Van der Waal’s gas equation for an adiabatic process and its Carnot engine efficiency

Kiran S. Kumar, Aravind P. Babu and M. Ponmurugan*
Department of Physics, School of Basic and Applied Sciences, Central University of Tamilnadu, Tiruvarur 610 005, Tamilnadu, India.

Abstract

There has been many studies on gases which obeys Van der Waal’s equation of state. However there is no specific and direct studies of Van der Waal’s gas which undergoes adiabatic processes are available in the undergraduate text books and also in literature. In an adiabatic process there is no heat energy exchange between the system and its surroundings. In this article, we find that the Van der Waal’s equation for the adiabatic process as
\[
\left( P + \frac{n^2 a}{V^2} \right) (V - nb)^\Gamma = \text{constant},
\]
where \( P \) is the pressure, \( V \) is the volume, \( n \) is the number of moles of the Van der Waal’s gas, \( a \) and \( b \) are Van der Waal’s constant and \( \Gamma \) is a factor which relates the specific heat at constant pressure and at constant volume. We use this relation explicitly and obtained the efficiency of a Carnot engine whose working substance obeys Van der Waal’s equation of state. Our simplest approach may provide clear idea to the undergraduate students that \( \Gamma \) is different from \( \gamma \) of the ideal gas for an adiabatic process. We also shown that the efficiency of the Carnot engine is independent of the working substance.

1 Introduction

In thermodynamics, heat and work are the form of energy transfer across the boundary between the system and its surroundings. If the boundary forbids the flow of heat energy between the system and the surroundings then the thermodynamic process which changes the state of the system is called as adiabatic process. If the energy transfer takes at a fixed system temperature then such a process is called as isothermal process. All heat engines make use of the mechanism of converting heat \( (Q) \) in to work \( (W) \), without involving any resultant change in the state of the system. It is a series of processes taking place in a cyclic manner in which the engine or system will be returned to its initial state.

During each of the processes, there may be a heat energy flow between the system and its surroundings. This comprises of a hot reservoir and a cold reservoir, which will be maintained at constant temperatures, and the working substance which exchanges heat. Therefore, the engine is said to operate between these two reservoirs. During a part of cycle performed by the working substance in an engine, some heat \( Q_H \) is absorbed from the hotter reservoir and a smaller amount of heat \( Q_L \) is rejected to the cooler reservoir during the another part of cycle. In a cyclic process, the system returns to the initial state with no change in internal energy \( (\Delta U = 0) \). So, from the first law of thermodynamics \( Q_{net} - W_{net} = 0 \), where \( Q_{net} = Q_H - Q_L \) is the net heat exchanged between the system and the surroundings and \( W_{net} = Q_{net} = Q_H - Q_L \) is the total or net work performed on the system. The engine efficiency \( \eta \) is defined as [1, 2, 3],

\[
\eta = \frac{\text{Net Work}}{\text{Heat absorbed}} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H} \tag{1}
\]
1.1 Carnot Cycle

Carnot proposed a thermodynamic cycle called as Carnot cycle which is a set of equilibrium reversible processes any thermodynamic system can perform [1, 2]. Initially the system or working substance in Carnot cycle is imagined to be in thermal equilibrium with a reservoir at lower temperature $T_L$. Four processes are then performed in the following order:

1. A reversible adiabatic process is performed in such a direction that the temperature rises to that of hotter reservoir temperature $T_H$. However, no energy in the form of heat flows in to or out of the system.

2. The working substance is maintained in contact with the reservoir at $T_H$ and a reversible isothermal process is performed in such a direction to the extent that $Q_H$ is absorbed from the reservoir. In this case the system is kept at a fixed reservoir temperature.

3. A reversible adiabatic process is performed in a direction opposite to that of the first process with no heat exchange. This is done until the temperature reaches $T_L$, temperature of the cooler reservoir.

4. A reversible isothermal process opposite to the direction of the second process is performed until the working substance reaches the initial state and $Q_L$ is rejected by the working substance. In this case the system is kept at a fixed reservoir temperature $T_L$.

Thus an engine in Carnot cycle operates between two reservoirs in a particular simple way. All the absorbed heat enters the system at a constant high temperature, namely that of a hotter reservoir. Also, all the rejected heat leaves the system at a constant low temperature, that of a cooler reservoir. Since all processes are reversible, the Carnot engine is a reversible engine. An engine which operates in this cycle is called as Carnot engine whose efficiency is always greater than any engines operated by the cycle other than Carnot cycle. Any hypothetical engine operated in this cycle is said to be ideal if it gives 100% efficiency. Since it is fact of experience that some heat is always rejected to cooler reservoir, the efficiency of actual engine is always less than the ideal and the Carnot engine [1, 2, 3].

