Halo Shapes From Weak Lensing: The Impact of Galaxy–Halo Misalignment

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ABSTRACT

We analyse the impact of galaxy–halo misalignment on the ability of weak lensing studies to constrain the shape of dark matter haloes, using a combination of the Millennium dark matter N-body simulation and different semi-analytic galaxy formation models, as well as simpler Monte Carlo tests. Since the distribution of galaxy–halo alignments is not known in detail, we test various alignment models, together with different methods of determining the halo shape. In addition to alignment, we examine the interplay of halo mass and shape, and galaxy colour and morphology with the resulting stacked projected halo shape. We find that only in the case where significant numbers of galaxy and halo minor axes are parallel does the stacked, projected halo axis ratio fall below 0.95. When using broader misalignment distributions, such as those found in recent simulations of galaxy formation, the halo ellipticity signal is washed out and would be extremely difficult to measure observationally. It is important to note that the spread in stacked halo axis ratio due to theoretical unknowns (differences between semi-analytic models, and between alignment models) are much bigger than any statistical uncertainty: It is naive to assume that, simply because ΛCDM predicts aspherical haloes, the stacked projected shape will be elliptical. In fact, there is no robust ΛCDM prediction yet for this procedure, and the interpretation of any such elliptical halo signal from lensing in terms of physical halo properties will be extremely difficult.

Key words: cosmology: dark matter – methods: N-body simulations – galaxies: haloes – gravitational lensing: weak

1 INTRODUCTION

Dark matter haloes are irregularly-shaped virialised clumps of collisionless matter. In the simplest model of the Universe that is most compatible with current observations (ΛCDM), dark matter dominates the mass budget and haloes form from the collapse and hierarchical merging of matter in overdense regions. Galaxies form from gas that originally clustered with the dark matter, following baryonic processes (e.g. radiative cooling, star formation, etc) that lead to structures with very different properties and behaviour to the nearly-self-similar dark matter haloes.

Theoretical properties of dark matter haloes are now very well established, following decades of research using N-body simulations and advances in computing power. This work includes characterising the distributions and time-dependence of various properties, and correlations between them. The physical properties include spin (angular momentum), shape, density profile and concentration, phase-space density profile, clustering, and the relationship to structures on larger and smaller scales. Recent examples of such studies using large-scale cosmological simulations include Shaw et al. (2006); Altay, Colberg & Croft (2006); Allgood et al. (2006); Harker et al. (2006); Hayashi, Navarro & Springel (2007); Hahn et al. (2007b,a); Bett et al. (2007, 2010); Macciò et al. (2007); Macciò, Dutton & van den Bosch (2008); Neto et al. (2007); Gao et al. (2008); Zhang et al. (2009); Davis & Natarajan (2009); Boylan-Kolchin et al. (2010); Muñoz-Cuartas et al. (2011); Wang et al. (2011); Ludlow et al. (2011); Prada et al. (2011) and the recent novel studies using principal component analysis (Skibba & Macciò 2011; Jeeson-Daniel et al. 2011). Halo properties were recently reviewed in Taylor (2011). Of particular interest in this paper are dark matter halo shapes, which are known to have a broad distribution, with a preference for prolateness.

Since “dark” matter is by definition transparent, it is very hard to measure these properties directly using standard direct astronomical observations. Methods that utilise gravitational lensing however are sensitive to the entire mass distribution, not just the radiating baryonic component. This has lead to gravitational lensing being proposed as a key technique for studying halo properties observationally (see e.g. the reviews of Hoekstra & Jain 2008; Massey, Kitching & Richard 2010; Hutner 2010).

Early work on measuring halo mass distributions using weak galaxy–galaxy lensing was performed by Kaiser & Squires (1993); Wilson, Cole & Frenk (1996a,b); Schneider & Bartelmann (1997); Schneider & Rix (1997). Following these, Natarajan & Refregier
(2000) proposed a technique for using weak gravitational lensing to measure the ellipticity of haloes (see also Brainerd & Wright 2000, 2002). Consider the shear signal from weak lensing of background (‘source’) galaxies, due to the mass associated with a foreground (‘lens’) galaxy. In practice, this galaxy–galaxy lensing shear signal will be far too weak to be detectable from single lens galaxies, so the signal from many lens systems needs to be stacked. If we compare the tangential shear either side of the lens galaxy image’s minor axis with that either side of its major axis, then we can obtain a measurement of the flattening of the surrounding matter distribution. However, if the ellipticities of halos and galaxies are not consistently aligned, the stacking procedure will result in this anisotropic shear signal being washed out.

This method was first used by Hoekstra, Yee & Gladders (2004), on data from the Red-sequence Cluster Survey. Assuming a model in which the lensing halo and galaxy ellipticities are related through $\epsilon_{\text{halo}} = f \epsilon_{\text{gal}}$, they found a best-fit value of $f = 0.77^{+0.18}_{-0.21}$ (68% confidence level), and claimed to exclude the possibility of spherical haloes at 99.5% confidence. Parker et al. (2007), using the CFHT Legacy Survey, measured the ratio of the tangential shear to be $0.76 \pm 0.10$, excluding spherical haloes at ~2$\sigma$ and implying (via Brainerd & Wright 2000) a halo ellipticity of ~0.3. They also attempted to select mostly elliptical galaxies, which resulted in a more significant detection of ellipticity.

Mandelbaum et al. (2006) performed a very thorough analysis using data from the Sloan Digital Sky Survey (SDSS), which included photometric redshifts and galaxy colours (not available to the other two studies). However, they did not manage to definitively detect halo ellipticity, although they found a hint at different alignments for different galaxy types. Their work showed how sensitive the results are to the models used for interpretation: If they assumed Gaussian errors with a power law density profile (as in Hoekstra, Yee & Gladders 2004), they found $f = 0.1 \pm 0.06$ and $f = -0.8 \pm 0.4$ for red and blue galaxies respectively; if they instead assumed non-Gaussian errors and a Navarro, Frenk & White (1997) density profile, they instead found $f = 0.60 \pm 0.38$ (reds) and $f = -1.4^{+1.7}_{-2.0}$ (blues), where negative numbers mean anti-alignment of mass and light.

Unambiguous detection of dark matter halo ellipticity has been seen as an important goal, because it offers the prospect of falsifying alternative theories of gravity, such as MOND/TeVeS (Milgrom 1983; Bekenstein 2004) or MOG/STVG (Moffat 2006; Moffat & Toth 2009a). Such theories suffer from being more theoretically and computationally challenging compared to simple collisionless matter in Newtonian gravity, which has resulted in their theoretical predictions being less developed at present. Nevertheless, the formalism for gravitational lensing has been developed both for TeVeS (Bekenstein 2004; Chiu, Ko & Tian 2006) and recently for STVG (Moffat & Toth 2009c). Predictions of lensing from MOND actually predate the relativistic description from TeVeS (Morlock & Turner 2001), and predictions for the equivalent counterpart of “halo” shapes in MOND was given in Milgrom (2001) and Sellwood & Kosowsky (2002). A robust prediction from TeVeS/MOND is that the lensing signal away from the lens galaxy should be isotropic. Thus any detection of ellipticity – regardless of whether it agrees with the predictions from ACDM simulations – would falsify TeVeS. However, this result will only be strictly true for a well-isolated lens galaxy, which is harder to establish in practice. The presence of mass from nearby galaxies can produce effects which go against our intuitive understanding of gravity, e.g. STVG violates Birkhoff’s theorem (Moffat & Toth 2009b), and can appear to fit the Bullet Cluster (Brownstein & Moffat 2007) although in that context, Springel & Farrar 2007 showed that neglecting the hydrodynamics of the baryons is also greatly misleading. Thus, interpreting the results of anisotropic shear measurements, whether circular or elliptical, should be done with caution.

The problem of galaxy–halo alignment is central to this work. There is, essentially, no robust prediction of the relative orientation of galaxies within their haloes from theory or simulation. This is not to say that it has not been measured, but that the physical processes involved vary significantly from simulation to simulation, and the number of objects studied is often still small ($\lesssim 10^3$) compared to the large statistical samples used in observations and dark matter simulations ($\gtrsim 10^6$). However, all simulation work has been consistent in predicting a reasonably broad distribution of galaxy–halo alignments, albeit with variation in the median angle. These include van den Bosch et al. (2002), van den Bosch, Abel & Hernquist (2003), Yoshida et al. (2003), Chen, Jing & Yoshikawa (2003), Sharma & Steinmetz (2005), Bailin et al. (2005), Gustafsson, Fairbairn & Sommer-Larsen (2006), Croft et al. (2009), Romano-Diaz et al. (2009), Bett et al. (2010), Hahn, Teyssier & Carollo (2010), and Deason et al. (2011).

The qualitative impact of galaxy–halo misalignment on the method of Natarajan & Refregier (2000) is intuitive and well-known, but it has not been considered quantitatively. On the other hand, different models of galaxy–halo alignment have been used for studies of the intrinsic alignment problem in galaxy–galaxy lensing (Heavens, Refregier & Heymans 2000; Heymans et al. 2004, 2006), and for modelling the satellite galaxy distribution when considering cluster lensing (Okumura, Jing & Li 2009; Okumura & Jing 2009). Furthermore, variation between the predictions of different galaxy formation simulations and models, and even from different methods of measuring shapes of simulated haloes, are rarely considered when observations are compared to “the” theoretical prediction. The complex systematic problems that can affect observations and prevent straightforward interpretation, are however very well studied (e.g. Brainerd & Wright 2000; Hoekstra, Yee & Gladders 2004; Mandelbaum et al. 2005, 2006; Howell & Brainerd 2010).

In this paper, we focus therefore on quantifying the impact of galaxy–halo misalignment on stacked projected halo shapes, using a range of different models for galaxies, halo shapes and alignment distributions to highlight the uncertainty in the theoretical prediction. We do not proceed to make a direct anisotropic shear prediction from our results, as this is already well studied (e.g. Howell & Brainerd 2010), and will only serve to reduce any ellipticity signal.

This paper is organised as follows. In section 2, we describe in detail the simulation and series of models we use. This includes the simulation and galaxy formation models (section 2.1), different methods of measuring halo shapes from simulations (§2.2), and the different alignment models we consider (§2.3). Section 3 describes simple Monte Carlo tests of the impact of our alignment models on distributions of halo shapes. We present our results in section 4, as series of axis ratios generated by stacking samples of projected halo shapes, showing how they depend on the distributions of halo and galaxy properties. We discuss our conclusions in section 5.
2 MODELLING THE IMPACT OF MISALIGNMENT

2.1 The simulation

We use the original\(^1\) Millennium Simulation (MS, Springel et al. 2005), a very large N-body cosmological dark matter simulation of the large-scale structure of a ΛCDM universe. This simulation resolves many millions of objects at each timestep, providing the statistical power for describing distributions of dark matter halo properties very precisely. The simulation is in a periodic box of length 500 h\(^{-1}\) Mpc, populated with over 10 billion collisionless dark matter particles (2160\(^3\)), each of mass \(m_p = 8.60657 \times 10^8 h^{-1} M_{\odot}\) and a gravitational softening length of 5.0 \(h^{-1}\) kpc. The simulation code used was L-GADGET-2, a version of the Tree-PM code GADGET-2 (Springel 2005) that was specially optimised for massively parallel computations and low memory consumption.

