THE DISTRIBUTION OF GALAXIES’ GRAVITATIONAL FIELD STEMMING FROM THEIR TIDAL INTERACTION

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ABSTRACT

We calculate the distribution function of astronomical objects’ (like galaxies and/or smooth halos of different kinds) gravitational fields due to their tidal interaction. For that we apply the statistical method of Chandrasekhar, used originally to calculate the famous Holtzmark distribution. We show that in our approach the distribution function is never Gaussian, its form being dictated by the potential of interaction between objects. This calculation permits us to perform a theoretical analysis of the relation between angular momentum and mass (richness) of the galaxy clusters. To do so, we follow the ideas of Catelan & Theuns and Heavens & Peacock. The main difference is that here we reduce the problem to a discrete many-body case, where all physical properties of the system are determined by the interaction potential \( V(r_f) \). The essence of reduction is that we use the multipole (up to quadrupole here) expansion of Newtonian potential so that all hydrodynamic, “extended” characteristics of an object, such as its density mass, are “integrated out,” leaving its “point-like” characteristics, such as mass and quadrupole moment. In that sense we do not distinguish between galaxies and smooth components such as halos. We compare our theoretical results with observational data.

Key words: galaxies: clusters: general

1. INTRODUCTION

The problem of galaxies and their structures’ formation is one of the objectives of modern extragalactic astronomy and cosmology. There are many scenarios of structure formation (Peebles 1969; Zeldovich 1970; Sunyaev & Zeldovich 1972; Doroshkevich 1973; Shandarin 1974; Wesson 1982; Efstathiou & Silk 1983; Dekel 1985), which are still important. This is because the new scenarios are essentially the modifications of the old ones and can be classified according to the classical ones. Revised and improved structure formation scenarios can be found in various papers (Lee & Pen 2000, 2001, 2002; Navarro et al. 2004; Mo et al. 2005; Bower et al. 2006; Trujillo et al. 2006; Brook et al. 2008; Paz et al. 2008; Vera-Ciro et al. 2011; Codis et al. 2012; Shandarin et al. 2012; Varela et al. 2012; Giahi-Saravani & Schäfer 2013). The crucial goal is to discriminate among different models of galaxies and their structures’ formation. The main controversy here is how galaxies acquire their angular moments, which yield subsequently the moments of galaxy clusters.

Currently the commonly accepted model is a spatially flat, homogeneous, and isotropic ΛCDM model of the universe. In such a model, the structures were formed from the primordial adiabatic, nearly scale-invariant Gaussian random fluctuations (Silk 1968; Peebles & Yu 1970; Sunyaev & Zeldovich 1970). The most popular galaxy formation scenario, the so-called hierarchic clustering model (Peebles 1969; Doroshkevich 1970; Dekel 1985), is based on this assumption. The numerical simulations (Bond et al. 1996; Springel et al. 2005; van de Weygaert & Bond 2008a, 2008b) confirm that such a mechanism could be realized in the universe. In this mechanism, the large-scale structure can appear from bottom to top as a consequence of gravitational interactions between galaxies. This means that galaxies are formed at the beginning with subsequent merger into larger clusters (structures). In this case, the galaxies’ spin angular momenta arise as a result of interaction with their neighbors. The original version of this model claims that the initial orientation of galaxy spins should be random. However, it had been shown later that the hierarchic clustering model admits naturally the so-called tidal torque mechanism. In this mechanism, galaxies have their angular momenta aligned owing to the coupling between the protogalaxy region and surrounding structure. The galaxy rotation in this mechanism is due to tidal interaction between galaxies (Wesson 1982; White 1984) based on the ideas of Hoyle (1951). The review of the tidal torque scenario is presented by Schaefer (2009). Note that while originally the tidal torque mechanism considers completely random distribution of spin angular momenta of galaxies, it has been shown later that the local tidal shear tensor can cause a local alignment of their rotational axes (Dubinski 1992; Catelan & Theuns 1996a; Lee & Pen 2000, 2001, 2002; Navarro et al. 2004). On the other hand, some authors, such as Brook et al. (2008), still argue that there is misalignment of angular momenta in the hierarchical clustering model.

Different scenarios make different predictions concerning distribution of their angular momenta and especially about orientation of galaxies in structures (for a review see Godłowski 2011a). Of course, the final test of a given scenario is to compare its predictions with observations. It is possible to conclude that the observed variations in angular momentum represent simple but fundamental constraints for any model of galaxy formation (Romanowsky & Fall 2012; Joachimi et al. 2015).

The investigations of galaxies’ alignment show that it depends on the mass of the parent structure. Generally, the groups of galaxies and the small galaxy clusters reveal almost no alignment, while we observe such alignment for rich galaxy clusters and superclusters. The alignment increases with mass of the structure (Godłowski et al. 2005, 2010; Aryal et al. 2007; Godłowski 2011a, 2011b, 2012).

Note that it is commonly agreed that there is no evidence for rotation of the groups and clusters of galaxies. That implies that such structures do not rotate (e.g., Regos & Geller 1989;
Diaferio & Geller 1997; Diaferio 1999; Rines et al. 2003; Hwang & Lee 2007); see, however, Kalinkov et al. (2005) for an opposing opinion. In this context, especially important is the result of Hwang & Lee (2007), who examined the dispersions and velocity gradients in 899 Abell clusters and found possible evidence for rotation in only six of them. Thus, any nonzero angular momentum in groups and clusters of galaxies should arise only from possible alignment of galaxy spins, and stronger alignment means larger angular momentum of such structures.

The aim of the present work is the theoretical analysis of the influence of tidal interaction between objects such as galaxies, their clusters, and smooth halos on their gravitational field distribution. For that we use the statistical method of Chandrasekhar (1943). To apply our result to observable quantities, we calculate the distribution function of the angular momenta. We do that for linear (corresponding to the Zeldovich 1970 approximation in the displacement field) and nonlinear regimes of fluctuation growth. As in our method, the parameters of a galaxy ensemble, such as their masses, radii, and volumes, enter the problem as parameters; our calculation permits us to trace the possible relation between angular momentum and mass (richness) of the galaxy clusters. The above statistical method reveals the fact that in the stellar systems, the derived distribution function cannot be Gaussian but rather belongs to the family of so-called heavy-tailed distributions (for details see, e.g., Kapur & Kesavan 1992). Moreover, choosing the cosmology on the basis of the corresponding Friedmann equation, our result permits us to trace the time evolution of the distribution function of angular momenta and its mean value $L$. In our approach, we can also derive the well-known empirical relation between mean galaxy cluster moment $L$ and its mass $M$, $L \sim M^{2/3}$.

The paper is organized as follows. In Sections 2 and 3, based solely on the quadrupole (tidal) interaction potential between astronomical objects, we calculate our universal (i.e., independent of the details of Lagrangian or Eulerian spaces) distribution function of gravitational fields $f(E)$. In Section 4, on the basis of the function $f(E)$, we calculate the distribution function of angular momenta $f(L)$ in both the linear and nonlinear Lagrangian approximation. We emphasize that as $f(E)$ does not depend on the Eulerian or Lagrangian picture, it can be used to calculate the distribution of any quantity (like momentum) of the astronomical objects (not only galaxy clusters but smooth components like halos as well) in any (linear or nonlinear) regime of fluctuation growth.

2. GENERAL FORMALISM

Unfortunately, the spin angular momentum is known only for very few galaxies and structures. Therefore, instead of the angular momentum by itself, the orientation of galaxies in each cluster is usually studied. This is the reason that here we are interested primarily in the absolute value of galaxies’ angular momenta.

