Integer Ambiguity Fixation Based on SC-PAR Algorithm

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Abstract. In terms of quality control of ambiguity estimation, the common partial ambiguity fixation algorithm is improved, and the SC-PAR (Single frequency Combined Partial Ambiguity Resolution) algorithm is proposed. After the algorithm fails to fix the full ambiguity, it filters the ambiguity subset step by step according to the number of continuous satellite lock epochs, satellite elevation angle, satellite signal-to-noise ratio, geometric precision factor, ambiguity variance and ambiguity precision attenuation factor, and searches Optimal ambiguity subset. According to the R-ratio value and the success rate index, the search results are jointly tested, and the remaining subsets are corrected with the subsets that pass the test. The results show that compared with the FAR and conventional PAR algorithms, the fixed rate of the SC-PAR algorithm is increased by 65.01\% and 27.97\%, respectively, and the accuracy is also significantly improved.

Keywords: RTK, quality control, ambiguity resolution

1. Introduction
With the advent of combined positioning technology, the number of available satellites is no longer an obstacle to high-precision solutions. Reliable estimation of ambiguity is a key issue in real-time kinematic positioning (RTK) [1], [2]. However, the joint solution of multiple systems makes the ambiguity dimension of the whole week increase sharply, which makes it difficult to estimate all ambiguities. This phenomenon is particularly prominent when the environment is poor [3]. Domestic and foreign scholars have shown that it is unnecessary to fix all the ambiguities at once, and the risk of failure of ambiguity fixation can be reduced by selecting a subset of the ambiguities for fixation [4]-[6]. Teunissen first proposed the concept of Partial Ambiguity Resolution (PAR), and proposed a fixed strategy to filter the ambiguity subset based on the success rate of Bootstrapping [7]. Some scholars have also put forward PAR algorithms based on satellite elevation angle, signal-to-noise ratio, continuous tracking epoch number, ADOP value, ambiguity variance and so on.

Based on the research of existing partial ambiguity fixation algorithms, this paper proposes a single frequency combined partial ambiguity resolution (SC-PAR) algorithm for quality control of ambiguity fixation [8], and combines the ambiguity confirmation method realizes the quality control of the ambiguity estimation process, and improves the ambiguity fixation rate and positioning accuracy.
2. Partial Ambiguity Resolution
The basic idea of partial ambiguity fixation is to divide the ambiguity into two subsets according to a certain criterion, and fix the easier subset first [9]. Specifically, if the ambiguity vector \( \tilde{a} \) is divided into two subsets \( [\tilde{a}_1 \quad \tilde{a}_2]^T \), its variance and covariance matrix can be expressed as:

\[
Q_{\tilde{a}} = \begin{bmatrix}
Q_{\tilde{a}_1, \tilde{a}_1} & Q_{\tilde{a}_1, \tilde{a}_2} \\
Q_{\tilde{a}_2, \tilde{a}_1} & Q_{\tilde{a}_2, \tilde{a}_2}
\end{bmatrix}
\]

(1)

Where, \( \tilde{a}_1 \) represents the selected ambiguity subset that is easier to fix, and \( \tilde{a}_2 \) represents the remaining ambiguity subset. When the LAMBDA method [10] is used to fix the subset \( \tilde{a}_1 \), the integer solution of \( \tilde{a}_1 \) can be used to modify \( \tilde{a}_2 \) and its variance covariance matrix \( Q_{\tilde{a}_2, \tilde{a}_2} \) to obtain the integer solution of \( \tilde{a}_2 \) and its fixed variance covariance matrix:

\[
\begin{align*}
\tilde{a}_2 &= \tilde{a}_2 - Q_{\tilde{a}_2, \tilde{a}_1} Q_{\tilde{a}_2, \tilde{a}_2}^{-1} (\tilde{a}_1 - \tilde{a}_1) \\
Q_{\tilde{a}_2, \tilde{a}_2} &= Q_{\tilde{a}_2, \tilde{a}_2} - Q_{\tilde{a}_2, \tilde{a}_1} Q_{\tilde{a}_2, \tilde{a}_2}^{-1} Q_{\tilde{a}_2, \tilde{a}_1}
\end{align*}
\]

