Nucleon spin decomposition with one dynamical gluon

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We solve for the light-front wave functions of the nucleon from a light-front quantum chromodynamics (QCD) effective Hamiltonian with three-dimensional confinement. We obtain solutions using constituent three quarks combined with three quarks and one gluon Fock components. The resulting light-front wave functions provide a good quality description of the nucleon’s quark distribution functions following QCD scale evolution. We present the effects from incorporating a dynamical gluon on the nucleon’s quark densities, helicity distribution and orbital angular momentum that constitutes the nucleon spin sum rule.

Introduction.—QCD is the accepted theory for the strong interactions [1], where nucleons are deemed as confined systems of quarks and gluons, together known as partons. However, it is not yet possible to predict, directly from QCD, the nucleon’s global static properties: mass, spin, size etc. This is due to insufficient knowledge of the nonperturbative aspects of QCD that account for colour confinement and chiral symmetry breaking. Successful theoretical frameworks for predicting some aspects of mass spectra and revealing partonic structures are the Dyson-Schwinger equations (DSEs) of QCD [2–4] and discretized space-time Euclidean lattice [5–8]. Complementing these, the Hamiltonian formulation of QCD quantized on the light front (LF) [9, 10] has advanced in a number of directions. Further enlightenment of nonperturbative QCD can be gained from LF holography [11–13]. Meanwhile, basis light-front quantization (BLFQ), which is that approach adopted here, provides a nonperturbative framework for solving relativistic many-body bound state problems in quantum field theories [14–25].

We adopt an effective LF Hamiltonian and solve for its mass eigenstates at the scales suitable for low-resolution probes within the framework of BLFQ [14]. With quark (q) and gluon (g) being the explicit degrees of freedom for the strong interaction, the Hamiltonian includes LF QCD interactions [9] relevant to constituent |qqq⟩ and |qqgg⟩ Fock sectors of the baryons with a complementary three-dimensional confinement [18]. After fitting Hamiltonian parameters to mass spectra, we compute the nucleon parton distribution functions (PDFs) from the wave functions attained as eigenvectors of the Hamiltonian. The PDFs encoding the nonperturbative structure of the nucleon in terms of the number densities of its confined constituents, are functions of the longitudinal momentum fraction (x) of the nucleon carried by the constituents. At leading twist, the complete spin structure of the nucleon is characterized in terms of three independent PDFs, namely, unpolarized, helicity, and transversity. With our truncated Fock space, we interpret our model as appropriate to a low energy scale and we employ QCD evolution of the PDFs to higher momentum scales to compare results with global analyses.

Two salient issues can be addressed with our approach. The first issue is the gluon density at a low energy scale that contributes to all the parton densities (sea and gluon) under QCD scale evolution. The second issue concerns the description of experimental data on the gluon helicity contribution ∆G to the nucleon spin sum rule: \( \frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L_q + L_g \), with quark helicity \( \frac{1}{2} \Delta \Sigma \), quark orbital angular momentum (OAM) \( L_q \), and gluon OAM \( L_g \) contributions. The RHIC spin program at BNL has revealed that ∆G is nonvanishing and likely sizable [26–30]. Together with the known quark helicity contribution ∆G ∼ 30%, the result manifests that the other three terms provide a significant fraction of the nucleon spin. Yet, there remain large uncertainties about the small-x contribution to ∆G defined as the first moment of the polarized gluon PDF: ∆G = \( \int_0^x dx \Delta G(x) \). For a recent review, see [31]. Resolving this matter is one of the prime goals of the future EIC [32]. Similar to the scale dependence of the angular momentum observables [33, 34], addressing these two issues demands a unified framework, such as we demonstrate here. In particular, we address the first issue by encapsulating properties of the nucleon at its model scale. Then, we address the second issue by applying QCD evolution to compare with available data across various other scales where we obtain agreement within sensible precision.

