Active Perception and Control from Temporal Logic Specifications

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Abstract—Next-generation autonomous systems must execute complex tasks in uncertain environments. Active perception, where an autonomous agent selects actions to increase knowledge about the environment, has gained traction in recent years for motion planning under uncertainty. One prominent approach is planning in the belief space. However, most belief-space planning starts with a known reward function, which can be difficult to specify for complex tasks. On the other hand, symbolic control methods automatically synthesize controllers to achieve logical specifications, but often do not deal well with uncertainty. In this work, we propose a framework for scalable task and motion planning in uncertain environments that combines the best of belief-space planning and symbolic control. Specifically, we provide a counterexample-guided-inductive-synthesis algorithm for probabilistic temporal logic over reals (PRTL) specifications in the belief space. Our method automatically generates actions that improve confidence in a belief when necessary, thus using active perception to satisfy PRTL specifications.

I. INTRODUCTION

Taskable and adaptive Intelligent Physical Systems (IPS) must not only take actions to satisfy high-level task specifications, but also change their behavior over time, learning from experience. These systems must formulate beliefs about the environment and explicitly act to improve confidence in these beliefs: this process is known as active perception [1].

Active perception can be formalized as a partially observable Markov decision process (POMDP) [2] since physical systems often follow the Markov assumption. POMDP planning involves a search for a policy that satisfies certain requirements. Since states are not directly observable, the policy is a mapping from history, a sequence of observations and actions, to new actions. History can be compactly represented as a probability distribution called the belief.

Various methods have been proposed for active perception via belief space planning, most of which search for a policy that maximizes the expected value of a reward function [3]–[8]. Specifying a reward function that guarantees completion of a complex task can be difficult, however. Using temporal logic specifications is often clearer and more intuitive. Of a complex task can be difficult, however. Using temporal logic specifications is often clearer and more intuitive. On the other hand, symbolic control

Furthermore, symbolic control approaches, which synthesize controllers to achieve logical specifications, can provide strong formal guarantees. Early efforts in symbolic control focused on discrete transition system models. Probabilistic extensions to these models focus on uncertainty arising from state transitions, and mature software tools have been developed to this end [9], [10]. In this work, we consider symbolic control of uncertain continuous systems. Specifically, we propose a provably correct framework for symbolic control in the belief space based on Probabilistic Temporal Logic over Reals (PRTL) specifications. We focus on PRTL in particular because it is defined over real-valued signals and for its simple, clean notation.

Existing synthesis methods for uncertain continuous systems are primarily based on mixed integer programming [11]–[13]. These methods provide satisfying controllers for a convex fragment of probabilistic signal temporal logic (PrSTL), another temporal logic for real-valued state variables. Other methods propose relaxations such as sampling-based optimization [14] and shrinking horizon model predictive control [15] to achieve (possibly non-convex) specifications. However, these approaches focus on robustness to uncertainty rather than active perception. This means that such algorithms do not plan to gather more information, though doing so may be necessary to achieve a specification over the belief.

We propose a PRTL controller synthesis algorithm for systems with perception and actuation uncertainty. By satisfying specifications defined over the belief space, our method incorporates active perception. This means that the system not only satisfies expressive temporal specifications, but also synthesizes actions to gain information when necessary. We prove that our approach is sound and probabilistically complete, and demonstrate its effectiveness with a simulated example.

The rest of this letter is organized as follows. First, we present the problem statement in Section II. Next, we present the details of our controller synthesis algorithm in Section III. We show that our approach is sound and probabilistically complete in Section IV. Section V provides an example of applying our methods to automated infrastructure inspection with a quadrotor, and Section VI concludes the paper.

