Surface anisotropy in a magnetic cylinder induced by the displacement of a vortex core

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In this article we investigate the induction of a surface anisotropy due to the displacement of the vortex core in a cylindrical nanostructure. In fact, the effect of the displacement of the vortex core in the dipolar energy can be modeled simply as a surface anisotropy of the form \(E_s = K_s \int_{S_m} dS (\hat{n} \cdot \hat{m})^2 / 2\). Moreover, the surface anisotropy constant \(K_s\) is proportional to the cylinder in-plane demagnetizing factor in the direction of the core deviation, \(N_s (L/R)\), i.e., \(K_s = \mu_0 M_0^2 R N_s (L/R)\), where \(R\) and \(L\) are the radius and the thickness of the cylinder, respectively. Our results show that the term of the nontrivial dipolar energy caused by the charges in the cylinder mantle can be replaced by a simple integral \(E_s\) that increases the efficiency of the numerical calculations in the analytical study of the displacement of the vortex core in magnetic vortices.

I. INTRODUCTION

Magnetic vortices are magnetic textures composed of a curling in-plane magnetization that turns out of plane at its center over a region known as the vortex core\textsuperscript{10}. These magnetic textures are very attractive both from a fundamental and applied point of view\textsuperscript{3,5,6} because they have no significant magnetic stray-fields, crosstalk between magnetic elements is avoided, enabling close packing required for potential technological applications\textsuperscript{9–13}. The magnetic vortices can be stabilized in cylindrical nanostructures, due to the edges, converting these nanostructures into potential candidates for nonvolatile memories\textsuperscript{14–16}, biomedicine\textsuperscript{17} and new logic operators\textsuperscript{18}.

On the other hand, the gyrotropic mode,\textsuperscript{14,15} which is the lowest frequency excitation of a magnetic vortex, is very different from the precessional modes that are typically seen in uniformly magnetized cylinders\textsuperscript{19,20}. For example, to the dipolar energy, \(E_d\), related with the surface charges of the cylinder covered by vortex core, \(E_d(s) = E_c + E_m(s)\), where \(E_c\) is associated to the magnetostatic potential, \(U_c\), related with the surface charges of the cylinder covers, which does not depend on the deviation \(s\), while \(E_m(s) = \mu_0 M_0 \int_0^s dV \hat{n} \cdot \hat{M}(\vec{r}) / (4\pi |\vec{r} - \vec{r}'|)\) corresponds to the dipolar energy associated with the surface charges of the mantle of the cylinder, with a magnetostatic potential given by \(U_m = \int_{S_m} dS' \hat{n}' \cdot \hat{M}(\vec{r}') / (4\pi |\vec{r} - \vec{r}'|)\), where \(S_m\) is the surface of the mantle of the cylinder. Thus,

\[U_m = \frac{M_0 C}{4\pi} \frac{s}{R} \int_0^{2\pi} d\phi' \mathcal{J}(\phi', s) \cos \phi' \left[ \ln \left( z + \sqrt{A + z^2} \right) - \ln \left( z - L + \sqrt{A + (L - z)^2} \right) \right] (1)\]
and

\[ E_m(s) = \frac{\mu_0 M_0^2}{4\pi} \frac{s}{R} \int_0^{2\pi} d\phi \int_0^{r_{\max}(\phi,s)} dr \int_0^{2\pi} d\phi' \mathcal{J}(\phi', s) \times r_{\max}(\phi', s) \cos \phi' \sin(\phi - \phi') \left[ \sqrt{A} - \sqrt{A + L^2} \right], \tag{2} \]

where we have considered the deviation of the core in the y-direction (as shown in Fig. 1), and have defined

\[ \mathcal{J}(\phi, s) = \sqrt{r_{\max}^2(\phi, s) + (\partial_\phi r_{\max}(\phi, s))^2}, \quad r_{\max}(\phi, s) = -s \sin \phi + \sqrt{R^2 - s^2 \cos^2 \phi} \quad \text{and} \quad A = (r \hat{r} - r_{\max}(\phi', s) \hat{r}')^2. \]

The integral expression given in Eq. (2) is equivalent to the infinite series expansion of Ref. [40]. \( E_m^{(\mu)}(s) = \sum_{j=1}^{\mu} E_m^{(j)}(s) \) in the limit \( \mu \to \infty \). The convergence of \( E_m^{(\mu)}(s) \) as \( \mu \) increases to the limit \( E_m^{(\infty)}(s) = E_m(s) \) is shown in Fig. 2 by plotting \( E_m^{(\mu)}(s) \) as a function of the core deviation aspect ratio \( s/R \), for \( \mu = 1, 2, 10, 50 \) and \( \mu \to \infty \), for a dot of \( R = 100 \) nm in radius and a) \( L = 10 \) nm and b) \( L = 50 \) nm in thickness. The curve for the limit \( \mu \to \infty \) was obtained using Eq. (2). The insets show a zoom of the curves for 0.98 \( \leq s/R \leq 0.985 \). Since the core deviation length is restricted by \( s < R - R_c \), the limit \( s = R \) (where the magnetic energy is singular) is not reached in this work.

