Robust gravitational wave burst detection and source localization in a network of interferometers using cross-Wigner spectra

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Abstract

We discuss a fast cross-Wigner transform-based technique for detecting gravitational wave bursts and estimating the direction of arrival, using a network of (three) non-co-located interferometric detectors. The performances of the detector as a function of signal strength and source location, and the accuracy of the direction of arrival estimation are investigated by numerical simulations. The robustness of the method against instrumental glitches is illustrated.

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(Some figures may appear in colour only in the online journal)

1. Introduction

The next generation of interferometric detectors of gravitational waves (henceforth GW) including AdLIGO [1], AdVirgo [2] and GEO-HF [3], hopefully to be followed soon by LCGT [4] and ACIGA [5], and eventually by ET [6], is expected to observe tens of events per year, opening the way to gravitational wave astronomy [7]. Identifying the direction of arrival (henceforth DOA) of the signals, and retrieving their shapes, will be a primary task in reconstructing the physics of the sources and their environments.

The possibility of retrieving the DOA from independent estimates of the signal arrival time at each detector was first suggested in [8], and further discussed in Saulson’s seminal book [9]. It was shown that three-interferometers are sufficient to retrieve the DOA up to a mirror-image ambiguity which can be solved in principle from the knowledge of the detectors’ directional responses. This method, often referred to as triangulation, was further elaborated by Sylvestre [10], Cavalier et al [11] and Merkovitz et al [12]. In [11], a Gaussian distribution was assumed.
for the (independent) arrival time estimation errors, and a $\chi^2$ minimization algorithm was accordingly proposed for retrieving the DOA, in the maximum likelihood spirit. In [12], it was shown that this method is affected by a systematic bias in the estimated DOA; a possible technique for removing the bias was discussed, and amplitude consistency tests for removing the mirror-image ambiguity were suggested. Fairhurst developed a similar analysis of the effect of arrival time estimation errors on the DOA estimation accuracy for the special case of chirping signals, including waveform and calibration errors [13, 14].

DOA estimation algorithms are already implemented in the coherent LIGO–Virgo pipelines for GW burst (henceforth GWB) detection [15, 16]. DOA estimation in coherent network data analysis was studied first by Krolak and Jaranowski [17], and then by Pai et al [18], as part of the waveform parameter estimation problem, with specific reference to chirping waveforms from coalescing binaries, in a Gaussian noise background. The conceptual foundations of coherent data analysis for unmodeled waveforms were laid out by Flanagan and Hughes [19], and further developed by Klimek et al [20, 21]. Gürsel and Tinto [22] first suggested the possibility of retrieving the DOA for unmodeled signals using null streams. This concept was analyzed in-depth by Schutz and Wen [23], and further exploited by Chatterji et al [24]. A Fisher-matrix-based analysis of arrival time estimation error in coherent network detection of modeled as well as unmodeled signals was made in Wen et al [25].

In this paper, we capitalize on the time shift and localization properties of the cross-Wigner–Ville (henceforth XWV) transform to introduce a new and conceptually simple GWB detection and DOA reconstruction algorithm, using a network of non-co-located interferometric detectors.

The Wigner–Ville transform is a well-known powerful tool for the analysis of non-stationary signals [26], whose potential in GW data analysis has been highlighted by several authors, under different perspectives [27–29]. Here we suggest its possible use as an effective tool for detecting GWBs, and estimating their DOA, which offers nice features in terms of performance and robustness against spurious instrumental/environmental transients (glitches).

Instead of using independent estimates of the arrival times at each detector, our DOA estimator uses data from (all) detector pairs to estimate the required propagation delays. In addition, it also provides an effective detection statistic, combining the data from all detectors in the network, at a remarkably light computational cost.

DOA reconstruction from arrival-time delay estimation in a network of sensors is a well-known problem in the technical literature on acoustics and radar (see, e.g., [30] for a broad review). The standard method for time-delay estimation in Gaussian noise is the (generalized) cross-correlation [31, 32], which is known to perform reasonably well for relatively large signal to noise ratios (SNRs) [33]. Remarkably, the correlation-based estimator offers worse performances compared to the XWV in the present context, as shown in section 5.2.

This paper is organized as follows. In section 2, we introduce the XWV transform and recall its time-shift properties, which are illustrated for the simplest case of sine-Gaussian (henceforth SG) GWBs. In the same section, we recall the relationship between arrival time delays and DOA. In section 3, we illustrate the proposed XWV transform-based DOA reconstruction algorithm. In section 4, we discuss the effect of noise in the data, and the related DOA reconstruction uncertainties. In section 5, we present the results of extensive numerical simulations, aimed at characterizing the performance of our XWV-based algorithm both as a detector and as a DOA estimator. The simulations are based on SG-GWBs, but the case of more realistic waveforms (including Dimmelmeier and binary merger waveforms) is also discussed. In section 6, we include a short discussion of the robustness of the proposed
2. Rationale: from cross-Wigner–Ville transforms to DOAs

In this section, we recall a relevant property of the XWV transform and illustrate it using ideal (SG) waveforms. We further recall the relationship between the arrival time delays and the DOA for a three-detector network, with special reference to the LIGO–Virgo observatory.

