Noncommutative Open String: Neutral and Charged

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Abstract

We review the quantization of open string in NS-NS background and demonstrate that its endpoint becomes noncommutative. The same approach allows us to determine the noncommutativity that arises for a charged open string in background gauge fields. While NS-NS background is relevant for “worldvolume” noncommutativity, a simple argument suggests that RR background is likely to be relevant for “spacetime” noncommutativity.

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1 Introduction

String theory has experienced remarkable progress in the last couple of years. One of the recent interests is the realization that noncommutative spacetime arises naturally in string and M-theory. The Matrix theory proposal \cite{1} conjecture that M theory can be defined by a supersymmetric quantum mechanics. Upon compactification, Matrix theory is described by a supersymmetric Yang-Mills living on the dual torus \cite{1,2}. The situation is however more complicated when there is a background field. It was proposed by Connes, Douglas and Schwarz \cite{3} that Matrix model compactified on a $T^2$ give rises to noncommutative SYM when there is a background field $C_{-12}$. Since Matrix model can be obtained by discretizing the supermembrane, one approach to obtain the Matrix model with background $C$-field is to discretize the supermembrane theory with a WZ coupling term \cite{4}. It turns out the resulting Matrix model is related to the original Matrix model without background field by a singular similarity transformation; and the Moyal product as well as the Seiberg-Witten map between commutative and noncommutative variables are correctly reproduced upon compactification \cite{4}.

From the string theory point of view, the above $C$-field background of M-theory corresponds to string theory with a NS-NS $B$-field background; and the noncommutativity over the D-brane worldvolume can be shown to arises from the open string point of view \cite{5,6,7,8,9,10,11,12}. Various aspects of noncommutative geometry in string theory were further examined in \cite{12}. See \cite{13,14} for a detail exposition of noncommutative geometry, with motivations and applications in physical problems.

In the following, we will follow the Hamiltonian approach taken in \cite{9} for open string quantization in background $B$-field. This has the advantage of being easily generalizable to the case of a charged open string in background gauge fields.

2 String Theory in Constant NS-NS Background

Consider a fundamental string ending on a Dp-brane in the presence of a $B$-field. The bosonic part of the action takes the form

$$S_B = \frac{1}{4\pi\alpha'} \int_{\Sigma} d^2\sigma [g^{\alpha\beta}G_{\mu\nu}\partial_{\alpha}X^\mu\partial_{\beta}X^\nu + \epsilon^{\alpha\beta}B_{\mu\nu}\partial_{\alpha}X^\mu\partial_{\beta}X^\nu] + \frac{1}{2\pi\alpha'} \int_{\partial\Sigma} d\tau A_i(X)\partial_{\tau}X^i,$$

(1)
where \( A_i, i = 0, 1, \ldots, p \), is the \( U(1) \) gauge field living on the D\( p \)-brane and the string background is \( G_{\mu \nu} = \eta_{\mu \nu}, \Phi = \text{constant}, H = dB = 0 \). We use the convention \( \eta^{\alpha \beta} = \text{diag}(-1, 1) \) and \( \varepsilon^{01} = 1 \) as in [9]. This can be in type 0 superstring, type II superstring, or in the bosonic string theory. If both ends of the string are attached to the same D\( p \)-brane, the last term in (1) can be written as

\[
-\frac{1}{4\pi \alpha'} \int \Sigma d^2 \sigma \epsilon^{\alpha \beta} F_{ij} \partial_\alpha X^i \partial_\beta X^j.
\]

Furthermore, consider the case \( B = \sum_{i,j=0}^p B_{ij} dX^i dX^j \), then the action (1) can be written as

\[
S_B = -\int d\tau L = \frac{1}{4\pi \alpha'} \int d^2 \sigma [g^{\alpha \beta} \eta_{\mu \nu} \partial_\alpha X^\mu \partial_\beta X^\nu + \epsilon^{\alpha \beta} F_{ij} \partial_\alpha X^i \partial_\beta X^j].
\]

Here

\[
F = B - dA = B - F
\]

is the modified Born-Infeld field strength and \( x^a_0 \) is the location of the D-brane. Indices are raised and lowered by \( \eta_{ij} = (-1, 1, \ldots, 1) \).

