A Quasi-staggered Scheme on Lattice

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Abstract Utilizing a picture of string and string spinors, we show a simpler version of staggered action. The advantage of this action is that in this action there always exist pair of quarks with different masses.
1 Introduction

There are different actions on the fermion lattice simulation, such as Wilson action\[2\], overlapped action\[3\], SLAC action\[4\]. The staggered action, proposed by Kogut and Susskind\[5\], stands in these actions. It is\[5, 6, 7\], of course, a very interesting action for its elegant treatment in solving the fermion doublers. There are arguments that it is really QCD\[8\]. However, a drawback of this action is that there are four degenerate fermions in this action. How to reduce the number of the degenerate fermion in dimension 4 lattice is a special topic on this action. Besides this, the spin-isospin symmetry is also an unexpected symmetry of staggered action, for this symmetry does not occur in standard continuum QCD.

the modern simulation to decrease the number of fermion in staggered action is based on the "rooting trick"\[5\]. However, the validity of "rooting trick" is under discussion. For instance, a fractional power of the determinant is, in general, not a legitimate operation in quantum field theory. Furthermore, the locality is not obvious in this trick.

Gamma matrix, particularly, \(\gamma_i\), can be written as direct product of two two-by-two matrices, \(\tau_A \otimes \tau_i\), where \(\tau_i\) is Pauli matrix and \(\tau_A\) is a fixed Pauli matrix, such as \(\tau_1, \tau_2\) and \(\tau_3\). After \(\tau_A\) has been adopted, we can set \(\gamma_0\) and \(\gamma_5\) as \(\tau_B \otimes \mathbf{1}_{2\times2}\) and \(\tau_C \otimes \mathbf{1}_{2\times2}\) respectively, provided \(A \neq B \neq C\). We shall use this interesting property of gamma matrices and a picture of string spinors, the notation of which is shown in the context, to make an attempt to reduce the fermion number in staggered action. The scheme adopted here is called as quasi-staggered scheme.

Section 2 is a list of results of structure of algebra which will been used in the scheme. The detail discussions of the quasi-staggered scheme are shown in section 3. Section 4 shows some basic properties of string spinor. Then we give a summary in section 5.

2 The algebraic structure of the two-component theory

Consider a four dimensional lattice theory with lattice spacing \(a_s = a_t = 1\). We assume that the lattice, which is divided into \(N^4\) grids, has a periodic conditions here for simplification.

On this lattice we have \(N^4\) string \(n = (n_0, n_1, n_2, n_3)\), in which we often define four points \(x(n) = (x_0, x_1)\). For these four points \(x\)'s, \((0, 0), (1, 0), (0, 1)\) and \((1, 1)\) stand for points \(n, n + b_0, n + b_1\) and \(n + b_0 + b_1\) respectively, where \(b_0 = h a_0/2\) and \(b_1 = \sum_{i=1}^{3} h a_i/2\) (At the moment we set parameter \(h = 1\)). To distinguish points belonging to the same string sometimes we also use \((n, x)\) to represent this points, for instance, \((n, (1, 0))\) stands for \(n + b_0\). Two-component spinors/tastes \(\varphi(x)\) and \(\bar{\varphi}(x)\) are defined on each of the four points.

Furthermore,

\[
c_0(x) = 1, \quad c_1(x) = (-1)^{x_0}
\]

are also defined on the link between points \(x\) and \(x + \vec{x}_0\) or between points \(x\) and \(x + \vec{x}_1\) (There are ambiguities in \(c_1\) when \(x_1 = 1\). In fact, all the variable links between points \(n + b_1\), i.e. \((n, (0, 1))\), and \(n + a_1, n + a_2\) or \(n + a_3\) are described as \(c_1((0, 1))\). The case is similar for \(c((1, 1))\). The importance is that \(c_1\) does not depend on \(x_1\). We shall show that this ambiguity is irrelevant in the next section).

