THREE-DIMENSIONAL SIMULATIONS OF STANDING ACCRETION SHOCK INSTABILITY IN CORE-Collapse Supernovae

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1. INTRODUCTION

Many efforts have been made to construct multidimensional models of core-collapse supernovae (see Woosley & Janka 2005, Kotake et al. 2006 for reviews), prompted by accumulated observational evidence that core-collapse supernovae are commonly globally aspherical (Wang et al. 1996, 2001, 2002). Various mechanisms for producing the asymmetry have been discussed, including convection (e.g., Herant et al. 1994; Burrows et al. 1995; Janka & Mueller 1996), magnetic field and rapid rotation (see, e.g., Kotake et al. 2006 for collective references), standing (stationary, spherical) accretion shock instability, or SASI (Blondin et al. 2003; Scheck et al. 2004; Blondin & Mezzacappa 2006; Ohnishi et al. 2006, 2007; Foglizzo et al. 2006), and g-mode oscillations of proto–neutron stars (Burrows et al. 2006). Most of these, however, have been investigated only with two-dimensional (2D) simulations.

Recently, a 3D study of SASI was reported by Blondin & Mezzacappa (2007). In 2D, the shock deformation by SASI is described with Legendre polynomials $P_l(\theta)$, or spherical harmonics $Y_{l,0}^m(\theta, \phi)$ with $m = 0$. Various numerical simulations have demonstrated unequivocally that the $l = 1$ mode is dominant and that a bipolar sloshing of the standing shock wave occurs, with pulsational strong expansions and contractions along the symmetry axis (Blondin et al. 2003; Scheck et al. 2004; Blondin & Mezzacappa 2006; Ohnishi et al. 2006, 2007). In 3D, on the other hand, Blondin & Mezzacappa (2007) perturbed a nonrotating accretion flow azimuthally and observed the dominance of a nonaxisymmetric mode with $l = 1, m = 1$, which produces a single-armed spiral in the later nonlinear phase. They claimed that this “spiral SASI” generates a rotational flow in the accretion flow (see also Blondin & Shaw [2007] for 2D computations in the equatorial plane), and that it may be an origin of pulsar spin.

However, many questions regarding 3D SASI still remain to be answered. Here we are particularly interested in investigating how the growth of SASI differs between 3D and 2D; in particular, the change in the saturation properties should be made clear. Another question is the generation of rotation in the accretion flow by SASI (Blondin & Mezzacappa 2007); its efficiency and possible correlation with the net linear momentum should be studied more in detail and will be the subject of a future paper (W. Iwakami et al. 2008, in preparation). We also note that neutrino heating and cooling were neglected and the flow was assumed to be isentropic in Blondin & Mezzacappa (2007), but the implementation of this physics is helpful in considering the implications for the shock revival and nucleosynthesis (Kifonidis et al. 2006).

In this paper, we have performed 3D hydrodynamic simulations, employing a realistic equation of state (Shen et al. 1998) and taking into account the heating and cooling of matter via neutrino emission and absorption on nucleons, as done in our 2D studies (Ohnishi et al. 2006, 2007). The inclusion of neutrino heating allows us to discuss how the critical luminosity for SASI-triggered

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explosion could be changed in 3D from those in 2D. To answer the questions raised above, we vary the initial perturbations as well as the neutrino luminosity, and compare the growth of SASI between 2D and 3D in detail, conducting a mode analysis for both the linear phase and the nonlinear saturation phase.

The plan of this paper is as follows. In § 2, we describe the models and numerical methods, show the main numerical results in § 3, and conclude the paper in § 4.

2. NUMERICAL MODELS

The numerical methods we employ are based on the code ZEUS-MP/2 (Hayes et al. 2006), which is a computational fluid dynamics code for the simulation of astrophysical phenomena, parallelized by the MPI (message-passing) library. The ZEUS-MP/2 code employs Eulerian hydrodynamics algorithms based on the finite-difference method with a staggered mesh. In this study, we have modified the original code substantially according to the prescriptions in our preceding 2D simulations (Ohnishi et al. 2006, 2007).

We consider spherical coordinates \((r, \theta, \phi)\) with the origin at the center of the proto–neutron star. The basic evolution equations describing accretion flows of matter attracted by a proto–neutron star and irradiated by neutrinos emitted from the proto–neutron star can be written as

\[
\frac{dp}{dt} + \rho \nabla \cdot \mathbf{v} = 0, \tag{1}
\]

\[
\rho \frac{dv}{dt} = -\nabla P - \rho \nabla \phi - \nabla \cdot \mathbf{Q}, \tag{2}
\]

\[
\rho \frac{d(e)}{dt} = -P \nabla \cdot \mathbf{v} + Q_E - \mathbf{Q} : \nabla \mathbf{v}, \tag{3}
\]

\[
\frac{dY_e}{dt} = Q_N, \tag{4}
\]

\[
\Phi = -\frac{GM_m}{r}, \tag{5}
\]

where \(\rho, v, e, P, Y_e,\) and \(\Phi\) are the density, velocity, internal energy, pressure, electron fraction, and gravitational potential, respectively, and \(G\) is the gravitational constant. The self-gravity of matter in the accretion flow is ignored. Here \(Q\) is the artificial viscous tensor, and \(Q_E\) and \(Q_N\) represent the heating/cooling and electron source/sink via neutrino absorptions and emissions by free nucleons, respectively. The Lagrangian derivative is denoted by \(d/dt \equiv \partial/\partial t + \mathbf{v} \cdot \nabla\). The tabulated realistic equation of state based on relativistic mean field theory (Shen et al. 1998) is implemented according to the prescription in Kotake et al. (2003). The mass accretion rate and the mass of the central object are fixed to be \(\dot{M} = 1 \, M_\odot \, \text{s}^{-1}\) and \(M_m = 1.4 \, M_\odot\), respectively. The neutrino heating is estimated under the assumptions that neutrinos are emitted isotropically from the central object and that the neutrino flux is not affected by local absorptions and emissions (see Ohnishi et al. 2006). We consider only the interactions of electron-type neutrinos and antineutrinos. Their temperatures are also assumed to be constant and are set to be \(T_v = 4 \, \text{MeV}\) and \(T_{\bar{v}} = 5 \, \text{MeV}\), typical values in the postbounce phase. The neutrino luminosity is varied in the range of \(L_v = (6.0 - 6.8) \times 10^{52} \, \text{ergs} \, \text{s}^{-1}\).

