Improving the Electric Response of a Cantilever Piezoelectric Energy Harvester by Constraining Tip Curvature

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Abstract. The paper studies the influence of tip curvature on the electrical response of a piezoelectric energy harvester, on a direction other than the charge generation one. Finite element analyses were conducted for the given piezoelectric transducer, which show an increase in the voltage output if blocking the tip curvature on yOz plane between two bars, leaving free only the deflection on xOz plane. The comparison was made between cantilever tip constrained with bars and cantilever with an equivalent proof mass respectively. The simulations in COMSOL Multiphysics are furthermore validated by experimental laboratory tests, confirming the increase of the electric output potential with ~8%. This occurs due to inducing higher stress into the structure by blocking its tip curvature. The frequency response, defined between base excitation and the output voltage, was determined using an impedance analyser. The bars also act as a proof mass, thus decreasing the resonant frequency optimally down to the vibration frequencies of an oil-free compressor’s downstream pipe on which the transducer will be mounted in order to harness the inherent vibrations and subsequently power wireless sensors.

1. Introduction
Piezoelectric energy harvesting from vibrations [1, 2] has emerged at the beginning of this century and is being thoroughly researched since, due to the highly efficient mechanic-to-electric energy conversion of piezoelectric materials. Even though advances in materials science enable a continuous improvement of the performances of energy conversion devices, most researches focus on ways of boosting the electric output without modifying the given material [3, 4]. The ultimate purpose is to harness enough energy to power wireless sensors or other low-power electronics, thus rendering these devices autonomous.

Piezoelectric harvesters and sensors use the direct piezoelectric effect. When subjected to mechanical stress, they produce electric charge, giving an alternating current response when vibrating. They have high sensitivity with vibrations frequency, generating peak voltage at resonance. Hence, for resonating
structures, the natural frequency has to be adjusted so as to match the fundamental frequency of the vibrations source [5].

Previous researches [6] have shown that for piezoelectric cantilevers with higher width to height ratios, a bending of the free tip subjected to vibrations occurs in yOz plane, besides the bending generating charge in xOz plane. It has been shown through numerical simulations in COMSOL Multiphysics, for several theoretic cases, that constraining this bending improves the electric response for cantilever structures with larger height to width ratios.

The present work is conducted on a physical piezoelectric harvester with a considerable width to height ratio, aiming to prove both through numerical simulations and experimentally that the discovered effect of constraining the free tip curvature in transversal plane has the desired effect of boosting the electric output. No literature references have been found about this effect, hence the novelty contribution of our research.

2. Piezoelectric harvester proposed

The piezoelectric harvester Midé PPA-4011 [7] studied is a quadmorph cantilever, consisting of 17 thin layers, out of which 4 piezoceramic wafers. Its overall dimensions are 71.0 mm x 25.4 mm x 1.32 mm (L x W x H), as shown in the CAD model in Figure 1a. Therefore, the width to height ratio is as high as 19.24, enough to exhibit the curvature effect occurring in vertical plane.

The piezoelectric layers, of thickness 0.15 mm (Figure 1b), are sandwiched between copper electrodes (0.03 mm thick), for conducting the electric charge to the terminals. Each of the four packs (copper, piezo, copper) is separated by FR4 (Flame Retardant Type 4 glass-reinforced epoxy laminate material) protection and insulation layers (0.08 mm thick). The PZT-5H (soft lead zirconate titanate) wafers are smaller in length and width than the other layers (46 mm x 20.8 mm relative to 65 mm x 25.4 mm). All the wafers are glued together by very thin epoxy resin layers (<0.02 mm). For simulation purposes, these epoxy layers can be neglected. The total mass of the beam is 7.6 g.

![Figure 1. Piezoelectric beam dimensions: (a) isometric view; (b) tip corner magnification](image)

Piezoelectric beams have a certain force-voltage relation. When the beam is bent by external stresses (mechanical, magnetic, electrical, etc.), a voltage is induced across the terminals, depending on the displacement produced. If the stress ceases its action upon the piezoelectric beam and this one becomes static in any position, no electric potential is generated furthermore, since there is no electrical dipole moment induced in the crystalline structure of the piezoelectric material. The polarization direction also plays an important role in the efficiency of the conversion into electric energy. The force should act in the polarization direction of the electrical dipoles [8].

