BSP-Net: Generating Compact Meshes via Binary Space Partitioning

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Abstract

Polygonal meshes are ubiquitous in the digital 3D domain, yet they have only played a minor role in the deep learning revolution. Leading methods for learning generative models of shapes rely on implicit functions, and generate meshes only after expensive iso-surfacing routines. To overcome these challenges, we are inspired by a classical spatial data structure from computer graphics, Binary Space Partitioning (BSP), to facilitate 3D learning. The core ingredient of BSP is an operation for recursive subdivision of space to obtain convex sets. By exploiting this property, we devise BSP-Net, a network that learns to represent a 3D shape via convex decomposition. Importantly, BSP-Net is unsupervised since no convex shape decompositions are needed for training. The network is trained to reconstruct a shape using a set of convexes obtained from a BSP-tree built on a set of planes. The convexes inferred by BSP-Net can be easily extracted to form a polygon mesh, without any need for iso-surfacing. The generated meshes are compact (i.e., low-poly) and well suited to represent sharp geometry; they are guaranteed to be watertight and can be easily parameterized. We also show that the reconstruction quality by BSP-Net is competitive with state-of-the-art methods while using much fewer primitives.

1. Introduction

Recently, there has been an increasing interest in representation learning and generative modeling for 3D shapes. Up to now, deep neural networks for shape analysis and synthesis have been developed mainly for voxel grids [14, 18, 49, 51], point clouds [1, 33, 34, 55, 56], and implicit functions [5, 13, 22, 28, 53]. As the dominant 3D shape representation for modeling, display, and animation, polygonal meshes have not figured prominently amid these developments. One of the main reasons is that the non-uniformity and irregularity of triangle tessellations do not naturally support conventional convolution and pooling operations [19]. However, compared to voxels and point clouds, meshes can provide a more seamless and coherent surface representation; they are more controllable, easier to manipulate, and are more compact, attaining higher visual quality using fewer primitives; see Figure 1.

For visualization purposes, the generated voxels, point clouds, and implicits are typically converted into meshes in post-processing, e.g., via iso-surface extraction by Marching Cubes [26]. Few deep networks can generate polygonal meshes directly, and such methods are limited to genus-zero meshes [17, 27, 45], piece-wise genus-zero [12] meshes, meshes sharing the same connectivity [11, 42], or meshes with very low number of vertices [7]. Patch-based approaches can generate results which cover a 3D shape with planar polygons [47] or curved [16] mesh patches, but their visual quality is often tampered by visible seams, incoherent patch connections, and rough surface appearance. It is difficult to texture or manipulate such mesh outputs.

Figure 1: (a) 3D shape auto-encoding by our network, BSP-Net, quickly reconstructs a compact, i.e., low-poly, 3D model – a polygonal mesh with a small number of polygons, which can be easily textured. The edges of the mesh can reproduce sharp details in the input (e.g., edges of the legs), yet still approximate smooth geometry (e.g., circular table-top). (b) State-of-the-art techniques regress an indicator function, which needs to be iso-surfaced, resulting in over-tessellated meshes; note how these models only approximate sharp details with smooth surfaces.
BSP-Net learns an implicit field: given \( n \) point coordinates and a shape feature vector as input, the network outputs values indicating whether the points are inside or outside. The construction of this implicit function is illustrated in Figure 2, and consists of three steps: (1) a collection of plane equations implies a collection of \( p \) binary partitions of space; see Figure 2-top; (2) an operator \( T_{p \times c} \) groups these partitions to create a collection of \( c \) convex shape primitives/parts; (3) finally, the part collection is merged to produce the implicit field of a single output shape.

Figure 3 shows the network architecture of BSP-Net corresponding to these three steps: (1) given the feature code, an MLP produces in layer \( L_0 \) a matrix \( P_{p \times 4} \) of canonical parameters that define the implicit equations of \( p \) planes: \( ax + by + cz + d = 0 \); these implicit functions are evaluated on a collection of \( n \) point coordinates \( x_{n \times 4} \) in layer \( L_1 \); (2) the operator \( T_{p \times c} \) is a binary matrix that enforces a selective neuron feed from \( L_1 \) to the next network layer \( L_2 \), forming convex parts; (3) finally, layer \( L_3 \) assembles the parts into a shape via either sum or min-pooling.

