Numerical simulation of the embedded discrete fractures by the finite element method

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Abstract. In this work we consider the embedded discrete fracture model (EDFM) for solution of the filtration problem in fractured porous media by the finite element method. The main idea of this model is that the thickness of the fractures is so small that the pressure on it practically does not change and we can reduce the spatial order of the fractures. Structured $nD$ and unstructured $(n-1)D$ grids are independently generated for matrix and fractures, respectively. The embedded fracture modelling strategy avoids the usual requirement that the discretization of the fracture domain conforms to the discretization of the matrix surrounding the fractures.

1. Introduction
When the direction and scale of the flow control several large fractures in oil reservoirs, standard models of double porosity [1-2] have a large error. In connection with this, modern methods of modeling fluid flow in fractured media were invented, among which the embedded discrete fractures method was set up. This method is based on the representation of fractures in one dimension below and on a structured grid for the matrix rock. The essence of the method is that the aperture of the fractures is much smaller than the sizes of the matrix blocks, due to which the fracture integral can be calculated as an integral along the line multiplied by its aperture (Fig. 1.1):

$$\int_{\Omega_f} f(x) dx = \alpha \int_E f(x) dx.$$ (1.1)

In addition, the grid of rocks and fractures are constructed independently as separate computational domains and in the same way the two systems are completely discretized independently, and the mass flow between them is strictly enforced through physics-derived coupling terms. In fact, EDFM is conceptually very similar to dual porosity model, but is able to maintain a more realistic representation of complex geologic features.
This model was first introduced in the works of Lee et al. [3-5]. It was built with the aim of avoiding the construction of complex unstructured meshes as in another similar model of discrete fractures (DFM) [6-10].

2. Problem statement
In this paper we consider the following connected system of two differential equations:

\[ c_m \frac{\partial p_m}{\partial t} - \text{div} \left( \frac{k_m}{\mu} \text{grad} p_m \right) + \delta_\alpha(x) \frac{k_s}{\mu} (p_m - p_f) = 0, \quad x \in \Omega_m \subset \mathbb{R}^n, \tag{2.1} \]

\[ c_f \frac{\partial p_f}{\partial t} - \text{div} \left( \frac{k_f}{\mu} \text{grad} p_f \right) + \frac{k_s}{\mu} (p_f - p_m) = 0, \quad x \in \Omega_f \subset \mathbb{R}^{n-1}, \tag{2.2} \]

where the subscript \( m \) and \( f \) represent the matrix and the fracture regions, respectively, \( c_m = \phi_m c_R \), \( c_f = \phi_f c_R \), \( \phi_m \) and \( \phi_f \) – porosity coefficients, \( c_R \) – medium compressibility, \( k_m \) and \( k_f \) – permeability coefficients, \( \mu \) – fluid viscosity, and \( k_s = 1/(1/k_m + 1/k_f) \). Here, the flow between fractures and porous media occurs only on the fractures, i.e.:

\[ \delta_\alpha(x) = \begin{cases} \alpha, & \text{fracture,} \\ 0, & \text{otherwise,} \end{cases} \tag{2.3} \]

where \( \alpha \) – fracture aperture.

To solve the Eq. (2.1) and (2.2), the following initial and boundary conditions are added:

\[ p_m(x, 0) = p_0(x), \quad x \in \Omega_m, \tag{2.4} \]

\[ p_f(x, 0) = p_0(x), \quad x \in \Omega_f, \tag{2.5} \]

\[ p_m(x) = g(x), \quad x \in \Gamma_m, \tag{2.6} \]

\[ p_f(x) = g(x), \quad x \in \Gamma_f. \tag{2.7} \]

3. Variation formulation
After spatial discretization by the finite element method, we obtain the following variational problem:

Find \( p_m \in V_m \) and \( p_f \in V_f \) such that

\[
\int_{\Omega_m} c_m \frac{p_m^{n+1} - p_m^n}{\tau} v_m \, dx + \int_{\Omega_m} \left( \frac{k_m}{\mu} \text{grad} \, p_m^{n+1}, \text{grad} \, v_m \right) \, dx + \int_{\Omega_m} \delta_\alpha(x) \frac{k_s}{\mu} (p_m^{n+1} - p_f^{n+1}) v_m \, dx = 0, \quad \forall v_m \in V_m, \tag{3.1}
\]
\[
\int_{\Omega_f} c_f \frac{p_m^{n+1} - p_f^n}{\tau} v_f \, dx + \int_{\Omega_f} \left( \frac{k_f}{\mu} \text{grad} p_m^{n+1}, \text{grad} v_f \right) \, dx + \\
+ \int_{\Omega_f} \frac{k_s}{\mu} (p_m^{n+1} - p_m^{n+1}) v_f \, dx = 0, \quad \forall v_f \in V_f,
\]

where the test and trial spaces are defined by
\[
V_m = \{ v \in H^1(\Omega); v = g \text{ on } \Gamma_m \}, \quad \tilde{V}_m = \{ v \in H^1(\Omega); v = 0 \text{ on } \Gamma_m \}, \quad V_f = \{ v \in H^1(\Omega); v = g \text{ on } \Gamma_f \}, \quad \tilde{V}_f = \{ v \in H^1(\Omega); v = 0 \text{ on } \Gamma_f \},
\]

