Quantum dynamics and state-dependent affine
gauge fields on CP(N-1)

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Abstract

Gauge fields frequently used as an independent construction additional
to so-called wave fields of matter. This artificial separation is of course
useful in some applications (like Berry’s interactions between the “heavy”
and “light” sub-systems) but it is restrictive on the fundamental level of
“elementary” particles and entangled states. It is shown that the linear
superposition of action states and non-linear dynamics of the local dynamical
variables form an oscillons of energy representing non-local particles
- “lumps” arising together with their “affine gauge potential” agrees with
Fubini-Study metric.

I use the conservation laws of local dynamical variables (LDV’s) during
affine parallel transport in complex projective Hilbert space $CP(N-1)$
for twofold aim. Firstly, I formulate the variation problem for the “affine
gauge potential” as system of partial differential equations [1]. Their so-
lutions provide embedding quantum dynamics into dynamical space-time
whose state-dependent coordinates related to the qubit spinor subjected to
Lorentz transformations of “quantum boosts” and “quantum rotations”.
Thereby, the problem of quantum measurement being reformulated as the
comparison of LDV’s during their affine parallel transport in $CP(N-1)$,
is inherently connected with space-time emergences. Secondly, the im-
portant application of these fields is the completeness of quantum theory.
The EPR and Schrödinger’s Cat paradoxes are discussed from the point of
view of the restored Lorentz invariance due to the affine parallel transport
of local Hamiltonian of the soliton-like field.

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1 Introduction

Space-time geometry was established during four revolutionary steps: Euclidean axiomatization of 3-geometry, summarizing previous mechanical hard body measurements, Galileo-Newton’s dynamics postulating absolute space and time structure, Einstein’s discovery of pseudo-Euclidean space-time 4-geometry which physically based on measurements by means of classical electromagnetic field, and Einstein’s discovery of pseudo-Riemannian space-time 4-geometry relies upon specification of same kind of measurements in gravitation field. One sees that the modeling the space-time geometry is boosted up by the development of measuring process in more general physical conditions.

Classical physics of system of material points frequently uses multi-dimensional generalization of space-time (configuration space), but such generalization deems to be merely artificial construction whereas space-time is treated fundamentally. If one, however, takes into account that the notion of material point has very limited sense in quantum theory then one will see just state space of quantum system has fundamental sense. It seems that the consistent development of quantum theory requires a new geometry of quantum state space and its new connection with space-time.

Probably the biggest mistake of our scientific paradigm is the assumption that we live in Universe coinciding with single 4D space-time curved locally by a matter distribution. This illusion is strongly supported by our every day experience and astronomy. Freely propagating quantum particles and macroscopic bodies clearly show that our “empty” space is absolutely transparent and the pseudo-Euclidean or pseudo-Riemannian space-time are good enough models for both macro- and microscopic physical theories. Nevertheless, there are some serious logical reasons to revise last assumption.

One of them is highly desirable agreement between quantum behavior and classical relativity (both special and general) that looks like the far distant future. All known attempts to reach the harmony (strings, super-gravity, e.g.) lead to unobservable predictions (say, super-partners) and requirements entirely to change the basis of our scientific paradigm: instead of the observer independence one uses so-called the anthropic principle, instead of Universe was invoked so-called Multiverse concept. I propose much more modest approach.

I assume that Universe is represented by infinite dimension action state space and the space-time being macroscopically observable as global pseudo-Riemannian manifold emerges due to projection during quantum measurement. It means that quantum measurement may be used as operational procedure for “marking” non-local quantum objects. I would like to recall that coordinatization of classical events by means of classical electromagnetic field is based on the distinguishability (separability), i.e. individualization of pointwise material points. However we loss the possibility to distinguish (non-local) quantum objects by means of quantum fields in space-time and, hence it is impossible directly to identify them in space-time. It seems to be reasonable to assume that space-time has “granular structure that respects Lorentz symmetry” only locally.
2 Space-time emergences due to the qubit-encoding of quantum measurement

Generally, it is important to understand that the problem of identification of physical objects is the root problem even in classical physics and that its recognition gave to Einstein the key to formalization of the relativistic kinematics and dynamics. Indeed, only assuming the possibility to detect locally an approximate coincidence of two pointwise events of a different nature it is possible to build full kinematic scheme and the physical geometry of space-time \[1\] [5]. As such the “state” of the local clock gives us local coordinates - the “state” of the incoming train. In the classical case the notions of the “clock” and the “train” are intuitively clear and approximately may be identified with material points or even with space-time points. This supports the illusion that material bodies present in space-time (Einstein emphasized that it is not so!). Furthermore, Einstein especially notes that he did not discuss the inaccuracy of the simultaneity of two \textit{approximately coinciding events} that should be overcame by some abstraction \[4\]. This abstraction is of course the neglect of finite sizes (and all internal degrees of freedom) of the both real clock and train. It gives the representation of these “states” by mathematical points in space-time. Thereby the local identification of two events is the formal source of the classical relativistic theory. However quantum object requires especial embedding in space-time and its identification with space-time point is impossible since the localization of quantum particles and generally even macroscopic space-time separation are state-dependent \[5, 6, 7, 8\]. Hence the identification of quantum objects requires a physically motivated operational procedure with corresponding mathematical description.

