Thermoelectric effect and Seebeck coefficient for hot and dense hadronic matter

Jitesh R. Bhatt, Arpan Das, and Hiranmaya Mishra

Theory Division, Physical Research Laboratory, Navrangpura, Ahmedabad 380009, India

We investigate the thermoelectric effect for baryon rich plasma produced in heavy ion collision experiments. We estimate the associated Seebeck coefficient for the hadronic matter. Using kinetic theory within relaxation time approximation we calculate the Seebeck coefficient of a hadronic medium with a temperature gradient. The calculation is performed for hadronic matter modeled by hadron resonance gas model with hadrons and resonance states up to a cutoff in the mass as 2.25 GeV. We argue that the thermoelectric current produced by such effect can produce magnetic field in heavy ion collision experiments.

I. INTRODUCTION

Transport coefficients are very important in characterizing how quickly interacting many-body systems would return to its equilibrium. Recently the study of transport coefficients of the strongly interacting matter created in the relativistic heavy-ion collider has attracted a lot of attention. The importance of transport coefficients like shear and bulk viscosity come in the context of the hydrodynamical evolution of the partonic medium produced in RHIC as well as in LHC. Knowledge of the coefficient of shear viscosity has become important in explaining the observed elliptic flow measurements at RHIC, which eventually led to the prediction of \( \eta/s \) very close to the well celebrated Kovtun-Son-Starinets [KSS] bound. While the coefficient of bulk viscosity is an important measure of trace anomaly (also the conformal symmetry) i.e. departure from \( (\epsilon - 3P)/T^4 \) behavior of the system where, \( \epsilon \) and \( P \) respectively denote the energy density and pressure of the medium. For the conformal symmetry, it is necessary to have \( \epsilon = 3P \) kind of behavior. For a conformal system trace of energy momentum tensor should be zero even at quantum level. Both \( \eta/s \) and \( \zeta/s \) as a function of temperature shows non monotonic behaviour near the critical temperature \( T_c \).

Recently there have been a lot of activities in understanding the behavior of quark-gluon plasma (QGP) in presence of an electromagnetic fields (see [11]). The transport coefficient which plays an important role in understanding the behaviour QGP in such a situation, also in the case of magnetohydrodynamics, is the electrical conductivity \( \sigma_{el} \). Transport coefficients for quark matter can be estimated using different approaches like perturbative QCD, effective models etc (see [15–36] and references therein). In order to distinguish the behavior of the deconfined partonic matter from hadronic matter it is necessary to know transport coefficients like viscosity and conductivities of the hadronic matter also. There are many different approaches in the literature for estimation of transport coefficients of the hadronic matter, e.g. the relaxation time approximation and Chapman-Enskog approximation of Boltzmann transport equation, Green-Kubo formalism, quantum molecular dynamics (UrQMD), hadron resonance gas model (HRG), etc.

In the present work, we study the thermoelectric behavior of the strongly interacting matter in heavy-ion collisions. We believe that such a study would be important for understanding the behavior of the system particularly with non vanishing baryon density. The phenomenon in which a temperature gradient in a conducting material is converted to electrical current and vice versa is known as thermoelectric effect. Seebeck effect in a conductor is manifestation of the fact that when there is a temperature gradient the charge carriers would diffuse towards the region of lower temperature. The diffusion stops from the electric field generated through the charges has established a strong enough field that stops further movement of the charges. The Seebeck coefficient is defined as the electric field produced in a conducting medium due to a temperature gradient when the electrical current is zero. In this work we study the Seebeck effect for hot and dense hadronic matter. It may be noted that in the usual condensed matter systems the thermoelectric effect requires only a temperature gradient, as the ions in the lattice are stationary. On the other hand for an electron positron plasma just having a temperature gradient is not enough to lead to any thermoelectric current. This will be similar in quark gluon plasma (QGP) with zero baryon density. However at finite baryon chemical potential there is a difference between positive and negative charged particles. Hence in the

---

*Electronic address: jeet@prl.res.in
†Electronic address: arpan@prl.res.in
‡Electronic address: hm@prl.res.in
presence of temperature gradient there will be net thermoelectric current driven by the temperature gradient. For the heavy-ion collisions at FAIR [39] one might expect a baryon-rich strongly interacting medium created. In this case, there may not be a neutrality of electric charge, but the thermalization of the strongly interacting medium is expected. The strongly interacting matter created in heavy-ion collisions can have large temperature gradient between the central and peripheral regions of the collisions. Thus by allowing the possibility of temperature gradient, one can argue that the electric current in the medium will not only depend on the given electric field but also on the temperature gradient. Keeping the above motivation in mind, we calculate the Seebeck coefficient of hadron resonance gas within the kinetic theory framework using the relaxation time approximation. We show that the pions in the strongly interacting medium also contribute to the Seebeck coefficient of the medium.