We also define a set of linear operators \(\Gamma_\rho\) (\(\rho = 1\) or \(2\)) which transform the subspace of functions \(\varphi(x)\) associated to sting \(n\) into itself:

\[
\Gamma_\rho \varphi(x) = c_\rho(x) \varphi(x + x_\rho), \quad x_\rho \text{ even},
\]
\[ \Gamma_\rho \varphi(x) = c_\rho (x - x_\rho) \varphi(x - x_\rho), \quad x_\rho \text{ odd}, \quad (2) \]

or

\[ \Gamma_\rho \varphi(x) = c_\rho (x) \varphi(x + (-)^{x_\rho}x_\rho). \quad (3) \]

Immediately, we have

\[ \Gamma_\rho^2 = 1, \quad \{ \Gamma_\rho, \Gamma_\sigma \} = 2 \delta_{\rho \sigma}. \quad (4) \]

We also define 2-by-2 matrices, for instance,

\[ \lambda_1 = \tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \lambda_0 = \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \lambda_5 = \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \quad (5) \]

This definitions are just for convention, other sequence of Pauli matrices is also suitable. Correspondingly, gamma matrices are defined as

\[ \gamma_i = \tau_1 \otimes \tau_i = \begin{pmatrix} 0 & \tau_i \\ \tau_i & 0 \end{pmatrix}, \quad \gamma_0 = \tau_3 \otimes 1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma_5 = \tau_2 \otimes 1 = \begin{pmatrix} 0 & -i1 \\ i1 & 0 \end{pmatrix}. \quad (6) \]

Choose an arbitrary path \( b_{\mu_1} + b_{\mu_2} + \cdots + b_{\mu_k} \) joining \( x^f(n) = (0,0) \) to \( x(n) \), i.e.

\[ x = x_{\mu_1} + x_{\mu_2} + \cdots + x_{\mu_k}, \quad (7) \]

we define \( p(x) \) as

\[ p(x) = c_{\mu_1}(x^f)\lambda_{\mu_1}c_{\mu_2}(x^f + x_{\mu_1})\lambda_{\mu_2} + \cdots + c_{\mu_k}(x - x_{\mu_k})\lambda_{\mu_k}. \quad (8) \]

Explicitly,

\[ p((0,0)) = 1, \quad p((1,0)) = \lambda_0, \quad p((0,1)) = \lambda_1, \quad p((1,1)) = -\lambda_0\lambda_1 = -i\lambda_5. \quad (9) \]

It is easy to verify that

\[ p(x)^{\alpha}_\beta p(y)^\beta_\alpha = 2\delta_{xy}, \quad \lambda_\rho \lambda_5 p(x)\lambda_5 \lambda_\rho = (-)^{x_\rho}p(x). \quad (10) \]

The algebra listed here is in fact a 2-dimensional version of the 4-dimensional algebra shown in reference [6]. Therefore we only list the results here, For the detail discussions, especially for the discussions in the case of four dimension, we refer to reference [6].

3 Formulae of the staggered action

Before the discussions of staggered action we should code the four \( \varphi(x) \)'s into a matrix \( \Psi \) associated to \( n \),

\[ \Psi^\alpha_\beta(n) = \frac{1}{\sqrt{2}} \sum_{x \in n} \varphi(x)p^\alpha_\beta(x), \quad \Psi^\beta_\alpha(n) = \frac{1}{\sqrt{2}} \sum_{x \in n} p^\beta_\alpha(x) \varphi(x). \quad (11) \]

\( \Psi \)'s are called string spinor thereinafter. Then

\[ \varphi(x) = \frac{1}{\sqrt{2}} \Psi^\alpha_\beta(n)p^\beta_\alpha(x), \quad \bar{\varphi}(x) = \frac{1}{\sqrt{2}} \Psi^\beta_\alpha(n)p^\alpha_\beta(x). \quad (12) \]
It should be emphasized here that $\Psi(n)$ is a $4 \times 2$ matrix since $\varphi(x)$’s are two-component spinors. The explicit forms of $\Psi$ and $\varphi(x)$ are

$$
\Psi(n) = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi((0,0)) + \varphi((1,0)) & \varphi((0,1)) + \varphi((1,1)) \\ \varphi((0,1)) - \varphi((1,1)) & \varphi((0,0)) - \varphi((1,0)) \end{pmatrix}, \tag{13}
$$

and

$$
\varphi((0,0)) = \frac{\Psi_1^1 + \Psi_2^3}{\sqrt{2}}, \quad \varphi((1,0)) = \frac{\Psi_1^1 - \Psi_2^3}{\sqrt{2}}, \quad \varphi((0,1)) = \frac{\Psi_1^2 + \Psi_2^4}{\sqrt{2}}, \quad \varphi((1,1)) = \frac{\Psi_1^2 - \Psi_2^4}{\sqrt{2}} \tag{14}
$$

respectively. For $\bar{\Psi}$ and $\bar{\varphi}$ the formulae are similar.

