Optimal Design of a Superconducting Motor for Electric-drive Aeropropulsion Based on Finite-Element Analysis and Genetic Algorithm

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Abstract. Electric-drive Aeropropulsion is one of the key technologies in design of future aircrafts because the electric motor drive has high efficiency and the this would reduce operational cost and pollutant emission. This paper discusses the optimization of electromagnetic design of superconducting motors. Finite element analysis, multi-objective genetic algorithm and analytical equations are applied in the optimization. Power density, power loss including AC loss of superconducting materials, and torque ripple are considered in the objective function. The results of optimization suggest that the application of superconducting motor is feasible for electric-drive aeropropulsion with power density higher than 16.4 kW/kg.

1. Electric-drive Aeropropulsion
A global reduction of the emissions of pollutants into the atmosphere is needed and a large part of the emissions come from transportation vehicles, including airplanes due to the nature of combustion-based propulsion system. With the application of Electric-drive Aeropropulsion (EA), operational cost, pollutant emission, and noise can be reduced largely due to the high efficiency of electric motor.

The overall cost of a passenger airline is assumed to reduce by 40% with EA [1]. Moreover, EA meets the following requirements: 1. Reduction of operational cost, 2. Low emissions of pollutants, and 3. Low noise.

However, conventional machines exhibit output power densities $D_p$ typically lower than 1 kW/kg and have reached an optimum in terms of performance [3], leaving little room for improvement.

On the contrary, superconducting rotating machines are destined to play a key role as the enabling technology for the EA because it is supposed to achieve $D_p$ comparable to turbine engines in excess of 10-20kW/kg [2]. So superconducting motor smaller and lighter than a gas turbine can be potentially built, which makes EA possible.

2. Design of Superconducting Motors

2.1. Introduction to Superconducting Motor
The concept of electrical ducted fan has been proposed that takes advantage of the high bypass ratio of engine and locally replace the engine core with a SuperConducting Motor (SCM) [2].
As Fig. 1 shows, a system configuration of EA consisting of turbine, generator AC/DC converter, DC Bus, DC/AC inverter, SCMs and ducted fans is considered. For that, we design SCMs used in this system. Energy source can be replaced with fuel cell when energy density of cell increases to be on a pair with that of current energy source.

Figure 1. Example of EA System Configuration

Figure 2. Structure of SCMs

Fig. 2 shows the structure of SCMs we considered. YBCO field coils in the rotor produce magnetic field in copper armature windings for Partially SuperConducting Motors (PSCMs) or in MgB2 armature windings for Fully SuperConducting Motors (FSCMs) in the stator. The motor has an air-core (i.e., non-magnetic) construction in the rotor and air gap windings in the stator. Liquid hydrogen (LH2) as the fuel, is also considered as coolant to cool down superconducting materials.

Table 1 lists the necessary variables for design and optimization.

| Parameters                                      | Symbols |
|-------------------------------------------------|---------|
| Inside radius of field winding (m)              | $R_1$   |
| Outside radius of field winding (m)             | $R_2$   |
| Inside radius of armature winding (m)           | $R_3$   |
| Average radius of armature winding (m)          | $r_0$   |
| Outside radius of armature winding (m)          | $R_4$   |
| Outside radius of back yoke (m)                 | $R_0$   |
| Thickness of back yoke (m)                      | $t$     |
| Winding pole angle (rad)                        | $\alpha$|
| Effective length (m)                            | $L_{ef}$|
| Mechanical gap (m)                              | $g_1$   |
| Air gap (m)                                     | $g_2$   |