The hadronic phase of the strongly interacting medium created in heavy-ion collisions are well described in terms of the hadron resonance gas (HRG) model, at chemical freeze-out [40, 41]. If one assumes strange and nonstrange particles freeze out in the same manner then HRG model has only two parameters of the hadron resonance gas (HRG) model, at chemical freeze-out [40, 41]. Naively one expects that a system of hadrons will be an interacting system and in general thermodynamics of interacting hadrons can be nontrivial. However, it can be shown that in the presence of narrow-resonances, the thermodynamics of interacting gas of hadrons can be approximated by the noninteracting gas of hadrons and resonances [43, 44]. Due to its simple structure and minimal parameters, HRG model has been well explored. One can improve upon the ideal HRG model e.g. including excluded volume HRG model. In this work, we will only discuss within the ideal hadron resonance gas model at finite temperature (T) and baryon chemical potential (µ) to estimate the Seebeck coefficient.

This paper is organized as follows, in section II we introduce the formalism of Seebeck coefficient from kinetic theory within relaxation time approximation. We also generalize it to multicomponent system. In section III we briefly discuss the HRG model and estimate the relaxation time within the same model. In section IV we present and discuss the results for Seebeck coefficient. Finally we summarize our work with an outlook in the conclusion section.

II. BOLTZMANN EQUATION IN RELAXATION TIME APPROXIMATION AND SEEBECK COEFFICIENT FOR MULTICOMPONENT SYSTEM

We consider here the linearized Boltzmann equation in relaxation time approximation. For a linear problem or weak external fields, the Boltzmann equation can be interpreted as a linear expansion of the distribution function around the equilibrium distribution function, hence \( f(\vec{k}) \) which denotes out of equilibrium distribution function, is not very far from equilibrium. Due to strong interaction equilibrium is achieved locally and electromagnetic force will take the system out of equilibrium. Using linear response approximation we write the Boltzmann equation as [45],

\[
\tilde{v}.\vec{\nabla} f_0 + \vec{F}.\vec{\nabla} f_0 = -\frac{f(\vec{r}, \vec{k}) - f_0(\vec{r}, \vec{k})}{\tau(\vec{k})} \equiv -\frac{f^{(1)}(\vec{r}, \vec{k})}{\tau(\vec{k})},
\]

where, \( f_0 \) denotes equilibrium distribution function, \( f \) denotes out of equilibrium distribution function, and \( \tau \) denotes the relaxation time of the system. The local equilibrium distribution function is considered to be of the following form [45]:

\[
f_0(\vec{r}, \vec{k}) = \frac{1}{1 + \exp\left(\frac{E(\vec{k}) - \mu(\vec{r})}{T}\right)},
\]

where \( E, T \) & \( \mu \) denote energy, temperature and chemical potential respectively, and they are functions of position vector \( \vec{r} \). Relaxation time encodes interaction processes of the microscopic theory and these interaction processes does not depend on the coordinate position. It is further assumed that cross-section of the local interaction is independent of spatial coordinates. Using Eq. (2) spatial gradient of distribution function can be written as,

\[
\vec{\nabla}_r f_0(\vec{r}, \vec{k}) = -\frac{f_0(\vec{r}, \vec{k})(f_0(\vec{r}, \vec{k}) - 1)}{T} \left( \vec{\nabla}_r \mu(\vec{r}) + \left( E(\vec{k}) - \mu(\vec{r}) \right) \vec{\nabla}_r \ln(T(\vec{r})) \right).
\]

To get the above equation we have used,

\[
\frac{\partial f_0(\vec{r}, \vec{k})}{\partial E(\vec{k})} = \frac{f_0(\vec{r}, \vec{k})(f_0(\vec{r}, \vec{k}) - 1)}{T}
\]
Similarly for the second term in the L.H.S of Eq. (1), momentum derivative of distribution function is given by,

\[
\nabla_k f_0(\vec{r}, \vec{k}) = \frac{\partial f_0(\vec{r}, \vec{k})}{\partial E(\vec{k})} \nabla E(\vec{k}) = \frac{f_0(\vec{r}, \vec{k})(f_0(\vec{r}, \vec{k}) - 1)}{T} \vec{v},
\]

where we have written, \( \nabla E(\vec{k}) = \vec{v} \).

Using Eq. (3) and Eq. (5) the Boltzmann equation (Eq. (1)) can be recast as, with the force \( \vec{F} = e \vec{E} \),

\[
\frac{f^{(1)}(\vec{r}, \vec{k})}{\tau(\vec{k})} = \frac{f_0(\vec{r}, \vec{k})(f_0(\vec{r}, \vec{k}) - 1)}{T} \vec{v} \cdot \left( (\vec{E} - T \vec{v}) \nabla \left( \frac{\mu}{T} \right) - \frac{E(\vec{k})}{T(\vec{k})} \nabla_{\vec{k}} T(\vec{k}) \right).
\]

From now on we omit the explicit functional dependence of distribution function \( f_0 \), chemical potential \( \mu \), temperature \( T \) and relaxation time \( \tau \) unless otherwise stated.

