Radiative $B - L$ symmetry breaking and the $Z'$ mediated SUSY breaking

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Abstract

We explore a mechanism of radiative $B - L$ symmetry breaking in analogous to the radiative electroweak symmetry breaking. The breaking scale of $B - L$ symmetry is related to the neutrino masses through the see-saw mechanism. Once we incorporate the U(1)$_{B-L}$ gauge symmetry in SUSY models, the U(1)$_{B-L}$ gaugino, $\tilde{Z}_{B-L}$ appears, and it can mediate the SUSY breaking ($Z'$-prime mediated SUSY breaking) at around the scale of $10^6$ GeV. Then we find a links between the neutrino mass (more precisely the see-saw or $B - L$ scale of order $10^6$ GeV) and the $Z'$-prime mediated SUSY breaking scale. It is also very interesting that the gluino at the weak scale becomes relatively light, and almost compressed mass spectra for the gaugino sector can be realized in this scenario, which is very interesting in scope of the LHC.

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1 Introduction

Based on the experimental data, now the evidence of the neutrino masses and flavor mixings are almost established, and this is also the evidence of new physics beyond the standard model. Interestingly, the neutrino mass and mixing properties have been revealed to be very different from those of the other fermions, namely, neutrino masses are very small and the flavor mixing angles are very large. A new physics must explain them.

Supersymmetry (SUSY) extension of the Standard Model (SM) is one of the attractive candidates for new physics [1]. This is one of the most promising way to solve the gauge hierarchy problem in the standard model. The experimental data support the unification of the three gauge couplings at the grand unified theory (GUT) scale $M_{\text{GUT}} \sim 2 \times 10^{16}$ GeV with the particle contents of the minimal supersymmetric standard model (MSSM) [2, 3]. If we use the see-saw mechanism [4], it can naturally explain the lightness of the neutrinos.

The experimental data suggests that the see-saw scale is much lower than the Planck scale or even the GUT scale. It is therefore natural to think the scale is related to the breaking of some symmetry. The simplest symmetry is the $B-L$ symmetry. In principle, the $B-L$ symmetry can be a global or a local symmetry. If we take it to be a global symmetry, its spontaneous breaking leads to the pseudo Nambu-Goldstone boson, majoron. Since several experiments give severe constraints on the majoron, it is natural to make it local gauge symmetry if we consider a higher ranked GUT such as SO(10) [5]. The spontaneous breaking of $B-L$ symmetry can be exploited by developing the vacuum expectation value (VEV) of a scalar multiplet $\Delta_1$ which carries $B-L=-2$. For the anomaly cancellation and to keep the low-energy supersymmetry, its counterpart $\Delta_2$ that has $B-L=+2$ has to be included into a theory. After the spontaneous breaking of this $B-L$ symmetry, it leads to a massive gauge boson, $Z_{B-L}$.

On the other hand, one of the most attractive features of the Minimal Supersymmetric extension of the Standard Model (MSSM), is the fact that it provides a mechanism for radiative breaking of the electroweak gauge $SU(2)_L \times U(1)_Y$ symmetry [6, 7, 8, 9, 10, 11]. The essential point for this mechanism is that the presence of the large top Yukawa coupling, which can dictate the Higgs mass squared driven to be negative at the weak scale. It is known that the radiative electroweak symmetry breaking (RESB) can take place if the top Yukawa coupling is large, such that $60 \text{ GeV} \lesssim M_t \lesssim 200 \text{ GeV}$, with the upper bound coming from the requirement that it remains in the perturbative range up to the GUT scale. It is interesting that the observed top quark mass found at the Tevatron was indeed at around $M_t = 175 \text{ GeV}$.

Then it is natural to think about the possibility to break $U(1)_{B-L}$ symmetry through the radiative corrections to the soft mass squared which is responsible for the VEV of the $U(1)_{B-L}$ breaking in analogous to the case of RESB in the MSSM. Here we explore such

\footnote{For the group theoretical aspects of SO(10), see for example, [5].}
a possibility by considering the renormalization group equations (RGEs) of the soft mass terms for the $B - L$ breaking sector. Our resultant $B - L$ breaking scale is found to be around $v_{B-L} \simeq 10^5$ GeV, that is in a sense quite appealing if we consider to incorporate the thermal leptogenesis scenario \[12\] in SUSY models because the gravitino problem \[13, 14\] put a severe constraint on the reheating temperature as $T_R \lesssim 10^6$ GeV for the gravitino mass of order $m_{3/2} \lesssim 100$ GeV \[15\].

Once we incorporate the $U(1)_{B-L}$ gauge symmetry in SUSY models, an extra $U(1)$ gaugino, $\tilde{Z}_{B-L}$ appears in addition to the extra gauge boson $Z_{B-L}$. It has recently been noticed that if there exist such an extra gaugino, it can mediate a SUSY breaking so as to induce the gaugino masses for each SM gauge group at the two loop level, while the scalar soft masses are generated at the one loop level \[19, 20\] \[4\]. The Z-prime mediated SUSY breaking is basically to use an extra $U(1)'$ vector multiplet as a field which communicates a SUSY breaking source with the visible sector. This setup is much more appealing and economical than the gauge-mediated SUSY breaking. In this mediation mechanism, it is not necessary to introduce some additional sector as a 'messenger field', that can be implemented into a theory just as a gauge multiplet associated with an extra $U(1)'$ gauge symmetry. We take such an extra $U(1)'$ as a $U(1)_{B-L}$ symmetry, and then, we can identify the messenger scale as the scale of $B - L$ symmetry breaking scale.