The MS uses a set of cosmological parameters that were chosen to be consistent with the results of the 2dFGRS (Percival et al. 2002) and WMAP-1 (Spergel et al. 2003). We write cosmological density parameters as \(\Omega_i(z) = \rho_i(z)/\rho_{crit}(z)\), in terms of the mass density \(\rho_i\) of component \(i\) and the critical density \(\rho_{crit}(z) = 3H(z)^2/(8\pi G)\), where the Hubble parameter is \(H(z)\). For the cosmological constant, total mass, and baryonic mass, the MS uses values of \(\Omega_m = 0.75\), \(\Omega_b = 0.25\), and \(\Omega_{\Lambda} = 0.045\). The present-day value of the Hubble parameter is parameterised in the standard way as \(H_0 = 100h\) km s\(^{-1}\) Mpc\(^{-1}\), where \(h = 0.73\). The spectral index is \(n = 1.0\) and the linear-theory mass variance in \(8 h^{-1}\) Mpc spheres at \(z = 0\) is given by \(\sigma_8 = 0.9\).

2.1.1 Semi-analytic models

Various halo and galaxy catalogues from the MS have been made publicly available through an online database\(^3\) (Leson & the Virgo Consortium 2006). They are based on two independent semi-analytic code development programmes, that of the ICC in Durham (based on the GALFORM model), and the MPA in Garching. While these models (and those of other groups) have been very successful in many regards, no model has yet matched the full distribution of galaxy properties at all luminosities, colours and redshifts simultaneously. The galaxy catalogues we shall use in this presentation are the main body of the halo (the most-massive substructure, MMSS), plus subhaloes.

The procedure starts with the simulation particles grouped by proximity, using the well-known Friends-of-Friends algorithm (FOF, e.g. Davis et al. 1985), with a linking length of \(b = 0.2\) times the mean interparticle separation (e.g. Porciani, Dekel & Hoffman 2002). Within each FOF group, self-bound substructures are identified using the SUBFIND program (Springel et al. 2001). This is itself a two-stage process, first identifying candidate substructures by finding peaks in the density field, then performing an iterative unbinding procedure, successively removing particles not gravitationally bound to the candidate (a minimum mass of 20 particles is imposed for substructures). This results in a set of FOF particle groups, each comprising some unbound particles ("fuzz") plus zero or more self-bound structures, usually divided conceptually into the main body of the halo (the most-massive substructure, MMSS), plus subhaloes.

Using the FOF/SUBFIND catalogues from different output snapshots in the simulation, merger trees are constructed, identifying structures in one snapshot with their progenitors and descendants in other snapshots (Harker et al. 2006). Our haloes are defined using information from the merger trees as a final stage of refinement. Firstly, the fuzz particles are excluded, leaving the basic halo as the set of bound structures originating from the same FOF group. Then subhaloes are subjected to a splitting algorithm, allowing them to be separated off from the original halo. This attempts to identify subhaloes that are spatially but not dynamically linked to the halo. For example, a subhalo might have been linked into a FOF group solely by fuzz particles (now excluded), or it could simply be flying past the main halo without yet becoming bound to it.

This halo definition, and the merger trees themselves, were originally designed for use with the GALFORM semi-analytic model (following Helly et al. 2003), and its application to the MS in the B06 model. B06 model (Bett et al. 2007) studied the effect of different halo definition algorithms, comparing haloes from this method with

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\(^1\) A second Millennium Simulation (MS-II) has since been performed, using the same number of particles in a smaller volume; see Boylan-Kolchin et al. (2009) for details.

\(^2\) One can write the equivalent mass-density of the cosmological constant \(A\) as \(\rho_h = A/\rho_{crit}(z)\).

\(^3\) http://gavo.mpa-garching.mpg.de/MyMillennium3/ and http://galaxy-catalogue.dur.ac.uk:8080/MyMillennium/
those from simply using FOF without refinement, and those defined by an overdensity criterion to give a spherical halo boundary at the virial radius. In addition to a visual comparison (in real and velocity space), they also compared halo spin, shapes and clustering. In terms of halo shapes, haloes defined by a spherical boundary were (unsurprisingly) biased towards spherical, and the simple FOF haloes had a much broader distribution of shapes than the merger tree haloes. It should be noted that, while further testing of different halo-finding algorithms is beyond the scope of this paper, the choice of algorithm will affect the results and should be borne in mind when interpreting results here and in other studies.

The DLB07 model uses a slightly different halo definition, omitting the splitting procedure outlined above. This means that, from the point of view of the DLB07 galaxies, a halo consists of all the bound structures associated with the parent FOF halo. These halo catalogues therefore have slightly fewer objects than the halo catalogues we use here (and were used in B06). However, since both halo catalogues are built up from the same set of Subfind structures, it is straightforward to identify galaxies from both models that are associated with the same corresponding dark matter structure.

2.1.3 Selecting halo–galaxy systems

In this paper, we are interested in the possibility of measuring the shapes of sub-cluster-mass haloes observationally. Therefore, we should attempt to use observational selection methods when picking objects for study from the raw halo catalogues. At the same time, it is important when working with N-body simulations to define and select objects for study carefully to guard against biases due to numerical effects. A technique commonly-used when selecting haloes from simulations is to attempt to exclude unvirialised systems. Bett et al. (2007) applied a cut on the halo energy ratio (to select haloes in ‘quasi-equilibrium’, as an approximation to virialisation), only accepting haloes with $|1 + 2T/U| < 0.5$, where $T$ is the kinetic and $U$ the potential energy. While Bett et al. (2007) studied halo shapes in the MS, a large part of that paper was focused on the halo spin parameter $\lambda$. As originally defined (Peebles 1969, 1971), $\lambda$ is really only valid for isolated, virialised haloes, so this cut played an important role in excluding invalid objects. More generally, a virialisation-based cut can help exclude haloes that are poorly defined, for example those that are currently undergoing a merger. In this case, the boundaries of the halo itself, and thus most of its other properties, are also poorly defined. However, aspects of our halo definition – excluding unbound particles, and splitting off dynamically separate subhaloes – go a long way towards solving these problems, such that the explicit cut in $|1 + 2T/U|$ only affects a relatively small minority of haloes (see Bett et al. 2007). Since such a cut would be very difficult to apply accurately in observational data, we choose to not apply it here.

Another important cut usually applied to simulations is on the minimum number of particles for a halo, to ensure that haloes are well-resolved. Bett et al. (2007) showed that the shapes of haloes in the MS realised with fewer than $\sim 10^3$ particles were biased away from spherical towards prolateness. We do not automatically apply this cut, but we will test its impact on our results. This is related, in principle, to the cut in galaxy magnitude we describe below.

The physical processes experienced by galaxies in clusters are different to those of galaxies in lower-mass haloes. Furthermore, the observational techniques used to study them are also different; the method we are concerned with here does not apply in the same way. We therefore exclude galaxy clusters, by applying an upper mass cut of $M < 10^{13}h^{-1}\text{M}_\odot$ (in practice, a particle-number cut at $N_p < 11619$). This cut is also difficult to perform observationally, but it could be approximated by, for example, excluding the brightest galaxies (presuming that they are BCGs), or excluding regions where the galaxy number density is high. Some methods are detailed in Brainerd (2005).

We are interested in the shape of haloes of individual galaxies. We therefore need to maintain a 1:1 relationship between galaxies and haloes: this means excluding satellite galaxies and subhaloes, and restricting ourselves to central galaxies only. (While the distinction between haloes and subhaloes is very important when analysing data from simulations, it is admittedly much harder to ascertain observationally.) Since the halo definition algorithm we use corresponds to that used in the B06 model, we shall base our analysis on that catalogue. This means that we can simply select galaxies from B06 that have been tagged as ‘centrals’ in the database (i.e. using Type = 0 in the B06 database table).

Galaxies from the DLB07 catalogue are selected by identifying the galaxies belonging to the same Subfind structures as the corresponding B06 galaxies. In most cases, these will also be central galaxies (as the MMSS of a B06 halo is likely to be the MMSS of a DLB07 halo). However, sometimes a halo identified in the B06 model (and our halo catalogues) will be considered to be a subhalo in the DLB07 model. This means that the associated galaxy could have evolved significantly differently to its B06 counterpart, as (in both models) central and satellite galaxies are treated differently. Nevertheless, each galaxy will still be the central galaxy of the same mass structure. While this means that our DLB07 galaxy sample is not the same as just selecting Type = 0 galaxies in the DLB07 database table, it should not have a very strong impact on the main results of this paper. Indeed, since this is effectively incorporating the theoretical uncertainty in determining satellites and central galaxies, it helps to overcome some of the artificialness of the Type = 0 selection, and mimic to some degree the difficulty in distinguishing centrals/satellites observationally. While a full study on the systematic differences between Type = 0 galaxies and field galaxies in general is beyond the scope of this paper, it is important to note that the halo mass function means that most galaxies ($\geq 80$ per cent) of a given brightness are centrals: Lower-mass haloes that can host galaxies of a given magnitude as centrals are always much more abundant than higher-mass haloes able to host them as satellites (although such galaxies are likely to differ systematically in other properties). Hence, the Type = 0 cut retains most galaxies of each magnitude.

In an effort to match observational samples as much as possible, we select (‘lens’) galaxies using a cut in apparent observer-frame $r$-band magnitude. While similar studies with the SDSS have selected galaxies with $r < 19$ (Mandelbaum et al. 2006), we take our limit from the upcoming KIDS survey, and use $r < 24.3$ (e.g. Kuijken 2006, 2010). This is admittedly a rather optimistic limit, but we want to avoid handicapping our data unnecessarily.

We perform our analysis on the redshift $z = 0.32$ data from the MS (output snapshot 52), again based on the expected median redshift of gravitational lenses in KIDS. At this redshift, our limiting apparent magnitude of $r_{\lim} = 24.3$ translates into an (observer-frame) absolute magnitude of $M_r = 5 \log_{10} h = r_{\lim} - 5 \log_{10} D(z) + 5 = -16.1$, where $D(z)$ is the luminosity distance in parsecs. We show the joint distributions of galaxy magnitude and halo mass for our two semi-analytic models in Fig. 1.

While, in reality, objects would be observed over a broad range of redshifts, it is more straightforward to use just a single snapshot
for our purposes. This gives us many millions of objects already, so we do not need to use other snapshots to improve our sample size. We do perform our analysis at other discrete redshifts however, and show these results in Appendix C.