(2)

For the single-frequency ambiguity solution, the currently commonly used PAR algorithm's strategy for selecting the optimal subset can be mainly divided into two categories:

The first category is in terms of satellites: satellites with poor accuracy are eliminated according to satellite elevation angle, signal-to-noise ratio, or the number of consecutive lock epochs. Before the PAR algorithm fixes the floating-point ambiguity, by setting the elevation angle threshold, the satellites below the threshold only participate in the floating-point solution calculation, and at the same time, it will always be in the continuous filtering iteration process; when the elevation angle is greater than the threshold When it is time, let it participate in the ambiguity resolution process.

The second category is in terms of ambiguity: sort according to the variance of ambiguity or the success rate of Bootstrapping or the size of ADOP, and iteratively eliminate ambiguity. Since the smaller the variance of the ambiguity, the higher the accuracy of the ambiguity, and the easier it is to be fixed. Therefore, filtering is performed according to the variance of the ambiguity after the correlation is reduced. This method can better reflect the accuracy of the ambiguity.

3. SC-PAR Algorithm
Because the subset selection strategies of the above two types of methods are usually relatively single and have their own drawbacks, this paper proposes an SC-PAR algorithm based on the two types of methods. The algorithm merges the two types of PAR algorithms and divides the ambiguity fixation algorithm into three stages. The first stage does not filter the ambiguity parameters, and fixes all the ambiguities. If the fixation fails, it enters the second stage, and if it succeeds, it returns to fixed solution.

In the second stage, with regard to satellites, the conventional screening strategy is improved to obtain and fix a better ambiguity subset. If the fixation is successful, the remaining ambiguity is corrected, and the final fixed solution of the ambiguity parameter and position parameter is calculated. If the second stage fixation fails, then the third stage is entered. Based on the ambiguity subset selected in the second stage, the ambiguity subset is further screened and fixed based on the ADOP value. In both stages, the distribution of selected satellites is considered, and the final subset is optimized as much as possible, so as to provide a fixed rate and accuracy of the solution. The specific steps of the new algorithm are as Fig. 1:
Figure 1. SC-PAR algorithm flow chart

1. Fix all ambiguities without dividing ambiguity subsets. Use the LAMBDA algorithm to search for all floating-point ambiguity parameters. A joint test of the R-ratio and Bootstrapping success rate indicators is carried out on the search results. If it passes the test, the fixation is considered successful,
and the fixation solution is calculated according to the least squares [11], [12].

Otherwise, go to the next step.

② Divide the ambiguity subsets at the satellite level. First, select the common-view satellites of all base stations and mobile stations whose number of consecutive locked epochs are greater than the set threshold, and then divide this set into 4 subsets according to the azimuth angle. Every 90° is a subset, and the 4 satellite subsets from 0° to 360° are denoted as $Q_1$, $Q_2$, $Q_3$, and $Q_4$, respectively;

③ For subset $Q_1$, eliminate 1 satellite whose elevation angle is lower than the threshold or whose signal-to-noise ratio is lower than the threshold (denoted as $s_i$). Record the set of all remaining satellites $S_i (i = 1, 2, 3, 4)$ (including other subsets) as $\Omega_i$. At this time, the principle of eliminating satellites is to make the GDOP value $g_i$ of $\Omega_i$ the lowest and less than the GDOP threshold $g_{th}$. Do the same for the other 3 subsets. Obtaining the rejected satellites $s_2$, $s_3$, and $s_4$, and the sets $\Omega_2$, $\Omega_3$, and $\Omega_4$, and their corresponding GDOP values $g_2$, $g_3$ and $g_4$, and use the set corresponding to the smallest GDOP value as the candidate set $H_1$, and record the GDOP value as $g_{min1}$;

④ The four eliminated satellites obtained in the third step are combined in pairs and removed from the original set $\Omega_0$ respectively. Calculate the GDOP of the remaining satellite set. Record the set $H_2$ corresponding to the smallest GDOP value less than the threshold $g_{th}$ as a candidate set, and its GDOP value as $g_{min2}$;