II. PRELIMINARIES

A. System

Consider the discrete-time linear control system

\[ x_{k+1} = Ax_k + Bu_k + \sqrt{R}w \]
\[ y_k = Cx_k + \sqrt{W_k}v, \quad v \sim N(0, I_n), \]

where \( x \in \mathbb{R}^n \) are the state variables, \( u \in \mathcal{U} \subseteq \mathbb{R}^m \) are the control inputs, \( y \in \mathbb{R}^p \) are the output variables, and \( A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}, C \in \mathbb{R}^{p \times n} \) are constant matrices. We assume that \( \mathcal{U} = \{ H_k u \geq c_k \} \) is a full-dimensional polytope and that \( (A, B) \) is stabilisable. This system is subject to uncorrelated Gaussian disturbances and noise with covariances \( R^T \geq 0 \) and \( W_k^T \geq 0 \), where \( W_k \) can be state dependent.
B. Belief State

Since only noisy observations $y_k$ are available in System \cite{1}, we must estimate the state $x_k$. To do so, the controller keeps track of a history of observations and actions. History can be compactly represented as a random process $b_k = P(x_k)$ known as the belief state \cite{2}. The belief state can be tracked using Bayesian filtering:

$$P(x_{k+1}) = \eta P(y_{k+1} | x_{k+1}, u_k) \int P(x_{k+1} | x, u_k) b_k,$$

where $\eta$ is a normalization constant \cite{2}.

In our case, we assume that the belief state is Gaussian with mean $m_k \in \mathbb{R}^n$ and covariance $\Sigma_k \in \mathbb{R}^{n \times n}$. Under this assumption, the belief dynamics can be derived by applying the Kalman filter \cite{1}:

$$m_{k+1} = K_k (y_k - C(f_k)) + F_k, \quad \Sigma_{k+1} = \Gamma_k - K_k C \Gamma_k, \quad \text{where} \quad F_k = A \Sigma_k A^T + R,$$

this means that the trajectory of System (1) fulfills the desired assumption by considering the maximum likelihood observation (MLO) assumption by considering the Kalman gain, and $\Gamma_k = A \Sigma_k A^T + R$.

Note that the belief update is a function of the measured observations $y_k$, which are unknown during planning. To address this issue, we follow \cite{17} using Bayesian belief space planning. This problem is formally defined by the following grammar:

$$m_{k+1} = \text{Am}_k + Bu_k, \quad \Sigma_{k+1} = \Gamma_k - K_k C \Gamma_k, \quad \text{where} \quad F_k = \text{Am}_k + Bu_k, \quad \Sigma_{k+1} = \Gamma_k - K_k C \Gamma_k.$$

Other approaches such as \cite{6, 7, 18} avoid the maximum likelihood observation (MLO) assumption by considering $m_{k+1}$ as a random variable, i.e., $m_{k+1} \sim N(f_k, K_k C \Gamma_k)$. While we focus on belief-space planning under the MLO assumption, we expect that extending our approach to more sophisticated belief dynamics will be relatively straightforward.

C. Probabilistic Temporal Logic over Reals

We adopt an extension of temporal logic over reals (RTL) \cite{19} to formally represent task requirements for System \cite{1}. We first define a run of this system as a sequence $\rho = b_0 u_0 y_0 b_1 \ldots$ with an initial belief state $b_0 \sim N(m_0, \Sigma_0)$. A sequence of belief states in a run $\rho$ is called a path: $\beta = b_0 b_1 b_2 \ldots$. This definition allows us to formally design specifications over $\beta$ using probabilistic temporal logic over reals (PRTL).

PRTL formulas are defined recursively over a finite set of predicates $\Pi$ according to the following grammar:

$$\varphi := \pi^\mu_0 | \neg \pi^\mu_0 | \varphi_1 \land \varphi_2 | \varphi_1 \lor \varphi_2 | \varphi_1 U \varphi_2 | \varphi_1 R \varphi_2,$$

where $\pi^\mu_0 \in \Pi$ is a probabilistic atomic predicate and $\varphi, \varphi_1, \varphi_2$ are PRTL formulas. Each predicate is determined by a tolerance $\epsilon \in [0, 1]$ and the sign of the function $\mu(\beta) = a - h^T x$ (where $h \in \mathbb{R}^n$ and $a \in \mathbb{R}$). We denote the fact that a path $\beta$ satisfies a PRTL formula $\varphi$ with $\beta \models \varphi$. Intuitively, this means that the trajectory of System (1) fulfills the desired properties encoded in the specification $\varphi$.

