CHIRAL HYBRID BAG MODEL
WITH THE BOSON FIELD INSIDE THE BAG

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Abstract

The three-phase version of the hybrid chiral bag model, containing the phase of asymptotic freedom, the hadronization phase as well as the intermediate phase of constituent quarks, is proposed. For this model the self-consistent solution, which takes into account the fermion vacuum polarization effects, is found in (1+1) D. Within this solution the total energy of the bag, including the one-loop contribution from the Dirac's sea, is studied as the function of the bag geometry under condition of nonvanishing boson condensate density in the interior region. The existence and uniqueness of the ground state bag configuration, which minimizes the total energy and contains all the three phases, are shown.

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1 Introduction

The idea to describe a hadron as a bounded region of space ("the bag") filled with quark and gluon fields appears nowadays to be one of the most natural ways of constructing the effective quantum field theory with total confinement of color objects [1-12]. The boundary conditions are chosen so as to confine the colored particles to the bag without breaking the relativistic covariance [1-3]. The very possibility of existence of such an object is related to the change of the vacuum structure inside the hadron. It is supposed that the non-perturbative vacuum inside the strongly interacting particle is completely destroyed, which produces an excess of energy proportional to the bag volume [1-5]. The stability of this system is provided by the valence quarks contribution to the total energy proportional to the inverse bag radius. Even the early MIT-bag model yielded consistent results for the mass spectrum and other static characteristics of hadrons with a relatively small set of free parameters [6-8]. Further development of this model has led to taking into account the effects of spontaneous chiral symmetry breaking one of the most important features of strong interactions at low energies, and to incorporate the meson fields into the theory which plays the role of Goldstone bosons [9-11]. (Depending on the number of light-quark flavors, it is either the triplet of $\pi$-mesons for SU(2) group or the octet of $\pi$- and $K$-mesons for SU(3).) As a result the most consistent approach to the description of hadronic structure based on the bag models has been elaborated within the framework of so-called Hybrid Chiral Models (HCM) [10-12]. The HCM treat the nucleon as the little bag with quarks and gluons confined inside, surrounded by a big cloud of virtual mesons. The latter can be described either by the "hedgehog" $\pi$-meson solution of the Skyrme model, or by some more complicated models including vector mesons [13-15].

On the whole, the models of such a type predict correct scales for various static characteristics of non-strange baryons. At the same time, the estimation of certain values may yield essential quantitative discrepancies, even the sign may be incorrect [12,16]. Moreover, the best HCM results have been obtained for such values of model free parameters (which are the vacuum pressure constant $B$, the coupling constant $\alpha_s$, the current quark masses, and the constant of Casimir energy $Z$), that differ from their values obtained by means of other methods [4,5]. These circumstances indicate that the current formulation of HCM suffers from a number of shortcomings.

Actually, the most subtle point in the HCM is the Cheshire Cat Principle (CCP) which is basic for parting space into regions with different phases inside of them [17]. The essence of CCP is the hypothesis [18] that the fermion theory inside of a bag and the boson theory outside are actually equivalent and can be transformed into each other via the bosonization procedure. Therefore none of physical properties of such bag depends actually on the choice of boundary surface, while the boundary conditions are determined from the bosonization equations [19]. However, the bosonization, as well as CCP, can be rigorously proved only in (1+1)D, while in the real (3+1)D world the solution for the bosonization problem is still absent. As a result, in the (3+1)-dimensional HCM based on CCP there exists a rather small set of observables (e.g., the topological charge), which do not really depend on the bag radius [20]. Moreover, the phenomenology of strong interactions predicts unambiguously the existence of the characteristic confinement scale about 0.5 fm, and so in the realistic (3+1)D models the CCP should be
strongly violated, regardless of the proof of bosonization.

Therefore it seems to be natural to modify HCM so as to avoid any relation to CCP and specific features associated with, such as an infinitely thin boundary surface between different phases and corresponding surface action. Such a modification appears to be possible provided the correlation between the different phases of a bag, which are now not assumed to be equivalent in the sense of bosonization, arises due to some interaction of the real status taking place in a finite region of space. The latter can be naturally treated as the region of the intermediate third phase of the bag [21]. The introduction of such an additional phase is useful also because it allows to introduce the chirally invariant mechanism of the quark mass generation, hence it can be considered to a certain extent as the phase of constituent quarks, while the original 2-phase model does not take into account their existence at all. Meanwhile, the concept of constituent quarks has been shown to be very efficient in the hadron spectroscopy. From this point of view the most attractive physically situation is the one, where the initially free, almost massless quarks (corresponding to large momentum transfer) transmute firstly into “dressed” due to interaction massive constituent quarks carrying the same quantum numbers of color, flavor, and spin, and only afterwards there emerges the purely mesonic colorless phase.

The first order approximation to such version of the bag is given by the 3-phase hybrid model with additional constituent quark phase instead of the boundary with vanishing radial extent [21]. This model allows to take into account three phases: the phase of asymptotic freedom with free massless quarks, the phase of constituent quarks which acquire an effective mass due to the chirally invariant interaction with the boson field in the intermediate region of finite size, and the hadronization phase where the creation of free quarks is suppressed by infinitely large mass, while the non-linear dynamics of boson field causes the appearance of the c-number boson condensate in the form of a classical soliton solution, which ultimately accounts for the quantum numbers of the whole bag.

In the present paper the toy model of such kind in (1+1)D is considered, where a single-flavor fermion field is coupled in a chirally invariant way to the real scalar field possessing the non-linear soliton solution in the exterior region. The self-consistent solution to the model equations, which takes into account the effects of fermion-vacuum polarization, is found. Within this solution the renormalized total energy of the bag is studied as a function of the bag geometry under condition of nonvanishing boson field in the interior region. It is shown that by suitable choice of the model parameters the configuration with minimal total bag energy and containing all three phases, exists and is unique.