This paper is organized as follows. In section 2 we give an explicit model having $B - L$ symmetry. In section 3, we explain the model of $Z_{B-L}$ mediated SUSY breaking mechanism, and apply it to the case of $B - L$ gauge symmetry. In section 4, some numerical analysis is performed to show some example SUSY mass spectra which is characteristic for this specific SUSY breaking mechanism. The last section is devoted for summary and discussions.

## 2 Radiative B-L breaking

The interactions between Higgs and matter superfields are described by the superpotential

\[
W = (Y_u)_{ij} U^c_i Q_j H_u + (Y_d)_{ij} D^c_i Q_j H_d + (Y_e)_{ij} E^c_i L_j H_d \\
+ \mu H_d H_u \\
+ (Y'_\nu)_{ij} \bar{N}^c_i L_j H_u + f_{ij} \Delta_1 \bar{N}_i \Delta_2 \\
+ \mu' \Delta_3 \Delta_4 ,
\]

where the indices $i, j$ run over three generations, $H_u$ and $H_d$ denote the up-type and down-type MSSM Higgs doublets, respectively.

After developing the VEV of the $B - L$ breaking field, $\langle \Delta_1 \rangle = v_{B-L}$, the right-handed neutrino obtains the Majorana mass as $M_N = f v_{B-L}$. And it gives a light neutrino mass

\[4\] The similar idea has also been suggested in \[16\].
through the see-saw mechanism as follows: \( M_\nu = m_D M_N^{-1} m_D^T \), where \( m_D = Y_\nu v \) (\( v = 174 \text{ GeV} \)) is the Dirac neutrino mass matrix.

The soft SUSY-breaking terms which is added to the MSSM soft mass terms are given by

\[
- \Delta L_{\text{soft}} = (m_N^2)_{ij} \tilde{N}_i^\dagger \tilde{N}_j + m_{\Delta_1}^2 |\Delta_1|^2 + m_{\Delta_2}^2 |\Delta_2|^2 + \left( (A_\nu)_{ij} \tilde{N}_i^\dagger \ell_j H_u + h.c. \right) \\
+ \left( A_f \right)_{ij} \Delta_1 \tilde{N}_i \tilde{N}_j + h.c. \\
+ \frac{1}{2} M_{\tilde{Z}_{B-L}} \tilde{Z}_{B-L} \tilde{Z}_{B-L} + h.c. 
\]

(2)

From Eqs. (1) and (2), the scalar potential relevant for the \( B - L \) breaking sector can be written as

\[
V(\Delta_1, \Delta_2) = (|\mu'|^2 + m_{\Delta_1}^2) |\Delta_1|^2 + (|\mu'|^2 + m_{\Delta_2}^2) |\Delta_2|^2 \\
+ \frac{1}{2} g_{B-L}^2 (|\Delta_1|^2 - |\Delta_2|^2)^2 , 
\]

(3)

where we have neglected the Yukawa coupling contributions to the scalar potential.

The minimalization condition of this potential leads to

\[
\frac{\partial V}{\partial \Delta_1^\dagger} = \left[ (|\mu'|^2 + m_{\Delta_1}^2) + \frac{1}{2} g_{B-L}^2 |\Delta_1|^2 \right] \Delta_1 = 0 , \\
\frac{\partial V}{\partial \Delta_2^\dagger} = \left[ (|\mu'|^2 + m_{\Delta_2}^2) + \frac{1}{2} g_{B-L}^2 |\Delta_2|^2 \right] \Delta_2 = 0 . 
\]

(4)

Therefore, the VEV of the \( B - L \) breaking field \( \Delta_1 \) is determined to be

\[
|\langle \Delta_1 \rangle|^2 = -\frac{2}{g_{B-L}^2} (|\mu'|^2 + m_{\Delta_1}^2) . 
\]

(5)

3 Z-prime mediation of SUSY breaking

Here we give a brief review of the Z-prime mediation of SUSY breaking [19, 20] by discussing the pattern of the soft SUSY breaking parameters, the masses of the \( Z' \)-ino and of the MSSM squarks and gauginos, which are the most robust predictions of this scenario. At the SUSY breaking scale, \( \Lambda_S \), SUSY breaking in the hidden sector is assumed to generate a SUSY breaking mass for the fermionic component of the \( U(1)_{B-L} \) vector superfield. Given details of the hidden sector, its value could be evaluated via the standard technique of analytical continuation into superspace [21]. In particular, the gauge kinetic function of the field
strength superfield $\mathcal{W}_{B-L}^\alpha$ at the SUSY breaking scale is

$$\mathcal{L}_{\tilde{Z}_{B-L}} = \int d^2\theta \left[ \frac{1}{g_{B-L}^2} + \beta_{B-L}^{hid} \ln \left( \frac{\Lambda_S}{M} \right) \right.$$

$$\left. + \beta_{B-L}^{vis} \ln \left( \frac{\Lambda_S}{M_{\tilde{Z}_{B-L}}} \right) \right] \mathcal{W}_{B-L}^\alpha \mathcal{W}_{B-L}^\alpha,$$

