Is pulsar timing a hopeful tool for detection of relic gravitational waves by using GW150914 data?

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Abstract The inflation stage has a behaviour as power law expansion like $S(\eta) \propto \eta^{1+\beta}$ where $\beta$ constrained on the $1+\beta < 0$. If the inflation were preceded by a radiation era, then there would be thermal spectrum of relic gravitational waves at the time of inflation. Based on this idea we find new upper bound on $\beta$ by comparison the thermal spectrum with strain sensitivity of single pulsar timing. Also we show that sensitivity curve of single pulsar timing may be hopeful tool for detection of the spectrum in usual and thermal case by using the GW150914 data.

1 Introduction

The inflation stage has a behaviour as power law expansion like $S(\eta) \propto \eta^{1+\beta}$. The $S$ and $\eta$ are scale factor and conformal time respectively and $\beta$ constrained on the $1+\beta < 0$ \cite{1,2}. If the inflation were preceded by a radiation era, then there would be thermal spectrum of gravitational waves at the time of inflation \cite{3–5}. Based on this idea we have shown in \cite{6} that there is some more chances for the detection of relic gravitational waves (RGWs) in usual and thermal spectrum by using the data of GW150914. On the other hand, there is an important procedure for direct detection of RGWs that called pulsar timing (PT). This procedure is based on the investigation of fluctuation of the pulses of the pulsars due to RGWs. The frequency range of PT is $10^{-9}–10^{-6}$ Hz \cite{7}. The authors in \cite{7} have found an upper bound on $\beta$ and shown that there is so low chance for the detection of RGWs by using the mentioned method. But we think that the upper bound on $\beta$ will modify if we use the thermal spectrum. Also the chance of detection of RGWs based on the strain sensitivity of PT will increase by using the obtained $\beta$ for GW150914 \cite{6} in usual and thermal case. Therefore the main purpose of this work is investigation of this detection. In the present work, we use the unit $c = \hbar = k_B = 1$.

2 The spectrum of gravitational waves in usual and thermal case

The perturbed metric for a homogeneous isotropic flat Friedmann–Robertson–Walker universe can be written as

$$ds^2 = S^2(\eta) \left( d\eta^2 - (\delta_{ij} + h_{ij})dx^i dx^j \right),$$

where $\delta_{ij}$ is the Kronecker delta symbol and $h_{ij}$ are metric perturbations with the transverse-traceless properties i.e; $\nabla_i h^i = 0$, $\delta^i_j h_{ij} = 0$. The gravitational waves are described with the linearized field equation given by

$$\nabla_\mu \left( \sqrt{-g} \nabla^\mu h_{ij}(x, \eta) \right) = 0.$$  (2)

The tensor perturbations have two independent physical degrees of freedom ($h^+, h^\times$) called polarization modes. We can write $h^+$ and $h^\times$ in terms of the creation ($a^+$) and annihilation ($a$) operators then,

$$h_{ij}(x, \eta) = \frac{\sqrt{16\pi l_{pl}}}{S(\eta)} \sum_p \int \frac{d^3k}{(2\pi)^{3/2}} e_p^i(k)$$

$$\times \frac{1}{\sqrt{2k}} \left[ \bar{a}_{k p} h_{ij}^p(\eta) e^{i k x} + a^+_{k p} h^*_{ij}^p(\eta) e^{-i k x} \right].$$  (3)

where $k$ is the comoving wave number with $k = |k|$, $l_{pl} = \sqrt{G}$ is the Planck’s length and $p = +, \times$ are polarization modes. The polarization tensors $e_p^i(k)$ are symmetric and transverse-traceless $k^i e_p^i(k) = 0$, $\delta^{ij} e_p^j(k) = 0$ and satisfy the conditions $e^i p (k) e^{* p}_i (k) = 2 \delta_{p p}$ and $e^{i p}_i (-k) = e^{i p}_i (k)$. Also the creation and annihilation operators satisfy $[a_p^p, a^*_p] = \delta_{p p} \delta^3 (k - k')$ and the initial vacuum state is defined as

1 Note: the upper bound on $\beta$ is $\beta \leq -1.8$ \cite{1}.

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Given the expanded context, it becomes clear that the text is discussing the analysis of perturbations in the universe, particularly focusing on the spectrum of relativistic gravitational waves (RGWs) and their comparison with the strain sensitivity of pulsar timing (PT) experiments. The analysis involves theoretical models and comparisons with observational data, such as the Planck satellite's observations. Key points include the use of the conformal time scale, the role of polarization states, and the modified amplitude in the spectrum of RGWs.

The text introduces equations to describe the perturbations, such as the modified amplitude $A_k^P(0) = 0$, and discusses the modified power spectrum $A = \frac{\Delta R(k_0)^{1/2}}{(1 + z_E)^{\frac{2 - \nu}{\nu}}\left(\frac{v_H}{v_0}\right)^{\beta}}$. It also references the thermal spectrum for the lower frequency and the enhancement of amplitude factors in non-vacuum states.

The analysis of the spectrum involves the calculation of noise spectral densities, which are crucial for understanding the detection capabilities of PT experiments. Figures illustrating these spectra are mentioned, with one showing the spectrum $h_{mc}(v)\sqrt{F/v}$ of RGWs compared to the strain sensitivity of PT.