Considering a working substance in a Carnot engine as an ideal gas, with no intermolecular interactions, the gas satisfies the ideal gas equation of state $PV = nRT$. Where $P$ is the pressure, $V$ is the volume, $T$ is the temperature of the system, $n$ is the number of moles and $R$ is the universal gas constant. The thermal efficiency of a Carnot engine whose working substance is an ideal gas is given by

$$\eta = 1 - \frac{T_L}{T_H}.$$  \hspace{1cm} (2)

Thus, a Carnot engine absorbing $Q_H$ amount of heat from the reservoir at higher temperature $T_H$ and rejecting $Q_L$ amount of heat to the reservoir at lower temperature $T_L$ has an efficiency that is independent of the nature of the working substance. The above result is obtained by using the ideal gas equation of state for the isothermal process $PV = constant$ and the ideal gas equation for the adiabatic process $PV^\gamma = constant$. Here, $\gamma$ is the ratio of the specific heat capacity at constant pressure to the specific heat capacity at constant volume. Further studies using various gas equations [4, 5, 6, 7] also showed that the efficiency of a Carnot cycle is independent of working substance and depends only on temperatures of reservoirs.

1.2 Van der Waal’s gas

By considering the intermolecular interactions, van der Waal proposed the equation of state for a real gas which is given by [2, 3, 5]

$$
\left(P + \frac{n^2a}{V^2}\right)(V - nb) = nRT,
$$  \hspace{1cm} (3)

where $a$ and $b$ are the Van der Waal’s constants. There has been several studies for Van der Waal’s equation of state [4, 6]. However, there is no specific and direct studies available in undergraduate textbooks and also in literature for the equation of Van der Waal’s gas.
der Waal’s gas subjected to an adiabatic process. In this paper we explicitly find that the Van der Waal’s equation for an adiabatic process as

\[
(P + \frac{n^2 a}{V^2})(V - nb)\Gamma = \text{constant},
\]

where \( \Gamma \) is a factor which relates the specific heat at constant pressure and at constant volume. We use the above relation and obtained the relation between the specific heat capacity at constant pressure and at constant volume. As similar to ideal gas approach we also used the above relation directly and showed that the efficiency of a Carnot cycle is independent of the working substance and depends only on the temperature of the reservoirs.

## 2 Van der Waal’s gas equation for an Adiabatic process

The entropy of pure substance can be considered as a function of any two variable \( T \) and \( V \) as

\[
S = S(T, V) \quad \text{(5)}
\]

\[
dS = \left( \frac{\partial S}{\partial T} \right)_V dT + \left( \frac{\partial S}{\partial V} \right)_T dV.
\]

Multiply both sides by \( T \) we get,

\[
TdS = T \left( \frac{\partial S}{\partial T} \right)_V dT + T \left( \frac{\partial S}{\partial V} \right)_T dV. \quad \text{(6)}
\]

For an adiabatic process \( dQ = TdS = 0 \), hence

\[
nC_V dT = -T \left( \frac{\partial P}{\partial T} \right)_V dV. \quad \text{(10)}
\]

Using Eq.(4), one can obtain

\[
\left( \frac{\partial P}{\partial T} \right)_V = \frac{nR}{(V - nb)}. \quad \text{(11)}
\]

Therefore Eq.(10) becomes

\[
nC_V dT = -T \frac{nR}{(V - nb)} dV \quad \text{(12)}
\]

\[
\frac{1}{T} dT = \frac{R}{C_V (V - nb)} dV. \quad \text{(13)}
\]

Integrating the above equation we get

\[
\ell n T = -\frac{R}{C_V} \ell n (V - nb) + \ell n z, \quad \text{(14)}
\]

where \( \ell nz \) is an integrating constant. Rearranging the above equation one can get

\[
T(V - nb)^{\frac{1}{\Gamma}} = z. \quad \text{(15)}
\]

Combining Eq.(3) and Eq.(14) one can obtain Van der Waal’s gas equation for an adiabatic process as

\[
\left( P + \frac{n^2 a}{V^2} \right)(V - nb)^\Gamma = K, \quad \text{(16)}
\]

where \( \Gamma = \frac{R}{C_V} + 1 \) and \( K = nRz = \text{a constant} \). It should be noted that, for real gas \( \Gamma \neq \gamma = \frac{C_P}{C_V} \) of an ideal gas, where \( C_P \) is the specific heat capacity at constant pressure. Thus, it would be interesting to obtain the relation between the \( C_P \) and \( C_V \) of the Van der Waal’s gas as follows.

