.47) × 10^−5 and B(B^0 → K*0γ) = (4.44 ± 0.35) × 10^−5. These decays have been analyzed recently [1,2,3,4], by taking into account O(α_s) corrections, henceforth referred to as the next-to-leading-order (NLO) estimates, in the large-energy-effective-theory (LEET) limit [5,6]. As this framework does not predict the decay form factors, which have to be supplied from outside, consistency of NLO-LEET estimates with current data constrains the magnetic moment form factor in B → K*γ in the range T_1^{K*}(0) = 0.27 ± 0.04. These values are somewhat lower than the corresponding estimates in the lattice-QCD framework, yielding 3.22 T_1^{K*}(0) = 0.32±0.04, and in the light cone QCD sum rule approach, which give typically T_1^{K*}(0) = 0.38±0.05 [2,4]. (Earlier lattice-QCD results on B → K*γ form factors are reviewed in [2].) It is imperative to check the consistency of the NLO-LEET estimates, as this would provide...
a crucial test of the ideas on QCD-factorization, formulated in the context of non-leptonic exclusive $B$-decays \[22\], but which have also been invoked in the study of exclusive rare $B$-decays \[13\].

The exclusive decays $B \to K^{*+}\ell^+\ell^-$, $\ell^\pm = e^\pm, \mu^\pm$ have also been studied in the NLO-LEET approach in \[23,24\]. In this case, the LEET symmetry brings an enormous simplicity, reducing the number of independent form factors from seven to only two, corresponding to the transverse and longitudinal polarization of the virtual photon in the underlying process $B \to K^{*+}\gamma^*$, called hereafter $\xi_{\perp}^{(K^*)}(q^2)$ and $\xi_{\parallel}^{(K^*)}(q^2)$. The same symmetry reduces the number of independent form factors in the decays $B \to \rho\ell\nu_\ell$ from four to two. Moreover, in the $q^2$-range where the large energy limit holds, the two set of form factors are equal to each other, up to $SU(3)$-breaking corrections, which are already calculated in specific theoretical frameworks. Thus, knowing $V_{ub}$ precisely, one can make theoretically robust predictions for the rare $B$-decay $B \to K^{*+}\ell^+\ell^-$ from the measured $B \to \rho\ell\nu_\ell$ decay in the SM. The LEET symmetries are broken by QCD interactions and the leading $O(\alpha_s)$ corrections in perturbation theory are known \[24,25,26\].

In this talk we present the results of \[1\], where a systematic analysis of the various independent helicity amplitudes in the decays $B \to K^{*+}\ell^+\ell^-$ and $B \to \rho\ell\nu_\ell$ were performed in the NLO accuracy in the large energy limit. We recall that a decomposition of the final state $B \to K^{*+}(\to K\pi)\ell^+\ell^-$ in terms of the helicity amplitudes $H_{\ell^\pm}^{L,R}(q^2)$ and $H_{0}^{L,R}(q^2)$, without the explicit $O(\alpha_s)$ corrections, was undertaken in a number of papers \[23,24\]. Combining the analysis of the decay modes $B \to K^{*+}\ell^+\ell^-$ and $B \to \rho\ell\nu_\ell$, we show that the ratios of differential decay rates involving definite helicity states, $R_{-}(s)$ and $R_{0}(s)$, can be used for testing the SM precisely.

## 2 General framework

At the quark level, the rare semileptonic decay $b \to s\;\ell^+\ell^-$ can be described in terms of the effective Hamiltonian

\[
\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu),
\]

where $\lambda_i = V_{ts}^\ast V_{tb}$ are the CKM matrix elements \[13\] and $G_F$ is the Fermi coupling constant. Following the notation and the convention used in ref. \[1\], the above Hamiltonian leads to the following free quark decay amplitude:\footnote{We put $m_s/m_b = 0$ and the hat denotes normalization in terms of the $B$-meson mass, $m_B$, e.g. $\hat{s} = s/m_B^2$, $\hat{m}_b = m_b/m_B$.}

\[
\mathcal{M} = \frac{G_F\alpha_s}{\sqrt{2}\pi} \lambda_i \left[ C_0 [\bar{s}\gamma_\mu L\bar{b}] [\bar{\ell}\gamma_\mu t] + C_{10} [\bar{s}\gamma_\mu L\bar{b}] [\bar{\ell}\gamma_\mu R\bar{b}] \right].
\]

Here, $L/R \equiv (1 \mp \gamma_5)/2$, $s = q^2$, $q_\mu = \frac{1}{2}\gamma_\mu\gamma^\nu$, and $q_\mu = (p_+ + p_-)_\mu$, where $p_\pm$ are the four-momenta of the leptons. Since we are including the next-to-leading corrections into our analysis, we will take the NLO $\overline{MS}$ definition of the $b$-quark mass $m_b \equiv m_0(\mu)$ and the Wilson coefficients in next-to-leading-logarithmic order given in Table 1 in \[1\].

