Decision analysis of international joint prevention and control of public health emergencies

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Abstract
COVID-19 has caused huge losses to countries around the world, and it will not end in a short time. The lack of motivation for international joint prevention and control is one of the important reasons for the global pandemic of COVID-19. How to improve the efforts and level of international joint prevention and control has become an urgent problem to be solved. Considering the long-term and dynamic nature of international joint prevention and control, the differential game method is used to compare and analyze the optimal decisions of countries in the three scenarios of spontaneous governance, external subsidies and internal cost sharing. The results show that the optimal prevention and control efforts of countries are negatively correlated with discount rates, prevention and control cost coefficients, decay rate and risk factors. It is positively correlated with the impact degree of social benefits, the impact degree of prevention and control efforts on the level of joint prevention and control, the distribution ratio of social benefits, and the impact degree of prevention and control level on social benefits. The prevention and control efforts, joint prevention and control level, social benefits and system benefits under spontaneous governance are the lowest and highest under the internal cost sharing. The internal cost sharing will only be carried out when social benefits distribution ratio obtained reach a certain threshold. This study provides decision-making support for the joint prevention and control of countries to defeat COVID-19 under the normalization of the epidemic.

Keywords COVID-19 · International cooperation · Differential game · Prevention and control efforts

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1 Introduction

Currently, the COVID-19 epidemic is still spreading globally and will not end in a short time. The outbreak of COVID-19 has had a strong impact on international politics, economy, culture and security. In order to curb the spread of COVID-19, more and more scholars have actively explored from the perspective of epidemic prevention. For example, some scholars have systematically discussed the application of game theory in epidemiology by combining epidemiology and game theory, so as to analyze people’s decision-making behavior more clearly (Tanimoto, 2019, 2021). However, due to the tragedy of the commons in the prevention and control of the epidemic, if a country relaxes its prevention and control measures and leads to the outbreak of the epidemic, other countries will also be negatively affected. If mankind wants to overcome the epidemic as soon as possible, it is particularly important to encourage countries and regions with different political systems and ideologies to actively participate in international cooperation in the fight against COVID-19.

International cooperation can improve the comprehensive prevention and control capabilities of various countries in response to the epidemic, but also promote the construction of a global health prevention and control system, thereby benefiting all countries. For example, since the outbreak of the epidemic in China, the United Nations and other international organizations, South Korea, Japan and other countries have given China a large amount of assistance, which has played an important role in encouraging China to win the victory of epidemic. With the improvement of the epidemic in China and the aggravation of the international epidemic, China has actively participated in global health governance. By providing economic assistance, medical materials and personnel support, and expanding the production and export of epidemic prevention supplies, China has helped developing countries and small and medium-sized countries with weak public health system improve their epidemic response capacity. However, some poorer countries, such as developing countries, have poor epidemic prevention capabilities and disproportionately undertake serious epidemic prevention and control, which has created obstacles to international cooperation in prevention and control. Therefore, it is of great significance to promote the vulnerable countries to improve the level of joint prevention and control and carry out international collaborative epidemic prevention.

The international epidemic prevention and control requires a lot of time and capital consumption, coupled with the free-riding psychology of some countries unable to fight the epidemic, leading to a lack of enthusiasm for participation. Relying solely on national initiatives is not conducive to international cooperation. It is an urgent need for cross-border cooperation and assistance from the World Health Organization and other international organizations (Brugnara & Marx, 2021). For example, in the prevention and control of COVID-19, the WHO not only provides an important platform for countries to cooperate in combating the epidemic, but also assists less-developed countries to strengthen health capacity building and improve epidemic prevention and control capacity. In fact, whether the external assistance provided by these international organizations is really effective. Whether there are more effective measures to promote the development of international joint prevention and control has become a concern and urgent issue.

Considering that international joint prevention and control is a continuous dynamic game process involving two or more parties, the level of joint prevention and control will be continuously updated and revised according to the situation of the epidemic. Therefore, it is necessary to consider the impact of the dynamic behavior of decision-making subjects
on international joint prevention and control. A differential game is a dynamic game problem that studies competition and cooperation between two or more parties over continuous time (Jørgensen & Gromova, 2016). Although some scholars have introduced it into the prevention and control of the epidemic, they consider the vertical game between the central government and local governments (Yin & Zhang, 2021). The international horizontal game is the key measure to curb the global pandemic. Therefore, this article uses the differential game method to study the problems of international cooperation in epidemic prevention, and discusses the decision-making problems of international joint prevention and control under different decision-making situations.

Based on the above analysis, the following questions are addressed: (1) What are the key factors affecting international joint prevention and control? (2) What is the impact of external interventions, such as the WHO on the prevention and control decision-making of international cooperation? (3) What is the impact of internal cost sharing among countries on the benefits of all parties and even the system? To answer these questions, this article constructs an international joint prevention and control system composed of countries (strong epidemic control capabilities) and countries (weak epidemic control capabilities). The optimal prevention and control decisions of various countries under spontaneous governance, external subsidies and internal cost sharing were compared and analyzed.

The contribution of this paper is mainly reflected in three aspects: (1) Since COVID-19 international cooperation is a long-term dynamic process, and differential games can solve continuous-time dynamic equilibrium solutions. The differential game model can better solve the problem of international cooperation in epidemic control; (2) A differential game model for the joint prevention and control of countries \( A \) and \( B \) is constructed. Based on this model, the impact of the prevention and control efforts of both parties on the level of joint prevention and control is discussed; (3) Two scenarios of external subsidy and internal cost sharing are designed to coordinate prevention and control efforts among countries, making them more willing to participate in international cooperation to fight the epidemic together.

This study is arranged as follows. In Sect. 2, the relevant literature is reviewed. Section 3 presents the problem. In Sect. 4, the strategy research under different decision-making situations is analyzed. And simulations are conducted in Sect. 5. Section 6 summarizes the main conclusions and points out the shortcomings.

2 Literature review

This article mainly studies the problem of international cooperation in the prevention and control of public health emergencies. The literature related to this problem is mainly carried out from three aspects: epidemic prevention and control, subsidies and differential games.

