Ground state cooling of micromechanical oscillators in the unresolved sideband regime induced by a quantum well

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(Dated: March 15, 2016)

We theoretically demonstrate the ground state cooling of a mechanical oscillator in an optomechanical cavity in presence of a quantum well, in the unresolved-sideband regime. Due to the presence of the quantum well, the cavity response gets modified and leads to asymmetric heating and cooling processes. The cooling rate of the mechanical resonator can potentially be enhanced by tuning the cavity-field detuning. It is demonstrated that, even when the cavity is in the unresolved-sideband regime, the effective interaction of the exciton and mechanical modes can bring the system back to effective resolved-sideband regime. Time evolution of the mean phonon number in the mechanical resonator is studied using the quantum master equation. The average phonon occupancy in the mechanical resonator tends to zero with time, exhibiting dynamic controllability of cavity dissipation.

PACS numbers: 42.50.Wk, 07.10.Cm, 42.50.Lc

I. INTRODUCTION

In recent years, the field of cavity optomechanics has been subjected to rapid exploration in terms of both theory and experiment [1-6]. The basic principle of coupling through radiation pressure force, as predicted in [7-9] can lead to Kerr type nonlinearity between the optical and mechanical modes [10-11]. The optomechanical interaction between optical and mechanical modes is not only a tool for readout of mechanical motion [12], but it also triggers the possibility of observing fundamental quantum effects in mesoscopic systems. Owing to the inherent nonlinearity, cavity optomechanics is a strong candidate for futuristic aspects of achievement of standard quantum limit [13], continuous variable entanglement of optical and mechanical modes [14], nonclassical state generation [15], quantum state transfer between different modes [16], optomechanically induced transparency [17], quantum nondemolition measurements [18] etc.

For observing quantum effects in optomechanical systems, ground state cooling of the mechanical oscillator is an essential condition [19, 20]. In an optomechanical system, the light scattered from the movable end mirror gives rise to Stokes and anti-Stokes sidebands. During the Stokes process, the mirror absorbs a quantum of energy from the cavity optical field, leading to heating of the mirror; whereas during the anti-Stokes process, the cavity field absorbs energy from the mirror resulting in cooling of the mirror. To obtain an effective cooling of the mechanical mirror, cooling rate should be higher than the heating rate. Therefore, in analogy to the laser cooling of ions in the strong binding regime [21], conventional cavity cooling of mechanical oscillators requires the condition of resolved-sideband regime, where the cavity mode decay rate is lower than the mechanical oscillator resonance frequency [19, 22]. However, in practical situations, for typical mechanical oscillators of frequency in the range of kHz-MHz; fulfilling this condition is a challenging task, that poses serious constraints experimentally. To relax this requirement, few different approaches have been suggested such as cooling using dissipative coupling [23-24], coupling with high-Q auxiliary cavity [25], hybrid atom- optomechanical systems [26], optomechanically induced transparency [27]. Here, in this paper, we consider the cavity cooling of a micromechanical mirror, with a quantum well (QW) having lower exciton decay rate, placed inside the optomechanical cavity. This type of solid-state systems has their own advantages over atomic cavity systems. Engineerable emission frequency, fixed position and potential for integration with cavities and waveguides using developed semiconductor fabrication techniques [28] make them unique tool for exploring optomechanics further [6, 29]. The same type of systems has been studied in the context of nonlinear effects like bistability and squeezing of the output field [30]. It is also predicted in such a system that the interaction between the exciton and mechanical modes through the cavity field may lead to entanglement between the two [31]. We explore the aspect of ground state cooling of the mechanical oscillator in the unresolved-sideband regime. In this regime, by coupling the mechanical oscillator to a quantum well placed at the anti-node of the cavity field, one can modify the noise spectrum. This illustrates mode structuring around the sidebands, due to the inhibition of the heating process while enhancing the cooling process through quantum interference. In order to study the cooling process in the mechanical oscillator, we use an effective exciton-phonon interaction model that is analogous to dark mode formation, which has been studied extensively in optomechanics, in connection to state transfer protocols[32]. The dark mode in optomechanics is similar to coherent-population trapped state or the dark state in atomic physics [33]. The similarity to the so-called stimulated Raman adiabatic passage

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(STIRAP) is also notable. In fact, STIRAP protocols are also studied extensively in the context of state transfer in optomechanical and electromechanical systems [32]. The paper is organized as follows. In Section II we describe the Hamiltonian of the system and derive the quantum Langevin equations for the system operators. Section III is devoted to the analysis of cooling of the mechanical oscillator, followed by conclusion of our work in Section IV.

