Modeling of stress-strain state of a pneumatic tire in Abaqus software at stationary rolling

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Abstract. The paper presents a step-by-step development of a finite element model of stationary rolling of a radial automobile tire on a surface, where friction is taken into account. The method of finite element modeling (FEM) is used to determine the influence of the method of setting the rolling of a pneumatic tire on its performance properties. The authors come to the conclusions about the advisability of using various methods for creating models of pneumatic tires and give recommendations for optimization of the numerical experiment.

1. Introduction

Currently, physical and mechanical properties of pneumatic tires by experimental methods are studied well enough and are actively used in predicting the performance characteristics of materials [1]. The growth of the car park and its modernization lead to increasing average speeds, traffic intensity and the growth rates of road accidents (RTA). In this regard, the problem of ensuring traffic safety on the road is urgent. Car manufacturers are constantly trying to improve their design in order to improve safety performance. However, they may not be effective enough if there is no reliable connection between the tire and the road due to their grip properties. An increase in vehicle speed leads to increased tire tread wear. When the tread is worn, the adhesion of the tire to the road surface decreases sharply, the braking distance of the car increases, the parameters of stability and controllability deteriorate. Therefore, the purpose of the research is to predict the performance properties of tires by numerical methods. The linear model of the dependence of stress on deformation of some materials can not be described under mechanical loading. Rubber belongs to this group [2]. It can have large deformations, that is why it is referred to the category of hyperelastic materials [3] and requires choosing a special model when performing engineering calculations.

Rubber is nonlinear; therefore, there are special models which can describe such hyperelastic behavior of materials [4]. Choosing a model is the main issue when performing calculations.

Figure 1 shows the deformation range for each mathematical model.
The Neo-Hookean model is the simplest one. It is one of the first to describe the behavior of the stress-strain state of rubber [5]. However, it is suitable only for small deformations (up to 30–40%) [6]. The two-component Mooney–Rivlin model is also widely used for deformations up to 100%, although it is not suitable for describing the influence of stiffness effects [7]. For deformations within 200%, it is better to use the five- and nine-component Mooney–Rivlin models [8]. The polynomial model also better describes cases of large deformations. Yо three-component model suits for describing the behavior of rubber at high elongations, but is not suitable for small deformations [9]. Moreover, the required material parameters are rather difficult to obtain. The Arruda-Boyce and Gent models are suitable for cases of both small and large elongations, but it is best to use them for deformations within 300% [10]. The Ogden's model is based on principal elongations and gives a very good curve fit, but requires a lot of computational resources and is used for deformations more than 700% [11]. The Blatz - Ko model is used for compressible foamed polyurethane rubbers [12]. Other highly compressible elastomeric foams are described by the hyperpenic model. The two-parameter Mooney-Rivlin model is best suited for solving the problem [13]:

\[ W = c_{10}(I_1 - 3) + c_{01}(I_2 - 3) + \frac{1}{d}(J_{el} - 1)^2 \]  

(1)

where the coefficient of incompressibility of the material \( d \) (Pa\(^{-1}\)) is calculated taking into account Poisson's ratio (\( \nu \)):

\[ d = \frac{1 - 2\nu}{c_{10} + c_{01}} \]  

(2)

where \( c_{10} \) and \( c_{01} \) (Pa) are the model coefficients.

Materials with highly elastic deformation (rubber resin, rubber, some plastics, and textiles, which are capable of significantly greater stretching under uniaxial loading than, for example, steel and various metals) show the linear dependence \( \sigma - \varepsilon \) only at very small initial strains. In general, despite the high reversibility of deformation, the dependence \( \sigma - \varepsilon \) for these materials is not linear and is usually not monotonic. Consequently, materials that do not correspond to the well-known Hookean position cannot be characterized by a single constant value of the modulus of longitudinal elasticity \( E \) calculated from the conditional stress \( \sigma \). In the section of nonlinear dependence, the modulus of the material \( E \) can be determined only in differential form. The local module, which is sometimes used, does not provide a structurally meaningful assessment of the material. The evaluation of rubber properties with a conventional stress, which is used in laboratory practice, corresponding to tension by 100, 300, or 500% against the initial length of the sample is also inconsistent. These so-called "modules" are only the ordinates of some intermediate points of the curve \( \sigma - \varepsilon \), but not material constants [14-19].
2. Materials and methods
The object of research was a radial automobile tire. This type of tire has the carcass threads arranged radially along the tire profile from one bead to the other. They are parallel to each other in all layers. Only the breaker has a diagonal design. The radial arrangement of the carcass threads does not allow the rubber to stretch strongly in the transverse direction, and the breaker keeps from the longitudinal movement. Such an arrangement of the carcass threads reduces pressure approximately by half. Therefore, it becomes possible to reduce the number of layers of cord, due to which the weight of radial tires is less than that of bias tires [20]. The carcass of radial tires is more elastic due to the lower thickness and has less internal friction. Consequently, a working tire of this design generates less heat [21]. It allows to increase tread thickness, tread depth and tire life. The breaker, on the other hand, is very rigid and practically inextensible in the radial direction. Radial tires also have greater stability of the shape of the contact patch with the road surface, create less rolling resistance, and thus provide lower fuel consumption [22].

