Absence of Relativistic Stars in $f(T)$ Gravity

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Abstract. In this work we investigate the structure of neutron stars in modified $f(T)$ gravity models. We find that, unlike the $f(R)$ models, the equations of motion put a rather strict constraint on the possible $f(T)$ functions. Specifically, after analyzing the problem in two different choice of coordinates with spherical symmetry, we conclude that relativistic neutron star solution in $f(T)$ gravity models is possible only if $f(T)$ is a linear function of the torsion scalar $T$, that is in the case of Teleparallel Equivalent of General Relativity.

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1. Introduction

One of the assumptions of the current paradigm of cosmology is the validity of Einstein’s theory of gravity in all scales: from phenomena we observe in our solar system, to the large scale structure of the universe. However, in recent years, data from distant supernovae Ia [1–4] are interpreted as evidence of late time acceleration in the expansion rate of the universe. Continuing to assume the validity of general relativity in all scales and best fit to observational data requires existence of a non–vanishing positive cosmological constant. There are several theoretical problems related with the existence of cosmological constant (see for example [5–8]), most important of which is the lack of a quantum theoretical method to calculate its inferred value from cosmological data. Several authors tried to avoid the problems of cosmological constant with alternative routes of explanations. To explain late time accelerated expansion one can either modify “wood” part, i.e. matter part, or modify “marble” part, i.e. geometric part, of the Einstein’s field equations. In the former approach one adds the energy–momentum component of dark energy with an equation of state $p/\rho \approx -1$ to the wood part of the Einstein’s equations. As opposed to this, in the latter approach, one modifies the theory of gravity, and this way changes the marble part of the Einstein’s equations. Such modifications could be in two fashions: one can either increase the number of degrees of freedom by adding new gravitational fields into the theory, or changes the form of the

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gridy action without introducing new fields. Metric or vierbein field still remains the only gravitational degree of freedom in the second approach.

One important family of modifications of Einstein–Hilbert action is the $f(R)$ theories of gravity (see reviews [9,12] and references therein). In such theories one uses a function of curvature scalar as the Lagrangian density. However, the field equations of $f(R)$ gravity models turn out to be 4th order differential equations in the metric formalism and therefore they are difficult to analyze. With a similar line of thought one can also modify teleparallel equivalent of general relativity [11]. This theory is defined on a Witzenböck space–time, which is curvatureless, but has a non–vanishing torsion. Lagrangian density is equivalent to the torsion scalar and the field equations of teleparallel gravity are exactly the same as Einstein’s equations in any background metric [13,16]. One can modify teleparallel gravity by having a Lagrangian density equivalent to a function of torsion scalar. This is first done in the context of Born–Infeld gravity [17,18], however it is possible to have any function $f(T)$. Then one has $f(T)$ theories of gravity [19]. These theories are more manageable compared to $f(R)$ theories, because their field equations are second order differential equations.

A modified gravity theory should be able to pass several tests before it can be considered a viable theory of gravity. In the weak gravity regime, such a theory should be compatible with solar system tests and table–top experiments. In cosmological scales, it should produce late time acceleration, be free of gravitational instabilities, and obey constraints of standard model of cosmology. Such a theory should do well also in strong gravity regime, for example it should have solutions of neutron stars with mass–radius relation inside the current observational bounds. In this paper we are analyzing $f(T)$ gravity in the strong field regime and test whether such theories could be viable theories of reality. To our surprise, we find that the field equations of $f(T)$ gravity theories are incompatible with the conservation equation of neutron star matter unless $f(T)$ is a linear function of torsion scalar $T$. This means that only teleparallel equivalent of general relativity could be a viable theory of gravity. Any modification of it, other than addition of a cosmological constant, will not have relativistic neutron star solution and therefore be at odds with the observations.

The plan of this paper is as follows. In the next section we will summarize main aspects and provide field equations of teleparallel and $f(T)$ theories of gravity. Then, in section (3), the field equations will be rewritten in a spherically symmetric background with a diagonal metric and it will be shown that they are incompatible with the conservation equation. Since $f(T)$ gravity theories are not Lorentz invariant [20,21], we are going to repeat our analysis also for a non-diagonal spherically symmetric metric in section (4). We are going to discuss implications of this result in the conclusions. In the appendix we will repeat the analysis of section (3) in the case of $f(R)$ gravity and show that for those theories field equations are compatible with the conservation equation.