At inference time, we feed the input to the network to obtain components of the BSP-tree, i.e., leaf nodes (planes \( P \)) and connections (binary weights \( T \)). We then apply classic Constructive Solid Geometry (CSG) to extract the explicit polygonal surfaces of the shapes. The mesh is typically compact, formed by a subset of the \( p \) planes directly from the network, leading to a significant speed-up over the previous networks during inference, and without the need for expensive iso-surfacing – current inference time is about 0.5 seconds per generated mesh. Furthermore, meshes generated by the network are guaranteed to be watertight, possibly with sharp features, in contrast to smooth shapes produced by previous implicit decoders [5, 22, 28].

BSP-Net is simultaneously trainable and interpretable. Importantly, the training of the network is self-supervised as no ground truth convex shape decompositions are needed. BSP-Net is trained to reconstruct all shapes from the training set using the same set of convexes constructed in layer \( L_2 \) of the network. This provides a natural correspondence between all the shapes at the level of the convexes. BSP-Net does not yet learn semantic parts. Grouping of the convexes into semantic parts can be obtained manually, or learned otherwise as semantic shape segmentation is a well-studied problem. Such a grouping only need to be done on one shape to propagate the semantic understanding to all shapes containing the same semantic parts.

**Contributions.**

- BSP-Net is the first deep generative network which directly outputs compact and watertight polygonal meshes with arbitrary topology and structure variety.
- The learned BSP-tree allows us to infer both shape segmentation and part correspondence.
- By adjusting the encoder of our network, BSP-Net can also be adapted for shape auto-encoding and single-view 3D reconstruction (SVR).
- To the best of our knowledge, BSP-Net is among the first to achieve structured SVR, reconstructing a segmented 3D shape from a single unstructured object image.
- Last but not the least, our network is also the first which can reconstruct and recover sharp geometric features.

Through extensive experiments on shape auto-encoding, segmentation, part correspondence, and single-view reconstruction, we demonstrate state-of-the-art performances by BSP-Net. Comparisons are made to leading methods on shape decomposition and 3D reconstruction, using conventional distortion metrics, visual similarity, as well as a new metric assessing the capacity of a model in representing sharp features. In particular, we highlight the favorable fidelity-complexity tradeoff exhibited by our network.
2. Related work

Large shape collections such as ShapeNet [2] and PartNet [30] have spurred the development of learning techniques for 3D data processing. In this section, we cover representative approaches based on the underlying shape representation learned, with a focus on generative models.

**Grid models.** Early approaches generalized 2D convolutions to 3D [6, 14, 24, 49, 50], and employed volumetric grids to represent shapes in terms of coarse occupancy functions, where a voxel evaluates to zero if it is outside and one otherwise. Unfortunately, these methods are typically limited to low resolutions of at most $64^3$ due to the cubic growth in memory requirements. To generate finer results, differentiable marching cubes operations have been proposed [26], as well as hierarchical strategies [18, 37, 43, 46, 47] that alleviate the curse of dimensionality affecting dense volumetric grids. Another alternative is to use multi-view images [25, 41] and geometry images [39, 40], which allow standard 2D convolution, but such methods are only suitable on the encoder side of a network architecture, while we focus on decoders. Finally, recent methods that perform sparse convolutions [15] on voxel grids are similarly limited to encoders.

**Surface models.** As much of the semantics of 3D models is captured by their surface, the boundary between inside/outside space, a variety of methods have been proposed to represent shape surfaces in a differentiable way. Amongst these we find a category of techniques pioneered by PointNet [33] that express surfaces as point clouds [1, 9, 10, 33, 35, 54, 56], and techniques pioneered by AtlasNet [16] that adopt a 2D-to-3D mapping process [48, 40, 45, 54]. An interesting alternative is to consider mesh generation as the process of estimating vertices and their connectivity [7], but these methods do not guarantee watertight results, and hardly scale beyond a hundred vertices.