In terms of epidemic prevention and control, some scholars pointed out that governments of various countries mainly use testing, isolation, blockade and other measures to block or reduce the disease spread, which can effectively mitigate the risk of infection (Bin et al., 2021; Carli et al., 2020; Chinazzi et al., 2020). Gao and Yu (2020) argued that only international cooperation can minimize the losses caused by the pandemic. However, the publicity and sharing of individual benefits in international relations have led to the “free-rider” behavior among cooperative members (Brown & Susskind, 2020). For example, some scholars believe that free-riding behavior can easily lead to negative emotions and
a sense of unfairness among cooperators, thereby affecting the cooperation between game subjects (Caparrós & Finus, 2020; Yong & Choy, 2021). Yang et al. (2021) pointed out that in the case of population mobility, regional cooperative governance can reduce the number of infections and maximize the overall benefit of the region. Considering that it is difficult for governments to cooperate voluntarily, it is necessary to promote international cooperation through incentive and coordination mechanisms (Amaya & De Lombaerde, 2021; Barrett, 2016; Carlson et al., 2021). Mamani et al. (2013), Enayati and Özaltın (2020) studied the inefficiency of influenza vaccine distribution among governments, and found that lack of coordination would lead to uneven distribution of influenza vaccines in different regions. In addition, Malik et al. (2021) found that the lack of cooperation among countries led to the large-scale spread of the epidemic in South Asia. It can be seen from the above literature that international cooperation plays an important role in controlling the spread of the epidemic. In particular, although Yang et al. (2021) pointed out that regional cooperation can maximize the overall benefits, but they did not take into account the long-term and dynamic nature of international joint prevention and control, nor did they propose a more effective incentive mechanism to ensure the realization of cooperative governance.

In terms of subsidies, Arefin et al. (2020) believed that subsidies, as an external incentive, can change the decision-making behavior of decision makers. Some scholars studied the inefficiency of influenza vaccine distribution through cost-sharing contracts. It is found that this mechanism can effectively improve the unbalanced distribution of influenza vaccine in domestic regions (Enayati & Özaltın, 2020; Mamani et al., 2013). Some scholars also designed different vaccine subsidy policies to evaluate which subsidy policy was the most beneficial to suppress disease transmission and found that the degree-dependent subsidy scheme outperforms the free ticket policy (Tanaka & Tanimoto, 2020; Tatsukawa et al., 2021). Kabir et al., (2021) proposed an epidemic model related to behavioral dynamics, and found that using bonuses for emergency rescue package at the individual level can reduce infection and improve the success of quarantine policy. Ma et al. (2022) found that the overall utility of patients can be greatly improved by dynamically implementing the bed subsidy plan at different stages of the COVID-19 pandemic. Shen et al. (2021) believed that government subsidies to consumers can promote the development of mask companies and improve social welfare. Nigro et al. (2021) pointed out that under COVID-19, retailers and suppliers can realize the coordinated development of the supply chain by applying for loans from financial institutions or retailers requesting suppliers to provide financial support. The above literatures mainly studied the effectiveness of subsidy policies from the perspective of government subsidies or enterprises subsidies. However, it did not consider how to formulate subsidy policies in international cooperation to encourage countries to actively participate in international cooperation. Based on this, this paper designs different subsidy scenarios to observe the impact of subsidy policies on countries’ participation in international cooperation.

Considering the long-term and dynamic nature of epidemic prevention and control, the differential game is an important method to effectively solve this problem. Some scholars use the differential game to study the problem of international environmental cooperation governance and found that when developed countries are willing to provide pollution control subsidies to developing countries, the amount of pollution will be reduced (Li & Chen, 2021; Xiao et al., 2022). Some scholars have also applied it in the supply chain. For example, Li (2020) and Zhang et al. (2013) studied the impact of internal cost subsidies on corporate social responsibility and advertising goodwill in the supply chain by constructing a differential game model composed of manufacturers and retailers. Zhu et al. (2021) studied the long-term dynamic cooperation of two-level supply chain composed of suppliers.
and manufacturers under government subsidies with mask output as the state variable. In addition, in terms of inter-government cooperation, Yin and Zhang, (2021) revealed the inter-city multi-agent emergency prevention and control mechanism by constructing a differential game model led by the central and local governments. It is found that the epidemic prevention and control efficiency is highest under the collaborative cooperation mode. According to the above literature, differential game is widely used in environmental pollution and supply chain coordination, but rarely in epidemic prevention and control. Although the literature (Yin & Zhang, 2021) studied the emergency prevention and control mechanism between inter-city multi-agents, they focused on the decision-making between the vertical central government and local governments. However, the decision-making of joint epidemic prevention and control between horizontal asymmetric countries has not been studied. Based on this, we established a differential game model to study the decision-making behavior of countries in international prevention and control, in order to provide decision support for international joint prevention and control.

3 Problem presentation

This paper considers the international joint prevention and control system composed of country A (strong ability in epidemic prevention and control) and country B (weak ability in epidemic prevention and control) to study the joint prevention and control strategies of the two governments. Among them, country A can provide necessary assistance to country B after effectively controlling its epidemic situation. For example, the United States and China, as the world’s largest developed and largest developing countries, should actively provide assistance to other countries with weaker prevention and control capabilities, and make greater contributions to the people of all countries in the early victory over the epidemic. The notations and definition are shown in Table 1.

During the outbreak of COVID-19, due to differences in the level of economic development, medical level, and concept, various countries invested different prevention and control efforts. For countries with high medical level and perfect prevention and control systems, scientific and effective prevention and control measures can be quickly and

| Notations | Definition |
|-----------|------------|
| $E_A, E_B$ | The prevention and control efforts of countries A and B |
| $k_A, k_B$ | The prevention and control cost coefficients of countries A and B |
| $\lambda_1, \lambda_2$ | The impact degree of the prevention and control efforts of countries A and B on the level of joint prevention and control |
| $\beta_1, \beta_2$ | The impact degree of the prevention and control efforts of countries A and B on social benefits |
| $\delta$ | The attenuation degree of the joint prevention and control level |
| $\gamma$ | The risk factor of the joint prevention and control level |
| $\eta$ | The impact degree of the prevention and control level on social benefits |
| $\omega$ | Social benefit distribution ratio $\omega \in (0, 1)$ |
| $\rho$ | Discount rate |
| $L(t)$ | The level of joint prevention and control at time $t$ |
| $D(t)$ | Social benefits brought by international joint prevention and control |
timely formulated and implemented to minimize the losses caused by the epidemic (Gao & Yu, 2020). Assume that the efforts of countries A and B to respond to COVID-19 are $E_A(t) \geq 0$ and $E_B(t) \geq 0$, respectively. $I_A(E_A(t))$, $I_B(E_B(t))$ represent the cost of prevention and control invested by countries A and B, such as the cost of a series of intervention measures such as testing, isolation, financial subsidies, and procurement of emergency prevention and control materials. Because the prevention and control measures invested by each country are proportional to their prevention and control efforts. As the workload increases, the cost of further prevention and control will also increase (Yin & Zhang, 2021). Therefore, it is assumed that the prevention and control costs invested by countries A and B at time $t$ are:

$$I_A(E_A(t)) = \frac{k_A}{2} E_A^2(t), \quad I_B(E_B(t)) = \frac{k_B}{2} E_B^2(t).$$

where $k_A$ and $k_B$, respectively, represent the epidemic prevention and control cost coefficients of countries A and B.