Nucleon wave functions from light-front QCD Hamiltonian.—The structural information of a bound state is encoded in the light-front wave functions (LFWFs), which are obtained by solving the eigenvalue problem of the Hamiltonian: \( P^− P^+ |\Psi⟩ = M^2 |\Psi⟩ \), where \( P^± = P^0 ± P^3 \) defines the longitudinal momentum \( (P^+) \) and the LF Hamiltonian \( (P^−) \) of the system, respectively, with \( M^2 \) being the mass squared eigenvalue. At fixed LF time \( (x^+ = t + z = 0) \), the nucleon state can be...
expressed as
\[ \Psi = \psi_{\langle qqq \rangle} |qqq\rangle + \psi_{\langle qqqq \rangle} |qqqq\rangle + \ldots, \]  
(1)

where the \( \psi_{\ldots} \) describe the probability amplitudes to attain different parton configurations in the nucleon. These amplitudes can be used to define the LFWFs either in coordinate or momentum space.

At the initial scale, where the baryons are defined in terms of \(|qqq\rangle\) and \(|qqqq\rangle\) components, we consider the LF Hamiltonian \( P^+ = P^+_{\text{QCD}} + P^+_C \), where \( P^+_{\text{QCD}} \) and \( P^+_C \) are respectively the LF QCD Hamiltonian that incorporates interactions relevant to those leading two Fock components and a model for the confining interaction. In LF gauge, with one dynamical gluon \[ P^+_{\text{QCD}} = \int d^2x \frac{d^2y}{d^2z} \left\{ \frac{1}{2} \bar{\psi} \gamma^\mu \left[ m_0^2 + (i\partial^\mu)^2 \right] \psi - \frac{1}{2} A_a^i \left[ m_0^2 + (i\partial^\mu)^2 \right] A_a^i + g_s \bar{\psi} \gamma_\mu T^a A_a^i \psi \right\}, \]  
(2)

where \( \psi \) and \( A^\mu \) represent the quark and gluon fields, respectively. \( T \) is the generator of the SU(3) gauge group in color space, and \( \gamma^\mu \) are the Dirac matrices. The first and second terms in Eq. (2) are the kinetic energies of the quark and gluon with bare mass \( m_0 \) and \( m \). While the gluon mass is zero in QCD, we permit a phenomenological gluon mass to fit the nucleon form factors (FFs) in our model. The last two are the vertex and instantaneous interactions with coupling \( g_s \). Following a Fock sector-dependent renormalization procedure developed for positronium in a basis embodying \(|e\bar{e}\rangle\) and \(|e\bar{e}\gamma\rangle\) and further employed for mesons, we produce the quark mass counter term \( \delta m \) and define \( m_0 = m_q + \delta m \), where \( m_q \) is the physical quark mass. In this initial work, we neglect antisymmetrization of identical quarks. Referring to Ref. [37], we allow an independent quark mass \( m_f \) in the vertex interaction.

We consider confinement in the leading Fock sector \[ P^+ P^- = \sum_{\{n,m\}} \psi^N(\{n,m\}) \prod_{i=1}^N \phi_{n_i m_i}(\vec{p}_{\perp i}, b), \]  
(4)

where \( \psi^N = 3(\{n_i\}) \) and \( \psi^{N-4}(\{n_i\}) \) are the components of the eigenvectors associated with the Fock sectors \(|qqq\rangle\) and \(|qqqq\rangle\), respectively, in the BLFQ basis implemented for diagonalizing the full Hamiltonian matrix.

All the calculations are performed with \( \{N_{\text{max}}, K\} = \{9,16.5\} \). We select the HO scale parameter \( b = 0.7 \text{ GeV} \), the UV cutoff for the instantaneous interaction \( b_{\text{inst}} = 3 \text{ GeV} \), and set our model parameters \( \{m_u, m_d, m_s, \kappa, m_f, g_s\} = \{0.31, 0.25, 0.50, 0.54, 1.80, 2.40\} \) (all are in units of \( \text{GeV} \) except \( g_s \)) by fitting the proton mass (\( M \)), electromagnetic properties, and its flavor FFs.

