Reliability Analysis Using Real Options Based on Hierarchical Bayesian Methods

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Abstract
Real option approaches are handled for reliability evaluations based on degradation models. In general, the reliability analysis is focused on a random variable of life data, and the Weibull distribution is used as a reliability modeling. However a degradation process is significant for making value of the reliability. In particular, stochastic process models are useful for representations of the degradation paths. In this paper, a hierarchical Bayesian VG (Variances Gamma) model is proposed for the representation of the degradation phenomenon, and the real option method based on a simulation is applied to the reliability evaluation using the model. It is also shown that other reliability approaches for the degradation analysis are handles based on GARCH (Generalized Autoregressive Conditional Heteroskedasticity) and SV (Stochastic Volatility) models, and a method of comparison for the models based on the information criterion EIC is proposed.

1 Introduction
Methodologies for the reliability analysis are handled based on random variables of life lengths. In particular, the extreme distributions are used as the models of the reliability phenomena. For example, the Weibull distribution is used for the reliability analysis, and accelerated models are expanded by using regression expressions for the scale parameter. As the other way, states of failure phenomena which are depended on the time proceeds are assumed, and then the stochastic models are addressed for the reliability inferences. In these cases, the homogeneous and nonhomogeneous Poisson processes are used as models for failure occurrence numbers. Maintenance policies are made decisions based on the renewal processes of the stochastic models. The estimations for the reliability models are performed using the maximum likelihood method, and also the Bayesian methods are applied to the estimations [1]. In the Bayesian methodologies, the prior distributions of experimental knowledge are handled for the estimations of the parameters. The hierarchical Bayesian modeling methodologies for complex failure phenomena are used to the analysis.

However, the general methodologies for the reliability analysis make evaluations only for fundamental of the failure phenomena based on observed random variables of lifetimes. In actual, the reliability predictions based on stochastic processes of degradation paths are needed to evaluate the reliability for failure modes. It is assumed that failures are occurred due to the exceedances of thresholds for the degradations [2]. In the case of the strength of materials, the observation data are the lengths of fatigue cracks. The fatigue is caused by the degradation which is occurred by the dislocation of the cell structure and the slip of the face. Therefore the degradation could not be observed. The filtering methods for the estimations might be applied to the problems. However we assume that the observed values as clacks are the degradations of phenomena, and the real option methodologies are effective to evaluate the random phenomena for the degradation predictions.

In this paper, real option approaches are handled for reliability evaluations based on degradation models. The degradation process is significant for making value of the reliability [3]. In particular, stochastic process models are useful for representations of the degradation paths. It is handled that a hierarchical Bayesian VG (Variances Gamma) model is proposed for the representation of the degradation phenomenon, and the real option method based on a simulation is applied to the reliability evaluation using the model [4]. It is also shown that other reliability approaches for the degradation analysis are handles based on GARCH (Generalized Autoregressive Conditional Heteroskedasticity) and SV (Stochastic Volatility) models. The parameters of these models are estimated based on the likelihood functions of the models, and the estimation methods are extended to applications of MCMC (Markov chain Monte Carlo) and MF (Monte Carlo filtering) methodologies. In addition, a method of comparison for the models and estimations based on the information criterion is proposed [5]. In particular, the Laplace’s method is applied for the evaluation of EIC based on the Bayesian estimation methods for these models [6].
2 Methodologies

2.1 Real Option
Let \( d_t \) be a state of the degradation at some time \( t \), and it assumed that \( d_t \) is continuous and \( t \) is discrete, \( t = 0, 1, 2, 3, \ldots \). We assumed that a failure occurs at \( d_t \geq K \), where \( K \) is a threshold for a mode. The probability of reliability is defined as \( P\{y_t \leq K|t \leq T\} \), where \( T \) is a conditional fixed time. The risk value at \( t = T \) is given by \( E\{\max(d_T - K, 0)\} \). According to the same way, the reliability value at \( t = T \) is given by \( E\{\max(K - d_T, 0)\} \). Note that these values are derived from the simulation method which is consisted by the real options based on the time series models and estimation methods using the observed degradation data, which means a transform \( y_t = \log(d_t/d_t-1) \) and \( y^{(n)} = [y_1, y_2, \cdots, y_n] \). Note that \( n \) is the number of the transformed data. Fig. 1 shows the abstract of the degradation modeling for the continuous time.

