Implementation of lesson study with rigorous mathematical thinking based on student worksheet to enhance the students’ mathematical critical thinking

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Abstract. This article is an implementation of lesson study with an RMT-based student worksheet which aims to enhance students' mathematical critical thinking skills. The subjects of this study were 32 students of class VII A in MTsN 1 South Lampung, who represented: (1) different mathematical learning styles; and (2) different self-efficacy. Research begins with a plan, then begins to implement and reflect. The instruments of this study were students' worksheets based on RMT integer and fraction topics, Mathematical Learning Style (MLS) questionnaire, self-efficacy questionnaire, test instrument of critical thinking, and observation sheet. This study aims to improve students' critical thinking skills with the help of Student Worksheets based on RMT in terms of MLS and student self-efficacy. Based on the results of the study, students with Self-Expressive Learning (SL) and Mastery Learning (ML) learning styles experienced a high increase in critical thinking skills with N-Gain scores of 0.73 and 0.71, respectively. While students with Understanding Learning (UL) and Interpersonal Learning (IL) learning styles have increased critical thinking skills in the medium category with N-Gain scores of 0.64 and 0.63. Students with high self-efficacy have increased high critical thinking skills. Students with intermediate self-efficacy experience an increase in mathematical critical thinking skills in the medium category.

1. Introduction
The ability of conceptual mathematics is very important and has a close relationship with other mathematical understanding abilities. A conceptual mathematics ability is an ability needed by students to develop further mathematical concepts. This was shown by [1] that most students could not do the assignments correctly because of the lack of understanding of the concepts. As stated by [2] that the student of good conceptual mathematics ability will make students easy to build relationships to understand new ideas and concepts. The importance of conceptual understanding ability leads students to achieve mathematical critical thinking skills. The mathematical critical thinking skills is a higher-order thinking ability based on the understanding concepts. Critical thinking skills affect student learning outcomes in mathematics [3]. Developing students' critical thinking skills is as important as helping the students to understand the knowledge in different content [4].

Critical thinking is thinking that is involved in solving problems, drawing conclusions, calculating possibilities, and making decisions [5]. Critical thinking is also the ability to analyze and evaluate evidence, identify questions, and build logical conclusions [6-10]. Critical thinking skills are important to develop, as stated by [11] that students in solving mathematical problems can be helped by mathematical critical thinking skills. Moreover, [12] states that by thinking critically, students will be encouraged to think independently and solve problems in school and everyday life. The need for critical mathematical thinking needs to be developed through learning in school [13-15]. Mathematical
critical thinking skills are important to develop so that the quality of learning is better and meaningful [6]. Interviews conducted by [17] in his research showed that students' critical thinking skills were still not optimal.

One of the causes of students' low mathematical critical thinking skills is teacher-oriented learning. This was stated by [16,18] that most learning processes in schools are teacher-oriented learning so that it only focuses on memorization and ignores other mathematical thinking skills that cause students to lack critical thinking. Besides, some teachers find it difficult to embed concepts in the learning process, so students find it difficult to understand concepts, especially for critical thinking [19]. One of the lessons that can improve teaching in various situations and levels is Lesson Study (LS). LS can increase students' cognitive involvement [20]. This is stated by [21], which explains the stages of LS and the factors that influence the improvement of the quality of learning. First, learning must have tangible goals. Second, learning is planned collaboratively by the teacher and observed by other teacher colleagues, documented, analyzed, discussed, and reflected. The LS implementation carried out to understand student learning based on teacher learning practices [22]. LS carried out in three stages, namely, plan, do and see. Before the planning stage, [21] revealed that the importance of an initial investigation in which teachers investigate the characteristics of students and their curriculum content. The planning stage, teachers, and researchers collaborate to plan, design, and goals learning based on student needs. In the do stage, the teacher implements the planned learning model. One team member doing learning in a classroom while other members observe the lesson to collect data [21]. Observations conducted to determine student activities in the form of interactions between students and other students, students with the topics presented in student worksheets, and students and teachers during the learning process. See (reflection) stage, where team members share data collected, analyzed, and discussed to considering the established lesson goals [21].

An important learning instrument in lesson study activities because of focus on how students learn [22]. One way to optimize student understanding is to use Student Worksheet (SW) as expressed by [23] that students can discuss and fill in worksheets well. SW is important in practical activities [24] and supports the learning process [25]. Critical thinking can be trained using SW [26]. As stated by [27-28] in his research, that SW can help improve students' critical thinking skills.

