EMITTING ELECTRONS SPECTRA AND ACCELERATION PROCESSES IN THE JET OF Mrk 421:
FROM THE LOW STATE TO THE GIANT FLARE STATE

DAHAI YAN1, LI ZHANG1, QIANG YUAN2, ZHONGHUI FAN1, AND HOUDUN ZENG1

1 Department of Physics, Yunnan University, Kunming 650091, Yunnan, China; lizhang@ynu.edu.cn
2 Key Laboratory of Particle Astrophysics, Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

Received 2012 September 24; accepted 2013 January 23; published 2013 February 26

ABSTRACT

We investigate the electron energy distributions (EEDs) and the acceleration processes in the jet of Mrk 421 through fitting the spectral energy distributions (SEDs) in different active states in the frame of a one-zone synchrotron self-Compton model. After assuming two possible EEDs formed in different acceleration models: the shock-accelerated power law with exponential cut-off (PLC) EED and the stochastic-turbulence-accelerated log–parabolic (LP) EED, we fit the observed SEDs of Mrk 421 in both low and giant flare states using the Markov Chain Monte Carlo method which constrains the model parameters in a more efficient way. The results from our calculations indicate that (1) the PLC and LP models give comparably good fits for the SED in the low state, but the variations of model parameters from low state to flaring can be reasonably explained only in the case of the PLC in the low state; and (2) the LP model gives better fits compared to the PLC model for the SED in the flare state, and the intra-day/night variability observed at GeV–TeV bands can be accommodated only in the LP model. The giant flare may be attributed to the stochastic turbulence re-acceleration of the shock-accelerated electrons in the low state. Therefore, we may conclude that shock acceleration is dominant in the low state, while stochastic turbulence acceleration is dominant in the flare state. Moreover, our result shows that the extrapolated TeV spectra from the best-fit SEDs from optical through GeV with the two EEDs are different. It should be considered with caution when such extrapolated TeV spectra are used to constrain extragalactic background light models.

Key words: acceleration of particles – BL Lacertae objects: individual (Mrk 421) – galaxies: active – galaxies: jets – gamma rays: galaxies – radiation mechanisms: non-thermal

Online-only material: color figures

1. INTRODUCTION

Blazars are the most extreme class of active galactic nuclei (AGNs). Their spectral energy distributions (SEDs) are characterized by two distinct bumps. The first bump, which is located at the low-energy band from radio through UV or X-rays, is generally explained by synchrotron emission from relativistic electrons in a jet that is closely aligned with the line of sight. The second bump, which is located at the high-energy band, could be produced by inverse Compton (IC) scattering of the relativistic electrons (the so-called leptonic model; e.g., Böttcher 2007). The seed photons for the IC process can be synchrotron photons (synchrotron self-Compton, SSC; Rees 1967; Maraschi et al. 1992) or external radiation fields (EC; Dermer & Schlickeiser 1993; Sikora et al. 1994). The hadronic model is an alternative explanation for the high-energy emissions from blazars (e.g., Mannheim 1993; Mücke et al. 2003; Dimitrakoudis et al. 2012; Dermer et al. 2012).

In the leptonic model, a power-law electron energy distribution (EED) with an exponential high-energy cutoff (PLC) or a broken power-law EED is commonly adopted (e.g., Tavecchio et al. 1998; Finke et al. 2008). The main justification for this EED approximation is that the non-thermal emissions from blazars can be described by a power-law spectrum, and the power-law EED can be naturally generated in the framework of shock acceleration (the Fermi I process; e.g., Baring 1997). In recent observations, however, the X-ray spectra of several blazars (such as Mrk 421, Mrk 501) show significant curvature, which are typically milder than an exponential cut-off (Massaro et al. 2004a, 2004b, 2006). Very recently, it has been found that gamma-ray emissions of many blazars can be successfully fitted with a log–parabolic (LP) spectrum (e.g., Aharonian et al. 2009; Abdo et al. 2010; Ackermann et al. 2011). The LP EED is then proposed to model the observed spectral curvature, and such LP EED can be generated in the stochastic turbulence acceleration scenario (the Fermi II process; e.g., Tramacere et al. 2009, 2011). Numerical simulation indicates that the stochastic acceleration process may play an important role in the formation of the particle spectrum in blazar jets (Virtanen & Vainio 2005). When the emission mechanisms are determined, the emitting EED can be reconstructed from the observed emission spectra. We can then investigate the acceleration processes acting in a blazar jet (e.g., Ushio et al. 2010; Garson et al. 2010).

