Keywords: Goldbach; De Polignac; Twin primes; Algebraic proof

Abstract
The present algebraic development begins by an exposition of the data of the problem. The definition of the primal radius $r > 0$ is: For all positive integer $x \geq 3$ exists a finite number of integers called the primal radius $r > 0$, for which $x + r$ and $x - r$ are prime numbers. The corollary is that $2x = (x + r) + (x - r)$ is always the sum of a finite number of primes. Also, for all positive integer $x \geq 0$, exists an infinity of integers $r > 0$, for which $x + r$ and $r - x$ are prime numbers. The conclusion is that $2x = (x + r) - (r - x)$ is always an infinity of differences of primes.

Introduction
There is a similarity between the assertion: “an even number is always the sum of two primes” and the assertion: “an even number is always the difference of two primes”. The present article gives the proof that the two assertions are the consequences of the same concept by the introduction of the notion of the primal radius.

The proof
Let us suppose that exists an integer $x \geq 3$ for which $2x$ is never the sum of two primes, then for all $p_1$ and $p_2$ primes, $3 \leq p_2 < p_1$, $2x = p_1 + p_2$, or $2x = p_1 + p_2 + 2b$, then $x = \frac{p_1 + p_2}{2} + b$.

But for all $p_1$, $p_2$, exists $b$, for which $x = \frac{p_1 - p_2}{2} + b$.

Let $x_1 = p_1 + 2b$, $x_2 = x_1 - 2b$, $x_3 = p_2 + 2b$, $x_4 = p_1 - 2b$.

We deduce that:

$x = \frac{p_1 + p_2}{2} - \frac{x_1 + x_2}{2} + b = \frac{x_1 - x_2}{2} + b$

$y = \frac{p_1 - p_2}{2} + b - \frac{x_1 + x_2}{2} = \frac{x_1 - x_2}{2} - \frac{x_1 - x_2}{2} - b$

$x_1 - x_2 = p_1 - p_2$

$y = \frac{x_1 - x_2}{2} + b - \frac{x_1 - x_2}{2} + b = \frac{x_1 - x_2}{2} + 3b - \frac{x_1 - x_2}{2} + b$

$x_1 + x_2 = p_1 + p_2$

$x_1 - x_2 = p_1 - p_2$
Lemma 1

The following formula

\[ x = \frac{p_1 + p_2}{2} + b = \frac{p_1 + x_1}{2} + 2b = \frac{p_1 + p_2}{2} = \frac{x_1 + x_2 + b}{2} \]

\[ y = \frac{p_1 - p_2}{2} + b = \frac{p_1 - x_2}{2} + b = \frac{p_1 - p_2}{2} = \frac{x_1 - x_2 - b}{2} \]

\[ = \frac{p_1 - x_2}{2} + 2b = \frac{x_1 - x_2}{2} + b = \frac{x_1 - x_2}{2} + 3b = \frac{x_1 - x_2 + b}{2} \]

Implies that there exist \( p_1 \) and \( p_2 \) prime numbers for which \( b = 0 \)

Proof of lemma 1

If \( x \) is prime \( 2x = x + x \) is the sum of two primes, then \( p_1 - p_2 \neq 0 \)

We will suppose firstly that \( (x_1 - x_2)(x_1 + x_2) \neq 0 \)

Let

\[ \frac{x_1 - x_2}{p_1 - p_2} = 1 + \frac{4b}{p_1 - p_2} \]

\[ \frac{x_1 - x_2}{x_1 - x_2} = 1 - \frac{4b}{x_1 - x_2} \]

We pose \( k = \frac{2b}{p_1 - p_2}, \quad k' = -\frac{2b}{x_1 - x_2} \)

\( k = 0 \Rightarrow b = 0 \), we have supposed \( b = 0 \)

\( \forall (x, y): \exists \varphi | x = \varphi y \)