‡ We will sometime use the shorter name, “teleparallel gravity,” instead of the longer name “teleparallel equivalent of general relativity” to describe the same theory.
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2. Field Equations of $f(T)$ Gravity

As is well known, general relativity is formulated on a pseudo–Riemannian manifold and its dynamical variable is the metric tensor defined on that manifold. Through metric tensor one defines the Levi–Civita connection, and then Riemann and the related tensors. Torsion tensor in a Riemannian space–time vanishes due to the symmetry properties of the Levi–Civita connection. If the torsion tensor is non-zero together with the Riemann tensor, then we have Riemann–Cartan space–time on which Einstein–Cartan theory of gravity is defined. In a sense, Riemannian space–time can be thought as a subclass of Riemann–Cartan space–time: by setting the torsion tensor to zero, Riemann–Cartan space–time is reduced to a Riemannian space–time. Another subclass is the Weitzenb¨ ock space–time which is obtained by setting the curvature tensor to zero instead. Then affine connections are no longer symmetric and the theory of gravity defined on a Weitzenb¨ ock space–time is called teleparallel gravity.

Dynamical field of teleparallel gravity is the vierbein field $e_{i}^{\mu}$ which is given in terms of the metric tensor as

$$\eta_{ij} = g_{\mu\nu}e_{i}^{\mu}e_{j}^{\nu},$$

where latin indices label coordinates of tangent space and greek indices label coordinates of the space–time. Both set of indices run over $(0, 1, 2, 3)$. Teleparallel gravity differ from the general relativity in the sense that it uses the curvatureless Weitzenb¨ ock connection

$$\Gamma_{\mu\nu}^{\rho} = e_{i}^{\rho}(\partial_{\nu}e_{i}^{\mu} - \partial_{\mu}e_{i}^{\nu}).$$

Then non–vanishing torsion is given by

$$T_{\mu\nu}^{\lambda} = e_{i}^{\lambda}(\partial_{\mu}e_{i}^{\rho} - \partial_{\nu}e_{i}^{\rho}).$$

To define the action for teleparallel gravity one defines two more tensors: one is the contorsion tensor given in terms of the torsion tensor,

$$K_{\mu\nu}^{\rho} = -\frac{1}{2}(T^{\mu\nu}_{\rho} - T^{\nu\mu}_{\rho} - T^{\mu\nu}_{\rho}),$$

and the other tensor is defined in terms of contorsion and torsion tensors as

$$S_{\mu\nu}^{\rho} = \frac{1}{2}(K^{\mu\nu}_{\rho} + \delta^{\mu}_{\rho}T^{\alpha\nu}_{\alpha} - \delta^{\nu}_{\rho}T^{\alpha\mu}_{\alpha}).$$

Then the torsion scalar is defined as

$$T = S_{\mu\nu}^{\rho}T^{\rho}_{\mu\nu}.$$  

Torsion scalar is used as the Lagrangian density in the action for teleparallel gravity

$$S = -\frac{1}{16\pi G} \int d^{4}x \ e \ T + S_{\text{matter}}$$

where $e = \text{det}(e_{\mu}^{i}) = \sqrt{-g}$ and $S_{\text{matter}}$ is the part of the action that describes matter fields interacting with the vierbein field. The variation of the action with respect to the vierbein leads to the equations of motion which are identical to the equations of motion of general relativity. Therefore this form of torsion gravity is equivalent to the general
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relativity. This theory is first proposed by Einstein [13, 14] and therefore it is rightfully called Einstein’s other gravity [22], new general relativity [15] or teleparallel equivalent of general relativity [16].

