Twelve-quark hypernuclei with $A = 4$ in relativistic quark-gluon model

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Hypernuclei $\,^{3}_2H, \,^{5}_4H, \,^{9}_8H, \,^{12}_9H$, where $Y = \Lambda, \Sigma^0, \Sigma^+_1, \Sigma^-_1$, $A = 4$ are considered using the relativistic twelve-quark equations in the framework of the dispersion relation technique. Hypernuclei as the systems of interacting quarks and gluons are considered. The relativistic twelve-quark amplitudes of hypernuclei, including $u, d, s$ quarks are constructed. The approximate solutions of these equations are obtained using a method based on the extraction of leading singularities of the amplitudes. The poles of the multiquark amplitudes allow us to determine the masses of hypernuclei with the atomic (baryon) number $A = B = 4$. The mass of state $\,^{3}_2H$ with the isospin projection $I_3 = \frac{1}{2}$ and the spin-parity $J^P = 0^+$ is equal to $M = 3922$ MeV. The mass of $\,^{3}_2\Lambda H$ $M = 4118$ MeV with the isospin projection $I_3 = 0$ and the spin-parity $J^P = 0^+$ is calculated. We predict the mass spectrum of hypernuclei with $A = 4$, which is valuable to further experimental study of the hypernuclei.

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I. INTRODUCTION.

The hypernuclear physics experimental program, and the difficulties arisen in accurately determining rates for low-energy nuclear reactions, warrant continued effort in the application of LQCD to nuclear physics [1,12].

In the limit of flavor $SU(3)$ symmetry at the physical strange quark mass with quantum chromodynamics (without electromagnetic interactions) the binding energies of a range of nuclei and hypernuclei with atomic number $A \leq 4$ and strangeness $|S| \leq 2$, including the deuteron, H-dibaryon, $^3He$, $^3\Lambda H$, $^4He$, $^4\Lambda H$ and $^4\Sigma H$ are calculated. From lattice QCD calculations performed with $n_f = 3$ dynamical light quark using an isotropic discretization, the nuclear states are extracted [13–15]. It is now clear that the spectrum of nuclei and hypernuclei changes dramatically from light quark masses.

In our recent paper [16] the relativistic six-quark equations are found in the framework of coupled-channel formalism. The dynamical mixing between the subamplitudes of hexaquark is considered. The six-quark amplitudes of dibaryons are calculated. The poles of these amplitudes determine the masses of dibaryons. We calculated the contribution of six-quark subamplitudes to the hexaquark amplitudes. The model in question has only three parameters: the cutoff parameter $\Lambda = 11$ and gluon coupling constants $g_0$ and $g_1$. These parameters are determined by the $\Lambda\Lambda$ and $\Lambda\Sigma$ masses. In our model the correlation of gluon coupling constants $g_0$ and $g_1$ is similar to the $S$-wave baryon ones [17].

In the previous paper [18], $^3He$ is considered. The relativistic nine-quark equations are derived in the framework of the dispersion relation technique. The dynamical mixing between the subamplitudes of $^3He$ is taken into account. The relativistic nine-quark amplitudes of $^3He$, including the $u$, $d$ quarks are calculated. The approximate solutions of these equations were obtained using a method based on the extraction of leading singularities of the amplitudes. The pole of the nonaquark amplitudes determined the mass of $^3He$.

The experimental mass value of $^3He$ is equal to $M = 2808.39$ MeV. The experimental data of the hypertriton mass is $M = 2991.17$ MeV. This model use only three parameters, which are determined by the following masses: the cutoff $\Lambda = 9.0$ and the gluon coupling constant $g = 0.2122$. The mass of the $u$-quark is $m = 410$ MeV, and the mass of strange quark $m_s = 607$ MeV, which takes into account the confinement potential (the shift mass is equal to 50 MeV) [19].

The relativistic nona-amplitudes of low-lying hypernuclei, including the three flavors ($u, d, s$) are calculated. The degeneracy of the isospin 0, 1, 2 is predicted in the lowest hypernuclei. It is the property of our approach. The low-lying hypernuclei with the spin-parity $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ are calculated. We calculated the masses and the binding energies of $12$ hypernuclei with $A = 3$. The binding energy is small for the states $\,^{3}_2H$, $\,^{3}_2\Lambda$, and $m\Lambda$. In the other cases, the binding energies are large, $\sim 50 – 100$ MeV. We have calculated only five systems of equations; therefore, the masses of hypernuclei are degenerated. We do not include the electromagnetic effect contribution.
TABLE I: Masses and binding energies of \(^4\)He. Parameters of model: \(\Lambda = 6.355\), \(g = 0.2122\), \(m = 410\ MeV\), \(m_s = 594\ MeV\).