II. MODEL AND THEORY

We consider an optomechanical cavity containing a Quantum Well (QW) placed at the antinode of the cavity field.

![Optomechanical cavity](image)

**FIG. 1.** (Color online) An optomechanical cavity containing a Quantum Well (QW) placed at the antinode of the cavity field.

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The first term \( H_{\text{free}} \) in Eq. (1) describes the free Hamiltonian of the system, given by

\[
H_{\text{free}} = \omega_c c^\dagger c + \omega_l d^\dagger d + \omega_m b^\dagger b, \quad \text{where,} \quad \omega_c, \omega_l \text{ and } \omega_m \text{ are the resonance frequencies of the cavity optical field, the QW excitons and the mechanical oscillator respectively. Since we are dealing with quasi-resonant coherent excitation, higher lying exciton states of the QW are neglected. The optomechanical interaction between the cavity mode and the mechanical oscillator is described by the second term,}
\]

\[
H_{\text{om}} = g_{\text{OM}} (b^\dagger c + c^\dagger b). \quad \text{Here } g_{\text{OM}} \text{ is the single-photon optomechanical coupling strength between the cavity field and the mechanical oscillator. The third term in Eq. (1) accounts for the coupling between the cavity mode and the exciton mode in the QW, given by}
\]

\[
H_{\text{drive}} = g (c^\dagger d^\dagger c + d^\dagger d c), \quad \text{with interaction strength } g.
\]

The last term represents the pump laser driving, given by

\[
H_{\text{drive}} = \varepsilon_p (c^\dagger e^{-i\omega_l t} + ce^{i\omega_l t}), \quad \text{with pump laser frequency } \omega_l \text{ and amplitude } \varepsilon_p = \sqrt{k_c \beta_{\text{in}}}; \quad \beta_{\text{in}} \text{ and } k_c \text{ being the input power and cavity decay rate respectively.}
\]

In the frame rotating with the input laser frequency \( \omega_l \), one obtains the Hamiltonian of the system as follows:

\[
H = -\Delta_c c^\dagger c - \Delta_d d^\dagger d + \omega_m b^\dagger b + g_{\text{OM}} c^\dagger c (b^\dagger + b) + g (c^\dagger d + d^\dagger c) + \varepsilon_p (c^\dagger + c)
\]

where, \( \Delta_c = \omega_l - \omega_c \) and \( \Delta_d = \omega_l - \omega_d \) are the detunings of the cavity mode and the exciton mode respectively. The time evolution of the system operators are given by nonlinear Heisenberg-Langevin equations:

\[
\dot{c} = (i\Delta_c - k_c) c - ig_{\text{OM}} (b^\dagger + b) - igd - i\varepsilon_p - \sqrt{k_c} \beta_{\text{in}}(t)
\]

\[
\dot{d} = (i\Delta_d - k_d) d - igc - \sqrt{k_d} \beta_{\text{in}}(t)
\]

\[
\dot{b} = (-i\omega_m - \gamma_m) b - ig_{\text{OM}} c^\dagger c - \sqrt{\gamma_m} \beta_{\text{in}}(t)
\]

where, \( k_c, k_d \) and \( \gamma_m \) are the decay rates of the optical mode, exciton mode and the mechanical mode respectively. \( \beta_{\text{in}}, d_{\text{in}} \) and \( b_{\text{in}} \) are the corresponding input vacuum noise operators with zero mean value and nonzero correlation functions given by:

\[
\langle c_{\text{in}}(t) c_{\text{in}}^\dagger(t') \rangle = \delta(t - t')
\]
\[ \delta c = \left( i\Delta'_c - \frac{k_c}{2} \right) \delta c - iG (\delta b^\dagger + \delta b) - igd - \sqrt{k_c c_{in}}(t) \]  
(5a)

\[ \delta d = \left( i\Delta_d - \frac{k_d}{2} \right) \delta d - ig\delta c - \sqrt{k_d d_{in}}(t) \]  
(5b)

\[ \delta \tilde{b} = \left( -i\omega_m - \frac{\gamma_m}{2} \right) \delta \tilde{b} - iG(\delta a^\dagger + \delta c) - \sqrt{\gamma_m b_{in}}(t) \]  
(5c)

