Electron subband degeneracy heat pump for cryogenic cooling

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An unconventional method of solid-state cryogenic cooling utilizing the electron subband degeneracy of semiconductor heterostructures is discussed in this Letter. The working principle of using the electrons as the working medium to perform cyclic heat pumping under electrostatic subband “expansion” and “compression” is discussed. The calculated base temperature can reach below dilution refrigeration temperatures with the fundamental limit set by electron-phonon interaction. Using an ultra-wide GaAs quantum well as an example, the cooling power per unit volume is estimated to reach 0.85 μW/cm³ at 25 mK with a hot-side temperature of 42.5 mK, suitable for applications such as quantum computing.

I. INTRODUCTION

Sustainable and efficient millikelvin cryogenic cooling is essential for supporting low-temperature physics experiments as well as cutting-edge applications such as quantum computing. To achieve millikelvin temperatures, one of the most common technologies is the ⁴He/³He dilution refrigerator. However, since ⁴He is a scarce and non-renewable resource, alternative approaches for achieving millikelvin temperatures are greatly desired. Furthermore, electron cooling via dilution refrigeration suffers from the notoriously weak coupling of electrons to cooled phonons at low temperatures, making it difficult to cool the electrons, themselves, to base temperature. A large Kapitza-like thermal resistance develops due to the vanishing electron-phonon coupling at lower temperatures, meaning that the base temperature of the electron system is frequently much higher than that of the nominal dilution refrigerator base temperature.

In this Letter, a novel approach for producing sustained cooling power that utilizes electrons directly as a cooling medium is proposed. Just as a dilution refrigerator expands the phase-space of ⁴He in the ⁴He/³He mixture from a low entropy-per-particle state (the concentrated phase) to a high-entropy-per-particle state (the dilute phase) to achieve cooling, this work proposes a novel solid state heat pump that will expand the entropy of a two-dimensional electron system (2DES) from one to multiple subbands to generate cooling power. As a direct means of cooling electrons, this work could realize record low electron temperatures not achievable in standard cryogenic systems. In this case, the weak coupling of phonons and electrons at extreme low temperature becomes an advantage, as it eliminates one possible source of heat leak from hot phonons to cold electrons. As such, this Letter has the potential to spur new discoveries in fundamental solid state physics. Realizing an active cooling mechanism at the coldest end of the temperature spectrum will improve the efficiency of existing cryogenic setups, increasing hold times and temperature stability by actively compensating heat leaks. The parasitic heat losses for the proposed solid-state refrigerator decrease faster at lower temperatures than the cooling power, so this refrigeration method has no fundamental low-temperature limit. Since coherence times typically increase with decreasing temperature, this may help to extend quantum coherence for quantum information storage.

II. DEVICE DESIGN

The solid state heat pump is based on a semiconductor heterostructure with deposited metal gates on both front and back surfaces. Rego and Kirczenow first theoretically proposed that by biasing the front and back gates, the electron subband degeneracy g of the 2DES can be increased without changing the electron density. As such, the temperature is decreased by the degeneracy factor ratio. Extending beyond this single-shot proposal, here a cyclic heat pump design is proposed analogous to a traditional Otto cycle, whereby instead of using a working medium of molecular gas manipulated by mechanical pistons and thermal switches, here a working medium of electrons in a quantum well is manipulated by electrostatic gates and electron heat-switches. Figure 1 shows a schematic of the proposed device, with the electrons that function as the working medium for refrigeration in the central region (purple) of a quantum well at variable temperature T_M. The front and back gates are biased to voltages V_F and V_B, which are referred to as the piston gates for compression and expansion in this Letter. As such, the piston gates serve to expand or compress the electrons from low subband degeneracy g_1 to high subband degeneracy g_2 and vice-versa. The heat switch gates biased with voltages V_H and V_C provide electrostatic depletion of the underlying electrons, so that electronic conduction of heat is inhibited periodically in the refrigeration cycle. When the heat switch gate on the hot side is negatively biased V_H << 0, the working medium of QW electrons are electrically separated from the heat sink, with analogous function for the heat switch gate biased with V_C which separates the working medium from the cold load. The various heat flows in Fig. 1 are indicated with arrows, indicating the predominant direction of heat flow under proper operation, where the solid black arrows...
III. WORKING PRINCIPLE

The cooling is generated by operating the working medium in an Otto cycle, which is comprised of adiabatic expansion, isochoric heating, adiabatic compression, and isochoric cooling. Figure 2 shows a T-S diagram. The two T(S) curves for degenerate electron systems with different subband degeneracy, g, can be obtained via Sommerfeld expansion and plotted in blue solid lines in Fig. 2.

\[
S = \frac{\pi m^* k_B^2 A_{QW}}{3h^2} gT = K gT,
\]

where \( g, m^*, k_B, A_{QW}, \) and \( T_e \) represent the electron subband degeneracy factor, effective mass, Boltzmann constant, quantum well area, and electron working medium temperature, respectively. A parameter \( K \) is defined, \( K = \pi m^* k_B^2 A_{QW} / (3h^2) \), which is material-dependent only through the electron effective mass \( m^* \) and geometry dependent only through the device area \( A_{QW} \). The four legs of the Otto cycle can be described in the following, with the inset diagrams of Fig. 1 illustrating the heat switch positions for each case. (A) Both heat switches are disconnected. The temperature of the working medium drops from \( T_H \) to \( (g_1/g_2)T_H \) via an adiabatic expansion by biasing the piston gates. (B) The right heat switch shown in Fig. 1 is connected, coupling the working medium to the cold load at \( T_C \), and heat is transferred away from the cold load to the working medium as the working medium undergoes isochoric heating. (C) Adiabatic compression by biasing the piston gates appropriately with both heat switches disconnected, and the temperature of the working medium is increased from \( T_C \) to \( (g_2/g_1)T_C \). (D) Finally, the working medium undergoes isochoric cooling from \( (g_2/g_1)T_C \) to \( T_H \), ejecting heat to the heat sink through the left heat switch in Fig. 1. To have positive cooling power, the following inequality must hold, \( T_C / T_H > g_1 / g_2 \).