Exclusive $B \to V$ transitions\footnote{For the $B \to K^{*+}\ell^+\ell^-$ decay, this region is identified as $s \leq \text{GeV}^2$.} are described by the matrix elements of the quark operators in Eq. \(3\) over meson states, which can be parameterized in terms of the full QCD form factors (called in the literature $A_0(q^2)$, $A_1(q^2)$, $A_2(q^2)$, $V(q^2)$, $T_1(q^2)$, $T_2(q^2)$, $T_3(q^2)$).

However, the factorization Ansatz enables one to relate in the restricted kinematic region the form factors in full QCD and the two corresponding LEET form factors, namely $\xi_{\perp}^{(V)}(q^2)$ and $\xi_{\parallel}^{(V)}(q^2)$ \[23,24\]:

\[
f_k(q^2) = C_{\perp} \xi_{\perp}^{(V)}(q^2) + C_{\parallel} \xi_{\parallel}^{(V)}(q^2) + \Phi_B \otimes T_k \otimes \Phi_V,
\]

where the quantities $C_i$ ($i = \perp, \parallel$) encode the perturbative improvements of the factorized part

\[
C_i = C_i^{(0)} + \frac{\alpha_s}{\pi} C_i^{(1)} + ...,
\]

and $T_k$ is the hard spectator kernel (regulated so as to be free of the end-point singularities), representing the non-factorizable perturbative corrections, with the direct product understood as a convolution of $T_k$ with the light-cone distribution amplitudes of the $B$ meson ($\Phi_B$) and the vector meson ($\Phi_V$). With this Ansatz, it is a straightforward exercise to implement the $O(\alpha_s)$-improvements in the various helicity amplitudes. For further details we refer to \[1\].

Lacking a complete solution of non-perturbative QCD, one has to rely on certain approximate methods to calculate the above form factors. We take...
the ones given in (19), obtained in the framework of Light-cone QCD sum rules. The corresponding LEET form factors $\xi_{\perp}^{(K)}(s)$ and $\xi_{\parallel}^{(K)}(s)$ are illustrated in ref. [1]. The range $\xi_{\perp}^{(K)}(s) = 0.28 \pm 0.04$ is determined by the $B \to K^\ast \gamma$ decay rate, calculated in the LEET approach in NLO order [3,4] and current data. This gives smaller values for $T_1(0)$ and $T_2(0)$ than the ones estimated with the QCD sum rules.

3 $B \to K^* \ell^+ \ell^-$ helicity analysis

Using the $B \to K^*(\to K \pi) \ell^+ \ell^-$ helicity amplitudes (24), namely $H_i^{(K)}(s)$ ($i = 0, \pm 1$), the dilepton invariant mass spectrum reads

$$\frac{d\mathcal{B}}{ds} = \tau_B \frac{\alpha^2 m_B^2}{384 \pi^5} \frac{\lambda^2}{m_B^2} \sum_{i=0, \pm 1} |H_i^{(K)}(s)|^2,$$

where $\lambda = \frac{1}{4} \left[(m_B^2 - m_{\ell}^2 - s)^2 - m_{\ell}^2 \right]$. Using the input parameters presented in (10), we have plotted in Fig. 1, the dilepton invariant mass spectrum $d\mathcal{B}/|H_i^{(K)}|^2/ds$ and the total dilepton invariant mass, showing in each case the leading order and the next-to-leading order results.

Figure 1: Various individual helicity contributions ($H_i^{(K)} = \frac{d\mathcal{B}/|H_i^{(K)}|^2}{ds} \times 10^{-7}$) and the sum ($\mathcal{H}_{(K)} = \frac{d\mathcal{B}}{ds} \times 10^{-7}$) to the dilepton invariant mass distributions for $B \to K^* \ell^+ \ell^-$ at NLO order (solid center line) and LO (dashed). The band reflects the theoretical uncertainties from input parameters.