After discretization, we obtain the following system:
\[
\begin{bmatrix}
A_m & -A_{mf} \\
-A_{mf} & A_f
\end{bmatrix}
\begin{bmatrix}
P_m \\
P_f
\end{bmatrix}
= 
\begin{bmatrix}
b_m \\
b_f
\end{bmatrix}
\]

where
\[
A_m = \left\{ a_{ij} = \int_{\Omega_m} c_m \phi_m^i \phi_m^j \, dx + \int_{\Omega_m} \left( \frac{k_m}{\mu} \text{grad} \phi_m^i, \text{grad} \phi_m^j \right) \, dx + \int_{\Omega_m} \delta_a(x) \frac{k_s}{\mu} \phi_m^i \phi_m^j \, dx \right\},
\]
\[
A_f = \left\{ a_{ij} = \int_{\Omega_f} c_f \phi_f^i \phi_f^j \, dx + \int_{\Omega_f} \left( \frac{k_f}{\mu} \text{grad} \phi_f^i, \text{grad} \phi_f^j \right) \, dx + \int_{\Omega_f} \frac{k_s}{\mu} \phi_f^i \phi_f^j \, dx \right\},
\]
\[
A_{mf} = \left\{ a_{ij} = \int_{\Omega_m} \delta_a(x) \frac{k_s}{\mu} \phi_m^i \phi_f^j \, dx \right\},
\]
\[
A_{fm} = \left\{ a_{ij} = \int_{\Omega_f} \frac{k_s}{\mu} \phi_f^i \phi_m^j \, dx \right\},
\]

and \(A_{mf} = A_{fm}^T\). Since the basis functions \(\phi_m\) are defined on mesh cells of the porous medium, to calculate the integrals in Eq.3.8. and Eq.3.9. their projections to \(\Omega_f\) are used. The basis functions will be the Lagrange polynomials of the first degree.

### 4. Numerical simulation

For approbation of this modelling technique we compare it with the double porosity model. To do this, we construct a grid with fractures with some aperture (Fig.4.1.) and discretization the equation for fractures will be produced with a coefficient of porosity equal to one. The remaining coefficients and the initial conditions will fully correspond to the EDFM problem (Tab.4.1). Note that the aperture of the fractures on the mesh Fig.4.1 is equal to the aperture \(\alpha\) in the EDFM.

| \(\phi_m\) | \(\phi_f\) | \(c_f\) | \(k_m\) | \(k_f\) | \(\mu\) | \(\alpha\) | \(p_0\) | \(g\) |
|---|---|---|---|---|---|---|---|---|
| 0.4 | 1 | 1E-9 Pa\(^{-1}\) | 1E-15 m\(^2\) | 1E-10 m\(^2\) | 2E-3 Pa\(\cdot\)s | 0.2 m | 10 Mpa | 1 Mpa |

Table 4.1. Initial data.

To solve the problem (2.1) – (2.7) by the EDFM method, we construct four successively refined meshes with fractures as shown in Fig.4.2. Time step we take \(\tau = 1\) day. To compare EDFM solutions with double porosity, we compare the solutions in the matrix rock, i.e.:
\[
||\varepsilon||_{L_2} = ||p_m^{DP} - p_m^{EDFM}||_{L_2}/||p_m^{DP}||_{L_2}, \quad ||\varepsilon||_{H} = ||p_m^{DP} - p_m^{EDFM}||_{H}/||p_m^{DP}||_{H},
\]
where $||u||_{L^2} = \sqrt{\int_{\Omega_f} \frac{k_m}{\mu} u^2 \, dx}$, $||u||_H = \sqrt{\int_{\Omega_f} \left( \frac{k_m}{\mu} \text{grad} u, \text{grad} u \right) \, dx}$ and $p_m^{DP}$ is solution by double porosity model and $p_m^{EDFM}$ by EDFM.

Fig. 4.1. Double porosity mesh with 541125 vertices and 1081483 cells.

Fig. 4.2. EDFM meshes.
In Fig.4.3. The comparison of solutions by the method EDFM and double porosity is shown at different times. As can be seen from the results, the flow velocity of the fluid is much higher at the fractures. Graphs of calculation error are shown in Fig.4.4., where the solutions on meshes mesh 3 and mesh 4 grids have higher accuracy. At the last moment of time, the error solutions in the norms $L_2$ and $H$ have the following values: on the mesh 1 – 3.3% and 24.2%, on the mesh 2 – 1.5% and 11.8%, on the mesh 3 – 0.9% and 4% and on the mesh 4 – 0.6% and 1.3%.

5. Conclusion
In this paper, another method for simulating embedded discrete fractures is presented. In this approach, the flow between the fractures and the porous medium is approximated by means of linear basis functions. It has been shown to be applicable and accurate in the modeling of oil reservoirs in which long fractures control the main flow of a fluid. This method has the advantage over the discrete fractures model (DFM) in that the mesh for the porous medium and for the fractures are constructed...
separately and do not need an appropriate thickening on the fractures, which leads to a reduction in the number of unknowns.

In the future, it is planned to improve this model and solve its three-dimensional problem. It is also planned to solve this problem by a generalized multiscale finite element method (GMsFEM).

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