Perfect cylindrical cloak under gyration, non-inertial effects make perfect cloak visible

Saeed Hasanpour Tadi\textsuperscript{a} and Babak Shokri\textsuperscript{b}

\textsuperscript{a}Laser and Plasma Research Institute, Shahid Beheshti University, Tehran, Iran; \textsuperscript{b}Physics Department of Shahid Beheshti University, Tehran, Iran

\textbf{ABSTRACT}

For the first time, electromagnetic wave interaction with a perfect non-inertial cloak is investigated. The electromagnetic detection of an ideal gyrating cloak, the effect of rotational speed on the scattering pattern, and the determination of rotational direction and rotational velocity magnitude are investigated in the present paper. It is shown that a rotating cloak is electromagnetically detectable due to the magneto-electric and Sagnac effects. These phenomena create bending in the passing wave pattern and generate cylindrical wavelets that are observable in the electromagnetic scattering pattern. It is shown, the gyrating cloak’s speed and direction can be determined using the amplitude of wavelets and bending location in the pulses passed because they increase with growing angular frequency. At low frequency gyration, the distortion of waveform pattern is humble and negligible; however, when the rotational frequency increases, the pattern distortion becomes huge, wavelets are formed, and their intensity grows. The direction of rotation can influence the scattering pattern; thus, the rotation direction could be identified with the scattering pattern analysis. The simulation is computed using the FDTD method by transforming Maxwell’s equations from the non-inertial framework to the lab framework.

\textbf{Introduction}

One of the interesting topics in the scope of electromagnetism is invisibility. After conceptualizing and introducing the principles of Transforming Optics (TO) \cite{1}, various work, such as perfect cloaking \cite{2}, hidden under carpet \cite{3} out of shell cloaking \cite{4}, multi-folded invisibility \cite{5} plasmonic coaking \cite{6} fishnet metamaterial \cite{7} were carried out. Opposite the invisibility, various techniques were recommended to detect and trace these targets. Opposite the invisibility, various techniques are recommended to detect and trace these targets. For example, transition and Cerenkov radiation are transmitted from an electron beam while interacts with the perfect and the carpet clock structures to electromagnetically detect the invisible target \cite{8, 9}.

Electromagnetic pulse interaction with the perfect cloak is another way to observe the cloaks. In this method, as time is a non-transformed parameter in transformation
optics, the cloaking structure can be detectable in the time-domain opposite of frequency-domain [8]. Besides, lateral shift [10], nonlinearity [11], and dispersive effects [12] can be utilized for tracking cloaking objects. All previous works have proposed the cloaking structures in the stationary state while motionless (rotation and displacement). Moreover, the effect of the motion on the invisible cloaks was studied and it was confirmed that Fizeau drag and dispersion effects at relativistic speeds could destroy the structure performance [13, 14]. Consequently, an invisible construction can be detectable in non-stationary states.

Opposite the previous surveys in which the cloaks were in the inertial state, this research, for the first time, the non-inertial effects of electromagnetic wave interaction with a perfect rotational cloak are investigated. A 2D cylindrical perfect cloak rotating around its axis with uniform angular velocity is in a non-inertial state without variations in its construction and symmetrical parameters over the time. Electromagnetic wave interaction with the inertial and the non-inertial structures creates different scattering patterns; this can be useful for tracking and measuring the speed and distance of invisible objects. Differences between the inertial and non-inertial states are caused by increasing the magneto-electric and the Sagnac effects that disturb cloaking performance when the angular frequency increases.

In magneto-electric materials, the electric displacement vector (D) depends on the electric and magnetic field strength. Furthermore, magnetic field flux is associated with magnetic field strength and electric field. Regarding magneto-electric coupling, these materials are called the bianisotropic medium [15]. This phenomenon is observed when a dielectric or magnetic structure rotates. According to the relativistic theory, electric and magnetic fields can be different for various observers. Thus from the inertial observer’s perspective, a gyrating dielectric becomes a bianisotropic structure [15]. From this point of view, an ideal cloak, which is not bianisotropic, becomes a bianisotropic structure under gyration. Under gyration, the transition from the anisotropic state to the bianisotropic state leads the cloak to be visible, while cloaks lose their performance by magneto-electric coupling [16].