Spherical polar coordinates are adopted. In the radial direction, the computational mesh is nonuniform, while the grid points are equally spaced in other directions. We use 300 radial mesh points to cover \(r_{\text{in}} \leq r \leq r_{\text{out}}\), where \(r_{\text{in}} \approx 50 \, \text{km}\) is the radius of the inner boundary, located roughly at the neutrino sphere, and \(r_{\text{out}} = 2000 \, \text{km}\) is the radius of the outer boundary, at which the flow is supersonic. A total of 30 polar and 60 azimuthal mesh points are used to cover the whole solid angle. In order to see if this angular resolution is sufficient, we have computed a model with the \(300 \times 60 \times 120\) mesh points and compared it to the counterpart with the \(300 \times 30 \times 60\) mesh points. As shown in Appendix B, the results agree reasonably well with each other in both the linear and nonlinear phases. Although the computational cost does not allow us to carry the convergence test further, we think that the resolution of this study is good enough.

We use an artificial viscosity of tensor type, described in Appendix A, instead of the von Neumann & Richtmyer type that was originally employed in ZEUS-MP/2. For 3D simulations with a spherical polar mesh, we find the former preferable to prevent the so-called carbuncle instability (Quirk 1994), which we observe around the shock front near the symmetry axis, \(\theta \approx 0, \pi\). With the original artificial viscosity, an appropriate dissipation is not obtained in the azimuthal direction for the shear flow resulting from the converging accretion, particularly when a fine mesh is used (Stone & Norman 1992). We have also applied this artificial viscosity to axisymmetric 2D simulations and reproduced the previous results (Ohnishi et al. 2006).

Figure 1 shows the radial distributions of various variables for the unperturbed flows. The spherically symmetric steady accretion flow through a standing shock wave is prepared in the same manner as in Ohnishi et al. (2006). Behind the shock wave, the electron fraction is less than 0.5 owing to electron capture, and a region of negative entropy gradient with positive net heating rates is formed for the neutrino luminosities \(L_v = (6.0 - 6.8) \times 10^{52} \, \text{ergs} \, \text{s}^{-1}\). The values of these variables on the ghost mesh points at the outer boundary are fixed to be constant in time, while on the ghost mesh points at the inner boundary they are set to be identical to those on the adjacent active mesh points, except for the radial velocity, which is fixed to the initial value at both the inner and outer boundaries.

In order to induce nonspherical instability, we have added a radial velocity perturbation, \(\delta v_r (r, \theta, \phi)\), to the steady spherically symmetric flow according to the equation

\[
v_r (r, \theta, \phi) = v_r^{1D} (r) + \delta v_r (r, \theta, \phi), \tag{6}
\]

where \(v_r^{1D} (r)\) is the unperturbed radial velocity. In this study, we consider three types of perturbations: (1) an axisymmetric \((l = 1, m = 0)\) single-mode perturbation,

\[
\delta v_r (r, \theta, \phi) \propto \frac{3}{4\pi} \cos \theta v_r^{1D} (r), \tag{7}
\]

(2) a nonaxisymmetric perturbation with \(l = 1\),

\[
\delta v_r (r, \theta, \phi) \propto \left[ \frac{3}{4\pi} \cos \theta - \frac{3}{8\pi} \sin \theta \cos \phi \right] v_r^{1D} (r), \tag{8}
\]

and (3) a random multi-mode perturbation,

\[
\delta v_r (r, \theta, \phi) \propto \text{rand} \times v_r^{1D} (r) \quad (0 \leq \text{rand} < 1), \tag{9}
\]

where “rand” is a pseudorandom number. The perturbation amplitude is set to be less than 1% of the unperturbed velocity. We note that there is no distinction between \(m = 1\) and \(-1\) modes when the initial perturbation is added only to the radial velocity, as is the case in this paper. To put it another way, the \(m = \pm 1\) modes contribute equally. Hence, they are referred to as the \(|m| = 1\) mode below. On the other hand, differences do show up,
for example, if random perturbations are added to the nonradial velocity components; this case is important in discussing the origin of pulsar spins proposed by Blondin & Mezzacappa (2007), and will be the subject of a forthcoming paper (W. Iwakami et al. 2008). We also note that the symmetry between $m = 1$ and $\pm 1$ modes is naturally removed if the unperturbed accretion flow is rotating (see Yamasaki & Foglizzo 2008). All the models presented in this paper are summarized in Table 1.

In the next section, we perform the mode analysis as follows. The deformation of the shock surface can be expanded as a linear combination of the spherical harmonic components $Y^m_l (\cos \theta) e^{im\phi}$:

$$R_S(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} c_l^m Y^m_l (\theta, \phi).$$  \hspace{1cm} (10)

where $Y^m_l$ is expressed by the associated Legendre polynomial $P^m_l$ and a constant $K^m_l$, given as

$$Y^m_l = K^m_l P^m_l (\cos \theta) e^{im\phi},$$  \hspace{1cm} (11)

$$K^m_l = \sqrt{\frac{2l + 1}{4\pi} \frac{(l - m)!}{(l + m)!}}.$$  \hspace{1cm} (12)

The expansion coefficients can be obtained as

$$c_l^m = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin \theta R_S(\theta, \phi) Y^{*m}_l (\theta, \phi),$$  \hspace{1cm} (13)

where the superscript asterisk (*) denotes complex conjugation.

### Table 1

| Model | Perturbation | Neutrino Luminosity $L_\nu$ (10$^{52}$ ergs s$^{-1}$) |
|-------|--------------|----------------------------------------------------|
| I ........ single-mode, $l = 1, m = 0$ | 6.0 |
| II .......... multi-mode, $l = 1, m = 0$ and $l = 1, |m| = 1$ | 6.0 |
| III .......... single-mode, $l = 1, m = 0$ with random perturbation at $t = 400$ ms | 6.0 |
| IV .......... random perturbation | 6.0 |
| V .......... random perturbation | 6.4 |
| VI .......... random perturbation | 6.8 |
| VII ........ axisymmetric random perturbation | 6.0 |
| VIII .......... axisymmetric random perturbation | 6.8 |
| IX .......... none | 6.0 |
| X .......... none | 6.8 |

3. RESULTS AND DISCUSSIONS

#### 3.1. Axisymmetric Single-Mode Perturbation

Before presenting the results of 3D simulations, we first demonstrate the validity of our newly developed 3D code. We compare the axisymmetric flows obtained by 2D and 3D simulations for the axisymmetric ($l = 1, m = 0$) single-mode perturbation. By 2D simulations we mean that axisymmetry is assumed and computations are done in the meridian section with all $\phi$ derivatives dropped, whereas in 3D simulations we retain all these derivatives, and a 3D mesh is employed. This validation is important because numerical errors may induce azimuthal motions even for the axisymmetric initial conditions. Hence, we have to confirm that azimuthal errors do not appear in the nonaxisymmetric simulation.

