Overlapping Community Detection using Dynamic Dilated Aggregation in Deep Residual GCN

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Abstract. Overlapping community detection is a critical problem in unsupervised machine learning on network-structured data. Some research has considered applying graph convolutional networks (GCN) to tackle the problem. However, the considered GCN is shallow and can not capture communities with larger diameters. Moreover, it is still open to incorporate dynamic dilated aggregation in deep GCNs for irregular graphs. In this study, we design a deep dynamic residual GCN (DynaResGCN) based on our dynamic dilated aggregation algorithm and a unified end-to-end encoder-decoder-based framework to detect overlapping communities. The deep DynaResGCN model is used as the encoder, whereas we incorporate the Bernoulli-Poisson (BP) model as the decoder. Consequently, we apply our overlapping community detection framework in a research topics dataset without having ground truth, a set of networks from Facebook having reliable (hand-labeled) ground truth, and in a set of very large co-authorship networks having empirical (not hand-labeled) ground truth. Our experimentation on these datasets shows significantly superior performance over many state-of-the-art methods for detecting overlapping communities in networks.

Keywords: Overlapping Community Detection · Deep GCN · Unsupervised Learning

1 Introduction

Network structured data are ubiquitous. Almost every domain of science and engineering has to deal with networks. The nodes of real-world networks often form special groups where link density is high, and the density of edges is low among these groups. This property of networks is called community structure [20]. Human society is composed of communities that are in many cases virtual due to the wide use of online social platforms. Indeed, social communities are of great importance to social scientists and have been studied thoroughly for a long time [14,64,25,41]. Communities in the global financial network play a key role in influencing the transitions of the global economy and financial system [9]. In protein-protein interaction networks, protein communities act as functional modules to perform specific tasks in cells [45,62,10]. As a consequence,
community detection is an essential tool to analyze different kinds of networks such as social networks, research topics networks, protein-protein interaction networks, ecological networks, financial networks, etc., and to understand their structure.

The communities in most real-world networks, especially social networks, overlap [70]. The same participant participates in different communities at the same time. Sometimes, a participant in a community might simultaneously be a different community member. A person in a social network is in his or her family’s community while he/she is also in the community of his or her coworkers. In reality, a node may belong to an unlimited number of communities in a social network. This overlapping node is critical for various reasons, as it might be used positively and negatively. For instance, in a social network context, a node belonging to two disputing communities might be potentially used to resolve the dispute. On the contrary, this node may disclose sensitive information about a community to other communities violating privacy ethics. Indeed, the overlap is an inherent characteristic of many real-world social networks [29]. Moreover, disjoint community detection induces a partition in the network that eventually breaks the communities [14]. Therefore, detecting overlapping communities in different networks is of utmost importance, and thus, it has attracted significant attention in the research community [66].

Many works in the literature are related to non-overlapping community detection [1,58]. In the case of non-overlapping community detection, node embedding approaches perform well [55,8]. However, most of these approaches are not well suited for overlapping community detection due to the lack of scalable and reliable clustering methods for high-dimensional vectors. A viable approach to resolve this issue is to generate the node embedding as community affiliation, which can be done effectively and efficiently using GCNs [50].

Among the classical approaches for overlapping community detection, label propagation algorithms are prominent [37]. For instance, SLPA is an overlapping community detection algorithm based on the idea of label propagation [67]. DEMON [15], a node-centric bottom-up overlapping community detection algorithm, also leverages ego network structure with overlapping label propagation. Other algorithmic approaches include seed-set expansion [3,18], evolutionary algorithms [36], and heuristics-based methods [47,17,21,34]. Some heuristics-based works considered game-theoretic concepts [62].

Many studies consider some kinds of matrix factorization along with probabilistic inference [69,41,31,30,33,59]. One of the earliest works in this regime is BigClam [69]. BigClam was designed for overlapping community detection in large networks [69]. BigClam uses gradient ascent to create an embedding, which is later used to determine nodes’ community affiliation. MNMF [60] uses modularity-based regularization with non-negative matrix factorization. DANMF uses a telescopic non-negative matrix factorization approach based on a deep auto-encoder to learn membership distribution over nodes [73]. Some works [77,2] performed factorization of modularity matrices using neural nets, while other works considered adversarial learning [28,11] or deep belief networks [26] to learn
community affiliation. However, they did not use graphs in their neural network architecture, which is crucial to achieving superior performance [50]. Finally, NOCD [50] used a graph neural networks (GNN) based encoder-decoder model to detect overlapping communities. Unlike NMF-based methods, it optimizes the weights of a GNN to generate a better community affiliation matrix.

Graph neural networks (GNN) are a class of specialized neural networks that can operate on graph-structured data [22,63,68]. The authors of NOCD [50] first developed a graph neural network-based approach for overlapping community detection. In that approach, they used a two-layer shallow graph convolutional network as an encoder to generate community embedding and a decoder based on the Bernoulli-Poisson model [69,74,54] to reconstruct the original graph from the embedding. However, a shallow GCN has essential limitations to capturing communities with a larger community diameter [19] (> 2) as it can aggregate information only up to two hops from a node. One solution to the issue is to consider a deep graph convolutional networks (DeepGCN) [32,12,57,35] instead of a shallow GCN. However, Li et al. [32] proposed the DeepGCN in the case of point cloud learning, where the considered graph is regular while the real-world networks are mostly irregular graphs (having variable-sized neighborhoods). The DeepGCN proposed by [57] is also specific to regular graphs where image pixels are considered nodes, while [12] completely ignores dilated aggregation. Moreover, [35] proposes a different approach that requires QR factorization with backpropagation, making it intractable for practical implementation. Thus, it is essential to design a deep GCN that can handle the irregular graphs to detect overlapping communities with a larger diameter.

Training deeper GCNs is challenging because of over-smoothing and vanishing gradient problems [32,12,57,35]. Previous works [32,57,12] overcame this challenge with the idea of residual connection and dilated aggregation borrowed from ResNet [24]. Residual connection means having direct connectivity from a layer’s input to that layer’s output, bypassing the non-linearity in the activation function. In GCNs, each node aggregates information from the neighboring nodes. It is called dilated aggregation if this aggregation mechanism aggregates information from some of the distant neighbors while skipping some of the nearest neighbors. In dilated aggregation, we may incorporate some randomization while considering neighbors, which introduces the concept of edge dynamicity. Dilated aggregation with edge dynamicity is called dynamic dilated aggregation. It is still challenging to introduce dynamic dilated aggregation in irregular graphs to train deep GCNs. Indeed, none of the existing works on deep GCN considered dilated aggregation in irregular graphs.

In this study, we design dynamic dilated aggregation mechanisms in irregular graphs. We incorporate our dynamic dilated aggregation schemes along with residual connection into the GCN to obtain a deep GCN that can learn from any graphs. Eventually, we apply our designed deep GCN to detect overlapping communities in different kinds of networks with different sizes. For overlapping community detection, we adopt an encoder-decoder-based approach similar to NOCD. Our encoder is a deep GCN model called DynaResGCN, which generates
the community embedding, and the decoder attempts to reconstruct the original graph.

