Intensity of gluon bremsstrahlung in a finite plasma

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The intensity of single gluon bremsstrahlung in a QCD plasma is evaluated with a Monte Carlo which solves the transport equation for a generic interaction with the medium. In particular the calculation is performed for a Debye screened potential and compared to the well known Gaussian/Fokker-Planck results. The full calculation including the first and last gluons show a qualitatively different behavior from the BDMPs result for any finite medium length. It is shown that the emission intensity is underestimated for the Gaussian approximation, compared to the Debye screened potential. This change can not be accounted for by a redefinition of the Gaussian parameter ($\hat{q}$).

I. INTRODUCTION

The formation of the quark-gluon plasma in high energy collisions is an open problem in QCD and a systematic study of the dependence of the medium characteristics on the energy loss of high energy partons and other probes of the medium are needed to fully understand it. The existence of multiple sources of scattering in a medium severely affects the way in which quanta are emitted from high energy particles. Local internal phases in the scattering amplitudes, growing with the traveled distance, regulate the amount of scatterers which can coherently participate into a single emission element. This interference effect, known as Landau-Pomeranchuk-Migdal (LPM) suppression [1, 2], leads to a substantial reduction of a radiation scenario naively described as an incoherent sum of single Bethe-Heitler [3] intensities.

Semi-infinite medium calculations of this phenomenon under the Fokker-Planck approximation have been introduced for QED in [4, 5] and subsequent extensions for QCD [6–8] have been developed. These results [9–13] have been formulated to account for the energy-loss mechanism originating the depletion of high transverse momentum particles in heavy ion collisions at RHIC and LHC due to a multiple scattering process with a medium, an indicative sign of QGP formation [14]. We have to note, however, that in the Fokker-Planck approximation, which naturally emerges when the number of collisions is large, the medium interaction is replaced by an effective one [6, 7]. This leads to a Gaussian momentum distribution which is valid for not too large deviations from the typical $p_t$ accumulated by the particle. For QCD plasmas, these conditions are not necessarily fulfilled. First, since media are never very large, the number of collisions can not be large. Second, realistic massless or massive/screened interactions [15], like the Coulomb or Debye potentials, respectively, have long $p_t$ tails substantially enhancing the emission intensity. The semi-infinite medium approximation, meanwhile, loses the relevant dependence with the medium length $l$. In this limit, the intensity becomes proportional to $l$, which produces an infinite suppression in the regime of vanishing phases and, therefore, the soft photon theorem [16] is not observed. While this approximation is adequate for large mediums it becomes critical for the energy-loss estimation at proton-proton or low centrality heavy ion collisions. Following these concerns several frameworks beyond the Gaussian approximation have been developed for finite QCD media [17–19], nuclei [20, 21], and for the case of heavy quarks [22, 23]. As expected these results have shown significant differences with the semi-infinite length calculations and they have produced reliable predictions at RHIC and LHC [24–26].

Taking these conditions into account we develop a formalism which is able to account for any interaction potential with the medium and incorporates gluon’s transverse spectrum dependence, if desired. In section II we briefly present the formalism for a general scenario, whereas in section III we restrict the intensity evaluation to the spectrum of gluons emitted after a first hard collision. Finally, we present the main conclusions.

II. RADIATION INTENSITY

We consider an ideal scenario in which an on-shell quark coming from infinity undergoes a multiple scattering process in a finite medium, and then goes to infinity. The amplitude of a single gluon emission can be written as

$$M = -i g_s \int d^4 x \bar{\Psi}_2(x) A^\mu_1(x) \gamma^\mu \Psi_1(x),$$

(1)

where $\Psi_{1,2}(x)$ represent the quark wave function before and after the emission, $A^\mu(x) = A^\mu_0(x) t_\alpha$ represents the emitted colored gluon, $t_\alpha$ are generators of SU(3) and $g_s = \sqrt{\alpha_s}$ is the coupling constant. Both quark and