Figure 2 shows the time evolutions of the average shock radius $R_S$ for 2D (Fig. 2a) and 3D (Fig. 2b) results. The average shock radius $R_S$ is obtained from the expansion coefficient $c_0^0$ in
equation (10) multiplied by $K_0^0$. The solid, dashed, and dotted lines correspond to the neutrino luminosities $L_\nu$ of 6.0, 6.4, and $6.8 \times 10^{52}$ ergs s$^{-1}$, respectively. One cannot expect a perfect agreement between two computations of the exponential growth of the instability followed by turbulent motions through mode coupling, as considered here; however, it is still obvious that the results of the 2D and 3D simulations agree in essential features. In particular, in both the 2D and 3D results, we see that the $R_S$ is settled to an almost constant value for $L_\nu = 6.0$ and $6.4 \times 10^{52}$ ergs s$^{-1}$; whereas it continues to increase for $L_\nu = 6.8 \times 10^{52}$ ergs s$^{-1}$.

Figure 3 presents the normalized amplitudes $|c_\ell^m/c_0^0|$ as a function of time for model I ($L_\nu = 6.0 \times 10^{52}$ ergs s$^{-1}$) for 2D (Fig. 3a) and 3D (Fig. 3b) results. The red, blue, black, and gray solid lines correspond to the modes of $(l, m) = (1, 0), (2, 0), (3, 0), \text{and} (4, 0)$, respectively. As can be seen, the time evolution can be divided into two phases. First is the linear growth phase, in which the amplitude of the initially imposed mode with $(l, m) = (1, 0)$ grows exponentially. This lasts for ~100 ms. Higher modes are also generated by mode couplings and grow exponentially during this phase. Then starts the second phase, which is characterized by the saturation of amplitudes of the order of 0.1. In this phase, the accretion flow becomes turbulent. It is interesting to note that in this nonlinear saturation phase the amplitude of the mode with $(l, m) = (2, 0)$ is almost of the same order as the initially imposed mode with $(l, m) = (1, 0)$.
and is dominant over other modes, which fact was observed in Ohnishi et al. (2006).

Most important of all, however, is the fact that none of the \( m \neq 0 \) modes are produced, implying that the perturbed flow retains axisymmetry, which is a necessary condition for the numerical study of nonaxisymmetric instability. Although the results of the 2D and 3D simulations are not identical, the essential features such as the linear growth rate of the initial perturbation \((l = 1, m = 0)\) and the production of other modes via nonlinear mode coupling, as well as the saturation levels in the nonlinear phase, are in good agreement for the two cases. The quantitative differences between the 2D and 3D results mainly originate from the difference in the time steps, which depend on the minimum grid width. Note that in addition to the 300 radial and 30 polar mesh points used for the 3D computations, 60 azimuthal mesh points are employed in the 3D simulations and, as a result, smaller time steps are usually taken for the 3D computations.

3.2. Nonaxisymmetric Perturbation with \( l = 1 \)

Now we discuss the results of genuinely 3D simulations, in which the nonaxisymmetric perturbation with \( (l = 1, |m| = 1) \) is added to the axisymmetric \((l = 1, m = 0)\) perturbation. Figure 4 shows the time evolutions of some of the normalized amplitudes \( |c^m_l/c_0^l| \) for model II with the neutrino luminosity \( L_\nu = 6.0 \times 10^{52} \text{ ergs s}^{-1} \). The red, yellow, blue, light blue, green, black, brown, violet, and pink solid lines denote the amplitudes of the modes with \((l, |m|) = (1, 0), (1, 1), (2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2), (3, 3)\), respectively. In the linear phase, the initially imposed modes with \((l, |m|) = (1, 0)\) and \((1, 1)\) grow exponentially, just as in the axisymmetric single-mode perturbation. It should be noted that the growth rate of the \((l, |m|) = (1, 1)\) mode is identical to that of the \((l, m) = (1, 0)\) mode. This is just as expected for the spherically symmetric background, for which modes with the same \( l \) but different \( m \) are degenerate in the linear eigenfrequency.

It can also be seen from Figure 4 that the modes with \((l, |m|) = (2, 0), (2, 1), (2, 2)\) are first produced by the nonlinear mode couplings of \((l, |m|) = (1, 0), (1, 1)\). Then the modes with \((l, |m|) = (3, 0), (3, 1), (3, 2), (3, 3)\) can be produced by the couplings of the first-order modes with \((l, |m|) = (1, 0), (1, 1)\) and the second-order modes with \((l, |m|) = (2, 0), (2, 1), (2, 2)\). Even higher order modes are produced subsequently toward the nonlinear saturation, but they are not shown here. Although the branching ratios should be investigated in more detail before coming to any conclusions, the above sequence of mode generation strongly suggests that the nonlinear coupling is mainly of quadratic nature.

In the nonlinear phase that begins at \( t \sim 200 \text{ ms} \), these mode amplitudes are saturated and settled into a quasi-steady state. As in the axisymmetric case, the \( l = 1 \) modes both with \( m = 0 \) and \( |m| = 1 \) are dominant over other modes in this stage for the nonaxisymmetric case. One thing to be noted here, however, is the fact that the saturated amplitudes for model II are lower than those for model I in general (see Fig. 3b). This is, we think, because the turbulent energy is shared by a larger number of modes in the nonaxisymmetric case than in the axisymmetric case. To demonstrate this more clearly, we have added a random perturbation to the radial velocity in the axisymmetric model I at \( t = 400 \text{ ms} \), by which time the axisymmetric nonlinear turbulence has fully grown. We refer to this model as model III. The time evolutions of the normalized amplitudes \( |c^m_l/c_0^l| \) for model III are shown in Figure 5. It is clear that the axisymmetric \( m = 0 \) mode amplitudes are reduced after the additional perturbation is given, and the nonaxisymmetric \( m 
eq 0 \) modes grow to saturation level at \( t \sim 450 \text{ ms} \). The power spectra of the turbulent motions will be discussed in more detail later.