In addition to galaxy magnitude, we shall also be looking at morphology and colour as ways of selecting objects to improve halo shape measurements. For morphology, we use the stellar-mass bulge-to-total ratio $B/T$. We classify galaxies according to whether they are bulge-dominated or disc-dominated: in particular, for convenience we describe those with $B/T > 0.5$ as “ellipticals”, and those with $B/T < 0.5$ as “discs”. The $B/T$ distribution from both semi-analytic models is strongly bimodal: there is a very strong peak for discs at $B/T \lesssim 0.005$, a much smaller but similarly narrow peak for ellipticals at $B/T \gtrsim 0.995$. There is also a significant but low-population set of intermediate-morphology galaxies, covering 34% of selected galaxies at $z \approx 0.3$ in the B06 model, and 27% for the DLB07 model. Parry, Eke & Frenk (2009) split the galaxy populations from these models into three samples ($B/T < 0.4$, $0.4 \lesssim B/T \lesssim 0.6$ and $B/T > 0.6$), but for our purposes simply splitting into two samples at $B/T = 0.5$ is sufficient.

We have tested three different measures of galaxy morphology in the B06 model: by stellar mass, by $r$-band magnitude, and by $g$-band magnitude. While the latter two correlate well with each other, they can scatter somewhat when compared to morphologies determined by stellar mass, with more galaxies appearing to have more intermediate morphologies when determined by magnitude. However, dividing our galaxy population simply into two broad morphological categories means that the vast majority of galaxies fall into the same category regardless of the measure used.

Following Strateva et al. (2001) (see also Baldry et al. 2004), Mandelbaum et al. (2006) make the division between “red” and “blue” galaxies at SDSS rest-frame $u-r = 2.22$. The semi-analytic models do not reproduce the observed colour distribution, although the colours are easily divided into red and blue samples. Empirically, we find that in the B06 model, we need to place that cut at rest-frame $u-r = 0.9$. For the DLB07 model, only the observer-frame magnitudes are available, meaning we cannot directly compare galaxy colours at different redshifts since a $K$-correction has not been applied. However, examination of the colour distributions at $z \approx 0.32$ suggests an empirical colour-cut of observer-frame $u-r \approx 3.5$. We show the colour distributions at different redshifts in Fig. 2.

It is important to note that galaxy colour and morphology are distinct, albeit related properties – see for example the comparison of galaxy morphologies and colour in the SDSS in Benson et al. (2007). We will discuss how the distributions of colour and morphology are related for the galaxy catalogues we use, in the Results section.

### 2.2 Shapes of dark matter haloes

Dark matter haloes are irregularly-shaped clumps of material, defined in principle by an envelope that demarcates either a given mass-density threshold, or – if one is more concerned with dynamic properties, such as virialisation – a threshold in phase-space density. In practice however, for ease of both definition and comparison between haloes, the shape of a halo is usually characterised by the ellipsoid defined by the eigenvectors and eigenvalues of a matrix describing the halo’s internal mass distribution. There are many ways to measure the mass distribution however, and different authors measure halo shapes from simulations using different matrices. Each of these methods have their own advantages and disadvantages, and not all are so relevant for observational studies. Observations do need to be compared with theoretical predictions from simulations however, and this can be complicated by the variation due to the range of methods used by theorists. Therefore, while an in-depth comparison of different methods of measuring halo shapes is outside the scope of this paper, we nevertheless elect to test four different shape tensors rather than picking just one, to highlight the variation in the theoretical predictions. We compare them briefly at the end of this subsection.

We describe the four halo shape algorithms we use below. Throughout, the tensor/vector components have indices $i$, $j$ = $\{1, 2, 3\}$, and the halo has $N$ particles indexed by $p$. The particles have positions $r_p = (r_{p1}, r_{p2}, r_{p3})^T$ with respect to the halo centre.

#### 2.2.1 The simple inertia tensor

The inertia tensor $I$ directly relates angular momentum $J$ and angular velocity $\omega$ through $J = I \omega$, and has components

\[
I_{ij} = \sum_{p=1}^{N} m_p \left(r_p^2 \delta_{ij} - r_p r_{p(i)} r_{p(j)}\right),
\]
where $\delta_{ij}$ is the Kronecker delta. Choosing a coordinate frame in which $I$ is diagonal (i.e. the eigenframe) is equivalent to finding the preferred axes of rotation of the object, i.e. the set of axes in which a torque about one does not induce a rotation about another. The axis directions are given by the eigenvectors, and the eigenvalues are the moments of inertia. The axis lengths $a \geq b \geq c$ are given by the square root of linear combinations of the moments of inertia per unit mass (e.g. Bett et al. 2007). These principal axes define the equivalent homogeneous ellipsoid that has the same moments of inertia – i.e. the same behaviour under rotations – as the halo itself. Axis ratios are usually denoted $s = c/a$, $q = b/a$, and $p = c/b$.

Unless these relations to $J$ and $\omega$ are directly relevant however, it is slightly simpler computationally to use the tensor of the quadrupole moments of the mass distribution, $M$, which has components

$$M_{ij} = \sum_{p=1}^{N} m_{p} r_{p,i} r_{p,j}. \tag{2}$$

This has the same eigenvectors as $I$, and the eigenvalues per unit mass give the squares of the ellipsoid axes lengths directly. The two tensors are related through $I_{ij} = \text{Tr}(M) \delta_{ij} - M_{ij}$, and the quadrupole tensor is often referred to as the inertia tensor in the literature (Binney & Tremaine 2008; Zhang et al. 2009; Bett et al. 2007, 2010). We shall refer to this as the simple inertia tensor $M_{\text{simp}}$ for brevity.

This tensor has been very widely used in the literature; other recent users include Faltenbacher et al. (2002), Kasun & Evrard (2005), Hopkins, Bahcall & Bode (2005), Shaw et al. (2006), Altay, Colberg & Croft (2006), Hahn et al. (2007b,a), Heller, Shlosman & Athanassoula (2007) Romano-Díaz et al. (2009), and Jeeson-Daniel et al. (2011).

2.2.2 The reduced inertia tensor

A commonly-used variation on the simple inertia tensor is to counterweight each particle by its distance from the centre, i.e. use the tensor with components

$$M_{ij} = \sum_{p=1}^{N} m_{p} r_{p,i} r_{p,j}. \tag{3}$$

This is done to remove bias due to, for example, large subhaloes located on the outskirts of the halo (Gerhard 1983). In this “reduced” inertia tensor (which we shall refer to as $M_{\text{red}}$), the particles are projected onto a unit sphere, and the shape measured is a description of the mass distribution in different directions; each particle contributes its mass equally. This can provide a better description of the “underlying” halo shape rather than just the distribution of subhaloes; whether or not one considers the subhaloes to be a distinctive aspect of the halo shape or an annoyance that needs to be removed depends on the study in question. For observational studies, the influence of the subhalo distribution is likely to be an important part of the measurement; furthermore, the weighting would be difficult to perform accurately. This method has been used recently by Bailin & Steinmetz (2005).

2.2.3 The iterative simple inertia tensor

In addition to the two preceding direct methods, iterative methods based on the same principles are also often used. The procedure we use is the following (e.g. Katz 1991):

(i) Compute the inertia tensor $M_{\text{simp}}$ using all the halo’s particles, yielding initial axis lengths $a, b, c$. This initial halo has a radius $R$.

(ii) Select the particles within the ellipsoid just defined, i.e. only the particles for which the elliptical distance satisfies

$$r_{p,i}^2 \equiv \frac{r_{p,i}^2}{q^2} + \frac{r_{p,j}^2}{s^2} + \frac{r_{p,k}^2}{c^2} \leq R^2 \tag{4}$$

where we use the axis ratios $q = b/a$ and $s = c/a$.

(iii) Using this new particle set, recompute $M_{\text{simp}}$.

This corresponds to keeping the major axis $a$ constant. An alternative is to keep the volume constant, using the condition $\frac{r_{p,i}^2}{q^2} + \frac{r_{p,j}^2}{s^2} + \frac{r_{p,k}^2}{c^2} \leq 1$.
The process is deemed to converge when, after a given iteration \( k \),
\[
\left| 1 - \frac{x_k}{x_{k-1}} \right| < 0.01 \quad \text{and} \quad \left| 1 - \frac{q_k}{q_{k-1}} \right| < 0.01.
\]

The process is deemed to have failed to converge if it takes more than 100 steps, or the shape ellipsoid comprises fewer than 10 particles. We denote the resulting tensor after convergence as \( M_{\text{Simple}} \).

This procedure (or close variants of it) is often used in situations where the set of particles comprising the object is unknown. For example, iterative shape-finding might be used as part of the halo-finding algorithm, so that the resulting halo has an ellipsoidal boundary that agrees with the measured shape exactly (within a given tolerance), rather than the shape ellipsoid being an approximation to the real shape of a pre-determined particle set. Another important usage case is when ellipsoidal density profiles are required, or just the shape profiles themselves. Rather than use spherical bins in halo radius, one uses the equivalent elliptical radii \( \hat{r} \), and the axis ratios \( q(\hat{r}) \) and \( s(\hat{r}) \) are computed iteratively in each bin.

Neither of these cases are relevant to us here. Our haloes are already defined using a more sophisticated method including particle proximity (the FOF step), binding energy (the \( \text{Sub} \) step) and substructure dynamics (the merger tree step), and is the same definition used for the \( \text{B06} \) galaxy model. (A consequence of this is that the halo shapes we measure are relatively crude approximations to the actual isodensity surfaces.) Furthermore, using shape profiles is beyond the scope of the present paper. However, an iterative scheme is still informative, as it may give a resulting halo shape that is more robust against numerical effects like dominance by very few particles. We include it here primarily to allow comparison between different methods used in the literature. This method has recently been used in Paz et al. (2006), Macciò et al. (2007); Macciò, Dutton & van den Bosch (2008), Muñoz-Cuartas et al. (2011), and Lau et al. (2011).

### 2.2.4 The iterative reduced inertia tensor

The iteration scheme described above can be used with a reduced inertia tensor, defined similarly to that in equation (3) (Dubinski & Carlberg 1991; Warren et al. 1992):
\[
M_{ij} = \sum_{j=1}^{N} \frac{r_{ij}^3}{r_p^2},
\]

where \( r_p \) is the elliptical distance defined in equation (4). We shall refer to the resulting tensor after convergence as \( M_{\text{Reduced}} \). This is, in fact, the most common way of using the reduced inertia tensor in practice, and has been recently used in Kazantzidis et al. (2004), Allgood et al. (2006) and Vera-Ciro et al. (2011).

### 2.2.5 Comparison

To illustrate the impact that these algorithms make on the halo shape measured in simulations, Fig. 3 shows the resulting axis ratios \( s = c/a \) as a function of halo mass (using haloes selected for our analysis of the \( \text{B06} \) model, i.e. at \( z = 0.32 \), and hosting a central galaxy from \( \text{B06} \) with \( r < 24.3 \)). The error bars on the medians are an estimate of their uncertainty, by analogy with the standard error on the mean of a Gaussian:
\[
\bar{\epsilon}_+ = \frac{X_{95} - X_{50}}{\sqrt{N}}, \quad \bar{\epsilon}_- = \frac{X_{50} - X_{95}}{\sqrt{N}},
\]

where \( X_i \) is the value at the \( i \)th percentile of the distribution in question, made up of \( N \) objects (\( X_{50} \) is the median). The error bars only become significant at high masses, where there are relatively few haloes in each mass bin.