At present, stress-strain state is described both by analytical and numerical methods. To obtain numerical data, the following software systems (PC) are used: Ansys, Abaqus, Nastran, etc. We have chosen the finite element modeling PC Simulia Abaqus from Dassault Systemes as a modeling tool due to:

1) wide opportunities for nonlinear analysis;
2) the ability to use an independent library of materials and elements.

There are many classifications of car tires. We use the following division:

- by sealing: chamber and non-chamber;
- by design: radial and diagonal;
- by size: small, medium, large;
- by type of profile: toroidal, wide-profile, low-profile, arched, pneumatic rollers.

In this classification, the most massive class is radial medium-sized pneumatic tires of toroidal profile, which is the object of study in [23]. The subject of the research is numerical methods for modeling complex mechanical systems.

Tests of rubber samples were carried out in accordance with GOST 270-75 "Method for determining elastic-strength properties in tension". The standard applies to rubber and establishes a method for determining elastic strength properties in tension in terms of: tensile strength, elongation at break, stress at a given elongation. The essence of the method is to stretch the samples at a constant rate to rupture and measure the force at given elongations, at the moment of rupture and elongation of the sample at the moment of rupture [24].

3. Results
Finite element modeling and visualization software packages are a convenient tool for the numerical study of tire characteristics and tire wear. In our research, a finite element model of a car tire was obtained by means of the Abaqus program complex for finite element strength calculations [25]. We used a pneumatic tire as a prototype, its structure is shown in figure 2.
Figure 2. Tire structure: 1) hermetic layer; 2) protector; 3) steel bead; 4) carcass; 5) breaker.

The geometric model of a radial car tire consists of five distinct parts: a rubber tread, metal rings, a nylon two-layer bead, a two-layer breaker, and a hermetic layer (figure 3).

Figure 3. The model and its components.

The nylon threads of the carcass bead are located perpendicular to each other in two composite reinforcing layers of the tire: along and across the axis of rotation of the tire. Similarly, the metal strands of the breaker, which are a composite in a rubber filler, are laid diagonally at an angle of 65 and -65 degrees to the axis of rotation of the tire.

SKI-3 rubber, the properties of which we obtained earlier [26], was taken as the material of the rubber filler of the tire. Non-linear properties of nylon thread are taken from literature sources. The properties of nylon threads and rubber are specified by a hyperelastic material model. The materials of the rim rings and metal threads of the breaker layers are taken to be linear. Determining the properties of the supporting beads is simplified by introducing the assumption that the geometry of the support is considered independent of the material surrounding it. Thus, the finite element mesh for the beads was constructed independently of the layers of the rubber components of the tire. For this, Abaqus provides surface elements, which help to describe the properties of support. The surface features had no structural properties and were only used to define the geometry of the beads. Such elements are embedded in 3D solid elements used to model the rubber components of a tire. At the same time, there are restrictions between the nodes of the mesh of the matrix of the rubber material and the nodes of the surface elements of the support. The use of embedded elements prevented the difficulties of mesh construction for support, such as very small elements between the layers. To select a mathematical model of a material, for example, rubber or nylon, we calculated the selected coefficients according to the results of experimental tensile data [27].
A 3-dimensional, 8-node hybrid finite element C3D8H was selected as the tire finite element (figure 4). M3D4H 4-point element was selected for bead and breaker components.

\[\text{Figure 4. Breakdown each tire component into finite elements.}\]

In the axisymmetric formulation, the geometry of the filler was divided by elements of the CGAX4 type: a 4-node bilinear axisymmetric element, further transformed with the help of the SYMMETRIC MODEL GENERATION module into a C3D8H element as a result of "turning" the axisymmetric model around the axis of rotation. Similarly, the MGAX1 type elements were transformed: a two-node linear axisymmetric membrane element into an M3D4H type element.

The number of elements for the axisymmetric model was 653, for the 3D model - 47881.

The inline element method was used to specify the inlining of an element or group of elements into "host" elements. An inline element algorithm was used to model the support. Geometric relationships between inline element nodes and host elements were found using Abaqus. If an inline element node was inside a host element, the translational degrees of freedom in the node were eliminated and the node became an inline node. The translational degrees of freedom of the built-in node were limited by the interpolated values of the corresponding degrees of freedom of the main element. Built-in elements could have rotational degrees of freedom. For modeling a car tire, the elements of the bead and breaker were built into the elements of the hermetic layer.

4. Discussion

The numerical experiment consisted of a dynamic rolling of a pneumatic tire model on an analytical plane with a simplified tread pattern. All parts of the model were combined into a prefabricated unit.

Static loading of the tire had three stages:

Stage 1 - tire inflation;
Stage 2 - pressing by shifting the analytical plane to the tire in a coordinate way in order to set the interaction in the contact zone;
Stage 3 - applying a weight load to the tire.