⑤ Compare $g_{min1}$ and $g_{min2}$, select the smaller set as the satellite set screened in the first stage, and record the ambiguity subset as $A_1$ and the remaining ambiguity subsets as $A_2$;

⑥ For $A_1$, lambda algorithm is used to search the ambiguity. The search results are jointly tested by r-ratio and bootstrapping success rate index. If it passes the inspection, the fixation is considered successful. The fixed solution is calculated, and the residual ambiguity is corrected by Equation 2;

⑦ If the sixth step fails to fix, $A_1$ will be sorted according to the ambiguity variance. The two ambiguities with large variance are recorded as $a_{max1}$ and $a_{max2}$ respectively. Calculate the GDOP value of the remaining satellite constellations after removing one ambiguity alone. If the GDOP value of the remaining satellite constellations is small after removing $a_{max1}$, $a_{max1}$ is classified into subset $A_2$. Otherwise, $a_{max2}$ is classified as subset $A_2$. Then, according to the variance covariance matrix of the remaining ambiguity subset of $A_1$, the adop value is calculated through equation 3. And record this subset and its adop value;

$$ADOP = \sqrt{\det Q_{aa}^{-1}}$$  (3)

Where, $m$ is the number of ambiguities.

⑧ If the ADOP value does not exceed the threshold, this subset is determined as the optimal ambiguity subset, and Step.9 is entered. If the ADOP value exceeds the threshold, three fuzziness $a_{max1}$, $a_{max2}$, and $a_{max3}$ with large variance are selected from the subset $A_1$. Then calculate the GDOP values of the remaining satellite constellations after eliminating the two ambiguities. Putting the combination of two Fuzziness with small GDOP value into subset $A_2$. And calculating the adop values of the remaining subset of $A_1$. Also record this subset and the adop values. And comparing the ADOP and the threshold. If the conditions are met, go to Step.9. If the conditions are still not met, compare the ADOP values at this time with the ADOP values in the previous step. Selecting the subset corresponding to the
smaller ADOP value as the optimal ambiguity subset and go to the next step;

⑨ Use the LAMBDA algorithm to search for the selected optimal ambiguity subset, and conduct a joint test of the R-ratio and Bootstrapping success rate indicators. If the fixation is successful, the final fixation solution is calculated. And using formula 2-2 to correct the remaining ambiguity. If the fixation fails, the floating point solution is saved.

4. Confirmation of Ambiguity

When confirming the ambiguity, this paper adopts the method of combining the R-ratio test and the determination of the ambiguity fixed success rate index. After the ambiguity is fixed, the R-ratio and Bootstrapping success rate index are jointly tested. Both the success rate of Bootstrapping and the R-ratio value are used as indicators for ambiguity testing. If all exceed the threshold, the fixation is considered to be successful; otherwise, the fixation is considered to be a failure, and the solution result of the current epoch adopts a floating point solution.

For the integer aperture estimation class, suppose that \( U \) represents the set of normalization domains for the entire integer aperture estimation, \( U_0 \) denotes the initial normalization domain, and \( U_a \) is the correct normalization domain. Then in the integer aperture estimation theory, the probabilities of the three states are:

\[
P_s = P(\hat{a} = a) = \int_{U_0} f_{\hat{a}}(x)dx, \quad \hat{a} \in U_a
\]

(4)

\[
P_f = \sum_{x \in \mathbb{Z} \setminus \{a\}} \int_{U_0} f_{\hat{a}}(x)dx, \quad \hat{a} \in U \setminus U_a
\]

(5)

\[
P_u = 1 - \int_{U_a} f_{\hat{a}}(x)dx, \quad \hat{a} \notin U
\]

(6)

Where, \( P_s \) is the probability that the ambiguity falls within the correct normalization domain, \( P_f \) is the probability that the ambiguity falls within the wrong normalization domain, \( P_u \) is the probability that the ambiguity does not fall within the normalization domain, and \( f_{\hat{a}}(x) \) is the probability density of the fixed solution residuals function. Then its final ambiguity estimation success rate is:

\[
P_{s|\hat{a}=z} = \frac{P_s}{P_s + P_f}
\]

(7)

For the integer estimation class, the success rate of ambiguity estimation can be expressed as [15]:

\[
P_s = P(\hat{a} = a) = \int_{S_a} f_{\hat{a}}(x)dx
\]

(8)

Where \( f_{\hat{a}}(x) \) is the probability density function of ambiguity and \( S_a \) is the correct normalization domain.