At the same time, the expected benefits brought by the prevention and control efforts of different countries are different. That is, the prevention and control efforts invested by different countries affect the level of international joint prevention and control, and it is a dynamic process over time $t$. In addition, with the improvement of the international joint prevention and control, the risk of COVID-19 infection is gradually reduced (Tuite et al., 2020). That is, the risk factor of COVID-19 infection is closely related to the level of prevention and control. Therefore, we used parameter $\gamma (\gamma > 0)$ to capture the risk of COVID-19 infection under international joint prevention and control, $\delta (\delta > 0)$ indicates the degree of attenuation of prevention and control level caused by factors such as the liberalization of Western countries and the decline in vaccine protection rates. Then the change of joint prevention and control level over time satisfies the following differential equation:

$$L(t) = \lambda_1 E_A(t) + \lambda_2 E_B(t) - \delta L(t) - \gamma L(t)$$

where the initial value is $L(0) = L_0 \geq 0$. $\lambda_1$ and $\lambda_2$, respectively, indicate the impact degree of the epidemic prevention and control efforts of countries A and B at time $t$ on the level of joint prevention and control.

In the process of COVID-19 prevention and control, the prevention and control efforts of various countries and the level of joint prevention and control affect the final prevention and control effect, that is, the benefits of joint prevention and control. Therefore, we assume that the social benefits brought by international joint prevention and control are a linear function of the prevention and control efforts and the level of joint prevention and control. Then the social benefits brought by international joint epidemic control at time $t$ are:

$$D(t) = M + \beta_1 E_A(t) + \beta_2 E_B(t) + \eta L(t)$$

where $M > 0$ represents the initial social benefit. $\beta_1 (\beta_1 > 0)$ and $\beta_2 (\beta_2 > 0)$ indicate the impact degree of the prevention and control efforts of countries A and B on social benefits. That is, the emergency management capacity of countries A and B. $\eta$ is the impact degree of the prevention and control level on social benefits.

It is assumed that the effect coefficients of social benefits brought by international joint prevention and control on the benefits of countries A and B are, respectively, $\omega$ and $1 - \omega$, where $\omega \in (0, 1)$. Both governments have the same discount rate $\rho (\rho > 0)$. 
4 International joint prevention and control decision analysis

The international joint prevention and control of COVID-19 is a long-term dynamic process. Both countries with strong epidemic control capabilities and weak epidemic control capabilities pursue profit maximization from a long-term perspective. Therefore, we have introduced the dynamic framework into international joint prevention and control. The differential game is used to study the optimal decision-making problem of countries under three scenarios: spontaneous governance, external subsidies of WHO, and internal cost sharing.

4.1 Equilibrium analysis under the spontaneous governance

In the case of spontaneous governance, countries $A$ and $B$ will independently choose their own optimal joint prevention and control levels at the same time to maximize their own interests. At this point, the objective functions of countries $A$ and $B$ are, respectively:

$$\Pi_A = \int_0^\infty e^{-\rho t} \{ \omega [M + \beta_1 E_A(t) + \beta_2 E_B(t) + \eta L(t)] - \frac{k_A}{2} E_A^2(t) \} dt$$

$$\Pi_B = \int_0^\infty e^{-\rho t} \{ (1 - \omega) [M + \beta_1 E_A(t) + \beta_2 E_B(t) + \eta L(t)] - \frac{k_B}{2} E_B^2(t) \} dt$$

where $E_A(t)$ and $E_B(t)$ are control variables, and $L(t)$ is the state variables. Because this model becomes very difficult to solve in the presence of dynamic parameters, the parameters in the model are assumed to be time-independent constants in the treatment method used by the reference (Jørgensen & Gromova, 2016). In addition, for the convenience of notation, the time unit $t$ will be omitted in the following text.

**Proposition 1** In the spontaneous governance model, the static feedback equilibrium strategies of countries $A$ and $B$ are, respectively:

$$E_A^* = \frac{\omega[(\rho + \delta + \gamma)\beta_1 + \lambda_1 \eta]}{(\rho + \delta + \gamma)k_A}$$  \hspace{1cm} (1)

$$E_B^* = \frac{(1 - \omega) [(\rho + \delta + \gamma)\beta_2 + \lambda_2 \eta]}{(\rho + \delta + \gamma)k_B}$$  \hspace{1cm} (2)

The optimal trajectory of the joint prevention and control level is:

$$L^*(t) = \frac{\lambda_1 \omega[(\rho + \delta + \gamma)\beta_1 + \lambda_1 \eta]}{(\delta + \gamma)(\rho + \delta + \gamma)k_A} + \frac{\lambda_2(1 - \omega)[(\rho + \delta + \gamma)\beta_2 + \lambda_2 \eta]}{(\delta + \gamma)(\rho + \delta + \gamma)k_B} + e^{(\delta + \gamma)t} \left( L_0 - \frac{\lambda_1 \omega[(\rho + \delta + \gamma)\beta_1 + \lambda_1 \eta]}{(\delta + \gamma)(\rho + \delta + \gamma)k_A} - \frac{\lambda_2(1 - \omega)[(\rho + \delta + \gamma)\beta_2 + \lambda_2 \eta]}{(\delta + \gamma)(\rho + \delta + \gamma)k_B} \right)$$ \hspace{1cm} (3)

The optimal social benefit of countries $A$ and $B$ and the system benefit are, respectively:
From the sufficient conditions of static feedback equilibrium, for any $L \geq 0$, it can be assumed that there exists HJB (Hamilton–Jacobi–Bellman) equation for a continuous bounded differential game benefit function $V_i(L)$, $i \in (A, B)$.

$$V_A^* = \frac{\omega \eta}{\rho + \delta + \gamma} - L + \frac{\omega M}{\rho} + \frac{\omega^2[(\rho + \delta + \gamma)\beta_1 + \lambda_1 \eta]^2}{2(\rho + \delta + \gamma)^2 k_A} + \frac{\omega(1 - \omega)[(\rho + \delta + \gamma)\beta_2 + \lambda_2 \eta]^2}{\rho(\rho + \delta + \gamma)^2 k_B}$$ (4)

$$V_B^* = \frac{(1 - \omega)\eta}{\rho + \delta + \gamma} - L + \frac{(1 - \omega)M}{\rho} + \frac{\omega(1 - \omega)[(\rho + \delta + \gamma)\beta_1 + \lambda_1 \eta]^2}{\rho(\rho + \delta + \gamma)^2 k_A} + \frac{(1 - \omega)^2[(\rho + \delta + \gamma)\beta_2 + \lambda_2 \eta]^2}{2(\rho + \delta + \gamma)^2 k_B}$$ (5)

$$V^* = \frac{\eta}{(\rho + \delta + \gamma)} - L + \frac{M}{\rho} + \frac{(2\omega - \omega^2)[(\rho + \delta + \gamma)\beta_1 + \lambda_1 \eta]^2}{2(\rho + \delta + \gamma)^2 k_A} + \frac{(1 - \omega^2)[(\rho + \delta + \gamma)\beta_2 + \lambda_2 \eta]^2}{2(\rho + \delta + \gamma)^2 k_B}$$ (6)

