Composition temperature-dependent g modes in superfluid neutron stars

E. M. Kantor,1⋆ and M. E. Gusakov1,2

1Ioffe Physical-Technical Institute of the Russian Academy of Sciences, Polytekhnicheskaya 26, 194021 St. Petersburg, Russia
2St. Petersburg State Polytechnical University, Polytekhnicheskaya 29, 195251 St. Petersburg, Russia

ABSTRACT

We demonstrate a possibility of existence of peculiar temperature-dependent composition g modes in superfluid neutron stars. We calculate the Brunt–Väisälä frequency for these modes, as well as their eigenfrequencies. The latter turn out to be rather large, up to ~500 Hz for a chosen model of a neutron star. This result indicates, in particular, that use of the barotropic equation of state may be not a good approximation for calculation of inertial modes even in most rapidly rotating superfluid neutron stars.

Key words: stars: interiors – stars: neutron – stars: oscillations (including pulsations).

1 INTRODUCTION

The aim of this Letter is to present some new results concerning the gravity oscillation modes (g modes) in neutron stars (NSs). It is generally accepted (e.g. Yakovlev, Levenfish & Shibanov 1999) that neutrons and protons in the NS cores become superfluid (SF) at temperatures \( T \lesssim 10^3–10^{10} \) K. Thus, here we concentrate on SF NSs. Until recently, all attempts to find g modes in such stars were unsuccessful (e.g. Lee 1995; Andersson & Comer 2001; Prix & Rieutord 2002). However, as we have shown (Gusakov & Kantor 2013), SF NS cores composed of neutrons (n), protons (p), and electrons (e) can harbour specific thermal g modes, whose frequencies (which are, typically, no more than a few Hz) depend on \( T \) and vanish at \( T = 0 \). In this Letter, we show that an admixture of additional particle species (e.g. muons) in the NS core leads to very peculiar temperature-dependent composition g modes. We discuss their properties, calculate their eigenfrequencies, which appear to be of the order of hundreds of Hz, and demonstrate that, although they depend on \( T \), they do not vanish in the limit \( T = 0 \).

2 DO COMPOSITION g MODES EXIST IN SF NSs?

Let us analyse if SF NS cores, composed of SF neutrons (n), possibly superconducting protons (p), electrons (e), and muons (\( \mu \)), can harbour composition g modes. We will follow the same reasoning as in Gusakov & Kantor (2013).

Consider two close points in NS core, 1 and 2, with the radial coordinate \( r = r_1 \) and \( r_2 \). Let \( A_1 \) and \( A_2 \) be the values of some thermodynamic quantity \( A \) (e.g. the energy density \( \varepsilon \) or pressure \( P \)) at points 1 and 2, respectively. Displace adiabatically a small fluid element, ‘attached’ to the normal liquid component (that is leptons and Bogoliubov excitations of baryons), upwards from point 1 to point 2. g-mode oscillations are only possible if the restoring force will appear that tends to return this fluid element back to point 1, or, equivalently, if the relativistic inertial mass density \( \rho = \varepsilon + P \) of the lifted element, \( \rho_{\text{lift}} \), will be larger than the equilibrium density \( \rho_2 \) at point 2, i.e. \( \rho_{\text{lift}} > \rho_2 \). If \( \rho_{\text{lift}} < \rho_2 \), then convection will take place.

To check this criterion, we present \( \rho \) as a function of four variables, say, \( P, \mu_n, x_{\text{eq}}, \) and \( x_S \). Here and below, \( n_i \) is the number density for particles \( i = n, p, \) and \( \mu \); \( \mu_n \) is the relativistic neutron chemical potential; \( S \) is the entropy density. The quantities \( P \) and \( \mu_n \) in a spherically symmetric SF NS satisfy two conditions of hydrostatic equilibrium (e.g. Gusakov & Andersson 2006; Gusakov & Kantor 2013; Gusakov et al. 2013): (i) \( \nabla P = -\nabla \rho \), and (ii) \( \nabla \mu_n = -\mu_n \nabla \phi \), where \( \phi(r) \) is the gravitational potential and we define \( \phi \equiv \frac{d}{dr} \) because all quantities of interest depend on \( r \) only. Thus, at point 2 both \( P \) and \( \mu_n \) of the lifted element adjust themselves to their equilibrium values \( P_2 \) and \( \mu_{n2} \), i.e. to the surrounding pressure and neutron chemical potential. The pressure adjusts by contraction/expansion of the fluid element, while \( \mu_n \) adjusts by the variation in the number of ‘SF neutrons’, which can freely escape from the fluid element, because their velocity differs from that of the ‘normal’ liquid component. At the same time, the quantities \( x_{\text{eq}} \) and \( x_S \) remain unaffected (\( x_{\text{eq}} = x_{\text{eq}1}; x_S = x_{\text{eq}1} \)), because electrons, muons, and entropy move with the same velocity (while beta-processes are slow and can be neglected). Hence, the restoring force arises if \( w(P_2, \mu_{n2}, x_{\text{eq}2}, x_S) < w(P_2, \mu_{n2}, x_{\text{eq}1}, x_S) \). Expanding \( w \) in Taylor series, we obtain