| hypernuclei | quark content | \(Q\) | \(I_3\) | \(I\) | \(J^P\) | mass, MeV | binding energy, MeV |
|-------------|---------------|-------|-------|-------|--------|-----------|-------------------|
| \(^4\)\(^\Lambda\)He \((ppn\Lambda)\) | uud udd udd uds | 2 | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(0^+\), \(1^+\) | 3922 | 10 |
| \(^4\)\(^\Sigma^-\)He \((ppn\Sigma^-)\) | uud udd udd uds | 2 | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(0^+\), \(1^+\) | 3922 | 87 |
| \(^4\)\(^\Sigma^+\)He \((ppn\Sigma^+)\) | uud udd uus uus | 3 | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(0^+\), \(1^+\) | 3904 | 101 |
| \(^4\)\(^\Xi^-\)He \((ppn\Xi^-)\) | uud udd uds uds | 1 | \(-\frac{1}{2}\) | \(\frac{1}{2}\) | \(0^+\), \(1^+\) | 3933 | 81 |
| \(^4\)\(^\Xi^+\)He \((ppn\Xi^+)\) | uud udd uss uus | 2 | 1 | \(1, 2\) | \(0^+\), \(1^+\) | 4106 | 25 |
| \(^4\)\(^\Omega^-\)He \((ppn\Omega)\) | uud udd uds uds | 1 | 0 | \(0, 1, 2\) | \(0^+\), \(1^+\) | 4121 | 17 |
| \(^4\)\(^\Omega^+\)He \((ppn\Omega)\) | uud udd uds uds | 1 | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(1^+\), \(2^+\) | 4293 | 196 |

TABLE II: Masses and binding energies of \(^4\)H hypernuclei.

| hypernuclei | quark content | \(Q\) | \(I_3\) | \(I\) | \(J^P\) | mass, MeV | binding energy, MeV |
|-------------|---------------|-------|-------|-------|--------|-----------|-------------------|
| \(^5\)\(^\Lambda\)H \((ppn\Lambda)\) | uud udd udd uds | 1 | \(-\frac{1}{2}\) | \(\frac{3}{2}\) | \(0^+\), \(1^+\) | 3922 | 12 |
| \(^5\)\(^\Sigma^-\)H \((ppn\Sigma^-)\) | uud udd udd uds | 1 | \(-\frac{1}{2}\) | \(\frac{3}{2}\) | \(0^+\), \(1^+\) | 3922 | 89 |
| \(^5\)\(^\Sigma^+\)H \((ppn\Sigma^+)\) | uud udd uds uds | 0 | \(\frac{3}{2}\) | \(\frac{3}{2}\) | \(0^+\), \(1^+\) | 3904 | 112 |
| \(^5\)\(^\Xi^-\)H \((ppn\Xi^-)\) | uud udd uus uus | 2 | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(0^+\), \(1^+\) | 3933 | 74 |
| \(^5\)\(^\Xi^+\)H \((ppn\Xi^+)\) | uud udd uus uus | 0 | -1 | \(1, 2\) | \(0^+\), \(1^+\) | 4106 | 34 |
| \(^5\)\(^\Omega^-\)H \((ppn\Omega)\) | uud udd uds uds | 1 | 0 | \(0, 1, 2\) | \(0^+\), \(1^+\) | 4121 | 12 |
| \(^5\)\(^\Omega^+\)H \((ppn\Omega)\) | uud udd uds uds | 0 | \(-\frac{1}{2}\) | \(\frac{3}{2}\) | \(1^+\), \(2^+\) | 4293 | 198 |

Hypernuclei spectroscopy is enjoying an experimental renaissance with ongoing and planned program at DAΦNE, FAIR, Jefferson Lab, J-PARC and Mainz providing motivation for enhanced theoretical efforts (for a recent review, see Ref. [20]).

In the chiral soliton approach using the bound state rigid oscillator version of the SU(3) quantization model [21, 22] the binding energies of neutron-rich are estimated.

Recently, a new direction of such studies, the studies of neutron-rich hypernuclei with strangeness \(S = -1\) got new impact due to discovery of the hypernucleus \(^8\)\(^\Lambda\)H (heavy hyperhydrogen) by the FINUDA Collaboration [23].

In the present paper, the hypernuclei \(^4\)^\(\Lambda\)He, \(^4\)^\(\Sigma^-\)He, \(^4\)^\(\Xi^-\)He, \(^4\)^\(\Sigma^+\)He, \(^4\)_Y\(^Y\)He, where \(Y = \Lambda, \Sigma_0, \Sigma_+\), \(\Sigma_-\); \(A = 4\) are considered.

TABLE III: Masses and binding energies of \(^4\)^\(\Sigma^+\)\(^Y\)He.
The approximate solutions of the twelve-quark equations are obtained using the method based on the extraction of leading singularities of the amplitude. The poles of these amplitudes determine the masses of the hypernuclei.

In Sec. II, the relativistic twelve-quark amplitudes, including the three flavors (u, d, s) are constructed. We derived the dynamical mixing between the subamplitudes of the hypernuclei. Sec. III is devoted to the calculation results for the masses and binding energies of \(^4H\), \(^4\Lambda\) and \(nnYY\) hypernuclei (Tables IV – VI).

In conclusion, the status of the considered model is discussed (Table VI).

