3D hydrodynamical simulation of accretion disk in binary star system using RKDG CFD solver

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Abstract. We present the results of 3D-hydrodynamical simulations of accretion disk in close binary star system. The model includes the optical star filling its Roche lobe, a gas stream emanating from the inner Lagrangian point of the binary system, and the accretion disc structure. A cold hydrogen gas stream is initially emitted towards a point-like gravitational centre. A stationary accretion disc is formed in about 5 orbital periods after the beginning of accretion. The model uses realistic cooling function for hydrogen. The simulation of accreting gas dynamics is performed using second order Runge — Kutta discontinuous Galerkin CFD solver for unstructured tetrahedral meshes. The monotonicity technique based on Hermite WENO solution reconstruction is implemented. Utilizing the hydrodynamical simulations the synthetic light curves of the system are calculated as the volume emission of optically thin layers along the line of sight. The simulations results are in a good agreement with observations.

1. Introduction
Mathematical modelling of plasma flow in binary semidetached star systems accretion disks provides a way to analyze and interpret incoming observations data [1]. Many features of binaries light curves appear to be related both to the structure of the gas flow in the disk itself and to the behavior of the jet propagating from the Lagrange point \(L_1\). Investigation of light curves especially for eclipsing systems gives information about system inclination, jet velocity at \(L_1\) point, shock or shock-less behavior of jet-disk interaction [2].

Accurate modelling of accretion flows necessitates to apply effective numerical methods capable to resolve solution discontinuities like shock waves and contact discontinuity. Taking into account the size of the calculation domain and the mesh detalization being used in astrophysical problems modeling, high-resolution methods are already in demand. The Runge — Kutta discontinuous Galerkin method shows good results in such problems [3, 4].

This paper is devoted to the application of the gas-dynamic solver based on the RKDG method to the three-dimensional modeling of the accretion disk. Basing on the computed gas dynamic parameters of the disk synthetic light curves were constructed in the vicinity of the eclipse for a binary system with a small donor. The modelled system is similar to the PHL1445 system considered in [5].

2. Mathematical model
The mathematical model of accreting plasma flow is based on the results of [1] with a little simplifications. It includes the set of hydrodynamic equations of ideal (inviscid) perfect hydrogen
gas with account for the Roche gravitational potential and radiative gas cooling, written in non-
inertial reference frame corotating with the binary orbital period:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0, \\
\frac{\partial \rho \vec{v}}{\partial t} + \nabla \cdot \left( \rho \vec{v} \vec{v} + p \hat{I} \right) = -\rho \nabla \Phi + 2\rho \vec{v} \times \vec{\Omega}, \\
\frac{\partial e}{\partial t} + \nabla \cdot (\vec{v} (e + p)) = -\rho \nabla \Phi \cdot \vec{v} - \Lambda(\rho, T),
\]

where \(\rho\) is the density, \(\vec{v}\) is the velocity vector, \(p\) is the pressure, \(e\) is the total gas energy per unit volume, \(\vec{\Omega}\) is the angular rotational velocity of the binary system, \(\Lambda(\rho, T)\) is the radiative gas cooling function,

\[
\Phi = -\frac{GM_1}{\|\vec{r} - \vec{r}_1\|} - \frac{GM_2}{\|\vec{r} - \vec{r}_2\|} - \frac{1}{2} \|\vec{\Omega} \times (\vec{r} - \vec{r}_c)\|^2,
\]

\(\vec{r}\) is the radius-vector, \(M_1, M_2\) are masses of the components, \(\vec{r}_1, \vec{r}_2\) are their radius-vectors, \(\vec{r}_c\) is the system barycentre radius-vector, \(G\) is the Newton gravity constant. The system of hydrodynamic equations is completed by the equation of state of perfect gas:

\[
\rho \varepsilon = p(\gamma - 1),
\]

where \(\varepsilon\) is the specific internal energy of gas.