\( x + y = (\varphi + 1)y = x_1 = 0, \quad x - y = (\varphi - 1)y = 0 \)

\( \forall (k, k'): \exists \alpha | k = \alpha k' \)

\[ \Rightarrow \frac{2b}{p_1 - p_2} = -\alpha \frac{2b}{x_1 - x_2} \]

\[ \Rightarrow x_1 - x_2 = \alpha(p_1 - p_2) = -x_2 - p_1 + p_2 = 4b = -(\alpha + 1)(p_1 - p_2) \]

\[ \Rightarrow b = \frac{-\alpha + 1}{4}(p_1 - p_2) \]

\[ \Rightarrow x = \frac{p_1 + p_2}{2} + b = \frac{p_1 + x_1}{2} + 2b = \frac{p_1 + p_2}{2} = \frac{x_1 + x_2 + b}{2} \]

\[ y = \frac{p_1 - p_2}{2} + b = \frac{p_1 - x_2}{2} + b = \frac{p_1 - p_2}{2} = \frac{x_1 - x_2 - b}{2} \]

Let

\[ \frac{x_1 + x_2}{p_1 + p_2} = 1 + \frac{4b}{p_1 + p_2} \]

\[ \frac{x_1 + x_2}{x_1 + x_3} = 1 - \frac{4b}{x_1 + x_3} \]
We pose \( m = \frac{2b}{p_1 + p_2}, \quad m' = -\frac{2b}{x_1 - x_1} \)

\( mm' = 0 \Rightarrow b = 0 \), we have supposed \( mm' \neq 0 \)

\( \forall (m, m'), \exists \beta \mid m = \beta m' \)

\[ \Rightarrow \frac{2b}{p_1 + p_2} = -\beta \frac{2b}{x_1 - x_1} \]

\[ \Rightarrow x_1 + x_3 = -\beta (p_1 + p_2) \Rightarrow x_1 + x_3 - p_1 - p_2 = 4b = -(\beta + 1)(p_1 + p_2) \]

\[ \Rightarrow b = \frac{-\beta + 1}{4} (p_1 + p_2) \]

\[ \Rightarrow \begin{align*}
  x &= \frac{p_1 + p_2 + b}{2} = \frac{p_1 + p_2 + \frac{-\beta + 1}{4} (p_1 + p_2)}{2} = \frac{1 - \beta}{4} p_1 + \frac{1 - \beta}{4} p_2 = \varphi - \frac{1}{\phi - 1} p_1 \\
  y &= \frac{p_1 - p_2 + b}{2} = \frac{p_1 - p_2 + \frac{-\beta + 1}{4} (p_1 + p_2)}{2} = \frac{1 - \beta}{4} p_1 - \frac{1 - \beta}{4} p_2 = \varphi - \frac{1}{\phi - 1} p_2 \\
  b &= \frac{-\alpha + 1}{4} (p_1 - p_2) = \frac{-\beta + 1}{4} (p_1 + p_2) \Rightarrow (\beta - \alpha) p_1 = -2 - \alpha - \beta) p_2
\end{align*} \]

But: \( (2k + 1)(2k' + 1) = 1 \Rightarrow 2k + 1 = k = 0 \Rightarrow (2k + 1)k' = -k \)

\[ \begin{align*}
  2k + 1 &= \frac{-k}{k'} = \frac{a(k + 1) + a'}{2} = \frac{a(k + 1) + a'}{2} = \frac{a}{a'} \left( k + 1 \right) + a' \\
  2k' + 1 &= \frac{-k'}{k} = \frac{c(k' + 1) + c'}{k} = \frac{c(k' + 1) + c'}{k} = \frac{c}{c'} \left( k' + 1 \right) + c'
\end{align*} \]

