Problems about Calculation in Calculating Table in the Tsinghua Bamboo Slips

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Abstract. At the end of 2013, Zhongxi Book Company published Bamboo Slips of the Warring States Period Reserved by Tsinghua University (4) [1]. In it, the content of Calculating Table is about mathematical calculations, including not only multiplication, but also division and square root. From the perspective of time, Calculating Table is not only earlier than the calculating table in Liye Bamboo Slips of Qin and others that have been discovered in China, but also earlier than the calculating tables of other countries. From the perspective of content, it integrates multiplication, division and square root, which is richer than any other calculating tables. Therefore, Calculating Table in The Tsinghua Bamboo Slips is an important discovery on calculation formulas, and it has a milestone significance in the history of mathematics.

Keywords: Calculating Table, Multiplication, Division, Square root

1. Introduction
At the end of 2013, Zhongxi Book Company published Bamboo Slips of the Warring States Period Reserved by Tsinghua University (4). In it, the content of Calculating Table is about mathematical calculations. There are 21 rows and 20 columns in total. After comparing it with other calculating tables, the organizer found: “Calculating Table is the earliest physical object of Chinese mathematics literature seen so far. It is not only older than the ancient decimal multiplication table that can be seen so far, its mathematics and calculation functions also surpass the ancient algorithm tables such as the Nine-Nine Multiplication Table of Liye Bamboo Slips of Qin and Zhangjiajie Nine-Nine Multiplication Table that discovered in the past. This is the first of its kind in China and even in the world. It is an amazing and major discovery, and it provides important first-hand information for understanding the application and popularization of mathematics in the pre-Qin period in China. Calculating Table not only can be used to multiply any two-digit number within 100, but also can perform complex operations such as division. The operation is very simple [1]” Although Calculating Table in The Tsinghua Bamboo Slips is an unearthed document in nature, it is a mathematical calculation formula in terms of content. Therefore, this paper combined history, archaeology, mathematics and other disciplines to analyze the multiplication, division and square root in Calculating Table in an interdisciplinary manner. At the same time, in order to facilitate a more intuitive expression, the author listed it at the end of the paper.
2. The Multiplication in Calculating Table

In mathematical calculations, multiplication and division are very common and practical calculation methods. Through the analysis of the entire structure of Calculating Table, it is found that it has the following three characteristics in terms of multiplication: First of all, the second column of the first row on the top of the table lists 1 to 9 and 10 to 90 from the left to the right, showing an increasing trend. The third row to the bottom penultimate row of the rightmost column of the table lists 90 to 10 and 9 to 1, showing a decreasing trend. The result of multiplying the two numbers in the row and column is exactly the number where the two intersect horizontally and vertically. In addition, this arrangement shows an obvious rule, that is, when the horizontal multiplier is below 10, it increases by 1 and when it is between 10 and 90, it increases by 10. When the vertical multiplicand is between 90 and 10, it decreases by 10, and when it is below 10, it decreases by 1.

Secondly, the threads not only exist as a way of stringing the table, but also connect the two rows of numbers in the vertical and horizontal directions for cross calculation, and the value obtained is the product. From the structure table of Calculating Table, the two threads are arranged horizontally below the top row of the table, and vertically arranged on the left side of the rightmost vertical column. From the perspective of shape, each grid has a hole, and the threads can be inserted into each hole in a horizontal and vertical manner, and then the entire structure table can be pierced. From the calculation result, the number crossed by the upper, right horizontal and vertical numbers passed by the threads is exactly the result of the multiplication of the two numbers. Mr. Li Junming pointed out: “This is the second functional area. In the multiplication operation, the product is obtained by the numbers in the vertical and horizontal directions of the threads [2].” Through this method, stringing the structure table and completing the product number operation are perfectly unified. And from the perspective of the structure, the single-digit multiplication and the ten-digit multiplication are combined on a table, which makes the calculation more direct. It is more convenient and faster to use, saving time and effort. It can be seen that the producer of Calculating Table was proficient in these two aspects of numerical calculations [3].