### 2.1 Relation between \( C_P \) and \( C_V \) for Van der Waal’s gas

The internal energy of a given system can be considered as a function of any two variable \( T \) and \( V \) as

\[
U = U(T, V) \quad \text{(16)}
\]

\[
dU = \left( \frac{\partial U}{\partial T} \right)_V dT + \left( \frac{\partial U}{\partial V} \right)_T dV.
\]

\[
\frac{\partial U}{\partial V} = \frac{\partial P}{\partial T} \quad \text{(17)}
\]

\[
\frac{1}{T} dT = \frac{R}{C_V (V - nb)} dV. \quad \text{(18)}
\]

\[
\ell n T = -\frac{R}{C_V} \ell n (V - nb) + \ell n z, \quad \text{(19)}
\]

\[
T(V - nb)^{\frac{1}{\Gamma}} = z. \quad \text{(20)}
\]
Using the above equation, the first law of thermodynamics in an infinitesimal form $dQ = dU + dW$ with $dW = -PdV$, can be written as

$$dQ = \left( \frac{\partial U}{\partial T} \right)_V dT + \left( \frac{\partial U}{\partial V} \right)_T dV - PdV. \quad (17)$$

In our study we have used the sign convention that the work done on the system is taken as positive and the work done by the system is taken as negative. For constant volume $dV = 0$, the above equation becomes,

$$nC_V = \left( \frac{\partial U}{\partial T} \right)_V. \quad (18)$$

Since $dQ/dT = nC_V$ for constant volume and $dQ/dT = nC_P$ for constant pressure, Eq.(17) can be rewritten as

$$dQ = nC_V dT + \left( \frac{\partial U}{\partial V} \right)_T dV - PdV \quad (19)$$

$$nC_P dT = nC_V dT + \left( \frac{\partial U}{\partial V} \right)_T dV - PdV \quad (20)$$

$$n(C_P - C_V) = \left\{ \left( \frac{\partial U}{\partial V} \right)_T - P \right\} \frac{dV}{dT}. \quad (21)$$

From the first thermodynamic potential $dU = TdS + PdV$ and using Eq.(17) and Eq.(8) at constant temperature one can obtain

$$\left( \frac{\partial U}{\partial V} \right)_T = T \left( \frac{\partial S}{\partial V} \right)_T + P \quad (22)$$

$$= T \left( \frac{\partial P}{\partial T} \right)_V + P. \quad (23)$$

Then Eq.(21) becomes

$$n(C_P - C_V) = T \left( \frac{\partial P}{\partial T} \right)_V \frac{dV}{dT}. \quad (24)$$

If $V$ is a function of $T$ and $P$, then change in $V$ for a constant pressure is

$$dV = \left( \frac{\partial V}{\partial T} \right)_P dT + \left( \frac{\partial V}{\partial P} \right)_T dP. \quad (25)$$

$$\frac{dV}{dT} = \left( \frac{\partial V}{\partial T} \right)_P. \quad (26)$$

From Eq.(3) on can obtain

$$\left( \frac{\partial P}{\partial T} \right)_V = \frac{nR}{(V - nb)}, \quad (27)$$

$$\left( \frac{\partial V}{\partial T} \right)_P = nRV^3(V - nb)G^{-1}, \quad (28)$$

where $G = V^3nRT - 2n^2a(V - nb)^2$.

Therefore Eq.(24) can be written as

$$C_P - C_V = \frac{R}{f_v}, \quad (29)$$

where

$$f_v = \left\{ 1 - \frac{2na}{V^3RT} (V - nb)^2 \right\}. \quad (30)$$

We have obtained the relation between the $C_P$ and $C_V$ of Van der Waals gas. Divide Eq.(29) throughout by $C_V$ we get

$$\frac{C_P}{C_V} = 1 + \frac{R}{f_vC_V} = 1 + \frac{\Gamma - 1}{f_v}. \quad (31)$$

Thus we have finally obtained the relation between $\gamma$ of ideal gas and $\Gamma$ of Van der Waal’s gas as

$$\gamma = 1 + \frac{\Gamma - 1}{f_v}. \quad (32)$$

There has been few studies to find out the efficiency of the Carnot cycle whose working substance is different from ideal gases [5, 6, 7]. In what follows, we employ simple approach to find out the efficiency of the Carnot engine whose working substance obeys the Van der Waal’s equation of state.

