Quark model description of the $NN^*(1440)$ potential

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We derive a $NN^*(1440)$ potential from a non-relativistic quark-quark interaction and a chiral quark cluster model for the baryons. By making use of the Born-Oppenheimer approximation we examine the most important features of this interaction in comparison to those obtained from meson-exchange models.

1. INTRODUCTION

Baryonic resonances play a major role in the understanding of reactions that take place in nucleons and nuclei in the so-called intermediate energy regime. In particular, the low-lying nucleonic resonances $\Delta(1232)$ and $N^*(1440)$, can be now analyzed in more detail due to the development of specific experimental programs in TJNAF, Uppsala...

In this context the transition, $NN \rightarrow NR$ ($R$ : resonance), and direct $NR \rightarrow NR$ and $RR \rightarrow RR$ interactions should be understood. Usually these interactions have been written as straightforward extensions of some pieces of the $NN \rightarrow NN$ potential with the modification of the values of the coupling constants, extracted from their decay widths. Though this procedure can be appropriate for the very long-range part of the interaction, it is under suspicion at least for the short-range part for which the detailed structure of the baryons may determine to some extent the form of the interaction. This turns out to be the case for the $NN \rightarrow N\Delta$ and $N\Delta \rightarrow N\Delta$ potentials previously analyzed elsewhere. It seems therefore convenient to proceed to a derivation of these potentials based on the more elementary quark-quark interaction.

This is the purpose of this talk: starting from a quark-quark non-relativistic interaction, we implement the baryon structure through technically simple gaussian wave functions and we calculate the potential at the baryonic level in the static Born-Oppenheimer approach. The $N^*(1440)$, the Roper resonance, considered as a radial excitation of the nucleon, is taken as a stable particle. For dynamical applications its width should be implemented through the coupling to the continuum.

We center our attention in the $NN^* \rightarrow NN^*$ potential where a complete parallelism with the $NN \rightarrow NN$ case can be easily established. Notice that the quark-quark interaction parameters are fixed (from the $NN \rightarrow NN$ case) and are kept independent of the baryons involved in the interaction. This eliminates the bias introduced in models at the
baryonic level by a different choice of effective parameters according to the baryon-baryon interaction considered (this effectiveness of the parameters may hide distinct physical effects).

2. THE $N N^*(1440)$ WAVE FUNCTION

The wave function of a two-baryon system, $B_1$ and $B_2$, with a definite symmetry under the exchange of the baryon quantum numbers is written as [3]:

$$\Psi^S_{B_1B_2}(\vec{R}) = \frac{A}{\sqrt{1 + \delta_{B_1B_2}}} \sqrt{\frac{1}{2}} \left\{ B_1 \left( 123; -\frac{\vec{R}}{2} \right) B_2 \left( 456; \frac{\vec{R}}{2} \right) \right\}_{ST} + (-1)^f \left\{ B_2 \left( 123; -\frac{\vec{R}}{2} \right) B_1 \left( 456; \frac{\vec{R}}{2} \right) \right\}_{ST},$$

being $A$ the six-quark antisymmetrizer given by:

$$A = (1 - \sum_{i=1}^{3} \sum_{j=4}^{6} P_{ij})(1 - \mathcal{P}),$$

where $\mathcal{P}$ exchanges the three quarks between the two clusters and $P_{ij}$ exchanges quarks $i$ and $j$.

If one projects on a state of definite orbital angular momentum $L$, due to the $(1 - \mathcal{P})$ operator in the antisymmetrizer the wave function $\Psi^S_{B_1B_2}(\vec{R})$ vanishes unless:

$$L + S_1 + S_2 - S + T_1 + T_2 - T + f = \text{odd.}$$

Since $S_1 = \frac{1}{2} = S_2$, $T_1 = \frac{1}{2} = T_2$, this fixes the relative phase between the two components of the wave function at Eq. (1) to be:

$$f = S + T - L + \text{odd.}$$

It is important to realize that for the $NN$ system $f$ is necessarily even in order to prevent the vanishing of the wave function. No such restriction exists for $NN^*$. Therefore, there are $NN^*$ channels ($f$ odd) with no counterpart in the $NN$ case.