We represent matter (both luminous and dark) as the Newtonian self-gravitating fluid embedded in the universe obeying the corresponding Friedmann equation. To obtain the tidal (i.e., shape-distorting) interaction between the astronomical objects, we, similarly to Poisson (1998), perform the multipole expansion of the Newtonian interaction potential between fluid elements. Truncating this expansion on the quadrupole terms, in the spirit of the article (Poisson 1998), we write the Hamiltonian (total classical energy) of interaction between galaxies in the form

$$\mathcal{H} = -G \sum_{ij} m_i V(r_{ij}) \equiv -GM^2 \sum_{ij} \frac{m_i m_j}{r_{ij}} V(r_{ij}),$$

$$V(r_{ij}) = \frac{1}{2} \frac{3 \cos^2 \theta_{ij} - 1}{r_{ij}^3},$$  \hspace{1cm} (1)

where $G$ is the gravitational constant, $M = \sum_{i=1}^{N} m_i$ is the total mass of the ensemble, $p_i = Q_i / (m_i M^2)$, $Q_i$ is the quadrupole moment of the $i$th galaxy, $m_i$ is its mass, $r_{ij} \equiv |r_{ij}|$, $r_{ij} = r_i - r_j$ is a relative separation between centers of galaxies, and $N$ is their number. The expression for $Q_i$ has the form (Poisson 1998)

$$Q_i = \int_{V_i} \rho_i(x) |x|^2 P_2 (\mathbf{s} \cdot \mathbf{x}) d^3x, \quad P_2(x) = \frac{1}{2} (3x^2 - 1),$$  \hspace{1cm} (2)

where $V_i$ is the volume of the $i$th galaxy and $\rho_i(x)$ is its mass density.

Note that expression (1) generalizes the two-particle result of Poisson (1998) on the ensemble of $N$ objects, splitting the interactions between them in a pairwise manner. Such splitting is customary in condensed matter physics, where the interacting many-body ensemble is represented by the sum of all possible pair interactions between particles $i$ and $j$ (e.g., $123 = 12 + 13 + 23$); see Majlis (2000) and references therein in the context of magnetic systems. The same procedure is also used in astronomy (Peacock 1999, p. 296).

The physics of interaction (1) is the following. Under the influence of the interaction with other astronomical objects, the shape of a given $i$th object changes, which inflicts the variation of its density field $\rho_i(x)$, Equation (2). As the objects are situated randomly and have random shapes, the mass $m_i$ and quadrupole moment $Q_i$ of an object (such as a galaxy, galaxy cluster, or smooth component) will vary randomly. This generates the random variations of the gravity field $E_{\text{quad}}$ from these quadrupoles. The latter field is a gradient of the potential (1) (divided by mass $m$) and has the form

$$E_{\text{quad}}(r) = i_i E_0 \frac{3 \cos^2 \theta - 1}{r^4}, \quad E_0 = \frac{GQ}{2},$$  \hspace{1cm} (3)

where $i_i$ is the unit vector in the direction of the radius vector.

As we cannot solve the random many-body problem (1) exactly, different approaches had been used for its approximate solution. The simplest approach in condensed matter physics is the so-called mean field approximation, where the fluctuating (electric, magnetic, elastic in condensed matter) field, acting on the specific particle, is substituted by some ensemble-averaged mean field (see, e.g., Majlis 2000). This approach does not take into account the particle clustering (i.e., two-, three-, four-particle cluster in the field of the rest of likewise clusters) and is not suitable to describe the systems with disorder, which is also our case. In the disordered case the adequate approach is a statistical method similar to that applied by Chandrasekhar (1943) to describe the fluctuations of the force acting on a specific star from the rest of the stellar system. In this method, the distribution function of a random gravitational field is introduced so that any observable quantity of an ensemble (orbital moment, energy, etc.) can be expressed by averaging the corresponding single-particle quantity with the above distribution function; see Stephanovich (1997),...
3. DISTRIBUTION FUNCTION OF QUADRUPOLAR FIELDS

According to the statistical method, very similar to that from Chandrasekhar (1943), the distribution function of random quadrupolar fields reads (see Stephanovich 1997; Semenov & Stephanovich 2003 and references therein)

\[
 f(E) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} e^{iE\rho} e^{-i\rho E} d\rho_x d\rho_y d\rho_z.
\]

(5)

The explicit averaging in Equation (5) is based on the fact that the mass and quadrupole moments of the galaxies in volume \( V \) obey the uniform distribution with probability density equal to \( 1/V \). This is equivalent to the fact that the fluctuations of the above galaxy parameters obey the Poisson distribution (Chandrasekhar 1943; see also Zolotarev 1986) for a purely mathematical treatment of this question. In this case the averaging for a single galaxy yields

\[
 \frac{1}{4\pi V} \int_V e^{-i\rho E} d^3\tau = \frac{1}{4\pi V} \int_V e^{-i\rho E(r)} d^3\tau,
\]

(6)

while for an ensemble of \( N \) galaxies the corresponding average is

\[
 e^{-i\rho E} \equiv \left[ e^{-i\rho E} \right]^N, \quad \text{i.e., the single-galaxy average, raised to power } N \text{ (Chandrasekhar 1943)}.
\]

We note here that the above averaging procedure implies that the astronomical objects are similar to each other. However, it can be shown (see Semenov & Stephanovich 2002 and references therein) that such an approach takes into account the pair clusters of galaxies or other spatially disordered constituents in the systems other than stellar. The higher-order clusters (three, four, etc., objects) can be taken into account along the lines of Ziman (1979), which would require solving the chain of kinetic equations for \( n = 3, 4 \), etc., bodies’ distribution functions. On the other hand, since the above procedure accounts exactly for pair clusters, the many-body clusters can be considered by splitting them into corresponding pairs. This means that our approach takes also the many-particle clustering into account. However, in the future, we are going to elaborate the above \( n \)-bodies procedure and compare it to our present results.

We have finally

\[
 e^{-i\rho E} = \lim_{N \to \infty, V \to \infty} \frac{1}{4\pi V} \int_V e^{-i\rho E(r)} d^3\tau \left[ e^{-i\rho E} \right]^N
\]

\[
 = \lim_{N \to \infty, V \to \infty} \left\{ 1 - \frac{1}{V} \int_V [1 - U(r)] d^3\tau \right\}^N
\]

\[
 = \lim_{N \to \infty} \left\{ 1 - \frac{n}{N} \int_V [1 - U(r)] d^3\tau \right\}^N
\]

\[
 \approx \exp \left[ \frac{-n}{N} \int_V [1 - U(r)] d^3\tau \right],
\]

\[
 U(r) = \frac{1}{4\pi} \int e^{-i\rho E} \sin \theta d\theta \sin \varphi = \frac{\sin \rho E(r)}{\rho E(r)},
\]

(7)

where \( E(r) \) is given by Equation (3).

The expression for \( U(r) \) signifies the averaging over random angle between vectors \( \rho \) and \( E \). Also, we denote \( N \) in Equation (7) to be the number of galaxies so that \( n = N/V = \text{const} \) is the density of galaxies in a given volume.

Combining Equations (5) and (7), we obtain the following expression for the distribution function:

\[
 f(E) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} e^{iE\rho} F(\rho) d^3\rho, \quad F(\rho)
\]

\[
 = n \int_V \left[ 1 - \frac{\sin \rho E(r)}{\rho E(r)} \right] d^3\tau.
\]

(8)

We see that \( F(\rho) \) is indeed the characteristic function for random gravitational fields’ distribution. Note that the galaxy clustering can be better considered if we assume that the density of galaxies is not a constant but rather \( n = n(r) \). Then, similar to the case of disordered magnetic semiconductors (see Equation (6b) of Semenov & Stephanovich 2003), the function \( n(r) \) should be put under the integral sign in Equation (8) to give

\[
 F(\rho) = \int_V n(r) \left[ 1 - \frac{\sin \rho E(r)}{\rho E(r)} \right] d^3\tau.
\]

(9)

If we specify the empirical dependence \( n(r) \), the characteristic function \( F(\rho) \) can be calculated only numerically.

Below we shall calculate this function without any assumptions analytically for the simpler case \( n = \text{const} \). One more way to consider the effects of clustering is to account for inhomogeneous distribution of masses (and/or quadrupolar moments) in the ensemble. This can be done along the lines of Chandrasekhar (1943), where the distribution function of masses \( \tau(m) \) had been introduced. In our case, however, the situation will be not that simple as we are dealing with more complicated parameters like the quadrupole moment of an object. In the present publication we do not consider this effect especially in view that there is large ambiguity in determination of function \( \tau(m) \) from the astronomical observation data. However, we are going to incorporate the dependence \( \tau(m) \) in our consideration in the future.