Electric current density is defined as,

\[
\vec{j} = \frac{eg}{(2\pi)^3} \int_{-\infty}^{\infty} \vec{v} f d^3k = \frac{eg}{(2\pi)^3} \int_{-\infty}^{\infty} \vec{v} f^{(1)} d^3k,
\]

here \( g \) is the degeneracy factor. In writing the second step we have used the fact that the equilibrium distribution function, being isotropic does not leads to a current. Similarly the heat current is defined as for example see Eq. 2.42 of [40],

\[
\vec{j}_Q = \frac{g}{(2\pi)^3} \int_{-\infty}^{\infty} (E - \mu) \vec{v} f^{(1)} d^3k.
\]

Using Eq. (9) in Eq. (7) and Eq. (8) for \( f^{(1)} \) electric and the heat current respectively given as,

\[
\vec{j} = -\frac{eg}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{f_0(f_0 - 1)}{T} \tau \vec{v} \left( (e \vec{E} - T \vec{v}) \left( \frac{\mu}{T} \right) - \frac{E}{T} \vec{v} T \right) d^3k.
\]

\[
\vec{j}_Q = -\frac{eg}{(2\pi)^3} \int_{-\infty}^{\infty} (E - \mu) \frac{f_0(f_0 - 1)}{T} \tau \vec{v} \left( (e \vec{E} - T \vec{v}) \left( \frac{\mu}{T} \right) - \frac{E}{T} \vec{v} T \right) d^3k.
\]

For an isotropic medium \( \vec{j}, \vec{j}_Q \) reduced to,

\[
\vec{j} = -\frac{eg}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{f_0(f_0 - 1)}{T} \tau \vec{v}^2 \left( \frac{e \vec{E} - T \vec{v}}{3} \left( \frac{\mu}{T} \right) - \frac{E}{T} \vec{v} T \right) d^3k,
\]

\[
\vec{j}_Q = -\frac{eg}{(2\pi)^3} \int_{-\infty}^{\infty} (E - \mu) \frac{f_0(f_0 - 1)}{T} \tau \vec{v}^2 \left( \frac{e \vec{E} - T \vec{v}}{3} \left( \frac{\mu}{T} \right) - \frac{E}{T} \vec{v} T \right) d^3k.
\]

Note that due to the presence of the external force in a specific direction, velocity of the particles will be different in different directions. But since the external force is considered to be small, the change in the velocity can be ignored. For later calculations, it is convenient to rewrite the momentum integration in Eq. (11), Eq. (12) in terms of integration over energies so that,

\[
\vec{j} = -\frac{2e}{3m} \int_{0}^{\infty} \frac{f_0(f_0 - 1)}{T} \tau E D(E) \left( (e \vec{E} - T \vec{v}) \left( \frac{\mu}{T} \right) - \frac{E}{T} \vec{v} T \right) dE,
\]

\[
\vec{j}_Q = -\frac{2e}{3m} \int_{0}^{\infty} (E - \mu) \frac{f_0(f_0 - 1)}{T} \tau E D(E) \left( (e \vec{E} - T \vec{v}) \left( \frac{\mu}{T} \right) - \frac{E}{T} \vec{v} T \right) dE,
\]
where density of states is defined as, $D(E)dE = \frac{\partial^2 \rho}{\partial E^2}$ and for particles in non relativistic limit $v^2 = \frac{2E}{m}$. The expressions for $\vec{j}$ and $\vec{j}_Q$ can be written in a compact manner, by defining the integral $\mathcal{L}_{ij}$ as, if we define the following integral,

$$\mathcal{L}_{ij} = \frac{2}{3m} \int \frac{f_0(f_0 - 1)}{T} E^i \tau^j D(E)dE,$$

where $i, j$ are not tensor indices, rather they denotes number of times $E$ and $\tau$ appear in the expression. In terms of $\mathcal{L}_{ij}$ one writes expressions for the currents as,

$$\vec{j} = e\mathcal{L}_{11}\left(e\vec{E} - T\vec{\nabla}\left(\frac{\mu}{T}\right)\right) - e\mathcal{L}_{21}\frac{\vec{\nabla}T}{T},$$

$$\vec{j}_Q = \left(\mathcal{L}_{21} - \mu\mathcal{L}_{11}\right)e\vec{E} - \left(\mathcal{L}_{31} - \mu\mathcal{L}_{21}\right)\frac{\vec{\nabla}T}{T}.$$  

One can further assume that that chemical potential has no spatial dependence. In this approximation $\vec{j}$ and $\vec{j}_Q$ becomes,

$$\vec{j} = e^2\mathcal{L}_{11}\vec{E} - \left(e^2\mathcal{L}_{11}\right)\frac{(\mathcal{L}_{21} - \mu\mathcal{L}_{11})}{e\mathcal{L}_{11}T}\vec{\nabla}T,$$

$$\vec{j}_Q = \left(\mathcal{L}_{21} - \mu\mathcal{L}_{11}\right)e\vec{E} - \left(\mathcal{L}_{31} - \mu\mathcal{L}_{21} + \mu^2\mathcal{L}_{11}\right)\frac{\vec{\nabla}T}{T}.$$  