\[
\frac{\partial w(P, \mu_n, x_{\text{eq}}, x_S)}{\partial x_{\text{eq}}} \nabla x_{\text{eq}} + \frac{\partial w(P, \mu_n, x_{\text{eq}}, x_S)}{\partial x_S} \nabla x_S < 0,
\]

where the last term depends on \( T \) and can be neglected in strongly degenerate npe\( \mu \)-matter. This Ledoux-type criterion is always

E-mail: kantor@mail.ioffe.ru

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satisfied in beta-equilibrated SF NSs. Thus, SF NS cores harbour convectively stable composition g modes.

3 SF OSCILLATION EQUATIONS

We will analyse linear oscillations of a spherically symmetric non-rotating NS with the metric

$$\text{d}s^2 = g_{\alpha\beta}\text{d}x^\alpha\text{d}x^\beta = -e^\nu \text{d}t^2 + e^\nu \text{d}r^2 + r^2(\text{d}\theta^2 + \sin^2\theta \text{d}\phi^2),$$

where $r$, $\theta$, and $\phi$ are the spatial coordinates in the spherical frame with the origin at the stellar centre; $t$ is the time coordinate; $v(r) = 2\phi(r)$ and $\lambda(r)$ are the metric coefficients. Here and below, $\alpha$ and $\beta$ are the space–time indices. In what follows, we assume that $g_{\alpha\beta}$ is not perturbed in the course of oscillations (Cowling approximation; see Cowling 1941). This approximation works very well for g modes (e.g. Gaertig & Kokkotas 2009). The equations governing oscillations of SF NSs can then be derived from

(i) energy–momentum conservation

$$T_{\alpha}^{\beta} = 0, \quad \text{where} \quad T_{\alpha}^{\beta} = (P + \epsilon) u^\alpha u^\beta + Pg^{\alpha\beta},$$

(ii) potentiality condition for the motion of SF neutrons

$$\partial_{\alpha}[u_{\alpha}\nu + \mu \nu] = \partial_{\beta}[u_{\alpha}\beta + \mu \partial_{\beta} u_{\nu}],$$

(iii) continuity equation for baryon currents $j^\mu_{(b)}$ (11)

$$(\partial_{\beta} j^\beta_{(b)}) = 0, \quad \text{where} \quad j^\mu_{(b)} = n_{(b)} u^\mu + Y_{ab} \partial_{\nu} u_{(b)}^\nu,$$

(iv) continuity equation for lepton currents $j^\mu_{(l)}$ (12)

$$(\partial_{\beta} j^\beta_{(l)}) = 0, \quad \text{where} \quad j^\mu_{(l)} = n_{(l)} u^\mu.$$  

Here $T^{\alpha\beta}$ is the energy–momentum tensor, $Y_{ab}$ is the (temperature-dependent) relativistic entrainment matrix (analogue of the SF density for mixtures; see, e.g., Gusakov & Andersson 2006; Gusakov, Kantor & Haensel 2009b). Here and below, indices $a$ and $b$ refer to baryon species ($i, k = n, p$) and summation over the repeated indices is assumed. Furthermore, $u^\mu$ is the four-velocity of normal liquid component, and the four-vectors $w^{(n)}$ and $w^{(p)}$ describe the SF degrees of freedom. They are related with each other and with $u^\mu$ by the quasi-neutrality condition, $Y_{ik} w^{(k)}_{\nu} = 0$, and by the ‘comoving frame’ condition, $u_{\nu} w_{\nu}^{(i)} = 0$ (see Gusakov et al. 2013 for details).

