QUANTUM GRAVITY AND STRING THEORY—
WHAT HAVE WE LEARNED?

Andrew Strominger

Department of Physics, University of California
Santa Barbara, CA 93106
Bitnet: andy@voodoo

*To appear in the proceedings of the Sixth Marcel Grossman Conference on General Relativity, June 24–29 (1991), Kyoto, Japan.
The last time I had the opportunity to speak at a Marcel Grossman meeting was six years ago in Rome, just following the exciting realization [1] that string theory might actually describe our universe. Since that time there has been an enormous amount of activity in the subject. Much of the progress has been rather technical and difficult to understand by those outside of the field. In this talk I am going to give a very non-technical distillation of some of this work. This is not meant to be a review. Rather I will focus on several results of the past six years with qualitative implications for the problem of quantum gravity, and begin with some perspective on that problem.

The Problem of Quantum Gravity

Quantum mechanics and general relativity are perhaps the two greatest achievements of twentieth-century physics. However the two theories are at odds: the standard recipe (quantum field theory) for quantizing a classical field theory fails when applied to general relativity. New ultraviolet divergences appear at every order in perturbation theory, requiring an infinite number of counterterms for renormalization. This is unacceptable: even if one could stomach a theory with an infinite number of free parameters, it is doubtful that the resulting theory is stable and unitary at the Planck scale.

This conflict is extremely fortunate for physicists. A glance at the history of physics, depicted in Figure 1, reveals that all great leaps in our understanding of the universe originated in such contradictions. For example, the ultraviolet catastrophe which occurs in the thermodynamics of classical electromagnetism led to the development of quantum mechanics.

The last problem in this figure, quantum gravity, was noted in the fifties by Pauli. It took three decades to arrive at superstring theory, the first—and only—plausible mathematical resolution of this problem to date. The basic idea is that all particles, on closer inspection \((10^{-33} \text{ cm})\), are actually tiny closed loops of string. Further, all the forces and
FIGURE 1. Contradiction Leads to Progress

particles of nature (photons, quarks, etc.) are simply different vibrational modes of the same fundamental string.

Let me list the main successes of superstring theory:

1. Perturbatively reconciles quantum mechanics and general relativity (probably, some
intricacies of higher order perturbation theory remain to be sorted out).

2. First example of a truly unified theory in which all particles and forces are different manifestations of a single object.

3. Could in principle describe our universe.
The main failure of superstring theory can be put in perspective by an elaboration of Figure 1. From Figure 2 we see that experiment played a key role in the resolution of past contradictions. Unfortunately, because of the enormous energies involved, it is difficult to conceive of an experiment which will aid in the reconciliation of quantum mechanics and general relativity. While no stone should be left unturned, it seems likely that we will have to live with this state of affairs for the foreseeable future (although optimists remain[2]).

While this is certainly sad, it does not prevent us from investigating the conceptual implications of string theory (assuming it is correct). One expects the successful reconciliation of quantum mechanics and general relativity to profoundly affect our view of the universe. Perhaps even if string theory is not physically correct there are useful lessons to be learned from such investigations.

In the following three subsections I will describe three such tantalizing themes which have recurred in investigations of string theory. These are

I) Duality and the existence of a fundamental shortest length,

II) Infinite numbers of local symmetries,

III) Quantum hair on black holes.

I. Duality/Minimal Length

In colliding particles at energy $E$ one probes distances of order

$$\Delta X \sim \frac{\hbar}{E},$$

in units where $c = 1$. This is not quite true for colliding strings because strings are extended objects. When thrown together at high energies they have a tendency to “squash out” to a size of order $G_N E$, where $G_N$ is Newton’s constant. This has been carefully analyzed in [3]. The distance one can really probe is therefore roughly

$$\Delta X \sim \frac{\hbar}{E} + G_N E,$$
and very short distances cannot be probed by going to very high energies. The minimum distance one can probe in any scattering experiment is in fact

\[ \Delta X_{min} \sim \sqrt{\frac{G_N}{\hbar}} = L_p, \]

where \( L_p \) is the Planck length. To a physicist what can’t be measured (even in principle) does not exist, therefore we should suspect that lengths less than \( L_p \) simply do not exist in string theory. This might also explain how string theory cures the ultraviolet divergences of quantum gravity.

