Double quarkonium production at high Feynman-$x$

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Abstract

In this paper we give estimates for the proton–proton cross sections into pairs of quarkonium states $J/\psi$, $\psi(2S)$, $\Upsilon(1S)$ and $\Upsilon(2S)$ at the scheduled AFTER@LHC energy of 115 GeV. The estimates are based on the intrinsic heavy quark mechanism which is observable for high values of $x_F$, a range outside the dominance of single parton and double parton scattering.

Keywords: Heavy Quark, Quarkonium, Intrinsic Heavy Quark Mechanism

1. Introduction

In the era of high luminosity and high energy accelerators the associated heavy quarkonium production plays a special role as a testing ground to study multiple parton scattering in a single hadron collision. Significant progress on the Double Parton Scattering (DPS) has been provided by the Tevatron and the LHC in measuring the productions of $J/\psi + W$ [1], $J/\psi + Z$ [2], $J/\psi + \text{charm}$ [3] and $J/\psi + J/\psi$ [4, 5, 6]. Therefore and for many other reasons, heavy quarkonium production is always a hot topic in high energy physics, as this kind of physics is an ideal probe for testing quantum chromodynamics.

Current colliders provide access only to the physics at low values of the Feynman parameter $x_F$. However, significant interest is given also for physics at high $x_F$ [7, 8, 9, 10, 11]. This region will be accessible at a future fixed-target experiment at the LHC (AFTER@LHC). In a recent paper, Jean-Philippe Lansberg and Hua-Sheng Shao discussed contributions of the DPS to the double-quarkonium production in the kinematic region of the AFTER@LHC [12]. However, as we learned from the low statistics NA3 experiment measurements of the double $J/\psi$ production [13, 14] and the observation of the double charmed baryons by the SELEX collaboration [15, 16, 17], the double intrinsic heavy quark mechanism can be the leading production mechanism [18, 19].
The existence of a non-perturbative intrinsic heavy quark component in the nucleon is a rigorous prediction of QCD. Intrinsic charm and bottom quarks are contained in the wavefunction of a light hadron – from diagrams where the heavy quarks are multiply attached via gluons to the valence quarks. In detail, the intrinsic heavy quark components are contributed by the twist-six contribution of the operator product expansion proportional to $1/m_Q^2$ [20, 21]. In this case, the frame-independent light-front wavefunction of the light hadron has maximum probability if the Fock state is minimally off-shell. This means that all the constituents are at rest in the hadron rest frame and thus have the same rapidity $y$ if the hadron is boosted. Equal rapidity occurs if the light-front momentum fractions $x = k^+/P^+$ of the Fock state constituents are proportional to their transverse masses, $x_i \propto m_{T,i} = (m_i^2 + \langle k_{T,i}^2 \rangle)^{1/2}$, i.e. if the heavy constituents have the largest momentum fractions. This features the BHPS model given by Brodsky, Hoyer, Peterson and Sakai for the distribution of intrinsic heavy quarks [22, 23].

In the BHPS model the wavefunction of a hadron in QCD can be represented as a superposition of Fock state fluctuations, e.g. $|h\rangle \sim |h_l\rangle + |h_l\rangle + |h_lQ\bar{Q}\rangle \ldots$, where $h_l$ is the light quark content, and $Q = c, b$. If the projectile interacts with the target, the coherence of the Fock components is broken and the fluctuation can hadronize. The intrinsic heavy quark Fock components are generated by virtual interactions such as $gg \to Q\bar{Q}$ where the gluons couple to two or more valence quarks of the projectile. The probability to produce such $Q\bar{Q}$ fluctuations scales as $\alpha_s^2(m_Q^2)/m_Q^2$ relative to the leading-twist production.