To translate the two-component theory into ordinary algebra of four-component spinors $\psi_i$, $i = 1, 2$, we define $\psi_i = \Psi_i$ and $\bar{\psi}^i = \bar{\Psi}^i$. Therefore,

$$
\bar{\psi}^i(n) \psi_i(n) = \Psi_j^j \Psi_k^k = \sum_{x \in n} \bar{\varphi}(x) \varphi(x),
$$

$$
\bar{\psi}^i(n) \psi_j(n) \lambda_{0ji} = \bar{\Psi}_k^i \Psi_k^j \lambda_{0ji} = \sum_{x \in n} \bar{\varphi}(x) \varphi(x + (-)^x \bar{x}_0)(-)^x. \tag{15}
$$

This equation is crucial. As we know, staggered action introduces spinor⊗taste interactions, which will lead to a spontaneous breaking of taste symmetry. However, these interactions are at higher order and we expect the symmetry breaking is not large, i.e. all the considered tastes are almost degenerate. In other words, it is difficult in standard staggered action to simulate quarks with different masses. If we ignore the spinor⊗taste interactions and weak interactions which may lead to admixture of different quarks, mass matrix $M$ with two taste will have a form of diagonal 2-by-2 matrix. Suppose $M$ can be decomposed as

$$
M = m_0 1_{2 \times 2} + m_1 \lambda_0,
$$

equations in (15) supplies a obvious way to simulate quarks with different masses, for the second equation in (15) can produce mass splitting of the two quark. The disadvantage of equation (15) is that to simulate quarks with different masses one should use two-component spinors at different position in the same string. After we introduce gauge fields, link variables which connect two different points, it seems that one should insert non-trivial link variables between this two-component spinors in the same string. We shall discuss this topic after equation (15).

The kinetic part of the action connects the $\Psi$’s associated with two different strings through the difference $\varphi(x + a_\mu) - \varphi(x - a_\mu)$. For difference operators, we define

$$
\nabla_\mu \Psi(n) = \frac{1}{2}(\Psi(n + a_\mu) - \Psi(n - a_\mu)),
$$

$$
\triangle_\mu \Psi(n) = \frac{1}{2}[\Psi(n + a_\mu) + \Psi(n - a_\mu) - 2\Psi(n)]. \tag{16}
$$

Then it is easy to verify

$$
I^0 = \text{Tr}[\bar{\Psi}(n) \gamma_0 \nabla_0 \Psi(n)] = \text{Tr}[\lambda_0 \lambda_5 \bar{\Psi}(n) \gamma_5 \triangle_0 \Psi(n)] \triangleq I_1^0 + I_2^0 + I_3^0
$$

$$
= \sum_x c_0(x)(-)^{x0} \bar{\varphi}(n,x) [\varphi(n, x + (-)^{x0} \bar{x}_0) - \varphi(n - (-)^{x0} a_0, x + (-)^{x0} \bar{x}_0)], \tag{17}
$$

where

$$
I_1^0 = \frac{1}{2} \text{Tr}[\bar{\Psi}(n) \gamma_0 \Psi(n + a_0) - \lambda_0 \lambda_5 \bar{\Psi}(n) \lambda_5 \Psi(n + a_0)]
$$
\[
\begin{align*}
I_2^0 &= \frac{1}{2} \text{Tr}[-\bar{\Psi}(n)\gamma_0\Psi(n-a_0) - \lambda_0\lambda_5\bar{\Psi}(n)\lambda_5\Psi(n-a_0)] \\
&= -[\bar{\varphi}(n, (0, 1))\varphi(n-a_0, (1, 1)) + \bar{\varphi}(n, (0, 0))\varphi(n-a_0, (1, 0))], \\
I_3^0 &= \frac{1}{2} \times 2\text{Tr}[\lambda_0\lambda_5\bar{\Psi}(n)\gamma_5\Psi(n)] \\
&= \bar{\varphi}(n, (0, 1))\varphi(n, (1, 1)) + \bar{\varphi}(n, (0, 0))\varphi(n, (1, 0)) \\
&- \bar{\varphi}(n, (1, 1))\varphi(n, (0, 1)) - \bar{\varphi}(n, (1, 0))\varphi(n, (0, 0)).
\end{align*}
\] (18)