Again, it breaks through the multiplication table people used in the past, which is "nine times nine is eighty-one". It also adds the multiplication of single digits and tens digits, and the multiplication of tens digits and tens digits. The biggest feature of the multiplication table of "nine times nine is eighty-one" is that the multiplier and the multiplicand are single digits, and the maximum product number obtained is eighty-one. The advantage of this calculating table is that, on the one hand, it provides people with the fastest calculation table for single digit multiplication, which is convenient for calculation at a glance. On the other hand, although it only shows single digit multiplication, it provides the most basic algorithm for the emergence of the multiplication table of tens digits. The algorithm and structure table of Calculating Table in The Tsinghua Bamboo Slips happens to combine the "multiplication table of nine times nine is eighty-one" and ten-digit multiplication. It not only makes the multiplication operation further in the range, but also lays the foundation for the subsequent multiplication operation in a larger range. Therefore, the multiplication of numbers from single digits to 100 can be calculated with this calculating table, and the results obtained are completely accurate, which also proves the reliability of the calculating table in multiplication calculation.

3. The Division and Square Root in Calculating Table

In addition to multiplication, Calculating Table in The Tsinghua Bamboo Slips can also be used for division and square root operations. By analyzing the rules of the structure of Calculating Table, it can be found that the division operation and the multiplication operation are similar, and roughly present the following characteristics: First, all numbers in the table except the ones of the top row and the right column can be used as divisors. According to the analysis in the previous statement, the numbers in the table except the ones of the top row and the right column can be used as multipliers or multiplicands, and the number at the intersection of the two numbers is the product number. Therefore, it can be concluded that the product number in the original multiplication operation becomes the divisor in the division operation, and the original multiplier and the multiplicand become the dividend
and the quotient. Judging the table from the rule of division operation, when the number in the top row of the table remains unchanged as the dividend, the larger the divisor in the table, the larger the quotient in the rightmost column; when the divisor in the table gets smaller, the smaller the quotient in the rightmost column. In other words, when the quotient is a fixed value, the divisor and the dividend are in a proportional relationship. The larger the divisor, the larger the dividend; the smaller the divisor, the smaller the dividend.

Secondly, the change of the value in the lower left area where the threads are in the division operation is directly related to the change of the number in the top row and the right column. When the number in the top row changes between 1-9, the value of the lower left area where threads are changes with the multiple of the number in the rightmost column. When the number in the top row changes between 10 and 90, the value in the lower left area of the threading position should increase ten times the value when the number in the top row changes between 1 and 9. When the numbers in the top row remain unchanged and the numbers in the right column gradually become smaller, the numbers in the lower left area where the threads are also show a trend of gradually becoming smaller. Then, this law of change is applicable in the calculation of the multiplier and the multiplicand, and the two are in a direct proportional relationship. At the same time, it is also applicable to the division operation, that is, when the number in the top row is fixed, the larger the number in the lower left area where the threads are, the larger the number in the rightmost column. Vice versa. When the number in the lower left area of the threading position is fixed, the larger the number in the top row, the smaller the number in the rightmost column. Vice versa.

In Calculation Table, whether it is multiplication or division, the laws reflected in it are consistent with some subsequent multiplication and division operations, and it also adds ½-related operations. There is a ½ in the upper left and lower right corners of the table. In the operations of multiplication and division, fractional operations like ½ are more common. On the one hand, it shows another mode of operation besides integer multiplication and division, which increases the range of digital calculations. On the one hand, from the actual calculations, the ½ in the table is only an approximate number, not a definite number. In other words, ½ can be replaced with other fractions according to the calculation rules in the table. In the multiplication operation, when the numerator in the fraction value is fixed, the larger the denominator, the smaller the product obtained by multiplying the fraction and the fixed multiplicand. Vice versa. Similarly, this rule of operation can also be used in the division operation, that is, when the numerator in the fraction value is fixed, the larger the denominator, the smaller the fraction, the greater the quotient obtained by dividing the fraction by the number in the lower left area where the threads are. Vice versa.

From the previous analysis, it can be seen that the multiplication and division algorithms in the table have completely exceeded the single-digit operation tables people later used. In particular, it also includes the multiplication and division of fractions, which is not available in other multiplication and division operations.

Although square root operation is not as common as multiplication and division in mathematical operations, it is still one of the indispensable calculation methods. If expressed by a calculation formula, that is \((a±b)^2=a^2±2ab+b^2\). In the square root formula, it involves the addition and square of two numbers, and the square of a number and the product of two numbers. They are inverse to each other. It can be seen that the square root not only includes addition and subtraction, but also the addition of number squares.

