Quantum states of indefinite spins: From baryons to massive gravitino

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Abstract

One of the long standing problems in particle physics is the covariant description of higher spin states. The standard formalism is based upon totally symmetric Lorentz invariant tensors of rank-K with Dirac spinor components, $\psi_{\mu_1...\mu_K}$, which satisfy the Dirac equation for each space time index. In addition, one requires $p^{\mu_1} \psi_{\mu_1...\mu_K} = 0$, and $\gamma^{\mu_1} \psi_{\mu_1...\mu_K} = 0$. The solution obtained this way (so called Rarita-Schwinger framework) describes the “has–been” spin-($K + \frac{1}{2}$) particles in the rest frame, and particles of uncertain (fuzzy) spin elsewhere. Problems occur when $\psi_{\mu_1...\mu_K}$ constrained this way are placed within an electromagnetic field. In this case, the energy of the has–been spin-($K + \frac{1}{2}$) state becomes imaginary (Velo-Zwanziger problem). Here I consider two possible avenues for avoiding the above problems. First I make the case that specifically for baryon excitations there seems to be no urgency so far for a formalism that describes isolated higher-spin states as all the observed nucleon and $\Delta(1232)$ excitations (up to $\Delta(1600)$) are exhausted by unconstrained $\psi$, $\psi_{\mu_1...\mu_3}$, and $\psi_{\mu_1...\mu_5}$, which originate from rotational and vibrational excitations of an underlying quark–diquark string. Second, I show that the $\gamma^{\mu_1} \psi_{\mu_1...\mu_K}(p) = 0$ constraint is a short-hand of: $-\frac{1}{2K+1} (\frac{1}{m^2} W^2 + (K^2 - \frac{1}{4})) \psi_{\mu_1...\mu_K} = \psi_{\mu_1...\mu_K}$, the covariant definition of the unique invariant subspace of the squared Pauli-Lubanski vector, $W^2$, that is a parity singlet and of highest spin-($K + \frac{1}{2}$) at rest.

I consider the simplest case of $K = 1$ and suggest to work in the sixteen dimensional vector space, $\Psi$, of the direct product of the four-vector, $A_\mu$, with the Dirac spinor, $\psi$, i.e. $\Psi = A \otimes \psi$, rather than keeping space-time and spinor indices separated and show that the “has–been” spin-3/2 piece is uniquely described by means of the second order equation $(-\frac{1}{3} \left( \frac{1}{m^2} W^2 + \frac{3}{4} \right) - 1) \Psi = 0$ without invoking any further supplementary conditions. In gauging the latter equation minimally and, in calculating the determinant, one obtains the energy-momentum dispersion relation. The latter turned out to be free from pathologies, thus avoiding the classical Velo-Zwanziger problem.

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I. PARTICLE STATES–REVISITED

The definition of particle states is at the very heart of contemporary quantum field theories. It is supposed to take its origin from frame-independent Casimir invariants of the Poincaré group as was first noticed by Wigner in his work of late thirties.\(^1\) Quantum states of free particles in a Poincaré covariant framework have been considered to transform for different inertial observers as

$$\Psi'(x) = \exp\left[ i (\epsilon^\mu P_\mu - \theta^{\mu\nu} M_{\mu\nu}) \right] \Psi(x).$$

(1)

Here, \(P_\mu\) and the totally antisymmetric tensor \(M_{\mu\nu}\) with \(\mu (\nu) = 0, 1, 2, 3\), are the generators of the Poincaré group which satisfy the well known Poincaré algebra:

$$[M_{\mu\nu}, M_{\rho\sigma}] = i (g_{\nu\rho} M_{\mu\sigma} - g_{\mu\rho} M_{\nu\sigma} + g_{\mu\sigma} M_{\nu\rho} - g_{\nu\sigma} M_{\mu\rho}),$$

(2)

$$[P_\mu, M_{\rho\sigma}] = i (g_{\mu\rho} P_\sigma - g_{\mu\sigma} P_\rho), \quad [P_\mu, P_\nu] = 0, \quad M_{\mu\nu} = S_{\mu\nu} + i x_\mu \partial_\nu - i x_\nu \partial_\mu,$$

(3)

where \(\epsilon^\mu\) and \(\theta^{\mu\nu}\) are continuous parameters, \(g_{\mu\nu}\) = diag(1, \(-1\), \(-1\), \(-1\)) is the metric tensor, while \(S_{\mu\nu}\) is the purely intrinsic part of \(M_{\mu\nu}\) which later on will be associated with spin. In the standard convention, \(P_\mu\) are the generators of the translation group, \(T_{1,3}\), in 1+3 time-space dimensions, while \(M_{\mu\nu}\) are related to the generators of boosts \((K_x, K_y, K_z)\) and rotations \((J_x, J_y, J_z)\) of the Lorentz group, \(SO(1,3)\), via

$$M_{01} = K_x, \quad M_{02} = K_y, \quad M_{03} = K_z,$$

$$M_{12} = J_z, \quad M_{13} = -J_y, \quad M_{23} = J_x.$$  

(4)

The Poincaré algebra has two invariant (Casimir) operators. These are the squared four-momentum, \(P^2\), on the one side, and the squared Pauli-Lubanski vector, \(W^2\), on the other side. The Pauli–Lubanski vector is defined as\(^2\)

$$W_\mu = -\frac{1}{2} \epsilon_{\mu\rho\sigma\tau} S^{\rho\sigma} P^\tau,$$

(5)

where \(\epsilon_{0123} = 1\). In terms of boost- and rotation generators, the Pauli-Lubanski vector for states characterized by the three momentum \(p\), is expressed as

$$W_\mu = \left( -S \cdot \mathbf{p}, -S E + (\mathbf{K} \times \mathbf{p}) \right),$$

(6)

Its squared (in covariant form) is calculated to be

$$W^2 = -\frac{1}{2} S_{\mu\nu} S^{\mu\nu} P^2 + S_{\mu\nu} P^\mu S^{\sigma\nu} P^\sigma.$$  

(7)
Note that for pure spin \((s, 0) \oplus (0, s)\) spaces it can be shown that \(W^2\) simplifies to

\[
W^2 = -\frac{1}{4} S^{\mu\nu} S_{\mu\nu} P^2.
\]  

(8)

In terms of Poincaré group invariants, the particle state definition accepted so far in the literature prescribes that particles must have definite masses, \(m\), and spins, \(s\), according to

\[
P^2 \Psi(x) = m^2 \Psi(x),
\]

\[
W^2 \Psi(x) = -s(s + 1) P^2 \Psi(x).
\]  

(9)

The latter equation imposes an essential restriction onto the representation spaces of the Poincaré group. In one of the possibilities, one can follow Wigner and take the view that particle states transform as classical unitary, and therefore infinite-dimensional, representations. However, according to Weinberg,\(^3\) also quantized non-unitary finite-dimensional representations of the Lorentz group of the type \((s, 0) \oplus (0, s)\) can be given particle interpretation because of unitarity of the corresponding particle creation and annihilation operators. The persisting particle definitions deny, therefore, existence of states without a definite mass or a definite spin.

The first quantum state to step out of the line was the physical neutrino of a particular flavor, \(f\). It is well known that such a neutrino ceases to have a well defined mass, a peculiarity that is manifest through the phenomenon of neutrino oscillations,\(^4\) a process of crucial importance for the baryogeneses in the Universe,

\[
|\nu_f\rangle = \sum_{i=1}^{3} U_{fi} |m_i\rangle.
\]  

(10)

Here, the mass eigenstates, \(|m_i\rangle\), although possible, serve solely as basis states as they are of indefinite flavor. The unitary \(3 \times 3\) matrix \(U_{fi}\) is the well known mixing matrix. At same time, in baryonic spectra, one sees resonances of different spins and parities coming together to narrow mass bands (see Figs. 1 and 2). In a recent analysis in Ref. [6] I argued that such crops fit exactly into finite dimensional non-unitary representations of the Lorentz group of the type \((\frac{K}{2}, \frac{K}{2}) \otimes [ (\frac{1}{2}, 0) \oplus (0, \frac{1}{2}) ]\) that are described (in the momentum space of interest) by means of a totally symmetric \(K\)-rank Lorentz tensor with Dirac spinor components, \(\psi_{\mu_1...\mu_K}(p)\). The quantum number \(K\) is associated with a Casimir invariant of the Lorentz– (rather than the Poincaré) algebra, which is determined as (see Ref. [7])
\[ C(\text{su}(2)_L \oplus \text{su}(2)_R) = \frac{1}{4}(S^2 - K^2). \] Its action upon \( \psi_{\mu_1...\mu_K}(p) \) amounts to,

\[ C(\text{su}_L(2) \oplus \text{su}_R(2)) \psi_{\mu_1...\mu_K}(p) = \frac{K}{2} \left( \frac{K}{2} + 1 \right) \psi_{\mu_1...\mu_K}(p). \] (11)

At rest, such states describe a family of mass degenerate spin-parity states. For concreteness, I here consider the case of \( K = 3 \) and unnatural parities (to be of interest in the following) where one finds the following spin- and parity sequence

\[
\psi_{\mu_1...\mu_3}(p) \xrightarrow{\text{rest}} \begin{array}{cccccccc}
1^- & 1^+ & 3^+ & 3^- & 5^- & 5^+ & 7^+ \\
\frac{1}{2} & \frac{1}{2} & \frac{3}{2} & \frac{3}{2} & \frac{5}{2} & \frac{5}{2} & \frac{7}{2}
\end{array}.
\] (12)

It is in particular interesting to study the action of the squared Pauli-Lubanski vector onto such spaces. In what follows I will show that \( W^2 \) splits \( \psi_{\mu_1...\mu_K}(p) \) into \( (K + 1) \) invariant subspaces.

To begin with I first consider the case of the rest frame where \( W^2 \) is particularly simple and equals \( -S^2 m^2 \). In order to characterize the \( W^2 \) invariant subspaces it is favorable to introduce the additional index \( \tau^\pm_l \) that takes the values \( \tau^\pm_l = (l \pm \frac{1}{2}) \left( l \pm \frac{1}{2} + 1 \right) \), and \( l = K, K - 1, K - 2, ..., 0 \). Notice, that \( \tau^+_l = \tau^-_{l+1} \). In the following I introduce the compact notation \( \tau_l \) as \( \tau_l := \tau^+_l = \tau^-_{l+1} \). As long states differing by one unit in \( l \) are of opposite parities, one finds the number of the parity degenerate \( W^2 \) invariant subspaces to equal \( K \) according to

\[ W^2 \psi_{\mu_1...\mu_K}(p) = -\tau_l m^2 \psi_{\mu_1...\mu_K}(p). \] (13)

The latter are constituted of resonances having \( l = 0, 1, ..., K - 1 \). There is only one parity singlet state, and it has \( l = K \). It is defined as \( W^2 \psi_{\mu_1...\mu_K}(p) = -\tau^+_K m^2 \psi_{\mu_1...\mu_K}(p) \). The \( W^2 \) eigenvalues are frame independent, and can be used to label the \( W^2 \) invariant subspaces in all inertial frames.

