Models for neutrino mass and physics beyond standard model

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In this work, we report on recent analysis of three-loop models of neutrino mass with dark matter. We discuss in detail the model of Krauss-Nasri-Trodden (KNT) [1], showing that it offers a viable solution to the neutrino mass and dark matter problems, and describe observable experimental signals predicted by the model. Furthermore, we show that the KNT model belongs to a larger class of three-loop models that can differ from the KNT approach in interesting ways.

I. INTRODUCTION

Models with radiative neutrino mass are of significant experimental interest. The inherent loop-suppression in such models allows the new physics responsible for neutrino mass to be lighter than otherwise expected. Such light new physics can be within experimental reach, either directly, through collider experiments, or indirectly, via e.g. searches for lepton flavor violating (LFV) effects. The loop suppression becomes more severe as the number of loops increases. Thus, models with three-loop masses are particularly interesting, as they generically require new physics at or around the TeV scale.

Here we present a class of models with radiative neutrino mass at the three-loop level [1-3]. We focus primarily on the KNT model [1] and report recent analysis showing that the model satisfies LFV constraints, such as $\mu \to e + \gamma$, and fits the neutrino oscillation data. Furthermore, the model contains a viable candidate for the dark matter (DM) in the universe, in the form of a light right-handed (RH) neutrino. We also show that a strongly first order electroweak phase transition can be achieved with a Higgs mass of $\approx 125$ GeV, as measured at the LHC [4, 5]. The model contains new charged scalars and we discuss their effect on the one-loop Higgs decay to neutral gauge bosons. Afterwards, we show that the KNT model belongs to a larger class of related three-loop models and briefly outline their features.

II. THE KNT MODEL

The KNT model [1] is an extension of the SM with three RH neutrinos, $N_i$, and two electrically charged scalars, $S_1^\pm$ and $S_2^\pm$, that are singlet under the $SU(2)_L$ gauge group. In addition, a discrete $Z_2$ symmetry is imposed on the model, under which $\{S_2, N_i\} \to \{-S_2, -N_i\}$, and all other fields are even. This symmetry plays two key roles, preventing a tree-level coupling between $N_R$ and the SM Higgs, which would otherwise induce tree-level neutrino masses, and ensuring that the lightest fermion $N_1$ is a stable DM candidate. The Lagrangian reads

$$\mathcal{L} = \mathcal{L}_{SM} + \left\{ f_{\alpha \beta} L^T_{\alpha} C v_2 L_{\beta} S_1^+ \right\} g_{\alpha \beta} N_i S_2^2 \ell_{\alpha R} + \frac{1}{2} m_{N_i} N_i C N_i + h.c \right\} - V(\Phi, S_1, S_2),$$

where $L_\alpha$ is the left-handed lepton doublet, $f_{\alpha \beta}$ are Yukawa couplings which are antisymmetric in the generation indices $\alpha$ and $\beta$, $m_{N_i}$ are the Majorana RH neutrino masses, $C$ is the charge conjugation matrix, and $V(\Phi, S_1, S_2)$ is the tree-level scalar potential, which is given by

$$V(\Phi, S_{1,2}) = \lambda \left( |\Phi|^2 \right)^2 - \mu^2 |\Phi|^2 + m_1^2 S_1^* S_1 + m_2^2 S_2^* S_2 + \lambda_1 S_1^* S_1 |\Phi|^2 + \lambda_2 S_2^* S_2 |\Phi|^2$$

$$+ \frac{\eta_1}{2} (S_1^* S_1)^2 + \frac{\eta_2}{2} (S_2^* S_2)^2 + \eta_3 S_1^* S_1 S_2^* S_2 + \{ \lambda_3 S_1 S_1 S_2^* S_2 + h.c \}.$$  (2)