2.1. Cross-Wigner–Ville transforms

The XWV transform built from two (analytic, complex) signals $\tilde{x}_{1,2}$ is given by [34, 35]:

$$W_{12}(t, f) = \int_{-\infty}^{\infty} d\theta \tilde{x}_{1}^*(t - \frac{\theta}{2}) \tilde{x}_{2}(t + \frac{\theta}{2}) \exp(-2\pi if\theta), \quad (1)$$

where $*$ denotes complex conjugation. We recall that the so-called analytic signal corresponding to a generic real-valued waveform $x(t)$ is

$$\tilde{x}(t) = x(t) + i\mathcal{H}[x](t), \quad (2)$$

where $\mathcal{H}[x](t)$ is the Hilbert transform [36]. For $x_1(t) = x_2(t) = x(t)$, equation (1) reduces to the well-known Wigner–Ville transform of $x(t)$.

2.2. Time-shift property of cross-Wigner–Ville transform

Let $T_{\theta}$ be the time-shift operator, such that

$$T_{\theta}[x] = x(t - \theta). \quad (3)$$

The following property of the XWV transform is easily proved:

$$W_{T_{\theta_1}x_1, T_{\theta_2}x_2}(t, f) = \exp[-2\pi if(\theta_2 - \theta_1)]W_{x_1,x_2}\left[t - \frac{\theta_2 + \theta_1}{2}, f\right]. \quad (4)$$

Hence, if $x_1$ and $x_2$ are the same waveform $x(t)$, except for having different amplitudes, and different time-shift (delays), one accordingly has

$$|W_{T_{\theta_1}x_1, T_{\theta_2}x_2}(t, f)| = C\left|W_{x_1,x_2}\left[t - \frac{\theta_2 + \theta_1}{2}, f\right]\right|, \quad (5)$$

where $C$ is an irrelevant (positive) constant.

2.2.1. SG GWBs

To illustrate the practical significance of equation (5) in the context of GW detection of unmodeled transients using a network of interferometers, we will refer here to SG waveforms, which have been widely used to model GWBs. More realistic transient waveforms will be considered in section 5.3. Consider two SG waveforms, with common carrier frequency $f_0$, time spread $T$, and initial phases $\phi_0$, peaked at $t_{1,2}$, with amplitudes $A_{1,2}$, respectively, namely

$$x_i(t) = A_i \cos\left[2\pi f_0(t - t_i) + \phi_0\right] \exp[-(t - t_i)^2/T^2], \quad i = 1, 2. \quad (6)$$

Under the assumption $f_0T \gg 1$, the analytic counterparts of (6) are asymptotically given by

$$\tilde{x}_i(t) \sim A_i \exp\left[2\pi if_0(t - t_i) + \imath\phi_0\right] \exp[-(t - t_i)^2/T^2], \quad i = 1, 2. \quad (7)$$
The XWV spectrum, equation (1), between $\tilde{x}_1$ and $\tilde{x}_2$ can be computed in the closed form, yielding

$$W_{12}(t, f) = W_{21}(t, f) = (2\pi)^{1/2}A_1A_2T \exp\left[-\frac{2\pi^2}{f^2}\left(t - \frac{t_1 + t_2}{2}\right)^2\right] \exp\left[-2\pi f (t_1 - t_2)\right].$$

(8)

It is seen that $|W_{12}(t, f)|$ is peaked at

$$t = \frac{t_1 + t_2}{2}, \quad f = f_0.$$  

(9)

2.2.2. Realistic waveforms. The above peak localization property of the XWV holds true not only for SG waveforms but essentially for all waveforms modeled by oscillatory transients with unimodal envelope, provided the product between the (instantaneous) carrier frequency and the envelope duration is a large number. Under this respect, the SG waveform is a kind of (worst) limiting case in view of its minimal spread property in the time–frequency plane. Indeed, the localization property can be more marked for other transient waveforms, e.g. those numerically generated for supernovas or mergers, as discussed in section 5.3.

2.3. XWV spectra, delays and DOAs

Let us confine ourselves for simplicity to the relevant case of the LIGO–Virgo network, which consists of the three large-baseline detectors located at Livingston, LA (USA), Hanford, WA (USA) and Cascina (Italy), henceforth denoted as L1, H1 and V, and labeled by the suffix $i = 1, 2, 3$, respectively. In the presence of a GWB, in view of equation (9), the three XWV spectra computed from the data gathered by the LIGO–Virgo network interferometers will be (scaled) replicas of the Wigner–Ville transform of the observed GWB, exhibiting magnitude peaks at

$$t = T_{ij} = \frac{\tau_i + \tau_j}{2}, \quad f = f_{ij} = f_0, \quad \{i, j\} = \{1, 2\}, \{1, 3\}, \{2, 3\},$$

(10)

where $\tau_i$ is the GWB arrival time at detector-$i$. The knowledge of the three $T_{ij}$ from the corresponding XWV peaks allows us to retrieve in principle two independent arrival-time delays, e.g.

$$t_{13} = \tau_1 - \tau_3 = 2(T_{12} - T_{23}), \quad t_{23} = \tau_2 - \tau_3 = 2(T_{12} - T_{13}),$$

(11)

from which the DOA, and hence the source location on the celestial sphere can be uniquely inferred, as shown in the next subsection.