### III. RESULTS AND DISCUSSION

To better understand the effect of the displacement of the vortex core in the cylinder, we have plotted in Fig. 3 the magnetostatic field \( \vec{H}_d = -\nabla U \), where \( U = U_c + U_m(s) \). In particular, Fig. 3 shows the magnetostatic field in the \( xy \) plane (top view of the cylinder), normalized to this plane, that is, \( \vec{H}^{(xy)}_d = [(\vec{H}_d \cdot \hat{x}) \hat{x} + (\vec{H}_d \cdot \hat{y}) \hat{y}] / \sqrt{(\vec{H}_d \cdot \hat{x})^2 + (\vec{H}_d \cdot \hat{y})^2} \), in a cylinder of \( R = 100 \) nm radius and \( L = 70 \) nm thickness, with a chirality \( \mathcal{C} = 1 \) and a core radius of \( R_c = 40 \) nm.

As we saw previously, the displacement of the vortex core produces surface charges in the mantle of the cylinder, charges that are associated with the appearance of a surface anisotropy, \( E_s = K_s \int_{S_m} dS (\hat{n} \cdot \hat{m})^2/2 \), given by,

\[ E_s = \frac{K_s L R}{2} \frac{s^2}{R^2} \int_0^{2\pi} d\phi \frac{\cos^2 \phi}{1 + s^2/R^2 - 2 \sin \phi s/R}, \tag{3} \]

where \( K_s \) corresponds to the surface anisotropy constant.

By equating \( E_s \) with the expansion in \( \mu \) of \( E_m^{(\mu)} \) at second order in \( s/R \), we have obtained an expression for \( K_s \) given by

\[ K_s = \mu_0 M_0^2 R N_\mu(L/R), \tag{4} \]

with \( N_\mu(\gamma) = \int_0^\infty \frac{dt}{T} \left( 1 - \frac{1-e^{-\gamma t}}{\gamma t} \right) J_1^2(t) \) being the cylinder in-plane demagnetizing factor in the direction of the deviation of the core. In this way, we have not only reached the conclusion that the displacement of the
vortex core induces a surface anisotropy in the cylinder, but also, we have obtained a simplified expression, $E_s$, for the dipolar energy of the mantle, $E_m$. It is worthwhile to mention that the results obtained in Eqs. (1), (2), (3) and (4) are valid for a general vortex Ritz model $\mathbf{m} = m_z(r)\mathbf{\hat{z}} + m_\phi(r)\mathbf{\hat{\phi}}$, with $m_\phi = 1$ for $r \geq R_c$ and $m_\phi = \sqrt{1 - m_z(r)^2}$ for $r < R_c$, independently of the particular shape of $m_z(r)$.

In order to compare $E_s$ with $E_m$, we have plotted in Fig. 4 both energy terms (over $M_0^2$) as a function of $s/R$ for a cylinder of radius (a) $R = 100$ nm and (b) $R = 200$ nm, for different thicknesses $L$. $E_s$ was calculated using Eq. (3) with $K_s$ given by the Eq. (4), which is shown as solid circles, whereas $E_m$ was obtained from Eq. (2) and is represented as hollow circles. The different $L$-curves are distinguished by different colors. From Fig. 4 it can be seen that $E_s$ has the same behavior as $E_m$, and that both energy terms are in the same order of magnitude for the entire range of parameters considered, both for $R = 100$ nm and $R = 200$ nm, regardless of the value of the thickness of the cylinder and its magnetic parameters.