The piezoelectric harvester operates in d31 mode, meaning that when stress is applied on x axis, charge is induced on z direction. Because a piezoelectric ceramic is anisotropic, physical constants relate to both the direction of the applied mechanical or electric force and the directions perpendicular to the
applied force. In this case, the two subscripts indicate the stress (force on the surface area of the element) and strain (change in element’s length) for elasticity. The direction of positive polarization usually coincides with z axis of the Cartesian system (Figure 2a) [9]. The piezoelectric beam is attached to a support in its rear clamping position, leaving the maximum length subjected to vibrations and therefore inducing less mechanical stresses in the beam and lengthening the lifetime of the device (Figure 2b).

![Figure 2. (a) Forces directions on a piezoelectric element [9]; (b) Piezo beam on support [10]](image)

The piezoelectric charge constant, \(d\), is the polarization generated per unit of mechanical stress (\(T\)) applied to the piezoelectric material in the case of direct piezoelectric effect (sensors and energy harvesters). For inverse piezoelectric effect (actuators), it is the mechanical strain (\(S\)) occurring in the piezoelectric material per unit of electric field applied. The first subscript indicates the polarization direction generated in the material when the electric field, \(E\), is zero or respectively the direction of the applied field strength. The second subscript stands for the direction of the applied stress or the induced strain, respectively.

Natural frequency (or eigenfrequency) is the frequency at which a system tends to oscillate in the absence of any driving or damping force. Forced vibrations occur when an oscillating object is excited by an external force or load to vibrate at a particular frequency. When the frequency of the external force is the same with that of the natural vibrations, resonance occurs [11]. Resonant frequencies induce large vibration amplitudes in the structure even if force inputs are low. That is why, usually, resonances ought to be avoided at all costs. However, when speaking about energy harvesters, resonant structures like piezoelectric cantilevers must be excited very close to resonant frequency in order to generate peak electric power due to the high tip deflections developed.

Natural frequencies depend on structure’s stiffness and total mass. As such, mechanical frequency can be changed by modifying one or more variables in the following equation characterising a cantilever with rectangular cross-section, constrained at one end.

\[
f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{3EI}{L^3m}}
\]

where: \(f_n\) [Hz] – natural frequency; \(k\) [N/m] or [kg/s\(^2\)] – stiffness constant; \(m\) [kg] – beam total mass; \(E\) [N/m\(^2\)] – Young’s modulus (equivalent one for composite structures); \(I\) [kg \cdot m\(^2\)] – moment of inertia of beam’s cross-section; \(L\) [m] – beam length.

Materials properties [7] are presented in Table 1 hereinafter. The piezoceramic material has no tensile yield strength or plastic deformation limit as it is not an elastic material, thus breaking directly when subjected to stresses over the ultimate tensile yield strength value. The transducer however should not be stressed over the lowest tensile strength supported by its materials, which is copper’s 33.3 MPa.
### Table 1. Transducer materials properties

| Parameter                                      | Symbol | Measuring unit | PZT-5H | Copper | FR4 | Structural steel |
|------------------------------------------------|--------|----------------|--------|--------|-----|-----------------|
| Piezoelectric charge (displacement) constants  | $d_{31}$ | [C/N·10^{-12}] | -320   | –      | –   | –               |
|                                                 | $d_{33}$ | [C/N·10^{-12}] | 650    | –      | –   | –               |
| Piezoelectric voltage constants                 | $g_{31}$ | [V·m/N·10^{-3}] | -9.5   | –      | –   | –               |
|                                                 | $g_{33}$ | [V·m/N·10^{-3}] | 19     | –      | –   | –               |
| Young’s modulus                                 | $E$    | [GPa]          | 63     | 110    | 26  | 200             |
| Relative permittivity                           | $\varepsilon$ | 1          | 3800   | 1      | 4.5 | 1               |
| Poisson’s ratio                                  | $\nu$  |                | 0.31   | 0.343  | 0.172 | 0.30 |
| Tensile yield strength                          | $TS$   | [MPa]          | –      | 33.3   | 340 | 250             |
| Ultimate tensile yield strength                 | $UTS$  | [MPa]          | 60     | 210    | 368 | 400             |
| Density                                         | $\rho$ | [kg/m$^3$]     | 7800   | 8930   | 1900 | 7850            |
| Mechanical quality factor                       | $Q$    |                | 30     | –      | –   | –               |