We find very little difference between the results of the Simple and the Simple Iterative shape tensors. The reduced tensor however yields significantly more spherical haloes at all masses, confirming the idea that much of a halo’s asphericity is due to the distributions of subhaloes and the outer mass distribution generally. The Iterative Reduced shape tensor produces very similar results to \( M_{\text{Simple}} \), but very slightly more spherical. Previous authors have found that, with the advent of simulations able to resolve a significant amount of substructure, iterative methods often failed to converge (e.g. Jing & Suto 2002; Bailin & Steinmetz 2005; Shaw et al. 2006). We find that 18.0 and 25.9 per cent of the selected haloes fail to converge for the \( M_{\text{Simple}} \) and \( M_{\text{Reduced}} \) tensors respectively – a significant amount, but a small fraction of the population (of 7 907 290 haloes). As seen in the figure, their loss does not bias the shape distribution significantly.

Detailed further discussion, and comparison of different shape-finding methods, can be found in Jing & Suto (2002), Springel, White & Hernquist (2004), Bailin & Steinmetz (2005), Vera-Ciro et al. (2011), and the recent paper dedicated to the subject by Zemp et al. (2011).

It is important to emphasize that, while all of these methods are commonly used in the literature to measure halo shapes from simulations, they are not all relevant for comparison with observational studies. In fact, it is the simple mass quadrupole moment tensor \( M_{\text{Simple}} \) that is the most directly related to the shear signal from weak lensing (e.g. Schneider & Bartelmann 1997). In the sense that the \( M_{\text{Simple}} \) is a more robust description of the same moments of inertia as \( M_{\text{Simple}} \), then it is also important to note if or when it yields significanitly different results. However, the “reduced” tensors \( M_{\text{Reduced}} \) and \( M_{\text{Reduced}} \), while providing very important measures of the physical halo shape, are much less accessible to observational tests by weak lensing. We include them here primarily to illustrate the systematic impact they have on the results, to aid comparison between observational studies and future theoretical predictions.

### 2.3 Modelling the orientation of galaxies

The orientation of galaxies with respect to their dark matter haloes is not tackled in current semi-analytic models of galaxy formation. We must therefore model galaxy–halo alignment ourselves.

We consider the central galaxy within a halo, where the galaxy minor axis \( c_{gal} \) is oriented in some direction \( \theta \) with respect to some characteristic halo vector \( v_h \), i.e. \( c_{gal} \cdot v_h = |c_{gal}| |v_h| \cos \theta \). In our model, we identify \( v_h \) with either the halo minor axis \( c_{gal} \) or the angular momentum \( J \). Note that \( c_{gal} \) and \( J \) themselves have an alignment distribution, which is not significantly correlated to other halo properties such as shape; see e.g. Bett et al. (2007) and Skibba & Macciò (2011). We can define a complete set of basis vectors for the halo (\( \hat{z}_h, \hat{r}_h \) & \( \hat{\phi}_h \)), identifying the ‘\( \hat{r} \)’-axis direction \( \hat{z}_h \) with that of \( v_h \). The other axes can be formed by rotations of 90◦ from that, following the right-hand rule. If \( v_h \) points at a polar angle \( \theta_h \) and azimuthal angle \( \phi_h \) (with respect to the simulation coordinate system, for example), then we have
\[
\hat{z}_h = \left\{ \begin{array}{c} \sin \theta_h \cos \phi_h \\ \sin \theta_h \sin \phi_h \\ \cos \theta_h \end{array} \right\}.
\]
Figure 3. Halo 3-D axis ratio $s = c/a$ vs mass for the four shape algorithms. Also plotted are the median values of $s$ in mass bins, with error bars given by equation (7). Dashed vertical lines show the masses corresponding to 1000 particles and 11619 particles, the latter being the upper mass limit we use in our analysis ($10^{13} h^{-1} M_\odot$).

\[
\begin{align*}
\hat{x}_h &= \begin{pmatrix} \sin(\theta_h + 90^\circ) \cos \phi_h \\ \sin(\theta_h + 90^\circ) \sin \phi_h \\ \cos(\theta_h + 90^\circ) \end{pmatrix} = \begin{pmatrix} \cos \theta_h \cos \phi_h \\ \cos \theta_h \sin \phi_h \\ -\sin \theta_h \end{pmatrix}, \\
\hat{y}_h &= \begin{pmatrix} \sin 90^\circ \cos(\phi_h + 90^\circ) \\ \sin 90^\circ \sin(\phi_h + 90^\circ) \\ \cos 90^\circ \end{pmatrix} = \begin{pmatrix} -\sin \phi_h \\ \cos \phi_h \\ 0 \end{pmatrix}. \tag{8}
\end{align*}
\]

Note that, if $v_h \equiv c_h$, then the plane spanned by the basis vectors $\hat{x}_h$ & $\hat{y}_h$ is parallel to that of the halo axes $a_h$ & $b_h$. However, we do not require that e.g. $\hat{x}_h$ and $a_h$ etc. are parallel, as our modelling of the galaxy orientation is based solely on the direction of $v_h$. The orientation of any given halo shape with respect to its $v_h$ (and hence $\hat{x}_h$ & $\hat{y}_h$) is fixed, and we do not need to specify it explicitly in our modelling.

In the same way as for our halo coordinates, we let $\theta$ and $\phi$ describe the polar coordinates giving the orientation of the galaxy minor axis $c_{\text{gal}}$, with respect to this halo reference frame. We choose the $\theta$ and $\phi$ by randomly sampling from different distributions. As we see no convincing physical reason for there being a preferred angle for $\phi$, we sample it from a uniform distribution between 0 and $2\pi$. However, we test four different models for the galaxy–halo alignment angle $\theta$, which we describe below.

Using these two angles we can define a set of basis vectors for the galaxy ($\hat{x}_{\text{gal}}, \hat{y}_{\text{gal}}, \hat{z}_{\text{gal}}$) in the same way as equations (8) above. However, if we consider the galaxy, like the halo, to be a triaxial ellipsoid, then we need a third angle $\xi$ to define the orientation of $a_{\text{gal}}$ and $b_{\text{gal}}$ on the $\hat{x}_{\text{gal}}-\hat{y}_{\text{gal}}$ plane. Like $\phi$, there is no convincing reason for there to be a strongly-preferred value of $\xi$, so we again randomly sample it from a uniform distribution over $0–2\pi$. It is important to note that $\xi$ is still significant even in the case of a disc galaxy with $a_{\text{gal}} = b_{\text{gal}}$. This is because we define our “image plane” later based on the galaxy’s $a_{\text{gal}}-c_{\text{gal}}$ plane, so $\xi$ has a strong impact on the orientation of the projected halo (see section 2.4). We give more mathematical details of the rotations involved in implementing our orientation model in Appendix B.

We now go on to describe the four models we use to provide distributions of the galaxy–halo alignment angle $\theta$. It should be noted that we do not expect that the “true” alignment distribution to match any of these models in detail. Rather, our intention is that they span the possibilities of galaxy–halo alignment, such that the impact of any given model can be easily understood in observational terms.

2.3.1 Parallel

In this model, we take the characteristic halo vector to be its minor axis ($v_h = c_h$), and set the galaxy minor axis to be perfectly aligned with it; i.e. the angle between $c_h$ and $c_{\text{gal}}$ is $\theta = 0$. This is the most optimistic, ‘best-case’ scenario for attempts to measure halo shape.

Note however that even in this case, due to our random sampling of $\phi$, $\xi$ and the inclination of the image plane (see later), the ellipticity of the projected shape can vary, and it can be misaligned with respect to the galaxy.

2.3.2 Uniform

In this case, the orientation of the galaxy with respect to the halo is uniformly distributed, i.e. the probability distribution of $\cos \theta$ is flat over the range $[-1, 1]$. This is the worst case scenario for halo shape measurements.
2.3.3 Fit to simulations

In the study of weak lensing with COMBO-17 data, Heymans et al. (2004) used a truncated Gaussian distribution to very roughly fit the galaxy–halo alignment from the simulations of van den Bosch et al. (2002), which used dark matter and non-radiative gas. In more recent years, the probability distribution for galaxy–halo alignment has been measured in more advanced hydrodynamic simulations, which include radiative cooling, star formation and feedback processes. Furthermore, we can fit them using functions more suited to the 3-D polar angle that we are measuring.

We model the galaxy–halo alignment based on the spin–spin alignment shown in Bett et al. (2010) (their fig. 17) and Deason et al. (2011) (the top-right panel in their fig. 3), which are based on the simulations of Okamoto et al. (2005) and the Gnap simulations (Crain et al. 2009), respectively. We assume that $e_{\text{gal}}$ is parallel to the galaxy spin axis (Libeskind et al. 2007; Bett et al. 2010). Bett et al. (2010) measure the orientation of their galaxies with respect to their parent halos in the galaxy formation simulation (‘DMG’) and also a dark matter-only resimulation of the same initial conditions (‘DMO’). We consider both here, giving us three different datasets in total: there are 431 galaxy–halo systems in the Deason et al. (2011) data, 99 systems from Bett et al. (2010) DMG, and 95 from their DMO simulation. Despite the differences in the physics used in the different simulations, we find that a Kolmogorov–Smirnov test fails to show a significant difference between the three datasets at a 5% significance level, i.e. they are consistent with having been drawn from the same distribution.

We use a Fisher (1953) distribution averaged over the azimuthal angle $\phi$ to characterise the alignment probability given by the data. (We describe this distribution in more detail in Appendix A.) The probability density function (PDF) is given by

$$P(\cos \theta) = \frac{\kappa}{2 \sinh \kappa} I_0(\kappa \sin \theta \sin \theta_0) \exp(\kappa \cos \theta - \cos \theta_0),$$  

in terms of the “mean” direction $\theta_0$ and the concentration $\kappa$, which we write in terms of the distribution width $\sigma = 1/\sqrt{\kappa}$. ($I_0$ is the zeroth-order modified Bessel function of the first kind.) We find that sufficiently accurate values for the mean direction and width are

$$\theta_0 = 0.0, \quad \sigma = 0.55.$$  

(10)

We show the three distributions and this fitted PDF in Fig. 4. Note that although the preferred direction $\theta_0 = 0$, the median value of $\theta$ for this distribution is actually 37.7°.

We therefore define our third galaxy–halo alignment model as the PDF given in equation (9) with the parameters given in equation (10), using the halo angular momentum as the reference vector $v_\text{h} = J$. Note that the distributions in the Parallel and Uniform alignment models are limiting cases of equation (9), for $\sigma \to 0$ and $\sigma \to \infty$ respectively (assuming $J$ and $e_h$ are parallel).