Cloaking objects with the transformation optics method are sensitive to small disturbances in their parameters [17], thus any variation in the electromagnetic parameters makes them detectable.

Another phenomenon that can be observed in gyrating structures is the Sagnac effect [18]. In fact, due to the difference between the optical path in the direction of rotation and its opposite, there is a phase difference between the two beams; this phase can be investigated for measuring the speed of rotation. The phase difference is related to the size of the circumferential area and its angular frequency [16].

As the electromagnetic pulse is split into two parts while passing the cloak (top and bottom sides), movement in two separate paths with different flying times creates a phase difference between these two parts, similar to the Sagnac phenomenon. The phase difference makes deformation in the electric field pattern compared to that in the inertial state when these two splits interfere. This issue can warp the pulse and damage the cloak. In the present study, the FDTD method is employed to simulate the impact of rotation on the gyrating 2D cylindrical cloak by transforming Maxwell’s equations from the non-inertial form to the inertial form. FDTD is one of the most widely applied computational methods in solving electromagnetic problems. This method is significant because of its
high speed and accuracy in problem solving. This method has been used to solve various problems in the interaction of electromagnetic waves with complex electromagnetic structures such as metamaterials, anisotropic materials, nanophotonics, plasma, biophotonics, and bioelectromagnetism. In the present paper, the effect of rotation on the perfect cloak’s structure is investigated using the upgraded FDTD method [19], which is used to simulate the Sagnac effect of a rotating optical waveguide. First a simple dielectric structure is simulated and then the FDTD equations are rewritten for the cloak structure because the perfect cloak has complex electromagnetics parameters. Making objects invisible with transformation optics methods needs a complex medium provided by metamaterial structures.

Material and method

Maxwell’s equations in a non-inertial framework can be rewritten as [19]

\[
\frac{\partial D}{\partial t} = \nabla \times H - J, \tag{1}
\]
\[
\frac{\partial B}{\partial t} = -\nabla \times E - M, \tag{2}
\]
\[
D = \varepsilon E - C^{-2} /Omega_1 \times r \times H, \tag{3}
\]
\[
B = \mu H + C^{-2} /Omega_1 \times r \times E, \tag{4}
\]

Where \( r = xe_x + ye_y + ze_z \), \( \Omega = \Omega e_z \), \( J = J_s + \sigma E \), \( M = M_s + \sigma m H \).

Here, we assumed that the medium rotates slowly around the z-axis with a uniform angular frequency \( \Omega \) while \( \Omega R \ll C \); \( R \) is the outer radius of the rotating medium and \( C \) is the speed of light in vacuum [19].

Maxwell’s equations remain invariant under coordinate transformation, whether the framework is inertial or non-inertial. In the first-order approximation of light speed, transformation between inertial (stationary) and non-inertial (rotating) frameworks is correlated with magneto-electric effect correction, which is shown in equations (3) and (4).

For the 2D cylindrical perfect clock electromagnetic scattering problem, permittivity and permeability are in the tensor form [2]

\[
\varepsilon_r = \mu_r = \frac{r - R_1}{r}, \tag{5}
\]
\[
\varepsilon_\psi = \mu_\psi = \frac{r}{r - R_1}, \tag{6}
\]
\[
\varepsilon_z = \mu_z = \frac{r - a}{r} \left( \frac{R_2}{R_2 - R_1} \right)^2, \tag{7}
\]

where \( R_1 \) and \( R_2 \) are inner and outer radius of the perfect cylinder cloak, respectively. For a uniform rotational motion, \( \Omega \) is constant, so \( \frac{\partial \Omega}{\partial t} = 0 \). Besides, a fixed axis in the rotating frame leads to \( \frac{\partial T}{\partial t} = 0 \). By transforming the cylindrical coordinate to the Cartesian ones,
permittivity and permeability tensors are given as below:

\[ \varepsilon_{xx} = \varepsilon_r \cos^2(\varphi) + \varepsilon_\varphi \sin^2(\varphi) \]  
\[ \varepsilon_{xy} = \varepsilon_{yx} = (\varepsilon_r - \varepsilon_\varphi) \sin(\varphi) \cos(\varphi) \]  
\[ \varepsilon_{yy} = \varepsilon_r \sin^2(\varphi) + \varepsilon_\varphi \cos^2(\varphi) \]  
\[ \mu_{xx} = \mu_r \cos^2(\varphi) + \mu_\varphi \sin^2(\varphi) \]  
\[ \mu_{xy} = \mu_{yx} = (\mu_r - \mu_\varphi) \sin(\varphi) \cos(\varphi) \]  
\[ \mu_{yy} = \mu_r \sin^2(\varphi) + \mu_\varphi \cos^2(\varphi) \]  

Substituting equations (3) and (4) into equations (1) and (2), assuming free space propagation (without electric and magnetic sources), considering \( \frac{\partial}{\partial z} = 0 \) for the 2D case, and making use of the tensor form of material properties for TM electromagnetic propagation, we can rewrite Maxwell’s equations as

\[
\frac{\partial H_x}{\partial t} = \left( \frac{\mu_{yy}}{\mu_{xx} \mu_{yy} - \mu_{xy} \mu_{yx}} \right) \times \left( -\frac{\partial E_z}{\partial y} - \sigma_m H_y - c^{-2}\Omega x \frac{\partial E_z}{\partial t} - \frac{\mu_{xy} \partial E_z}{\mu_{yy}} - c^{-2}\Omega y \frac{\mu_{xy} \partial E_z}{\mu_{yy}} + \frac{\mu_{xy}}{\mu_{xx}} \sigma_m H_y \right),
\]

\[
\frac{\partial H_y}{\partial t} = \left( \frac{\mu_{xx}}{\mu_{xx} \mu_{yy} - \mu_{xy} \mu_{yx}} \right) \times \left( -\frac{\partial E_z}{\partial x} \sigma_m H_y - c^{-2}\Omega y \frac{\partial E_z}{\partial t} - \frac{\mu_{xy} \partial E_z}{\mu_{xx}} \frac{\partial E_z}{\partial y} - c^{-2}\Omega x \frac{\mu_{xy} \partial E_z}{\mu_{xx}} + \frac{\mu_{xy}}{\mu_{xx}} \sigma_m H_y \right),
\]

\[
\frac{\partial E_z}{\partial t} = \frac{\partial H_x}{\partial x} - \frac{\partial H_y}{\partial y} - \sigma E_z - c^{-2}\Omega y \frac{\partial H_y}{\partial t} - c^{-2}\Omega x \frac{\partial H_x}{\partial t}.
\]

Combining equations (14) and (15) with equation (16) and neglecting \( c^{-4} \) terms leads to:

\[
\left( \frac{\partial E_z}{\partial t} + \frac{\sigma}{\varepsilon_{zz}} E_z \right) = \frac{1}{\varepsilon_{zz}} \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - c^{-2}\Omega y \left( \frac{\mu_{xx}}{\mu_{xx} \mu_{yy} - \mu_{xy} \mu_{yx}} \right) \right)
\]

\[
\times \left( -\frac{\partial E_z}{\partial x} \sigma_m H_y - \frac{\mu_{xy}}{\mu_{xx}} \frac{\partial E_z}{\partial y} + \frac{\mu_{xy}}{\mu_{yy}} \sigma_m H_x \right) - c^{-2}\Omega x \left( \frac{\mu_{yy}}{\mu_{xx} \mu_{yy} - \mu_{xy} \mu_{yx}} \right)
\]

\[
\times \left( -\frac{\partial E_z}{\partial y} - \sigma_m H_y - \frac{\mu_{xy}}{\mu_{yy}} \frac{\partial E_z}{\partial x} + \frac{\mu_{xy}}{\mu_{yy}} \sigma_m H_x \right) \right)
\]