One obtains the equations of motion

\[
(\partial_\tau^2 - \partial_\sigma^2) X^\mu = 0
\]

and the boundary conditions at \( \sigma = 0, \pi \):

\[
\partial_\sigma X^i + \partial_\tau X^j \mathcal{F}_j^i = 0, \quad i, j = 0, 1, \ldots, p,
\]

\[
X^a = x^a_0, \quad a = p + 1, \ldots, D.
\]

The mode expansion that solve (3) is

\[
X^k = x^k_0 + (p^k_0 \tau - p^k_0 \mathcal{F}^k_0) + \sum_{n \neq 0} \frac{e^{-in\tau}}{n} (ia^k_n \cos n\sigma - a^k_n \mathcal{F}^k_j \sin n\sigma).
\]

This implies that the canonical momentum \( 2\pi \alpha' P^k(\tau, \sigma) = \partial_\tau X^k + \partial_\sigma X^j \mathcal{F}^k_j \) has the expansion

\[
2\pi \alpha' P^k(\tau, \sigma) = \{p^l_0 + \sum_{n \neq 0} a^l_n e^{-in\tau} \cos n\sigma \} M^k_l,
\]

\(^2\) With slight modification, the considerations here can also be applied to study open string ending on a D-brane in type I string theory. There a quantized \( B \)-field appears [15] and natively one expect that a noncommutative gauge theory with \( SO \) or \( SP \) gauge group to appear. A more careful analysis shows however that the resulting gauge theory is equivalent to one without deformation by doing a field redefinition. It remains a challenge to find out how to define noncommutative gauge theory with gauge group other than \( U(N) \) [12] and in what setting of string theory they arise. I am grateful to Bogdan Morariu and Bruno Zumino for carrying out this analysis together.
where $M_{ij} = \eta_{ij} - F^k_i F^j_k$.

The constraint (7) is standard. We will be mainly interested in the constraint (6). As demonstrated in [9], the BC (6) implies that

$$2\pi\alpha' P^k(\tau, 0) F^i_k = -\partial_\sigma X^j(\tau, 0) M^i_j. \quad (10)$$

It follows that

$$2\pi\alpha'[P^k(\tau, 0), P^j(\tau, \sigma')] F^i_k = -\partial_\sigma [X^k(\tau, \sigma), P^j(\tau, \sigma')] F^i_k, \quad (11)$$

$$2\pi\alpha'[P^k(\tau, 0), X^j(\tau, \sigma')] F^i_k = -\partial_\sigma [X^i(\tau, \sigma), X^j(\tau, \sigma')] F^i_k. \quad (12)$$

These simple relations show that the standard canonical commutation relations for $F = 0$,

$$[X^i(\tau, \sigma), P^j(\tau, \sigma')] = i\delta^i_j \delta(\sigma, \sigma'), \quad (13)$$

$$[P^i(\tau, \sigma), P^j(\tau, \sigma')] = 0, \quad (14)$$

$$[X^i(\tau, \sigma), X^j(\tau, \sigma')] = 0, \quad (15)$$

are not compatible with the boundary condition (6) when $F \neq 0$. Since the modified boundary condition (6) occurs only at the boundary, it is clear that the commutation relations (13)-(15) are modified only there.

Following the procedure of [16], one finds that the symplectic form is

$$\Omega = \int_0^\pi d\sigma dP_\mu dX^\mu, \quad (16)$$

because the modifications to (13)-(15) occur on a measure zero set and so do not modify the familiar form of $\Omega$. To determine how the commutation relations are modified, we evaluate (14) for the mode expansions (8) and (9) to get the Poisson structure for the modes. To be consistent, the resulting expression should be $\tau$-independent. Using (5) and (6), it is easy to check that this is indeed the case. Substituting the mode expansions (8), (9), one obtains

$$\Omega = \frac{1}{2\alpha'} \left\{ M_{ij} d\rho^i_0(dx^j_0 + \frac{\pi}{2} F^j_k d\rho^k_0) + \sum_{n>0} \frac{-i}{n} (M_{ij} da^i_n da^{-i}_{-n} + da^a_n da^a_{-n}) \right\}, \quad (17)$$