2.3.1. DOA from delays. The DOA is easily retrieved from the arrival-time delays using the reference system sketched in figure 1, whose origin is the circumcenter $O$ of the triangle whose vertices are (1) LIGO-Livingston (L1), (2) LIGO Hanford (H1) and (3) Virgo (V), and whose $x$-axis goes, e.g., through L1. In this reference system, the three detectors have spherical polar coordinates

$$(R_i = R, \theta_i = \pi/2, \phi_i), \quad i = 1, 2, 3$$

(12)

5 We implicitly assume the interferometers’ transfer functions as being frequency independent throughout the useful band of the sought signals.
Figure 1. Detectors’ plane and reference system.

and are located at

$$\vec{r}_i = R(\hat{u}_x \cos \varphi_i + \hat{u}_y \sin \varphi_i), \quad i = 1, 2, 3,$$

where $\varphi_1 = 0$ by construction, and $R$ is the radius of the circumference through L1, H1 and V.

Let the source polar coordinates and vector position be $\vartheta = \vartheta_s$, $\varphi = \varphi_s$ and

$$\vec{r} = \rho(\sin \vartheta \cos \varphi \hat{u}_x + \sin \vartheta \sin \varphi \hat{u}_y + \cos \vartheta \hat{u}_z),$$

respectively, where $\rho$ is the distance of the source from $O$. Under the obvious assumption where $\rho \gg R$, one has

$$|\vec{r} - \vec{r}_i| \sim \rho - R \sin \vartheta_s \cos (\varphi_s - \varphi_i), \quad i = 1, 2, 3,$$

whence the delays between the wavefront arrival times at the detectors are

$$t_{ij} = \tau_i - \tau_j = c^{-1}R \sin \vartheta_s [\cos(\varphi_s - \varphi_j) - \cos(\varphi_s - \varphi_i)],$$

where $c$ is the speed of light in vacuum. From the ratio $\xi = t_{13}/t_{23}$, one may accordingly retrieve $\varphi_s$ as follows:

$$\varphi_s = -\tan^{-1} \left[ \frac{(1 - \xi) \cos \varphi_3 + \xi \cos \varphi_2 - \cos \varphi_1}{(1 - \xi) \sin \varphi_3 + \xi \sin \varphi_2 - \sin \varphi_1} \right].$$

Once $\varphi_s$ has been computed, it can be used in (any of) equations (16) to retrieve $\vartheta_s$. Note that the delays (16) do not change upon letting $\vartheta_s \rightarrow \pi - \vartheta_s$, yielding the source mirror image with respect to the detectors’ plane. The above mirror-image ambiguity in a three-detector network is well known\(^6\) [9].

2.4. XWV spectra of noise

As a preparation for the next sections, it is important to characterize the key features of the XWV spectrum of independent, pure stationary Gaussian noise streams (the effect of instrumental transients, also known as glitches, will be discussed in section 6). In this case, the time–frequency levels in the XWV spectrum will be random, and their statistical distribution, in view of the assumed noise stationarity, will be the same for all (discrete) times.

The first two moments of the above distribution can be computed analytically with relative ease [37]. In particular, for all (discrete) frequencies, the average value is zero, and the variance exhibits a piecewise linear dependence on frequency, as sketched in figure 2. The maximum variance occurs at $f = f_s/2$, where $f_s$ is the sampling frequency, and its value depends on the

\(^6\) The mirror-image ambiguity can be resolved, in principle, from the knowledge of the detectors’ pattern functions, featuring different responses in the $\vartheta = \vartheta_s$ and $\vartheta = \pi - \vartheta_s$ directions.
details of the XWV implementation (size, windowing), and the noise level in the data streams (see the appendix for details). It is thus expedient to equalize the XWV time–frequency levels, so as to obtain a uniform (flat) XWV spectrum for pure-noise data streams. To this end, we merely scale the XWV level in each time–frequency pixel to the (computed) standard deviation of the XWV level in that pixel.

3. Estimating DOAs from discrete XWV spectra of noisy data

In practice, the XWV spectra will be computed in discrete form [34], yielding two-dimensional (complex) arrays, rather than continuous time–frequency functions over $\mathbb{R}^2$.

To minimize the effect of time-discretization error, it is convenient to estimate the independent delays corresponding to the largest available baselines, i.e., in our case, $t_{13}$ (L1-V) and $t_{23}$ (H1-V).

Also, in the presence of noise, equations (11) used in (16) will provide a mere estimate of the DOA, whose quality will basically depend on the available SNR, which affects the accuracy whereby the XWV peaks can be identified.