2 Lagrangian and Equations of Motion

The division of space into phases is performed by means of the system of subsidiary fields $\theta(x)$ according to the method of ref. [3, 22]. The essence of this method may be explained as follows. Let us consider the Lagrangian of the form:

$$\mathcal{L}_0 = \frac{1}{2} (\partial_\mu \phi)^2 - \phi V(\phi) + \frac{1}{2} (\partial_\mu \theta)^2 - g_0^2 W(\theta), \quad (2.1)$$

where the coupling constant $g_0$ of self-interaction of field $\theta$ is assumed to be large enough to neglect the matter fields $\phi$ in the dynamics of $\theta$ to the leading order, and thereafter to
use $\theta$ as background fields for the dynamics of $\phi$'s \cite{21,22}. One can obviously construct the Lagrangian containing as many fields $\theta(x)$ as needed with appropriate self-interaction, which will determine (almost) rectangular division of space into regions corresponding to different phases, while the Lorentz-covariance will be broken only spontaneously, namely on the level of solutions of equations of motion. The latter circumstance allows one to use the framework of covariant group variables \cite{23} in order to restore the covariance. Assuming further that subsidiary fields $\theta(x)$ have already formed the required bag configuration, let us start with the following Lagrangian:

$$
\mathcal{L} = \bar{\psi}i\partial\psi + \frac{1}{2} (\partial_{\mu}\phi)^2 - \theta(x_1 < |x| < x_2) \left( \frac{M}{2} \left[ \bar{\psi}, e^{i\gamma^\mu\phi} \psi \right] - \theta(|x| > x_2) \left( \frac{M_0}{2} \left[ \bar{\psi}, e^{i\gamma^\mu\phi} \psi \right] + V(\phi) \right) \right),
$$

(2.2)

The commutator of fermion fields in terms, describing the chiral fermion-boson coupling, ensures the charge conjugation symmetry of the model.

So, in initial stage we have the theory of two fields, the spinor field $\psi$ and the boson field $\phi$. These fields are free and massless in the region I: $\{|x| < x_1\}$; in the region II: $\{x_1 \leq |x| \leq x_2\}$ the field $\phi$ interacts in a chirally invariant way with fermions resulting in emergence of effective fermion mass $M$; and in the region III: $\{|x| > x_2\}$ the effective fermion mass increases up to $M_0$ and the self-interaction of $\phi$ switches on, providing the appearance of a soliton solution for the boson field in this region. Note that in this model the vacuum pressure term appears to be redundant, since due to the existence of the intermediate phase the Dirac’s sea polarization behaves very specifically and ensures itself the required “inward pressure”. Moreover, in our model there is no special need in “valence” fermions, since it is the boson condensate in the form of the topological soliton, which accounts now for all the bag’s quantum numbers, represented by the topological “baryon number” in this simplest case. The latter point makes it difficult to explain the need for the vacuum pressure term in this model too.

In order to provide the confinement of fermions we assume the mass $M_0$ to be very large \cite{24}, what leads to the dynamical suppressing the fermion field in the exterior region III. At the same time, in the region of asymptotic freedom we have free (decoupled from fermions) massless scalar field and so the nonvanishing density of the boson condensate. This situation does not contradict the general concept of bag models, and can be treated as one of possible versions of the 3-phase model. Another possibility with vanishing scalar field in inner region has been studied in detail in ref. \cite{21}. Let us assume also, that the solution of equations of motion for the boson field is an odd topological soliton. The case of an even soliton with nonvanishing scalar field inside is of no interest, since in this case the trivial solution (scalar field is a constant equal to one of minima of self-interaction potential $V(\phi)$) is the only energetically preferable.

Let us consider now the behavior of fields in detail. According to the general approach accepted in hybrid models we consider the boson field in mean-field approximation, i.e. it is assumed to be a $\gamma$-number field. Neglecting temporarily the explicitly Lorentz-covariant description, we will consider the center-of-mass system of the bag, where $\phi(x)$ is stationary classical field being a background for evolution of fermions. The equations of motion read:

in region I

$$
i\partial\psi = 0, \quad (2.3a)$$
\[ \varphi'' = 0, \quad (2.3b) \]

in region II

\[ \left( i \partial - M e^{i \gamma_5 \varphi} \right) \psi = 0, \quad (2.4a) \]

\[ \varphi'' = i g \frac{M}{2} \langle \left[ \bar{\psi}, \gamma_5 e^{i \gamma_5 \varphi} \psi \right] \rangle, \quad (2.4b) \]

and in region III

\[ \left( i \partial - M_0 e^{i \gamma_5 \varphi} \right) \psi = 0, \quad (2.5a) \]

\[ -\varphi'' + V'(\varphi) = 0, \quad (2.5b) \]

where \( \langle \cdots \rangle \) in eq. (2.4b) stands for the expectation value in a given bag state. To simplify the calculations, we put further \( g = 1 \), because the dependence on it can easily be restored by means of the substitution \( \varphi \to \varphi/g \). Then the spectral problem for fermionic wave functions \( \psi_\omega \) with definite energy \( \omega \) reads

\[ \omega \psi_\omega = -i \alpha \psi'_\omega + \beta e^{i \gamma_5 \varphi} [M \theta(x_1 < |x| < x_2) + M_0 \theta(|x| > x_2)] \psi_\omega, \quad (2.6) \]

To complete the formulation of the spectral problem, the following boundary conditions have to be imposed

\[ \pm i \gamma \psi_\omega(\pm x_2) + e^{i \gamma_5 \varphi(\pm x_2)} \psi_\omega(\pm x_2) = 0, \quad (2.7) \]

provided with the condition of continuity for \( \psi(x) \) on boundaries separating regions I and II. Note that the boundary conditions (2.7) are actually the standard chiral boundary conditions for the hybrid models [9-21]. However, they arise now as a direct consequence of an infinite mass of fermions in region III, rather than from the local surface action, which is not completely correct [21]. In region I equation (2.6) is the equation for free massless fermions

\[ \omega \psi_I = -i \alpha \psi'_I, \quad (2.8) \]

while in the intermediate region II one has

\[ \omega \psi_{II} = -i \alpha \psi'_{II} + \beta M e^{i \gamma_5 \varphi} \psi_{II}. \quad (2.9) \]

Conditions of wavefunction's continuity on the boundary between I and II are given by

\[ \psi_I(\pm x_1) = \psi_{II}(\pm x_1), \quad (2.10) \]

while at points \(|x| = x_2\) the wavefunctions satisfy the conditions (2.7). By this the field \( \varphi \) in eq. (2.9) is not arbitrary but has to be determined self-consistently from eq. (2.4b) with appropriate continuity conditions imposed on the field and its derivatives at points \(|x| = x_{1,2}\).

