Transient simulation of dynamic buckle propagation of a shallow cylinder under external pressure with damping

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Abstract. This paper presents a finite element simulation of transient buckle propagation of shallow elastic cylinder under time dependent external pressure load considering the effect of damping. We conclude that by assigning transient pressure a value slightly smaller or larger than static buckle propagation pressure the buckle propagation directions can be reversed.

1. Introduction
Shallow arches are typical structures experiencing snap-through buckling under external pressure. When non-shallow arch is buckling under external pressure, the membrane strain is very small, so that the mid-surface is often assumed inextensible [1]. For a shallow arch with two ends fixed, the membrane strain is not negligible. The pressure would first increase from zero to a limit value and then decrease to a small value after which the pressure would increase due to membrane stretching [2]. This up-down-up feature enables the buckle propagation of long panels with shallow arch-shaped cross-section. The response of 2D shallow arch under external pressure is a classic problem which has attracted many studies. Early researchers are Timoshenko [1], Schreyer and Masur [3], Gjelsvik [4], Fung and Kaplan [5]. Schreyer and Masur [3] obtained closed-form solution for the collapse of shallow arch under external pressure. Schreyer [6] considered the effect of imperfection following an experimental research by Cheung [7]. Recently, Pi et al. [8, 9] considered various boundary conditions and analytically solved the problem of non-linear buckling and post-buckling of shallow arches with rotational end restraints. Ha et al.[10] used functional analysis to study the existence, uniqueness and parameter dependence of the solutions of a weakly damped shallow arch under dynamic load. In contrast to the case of 2D arch, the long panel with shallow arch-shape cross-section has not fully investigated.

In addition to local buckling phenomenon, long panel would experience an instability propagation often called buckle propagation. For instance, Kyriakides [2] numerically and experimentally analyzed the quasi-static buckle propagation of a locally collapsed long panel. Power and Kyriakides [11] further numerically researched the quasi-static buckle propagation arrest by circumferential stiffeners. This paper presents a transient FEA analysis of damped buckle propagation of long panel and illustrates how buckle propagation direction could be reversed by change the imposed pressure.
2. Problem description and FEM
A schematic of shallow long panel under external pressure is shown in Fig. 1. The panel is symmetric with Y axis with radius $R$ and thickness $t_0$. External pressure $p$ is loaded uniformly. The shallowness of an panel is determined by the following parameter [12]:

$$\lambda = \theta^2 R / t_0$$  

(1)

Where $\lambda$ is the shallowness parameter. In this paper $\lambda = 12$, $R = 1m$, $R / t_0 = 480$, density $\rho = 7700kg/m^3$, elastic modulus $E = 206GPa$, Poisson’s ratio $\mu = 0.3$. The material model parameters are chosen the same as those of steel. $\theta$ could be determined by Eq.(1). The boundary conditions are as shown in Fig. 1 where “free” means free boundary, “X-symmetric” means symmetry about OYZ plane, and “fixed” means fixed boundary (fixing all rotation and translation DOFs). The long panel has length $L$. Under the external pressure, the “free” boundary condition is imposed in order to introduce some imperfections into this model and it is expected that the structure first collapses on this boundary as the weakest location. Due to time costing nature of dynamic analysis, we have remeshed the structure and take $L = 10m$ to ensure the steady propagation could be achieved. The mesh assigns uniformly 1000 elements in the axial direction and 20 elements in the circumferential direction. The pressure should be larger than 0.17MPa to initiate the collapse in Fig. 1. So a non-uniform external pressure is imposed on the panel by the following analytical expression:

$$p = \text{floor}(-Z / L_{\text{dis}}) \cdot p_1 + p_2$$  

(2)

Where floor($g$) means the largest integer not greater than $g$, $p_1 = 0.1MPa$, $p_2 = 0.1MPa$ and $L_{\text{dis}} = 0.2m$.