We note here that distribution function \( f(E) \) in Equation (8) is by no means Gaussian. We will show that function (8) does...
not admit a Gaussian limit so that for the problem of interaction between gravitational quadrupoles (and actually higher-order terms like octupoles, etc.) the distribution function of gravitational fields and angular momenta is never Gaussian. To demonstrate that, we note that it had been shown earlier for condensed matter systems (ferroelectrics in Stephanovich 1997 and magnetic semiconductors in Semenov & Stephanovich 2003) that the Gaussian limit corresponds to large density $n \to \infty$ of electric dipoles (ferroelectrics) or spins (magnetic systems).

It had also been shown by Stephanovich (1997) and Semenov & Stephanovich (2003) that the limit $n \to \infty$ corresponds to the small Fourier variable $\rho$ in Equation (8). This means that to obtain the Gaussian limit of the distribution function (8), we should expand its characteristic function $F(\rho)$ in small $\rho$. In first nonvanishing (Gaussian) approximation in small $\rho$ this procedure yields

$$F_G(\rho) = \frac{n \rho^2}{6} \int_V E^2(r) d^3r.$$  

(10)

To derive Equation (10), we use the expansion $\sin x/x \approx 1 - x^2/6 + \ldots$ valid at $x \to 0$. The explicit rewriting of Equation (10) with respect to Equation (3) yields

$$F_G(\rho) = \frac{1}{3} \pi n \rho^2 \int_0^\infty \left( 3 \cos^2 \theta - 1 \right)^2 \sin \theta d\theta \int_0^\infty \frac{dr}{r^6},$$  

(11)

where the last integral over $r$ is divergent at small $r$. We note here that in the solids the Gaussian limit exists (i.e., the integral (11) becomes convergent) as a result of the presence of short-range terms $\sim \exp(-r/\xi)$ ($\xi$ is the so-called correlation radius, defining the range of interaction) in the interaction potential between dipoles or spins. As the exchange interaction between spins is always of short range, this feature is peculiar to magnetic systems; see Semenov & Stephanovich (2003) and references therein. This means that as a result of long-range quadrupole-quadrupole interaction between galaxies, $\sim r^{-4}$, the distribution function of their fields and/or orbital moments (see below) is never Gaussian.

The explicit expression for $F(\rho)$ reads

$$F(\rho) = n \int_0^\infty r^2 dr \left[ \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\varphi \right] \times \frac{\rho \rho_0 \left( 3 \cos^2 \theta - 1 \right) \left( 3 \cos^2 \theta - 1 \right)}{\rho \rho_0 \left( 3 \cos^2 \theta - 1 \right) \left( 3 \cos^2 \theta - 1 \right)}.$$  

(12)

We first perform the integration over $r$ in Equation (12). For that we denote $b/r^4 = x(b = \rho \rho_0 (3 \cos^2 \theta - 1))$ to obtain

$$K = \int_0^\infty x^2 dx \left[ 1 - \sin \frac{Q}{x} \frac{Q}{x} \right] = \frac{1}{4} b^{3/4} \int_0^\infty dx x^{7/4} \left[ 1 - \sin \frac{x}{x} \right].$$  

(13)

The last integral can be calculated analytically to give

$$\int_0^\infty dx x^{7/4} \left[ 1 - \sin \frac{x}{x} \right] = \frac{8}{21} \sqrt{2} \sqrt{2} \Gamma \left( \frac{1}{4} \right) \approx 1.05711302,$$

where $\Gamma(x)$ is the Gamma-function (Abramowitz & Stegun 1972). This finally yields

$$K = 0.264278 |b|^{3/4} = A |b|^{3/4}.$$  

(14)

With respect to Equation (14) we have

$$F(\rho) = 2\pi n A (\rho \rho_0)^{3/4} \int_0^\infty \sin \theta d\theta \left[ 3 \cos^2 \theta - 1 \right]^{3/4}.$$  

(15)

The auxiliary integral $K_1$ can be calculated numerically to give

$$K_1 = \int_{-1}^1 dx \left[ 3x^2 - 1 \right]^{3/4} = 2 \int_{-1}^1 dx \left( 3x^2 - 1 \right)^{3/4} = 1.581940777.$$  

(16)

With respect to Equation (16), $F(\rho)$ assumes the form

$$F(\rho) = 2\pi n \cdot 0.41807255 \cdot (\rho \rho_0)^{3/4}.$$  

(17)

Substitution of Equation (17) into Equation (8) gives

$$f(E) = \frac{1}{2\pi^2 E} \int_0^\infty \rho \rho' \rho \rho'' \sin \rho E d\rho,$$

$$\rho' = 2\pi n \cdot 0.41807255 \cdot E^{3/4}. $$  

(18)

Expression (18) is the main theoretical result of the present paper, constituting the final answer for the distribution function of gravitational fields’ moduli. It is seen that distribution function (18) depends parametrically on the galaxies’ density $n$, as well as on the average galaxy quadrupole moment. To the best of our knowledge, neither distribution function (18) nor its explicit dependence on $n$ and $Q$ has been known previously. The integral (18) can be calculated only numerically.

The normalization condition for distribution function (18) looks like

$$4\pi \int_0^\infty E^2 f(E) dE = 1.$$  

(19)

We check explicitly

$$I = 4\pi \int_0^\infty E^2 f(E) dE = \frac{2}{\pi} \int_0^\infty E \sin \rho E dE \int_0^\infty \rho d\rho e^{-\alpha \rho^4/4}.$$  

With respect to the relation $\int_0^\infty E \sin \rho E dE = -\pi \delta' (\rho)$, the integral $I$ can be calculated as

$$I = -2 \int_0^\infty \rho \delta' (\rho) d\rho e^{-\alpha \rho^4/4} = \frac{d}{d\rho} \left[ \rho e^{-\alpha \rho^4/4} \right]_{\rho=0} = 1.$$  

(20)

To derive the result (20), we use the following identity for the $n$th derivative of the Dirac $\delta$-function:

$$\int_{-\infty}^\infty f(x) \delta'^{(n)}(x) dx = -\int_{-\infty}^\infty \frac{df}{dx} \delta^{(n-1)}(x) dx.$$  

We finally mention the difference between our approach to the problem of angular moments’ distribution and that of Catelan & Theuns (1996a) and Heavens & Peacock (1988). The main difference is that we consider the discrete many-body problem, stemming from multipole expansion (up to quadrupole here) of the Newtonian interaction potential between fluid elements. In such an approach, all physical properties of the system are determined by the interaction potential $V(r_{ij})$ given by Equation (1). In the case of disorder, the distribution
function of random gravitational fields is also completely determined by the form of potential $V(r_i)$. To derive the distribution function of random gravitational fields, we use the statistical method of Chandrasekhar (1943). As we have shown above, within this method, the distribution function is never Gaussian for any long-range potential, obtained as the multipole expansion of a Newtonian one. At the same time, all previous approaches postulated (rather then derived) the distribution function in Gaussian form, which, in our opinion, does not reflect the physical nature of long-range gravitational multipole interaction, which generates distribution functions with long tails.

3.1. Numerical Calculation of $f(E)$: Dimensionless Variables

Following Chandrasekhar (1943) for the case of the Holtzmark function, we introduce the dimensionless variables $\rho E = x$ and $\beta = E/\alpha^{3/4}$. In these variables the integral (18) renders to

$$ f(\beta) = \frac{1}{2\pi^2\beta^4} \int_0^\infty x \sin x \exp\left[-\left(\frac{x}{\beta}\right)^{3/4}\right] dx = \frac{H(\beta)}{4\pi\beta^2\alpha^3}, \quad \beta > 0. \quad (21) $$

The physical meaning of the function $H(\beta)$ is that as it is proportional to $\beta^2 f(\beta)$, it is just the integrand in Equation (19), being the effective one-dimensional distribution function of random fields. In other words, the normalization condition for $H(\beta)$ assumes the one-dimensional form

$$ \int_0^\infty H(\beta) d\beta = 1. \quad (23) $$

In this case, the average value of dimensionless random field $\beta$ reads

$$ \bar{\beta} = \int_0^\infty \beta H(\beta) d\beta. \quad (24) $$

Function $H(\beta)$ will be calculated numerically below.

