ON NON-MARKOVIAN NATURE OF STOCK TRADING$^1$

Andrei Leonidov

(a) Theoretical Physics Department, P.N. Lebedev Physics Institute, 119991 Leninsky pr. 53, Moscow, Russia

(b) Netcominvest Financial Investment Company, 109017 Profsoyznaya 3, Moscow, Russia

(c) Institute of Theoretical and Experimental Physics, 117259 B. Cheremushkinskaya 25, Moscow, Russia

Abstract

Using a relationship between the moments of the probability distribution of times between the two consecutive trades (intertrade time distribution) and the moments of the distribution of a daily number of trades we show, that the underlying point process generating times of the trades is an essentially non-markovian long-range memory one. Further evidence for the long-range memory nature of this point process is provided by the powerlike correlation between the intertrade time intervals. The data set includes all trades in EESR stock on the Moscow International Currency Exchange in January 2003 - September 2003 and in Siemens, Commerzbank and Karstadt stocks traded on the Xetra electronic stock exchange of Deutsche Boerse in October 2002.

$^1$Presented at ”Applications of Physics in Financial Analysis”, Warsaw, 13-15 November 2003
One of the hot topics in the new field of econophysics [2, 3, 5] is working out a parsimonious description of price dynamics. An ultimate challenge is to describe the whole price formation process starting from intentions of market participants expressed in terms of buy or sell orders up to the bid-ask annihilation leading to an appearance of the trade at given time, price and volume. Below we shall confine our consideration to the "intermediate" level, where one looks only at the dynamics of accomplished trades. The material of this talk is based on [1].

Among the most important characteristics of financial markets is their activity. On the trade-by-trade basis one can think of two possible ways of characterizing it and thus setting the clock measuring the operational time. The first possibility is to follow the volume traded [10], the second is to analyze the temporal pattern of trading operations. Below we shall follow the latter route. The importance of effects related to the varying trading frequency is well recognized. One traditional topic of great practical interest is a study of seasonality effects characterized by different timescales [4]. On a more fundamental level, in [7] it was argued that long-range correlations in trading frequency [6, 7] directly induce the observed long-range correlations of volatility.

Technically the influence of the temporal trading pattern on price dynamics can be taken into account by considering the price generating process \( P(t) \) as being subordinate [9] to the directing point process \( T(t) \) that, in turn, generates the times of the trades \( t_1 < t_2 < \cdots \), so that \( P(t) = P(T(t)) \). Depending on the properties of \( T(t) \), probabilistic price dynamics for \( P(t) \) can be described by differential (in time) or integral equations. More specifically, one can distinguish the following generic types of \( T(t) \):

- **Type 1.** The time at which the \( n \)-th point (trade) takes place is completely independent of the times at which the previous \( n - 1 \) points were generated. In this case one deals with the Poisson distribution of the number of trades in any fixed time interval. This point process is fully characterized by an exponential probability distribution for the intertrade time \( \tau = t_n - t_{n-1} \), \( \psi(\tau) = \tau_0^{-1} \exp(-\tau/\tau_0) \). The evolution equations are then differential with respect to time, and trading frequencies in non-overlapping time windows are independent.

- **Type 2.** The time of the \( n \)-th trade is correlated with that of the previous \( n - 1 \)-th trade, so that the point process has a unit memory depth (and is thus, by definition, markovian). The point process \( T(t) \) is still fully characterized by some non-exponential intertrade time probability distribution \( \psi(\tau) \), but evolution equations are no
longer differential, but integral ones, of continuous time random walk (CTRW) type. The financial applications of CTRW were discussed in [12, 13, 14, 15, 16, 17, 18]. The correlation between the number of trades in non-overlapping time windows can in principle be computed [8], but the resulting expressions are quite involved.

- **Type 3.** The time of the n-th trade depends on $r > 1$ times of the previous trades $t_{n-1}, t_{n-2}, ..., t_{n-r}$. In this case the process is a long-range memory non-markovian one with a memory depth equal to $r$, and the corresponding evolution equations are complicated integral equations with kernels depending on $r - 1$ arguments. If all temporal scales are effectively involved, $r$ is infinite.