The damping force is imposed by attaching each node an engineering feature named “dashpot” [13, 14, 15] and the damping effect is realized by imposing an additional load per unit area

$$\tilde{p}_{\text{damping}} = -C_f (\tilde{v} \cdot \tilde{e}_r) \tilde{e}_r$$

where $C_f$ means a damping coefficient and in this paper, $C_f = \rho_w c_w / 10$  

where $\rho_w$ is density of water(1000k/m$^3$) and $c_w$ is sound speed in water(1500m/s) and $\tilde{v}$ means the velocity vector and $\tilde{e}_r$ is the unit vector in positive $Y$ direction (refer to Yan[15] for technical details).

![Figure 1. Schematic of long shallow panel](image)

Eq. (2) ensures the initiation of collapse locally and the local buckle would propagation axially. Fig. 2 shows the deformation history of leading generator for various times. Fig. 3 gives the deformation history of the panel for various times.
From Fig. 2 and Fig. 3, due to non-uniform distribution of pressure the collapse is initiated in one end and a local buckle develops. The buckle then propagates forward steadily. From Fig. 4, the velocity magnitude is only large for the transitional part while the velocity of the rest is very small relatively.
3. Forward and backward buckle propagation

In this section we show that the buckle could be travelling forward or backward depending on the pressure relative to the quasi-static pressure. The repeated motion of forward and backward propagation might be utilized to build interesting mechanical devices to for example transmit fluid. In this section we show this idea by investigating the transient direction reversal of buckle propagation. The basic idea is that initially a pressure larger than the quasi-static pressure is prescribed and as a result steady forward buckle propagation is developed which process is followed by the action of prescribing a pressure suddenly a pressure smaller than the quasi-static pressure and the transient details of reversal phenomenon are studied.

In order to accomplish the above task, we input the following load history in transient dynamic analysis by FEM. Three pressure loads are applied to the panel:

- **Load1**: an exponentially decaying locally distributed pressure by Eq. (3).
  \[
  p_1(Z,t) = p_1 \cdot \text{floor}\left(-\frac{Z}{9.8}\right) \cdot \exp\left(-\frac{t}{\overline{T}}\right)
  \]
  where \( p_1 = 0.1 \text{MPa} \) as the above and \( \overline{T} = 0.2 \text{s} \).

- **Load2**: a uniform constant pressure \( p_2 \) same as the quasi-static propagation pressure 0.068MPa and instantaneously applied at \( t = 0 \text{s} \).

- **Load3**: a time varying uniform pressure with the following analytical expression:
  \[
  p_3(t) = \begin{cases} 
  0.02; & 0 \leq t \leq 1 \\
  0.02 - 4(t-1); & 1 < t \leq 1.01 \\
  -0.02; & t > 1.01 
  \end{cases}
  \]
  where \( p_3 \) is in unit MPa and \( t \) is in unit second.

Roughly speaking, \( p_3 \) is kept constant as 0.02MPa for initial phase and suddenly changed to -0.02MPa. The Load1 is used to initiate the collapse. Total pressure load is the sum of these three loads. Fig. 5 describes the deformation history of the leading generator, i.e., AC line in Fig.1 in the reversal process.

![Figure 5](image)

**Figure 5.** Deformation history of leading generator in the reversal process(time in seconds)

After the steady propagation has been developed, at time=1s, the uniform buckled part bounces back slightly and the buckle propagation is in the backward direction. This figure vividly verifies the existence of backward propagation and it could be realized only by assigning a smaller pressure. This repeated motion of “forward-backward-forward-…” propagation makes it possible in principle to be utilized as
a mechanical device to transmit fluid. But before the design of this device we should remedy one flaw: although not shown in Fig. 5, after backward propagating buckle reaches the “free” end (see Fig. 1) the structure bounces back to its initial configuration dynamically. This “bounce back” phenomenon prevents the structure from redeveloping a forward propagation. For possible engineering application, this flaw could be fixed by introducing a displacement imperfection to the structure as follows: move the point A in Fig. 1 downward in −Y direction to a position with Y=-0.02m and fix this point ever since to avoid the bounce back.