According to the forms (1) and (2), the posterior distribution of \( \lambda_t \) denoted by \( p(\lambda_t|y_t) \) is given by
\[
p(\lambda_t|y_t) = \frac{f(y_t|\lambda_t)g(\lambda_t|a, \Phi_t)}{\int_0^\infty f(y_t|\lambda_t)g(\lambda_t|a, \Phi_t)d\lambda_t} \]
\[
= \frac{S_t^{a+\frac{1}{2}}}{\Gamma(a + \frac{1}{2}) \left( \frac{1}{S_t} \right)^{\alpha+1}} \exp \left[ - \frac{S_t}{\lambda_t} \right],
\]
where \( S_t = y_t^2/2 + \Phi_t \), and the estimate \( \hat{\lambda}_t \) is assigned by \( IG\{a+1/2, S_t\} \) with \( E\{\lambda_t|y_t\} = S_t/(a - 1/2) \).

We denote the parameters of the hierarchical Bayesian VG model as \( \theta = [a, p_t] \), and the estimate \( \hat{\theta} \) is derived from the maximum likelihood method. The likelihood \( l_m(\theta|y^{(n)}) \) is given by
\[
l_m(\theta|y^{(n)}) = \prod_{t=1}^n \int_0^\infty f(y_t|\lambda_t)g(\lambda_t|a, \Phi_t)d\lambda_t
\]
\[
= \left( \frac{1}{\sqrt{2\pi}} \right)^n \left( \frac{\Gamma(a + \frac{1}{2})}{\Gamma(a)} \right)^n \prod_{t=1}^n \left( \frac{\Phi_t}{S_t} \right)^a \left( \frac{1}{S_t} \right)^{\frac{n}{2}}.
\]

According to \( \log E\{\lambda_t|y_t\} \) of the form (3), \( q_t \), \( t = 1, 2, \cdots, n \) are given by
\[
q_t = \log \frac{S_t}{a - \frac{1}{2}}.
\]

We assume the prior distribution \( \pi(a|c, n) \) which is given by
\[
\pi(a|c, n) = \left( \frac{\Gamma(a + 1)}{\Gamma(a + \frac{1}{2})} \right)^n \exp \left[ -ac \right],
\]
where \( c \) and \( n \) are parameters. The posterior distribution of \( a \) denoted by \( p(a|y^{(n)}) \) is given as
\[
p(a|y^{(n)}) = \frac{l_m(\theta|y^{(n)})\pi(a|c, n)}{\int_0^n l_m(\theta|y^{(n)})\pi(a|c, n)da} = \frac{(\gamma + c)^{n+1}}{\Gamma(n + 1)} a^n \exp[-a(\gamma + c)],
\]
where \( \gamma = \log \prod_{t=1}^n (S_t/\Phi_t) \). The expectation of the posterior distribution is the gamma distribution and the expectation is given as \( E\{a|y^{(n)}\} = (n + 1)/(\gamma + c) \). The parameters of the hierarchical Bayesian VG model are estimated by the maximum likelihood method based on the form (4). However in the maximization, the parameter \( a \) is assigned by \( E\{a|y^{(n)}\} \).

2.2 Hierarchical Bayesian VG Model
The hierarchical Bayesian VG model for the time series is given by
\[
y_t = \zeta_t \exp \left[ \frac{q_t}{2} \right], \quad \lambda_t = \exp[q_t],
\]
where \( \zeta_t \) is generated by the standard normal distribution \( N(0,1) \), and \( \lambda_t \) is given by the inversed gamma distribution \( IG\{a, \Phi_t\} \). Note that \( a \) is the shape parameter and \( \Phi_t = \exp[p]\\zeta_7 \), where \( \zeta_t \) means environmental factors denoted by a vector, and \( p \) is a parameter vector. Note that \( \tau \) means the transfer of a vector and a matrix. The probability density functions of \( y_t \) and \( \lambda_t \) which are denoted by \( f(y_t|\lambda_t) \) and \( g(\lambda_t|a, \Phi_t) \), respectively, are given by
\[
f(y_t|\lambda_t) = \frac{1}{\sqrt{2\pi}\lambda_t} \exp \left[ -\frac{y_t^2}{2\lambda_t} \right],
\]
\[
g(\lambda_t|a, \Phi_t) = \frac{\Phi_t^a}{\Gamma(a)} \left( \frac{1}{\lambda_t} \right)^{\alpha+1} \exp \left[ -\frac{\Phi_t}{\lambda_t} \right].
\]

