Finite element modelling of unbonded post-tensioned simply supported beam

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Abstract. This paper describes an experience of designing of some finite element models of unbonded post-tensioned simply supported concrete beam. It was purposed to evaluate a bending moment capacity, crack resistance, increment of pre-stressing force in reinforcement and deflection of models. Their behaviour under gradual loading was in the spotlight as well. There is short information about preliminary performed experimental study of beams. Parameters of physical models of beams obtained from experiment were used as initial data while finite element models were being created. Discussion of results obtained is listed. Their comparison with an experimental data is done.

1. Introduction
In recent decades, systems of residential and public buildings with a monolithic frame and flat slabs have become more widespread, due to their high technical and economic figures. In addition, there is an obvious request from the customer to increase the pitch of vertical load-bearing elements. In designing floor slabs with large spans, reinforcement consumption of the structural solution increases significantly, the thickness of the plate increases, which leads to an increase of the dead weight of the structure. The most effective solution to this problem is to prestress slabs at the construction site. A technology that allows to do so is known as post-tension. It is believed that in thin slabs post-tensioning with unbonded reinforcement is optimal.

Modern architectural trends are aimed at creating a unique and expressive appearance of the building. Monolithic reinforced concrete is a material that allows architects to obtain the desired shape of the facades and flexible layout of the interior space. Slabs of complex shapes in plan and cantilevers with relatively large overhangs are often used in buildings nowadays. Because of such diversity of structural forms, a calculation and designing of modern buildings requires, as a rule, the usage of computer systems based on finite element method (FEM).

It should be noted that specialized software for calculation of post-tensioned reinforced concrete structures has been developed. However, these programs are expensive and focused only on solving problems related to post-tensioning. In conditions when the post-tensioning itself is represented to a very insignificant degree in the Russian market, widespread use of such software by design organizations seems unlikely.

2. Existing works
Various scientists and researchers have studied an applicability of universal FEM programs to the analysis of unbonded post-tensioned flexural elements.
In the work of T. Kang [1], unbonded beam was simulated in the ABAQUS software package, and in the works of F. Jnaid [2] and J. Kim [3] in the ANSYS program. In all the mentioned works, results of calculating the model were compared with results obtained from experimental tests of beams. In [1] and [2], almost full compliance was observed between a behavior of finite element model and physical model of a beam. In [3], the discrepancy between the results of experiment and model calculation ranged from 35% in an initial stage of construction behavior to 7% in stage of work preceding the failure. Authors explain a marked difference in the results at an initial stage of construction’s behavior by fact that behavior of an experimental beams was not completely elastic when the first loading steps were applied.

These studies generally show a good applicability of powerful universal FEMs software to predicting an ultimate bending capacity, deformability and crack resistance of unbonded beams. However, their use may be limited in the construction industry in general and in Russia in particular due to their complexity and high cost.

On the Russian market SCAD and Lira-SAPR programs can be considered as most common FEM software. Modeling of a post-tensioned one-way floor slab in SCAD was performed in [4]. Also, a number of practical techniques for modeling and analysis of post-tensioned structures in this software are considered in the monograph [5].

3. Short description of physical models of beams
For studying of unbonded post-tensioned beams behavior some specimen were made. Their cross section was taken equal to 100 mm in height and 200 mm in width. Length of samples was 1985 mm. The samples have had pinned contact at one end, and a rolling contact at the other end. Space between supports was 1920 mm in average. Post-tensioning was made by two 5 mm diameter reinforcement wires (rods) with tensile strength equals to 176 kN per square cm. Reinforcement was enclosed in a plastic shell in order to exclude its contact with concrete. Load on the samples was applied at two points along the length of a span by hydraulic jack through a traverse. Distance between points of application of loads was taken equal to 610 mm. Samples were tested in two series. In the first, the concrete strength was lower, and the pre-compression force was higher than in the second one. The samples and their parameters are described in more detail in [6] and [7]. A general view of sample in the test bench is shown in figure 1, a.

4. Description of finite element models
Finite element models of unbonded post-tensioned beams were performed in SCAD 21 and Lira-SAPR 2015.

4.1. Geometry and bearing of FE models, performed by volumetric finite elements
Sizes of models were taken in accordance with data of the experimental study equal to b×h×l=200×100×2000 mm.

In both programs, concrete part of the structure was composed by volumetric finite elements in form of cubes with an edge size of 25 mm. In areas of the beam adjacent to holes for the placing of prestressed reinforcement, the shape and dimensions of the volumetric finite elements were accordingly changed. When modeling in Lira-SAPR, finite elements of type 236 (physically non-linear universal spatial eight-node isoparametric FE) were used. In SCAD finite element of type 36 (eight-node isoparametric FE) was chosen.

