Methodology for calculating the stress-strain state of robotic system made of composite materials

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Abstract. The methodology for calculating the stress-strain state of robotic systems made of composite material under dynamic load is considered. The problem is solved by the finite element method. Formulas for determining the reduced stiffness of a multilayer composite material are given depending on the location of the base of the composite in layers, as well as in three-layer structures consisting of external bearing layers and a layer of aggregate between them. The method of placing the base layers of a composite material in the structure under study along the lines of maximum stress is considered. Particular attention is paid to the accounting of gearboxes, gears and support bearings in robotic systems. The results obtained in relation to the multi-stage dynamic stand, made of composite material.

Keywords: composite materials, robotic systems, multi-stage stands, finite element method, stress-strain state.

1. Introduction
Composite materials are widely used in recent years due to the ability to change the characteristics of the material depending on the location of the base of the composite material. High strength characteristics and a relatively small specific gravity of composite materials made it possible to use them in various areas of technology: aircraft manufacturing, shipbuilding, automotive, etc [1]. At the same time, the use of composite materials in robotics is not enough. This is primarily due to the presence in these designs of gear rims, gearboxes, bearing supports made of homogeneous materials and therefore not compatible with the main composite structure. In connection with the use of composite materials in various fields of technology, various methods are being developed for the calculation of multilayer composite structures. Modern numerical methods are used, mainly the finite element method [2, 3]. Currently, there are a sufficient number of computer-aided design systems for calculating structures made of composite materials [4], but given the many factors that affect the bearing capacity of composite structures, the method for calculating composite materials is being developed and improved [5-7].

Consideration of the stress-strain state of a robotic structure containing gearboxes, bearings and gears is a complex task. But at the same time, taking into account their interaction allows to take into account the work of all parts of the structure during operation. Therefore, the solution to this problem is an important and relevant topic.

Consider the definition of the stress-strain state of structures made of composite materials on the basis of a multi-stage stand of semi-natural modeling (hereinafter referred to as a stand). The use of composite material in the stands has not yet been applied. At the same time, the main characteristic of the stand performance is small inertial characteristics and maximum rigidity, which largely depend on the specific strength of the material. Composite material has a higher specific strength compared to homogeneous metals. For example, the specific strength of magnesium alloy, the most fully used material in the manufacture of the stand is 12-15 kilometers, titanium - 26-40 kilometers, while this indicator for composite material is 50-100 kilometers.
The main disadvantage of the composite material is its low rigidity in the direction that does not coincide with the base of the composite. At the same time, the composite material is multi-layered; therefore, varying the layer-by-layer direction of the base of the composite can achieve a sufficient rigidity of the structure [8]. Consideration of the stress-strain state of a robotic structure containing gearboxes, bearings and gears is a complex task. But at the same time, taking into account their interaction allows to take into account the work of all parts of the structure during operation.

Methods for counting bearings, gears and especially gearboxes in robotics are not yet fully developed. Therefore, the methods of accounting for such elements in robotic systems are important and relevant.

2. Method and solution building
In this paper, the stand made from composite material is considered. The stand includes both shell elements of different curvature, as well as parts that are not amenable to direct approximation by finite elements, such as, for example, bearing supports, gearboxes, and toothed rims. Therefore, when approaching the bearing supports, a rod structure was applied with rigidity equal to that of the bearing support. The gearbox was replaced with a core system with stiffness that matched the rigidity of the gearbox; the same procedure was used for gears [9].

The basic design of the stand was approximated by multilayer shell finite elements of different curvature, including three-layer finite elements with external carrier layers and a filler between them. The filler prevented the approach of the bearing layers and perceived the shear stresses.

Figure 1 shows a stand approximated by finite elements.

2.1. Stress-strain equations
The characteristics of a composite material having layers with a different arrangement of the base, in the method of reduced stiffness, can be obtained as follows.