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gluon are subject to the external field of the medium, then at $x$ they are a superposition of a free state plus a set of scattered states, of the form

$$\Psi_1(x) = \int \frac{d^3p}{(2\pi)^3} S_q(p, p_0; z, 0) \Psi(p, x),$$

where $\Psi(p, x) = \mathcal{N}(p) u(p) e^{-ip\cdot x}$ is a free quark and $\mathcal{N}(p) = \sqrt{m/p^3}$. Similarly for the gluon

$$A_\mu(x) = \int \frac{d^3k}{(2\pi)^3} S_g(k, k_f; z, l) A^{(0)}_{\mu}(k, x),$$

where $A^{(0)}_{\mu}(k, x) = \mathcal{N}(k) \epsilon_\mu(k) e^{-ik\cdot x}$ is a free gluon, $\mathcal{N}(k) = \sqrt{2\pi/\omega}$ the normalization and $\epsilon_\mu(k)$ its polarization.

FIG. 1: Diagrammatic representation of the discretized parton paths $p_i$ for the quark and $k_i$ for the gluon, for $i = 0, 1, \ldots, n$ steps. Here $D^3p/(2\pi)^3$ is a shorthand notation for $d^3p_1/(2\pi)^3 \ldots d^3p_{n-1}/(2\pi)^3$. The square amplitude can be averaged over the transverse medium coordinates of each sheet and integrated in the final gluon solid angle, giving the intensity of emission of a gluon within an energy interval from $\omega$ to $\omega + d\omega$, per unit of medium transverse size, in a total length $l$

$$\frac{dI(l)}{d\omega} = \alpha_s \frac{N_c^2 - 1}{2N_c} \beta_k^2 \omega^2 \int d\Omega k_0 \left( \prod_{k=0}^{n-1} \int \frac{d^3k_j}{(2\pi)^3} \right) \left( \prod_{k=1}^{n} \phi(\delta k_i, \delta z_i) \right) \left( h^n(y) \sum_{i=1}^{n} \delta_i^2 + h^s(y) \sum_{i=1}^{n} \delta_i^2 \right)$$

where $y = \omega/E_q$ and the functions $h^n(y)$ and $h^s(y)$ are the kinematical weights appearing in the diagonal and non-diagonal sum in spins and helicities of the squared
emission vertices, at \( z_i \) and \( z_f \). In the high energy limit
\[
 h^n(y) = \frac{1 + (1 - y)^2}{2}, \quad h^s(y) = \frac{y^2}{2}.
\]
The \( \delta^i \) can be interpreted as a classical current, representing spin preserving amplitudes and is given by
\[
 \delta^n_i = p(0) \times \left( \frac{\hat{k}(z_{i+1})}{k_{\mu}(z_{i+1})p^\mu(0)} - \frac{\hat{k}(z_i)}{k_{\mu}(z_i)p^\mu(0)} \right) e^{i\varphi_i}, \quad (9)
\]
whereas the spin flipping amplitudes produce
\[
 \delta^s_i = \left( \frac{p^\mu(0)}{k_{\mu}(z_{i+1})p^\mu(0)} - \frac{p^\mu(0)}{k_{\mu}(z_i)p^\mu(0)} \right) e^{i\varphi_i}, \quad (10)
\]
and \( \varphi_i = 1/E_q \sum_{k=i}^n \delta z_k k_{\mu}(z_k)p^\mu(0) \). Once integrated along \( \hat{k}(z) \) with the elastic weights, they produce two contributions of the same order. The spin flipping part, however, only becomes relevant for gluon energies \( \omega \) of the order of \( E_q \) due to the form of \( h^n(y) \), in accordance with the classical behavior at the infrared divergence. In Eq. (8) we are assuming \( \omega \ll E_q \) and therefore, we will neglect the contribution of \( h^s(y) \) in what follows. Within the same approximation one can assume that the quark momenta is frozen \( p_\mu(z) \sim p_\mu(0) \).