We write $\beta \models \varphi$ if the path $b_k b_{k+1} \ldots$ satisfies $\varphi$. Formally, the following semantics define the validity of a formula $\varphi$ with respect to the path $\beta$, where $\beta \models \varphi$ if and only if $\beta \models \varphi_0 \varphi$ and:

- $\beta \models \varphi_0 \pi^\mu_0$ if and only if $P(\mu(x_k) \geq 0) > 1 - \epsilon$, $\beta \models \varphi_0 \neg \pi^\mu_0$ if and only if $P(\mu(x_k) \geq 0) > 1 - \epsilon$, $\beta \models \varphi_0 \varphi_1 \land \varphi_2$ if and only if $\beta \models \varphi_0 \varphi_1$ and $\beta \models \varphi_0 \varphi_2$, $\beta \models \varphi_0 \varphi_1 \lor \varphi_2$ if and only if $\beta \models \varphi_0 \varphi_1$ or $\beta \models \varphi_0 \varphi_2$, $\beta \models \varphi_0 \varphi_1 U \varphi_2$ if and only if $\exists k' \geq k$ s.t. $\beta \models \varphi_0 \varphi_1$ and $\forall k' \leq k'' \leq k' \beta \models \varphi_0 \varphi_2$.

D. Problem Formulation

The problem of task and motion planning with active perception for System \cite{1} can now be formulated in terms of a PRTL specification and belief space dynamics. This problem is formally defined as follows:

**Problem 1.** Given a stochastic system \cite{1}, a PRTL formula $\varphi$, and a prior belief state $b_0 \sim N(m_0, \Sigma_0)$, determine whether there exists a path $\beta$ of System \cite{1} that satisfies $\varphi$ and return the corresponding control inputs $u_0 u_1 u_2 \ldots$.

III. PROPOSED APPROACH

Our proposed approach, iterative deepening probabilistic temporal logic over reals (idPRTL), is illustrated in Figure \cite{4}. We formulate the planning problem as an existence model checking problem and use counterexample-guided synthesis \cite{20} to find a path that satisfies PRTL specification $\varphi$.

Specifically, two interacting layers, discrete and continuous, work together to overcome nonconvexities in the logical specification. At the discrete layer, existential bounded model checking (BMC) for an abstraction of System \cite{1} acts as a proposer, generating a discrete path that satisfies the specification. Satisfying discrete plans are passed to the continuous layer, which acts as a teacher. At the continuous layer, a sampling-based search is applied to check whether a discrete plan is feasible. If the feasibility test does not pass, a counterexample is provided to update the abstraction. This forces the discrete layer to propose a different discrete path. This process repeats until either a feasible plan is found or there is no satisfying discrete plan.

A. Discrete Existential Model Checking

We aim to find a path $\beta$ of System \cite{1} that satisfies a PRTL formula. To do so, we propose a finite abstraction that captures the requirements of $\varphi$. The finite nature of this abstraction allows us to use counterexample-guided-synthesis to find a dynamically feasible discrete path. We propose this abstraction as a deterministic Kripke structure:

**Definition 1.** \cite{21} A Kripke structure is a tuple $M = (S, I, \rightarrow, \mathcal{L})$, where $S$ is a finite set of states, $I \subseteq S$ are initial states,
Fig. 1: Bounded Existential Model Checking (BMC) proposes runs of an abstract system, while sampling-based feasibility search finds a corresponding run of the actual system. Feasibility search uses belief-space planning methods, incorporating active perception.

