A New Scenario for String Unification

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Abstract

We present a new scenario for gauge coupling unification in flipped SU(5) string models, which identifies the $M_{32}$ scale of SU(3) and SU(2) unification with the empirical $M_{\text{LEP}} \sim 10^{15–16}$ GeV scale, and the $M_{51}$ scale of SU(5) and U(1) unification with the theoretical $M_{\text{string}} \sim 5 \times 10^{17}$ GeV string unification scale. The vacuum shift necessary for the cancellation of the anomalous $U_A(1)$ and an SU(4) hidden sector with fractionally-charged particles, play a crucial role in the dynamical determination of all intermediate mass scales in this scenario.
The convergence of the Standard Model gauge couplings, when extrapolated to very high energies in the context of supersymmetric theories, has received a great deal of attention ever since it was first observed [1], and especially since the advent of the LEP era [2]. In the simplest and best studied scenarios, only the particles in the Standard Model and their superpartners are included in the evolution of the gauge couplings, which are seen to converge at the scale

\[ M_{\text{LEP}} \sim 10^{15-16} \text{ GeV} \],

above which a larger structure must be revealed. As compelling as this simple result may be, it is more than one order-of-magnitude lower than the scale at which the gauge couplings should unify [3] (to lowest order) in the context of superstring models [4],

\[ M_{\text{string}} = 5 \times g \times 10^{17} \text{ GeV} \],

where \( g \) is the unified gauge coupling at this scale. This discrepancy of scales (which may enhance the reliability of the low-energy effective theory description) has been taken seriously by string model builders, as \( M_{\text{LEP}} \) is an empirical result, whereas \( M_{\text{string}} \) is an actual theoretical prediction. Early on it was pointed out that reconciliation of these two scales required to supplement the particle content of the Minimal Supersymmetric Standard Model (MSSM) with new intermediate-scale particles [5]. Alternatively, it was proposed that threshold corrections from the infinite tower of massive string states shifted \( M_{\text{string}} \) down to effectively coincide with \( M_{\text{LEP}} \) [6]. The latter scenario is now disfavored, as it requires (large) values of the (moduli) fields that parametrize the threshold corrections, which are hard to obtain in actual string models [7, 8]. Moreover, this scenario appears to push “gravity” down to scales uncomfortably lower than the Planck mass, and still requires the addition of new particles beyond the MSSM [8]. Further generic alternatives may exist [9], but these are yet to be realized in realistic string models.

We then see that all known scenarios of string unification predict the existence of new intermediate-scale particles in the observable sector, a property quite common among actual string models, such as those based on the gauge groups SU(5) × U(1) (“flipped” SU(5)), or SU(3) × SU(2) × U(1) (“standard-like” models), or SU(4) × SU(2) × SU(2) (“Pati-Salam” models), or SU(3)³. Of these possible gauge groups, only flipped SU(5) unifies the SU(3) and SU(2) non-abelian factors of the Standard Model gauge group (at the scale \( M_{32} \)), and therefore can in principle reproduce the “observed” \( M_{\text{LEP}} \) scale (identified with \( M_{32} \)), above which SU(5) × U(1) is revealed. Note that this result is unaffected (to lowest order) by the introduction of the intermediate-scale representations, if these come in complete SU(5) multiplets as we advocate below. The SU(5) and U(1) gauge couplings unify at the scale \( M_{51} \), which we consistently identify with \( M_{\text{string}} \). In the case of the other gauge groups, unification must occur at \( M_{\text{string}} \) leaving no room for \( M_{\text{LEP}} \), which must be regarded as an accidental result.

In this Letter we present a new scenario for string unification in the context of flipped SU(5) models, following the guidelines just described. Such models have
been derived from string [10, 11] and, through detailed first-principles calculations
[12, 13, 14], have been shown to possess many interesting properties, which even
though not of crucial importance for the subsequent discussion, do motivate further
the consideration of this class of models. The latest incarnation of the string model
[11] includes three generations of quarks and leptons, an SU(5)×U(1) observable
gauge group, an SO(10)×SU(4) hidden sector gauge group, vanishing vacuum energy
at tree-level (\(V_0 = 0\)), and vanishing quadratically-divergent one-loop correction to
the vacuum energy (\(Q = 0\)) in the shifted vacuum where the anomalous U\(_A\)(1) is
cancelled to ensure unbroken supersymmetry at the string scale. Moreover, in this
vacuum shifting the SU(5)×U(1) symmetry is broken, and one is able to find naturally
solutions [14] with \(M_{32} \sim M_{\text{LEP}}\), as advocated above.