3.2. Asymptotics of Distribution Function $f(\beta)$

We begin with asymptotics of $H(\beta)$. At $\beta \to 0$ we make substitution $x/\beta = t$ to obtain from Equation (22)

$$ H(\beta) = \frac{2\beta}{\pi} \int_0^\infty t \sin t \ e^{-t^{1/4}} dt \cong \frac{2\beta}{\pi} \int_0^\infty t \left(\beta - \frac{1}{6} (\beta t)^3 + \ldots\right) e^{-t^{1/4}} dt. \quad (25) $$

The first term of the expansion (25) yields

$$ H(\beta \to 0) = \frac{2\beta^3}{\pi} \int_0^\infty t^2 e^{-t^{1/4}} dt = \frac{8\beta^2}{3\pi} \int_0^\infty z^3 e^{-z^2} dz = \frac{48}{3\pi} \beta^2. \quad (26) $$

At $\beta \to \infty$ we expand Equation (22) over the small parameter $1/\beta$. We obtain in the first approximation

$$ H(\beta \to \infty) = \frac{2}{\pi\beta} \lim_{\beta \to 0} \int_0^\infty x \sin x \left(1 - \frac{x^{3/4}}{\beta^{3/4}}\right) e^{-\beta x} dx $$

$$ = - \frac{2}{\pi\beta^{3/4}} \lim_{\beta \to 0} \int_0^\infty x^{7/4} \sin x e^{-\beta x} dx $$

$$ = \frac{2}{\pi\beta^{3/4}} \Gamma(11/4) \cos \frac{\pi}{8} \approx 0.945972642/\beta^{-7/4}. \quad (27) $$

To derive expression (27), we take into account that

$$ \lim_{\beta \to 0} \int_0^\infty x \sin x e^{-\beta x} dx = 0. $$

Having asymptotics $H(\beta)$, we calculate those for $f(\beta)$ with the help of relation (21):

$$ f(\beta) = \left\{ \begin{array}{ll} \frac{4}{\pi\alpha^2} & , \beta \to 0 \\ \frac{2015 September 10}{4\pi\alpha^2} & , \beta \to \infty. \end{array} \right. \quad (28) $$

Here $\alpha$ is given by Equation (18). The asymptotics (28) shows that $f(\beta)$ does not depend on $\beta$ at small $\beta$ and decays at large $\beta$. This shows that although the normalization integral is convergent (we recollect that the normalization condition for $f(\beta)$ looks like $\int_0^\infty \beta^2 f(\beta) d\beta = 1$), already the first moment does not exist. This can also be seen from large $\beta$ asymptotics of $H(\beta)$ given in Equation (27). Expression (28) is a confirmation of the fact that function $f(\beta)$ belongs to the class of heavy-tailed distributions.

4. DISTRIBUTION FUNCTION OF ANGULAR MOMENTA

Our aim is to calculate the distribution function of galaxies’ angular momenta $L_i$. For that we need a relation between the angular momentum $L$ of a galaxy and its gravitational field $E_{\text{quad}}(r)$, given by Equation (3). The expression for components $L_\alpha (\alpha = x, y, z)$ of $L$ has been derived in the form of a perturbation series in the small Lagrangian coordinate $q$. The first-order terms are defined by Equation (11) of Catelan & Theunis (1996a), while second-order ones are defined by Equation (28) of the follow-up article (Catelan & Theunis 1996a). Both expressions have identical structure and can be written in the form

$$ L_\alpha^{(i)} = \xi_{\alpha\beta} \xi_{\beta\gamma} L_{\gamma\gamma}, \quad \alpha, \beta, \gamma, \sigma = x, y, z, \quad (29) $$

where index $i = 1, 2$ denotes the order of the perturbation theory, $\xi_{\alpha\beta}$ is the Levi-Civita symbol, $E_{\beta\gamma}$ are components of quadrupole (tidal) field (3), and $L_{\gamma\gamma}$ are components of the inertia tensor. Accordingly, functions $f_1(t) = a^2(t) \dot{D}(t)$ and $f_2(t) = \dot{E}(t)$ (dot means time derivative) are known functions of time, calculated from the differential equations, derived in $i$th order of perturbation theory (Bouchet et al. 1992). The explicit forms of these equations read

$$ t_0^2 \dot{D}(t) + a(t) D(t) = 0, \quad (30) $$

$$ t_0^2 \dot{E}(t) + a(t) E(t) = -a(t) D(t)^2, \quad (31) $$
where \(0 \leq t < \infty\) is dimensional physical time and \(t_0\) is some characteristic time, depending on the cosmological model considered. We will choose this quantity below. The dimensionless function \(a(t)\) (so-called scale factor) is determined from a zero-order perturbative equation, which indeed is the first Friedmann equation, depending again on the chosen cosmological model. The general form of this equation reads

\[
\frac{H^2}{H_0^2} = \Omega_0 a^{-4} + \Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda.
\]  

(32)

Here \(H = \dot{a}/a\) is the Hubble parameter \((\dot{a} \equiv da/dt)\), \(H_0\) is the Hubble constant, and \(\Omega_i\) \((i = R, M, k, \Lambda)\) are the corresponding density parameters taken at the present time, when \(a(t) = 1\). Specifically, \(\Omega_k\) is radiation density, \(\Omega_M\) is matter (dark plus baryonic) density, \(\Omega_k\) is so-called spatial curvature density, and \(\Omega_\Lambda\) is cosmological constant or vacuum density, \(\Omega_\Lambda = \Lambda/(3H_0^2)\), where \(\Lambda\) is cosmological and \(H_0\) is the Hubble constant. For our calculations of the distribution function of angular momenta, we will choose the flat ΛCDM model of the universe, keeping in Equation (32) only the \(\Omega_M\) and \(\Omega_\Lambda\) terms, \(\Omega_M + \Omega_\Lambda = 1\).

Although the components of the gravitational tidal (shear) field \(\tilde{E}\) are different in the first and second orders of perturbation theory, for our purposes it is sufficient to consider them to be the same as they are simply the arguments of distribution function (18). Denoting \(b(t) = f(t)\) and omitting index \(i\) in the components of tidal field \(E\), we can rewrite relation (29) explicitly:

\[
L_x/b(t) = E_{xx} I_{xz} + E_{xy} I_{zy} - E_{xz} I_{xy} - E_{zy} I_{yx} - E_{xx} I_{zy} - E_{xy} I_{xz} - E_{xz} I_{xy} - E_{zy} I_{xx},
\]

\[
L_y/b(t) = E_{xx} I_{xy} + E_{xy} I_{yy} - E_{xx} I_{yy} - E_{xy} I_{yx} - E_{xx} I_{xy} - E_{xy} I_{xx} - E_{xx} I_{yy} - E_{xy} I_{xx},
\]

\[
L_z/b(t) = E_{xx} I_{xz} + E_{xy} I_{yz} - E_{xx} I_{yz} - E_{xy} I_{zx} - E_{xx} I_{xz} - E_{xy} I_{yz} - E_{xx} I_{yz} - E_{xy} I_{zx}.
\]

(33)

Taking into account the symmetry relations \(L_{ab} = L_{ba}\) and \(E_{ab} = E_{ba}\) and leaving only \(E_{xz}\), we obtain from Equation (33)

\[
L_x = b(t) E_{xx} I_{xz},
\]

\[
L_y = b(t) E_{xx} I_{xy},
\]

\[
L_z = 0.
\]

\[
L = \sqrt{L_x^2 + L_y^2 + L_z^2} = b(t) E_{xx} \sqrt{I_{xz}^2 + I_{xy}^2} \equiv L_0 E,
\]

\[
L_0 = L_0(t) = f_0(t) \sqrt{I_{xz}^2 + I_{xy}^2}.
\]

(34)

Expression (34) constitutes the linear relation between angular momentum and tidal field moduli in both linear \((i = 1)\) and nonlinear \((i = 2)\) regimes.