In the Appendix A, the coefficients of the coupled equations reduced amplitude \(\alpha_4\) \((I_3 = \frac{1}{2}, J^P = 0^+ \frac{3}{2}H\)) are given.

| hypernuclei | quark content | \(Q, I^s\) | \(I\) | \(J^P\) | mass, MeV | binding energy, MeV |
|-------------|---------------|-------------|-------|--------|------------|--------------------|
| \(^4\Lambda\) \(H\) \((p\Lambda\Lambda)\) | uud udd uds uds | 1 0 | 0, 1 | 0+, 1+ | 4118 | -8 |
| \(^4\Lambda\Sigma^0\) \(H\) \((p\Lambda\Sigma^0)\) | uud udd uds uds | 1 0 | 0, 1, 2 | 0+, 1+, 2+ | 4118 | 69 |
| \(^4\Sigma^0\Sigma^0\) \(H\) \((p\Sigma^0\Sigma^0)\) | uud udd uds uds | 1 0 | 0, 1, 2, 3 | 0+, 1+ | 4118 | 146 |
| \(^4\Sigma^+\Sigma^+\) \(H\) \((p\Sigma^+\Sigma^+)\) | uud udd uds uds | 3 2 | 2, 3 | 0+, 1+ | 4081 | 175 |
| \(^4\Sigma^-\Sigma^-\) \(H\) \((p\Sigma^-\Sigma^-)\) | uud udd uds uds | -1 2 | 2, 3 | 0+, 1+ | 4081 | 193 |
| \(^4\Lambda\Sigma^+\) \(H\) \((p\Lambda\Sigma^+)\) | uud udd uds uus | 2 1 | 1, 2 | 0+, 1+, 2+ | 4099 | 84 |
| \(^4\Lambda\Sigma^-\) \(H\) \((p\Lambda\Sigma^-)\) | uud udd uds uds | 0 1 | 1, 2 | 0+, 1+, 2+ | 4099 | 93 |
| \(^4\Sigma^+\Sigma^-\) \(H\) \((p\Sigma^+\Sigma^-)\) | uud udd uus uds | 1 0 | 1, 2, 3 | 0+, 1+, 2+ | 4109 | 156 |

**TABLE V: Masses and binding energies of \(nnYY\).**

| hypernuclei | quark content | \(Q, I^s\) | \(I\) | \(J^P\) | mass, MeV | binding energy, MeV |
|-------------|---------------|-------------|-------|--------|------------|--------------------|
| \(nn\Lambda\) | udd udd uds uds | 0 1 | 1 | 0+ | 4127 | -15 |
| \(nn\Lambda\Sigma^0\) | udd udd uds uds | 0 -1 | 1, 2 | 0+, 1+ | 4127 | 62 |
| \(nn\Sigma^0\Sigma^0\) | udd udd uds uds | 0 0 | 1, 2, 3 | 0+ | 4127 | 139 |
| \(nn\Sigma^+\Sigma^+\) | udd udd uds uds | 2 1 | 1, 2, 3 | 0+ | 4150 | 108 |
| \(nn\Sigma^-\Sigma^-\) | udd udd uds uds | -2 3 | 3 | 0+ | 4008 | 268 |
| \(nn\Sigma^+\Sigma^-\) | udd udd uds uds | 0 1 | 1, 2, 3 | 0+, 1+ | 4114 | 71 |
| \(nn\Lambda\Sigma^-\) | udd udd uds uds | -1 2 | 1, 2, 3 | 0+, 1+ | 4086 | 108 |
| \(nn\Sigma^+\Sigma^-\) | udd udd uds uds | 0 1 | 1, 2, 3 | 0+, 1+ | 4105 | 162 |

**TABLE VI: Binding energies of low-lying nuclei and hypernuclei (NPLQCD Collaboration).**

| State | \(A\) | \(S\) | \(I\) | \(J^P\) | binding energy \((B^\infty)\), MeV |
|-------|-------|-------|-------|-------|----------------------------------|
| \(\frac{3}{2}H\) \(e\) | \(4\) | 0 | 0 | 0+ | 107 |
| \(\frac{3}{2}H\) \(e\) | \(4\) | -1 | \(\frac{1}{2}\) | 0+ | 107 |
| \(\frac{1}{2}H\) | \(4\) | -1 | \(\frac{1}{2}\) | 0+ | 107 |
| \(\frac{1}{2}\Lambda\) \(H\) | \(4\) | -2 | 1 | 0+ | 156 |
| \(\frac{3}{2}\Lambda\) \(H\) | \(4\) | -2 | 0 | 0+ | 156 |
| \(\Lambda\Lambda\) | \(4\) | -2 | 1 | 0+ | 156 |
The Appendix B shown the graphical equations of the reduced amplitude $A_4^{uu1^+1^+1^+1^+0^0}$ ($I_3 = \frac{1}{2}$, $J^P = 0^+$ $\Lambda He$). The interaction of quarks and gluons is described with the functions $I_1 - I_{46}$ in Appendix C.

**II. TWELVE QUARK AMPLITUDES OF HYPERNUCLEI.**

We derive the relativistic twelve-quark equations in the framework of the dispersion relation technique. The planar diagrams are used; the other diagrams are neglected due to the rules of $1/N_c$ expansion [24–26].