2.3 GARCH Type Model
The GARCH(1,1) model is given as
\[
y_t = \eta_t \sqrt{\sigma_t^2},
\]
\[
\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2,
\]
where \( \eta_t \) is assumed as the i.i.d. with the standard normal distribution \( N(0,1) \), and \( \alpha_0 > 0, \alpha_1 > 0, \beta_1 > 0 \).
because $\sigma^2_t > 0$, $t = 1, 2, \ldots$. If $\beta_1 = 0$, the form (6) means the ARCH(1) model. The GARCH model involves the ARCH model and we represent these models as the GARCH type model.

The parameters of the GARCH type model is denoted by $\theta$. In the case of the GARCH(1,1), the parameter vector is given by $\theta = [\alpha_0, \alpha_1, \beta_1, \gamma_1]^{T}$. The form (5) gives the probability density function $f(y_t|\sigma^2_t)$ as

$$f(y_t|\sigma^2_t) = \frac{1}{\sqrt{2\pi}\sigma_t} \exp\left[-\frac{y^2_t}{2\sigma_t^2}\right].$$

The maximum likelihood method enables to estimate the parameters of the GARCH type model. The likelihood function $l(\theta|y^{(n)})$ of the GARCH(1,1) is given as

$$l(\theta|y^{(n)}) = \prod_{i=1}^{n} f(y_t|\sigma^2_t),$$

where $\sigma^2_t, t = 2, 3, \ldots, n$ is given by the form (6) and $\sigma^2_t$ is assigned initially. The maximum likelihood estimate is given by maximization of $l(\theta|y^{(n)})$.

### 2.4 SV Model

The SV model is given by

$$y_t = \varepsilon_t \exp\left[\frac{h_t}{2}\right],$$

$$h_t = \mu + \phi(h_{t-1} + \mu) + \omega_t,$$

where $\varepsilon_t$ and $\omega_t$ are drawn by the normal distribution $N(0, 1)$ and $N(0, \sigma^2_w)$, respectively, and $|\phi| < 1$.

The translation of the stochastic volatility $h_t$ is given as $\tau_t = \exp[h_t]$. The probability density functions of the $y_t$ and $\tau_t$ are denoted as $f(y_t|\tau_t)$ and $g(\tau_t|\mu, \phi, \sigma^2_w, \tau_{t-1})$ respectively. These are given as

$$f(y_t|\tau_t) = \frac{1}{\sqrt{2\pi}\tau_t} \exp\left[-\frac{y^2_t}{2\tau_t}\right],$$

$$g(\tau_t|\mu, \phi, \sigma^2_w, \tau_{t-1}) = \frac{1}{\sqrt{2\pi\sigma^2_w} \tau_t} \exp\left[-\frac{(\log \tau_t - \mu - \phi(\log \tau_{t-1} + \mu))^2}{2\sigma^2_w}\right].$$

The forms (10) and (11) mean the normal and lognormal distributions, respectively. The parameters $\mu, \phi, \sigma^2_w$ have to be estimated based on the data $y^{(n)}$. The parameter vector $\theta = [\mu, \phi, \sigma^2_w]^{T}$, and the marginal likelihood of the SV model denoted by $l_m(\theta|y^{(n)})$ is given as

$$l_m(\theta|y^{(n)}) = \prod_{i=1}^{n} \int_{0}^{\infty} f(y_t|\tau_t)g(\tau_t|\theta, \tau_{t-1})d\tau_t = \int_{0}^{\infty} \cdots \int_{0}^{\infty} p(\tau_1, \cdots, \tau_n|\theta)d\tau_n \cdots d\tau_1,$$

where $p(\tau_1, \cdots, \tau_n|\theta)$ is the joint distribution of $\tau_1, \tau_2, \cdots, \tau_n$. The form (12) is not closed, and then the form is not solved analytically. It is difficult to get the maximum likelihood estimate by the marginal likelihood $l_m(\theta|y^{(n)})$.