Our objective is summarized as follows:

– Detecting overlapping communities with higher community diameters effectively
– Designing a DeepGCN model

In Table 1, we provide a conceptual comparison of our method with the relevant literature.

| Algorithm/Model          | Overlapping community detection | Large community diameter | Residual connection | Edge dynamicity | Dil.Agg. in irregular graphs | Deep GCN model | Reference |
|--------------------------|---------------------------------|--------------------------|---------------------|----------------|-------------------------------|----------------|-----------|
| DynaResGCN + BP          | ✔                               | ✔                        | ✔                   | ✔              | ✔                             | N/A            | [50]      |
| NOCD                     | ✔                               | ✔                        | ✔                   | ✔              | ✗                             |                | [70]      |
| BigClam                  | ✔                               | ✔                        | ✗                   | ✗              | ✗                             |                | [69]      |
| SLPA                     | ✔                               | ✔                        | ✔                   | ✔              | ✗                             |                | [67]      |
| DEMON                    | ✔                               | ✔                        | ✔                   | ✔              | ✗                             |                | [15]      |
| DANMF                    | ✔                               | ✔                        | ✗                   | ✗              | ✗                             |                | [73]      |
| CDE                      | ✔                               | ✔                        | ✗                   | ✗              | ✗                             |                | [33]      |
| EPM                      | ✔                               | ✔                        | ✗                   | ✗              | ✗                             |                | [74]      |
| CESNA                    | ✔                               | ✔                        | ✗                   | ✗              | ✗                             |                | [50]      |
| SVNMF                    | ✔                               | ✔                        | ✗                   | ✗              | ✗                             |                | [71]      |
| SNetOC                   | ✔                               | ✔                        | ✗                   | ✗              | ✗                             |                | [54]      |
| DeepWalk with K-means    | ✔                               | ✔                        | ✗                   | ✗              | ✗                             |                | [42]      |
| Graph2Gauss with K-means | ✔                               | ✔                        | ✗                   | ✗              | ✗                             |                | [5]       |
| DeepGCN                  | ✗                               | N/A                      | ✔                   | ✔              | ✗                             | N/A            | [32]      |
| GCNII                    | ✗                               | N/A                      | ✔                   | ✔              | ✗                             | N/A            | [57]      |
| Krylov GCN               | ✗                               | N/A                      | N/A                 | N/A            | N/A                           | ✔              | [43]      |

(1) Dilated aggregation in the graphs having variable-sized neighborhoods.

For evaluation, we consider a large research topics network [6], a set of Facebook datasets [39] having reliable ground truth information, and a set of very large co-authorship networks (nodes > 10K) [50] as an additional benchmark. In the case of the topics network, ground truth information related to overlapping communities does not exist. Therefore, we have to rely on unsupervised metrics called fitness functions or quality measures such as conductance, clustering coefficient, density, and coverage [38,69] to determine the quality of the detected overlapping communities. In addition to the evaluation metrics, we generate heatmaps or colormaps to illustrate community overlap. We compare our results in topics network with existing overlapping community detection approaches, such as NOCD, BigClam, SLPA, DANMF, and DEMON. Our methods show superior performance over most of the considered existing methods in terms of both quality metrics and visualization. In the case of the datasets having ground truth information, we compare with exactly the same baselines as in NOCD considering both node features and without node features in terms of normalized mutual information (NMI) [40]. Our method, DynaResGCN, achieves a clear
superior performance over all the baselines in almost every case in terms of NMI.

In summary, our contributions are as follows:

– Development of a deep residual GCN (DynaResGCN) model based on dynamic dilated aggregation in irregular graphs
– Design of an effective and scalable overlapping community detection framework based on DynaResGCN and Bernoulli-Poisson model
– Detection of overlapping communities in real-world networks outperforming many state-of-the-art methods

The remainder of the paper is organized as follows. In Section 2, we introduce preliminary terminologies and some of the prominent related works. We describe our methodology in Section 3. Section 4 deals with our experimentation. In section 5, we present our results and discuss our insights with overall findings. Finally, Section 6 provides our conclusions.

2 Preliminaries

In this section, we introduce the necessary notations and terminologies that will be used throughout the paper and discuss prominent related works briefly.

2.1 Notations and Terminologies

We consider an undirected graph as $G = (V, E)$, where $V$ is the set of vertices and $E$ is the set of edges. We represent the adjacency matrix as $A \in \{0, 1\}^{n \times n}$, the number of vertices as $n$, the set of vertices as $V = \{1, \ldots, n\}$, and the set of edges as $E = \{(u, v) \in V \times V : A_{uv} = 1\}$. For each node, we may consider a $d$-dimensional feature vector. We denote the feature matrix as $X \in \mathbb{R}^{n \times d}$. If no feature is associated with a node, $X$ is an identity matrix. Let us assume $C$ is the set of communities and the cardinality of $C$ is $k$. In the case of overlapping community detection, we assign each node to some of the communities in $C$. However, for each node, there is a strength associated with each different community. We define this strength as the community affiliation of the node. For each node, there is a $k$-dimensional community affiliation vector. We assign the community affiliation as a non-negative real number. Therefore, we can obtain a community affiliation matrix as $F \in \mathbb{R}^{n \times k}$. For a node $u$, we denote the membership strength of community $c$ as $F_{uc}$. We use a threshold to obtain a binary community affiliation matrix as $F \in \{0, 1\}^{n \times k}$. Therefore, each node can be assigned to multiple communities. It is also possible that some nodes have no communities at all. The following text describes the basic concepts related to our paper.

Artificial neural networks are a class of machine learning models. The smallest computing unit of a neural network is a neuron. The input of a neuron is a vector. A neuron is a parameterized function whose output varies with different parameters for the same input. These neurons connected in a computational
graph are called a neural network. For more information, readers may study the overview article by Schmidhuber [49].

A convolutional neural network (CNN) is a class of neural networks based on convolution operation. We apply CNN on image structured data, which can be represented as an 8-regular graph where each pixel is a node and each pixel is connected to eight neighboring pixels. However, these classical CNNs are not able to process general graphs. Special treatment is required to perform convolution on general graphs.

Graph neural networks (GNN) is designed to handle the inability of classical neural networks (NN) to process general graphs. To alleviate this limitation of NN, Scarselli et. al. proposed a graph neural network model [48]. According to this model, a label vector and a feature vector are associated with each graph node. This model predicts the class of the graph or the classes of the nodes of the graph provided the labels and feature vectors for all nodes.

Graph convolutional networks (GCN) are GNNs that can perform convolution operations in general graphs. In the case of GCNs, each node aggregates the feature vectors of its neighboring nodes to achieve a richer feature vector. For each vertex $u$, a $d$-dimensional feature vector is considered as $h_u \in \mathbb{R}^d$. The whole graph $G$ can be represented by concatenating the feature vectors of all the nodes as $h_G = [h_{u_1}, h_{u_2}, \ldots, h_{u_n}]^T \in \mathbb{R}^{n \times d}$. A general graph convolution operation is defined based on two operations: an aggregation operation and an update operation,

$$G_{l+1} = Update(Aggregate(G_l, W_{agg}^l), W_{update}^l)$$

where $G_l$ is the input graph with features at the $l$-th layer and $G_{l+1}$ is the output graph with updated features at the $(l+1)$-th layer. For the aggregation operation, the learnable weights matrix is $W_{agg}$ and for the update operation, the learnable weights matrix is $W_{update}$.

Shallow GCNs are GCNs having small numbers of layers (< 3). In previous work, NOCD used a two-layered shallow GCN that was first proposed by Kipf and Welling [63]. According to NOCD, the GCN model is defined as follows:

$$F := GCN_{\theta}(A, X) = ReLU(\hat{A}(ReLU(\hat{A}XW^{(1)}))W^{(2)})$$

Here, $\hat{A} = \hat{D}^{-1/2} \hat{A} \hat{D}^{-1/2}$ is the normalized adjacency matrix, and $\hat{A} = A + I_N$ is the adjacency matrix considering self-loops. The diagonal degree matrix of $\hat{A}$ is $\hat{D}_{ii} = \sum_j \hat{A}_{ij}$. Here, the ReLU function works as a non-linear update function and aggregation is done through a weighted sum of the features of neighbors. This weight is a learnable parameter.

### 2.2 Overlapping Community Detection Methods

In the following, we include most of the prominent related works which we have used for comparison and evaluation.
SLPA The Speaker-listener Label Propagation Algorithm (SLPA) \[67\] is a general framework to analyze overlapping communities in social networks. According to this algorithm, nodes exchange information (label) based on dynamic interaction rules. This framework is designed to analyze both individual overlapping nodes and the whole community. SLPA is an extension of the previous label propagation algorithm (LPA) \[43\]. Each node in the LPA has a single label. This label is iteratively updated by the majority of labels in the neighborhood. After completion of the algorithm, non-overlapping (disjoint) communities are discovered. To allow overlap, each node is allowed to have multiple labels.