2.3.4 Split distribution

Finally, we use a model for galaxy–halo alignment that explicitly differentiates between different types of galaxies.

The strong link between the angular momenta of gas and dark matter, and the formation of disc galaxies, leads us to link their alignment to the halo angular momentum. On the other hand, elliptical galaxies are usually considered to have formed through mergers, which will randomise their orientation (e.g. Scannapieco et al. 2009; Romano-Díaz et al. 2009; Bett & Frenk 2011, and references therein). However, the galaxy will subsequently accrete further material, which could come from certain directions preferentially (e.g. along filaments). The same is true for the halo, and we can consider the same directed merger and accretion events that determine the halo shape to influence the galaxy shape in a similar way. This suggests that it is reasonable to set bulge-dominated galaxies to be aligned to their halo. Observational studies have given evidence for good alignment of early-type galaxies and their haloes (e.g. Kochanek 2002, 2006; Cypriano et al. 2004; Ferreras, Saha & Burles 2008; but see Okumura, Jing & Li 2009).

Heavens, Refregier & Heymans (2000) introduced a simple model for galaxy–halo alignment, in which elliptical galaxies were (implicitly) co-aligned with their halo, and disc galaxies were aligned parallel to their halo’s angular momentum vector. This model was also used in conjunction with the Millennium Simulation in the work on satellite galaxy alignments of Agustsson & Brainerd (2010). Heymans et al. (2004, 2006) extended this alignment model to allow for a misalignment distribution around the halo angular momentum vector, following the results from van den Bosch et al. (2002) as already discussed.

Given the obvious similarities between our alignment models and that of Heymans et al. (2004), we construct our Split model in the same way. Using the stellar-mass bulge-to-total ratio $B/T$ as a physical measure of galaxy morphology, we apply the Fitted alignment model for galaxies with $B/T \leq 0.5$ (i.e. using equations (9) & (10)) to sample $\theta$ with respect to the halo angular momentum, for disc-dominated galaxies), and the Parallel alignment model for galaxies with $B/T > 0.5$ (i.e. $\theta = 0$ with respect to the halo shape, for bulge-dominated galaxies).

2.4 The image plane

We consider our galaxy–halo systems as lenses, and that, when stacked, their mass distributions will be measurable through weak lensing of the shapes of background source galaxies. We don’t need to actually perform the lensing itself, as we are most interested in how galaxy–halo alignment affects the projected mass distribution; for our purposes, the lensing process would mostly serve to add noise to the halo ellipticity signal, making the stacked halo appear more circular.

We assume that the observer will try to align their lens galaxies in an image plane such that minor axis of the galaxy is parallel to the image $y$-axis, and the galaxy major axis is parallel to the image $x$-axis. As galaxies will not be exactly edge-on when observed on the sky, we have to allow for some variation in inclination angle $\zeta$, which we define as a rotation about the galaxy major axis (the image $x$-axis), such that the image-plane normal vector is rotated above/below the galaxy intermediate axis; the galaxy minor axis is rotated in front of or behind the image plane, no longer parallel to the image $y$-axis. We sample $\zeta$ from a uniform distribution over the range $\pm 30°$.

Note that, even in the case of perfect galaxy–halo alignment in 3-D ($\theta = 0$), the combination of the non-zero azimuthal angle $\phi$, galaxy orientation $\xi$, and image-plane inclination $\zeta$ results in misalignment between the projected galaxy and halo, and variation in alignment.
the ellipticity of the projected halo itself. Perfect alignment in 3-D need not mean perfect alignment in projection.

2.5 Stacking

In observations, in order to obtain a measurable signal above the noise from single measurements, the shear signal from many galaxy images must be stacked, with the resulting shape being that of the net mass distribution. In practice, it is sensible to scale the galaxy–halo systems to ensure that they are compared fairly, and the signal does not become dominated by very few systems. This might be done according to some spatial scale on the galaxy images, or more directly by luminosity. In our case, care must be taken to use an appropriate weighting when summing (stacking) the shape tensors of haloes. The shape tensors we use are weighted differently: $\mathbf{M}_{\text{simp}}$ and $\mathbf{M}_{\text{simp h}}$ scale with halo mass and square radius (and are thus their sum is very susceptible to dominance by high-mass haloes), whereas $\mathbf{M}_{\text{shdu}}$ and $\mathbf{M}_{\text{shdu h}}$ just scale with halo mass.

We choose to stack halo shapes weighting by galaxy $r$-band luminosity $L_r$, with respect to some constant reference luminosity $L_{r,0}$. For a given shape tensor definition $\mathbf{M}$,

$$\mathbf{M}^{\alpha\beta} = \sum_s \frac{L_{r,s}}{L_{r,0}} \mathbf{M}_{s},$$

where the sum is over selected galaxy–halo systems $s$. The choice of $L_{r,0}$ is not important. Since we obtain luminosities from magnitudes, $M_{r,0} - M_r = -2.5 \log_{10}(L_{r,0}/L_r)$, we simply choose $M_{r,0} = 0$ such that $L_{r,0}/L_r = 10^{M_r/2.5}$. Note that this will not be possible observationally if redshift information is not available. Instead, the strong weighting of the shape measurement towards large haloes would be retained. Even with photometric redshifts, such an observational study might choose to calculate the halo shapes in luminosity bins, rather than use luminosity to scale the data from each lens (Mandelbaum et al. 2006).

2.6 Summary

We have four alignment models (Parallel, Uniform, Fitted and Split) that define the galaxy–halo alignment angle $\theta$, together with random sampling for the azimuthal angle $\phi$, galaxy orientation $\xi$ about its minor axis, and (over a restricted range) the image plane inclination $\zeta$. We also use four methods for measuring halo shape (by the tensors $\mathbf{M}_{\text{simp}}$, $\mathbf{M}_{\text{simp h}}$, $\mathbf{M}_{\text{shdu}}$ and $\mathbf{M}_{\text{shdu h}}$). We are using a single algorithm for defining the haloes, and the publicly-available results from two semi-analytic galaxy formation models (DLB07 and B06).

After assigning values for $\theta$, $\phi$, $\xi$, $\zeta$, we rotate the halo shape matrix in question into the image plane. We obtain the eigenvalues and eigenvectors of its projection onto the image plane, giving us the projected halo shape axes $a_\parallel$ and $a_\perp$. We can measure the circularity of the haloes in projection through the axis ratio $q_{\parallel\parallel} = a_\parallel/a_\perp$. Note that for the Parallel, Uniform and Fit alignment models there is no link between galaxy properties and alignment. However, since both halos and galaxy properties depend on the merger history of the halo, it is possible that halo shapes can be correlated to galaxy properties: in principle, one could be able to select galaxies that preferentially have less-spherical haloes.

3 MONTE CARLO TESTS

To directly test the impact of our alignment models and the halo shape distribution on the resulting stacked shapes, we perform Monte Carlo experiments to construct a halo–galaxy sample, without using the simulation or semi-analytic model.

To generate a halo population, we sample the 3-D axis ratio $s$ from a Gaussian probability distribution based on the results of Allgood et al. (2006). We take the standard deviation of the Gaussian to be $s = 0.1$, and take the mean to be

$$\langle s \rangle = \alpha \left( \frac{M_r}{M_*} \right)^{\beta}.$$

where $\alpha = 0.54$, $\beta = -0.050$, and $M_*$ is the halo mass. The characteristic mass $M_*(z)$ is given by

$$\log_{10} \left[ \frac{M_*}{(h^{-1}M_\odot)} \right] = A - B \log_{10}(1 + z) - C (\log_{10}(1 + z))^2 \quad (13)$$

with $A = 12.9$, $B = 2.68$, and $C = 5.96$. For the purposes of these tests, we take a constant halo mass $M = 10^{12} h^{-1}M_\odot$. Using our standard redshift of $z = 0.32$, we obtain $M_*(z) = 3.09 \times 10^{12} h^{-1}M_\odot$, leading to a distribution with a mean sphericity of $\langle s \rangle = 0.571$. For each value of $s$, we assign an intermediate axis ratio of $q = b/a = (1 + s)/2$. 

![Figure 4. Galaxy–halo alignment distributions, from Deason et al. 2011 (their fig. 3) and Bett et al. 2010 (their fig. 17). Angles are between the halo and galaxy angular momentum vectors. The black curve is the azimuthally-averaged Fisher distribution given by equations (9) & (10). Medians are marked with vertical dashed lines.](image)
Using this halo shape distribution, we then generate samples of projected halo shapes, each comprising $10^6$ objects. We generate one sample each using the Parallel and Uniform alignment models, and a series of samples based on the Fitted model. In the latter case, we choose a different value of the alignment distribution width $\sigma$ for each sample. We do not model the halo angular momentum, and instead take the alignment distribution to always be with respect to the halo shape. We retain the variability in image plane alignment instead of galaxy luminosity.

We stack these projected haloes directly, giving $M_{\text{tot}} = \sum_i M_i$; since the haloes are all the same size, we need not (and cannot!) weight by galaxy luminosity.

The results, showing how the resulting stacked halo shape depends on the alignment distribution width, are shown in Fig. 5. The stacked halo shape quickly changes from $q_{pr} \approx 0.68$ for Parallel alignment, through $q_{pr} > 0.9$ for $\sigma \geq 0.6$, and converging to the result from the Uniform distribution by $\sigma \approx 2$.

We have also investigated the joint impact of the original halo shape distribution and the alignment distribution width. For this, we did not sample halo shapes from a Gaussian, but instead set them all to a fixed value $s$. The orientation distributions were randomly sampled in the same way as before, for a grid of values of $s$ and $\sigma$. The results are shown in Fig. 6. It shows that, as expected, the sphericity of the halo population is largely immaterial, unless the alignment distribution has $\sigma \leq 0.5$. Even in that case, one needs a strongly aspherical shape distribution, with $s \leq 0.3$ in order to get a stacked shape of $q_{pr} \leq 0.8$.

These Monte Carlo tests have shown quantitatively the sensitivity of the stacked halo shape on the form of the galaxy–halo alignment distribution. Thus we expect that, if galaxies are aligned randomly in their haloes, or even if they are aligned as found in recent hydrodynamic simulations, then the stacked halo shape will be $\geq 0.9$.

4 RESULTS

The results for the axis ratios $q_{pr} = b_{pr}/a_{pr}$ of the stacked projected halo shapes are shown in Fig. 7 for the B06 model, and Fig. 8 for the DLB07 model. Each point represents the result for a given combination of models, with the different columns showing the effect of different selection criteria. We now go on to examine these results in detail.

A quick glance confirms that the primary factor in determining the measured stacked halo shape is the galaxy–halo alignment distribution. When the Uniform model is used (triangles in the plots), the stacking process washes out any intrinsic halo ellipticity, and the stacked halo is circular. The maximum deviation from circularity comes when the Parallel alignment model is applied, as this allows the maximal contribution from all haloes towards the final shape.

