Doublet-Triplet Splitting in Supersymmetric SU(6) By Missing VEV Mechanism

Z. Chacko and Rabindra N. Mohapatra
Department of Physics, University of Maryland, College Park, MD 20742, USA

We present a realistic supersymmetric SU(6) model which implements doublet-triplet splitting by the missing vev mechanism. The model makes use of only the simplest representations, requires no fine tuning of parameters and maintains coupling constant unification as a prediction. Fermion masses also emerge in a very straightforward manner. This is the first time that the missing vev mechanism has been realized in the context of SU(6).

UMD-PP-99-019 e-mail address: rmohapat@physics.umd.edu

I. INTRODUCTION

Satisfactory resolution of the Higgs mass problem as well as the radiative origin of electroweak symmetry breaking in the supersymmetric extension of the standard model (MSSM) has rightly focused the interest of the particle theory community in understanding the high energy origin of the MSSM. The unification of the gauge couplings for the MSSM particle content gives rise to the natural speculation that the high energy theory may indeed be a supersymmetric grand unified theory [1,2] based on some simple group. Some popular groups which have been investigated are: SU(5), SO(10), E6, and SU(6). In this paper, we will concentrate on the SU(6) model.

A key problem of all SUSY GUTs is how to split the weak MSSM doublets from the color triplet fields that accompany them as part of the representation of the GUT symmetry. This is an essential aspect of SUSY GUTs since both coupling constant unification and suppression of proton decay require that the MSSM doublets \(H_u\) and \(H_d\) be at the weak scale whereas the triplets which mediate proton decay must have GUT scale mass. This is the famous doublet-triplet splitting problem (DTS).

In the simplest grand unifying group SU(5) natural implementation of doublet triplet splitting seems to require the use of the relatively large representations \(50, \overline{50}\) and \(75\). The search for a simpler solution has led various authors to consider the extension of the gauge symmetry to SU(6) which allows for more possibilities. Some ideas that have received considerable attention are: (i) the “GIFT” mechanism, where the light Higgs doublets emerge as the pseudo-Goldstone bosons of a broken global symmetry of the superpotential [3] and (ii) the sliding singlet mechanism [4], where the desired vev pattern follows automatically from the conditions of the supersymmetric minima condition. Although the “GIFT” mechanism is very interesting as an idea, it leads to complications while trying to construct realistic models, specially in obtaining masses for the quarks and leptons. Future developments in implementing this elegant idea are awaited. On the other hand, the sliding singlet mechanism, which is perhaps the most straightforward of all doublet-triplet splitting mechanisms has its own shortcomings. After this was first proposed in the context of SU(5) model, it was realized that it runs into severe difficulties due to radiative corrections which actually lift the MSSM doublet masses to an intermediate scale [5]. It was shown by Sen [6] that if instead one considers an embedding of the SU(5) model into the SU(6) group, then the problems associated with radiative instability of doublet-triplet splitting can be cured. The observation of Sen also made it clear that the SU(6) model may have certain technical advantages over other GUT groups as far as understanding the DTS problem goes. This version of the SU(6) model however is ruled out by the present data on \(\sin^2\theta_W\). An interesting variation of Sen’s idea which cures this difficulty has been discussed in a recent paper [9].

In this paper we show that the missing vev mechanism which has been successfully implemented in supersymmetric SO(10) can also be realized in a simple manner in SU(6), providing yet another way to solve the DTS problem in these models. We present a realistic model which makes use of this idea to implement DTS without finetuning of any parameters and which maintains coupling constant unification as a prediction. The Higgs sector of the model is quite economical and makes use of only the simplest representations; namely two adjoints \((35s)\), two \(6 + \overline{6}\) pairs and a singlet field. Fermion masses also emerge in a more straightforward manner than in earlier schemes.

