Numerical analysis of collapse load and specific strength for moderate-walled Ti6Al4V-laminate composite submersible vessel under external hydrostatic pressure

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Abstract. The effects of geometrical imperfections on the collapse load and specific strength of moderate-walled submersible vessel made of Ti6Al4V-laminate composites and Ti6Al4V alloy, respectively, have been investigated in this study. Two kinds of imperfections, which are the initial ovality along cross-section direction and the winding-induced waves along longitudinal symmetric axis, are considered. Two numerical analyses including the non-linear finite element (FE) and elastic-plastic buckling analysis are performed by the commercial software MSC Marc/Mant 2005. The obtained results indicate that initial imperfections have an obvious influence on the collapse loads of both the Ti6Al4V-laminate composite submersible vessel and the Ti6Al4V one. The collapse load is mainly governed by the elastic-plastic buckling critical load, which is conservative and safe for a vessel to bear the admissible failure load under the deepwater. Moreover, the Ti6Al4V-laminate composite submersible vessel has a better capability to resist imperfections than the Ti6Al4V one.

1. Introduction

Deepwater submersibles are typically constructed by Ti6Al4V alloy for their high strength. However, these submersibles consume plenty of energy due to their high ratios of weight to displacement, which is disadvantageous for their long endurance requirement under the deepwater. It is well-known that the composite materials have the high specific strength, ratios of low weight to displacement, and good corrosion resistance, and thus they have become potential choices for submersible vessels which are generally fabricated by using multilayered composites with metallic materials such as Ti6Al4V alloy wrapped outside and closed at their ends [1]. The existing numerical and experimental studies have shown that vessels can be considered to be relatively free of other loads but an external hydrostatic pressure when they reach the deepwater. The collapse strength and elastic buckling phenomenon of vessels are the most important considerations in their design.

Many important factors strongly influence the collapse strength and buckling of a vessel. The initial geometric imperfections are usually assumed to be a main source of buckling load reductions for both isotropic [3] and composite [4] cylinders under hydrostatic pressure, which lead to considerable discrepancies (sometimes exceeding more than 50%) between the experimental results and those from perfect cylinder model predictions [5]. The effects of initial geometric imperfection
have been theoretically analyzed based on non-linear shell theory for the loadings cases of pure torsion [6] and combined axial compression and torsion [7]. The influence of the winding-induced longitudinal thickness imperfection on the elastic buckling load of laminated cylinders has also been investigated [8], and the comprehensive reviews for the buckling and post-buckling analyses of shells have been reported[9]. However, a thin-walled shell, whose ratio of diameter $D$ and wall-thickness $t$ is below 10, is used as a main model in most investigations, in which the transverse shear effect is always ignored. In fact, vessels with $D/t$ between 10 and 40, which belong to the moderate-walled, are widely used for the deepwater applications nowadays. In this case, the collapse pressure is largely determined by the inelastic behavior of materials, so it is difficult to derive an ideal non-linear equation with material anisotropy. Tamano [10] and Yeh [11] used their empirical equations to analyze only the effects of longitudinal geometric imperfections on the collapse strength of vessel.

However, no attempt has been made to investigate the influence of both the above imperfections so far. Thus, a moderate-walled self-service exploring submersible vessel is chosen as a research subject in this study to study the effects of both the initial longitudinal and transverse imperfections on the elastic-plastic collapses load and specific strengths of moderate-walled submersible vessel. The collapse loads of imperfective vessel with shear deformations are determined using the non-linear FE method. Two kinds of numerical analyses are performed by using the commercial software MSC Marc/Mant 2005[12]. The first is the non-linear FE analysis, in which the branching load is taken as the collapse load in the pressure-deformation curve. The second is the elastic-plastic buckling analysis, in which the mode corresponding to the lowest eigenvalue load shall determine the actual collapse mode of the vessel.