In the Parallel case, there are significant differences caused by the different halo-shape algorithms. When $M_{\text{base}}$ is used, the result is furthest from circular, with larger axis ratios generated when $M_{\text{base}}$ is used. The iterative methods give moderated values of these extremes: using $M_{\text{simple}}$ yields slightly more circular haloes than $M_{\text{base}}$, and using $M_{\text{radler}}$ yields slightly less circular haloes than $M_{\text{base}}$. This difference is due to the different implicit weighting that these methods give to haloes when stacking. The reduced tensors have their dependence on halo size (radius) scaled out, so that haloes contribute proportionally to their mass (which we then reduce by counterweighting by luminosity). The simple inertia tensors however retain the additional square-radius dependence. This means that the stacked halo results are much more strongly dominated by high-mass objects in the simple inertia tensor case, but

Figure 5. Shape of stacked, projected haloes generated by sampling their 3-D shapes from a Gaussian, then sampling their orientation with respect to an image plane in the usual way; see text for details. The continuous line joins the results from alignment distributions with a range of widths $\sigma$ (see equation 9), with the additional points (the circle and triangle) representing the limiting cases of $\sigma \to 0$ and $\sigma \to \infty$ respectively.

Figure 6. The contours and shading show the stacked axis ratio $q_{pr}$ from random-sampling halo orientations in the usual manner, from distributions with width $\sigma$, and haloes with 3-D axis ratio $s$. The lowest contour (in the red region) is at $q_{pr} = 0.35$; the other contours increase in steps of 0.05.
Figure 7. Axis ratios of the resulting projected shape tensors computed from stacking haloes selected according to different criteria. This plot uses our standard redshift of $z = 0.32$ and the B06 semi-analytic model. Each of the four panels gives the results from using a different halo shape algorithm, and each symbol type gives the result from different galaxy–halo alignment models. The first column in each panel (“All”) gives the results for all selected haloes with $M < 10^{13} h^{-1} M_\odot$. The next column adds a restriction at low masses, excluding haloes comprising < 1000 particles. The third and fourth columns split the halo population from the “All” column by colour and morphology respectively, with the blue symbols representing the blue/disc case, and red symbols representing the red/elliptical case. The colour cut is made at rest-frame $u - r = 0.9$, and the morphology cut is made at $B/T = 0.5$.

Figure 8. As Fig. 7, but using the DLB07 semi-analytic model. The colour-cut is therefore made at observer-frame $u - r = 3.5$. 
are more evenly weighted in the reduced case. How this effects the results depends on how the intrinsic halo shape distribution varies with mass for the different algorithms, which we showed earlier in Fig. 3: higher mass haloes tend to be less spherical.

A numerical artefact, present in N-body simulations such as the MS, is that haloes consisting of a small number of particles tend appear systematically less spherical than those with many particles. A lower limit of around 300 particles was suggested for the MS in Bett et al. (2007) to ensure accurate halo shapes. In Figs. 7 and 8, we compare the results from all haloes and those with at least 1000 particles (in all cases, our upper mass limit of $M < 10^{13} h^{-1} M_{\odot}$ applies). We see that excluding the low-mass haloes makes only a very small difference to the stacked result: For the $M_{\text{Smp}}$ and $M_{\text{Smppl}}$ algorithms, high-mass haloes dominate the stacking anyway, and for $M_{\text{Rdu}}$ and $M_{\text{Rdupl}}$, the fact that the numerical biasing at low masses is in the same direction as the natural trend going to high masses leaves negligible net effect.

If, without numerical constraints, haloes in fact continue to get more spherical towards lower masses (as suggested by Macciò, Dutton & van den Bosch 2008 and Muñoz-Cuartas et al. 2011), then the effect of retaining lower masses in the stacking would be more important: when using the reduced inertia tensor, the result for “all” haloes in our figure would be more circular.

### 4.1 Split alignment and the morphological mix

The Split alignment model shows the greatest variation among the models and selections tested. Since in this model the galaxy–halo alignment depends explicitly on galaxy morphology, the stacked results when selecting by morphology are entirely predictable: For elliptical galaxies, the result mirrors that from the Parallel alignment model, whereas for disc galaxies it mirrors that of the Fitted alignment model. There is still a significant number of disc galaxies however, and their Fitted alignment model means that the net stacked shape is more circular than if the Parallel alignment model was used alone. In the DLB07 model, the galaxy morphologies are even more mixed, with the additional misalignment resulting in a more circular stacked result.

On the other hand, if a reduced shape tensor is used, then in all cases the haloes are weighted more equally. Those dominant by number are at low masses, which are dominated by disc galaxies, using the Fitted alignment distribution. Because the steep drop in the fraction of disc-dominated galaxies happens at lower masses in the B06 model, that model has fewer disc galaxies and results in a less circular stacked halo shape.

When considering how the Split model operates when selecting systems by galaxy colour, we need to examine the relationship between colour and morphology. Although they are classically seen to correlate well (e.g. de Vaucouleurs 1961; Simien & de Vaucouleurs 1986; Strateva et al. 2001; Bell et al. 2004), both theoretical and observational studies have shown the relationship to be not straightforward, and based on detailed processes occurring during galaxy formation and evolution (Croton et al. 2006; Benson et al. 2007; Deng et al. 2007; Guo et al. 2011). We show the relationships between galaxy colour and morphology for our models in Figs. 10 & 11.

In the B06 model (Fig. 10), it is clear that blue galaxies are mainly discs, and disc galaxies are mainly blue. Elliptical galaxies have a broader range of colours, albeit dominated by red galaxies. This means that, in the stacked halo results, selecting just blue galaxies yields a much more circular shape than selecting just red galaxies, although the mixing between colour and morphology means that the situation in both cases is less extreme than when selecting by morphology directly.

For the DLB07 model (Fig. 11), the population remains dominated by disc galaxies when either blue or red galaxies are selected, although it is to a lesser extent in the red case. Similarly, both the disc galaxy and elliptical galaxy populations are dominated by blue galaxies, with the elliptical galaxy population having a significant red galaxy presence too. This results in a much smaller difference between the stacked halo shapes when selecting just red and just blue galaxies, when compared to the results from the B06 model; in particular, the result for the red population is significantly more circular. However, the tight link between blue and disc galaxies means that for both the B06 and DLB07 model, selecting blue galaxies yields the more circular stacked halo.

| Model | Shapes | All | $N_p \geq 1000$ | Blue | Red | Disc | Elliptical |
|-------|--------|-----|----------------|------|-----|------|------------|
| B06   | $M_{\text{Smp}}, M_{\text{Rdu}}$ | 7866537 | 443053 | 7064928 | 801609 | 6890492 | 976045   |
|       | $M_{\text{Smppl}}$ | 6454412 | 320280 | 5869870 | 584542 | 5738275 | 716137   |
|       | $M_{\text{Rdupl}}$ | 5830433 | 318931 | 5249670 | 580763 | 5116628 | 713805   |
| DLB07 | $M_{\text{Smp}}, M_{\text{Rdu}}$ | 10710174 | 442862 | 10460421 | 249753 | 10241493 | 468681   |
|       | $M_{\text{Smppl}}$ | 9034263 | 320177 | 8842123 | 192140 | 8660850 | 373413   |
|       | $M_{\text{Rdupl}}$ | 7887973 | 318797 | 7707025 | 180948 | 7542620 | 345353   |

Table 1. The number of galaxy–halo systems for the different models and selections in Figs. 7 & 8. Parry, Eke & Frenk (2009) have studied in detail the origin and evolution of galaxy morphologies in the B06 and DLB07 models.

This is reflected in the results for the stacked halo shape. When the $M_{\text{Smp}}$ tensor is used, the stacked shape is weighted towards the high-mass haloes. In the B06 model, this means that the majority of strongly-contributing haloes host elliptical central galaxies, with the Parallel alignment model. There is still a significant number of disc galaxies however, and their Fitted alignment model means that the net stacked shape is more circular than if the Parallel alignment model was used alone. In the DLB07 model, the galaxy morphologies are even more mixed, with the additional misalignment resulting in a more circular stacked result.
Figure 9. Distribution of galaxy morphologies as a function of halo mass, for the B06 and DLB07 semi-analytic models. The left panel shows the proportion of galaxies at each mass that have $B/T < 0.5$ (i.e. discs), with Poisson error bars. The middle and right panels show the (normalised) histograms of the different galaxy samples. Note that the plots extend beyond the upper mass limit of $10^{13} h^{-1} M_\odot$ that we use in our analysis, and below the lower limit of 1000 particles that we consider; both limits are marked with vertical dashed lines.

Figure 10. Distribution of galaxy morphologies as a function of colour, for the B06 model. The top panel gives the fraction at each colour that are disc-dominated, with Poisson error bars. The bottom panel shows colour histograms for each population sample. The colour cut at rest-frame $u - r = 0.9$ is marked with a vertical dashed line.

Figure 11. As Fig. 10, but for the DLB07 semi-analytic model; colours are therefore observer-frame. The colour-cut is marked in the lower panel at $u - r = 3.5$. 
4.2 Parallel & Fitted alignment, and galaxy–halo correlations

The results in Figs. 7 & 8 for the Parallel model (and to a lesser extent the Fitted model) also depend on the colour and morphological selection, with red or elliptical galaxies resulting in a more circular stacked halo than blue or disc galaxies. In this case, the difference is not due to the alignment model, but an intrinsic correlation between galaxy colour/morphology and halo shape.

Figure 12 shows the joint distribution of 3-D halo sphericity \( s \) from the \( M_{\text{smp}} \) and \( M_{\text{rdu}} \) tensors, against \( u-r \) colour for the B06 and DLB07 semi-analytic models. The projected halo shapes in these cases, for the Parallel alignment model, are shown against colour in Fig. 13. We can see that the colour distribution is very broad for any given halo shape, although the haloes of blue galaxies have a less-spherical median shape than those of the red galaxies. Similarly, we plot projected axis ratio histograms in Fig. 14, cut by galaxy morphology: elliptical galaxies have slightly more circular projected haloes in the median than disc galaxies. It is important to note that the median shape in a distribution is not the same as the stacked halo shape from the same sample of haloes, because the stacking process weights haloes differently.

While most of the results for Parallel alignment are very much consistent between the B06 and DLB07 models, the projected shapes for blue (and disc-dominated) galaxies using the \( M_{\text{smp}} \) tensor are noticeably less circular for the B06 model. This is again due to the \( M_{\text{smp}} \) shapes being dominated by the very largest haloes: In the B06 model, there are very few large haloes hosting blue central galaxies (just 242 systems with masses \( 10^{12.5} - 10^{13} M_{\odot} \)), whereas with the DLB07 model the blue population extends to much higher masses (in the same mass bin, there are 29 905 haloes). We show the distributions of galaxies of different colours,
as functions of their parent halo mass, in Fig. 15. The figure shows clearly how the highest-mass haloes (up to $10^{13} \, h^{-1} M_{\odot}$) hosting blue galaxies in the B06 model have a lower median projected axis ratio than those in the DLB07 model. Furthermore, despite the significant differences in the distributions of red galaxies between the B06 and DLB07 models, the medians as a function of mass are very similar.