3.3. Random Multi-Mode Perturbations

3.3.1. Dynamical Features and Critical Luminosity

Having understood the elementary processes of the linear growth and nonlinear mode couplings in the previous section, we now discuss the 3D SASI induced by random multi-mode perturbations, which are supposed to be closer to reality. In this subsection, we show the typical dynamics, paying attention to the time evolution of the shock wave, and hence to the critical luminosity, at which the stalled shock is revived.

Figure 6 shows the time variations of the average shock radius \( R_s \) for models IV, V, and VI, with neutrino luminosities \( L_\nu = 6.0, 6.4, \) and \( 6.8 \times 10^{52} \text{ ergs s}^{-1} \), respectively. It is found that for \( L_\nu = 6.0 \) and \( 6.4 \times 10^{52} \text{ ergs s}^{-1} \), the shock settles to a quasi-steady state after the linear growth, whereas it continues to expand for \( L_\nu = 6.8 \times 10^{52} \text{ ergs s}^{-1} \), which is also true for the axisymmetric counterpart displayed in the right panel of Figure 6 for comparison. This implies that the critical neutrino luminosity is not much affected by the change from 2D to 3D. In the following discussion we refer to the models with \( L_\nu = 6.0 \) and \( 6.4 \times 10^{52} \text{ ergs s}^{-1} \) as the non-explosion models, and the model with
$L_\nu = 6.8 \times 10^{52} \text{ ergs s}^{-1}$ as the explosion model, and we look into their dynamical features in turn. The mode analyses will be done in §3.3.2 and 3.3.3.

Figure 7 shows the side views of the iso-entropy surfaces together with the velocity vectors in the meridian section at four different times for the non-explosion model with $L_\nu = 6.0 \times 10^{52} \text{ ergs s}^{-1}$ (model IV). The hemispheres ($0 \leq \phi \leq \pi$) of eight iso-entropy surfaces are superimposed on one another, and the outermost surface almost corresponds to the shock front. The initial perturbations grow exponentially in the linear phase (Fig. 7a; $t = 40$ ms). At the end of the linear phase, a blob of high entropy is formed, which subsequently grows (Fig. 7b; $t = 93$ ms). High-entropy blobs are continuously generated, and the nonlinear phase begins (Fig. 7c; $t = 100$ ms). Circulating flows occur inside the blobs, while high-velocity accretions onto a proto-neutron star surround the blobs. These flows expand and shrink repeatedly, being distorted, split, and merged with each other inside the shock wave (Fig. 7d; $t = 350$ ms). Reflecting these complex motions, the shock surface oscillates in all directions but remains almost spherical for the nonaxisymmetric model. This is in sharp contrast to the axisymmetric case, in which large-amplitude oscillations occur mainly in the direction of the symmetry axis (Blondin et al. 2003; Ohnishi et al. 2006).

Figure 8 displays snapshots of the iso-entropy surfaces and the velocity vectors for the explosion model with $L_\nu = 6.8 \times 10^{52} \text{ ergs s}^{-1}$ (model VI). As in the non-explosion model, the sequence of events starts with the linear growth of the initial perturbation inside the shock wave (Fig. 8a; $t = 40$ ms). In the explosion model, however, many high-entropy blobs emerge simultaneously much earlier on (Fig. 8b; $t = 70$ ms) than in the non-explosion model. These blobs then repeatedly expand and contract, being distorted, split, and merged together just as in the non-explosion counterpart (Fig. 8c; $t = 80$ ms). After that, some of the blobs get bigger, absorbing other blobs (Fig. 8d; $t = 350$ ms) and, as a result, the shock radius increases almost continuously up to the end of the computation ($t = 400$ ms), as already demonstrated in Figure 6a. At this point, the shock surface looks ellipsoidal rather than spherical. However, the major axis is not necessarily aligned with the symmetry axis, nor is the flow symmetric with respect to this major axis.

### 3.3.2. Mode Spectra

Next we look into the spectral intensity in more detail. As a standard case, we take model IV with $L_\nu = 6.0 \times 10^{52} \text{ ergs s}^{-1}$. Figure 9 shows the time evolutions of the normalized amplitudes $|c_{lm}^\nu/c_0^\nu|$ with $m = 0$ and compares them with the axisymmetric counterparts in model VII. Note that the $m \neq 0$ modes also exist in the nonaxisymmetric model; these are not shown in the figure but will be discussed in the following paragraphs. As can be clearly seen, the amplitude of each mode grows exponentially until $\sim 100$ ms, which is the linear phase. Note in particular that the growth rate and oscillation frequency for the $l = 1$ mode are the same as those obtained for the model with the single-mode perturbation. After $\sim 100$ ms, the mode amplitudes are saturated and the evolution enters the nonlinear phase, with a clear dominance of the modes with $l = 1, 2$. It is also evident in the figure that the saturation level is lower in general for the nonaxisymmetric case than for the axisymmetric one, which confirms the claim in the previous section based on the results for the single-mode perturbation.

In Figure 10, we display snapshots at four different times of the spectral distributions for both the nonaxisymmetric (left panels) and axisymmetric (right panels) cases. The top panels correspond to the linear phase, in which the intensity is distributed rather uniformly over all modes. As time passes, however, the amplitudes of low-$l$ modes grow much more rapidly than those with higher $l$ (second and third rows). One can see a similarity in the time evolutions between the nonaxisymmetric and axisymmetric models. It should be noted again that there is no superiority in the linear growth rate among different $m$ modes in the nonaxisymmetric case. Since the amplitudes of different modes are oscillating in time with some phase lags, the mode with $(l, |m|) = (1, 0)$ is largest at one instance (Fig. 10b; $t = 30$ ms), whereas the mode with $(l, |m|) = (1, 1)$ has the greatest amplitude at another instance (Fig. 10b; $t = 60$ ms). On average, however, none of them is superior to others. This is also true of the explosion models described in the next section. After the nonlinear phase starts...
(bottom panels), the growths of all modes are saturated, and the spectral distributions are settled to be quasi-steady. It is again obvious in these panels that the low-$l$ modes are dominant in the nonlinear phase, and the saturation level is lower in the non-axisymmetric case.

