Rigorous Mathematical Thinking Approach to Enhance Students’ Mathematical Creative and Critical Thinking Abilities

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Abstract. The ability of mathematical creative and critical thinking are two abilities that need to be developed in the learning of mathematics. Therefore, efforts need to be made in the design of learning that is capable of developing both capabilities. The purpose of this research is to examine the mathematical creative and critical thinking ability of students who get rigorous mathematical thinking (RMT) approach and students who get expository approach. This research was quasi experiment with control group pretest-posttest design. The population were all of students grade 11th in one of the senior high school in Bandung. The result showed that: the achievement of mathematical creative and critical thinking abilities of student who obtain RMT is better than students who obtain expository approach. The use of Psychological tools and mediation with criteria of intentionality, reciprocity, and mediated of meaning on RMT helps students in developing condition in critical and creative processes. This achievement contributes to the development of integrated learning design on students’ critical and creative thinking processes.

1. Introduction
Thinking is an activity that people do in everyday life. Human thinking ability is also a very important asset in placing oneself, solve problems, or just to undergo daily activities. Thinking also means a process to understand something that is experienced or find a way out of the problems faced. The way is obtained by using the things that have been known before. The process of thinking is an event of mixing, matching, combining, exchanging, and sorting on previous concepts, perceptions, and experiences [1]. Thus, the ability of thinking is a very important to be developed by each individual.

In the face of the challenges of the 21st century, every individual is faced with a condition that forces them to expend their thinking ability to solve every problem that will arise. Ability to think that is needed at this time are familiar with Higher Order Thinking (HOT). HOT is a thinking skill that goes beyond just memorizing facts, repeating concepts, or implementing procedures [2]. HOT requires us to do something with the facts. We must understand them, connect them, categorize them, manipulate them, and put them together in a new way. Further understanding of the facts is used in finding new solutions to new problems. The two thinking ability that are included in the HOT are critical thinking and creative thinking ability. As King [3] suggests that HOT contains several
capabilities of critical thinking, logical, reflective, metacognitive, and creative thinking. Two of these abilities can be developed in school learning one of them is on learning mathematics.

Based on the above description can be said that the ability to think mathematical creative and critical needs to be developed in the learning in schools therefore need a learning approach that is able to develop both of them. One approach to learning that can develop students' thinking ability is the Rigorous mathematical thinking (RMT) approach. RMT is a learning approach based on two main theories: theory psychological tools from Vygotsky and theory of mediated learning from Feuerstein. This approach emphasizes the interaction and mediation between teachers and students resulting in a good understanding of the material presented to further transform and conceptualise and emerge into bound and interrelated ideas [4].

Theory psychological tools is a theory of Vygotsky which reveals that the use of psychological tools in learning can help students in organizing and integrating cognitive functions with basic concepts that can support their generalizations and mathematical abstractions. Such psychological tools can be symbols, tables, diagrams, drawings, maps, graphs, and coding. With the tools of psychology is expected to be able to bridge students in understanding a problem and simplify in relating experiences they have passed with new experiences they get.

Mediated Learning Experience (MLE) reflects the quality of the interaction between the student, the material, and the mediator or teacher itself. The quality will be obtained if and only if some MLE criteria are implemented. These criteria are intentionality & reciprocity, transcendence, and mediation of meaning. Intentionality & reciprocity means that mediation by the teacher as a mediator should make the students retain their attention and also make the material easily digested by the students. Transcendence means that mediation does not limit their interaction with the material. Mediation of meaning means that the interaction must be meaningful and become a good experience for the students.

Based on the description above, RMT approach can be an alternative approach to learning to help students not only in linking the knowledge that has been previously owned with the problems that are facing but also develop new ideas in finding an appropriate solution in solving mathematical problems. This will certainly affect students' critical and creative thinking ability. Formulation of the problems in this research is: 1) is the achievement of mathematical critical thinking ability of student who obtain RMT better than students who obtain expository approach?; 2) is the achievement of mathematical creative thinking ability of student who obtain RMT better than students who obtain expository approach?

2. Theory

2.1 Mathematical critical thinking

The ability to think critically can have different definitions of different characters. According to Ennis [7] critical thinking is reasonable reflective thinking focused on deciding what to believe or do. Furthermore, Bailin [8] reveals that critical thinking is thinking that is goal-directed and purposive, “thinking aimed at forming a judgment where the thinking itself meets standards of adequacy and accuracy. While Fisher [9] argues that critical thinking is explaining what is thought. Based on the above definition, critical thinking can be interpreted as the ability in making decisions about seeking truth or clarity of a thing in a context or problem.

Krulik and Rudnick in Sabandar [10] argue that being included in critical thinking skills in mathematics is to examine, question, connect, evaluate all aspects of a situation or a mathematical problem. Correspondingly, O’dafer and Thornquist [6] argue that critical thinking in mathematics is the process of using the mind effectively to assist one in organizing, evaluating and applying decisions about what to believed or do.

