Fluctuation Spectrum from a Scalar-Tensor Bimetric Gravity Theory

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Abstract

Predictions of the CMB spectrum from a bimetric gravity theory (BGT) [1] are presented. The initial inflationary period in BGT is driven by a vanishingly small speed of gravitational waves $v_g$ in the very early universe. This initial inflationary period is insensitive to the choice of scalar field potential and initial values of the scalar field. After this initial period of inflation, $v_g$ will increase rapidly and the effects of a potential will become important. We show that a quadratic potential introduced into BGT yields an approximately flat spectrum with inflation parameters: $n_s = 0.98$, $n_t = -0.027$, $\alpha_s = -3.2 \times 10^{-4}$ and $\alpha_t = -5.0 \times 10^{-4}$, with $r \geq 0.014$.

1 Introduction

Recently we have introduced a cosmological model [2, 1], based on a scalar-tensor bimetric gravitational theory (BGT) in which the relative propagation velocities of gravitational and matter disturbances is determined dynamically. In the resulting diffeomorphism invariant theory there is a prior-geometric relationship between two metrics that involves the gradient of a scalar field. Similar models based on a vector field have been published [3, 4]. Due to its role in producing a theory with multiple light cones (or birefringent spacetimes), the scalar field is referred to as the “biscalar” field. Depending on the choice of frame, spacetimes may either be viewed as having a fixed speed of light and a dynamically-determined speed of gravitational disturbances, or as having a fixed speed of gravitational waves and a dynamical speed of light [2, 6, 7].

The BGT cosmological model was shown to possess solutions that had sufficient inflation to solve the horizon and flatness problems, and in order to bring it in line with current observations on the CMB radiation, we now demonstrate that it is capable of generating an approximately

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scale invariant primordial scalar perturbation spectrum. To accomplish this we will adopt the
standard scenario from slow-roll inflation models, assuming an adiabatic quantum vacuum for the
perturbative scalar and tensor modes. To accomplish this and at the same time retain the usual
picture of (pre-)heating following inflation [6], we will introduce a mass for the biscalar field.

While this would seem to be a small deviation from slow-roll inflation, because the speed of
gravitational wave propagation is less than the speed of light during inflation, the physics of the
tensor modes is altered. In particular, there is no simple consistency relation between the scalar
mode spectral index and the ratio of tensor and scalar spectra. At the same time, the biscalar field
potential is strongly suppressed during an earlier period of inflation, much as if a coupling constant
“ran” to zero in the very early universe. So, unlike standard slow-roll inflation scenarios, it would
be difficult to argue that any particular value for the biscalar field (or the biscalar field potential
energy) in the very early universe is “unnatural”.

Following this initial period of inflation, the biscalar field potential will become important, and
as the biscalar field approaches the bottom of the potential, the universe is capable of going through
another period of inflation, resembling that of chaotic inflation models [7, 8]. It is during this period
that we expect that the primordial seeds of structure formation are generated from the quantum
vacuum of the biscalar field \( \phi \), the results of which we describe in this paper.

To begin with, we will briefly review the structure of the BGT model in Section 2. Then, we
shall examine the very early universe in Section 3 and determine the conditions under which the
model possesses inflating solutions with a constant speed gravitational wave propagation \( v_g < c \).
In Section 4, we determine the inflation parameters for a model in which vacuum fluctuations of
the biscalar field \( \phi \) produce the primordial seeds of the CMB spectrum, and constrain the model
parameters to give the observed density profile \( \delta \rho/\rho \sim 10^{-5} \). We end in Section 5 with concluding
remarks.

### 2 The model

The model that we introduced in [3, 4] consisted in a minimally-coupled, self-gravitating biscalar
field \( \phi \) coupled to matter through what we refer to as the “matter metric”:

\[
\dot{g}_{\mu\nu} = g_{\mu\nu} + B \partial_\mu \phi \partial_\nu \phi. \tag{1}
\]

The action takes the form

\[
S = S_{\text{grav}} + S_{\phi} + \hat{S}_{\text{M}}, \tag{2}
\]

where

\[
S_{\text{grav}} = -\frac{1}{\kappa} \int d\mu (R[g] + 2\Lambda), \tag{3}
\]

\( \kappa = 16\pi G/c^4 \), \( \Lambda \) is the cosmological constant, and we employ a metric with signature \((+,-,-,-)\).
We will write, for example, \( d\mu = d^4x \sqrt{-g} \) and \( \mu = \sqrt{-g} \) for the metric density related to the
gravitational metric \( g_{\mu\nu} \), and similar definitions of \( d\hat{\mu} \) and \( \hat{\mu} \) in terms of the matter metric \( \hat{g}_{\mu\nu} \). A
useful result for two metrics related by (3) is that the determinants are related by:

\[
\mu = \sqrt{K \hat{\mu}}, \tag{4}
\]

