Stable and Robust Alternating Current Power Source Control Technique Applied to Ultraprecision Machining of Aluminum Alloy Materials

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Abstract. This paper develops a stable, robust control technique for AC power sources in ultraprecision machining of aluminum alloy materials that maintains low total harmonic distortion output-voltage and fast transient under uncertain interferences. The proposed control technique associates the properties of the finite-time variable structure controller (FTVSC) with the gray Bernoulli model (GBM). The FTVSC provides finite system state convergence time with non-singular advantages, which is different from the asymptotic convergence of the traditional variable structure controller in infinite time. However, once there is a highly uncertain disturbance, there will be chatter around FTVSC. The chatter produces high voltage harmonics in the AC power source output, which reduces the stability of ultraprecision machining of aluminum alloy materials. The GBM is used to eliminate the chatter and produce a high performance AC power source. Experimental results on a high-performance AC power source show that the proposed stable and robust control technique can result in high quality AC output voltage against system uncertainties. Since the proposed control technique is easier to implement than the existing control technique, and has high-precision tracking, fast convergence speed and efficient calculation, this paper will cause an interest in researchers of related aluminum alloy materials applications.

1. Introduction
Alternating current (AC) power supplies have been used as key components in ultraprecision machining of aluminum alloy materials, such as forging, drawing and extrusion [1-4]. High-performance AC power supplies should include: (1) Output voltage waveform with low harmonic distortion for linear or non-linear cyclic loads. (2) Rapid transient when sudden load changes occur. (3) Steady state error should be as small as possible. In order to obtain requisitions, the proportional-integral controller performs well under constant load conditions; however, when the plant is disturbed by fluctuating loads, good performance cannot be met [5], [6]. Nonlinear control strategies such as repetitive control, deadbeat control and optimal state feedback, are often used [7-11]. However, they have complex algorithms and are difficult to implement. Variable structure controller (VSC) is a robust method for controlling nonlinear systems in which system uncertainties occur [12-17]; many VSCs have been developed for AC power supplies. A single sliding surface function is used to perform all control objects, but the transient and steady state are not good [18]. A multi-loop control is proposed through the improvement of the [18], but there is a steady-state error [19]. A discrete variable structure control scheme for the servo system is proposed [20], and it is modified for the use of the PWM inverter [21]. Although a fast transient response is obtained, the output voltage waveform is distorted because the step size of the discrete reference sinusoid is greater than the sampling period. In addition, a discrete feedforward sliding mode control scheme is developed that shows fast transient and good steady-state response [22]. Unfortunately, due to the rated load deviation, the responsibility of the VSC becomes heavier. A discretized version of the integral VSC scheme is introduced. Unless a smooth function is used, the chatter still exists [23]. A cheap 8-bit microcontroller for the outer loop replaces the classical analog dual-loop technique [24]. A first-order Takagi-Sugeno fuzzy controller is proposed to synthesize and analytically design a hybrid PWM and sliding-mode global nonlinear control for boost switching regulators [25]. A Type-2 fuzzy sliding mode
controller is developed to handle the rule uncertainty when the operation is extremely uncertain and/or the membership level cannot be accurately determined [26]. A sliding mode fuzzy logic controller is proposed to obtain the better start-up and steady state performance [27]. As described above [18-23], the use of a linear sliding surface produces an infinite time convergence problem. Although satisfactory results for transients and steady-states are given in [24-27], they have high control complexity, require precise system parameters and need more calculations. In order to improve the convergence speed, a finite-time variable structure controller (FTVSC) using a nonlinear sliding surface is developed instead of a linear sliding surface [28-35]. Compared with the control based on the linear sliding surface, the FTVSC can drive the tracking error to converge to zero in a finite time [36-42]. However, in the event of highly uncertain disturbances, the chatter occurs and the AC power source output produces high harmonic distortion, resulting in thermal breakdown, which reduces the stability and reliability of ultra-precision machining of aluminum alloy materials. Based on past and present dynamic system data, a computationally efficient gray Bernoulli model (GBM) is used to describe and analyze future trends in serial numbers [43-48]. The GBM improves the accuracy of the classic grey prediction model and has been successfully applied in many areas of engineering [49-53]. Therefore, the mathematically simple and accurately prediction GBM is used to eliminate the chatter while the system uncertainty boundary is overestimated. Through the combination of FTVSC and GBM, the tracking error between the desired output and the actual output can be minimized, and the ultraprecision machining of the aluminum alloy material produces low total harmonic distortion, fast transient, reduced chattering and less steady state error. Experimental results are provided to verify the applicability of the proposed technique.