We compute the electromagnetic radii of the proton from the slope of the Sachs FFs and find its charge radius \( \sqrt{\langle r_E^2 \rangle} = 0.85 \pm 0.01 \text{ fm} \) and the magnetic radius \( \sqrt{\langle r_M^2 \rangle} = 0.88 \pm 0.07 \text{ fm} \), which agree with experimentally measured data \( \sqrt{\langle r_E^2 \rangle}_{\text{exp}} = 0.840_{-0.003}^{+0.002} \text{ fm} \) and \( \sqrt{\langle r_M^2 \rangle}_{\text{exp}} = 0.849_{-0.003}^{+0.003} \text{ fm} \). We obtain the magnetic moment of the proton, \( \mu_p = 2.443 \pm 0.027 \), close to the recent lattice QCD simulations: \( \mu_p^{\text{lat}} = 2.43(9) \).
while the experimental value of the magnetic moment is $\mu^\text{exp}_p = 2.79$ [45].

Parton distribution functions.—With our resulting LFWFs, the proton’s valence quarks and gluon unpolarized and helicity PDFs are given by

$$f(x) = \int_N \psi_{1,1,1,1} (x_i; \vec{p}, \lambda_i) \delta(x - x_i),$$

and

$$\Delta f(x) = \int_N \lambda_1 \psi_{1,1,1,1} (x_i; \vec{p}, \lambda_i) \delta(x - x_i),$$

respectively, with $f \equiv q, g$. We use the abbreviation $\int_N \equiv \sum_{N, \lambda_i} \prod_{i=1}^N \int \frac{dx_i d^2 \vec{p}_i}{16\pi^2} \delta(1 - \sum x_i) \delta^2(\sum \vec{p}_i)$ and $i = q, g$ labels the valence quarks and gluon, respectively and $\lambda_1 = 1 (-1)$ for the struck parton helicity. At our model scale the PDFs for the valence quarks are normalized as $\int_0^1 q(x) dx = n_q$, with $n_q$ being the number of quarks of flavor $q$ in the proton and those PDFs together with the gluon PDF complete the momentum sum rule: $\int_0^1 \sum_i x_i f_i(x) dx = 1$.

To evolve our PDFs from our model scale ($\mu_0^2$) to a higher scale ($\mu^2$), we numerically solve the NNLO DGLAP equations [46–48] of QCD using the HOPPET [49]. We determine the initial scale by requiring the result after evolution to generate the total first moments of the valence quarks PDFs from the global QCD analysis, $\langle x \rangle_u + \langle x \rangle_d = 0.3742$ at 10 GeV$^2$ [50]. This yields $\mu_0^2 = 0.23 \sim 0.25$ GeV$^2$, and we then evolve our initial PDFs to the relevant experimental scales.

Figure 1 shows our results for the proton unpolarized PDFs at $\mu^2 = 10$ GeV$^2$, where we compare the valence quarks and gluon distributions after QCD evolution with the NNPDF3.1 [51] and MMHT [52] global fits. A similar comparison can be made with the other global fits of the quark and gluon PDFs [54–58]. We also include the proton PDFs previously obtained from a LF effective Hamiltonian [23] based on a valence Fock representation for comparison. The error bands in our evolved distributions are reflected from an adopted 10% uncertainty in our model scale. We find a good consistency between our prediction for the proton’s valence quark distributions and the global fit. The ratio $d^u(x)/u^v(x)$ is also in reasonable agreement with the extracted data from the MARATHON experiment at JLab [62]. A robust method for analysing and extrapolating JLab MARATHON data results in the proton valence-quark ratio: $\lim_{x \to 1} d^u(x)/u^v(x) = 0.230 \pm 0.057$ [63]. We predict the value of $d^u(x)/u^v(x) = 0.225 \pm 0.025$ at $x \to 1$ which agrees with the extrapolated result.