As the relation between gravitational field modulus and angular moment (34) is linear, the shape of the distribution function of angular moments \(f(L)\) is similar to that of gravitational fields. The explicit transition from \(f(E)\) in Equation (18) to \(f(L)\) can be accomplished by combining expression (34) and the known relation from the theory of probability

\[
f(L) = f[E(L)] \left| \frac{dE}{dL} \right|,
\]

(35)

which yields

\[
f(L) = \frac{1}{2\pi^2 L_0} \int_0^\infty \rho e^{-\rho^2/L_0^2} \sin \left( \frac{L}{L_0(t)} \rho \right) d\rho,
\]

(36)

where \(L_0(t)\) is given by expression (34). Passing to dimensionless variables

\[
\rho L/L_0 = x, \quad \lambda = L/L_0 \alpha^3/3
\]

(37)

generates the following pair of functions similar to the case of gravitational fields’ distribution (21) and (22):

\[
f(\lambda) = \frac{1}{2\pi^2 \lambda^4 L_0} \int_0^\infty x \sin x e^{\left[ -\left( \frac{x}{\lambda} \right)^{3/4} \right]} dx
\]

\[
= \frac{H(\lambda)}{4\pi^2 \lambda^4 L_0},
\]

(38)

\[
H(\lambda) = \frac{2}{\pi \lambda} \int_0^\infty x \sin x e^{\left[ -\left( \frac{x}{\lambda} \right)^{3/4} \right]} dx.
\]

(39)

In this case the effective 1D distribution function \(H(\lambda)\) is similar to that from the gravitational fields (22) and we have the asymptotics (28) (divided by \(L_0\)) for the distribution function of momenta.

The effective 1D distribution of fields or momenta is shown in Figure 1. It is seen that this function has a characteristic bell shape and is asymmetric. Asymptotics (27) shows that the integral, defining the mean value of the galaxy orbital moment (24), is divergent. To estimate the most probable value of the orbital moment, we calculate \(\lambda_{\text{max}}\), corresponding to the maximum of distribution function \(H(\lambda)\); see Figure 1 for details. This situation is typical for so-called heavy-tailed distributions like the Cauchy one, \(f(x) \sim (a^2 + x^2)^{-1}\). Such distributions can be met very frequently in all branches of physics, dealing with random processes and ranging from condensed matter physics to chemical kinetics and econophysics (see, e.g., Kapur & Kesavan 1992). As Cauchy and many other heavy-tailed distributions do not admit a first moment (corresponding integral is divergent), the maximum of such a probability density function is usually taken as a measure of its mean value.

The analysis of this mean value will permit us to derive some useful relations, which earlier had been taken as empirical ones.
To perform this analysis, we adopt the simplest possible cold dark matter (CDM) cosmology in the first order of perturbation theory, where $a(t) = D(t) = (t/t_0)^{2/3}$ (Doroshkevich 1970) so that $L_0 = \frac{\kappa_2}{t_0^{2/3}}$, $\tau = t/t_0$, $I = I_{\kappa_2}^2 + I_{\kappa_2}^2$. The solution of the equation $dH/d\lambda = 0$ reads

$$\lambda_{\text{max}} = 0.60273063.$$  

(40)

In dimensional units (37) we have from Equation (40)

$$L_{\text{max}} = 0.60273063 \cdot (2\pi \cdot 0.41807255)^{4/3} n^{1/3} L_0 \frac{GQ}{2}$$

$$= 1.0922864 \left(\frac{N}{V}\right)^{4/3} L_0 GQ.$$  

(41)

$$L_{\text{max}} = 0.7281884 \left(\frac{N}{V}\right)^{4/3} \frac{t}{t_0^2} G\kappa_2 m^2,$$  

(42)

where $\kappa$ is a constant of order unity. To derive Equation (42), we estimate (on the basis of Equation (2)) both galaxy quadrupole moment $Q$ and its mean inertia moment $I$ as being proportional to $mR^2$, where $m$ is galaxy mass and $R$ is its mean radius. If we estimate volume $V$ as $V = R^3$, we see that $R$ cancels out in Equation (42) so that we obtain

$$L_{\text{max}} \sim (t/t_0)^2 m^{N^{4/3}}.$$  

On the other hand, we can suppose that volume $V$ is related to the galaxy cluster so that its value $V = R_0^3$, where $R_0$ is a mean radius of the cluster, which is proportional to the autocorrelation radius (Peebles 1973, 1980; Lin et al. 1996; Tucker et al. 1997; Peacock 1999, p. 296; Longair 2008). Although $n$ is still a constant for any particular cluster, it varies from cluster to cluster with increasing richness $N$. In this case we may rewrite $N = M/m$ to obtain the different form of expression for $L_{\text{max}}$,

$$L_{\text{max}} \sim \frac{t}{t_0^2} \left(\frac{R}{R_0}\right)^4 m^{2/3} M^{4/3},$$  

(43)

which does not contain $M$. It seems formally that Equation (43) implies that $L_{\text{max}} \sim t$, but the time evolution of galaxy radius $R$ and mean cluster radius $R_0$ being very complex astrophysical processes (Longair 2008), can complicate real time dependence $L_{\text{max}}(t)$ a lot. This question needs to be studied additionally.

Equation (42) shows that the mean orbital moment of a galaxy is proportional to $N^{4/3}$, where $N$ is the number of galaxies. The dependence $L_{\text{max}}(N) = \kappa_2 N^{4/3}$ ($\kappa_2 = \kappa G m^2/(t_0^2 V^{4/3})$) given by Equation (43) is shown in Figure 2. It is seen that in our model of constant galaxies’ density $n = \text{const}$, the systems with a larger number of galaxies have larger angular momenta. Below, comparing the linear and nonlinear regimes of fluctuation growth, we will show that the assumption of constant density may safely be used for qualitative analysis of angular momentum acquisition.

Let us finally pay attention that the dependence between mean momentum of galaxy cluster and its mass comes from parametrical dependence of $L_{\text{max}}$ on galaxy mass, volume, and quadrupolar moment. This same dependence can be identically rewritten in different ways. Then, if we assume that certain parameters are constant, we obtain different dependencies of $L_{\text{max}}$ on not only cluster mass $M$ but also mass and density of galaxies, as well as number of galaxies $N$. For instance, in

![Figure 2](image-url)  

Figure 2. Dependence of average galaxy angular moment $L_{\text{max}} = \kappa_2 N^{4/3}$ on number of galaxies $N$ in a stellar system, $\kappa_2 = \kappa G m^2/V^{4/3}$.

Equation (42), at $R = \text{const}$ $L_{\text{max}}$ scales as a square of galaxy mass $m$, in contrast to relation (43). This shows that observational verification of the dependencies $L_{\text{max}}(N, m)$ would permit making an unambiguous conclusion about the constancy of a particular stellar parameter.

4.1. Time Dependence of Distribution Function in $\Lambda$CDM Model

The time evolution of distribution function (39) can be obtained with respect to the definition of $\lambda$ given by Equation (37) and subsequently $L_0$ given by Equation (34). The dependence $L_0(t)$ generates substitution $\lambda \rightarrow \lambda f_i(\tau)$, where $\tau = t/t_0$ so that we have from Equation (39)

$$H(\lambda, \tau) = 2 \pi \lambda \int_0^\infty x \sin x \exp \left[-\frac{x f_i(\tau)}{\lambda}\right] \frac{x^3}{\lambda} dx, \ i = 1, 2.$$  

(44)

To derive Equation (44), we take into account that there is an additional coefficient $L_0$ in the denominator of Equation (38) containing $f_i(\tau)$ so that there is no $f_i(\tau)$ before the integral in Equation (44). To obtain $f_{1,2}(\tau)$ in the $\Lambda$CDM model, we begin with the determination of $a(\tau)$ from Friedmann Equation (32), which reads

$$\frac{da}{dt} = H_0 \sqrt{\Omega_0 a^2 + 1 - \frac{\Omega_\Lambda}{a}}.$$  

(45)

The solution of Equation (45) has the form

$$a(\tau) = \alpha \sin^{2/3}(t/t_0), \ \alpha = \left(1 - \frac{\Omega_\Lambda}{\Omega_\Lambda}\right)^{1/3}, \ t_0 = \frac{2}{3H_0\sqrt{\Omega_\Lambda}}.$$  

(46)

Now, the function $f_{1,2}(\tau) = a^{2}(\tau)D'(\tau)$ ($D' = dD/d\tau$), where $D(\tau)$ can be found numerically from Equation (30). Accordingly, in the nonlinear regime, the function $f_{1,2}(\tau)$ should be found numerically from Equation (31).

