Scaling properties of the magnetic-field-induced specific heat of superconducting UBe$_{13}$

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We report on measurements of the low-temperature specific heat $C_{\mu}(T,B)$ of the unconventional superconductor UBe$_{13}$ in magnetic fields up to 7 T. At $B \approx 2$ T, a substantial change in the magnetic-field dependence of the temperature derivative of the magnetic-field induced contribution to the electronic specific heat $C_{\mu}(T,B)$, resulting from the flow of supercurrents around the vortices, is observed, suggesting a crossover between two different regions in the superconducting phase diagram of UBe$_{13}$. For fields $B > 2$ T, $C_{\mu}(T,B)$ exhibits a scaling behavior with respect to $TB^{-1/2}$, which provides substantial evidence for the existence of point nodes in the quasiparticle excitation spectrum of the superconductor.

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Ever since the discovery of the heavy-electron superconductor UBe$_{13}$, it has been proposed that the superconducting state is an axial spin-triplet state. For small gap values, i.e., at and close to the $T_c$ temperature, the behavior of $C_{\mu}(T,B)$ of a superconductor with gap-nodes strongly depends on the topology of the gap nodes. In the limit $k_B T \ll \Delta$ and $B \ll B_c$, the DOES at zero energy in the case of point nodes, is given by $N(0)/N_F \sim B/B_c \ln(B_c/B)$ for arbitrary direction of the magnetic field, or $N(0)/N_F \sim B/B_c^{1/2}$, if the field is exactly parallel to the orientation of the nodes, respectively. For line nodes, $N(0)/N_F \sim (B/B_c)^{1/2}$, independent of the direction of the magnetic field. Here $N_F$ is the DOS at the Fermi energy $E_F$. This leads, in the limit of $T \approx T_c$, to an additional linear-in-$T$ term to $C_{\mu}$ of the form

$$\frac{C_{\mu}(T,B)}{T} \sim \frac{N(0)}{N_F}. \quad (1)$$

A more general way to analyze the magnetic-field induced specific heat has recently been discussed by Volovik and by Simon and Lee. They showed that the magnetic-field induced contribution to the specific heat $C_{\mu}(T,B)$ obeys a scaling behavior, with respect to the scaling parameter $x \sim (T/T_c) \sqrt{B_c/B}$, of the form $C_{\mu}(T,B) \sim B^{-1/2} f(x)$. This scaling may be rewritten as

$$\frac{C_{\mu}(T,B)}{T^{2-D} B^{1/2}} \sim F(x), \quad (2)$$

where $D$ denotes the dimension of the nodes, i.e., $D = 0$ for point nodes and $D = 1$ for line nodes. $f(x)$ is a universal scaling function. According to Eq. (1), its asymptotic behavior for $x \approx 1$ is of the form $f(x) \sim x^{-D-1}$. We note that in the case of point nodes an additional term due to the logarithmic term in the DOES, which would invalidate the scaling relation, enters Eq. (2). Nevertheless, it can be shown that in materials with cubic symmetry, this term may well be small enough to not significantly affect the scaling relation given in Eq. (2).

In order to test these scaling predictions in connection with an unconventional and strongly type-II heavy-electron superconductor, the specific heat $C_{\mu}(T,B)$ of a small piece of polycrystalline UBe$_{13}$ ($\approx 50$ mg) has been measured in...
various temperature ranges between 0.08 and 0.38 K and in external magnetic fields between 0 and 7 T, using a thermal-relaxation technique in a dilution refrigerator. Special care was taken for the calibration of the RuO$_2$ thermometer attached to the sapphire disk serving as the sample platform in external magnetic fields. For this purpose, a calibrated temperature sensor was mounted inside a multilayered superconducting magnetic shield, which was located outside of the core of the magnet solenoid, but was thermally shortcut to the calorimeter. The specific heat of the sample platform alone has been measured in a separate experiment and was only a few ppm of the total measured specific heat at all temperatures and all fields. The absolute accuracy of the calorimeter is better than 3% at the lowest temperatures and further improves with increasing temperature.

This experimental setup allows for $C_p$-measurement scans either parallel to $B$ in the $T-B$ phase diagram by varying the temperature in a fixed magnetic field or vice versa, thus providing a good sensitivity to thermodynamic features in any direction of the phase diagram.