2. Modelling of AC power source

Figures 1 and 2 show the structure of two AC power sources for ultraprecision machining of aluminum alloy materials. In order to reduce the switching loss, Figure 2 is used, and its equivalent circuit is shown in Figure 3. The load can be linear/nonlinear to test the performance of the buck inverter. The output of the buck inverter is designed to provide 110 Vrms at 60 Hz frequency.
From the Figs. 2 and 3, the state space equation can be expressed as

\[
\begin{bmatrix}
\frac{\dot{v}_c}{v_c} \\
\frac{\dot{v}_r}{v_r}
\end{bmatrix} = 
\begin{bmatrix}
0 & 1/LC \\
-1/LC & -1/RrCc
\end{bmatrix}
\begin{bmatrix}
v_c \\
v_r
\end{bmatrix} + 
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\begin{bmatrix}
0 \\
\frac{v_c}{RrCc}
\end{bmatrix}
\begin{bmatrix}
u_c \\
u_r
\end{bmatrix}
\]

(1)

where \( v_c = v_{c1}, \quad L_1 = L \) and \( C_1 = C \).

Set \( v_c \) be output voltage, \( v_r \) be the desired AC waveform, \( e_1 = v_c - v_r \) stands for voltage error. Then, define \( \dot{e}_1 = e_2, \quad a_1 = b = 1/LC, \) and \( a_2 = 1/RrCc \). The parameter \( a_1 \) is known, but the parameter \( a_2 \) is uncertain because it depends on the load conditions. In fact, the load conditions are unpredictable, but the load variation range is limited by the inverter design. Therefore, the parameter \( a_2 \) are limited to \( \bar{a}_2 - \Delta a_2 < a_2 < \bar{a}_2 + \Delta a_2 \), where the bar on the character symbol stands for the nominal value and the symbol \( \Delta \) represents the parameter change.

As can be seen from the (1), the control signal \( u \) must be designed to converge \( e_1 \) and \( e_2 \) to zero. Therefore, when GBM is used to reduce the chatter, the FTVSC can drive system tracking error to converge to zero for a finite time, thereby ensuring closed-loop stability and producing a higher performance AC output voltage. The design philosophy of this proposed control technique is to improve the classic VSC by introducing the FTVSC and the GBM to solve the problems of the infinite time convergence and the chatter. Finally, the AC power source will provide powerful performance for ultraprecision machining of aluminum alloy materials.

3. Design of control technique

A finite time variable structure controller with a non-linear sliding surface is defined as

\[
s = e_1 + ((1/\rho) \cdot e_2^\xi)
\]

(2)

where \( \rho > 0, \quad 1 < \xi < 2 \), and a sliding-mode reaching entry \( \dot{s} = -\phi_1|s|^\phi \text{sign}(s) - \phi_2|s|^\phi \text{sign}(s) \) is used.

Therefore, the control law \( u \) can be written as

\[
u(t) = u_{eq}(t) + u_f(t)
\]

(3)

with

\[
u_{eq}(t) = b^{-1}\left[a_1e_1 + a_2e_2 - \frac{\rho}{\xi}e_2^{\xi-\xi}\right]
\]

(4)

\[
u_f(t) = -b^{-1}\left[\phi_1|s|^\phi \text{sign}(s) + \phi_2|s|^\phi \text{sign}(s)\right]
\]

(5)

where \( \phi_1, \phi_2 > 0, \quad 0 < \phi < 1 \), \( u_{eq} \) represents the equivalent control that determines the dynamics of the system, and \( u_f \) indicates the sliding control that prevents system uncertainty. Therefore, the system will be driven to \( s = 0 \) and converged in a finite time; however, if highly uncertain disturbances are applied, the chatter will still occur and the system (1) will not be able to achieve accurate tracking performance. Therefore, the control signal \( u(t) \) (3) is modified by adding a gray Bernoulli model (GBM) \( u_{gbm} \), which reduces the chatter in the AC power source system. The modeling steps for the GBM are as follows.

Step 1: Input the original sample data sequence

Represent the original data sequence as

\[
X^{(0)} = \{x^{(0)}(k), k = 1, 2, \ldots, n
\]

(6)

where \( x^{(0)} \) stands for the set of \( n \) original sample data.

Step 2: Use accumulated generating operation (AGO)

By taking the AGO on \( x^{(0)} \), we can written the first-order AGO sequence as

\[
X^{(1)} = \{x^{(1)}(k), k = 1, 2, \ldots, n
\]

(7)
where $x^{(l)}(k) = \sum_{i=1}^{k} x^{(l)}(i), k = 1, 2, \ldots, n$.

Step 3: The grey differential equation for constructing GBM is

$$x^{(l)}(k) + \alpha z^{(l)}(k) = \beta [z^{(l)}(k)]^\sigma$$

In addition, the whitening differential equation is expressed as

$$\frac{dX^{(l)}}{dt} + \alpha X^{(l)} = \beta [X^{(l)}]^\sigma$$

where $z^{(l)}(k) = g x^{(l)}(k) + (1-g) x^{(l)}(k-1), k = 2, 3, \ldots, n, g \in [0,1]$ denotes the generating coefficient of the background value, and $\sigma$ is an any real number precluding $\sigma = 1$.