Blazars are well known for their rapid and large-amplitude variability at all wavebands, most prominently at keV and TeV energies. The relations (including correlation and time lag) between variabilities at different wavelengths are crucial for constraining jet models (e.g., Sokolov et al. 2004; Fossati et al. 2008; Katarzyński & Walczewska 2010; Böttcher & Dermer 2010). For instance, if the hard lag is observed, an acceleration process could be considered in the model. Different flare patterns indicate different causes of the flare, such as a change in the injection rate and change in the acceleration process, etc. (e.g., Kirk et al. 1998; Kataoka et al. 2000; Graff et al. 2008; Moraitis & Mastichiadis 2011; Chen et al. 2011).

Mrk 421 is the closest known (redshift \( z = 0.031 \)) and the first very high energy (VHE) blazar (Punch et al. 1992). It is classified as a high-peaked BL Lac according to its synchrotron peak location. Mrk 421 is the main target of multi-wavelength monitoring campaigns. There are a large number of publications on the multi-wavelength observations of this source. Its SED and
relation of variabilities at different bands are intensively studied
(e.g., Blażejowski et al. 2005; Albert et al. 2007; Acciari et al.
2009, 2011; Aleksić et al. 2010; Bartolli et al. 2011). From these
studies, we learned that TeV flare is often correlated with
keV flare; however, such relation is very complex. For example,
the TeV flux increases more than quadratically with respect to
the X-ray one, and there is a time lag between TeV and keV
flares. Although the SED of Mrk 421 is commonly well fitted
by a one-zone SSC model, the complex variability behaviors
remind us that the realistic model is more complicated. Lately,
the multi-wavelength campaign showed evidence for Mrk 421
being in the low/quiescent state from 2009 January 19 to 2009
June 1 (Abdo et al. 2011). During this campaign, no significant
flare activity was seen and the measured VHE flux is among
the lowest fluxes recorded by MAGIC. Abdo et al. (2011)
therefore claimed that the unprecedented complete, 4.5 month
average SED observed during this campaign can be considered
an excellent proxy for the low/quiescent state SED of Mrk 421.
Several months later, Mrk 421 was found to undergo one of its
brightest outbursts at the X-ray and gamma-ray bands on 2010
February 17 (Shukla et al. 2012). During this flaring, the flux
correlation at the X-ray and TeV band was observed (Shukla
et al. 2012), and intra-night variability at the GeV–TeV band
was found (Galante et al. 2011; Shukla et al. 2012; Raue et al.
2012).

In this paper, we investigate the EEDs and the acceleration
processes in the jet of Mrk 421 in the low state and giant flare
state. To achieve our aim, we assume two electron distributions,
both well motivated by the current particle acceleration models:
the shock-accelerated PLC EED and the stochastic-turbulence-
accelerated LP EED, to model the SEDs in the framework of
a one-zone SSC model. To more efficiently constrain the
model parameters and better distinguish between the models,
we employ the Markov Chain Monte Carlo (MCMC) method
instead of a simple χ²-minimization procedure to investigate
the high-dimensional model parameter space systematically.
The emission models and MCMC method are briefly described
in Section 2. In Section 3, we report our results. Finally in
Section 4, we summarize our discussions and conclusions.

2. EMISSION MODEL AND MCMC METHOD

2.1. Emission Model

The one-zone SSC assumes that non-thermal radiation is
produced by both synchrotron radiation and SSC process
in a spherical blob filled with uniform magnetic field (B),
which is moving relativistically at a small angle to our line
of sight, and the observed radiation is strongly boosted by a
relativistic Doppler factor δD. The radius of the emitting blob is
R = (t_v minδDc/1 + z), where t_v min is the minimum variability
timescale. Here, quantities in the observer’s frame are unprimed,
and quantities in the comoving frame are primed. Note that the
magnetic field B is defined in the comoving frame, despite being
unprimed. We use the methods given in Finke et al. (2008)
to calculate synchrotron and SSC fluxes.