Therefore the reason of deep conflict between general relativity (GR) and quantum theory (QT) lies in space-time non-locality of quantum systems (say, so-called entangled EPR pears), and as a consequence, the problem of identification and comparison (measurement) of holistic quantum states by the intrinsic means of quantum theory. Besides this there are the set of pure “quantum paradoxes” (EPR, Zeno, “interaction-free measurement”, etc., that require adequate (in fact - deterministic) description of non-local quantum objects. On the technical level \textit{determinism} is equal to \textit{locality} but in quantum area only functional, i.e. state space localization of quantum states is possible \[9\]. The fundamental question is as follows: how to \textit{translate} the state space localization into ordinary space-time deterministic predictions (science predictions have a price only in this case). I will show that technically this question may be formulated as follows: \textit{what is the most natural manner of embedding quantum dynamics (in state space) into dynamics of the two-level detector}. My answer is based on the two postulates:

1. “Super-relativity” : coset deformation of quantum state may be created and compensated by some physical fields

2. “Dynamical space-time structure emergence” : quantum measurement of the local dynamical variables may be encoded by the local Lorentz
transformations of the qubit spinor representing the state of two-level detector into local space-time. Thereby local space-time structure accompanying quantum dynamics may be established.

I formulate the quantum measurement problem as a comparison of quantum LDV’s (instead of the comparison of quantum states) with help of their affine parallel transport in \( CP(N-1) \). Therefore one should deal with geometry of the quantum state space, since we face with the measurement problem by means of quantum fields. I would like to emphasize: by means of quantum fields rather in sense of de Broglie - Schrödinger than by the means of secondary quantized fields in the sense of Dirac. We should remember that Schrödinger wrote his equation initially just not in space-time but in the configuration space of a system of material points [10]. However, material point is good and very prolific abstract notion in classical theory but this notion is terribly misleading in quantum area.

3 Oscillons of the action states

Each action state of quantum system is a quantum motion in some “dynamical order” defined, say, be some Lagrangian or action functional. The state vector of this quantum motion thereby may be treated as “order parameter” belonging to Hilbert space - “the space of the order parameter”. This motion appears as excitations of global vacuum defined by the self-consistent cosmic potential \( \Phi_U = c^2 \) of Universe that may locally oscillate and create quantum particles, solitons, unparticles, etc., under some conditions that should be especially established. Thus cosmic potential \( \Phi_U = c^2 \) forms some global vacuum \( |\Phi_U> = |\hbar 0> \) whose perturbation by the action operator

\[
\hat{S} = \hbar A(\hat{\eta}^+ \hat{\eta}),
\]

creates matter in some superposition (generalized coherent) state

\[
|F> = \sum_{a=0}^{\infty} f^a |h a>,
\]

where \( |h a> = (a!)^{-1/2}|\hbar \eta>^a |h 0> \) constituting \( SU(\infty) \) multiplete of the Planck’s action quanta operator \( \hat{S}_P = \hbar \hat{\eta}^+ \hat{\eta} \) with the spectrum \( S_a = \hbar a \) in the separable Hilbert space \( \mathcal{H} \). Formally these oscillations may be represented by the superposition in infinite dimension manifold of the Planck’s oscillators of action. Then well known relation of Einstein-de Broglie being rewritten as follows \( \frac{m}{\omega} = \frac{\hbar}{2} \) shows that only fixed relations of mass to frequency are acceptable on the fundamental level. Then the both mass \( m \) and frequency \( \omega \) should arise as intrinsically quantum values but not as in fact free classical parameters. One may think about this model as some abstract realization of the de Broglie ensemble of “weights suspended on springs” serves as “a crude analogue to a parcel of energy” [11]. In the de Broglie model the non-homogeneous distribution of the weights on the disk was given “by hands”, but I seek to find equations naturally describing similar distribution.
I assume that action states $|F>$ correspond to extremals of some least action problem and describe stationary quantum motion. These states do not gravitate since they don’t posses mass/energy. Therefore their linear superposition is robust and the rays of these generalized coherent states (GCS) will be main building blocks of the model. Only velocities of variation of these states given by local dynamical variables correspond materialized particles, etc. For simplicity I will discuss here $N$-dimension version of the model.

4 Geometry of the quantum evolution and/or measurement

Let me assume that “ground state” $|G> = \sum_{a=0}^{N-1} g^a |ha>$ is a solution of some the least action problem. Since any action state $|G>$ has isotropy group $H = U(1) \times U(N)$ only the coset transformations $G/H = SU(N)/S[U(1) \times U(N − 1)] = CP(N − 1)$ effectively act in $H$. Therefore the ray representation of $SU(N)$ in $C^N$, in particular, the embedding of $H$ and $G/H$ in $G$, is a state-dependent parametrization. Hence, there is a diffeomorphism between the space of the rays marked by the local coordinates in the map $U_j : \{|G>,|g^j| \neq 0\}, j > 0$

$$\pi^j = \begin{cases} \frac{g^j}{g^j} & \text{if } 1 \leq i < j \\ \frac{g^{j+1}}{g^j} & \text{if } j \leq i < N − 1 \end{cases}$$

and the group manifold of the coset transformations $G/H = SU(N)/S[U(1) \times U(N − 1)] = CP(N − 1)$. This diffeomorphism is provided by the coefficient functions $\Phi^a_i$ of the local generators (see below).