**Implicit models.** A very recent trend has been the modeling of shapes as a learnable indicator function [5, 22, 28], rather than a sampling of it, as in the case of voxel methods. The resulting networks treat reconstruction as a classification problem, and are universal approximators [20] whose reconstruction precision is proportional to the network complexity. However, at inference time, generating a 3D model still requires the execution of an expensive iso-surfacing operation whose performance scales cubically in the desired resolution. In contrast, our network directly outputs a low-poly approximation of the shape surface.

**Shape decomposition.** BSP-Net generates meshes using a part-based approach, hence techniques that learn shape decompositions are of particular relevance. There are methods that decompose shapes as oriented boxes [44, 31], axis aligned gaussians [13], super-quadrics [32], or a union of indicator functions, in BAE-NET [4]. The architecture of our network draws inspiration from BAE-NET, which is designed to segment a shape by reconstructing its parts in different branches of the network. For each shape part, BAE-NET learns an implicit field by means of a binary classifier. In contrast, BSP-Net explicitly learns a tree structure built on plane subdivisions for bottom-up part assembly.

Another similar (still unpublished) work is CvxNet [8], which decomposes shapes as a collection of convex primitives. However, BSP-Net differs from CvxNet in several significant ways: (1) we target low-poly reconstruction with sharp features, while they target smooth reconstruction; (2) their network always outputs $K$ convexes, while the “right” number of primitives is learnt automatically in our method; (3) our optimization routine is completely different from theirs, as their compositional tree structure is hard-coded.

**Structured models.** There have been recent works on learning structured 3D models, in particular, linear [58] or hierarchical [23, 57, 29, 31] organization of part bounding boxes. While some methods learn part geometries separately [23, 29], others jointly embed/encode structure and geometry [52, 12]. What is common about all of these methods is that they are supervised, and were trained on shape collections with part segmentations and labels. In contrast, BSP-Net is unsupervised. On the other hand, our network is not designed to infer shape semantics; it is trained to learn convex decompositions. To the best of our knowledge, there is only one prior work, Im2Struct [31], which infers part structures from a single-view image. However, this work only produces a box arrangement; it does not reconstruct a structured shape like BSP-Net.

**Binary and capsule networks.** The discrete optimization for the tree structures in our network bears some resemblance to binary [21] and XNOR [36] neural networks. However, only one layer of BSP-Net employs binary weights, and our training method differs, as we use a continuous relaxation of the weights in early training.

Further, as our network can be thought of as a simplified scene graph, it holds striking similarities to the principles of capsule networks [38], where low-level capsules (hyperplanes) are aggregated in higher (convexes) and higher (shapes) capsule representations. Nonetheless, while [38] addresses discriminative tasks (encoder), we focus on generative tasks (decoder).

3. Method

We seek a deep representation of geometry that is simultaneously trainable and interpretable. We achieve this task by devising a network architecture that provides a differentiable Binary Space Partitioning tree (BSP-tree) representa-
tion\(^1\). The representation is easily trainable as it encodes geometry via implicit functions, and interpretable because its outputs are a collection of convex polytopes. While we generally target 3D geometry, we employ 2D examples to explain the technique without any loss of generality, and the code to reproduce all of our 2D experiments can be found in our additional material. We achieve our goal via a network containing three main modules, and acts on feature vectors extracted by an encoder corresponding to the type of input data (e.g. the features produced by ResNet for images or 3D CNN for voxels). In more details, a first layer that extracts hyperplanes conditional on the input data, a second layer that groups hyperplanes in the form of half-spaces to create parts (convexes), and a third layer assembles parts together to reconstruct the overall object; see Figure 3.