Let us start our analysis of the character of the process generating the times of trades with studying the normalized correlation of intertrade time intervals $\tau_i$ and $\tau_{i+d}$ separated by $d - 1$ intermediate intervals

$$C_r(d) = \frac{\langle \tau_k \tau_{k+d} \rangle - \langle \tau \rangle^2}{\langle \tau^2 \rangle} \quad (1)$$

The correlation is computed by first averaging over all intervals separated by a fixed number of intervals for a given day and then averaging over all days. The correlation (1) is shown, together with errorbars characterizing its variation on the day-to-day basis, in Fig. From Fig. one can conclude that the series of intertrade time intervals possesses long-memory properties. The correlation function of intertrade time intervals $C_r(d)$ is slowly (power-like) decaying, $C_r(d) \simeq 0.15/d^{0.29}$, which is typical for long-range memory processes.

Another way of establishing the long-range memory nature of the point process generating the times of the trades is to consider specific combinations of the moments of trade multiplicity distribution in some given time interval $\Delta T$ and the moments of the intertrade time $\tau$ [8]. More specifically, for the processes of Type 1 and Type 2 there exist, for some given large time interval $\Delta T$, a relation between the first two moments of the distribution of the number of trades within this interval $\langle N_{\Delta T} \rangle$ and $\langle N^2_{\Delta T} \rangle$ and the first two moments of the intertrade time distribution $\psi_1 \equiv \langle \tau \rangle$ and $\psi_2 \equiv \langle \tau^2 \rangle$ [8]:

$$\langle N^2_{\Delta T} \rangle - \langle N_{\Delta T} \rangle^2 = \frac{\psi_2 - \psi_1^2}{\psi_1^2} \Delta T \equiv \frac{\psi_2 - \psi_1^2}{\psi_1^2} \langle N_{\Delta T} \rangle \quad (2)$$

Corrections to (2) are of order $1/\Delta T$. Equivalently,

$$\rho_N \equiv \frac{\langle N^2_{\Delta T} \rangle - \langle N_{\Delta T} \rangle^2}{\langle N_{\Delta T} \rangle} = \frac{\psi_2 - \psi_1^2}{\psi_1^2} \equiv \rho_\tau \quad (3)$$
Figure 1: Normalized correlation of intertrade time intervals in tick time. Solid line shows a powerlike fit of the form $0.15/d^{0.29}$. Upper and lower lines show the standard deviation of the measured correlation characterizing its variation on the day-to-day basis.

so that if the underlying point process $T(t)$ is indeed of Type 1 or Type 2, one should have $\rho_N/\rho_\tau = 1$. Let us note, that in the simplest poissonian case $\rho_N = \rho_\tau = 1$.

Let us now choose $\Delta T = 1$ day and consider all trades in EESR in the period January 2003 - September 2003 and SIE, CBK and KAR in October 2002. Relevant information is summarized in Table 1:

Table 1

|       | $\langle N_{\Delta T} \rangle$ | $\langle \tau \rangle$ (seq) | $\rho_\tau$ | $\rho_N$ | $\rho_N/\rho_\tau$ |
|-------|-------------------------------|-------------------------------|-------------|---------|-------------------|
| EESR  | 6823                          | 4.5                           | 10.3        | 492.8   | 109.4             |
| SIE   | 4364                          | 9.1                           | 3.8         | 124.8   | 32.8              |
| CBK   | 1856                          | 21.3                          | 3.1         | 274.4   | 88.5              |
| KAR   | 373                           | 104.1                         | 4.5         | 39.7    | 8.8               |
From Table 1 it is clear, that poissonian Type 1 and CTRW Type 2 processes are excluded as candidate point processes generating the times of the trades. This leaves us with the only remaining possibility of Type 3 non-markovian long-range memory process. This conclusion is in obvious agreement with the long-range correlation between the intertrade time intervals for EESR shown in Fig. (1).

To provide some statistical background to this conclusion, let us consider a simulation of the point process having exactly the same distribution over intertrade times as the EESR data shown in Fig. (2), but otherwise no additional correlations between the intertrade time intervals. The considered number of trade intervals $\Delta T$ was equal to the number of trading days in the EESR data. This simulation effectively reconstructs a CTRW process possessing the observed distribution over intertrade times.