which is explicitly time independent \footnote{This corrects an irrelevant step in [4].}. Eqn. (17) implies the following commutation relations for the modes

$$[a^i_n, x^0] = [a^i_n, p_0^0] = 0, \quad [a^i_n, a^j_n] = 2\alpha' m M^{-1} \delta_{m+n}, \quad (18)$$

$$[p_0^0, p_0^0] = 0, \quad [x^i_0, p_0^0] = i2\alpha' M^{-1} x^i_0, \quad [x^i_0, x^j_0] = i2\pi\alpha'(M^{-1} F)^{ij}, \quad (19)$$
and in turn implies

\begin{align}
[P^i(\tau, \sigma), P^j(\tau, \sigma')] &= 0, \\
[X^k(\tau, \sigma), X^l(\tau, \sigma')] &= \begin{cases} 
\pm 2\pi i \alpha' (M^{-1} F)^{kl}, & \sigma = \sigma' = 0 \text{ or } \pi, \\
0, & \text{otherwise},
\end{cases} \\
[X^i(\tau, \sigma), P^j(\tau, \sigma')] &= i\eta^{ij} \delta(\sigma, \sigma'),
\end{align}

where \(\delta(\sigma, \sigma')\) is the delta function on \([0, \pi]\) with vanishing derivative at the boundary, \(\delta(\sigma, \sigma') = \frac{1}{\pi} \left(1 + \sum_{n \neq 0} \cos n\sigma \cos n\sigma'\right)\). Thus we see that the string becomes noncommutative at the endpoint, i.e. the D-brane becomes noncommutative. Note that the noncommutativity depends on quantity defined on the D-brane. The relation (21) is manifestly local.

We finally remark that since the ghost system is not sensitive to the presence of \(F\); their boundary condition and hence their central charge is not modified by \(F\). Therefore to be free from conformal anomaly, the matter system must have a central charge independent of \(F\). This is indeed so since the normal-ordered Virasoro generators are

\begin{align}
L_k &= \frac{1}{4\alpha'} \sum_{n \in \mathbb{Z}} \left( M_{ij} a^i_{k-n} a^j_n + a^q_k a^a_n \right)
\end{align}

and they satisfy the standard Virasoro algebra [9]

\begin{align}
[L_m, L_n] &= (m - n)L_{m+n} + \frac{d}{12} m(m^2 - 1) \delta_{m+n}, \quad d = \text{spacetime dimension},
\end{align}

with a central charge unmodified by \(F\).

### 3 Charged String

We now come to the case of of a charged open string in background fields or an open string ending on two different D-branes with different worldvolume field strengths. Most part of the analysis has already appeared in [17]. We will go over some of the salient features. The method we used in sec. 2 can be easily applied here. Consider an open string with charges \(q_1\) and \(q_2\) at the endpoints, the action is \((\alpha' = 1/2)\)

\begin{align}
S &= \frac{1}{2\pi} \int d\tau d\sigma (\dot{X}_\mu X^\mu - X'_\mu X'^\mu) - \frac{1}{\pi} \int d\tau (q_1 A_i \dot{X}^i(\sigma = 0) + q_2 A_i \dot{X}^i(\sigma = \pi))
\end{align}
and the boundary conditions are
\begin{align}
X^i' &= q_1 F^i_j \dot{X}^j, \quad \sigma = 0, \\
X^i' &= -q_2 F^i_j \dot{X}^j, \quad \sigma = \pi.
\end{align}
(26)

We will concentrate on a $2 \times 2$ block of $F$,
\begin{equation}
F = \begin{pmatrix} 0 & f \\ -f & 0 \end{pmatrix}.
\end{equation}
(28)