3 Self-consistent solution of bag equations

The essential feature of this bag configuration is the fact, that the self-consistent equations (2.4) in the closed intermediate region II of finite size \( d = x_2 - x_1 \) possess a simple and physically meaningful solution, which would be unacceptable if these equations were considered in the
infinite space. In order to obtain this solution, we perform in the region II the chiral Skyrme rotation

\[ \psi = \exp(-i\gamma_5 \varphi/2)\chi, \]  

(3.1)

hence equation (2.9) and the boundary condition (2.7) transform into

\[ (\omega - \frac{1}{2} \varphi')\chi_\omega = -i\alpha \chi'_\omega + \beta M \chi_\omega, \]  

(3.2)

\[ \pm i\gamma^1 \chi_\omega(\pm x_2) + \chi_\omega(\pm x_2) = 0 \]  

(3.3)
correspondingly.

It follows from (3.2) and (3.3) that if we assume the linear behavior of scalar field in the region II, namely

\[ \varphi' = \text{const} = 2\lambda, \]  

(3.4)

then equation (3.2) becomes the equation for free massive fermions:

\[ \nu \chi = -i\alpha \chi' + \beta M \chi \]  

(3.5)

with eigenvalues \( \nu = \omega - \lambda \). Therefore, as we expected, the massless (in the region I) fermions acquire the mass \( M \) in the region II due to the coupling to field \( \varphi \), and so the intermediate phase emerges which describes massive quasifree “constituent quarks”. Continuity conditions for the scalar field, which should be an odd function, lead us to the unique solution

\[ \varphi(x) = 2\lambda x. \]  

(3.6)

Equation (3.5) obviously possesses the sign symmetry \( \nu \rightarrow -\nu \), which corresponds to the unitary transformation of fermionic wave function

\[ \chi \rightarrow \tilde{\chi} = i\gamma_1 \chi. \]  

(3.7)

The important point here is, that the axial currents

\[ j_5 = i\bar{\psi} \gamma_5 e^{i\gamma_5 \varphi} \psi = i\chi^+ \gamma_1 \chi \]  

(3.8)

coincide for these sign-symmetric states

\[ j_5 = i\chi^+ \gamma_1 \chi = i\tilde{\chi}^+ \gamma_1 \tilde{\chi} = j_5. \]  

(3.9)

In general, however, one cannot derive the sign symmetry for the fermion spectrum in our case from the sign symmetry \( \nu \leftrightarrow -\nu \) in eq. (3.5), because the latter is true only in the region II, while the spectrum has to be determined from the Dirac equation on the unification of the regions I + II. The straightforward solution of equations (2.8-9) with account of boundary conditions (2.7) and constraint (2.10) gives the following equation for spectrum:

\[ \exp(4i\omega x_1 - 2i\varphi_1) = \frac{1 - e^{-2ikd \frac{M-i(v+k)}{M-i(u-k)}}}{1 - e^{-2ikd \frac{M+i(v-k)}{M+i(v+k)}}} \frac{1 - e^{2ikd \frac{M-i(v-k)}{M-i(u-k)}}}{1 - e^{2ikd \frac{M+i(v+k)}{M+i(v-k)}}}, \]  

(3.10)
where $v^2 = k^2 + M^2$ and $\varphi_1 = \varphi(x_1)$. Analysing equation (3.10) one easily finds that the fermionic spectrum reveals the symmetry $v \leftrightarrow -v$, if

$$4\lambda x_1 - 2\varphi_1 = \pi s ,$$  
(3.11)

where $s$ is integer, since for such values of the derivative of the field $\varphi(x)$ the l.h.s. of equation (3.10) reduces to $(-1)^s \exp(4\nu x_1)$. However, in our case $\varphi_1 = 2\lambda x_1$, hence equation (3.11) leads to the single possibility $s = 0$, while the parameter $\lambda$ remains arbitrary. This result differs crucially from the case considered in [21], where $\varphi_1 = 0$ by virtue of vanishing of the boson condensate in region $I$, and so the set of solutions with different $s \neq 0$ emerges. In the latter case, however, equation (3.11) produces the non-trivial relation between $\lambda$ and $x_1$.

According to eq. (2.4b), in region $II$ $\varphi''(x)$ is determined by the v.e.v. of the C-odd axial current

$$J_5 = \frac{1}{2} \left[ \bar{\psi} i\gamma_5 e^{i\sigma \phi} \psi \right] = \frac{1}{2} \left[ \chi^+ i\gamma_1 \chi \right] ,$$  
(3.12)

with $\chi$ being now the secondary-quantized Dirac field in chiral representation (3.1)

$$\chi(x, t) = \sum_n b_n \chi_n(x)e^{-i\omega_n t} ,$$  
(3.13)

while $\chi_n(x)$ are the normalized solutions of the corresponding Dirac equation, and $b_n, b_n^+$ are fermionic creation-annihilation operators, which obey the canonical commutation relations

$$\{b_n, b_{n'}^+\} = \delta_{nn'} , \quad \{b_n, b_{n'}\} = 0 .$$  
(3.14)

The average over the given bag’s state includes, by definition, the average over the filled sea of negative energy states $\omega_n < 0$ + possible filled valence fermion states with $\omega_n > 0$, which are dropped for the moment because their status is specially discussed below. Finally,

$$\langle J_5 \rangle = \langle J_5 \rangle_{\text{sea}} = \left( \frac{1}{2} \sum_{\omega_n < 0} -\frac{1}{2} \sum_{\omega_n > 0} \right) \chi_n^+ i\gamma_1 \chi_n .$$  
(3.15)