The integral cooling function for pure hydrogen was calculated as a function of gas density and temperature assuming local thermodynamical equilibrium (i.e. the Saha-Boltzmann distribution of atomic level populations) [1]. The approximation used in this work is shown in Fig. 1

3. Numerical scheme
As the computational domain \(D\) and mesh should be adopted to accreting plasma flow specific, we use the unstructured tetrahedral mesh in \(D\) which is fine near the accretor or \(L_1\) point and coarse at a distance of them. Each mesh cell is denoted as \(I_j, j = 1, n_{\text{cells}}, n_{\text{cells}}\) — number
of cells. Define the space of piecewise-continuous functions \( V_h^k = \{ p : p|_{I_j} \in P_k(I_j) \} \), which are polynomials of degree at most \( k \) with respect to \( x, y \) and \( z \) defined on \( I_j \). In this paper we use the linear polynomials with orthonormal basis functions on each cell. Basis can be constructed using orthogonal system of polynomials

\[
P_0 = 1, \ P_1 = \frac{\sqrt{5}}{\sqrt{3}} (4z - 1), \ P_2 = \frac{\sqrt{10}}{\sqrt{3}} (z + 3y - 1), \ P_3 = \sqrt{10} (z + y + 2x - 1)
\]

set on standard tetrahedron with vertices \( \{(0,0,0), (1,0,0), (0,1,0), (0,0,1)\} \).

Write the system (1)–(3) in conservative form

\[
\frac{\partial U}{\partial t} + \nabla \cdot \tilde{F}(U) = \Psi(U),
\]

where \( U = (\rho, \rho \vec{v}, e) \) is conservative variables vector, \( \tilde{F}(U) \) is flux vectors matrix and \( \Psi(U) \) is gravitational and cooling terms vector.

Following discontinuous Galerkin method [3] system (6) should be multiplied by the test function \( v(x, y, z) \in V_h^k \) and integrated over the cell \( I_j \). We define the approximate solution as

\[
U_h(x, y, z, t) = \sum_{I_j} \sum_{s=0}^{3} U_j^{(s)}(t) \varphi_j^{(s)}(x, y, z),
\]

where \( \{ \varphi_j^{(s)}(x, y, z) \}_{s=0}^{3} \) is the basis in \( V_h^k \), defined for each cell \( I_j \). The resulting system of ordinary differential equations for \( U_j^{(s)}(t) \) if we take \( v = \varphi_i^{(k)} \) is

\[
\frac{dU_j^{(k)}}{dt} = -\int_{\partial I_i} \varphi_i^{(k)} \left( \tilde{F} \cdot \hat{n} \right)_h dS + \int_{I_i} \tilde{F} \cdot \left( \nabla \varphi_i^{(k)} \right) dV + \int_{I_i} \Psi \varphi_i^{(k)} dV, \quad i = 1, n_{\text{cells}}, \quad k = 0, 3
\]

where \( \left( \tilde{F} \cdot \hat{n} \right)_h \) is the numerical flux between neighbouring cells computed in the surface where the numerical solution is discontinuous. Each integral is computed using suitable Gauss quadrature formula, so the numerical flux should be computed only in quadrature points on each surface. Here we use HLLC numerical flux [6].

The ODE system (7) is integrated using the second order explicit TVD Runge — Kutta method [7]:

\[
U^* = U^n + \tau L_h(U^n),
\]

\[
U^{n+1} = \frac{1}{2} U^n + \frac{1}{2} U^* + \frac{\tau}{2} L_h(U^*),
\]

where \( \tau \) is the time step, \( L_h(U) \) is the operator of right-hand side in (7), \( U^n \) and \( U^{n+1} \) are the values of solution at the current and next time points, respectively.

### 3.1. Solution monotization

The use of high order numerical schemes for hyperbolic systems leads to spurious oscillations in numerical solution in proximity to strong discontinuities due to the decreasing of scheme numerical viscosity. The problem grows worse in case of equations system with gravitational and radiative cooling terms. At the same time, the use of unstructured tetrahedral meshes and necessity of computational algorithm parallelization significantly limit the range of monotization techniques which can be applied in numerical scheme. As the monotization process can be divided into to steps, namely locating “troubled cells” and applying limiter to higher moments of solution, each step should invoke solution only on compact stencil, as it general RKDG scheme does. In our work we use the following approach:
Figure 2. Acceleration of computations: black line is ideal (linear) acceleration, blue line is code results.

- the “troubled cell” indicator is based on the accuracy of RKDG numerical solution extrapolation from one cell to another coherent to method accuracy order in case of smooth solution [8]; testing variables are density and energy;
- the limiter is applied component-by-component and in every Runge — Kutta stage;
- we use simple WENO (WENO_S) limiter [9] as it use only data from 4 neighbouring cells to reconstruct solution in the troubled cell.

