STRUCTURE OF THE NUCLEON FROM ELECTROMAGNETIC SPACE-LIKE AND TIME-LIKE FORM FACTORS

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Abstract

Recent experimental data on space-like and time-like form factors of the nucleon are reviewed in light of a model of the nucleon with an intrinsic (quark-like) structure and a meson cloud. The analysis points to the astonishing result that the proton electric space-like form factor vanishes at \( Q^2 \approx 8 \text{ (GeV/c)}^2 \) and becomes negative beyond that point. The intrinsic structure is estimated to have a r.m.s. radius of \( \sim 0.34 \text{ fm} \), much smaller than the proton r.m.s. radius \( \sim 0.82 \text{ fm} \). The calculations are in perfect agreement with space-like proton data, but deviate drastically from space-like neutron data at \( Q^2 > 1 \text{ (GeV/c)}^2 \). Analysis of the time-like data appears to indicate excellent agreement with both proton and neutron data in the entire range of measured \( q^2 = -Q^2 \) values.

INTRODUCTION

Electromagnetic form factors have played a crucial role in understanding the structure of composite particles. A particularly important composite particle is the nucleon, which forms the basis upon which all matter is built. Studies of the structure of the nucleon with electromagnetic probes begun in the late 50’s and early 60’s when Hofstadter and collaborators demonstrated that the nucleon was not point-like with a (proton) root-mean square radius \( \langle r_p^2 \rangle^{1/2} \approx 0.75 \text{ fm} \). In the 1970’s many experiments were performed, showing that the neutron was a complex particle with a negative r.m.s. radius and \( dG_{E_n}/dQ^2 \approx 0.50 \text{ (GeV/c)}^2 \). In 1973, it was suggested that the nucleon has a two component structure with an intrinsic part with form factor \( g(Q^2) \) and a meson cloud parametrized in terms of vector mesons, \( (\rho, \omega, \varphi) \). In the late 1970’s the non-relativistic quark-model was used to describe the properties of hadrons. It was soon realized that this model cannot describe form factors in a consistent way. Also in the late 1970’s, QCD emerged as the theory of strong interactions. In a perturbative approach, p-QCD, the asymptotic behavior of the form factors can be derived, yielding the large \( Q^2 \) behavior of the nucleon form factors to be \( \propto \frac{1}{Q^2} \). In the 1980’s, experimental groups noted that all form factors, except \( G_{E_n} \), could be described by the empirical dipole form \( G_D(Q^2) = 1/(1 + \frac{Q^2}{r_D^2})^2 \). These observations culminated in the SLAC experiment NE11 on the ratio \( \mu_p G_{E_p}/G_{M_p} \) that appeared to be consistent with scaling up to 10 \( \text{(GeV/c)}^2 \). However, in 2000-2002 experiments performed at TJNAF using the recoil polarization method have shown the astounding result that the ratio of proton electric to proton magnetic form factor decreases dramatically with \( Q^2 \), inconsistent with scaling. In the first part of this article, the present situation on electromagnetic form factors of the nucleon in the space-like region will be reviewed.

For relativistic systems, one has access to the time-like part of the form factors. Studies of this part begun in the late 60’s and early 70’s through processes \( p \bar{p} \to e^+e^- \) and \( e^+e^- \to p \bar{p} \). The first positive result on the time-like form factors of the proton was obtained in 1972 at Frascati. Several other experiments where subsequently performed culminating in the Fermilab experiment E835. In 1998 the first measurement of the neutron time-like form factor was reported. This measurement appeared to be in disagreement with simple quark model extensions to large \( Q^2 \), where \( G_{M_n}/G_{M_p} = -2/3 \). In the second part of this article, the present situation on the time-like electromagnetic form factors of the nucleon will be reviewed.