**Proof:** From the sufficient conditions of static feedback equilibrium, for any $L \geq 0$, it can be assumed that there exists HJB (Hamilton–Jacobi–Bellman) equation for a continuous bounded differential game benefit function $V_i(L)$, $i \in (A, B)$.

$$\rho V_A = \max_{E_A} \{\omega[M \beta_1 E_A + \beta_2 E_B + \eta L] - \frac{k_A}{2} E_A^2 + V_A'(\lambda_1 E_A + \lambda_2 E_B - \delta L - \gamma L)\}$$ (7)

$$\rho V_B = \max_{E_B} \{(1 - \omega)[M \beta_1 E_A(t) + \beta_2 E_B(t) + \eta L(t)] - \frac{k_B}{2} E_B^2(t) + V_B'(\lambda_1 E_A + \lambda_2 E_B - \delta L - \gamma L)\}$$ (8)

Take the first-order partial derivative of Eqs. (7) and (8) with respect to $E_A, E_B$ and equal to zero. It is easy to obtain:

$$E_A = \frac{\beta_1 \omega + \lambda_1 V_A'}{k_A}$$ (9)

$$E_B = \frac{(1 - \omega)\beta_2 + \lambda_2 V_B'}{k_B}$$ (10)

Incorporate Eqs. (9) and (10) into Eqs. (7) and (8), simplify and sort out:

$$\rho V_A = (\omega \eta - (\delta + \gamma)V_A')L + \omega M + \frac{(\beta_1 \omega + \lambda_1 V_A')^2}{2k_A} + \frac{(\beta_2 \omega + \lambda_2 V_A')(\lambda_2 V_B' + (1 - \omega)\beta_2)}{k_B}$$ (11)

$$\rho V_B = ((1 - \omega)\eta - (\delta + \gamma)V_B')L + (1 - \omega)M + \frac{(\beta_2(1 - \omega) + \lambda_1 V_B')\omega \beta_1 + \lambda_1 V_A'}{k_A} + \frac{(\lambda_2 V_B' + (1 - \omega)\beta_2)^2}{2k_B}$$ (12)

From Eqs. (11) and (12), we can see that the linear optimal function of $L$ is the solution of the HJB equation. Let

$$V_A = h_1 L + b_1, \quad V_B = h_2 L + b_2$$ (13)

where $h_1, h_2, b_1$ and $b_2$ are constant. Incorporating Eq. (13) into Eqs. (11) and (12), it is easy to obtain:

$$\rho(h_1 L + b_1) = (\omega \eta - (\delta + \gamma)V_A')L + \omega M + \frac{(\beta_1 \omega + \lambda_1 V_A')^2}{2k_A} + \frac{(\beta_2 \omega + \lambda_2 V_A')(\lambda_2 V_B' + (1 - \omega)\beta_2)}{k_B}$$ (14)
\[ \rho(h_2L + b_2) = ((1 - \omega)\eta - (\delta + \gamma)V_B')L + (1 - \omega)M \]
\[ + \frac{(\lambda_2V_B' + (1 - \omega)\beta_2)^2}{2k_B} + \frac{\beta_2(1 - \omega) + \lambda_1V_B'}{k_A} \]  
(15)

The parameters of the optimal benefit function can be calculated as
\[
\begin{aligned}
    h_1 &= \frac{\omega\eta}{\rho + \delta + \gamma} \\
    b_1 &= \frac{\omega M}{\rho} + \frac{\omega^2[(\rho + \delta + \gamma)\beta_1 + \lambda_1\eta]^2}{2(\rho + \delta + \gamma)^2k_A} + \frac{\omega(1 - \omega)[(\rho + \delta + \gamma)\beta_2 + \lambda_2\eta]^2}{\rho(\rho + \delta + \gamma)^2k_B}
\end{aligned}
\]
(16)

\[
\begin{aligned}
    h_2 &= \frac{(1 - \omega)\eta}{\rho + \delta + \gamma} \\
    b_2 &= \frac{(1 - \omega)M}{\rho} + \frac{\omega(1 - \omega)[(\rho + \delta + \gamma)\beta_1 + \lambda_1\eta]^2}{\rho(\rho + \delta + \gamma)^2k_A} + \frac{(1 - \omega)^2[\beta_2(\rho + \delta + \gamma) + \lambda_2\eta]^2}{2\rho(\rho + \delta + \gamma)^2k_B}
\end{aligned}
\]
(17)

Substituting Eqs. (16) and (17) into Eqs. (9), (10) and (11), (12), the optimal prevention and control efforts (1) and (2) and optimal social benefits (4-6) of affected countries A and B can be obtained, respectively.

**Proposition 1** is proved.

**Corollary 1** In the spontaneous governance model, the optimal prevention and control efforts \( E_A^* \) and \( E_B^* \) of countries A and B are negatively correlated with the discount rate \( \rho \), the prevention and control cost coefficients \( k_A \) and \( k_B \), the decay rate \( \delta \), and the risk factor \( \gamma \). It is positively correlated with impact degree \( \beta_1 \) and \( \beta_2 \) of social benefit, the impact degree of prevention and control efforts on the joint prevention and control level \( \lambda_1 \) and \( \lambda_2 \), the distribution ratio \( \omega \) of social benefits, and the impact degree \( \eta \) of the prevention and control level on social benefits.

It can be seen from Corollary 1 that when engaged in international joint prevention and control, countries A and B will comprehensively consider its cost, the improvement effect of joint prevention and control levels, and the improvement degree of social benefits to determine the optimal prevention and control efforts. When the investment in prevention and control efforts can greatly improve the level of joint prevention and control, both sides will actively invest in prevention and control efforts. At the same time, the prevention and control efforts of countries A and B are also constrained by their prevention and control costs. The higher cost of prevention and control will reduce the willingness of both parties to invest in prevention and control efforts. In addition, the optimal prevention and control efforts of the two countries are positively related to the distribution proportion of social benefits.