\( \forall (a, a', c, c') \), particularly \( (a, a', c, c') = \delta k, \quad a' = \delta c \)

\[ \Rightarrow (k - k') = \frac{2}{2k' + 1} \]

\[ \Rightarrow k - k' = \frac{2ac(k + 1)(k - 1) + 2ac + a'c}{(ac + 2ac')^2 + 2ac + a'c} \]

\[ \Rightarrow \begin{align*}
  k &= k' \Rightarrow a = 1 = \frac{a}{a'} \left( \frac{3 + a}{3 + a} \right) = a' \\
  \text{trivial solution: it is impossible.}
\end{align*} \]

\[ \begin{align*}
  k + k' &= \Rightarrow \frac{2}{2k' + 1} = 0 = 2ac + a + a'c + 2a'c - 2dkk' \\
  2k' + 1 &= \frac{2ac + a + a'c + 2a'c - 2dkk}{(ac + 2ac')e + 2ac + a'c + 2a'c)k' + ac + a'c + a'c}
\end{align*} \]

Also \( (2m + 1)(2m' + 1) = 1 \Rightarrow 2mm' + m + m' = 0 \Rightarrow (2m + 1)m' = -m \)

\[ \begin{align*}
  2m + 1 &= \frac{m}{m'} = \frac{a(m + 1) + a'}{2} = \frac{a(m + 1) + a'}{2} = \frac{a}{a'} \left( m + 1 \right) + a' \\
  2m' + 1 &= \frac{m}{m'} = \frac{c(m + 1) + c'}{2} = \frac{c(m + 1) + c'}{2} = \frac{c}{c'} \left( m + 1 \right) + c'
\end{align*} \] \( \forall (a, a', c, c') \)

\[ \begin{align*}
  2m + 1 &= \frac{(a + 2a')m + a + a')(m + c + c')
  2m' + 1 &= \frac{(c + 2c')m' + c + c')(m' + a' + a')
\end{align*} \]

\[ \begin{align*}
  (ac + 2ac')^2 + 2ac + a'c + 2a'c)k' + ac + a'c + a'c
  2m + 1 &= \frac{2}{2m' + 1} = 1 - (m - m') = \frac{2}{2m' + 1}
\end{align*} \]
Another proof: let u, u', v, v' verifying

\( (\sigma, \sigma', c, c') \) with \( \sigma' = \delta k^2 = \gamma m^2 \), \( a' c = \delta k^2 = \gamma m^2 \)

\[ (m - m') = \frac{2}{2m + 1} \]

\[ = \frac{2a cm + 2(-\gamma m') m^2 + (2a + ac' + 2a' c')(m - m') + 2ac' m - 2ac' m'}{(2a + ac') m^2 + (2a + 3ac' + a' c' + 2a' c') m' + ac' + ac' + a' c' + a' c' + a' c'} \]

But

\[ \frac{2}{2k + 1} = \frac{2a cm + 2ac' + ac' + a' c' + 2a' c' - 2\delta k^2}{(ac + 2ac') m^2 + (2a + 3ac' + a' c' + 2a' c') m' + ac' + ac' + a' c' + a' c'} \]

For \( (\sigma, \sigma', c, c') \) with \( \sigma' = \delta k^2 = \gamma m^2 \), \( a' c = \delta k^2 = \gamma m^2 \)

\[ \Rightarrow 2 \]

\[ = \frac{-2ac cm + 2ac' + ac' + a' c' + 2a' c' - 2\gamma m^2}{(ac + 2ac') m^2 + (2a + 3ac' + a' c' + 2a' c') m' + ac' + ac' + a' c' + a' c'} \]

\[ = \frac{-2ac cm + 2(ac + 3ac' + a' c' + 2a' c') m' + ac + ac' + a' c' + a' c'}{(ac + 2ac') m^2 + (2a + 3ac' + a' c' + 2a' c') m' + ac' + ac' + a' c' + a' c'} \]

\[ = \frac{-2ac cm + 2(ac + 3ac' + a' c' + 2a' c') m' + ac + ac' + a' c' + a' c'}{(ac + 2ac') m^2 + (2a + 3ac' + a' c' + 2a' c') m' + ac' + ac' + a' c' + a' c'} \]

\[ \Rightarrow \gamma' = \gamma \Rightarrow a' c' = \frac{2}{m - m'} \frac{\gamma m^2}{m - m'} \Rightarrow a' c' = \frac{2k^2}{k^2} \frac{\gamma m^2}{m - (m + 1)^2} = \beta^2 \]