\[ \rightarrow \subseteq S \times S \text{ is a transition relation, and } \mathcal{L} : S \rightarrow 2^{AP} \text{ is a labeling function which maps to atomic propositions } AP. \]

To define a Kripke abstraction for System 1, first consider the subspace \( P_i = \{ \{ b \mid \pi^\alpha \in \Pi, \pi^\alpha \}\} \), which defines a region of the belief space where all atomic propositions \( \Pi_i \subseteq \Pi \) hold. Considering all subsets of \( \Pi \), we can construct the space \{P\_1, P\_2, ..., P\_m\}. We now define the following abstraction:

**Definition 2.** The abstraction of a belief state \( b \) is the set of all subspaces \( P_i \) that contain \( b \):

\[ \alpha(b) = \{ P_i \in \{ P_1, \ldots, P_m \} \mid b \in P_i \} \]

Each subspace \( P_i \) is convex. This is because each predicate \( \pi^\alpha \leq h^\alpha \) can be translated to a convex constraint:

\[ \pi^\alpha \leq h^\alpha = P(a - h^\alpha x_0 \geq 0) > 1 - \epsilon = P(\nu \leq -h^\alpha m_k - a < h^\alpha h^\alpha) < \epsilon = \pi^\alpha \leq h^\alpha \]

where \( \nu \sim N(0, I_n) \) is a standard Gaussian random variable and \( \Phi(z) \) is its cumulative distribution function (CDF). The volume of each \( P_i \) can be bounded with a convex polytope, which we denote \( E[P_i] \).

**Definition 3.** Given a PRTL formula \( \varphi, M_\varphi = (\mathcal{P}, P_0, \rightarrow, \mathcal{L}) \) is a Kripke structure where \( \mathcal{P} = \{ P_1, \ldots, P_m \} \) are a finite set of convex partitions of the belief space, \( P_0 = \{ \alpha(b) \} \), \( (\mathcal{P}, P_0) \rightarrow \) if and only if \( (E[\mathcal{P}] \cup E[P_0]) \cap (E[P] \cup E[P_0]) \neq \emptyset \) with \( \mathcal{P} = \{ P_i \in \mathcal{P} | \mathcal{L}(P_i) = \mathcal{L}(P) \text{ and } E[P] \cap E[P_0] \neq \emptyset \} \), \( AP = \{ \Phi_1, \ldots, \Phi_m \} \) are state subformulas of \( \varphi \), and \( \Phi_i \in \mathcal{L}(P) \text{ if and only if } b \in \Phi_i \text{ for all } b \in \mathcal{P} \).

Note that any PRTL formula \( \varphi \) has an unique corresponding formula \( \tilde{\varphi} \) over the subformulas \( \tilde{\varphi} \). For example, for a formula \( \varphi = (\pi^\alpha_1 \land \pi^\alpha_2)U(\pi^\alpha_3 \lor \pi^\alpha_4) \), \( AP = \{ \pi_1, \pi_2 \} \) is the set of atomic propositions of \( M_\varphi \), where \( \pi_1 = \pi^\alpha_1 \land \pi^\alpha_2 \text{ and } \pi_2 = \pi^\alpha_3 \lor \pi^\alpha_4 \). Consequently, the simplified PRTL formula \( \tilde{\varphi} = \pi_1 U \pi_2 \).

**Theorem 1.** The abstraction of every path \( \beta \) in System 1 that satisfies a PRTL formula \( \varphi \), i.e., \( \alpha(b_0) \alpha(b_1) \alpha(b_2) \ldots \), is a path in \( M_\varphi \) that satisfies the corresponding \( \tilde{\varphi} \) over \( AP \).