The current generates a twelve-quark system. The correct equations for the amplitude are obtained by taking into account all possible subamplitudes. Then one should represent a twelve-particle amplitude as a sum of 66 subamplitudes:

$$A = \sum_{i<j}^{12} A_{ij}.$$  \hspace{1cm} (1)

This defines the division of the diagrams into groups according to the certain pair interaction of particles. The total amplitude can be represented graphically as a sum of diagrams. It allows us to consider only one group of diagrams and the amplitude corresponding to them, for example $A_{12}$. The relativistic generalization of the Faddeev-Yakubovsky approach is used [27, 28]. In our case, the hyperhydrogens and hyperheliums consist of the similar equations and therefore, the masses and other properties of these states are resembling. The quark content of all states is given in the Tables I–V. The pairwise interaction of all twelve quarks is taken into account. The set of diagrams associated with the amplitude $A_{12}$ can further be broken down into some groups corresponding to subamplitudes.

In order to represent the subamplitudes $A_{ij}$ in the form of a dispersion relation it is necessary to define the amplitudes of quark-quark interactions $b_{ij}(s_{ik})$. The pair quarks amplitudes $qq \rightarrow qq$ are calculated in the framework of the dispersion $N/D$ method with the input four-fermion interaction with quantum numbers of the gluon [29]. We use the results of our relativistic quark model [29] and write down the pair quark amplitudes in the form:

$$b_{ij}(s_{ik}) = \frac{G_n^2(s_{ik})}{1 - B_{ij}(s_{ik})},$$  \hspace{1cm} (2)

$$B_{ij}(s_{ik}) = \int_{(m_i + m_k)^2}^{\Lambda_n^2} \frac{ds'_{ik}}{\pi} \frac{\rho_n(s_{ik})G_n^2(s'_{ik})}{s'_{ik} - s_{ik}}.$$  \hspace{1cm} (3)

Here, $s_{ik}$ is the two-particle subenergy squared. $G_n(s_{ik})$ are the quark-quark vertex functions (Table VII). $B_{ij}(s_{ik})$, $\rho_n(s_{ik})$ are the Chew-Mandelstam functions with cutoff $\Lambda_n$ (30 and the phase space, respectively:

$$\rho_n(s_{ik}, J^P) = \left(\alpha(J^P, n) \frac{s_{ik}}{(m_i + m_k)^2} + \beta(J^P, n) + \delta(J^P, n) \frac{(m_i - m_k)^2}{s_{ik}}\right) \times \frac{\sqrt{(s_{ik} - (m_i + m_k)^2)(s_{ik} - (m_i - m_k)^2)}}{s_{ik}}.$$  \hspace{1cm} (4)

The coefficients $\alpha(J^P, n)$, $\beta(J^P, n)$, and $\delta(J^P, n)$ are given in Table VII. Here $n = 1$ corresponds to $qq$-pairs with $J^P = 0^+$, $n = 2$ describes the $qq$ pairs with $J^P = 1^+$. In our model the interacting quarks do not produce a bound states; therefore the integration is carried out from the threshold $(m_i + m_k)^2$ to the cutoff $\Lambda_n$.

Let us extract singularities in the coupled equations and obtain the reduced amplitudes $\alpha_i$.

At first, we obtain the system of 20 equations for the $\frac{4}{2}He$ with the isospin projection $I_3 = 0$, the spin-parity $J^P = 0^+$ ($ppnn$). The reduced amplitudes $\alpha_1$ are determined by the channels: $1uu$, $0ud$, $1dd$. The $\alpha_2$ are constructed as $1uu1uu1uu$, $1uu0ud1uu$, $1uu1dd1uudd0ud1dd0ud0ud1dd1dd1dd$. The reduced amplitudes $\alpha_3$ are the following:

$$1uu1uu0ud, 1uu1uu1dd, 1uu0ud1dd1uu1dd1dd, 1uu1dd1dd, 0ud1dd1dd.$$  \hspace{1cm} (5)
The \( \alpha_4 \) are just:

\[
1^{uu}1^{uu}0^{ud}0^{ud}, \quad 1^{uu}1^{uu}0^{ud}1^{dd}, \quad 1^{uu}1^{uu}1^{dd}1^{dd},
\]

\[
1^{uu}0^{ud}0^{ud}1^{dd}, \quad 1^{uu}0^{ud}1^{dd}1^{dd}, \quad 0^{ud}0^{ud}1^{dd}1^{dd}.
\]

(6)

Here the \( \alpha_1 \) are determined by the diquarks, the \( \alpha_2 \) includes the two diquarks and eight quarks. \( \alpha_3 \) defines the three diquarks and six quarks. The \( \alpha_4 \) allows us to consider the \( ppm \) (\( uud uud udd udd \)) \( \frac{3}{2}He \) state.

Then we calculate the solution of 33 equations for the hypernucleus \( \frac{3}{2}He \) (\( ppm \)) with the isospin projection \( I_3 = \frac{1}{2} \) and the spin-parity \( J^P = 0^+ \) (Table III).