As can be seen from Fig. 1, the total decay rate is dominated by the contribution from the helicity $|H_+|$ component, whereas the contribution proportional to the helicity amplitude $H_0(s)$ is negligible. The next-to-leading order correction to the lepton invariant mass spectrum in $B \to K^* \ell^+ \ell^-$ is significant in the low dilepton mass region ($s \leq 2 \text{ GeV}^2$), but small beyond that shown for the anticipated validity of the LEET theory ($s \leq 8 \text{ GeV}^2$). Theoretical uncertainty in our prediction is mainly due to the form factors, and to a lesser extent due to the parameters $\lambda_{B,\pm}^{-1}$ and the $B$-decay constant, $f_B$.

Besides the differential branching ratio, $B \to K^* \ell^+ \ell^-$ decay offers other distributions (with different combinations of Wilson coefficients) to be measured. An interesting quantity is the Forward-Backward (FB) asymmetry defined in (21)

$$\frac{dA_{FB}}{ds} = - \int_0^{-\hat{u}(s)} d\hat{u} \frac{d^2 \Gamma}{d\hat{u} ds} + \int_{-\hat{u}(s)}^0 d\hat{u} \frac{d^2 \Gamma}{d\hat{u} ds}.$$  \hspace{1cm} (6)

Where the kinematic variable $\hat{u} \equiv (\hat{p}_B - \hat{p}_-)^2 - (\hat{p}_B - \hat{p}_+)^2 \parallel$. Our results for FBA are shown in Fig. 2 in the LO and NLO accuracy.

Figure 2: FB-asymmetry at NLO order (solid center line) and LO (dashed). The band reflects the theoretical uncertainties from the input parameters.

At the LO the location of the FB-asymmetry zero is $s_0 \simeq 3.4 \text{ GeV}^2$, which is substantially shifted at the NLO by ~ 1 GeV$^2$ leading to $s_0 \simeq 4.7 \text{ GeV}^2$. We essentially confirm the results obtained in the NLO-LEET context by Beneke et al. [14].

4 $B \to \rho \ell \nu_{\ell}$ helicity analysis

The helicity amplitudes for $B \to \rho(\to \pi^\pm \pi^-) \ell \nu_{\ell}$, namely $H_i^{(\rho)}(s)$ ($i = 0, \pm 1$), can in turn be related to the two axial-vector form factors, $A_1(s)$

$$\frac{d\mathcal{B}}{ds} \propto \sum_{i=0, \pm 1} |H_i^{(K)}|^2 \propto 10^{-7}, \frac{d\mathcal{B}}{ds} \propto 10^{-7}, \frac{d\mathcal{B}}{ds} \propto 10^{-7}, \frac{d\mathcal{B}}{ds} \propto 10^{-7}.$$

which are bounded as $(2\hat{m}_\ell)^2 \leq \hat{s} \leq (1 - \hat{m}_K^2)^2$, $\hat{u}(\hat{s}) \leq \hat{u} \leq \hat{u}(\hat{s})$, with $\hat{m}_\ell = m_\ell/m_B$ and $\hat{u}(\hat{s}) = \frac{2}{\hat{m}_\ell^2} \sqrt{\lambda(1 - \frac{\hat{m}_K^2}{\hat{s}})}$, $\lambda = \frac{1}{4} \left[(m_B^2 - m_{\ell}^2 - s)^2 - m_{\ell}^2 \right]$. $\lambda = \frac{1}{4} \left[(m_B^2 - m_{\ell}^2 - s)^2 - m_{\ell}^2 \right]$. $\lambda = \frac{1}{4} \left[(m_B^2 - m_{\ell}^2 - s)^2 - m_{\ell}^2 \right]$.
and \( A_2(s) \), and the vector form factor, \( V(s) \), which appear in the hadronic current \([22]\). The \( B \to \rho \to \pi^+ \pi^- \ell^+ \ell^- \) total branching decay rate \([21]\) can be expressed in terms of the corresponding helicity amplitudes as \([33\), 32]\):

\[
\frac{d\mathcal{B}}{ds} = \frac{s \sqrt{\lambda}}{36 \pi \rho^2} |V_{ub}|^2 \sum_{i=0, \pm 1} |H_i^{(\rho)}(s)|^2,
\]

\[
= \frac{d\mathcal{B}|H_0^{(\rho)}|^2}{ds} + \frac{d\mathcal{B}|H_{\pm}^{(\rho)}|^2}{ds} + \frac{d\mathcal{B}|H_{-\pm}^{(\rho)}|^2}{ds}.
\] (7)

The contributions from \( |H_{-}^{(\rho)}(s)|^2 \), \( |H_{+}^{(\rho)}(s)|^2 \), \( |H_{0}^{(\rho)}(s)|^2 \) and the total are shown in Fig. 3. Contrary to the \( B \to K^* \ell^+ \ell^- \) decay rate, the \( B \to \rho \ell \nu \ell \) decay is dominated by the helicity-0 component. The impact of the NLO correction on the various branching ratios in \( B \to \rho \ell \nu \ell \) is less significant than in the \( B \to K^* \ell^+ \ell^- \) decay, reflecting the absence of the penguin-based amplitudes in the former decay.