The core search method used in this paper is lambda algorithm. Bootstrapping success rate is the lower limit of integer least squares success rate. Therefore, it can be used as the approximate value of fuzzy fixed success rate, that is:
\[ P_s \approx \prod_{i=1}^{m} \left( 2 \Phi \left( \frac{1}{2\sigma_{s_i}} \right) - 1 \right) \]  

(9)

5. Example analysis

In order to analyze and verify the effectiveness of the proposed algorithm, actual measured data are selected for experimental verification. The data of the reference station and mobile station selected in this example are from a landslide hidden danger monitoring area in Guangxi Zhuang Autonomous Region, using a BDS/GPS dual-mode single-frequency receiver for data collection, with a total of 2735 epochs.

The FAR algorithm, the conventional PAR algorithm and the SC-PAR algorithm proposed in this paper are used to solve the data. Among them, the FAR algorithm only sets the cut-off angle of the satellite elevation angle (in this case, it is set to 15°). Satellites with an elevation angle lower than this value do not participate in the construction of the double-difference equation. The conventional PAR algorithm is based on the FAR algorithm, and uses a simple combination of satellite elevation angle and the number of continuous lock epochs to filter the ambiguity subset, and achieve partial ambiguity fixation. Among them, the satellite elevation threshold allowed to participate in the ambiguity resolution is set to 20°, and the number of continuous lock epochs is set to 10. The SC-PAR algorithm's satellite elevation angle and continuous lock epoch threshold settings are the same as conventional PAR. The signal-to-noise ratio threshold is set to 35dBHz, and the GDOP threshold is set to 2.0. When the number of ambiguities is within the interval, the ADOP threshold is set to 0.14; when the number of ambiguities is greater than 15, the ADOP threshold is set to 0.13. In addition, when confirming the ambiguity search results of the three methods, the R-ratio test is combined with the Bootstrapping success rate index. Bootstrapping success rate index is set to 99.5%. The R-ratio test threshold is set to 3.0.

![Figure 2. FAR Algorithm Solution Result](image1)

![Figure 3. Timing Diagram of the Number of Available Common-View Satellites](image2)
The solution results of far algorithm are shown in Figure 2, in which the fixed solution is represented by red solid points and the floating-point solution is represented by green solid points. It can be seen from the figure that due to the complex environment around the mobile station, a large number of floating-point solutions are generated when the ambiguity is fixed by far algorithm. Fig. 3 shows the sequence diagram of the number of available common view satellites. It can be seen from the figure that the number of satellites changes sharply in the first half of the data, indicating that the signals of some satellites may be disturbed or damaged by surrounding trees, hillside and other obstacles block, which affects the signal quality, causes the frequent increase and decrease of the number of available satellites, and then affects the fixation of the overall ambiguity.

The solution result of the conventional PAR algorithm is shown in Figure 4. It can be seen from the figure that after the partial ambiguity fixation algorithm is adopted, the number of floating-point solutions is significantly reduced. Especially in the first half of the data, the epoch of the floating-point solution appears in the FAR algorithm. Almost all of the PAR algorithms are fixed solutions, which verifies the effectiveness of the partial ambiguity fixed algorithm. However, a large number of floating-point solutions still appeared in the second half of the data, indicating that the conventional PAR algorithm can not well eliminate all satellites that affect the ambiguity resolution.