The PLC electron distribution formed in the Fermi I acceleration
process is

\[ N'(y') = K'_e y'^{-s} \exp\left(\frac{-y'}{y'^c}\right) \text{ for } y'_\text{min} \leq y' \leq y'_\text{max}, \]

where \( K'_e \) is the normalization of the EED, \( s \) is the electron
energy index, and \( y'^c \) is the electron cut-off energy. \( y'_\text{min} \) and \( y'_\text{max} \)
are the electron minimum energy and electron maximum energy,
respectively. In this model, there are eight free parameters;
five of them specify the EED \( (K'_e, y'_\text{min}, y'^c, y'_\text{max}, s) \) and the other
description 3 the global properties of the emitting region
\( (B, R^3, \delta_D) \).

The LP electron distribution generated in the Fermi II acceleration
process is

\[ N'(y') = K'_e \left\{ \left(\frac{y'}{y'^c}\right)^{-s} \right\} \text{ for } y' \leq y'_\text{min}, \]

where \( r \) is the curvature term of EED (Massaro et al. 2006).
In this model, \( r \) is an additional parameter besides the eight
mentioned above.

2.2. MCMC Method

The MCMC method, based on Bayesian statistics, is well
suited for high-dimensional parameter space investigation,
which is superior to the grid approach with a more efficient
sampling of the parameter space of interest. The Metropolis–Hastings sampling algorithm is adopted to
determine the jump probability from one point to the next in parameter space (Mackay 2003). The algorithm ensures that the
probability density functions of model parameters can be asymptotically approached with the number density of samples. In the
following fitting, we will run single chains using the Raferty & Lewis (1992) convergence diagnostics, and we assume flat
priors in the model parameter spaces. A brief introduction to
the basic procedure of MCMC sampling can be found in
Fan et al. (2010). For more details about the MCMC method, please
refer to Neal (1993), Gamerman (1997), and Mackay (2003).
Since the code we used in this paper (Liu et al. 2012) is adapted
from COSMOMC, we refer the reader to the Web site³ and to
Lewis & Bridle (2002 and references therein) for a detailed
explanation of the code about the sampling options, convergence
criteria, and statistical quantities.

3. MODELING RESULTS

3.1. Modeling the SED in Low State

In this case, we adopt the optical–UV to X-ray data observed by
Swift/UVOT/RXT/BAT and the Fermi-LAT gamma-ray data
reported in Abdo et al. (2011). For the Fermi-LAT data, the
last two data points are not included in our modeling due to
their very large uncertainties. Instead, we use the first two data
points measured by MAGIC since the extragalactic background
light (EBL) absorption on the flux at such energy band is
negligible. Because EBL absorption has an effect on the other
fluxes measured by MAGIC, we also do not take them into
account in our modeling. The absorption effect of EBL will be
discussed in Section 3.3. The SMA data at 2.3 × 10^{11} Hz are
used to constrain \( y'_\text{min} \). We fix \( y'_\text{min} = 700 \) for the PLC case,
\( y'_\text{min} = 1200 \) for the LP case, and \( y'_\text{max} = 10^8 \) for both cases.

For the PLC electron distribution, the one-dimensional (1D)
probability distributions and two-dimensional (2D) confidence
regions (at 1σ and 2σ confidence levels) of the model parameters,
and the SED are shown in Figure 1. The fitting parameters
are summarized in Table 1 with the reduced Chi² of 1.01
for 27 d.o.f. (Note that a relative systematic uncertainty of