The sample has been cut from a piece of material used in a previous investigation of the specific heat. The specific heat in zero field and above 0.07 K is well described by $C_p = \gamma_0 T + \beta_c T^3$. Earlier measurements of the specific heat of UBe$_{13}$ had indicated that $C_p$ is sample dependent such that the apparent linear term $\gamma_0 T$ arises from resonant scattering at impurities, imperfections, etc. The fit parameters $\gamma_0$ and $\beta_c$ of the zero-field data presented here are consistent with those obtained in earlier measurements.

In Fig. 1, we show the low-temperature specific heat as measured for 0.08≤$T$≤0.20 K and 0≤$B$≤7 T, as (a) a function of temperature at constant magnetic fields and (b) at constant temperatures vs field. The slight upturn at low $T$ and high $B$ is due to the nuclear Zeeman contribution of the Be atoms.

For field scans at constant $T$ [Fig. 1(b)], $C_p(B)$ shows a broad shoulderlike feature centered around $B=3$ T. In a recent study of the specific heat of UBe$_{13}$ in magnetic fields at somewhat higher temperatures, similar features of $C_p(T,B)$ at $B=2$ T have been reported and were interpreted as an indication for the occurrence of a second phase in superconducting UBe$_{13}$. In Ref. 20, the results of thermal expansion and specific heat measurements have been combined to show the existence of an additional feature in the phase diagram of UBe$_{13}$. Below, we find further evidence for the existence of an additional feature in the phase diagram of UBe$_{13}$, which might be related to the shoulderlike feature, but which appears at slightly different $B$.

In this work, we are mainly interested in the magnetic-field induced electronic contribution to the specific heat, $C_H(T,B)$. The Be nuclei carry a nuclear spin, causing a magnetic field dependent and, especially at high fields and low temperatures, non-negligible contribution to the specific heat, $C_N(T,B)$, which is proportional to $B^2 T^{-2}$ and is discussed in great detail in Ref. 21. Aiming only at the magnetic-field induced electronic contribution, we consider

$$C_H(T,B) = C_p(T,B) - C_p(T,0) - C_N(T,B),$$

where $C_p(T,B)$ is the measured specific heat, $C_N(T,B)$ the evaluated nuclear contribution and $C_p(T,0)$ the measured specific heat in zero field. This procedure for obtaining the magnetic-field induced electronic contribution, $C_H(T,B)$, to be discussed below does not depend on any fit parameters or unknown background contributions and, therefore, leads to reliable results of $C_H(T,B)$.

According to Eq. (1), in the limit of $T\ll T_c$ and $B\ll B_{c2}$ the magnetic-field induced electronic specific heat should vary linearly with $T$. Thus, we have plotted $C_H(T,B)/T$ vs $T$ measured at constant fields in Fig. 2. The discrepancy between the theoretical predictions and the experimental data is obvious. The model leading to Eq. (1) is expected to be valid only in the limit where $k_B T \ll \Delta E_D$, i.e., $x \equiv T/T_c \sqrt{(B_{c2}/B)} < 1$. Setting the magnetic field to $B = B_{c2}/5$, which is already at the upper limit of the valid regime, the condition $x \approx \frac{1}{2}$ can only be fulfilled if $T/T_c \ll 0.09$. The critical temperature of the present material is $T_c = 0.91$ K. Thus, for the model to be applicable, the temperature has to be below 0.08 K, the lower limit of the temperature range covered in these experiments. This upper limit of the temperature decreases further with decreasing magnetic field and therefore, in this model’s context and using these data no conclusions concerning the topology of possible gap nodes can be drawn.

FIG. 1. Representative data of the as-measured total specific heat of UBe$_{13}$ (a) vs $T$ in constant magnetic fields and (b) vs magnetic field at constant temperatures.