Notice that interacting two-component spinors in \(I_3^0\) belong to the same string. That is, this interaction between spinors is self interaction of the string spinor \(\Psi\). Here we meet the same puzzle as we treat mass problem in equation (15). For interactions between two two-component spinors, if these two two-component spinors belong to the same string, they correspond to couplings between different components in the same string spinor. We can regard the string spinor as two ordinary four-component spinors at the same spatial-time point, then such couplings are in fact the ones between components of ordinary spinors in the same spatial-time point. We shall call such couplings as self-couplings thereinafter. For instance, in equation (15), the couplings are \(\bar{\varphi}(n, (0, 1))\varphi(n-a_0, (1, 1))\varphi(n-a_0, (1, 0))\). We can regard the string spinor as two ordinary four-component spinors at the same spatial-time point, then such couplings are in fact the ones between components of ordinary spinors in the same spatial-time point. We shall call such couplings as self-couplings thereinafter. For instance, in equation (15), the couplings are \(\bar{\varphi}(n, (0, 1))\varphi(n-a_0, (1, 1))\varphi(n-a_0, (1, 0))\).

This property of self-coupling makes that the value of \(h\) in the definition of \(b_0\) and \(b_1\) irrelevant. This can be seen that all spinors connected by self-couplings, which are irrelevant to covariant difference operators, will be interpreted as components of the two ordinary entangled four-component spinors, since they belong to the same string. Therefore, we can choose \(h \to 0\) in the simulation. After such choice, the four points in the string, represented by \(x\), will tend to the same point \(n\). Since that, we can always choose link variables, which connect these four points in the same string, as unitary. In other words, the insertion of gauge fields in self-coupling is not needed.

The needlessness of the insertion can also be seen as follows. In the staggered action, quarks reflect the movement of string spinors and they can not be regarded as point particles. In this sense parameter \(h\) determines the sizes of expansions of quarks. On the contrary, in standard QCD, quarks are treated as point particles and they have no inherent structure. Therefore, finite parameter \(h\) in staggered action presents the deviation from standard QCD. At fixed lattice spacing, the more larger \(h\) is, the more sharper the deviation becomes. However, at the fashion scalar, this deviation should be very small, had this deviation existed. Therefore, it seems that \(h \ll 1\) in the simulation, due to the experiments. This constraint makes that the gauge field between points connected by self-coupling are weaker than the gauge field between points connected by other interactions, because of the asymptotic behavior of QCD. In other words, the insertion of gauge fields between point connected by self-coupling is needless.

In this scheme, therefore, for each string, four \(x\)'s are only used to distinguish different two-component spinors. In other words, \(x\)'s belong to inner space and they are irrelevant to spatial-time points.

Since two-component spinors belong to the same string spinor (matrix) are in the same
lattice simulations. At this case, especially for including self-couplings, even only for maturity of the ory consideration or for the future to set large tadpole correction. Also perform a tadpole improvement here, since the insertion of three link variables leads to smearing process, which is commonly used in many lattice simulations. Surely one should between these points. One should make an average between these insertions. This is just a case we need insert three link variables between these points. However, there are eight variable between these point. The second is the coupling between points with different spatial such as coupling between (\(i\), (1, 0)) \(\tau_i\) \(\varphi(n, (1, 0))\) \(\tau_i\) \(\varphi(n, (1, 1))\) - \(\varphi(n, (0, 1))\) \(\tau_i\) \(\varphi(n, (0, 1))\).

\[ I_1 = \frac{1}{2} \sum_{i \in \mathbb{Z}} \varphi(n, x) c_i \tau_i \varphi(n, x + \vec{x}_i) \]

Similarly, if we define \(\varphi(n, x) = \varphi(n \pm a_i, (0, x))\) in \(I^0\), \(I^i\) can also be written as

\[ I^i = \sum_{x \in \mathbb{Z}} \varphi(n, x) c_i \tau_i \varphi(n, x + \vec{x}_i) \]