Now we turn our attention to the quasi-steady turbulence in the nonlinear phase. Shown as a function of $l$ in Figure 11 are the power spectra $|c_m^l/c_0^l|^2$ averaged over the interval from $t = 150$ to 400 ms. The nonaxisymmetric and axisymmetric cases are shown by open and filled circles, respectively. In the left panel, modes with different $m$ are plotted separately, whereas they are summed in the right panel. We find that the time-averaged power spectra are not very different among the modes with the same $l$ but different $m$. This implies that the equipartition of the turbulence energy is nearly established among modes with the same $l$. This will have important ramifications for the origin of pulsar spin, and will be discussed in our forthcoming paper (W. Iwakami et al. 2008, in preparation).

The right panel of Figure 11 demonstrates that the time-averaged power spectrum summed over $m$ for the nonaxisymmetric case is
not very different from that for the axisymmetric counterpart. This means that the total turbulence energy does not differ very much between the axisymmetric and nonaxisymmetric cases. As a result, the power of each mode in the nonaxisymmetric case is smaller by roughly a factor of \(2l + 1\) than that of the same \(l\) mode in the axisymmetric case. This is the reason why we have observed that the saturation level of the nonlinear SASI is smaller in the nonaxisymmetric case than in the axisymmetric case and the shock surface oscillates in all directions with smaller amplitudes in the nonaxisymmetric flow, whereas it sloshes in the direction of the symmetry axis with larger amplitudes in the axisymmetric case. The difference in the fluctuations of the average shock radius seen in Figures 6a and 6b can also be explained in the same manner.

Another interesting feature observed in Figure 11 is the fact that the time-averaged power spectra obey a power law at \(l \gtrsim 10\) for both the nonaxisymmetric and axisymmetric models. Two straight lines in Figure 11a are the fits to the data for \(l \gtrsim 10\), each obtained for the axisymmetric and nonaxisymmetric models. The powers are found to be \(-5.7\) and \(-4.3\) for the nonaxisymmetric
and axisymmetric cases, respectively. Although the difference is almost unity, which originates from the difference in multiplicity of the modes with the same $l$, the slope excluding this effect is still a little bit steeper in the nonaxisymmetric case, as seen in Figure 11b. At the moment we do not know if this difference in slope is real or not, and the power itself also remains to be explained theoretically.

### 3.3.3 The Explosion Models

So far, we have been discussing the non-explosion models, in which the SASI is saturated in the nonlinear phase and is settled to a quasi-steady state. In considering the possible consequences of SASI after a supernova explosion occurs, however, it is also interesting to investigate the explosion models, in which a shock revival appears as a result of SASI.

Figure 12 shows the time evolutions of the normalized amplitudes $|c_l^m/c_0^m|$ for the explosion model (model VI) and compares them with the axisymmetric counterpart (model VIII). One feature shared by both the axisymmetric and nonaxisymmetric explosion models is that the nonlinear stage is divided into two phases. The earlier phase is quite similar to the nonlinear phase that we have seen so far for the non-explosion models. The later phase, on the other hand, has the distinctive feature that the $l = 1$ mode becomes much more remarkable than other modes, and the oscillation period tends to be longer as the shock radius gets larger and the advection-acoustic cycle, supposedly an underlying mechanism of the instability, takes longer. The later prominence of the $l = 1$ mode is intriguing, and will be important to any quantitative discussion of the pulsar kick velocity. A theoretical account, however, is yet to be given. The differences between the nonaxisymmetric and axisymmetric models, such as the saturation level, are carried over to the explosion models.

The power spectra averaged over time intervals of $100 \leq t \leq 200$ ms and $250 \leq t \leq 400$ ms are presented in Figure 13a and 13b, respectively. As in the non-explosion models, the equipartition among different $m$ modes is again established for the explosion models. This is true even in the later nonlinear phase, as seen in the right panel of the figure. As a result, the saturation level is smaller for the nonaxisymmetric case than for the axisymmetric case, as mentioned above. The power spectra in the earlier nonlinear phase look very similar to those found in the non-explosion models, with the power law being satisfied for $l \gtrsim 10$. In the later nonlinear phase, on the other hand, the power law is extended to much lower $l$. This is related to the late-time prominence of the $l = 1$ mode, as mentioned above. In fact, the amplitudes in the lower $l$ portion of the power spectrum are enhanced in $250 \leq t \leq 400$. The mechanism of this amplification remains to be understood, but it might be related to the volume-filling thermal convection advocated for the late stage of convective instability in a supernova core by Kifonidis et al. (2006). As a result of this evolution of the spectrum, the shock tends to be ellipsoidal as the shock expands.

#### 3.4 Neutrino Heating

The SASI is supposed to be an important ingredient not only for a pulsar’s proper motion and spin, but also for the explosion mechanism itself. In this section, we look into neutrino heating in nonaxisymmetric SASI.

In Figure 14 we show the color contours of the net heating rate in the meridian section for the non-explosion model with $L_\nu = 6.0 \times 10^{52}$ ergs s$^{-1}$ (nonaxisymmetric model IV and axisymmetric model VII), as well as for the nonaxisymmetric explosion model with $L_\nu = 6.8 \times 10^{52}$ ergs s$^{-1}$ (model VI). In the early phase, a cooling region with negative net heating rates exists around the proto-neutron star, while the heating region extends over the cooling region up to the stalled shock wave in all cases. As time passes and the SASI grows, a pocket of regions with positive but relatively low net heating rates appear. These regions correspond to the high-entropy blobs (high-entropy rings for the axisymmetric case), where the circulating flow exists, as observed in Figures 7b, 7c, and 7d. Since the neutrino emission in these regions is more efficient than in the surroundings, the net heating rate is rather low.