³ http://cosmologist.info/cosmomc/
be seen that a very good constraint is derived for the spectral optical–UV–X-rays data reported in Abdo et al. 2011.) It can
rewritten as $(v F_\nu) \propto (K'_\nu / \gamma'_c v'_c)^{2} E'_s$; therefore the positive correlation between $K'_\nu$ and $\gamma'_c$ is expected from the constraint of the observed synchrotron peak flux. As for the positive correlation between $K'_\nu$ and $t_{v, \text{min}}$, since the IC peak flux $(v F_\nu)_{\nu}$ can be estimated using the observed gamma-ray flux and $(v F_\nu)_{\nu} \propto K'_\nu / v'_c$, and then $K'_\nu / t_{v, \text{min}} \propto [(v F_\nu)_{\nu} / (v F_\nu)] [v'_c / v'_s]$, where $t_{v, \text{min}} B \delta_D^3 = \text{constant}$ and $B \delta_D \propto v'_2 / v'_s$ (Tavecchio et al. 1998) are used. Therefore, the positive correlation between $K'_\nu$ and $t_{v, \text{min}}$ can be predicted according to the observations.

For the LP electron distribution, the parameter distributions and SED are shown in Figure 2. The fitting parameters are listed in Table 1. In this case, we have $\chi^2 = 0.97$ (for 26 d.o.f.). The curvature term $r$ is well constrained. It seems that $B$ can be better constrained than in the PLC model. The constraints on the other parameters are similar to that derived in the PLC model. The explanations for the correlations in the case of PLC still hold in the case of LP except for the correlation between $B$ and $\gamma'_c$. In the LP case, the correlation between $B$ and $\gamma'_c$ becomes weak, which is caused by the effect of $r$. This correlation between $B$ and $r$ is caused by the well-observed spectrum around the synchrotron peak.

Comparing the results derived in the two EEDs, it can be found that the global properties of the emitting blob $(B, \delta_D, R')$ derived in the two cases are comparable. Obviously, we cannot distinguish between the PLC and LP electron distributions in the low state based on the above results (see Figures 1 and 2 as well as Table 1).
1.35 3 6 6.2 5.8 2.4 1
0.04 7.98 0.08 110.08
3.63 ± 0.21 0.17 ± 0.03 2.02 ± 0.04 3.79 ± 0.26 2.67 ± 0.32 2.66 ± 0.32 3.44 ± 0.20 61.75
0.04 7.36–8.57 (<3.07)

Figure 2. Same as in Figure 1 but with the LP electron distribution.

(A color version of this figure is available in the online journal.)

Figure 3. Modeling the SED in the giant flare state with PLC electron distribution. The curves are the same as that in Figure 1. For the low energy component of the SED, the symbols denote the data from the SPOL CCD Imaging/Spectropolarimeter at Steward Observatory (squares), the XRT data (circles), and the PCA data (triangles); at the high energy component, the symbols denote LAT data (circles) and HAGAR data (triangle). Please see Shukla et al. (2012) for more details about the data sets.

(A color version of this figure is available in the online journal.)

Table 2
Fitting Parameters for the SED in the Giant Flare State

| Model | $\gamma_1$ ($10^2$) | $K_1^c$ ($10^{33}/10^{42}$) | $s$ | $r$ | $B$ (0.01 G) | $\tau_{\nu, \text{min}}$ ($10^4$ s) | $\delta_D$ (10) | $U'_f/U'_B$ |
|-------|-----------------|-----------------|-----|-----|--------------|-----------------|-------------|-------------|
| PLC model | 5.82 ± 0.14 | 0.04 ± 0.01 | 1.65 ± 0.01 | ... | 1.07 ± 0.06 | 7.98 ± 0.55 | 2.99 ± 0.08 | 110.08 |
| 68% limit | (5.66–5.97) | (<0.04) | (1.64–1.66) | ... | (<1.14) | (7.36–8.57) | (<3.07) |
| LP model | 3.63 ± 0.21 | 0.17 ± 0.03 | 2.02 ± 0.04 | 3.79 ± 0.26 | 2.67 ± 0.32 | 2.66 ± 0.32 | 3.44 ± 0.20 | 61.75 |
| 68% limit | (3.42–3.86) | (0.14–0.20) | (1.99–2.06) | (3.51–4.08) | (2.30–3.03) | (2.30–3.02) | (3.22–3.66) |