If \( \alpha = -\beta \Rightarrow (\beta - \alpha) p_1 = -2 \Rightarrow p_1 = \frac{p_2}{p_1} \Rightarrow (-2 - \alpha - \beta) p_2 = -2 p_2 \Rightarrow \alpha = -\beta = \frac{p_2}{p_1} \)

\[ b = \left(\frac{p_2 - p_1}{4}\right) \frac{p_2}{p_1} \]

\[ \Rightarrow 4b p_1 = p_2^2 \Rightarrow (4b p_1) p_2 = \frac{p_2^2}{p_1} \]

\[ \Rightarrow 4b p_1 = p_2^2 \Rightarrow (4b p_1) p_2 = \frac{p_2^2}{p_1} \]

Another proof: let u, u', v, v' verifying

\[ \left(\frac{1}{p_1 - v p_2} - \frac{1}{u' p_1 + v' p_2}\right) p_1 = \left(\frac{1}{u p_1 - v p_2} + \frac{1}{u' p_1 + v' p_2}\right) p_2 \]

\[ \Rightarrow (u' - u) p_1^2 + (v' + v) p_2 = (u + u') p_1 p_2 + (v' - v) p_2^2 \]
Thus

\[(u' + u - v)^2 p_1 p_2 + (u - u')^2 p_2^2 = (v - v')^2 p_2^2\]

We pose with a different of zero

\[u' + u - v = a(\alpha - \beta)\]

\[\Rightarrow ((v - v') - a(2 + \alpha + \beta)) p_2^2 = (u - u') p_1^2\]

Let

\[v - v' = a(2 + \alpha + \beta) = (2 + \alpha + \beta)^2\]

\[u - u' = (\alpha - \beta)^2\]

\[\Rightarrow u' p_1 + v' p_2 = (u - (\alpha - \beta)^2) p_1 + (v - a(2 + \alpha + \beta) - (2 + \alpha + \beta)^2) p_2\]

\[= u p_1 + v p_2 - (\alpha - \beta)^2 p_1 - a(\alpha - \beta) p_1 - (2 + \alpha + \beta)(\alpha - \beta) p_1\]

Let \(u p_1 = -v p_2\), hence

\[\left(\frac{1}{u p_1 - v p_2} - \frac{1}{u' p_1 + v' p_2}\right) \frac{p_1}{p_2} = \left(\frac{1}{u p_1 - v p_2} + \frac{1}{u' p_1 + v' p_2}\right) \frac{p_1}{p_2}\]

\[\Rightarrow \frac{1}{2u} + \frac{(\alpha - \beta)^2 + a(\alpha - \beta) + (2 + \alpha + \beta)(\alpha - \beta)}{p_1}\]

\[= \frac{1}{2u} - \frac{(\alpha - \beta)^2 + a(\alpha - \beta) + (2 + \alpha + \beta)(\alpha - \beta)}{p_2}\]

\[\Rightarrow \frac{p_1 - p_2}{2u} = -p_1 \frac{1}{(2 + \alpha + \beta)(2 \alpha + a + 2)} \frac{1}{(\alpha - \beta)(2 \alpha + a + 2)}\]

Let \(2u = \alpha - \beta\) hence

\[\frac{1}{\alpha - \beta} - \frac{1}{\alpha + \beta + 2} = \frac{1}{2 + \alpha + \beta}(2 \alpha + a + 2) \frac{1}{(\alpha - \beta)(2 \alpha + a + 2)}\]

\[\Rightarrow 2 \alpha + a + 3 = \frac{2 \alpha + a}{2 + \alpha + \beta}\]

It means \(b = 0\) thus

\[x + y = p_1, \quad x - y = p_2\]

the primal radius. As there is the condition \(p_2 < x < p_1\),

there is not an infinity of \(p_2, p_1\).