The basic idea behind our model is to obtain a pattern of vevs for one of the adjoints \(S\) of the form:

\[
< S > = \text{const.} \begin{pmatrix}
-3 \\
1 \\
1 \\
1 \\
0 \\
0
\end{pmatrix}
\] (1)
where the lower $5 \times 5$ block corresponds to the fields which transform under the SU(5) which contains the standard model. Then the two $SU(2)_L$ doublets in one of the $6 + \bar{6}$ pairs (denoted by $h + \bar{h}$) remain massless and become the MSSM doublets via a coupling of the form $Sh\bar{h}$ without any finetuning of parameters. Since the adjoint $S$ does not break $SU(6)$ all the way down to the standard model (it leaves an extra U(1)), we will need additional Higgs fields, which are the other the $6 + \bar{6}$ pair. It is important to point out that a similar scheme is not feasible in the case of the SU(5) model since the adjoint must be traceless and the pattern of symmetry breaking implies that there be only two different vevs along the diagonal.

II. SYMMETRY BREAKING AND DOUBLET-TRIPLET SPLITTING VIA MISSING VEV

We now describe the Higgs sector of the model which breaks SU(6) down to the standard model while keeping supersymmetry intact and show how in this process doublet-triplet splitting is realized. The Higgs sector of the model as already mentioned consists of two adjoints $\Sigma, \bar{\Sigma}$ ($35$-dim.), two pairs of $6 + \bar{6}$ denoted by $h + \bar{h}$ and $H + \bar{H}$ and a singlet $Y$. We will assume the MSSM doublets to emerge from $h + \bar{h}$ pair. We choose the Higgs superpotential to have the form:

$$W_{Higgs} = Y(\Sigma^2 - \bar{\Sigma}^2) + (\Sigma^3 + \bar{\Sigma}^3) + H(\Sigma + \bar{\Sigma})\bar{H} + M H \bar{H} + h(\Sigma + \bar{\Sigma})\bar{h}$$

(2)

This superpotential is invariant under the discrete symmetry:

$$\Sigma \leftrightarrow \bar{\Sigma}; Y \leftrightarrow -Y; H \leftrightarrow -H; \bar{H} \leftrightarrow -\bar{H}$$

(3)

Fields not included in the above equation are invariant under the discrete symmetry. Freedom to rescale the fields has been used to set all the dimensionless coupling parameters in the superpotential to one.

We are interested in the supersymmetric vacua of the following form:

$$< H > = < \bar{H} > = \begin{pmatrix} v \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$< \Sigma > = \begin{pmatrix} a_1 \\ a_2 I_3 \\ a_5 I_2 \end{pmatrix}$$

$$< \bar{\Sigma} > = \begin{pmatrix} \bar{a}_1 \\ \bar{a}_2 I_3 \\ \bar{a}_5 I_2 \end{pmatrix}$$

(4)

Here $I_n$ stands for an $n \times n$ unit matrix. These vev’s break SU(6) down to $SU(3) \times SU(2) \times U(1)$. Now let us note that if the vacuum conditions are such that $a_5 = -\bar{a}_5$, then $\Sigma + \bar{\Sigma}$ will have the vev form of $< S >$ in Eq. (1) and the last term in the superpotential in Eq.(2) then implies that the weak doublets in $h + \bar{h}$ are massless after the SU(6) breaking. We now show that vacuum conditions for maintaining supersymmetry indeed imply that $a_5 = -\bar{a}_5$.

The equations of motion for the various fields are:

$$M + (a_1 + \bar{a}_1) = 0$$

$$\Sigma_i a_i^2 = \Sigma_i \bar{a}_i^2$$

$$2ya_i + \alpha + 3a_i^2 + v^2\delta_i^1 = 0$$

$$-2ya_i + \bar{\alpha} + 3\bar{a}_i^2 + v^2\delta_i^1 = 0$$

(5)

where $\alpha$ and $\bar{\alpha}$ are the Lagrange multipliers that are needed to guarantee the tracelessness of the $\Sigma$ and $\bar{\Sigma}$ vevs. Summing the last two equations in the Eq. (5) and subtracting and remembering that $\Sigma_i a_i = \Sigma_i \bar{a}_i = 0$, we get $\alpha = \bar{\alpha}$. Then subtracting the same two last equations of Eq.(5), we get:

$$(a_i + \bar{a}_i)(a_i - \bar{a}_i + \frac{2}{3}y) = 0$$

(6)
This implies that either

\[ a_i = -\bar{a}_i \]

or

\[ \bar{a}_i = a_i + \frac{2}{3}y \]  