2. Analytical model

2.1. Geometries and material properties

Most classically investigated thin cylinders subjected to internal pressure so far are based on 55° orthotropic sequences. However, a stress distribution for a moderate-walled vessel is definitely different with that for a thin-walled shell [11] and transverse shear effects cannot be ignored. Hence the special configurations of laminates are made in this study as shown in Figure 1. The vessel is made of Ti6Al4V alloy in the inner layer (15 mm), stiffener and closed ends, attached by some Carbon Fiber Reinforced Polymer (CFRP) composites with different stacking sequences in the middle-layer (15 mm) and the outer layer (8.25 mm) to resist the bending and twisting deformations (The details of stacking sequences are not shown in Figure 1 because of its current application for a patent). The length of the vessel is $l = 1400$ mm, the inner diameter of the cylinders is 280 mm, the wall thickness is 30 mm, and the space of each ring stiffener is 100 mm. The ratio of the outer-diameter $D$ to the wall's thickness $t$ of the vessel is 11.3, indicating that this vessel is a moderate-walled one.

![Figure 1. Stacking sequence for a deepwater vessel made of the Ti6A14V-laminated composites (half of the model).](image)

The material properties of the Ti6A14V-laminate composites are listed in Table 1, which are calculated by using the classical Mori-Tanaka method. $E$, $v$, $G$ denote Young’s modulus, Poisson’s
ratio, and shear modulus, respectively; subscript 1 represents the fiber directions, the subscript 2 and 3 are the directions orthogonal to fiber direction. Hill’s failure criteria are used for laminate composites. A simple boundary condition is to clamp either of the centers of two semi-spheres, which is selected for a free vessel to avoid rigid displacement.

\[
x = D / 2, \quad w = 0, \quad \frac{\partial w}{\partial x} = 0
\]

(1)

| Stacking sequence | \(E_{11}\) (MPa) | \(E_{22}\) (MPa) | \(E_{33}\) (MPa) | \(G_{11}\) (MPa) | \(G_{23}\) (MPa) | \(G_{13}\) (MPa) |
|-------------------|-----------------|-----------------|-----------------|----------------|----------------|----------------|
| \([0_2/90_s]\)   | 6445.4          | 91404.6         | 10544.8         | 5154.8         | 4566.8         | 4566.8         |
| \([0_3/90_s]\)   | 91404.6         | 64454.6         | 10544.8         | 5154.8         | 4566.8         | 4566.8         |
| \([45/-45_s]\)   | 40305.4         | 40305.4         | 10544.8         | 37151.8        | 4566.8         | 4566.8         |
| \([/0/90/45/-45_s]\) | 61600.1       | 61600.1         | 10544.8         | 21153.8        | 4713.8         | 4713.8         |

Note: The subscript 2 and 3 represent the layers of fiber with the same angle, and the subscript ‘s’ means that the stacking sequence is symmetric.

2.2. Constitutive behavior and non-linear buckling analysis

Since a metallic material has usually a strong plastic behavior, the non-linear constitutive equation is very crucial for accurate prediction of the collapse process of a submersible vessel, and the stress-strain relationship is calculated by Ref. [13]

\[
\sigma = E \varepsilon \quad \text{(Elastic stage)}
\]

(2)

\[
\sigma - \sigma_p = \frac{E(\varepsilon - \varepsilon_p)}{\sqrt{1 + b^2(\varepsilon - \varepsilon_p)^2}} \quad \text{(Plastic stage)}
\]

(3)

and

\[
b^2 = \frac{E^2}{(\sigma_{0.2} - \sigma_p)^2 - (\varepsilon_{0.2} - \varepsilon_p)^2}
\]

(4)

Where \(\sigma\) and \(\varepsilon\) is a stress and strain, respectively. \(\sigma_{0.2}\) is the stress for 0.2% strain \(\varepsilon_{0.2}\), \(\sigma_p\) is the proportional limitation, and \(\varepsilon_p\) is the strain corresponding to \(\sigma_p\). All the material parameters are listed in Table 2.

| \(E\) (MPa) | \(\nu\) (MPa) | \(\sigma_p\) (MPa) | \(\varepsilon_p\) (MPa) | \(\sigma_{0.2}\) (MPa) | \(\varepsilon_{0.2}\) (MPa) |
|------------|---------------|-----------------|----------------|----------------|----------------|
| 110544     | 0.3           | 733.04          | 0.00663        | 833            | 0.00954        |

Since the large deformation, material non-linearity, and collapse load of submersible vessels are governed by the non-linear equations of the stress, strain, and displacement. The non-linear buckling analysis is performed by the commercial code MSC Marc/Mant 2005. A nonlinear static analysis with gradual increase in loads is employed to seek a load level at which the structure becomes unstable. The effective way of 3D hexahedral iso-parameter elements with large displacement and large strain additive is used to determine the ultimate collapse load and to analyze the shear deformation. 26464 elements are employed to mesh the model (Figure1) and the mesh density has been set-up after conducting convergence tests. In order to find a suitable size that leads to reliable results with a reasonable computational time, the arc length method with automatic time step is used in this study.