If \((x_1 - x_2)(x_1 + x_2) = 0\), \(x_1 = 0\), \(x_1 = -x_2\).

Let \(\frac{a + b}{p_1 + p_2} = \frac{1}{p_1 - p_2}\)

\[\frac{p_1 + p_2}{x_1 + x_2} = \frac{1}{x_1 - x_2}\]

\[k = \frac{2b}{p_1 + p_2} = \frac{2b}{x_1 + x_2}\]

\[\frac{x_1 - x_2}{p_1 - p_2} = \frac{1}{x_1 - x_2}\]

\[m = \frac{2b}{p_1 - p_2}, \quad m' = \frac{2b}{x_1 - x_2}\]

With the same reasoning and calculus \(\Rightarrow b = 0\) But \(b\) can not be equal to zero in all cases, it means there is an impossibility related to the fact that the conjecture is indecidable and if it is so, it is true! Because, we would find in the case it is indecidable and false with the computer the 2x different of all sum of primes and it is contradictory!
Now, if we suppose that for all $p_2$, $p_1$ primes, exists $x \mid 2x = p_1 - p_2$

$$x = \frac{p_1 - p_2}{2} + b, \quad y = \frac{p_1 + p_2}{2} + b,$$

with the same reasoning, the same calculus but replacing $x$ by $y$ and $y$ by $x$, we prove that $b = 0$, which means that for all positive integer $x$, exists $p_1, p_2$ for which $x = \frac{p_1 - p_2}{2}$, if we pose $y = \frac{p_1 + p_2}{2}$, $x + y = p_1$, $y - x = p_2$.

Let us prove it. Let us suppose that exists an integer $x \geq 0$ for which $2x$ is never the difference of two primes, then for all $p_1$ and $p_2$ primes, $p_1 - p_2 = 2x$, or $2x = p_1 - p_2 + 2b$, $p_1 = p_2 + 2b$, then $x = \frac{p_1 - p_2}{2} + b$.

But for all $p_1, p_2$ exists $y$, for which

Let

$$x_1 = p_1 + 2b, \quad x_2 = p_2 - 2b, \quad x_3 = p_2 + 2b, \quad x_4 = p_1 - 2b$$

We deduce that

$$y = \frac{p_1 + p_2}{2} + b = \frac{p_1 - x_2}{2} + 2b = \frac{x_1 + p_2}{2} = \frac{x_1 + x_2}{2} + b$$

$$= \frac{p_1 - p_2}{2} = \frac{x_1 + x_2}{2} - b = \frac{x_1 - x_2}{2} + b = \frac{x_1 + x_2}{2} + 3b$$

$$= \frac{p_1 - p_2}{2} + b = \frac{x_1 - x_2}{2} + b = \frac{x_1 + x_2}{2} + 3b = \frac{x_1 - x_2}{2} + b$$

$$x_1 + x_2 = p_1 + p_2$$

$$x_1 - x_2 = p_1 - p_2$$

Lemma 2

The following formula

$$y = \frac{p_1 + p_2}{2} + b = \frac{p_1 + x_2}{2} + 2b = \frac{x_1 + p_2}{2} = \frac{x_1 + x_2}{2} + b$$

$$= \frac{p_1 - p_2}{2} = \frac{x_1 + x_2}{2} - b = \frac{x_1 - x_2}{2} + b = \frac{x_1 + x_2}{2} + 3b$$

$$x = \frac{p_1 - x_3}{2} + 2b = \frac{x_1 - x_3}{2} + b = \frac{x_1 - x_2}{2}$$

$$= \frac{p_1 - x_3}{2} + 2b = \frac{x_1 - x_3}{2} + b = \frac{x_1 - x_2}{2}$$

Imply that exist $p_1$ and $p_2$ prime numbers for which $b = 0$

Proof of lemma 2

If $x$ is prime $0 = x - x$ is the sum of two primes, then $p_1 - p_2 \neq 0$.

