Inelastic nucleon contributions in \((e, e')\) nuclear response functions

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Abstract

We estimate the contribution of inelastic nucleon excitations to the \((e, e')\) inclusive cross section in the CEBAF kinematic range. Calculations are based upon parameterizations of the nucleon structure functions measured at SLAC. Nuclear binding effects are included in a vector-scalar field theory, and are assumed have a minimal effect on the nucleon excitation spectrum. We find that for \(q \lesssim 1\) GeV the elastic and inelastic nucleon contributions to the nuclear response functions are comparable, and can be separated, but with roughly a factor of two uncertainty in the latter from the extrapolation from data. In contrast, for \(q \gtrsim 2\) GeV this uncertainty is greatly reduced but the elastic nucleon contribution is heavily dominated by the inelastic nucleon background.

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1 Introduction

Inelastic scattering of electrons from nuclear targets has long been a tool for the study of nuclear structure. For example, the Coulomb response has been experimentally determined with a view to testing the Coulomb sum rule against models of nuclear structure. Most data and analyses have been obtained for three-momentum transfers \( q < 550 \text{ MeV}/c \),\([1] – [7]\) although some recent work has been done for \( q \sim 1 \text{ GeV}/c \).\([8]\) At the lower \( q \) and energy transfers \( \omega (\sim q) \) the nuclear response involves almost entirely nucleons without internal excitation: the \( \Delta \)-resonance is excited at higher \( \omega \), but contributes mainly to the transverse response, and therefore has less effect on the Coulomb response.

Interest in extending the study of the nuclear \((e, e')\) response to higher \( q \) has developed in recent years, especially with the advent of the Continuous Electron Beam Accelerator Facility (CEBAF), with beam energies of \( \omega \sim 4 \text{ GeV} \) or more. This has led to the study of relativistic effects in the nuclear response which become important for \( q \gtrsim 1 \text{ GeV}/c \).\([9] – [16]\)

But for such momentum transfers the probability of exciting internal states of the nucleons becomes increasingly important. It is the purpose of this paper to make an estimate of the magnitude of the contribution of internal excited states of the nucleon to the nuclear response based on measured values of the inelastic nucleon structure functions and a Fermi gas model of the nuclear target. The contribution of inelastically excited nucleons may be considered the background to the nuclear response function with non-excited (elastic) nucleons; it is the latter which is usually compared to models of nuclear structure. Our estimates are for \( 1 \leq q \leq 4 \text{ GeV}/c \), with the appropriate ranges of \( \omega \).

Electroexcitation of nucleons has been studied at the Stanford Linear Accelerator (SLAC) and elsewhere in this momentum range, but more recently at much higher momenta. We make use of systematic fits\([17, 18]\) to SLAC data, which include some of our range of interest, or lie close enough for extrapolation. We ignore the details of the low energy resonances \([19]\) in the relevant range, which are only approximately included in one of the parameterizations\([17]\) and not at all in the second\([18]\). We concentrate on the magnitude and shape of the background from smoothly varying fits to the nucleon structure functions. Resonances should of course be added, but this will require more complete data separating the longitudinal and transverse contributions.

The paper begins with a review of the basic formalism for \((e, e')\) on complex targets, which defines the structure (or response) functions for nucleon or nuclear targets. In Section 3 we formulate a simple impulse model for the response of a nuclear target based on a Fermi gas of bound nucleons. This follows earlier treatments of Fermi smearing, e.g., by Bodek and Ritchie,\([20]\) but with some detailed differences in the prescription for going off shell. The treatment of binding effects continues in Section 4. The parametric fits to the nucleon
structure functions appear in Section 5. Finally, we present our estimates for the contribution of nucleon inelastic (internally excited) response functions in Section 6, and discuss these results in Section 7.

2 Basic formalism

In this section we summarize the basic formalism for \((e, e')\) scattering from a nuclear target, which may be either a single nucleon or an \(A\)-body nucleus. The following sections give extensions for bound nucleons. In the Born (one-photon exchange) approximation one can write the differential cross section

\[
\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{q^4} \frac{|k'|}{|k|} L_{e}^{\mu\nu} W_{\mu\nu},
\]

where \(L_{e}^{\mu\nu}\) is a lepton tensor describing incoming and outgoing plane-wave electron states, \(k\) and \(k'\) represent the initial and final electron four-momenta, and \(q \equiv k - k'\) is the four-momentum transferred to the target via virtual photon exchange.

Let \(p\) and \(p'\) represent the initial and final four-momenta of the target. In principle the target (response) tensor \(W_{\mu\nu}\) is a function of all three variables \(p\), \(p'\) and \(q\), however, conservation of four-momentum can be used to eliminate any one of these; it is conventional to eliminate \(p'\). The most general form of \(W_{\mu\nu}\) satisfying Lorentz invariance, gauge invariance and parity can then be written\[21\]

\[
W_{\mu\nu}(p, q) = W_1 \left[ -g_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{q^2} \right] + W_2 \left[ \frac{p_{\mu}}{M} - \frac{p \cdot q}{M} \frac{q_{\nu}}{q^2} \right] \left[ \frac{p_{\nu}}{M} - \frac{p \cdot q}{M} \frac{q_{\mu}}{q^2} \right],
\]

where the tensor behavior of \(W_{\mu\nu}\) under Lorentz transformations is described entirely by the quantities in square brackets. The scalar structure functions \(W_1\) and \(W_2\) depend only on scalar combinations of \(p\) and \(q\) (and the target mass \(M\)), from which one can form three scalar combinations: \(p^2\), \(q^2\) and \(p \cdot q\), or equivalently \(Q^2 \equiv -q^2\), \(\nu \equiv p \cdot q / M\) and \(W^2 \equiv (p+q)^2\). For an on-shell target \(p^2 = M^2\) so only two independent variables remain. A conventional choice for nucleons, which we adopt, is \(Q^2\) and \(\nu\).

For a nuclear target at rest in the laboratory frame, inserting (2.2) into (2.1) leads to

\[
\frac{d^2\sigma}{d\Omega dE'} = \frac{d\sigma_{sl}}{d\Omega} \left[ \frac{Q^2}{q^2} W_L^A(\omega, q) + \left( \frac{1}{2} \frac{Q^2}{q^2} + \tan^2 \theta / 2 \right) W_T^A(\omega, q) \right],
\]

where the superscript \(A\) refers to the \(A\)-body nucleus. The longitudinal \((W_L^A)\) and transverse \((W_T^A)\) structure (or response) functions are defined

\[
W_L^A(\omega, q) \equiv \frac{q^2}{Q^2} W_2^A(\omega, q) - W_1^A(\omega, q),
\]

\[3\]
\[ W^A_T(\omega, q) \equiv 2 \times W^A_1(\omega, q). \] (2.5)

Equations (2.3)–(2.5) have been written as functions of the lab variables \( \omega \) (energy transfer) and \( q \) (three-momentum transfer) with \( q^\mu = (\omega, q) \), as is standard for analyzing the nuclear response. In these expressions the target mass \( M \) which appears in (2.2) has dropped out since \( p_0 = M \) for an on-shell target at rest. Note also that in this case \( \nu = \omega \).