**Corollary 2** Under spontaneous governance, the optimal trajectory of the joint prevention and control level \( L(t) \) presents a monotonic characteristic. That is, taking
\[
L_0 = \frac{\lambda_1(\rho + \delta + \gamma)\beta_1 + \lambda_1\eta}{(\delta + \gamma)(\rho + \delta + \gamma)k_A} + \frac{\lambda_2(1 - \omega)[(\rho + \delta + \gamma)\beta_2 + \lambda_2\eta]}{(\delta + \gamma)(\rho + \delta + \gamma)k_B}
\]

as the critical point, when
L_0 < \frac{\lambda_1 \omega ((\rho + \delta + \gamma) \beta_1 + \lambda_1 \eta)}{(\delta + \gamma)(\rho + \delta + \gamma) k_A} + \frac{\lambda_2 (1 - \omega)((\rho + \delta + \gamma) \beta_1 + \lambda_1 \eta)}{(\delta + \gamma)(\rho + \delta + \gamma) k_B}, \text{ the level of joint prevention and control increases monotonously with time; When } L_0 > \frac{\lambda_1 \omega ((\rho + \delta + \gamma) \beta_1 + \lambda_1 \eta)}{(\delta + \gamma)(\rho + \delta + \gamma) k_A} + \frac{\lambda_2 (1 - \omega)((\rho + \delta + \gamma) \beta_1 + \lambda_1 \eta)}{(\delta + \gamma)(\rho + \delta + \gamma) k_B}, \text{ the level of joint prevention and control decreases monotonously over time; When } L_0 = \frac{\lambda_1 \omega ((\rho + \delta + \gamma) \beta_1 + \lambda_1 \eta)}{(\delta + \gamma)(\rho + \delta + \gamma) k_A} + \frac{\lambda_2 (1 - \omega)((\rho + \delta + \gamma) \beta_1 + \lambda_1 \eta)}{(\delta + \gamma)(\rho + \delta + \gamma) k_B}, \text{ the level of joint prevention and control is constant and does not change over time.}

From Corollary 2, there is a threshold for the level of joint prevention and control. Only when the initial prevention and control level at \( t = 0 \) is less than the threshold, the joint prevention and control level will increase monotonically with time. When the initial prevention and control level is higher than the threshold, the joint prevention and control level cannot be improved, which is not conducive to countries investing in higher prevention and control efforts to curb the spread of the epidemic.

According to Proposition 1 and Corollaries 1 and 2, under the spontaneous governance model, countries A and B, as relatively independent subjects of epidemic prevention and control, choose the optimal prevention and control efforts based on the principle of maximizing their own interests. It is difficult for the system to achieve a high level of joint prevention and control through pure spontaneous management. It cannot effectively curb the international spread of public health events such as COVID-19. The next section will focus on the impact of external subsidies such as the WHO on the prevention and control decisions of both countries.

4.2 Equilibrium analysis under external subsidies

In order to encourage countries to actively fight the epidemic, the WHO and other external forces have provided funds, materials and other assistance to countries affected by the epidemic to improve their prevention and control level. It is assumed that the subsidy level depends on the increase of the joint prevention and control level, that is, the function \( f(L) \) of the joint prevention and control level. \( \sigma \) is the subsidy coefficient of the WHO. In order to simplify the calculation process, let \( f(L(t)) = L(t) \). At this time, the objective functions of countries A and B are:

\[
\prod_A = \int_0^\infty e^{-\rho t} \left\{ \omega [M + \beta_1 E_A(t) + \beta_2 E_B(t) + \eta L(t) + \sigma L] - \frac{k_A}{2} E_A^2(t) \right\} dt
\]

\[
\prod_B = \int_0^\infty e^{-\rho t} \left\{ (1 - \omega) [M + \beta_1 E_A(t) + \beta_2 E_B(t) + \eta L(t) + \sigma L] - \frac{k_B}{2} E_B^2(t) \right\} dt
\]

**Proposition 2** In the external subsidies such as the WHO, the optimal prevention and control efforts of countries A and B are:

\[
E^{**}_A = \frac{\omega \left[ (\rho + \delta + \gamma) \beta_1 + \lambda_1 (\eta + \sigma) \right]}{(\rho + \delta + \gamma) k_A}
\] (18)
The optimal trajectory of the joint prevention and control level is:

\[
L^{\ast}(t) = \frac{\lambda_1 \omega[(\rho + \delta + \gamma)\beta_1 + \lambda_1(\eta + \sigma)]}{(\delta + \gamma)(\rho + \delta + \gamma)k_A} + \frac{\lambda_2(1 - \omega)[(\rho + \delta + \gamma)\beta_2 + \lambda_2(\eta + \sigma)]}{(\delta + \gamma)(\rho + \delta + \gamma)k_B} + e^{(\delta + \gamma)t}(L_0 - \frac{\lambda_1 \omega[(\rho + \delta + \gamma)\beta_1 + \lambda_1(\eta + \sigma)]}{(\delta + \gamma)(\rho + \delta + \gamma)k_A} - \frac{\lambda_2(1 - \omega)[(\rho + \delta + \gamma)\beta_2 + \lambda_2(\eta + \sigma)]}{(\delta + \gamma)(\rho + \delta + \gamma)k_B})
\]

(20)

The optimal social benefit of countries A and B and the system benefit are, respectively:

\[
V^{\ast}_A = \frac{\omega(\eta + \sigma)}{\rho + \delta + \gamma}L + \frac{\omega M}{\rho} + \frac{\omega^2[(\rho + \delta + \gamma)\beta_1 + \lambda_1(\eta + \sigma)]^2}{2\rho(\rho + \delta + \gamma)^2k_A} + \frac{\omega(1 - \omega)[(\rho + \delta + \gamma)\beta_2 + \lambda_2(\eta + \sigma)]^2}{\rho(\rho + \delta + \gamma)^2k_B}
\]

\[
V^{\ast}_B = \frac{(1 - \omega)(\eta + \sigma)}{\rho + \delta + \gamma}L + \frac{(1 - \omega)M}{\rho} + \frac{\omega(1 - \omega)[(\rho + \delta + \gamma)\beta_1 + \lambda_1(\eta + \sigma)]^2}{\rho(\rho + \delta + \gamma)^2k_A} + \frac{(1 - \omega)^2[(\rho + \delta + \gamma)\beta_2 + \lambda_2(\eta + \sigma)]^2}{2\rho(\rho + \delta + \gamma)^2k_B}
\]

\[
V^{\ast} = \frac{(\eta + \sigma)}{\rho + \delta + \gamma}L + \frac{M}{\rho} + \frac{(2\omega - \omega^2)[(\rho + \delta + \gamma)\beta_1 + \lambda_1(\eta + \sigma)]^2}{2\rho(\rho + \delta + \gamma)^2k_A} + \frac{(1 - \omega^2)[(\rho + \delta + \gamma)\beta_2 + \lambda_2(\eta + \sigma)]^2}{2\rho(\rho + \delta + \gamma)^2k_B}
\]

(23)

The proof process is similar to Proposition 1, so it is omitted.

**Corollary 3** In the external subsidies such as the WHO, the optimal prevention and control efforts of countries A and B are greater than those under spontaneous governance. It is positively correlated with the subsidy coefficient \( \sigma \) of the WHO.

This shows that external subsidies have shared the prevention and control costs of the two countries, reduced their prevention and control pressures, improved the prevention and control efforts of the two countries, and finally improved the level of joint prevention and control. This means that external subsidies can encourage the two countries to prevent and control the epidemic, and lay the foundation for the establishment of joint prevention and control between the two countries.