Seebeck coefficient $S$ is determined by setting $\vec{j} = 0$, so that the electric field gets related with the temperature gradient as

$$\vec{E} = \left(\frac{\mathcal{L}_{21} - \mu\mathcal{L}_{11}}{e\mathcal{L}_{11}T}\right)\vec{\nabla}T \equiv S\vec{\nabla}T,$$

From Eq. (18) electrical conductivity can be identified as,

$$\sigma_{el} = e^2\mathcal{L}_{11}.$$  

Hence the electric current can be expressed in terms of $\sigma_{el}$ and $S$ as

$$\vec{j} = \sigma_{el}\vec{E} - \sigma_{el}S\vec{\nabla}T.$$  

In a similar way, when $\vec{\nabla}\mu = 0$, the heat current can be expressed as

$$\vec{j}_Q = T\sigma_{el}S\vec{E} - k_0\vec{\nabla}T,$$

where,

$$k_0 = \frac{1}{T} \left(\mathcal{L}_{31} - 2\mu\mathcal{L}_{21} + \mu^2\mathcal{L}_{11}\right).$$

Using Eq. (22) and Eq. (23), heat current $\vec{j}_Q$ can be expressed in terms of electric current $\vec{j}$,

$$\vec{j}_Q = TS\vec{j} - (k_0 - T\sigma_{el}S^2)\vec{\nabla}T.$$
From Eq. (25) we can identify the Peltier coefficient and thermal conductivity respectively,

$$\Pi = TS,$$  \hspace{1cm} (26)

$$k = k_0 - T\tau e S^2.$$  \hspace{1cm} (27)

Seebeck coefficient as given in Eq. (20) is a standard result for condensed matter systems \cite{46} and it is obtained by considering a single species of charged particle. However for the case of heavy-ion collisions there can be multiple charged particle species and we need to generalize above result. The total Seebeck coefficient of the system can not be given by Eq. (20). For multiple species case, the total electric current is a vector sum of the currents due to different species and thus one writes:

$$\vec{j} = j_{(1)} + j_{(2)} + j_{(3)} + \ldots = \sum_i j_{(i)},$$  \hspace{1cm} (28)

with $j_{(i)}$ being electrical current for the $i^{th}$ species,

$$j_{(i)} = e_{(i)}^2 \mathcal{L}_{11} \vec{\varepsilon} - \frac{e_{(i)}^2}{T} \left( \mathcal{L}_{21} - \mu \mathcal{L}_{11} \right).$$  \hspace{1cm} (29)

Hence,

$$\vec{j} = \left( e_{(1)}^2 \mathcal{L}_{11}^{(1)} + e_{(2)}^2 \mathcal{L}_{11}^{(2)} + \ldots \right) \vec{\varepsilon} - \left( \frac{e_{(1)}^2}{T} \left( \mathcal{L}_{21}^{(1)} - \mu \mathcal{L}_{11}^{(1)} \right) + \frac{e_{(2)}^2}{T} \left( \mathcal{L}_{21}^{(2)} - \mu \mathcal{L}_{11}^{(2)} \right) + \ldots \right) \nabla T.$$  \hspace{1cm} (30)

The Seebeck coefficient of the multi-species system can now be defined as,

$$S = \frac{\sum_i e_{(i)}^2 \mathcal{L}_{21}^{(i)} - \mu \mathcal{L}_{11}^{(i)}}{\sum_i e_{(i)}^2 \mathcal{L}_{11}^{(i)}} = \frac{\sum_i S^{(i)} e_{(i)}^2 \mathcal{L}_{11}^{(i)}}{\sum_i e_{(i)}^2 \mathcal{L}_{11}^{(i)}},$$  \hspace{1cm} (31)

where, we have defined the Seebeck coefficient of each species as $S^{(i)} = \left( \mathcal{L}_{21}^{(i)} - \mu \mathcal{L}_{11}^{(i)} \right) / e_{(i)} \mathcal{L}_{11}^{(i)} T$. Thus the total Seebeck coefficient $S$ of the medium is a weighted average of the Seebeck coefficients of the individual species. Thus estimating the Seebeck coefficient of a system of charged particles reduced to calculating the Seebeck coefficient of each species which is written in terms of relaxation time through the integrals $\mathcal{L}_{ij}$. While in general the relaxation time is energy dependent we shall replace it by energy averaged relaxation time, so that we can take the $\tau$ dependence out of the energy integral in Eq. (13) for $\mathcal{L}_{ij}$. In HRG model first we calculate Seebeck coefficient, $\mathcal{L}_{11}$ and $\mathcal{L}_{21}$ of each charged particle, after that we use Eq. (31) to estimate the Seebeck coefficient of the entire system.