Since real photons are purely transverse, \( W^A_L = 0 \) at \( Q^2 = 0 \), i.e., at \( \omega = |q| \). In order to analyze nuclear \((e, e')\) data for the purpose of extracting two-body correlation functions \([15, 16]\) the function \( W^A_L \) must be evaluated over the entire range \( 0 \leq \omega \leq |q| \). Its behavior near \( Q^2 = 0 \) is therefore an important qualitative feature which must be maintained.

### 3 Fermi smearing

We are primarily interested in estimating the contribution of internal excited states of the nucleon to the inelastic nuclear response, compared to the nuclear response in which nucleons are not excited internally. For this comparison we need a model in which we can express the \( A \)-body target tensor \( W^A_{\mu\nu}(\omega, q) \) in terms of \( W^\sigma_{\mu\nu}(p, q) \), the corresponding 1-body tensor for constituent nucleons in the target of isospin projection \( \sigma \). We also need quantitative information on the nucleon structure function, for both elastic and inelastic nucleon kinematics.

Any realistic model which describes an \( A \)-body nucleus in terms of its constituent nucleons must include at least Fermi motion and nuclear binding effects. The simplest model is the plane-wave impulse approximation (PWIA) in which final state interactions are ignored. This should be adequate to provide a reasonable estimate of the inelastic nucleon background. Within the PWIA, West\([22]\) has shown that the \( A \)-body response tensor in the laboratory is given by

\[ W^A_{\mu\nu}(\omega, q) = 2 \sum_\sigma \int \frac{d^3p}{(2\pi)^3} \frac{n_\sigma(p)}{E_p/M} W^\sigma_{\mu\nu}(p, q), \] (3.1)

where \( E_p = \sqrt{p^2 + M^2} \) and \( M \) is the nucleon mass. The momentum distribution \( n_\sigma(p) \) for nucleon species \( \sigma \) is normalized to \( N_\sigma/2 \), where \( N_p = Z \) and \( N_n = N \), and the overall factor of 2 reflects a sum over both spin states. The factor \( E_p/M \) is required to preserve the phase space volume of \( d^3p \) under Lorentz transformations\([23]\).

For a free Fermi gas we would put (2.2) on the right-hand side of (3.1), however, expression (2.2) was derived for on-shell targets and is not strictly valid for bound nucleons. A complete

\[ 1 - n_\sigma(p + q) \]. See \([D3]\) and \([D4]\).
specification of $W_{\mu\nu}^\sigma$ for the general off-shell case is outside the scope of this paper because it requires knowledge of nucleon dynamics in the medium. We will simply assume that (2.2) is a valid starting point for an off-shell extension. Binding effects then enter $W_{\mu\nu}^\sigma$ in two ways: 1) through the initial-state nucleon energy $p_0$ which appears explicitly in the square brackets in (2.2), and 2) implicitly through the scalar functions $W_1^\sigma$ and $W_2^\sigma$. We have already stated that on shell $W_1^\sigma$ and $W_2^\sigma$ are functions only two variables, which we take to be $Q^2$ and $\nu$, and that for a bound nucleon $W_1^\sigma$ and $W_2^\sigma$ can in principle depend on three variables, e.g., $Q^2$, $\nu$ and $W^2$. Since experimental data for $W_{\mu\nu}^\sigma$ is available only on shell, a minimal procedure is to assume that off shell $W_1^\sigma$ and $W_2^\sigma$ are also functions of only two variables. In principle there exists considerable freedom to choose which two, however, in what follows we will show that $Q^2$ and $\nu$ is the natural choice in order to satisfy the requirement that $W_L^A = 0$ at $Q^2 = 0$.

To proceed we insert (2.2) into each side of (3.1). With the z-axis chosen along $q$ and assuming spherical symmetry for $n(p)$, equating tensor components leads to

$$W_1^A(\omega, q) = 2 \sum_\sigma \int \frac{d^3p}{(2\pi)^3} \frac{n_\sigma(p)}{E_p/M} \left[ W_1^\sigma(Q^2, \nu) + \left( \frac{p_\perp}{M} \right)^2 W_2^\sigma(Q^2, \nu) \right],$$

(3.2)

$$W_2^A(\omega, q) = 2 \sum_\sigma \int \frac{d^3p}{(2\pi)^3} \frac{n_\sigma(p)}{E_p/M} \left[ \left( 1 + \frac{p_\perp}{M} \frac{Q^2}{|q|} \right)^2 \left( \frac{\nu}{\omega} \right)^2 + \left( \frac{p_\perp}{M} \right)^2 Q^2 \right] W_2^\sigma(Q^2, \nu).$$

(3.3)

Inserting (3.2) and (3.3) into (2.4) and (2.5) leads to

$$W_L^A(\omega, q) = 2 \sum_\sigma \int \frac{d^3p}{(2\pi)^3} \frac{n_\sigma(p)}{E_p/M} \left[ \frac{Q^2}{Q^2} \left( 1 + \frac{p_\perp}{M} \frac{Q^2}{|q|} \right)^2 \left( \frac{\nu}{\omega} \right)^2 W_2^\sigma(Q^2, \nu) - W_1^\sigma(Q^2, \nu) \right],$$

(3.4)

$$W_T^A(\omega, q) = 2 \times 2 \sum_\sigma \int \frac{d^3p}{(2\pi)^3} \frac{n_\sigma(p)}{E_p/M} \left[ W_1^\sigma(Q^2, \nu) + \left( \frac{p_\perp}{M} \right)^2 W_2^\sigma(Q^2, \nu) \right].$$

(3.5)

From (3.4) it is easily verified that the condition $W_L^A = 0$ at $Q^2 = 0$ will be satisfied if

$$\lim_{Q^2 \to 0} \left[ \frac{\nu^2}{Q^2} W_2^\sigma(Q^2, \nu) - W_1^\sigma(Q^2, \nu) \right] = 0.$$  

(3.6)

To understand the implications of (3.6) we first observe that the factor $\nu^2/Q^2$ originates from the $p \cdot q$ terms in (2.2). Although the choice of variables on which $W_1^\sigma$ and $W_2^\sigma$ depend off shell is in principle arbitrary, this work is based on the assumptions that 1) $W_1^\sigma$ and $W_2^\sigma$ depend on only two variables, and 2) $W_1^\sigma$ and $W_2^\sigma$ depend on the same two variables.