Similar to Corollary 1, the optimal prevention and control efforts \( E_A^{\ast\ast} \) and \( E_B^{\ast\ast} \) of countries A and B are negatively correlated with the discount rate \( \rho \), the prevention and control cost coefficients \( k_A \) and \( k_B \), the decay rate \( \delta \), and the risk factor \( \gamma \). It is positively correlated with impact degree \( \beta_1 \) and \( \beta_2 \) of social benefit, the impact degree of prevention and control efforts on the joint prevention and control level \( \lambda_1 \) and \( \lambda_2 \), the distribution ratio \( \omega \) of social benefits, and the impact degree \( \eta \) of the prevention and control level on social benefits.
4.3 Equilibrium analysis under internal cost sharing

On the basis of external subsidies, in order to further encourage country B to improve its prevention and control efforts, country A will provide necessary assistance to country B to improve its epidemic prevention and control capacity. At this time, a Stackelberg game relationship is formed between the two parties. Country A first determines the optimal prevention and control effort and sharing ratio. Then Country B determines the optimal prevention and control effort. Suppose that the proportion of country A to bear the prevention and control cost of country B is $s(t)(0 \leq s(t) < 1)$, then the objective functions of countries A and B are:

\[
\prod_A = \int_0^\infty e^{-\rho t}\{\omega(M + \beta_1 E_A(t) + \beta_2 E_B(t) + \eta L(t) + \sigma L) - \frac{k_A}{2} E_A^2(t) - \frac{k_B}{2} s E_B^2(t)\} dt
\]

\[
\prod_B = \int_0^\infty e^{-\rho t}\{(1 - \omega)(M + \beta_1 E_A(t) + \beta_2 E_B(t) + \eta L(t) + \sigma L) - \frac{k_B}{2}(1 - s) E_B^2(t)\} dt
\]

**Proposition 3** On the basis of external subsidies, country A shares part of the epidemic prevention and control costs of country B the optimal prevention and control efforts of countries A and B are:

\[
E^{***}_A = \frac{\omega[(\rho + \delta + \gamma)\beta_1 + \lambda_1(\eta + \sigma)]}{(\rho + \delta + \gamma)k_A} \quad (24)
\]

\[
E^{***}_B = \frac{(1 + \omega)[(\rho + \delta + \gamma)\beta_2 + \lambda_2(\eta + \sigma)]}{2(\rho + \delta + \gamma)k_B} \quad (25)
\]

\[
\begin{cases} 
  s = \frac{(3\omega-1)}{(\omega+1)}, & \omega > 1/3 \\
  0, & \omega \leq 1/3 
\end{cases} \quad (26)
\]

The optimal trajectory of the joint prevention and control level is:

\[
L^{**}(t) = \frac{\lambda_1 \omega[(\rho + \delta + \gamma)\beta_1 + \lambda_1(\eta + \sigma)]}{(\delta + \gamma)(\rho + \delta + \gamma)k_A} + \frac{\lambda_2(\omega + 1)[(\rho + \delta + \gamma)\beta_2 + \lambda_2(\eta + \sigma)]}{2(\delta + \gamma)(\rho + \delta + \gamma)k_B}
\]

\[
+ e^{(s+\gamma)t}L_0 - \frac{\lambda_1 \omega[(\rho + \delta + \gamma)\beta_1 + \lambda_1(\eta + \sigma)]}{(\delta + \gamma)(\rho + \delta + \gamma)k_A} - \frac{\lambda_2(\omega + 1)[(\rho + \delta + \gamma)\beta_2 + \lambda_2(\eta + \sigma)]}{2(\delta + \gamma)(\rho + \delta + \gamma)k_B} \quad (27)
\]

The optimal social benefit of countries A and country B and the system benefit are, respectively:

\[
V_A(L) = \frac{\omega(\eta + \sigma)}{(\rho + \delta + \gamma)}L + \frac{\omega M}{\rho} + \frac{\omega^2[(\rho + \delta + \gamma)\beta_1 + \lambda_1(\eta + \sigma)]^2}{2(\rho + \delta + \gamma)^2k_A}
\]

\[
+ \frac{(\omega + 1)^2[(\rho + \delta + \gamma) + \lambda_2(\eta + \sigma)]}{8(\rho + \delta + \gamma)^2k_B} \quad (28)
\]
\( V_B(L) = \frac{(1 - \omega)(\eta + \sigma)}{(\rho + \delta + \gamma)} L + \frac{(1 - \omega)M}{\rho} + \omega(1 - \omega)[(\rho + \delta + \gamma)\beta_1 + \lambda_1(\eta + \sigma)]^2 \)

\[ \rho V_B = \max_{E_B} \{(1 - \omega)[M + \beta_1 E_A + \beta_2 E_B + \eta L + \sigma L] - \frac{k_B}{2}(1 - s)E_B^2 + V_B'(\lambda_1 E_A + \lambda_2 E_B - \delta L - \gamma L)\} \]

Taking the first-order partial derivative in Eq. (31) with respect to \( E_B \) and equating to zero, we can get:

\[ E_B = \frac{(1 - \omega)\beta_2 + \lambda_2 V_B'}{k_B(1 - s)} \]

In order to maximize its own benefits, the rational country \( A \) will choose its own prevention and control efforts according to Eqs. (32). At this time, the HJB equation of country \( A \) is:

\[ \rho V_A = \max_{E_A} \{\omega[M + \beta_1 E_A + \beta_2 E_B + \eta L + \sigma L] - \frac{k_A}{2}E_A^2 - \frac{k_B}{2}sE_B^2 + V_A'(\lambda_1 E_A + \lambda_2 E_B - \delta L - \gamma L)\} \]

Substituting Eq. (32) into (33) and solving the first-order conditions of \( E_A \) and \( s \), we can get:

\[ E_A = \frac{\beta_1 \omega + \lambda_1 V_A'}{k_A} \]

\[ s = \frac{2\beta_2(1 - \omega)\beta_2 + 2\lambda_2 V_A'}{2\beta_2(1 - \omega) + 2\lambda_2 V_A'}B > A \]

\[ s = 0, B \leq A \]

where \( V_A' = \partial V_A/\partial L, \ V_B' = \partial V_B/\partial L \).