### III. HADRON RESONANCE GAS MODEL

The central quantity in hadron resonance gas model is the thermodynamic potential which is given by \cite{30},

$$\log Z(\beta, \mu, V) = \int dm \left( \rho_M(m) \log Z_b(m, V, \beta, \mu) + \rho_B(m) \log Z_f(m, V, \beta, \mu) \right),$$  \hspace{1cm} (32)

where, the gas of non-interacting point like hadrons and their resonances is contained in the volume $V$ at a temperature $T = 1/\beta$ and baryon chemical potential $\mu$. $Z_b$ and $Z_f$ corresponds to the partition functions of free bosons (mesons) and fermions (baryons) respectively with mass $m$. Spectral densities denoted by $\rho_{B/M}(m)$ encodes the hadron properties. In the standard scenario, in HRG model all the hadrons and their resonances below a certain mass cutoff $\Lambda$ are taken to estimate the thermodynamic potential. This is achieve by taking the spectral density $\rho_{B/M}(m)$ as,

$$\rho_{B/M}(m) = \sum_i g_i \delta(m - M_i),$$  \hspace{1cm} (33)
where the sum is taken over all the hadron and resonance states up to a mass scale less than the cutoff $\Lambda$. In the above equation $M_i$ are the masses of the known hadrons and resonances and $g_i$ is the corresponding degeneracy, which includes spin and isospin quantum numbers. Although in this work we have used the discrete spectrum, it is important to mention that HRG model including discrete particle spectrum can explain lattice QCD data for trace anomaly up to temperature $T \sim 130$ MeV \cite{42}. Including Hagedorn spectrum along with the discrete spectrum for the spectral function can explain lattice QCD data for trace anomaly up to $T \sim 160$ MeV \cite{17}. Once the partition function of the HRG model is known from Eq.\ref{22}, thermodynamic quantities like pressure, energy density, number density etc. can be calculated using standard thermodynamic relations. For details of thermodynamics of HRG model, see e.g. Ref.\cite{40}. In terms of discrete spectral function Eq.\ref{20}, the integrals $L_{11}$ and $L_{21}$ for each species as in Eq.\ref{15} in Boltzmann approximation can be expressed as,

$$ L_{11}^i = \frac{\tau_i g_i}{2\pi^2 T} \int_0^{-\infty} \frac{k^4}{k^2 + m_i^2} \exp \left( -\frac{\sqrt{k^2 + m_i^2} - \mu B^i}{T} \right) dk, \tag{34} $$

and,

$$ L_{21}^i = \frac{\tau_i g_i}{2\pi^2 T} \int_0^{-\infty} \frac{k^4}{\sqrt{k^2 + m_i^2}} \exp \left( -\frac{\sqrt{k^2 + m_i^2} - \mu B^i}{T} \right) dk, \tag{35} $$

where $B^i$ is the baryon number of the $i-$th species. The Boltzmann approximation for baryons is a reasonable approximation as long as $m_{\text{nucleon}} - \mu \geq T$. In this work we have considered hadrons and resonances for which $m_i \leq \Lambda \approx 2.25$ GeV. The energy averaged relaxation time ($\tau_a$), assuming hard sphere scattering can be estimated as \cite{30},

$$ \tau_a^{-1} = \sum_b n_b \langle \sigma_{ab} v_{ab} \rangle, \quad b \neq a \tag{36} $$

where $n_a$ and $\langle \sigma_{ab} v_{ab} \rangle$ represents number density and thermal averaged cross section respectively. The thermal averaged cross section for the scattering process $a(p_a) + b(p_b) \rightarrow a(p_c) + b(p_d)$ is given as, assuming hard sphere scattering \cite{48},

$$ \langle \sigma_{ab} v_{ab} \rangle = \frac{\sigma}{8 T m_a^2 m_b^2 K_2(m_a/T) K_2(m_b/T)} \int_{(m_a + m_b)^2}^{\infty} ds \times \frac{[s - (m_a - m_b)^2]}{\sqrt{s}} \times [s - (m_a + m_b)^2] K_1(\sqrt{s}/T), \tag{37} $$

where $\sigma = 4\pi r_h^2$ is the total scattering cross section for the hard sphere.

IV. RESULTS AND DISCUSSIONS

As mentioned earlier for the hadron resonance gas model we shall include all the hadrons and resonances upto a mass cutoff $\Lambda = 2.25$ GeV and include all the mesons and baryons listed in Ref.\cite{40}. The other parameter is the radii of the hard spheres. We have chosen an uniform radius of $r_h = 0.3$ fm for all the mesons and baryons \cite{48,50}. We have estimated Seebeck coefficient of each species using Eq.\ref{29}, Eq.\ref{44} and Eq.\ref{55} for baryon chemical potential $\mu = 60$ MeV, 80MeV, 100MeV and 150 MeV. For each value of chemical potential we have varied temperature from 80MeV to 160 MeV. Knowing the Seebeck coefficient, $L_{11}$ and $L_{21}$ of each species, Seebeck coefficient of the entire system can be found from Eq.\ref{31}. However it is instructive to discuss the Seebeck coefficient for a single species as given by Eq.\ref{29} which can be written as,