where

\[
K = 1 - B \hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi. \tag{5}
\]
The minimally-coupled biscalar field action is given by
\[ S_\phi = \frac{1}{\kappa} \int d\mu \left[ \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right], \] (6)
where the scalar field \( \phi \) has been chosen to be dimensionless. The gravitational field equations can be written as
\[ G^{\mu\nu} = \Lambda g^{\mu\nu} + \frac{\kappa}{2} \left( T^{\mu\nu}_\phi + \frac{1}{\sqrt{K}} \mathring{T}^{\mu\nu} \right), \] (7)
where \( \mathring{T}^{\mu\nu} \) is the matter energy-momentum tensor and
\[ T^{\mu\nu}_\phi = \frac{1}{\kappa} \left[ g^{\mu\alpha} g^{\nu\beta} \partial_\alpha \phi \partial_\beta \phi - \frac{1}{2} g^{\mu\nu} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + g^{\mu\nu} V(\phi) \right]. \] (8)
The field equations for the scalar field, written here in terms of matter covariant derivatives \( \mathring{\nabla}_\mu \), are given by
\[ \mathring{g}^{\mu\nu} \mathring{\nabla}_\mu \mathring{\nabla}_\nu \phi + KV'[\phi] = 0. \] (9)
In the latter, we have defined the biscalar field metric \( \mathring{g}^{\mu\nu} \) by
\[ \mathring{g}^{\mu\nu} = \hat{g}^{\mu\nu} + \frac{B}{K} \mathring{\nabla}_\mu \phi \mathring{\nabla}_\nu \phi - \kappa B \sqrt{K} \mathring{T}^{\mu\nu}. \] (10)
The matter model is constructed solely using the metric (1), resulting in the identification of \( \hat{g}_{\mu\nu} \) as the metric that provides the arena on which matter fields interact. Therefore the conservation of energy-momentum takes on the form:
\[ \mathring{\nabla}_\nu \left( \mathring{\mu} \mathring{T}^{\mu\nu} \right) = 0, \] (11)
as a consequence of the matter field equations only [1]. It is the matter covariant derivative \( \mathring{\nabla}_\mu \) that appears here, which is the metric compatible covariant derivative determined by the matter metric: \( \mathring{\nabla}_\alpha \hat{g}_{\mu\nu} = 0 \). In this work, we will assume a perfect fluid form for the matter fields
\[ \mathring{T}^{\alpha\beta} = \left( \rho + \frac{p}{c^2} \right) \hat{u}^\alpha \hat{u}^\beta - p \mathring{\hat{g}}^{\alpha\beta}, \] (12)
with \( \mathring{\hat{g}}_{\mu\nu} \hat{u}^\mu \hat{u}^\nu = c^2 \).

3 Cosmology
We will now specialize to a cosmological setting, imposing homogeneity and isotropy on spacetime and writing the matter metric in comoving form as
\[ \mathring{g}_{\mu\nu} = \text{diag}(c^2, -R^2(t)\gamma_{ij}), \] (13)
with coordinates \((t, x^i)\) and spatially flat \((k = 0)\) 3-metric \( \gamma_{ij} \) on the spatial slices of constant time and \( c \) is the constant speed of light. In the following, we shall restrict our attention to the variable speed of gravitational waves frame [3]. The matter stress-energy tensor (using \( \hat{u}^0 = c \)) is given by
\[ \mathring{T}^{00} = \rho, \quad \mathring{T}^{ij} = \frac{p}{R^2} \gamma^{ij}. \] (14)
This leads to the conservation laws (an overdot indicates a derivative with respect to the time variable \( t \), and \( H = \dot{R}/R \) is the Hubble function):

\[
\dot{\rho} + 3H \left( \rho + \frac{p}{c^2} \right) = 0.
\]

(15)

It is useful at this point to introduce the following quantities derived from the constant \( B \) which will appear throughout this work:

\[
H_B^2 = \frac{c^2}{12B}, \quad \rho_B = \frac{1}{2\kappa c^2 B}.
\]

(16)

The latter is obtained by requiring that \( H_B^2 = \frac{1}{6\kappa c^4 \rho_B} \), and we have defined the dimensionless parameter:

\[
\xi_B^2 = \frac{\ell_P^2}{B},
\]

(17)

where the Planck length is defined by \( \ell_P = \sqrt{\hbar G/c^3} \). In [1], we described the parameters required to generate sufficient inflation from a finite time after the “quantum gravity epoch”. Just after this, spacetime contained an approximately constant energy density \( \rho_B \) defined by (16). This motivated the identification \( \rho_B \approx \rho_P = c^5/(\hbar G^2) \), which would correspond to \( \xi_B^2 = 32\pi \).

From (5) we have

\[
K = 1 - \frac{\dot{\rho}^2}{12H_B^2},
\]

(18)

and using the relation (1), the gravitational metric is found to be

\[
g_{\mu\nu} = \text{diag}(v_g^2, -R^2\gamma_{ij}),
\]

(19)

where \( v_g = cK^{1/2} \leq c \) is the gravitational wave speed. From the definition (10) we find

\[
\bar{g}^{\mu\nu} = \text{diag} \left[ \frac{1}{v_g^2} \left( 1 - K^{3/2} \frac{\rho}{2\rho_B} \right), -\frac{1}{R^2\gamma_{ij}} \left( 1 + \sqrt{K} \frac{\rho}{6\rho_B} \right) \right].
\]

(20)

The Friedmann equation \( (R_{00} + K \gamma^{ij} R_{ij}) \) is given by

\[
H^2 + \frac{k c^2 K}{R^2} = \frac{1}{3} K c^2 \Lambda + H_B^2(1 - K) + 2H_B^2K\rho_B + \frac{1}{6\kappa c^4 K^{3/2} \rho}.
\]

(21)

In [2, 1] we showed that very soon after an initial period of inflation during which \( \sqrt{K} \rho \approx 2\rho_B \), we will have \( \rho \ll \rho_B \) and so we will have \( \bar{g}_{\mu\nu} \approx g_{\mu\nu} \). In this limit, the biscalar field equation (9) can be written in terms of \( K \) as

\[
\dot{K} - 6K(1 - K)H - 2K^2 \dot{V}_B = 0,
\]

(22)

where we have introduced the dimensionless potential: \( V_B = BV \) and \( \dot{V}_B = V_B' \dot{\phi} \). From here it is clear that while \( K \approx 0 \) the effect of the biscalar field potential is suppressed, during which time we have a second period of inflation as described previously [1].