In the single-stage Otto cycle calculation below, we determine that the cooling power per expansion cycle when repeated at MHz frequencies can drive enough heat flow to both compensate for back-flow of heat through substrate phonons as well as achieve excess cooling power necessary to operate as a refrigerator, such as to overcome heat leaks from multiple coaxial electrical leads attached to the device targeted for refrigeration. For a given operating frequency \( f \) and heat sink temperature \( T_{HI} \), the cooling power can be calculated from the...
Electrons, on the other hand, are expected to equilibrate rapidly among themselves and therefore have uniform temperature $T_e$ across the width of the quantum well in Eq. (3).

Other heating terms in the problem are negligibly small. The only source of Joule heating $J$ comes from electrons depleting and repopulating the heat-switch gates, which can be made negligible by putting a reservoir gate immediately next to the heat-switch gate whose accumulated charge can compensate for the depleted charge under the heat switch. Similarly, calculations using a 100 μm thinned substrate predict a phonon heat flow $Q_B$ from the hot side to the cold side which is negligible compared to the electron-phonon heat transfer. This can be achieved by thinning the sample via thinning the wafer or using remote epitaxial grown wafers, thus reducing the cross-sectional area for the substrate phonon back-flow. The upper bound for the frequency of operation will be set by the time delay associated with thermal equilibration of the central QW electrons with the cold load or the heat sink. But by adjusting the gate aspect ratio, this upper cutoff can be increased arbitrarily by reducing the piston gate length $L$ while keeping the area $A_{QW}$ constant.

FIG. 3. Self-consistent Hartree calculations of (a) the asymmetric, imbalanced and (b) symmetric, balanced condition for a 150 nm-wide GaAs QW as controlled by front and back piston gates. For the imbalanced QW only the ground state wavefunction at energy $E_0$ is occupied, resulting in a subband degeneracy of $g_1 = 1$. The balanced QW, on the other hand, has both $E_{0,1}$ and $E_{2,3}$ degenerate subbands occupied, resulting in a degeneracy of $g_2 = 4$ with fourfold subband degeneracy.

IV. MATERIAL AND DEVICE PARAMETERS

We calculate the viability of net cryogenic cooling power using GaAs quantum well as an example. A ultra-wide
quantum well of width $d_{QW}$ is chosen to maximize the ratio of degeneracy factors $g_1$ and $g_2$. Figure 3 shows a self-consistent Hartree calculation of quantum well subband energies. When imbalanced with a large positive front-gate and negative back-gate voltage, a sharp triangular quantum well is formed, shown in Fig. 3a. A density of $10^{12}$ cm$^{-2}$ electrons in this GaAs quantum well with an effective mass of $m^* = 0.067m_0$ will populate only the lowest subband, with energy $E_0$ in Fig. 3b. Note that the next nearest subband $E_1$ is above the Fermi energy and therefore unpopulated. When this same density is balanced by compensating front and back piston gate voltages to a "square"-well configuration, the Coulomb repulsion of the electrons results in two symmetric triangular wells at the opposite corners of what would have been the bare square-well potential, Fig. 3b. These electrons together populate four subbands: the ground states $E_0$, $E_1$ and the first excited subband states $E_2$, $E_3$ of the right and left triangular wells, respectively. Thus, the same density of electrons in the wide quantum well can be tuned from $g_1 = 1$ to $g_2 = 4$ occupancy.

V. COOLING POWER

The cooling power of the Otto cycle will now be estimated according to the model above. With a fixed heat sink temperature of $T_H = 42.5$ mK, Fig. 4 shows the calculated total cooling power $P_C$ (black line), the estimated heat loss through electron-phonon coupling $Q_{e-ph}$ (dotted line) and the net cooling power $P_{net} = P_C - Q_{e-ph}$ (blue line) as a function of the cold-side temperature $T_C$ for a subband degeneracy refrigerator operating at a cycling frequency of $f = 108.9$ MHz. Sample dimensions for the simulation are for $A_{QW} = 1$ cm$^2$ area front/back gates with a $t = 100$ μm thick GaAs substrate. The result shows that this simple, single stage cooler can yield 8.5 nW of net cooling power per 1 cm$^2$ of device area. Converting this into cooling power density per unit volume, this translates to 0.9 μW/cm$^3$ at $T_C = 25$mK of volumetric cooling power density. Such volumetric cooling could be achieved, for example, by vertically stacking 100 devices, each 100 μm thick, electrically connected in parallel. The potential for creating compact, in-situ solid-state refrigeration with this method becomes clear.

VI. DISCUSSIONS

Subband degeneracy cooling is scalable and can be enhanced by integrating multiple cooling stages. Our single-stage operation is demonstrated for a heat sink at 42.5 mK, but multiple stages will be able to yield higher hot-side temperatures. The lateral design of the subband degeneracy refrigerator is particularly suited for cascading, allowing for simple, monolithic, planar cascade designs where the heat sink of the former stage is the cold load of the latter stage.

Further performance enhancements can be envisioned with multistage cascades by pairing with other solid state cooling mechanisms such as superconducting-insulating-normal metal junction cooling. Eventually, and all-solid state platform is envisioned which may serve as a substitute for a dilution refrigerator.

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