Introducing $X_\pm = \frac{1}{\sqrt{2}} (X_1 \pm i X_2)$, the boundary conditions are diagonalized
\begin{align}
X_+^i &= -i \alpha \dot{X}_+, \quad \sigma = 0; \quad X_-^i = i \beta \dot{X}_+, \quad \sigma = \pi,
\end{align}
(29)

with $\alpha = q_1 f$, $\beta = q_2 f$. In the gauge $A_i = -\frac{1}{2} F_{ij} X^j$, the conjugated momentum are
\begin{align}
\pi P_- &= \dot{X}_+ - i \frac{1}{2} X_+ [\alpha \delta(\sigma) + \beta \delta(\pi - \sigma)], \\
\pi P_+ &= \dot{X}_- + i \frac{1}{2} X_- [\alpha \delta(\sigma) + \beta \delta(\pi - \sigma)].
\end{align}
(30)

The same argument as in the previous section shows that the standard canonical commutation relations have to be modified at the boundary due to the boundary conditions.

One can expands $X_\pm$ as
\begin{align}
X_+ &= x_+ + i \sum_{n > 0} a_n \psi_n - i \sum_{m \geq 0} b_m^\dagger \bar{\psi}_{-m}, \\
X_- &= x_- + i \sum_{m \geq 0} b_m \bar{\psi}_{-m} - i \sum_{n > 0} a_n^\dagger \bar{\psi}_n,
\end{align}
(32)

where we have taken into account $X_+^{\dagger} = X_+$ and $x_+ = x_-^{\dagger}$ and the normalized mode functions for any integer $n$ is given by
\begin{equation}
\psi_n = \frac{1}{|n - \epsilon|^{1/2}} \cos[(n - \epsilon)\sigma + \gamma] e^{-i(n - \epsilon)\tau},
\end{equation}
(33)

with $\epsilon = \frac{1}{\pi} (\gamma + \gamma')$ and $\gamma = \tan^{-1} \alpha$, $\gamma' = \tan^{-1} \alpha'$. $\psi_n$ and the constant mode form a complete basis. $\psi_n$'s satisfies the boundary condition (29) and the orthogonality condition
\begin{equation}
\langle \psi_m, \psi_n \rangle = \delta_{mn} \text{sign}(m - \epsilon), \quad \langle 1, \psi_n \rangle = 0,
\end{equation}
(34)

where the inner product is defined by
\begin{equation}
\langle f, g \rangle = \frac{1}{\pi} \int_0^\pi d\sigma \bar{f}(\tau, \sigma) [i \frac{\partial}{\partial \tau} + \alpha \delta(\sigma) + \beta \delta(\pi - \sigma)] g(\tau, \sigma).
\end{equation}
(35)
The symplectic form is given by

$$\Omega = \int d\sigma \left( dP_+ dX_+ + dP_- dX_- \right)$$

and it is straightforward to show that it is time independent. In terms of $\langle , \rangle$, $\Omega$ can be written compactly as

$$\Omega = i \langle dX_+, dX_+ \rangle.$$  (37)

Using (34), it is then easy to get

$$-i \Omega = dx_- dx_+ \frac{\alpha + \beta}{\pi} + \sum_{n>0} da_n^\dagger da_n + \sum_{m\geq 0} db_m^\dagger db_m.$$  (38)

This implies the nonvanishing commutation relations

$$[x_+, x_-] = \frac{\pi}{\alpha + \beta},$$

$$[a_k, a_n^\dagger] = \delta_{nk}, \quad k, n > 0,$$  (40)

$$[b_l, b_m^\dagger] = \delta_{lm}, \quad l, m \geq 0,$$  (41)

with all the other commutators zero. These relations are exactly those obtained in [17]. However, as we will see now, the commutation relation for $[X_+(\tau, \sigma), X_-(\tau, \sigma')]$ are different. Substituting (39)-(41) back into (32), one finds

$$[X_+(\tau, \sigma), X_-(\tau, \sigma')] = J(\sigma, \sigma'),$$  (42)

where

$$J(\sigma, \sigma') = \frac{\pi}{\alpha + \beta} + \sum_{n=-\infty}^{\infty} \frac{1}{n - \epsilon} \cos((n - \epsilon)\sigma + \gamma) \cos((n - \epsilon)\sigma' + \gamma).$$  (43)