2.2. Materials for SCMs

We choose High-Temperature Superconductor (HTS) YBCO wire, as the field coils because YBCO wire has high critical current density even among high external magnetic field. Copper or MgB2 superconducting wire shown in Fig. 3, are chosen as armature winding due to the potential of reduction of AC loss due to multi-filament structure of MgB2 wire, and back yoke uses low high-frequency iron-loss steel sheet. The operation conditions of the materials are shown in Table 2. The $J_{ave}$ is the average current density considered for the initial design. In optimization, $J_{ave}$ of copper depends on the area of armature winding and $J_{ave}$ of MgB2 depends on the area of armature winding and external magnetic flux density.
Table 2. Materials and Operation Conditions

| Parts      | Materials | Operation Temperature (K) | $J_{ave}$ (A/mm²) |
|------------|-----------|---------------------------|-------------------|
| Field winding | YBCO      | 20                        | 177               |
| Armature winding I | Copper | 293                       | 7.5               |
| Armature winding II | MgB₂  | 20                        | 48                |
| Back yoke   | 20JNEH1200 | 293                      | /                 |

Figure 3. Cross Section of MgB₂ Wire

Figure 4. (1). Field Coil Racetrack Structure; (2). Armature Winding Structure

2.3. Differences between PSCM and FSCM

For PSCM, the superconducting material YBCO wire is for DC application, so the Joule loss of copper armature winding: $W_{Cu}$, is considered as loss. Because only field winding of PSCM is superconducting material, cryogenic chamber is set outside the rotor, in which LH₂ flows to cool down the field winding.

For FSCM, loss due to current leads of both field coils and armature windings $W_{Lead}$, 100 times AC loss of MgB₂ at 20K for AC application: $100 \times W_{AC,20K}$, and $W_{Fe}$ are considered as loss. Because both of field winding and armature winding of FSCM are superconducting materials, cryogenic chamber is set outside the rotor, and outside the stator, respectively. LH₂ flows to cool down the field winding and armature windings in the 2 chambers and back yoke is isolated to avoid cooling back yoke.

The $W_{Lead}$ are considered as a resistance 0.02Ω for YBCO field coils and MgB₂ armature windings [7]. Because of Joule loss, average current densities: $J_{ave}$ of copper armature winding are limited to 7.5 (A/mm²) (with liquid cooling in ducts) respectively. However, $J_{ave}$ of MgB₂ wire can achieve 48 (A/mm²) that creates higher interlinkage flux, which realizes higher $D_{p}$.

2.4. Specifications of SCMs

The SCMs feasible for EA should meet the following requirements: 1. Power: 1 to 30 MW, 2. Revolving Speed: 2000 to 12000 rpm, 3. Number of pole pairs: 2 to 4. One near-term challenge of National Aeronautics and Space Administration (NASA) is to design a 4MW FSCM with $D_{p} \geq 16.4 \text{kW/kg}$ and efficiency $\geq 99\%$ [5].

The target of the motor output power is 3.0, 4.0 and 5.0 MW class in this paper. If the jet engines in airplane for around 180-passenger class are replaced with electrical motors, the overall output capacity should be 44 MW. Therefore, a distributed EA needs 15, 11 and 9 motors, respectively. The phase voltage of the motors is set to be 1000 V that is among the range between ± 4.5 kVDC at the DC bus
in Fig. 1 [4]. Common specifications for different SCM cases are shown in Table 3. The operation for YBCO and MgB$_2$ is 20 K, and we plan to use liquid hydrogen for the coolant.

### Table 3. Specifications of SCMs

| Parameters               | PSCM | FSCM |
|-------------------------|------|------|
| Output Power $P$ (MW)   | 3.0  | 4.0  | 5.0  |
| Line Voltage $U_L$ (V)  | 1732 |      |
| Line Current $I_L$ (A)  | 1177 | 1569 | 1961 |
| Revolving Speed $N$ (rpm)| 5000 |      |
| Number of pole pairs $p$| 2    |      |
| Wires of Armature Windings             | Copper | MgB$_2$ |

2.5. **Analytical Design and Equations**

The field coil structure is assumed as racetrack structure and armature winding structure is assumed as similar structure in AC loss and $D_p$ calculation, as shown in Fig. 4. The average length of the winding: $L_{ave}$ is expressed as following:

$$L_{ave} = 2L_{ef} + 2(D_{pitch} - 2R_b) + 2\pi R_b$$  \hspace{1cm} (1)$$

where $D_{pitch}$: winding pitch and $R_b$: bending radius, respectively. We assumed $R_b$ of YBCO as $D_{pitch}/2$, MgB$_2$ windings as 60 mm [6], and that of Cu as 20 mm.