We note here that in the Einstein–de Sitter model $f_1(\tau) = (2/3)\tau$ and $f_2(\tau) = (-4/7)\tau^{-1/3}$ (Doroshkevich 1970;
Figure 3. Time evolution of effective 1D distribution function \( H(\lambda, \tau) \) in CDM model. Left panel: linear regime \( \lambda_{\text{max}} = 0.6027f(\tau) = 0.4018\tau \); middle panel: nonlinear regime \( \lambda_{\text{max}} = 0.6027 \left| f(\tau) \right| = 0.3444\tau^{1/3} \) (as \( f(\tau) < 0 \), everywhere here we take its modulus). Figures near curves correspond to dimensionless time \( \tau = t/t_0 \). Right panel: dependence \( \lambda_{\text{max}}(\tau) \) for CDM and \( \Lambda \)CDM models. Solid lines correspond to linear regime, dashed lines to nonlinear regime. Mind the logarithmic scale on the vertical axis, needed to place the curves of CDM and \( \Lambda \)CDM models having very different growth rates. For the \( \Lambda \)CDM model we choose 
\[
\alpha = \left( \frac{1 - \Omega_{\Lambda}}{\Omega_{\Lambda}} \right)^{1/3} = 1.
\] 

Catelan & Theuns 1996b). Also, the maximum of function (44) occurs at
\[
\lambda_{\text{max}}(\tau) \approx 0.602730263f(\tau),
\] 
where \( \lambda_{\text{max}} \approx 0.602730263 \) is a maximum of time-independent function \( H(\lambda) \) given by Equation (40). It is seen that functions \( f_2(\tau) \), related to the second perturbative corrections, are negative. Substitution of such a function into the exponent of integrand (44) generates the imaginary part, which does not change the behavior of \( H(\lambda, \tau) \) qualitatively. That is why for the purpose of comparison of the linear and nonlinear regimes of fluctuation growth, everywhere we use the moduli of the functions \( f_2(\tau) \) both in CDM and in \( \Lambda \)CDM models.

The dependencies \( H(\lambda, \tau) \) (44) for the Einstein–de Sitter CDM model with the above \( f(\tau) \) are shown in Figure 3. It is seen that at time increase the distribution function diminishes, while at zero time it goes to infinity. As time (figures near curves in Figure 3) grows, the maximum \( \lambda_{\text{max}}(\tau) \) shifts toward larger \( \lambda \) so that the whole distribution function “blurs” at large times. This is related to the fact that functions \( f(\tau) \) enter the exponent in integrand (44). The comparison of the left and middle panels of Figure 3 shows that the behavior of \( H(\lambda, \tau) \) is qualitatively similar in linear and nonlinear regimes of fluctuation growth. This means that for qualitative analysis we may safely use the linear regime. To further demonstrate that, in the right panel of Figure 3, we report the dependence \( \lambda_{\text{max}}(\tau) \) given by Equation (47). It is seen that in both linear and nonlinear regimes this function grows with time, although in the \( \Lambda \)CDM model this growth is much faster, so that in order to show both CDM and \( \Lambda \)CDM curves in one panel, we use the logarithmic scale.

The results of numerical calculations in the \( \Lambda \)CDM model are reported in Figure 4. It is seen that the qualitative behavior of \( H(\lambda, \tau) \) is similar to that for the CDM model. Here, however, we can trace the variation of \( H(\lambda, \tau) \) with parameter \( \alpha \). It is seen that as parameter \( \alpha \) increases, the distribution function decreases—for the similar times the distribution function is smaller for larger \( \alpha \). Similar to the CDM model, the maximum of the distribution function shifts toward larger \( \lambda \) at time growth. The above regularities are the same for linear and nonlinear regimes, while the values of distribution functions in the \( \Lambda \)CDM model for the same \( \alpha \) are smaller in the nonlinear regime. This is related to the fact that function \( f_2(\tau) \) grows much faster than \( f_1(\tau) \) in the \( \Lambda \)CDM model. Also, as has been noticed in many references (see, e.g., Catelan & Theuns 1996a and references therein), that first-order perturbation result (linear regime) corresponds to the so-called Zeldovich approximation, which is approximately valid also for the nonlinear situation. This shows once more that for qualitative discussion of the time dependence of the distribution function \( H(\lambda, \tau) \) we can safely use the linear approximation, based on the function \( f_1(\tau) \) only. It should be noted that parameter \( t_0 \) in Equation (46) is of order of the Hubble time \( 1/H_0 \). The comparison of the curves for different \( \tau = t/t_0 \) in Figure 4 shows that the relaxation time is very long. This is in agreement with the fact that clusters are known to be dynamically young objects, i.e., with crossing timescales that are non-negligible with respect to the age of the universe at the time of their formation. The fact that the relaxation time in Figure 4 is very long suggests that the dependence of the effective 1D distribution function \( H(\lambda, \tau) \) (and average angular momentum as a result) on the redshift should be weak. Also, the comparison of Figures 3 and 4 shows that the distribution of...
spin parameters of galaxy clusters depends on the cosmological parameters and generally speaking is weak.

5. DISCUSSION: RELATION TO OBSERVATIONAL RESULTS

Our theoretical results about the distribution function of momenta can shed some light on the problem of galaxies’ orientation in clusters. Our main message is that although the gravitational interaction between galaxies is of long-range character, the observations (which we will discuss below) may evidence that there is additional short-range intergalaxy interaction with characteristic radius $r_c$. This means that if the distance $r$ between two galaxies is smaller than $r_c$, they are correlated and have their orbital moments aligned. This is the case for the dense (rich) galaxy clusters, which, by this virtue, have a high degree of orbital moment alignment. In the opposite situation of poor clusters, where the intergalaxy distance $r > r_c$, the long-range multipole interaction of alternating sign dominates and the alignment of the orbital moments is absent. We speculate that this situation resembles that in diluted magnetic systems, where the presence of short-

Figure 4. Time evolution of effective 1D distribution function $H(\lambda, \tau)$ in the $\Lambda$CDM model for different $\alpha$. Upper panels: linear regime; lower panels: nonlinear regime. The notations are the same as in Figure 3.
range exchange interaction between magnetic spins promotes long-range magnetic order, which is characterized by macroscopic spin alignment; see Stephanovich (1997), Semenov & Stephanovich (2003), and references therein for details.

The aforementioned statistical method permits us to account for this situation if we add the (empirical) short-range interaction term to the initial potential (3). In this case, the distribution function of random fields would depend on the above average value of angular momentum $L_{\text{max}} = L_{av}$ as a parameter (see Stephanovich 1997; Semenov & Stephanovich 2003) so that the self-consistent equation for $L_{av}$ of the form

$$L_{av} = \int L(E)f(E, L_{av})d^3E \quad (48)$$

can be derived. Here $f(E, L_{av})$ is the distribution function of random gravitational fields, which substitutes expression (18) in the case of inclusion of the possible short-range interaction term. In such a case, for finite $r_s$, the distribution function decays at $E \to \infty$ faster than Equation (18), so that integral (48) converges. As now the total interaction potential includes both luminous and dark matter, Equation (48) permits us to address the question about alignment of subdominant galaxies, when most of cluster angular momentum is in the smooth dark matter halo component. Observationally, it is relatively easy to analyze the orientation of angular momenta in the luminous matter, i.e., in real galaxies and their clusters. With dark matter (sub)halos this is not so easy. One should not, however, forget that there are relations between the properties of luminous and dark matter. Several articles (e.g., Paz et al. 2008; Bett et al. 2010; Kimm et al. 2011; Varela et al. 2012) show clear observational evidence of the relation between the orientation of dark matter halos and galaxies; see, however, Tenneti et al. (2015) for an opposing opinion. The assumption that angular momentum of the luminous matter traces that of the associated dark matter (sub)structures (e.g., angular momentum totally aligned, one galaxy per subhalo) allows us to conclude that angular momentum alignment of galaxies gives us information about similar alignment in dark matter. In other words, the properties of angular momentum of luminous matter (like real galaxies) give us information about those of dark matter (sub)structures.