Here, two very peculiar properties of massive \( \psi_{\mu_1...\mu_K}(p) \) spaces show up. To see them, one has to go back to \( (\frac{K}{2}, \frac{K}{2}) \). The simplest one, with \( K = 1 \), was considered extensively in Ref. [8]. Massive \( (\frac{1}{2}, \frac{1}{2}) \)'s (generically denoted by \( A_\mu(p) \)) are associated with gauge bosons in theories with spontaneously broken local gauge symmetries. Such representation spaces are spanned by four basis vectors. Three of them are divergence-less. These are the two transversal degrees of freedom (d.o.f.) giving rise to left- and right-handed circularly polarized gauge bosons, on the one side, and the longitudinal polarization vector, on the other. Finally, the divergence-full time-like degree of freedom is necessary to ensure completeness within the space under consideration.
1. The expel of the time-like degree of freedom from \((\frac{1}{2}, \frac{1}{2})\): All massive gauge theories, be they Abelian or non-Abelian, are based upon Proca’s equation, \(p^\mu A_\mu(p) = 0\). In so doing one expels the time-like polarization vector which seems favorable because its norm, in being of sign opposite to one of the three remaining d.o.f., brings in unwanted imaginary masses after quantization. Amazingly, at a later stage, the isolated degree of freedom has to reenter the theory through the back-door, in order to provide the Stückelberg term to the propagator of massive gauge bosons as required for renormalizability. Nonetheless, physical reality at classical level is still well designed by means of Proca’s equation alone in so far as the time-like degree of freedom does not contribute to the forces, i.e. to gradients of the gauge fields, an observation first reported in Ref. [10]. The lesson to be learned from the above considerations can be formulated as following. Whenever we tailor a “slim cut” for a representation space with the conviction to better serve physical reality at the classical level, the expelled degree of freedom leave its footprint at the quantum level in form of troublesome divergences disturbing renormalization. It is at that level that the disregarded degree of freedom has to be brought back to existence in terms of somewhat artificially furnished renormalization schemes.

2. Indefinite spin in \((\frac{1}{2}, \frac{1}{2})\): None of the four degrees of freedom within the massive \((\frac{1}{2}, \frac{1}{2})\) carries a definite intrinsic spin. This is so because the commutator between \(W^2\) and \(S^2\) does not vanish co-variantly and, in effect, \(W^2\) invariant subspaces in \((1/2,1/2)\) are no longer \(S^2\) eigenstates. In Ref. [11], we calculated \([W^2, S^2] = -4iE K \cdot p\). In order to distinguish \(W^2\) for \((s,0) \oplus (0,s)\) representations, from \(W^2\) for \((\frac{K}{2}, \frac{K}{2})\) representations, we introduced in Ref. [11] the notion \(\tilde{W}^2\). After all, in general, \(\psi_{\mu_1...\mu_K}(p)\) stand as examples for states of indefinite (fuzzy) spins.

Therefore, with the physical neutrino of a fixed flavor, a particle of fuzzy mass, and the baryon resonances of the type \(\psi_{\mu_1...\mu_K}(p)\), particles of fuzzy spin, we here have at hand two contra-examples to what a particle should be according to trade-mark definitions. And yet, one can not deny particle status neither to neutrino, nor to baryon resonances. In this sense, extending the particle definition seems inevitable. The extension can be such as to allow for the possibility to use besides the Poincaré invariants \(p^2\) and \(W^2\) also Lorentz covariant quantum numbers for labeling the states. For example, \(\psi_{\mu_1...\mu_K}(p)\) could
be primarily characterized by $K$ before getting down to $\tau_i^\pm$ (see Section 4 below for more details).

Before proceeding further, a comment on the impact of the time-like degree of freedom in $(\frac{1}{2}, \frac{1}{2})$ (here denoted by $\epsilon_4(p)$) onto the fermionic degrees of freedom in the product space of the four-vector spinor $\psi_\mu(p)$ is in order. In momentum space, the fermionic degrees of freedom that take their origin from $\epsilon_4(p)$ are $\epsilon_4(p)u_h(p)$, and $\epsilon_4(p)v_h(p)$, respectively, where $u_h(p)$ and $v_h(p)$ are in turn Dirac's particle- and anti-particle spinors of momentum $p$ and helicity $h$. Because of the opposite sign of the norm of $\epsilon_4(p)$ relative to the norm of the remaining three divergenceless basis vectors (here denoted by $\epsilon_i(p)$ with $i = 1, 2, 3$), one encounters the situation that if, say, $\epsilon_i(p)u_h(p)$ are to be associated with particles, $\epsilon_4(p)u_h(p)$ has to be associated with an anti-particle. Thus it looks like $\psi_\mu(p)$ joins particles of opposite fermionic numbers which is likely to give rise to inconsistencies during propagation. The way out of this is the following.

First one has to clearly single out the case of totally neutral massive $\psi_\mu(p)$ fields, such like the massive gravitino. In contrast to matter fermions, gauge fermions can not be endowed by any fermion numbers (charges) that need to be conserved. Gravitino and anti-gravitino should be identical and not distinct neither through opposite charges, nor through opposite parities, contrary to usual fermions. This is the price one has to pay for the unrestricted back and forth exchange of fermions among other fields, where the vertex form should not change in depending on whether the gauge fermion under consideration has been emitted as a particle, or, absorbed as an anti-particle. Therefore, one is no longer interpretation bound to associate $u_h(p)$ with particles, while $v_h(p)$ with anti-particles. Stated differently, $\epsilon_4(p)v_h(p)$ is as good a particle as is $\epsilon_i(p)u_h(p)$. Therefore, massive gravitino presents itself as a $\psi_\mu(p)$ wholeness of sixteen physical degrees of freedom, an observation already reported in our previous work in Ref. [11].

Next, if one is to consider, say, the N(1440) state as a part of the baryonic $\psi_\mu(p)$, then one indeed would have to describe it in terms of $\epsilon_4(p)v_h(p)$. Simultaneously, $N(1520)$ and $N(1535)$ would require $\epsilon_i(p)u_h(p)$. In order to avoid confusion with the baryon number conservation, one could look on $\epsilon_i(p)\psi(p)$, and $\epsilon_4(p)\psi(p)$ from the following independent
perspectives. One can use

\begin{align}
(p'\gamma_{\nu} - m)u_\mu(p) &= 0 , \\
p^\nu u_\mu(p) &= 0 ,
\end{align}

(14)

(15)

to pick up the \((D_{13} - S_{11})\) sub-cluster from \(\psi_\mu(p)\). In Refs. [6] the \((D_{13} - S_{11})\) cluster propagator was given as

\begin{equation}
S_{D_{13}S_{11}}^{\mu\nu} = \frac{(-g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2})(p^\lambda\gamma_\lambda + m)}{2m(p^2 - m^2)}.
\end{equation}

(16)

The \(P_{11}\) (Roper) resonance needs to be treated independently, say, by exploiting once again Dirac’s equation, however, this time supplemented by a different auxiliary condition that removes the parity degeneracy of the \(W^2\) invariant subspace under consideration (compare also Ref. [8]):

\begin{align}
(p'\gamma_{\nu} - m)u_\mu(p) &= 0 , \\
W^\mu u_\mu(p) &= 0 .
\end{align}

(17)

(18)

The propagator of the Roper resonance resulting from Eqs. (17), (18) would be

\begin{equation}
S_{\mu\nu}^{Roper} = \frac{(-p_\nu p_\mu)(p^\lambda\gamma_\lambda + m)}{2m(p^2 - m^2)}.
\end{equation}

(19)

Eventually, the Roper resonance could be treated independently from Eqs. (17), (18), and (19) by means of the Dirac equation alone.

A different perspective on \(N(1520) - N(1535)\) appears in recalling that the sixteen dimensional vector representing the direct product of the four-vector with the Dirac spinor, \(\Psi(p) = A_\mu(p) \otimes \psi(p)\), is reducible into

\begin{equation}
\Psi(p) : \left( \frac{1}{2}, \frac{1}{2} \right) \otimes \left[ \left( \frac{1}{2}, 0 \right) \oplus \left( 0, \frac{1}{2} \right) \right] \longrightarrow \left[ \left( \frac{1}{2}, \frac{1}{2} \right) \oplus \left( \frac{1}{2}, 1 \right) \right] \oplus \left[ \left( \frac{1}{2}, 0 \right) \oplus \left( 0, \frac{1}{2} \right) \right] .
\end{equation}

(20)

In this case, the Roper has clearly to be attached to the Dirac piece, while the \(D_{13} - S_{11}\) sub-cluster has to be mapped onto, \(\left[ \left(1, \frac{1}{2} \right) \oplus \left( \frac{1}{2}, 1 \right) \right]\). There are different ways to obtain the wave equations for the latter representations (see Napsuciale’s talk in Ref. [12]). In one of the possibilities one may consider the wave equation for \(\Psi(p)\) as the direct product of the
wave equations for the four vector, presented in our previous work,\(^8\) on the one side, and the Dirac equation, on the other,

\[
(\Lambda_{\mu\nu}p^\mu p^\nu \pm m^2 1_4) \otimes (\gamma^\eta p_\eta \mp m 1_4) \Psi(p) = 0,
\]

(21)

with the matrices \(\Lambda_{\mu\nu}\) from Ref. [8]. Now, by means of an appropriate similarity transformation, the \(16\times16\) dimensional matrix in front of \(\Psi(p)\) can be block-diagonalized as to obtain a \(12\times12\) matrix for \([\frac{1}{2}, \frac{1}{2}) \oplus (\frac{1}{2}, 1)\], (and thereby the corresponding wave equation) on top of Dirac’s \(4\times4\) matrix \((p^\mu \gamma_\mu \mp m 1_4)\) for \((\frac{1}{2}, 0) \oplus (0, \frac{1}{2})\). The disadvantage of this scheme lies in the loss of separation between space-time– and spinor indices, on the one side, and in shoveling problems of indefinite metrics onto the 12 component rest space, on the other.