### 3 Carnot engine efficiency for Van der Waal’s gas

Efficiency of the Carnot engine, $\eta$, is defined as the ratio of the net work done to the heat absorbed in the Carnot cycle. As discussed earlier, this reversible cycle consist of four processes such as i) adiabatic compression ii) isothermal expansion iii) adiabatic expansion and iv) isothermal compression. In order to find out $\eta$, we calculate the total work done during the Carnot cycle as follows.
3.1 Work done in an adiabatic compression

In this process, the volume changes from \( V_1 \) to \( V_2 \) \((V_1 > V_2)\), the pressure changes from \( P_1 \) to \( P_2 \) and the temperature from \( T_L \) to \( T_H \). There is no heat exchange between the system and the surroundings. The work done during this process \( W_1 \) is given by,

\[
W_1 = - \int_{V_1}^{V_2} PdV.
\]  

(33)

The Van der Waal’s equation for an adiabatic process obtained in Eq. (15) as

\[
(P + \frac{n^2a}{V^2})(V - nb)^\Gamma = K. \tag{34}
\]

Substitute Eq. (35) in Eq. (33) and integrating, we get

\[
W_1 = - \frac{K(V - nb)_{1 - \Gamma}^{V_2} - n^2a}{(1 - \Gamma)V_1}.
\]

(36)

The Van der Waal’s equation of state for the system in the initial state \((P_1, V_1)\) at \( T_L \) is

\[
(P_1 + \frac{n^2a}{V_1^2})(V_1 - nb) = nRT_L
\]

(37)

and for the system in the final state \((P_2, V_2)\) at \( T_H \) as

\[
(P_2 + \frac{n^2a}{V_2^2})(V_2 - nb) = nRT_H.
\]

Substitute Eq. (36) and Eq. (37) in the above equation, we get

\[
nRT_L(V_1 - nb)^\Gamma - 1 = nRT_H(V_2 - nb)^\Gamma - 1 \equiv K. \tag{39}
\]

Therefore,

\[
K(V_1 - nb)^{1 - \Gamma} = nRT_L \tag{40}
\]

\[
K(V_2 - nb)^{1 - \Gamma} = nRT_H \tag{41}
\]

and hence the work done in an adiabatic compression becomes

\[
W_1 = \frac{nR(T_L - T_H)}{1 - \Gamma} - \frac{n^2a}{V_2} + \frac{n^2a}{V_1}. \tag{42}
\]

3.2 Work done in an isothermal expansion

In this process, the system undergoes volume expansion \( V_2 \) to \( V_3 \) and the pressure change from \( P_2 \) to \( P_3 \), while the temperature remains constant at \( T_H \). During this process the system absorbs \( Q_H \) amount of heat energy from the hot reservoir, then the work done,

\[
W_2 = - \int_{V_2}^{V_3} PdV. \tag{43}
\]

For a reservoir temperature \( T_H \), Eq. (43) can be written as,

\[
P = \frac{nRT_H}{V - nb} - \frac{n^2a}{V^2} \tag{44}
\]

Substitute the above equation in Eq. (33) and integrating, we get

\[
W_2 = nRT_H \ln \left( \frac{V_2 - nb}{V_3 - nb} \right) - \frac{n^2a}{V_3} + \frac{n^2a}{V_2}. \tag{45}
\]

From the Van der Waal’s equation of state \((Eq. 38)\), we can relate the system in the initial state \((P_2, V_2)\) and final state \((P_3, V_3)\) at a fixed temperature \( T_H \) as

\[
\left( P_2 + \frac{n^2a}{V_2^2} \right) (V_2 - nb) = \left( P_3 + \frac{n^2a}{V_3^2} \right) (V_3 - nb). \tag{46}
\]

Substitute Eq. (41) in Eq. (48), then

\[
\frac{P_1 + \frac{n^2a}{V_1^2}}{P_3 + \frac{n^2a}{V_3^2}} = \frac{(V_3 - nb)(V_1 - nb)^{-\Gamma}}{(V_2 - nb)^{1 - \Gamma}}. \tag{47}
\]
3.3 Work done in an adiabatic expansion

As similar to adiabatic compression, the heat exchange in an adiabatic expansion is zero. As the system expands from $V_3$ to $V_4$, the pressure changes from $P_3$ to $P_4$, and the temperature changes from $T_H$ to $T_L$, then the work done $W_3$ during this process is given by,

$$W_3 = - \int_{V_3}^{V_4} PdV.$$  (48)