The reduced amplitudes \( \alpha_1, \alpha_2 \) with the one \( s \)-quark are similar to:

\[
\alpha_1^{uu}, \alpha_1^{dd}, \alpha_1^{0^{ud}0^{ud}}, \alpha_1^{0^{uu}0^{ud}}, \alpha_1^{0^{dd}0^{ud}}, \alpha_1^{0^{ud}0^{dd}}, \alpha_1^{0^{dd}0^{dd}}; \tag{7}
\]

\[
\alpha_2^{uu}u^{uu}, \alpha_2^{uu}u^{dd}, \alpha_2^{uu}0^{ud}0^{ud}, \alpha_2^{uu}0^{uu}0^{ud}, \alpha_2^{uu}0^{dd}0^{dd}, \alpha_2^{uu}0^{ud}0^{dd}, \alpha_2^{uu}0^{dd}0^{ud}, \alpha_2^{uu}0^{dd}0^{dd}. \tag{8}
\]

We have to add the reduced amplitudes \( \alpha_3, \alpha_4 \)

\[
\alpha_3^{uu}u^{uu}u^{dd}, \alpha_3^{uu}u^{uu}0^{ud}0^{ud}, \alpha_3^{uu}u^{uu}0^{uu}0^{ud}, \alpha_3^{uu}u^{uu}1^{dd}1^{dd}, \tag{9}
\]

\[
\alpha_4^{uu}u^{uu}1^{dd}0^{ud}0^{ud}, \alpha_4^{uu}u^{uu}1^{dd}0^{uu}0^{ud}, \alpha_4^{uu}u^{uu}1^{dd}0^{dd}0^{ud}, \alpha_4^{uu}u^{uu}1^{dd}0^{ud}0^{dd}, \alpha_4^{uu}u^{uu}1^{dd}0^{dd}0^{dd}. \tag{10}
\]

If we consider the coupled equations corresponded to the reduced amplitudes with two \( s \)-quarks (\( \frac{4}{2} \Lambda \Lambda, \frac{4}{2} \Lambda \Sigma, \frac{4}{2} \Sigma \Sigma \)), we use 33 equations (Tables III, IV). The reduced amplitudes \( \alpha_1, \alpha_2 \) are equal to:

\[
\alpha_1^{uu}, \alpha_1^{dd}, \alpha_1^{0^{ud}0^{ud}}, \alpha_1^{0^{uu}0^{ud}}, \alpha_1^{0^{dd}0^{ud}}, \alpha_1^{0^{ud}0^{dd}}, \alpha_1^{0^{dd}0^{dd}}; \tag{11}
\]

\[
\alpha_2^{uu}u^{uu}, \alpha_2^{uu}u^{dd}, \alpha_2^{uu}0^{ud}0^{ud}, \alpha_2^{uu}0^{uu}0^{ud}, \alpha_2^{uu}0^{dd}0^{dd}, \alpha_2^{uu}0^{ud}0^{dd}, \alpha_2^{uu}0^{dd}0^{ud}, \alpha_2^{uu}0^{dd}0^{dd}. \tag{12}
\]

The reduced amplitudes \( \alpha_3, \alpha_4 \) are following:

| \( n \) | \( J^P \) | \( G_n^{\pm}(s_{kk}) \) | \( \alpha_n \) | \( \beta_n \) | \( \delta_n \) |
|---|---|---|---|---|---|
| 1 | 0\(^+\) | \( \frac{4\alpha}{3} - \frac{8g m^2}{(3\alpha)} \) | \( \frac{1}{2} \) | \( -\frac{1}{2} (m_k - m_l)^2 \) | 0 |
| 2 | 1\(^+\) | \( \frac{2\alpha}{3} \) | \( \frac{1}{2} \) | \( -\frac{1}{2} (m_k + m_l)^2 \) | 0 |
where $i$ and $j$ correspond to the diquarks with the spin-parity $J^P = 0^+, 1^+$.

We used the contributions of the functions (C1) – (C25) in the Appendix C. The other functions $I_i$ are small.
III. **CALCULATION RESULTS.**

The poles of the reduced amplitudes $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ correspond to the bound states and determine the masses of low-lying hypernuclei with the atomic number $A = 4$ (Tables I – V).

The experimental mass value of $\frac{1}{2}^4H\alpha$ is equal to $M = 3727.4\text{MeV}$. The experimental data of the hypernucleus $\frac{1}{2}^4H\alpha\left(\frac{1}{2}^4H\right)$ is $M = 3922\text{MeV}$.

Our model uses only three parameters, which are determined by the following masses: the cutoff $\Lambda = 6.355$ and the gluon coupling constant $g = 0.2122$. The mass of the $u, d$ quarks is equal to $m = 410\text{MeV}$, and the mass of strange quark is $m_s = 594\text{MeV}$, which takes into account the confinement potential (the shift mass is equal to $37\text{MeV}$).

We used (as input) the hypernucleus $\frac{1}{2}^4He\left(ppn\Lambda\right)$. This mass is equal to $M = 3922\text{MeV}$ with the binding energy $B = 10\text{MeV}$. The similar equations give rise to the $\frac{1}{2}^4H\left(ppn\Lambda\right)$ with the mass $M = 3922\text{MeV}$ and the binding energy $B = 12\text{MeV}$ (Tables I – II). In the case of the hypernuclei $\frac{4}{2}^2He$ and $\frac{4}{2}^6He$ with the mass $M = 4121\text{MeV}$ the binding energy $B = 12\text{MeV}$ and $B = 17\text{MeV}$ are calculated. The interesting results we obtained for the states $\frac{4}{2}^2He$ and $\frac{4}{2}^6He$ and the mass $M = 4106\text{MeV}$. The binding energies are equal to $25\text{MeV}$ and $34\text{MeV}$, respectively.