Step 4: Solve the estimated parameters $\alpha$ and $\beta$ by the least square method below.

$$\begin{bmatrix} -z^{(l)(2)} & [z^{(l)(2)}]^\sigma \\ -z^{(l)(3)} & [z^{(l)(3)}]^\sigma \\ \vdots & \vdots \\ -z^{(l)(n)} & [z^{(l)(n)}]^\sigma \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} x^{(l)(2)} \\ x^{(l)(3)} \\ \vdots \\ x^{(l)(n)} \end{bmatrix}$$

where $D = \begin{bmatrix} -z^{(l)(2)} \\ -z^{(l)(3)} \\ \vdots \\ -z^{(l)(n)} \end{bmatrix}$ and $M = \begin{bmatrix} x^{(l)(2)} \\ x^{(l)(3)} \\ \vdots \\ x^{(l)(n)} \end{bmatrix}$.

Step 5: The solution of the (10) is generated as

$$\hat{x}^{(l)}(k) = [x^{(l)(1)}]^{1-\sigma} - \frac{\beta}{\alpha} e^{-\alpha(l-\sigma)(k-1)} + \frac{\beta}{\alpha} [l]^{1-\sigma}$$

where ‘$\hat{}$’ denotes predicted value, $\sigma \neq 1$, and $k = 1, 2, \ldots, n$.

Step 6: Use inverse accumulated generating operation (IAGO)

The data sequence $\hat{x}^{(l)}(k)$ using the IAGO can be estimated as

$$\hat{x}^{(l)(1)} = x^{(l)(1)}$$

$$\hat{x}^{(l)(k)} = \hat{x}^{(l)(k)} - \hat{x}^{(l)(k-1)}, k = 2, 3, \ldots, n$$

Therefore, the control law of the (3) is rewritten as

$$u(k) = u_{eq}(k) + u_{gbm}(k)$$

where the added compensation component is gray Bernoulli control $u_{gbm}$, which can eliminate the chattering.

$$u_{gbm}(k) = \begin{cases} 0 & \text{if } |k(k)| < \tau \\ \psi \text{sign}(s(k)\hat{s}(k)) & \text{if } |k(k)| \geq \tau \end{cases}$$

where $\hat{s}(k)$ represents for the predicted value of $s(k)$, $\psi$ is a constant, and $\tau$ symbols the system boundary. The sign function in the (15) is replaced by a continuous function, i.e., hyperbolic tangent function.

4. Experiments

The proposed system parameters are listed in Table 1. Figure 4 shows the simulated output voltage waveform of an AC power source controlled by the proposed technique under a rectifier load, including full-wave rectified with a parallel resistor 12Ω and a capacitive filter 300μF. The output voltage has very low output voltage distortion (%THD is 0.91%) while the current suddenly rises. The simulated performance of a classic VSC (diode bridge with a capacitive filter 300μF and 12Ω resistive load) with a rectified load is shown in Figure 5. The simulated output voltage is hardly sinusoidal and produces high distortion (the %THD of the output voltage is 6.84%). To test the transient characteristics of AC power sources in ultraprecision machining of aluminum alloy materials, Figures 6 and 7 show the experimental output voltage obtained using the proposed technique and the classic VSC under step changes in load (from no load to full load) at a 90 degree
firing angle, respectively. A detailed examination of the waveforms shows that Figure 6 has a slight voltage dip and a fast recovery steady-state response. However, the classic VSC system as shown in Figure 7 produces unsatisfactory transient response and delayed recovery time. It has been well established that the proposed technique does produce higher tracking accuracy, lower harmonic distortion and faster convergence than classic VSC. Therefore, the finite time convergence of the system state is achieved by the FTVSC, and the chatter is reduced by the GBM. The performance of the proposed AC power source has been significantly improved, which is very suitable for the application of ultraprecision machining of aluminum alloy materials.

| Table 1. System Parameters. |
|-----------------------------|
| DC-link voltage             | $V_s = 200$ V          |
| Output voltage and frequency| $v_o = 110$ V rms, $f = 60$ Hz |
| Filter inductor             | $L = 1.1$ mH           |
| Filter capacitor            | $C = 12$ $\mu$F        |
| Resistive load              | $R_L = 12$ $\Omega$    |
| Switching frequency         | $f_s = 18$ kHz         |

**Figure 4.** Output voltage for the proposed technique under rectifier load (100V/div; 5ms/div).

**Figure 5.** Output voltage for the classic VSC under rectifier load (100V/div; 5ms/div).

**Figure 6.** Output voltage for the proposed technique under step change in load (100V/div; 5ms/div).
5. Conclusions
This paper designs the FTVSC with the GBM to improve AC power sources in ultraprecision machining of aluminum alloy materials. The proposed technique not only has the robustness of the classic VSC, but also provides finite time convergence of the system state and reduces the chatter. According to theoretical analysis, simulation and experimental results, the effectiveness of the proposed technique is proved and suitable for ultra-precision machining of aluminum alloy materials. In addition, the proposed technique can be used for more complex three-phase AC power sources for machining applications in future research.

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