According to the Drell-Yan-West relation [59, 60], at large $\mu^2$ the valence quark distributions fall off at large $x$ as $(1 - x)^p$, where $p$ is associated to the number of valence quarks and for the proton $p = 3$. We find that the up quark unpolarized PDF falls off at large $x$ as $(1 - x)^{3.2 \pm 0.1}$, whereas for the down quark the PDF exhibits $(1 - x)^{3.5 \pm 0.1}$. Our findings support the perturbative QCD prediction [61]. We observe that the gluon PDF is suppressed at low-$x$ and moves towards the global fits [51, 52] with the addition of the dynamical gluon, while the distribution for $x > 0.05$ is in reasonable agreement with the global fit.

Figure 2 shows the helicity PDFs of the up quarks $\Delta u(x)$ and down quarks $\Delta d(x)$, at the scale $\mu^2 = 3$ GeV$^2$. We find that our helicity PDFs for both the up and down quarks are reasonably consistent with the experimental data from COMPASS [53]. For comparison, we also in-
include the quark helicity PDFs previously obtained from a LF effective Hamiltonian based on a valence Fock representation [23]. We notice that $\Delta u(x)$ for the up quark improves significantly at small-$x$ region with our current treatment for the nucleon with dynamical gluon.

Utilizing the $|qqqq\rangle$ LFWF amplitudes, we compute the gluon polarized distribution that constitutes the proton spin structure. We present the gluon helicity PDF at the scale $\mu^2 = 1 \text{ GeV}^2$ in Fig. 3, where we compare our prediction with the global analyses by the JAM [64] and the NNPDF Collaborations [28]. We observe a fair agreement between our prediction and the global fits at small-$x$, whereas our gluon helicity distribution at large-$x$ falls faster than that of the NNPDFpol1.1 analysis. The BLFQ prediction shows somewhat better agreement with the JAM results. Note that there still remain large uncertainties both in the large-$x$ domain and especially in the small-$x$ domain where even the sign is uncertain [65].

In Fig. 4, we compare the $\Delta g(x)/g(x)$ ratio obtained from our BLFQ approach with data extracted from high $p_T$ hadrons in the LO analyses [66, 67] and from the open charm production in the NLO analysis [68] at COMPASS, from high $p_T$ hadrons in the LO analyses by the SMC at CERN [69] and at the HERMES experiment [70]. We find a reasonable agreement with the COMPASS data considering the large experimental uncertainties.

The partonic helicity contributions to the proton spin are given by the first moment of the helicity distributions. Our current analysis yields that, at the model scale, the quark contribution is $\frac{1}{2} \Delta \Sigma = 0.359 \pm 0.002$ to the proton spin, where the up quark contribution, $\frac{1}{2} \Delta \Sigma_u = 0.438 \pm 0.004$, strongly dominates over the down quark contribution, $\frac{1}{2} \Delta \Sigma_d = -0.080 \pm 0.002$. The gluon contribution, $\Delta G = 0.131 \pm 0.003$, is sizeable to the proton spin.

A recent analysis with updated data sets and PHENIX measurement [71] yielded $\Delta G = 0.2$ with a constraint: $-0.7 < \Delta G < 0.5$ for $x_g \in [0.02, 0.3]$. Excluding the $x_g < 0.05$ domain, the value of $\Delta G = \int_{x_0}^{x_f} dx \; \Delta g(x) = 0.23(6)$ [28] and $\Delta G = \int_{x_0}^{x_f} dx \; \Delta g(x) = 0.19(6)$ [27] were extracted. The lattice QCD simulations predicted $\Delta G = 0.251(47)(16)$ at the physical pion mass [72]. Future measurements of $\Delta g(x)$ in the $x_g < 0.02$ are required to decrease the uncertainty in $\Delta G$. Fortunately, the upcoming EIC [32, 73] aims to accurately measure the gluon helicity distribution, particularly in the small-$x$ region and provide rigorous limits on the gluon polarized distribution.