2Technically speaking, the proton tensor $W_p^{\mu\nu}$ is measured on shell by scattering from free protons, while the neutron tensor $W_n^{\mu\nu}$ is determined by scattering from deuterons and extracting neutron contributions using theoretical arguments.
Consequently (3.6) will be satisfied only if $W_1^\sigma$ and $W_2^\sigma$ are chosen to depend on $Q^2$ and $\nu$ off shell. In contrast Bodek and Ritchie\cite{20}, in a study of Fermi smearing at high $Q^2$, assumed $W_1^\sigma$ and $W_2^\sigma$ to be functions of $Q^2$ and $W$ off shell, which is equivalent to choosing $Q^2$ and $\nu$ and evaluating $\nu$ at the point $\nu_W \equiv (W^2 - M^2 + Q^2)/2M$. Since off shell $\nu_W \neq \nu$ in general, this prescription will not satisfy (3.6) and implies $W_A^\sigma \neq 0$ at $Q^2 = 0$. We therefore believe that their prescription is not useful when seeking an off-shell extrapolation valid at low $Q^2$.

(Except for the region $Q^2 \simeq 0$, the effect of the two choices differs little.)

When considering inelastic nucleons on shell it is conventional to express $W_1^\sigma(Q^2, \nu)$ in terms of $W_2^\sigma(Q^2, \nu)$ and a third function $R_\sigma(Q^2, \nu)$, which is proportional to the ratio $W_L^\sigma/W_T^\sigma$:

$$W_1^\sigma(Q^2, \nu) = \frac{1 + \nu^2/Q^2}{1 + R_\sigma(Q^2, \nu)} W_2^\sigma(Q^2, \nu) .$$

(3.7)

With the above assumptions for $W_1^\sigma$ and $W_2^\sigma$, inserting (3.2)–(3.7) into (2.4) and (2.5) gives

$$W_A^L(\omega, q) = 2 \sum_\sigma \int \frac{d^3p}{(2\pi)^3} \frac{n_\sigma(p)}{E_p/M} \left[ \frac{q^2}{Q^2} \left( \frac{1 + p_z Q^2}{M |q| \nu} \right)^2 \left( \frac{\nu}{\omega} \right)^2 - \frac{1 + \nu^2/Q^2}{1 + R_\sigma(Q^2, \nu)} \right] W_2^\sigma(Q^2, \nu) ,$$

(3.8)

$$W_A^T(\omega, q) = 2 \times 2 \sum_\sigma \int \frac{d^3p}{(2\pi)^3} \frac{n_\sigma(p)}{E_p/M} \left[ \frac{1 + \nu^2/Q^2}{1 + R_\sigma(Q^2, \nu)} + \left( \frac{p_x}{M} \right)^2 \right] W_2^\sigma(Q^2, \nu) .$$

(3.9)

From (3.7) we see that in order to satisfy (3.6) we must have $R_\sigma(Q^2, \nu) = 0$ at $Q^2 = 0$. In Section 5 we provide a parameterized form for $R_\sigma(Q^2, \nu)$ which has this property.

### 4 Spectator models of nuclear binding

In order to evaluate (3.8) and (3.9) for bound nucleons we must specify a particular model which determines the off-shell kinematics. In conventional terminology, a “spectator” model specifies the energy $p_0$ of a bound nucleon in terms of its three-momentum $p$, which is related to the excitation energy of the recoiling residual $(A-1)$-body target nucleus (spectator). This in turn determines the off-shell value of the variable $\nu = p \cdot q/M$. As in the PWIA of (3.1), a spectator model is based on the assumption that nuclear interactions enter only the initial state; the final excited nucleon state is free. In this section we discuss three such models for inelastic nucleon response.

Bodek and Ritchie\cite{20} have investigated the effects of Fermi motion and nuclear binding beginning with (3.1). In that work they assumed a nucleon energy of the form
\[ p_0 = M_A - \sqrt{p^2 + M_{A-1}^2}, \]  

(4.1)

where \( M_A = A(M - E/A) \) is the mass of the \( A \)-body target nucleus and \( M_{A-1} = M_A - M \) is the mass of the recoiling spectator nucleus after nucleon knockout. This form is a direct generalization of the expression for dissociation of the deuteron, as given by Atwood and West[23], and corresponds roughly to the “separation” energy, i.e., the energy required to remove a particle from the least bound state in the target. Compared to the free nucleon energy \( p_0 = E_p \), which is bounded by \( M \leq E_p \leq M + 34.5 \) MeV on the range \( 0 \leq |p| \leq p_F \), (4.1) has relatively weak dependence on \( p \), being bounded by \( M \leq p_0 \leq M - 0.6 \) MeV over the same range. Expression (4.1) has two peculiar features when applied to a many-body system: 1) \( p_0 \) decreases with increasing \( p \), and 2) \( p_0 \) includes essentially no binding effects, since \( p_0 = M \) at \( p = 0 \) and for \( p \neq 0 \) the binding energy \( E/A \) enters only negligibly. Furthermore, because (4.1) corresponds to the separation energy it does not account for the possibility that the ejected nucleon was initially deeply bound.

A simple modification of (4.1) is to account for the variation of \( p_0 \) with \( p \) using a self-consistent potential model. We employ a relativistic mean field model based on quantum hadrodynamics (QHD) [24], a relativistic quantum field theory of hadronic matter with Lorentz vector and scalar meson interactions. In this model the energy of a bound nucleon is given by

\[ p_0 = V_0 + \sqrt{p^2 + M^*}, \]  

(4.2)

where \( M^* \equiv M + S \) is the effective nucleon mass, \( S < 0 \) represents an attractive scalar field and \( V_0 > 0 \) represents a repulsive, time-like vector field. The binding effects are much greater than in (4.1). The energy \( p_0 \) in (4.2) is bounded by \( M - 76 \) MeV \( \leq p_0 \leq M - 24 \) MeV over the range \( 0 \leq |p| \leq p_F \), i.e., nucleons in the Fermi sea are bound by approximately 50 MeV. Furthermore, \( p_0 \) has the expected behavior, i.e., increases with increasing \( p \), and correctly accounts for the average binding energy for nucleons bound deeply in the Fermi sea. In this sense (4.2) corresponds to the “removal” energy from occupied orbitals in the target.