The “ground state” $|G> = \sum_{a=0}^{N-1} g^a |ha>$ may be expressed in local coordinates as follows: for $a = 0$ one has

$$g^0(\pi^1_{(p)},\ldots,\pi^{N-1}_{(p)}) = (1 + \sum_{s=1}^{N-1} |\pi^s_{(p)}|^2)^{-1/2}$$

and for $a : 1 \leq a = i \leq N − 1$ one has

$$g^i(\pi^1_{(p)},\ldots,\pi^{N-1}_{(p)}) = \pi^i_{(p)}(1 + \sum_{s=1}^{N-1} |\pi^s_{(p)}|^2)^{-1/2}.$$ 

Then the velocity of the ground state evolution relative “world time” $\tau$ is given by the formula

$$|\Psi> \equiv |T> = \frac{d|G>}{d\tau} = \frac{\partial g^a}{\partial \pi^i} \frac{d\pi^i}{d\tau} |ha> + \frac{\partial g^a}{\partial \pi^s} \frac{d\pi^s}{d\tau} |ha>$$

$$= |T_i \frac{d\pi^i}{d\tau} + |T_{si} \frac{d\pi^{si}}{d\tau} = H^i |T_i > + H^{si} |T_{si} >,$$

is the tangent vector to the evolution curve $\pi^i = \pi^i(\tau)$, where

$$|T_i > = \frac{\partial g^a}{\partial \pi^i} |ha > = T^a_i |ha >, \quad |T_{si} > = \frac{\partial g^a}{\partial \pi^{si}} |ha > = T^a_{si} |ha >.$$
Thereby state vector $|\Psi\rangle \equiv |T\rangle$ giving velocity of evolution, is represented by the tangent vector to the projective Hilbert space $CP(N-1)$ where instead of an arbitrary parameters $X = (x_1, ..., x_p)$ of the Hamiltonian $I$ use intrinsic local projective coordinates $\pi^k_{(j)} = \frac{d}{d\theta^j}$ of the quantum states. Note, it is close (but not identical) to the Berry’s intuitive analogy between the tangent vector $\vec{e}$ to the some sphere $S^2$ and the state vector $|\phi(X)\rangle = |\phi(x_1, ..., x_p)\rangle$ whose time dependence is generated by the periodic Hamiltonian $\hat{H}(X(t)) : X(T) = X(0)$. The parallel transport of $|\Psi\rangle$ is required to be in agreement with the Fubini-Study metric

$$G_{ik^*} = [(1 + \sum |\pi|^2)\delta_{ik} - \pi^{*i}\pi^{k}](1 + \sum |\pi|^2)^{-2}. \quad (8)$$

Then the affine connection

$$\Gamma^i_{mn} = \frac{1}{2} G^{ip^*} \left( \frac{\partial G_{mp^*}}{\partial \pi^n} + \frac{\partial G_{p^*n}}{\partial \pi^m} \right) = -\frac{\delta^{i^*}_{m}\pi^{n^*} + \delta_{m}^{i^*}\pi^{m^*}}{1 + \sum |\pi|^2} \quad (9)$$

takes the place of the gauge potential of the non-Abelian type playing the role of the covariant instant renormalization of the dynamical variables during general transformations of the quantum self-reference frame [12]. Berry’s parallel transport is not affine and this acts in the fibre bundle of local reference frame rather in the tangent bundle.

Velocity of the $|\Psi\rangle$ variation is given by the equation

$$|A\rangle = \frac{d|\Psi\rangle}{d\tau} = (B_{ik} H^i d\pi^k_{\tau} + B_{ik^*} H^{i^*} d\pi^{k^*}_{\tau} + B_{i^*k} H^i d\pi^k_{\tau} + B_{i^*k^*} H^{i^*} d\pi^{k^*}_{\tau})|N\rangle$$

$$+ \left( \frac{dH^s}{d\tau} + \Gamma^s_{ik} H^i d\pi^k_{\tau} \right)|T_s\rangle + \left( \frac{dH^{*s}}{d\tau} + \Gamma^{*s}_{i^*k^*} H^{i^*} d\pi^{k^*}_{\tau} \right)|T^{*s}\rangle, \quad (10)$$

where I introduce the matrix $\tilde{B}$ of the second quadratic form whose components are defined by following equations

$$B_{ik}|N\rangle = \frac{\partial |T_i\rangle}{\partial \pi^k} - \Gamma_{ik}^s |T_s\rangle, \quad B_{ik^*}|N\rangle = \frac{\partial |T_{i^*}\rangle}{\partial \pi^{k^*}}$$

$$B_{i^*k}|N\rangle = \frac{\partial |T_{i^*}\rangle}{\partial \pi^k} - \Gamma_{i^*k}^{*s} |T^{*s}\rangle, \quad B_{i^*k^*}|N\rangle = \frac{\partial |T^{*s}\rangle}{\partial \pi^{k^*}} - \Gamma_{i^*k}^{*s} |T^{*s}\rangle \quad (11)$$

through the state $|N\rangle$ normal to the “hypersurface” of the ground states. Assuming that the “acceleration” $|A\rangle$ is gotten by the action of some linear Hamiltonian $\hat{H}$ describing the evolution (say, during a measurement), one has the “Schrödinger equation of evolution”

$$\frac{d|\Psi\rangle}{d\tau} = -i\hat{H}|\Psi\rangle = (B_{ik} H^i d\pi^k_{\tau} + B_{ik^*} H^{i^*} d\pi^{k^*}_{\tau} + B_{i^*k} H^i d\pi^k_{\tau} + B_{i^*k^*} H^{i^*} d\pi^{k^*}_{\tau})|N\rangle$$
be achieved by the annihilation of the tangential component

The minimization of the \( |\Psi| \) of vector field. The last equations in (13) shows that the affine parallel transport i.e. under the condition of the affine parallel transport of the Hamiltonian \( \hat{H} \) of second-rank tensor \( CP \) to another one which is physically distinguishable. Thereby the unitary evolution of the action amplitudes generated by (12) leads in general to the non-

GCS to another one which is physically distinguishable. The reby the unitary evolution of the tangent vector to \( CP \) in invariant character.