Besides, the appropriate learning management is needed so that students can understand the mathematics concepts. Teacher creativity is needed to make mathematics learning innovation to make the students understand concepts well [22]. Thus, the implementation of LS carried out with the assistance of the SW based on Rigorous Mathematical Thinking (RMT). RMT is a learning approach based on two theories: the theory of psychological tools by Vygotsky and the theory of mediation learning by Feuerstein [29].

Psychological tools can be in the form of symbols, pictures, tables, graphs, or others that have special meaning to bridge students in understanding concepts. The RMT approach emphasizes interaction and mediation between teachers and students, which results in a good understanding of the material presented, conceptualize, and emerge into bound and interrelated ideas [13]. RMT emphasizes the need for the maturity of concepts and prerequisite materials in learning by providing schemes [29].

Based on the description, the implementation of a lesson study assisted by RMT-based SW can be an alternative to enhance students' mathematical critical thinking skills. Through RMT-based, SW expected to help students to understand concepts properly so that it has an impact on increasing students' mathematical critical thinking skills.

2. Method
This study describes the implementation of a lesson study assisted by RMT-based SW. This study aims to enhance students' mathematical critical thinking skills. Subjects in this study consisted of 32 students of class VII A at MTsN 1 South Lampung. Setting subjects in this study is based on MLS and self-efficacy students. The teacher who did the lesson study is a mathematics teacher from MTsN 1 South Lampung, who has been teaching for five years. In this study, interactions between students and RMT-based SW and enhancement in students' mathematical critical thinking skills will be explained.
Students divided into eight groups; one group consists of four students with a Mathematical Learning Style (MLS) and a different level of self-efficacy.

The instruments in this study were observation sheets, MLS questionnaires, self-efficacy questionnaires, RMT-based SW, and tests of mathematical critical thinking skills. The observation sheet used as a preliminary study and needs that were by the problems of VII grade students of MTsN 1 South Lampung. MLS questionnaire to determine students' MLS and aims to group students to be analyzed the enhancement of their mathematical critical thinking skills. RMT-based SW as mediation in mathematics learning to enhance students' mathematical critical thinking skills. Mathematical critical thinking skills test to find out the enhancement of students' mathematical critical thinking skills.

This research was conducted by following the stages of lesson study activities, namely: (1) plan stage: teachers and researchers design learning activities as well as RMT-based SW and critical thinking ability test, (2) do stage: a teacher teaching model and researcher as an observer, (3) stage see: model teacher and researcher together reflect on learning outcomes and analyze student work outcomes.

The data in this study are quantitative, namely, the value of students' mathematical critical thinking skills. The test technique measures the value of students' mathematical critical thinking skills. To see an increase in students' critical thinking skills, an N-gain score used. According to [30], the gain factor can be formulated as follows: $g = \frac{\text{Final Score} - \text{Initial Score}}{\text{Max Score} - \text{Initial Score}}$. The results of N-Gain coverage interpreted using the classification from [31] as in the following table.

| $N$-Gain (g) | Category |
|--------------|----------|
| $g \geq 0.7$ | High     |
| $0.3 \leq g < 0.7$ | Medium |
| $g < 0.3$ | Low      |

Mathematical Learning Style (MLS) in this study is according to [32], which categorized into Mastery Learning (ML), Self-Expressive Learning (SL), Interpersonal Learning (IL), and Understanding Learning (UL). These four learning styles have different characteristics. Students who usually do things independently and often faced with problem-solving (non-routine problems), then they have the potential to have MLS SL. Students who are accustomed to having to collaborate with peers have the potential to become students with MLS IL. Students who are accustomed to waiting for explanations and work procedures step by step until having the potential to have MLS ML. Students who are accustomed to asking and giving reason related to the concepts given and are usually dealing with problem-solving problems have the potential to have MLS UL. The MLS questionnaire consists of 40 statements, each of the ten statements representing the four MLS.