The subject is currently under investigation. Nonetheless, by means of Eqs. (14), (15), and (18) all the \(\tilde{W}^2\)-invariant subspaces of \(\psi_\mu(p)\) (including the parity degenerate one) are described as physical and the idea of \(\tilde{\psi}_\mu(p)\) (be it \(u_\mu(p)\), or, \(v_\mu(p)\)) as a particle superior of states characterized by unique \(SU(2)\) spins, and unique fermion numbers, still preserves viability.

The present lecture devotes itself to reviewing theory and phenomenology of \(\psi_{\mu_1...\mu_K}(p)\) states. The presentation is organized as follows. In the next Section I present existing observations on baryon spectra. In Section III I compare different ideas on data interpretation. There I also review the model of a rotating and vibrating quark-diquark system\(^{13}\) and the agreement of its predictions with observations. Also there, in following Refs. [14]– [17], I review QCD inspired motivations for legitimacy of a quark-diquark configuration in baryon structure. Section IV is devoted to the auxiliary condition \(\gamma^\mu \psi_\mu(p)\) of the Rarita-Schwinger framework\(^{18}\) for the four-vector-spinor. There I show that \(\gamma^\mu \psi_\mu(p) = 0\) is a short-hand from the general definition of the parity singlet \(\tilde{W}^2\)-invariant subspace \(\tilde{\psi}_\mu^+(p)\) by means of the covariant projector, \(-\frac{1}{3}(\frac{1}{m^2}\tilde{W}^2 + \frac{2}{3})\tilde{\psi}_\mu^+(p) = \psi_\mu^+(p)\). I suggest solving the latter equation for \(\tilde{\psi}_\mu^+(p)\), which is of second order in the momenta and does not need any further supplementary conditions (provided, \(\psi_\mu(p)\) is considered as a 16 dimensional vector) rather than using the Rarita-Schwinger set of linear differential equations \((p^\nu \gamma_\nu - m)\psi_\mu(p) = 0,\) and \(p^\mu \psi_\mu(p) = 0,\) as supplemented by \(\gamma^\mu \psi_\mu(p) = 0.\) In so doing, one derives in Section IV the associated Lagrangian and verifies that our scheme does not suffer the Velo-Zwanziger\(^{19}\) problem of complex energy in the presence of an electromagnetic field and acausal propaga-
tion. The paper ends with a brief outlook.

II. SPECTRA OF LIGHT-QUARK BARYONS

A. Observations

Understanding the spectrum of the most simplest composite systems has always been a key point in the theories of the micro-world. Recall that quantum mechanics was established only after the successful description of the experimentally observed regularity patterns (such like the Balmer- series) in the excitations of the hydrogen atom. Also in solid state physics, the structure of the low–lying excitations, be them without or with a gap, has been decisive for unveiling the dynamical properties of the many-body system– ferromagnet versus superconductor, and the relevant degrees of freedom, magnons versus Cooper pairs.

In a similar way, the regularity patterns of the nucleon excitations are decisive for uncovering the relevant subnucleonic degrees of freedom and the dynamical properties of the theory of strong interaction– the Quantum Chromo- Dynamics.

Despite its long history, amazingly, the structure of the nucleon spectrum is far from being settled. This is due to the fact that the first facility that measured nucleon levels, the Los Alamos Meson Physics Facility (LAMPF) did not find all the states that were predicted by the excitations of three quarks. Later on, the Thomas Jefferson National Accelerator Facility (TJNAF) was designed (among others) to search for those “missing resonances”. At present, all data have been collected and are awaiting evaluation.

In a series of papers\cite{6} a new and subversive look on the reported data in Ref. [22] was undertaken. There I drew attention to the “Come-Together” of resonances of different spins and parities to narrow mass bands in the nucleon spectrum and, its exact replica in the $\Delta$ spectrum (see Figs. 1 and 2).

The first group of states consists of two spin-$\frac{1}{2}$ states of opposite parities and a parity singlet spin-$\frac{3}{2}^-$. The second group has three parity degenerate states with spins varying from $\frac{1}{2}^\pm$ to $\frac{5}{2}^\pm$, and a single spin-$\frac{7}{2}^+$ state. Finally, the third group has five parity degenerate states with spins ranging from $\frac{1}{2}^\pm$ to $\frac{9}{2}^\pm$, and a single spin $\frac{11}{2}^+$ state (see Ref. [23] for the complete $N$ and $\Delta$ spectra). A comparison between the $N$ and $\Delta$ spectra shows that they are identical up to two “missing” resonances on the nucleon side (these are the counterparts
of the $F_{37}$ and $H_{3,11}$ states of the $\Delta$ excitations) and up to three “missing” states on the $\Delta$ side (these are the counterparts of the nucleon $P_{11}$, $P_{13}$, and $D_{13}$ states from the third group). The $\Delta(1600)$ resonance which is most probably and independent hybrid state, is the only state that at present seems to drop out of our systematics.

B. Ideas

The existence of exactly same nucleon- and $\Delta$ crops of resonances raises several questions:

1. Are we facing here a new type of symmetry which was not anticipated by any model or theory before?

2. Is the clustering pattern a pure accident?

3. Or, is it an artifact of spectra incompleteness?

The oldest idea favors the possibility of spectrum incompleteness and counts on the discovery of “missing” resonances that are supposed to restore the uniform distributions of excitations in the spectra in accordance with the predictions of the three-quark model. A more recent idea puts the emphasis on the tendency of resonances to pair and interpret the pairing as a signal for manifest chiral symmetry.

Indeed, it is hardly to overlook the existence of parity couples

\[
\begin{align*}
N\left(\frac{1}{2}^+; 1440\right) - N\left(\frac{1}{2}^-; 1535\right), & \quad \Delta\left(\frac{1}{2}^+; 1900\right) - \Delta\left(\frac{1}{2}^-; 1910\right), \\
N\left(\frac{3}{2}^+; 1720\right) - N\left(\frac{3}{2}^-; 1700\right), & \quad \Delta\left(\frac{3}{2}^+; 1920\right) - \Delta\left(\frac{3}{2}^-; 1940\right), \\
N\left(\frac{5}{2}^+; 1675\right) - N\left(\frac{5}{2}^-; 1680\right), & \quad \Delta\left(\frac{5}{2}^+; 1905\right) - \Delta\left(\frac{5}{2}^-; 1930\right), \\
N\left(\frac{9}{2}^+; 2200\right) - N\left(\frac{9}{2}^-; 2250\right), & \quad \Delta\left(\frac{9}{2}^+; 2300\right) - \Delta\left(\frac{9}{2}^-; 2400\right),
\end{align*}
\]

and escape the temptation to interpret them as parity doublets. The tendency of resonances to form parity couples was realized in the very early days of nucleon spectroscopy. It would be a long list of literature would one want to compile a complete bibliography on that subject. I only like to mention at this place work by Dashen, Dönau and Reinhardt, Iachello, and Robson. Yet, new is often what has been only well forgotten and eagerly
awaiting for a remake. In 1992 I was still under the very strong impression, the 1990 Nobel prize to E. J. Corey for his achievements on the synthesis of chiral drugs had on me, and very much enthusiastic about chirality in biology. The chiral symmetry of laboratory manufactured chemical (better, stereo-chemical) substances was the father of my wish to find their counterpart in the particle world and what could be better suited for that but the resonant parity couples. In a 1992 preprint Ref. [28] I wrote:

“In studying chiral symmetry of strong interacting particle systems one can not oversee their analogy to enantiomorphic inorganic systems considered in stereochemistry. The approximate duplication of the nucleon excitations with respect to parity observed above 1.3 GeV can be viewed as a kind of enantiomorphism at the resonance particle level.”

In my lectures on chiral symmetry at the Indian Summer School 1992 in Sassava, former Czechoslovakia I restricted myself to the more factual expression for same reality by saying that

“Chiral symmetry is indeed realized (at least partly) in the excitation spectrum of the non-strange baryons with masses higher but 1.3 GeV, where the duplication of the resonances with respect to parity is well pronounced”.

The parity doublet “sound-track” was taken on in good spirit by D. O. Riska and found extensive coverage in his lectures at the Swieca school on nuclear physics in Brazil early 1993. The years that followed are a symptomatic example for a hasty and uncritical remake of an idea by various groups, and its subsequent promotion to the ultimate truth, for the good or bad.31−35 Although questioning seems to be out of question in the scene, the unprejudiced reader is invited to have his/her own independent look onto the following questions:

1. What drives parity couples of different spins to share same narrow mass bands?

2. Can manifest chiral symmetry be realized “partly”, or “approximately”? Stated differently, does the systematic occurrence of “unpaired” states like \(N\left(\frac{3}{2}^-; 1520\right), \Delta\left(\frac{7}{2}^+; 1950\right), \Delta\left(\frac{3}{2}^-; 1650\right), \Delta\left(\frac{11}{2}^+; 2420\right)\) destroy chiral symmetry?

3. In Ref. [27], a comment on Iachello’s paper Ref. [26] on parity doublets in hadron spectra, Robson warns that the separation between angular momentum and intrinsic spin performed in the contemporary quark models, is incompatible with relativity.
The subsequent Section attends to these questions.

III. BARYON SYSTEMATICS IN TERMS OF $SU(2) \otimes O(4)$ MULTIPLET

As already mentioned in Section I, the excitations of the light-quark baryons fit exactly into multiplets of the type $(\frac{3}{2}, \frac{3}{2}) \otimes \left[ (\frac{1}{2}, 0) \oplus (0, \frac{1}{2}) \right]$. There are two possible avenues in treating such multiplets. The first leads over the internal quark baryon dynamics and ends up in looking upon such states as composite systems characterized by a four dimensional angular momentum $K$. The second goes over relativistic states transforming co-variantly from one inertial frame to an other as totally symmetric Lorentz tensors of rank-$K$ with Dirac spinor components. This is akin to the modeling of the nucleon structure when one first couples the three spin-$\frac{1}{2}$ constituent quarks to a spin-$\frac{1}{2}$ nucleon, and then considers the nucleon as covariant $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ Lorentz state (up to form factors).

The present Section is devoted to the first avenue while the next Section deals with the second option.