If we want to understand the quantum origins of the CMB spectrum and at the same time realize (pre-)heating as the decay of oscillations of the biscalar field about a minimum of its potential, then we must add a nontrivial potential to the model. Rolling towards the minimum of this potential will
cause inflation similar to that appearing in chaotic inflation scenarios, during which time modes of interest are being generated. The biscal field $\phi$ therefore provides both a mechanism to produce inflation as the speed of gravitational waves $v_g \to 0$, and in addition the fluctuations of $\phi$ on this inflating background produces the CMB spectrum. This is the scenario developed in this work.

An alternative scheme would be to separate inflation from the production of the CMB spectrum. Assuming that inflation is produced as described in [1], we could introduce a second scalar field $\psi$ in the matter model, with quantum fluctuations of $\psi$ generating the primordial quantum fluctuations that determine the CMB spectrum after decoupling. Since $v_g \to 0$ during inflation, quantum effects of the biscal and gravitational field would be suppressed, allowing these scalar matter modes to dominate. This mechanism would be independent of a biscal field potential and does not depend on the initial values of $\phi$, and so we anticipate that the difference between the initial BGT inflation and the standard slow-roll inflation mechanism can remove the severe fine tuning problem of the latter models, caused by an unnaturally small coupling constant $g \sim 10^{-13} - 10^{-14}$ required to fit the observed density profile, $\delta \rho/\rho \sim 10^{-5}$. We leave the details of such a model to a later publication.

3.1 The Very Early Universe

We have previously demonstrated that if the potential $V(\phi)$ is not important in the very early universe, then the field equations require that an infinite amount of inflation takes place as $R \to 0$ [1]. In these solutions we found that $R \sim \exp(H_B t)$ (which is the origin of the definitions (16)), and $K \sim R^6$ or $K \sim R^8$. In either case, since $K \to 0$ implies that $\phi \to \text{const}$ as $R \to 0$, the fact that the biscal field potential enters into (21) and (22) multiplied by a power of $K$ implies that its contribution to both will be vanishingly small. Clearly this does not rule out other possibilities (for example, that $V \gtrsim 1/R^{12}$ as $R \to 0$), but it is a good indication that such behavior is likely in the very early universe. The result that the onset of the initial inflationary period is essentially independent of the potential $V_B$, demonstrates an important difference of BGT from the standard inflationary models [6], based on a slow-roll approximation for the inflaton potential.

At the same time, the matter contribution to the Friedman equation is scaled by a factor of $K^{3/2}$ with $K^{3/2} \rho \approx \text{constant}$ during this period. In this way the effect of matter “saturates”, and increasingly large mass densities do not cause increasingly large curvatures; an intriguing result. Given this, it is likely that this model will have something to say about primordial black hole production. Another intriguing aspect of the vanishingly small speed of gravitational waves $v_g$ is that the exchange of gravitons between two gravitationally interacting bodies is suppressed in the very early universe, so that gravitating particles become effectively asymptotically free. This could have important consequences for quantum gravity.

We can imagine a scenario in which a more fundamental quantum theory singles out an initial state with $K \approx 0$, so that the biscal field $\phi$ is initially rolling at a fixed rate $\dot{\phi} = \sqrt{12} H_B$ but of no particular value, for effectively there is no potential energy associated with it. Different regions of the universe undergo the same type of early inflation, regardless of the value of the biscal field, but as $K$ (and therefore $v_g$) becomes appreciable, the universe is separated into different regions. In regions where the biscal field $\phi$ emerges close to the bottom of the potential, no additional inflation occurs and presumably no (pre-)heating occurs, and these regions remain largely empty of matter (other than the radiation that is initially present in our model). In regions where the biscal field emerges farther up the potential, early inflation may continue for a while before a period of slow-roll-type inflation takes over, following which the universe is (pre-)heated.

In the second period of inflation, the attractor solutions are not necessarily slow-roll (although for models which generate sufficient inflation, they effectively are during the time when modes are
generated and leave the horizon), rather they are constant-roll, i.e., \( \dot{\phi} \approx \) constant. For the biscalar field this has additional meaning: constant \( \dot{\phi} \) implies that \( K \approx K_c \) is constant, and therefore from (13) and (21) the speed at which gravitational waves travel is \( v_g < c \). Therefore, while the scalar mode spectrum alone is unlikely to be distinguishable from some particular slow-roll model of inflation, the tensor modes have a different propagation velocity and new physics is expected.