Critical thinking ability in mathematics or critical mathematical thinking ability include the ability of students to identify a concept and provide reasons for the use of the concept, generalize the concept based on observed data, analyze algorithms in solving problems, and problem-solving ability.
2.2 Mathematical creative thinking
Sumarmo [11] states that creative thinking contains aspects of cognitive, affective, and metacognitive skills. First, cognitive skills include the ability to: identify problems and opportunities, make good and different questions, identify relevant and irrelevant data, productive problems and opportunities, generate many ideas (fluency), different ideas (flexibility), and New products or ideas (originality), examine and assess relationships between choices and alternatives, change old mindsets and habits, develop new relationships, expand and update plans or ideas. Second, the affective skills contained in creative thinking include attitudes in: feeling problems and opportunities, tolerating uncertainty, understanding the environment and creativity of others, being open, taking risks, building self-confidence, self-control, curiosity, stating and respond to feelings and emotions, and anticipate something unknown. Third, the metacognitive abilities contained in creative thinking include the ability to: design strategies, set goals and decisions, predict from incomplete data, understand creativity and things that others do not understand, diagnose incomplete information, create multiple balance, regulate emotions, and advance the elaboration of problem solutions and plans.

The ability to think creative in mathematics involves students' thinking skills when faced with mathematical problems characterized by fluency in expressing varied and fluent ideas, generating ideas of flexibility, generating new and innovative mathematical ideas (originality), and develop or expand an idea and elaborate the ideas.

2.3 Rigorous Mathematical Thinking (RMT)
Rigorous Mathematical Thinking (RMT) is a learning approach developed by James T Kinard and Alex Kozulin. RMT's approach is based on two main theories: theory psychological tools from Vygotsky and theory of mediated learning from Feuerstein. Psychological tools in question can be gestures, symbols, images, graphics, or artefacts that have a special meaning in one's culture in society. Mediated learning experience (MLE) is a mediation process conducted by the teacher to the students during the learning takes place. MLE has three principles: (1) intentionality and reciprocity, (2) transcendence, and (3) mediation of meaning.

Kinard & Kozulin [5] formulated a RMT approach in classroom learning with three phases that included six processes:
Phase I: Cognitive Development
a. The learner is mediated to appropriate the models in the cognitive tasks as general psychological tools based on their structure/function relationship.
b. The learner is mediated to perform the cognitive tasks through the use of the psychological tools to construct higher order cognitive processes.
Phase II: Content as Process development
c. The learner is mediated to systemically build basic essential concepts needed in mathematics from everyday experiences and language.
d. The learner is mediated to discover and formulate the mathematical patterns and relationships in the cognitive exercise.
e. The learner is mediated to appropriate mathematically specific psychological, based on their unique structure/function relationships.
Phase III: Cognitive Conceptual Construction Practice
f. The learner is mediated to practice the use of each mathematically specific psychological tool to organize and orchestrate the use of cognitive function to construct mathematical conceptual understanding.

3. Experimental Method
This research was quasi experimental consisting of two groups: experimental group and control group. The experimental group is a group of students in a class whose lessons by the Rigorous Mathematical Thinking (RMT) approach. While the control group is a group of students in a class whose lessons by
expository approach. Population in this research were all student of grade 11th in one of high school in Bandung academic year 2016/2017. The sampling technique used purposive sampling to determine the experimental group and control group. This design is used with individual considerations in both groups are not randomly selected but have been formed before and researchers do the treatment in accordance with the existing schedule in school. The instrument used to collect data were the mathematical creative thinking ability test, mathematical critical thinking ability test, and observation sheet. The data obtained is analyzed using the mean difference test, which are the t-test and Mann-Whitney U test.

4. Result and Discussion
The purpose of this research is to examine the mathematical creative and critical thinking abilities of students who get RMT approach and students who get expository approach. Descriptive analysis of students' mathematical creative and critical abilities is presented in the table below.

Table 1. Descriptive analyzes of mathematical critical thinking ability

|          | RMT          | Expository   |
|----------|--------------|--------------|
| Pretest  | Posttest     | Pretest      | Posttest     |
| $x$      | 9.58         | 27.58        | 9.75         | 20.92        |
| $s$      | 3.73         | 6.07         | 3.81         | 6.78         |

Based on Table 1, it appears that the average pretest grade of the experimental group is lower than the control group by 0.17. As for the posttest score, the experimental group is higher than the control group by 6.66. From this, it can be seen that the descriptive posttest result of mathematical critical thinking ability of experiment group is better than control group.

Table 2. Descriptive analyzes of mathematical creative thinking ability

|          | RMT          | Expository   |
|----------|--------------|--------------|
| Pretest  | Posttest     | Pretest      | Posttest     |
| $x$      | 7.67         | 23.33        | 7.25         | 18.67        |
| $s$      | 3.10         | 7.07         | 2.94         | 5.10         |

Based on Table 2, it appears that the average pretest of the experimental group is higher than the control group by 0.42. As for the posttest score, the experimental group is higher than the control group by 4.66. From this it can be seen that the descriptive posttest results of mathematical creative thinking ability experimental group is better than control group.