This view gains support from some fascinating work on a lattice formulation of string theory. In reference [4], a regulated version of bosonic string theory is defined in which the string is composed of discrete bits joining neighboring points of a spacetime lattice. A certain measure is defined for summing over all string configurations and the theory is studied as a function of the string tension, \( T \) (but the spacetime lattice spacing \( a \) is not varied). Astonishingly, it is found that for a range of values of \( T \) (measured in lattice units) the spectrum of this lattice theory agrees exactly with that of continuum bosonic string theory, even for finite \( a \). Thus an exact description of bosonic string theory can be obtained \textit{without} any short distance degrees of freedom!

These observations tie in neatly with another well-known phenomenon in string theory: duality. To understand duality in string theory, let us first consider ordinary Kaluza-Klein compactification of \( d = 10 \) gravity on a 6-torus of radius \( R \). At distances much greater than \( R \), the universe is effectively four-dimensional. It may be described by the effective action

\[ S_{\text{eff}} = \frac{1}{G_N} \int d^4x \sqrt{-g} \left( R - \sum_{i=1}^{\infty} \left( (\nabla \phi_i)^2 + (m_{ik}^{kk})^2 \phi_i^2 \right) + \cdots \right). \]

In addition to several massless fields, there is an infinite tower of massive Kaluza-Klein fields, \( \phi_i \), which are relics of the underlying ten-dimensional physics. The masses \( m_{ik}^{kk} \) of
these Kaluza-Klein fields are proportional to the eigenvalues of the spin-two operator govern-ning linearized fluctuations of the metric about the flat 6-torus of radius $R$. Dimensional analysis then implies that the masses vary with $R$ as

$$m_{kk}^i \sim \frac{1}{R}.$$ 

As $R \to \infty$, an infinite number of fields move down to zero mass in an attempt to recover the continuous spectrum of ten-dimensional flat space.

Since string theory resembles a field theory at large distances, these Kaluza-Klein modes also appear in a 6-torus compactification of ten-dimensional string theory. However in this case it is not the whole story. In addition there are “winding” modes, depicted in Figure 3, corresponding to a single string which wraps (perhaps many times) around the 6-torus. From the four-dimensional perspective, this appears as a particle with mass given by the length of the string ($\sim R$) times the string tension $T$, which is roughly $L_p^{-2}$:

$$m_{w}^i \sim \frac{R}{L_p^2}.$$
We see that these modes vary with $R$ in the opposite fashion of the Kaluza-Klein modes: they become light at very small $R$ when only a short string is required to wrap around the torus. In fact it was shown in [5] that the spectrum of masses is invariant under the duality transformation:

$$R \leftrightarrow L_p^2/R.$$ 

While this may seem rather peculiar, it is even more surprising that this duality extends to interactions. Indeed it was shown in [6] that

*No physical experiment can distinguish a compactification of radius $R$ from one of radius $L_p^2/R$.*

Again to a physicist that which cannot be measured does not exist, so we must conclude that a very small torus is the same thing as a very large torus.

At this point you might raise the following objection: “Surely if the torus is 10 meters across, I can simply go in with my ruler and measure its radius, thereby distinguishing it from a torus of radius $L_p^2/10 \, m = 10^{-71} \, m$!” The problem with this procedure is that there are two kinds of rulers: those constructed from Kaluza-Klein modes and those constructed from winding modes. There will be no invariant way to determine which kind of ruler you have used, and accordingly whether you have actually measured $R$ or $L_p^2/R$.

This duality between long and short distances is not confined to the 6-torus, and has been discovered in a wide variety of different situations. This leads one to believe that in some sense long and short distances should be identified in string theory. Clearly this will require a fundamental revision in our usual notion of a spacetime continuum. Finding the proper notion to replace the spacetime continuum and describing this duality is one of the exciting current problems in string theory.