The self-efficacy questionnaire consists of 15 statements and uses a Likert scale with four choices of answers to existing statements, namely: Strongly Agree (SA), Agree (A), Disagree (D), and Strongly Disagree (SD). The instrument reliability calculation uses the Cronbach Alpha-model with a correlation coefficient of 0.864. This shows that the self-efficacy scale has good reliability.

| Answer's Alternative | Score |
|----------------------|-------|
| Strongly Agree (SA)  | 4     |
| Agree (A)            | 3     |
| Disagree (D)         | 2     |
| Strongly Disagree (SD)| 1    |

Table 1. $N$-Gain’s Category (g)

Table 2. Alternative Questionnaire Answer Scores
Student Worksheets (SW) used on an RMT basis. There are two SW, namely SW, with Integers material and SW with Fractional material. The students' mathematical critical thinking skills test consists of problem descriptions consisting of 5 items, including integer and fraction material. The test instruments first tested for reliability, and the validity of the results are presented in Table 3 below:

| Number | Test   | Validity | Reliability | Decision |
|--------|--------|----------|-------------|----------|
| 1      | 0,7863 (High) |         |             | Valid    |
| 2      | 0,5347 (Medium) |      | 0.76524     | Valid    |
| 3      | 0,7521 (High)  |      | High        | Valid    |
| 4      | 0,7334 (High)  |      |             | Valid    |
| 5      | 0,8274 (High)  |      |             | Valid    |

Documentation used to strengthen data obtained from photographs, videos, and notes during the research process.

3. Result and Discussion
Based on the observation of researchers from the results of the questionnaire, students with moderate ability tend to have difficulties in integer and fraction topics and do not memorize multiplication tables 1-10. Based on the results of the questionnaire, although there are many students of moderate and low ability who find it easy on integer and fraction material, in reality, students still often make mistakes. This based on the results of completing the mathematics teacher interview sheet, which states that students still often make mistakes in integer and fraction counting operations.

![Figure 1. Completion of Observation Sheet](image)

Student learning styles determined from the largest score in each statement category. The research sample that will be explained in this discussion is one group consisting of MLS ML, SL, UL, and IL, as well as the diversity of self-efficacy. Based on the results of the questionnaire analysis, the results of the classification of MLS and self-efficacy of class VII A students presented in Table 4 below.

| No. | Student’s Code | Student’s Score | MLS’s Classification | Self-Efficacy’s Score | Self-Efficacy’s Classification |
|-----|----------------|-----------------|----------------------|----------------------|------------------------------|
| 1.  | S-1            | 18 20 8 10      | LS SL                | 82,67                | High                         |
| 2.  | S-2            | 8 8 10 9        | LS UL                | 58,33                | Medium                       |
| 3.  | S-3            | 17 16 12 16     | LS ML                | 78,33                | High                         |
| 4.  | S-4            | 16 20 10 16     | LS IL                | 51,67                | Medium                       |
Cycle I Research Results
The stage of the plan is following activities: (1) Arranging the learning plan on integer and RMT-based SW so the students can understand the concept well; (2) Making a mathematical critical thinking ability test instrument (pre-test and post-test) to see an increase in students' mathematical critical thinking abilities.

The do phase has conducted a pre-test before learning activities. Pre-test results on subjects with self-efficacy are showing a low understanding of their concepts in integer and fraction topics. Based on interviews with S-4, subjects said that the difficulty experienced was doing integer and fraction counting operations, S-4 also did not memorize the multiplication tables 1 to 10. When working on SW in groups, S-1 and S-3 tended to be active in discussions and work on the problems contained in the SW. S-2 and S-4 tend to be quiet and observing. Teachers who saw this happen motivated students, and students are actively involved in working on the worksheet. S-1 and S-3 help S-2 and S-4 in understanding the instructions given to SW. S-3 tends to ask questions actively and pay attention to every teacher's explanation carefully.

Table 5. Student Activities in the Cycle I Learning Process

| Parameter                  | Description                                                                 |
|----------------------------|----------------------------------------------------------------------------|
| Students doing assignments | All groups work on assignments.                                             |
| Collaboration in groups    | At the time of the discussion, some groups had not shown good cooperation. During the discussion, only one or two students were active in the group. |
| Interaction between students | Interaction between students in the discussion went quite well. Students with high self-efficacy appear to help other students in working on student worksheets |
| Student attention in learning | Some students in learning are enthusiastic, and some are not. Some pay attention to the teacher's explanation in answering other students' questions, and some do not. |
| Students ask questions     | Some students ask questions related to what contained in student worksheets. |

As stated in the first cycle, I have not been satisfying in terms of student responses and interactions. The results of student worksheets are also not as expected. Some groups still need improvement in understanding the instructions and questions contained in the student worksheet.