According to the Berenger’s solution for the Perfect Matched Layer (PML) \([20, 21]\), \( E_z \) splits into two components \( E_{zx} \) and \( E_{zy} \). Therefore, equation (17) can be written as:

\[
\frac{\partial E_{zx}}{\partial t} + \frac{\sigma}{\varepsilon_{zz}} E_{zx} = \frac{1}{\varepsilon_{zz}} \left( \frac{\partial H_y}{\partial x} - c^{-2}\Omega y \left( \frac{\mu_{xx}}{\mu_{xx} \mu_{yy} - \mu_{xy} \mu_{yx}} \right) \frac{\partial E_z}{\partial x} - \sigma_m H_y \right)
\]

\[
- c^{-2}\Omega x \left( \frac{\mu_{yy}}{\mu_{xx} \mu_{yy} - \mu_{xy} \mu_{yx}} \right) \left( -\frac{\mu_{xy}}{\mu_{yy}} \frac{\partial E_z}{\partial x} + \frac{\mu_{xy}}{\mu_{yy}} \sigma_m H_y \right),
\]
Equations (14), (15), (18), (19) are solved with discretization based on the Yee lattice and leapfrog scheme, using exponential stepping for the rotating 2D cylindrical perfect cloak in free space. Out of cloaking structure the non-diagonal components of permittivity and permeability are zero and diagonal elements are equal to 1. Moreover, out of cloaking structure, $\Omega = 0$ for the inertial framework.
Figure 3. Electric fields amplitude at \( x = 5 \) cm in for \( \Omega = 0–10^9 \text{ rad s}^{-1} \) (a), amplitude differences between stationary state and \( \Omega = 10^9 \text{ rad s}^{-1} \) (b).

**Result and discussion**

This section presents the simulation results of the interaction of electromagnetic waves with a rotating 2D perfect cloak. As mentioned, the interaction of an electromagnetic pulse with a cloak structure in the time domain can make it visible. In this situation, the direction of the rotation and the cloak structure’s angular velocity can be measured according to the electric or magnetic field phase shift, as well as the Sagnac effect.

The simulation’s geometry and the 2D cylindrical perfect cloak placed in the center of simulation medium and gyrating around \( z \)-axis is depicted in Figure 1. Here, \( R_1 = 1 \) cm and \( R_2 = 2 \) cm is the interior and exterior boundary of the cloak, respectively. Background medium is free space. Since only the cloak gyrates and the background is in the rest frame,
then $\Omega = 0, \varepsilon_{ij} = \mu_{ij} = 0$ if $i \neq j$ and $\varepsilon_{ij} = \mu_{ij} = 1$ if $i = j$ outside the cloak. Thus, outside the cloak, equations (17) and (18) change to the simpler equations without magneto-electric coupling terms. Moreover, $\sigma = 0$ entire the simulation medium and the cloak structure, except at boundaries of simulation medium, which act as PML.

**Plane wave interaction with stationary and non-stationary cloak**

Continuous plane waves of 20 GHz frequency interact with the cloak displayed in Figure 2. The cloak is gyrating anti-clockwise with $\Omega = 0 - 10^9 \text{ rad s}^{-1}$. By increasing angular frequency, plane wave scattering pattern is destroyed due to time-domain affected scattering and Sagnac effect. As shown in Figure 2(a) and (b), the plane wave pattern does not significantly change after passing through the cloak and maintains its shape. As the angular frequency increases, the passing wave pattern is disturbed due to the increasing rotation term in the Maxwell equations (growth of the magneto-electric), as seen in Figure 2(c) and (d). In Figure 2(d), a large perturbation is clearly apparent in the scattering pattern. Although the cloak has cylindrical symmetry, the distorted pattern is nonsymmetrical. The electromagnetic fields are bent within the cloak structure because of the anti-clockwise gyration. The pattern’s asymmetry depends on rotation direction, while the magnitude velocity determines the disturbance’s intensity.
This effect could be employed to detect the non-dispersive of the perfect cloak when the structure is accelerating.

Figure 3 demonstrates the electric field received in a line parallel to the y-axis located in $x = 5$ cm. The phase difference of the electric field on the left and right of the curve is created because of the asymmetry in the magneto-electric coupling and Sagnac effect. Since the direction of the cloak velocity at the top and the bottom side is opposite, the continuous plane wave scattering pattern is deformed by increasing $\Omega$, as mentioned before in the Sagnac effect. Phase retardation and warped electric field at $x = 3$ is shown in Figure 3 (red circles).