BigClam BigClam (Cluster Affiliation Model for Big Networks) was originally designed for large networks \[69\]. BigClam is a probabilistic generative model for graphs to capture network community structure based on the community distribution of each node. The whole idea of BigClam is based on a bipartite community affiliation network where the generative model is based on the fact that the higher number of communities are shared by two nodes, the greater the probability of connecting these two nodes with an edge. Community detection using the BigClam model is the reverse problem of generating the graph. The community affiliation matrix $F$ is determined by the underlying graph $G(V, E)$. The number of communities $k$ is given, and the BigClam model finds the most likely affiliation matrix $\hat{F}$ maximizing log-likelihood.

DEMON DEMON stands for Democratic Estimate of the Modular Organization of a Network \[15\], an algorithm designed to discover communities in complex networks. In this approach, each node gives community labels to its neighboring nodes. Thus, every node obtains community labels from its neighboring nodes. This approach is called a democratic approach because each node can vote for every neighboring node. Then, the communities are discovered from the labels for each node. Since one node can have multiple labels, one node can belong to multiple communities. After obtaining labels from the neighborhood, the communities can be discovered using different merging algorithms.

MNMF Wang et al. \[60\] proposed the Modularized Nonnegative Matrix Factorization (M-NMF) model. In this approach, the authors designed a community-preserving matrix factorization scheme. The modularity-based community detection model and NMF-based representation learning model are jointly optimized in a unified learning framework. Basic intuition indicates the nodes belonging to the same community should have similar representations. The proposed M-NMF model is optimized to preserve both pairwise node similarity and community structure. For pairwise node similarity, the first-order and second-order proximities of nodes are incorporated to learn representations using matrix factorization. A modularity constraint term is incorporated to detect communities.

DANMF DANMF \[73\] (Deep Auto-encoder like Non-negative Matrix Factorization) is a variant of NMF. NMF-based approaches factorize the adjacency
matrix $A$ into two non-negative matrices $U$, $V$, where $A \sim UV (U \geq 0, V \geq 0)$. Normally, the columns of $V$ represent the community strength of the nodes of the network. The authors of DANMF proposed a hierarchical matrix factorization scheme. Inspired by deep auto-encoders, the adjacency matrix is factorized into multiple levels. DANMF is like auto-encoders where there is an encoder component and a decoder component. Both components have deep structures. The encoder part transforms the original adjacency matrix to a community affiliation matrix and the decoder part reconstructs the original adjacency matrix. Unlike deep-autoencoders, the loss function in DANMF is unified with both the encoder and decoder.

**NOCD** NOCD stands for neural overlapping community detection [50]. The model consists of an encoder and a decoder. The encoder is a shallow graph convolution network (GCN) with two layers. The decoder is based on the Bernoulli-Poisson model [69]. The output of the encoder is a community affiliation matrix. The decoder attempts to reconstruct the original graph. Finally, a loss is generated based on the reconstruction error. This loss is used to train the GCN model. We use a similar framework in this study.

We also compare our method with some other related works including EPM [74], SNetOC [54], CESNA [71], SNMF [59], CDE [33], DeepWalk [42] with Non-exhaustive Overlapping $K$-means [65] (DW/NEO), and Graph2Gauss [6] with Non-exhaustive Overlapping $K$-means [65] (G2G/NEO).

## 3 Our Methodology

Communities are defined on a graph as the dense sub-graphs of the graph. According to the most recent research, a community is a group of nodes. These nodes have a higher probability of forming edges within the group than forming edges with nodes outside of the group [16,50]. We adopt this definition of a community to develop our framework of overlapping community detection. We consider a graph generative model $p(G|F)$ for a graph $G$ [69,50]. According to this model $p(G|F)$, we can generate the graph $G$ for a given $F$. However, overlapping community detection is exactly the opposite task. We have to find the unobserved affiliation matrix $F$ for a given graph $G$ [69,50]. We can optimize $F$ by minimizing some loss function based on the model $p(G|F)$ [69]. On the contrary, we can feed the graph $G$ into a GNN; then we can optimize the GNN to generate a more accurate $F$. In this paper, we follow the GNN-based approach to optimize $F$.

We consider an unsupervised approach in this study. Essentially, we design an encoder and decoder-based approach to tackle the problem of overlapping community detection (Fig. 1). The encoder takes a graph with adjacency matrix $A$ and feature matrix $X$ as input and generates the community affiliation matrix $F$ as output. The decoder takes $F$ as input and produces a loss. We use the loss generated by the decoder to train the encoder. Indeed, we use a deep GCN-based encoder and the Bernoulli-Poisson decoder. To achieve deep GCNs, we introduce the concept of dynamic dilated aggregation successfully in the case
of irregular graphs. We also incorporate the idea of residual connection in our model. We term our deep GCN model DynaResGCN. In a nutshell, we unify the idea of dynamic edges, dilated aggregation, and residual connection in an encoder-decoder-based framework for overlapping community detection. Next, we describe our approach to achieving the Deep GCN (DynaResGCN) encoder, the corresponding Bernoulli-Poisson decoder to train the Deep-GCN encoder, and the training algorithm.

### 3.1 Deep Dynamic Residual GCN Encoder

A deep-GCN encoder is a GNN with many layers having the capability of effective learning. In this study, we introduce a deep-GCN encoder for overlapping community detection in graphs. We incorporate the idea of residual connections, dynamic edges, and dilated aggregation in a single framework called DynaResGCN (Fig. 2). This figure explains the whole architecture of the deep DynaResGCN encoder where message passing of GNN is shown for only a single node (for simplicity). At each layer, information is aggregated from a fixed number of random nodes. Some of these nodes are chosen from the first hop and others are chosen from the second hop neighbors based on the dynamic dilated aggregation algorithm. The descriptions of each part of the architecture are provided in the following text.

**Residual Connection** In this study, we incorporate residual connections into the GCN encoder for overlapping community detection. If $\hat{A}$ is the normalized adjacency matrix, $X_l$ and $W_l$ are the feature matrix and learnable weight matrix at layer $l$ respectively, then computation of the GCN encoder at layer $l$ is expressed as:

$$X_{t+1} = ReLU(\hat{A}X_l W_l)$$

In our study, we incorporated residual connection at layer $l$, as follows:

$$X_{l+1} = ReLU(\hat{A}X_l W_l) + \hat{A}X_l$$

$$= X_{l+1}^{res} + \hat{A}X_l$$

(1)
The additional term in equation (1) is $\hat{A}X_l$. This term connects the input $X_l$ with the output $X_{l+1}$. Consequently, a skip connection is created in the computation path, which enables a smooth flow of gradients for solving the vanishing gradient problem.

**Dilated Aggregation** Dilated aggregation was applied to GNNs for point cloud learning [32]. In the point cloud, a neighborhood is constructed for each point using the k-nearest neighbor (k-NN) approach [32]. Then, dilation is applied through sub-sampling from the fixed neighborhood [32]. For instance, if $k = 10$, then the nearest neighborhood of size 10 is constructed from the point cloud. The nearest neighbors can be ordered for each point based on the distance. To perform dilation, every even-numbered neighbor can be skipped, and every odd-numbered neighbor can be considered. Thus, aggregation for information could be done from the first, third, fifth, seventh, and ninth neighbors. Therefore, this aggregation is called dilated aggregation. Using dilated aggregation, it is possible to reach distant neighbors without increasing the number of neighbors considered for aggregation.

However, there is no flexibility in the neighborhood size in general graphs. For each node, the neighborhood is defined according to the graph structure. In fact, the size of the neighborhood in general graphs varies widely. Therefore, the fixed-sized k-NN approaches are not applicable. An adaptive approach that accounts for the variable neighborhood size must be designed. It is also possible to define the neighborhood of the general graphs based on proximity. For instance, we can define first-order proximity based on the first-hop neighbors of a node. Accordingly, second-order proximity may be defined considering all first- and second-hop neighbors. If we consider the first-order proximity and sub-sample from the first-hop neighbors, there is a possibility of information loss due to losing a number of first-hop neighbors. In the case of higher-order proximity, the neighborhood size can be large and intractable in practical cases. Moreover, a large number of neighbors can boost over-smoothing. Over-smoothing means...
very similar features for all the nodes in such a way that all the nodes lose
distinguishability. To tackle these challenges, we designed dilated aggregation
algorithm, which can be combined with the concepts of dynamic edges.