Layer 1: hyperplane extraction. Given a feature vector \( f \), we apply a multi-layer perceptron \( P \) to obtain plane parameters \( P_{p \times 4} \), where \( p \) is the number of planes – i.e. \( P = P_\omega(f) \). For any point \( x = (x, y, z, 1) \), the product \( D = xP^T \) is a vector of signed distances to each of the \( p \) planes – the \( i-th \) distance is negative if the \( x \) is inside and positive if inside the \( i-th \) plane.

Layer 2: hyperplane grouping. To group hyperplanes into geometric primitives we employ a binary matrix \( T_{p \times c} \). Via a max-pooling operation we aggregate input planes to form a set of \( c \) convex primitives:

\[
C^+(x) = \max(D_iT_{ij}) \begin{cases} < 0 & \text{inside} \\ > 0 & \text{outside} \end{cases} \tag{1}
\]

Note that during training the gradients would flow through only one (max) of the planes. Hence, to ease training, we employ a version that replaces max with summation:

\[
C^+(x) = \sum_i \text{relu}(D_i)T_{ij} \begin{cases} = 0 & \text{inside} \\ > 0 & \text{outside} \end{cases} \tag{2}
\]

Layer 3: shape assembly. The third layer groups convexes to create a (potentially non-convex) output shape via min-pooling:

\[
S^+(x) = \min_{j} (C^+(x)) \begin{cases} = 0 & \text{inside} \\ > 0 & \text{outside} \end{cases} \tag{3}
\]

We would like to note that the use of \( C^+ \) in the expression above is intentional. We avoid using \( C^+ \) due to the lack of a memory efficient implementation of the operator in TensorFlow 1.

Again, to facilitate learning, we distribute gradients to all convexes by resorting to a (weighted) summation:

\[
S^+(x) = \left[ \sum_j W_j[1 - C^+(x)] \right]_{[0,1]} \begin{cases} = 1 & \text{in} \\ 0, 1 \end{cases} \approx \text{out} \tag{4}
\]

where \( W_{c \times 1} \) is a weight vector, and \([\cdot]_{[0,1]} \) performs clipping. During training we will enforce \( W \approx 1 \). Note the inside/outside status here is only approximate. For example, when \( W = 1 \), and all \( C_j^+ = 0.5 \), one is outside of all convexes, but inside their composition.

Two-stage training. Losses evaluated on (4) will be approximate, but have better gradient than (3). Hence, we develop a two-stage training scheme where: \( \odot \) in the continuous phase, we try to keep all weights continuous and compute an approximate solution via \( S^+(x) \) – this would generate an approximate result as can be observed in Figure 4 (b); \( \odot \) in the next discrete phase, we quantize the weights and use a perfect union to generate accurate results by fine-tuning on \( S^+(x) \) – this creates a much finer reconstruction as illustrated in Figure 4 (c,d).

3.1. Training Stage 1 – Continuous

We initialize \( T \) and \( W \) with random zero-mean Gaussian noise having \( \sigma = 0.02 \), and optimize the network via:

\[
\arg\min_{\omega, T, W} L^+_{rec} + L^+_{T} + L^+_{W} \tag{5}
\]

Given query points \( x \), our network is trained to match \( S(x) \) to the ground truth indicator function, denoted by \( F(x|G) \), in a least-squares sense:

\[
L^+_{rec} = \mathbb{E}_{x \sim G} [(S^+(x) - F(x|G))^2] \tag{6}
\]

where \( x \sim G \) indicates a sampling that is specific to the training shape \( G \) – including random samples in the unit box as well as samples near the boundary \( \partial G \); see [5]. An edge between plane \( i \) and convex \( j \) is represented by \( T_{ij} = 1 \), and the entry is zero otherwise. We perform a continuous relaxation of a graph adjacency matrix \( T \), where we require its values to be bounded in the \([0,1]\) range:

\[
L^+_{T} = \sum_{t \in T} \max(-t, 0) + \sum_{t \in T} \max(t - 1, 0) \tag{7}
\]