**Orbital angular momentum.**—Another important quantity in our understanding of the nucleon spin is the OAM. The canonical OAM in the light-front gauge is computed using generalized transverse momentum dependent distributions (GTMDs) as [74–76]

$$L_z^i = -\int dx d^2 \vec{p}_\perp \frac{\vec{p}_\perp^2}{M^2} F_{1,A}^i(x, 0, \vec{p}_\perp^2, 0, 0),$$  \hspace{1cm} (7)

with $F_{1,A}^i(x, \xi, \vec{p}_\perp^2, \vec{q}_\perp, q_z)$ being one of the GTMDs for the unpolarized parton [77, 78], where $q$ is the momentum transfer and the skewness variable $\xi$ represents the momentum transfer in the longitudinal direction. The GTMD can be expressed in terms of LFWFs as

$$F_{1,A}^i \sqrt X N \int dx \frac{\bar{N}}{2} \bar{\epsilon}_{\perp} F_{1,A}^i(x', \vec{p}_\perp', \vec{q}_\perp') \Lambda \Psi_{x', \vec{p}_\perp', \lambda'} \Psi_{x, \vec{p}_\perp, \lambda} \delta^2 (\vec{p}_\perp - \bar{\vec{p}}_{\perp} + \bar{\vec{q}}_{\perp}) \delta (x - x'),$$  \hspace{1cm} (8)

where $\bar{\epsilon}_{\perp}^2 = 1$ is used.

We predict that $L^u_z = 0.0327 \pm 0.0013$, $L^d_z = -0.0114 \pm 0.0004$, and $L^g_z = -0.0065 \pm 0.0005$. Note that nothing
is known about $L(x)$ experimentally at the present time. Nonetheless, the gluon OAM can be extracted experimentally from the double spin asymmetry in diffractive dijet production [75], while the quark OAM can be measured in the exclusive double Drell-Yan process [79]. Our calculation provides a prediction of the expected data for the quark and gluon helicities and their OAM from the future experiments as well as baselines for the theoretical investigations with higher Fock components.

Conclusion and outlook.—In this letter, we have solved for the first time the light-front QCD Hamiltonian for the proton within the combined constituent three quarks ($\langle qqg \rangle$) and three quarks and one gluon ($\langle qqqg \rangle$) Fock spaces. Together with a three dimensional confinement in the leading Fock sector, the LFWFs obtained as the eigenvectors of this Hamiltonian in Basis Light Front Quantization were employed to compute the proton initial PDFs. The PDFs at a higher scale have been generated based on the NNLO DGLAP equations and we find reasonable agreement with the global fits for the unpolarized valence quark and gluon distributions. We have predicted that $d^v/u^v = 0.225 \pm 0.025$ at $x \to 1$ which agrees with the recent analysis from MARATHON experiment yielding $\lim_{x \to 1} d^v/u^v = 0.230 \pm 0.057$ [63].

We have calculated the quark and gluon helicity distributions and their OAM that constitutes the proton spin sum rule. We have observed a good consistency between our predictions for the polarized distributions and the experimental data and/or the global fits. The gluon helicity asymmetry is found to be in fair agreement with the COMPASS data. With one dynamical gluon, we have predicted that the gluon helicity ($\Delta G$) contributes 26%, while the contribution from quark helicity ($\Delta \Sigma$) is 72% to the proton spin. The contributions from the OAM to the proton spin are: $L_x^x = 0.0327 \pm 0.0013$, $L_y^x = -0.0114 \pm 0.0004$, and $L_z^x = -0.0065 \pm 0.0005$. Experimentally, there remain large uncertainties in $\Delta g(x)$, including even the sign, especially in the small-$x$ domain. Future measurements of $\Delta g(x)$ in the $x_g < 0.02$ would be most valuable to constrain $\Delta G$. On the other hand, nothing is known about OAM experimentally at the moment. Resolving these issues are major goals of the future EIC [32].

The obtained LFWFs can be further employed to compute the quark and gluon GPDs, TMDs, Wigner distributions as well as the double parton correlations etc., in the nucleon. On the other hand, the present calculation can be straightforwardly extended to higher Fock sectors to incorporate, for example, sea quarks and multi-gluons configurations as well.

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