It is important to note that the correlation between galaxy properties and halo shape is relatively weak, and (as shown from the Monte Carlo tests earlier, Fig. 6) only plays a role when the impact of galaxy–halo misalignment is strongly reduced. Applying the Fitted distribution provides an example of an intermediate case: while there is still some variation between the results for red/elliptical and blue/disc galaxies, the circularising effect of galaxy–halo misalignment means that these differences are negligible. Indeed, the stacked halo results are sufficiently circular under the Fitted distribution ($q_{pr} \geq 0.95$) that it would be extremely difficult for a weak lensing study to measure any significant ellipticity.

5 DISCUSSION AND CONCLUSIONS

Natarajan & Refregier (2000) and Brainerd & Wright (2000) described a technique for measuring halo ellipticity using the azimuthal variation in the tangential shear signal. Since this is a weak variation on top of the already weak shear signal, any measurement would require very large numbers of galaxies, from large surveys. In this paper, we have shown quantitatively, for a range of possible models, that a measurement of anisotropy would be extremely difficult indeed. For example, using a model based on current hydrodynamic galaxy formation simulations yields stacked halo axis ratios of $q_{pr} \geq 0.95$, which would be extremely difficult to observe.

The main problem, as expected, is the galaxy–halo misalignment. Any intrinsic correlations between halo shape and galaxy properties are only relevant in the case of unrealistically perfect alignment. While this can be seen in our main results from using the Millennium Simulation, it is very clearly demonstrated in just using simple Monte Carlo tests.

It is important to note, when considering observations related to galaxy–halo alignment, that the physically relevant angle $\theta$, as measured from simulations, is the alignment in 3-D. Due to the random orientations of the other angles in the system (e.g., galaxy minor axis polar angle $\phi$, galaxy orientation $\xi$, image plane inclination $\zeta$), even parallel alignment with $\theta = 0$ does not necessarily lead to perfect alignment of the observationally relevant angle, between the projected axes.

In both of the semi-analytic galaxy formation models we test here, we find that blue or disc-dominated galaxies tend to reside in less-spherical haloes. However, if we assume that elliptical galaxies are aligned more closely to their halo than disc galaxies (following e.g., Heymans et al. 2004), then it is selecting red or elliptical galaxies that yields a more elliptical stacked halo in projection. In our work, this of course occurs by construction, and further work on galaxy–halo alignment in simulations is required to see how plausible this is in practice. However, the alignment distributions we use span the range of possibilities, and, in conjunction with our Monte Carlo tests and the colour/morphology distributions of modelled galaxies, the result of any arbitrary alignment distribution can be estimated.

The work presented in this paper can be seen as a dark matter-theoretical counterpart to the paper of Howell & Brainerd (2010), who performed a thorough study of difficulties with measuring anisotropic shear from a lensing-theoretical standpoint. Our distributions for the projected halo axis ratio (Figs. 12–15) derive from more complex modelling for example, and show the possible variability due to galaxy type, but Howell & Brainerd (2010) take a given distribution through a Monte Carlo lensing process, demonstrating that multiple deflections of background galaxies can also have a catastrophic effect on the anisotropic shear signal.

Our results may lead one to wonder how it was that both Hoekstra, Yee & Gladders (2004) and Parker et al. (2007) managed to claim a measurement of halo ellipticity. It should first be mentioned that their detections are relatively weak anyway, with their halo ellipticity measurements being at the $1–2 \sigma$ level (although Hoekstra, Yee & Gladders 2004 rule out spherical haloes at the $3 \sigma$ level). Possible sources of the discrepancy include underestimation of systematic errors in the modelling and interpretation of the data, not helped by the lack of redshift and colour/morphology data in both of these studies. Mandelbaum et al. (2006) discuss in some detail the discrepancy between their non-detection and the results of Hoekstra, Yee & Gladders (2004). It is clear that, regardless of the physics-related problems highlighted in the present work, con-
trolling systematics in observational studies such as these is very difficult.

We have already mentioned the difficulty in using this method as a way of distinguishing between $\Lambda$CDM and modified gravity theories. The basic idea – $\Lambda$CDM predicts non-spherical haloes, but modifications to gravity without dark matter predict spherical symmetry – is based on a na"ıve understanding of practical issues in both the $\Lambda$CDM and modified gravity case. With $\Lambda$CDM, systematic effects in the lensing and stacking procedure can easily render the net signal isotropic, and as we have shown, our poor knowledge of the relationship (and in particular the alignment) between galaxies and their parent haloes provides much of the uncertainty. In the case of galaxies in modified gravity theories, all the aforementioned problems with our understanding of the baryonic physics still apply, but in the context of gravity laws that are more complex and less well understood. There has been some simulation work with gas dynamics in MOND (Tiret & Combes 2008), but no full simulation of galaxy formation in a cosmological context, with any star formation or feedback (simulations in $\Lambda$CDM have shown how significant an impact these processes have on the resulting galaxies). In the context of STVG, the gravity law is more difficult to work with, and numerical simulations are still in their infancy (see e.g. Moffat & Toth 2010). Thus, we do not believe that statistical analysis of stacked, projected lens galaxies can be used to discriminate between $\Lambda$CDM and alternative theories, simply because we lack robust predictions from either case.

Consequently, in this paper, we have not gone as far as to make a prediction for observations, as the theoretical uncertainty is still too large. In the future however, if the galaxy formation models reach better convergence and can offer statistical predictions of galaxy–halo alignment, then a study such as ours could be advanced further to make such an observational prediction. In that case, certain other effects would need to be taken in to account. We have been able to neglect these here, as they are all secondary to the main misalignment difficulty.

Firstly, when computing shapes from simulations to compare with observations, then it would be more appropriate to use the mass within a given radius. This is (arguably) not the same as the shape of the halo, which is a dynamically relaxed physical structure, rather than a geometrically-defined overdense region. However, an observation such as this has no practical way of accessing the dynamical information necessary to define a halo, and so there is no need to do so in simulations for this purpose either\textsuperscript{6}. Ideally, the mass distribution as a function of radius would be generated from the simulations, as it has been shown that halo ellipticity is not constant with radius. The distinction between central and satellite galaxies could also be relaxed, and the shape of the mass distribution around each (lens) galaxy could be computed, allowing for selection criteria that more closely match those in observational studies. (The problem of which galaxy is at the centre of a halo is not limited to simulations; for example, Skibba et al. (2011) have shown that the brightest galaxies in haloes are often not in the centre, and this should be taken into account when simulating observations.)

When considering the inner halo shape however, it becomes vital to consider the impact of baryonic processes.

While dark matter-only haloes are triaxial with a tendency for prolateness, becoming more prolate towards the centre (e.g. Bett et al. 2007; Hayashi, Navarro & Springel 2007), haloes that have had a galaxy form in the centre are overall more spherical, with a tendency towards oblateness (Kazantzidis et al. 2004; Kazantzidis, Abadi & Navarro 2010; Bailin et al. 2005; Berentzen & Shlosman 2006; Gustafsson, Fairbairn & Sommer-Larsen 2006; Debattista et al. 2008; Tissera et al. 2010; Machado & Athanassoula 2010; Abadi et al. 2010; Lau et al. 2011). This is likely to make halo shapes more difficult to measure. On the other hand, strong lensing studies have suggested that mass and light are well aligned in the inner regions of the halo (Kochanek 2002, 2006; Minor & Kaplinghat 2008).

It would be important to measure the alignment distribution from a statistically large sample of objects, and over a range of time steps. It is known that both halo and galaxy orientations vary in time even outside major mergers (e.g. Scannapieco et al. 2009; Romano-Díaz et al. 2009; Bett 2010; Bett & Frenk 2011), so the relative orientations of a few galaxies and haloes at a single redshift might not be at all robust, regardless of how well-resolved they are spatially, or how realistic the baryonic physics in the simulation is.

Eventually, realistic mock-observations would need to be produced, using ray tracing through the simulation (e.g. Hilbert et al. 2009) so that a realistic background source population and the effect of multiple deflections are included, as it has been shown that these have a significant impact on shear measurements (Brainerd 2010; Howell & Brainerd 2010; Hoekstra et al. 2011).

While our results do not seem to give much cause for optimism in measuring shapes using weak lensing, it should be pointed out that we are only considering one method, for the shape distribution of non-cluster haloes. Many other methods of measuring halo shapes are possible, and indeed are actively pursued. Furthermore, the ideal test of $\Lambda$CDM is to measure the increasing asphericity of haloes with increasing mass, and thus the shapes of cluster haloes are particularly important. Evans & Bridle (2009) applied essentially the same technique as Natarajan & Refregier (2000), but on clusters rather than field galaxies. They managed to measure a projected halo axis ratio of $q_e = 0.48^{+0.14}_{-0.10}$ with 1$\sigma$ errors, ruling out a circular shape at 99.6% confidence. Using clusters has the practical advantage that the cluster member galaxies can be used for alignment. Much work has been done on the alignment of the satellite galaxy distribution, both for measuring cluster halo shapes and as another test of dark matter, observationally and in simulations (e.g. Kuhlen, Diemand & Madau 2007; Faltenbacher et al. 2008; Libeskind et al. 2009; Knebe et al. 2010; Yang et al. 2006; Wang et al. 2008; Agustsson & Brainerd 2010, and references therein). Hopkins, Bahcall & Bode (2005) also investigated the distribution of projected halo shapes, for simulated cluster-mass haloes as a function of redshift. Other cluster-based lensing methods attempt to map the shape directly (e.g. Cypriano et al. 2004; Ferreras, Saha & Burles 2008; Oguri et al. 2010; Deb et al. 2010, in the latter case finding $q_e = 0.54 \pm 0.04$ at 1$\sigma$), or use Markov Chain Monte Carlo methods to fit triaxial models (Corless, King & Clowe 2009). Lensing flexion has recently been proposed as another method for studying galaxy-scale haloes (Er & Schneider 2011; Er et al. 2011). Non-lensing methods for studying halo shape include studying the distribution of HI in disk galaxies (e.g. Banerjee & Jog 2008; O'Brien, Freeman & van der Kruit 2010, and references therein).