The phases and denominators appearing above are
\[
 k_{\mu}(z_i)p^\mu(0) = (1 - \beta(\omega)) \omega + \frac{\delta \hat{k}^2(z_k)}{2\omega \beta(\omega)} \approx \frac{m_g^2 + k_z^2(z_k)}{2\omega} \quad (11)
\]
The gluon’s velocity is taken as \( \beta(\omega) = 1 - m_g^2/\omega^2 \), where a plasma mass \( m_g \) has been introduced to take into account possible medium effects in the gluon dispersion relation. The gluon effective mass can be considered of the order of \( \mu_d \) [27]. The color matrices factor out of the elastic amplitudes and they can be averaged independently over color configurations at the vertex in \( z \). Finally, the elastic weights in (8) are calculated by an incoherent average of the elastic amplitudes squared, the dominant term is given by the factor \( S_y S_y \), which gives in a layer of thickness \( \delta z \)
\[
 \phi(\delta \hat{k}, \delta z) = e^{-n_0 \delta z \sigma_{gg}} (2\pi)^3 \delta^3(\delta \hat{k}) + 2\pi \delta(\delta k^\mu) S_2(\delta \hat{k}, \delta z). \quad (12)
\]
The first contribution represents the no collision probability within the layer of thickness \( \delta z \) and density \( n_0(z) \) times the forward distribution. The quantity \( \lambda^{-1} = \sigma_{gg} n_0(z) \) is the mean free path of the gluon, \( \sigma_{gg} \) its elastic cross section with a single scattering center. The second contribution represents the collisional distribution in case of collision. It can be shown to satisfy a Molière’s transport equation whose solution reads
\[
 \Sigma_2(q, \delta z) = \int d^2 x e^{-q \cdot x} e^{-n_0 \delta z \sigma_{gg}} \left( e^{n_0(z) \delta z \sigma_{gg}(|x|)} - 1 \right), \quad (13)
\]
and where the Fourier transform of the squared single elastic amplitude reads at first order in the coupling
\[
 \sigma_{gg}(|x|) = \frac{4\pi \alpha_s^2}{\beta^2(\omega) \mu_d^2} T_f \mu_d(|x|) K_1(\mu_d(|x|)), \quad (14)
\]
where \( T_f = 1/2 \) is the first order Casimir for SU(3). Using (14) the single elastic cross section required at (13) is given at leading order by
\[
 \sigma_q = \frac{4\pi \alpha_s^2}{\beta^2(\omega) \mu_d^2} T_f. \quad (15)
\]
The convolution of (12) over a step \( \delta l \) produces rules for momentum additivity. For a step verifying \( \delta l < \lambda_{gg} = 1/n_0 \sigma_{gg} \) and constant density an expansion in small \( \delta l \lambda_{gg}^{-1} \) is enough and the total scattering distribution reduces to the incoherent superposition of the single distributions for the matter in \( \delta l \). The squared momentum change in \( \delta l \) is equal to a momentum change in a single scattering,
\[
 \langle \delta k^2(\delta l) \rangle = \mu_d^2 \left( 2 \log \left( \frac{2\omega}{\mu_d} \right) - 1 \right) = \mu_d^2 \eta(\omega), \quad (15)
\]
where \( \eta(\omega) \) accounts for the long tail correction of the Debye potential. For arbitrary distances \( l \), using equation (13) it can be shown that the squared momentum change is additive in the traveled length. Indeed
\[
 \frac{\partial}{\partial l} \langle \delta k^2(l) \rangle = n_0 \sigma_{gg} \langle \delta k^2(\delta l) \rangle = 2 \dot{q}, \quad (16)
\]
where we have defined the transport coefficient \( \dot{q} \). Since the single momentum change depends on the gluon’s energy, \( \dot{q} \) has to be fixed, in principle, for each gluon’s energy.