![Intertrade time distribution EESR 200301-200309](image)

Figure 2: Intertrade time distribution for EESR 200301-200309. Solid line shows a fit of the form $\exp\{-\ln(\tau)^{1.5} - 1\}$

\(^2\)The distributions over intertrade time intervals were studied in a number of papers \[14, 17, 18\]
The simulation results are shown in Table 2:

| Und            | $\langle N_{\Delta T} \rangle$ | $\langle \tau \rangle$ (seq) | $\rho_\tau$ | $\rho_N$ | $\rho_N/\rho_\tau$ |
|----------------|-------------------------------|-------------------------------|-------------|--------|---------------------|
| EESR (data)    | 6823                          | 4.5                           | 10.3        | 492.8  | 109.4               |
| EESR (sim.)    | 6816                          | 4.5                           | 12.4        | 8.2    | 0.66                |

We see, that although due to statistical limitations (finite size corrections, etc.) the simulation does not reproduce the theoretical value of $\rho_N/\rho_\tau = 1$, it is still close to it, while missing the experimental ratio by two orders of magnitude.

**Conclusion**

We conclude that the point process generating the times of trades is of long-range memory non-markovian nature.

This research was supported by RFBR grant 02-02-16779 and Scientific School Support grant 1936.2003.02.

**References**

[1] A. Leonidov, “Long Memory in Stock Trading”; [arXiv:cond-mat/0303222]; *International Journal of Theoretical and Applied Finance*, to appear

[2] R.N. Mantegna and H.E. Stanley, *An Introduction to Econophysics*, Cambridge, 2000

[3] J.-P. Bouchaud and M. Potters, *Theory of Financial Risks*, Cambridge, 2001

[4] M.M. Dracorogna, R. Gencay, U. Muller, R.B. Olsen and O.V. Pictet, *An Introduction to High Frequency Finance*, Academic Press, 2001

[5] D. Sornette, *Why Stock Markets Crash*, Princeton, 2003

[6] G. Bonanno, F. Lillo and R.N. Mantegna, *Dynamics of the Number of Trades in Financial Securities*, Physica A **280** (2000), 136-141; [arXiv:cond-mat/9912006]

[7] V. Plerou, P. Gopikrishnan, L. Amaral, X. Gabaix and H.E. Stanley, *Economic Fluctuations and Diffusion*, Phys. Rev. **E62** (2000) (Rapid Comm.), R3023 [arXiv:cond-mat/9912051].
[8] C. Domb, *The Statistics of Correlated Events - I.*, Phil. Mag. 41 (1950), 969-982.

[9] W. Feller, *An Introduction to Probability Theory and Its Applications*, Vol. II, Wiley and Sons, 1966

[10] P. Clark, *A subordinated stochastic process model with finite variance for speculative prices*, Econometrica 41 (1973), 135-159

[11] E. Montroll and M. Shlesinger, *The Wonderful World of Random Walks*, in *Nonequilibrium Phenomena II, From Stochastics to Hydrodynamics, Studies in Statistical Mechanics*, North Holland, Amsterdam, 1984

[12] E. Scalas, R. Gorenflo and F. Mainardi, *Fractional calculus and continuous-time finance*, Physica A284 (2000), 376-384; [arXiv:cond-mat/0001120](http://arxiv.org/abs/cond-mat/0001120)

[13] F. Mainardi, M. Raberto, R. Gorenflo and E. Scalas, *Fractional calculus and continuous-time finance II: the waiting-time distribution*, Physica A287 (2000), 468-481; [arXiv:cond-mat/0012155](http://arxiv.org/abs/cond-mat/0012155)

[14] M. Raberto, E. Scalas and F. Mainardi, *Waiting-times and returns in high-frequency financial data: and empirical study*, Physica A314 (2002), 749-755; [arXiv:cond-mat/0203596](http://arxiv.org/abs/cond-mat/0203596)

[15] J. Masoliver, M. Montero and G.H. Weiss, *A continuous time random walk model for financial distributions*, [arXiv:cond-mat/0210513](http://arxiv.org/abs/cond-mat/0210513)

[16] J. Masoliver, M. Montero, J. Perello and G. Weiss, *The CTRW in finance: Direct and inverse problem*, [arXiv:cond-mat/0308017](http://arxiv.org/abs/cond-mat/0308017)

[17] E. Scalas, R. Gorenflo, F. Mainardi, M. Mantelli and M. Raberto, *Anomalous waiting times in high-frequency financial data*, [arXiv:cond-mat/0310305](http://arxiv.org/abs/cond-mat/0310305)

[18] P. Repetowicz and P. Richmond, *Modeling of waiting times and price changes in currency exchange data*, [arXiv:cond-mat/0310351](http://arxiv.org/abs/cond-mat/0310351)