Radial components $B_r$ and tangential field components $B_\theta$ of magnetic flux density of 4-pole motor in the region at angular location $\theta$ and radius $\rho$ of the field winding are given by (2)-(3) and those of armature winding are given by (4)-(5), respectively [7], as a function of $\rho$ and $\theta$. Fig. 2 shows necessary variables for SCM analytical design and optimization.

From field coils:

$$B_r(\rho, n) = \frac{\mu_0 J_r 2 \cos(n \alpha)}{\pi n} \cdot (R_2^2 + 2n - R_1^2) \cdot \left\{ \rho^{-2n-1} - \frac{\rho^{2n-1}}{(R_0 - \xi)^{4n}} \right\} \cdot \cos(2n\theta)$$  \hspace{1cm} (2)$$

$$B_\theta(\rho, n) = \frac{\mu_0 J_r 2 \cos(n \alpha)}{\pi n} \cdot (R_2^2 + 2n - R_1^2) \cdot \left\{ \rho^{-2n-1} - \frac{\rho^{2n-1}}{(R_0 - \xi)^{4n}} \right\} \cdot \sin(2n\theta)$$  \hspace{1cm} (3)$$

From armature windings:

$$B_r(\rho, n) = \begin{cases} \frac{\mu_0 J_a 2 \cos(n \alpha)}{\pi n} \cdot \left[ \ln \left( \frac{R_a}{R_0} \right) + \frac{R_2^2 + 2n - R_1^2}{(R_0 - \xi)^{4n}} \right] \cdot \rho^{2n-1} \cdot \cos(2n\theta), & \text{if } R_2 \leq \rho \leq R_3 \\
\frac{\mu_0 J_a 2 \cos(n \alpha)}{\pi n} \cdot \left\{ 1 - \frac{R_2^2}{(R_0 - \xi)^{4n}} + \frac{2n + 2}{\rho} \cdot \ln \left( \frac{R_a}{\rho} \right) \right\} \cdot \rho \cdot \cos(2n\theta), & \text{if } R_3 \leq \rho \leq R_4 \end{cases}$$  \hspace{1cm} (4)$$

$$B_\theta(\rho, n) = \begin{cases} \frac{\mu_0 J_a 2 \cos(n \alpha)}{\pi n} \cdot \left[ \ln \left( \frac{R_a}{R_0} \right) + \frac{R_2^2 + 2n - R_1^2}{(R_0 - \xi)^{4n}} \right] \cdot \rho^{2n-1} \cdot \sin(2n\theta), & \text{if } R_2 \leq \rho \leq R_3 \\
\frac{\mu_0 J_a 2 \cos(n \alpha)}{\pi n} \cdot \left\{ 1 - \frac{R_2^2}{(R_0 - \xi)^{4n}} - (2n + 2) \cdot \ln \left( \frac{R_a}{\rho} \right) \right\} \cdot \rho \cdot \sin(2n\theta), & \text{if } R_3 \leq \rho \leq R_4 \end{cases}$$  \hspace{1cm} (5)$$

where $\rho$: radius, $n$: odd number, $J_r$: field coil current density, $J_a$: armature winding current density, and $\theta$: angular location, respectively.
Equation (2)-(5) are used in analytical design and AC loss calculation. Fig. 5 shows airgap magnetic field distribution of one case. Both of the radial component and tangential component have good agreement.

3. Optimization on SCMs
In the case of SCM for EA, the output power density \( D_p \), electrical loss \( W_{loss} \) and torque ripple \( T_r \) are the main concerns. To acquire a model with balance of \( D_p \), \( W_{loss} \) and \( T_r \), we optimized analytical design with FEM and Multi-Objective Genetic Algorithm (MOGA). FEM analysis is carried out with J-MAG ®.

