Implications of $\mu - \tau$ Flavored CP Symmetry of Leptons

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We discuss gauge models incorporating $\mu - \tau$ flavored CP symmetry (called CP$^{\mu\tau}$ in the text) in combination with $L_\mu - L_\tau$ invariance to understand neutrino mixings and discuss their phenomenological implications. We show that viable leptogenesis in this setting requires that the lightest right-handed neutrino mass must be between $10^9 - 10^{12}$ GeV and for effective two hierarchical right-handed neutrinos, leptogenesis takes place only in a narrower range of $5 \times 10^{10} - 10^{12}$ GeV. A multi-Higgs realization of this idea implies that there must be a pseudoscalar Higgs boson with mass less than 300 GeV. Generically, the vev alignment problem can be naturally avoided in our setting.

I. INTRODUCTION

Phenomenal success of experimental research in neutrino physics in the last two decades have led not only to unequivocally establishing that neutrinos have mass but also to an almost complete determination of flavor mixings between the different lepton generations. The missing parts are: (i) the Dirac CP phase, (ii) neutrino mass hierarchy and (iii) a knowledge of whether neutrinos are Majorana or Dirac fermions. Assuming that there are no extra sterile neutrinos, the discovery of the CP phase for neutrinos would put flavor information on leptons on the same footing as quarks. If neutrinos are Majorana fermions, there would be two more phases present in the flavor space and for complete information, one will need information on them. The latest global fits [1, 2] of neutrino parameters point to a preference for a negative value for the Dirac CP phase, $-\pi < \delta_{CP} < 0$. A key focus of experimental research in neutrino physics at the moment is therefore to determine the Dirac CP phase in addition to answering the question of whether neutrinos are Dirac or Majorana particles and their mass hierarchy. An additional motivation to determine the Dirac CP phase comes from its possible connection to understanding the origin of matter and antimatter asymmetry in the universe via leptogenesis [3]. While it is well known that non-observation

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of a non-zero Dirac CP phase does not preclude leptogenesis, its observation can nonetheless provide
important insight into the latter [4].

On the theory front, understanding of the lepton mixing angles $\theta_{ij}$ has been one of the two
major driving forces of much of the research in this field, the other being to probe the scale of
neutrino masses. In the former case, symmetries have been used as a main tool, motivated by the
observation that mixing angles $\theta_{23} \sim \frac{\pi}{4}$ and $\sin \theta_{12} \sim \frac{1}{\sqrt{3}}$, suggesting their possible group theoretic
origin [5]. Among the very first symmetries studied for neutrinos is the $\mu$-$\tau$ exchange symmetry [6],
which not only predicted maximal $\theta_{23}$ but also that $\theta_{13} = 0$. Many other symmetries such as $S_4$,
$A_4$, $\Delta(3n^2)$, etc., were considered later on. The so-called tri-bi-maximal (TBM) mixing pattern [7]
which embodied all these three features, i.e., $\theta_{23} \sim \frac{\pi}{4}$, $\sin \theta_{12} \sim \frac{1}{\sqrt{3}}$ as well as $\theta_{13} = 0$, together
with the symmetry techniques to obtain this pattern, gave a big boost to this approach. Discovery
of a non-zero and large value for $\theta_{13}$ [8] was a turning point in this research since it ruled out
the tri-bi-maximal mixing pattern. Since then, many attempts have been made to combine flavor
symmetries with CP transformation to accommodate a non-zero $\theta_{13}$ while trying to predict the
Dirac CP phase [9–12], sometimes without imposing CP explicitly [13, 14].

In this paper, we pursue this line of research and consider a simple approach based on a general-
ized definition of CP transformation that mixes it with $\mu$-$\tau$ exchange (called $C_{\mu\tau}$ from now on) [9].
This symmetry is known to accommodate a non-zero $\theta_{13}$ while at the same time predicting a Dirac
CP phase $\delta \sim \pm 90^\circ$ [9, 13] if the charged lepton mass matrices are taken diagonal. There are also
models where one has deviations from the exact $C_{\mu\tau}$ limit [15]. A key challenge to building such
models has been that in the $C_{\mu\tau}$ symmetry limit, the muon and tau lepton Yukawa couplings
are degenerate, leading to same masses. In Ref. [9], explicit soft breaking terms were introduced
to generate the $\mu\tau$ mass splitting. Another uncomfortable feature of these models has been its
apparent inability to explain the origin of matter via leptogenesis [9]. We address both these issues
in this paper. Our goal is to present a model where starting with a high scale symmetry, we find
a low energy effective theory where the neutrino sector maintains exact $C_{\mu\tau}$ symmetry whereas
in the charged lepton sector, the symmetry is spontaneously broken so as to allow the muon and
tau masses to be different. We give two examples: one with an extended Higgs sector and another
with an extension involving heavy vector like fermions. The former has interesting implications for
Higgs physics that we discuss below. We also show that there exists a limited range of seesaw scales
where successful leptogenesis can take place, when lepton flavor effects are taken into account.