**Proof.** We will prove the theorem by induction. First, note that \( \{ \pi^\alpha_i \mid \beta = \pi^\alpha_i \} \subseteq \alpha(b_k) \). If a trace in \( M_\varphi \) satisfies \( \tilde{\varphi} \), subsets \( \tilde{P}_k \subseteq \mathcal{P}_k \) also satisfy \( \tilde{\varphi} \) since \( \mathcal{L}(\tilde{P}_k) = \mathcal{L}(P_k) \). Moreover, by Definition 3, \( \alpha(b_0) = \mathcal{P}_0 \). Finally, if there exists \( u_k \in U \) such that \( b_{k+1} = f(b_k, u_k, y_k) \), then \( \alpha(b_k) \alpha(b_{k+1}) \).

We will prove this claim by contradiction. Assume that there exists \( u_k \in U \) such that \( b_{k+1} = f(b_k, u_k, y_k) \), but \( \alpha(b_k) \alpha(b_{k+1}) \). Consider the case of two subformulas \( \varphi_1 \) and \( \varphi_2 \) combined with a temporal operator in \( \varphi \). In this case, there exists an instant \( t_k \leq t_k+1 \) such that \( \beta \not\models (\varphi_1 \varphi_2) \), \( \beta \not\models (\varphi_1 \varphi_2) \), and \( \beta \not\models (\varphi_1 \varphi_2) \). But this violates the PRTL semantics. Therefore, this path cannot satisfy a PRTL formula.

Given an abstraction \( M_\varphi \) and an abstracted formula \( \tilde{\varphi} \), we can use BMC tools such as NuSMV to find a path \( \mathcal{P}_K = \mathcal{P}_0 \ldots \mathcal{P}_{L-1}(\mathcal{P}_L \ldots \mathcal{P}_R)^\oplus \) that satisfies \( \tilde{\varphi} \) in \( (K, L) \)-loop form. Such tools find a satisfying discrete path if one exists, and otherwise indicate that no such path exists.

**B. Feasibility Search**

Existential Model Checking generates a discrete path \( \mathcal{P}_K \) which satisfies \( \tilde{\varphi} \). Given such a path, feasibility search looks for a dynamically feasible path \( \beta \) corresponding to \( \mathcal{P}_K \). This problem can be encoded in the following feasibility problem:

\[ \text{find } \beta \]

\[ \text{s.t. } m_0 = m_0, \Sigma_0 = \Sigma_0, \]

\[ \text{if } L \leq k \text{ then } m_H = m_{(L,H)}, \Sigma_H = \Sigma_{(L,H)}, \]

\[ \forall k = 1..H : m_{k+1} = Am_k + Bu_k, \]

\[ \Sigma_{k+1} = K(\Sigma_k + K(\delta - 1)) \]

\[ h_0^\top m - \Phi^{-1}(\epsilon I(k)) \| \sqrt{h} \|_2 < a_0(k), \]

where \( \| \) is an increasing function that maps the instant \( k \) to the relevant constraints \( \mathcal{P}_i \in \mathcal{P}_K \). In formulating this problem, we use the MLO belief dynamics since we do not have access to future observations, as discussed in Section II.

This simplifying assumption renders the feasibility problem computationally tractable, but limits the completeness of our approach (see Section IV).

Even with the MLO belief dynamics, the feasibility problem is non-convex. For this reason, we turn to rapidly exploring random tree (RRT) search. We propose modified sampling (SAMPLE) and steering (PROPAGATE) strategies which push the search towards trajectories within the discrete path \( \mathcal{P}_K \).

Furthermore, we assume that the noise covariance \( W_k \) may be state dependent. In this case, a sparse search is beneficial as it quickly samples from many nearby states. Thus we draw on techniques from SPARSE-RRT. This includes the function BESTNEAREST, which searches for active vertices \( \delta \)-near the sampled state, and the function DRAIN, which removes new vertices that are too close to other active vertices.