In a previous research [10] we calculated, for the given piezoelectric harvester, Midé PPA-4011, the cross-sectional stiffness $K = 0.223 \text{ kg·m/s}^2$. The equivalent density of the beam is $\rho_{eqv} = 4469 \text{ kg/m}^3$ and the efficient length subjected to vibrations in the rear clamping position on support is $L_{eff} = 54 \text{ mm}$ (see Figure 2b). The theoretical fundamental frequency is calculated, using the formula in [12], as:

$$f_n = \frac{1}{2\pi \sqrt{\frac{K}{\rho_{eqv} A L_{eff}^4}}} = 241.68 \text{ Hz}$$

(2)

3. Finite element analysis in COMSOL Multiphysics

The FEM (Finite Element Method) simulation was realised in COMSOL Multiphysics 5.3a, in Structural Mechanics module – Piezoelectric Devices (Figure 3). The microscale behaviour is evaluated using two interconnected physics:

- **Solid Mechanics**: Materials are assigned and initial and boundary conditions are set. A base acceleration of 1g (9.81 m/s$^2$) was prescribed on the inferior side of the bottom clamping bar. PZT-5H layers are declared as piezoelectric.
- **Electrostatics**: Electric charge conservation is set for the piezoelectric layers. Terminal and ground are also declared on the faces of the piezoelectric wafers. Zero charge was considered for the terminals, supposing infinite impedance.

The physics are coupled through Multiphysics – Piezoelectric effect, allowing simultaneous numerical solving of all the equations from solid mechanics, electrostatics and the constitutive equations of piezoelectric material.

An Eigenfrequency study was conducted, which is a static study, since the effect of constraining the tip was observed in static conditions in previous research [6]. The aim is to prove that a difference does exist between constrained tip and concentrated tip mass. The experimental tests performed in dynamic conditions may show a more significant difference.

Typically, for harvesters operating in air, the external damping excitation is negligible comparing to the inertial excitation term. It was shown in a dimensionless basis that, in the absence of a tip mass, the...
amount of modal forcing due to external damping term is less than 5% of the total modal base excitation force if the component of modal damping ratio due to external damping is less than 2.5% [2].

Figure 3. Piezoelectric FEM modelling in COMSOL Multiphysics

For free tip conditions, the Eigenfrequency study renders a fundamental frequency of 242.78 Hz and a total displacement of 0.16 mm (Figure 4a) with a generated maximum voltage of 3.44 V, as one can observe from Figure 4b below. The frequency is very close to the analytical one. Both subsequent cases with concentrated mass and constraining tip bars are simulated with the same initial and boundary conditions, with the same mesh with tetrahedral elements of default normal size.

Figure 4. Cantilever with free tip (a) Displacement and (b) Electric potential

In order to match the experimental conditions, the equivalent added masses and constraining bars modelled in COMSOL Multiphysics were designed so as to be $m = 4.0$ g. Considering structural steel
of density $\rho = 7850 \text{ kg/m}^3$, for the first case with concentrated mass, a cylinder of radius $r = 4 \text{ mm}$ was considered, its height being calculated according to relation (3).

$$V = \frac{m}{\rho} \Rightarrow \pi r^2 h = \frac{m}{\rho} \Rightarrow h = \frac{m}{\rho \pi r^2} \Rightarrow h = 10.14 \text{ mm}$$  \hspace{1cm} (3)

The total displacement obtained at the fundamental frequency of 146.07 Hz is 0.15 mm (Figure 5a), giving an electric potential of 3.41 V (Figure 5b).

For the case of constrained tip between bars, considering the same structural steel material assigned, of density $\rho = 7850 \text{ kg/m}^3$, and two bars of length 25.4 mm and width 7 mm, the height of one bar was calculated according to Equation (4) below.

$$V = \frac{m}{\rho} \Rightarrow 2(L \cdot W \cdot H) = \frac{m}{\rho} \Rightarrow H = \frac{m}{2 \cdot \rho L W} \Rightarrow H = 1.43 \text{ mm}$$  \hspace{1cm} (4)

The total displacement obtained at the fundamental frequency of 146.65 Hz is 0.14 mm (Figure 6a), rendering an electric potential of 3.44 V (Figure 6b).
The frequency also increases slightly when constraining the tip, due to adding thickness to the beam (Equation (1)). Hence, the cross-section inertia momentum increases since the cross-section subjected to vibrations is larger. The electric potential is expected to increase more in dynamic tests conditions. A more noticeable frequency increase is also expected.