(6)

where $M$ is the messenger scale, which we have assumed to be around the SUSY breaking scale, $M \sim \Lambda_S$. $\beta_{B-L}^{hid}$ and $\beta_{B-L}^{vis}$ are $\beta$-functions induced by $U(1)_{B-L}$ couplings to hidden and visible sector fields, respectively. Using analytical continuation, we replace $M$ with $M + \theta^2 F$, where $F$ is the SUSY breaking order parameter. We obtain the $\tilde{Z}_{B-L}$ mass as $M_{\tilde{Z}_{B-L}} \sim g_{B-L}^2 \beta_{B-L}^{hid} F/M$. We assume that the $U(1)_{B-L}$ gauge symmetry is not broken in the hidden sector. And we assume some sequestering mechanism so that only the $B - L$ gaugino obtains a leading order mass term while the threshold corrections to the squarks and sleptons are only arisen at the next leading order as similar to the case of the gaugino mediation, where the $B - L$ gaugino lives in the bulk in a five dimensional setup while squarks and sleptons are put on the brane. In such a case, only the $B - L$ gaugino obtains a mass while the scalar masses receive negligible threshold corrections at the lowest order since they receive volume suppression.

Since all the chiral superfields in the visible sector are charged under $U(1)_{B-L}$, so all the corresponding scalars receive soft mass terms at 1-loop of order

$$m_{\tilde{q}_i}^2 = \frac{8}{9} \alpha_{B-L} M_{\tilde{Z}_{B-L}}^2 \ln \left( \frac{\Lambda_S}{M_{\tilde{Z}_{B-L}}} \right),$$

$$m_{\tilde{\ell}_i}^2 = \frac{8}{4} \alpha_{B-L} M_{\tilde{Z}_{B-L}}^2 \ln \left( \frac{\Lambda_S}{M_{\tilde{Z}_{B-L}}} \right),$$

(7)

where $\alpha_{B-L} = g_{B-L}^2/(4\pi)$ and $Q_{B-L}^f$ is the $U(1)_{B-L}$ charge of $f$.

The MSSM gaugino masses, however, can only be generated at 2-loop level since they do not directly couple to the $U(1)_{B-L}$,

$$M_a = 4 c_a \frac{\alpha_{B-L}}{4\pi} \frac{\alpha_a}{4\pi} M_{\tilde{Z}_{B-L}} \ln \left( \frac{\Lambda_S}{M_{\tilde{Z}_{B-L}}} \right),$$

(8)

where $(c_1, c_2, c_3) = \left( \frac{22}{35}, 4, \frac{4}{3} \right)$.

Since these gaugino masses are proportional to $c_a$, we expect that the gluino will typically be lighter than the others at $\mu = M_{\tilde{Z}_{B-L}}$, so the resultant mass spectra of the gauginos are relatively compressed than the other mediation mechanisms.
From the discussion above, we see that the gauginos are considerably lighter than the sfermions. Taking $M_a \simeq 100$ GeV, we find

$$M_{Z_{B-L}} \ln \left( \frac{\Lambda_S}{M_{Z_{B-L}}} \right) \simeq 10^4 \text{ TeV}$$

(9)

and

$$m_f \simeq 10^{-1} M_{Z_{B-L}} \simeq 10^5 \text{ GeV}.$$  

(10)

Hence, in this scheme of Z-prime mediation, all the sfermion masses become very heavy at around $10^5$ GeV, while the gauginos are kept at around the weak scale, $M_a \simeq 100$ GeV, which can in principle provide a natural candidate of the dark matter.

In our choice of parameters, the gravitino mass is given by

$$m_{3/2} = \frac{\Lambda_S^2}{\sqrt{3} M_{Pl}} = \{24 \text{ keV}, \ 2.4 \text{ MeV}, \ 240 \text{ MeV}\}.$$  

(11)

for $\Lambda_S = \{10^7, \ 10^8, \ 10^9\}$ GeV. Hence the gravity mediation contribution to the gaugino masses is much suppressed, and is well negligible compared to the Z-prime mediated contribution.

4 RGEs and its numerical evaluations

Now we consider the RGEs and analyze the running of the scalar masses $m_{\Delta_1}^2$ and $m_{\Delta_2}^2$. The key point for implementing the radiative $B-L$ symmetry breaking is that the scalar potential $V(\Delta_1, \Delta_2)$ receives substantial radiative corrections. In particular, a negative (mass)$^2$ would trigger the $B-L$ symmetry breaking. We argue that the masses of Higgs fields $\Delta_1$ and $\Delta_2$ run differently in the way that $m_{\Delta_1}^2$ can be negative whereas $m_{\Delta_2}^2$ remains positive. The RGE for the $B-L$ coupling and mass parameters can be derived from the general results for SUSY RGEs of Ref. [18].

For the RGEs of the Yukawa couplings, we consider to include the additional contribution from the $U(1)_{B-L}$ gauge sector.

$$16\pi^2 \mu \frac{d}{d\mu} Y_A = [\text{MSSM + see-saw}] + \delta_{A\nu} Y_\nu f^\dagger f - 2 a_A g_{B-L}^2 Y_A ,$$

(12)

where $(a_u, a_d, a_\nu, a_e) = (\frac{2}{3}, \frac{2}{3}, 2, 2)$, and the [MSSM + see-saw] part of the RGEs can be found in the Appendix.