Here $\Phi$ denotes the SM Higgs doublet.
The neutrino mass matrix elements, arising from the three-loop diagram in Fig. 1, are given by

\[(M_\nu)_{\alpha\beta} = \frac{\lambda_\alpha m_{\ell_\alpha} m_{\mu}}{4\pi^2} f_{\alpha\beta} f_{\mu\nu} g_{ij} g_{k\ell} F \left( m_{N_i}^2/m_{S_j}^2, m_{N_k}^2/m_{S_{\ell\nu}}^2 \right), \]

where \(\rho, \kappa (= e, \mu, \tau)\) are the charged leptons flavor indices, \(i = 1, 2, 3\) denotes the three RH neutrinos, and the function \(F\) is a loop integral which is \(O(1)\) \([8]\). It is interesting to note that, unlike the conventional seesaw mechanism, the radiatively generated neutrino masses are directly proportional to the charged lepton and RH neutrino masses, as well as being loop-suppressed.

The Lagrangian \([8]\) induces flavor violating processes such as \(\ell_\alpha \to \gamma \ell_\beta\) for \(m_{\ell_\alpha} > m_{\ell_\beta}\), generated at one loop via the exchange of the charged scalars \(S_{12}^\pm\). The branching ratio of such process is given by

\[B(\ell_\alpha \to \gamma \ell_\beta) = \frac{\alpha_{em} v^4}{384\pi} \left\{ \left| f_{\alpha\alpha} f_{\beta\beta}^* \right|^2 \frac{m_{S_1}^2}{m_{S_1}^4} + \frac{36}{m_{S_2}^4} \sum_{\tau} g_{i\alpha} g_{i\beta}^* F_2 \left( m_{S_2}^2/m_{S_2}^4 \right)^2 \right\},\]

with \(\alpha \neq \beta, \alpha_{em}\) being the fine structure constant and \(F_2(x) = (1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x)/6(1 - x)^4\). For the case of \(\ell_\alpha = \ell_\beta = \mu\), this leads to a new contribution to the muon anomalous magnetic moment, \(\delta a_\mu\).

In our scan of the parameter space of the model we impose the experimental bound \(B(\mu \to e\gamma) < 5.7 \times 10^{-13}\) \([9]\); and use the allowed values for the neutrino mixing \(s_{12}^2 = 0.320^{+0.016}_{-0.017}, s_{23}^2 = 0.435^{+0.003}_{-0.003}, s_{13}^2 = 0.025^{+0.003}_{-0.003}\), and the mass squared difference \(|\Delta m^2_{31}| = 2.55_{-0.09}^{+0.06} \times 10^{-3}\text{ eV}^2\) and \(\Delta m^2_{21} = 7.62_{-0.19}^{+0.19} \times 10^{-5}\text{ eV}^2\) \([8]\).

**B. Dark matter**

An immediate implication of the \(Z_2\) symmetry is that that lightest right handed neutrino, \(N_1\), is stable, and hence a candidate for dark matter (DM). The \(N_1\) number density get depleted through the process \(N_1 N_1 \to \ell_\alpha \ell_\beta\) via the \(t\) and \(u\) channels exchange of \(S_{12}^\pm\). In the non-relativistic limit, the total annihilation cross section is given by

\[\sigma_{N_1 N_1} v_r \simeq \sum_{\alpha, \beta} |g_{i\alpha} g_{i\beta}^*|^2 \frac{m_{N_i}^2 (m_{S_2}^2 + m_{S_3}^2)}{48\pi (m_{S_2}^2 + m_{S_3}^2)} v_r^2,\]

with \(v_r\) is the relative velocity between the annihilation \(N_1\)’s. The relic density after the decoupling of \(N_1\) can be obtained by solving the Boltzmann equation, and it is approximately given by

\[\Omega_{N_1} h^2 \simeq \frac{1.28 \times 10^{-2}}{\sum_{\alpha, \beta} |g_{i\alpha} g_{i\beta}^*|^2} \left( \frac{m_{N_1}}{135 \text{ GeV}} \right)^2 \frac{(1 + m_{S_2}^2/m_{N_1}^2)^4}{1 + m_{S_3}^2/m_{N_1}^2},\]

where \(< v_r^2 > \simeq 6/M_{Pl} \simeq 6/25\) is the thermal average of the relative velocity squared of a pair of two \(N_1\) particles, \(M_{Pl}\) is planck mass; and \(g_*(T_f)\) is the total number of effective massless degrees of freedom at the freeze-out temperature \(T_f \sim m_{N_1}/25\).
The charged scalar masses $m_{S_1}$ (red) and $m_{S_2}$ (green) versus the lightest RH neutrino mass, where the consistency with the neutrino data, LFV constraints and the DM relic density have been imposed.