We consider small non-radial perturbations of the equations \(\text{exp}(i\omega) Y_{ab}(\theta, \phi)\), where $Y_{ab}$ is the spherical harmonic. In linear approximation, they reduce to the following system

\[
\begin{aligned}
\left( g_{\mu\nu} \frac{\partial n_{\nu}}{\partial P} + g_{\mu\nu} \frac{\partial n_{\nu}}{\partial n_{\mu}} - \frac{\partial n_{\nu}}{\partial \chi_{\mu}} \nabla \chi_{\nu} \right) \xi_{(b)} & = \frac{n_{b}}{e^{\nu/2}r^2} \frac{\delta}{\partial r} \left( e^{\nu/2}r^2 \xi_{(b)} \right) + \frac{n_{b}}{r^2 \omega^2 (P + \epsilon)} \frac{\partial}{\partial r} \delta P, \\
\frac{\partial \xi_{(b)}}{\partial x_{\mu}} - \frac{\omega^2 n_{b}}{e^{\nu} e^\nu} \xi_{(b)} + \frac{\delta \delta P}{\partial r} & = 0, \\
\end{aligned}
\]

where $\delta$ denotes Eulerian perturbation and $g = \nabla \phi = \nabla v/2$. In equations (7)–(10), the quantities $n_b$, $n_e$, and $\omega$ are functions of $P$, $\mu_b$, and $x_{cb}$; their dependence on $x_{cs}$ (or, equivalently, on $T$) is ignored. Radial components $\xi_{(b)}^r$ and $\xi_{(b)}^{\nu}$ of Lagrangian displacements for the normal liquid component and baryons are defined by

\[
\begin{aligned}
u & = -i \omega e^{-\nu/2} \xi_{(b)}^r, \\
U_{(b)} & = i \omega e^{-\nu/2} \xi_{(b)}^{\nu},
\end{aligned}
\]

where $U_{(b)}^\mu = \frac{j_{(b)}^\mu}{n_{(b)} / n_b}$ is the baryon four-velocity. The parameter $\gamma$ in equations (7)–(10) depends on $T$ (through the elements of the matrix $Y_{ab}$) and equals

\[
y = \frac{n_b Y_{pp}}{\mu_b (Y_{nn} + Y_{pp} - Y_{nn}^{\mu})} - 1 > 0.
\]

In the low-temperature limit (12) gives $\gamma \approx n_{p}/n_{n}$. This estimate follows from the sum rule $\mu_e Y_{ei} = n_e$ and the fact that $Y_{pp}$ is noticeably smaller than $Y_{nn}$ and $Y_{pp}$, and hence can be neglected in (12) (Gusakov, Kantor & Haensel 2009a).

To derive equations (7)–(10), we used the thermodynamic relation $P + \epsilon = \mu_b n_b$, the hydrostatic equilibrium conditions (i) and (ii) from Section 2, and expressed $\delta x_{\mu}$ as

\[
\delta x_{\mu} = -\xi_{(b)}^{\nu} \nabla x_{\mu}.
\]

by employing the continuity equations (6) for $l = e, \mu$.

4 LOCAL ANALYSIS

Let us analyse short-wave perturbations of the system (7)–(10), proportional to $\text{exp}(i\omega t) \text{exp}[\frac{k}{r} (r') \nu] Y_{ab}$, where the wavenumber $k$ of a perturbation weakly depends on $r$ ($k \gg |d\ln k/\text{d}r|$, WKB approximation). Solving equations (7)–(10), we find the standard (see McDermott, van Horn & Scholl 1983) short-wave g mode dispersion relation,

\[
\omega^2 = N^2 \frac{l(l+1)e^\nu}{(l+1)e^\nu + k^2 r^2},
\]

where

\[
N^2 = -\frac{g}{\mu_b n_b} e^{\nu} \left( \frac{1 + y}{y} \right) \frac{\text{d}w(P, \mu_e, x_{cb})}{\text{d}x_{cb}} \nabla x_{cb}
\]

is the corresponding Brunt–Väisälä frequency squared. The stability condition for these g modes, $N^2 > 0$, coincides with the inequality (1). Note that, in contrast to composition g modes in non-SF NS matter (which are independent of $T$, because the matter is strongly degenerate), $N^2$ for composition g modes in SF npea–matter strongly depends on temperature through the parameter $\sqrt{(T + y)/y}$ (see Fig. 3 below).

