Is the additional increase of star luminosity due to partial mixing real?

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Abstract  The partial mixing of matter between the radiative envelope and convective core in an early B-type star produces an additional increase of star luminosity during main sequence evolution. High quality data on stellar mass and luminosity defined from studies of detached double-lined eclipsing binaries are used to check the existence of such additional increase. It is shown that the additional luminosity increase does not contradict observed high quality data, if the intensity of partial mixing is restricted by the observed increase in surface helium content.

Key words: stars: structure and evolution

1 INTRODUCTION

One of the observed properties of single early B-type main sequence stars is an increase in surface helium abundance with stellar age. According to Lyubimkov et al. (2004), the change in number density ratio, $N_{\text{He}}/N_{\text{H}}$, reaches $(26 \pm 10)\%$ in the surfaces of stars with masses $4 < M/M_\odot < 12$ by the end of main sequence evolution. Here $N_{\text{H}}$ and $N_{\text{He}}$ are the number densities of hydrogen and helium respectively. In stars with masses $12 < M/M_\odot < 19$ this change may be as large as 67%. According to Huang & Gies (2006b), such change among stars with masses $8 < M/M_\odot < 16$ is $(23\pm13)\%$. The helium enrichment process acts more strongly in faster rotators (Huang & Gies 2006b). The increase in surface helium abundance among binary star components may be greater than among single stars, as much as twice (Lyubimkov et al. 1995, 1996, 1997; Tarasov et al. 1995).

The partial mixing of matter between the convective core and the radiative envelope in a rotating star may be a possible mechanism for surface helium enrichment (Staritsin 2014a,b, 2017a,b). A rotating star is the site of hydrodynamic processes of transport such as meridional circulation and shear turbulence. In the case of moderately rapid rotation, the intensity of shear turbulence is enough to remove some amount of hydrogen from the radiative envelope to the layer with variable chemical composition. Semi-convective mixing begins to operate in that layer as a result of the hydrogen content increase, so hydrogen flows from the radiative envelope through the semi-convective zone to the convective core. Helium flows in the opposite direction. As a consequence, some amount of helium is removed from the convective core to the radiative envelope. The result is that the surface helium content increases. Another consequence is that some amount of hydrogen is removed from the radiative envelope to the convective core. This leads to both the synthesis of an additional amount of helium and an additional increase of star luminosity during the main sequence evolution (Fig. 1). Partial mixing of material in the star’s interior may influence the mass-luminosity relation of main sequence stars. Such influence is stronger for faster rotators. In this paper, we compare the theoretical mass-luminosity relation with the observational one. To produce the observational relation, we use high quality data on stellar masses and radii defined for detached double-lined eclipsing binaries.

2 EMPIRICAL MASS-LUMINOSITY RELATION

In order to get the empirical mass-radius and mass-luminosity relations, a subset of close binary components with masses $7 \leq M/M_\odot \leq 23$ was selected from the catalog of absolute elements collected by Malkov (2007). The data on mass, radius and luminosity of components compiled in that catalog are based on research about detached double-lined eclipsing binaries. There are 27 components with masses from this specific interval for mass in the cata-
The spin velocity of the binary components taken from the rotational disturbance, the binary components are influenced by mutual tidal interaction. This tidal interaction may give rise to both thermally (Tassoul & Tassoul 1982) and mechanically (Tassoul 1987; Tassoul & Tassoul 1990) driven circulation. The powers of mixing processes generated by the rotational disturbance and tidal interaction are characterized by centrifugal acceleration and tidal acceleration respectively. The ratio of centrifugal acceleration for the component $a_{\text{spin}}$ to the tidal one $a_{\text{tidal}}$

$$\frac{a_{\text{spin}}}{a_{\text{tidal}}} = \frac{M + M_{\text{sec}}}{2M_{\text{sec}}} \left(\frac{P_{\text{orb}}}{P_{\text{spin}}}\right)^2$$

(3)

does not differ much from unity in a binary system composed of stars with approximately equal mass in the case of synchronization of spin and orbital rotation. Here $M$ and $M_{\text{sec}}$ are mass of the component and period of its spin rotation respectively, $M_{\text{sec}}$ is the mass of the second star and $P_{\text{orb}}$ is orbital period of the binary system. The tidal interaction may enlarge the intensity of partial mixing in the interiors of binary components as much as twice. The slowly rotating component of a binary star may be mixed as much as a single star in the case of moderately rapid rotation.

### 3.3 Evolution of Binary Components with Partial Mixing

Mixing induced by spin rotation may be reduced to a one-dimensional problem (Zahn 1992). This is not the case for tidally induced mixing. The hydrogen burning stage of binary components with masses of $6M_\odot$, $8M_\odot$, $12M_\odot$, $16M_\odot$, $20M_\odot$ and $24M_\odot$ was calculated using the parametrical description of partial mixing of matter between the convective core and the radiative envelope of the star (Staritsin 2018). The change in number density ratio by the end of main sequence evolution, $\Delta(N_{\text{He}}/N_{\text{H}})$, was chosen as a parameter. The possible values of that parameter are restricted by the observed increase in surface helium content (Huang & Gies 2006b; Lyubimkov et al. 1995, 1996, 1997, 2004; Tarasov et al. 1995). The usual overshooting parameter $\alpha$ was applied to determine the position of the convective core boundary. This parameter is restricted by the observed main sequence width of open clusters (Schaller et al. 1992; Meynet et al. 1993). Five variants of stellar evolution were calculated: $(\Delta(N_{\text{He}}/N_{\text{H}}), \alpha) = \{(0\%, 0.05), (40\%, 0.05), (80\%, 0.05), (0\%, 0.15), (0\%, 0.25)\}$. These calculated variants of stellar evolution encompass the helium enlargement of radiative envelopes produced by the hydrodynamic calculation of single star evolution (Staritsin 2017a).
3.4 Mean Relative Age of the Subset of Binary Components