3.2. Modeling the SED in Flare State

For the SED in flaring state, the Swift/RXT, RXTE/PCA, Fermi-LAT; and HAGAR data reported in Shukla et al. (2012) are adopted. We also fix $\gamma_{\min}$ and $\gamma_{\max}$ as we did in Section 3.1. The parameter distributions and SED fitting derived with PLC EED are shown in Figure 3. The fitting parameters are listed in Table 2. The resulting $\chi^2 = 0.79$ for 318 d.o.f. is below unity after a relative systematic uncertainty of 8% was added. The fittings at optical and GeV bands are bad. $\gamma_1'$ and $s$ are well constrained, and $\tau_{\nu, \text{min}}$ can be constrained to a relatively small range. However, the parameters $K_1^c$, $\delta_D$, and $B$ are poorly constrained. We cannot obtain meaningful distribution ranges of the three parameters, and thus only the 68% upper limits are reported in Table 2. Unfortunately, the variability timescale (~1 day) required in this PLC model contradicts the observed intra-day variability at GeV–TeV bands (Shukla et al. 2012; Raue et al. 2012).

Compared to the results in the PLC case, it can be seen from Figure 4 that the fittings with LP EED are significantly improved with $\chi^2 = 0.50$ (for 317 d.o.f.). Besides, relatively better constraints are obtained for all parameters, especially for $s$, $r$, $\gamma_1'$.
and $K'$. Furthermore, the LP model with required $t_{\text{min}} \sim 8$ hr can accommodate the intra-day/night variability observed at GeV–TeV bands. It should be noted that the interpretations of the correlations of parameters in Section 3.1 still hold here.

### 3.3. From Low State to Giant Flare State

After determining the EED in the flare state, we can discuss the EED in the low state by investigating the variations of model parameters with activities. Tramacere et al. (2007, 2009) studied the SEDs of Mrk 421 in different states in the frame of the SSC model and found that there is a negative correlation between the curvature parameter of the radiation spectrum $b \approx r/5$ and $E_\gamma$, which is expected from the stochastic acceleration mechanism. Tramacere et al. (2011) pointed out that the observed negative correlation can be explained by the variation of the momentum diffusion coefficient $D_p(\gamma) = D_p(\gamma/\gamma_0)^{q/\gamma}$, where $q$ is the turbulence spectrum index (note that a larger value of $D_p$ implies a higher acceleration rate), or the fact that the corresponding EEDs are far from equilibrium, where the acceleration dominates over the radiative cooling. They also suggested that the curvature increases as the radiative cooling becomes important and EED is approaching equilibrium during the evolution of EED. From Tables 1 and 2, if we assume that EEDs in low and giant flare states both have LP shapes, we find that $\gamma'_c = 1.47 \times 10^5$, $B = 2.05 \times 10^{-2}$ G, $\delta_D = 34.4$, and $r = 1.68$ in the low state, and $\gamma'_c = 3.63 \times 10^5$, $B = 2.67 \times 10^{-2}$ G, $\delta_D = 33.4$, and $r = 3.79$ in the giant flare state. Therefore, the ratios of synchrotron peak energies and the curvature parameters in two states are $[E_{\gamma,\text{giant}}/E_{\gamma,\text{low}}] = [\gamma'_c^2 B_0]_{\text{giant}}/[\gamma'_c^2 B_0]_{\text{low}} \approx 7.7$ and $[r]_{\text{giant}}/[r]_{\text{low}} \approx 2.3$, which means that $E_\gamma$ increases with $r$. This scenario is not compatible with a purely acceleration-dominated transition, but the increase in the value of the curvature hints that during the high state, the EED is at equilibrium or very close, and that the cooling is dominating over the acceleration. However, many other analyses (e.g., Becker et al. 2006; Stawarz & Petrosian 2008; Tramacere et al. 2011) pointed out that in the high state if the EED is close to or at equilibrium, the PLC EED would fit the SED better, while our results (Figures 3 and 4) show that in the high state the LP EED fits the SED better. Therefore, the above assumption of EEDs in low and giant flare states having LP shapes is not correct.