A simple algorithm for seeking peaks in the three LIGO–Virgo XWV spectra which are consistent with the constraints

$$
\begin{align*}
|t_{23}| &= 2 |T_{12} - T_{13}| \leq c^{-1} |\tilde{r}_{23}| \\
|t_{13}| &= 2 |T_{23} - T_{12}| \leq c^{-1} |\tilde{r}_{13}| \\
|t_{12}| &= 2 |T_{23} - T_{13}| \leq c^{-1} |\tilde{r}_{12}|
\end{align*}
$$

expressing the obvious requirements that the wavefront propagation delay between two detectors cannot exceed the limiting value corresponding to propagation along the line-of-sight direction between the detectors can be now formulated. The algorithm uses the three (discrete, noisy) XWV spectra to construct a grid in the time-delay plane $(t_{13}, t_{23})$, and assign
different levels $R$ to its nodes:

initialize all time-delay grid node levels to zero

for all time-delay frequency pairs $(T_{12}, f_{12})$ in $W_{12}$

for all time-delay frequency pairs $(T_{13}, f_{13})$ in $W_{13}$ such that:

2$|T_{12} - T_{13}| \leq c^{-1}|\vec{r}_{12}|$ and $f_{13} = f_{12}$

for all time-delay frequency pairs $(T_{23}, f_{23})$ in $W_{23}$ such that:

2$|T_{12} - T_{23}| \leq c^{-1}|\vec{r}_{12}|$ and $f_{23} = f_{12}$

accumulate level $R = R + |W_{12}(T_{12}, f_{12})W_{13}(T_{13}, f_{13})W_{23}(T_{23}, f_{23})|$

at grid node $\{t_{13} = 2(T_{12} - T_{23}), t_{23} = 2(T_{12} - T_{13})\}$

end for

end for

end for

A candidate DOA is obtained by taking the highest-level grid node in the $(t_{13}, t_{23})$ plane subset defined by the further constraint\(^7\)

$|t_{13}| = |t_{13} - t_{23}| \leq c^{-1}|\vec{r}_{12}|.$

The highest level in the grid can be used both as an estimator of the DOA, and as a detection statistic, whose performances will be discussed in section 5.1.

Note that the proposed algorithm is coherent, in the sense that it produces a single detection statistic by combining the data from all detectors in the network. It also inherits the typical features of coincident tests: the outermost loop enforces frequency consistency, while the two inner loops enforce time-delay admissibility.

Note also that the XWV spectra will display sensible peaks only if the waveforms gathered by the different detectors are consistent in shape. This suggests that the algorithm will be robust against (independent) instrumental disturbances, as further illustrated in section 6.

3.1. LIGO–Virgo network directional response under the XWV-based algorithm

In the absence of noise, the above algorithm will produce a peak in the time-delay grid whenever a GWB is observed by the LIGO–Virgo network. The peak will be located at a node whose time-delay coordinates correspond to the DOA $(\theta_s, \phi_s)$. This peak will be well localized, provided the duration of the transient signal is substantially shorter than the minimum graviton flight time between detectors.

In view of the bilinear nature of the XWV, it can be argued that the peak height will be proportional to the squared product of the three detectors’ pattern functions along that direction. This quantity, normalized to its maximum, and denoted henceforth as $\Phi(\theta, \phi)$ describes the directional response of the proposed GWB detector/DOA estimator, and is plotted in figure 3 for the LIGO–Virgo network, for circularly polarized GWs. We checked numerically that the expected (normalized) levels of the time-delay grid peaks reproduce those computed from the function $\Phi$ for each DOA in a $\theta, \phi$ grid of $50 \times 100$ points (using $10^3$ noise realization for each DOA).

7 As shown in section 4, the bound in (19) can be made slightly tighter.
Figure 3. Normalized directional response of the proposed network detection statistic (H1–L1–V network, circular polarization).

Figure 4. Angular fraction of the celestial sphere where normalized directional response exceeds a given level $\Phi_{\text{min}}$.

The quantity

$$\frac{\Omega[\Phi_{\text{min}}]}{4\pi} = \frac{1}{4\pi} \int_{\Phi(\theta, \phi) > \Phi_{\text{min}}} \sin \theta \, d\theta \, d\phi$$

(20)

expresses the fraction of the (unit) celestial sphere where the (normalized) directional response of the proposed detector/estimator exceeds the threshold value $\Phi_{\text{min}}$ and is displayed in figure 4.
It is seen that roughly 50% of the celestial sphere is covered with $\Phi_{\text{min}} \geq 0.2$ by the LIGO–Virgo network, using the proposed algorithm.

4. DOA reconstruction uncertainties

Uncertainties in the DOA reconstruction stem from a twofold origin: the discreteness of the time-delay grid, due to the discrete implementation of the XWV spectra (finite-time resolution), and the additive noise in the data (see the discussion in section 5.2).