In our design, we have to handle two rival issues. Firstly, dilation in first-
order proximity results in information loss. On the contrary, the neighborhood
size increases at a quadratic rate when second-order proximity is considered.
To achieve a fair balance between these rival issues, we choose the augmented
neighborhood size of a node as \(2 \times m\) where \(m\) is the degree of the node. Then, we
choose \(m\) nodes from the first hop and the remaining \(m\) nodes from the second
hop. We design an augmentation algorithm (Algorithm 1) to choose the nodes
from the second-hop neighbors. Finally, we introduce dilation in this augmented
neighborhood. We have designed a simple but effective dilated aggregation scheme.
The dilation scheme is to sub-sample 50\% nodes randomly from the augmented
neighborhood. After sub-sampling, each node would have \(m\) neighbors effectively.
Thus, we do not increase the effective neighborhood size while we can aggregate
information from the distant nodes.

Let us consider an \(m\)-regular graph. In this case, first-order proximity considers
a neighborhood of size \(m\). However, the second-order proximity would consider
\(m^2 + m\) neighbors. Our approach considers an augmented neighborhood of size
\(2 \times m\), which is not much larger. To implement dilation, we sample 50\% nodes
from this augmented neighborhood. Therefore, the effective number of neighbors
is \(2 \times m \times 0.5 = m\), which is the same as the number of original first-hop neighbors.
Therefore, the possibility of information loss due to dilation is diminished with
the advantage of a larger receptive field.

**Dynamicity of Edges** The dilated aggregation procedures inherently lose some
first-hop neighbors. When the same dilated neighborhood is considered at every
GNN layer, some first-hop neighbors are never explored for each node. This fixed
neighborhood at every layer effectively alters the original graph. Moreover, this
introduces over-smoothing as the information from the same neighbors is aggre-
gated at each layer. Consequently, learning is not possible. To remedy this issue,
we consider different sub-samples of neighbors from the augmented neighborhood
at every different layer. Therefore, the neighborhood is different at every layer.
In essence, the edge set of the graph becomes dynamic. Interestingly, the concept
of dynamic edges and dilated aggregation can be unified. As a result, learning
becomes stable and better than static approaches \([61][56][51]\). In the case of point
clouds, the nearest neighbor graph is dynamically constructed after every layer
\([61][56]\). To introduce this concept into the DynaResGCN encoder, the adjacency
matrix is constructed differently at each layer based on the augmentation algo-
rithm and dilation scheme. Let us assume \(A_l\) is the adjacency matrix at layer \(l\)
where \(A_l\) differs in different layers. Therefore, the computation at layer \(l\) defined
in \([1]\) is updated to the following

\[
X_{l+1} = ReLU(\hat{A}_lX_lW_l) + \hat{A}_lX_l \\
= X_{l+1}^{res} + \hat{A}_lX_l
\]
Algorithm 1 Graph Augmentation: Input $G = (V, E)$

1: Let $E' = E$
2: for each node $u$ in the graph do
3:   Let $S_u$ be the set of original neighbors of $u$
4:   Let $S'_u$ be the set of augmented neighbors of $u$
5:   Initialize $S'_u$ with the elements of $S_u$
6:   for each node $v \in S_u$ do
7:      Obtain a random node $w \not\in S'_u$ from the neighbors of $v$ ($w \in S_v$)
8:      Add $w$ to $S'_u$
9:      Add edge $(u, w)$ to $E'$
10: Return $G' = (V, E')$ considering augmented neighborhood

Dynamic Dilated Aggregation The ideas of residual connection, dilated aggregation, and dynamic edges are unified in a single framework by the equation \[ (2) \]. This section defines the augmentation algorithm and dilated aggregation techniques. The adjacency matrices $\hat{A}_l$ are generated using these algorithms. Our augmentation algorithm is described in the Algorithm [1] The random augmentation algorithm is applicable to any graph. In Figure [3] we show graph augmentation and random sub-sampling of neighbors for a single node. In the following text, we describe a dilated aggregation scheme based on the sub-sampling/dilation of an augmented neighborhood.

Random Dynamic Dilated Aggregation: For random dynamic dilated aggregation, we first apply Algorithm [1]. All the first-hop neighbors are considered in this algorithm. Additionally, for each first-hop neighbor, a new node from
the second-hop neighborhood, connected to the first-hop node, is considered. Now, we randomly sample 50% of the nodes from the augmented neighbors at each layer. The sampling at each layer differs from other layers, allowing for exploring different neighborhoods. Thus, we employ the concept of dynamic edges in terms of dynamic neighborhoods with dilated aggregation. This random dynamic dilated aggregation helps resolve the over-smoothing problem [32].

3.2 Bernoulli-Poisson Decoder

The final layer of the GCN encoder generates the affiliation matrix, $F$. To train the GCN, we need to calculate the loss. To generate the loss, we exploit the Bernoulli-Poisson model. According to this model, for each node $u$, there is a community affiliation vector $F_u$ with dimension $k$. This model generates the adjacency matrix $A$, for a given community affiliation matrix $F$, as follows:

$$A_{uv} \sim \text{Bernoulli}(1 - \exp(-F_u F_v^T))$$

For a given graph with adjacency matrix $A$ and feature matrix $X$, we generate the community affiliation matrix using the DynaResGCN encoder as follows:

$$F = \text{DynaResGCN}_\theta(A, X)$$

Now, according to the BP model, the negative log-likelihood is [50]

$$-\log p(A|F) = -\sum_{(u,v) \in E} \log(1 - \exp(1 - F_u F_v^T)) + \sum_{(u,v) \notin E} F_u F_v^T.$$

Due to the sparsity of many real-life graphs, non-edges are much more common than edges. To overcome this issue, NOCD proposed balancing the terms of the negative log-likelihood according to the standard technique for imbalanced data [23] as

$$\mathcal{L}(F) = -E_{(u,v) \sim P_E} \left[ \log(1 - \exp(-F_u F_v^T)) \right] + E_{(u,v) \sim P_N} \left[ F_u F_v^T \right],$$

(3)

where uniform distributions over edges and non-edges are represented by $P_E$ and $P_N$, respectively. In our case, we follow the same approach. To learn the community affiliation matrix $F$, the authors of BigClam directly optimized the affiliation matrix $F$. Unlike these traditional approaches, we optimize the parameters of the GNN to obtain a more accurate affiliation matrix. The equation [3] generates the loss. This loss is back-propagated in the GNN encoder to learn the parameters $W_l$. Next, we describe our training algorithm.

3.3 Training Algorithm

We consider $\tau$ as the total number of epochs for simplicity. In our implementation, we consider the early stopping criterion when the learning becomes stable. Our training algorithm, Algorithm 2, is as follows:
Algorithm 2 Training Algorithm

1: Perform graph augmentation (random)
2: Build the GNN encoder with a predefined number of layers $n$
3: For each layer, generate $\hat{A}_l$ based on the dilated aggregation scheme
4: for each epoch $e$ in $\tau$ do
5: for each layer $l$ in total $p$ layers do
6: $X_{l+1} = \text{Relu}(\hat{A}_lX_lW_l) + \hat{A}_lX_l$
7: Get $F = X_p$ from the output of the GNN
8: Calculate loss and update parameters

4 Experimentation

In this study, we have extended the codebase of NOCD to implement our ideas. We have two types of datasets: 1) without ground truth and 2) with ground truth. The research topics dataset ($n \approx 6K$) [6] is a medium-sized dataset without ground truth information. All other datasets have ground-truth information [39, 50]. These datasets with ground truth are of two types: 1) small ($n < 800$) datasets (Facebook datasets [39]), and 2) very large ($n > 20K$ up to $65K$) datasets [50].

We vary the number of layers from 10 to 90 in steps of 10 for the topics dataset. However, for the small datasets, we vary the number of layers starting from 2 up to 15 in the set $\{2, 3, 5, 7, 10, 15\}$. For the very large datasets, we vary the number of layers up to 50 in the set $\{2, 3, 5, 7, 10, 15, 20, 30, 40, 50\}$. We also consider network widths of 16, 32, 64, or 128. We determine the width for a dataset and use it for all other cases.