In both models, the reinforcement was modeled by bar finite elements 25 mm long. Finite elements of type 5 were used in the SCAD program and type 10 in Lira-SAPR. The cross section of these rods was set from rolled products with a diameter of 5 mm.

The modeling of reinforcement pressure on concrete part of the structure was made by combining vertical movements of the relative nodes of the concrete part of the model and the finite elements of the reinforcement along the length of the beam. In addition, the reinforcement was fixed at the ends.
Reinforcing bars were displaced downward by 20 mm from the center of gravity of the concrete section of the beam.

At a distance of 625 mm from each end on the upper face of the beam, steel plates 10 mm thick and 100x200 mm in size were fixed. Similar plates were used in an experimental study of beams. The plates distributed the concentrated external load and reduced the stresses concentration in the concrete.

In both programs, the nodes of the lower face of the model at a distance of 25 mm from each end along the entire width of the beam were fixed. On the left, all movements except UY were limited whereas on the right was the same except X and UY.

4.2. Geometry and bearing of FE model, presented by bar finite elements

Because Lira-SAPR provides an advanced analysis of capabilities of physically non-linear bar finite elements, an additional model of a beam by finite elements of type 210 (physically non-linear universal spatial bar FE) was made in this software. Section parameters equal to the section parameters of the beam considered in the experiment were set to these finite elements. The splitting of the model along the length was performed in increments of 100 mm.

Reinforcing bars were modeled similarly to the scheme performed by volumetric finite elements. They were placed 20 mm down from the center of gravity of the concrete section on solid bodies (special type of finite elements in considered software).

The characteristics of the connections were remained similar; however, they were given to the end nodes of the model.

4.3. Characteristics of materials

In SCAD characteristics of concrete of physical models of beams were set to volumetric elements:

- modulus of elasticity: 3249,49 kN/cm²;
- volume weight: 22,96 kN/m³;
- Poisson's ratio: 0,2.

The material of the reinforcing rods was set as «high-quality steel».

In the Lira-SAPR program physical nonlinearity of concrete was modeled. For this, a state diagram was set in the form of an exponential function (law No. 15 in Lira-SAPR «exponential deformation law for reinforced concrete»). Values corresponding to the characteristics of concrete of the fabricated physical models of beams were set in the parameters of this law: initial compression modulus $E_{0(-)} = 3249$ kN/cm², initial tensile modulus $E_{0(+)} = 1900$ kN/cm², compressive strength $\sigma_{(i)} = 2,68$ kN/cm², tensile strength $\sigma_{(t)} = 0,19$ kN/cm², ultimate compressive strain $\varepsilon_{(i)} = 0,002$, ultimate tensile strain $\varepsilon_{(t)} = 0,0001$. In this way, the physical nonlinearity of concrete was taken into account in finite element models of beam performed both the volumetric and rod finite elements.

The material of reinforcing bars in Lira-SAPR was adopted in the form of steel 09G2S.

In the model made of rod FE, a certain amount of fictitious reinforcement with adhesion to concrete was added. This allowed to evaluate the width and depth of crack opening. Parameters of this reinforcing material were adopted corresponding to the characteristics of high-strength reinforcing wire used for prestressing in this work. The behavior of the reinforcing material was described by a state diagram in the form of an exponential function (the law of nonlinear deformation No. 11 in Lira-SAPR "exponential"). Parameters for this function were adopted in the following way: initial compression modulus $E_{0(-)} = 20000$ kN/cm², initial tensile modulus $E_{0(+)} = 20000$ kN/cm², compressive strength $\sigma_{(i)} = 40,00$ kN/cm², tensile strength $\sigma_{(t)} = 176$ kN/cm², ultimate compressive strain $\varepsilon_{(i)} = 0,03$, ultimate tensile strain $\varepsilon_{(t)} = 0,03$. Some of these parameters were set in accordance with the mechanical characteristics of the wire, determined experimentally whereas some of them were accepted according to recommendations [8]. In order for the reinforcing material not to affect the nature of the element under study, its cross-sectional area was set to the minimum allowable by the program.
4.4. Loads
In models there were loads from the dead weight of all elements, a force of post-tensioning, as well as an external transverse load.

The self-weight of the elements was set automatically – software complexes calculated the weight of FEs using the known parameters of their materials.