The static results are not trustworthy though during the phenomena occurring in dynamic conditions of vibrations, but can show if there is any difference between the two configurations. Nevertheless, it is true that the displacement would be a little smaller when constraining the curvature, because otherwise the corners deflect more than the area around the median line. The electric potential also increases slightly comparing to concentrated mass, as higher strain is induced in the piezoelectric material when blocking its motion on transversal plane.

From quasi-static point of view, the above simulations show small differences in the electric potential generated between the case when free tip without seismic mass and the case with additional tip mass but with unobstructed deformations (free curvature in transversal plane) and the case with additional tip mass in the form of blocking elements of the curvature in transversal plane. Larger von Mises stresses are induced in the beam by blocking tip curvature, thus explaining the higher electric potential comparing to concentrated mass. Being a static evaluation, it does not consider oscillations over a certain period of time. The case with concentrated mass is also the case recommended by the manufacturer for improving the electric response.

The quasi-static studies are not reliable, being recommended to use the transducer very close to its resonant frequency. The simulations realized in the vicinity of resonance have indicated that the best case is that of additional mass in the form of bars constraining the tip curvature in transversal plane, superior to the case with concentrated mass.

The piezoelectric cantilever without any proof mass whatsoever would render the weakest voltage response in real dynamic conditions. An additional tip mass increases tip deflection, inducing higher deformations in the piezoelectric crystalline structure, and implicitly increasing the generated voltage.

4. Experimental tests
For the validation of the effect of blocking tip curvature discovered through FEM simulations, experimental tests have also been conducted, which are presented hereinafter.

The experimental setup in Figure 7 consists of:

- Dynamic FFT (Fast Fourier Transform) signal analyser with embedded functions generator (Stanford SR785);
- Power amplifier with sine output power of 120 VA (RMS value), high signal-to-noise ratio (SNR > 90 dB), and signal amplification in direct current up to 20 kHz (TIRA BAA 120);
- Shaker table (TIRA S 513) driven by the signal analyser via power amplifier and connected to the output channel of the analyser;
- Piezoelectric energy harvesting system, with terminal output connected to input channel 2 of the analyser;
- Accelerometer (Brüel&Kjær type 4507-B-006), mounted on shaker table and used as reference input signal, connected to input channel 1 of the analyser. Being an ICP (Integrated Circuit Piezoelectric) accelerometer, with built-in preamplifier, the ICP type power supply was activated for this channel.
4.1. Free tip cantilever configuration
The first set of tests was conducted considering no additional proof mass (Figure 8a), in no load conditions, leaving the tip free to vibrations. A swept-sine signal was generated from the analyser, passing through the power amplifier in order to excite the shaker table. The frequency domain analysed was established so as to include the resonance frequency, between 150 Hz and 250 Hz. An accelerometer is used for measuring the base vibrations in real time and avoiding the errors which could arise from relying solely on the correspondence between the power signal and generated vibrations. The voltage response was displayed on signal analyser and can be seen in Figure 8b.

Figure 7. Experimental setup

Figure 8. (a) Piezoelectric transducer in no load conditions and (b) Signal analyser voltage response
The voltage output peak obtained without seismic mass was $V_0 = 334.5 \text{mV/(m/s}^2\text{)}$, or 3.28 V/g, at the resonant frequency of ~214 Hz. It is noteworthy to mention that the piezoelectric element is very sensitive to clamping conditions, fastening elasticity on the mobile platform of the vibrations generator, and various other factors which can affect the electric response. The support was attached to the mobile platform with a double-sided strong adhesive tape. Considering this, we mounted the seismic masses configurations hereinafter keeping the system attached on the shaker table, trying to maintain the test conditions unchanged in order to have comparable results.

4.2. Concentrated seismic mass

The total mass of the elements used was 4.0 g, weighted with a precision of 0.1 g. The mass was established so as to match the total mass of the constraining elements in the subsequent section. The testing configuration on shaker table is shown in Figure 9a.