A. The quark version of the diatomic rovibron model and the clustering in baryon spectra

Baryons in the quark model are considered as constituted of three quarks in a color singlet state. It appears naturally, therefore, to undertake an attempt of describing the baryonic system by means of algebraic models developed for the purposes of triatomic molecules, a path already pursued by Refs. [36]. There, the three body system was described in terms of two vectorial ($\vec{p}^+$) and one scalar ($s^+$) boson degrees of freedom that transform as the fundamental $U(7)$ septet. In the dynamical symmetry limit

$$U(7) \longrightarrow U(3) \times U(4),$$

the degrees of freedom associated with the one vectorial boson factorize from those associated with the scalar boson and the remaining vectorial boson. Because of that the physical states constructed within the $U(7)$ IBM model are often labeled by means of $U(3) \times U(4)$ quantum numbers. Below we will focus on that very sub-model of the IBM and show that its excited modes exactly accommodate the K–clusters from above and thereby the LAMPF data on the non-strange baryon resonances.
The dynamical limit $U(7) \rightarrow U(3) \times U(4)$ corresponds to the quark–diquark approximation of the three quark system, when two of the quarks reveal a stronger pair correlation to a diquark (Dq), while the third quark (q) acts as a spectator. The diquark approximation turned out to be rather convenient in particular in describing various properties of the ground state baryons. Within the context of the quark–diquark (q-Dq) model, the ideas of the rovibron model, known from the spectroscopy of diatomic molecules, can be applied to the description of the rotational-vibrational (rovibron) excitations of the q–Dq system.

### B. Rovibron Model for the Quark–Diquark System

In the rovibron model (RVM) the relative q–Dq motion is described by means of four types of boson creation operators $s^+, p_1^+, p_0^+$, and $p_{-1}^+$ (compare). The operators $s^+$ and $p_m^+$ in turn transform as rank-0, and rank-1 spherical tensors, i.e. the magnetic quantum number $m$ takes in turn the values $m = 1$, $0$, and $-1$. In order to construct boson-annihilation operators that also transform as spherical tensors, one introduces the four operators $\tilde{s} = s$, and $\tilde{p}_m = (-1)^m p_{-m}$. Constructing rank-$k$ tensor product of any rank-$k_1$ and rank-$k_2$ tensors, say, $A_{m_1}^{k_1}$ and $A_{m_2}^{k_2}$, is standard and given by

$$[A^{k_1} \otimes A^{k_2}]_m^k = \sum_{m_1, m_2} (k_1 m_1 k_2 m_2|km) A_{m_1}^{k_1} A_{m_2}^{k_2}.$$  \hspace{1cm} \text{(23)}

Here, $(k_1 m_1 k_2 m_2|km)$ are the well known $O(3)$ Clebsch-Gordan coefficients.

Now, the lowest states of the two-body system are identified with $N$ boson states and are characterized by the ket-vectors $|n_s n_p l m\rangle$ (or, a linear combination of them) within a properly defined Fock space. The constant $N = n_s + n_p$ stands for the total number of $s$- and $p$ bosons and plays the rôle of a parameter of the theory. In molecular physics, the parameter $N$ is usually associated with the number of molecular bound states. The group symmetry of the rovibron model is well known to be $U(4)$. The fifteen generators of the associated $su(4)$ algebra are determined as the following set of bilinears

$$A_{00} = s^+ \tilde{s}, \quad A_{0m} = s^+ \tilde{p}_m, \quad A_{m0} = p_m^+ \tilde{s}, \quad A_{mm'} = p_m^+ \tilde{p}_{m'}.$$  \hspace{1cm} \text{(24)}
The $u(4)$ algebra is then recovered by the following commutation relations

$$[A_{\alpha\beta}, A_{\gamma\delta}]_+ = \delta_{\beta\gamma} A_{\alpha\delta} - \delta_{\alpha\delta} A_{\gamma\beta}.$$  \hfill (25)

The operators associated with physical observables can then be expressed as combinations of the $u(4)$ generators. To be specific, the three-dimensional angular momentum takes the form

$$L_m = \sqrt{2} [p^+ \otimes \tilde{p}]_m^1.$$  \hfill (26)

Further operators are $(D_m)$ and $(D'_m)$ defined as

$$D_m = [p^+ \otimes \tilde{s} + s^+ \otimes \tilde{p}]_m^1,$$  \hfill (27)

$$D'_m = i[p^+ \otimes \tilde{s} - s^+ \otimes \tilde{p}]_m^1,$$  \hfill (28)

respectively. Here, $\vec{D}$ plays the rôle of the electric dipole operator.

Finally, a quadrupole operator $Q_m$ can be constructed as

$$Q_m = [p^+ \otimes \tilde{p}]_m^2, \quad \text{with} \quad m = -2, ..., +2.$$  \hfill (29)

The $u(4)$ algebra has the two algebras $su(3)$, and $so(4)$, as respective sub-algebras. The $su(3)$ algebra is constituted by the three generators $L_m$, and the five components of the quadrupole operator $Q_m$. Its $so(4)$ sub-algebra is constituted by the three components of the angular momentum operator $L_m$, on the one side, and the three components of the operator $D'_m$, on the other side. Thus there are two exactly soluble RVM limits that correspond to the two different chains of reducing $U(4)$ down to $O(3)$. These are:

$$U(4) \supset U(3) \supset O(3), \quad \text{and} \quad U(4) \supset O(4) \supset O(3),$$  \hfill (30)

respectively. The Hamiltonian of the RVM in these exactly soluble limits is then constructed as a properly chosen function of the Casimir operators of the algebras of either the first, or the second chain. For example, in case one approaches $O(3)$ via $U(3)$, the Hamiltonian of a dynamical $SU(3)$ symmetry can be cast into the form:

$$H_{su(3)} = H_0 + \alpha C_2 (su(3)) + \beta C_2 (so(3)).$$  \hfill (31)

Here, $H_0$ is a constant, $C_2 (su(3))$, and $C_2 (so(3))$ are in turn the quadratic (in terms of the generators) Casimirs of the $su(3)$, and $so(3)$ algebras, respectively, while $\alpha$ and $\beta$ are constants, to be determined from data fits.
A similar expression (in obvious notations) can be written for the RVM Hamiltonian in the $U(4) \supset O(4) \supset O(3)$ exactly soluble limit:

$$H_{so(4)} = H_0 + \tilde{\alpha} C_2 (so(4)) + \tilde{\beta} C_2 (so(3)) .$$  \hspace{1cm} (32)

The Casimir operator $C_2 (so(4))$ is defined accordingly as

$$C_2 (so(4)) = \frac{1}{4} \left( \vec{L}^2 + \vec{D}'^2 \right)$$  \hspace{1cm} (33)

and has an eigenvalue of $\frac{K^2}{2} \left( \frac{K^2}{2} + 1 \right)$. In molecular physics, only linear combinations of the Casimir operators are used, as a rule. However, as known from the hydrogen atom,\textsuperscript{41} the Hamiltonian is determined by the inverse power of $C_2 (so(4))$ according to

$$H_{Coul} = f \left( -\frac{1}{4} C_2 (so(4)) - 1 \right)^{-1}$$  \hspace{1cm} (34)

where $f$ is a parameter with the dimensionality of mass. This Hamiltonian predicts the energy of the states as $E_K = -f/(K + 1)^2$ and does not follow the simple linear pattern (see also Eq. (32)).

In order to demonstrate how the RVM applies to baryon spectroscopy, let us consider the case of q-Dq states associated with $N = 5$ and for the case of a $SO(4)$ dynamical symmetry. From now on we shall refer to the quark rovibron model as qRVM. It is of common knowledge that the totally symmetric irreps of the $u(4)$ algebra with the Young scheme $[N]$ contain the $SO(4)$ irreps $(\frac{K}{2}, \frac{K}{2})$ with

$$K = N, N - 2, ..., 1 \text{ or } 0 .$$  \hspace{1cm} (35)

Each one of these $SO(4)$ irreps contains $SO(3)$ multiplets with three dimensional angular momenta

$$l = K, K - 1, K - 2, ..., 1, 0 .$$  \hspace{1cm} (36)

In applying the branching rules in Eqs. (35), (36) to the case $N = 5$, one encounters the series of levels

$$K = 1 : \quad l = 0, 1 ;$$

$$K = 3 : \quad l = 0, 1, 2, 3 ;$$

$$K = 5 : \quad l = 0, 1, 2, 3, 4, 5 .$$  \hspace{1cm} (37)
The parity carried by these levels is $\eta(-1)'$ where $\eta$ is the parity of the relevant vacuum. In coupling now the angular momenta in Eq. (37) to the spin-$\frac{1}{2}$ of the three quarks in the nucleon, the following sequence of states is obtained:

\[
\begin{align*}
K = 1 : & \quad \eta J^\pi = \frac{1^+}{2}, \frac{1^-}{2}, \frac{3^-}{2} ; \\
K = 3 : & \quad \eta J^\pi = \frac{1^+}{2}, \frac{1^-}{2}, \frac{3^-}{2}, \frac{3^+}{2}, \frac{5^+}{2}, \frac{5^-}{2}, \frac{7^-}{2} ; \\
K = 5 : & \quad \eta J^\pi = \frac{1^+}{2}, \frac{1^-}{2}, \frac{3^-}{2}, \frac{3^+}{2}, \frac{5^+}{2}, \frac{5^-}{2}, \frac{7^-}{2}, \frac{7^+}{2}, \frac{9^-}{2}, \frac{11^-}{2}.
\end{align*}
\]

Therefore, rovibron states of half-integer spin transform according to \((K, \frac{K}{2}) \otimes [(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})]\) representations of \(SO(4)\). The isospin structure is accounted for pragmatically through attaching to the K–clusters an isospin spinor $\chi^I$ with $I$ taking the values $I = \frac{1}{2}$ and $I = \frac{3}{2}$ for the nucleon, and the $\Delta$ states, respectively. As illustrated by Figs. 1 and 2, the above quantum numbers cover both the nucleon and the $\Delta$ excitations.

Note that in the present simple version of the rovibron model, the spin of the quark–diquark system is $S = \frac{1}{2}$, and the total spin $J$ takes the values $J = l \pm \frac{1}{2}$ in accordance with Eqs. (37) and (38). The strong relevance of same picture for both the nucleon and the $\Delta(1232)$ spectra (in $\Delta(1232)$ the diquark is, however, in a vector-isovector state) hints onto the dominance of a scalar (pseudoscalar) diquark for both the excited nucleon– and $\Delta(1232)$ states. This situation is reminiscent of the $^210$ configuration of the $70(1^-)$plet of the canonical $SU(6)_{SF} \otimes O(3)_L$ symmetry where the mixed symmetric/antisymmetric character of the $S = \frac{1}{2}$ wave function in spin-space is compensated by a mixed symmetric/antisymmetric wave function in coordinate space, while the isotriplet $I = \frac{3}{2}$ part is totally symmetric.

We here will leave aside the discussion of the generic problem of the various incarnations of the IBM model regarding the symmetry properties of the resonance wave functions to a later date and rather concentrate in the next subsection onto the “missing” resonance problem.