3.2 Solutions with Fixed Light Cones

We are looking for solutions that have a constant \( K = K_c \) which implies that \( v_g < c \) is constant. From (22) this implies that

\[
3(1 - K_c)H + K_c \dot{V}_B = 0. \tag{23}
\]

Using (18) to write this in terms of \( K \) only (assuming that \( \dot{\phi} > 0 \)) and (21) to replace \( H \), this can be written as (assuming that \( \Lambda = 0 \)):

\[
\frac{1 - K_c}{2K_c} + V_B = \frac{2K_c}{3(1 - K_c)} (V_B')^2, \tag{24a}
\]

which must hold at the beginning of the constant \( K = K_c \) solution which we define to begin at \( \phi = \phi_c \) and \( R = R_c \). Taking the time derivative of (24a), and defining the dimensionless mass \( m_c \) of the biscalar field near these solutions, we find

\[
V_B'' = \frac{3(1 - K_c)}{4K_c} = m_c^2. \tag{24b}
\]

Further time derivatives will require the vanishing of higher order derivatives of the biscalar potential. The two conditions (24) imply a quadratic potential of the form:

\[
V_B = m_c^2 \left[ \frac{1}{2} (\phi - \phi_c)^2 - 2 \sqrt{2} \tilde{N}_c (\phi - \phi_c) + 4 \tilde{N}_c \right], \quad \tilde{N}_c = N_c + \frac{1}{6}, \tag{25}
\]

where \( N_c \) is a parameter that will turn out to determine the number of e-folds of inflation that will take place if the biscalar field begins at \( \phi = \phi_c \) in this potential, and \( \tilde{N}_c \) is introduced for convenience.

The condition (23) determines the potential as a function of the scale factor

\[
V_B = 4m^2_c \left[ N_c - \ln \left( \frac{R}{R_c} \right) \right], \tag{26}
\]

where we have used the definition (24b) as well as \( V_B(\phi_c) = 4m^2_c N_c \), which is consistent with the result from (25). Using this, the Friedmann equation becomes

\[
H^2 = 6H^2_B (1 - K_c) \left[ \tilde{N}_c - \ln \left( \frac{R}{R_c} \right) \right], \tag{27}
\]

and the Hubble function at \( t_c \) is given by \( H^2_c = 6H^2_B (1 - K_c) \tilde{N}_c \). Integrating this, we find the solutions

\[
R = R_c \exp \left[ -\frac{3}{2} H^2_B (1 - K_c) (t - t_c - 2 \tilde{N}_c / H_c)^2 + \tilde{N}_c \right], \tag{28}
\]

where an arbitrary sign has been chosen so that \( R \) is increasing at \( R_c \).

This solution is Gaussian in time, and so it will reach a maximum in \( R \), following which it begins to contract. This cannot be correct though, since at some point this means that the potential has
to become negative (recall that \( H > 0 \) for \( V_B > 0 \) and for \( R \) to decrease then \( H = 0 \) somewhere). If we define \( R_f \) to be the point at which \( V_B = 0 \), then from the second form of (26) we see that

\[
R_f = R_c e^{N_c},
\]

which is the motivation for introducing the constant \( N_c \) into the potential. Although we shall not do so here, it is a straightforward matter to show that the solutions (23) are attractors, with nearby solutions converging by a factor \( \exp(-3N_c) \) over the period of inflation.

Evaluating the Hubble function at the same time gives

\[
H_f^2 = (1 - K_c)H_B^2, \tag{30}
\]

using which, we can obtain

\[
\frac{R_c^2 H_c^2}{R_f^2 H_f^2} = \frac{1}{6N_c} e^{2N_c}. \tag{31}
\]

Noting that the condition for sufficient inflation to solve the horizon and flatness problems is (assuming instantaneous (pre-)heating and standard physics, following inflation [10]):

\[
\frac{R_c^2 H_c^2}{R_f^2 H_f^2} = \frac{1}{6N_c} e^{2N_c} 10^{-60} z_{de} \left( \frac{T_P}{T_f} \right)^2 \gg 1, \tag{32}
\]

where we have assumed that following \( R_f \) the universe matches on to the standard big bang. In this, \( z_{de} = 4.3 \times 10^4 \Omega h^2 \) is the redshift at decoupling, \( 0.4 \leq h = H_0/100 \leq 1 \), \( T_P = \sqrt{\hbar c^5/G/k_{Bolz}} \) is the Planck temperature, \( (k_{Bolz} \text{ is the Boltzmann constant}) \), and \( T_f \) is the temperature at the end of inflation, where typically \( T_f/T_P \in (10^{-5}, 1) \). This gives the usual result that \( N_c \gtrsim 60 \) yields an adequate inflation factor to solve the horizon and flatness problems [10]. However, we note that since this period is preceded by an earlier epoch of inflation, this is not the critical issue. What is more important is that there be enough inflation for modes to be created in an adiabatic vacuum state, which is the case if this is satisfied.