Evaluation of (8) can be accomplished by taking the \( \delta z \to 0 \) limit, either by using a Boltzmann transport equation or equivalently by integrating the kinetic phase with the elastic weights (12), producing a path integral. In both cases, by using the Fokker-Planck/Gaussian approximation for (13) Migdal’s result [5] for QCD matter is found [6–8].

We have built a Monte Carlo program where (8) is evaluated as a sum over discretized paths. At each step the potential distribution is sampled and the gluon’s path is built. The quark is also allowed to interact with the medium (so that its momentum does change). Phases are calculated with the exact kinematical expression. In this way one can calculate the gluon emission distribution for any given potential, in particular it has been calculated for the Debye potential and for the Gaussian approximation. A similar Monte Carlo was made for QED [28] whose results reproduce the path integral limit for the Gaussian potential and agree with Migdal’s expression for \( l \to \infty \), although our results are valid for an arbitrary size of the medium. In a typical run, the step size is taken as 0.01\( \lambda_{gg} \), so that for medium size of \( l \sim 5 \text{ fm} \) we have \( \gtrsim 10^4 \) steps. Paths are calculated for an array of
\( \sim 100 \) frequencies of the gluon and \( \sim 800 \) different emission angles. For these values, we run \( \approx 10^4 \) simulations and average over them. This takes \( \approx 24 \) h of CPU time in a PC. We have checked that the size of the grid and the number of simulations are enough for a statistical uncertainty less than 10% in all cases. On the other hand,

![FIG. 2: Differential spectrum for gluons of \( m_g = 0.45 \) GeV for a medium of \( l = 1 \) fm and \( n_0 T_f = 8 \) fm\(^{-3} \) (equivalently \( \hat{q} = 1 \) GeV\(^2/\)fm). We show calculations for our Monte Carlo results both for the Debye potential (D), using the total intensity (8) (circles) and the inner intensity (24) (pentagons), and for the Fokker-Planck (G) approximation, using the total intensity (squares) and the inner intensity (diamonds). Also shown is our approximation (20) (solid line) and the BDMPS result (dot-dashed line). The coherent limits both for the Fokker-Planck approximation (dotted dark line) and the Debye potential (dotted light line) are also shown.

we can take a simpler approach if we note that the single radiation elements appearing in the sums (9) and (10) are interpretable as single Bethe-Heitler amplitudes for a gluon emitted at \( z_i \) due to the medium in \( \delta z \), with a phase which produces interferences in the squared amplitude. Each diagram appears twice, representing the possible change of momentum or just before the \((i+1)\)th, except those two cases where gluons are emitted before and after the first and last available momentum changes. When the medium is removed, \( l = 0 \), following (12), momentum homogeneity \( k(z) = k(l) \) cancels the sums (9) and (10) and the radiation vanishes. For small media \( l \lesssim \lambda gq \) internal sum cancels and we are left with the first and the last terms only which reproduces the single Bethe-Heitler spectrum for a \( gg \to qgq \) process. By taking the limit \( m_g \to 0 \) and neglecting the spin flip suppressed part, the Bertsch-Gunion formula is recovered [29].

![FIG. 3: Differential spectrum for gluons of \( m_g = 0.45 \) GeV for a medium of \( l = 5 \) fm and \( n_0 T_f = 8 \) fm\(^{-3} \) (equivalently \( \hat{q} = 1 \) GeV\(^2/\)fm). We show calculations for our Monte Carlo results both for the Debye potential (D), using the total intensity (8) (circles) and the inner intensity (24) (pentagons), and for the Fokker-Planck (G) approximation, using the total intensity (squares) and the inner intensity (diamonds). Also shown is our approximation (20) (solid line) and the BDMPS result (dot-dashed line). The coherent limits both for the Fokker-Planck approximation (dotted dark line) and the Debye potential (dotted light line) are also shown.

weight (12), of the order of 1, a condition which reads

\[
\varphi_i^2 = \frac{1}{E_q} \int_{z_i}^{z_f} dz k_\mu(z) p^\mu(0) \approx \frac{m_g^2}{2w^2} \delta l + \frac{\hat{q}}{2w}(\delta l)^2 = 1. \tag{17}
\]