**II. Infinite Symmetry**

String theory can be thought of as a field theory with an infinite number of particles,
one for each vibrational mode of the string. The massless modes (e.g. graviton, photon) are associated with local gauge symmetries. Since these are unified with the massive modes by the string, one expects local symmetries associated with the infinite tower of massive modes as well. Of course since these modes are massive, the infinite set of local symmetries must be spontaneously broken.

An infinite number of local symmetries might also explain, as argued in [7], the high energy behavior of scattering amplitudes. For large center-of-mass energy $\sqrt{s}$, the amplitude $A(s)$ is exponentially suppressed:

$$A(s) \to e^{-s/M_p^2}.$$ 

where $M_p = \sqrt{\hbar/G_N}$ is the Planck mass. This suppression may be due to high-energy symmetry restoration: the S-matrix near infinite $s$ is constrained by an infinite number of symmetries which force it exponentially to zero.

In ordinary field theories, spontaneously broken symmetries lead to spontaneously broken Ward identities among the Feynman diagrams. The Feynman diagrams of string theory (in the BRST formalism) do indeed exhibit an infinite number of such identities. This provides our most concrete understanding of the infinite string symmetries.

But this is not good enough. In ordinary gauge theories, the Ward identities can be derived beginning with a gauge-invariant action, and the nature of the symmetries is better understood as an invariance of this original action than as identities among Feynman diagrams. A similar understanding is desirable for the infinite string symmetries.

Fundamentally new ideas are probably required before such an understanding is obtained, but some partial progress has been made in the context of “string field theory”. While it is generally believed for a variety of reasons that this is ultimately the wrong direction, the partial results in this direction nevertheless do much to clarify the nature of the problem and possible solutions, as follows. The infinite number of fields in string
theory can be assembled into one very large “string field,” an infinite multiplet, which we shall denote $A$. The action can then (for some string theories) be elegantly written in the Chern-Simons form: \[ S = \int (AQA + \frac{2}{3} A^3) \]

where the $Q$ is a (nilpotent) generalization of the spacetime laplacian, $\int$ is a generalization of integration and the last term is the interactions. This action is invariant under the symmetry

$$\delta_\epsilon A = Q\epsilon + [\epsilon, A].$$

Since $\epsilon$, like $A$, is a string field this amounts to an infinite number of ordinary spacetime symmetries.

This construction partially realizes the goal of representing the string symmetries in a simple way. However, it is still not quite satisfactory because of the homogeneous term $Q\epsilon$ in the symmetry transformation law for $\delta_\epsilon A$. This means that the vacuum state $A = 0$ spontaneously breaks all symmetries for which $Q\epsilon \neq 0$, which includes the symmetries associated to the massive string modes. This is like expanding the standard model lagrangian about the broken symmetry minimum of the Higgs potential. The nature of the symmetry is much more evident when the lagrangian is expanded about the state of unbroken symmetry.

*Is there a state of unbroken symmetry in string theory?* This is a fascinating question. Some light has been shed on it by the discovery [9] that a redefinition of the string field $A$

$$\tilde{A} = A - A_0,$$

where $A_0$ is a certain classical solution of $S$, leads to the action

$$S[\tilde{A}] = \frac{2}{3} \int \tilde{A}^3$$
and gauge transformation law
\[ \delta_\epsilon \tilde{A} = [\epsilon, A]. \]

The vacuum \( \tilde{A} = 0 \) is then the state of unbroken string symmetry since it is left invariant by all symmetry transformations. It is further of interest to note that, as \( Q \) has disappeared in this reformulation, there is no reference to spacetime geometry in \( S[\tilde{A}] \). This suggests that the notion of spacetime appears only as a result of spontaneous breakdown of the infinite string symmetry!

While tantalizing, the above description is beset by possibly incurable difficulties which are too technical to describe here. Most notably, the state \( \tilde{A} = 0 \) is represented in a singular fashion. Witten’s “topological field theory” [10] attempts to find a better description of this state of unbroken symmetry, but has so far not fully succeeded. While it is generally believed that string theory contains an infinite number of local symmetries, their proper description is yet to be found.