The three hypernuclei $\frac{1}{2}^4AHe$, $\frac{1}{2}^2AHe$ and $nn\Lambda\Lambda$ possess the negative binding energies $B_1 = -8\text{MeV}$, $B_2 = -19\text{MeV}$ and $B_3 = -15\text{MeV}$, respectively (Tables III – V). These state masses $M_1 = 4118\text{MeV}$, $M_2 = 4127\text{MeV}$ and $M_3 = 4127\text{MeV}$ are located near the energy thresholds. For the other nuclei the binding energies are large; $\sim 50 - 150\text{MeV}$ (Tables I – V).

We predict the eight hypernuclei with one $s$-quark. These states are determined by the isospin $I = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ and the spin-parity $J^P = 0^+, 1^+$. Then the 38 hypernuclei with two $s$-quarks are calculated. We obtained the two heavy hypernuclei $\frac{1}{2}^4H\left(ppn\Omega\right)$ and $\frac{1}{2}^4He\left(ppn\Omega\right)$ with the mass equal to $M = 4293\text{MeV}$ and the binding energies $B_1 = 198\text{MeV}$ and $B_2 = 196\text{MeV}$, respectively. We derived only 16 different system equations, therefore the masses of hypernuclei with $A = 4$ are degenerated. The electromagnetic effect is not included. The estimation of theoretical error is equal to $1\text{MeV}$. This result has been obtained with the choice of the parameters of the model. The spectrum of nuclei and hypernuclei changes from light quark masses. In the recent work [31] on the $H$- dibaryon and nucleon-nucleon scattering length one shown this for even larger systems.

IV. **CONCLUSIONS.**

The binding energies of a range of nuclei and hypernuclei with the atomic number $A = 4$ and strangeness $S \leq 2$, including $^4He, \frac{1}{2}^4H$ and $\frac{1}{2}^4AHe$ are calculated in the limit of flavor $SU(3)$ symmetry at the physical strange quark mass with quantum chromodynamics (without electromagnetic interactions) [13]. Infinite volume binding energies $B^\infty(\frac{3}{2}^4H), B^\infty(\frac{1}{2}^4He), B^\infty(\frac{1}{2}^4AHe)$ are given in Table V.

In present paper the hypernucleus $\frac{1}{2}^4AHe$ with the mass $M = 4118\text{MeV}$ and the binding energy $B_1 = -8\text{MeV}$ there is near threshold. This state is similar to the multiquark systems with the small decay width: the tetraquarks, pentaquarks and hexaquarks [32]. The experimental results on this state are not definite. The interactions of nucleons in the nuclei can be considered using the phenomenological nucleon-nucleon potential, which will be able to construct in the first principles QCD. We believe that the nucleon-nucleon potentials describe only small part of the quark-gluon interactions in the hadrons. In this case the new approach will be able to consider the low-lying hypernuclei and will allows us to construct the nonnucleon systems.

The all baryon using the strange quarks have the short life time. In the nucleon medium their life time do not change essentially. Therefore, the systems of 6, 9, 12 quarks are shortlivity. These results was discovered by the experimental data. In the case of hadrons we will able to shown that the rescattering hadron to be forbidden space corresponds, to the interaction with the massive particles, which consist of some quarks. The meson exchange model allows us to obtain the only reduced picture of baryon interactions. The new approach are constructed using the quark-gluon structure of hypernuclei.

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Appendix A

For instance, we consider the reduced amplitudes in the Appendix A: $\alpha_1^{uu}$ (page 10), $\alpha_2^{uu1^{uu1}}$ (page 11), $\alpha_3^{uu1^{uu1}dd}$ (page 12) and $\alpha_4^{uu1^{uu1}dd0^{dd}ds}$ (page 13).