And the RGE of the Yukawa coupling relevant for the right-handed neutrino mass is written by

$$16\pi^2 \mu \frac{d}{d\mu} f = 4 \text{Tr}[f^\dagger f]f + 2 (Y_\nu^\dagger Y_\nu) f + 2 f (Y_\nu^\dagger Y_\nu)^T - 12 g_{B-L}^2 f .$$

(13)
The RGEs of the MSSM gauge couplings are the same as MSSM, while the RGE of the $U(1)_{B-L}$ gauge coupling is given by

$$16\pi^2 \mu \frac{dg_{B-L}}{d\mu} = b_{B-L} g_{B-L}^3,$$

where $b_{B-L} = 24$.

For the RGEs of the gaugino masses, it can be written as follows.

$$16\pi^2 \mu \frac{dM_{\tilde{Z}_{B-L}}}{d\mu} = 2b_{B-L} g_{B-L}^3 M_{\tilde{Z}_{B-L}},$$

$$16\pi^2 \mu \frac{dM_a}{d\mu} = [\text{MSSM} + \text{see-saw}] + \frac{4c_0g_0^2}{16\pi^2} g_{B-L}^2 M_{\tilde{Z}_{B-L}},$$

where $(c_0) = \left(\frac{92}{15}, 4, \frac{4}{3}\right)$.

For the RGEs of the A-terms, it can be written as follows.

$$16\pi^2 \mu \frac{d\tilde{A}_A}{d\mu} = [\text{MSSM} + \text{see-saw}] - 2a_A g_{B-L}^2 (\tilde{A}_A - 2M_{\tilde{Z}_{B-L}} Y_A),$$

where $\tilde{A}_A = A_A Y_A$. The RGE of the $A_f$-term can be written as

$$16\pi^2 \mu \frac{d\tilde{A}_f}{d\mu} = (9 \text{Tr}[f^\dagger f] + 2 \text{Tr}[Y^\dagger \nu Y\nu]) \tilde{A}_f + 8 f Y^\dagger \nu \tilde{A}_\nu.$$

The RGEs of the soft scalar masses are given by

$$16\pi^2 \mu \frac{dm_{\Delta_1}^2}{d\mu} = 2 \text{Tr}[f^\dagger f] m_{\Delta_1}^2 + 4 \text{Tr}[f^\dagger m_N^2 f] - 32 g_{B-L}^2 |M_{\tilde{Z}_{B-L}}|^2,$$

$$16\pi^2 \mu \frac{dm_{\Delta_2}^2}{d\mu} = -32 g_{B-L}^2 |M_{\tilde{Z}_{B-L}}|^2,$$

$$16\pi^2 \mu \frac{dm_f^2}{d\mu} = [\text{MSSM} + \text{see-saw}] - 8g_{B-L}^2 (Q_{B-L}^f)^2 |M_{\tilde{Z}_{B-L}}|^2.$$

where $Q_{B-L}^f$ is the $B-L$ charge of each chiral multiplet $f = Q, U^c, D^c, L$ and

$$16\pi^2 \mu \frac{dm_{\tilde{N}_e}^2}{d\mu} = [\text{MSSM} + \text{see-saw}] + \left(m_{\tilde{N}_e}^2 f^\dagger f + f^\dagger f m_{\tilde{N}_e}^2\right)$$

$$+ 2 \left(f^\dagger m_{\tilde{N}_e}^2 f + m_{\Delta_1}^2 f^\dagger f + \tilde{A}_f^\dagger \tilde{A}_f\right) - 8g_{B-L}^2 (Q_{B-L}^f)^2 |M_{\tilde{Z}_{B-L}}|^2.$$

For the RGEs of the $\mu'$-term, it can be written as follows.

$$16\pi^2 \mu \frac{d\mu'}{d\mu} = (\text{Tr}[f^\dagger f] - 16g_{B-L}^2) \mu'.$$
In the numerical analysis we take input all the soft SUSY breaking parameters to be zero at the SUSY breaking scale, in which the SUSY breaking scale is varied in the range, $\Lambda_S = 10^7 - 10^9$ GeV,

$$\tilde A_A = 0, \ m_{\tilde f} = 0, \ M_a = 0$$

and use the following inputs

$$M_{\tilde Z_{B-L}} = 8.7 \times 10^5 \text{GeV}, \ f = 4, 5, 6, 7, \ g_{B-L} = 0.5 .$$

Note that $\tilde Z_{B-L}$ has to be decoupled at the mass scale $M_{\tilde Z_{B-L}}$. Here some comments are in order for the above choices of parameters. For the large values of the Yukawa coupling $(f)$, it blows up before reaching the GUT scale, so we have to have a cutoff scale below the GUT scale. However, since our motivation to consider a model with $B-L$ gauge symmetry is to find a relation to the origin of the neutrino masses via the see-saw mechanism. So, we do not assume a simple SU(5) like GUT picture. In fact, if one consider the $B-L$ gauge symmetry, the naive GUT picture would be broken at an intermediate scale while there is a possibility to realize a grand unification with an intermediate scale.

Using these inputs, in Fig. 1 we plot the evolution of the gaugino masses $M_{1,2,3}$ from the SUSY breaking scale to the weak scale. In these plots, we varied the SUSY breaking scale as $10^8$ GeV and $10^9$ GeV. It is very interesting that the gluino at the $\tilde Z_{B-L}$ scale is given as the lightest gaugino, that is very different from most of the other models of SUSY breaking mediation. For that reason, the gluino at the weak scale becomes relatively light, and almost compressed mass spectra for the gaugino sector can be realized in this scenario, which is very interesting in scope of the LHC.