As in the case of general relativity, this action might be modified by having a function of torsion scalar, $f(T)$, as the Lagrange density:

$$S = -\frac{1}{16\pi G} \int d^4x \sqrt{g} f(T) + S_{\text{matter}}.$$  \hspace{1cm} (8)

Then we have $f(T)$ theories of gravity similar to $f(R)$ theories of gravity. As stated in the introduction, this is a new set of modified gravity theories which might have the potential to answer some unresolved questions in the contemporary cosmology. The variation of the action for $f(T)$ gravity with respect to the vierbein leads to the following field equations:

$$e^\rho_i S_{\mu\nu}^\rho \partial_\mu f_T + e^{-1} \partial_\mu (e e_i^\rho S_{\mu\nu}^\rho) f_T + e_i^\rho T_{\mu\nu}^\lambda S_\lambda^\nu f_T - \frac{1}{4} e_i^\nu f = -4\pi e_i^\lambda T_{\lambda}^\nu$$ \hspace{1cm} (9)

where $T_{\mu\nu}$ is the energy-momentum tensor of the particular matter, whereas $f_T$ and $f_{TT}$ represent first and second derivatives of $f(T)$ with respect to the torsion scalar $T$, respectively. Note that we are setting $c = 1$ and $G = 1$ here and for the rest of this paper.

3. Contradiction with the Conservation Equation

To describe relativistic neutron stars in a theory of gravity, one usually starts with two assumptions: 1) the spherically symmetric metric of neutron star has diagonal structure,

$$ds^2 = -e^{2\Sigma(r)} dt^2 + e^{2\Lambda(r)} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2,$$ \hspace{1cm} (10)

and 2) matter inside the neutron star is a perfect fluid which has a diagonal energy–momentum tensor in the rest frame of the matter,

$$T_{\mu}^\nu = \text{diag}(-\rho, p, p, p)$$ \hspace{1cm} (11)

where $\rho$ and $p$ are the energy density and pressure of the fluid, respectively. Matter functions, $\rho$ and $p$, and metric functions, $\Sigma$ and $\Lambda$, are taken independent of time, which means that the system is in equilibrium, and due to spherical symmetry they are functions of $r$ only.

Energy–momentum tensor is assumed to be covariantly constant in general relativity, which leads to

$$\frac{dp}{dr} = - (\rho + p) \frac{d\Sigma}{dr}.$$ \hspace{1cm} (12)

This equation should hold identically for all systems described with the metric (10). This is an equation of hydrostatic equilibrium and global aspects of neutron stars, such as mass–radius relation, can be determined from it, if the metric function $\Sigma(r)$ is known. For a spherically symmetric object in general relativity, this is one of the Tolman–Openheimer–Volkov (TOV) equations after $\Sigma(r)$ is solved from the field equations.
The vierbein field derived from the metric (10) is
\[ e_0^0 = e^{\Sigma(r)}, \quad e_1^1 = e^{\Lambda(r)}, \quad e_2^2 = r, \quad e_3^3 = r \sin \theta. \quad (13) \]
With these values, the determinant of vierbein becomes
\[ e = \sqrt{-g} = r^2 \sin \theta e^{(\Sigma+\Lambda)}, \]
and the torsion scalar in this background is found as
\[ T = -\frac{2}{r} (2\Sigma' + \frac{1}{r}) e^{-2\Lambda}, \quad (14) \]
where prime denotes the derivative with respect to \( r \).
Substituting these values into the field equations of modified gravity (9), we obtain modified equations of motion for \( i = \nu = 0 \):
\[ 16\pi \rho = -\frac{4}{r} e^{-2\Lambda} T'^{fT} + \left( \frac{2}{r^2} + 2T + \frac{4}{r} (\Sigma' + \Lambda') e^{-2\Lambda} \right) f_T - f \quad (15) \]
and for \( i = \nu = 1 \):
\[ 16\pi p = -\left( \frac{2}{r^2} + 2T \right) f_T + f. \quad (16) \]
The other two components of the field equations are the same and are given as
\[ 16\pi p = 2(\Sigma' + \frac{1}{r}) e^{-2\Lambda} T'^{fT} \]
\[ + 2e^{-2\Lambda} (\Sigma'' + \Sigma' (\Sigma' - \Lambda') + \frac{1}{r^2} (3\Sigma' - \Lambda') \right) f_T + f. \quad (17) \]
One can check that these equations become the field equations of general relativity in the limit that \( f(T) = T \).
Now we would like to check the compatibility of these field equations with the conservation equation. Even though the connection on Wietzenb"ock space–time is different than Levi–Civita connection, the conservation equation turns out to be the same as the one on Riemannian space–time (12). The form of the conservation equation is independent of the particular \( f(T) \) theory that describes the gravity. It depends just on the particular form of the background metric and the particular form of the matter. So if one finds a discrepancy between field equations and the conservation equation, this means that the particular \( f(T) \) theory is not valid in the background considered.
To obtain the conservation equation (12) from the field equations (15,17) we do the following calculations: 1) Take the derivative of (16) with respect to \( r \), 2) Subtract (16) from (17) and then multiply the result with \( 2/r \), 3) Add (15) and (16) and then multiply the result with \( \Sigma' \), 4) Subtract the result of step 2 from the result of step 3, and then substitute it into the result of step 1. After these calculations one obtains
\[ \frac{dp}{dr} = -(\rho + p) \frac{d\Sigma}{dr} + \frac{1}{8\pi r^2} \frac{dT}{dr} f_{TT}. \quad (18) \]
This equation is in contradiction with the conservation equation (12) which should be satisfied in all cases. Therefore, to eliminate the contradiction, second term on the right hand side of (18) must vanish:
\[ \frac{dT}{dr} \frac{d^2 f(T)}{dT^2} = 0. \quad (19) \]
This equation puts a rather strict constraint on the possible $f(T)$ functions. In fact, the only possible $f(T)$ function is the linear one. As a result, among the $f(T)$ theories of gravity, only teleparallel equivalent of general relativity is possible in the case of spherically symmetric problem.