In Fig. 1 the coefficient of the term $I_{29}(1^{uu1^{uu1}dd0^{dd}ds}1^{uu})\alpha_1^{uu}$ (page 10) is equal to 8, that is the number $8 = 2$ (the permutation of particles 1 and 2) $\times 2$ (the permutation of pairs (12) and (34)) $\times 2$ (we can replace the 9-th $u$-quark with the 10-th $u$-quark); the coefficient of the term $I_{30}(1^{uu1^{uu1}dd0^{dd}ds}1^{uu})\alpha_1^{uu}$ (page 10) is equal to 4, that is the number $4 = 2$ (the permutation of particles 1 and 2) $\times 2$ (the permutation of particles 3 and 4); the coefficient of the term $I_{31}(1^{uu1^{uu1}dd0^{dd}ds}1^{uu1^{uu}})\alpha_1^{uu}$ (page 11) is equal to 4, that is the number $4 = 2$ (the permutation of pairs (12) and (78)) $\times 2$ (we can replace the 9-th $u$-quark with the 10-th $u$-quark); the coefficient of the term $I_{32}(1^{uu1^{uu1}dd0^{dd}ds}1^{uu1^{uu}})\alpha_2^{uu1^{uu}}$ (page 11) is equal to 8, that is the number $8 = 2$ (the permutation of particles 1 and 2) $\times 2$ (the permutation of particles 3 and 4) $\times 2$ (we can replace the 9-th $u$-quark with the 10-th $u$-quark); the coefficient of the term $I_{37}(1^{dd1^{uu1^{uu}}0^{dd}ds1^{uu1^{uu}}})\alpha_3^{uu1^{uu1}dd}$ (page 12) is equal to 16, that is the number $16 = 2$ (the permutation of pairs (34) and (56)) $\times 2$ (we can replace the 10-th $u$-quark with the 11-th $u$-quark) $\times 2$ (the permutation of particles 1 and 2) $\times 2$ (we can replace the 9-th $u$-quark with the 12-th $u$-quark); the coefficient of the term $I_{38}(1^{uu1^{uu1}dd0^{dd}ds1^{uu1^{uu}}})\alpha_3^{uu1^{uu1}0^{dd}}$ (page 12) is equal to 32, that is the number $32 = 2$ (the permutation of particles 1 and 2) $\times 2$ (the permutation of particles 3 and 4) $\times 2$ (the permutation of pairs (12) and (34)) $\times 2$ (we can replace the 9-th $u$-quark with the 11-th $u$-quark) $\times 2$ (we can replace the 10-th $u$-quark with the 12-th $d$-quark) $\times 2$ (we can replace the 9-th $u$-quark with the 10-th $d$-quark) $\times 2$ (we can replace the 11-th $d$-quark with the 12-th $d$-quark); the coefficient of the term $I_{46}(1^{uu0^{ds}ds1^{uu1^{uu1}dd0^{dd}ds}}1^{uu1^{uu1}dd0^{dd}ds})\alpha_4^{uu1^{uu1}dd0^{dd}ds}$ (page 13) is equal to 8, that is the number $8 = 2$ (the permutation of pairs (12) and (56)) $\times 2$ (we can replace the 9-th $u$-quark with the 10-th $u$-quark) $\times 2$ (we can replace the 11-th $d$-quark with the 12-th $d$-quark).

The similar approach allows us to take into account the coefficients in all the diagrams and equations.
Appendix B

\[ \alpha_4^{ued\mu_1\mu_3 \bar{d}\bar{d}q_{\bar{d}s}} = \lambda \]

\[ 8I_{29}(1^{u}u\mu_1^{1}u\mu_1^{1}d\bar{d}q_{\bar{d}s}1^{u}u\mu_1) \alpha_1^{uu} \]

\[ 4I_{30}(1^{u}u\mu_1^{1}d\bar{d}q_{\bar{d}s}1^{u}u\mu_1) \alpha_1^{uu} \]
Fig. 1. The graphical equations of the reduced amplitude $\alpha_4^{1_u u_1 d \bar{d} 0 \bar{u} d \bar{s} 1 u u_1 d \bar{d} 0 \bar{u} d} \times H e \ ppn \Lambda$ with the isospin projection $I_3 = \frac{1}{2}$ and the spin-parity $J^P = 0^+$ (Table I).
Appendix C: Some useful formulae.

We used the functions $I_1$, $I_2$, $I_3$, $I_4$, $I_5$, $I_6$, $I_7$, $I_8$, $I_9$, $I_{11}$, $I_{12}$, $I_{13}$, $I_{14}$, $I_{15}$, $I_{16}$, $I_{18}$, $I_{23}$, $I_{24}$, $I_{29}$, $I_{30}$, $I_{31}$, $I_{32}$, $I_{37}$, $I_{38}$, $I_{46}$.

\[
I_1(ij) = \frac{B_j(s_{13}^{13})}{B_i(s_0^{12})} \frac{(m_1+m_2)^2 \Lambda_i}{(m_1+m_2)^2} \int \frac{d^4 s_{12}^G s_{12}^{12} G_i^2(s_0^{12}) \rho_i(s_{12}^{12})}{s_{12}^{12} - s_0^{12}} \int \frac{dz_1(1) + 1}{2} \frac{1}{1 - B_j(s_{13}^{13})}, \quad (C1)
\]

\[
I_2(ijk) = \frac{B_j(s_{13}^{13}) B_k(s_{24}^{24})}{B_i(s_0^{12}) B_j(s_0^{12})} \frac{(m_1+m_2)^2 \Lambda_i}{(m_1+m_2)^2} \int \frac{d^4 s_{12}^G s_{12}^{12} G_i^2(s_0^{12}) \rho_i(s_{12}^{12})}{s_{12}^{12} - s_0^{12}} \int \frac{dz_1(1) + 1}{2} \frac{1}{1 - B_j(s_{13}^{13})} \frac{1}{1 - B_k(s_{24}^{24})}, \quad (C2)
\]