Concerning the \( B \to \rho \ell \nu \ell \) form factors, one has to consider the SU(3)-breaking effects in relating them to the corresponding form factors in \( B \to K^* \ell^+ \ell^- \). For the form factors in full QCD, they have been evaluated within the QCD sum-rules \([34\), 33]\). These can be taken to hold also for the ratio of the LEET form factors. Thus, we take

\[
\xi_{-\pm ||}(s) = \frac{\xi_{K^*}^{(\rho)}(s)}{\xi_{SU(3)}}.
\] (8)

While admitting that this is a somewhat simplified picture, as the effect of SU(3)-breaking is also present in the \( s \)-dependent functions, but checking numerically the functions resulting from Eq. \([8]\) with the ones worked out for the full QCD form factors in the QCD sum-rule approach in \([20]\), we find that the two descriptions are rather close numerically in the region of interest of \( s \).

### 5 Determination of \( |V_{ub}|/|V_{ts}| \)

The measurement of exclusive \( B \to \rho \ell \nu \ell \) decays is one of the major goals of B physics. It provides a good tool for the extraction of \( V_{ub} \), provided the form factors can be either measured precisely or calculated from first principles, such as the lattice-QCD framework. To reduce the non-perturbative uncertainty in the extraction of \( V_{ub} \), we propose to study the ratios of the differential decay rates in \( B \to \rho \ell \nu \ell \) and \( B \to K^*\ell^+\ell^- \) involving definite helicity states. These \( s \)-dependent ratios \( R_i(s) \), \((i = 0, -1, +1)\) are defined as follows:

\[
R_i(s) = \frac{d\Gamma_{B \to K^* \ell^+ \ell^-}/ds}{d\Gamma_{B \to \rho \ell \nu \ell}/ds}.
\] (9)

The ratio \( R_-(s) \) suggests itself as the most interesting one, as the form factor dependence essentially cancels. From this, one can measure the ratio \( |V_{ts}|/|V_{ub}| \). In Fig. 4, we plot \( R_-(s) \) and \( R_0(s) \) for three representative values of the CKM ratio \( R_0 = |V_{ub}|/|V_{tb}V_{ts}^*| = |V_{ub}|/|V_{cb}| = 0.08, 0.094, \) and 0.11. However, as we remarked earlier, the ratio \( R_-(s) \) may be statistically limited due to the dominance of the decay \( B \to \rho \ell \nu \ell \) by the Helicity-0 component. Hence, we also show the ratio \( R_0(s) \), where the form factor dependence does not cancel. For the
LEET form factors used here, the compounded theoretical uncertainty is shown by the shaded regions. This figure suggests that high statistics experiments may be able to determine the CKM-ratio from measuring $R_0(s)$ at a competitive level compared to the other methods en vogue in experimental studies.

6 SUSY effect in $B \to K^* \ell^+ \ell^-$

In order to look for new physics in $B \to K^* \ell^+ \ell^-$, we propose to study the ratio $R_{(0,-)}(s)$, introduced in Eq. (8). To illustrate generic SUSY effects in $B \to K^* \ell^+ \ell^-$, we note that the Wilson coefficients $C_7^{\text{eff}}, C_8^{\text{eff}}, C_9$ and $C_{10}$ receive additional contributions from the supersymmetric particles. We incorporate these effects by assuming that the ratios of the Wilson coefficients in these theories and the SM deviate from 1. These ratios are defined as follows:

$$r_k(\mu) = \frac{C_k^{\text{SU(SU(3)}}}{C_k^{\text{SM}}} , \quad (k = 7, \cdots, 10). \quad (10)$$

They depend on the renormalization scale (except for $C_{10}$), for which we take $\mu = m_{b,\text{pole}}$. For the sake of illustration, we use representative values for the large(small)-tan $\beta$ SUGRA model, in which the ratios $r_7$ and $r_8$ actually change (keep) their signs. The supersymmetric effects on the other two Wilson coefficients $C_9$ and $C_{10}$ are generally small in the SUGRA models, leaving $r_9$ and $r_{10}$ practically unchanged from their SM value. To be specific, we take $r_7 = -1.2, \ r_8 = -1, \ r_9 = 1.03, \ r_{10} = 1.0$ ($r_7 = 1.1, \ r_8 = 1.4, \ r_9 = 1.03, \ r_{10} = 1.0$).