Note that this is more effective than using a sigmoid activation, as its gradients do not vanish. Further, we would like \( W \) to be close to \( 1 \) so that the merge operation is a sum:

\[
L^+_{W} = \sum_j |W_j - 1| \tag{8}
\]

However, we remind the reader that we initialize with \( W \approx 0 \) to avoid vanishing gradients in early training.
Figure 4: Evaluation on 2D examples – Training the auto-encoder on the synthetic 2D dataset. We show the auto-encoding results, and highlight the mistakes made in Stage 1 with red circles, which are resolved in Stage 2. We further show the effect of enabling the (optional) overlap loss. Notice that in the visualization we use different (possibly repeating) colors to indicate different convexes.

Figure 5: Examples of $L_2$ output – A few convexes from the first shape in Figure 4, and the planes to construct them. Note how many planes are unused.

3.2. Training Stage 2 – Discrete

In the second stage, we first quantize $T$ by picking a small value $\lambda=0.01$, and then assign $t=(t>\lambda)?1:0$, then we fine-tune the network via:

$$\arg\min_{\omega} \mathcal{L}_{\text{recon}}^* + \mathcal{L}_{\text{overlap}}^*$$

where we ensure the shape is well reconstructed via:

$$\mathcal{L}_{\text{recon}}^* = \mathbb{E}_{x \sim G} [F(x|G) \cdot \max(S^*(x), 0)]$$

$$+ \mathbb{E}_{x \sim G} [(1 - F(x|G)) \cdot (1 - \min(S^*(x), 1))]$$

The above loss function pulls $S^*(x)$ towards 0 if $x$ should be inside the shape; it pushes $S^*(x)$ beyond 1 otherwise. Optionally, we can also discourage overlaps between the convexes. We first compute a mask $M$ such that $M(x)=1$ if $x$ is contained in more than one convex, and then evaluate:

$$\mathcal{L}_{\text{overlap}}^* = -\mathbb{E}_{x \sim G} [M(x)S^*(x)]$$

3.3. Algorithmic and training details

In our 2D experiments, we use $p=256$ planes and $c=64$ convexes. We use a simple 2D convolutional encoder where each layer downsamples the grid by half, and doubles the number of feature channels. It takes a volume of size $64^3$ as input. The encoder for images is ResNet-18 without pooling layers that receives images of size $128^2$ as input. All encoders produce feature codes $|f|=256$. The dense network $P_\omega$ has widths $\{512, 1024, 2048, 4p\}$ where the last layer outputs the plane parameters.

When training the auto-encoder for 3D shapes, we adopt the progressive training from [5], on points sampled from grids that are increasingly denser ($16^3$, $32^3$, $64^3$). In Stage 1, we train the network on $16^3$ grids for 8 million iterations with batch size 36, then $32^3$ for 8 million iterations with batch size 36, then $64^3$ for 8 million iterations with batch size 12. In Stage 2, we train the network on $64^3$ grids for 8 million iterations with batch size 12.

For single-view reconstruction, we also adopt the training scheme in [5], i.e., train an auto-encoder first, then only train the image encoder of the SVR model to predict latent codes instead of directly predicting the output shapes. We train the image encoder for 1,000 epochs with batch size 64. We run our experiments on a workstation with an Nvidia GeForce RTX 2080 Ti GPU. When training the auto-encoder (one model on the 13 ShapeNet categories), Stage 1 takes about 3 days and Stage 2 takes about 2 days. Training the image-encoder requires 1 additional day.

4. Results and evaluation

We study the behavior of BSP-Net on a synthetic 2D shape dataset (Section 4.1), and evaluate our auto-encoder (Section 4.2), as well as single view reconstruction (Section 4.3) compared to other state-of-the-art methods.

4.1. Auto-encoding 2D shapes

To illustrate how our network works, we created a synthetic 2D dataset. We place a diamond, a cross, and a hollow diamond with varying sizes over $64 \times 64$ images; see...
Figure 6: **Segmentation and correspondence** – Semantics implied from autoencoding by BSP-Net. Colors shown here are the result of a manual grouping of learned convexes. The color assignment was performed on a few shapes: once a convex is colored in one shape, we can propagate the color to the other shapes by using the learnt convex id.