In each group, the partial internal sum between \( z_i \) and \( z_f \) at (9) or (10) cancels, since their relative phase is negligible using condition (17). The elements in the group act coherently between themselves, but they incoherently interfere with any other group due to condition (17). Using (17) this defines a coherence length modulated by \( \omega \), we call \( \delta l(\omega) \), hence

\[
\delta l(\omega) = \frac{m_g^2}{2\hat{q}} \left( \sqrt{1 + \frac{8\hat{q}\omega}{m_g^4}} - 1 \right), \quad \omega \leq \omega_c, \tag{18}
\]

and \( \delta l(\omega) = l \) for \( \omega > \omega_c \), where \( \omega_c \) verifies \( l(\omega_c) = l \), i.e. \( \omega_c \sim (\hat{q}l + m_g^2)l \). Over the length \( \delta l(\omega) \) the scattering centers are not resolved due to the negligible accumulated phase change. This part of the medium acts like a single scatterer of an equivalent charge \( n_0 \delta l(\omega) \). Since there are \( l/\delta l(\omega) \) of these partial sums acting incoherently, we can write for the total intensity

\[
\frac{dI(l)}{d\omega} = \frac{l}{\delta l(\omega)} \alpha_s C_f \beta^2(\omega) \omega^2 \int \frac{d^2k_2}{(2\pi)^2} \int \frac{d^2k_0}{(2\pi)^2} \left( h^a(y) \delta k^a_l + h^a(y) \delta k^a_i \right)^2 \Sigma_2(\delta k(z_i), \delta l(\omega)), \tag{19}
\]

The above equation is a very good approximation in the frequency interval \( (m_g, \omega_s) \) in which the gluon completely
resolves each of the single scattering centers and in the interval \((\omega_c, E_q)\) in which the gluon stops being able to resolve any internal structure of the medium. These values are given by \(\delta l(\omega_s) = \lambda g_q\) and \(\delta l(\omega_s) = l\), their phases satisfying \(\varphi_0^2(\omega_s) = 1\) and \(\varphi_0^2(\omega_s) = 1\). Using (18) \(\omega_s = m_g^2/\bar{q}\). For \(\omega > \omega_c\) the entire medium acts coherently like a single scatterer with an equivalent charge contained in the length \(l\) following a Bethe-Heitler power law \(1/\omega\). Radiation intensity in this interval depends on the medium length and energy loss is dominated by these gluons. For frequencies below \(\omega_c\), gluon resolution power starts to decouple the medium in groups of charges of big size, so radiation grows as \(l/\delta l(\omega) \sim \sqrt{\bar{q}/\omega}\) times a Bethe-Heitler power law \(1/\omega\), with a slow logarithmic change decrease \(\omega/\bar{q}\). This decoupling saturates at \(\omega_c\), when the coherence length acquires the minimum length required to produce radiation, given by \(\lambda g_q\). However, suppression due to a vanishing velocity \(\beta(\omega)\) rapidly cancels the enhancement. Using (19) we then find for the intensity produced after traversing a length \(l\)

\[
\frac{dI(l)}{d\omega} = \frac{\alpha_s C_f}{\pi^2} \frac{l}{\delta l(\omega)} \int_0^\pi d\theta \sin(\theta) F(\theta) \Sigma_2(\delta \mathbf{k}, \delta l(\omega)),
\]

with \(|\delta \mathbf{k}| = 2\beta(\omega)\omega \sin(\theta/2)\) and

\[
F(\theta) = \frac{1 - \beta^2(\omega) \cos\theta}{[2\beta(\omega) \sin(\theta/2) \sqrt{1 - \beta^2(\omega) \cos^2(\theta/2)} + \beta(\omega) \sin(\theta/2)]} - 1.
\]