To simulate the pre-compression force, a negative temperature effect was applied to the reinforcing bars. Its value was calculated according to the well-known formulas of mechanics of materials in such a way that the compression force in the finite element model turned out to be equal to the compression force in the physical models of the beams of both series. The coefficient of thermal expansion of steel was taken equal to \( \alpha = 1.2 \times 10^{-5} \). When assessing the magnitude of the temperature effect, pre-stress losses due to an elastic shortening of concrete were taken into account.

To analyze the behavior of the structure with a gradual increase in the external load, a number of loads with different intensities were set in SCAD.

In Lira-CAD, it is possible to perform the calculation with step-by-step method. In this regard, the external load was applied in steps of 0.981 kN.

External loads in the volumetric FEs models were set distributed over the above described steel plates.

In the model performed by bar finite elements in the nodes spaced 600 mm from the ends to the center of the span, concentrated forces were applied. Their increment was set in steps of 0.491 kN.

Finite element models of beam are shown in figure 1, b, c and d.

**Figure 1.** Models of post-tensioned unbonded beam: a – physical model, placed into test device; b – finite element model of beam in SCAD; c – FEM of beam in Lira-SAPR, performed by volumetric finite elements; d – FEM of beam in Lira-SAPR, performed by bar finite elements

5. Results of calculation
There are results of calculation of models and their discussion in this section.

5.1. Results in SCAD software
The preliminary calculations showed that the calculation of the beam in the SCAD without taking into account the physically non-linear work of concrete leads to results that are not interesting in view of this study. Behavior of model was linear on every reasonable level of load, so it was impossible to detect moment of crack appearance. Deflection of the model was too small even under quite a number of loads.

5.2. Results in Lira-SAPR: model, performed by volumetric finite elements
In accordance with the calculation protocol, under the action of concentrated loads of 6.54 kN for a model with parameters corresponding to the strength of concrete and the compression force of the first
series of samples, and 7.85 kN for a model with parameters corresponding to the second series of samples, in several finite elements of the most stretched zone of the beam there was achieved stress-strain state, which can correspond to the level of tensile strength according to the adopted state diagram of concrete. These finite elements were turned off from further model’s work. By analogy with the physical model, at this stage we can talk about the formation of cracks in the stretched zone of the finite element model.

Appearance of the stress-strain state corresponding to the exhaustion of the compressive strength of the finite elements in the most compressed part of the beam occurred at a load of about 17 kN at each point for a model with parameters of the first series of physical samples and at a load of 15.7 kN for a model with parameters of the second series.

The value of the force in the reinforcement at the moment of shutting down of the finite elements of upper face of the model turned out to be 48.5 kN in each bar for a model with parameters of the first series of samples and 39.8 kN for a model with parameters of the second series. Thus, increment of post-tensioning force to the moment of destruction of the beam was 21.0 kN and 15.3 kN in each wire for model with parameters of the first and second series of samples, respectively. Wherein stresses were obtained in the reinforcement as an excess of its tensile strength.

Based on the analysis of results, a curve showing the dependence of the beam deflection in the middle of the span on the magnitude of the applied load was depicted (figure 2).

5.3. Results in Lira-SAPR: model, performed by bar finite elements

In the advanced analysis mode, it was determined that under a load of 4.9 kN and 4.41 kN at each point for models with parameters of the first and second series of samples, respectively, hairy cracks are formed in the beam.

The breaking load for the beam was 6.87 kN and 7.36 kN at each point for models with parameters of the first and second series of samples.

At the time of fracture, the forces in each reinforcing bar turned out to be 34.3 kN and 36.4 kN for models with parameters of the first and second series of samples. Consequently, the increment of forces in the reinforcement was 6.8 kN and 11.9 kN per wire in the first and second group of parameters.

Based on the results of the step-by-step calculation, a curve showing the dependence of the deflection of the element on the level of the load arriving at each point was pictured (figure 2).

![Figure 2](image-url)

**Figure 2.** Dependences "deflection-load" for models of volumetric and bar finite elements: a - with the parameters of the first series of physical models; b - with the parameters of the second series of physical models.
From figure 2, it can be seen that the breaking load on the model from volume elements is about two times higher than the breaking load calculated in the model from rods. The magnitude of the deflections in both models was approximately equal when calculating the models with the parameters of the first series of field samples. When calculating the model with the parameters of the second series of samples, the maximum deflection in the model of bar finite elements is approximately two times greater than the deflection in the model of volumetric FEs.

6. Comparison of calculated results with an experimental data

When performing an experimental study of unbonded beams, moment of crack formation, moment of stress in the reinforcement, dependence of the deflection on the magnitude of the external load and the moment of fracture were established.