There are two important differences between original Berry’s formula referring to arbitrary parameters and this 2-form in local coordinates inherently related to eigen-problem.

1. The \( V_{ik*}(\pi^i) \) is the singular-free expression.

2. It does not contain two eigen-values, say, \( E_n, E_m \) explicitly, but implicitly \( V_{ik*} \) depends locally on the choice of single \( \lambda_p \) through the dependence in local coordinates \( \pi_{i(p)}^* \). In some sense it looks like “degeneration” but now the reason of the anholonomy larks in the curvature of \( CP(N-1) \) and therefore it has invariant character.

\[
\begin{align*}
+ \left( \frac{dH^s}{d\tau} + \Gamma_{ik}^*H^i d\pi^k \right) |T_s > + \left( \frac{dH^{ss}}{d\tau} + \Gamma_{i^*k^*}^{ss} H^{i^*} d\pi^{k^*} \right) |T_{ss} > .
\end{align*}
\]

\[
(12)
\]

I should emphasize that “world time” is the time of evolution from the one \( N \) to another one which is physically distinguishable. Thereby the unitary evolution of the action amplitudes generated by (12) leads in general to the non-

\[
\begin{align*}
< N | \hat{H} | \Psi > = i (B_{ik} H^i \frac{d\pi^k}{d\tau} + B_{ik^*} H^i d\pi^{k^*} + B_{i^*k} H^i \frac{d\pi^k}{d\tau} + B_{i^*k^*} H^i d\pi^{k^*} ) ,
\end{align*}
\]

\[
< \Psi | \hat{H} | \Psi > = i G_{p^*s} ( \frac{dH^s}{d\tau} + \Gamma_{ik} H^i d\pi^k ) H^{p^*} + i G_{ps} ( \frac{dH^{ss}}{d\tau} + \Gamma_{i^*k^*} H^{i^*} \frac{d\pi^{k^*}}{d\tau} ) H^p
\]

\[
= i < \Psi | \frac{d}{d\tau} | \Psi > .
\]

(13)

The minimization of the \( |A > \) under the transition from point \( \tau \) to \( \tau + d\tau \) may be achieved by the annihilation of the tangential component

\[
\frac{dH^s}{d\tau} + \Gamma_{ik}^* d\pi^k = 0 , \quad \frac{dH^{ss}}{d\tau} + \Gamma_{i^*k^*}^{ss} d\pi^{k^*} = 0
\]

(14)

i.e. under the condition of the affine parallel transport of the Hamiltonian vector field. The last equations in (13) shows that the affine parallel transport of \( H^i \) agrees with Fubini-Study metric (8) leads to Berry’s “parallel transport” of \( |\Psi \rangle \). Geometrically this picture corresponds to the special choice of the moving reference frame \( \{ N > , |T_1 > , , . . . , |T_{N-1} > , |T_1^* > , , . . . , |T_{N-1}^* > \} \) on \( CP(N-1) \) that only “longitudinal” component along \( N > \) is alive.

The Berry’s formula (1.24) [13] being applied to the action state vector in the local coordinates \( \pi^i \), i.e [4], [5] gives following equation for antisymmetric second-rank tensor

\[
V_{ik*}(\pi^i) = \Im \sum_{a=0}^{N-1} \frac{1}{2} \left\{ \frac{\partial g^{aa*}}{\partial \pi^i} \frac{\partial g^{a}}{\partial \pi^k} - \frac{\partial g^{aa*}}{\partial \pi^k} \frac{\partial g^{a}}{\partial \pi^i} \right\} = \Im \sum_{a=0}^{N-1} \left\{ T_{i}^a T_{k}^{*a} - (T_{k}^a)^{*} T_{i}^{a} \right\}
\]

\[
= -\Im [(1 + \sum |\pi^i|^2) \delta_{ik} - \pi^i \pi^k](1 + \sum |\pi^i|^2)^{-2} = -\Im G_{ik*} .
\]

(15)

It is simply the imaginary part of the Fubini-Study quantum metric tensor. There are two important differences between original Berry’s formula referring to arbitrary parameters and this 2-form in local coordinates inherently related to eigen-problem.

1. The \( V_{ik*}(\pi^i) \) is the singular-free expression.

2. It does not contain two eigen-values, say, \( E_n, E_m \) explicitly, but implicitly \( V_{ik*} \) depends locally on the choice of single \( \lambda_p \) through the dependence in local coordinates \( \pi_{i(p)}^* \). In some sense it looks like “degeneration” but now the reason of the anholonomy larks in the curvature of \( CP(N-1) \) and therefore it has invariant character.
I formulate the problem to find field equations for the $SU(N)$ parameters $\Omega^\alpha$ leading to the affine parallel transport of the Hamiltonian field $H^i = \hbar \Omega^\alpha \Phi^i_\alpha$, where

$$\Phi^i_\sigma = \lim_{\epsilon \to 0} \epsilon^{-1} \left\{ \frac{\exp(i\epsilon \lambda_\sigma)|g^m|}{\exp(i\epsilon \lambda_\sigma)|g^m|} - g^i \right\} = \lim_{\epsilon \to 0} \epsilon^{-1} \{ \pi^i_1(\epsilon \lambda_\sigma) \}$$  \hspace{1cm} (16)

[12]. These field equations of motion for quantum system whose ‘particles’ do not exist a priory but they are becoming during the evolution. But first of all we should to introduce the notion of the “dynamical space-time” which arises due to the natural evolution or the objective measurement of some dynamical variable.