C. Observed and “Missing” Resonance Clusters within the Rovibron Model

The comparison of the states in Eq. (38) with the reported ones in Figs. 1 and 2 shows that the predicted sets reproduce exactly the quantum numbers of the non-strange baryon excitations with masses below $\sim 2500$ MeV, provided, the parity $\eta$ of the vacuum changes.
from scalar ($\eta = 1$) for the $K = 1$, to pseudoscalar ($\eta = -1$) for the $K = 3, 5$ clusters. A pseudoscalar “vacuum” can be modeled in terms of an excited composite diquark carrying an internal angular momentum $L = 1^-$ and maximal spin $S = 1$. In one of the possibilities the total spin of such a system can be $|L - S| = 0^-$.

To explain the properties of the ground state, one has to consider separately even $N$ values, such as, say, $N' = 4$. In that case another branch of excitations, with $K = 4, 2$, and 0 will emerge. The $K = 0$ value characterizes the ground state, $K = 2$ corresponds to $(1, 1) \otimes \left[ (\frac{1}{2}, 0) \oplus (0, \frac{1}{2}) \right]$, while $K = 4$ corresponds to $(2, 2) \otimes \left[ (\frac{1}{2}, 0) \oplus (0, \frac{1}{2}) \right]$. These are the multiplets that we will associate with the “missing” resonances predicted by the rovibron model. In this manner, reported and “missing” resonances fall apart and populate distinct $U(4)$- and $SO(4)$ representations. In making observed and “missing” resonances distinguishable, reasons for their absence or, presence in the spectra are easier to be searched for. As to the parity of the resonances with even $K$’s, there is some ambiguity. As a guidance one may consider the decomposition of the three-quark ($q^3$) Hilbert space into Lorentz group representations as performed in Ref. [42]. There, two states of the type $(1, 1) \otimes \left[ (\frac{1}{2}, 0) \oplus (0, \frac{1}{2}) \right]$ were found. The first one arose out of the decomposition of the $q^3$-Hilbert space spanned by the $1s - 1p - 2s$ single-particle states. It was close to $\left( \frac{1}{2}, \frac{1}{2} \right) \otimes \left[ (\frac{1}{2}, 0) \oplus (0, \frac{1}{2}) \right]$ and carried opposite parity to the latter. It accommodated, therefore, unnatural parity resonances. The second $K = 2$ state was part of the $(1s - 3s - 2p - 1d)$- single-particle configuration space and was closer to $\left( \frac{3}{2}, \frac{3}{2} \right) \otimes \left[ (\frac{1}{2}, 0) \oplus (0, \frac{1}{2}) \right]$. It also carried opposite parity to the latter and accommodated natural parity resonances. Finally, the $K = 4$ cluster $\left( 2, 2 \right) \otimes \left[ (\frac{1}{2}, 0) \oplus (0, \frac{1}{2}) \right]$ emerged in the decomposition of the one-particle-one-hole states within the $(1s - 4s - 3p - 2d - 1f - 1g)$ configuration space and carried also natural parity, that is, opposite parity to $\left( \frac{5}{2}, \frac{5}{2} \right) \otimes \left[ (\frac{1}{2}, 0) \oplus (0, \frac{1}{2}) \right]$. In accordance with the above results, we here will treat the $N = 4$ states to be all of natural parities and identify them with the nucleon ($K = 0$), the natural parity $K = 2$, and the natural parity $K = 4$–clusters.

The unnatural parity $K = 2$ cluster from Ref. [42] could be generated through an unnatural parity $N = 2$ excitation mode. However, this mode would require manifest chiral symmetry up to $\approx 1550$ MeV which contradicts at least present data. With this observation in mind, we here will restrict ourselves to the consideration of the natural parity $N = 4$ clusters. In this manner the unnatural parity $K = 2$ state from Ref. [42] will be dropped out from the current version of the rovibron model. From now on we will refer to the excited $N = 4$
states as to “missing” rovibron clusters.

Now, the qRVM Hamiltonian that fits masses of the reported cluster states is given by the following function of $C_2 (so(4))$

$$H_{qRVM} = H_0 - f_1 \left( 4C_2 (so(4)) + 1 \right)^{-1} + f_2 C_2 (so(4)). \quad (39)$$

The states in Eq. (38) are degenerate and the dynamical symmetry is $SO(4)$. Here, the parameter set has been chosen as

$$H_0 = M_{\Delta N} + f_1, \quad f_1 = 600 \text{ MeV}, \quad f_2^N = 70 \text{ MeV}, \quad f_2^\Delta = 40 \text{ MeV}. \quad (40)$$

Thus, the $SO(4)$ dynamical symmetry limit of the qRVM picture of baryon structure motivates existence of quasi-degenerate crops of resonances in both the nucleon- and $\Delta$ baryon spectra. In Table I we list the masses of the K–clusters concluded from Eqs. (39), and (40). Preliminary indications for “missing” resonances have been reported recently, for example, in Refs. [43], and [44].

The data on the $\Lambda$, $\Sigma$, and $\Omega^-$ hyperon spectra are still far from being as complete as those of the nucleon and the $\Delta$ baryons and do not allow, at least at the present stage, a conclusive statement on relevance or irrelevance of the rovibron picture. The presence of the heavier strange quark can significantly influence the excitation modes of the $q^3$-system. In case, the presence of the $s$ quark in the hyperon structure is essential, the $U(4) \supset U(3) \supset O(3)$ chain can be favored over $U(4) \supset O(4) \supset O(3)$ and a different clustering motif can appear here. For the time being, this issue will be dropped out of the present consideration.

D. Spin and quark–diquark in QCD

The necessity for having a quark–diquark configuration within the nucleon follows directly from QCD arguments. In Refs. [14], [15], [16] the notion of spin in QCD was re-visited in connection with the proton spin puzzle. As it is well known, the spins of the valence quarks are by themselves not sufficient to explain the spin-$\frac{1}{2}$ of the nucleon. Rather, one needs to account for the orbital angular momentum of the quarks (here denoted by $L_{QCD}$) and the angular momentum carried by the gluons (so called field angular momentum, $G_{QCD}$):

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_{QCD} + G_{QCD}$$

$$= \int d^3x \left[ \frac{1}{2} \bar{\psi} \gamma_5 \psi + \psi^\dagger (\vec{x} \times (-\vec{D})) \psi + \vec{x} \times (\vec{E}^a \times \vec{B}^a) \right].$$

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In so doing one encounters the problem that neither $L_{QCD}$, nor $G_{QCD}$ satisfy the spin $su(2)$ algebra. If at least $(L_{QCD} + G_{QCD})$ is to do so,

$$\left[ (L^i_{QCD} + G^i_{QCD}) , (L^j_{QCD} + G^j_{QCD}) \right] = i\epsilon^{ijk} (L^k_{QCD} + G^k_{QCD}), \quad (41)$$

then $E^{i,a}$ has to be restricted to a chromo-electric charge, while $B^{i,a}$ has to be a chromo-magnetic dipole,

$$E^{i,a} = \frac{g x'^i}{r^3} T^a, \quad (42)$$

$$B^{i,a} = \left( \frac{3x'^i x'^lm^l}{r^5} - \frac{m^i}{r^3} \right) T^a, \quad (43)$$

where $x'^i = x^i - R^i$. In Singleton’s contribution to this meeting (see Ref. [15]) one reads that the diquark gives the color Coulomb fields, while the quark gives the color magnetic dipole field. In terms of color and flavor degrees of freedom, the nucleon wave function indeed has the required quark–diquark form:

$$| p^\uparrow \rangle = \epsilon^{ijk} \frac{\sqrt{18}}{\sqrt{18}} \left[ u^+_i d^+_j - u^+_i d^+_j \right] u^+_k | 0 \rangle. \quad (44)$$

A similar situation appears when looking for covariant QCD solutions in form of a membrane with the three open ends being associated with the valence quarks. When such a membrane stretches to a string, so that a linear action (so called gonihedric string) can be used, one again encounters that very $K$-cluster degeneracies in the excitations spectra of the baryons, this time as a part of an infinite tower of states. The result was reported by Savvidy in Ref. [17]. Thus the covariant spin-description provides an independent argument in favor of a dominant quark-diquark configuration in the structure of the nucleon, while search for covariant resonant QCD solutions leads once again to infinite $K$ cluster towers. The quark-diquark internal structure of the baryon’s ground states is just the configuration, the excited mode of which is described by the rovibron model and which is the source of the $\left( \frac{K}{2} , \frac{K}{2} \right) \otimes \left[ \left( \frac{1}{2} , 0 \right) \oplus \left( 0 , \frac{1}{2} \right) \right]$ pattern.

**E. Manifest chiral symmetry and baryon spectra: parity doublets versus $\psi_{\mu_1...\mu_K}(p)$ states**

At that stage we are ready to answer the questions posed to the end of Section II.
1. Several parity couples of different spins come together to the same narrow mass band because baryon excitations are rotational-vibrational modes of an excited quark-diquark string, be the diquark scalar (in the respective observed $\psi_\mu(p)$, and the “missing” $\psi_{\mu_1\mu_2}(p)$, and $\psi_{\mu_1...\mu_4}(p)$), or pseudoscalar (in the observed $\psi_{\mu_1...\mu_3}(p)$, and $\psi_{\mu_1...\mu_5}(p)$), respectively. Each $K$ state consists of $K$ parity couples and a single unpaired “has–been” spin-$J = K + \frac{1}{2}$ at rest. The parity couples should not be confused with parity doublets. The latter refer to states of equal spins, residing in distinct Fock spaces built on top of opposite parity (scalar, and pseudoscalar) vacua with $\Delta l = 0$. The parity degeneracy observed in the baryon spectra is an artifact of the belonging of resonances to $\left(\frac{K}{2}, \frac{K}{2}\right) \otimes \left(\frac{1}{2}, 0\right) \oplus \left(0, \frac{1}{2}\right)$, in which case the opposite parities of equal spins originate from underlying angular momenta differing by one unit, i.e. from $\Delta l = 1$. Within the $K$–cluster scheme, the unpaired states are no longer of an uncertain status, but necessary for the completeness of the classification scheme. The states of equal spins but opposite parities do not form parity doublets because the underlying internal angular momenta differ by one unit (i.e. $\Delta l = 1$), while parity doublets require $\Delta l = 0$.

The above considerations show that a $K$-mode of an excited quark-diquark string (be the diquark a scalar, or, pseudoscalar) represents an independent entity (particle?) in its own rights which deserves its own name. To me the different spin facets of the $K$–cluster pointing into different “parity directions” as displayed in Fig. 3 look like barbs. That’s why I suggest to refer to the $K$-clusters as barbed states to emphasize the aspect of alternating parity. Barbs could also be associated with thorns (Spanish, espino), and espinons could be another sound name for $K$-clusters.