This two equations are very similar to the standard staggered action. Utilizing equation (15) - (22), one can easily construct action, in which there are two fermions with different masses. In the form of string spinors, the action is as

\[ S = \sum_n Tr[m_0 \bar{\Psi}(n)\Psi(n) + m_1 \bar{\Psi}(n)\Psi(n) + \bar{\Psi}(n)\lambda_{\mu} \nabla_{\mu} \Psi(n) - \lambda_{\mu}(a_1, \mu) \lambda_{\mu} \bar{\Psi}(n)\Psi(n), \]

where \(\lambda_{\mu} = \bar{\Psi}(n)\lambda_5 \Psi(n)\lambda_5 \nabla_{\mu} \Psi(n)\]
4 Basic dynamics of string spinors

When $h \to 0$, we have a string structure in each "point" (string) in this scheme. In each "point" of QCD, there is a curling string, which connected four two-component spinors. Since the string is very small, it seems that there exists interaction between these spinors. However, the force should decay very sharply with the increase of distance, for the interaction between spinors on string is adjoining. An extreme case is that we choose $h = 0$ directly. At this time the string is living on an extra dimension.

This interaction is not QCD. We first notice there is a basic symmetry of this interaction. That is, for $I_0$ and $I_i$, there is a symmetry under the following discrete transformation,

$$
\begin{align*}
\Psi(n) & \to i\gamma_5 \Psi(n) \lambda_5, \\
\bar{\Psi}(n) & \to i\lambda_5 \bar{\Psi}(n) \gamma_5.
\end{align*}
$$

(24)

Notice that the mass splitting term in (15) is also invariant under this transformation. We are able to rewrite this transformation in the next form,

$$
\begin{align*}
\varphi((1, 0)) & \to -\varphi((1, 0)), \varphi((0, 1)) \to -\varphi((0, 1)), \\
\bar{\varphi}((0, 0)) & \to -\bar{\varphi}((0, 0)), \bar{\varphi}((1, 1)) \to -\bar{\varphi}((1, 1)),
\end{align*}
$$

(25)

with other variables, $\varphi((1, 1)), \varphi((0, 0)), \bar{\varphi}((1, 0))$, and $\bar{\varphi}((0, 1))$, invariance. Or

$$
\begin{align*}
\varphi((1, 1)) & \to -\varphi((1, 1)), \varphi((0, 0)) \to -\varphi((0, 0)), \\
\bar{\varphi}((0, 1)) & \to -\bar{\varphi}((0, 1)), \bar{\varphi}((1, 0)) \to -\bar{\varphi}((1, 0)),
\end{align*}
$$

(26)

with other variables invariance.

At both case ($h \to 0$ and $h = 0$), the string spinor, $\Psi$, has a inherent structure. To describe this structure one should find the dynamical variables of the string and spinors and the Lagrangian of dynamical variables. One may choose the four tastes described as $\varphi(x)$ for fermion freedom. To study dynamics of fermion one should insert interaction between them, which is a gauge interaction determined by boson freedom. Notice the role of $c_\mu$ in equations (21) and (22), one may consider that $c_\mu$ just reflects this interaction. In other words, this interaction is possibly described by $U(1)$ theory on the string, the topology of which is also $U(1)$.

The Lagrangian, which describes movement dynamics of $\varphi(x)$’s and $A_\rho$’s ($A_\rho$ are defined through link variables, $c_\rho = e^{i \int_{\text{path}} A_\rho}$), should satisfy the constraint shown in equations (25) and (26). This means there exist only adjoining interactions between this four tastes/spinors on the string. The interaction which connects tastes on diagonal points, such as interaction $\bar{\varphi}((0, 0))\varphi((1, 1))$, can not occur.

Since $c_\rho = e^{i \int_{\text{path}} A_\rho}$ is a link variable which connects the adjoining tastes, it seems that the value of $c_\rho$ is arbitrary, provided it satisfies $c_\rho(x)c_\rho^*(x) = 1$. It is obvious that one can always choose a gauge to make $c_\rho$ satisfy equation (11). Therefore, the choice of $c_\rho$ in equation (11) can be regarded as a special gauge fixing on $c_\rho$. However, a gauge independent variable

$$
c_{\text{string}} = c_1((0, 0))c_0((0, 1))c_1^*((1, 0))c_0^*((0, 0)) = -1
$$

(27)

for string implies that boson freedom $A_\rho$ is not arbitrary fluctuating. In fact, this identity implies that for each string (plaquette) we have

$$
\oint d\mathbf{l} \cdot \mathbf{A} \equiv (2k + 1)\pi,
$$

(28)
where $k$ is an integer. Since $A_\rho$ plays as a phase factor of $c_\rho$ and the couplings between different $\varphi$ is not $A_\rho$ but $c_\rho$, different $k$ corresponds to the same dynamics of string. We choose $k = 0$ here. Therefore, there exists a constraint on boson freedom $A_\rho$.