We first compute $J(\sigma, \sigma')$ for $\sigma = \sigma' = 0$ or $\pi$. Using the identity

$$\sum_{n=1}^{\infty} \frac{2\epsilon}{\epsilon^2 - n^2} + \frac{1}{\epsilon} = \pi \cot \pi \epsilon, \quad \epsilon \neq \text{integer},$$  (44)

one obtains $J(0, 0) = \pi \alpha/(1 + \alpha^2)$, $J(\pi, \pi) = \pi \beta/(1 + \beta^2)$. As for $J(\sigma, \sigma')$ for other values of $\sigma, \sigma'$, it is not hard to show that it is zero. To see this, we first remark that it is straightforward to show that $\frac{\partial}{\partial \sigma} J = 0$ for $\sigma, \sigma'$ not both 0 or $\pi$. Therefore $J$ is a constant for this range of $\sigma, \sigma'$ and hence it is sufficient to calculate $J(0, \pi)$. The latter is equal to

$$J(0, \pi) = \frac{\pi}{\alpha + \beta} + \cos \gamma \cos \gamma' \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{n - \epsilon}.$$  (45)
Now it is known that for any meromorphic function $h(z)$ with poles $a_1, \cdots, a_m$ (not integers) and with $z = \infty$ a zero of order $p \geq 2$, the following holds

$$\lim_{N \to \infty} \sum_{n = -N}^{N} (-1)^n h(n) = -\pi \sum_{k=1}^{m} \text{Res} \left( \frac{h(z)}{\sin \pi z}, a_k \right).$$

(46)

Applying this for $h = 1/(z^2 - \epsilon^2)$, we get $J(0, \pi) = 0$. Therefore we obtain finally

$$[X_+(\tau, \sigma), X_-(\tau, \sigma')] = \begin{cases} \frac{\pi \alpha}{1+\sigma^2}, & \sigma = \sigma' = 0, \\ \frac{\pi \beta}{1+\beta^2}, & \sigma = \sigma' = \pi, \\ 0, & \text{otherwise}, \end{cases}$$

(47)

The geometrical meaning is clear: the noncommutativity is localized at the endpoints and is determined by the field strength there. If we consider an open string ending on two different D-branes, the same results (47) are obtained; and they agree with the commutation relations (21) obtained by quantizing individual open string with both ends ended on each same D-brane. This is consistent with the fact that the noncommutativity is a property of the D-brane and does not depend on what probe we use to see the noncommutativity.

The results (47) can also be obtained by carrying out the Dirac constrained quantization with the boundary conditions treated as constraints [8, 10]. The procedure was carried out at the level of fields in [11]. We mention that one may also express the boundary conditions as constraints on the modes and carry out the constrained quantization [19].

One may try to recover these results using the approach of worldsheet perturbation [11], but it is difficult to proceed since now the perturbation due to the charges cannot be written as a worldsheet action unless one is willing to use delta function. It is also difficult to proceed in the line of [12] as the Green function that satisfies different BC at the two end points is not available. The advantage of the present approach is apparent, both the neutral and charged string can be treated uniformly.

Two interesting applications of the charged string system are the studies of creation of open string in electric field and D-brane scattering [20]. Notice that in these calculations, only the relations (39)-(41) are used, but not (47).
4 Outlooks

The kind of noncommutativity that appears in string theory so far are on the worldvolume of brane and can be called “worldvolume” noncommutativity. Physically this kind of noncommutativity arises from the open string interaction and has nothing to do with gravity. This is to be contrasted with another kind of noncommutativity that is due to quantum gravity effects at small distance scale. We refer to this as “spacetime” noncommutativity. It is often believed that at the Planck scale, spacetime will become fuzzy since the quantum fluctuation of the geometry cannot be ignored anymore. Since string theory provide a consistent treatment of quantum gravity, it would be very interesting to understand this better from within string theory. Consider a D-string with a NS-NS flux along it. Doing a S-duality turns the NS-NS flux into a RR flux and the original noncommutative D-string into a fundamental string with a noncommutative worldsheet [21]. It seems RR background is likely to be relevant for “spacetime” noncommutativity. String theory in RR background is notoriously difficult, see [22, 23] however for some proposals.

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