Table 1: **Surface reconstruction** quality and comparison for 3D shape autoencoding. Best results are marked in bold.

|       | CD   | NC   | LFD  |
|-------|------|------|------|
| VP [44] | 2.259 | 0.683 | 6132.74 |
| SQ [32] | 1.656 | 0.719 | 5451.44 |
| BAE [4]  | 1.592 | 0.777 | 4587.34 |
| Ours     | 0.447 | 0.858 | 2019.26 |
| Ours + L∗overlap | 0.448 | 0.858 | 2030.35 |

Figure 7: **Segmentation and reconstruction / Qualitative**

Table 2: **Segmentation**: comparison in per-label IoU.

|       | plane | car | chair | lamp | table | mean |
|-------|-------|-----|-------|------|-------|------|
| VP [44] | 37.6 | 41.9 | 64.7 | 62.2 | 62.1 | 56.9 |
| SQ [32] | 48.9 | 49.5 | 65.6 | **68.3** | 77.7 | 66.2 |
| BAE [4]  | 40.6 | 46.9 | 72.3 | 41.6 | 68.2 | 59.8 |
| Ours     | 74.2 | 69.5 | 80.9 | 52.3 | **90.3** | 79.3 |
| Ours + L∗overlap | **74.5** | **69.7** | **82.1** | 53.4 | **90.3** | **79.8** |
| BAE∗ [4] | 75.4 | 73.5 | 85.2 | 73.9 | 86.4 | 81.8 |

4.2. Auto-encoding 3D shapes

For 3D shape autoencoding, we compare BSP-Net to a few other shape decomposition networks: Volumetric Primitives (VP) [44], Super Quadrics (SQ) [32], and Branched Auto Encoders (BAE) [4]. Note that for the segmentation task, we also evaluate on BAE∗, the version of BAE that uses the values of the predicted implicit function, and not just the classification boundaries – please note that the surface reconstructed by BAE and BAE∗ are identical. Since all these methods target shape decomposition tasks, we train single class networks, and evaluate segmentation as well as reconstruction performance. We use the ShapeNet (Part) Dataset [2], and focus on five classes: airplane, car, chair, lamp, and table. For the car class, since none of the networks separates surfaces (as we perform volumetric modeling), we reduce the parts from (wheel, body, hood, roof) → (wheel, body); and analogously for lamps (base, pole, lampshade, canopy) → (base, pole, lampshade) and tables (top, leg, support) → (top, leg).

As quantitative metrics for reconstruction tasks, we report symmetric Chamfer Distance (CD, scaled by ×1000) and Normal Consistency (NC) computed on 4k surface sampled points. We also report the Light Field Distance (LFD) [3] – this is the only metric that directly correlates...
to visual appearance. For segmentation tasks, we report the typical mean per-label Intersection Over Union (IOU).

**Segmentation.** Table 2 shows the per category segmentation results. As we have ground truth part labels for the point clouds in the dataset, after training each network, we obtain the part label for each primitive/convex by voting: for each point we identify the nearest primitive to it, and then the point will cast a vote for that primitive on the corresponding part label. Afterwards, for each primitive, we assign to it the part label that has the highest number of votes. We use 20% of the dataset for assigning part labels, and we use all the shapes for testing. At test time, for each point in the point cloud, we find its nearest primitive, and assign the part label of the primitive to the point. In the comparison to BAE, we employ their one-shot training scheme [4, Sec.3.1]. Note that BAE-NET* is specialized to the segmentation task, while our work mostly targets part-based reconstruction; as such, the IoU performance in Table 2 is an upper bound of segmentation performance.

Figure 6 shows semantic segmentation and part correspondence implied by BSP-Net autoencoding, showing how individual parts (left/right arm, left/right leg, etc) are corresponded. In our method, all shapes are corresponded at the primitive (convexes) level. To reveal shape semantics, we manually group convexes that belong to the same semantic part and assign them the same color. Note again that such color assignment is done on each convex once, and propagated to all the shapes.