Let us emphasize, that in (3.15) the division of fermions into sea and valence ones is made in correspondence with the sign of their eigen-frequencies $\omega_n$, which differ from sign-symmetric $\nu_n$ by the shift in $\lambda$

$$\omega_n = \nu_n + \lambda ,$$  
(3.16)

and so do not possess the symmetry $\omega \leftrightarrow -\omega$. However, if we suppose additionally that $\nu_n$ and $\lambda$ are such that for all $n$ the signs of $\nu_n$ and $\omega_n$ coincide, i.e. after shifting by $\lambda$ none of $\nu_n$’s changes its sign, then the condition $\omega_n > 0$ in eq. (3.15) will be equivalent to the condition $\nu_n > 0$. Hence

$$\langle J_5 \rangle_{\text{sea}} = \left( \frac{1}{2} \sum_{\nu_n < 0} -\frac{1}{2} \sum_{\nu_n > 0} \right) \chi_n^+ i\gamma_1 \chi_n = 0$$  
(3.17)

by virtue of relation (3.9). In turn, it means that eq. (2.4b) in region $II$ reduces to $\varphi'' = 0$, which is in excellent agreement with the assumption that $\varphi'(x) = \text{const}$ in region $II$. In other words, we obtain the solution of coupled equations (2.4) in region $II$ in the form of the linear function (3.6) for the scalar field, and equation (3.16) for the fermion energy spectrum, where $\nu_n$ is defined from eq. (3.10) after replacing the l.h.s. to $\exp(4i\nu x_1)$. 

7
There are the following keypoints, that make this solution meaningful. The first is the finiteness of intermediate region size $d$, because for infinite region $II$ the solution (3.6) is obviously unacceptable. In our case, however, the size of the intermediate region is always finite by construction, and the boson field $\varphi(x)$ acquires the solitonic behavior in region $III$ due to the self-interaction $V(\varphi)$. Here the following circumstance manifests again: in $(1+1)$D the chiral coupling $\bar{\psi}\gamma^\mu\gamma_{\mu}\psi$ itself cannot cause the solitonic behavior of the scalar field by virtue of the effects of fermion-vacuum polarization only, i.e. without additional self-interaction of bosons [25].

Another important circumstance is the calculation of the Dirac sea average of axial current $J_5$ by means of the symmetry $\nu \leftrightarrow -\nu$. The point is that for such kinds of averages another consistent definition is possible, namely, via the so-called $\eta$-invariant, which is often used for the study of fermion vacuum polarization effects [26]:

$$\langle J_5 \rangle_{\text{sea}} = \lim_{\eta \to 0} \left[ \frac{1}{2} \sum_{\omega_n \leq 0} e^{-|\omega_n|\eta} \chi_n^+ i\gamma_1 \chi_n - \frac{1}{2} \sum_{\omega_n > 0} e^{-\omega_n\eta} \chi_n^+ i\gamma_1 \chi_n \right]. \quad (3.18)$$

Since the spectrum of $\omega_n$ is not symmetric, the expression (3.18) doesn’t vanish, in contrast to the average (3.17). The expression (3.18) with connection between $\omega_n$ and $\nu_n$ taken into account, transforms into

$$\lim_{\eta \to 0} \left[ \sinh \eta \left( \sum_{\nu_n > 0} e^{-\nu_n\eta} \chi_n^+ i\gamma_1 \chi_n \right) \right], \quad (3.19)$$

and for $\eta \to 0$ the sum in (3.19) diverges as $1/\eta$, so that $\langle J_5 \rangle$ appears to be proportional to $\lambda$ and doesn’t vanish unless $\lambda \neq 0$. The same makes the r.h.s. of eq. (2.4b), and so the linear functions will not be the solution of this equation in region $II$.

Moreover, this is true for other fermionic averages too, in particular for the $\nu$ c.v. of the fermion charge. In our approach, by virtue of discreteness of the fermionic spectrum, the vacuum value of the charge is zero, in analogy to the case of the axial current. Namely, if the $C$-odd expression for the charge

$$Q = \frac{1}{2} \int dx \left[ \bar{\psi}^+ \psi^+ \right], \quad (3.20)$$

is used, then under the same conditions of sign-correspondence between $\omega_n$ and $\nu_n$ for each $n$, one obtains for the average of $Q$ the following

$$\langle Q \rangle = \langle Q \rangle_{\text{sea}} = \frac{1}{2} \sum_{\omega_n < 0} - \frac{1}{2} \sum_{\omega_n > 0} = \frac{1}{2} \sum_{\nu_n < 0} - \frac{1}{2} \sum_{\nu_n > 0} = 0. \quad (3.21)$$

However, if $\langle Q \rangle_{\text{sea}}$ is defined via $\eta$-invariant, then one obtains

$$\langle Q \rangle_{\text{sea}} = \lim_{\eta \to 0} \left[ \frac{1}{2} \sum_{\omega_n < 0} e^{-|\omega_n|\eta} - \frac{1}{2} \sum_{\omega_n > 0} e^{-\omega_n\eta} \right] \neq 0 \quad (3.22)$$

for $\lambda \neq 0$ by virtue of absence of the symmetry $\omega \leftrightarrow -\omega$. Nethertheless, there are serious reasons to consider the relations (3.17) and (3.21) as the most adequate way of calculating the sea average in our problem. First, $\eta$-invariant is actually the measure of the Hamiltonian asymmetry, rather than the fermion charge of the ground state. Second, the other regularization
schemes besides of thermal regularization are possible for divergent sums such as $\langle J_3 \rangle$ and $\langle Q \rangle$. Also, there is no warranty that the regularized expressions (3.18) and (3.22) would produce the correct results for $\eta \rightarrow 0$. It is easy to give an example, when the dependence on a parameter in the sum or integral is not continuous, in particular, $F(k) = \int_{0}^{\infty} dx \sin kx/x = \pi/2$ for any $k > 0$, but $F(0) = 0$, i.e. in this case the limit of the integral for $k \rightarrow 0$ and it’s exact value at $k = 0$ are different.