For luminous matter, it is possible to consider the relation between the angular momentum and the mass of a structure as based on the observational data (Godlowski et al. 2005; Godłowski 2011a). It is possible to investigate how this image varies with the mass of galaxy clusters, beginning with the simplest ones, i.e., galaxy pairs. These investigations show that their angular momentum originates mainly from the orbital motion of galaxies (Karachentsev & Mineva 1984a, 1984b; Mineva 1987). Helou (1984), examining a sample of 31 galaxy pairs, found that an “anti-alignment” of these galaxies’ spins occurs. Parmopyevsky et al. (1997) recognized a weak alignment in physical pairs of galaxies. Alignment in pairs of spiral galaxies was also discerned by Pestana & Cabrera (2004). Intrinsic spin alignment in galaxy pairs has independently been confirmed by Heymans et al. (2004) within their research on weak gravitational lensing, where it was necessary to estimate and remove the effects related to alignment of galaxy orientations. Also, the analysis of positions of the Milky Way’s companions shows their nonrandom distribution (they are located perpendicularly to the Milky Way’s disk), which can be regarded as their orbital alignment. Galaxies within compact groups rotate on prolate orbits about the group’s center (Tovmassian et al. 2002), which contributes to the system’s total angular momentum. Yang et al. (2006) found—while Sales & Lambas (2009) and Wang et al. (2009, 2010) confirmed it—that the companions of central red galaxies are aligned along their large axes. A similar result has been obtained by Ibata et al. (2013, 2014) in the two papers about the ordering of satellite’s orbits around M31. Their latest article (Ibata et al. 2015) also corroborates this result. Thus, it can be maintained that structures like galaxies and their companions, pairs, and compact groups of galaxies have a nonzero net angular momentum related mostly to their orbital motion. One should not forget, however, that the situation is more complicated in more massive structures. As there is no evidence for rotation of the groups and clusters of galaxies (see, e.g., Regos & Geller 1989; Diaferio & Geller 1997; Diaferio 1999; Rines et al. 2003; Hwang & Lee 2007), it is clear that angular momentum of such structures is related primarily to the alignment of constituents’ spins.

We should note here that there are many (seemingly contradictory) observational results regarding the alignment (or misalignment) of galaxies’ angular momenta in the literature, beginning with the paper of Thompson (1976), who found an alignment in the galaxy orientations in the Virgo and A2197 clusters. The evidence for alignment of galaxies belonging to the Virgo cluster had been found by Helou & Salpeter (1982) and MacGillivray & Dodd (1985). Nonrandom galaxy orientation was found in very rich galaxy clusters (Djorgovski 1983; Godłowski et al. 1998; Wu et al. 1998; Baier et al. 2003; Kitzbichler & Saurer 2003; Aryal et al. 2007; Godłowski 2012). The alignment of galaxy planes was also found in A1689 (Hun et al. 2010; Hung & Ebeling 2012). This result is important as A1689 is the most distant cluster where the alignment has been found until now.

There are also contradictory results. For instance, Bukhari (1988), Bukhari & Cram (2003), and Hofman et al. (1989) studied the orientation of galaxies within clusters and did not find any alignment. The same result has been obtained by Aryal & Saurer (2005) in the investigations of three Abell clusters of richness class zero. During studies of the isolated Abell groups (Flin & Olowin 1991; Trevese et al. 1992; Kim 2001; Niederste-Ostholt et al. 2010), only a rudimentary alignment was found and related only to the brightest cluster members. The alignment has also not been found during analysis of Tully’s groups of galaxies belonging to the Local Supercluster (Godłowski & Ostrowski 1999; Godłowski et al. 2005; Godłowski 2011b). Summarizing above observations of angular momenta misalignment, we can conclude that such misalignment had been obtained for less massive structures like group and poor galaxy clusters.

To check the hypothesis that the alignment of galaxies’ angular momenta increases with the cluster richness, Godłowski et al. (2010) examined the orientation of galaxies in clusters both qualitatively and quantitatively. The analysis of the spatial orientations of galaxies in the 247 optically selected rich Abell clusters, having in the considered area at least 100 members, has been performed by Godłowski et al. (2010). The structures have been taken from the PF catalog (Panko & Flin 2006). The statistical analysis, based on linear regression, permitted us to conclude that cluster angular momenta increase
with their numerousness. Note, however, that the relatively small statistical sample of 247 clusters, analyzed by Godłowski et al. (2010), does not give a possibility to discriminate between the linear dependence tested by Godłowski et al. (2010), the dependence $L_{\text{max}} \sim M^{2/3}$ (Catelan & Theuns 1999a), and our result (43) that mean angular momentum is proportional to $M^{1/3}$ (stemming from $N^{4/3}$). However, such detailed analysis would be possible if a larger statistical sample of galaxy clusters were available.

The above results show clearly that galaxy clusters have a nonzero net angular momentum related mostly to their orbital motion. For more massive structures there is a lack of alignment of the orientation of galaxies for group and poor galaxy clusters, while there is evidence for alignment for the rich clusters of galaxies (Godłowski et al. 2005; see also Godłowski 2011a for a later, improved analysis). It has been suggested that degree of alignment increases with cluster richness and as a result the cluster angular momentum increases with its numerousness. Here we emphasize once more that it has been found in the Abell 754 cluster (Baier et al. 2003), but the direction (relative to the main cluster plane) of the observed galaxy ordering is perpendicular to that for Abell 754. The interpretation of the above orientation difference has been presented by di Fazio & Flin (1988) in terms of two different types of galaxy clusters: oblate and prolate. One more interpretation can be made on the basis of the tidal interaction scenario. Namely, it has been observed by Paz et al. (2008) that in large-scale structure the direction of angular momenta (relative to its main plane) of constituting objects depends on the structure mass. The same result has been obtained by Trujillo et al. (2006) and Varela et al. (2012). The newest analysis (Codis et al. 2012) (based on the earlier studies of Lee & Pen 2000; Sugerman et al. 2000; Bailin & Steinmetz 2005; Aragon-Calvo et al. 2007; Hahn et al. 2007; Paz et al. 2008; Zhang et al. 2009) on the dark matter halos’ angular momentum orientation also confirms the above result. The studies of galaxies’ angular momenta ordering in large scale had been fulfilled by Paz et al. (2008) and Zhang et al. (2013), who use the data from the Sloan Digital Sky Survey catalog. They found that galaxies’ angular momenta align perpendicularly to the large-scale structures’ planes. The latter effect has not been observed for the structures with relatively small masses. These results agree well with the simulations of Paz et al. (2008) based on the tidal interaction mechanism. Jones et al. (2010) have found that the spins of spiral galaxies in the cosmic web have a tendency to align along the filaments’ axes, which has been interpreted as the “fossil” evidence of the effects of long-range tidal interactions.

The other interesting problem is the possible time evolution of galaxy clusters’ alignment. Assuming the Einstein–de Sitter model (Doroshkevich 1970), in Figure 3 we report the time evolution of the function $H(\lambda)$. It is interesting that at time growth the distribution function goes to zero. This means that older structures (clusters) should have more scattered observed values of angular momenta than younger ones. At the same time, Equation (43) shows that mean angular momentum of the clusters should increase in time.

The latter result is obtained in the Einstein–de Sitter model but is still valid for any similar cosmological models. The predictions will be available to verify when we get better data concerning alignment in galaxy clusters. Note, however, that even now we have observational results suggesting the possible evolution of alignment with redshift. There are, for example, the results of Song & Lee (2012), who found that the alignment profile of cluster galaxies drops faster at higher redshifts.