2. Chiral symmetry realization within the $K$-cluster scenario means having coexisting scalar and pseudoscalar diquarks (“vacua”), and consequently coexisting $K$-clusters of both natural and unnatural parities. Stated differently, if TJNAF is to supplement the unnatural parity LAMPF “espinons” with $K=3$, and 5 by the natural parity $K=2,4$ ones, then we will have manifest mode of chiral symmetry in the baryonic spectra. The total number of ordinary-spin states in our scenario needs not be multiple of two, as it should be in case of parity doubling.

3. In response to Robson’s warning I here emphasize isomorphism between non-relativistic rovibron excitations and relativistic Lorentz group representations of the
type
\[
\left( \frac{K}{2}, \frac{K}{2} \right) \otimes \left[ \left( \frac{1}{2}, 0 \right) \oplus \left( 0, \frac{1}{2} \right) \right] \simeq F^K(q^2) \psi_{\mu_1 \ldots \mu_K}(\mathbf{p}), \tag{45}
\]

where \( F^K(q^2) \) stands for an appropriate form factor (or, a set of such). Mapping non-relativistic \( O(4) \) levels onto covariant \( \psi_{\mu_1 \ldots \mu_K}(\mathbf{p}) \) objects is not more and not less justified but mapping Eq. (44) onto, \( F(q^2) \left( \frac{1}{2}, 0 \right) \oplus \left( 0, \frac{1}{2} \right) \), where again, we denote form factors generically by \( F(q^2) \).

\section*{F. Electromagnetic de-excitation modes of rovibron states}

In Ref. [13] we presented the four dimensional Racah algebra that allows to calculate transition probabilities for electromagnetic de-excitations of the rovibron levels. The interested reader is invited to consult the quoted article for details. Here I restrict myself to reporting the following two results.

1. All resonances from a \( K \)-mode have same widths.

2. As compared to the natural parity \( K = 1 \) states, the electromagnetic de-excitations of the unnatural parity \( K = 3 \) and \( K = 5 \) rovibron states appear strongly suppressed.

To illustrate our predictions I compiled in Table 2 below data on experimentally observed total widths of resonances belonging to \( K = 3 \), and \( K = 5 \).

Table 2 clearly shows that resonances belonging to same \( K \)-mode have same widths. The suppression of the electromagnetic de-excitation modes of unnatural parity states to the nucleon (of natural parity) is shown in Table 3.

The suppression under discussion is due to the vanishing overlap between the scalar diquark in the latter case, and a pseudo-scalar one, in the former. Non-vanishing widths can signal small admixtures from natural parity states of same spins belonging to even \( K \) number states from the “missing” resonances. For example, the significant value of \( A_5^0 \) for \( N \left( \frac{5}{2}^+; 1680 \right) \) from \( K = 3 \) may appear as an effect of mixing with the \( N \left( \frac{5}{2}^+; 1612 \right) \) state from the natural parity “missing” cluster with \( K = 2 \),

\[
|J^\pi = \frac{5}{2}^+; m_\frac{5}{2}\rangle = \sqrt{1 - \alpha^2} |N = 5; K = 3; 0^-; l = 2^-; \frac{5}{2}^+, m_\frac{5}{2}\rangle + \alpha |N' = 4; K = 2; 0^+; l = 2^+, \frac{5}{2}^+, m_\frac{5}{2}\rangle.	ag{46}
\]
In first approximation, the mixing amplitude $\alpha$ that determines the transition matrix element

$$\alpha \langle N' = 4; K = 2; 0^+; l = 2^+; \frac{5^+}{2}, m_{\frac{5}{2}} | T^{(1,1)^{2m}} | \text{gst}[N' = 4; K = 0; 0^+; l = 0^+; \frac{1^+}{2}, m_{\frac{1}{2}}] \rangle, \quad (47)$$

between the $K$ mixed state and the nucleon can be considered to be same for all resonances belonging to the cluster under consideration (in the notations of Ref. [13]).

This gives one the idea to use helicity amplitudes to extract “missing” states.

**IV. PATHOLOGY-FREE LAGRANGIANS FOR $W^2$ INVARIANT SUBSPACES**

In this Section I focus onto the direct product space between the Lorentz vector, $(\frac{1}{2}, \frac{1}{2})$, generically denoted by $A_{\mu}(p)$, and a Dirac spinor, $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$:

$$\psi_{\mu}(p) = \left( \frac{1}{2}; \frac{1}{2} \right) \otimes \left[ \left( \frac{1}{2}; 0 \right) \oplus \left( 0; \frac{1}{2} \right) \right] := A_{\mu}(p) \otimes \psi(p). \quad (48)$$

Apparently, for charged particles, $\psi_{\mu}$ satisfies the Dirac equation for any space-time index

$$(p^\nu \gamma_\nu - m)\psi_{\mu}(p) = 0. \quad (49)$$

Spin- and parity content of $\psi_{\mu}(p)$ at rest is given by

$$\psi_{\mu}(p) : \left( \frac{1^+}{2}; \frac{1^-}{2}, \frac{3^-}{2} \right), \quad (50)$$

where the example refers to natural parity. It is one of the long standing dreams of theoreticians to have wave equations for particles with–higher spins that satisfy the following four criteria:

1. The wave equations are of pure arbitrary spin.

2. The wave equations are linear in the momenta.

3. The wave equations allow for the direct construction of vertices between the higher–spin state on the one side, and the nucleon–photon/pion system, on the other, a demand best realized in terms of a separation between Lorentz (i.e. space-time) and Dirac (i.e. spinorial) indices.

4. The wave equations do not suffer pathologies like having only non-standard energy-momentum dispersion relations and propagators.
The Rarita-Schwinger framework in Eqs. (14), (15), and (18) satisfies the second and third criteria and violates the first and fourth one.

In the following I shall present an alternative to the Rarita-Schwinger framework that satisfies the third and fourth criteria, and does not satisfy the first and second ones. It has been shown by Ahluwalia et al. in Refs. [45] that pure spin \((s, 0) \oplus (0, s)\) states necessarily require wave equations that are of the order \(p^{2s}\) in the momenta. With that observation in mind, the first criterion becomes irrelevant as it can not be fulfilled at all. The avenue toward our goal leads over covariant projectors onto invariant subspaces of the squared Pauli-Lubanski vector. In the following subsections I shall first calculate the Pauli-Lubanski vector in \(\psi_\mu(p)\), and then its squared. Afterward I shall present the construction of covariant projectors onto the \(W^2\) invariant subspaces and emphasize that the \(\gamma^\mu\psi_\mu(p) = 0\) subsidiary condition of the Rarita-Schwinger framework is a short-hand from the more general definition of the unique \(W^2\) invariant subspace that is a parity singlet and of highest spin at rest. Using that very definition as the principal equation will turn out to be quite favorable. In so doing one ends up with a wave equation that is quadratic in the momenta (for \(\psi_\mu(p)\) treated as a 16 dimensional column vector, \(\Psi(p)\)) and has a correct energy-momentum dispersion relation in the presence of a magnetic field thus avoiding the classical Velo-Zwanziger problem.

A. The Pauli-Lubanski vector in \(\psi_\mu(p)\)

In order to construct the Pauli-Lubanski vector for \(A_\mu(p) \otimes \psi(p)\) we first write down the generators in \((\frac{1}{2}, 0) \oplus (0, \frac{1}{2})\)

\[
S^{(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})}_{\nu \rho} = \frac{1}{2} \sigma_{\nu \rho} , \quad \sigma_{\nu \rho} = i \frac{1}{2} [\gamma_\nu, \gamma_\rho] , \quad (51)
\]

where \(\gamma_\mu\) are the Dirac matrices. The Lorentz generators in the four-vector \((\frac{1}{2}, \frac{1}{2})\) space are obtained from those of the right-handed \((\frac{1}{2}, 0)\), and left-handed \((0, \frac{1}{2})\) spinors in noticing that \((\frac{1}{2}, \frac{1}{2})\) is the direct product of \((\frac{1}{2}, 0)\) and \((0, \frac{1}{2})\). With that in mind, one finds

\[
S^{11}_{ij} = 1_2 \otimes (i\sigma_i) + (-i\sigma_i) \otimes 1_2 , \quad S^{11}_{ij} = i\epsilon_{ijk}(1_2 \otimes \sigma_k + \sigma_k \otimes 1_2) , \quad (52)
\]

where \(\sigma_k\) are the standard Pauli matrices. Now the generators in \(\psi_\mu(p)\) are cast into the form (compare Ref. [47] )

\[
S_{\nu \rho} = S^{11}_{\nu \rho} \otimes 1_4 + 1_4 \otimes \frac{1}{2} \sigma_{\nu \rho} \quad . \quad (53)
\]
B. The squared Pauli-Lubanski vector in $\psi_\mu(p)$

Insertion of Eq. (53) into Eq. (5) shows that the Pauli-Lubanski vector in the product space equals

$$\tilde{W}_\mu^{(\frac{1}{2})} \otimes \tilde{W}_\mu^{(\frac{1}{2})} = \tilde{W}_\mu^{(\frac{1}{2})} \otimes 1_4 + 1_4 \otimes \tilde{W}_\mu^{(\frac{1}{2})}.$$

Apparently, $\tilde{W}^2$, be they in $\psi_\mu(p)$, or the sixteen dimensional column vector, $\Psi(p)$, split these spaces into covariant sectors associated with different $\tilde{W}^2$ covariant eigenvalues according to

$$(\tilde{W}_\mu^{(\frac{1}{2})} \otimes 1_4 + 1_4 \otimes \tilde{W}_\mu^{(\frac{1}{2})})^2 \Psi^{\tau^\pm}(p) = -\tau^\pm m^2 \Psi^{\tau^\pm}(p),$$

where, again, $\tau^\pm = (l \pm \frac{1}{2})(l \pm \frac{1}{2} + 1)$, and $l = K, K - 1, ..., 1, 0$. The parity non-degenerate $\tau^+ = 15/4$ sector describes a boosted “has-been” spin-$\frac{3}{2}^-$ state in the rest frame and a state of indetermined spin elsewhere. Equation (55) is equivalently rewritten to

$$(\tilde{W}_\mu^{(\frac{1}{2})} \otimes 1_4)^2 + (1_4 \otimes \tilde{W}_\mu^{(\frac{1}{2})})^2 + 2(1_4 \otimes \tilde{W}_\mu^{(\frac{1}{2})}) \cdot (1_4 \otimes \tilde{W}_\mu^{(\frac{1}{2})}) \Psi^{\tau^\pm}(p) = -\tau^\pm m^2 \Psi^{\tau^\pm}(p),$$

where $\beta_l$ is defined as

$$[\tilde{W}_\mu^{(\frac{1}{2})} \otimes 1_4]^2 A(p) \otimes \psi(p) = -\beta_l m^2 A(p) \otimes \psi(p),$$

while

$$[\tilde{W}_\mu^{(\frac{1}{2})} \otimes 1_4]^2 A(p) \otimes \psi(p) = -\frac{3}{4} m^2 A(p) \otimes \psi(p).$$