If we assume that (pre-)heating happens instantaneously, then using (30) the energy density in the biscalar field \( \phi \) is transferred completely into the radiation field and we have

\[
\frac{1}{6} \kappa e^4 K_c^2 \rho_f \approx (1 - K_c)H_B^2. \tag{33}
\]

Relating this to the temperature at the end of inflation, we obtain

\[
\left( \frac{T_P}{T_f} \right)^2 = \frac{\rho_P}{\rho_f} \approx 4\sqrt{2\pi} \frac{K_c^3}{\xi_B^{1/2} - K_c}, \tag{34}
\]

which determines the end of inflation given \( K_c \) and \( B \). We observe that a larger value of \( K_c \) tends to push the end of inflation to lower temperatures, whereas if \( K_c \) is small enough then (pre-)heating will “boost” the radiation density suddenly, and, depending on the parameters, the physics of (pre-)heating could be drastically altered due to the presence of \( \rho \) in \( g_{\mu\nu} \) in (21).

Using \( R_f \) and the solution for \( R \), we can determine that during this period of inflation the biscalar field rolls a distance

\[
\Delta \phi = 2\sqrt{2N_c} \left( 1 - \frac{1}{\sqrt{6N_c}} \right), \tag{35}
\]
and during this time it is rolling linearly in time
\[ \phi = \phi_c + 2H_B \sqrt{3(1 - K_c)}(t - t_c). \] (36)

The scale factor can be written explicitly in terms of the biscalar field
\[ R = R_c \exp \left[ -\frac{1}{8}(\phi - \phi_c)^2 + \sqrt{\frac{N_c}{2}}(\phi - \phi_c) \right]. \] (37)

Evaluating the potential explicitly in terms of the biscalar field results in the potential (25). Note though that there is no dependence on \( m_c \) in (37), so all solutions with a given number \( N_c \) of e-folds of inflation will traverse the same path in the \( R - \phi \) plane, but will do so at potentially different rates, determined by \( m_c \).

4 Inflation Parameters

Using the solution obtained in the previous section, we can now determine the inflation parameters. In the usual scenario, large scale structure is linked to primordial quantum fluctuation of the inflaton field. Following this, we consider quantum fluctuations of the biscalar field \( \phi \), and determine the resulting spectrum. The analysis of linearized perturbations of the field equations (7) and (9) is straightforward but rather involved, and we will merely quote the required results, relegating most of the details to a future publication.

The mechanism is centred on the standard form of a quantum scalar field [11]:
\[ \delta \hat{\phi}(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^2} \left[ a_k \delta \hat{\phi} e^{i\vec{k} \cdot \vec{x}} + a_k^\dagger \delta \hat{\phi}^* e^{-i\vec{k} \cdot \vec{x}} \right], \] (38a)

where the ladder operators satisfy: \([a_k, a_{k'}] = 0 = [a_k^\dagger, a_{k'}^\dagger]\), and \([a_k, a_k^\dagger] = \delta^3(\vec{k} - \vec{k'})\). We require that these quantum field operators satisfy the equal time commutation relations (note that here the scalar field is dimensionless):
\[ [\delta \hat{\phi}(t, \vec{x}), \delta \hat{\phi}(t, \vec{x}')] = 0 = [\dot{\delta} \hat{\phi}(t, \vec{x}), \dot{\delta} \hat{\phi}(t, \vec{x}']], \] (38b)
\[ [\delta \hat{\phi}(t, \vec{x}), \dot{\delta} \hat{\phi}(t, \vec{x}')] = \frac{i\hbar c^2 K \sqrt{K}}{R^3} \delta^3(\vec{x} - \vec{x}'), \] (38c)

where the additional factor of \( K \) results from the form of the metric \( g_{\mu\nu} \).

The mode functions of a free quantum field in an approximately flat spacetime (expanded in a small time about some \( R = R_i \)) that satisfies our conditions would have the form:
\[ \delta \hat{\phi} = \sqrt{\frac{\hbar c^2 K \sqrt{K}}{2\omega_k R_i^3}} \left( C_+ e^{-i\pi/4} e^{i\omega_k t} + C_- e^{i\pi/4} e^{-i\omega_k t} \right), \] (39a)

where
\[ |C_-|^2 - |C_+|^2 = 1, \quad \rightarrow \quad C_- = 1, \quad C_+ = 0, \] (39b)

with the latter being the conventional adiabatic vacuum.

The second ingredient is the matched WKB approximation of the solution of an equation of the form [12]:
\[ \partial_t^2 \Sigma + U \Sigma = 0, \] (40a)
where there is a simple turning point at \( t = t_k \) defined by \( U(t_k) = 0 \), and \( U > 0 \) for \( t < t_k \), and \( U < 0 \) for \( t > t_k \). In this case, the matched WKB approximation is given by

\[
\tilde{\Sigma}_i = |U|^{-\frac{1}{4}} \left( \tilde{\Sigma}^+_i e^{i\theta_i} + \tilde{\Sigma}^-_i e^{-i\theta_i} \right), \quad t < t_k, \quad \text{(40b)}
\]

\[
\tilde{\Sigma}_o = |U|^{-\frac{1}{4}} \left( \tilde{\Sigma}^+_o e^{\theta_o} + \tilde{\Sigma}^-_o e^{-\theta_o} \right), \quad t_k < t, \quad \text{(40c)}
\]

with

\[
\theta_i = -\int_{t}^{t_k} dt \sqrt{U}, \quad \theta_o = \int_{t_k}^{t} dt \sqrt{U}. \quad \text{(40d)}
\]