Alternatively, we still assume that the EEDs in low and giant flare states both have LP shapes and that the different states are caused by the variation of the momentum diffusion coefficient. From Tables 1 and 2, it can be found that $\gamma'_c$ and $r$ increases by a factor of $\sim 2$, while $\delta_D$ and $B$ almost remain constant from the low state to the flare state. Since $\gamma'_c$ is estimated using the condition $t_{\text{acc}}(\gamma) = t_{\text{cool}}(\gamma)$, where $t_{\text{acc}}(\gamma) \propto D_p(\gamma/\gamma_0)^{q/\gamma}$ (Tramacere et al. 2011) is the acceleration time and $t_{\text{cool}}(\gamma) \propto 1/\gamma$ (where the KN effect of IC scattering is neglected) is the radiative cooling time, we then have $\gamma'_c \propto D_p(\gamma_0)^{1-3q}$, and $\gamma'_c \propto D_p(\gamma_0)$ in the hard-sphere approximation ($q = 2$). Hence, $\gamma'_c$ increases with $D_p$. On the other hand, $r$ is inversely proportional to the momentum diffusion coefficient, i.e., $r \propto D_p^{-1}$ (e.g., Tramacere et al. 2011; Massaro et al. 2011). Therefore, the increase of $D_p$ cannot result in the increases of both $\gamma'_c$ and $r$.

Since the SED in the giant state using the LP EED can be fitted better in comparison with that using the PLC EED, and the intra-day/night variability observed at GeV–TeV bands can be accommodated in the LP case (see Section 3.2 and Tables 1 and 2), the EED in the low state may have the PLC shape and the EED in the giant flare state may have the LP shape here.

### 3.4. The EBL Absorption

In this section, we investigate the EBL absorption. As an example we take the SED in the low state. Since the EBL absorption becomes important when $E > \sim 2$ TeV for Mrk 421 ($z = 0.031$), we compare the TeV spectra predicted by both PLC and LP best-fit models, which is shown in the left panel of Figure 5. It can be seen that the TeV fluxes calculated by the PLC and LP best-fit models are different when $E > 2$ TeV. Note that the predicted TeV flux is intrinsic flux, and therefore, the optical depth of TeV photons with energy $E$ for EBL absorption is given by

$$\tau_{\gamma\gamma}(E) = \ln(f_{\text{int}}(E)/f_{\text{obs}}(E)), \quad (3)$$

where $f_{\text{int}}$ is the TeV flux calculated by our best-fit model from optical through GeV and $f_{\text{obs}}$ is the measured TeV flux. This optical depth can then be compared to the optical depth calculated for the various EBL models (e.g., Mankuzhiyil et al. 2010).
We use the two extrapolated intrinsic TeV fluxes (see Figure 5) to calculate $\tau_{\gamma\gamma}$. The results are shown in the right panel of Figure 5. Obviously, the values of $\tau_{\gamma\gamma}$ calculated by using the TeV fluxes derived in the LP model are almost twice those in the PLC model when $E > 2$ TeV. In the right panel of Figure 5, for comparison, we also show $\tau_{\gamma\gamma}$ calculated by three kinds of EBL models: the high level one (e.g., Finke et al. 2010), the middle level one (e.g., Franceschini et al. 2008; Gilmore et al. 2012), and the low level one (e.g., Kneiske & Dole 2010). It can be seen that there are discrepancies not only for the values of $\tau_{\gamma\gamma}$ among various EBL models but also for those among each EBL model and our results. Therefore, when such a method, in which an intrinsic TeV spectrum is obtained from the extrapolation of the best-fit spectrum from the optical through GeV, is used to constrain the EBL models, the emission model of the source should be well determined first, or at least, an alternative model should be taken into account to compare. Here, due to the large uncertainties of the measured TeV data, we cannot obtain meaningful constraints on EBL models or emission models.

As pointed out in Massaro et al. (2006) and Tramacere et al. (2009), our results distinctly show that the curvature in the TeV spectrum can indeed affect the constraints on the EBL models. In our fitting, we do not consider the data above 1 TeV; however, as discussed in Tramacere et al. (2011) the property of the curvature in the TeV spectrum is more complex. Especially in the extreme KN regime, the curvature of the IC emission is larger compared to the Thomson regime, and it is almost equal to that of the EED (Tramacere et al. 2011). Therefore, the value of $r$ may be slightly underestimated in the LP case although $r$ is mainly constrained by synchrotron emissions here, and this TeV flux may be overestimated. Consequently, a subtle bias in $\tau_{\gamma\gamma}$ may be introduced. Therefore, it is suggested that in order to constrain the EBL more precisely, the complete TeV data should be taken into account in the fitting. In this paper, for our purposes and due to the large errors of TeV data, our conclusion on the EBL constraint is still robust.