In the Gaussian/Fokker-Planck approximation for \(\Sigma_2(\delta \mathbf{k}, \delta l)\) we find in the coherence plateau where \(\omega dI/d\omega\) is constant analytical expressions for the asymptotic limits of very large and very small mediums. An expression which interpolates between them can be found

\[
i_0(l) \equiv \frac{dI(l)}{d\omega} = \frac{2}{\pi} \frac{\alpha_s C_f}{2} \frac{1 + \eta_c}{A + \eta_c} \log(1 + A \eta_c),
\]

where \(A = e^{-(1+\gamma)}, \gamma\) is Euler’s constant, and \(\eta_c = 2\bar{q}l/m_g^2\) is a dimensionless number which is a measure of the number of collisions.

In Figures 2 and 3 we show the results of our calculations for a medium density of \(n_0 T_f = 8\) fm\(^{-3}\) and a gluon mass of \(m_g = 0.45\) GeV, for a medium length of \(l = 1\) fm and \(l = 5\) fm, respectively, for both the Gaussian approximation and the Debye potential. We see that, for the same parameters, the Debye potential produces more radiation. The difference can be cast approximately into a redefinition of \(\bar{q}\) but at the cost of making it medium size, and Debye mass, dependent. Also shown is our estimation of the intensity using the approximated expression (20) for the Gaussian case. As it can be seen, the approximation is rather good, specially at large frequencies. We show also the result obtained by neglecting altogether the phases in the calculation (labeled as coherent limit in the figure). In Figures 4 and 5 we show the same results for a medium density of \(n_0 T_f = 1\) fm\(^{-3}\) and a gluon mass of \(m_g = 0.15\) GeV and a medium length of \(l = 1\) fm and \(l = 5\) fm, respectively.

In Figure 6 we show the asymptotic emission intensity \(i_0(l)\) as a function of the medium length for the Gaussian approximation and for the Debye potential for different Debye masses (keeping the effective \(\bar{q}\) constant). As can be seen in the figure the ratio between the Debye and the Gaussian intensities is not constant so that one can not redefine a Gaussian \(\bar{q}\) independently of the medium properties.

**III. Radiation Intensity After a First Collision**

We now consider a more realistic case where the parton suffers a first hard collision. We assume here that this scenario can be approximated within our formalism taking into account gluons emitted after the first momentum change. This corresponds, in the high energy limit, to a restriction of the \(z\)-integration in (1) to the interval \([0, +\infty)\). The square of this truncated amplitude can then be splitted into three terms

\[
\left| \int_0^\infty \right|^2 = \int_0^l \int_0^l + \int_0^\infty \int_0^\infty + 2 \Re \int_0^l \int_0^\infty,
\]

namely the intensity of a “medium” part, the intensity of the last leg, and their interference. Since the last leg intensity appears now impaired in the absence of a first
FIG. 5: Differential spectrum for gluons of $m_g = 0.15$ GeV for a medium of $l = 5$ fm and $n_0 T_f = 1$ fm$^{-3}$ (equivalently $\bar{q} = 0.12$ GeV$^2$/fm). We show calculations for our Monte Carlo results both for the Debye (D) potential, using the total intensity (8) (circles) and the inner intensity (24) (pentagons), and for the Fokker-Planck (G) approximation, using the total intensity (squares) and the inner intensity (diamonds). Also shown is our approximation (20) (solid line) and the BDMPS result (dot-dashed line). The coherent limits both for the Fokker-Planck approximation (dotted dark line) and the Debye approximation (dotted light line) are also shown.

FIG. 6: Asymptotic emission intensity $\omega dI/d\omega$ as a function of the medium length for density $n_0 T_f = 8$ fm$^{-3}$ for the Gaussian approximation (continuous black line) and for the Debye potential for different Debye masses, as marked, $m_g = 0.45$ GeV. Also shown the large and small lengths approximations. Vertical dashed lines mark the transition between large and small media for each Debye mass, $\eta_c = 2 q_l / m_g^2 = 1$.