6. CONCLUSIONS

The main physical result of the present paper is the calculation of the distribution function of the gravitational fields of astronomical objects like galaxy ensembles and smooth halos based solely on the tidal interaction between constituting elements. We show that for tidal (quadrupolar) interaction the distribution function cannot be Gaussian, its explicit form being presented by Equation (18). We emphasize here that the derived distribution function of gravitational fields does not depend on the specific Eulerian or Lagrangian description of Newtonian matter and thus can be used to calculate virtually any observable characteristic of the stellar ensemble. As an example, we use the above distribution function to calculate the distribution of angular momenta. From the astronomical interpretation point of view, it is important that for a particular cluster with richness $n$ we expect not a single value of angular momentum but the range of allowed values described by the probability function. As the distribution function (18) slowly decays at infinity, its first moment does not exist because the corresponding integral is divergent. To calculate the mean value of angular momentum in this situation, we assume that the maximal value of distribution function $L_{\text{max}}$ gives the desired quantity. We note that such a procedure is usual while dealing with so-called heavy-tailed distributions (see, e.g., Kapur & Kesavan 1992 and references therein).

As masses, radii, and number of astronomical objects (for instance, galaxies) enter Equation (18) as parameters, we were able to show that the mean value of angular momentum $L_{\text{max}}$ for a particular galaxy cluster of mass $M$ is proportional to $M^{3/5}$, thus corroborating the well-known (see Schaefer 2009 and references therein) empirical result. Our other result $L_{\text{max}} \sim N^{4/3}$ (or $M^{3/5}$, Equation (43)) has also its astronomical interpretation that larger (richer) clusters of galaxies have higher angular momentum; see also the discussion below. The observational discrimination between the dependencies $M^{5/3}$ and $M^{3/5}$ would be possible when a larger statistical sample of galaxy clusters is available. The parametric time dependence of $L_{\text{max}}$ via functions $a(t)$ and $D(t)$ in the Einstein–de Sitter model permits
us to trace its time evolution. This shows that our approach to
derivation of the distribution function of galaxies’ angular
momenta gives physically reasonable answers. We have also
analyzed the time evolution of the distribution function. It is
reported in Figure 3 and shows that the distribution function is
flattening with time.

The relation between angular momentum and mass of the
structures has been extensively analyzed theoretically (Mur-
daryan 1975; Wesson 1979, 1981, 1983; Carrasco et al. 1982;
Sistero 1983; Brosche 1986; Mackrossan 1987; Paz et al. 2008); see Schaefer (2009) for review. This relation has
usually been presented empirically in the form \( L_{\text{max}} \sim M^{2/3} \).
From the point of view of modern scenarios of galaxies and
their structures’ formation, an increase of angular momentum
with the cluster richness could be explained only in the tidal
force scenario in the hierarchical clustering model (Heavens &
Peacock 1988; Noh & Lee 2006a, 2006b; Hwang & Lee 2007)
and in the Li model (Li 1998; Godlowski et al. 2005). One
should note, however, that the value of the universe rotation,
required by Li (1998), is too large as compared to the
anisotropy found in cosmic microwave background radiation.

The increase of the galaxies’ angular momentum with mass of
the structure was found observationally during analysis of the
alignment of galaxies in clusters (Godlowski et al. 2005, 2010; Aryal et al. 2007; Godlowski 2011a, 2011b, 2012). Since
it is commonly agreed that groups and clusters of galaxies do
not rotate (Regos & Geller 1989; Diaferio & Geller 1997;
Diaferio 1999; Rines et al. 2003; Hwang & Lee 2007), any
possible nonzero angular momentum of such structures should
arise only from possible alignment of galaxy spins, and
stronger alignment means greater angular momentum of such
structures. Generally, there is no evidence for a nonzero
angular momentum of galaxies or group and poor galaxy
clusters, while we observe such alignment for rich clusters of
galaxies and superclusters. We speculate that this phenomenon
may occur owing to some additional short-range interaction
between galaxies such that in rich clusters the galaxies are
correlated as they fall in the range of this interaction and hence
have their angular moments aligned. In such situations there
should be some critical richness \( n_{cr} \), related to the interaction
range \( r_c \), such that only clusters of richness \( n > n_{cr} \) would have
their spins aligned. We note here that such a physical picture is
common for disordered magnets and ferroelectrics (see Ste-
phanovich 1997; Semenov & Stephanovich 2003 and refer-
ences therein). We postpone the quantitative investigation of
this interesting question for future publications. The above
scenario can we applied to the problem of the possible merger
of galaxies into larger objects. The corresponding results will
also be published elsewhere.

The problem of the merger of galaxies in a cluster is related
to that of the role of a much more massive central galaxy. This
problem, in turn, is due to the fact of (generally speaking
random) interaction between cluster members and dynamic
evolution of the nearby (to a specific cluster member)
structures. The problem of dynamic evolution can be studied
by the combined examination of the mutual orientation of
galaxies in clusters and the Binggeli effect (Binggeli 1982;
Struble & Peebles 1985; Chambers et al. 2000; Hashimoto
et al. 2008; Godlowski & Flin 2010). More specifically, this
can be done by two methods. The first is Binggeli effect
studies, i.e., the investigation of the relation between positions
of great axes in groups or clusters of galaxies and directions
toward their neighbors. The second one is the study of mutual
orientation of the brightest galaxy (and other bright galaxies) in
a structure relative to the position of cluster great axes or even
examination of structure ellipticity redshift dependence,
especially in the enlarged observational samples. The analysis
of the differences between position angles of the Tully’s groups
of galaxies belonging to the Local Supercluster shows (God-
lowski & Flin 2010) that there exists the alignment of the line
joining two brightest galaxies with both the position angle of
the parent group and the direction toward Virgo cluster center.
This analysis reveals the following picture of the structure
formation. The two brightest (most massive) galaxies were
formed first. They originated in the filamentary structure
directed toward the center of the protocluster. This is the place
where the Virgo cluster center is located now. Owing to
gravitational clustering, the groups were formed in such a
manner that galaxies follow the line determined by the two
brightest (most massive) objects. Therefore, the alignment of
the structure position angle and the line joining two brightest
galaxies is observed. The other groups were formed on the
same or a nearby filament. This shows the particular role
played by the more massive (brighter) galaxies. In our future
investigations, we will analyze the particular role played by
the central (most massive) galaxies quantitatively.

One should note that the theoretical and observational
analysis of galaxy alignment in clusters is also very important
from the point of view of weak gravitational lensing (Troxel &
Ishak 2014; Joachimi et al. 2015). There is mutual influence of
the orientation of galaxies and weak gravitational lensing. It
should be pointed out as well that the examination of galaxies’
orientations is also meaningful owing to one of the outcomes
of the activity of the gravitational lensing effect (Heavens
et al. 2000; Schneider 2006), which is the alignment of the
galaxies’ images. Such alignment, in several Mpc scale, is also
expected in the case of cosmic shear existence. Crittenden et al.
(2001) proved that at least in the scenario of tidal interactions,
the effects of alignment can be distinguished from the effects of
weak gravitational lensing. Taking both of these effects into
consideration (in appropriate proportions) is of paramount
importance for mapping the mass distribution with weak-
lensing techniques, and vice versa: weak-lensing-induced
shape deformations are important for studies of the intrinsic
orientation of galaxies within structures. Therefore, weak-
lensing studies will allow investigating mass distribution in
clusters, which is important for studies of dark matter in them.

Let us finally summarize the simplifying assumptions made
in the above calculation of the distribution function. We
assume that the autocorrelation radius is constant and is the
same for each cluster (Peebles 1973, 1980; Lin et al. 1996;
Tucker et al. 1997; Peacock 1999, p. 296; Longair 2008). Also,
we treated cluster density as a parameter rather than a function
of interobject (intergalaxy) distance. We note here that as time
(via the functions \( f_{1,2}(t) \), calculated in CDM and \( \Lambda \)CDM
models in linear and nonlinear fluctuation growth regimes)
enters the distribution function as a parameter, our approach is
valid for any cosmological model—from conventional Ein-
stein–de Sitter CDM (Doroshkevich 1970) to \( \Lambda \)CDM. The
most important simplification is to assume that all galaxie-
s have equal masses. In the future studies we plan to consider a
more realistic situation, introducing \( n = n(r) \), the real (i.e.,
extracted from observational data) galaxy mass distribution
function and the short-range term in the potential of interaction
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