In Fig. 2 we plot the SUSY breaking scale, $\Lambda_S$, dependence of the gaugino masses $M_{1,2,3}$ at the weak scale. It simply shows that raising the SUSY breaking scale corresponds to the increase of the gaugino masses at the weak scale.

In Fig. 3 we plot the gauge coupling constant, $g_{B-L}$, dependence of the gaugino masses at the weak scale. Here the gauge coupling constant, $g_{B-L}$ is given at a given SUSY breaking scale $\Lambda_S = 10^9$ GeV.

The evolutions of the soft mass squared for the fields $\Delta_1$ and $\Delta_2$ are plotted in Fig. 4 and Fig. 5 for a given SUSY breaking scale as $\Lambda_S = 10^9$ GeV. In Fig. 4 from top to the bottom curves, we varied the value of $f$ as $f = 4, 5, 6, 7$. For example, for the case of $f = 5$, the soft mass squared for the fields $\Delta_1$ goes across the zeros at the scale about $10^6$ GeV toward negative value, that is nothing but the realization of the radiative symmetry breaking of U(1)$_{B-L}$ gauge symmetry. The running behavior in Fig. 4 can be understood in the following way. At first, starting from the high energy scale, the soft mass squared increases because of the gauge coupling contributions, and decrease of the mass squared is caused by the Yukawa coupling that dominate over the gauge coupling contribution at some scale. Next, since at the mass scale of $\tilde Z_{B-L}$, it is decoupled from the RGEs, there are only
the Yukawa coupling contributions to the soft mass squared which rapidly decreases to across the zeros. Therefore, the radiative $B - L$ symmetry breaking can naturally be realized.

The see-saw scale, which is found to be at $v_{B-L} = 10^5$ GeV, hence the right-handed neutrino obtains a mass of $M_N = f v_{B-L} = 5 \times 10^5$ GeV. This scale of the right-handed neutrino is nice for the thermal leptogenesis to be viable in supersymmetric models with gravity mediation.

5 Summary

We have shown that a mechanism of radiative $B - L$ symmetry breaking can work in analogous to the RESB. The breaking scale of $B - L$ symmetry is related to the neutrino masses through the see-saw mechanism. Once we incorporate the $U(1)_{B-L}$ gauge symmetry in SUSY models, the $U(1)_{B-L}$ gaugino, $\tilde{Z}_{B-L}$ can provide all the soft masses in the MSSM. Then we find a link between the neutrino mass (more precisely the see-saw or $B - L$ scale of order $10^5$ GeV) and the Z-prime mediated SUSY breaking scale. In this scheme of Z-prime mediation, all the sfermion masses become very heavy at around $10^5$ GeV, while the gauginos are kept at at around the weak scale, $M_a \simeq 100$ GeV. It is also very interesting that the gluino at $\tilde{Z}_{B-L}$ scale is given as the lightest gaugino, that is very different from most of the other models of SUSY breaking mediation. For that reason, the gluino at the weak scale becomes relatively light, and almost compressed mass spectra for the gaugino sector can be realized in this scenario, which is very interesting in scope of the LHC.

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A RGEs in the MSSM with right-handed neutrinos

A.1 The 2-loop RGE for the gauge couplings

\[ 16 \pi^2 \frac{d}{d \mu} g_1 = \frac{33}{5} g_1^3 + \frac{g_1^3}{16 \pi^2} \left( \frac{199}{25} g_1^2 + \frac{27}{5} g_2^2 + \frac{88}{5} g_3^2 \right) , \]  
\[ 16 \pi^2 \frac{d}{d \mu} g_2 = g_2^3 + \frac{g_2^3}{16 \pi^2} \left( \frac{9}{5} g_1^2 + 25 g_2^2 + 24 g_3^2 \right) , \]  
\[ 16 \pi^2 \frac{d}{d \mu} g_3 = -3 g_3^3 + \frac{g_3^3}{16 \pi^2} \left( \frac{1}{5} g_1^2 + 9 g_2^2 + 14 g_3^2 \right) . \]  

Here \( g_2 \equiv g \) is the \( SU(2)_L \) gauge coupling constant and \( g_1 \equiv \sqrt{\frac{5}{3}} g' \) is the \( U(1) \) gauge coupling constant with the GUT normalization (\( g_1 = g_2 = g_3 \) at \( \mu = M_{\text{GUT}} \)).