The $CP(N - 1)$ points serves as discriminators of physically distinguishable quantum GCS. Let me assume that GCS described by the local coordinates $(\pi^1, ..., \pi^{N-1})$ and the coordinates $(\pi^1 + \delta \pi^1, ..., \pi^{N-1} + \delta \pi^{N-1})$ correspond to the GCS displaced due to measurement or evolution.

Local coordinates of the GCS gives the a firm geometric tool for the description of quantum dynamics during interaction which used for a measuring process or evolution. The question that I would like to raise is as follows: what “classical field”, i.e. field in space-time, corresponds to the transition from the original to the displaced GCS? In other words I would like to find the measurable physical manifestation of the GCS in the form of “field shell”, its space-time shape and its dynamics. The GCS dynamics will be represented by (energy) frequencies distribution that are not a priori given, but are defined by some field equations which should established by means of variation problem applied to operators represented by tangent vectors to $CP(N - 1)$.

In order to build the qubit spinor $\eta$ of the quantum question $\hat{Q}$ [20] in the local basis $\{|N>, |T_1>, ..., |T_{N-1}>, |T_1\ast>, ..., |T_{N-1}\ast>\}$ for the measurement of the Hamiltonian $H$ at corresponding GCS I will use following equations

$$\eta = \left( \begin{array}{c} \alpha(\pi^1, ..., \pi^{N-1}) \vspace{0.1cm} \\ \beta(\pi^1, ..., \pi^{N-1}) \end{array} \right) = \left( \begin{array}{c} <N|H|\psi> \vspace{0.1cm} \\ <N|N> \vspace{0.1cm} \\ <\psi|H|\psi> \vspace{0.1cm} \\ <\psi|N> \end{array} \right)$$  \hspace{1cm} (17)

Then from the infinitesimally close GCS $(\pi^1 + \delta^1, ..., \pi^{N-1} + \delta^{N-1})$, whose shift is induced by the interaction used for a measurement, one get a close spinor $\eta + \delta \eta$ with the components

$$\eta + \delta \eta = \left( \begin{array}{c} \alpha(\pi^1 + \delta^1, ..., \pi^{N-1} + \delta^{N-1}) \vspace{0.1cm} \\ \beta(\pi^1 + \delta^1, ..., \pi^{N-1} + \delta^{N-1}) \end{array} \right) = \left( \begin{array}{c} <N|H'|\psi> \vspace{0.1cm} \\ <N|N> \vspace{0.1cm} \\ <\psi|H'|\psi> \vspace{0.1cm} \\ <\psi|N> \end{array} \right)$$  \hspace{1cm} (18)

Here $\hat{H} = \hbar \Omega^\alpha \lambda_\alpha$ is the lift of Hamiltonian vector field $H^i = \hbar \Omega^\alpha \Phi^i_\alpha$ from $(\pi^1, ..., \pi^{N-1})$ and $\hat{H}' = \hbar (\Omega^\alpha + \delta \Omega^\alpha) \lambda_\alpha$ is the lift of the parallel transported Hamiltonian vector field $H^i = \hbar \Omega^\alpha \Phi^i_\alpha$ (see below) from the infinitesimally close point $(\pi^1 + \delta^1, ..., \pi^{N-1} + \delta^{N-1})$ back to the $(\pi^1, ..., \pi^{N-1})$ into the adjoint representation space.
Now one should find how the affine parallel transport connected with the variation of coefficients $\Omega^\alpha$ in the dynamical space-time associated with quantum question $\hat{Q}$.

The covariance relative transition from one GCS to another

$$\left(\pi^j_1, \ldots, \pi^j_{N-1}\right) \rightarrow \left(\pi'^j_1, \ldots, \pi'^j_{N-1}\right)$$

and the covariant differentiation (relative Fubini-Study metric) of vector fields provides the objective character of the “quantum question” $\hat{Q}$ and, hence, the quantum measurement. This serves as a base for the construction of the dynamical space-time as it will be shown below.

These two infinitesimally close spinors $\eta$ and $\eta + \delta\eta$ may be connected with infinitesimal “Lorentz spin transformations matrix” [15]

$$L = \begin{pmatrix}
1 - \frac{i}{2}\tau(\omega_3 + ia_3) & -\frac{i}{2}\tau(\omega_1 + ia_1 - i(\omega_2 + ia_2)) \\
-\frac{i}{2}\tau(\omega_1 + ia_1 + i(\omega_2 + ia_2)) & 1 - \frac{i}{2}\tau(-\omega_3 - ia_3)
\end{pmatrix}.$$ (20)

Then accelerations $a_1, a_2, a_3$ and angle velocities $\omega_1, \omega_2, \omega_3$ may be found in the linear approximation from the equation

$$\eta + \delta\eta = L\eta$$

as functions of the qubit spinor components of the quantum question depending on local coordinates $(\pi^1, \ldots, \pi^{N-1})$ involved in the $\delta\Omega^\alpha$ throughout field equations (25).