In Calculating Table in The Tsinghua Bamboo Slips, there are also operations on the square roots of the units and tens digits. From the perspective of the entire structure of the table, the perfect square number is located on the diagonal of the table, so when performing square root operations, the diagonal is needed. For the square root operation in this table, it is very important to meet two points: the number on the diagonal should be smaller than the square root, and it should also correspond to the number closest to the square root on the diagonal. On this basis, intersecting it with the diagonal through the thread in the top row and the rightmost column, and then using the square root formula to get the final square root number. Li Junming pointed out: "The fractions seen in Calculating Table are
only ½ or numbers with ½ and ¼ of its product, so its calculation range is mainly integers, and only involves the most commonly used fractions in social life at that time, indicating that Calculating Table has strong practicability. Obviously, Calculating Table is very suitable for calculating the land area (including fields and homesteads), tax rates, labor productivity, and even commodity transactions, which were commonly used by the government and private life at that time [4].” Therefore, Calculating Table in The Tsinghua Bamboo Slips can not only perform mathematical calculation functions, but also can be widely used in daily life.

4. The Meaning of Calculating Table for Calculation

Although Calculating Table in The Tsinghua Bamboo Slips is an unearthed document in nature, it is a mathematical calculation formula in terms of content. There is indeed a certain difference between the two, but combining the unearthed documents with mathematics, relevant research can still be carried out. Through the analysis above, it can be seen that Calculating Table is of great significance in actual calculations.

Firstly, it appeared relatively early. The early or late appearance of the calculation formula is a symbol of the wisdom of a nation, and can be used in the actual life of then and later generations. The Nine-Nine Arithmetic Table found in Liye Bamboo Slips of Qin in Hunan Province appeared in more than 200 BC, while Calculating Table in The Tsinghua Bamboo Slips was around 305 BC. Li Xueqin pointed out: "Calculating Table in The Tsinghua Bamboo Slips is the earliest practical calculation tool seen so far [5]." This shows that the ancients had mastered the laws of multiplication and division and square root more than two thousand years ago, which embodies the superb wisdom of the ancients.

Secondly, it has rich content. On the whole, Calculating Table in The Tsinghua Bamboo Slips not only appeared earlier, but also has richer content that those appeared later. Mr. Feng Lisheng pointed out: "Calculating Table applied the decimal counting method, and used the commutative law of multiplication, the distribution law of multiplication to addition, and fractions. It can be used not only for two-digit multiplication, but also for division. It can also perform certain operations on fractions ½ or mixed numbers containing ½. …. Calculating Table is the direct evidence of the development of mathematics and computing technology in the pre-Qin period. It is not only earlier than Book of Calculation in Zhangjiashan Bamboo Slips of Han and Numbers in The Bamboo Slips of Qin collected by Yuelu Academy, but also contains content that is not in the above slips. It is an important historical material for understanding the mathematical level in pre-Qin period [6].” Even when looking at the whole world, the table is earlier than the calculating tables of ancient Babylon and so on, and is rich in content and more practical [7].

5. Conclusion

Calculating Table in The Tsinghua Bamboo Slips integrated multiplication, division and square root algorithms into a structural table, which is convenient for users to use. Moreover, it is not only the earliest mathematical table found in China, but also more practical than any other ancient arithmetic tables in other countries [8]. It demonstrated the achievements and wisdom of the ancients in mathematics as well.

Attached List:

| 1/2 | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|     | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
| 45  | 9   | 1   | 2   | 4   | 5   | 6   | 7   | 8   | 9   |
|     | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |

| 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|     | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |

| 3   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
|     | 6   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 0   |
| 1/2 | 3 6 9 1 1 1 2 2 2 3 6 9 1 1 1 2 2 2 0 0 0 2 5 8 1 4 7 0 0 0 0 0 0 0 |
|-----|-----------------------------------------|
| 1   | 2 4 6 8 1 1 1 1 1 2 4 6 8 1 1 1 1 0 2 4 6 8 0 0 0 0 2 4 6 8 0 0 0 0 |
| 1/2 | 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6 7 8 9 0 0 0 0 0 0 0 0 |