Finally,

$$2(\tilde{W}_\mu^{\frac{1}{2}} \otimes 1_4) \cdot (1_4 \otimes \tilde{W}_\mu^{(\frac{1}{2})}) \Psi^{\tau^\pm}(p) = -\alpha^\pm m^2 \Psi^{\tau^\pm}(p)$$

Obviously, equation (56) is satisfied if

$$\alpha^\pm = \tau^\pm - \beta_l - \frac{3}{4}.$$

At this stage is important to recall that for any $\mu$, the component $W_\mu^{(\frac{1}{2})}$ is represented by a 4 $\times$ 4 matrix whose elements are again labeled by (two) Lorentz indices, $W_\mu^{(\frac{1}{2})}$, as the components of the four dimensional $(\frac{1}{2})$ vectors are labeled by a Lorentz index, i.e. $A_\mu(p)$.
The non-relativistic counterpart of Eq. (63) reads in the Rarita-Schwinger framework for a has–been spin-3/2 particle is favorably replaced by the subsequent Section I will exploit this fact in order to show that the set of three equations of \( \Psi \) work is a short-hand of the general covariant definition of the single-parity has-been spin-

\[
\begin{align*}
2 \left[ W_{\mu}^{\frac{1}{2}} \otimes 1_4 \right] & \left[ 1_4 \otimes \left( -\frac{1}{4} \gamma_5 [\not{\phi}, \gamma^\nu] \right) \right] \Psi^{\frac{7}{2}} (p) = -m^2 \alpha_1^+ \Psi^{\frac{7}{2}} (p), \\
\left[ W_{\mu}^{\frac{1}{2}} \otimes 1_4 \right] & \left[ 1_4 \otimes \gamma_5 (\not{\phi} \gamma^\nu - \gamma^\nu \not{\phi}) \right] \Psi^{\frac{7}{2}} (p) = 2m^2 \alpha_1^+ \Psi^{\frac{7}{2}} (p), \\
\left[ W_{\mu}^{\frac{1}{2}} \otimes 1_4 \right] & \left[ 1_4 \otimes \gamma_5 (2g_{\alpha \nu} p_\lambda - 2\gamma^\nu \not{\phi}) \right] \Psi^{\frac{7}{2}} (p) = 2m^2 \alpha_1^+ \Psi^{\frac{7}{2}} (p), \\
\left[ \frac{1}{2} \not{\psi} \cdot p \otimes 1_4 \right] & \left[ 1_4 \otimes \gamma_5 \right] \Psi^{\frac{7}{2}} (p) + \left[ \frac{1}{2} \not{\psi} \otimes 1_4 \right] \left[ 1_4 \otimes \gamma_5 \right] m \Psi^{\frac{7}{2}} (p) = m^2 \alpha_1^+ \Psi^{\frac{7}{2}} (p).
\end{align*}
\]

where we used \( \frac{1}{2} \not{\psi} \cdot p = 0 \).

In going back to standard \( \psi^{\pm}_\mu (p) \) notation, the last equation takes the form

\[
\left[ W_{\mu}^{\frac{1}{2}} \otimes 1_4 \right] \left[ 1_4 \otimes \gamma_5 \right] \psi^{\pm}_\eta (p) = m\alpha_1^+ \psi^{\pm}_\mu (p).
\]

In taking now Lorentz contraction of both sides of Eq. (62) by \( \gamma^\mu \), one encounters the following constraint for the \( \gamma \cdot \psi^{\frac{7}{2}} (p) \) spinor

\[
\frac{1}{m\alpha_1^+} \gamma^\mu \left( \left[ W_{\mu}^{\frac{1}{2}} \otimes 1_4 \right] \left[ \gamma \gamma_5 \otimes 1_4 \right] \right) \psi^{\frac{7}{2}} (p) = \gamma \cdot \psi^{\frac{7}{2}} (p).
\]

The non-relativistic counterpart of Eq. (63) reads

\[
(1 + \frac{2}{\alpha_1^+}) \sigma \cdot \bar{\psi}^{\pm}_\gamma (0) = 0.
\]

For \( \alpha_1^+ = 1 \) (related to \( \tau_1^+ = 15/4 \)) one finds \( \sigma \cdot \bar{\psi}^{15}_\gamma (0) = 0 \) (corresponding to \( \gamma \cdot \psi^{15}_\gamma (p) = 0 \)) while for \( \alpha_1^- = -2 \) (related to \( \tau_1^- = 3/4 \)), where the numerical factor in (64) vanishes, one encounters \( \sigma \cdot \bar{\psi}^4_\gamma (0) \neq 0 \) (corresponding to \( \gamma \cdot \psi^4_\gamma (p) \neq 0 \)).

Equation (63) shows that the second auxiliary condition of the Rarita-Schwinger framework is a short-hand of the general covariant definition of the single-parity has-been spin-3/2 of \( \psi_\mu (p) \) as an invariant subspace of the squared Pauli-Lubanski vector given in Eq. (54). In the subsequent Section I will exploit this fact in order to show that the set of three equations in the Rarita-Schwinger framework for a has–been spin-3/2 particle is favorably replaced by a second order equation.
C. Projectors onto $\tilde{W}^2$ invariant subspaces

On mass shell, $p^2 = m^2$, equations of the type of Eq. (55) are equivalently cast into the form (compare Ref. [47])

$$ P_{15}^\pm(p) \Psi_{15}^\pm(p) = \Psi_{15}^\pm(p), $$
$$ P_{15}^\pm(p) = -\frac{1}{3} \left[ \frac{1}{m^2} \tilde{W}^2 + \frac{3}{4} (1 \otimes 1) \right], $$

(65)

and

$$ P_{3}^\pm(p) \Psi_{3}^\pm(p) = \Psi_{3}^\pm(p), $$
$$ P_{3}^\pm(p) = \frac{1}{3} \left[ \frac{1}{m^2} \tilde{W}^2 + \frac{15}{4} (1 \otimes 1) \right]. $$

(66)

In favor of transparent notations, I here suppress the upper label \((1/2, 1/2) \otimes (1/2, 0) \oplus (0, 1/2)\) in the squared Pauli-Lubanski vector in $A(p) \otimes \psi(p)$.

The non-relativistic version of these projectors has found application in large $N_c$ baryon physics.\(^{48}\) It is easily verified that the operators $P_{15}^\pm(p)$, and $P_{3}^\pm(p)$ are indeed projectors onto the $\tilde{W}^2$ invariant subspaces with $\tau_1^+ = \frac{15}{4}$ (corresponding to $(1 + \frac{1}{2})(1 + \frac{1}{2} + 1)$), on the one side, and onto $\tau_1^- = \frac{3}{4}$ (corresponding to $(1 - \frac{1}{2})(1 - \frac{1}{2} + 1)$), and $\tau_0^\pm = \frac{3}{4}$ (corresponding to $(0 \pm \frac{1}{2})(0 \pm \frac{1}{2} + 1)$) respectively, on the other side, i.e.

$$ \left[ P_{15}^\pm(p) \right]^2 = P_{15}^\pm(p), \quad \left[ P_{3}^\pm(p) \right]^2 = P_{3}^\pm(p), $$
$$ P_{15}^\pm(p) + P_{3}^\pm(p) = 1_{16}, \quad P_{15}^\pm(p) P_{3}^\pm(p) = 0. $$

(67)

Now, in making use of Eq. (7), where we replaced the operators $P^\mu$ by $p^\mu$, i.e. by numbers, and in setting $p^2 = m^2$, Eq. (67) can be cast into the form

$$ P_{15}^\pm(p) = -\frac{1}{3} \left[ -\frac{1}{2} S^\mu S_\mu + \frac{1}{m^2} S_\mu p^\mu S^\mu p_\mu + \frac{3}{4} (1 \otimes 1) \right], $$
$$ P_{3}^\pm(p) = \frac{1}{3} \left[ -\frac{1}{2} S^\mu S_\mu + \frac{1}{m^2} S_\mu p^\mu S^\mu p_\mu + \frac{15}{4} (1 \otimes 1) \right]. $$

(68)

Recall that the $\tau_1^+ = \frac{15}{4}$ sector of $\Psi(p)$ is a parity singlet.

D. Propagators and Lagrangians for $\tilde{W}^2$ invariant subspaces

In having favored the 16 dimensional vector column space $\Psi(p)$ over $\psi_\mu(p)$, we gained the advantage that the Dirac equation has been automatically accounted for by means of
the definition of the Lorentz generators within $\Psi(p)$, i.e. through the second term on the rhs in Eq. (53). In so doing, one arrives at a wave equation for the $\tau_1^+ = 15/4$ sector of the four-vector–spinor that is quadratic in the momenta and reads

$$-\frac{1}{3} \left[ -\frac{1}{2} S_{\mu\nu} S^{\mu\nu} \frac{p^2}{m^2} + S_{\mu\nu} p^\mu S^{\sigma\nu} \frac{p_\sigma}{m^2} + \frac{3}{4} (1_4 \otimes 1_4) \right] \Psi_{15}^m(p) = \Psi_{15}^m(p).$$  

(69)

The propagator associated with Eq. (69) is now deduced as the following $16 \times 16$ matrix:

$$S_{15}^m(p) = \tilde{P}_{15}^m(p) \frac{p^2 - m^2}{2m^2 S^{\mu\nu} S_{\mu\nu} - 4 S_{\mu\nu} p^\mu S^{\rho\nu} p_\rho - 3 (1_4 \otimes 1_4)}.$$  

(70)

It directly verifies that Eq. (69) is obtained from the following Lagrangian

$$\mathcal{L}_{15} = \bar{\Psi}_{15}^m(p) \left[ 2m^2 S^{\mu\nu} S_{\mu\nu} - 4 S_{\mu\nu} p^\mu S^{\rho\nu} p_\rho \right] \Psi_{15}^m(p) - 15m^2 (1_4 \otimes 1_4) \bar{\Psi}_{15}^m(p) \Psi_{15}^m(p),$$  

$$\bar{\Psi}(p) \Psi(p) = \left[ A_\mu(p) g^{\mu\nu} \otimes \bar{\psi}(p) \right] \left[ \psi(p) \otimes A_\nu(p) \right].$$  

(71)

E. Energy-momentum dispersion relations for $\tilde{W}^2$ invariant subspaces in the presence of electromagnetic fields

In Ref. [50] we studied the energy–momentum dispersion relation of Eq. (69) in the presence of a simple magnetic field oriented along the $z$ axis, here denoted by $B_z$. We also took for the sake of simplicity of the calculation the $z$ axis along $p$. With the help of the symbolic code Mathematica we then calculated the appropriate determinant and, in nullifying it, found the energy-momentum dispersion relation to be

$$E^2 = (p_z - eB_z)^2 + m^2,$$  

(72)

and therefore free from the Velo-Zwanziger problem of complex energy in the background of a magnetic field. The associated interacting propagators can now be obtained in the standard way by replacing $\not{p}$ through $\not{p} := (p^\mu - eA^\mu) \gamma_\mu$. In having done so, we have produced a pathology-free propagating $\tau_1^+ = 15/4$ sector in the presence of an electromagnetic field. In this it was possible to avoid the Velo-Zwanziger problem.
The $\tau^+_1 = \frac{15}{4}$ sector corresponds to the “has–been” spin-$\frac{3}{2}$ piece of $\psi_\mu(0) \simeq \Psi(0)$ at rest, which, after boosting, ceases to be $S^2$ eigenstate. For a particle in the background of a magnetic field we therefore obtained a pathology-free covariant description.