**Gaussian and Gaussian sine wave scattering**

To accurately analyze the gyration effect on the scattering pattern, Gaussian and Gaussian sine scattering from the cloak is simulated. Figure 4 illustrates the scattering pattern of the Gaussian sine pulse from the stationary and gyrating cloak. As compared to the stationary state, the electric field pattern changes after passing through the cloak because of wavelets’ production and bending in the passing pulse. Accordingly, in time domain a cylindrical wave is reflected from the cloak when an electromagnetic pulse interacts with a perfect cloak, which is accounted as a technique for detecting the cloak in the time-domain [10].
These cylindrical waves can be observed in Figure 4. However, while the angular frequency increases, tiny wavelets are formed, which are different from the scattering waves reported in [10]. Moreover, the wavelet tails’ size increases as the angular frequency increases.

To clarify better, the amplitude of the electric field is plotted logarithmically in Figure 5. In the stationary state ($\Omega = 0$), a perfect cylindrical scattering wave is apparent, while wavelets and bending in the passing pulse are absent. With increasing the angular frequency ($\Omega$), in the non-steady-state cloaks, magneto-electric effects generate wavelets (arrows in Figure 5) and the transient pulse is warped due to the Sagnac effect. This phase retardation in the electromagnetic pulse is huge near the line $y = 0$, and by distance away from it, the deformation decreases.

Measuring the electric field at (2.5, 0) coordinate over time provides results for investigating the influence of rotation on passing wave pattern, as shown in Figure 6. The shifts in main peaks of the pulse show the bending in Gaussian sine pulse (red circles) while at time 480–500 ns, wavelets appear at this point.

Also, the interaction of the Gaussian pulse with the rotating cloak is shown in Figure 7. Similar to Gaussian sine scattering, in which wavelets are produced and increased with
Figure 7. Logarithmic electric field of the Gaussian pulse interacting with rotating Cloak.

Figure 8. Electric field amplitude in line $x = 3$ at 471 ns.
Figure 9. Electric field amplitude difference between stationary and gyrating states in line $x = 3$ at 471 ns.

Increasing angular frequency, Gaussian plane wave pulse is warped. However, it is not as clear as in the Gaussian sine scattering pattern.

By probing a Gaussian pulse electric field in $x = 3$ cm at 471 ns after starting the simulation, we obtain curvatures plotted in Figure 8. Also, the differences between stationary and
gyrating states are plotted in Figure 9. As shown in Figures 8 and 9 these differences change with $\Omega$, which suggests a new measurement method for detecting the speed of rotation.

The asymmetry in the curves, as mentioned before, is due to the rotation of the structure. The phase difference between the two sides of the diagram in Figure 9 increases with increasing the angular frequency, while the peaks’ location has not changed. Only the shape of the curves is reversed by changing the direction of rotation that is shown in the next section.

**Detecting the direction of rotation**

An important point that can be determined through the Gaussian pulse interaction with the gyrating cloaks is the direction of rotation. Due to the Sagnac and magneto-electric coupling effects, an asymmetry forms in the electric field scattering pattern. This asymmetry is presented in Figure 10. This figure shows a tail that forms in the scattering pattern for the clockwise and counter-clockwise rotating cloaks (showing by arrows), which occurs at different locations (top and bottom side). When the angular frequency increases, this tail’s length becomes more extensive and more transparent, as seen in Figure 7. The location and direction of this tail indicate the magnitude of the gyration’s speed and direction, as shown in Figure 10.

**Conclusion**

In the present paper, we proved that a 2D perfect cylindrical cloak could be electromagnetically detected in the time domain. When a cloak is gyrating, the magneto-electric and Sagnac effects cannot be neglected. Hence, the scattering pattern destruction could determine the location of the cloak. Besides, the strength of pulse bending, and the size of the cylindrical wavelets produced in the rotating cloak permit us to measure the angular frequency and rotation direction. The present study’s achievement, compared to previous works, is to examine the Sagnac and Magneto-electric effects in the rotating cloaks. As a result of these effects, the deformation of the electric field pattern and wavelet generation can be applied to measure the speed and direction of rotation.

**Disclosure statement**

No potential conflict of interest was reported by the author(s).

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