Crucial to the string unification program are a pair of (\(10, 10\)) SU(5) repre-
sentations, in addition to those required for SU(5)×U(1) symmetry breaking, with
intermediate-scale masses \(M_{10}\). In fact, in the class of fermionic string models that
we study, the vanishing of the tree-level vacuum energy (\(V_0 = 0\)) appears inextricably
correlated to the existence of the extra (\(10, 10\)) pair [13]. Also important are the
(\(4, \overline{4}\)) representations of the hidden SU(4) gauge group, which have fractional electric
charges (\(\pm \frac{1}{2}\)), and are either heavy or become confined into integrally-charged “cryp-
tons” at the SU(4) confinement scale \(\Lambda_4\) [15]. Moreover, a condensate of such hidden
sector fields provides the mass scale that determines \(M_{10}\) dynamically. The novelty
in our approach is that all intermediate scales, including \(M_{32} \leftrightarrow M_{\text{LEP}}\) and \(M_{10}\), are
generated dynamically in a self-consistent fashion.

The top-down scenario for string unification that we envision consists of the
following steps:

(i) \(Q = M_{\text{string}}\): The SU(5), U(1), SO(10), and SU(4) gauge couplings are unified at
the common value \(g\). String threshold corrections, that may shift \(M_{\text{string}}\), are
expected to be very small in this class of models [8].

(ii) \(\Lambda_{10} < Q < M_{\text{string}}\): The hidden SO(10) group evolves according to the one-loop
beta function \(\beta_{10} = -24 + N_{10}\), where \(N_{10}\) is the number of SO(10) decaplets,
and confines at the scale \(\Lambda_{10} = M_{\text{string}} e^{8\pi^2/g^2\beta_{10}} \sim 10^{15} - 16\) GeV.

(iib) \(\Lambda_4 < Q < M_{\text{string}}\): The hidden SU(4) group evolves according to the one-loop
beta function \(\beta_4 = -12 + \frac{1}{2}N_4 + N_6\), where \(N_4\) is the number of \(4\) and \(\overline{4}\) fields,
and \(N_6\) is the number of \(6\) fields, and confines at the scale
\[
\Lambda_4 = M_{\text{string}} e^{8\pi^2/g^2\beta_4} .
\]  (3)
This scale depends on the detailed spectrum of \(4, \overline{4}, 6\) particles and on \(g\). For
concreteness, we will assume the typical case of \(N_6 = 0\) and \(N_4 = 0, 2, 4\).

(iii) \(M_{32} < Q < M_{\text{string}}\): The SU(5) and U(1) gauge groups evolve according to the
following one-loop beta functions [17]
\[
\begin{align*}
b_5 &= -15 + 2N_g + \frac{1}{2}N_5 + \frac{3}{2}N_{10} = -2 , \quad (4) \\
b_1 &= 2N_g + \frac{1}{2}N_5 + \frac{1}{4}N_{10} + \frac{5}{2}N_4 = 8 + \frac{3}{8}N_4 , \quad (5)
\end{align*}
\]
where \(N_g = 3\) is the number of generations, \(N_5 = 2\) is the number of Higgs pentaplets \((h, \bar{h})\), \(N_{10} = 4\) is the number of Higgs decaplets (two pairs of \((10, \bar{10})\)), and \(N_4\) is the number of (light) hidden \(4, \bar{4}\) fields, as in item (iib). (The hidden fields are SU(5) singlets and do not affect the running of SU(5).) Symmetry breaking down to the Standard Model gauge group occurs at the scale \(M_{32}\), as triggered by the the vevs \(\langle \nu^c_H \rangle = \langle \nu^c_{\bar{H}} \rangle\) of the neutral components of the \((10, \bar{10})\) Higgs representations.