3.1. AC Loss Calculation
When considering the calculation of AC loss of FSCM, we consider the hysteresis loss: \( W_h \) and coupling loss: \( W_c \). The \( W_h \) is expressed as following [8]:

\[
W_h = L_a \cdot n_f \cdot \pi \cdot R^2_0 \cdot q_h
\]

(6)

\[
q_h = \frac{16}{3\pi} \cdot f \cdot B_{max} \cdot r_{one} \cdot J_c
\]

(7)

\[
L_a = m \cdot p \cdot L_{ave} \cdot n_{para}
\]

(8)

where \( L_a \): MgB\(_2\) wire length, \( n_f \): number of MgB\(_2\) filaments, \( r_{one} \): filament radius, \( q_h \): hysteresis loss per unit volume, \( f \): frequency, \( B_{max} \): maximal magnetic flux density on armature winding, \( J_c \): critical current of MgB\(_2\) wire, \( m \): number of phase and \( n_{para} \): number of parallel connected MgB\(_2\) wires, respectively.

The \( W_c \) is expressed as following [9]:

\[
W_c = L_a \cdot \pi \cdot r^2_w \cdot q_c
\]

(9)

\[
q_c = \left( \frac{r_{fil}}{r_w} \right)^2 \cdot \left( \frac{2\pi f B_{max}^2}{\mu_0} \right)^2 \cdot \frac{r^2_w}{\mu_0}
\]

(10)

\[
\tau_{se} = \frac{1}{2} \cdot \left( \frac{L_p}{2\pi} \right)^2 \cdot \mu_0 \cdot \sigma_{eff}
\]

(11)

where \( r_w \): MgB\(_2\) wire radius, \( r_{fil} \): radius of filament region, \( \tau_{se} \): time constant, \( L_p \): twist pitch and \( \sigma_{eff} \): transverse conductivity, respectively. The characteristics of MgB\(_2\) wires are based on samples [9] and commercial products [10], and the properties are listed in Table 4.
According to the result of PSCM and FSCM, thus, with 50
g=4.

and the other bounds referring to initial importance of the three properties are considered the same.

parameters by weighted sum approaches, thus:

Set the variable vector \( \mathbf{X} \)

\[
\mathbf{X} = \begin{bmatrix} R_0, t_4 \left( \frac{a_4 - R_3}{2} \right), (R_3 - R_2), L_{ef}, \alpha \end{bmatrix}
\]

(12)

The cost function \( Z \), consists of \( D_p, W_{loss} \), and \( T_r \) and we adjust the importance of each parameter by weighted sum approaches, thus:

\[
Z(\mathbf{X}) = w_1 \cdot \frac{1}{D_p(\mathbf{X})} + w_2 \cdot W_{loss}(\mathbf{X}) + w_3 \cdot Tr(\mathbf{X})
\]

(13)

where \( w_1, w_2, \) and \( w_3 \) are the weights of \( D_p, W_{loss} \), and \( T_r \), which denote the importance of the parameters, respectively. We set the ratio of weights \( w_1: w_2: w_3 \) to be 1:1:1, which means the importance of the three properties are considered the same.

Thus, the optimization is to minimize \( Z(\mathbf{X}) \) subject to bound constraints:

\[
X^l_c \leq X_c \leq X^u_c \quad (c = 1, 2, \ldots, 6)
\]

(14)

where \( X^l_c \) and \( X^u_c \) arfe the lower and upper bounds of a design variable, respectively.

We set the upper bounds of \( R_0 \) and \( L_{ef} \) as around half of the current turbofan engine IAE V2500, and the other bounds referring to initial design and our previous optimization result as following:

\[
\overline{X}^l_c = \begin{bmatrix} 200 \text{ mm}, 5 \text{ mm}, T_a \text{ (mm)}, 4 \text{ mm}, 400 \text{ mm}, 10^{-3} \end{bmatrix}
\]

(15)

\[
\overline{X}^u_c = \begin{bmatrix} 400 \text{ mm}, 30 \text{ mm}, T_a \text{ (mm)}, 50 \text{ mm}, 1500 \text{ mm}, 40^{-3} \end{bmatrix}
\]

(16)

where \( T_a \) is the thickness of one-layer armature winding that varies with \( I_L \) and \( J_{ave} \). While \( g_1 \) and \( g_2 \) are set to be constant, 10 mm and 40 mm, respectively [12].