A.2 The 1-loop RGE for the Yukawa couplings

\[ 16 \pi^2 \frac{d}{d \mu} Y_u = Y_u \left[ \left\{ -\frac{13}{15} g_1^2 - \frac{16}{3} g_2^2 + \frac{1}{3} \text{Tr}(Y_u^\dagger Y_u) + \text{Tr}(Y_\nu^\dagger Y_\nu) \right\} \right] \begin{bmatrix} 1 & 3 & x & 3 \end{bmatrix} \]  
\[ + 3 \left( Y_u^\dagger Y_u \right) \left( Y_\nu^\dagger Y_\nu \right) \]  
\[ \left[ \begin{bmatrix} 1 & 3 & x & 3 \end{bmatrix} \right] , \]  
\[ 16 \pi^2 \frac{d}{d \mu} Y_d = Y_d \left[ \left\{ -\frac{7}{15} g_1^2 - \frac{16}{3} g_2^2 + \frac{1}{3} \text{Tr}(Y_d^\dagger Y_d) + \text{Tr}(Y_e^\dagger Y_e) \right\} \right] \begin{bmatrix} 1 & 3 & x & 3 \end{bmatrix} \]  
\[ + 3 \left( Y_d^\dagger Y_d \right) \left( Y_e^\dagger Y_e \right) \]  
\[ \left[ \begin{bmatrix} 1 & 3 & x & 3 \end{bmatrix} \right] , \]  
\[ 16 \pi^2 \frac{d}{d \mu} Y_\nu = Y_\nu \left[ \left\{ -\frac{3}{5} g_1^2 - \frac{1}{3} \text{Tr}(Y_u^\dagger Y_u) + \text{Tr}(Y_\nu^\dagger Y_\nu) \right\} \right] \begin{bmatrix} 1 & 3 & x & 3 \end{bmatrix} \]  
\[ + 3 \left( Y_\nu^\dagger Y_\nu \right) \left( Y_\nu^\dagger Y_\nu \right) \]  
\[ \left[ \begin{bmatrix} 1 & 3 & x & 3 \end{bmatrix} \right] , \]  
\[ 16 \pi^2 \frac{d}{d \mu} Y_e = Y_e \left[ \left\{ -\frac{9}{5} g_1^2 - \frac{1}{3} \text{Tr}(Y_d^\dagger Y_d) + \text{Tr}(Y_e^\dagger Y_e) \right\} \right] \begin{bmatrix} 1 & 3 & x & 3 \end{bmatrix} \]  
\[ + 3 \left( Y_e^\dagger Y_e \right) \left( Y_\nu^\dagger Y_\nu \right) \]  
\[ \left[ \begin{bmatrix} 1 & 3 & x & 3 \end{bmatrix} \right] . \]
A.3 The 2-loop RGE for the gaugino masses

A.3.1 The 2-loop RGE for the gaugino masses

\[16\pi^2 \frac{d}{d\mu} M_1 = \frac{66}{5} g_1^2 M_1 \]
\[+ \frac{2g_1^2}{16\pi^2} \left\{ \frac{199}{5} g_1^2 (2M_1) + \frac{27}{5} g_2^2 (M_1 + M_2) + \frac{88}{5} g_3^2 (M_1 + M_3) \right\} , \quad (30)\]

\[16\pi^2 \frac{d}{d\mu} M_2 = 2 g_2^2 M_2 \]
\[+ \frac{2g_2^2}{16\pi^2} \left\{ \frac{9}{5} g_1^2 (M_1 + M_2) + 25 g_2^2 (2M_2) + 24 g_3^2 (M_2 + M_3) \right\} , \quad (31)\]

\[16\pi^2 \frac{d}{d\mu} M_3 = -6 g_3^2 M_3 \]
\[+ \frac{2g_3^2}{16\pi^2} \left\{ \frac{11}{5} g_1^2 (M_1 + M_3) + 9 g_2^2 (M_2 + M_3) + 14 g_3^2 (2M_3) \right\} . \quad (32)\]
A.4 The 1-loop RGE for the soft SUSY breaking mass terms

\[
16\pi^2 \mu \frac{d}{d\mu} (m^2_{\tilde{q}})_{ij} = - \left( \frac{2}{15} g_1^2 |M_1|^2 + 6g_2^2 |M_2|^2 + \frac{32}{3} g_3^2 |M_3|^2 \right) \delta_{ij} + \frac{1}{5} g_1^2 S \delta_{ij} \\
+ \left( m_{\tilde{q}}^2 Y_u^\dagger Y_u + m_{\tilde{q}}^2 Y_d^\dagger Y_d + Y_u^\dagger Y_u m_{\tilde{q}}^2 + Y_d^\dagger Y_d m_{\tilde{q}}^2 \right)_{ij} \\
+ 2 \left( Y_u^\dagger m_{\tilde{u}}^2 Y_u + m_{H_u}^2 Y_u^\dagger Y_u + A_u^\dagger A_u \right)_{ij} \\
+ 2 \left( Y_d^\dagger m_{\tilde{d}}^2 Y_d + m_{H_d}^2 Y_d^\dagger Y_d + A_d^\dagger A_d \right)_{ij}, \tag{33}
\]

\[
16\pi^2 \mu \frac{d}{d\mu} (m^2_{\tilde{u}})_{ij} = - \left( \frac{32}{15} g_1^2 |M_1|^2 + \frac{32}{3} g_3^2 |M_3|^2 \right) \delta_{ij} - \frac{4}{5} g_1^2 S \delta_{ij} \\
+ 2 \left( m_{\tilde{u}}^2 Y_u^\dagger Y_u + m_{H_u}^2 Y_u^\dagger Y_u \right)_{ij} \\
+ 4 \left( Y_u^\dagger m_{\tilde{u}}^2 Y_u + m_{H_u}^2 Y_u^\dagger Y_u + A_u^\dagger A_u \right)_{ij}, \tag{34}
\]

\[
16\pi^2 \mu \frac{d}{d\mu} (m^2_{\tilde{d}})_{ij} = - \left( \frac{8}{15} g_1^2 |M_1|^2 + \frac{32}{3} g_3^2 |M_3|^2 \right) \delta_{ij} + \frac{2}{5} g_1^2 S \delta_{ij} \\
+ 2 \left( m_{\tilde{d}}^2 Y_d^\dagger Y_d + m_{H_d}^2 Y_d^\dagger Y_d \right)_{ij} \\
+ 4 \left( Y_d^\dagger m_{\tilde{d}}^2 Y_d + m_{H_d}^2 Y_d^\dagger Y_d + A_d^\dagger A_d \right)_{ij}, \tag{35}
\]