We will suppose firstly that $(x_1 - x_2)(x_1 + x_2) = 0$.
Let
$$\frac{x_1 - x_2}{y_1 - y_2} = \frac{p_1 - p_2 + 4b}{p_1 - p_2} = 1 + \frac{4b}{p_1 - p_2}$$
$$\frac{p_1 - p_2}{y_1 - y_2} = \frac{x_1 - x_2 - 4b}{x_1 - x_2} = 1 - \frac{4b}{x_1 - x_2}$$

We pose
$$k = \frac{2b}{p_1 - p_2}, \quad k' = -\frac{2b}{x_1 - x_2}$$

kk' = 0 \Rightarrow b = 0, \text{ we have supposed } kk' \neq 0

\forall (x, y), \exists \varphi \mid y = \varphi x

x + y = (\varphi + 1)x = x_1 \neq 0, \quad y - x = (\varphi - 1)x = p_2 \neq 0

\forall (k, k'), \exists \alpha \mid k = \alpha k'

\Rightarrow \frac{2b}{p_1 - p_2} = -\frac{2b}{x_1 - x_2}

\Rightarrow x_1 - x_2 = -\alpha (p_1 - p_2) \Rightarrow x_1 - x_2 - p_1 + p_2 = 4b = -(\alpha + 1)(p_1 - p_2)

\Rightarrow b = -\frac{\alpha + 1}{4}(p_1 - p_2)

\Rightarrow y = \frac{p_1 + p_2}{2} + b = \frac{p_1 + p_2}{2} - \frac{\alpha + 1}{4}(p_1 - p_2) = \frac{(1 - \alpha)p_1 + (\alpha + 3)p_2}{4} - \frac{\varphi}{\varphi - 1}p_2

x = \frac{p_1 - p_2}{2} + b = \frac{p_1 - p_2}{2} - \frac{\alpha + 1}{4}(p_1 - p_2) = \frac{(1 - \alpha)p_1 - (\alpha + 3)p_2}{4} + \frac{1}{\varphi - 1}p_2

Let
$$\frac{x_1 + x_2}{y_1 + y_2} = \frac{p_1 + p_2 + 4b}{p_1 + p_2} = 1 + \frac{4b}{p_1 + p_2}$$
$$\frac{p_1 + p_2}{y_1 + y_2} = \frac{x_1 + x_2 - 4b}{x_1 + x_2} = 1 - \frac{4b}{x_1 + x_2}$$

We pose
$$mm' = 0 \Rightarrow b = 0, \text{ we have supposed } mm' \neq 0$$

\forall (m, m'), \exists \beta \mid m = \beta m'

\Rightarrow \frac{2b}{p_1 + p_2} = -\beta \frac{x_1 + x_2}{y_1 + y_2}

\Rightarrow x_1 + x_2 = -\beta (p_1 + p_2) \Rightarrow x_1 - x_2 - p_1 + p_2 = 4b = -(\beta + 1)(p_1 + p_2)

\Rightarrow b = -\frac{\beta + 1}{4}(p_1 + p_2)