The large deformation is taken by considering the non-linear stiffness matrix \([K_{NL}]\) and the analyses are performed like usual static non-linear problems using an incremental procedure integrated with an iterative one. In the incremental procedure, the displacement, which grows linearly with a fictitious time (varying from 0 to 1), is applied by increment. The stiffness matrix is updated at the end of each
increment in order to take into account the evolution of the non-linear phenomena. For each increment, \( k \) the equilibrium is expressed as follows:

\[
([K_L] + [K_{NL}])_k \{\Delta d_k\} = \{\Delta F_{e,k}\}
\]  

(5)

Where \([K_L]\), \([\Delta d_k]\) and \([\Delta F_{e,k}]\) are the linear stiffness matrix, the vectors of nodal incremental displacements, and loads, respectively.

Owing to the assumption of linearity in the increment, the external load system given by Eq. (5) is not equilibrated by internal load system, \([\Delta F_{i,k}]\) is equal to:

\[
\{\Delta F_{i,k}\} = \int_B [B]^T \{\sigma\} dV
\]

(6)

Where \([B]\) is a matrix that links strains in the element to nodal displacements and \([\sigma]\) is a stress vector. A modified Newton-Raphson iterative procedure with non-positive definite solution technique is used to solve non-linear equations with collapse loads, in which the tangent stiffness matrix is updated each iteration in order to reach an equilibrium between external loads and internal reactions with required tolerance \(\varepsilon\)

\[
\{\Delta F_{e,k}\} - \{\Delta F_{i,k}\} \leq \varepsilon
\]

(7)

In a non-linear procedure, it is possible to examine the behaviors of the structure during the entire range of the applied load, which includes the pre-buckling, buckling, and post-buckling phases.

2.3. Elastic-plastic buckling analysis of critical load

An elastic-plastic buckling eigenvalue analysis is performed to determine the critical loads and the condition of neutral equilibrium between external loads and internal reactions is searched to solve the equation:

\[
([K_0] + \lambda_i[K_\sigma])\{d\}_i = 0
\]

(8)

Where \([K_0]\) is a stiffness matrix for small and large deformation range, which is controlled by 'the types of buckling problems' in MSC Marc/Mant 2005. Since the structure keeps a curve for the imperfection of vessel, the large deflection and yielding are considered for the calculation in this study. \([K_\sigma]\) is a geometric stiffness matrix corresponding to a reference load; \(\lambda_i\) is an eigenvalue that is the load factor to multiply the reference load to obtain the critical value, only the smallest value is of practical interest; and \([d]\) is a displacement vector.

The determination of the eigenvalues is performed by the elastic-plastic buckling FE model with large displacement and Lanczos method, and the plastic constitutive equations described in Eq. (2) to (4) are also used in the process.

2.4. Modeling of geometric imperfections along cross-section direction

According to the Timoshenko's assumption, the initial ovality \(u\) is introduced by the following equations:

\[
r = R + u \cos(2\theta) \quad (\theta = 0.180)
\]

(9)

and

\[
u = \alpha \times R
\]

(10)

Where \(r\) is a radius of imperfective vessel, \(R\) is an inner radius of the original vessel, \(\alpha\) is an imperfective coefficient, and \(\theta\) is an angular coordinate measured from the centre of the vessel as shown in Figure 2 (d). It is assumed that all the geometrical variables involved in the analysis are uniform as shown in Figure 2 because the geometric variables and imperfections are found not to substantially vary along longitudinal direction of the vessel. The imperfections are introduced to allow the buckling to occur although they are not plainly visible.
2.5. Modeling of geometric imperfection along longitudinal direction

For the imperfections along longitudinal direction, Guggenberger gave a FE analysis for the deep single longitudinal initial dent [6]. In this study, three kinds of typical axisymmetric waves are taken as the initial imperfections:

1. The centerline path is sinusoidal function in x-z plane, given along x axis.

\[ z = A \sin\left(\frac{2\pi x}{L}\right) \]  \hspace{1cm} (11)

where \( A \) is an amplitude of the waviness and \( L \) is the length of the vessel.