Following Refs. [18, 22, 23], the general formula for the probability distribution of an $n$-particle intrinsic heavy quark Fock state as a function of the momentum fractions $x_i$ and the transfer momenta $k_{T,i}$ can be written as

$$\frac{dP_i}{\prod_{i=1}^{n} dx_i d^2 k_{T,i}} \propto \alpha_s^4(M_{QQ}) \delta\left(\sum_{i=1}^{n} k_{T,i}\right) \delta\left(1 - \sum_{i=1}^{n} x_i\right) \left(m_h^2 - \sum_{i=1}^{n} m_{T,i}^2/x_i\right)^2,$$  \hspace{1cm} (1)

where $m_h$ is the mass of the initial hadron. The probability distribution for the production of two heavy quark pairs is given by

$$\frac{dP_{Q_1Q_2}}{\prod_{i=1}^{n} dx_i d^2 k_{T,i}} \propto \alpha_s^4(M_{Q_1Q_1}) \alpha_s^4(M_{Q_2Q_2}) \delta\left(\sum_{i=1}^{n} k_{T,i}\right) \delta\left(1 - \sum_{i=1}^{n} x_i\right) \left(m_h^2 - \sum_{i=1}^{n} m_{T,i}^2/x_i\right)^2.$$  \hspace{1cm} (2)

If one is interested in the calculation of the $x$ distribution, one can simplify the formula by replacing $m_{T,i}$ by the effective mass $\tilde{m}_i = (m_i^2 + \langle k_{T,i}^2 \rangle)^{1/2}$ and
neglecting the masses of the light quarks,

$$\frac{dP_{iQ_1Q_2}}{\prod_{i=1}^{n} dx_i} \propto \alpha_s^4(M_{Q_1\bar{Q}_1})\alpha_s^4(M_{Q_2\bar{Q}_2}) \frac{\delta(1-\sum_{i=1}^{n} x_i)}{\left(\sum_{i=1}^{n} \hat{m}_{T,i}/x_i\right)^2}.$$  

(3)

The $x_F$ distribution for the double quarkonium production $X_1 + X_2$ (with $X_i = J/\psi, \psi(2S), \Upsilon(1S), \Upsilon(2S), \ldots$) is then given by [18]

$$\frac{dP_{iQ_1Q_2}}{dx_{X_1,X_2}} = \int \prod_{i=1}^{n} dx_i dx_{X_1} dx_{X_2} \frac{dP_{iQ_1Q_2}}{\prod_{i=1}^{n} dx_i} \delta(x_{X_1,X_2} - x_{X_1} - x_{X_2})$$

$$\times \delta(x_{X_1} - x_{Q_1} - x_{\bar{Q}_1}) \delta(x_{X_2} - x_{Q_2} - x_{\bar{Q}_2}).$$  

(4)

The BHPS model assumes that the vertex function in the intrinsic heavy quark wavefunction is varying relatively slowly. The particle distributions are then controlled by the light-cone energy denominator and the phase space. The Fock states can be materialized by a soft collision in the target which brings the state on shell. The distribution of produced open and hidden charm states will reflect the underlying shape of the Fock state wavefunction.

In this paper we investigate the double intrinsic heavy quark mechanism for the double-quarkonium production in the high Feynman-$x$ region at the AFTER@LHC experiment. In this particular case the production of the double quarkonium plays a special role as it provides the direct access to extract the double heavy quark probabilities $P_{icc}$, $P_{icb}$, and $P_{ibb}$. To the best of our knowledge the $x_F$ distribution for double-quarkonium production in proton beam events has not yet been measured (cf. also a comment at the end of the third paragraph in the Introduction of Ref. [18]). Therefore, our estimates cannot be compared to existing data but wait for future confirmation by experiments like AFTER@LHC, for which we give numerical values. As an innovative element, for our analysis we use the color evaporation model, applied also to excited $2S$ states. Finally, in the conclusions we discuss why existing LHC measurements cannot be interpreted as non-evidence of the intrinsic heavy quark mechanism.