Eqs. (5a)-(5c) can be solved in the frequency domain to obtain the expression for \( \delta \tilde{b}(\omega) \) as follows:

\[ \delta \tilde{b}(\omega) = \frac{\sqrt{\gamma_m b_{in}}(\omega) - i\sqrt{K_c A(\omega) - K_d B(\omega)}}{i\omega - i\{\omega_m + \Sigma(\omega)\} - \frac{\gamma_m}{2}} \]  
(6)

where,

\[ A(\omega) = G[\chi(\omega) c_{in}(\omega) + \chi^*(\omega) c_{in}^\dagger(\omega)] \]
\[ B(\omega) = gG[\chi(\omega) \chi_d(\omega) d_{in}(\omega) - \chi^*(\omega)] \]

\[ \chi_d(\omega) c_{in}(\omega) + \chi^*(\omega) c_{in}^\dagger(\omega) \]

Here, \( \Sigma(\omega) = -iG^2[\chi(\omega) - \chi^*(\omega)] \) is the optomechanical self-energy, where \( \chi(\omega) = \frac{[\chi_c(\omega)]^{-1} + g^2 \chi_d(\omega)]}{\gamma_m} \) is the total response function of the optomechanical cavity with the QW. \( \chi_c(\omega) = [-i (\omega + \Delta_c) + \frac{\hbar^2}{2}]^{-1} \), \( \chi_d(\omega) = [-i (\omega + \Delta_d) + \frac{\hbar^2}{2}]^{-1} \) and \( \chi_m(\omega) = [-i (\omega - \omega_m) + \frac{\hbar^2}{2}]^{-1} \) are the response functions of the optical mode, the exciton mode and the mechanical mode respectively. The radiation pressure force, in an optomechanical system, arising due to the interaction term, \( H_{int} \), is given by \( F = -\frac{\partial H_{int}}{\partial x} \). Using this, the radiation pressure force for our system is estimated as \( F = -G \frac{\delta c^\dagger + \delta c}{x_{ZPF}} \), where \( x_{ZPF} \) is the zero-point fluctuation of the mechanical oscillator. The quantum noise spectrum is calculated using: 

\[ S_{FF}(\omega) = \int dt e^{i\omega t} \langle F(t) F(0) \rangle \]  
[6]

The spectral density, in our system is calculated to be:

\[ S_{FF}(\omega) = \frac{G^2 |\chi(\omega)|^2}{x_{ZPF}^2} [k_c + k_d a^2] \chi_d(\omega)^2 \]  
(7)

The cooling rate of the mechanical resonator is given by \( \dot{A}_c = S_{FF} (\omega_m)x_{ZPF} \), while the heating rate is given by \( \dot{A}_h = S_{FF} (-\omega_m)x_{ZPF} \). Due to the dynamical back-action induced by the radiation pressure force, the spring
constant (and thereby the effective oscillation frequency) and the damping rate of the mechanical oscillator get modified. The extra damping of the mechanical oscillator due to the optomechanical interaction is given by, \( \gamma_{OM} = A_- - A_+ = -2Im(\Sigma(\omega_m)) \) and the mechanical frequency shift is given by, \( \delta \omega_m = Re(\Sigma(\omega_m)) \).

In Fig. 3, we have plotted the noise spectrum for different values of \( \gamma \) are: \( \gamma_m = 10^{-2} \omega_m, \ k_c = 10^2 \omega_m, \ g = 100 \omega_m, \ G = 50 \omega_m, \ \Delta_d = 0.5 \omega_m. \)

FIG. 4. (Color online) Plot of steady-state cooling limit as function of (a) \( \Delta'_c/\omega_m \), with \( \kappa_d = \omega_m \) and \( n_{th} = 10^4 \). The red solid line denotes the final phonon number in the mechanical oscillator for a generic optomechanical cavity, whereas the blue dotted line shows the phonon number in presence of the QW in the cavity; (b) \( n_{th} \) with \( \Delta_c = 10^4 \omega_m \). Other unspecified parameters are: \( \gamma_m = 10^{-2} \omega_m, \ k_c = 10^2 \omega_m, \ g = 100 \omega_m, \ G = 50 \omega_m, \ \Delta_d = 0.5 \omega_m \).