5 BRUNT–VÄISÄLÄ FREQUENCY

In all calculations, we employ Heiselberg & Hjorth-Jensen (1999) parametrization of APR (Akmal, Pandharipande & Ravenhall 1998).
equation of state (EOS) in the core. This EOS allows for muons (npeμ-composition), which appear first at $n_\mu \approx 0.133 \, \text{fm}^{-3}$.

All numerical results here are obtained for an NS with the mass $M = 1.4 M_\odot$. The circumferential radius for such a star is $R \approx 12.1 \, \text{km}$; the central density is $\rho_c = 9.47 \times 10^{14} \, \text{g} \, \text{cm}^{-3}$. The threshold for muon appearance lies at a distance $r \approx 10.7 \, \text{km}$ from the centre.

We consider two models of nucleon SF: model I (simplified) and model II (more realistic, see Fig. 1). In both models, the redshifted proton critical temperature is constant over the core, $T_{\nu p}^\infty \equiv T_{\nu p} \equiv 5 \times 10^9 \, \text{K}$. In model I, the redshifted neutron critical temperature is also constant over the core, $T_{\nu n}^\infty \equiv T_{\nu n} \equiv 6 \times 10^9 \, \text{K}$, while in model II $T_{\nu n}^\infty$ increases with the density $\rho$ from the value $T_{\nu n} \approx 5 \times 10^9 \, \text{K}$ at the core–crust interface to the maximum value $T_{\nu n}^\infty \approx 6 \times 10^9 \, \text{K}$ at $\rho = 10^{15} \, \text{g} \, \text{cm}^{-3}$ (this density is larger than $\rho_c$). Our model II agrees with the results of some microscopic calculations (e.g., Baldo et al. 1998). For simplicity, we assume that neutrons in the crust are non-SF. This assumption should not affect SF composition g modes significantly.

Fig. 2 presents Brunt–Väisälä frequency $N$, given by equation (15), as a function of radial coordinate $r$. Solid line shows $N(r)$ for SF npeμ-matter, calculated in the low-temperature limit (i.e., assuming $T^\infty \ll T_{\nu n}^\infty$, $T_{\nu p}^\infty$, where $T^\infty = T_{\nu n}^{\nu n / 2}$ is the redshifted temperature). Dashed line shows the Brunt–Väisälä frequency $N_{\text{nsf}}$ of non-SF matter ($T^\infty > T_{\nu n}^\infty$) in the NS core. It equals (see equation (16), where $c_{1 \mu}^2 = \nu P/(\mu_\mu n_\mu)$, $\gamma = (n_\mu / P) \delta P(n_\mu, n_n, n_\nu, n_b)/\delta n_\mu$ is the adiabatic index, and $c_{2 \mu}^2 = \nabla P/(\mu_\mu \nabla n_\mu)$. [Note that $N$ in Figs 2 and 3 is given in kHz, while equations (15) and (16) give circular frequency.] In the very vicinity of muon threshold, $N$ sharply falls to zero (because $\nabla n_\mu = 0$ at the threshold), while $N_{\text{nsf}}$ decreases only slightly and then grows again. Generally, $N$ is severalfold higher than $N_{\text{nsf}}$. The reason for that is the dimensionless factor $\sqrt{1 + \gamma / \nu}$ [see equation (15)], which, in the low-temperature limit, can be estimated as $\sqrt{\nu / \nu} \sim 3$ (see Section 3), and increases with $r$. This factor also depends on $T$ (since $\gamma$ depends on $T$, see equation (12)), and monotonically decreases with increasing $T$, approaching 1 at $T^\infty = T_{\nu n}^\infty$. Solid line in Fig. 3 illustrates this dependence and represents $N$ at $r/R = 0.6$ as a function of $T^\infty$ for model I of baryon SF. Dashed line shows the frequency $N_{\text{nsf}}$ of non-SF npeμ-matter at the same distance.

6 BOUNDARY CONDITIONS

To calculate the eigenfrequencies of global stellar oscillations, one has to solve equations (7)–(10) with the appropriate boundary conditions, which should be imposed at the stellar centre and surface, as well as at the interface between SF and non-SF regions.

Note that both $T_{\nu n}^\infty$ profiles adopted here ensure that either the star will be non-SF or it will consist of two layers, SF internal layer (where neutrons are SF) and non-SF external layer (where neutrons are non-SF).