Simply using (4.2) to evaluate \( \nu \) in (3.8) and (3.9) is not consistent, however, because (2.2) was derived for a nucleon of mass \( M \). In the mean field QHD theory, a bound nucleon acquires an effective mass \( M^* \), on which its Lorentz transformation properties are based. For a nucleon of mass \( M^* \) (2.2) must be replaced by

\[ W_{\mu
u}^\sigma(p, q) = W_1^\sigma\left[-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2}\right] + W_2^\sigma\left[\frac{p_{\mu}^*}{M^*} - \frac{p_{\mu}^* \cdot q q_{\mu}}{M^* q^2}\right] \left[\frac{p_{\nu}^*}{M^*} - \frac{p_{\nu}^* \cdot q q_{\nu}}{M^* q^2}\right], \]  

(4.3)
where \( p^* \equiv (E_p^*, p) \) and \( E_p^* \equiv \sqrt{p^2 + M^2} \). (Notice that the vector field \( V_0 \) does not enter the definition of \( p^* \), just as it does not enter a spinor \( u_s(p) \) describing a Dirac plane-wave.) It is then natural to introduce the variable \( \nu^* \equiv p^* \cdot q/M^* \). Repeating the arguments of Section 3, we assume that off-shell \( W_1^\sigma \) and \( W_2^\sigma \) are given by the on-shell functions evaluated at \( Q^2 \) and \( \nu^* \). This leads to the following expressions, which replace (3.8) and (3.9):

\[
W_A^A(\omega, q) = 2 \sum_\sigma \int \frac{d^3p}{(2\pi)^3} \frac{n_\sigma(p)}{E_p^*/M^*} \left[ \frac{Q^2}{Q^2} \left( 1 + \frac{p_+ Q^2}{q|\nu^*|} \right)^2 \left( \frac{\nu^*}{\omega} \right)^2 - \frac{1 + \nu^*|q^2/Q^2|}{1 + R_\sigma(Q^2, \nu^*)} \right] W_2^\sigma(Q^2, \nu^*),
\]

\[
W_T^A(\omega, q) = 2 \times 2 \sum_\sigma \int \frac{d^3p}{(2\pi)^3} \frac{n_\sigma(p)}{E_p^*/M^*} \left[ \frac{1 + \nu^*|q^2/Q^2|}{1 + R_\sigma(Q^2, \nu^*)} + \left( \frac{p_+}{M^*} \right)^2 \right] W_2^\sigma(Q^2, \nu^*).
\]

These are the primary formulae used in our calculations of the inelastic nucleon background. These expressions are also used to compute the elastic nucleon response, i.e., quasielastic peak, for a bound Fermi gas. Numerical results, along with essential differences in interpretation, are given in Section 6.

5 Nucleon structure functions

In this section we describe the forms of the nucleon structure functions \( W_\mu^\nu \) used in our calculations of the inelastic nucleon background. We rely on two parametric fits [17, 18] which characterize \((e, e')\) data obtained over a large kinematic range at SLAC, but which unfortunately do not cover all of the lower energy and momentum transfers available at CEBAF. We therefore must extrapolate these fits to this lower range, which introduces some uncertainty into the resulting estimates, as we shall see by comparing the results for different fits.

The two experimental structure functions are often presented in terms of \( W_2^\sigma \) and \( R_\sigma \), as in (3.7), and given in terms of the invariant variables \( Q^2 \) and \( \nu \), or \( Q^2 \) and \( x \equiv Q^2/2M\nu \); the latter choice is of interest for describing scaling behavior at high \( x \). Data for the proton functions comes directly from \(^1\text{H}(e, e')X\), while the neutron functions must be extracted from \(^2\text{H}(e, e')X\) data using theoretical assumptions about the momentum distribution and binding effects in the deuteron.

Bodek et. al. [17] fit data for \( W_2^\sigma \) in the kinematic range \( 1.0 \leq Q^2 \leq 20 \text{ GeV}^2 \) and \( 0.1 \leq x \leq 0.77 \). They assume constant \( R_p = R_n = 0.18 \), which allows (2.3) to be expressed entirely in terms of \( W_2^\sigma \). Following a treatment developed by West [22, 23], they use deuteron wavefunctions to account for Fermi smearing and binding effects in extracting the neutron.
structure function. The parameterization for both proton and neutron structure functions is then given in the form

$$W^\sigma_2(Q^2, \nu) = B(Q^2, W) \tilde{W}^\sigma_2(Q^2, \nu), \quad (5.1)$$

where $B(Q^2, W)$ is a modulating function for low energy excitations including threshold and resonant behavior, as a function of final invariant mass $W = (M^2 + 2M\nu - Q^2)^{1/2}$. We eliminate the resonant terms from this function since they do not appreciably modify the estimates, and since the value of $R$ is expected to vary from resonance to resonance, contrary to the assumption of constant $R_p = R_n$. Without resonances, $B(Q^2, W)$ vanishes below the $\pi$-production threshold $W \approx 1.07$ GeV, increases with increasing $W$ to a second threshold near $W \approx 1.74$ GeV, and reaches unity for $W > \sim 2M$; the functional form of $B(Q^2, W)$ is given in Appendix A. The function $\tilde{W}^\sigma_2$ in (5.1) is a smoothly varying function of $Q^2$ and $\nu$, and is also given in Appendix A. Its functional form was chosen to illustrate the behavior of $W^\sigma_2(Q^2, \nu)$ in a certain scaling variable (see Ref. [17]).

Whitlow [18] fits $W^\sigma_2$ in the kinematic range $0.6 \leq Q^2 \leq 30$ GeV$^2$ and $0.06 \leq x \leq 0.9$, using all SLAC data available at the time of publication. He first extracts $R_\sigma(Q^2, x)$ using only data for which $W > 2$ GeV to exclude the resonance region. He finds $R_p = R_d$, and therefore $R_p = R_n$, to within experimental errors ($\sim 5\%$), in good agreement with theoretical predictions based on QCD [26]. However, noting that QCD predictions [26, 27] systematically underestimate the data, he provides a parameterization of the experimentally determined $R_\sigma$ in the form

$$R_\sigma(Q^2, x) = \frac{1}{3} \left[ R_a(Q^2, x) + R_b(Q^2, x) + R_c(Q^2, x) \right], \quad \text{for } Q^2 \geq 0.3, \quad (5.2)$$

where $R_a$, $R_b$ and $R_c$ are given in Appendix B.

For fixed $x$ and $Q^2 \gg 1$ GeV$^2$, the experimental data indicates that $R_\sigma \approx 0.2$ and decreases gradually as $Q^2$ increases. For $Q^2 \ll 1$ GeV$^2$, $R_\sigma$ decreases rapidly and tends to zero, as required to ensure $W_L = 0$ at $Q^2 = 0$. In contrast, if (5.2) is evaluated for $Q^2 < 0.3$ GeV$^2$ it will diverge as $Q^2 \to 0$. To make reasonable estimates, therefore, we extrapolate $R_\sigma(Q^2, x)$ down to $Q^2 = 0$ by assuming a simple analytic form which behaves properly at all $Q^2$. A theoretically motivated form [28] is

$$R_\sigma(Q^2, x) = \frac{a_R(x) Q^2}{b_R(x) + Q^4}, \quad \text{for } Q^2 < 0.3, \quad (5.3)$$

where $a_R(x)$ and $b_R(x)$ are chosen such that (5.3) smoothly matches (5.2) at $Q^2 = 0.3$ GeV$^2$. We have verified that the results in Section 6 are insensitive to the detailed form of this continuation, provided it matches (5.2) and tends smoothly to zero at $Q^2 = 0$.