SPACE-LIKE FORM FACTORS

Two basic principles play a crucial role in the analysis of electromagnetic form factors of the nucleon. The first of these is relativistic invariance. This principle fixes the form of the nucleon current to be

\[ J^\mu = F_1(Q^2)\gamma^\mu + \frac{\kappa}{2M_N} F_2(Q^2)i\sigma^{\mu\nu}q_\nu \]  

(1)

where \( F_1(Q^2) \) and \( F_2(Q^2) \) are the so-called Dirac and Pauli form factors and \( \kappa \) is the anomalous magnetic moment. This symmetry is expected to be exact. The second is isospin invariance. Although this symmetry is not exact, being of dynamical origin, it is expected to be only slightly broken in a realistic theory of strong interaction. Isospin invariance leads to the introduction of isoscalar, \( F_1^S \) and \( F_2^S \), and isovector, \( F_1^V \) and \( F_2^V \), form factors, and hence to relations among proton and neutron form factors. The observed Sachs form factors, \( G_E \) and \( G_M \) can be obtained by the relations

\[ G_{M_p} = (F_1^S + F_1^V) + (F_2^S + F_2^V) \]
\[ G_{E_p} = (F_1^S + F_1^V) - \tau (F_2^S + F_2^V) \]
\[ G_{M_n} = (F_1^S - F_1^V) + (F_2^S - F_2^V) \]
\[ G_{E_n} = (F_1^S - F_1^V) - \tau (F_2^S - F_2^V) \]  

(2)

with \( \tau = Q^2/4M_N^2 \). These relations also satisfy another constraint, namely the kinematical constraint \( G_E(-4M_N^2) = G_M(-4M_N^2) \). This constraint is of crucial
importance in the time-like region, while playing a minor role in the space-like region.

Different models of the nucleon correspond to different assumptions for the Dirac and Pauli form factors. In 1973 a model of the nucleon in which the external photon couples to both an intrinsic structure, described by the form factor \( g(Q^2) \), and a meson cloud, treated within the framework of vector meson (\( \rho, \omega \) and \( \varphi \)) dominance, was suggested. In this model the Dirac and Pauli form factors are parametrized as

\[
F_1^S(Q^2) = \frac{1}{2} g(Q^2)[(1 - \beta_\omega - \beta_\varphi) + \beta_\omega \frac{m_\omega^2}{m_\omega^2 + Q^2} + \beta_\varphi \frac{m_\varphi^2}{m_\varphi^2 + Q^2}]
\]

\[
F_1^V(Q^2) = \frac{1}{2} g(Q^2)[(1 - \beta_\rho) + \beta_\rho \frac{m_\rho^2}{m_\rho^2 + Q^2}]
\]

\[
F_2^S(Q^2) = \frac{1}{2} g(Q^2)[(1 - \alpha_\varphi) - \alpha_\varphi \frac{m_\varphi^2}{m_\varphi^2 + Q^2}]
\]

\[
F_2^V(Q^2) = \frac{1}{2} g(Q^2)[3.706 \frac{m_\rho^2}{m_\rho^2 + Q^2}]
\]

In a model of the nucleon in which the external photon couples to both an intrinsic structure, described by the form factor \( g(Q^2) \), the magnetic form factor of the proton is limited to the ratio \( \mu_p G_{E_p}/G_{M_p} \). Consider the magnetic form factor, \( G_{M_p} \). For convenience of display, normalize it to the so-called dipole form factor, \( G_D = (1 + Q^2/\rho^2)^{-2} \). The data in the interval \( 0 \leq Q^2 < 10 \text{ (GeV/c)}^2 \) are plotted in Fig. 2. They show an oscillation, crossing the value one at \( Q^2 \sim 0.6 \text{ (GeV/c)}^2 \) and again at \( \sim 6 \text{ (GeV/c)}^2 \). The calculation is in excellent agreement with the data, with crossing points at precisely the same values \( \sim 0.6 \) and 6 (GeV/c)^2. The observed oscillation is proof that vector meson (with masses \( \mu^2 \sim 0.5 - 1.0 \text{ (GeV/c)}^2 \)) components are important. Without \( \rho \) meson component, the form factor should behave smoothly (see Fig. 3 of [8]).

**The magnetic form factor of the neutron**

Having established the structure of the proton, I now come to that of the neutron. This is dictated by isospin invariance. Measurements of the neutron form factors are obscured by the knowledge of the wave functions of deuterons or He^3. Older measurements are either in disagreement (for \( Q^2 > 1 \text{ (GeV/c)}^2 \)) or in marginal agreement (\( Q^2 < 1 \text{ (GeV/c)}^2 \)).
Figure 1: The measured ratio $\mu_p G_{Ep}/G_{Mp}$ compared with the 1973 prediction. Ref. [2]: open square. Ref. [3]: filled circle.