In SAMPLE, the probability of sampling a belief state in the convex space \( \mathcal{P}_k \) is inversely proportional to the number of belief states in \( V_{active} \) that are in \( \mathcal{P}_k \). This encourages sampling points in those \( \mathcal{P}_k \) that are relatively unexplored.
Algorithm 1 fSearch($\mathcal{P}_K, N$)

1: $V_{act} \leftarrow \{b_0\}; V_{inact} \leftarrow \emptyset; E \leftarrow \emptyset; i \leftarrow 0$
2: $V = V_{act} \cup V_{inact}; G = \{V, E\}$
3: for $i = 1..K$ do
4: $b_{rand} \leftarrow \text{Sample}(V_{act}, \mathcal{P}_K)$
5: $(b_{near}, k) \leftarrow \text{BestNearest}(V_{act}, b_{rand}, \mathcal{P}_K, \delta_{near})$
6: $(b_{new}, u) \leftarrow \text{Propagate}(b_{near}, \mathcal{P}_K, \mathcal{P}_{k+1})$
7: $V_{act} \leftarrow V_{act} \cup \{b_{new}\}$
8: $E \leftarrow E \cup \{(b_{near}, u, b_{new})\}$
9: $\text{Drain}(\delta_{\text{drain}}, b_{new}, G)$
10: if $b_{new} \in \mathcal{P}_{k\text{near}+1}$ then
11: $k_{last} \leftarrow k_{last} + 1$
12: $(\beta, u) \leftarrow \text{FeasRun}(G, V_{act})$
13: return $(\beta, u)$

The basic idea used in Propagate, presented as Algorithm 2, is to account for the convexity of the constraints. Since the belief dynamics are nonlinear and underactuated, we (1) sample a belief mean from $\mathcal{P}_k \cup \mathcal{P}_{k+1}$[10], (2) generate a run towards the sampled mean (lines 11-14), and (3) generate a run that minimizes the belief covariance using the first run as the nominal run (lines 15-18). The first two steps essentially ensure that a trajectory is found that transitions from $\mathcal{P}_k$ to $\mathcal{P}_{k+1}$. The last step encodes active perception: we find nominal trajectories that minimize uncertainty in the belief.

In the case of a loop (line 1) the basic idea is to close the loop by solving the following optimization problem:

$[\mathbf{u}_0, \ldots, \mathbf{u}_{n-1}] = \arg\min \left[ \begin{array}{c} \mathbf{u}_0 \cdot u_0, \ldots, \mathbf{u}_{n-1} \cdot u_{n-1} \end{array} \right]$

s.t. $\mathbf{m}_{\text{final}} - A^n \mathbf{m}_{\text{near}} = C[u_0, \ldots, u_{n-1}]^T$

where $C$ is the controllability matrix and a closed-form solution is given on line 3 of Algorithm 2.

Algorithm 1 has similar properties to SPARSE-RRT, including asymptotic near-optimality:

**Theorem 2.** Given a discrete path $\mathcal{P}_K$ in $K$-loop form, Algorithm 1 finds a path $\beta_K$ of System 1 only if Eq. 2 has a solution. Moreover, if there exists an optimal path $\beta^* \in \mathcal{P}_K$ of System 1, Algorithm 2 will eventually generate a solution to Eq. 2.

**Proof.** SPARSE-RRT [23] assumes that (1) the state space is sampled uniformly and (2) Propagate randomly selects controls of propagation. It is easily seen that Assumption (1) is asymptotically guaranteed. Assumption (2) is guaranteed by randomly selecting target states $b_{final}$ and duration $T$. Thus our modified propagation algorithm gives the dispersion necessary to achieve asymptotic optimality. Therefore, from [23, Lemma 2], Algorithm 1 will eventually generate a path $\beta$ in $\mathcal{P}_K$, given that there exists an optimal path $\beta^*$ in $\mathcal{P}_K$. The proof of soundness is trivial since Propagate only generates valid path segments.

Feasibility search returns a trajectory (a sequence of states and control inputs) that satisfies the given PRTL specification.