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A more general approach to analyze the topology of possible gap nodes which is not only valid in the limit $\delta t \ll 1$ but for all $\delta t$, is provided by the universal scaling relation given in Eq. (2). According to Eq. (2), $C_H(T,B)/T^2 - DB^{1/2}$ should be proportional to a universal function of the parameter $x \sim T/T_c \sqrt{(B_{c2}/B)}$. Although this scaling relation has theoretically only been demonstrated to be valid for $T \ll T_c$ and $B \ll B_{c2}$, we have plotted our entire $C_H(T,B)$ data set versus $y = T/\sqrt{B}$, setting $D = 0$ and $D = 1$, respectively. The result for $D = 0$ is shown in Fig. 3. While the open symbols represent the magnetic field scans at constant $T$, the full symbols correspond to the temperature scans at constant $B$. We may separate the investigated field regime into a low-field part ($B < B^* \approx 2$ T) and a high-field part. Inspecting Fig. 3 it turns out that all the data collapse onto a single common curve, except for those obtained at $B < B^*$, where the scaling behavior according to Eq. (2) is not obeyed at all. We note that in Fig. 3 also data for fields up to $B \approx B_{c2}/2$ are shown, but even for the highest fields applied here, the data reveal the scaling behavior. This scaling with respect to the scaling variable $T/\sqrt{B}$ is rather unique and implies the occurrence of nodes in the superconducting energy gap. Since this scaling is appropriate for $D = 0$, we conclude that for $B > B^*$ the energy-gap function of superconducting UBe$_{13}$ exhibits point nodes. Hence, we assume that the scaling prediction of Eq. (2) is more robust than previously expected and holds up to $B < B_{c2}/2$. In the region of the $B - T$ phase diagram of superconducting UBe$_{13}$ where $B < B^*$ and at the lowest temperatures, we cannot make any claims about the topology of possible gap nodes. We note, however, that the broad shoulderlike feature reported in Fig. 1(b) is most probably related to this distinct change in the scaling behavior as may be seen in Fig. 3. In the investigated temperature range, some type of crossover from $B < B^*$ to $B > B^*$ at $B^* \approx 2$ T is also apparent, if we inspect the temperature derivative of $C_H(T,B)$ and display it in a plot of $\partial C_H/\partial T|_{T=\text{const}}$ vs $B$ (Fig. 4). For $B < B^*$, the data are rather well described by a power law as indicated by the solid line in Fig. 4, whereas for $B > B^*$, the behavior is distinctly different. At present, the origin of this crossover at 2 T remains unclear.

As a test, the same procedure, but setting $D = 1$, has been applied to both sets of data. No evidence for a scaling behavior in the whole investigated magnetic field and temperature range has been obtained, as may be seen in the inset of Fig. 3.

In the mixed state of a superconductor, a contribution to the specific heat might also arise from the low lying excitations localized near the core of the vortex. This problem has been discussed for $s$-wave superconductors by Caroli et al. 22.
and by Bardeen et al.\textsuperscript{23} According to that work, the lowest excitation level is expected at $E_0 \approx \Delta^2/E_F$, where $\Delta$ denotes the energy gap far away from the vortex. It is well established\textsuperscript{4,24} and has recently been confirmed\textsuperscript{25} that UBe$_{13}$ is a strong coupling superconductor, which implies that the energy-gap amplitude is substantially larger than the BCS weak-coupling value of $\Delta \approx 1.76 k_BT_c$. The Fermi-energy of UBe$_{13}$, given by $E_F/k_B$, is of the order of 10 K.\textsuperscript{4} Using these values and the predictions for $s$-wave superconductors, we find the lowest level of the low-energy excitations in the vortex core to be at significantly higher energies than the typical thermal energies $k_BT$ of this experiment. Furthermore, the observed scaling behavior with respect to $TB^{-1/2}$ is, as discussed above, rather unique, and cannot be associated with the mentioned vortex-core contribution.\textsuperscript{16}

In conclusion we note, that for magnetic fields $B > B^*$, the magnetic-field induced contribution to the specific heat exhibits a scaling behavior with respect to the scaling parameter $T/\sqrt{B}$ which, according to Eq. (2) for $D = 0$, suggests the existence of pointlike nodes in the superconducting energy gap of UBe$_{13}$. No scaling could be established for the superconducting state of UBe$_{13}$ at low temperatures and $B < B^*$. A crossover behavior at $B = B^*$ is also observed in the $\partial C_H/\partial T$ vs $B$ data. The same data set gives no similarly convincing evidence for the existence of nodes with $D > 0$, in any regime of the $B-T$ phase diagram.

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