2. Double-quarkonium production cross section

The production cross section of the quarkonium can be obtained as an application of the quark–hadron duality principle known as color evaporation model (CEM) [24]. In this model the cross section of quarkonium are
obtained by calculating the production of a $Q\bar{Q}$ in the small invariant mass interval between $2m_Q$ and the threshold to produce open heavy-quark hadrons, $2m_H$. The $Q\bar{Q}$ pair has $3\times3 = (1+8)$ color components, consisting of a color-singlet and a color-octet. Therefore, the probability that a color-singlet is formed and produces a quarkonium state is $1/(1+8)$, and the model predicts

$$
\sigma(Q\bar{Q}) = \frac{1}{9} \int_{2m_Q}^{2m_H} dM_{Q\bar{Q}} \frac{d\sigma_{Q\bar{Q}}}{dM_{Q\bar{Q}}} = \frac{1}{9} \int_{4m_Q^2}^{4m_H^2} dM_{Q\bar{Q}}^2 \frac{d\sigma_{Q\bar{Q}}}{dM_{Q\bar{Q}}^2},
$$

where $\sigma_{Q\bar{Q}}$ is the production cross section of the heavy quark pairs and $\sigma(Q\bar{Q})$ is a sum of production cross sections of all quarkonium states in the duality interval. For example, in case of charmonium states one has

$$
\sigma(Q\bar{Q}) = \sigma(J/\psi) + \sigma(\psi(2S)) + \ldots.
$$

According to a simple statistical counting, the fraction of the total color-singlet cross section into a quarkonium state is given by

$$
\sigma(X) = \rho_X \cdot \sigma(Q\bar{Q})
$$

$$
(\ X = J/\psi, \psi(2S), \ldots) \text{ with}
$$

$$
\rho_X = \frac{2J_X + 1}{\sum_i(2J_i + 1)},
$$

where $J_X$ is the spin of the quarkonium state $X$ and the sum runs over all quarkonium states. In case of the $J/\psi$ meson the calculation gives

$$
\rho_{J/\psi} \approx 0.2.
$$

This statistical counting rule works well for $J/\psi$ but not so well for other charmonium states, even not for $\psi(2S)$. Instead, in this paper we use the fact that a quarkonium production matrix element is proportional to the absolute square of the radial wave function at the origin [25], so that

$$
\sigma(J/\psi) : \sigma(\psi(2S)) \approx |R_{J/\psi}(0)|^2 : |R_{\psi(2S)}(0)|^2.
$$

The absolute square of the radial wave function $R_X(0)$ of the quarkonium state $X = J/\psi, \psi(2S), \ldots$ at the origin is determined by the leptonic decay rate [26]

$$
\Gamma(X \to e^+e^-) = \frac{4N_c\alpha^2e^2}{3} |R_X(0)|^2 \left(1 - \frac{16\alpha_s}{3\pi}\right),
$$

$$
\left(1 - \frac{16\alpha_s}{3\pi}\right),
$$

(10)
where $N_c = 3$ is the number of quark colors, $e_Q$ is the electric charge of the heavy quark, and $M_X$ is the mass of the quarkonium state $X$. Splitting $\sigma(Q\bar{Q})$ up into the different quarkonium states one can obtain the corresponding production cross sections.

According to the intrinsic heavy quark mechanism the production cross section $\sigma(Q\bar{Q})$ of a $Q\bar{Q}$ pair in the duality interval is given by [18]

$$\sigma^{iQ}(Q\bar{Q}) = f_{Q\bar{Q}/p}^{iQ} \cdot P_{iQ} \cdot \sigma_{pp}^{inel} \cdot \frac{1}{9} \frac{\mu^2}{4\hat{m}_Q}, \quad (11)$$

where $\mu \approx 0.2$ GeV denotes the soft interaction scale parameter, $f_{Q\bar{Q}/p}^{iQ}$ is the fragmentation ratio of the $Q\bar{Q}$ pair written as

$$f_{Q\bar{Q}/p}^{iQ} = \int_{4m_{2Q}^2}^{4m_{2H}^2} dM_{Q\bar{Q}}^2 \frac{dP_{iQ}}{dM_{Q\bar{Q}}^2} \left/ \int_{4m_{2Q}^2}^{s} dM_{Q\bar{Q}}^2 \frac{dP_{iQ}}{dM_{Q\bar{Q}}^2} \right., \quad (12)$$

and the inelastic proton–proton cross section $\sigma_{pp}^{inel}$ in the region of $\sqrt{s} \geq 100$ GeV is obtained by the approximation [27]

$$\sigma_{pp}^{inel} = 62.59 \hat{s}^{-0.5} + 24.09 + 0.1604 \ln(\hat{s}) + 0.1433 \ln^2(\hat{s}) \text{ mb}, \quad (13)$$

where $\hat{s} = s/2m_p^2$. At the AFTER@LHC energy $\sqrt{s} = 115$ GeV, one obtains $\sigma_{pp}^{inel} = 28.4$ mb.