We will assume the three-quark wave function for the quark clusters at a position $\vec{R}$ to be given by

$$|N\rangle = |[3](0s)^3\rangle,$$

$$|N^*\rangle = \sqrt{\frac{2}{3}}|[3](0s)^2(1s)\rangle - \sqrt{\frac{1}{3}}|[3](0s)(op)^2\rangle,$$

explicitly,

$$N(\vec{r}_1, \vec{r}_2, \vec{r}_3; \vec{R}) = \prod_{n=1}^{3} \left( \frac{1}{\pi b^2} \right)^{3/4} e^{-\frac{(\vec{r}_n - \vec{R})^2}{2b^2}} \otimes [3]_{ST} \otimes [1^3]\rangle,$$
and

\[ N^*(\vec{r}_1, \vec{r}_2, \vec{r}_3; \vec{R}) = \left( \sqrt{\frac{2}{3}} \phi_1 - \sqrt{\frac{1}{3}} \phi_2 \right) \otimes [3]_{ST} \otimes [1^3]_{C} \, , \]  

(8)

being

\[ \phi_1 = \frac{\sqrt{2}}{3} \left( \frac{1}{\pi b^2} \right)^{9/4} \sum_{k=1}^{3} \left[ \frac{3}{2} - \frac{(\vec{r}_k - \vec{R})^2}{b^2} \right] \prod_{i=1}^{3} e^{-\frac{(\vec{r}_i - \vec{R})^2}{2b^2}} \, , \]  

(9)

and

\[ \phi_2 = -\frac{2}{3} \left( \frac{1}{\pi b^2} \right)^{9/4} \sum_{j<k=1}^{3} (\vec{r}_j - \vec{R}) \cdot (\vec{r}_k - \vec{R}) \prod_{i=1}^{3} e^{-\frac{(\vec{r}_i - \vec{R})^2}{2b^2}} \, , \]  

(10)

where \([3]_{ST}\) and \([1^3]_{C}\) stand for the spin-isospin and color part, respectively. The validity of the harmonic oscillator wave functions to calculate the two-baryon interaction has been discussed in ref. \[4\].

The quark-quark potential we use can be written in terms of the interquark distance \(\vec{r}_{ij}\) as:

\[ V_{qq}(\vec{r}_{ij}) = V_{CON}(\vec{r}_{ij}) + V_{OGE}(\vec{r}_{ij}) + V_{OPE}(\vec{r}_{ij}) + V_{OSE}(\vec{r}_{ij}) \, , \]  

(11)

where \(V_{CON}\) stands for the confining potential, and \(V_{OGE}\), \(V_{OPE}\), and \(V_{OSE}\) for one-gluon, one-pion and one-sigma exchange potentials, respectively. The expression of these potentials has been very much detailed elsewhere \[5\].

The baryon-baryon potential is obtained as the expectation value of the energy of the six-quark system minus the self-energy of the two clusters. The presence of the antisymmetrization in the two-baryon wave function has also an important dynamical effect, the baryon-baryon potential contains quark-exchange contributions where the interaction takes place between two baryons that exchange a quark.

3. RESULTS

In Figure \[3\] we show the results for the \(NN^*\) potential in terms of the interbaryon distance \(R\) for two channels: \(^1S_0(T = 0)\), which is forbidden in the \(NN\) system, and the \(^1S_0(T = 1)\), which is allowed in the \(NN\) system. In this last case, the result is quite close to the corresponding channel in the \(NN\) system, a consequence of the near to identity similarity of \(N\) and \(N^*\). As can be seen, the behavior in the two previous channels is completely different such that it could not be obtained by a simple rescaling of the vertex coupling constants from one case to the other. In order to emphasize the effects of quark antisymmetrization, we have compared to a direct potential without quark-exchange contributions. We have also separated the contribution of the different terms of the quark-quark potential in Eq.

As general features of the results we may remark that the OPE interaction determines the very long range behavior (\(R > 4 \text{ fm}\)), the OPE altogether with the OSE are responsible for the long-range par (1.5 \(\text{ fm} < R < 4 \text{ fm}\), and OPE, OSE and OGE added to quark-exchange determine the attractive or repulsive character of the interaction at the intermediate- and short-range.
Certainly data on $NN^* \rightarrow NN^*$ phase shifts can be only obtained indirectly and no direct experimental test of our results can actually be performed. Nonetheless, our results should help to a better understanding of baryonic processes at a microscopic level and serve as a guide when dealing with reactions where some indicative predictions are needed in theoretical as well as in experimental studies. The elastic $\pi d$ scattering above the Roper threshold as well as the breakup of the deuteron into $NN^*$ channels, although not available for the moment, should serve as a test of the results we have derived.

A transition potential $NN \rightarrow NN^*$ can also be derived within the same framework. Although this transition does not show forbidden channels, the quantum numbers are fixed by the $NN$ system, the quark model provides a parameter-free prediction. This potential can be tested in several reactions [6]. We have determined the $NN^*(1440)$ probability on the deuteron by means of a multichannel calculation including: $^3S_1^{NN}$, $^3D_1^{NN}$, $^3S_1^{\Delta\Delta}$, $^3D_1^{\Delta\Delta}$, $^7D_1^{\Delta\Delta}$, $^7G_1^{\Delta\Delta}$, $^3S_1^{NN^*(1440)}$, and $^3D_1^{NN^*(1440)}$, finding for the Roper components 0.003% and 0.024%, respectively, much lower than the $\Delta\Delta$ ones ($\approx 0.25\%$).

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