The proton structure function $W^p_2$ is parameterized in the form
\[ W^p_2(Q^2, x) = \frac{F^{th}(x)}{Q^2/2Mx} \left[ 1 + \lambda_1(x) \ln \frac{Q^2}{A(x)} + \lambda_2(x) \ln^2 \frac{Q^2}{A(x)} \right], \quad (5.4) \]

where explicit forms for the functions appearing here are given in Appendix C. The dominant behavior of (5.4) is determined by the function \( F^{th}(x) \), which is a power series in \((1 - x)\) and therefore tends to zero at the elastic nucleon threshold at \( x = 1 \). The neutron structure function is parameterized in terms of the ratio

\[ \frac{W^n_2}{W^p_2} \approx \left( \frac{W^n_2}{W^p_2} \right)_S = P_1(x) + P_2(x) \left[ \ln Q^2 \right], \quad (5.5) \]

where the subscript \( S \) refers to Fermi smearing effects in the deuteron, which Whitlow points out are negligible for \( x < 0.5 \). In this region the data looks almost linear in \( x \), although strong curvature is apparent for \( 0.5 < x < 0.7 \). This curvature is most likely an artifact of Fermi smearing in the deuteron. Whitlow gives two different parameterizations of the functions \( P_1(x) \) and \( P_2(x) \), one essentially linear in \( x \) and one which shows some curvature corresponding to this Fermi motion effect. Bodek et. al. [17] have extracted the non-smeared ratio \( W^n_2/W^p_2 \) and find linear \( x \)-dependence over their whole range of data. For the purpose of our estimates, therefore, we choose the more linear of Whitlow’s two parameterizations even though it was fit only for \( x < 0.5 \).

Before proceeding to our estimate of the inelastic nucleon background, it is useful to point out some general features of the fits to \( W^p_2 \), since these largely determine the behavior of the nuclear response functions (4.4) and (4.5). Although data for \( W^p_2 \) are shown in the original references, they are typically shown at higher energies and in terms of invariant variables, e.g., \( Q^2 \) and \( x \). Figures 1–3 show \( W^p_2 \) and \( W^n_2 \) in laboratory variables \( \omega \) and \( |q| \), for free nucleons at rest and for three-momentum transfers \( |q| = 1, 2 \) and 4 GeV/c. The energy range shown is \( E_q - M \leq \omega \leq |q| \), i.e., from the position of the elastic nucleon peak to the absolute upper limit attainable by electron scattering.

We first focus on protons. At these relatively low momenta, the fit by Bodek et. al. (B) is significantly smaller than that by Whitlow (\( \Lambda_{12} \)), with nearly a factor of two difference for \( |q| = 1 \) GeV/c. This difference decreases with increasing \( |q| \), and by \( |q| = 4 \) GeV/c the two fits are nearly indistinguishable. At \( |q| = 2 \) GeV/c, B shows the second (higher) threshold behavior of (A3), while at \( |q| = 1 \) GeV/c the values of the final invariant mass \( W \) are such that all \( \omega \) lie entirely below the second threshold. The behavior of B and \( \Lambda_{12} \) at the endpoints differs qualitatively. At the lower limit, i.e., the elastic threshold, \( \Lambda_{12} \) vanishes at \( \omega = \sqrt{q^2 + M^2} - M \) (see discussion following (5.4)), while B vanishes at the first inelastic threshold \( \omega = \sqrt{q^2 + W^2} - M \) with \( W \approx 1.07 \) GeV/c. At the upper limit B vanishes precisely at \( \omega = |q| \), while the logarithmic behavior of \( \Lambda_{12} \) in \( Q^2 \) must be cut
off to allow only positive values, causing $W_p^2$ to vanish slightly prior to $\omega = |q|$. Generally speaking, the fits to $W_n^2$ follow similar trends, although with different numerical values. For both fits the neutron-to-proton ratio $W_n^2/W_p^2 < 1$. Compared to the B fit, this ratio in the $\Lambda_{12}$ fit, i.e., that given by (5.3) is $\sim 40\%$ smaller at the lower limit and $\sim 10\%$ larger at the upper limit for $|q| = 1 \text{ GeV/c}$, and is only $\sim 25\%$ smaller at the lower limit and $\sim 5\%$ larger at the upper limit for $|q| = 2 \text{ GeV/c}$. In Fig. 1, the difference in shape between $W_n^2$ and $W_p^2$ in the $\Lambda_{12}$ fit is reflective of the fact that at lower $|q|$ the slope of the ratio $W_n^2/W_p^2$ is greater than that in the B fit.

6 Numerical results

In this section we present numerical results for the inelastic nucleon background obtained by using (4.4) and (4.5) with the fits to $W_\sigma^2$ and $R_\sigma$ given in Section 5. We assume a simple Fermi gas momentum distribution $n_\sigma(p) = \theta(p_F - |p|)$ with $p_F = 0.257 \text{ fm}^{-1}$, corresponding roughly to $^{56}\text{Fe}$. The free nucleon mass $M = 938.92 \text{ MeV}$. To account for nuclear binding we use a reduced mass $M^*/M = 0.648$, corresponding to the mean field theory of Ref. [24] for the same value of $p_F$. This value of $M^*$ represents the smallest reduced mass which can reasonably be expected in a real nucleus, so that comparing this result with that for free nucleons demonstrates the range of results which can be expected in this model. In all cases where $W$ and $x$ must be replaced by the variables $Q^2$ and $\nu$ in the functions $W_\sigma^2$ and $R_\sigma$, as mentioned in Section 5, these replacements have been made in the on-shell functions before letting $\nu \to \nu^*$. 

In what follows the inelastic background will be shown compared to the corresponding quasielastic peak at the same three-momentum transfer, whose calculation for free nucleons is summarized in Appendix D. To include the effects of nuclear binding on the quasielastic peak we follow the theoretical treatment of Ref. [16], which is described at the end of Appendix D. This leads to the standard expressions for quasielastic scattering from a Fermi gas composed of nucleons of mass $M^*$, for which binding effects enter both initial and final nucleon states. Note that this differs from the spectator models described in Section 4, for which binding effects enter final states only indirectly, since there is assumed to be no final state interaction of the inelastically excited nucleon. Therefore, the comparison between the elastic and inelastic nucleon contributions computed here is given only to provide a reasonable estimate of their relative sizes and positions.

