Influence of magnification on extraction efficiency in laser resonators with non-overlapping beams

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Abstract. The magnification and the Fresnel number determine the mode profile and losses in a bare unstable resonator. Upon inclusion of gain, both the beam pattern and the reflectivity are changed, more than in a stable cavity, because the counter-propagation intensities differ spatially and saturate the amplifier in a way that alters the mode profile, the reflectivity and the conditions of optimal operation. In this paper we present a numerical study of two types of cavities and compute the mode profile and losses in presence of an amplifier that saturates homogeneously. We compare these results with experimental data obtained on a TEA CO₂ laser.

1. Introduction

Unstable resonators have been extensively studied since the seminal paper of Siegman [1]. These resonators have some desirable properties: large volume mode, single transverse mode operation, lower sensitivity to misalignment and good beam quality. Their main drawback is the rather high losses, which hinders the use in low gain amplifiers. Early studies addressed computations on mode structure and losses in bare unstable resonators. In 1974 and 1975 Siegman and Siklas published the first studies on a loaded unstable resonator [2, 3]. The results showed an unstable resonator with gain has an effective reflectivity lower than bare cavity value and the figure depends on the excess gain over threshold. The behavior, although difficult to compute, can be easily understood on simple physical grounds. In an unstable resonator the two counter-propagating waves occupy different volumes, leading to a strong position-dependent gain that prevents us from employing a model similar to Rigrod’s in stable resonators [4]. The outer region of the output beam is amplified more than the central zone; thus reducing the effective reflectivity. In large Fresnel-number unstable resonators the counter propagating beams still resemble spherical waves; thus an approximate analytic analysis can be carried out. A similar situation arises when a Gaussian output mirror is employed. On the other hand, hard-edge cavities with low-to-medium
Fresnel number demand numerical simulations. The results depend on the optical configuration and are less general than those obtained in stable cavities.

In this paper we analyze, numerically, the influence of an active medium on mode pattern and effective reflectivity in two types of cavities we have previously studied neglecting the presence of gain [5,6]. These simulations attempt, among other goals, to determine the conditions of optimum coupling and compare the performance of a given unstable resonator with a stable one.

We consider two cases, the self-filtering unstable resonator [7] (SFUR) and the dot mirror cavity [8]. Even though the latter is a stable one, the energy is coupled out through diffraction in a way similar to the unstable case. These resonators have a medium-to-low Fresnel number and produce very clean, near-gaussian, lowest order mode which leads to better beam quality. In addition, lower losses per round trip (compared to other designs) can be attained. We present computed mode patterns in loaded resonators as well as measured effective reflectivity in a TEA CO2 laser.

2. Experimental differences between bare and loaded cavities.

In previous papers, we reported on the extension of the SFUR resonator to a ring cavity [5] and the characteristics of the dot mirror when the Fresnel number is outside the range originally conceived by Pax and Weston [6]. The SFUR cavity is a confocal unstable one that belongs to the negative branch. Two concave mirrors of focal lengths \( f_l \) and \( f_s \) \( (f_l > f_s) \) define a geometrical magnification \( |m| = f_l / f_s \). An aperture, placed at the common focal point, provides spatial filtering and outcoupling. The dot mirror cavity is a semiconfocal stable one with a concave mirror of focal length \( f \) and a flat one placed at the focal point. The plane mirror’s radius \( a \) is smaller than the spot size of the associated Gaussian beam \( w_0 \). Due to diffraction effects some energy is coupled out around the plane mirror, in a way similar to unstable cavities. This way, a pseudo magnification can be defined in terms of the Fresnel number \( N_F = a^2 / f \lambda \) in the following way: \( m = 0.61 / N_F \) [6].

Figure 1 shows a schematic of both resonators. Following the traditional approach, we consider two counter-propagating waves whose intensities are \( I^+ \) and \( I^- \). The difference \( I^+ - I^- \), evaluated at the filtering aperture (SFUR) or the dot mirror, gives the output intensity \( I_{out} \). The dashed box represents an amplifying medium of length \( L_a \).

![Figure 1. Sketch of the SFUR (left) and the dot mirror (right)](image)

We define the effective transmissivity \( T_{ef} \) as the ratio of the integral of \( I_{out} \) on the output plane (total output power) to the integral of \( I^- \) on the same plane. The latter can be measured by inserting, inside the resonator, a low-reflectivity (1%) beam splitter that samples \( I^- \). With lossless mirrors the effective reflectivity is computed as \( R_{ef} = 1 - T_{ef} \). This procedure, defined for a continuous-wave laser, can be extended to a pulsed one if the respective powers are replaced with energies.