The goodness of fit of $E_s$ to the dipolar energy term $E_m$ is estimated for the numerical data using the coefficient of determination $R^2$,

$$R^2 = 1 - \frac{\sum_i [E_s(s_i) - E_m(s_i)]^2}{\sum_i [E_m(s_i) - \langle E_m \rangle]^2}, \quad (5)$$

where $\langle E_m \rangle$ is the average value of $E_m(s_i)$. We have found a high accuracy fit when $s/R \leq 0.5$ for both radii, independently of the thickness $L$. The detailed values of the $R^2$-coefficient are given in Table I. From these data it can be mentioned that, when $s/R \leq 0.5$, the fit explains at least $99.99\%$ of the total variation in the data about the average of $E_m(s_i)$, and the accuracy fit of $E_s$ increases as the dot thickness decreases. Moreover, since the $R^2$ could be misleading for non linear regression, we have also computed the standard error deviation of the data founding typical errors between 1 per thousand and 1 per ten thousand, in the scale of Fig. 4.

In Fig. 5 we show the surface anisotropy constant $K_s$ (over $M_0^2$) as a function of the cylinder aspect ratio, $L/R$, for different radii. This constant has been obtained from Eq. 4. From this figure we can obtain the value of the surface anisotropy constant $K_s$ for cylinders with different magnetic and geometric parameters. It is important to note that $K_s$ increases as both the radius and the aspect ratio of the cylinder increase.

Figure 6 shows the displacement of the core, $s/R$, which minimizes the total energy, $E(s)$, which includes not only the dipolar energy, but also the exchange and Zeeman energies obtained from Ref. [40]. In this case we have considered cobalt cylinders with a saturation magnetization of $M_0 = 1.4 \times 10^6$ A/m and an exchange stiffness constant of $A = 3 \times 10^{-11}$ J/m, with radii (a) $R = 100$ nm and (b) $R = 200$ nm, for different thicknesses (to calculate the total energy we have used the particular vortex Ritz model of Ref[1], which was also

![Image](image.png)

FIG. 3. (Color online) Magnetostatic field in the $xy$ plane, $\mathbf{h}_{\text{ms}}$, in a cylinder of $R = 100$ nm radius and $L = 70$ nm thickness, with a chirality $C = 1$ and a core radius of $R_c = 40$ nm. (a) shows a vortex without displacement, that is, centered on the cylinder, while (b) shows a displaced vortex core. The + and − signs are guide to the eyes to understand the field lines behavior using the dumbbell picture.[22]

![Image](image.png)

FIG. 4. (Color online) Comparison of the behavior of $E_s$ given by the Eq. (3) with $E_m$ given by the Eq. (2) as a function of $s/R$ for cylinders with different thicknesses $L$ and radii of (a) $R = 100$ nm and (b) $R = 200$ nm. Hollow and solid circles correspond to the $E_m$ and $E_s$ curves, respectively.

| $R^2$   | $R = 100$ nm | $R = 200$ nm |
|---------|-------------|-------------|
| $L = 10$ nm | 0.999993 | 0.999997 |
| $L = 20$ nm | 0.999988 | 0.999993 |
| $L = 30$ nm | 0.999980 | 0.999992 |
| $L = 40$ nm | 0.999974 | 0.999988 |
| $L = 50$ nm | 0.999968 | 0.999983 |
| $L = 60$ nm | 0.999963 | 0.999980 |
| $L = 70$ nm | 0.999957 | 0.999976 |

TABLE I. Coefficient of determination $R^2$ of the fit for $s/R \leq 0.5$. |
FIG. 5. (Color online) surface anisotropy constant $K_s$ (over $M_0^2$) as a function of the cylinder aspect ratio, $L/R$, for different radii.

The circles were obtained by considering the contribution of the mantle, $E_m$, while the lines were obtained using the approximation of the surface energy, $E_s$. When the vortex is the stable magnetic configuration, the circles are solid, whereas when a uniform state is stable (and therefore the vortex is unstable), the circles are hollow. It can be seen a very good agreement between circles and lines data when the vortex state is stable.

IV. CONCLUSIONS

In conclusion, we have shown that the effect of the displacement of the vortex core in the dipolar energy can be modeled simply as a surface anisotropy of the form $E_s = K_s \int_{S_m} dS \cdot (\hat{n} \cdot \hat{m})^2 / 2$. Interestingly, $E_s$ estimates the behavior of dipolar energy quite well over the entire range of parameters investigated, and in particular, with high accuracy for $s/R \leq 0.5$. Moreover, we have found that the surface constant is proportional to the cylinder in-plane demagnetizing factor in the direction of the core deviation, $N_y(L/R)$, i.e., $K_s = \mu_0 M_0^2 R N_y(L/R)$, where $R$ and $L$ are the radius and the thickness of the cylinder, respectively. This expression can be used to calculate the surface constant of cylinders with different magnetic and geometrical parameters.

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