Figure 4 shows the longitudinal (a) and transverse (b) nuclear response functions at three-momentum transfer $|q| = 1 \text{ GeV/c}$. Thin curves are for free nucleons with mass $M$ and thick curves are for bound nucleons with mass $M^*$. As seen in Fig. 1, at this three-momentum the fit by Bodek et. al. (B) implies a background roughly half as large as that
implied by Whitlow’s fit ($\Lambda_{12}$). (Note that the energy range extends slightly lower than that in Fig. 1 to accommodate Fermi broadening.) As in Fig. 1, $B$ vanishes at exactly $\omega = |q|$, while $\Lambda_{12}$ vanishes at slightly lower $\omega$ due to the logarithmic behavior of (5.4). Note that although the inelastic nucleon results derived from $B$ and $\Lambda_{12}$ differ, they are of the same magnitude as the quasielastic peak and displaced sufficiently that respective peaks appear at distinct energies. It therefore seems possible to effect a separation of elastic and inelastic nucleon contributions in this momentum range using these data fits, albeit with substantial errors reflecting the difference between the fits.

The reduced mass $M^*$ affects the quasielastic peak much more than the inelastic background. To understand this result we first note that Fermi motion is treated identically in the two calculations, and that in (4.3) the effective mass always enters the coefficients of $W_1^\sigma$ and $W_2^\sigma$ through the ratio $p_0^*/M^* = E_0^*/M^* \sim 1$. Thus differences in sensitivity to the value of $M^*$ originate in the nucleon structure functions $W_1^\sigma$ and $W_2^\sigma$. We have chosen to evaluate the inelastic nucleon structure functions in terms of $Q^2$ and $\nu^* \sim \omega$, hence the inelastic nucleon background is not strongly sensitive to the value of $M^*$. It is in this sense that our background calculation includes binding effects only indirectly in the final state. In contrast, in order to match standard treatments the elastic nucleon structure functions (D1) and (D2) have explicit factors of $\tau^* \equiv Q^2/4M^{*2}$, which scales quadratically with the reduced mass $M^*$. Hence the size and location of the quasielastic peak is more sensitive to the value of $M^*$ than is the inelastic nucleon background. This is clearly an artifact of the particular model we have used for the quasielastic peak, and would change with any modification of the theory for either elastic or inelastic nucleons. Such changes are not of interest here, since our main goal is to provide a reasonable estimate the inelastic background and determine how reliably these parameterizations of the SLAC data can be extended to the CEBAF kinematic range.

Figure 5 shows the longitudinal (a) and transverse (b) nuclear response functions at three-momentum transfer $|q| = 2 \text{ GeV}/c$. The curve labeling is the same as in Fig. 4. The threshold behavior of $B$, which can be seen in Fig. 2, is not visible in Fig. 5 because of Fermi smearing. The most notable difference with Fig. 4 is that the inelastic background is much larger compared to the quasielastic peak. The scales of the plots show that this is due mainly to the rapid decrease of the dipole form factor (D7), and due to a lesser extent to a slight increase in the inelastic background. Thus separating elastic and inelastic nucleon contributions from experimental data will be much more challenging for $|q| \gtrsim 2 \text{ GeV}/c$ – an effect which has long been recognized but not quantitively investigated. However, this difficulty is partly compensated by the fact that, as noted in Fig. 2, the two inelastic fits $B$ and $\Lambda_{12}$ are in much closer agreement, and therefore the inelastic background can be evaluated with more confidence. Results for $|q| = 4 \text{ GeV}/c$ are not shown because by this
point the dipole form factor has totally suppressed the elastic peaks relative to the inelastic background.

7 Summary and conclusions

The study of nuclear structure by \((e, e')\) reactions at a few GeV, the CEBAF kinematic range, requires the removal of the background resulting from inelastic excitation of single nucleons. In this paper we provide an estimate of this background based on two different parameterizations of inelastic nucleon structure functions measured at SLAC. We then compare the sizes and positions of the inelastic nucleon background to those of the quasielastic peak, which represents the dominant contribution of elastic nucleons to inelastic nuclear excitation. We assess how confidently the available SLAC fits can be applied in the CEBAF kinematic range from the difference between the results for each fit over that kinematic range.

We make a number of assumptions which allow us to relate inelastic nucleon excitations in the nuclear target to those of free nucleons; these all involve allowing only “minimal” effects of the nucleus on internal nucleon excitations. First we use the PWIA in (3.1), which introduces Fermi motion but does not include final state interactions for the excited nucleon. Second we assume that the nucleon response tensor \(W_{\sigma uv}^\sigma\) for a bound nucleon, given in (1.3) is of the same tensor form as that for a free nucleon, given in (2.2), but modified for a nucleon with effective mass \(M^*\). Third we assume that the nucleon structure functions \(W_{\sigma 1}^\sigma\) and \(W_{\sigma 2}^\sigma\) for bound nucleons are equal to the free-nucleon structure functions evaluated at the same value of \(Q^2\) and at \(\nu = \nu^*\) (see discussion leading to (4.4) and (4.5)). Thus binding effects enter the enter final states in (4.3) only indirectly, which is consistent with ignoring final state interactions in (3.1). This rule for off-shell extrapolation has good behavior at \(Q^2 = 0\), as seen in (3.6).

Our calculation of the inelastic response functions uses parameters based on the structure of \(^{56}\text{Fe}\), but with a sharp Fermi distribution. There is little sensitivity of the inelastic nucleon background to the choice of \(M^*\), which enters only minimally. In contrast, our model for the quasielastic peak, which does have final state interactions, is more sensitive to \(M^*\), but this does not significantly affect our comparison of the relative size and position of the two contributions. We find that for \(|q| \lesssim 1\ \text{GeV/c}\) the elastic and inelastic nucleon contributions are comparable in size, and can be separated, but in the inelastic nucleon response there is an uncertainty of roughly a factor of two resulting from disagreement between the SLAC fits in this kinematic range. For \(|q| \gtrsim 2\ \text{GeV/c}\) improved agreement between the two extrapolations raises the certainty of the background calculation, but the momentum dependence of the Sachs form factors quenches the elastic nucleon contribution with increasing \(|q|\).

Our estimates of the inelastic nucleon background show that with the presently avail-
able SLAC fits it is feasible to extract nuclear information from nuclear \((e,e')\) data for \(|q| \lesssim 1\ \text{GeV/c}\). However, the present accuracy does not make this sufficiently useful, since many interesting effects (e.g., of correlations) make rather small contributions to the response functions. What is needed is more complete data and analysis on nucleon structure functions at lower \(Q^2\), with full separation of \(W_1^\sigma\) and \(W_2^\sigma\), i.e., \(R_\sigma\). Clearly resonances are important in this kinematic region, and should be included in the inelastic nucleon background. In contrast, for \(|q| \gtrsim 2\ \text{GeV/c}\) separating the inelastic nucleon background can be done with more confidence, but the momentum dependence of the Sachs form factors makes identification of the elastic nucleon response difficult. It seems that this situation can not be improved by making more precise measurements of the inelastic nucleon response functions, since the existing fits are already in good agreement in this kinematic range. For some purposes, the use of \((e,e'p)\) and \((e,e'n)\) experiments would reduce the problem of the background, but these reactions are usually less complete kinematically than \((e,e')\), and are more sensitive to final state interactions. These features limit their usefulness in sum rule studies.