4. DISCUSSION AND CONCLUSIONS

Using the MCMC method, we study the SED of Mrk 421 in different states for two kinds of EEDs with clear physical meanings: PLC and LP EEDs. Our results indicate that the EED in the giant flare state has the LP shape and the stochastic turbulence acceleration is dominant, while in the low activity state the EED may have the PLC shape and shock acceleration may play a more important role. This giant flare may be attributed to the re-acceleration of these electrons with PLC shape through the stochastic process. Basically, we can understand this process from the low state to the flare state in the scenario proposed by Virtanen & Vainio (2005); electrons are injected at the shock front, and then are accelerated at the shock by the Fermi I process and subsequently by the stochastic process in the downstream region. Here, we specify this scenario according to our results that the Fermi-I-process-accelerated electrons are continuously injected into the emitting blob, and the SSC radiation from the steady EED (PLC shape) in the emitting blob is responsible for the emission of Mrk 421 in the low state. Subsequently, the electrons with PLC shape are re-accelerated by the stochastic process, and the EED with a significant curvature at high energies (LP) is formed, the radiation of which contributes to the emission in the flare state. A requirement in this scenario is that the magnetic field turbulence spectrum must be in the case of the so-called hard-sphere approximation with spectrum index $q = 2$. This is because for cases of Kolmogorov turbulence ($q = 5/3$) and Kraichna ($q = 3/2$) turbulence, acceleration efficiency depends on the electron energy ($\gamma'$), so that the acceleration efficiency for electrons with $\gamma' \sim 10^5$ is very low and the escape would be dominant over the acceleration of electrons (e.g., Becker et al. 2006). This scenario including the acceleration process also can account for the spectra hardening in flaring (e.g., Kirk et al. 1998). This scenario can be examined and the details of the flare can be studied in the time-dependent model. We will study them in the model including the acceleration process (e.g., Kusunose et al. 2000; Katarzyński et al. 2006; Yan et al. 2012) in a subsequent work.

We note that the jet of Mrk 421 appears to be particle dominated (see $U'/U'_0$ in Tables 1 and 2), which is consistent with the result derived by Mankuzhiyil et al. (2011). Finally, we stress the caveat on EBL constraints we derived. The best-fit spectrum from the optical through GeV is often extrapolated into the TeV regime, and is considered to be the intrinsic TeV spectrum. However, our results show that with different emission models, the
best-fit SEDs from the optical through GeV give different TeV spectra. Therefore, it should be considered with caution when such extrapolated TeV spectra are used to constrain the EBL models or the redshift of the source. The accurately measured such extrapolated TeV spectra are used to constrain the EBL spectra. Therefore, it should be considered with caution when best-fit SEDs from the optical through GeV give different TeV 

We thank David Paneque, Amit Shukla, and Justin Finke for sending us the observed data sets we used in this paper and the anonymous referee for his/her very constructive comments. We acknowledge the use of CosRayMC (Li et al. 2012) adapted from the COSMOMC package (Lewis & Bridle 2002). This work is partially supported by the 973 Programme (2009CB824800) and by the Yunnan Province under a grant 2009 OC. Q.Y. acknowledges the support of National Natural Science Foundation of China under grant No. 10963004. Z.H.F. acknowledges the support of National Natural Science Foundation of China under grant No. 10963004.