**4. Results and Discussion**

Thus, with 50-generation MOGA, we got optimal results of PSCM and FSCM as shown in Table 5. According to the result of PSCM and FSCM, we designed motors with high \( D_p \), among which, 5.0

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**Table 4. Properties of MgB\(_2\) Wires**

| Component         | Fill Factor of MgB\(_2\) | \( J_c \) (20 K, 2.0 T) | \( \sigma_{ef} \) |
|-------------------|--------------------------|--------------------------|-------------------|
| Copper            | 0.15                     | 1780 A/mm\(^2\)         | 1.36 \times 10^6\ S/m |

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3.2. Motor Mass Calculation

In the calculation of \( D_p \), the mass densities of copper, MgB\(_2\) wire, YBCO wire and 20JNEH1200 are 8960 kg/m\(^3\), 7995 kg/m\(^3\), 8940 kg/m\(^3\) and 7650 kg/m\(^3\), respectively. The mass of other parts including bearing and supporting parts are considered as 25% of the field coils, armature windings and back yoke [11].

3.3. Objective of Optimization

The objective of optimization is to optimize the SCM design to be with \( D_p \geq 16.4 \text{ kW/kg}, W_{loss} \leq 1\% \) for PSCM or \( W_{loss} \leq 10\% \) for FSCM, and \( T_r \leq 4\% \).

3.4. Cost Function and Bound Constraints

Set the variable vector \( \mathbf{X} \)

\[
\mathbf{X} = \begin{bmatrix} R_0, t_4 \left( \frac{a_4 - R_3}{2} \right), (R_3 - R_2), L_{ef}, \alpha \end{bmatrix}
\]

(12)

The cost function \( Z \), consists of \( D_p, W_{loss} \), and \( T_r \) and we adjust the importance of each parameter by weighted sum approaches, thus:

\[
Z(\mathbf{X}) = w_1 \cdot \frac{1}{D_p(\mathbf{X})} + w_2 \cdot W_{loss}(\mathbf{X}) + w_3 \cdot Tr(\mathbf{X})
\]

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where \( w_1, w_2, \) and \( w_3 \) are the weights of \( D_p, W_{loss} \), and \( T_r \), which denote the importance of the parameters, respectively. We set the ratio of weights \( w_1: w_2: w_3 \) to be 1:1:1, which means the importance of the three properties are considered the same.

Thus, the optimization is to minimize \( Z(\mathbf{X}) \) subject to bound constraints:

\[
X^l_c \leq X_c \leq X^u_c \quad (c = 1, 2, \ldots, 6)
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We set the upper bounds of \( R_0 \) and \( L_{ef} \) as around half of the current turbofan engine IAE V2500, and the other bounds referring to initial design and our previous optimization result as following:

\[
\overline{X}^l_c = \begin{bmatrix} 200 \text{ mm}, 5 \text{ mm}, T_a \text{ (mm)}, 4 \text{ mm}, 400 \text{ mm}, 10^{-3} \end{bmatrix}
\]

(15)

\[
\overline{X}^u_c = \begin{bmatrix} 400 \text{ mm}, 30 \text{ mm}, T_a \text{ (mm)}, 50 \text{ mm}, 1500 \text{ mm}, 40^{-3} \end{bmatrix}
\]

(16)

where \( T_a \) is the thickness of one-layer armature winding that varies with \( I_L \) and \( J_{ave} \). While \( g_1 \) and \( g_2 \) are set to be constant, 10 mm and 40 mm, respectively [12].
MW PSCM, 4.0 MW FSCM and 5.0 MW FSCM surpass 16.4 kW/kg. And, all of the PSCMs meet the requirement of low \( W_{\text{loss}} \), and even 5.0 MW FSCM, which has the highest loss, almost meets the requirement of 10 \%. So, the optimization improves the properties with a good balance.

The FSCMs have higher \( D_p \) than PSCMs of the same output power, while much higher loss considering cryogenic loss and loss from the current lead connecting the terminal of MgB2 armature winding.