**Reconstruction quality and comparisons.** BSP-Net achieves significantly better reconstruction quality, while maintaining a high segmentation accuracy; see Table 1 and Figure 7, where we color each primitive with respect to its inferred part label. Note that BAE-NET was designed for segmentation, thus produces poor-quality part-based 3D reconstructions. Note how our method is capable of representing complex parts such as legs of swivel chairs in Figure 7, while none of the other methods can.

### 4.3. Single view reconstruction (SVR)

We compare our method with AtlasNet [16], IM-NET [5] and OccNet [28] on the task of single view reconstruction. We report quantitative results in Table 3 and Table 4, and qualitative results in Figure 8. We use the 13 categories in ShapeNet [2] that have more than 1,000 shapes each, and the rendered views from 3D-R2N2 [6]. We train one model on all categories, using 80% of the shapes for training and 20% for testing, in a similar fashion to AtlasNet [16]. For other methods, we download the pre-trained models released by the authors. Since the pre-trained OccNet [28] model has a different train-test split than others, we evaluate it on the intersection of the test splits.

**Edge Chamfer Distance (ECD).** To measure the capacity of a model to represent sharp features, we introduce a new metric. We first compute an “edge sampling” of the surface by generating $16k$ points $S = \{s_i\}$ uniformly distributed on the surface of a model, and then compute sharpness as: $$\sigma(s_i) = \min_{j \in N(\varepsilon)(s_i)} [n_i \cdot n_j],$$ where $N(\varepsilon)(s)$ extracts the indices of the samples in $S$ within distance $\varepsilon$ from $s$, and $n$ is the surface normal of a sample. We set $\varepsilon = 0.01$, and generate our edge sampling by retaining points such that $\sigma(s_i) < 0.1$; see Figure 8. Given two shapes, the ECD be-
Table 3: Single view reconstruction – comparison to the state of the art. Atlas25 denotes AtlasNet with 25 square patches, while Atlas0 uses a single spherical patch. Subscripts to OccNet and IM-NET show sampling resolution. For fair comparisons, we use resolution 32^3 so that OccNet and IM-NET output meshes with comparable number of vertices and faces.

Table 4: Low-poly analysis – The dataset-averaged metrics in single view reconstruction. We highlight the number of vertices #V and triangles #F in the predicted meshes.

original resolutions used by the authors. Note the average number of polygons inferred by our method is 655 (recall #polygons ≤ #triangles in polygonal meshes).

5. Conclusions

We introduce BSP-Net, an unsupervised method which can generate compact and structured polygonal meshes in the form of convex decomposition. Our network learns a BSP-tree built on the same set of planes, and in turn, the same set of convexes, to minimize a reconstruction loss for the training shapes. These planes and convexes are defined by weights learned by the network. Compared to state-of-the-art methods, meshes generated by BSP-Net exhibit superior visual quality, in particular, sharp geometric details, when comparable number of primitives are employed.

Limitations and future works. The main limitation of BSP-Net is that it can only decompose a shape as a union of convexes. Convex structures, e.g., a teacup or ring, have to be decomposed into many small convex pieces, which is unnatural and wasting a considerable amount of representation budget (planes and convexes). A better way to represent such shapes is to do a difference operation rather than union. How to generalize BSP-Net to express a variety of CSG operators is an interesting direction for future work.

Current training times for BSP-Net are quite significant (inference is fast): 6 days for 4,096 planes and 256 convexes for the SRV task trained across all categories. While most shapes only need a small number of planes to represent, we cannot reduce the total number of planes as they are needed to well represent a large set of shapes. It would be ideal if the network can adapt the primitive count based on the complexity of the input shapes; this may call for an architectural change to the network.

While its applicability to RGBD data could leverage the auto-decoder ideas explored by [22], the generalization of our method beyond currated datasets [2], and the ability to train from only RGB images is of critical importance.
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