In Fig. 2, we plot the allowed mass range for $(m_{N_1}, m_{S_i})$ that gives the observed dark matter relic density. As seen in the figure, the neutrino experimental data, the bound on LFV process, combined with the relic density, seems to prefer $m_{S_1} > m_{S_2}$ for large regions of parameter space. However, the masses of both the DM and the charged scalar $S_2^\pm$ are bounded from above, with $m_{N_1} < 225$ GeV and $m_{S_2} < 245$ GeV, respectively.

### C. Electroweak Phase Transition

Although the SM has all the qualitative ingredients for electroweak baryogenesis, the amount of matter-antimatter asymmetry generated is too small. One of the reasons has to with the fact that the electroweak phase transition (EWPT) is not strongly first order, which is necessary to suppress the sphaleron processes in the broken phase. The strength of the EWPT can be improved if there are new scalar degrees of freedom around the electroweak scale coupled to the SM Higgs, which is the case in the KNT model.

The investigation of the scalar effective potential reveals that, within the allowed parameter space of the model, the strength of the electroweak phase transition (EWPT) can be first order [6]. We find that if the one-loop corrections to the Higgs mass are sizeable, then the condition $\upsilon(T_c)/T_c > 1$ for having a first order EWPT can be realized while keeping the Higgs mass around 125 GeV. The reason for this being that the extra charged singlets affect the dynamics of the SM scalar field VEV around the critical temperature [10]. In Fig. 3, we show the plot for $\upsilon(T_c)/T_c$ versus the critical temperature and one observes that a strongly first order EWPT is possible while the critical temperature lies around 100 GeV.

![FIG. 3: The critical temperature is presented versus the quantity $\upsilon_c/T_c$.](image-url)
variant models employ larger multiplets. The resulting models share key similarities with the KNT model, with a new scalar $S_Z$ model, the DM is part of a problems. It predicts new physics that can be probed at colliders and precision experiments. Within the KNT inner loop.

It is perhaps not surprising, therefore, to learn that the KNT model forms part of a mechanism can be generalized to a type-III variant with $\sqrt{E}$ such as the ILC and CLIC, with a collision energy $P$ kinematic variables for different CM energy:

We use LanHep and CalcHep to simulate our model and generate the differential cross section and the relevant Relevant cuts for the process $N \to e^+ e^- \to E_{\text{miss}} + e^- \mu^+$ for different CM energies. Here we concentrate on the process $e^+ e^- \to E_{\text{miss}} + e^- \mu^+$ is the missing invariant mass.

D. KNT at High Energy Lepton colliders

Since the RH neutrinos couple to the charged leptons, one expects them to be produced at $e^- e^+$ colliders, such as the ILC and CLIC, with a collision energy $\sqrt{s}$ of few hundreds of GeV up to a TeV. If the produced pairs are the form $N_{2,3}N_{2,3}$ or $N_1N_{2,3}$, then $N_{2,3}$ will decay into a charged lepton and $S_2^\pm$, and subsequently $S_2^\pm$ decays into $N_1$ and a charged lepton. In addition, the SM neutrinos will be also produced via the decay of the charged scalar $S_1$. Here we concentrate on the process $e^+ e^- \to e^- \mu^+ + E_{\text{miss}}$ [11], where the missing energy corresponds to any state in the set $E_{\text{miss}} \subset \{\nu_\ell \nu_\ell, \nu_\ell \bar{\nu}_\ell, \nu_\ell \nu_\ell, \nu_\ell \bar{\nu}_\ell, \nu_\ell \nu_\ell, \nu_\ell \bar{\nu}_\ell, N_{i}N_{k}; i, k = 1, 2, 3\}$. The total expected cross section of the processes $e^+ e^- \to e^- \mu^+ + E_{\text{miss}}$ is represented by $\sigma^{EX}$, while $\sigma(E_{\text{miss}})$ denotes the cross section of different subprocesses. As a benchmark, we consider