\Rightarrow y = \frac{p_1 + p_2}{2} + b = \frac{p_1 + p_2}{2} - \frac{\beta + 1}{4}(p_1 + p_2) = \frac{(1 - \beta)(p_1 + p_2)}{4} - \frac{\varphi}{\varphi - 1}p_2

x = \frac{p_1 - p_2}{2} + b = \frac{p_1 - p_2}{2} - \frac{\beta + 1}{4}(p_1 + p_2) = \frac{(1 - \beta)p_1 - (\beta + 3)p_2}{4} + \frac{1}{\varphi - 1}p_2

b = -\frac{\alpha + 1}{4}(p_1 - p_2) = -\frac{\beta + 1}{4}(p_1 + p_2) \Rightarrow (\beta - \alpha)p_1 = (2 - \alpha - \beta)p_2
But : \((2k+1)(2k'+1)=1 \Rightarrow 2k+k+k'=0 \Rightarrow (2k+1)k'=-k\)

\[
\begin{align*}
2k+1 &= \frac{-k}{k'} = \frac{a(k+1)+a'(2k+1)}{a(k'+1)+a'} \\
2k'+1 &= \frac{-k'}{k} = \frac{c(k'+1)+c'(2k'+1)}{c(k+1)+c'} \\
2k+1 &= \frac{(a+2a')k + a + a'(ck + c + c')}{ck + a + a'} \\
2k'+1 &= \frac{(c + 2c')k' + c + c')(ak + a + a')}{(ac + 2ac')k + ac + ac' + a'c + a'c'} \\
\alpha(a, a', c, c') \text{ particularly, } (a, a', c, c') &\Rightarrow \alpha' = \delta k^2, \quad a'c = \delta k^2 \\
\Rightarrow \frac{(k-k')}{2k+1} &= \frac{2}{2k+1} \\
&= \frac{a(c(k-k'))(2k+1) + (2ac + ac' + a'c + 2a'c')(k-k') + 2\delta k^2 - 2\delta k^2}{(ac + 2ac')k + ac + ac' + a'c + a'c'} \\
&= \frac{2\delta k^2}{ac + 2ac' + 2a'c' - \delta k^2} \\
k = k' \Rightarrow \alpha = 1 \Rightarrow y = \frac{(1-\alpha)p_1 + (3-\alpha)p_2}{4} = p_3 \text{ it is impossible.} \\
k - k' \neq 0 \Rightarrow \frac{2}{2k+1} = \frac{-2\delta k^2 + 2a + ac + a'c + 2a'c}{ac + 2ac' + 2a'c' - \delta k^2} \\
\text{Also } (2m+1)(2m'+1)=1 \Rightarrow 2mm' + m + m' = 0 \Rightarrow (2m+1)m = m \\
2m+1 &= \frac{-m}{m+1} = \frac{-a(m+1) + a'(2m+1)}{a(m+1) + a'} \\
2m'+1 &= \frac{-m'}{m+1} = \frac{c(m+1) + c'(2m'+1)}{c(m+1) + c'} \\
2m+1 &= \frac{((a+2a')m + a + a'c + a'c')}{(ac + 2ac')m + ac + ac' + a'c + a'c'} \\
2m'+1 &= \frac{(c + 2c')(m+1) + c + c'(m+1)}{(ac + 2ac')m + ac + ac' + a'c + a'c'} \\
\alpha(a, a', c, c') \text{ particularly, } (a, a', c, c') &\Rightarrow \alpha' = \gamma m^2, \quad a'c = \gamma m^2 \\
\Rightarrow \frac{(m-m')}{2m+1} &= \frac{1}{2m+1} \\
&= \frac{a(m + m')(m-m') + 2(a + ac + a'c + 2a'c')(m-m') + 2\gamma m^2 - 2\gamma m^2}{(ac + 2ac')m + ac + ac' + a'c + a'c'} \\
&= \frac{-2a(m-m') + 2(a + ac + a'c + 2a'c')(m-m') + 2\gamma m^2 - 2\gamma m^2}{(ac + 2ac')m + ac + ac' + a'c + a'c'} \\
m - m' \Rightarrow \beta = 1 \Rightarrow y = \frac{1-\beta}{2m+1} = 0, \text{ it is impossible}
Another proof: let \( u, u', v, v' \) verifying

\[
\begin{align*}
\text{Thus} & \\
\text{We pose with a different of zero} & \\
\text{Let} & \\
\text{RETRACTED} & \\
\end{align*}
\]