The coefficients are related by

\[
\tilde{\Sigma}^+_0 = e^{i\pi/4} \tilde{\Sigma}^+_i - e^{-i\pi/4} \tilde{\Sigma}^-_i, \quad \tilde{\Sigma}^-_0 = \frac{1}{2} e^{i\pi/4} \tilde{\Sigma}^-_i - \frac{1}{2} e^{-i\pi/4} \tilde{\Sigma}^+_i. \quad \text{(40e)}
\]

### 4.1 Scalar Mode Perturbations

Considering scalar mode perturbations of the system (7), it can be shown that perturbations of the biscal field can be determined from

\[
\delta \phi = \frac{4}{\dot{\phi} \sqrt{R}} \left( \partial_t \Sigma + \frac{1}{2} H \Sigma \right), \quad \text{(41)}
\]

with \( \Sigma \) satisfying an equation of the form (40a) with

\[
U = \frac{K_c e^2 k^2}{R^2} + \frac{3}{2} \dot{H} - \frac{1}{4} H^2. \quad \text{(42)}
\]

Before the mode has left the horizon, the WKB solution (40) can be matched to a flat spacetime solution of the form (39), to give

\[
\tilde{\Sigma}^\pm_i = \phi \frac{\sqrt{\frac{hc^2 \kappa}{2e^2 k^2 \sqrt{K_c}}} \sqrt{R}}{4} C^\pm e^{\mp i\pi/4}. \quad \text{(43)}
\]

Once the mode has moved outside the horizon, only the increasing mode of \( \Sigma \) will be important, and using the above result with (40d), we therefore have

\[
\tilde{\Sigma}_o \approx \phi \frac{\sqrt{\frac{hc^2 \kappa}{2e^2 k^2 \sqrt{K_c}}} \sqrt{R}}{4} \sqrt{\frac{R}{HR_k}} (C_+ - C_-). \quad \text{(44)}
\]

In deriving this form, we have used the fact that very soon after the turning point \( U \approx (H/4)^2 \), and so we obtain

\[
\theta_o \approx \frac{1}{2} \int_{t_k}^{t} dt \dot{H} = \frac{1}{2} \ln(R/R_k). \quad \text{(45)}
\]

Here, \( R_k \) is defined by the turning point condition \( U = 0 \) (using \( H^2 \gg \dot{H} \)):

\[
\frac{\sqrt{K_c e^2 k^2}}{R_k^2} = \frac{1}{4} H_k^2. \quad \text{(46)}
\]
Near the turning point $\partial_t \Sigma \ll H\Sigma$, we find that
\[
\delta \phi \approx \frac{4}{\phi} H \Psi = \sqrt{\frac{\hbar c^2 K}{\sqrt{K_c}e^{2K}}} \sqrt{\frac{1}{HR_k}} (C_+ - C_-). \tag{47}
\]

This result contains a few important features. Since it “runs” slightly due to the presence of $H$, we merely evaluate it at the end of inflation: $H \approx H_f$ defined in (30). This is not strictly correct because near $R_f$, $\dot{H}$ is no longer negligible, but it will have the correct order of magnitude. Also, because $H$ is not constant during inflation, the solution of (46) does not just lead to $R_k \propto k$ and a trivially scale invariant spectrum. Instead, we find that $(W(-1, x)$ is the branch of the Lambert $W$-function appropriate for $-1 \ll x < 0)$:
\[
\left(\frac{R_k}{R_c}\right)^2 = e^{2\hat{N}_c} \frac{-\lambda^2}{W(-1, -\lambda^2)} \approx e^{2\hat{N}_c} \frac{-\lambda^2}{\ln(\lambda^2)}. \tag{48}
\]

We have used the fact that (using (4/3) $\exp(-1/3) \approx 1$, $4\sqrt{2\pi} \approx 10$ and (34)):
\[
\lambda^2 = \frac{4K_c}{3(1 - K_c)} \frac{c^2 k^2}{H_B^2 R_c^2} e^{-2\hat{N}_c} \approx \frac{K_c}{\hat{N}_c} \frac{c^2 k^2}{\xi B \sqrt{1 - K_c} R_0^2 H_0^2} 10^{-59} \zeta_{\text{dec}}, \tag{49}
\]
is small to keep only the asymptotic form of the solution.

We now have
\[
R_k \approx \frac{ck}{H_B} \sqrt{\frac{4K_c}{3(1 - K_c)}} \frac{1}{\sqrt{-\ln(\lambda^2)}}, \tag{50}
\]
which gives the scalar spectrum (evaluated at the end of inflation):
\[
\mathcal{P}_{\delta \phi} = \frac{k^3}{2\pi^2} |\delta \phi|^2 \approx \frac{1}{\sqrt{3\pi}} \xi_B^2 \frac{1 - K_c}{K_c^2} [-\ln(\lambda^2)] |C_+ - C_-|^2. \tag{51}
\]

Following inflation and (pre-)heating, we expect that we can match this result to standard post-inflation physics, and therefore using the fact that at the end of inflation we have
\[
\frac{H_f^2}{\phi^2} \approx \frac{1}{12}, \tag{52}
\]
the spectrum of curvature perturbations is determined by (41):
\[
\mathcal{P}_R = \frac{H_f^2}{\phi^2} \mathcal{P}_{\delta \phi} \approx \frac{\xi_B^2}{12\sqrt{3\pi}} \frac{1 - K_c}{K_c^2} [-\ln(\lambda^2)] |C_+ - C_-|^2. \tag{53}
\]