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Appendix A: Fit to \(W_2^\sigma(Q^2,\nu)\) by Bodek et. al.

In Ref. [17] the proton and neutron structure functions are parameterized in the form

\[
\bar{W}_2^\sigma(Q^2,\nu) = \left[ \frac{Q^2}{2M\nu} \frac{2M\nu + c_1}{Q^2 + c_2} \right] \sum_{n=3}^{7} c_{n\sigma} \left[ 1 - \frac{Q^2 + c_2}{2M\nu + c_1} \right]^n,
\]

where \(c_1 = +1.6421\) and \(c_2 = +0.3764\). The remaining parameters \(c_{n\sigma}\) are listed in Table 1.

| \(c_{n\sigma}\) | \(\sigma = p\)  | \(\sigma = n\)  |
|----------------|----------------|----------------|
| \(c_{3\sigma}\) | +0.2562        | +0.0640        |
| \(c_{4\sigma}\) | +2.1785        | +0.2254        |
| \(c_{5\sigma}\) | +0.8978        | +4.1062        |
| \(c_{6\sigma}\) | −6.7162        | −7.0786        |
| \(c_{7\sigma}\) | +3.7557        | +3.0549        |
The modulating function $B(Q^2, W)$ is given by

$$B(Q^2, W) = \tilde{B}(W) \left[ 1 + \left( 1 - \tilde{B}(W) \right) \left( b_6 + b_7(x - b_8)^2 \right) \right], \quad (A2)$$

where $x = Q^2/(W^2 - M^2 + Q^2)$ and $\tilde{B}(W)$ is defined

$$\tilde{B}(W) = \theta(W - b_1) b_2 \left[ 1 - e^{-b_3(W - b_1)} \right] + \theta(W - b_4) (1 - b_2) \left[ 1 - e^{-b_5(W^2 - b_6)} \right]. \quad (A3)$$

The coefficients $b_i$ are listed in Table 2.

| $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | $b_6$ | $b_7$ | $b_8$ |
|-------|-------|-------|-------|-------|-------|-------|-------|
| +1.0741 | -0.7553 | +3.3506 | +1.7447 | +3.5102 | -0.5999 | +4.7616 | +0.4117 |

Table 2. Numerical parameters for $B(Q^2, W)$, supplied by Bodek [23].

Appendix B: Fit to $R(Q^2, x)$ by Whitlow

The functions appearing in (B.2) are given by

$$R_a(Q^2, x) = \frac{a_1}{\ln(Q^2/0.04)} \Theta(Q^2, x) + \frac{a_2}{\left( Q^8 + a_3 \right)^{1/4}}, \quad (B1)$$

$$R_b(Q^2, x) = \frac{b_1}{\ln(Q^2/0.04)} \Theta(Q^2, x) + \frac{b_2}{Q^2} + \frac{b_3}{\left( Q^4 + 0.3^2 \right)}, \quad (B2)$$

$$R_c(Q^2, x) = \frac{c_1}{\ln(Q^2/0.04)} \Theta(Q^2, x) + \frac{c_2}{\sqrt{(Q^2 - 5(1 - x)^5)^2 + c_3^2}}, \quad (B3)$$

where

$$\Theta(Q^2, x) = 1 + 12 \left( \frac{Q^2}{Q^2 + 1} \right) \left( \frac{0.125^2}{x^2 + 0.125^2} \right), \quad (B4)$$

and the coefficients appearing in (B1)–(B3) are listed in Table 3.

| $a_1$ | $a_2$ | $a_3$ | $b_1$ | $b_2$ | $b_3$ | $c_1$ | $c_2$ | $c_3$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| +0.0672 | +0.4671 | +1.8979 | +0.0635 | +0.5747 | -0.3534 | +0.0599 | +0.5088 | +2.1081 |
Table 3. Numerical parameters for $R(Q^2, x)$, taken from Whitlow [18].

Appendix C: Fit to $W_2^\sigma(Q^2, x)$ by Whitlow

The functions appearing in (5.4) are given by

$$F_{2}^{\text{thr}}(Q^2, x) = \sum_{i=1}^{5} d_i \ (1 - x)^{i+2}, \quad (C1)$$

$$\lambda_1(x) = \sum_{i=0}^{3} d_{i+9} \ x^i, \quad (C2)$$

$$\lambda_2(x) = \begin{cases} \sum_{i=0}^{2} d_{i+6} \ x^i & \text{if } Q^2 < A(x), \\ 0 & \text{otherwise}, \end{cases} \quad (C3)$$

$$A(x) = 1.22 \ e^{3.2x}, \quad (C4)$$

where the coefficients $d_i$ are listed in Table 4.

| $d_1$  | $d_2$  | $d_3$  | $d_4$  | $d_5$  | $d_6$  | $d_7$  | $d_8$  | $d_9$  | $d_{10}$ | $d_{11}$ | $d_{12}$ |
|-------|-------|--------|-------|-------|-------|-------|-------|-------|----------|----------|----------|
| +1.417 | -0.108 | +1.486 | -5.979 | +3.524 | -0.011 | -0.619 | +1.385 | +0.270 | -2.179 | +4.722 | -4.363 |

Table 4. Parameters for Whitlow’s $\Lambda_{12}$ fit to $W_2^\sigma(Q^2, x)$, taken from Ref. [18].

The functions $P_{1}(x)$ and $P_{2}(x)$ appearing in (5.5) are given by

$$P_{1}(x) \simeq 0.9498 - 0.9706 \ x + 0.3102 \ x^2, \quad (C5)$$

$$P_{2}(x) \simeq -0.0146, \quad (C6)$$

where the numerical values are taken from Ref. [18].