The threshold to consider a node in a community was also varied in the set $\{0.05, 0.10, 0.125, 0.20, 0.30, 0.35, 0.40, 0.50\}$. We find the best model depth and threshold for each dataset on each of our considered methods through grid search. Then, we run 50 iterations with different initialization for the best hyper-parameter set. Finally, we report the mean value of the respective metric over these 50 iterations. In the topics network, we set the number of communities to 100 which is determined based on searching in the set $\{25, 50, 100, 200\}$ and considering the best performing one. For the datasets having ground truth, we set the number of communities as it is in the ground truth (similar to NOCD). We also perform a statistical test to determine the statistical significance in the case of the datasets having ground truth. The following text elaborates on our baselines, datasets, metrics, statistical testing procedure, and experiment setup. Our code is available at [https://github.com/buet-gd/Deep-DynaResGCN-community](https://github.com/buet-gd/Deep-DynaResGCN-community).

4.1 Baselines

In the case of the topics dataset, we use SLPA [67], DEMON [15], MNMF [60], BigClam [69], DANMF [73], and NOCD [50] as the baselines. We use their publicly available code for NOCD [50]. For other baselines, we use the implementations in the CDlib python library with default parameters [46]. In the case of other datasets with ground truth, we compare our methods with the
same baselines used in the NOCD [50]. Our method DynaResGCN-G denotes the DynaResGCN method without considering node attributes, while DynaResGCN-X denotes the same method considering node attributes. As we consider exactly the same baselines on the same datasets as described in NOCD [50], we report the results already published in [50]. These baselines include BigClam [69], EPM [74], SNetOC [54], CESNA [71], SNMF [59], CDE [33], DeepWalk [42] with Non-exhaustive Overlapping K-means [65] (DW/NEO), and Graph2Gauss [5] with Non-exhaustive Overlapping K-means [65] (G2G/NEO).

4.2 Datasets

All of our datasets are described in Table 2. We use a topics graph dataset [6] that is based on various research topics. We can observe a small portion of the topics network in Figure 4. Every topic is a node in the graph. An edge connects two topics if a researcher works on both topics. One of the main motivations for using the topics network is using visualization to understand overlapping community structures. Most works in the literature evaluated the results of experiments on community detection by just observing the values of the evaluation metrics. In fact, the similarity/dissimilarity between two different topics is evident from the names of the topics. For instance, if we consider computer science and algorithms, then it is clearly evident that they should belong to the same community. On the contrary, if we consider bio-chemistry and graph theory, then it is also obvious that they should not belong to the same community. Thus, we can generate a heatmap of similar topics group and dissimilar topics group, which would help us understand the results visually. This heatmap is explained later. We also consider very large co-authorship networks compiled by authors of NOCD from Microsoft.

Table 2: Our considered datasets

| Dataset           | Nodes | Edges | MCD | Connectivity | Ground truth | Comment                        |
|-------------------|-------|-------|-----|--------------|---------------|---------------------------------|
| Topic             | 5947  | 26695 | N/A | ✔            | Not available | [6]                             |
| Facebook 348      | 227   | 6384  | 4   | ✗            | Hand-labeled | Social network [39]             |
| Facebook 414      | 159   | 3386  | 2   | ✗            | Hand-labeled | Social network [39]             |
| Facebook 686      | 170   | 3312  | 5   | ✗            | Hand-labeled | Social network [39]             |
| Facebook 698      | 66    | 540   | 2   | ✗            | Hand-labeled | Social network [39]             |
| Facebook 1684     | 692   | 28048 | 5   | ✗            | Hand-labeled | Social network [39]             |
| Facebook 1912     | 755   | 60050 | 3   | ✗            | Hand-labeled | Social network [39]             |
| Computer Science  | 21957 | 193500| INF | ✔            | Empirical    | Co-authorship network [50]      |
| Engineering       | 14297 | 98610 | INF | ✔            | Empirical    | Co-authorship network [50]      |
| Medicine          | 63282 | 1620828| INF| ✔            | Empirical    | Co-authorship network [50]      |

(1) MCD stands for the maximum community diameter. N/A refers to that ground truth is unavailable. INF refers that every community-induced sub-graph in the corresponding dataset is a disconnected graph. Therefore, MCD is undefined and theoretically infinite.

(2) Connectivity says whether the whole graph is connected.

(3) Community assignment is not hand-labeled by a human agent.
Academic Graph [50]. The communities are assigned roughly in these datasets based on the respective research areas. In this case, hand-labeled ground truth is not available. Additionally, every community-induced subgraph is a disconnected graph, while the entire dataset is a connected graph. Thus, these datasets are highly unreliable. However, these datasets may help understand the different methods’ scalability limits.

Finally, we consider real-world social network datasets [39]. These are small datasets with reliable ground truth where communities are hand-labeled. Using these datasets, we can estimate community recovery with much better reliability.

In essence, we experiment on datasets having different sizes with the number of nodes ranging from 66 up to 65K. Along with very different-sized datasets, we use datasets with hand-labeled ground truth, datasets with empirically assigned ground truth, and datasets without ground truth, which ensure the evaluation procedure’s robustness, reliability, and strength.

4.3 Evaluation Metric and Visualization

In the case of the topics dataset, we consider different quality measures or fitness functions to evaluate the detected communities. These quality measures include conductance [69], clustering coefficient, density, and coverage. In the case of the topics network, we also use heatmaps to understand the overlapping tendencies of various communities. In the case of other datasets, we use normalized mutual information (NMI) [40] to measure the similarity between ground-truth communities and predicted communities. In the case of the Facebook datasets, we visualize the largest detected community to understand the quality of the detection. To understand the definitions below, let us assume $S$ is the set of nodes in a single community. $S_1, S_2, \ldots, S_k$ denotes the set of nodes in different
communities. $F$ is the affiliation matrix defined earlier. In the following text, we describe each of the measures and also the visualization processes.

**Conductance:** We consider the average conductance of the detected communities (weighted by community size); The lower the conductance, the better the performance. In fact, conductance covers the intuitive definition of community [69].

$$\text{Outside}(S) = \sum_{u \in S, v \notin S} A_{uv}$$
$$\text{Inside}(S) = \sum_{u \in S, v \in S, v \neq u} A_{uv}$$
$$\text{Conductance}(S) = \frac{\text{Outside}(S)}{\text{Inside}(S) + \text{Outside}(S)}$$
$$\text{AvgConductance}(S_1, ..., S_k) = \frac{1}{\sum_i |S_i|} \sum_i \text{Conductance}(S_i) \cdot |S_i|$$

**Coverage:** Coverage refers to the percentage of edges explained by at least one community i.e. if $(u, v)$ is an edge and both nodes share at least one community, then this edge is explained by at least one community; the higher the density, the better the performance.

$$\text{Coverage}(S_1, ..., S_k) = \frac{1}{|E|} \sum_{u,v \in E} 1[F_u^T F_v > 0]$$

**Density:** The average density of the detected communities (weighted by community size); the higher the density, the better the performance.

$$\rho(S) = \frac{\text{number of existing edges in } S}{\text{number of possible edges in } S}$$
$$\text{AvgDensity}(S_1, ..., S_k) = \frac{1}{\sum_i |S_i|} \sum_i \rho(S_i) \cdot |S_i|$$

**Clustering Coefficient:** The average clustering coefficient of the detected communities (weighted by community size); a higher clustering coefficient is better.

$$\text{ClustCoef}(S) = \frac{\text{number of existing triangles in } S}{\text{number of possible triangles in } S}$$
$$\text{AvgClustCoef}(S_1, ..., S_k) = \frac{1}{\sum_i |S_i|} \sum_i \text{ClustCoef}(S_i) \cdot |S_i|$$

**Normalized Mutual Information (NMI):** It is an information similarity measure between the ground truth communities and the detected communities [40]. We use this measure when we have ground truth information.
Heatmap Visualization: In the case of the topics dataset, we know the names of the topics that are similar and the names of the topics that are dissimilar. To generate each heatmap, we consider a total of nine topics. Among these nine topics, every three topics are similar. Thus, we have three groups of similar topics. However, two topics from two different groups should be dissimilar. Then, for each topic, we find the community with the strongest affiliation (as described earlier, we generate community strength for each community for each node). Now, we know the strongest community for each topic. Therefore, we have nine communities, corresponding to the nine topics. Then, we find the number of overlapped nodes for each pair of the communities, which can be presented as an overlapping matrix where communities are indexed in the same order, both in rows and columns. Finally, we normalize this overlapping matrix. This normalized overlapping matrix is shown as a heatmap. Each cell of the heatmap shows the overlapping strength between the corresponding communities. The higher the overlapping, the darker the cell.