Figure 2: Experimental values $G_{Mp}/\mu_p G_D$ compared with calculation. Ref.[14]: open square. Ref.[15]: filled circle. Ref.[1]: filled diamond.

Figure 3: Recent experimental values for $G_{Mn}/\mu_n G_D$ compared with calculation. Ref. [16]: open square. Ref.[17]: filled circle.

The electric form factor of the neutron

A similar situation occurs for new (1999) data for the electric form factor $G_{En}$ by Herberg et al [18], Passchier et al [19], Ostrick et al [20], Rohe et al [21], Zhu et al [22]. These are in fair agreement with the calculation as shown in Fig. 4. For $Q^2 \geq 1$ (GeV/c)^2 the calculation is in disagreement with new unpublished data. While the data remain close to 0.05, the calculation keeps decreasing and crosses zero at $\sim 1.4$ (GeV/c)^2. It would be of the utmost importance to measure $G_{Mn}$ and $G_{En}$ at $Q^2 \geq 1GeV^2$ in a as much as possible model independent way. A measurement of the ratio $\mu_n G_{En}/G_{Mn}$ similar to that done for the proton, perhaps using the reaction $d(e,e'n)p$ [23], will be of great value. Similar observations can be made for $G_{En}$. In present analyses this form factor is even more sensitive to models than $G_{Mn}$.

SCALING LAWS

Another important question is the extent to which the new data support scaling laws [24]. The parametrization of Eq.(3) is consistent with scaling laws expected from perturbative QCD, $F_1 \sim 1/Q^4$, $F_2 \sim 1/Q^6$ except for $F_2^V$ whose asymptotic behavior ($Q^2 \to \infty$) is

$$F_2^V(Q^2) \to \frac{3.706}{2\pi^2 Q^6} \frac{m_\rho^2 + 8F_\rho m_\pi/\pi}{1 + \frac{\Gamma_\rho}{m_\rho} \ln 2\sqrt{\frac{Q^2}{4m_\pi^2}}} ,$$

that is with a weak logarithm dependence due to the effective $\rho$ mass induced by the $\rho$ width. The scaling properties of $F_1$ and $F_2$ are determined by the only length scale in the problem, namely the size of the intrinsic quark structure, $1/\gamma$. In order to have a quantitative estimate of the value of $Q^2$ at which scaling is reached, I shall use the following definition: a function $f(z)$ is said to be $x\%$ scaled when its
value is \(x\%\) of the asymptotic value \(f_{as}(z)\). The value at which this condition is met is the solution of the equation \(| f(z) | = x \cdot f_{as}(z)\). For the form factors \(F_1^p, F_2^p, F_2^S\) and with minor modifications also for \(F_1^n\), scaling properties are determined by the function \(g(Q^2)\). Using the value \(\gamma = 0.25 \text{ GeV}/c^2\), one obtains an estimate of scaling properties. The function \(g(Q^2)\) is 80% scaled at \(Q^2 \geq 34 \text{ (GeV)/c}^2\). This value is much larger than conventionally believed, \(Q^2 \sim 4 \text{ (GeV)/c}^2\). (The dipole form \(G_D(Q^2)\) is 80% scaled at \(Q^2 \sim 6 \text{ (GeV)/c}^2\).) The situation for the scaling properties of the form factors \(G_E\) and \(G_M\) is more complex. The parametrization of Eq.(3) is consistent, apart from a weak logarithm dependence, with the scaling laws of perturbative QCD, \(G_E \sim G_M \sim 1/Q^4\). However, relativity introduces here another scale, \(4M_N^2 = 3.52 \text{ (GeV)/c}^2\), and, independently from the actual value of the size scale \(\gamma\), relativistic invariance requires that scaling is not reached unless \(Q^2 \geq 2\) is greater than a few times \(4M_N^2\). (This is particularly so for the electric form factors.) To check scaling properties it would be of utmost importance to measure the ratio \(\mu_p G_{E_n}/G_{M_p}\) with the recoil polarization method beyond 10 \text{ (GeV)/c}^2.\]