Figure 6. Deformation history of leading generator in reversal process with time in seconds

In Figure 6, the initial configuration has one point fixed and the Load1 is disregarded in this case and the imposed load is the sum of Load2 and Load3. The j-line (solid line) and k-line (dot line) are difficult to distinguish since they are almost overlapping since the propagation has stopped then.

Figure 7. Deformation history of leading generator in reversal process with time in seconds

If the Load3 by Eq. (4) is replaced corresponding load by Eq. (5), the forward propagation would reach another end and we finally realize the robust repeated motion by only prescribing the pressure as in Fig. 7. Another issue of concern is the damping parameter $\rho_f c_f$. In the above transient analysis, $\rho_f c_f = \frac{1}{10}\rho_w c_w$. What would happen if $\rho_f c_f = \frac{1}{00}\rho_w c_w$ is prescribed? The repeated motion pattern is identical but the propagation is much faster. So basically there are two ways to control the propagation velocity: adjusting the pressure load and change the damping parameter of this system.
4. Conclusions
In this paper, we investigate the effect of dynamic buckle propagation of long shallow panel considering damping. Main conclusions are briefly listed hereinafter:

With damping the dynamic propagation could be controlled to maintain a repeated forward-backward motion, by assigning external pressure slightly higher or smaller than static propagation pressure periodically and the buckle could propagate forward and backward in a controlled manner. This phenomenon may be useful to develop fluid transmitting devices.

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References
[1] Timoshenko S P, Gere J M, 1961. Theory of elastic stability. McGrawHill-Kogakusha Ltd, Tokyo.
[2] Kyriakides S, Arseculeratne R. Propagating instabilities in long shallow panels[J]. Journal of engineering mechanics, 1993, 119(3): 570-583.
[3] Schreyer H L, Masur E F. Buckling of shallow arches[J]. Journal of the Engineering Mechanics Division, 1966, 92(4): 1-20.
[4] Gjelsvik A, Bodner S R. The energy criterion and snap buckling of arches[J]. J. Eng. Mech. Div, 1962, 88(5): 87-134.
[5] Fung Y C, Kaplan A. Buckling of low arches or curved beams of small curvature[J]. 1952.
[6] Schreyer H L, the effect of initial imperfections on the buckling load of shallow circular arches. J. Appl. Mech. Trans. 1972, June:445-450.
[7] Cheung, M. C. and Babcock, C.D., An energy approach to the dynamic buckling of arches. J Appl. Mech. Trans. ASME, 92, Dec, 1012-1018.
[8] Pi Y L, Bradford M A. Non-linear in-plane postbuckling of arches with rotational end restraints under uniform radial loading[J]. International Journal of Non-Linear Mechanics, 2009, 44(9): 975-989.
[9] Pi Y L, Bradford M A. Non-linear buckling and postbuckling analysis of arches with unequal rotational end restraints under a central concentrated load[J]. International Journal of Solids and Structures, 2012, 49(26): 3762-3773.
[10] Ha J, Gutman S, Shon S, et al. Stability of shallow arches under constant load[J]. International Journal of Non-Linear Mechanics, 2014, 58: 120-127.
[11] Power T L, Kyriakides S. Circumferential stiffeners as buckle arrestors in long panels[J]. International journal of solids and structures, 1996, 33(13): 1837-1851.
[12] Kyriakides, S., "Propagating Instabilities in Structures." In Advances in Applied Mechanics, Ed. by J.W. Hutchinson and T.Y. Wu, Vol. 30, pp. 67-189, Academic Press, Boston, MA, Oct. 1993.
[13] Hibbitt H, Karlsson B, Sorensen P, 2011. Abaqus analysis users manual version 6.10. Dassault Systèmes Simulia Corp.: Providence, RI, USA.
[14] Yan S T, Zhu Y F, Jin Z J, et al. 2014. Buckle Propagation Analysis of Deep-Water Petroleum Transmission Pipelines with Axial Tension, Advanced Materials Research. 1008: 1134-1143.
[15] Yan S T, Theoretical Buckling Analysis of Externally Pressurized Subsea Pipelines and Its Buckle Propagation Analysis[D]. Zhejiang University, 2017.