(iv) \(M_{10} < Q < M_{32}\): The SU(3), SU(2), and U(1)\(_Y\) gauge couplings evolve according to the one-loop beta functions [16]

\[
\begin{pmatrix}
  b_Y \\
  b_2 \\
  b_3
\end{pmatrix} = - \begin{pmatrix}
  0 & 6 \\
  2 & 2 \\
  9 & 8
\end{pmatrix} + N_g \begin{pmatrix}
  \frac{3}{10} \\
  0 \\
  \frac{3}{5}
\end{pmatrix} + N_2 \begin{pmatrix}
  \frac{3}{5} \\
  \frac{2}{5} \\
  0
\end{pmatrix} + N_3 \begin{pmatrix}
  1 \\
  0 \\
  1
\end{pmatrix} + N_{32} \begin{pmatrix}
  \frac{1}{10} \\
  0 \\
  1
\end{pmatrix} + N_4 \begin{pmatrix}
  \frac{3}{5} \\
  0 \\
  0
\end{pmatrix},
\]

where \(N_g = 3\), \(N_2 = 2\) is the number of light Higgs doublets, \(N_3 = 2\) is the number of \(D^c, \bar{D}^c\) fields from the extra \((10, \bar{10})\), and \(N_{32} = 2\) the corresponding number of \(Q, \bar{Q}\). Here \(N_4\) is the number of \(4, \bar{4}\) hidden fields (as in item (iib)), which decouple from the evolution at the scale \(\Lambda_4\) (higher than \(M_{10}\)). We obtain:

\[b_Y = \frac{36}{5} + \frac{3}{5} N_4,\ b_2 = 4,\ b_3 = 0\]  

Because of the non-standard embedding of the electric charge generator in SU(5), the gauge couplings at \(M_{32}\) are related via \(25/\alpha_Y = 1/\alpha_5 + 24/\alpha_1\) [16]. We first adjust \(M_{10}\) to obtain string unification at \(M_{\text{string}}\), given values of the low-energy gauge couplings and \(N_4\), and later check the consistency of this calculation against the dynamically determined value of \(M_{10}\) (given in terms of \(\Lambda_4\), as discussed below).

(v) \(M_Z < Q < M_{10}\): The extra \((10, \bar{10})\) representations decouple at \(M_{10}\) and the SU(3), SU(2), and U(1)\(_Y\) gauge couplings evolve according to the one-loop beta functions in Eq. (6). With the traditional values \(N_g = 3\), \(N_2 = 2\), and \(N_3 = N_{32} = 0\), we obtain the usual result \(b_Y = \frac{33}{5}\), \(b_2 = 1\), and \(b_3 = -3\).

The above general scenario requires the dynamical generation of three mass scales: \(M_{32}, M_{10}, \Lambda_4\). The generation of the latter scale is well known, and was discussed in item (iiib) above. Generation of the scale of unified symmetry breaking \(M_{32}\) has remained a puzzle in string model-building. Here we advocate a mechanism of pure stringy origin. Many (if not all) realistic string models possess a U(1) factor in the gauge group whose trace over the massless string states does not vanish, \(i.e.,\) an “anomalous” \(U_A(1)\). It has been long known [17] that this anomaly is simply the result of truncating the low-energy effective theory to the massless spectrum, and it is not present in the full string theory. Its presence affects the D-term contribution to the scalar potential from this U(1) gauge symmetry, and breaks supersymmetry in the “original” vacuum. Supersymmetry can be easily restored by sliding to a nearby vacuum.

\[\text{In our numerical calculations below we include the full two-loop beta function coefficients [16], which smooth out this zero-slope behavior.}\]
vacuum, which is still a consistent solution of string theory, and which is parametrized by vevs of scalar fields of typical magnitude \( M_{\text{string}} \) or lower. In particular, the set of shifted scalar fields includes those that break the \( \text{SU}(5) \times \text{U}(1) \) gauge symmetry, which then become dynamically determined. In specific models one finds that \( M_{32} \approx g \langle \nu H \rangle \sim M_{\text{LEP}} \) can be readily obtained [14].