2. The centerline path is half sinusoidal function in x-z plane, given along x axis.

\[ z = \frac{A}{2} \sin\left(\frac{\pi x}{L}\right) \]  \hspace{1cm} (12)

3. The centerline path is hyperbola function in x-z plane, given along x axis.

\[ \frac{x^2}{b^2} + \frac{z^2}{a^2} = 1 \]  \hspace{1cm} (13)

Here \( a = 137.2 \) mm, \( b = 3447.28 \) mm. The amplitude \( A \) in drafts of Figure 3 for three kinds of imperfections is 2.8 mm.

![Figure 3](image)

Figure 3. Three kinds of imperfections along longitudinal direction (a) original, (b) sinusoidal function, (c) half-sinusoidal function and (d) hyperbola function.

3. Results and discussion

The submersible vessel is investigated in this study for one shipbuilding company. It is required that the provided admissible failure pressure under the deepwater is 147.784 MPa (15.08 kgf/mm\(^2\)), and the depth under the water is 10000 m.

The comparison is made for the elasticity and plasticity buckling loads of perfect vessels. The elastic critical pressure of the Ti6Al4V vessel is 18.5846 Kgf/mm\(^2\) (182.13 MPa), but its plastic critical pressure is only 15.82 Kgf/mm\(^2\) (155.083 MPa). By contrast, the counterparts of the Ti6Al4V-composites are 17.4001 Kgf/mm\(^2\) (170.52 MPa) and 16.19 Kgf/mm\(^2\) (158.694 MPa), respectively.

Since the buckling loads predicted by elasticity analysis are much larger than those predicted by plastic analysis, it is unsafe for the purpose of design to apply the elasticity buckling load by elasticity analysis and the plasticity bucking analysis is performed directly for imperfect vessels.

The global and local deformations of the vessel with an imperfection \( (u = -4.06 \text{ mm}) \) along cross-section direction are illustrated in Figure 4(a) and (b). The node 17640 is located at the position of long radius of the cylinder with the largest ovality \( u \). The typical non-linear evolution process is shown in Figure 5, in which the complex non-linear post-buckling phenomenon can be observed after it...
reaches the peak value. For the increment of external pressure, the measured pressure increases until it reaches the maximum pressure, and then the decrease in the pressure can be observed with a larger increase of deformation. The maximum non-linear ultimate strength is 158.584 MPa (16.182 kgf/mm²), which is the peak load when the abrupt change occurs in the load-deformation curve. Several values of $\alpha$ are calculated by the same way. The typical local deformations are shown in Figure 4(c)-(f), in which all the deformations are amplified 20 times than their original values. With an increase of $u$, the collapse loads decrease gradually, but all of them are higher than the admissible failure pressure under the deepwater (147.784 MPa), even higher than the critical load of perfect vessel (158.074 MPa), which indicates that the circumferential stiffeners are effective to enhance the circumferential strength of the submersible vessel to resist the local buckling, so the larger collapse load is obtained from the non-linear FE analysis with the imperfection along cross-section direction. The calculation of elastic-plastic buckling is also performed for the same $\alpha$ value.

![Figure 4](image)

**Figure 4.** Typical global and local deformations for cross-section imperfection (non-linear analysis) (a) global deformation for $u=-4.06$ mm, (b) local deformation for $u=-4.06$ mm, (c) local deformation for $u=-3.6$ mm, (d) local deformation for $u=-2.8$ mm, (e) local deformation for $u=-1.4$ mm and (f) local deformation for $u=1.4$ mm.

![Figure 5](image)

**Figure 5.** (a) position of point A, (b) the non-linear evolution process of point A and (c) local magnification of (b).
The typical global deformations are shown in Figure 6, which are completely different from those by non-linear analysis. Their ultimate collapse loads are listed in Table 3, which shows that the buckling critical load is lower than the collapse load predicted by non-linear analysis, but all of the collapse loads are still higher than 147.784 MPa (15.08 kgf/mm²). In fact, for elastic-plastic buckling analysis, the non-linear behavior in the pre-buckling phase has been considered in the calculation of critical load by using Eq. (10), thus the elastic-plastic analysis can give a more conservative prediction than that given by a branching load in the non-linear FE analysis, and the submersible vessel is safe to bear the failure pressure under the depth of 10000 m.