4.4 Statistical Significance Test

We use NMI in the case of the datasets having ground truth. We use a statistical t-test to measure the significance of the difference in mean NMI between NOCD and DynaDesGCN. We perform a standard t-test with 95% confidence (significance α = 0.05) to test whether the difference between two means from two different distributions is statistically significant. Let \( \bar{x}_1 \) be the mean over \( n_1 = 50 \) (as we run 50 independent initializations for each method) independent samples from one method where \( s_1 \) is the standard error and \( \bar{x}_2 \) is the mean over \( n_2 = 50 \) different samples from another method where \( s_2 \) is the standard error. Then, our null hypothesis \( (H_0) \) is that the difference in the mean is zero \( (\mu_1 = \mu_2) \) or statistically insignificant. The alternative hypothesis is that the difference in the mean is statistically significant \( (\mu_1 \neq \mu_2) \). Now, we calculate the standard t-statistic as follows:

\[
t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}
\]

For a t-test with a significance level of \( \alpha = 0.05 \), we calculate upper sided critical value from the t-distribution with degrees of freedom 49 (min\( (n_1 - 1, n_2 - 1) \)). Let us assume \( t^* \) is the \( \alpha/2 \) upper critical value \(( (1 - \alpha/2) \times 100 = 97.5th \) percentile). If the absolute t-statistic is greater than the \( t^* \), then we reject the null hypothesis and state that the difference of means is statistically significant.

4.5 Experiment Setup

We perform our experimentation on NVIDIA GeForce RTX GPU in the Linux platform with 4GB RAM. We have implemented our codes in the PyTorch library with Python version 8. We use weight decay rate as 0.01, learning rate 0.001, batch-normalization, and stochastic loss based on the sampled edges of
the original graph during training. We also use the early stopping criterion for training with a maximum 500 epochs.

5 Results and Discussion

A previous study [50] demonstrated the effectiveness of graph neural networks for detecting overlapping communities. They showed that GNNs perform better than free variable optimization or other direct methods such as BigClam [69], EPM [74], SNetOC, [54] etc. In this study, we aim to develop a better GNN-based method to detect overlapping communities with higher detection quality. Specifically, we design a deep GCN encoder considering dynamic dilated aggregation for the overlapping community detection model. Eventually, we achieve significantly better performance and detection quality than related studies. For evaluation, we use three different types of datasets. In the following text, we describe our results.

5.1 Dataset without Ground Truth

Table 3 compares our various methods' performance with the existing methods on topics dataset [6] which has no ground truth. In practice, there is no sufficiently large dataset with reliable overlapping ground truth information. Overlapping ground truth information is also more challenging because of subjectivity. It is not impossible to have different ground truths for the same dataset if the dataset is labeled based on two different perspectives. Moreover, the amount of overlap between two communities is also subjective. As a result, it is necessary to evaluate different methods on datasets having no ground truth with reliable evaluation metrics. It is also good to assess with some auxiliary metrics to detect the corner cases. Therefore, we use conductance as the main evaluation metric [69], as well as three auxiliary metrics to determine whether the main metric (conductance) is actually performing well. Finally, we attempt to visualize overlapping tendencies using a heatmap.

| Method     | Conductance (↓) | Clustering coefficient (↑) | Density (↑) | Coverage (↑) |
|------------|-----------------|--------------------------|-------------|--------------|
| DynaResGCN | **0.1807**      | 0.00037                  | 0.00781     | 0.9986       |
| NOCD [50]  | 0.3049          | 0.00016                  | 0.01135     | **0.9999**   |
| SLPA [67]  | 0.3058          | **0.00217**              | **0.04867** | 0.8273       |
| BigClam [69]| 0.2129          | 0.00000                  | 0.00175     | 0.8698       |
| DANMF [73] | 0.2753          | 0.00008                  | 0.00869     | 0.7250       |
| DEMON [15] | 0.4873          | 0.00151                  | 0.04199     | 0.8965       |
| MNMF [60]  | 0.3965          | 0.00009                  | 0.00907     | 0.6179       |
Fig. 5: Overlapping tendency of selected topics from the topics dataset. The darker the cell, the higher the overlap.
We compare our method *DynaResGCN* with NOCD [50], SLPA [67], BigClam [69], DANMF [73], DEMON [15], and MNMF [60] on the topics dataset (Table 3). Our method DynaResGCN performs the best in terms of conductance and also comparatively well in terms of other auxiliary metrics. Indeed, conductance is the most important metric because it captures the basic definition of a community [69]. BigClam has the closest performance to DynaResGCN in terms of conductance. However, in terms of clustering coefficient and density, BigClam performs the worst. The performance of BigClam is also not good in terms of coverage. Therefore, it is a degenerate case.

Instead of relying on just numbers, we attempt to understand overlapping community detection through a heatmap visualization approach. We choose three groups of topics to create heatmaps based on the topic network dataset. The first group consists of *graph theory, algorithms, and theoretical computer science (TCS)*. The second group consists of *music, emotion, and cognition*. The third group consists of *electro-chemistry (EC), material science (mat. science), and chemistry*. In the heatmap in Figure 5, the topics are arranged in order of the first group, second group, and third group.

The heatmaps show the effectiveness of each method for detecting overlapping regions. It is visible from the heatmaps that the GNN-based models are very effective for detecting overlapping communities. It is also evident from the heatmaps that the DynaResGCN-based models better detect overlapping communities than the shallow models. There is an interesting observation related to the BigClam model. Based on conductance, BigClam performed very well. However, in terms of the other metrics, BigClam performed poorly. The heatmaps clearly reveal that the performance of BigClam based on conductance is a corner case or degenerate case, where conductance does not reflect the true phenomenon. Therefore, based on the heatmaps, we confidently conclude that if a method performs well in terms of conductance and also in all other metrics, only then we can refer that the method is a winning method.

### 5.2 Datasets with Ground Truth

We compare our best method, the DynaResGCN model, on a set of hand-labeled networks and on a set of very large graphs with empirically assigned ground truth. In both of the cases, we compare DynaResGCN and NOCD with attributes of the nodes and without attributes of the nodes in terms of NMI. To evaluate in terms of NMI, we require ground truth. The methods DynaResGCN-G and NOCD-G are compared without considering the attributes of the nodes in Table 4 and the details on DynaResGCN-G are described in Table 5. On the other hand, the methods DynaResGCN-X and NOCD-X are compared considering node attributes in Table 6 and details on DynaResGCN-X are described in Table 7. We explain the results in the following text:

**Dataset with Hand-labeled Ground Truth:** These are small graphs from Facebook [39] with reliable (hand-labeled) ground truth. From Table 4 and Table 6, it is clearly evident that our method DynaResGCN significantly outperforms the nearest best method NOCD in all of the Facebook networks irrespective
of the node attributes. In fact, only in the case of the dataset Facebook 414, NOCD-X has a comparative performance with DynaResGCN-X. DynaResGCN outperforms all the other methods in these datasets with ground truth by a large margin.

One of the key implications of our method is the robustness to node features. In fact, in some cases, DynaResGCN-G outperforms DynaResGCN-X. For instance, in the datasets including Facebook 348, Facebook 698, Facebook 1684, and Facebook 1912, DynaResGCN-G is surprisingly robust, outperforming all the methods even with node attributes. This implies that our method can exploit structural information lying in the graph very well. From Table 4 and Table 7, it is prominent that different datasets have different model depths and thresholds, which in turn proves that model depth and thresholds are subjective to the datasets.

Achieving state-of-the-art performance from deeper models implies that our method can train deep GCNs successfully in general irregular graphs. This proves that DynaResGCN can resolve the over-smoothing problem and vanishing gradient problem. It also proves that we effectively incorporate the concepts of residual connection, dilated aggregation, and dynamic edges in our DynaResGCN model.