The above result seems peculiar to relativistic star solution in $f(T)$ gravity. For instance, in the case of $f(R)$ gravity we do not find a similar contradiction. There, if one follows the same steps as described in the paragraph preceding equation (18), one obtains exactly the conservation equation. The details of the calculation for $f(R)$ gravity is presented in the Appendix A.

One might argue against the conclusion of this section on the grounds that the $f(T)$ theories of gravity lack Lorentz invariance [20,21] and therefore result of this section could be just the artifact of the frame used. In another frame we might not find the same result and relativistic star solution could be natural there. If that is so, then we have very strong frame dependent physics: in one frame we have a universe without stars and in the other with stars. Such a frame dependent physics seems most bizarre. We are against this line of thought. As it is commented in [20], because of absence of local Lorentz invariance one cannot choose vierbein field, but has to determine it from the field equations. However, this is computationally very complicated. Even though we might not be able to check the possibility of relativistic star solution in all choices of frame corresponding to spherically symmetric background and find if relativistic star solution is possible in one of them, we would like to repeat our analysis for a non–diagonal spherically symmetric metric to see if the same kind of conclusion could be made there. The non–diagonal metric we are going to use is Gullstrand–Painlevé type. A similar metric is used recently in [23]. There, it is firstly shown that, in the case of Friedmann–Robertson–Walker metric, depending on the choice of frame, one obtains sets of field equations that give contradictory physics. Then, the generalized Gullstrand–Painlevé metric is used to search spherically symmetric static solutions. Curiously, it is shown, in the mentioned coordinate system [23], that the Reissner–Nordström solution does not exist in $f(T)$ gravity theories other than the teleparallel equivalent of general relativity. This result is similar to our result about the absence of relativistic stars in $f(T)$ theories of gravity.

4. The Case of a Non–Diagonal Frame

To see if the result of the previous section still holds in the case of a non–diagonal spherically symmetric metric, we now work with a Gullstrand-Painlevé type metric

$$ds^2 = -\beta^2 dt^2 + \delta_{ab}(\beta \sqrt{\alpha} x^a_r dt + dx^a)(\beta \sqrt{\alpha} x^b_r dt + dx^b),$$

where latin indices from the beginning of alphabet run over (1, 2, 3) and $r = \sqrt{x^a x_a}$ is the radial coordinate. Metric functions, $\alpha$ and $\beta$, depend on $r$ only. Time coordinate of this metric is conformally Cartesian and space coordinates $x^a$ fully span the range
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$(-\infty, +\infty)$. In order to see the relation of this metric with $\text{(10)}$ we write it in terms of spherical coordinates:

$$ds^2 = -\beta^2(1 - \alpha)dt^2 + 2\beta\sqrt{\alpha}dt dr + dr^2 + r^2d\Omega^2.$$  \hfill (21)

This form of metric is related to a diagonal form of metric via a Lorentz transformation done only in the time direction:

$$\beta dt = \beta \tilde{t} + \sqrt{\alpha} \frac{1}{1 - \alpha} dr.$$  \hfill (22)