Algorithm 2 Propagate($b_{near}, \mathcal{P}_k, \mathcal{P}_{k+1}$)

1: if in loop then
2: $b_{final} \leftarrow \text{Sample}(V_{act}, \mathcal{P}_{k+1})$
3: $[\mathbf{u}_0, \ldots, \mathbf{u}_{n-1}] \leftarrow C^{-1}(m_{final} - A^n m_{near})$
4: for $k = 0..n - 1$ do
5: $u_{new,k} \leftarrow \arg\min \| u + \mathbf{u}_k \|_1$
6: $b_{new,k+1} \leftarrow \mathbf{f}(b_{near,k}, u_{new,k})$
7: return $(b_{new,n}, u_{new})$
8: else
9: $(m_{final,T}) \leftarrow \text{Sample}(\mathcal{P}_k, \mathcal{P}_{k+1})$
10: $b_{final} \leftarrow N(m_{final,T}, \mathbf{0}_{n \times n})$
11: $F \leftarrow \text{LQR}(A, B, Q, R), b'_{0} \leftarrow b_{near}$
12: for $k = 0..T - 1$ do
13: $u_{k} \leftarrow \arg\min \| u + F(k_{b}' - m_{final}) \|_1$
14: $b'_{k+1} \leftarrow \mathbf{f}(b'_{k}, u_{k})$
15: $F_k \leftarrow \text{B-LQR}(T, f, Q, Q_f, R), b'_{new,0} \leftarrow b_{near}$
16: for $k = 0..T - 1$ do
17: $u_{new,k} \leftarrow \arg\min \| u + F(k_{b}' - b_{final}) \|_1$
18: $b_{new,k+1} \leftarrow \mathbf{f}(b'_{new,k}, u_{k})$
19: return $(b_{new,n}, u_{new})$

Since the feasibility problem is solved with sampling-based search, this trajectory is not necessarily unique; however, it is guaranteed to satisfy the specification.

C. Iterative Deepening Search

If feasibility search does not find a satisfying run, we consider the given abstract path $\mathcal{P}_K$ to be infeasible. We then use $\mathcal{P}_K$ as a counterexample, and generate a new discrete path. Specifically, we generate a new Kripke structure by taking the product of $M_e$ and the complement of $\mathcal{P}_K$. In this way, new discrete paths found by BMC over the new Kripke structure will avoid the infeasible counterexample.

Algorithm 3 idPRTL

1: $(M_z, \hat{\varphi}) \leftarrow \text{abstract}(\varphi, \text{sys})$
2: while $\text{BMC}(M_z, \hat{\varphi}) \neq \emptyset$ do
3: $\mathcal{P}_K \leftarrow \text{BMC}(M_z, \hat{\varphi})$
4: $(\beta, u) \leftarrow \text{fSearch}(\mathcal{P}_K)$
5: if feasible then
6: return $(\beta, u)$
7: $M_z \leftarrow M_z \times \text{kripke}(\mathcal{P}_K)$
8: return infeasible

This process, outlined in Algorithm 3, continues until either a satisfying trajectory of System 1 is found or BMC indicates that there is no satisfying discrete path. We call this Iterative Deepening PRTL because it is inspired by iterative deepening graph search [24]. Each iteration of BMC is like a depth-limited-depth-first search over the space of possible discrete plans. Repeating this procedure with the new product Kripke structure is analogous to increasing the depth of the search.
IV. Soundness and Completeness

First, we prove the soundness of our approach as follows:

**Theorem 3.** Given a stochastic system (1), a PRTL formula \( \varphi \), and a prior belief state \( b_0 \sim \mathcal{N}(m_0, \Sigma_0) \), Algorithm 3 finds a path \( \beta_H \) only if \( \beta_H \models \varphi \).