The quasi-likelihood is useful for the estimation based on the Kalman filter and a state space modeling of SV. The state space model for the forms (8) and (9) is given as

$$x_t = h_t + \xi_t,$$

$$h_{t+1} = \mu(1-\psi) + \psi h_t + \omega_t,$$

where $x_t = \log y^2_t$, $\xi_t = \log \varepsilon^2_t$, and $\xi_t$ is distributed approximately by the normal distribution with the mean $m_\xi = -1.2707$ and the variance $v_\xi = 4.93$ based on the chi square distribution with the degree of freedom 1 and logarithm transfer. The Kalman filter algorithm is applied as

$$h_{t+1|t} = \mu(1-\psi) + \psi h_{t|t-1} + K_t v_t,$$

$$K_t = \psi P_{t|t-1} F^{-1},$$

$$P_{t+1|t} = \psi P_{t|t-1}(\psi - K_t)^{-1},$$

$$F_t = \psi P_{t|t-1} + v_t,$$

$$v_t = x_t - \psi h_{t|t-1} - m_\xi,$$

where the notation $***$ means the value at ** with the conditional value given at *. Therefore $K_t$ is the Kalman gain, $P_{t+1|t}$ is the variance of the Kalman filter, and $F_t$ is the corresponding mean square error and $v_t$ is the one-step-ahead prediction error. The quasi likelihood of the SV model is denoted by $l_q(\theta|y^{(n)})$ and it is given as

$$l_q(\theta|y^{(n)}) = \prod_{t=1}^{n} \frac{1}{\sqrt{2\pi F_t}} \exp\left[-\frac{v^2_t}{2F_t}\right].$$

The maximum likelihood estimate is given by the maximization of $l_q(\theta|y^{(n)})$. However the approximation to the normal distribution is not good performance at small sample situations.

### 3 Bayesian Estimation

#### 3.1 MCMC

The Bayesian estimation is performed based on the posterior distribution. The hierarchical Bayesian VG, the GARCH type, and the SV models have their parameters, and the parameters are not only estimated by the maximum likelihood methods but also the Bayesian methods.

The Gibbs sampling is used to estimate of the posterior distribution. Let a parameter vector $\theta$ be divided as $\theta = [\theta_1, \theta_2, \cdots, \theta_k]$, and the posterior distribution is denoted by $p(\theta_1, \theta_2, \cdots, \theta_k|y)$, the conditional posterior distributions could be given as $p(\theta_i|\theta_{<i})$, $i = 1, 2, \cdots, k$. The notation $\theta_{<i}$ means the parameter vector excluding $\theta_i$, that is, $\theta_{<i} = [\theta_1, \theta_2, \cdots, \theta_{i-1}, \theta_{i+1}, \cdots, \theta_k]$. The
Gibbs sampling algorithm is performed as the following repeats.  
\begin{align*}
    j &= 1, 2, \ldots, M, \\
    \theta^{(j)}_1 &\sim p(\theta|\theta^{(j-1)}_2, \theta^{(j-1)}_3, \ldots, \theta^{(j-1)}_k), \\
    \theta^{(j)}_2 &\sim p(\theta|\theta^{(j)}_1, \theta^{(j-1)}_3, \ldots, \theta^{(j-1)}_k), \\
    \theta^{(j)}_3 &\sim p(\theta|\theta^{(j)}_1, \theta^{(j)}_2, \theta^{(j-1)}_4, \ldots, \theta^{(j-1)}_k), \\
    &\quad \vdots \\
    \theta^{(j)}_k &\sim p(\theta|\theta^{(j)}_1, \theta^{(j)}_2, \ldots, \theta^{(j-1)}_k) \\
\end{align*}
(16)

It is considered that the samples \( \theta^{(i)}_j \), \( i = 1, 2, \ldots, k \) are distributed by the posterior distribution \( p(\theta_1, \theta_2, \ldots, \theta_k|y) \) for the enormous number of \( j \). In actual, the burn-in period \( B \) is assumed and the samples for \( j = B + 1, B + 2, \ldots, M \) are used for the estimation of the posterior distribution.