With this transformation we obtain a more familiar form of the metric

$$ds^2 = -\beta^2(1 - \alpha)\tilde{t}^2 + \frac{1}{1 - \alpha}dr^2 + r^2d\Omega^2.$$  \hfill (23)

The vierbein field $e^i_\mu$ derived from the metric $\text{(20)}$ is

$$e^0_0 = \beta, \quad e^a_0 = \beta \sqrt{\alpha} \frac{r^a}{r}, \quad e^b_a = \delta^b_a.$$  \hfill (24)

The determinant of vierbein field is $e = \sqrt{-g} = \beta$ and torsion scalar is found equal to

$$T = \frac{2\alpha}{r} \left( \frac{2\beta'}{\beta} + \frac{\alpha'}{\alpha} + \frac{1}{r} \right).$$  \hfill (25)

Substituting these values into the field equations of modified gravity $\text{(9)}$ and using again energy–momentum tensor of perfect fluid, we obtain modified equations of motion for $a = \nu = 0$:

$$16\pi \rho = 2T f_T - f,$$  \hfill (26)

for $a = 0$, $\nu = i$:

$$16\pi p = \left( \frac{4\beta'}{\beta} - 2T \right) f_T + f,$$  \hfill (27)

and for $a = i$, $\nu = i$:

$$48\pi p = \left( \frac{4\beta'}{\beta} - \frac{2\alpha}{r} - rT \right) T' f_{TT} - \frac{1}{\beta} \frac{d}{dr} (\beta rT - 4\beta') f_T$$

$$+ \left[ \frac{2\alpha}{r} \left( \frac{\beta'}{\beta} + \frac{4\beta'}{\alpha \beta} \right) - 5T \right] f_T + 3f.$$  \hfill (28)

Unlike the case of diagonal metric, there is one more non-vanishing component of field equation here: for $a = i$, $\nu = 0$ we have

$$\frac{d}{dr} \left( \frac{f_T}{\beta} \right) = 0.$$  \hfill (29)

This equation already requires that $f(T)$ must be a linear function of $T$. However, we would like to check the compatibility of the field equations with the conservation equation as in the previous section to see if we reach the same conclusion.

In order to check the compatibility of the field equations with the conservation equation we perform the following calculations: first, we take derivative of equation $\text{(27)}$ with respect to $r$. Then, we subtract equation $\text{(28)}$ from three times the equation $\text{(27)}$ and multiply the result with $1/r$. After that, we subtract results of previous steps
from each other. Lastly, we use equations (26) and (27) and the expression for torsion scalar (25) to simplify the result of third step. After these calculations we obtain

$$\frac{dp}{dr} = -\frac{\beta'}{\beta}(\rho + p) - \frac{\alpha\beta}{8\pi r} \left( \frac{2\beta'}{\beta} + \frac{\alpha'}{\alpha} \right) \frac{d}{dr} \left( \frac{f_T}{\beta} \right).$$

(30)

The form of the conservation equation in the chosen Gullstrand–Painlevé coordinates turns out to be

$$\frac{dp}{dr} = -\frac{\beta'}{\beta}(\rho + p).$$

(31)

Hence we reach the same conclusion as in the previous section. Only possible form of \(f(T)\) function is the linear one and therefore other than teleparallel equivalent of general relativity no \(f(T)\) theory of gravity is possible in the case of spherically symmetric problem.

5. Conclusions

In this paper, we aimed to analyze the \(f(T)\) theories of gravity in the strong field regime. \(f(T)\) gravity has 2nd order field equations compared to 4th order field equations of \(f(R)\) gravity. Therefore it is an easier theory to study. However, it is shown in [20, 21] that the action of \(f(T)\) gravity is not Lorentz invariant. This means that the gravitational effects in \(f(T)\) theories are frame dependent and different choices of frame will result in different forms of field equations. This strong background dependence of field equations of \(f(T)\) gravity is recently discussed in [23, 24].