$$f_{\mu\mu} = -(4.97 + i1.41) \times 10^{-2}, \quad f_{\ell\ell} = 0.106 + i0.0859, \quad f_{\ell\tau} = (3.04 - i4.72) \times 10^{-6},$$

$$g_{\ell\alpha} = 10^{-2} \times \begin{pmatrix} 0.23249 + i0.3252 & 0.0053 + i0.7789 & 0.4709 + i1.47 \\ 1.099 + i1.511 & -1.365 - i1.003 & 0.6532 - i0.1845 \\ 122.1 + i178.4 & -6.398 - i6.656 & -10.56 + i6.56 \end{pmatrix},$$

$$m_{N_i} = \{162.2 \text{ GeV}, 182.1 \text{ GeV}, 209.8 \text{ GeV}\}, \quad m_{S_1} = \{914.2 \text{ GeV}, 239.7 \text{ GeV}\}. \quad (7)$$

We use LanHep and CalcHep to simulate our model and generate the differential cross section and the relevant kinematic variables for different CM energy: $E_{\text{CM}} = 250, 350, 500 \text{ GeV}$ and 1 TeV, with unpolarized beams at first; and then we consider the polarized beams with $P(e^-, e^+) = [-0.8, +0.3]$ and/or $P(e^-, e^+) = [+0.8, -0.3]$. Imposing the appropriate cuts given in Table-I, we show in Fig. the dependence of the significance on the accumulated luminosity with and without polarized beams for the considered CM energies. We clearly see that for a polarized beam, the signal can be observed even with relatively low integrated luminosity. For example, at $E_{\text{CM}} = 250 \text{ GeV}$, the 5 $\sigma$ required luminosity is 150 $fb^{-1}$ for polarized beam as compared to 700 $fb^{-1}$ without polarization.

### III. A CLASS OF THREE-LOOP MODELS WITH DARK MATTER

The KNT model is a simple theory that is capable of simultaneously addressing the neutrino mass and DM problems. It predicts new physics that can be probed at colliders and precision experiments. Within the KNT model, the DM is part of a $Z_2$-odd sector that propagates in the inner loop of the neutrino mass diagram (see Figure 1). One might wonder whether generalizations of the KNT model exist. It is well known that the seesaw mechanism can be generalized to a type-III variant with $SU(2)$ triplet fermions [12], and a further variant with quintuplet fermions [13]. It is perhaps not surprising, therefore, to learn that the KNT model forms part of a larger class of theories that similarly achieve neutrino mass at the three-loop level, with DM propagating in the inner loop.

The generalized models require that the SM be extended to include the charged singlet scalar $S_1^+ \sim (1, 1, 2)$, a new scalar $S_2$, and three real fermions $N_R$. However, instead of taking $S_2$ and $N_R$ as $SU(2)_L$ singlets, the variant models employ larger multiplets. The resulting models share key similarities with the KNT model, with the neutrino mass diagram retaining the exact form in Figure 1 and the DM remaining as the lightest fermion $N_R$. However, there are also some interesting differences, relative to the KNT model. Firstly, the DM is now
part a larger multiplet with non-trivial gauge interactions. This modifies expectations for the DM mass and allows new DM interactions. Secondly, there are interesting consequences for the $Z_2$ symmetry, which depend on the specific details of the model, as we now outline:

- The triplet model: Here $S_2 \sim (1,3,2)$ and $N_R \sim (1,3,0)$ are taken as SU(2)$_L$ triplets. Similar to the KNT model, the triplet model requires a $Z_2$ symmetry with action $\{S_2, N_R\} \rightarrow \{-S_2, -N_R\}$ to prevent tree-level neutrino masses and ensure a stable DM candidate. The DM is the neutral component of the lightest triplet fermion and has extra Yukawa interactions mediated by $g_{\alpha} \neq 0$ in Eq. 4. Consequently its mass increases to $M_{DM} \approx 3$ TeV.