$$ S^i = \frac{1}{e} \frac{I_{21}}{I_{11}} - \frac{\mu B^i}{eT}, \tag{38} $$

where the integral $I_{21}$ and $I_{11}$ are given by,

$$ I_{11} = \int_0^{\infty} \frac{\hat{k}^4}{\hat{k}^2 + \hat{m}^2} \exp(-\sqrt{\hat{k}^2 + \hat{m}^2} - \hat{\mu} B^i) dk, \tag{39} $$
and,

\[ I_{21} = \int_0^\infty \frac{k^4}{\sqrt{k^2 + \hat{m}^2}} \exp(-\sqrt{k^2 + \hat{m}^2 - \mu B^i}) \, dk, \]  

(40)

where, \( \hat{k} = k/T, \hat{m} = m/T \) and \( \mu = \mu/T \). Let us note that the individual Seebeck coefficient \( S^{(i)} \) as defined in Eq.(31) is independent of the corresponding relaxation time as it cancels out from the numerator and denominator. On the other hand the total Seebeck coefficient of the system is dependent on the relaxation time of individual hadrons through the \( \mathcal{L}^{(i)}_{11} \)'s as in may be observed in Eq.(31).

To discuss the behaviour of the Seebeck coefficient we have plotted the ratio \( I_{21}/I_{11} \), for pion, i.e. for \( B^i = 0 \), in Fig.(1).

![Fig. 1: Behaviour of \( I_{21}/I_{11} \) of pion as a function of \( m/T \). From the figure it is clear that as the temperature increases \( I_{21}/I_{11} \) decreases. From the behaviour of \( I_{21}/I_{11} \) one can understand the behaviour of Seebeck coefficient of pions as well as for other mesons. Note that baryon chemical potential plays no role in the Seebeck coefficient for mesons.](image)

We next show the variation of the total Seebeck coefficient for the hadronic medium of Eq.(31) with temperature (T) for different values of baryon chemical potential (\( \mu \)) in Fig.(2). The behaviour of the Seebeck coefficient as a function of baryon chemical potential can be understood from Eq.(31) and Eq.(20). Seebeck coefficient of the particle and the associated anti particle is same but opposite in sign due to the explicit presence of the electric charge in \( S^{(i)} \). Thus in the numerator of the Eq.(31) mesons do not contribute. Hence only the baryons contribute to the in the numerator of the Eq.(31). The mesons contributes in the denominator of the Eq.(31). This is because in the denominator particles and antiparticles do not cancels out. In the denominator of the Eq.(31) mesons also take part because in this case contribution from the particle and anti particle does not cancel out.

To understand the behavior of the Seebeck coefficient with baryon chemical potential, let us first note that for the temperature (T), baryon chemical potential (\( \mu \)) range considered here the dominant contribution to the Seebeck coefficient arises from protons. The contribution from other higher mass baryons are thermally suppressed. Behavior of proton Seebeck (\( S^{(p)} \)) shown in Fig.(3), which decreases with chemical potential (\( \mu \)). However the quantity \( \mathcal{L}^{(p)}_{11} \) for proton increases with chemical potential (\( \mu \)). This increasing behaviour of \( \mathcal{L}^{(p)}_{11} \) with \( \mu \) is rather fast enough to make the product \( S^{(p)} \mathcal{L}^{(p)}_{11} \) increasing with \( \mu \). This make the numerator for the total Seebeck coefficient in Eq.(31) increases with chemical potential. This apart, for the denominator in Eq.(31) the dominant contribution arises for the pions. \( \mathcal{L}^{(pion)}_{11} \) decreases with chemical potential as may be seen in left panel of Fig.(4). This decreases is due to the decrease of relaxation time for pions with increase in baryon chemical potential, through the hard sphere scattering. Taken together this explains the behaviour of the total Seebeck coefficient with baryon chemical potential (\( \mu \)), as be seen in Fig.(2).
FIG. 2: Behaviour of Seebeck coefficient of hadron resonance gas as a function of temperature and baryon chemical potential. We have used temperature range 80 MeV to 160 MeV, because degrees of freedom of HRG model are hadrons and resonances. We have also taken the range of baryon chemical potential from 60 MeV to 150 MeV. In this calculation we have taken into account all the hadrons and resonances having mass up to 2.25 GeV.