Cycle II Research Results
Based on the results of the reflex cycle I, the plan phase in cycle II is more focus on improvement strategies on aspects: (1) improvement of student worksheets by paying attention to psychological tools to bridge students in understanding the concept of material, (2) teachers are more active mobile, pay attention to the course of group discussions, so each student is actively involved in learning.

The do phase of the second cycle of the material studied is fractions. Students can already use psychological tools to understand the concept of fractions contained in SW. Some groups are also actively involved in discussions and working on questions given in SW. Student activities had increased compared to cycle I. Before the learning activities ended, a post-test carried out with the same questions in the previous pre-test. Before the learning activities ended, a post-test done with the same questions in the last pre-test.

The phase sees the results of reflection on learning activities. The post-test results showed enhancement compared to the results of the previous pre-test. The sample meets the indicators of critical thinking: (1) focus, write information and questions correctly, (2) clarity, write mathematical symbols and answers correctly, (3) conclusions, write the right reasons and conclusions.

The enhancement of students' mathematical critical thinking skills can be seen in the following tables.
Table 6. N-Gain Students' Critical Mathematical Thinking Ability

| Student's Code | Pre-test | Post-test | N-gain |
|----------------|----------|-----------|--------|
| S-1            | 72,5     | 92,5      | 0,73   |
| S-2            | 37,5     | 77,5      | 0,64   |
| S-3            | 65       | 90        | 0,71   |
| S-4            | 45       | 80        | 0,63   |

Based on the value of N-gain, the increase in mathematical critical thinking skills in S-1 and S-3 is in the high category. In contrast, the increase in mathematical critical thinking skills in S-2 and S-4 is in the medium category. The enhancement of students' mathematical critical thinking skills seen from students' MLS presented in the form of the following bar diagram.

![Figure 2. Student’s Mathematical Critical Thinking Ability](image)

According to [32], subject with LS SL can be sensitive to patterns, symmetry, logic, and mathematical aesthetics and solve problems in design and modeling. At the same time, ML can use numbers to compute. The ability of SL to construct ideas and solve problems logically is the ability to think critically [6]. According to [32], ML's ability to absorb information concretely is an indicator of critical thinking, namely, focus [33]. Another indicator of critical thinking in line with ML is clarity [33]. Subjects with LS UL and IL, namely S-2 and S-4, experienced an increase in critical thinking ability in the medium category. UL learns through question and answer, reasoning, and evaluation learning with logical standards and the use of evidence while IL likes the ML focuses on concrete [33]. The enhancement of students' mathematical critical thinking ability in terms of students' self-efficacy, as is shown in the following bar diagram.

![Figure 3. The Relationship Between Self-Efficacy and Students' Mathematical Critical Thinking](image)
The diagram shows that students with high self-efficacy experience an increase in high mathematical critical thinking ability. Students with self-efficacy have increased mathematical critical thinking skills. Based on these results, there is a linear relationship between self-efficacy and students' mathematical thinking abilities [34].

Based on the description of the results and discussion, the implementation of lesson study with RMT-based SW can enhance students' mathematical critical thinking skills. As stated by [10] in his research that learning with rigorous can enhance students' mathematical critical thinking skills. Data analysis of critical thinking skills improvement carried out by [10] based on Initial Mathematical Ability (IMA) and learning models (RMT and contextual learning). The results showed that the average increase in mathematical critical thinking ability through RMT learning was higher than contextual learning. Other research conducted by [13] shows that students' mathematical critical thinking skills through learning with the RMT approach are better than learning with an expository approach. The results of the study average post-test scores on RMT learning were better than expository learning at 27.58> 20.92.

4. Conclusion
Based on the results of the study, it can be concluded that the implementation of LS with SW based on RMT can enhance students' mathematical critical thinking skills. Viewed from the MLSs, students with SL and ML learning styles have increased high critical thinking skills with N-Gain scores, respectively are 0.73 and 0.71. Students with UL and IL learning styles experienced an increase in critical thinking skills in the medium category with N-Gain scores, respectively 0.64 and 0.63. Viewed from the level of students' self-efficacy, students with high self-efficacy have increased high critical thinking skills. At the same time, students with self-efficacy are experiencing an increase in mathematical critical thinking skills in the medium category.

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