The connection matrix of stresses and strains for an orthotropic material, the orthotropic axes of which coincide with the axes of coordinates, in the plane-stressed state have the form

\[
\sigma = (E) \varepsilon,
\]

where

\[
(E) = \begin{pmatrix}
Q_{11} & Q_{12} & 0 \\
Q_{21} & Q_{22} & 0 \\
0 & 0 & Q_{66}
\end{pmatrix},
\]

\[
\varepsilon^T = (\varepsilon_s, \varepsilon_\theta, \varepsilon_{s\theta}),
\]

\[
\sigma^T = (\sigma_s, \sigma_\theta, \sigma_{s\theta}),
\]

\[
Q_{11} = E_s/(1 - \nu_{s\theta}\nu_{s\theta}), \quad Q_{12} = \nu_{s\theta}E_s/(1 - \nu_{s\theta}\nu_{s\theta}),
\]
\[ Q_{21} = v_\theta E_\phi/(1 - v_\theta v_\phi), \quad Q_{22} = E_\phi/(1 - v_\theta v_\phi), \quad Q_{66} = G_{66}. \]

Rotation of the axes of coordinates at an angle \( \theta \) converts the matrix of elastic coefficients to the form

\[
(\tilde{E}) = \begin{pmatrix}
\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\
\bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} \\
\bar{Q}_{61} & \bar{Q}_{62} & \bar{Q}_{66}
\end{pmatrix},
\]

where

\[
\bar{Q}_{11} = c^4 Q_{11} - s^4 Q_{22} + 2(Q_{12} + 2Q_{66}) s^2 c^2,
\]
\[
\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66}) s^2 c^2 + (s^2 + c^2)Q_{22},
\]
\[
\bar{Q}_{16} = (c^2 Q_{11} - s^2 Q_{12} + (Q_{12} + 2Q_{66}) (s^2 - c^2)) sc,
\]
\[
\bar{Q}_{22} = s^4 Q_{11} - c^4 Q_{22} + 2(Q_{12} + 2Q_{66}) s^2 c^2,
\]
\[
\bar{Q}_{26} = (s^2 Q_{11} - c^2 Q_{12} - (Q_{12} + 2Q_{66}) (s^2 - c^2)) sc,
\]
\[
\bar{Q}_{66} = (Q_{11} - 2Q_{12} + Q_{22}) s^2 c^2 + (s^2 - c^2) Q_{66}, \quad s = \sin \theta, \quad c = \cos \theta.
\]

Layer deformations at a distance \( z \) from the middle surface take the form

\[ \varepsilon = \varepsilon^0 + z \chi^0, \]

where \( \varepsilon^0 \) are the deformations of the middle surface, \( \chi^0 \) are the curvature changes.

Substituting these relations into equation (1) we get

\[ \sigma = (\tilde{E}) \varepsilon^0 + z(\tilde{E}) \chi^0. \]

If the stresses are expressed through normal efforts and moments

\[
N = \int_{-h/2}^{h/2} \sigma dz, \quad N^T = (N_s, N_\theta, N_{s\theta}), \quad M = \int_{-h/2}^{h/2} \sigma z dz, \quad M^T = (M_s, M_\theta, M_{s\theta}).
\]  \( (2) \)

Here is designated \( N \) – are the membrane forces, \( M \) – are the bending moments.

Conducting the integration of expressions (2) we have

\[
\begin{pmatrix}
N \\
M
\end{pmatrix} = \begin{pmatrix}
A & B \\
B & D
\end{pmatrix}\begin{pmatrix}
\varepsilon^0 \\
\chi^0
\end{pmatrix},
\]

\[
(A_{ij}, \ B_{ij}, \ D_{ij}) = \int_{-h/2}^{h/2} Q_{ij}(1, z, z^2) dz, \ (i, j = 1, 2, 3).
\]

Denoted here \( A_{ij} = \sum_{k=1}^{n} \bar{Q}_{ij}(h_k - h_{k-1}), \ i, j = 1, 2, 6, \)
\[
B_{ij} = \sum_{k=1}^{n} \bar{Q}_{ij}(h^2_k - h^2_{k-1}), \ i, j = 1, 2, 6, \quad D_{ij} = \sum_{k=1}^{n} \bar{Q}_{ij}(h^3_k - h^3_{k-1}), \ i, j = 1, 2, 6.
\]  \( (3) \)

The designations in the formula (3) are shown in figure 2.