FIG. 3: Left Plot: $L_{11}$ of proton as function of baryon chemical potential at temperature $T = 100$ MeV. This plot shows $L_{11}$ of proton increases with baryon chemical potential for a given temperature. This behaviour of $L_{11}$ has been seen for the range of baryon chemical potentials and temperature taken in the calculation. Right Plot: Variation of proton Seebeck coefficient as function of baryon chemical potential at temperature $T = 100$ MeV. This plot shows Seebeck coeff. of proton decreases with baryon chemical potential for a given temperature. This behaviour of Seebeck coefficient has been seen for the range of baryon chemical potentials and temperature taken in the calculation.
FIG. 4: Left Plot: $L_{11}$ of pion as function of baryon chemical potential at temperature $T = 100\text{MeV}$. This plot shows $L_{11}$ of pion decreases with baryon chemical potential for a given temperature. This behaviour of $L_{11}$ has been seen for the range of baryon chemical potentials and temperature taken in the calculation. This decrease in the $L_{11}$ of pion is very important for the behaviour of total Seebeck coefficient of HRG model. Right Plot: $L_{11}$ of pion as function of temperature at baryon chemical potential $\mu = 60\text{MeV}$. This plot shows $L_{11}$ of pion decreases with temperature for a given chemical potential. This behaviour of $L_{11}$ has been seen for the range of baryon chemical potentials and temperature taken in the calculation.

In a similar way one can understand the temperature dependence of Seebeck coefficient for the system of hadrons and resonances. With increasing temperature for a fixed chemical potential $L_{11}^{(p)}$ of proton increases faster than the slow decreases of its Seebeck coefficient as may be seen in Fig.(5), making the numerator of Eq. (31) increases with $T$ for fixed $\mu$. This apart $L_{11}^{(\pi)}$ of pion also decreases with temperature for fixed baryon chemical potential as seen in the right panel of Fig.(5). This leads to the increasing behaviour of Seebeck coefficient as a function of temperature for a given baryon chemical potential, as seen in Fig.(2).

It might be relevant here to note that the thermoelectric current produced due to the temperature gradient can produce magnetic field in the heavy ion collision experiments. One can estimate the order of magnitude of the magnetic field produced due to this thermoelectric current. The magnitude of electric current density produced by the temperature gradient can be expressed as,

$$j = \sigma_{el} S \nabla T, \quad (41)$$

It may be seen in Fig.(2) the order of magnitude of total Seebeck coefficient of the hadron resonance gas can be of $\mathcal{O}(1)$ for temperature $T = 120$ MeV. The electrical conductivity of hot pion gas can be taken to be of the order of $\sigma_{el}/T \sim 0.01 \text{[51]}$. Therefore for the temperature of the order of 100 MeV, electrical conductivity is of $\mathcal{O}(1)$ MeV. If we assume that the system size is about 20 fm and the temperature difference between central and peripheral region to be of the order of 100 MeV then the temperature gradient is of the order of $10^3$ MeV. This leads to electrical current density is of the order of $10^3$ MeV. If we take the cross-sectional area to be $(20 \text{ fm})^2$, then the electrical current is $\sim 10$ MeV. Magnetic field generated by the current $I$ can be given as, $B = \frac{I}{2\pi r}$. So, for $r \sim 20$ fm, the magnetic field is $\sim 16 \text{ MeV}^2 \sim 10^{-3} m_0^2$. Note that this magnetic field is not remnant of the initial magnetic field produced in heavy ion collisions, rather the source of this magnetic field is the current produced due to the temperature gradient in a baryon rich plasma.

V. CONCLUSION

In this work, we have attempted to study the thermoelectric effect of a thermalized hadronic medium with a temperature gradient. We have estimated the corresponding Seebeck coefficient of hot hadronic matter within hadron resonance gas model (HRG). Thermoelectric effect necessarily requires a temperature gradient which is achievable...
in heavy ion collision experiment due to the temperature difference in the central and peripheral part of the fireball produced in these collisions. One of the important outcomes of this calculation is that, for a baryon free plasma, contributions in total Seebeck coefficient due to the mesonic degrees of freedom cancel out. This happens because of the fact that each meson particle comes with its antiparticle with opposite charge leading to cancellation of the corresponding Seebeck coefficient for the charged mesons. However, in a baryon rich plasma, the contributions to the total Seebeck coefficient due to the baryons do not cancel out. Total Seebeck coefficient of thermalized hadron resonance gas increases with increasing temperature for fixed baryon chemical potential and increases with baryon chemical potential for fixed temperature. It is important to note that the formalism we are using is a non-relativistic one. It will be important to study the thermoelectric effect in a relativistic formalism, particularly for QGP medium. Electrical current produced due to the temperature gradient can be a source of magnetic field. According to our estimate the strength of the magnetic field can be $\sim 10^{-3}m_\pi^2$. However, the magnetic field so produced through thermoelectric effect crucially depends upon the temperature gradient, the thermal profile of electrical conductivity and the Seebeck coefficient. In this work, we have discussed the formalism to calculate the thermoelectric coefficient in the case of spatially uniform baryon chemical potential. In a more general scenario, there can be a spatial variation of baryon chemical potential. In that case there can be current generation driven by the chemical potential gradient. Although we cannot make any comment at present on the current generation due to both the temperature gradient and chemical potential gradient, it will never the less be interesting to study these effects. In particular current generation due to a chemical potential gradient might be interesting for high baryon density matter.