From this result, we can find the spectral index (3):
\[
n_s = 1 + \frac{d \ln \mathcal{P}_R}{d \ln k} = 1 + \frac{1}{\ln(\lambda)}, \quad \alpha_s = \frac{d n_s}{d \ln k} = -\frac{1}{\ln(\lambda)^2}. \tag{54}
\]

Furthermore, we have
\[
\delta_H = \frac{2}{5} \sqrt{\mathcal{P}_R} \approx \xi_B \frac{2}{5\sqrt{12\pi}} \frac{\sqrt{1 - K_c}}{K_c^\frac{3}{2}} \sqrt{-\ln(\lambda^2)} |C_+ - C_-|. \tag{55}
\]
4.2 Tensor Modes

The analysis of the tensor modes proceeds in the same way, although this time with the tensor metric fluctuation: \( \delta g_{ij} = h_{ij} = \delta g_{ij} \), and the transverse and traceless quantities: \( \gamma^{ij} h_{ij} = 0 \) and \( \nabla^i h_{ij} = 0 \). The modes are represented in terms of the usual \( I = \times, + \) polarization tensors \( e^I_{ij} \) [13]:

\[
 h_{ij} = \frac{K}{R^2} e^I_{ij} \Sigma_I, \tag{56}
\]

where each of the \( \Sigma_I \) satisfy an equation of the form (40a) with

\[
 U = \frac{K_c c^2 k^2}{R^2} - \frac{3}{2} \dot{H} - \frac{9}{4} H^2. \tag{57}
\]

The same procedure as described above leads to

\[
 \tilde{\Sigma}_I^\pm = \sqrt{\frac{1}{2} \hbar c^2 \kappa C^\pm} e^{\mp i\pi/4}, \tag{58}
\]

where the \( C^\pm \) again satisfy (39b). Outside the horizon we now have

\[
 U \approx \frac{9}{4} H^2, \quad \theta_o \approx \frac{3}{2} \ln(R/R_k), \tag{59}
\]

and so well outside the horizon the modes have the form:

\[
 \tilde{\Sigma}_o \approx \sqrt{\frac{\hbar c^2 \kappa R^3}{3H R_k^3}} (C_+ - C_-). \tag{60}
\]

Again these modes are slowly increasing, so we evaluate them at the end of inflation (30). In this case, the turning point is determined by

\[
 \sqrt{\frac{K_c c^2 k^2}{R^2_k}} = \frac{9}{4} H^2_k, \tag{61}
\]

which has the same solution as (46) with \( \lambda \) replaced by \( 3\lambda \).

Putting these results together, we find for each mode that

\[
 \mathcal{P}_h = \frac{k^3}{2\pi^2} |h_{ij}|^2 \approx \xi_B^2 \frac{3\sqrt{3}}{4\pi} \left( 1 - \frac{K_c}{K_c} \left[ -\ln(9\lambda^2) \right]^{\frac{3}{2}} \right) |C_+ - C_-|^2, \tag{62}
\]

using which we can determine the tensor spectral index:

\[
 n_t = \frac{d \ln(\mathcal{P}_h)}{d \ln k} = \frac{3}{2 \ln(3\lambda)}, \quad \alpha_t = \frac{d n_t}{d \ln k} = -\frac{3}{2 \ln(3\lambda)^2}. \tag{63}
\]

Finally, the ratio of the tensor to scalar mode contributions can be determined using (55) and (62). To use the latter result, we have ignored the running of \( \lambda \) with \( k \) and we have taken \( c_t \approx 1 \) (see e.g., the discussion in Section 7.7.2 of [1]).

\[
 r \approx \xi_B \sqrt{\frac{25\sqrt{6}}{8\pi} \left( 1 + \frac{48\pi^2}{385} \right) \sqrt{1 - K_c K_c^{-\frac{3}{2}} \left[ -\ln(9\lambda^2) \right]^{\frac{3}{2}} \left[ -\ln(\lambda^2) \right]^{\frac{3}{2}}}}. \tag{64}
\]
4.3 Observational Constraints

Assuming an adiabatic vacuum, \(C_+ = 0\) and \(C_- = 1\), \(z_{\text{dec}} = 4.0 \times 10^4\), and evaluating everything at the pivot point \(c k_{\text{pivot}} = 7 R_0 H_0\), from (55) we obtain [1]:

\[
\delta_H \approx \xi_B \frac{\sqrt{1 - K_c}}{20K_c^3} \sqrt{-\ln(\lambda^2)} \approx 1.91 \times 10^{-5},
\]

(65a)

for

\[
\lambda^2 \approx 2 \times 10^{-53} \frac{K_c^7}{\xi_B \sqrt{1 - K_c}}.
\]

(65b)

Satisfying (65) allows one to determine \(\xi_B\) in terms of \(K_c\). Once this is done, we find that we can restrict \(K_c\) to lie in the range

\[
K_c \in [2.0 \times 10^{-15}, 1.0 - 1.3 \times 10^{-13}],
\]

(66)

by requiring that \(T_f \leq T_P\) (giving the lower limit) and \(\xi_B \leq 32\pi\) (giving the upper limit). That is, we require that the temperature at the end of inflation is below the Planck temperature and the fundamental length \(\sqrt{B}\) appearing in (1) is not less than the Planck length. As we have discussed elsewhere [2, 1], it is far from clear how to define a fundamental scale that plays the role of the Planck length in this type of model. Nevertheless, these seem like reasonable conditions.