The Gibbs sampling is performed by the Monte Carlo simulation based on the conditional posterior distributions. However, it is possible that the simulation is not permitted due to the form type of the conditional distribution. The MH (Metropolis-Hastings) method is used to simulate random variables based on any distribution. Let \( \theta = \theta^{(i)} \) be given, and \( \theta' \) is assigned as \( \theta^{(i+1)} \) which is proposed by \( q(\theta|\theta') \). The algorithm is handled as following, that is
\[
    \alpha(\theta, \theta') = \left\{ \begin{array}{ll} 
    \min \left( \frac{p(\theta')q(\theta|\theta')}{p(\theta)q(\theta'|\theta)}, 1 \right), & p(\theta)|q(\theta', \theta) > 0, \\
    1, & p(\theta)|q(\theta', \theta) = 0,
    \end{array} \right.
\]

and \( u \) is generated by the normal distribution \( N(0, 1) \). If \( \alpha(\theta, \theta') < u \) then \( \theta' \) is accepted, not if then it is rejected. Using the substitute distribution \( q(\theta|\theta') \), the value is generated according to any conditional distribution \( p(\theta') \), if the \( \alpha(\theta, \theta') \) is accepted. We use the MH method in the algorithm of Gibbs sampling.

### 3.2 Monte Carlo Filtering

The filtering methods are used to estimate the parameters of the stochastic volatility models. The Kalman Filtering is base of these methods. However the Kalman filter is the algorithm for the Gaussian innovation distributions for the state space modeling. Therefor the modeling of based on the non-Gaussian distribution, it is made possible to apply the sequential of Monte Carlo methods. In this section, we describe the theses filtering methods in detail.

The state estimation and prediction are as follow. The state space representation for the stochastic volatility models is given as
\[
    y_t \sim f(y_t|\theta_y, \theta_h), \\
    h_t \sim g(h_t|\theta_y, h_{t-1}),
\]
(17)
where \( \theta_y \) and \( \theta_h \) are the parameters of the observation and system equations, respectively. Let \( \theta \) denote the parameter of the state space model, that is, \( \theta^T = [\theta_y^T, \theta_h^T] \). The filtering and prediction of the Bayesian frame work are given as follows.
\[
    \begin{align*}
    p(h_t|y^{(t-1)}, \theta) &= \int_0^\infty g(h_t|h_{t-1}, \theta_h)p(h_{t-1}|y^{(t-1)}, \theta)dh_{t-1}, \\
    p(h_t|y^{(t)}, \theta) &= \frac{f(y_t|h_t, \theta_h)p(h_t|y^{(t-1)}, \theta)}{p(y_t|y^{(t-1)}, \theta)}, \\
    p(y_t|y^{(t-1)}, \theta) &= \int_0^\infty f(y_t|h_t, \theta_h)p(h_t|y^{(t-1)})dh_t,
\end{align*}
\]
The algorithm of MCF is given by \( i = 1, 2, \ldots, n \),
\[
    \begin{align*}
    k &= 1, 2, \ldots, m, \\
    \omega^{(k)}_i &= q(\omega), \\
    p^{(k)}_i &= g(\omega^{(k)}), \\
    \alpha^{(k)}_i &= r \left( f^{-1}(y^{(k)}_i) \right) \left( \frac{\partial f^{-1}}{\partial y} \right),
\end{align*}
\]
where \( \{f_0^{(1)}, f_1^{(1)}, \ldots, f_0^{(m)}\} \) are initial values, and \( \{f_1^{(1)}, f_2^{(1)}, \ldots, f_1^{(m)}\} \) are offered from \( \{p_1^{(1)}, p_2^{(1)}, \ldots, p_1^{(m)}\} \) with m resampling repetitions based on the probability \( \{\alpha^{(1)}, \alpha^{(2)}, \ldots, \alpha^{(m)}\} \). The log likelihood function likelihood \( l(\theta|y^{(n)}) \) is approximately given by
\[
    \log l(\theta|y^{(n)}) \approx \sum_{i=1}^n \log \left( \sum_{k=1}^m \alpha^{(k)}_i \right) - n \log m.
\]

### 3.3 EIC

The Kullback-Leibler (KL) information is denoted by \( KL(g: f) \), and the random variables \( X, U \) have the same distribution as the probability model for \( y^{(n)} \). Then \( KL(g: f) \) is given by
\[
    KL(g: f) = \int_0^\infty \log \frac{g(x|\theta)}{f(x|\theta)} \cdot g(x|\theta_\theta)dx \\
    = \int_0^\infty \log g(x|\theta_\theta)g(x|\theta)dx \\
    - \int_0^\infty \log f(x|\theta)g(x|\theta_\theta)dx,
\]
where \( g(x|\theta_\theta) \) is the probability density function for the true model, and \( f(x|\theta) \) is the probability density function for the fitted model. When \( KL(g: f) \geq 0 \) is close to 0, the fitted model means to be close to the true model.