Let \( B = 2\beta_2(1 - \omega) + 2\lambda_2 V_A' \). \( A = \beta_2(1 - \omega) + \lambda_2 V_B' \). Substitute Eqs. (34) and (35) into Eqs. (31) and (33), and simplify to get:

\[ \rho V_A = [\omega(\eta + \sigma) - (\delta + \gamma)V_A']L + \omega M + \frac{(\omega \beta_1 + \lambda_1 V_A')^2}{2k_A} + \frac{(\beta_2(1 + \omega) + \lambda_2(2V_A' + V_B')^2}{8k_B} \]
\[
\rho V_B = \left[ (1 - \omega)(\eta + \sigma) - (\delta + \gamma)V'_B \right] L + (1 - \omega)M + \frac{(\omega\beta_1 + \lambda_1 V'_A)((1 - \omega)\beta_1 + \lambda_1 V'_B)}{k_A} \\
+ \frac{[2\omega\beta_2 + (1 - \omega)\beta_2 + \lambda_2 V'_B + 2\lambda_2 V'_A][(1 - \omega)\beta_2 + \lambda_2 V'_B]}{4k_B}
\] (37)

From the structure of Eqs. (36) and (37), it can be seen that the linear optimal social benefit of \( L \) is the solution of the HJB equation. Let

\[
V_A(L) = p_1L + p_2, \quad V_B(L) = q_1L + q_2
\] (38)

where \( p_1, p_2, q_1 \) and \( q_2 \) are constant. Take the derivative of \( L \) in Eq. (38) and put it into Eq. (36) and (37). The parameters of the optimal social benefit can be obtained as follows:

\[
\begin{align*}
p_1 &= \frac{\omega(\eta + \sigma)}{\rho + \delta + \gamma} \\
p_2 &= \frac{\omega M}{\rho} + \frac{\omega^2[(\rho + \delta + \gamma)\beta_1 + \lambda_1(\eta + \sigma)]^2}{2(\rho + \delta + \gamma)^2k_A} + \frac{(1 + \omega^2)[\beta_2(\rho + \delta + \gamma) + \lambda_2(\eta + \sigma)]^2}{8(\rho + \delta + \gamma)^2k_B}
\end{align*}
\] (39)

\[
\begin{align*}
q_1 &= \frac{(1 - \omega)(\eta + \sigma)}{\rho + \delta + \gamma} \\
q_2 &= \frac{(1 - \omega)M}{\rho} + \frac{\omega(1 - \omega)[(\rho + \delta + \gamma)\beta_1 + \lambda_1(\eta + \sigma)]^2}{(\rho + \delta + \gamma)^2k_A} + \frac{(1 - \omega^2)[(\rho + \delta + \gamma)\beta_2 + \lambda_2(\eta + \sigma)]^2}{4(\rho + \delta + \gamma)^2k_B}
\end{align*}
\] (40)

By introducing Eqs. (39) and (40) into Eqs. (32), (34) and (35), it can be obtained that the optimal prevention and control efforts and the optimal cost sharing ratio of the two countries are Eqs. (24), (25) and (26) and the optimal social benefits (28)–(30).

**Corollary 4** It can be seen from Proposition 3 that when the distribution ratio of social benefit obtained by country A is higher than a certain threshold \((\omega > 1/3)\), the country A will subsidize country B. The optimal sharing ratio \( s \) is positively correlated with \( \omega \). (Take the partial derivative of the coefficient in \( s \), and the proof is omitted).

This means that only when the social benefits obtained by country A reach a certain threshold \((\omega > 1/3)\), the part of the prevention and control costs of country B will be shared. The greater the social benefits obtained by the country A, the greater the subsidy ratio. This also shows that country A can adjust the proportion of the cost of country B through its dominant position, and then mobilize country B to invest in the optimal prevention and control efforts.

Other parameters are similar to Corollary 1, so they are omitted.

### 4.4 Comparison and analysis

This section compares and analyzes the optimal prevention and control efforts, joint prevention and control level, and social benefits under the three modes of spontaneous governance, external subsidies and internal cost sharing, so as to provide decision support for
promoting international joint prevention and control. When the distribution ratio of social benefit obtained by country $A$ is a certain threshold ($1/3 < \omega < 1$), the following important propositions can be obtained:

**Proposition 4.** The optimal prevention and control efforts of countries $A$ and $B$ are $E^*_A < E^{**}_A = E^{***}_A$ and $E^*_B < E^{**}_B < E^{***}_B$.

Proposition 4 shows that external subsidies can improve the prevention and control efforts invested by both countries compared with the spontaneous governance model. This shows that external subsidies such as the World Health Organization can help ease the pressure on the affected countries to fight the epidemic and help member states invest more in prevention and control efforts. It will play an important role in promoting the establishment of an international joint prevention and control mechanism. On the basis of external subsidies, when $\omega > 1/3$, the optimal prevention and control efforts of country $A$ will remain unchanged in internal cost sharing, while the optimal prevention and control efforts of country $B$ increase.

This shows that under external subsidies, the optimal prevention and control efforts of country $A$ have reached the optimal level. At this time, in order to prevent the domestic epidemic from being imported from abroad, it is urgent to mobilize the enthusiasm of epidemic prevention and control efforts of country $B$, so as to achieve greater social benefits. For example, since the outbreak of the COVID-19, after the domestic epidemic has been basically controlled, China has actively dispatched anti-epidemic medical expert teams and aided anti-epidemic materials to Iraq, Pakistan and other developing countries to help them control the epidemic as soon as possible and improve their public health governance capacity.

**Proposition 5.** The joint prevention and control level is $L^*(t) < L^{**}(t) < L^{***}(t)$.

Proposition 5 shows that under the three modes of spontaneous governance, external subsidies and internal cost sharing, the level of joint prevention and control has gradually increased. This is because the international joint prevention and control level is affected by the prevention and control efforts between the two countries. Through Proposition 4, it can be found that with the implementation of external subsidies and internal cost sharing, the members of the system are encouraged to invest in higher prevention and control efforts. At this time, the international joint prevention and control level gradually improves, and finally reaches the optimal under certain conditions.

**Proposition 6.** The optimal social benefit of countries $A$ and $B$ are $V^*_A(t) < V^{**}_A(t) < V^{***}_A$ and $V^*_B(t) < V^{**}_B(t) < V^{***}_B$. The system benefits are $V^*(t) < V^{**}(t) < V^{***}(t)$.

Proposition 6 shows that, compared with the spontaneous governance model, the external subsidies can encourage countries $A$ and $B$ to increase their prevention and control efforts. When $\omega > 1/3$, internal cost sharing is superior to the first two models, which maximizes the social benefits of all parties and realizes the Pareto improvement of all parties.

Through the analysis of Propositions 4–6, it can be found that external subsidies are conducive to improving joint prevention and control efforts, further improving joint prevention and control level and social benefits. This finding is consistent with the actual
situation. Importantly, we also find that on the basis of external subsidies such as the WHO, when certain conditions are met, the WHO and other international organizations actively coordinate country A to share part of the prevention and control costs of country B. That is to say, increasing the internal interaction between international governments can improve the overall benefits of both countries and the system, and achieve Pareto improvement. It has important practical guiding significance for the promotion of international joint prevention and control.