In order to translate the effect of systematic and statistical errors in the estimated delays into uncertainty ranges in the estimated DOAs, it is expedient to introduce the projection which maps the DOA polar angles $(\vartheta_s, \phi_s)$ into a point $(x_s, y_s)$ of the disk (with center $O$ and radius $R$) going through the detector, namely

$$
x_s = R \sin \vartheta_s \cos \phi_s, \
y_s = R \sin \vartheta_s \sin \phi_s.
$$

The formula which relates the $(x_s, y_s)$ projection to the arrival-time delays is obtained from equation (16):

$$
\begin{align*}
x_s &= c(t_{13} - t_{23}) \sin \varphi_3 - t_{13} \sin \varphi_2 \\
y_s &= c(t_{13} \cos \varphi_2 - (t_{13} - t_{23}) \cos \varphi_3 - t_{23}) - \sin(\varphi_2 - \varphi_3) + \sin \varphi_2 - \sin \varphi_3
\end{align*}
$$

Equation (22) is a linear transformation, relating not only the coordinates $(x_s, y_s)$ to the delays $(t_{13}, t_{23})$ but also the uncertainties $\delta x_s, \delta y_s$ to the delay errors $\delta t_{13}$ and $\delta t_{23}$. Thus, under the simplest assumption where these latter are independent and identically distributed, the uncertainty region in the $(x_s, y_s)$ plane is an ellipse. Notably, the shape of this latter is translation invariant across the circle $x_s^2 + y_s^2 \leq R^2$, i.e. DOA independent.

The ratio between the uncertainty areas in the $(x_s, y_s)$ and $(t_{13}, t_{23})$ planes is given by the Jacobian of the transformation (22), namely

$$
J = c^2 \frac{\sin(\varphi_3 - \varphi_2)}{\sin \varphi_3 - \sin \varphi_2 + \sin(\varphi_2 - \varphi_3)}
$$

which is also DOA-independent.

By back-projecting the uncertainty ellipse onto the sphere of radius $R$ centered at $O$, we obtain a DOA-dependent uncertainty region. This is illustrated in figure 5 for a few representative cases. The ratio between the area of the uncertainty region on the celestial sphere, and the area of the uncertainty ellipse in the $(x_s, y_s)$ plane is displayed in figures 6(a) and (b) as a function of $\varphi_s$ for various values of $\vartheta_s$. For $\vartheta_s \sim 0$, this ratio is close to unity, whatever $\varphi_s$, as shown in figure 6(a). On the other hand as $\vartheta_s \to \pi/2$, the ratio blows up, and its dependence on $\varphi_s$ is shown in figure 6(b). Such behavior has been noted in [11, 13].

5. Numerical experiments

In order to check the performance of the proposed algorithm, we run a series of Monte Carlo simulations. The simulations use time-discretized GWBs and glitches injected into white (independent) random Gaussian sequences to represent the three interferometer data. In the case of GWBs, the delays are chosen according to the assumed source location.

Our XWV engine uses data chunks 2048 time samples wide to produce a $1024 \times 1024$ time–frequency node XWV transform, using the Pei–Yang fast algorithm [38]. The sampling
frequency is 4 KHz. The data are decomposed accordingly into half-overlapping chunks 2048 time samples wide (we use a plain rectangular windowing function), in order to use a fixed number of time samples to compute each time–frequency sample. The resulting discrete XWV spectrum spans the time range between samples 513 and 1536, and the frequency range between 0 and 1000 Hz. As already mentioned, the XWV values are equalized so that in the absence of signals their first and second moments are 0 and 1, respectively.

Now, even in the absence of a signal, the levels produced by our algorithm in the time-delay plane \((t_{13}, t_{23})\) grid nodes will be non-uniform, due to the different number of (noisy) time–frequency XWV values mapped into each node. The average and standard deviation in the \((t_{13}, t_{23})\) plane for pure-noise (stationary, Gaussian) data are shown in figure 7. We accordingly equalize the time-delay grid levels by subtracting the average and dividing the result by the standard deviation, so that in the absence of signals, the grid-node levels in the time-delay plane will have zero average and unit variance.

For each injected waveform, we generated \(10^4\) different noise realizations to test the statistical properties of the proposed algorithm, both as a detector and as a DOA estimator. The waveforms were parameterized by their intrinsic SNR, defined by

\[
\delta_h = \frac{h_{\text{rss}}}{N} = \left\{ \int \left[ h_x^2(t) + h_y^2(t) \right] dt \right\}^{1/2} / N,
\]

\(N\) being the (two-sided) power spectral density of the stationary white (whitened) Gaussian noise component, assumed for simplicity the same in all detectors (the effect of glitches will be discussed in section 6).

9 Note that when using analytic signals for computing discrete versions of the XWVT, the minimum sampling rate must be twice the Shannon rate [34].