(7)

We are interested in the vacuum where the first of the equations in Eq.(7) is satisfied by \( a_5 \) whereas \( a_{1,2} \) satisfy the second of the Eq. (7). Then \( \Sigma a_i = \Sigma \bar{a}_i = 0 \) implies that

\[ a_5 = \frac{2}{3}y \]  

(8)

The equation of motion for \( \Sigma \) in Eq.(5) is quadratic which then implies that

\[ a_2 + a_5 = -\frac{2}{3}y \]  

leading to \( a_2 = -\frac{4}{3}y \). The trace condition then implies that \( a_1 = \frac{8}{3}y \). The first equation in Eq.(5) then determines \( y = -M/6 \) and \( v^2 = -\frac{2}{9}M^2 \). Thus all the vevs are determined in terms of the mass parameter \( M \) of the superpotential. The vevs of the 35s are given by:

\[
<\Sigma> = -\frac{M}{9} \begin{pmatrix}
4 & -2 & -2 \\
-2 & 1 & 1 \\
-2 & 1 & 1
\end{pmatrix}
\]

\[
<\bar{\Sigma}> = -\frac{M}{9} \begin{pmatrix}
5 & -1 & -1 \\
-1 & -1 & -1 \\
-1 & -1 & -1
\end{pmatrix}
\]

(10)

It is now obvious that \( <\Sigma + \bar{\Sigma}> \) has the desired form and the doublets in \( h + \bar{h} \) are now massless whereas the triplets in those multiplets have GUT scale mass thus solving the DTS problem of SU(6) in a natural manner.

III. FERMION MASSES IN SU(6)

It is relatively straightforward to incorporate fermions into the model and get desired masses for them. Following a procedure identical to that in Ref. [11], we add the fields \( \bar{P}_{1,2} \) belonging to \( \bar{6} \), \( N_j \) belonging to \( 15 \) dim. representation, an extra \( 15 + \bar{15} \) pair denoted by \( Z + \bar{Z} \) and a pair of singlets \( T \) and \( \bar{T} \). (Here \( j \) is the generation index.) The known fermions live in \( N \) and \( \bar{P}_2 \). The part of the superpotential responsible for fermion masses is given by

\[
W_{\text{fermion}} = \lambda_i^k N_j \bar{H} \bar{P}_{1k} + Y_d^j \bar{N}_i \bar{P}_{2k} \bar{h} + T \bar{H} \bar{P}_1 + \bar{T} \bar{H} \bar{P}_2 \\
+ Y_u^j N_j N_k Z + M' Z \bar{Z} + \bar{Z} \bar{H} \bar{h}
\]  

(11)

The discrete symmetry of Eq. (2) can be extended to this part of the superpotential as follows:

\[
\bar{P}_1 \rightarrow i\bar{P}_1; \bar{P}_2 \rightarrow -i\bar{P}_2; N \rightarrow iN; \\
Z, \bar{Z} \rightarrow -Z, -\bar{Z}; T \rightarrow iT; \bar{T} \rightarrow -iT
\]  

(12)

The role of the various terms in the superpotential \( W_{\text{fermion}} \) are as follows. The term \( \bar{N} \bar{H} \bar{P} \) along with the terms containing the singlets decouple one of the 6 fields and the unwanted fields in the 15 to the GUT scale, leaving three light generations as desired. The \( Y_d \) term gives mass to the down type quarks and the charged leptons. The \( M' \) and the \( \bar{Z} \bar{H} \bar{h} \) terms combine to have the MSSM \( H_u \) as an admixture of the \( Z \) and the \( h \) field. Thus after SUSY breaking the up-vev will reside partly in both these fields. Now it is easy to see that the \( NNZ \) coupling give masses to the up type quarks. Upto now the fermion masses are exactly as in the case of SUSY SU(5) and therefore are plagued by
the “bad” relation \( m_e/m_\mu = m_d/m_s \). This can however be cured by a nonrenormalizable term in the superpotential of the form:

\[
W_{nr} = \frac{\lambda'_{jk}}{M_p} N_j (\Sigma + \bar{\Sigma}) \bar{P}_{2k} \bar{h}
\]  