5 Numerical simulation analysis

In this study, the optimal prevention and control efforts, joint prevention and control level and social benefits of countries A and B depend on the parameter selection in the model. Due to the difficulty in obtaining relevant data, we studied in detail the basic method of setting simulation parameters in the literature (Li, 2020; Wang et al., 2019), and assigned the parameters according to the parameter assumptions in this paper. Assume that the cost coefficient of prevention and control invested by countries with strong epidemic control capabilities is $k_A = 3$, and the impact degree of prevention and control efforts on the level of joint prevention and control is $\lambda_1 = 2$. Due to the restriction of economic development level, the investment cost of prevention and control in countries with weak epidemic control capacity is lower than that in countries with strong epidemic control capacity, denoting $k_B = 2$. The influence of prevention and control efforts on the joint prevention and control level is recorded as $\lambda_2 = 1$. The assignment of other parameters in the study can be set as $\delta = 0.1, \gamma = 0.2, \beta_1 = 1, \beta_2 = 1, \eta = 0.3, M = 2, \omega = 0.6$.

5.1 Comparative analysis of prevention and control efforts

In order to compare and analyze the optimal prevention and control efforts of countries A and B in the three decision-making scenarios, the above parameters are put into the prevention and control efforts function under the three decision-making scenarios to obtain the specific parameter values, as shown in Table 2.

According to Table 2, it can be seen that the optimal prevention and control efforts of countries A and B are the lowest under spontaneous governance. The optimal prevention and control efforts of both countries have been improved under external subsidies. In the internal cost sharing model, the epidemic prevention and control efforts of country A are consistent with those under external subsidies. The prevention and control efforts of country B are optimal. This result is consistent with the conclusions of Propositions 4.

| Table 2 | Optimal prevention and control efforts of the two countries |
|---------|----------------------------------------------------------|
| Decision situation | $E_A$ | $E_B$ |
| Spontaneous governance | 0.75 | 0.55 |
| external subsidy | 2.25 | 1.05 |
| internal cost sharing | 2.25 | 2.10 |
| Comparison and analysis | $E_A^* < E_A^{**} = E_A^{***}$ | $E_B^* < E_B^{**} < E_B^{***}$ |
5.2 Comparative analysis of optimal joint prevention and control level

Based on these parameters and the models solved in the previous section, MATLAB is used to obtain the optimal trajectory comparison chart of the joint prevention and control level in the three scenarios, as shown in Fig. 1.

It can be seen from Fig. 1 that the optimal trajectory of the joint prevention and control level tends to be stable with the increase of time. The joint prevention and control level of countries A and B is the lowest under spontaneous governance and highest under the internal cost sharing, which is consistent with the conclusion of Proposition 5. In addition, under the spontaneous governance, the growth rate of joint prevention and control level is the slowest and fastest under the internal cost sharing. With the improvement of epidemic prevention and control efforts of countries A and B, the joint prevention and control level of both countries has also reached the optimal state.

5.3 Comparative analysis of optimal social benefits

As shown in Figs. 2 and 3, the social benefits for countries A and B are compared among three game models. The social benefits of countries A and B are the lowest under spontaneous governance. When external subsidies are introduced, the social benefits of both sides of the game are improved. When \( \omega > 1/3 \), country A shares part of the joint prevention and control costs of country B. The social benefits of both parties are higher than those from the first model. This is consistent with the conclusion of Proposition 6. In addition, it is found that the social benefits of countries A and B gradually increase and tend to be stable in the three game scenarios.
5.4 Comparative analysis of optimal system benefits

Figure 4 describes the changes of the system benefits over time under the three game models. The results for external subsidies are superior to those of the spontaneous governance model, and the internal cost sharing model delivered the optimal results, which is consistent with the conclusion of Proposition 6. In addition, it is found that
the total benefits of the system in the three scenarios tend to be stable with the increase of time, which is caused by the stabilization of epidemic prevention and control efforts and joint prevention and control level with the increase of time.

6 Conclusions

Considering the long-term and dynamic nature of international joint prevention and control, the differential game method is used to compare and analyze the optimal decisions of countries in the three scenarios of spontaneous governance, external subsidies and internal cost sharing. The main results are as follows:

(1) The prevention and control efforts invested by countries A and B are negatively correlated with discount rates, prevention and control cost coefficients, decay rate and risk factors. It is positively correlated with the impact degree of social benefits, the impact degree of prevention and control efforts on the level of joint prevention and control, the distribution ratio of social benefits, and the impact degree of prevention and control level on social benefits.

(2) When the social benefit obtained by the country A exceeds a certain threshold ($\omega=1/3$), it will subsidize the country B. The subsidy ratio depends on the social benefit ratio obtained by the country A. As an effective incentive mechanism, the subsidies from country A can promote the joint efforts of countries A and B to control the epidemic.

(3) From the perspective of prevention and control efforts, the prevention and control efforts are the lowest in spontaneous governance. The external subsidies can effectively increase the investment in prevention and control efforts of both countries. When $\omega > 1/3$, compared with the external subsidies, the prevention and control efforts of countries with strong epidemic control capabilities remain unchanged, while those with weak epidemic control capabilities increase.
From the perspective of joint prevention and control level and social benefits, the external subsidies model is superior to the spontaneous governance model. It can effectively improve joint prevention and control level and social benefits. When $\omega > 1/3$, the internal cost sharing model maximizes the joint prevention and control level and social benefits. It is conducive to the rapid control of the spread of the epidemic.

This research provides important management implications for joint epidemic prevention among international governments: First, external subsidies can effectively stimulate international cooperation in epidemic prevention. It can not only improve the prevention and control efforts and level of countries, but also improve the social benefits of countries and the overall benefits of the system. Therefore, international organizations such as the United Nations and the WHO should actively give full play to their functional advantages and promote the smooth implementation of joint epidemic prevention and control in the international community. Second, countries with strong epidemic control capabilities provide financial, technical and personnel assistance to countries with weak epidemic control capabilities. It can not only reduce the burden of fighting the epidemic of countries with weak epidemic control capabilities, but also reduce their free-rider mentality and encourage them to actively participate in international cooperation on the epidemic. It is conducive to the formation of an international anti-epidemic cooperation situation. Finally, in response to the COVID-19 epidemic, countries should work together to share information on the epidemic and formulate effective joint epidemic prevention policies. The only way to defeat the epidemic at an early date is to jointly provide necessary assistance to less developed countries to enhance their capacity for epidemic prevention and control and prevent its large-scale spread.

The shortcoming of this study is that in the process of international joint prevention and control, the important role played by enterprises, charities, social groups and other non-governmental organizations in the process of international cooperation needs to be further studied. Due to the high transmissibility and long incubation period of COVID-19, the prevention and control efforts invested by various countries are closely related to the number of infected cases and the scope of spread. Therefore, how to further analyze the evolution process of the epidemic through the epidemic model also needs further research. In addition, most of the factors taken into consideration in the international joint prevention and control model are macro factors. The micro factors such as physiotherapy resources and the impact of international cooperation on the economy also need to be further studied.

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