10
The results of our simulations are summarized below.

5.1. Detection performance

The performance of our algorithm as a detector is illustrated in figures 8 and 9. The detection statistic is the level of the highest peak in the time-delay grid. Figures 8(a) and (b) display the false alarm (continuous line) and false dismissal probabilities (the dashed lines) corresponding...
to different values of the intrinsic SNR ($\delta h$) as functions of the detection threshold $\gamma$ for DOAs corresponding to the maximum ($\delta = 0.705$ rad, $\varphi = 5.073$ rad) and the minimum ($\delta = 0.800$ rad, $\varphi = 1.100$ rad) of the network pattern function in figure 3. Figures 9(a) and (b) show the receiver operating characteristics, i.e. the detection probability versus the false alarm probability, for fixed values of the intrinsic SNR, $\delta h$, for a DOA corresponding to the maximum of the network pattern function in figure 3.

5.2. DOA estimation performance

As already mentioned, the finite-time resolution implies that the estimated delays are affected by a systematic uncertainty which can be twice the XWV time step $\delta t$. The noise in the data entails that estimated delays spread around the actual delays in a signal-to-noise-ratio-dependent way. This is illustrated in figures 10 and 11. The estimate is always unbiased whenever the signals are shorter than the minimum graviton flight time between the detectors. Figure 10 displays the standard deviation of the estimated delays (average between the two) as a function of the intrinsic SNR for DOAs corresponding to the maximum ($\delta = 0.705$ rad, $\varphi = 5.073$ rad) and the minimum ($\delta = 0.800$ rad, $\varphi = 1.100$ rad) of the network pattern function in figure 3. In a log–log scale, both curves show the same slope, corresponding to an exponent $\approx -1.4$. Figure 11(a) displays the empirical distribution of the estimated delays for $10^4$ different noise realizations for a DOA corresponding to the maximum of the network pattern function in figure 3 for two different values of the intrinsic SNR.

It is interesting to compare figure 11(a) with figure 11(b), where a standard correlation-based time-delay estimator [39] has been used to retrieve the two propagation delays. Our XWV-based estimator is seen to offer distinctly better performances.

5.3. Realistic waveforms

As anticipated in section 2, the XWV transform peak localization property holds not only for SG waveforms but also for general transient waveforms. This is further illustrated in figures 12–15.

Figure 12 (top) shows two copies of a typical supernova GWBs, belonging to the family computed by Dimmelmaier et al [40], with a time shift of 82 time samples (corresponding to 20.5 ms at our sampling frequency), together with their XWV transform. The XWV is
Figure 8. False alarm and false dismissal probability versus threshold for various values of the intrinsic SNR ($\delta h$ in equation (24)). Source at maxima (a) and minima (b) of the pattern function in figure 3.

Figure 9. ROCs for various values of the intrinsic SNR ($\delta h$ in equation (24)). Source at the maxima of the pattern function in figure 3.

identical to the Wigner transform of the GWB waveform, except for the time shift given by equation (5). Accordingly, the XWV peak in figure 12 (bottom) is localized at the midpoint between the peak times of the two waveforms in figure 12 (top). Figure 13 (left) shows the time-delay grid histogram when this waveform is emitted by a source located in the direction of maximum network sensitivity in figure 3 for $\delta h = 100$. It can be seen that, not unexpectedly, the localization properties in the delay plane are even better than for Gaussian waveforms in view of the larger time-bandwidth figure of the Dimmelmaier waveform.

Figures 14 and 15 are similar to figures 12 and 13, except that the waveform here is that of a typical binary merger [41].
Figure 10. Standard deviation (in time bins) of estimated arrival times versus intrinsic SNR ($\delta_h$ in equation (24)). Source at maxima (dashed line) and minima (solid line) of the pattern function in figure 3.

Figure 11. (a) Normalized binning (histogram) of estimated time delays for two representative values of the intrinsic SNR: $b_0=15$ (left) and $b_0=35$ (right). Source at the maximum of the pattern function in figure 3. (b) Same as (a), but a standard correlation-based time delay estimator has been used to retrieve the propagation delays $t_{12}, t_{13}$.
Figure 12. Supernova core-collapse GWBs according to Dimmelmaier (top) and XWV transform (bottom).

Figure 13. Normalized density (left) and surface plot (right) of time-delay grid levels. Supernova core-collapse GWB (Dimmelmaier). Source at the maximum of the pattern function in figure 3.
Figure 14. Binary merger GWBs according to Baker (top) and XWV transform (bottom).

Figure 15. Normalized density (left) and surface plot (right) of time-delay grid levels. Binary merger GWB (Baker). Source at the maximum of the pattern function in figure 3.
6. Glitch rejection

By construction, the proposed detection/localization algorithm should be robust against spurious transients of environmental/instrumental origin (glitches).

We may expect that glitch-induced false detection may occur only in the (unlikely) case where each detector shows a glitch in the analysis window, such that the mutual delays are consistent with an acceptable DOA, and the (independent) glitch waveforms are consistent in shape.

In order to illustrate these features, we consider first the no-GWB case where a glitch occurs in the data of each of the three interferometers, the three glitches being different, but with delays consistent with an admissible DOA.