Let us consider further an adiabatic process of changing the gradient of the boson field $\lambda$ in region II, starting from $\lambda = 0$. In the initial moment $\omega_n = \nu_n$, so any C-odd fermionic sea average obviously disappears: $\langle \ldots \rangle_{\text{sea}} \equiv 0$ for any reasonable regularization scheme. Now let us take into account that in our case the fermionic spectrum is discrete and depends on $\lambda$ continuously, so for $\lambda$ small enough all the $\omega_n$’s keep their signs. Then from general grounds $\langle \ldots \rangle_{\text{sea}}$ should vanish still, although the spectrum $\omega_n$ is already non-symmetric and so the $\eta$-invariant is nonzero. Therefore it is reasonable to assume that in this case the thermal regularization can yield physically inconsistent results for C-odd quantities such as the axial current and the fermion charge of the vacuum. It is worth-while to note the essential difference of this situation from the case of infinite space, when the spectrum is continuous. In the latter case, the density of states changes for any small change of $\lambda$, and so all the averages change too, what gives rise to the effect of induced fermion numbers [26].

Finally, it is possible to check the self-consistency of solution (3.6) numerically. In this case the problem is solved on a lattice, and so the number of degrees of freedom, as well as the number of fermion levels, is finite, so one doesn’t need any regularization at all. The results of these calculations show that the linear function (3.6) and the vanishing averages are the only self-consistent solution to this problem.

The interpretation of this result is as following. In our case the boson field has no discontinuities anywhere (it doesn’t vanish in region I, too), and is topologically equivalent to the odd soliton which would take place in the absence of fermions due to the self-interaction $V(\varphi)$ only. Therefore, the topological number does not depend on the existence and sizes of the spatial regions with fermions (I and II). On the other hand, the baryon charge of the hybrid bag is, by definition, the sum of the topological charge of the boson soliton and the fermion charge of the bag interior. The latter one is zero in our case, hence the baryon charge is determined completely by the topological charge of the boson field, and so does not depend on the sizes of regions I and II, in correspondence with the general ideology of hybrid models. Therefore, in our approach the “hadron” is a composite particle, which consists of the boson soliton, to which the fermion bag is coupled through the interaction in the intermediate region II. Note also, that although quantum numbers of such a composite particle are completely determined by the soliton, it doesn’t mean that the filled fermion levels with positive energy should not exist at all. This could take place for small enough values of the parameter $\lambda$ only. If $\lambda$ increases, the negative levels $\omega_n = -|\nu_n| + \lambda$ will unavoidably move into the positive part of the spectrum. The change of sign of each such level will decrease $\langle Q \rangle_{\text{sea}}$ by one unit of charge, but if we fill the emerging positive level with the valence fermion, then the sum $Q_{\text{val}} + Q_{\text{sea}}$ remains unchanged. Analogously, the total axial current will be equal to $J_{\text{val}} + J_{\text{sea}}$ and will not change too, which ensures the vanishing r.h.s. of (2.4b) and so preserves the status of linear function (3.6) as the
self-consistent solution of the field equations. In other words, by definition the ground state of the bag is a state in which all levels with $\nu_n < 0$ are filled (the inequality is strong, because there are no levels with $\nu = 0$ in the considered case). The existence or absence of valence fermions in such a construction of the ground state of the bag depends actually on the relation between $\lambda$ and $|\nu_n|_{\text{min}}$, and so appears to be a dynamical quantity like the other parameters of the bag (the size and mass), which are determined from the total energy minimization procedure.

4 The Total Energy of the Bag

On the unification of $I + II$ the boson soliton takes the form of the linear function (3.6). This function (after rescaling $\varphi \to \varphi / g$) is sewn together with the soliton solution of eq. (2.5b) in the exterior region by means of the continuity conditions for the field and its derivative. To preserve the generality of consideration, in region $III$ we will use the asymptotic expansion of the soliton solution of eq. (2.5b) for large $|x|$, namely

$$\varphi_{\text{sol}}(x) = \frac{\pi}{g} \left( 1 - Ae^{-mx} \right), \quad x > x_2,$$

with $m$ being the meson mass in the exterior region of the bag, while for $x < -x_2$ $\varphi_{\text{sol}}(x)$ is determined by oddness. The factor $\pi / g$ means that we deal actually with a phase soliton with topological charge being multiple of $2\pi / g$, since it is the period of the initial chiral interaction $\bar{\psi} \exp(i \gamma_5 g \varphi) \psi$. The constant $A$ is determined from the continuity condition for boson field at points $x = \pm x_2$. Note that in the exterior region there is no chiral invariance due to the phenomenology of strong interactions on the one hand, and on the other due to the specific features of $(1+1)$D scalar models, which make the presence of the meson mass the necessary condition for the required soliton profile to be formed.

The continuity conditions at $x = \pm x_2$ give

$$2\lambda x_2 = \pi \left( 1 - Ae^{-mx_2} \right), \quad (4.2a)$$

$$2\lambda = \pi m A e^{-mx_2}. \quad (4.2b)$$

Then one finds the relation between the parameter $\lambda$ and the bag’s size $x_2$:

$$2\lambda = \pi - \frac{m}{mx_2 + 1}, \quad (4.3)$$

whence the total energy of boson soliton can be represented as

$$E_\varphi = \frac{\pi^2}{g^2} \frac{m}{mx_2 + 1}. \quad (4.4)$$

The total energy of the bag consists of $E_\varphi$ and of the fermionic contribution $E_\psi$

$$E_{\text{bag}} = E_\varphi + E_\psi. \quad (4.5)$$

It is obvious from (4.4), that the energy of the boson field is a smooth function, which decreases for $x_2 \to \infty$, producing no vacuum pressure in spite of the fact, that the gradient of $\varphi$ in the domain $I + II$ yields the constant positive contribution to the energy density $\frac{1}{2} \varphi'^2 = 2\lambda^2$, which
could be identified with the vacuum pressure $B$ in the standard HCM. Actually, it is an artifact of one spatial dimension in our problem: when the bag’s size increases, the gradient of $\varphi$ in $I + II$ will always decrease in any number of space dimensions by virtue of (4.3), while the volume of domain $I + II$ in 1(space)D increases only linearly and so cannot compensate the decreasing of $\lambda$, which takes place in 2- and 3-(space)D. So in (1+1)D the non-trivial dependence of the total energy $E_{bag}$ of the bag on the model parameters could originate only from the fermion contribution $E_\psi$, which is generally the sum of the Dirac’s sea of filled negative energy states and of positive energy valence fermions

$$E_\psi = E_{val} + E_{sea} .$$

For the ground state of the bag described above, the sum (4.6) can be reduced to the single universal expression by taking into account, that the charge conjugation symmetry dictates the following definition of the Dirac’s sea energy [25,27]

$$E_{sea} = \frac{1}{2} \sum_{\omega_n < 0} \omega_n - \frac{1}{2} \sum_{\omega_n > 0} \omega_n .$$