A universal (direct) approach is to build a full 3D tire model ready for all types of testing. It is adapted to rapidly changing boundary and initial conditions, geometry and properties. The disadvantages of this method are the heaviness of the model (15000-20000 elements) and the duration of the calculation (from 15-20 minutes for static problems to 2 hours for dynamic problems).

At the same time, Dassault Systemes proposes a methodology for the numerical test of tire rolling, which has several stages:

- building a tire model in an axisymmetric setting;
- using the Dassault Systemes plugin, which implements the algorithm of actions;
- transformation of the section into a three-dimensional view in an axisymmetric setting, pressing it to the surface;
- obtaining a complete static three-dimensional model of a tire with an axisymmetric stress-strain state;
- obtaining a stress-strain state (SSS) during steady rolling (Steady State Transport).

Steady-state rolling occurs at a constant speed of the wheel with a coefficient of friction between the wheel and the surface of 0.5. It is usually calculated on a simplified model, where the part of the
tire (fitted to the wheel together with the rings) is cut off. It is replaced by its binding to the reference point (RP).

Analysis of the experimental results showed a small deviation between the test data of the full 3D model and two variants of axisymmetric models transformed using the plug-in. It allows to use all three methods of constructing the wheel. Taking into account the large amount of time spent on the numerical calculation of the three-dimensional model, with simple shapes of the tire tread, which can be obtained by rotating the sectors around the tire axis, it is more economical to use the axisymmetric model transformed with a help of plug-in. For a complex tread, without the possibility of obtaining sectors by axial rotation of a flat section, it is possible to use only a complete construction of a three-dimensional tire model.

Modeling results are shown in figure 5. The solution to the problem of stationary tire rolling on a flat surface with allowance for friction was carried out in a hybrid Euler-Lagrangian formulation.

Figure 5. Results of the tire rolling modeling stages.

To study the effect of each type of load on the stress-strain state, we set several loading modes, where the first was a test one, and in each subsequent one we changed only one tire load parameter. Loading data is presented in table 1.

| Parameters Dimension | Inflation MPa | Pressure N | Angular velocity rad·s⁻¹ | Linear velocity mm·s⁻¹ |
|----------------------|---------------|------------|--------------------------|----------------------|
| 1                    | 0.3           | 3300       | 74                       | 22222                |
| 2                    | 0.2           | 3300       | 74                       | 22222                |
| 3                    | 0.4           | 2800       | 74                       | 22222                |
| 4                    | 0.3           | 3300       | 50                       | 15000                |
|                      |               | 3800       |                          | 30000                |

For each loading mode, a numerical experiment was performed in all three stages. The value of the maximum contact pressure in the tire was selected as the measured value. By measuring the value of the parameter for each experiment, we obtained a set of values of the contact pressure of numerical experiments. To assess the degree of influence of each parameter on the magnitude of the contact stress, we took the change in each parameter in percentage terms, constant for each experiment, and evaluated the change in voltage in the contact zone. Taking as a basis the change in the tire pressure value, which was about 15%, the boundaries of the considered sections were obtained for the remaining parameters. The calculation results are shown in table 2. Putting the values of significant areas along the Ox axis, we obtained the range of variation of the contact pressure values for each experiment (figure 6).
Table 2. Results of calculating the boundaries of significant areas.

| Contact pressure, MPa | Inflating tires, MPa | Area boundaries, MPa |
|-----------------------|----------------------|----------------------|
| 0.887                 | 0.2                  | 0.254                |
| 0.816                 | 0.3                  | 0.3                  |
| 0.736                 | 0.4                  | 0.345                |

| Contact pressure, MPa | Tire pressure, N | Area boundaries |
|-----------------------|-----------------|-----------------|
| 0.641                 | 2800            | 1.178           |
| 0.816                 | 3300            | 1               |
| 0.95                  | 3800            | 0.868421        |

| Contact pressure, MPa | Movement speed, rad·s⁻¹ | Area boundaries, rad·s⁻¹ |
|-----------------------|--------------------------|--------------------------|
| 0.824                 | 50                       | 62.787                   |
| 0.816                 | 74                       | 74                       |
| 0.732                 | 100                      | 85.212                   |

The analysis of the data made it possible to prove the greatest influence of the weight load parameter on the change in contact pressure in the zone of contact between the tire and the road surface. The pressure values with a change in axle load by 15% more than 3 times exceeded the change with a similar change in the parameters of inflation or movement speed, which corresponds to the data presented in [28].
5. Conclusion
During our research, we:
1. Revealed the main structural and physical-mechanical features of the car tire.
2. Investigated the elastic properties of SKI-3 rubber, the results were approximated by the Mooney-Rivlin hyperelastic material model.
3. Developed a finite element model of quasi-static rolling of a car tire on a flat surface.
4. Revealed the greatest influence of the axial pressure parameter on the pressure in the zone of contact of the tire with the surface. A qualitative comparison of the modeling results with other studies showed the correlation of the results.

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