Substitute Eq. (35) for an adiabatic process in the above equation, we get

$$W_3 = \frac{K(V_3 - nb)^{1-\Gamma} - K(V_4 - nb)^{1-\Gamma}}{1-\Gamma} - \frac{n^2a}{V_4} + \frac{n^2a}{V_3}.$$  (49)

The Van der Waal’s equation of state for the system in the initial state ($P_3, V_3$) at $T_H$ is

$$\left( P_3 + \frac{n^2a}{V_3^2} \right) (V_3 - nb) = nRT_H$$  (50)

and for the system in the final state ($P_4, V_4$) at $T_L$ as

$$\left( P_4 + \frac{n^2a}{V_4^2} \right) (V_4 - nb) = nRT_L.$$  (51)

From the Van der Waal’s equation (Eq.34) for the adiabatic process, we can relate the system in the initial state ($P_3, V_3, T_H$) and final state ($P_4, V_4, T_L$) as

$$\left( P_3 + \frac{n^2a}{V_3^2} \right) (V_3 - nb)^\Gamma = \left( P_4 + \frac{n^2a}{V_4^2} \right) (V_4 - nb)^\Gamma.$$  (52)

Substitute Eq. (49) and Eq. (50) in the above equation, we get

$$nRT_H(V_3 - nb)^\Gamma^{-1} = nRT_L(V_4 - nb)^\Gamma^{-1} \equiv K.$$  (53)

Therefore,

$$K(V_3 - nb)^{1-\Gamma} = nRT_H$$  (54)

and hence the work done in an adiabatic expansion becomes

$$W_3 = \frac{nR(T_H - T_L)}{1-\Gamma} - \frac{n^2a}{V_4} + \frac{n^2a}{V_3}.$$  (55)

3.4 Work done in an isothermal compression

In this process, the pressure changes from $P_4$ to $P_1$, the volume changes from $V_4$ to $V_1$ and $Q_L$ amount of heat energy rejected to a cold reservoir at constant temperature $T_L$. Work done during this process $W_4$ is given by

$$W_4 = - \int_{V_4}^{V_1} PdV.$$  (56)

For a reservoir temperature $T_L$, Eq. (34) can be written as,

$$P = \frac{nRT_L}{V - nb} - \frac{n^2a}{V^2}.$$  (57)

Substitute the above in equation in Eq. (56) and integrating, we get

$$W_4 = nRT_L \ln \left( \frac{V_4 - nb}{V_1 - nb} \right) - \frac{n^2a}{V_1} + \frac{n^2a}{V_4}.$$  (58)

From the Van der Waals equation of state (Eq.34), we can relate the system in the initial state ($P_4, V_4$) and final state ($P_1, V_1$) at a fixed temperature $T_L$ as

$$\left( P_4 + \frac{n^2a}{V_4^2} \right) (V_4 - nb) = \left( P_1 + \frac{n^2a}{V_1^2} \right) (V_1 - nb).$$  (59)

Substitute Eq. (59) in Eq. (51), then

$$\frac{P_3 + \frac{n^2a}{V_3^2}}{P_1 + \frac{n^2a}{V_1^2}} = \frac{(V_1 - nb)(V_3 - nb)^\Gamma}{(V_4 - nb)^{1-\Gamma}}.$$  (60)

3.5 Efficiency of the engine

The net work done $W_{net}$ for one complete cycle is $W_{net} = W_1 + W_2 + W_3 + W_4$. Therefore, the total work done for the Carnot cycle is given by

$$W_{net} = w_H + w_L.$$  (61)

where

$$w_H = nRT_H \ln \left( \frac{V_2 - nb}{V_3 - nb} \right)$$  (62)

$$w_L = nRT_L \ln \left( \frac{V_4 - nb}{V_1 - nb} \right).$$  (63)
According to the first law of thermodynamics $Q_{\text{net}} - W_{\text{net}} = \Delta U$, where $Q_{\text{net}} = Q_H - Q_L$ is the net heat energy exchange between the system and the reservoir and $\Delta U$ is the change in the internal energy. Here we have used the sign convention that the heat energy flow in to the system is taken as positive and the heat energy flow out of the system is taken as negative. For a cyclic process, the change in internal energy $\Delta U$ is zero. Therefore, the net heat energy exchange between the system and the reservoir in a given cycle is completely converted into net work done which is given by

$$ W_{\text{net}} = Q_H - Q_L. \quad (64) $$

In the Carnot cycle, all the absorbed heat $Q_H$ enters the system at a constant high temperature $T_H$ and all the rejected heat $Q_L$ leaves the system at a constant low temperature $T_L$. Hence, comparing Eq. $\text{(61)}$ and Eq. $(64)$, one can identify

$$ Q_H = |w_H| = nRT_H \ln \left( \frac{V_2 - nb}{V_3 - nb} \right), \quad (65) $$

$$ -Q_L = |w_L| = nRT_L \ln \left( \frac{V_4 - nb}{V_1 - nb} \right). \quad (66) $$