In the search of spherically symmetric relativistic star solution in \(f(T)\) gravity, we found that the field equations are incompatible with the conservation equation except in the case of linear functional form of \(f(T)\). In other words, relativistic star solutions are possible only in the case of teleparallel equivalent of general relativity. We reached to the same conclusion in two different frames: one in the case of a diagonal metric and the other in the case of a non-diagonal metric. This conclusion of the present paper points out to a serious problem. Even though \(f(T)\) theories of gravity might explain late time acceleration of the universe [19, 22, 25, 26], they cannot be a viable theory of gravity due to unsatisfactory behavior in the strong field regime.

We would like to note here that, in the case of Friedmann–Robertson–Walker cosmological model, the continuity equation is compatible with the field equations [24] independent of the chosen form of \(f(T)\) function. Thus this paper demonstrates the importance of testing alternative theories of gravity by neutron star physics. Tests in the strong field regime complement the solar system and the cosmological tests which are done to decide if a particular theory of gravity could be a candidate of a viable theory of reality.

Lastly, we should mention that a locally Lorentz invariant version of \(f(T)\) gravity is proposed in [20, 21] and its implications for cosmology is investigated in [27]. Investigating the neutron star solutions in such a theory would be an interesting problem that we are planning to do as a future work.
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Appendix A. Conservation Equation in $f(R)$ Gravity

Following [28] we define

$$f(R) = R + \alpha h(R) + \mathcal{O}(\alpha^2) \quad (A.1)$$

where $h(R)$ is an arbitrary function of $R$ and action is given by

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} f(R) + S_{\text{matter}} \quad (A.2)$$

Variation of the action with respect to the metric yields the field equations

$$(1 + \alpha h_R) G_{\mu
u} - \frac{1}{2} \alpha (h - h_R R) g_{\mu
u} - \alpha (\nabla_\mu \nabla_\nu - g_{\mu\nu} \Box) h_R = 8\pi T_{\mu\nu} \quad (A.3)$$

where $h_R$ is derivative of $h(R)$ with respect to $R$. Keeping only first order terms in $\alpha$, one can write $tt$ component of the field equation [28] as

$$8\pi \rho_\alpha = \frac{1}{r^2} - \frac{1}{r^2} (1 - 2r \Sigma'_\alpha) e^{-2\Lambda_\alpha} + \alpha \frac{h_R}{r^2} (1 - (1 - 2r \Lambda') e^{-2\Lambda})$$

$$+ \alpha \frac{1}{2} (h - h_R R) - \alpha \left( \frac{h'_R}{r} (2 - r \Lambda') + h''_R \right) e^{-2\Lambda}, \quad (A.4)$$

and $rr$ component [28] as

$$8\pi p_\alpha = -\frac{1}{r^2} + \frac{1}{r^2} (1 + 2r \Sigma'_\alpha) e^{-2\Lambda_\alpha} - \alpha \frac{h_R}{r^2} (1 - (1 + 2r \Sigma') e^{-2\Lambda})$$

$$- \alpha \frac{1}{2} (h - h_R R) + \alpha \frac{h'_R}{r} (2 + r \Sigma') e^{-2\Lambda}. \quad (A.5)$$

The other two components of the field equations are the same and given as

$$8\pi p_\alpha = (1 + \alpha h_R) \left( \Sigma''_\alpha + \Sigma'^2_\alpha - \Sigma'_\alpha \Lambda'_\alpha - \frac{1}{r} (\Sigma'_\alpha - \Lambda'_\alpha) \right) e^{-2\Lambda_\alpha}$$

$$- \alpha \frac{1}{2} (h - h_R R) + \alpha \left( \frac{h'_R}{r} (1 + r (\Sigma' - \Lambda')) + h''_R \right) e^{-2\Lambda}. \quad (A.6)$$

where functions with subscript $\alpha$ have perturbative expansion in $\alpha$ as $\Sigma_\alpha = \Sigma + \alpha \Sigma_1 + \ldots$, etc. Zeroth order functions, which are solutions of Einstein equations, are written without subscript.

Following the same steps performed in the case of $f(T)$ gravity one ends up with

$$\frac{dp_\alpha}{dr} = -(\rho_\alpha + p_\alpha) \frac{d\Sigma_\alpha}{dr} \quad (A.7)$$

which is perturbative form of conservation equation and it is satisfied regardless of the form of $f(R)$ function. Therefore, one do not find any constraint on possible $f(R)$ functions from the field equations in $f(R)$ gravity.
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