\[
16\pi^2 \mu \frac{d}{d\mu} (m^2_{\tilde{e}})_{ij} = - \left( \frac{6}{5} g_1^2 |M_1|^2 + 6g_2^2 |M_2|^2 \right) \delta_{ij} - \frac{3}{5} g_1^2 S \delta_{ij} \\
+ \left( m_{\tilde{e}}^2 Y_e^\dagger Y_e + m_{\tilde{e}}^2 Y_\nu^\dagger Y_\nu + Y_e^\dagger Y_e m_{\tilde{e}}^2 + Y_\nu^\dagger Y_\nu m_{\tilde{e}}^2 \right)_{ij} \\
+ 2 \left( Y_e^\dagger m_{\tilde{e}}^2 Y_e + m_{H_\nu}^2 Y_e^\dagger Y_e + A_e^\dagger A_e \right)_{ij} \\
+ 2 \left( Y_\nu^\dagger m_{\tilde{e}}^2 Y_\nu + m_{H_\nu}^2 Y_\nu^\dagger Y_\nu + A_\nu^\dagger A_\nu \right)_{ij}, \tag{36}
\]

\[
16\pi^2 \mu \frac{d}{d\mu} (m^2_{e})_{ij} = - \frac{24}{5} g_1^2 |M_1|^2 \delta_{ij} + \frac{6}{5} g_1^2 S \delta_{ij} + 2 \left( m_{e}^2 Y_e^\dagger Y_e + Y_e^\dagger Y_e m_{e}^2 \right)_{ij} \\
+ 4 \left( Y_e^\dagger m_{\tilde{e}}^2 Y_e + m_{H_\nu}^2 Y_e^\dagger Y_e + A_e^\dagger A_e \right)_{ij}, \tag{37}
\]

\[
16\pi^2 \mu \frac{d}{d\mu} (m^2_{\nu})_{ij} = 2 \left( m_{\nu}^2 Y_\nu^\dagger Y_\nu + Y_\nu^\dagger Y_\nu m_{\nu}^2 \right)_{ij} + 4 \left( Y_\nu^\dagger m_{\nu}^2 Y_\nu + m_{H_\nu}^2 Y_\nu^\dagger Y_\nu + A_\nu^\dagger A_\nu \right)_{ij}. \tag{38}
\]
where

\[ S \equiv \text{Tr}(m_{q}^{2} + m_{d}^{2} - 2m_{u}^{2} - m_{t}^{2} + m_{e}^{2}) - m_{H_{u}}^{2} + m_{H_{d}}^{2} . \]  

**A.5 The 1-loop RGE for the soft SUSY breaking A-terms**

\[
16\pi^{2}\mu \frac{d}{d\mu}(m_{H_{u}}^{2}) = - \left( \frac{6}{5}g_{1}^{2}|M_{1}|^{2} + 6g_{2}^{2}|M_{2}|^{2} \right) + \frac{3}{5}g_{1}^{2}S + 6\text{Tr}\left(m_{q}^{2}Y_{u}^{\dagger}Y_{u} + Y_{u}^{\dagger}(m_{u}^{2} + m_{H_{u}}^{2})Y_{u} + A_{u}^{\dagger}A_{u} \right) \\
+ 2\text{Tr}\left(m_{t}^{2}Y_{\nu}^{\dagger}Y_{\nu} + Y_{\nu}^{\dagger}(m_{\nu}^{2} + m_{H_{u}}^{2})Y_{\nu} + A_{\nu}^{\dagger}A_{\nu} \right), \tag{39}
\]

\[
16\pi^{2}\mu \frac{d}{d\mu}(m_{H_{d}}^{2}) = - \left( \frac{6}{5}g_{1}^{2}|M_{1}|^{2} + 6g_{2}^{2}|M_{2}|^{2} \right) - \frac{3}{5}g_{1}^{2}S + 6\text{Tr}\left(m_{t}^{2}Y_{d}^{\dagger}Y_{d} + Y_{d}^{\dagger}(m_{d}^{2} + m_{H_{d}}^{2})Y_{d} + A_{d}^{\dagger}A_{d} \right) \\
+ 2\text{Tr}\left(m_{t}^{2}Y_{e}^{\dagger}Y_{e} + Y_{e}^{\dagger}(m_{e}^{2} + m_{H_{d}}^{2})Y_{e} + A_{e}^{\dagger}A_{e} \right), \tag{40}
\]

\[
\frac{d}{d\mu}A_{u_{ij}} = \left\{ -\frac{13}{15}g_{1}^{2} - 3g_{2}^{2} - \frac{16}{3}g_{3}^{2} + 3\text{Tr}(Y_{u}^{\dagger}Y_{u}) + \text{Tr}(Y_{\nu}^{\dagger}Y_{\nu}) \right\} A_{u_{ij}} \\
+ 2 \left\{ \frac{13}{15}g_{1}^{2}M_{1} + 3g_{2}^{2}M_{2} + \frac{16}{3}g_{3}^{2}M_{3} + 3\text{Tr}(Y_{u}^{\dagger}A_{u}) + \text{Tr}(Y_{\nu}^{\dagger}A_{\nu}) \right\} Y_{u_{ij}} \\
+ 4(Y_{u}^{\dagger}Y_{u}A_{u})_{ij} + 5(A_{u}Y_{u}^{\dagger}Y_{u})_{ij} + 2(Y_{u}Y_{d}^{\dagger}A_{d})_{ij} + (A_{u}Y_{d}^{\dagger}A_{d})_{ij} , \tag{42}
\]