We also attempt to understand the results from the Facebook dataset by visualizing the largest cluster. Interestingly, the GNN-based methods can clearly identify the largest cluster. Among the GNN-based methods, the DynaResGCN-X model identified the largest cluster more clearly (Figure 6a). The NOCD-X (GNN-based) model can identify the largest cluster in a fairly good manner. However, in Figure 6b, we mark some part that should be in the largest cluster but is excluded from the largest cluster by NOCD. Other methods like SLPA and DEMON clearly fail to identify the largest cluster as in Figure 6c, 6d. It is evident that the DynaResGCN-based method is clearly the best method to identify overlapping communities in networks.

Datasets with Empirical Ground Truth: These datasets include Computer Science, Engineering, and Medicine. These are very large datasets having nodes up to 65K. Reliable ground truth information is not available for these datasets. As these are co-authorship networks, ground truth is roughly assigned

Table 4: Results for community recovery in terms of overlapping NMI without node attributes using 50 different initializations.

| Dataset     | BigCLAM | CESNA | DPRT | EPM | G2G/NEO | NOCD-G | NOCD-X | DynaResGCN-G | Significant |
|-------------|---------|-------|------|-----|---------|--------|--------|--------------|-------------|
| Facebook 414| 26.0    | 29.4  | 6.5  | 24.8| 24.8    | 24.8   | 21.7   | 17.2         | 34.7        |
| Facebook 414| 48.3    | 50.3  | 17.5 | 52.8| 32.5    | 40.9   | 32.3   | 32.3         | 56.3        |
| Facebook 686| 13.3    | 13.3  | 3.1  | 10.6| 13.5    | 11.6   | 11.8   | 5.6          | 32.3        |
| Facebook 1684| 45.6   | 39.4  | 9.2  | 44.9| 31.6    | 31.6   | 40.1   | 4.6          | 49.3        |
| Facebook 1912| 32.7   | 28.0  | 6.8  | 26.1| 28.8    | 13.0   | 37.2   | 9.9          | 34.7        |
| Computer Science| 21.4  | 21.2  | 9.8  | 21.4| 15.5    | 23.4   | 20.8   | 16.0         | 36.8        |
| Engineering   | 7.9     | 24.3  | DNF  | DNF | DNF     | DNF    | 10.1   | 4.7          | 33.4        |
| Medicine      | 0.0     | 14.4  | DNF  | DNF | DNF     | DNF    | 4.9    | 5.3          | 28.8        |

(1) The significance of the mean difference of NMI between NOCD-G and DynaResGCN-G is determined with 95% confidence based on the standard t-test.
Table 5: DynaResGCN-G experimental details and results

| Dataset     | Mean NMI | Std error (NMI) | Layers | Threshold |
|-------------|----------|-----------------|--------|-----------|
| Facebook 348| 39.80    | ±0.2            | 7      | 0.40      |
| Facebook 414| 58.02    | ±3.2            | 2      | 0.50      |
| Facebook 686| 25.40    | ±1.9            | 7      | 0.35      |
| Facebook 698| 51.00    | ±3.3            | 5      | 0.50      |
| Facebook 1684| 44.30   | ±2.6            | 3      | 0.50      |
| Facebook 1912| 40.10   | ±1.7            | 3      | 0.50      |
| Computer Science| 37.4    | ±1.7            | 15     | 0.50      |
| Engineering | 37.3     | ±1.2            | 15     | 0.35      |
| Medicine    | 37.3     | ±1.1            | 15     | 0.35      |

Table 6: Results for community recovery in terms of overlapping NMI with node attributes using 50 different initializations.

| Dataset     | BigCLAM | CESNA | EPM | SNetOC | CDE | SNMF | DW/NEO | G2G/NEO | NOCD-X | DynaResGCN-X | Significant |
|-------------|---------|-------|-----|--------|-----|------|--------|---------|--------|---------------|-------------|
| Facebook 348| 34.8    | 29.4  | 6.5 | 24.0   | 6.2 | 13.5 | 17.2   | 36.4    | 39.8   | ✔             |             |
| Facebook 414| 48.3    | 50.3  | 17.5| 52.0   | 28.7| 32.5 | 40.9   | 32.3    | 59.8   | ✔             | ✔           |
| Facebook 686| 13.8    | 13.3  | 3.1 | 10.6   | 13.5| 11.6 | 11.8   | 5.6     | 21.0   | ✔             |             |
| Facebook 698| 45.6    | 39.4  | 9.2 | 44.9   | 31.6| 28.0 | 40.1   | 2.6     | 41.7   | 50.1          | ✔           |
| Facebook 1684| 32.7   | 28.0  | 6.8 | 26.1   | 28.8| 13.0 | 37.2   | 9.9     | 26.1   | 41.0          | ✔           |
| Facebook 1912| 21.4   | 21.2  | 9.8 | 21.4   | 15.5| 23.4 | 20.8   | 16.0    | 35.6   | 39.6          | ✔           |
| Computer Science| 33.8    | DNF   | DNF | DNF    | 9.4 | 3.2  | 31.2   | 50.2    | 46.0   |               |             |
| Engineering | 7.9     | 24.3  | DNF | DNF    | 10.1| 4.7  | 33.4   | 39.1    | 39.0   |               |             |
| Medicine    | 0.0     | 14.4  | DNF | DNF    | 4.9 | 5.5  | 28.8   | 37.8    | 40.0   |               |             |

(1) The significance of the mean difference of NMI between NOCD-G and DynaResGCN-G is determined with 95% confidence based on the standard t-test.

Table 7: DynaResGCN-X experimental details and results

| Dataset     | Mean NMI | Std error (NMI) | Layers | Threshold |
|-------------|----------|-----------------|--------|-----------|
| Facebook 348| 39.8     | ±0.2            | 7      | 0.35      |
| Facebook 414| 59.0     | ±3.1            | 3      | 0.50      |
| Facebook 686| 27.3     | ±2.0            | 5      | 0.125     |
| Facebook 698| 50.1     | ±2.3            | 7      | 0.50      |
| Facebook 1684| 41.0    | ±2.1            | 7      | 0.50      |
| Facebook 1912| 39.6    | ±1.7            | 3      | 0.50      |
| Computer Science| 39.0    | ±3.3            | 40     | 0.35      |
| Engineering | 39.0     | ±2.2            | 2      | 0.35      |
| Medicine    | 40.0     | ±2.1            | 2      | 0.35      |
based on the research area. In this case, DynaResGCN outperforms NOCD by a large margin when node attributes are ignored. When node attributes are considered, NOCD can outperform DynaResGCN only in a single case. In most other cases, DynaResGCN significantly outperforms NOCD. Moreover, it also proves that DynaResGCN is much more robust to node attributes. In fact, in some cases, DynaResGCN can reach the best performance even without considering node attributes. Indeed, we experiment with these datasets to test the scalability limits of our methods. Surprisingly, our DynaResGCN-based methods are highly scalable with very deep models. Table 8 shows that our methods can converge in less than 2 minutes in almost every dataset.

5.3 Computational Efficiency and Execution Time

The execution times for NOCD and DynaResGCN are reported in Table 8. In this table, pre-processing time refers to the time for graph augmentation before training. In the case of the DynaResGCN method, the execution time is reported for the optimal number of layers for each dataset as it is in Table 5. For the smaller datasets from Facebook, the execution time for DynaResGCN is less than 15 seconds, even with a deep model having seven layers. In the case of large
Table 8: Comparison of the execution time of NOCD and DynaResGCN without node attributes. The time of DynaResGCN is reported for the best number of layers as in table 5. All the times are reported in seconds.

| Dataset       | NOCD | DynaResGCN |
|---------------|------|------------|
|               | training | pre-processing | training |
| Facebook 448  | 6    | 12          | 12        |
| Facebook 414  | 5    | 5           | 5         |
| Facebook 686  | 5    | 8           | 8         |
| Facebook 698  | 6    | 9           | 9         |
| Facebook 1684 | 6    | 8           | 8         |
| Facebook 1912 | 10   | 12          | 12        |
| Computer Science | 13 | 109        | 113       |
| Engineering   | 10   | 63          | 65        |
| Medicine      | 28   | 759         | 808       |

datasets up to 65 thousand nodes, the execution time of DynaResGCN is less than 15 minutes with a deep model up to 15 layers. In fact, the execution time of DynaResGCN is less than 2 minutes, even with 15 layers except for the largest dataset (Medicine). The reason is actually behind our design of deep GCN, where we consider at most two hops of the neighborhood, making our model efficient.