To this end, we used a set of $N = 7$ visually different waveforms, shown in figure 16 (top), from the catalogue of ‘typical’ LIGO glitches compiled by Saulson [42]. All glitches in the set were scaled to the unit norm, and time shifted so as to bring their envelope peaks to coincidence. The (normalized) pairwise correlation coefficient of the selected glitches, which provides some quantitative measure of their (dis)-similarity, does not exceed 0.62, with mean and median values of 0.195 and 0.105, respectively. The correlation coefficient histogram is shown in figure 16 (bottom left).

From this glitch set, we formed (all) 35 triplets of different waveforms and computed the related X-Wigner transforms and the peak levels in the time-delay grid produced by our algorithm. For each glitch-triplet $(g_1, g_2, g_3)$, we also computed the geometric mean of the time-delay grid peak levels for the three cases where the data from all interferometers contain the same waveform $g_i$, $i = 1, 2, 3$. This quantity was used to re-scale the time-delay grid peak level for that glitch-triplet.

The histogram of the rescaled time-delay grid peak levels for the 35 different glitch-triplets considered is shown in figure 16 (bottom right). The largest (scaled) peak level was 0.51, with a median value of 0.12 and a mean of 0.17.

These, admittedly limited, results illustrate the waveform-consistency test capabilities of the proposed algorithm.

We next consider the case where GWBs and glitches co-exist in the data. In these further simulations, we used SG glitches and GWBs, with glitch parameters (center frequency, peak position and carrier frequency) generated randomly and independently in each detector. The pertinent results are illustrated in figures 17–20. These figures show the noisy waveforms (left column), the density maps of the XWV transforms (mid column) and the (normalized) level map in the $(t_1, t_13)$ time-delay grid.

In figure 17, we consider the simplest case where the GWB data in a single detector (H1 in this case) are corrupted by a single glitch in the analysis window. The GWB SNRs are 22.16 (L1), 23.13 (H1) and 31.85 (V) for a (circularly polarized) source with $\delta_h = 50$ at $\vartheta = 2.58$ rad and $\varphi = 2.71$ rad. The glitch SNR in H1 is 22.59. The glitch shows up clearly in the H1 data, and produces evident artifacts in the H1-V and H1-L1 XWV spectra. Nonetheless, its effect on the time-delay level map is almost negligible.

In figure 18, we consider the (unlikely) case where the data in each detector are corrupted by (single) glitches in the analysis window, with SNR values of 11.5 (L1), 12.00 (H1) and 16.53 (V). None of the glitches has a significant overlap with the GWBs; nonetheless, they produce artifacts in all XWV transforms. Also in this case the effect of these artifacts on the detection/localization properties is negligible, as seen from the time-delay level map.

10 These correspond to a source equidistant from all detectors radiating the waveform $g_i$, with all detectors exhibiting the same response in the source direction. The last assumption is unrealistic, but this is irrelevant for the present purpose.
Figure 16. Top: set of typical glitches (amplitude versus time bin) from [42], scaled to unit energy. Bottom-left: histogram of related pairwise correlation coefficients. Bottom-right: histogram of normalized time-delay grid-peak levels for triplets of different glitches.
Figure 17. GWB and single glitch (nonoverlapping) in one detector (HI). Left: waveforms; middle: XWV spectra; right: normalized density plot of time-delay grid levels.
Figure 18. GWB and single glitch (non overlapping) in each detector. Left: waveforms; middle: XWV spectra; right: normalized density plot of time-delay grid levels.
Figure 19. GWB and single glitch (overlapping) in one detector (H1). Left: waveforms; middle: XWV spectra; right: normalized density plot of time-delay grid levels.
Figure 20. GWB and single glitch (overlapping) in each detector. Left: waveforms; middle: XWV spectra; right: normalized density plot of time-delay grid levels.
Not unexpectedly, the localization performance deteriorates significantly in the rather extreme situation where glitches overlap the GWBs in the data. When this happens in such a way that true peaks in the XWV transforms are no longer resolvable from spurious ones, the time-delay level map topography is substantially blurred in the neighborhood of the true delays, resulting into more or less severe localization errors. This is illustrated in figures 19 and 20.

In figure 19, we have a (single) GWB-overlapping glitch in H1 with SNR = 16.28. In figure 20, each detector is affected by a GWB-overlapping glitch. The glitch SNR values in figures 19 and 20 are the same as those in figures 17 and 18. A sensible distortion in the XWV transforms is observed, entailing a sensible error in the estimated delays.

7. Conclusions

We presented a simple, computationally light and fast algorithm for detecting short unmodeled GWBs in a network of three interferometric GW detectors, and estimating the related DOA, based on XWV spectra. The algorithm is reasonably performant, and nicely robust against spurious transients (glitches) of instrumental origin corrupting the (otherwise Gaussian) detector noise floor. It does not provide waveform reconstruction; this, however, can be accomplished in principle off-line, once the DOA has been estimated.

Generalization to larger networks and other potentially interesting waveforms (e.g. chirps) is relatively straightforward. Such extensions will be explored in a forthcoming paper.