The positive term of \( KL(g: f) \) is unknown, but the negative term means \( n \) times the expected mean log likelihood which is denoted by \( ELL \) and given by
\[
    ELL = \frac{n}{o} \left\{ \log l(\hat{\theta}(x)|x) \right\}
\]
\[
    = \frac{n}{o} \left\{ \int \log f(u|\theta(x))g(u|\theta_\theta)du \right\}.
\]
However \( \log l(\hat{\theta}(y^{(n)})|y^{(n)}) \) is an estimate of \( ELL \), then the bias correction is necessary to the estimation of
ELL. EIC consists of the bias correction based on the data $y^{(n)}$ and the bootstrap samples, and the bias $C^*$ is given by

$$C^* = E_X \cdot \left\{ \log l(\hat{\theta}_A(x^*)|x^*) - l(\hat{\theta}_A(y^{(n)})) \right\} + \left\{ E_U \cdot \left\{ \log l(\hat{\theta}_A(y^{(n)})|u^*) \right\} - E_U \cdot \left\{ \log l(\hat{\theta}_A(x^*)|u^*) \right\} \right\}$$

where $X^*, U^*$ are bootstrap samples based on $y^{(n)}$ and $\hat{\theta}_A(x)$ is any estimator based on the data $x$. In this paper, the bootstrap samples are driven by the fitted model and estimated parameters under $y^{(n)}$. EIC is defined as

$$EIC = -2 \times \log l(\hat{\theta}_A(y^{(n)})) + 2 \times C^*$$

In the Bayesian methodology, the values of EIC are calculated based on the marginal likelihood functions, and the estimate is $\hat{\theta}_A = \hat{\theta}_{MCMC}$. Note that $\hat{\theta}_{MCMC}$ means the estimate based on the MCMC of the Bayesian methodology, and EIC is derived by $\log l(\hat{\theta}_{MCMC}(y^{(n)}))$. However, the MCMC and the bootstrap methods need an enormous amount of repetitions in the numerical calculation for EIC. Therefore, in this paper, the Laplace’s method is also used for the application of EIC with the MCMC method. Calculating the bootstrap evaluation for $C^*$, the posterior mode $\hat{\theta}_{BMO}$ is used. According to the Laplace’s method, the marginal likelihood $l_m(\theta_h|y^{(n)})$ is given by

$$l_m(\theta_h|y^{(n)}) = \int_\Theta l(\theta|y^{(n)}) \pi(\theta) d\theta$$

$$\approx l(\hat{\theta}_{BMO}|y^{(n)}) \pi(\hat{\theta}_{BMO}|\theta_h) \times \left( \frac{(2\pi)^{V/2}}{n^{V/2}} \right) \frac{1}{i(\hat{\theta}_{BMO})}$$

where $V$ is the dimension of $\theta$, and $\theta_h$ means the hype parameter. The description of $i(\hat{\theta}_{BMO})$ is given by

$$i(\hat{\theta}_{BMO}) = -\frac{\partial^2 \log l(\hat{\theta}(y^{(n)}) \pi(\hat{\theta}|\theta_h))}{\partial \theta \partial \theta^T} \bigg|_{\theta = \hat{\theta}_{BMO}}$$

### 4 Concluding Remarks

In this paper, a hierarchical Bayesian VG (Variances Gamma) model has been proposed for the representation of the degradation phenomenon, and other reliability approaches for the degradation analysis have been handled based on the GARCH type and the SV models. We have showed the applications of the maximum likelihood, the Bayesian method with MCMC and the estimation methods using the Kalman filter and MCF for these models. The real option method based on a simulation has been applied to the reliability evaluation using these models. We have proposed a method of comparison for the models and the estimation methods based on the information criterion EIC. In particular, it has been shown that the Laplace’s method is useful for the evaluation of the EIC based on the Bayesian estimation methods for these models. However the application of EIC using an approximation method for the degradation model with MCF was not proposed in the paper. The problem would be solved in the future.

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