Over this range, several of the parameters are rather insensitive:

\[
n_s \in [0.982, 0.987], \quad n_t \in [-0.027, -0.019], \quad \alpha_s \in [-3.2, -1.6] \times 10^{-4}, \quad \alpha_t \in [-5.0, -2.5] \times 10^{-4}.
\]

(67a, 67b)

In all cases, the lower limit corresponds to the upper limit of \(K_c\). Note that the very small tilt rules out cold dark matter models within this scenario [14], and the small values of \(\alpha_s\) and \(\alpha_t\) imply that any running of the spectral indices should be unobservable in data from the MAP and Plank satellite missions [13].

The remaining parameters display similar trends, all of them being roughly constant for \(K_c \gtrsim 10^{-7}\) with the values

\[
r \approx 0.014, \quad T_f \approx 0.0019 T_P, \quad m_c\xi_B \approx 0.0018.
\]

(68a)

They all possess exponential behavior for smaller \(K_c\), and at the lower limit of (66) have the values:

\[
r \approx 77, \quad T_f \approx T_P, \quad m_c\xi_B \approx 3.1 \times 10^{-5}.
\]

(68b)

The combination \(m_c\xi_B\) determines the length scale appearing in the biscalare potential, from which we find that the biscalare field mass is \(m_c = 2.2 \times 10^{16}\) GeV, decreasing to \(m_c = 3.8 \times 10^{14}\) GeV at the lower limit of (66). The observation of CMB polarization from the MAP and Plank satellite missions [16] should constrain \(r\), effectively fixing the remaining parameters of the model. Note the lack of a simple ‘consistency relation’ between \(r\) and \(n_s\) that is typical of inflaton models. For a quadratic inflaton potential one would have \(r = 7(1 - n_s)/2\), and so even in the small \(K_c\) limit the tensor modes are larger by a factor \(\approx 2\).

The most complicated behavior is displayed by \(\xi_B\), which near \(K_c \approx 1\) is increasing exponentially to reach \(\xi_B = 32\pi\) at the upper limit in (66). In the intermediate regions, it is relatively constant at \(\xi_B \approx \exp(-10) \approx 4.5 \times 10^{-5}\), and then approaches its lower limit \(\xi_B \approx 10^{-10}\) at the lower limit of
If we choose $B = 1/\ell_P^2$ as suggested in [1], then we are in the regime where $\xi_B$ is approximately constant, and we find the parameter set:

$$1 - K_c \approx 1.3 \times 10^{-9}, \quad m_c \xi_B \approx 0.0018, \quad \xi_B \approx 1, \quad T_f/T_P \approx 0.0019,$$

$$n_s \approx 0.982, \quad n_t \approx -0.027, \quad \alpha_s \approx -3.2 \times 10^{-4}, \quad \alpha_t \approx -5.0 \times 10^{-4}, \quad r \approx 0.014.$$  \hspace{1cm} (69a) (69b)

These predictions are consistent with observational results [17] and in particular with the recent MAXIMA results [18, 19].

5 Conclusions

We have investigated a model of the early universe based on a bimetric gravitational theory with a biscalar field $\phi$, which has an initial period of inflation when the speed of gravitational waves $v_g \to 0$. This period is insensitive to the potential $V(\phi)$ and to the initial values of $\phi$. Two possible models for the generation of vacuum fluctuations present themselves: (i) the biscalar field $\phi$ both initiates inflation and its quantum vacuum fluctuations serve to generate the CMB spectrum after decoupling, (ii) the biscalar field $\phi$ produces an initial period of inflation as $v_g \to 0$ but the quantum vacuum fluctuations of a matter scalar field $\psi$, on the initial inflating background, generates the seeds of the CMB spectrum after the fluctuations cross the horizon.

We have concentrated on calculating the predictions of model (i), in which a second period of inflation begins when $v_g$ increases towards the measured speed of light $c$ and the potential $V(\phi)$ begins to play a significant role. This second period of inflation is characterized by a “constant-roll” of $\phi$ towards the bottom of the potential. A calculation yields predictions for the parameters describing the CMB spectrum, which agree with current observations. In a subsequent paper, we will obtain predictions for model (ii) and we anticipate that in this scenario a fine tuning of the $\psi$ scalar field mass and coupling constant will not occur, in contrast to the standard slow-roll inflaton models of inflation.

A significant aspect of both models (i) and (ii) is that the initial period of inflation, mediated by $v_g \to 0$, suggests a lack of sensitivity to any potential transplanckian physics influence. Such an influence could lead to a lack of predictability of the early universe cosmologies, as appears to be the case for standard slow-roll inflationary models [20]. It is also important to note that the predictions of BGT for the scalar-tensor fluctuation modes do not necessarily coincide with the standard inflaton inflationary models. Perhaps, future observations of the CMB spectrum, using more accurate data, will be able to distinguish between the BGT predictions and other inflationary scenarios.

We are now in a position to claim that bimetric gravitational theories can produce predictions in agreement with the CMB spectrum observations, and therefore provide an alternative scenario to the standard slow-roll inflationary models.

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