Acknowledgements

The idea discussed in the present investigation arose during a visit of one of the author’s (HM) to the research group of Prof. Ajit M. Srivastava at Institute of Physics Bhubaneswar. The authors would like to thank Ajit. M. Srivastava for suggesting the problem and members of his research group for subsequent extensive discussions. The authors would also like to thank Sabyasachi Ghosh, Abhishek Atreya, Aman Abhishek, Chowdhury Aminul Islam, Rajarshi Ray for many discussions during working group activities at WHEPP 2017, IISER Bhopal. We thank Guruprasad
Kadam for useful comments on the manuscript.

[1] U. W. Heinz and R. Snellings, Annu. Rev. Nucl. Part. Sci. 63, 123-151, 2013.
[2] P. Romatschke and U. Romatschke, Phys.Rev.Lett. 99, 172301 (2007).
[3] P.K. Kovtun, D.T.Son and A.O.Starinets, Phys.Rev.Lett. 94, 111601 (2005).
[4] A.Dobado and J. M. Torres-Rincon, Phys. Rev.D86, 074021 (2012).
[5] C. Sasaki and K.Redlich, Phys. Rev.C79, 055207 (2009).
[6] C. Sasaki and K.Redlich, Nucl. Phys.A832, 62 (2010).
[7] F. Karsch, D. Kharzeev, and K. Tuchin, Phys. Lett. B663, 217 (2008).
[8] S. I. Finazzo, R. Rougemont, H. Marrochio, J. Noronha, JHEP 1502, 051 (2015).
[9] A. Wiranata and M. Prakash, Nucl. Phys.A830, 219C-222C (2009).
[10] S. Jeon and L. Yaffe, Phys.Rev. D53, 5799-5809 (1996).
[11] “Strongly interacting Matter in Magnetic field”, edited by D. Kharzeev, K. Landsteiner, A. Schmitt and H. Yee, Lecture Notes in Physics vol 871, Springer-Verlag Berlin Heidelberg 2013.
[12] K.Tuchin, Phys.Rev.C83, 017901 (2011); Phys. Rev. C82, 034904 (2010).
[13] F. Karsch, D. Kharzeev, and K. Tuchin, Phys. Lett. B663, 217 (2008).
[14] A. Wiranata and Madappa Prakash, Phys. Rev. C96, 074902 (2012).
[15] C. Sasaki and K.Redlich, Nucl. Phys.A832, 62 (2010).
[16] C. Sasaki and K.Redlich, Nucl. Phys.A830, 219C-222C (2009).
[17] S. I. Finazzo, R. Rougemont, H. Marrochio, J. Noronha, JHEP 1502, 051 (2015).
[18] A. Wiranata and M. Prakash, Nucl. Phys.A830, 219C-222C (2009).
[19] S. Jeon and L. Yaffe, Phys.Rev. D53, 5799-5809 (1996).
[20] “Strongly interacting Matter in Magnetic field”, edited by D. Kharzeev, K. Landsteiner, A. Schmitt and H. Yee, Lecture Notes in Physics vol 871, Springer-Verlag Berlin Heidelberg 2013.
[21] K.Tuchin, Phys.Rev.C83, 017901 (2011); Phys. Rev. C82, 034904 (2010).
[22] F. Karsch, D. Kharzeev, and K. Tuchin, Phys. Lett. B663, 217 (2008).
[23] A. Wiranata and Madappa Prakash, Phys. Rev. C96, 074902 (2012).
[24] C. Sasaki and K.Redlich, Nucl. Phys.A832, 62 (2010).
[25] C. Sasaki and K.Redlich, Nucl. Phys.A830, 219C-222C (2009).
[26] S. Jeon and L. Yaffe, Phys.Rev. D53, 5799-5809 (1996).
[27] “Strongly interacting Matter in Magnetic field”, edited by D. Kharzeev, K. Landsteiner, A. Schmitt and H. Yee, Lecture Notes in Physics vol 871, Springer-Verlag Berlin Heidelberg 2013.
[28] K.Tuchin, Phys.Rev.C83, 017901 (2011); Phys. Rev. C82, 034904 (2010).
[29] F. Karsch, D. Kharzeev, and K. Tuchin, Phys. Lett. B663, 217 (2008).
[30] A. Wiranata and Madappa Prakash, Phys. Rev. C96, 074902 (2012).
[31] C. Sasaki and K.Redlich, Nucl. Phys.A832, 62 (2010).
[32] C. Sasaki and K.Redlich, Nucl. Phys.A830, 219C-222C (2009).
[33] S. Jeon and L. Yaffe, Phys.Rev. D53, 5799-5809 (1996).
[34] “Strongly interacting Matter in Magnetic field”, edited by D. Kharzeev, K. Landsteiner, A. Schmitt and H. Yee, Lecture Notes in Physics vol 871, Springer-Verlag Berlin Heidelberg 2013.