The oscillations of the internal SF layer are governed by equations (7)–(10), while non-SF matter oscillations are described by the following equations (see, e.g., McDermott et al. 1983; Reisenegger & Goldreich 1992),

$$N_{\text{nsf}}^2 \equiv g^2 \left( \frac{1}{c_{1 \mu}^2} - \frac{1}{c_{2 \mu}^2} \right) e^{\lambda / \nu},$$

(16)
For simplicity, we will treat matter in the crust as a one-component liquid and will ignore the density discontinuities there (this is equivalent to vanishing $N_{in}$ in the crust).

One formulates the following boundary conditions for equations (7)–(10) and (17)–(18).

1. Existence of the solution to equations (7)–(10) implies that at the stellar centre
   \[ \xi' \propto r^{-1}, \quad \xi_{b0}' \propto r^{-1}, \quad \delta P \propto r^1, \quad \delta \mu_n \propto r^1. \]  
\[ \tag{19} \]

2. The continuity of electron (or muon) current as well as the continuity of energy and momentum currents through the SF/non-SF interface results in
   \[ \xi_{b0}^e(r_0 - 0) = \xi_{b0}^e(r_0 + 0), \]  
\[ \tag{20} \]

\[ \delta P(r_0 - 0) = \delta P(r_0 + 0), \]  
\[ \tag{21} \]

\[ \xi_{b0}^o(r_0 - 0) = \xi_{b0}^o(r_0 - 0), \]  
\[ \tag{22} \]

where $r_0$ is the radial coordinate of the interface.

3. Vanishing of the pressure $P$ at the stellar surface means
   \[ \Delta P = \delta P + \nabla P \xi_{b0} = 0. \]  
\[ \tag{23} \]

Solution to oscillation equations with these boundary conditions allows one to determine stellar eigenfrequencies in the Cowling approximation.

\section*{7 g Mode Eigenfrequencies}

Solid lines in Fig. 4 present the eigenfrequencies $\nu = \omega/(2\pi)$ of the first four quadrupolar ($l = 2$) g modes in SF NS as functions of $T_\infty$ for the SF model I.

As local analysis shows, the temperature dependence $\nu(T_\infty)$ is driven by the parameter $\sqrt{(1 + \gamma)/(1 + 2\gamma)}$, which strongly changes in the range $0.1 T_{cs}^{\infty} < T_\infty < T_{cs}^{\infty}$ (see Section 5).

As a result, the eigenfrequencies vary from their high asymptotic low-temperature values (at $T_\infty \lesssim 5 \times 10^9$ K) down to zero at $T_\infty = T_{cs}^{\infty}$.\footnote{The fact that eigenfrequencies vanish at $T_\infty \rightarrow T_{cs}^{\infty}$ for SF model I may seem strange, because $\mathcal{N}$ in this limit tends to a finite value [see equation (15), where $\gamma \rightarrow \infty$ as $T_\infty \rightarrow T_{cs}^{\infty}$]. However, one can show that in the vicinity of $T_{cs}^{\infty}$ g mode turns into a specific p mode, which is absent at low $T_\infty$ and vanishes at $T_\infty = T_{cs}^{\infty}$ (see our subsequent publication for more details on the asymptotic behaviour of g modes). This transformation is seen as a sharp bend in the spectrum near $T_{cs}^{\infty}$.}

When neutron SF disappears at $T_\infty > T_{cs}^{\infty}$, NS harbours ordinary temperature-independent composition g modes (Reisenegger & Goldreich 1992), which we call normal. Their eigenfrequencies are shown by dashed lines. Finally, dot–dashed line in Fig. 4 presents $\nu$ for the fundamental $l = 2$ g mode calculated for a star of the same mass and for the same EOS, but under an assumption that there are no muons in the NS core (npe-matter).