(13)

Let us now turn our attention to the prediction for proton lifetime in our model. As the model has been constructed so far, proton lifetime is same as in the minimal SUSY SU(5) model i.e. the Higgsinos whose exchange leads to proton instability must be heavier than the GUT scale for proton lifetime to be compatible with present experiments. It may therefore be useful to see whether by simple modification of the model, one can have suppression of proton decay. Indeed it turns out as we show now that a weak suppression is not hard to achieve in our model without effecting its other attractive features. All we have to do is to supplement the Higgs spectrum by adding another pair of \( 6 + \bar{6} \) pair (denoted by \( h' + \bar{h}' \)) and replacing the \( h(\Sigma + \bar{\Sigma})h \) term in the \( W_{Higgs} \) to the following form:

\[
W'_{Higgs} = h(\Sigma + \bar{\Sigma})h' + h'(\Sigma + \bar{\Sigma})h + M''h'h'
\]  

(14)

The proton decay amplitude now has an extra suppression factor proportional to \( M''/M_U \) where \( M_U \) is the GUT scale. If we keep this factor to be somewhat less than one then the proton lifetime will be suppressed by a decent factor over the SUSY SU(5) prediction. There is of course a price for this i.e. it will generate some light fields just below the scale \( M_U \). This will slightly enhance the threshold effects with a marginal effect on coupling constant unification.

It is conceivable that the theory as written down or some straightforward generalization thereof is the most general possible that is consistent with some set of discrete symmetries. However since the nonrenormalization theorem of supersymmetry protects the superpotential from radiative corrections this is not necessary and we do not pursue the matter further here.

In conclusion, we have demonstrated how the missing vev mechanism to realize doublet-triplet splitting can be implemented in the supersymmetric SU(6) grand unification model without any fine tuning of parameters. This is the first time such a model has been constructed. We have also shown how a realistic fermion spectrum and weak suppression of proton decay can be achieved consistently within this scenario.

This work is supported by the National Science Foundation under grant no. PHY-9802551.

[1] J. C. Pati and A. Salam, Phys. Rev. D 10, 275 (1974); H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974); H. Georgi, H. Quinn and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).
[2] For a recent review, see R. N. Mohapatra, TASI97 lectures, hep-ph/9801235.
[3] K. Inoue, A. Kakuto and T. Takano, Prog. Theor. Phys. 75, 664 (1986); A. Anselm and A. Johansen, Phys. Lett. B200, 331 (1988); Z. Berezhiani and G. Dvali, Sov. Phys. Lebedev Inst. Report 5, 55 (1989).
[4] E. Witten, Phys. Lett. B105, 267 (1981); D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B113, 151 (1982); S. Dimopoulos and H. Georgi, Phys. Lett. B117, 287 (1982); L. Ibáñez and G. G. Ross, Phys. Lett. B110, 215 (1982).
[5] S. Dimopoulos and F. Wilczek, Preprint NSF-ITP-82-07; K. S. Babu and S.M. Barr, Phys. Rev. D 48, 5354 (1993).
[6] K. S. Babu and S.M. Barr, Phys. Rev. D 50, 3529 (1994); K. S. Babu and S.M. Barr, Phys. Rev. D 51, 2463 (1995); K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. 74, 2418 (1995); S.M. Barr and S. Raby, Phys. Rev. Lett 79, 4748 (1997); Z. Chacko and R. N. Mohapatra, hep-ph/9808 4.
[7] For other recent work on the sliding singlet mechanism, see Y. Chikira, N. Haba and Y. Mimura, hep-ph/9804273.
[8] A. Sen, Phys. Lett. B148, 65 (1984).
[9] S. Barr, Phys. Rev. D 57, 190 (1998); G. Dvali, Phys. Lett. 324 B, 59 (1994).
[10] R. Arnowitt and P. Nath, Phys. Rev. D 49, 1479 (1994); J. Hisano, H. Murayama and T. Yanagida, Nucl. Phys. B402, 46 (1993).
[11] Z. Chacko, Markus A. Luty, Eduardo Ponton, hep-ph/9806398.