- The quintuplet model: This model employs the multiplets $S_2 \sim (1,5,2)$ and $N_R \sim (1,5,0)$. Unlike the KNT model and the triplet model, the quintuplet variant does not require a $Z_2$ symmetry to prevent tree-level neutrino mass. It is therefore a viable model of radiative neutrino mass, independent of DM considerations. Interestingly, the $Z_2$ symmetry $\{S_2, N_R\} \rightarrow \{-S_2, -N_R\}$ is broken by a single coupling $\lambda$ in the model; for technically-natural values of $\lambda \ll 1$, one obtains a long-lived DM candidate, while in the limit $\lambda \rightarrow 0$ the $Z_2$ symmetry becomes exact and the DM is absolutely stable. The quintuplet model is therefore a model of radiative neutrino mass, with or without DM, depending on the region of parameter space considered. Due to the gauge interactions, the case with DM requires $M_{DM} \approx 10$ TeV.

- The septuplet model: This case has $S_2 \sim (1,7,2)$ and $N_R \sim (1,7,0)$, and is of particular interest. Similar to the quintuplet model, the $Z_2$ symmetry is not required to prevent tree-level neutrino mass. However, different from the quintuplet model, the septuplet model automatically possesses an accidental $Z_2$ symmetry with action $\{S_2, N_R\} \rightarrow \{-S_2, -N_R\}$, and therefore always contains a stable DM candidate. This gives a common description for neutrino mass and DM without requiring any new symmetries. The DM is relatively heavy, with $M_{DM} \approx 20 - 25$ TeV, due to the Sommerfeld enhancement induced by SU(2) gauge boson exchange.

Detailed studies show that the variant models have large regions of viable parameter space that fit the neutrino oscillation data, reproduce the observed DM relic density, and satisfy experimental constraints from, e.g., LFV searches. In each of the variant models, the DM is a heavy Majorana fermion and the mass for $S_2$ should obey $M_2 > M_{DM}$, placing both multiplets beyond the reach of current colliders. However, the scalar $S_1$ can retain an $\mathcal{O}(100)$ GeV mass in all cases, offering the best chance for testing the models at colliders. The models with larger $SU(2)_L$ multiplets also generate sizable contributions to LFV processes like $\mu \rightarrow e + \gamma$. In fact, these are typically enhanced relative to the KNT case, improving the prospects for testing the model in future LFV searches. Due to the new gauge interactions for the DM, direct-detection prospects also improve for the models with larger multiplets.

There are additional models of neutrino mass with DM that are related to the KNT model. For example, one can replace the internal SM lepton in Figure 4 with down-type quarks, $e_L, \nu \rightarrow d_L, \nu$, and take $S_1 \sim (3,1,2/3)$ and $S_2 \sim (3,1,-2/3)$ as colored multiplets. Further colored variants are also possible. Alternatively, one may replace the real (Majorana) fermion $N_R$ with a complex (Dirac) fermion and consider...
FIG. 5: Three-loop diagram for neutrino mass in a variant model with $S_1 \sim (1,2,1)$, $S_2 \sim (1,2,3)$ and $F \sim (1,2,-1)$. The beyond-SM multiplets $S_{1,2}$ and $F$ are taken odd under a $Z_2$ symmetry and the dark matter belongs to the inert doublet $S_1$.

a modified topology for the loop-diagram, with the DM being an inert scalar (see Figure 5, for an example). A systematic description of these variant models appears in Ref. [17].

IV. CONCLUSION

Models with radiative neutrino mass offer a promising way to experimentally discern the new physics responsible for the origin of neutrino mass. Three-loop models are particularly interesting as the new physics is generically expected at or around the TeV scale. Furthermore, such models can provide viable DM candidates, thereby solving both the neutrino mass and DM problems. Here we reported recent analysis of the KNT model, which show the model to be a viable theory for neutrino mass and DM that can be tested at collider experiments. We also showed that the KNT model belongs to a larger class of three-loop models that can generate similarly interesting observable effects.

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