\[ \text{Figure 2. Multi-layer composite material.} \]

Consider the three-layer structure of a composite material, which consists of two external carrier layers and a layer of aggregate between them. For carrier layers apply the equations discussed above. The displacements of the filler are defined as the arithmetic average of the displacements of the carrier layers. Angle of rotation as the difference in the displacements of the bearing layers divided by the thickness of the composite material.
Figure 3 shows a three-layer composite structure with two outer carrier layers and a layer of aggregate between them. The displacements of the bearing layers are denoted by \( u_1 \) and \( u_2 \), the angles of rotation through \( \varphi_1 \) and \( \varphi_2 \), the thicknesses are indicated as \( h_1 \) and \( h_2 \). For the filler, the thickness is equal to \( h_0 \) and the displacement of the neutral line \( u_0 \).

The displacements of the neutral line of the aggregate and the angle of rotation of the normal during deformation are determined from the relations:

\[
u_0 = (\bar{u}_1 + \bar{u}_2)/2, \quad \varphi_0 = (\bar{u}_1 - \bar{u}_2)/h_3, \quad \bar{u}_1 = u_1 - h_1\varepsilon_{13}/2, \quad \bar{u}_2 = u_2 + h_2\varepsilon_{23}/2 \]

where \( u_0 \) is the displacement and \( \varphi_0 \) is the rotation angle of the neutral filler line, \( r \) are the shell radius, \( \varepsilon_{13}, \varepsilon_{23} \) - are the rotation angles around the coordinate lines, \( \varphi_s, \varphi_\theta \) are the angles of inclination of the meridian to the axis of the shell and the angle in the circumferential direction

\[
\varepsilon_{13} = \left( \frac{\partial w}{\partial s} + u \frac{\partial \varphi}{\partial s} \right); \quad \varepsilon_{23} = -\frac{1}{r} \left( \frac{\partial w}{\partial \theta} - v \sin \varphi \right).
\]

Here denoted \( u \) is the indicated by displacement in the meridional direction, \( w \) is the displacement in the normal direction, \( v \) is the displacement in the circumferential direction of the shell.

Displacements in the placeholder at a distance \( z \) from the neutral axis can be written as

\[
v_0(z) = v_0 + z\varphi_0 = 0.5(v_1 - h_1\varepsilon_{13}/2 + v_2 + h_2\varepsilon_{23}/2) + z(v_1 - h_1\varepsilon_{13}/2 - v_2 - h_2\varepsilon_{23}/2)/h_3, \\
u_0(z) = u_0 + z\varphi_0 = 0.5(u_1 - h_1\varepsilon_{13}/2 + u_2 + h_2\varepsilon_{23}/2) + z(u_1 - h_1\varepsilon_{13}/2 - u_2 - h_2\varepsilon_{23}/2)/h_3, \\
w_0(z) = w_1 + z(w_1 - w_2)/h_3.
\]

Thus, one can consider a model of a shell structure consisting of several layers.

The stand design was considered as a three-layer structure with two external carrier layers and filler between them. The filler perceived only shear stresses. To determine the location of the base of the composite material along the lines of maximum stress in the first approximation, a stand of a homogeneous material was calculated. As a result of the calculation, the location of the maximum voltage lines was determined. At the second stage, a stand made of composite material was considered. The basis of the composite material was located mainly along the lines of maximum stress obtained by calculating the stress-strain state of the stand from a homogeneous material. Subsequently, the arrangement of the composite material was varied to achieve its best position. As a result of the iterations, the optimal location of the composite material was determined from the point of view of the minimum stress-strain state of the stand under the load. Thus, it is possible to determine the arrangement of the layers of the composite material for their best arrangement in terms of the stress-strain state.
2.2. Matrix of the resolving equation

To determine the matrix of the resolving equation, it is necessary to determine the potential energy of the structure deformation. The potential strain energy for a multi-layer composite construction is separated from the equation:

\[ U = 0.5 \iint [\varepsilon^o (A) \varepsilon^o + \chi^o (B) \varepsilon^o + \varepsilon^o (B) \chi^o + \chi^o (D) \chi^o] dA. \]

Here notation: \((A), (B), (D)\) are membrane, bending-membrane, and bending rigidity matrices of the layers, \(\varepsilon^o = (\varepsilon^o_\sigma, \varepsilon^o_\theta, \varepsilon^o_{\sigma \theta})\) are strains in the middle surface plane, \(\chi^o = (\chi^o_\sigma, \chi^o_\theta, \chi^o_{\sigma \theta})\) are curvatures, and \(dA = Rd\theta \) is the area of integration.