Hence the infinitesimal Lorentz transformations define small “space-time” coordinates variations. It is convenient to take Lorentz transformations in the following form $ct' = ct + (\vec{x}\vec{a})d\tau$, $\vec{x}' = \vec{x} + c\vec{a}d\tau + (\vec{\omega} \times \vec{x})d\tau$, where I put $\vec{a} = (a_1/c, a_2/c, a_3/c)$, $\vec{\omega} = (\omega_1, \omega_2, \omega_3)$ [15] in order to have for $\tau$ the physical dimension of time. The expression for the “4-velocity” $V^\mu$ is as follows

$$V^\mu = \frac{\delta x^\mu}{\delta\tau} = (\vec{x}\vec{a}, ct\vec{a} + \vec{\omega} \times \vec{x}).$$ (22)

The coordinates $x^\mu$ of points in dynamical space-time serve in fact merely for the parametrization of deformations of the “field shell” arising under its motion according to non-linear field equations [11] [14].

5 Field equations in the dynamical space-time

The energetic packet - “lump” associated with the “field shell” is now described locally by the Hamiltonian vector field $\vec{H} = \hbar\Omega^\alpha\Phi^\dagger_\alpha \frac{\partial}{\partial \pi^\alpha} + c.c.$ Our aim is to find the wave equations for $\Omega^\alpha$ in the dynamical space-time intrinsically connected with the objective quantum measurement (evolution).

At each point $(\pi^1, \ldots, \pi^{N-1})$ of the $CP(N-1)$ one has an “expectation value” of the $\vec{H}$ defined by a measuring device. But a displaced GCS may by reached along one of the continuum paths. Therefore the comparison of two vector
fields and their “expectation values” at neighboring points requires some natural rule. The comparison makes sense only for the same lump represented by its “field shell” along some path. For this reason one should have an identification procedure. The affine parallel transport in $CP(N-1)$ of vector fields is a natural and the simplest rule for the comparison of corresponding “field shells”.

The dynamical space-time coordinates $x^\mu$ may be introduced as the state-dependent quantities, transforming in accordance with the functionally local Lorentz transformations $\delta x^\mu = V^\nu \delta \tau$ depend on the transformations of local reference frame in $CP(N-1)$ as it was described in the previous paragraph.

Let me discuss now the self-consistent problem

$$V^\mu \frac{\partial \Omega^\alpha}{\partial x^\mu} = -(\Gamma^m_{mn} \Phi^\alpha_\beta + \frac{\partial \Phi^\alpha_\beta}{\partial \pi^n}) \Omega^\alpha \Omega^\beta, \quad \frac{d\pi^k}{d\tau} = \Phi^k_\beta \Omega^\beta$$

arising under the condition of the affine parallel transport

$$\frac{\delta H^k}{\delta \tau} = \hbar \frac{\delta (\Phi^k_\alpha \Omega^\alpha)}{\delta \tau} = 0$$

of the Hamiltonian vector field.

The simplest case of $CP(1)$ dynamics assumes $1 \leq \alpha, \beta \leq 3, \quad i, k, n = 1$. The quasi-linear field equations in the case of the spherical symmetry being split into the real and imaginary parts take the form

$$(r/c)\omega_t + ct\omega_r = -2\omega \gamma F(u, v),$$
$$(r/c)\gamma_t + ct\gamma_r = (\omega^2 - \gamma^2) F(u, v),$$
$$u_t = U_1(u, v, \omega, \gamma),$$
$$v_t = U_2(u, v, \omega, \gamma),$$

where $F(u, v), U_1(u, v, \omega, \gamma), U_2(u, v, \omega, \gamma)$ well defined functions and $\pi = u + iv$.

6 Reality, EPR and Schrödinger’s Cat paradoxes

Reality is very wide philosophical category; it is even wider than the notion of matter. The definition of such kind of category is very problematic. The all philosophical battles between materialism and idealism concern just the criterion of reality. Particularly, macroscopic physical reality has a long history too. Contradictable development of quantum physics evoked new attempts of subjective idealism or/and even agnosticism to take over materialism and objective character of the scientific description of “reality”. Such philosophy sharply contradicted to Einstein’s point of view. He was sure that the objective description (independent from observer) of quantum phenomenon without any reference to the agnostic “uncontrol perturbation” should be achieved. Pursing this aim, Einstein, Podolsky and Rosen were insisted to give the definition of “element of physical reality” [16].
In order to understand the character of difficulties discovered in EPR article it will be interesting to recall firstly the Einstein’s point of view on “reality” [17]. Einstein, discussing reality of gravitation field, notes that distinguishing “real” and “non-real” has no meaning. He proposed instead to distinguish proper values of physical system (invariants) and values depending on coordinate description.

Nevertheless, Einstein in EPR article avoided his own point of view and he together with co-authors came to definition of the “element of physical reality”. This definition leads to conclusion that whether ordinary quantum scheme is not complete or some space-like interaction (“spooky action”) between spatially distinguished observers is “real”. “No reasonable definition of reality could be expected to permit this” [16]. In fact locality-separability [18] contradicts to link of eigenfunctions-eigenvalues prescribed by standard quantum approach denying necessity to know dynamics of quantum transitions from one stationary state to another. This is the source of the indeterminism and primitive projective postulate treating measuring process as instant process that leads to simultaneous “knowledge” of quantum state of any (even remote) subsystem. In fact one has not knowledge but only “believe”. There are however a number of evidences of quantum long-range correlations, i.e. some “outlandish reality” denied by EPR does exist! This paradox may be resolved if we return to Einstein’s initial point of view. Namely, applying his approach to quantum physics one needs to distinguish functional (state-dependent) invariants and values depending on representation (choice of the functional basis). Probably the absence of this detail is one of the main reasons of Einstein’s complaint about EPR text written by Podolsky [18].