The text concludes with remarks on the lower frequency spectrum of RGWs, with a discussion on the significance of the obtained $\beta$ values and their implications for the detection of gravitational waves.
Fig. 2 The spectrum $h_{mc}(\nu)\sqrt{F/\nu}$ of RGWs in usual case (solid lines) and thermal case (dashed lines) for two obtained $\beta$ based on GW150914 data [6] compared to the strain sensitivity of PT (green line)

chance for detection of RGWs for the spectrum in usual and thermal case.

Therefore based on our results, the PT may be more hopeful tool for the detection of RGWs.

4 Discussion and conclusion

The thermal spectrum of RGWs causes an enhanced amplitude that called 'modified amplitude'. The obtained results based on the thermal spectrum cause new upper bound on $\beta$ which can give us more information about the inflation. Also using the obtained amounts of $\beta$ based on the GW150914 data tell us that PT may be more hopeful tool for detection of RGWs in usual and thermal case.

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Appendix A: extra-dimensional scenario and thermal gravitons

Cosmology with extra dimensions has been motivated by Kaluza and Klein (KK) [20–22]. The modern scenarios involving extra dimensions are being explored in particle physics, with most models possessing either a large volume or a large curvature. Although there exist different models of extra dimensions, there are some general features and signals common to all of them. In the presence of $D$ extra spatial dimensions, the $(3 + D + 1)$-dimensional action for gravity can be written as

\[ S = \int d^4X \left[ \int d^Dy \sqrt{-g_D} \left( \frac{R_D}{16\pi G_D} + \sqrt{-g} L_m \right) \right], \]  

(A.1)

where

\[ G_D = G_N \frac{m_{pl}^2}{m_D^{2+D}}, \]  

(A.2)

and $m_{pl}$ is Planck mass, $g$ is the four-dimensional metric, $G_N$ is Newton’s constant, $g_D$, $G_N$ and $R_D$ denote the higher dimensional counterparts of the metric, Newton’s constant, and the Ricci scalar, respectively. The factor $m_D$ is the fundamental scale of the extra-dimensional theory.

Since the gravitational interactions are not strong enough to produce a thermal gravitons at temperatures below the Planck scale ($m_{pl} \sim 1.22 \times 10^{19}$ GeV), the standard inflationary cosmology predicted the existence of the cosmic gravitational wave backgrounds which are non-thermal in nature. However if the universe contains extra dimensions that can generate the thermal gravitational waves, then its shape and amplitude of the stochastic cosmic background of gravitational waves (CGWB) may change significantly. This can happen when energies in the universe are higher than the fundamental scale $m_D$. The gravitational coupling strength increases significantly as the gravitational field spreads out into the full spatial volume. Instead of freezing out at $O(m_{pl})$, as in $3+1$ dimensions, gravitational interactions freeze-out at $\sim O(m_D)$. If the gravitational interactions become strong at an energy scale below the reheat temperature ($m_D < T_{RH}$), gravitons get the opportunity to thermalize, creating a thermal CGWB. The creation of a thermal CGWB if ($m_D < T_{RH}$), is unchanged by the type of extra dimensions chosen [9].

Thus, if extra dimensions do exist and the fundamental scale of those dimensions is below the reheat temperature, a relic thermal CGWB ought to exist today. Compared to the relic thermal photon background (CMB), a thermal CGWB
would have the same shape, statistics and high degree of isotropy and homogeneity. The energy density ($\rho_g$) and fractional energy density ($\Omega_g$) of a thermal CGWB are

$$\rho_g = \frac{\pi^2}{15} \left(\frac{3.91}{g_*}\right)^{4/3} T_{CMB}^4,$$

(A.3)

$$\Omega_g = \frac{\rho_g}{\rho_c} \simeq 3.1 \times 10^{-4} (g_*)^{4/3},$$

(A.4)

where $\rho_c$ is the critical energy density today, $T_{CMB}$ is the present temperature of the CMB and $g_*$ is the number of relativistic degrees of freedom at the scale of $m_D$. $g_*$ is dependent on the particle content of the universe, i.e. whether (and at what scale) the universe is supersymmetric, has a KK tower, etc. Other quantities, such as the temperature ($T$), peak frequency ($\nu$), number density ($n$), and entropy density ($s$) of the thermal CGWB can be derived from the CMB if $g_*$ is known, as

$$n_g = n_{CMB} \left(\frac{3.91}{g_*}\right), \quad s_g = s_{CMB} \left(\frac{3.91}{g_*}\right),$$

(A.5)

$$T_g = T_{CMB} \left(\frac{3.91}{g_*}\right)^{1/3}, \quad \nu_g = \nu_{CMB} \left(\frac{3.91}{g_*}\right)^{1/3}.$$  

(A.6)

These quantities are not dependent on the number of extra dimensions, as the large discrepancy in size between the three large spatial dimensions and the $D$ extra dimensions suppresses those corrections by at least a factor of $\sim 10^{-29}$. If $m_D$ is just barely above the scale of the standard model, then $g_* = 106.75$. The thermal CGWB then has a temperature of $T_g = 1.52 \times 10^{25} Mpc^{-1} \simeq 0.905 \, K$ and a peak frequency of 19 GHz [9].

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