The eigenfrequencies of the first four $l = 2$ g modes for the SF model II are presented in Fig. 5 by solid lines. At low $T_\infty$, g modes demonstrate SF-like behaviour. At $T_\infty \lesssim 5 \times 10^9$ K, their eigenfrequencies coincide with those for model I, because in that case $T_\infty \ll T_{cs}^{\infty}$ in the whole NS core for both models. When $T_\infty$ is (roughly) proportional to the size of SF region $r_0$ [see equation (14)], which tends to zero at $T_\infty \rightarrow T_{cs}^{\infty}$, normal g modes (dashed lines) are calculated under an upper assumption that SF matter of NSs does not support g modes [i.e. the underlined terms in equations (7)–(10),

\begin{center}
\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{Spectrum of quadrupolar ($l = 2$) g modes versus $T_\infty^{\infty} \equiv T_\infty^{\infty}/(10^9 \text{K})$ for model I of nucleon SF. Solid/dashed lines show eigenfrequencies $\nu$ (in Hz) for the first four g modes in SF/non-SF NS with npe$m$ core composition; dot–dashed line shows $\nu \approx 127$ Hz for the fundamental $l = 2$ g mode in non-SF NS with npe core composition. Dotted line indicates (constant over the core) $T_{cs}^{\infty}$ for neutrons.}
\end{figure}
\end{center}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{The same as Fig. 4, but for model II of nucleon SF. Dotted line indicates $T_{cs}^{\infty}$ in the stellar centre ($T_{cs}^{\infty}$). Thin dashed and dot–dashed lines show artificially decoupled normal g modes and SF g modes (see text for details).}
\end{figure}
The frequencies of normal g modes vanish at \( T^\infty \to 5 \times 10^7 \) K for the same reason: \( \nu \) for the normal g modes is proportional to the size of non-SF region in the core, which decreases with decreasing \( T^\infty \) and becomes zero at \( T^\infty = 5 \times 10^7 \) K.

8 SUMMARY AND OUTLOOK

We showed that specific composition g modes can propagate in SF NS cores composed of npe-matter with admixture of muons (or some other non-SF particle species), provided that \( \nabla x_{cu} \equiv \nabla (n_p/n_e) \neq 0 \). The most peculiar feature of these g modes is that their eigenfrequencies \( \nu \) (and the corresponding Brunt–Väisälä frequency \( \lambda^2 \)) are strong functions of temperature \( T^\infty \). They depend on \( T^\infty \) through the parameter \( \sqrt{1 + \gamma} \), which is in turn a function of the temperature-dependent entrainment matrix \( \lambda^2 \), see equation (12). Since at \( T^\infty \to 0 \) this parameter is \( \approx \sqrt{n_p/n_e} \sim 3 \) (see Section 5), the frequencies \( \nu \) (and \( \lambda^2 \)) of SF NSs can exceed, in this limit, the corresponding frequencies of non-SF NSs by a factor of few. We illustrated our results by finding solutions to oscillation equations (7)–(10) and (17)–(18) for a particular SF NS with \( M = 1.4 \text{ M}_\odot \) and APR EOS in its core. We found that in the limit \( T^\infty \to 0 \) (i.e. larger by a factor of 2.43 than the corresponding \( \nu \approx 190 \text{ Hz} \) in a non-SF NS with npe-\( \mu \) core composition, and by a factor of 3.64 than the corresponding \( \nu \approx 127 \text{ Hz} \) in a non-SF NS with npe core composition). At finite \( T^\infty \), the calculations were made for two models (I and II) of nucleon SF. In particular, for the more realistic model II, we showed that \( \nu \) first decreases with increasing \( T^\infty \) but then, close to a temperature \( T_{\text{cmax}} \), at which neutron SF disappears, it starts to grow up reaching, at \( T^\infty = T_{\text{cmax}} \), the value of \( \nu \) for ordinary composition g modes, first discussed by Reisenegger & Goldreich (1992, see Fig. 5).

As we showed, the frequencies of SF g modes can reach the values \( \nu \sim 500 \text{ Hz} \). That is, they can be of the order of the spin frequencies of the most rapidly rotating NSs (e.g. NSs in low-mass X-ray binaries). This may have a strong impact on the properties of the so-called inertial (or, more precisely, inertial-gravity) modes in rotating NSs (e.g. Passamonti et al. 2009). As a result, the latter modes may be very different from their cousins in barotropic NSs (for which \( \lambda^2 \) = 0). This could mean that the approximation of a barotropic NS is too rough, e.g. for the analysis of r-mode saturation amplitude (Bondarescu, Teukolsky & Wasserman 2007), resulting from the resonance interaction of r mode with a couple of inertial modes. At the end, we would like to note that, although in this Letter we discussed SF composition g modes in the context of NSs, they can, in principle, be observed in laboratory experiments (e.g. with ultra-cold atoms or with liquid He II), provided that one has a mixture of an SF and two non-SF species.

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