Based on the above preliminary results, we suggest that the proposed algorithm may be used as a quick-and-(not-so)-dirty online data-sieving tool. A quantitative comparison with existing GWB detection/DOA estimation algorithms in terms of efficiency and computational burden will be the subject of future investigation.

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Appendix. XWV moments

Let

\[ W_{f,g}(n,m) = \sum_{k=-L}^{L} f(n+k)g^*(n-k) \exp[-4i\pi^{-1} \pi mk] \]  \hspace{1cm} (A.1)

be the discrete version of the XWV spectrum, where \( n \) and \( k \) are the discrete time and frequency index. Note that, formally,

\[ W_{f,g}(n_0, m) = DFT_x(2m), \]  \hspace{1cm} (A.2)

where \( x = f(n_0 + k)g^*(n_0 - k) \). This shows that while the time index \( n \) spans the range \((-L, L) \cap \mathbb{N}\), the frequency index \( m \) spans the range \((-L/2, L/2) \cap \mathbb{N}\).

Let

\[ \tilde{\nu} = \nu + i\mathcal{H}\nu \]  \hspace{1cm} (A.3)

be the analytic version of the background noise, and denote as \( \nu(k) \) and \( \nu_H(k) \) the (real-valued) samples of the noise and its Hilbert transform.
For zero average Gaussian white noise with (two-sided) power spectral density \( W_0 \), the spectral power density of analytic noise is
\[
S_{\nu,\nu} = 4W_0 U[\theta(m)],
\]
where \( U(\cdot) \) is Heaviside’s function and \( \theta = 2\pi m/N \in (-\pi, \pi) \), \( N = 2L + 1 \) being the number of DFT frequency samples.

It is a simple task to show that the first moment of the XWV is zero, in view of the assumed independence of the (Gaussian) noises in different detectors.

We now compute the second moment, namely
\[
\sigma^2(n, m) = E\left[ W_{\nu,\mu}[n, \theta(m)]W_{\nu,\mu}^*[n, \theta(m)] \right]
\]
\[
= \sum_{k=-L}^{L} \sum_{p=-L}^{L} E[v_n(n + k)v_n^*(n + p)\mu_{\mu}(n - p)]e^{-i2(k-p)\theta(m)}
\]
\[
= \sum_{k=-L}^{L} \sum_{p=-L}^{L} E[v_n(n + k)v_n^*(n + p)]E[\mu_{\mu}(n - p)\mu_{\mu}^*(n - k)]e^{-i2(k-p)\theta(m)}
\]
\[
= \sum_{k=-L}^{L} \sum_{p=-L}^{L} R^2_{\nu,\nu}(k - p)e^{-i2(k-p)\theta(m)},
\]
where \( v_n \) and \( \mu_{\mu} \) are built from independent, zero average, white (whitened) Gaussian noises pertinent to different detectors, but assumed as having the same power spectral density \( W_0 \), and \( R_{\nu,\nu}(h - k) = E[v_n(h)v_n^*(k)] \) denotes the autocorrelation function of analytic noise. The inner summation in (A.5) can be extended to \( \infty \), and the Wiener–Khinchin theorem can be invoked to prove that
\[
\sigma^2(n, m) \approx \sum_{k=-L}^{L} \sum_{p=-\infty}^{\infty} R^2_{\nu,\nu}(k - p)e^{-i2(k-p)\theta(m)}
\]
\[
= \sum_{k=-L}^{L} S_{\nu,\nu}[2\theta(m)] * S_{\nu,\nu}[2\theta(m)]
\]
\[
= 16W_0^2 \sum_{k=-L}^{L} \left| \frac{\theta}{\pi} \right| = (2L + 1)6W_0^2 \left| \frac{\theta}{\pi} \right| = 32W_0^2 |m|,
\]
where \( m \in [-L/2, L/2] \cap N. \)

References

[1] http://www.advancedligo.mit.edu
[2] https://wwwcascina.virgo.infn.it/advirgo
[3] Willke B et al 2006 The GEO-HF project Class. Quantum Grav. 23 S207
[4] http://gw.icrr.u-tokyo.ac.jp/lcgt
[5] http://www.anu.edu.au/physics/ACIGA
[6] http://www.et-gw.eu
[7] Anderson N and Kokkotas K D 2005 Gravitational wave astronomy: the high frequency window Lecture Notes Phys. 653 255
[38] Pei S C and Yang I I 1992 Computing pseudo-Wigner distribution by the fast Hartley transform *IEEE Trans. Signal Proc.* **40** 2346

[39] Marple S L 1999 Estimating group delay and phase delay via discrete time analytic cross correlation *IEEE Trans. Signal Proc.* **47** 2604

[40] Dimmelmeier H, Ott C D, Marek A and Janka H T 2008 Gravitational wave burst signal from core collapse of rotating stars *Phys. Rev. D* **78** 064056

[41] Baker J G *et al* 2007 Comparisons of binary black hole merger waveforms *Class. Quantum Grav.* **24** S25

[42] Saulson P 2007 Listening to glitches *LIGO Document G070548*