Assuming, that the sign preserves under the transition from $\omega_n$ to $\nu_n$ for all $n$, and so there are no valence fermions in the ground state of the bag (in order to provide the vanishing v.e.v. for the charge and axial current), one finds from (4.7)

$$E_\psi = E_{sea} = \frac{1}{2} \sum_{\nu_n > 0} (-\nu_n + \lambda) - \frac{1}{2} \sum_{\nu_n > 0} (\nu_n + \lambda) = - \sum_{\nu_n > 0} \nu_n .$$

If the parameter $\lambda$ appears to be large enough, so that the initially negative level $\omega_n = -|\nu_n| + \lambda$ changes its sign and turns into the filled valence state, it is convenient to calculate $E_\psi$ in two steps. First, we consider the contribution from all states with $|\nu_n| > \lambda$ to $E_{sea}$, which in analogy to (4.8) reads

$$E_{sea}' = - \sum_{\nu_n > \lambda} \nu_n .$$

For this expression the energy of the valence fermions $E_{val} = -|\nu_n| + \lambda$ and the contribution of the positive levels with $\omega_n = \pm |\nu_n| + \lambda$ should be added, which yields

$$E_\psi = -|\nu_n| + \lambda - \frac{1}{2} [(-|\nu_n| + \lambda) + (|\nu_n| + \lambda)] + E_{sea}' = - \sum_{\nu_n > 0} \nu_n ,$$

i.e. the same eq. (4.8) as we have got for the energy of fermions without filled valence states.

It is convenient to introduce a set of new parameters, in terms of which the total energy of the bag will be expressed in the most appropriate form. First of all, we introduce the dimensionless quantities

$$\alpha = 2Mx_1 , \quad \beta = 2Md , \quad \rho = 2Mx_2 ,$$

and consider eq. (3.10) in their terms. This equation has two branches of roots. The first one corresponds to real $k$ and in terms of parameters $\alpha$ and $\beta$ is determined from

$$\tan \left( \alpha \sqrt{1 + x^2} \right) = \frac{x}{\sqrt{x^2 + 1}} \frac{x \cos \beta x + \sin \beta x}{1 - \cos \beta x + x \sin \beta x} ,$$

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where the unknown quantity is the dimensionless \( x \) defined through 
\[ k = Mx, \quad \nu = M\sqrt{1 + x^2}, \]

The real roots \( x_n \) belong to the half-axis \( 0 \leq x_n < \infty \), since the fermionic wave functions are actually the standing waves in a finite spatial box with degeneracy in the sign of \( k \), while the corresponding frequencies \( \nu_n \) lie in the interval \( M \leq \nu_n < \infty \).

The second branch corresponds to imaginary \( k = iMx, \ \nu = M\sqrt{1 - x^2} \), \( 0 \leq x \leq 1 \), and can be derived from (4.12) by means of the analytical continuation

\[
\tan \left( \alpha \sqrt{1 - x^2} \right) = \frac{x \cosh \beta x + \sinh \beta x}{\sqrt{1 - x^2} \cosh \beta x + x \sinh \beta x - 1}.
\]  

For this branch \( 0 < \nu_n < M \).

Therefore, \( \nu_n \) and \( E_\psi \) appear to be the functions of two independent dimensionless parameters \( \alpha \) and \( \beta \), the sum of which defines the dimensionless size of the confinement domain \( \rho \)

\[ \alpha + \beta = \rho. \]  

Proceeding further, it is convenient to extract the mass of a “constituent quark” \( M \) from the sea energy and fermion frequencies as a dimensional factor:

\[ \varepsilon_n = \nu_n / M = \sqrt{1 + x_n^2}, \]  

hence \( E_\psi = -M \sum_n \varepsilon_n \). The dimensionless relation of the two model parameters is

\[ \mu = m/2M, \]  

and the dimensionless total energy is \( E_{\text{bag}} = E_{\text{bag}}/M \). The latter is given by

\[ E_{\text{bag}} = E_\psi(\alpha, \beta) + \frac{\pi^2}{g^2 \mu \rho + 1}. \]  

Let us note, that in this case the independent model parameters are \( \alpha, \beta \), which define the sizes of the inner bag’s regions, while \( \rho \) is determined from (4.14). At this point the present version of 3-phase model differs from the one considered in [21], where \( \alpha, \beta \) are defined unambiguously by \( \mu \) and \( \rho \). So the total bag’s energy depends ultimately on the following dimensionless parameters: \( \mu, g, \alpha, \beta \), where the parameter \( \mu \) is fixed by the ratio of the masses \( m \) and \( M \), while the optimal values of \( \alpha, \beta \) for the ground state should be found from the condition of minimum of the total energy for given \( \mu \).

The dimensionless fermion sea energy \( E_\psi \), which obviously diverges in the upper limit, should be regularized, so a consistent renormalization procedure is required. First, let us consider the asymptotics of the roots of eq. (4.12) in the UV-domain, when \( x_n \gg 1 \). It is convenient to rewrite eq. (4.12) as

\[
\sin \alpha \sqrt{1 + x^2} = \frac{1}{2} (\sqrt{1 + x^2} + x) \sin \left( \alpha \sqrt{1 + x^2} + \beta x + \gamma \right) + \frac{1}{2} (\sqrt{1 + x^2} - x) \sin \left( \alpha \sqrt{1 + x^2} - \beta x - \gamma \right),
\]

where \( \gamma = \arctan x \). One can show that

\[
x_n(\alpha, \beta) = \frac{\pi/2 + \pi n}{\rho} + \frac{(-1)^{n+1} \sin [(\pi/2 + \pi n)\alpha/\rho] + 1 - \alpha/2}{\pi/2 + \pi n} + O(1/n^2).
\]
In the expression (4.19) the first term leads to the quadratic and linear divergences in \( \Sigma \varepsilon_n \), and the second one contains the logarithmic divergence, while the term with the sine does not yield any divergence at all. Therefore, the UV-regularization consists in the compensation of the first term and the divergent part of the second term in the asymptotic expression (4.19). As a first step we use the obvious fact, that only the difference between two energies is physically meaningful, rather than the energy itself. It could seem, that the most natural way is to choose the energy of the free fermions sea in the similar “volume” \( \rho \) as the reference point for \( \varepsilon_\phi \). However, it has been shown in [21], that this subtraction fails in our case because this energy appears to be infinitely larger than any configuration with \( \beta \neq 0 \), even after any possible counterterms are added. Such an infinite energy barrier between the regularized \( \varepsilon_\phi(\alpha, \beta) \) and the free fermion sea corresponds perfectly well to the intuitive feeling that the free fermions can hardly be a reasonable first approximation to the confinement problem.