5.4 Discussion

The following text discusses our insights learned from the experimentation and results. We also attempt to find out our limitations and subsequent future directions.

![Maximum community diameter (MCD) with the best model depth](image)

**Fig. 7: Maximum community diameter (MCD) with the best model depth.**

**Model Depth:** From the experiments on the different datasets, we find that the best model depth is subjective to the dataset. Sometimes, we find that the smaller datasets have the best performance with a smaller number of layers, and comparatively larger datasets achieve the best results with a large number of
layers. The reason behind this is that the smaller datasets often have smaller community diameters while the larger datasets have larger community diameters. It is evident that the deeper the GCN model, the more distant neighbors are aggregated. Therefore, the deep model can aggregate information from distant neighbors, which is required to discover communities with a larger diameter. Accordingly, when the community diameter is smaller, we should explore closer neighbors which requires less deep models. It is clearly evident in Figure 7 that the best model depth is highly correlated with the maximum community diameter (MCD). Even in some cases model depth is exactly the same as MCD. The key insight here is that we should not define a fixed depth for the model beforehand, rather we should vary the depth of the model depending on the nature of the dataset.

**Dynamic Dilated Aggregation:** However, then one question arises why should we need the small depth DynaResGCN model for the smaller datasets? In fact, the DynaResGCN model achieves superior performance than the NOCD model in the case of smaller datasets also. Here comes the power of the dynamic dilated aggregation scheme in the DynaResGCN model. Because of the dilated aggregation in the DynaResGCN model, it can aggregate information from the second hop neighbors in one layer of the GCN without increasing the effective number of neighbors (explained earlier). The dynamicity comes from the random sampling of the augmented neighbors, which allows exploring a number of new neighbors at every layer as we perform a new sub-sampling of the neighbors at every layer. Therefore, the model gets some new information at every layer. In fact, the dilated aggregation mechanism also helps to alleviate the over-smoothing problem [32]. Consequently, dynamic dilated aggregation is one of the key ideas to achieving better performance in the DynaResGCN model.

**Residual Connection:** Although the dynamic dilated aggregation mechanism solves the over-smoothing problem and gives better learning capability, the deep GCN models would still fail to learn because of the vanishing gradient problem [68,32]. The residual connection provides the skip connections in the computation path of the deep neural networks. This direct computation path allows a smooth backward flow of the gradients from the last layer to the beginning layers. Consequently, the gradients do not vanish when they reach the beginning layers. Therefore, the residual connection is essential to train deep GCN models which we use in our deep DynaResGCN model.

**Network Property:** Another important thing is the property of the network being studied. For example, if the network is very dense (almost fully connected), then no method can find a community structure in the network because the network has essentially no community structure. This property of a network is called the information-theoretic detection threshold [134], which is well defined in the stochastic block model of community detection [2]. In general, if the graph being studied is almost regular, then there will be no community structure at all; eventually, no algorithm can detect a community structure. Therefore, when we study community detection in some networks, we should also consider the
network property of the detection threshold, though it is well defined only for the special stochastic block model [2].

**Evaluation Metric:** The metric to determine the quality of the detected community is also crucial. In fact, using only a single metric is always highly risky because if the metric fails due to a corner case, there is no way to detect this failure. Therefore, having multiple metrics to detect different characteristics of the detected communities is of utmost importance. We alleviate this issue using multiple metrics that capture different characteristics of the detected communities. We refer to a method winning only when it performs better in terms of all the metrics considered.

**Threshold Sensitivity:** We have already discussed that every dataset is different from every other dataset. Therefore, the cut-off value (threshold) of the community strength (community affiliation) to determine the community belonging (whether a node belongs to a community) should be varied based on the dataset. Using the same threshold for every dataset is not a clever idea. In our experiments, we vary the threshold in a specified range and determine the best threshold for every dataset. We can observe the best threshold for every dataset in Table 5 and Table 7. In these tables, we can observe that the best threshold is not the same for all datasets. Therefore, it is necessary to consider variable thresholds in order to achieve the best result from every dataset. The key finding here is that the threshold value to determine the community belonging is subjective to the dataset being considered.

**Robustness:** From the results in Table 4 and Table 6, it is clearly evident that the DynaResGCN method is much more robust to node features than that of NOCD or other methods. In fact, the performance of DynaResGCN-G is much better than NOCD-G. We consider a one-hot encoded feature vector for every node when we do not have specific node features. As a result, the feature vector for each node is unique, which helps to achieve a deeper model without over-smoothing. When we have node features, the deeper model does not work well in the case of very large graphs. The most probable reason is the over-smoothing due to similar feature vectors of many nodes. Therefore, DynaResGCN achieves better performance without node features due to lower over-smoothing, and it also performs better with node features due to additional information coming from node features. These two advantages in the two cases make the DynaResGCN method much more robust than related methods.

**Statistical Significance:** Our best method, DynaResGCN, is significantly better than the nearest best method in almost every case. We measure the significance based on a standard t-test [53,27] with 95% confidence. In fact, very few works in the literature provided statistical significance for their results. Therefore, we can provably state that DynaResGCN is a significantly better method than many state-of-the-art methods for overlapping community detection.

**Scalability:** GCN-based methods are extremely scalable for overlapping community detection. The execution time for DynaResGCN is less than 15 seconds, even with seven layers for small datasets. In this case, DynaResGCN does not impose any extra burden on the execution time. For datasets up to 22
thousand nodes, DynaResGCN took less than 2 minutes of execution time with a deep model of up to 15 layers. In fact, the execution time of DynaResGCN is not more than 15 minutes, even with a very large dataset (65K nodes) with the deep model. Therefore, our method, DynaResGCN, is highly scalable even with the very deep model. As a result, the GCN-based deep methods would be much more suitable than any other method in practice where real-time overlapping community detection is required and in the case where networks are very dynamic, such as social networks.

Limitation and Future Directions: First of all, it is crucial to incorporate the information from edge weights in the case of weighted graphs. Therefore, there are scopes to design methods and algorithms to incorporate information lying in the edge weights in a meaningful and sophisticated way. In the very large datasets with node attributes, the DynaResGCN-based method achieves the best performance with only two layers in some cases. This implies that the DynaResGCN model can not perfectly resolve the over-smoothing problems. As a result, in some cases, deeper models do not outperform. Resolving the over-smoothing problem in better ways would be an interesting future research direction. In this study, we focus on improving the encoder part of the whole framework. There are scopes to research on improved decoders considering different properties of community. Finally, we apply our developed methods and algorithms in a completely unsupervised setting. It would be interesting to incorporate our ideas, methods, and algorithms in supervised settings of graph learning.

6 Conclusion

In this study, we develop dynamic dilation aggregation-based deep residual graph convolutional network (DynaResGCN) to detect overlapping communities in graphs. We resolve several challenges to incorporate dilated aggregation, dynamic edges, and residual connection in the general irregular graphs for overlapping community detection tasks. Eventually, our methods significantly outperform most of the prominent existing works. Future works should investigate the challenges of heterogeneous graphs where heterogeneity comes from both edges and nodes. The challenges of GNNs in very large networks, such as the whole Facebook or Twitter networks, are also worthy of investigation in the context of overlapping community detection. Moreover, consideration of edge-weights in a meaningful way can be an interesting future research direction. This work has many different applications, ranging from bioscience to social science and any tasks where there is a network structure.

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Data Availability Statement  The research topics dataset analyzed during the current study is available in the corresponding repository at https://github.com/enggiqbal/mlgd/tree/master/data/datasets/topics/original. Other analyzed datasets are available in the NOCD repository at https://github.com/shchur/overlapping-community-detection/tree/master/data and our code is available at https://github.com/buet-gd/Deep-DynaResGCN-community.
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