FIG. 7: Ratio of the bremsstrahlung intensity for the Debye potential compared to the Gaussian approximation for two gluon masses. We show the results for $n_0 T_f = 8$ fm$^{-3}, \mu_d = 0.45$ GeV and $l = 5$ fm as the dark dot-dashed line, and $l = 1$ fm as the light dot-dashed line, for $n_0 T_f = 1$ fm$^{-3}, \mu_d = 0.15$ GeV and $l = 5$ fm as the dark dotted line and $l = 1$ fm as the light dotted line. Shadow bands show the statistical uncertainty.

which is interpretable as a probability only when $l \to \infty$, corresponding in this limit to the semi-infinite medium evaluation of (8).

In Figures 2, 3, 4 and 5 the evaluation of (24) is shown together with the BDMPS result [9, 30]. Neither (24) nor the BDMPS calculation consider the emission in the coherent limit, therefore the intensity goes to zero for large frequencies. Whereas this prescription is correct for the infinite length limit, it is assumed that for finite media the neglected terms are reabsorbed into the structure and fragmentation functions. As can be seen in the figures, our results for the Gaussian case agree with the BDMPS results at high energy, as expected. At low energy, the kinematical restriction in the $k_t$ integration and the effect of the mass of the gluon make the intensity to decrease. As before, the calculation done with the Debye potential gives a larger emission intensity than the Gaussian case, for the same parameters. This can be seen in figure 7, were we shown the ratio of the intensity for the Debye calculation to the Gaussian calculation as a function of the energy of the gluon emitted. This ratio is not a constant and depends on the mass of the gluon going from $\sim 1$ ($\sim 1.5$) at low energies for $m_g = 0.45$ GeV ($m_g = 0.15$ GeV) to $\sim 2.2$ ($\sim 8$) at larger energies. Therefore a change on the value of the Gaussian $\bar{q}$ can fit the Debye results but in a qualitative way only. For instance a change of $\bar{q} \to 3.5 \bar{q}$ fits the result for the case $n_0 T_f = 8$ fm$^{-3}, \mu_d = 0.15$ GeV and large lengths ($l \gtrsim 3$ fm) with a maximum 20% error. For Debye masses $\mu_d = 0.45$ GeV the scale factor is 2.8 with a larger error of $\sim 40\%$. In general, as expected, for small lengths the Debye result can not be fit with a redefinition of $\bar{q}$.
IV. CONCLUSIONS

We have developed a Monte Carlo method which is able to calculate the gluon bremsstrahlung for realistic Debye screened interactions. A semi-analytical approximation has been also estimated which helps to qualitatively understand the LPM effect in QCD. The Fokker-Planck approximation is shown to underestimate the emission intensity, and the dilution cannot be cast into a redefinition of \( \hat{q} \), independently of gluon's energy or medium size. If we consider the total intensity (8) the finite size of the medium translates into a length-dependent coherent term which would dominate the energy loss as \( \Delta \propto E_q \) for large quark energies. This was already found in [31] in the case of cold nuclear matter and applied to the azimuthal asymmetries in pA collisions in [32]. We found an expression for the intensity in the coherent regime which confirms the same behavior in the small opacity limit, linear in length and density. In contrast, if we consider only the gluons emerging from the inner legs (24) energy loss is dominated by the emission of small energy gluons and we recover the BDMPS result except for kinematical restrictions and length effect. In this scenario one should expect a qualitatively different behavior since the coherent plateau, related to the initial and final state radiation, is missing. A study of the full amplitude including the interferences between the initial and final emissions was made in [33].

In both scenarios we found that the emission is suppressed at small gluon energies and is largest at around \( \omega_c (\sim 5\text{-}10 \text{ GeV}) \) as a result of medium incoherence. The shape and depth of this intensity gives information on the medium properties. A large width of the intensity is a sign of a large length of the medium, whereas a narrow rise indicates a high density medium or large gluon masses.

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