As a result, in this case there is no unique prescription for the choice of a subtraction point in the renormalization of \( \varepsilon_\phi(\alpha, \beta) \), which is actually the common feature for the majority of bag models [10,12,28]. In the “classical” renormalization procedure, the uncertainty in the choice of a subtraction point is cancelled by fixing the physical values for the corresponding number of parameters. For obvious reasons, we won’t do that in our “toy” (1+1)D model, but will instead consider the most straightforward approach to the compensation of divergences in the sum (4.10), which preserves the continuous dependence of the result of subtraction on the model parameters. This approach is based on the subtraction from \( \Sigma \varepsilon_n \) the other sum with the same summation index \( n \), the common term of which coincides exactly with the divergent part of asymptotics (4.19), resulting in the finite quantity

\[
\tilde{\varepsilon}_\phi = -\sum_n \varepsilon_n - \left(\frac{\pi/2 + \pi n}{\rho} + \frac{1 + \beta/2}{\pi/2 + \pi n}\right).
\]  

This method requires no counterterms because all the divergencies are already cancelled by the subtracted sum. Of course, to some extent the physical meaning of such procedure is lost. However, it should be emphasized, that it is only the (1+1)D case, when the theory with coupling \( \mathcal{L}_1 = G \bar{\psi}(\sigma + i\gamma_5 \pi) \psi \) is (super)renormalizable and any counterterm has explicit physical meaning. For higher space dimensions this is already not true, and so the procedure of compensation of divergences in the energy based on (4.20) should not be considered as having no motivation. For a more detailed discussion on the extraction of finite part from the divergent Dirac’s sea energy see refs. [28-31].

Proceeding further, let us turn to the study of the total bag’s energy

\[
\varepsilon_{bag} = \tilde{\varepsilon}_\phi(\alpha, \beta) + \frac{\pi^2}{g^2} \frac{2\mu}{\mu \rho + 1}
\]  

as a function of parameters \( \alpha, \beta \). The analysis of the contribution of convergent logarithmic part from the sine-term in the asymptotic expression (4.19) to \( \tilde{\varepsilon}_\phi \) yields the first feature of \( \varepsilon_{bag} \). Let us transform this contribution to the form

\[
\left(\tilde{\varepsilon}_\phi\right)_{bag}(\alpha, \beta) = \frac{1}{\pi} \sum_{n \geq 1} (-1)^n \sin(\pi \alpha/\rho(n + 1/2)) / n + 1/2
\]  

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and then use the well-known relation

\[ \sum_{n=0}^{\infty} \frac{(-1)^n \sin[z(n+1/2)]}{n+1/2} = \ln \tan(\pi/4 + z/4) \, . \]  

(4.23)

It is easy to see, that the sums (4.22) and (4.23) have the similar common term, and the sum (4.23) diverges as \(-\ln(\pi - z)\) when \(z \to \pi\). Hence, for \(\pi \alpha / \rho \to \pi\), which means either \(\beta \to 0\), or \(\alpha \to \infty\) for finite \(\beta\), the sum (4.22) will show the similar behavior, namely

\[ (\tilde{\mathcal{E}}_\psi)_{\text{bag}}(\alpha, \beta) \to \frac{-1}{\pi} \ln(\beta/\alpha) \, , \quad \beta/\alpha \to 0. \]  

(4.24)

Therefore, both the regularized fermion energy (4.20) and the total bag’s energy possess the logarithmic singularity for \(\beta \to 0\), and at the same time the logarithmic increase for \(\alpha \to \infty\) and finite \(\beta\). Note that although the possibility of a smooth transition to a 2-phase configuration, when \(d \to 0\), exists formally for the initial Lagrangean (2.2), actually it disappears due to the singularity for \(\beta \to 0\). In other words, in such 3-phase model the radial extent of the intermediate phase can be sufficiently small, but not zero, what corresponds to the general physical treatment of the structure of many-phase systems.

\(\mathcal{E}_{\text{bag}}\) will also grow for \(\beta \to \infty\) and finite \(\alpha\). Because the increase of \(\beta\) yields the increase of \(\rho\), in this case \(\pi \alpha / \rho \to 0\), hence the logarithmic term (4.22) becomes negligibly small, and the main contribution is provided by the next terms in expansion in \(1/n\). However, it is technically more convenient to use the fact that for \(\rho \to \infty\) the fermionic spectrum becomes quasicontinuous with the exception of a small vicinity of Fermi surface with zero energy, which allows to transform the sums over \(x_n\) into integrals over \(dx\). In particular, the analysis of distribution of the roots of eq. (4.12) shows, that for this limit \(\sum_n \bar{\varepsilon}_n\) is approximated by the following (divergent) integral

\[ \sum_n \bar{\varepsilon}_n \to \frac{1}{\pi} \int dx \sqrt{1 + x^2} \times \]  

\[ \times \left[ \beta + \frac{1}{1 + x^2} + \frac{x^2}{\alpha x^2 + \sin^2(\alpha \sqrt{1 + x^2})} - \frac{\sin(\alpha \sqrt{1 + x^2}) \cos(\alpha \sqrt{1 + x^2})}{\sqrt{1 + x^2} \left( x^2 + \sin^2(\alpha \sqrt{1 + x^2}) \right)} \right] . \]  

(4.25)