The matrix of the resolving equation or the stiffness matrix for a multilayer composite construction is obtained by differentiating the potential deformation energy by generalized displacements

\[ \frac{\partial U}{\partial q} = (K)q, \]

where \(U\) is the strain energy of the structure, and \(q\) is the displacements at the nodes.

2.3. External force vector

To determine the vector of external forces, it is necessary to find the work of external forces on possible displacements. The work of external forces can be determined from the ratio

\[ W = \int P u dA, \]

where \(P, u\) are the vectors of external forces and the vector of displacements.

The vector of external forces is determined by differentiating the work of external forces by generalized displacements

\[ Q = - \frac{\partial W}{\partial q}. \]

2.4. System of resolving equations

The definition of the stress-strain state of a structure is reduced to solving a system of linear equations derived from the Lagrange equations [9]

\[ (K)q = Q. \]
Here \((K)\) is the stiffness matrix, \(Q\) is the vector of external forces, and \(q\) is a generalized displacement.

As a result of the solution of equation (4), a picture of the stress-strain state of the stand of the semi-natural modeling was obtained upon striking the locking (Figure 4). The applied force was determined based on the D'Alembert's principle, and was applied to the locking device of the stand.

3. Results

The stress-strain state of a robotic structure under the action of an external load has been studied. A method for determining the characteristics of a multilayer composite material is presented. Dependencies are obtained for determining displacements in a three-layer composite shell consisting of external carrier layers and a filler layer between them. The method of determining and positioning the base of the composite material along the lines of maximum stress is considered. The results are shown in the figures. Gears, bearings and gearboxes approximated by the method proposed by the author. The results obtained are in good agreement with the experimental results.

4. Conclusion

The study showed that the use of composite material for robotic systems is important and relevant. One of the main characteristics of robotic systems are inertial characteristics, which are the best in composite materials. The lack of rigidity of products made of composite materials is compensated for by the correct positioning of the base of the composite material along the lines of maximum stresses. To find the direction of the maximum stresses of the structure, it is necessary to calculate the stress-strain state of the structure from a homogeneous material. Determine the lines of maximum stress and then place the basis of the composite material in the composite structure along these lines. A new approach to the approximation of such elements as support bearings, gearboxes, gears is proposed. The proposed method compares favorably with the methods used in [10-12], where the bearings are directly approximated by finite elements, which leads to an unjustified increase in the number of finite elements. Errors in the calculation of large systems of equations negate the direct approximation of bearings. Moreover, gearboxes that have a direct approximation do not lead to satisfactory results.

In this paper, the rigidity of the gearbox is calculated separately and the gearbox is replaced by a system of connecting kernels with similar rigidity. Also gears, bearings are approximated by the structure of the kernels, the rigidity of which coincides with these elements. The convergence of the research results was checked by thickening the finite element mesh. If the results did not differ by more than 2%, the results of the study were considered reliable. Comparison of the results of the study of the stress-strain state with the available experimental results showed that the use of composite material in robotic systems is a promising direction.

Calculations of a robotic system using a bench as an example showed that the mass of the composite structure as compared to the magnesium alloy traditionally used in the production of robotic systems decreases by at least 5-7%, and, accordingly, the inertia characteristics decrease. Thus, the robotic system reacts faster to the signal and improves the positioning accuracy by reducing the inertial forces of the channels of the robotic system.

The calculations of the robotic system on the example of the stand showed that the proposed method of calculating systems containing gearboxes, gears and support bearings, gives good results that reliably reflect the behavior of structures in working conditions. The proposed method of calculating the stress-strain state of robotic structures can be used for structures made of composite and homogeneous materials.

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