Appendix D: Elastic nucleon structure functions

The elastic structure functions for free nucleons may be written

$$W_1^\sigma(Q^2, \nu) = \tau \ G_{M\sigma}^2(Q^2) \ \delta\left(\nu - \frac{Q^2}{2M}\right), \quad (D1)$$
\[ W_2^\sigma(Q^2, \nu) = \frac{G_{E\sigma}(Q^2) + \tau G_{M\sigma}(Q^2)}{1 + \tau} \delta\left(\nu - \frac{Q^2}{2M}\right), \]  
(D2)

where \( G_{E\sigma}(Q^2) \) and \( G_{M\sigma}(Q^2) \) are the Sachs electric and magnetic form factors, and \( \tau \equiv Q^2/4M^2 \). Inserting (D1) and (D2) into (3.4) and (3.5) leads to

\[ W^A_L(\omega, q) = \frac{Q^2}{q^2} \times 2 \sum_\sigma \int \frac{d^3p}{(2\pi)^3} f_\sigma(p) \frac{L_{00}\sigma(p, q; Q^2)}{4E_pE_{p+q}} \delta(\omega + E_p - E_{p+q}), \]  
(D3)

where

\[ L_{00}\sigma(p, q; Q^2) = \frac{G_{E\sigma}(Q^2)}{1 + \tau} (E_p + E_{p+q})^2 + \frac{G_{M\sigma}(Q^2)}{1 + \tau} \left[ \tau (E_p + E_{p+q})^2 - (1 + \tau)q^2 \right], \]  
(D5)

\[ L_{11}\sigma(p, q; Q^2) = \frac{G_{E\sigma}(Q^2)}{1 + \tau} (2p_x)^2 + \frac{G_{M\sigma}(Q^2)}{1 + \tau} \left[ \tau (2p_x)^2 + (1 + \tau)Q^2 \right]. \]  
(D6)

The factor \( f_\sigma(p) \equiv n_\sigma(p) \left[ 1 - n_\sigma(p + q) \right] \) in (D3) and (D4) provides Pauli blocking for final states, and must be included when starting from (3.1) (see footnote 1). Expressions (D3) and (D4) can be evaluated numerically by rewriting the \( \delta \)-function as a \( \theta \)-function which restricts the polar angle \( \theta \equiv \cos^{-1}[p \cdot q/|p||q|] \).

In numerical calculations we take the Sachs form factors to be

\[ G_{E_p}(Q^2) = (1 + Q^2/0.71 \text{ GeV}^2)^{-2}, \]  
(D7)

\[ G_{M_p}(Q^2) = (1 + \kappa_p)G_{E_p}(Q^2), \]  
(D8)

\[ G_{M_n}(Q^2) = \kappa_nG_{E_p}(Q^2), \]  
(D9)

\[ G_{E_n}(Q^2) = 0. \]  
(D10)

where \( \kappa_p = +1.79 \) and \( \kappa_n = -1.91 \) are the proton and neutron anomalous magnetic moments, respectively.

For nucleons of mass \( M^* \) the above expressions must be modified. This is accomplished in part by letting \( E_p \to E^*_p, E_{p+q} \to E^*_p + q \) and \( \tau \to \tau^* \) in expressions (D3)–(D6). While these changes must be made to account for the modified kinematics of nucleons with a reduced mass \( M^* \), there is some ambiguity in the treatment of the Sachs form factors which are fit only to free nucleon data. In this work we adopt Model G of Ref. [14], in which we assume that the Sachs electric and magnetic form factors are unmodified in the nuclear medium.
References

[1] Z.-E. Meziani, Nucl. Phys. **A446**, 113 (1985).

[2] R. Altemus, A. Cafolla, D. Day, J.S. McCarthy, R.R. Whitney and J.E. Wise, Phys. Rev. Lett. **44**, 965 (1980).

[3] P. Barreau *et al.*, Nucl. Phys. **A358**, 287 (1981).

[4] P. Barreau *et al.*, Nucl. Phys. **A402**, 515 (1983).

[5] M. Deady *et al.*, Phys. Rev. C **28**, 631 (1983).

[6] Z.-E. Meziani *et al.*, Phys. Rev. Lett. **52**, 2130 (1984).

[7] M. Deady, C.F. Williamson, P.D. Zimmerman, R. Altemus and R.R. Whitney, Phys. Rev. C **33**, 1897 (1986).

[8] Z.-E. Meziani *et al.*, Phys. Rev. Lett. **69**, 41 (1992).

[9] J. D. Walecka, Nucl. Phys. **A399**, 387 (1983).

[10] T. Matsui, Phys. Lett. **132B**, 260 (1983).

[11] G. Do Dang, M. L’Huillier, Nguyen Van Giai and J. W. Van Orden, Phys. Rev. C **35**, 1637 (1987).

[12] C. J. Horowitz, Phys. Lett. **208B**, 8 (1988).

[13] T. W. Donnelly, E. L. Kronenberg and J. W. Van Orden, Nucl. Phys. **A494**, 365 (1989).

[14] T. De Forest, Jr., Nucl. Phys. **A414**, 347 (1984).

[15] T.C. Ferrée and D.S. Koltun, Phys. Rev. C **49**, 1661 (1994).

[16] D.S. Koltun and T.C. Ferrée, Phys. Rev. C **52**, 901 (1995).

[17] A. Bodek et. al., Phys. Rev. D **20**, 1471 (1979).

[18] L.W. Whitlow, Ph.D. Thesis, SLAC Report 357 (1990).

[19] S. Stein et. al., Phys. Rev. D **12**, 1884 (1975).

[20] A. Bodek and J.L. Ritchie, Phys. Rev. D **23**, 1070 (1981).

[21] S.D. Drell and J.D. Walecka, Ann. Phys. (N.Y.) **28**, 18 (1964).
[22] G.B. West, Ann. Phys. (N.Y.) 74, 646 (1972).

[23] W.B. Atwood and G.B. West, Phys. Rev. D 7, 773 (1973).

[24] B.D. Serot and J.D. Walecka, Adv. Nucl. Phys. 16, 1 (1986).

[25] A. Bodek, personal communication.

[26] G. Altarelli and G. Martinelli, Phys. Lett. 76B, 89 (1978).

[27] H. Georgi and H.D. Politzer, Phys. Rev. D 14, 1829 (1976).

[28] Un-Ki Yang, personal communication.
Figure Captions

FIG. 1. Fits to nucleon structure functions $W_p^2$ and $W_n^2$ in laboratory variables for three-momentum transfer $|q| = 1 \text{ GeV/c}$. Fits by Whitlow ($\Lambda_{12}$) for the proton (solid) and neutron (dashed), and fits by Bodek et. al. (B) for the proton (dot-dashed) and neutron (dotted).

FIG. 2. Same as Fig. 1 except for $|q| = 2 \text{ GeV/c}$.

FIG. 3. Same as Fig. 1 except for $|q| = 4 \text{ GeV/c}$.

FIG. 4. Nuclear response functions (per nucleon) for three-momentum transfer $|q| = 1 \text{ GeV/c}$: (a) longitudinal $W_L^A(\omega, q)/A$, and (b) transverse $W_T^A(\omega, q)/A$. Inelastic nucleon background based on fits by Whiltow ($\Lambda_{12}$) (solid curves) and by Bodek et. al. (dashed curves) are shown, along with the quasielastic peak (dotted curves). Thin lines are for free nucleons with $M^*/M = 1$, and thick lines are for interacting nucleons with $M^*/M = 0.648$.

FIG. 5. Same as Fig. 4 except for $|q| = 2 \text{ GeV/c}$. The inserts show enlarged views of the quasielastic peak region.
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