Table 1 shows a comparison of these parameters obtained from the analysis of models in Lira-SAPR, with data recorded in the experiment.

As follows from the data in Table 1, the finite element model performed in Lira-SAPR from volumetric FEs showed a significantly overestimated result in the bending capacity and crack resistance in comparison with the experimental results.

Table 1. Comparison of calculated results with an experimental data

| Parameters                  | Samples of the first series | Samples of the second series |
|-----------------------------|-----------------------------|-------------------------------|
|                             | Model by volumetric FEs     | Model by bar FEs              | Experimental study | Model by volumetric FEs | Model by bar FEs | Experimental study |
| Ultimate load, $F, kN$      | 17,00                       | 6,87                         | 5,23               | 15,70                     | 7,36             | 5,65              |
| Bending moment capacity, $M_{ult}, kN\cdot cm$ | 1043,26                     | 435,08                       | 340,54             | 964,86                    | 464,55           | 360,02            |
| Force increment in reinforcement, $\Delta P, kN$ | 42,00                       | 13,60                        | 2,15               | 30,60                     | 23,80            | 3,06              |
| Cracking moment, $M_{crc}, kN\cdot cm$ | 415,46                      | 317,37                       | 280,70             | 493,98                    | 287,94           | 316,18            |
| Short-term deflection, $f, mm$ | 81,10                       | 68,10                        | 23,72              | 58,40                     | 123,00           | 26,70             |

The calculation of the model in which the beam was modeled by rod physically nonlinear FEs shows a more acceptable results for both series of samples. In particular, the moment of crack formation in calculating the model with parameters of beams of the first series leads to an overestimated result by 13%, and in the case of beams of the second series, by about 10% underestimated. This discrepancy is in general at an acceptable level. The bending capacity according to the calculation of the bar FEs model is approximately 28% overstated for both the first and second series of samples.

When the parameters of the samples of the first series were established in the model from the rod FEs, the deflection was 68.1 mm, and for the parameters of the beams of the second series, 123 mm. In the case of a model by volumetric finite elements, the corresponding deflections were 81.1 mm and 58.4 mm. As regards the experimental data, a significant, almost unacceptable, overestimation of the deflection of the structure in both models is observed. It is likely that due to excessive deflections, values of the increment of forces in the reinforcement that are contrary to the experimental data were also obtained. They are especially overpriced in the case of a model by volumetric elements.

Graphs that compare the experimental curves of the dependence of the deflection on the load with the curves obtained from the calculation of both models in Lira-SAPR are shown below (figure 3).

It can be seen from the graphs that in the area of conditionally elastic work, the behavior of both calculation models in Lira-SAPR is in good agreement with the actual nature of the work of the
samples. Further, there is a noticeable deviation in the nature of the operation of the model performed by volumetric elements from the actual behavior of the samples. At the same time, a model made from bar finite elements shows a very good qualitative agreement with experimental curves.

![Comparison of experimental and calculated curves "deflection-load"
](image)

**Figure 3.** Comparison of experimental and calculated curves "deflection-load": a - for the first series of samples; b - for the second series of samples

It can be assumed that inconsistencies in the results of calculating the models in Lira-SAPR with the experimental data could arise due to the incomplete correspondence of the accepted concrete's "strain-stress" function with respect to the actual nature of its work. In addition, a number of concrete parameters of the beam samples were not experimentally determined and were adopted in accordance with the requirements of [9], which could also affect the accuracy of the calculation.

7. Conclusion
The performed modeling of the beam in software showed that Lira-SAPR is quite applicable for the analysis of unbonded post-tensioned beams because it contains physically non-linear finite elements. SCAD software can be acceptable on cases, when only elastic work of construction is interested for researcher.

In the field of conditionally elastic work, both models of the Lira-SAPR system show satisfactory agreement with the experimental results. However, in the study of elements in which crack formation and development is assumed, the use of volume FEs in Lira-SAPR for modeling the beam leads to unreliable results regarding the nature of the work of samples. A model made by rod FEs, although it has a tendency to overestimate the results, shows a very good agreement with the experimental data.

In conclusion, it can be noted that the implementation of beam models in Lira-SAPR with the basic skills of working with the software does not cause difficulties. At the same time, creating a model from rod FEs in Lira-SAPR is less time-consuming, and the calculation results of such model show noticeably better agreement with the experimental data. The calculation accuracy in this case is quite acceptable for a preliminary assessment of the work of unbonded post-tensioned beams.
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