Now considering the effect of the cavity mode as perturbation. Integrating Eqs. (5a)-(5c), we get the time dependent form of the operators as follows:

\[
\delta c(t) = \delta c(0) \exp\left(i \Delta'_c t - \frac{k_c}{2} t\right) + \exp\left(i \Delta'_d t - \frac{k_c}{2} t\right) \\
\int_0^t \left[ -i G \delta d(\tau) - i G \delta b(\tau) - i g \delta d(\tau) - \sqrt{\kappa_c} c_{in}(\tau) \right] \exp\left(-i \Delta'_d \tau + \frac{k_c}{2} \tau\right) d\tau
\]

\[
\delta d(t) = \delta d(0) \exp\left(i \Delta_d t - \frac{k_d}{2} t\right) + \exp\left(i \Delta_d t - \frac{k_d}{2} t\right) \\
\int_0^t \left[ -i g \delta c(\tau) - \sqrt{\kappa_d} c_{in}(\tau) \right] \exp\left(-i \Delta_d \tau + \frac{k_d}{2} \tau\right) d\tau
\]

\[
\delta b(t) = \delta b(0) \exp\left(-i \omega_m t - \frac{\gamma_m}{2} t\right) + \exp\left(-i \omega_m t - \frac{\gamma_m}{2} t\right) \\
\int_0^t \left[ -i G \delta c(\tau) - i G \delta b(\tau) - \sqrt{\kappa_m} b_{in}(\tau) \right] \exp\left(i \omega_m \tau + \frac{\gamma_m}{2} \tau\right) d\tau
\]

Now considering the effect of the cavity mode as perturbation, the expressions for the time dependent exciton mode and mechanical mode operators are approximated to be as follows:

\[
\delta d(t) \approx \delta d(0) \exp\left(i \Delta_d t - \frac{k_d}{2} t\right) + D_{in}(t)
\]

\[
\delta b(t) \approx \delta b(0) \exp\left(-i \omega_m t - \frac{\gamma_m}{2} t\right) + B_{in}(t)
\]

where the effect of the cavity mode is included in the
noise terms $D_{in}(t)\text{ and }B_{in}(t)$ . Substituting Eqs. (9a) and (9b) back into Eq. (8a) and under the assumptions $|\Delta_c'\gg \Delta_d, k_c \gg (k_d, \gamma_m)$, we obtain:

$$\delta c(t) = \frac{iG(\delta b(t) + \delta b^\dagger(t))}{-i\Delta_c' + \frac{k_d}{2}} - \frac{ig\delta d(t)}{-i\Delta_c' + \frac{k_d}{2}} + \delta c(0) \exp \left(\frac{i\Delta_c' t - k_c}{2}\right) + C_{in}(t)$$

$$(10)$$

And substituting $\delta c(t)$ into Eqs. (5b) and (5c), we get the equation for the exciton mode as:

$$\delta d = \left(\frac{i\Delta_d - \frac{k_d}{2}}{\Delta_d - \eta_2 \Delta_c'}\right) \delta d + ig \left[\frac{iG(\delta b(t) + \delta b^\dagger(t))}{-i\Delta_c' + \frac{k_d}{2}} + \frac{ig\delta d(t)}{-i\Delta_c' + \frac{k_d}{2}}\right]$$

$$- \sqrt{k_d d_{in}(t)} - ig[\delta c(0) \exp \left(\frac{i\Delta_c' t - k_c}{2}\right) + C_{in}(t)]$$

$$(11)$$

Comparing this with the generic single-cavity optomechanical system, we can derive the parameters for the effective exciton-mechanical mode interaction as $\Delta_{eff} = \Delta_d - \eta_2 \Delta_c'$, $k_{eff} = k_d + \eta^2 k_c$, $G_{eff} = \eta G$, where $\eta = g/\sqrt{\Delta_c^2 + (k_d/2)^2}$ that can be approximated to $\eta = g/\Delta_c'$ for $\Delta_c' \gg k_c$. Analogous to the single cavity optomechanical system, the steady-state cooling limits are approximated as $n_{eff} = n_{classical} + n_{quantum}$ [34, 37]. Here, $n_{classical} = \frac{4G^2_{eff} + k_{eff}^2}{4G^2_{eff} k_{eff}} \gamma_m n_{th} \approx \frac{k_{eff}}{4G_{eff}} \gamma_m n_{th}$ is the classical steady-state cooling limit and $n_{quantum} = \frac{k_{eff}^2 + 8G^2_{eff}}{16(\omega_m^2 - 4G^2_{eff})}$ is the quantum limit of cooling for effective resolved sideband ($k_{eff} \ll \omega_m$) and the effective weak coupling regime ($G_{eff} < k_{eff}$). In Figs. 4(a)-4(b) the variations of steady-state cooling limit as function of normalized cavity detuning and the thermal phonon number are shown. As seen from Fig. 4(a), ground state cooling is not possible for a generic optomechanical cavity in the unresolved-sideband regime. Nevertheless, in case of the QW coupled system, ground state cooling can be achieved for a range of high cavity detuning near $10^4 \omega_m$. For large detuning, dark modes with respect to the low-Q cavity mode (c) are formed via linear combination of the mechanical mode (b) and high-Q exciton mode (d). This dark-mode is responsible for an effective cooling [32]. It is worth to be noted that in case of a single optomechanical cavity without the QW, cooling occurs in the red detuned regime only. But, with the QW inside the cavity, the damping is significantly enhanced in the blue-detuned regime as illustrated in Fig. 3. The effective detuning term $\Delta_{eff}$ contains both $\Delta_c'$ and $\Delta_d$. Hence, it is possible to tune these blue-detuned terms to get a red-detuned $\Delta_{eff}$ at high value of $\Delta_c'$, that results in cavity cooling. Fig. 4(b) depicts the variation of the steady-state cooling limit as a function of the bath phonon number for different values of $k_d$. The plots show that, for ground state cooling of the mechanical resonator, high values of bath phonon number is tolerable. For example, for $k_d = \omega_m$, the maximum tolerable bath phonon number is approximately equal to $37 \times 10^3$. It is also to be noted that more bath phonon number is tolerable for ground state cooling with lower decay rate of the QW excitons.