Some evolutionary sequences and all binary components taken from the selected subset are shown on Figure 2. It is obvious that the stars staying in the late stage of hydrogen burning are absent in the selected subset of binary components. For each calculated variant of stellar evolution, we determine the relative age \( t/t_{MS} \) of each binary component using its mass and radius. Here \( t \) is the age of the component and \( t_{MS} \) is its lifetime on the hydrogen burning stage. Then we calculate the mean value of relative age for the subset of components for each variant of stellar evolution. The mean values of relative age are 0.40, 0.43, 0.47, 0.38 and 0.37 for considered variants of stellar evolution respectively.

The theoretical mass-radius and mass-luminosity relations are built for the mean relative age of the subset of binary components (Fig. 3). For each evolutionary track, the mean values of luminosity and radius are determined according to the formula

\[
\overline{\lg \xi} = \sum_{j=1}^{10} w_j \overline{\lg \xi_j},
\]

where \( \xi = L \) and \( \xi = R \) respectively. \( \overline{\lg \xi_j} \) is the mean value calculated using evolutionary sequences over the interval

\[
0.1(j - 1) \leq t/t_{MS} \leq 0.1j.
\]

\( w_j \) is the relative number of components satisfying the interval in Equation (5). Due to the method applied, the theoretical mass-radius relations for different variants of evolution do not differ much from the observed relation, Equation (1), and from each other (Staritsin 2018).

4 DISCUSSION

The theoretical mass-luminosity relation is influenced by the partial mixing of matter between the radiative envelope and convective core of a star (Fig. 3a). The deviation of the theoretical mass-luminosity relations from the observed relation is no more than one standard deviation. This is because both the number of binary star components with known absolute elements is not too many and the mean value of relative age of those components is not too much. The additional luminosity increase of stellar models produced by the evolution with partial mixing of matter between the radiative envelope and the convective core of the star does not contradict the observed high quality data.

The theoretical mass-luminosity relation is practically not influenced by the additional mixing at the convective core boundary (Fig. 3b). This is because the additional luminosity increase gained by a star by the mean relative age of a subset of components depends only moderately on the overshooting parameter \( \alpha \). The overshooting parameter should be determined using the observed width of the main sequence based on data on absolute elements of binary components (Popova & Tutukov 1990).

5 CONCLUSIONS

The partial mixing of matter between the radiative envelope and the convective core of a single star, introduced in Staritsin (2014a,b, 2017a,b) to explain the observed increase in surface helium content in main sequence early type B–stars, gives rise to an additional luminosity increase during the hydrogen burning stage. The additional luminosity increase grows with evolution of the star, and it is larger among rapid rotators. The components of binary systems with known absolute elements are characterized by lower spin rotation. However, there are additional mixing processes in the interiors of components produced by tidal interaction. Such additional mixing processes may enlarge the intensity of partial mixing in the binary star components. The mean age of the components with known absolute elements is approximately equal to half of the main sequence lifetime. The additional luminosity increase produced that time does not contradict the observed luminosity of binary components. This conclusion does not depend on core overshooting, if the overshooting parameter is restricted by the observed main sequence width of

Fig. 2 The evolutionary tracks for the stars with masses 6, 12 and 24 \( M_\odot \) for the variants \( \Delta(N_{He}/N_{H}),\alpha = (0, 0.05) \) (solid lines), \( (40\%, 0.05) \) (dashed lines) and \( (0, 0.15) \) (dot-dashed lines). ZAMS and TAMSS are also shown (dotted lines). Filled circles represent primary components and open circles correspond to secondary components. The data on tracks are taken from Staritsin (2018). The data on component elements are taken from Malkov (2007).
open clusters. The largest additional increase of luminosity is produced at the end of main sequence evolution. Unfortunately, the subset of components with known absolute elements does not contain any information about the last stage of core hydrogen burning.

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*Fig. 3* The mass-luminosity relation for the variants (a) $(\Delta (N_{\text{He}}/N_{\text{H}}), \alpha) = (0, 0.05)$ (solid line), (40%, 0.05) (dashed line) and (80%, 0.05) (dot-dashed line), (b) (0, 0.05) (solid line) and (0, 0.25) (dot-dashed line). Filled symbols represent primary components and open symbols correspond to secondary components. Observed error bars are shown in cases when the error bar exceeds the size of a symbol. Triangles signify cases when the error bar is unknown. Upper and lower standard deviations from empirical relation Eq. (2) are shown by dotted lines. The data on component elements are taken from Malkov (2007).