Functional invariant reflects the objective character of quantum state and its symmetries. In EPR example the correlations arise due to space-time symmetries and corresponding conservation laws. They have a statistical character since the dynamical nature of the entanglement is hidden under standard QM approach. Dynamical nature of entanglement related to conservation laws for LDV’s in state space. Quantum measurement of LCD’s being understood as set of answers on quantum questions “yes/no” create local dynamical space-time structure [1,12,14,19,20]. Thereby instead of EPR criterion of reality one may search invariants of entangled state and quantities depending of its representation. It is clear that spatially non-local description of the entangled (extended) state could not be relativistically invariant. Namely, detection of two parts of single extended system (two remote solitons and even single “big” soliton) are relativistically non-invariant procedure depending on the choice of spatial reference frame. A long time this fact was the main argument against non-local quantum field theory. However there are no natural reasons for requirements of conservation of the global causal relations and space-time locality in self-interacting extended systems (like well known paradoxically formulated general question about priority of chicken or egg). For such kind of quantum systems, the relativity should be accompanied by super-relativity to the choice of functional reference frame [1,12,9]. Namely, broken Lorentz symmetry widely discussed now (see, say, [21]), should be locally restored with
help the affine parallel transport of the local Hamiltonian in the projective
Hilbert state space that leads to extended soliton-like solutions [1, 14].

The EPR article appeals to intuitively clear picture of good localized particles
which after interaction may be treated as non-interacting and hence independent
after short time. Einstein and his co-authors in 1935 did not have such non-local self-interacting objects as solitons. But now soliton solution of good defined quantum system with spatially extended quantum state may replace
this intuitive picture at least in the framework of some model. This leads to
new “outlandish reality” in physics. I would like to show the difference between
such non-local objects and EPR original entangled states.

Let me assume that two entangled particles are described by some two-soliton solution, say of the SG equation as follows

$$\Psi(x, t) = 4 \arctan \frac{e^{\eta_1} + e^{\eta_2}}{1 + e^{\eta_1}e^{\eta_2}},$$

(26)

where \( \eta_r = \kappa_r x - \nu_r t + \eta_r^0 \), and \( r = 1, 2 \). This solution describes configuration with energy concentrated in two different space-time points but it depends on single pair \((x, t)\) and may be formally rewritten as series in some functional basis

$$\Psi(x, t) = \sum_{n=1}^{\infty} \psi^n u_n(x, t),$$

(27)

where \( u_n(x, t) \) are eigen-functions of dynamical variable \( A \). This decomposition sharply differs from EPR bi-local wave-function

$$\Psi(x_1, x_2) = \sum_{n=1}^{\infty} \psi_n(x_2)u_n(x_1).$$

(28)

If one uses different basis \( v_n(x, t) \) consists of eigen-functions of dynamical variable \( B \), one will have the decomposition

$$\Psi(x, t) = \sum_{n=1}^{\infty} \phi^n v_n(x, t).$$

(29)

First, it does not mean, however, that \( u_n(x, t) \) describes one of the “hump” and that Fourier components \( \psi^n \) describes the state of the second “hump”. Second, one could not, therefore, conclude that the alternative measurement of dynamical variable \( B \) having eigen-functions \( v_n(x, t) \) leads to the conclusion that the second “hump” will be in the state \( \phi^n \). We have here the dynamical entanglement for self-interacting non-separable soliton system. Therefore the EPR conclusion about instant dependence of “reality” from remote measurement for such kind of entanglement is not applicable but global Lorentz invariance will be broken. Non-locality of such “elementary” particles leading to breakdown of Lorentz symmetry may be compensated by affine gauge field associated with parallel transport of the local Hamiltonian. Phenomenologically it may appear
as new states or particles resulting “deformation” of the Hamiltonian during the parallel transport and the continuous “measurement” provided by “quantum boosts” and “quantum rotations” of local Lorentz reference frame. In other words: in order to avoid the contradiction with causality, the local Lorentz reference frame should be adapted during “scanning along lump”. Such local Lorentz reference frame has been built above whose “quantum boosts” and “quantum rotations” are defined by formulas (21) and (25). The parallel transport of the local Hamiltonian provides the “self-identity” of extended object, i.e. the affine gauge fields couple the soliton-like system (25) discussed in [1, 14].

Assuming \( \omega = \rho \cos \psi, \gamma = \rho \sin \psi \) and then \( \omega^2 + \gamma^2 = \rho^2 = \text{constant} \), the two first PDE’s may be rewritten as follows:

\[
\frac{r}{c} \psi_t + ct \psi_r = F(u, v) \rho \cos \psi. \tag{30}
\]

The one of the exact solutions of this quasi-linear PDE is

\[
\psi_{\text{exact}}(t, r) = \arctan \left( \frac{\exp(2cF(u, v)f(r^2 - c^2t^2))(ct + r)^2F(u, v) - 1}{\exp(2cF(u, v)f(r^2 - c^2t^2))(ct + r)^2F(u, v) + 1} \right). \tag{31}
\]

where \( f(r^2 - c^2t^2) \) is an arbitrary function of the interval.