Suppose the string is living on an extra dimension. Since the interaction is $U(1)$ gauge theory defined on manifold $U(1)$, it is interesting to study the topology behavior of $A_\rho$. Then it is natural to define winding number, which is topological invariant under infinitesimal gauge transformation, in the manifold $U(1)$ as

$$n = \frac{1}{2\pi} \oint d1 \cdot A = \frac{1}{4\pi} \int_{\text{string}} ds \epsilon_{\rho\lambda} F_{\rho\lambda},$$

(29)

where the second integrand is over the area encircled by the string in inner space and $F_{\rho\lambda} = \partial_\rho A_\lambda - \partial_\lambda A_\rho$, $\rho, \lambda = 1, 2$. The choice of $k = 0$ in equation (28) means that $n = \frac{1}{2}$. There is no connection between $c_\rho$ and topological quantity, winding number, in standard staggered action, whereas in our scheme there exists a deep connection between them. We think that the dynamics of the string, especially the nontrivial topology behavior, reveals that it is worth studying furthermore. For instance, whether there exists a relation between nontrivial topology of the string and the broken of chiral symmetry and how the relation occurs, if the existent is positive.

There is subtlety in rotation. In standard staggered action they rotation symmetry is very complex. On lattice this symmetry includes not only the usual hypercubic group, the subgroup of the continuous rotation group, but also the spin-isospin mixing $[10]$. The hypercubic symmetry guarantees that the rotation symmetry is restored at the limit lattice spacing $a \to 0$ (Suppose we only give our attention to the rotation in spatial), while the other part of the symmetry, the spin-isospin symmetry, does not occur in standard QCD. There is a small symmetry in our action. Particularly, it seems that the system is not invariant under transformations in cubic group. However, as revealed in reference [10], $c_\rho$ is spin-zero field.

We furthermore consider that the string, represented by series of $c_\rho$'s, is also invariant under the rotation, including its subgroup, cubic group. It is more obvious if we think the string is living on extra dimension. Since the string, especially, the orient of string, is invariant under rotation, the relative positions of the four tastes are also invariant, that is, there is no spin-isospin mixing under cubic transformation in our action. Therefore, the only symmetry of our action is the cubic group, which excludes the spin-isospin mixing. This can also be seen in action (23). If the cubic rotations exclude the interchange of spin-isospin, that is, $\psi^1$ can not change to $\psi^2$, or vice versa, under cubic transformation (This means that, $\psi^1$ and $\psi^2$ transform as a independent four-component spinor under cubic transformation), the system is invariant under the cubic group. The cubic group guarantees that the continuous rotation invariant is restored in the continuum limit. In this scheme we have discarded the spin-isospin mixing symmetry which does not occur in standard QCD.

In summary, each point of QCD is described as a string, on which live four fermion tastes. Boson freedom, $A_\rho$ (or $c_\rho$), and fermion freedom, four fermion tastes $\varphi(x)$, make up of the complete variables of string. But fermion freedom and boson freedom themselves are not observable quantity. Observable quantities are described as the dynamics of these freedoms. For instance, the two types of quarks are determined by the dynamics of the variables.
5 Summary

In this note we show a quasi-staggered action, which preserves the cube symmetry. This action regards that the point of QCD has an inherent structure. Each point of QCD is in fact a string, which connects four interacting tastes with spin 1/2. However, these tastes themselves are not observed spinors. The observed fermions (quarks) are the eigen-models of the dynamics of the tastes. One byproduct is that quarks should be in pair in this action. However, it is not needed to require quarks, which occur in pair, be degenerate, that is, masses of the pair of quarks can be different. This is a significant property of this action. The other byproduct is that the spin-isospin interchange symmetry does not occur in this scheme.

The string structure of the "point" of vacuum in this action is an amusing picture. We possibly meet a bridge between staggered action in QCD and a more modern physics, string or superstring theory\[9\]. It is interesting to compare the similarities and differences between the picture of string adopted here and that in standard string theory.

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