Our scenario also requires the dynamical generation of the \( M_{10} \) scale, which provides masses to the extra \((10,10)\) representations. We propose to obtain this scale through a non-renormalizable superpotential coupling of the form

\[ \lambda (10) (\bar{10}) (4) (\bar{4}) \frac{1}{M}, \]

as generically available in this class of string models, where \( M \approx 10^{18} \text{ GeV} \) is the appropriate scale [12], and \( \lambda \lesssim 1 \) is expected. Assuming that in the \( \text{SU}(4) \) condensation process the \( N_4 \) hidden fields obtain masses \( \mathcal{O}(\Lambda_4) \), the \( \langle 4\bar{4} \rangle \) condensate is estimated to be \( \langle 4\bar{4} \rangle \sim \Lambda_4^2 \) and thus

\[ M_{10} \sim \lambda \frac{\langle 4\bar{4} \rangle}{M} \sim \frac{\Lambda_4^2}{M}. \]  

Whether the above scenario for string unification is realistic or not can be determined by following the evolution of the gauge couplings from the bottom up: starting from the well measured Standard Model gauge couplings and running up to the string scale, using the two-loop renormalization group equations. In practice, \( M_{32} \) is determined by the low-energy gauge couplings to be close to \( M_{\text{LEP}} \), irrespective of the value of \( M_{10} \), whereas \( M_{\text{string}} \) depends on \( M_{10} \). We first adjust \( M_{10} \) to obtain string unification, and then check the consistency of our procedure against our dynamical prediction for \( M_{10} \) in Eq. (8).

In Fig. 1 we show the calculated values of \( M_{31} \leftrightarrow M_{\text{string}}, M_{32}, \Lambda_4, \) and \( M_{10} \) as a function of \( \alpha_s(M_Z) \), calculated to two-loop precision.\(^2\) The realistic case requires \( N_4 \neq 0 \) to be able to generate \( M_{10} \) dynamically. Different values of \( N_4 \) affect \( M_{10} \), as shown in the figure for \( N_4 = 0, 2, 4 \). Comparing these calculated values of \( M_{10} \) with those predicted from Eq. (8) (dashed lines in Fig. 1), shows that \( N_4 = 2 \) for \( \alpha_s = 0.116 \) works rather well: \( M_{10} \sim 10^9 \text{ GeV} \). It is most interesting that such self-consistency checks work at all, and that they can constrain the spectrum of hidden sector states with observable-sector quantum numbers. We finally put all pieces together and show in Fig. 2 the running of the gauge couplings for the favored values \( \alpha_s = 0.116 \) and \( N_4 = 2 \), with all the scales as indicated.

Some phenomenological aspects of this type of scenario (restricted to \( Q < M_{32} \)), including the (small) effects of light and GUT thresholds and a possibly observable proton decay signal into \( e^+\pi^0 \) at Superkamiokande, have been recently explored in Ref. [18]. The above scenario should motivate further flipped SU(5) string model-building along these lines.

\(^2\)Note that \( \Lambda_4 \) does not decrease with increasing \( N_4 \) (as naively expected) because of its dependence on \( g \), which needs to be self-consistently determined (in the bottom-up approach) for every choice of \( N_4 \), and which increases with \( N_4 \). This is also the source of the \( \alpha_s \) dependence of \( \Lambda_4 \).
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Figure 1: The calculated values of the SU(5) and U(1) unification scale ($M_{51}$, identified with $M_{\text{string}}$), the SU(5) unification scale ($M_{32}$, identified with $M_{\text{LEP}}$), the SU(4) confinement scale $\Lambda_{4}$, and the intermediate scale $M_{10}$, as a function of $\alpha_{s}(M_{Z})$ for $N_{4} = 0, 2, 4$ (indicated in parenthesis). Dashed lines display estimates of the dynamical prediction for $M_{10}$. Note that $\alpha_{s} = 0.116$ and $N_{4} = 2$ work rather well. (All masses in GeV.)
Figure 2: The running of the gauge couplings for the preferred values $\alpha_s(M_Z) = 0.116$ and $N_4 = 2$. One obtains $M_{10} = 1.8 \times 10^9$ GeV, $M_{32} = 8.7 \times 10^{15}$ GeV, $M_{51} = 4.4 \times 10^{17}$ GeV, $\Lambda_4 = 3.9 \times 10^{13}$ GeV, and $g = 0.88$. This value of $M_{10}$ agrees rather well with the dynamical prediction $M_{10} \sim 10^9$ GeV.