In both cases, the values of \( R_{ef} \) we obtained were always below the theoretical predictions, and the difference increased as the gain rose. However, when we operated the laser near the threshold we registered a quantity close to the expected result. Figures 2 and 3 present some experimental results that follow the same tendency: the higher the gain, the lower the measured reflectivity.
To quantify the dependence of reflectivity on gain we resorted to the Fox and Li method [7] but, instead of propagating the beam in a round trip, we applied successive steps of free propagation and lumped saturated amplification. From a formal point of view, the Fox and Li method is only valid in a continuous wave laser; nevertheless, a TEA CO₂ laser provides a pulse that may last as long as 1 μs (including the Nitrogen “tail”). Therefore, a laser pulse includes many round trips and the Fox and Li method provides fair results. We considered a homogenously broadened medium of small signal intensity gain \( \alpha_0 \), saturation intensity \( I_s \) and length \( L_a \). We did not take into account interference effects between the counter propagating beams \( I^+ \) and \( I^- \). Therefore, the medium saturates by the simple addition of intensities; this fact determines a saturated gain: \( \alpha = \alpha_0 \left[ 1 + (I^+ + I^-)/I_s \right] \). We divided the
amplifier in \( N_s \) sections, propagated the beams a distance \( \Delta z = L_a/N_s \) by means of the spectral method and adjusted the amplitudes using a saturated gain computed with the intensities of the previous round trip. The initial amplitudes were set to a value well below saturation and the spatial pattern to that of the bare cavity. We increased \( N_s \) until further increments did not bring a noticeable change in beam pattern and reflectivity. For most cases \( N_s \approx 10 \) proved to be large enough. The repetitive procedure stopped when the output power fractional change was less than \( 10^{-3} \) in a round trip.

This problem has several independent parameters: small signal gain, position and length of the amplifier, etc. Hence, we will restrict the analysis to a set of values close to the ones we had in our experiments. Nevertheless, the qualitative characteristics are similar and general tendencies can be derived.

Figures 4 and 5 depict the beam profile in a bare \((G_0=\alpha_0L_a=0)\) and loaded \((G_0 > 0)\) resonator and the magnification as a parameter. The position has been normalized to the aperture (SFUR) or radius (Dot mirror) \( a \). The presence of the amplifying medium determines an increased energy content of the outer lobes; this accounts for the smaller effective reflectivity for more energy is coupled out. In addition, it can be seen the loaded cavity mode approaches the bare condition either when the total gain \( G_0 \) is low, or the counter propagating beams highly overlap (low magnification \( m \)), or the losses are high. At intermediate situations a noticeable distortion appears; this may lead to confusion when computing the optical efficiency. These simulations assumed the amplifying medium has no transverse limits; however, the gain zone has some finite extent.

![Figure 4](image)

**Figure 4.** Electric field pattern versus normalized position for a SFUR resonator.
Figure 5. Electric field pattern versus normalized position for a dot mirror resonator.

We repeated the procedure but limited the width of the amplifying volume to the size of the central lobe in a bare cavity. Even though this is an arbitrary option, it fairly represents several lasers. Figures 6 and 7 illustrate the new beam profiles. As expected, the secondary lobes have lower amplitude and the effective reflectivity is larger. This feature is less noticeable in the SFUR resonator for the beam is filtered twice in each round trip.

Figure 6. Field pattern with (red) and without (blue) spatially limited gain for a SFUR resonator.
The effective reflectivity can be seen in figures 8 and 9. To ease comparison the values have been normalized to the bare cavity figure \((R_e/R_0)\). The general behavior of the curves has been commented previously. It is interesting to note that, for some values of \(G_0\) and \(m\), the loaded cavity reflectivity can be as low as one-half the value of the bare cavity. This reduction is smaller when the gain zone has a limited width.

These results allow us analyzing the optical power extraction process from a different perspective. In a uniform intensity laser the extraction efficiency \(\eta\) is computed as the ratio of the output intensity \(I_{out}\) to the maximum available intensity \(I_{max}\) from a highly saturated amplifier \((I_{max} = G_0I_e)\). In our case the extraction efficiency is better computed in terms of total optical power because the intensity pattern is not uniform. To ease comparison, we imagine a stable cavity that operates in a multimode
(“flat top”) regime, and whose size equals the width of the main lobe of the “unstable” resonator.

![Figure 9](image)

**Figure 9.** Normalized effective reflective versus magnification for a dot mirror resonator.

![Figure 10](image)

**Figure 10.** Extraction efficiency versus effective reflectivity for a SFUR resonator.

Figures 10 and 11 show the extraction efficiency of the SFUR (a) and the dot mirror cavity (b). To highlight the presence of a maximum, we included a non-saturable loss of 20%.

The results differ from those obtained by other authors [9] who plotted the extraction efficiency versus the bare–cavity reflectivity. “Unstable” resonators can be more efficient than the stable, multimode cavity when compared at equal effective reflectivity. Unfortunately, the latter depends, in a complex way, on geometrical magnitudes and the amplifier gain.
3. Conclusions

Even though the effective reflectivity in a loaded unstable resonator differs from that of a stable one has been known for a long time, the analysis of the extracted energy can be carried out in different ways. The standard approach, based on the bare cavity reflectivity leads to a superiority of the stable resonator. However, when the effective reflectivity is taken into account, the unstable resonator gives better results.

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