It is important to have an idea about the optimum range of the optomechanical coupling $G$ and the exciton-cavity coupling $g$ to be used for efficient mechanical mode cooling. These couplings are fixed by the material at the fabrication stage. Though the optomechanical coupling $G$ also depends on the input laser power, it is difficult to tune the couplings at a later stage.
Equations for this, we solve a linear system of differential equations. For this, we solve a linear system of differential equations of the form:

\[ \frac{\partial}{\partial t} \langle \hat{n}_i \rangle = \text{Tr} (\hat{\rho} \circ \hat{\rho}_i) = \sum_{m,n} \mu_{m,n} \langle \hat{o}_m \hat{o}_n \rangle, \]

where, \( \hat{o}_i, \hat{o}_j, \hat{o}_m, \hat{o}_n \) are one of the operators: \( \hat{b}^\dagger \hat{b}, \hat{c}^\dagger \hat{c}, \hat{d}^\dagger \hat{d}, \hat{b}, \hat{c}, \hat{d}, \hat{\rho}, \hat{\rho}_1, \hat{\rho}_2 \), and \( \mu_{m,n} \) are the corresponding coefficients. Solving these, we can find out the mean values of all the time-dependent second-order moments: \( \langle \hat{b}^\dagger \hat{b} \rangle, \langle \hat{b}^\dagger \hat{c} \rangle, \langle \hat{c}^\dagger \hat{b} \rangle, \langle \hat{c}^\dagger \hat{c} \rangle, \langle \hat{d}^\dagger \hat{d} \rangle, \langle \hat{b}^\dagger \hat{d} \rangle, \langle \hat{c}^\dagger \hat{d} \rangle, \langle \hat{d}^\dagger \hat{d} \rangle \). In Fig. 6, we show the time evolution of the mean phonon number. The environmental phonon number is assumed to be 10^4. The cavity is considered to be in highly unresolved sideband regime. Initially the phonon number in the mechanical resonator is equal to the environmental phonon number. All other second order moments are initially zero. The plot shows that with increasing time, the average phonon occupancy in the mechanical resonator is cooled down to below 1. This indicates ground state cooling of the resonator mode in the highly unresolved sideband regime.

IV. CONCLUSION

In conclusion, we have studied the sideband cooling of a mechanical resonator in an optomechanical cavity containing a quantum well. It is worthwhile to note that such semiconductor structures, with well-developed semiconductor fabrication techniques, are easily integrable with cavities and waveguides making them a unique tool for exploiting optomechanics. The exciton and mechanical modes are not coupled directly, but their interaction with the cavity optical field gives rise to an indirect coupling between them. This specific configuration of the system can lead to cooling of the mechanical oscillator in the unresolved sideband regime. Due to the presence of the high-Q element in the cavity, the noise spectrum is modified and leads to asymmetric cooling and heating rates. Even when the cavity is in the highly unresolved-sideband regime, the effective interaction between the exciton and mechanical modes can bring the system back to effective resolved-sideband regime. Hence the requirement of the resolved-sideband condition for cooling is relaxed significantly. The cooling rate of the mechanical resonator can be enhanced by tuning the cavity-field detuning. The time evolution of the mean phonon number in the mechanical resonator is studied using the quantum master equation. It is found that, with increasing time, the average phonon occupancy in the mechanical resonator tends towards zero exhibiting dynamic controllability of cavity dissipation. This might open up the possibility of manipulation of semiconductor integrated mechanical systems in the quantum mechanical regime.

Acknowledgements

B. Sarma would like to thank MHRD, Government of India for a research fellowship.

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