2.1. Double-charmonium production from $| uudc\bar{c}\bar{c} \rangle$

The double-charmonium production cross section $\sigma(c\bar{c} + c\bar{c})$ from the Fock state $| uudc\bar{c}\bar{c} \rangle$ can be written obviously as

$$\sigma^{icc}(c\bar{c} + c\bar{c}) = (f_{c\bar{c}/p}^{icc})^2 P_{icc} \sigma_{pp}^{inel} \frac{1}{9} \frac{\mu^2}{4\hat{m}_c}, \quad (14)$$

where the fragmentation ratio $f_{Q\bar{Q}/p}^{iQ_1Q_2}$ is obtained as

$$f_{Q\bar{Q}/p}^{iQ_1Q_2} = \int_{4m_{2Q}^2}^{4m_{2H}^2} dM_{Q\bar{Q}}^2 \frac{dP_{iQ_1Q_2}}{dM_{Q\bar{Q}}^2} \left/ \int_{4m_{2Q}^2}^{s} dM_{Q\bar{Q}}^2 \frac{dP_{iQ_1Q_2}}{dM_{Q\bar{Q}}^2} \right.. \quad (15)$$

In this case ($Q = c$, $H = D$) we use $m_c \approx 1.3$ GeV for the mass of $c$ quark, $\hat{m}_c = 1.5$ GeV for the effective transverse $c$-quark mass, and $m_D = 1.87$ GeV.
for the mass of the $D$ meson. For the integrated probability distribution we take the value $P_{icc} \simeq 0.002$ [18].

Combining Eqs. (14) and (15), we may expect the double-charmonium production cross section to be

$$\sigma^{icc}(c\bar{c} + c\bar{c}) \approx 1.5 \times 10^2 \text{ pb}.$$  

Analyzing the values of the radial wave functions at the origin [26], one finds

$$\sigma(J/\psi + J/\psi) : \sigma(J/\psi + \psi(2S)) : \sigma(\psi(2S) + \psi(2S)) \approx 1 : 0.65 : 0.43$$

Taking into account Eq. (8) and the generalization of Eq. (6),

$$\sigma(X_1 + X_2) = \rho_{X_1}\rho_{X_2} \cdot \sigma(Q\bar{Q} + Q\bar{Q}),$$

one obtains

$$\sigma^{icc}(J/\psi + J/\psi) \approx 6.0 \text{ pb}$$
$$\sigma^{icc}(J/\psi + \psi(2S)) \approx 3.9 \text{ pb}$$
$$\sigma^{icc}(\psi(2S) + \psi(2S)) \approx 2.6 \text{ pb}$$  (17)

2.2. Associated charmonium–bottomonium production from $|uudc\bar{c}\bar{b}\rangle$

Following Refs. [28, 29], the associated charmonium–bottomonium production cross section is given by

$$\sigma^{icb}(c\bar{c} + b\bar{b}) = f^{icb}_{c\bar{c}/p} f^{icb}_{b\bar{b}/p} P_{icb} \sigma_{pp}^{incl} \frac{1}{9} \frac{\mu^2}{4\hat{m}_b^2} \left( \frac{\hat{m}_c}{\alpha_s(M_{bb})} \right)^4.$$  (18)

Applying Eq. (15) to this case ($Q = b$, $H = B$) we use $m_b \approx 4.2$ GeV for the mass of the $b$ quark, $\hat{m}_b = 4.6$ GeV for the effective transverse $b$-quark mass, and $m_B = 5.3$ GeV for the mass of the $B$ meson. The value of $P_{icb}$ is unknown at this moment but we assume it to be approximately equal to $P_{icc}$. Finally, we calculate the associated charmonium–bottomonium production cross section to be

$$\sigma^{icb}(c\bar{c} + b\bar{b}) = 0.35 \text{ pb}.$$  (19)