\[
\frac{d}{d\mu}A_{d_{ij}} = \left\{ -\frac{7}{15}g_{1}^{2} - 3g_{2}^{2} - \frac{16}{3}g_{3}^{2} + 3\text{Tr}(Y_{d}^{\dagger}Y_{d}) + \text{Tr}(Y_{e}^{\dagger}Y_{e}) \right\} A_{d_{ij}} \\
+ 2 \left\{ \frac{7}{15}g_{1}^{2}M_{1} + 3g_{2}^{2}M_{2} + \frac{16}{3}g_{3}^{2}M_{3} + 3\text{Tr}(Y_{d}^{\dagger}A_{d}) + \text{Tr}(Y_{e}^{\dagger}A_{e}) \right\} Y_{d_{ij}} \\
+ 4(Y_{d}^{\dagger}Y_{d}A_{d})_{ij} + 5(A_{d}Y_{d}^{\dagger}Y_{d})_{ij} + 2(Y_{d}Y_{u}^{\dagger}A_{u})_{ij} + (A_{d}Y_{u}^{\dagger}A_{u})_{ij} , \tag{43}
\]

\[
\frac{d}{d\mu}A_{e_{ij}} = \left\{ -\frac{9}{5}g_{1}^{2} - 3g_{2}^{2} + 3\text{Tr}(Y_{d}^{\dagger}Y_{d}) + \text{Tr}(Y_{e}^{\dagger}Y_{e}) \right\} A_{e_{ij}} \\
+ 2 \left\{ \frac{9}{5}g_{1}^{2}M_{1} + 3g_{2}^{2}M_{2} + 3\text{Tr}(Y_{d}^{\dagger}A_{d}) + \text{Tr}(Y_{e}^{\dagger}A_{e}) \right\} Y_{e_{ij}} \\
+ 4(Y_{e}^{\dagger}Y_{e}A_{e})_{ij} + 5(A_{e}Y_{e}^{\dagger}Y_{e})_{ij} + 2(Y_{e}Y_{d}^{\dagger}A_{d})_{ij} + (A_{e}Y_{d}^{\dagger}A_{d})_{ij} , \tag{44}
\]

\[
\frac{d}{d\mu}A_{u_{ij}} = \left\{ -\frac{3}{5}g_{1}^{2} - 3g_{2}^{2} + 3\text{Tr}(Y_{u}^{\dagger}Y_{u}) + \text{Tr}(Y_{\nu}^{\dagger}Y_{\nu}) \right\} A_{u_{ij}} \\
+ 2 \left\{ \frac{3}{5}g_{1}^{2}M_{1} + 3g_{2}^{2}M_{2} + 3\text{Tr}(Y_{u}^{\dagger}A_{u}) + \text{Tr}(Y_{\nu}^{\dagger}A_{\nu}) \right\} Y_{u_{ij}} \\
+ 4(Y_{\nu}^{\dagger}Y_{\nu}A_{\nu})_{ij} + 5(A_{\nu}Y_{\nu}^{\dagger}Y_{\nu})_{ij} + 2(Y_{\nu}Y_{d}^{\dagger}A_{d})_{ij} + (A_{\nu}Y_{d}^{\dagger}A_{d})_{ij} . \tag{45}
\]
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Figure 1: The evolution of the gaugino masses from the SUSY breaking scale to the $B - L$ breaking scale. The red line shows the running of the gluino mass, the green line is the running of the SU(2) gaugino mass, and the blue corresponds to the running of the U(1)$_Y$ gaugino mass.
Figure 2: The SUSY breaking scale, $\Lambda_S$, dependence of the gaugino masses at the weak scale. Again, the red line shows the running of the gluino mass, the green line is the running of the SU(2) gaugino mass, and the blue corresponds to the running of the U(1)$_Y$ gaugino mass.
Figure 3: The gauge coupling constant, $g_{B-L}$, dependence of the gaugino masses at the weak scale. Here the gauge coupling constant, $g_{B-L}$ is given at a given SUSY breaking scale $\Lambda_S = 10^9$ GeV. Again, the red line shows the running of the gluino mass, the green line is the running of the SU(2) gaugino mass, and the blue corresponds to the running of the $U(1)_Y$ gaugino mass.
Figure 4: The evolution of the soft mass squared for the field $\Delta_1$ from the SUSY breaking scale to the $B-L$ breaking scale. In this plot, we take the SUSY breaking scale as $\Lambda_S = 10^9$ GeV, $M_{\tilde{Z}_{B-L}} = 8.7 \times 10^5$ GeV and $g_{B-L} = 0.5$. In this figure, from top to the bottom curves, we varied the value of $f$ as $f = 4, 5, 6, 7$. 
Figure 5: The evolution of the soft mass squared for the field $\Delta_2$ from the SUSY breaking scale to the $B-L$ breaking scale. In this plot, we take the SUSY breaking scale as $\Lambda_S = 10^9$ GeV, $M_{\tilde{Z}_{B-L}} = 8.7 \times 10^5$ GeV and $g_{B-L} = 0.5$. 