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Let \( u_1 = -vp_2 \) hence
\[
\left( \frac{1}{u_1 - vp_2} - \frac{1}{u_1 p_1 + vp_2} \right) p_1 = \left( \frac{1}{u_1 p_1 - vp_2} + \frac{1}{u_1 p_1 + vp_2} \right) p_2
\]
\[
\Rightarrow \left( \frac{1}{2u} (\alpha - \beta)^2 + \alpha(\alpha - \beta) + (2 + \alpha + \beta)(\alpha - \beta) \right) p_1
\]
\[
= \left( \frac{1}{2u} (\alpha - \beta)^2 + \alpha(\alpha - \beta) + (2 + \alpha + \beta)(\alpha - \beta) \right) p_2
\]
\[
\Rightarrow \frac{p_1 - p_2}{2u} = -p_1 \left( \frac{1}{(2 + \alpha + \beta)(2\alpha + a + 2)} + \frac{1}{(\alpha - \beta)(2\alpha + a + 2)} \right)
\]
Let \( 2u = \alpha - \beta \) hence
\[
\frac{1 - 1}{\alpha - \beta} \Rightarrow \frac{2\alpha + a + 3}{\alpha - \beta} = \frac{2\alpha + a + 1}{2 + \alpha + \beta} \quad \forall \alpha \Rightarrow \alpha = \beta = -1
\]
It means \( b=0 \) thus
\[
x + y = p_1, \quad y - x = p_2
\] are primes \( y = r \) is the primal radius. As there is no condition, there is an infinity of \( p_1, p_2 \).

If \( (x_1 - x_2)(x_1 + x_2) = 0 \Rightarrow (x_1 + x_2)(x_1 - x_2) = 0 \)

Let
\[
\frac{x_1 + x_2}{p_1 + p_2} = 1 - \frac{4b}{p_1 + p_2}
\]
\[
\frac{x_1 + x_2}{x_1 + x_2} = 1 + \frac{4b}{x_1 + x_2}
\]
\[
\frac{x_1 - x_2}{p_1 - p_2} = 1 - \frac{4b}{p_1 - p_2}
\]
\[
\frac{x_1 - x_2}{x_1 - x_2} = 1 + \frac{4b}{x_1 - x_2}
\]
\[
\frac{p_1 - p_2}{p_1 - p_2} = 1 - \frac{4b}{p_1 - p_2}
\]
\[
\frac{p_1 - p_2}{x_1 - x_2} = x_1 - x_2
\]

With the same calculus and reasoning, it implies that \( b = 0 \). But \( b \) can not be equal to zero in all cases, it means there is an impossibility related to the fact that the conjecture is indecidable and if it is so, it is true! Because, we would find in the case it is indecidable and false with the computer the 2x difference of all sum of primes and it is contradictory!

For \( 2x = p_1 - p_2 \) is a difference of an infinity of couples of primes. There is an infinity of consecutive primes. And for all \( x \geq 2104 \) exists \( p_1 = p_2 + 2 \) primes for which \( 2x = p_1 + p_2 \)

**Conclusion**

The notion of the primal radius as defined in this study allows to confirm that for all integer \( x \geq 3 \) exists a number \( r > 0 \) for which \( x + r \) and \( x - r \) are primes and that for all integer \( x \geq 0 \) exists a number \( r > 0 \) for which \( x + r \) and \( r - x \) are primes and that exists an infinity of such primes. \( r \) is called the primal radius. The corollary is the proof of the Goldbach conjecture and de Polignac conjecture which stipulate, the first that an even number is always the sum of two prime numbers, the second that an even number is always the difference between two primes and that there is an infinity of such couples of primes. Another corollary is the proof of the twin primes conjecture which stipulates that there is an infinity of consecutive primes.
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