For PSCM and FSCM, the \( D_p \) increases as \( P \) increases because the thickness of field winding increases, which causes a higher airgap magnetic flux density \( B_g \). And, as \( P \) increases, the \( W_{\text{loss}} \) of PSCM decreases while that of FSCM increases because the \( W_{\text{Cu}} \) changes little, \( W_{\text{Lead}} \) remains the same and \( W_{\text{Fe}} \) decreases much when the volume of back yoke decreases, and because the \( P \) increases, the loss per output power decreases. However, the \( L_a \) of MgB2 increases with \( P \) and since the \( W_{\text{AC}} \) accounts for the vast majority of loss of FSCM, the \( W_{\text{loss}} \) increases as \( P \) raises.

Among the variables, the thickness of back yoke, \( t \) and the thickness of field coils, thus \( R_2-R_1 \), has the most effect on the \( D_p \) and \( W_{\text{loss}} \) because, when we keep the \( L_{\text{ef}} \) constant, the increase of \( t \) will increase the volume of back yoke, which cause the increase of \( W_{\text{Fe}} \) and mass simultaneously.

### Table 5. Optimal Result of SCMs

| \( w_1: w_2: w_3 \) | 1:1:1 |
|----------------------|-------|
| **Type**              | PSCM  | FSCM  |
| \( P \) (MW)          | 3.0   | 3.0   | 3.0   |
| \( D_p \) (kW/kg)     | 5.42  | 5.85  | 5.85  |
| \( W_{\text{loss}} \)%| 0.54  | 0.69  | 0.65  |
| \( T_r \)%            | 1.10  | 0.75  | 0.65  |
| \( W_{\text{AC}}@20 \text{ K} \) (W) | /  | /  | 1697.47 |
| \( B_g \) (T)         | 1.61  | 2.14  | 1.34  |
| \( R_1 \) (mm)        | 166.41| 211.29| 272.54|
| \( R_2 \) (mm)        | 185.86| 236.31| 286.07|
| \( R_3 \) (mm)        | 225.86| 276.31| 326.07|
| \( R_4 \) (mm)        | 237.20| 284.47| 332.61|
| \( R_0 \) (mm)        | 271.02| 301.09| 366.21|
| \( t \) (mm)          | 23.82 | 6.62  | 23.59 |
| \( \alpha \) (deg)    | 25.57 | 27.81 | 26.22 |
| \( L_{\text{ef}} \) (mm) | 982.48| 671.45| 743.37|

Among the 6 types of optimized motor, the 5.0 MW PSCM meets the requirements best, so the magnetic field distribution of it is shown in Fig. 6, which illustrates the high airgap magnetic field density is the key reason of high \( D_p \) of PSCM. The external field of field coils is below 4 T, which ensures the application and high current density of YBCO wires. Change of 5 MW PSCMs \( D_p \) and \( W_{\text{loss}} \) with \( t \) and thickness of field winding are shown on Fig. 7 and Fig. 8, which help us understand the trend of the change of the two key properties.

### 5. Conclusion

We have carried out electromagnetic design of 5000-rpm PSCM and FSCM with output power 3.0 MW, 4.0 MW and 5.0 MW for EA. Using real samples of superconducting wires, we calculated AC loss and realized low AC loss \( \leq 1 \% \). With FEM and MOGA, we optimized design and got model that minimized cost function. And, we succeeded to optimize PSCM that meet near-term challenge for EA with \( D_p = 17.12 \text{ kW/kg} \), \( W_{\text{loss}} = 0.33 \% \) and \( T_r = 0.75 \% \). We optimized FSCM to higher \( D_p \) and around 10\% \( W_{\text{loss}} \), which is low to P but is still hard to take the heat.
With optimization, we understand the dependence of variables on the properties we need to optimize, and we testified the effectiveness of MOGA and weighted sum approaches in the optimization of SCMs for EA. As the next step, we plan to adjust the weights to get higher $D_p$ for PSCM and lower $W_{loss}$ for FSCM.

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