III. Quantum Hair

Consider the action
\[ S = \frac{1}{G_N} \int d^4x \sqrt{-g} (R - H_{\mu\nu\lambda}H^{\mu\nu\lambda} + \cdots) \]
\[ H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} \]

The axion field strength \( H \) describes one pseudoscalar degree of freedom \( a \) defined by \( H_{\mu\nu\lambda} = \epsilon_{\mu\nu\lambda}^\rho \partial_\rho a \). This arises as part of the low-energy effective action for string theory, but most of the comments of this section are more general and pertain to any action with axions and gravity. The action \( S \) has the solution
\[ g_{\mu\nu} = g_{\mu\nu}^s \]
\[ H_{\mu\nu\lambda} = 0 \]
\[ B_{\mu\nu} = q\epsilon_{\mu\nu} \]

where \( g_{\mu\nu}^s \) is the standard Schwarzchild black hole metric and the two-form \( \epsilon \) is tangent to the two spheres of constant \( r, t \) and normalized so that

\[ \int_{S^2} B = q \]

for any two-sphere surrounding the black hole.

Classically, the “hair” \( q \) is unobservable (since it does not enter into the field strength) and therefore uninteresting. However, as argued in [11], it is quantum mechanically detectable. Strings (either fundamental or solitonic) couple to \( B_{\mu\nu} \) via the action

\[ S_B = \frac{T}{2} \int_{\Sigma} \epsilon^{ij} \partial_i X^\mu \partial_j X^\nu B_{\mu\nu} \]

(where \( T \) is the string tension) which is just the integral of \( B \) over the string worldsheet \( \Sigma \). It follows that for a string which encircles a black hole

\[ S_B = Tq \]

and \( S_B \) is otherwise zero. One then concludes that [11]

A string thrown around a hairy black hole gets a phase \( e^{iTq/\alpha' h} \). This phase is quantum mechanically detectable through interference experiments.

Classically, the no-hair theorems assert that the final states of a black hole are labelled by just a few parameters, such as charge, mass and angular momentum. Quantum mechanically we now find that additional quantum numbers are required. This basic notion of quantum hair on black holes has been further developed and elaborated in a variety of contexts, most notably that of discrete gauge theories[12].

An important feature of quantum hair is that it persists even if the axion gets a mass through spontaneous symmetry breaking†[13].

† As in the Cremmer-Scherk mechanism. It does not persist for explicit symmetry breaking in which case domain walls will form and confine strings.
The existence of quantum hair is relevant to issues surrounding Hawking radiation. According to Hawking, a black hole loses its mass by radiation of a thermal spectrum of particles. This may be described as a pair creation process in which one particle goes down the black hole and the other escapes to infinity. The black hole eventually loses most (or all) of its mass and settles down to a final state classically characterized by just a few parameters such as mass or charge. This final state can not carry any information about what went down the hole and so, Hawking argues, there is a net loss of information and quantum coherence.

This argument is affected by the possibility of quantum hair. We have just learned that quantum mechanically there is additional information contained in the final state of the black hole. However, since an arbitrarily large amount of information can fall into the black hole, a qualitative effect on the question of coherence loss could occur only in a theory with infinite varieties of quantum hair. Since quantum hair is associated with (spontaneously broken) local symmetries, infinite varieties of quantum hair might be expected in a theory with infinite varieties of local symmetries. But we have just learned that string theory may be precisely such a theory. This led Schwarz [14] to the following bold conjecture:

*The infinite string symmetry leads to infinite varieties of quantum hair on black holes. This hair encodes complete information about what went down the black hole, and no information is lost in the process of Hawking evaporation.*

Clearly string theory is providing interesting perspectives on the many conceptual issues arising in the reconciliation of quantum mechanics and general relativity.

**TOY STRINGS**

It used to be thought that strings only made sense in 10 or 26 spacetime dimensions. However, it was realized relatively recently[15] that in fact strings can be made mathematical sense of in any number of dimensions, although they have some unphysical properties
away from the critical dimensions of 10 or 26. Nevertheless, these so-called “non-critical” strings can provide interesting theoretical laboratories for investigating questions in quantum gravity.

In particular there has been much recent activity in the study of string theories in two or fewer spacetime dimensions. The amazing “matrix model” techniques lead to a closed form solution[16] for many of these theories to all orders in quantum perturbation theory—in some cases even non-perturbatively!