Another prediction from perturbative QCD is that the ratio \(G_{M_p}/G_{M_n}\) approaches zero from the negative side for large \(Q^2\),

\[
\frac{G_{M_p}}{G_{M_n}} \to 0^-
\]

as a power of \(\ln(Q^2/A^2)\). The predictions of the model discussed here are \(G_{E_p} \to -4.08/Q^4, G_{M_p} \to 0.9120/Q^4,\) and \(G_{E_n} \to -10.86/Q^4, G_{M_n} \to -4.33/Q^4\) from which one can obtain

\[
\frac{G_{M_p}}{G_{M_n}} \to -0.21.
\]

The electric values have been obtained by estimating the logarithm dependence at \(Q^2 = 100 \text{ (GeV)/c}^2\). Checking this prediction requires the measurement of \(G_{M_n}\) at large \(Q^2\). Both the pQCD result and the 1973 result are in disagreement with the \(SU(6)\) value \(-3/2\) often used in experimental analyses.

The extent to which dimensional scaling is valid has been in recent years the subject of many investigations. It has been suggested that the appropriate scaling variable is \(QF_{2p}(Q^2)/F_{1p}(Q^2)\) instead of \(Q^2 F_{2p}(Q^2)/F_{1p}(Q^2)\). Using Eq.(3) one can easily calculate \(QF_{2p}(Q^2)/F_{1p}(Q^2)\). From this calculation one can see that the quantity \(QF_{2p}(Q^2)/F_{1p}(Q^2)\) remains flat in the interval \(2 \leq Q^2 \leq 10 \text{ (GeV)/c}^2\) and drops from there on, especially after dimensional scaling is reached at \(Q^2 \geq 34 \text{ (GeV)/c}^2\), Fig.5.

The scaling with \(Q\) is thus accidental and appropriate only to the intermediate region.

### STABILITY AGAINST PERTURBATIONS

The new data clearly point out that the structure of the proton is rather complex and that it contains at least two components. The data appear to be in agreement in the entire measured range with a calculation in which the two
component are an intrinsic structure, presumably $q^3$, and a meson cloud, $q^2 q\bar{q}$, the latter being expressed through vector mesons ($\rho, \omega, \varphi$). The situation for the neutron is different. The new data are in agreement with the 1973 calculation up to 1 (GeV/c)^2. From there on, they appear to be in disagreement with the new (unpublished) data [27]. One can inquire whether addition of other ingredients changes this conclusion. There are three contributions that can be analyzed easily.

(i) The role of additional vector mesons, $\rho(1450), \omega(1390), \varphi(1680)$ [28].

(ii) The addition of an intrinsic piece to the Pauli form factor $F_{2V}$. This can be done by the replacement

$$3.706 \left( \frac{m_\rho^2}{m_\rho^2 + Q^2} \right) \rightarrow (3.706 - \alpha_\rho) \left( \frac{1}{1 + \gamma Q^2} \right) + \alpha_\rho \left( \frac{m_\rho^2}{m_\rho^2 + Q^2} \right)$$

(11)

The additional piece must be of this type to insure the proper behavior of $F_{2V}$ for $Q^2 \rightarrow 0$ and $Q^2 \rightarrow \infty$.

(iii) The role of the widths of $\omega, \varphi$ as well as the effect of changing the width of the $\rho$ meson from the value used in [3].

The qualitative features are not affected by these changes, although quantitatively one can make some improvements on the form factor of the neutron. However, because of isospin invariance, an improvement in the neutron form factors produces a deterioration in the description of the proton data. It does not appear that the problem of the neutron form factor at large $Q^2$ can be solved with these changes. To solve this problem one needs to introduce terms which act only on the neutron, that is terms with $F_S = -F_V$. Work in this direction is in progress.

One can also check whether the logarithmic dependence of perturbative QCD

$$Q^2 \rightarrow Q^2 \ln \left( \frac{\Lambda^2 + Q^2}{\Lambda^2_{QCD}} \right)$$

(12)

with $\Lambda = 2.27$ GeV/c and $\Lambda_{QCD} = 0.29$ GeV/c [29] produces major changes in the conclusions. This does not appear to be the case at least up to $Q^2 = 10$ (GeV/c)^2.

