Analysis of the impact of shallow subway train traffic on urban development

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Abstract. The article discusses the process of propagation of vibrations in a soil massif from a moving subway train. With the help of previously performed measurements of ground surface vibrations above a shallow metro line, it was found that the electrical insulating joint of rail strings is a source of increased vibrations when a train enters the station from a tunnel, which can lead to an excess of vibration acceleration levels in residential and industrial buildings limited by sanitary standards. The main types of wheel and rail defects are considered. The propagation of vibrations in unlimited space from a moving subway train in a tunnel, represented by an absolutely rigid body, loaded with dynamic action, taking into account the shock impact at the rail joint, has been studied. The dependences of the parameters of the wave field in infinite space and the displacement of the points of the tunnel on the frequency of the forcing action are found. It is noted that the parameters of the wave field decrease with an increase in the frequency of the external forcing action. The results obtained can be used to solve the problem of the propagation of vibrations in a confined space from moving subway trains.

1. Introduction
The metro is an integral part of the transport network of any modern metropolis. Ease of movement between different parts of the city, bypassing traffic jams on public roads, cannot be overestimated. The location of future metro lines is closely related to the development of new territories and is necessarily reflected in the master plan for the city's development. Subway tunnels, depending on the method of construction and hydrogeological conditions, can be located at different depths, practically independent of the building on the day surface. The underground is a complex movable mechanical system, which can be simplified to the following constituent elements: rolling stock - railroad bed - ballast - tunnel lining. Each of these components in the process of manufacturing and operation acquires a certain random number of various defects and damages, the interaction of which leads to the formation of an increased vibration background. The tunnel, thus, becomes the emitter of a wave field propagating radially in the soil massif, which, attenuation with distance, collides with the nearby building structures of buildings and structures. A number of works have been devoted to the propagation of vibrations from the subway in domestic science, they have in common the presentation of a moving load on a railroad bed in the form of a certain periodically repeating average weight of a car. This approach allows you to simplify mathematical calculations and, with a certain accuracy, estimate the vibration parameters of a tunnel or soil mass. However, both domestic and foreign researchers note several main factors affecting the formation of vibration from a moving train, namely: defects in wheels and rails, as well as a wheel
hitting an obstacle - a rupture of rail strings (the so-called electrical insulating joint) in one level or vertically offset to form a step. Figure 1 shows an example of a junction of rails.

Figure 1. An example of junctions of rails.

The following works [1, 2, 3] are devoted to the issues of numerical simulation of the interaction of a wheel and a rail during their impact. The authors describe the physics of the process in sufficient detail and determine with sufficient accuracy the arising internal forces both in the rail and in the wheel. For example, [4] propose, by solving a linear partial differential equation of beam vibrations on a viscoelastic basis, to determine the significant components of the stress-strain state at a specific point of the rail for a given law of dynamic action.

A number of studies are devoted to solving problems of the propagation of an oscillatory front from a metro tunnel as a "radiator", among which the work of M.A. Dashevsky [5], describing in detail the application of the method of successive wave approximations together with the method of compensating loads. This method makes it possible to find a solution for a half-plane with a free boundary, simulating a day surface, using a combination of certain solutions for an unbounded domain in a flat formulation of the problem. In [6], the process of propagation of a wave field in a soil massif under the action of a periodic load on a tunnel is considered, and the wave interaction of the tunnel structures with the surrounding soil environment is also taken into account. As an external influence, the author adopted a loading scheme obtained by replacing concentrated forces from car bogies with a uniformly distributed averaged load on a section bounded by a railroad bed. In the article [7] it is shown that the impact of the composition on the electrical insulating joint leads to the formation of increased vibrations.

2. Determination of wave field parameters

Following the logic of works [5, 6], consider a rigid inclusion in the form of a washer, "soldered" into an elastic space, loaded with an average dynamic shock load.

Figure 2. Diagram of load application to the tunnel.
The force of the impact interaction of the wheel and the rail can be written as follows \[ P = P_0 \sin \frac{\pi}{\tau} \tag{1} \]

where \( P_0 \) is the amplitude value of the impact force.

Impact impulse

\[ I_y = m o d \tag{2} \]

where \( d \) is the width of the insulating joint.

Using the Hertz theorem describing contact interactions \[ P_0 = k \frac{51.2}{4\mu} \left( \frac{S1^2}{4m} \right)^{\frac{3}{2}} \tag{3} \]

where \( k = \frac{4}{3\pi} \frac{\sqrt{R}}{\delta_1 + \delta_2} \), \( R \) - radius of the wheel rolling circle, \( \delta_1, \delta_2 \) - material characteristics.

For this and subsequent problems, the main dependencies are derived from the Lamé equation of motion of points of a continuous elastic medium

\[ \mu \cdot \Delta \vec{u} + (\lambda + \mu) \cdot \text{grad} \left( \text{div} \vec{u} \right) = \rho \frac{\partial^2 \vec{u}}{\partial t^2} \tag{4} \]

where \( \vec{u} \) is the displacement vector, \( \lambda, \mu \) are the Lamé constants, \( \rho \) is the density of the elastic medium.