\[
I_3(ijk) = \frac{B_k(s_{24}^{24})}{B_i(s_0^{12}) B_j(s_0^{12})} \frac{(m_1+m_2)^2 \Lambda_i}{(m_1+m_2)^2} \int \frac{d^4 s_{12}^G s_{12}^{12} G_i^2(s_0^{12}) \rho_i(s_{12}^{12})}{s_{12}^{12} - s_0^{12}} \int \frac{dz_1(1) + 1}{2} \frac{1}{1 - B_k(s_{24}^{24})}, \quad (C3)
\]

\[
I_4(ijk) = I_1(ijk), \quad (C4)
\]

\[
I_5(ijkl) = I_2(ijkl), \quad (C5)
\]

\[
I_6(ijkl) = I_1(ijk) \times I_1(jkl), \quad (C6)
\]

\[
I_7(ijkl) = \frac{B_k(s_{24}^{24}) B_l(s_{45}^{45})}{B_i(s_0^{12}) B_j(s_0^{12}) B_k(s_0^{12})} \frac{(m_1+m_2)^2 \Lambda_i}{(m_1+m_2)^2} \int \frac{d^4 s_{12}^G s_{12}^{12} G_i^2(s_0^{12}) \rho_i(s_{12}^{12})}{s_{12}^{12} - s_0^{12}} \int \frac{dz_1(1) + 1}{2} \frac{1}{2} \frac{1}{1 - B_k(s_{24}^{24})} \frac{1}{1 - B_l(s_{45}^{45})}, \quad (C7)
\]

\[ I_8(ijklm) = \frac{B_k(s_{15}^2) B_l(s_{23}^2) B_m(s_{46}^2)}{B_i(s_{12}^2) B_j(s_{34}^2)} \frac{\left(m_1 + m_2\right)^2}{\Lambda_1} \int \frac{ds_{12}' G_i^2(s_{02}^2) \rho_i(s_{12}')}{\pi (s_{12}' - s_{02}^2)} \]
\[ \times \frac{\left(m_3 + m_4\right)^2}{\Lambda_1} \int \frac{ds_{34}' G_j^2(s_{04}^2) \rho_j(s_{34}')}{\pi (s_{34}' - s_{04}^2)} \]
\[ \times \frac{1}{2\pi^2} \frac{1}{\pi^2} \frac{1}{\pi^2} \frac{1}{\pi^2} \int_{-1}^{+1} \frac{dz_1(8)}{2} \int_{-1}^{+1} \frac{dz_2(8)}{2} \int_{-1}^{+1} \frac{dz_3(8)}{2} \int_{-1}^{+1} \frac{dz_4(8)}{2} \int_{-1}^{+1} \frac{dz_5(8)}{2} \int_{-1}^{+1} \frac{dz_6(8)}{2} \]
\[ \times \frac{1}{\sqrt{1 - z_1^2(8) - z_2^2(8) - z_4^2(8) + 2z_1(8)z_4(8)}} \]
\[ \times \frac{1}{\sqrt{1 - z_3^2(8) - z_5^2(8) - z_6^2(8) + 2z_3(8)z_6(8)}} \]
\[ \times \frac{1}{1 - B_k(s_{15}')} \frac{1}{1 - B_l(s_{23}')} \frac{1}{1 - B_m(s_{46}')} \]
\[ (C8) \]
\[ I_9(ijkl) = I_3(ijl), \]
\[ (C9) \]
\[ I_{11}(ijklm) = I_1(ik) \times I_2(jlm), \]
\[ (C10) \]
\[ I_{12}(ijkl) = I_1(il), \]
\[ (C11) \]
\[ I_{13}(ijklm) = I_2(ilm), \]
\[ (C12) \]
\[ I_{14}(ijklm) = I_1(il) \times I_1(jm), \]
\[ (C13) \]
\[ I_{15}(ijklm) = I_2(ilm), \]
\[ (C14) \]
\[ I_{16}(ijklmn) = I_8(ijlmn), \]
\[ (C15) \]
\[ I_{18}(ijklmn) = I_1(il) \times I_2(jmn), \]
\[ (C16) \]
\[ I_{23}(ijklmn) = I_2(ikl) \times I_2(jmn), \]
\[ (C17) \]
\[ I_{24}(ijklmnp) = I_2(ilm) \times I_2(jnp), \]
\[ (C18) \]
\[ I_{29}(ijklm) = I_1(im), \]
\[ (C19) \]
\[ I_{30}(ijklm) = I_3(ijm), \]
\[ (C20) \]
\[ I_{31}(ijklmn) = I_2(imn), \]
\[ (C21) \]
\[ I_{32}(ijklmn) = I_1(im) \times I_1(jn), \]
\[ (C22) \]
\[ I_{37}(ijklmnp) = I_1(im) \times I_2(jnp), \]
\[ (C23) \]
\[ I_{38}(ijklmnp) = I_8(ijmnp), \tag{C24} \]
\[ I_{46}(ijklmnpq) = I_2(inn) \times I_2(jpq). \tag{C25} \]

Here \( i, j, k, l, m, n, p, q \) correspond to the diquarks with the spin-parity \( J^P = 0^+, 1^+ \).

The other functions \( I_i \) can be neglected. The contributions of these functions are smaller of few orders as compared the functions \( \text{(C1)} \) – \( \text{(C25)} \). We do not take into account these functions in the systems of coupled equations.