For the subtracted sum in (4.20) one can easily find

\[ \sum_n \left( \frac{n/2 + \pi n}{\rho} + \frac{1 + \beta/2}{\pi/2 + \pi n} \right) \to \frac{\rho}{\pi} \int dx \left( x + \frac{1 + \beta/2}{\rho x} \right) . \]  

(4.26)

The integrals (4.25) and (4.26) have the same divergent part

\[ \frac{1}{\pi} \int dx \left( px + 1/x + \beta/2x \right) , \]

so their difference yields a converging integral, in agreement with the subtraction procedure. The leading term of the intergrand in this difference, taken with the (correct) inverse sign, is \(\beta/8\pi x^3\). This finally leads to the emergence of the positive, proportional to \(\beta\), contribution to \(\tilde{\mathcal{E}}_\psi\), and correspondingly to \(\mathcal{E}_{\text{bag}}\). So for \(\alpha \to 0\) and finite \(\beta\) there is no singularity in \(\tilde{\mathcal{E}}_\psi\) and \(\mathcal{E}_{\text{bag}}\), which is a direct consequence of the analysis performed in ref.[21], where it has been shown that the renormalized \(\tilde{\mathcal{E}}_\psi\)'s for \(\alpha = 0\) and finite \(\alpha \neq 0\) differ always by a finite value. Therefore
it is the soliton energy $E_\phi$ and so the current values of parameters $\mu$ and $g$, which play the main role in this domain. More exactly, for $\mu$ being not too small, the dependence of $E_\phi$ on $g$ will be such that for sufficiently small $g$ the sharp enough increase of $E_{bag}$ takes place for $\alpha \to 0$ and finite $\beta$, providing the existence of a well-defined minimum. On the contrary, the sufficient increase of $g$ (or $\mu \to 0$) could make $E_\phi$ to be negligibly small (and almost constant) for any bag’s size, and then the minimum in the total energy disappears.

The numerical calculation confirms completely such qualitative predictions for the behavior of $E_\phi$ and $E_{bag}$. However, due to the absence of singularity in $E_{bag}$ for $\alpha = 0$ the question about the existence of the minimum of the total bag’s energy considered as a function of $\alpha, \beta$, and therefore the existence of a stable ground state of the bag itself for given $\mu$ and $g$ can be answered only numerically. Such a calculation has been performed for $\mu = 0.25$, which corresponds approximately to the ratio $\frac{m_\pi}{2m_Q}$, where the constituent quark mass is assumed to be equal to 300 MeV, and for $g = 1$. The qualitative behavior of $E_{bag}$ as a function of $\alpha, \beta$ is shown in Fig.1, from which one can easily see, that the total energy reveals the unique minimum for non-zero values of $\alpha, \beta$. In Fig. 2 the equal energy curves are shown, which allows to observe this minimum of energy more explicitly. In the case under consideration the values of $\alpha$ and $\beta$ in the minimum differ by two orders, i.e. the size of the intermediate region appears to be essentially smaller than that of the inner one. Therefore to some extent it can be treated as a smeared boundary between the phase of asymptotic freedom and the purely colorless phase. In Fig. 3 the scheme of fermionic levels in the vicinity of zero energy is presented for this configuration, which shows that in this case the ground state contains one filled valence level with positive energy.

5 Conclusion

The aim of the present study is the construction of a consistent model of a hybrid chiral bag, in which the exact equivalence between fermionic and bosonic (meson) phase is not assumed. Our results show that such a model can be actually formulated in a quite consistent manner, and to a certain extent could be a more effective way of description of low-energy hadron physics compared to the traditional HCM.

First, let us note, that the initial formulation of the model is a local field theory, and in spite of the variety of classical solutions one needs to deal with, the covariance is broken only spontaneously, and so can be restored by means of the methods of refs. [23] using the covariant group center-of-mass variables for a localized quantum-field system. Besides this, the positive points of this approach are: the more correct derivation of chiral boundary conditions, by which any term in the initial Lagrangian has exact physical meaning; the presence of an intermediate phase describing quasilocal massive “constituent” quarks; physically acceptable behavior of the total energy of the bag as a function of it’s geometry. Moreover, in this model the condition of fermion confinement, incorporated into it from the very beginning, shows up more explicitly. It manifests, in particular, in the fact that there is no need in the term with vacuum pressure $B$, which in the standard approach is inserted into the model by means of some extra assumptions, since in our case the Dirac’s sea polarization itself produces the infinite increase of energy at large distances.
The important question of the choice of method of calculation of the Dirac's sea averages for fermion bags should also be emphasized. The method we used is based on the discreteness of the fermion energy spectrum, which by means of quite obvious considerations leads to a very simple solution of the self-consistent equations of the bag in the intermediate region. Note, however, that in spite of the arguments in favor of such a method of calculations, we cannot completely reject alternative methods like thermal regularization. The question of which one is more adequate to the physics of the problem should be answered only by means of a detailed study of realistic models.

Compared to ref.[21], the specific feature of the considered model is that the condition of nonvanishing density of the boson condensate in the interior region affects crucially the amount of possible bag configurations, which provide the local minima of the energy. In ref. [21] the boson field vanishes in the inner region, which leads to an infinite set of such configurations with the same topology and infinitely increasing size and energy with the main difference between them being the value of gradient $\lambda$ of the boson field in the intermediate region. In the present case, there exists no more than one such configuration, and the excited bag states of greater energy can be obtained only by adding a number of valence fermions. So within the framework of 3-phase modification of a hybrid model there are possible quite different versions of description of composite particles like hadrons and their excitations.

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Fig. 1: The total energy of the bag $E_{bag}(\alpha, \beta)$ as a function of dimensionless bag parameters $\alpha, \beta$ for $\mu = 0.25$ and $g = 1$. 
Fig. 2: The curves of equal energy for $\mathcal{E}_{\text{bag}}(\alpha, \beta)$ for $\mu = 0.25$. The minimum is clearly seen for $\alpha \simeq 40$ and $\beta \simeq 0.34$. The values of energy in the minimum are $\mathcal{E}_{\text{bag}} = 2.41$, $\mathcal{E}_\psi = 1.96$ and $\mathcal{E}_\varphi = 0.45$. 
Fig. 3: The scheme of fermionic levels for the ground state bag configuration.