In this section we calculate only the production cross section for the ground states,

$$\sigma^{icb}(J/\psi + \Upsilon(1S)) \approx 14 \text{ fb}.$$  (20)
2.3. Double-bottomonium production from $|uudb\bar{b}\bar{b}\rangle$

We already have all ingredients for the calculation of the production cross section of the double-bottomonium states except for $P_{ibb} = (\hat{m}_c/\hat{m}_b)^2 \cdot P_{icb}$, so the numerical value will be

$$\sigma^{ibb}(b\bar{b} + b\bar{b}) = 0.03 \text{ pb},$$

(21)

and the cross sections for the particular double-bottomonium states are given by

$$\sigma^{ibb}(\Upsilon(1S) + \Upsilon(1S)) \approx 1.2 \text{ fb}$$
$$\sigma^{ibb}(\Upsilon(1S) + \Upsilon(2S)) \approx 0.6 \text{ fb}$$
$$\sigma^{ibb}(\Upsilon(2S) + \Upsilon(2S)) \approx 0.3 \text{ fb}$$

(22)

Figure 1: The histogram shows the $x_F$ distribution of the $J/\psi$ pair due to the double intrinsic heavy quark mechanism in arbitrary units.
3. Conclusions

In this paper we investigated the contribution of the double intrinsic heavy quark mechanism to the production of a quarkonium pair. It is clear that Single Parton Scattering (SPS) and Double Parton Scattering (DPS) provide the main contributions to the double quarkonium production cross section. However, both these contributions are vanishing fast with increasing Feynman parameter $x_F$. On the other hand, the contribution from the double intrinsic heavy quark mechanism mainly grows with $x_F$ (see Fig. [1]). If one considers proton–proton collisions in the center-of-mass frame, one can distinguish between charm production at positive $x_F$ coming from the intrinsic heavy in the beam proton and negative $x_F$ coming from the intrinsic heavy in the nucleons of the target. As it shown in Ref. [12], the DPS contribution starts at $x_F = -0.5$. This is the region where the double intrinsic heavy quark mechanism is from the target and contributes on the average. On the other hand, the double intrinsic charm becomes the leading production mechanism at high $x_F$, $\langle x_{\psi\psi}\rangle \simeq 0.64$ [18].

Another interesting aspect to be discussed is $\sigma(\psi + J/\psi)/\sigma(J/\psi)$. The only result for this ratio with access to high values of $x_F$ was provided by the NA3 experiment and was found to be $(3 \pm 1) \times 10^{-4}$ with 150 and 280 GeV/c pion [13] and 400 GeV/c proton [14] beams. The same ratio measured by the LHCb Collaboration is found to be $(5.1 \pm 1.0 \pm 0.6^{+1.2}_{-1.0}) \times 10^{-4}$ [4]. This result can be interpreted wrongly as non-evidence for the intrinsic heavy quark mechanism. However, the traditional $q\bar{q}$ annihilation mechanism and the leading gluon-gluon fusion mechanism for LHCb are not in good agreement with the NA3 data (cf. the discussion in Ref. [29]) which shows that perturbative QCD can explain neither the NA3 cross section nor the $x_F$ distribution. Compared to this, the double intrinsic heavy quark mechanism reproduces $x_F$ dependencies very well [18], at least for the case measured by NA3, namely the case of pion-nucleon scattering [13] (cf. Fig. 2).

Current experimental knowledge does not give us much information about the main contribution of the double intrinsic heavy quark mechanism. In our calculations we use $P_{icc}$ from data with low statistics and provide other formal assumptions. However, the key feature of AFTER@LHC is the access to high Feynman-$x$. Therefore, the measurement of the double quarkonium production can provide more accurate data and shed more light on the role of the intrinsic heavy quark mechanism.
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Figure 2: $x_F$ distribution for (a) $\pi N \rightarrow \psi \psi$ and (c) $p N \rightarrow \psi \psi$ ($x_F \in [0, 1]$). The plots are taken from Ref. [18]. Shown are NA3 $\pi^- N$ data at 150 and 280 GeV/c [13] (histograms) and estimates of the intrinsic heavy quark mechanism (solid curve).