The comparison of the collapse loads between the Ti6Al4V-laminate composite submersible vessel and the Ti6Al4V one is made by using the elastic-plastic buckling analysis. The geometry of the metallic vessel is identical with that of the Ti6Al4V-laminate composite vessel as shown in Figure 1 and the laminated composites are substituted by the Ti6Al4V alloy. The specific strength (the critical load for per unit weight) is listed in Table 3. It can be observed that the Ti6Al4V-laminated composite vessel with cross-section imperfection can not only continue to bear the failure loads under the deepwater but also offer a higher specific strength than the Ti6Al4V one with the same imperfections, which indicates that the effect of geometric imperfections on the specific strength become less obvious for the moderate-walled Ti6Al4V-laminated composite vessel, and the stacking sequence in this study is effective to resist the non-linear buckling.

| Imperfection \(u\) (mm) | 0 | -4.06 | -3.6 | -2.8 | 1.4 |
|-------------------------|---|-------|------|------|-----|
| Non-linear collapse load (MPa) | --- | 158.58 | 163.17 | 163.542 | 163.111 |
| Plastic buckling collapse load (MPa) | 158.694 | 153.463 | 153.996 | 154.385 | 157.154 |
| Ti6Al4V-composite vessel specific strength (MPa) | 0.6948 | 0.6719 | 0.6742 | 0.6759 | 0.6881 |
| Ti6Al4V-vessel specific strength (MPa) | 0.5761 | 0.5616 | 0.5687 | 0.5639 | 0.5723 |

Following the same numerical process, the non-linear FE and elastic-plastic buckling analyses are performed for the submersible vessel with longitudinal imperfections. The typical global deformations and the changes in cross-section are shown in Figure 7. Since the longitudinal axisymmetric waves generate the global geometrical imperfection along thickness direction on the moderate-walled vessel, it is difficult to produce a local buckling and the plastic flowing is a main failure mode in this case. On the other hand, the buckling loads obtained from the elastic-plastic buckling analysis are obviously lower than those by non-linear FE analysis as shown in Table 4, so the ultimate collapse load is also governed by the elastic-plastic buckling critical load, which is higher than the admissible failure load that the vessel must bear under the deepwater.
The same comparison is also made for the Ti6A14V-laminated composite vessel and Ti6A14V one with the other longitudinal imperfections. The results are listed in Table 4. It can be seen that the specific strength of the Ti6A14V-laminated composite vessel is higher than that of the Ti6A14V one. Obviously, the specific strength of the Ti6A14V-laminated composite submersible vessel is insensitive to the imperfections along longitudinal direction.

Table 4. Collapse loads and specific strength for imperfection along longitudinal direction.

| Imperfection mode                  | Sinusoidal | Half-sinusoidal | Hyperbola |
|-----------------------------------|------------|----------------|-----------|
| Non-linear collapse load (MPa)    | 159.583    | 165.816        | 159.172   |
| Plastic buckling collapse load (MPa) | 161.377    | 156.398        | 150.263   |
| Ti6A14V-composite vessel specific strength (MPa) | 0.7125     | 0.6909         | 0.6635    |

Figure 7 Global and local deformations for longitudinal imperfection (non-linear analysis)  
(a) global deformation for sinusoidal imperfection, (b) sinusoidal imperfection, (c) half-sinusoidal imperfection and (d) hyperbola imperfection

4. Conclusions
The effects of geometrical imperfections on the collapse load and specific strength of moderate-walled submersible vessel made of Ti6A14V-lamine composites and Ti6A14V alloy, respectively, have been investigated by the non-linear FE and elastic-plastic buckling analyses in this study. The results show that the initial imperfections have an obvious influence on the buckling load, but all of the obtained ultimate collapse loads are higher than the admissible failure load under the deepwater. The collapse load of a submersible vessel is mainly governed by the elastic-plastic buckling critical load, which is conservative and safe for a submersible vessel. The stacking sequences selected in this study are effective to keep better specific strength of the Ti6A14V-lamine composite submersible vessel than that of the Ti6A14V one with the same imperfections along cross-section and longitudinal directions. Thus it is theoretically reasonable to fabricate one submersible vessel using the Ti6A14V-laminated composites instead of Ti6A14V alloy.

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