**Proof.** Assume that Algorithm 3 finds a path \( \beta_H \). From Theorem 2, this run is in a discrete path that satisfies the specification. From Theorem 4, the trace of this run is a trace of \( M_\varphi \) which satisfies the formula \( \varphi \). Thus \( \beta_H \) is a path of System (1) that satisfies \( \varphi \).

We prove probabilistic completeness as follows, by showing that the nonexistence of a solution of Algorithm 3 is evidence that no solution exists in the following sense:

**Remark 3 (Complexity).** The complexity of 1DPRTL depends on the PRTL formula complexity and the parameter \( N \). First, the worst case number of symbols in the abstracted system (Def. 5) is exponential in number of predicates, \( O(2^{11}) \). State-of-the-art BMC solvers are linear in the number of symbols and the length of bound \( K \), \( O(K) \). Finally, the complexity of each feasibility search query is \( O(N \log N) \).

V. Example

In this section, we apply our planning framework to automated infrastructure inspection with a quadrotor. The inspection task involves collecting images of key points. While the locations of these key points are often known a priori, imperfect localization requires that the quadrotor must maintain a belief over its location. Furthermore, when the quadrotor is directly over the power line it can use a camera for precise localization, while it must otherwise rely on less accurate GPS.

A simple inspection task might be articulated as follows: “Staying away from the power line, take pictures of the top of two poles and return to the charging station.” This task, illustrated in Figure 2, can be written in PRTL as follows:

\[
\varphi = [\Box \varphi_{safe} \land \Box (\varphi_{pole1} \lor \varphi_{pole2})] \land \Box \neg \varphi_h,
\]

where \( \varphi_{safe} \) denotes avoiding collisions with the powerline (grey box in Figure 2), \( \varphi_{pole1} \) indicates flying over the powerline, \( \varphi_{pole2} \) indicate visiting the key points, and \( \varphi_h \) indicates returning “home” to the charging station. All predicates are defined with \( \epsilon = 0.05 \).

For the quadrotor dynamics, we follow [28] in modeling the quadrotor with a chain of integrators using differential flatness. We model the observations \( y \) as follows, where \( x = [x, y, z]^\top \) is the position of the quadrotor:

\[
y = x + w(x) = x + \mathcal{N}(0, min(\sigma^2_{gps}, \sigma^2_{camera}(x)))
\]

\[
\sigma^2_{camera}(x) = (y - \bar{y}_{center})^4 + (z - z_{top})^4 + \sigma^2_{min}
\]

Essentially, the quadrotor has low localization uncertainty while it is directly over the power line \( (\sigma^2_{camera}) \), but high localization uncertainty otherwise \( (\sigma^2_{gps}) \). After specifying an initial belief, 1DPRTL returns the trajectory shown in blue, where the shaded region indicates the covariance. We know from Theorem 3 that this trajectory satisfies the specification.
Fig. 2: A quadrotor performing an inspection task must visit two key points (green) and avoid an obstacle (gray) before returning to a charging station (red). The belief trajectory (blue) indicates the use of active perception: the quadrotor flies high above the powerline to improve confidence in its belief before approaching the key points.

Figure 2 illustrates three important features of our approach. First, the resulting trajectory satisfies the specification, as both points of interest (green) are visited before returning to the charging station (red). Second, the specification (5) is non-convex, as it contains a disjunction and nested temporal operators. This means that existing sampling-based approaches such as [14] cannot be used in this scenario. Finally, the resulting trajectory illustrates the use of active perception. The quadrotor first flies a conservative distance above the powerline, approaching it from above so that the camera can be used to reduce uncertainty before moving close to the key points.

VI. CONCLUSION AND FUTURE WORK

We presented a framework for controller synthesis from PRTL specifications in the belief space that combines the advantages of Bounded Model Checking and sampling-based motion planning. Our framework allows for active perception in complex tasks involving nonconvex PRTL specifications. We demonstrated the efficacy of our approach on a simulation of a quadrotor power line inspection task.

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