![Figure 9. (a) Harvester configuration with concentrated tip mass; (b) Signal analyser response screen](image)

Adding tip mass increases vibrations amplitude, hence boosting the electric response due to higher deformations, which induce higher stress in the piezoelectric material. The electric potential was more than double when adding the 4 g concentrated mass, a voltage peak of $V_1 = 776.1 \text{mV/(m/s}^2\text{)}$ or 7.611 V/g being generated (Figure 9b above). The increase relative to free tip configuration was calculated as:

$$V_{0 \rightarrow 1}[\%] = \frac{(V_1 - V_0)}{V_0} \cdot 100 \approx 132\%$$

(5)

In the same time, according to Equation (1), a decrease in the natural frequency occurs as well, down to ~108.38 Hz.

4.3. Constrained tip between bars

The interesting case was assessing the electric output while constraining the tip curvature in transversal vertical plane. For this, we blocked the tip between two clamping bars, as shown in Figure 10a. The total mass of the elements used was 4.0 g, weighted carefully with a precision of 0.1 g in order to be the same mass as for concentrated unobstructed curvature.

The maximum voltage output recorded during this set of measurements was $V_2 = 836.7 \text{mV/(m/s}^2\text{)}$ or ~8.205 V/g, at 110.58 Hz, as shown in Figure 10b. As expected, blocking tip curvature on transversal plane increases the electric potential response.
The transducer behaviour has a more effective energy conversion by blocking its buckling on transversal plane that hinders obtaining maximum electric response with the piezoelectric beam. The response increase in terms of voltage peak from the concentrated mass between $V_2 = 836.7 \text{ mV/(m/s}^2\text{)}$ and concentrated proof mass case $V_1 = 776.1 \text{ mV/(m/s}^2\text{)}$ is:

$$V_{1\rightarrow2}[\%] = \frac{(V_2 - V_1)}{V_1} \cdot 100 \equiv 7.8\%$$

(6)

The increase percent from the free tip case is thus of $\sim 150\%$.

The frequency also increases with about 2 Hz when constraining the tip, even though the elements used have the same value. This occurs due to adding thickness to the beam. Hence, the cross-section inertia momentum increases as the tip cross-section subjected to vibrations is larger (equation (1)). As expected, the differences are much more significant in dynamic real conditions than in the static Eigenfrequency study in COMSOL Multiphysics.

5. Conclusions
The paper presents a unique way of boosting the performances of a piezoelectric energy harvester regarding its electric potential output. Thus, the previous theoretic research was validated and the findings were confirmed experimentally.

We showed that as the height to width ratio is higher, inducing a more significant tip buckling in vertical transversal plane, the more significant the influence of constraining the tip curvature is upon the electric output. Adding a proof mass increases vibrations amplitude anyway and hence the output voltage. Therefore, in order to have relevant experimental results, the comparison had to be made between cantilever tip constrained with bars and cantilever with an equivalent concentrated proof mass, of 4.0 g.

The free transducer, in no load conditions, gives a maximum output voltage of 3.28 V/g at a resonant frequency of 214 Hz. By adding the concentrated proof mass, the increase is with 132%, up to 7.611 V/g. Implicitly, the resonant frequency decreases to 108.38 Hz. By constraining the tip curvature in yOz plane, 8.205 V/g were generated at 110.58 Hz.

Thus, the peak voltage at resonance increased with 7.8% comparing to additional concentrated tip mass of the same value, with a total increase from the free tip beam with 150%.
The Eigenfrequency study in COMSOL Multiphysics showed that differences do exist between the three cases analysed, which determined a further investigation through experimental tests. These ones showed quite important improvements in the electric response. An almost 8% performance boost may not be remarkable for other systems, but for a piezoelectric crystal it is quite a significant electric output boost obtained without modifying the material.

6. Appendix
In order to have a clearer view on the results obtained, we include the following summarizing table.

Table A2. Summarizing results

| Configuration                  | No load   | Concentrated proof mass | Constrained tip |
|-------------------------------|-----------|-------------------------|-----------------|
| FEM simulation Eigenfrequency | 242.78 Hz | 146.07 Hz               | 146.65 Hz       |
| FEM simulation Eigenfrequency voltage | 3.44 V/g | 3.41 V/g               | 3.44 V/g       |
| Experimental resonant frequency | 214.03 Hz | 108.38 Hz               | 110.58 Hz       |
| Experimental voltage response | 3.280 V/g | 7.611 V/g              | 8.205 V/g      |

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