In Fig. 5, we present a comparative study of the SM and SUGRA partial distribution for $H^-$ and $H_0$. In doing this, we also show the attendant theoretical uncertainties for the SM, worked out in the LEET approach. For these distributions, we have used the form factors from [13] with the SU(3)-symmetry breaking parameter taken in the range $\xi_{SU(3)} = 1.3 \pm 0.06$.

From Fig. 5 left-hand plots, where $r_k > 0$, it is difficult to work out a signal of new physics from the SM picture. There is no surprise to be expected, due to the fact that in this scenario the corresponding ratio $r_k$ is approximatively one, which makes the SUGRA picture almost the same as in the SM one. However, Fig. 5 right-hand plots with $(r_7, \ r_8) < 0$ illustrates clearly that despite non-perturbative uncertainties, it is possible, in principle, in the low $s$ region to distinguish between the SM and a SUGRA-type models, provided the ratios $r_k$ differ sufficiently from 1.

7 Summary

In this talk, we have reported an $O(\alpha_s)$-improved analysis of the various helicity components in the decays $B \to K^* \ell^+ \ell^-$ and $B \to \rho \ell \nu$, carried out in the context of the Large-Energy-Effective-Theory. The results presented here make use of the form factors calculated in the QCD sum rule approach. The LEET form factor $\xi_{\perp}(K^*)(0)$ is constrained by current data on $B \to K^* \gamma$. As the theoretical analysis is restricted to the lower part of the dilepton invariant mass region in $B \to K^* \ell^+ \ell^-$, typically $s \leq 8 \text{ GeV}^2$, errors in this form factor are not expected to severely limit theoretical precision. This implies that distributions involving the $H^- (s)$ helicity component can be calculated reliably. Precise measurements of the two LEET form factors $\xi_{\perp}(\rho)(s)$ and $\xi_{\parallel}(\rho)(s)$ in the decays $B \to \rho \ell \nu$ can be used to largely reduce the residual model dependence. With the assumed form factors, we have worked out a number of single (and total) distributions in $B \to \rho \ell \nu$, which need to be confronted with data. We also show the $O(\alpha_s)$ effects on the FB-

\[ \text{Figure 5: The Ratio } R_{(0,-)}(s) \text{ in the SM with } R_6 = 0.094, \xi_{SU(3)} = 1.3, \xi_{\perp}(K^*)(0) = 0.28 \text{ and in SUGRA, with } (r_7, \ r_8) = (1.1, 1.4) \text{ (left-hand plots) and } (r_7, \ r_8) = (-1.2, -1) \text{ (right-hand plots). The SM and the SUGRA contributions are represented respectively by the shaded area and the solid curve. The shaded area depicts the theoretical uncertainty on } \xi_{SU(3)} = 1.3 \pm 0.06 \text{ and on } \xi_{\perp}(K^*)(0) = 0.28 \pm 0.04. \]

\[ \text{We thank Enrico Lunghi for providing us with these numbers.} \]
asymmetry, confirming essentially the result found in [4]. Combining the analysis of the decay modes $B \rightarrow K^*\ell^+\ell^-$ and $B \rightarrow \rho\ell\nu$, we show that the ratios of differential decay rates involving definite helicity states, $R_+ (s)$ and $R_0 (s)$, can be used for testing the SM precisely. We work out the dependence of these ratios on the CKM matrix elements $|V_{ub}|/|V_{ts}|$.

We have also analyzed possible effects on these ratios from New Physics contributions, exemplified by representative values for the effective Wilson coefficients in SUGRA models. Finally, we remark that the current experimental limits on $B \rightarrow K^*\ell^+\ell^-$ decay (and the observed $B \rightarrow X_s\ell^+\ell^-$ and $B \rightarrow K\ell^+\ell^-$ decays) are already probing the SM-sensitivity. With the integrated luminosities over the next couple of years at the $B$ factories, the helicity analysis in $B \rightarrow \rho\ell\nu$ and $B \rightarrow K^*\ell^+\ell^-$ decays presented here can be carried out experimentally.

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