In this paper, we have presented a quantitative analysis of the impact of galaxy–halo misalignment on the possibility of measuring

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\textsuperscript{6} It should be noted that in analyses of simulations alone, the distinct ideas of measuring the shape of an overdensity contour colocated on the same density peak as a halo, and the shape of the dynamically-defined halo itself are often conflated.
halo shapes via weak lensing in stacked images. We have tested a series of alignment models, spanning the range from perfect alignment (in 3-D) to uniformly-distributed alignment. As intermediate models, we included a fit to recent hydrodynamic simulations of galaxy formation, and a distribution that explicitly differentiates between galaxy morphologies. Our results have shown that, for there to be a reasonable possibility for shapes to be measured, a significant fraction of the lens galaxies must have close to perfect alignment, which seems physically implausible. Using simple Monte Carlo models, we have quantified how well-aligned the galaxies have to be in their haloes before the intrinsic shape distribution becomes measurable. For our results using the Millennium Simulation, we have also tested the impact of using different models of galaxy formation, and different ways of measuring haloes in simulations. These illustrate some of the difficulties in applying results from current simulations directly to models: there simply is not a single robust quantitative prediction from ΛCDM for halo shape measurements using this method. Since the same is true for alternative theories without dark matter, this method cannot yet be used to falsify one or the other.

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APPENDIX A: THE AZIMUTHALLY-AVERAGED FISHER DISTRIBUTION

The probability distribution we use in section 2.3.3 to define our Fitted alignment distribution is based on the Fisher (1953) distribution,

\[ P_\phi(v, v_0, \kappa) = \frac{\kappa}{\sinh \kappa} \exp(\kappa v \cdot v_0), \]

(A1)

where the probability density function (PDF) is given in terms of the unit vector random variable \( v \), the mean direction unit vector \( v_0 \), and the concentration \( \kappa \); the latter is often written in terms of the width of the distribution \( \sigma \) through \( \kappa = 1/\sigma^2 \). This is the 3-D case of the more general von Mises–Fisher family of distributions, and is often used as more mathematically tractable approximation to a wrapped Normal distribution (see Mardia & Jupp 2000 for more details).

If we write our vectors in a cartesian basis in terms of spherical polar coordinates, and (for narrow widths the PDF tends to a delta function related to the zeroth-order modified Bessel function of the first kind, \( I_0(x) \)),

\[ v = c_{gal} = \left(\begin{array}{c} \sin \theta \cos \phi \\
\sin \theta \sin \phi \\
\cos \theta \end{array}\right), \quad v_0 = \left(\begin{array}{c} \sin \theta_0 \cos \phi_0 \\
\sin \theta_0 \sin \phi_0 \\
\cos \theta_0 \end{array}\right). \]

(A2)

Normalisation of the PDF is over the surface of the unit sphere \( \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \sin \theta d\theta d\phi = 4\pi \), so we can write the PDF in terms of \( \theta \) and \( \phi \) as:

\[ P_\phi(\theta, \phi) = \frac{\kappa}{4\pi \sinh \kappa} e^{(\cos \theta \cos \phi_0 \sin \theta \sin \phi_0 + \sin \theta_0 \cos (\phi - \phi_0)) \sin \theta}, \]

(A3)

such that the normalisation integral is

\[ \int_0^\pi \int_{-\pi}^{\pi} P_\phi(\theta, \phi) \ d\phi \ d\theta = 1. \]

(A4)

However, in our case, we are only interested in the angle \( \theta \) between our two vectors, so we have to integrate the Fisher distribution over all values of the azimuthal angle \( \phi \). The \( \phi \)-integral is in fact related to the zeroth-order modified Bessel function of the first kind, \( I_0(x) \),

\[ \int_0^{2\pi} \exp[k \sin \theta \sin \theta_0 \cos(\phi - \phi_0)] \ d\phi = 2\pi I_0(k \sin \theta \sin \theta_0). \]

(A5)

so we can write our azimuthally-averaged Fisher distribution as

\[ P(\theta) = \frac{\kappa}{2 \sinh \kappa} I_0(k \sin \theta \sin \theta_0) \exp(\cos \theta \cos \theta_0) \sin \theta. \]

(A6)

To aid comparison with the uniform distribution, we shall actually normalise in \( \cos \theta \) instead of \( \theta \), so the final PDF that we use is:

\[ P(\cos \theta) = \frac{k}{2 \sinh \kappa} I_0(k \sin \theta \sin \theta_0) \exp(\cos \theta \cos \theta_0), \]

(A7)

which is equation (9).

Note that for large widths \( \sigma \approx 10 \), the PDF tends to the uniform distribution, and for narrow widths the PDF tends to a delta function spike at \( \theta_0 \).

The routine we use to sample from this distribution is based on that given in the Pknn library\(^7\) of John Burkardt, which in turn is based on Fisher, Lewis & Embleton (2003).

APPENDIX B: ROTATION AND PROJECTION

Here, for completeness, we give further details of our method for defining the rotations and projection involved in our galaxy–halo alignment model described in section 2.3. Consider a reference frame \( S \), and a second frame \( S' \) that is a rotation of \( S \). If we write the cartesian basis vectors of \( S' \) in terms of those of \( S \), e.g. \( \mathbf{e} = (\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3) \), then we can simply write down the rotation matrix that transforms from \( S \) to \( S' \):

\[ R = \begin{pmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \end{pmatrix} \]

(B1)

\(^7\) http://people.sc.fsu.edu/~jburkardt/cpp_src/prob/prob.html
Thus, given the basis vectors for the halo coordinate frame in terms of the simulation coordinates, given in equation (8), we can write down the rotation matrix for transforming from the simulation coordinates into these halo-based coordinates:

\[
R_h = \begin{pmatrix}
\cos \theta_h \cos \phi_h & \cos \theta_h \sin \phi_h & -\sin \theta_h \\
-\sin \phi_h & \cos \phi_h & 0 \\
\sin \theta_h \cos \phi_h & \sin \theta_h \sin \phi_h & \cos \theta_h
\end{pmatrix}
\]  

(B2)

In this halo frame, we define the orientation of the galaxy by first specifying a minor axis vector direction given by \( \theta \) and \( \phi \). We can thus rotate into a similar galaxy-vector-based frame using a matrix \( R_{gal} \) identical in form to equation (B2). However, the orientation of the galaxy about its minor axis must also be specified, by a further rotation by the angle \( \xi \). Defining \( \xi \) such that setting \( \xi = 0 \) makes \( R_{gal} \) have the same form as \( R_h \), means that we can write down the full rotation matrix as

\[
R_{gal} = \begin{pmatrix}
c_\phi c_\xi - s_\phi s_\xi & c_\phi s_\xi + s_\phi c_\xi & -s_\phi \\
c_\phi c_\xi - s_\phi s_\xi & c_\phi s_\xi + s_\phi c_\xi & s_\phi \\
s_\phi c_\xi & s_\phi s_\xi & c_\phi
\end{pmatrix}
\]  

(B3)

where we have used \( c_\chi \) and \( s_\chi \) as shorthands for \( \cos \chi \) and \( \sin \chi \), for brevity.

The image plane (section 2.4) is based on a projection of the galaxy major and minor axes, corresponding to the image \( x \) and \( y \) axes respectively, but also allowing for a rotation of \( \zeta \) about the major axis. This means that, to rotate into the image frame from the galaxy frame, we use

\[
R_{img} = \begin{pmatrix}
1 & 0 & 0 \\
0 & -\sin \zeta & \cos \zeta \\
0 & -\cos \zeta & -\sin \zeta
\end{pmatrix}
\]  

(B4)

Putting these together, a vector in the simulation frame \( p^\text{sim} \) can be transformed into the image frame simply by

\[
p^\text{img} = R_{img} R_{gal} p^\text{sim}
\]  

(B5)

In practice, we will have the halo mass distribution matrix \( M \) (see section 2.2), measured in the simulation frame. We therefore transform this into the image frame by

\[
M^\text{img} = (R_{img} R_{gal} R_h) M (R_{img} R_{gal} R_h)^{-1}
\]  

(B6)

The projected mass distribution in the image plane is then the top-left 2 \( \times \) 2 submatrix of \( M^\text{img} \). The eigenvalues and eigenvectors of this can then easily be found, with the axis lengths of the projected halo ellipse being given by the square root of the eigenvalues.

APPENDIX C: RESULTS FROM OTHER REDSHIFTS

We show here stacked projected halo shapes at different redshifts. They show essentially the same dependences on halo and galaxy properties as have already been illustrated, but nevertheless give an idea of another parameter that can have a significant quantitative impact on the results.

We choose \( z \approx 0.50 \) (MS snapshot 48) as a higher redshift for analysis. The results for the B06 and DLB07 models are shown in Figs. 1C and 1B2 respectively. There is very little change from the \( z = 0.32 \) results in Figs. 7 & 8. When the Split distribution is used, the results for “All” haloes are noticeably less circular, implying a greater proportion of elliptical galaxies. This might initially be seen to contradict the findings of Parry, Eke & Frenk (2009); they showed that, at higher redshifts there should be more disc galaxies, and fewer ellipticals. However, they found that this is strongest for the DLB07 model (where we see less change) and very weak for B06 (where we see the largest difference). The key is that while Parry, Eke & Frenk (2009) select bright galaxies at each redshift, selecting by K-band absolute magnitude \( M_K \sim 5 \log_{10} h = -22.17 \), we are using a deeper cut in apparent magnitude: at this redshift, we are selecting galaxies brighter than \( M_r \sim 5 \log_{10} h = -17.3 \). Thus, we are sampling more low-mass systems than Parry, Eke & Frenk (2009) (at all redshifts), but fewer than we were at \( z = 0.32 \). Our sample has proportionally more higher-mass systems (the biggest difference in shape is with \( M_{140} \)), which tend to host elliptical galaxies. The changes among the bright galaxy population seen in

Figure C1. As Fig. 7 (i.e. using B06), but using redshift \( z \approx 0.5 \).

Figure C2. As Fig. 8 (i.e. using DLB07), but using redshift \( z \approx 0.5 \).
that a colour cut is no longer an efficient way of selecting the more aspherical haloes.

Taken together, these results emphasise those from the main body of the paper. Under the alignment model designed to fit recent galaxy formation simulations, the alignment is sufficiently poor that very little changes the result. However, if there is a significant population with very good alignment, then the resulting stacked shape will depend sensitively on the mass, shape, colour and morphology distribution of the galaxy–halo systems; all of which depends on redshift, and, at present, the galaxy model used.

**Figure C3.** As Fig. 7 (i.e. using B06), but using redshift \( z \approx 0.17 \).

**Figure C4.** As Fig. 8 (i.e. using DLB07), but using redshift \( z \approx 0.17 \).

Parry, Eke & Frenk (2009) are secondary to the overall change in galaxy demographic at higher redshift.

Figs. C3 & C4 show our results at a lower redshift, \( z \approx 0.17 \) (MS snapshot 56; here, our selection cut is at \( M_r - 5 \log_{10} h = -14.6 \)). In this case, the results from the B06 model show a noticeably less circular shape for blue and disc galaxies. Since this is only significant for the Parallel alignment model, it must be due to a greater intrinsic correlation between elliptical haloes and blue/disc-dominated galaxies at this redshift. Furthermore, the effect is not apparent in the DLB07 model. The colour distribution in DLB07 is much broader, particularly for blue galaxies (see Fig. 2), such