3.2. Numerical code

Considered algorithm is implemented in parallel numerical code for cluster computational systems. The computational domain is decomposed to number of subdomains and computations are performed in every subdomain separately. On every stage of Runge — Kutta integration the interchange between subdomains is realized. The code is verified using number of well-known test problems like Sod problem (cylindrical and spherical multidimensional cases), blast wave propagation etc. [6]. The acceleration of computations on K-100 cluster (KIAM RAS) for accretion disk problem is presented in Fig. 2.

4. Results

The semidetached binary star system with donor mass \( M_{\text{don}} = 0.05M_\odot \) and accretor mass \( M_{\text{acc}} = 0.73M_\odot \) and orbital period \( P_{\text{orb}} = 76.3 \text{ min.} \) is modelled. Here and later \( M_\odot \) and \( R_\odot \) are solar mass and radius, respectively. The characteristic scales of problem variables are: density scale is \( \rho_0 = 10^{-8} \text{ g/sm} \), temperature scale is \( T_0 = 3500 \text{ K} \), distance between star components is \( L = 0.54629R_\odot \). The origin of a non-inertial reference frame is set at the centre of the donor star, the axis \( Ox \) passes through the binary components centres. The computational domain is the hollow ellipsoid of revolution with center at the accretor (\( x = 1 \)), major semiaxis \( a = 0.746466L \) and minor semiaxis \( b = 0.25L \). Inner boundary of domain is spherical with center in accretor center and radius equal to 0.08L. Computational domain size is taken to locate the \( L_1 \) point in the outer boundary. Near this point neighbourhood with radius \( r_{\text{jet}} = 0.01L \) the jet is set as
gas inflow with density $\rho = 12\rho_0$, temperature $T_{jet} = 2100$ and initial velocity $v_{jet} = 41.5 \text{km/s}$. Starting values of variables in computational domain are modeling vacuum and are taken to be $\rho_{init} = 10^{-5}\rho_0$ and $T_{init} = 10^{-4}T_0$.

After five orbital periods after jet arises near the Lagrangian point the quasistationary regime of accreting plasma flow is achieved. Inside the computational domain the accretion disk with average thickness 0.04$L$ and radius 0.4$L$ is formed. Fig. 3 shows the 3d visualization of donor, accretor and the accretion disk based on the values of emitted radiative energy. Fig. 4 contains the pictures of density and radiative energy extraction rate in equatorial section of disk. One can see that jet plasma penetrates inside the disk without shock wave formation. At the same time the disk plasma after one cycle of rotation about accretor comes into collision with jet flow.

**Figure 3.** 3d view of accretion disk, donor and accretor colored by values of emitted radiative energy.

**Figure 4.** Density and emitted radiative energy distribution in equatorial section of computational domain in moment $t = 5P_{orb}$. 
and forms the dense and hot shock wave, so called “hot line”.

Applying technique described in [1] the synthetic light curves for system inclination $87^\circ$ are calculated. The donor star surface is modelled to be opaque with temperature $T = 2100$K. Inside the computational domain the accretor is modelled as white dwarf with thin and bright circumstellar disk. Fig. 5 shows a fragment of the light curve in the vicinity of the eclipse. It can be seen that near eclipse the curve becomes asymmetric. This is due to the fact that, along with the white dwarf, the contribution to the brightness of the system is made by the hot line, whose visibility is significantly different before and after the eclipse. This effect is close to discussed in [5], but our results are based on the hydrodynamical model.

5. Conclusions
A three-dimensional mathematical model of the accretion disk in a semidetached binary star system with a small donor is considered. The model includes the equations of gas dynamics taking into account gravitation and radiation cooling of gas. A numerical solution was obtained using a parallel CFD solver based on the second-order RKDG method for unstructured tetrahedral meshes. Modeling shows that accretion disk is formed during first 5 orbital periods after jet flow arises. The jet-disk system contains a dense and hot area in the place of disk and jet interaction, so called “hot line”. This line visibility and it’s picture plane projection size determine the asymmetry of the light curve in the vicinity of the eclipse.

Acknowledgments
The work of V.V. Lukin is supported by Russian Science Foundation (proj. 17-79-20445).

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