Although the critical neutrino luminosities are not very different between the nonaxisymmetric and axisymmetric cases, the spatial distributions of neutrino heating are different. In the axisymmetric
Fig. 10.—Normalized amplitudes $|c_i^m/c_0^0|$ for model IV (left panels) and model VII (right panels) at different times. Note that the time-averaged values are plotted in panels d and h. [See the electronic edition of the Journal for a color version of this figure.]
case, the shock wave oscillates up and down, whereas it moves in all directions in the nonaxisymmetric case. The oscillation amplitudes are larger in the axisymmetric than in the nonaxisymmetric case in general, as repeatedly mentioned. Reflecting this difference in the shock motions, the neutrino heating is enhanced mainly in the polar regions in the axisymmetric case, while in the nonaxisymmetric case the heating rate is affected by SASI chiefly around the high-entropy blobs. These effects are clearly seen in Figure 14 (see also Fig. 7).

As described in § 3.3.1, the generation of the high-entropy blobs starts around the end of the linear phase and continues during the nonlinear phase. Although the blobs repeatedly expand and contract, sometimes merging or splitting, the turbulent motions together with the neutrino heating finally settle to a quasi-steady state in the non-explosion model. For the explosion model, on the other hand, the number and volume of high-entropy blobs increase much more quickly, and as a result, the heating region prevails, pushing the shock wave outward and narrowing the cooling region near the proto-neutron star.

Figure 15 shows the time evolutions of the net heating rates integrated over the gain region inside the shock wave. For the non-explosion model (model IV; solid line), the volume-integrated heating rate grows until $t \sim 150$ ms, but is saturated thereafter, whereas it increases continuously for the explosion model...
These behaviors of the volume-integrated heating rate are in accord with the time evolutions of the average shock radius, as shown in Figure 6a. It is clear from the figure that the heating rates are mainly affected by SASI during the nonlinear phase. For comparison, the spherically symmetric counterparts, models IX with \( L_{\nu} = 6.0 \times 10^{52} \text{ ergs} \text{ s}^{-1} \) and X with \( L_{\nu} = 6.8 \times 10^{52} \text{ ergs} \text{ s}^{-1} \), are also presented as dashed and dotted lines, respectively. Note that the last model does not lead to shock revival even with this high neutrino luminosity. It is found that the SASI enhances the neutrino heating for both the explosion and non-explosion models.

It is understandably a difficult task to make a clean comparison with more realistic simulations, since they are highly dynamical, and the extraction of an unperturbed background is not easy. The SASI they observe is more often than not nonlinear from the beginning. We do, however, point out that the heating rate of our model IV is similar to what was observed in the nonrotating model of Marek & Janka (2007; see the line for HW-2D in the middle panel of Fig. 7). After \( 1 \sim 200 \text{ ms} \), our results with simplified physics are consistent with their results taking more detailed physics into account, at least for the nonlinear phase. The linear phase is more difficult to compare, since their models probably have no linear phase at all. This clearly demonstrates the complimentary roles of the idealized toy models and detailed simulations.

4. CONCLUSIONS

In this paper we have studied the nonaxisymmetric SASI by 3D hydrodynamical simulations, taking into account a realistic EOS and neutrino heating and cooling. We have added various nonaxisymmetric perturbations to spherically symmetric steady flows that accrete through a standing shock wave onto a proto-neutron star, being irradiated by neutrinos emitted from the proto-neutron star. Mode analysis has been done for the deformation of the shock surface by the spherical harmonics expansion. After confirming that our 3D code is able to reproduce for the axisymmetric perturbations the previous results for 2D SASI that we obtained in Ohnishi et al. (2006), we have done genuinely 3D simulations and found the followings.

First, the model of the initial perturbation with \( (l, |m|) = (1, 0) \) and \( (1, 1) \) has demonstrated that the nonaxisymmetric SASI proceeds much in the same manner as the axisymmetric SASI: the linear phase, in which the initial perturbation grows exponentially, lasts for about \( 100 \text{ ms} \) and is followed by a nonlinear phase, in which various modes are produced by nonlinear mode coupling, and their amplitudes are saturated. It has been found that the critical neutrino luminosity, above which shock revival occurs, is not very different between 2D and 3D. For neutrino luminosities lower than the critical value, the SASI settles to a quasi-steady turbulence. We have found that the saturation level of each mode in the nonaxisymmetric SASI is lower in general than that of its axisymmetric counterpart. This is mainly due to the fact that the number of the modes contributing to the turbulence is larger for the nonaxisymmetric case. The sequence of mode generation, on the other hand, strongly suggests that nonlinear mode coupling is chiefly quadratic in nature.

Second, the simulations with random multi-mode perturbations imposed initially have shown that the dynamics in the linear phase is essentially a superposition of those of single modes. Toward the end of the linear phase, high-entropy blobs are generated continuously and grow, starting the nonlinear phase. We have observed that these blobs repeatedly expand and contract, merging and splitting from time to time. In the non-explosion models, these nonlinear dynamics lead to the saturation of mode amplitudes and quasi-steady turbulence. For the explosion models, on the other hand, the production of blobs proceeds much more quickly, followed by an oligarchic evolution, with a relatively small number of large blobs absorbing smaller ones, and as a result the shock radius increases almost monotonically. The spectral analysis has clearly demonstrated that low-\( l \) modes are predominant in the nonlinear phase, just as in the axisymmetric case. We have also shown that equipartition is nearly established among different \( m \) modes in the quasi-steady turbulence, and that the spectrum summed over \( m \) in the nonaxisymmetric case is quite similar to the axisymmetric counterpart. This implies that the larger number of modes in the nonaxisymmetric case is the main reason why the amplitude of each mode is smaller in 3D than in 2D. The power spectrum is approximated by a power law.
Fig. 14.—Color contours of the net heating rate in the meridian section for model IV (top panels), model VII (middle panels), and model VI (bottom panels) at different times. All panels have the same spatial scale. [See the electronic edition of the Journal for a color version of this figure.]
for } l \geq 10. \text{ Although the slope is slightly steeper for the nonaxisymmetric models, whether the difference is significant or not is unknown at present.}

We have seen in the explosion models, on the other hand, that the oscillation period of each mode becomes longer in the late nonlinear phase, as the shock radius gets larger and the advection-acoustic cycle becomes longer. What is more interesting is the fact that in this late phase the dominance of low } l \text{ modes becomes even more remarkable. Although this may be related to the volume-filling thermal convection, the mechanism has yet to be revealed. This spectral evolution leads to the global deformation of the shock surface to an ellipsoidal shape, whose major axis is not, however, necessarily aligned with the symmetry axis.}