Admittedly, one has not produced pure-spin propagators. Nonetheless, a formalism was created that at least allows for the covariant description of a propagating “has–been” spin-$\frac{3}{2}$ piece of the vector spinor. It may be useful for the covariant description of the $\Delta(1232)$ state, although the $\Delta(1232)$ resonance, strictly speaking, does not belong to $\psi_\mu(p)$, unless, some still undiscovered $\Delta$ state of spin-$\frac{1}{2}^+$ happens to lie hidden in the vicinity of 1200 MeV.

The trouble with the subsidiary conditions of the Rarita-Schwinger framework has been that after gauging, Eq. (65) does no longer reduce to $\gamma^\mu \psi_\mu(p) = 0$ for the $\tau^+_1 = \frac{15}{4}$ sector. In using the full covariant projector, it was possible to circumvent the classical Velo-Zwanziger problem in the covariant description of “has–been” spin-$\frac{3}{2}$ field. As long as the tensors $\psi_{\mu_1...\mu_K}(p)$ are totally symmetric, the resolution of the problems related to any one of the indices automatically applies to all of them. Finally, that we worked in the sixteen dimensional vector column space of $\Psi(p)$ rather than in $\psi_\mu(p)$ did not restrict generality at all. Indeed, in order to construct vertices between $\Psi^{15}_\pi(p)$ and, say, the nucleon-pion system, it is sufficient to introduce the sixteen dimensional nucleon-pion vector $\Psi^{\pi N}(p') = q^{\pi}_\mu \otimes \psi_N(p^N)$ with $p'_\mu = q^{\pi}_\mu + p^N_\mu$, and $q^{\pi}_\mu$, and $p^N_\mu$ standing in turn for pion- and nucleon momentum, and consider its overlap with $\Psi^{15}_\pi(p)$. In one of the possibilities, one finds

$$\bar{\Psi}^{15}_\pi(p) \left( 1 \otimes \gamma^\mu \gamma^5 \right) \Psi^{\pi N}(p')$$

(73)

Vertices will be presented elsewhere.

V. PERSPECTIVES

I reviewed the classification scheme for (non-strange) baryon spectra in terms of $(\frac{K}{2}, \frac{K}{2}) \otimes \left[ \left( \frac{1}{2}, 0 \right) \oplus \left( 0, \frac{1}{2} \right) \right]$ “barbed” states (espinons), be they the non relativistic O(4) rotational and vibrational modes of a quark-diquark string, or, their covariant counterparts $\psi_{\mu_1...\mu_K}(p)$ (up to form factors).

I briefly stressed on recent QCD analyzes hinting on a quark-diquark structure of the ground state baryons. I argued that a quark-(scalar diquark) string gives rise to natural parity “espinons”, while a quark-(pseudoscalar diquark) gives rise to unnatural parity lev-
els. The nucleon ground state and the first “espinon” $\psi_\mu(p)$ were shown to belong to a quark-(scalar diquark) configuration, while $\psi_{\mu_1...\mu_3}(p)$, and $\psi_{\mu_1...\mu_5}(p)$ belonged to a quark-(pseudoscalar diquark) configuration. Despite that the $\Delta(1232)$ ground state is well described by means of a quark-(axial-vector) diquark string, in its excited states, the angular momentum of the diquark seems to change. I formulated chiral symmetry in baryon spectra in terms of $\psi_{\mu_1...\mu_K}(p)$ and left the decision for the TJNAF “missing resonance” program. Finding even one of the natural parities “espinons” $\psi_{\mu_1\mu_2}(p)$, or, $\psi_{\mu_1...\mu_4}(p)$ would speak in favor of chiral symmetry in the manifest Wigner-Weyl mode there.

As to the present status of affairs, I argued that one does not observe genuine parity doublets, i.e. states having equal spins but residing in different Fock spaces, i.e. spaces built on top of vacua distinct through opposite parities. The occurrence of exactly $K$ states of equal spins and opposite parities in same mass region is nothing but an artifact of the parity degeneracy of the $K$ invariant subspaces of the squared Pauli-Lubanski vector, $W^2$, in $\psi_{\mu_1...\mu_K}(p)$, a state residing as a whole, with all its “has–been” lower spin components in same Fock space, i.e. built on top of same vacuum of either positive (scalar diquark), or, negative (pseudoscalar diquark) parities. Within this context, I also clearly justified existence of parity singlet states as the $W^2$ sectors of the highest absolute value.

I reviewed properties of the squared Pauli-Lubanski vector $W^2$ in the direct products space $(\frac{1}{2}, \frac{1}{2}) \otimes \left[ (\frac{1}{2}, 0) \oplus (0, \frac{1}{2}) \right]$, which corresponds to the well known four-vector-spinor $\psi_\mu(p)$. The squared Pauli Lubanski vector was shown to split $\psi_\mu(p)$ in two invariant subspaces according to $(W^2)^{\delta_\eta}_\gamma \psi_\gamma^\pm(p) = -\tau^\pm_{\mu} m^2 \psi_\eta^\mp(p)$ with $l = 1, 0$ and $\tau^+_1 = \frac{15}{4}, \tau^-_1 = 3/4$, and $\tau^+_0 = \frac{3}{4}$, respectively. I outlined construction of covariant projectors and onto these subspaces, focused on the parity non-degenerate $W^2$ invariant subspace (the has-been spin-$\frac{3}{2}$ at rest) and showed that it is favorably described by a wave equation that does not suffer the Velo-Zwanziger problem of complex energies in the presence of a magnetic field. In this way a formalism was created for a covariant and pathology-free description of “has–been” higher-spin states at rest to the cost of giving up the demand on Lagrangians linear in the momenta.
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Figs. 1 and 2. Summary of the data on $N$ and $\Delta$ resonances. The breaking of the mass degeneracy for each of the clusters at about 5% may in fact be an artifact of the data analysis, as has been suggested by Höhler.\textsuperscript{5} The filled circles represent known resonances, while the sole empty circle corresponds to a prediction.
Fig. 3  $K$-excitation mode of a quark-diquark string: barbed states (espinons).
Table 1

TABLE I: Predicted mass distribution of observed (obs), and missing (miss) rovibron clusters (in MeV) according to Eq. (34,35). The sign of $\eta$ in Eq. (3) determines natural- ($\eta = +1$), or, unnatural ($\eta = -1$) parity states. All $\Delta$ excitations have been calculated with $f_2 = 40$ MeV rather than with the nucleon value of $f_2 = 70$ MeV. The experimental mass averages of the resonances from a given K–cluster have been labeled by “exp”. The nucleon and $\Delta$ ground state masses $M_N$ and $M_\Delta$ were taken to equal their experimental values.

| K sign | $\eta$ | $N^{\text{obs}}$ | $N^{\text{exp}}$ | $\Delta^{\text{obs}}$ | $\Delta^{\text{exp}}$ | $N^{\text{miss}}$ | $\Delta^{\text{miss}}$ |
|--------|--------|-----------------|-----------------|----------------|----------------|----------------|----------------|
| 0      | +      | 939 939 1232 1232 | 1232            |
| 1      | +      | 1441 1498 1712 1690 | 1612 1846 |
| 2      | +      | 1764 1689 1944 1922 | 1935 2048 |
| 3      | -      | 2135 2102 2165 2276 |              |
Table 2

| K   | Resonance          | width [in GeV] |
|-----|--------------------|----------------|
| 3   | $N\left(\frac{1}{2}^{-}; 1650\right)$ | 0.15           |
| 3   | $N\left(\frac{1}{2}^{+}; 1710\right)$ | 0.10           |
| 3   | $N\left(\frac{3}{2}^{+}; 1720\right)$ | 0.15           |
| 3   | $N\left(\frac{3}{2}^{-}; 1700\right)$ | 0.15           |
| 3   | $N\left(\frac{5}{2}^{-}; 1675\right)$ | 0.15           |
| 3   | $N\left(\frac{5}{2}^{+}; 1680\right)$ | 0.13           |
| 5   | $N\left(\frac{3}{2}^{+}; 1900\right)$ | 0.50           |
| 5   | $N\left(\frac{3}{2}^{+}; 2000\right)$ | 0.49           |
Table 3

TABLE III: Helicity amplitudes of resonance clusters

| K parity of the spin-0 diquark | Resonance         | $A^p_{\frac{1}{2}}$ | $A^2_{\frac{3}{2}}$ [in $10^{-3}$GeV$^{-\frac{3}{2}}$] |
|--------------------------------|------------------|---------------------|---------------------------------------------------|
| 3                             | $N \left( \frac{1}{2}^+; 1710 \right)$ | 9 ±22               |                                                   |
| 3                             | $N \left( \frac{3}{2}^+; 1720 \right)$ | 18±30               | -19±20                                            |
| 3                             | $N \left( \frac{3}{2}^-; 1700 \right)$ | -18±30              | -2±24                                             |
| 3                             | $N \left( \frac{3}{2}^-; 1675 \right)$ | 19 ±8               | 15±9                                              |
| 3                             | $N \left( \frac{3}{2}^+; 1680 \right)$ | -15±6               | 133±12                                            |
| 1                             | $N \left( \frac{3}{2}^-; 1520 \right)$ | -24±9               | 166± 5                                            |