According to the Helmholtz theorem \[ \nabla \phi + \nabla \times \psi = 0 \tag{5} \]

where \( \phi \) is the scalar potential \( \psi \) is the vector potential.

Thus, the problem of determining the displacement field is reduced to solving the following set of equations for unknown potentials

\[
\begin{aligned}
\Delta \phi &= \frac{1}{a^2} \frac{\partial^2 \phi}{\partial t^2} \\
\Delta \psi &= \frac{1}{b^2} \frac{\partial^2 \psi}{\partial t^2}
\end{aligned}
\tag{6}
\]

where \( a^2 = \frac{\lambda + 2\mu}{\rho} \), \( b^2 = \frac{\mu}{\rho} \) are the velocities of longitudinal and transverse waves in an elastic medium.

In this and subsequent problems, along with the Cartesian one, a polar coordinate system is used, the origin of which coincides with the center of an inclusion soldered into the elastic space.

In accordance with \[ 11 \], elastic displacements of points can be written as follows

\[
\begin{aligned}
u_r &= \frac{\partial \phi}{\partial r} + \frac{1}{r} \frac{\partial \psi}{\partial \theta} \\
u_\theta &= \frac{1}{r} \frac{\partial \phi}{\partial \theta} - \frac{\partial \psi}{\partial r}
\end{aligned}
\tag{7}
\]

Using the known relations for the components of the linear theory of elasticity \[ 12 \] and the results \[ 6 \], we write, with some refinements, the expressions for stresses, expressed in terms of potentials in a polar coordinate system.
\[ \begin{align*}
\sigma_r &= -\rho a^2 k_r^2 \varphi - 2\rho b^2 \left( \frac{\partial \varphi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \varphi}{\partial \theta^2} + \frac{1}{r} \frac{\partial \psi}{\partial \theta} - \frac{\partial^2 \psi}{\partial r \partial \theta} \right), \\
\sigma_\theta &= -\rho a^2 k_\theta^2 \varphi + \frac{2\rho b^2}{r} \left( \frac{r}{2} \frac{\partial^2 \varphi}{\partial \theta^2} + \frac{1}{r} \frac{\partial \psi}{\partial \theta} - \frac{\partial^2 \psi}{\partial r \partial \theta} \right), \\
\sigma_{r\theta} &= \tau_{r\theta} = \frac{4\rho b^2}{r} \left( \frac{\partial^2 \varphi}{\partial r \partial \theta} - \frac{1}{r} \frac{\partial \psi}{\partial \theta} + \frac{r}{2} k_r^2 \psi + \frac{\partial \psi}{\partial r} + \frac{1}{r} \frac{\partial^2 \psi}{\partial \theta^2} \right),
\end{align*} \]

(8)

where: \( k_r^2 = \omega^2 / a^2 = \frac{\rho \omega^2}{\lambda + 2\mu} \), \( k_\theta^2 = \omega^2 / b^2 = \frac{\rho \omega^2}{\mu} \).

In general form, solutions of the Helmholtz equations for retarded potentials [11] can be written in the form

\[
\varphi = \varphi_1 \cos \theta = L_1 H_1^{(2)}(k_r r) \cos \theta \\
\psi = \psi_1 \sin \theta = Q_1 H_1^{(2)}(k_\theta r) \sin \theta
\]

(9)

where: \( L_1, Q_1 \) are the required coefficients that fully characterize the wave field in unlimited space.

Using the boundary conditions for the plane problem of radiation of cylindrical waves into an unbounded plane [5], we can write the following system of equations for the coefficients \( L_1, Q_1 \).

\[
L_1 \left( \alpha H_0^{(2)}(k_r R) - \frac{\alpha}{k_r R} H_1^{(2)}(k_r R) - \frac{\beta \omega^2}{R} H_2^{(2)}(k_r R) \right) + Q_1 \left( \frac{\alpha}{R} H_1^{(2)}(k_r R) + \frac{\beta \omega^2}{R} H_2^{(2)}(k_r R) \right) + 2P_0 = 0
\]

(10)

3. Results and discussions

Having solved the system of equations (10) in the mathematical software with respect to \( L_1, Q_1 \), we will construct graphs of the dependence of the parameters of the wave field in infinite space and the radial displacement of the tunnel on the change in the frequency of the external influence (figures 3, 4).

**Figure 3.** Dependence of the parameters of the wave field on the frequency of the external influence.
Note that figure 3 shows that the intensity of the wave field decreases with increasing frequency of the external influence.

4. Conclusions
The studies carried out within the framework of the set task allow us to formulate the following conclusions:

- the impact at the junction of the rails induces all modes of vibration, but the largest are vibrations with frequencies close to 31.5 Hz and 63 Hz;
- vibration levels, measured in octaves 31.5 and: 63 Hz, at points located above the metro line, significantly exceed the permissible according to the Sanitary Standards of the Russian Federation;
- with an increase in the frequency of external influence, the intensity of the wave field in infinite space decreases;
- the obtained coefficients $L_i$, $Q_i$, characterizing the wave field in unbounded space, will be further used to solve the problem of the propagation of oscillations induced by a tunnel in a semi-infinite space.

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