Finally, we have presented the neutrino heating in 3D SASI. It has been shown that the volume-integrated heating rate is affected mainly in the nonlinear phase. A comparison with symmetric counterparts has confirmed that the SASI also enhances neutrino heating in the nonaxisymmetric case. Although the critical neutrino luminosity in the nonaxisymmetric SASI is not much changed from that for the axisymmetric case, the spatial distribution of neutrino heating is different in the nonlinear phase. Relatively narrow regions surrounding high-entropy blobs are efficiently heated for the nonaxisymmetric case, while wider regions near the symmetry axis are heated strongly, in accord with the sloshing motion of shock wave along the symmetry axis, for the axisymmetric case. For the non-explosion models, the high-entropy blobs produced by neutrino heating occupy the inside of shock wave, repeatedly expanding and contracting and being intermittently split and merged, but the flow finally settles to a quasi-static state. For the explosion models, on the other hand, the high-entropy blobs are generated much more quickly and extend the heating region, pushing the shock outward and narrowing the cooling region near the proto-neutron star.

In this paper we have not discussed the instability mechanism, which is still controversial at the moment. Based on our previous work (Ohnishi et al. 2006), we prefer the advection-acoustic cycle proposed by Foglizzo (2001, 2002) to the purely acoustic cycle discussed by Blondin & Mezzacappa (2006). It is, however, fair to mention that most of our models have ratios of the shock radius to the inner boundary radius for which the recent analysis by Laming (2007) predicts the operation of a pure acoustic cycle. Since his analysis includes some approximations, it is certainly not the final verdict in our opinion, and further investigations are needed.

In the present study we have not observed a persistent segregation of angular momentum in the accretion flow, such as found by Blondin & Mezzacappa (2007) and Blondin & Shaw (2007), although instantaneous spiral flows are frequently seen. As discussed above, equipartition is nearly established among different } m \text{ modes in our models. It should be emphasized here again, however, that we have added the initial perturbations only to the radial velocity in this study, and as a result, modes with } m = \pm 1 \text{ are equally contributing. We defer the analysis of models with initial nonaxisymmetric perturbations added also to the azimuthal component of the velocity to a forthcoming paper (W. Iwakami et al. 2008, in preparation), in which we will also discuss a possible correlation between the kick velocity and spin of neutron stars if they are indeed produced by the 3D SASI. Many questions regarding the SASI still remain to be studied; in particular, the influences of rotation and magnetic field are among the top priorities, and will be addressed elsewhere in the near future.

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APPENDIX A

TENSOR ARTIFICIAL VISCOSITY

Here we present the tensor-type artificial viscosity, } Q \text{, that we use in this paper. It is a direct extension to 3D of that employed by Stone & Norman (1992) for 2D simulations. Following the notations of Stone & Norman (1992), } Q \text{ can be written as}

\[ Q = \begin{cases} 
\frac{l^2}{2} \rho \mathbf{\nabla} \cdot \mathbf{v} \left[ \mathbf{\nabla}_{\mathbf{v}} - \frac{1}{3} (\mathbf{\nabla} \cdot \mathbf{v}) \mathbf{e} \right] & \text{if } \mathbf{\nabla} \cdot \mathbf{v} < 0, \\
0 & \text{otherwise},
\end{cases} \tag{A1} \]
where \( I \) denotes a constant with a dimension of length, \( \nabla \mathbf{v} = (v_i, v_j, v_k) \) is the symmetrized velocity-gradient tensor, and \( \mathbf{e} \) is the unit tensor. Dropping the off-diagonal components, we utilize only the diagonal components of \( \mathbf{Q} \) in this study, which are written in finite difference form as

\[
Q_{11,i,j,k} = l^2_{i,j,k} d_{i,j,k} \left( \nabla \cdot \mathbf{v} \right)_{i,j,k} \left[ (\nabla v_{(11)})_{i,j,k} - \frac{1}{3} (\nabla \cdot \mathbf{v})_{i,j,k} \right], \\
Q_{22,i,j,k} = l^2_{i,j,k} d_{i,j,k} (\nabla \cdot \mathbf{v})_{i,j,k} \left[ (\nabla v_{(22)})_{i,j,k} - \frac{1}{3} (\nabla \cdot \mathbf{v})_{i,j,k} \right], \\
Q_{33,i,j,k} = l^2_{i,j,k} d_{i,j,k} (\nabla \cdot \mathbf{v})_{i,j,k} \left[ (\nabla v_{(33)})_{i,j,k} - \frac{1}{3} (\nabla \cdot \mathbf{v})_{i,j,k} \right],
\] (A2)

where \( d_{i,j,k} \) stands for the density at the site specified by \((i,j,k)\), and \( l_{i,j,k} \) is written as

\[
l_{i,j,k} = \max (C_2 dx_1 a_i, C_2 dx_2 a_j, C_2 dx_3 a_k).
\] (A3)

Here \( C_2 \) is a dimensionless constant controlling the number of grid points, over which a shock is spread, and \( dx_1 a_i, dx_2 a_j, \) and \( dx_3 a_k \) are the grid widths of the “\( a \)-mesh” at the \( i,j \), and \( k \)th grid points in the \( r, \theta, \) and \( \phi \) directions, respectively. The \( a \)-mesh and \( b \)-mesh are distinguished in ZEUS-MP2 and are defined on the cell edge and cell center, respectively (see Stone & Norman 1992 for more details).