TIME-LIKE FORM FACTORS

By an appropriate analytic continuation in the complex plane, the form factor of Eq.(3) can be used to analyze form factors in the time-like region. These can be and have been experimentally obtained in the reactions $p\bar{p} \rightarrow e^+ e^-$ and $e^+ e^- \rightarrow p\bar{p}$. Two ingredients are needed to study time-like form factors: (i) an appropriate analytic continuation of the intrinsic form factor and (ii) an analytic continuation of the vector meson components. For the intrinsic part, a simple analytic continuation of the intrinsic form factor, $g(Q^2)$, that takes into account the complex nature of the $p\bar{p}$ interaction is

$$g(q^2) = \frac{1}{(1 - \gamma e^{i\theta} q^2)^2}$$

(13)

where $q^2 = -Q^2$. The parameter $\gamma$ is the same as in the space-like region, but there is now a phase $\theta$. The width of the $\omega$ and $\varphi$ mesons is small and can be neglected. For the $p\rho$ meson, one needs to replace

$$F_{2V} \rightarrow \left( \frac{m_\rho + 8 \pi \Gamma_\rho m_\rho}{m_\rho^2 - q^2} \right) \left[ \frac{4m_\pi^2 - q^2}{m_\pi} \Gamma_\rho \alpha(q^2) + i4m_\pi \Gamma_\rho \beta(q^2) \right]$$

(14)

where

$$\alpha(q^2) = \frac{2}{\pi} \left[ \frac{q^2 - 4m_\pi^2}{q^2} \right]^{1/2} \ln \left( \frac{\sqrt{q^2 - 4m_\pi^2} + \sqrt{q^2}}{2m_\pi} \right)$$

(15)

The replacement (14) and (15) applies to $q^2 \geq 4m_\pi^2$ and should be compared with (6) and (7) that applies to $Q^2 \geq 0$. Using the same parameters of the 1973 calculation and adjusting the angle $\theta$, one obtains the proton form factor $|G_{M_p}|$ shown in Fig. 6. Here $\theta \simeq 53^\circ$. The calculation is compared with data from [3, 5, 11, 22, 33, 34, 35]. The large $q^2$ values [35, 36], extracted under the assumption $|G_E| = |G_M|$, have been corrected with the calculated $|G_E|$ values [35]. Apart from the threshold behavior, presumably due to a subthreshold resonance, the overall agreement is good. (The question of a subthreshold resonance will be discussed in a forthcoming paper [35].) Without further parameters one can now calculate the neutron form factor $|G_{M_n}|$. A comparison with experiment [6] is shown in Fig. 7. The agreement is astonishing. In the same figure the dipole form factors, $\mu_G D$ and $\mu_n G_D$, are shown. One can see that the experimental data for $|G_{M_p}|$ in the region $q^2 \simeq 4 - 6$ (GeV/c)^2 are a factor of 2 larger than the dipole, and those for $|G_{M_n}|$ are a factor of 4 larger than it.

CONCLUSIONS

The main conclusions that one can draw from the analysis of recent experimental data on electromagnetic form factors are:

(i) The proton appears to have a complex structure with at least two components, an intrinsic component (valence quarks) and a meson cloud ($q\bar{q}$ pairs). The size of the intrinsic structure is r.m.s. $\simeq 0.34$ fm.

(ii) Perturbative QCD is not reached in the proton up to $Q^2 \sim 10$ (GeV/c)^2. Physics up to this scale is dominated by a mixture of hadronic and quark components.

(iii) Symmetry (in particular relativistic invariance), rather than detailed dynamics, appears to be the determining factor in the structure of the proton.
The situation appears to be different for the neutron. Here recent experimental data up to $1 \, (\text{GeV/c})^2$ are consistent with isospin invariance and the structure of the proton, while preliminary data at $Q^2 \geq 1 \, (\text{GeV/c})^2$ appear to indicate that either isospin invariance is broken or that additional components play a role. It would be of the utmost importance to understand this discrepancy.

An analysis of the form factors in the time-like region indicates that both proton and neutron data are consistent with the two component picture introduced in 1973, in particular the observed ratio $|G_{M_n}/G_{M_p}| \sim 2$, appears to be a consequence of this structure and is easily explained. In order to understand this basic point of the structure of the nucleon, it would be of utmost importance to remeasure $G_{M_n}$ with better accuracy, possibly extending the measurement to larger $Q^2$ values.

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