A related - and even more recent - development is the discovery[17] of an exact classical solution of non-critical string theory corresponding to a two-dimensional black hole. This opens the possibility of investigating the fascinating issues surrounding the problem of Hawking radiation in the context of a consistent quantum theory of gravity.

These are striking discoveries. I would not have suspected that it would prove possible to obtain such complete analytic understanding of these toy string models. However, I think that the full benefit of these recent technical breakthroughs is yet to be reaped. Can any light be shed on the physical questions discussed in the previous sections? Currently there is much activity in this subject, with encouraging progress. Perhaps I will be fortunate enough to report on this at the seventh Marcel Grossman meeting.

CONCLUSION

Barring enormous luck or inspiration, string theory is unlikely to be experimentally verified or disproved in the foreseeable future (although optimists remain [2]). Nevertheless it provides a rich and fascinating model to study the conceptual revolution in our view of the universe which will inevitably accompany the unification of quantum mechanics and general relativity. On this issue progress is being made in leaps and bounds.
ACKNOWLEDGEMENTS

I would like to thank the organizers for the invitation to speak. This work was supported in part by DOE grant DE-FG03-91ER40168.

REFERENCES

1. M. B. Green and J. H. Schwarz, Phys. Lett. 149B, (1984) 117.

2. J. H. Schwarz, Caltech preprint, CALT-68-1740, Ginsparg#9108022 (1991).

3. T. Yoneya in “Wandering in the Fields” (World Scientific: Singapore (1987)); D. J. Gross and P. F. Mende, Phys. Lett. 197B (1987) 129, D. Amati, M. Ciafaloni and G. Veneziano, Phys. Lett. 216B (1989) 41.

4. I. Klebanov and L. Susskind, Nucl. Phys. B309 (1988) 175.

5. M. B. Green, J. H. Schwarz and L. Brink, Nucl. Phys. 198B (1982) 474; K. Kikkawa and M. Yamasaki, Phys. Lett. 149B (1984) 357; N. Sakai and I. Senda Prog. Theor. Phys. Suppl. 75 (1986) 692.

6. V. P. Nair, A. Shapere, A. Strominger and F. Wilczek, Nucl. Phys. 287B, 402 (1987).

7. D. J. Gross, Phys. Rev. Lett. 60 (1988) 1229.

8. E. Witten, Nucl. Phys. 268B, 353 (1986).

9. H. Hata, K. Itoh, T. Kugo, H. Kunitomo and K. Ogawa, Phys. Lett. 175B, (1986) 138; G. Horowitz, J. Lykken, R. Rohm and A. Strominger, Phys. Rev. Lett. 57 (1986) 283; A. Strominger, Nucl Phys. 294B (1987) 93.

10. E. Witten, Comm. Math Phys. 117, (1988) 353.
11. M. Bowick, S. Giddings, J. Harvey, G. Horowitz and A. Strominger, Phys. Rev. Lett. 61 (1988) 2823.

12. L. M. Krauss and F. Wilczek, Phys. Rev. Lett. 62 (1989) 301; J. Preskill, Caltech Preprint, CALT-68-1671 (1990) and references therein.

13. F. Wilczek (private communication, unpublished); T. J. Allen, M. J. Bowick and A. Lahiri, Phys. Lett. 237B (1990) 47.

14. J. H. Schwarz, Caltech preprint, CALT-68-1728 (1991).

15. R. Myers, Phys. Lett. B199 (1988) 371; J. Polchinski, Nucl. Phys. B324 (1989) 123.

16. E. Brezin and V. Kazakov, Phys. Lett. B236 (1990) 14; M. Douglas and S. Shenker, Nucl. Phys. B335 (1990) 635; D. Gross and A. Migdal, Phys. Rev. Lett. 64 (1990) 127.

17. E. Witten, Phys. Rev. D44 (1991) 314; I Bars and D. Nemeschansky, Nucl. Phys. B348 (1991) 89, S. Elitzur, A. Forge and E. Rabinovici, Nucl. Phys. B359 (1991) 581; G. Mandal, A. M. Sengupta and S. R. Wadia Lett. Mod. Phys. A6 (1991) 1685.