The efficiency of the engine obtained from Eq. $(11)$ as

$$ \eta = 1 - \frac{Q_L}{Q_H} = 1 + \frac{T_L \ln \left( \frac{V_1 - nb}{V_1 - nb} \right)}{T_H \ln \left( \frac{V_2 - nb}{V_2 - nb} \right)}. $$

The above equation can be rewritten as

$$ \eta = 1 - \frac{T_L \ln \left( \frac{V_4 - nb}{V_4 - nb} \right)}{T_H \ln \left( \frac{V_3 - nb}{V_2 - nb} \right)}. \quad (67) $$

In order to simply the above equation, substitute Eq. $(60)$ in Eq. $(11)$ and rearranging, we get

$$(V_1 - nb)^{1-\Gamma}(V_3 - nb)^{1-\Gamma} = (V_2 - nb)^{1-\Gamma}(V_4 - nb)^{1-\Gamma}$$

$$ \left( \frac{V_3 - nb}{V_2 - nb} \right)^{1-\Gamma} = \left( \frac{V_4 - nb}{V_1 - nb} \right)^{1-\Gamma}. $$

So,

$$ \left( \frac{V_3 - nb}{V_2 - nb} \right) = \left( \frac{V_4 - nb}{V_1 - nb} \right). \quad (68) $$

Therefore, Eq. $(67)$ reduces to

$$ \eta = 1 - \frac{T_L}{T_H}. \quad (69) $$

Thus, we used Van der Waal’s gas as a working substance and obtained the efficiency of the Carnot engine which is independent of the working substance.

### 4 Conclusion

In summary, we have obtained the Van der Waal’s equation for the adiabatic process as $(P + \frac{\gamma a}{V^2})(V - nb)^\gamma = constant$. Our result explicitly shows that $\Gamma$ of the Van der Waal’s gas is different from $\gamma$ of ideal gas for adiabatic process. This equation has been used directly in Carnot cycle for the adiabatic process and shown that the efficiency of the Carnot engine is independent of the working substance. For this calculation we have used the simple approach as similar to the ideal gas usually found in the undergraduate text books. With this we find out the efficiency of the Carnot engine whose working substance obeys Van der Waal’s equation of state $(P + \frac{\gamma a}{V^2})(V - nb) = nRT$. Using the above two equations, we have also shown that the alternative Van der Waal’s equation for an adiabatic process is $T(V - nb)^{\gamma - 1} = constant$. We can also write the above relation as $(P + \frac{\gamma a}{V^2}) T^{\gamma - 1} = constant$. 

It should be noted that $Q$ can also be obtained from other approaches. Using Eq. $(23)$ and Eq. $(27)$ for an isothermal process, Eq. $(11)$ becomes

$$ dQ = T \left( \frac{\partial P}{\partial T} \right)_V dV = \frac{nRT}{(V - nb)} dV. $$

Integrating the above equation from the initial volume $V_i$ to the final volume $V_f$, one can obtain the amount of heat transferred between the system and the surroundings as

$$ Q = nRT \ln (V - nb)|_{V_f}^{V_i}. $$
*Corresponding author: ponphy@cutn.ac.in

References

[1] M. W. Zemansky, and R. H. Dittman, *Heat and Thermodynamics*, (McGraw-Hill, New York, 1997).

[2] F. W. Sears, and G. L. Salinger, *Thermodynamics, Kinetic Theory, and statistical Thermodynamics* (Addison-Wesley, Philippines, 1975).

[3] D. V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, Cloth, 1999).

[4] R. J. Tykodi and E. Hummel, Am. J. Phys. **41**, 340-343 (1972).

[5] D. C. Agrawal and V. J. Menon, Eur. J. Phys. **11**, 86-90 (1990).

[6] P. C. Tjiang and S. H. Sutanto, Eur. J. Phys. **27**, 719-726 (2006).

[7] J. G. Ayala and F. A. Brown, Eur. J. Phys. **34**, 273-289 (2013).