REFERENCES

Abdo, A. A., Ackermann, M., et al. 2010, ApJ, 710, 1271
Abdo, A. A., Ackermann, M., Ajello, M., et al. 2011, ApJ, 736, 131
Acciari, V. A., Aliu, E., Aune, T., et al. 2009, ApJ, 703, 169
Acciari, V. A., Aliu, E., Aune, T., et al. 2011, ApJ, 738, 25
Aharonian, F., Akhperjanian, A. G., Anton, G., et al. 2009, A&A, 502, 749
Ackermann, M., Ajello, M., Allafort, A., et al. 2011, ApJ, 743, 171
Albert, J., Aliu, E., Anderhub, H., et al. 2007, ApJ, 663, 125
Aleksić, J., Anderhub, H., Antonelli, L. A., et al. 2010, A&A, 519A, 32

Galante, N. VERITAS Collaboration 2011, arXiv:1109.6059
Gamerman, D. 1997, Markov Chain Monte Carlo: Stochastic Simulation for Bayesian Inference (London: Chapman and Hall)

Graff, P. B., Georganopoulos, M., Perlman, E. S., & Kazanas, D. 2008, ApJ, 689, 68

Kataoka, J., Takahashi, T., Makino, F., et al. 2000, ApJ, 528, 243
Katarzyński, K., Ghisellini, G., Mastichiadis, A., Tavecchio, F., & Maraschi, L. 2006, A&A, 453, 47
Katarzyński, K., & Walczewska, K. 2010, A&A, 510A, 63
Kirk, J. G., Rieger, F. M., & Mastichiadis, A. 1998, A&A, 333, 452
Kneiske, T. M., & Dole, H. 2010, A&A, 515A, 19
Kusunose, M., Takahara, F., & Li, H. 2000, ApJ, 536, 299

Lewis, A., & Bridle, S. 2002, PhilD, 66, 103511
Liu, J., Yuan, Q., Bi, X. J., Li, H., & Zhang, X. M. 2012, PhilD, 85, d3507
Mackay, D. J. C. 2003, Information Theory, Inference and Learning Algorithms (Cambridge: Cambridge Univ. Press)
Mankuzhiyil, N., Ansoldi, S., Persic, M., & Tavecchio, F. 2011, ApJ, 733, 14
Mankuzhiyil, N., Persic, M., & Tavecchio, F. 2010, ApJL, 715, L16
Mannheim, K. 1993, A&A, 269, 67
Maraschi, L., Ghisellini, G., & Celotti, A. 1992, ApJL, 397, L5
Massaro, E., Perri, M., Giommi, P., & Nesci, R. 2004a, A&A, 413, 489
Massaro, E., Perri, M., Giommi, P., & Nesci, R., & Verrecchia, F. 2004b, A&A, 422, 103
Massaro, E., Tramacere, A., Perri, M., Giommi, P., & Tosti, G. 2006, A&A, 488, 861
Moraitis, K., & Mastichiadis, A. 2011, A&A, 525A, 40
Mücke, A., Protheroe, R. J., Engel, R., et al. 2003, Ap, 18, 593
Neal, R. M. 1993, Probabilistic Inference Using Markov Chain Monte Carlo Methods (Canada: Department of Computer Science, Univ. Toronto)

Per, M., Akeroj, C. W., Crawley, M. F., et al. 1992, Natur, 358, 477
Rafferty, A. E., & Lewis, S. M. 1992, StaSc, 7, 493
Raue, M., et al. the HESS Collaboration 2012, JPhCS, 355, 2001
Rees, M. J. 1967, MNRAS, 137, 429
Shukla, A., Chitnis, V. R., Vishwanath, P. R., et al. 2012, A&A, 541A, 140
Sikora, M., Begelman, M. C., & Rees, M. J. 1999, ApJ, 421, 153
Sokolov, A., Marscher, A. P., & McHardy, I. M. 2004, ApJ, 613, 725
Stawarz, Ł., & Petrosian, V. 2008, ApJL, 681, 1725
Tavecchio, F., Maraschi, L., & Ghisellini, G. 1998, ApJ, 509, 608
Tramacere, A., Massaro, F., & Cava, A. 2007, ApJ, 666, 21
Tramacere, A., Giommi, P., Perri, M., Verrecchia, F., & Tosti, G. 2009, A&A, 501, 879
Tramacere, A., Massaro, E., & Taylor, A. M. 2011, ApJ, 739, 66

Ushio, M., Stawarz, Ł., Takahashi, T., et al. 2010, ApJ, 724, 1509
Virtanen, J., & Vainio, R. 2005, ApJ, 621, 313
Yan, D. H., Zeng, H. D., & Zhang, L. 2012, MNRAS, 424, 2173