The solution result of the new algorithm is shown in Figure 5. It can be seen from the figure that almost all solutions are fixed solutions during the entire observation period. In the second half of the data, compared to the conventional PAR algorithm, the epochs of the floating-point solution appearing in the conventional PAR algorithm are all fixed solutions in the SC-PAR algorithm. This shows that the SC-PAR algorithm can further identify satellites that have an impact on the ambiguity fixation, thereby improving the ambiguity fixation rate.

Table 1 statistics the fixed rate of the solution results after using the three algorithms respectively. The fixed rates of far algorithm, conventional par algorithm and SC-PAR algorithm are 34.88%, 71.92% and 99.89% respectively. From the data statistics, the fixed rate is greatly improved after the partial ambiguity fixing algorithm is adopted. Compared with far algorithm, after using conventional par algorithm, the fixed rate is increased by 37.04%. After using SC-PAR algorithm, the fixed rate is increased by 65.01%.

Table 2 shows the overall accuracy statistics of the solution results of different algorithms. Combined with Table 1, it can be seen that the existence of floating-point solutions greatly reduces the overall accuracy level of the calculation results. After adopting the partial ambiguity fixation algorithm, whether it is the conventional PAR algorithm or the SC-PAR algorithm, the accuracy in all directions has been improved. The improvement of SC-PAR algorithm is particularly obvious. After using the SC-PAR algorithm, the east and north accuracy of the solution results are both within 1cm, and the elevation
accuracy can reach about 1.5cm. This further shows that in a complex environment, the partial ambiguity fixation algorithm can effectively control the quality of the whole-week ambiguity solution process, thereby improving the overall accuracy and fixation rate of the solution result, and it also reflects the SC-PAR algorithm superiority.

Table 1. Fixed Rate Statistics of Each Algorithm

| Ambiguity fixation algorithm | FAR     | Conventional PAR | SC-PAR algorithm |
|------------------------------|---------|------------------|------------------|
| Fixed number of successful epochs | 954     | 1967             | 2732             |
| Fixed number of failed epochs   | 1781    | 768              | 3                |
| Fixed rate                     | 34.88%  | 71.92%           | 99.89%           |

Table 2. Accuracy Statistics of the Solution Results of Each Algorithm (2sigma)

| Ambiguity fixation algorithm | Eastward (m) | North (m) | Elevation (m) | Horizontal (m) |
|------------------------------|--------------|-----------|--------------|----------------|
| FAR                          | 0.0581       | 0.0263    | 0.0987       | 0.0661         |
| Conventional PAR             | 0.0470       | 0.0152    | 0.0837       | 0.0514         |
| SC-PAR algorithm             | 0.0061       | 0.0088    | 0.0153       | 0.0102         |

6. Conclusion

This paper mainly adopts the combined inspection method of R-ratio and ambiguity fixed success rate index in the ambiguity confirmation stage to confirm the ambiguity obtained by searching, and proposes an SC-PAR algorithm. Finally, a comparative experiment was carried out on the FAR algorithm, the conventional PAR algorithm and the SC-PAR algorithm. The experimental results show that compared with all ambiguity fixation algorithms, partial ambiguity fixation algorithms can effectively improve the overall fixation rate and accuracy of the solution results. It reflects the effectiveness of the partial ambiguity fixation algorithm in quality control. From the statistical point of view of the experimental results: in terms of the fixation rate, compared with all the ambiguity fixation algorithms, the fixation rate of the conventional PAR algorithm has increased by 37.04%, and the fixation rate of the SC-PAR algorithm has increased by 65.01%; in the solution results in terms of overall accuracy, affected by the floating point solution, the horizontal accuracy of all fuzzy fixed algorithms is about 6.6cm, and the elevation accuracy is about 9.8cm. Compared with the FAR algorithm, the accuracy of the conventional PAR algorithm is slightly improved, the horizontal accuracy is about 5cm, and the elevation accuracy is about 8cm. The accuracy of the SC-PAR algorithm in all directions has been significantly improved, the east and north accuracy can reach within 1cm, and the elevation accuracy can reach about 1.5cm. It can be seen that in a complex environment, the SC-PAR algorithm can greatly improve the fixed rate of the whole-week ambiguity solution and the accuracy of the overall solution result.

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