After knowing the descriptive statistics, then is doing inferential statistical analysis. Based on data posttest critical mathematical thinking ability known that the data is normal distribution but not homogeneous. Therefore, the average difference test used is using the t-test. The test results are presented in the table below.

Table 3. T-test mathematical critical thinking ability

| Test statistics | Posttest |
|-----------------|----------|
| $t$             | 3.517    |
| df              | 46.777   |
| Asymp Sig. (2-tailed) | 0.001   |

Based on Table 3, it appears that the significance value (sig 2 tailed) is 0.001, because the test used is 1-tailed test then the significance value (sig 1-tailed) is $0.5 \times 0.001 = 0.0005$. Note that the significance value is less than $\alpha = 0.05$ so the decision taken is reject $H_0$. It can be concluded that at
the specified level of significance, the achievement of students’ mathematical critical thinking abilities that get the RMT approach is better than that of students who have an expository approach.

In data postes mathematical creative thinking ability, note that the data is not normally distributed. Therefore, the test of average difference used is using Mann-Whitney U test. The test results are presented in the table below.

| Test statistics | Posttest |
|-----------------|---------|
| Mann-Whitney U   | 199,000 |
| Z                | -2.040  |
| Asymp Sig. (2-tailed) | 0.041  |

Based on Table 4, it appears that the significance value (sig 2 tailed) is 0.041, because the test used is 1-tailed test then the significance (sig 1 tailed) is 0.5 x 0.041 = 0.0205. Note that the significance value is less than α = 0.05 so the decision taken is reject H₀. It can be concluded that at the specified level of significance, the achievement of students' mathematical creative thinking abilities that get the RMT approach is better than that of students who get the expository approach.

Based on the results of this research that has been presented, there are two characteristics on the RMT approach that is able to develop students’ mathematical critical and creative thinking ability. First, the use of psychological tools as a tool in helping students understand and associate the mathematical concepts learned. Using psychological tools in classroom math learning such as symbols, graphs, drawings, or diagrams that represent a concept will help students in generalizations and mathematical abstractions. This will certainly facilitate students in understanding a concept and connect ideas that they are new to know with the knowledge already owned previously. Aspects of mathematical critical thinking ability that can be developed by using this psychological tool that is the problem-solving ability. While on the mathematical creative thinking ability is fluency in expressing varied and fluent ideas (fluency).

Secondly, learning with RMT approach promotes good mediation between teacher and student. Mediation is in accordance with the concept of mediation that exist in the theory of mediated learning developed by Feuerstein. Mediation in RMT is done by criteria that is intentionality & reciprocity, transcendence, and mediation of meaning. Intentionality & reciprocity mediation can create exciting classroom conditions and focus on the thinking process that puts students active on the material presented. In this mediation, teachers are intensely facilitating students to focus in through every learning activity and students responding or responding positively. In this mediation, the aspect of critical thinking ability that can be developed is to identify a concept and provide a reason for the use of the concept. While the ability to think creatively generate ideas that vary (flexibility). This is supported by the theory put forward by Piaget, the knowledge constructed by the students formed through interaction with the environment through the process of assimilation (process of using or transforming the environment so that it can be placed in preexisting cognitive structure) and accommodation (process of changing cognitive structure in order to accept something from the environment) [13]. Transcendence criteria can make students more explore what has been done with the guidance of teachers as a mediator for subsequent students are able to be active in making their own learning strategies in building many ideas in solving problems. Aspects of critical thinking ability that can be developed that is analyzing the algorithm in solving the problem. While the ability to think creatively is develop or expand an idea and elaborate the ideas. This is in line with the theory of Vygotsky [12] that in achieving ZPD students need assistance (scaffolding) so that the process of assimilation and accommodation going well. Mediation of meaning criteria can make a learning process meaningful. This is done in order to arise a belief that the material taught is directly related to the life and experience of the students. Aspects of critical thinking ability that can be developed is to generalize the concept based on observed data. While on the ability of creative thinking that is
generating a new and innovative mathematical ideas (originality). This is in line with the Ausubel meaningful learning theory, Ausubel believes that meaningful learning involves a recognition of the link between concepts, it has the privilege of being transferred to long term memory and new information is meaningful to extent that it can related to what is already known [14].

Based on these two characteristics, the existence of psychological tools as a tool to bridge the students in connecting ideas about new concepts of previous information have been studied and three mediation criteria that will help students become active, independent, and interested to follow the learning process has a big role to what obtained by the experimental group.

5. Conclusion
Based on the result and discussion, it can be concluded that: the achievement of mathematical critical thinking ability of student who obtain RMT is better than students who obtain expository approach; 2) the achievement of mathematical creative thinking ability of student who obtain RMT is better than students who obtain expository approach.

Teachers can apply learning mathematics with RMT approach to develop students' mathematical creative and critical thinking ability. For readers, further research is needed to observe the implementation of RMT in developing other mathematical ability at other educational levels as well.

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