The artificial viscous stress \(-\nabla \cdot \mathbf{Q}\) and artificial dissipation \(-\mathbf{Q} : \nabla \mathbf{v}\) in the momentum equation (2) and energy equation (3) are given as

\[
(\nabla \cdot \mathbf{Q})_i = \frac{\partial}{g_2^2 g_{31}} \left( g_2^2 g_{31} (Q_{11})_i \right), \\
(\nabla \cdot \mathbf{Q})_j = \frac{\partial}{g_2^2 g_{31}} \left( g_2^2 g_{31} (Q_{22})_j \right) + \frac{Q_{11}}{g_2 g_{32}} \frac{\partial g_{32}}{\partial x_2}, \\
(\nabla \cdot \mathbf{Q})_k = \frac{\partial Q_{33}}{g_2 g_{32}} \frac{\partial x_3}{\partial x_3},
\] (A4)

\[
\mathbf{Q} : \nabla \mathbf{v} = l^2 \rho \nabla \cdot \mathbf{v} \frac{1}{3} \left[ \left( \nabla v_{(11)} - \nabla v_{(22)} \right)^2 + \left( \nabla v_{(11)} - \nabla v_{(33)} \right)^2 + \left( \nabla v_{(22)} - \nabla v_{(33)} \right)^2 \right],
\] (A5)

where \( g_2 = r, g_{31} = r, \) and \( g_{32} = \sin \theta \) are the metric components for the spherical coordinates, \((x_1, x_2, x_3) = (r, \theta, \phi)\) (see again Stone & Norman 1992). These equations are discretized as

\[
(\nabla \cdot \mathbf{Q})_{1,i,j,k} = \frac{g_2 b_i g_{31} b_j Q_{11,i,j,k} - g_2 b_i g_{32} b_j - g_{31} b_i Q_{11,i-1,j,k}}{g_2^2 g_{31} a_i dx_1 b_i}, \\
(\nabla \cdot \mathbf{Q})_{2,i,j,k} = \frac{g_3 b_i g_{32} b_j Q_{22,i,j,k} - g_2 b_i g_{32} b_j - g_{31} b_j Q_{22,i,j-1,k}}{g_2^2 g_{31} a_j dx_2 b_j} + \frac{Q_{11,i,j,k} + Q_{11,i-1,j,k}}{2 g_2 b_i g_{32} a_j} \frac{\partial g_{32} a_j}{\partial x_2}, \\
(\nabla \cdot \mathbf{Q})_{3,i,j,k} = \frac{Q_{33,i,j,k} - Q_{33,i,j,k-1}}{g_2 b_i g_{32} b_j dx_3 b_k}.
\] (A6)

\[
(\mathbf{Q} : \nabla \mathbf{v})_{i,j,k} = l^2 d_{i,j,k} (\nabla \cdot \mathbf{v})_{i,j,k} \frac{1}{3} \left\{ \left[ (\nabla v_{(11)})_{i,j,k} - (\nabla v_{(22)})_{i,j,k} \right]^2 + \left[ (\nabla v_{(11)})_{i,j,k} - (\nabla v_{(33)})_{i,j,k} \right]^2 + \left[ (\nabla v_{(22)})_{i,j,k} - (\nabla v_{(33)})_{i,j,k} \right]^2 \right\},
\] (A7)

where \( g_2 a_i, g_3 a_j, \) and \( g_3 a_k \) are defined on the \( a \)-mesh, and \( g_2 b_i, g_3 b_j, \) and \( g_3 b_k \) are defined on the \( b \)-mesh. The terms \( dx_1 b_i, dx_2 b_j, \) and \( dx_3 b_k \) represent the width of the \( b \) mesh at the \( i,j \), and \( k \)th grid points in the \( r, \theta, \) and \( \phi \) directions, respectively. Finally, the velocity-gradient tensor \((\nabla \mathbf{v})_{i,j,k} = (\nabla v_{(11)})_{i,j,k}, (\nabla v_{(22)})_{i,j,k}, \) \((\nabla v_{(33)})_{i,j,k}\) are given by

\[
(\nabla v_{(11)})_{i,j,k} = \frac{v_{1,i+1,j,k} - v_{1,i,j,k}}{dx_1 a_i}, \\
(\nabla v_{(22)})_{i,j,k} = \frac{v_{2,i,j+1,k} - v_{2,i,j,k}}{g_2 b_i dx_2 a_j} + \frac{v_{1,i,j,k} + v_{1,i+1,j,k}}{2 g_2 b_i} \frac{\partial g_{32} a_j}{\partial x_2}, \\
(\nabla v_{(33)})_{i,j,k} = \frac{v_{3,i,j,k+1} - v_{3,i,j,k}}{g_2 b_i g_3 b_j dx_3 a_k} + \frac{v_{2,i,j+1,k} + v_{2,i,j+1,k}}{2 g_3 b_i g_3 b_j} \frac{\partial g_{32} a_j}{\partial x_2} + \frac{v_{1,i,j,k} + v_{1,i+1,j,k}}{2 g_3 b_i} \frac{\partial g_{31} b_j}{\partial x_1}.
\] (A8)
In order to see if the numerical resolution employed in the main body is sufficient, we increase the number of angular grid points and compare the results. Figure 16a shows the time evolutions of the normalized amplitudes \[ \frac{|c_l^m/c_0^0|}{t} \] in the linear phase. In this comparison, we impose the \( l = 1, |m| = 1 \) perturbation initially. The models with \( 300 \times 30 \times 60 \) and \( 300 \times 60 \times 120 \) mess points are referred to as MESH0 and MESH1, respectively. (b) Time-averaged power spectra \[ c_l^m/c_0^0 \] The average is taken over \( 150 \leq t \leq 400 \) ms. In this comparison, the random multi-mode perturbation is imposed. [See the electronic edition of the Journal for a color version of this figure.]

**APPENDIX B**

**NUMERICAL CONVERGENCE TESTS**

In order to see if the numerical resolution employed in the main body is sufficient, we increase the number of angular grid points and compare the results. Figure 16a shows the time evolutions of the normalized amplitudes \[ \frac{|c_l^m/c_0^0|}{t} \] in the linear phase. In this comparison, we impose the \( l = 1, |m| = 1 \) perturbation initially. We refer to the model with \( 300 \times 30 \times 60 \) mess points as MESH0, and to that with \( 300 \times 60 \times 120 \) grid points as MESH1 in the figure. We find that the linear growth rates agree with each other reasonably well, although the coarser mesh slightly overestimates the growth time. Figure 16b presents the power spectra \[ c_l^m/c_0^0 \] that are time-averaged over the nonlinear phase. The random perturbation is imposed in this case. It is again clear that the results for MESH0 are in good agreement with those for MESH1.

It should be mentioned that for MESH0 it takes 32 parallel processors about 1.5 days to compute the evolution up to \( t = 400 \) ms, while MESH1 needs 22 days even for 128 parallel processors. This is partly because of the difference in the Courant numbers, which are set to 0.5 for MESH0 but 0.1 for MESH1 to better use the tensor-type artificial viscosity. Although this severe limit of CPU time does not allow us to do more thorough convergence tests, we think, based on the results of the tests shown above, that our results given in this paper are credible.

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