In order to keep physical interpretation of these equations I will find the stationary solution for (30). Let me put \( \xi = r - ct \). Then one will get ordinary differential equation

\[
\frac{d\Psi(\xi)}{d\xi} = -F(u, v) \rho \frac{\cos \Psi(\xi)}{\xi}. \tag{32}
\]

Two solutions

\[
\Psi(\xi) = \arctan(\frac{\xi - 2M e^{-2CM} - \frac{1}{2}}{\xi - 2M e^{-2CM} + 1 + \frac{2\xi - 2M e^{-2CM}}{\xi - 2M e^{-2CM} - 1}}), \tag{33}
\]

where \( M = F(u, v) \rho \) are concentrated in the vicinity of the light-cone looks like solitary waves. Here we have example of relativistic non-local solution arose due to restoration of the Lorentz symmetry.

This solution is localizable in some functional space. I would like to emphasize that locality is formally achievable in any appropriate functional state space [9], but we wish to have the state space capable to distinguish dynamically different states of non-local in space-time solutions. The projective Hilbert state space in local coordinates apparently distinguishes states with different amplitudes and/or phases.

### 7 Summary

Taking into account the main target of Einstein which is not perfectly realized in the EPR article [18, 16], the uncontrollable perturbation during a quantum measurement has been replaced to the coset structure \( G/H = SU(N)/S[U(1) \times \ldots] \).
In the framework of this model I may conclude:

1. Quantum measurement consists of two procedures: the comparison of the local dynamical variables with the help of affine parallel transport in $CP(N-1)$ and the qubit-encoding of the quantum measurement.

2. Space-time emergences due to the qubit-encoding of the quantum measurement of local dynamical variables in projective Hilbert space.

3. Global Lorentz symmetry is broken and local unitary transformations of the affine parallel transport of the local Hamiltonian in state space should restore it.

4. The superposition principle is realized locally in tangent space $T_\pi CP(N-1)$ at the solution of variational problem.

5. Identification of coset transformations $SU(N)/S[U(1) \times U(N-1)]$ and $CP(N-1)$ manifold of GCS is dynamical, i.e. state dependent. This means that a concrete form of the Cartan decomposition is state dependent, functionally local. Therefore vector fields representing local dynamical variable has invariant, physically objective character.

6. Schrödinger’s Cat [22] paradox may be treated as a “compact version of the EPR argument for incompleteness” [18]. Furthermore, the Cat plays the role of two-level system (logical spin 1/2 [12], or qubit) whose 2 states “yes” and “no” fix a result of “measurement”. The continuous description of quantum evolution in $CP(N-1)$ and the covariant differentiation of local Hamiltonian provide the instantaneous “collapse” by local projection onto tangent space $T_\pi CP(N-1)$. Following procedure of encoding the result of the “measurement” by the lift in the tangent fibre bundle leads to the local dynamical space-time structure provided by moving local Lorentz reference frame.

7. The geometric formulation of QM being taken not as embellishment but as serious reconstruction, paves the way to new physical interpretation resolving old paradoxes (EPR, Schrödinger’s Cat), namely: standard QM is incomplete and non-local. It requires reformulation in accordance with super-relativity like the classical mechanics was reformulated in accordance with Lorentz invariance of Maxwell equations.

References

[1] P. Leifer, Annales de la Fondation Louis de Broglie, 32, (1) 25 (2007).
[2] R. Penrose, *The Road to Reality*, Alfred A.Knopf, New-York, (2005).
[3] Y. Bonder, arXiv:0801.2919v1 [gr-qc].
[4] A. Einstein, Ann. Phys. 17, 891 (1905).
[5] A. Einstein, Ann. Phys. 49, 769 (1916).
[6] T.D. Newton and E.P. Wigner, Rev. Mod. Phys., 21, 400 (1949).
[7] G.C. Hegerfeldt, Phys.Rev. D10, 3320 (1974).
[8] Y. Aharonov, et al., Phys.Rev. A57, 4130 (1998).
[9] A.A. Kryukov, Found. Phys. 36, 175 (2006).
[10] E. Schrödinger, Ann. Physik, 79, 361 (1927).
[11] L. de Broglie, On the Theory of Quanta, A translation of “Rechercher sur la Théorie des Quanta” (Ann. de Phys., 10e série, t.III (Janvier-Février 1925). by A.F. Kracklauer, 2004.
[12] P. Leifer, Found. Phys. 27, (2) 261 (1997).
[13] M.V. Berry, Quantum Adiabatic Anholonomy, Lectures.
[14] P. Leifer, arXiv:gr-qc/0503083
[15] C.W. Misner, K.S. Thorne, J.A. Wheeler, Gravitation, W.H. Freeman and Company, San Francisco, 1973.
[16] A. Einstein, B. Podolsky and N. Rosen, Phys.Rev. 47, 777 (1935).
[17] A.Einstein, Naturwiss., 6, 697 (1918).
[18] A. Fine, “The Einstein-Podolsky-Rosen Argument in Quantum Theory”, http://plato.stanford.edu/entries/qt-epr
[19] P. Leifer, Found.Phys.Lett., 18, (2) 195 (2005).
[20] P. Leifer, JETP Letters, 80, (5) 367 (2004).
[21] V.A. Kostelecký, arXiv:0802.0581v1 [gr-qc].
[22] E. Schrödinger, Naturewissenschafen 23: pp.807-812; 823-828; 844-849 (1935).