As a part of this investigation, we also identify the combination of family lepton numbers
$\mathbb{L}_\mu - \mathbb{L}_\tau$ [16] (which we denote as $U(1)_{\mu-\tau}$) as the largest natural abelian symmetry that can be
imposed in conjunction with CP\textsuperscript{\mu\tau}, thus providing the simplest example of combining an abelian symmetry with CP, yet with predictive CP violation at low energies. We arrive then at a natural setting where \( G_l = U(1)_{\mu-\tau} \) can be the residual symmetry of the charged lepton sector (ensuring diagonal mass matrix) and \( G_\nu = \mathbb{Z}_2^{CP} \), generated by CP\textsuperscript{\mu\tau}, is the residual symmetry of the neutrino sector. Because of the properties of \( U(1)_{\mu-\tau} \) and CP\textsuperscript{\mu\tau}, these residual symmetries can be maintained separately in each sector without perturbing interactions in the scalar potential, thus avoiding the vev alignment problem of flavor symmetry models with larger nonabelian groups.

New results of the paper are: (i) construction of a model with natural residual symmetries \( G_l \) and \( G_\nu \) but without soft breaking of CP\textsuperscript{\mu\tau}; (ii) discussion of how one can implement successful leptogenesis in these models and constraints imposed by it on the seesaw scale and (iii) implications for neutrino-less double beta decay and Higgs physics.

This paper is organized as follows: in sec. II, we review the consequences of CP\textsuperscript{\mu\tau} on the neutrino mass matrix and PMNS. Sections III and IV present general consequences of CP\textsuperscript{\mu\tau} symmetry on neutrino-less double beta decay and leptogenesis. In sec. V, we introduce the generalized CP like symmetries and show how CP\textsuperscript{\mu\tau} symmetry emerges as the trivial automorphism of gauged \( U(1)_{\mu-\tau} \) symmetry. We then present a multi-Higgs implementation of the symmetry in sec. VI, together with some phenomenological implications. Our paper is summarized in sec. VII. The appendices contain the proof of the uniqueness of CP\textsuperscript{\mu\tau}, the CP\textsuperscript{\mu\tau} symmetry in the real basis and another realization of the idea where \( G_l \times G_\nu \) is exact at high energies, which uses heavy vector like fermions instead of extra weak scale Higgs doublets.

II. MAXIMAL  \( \theta_{23} \) AND DIRAC CP PHASE FROM CP\textsuperscript{\mu\tau}: A REVIEW

The latest global fits \cite{1, 2} of neutrino parameters still allows maximal atmospheric angle \( \theta_{23} = 45^\circ \) within \( 2\sigma \) and also point to a preference for negative values for the Dirac CP phase, \( -180^\circ < \delta_{CP} < 0 \). It was pointed out in \cite{9} that maximal \( \theta_{23} \) and maximal \( \delta_{CP} \), i.e.,

\[
\theta_{23} = \pi/4 \quad \text{and} \quad \delta_{CP} = \pm \pi/2 ,
\]

(1)

follow from the neutrino mass matrix invariant under CP\textsuperscript{\mu\tau} symmetry. In the flavor basis (fixed by some \( G_l \)), it corresponds to the relation:

\[
X^T M_\nu X = M_\nu^* ,
\]

(2)
where

\[ X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \]  

(3)

Clearly, this symmetry can be implemented in the neutrino sector as the composition of $\mu\tau$ interchange symmetry with CP conjugation. We will show a simple and natural setting where this symmetry survives in the neutrino sector but is broken in the charged lepton sector.

Let us review some aspects of CP$^{\mu\tau}$. First, the symmetry (2) implies a neutrino mass matrix of the form [9, 13]

\[ M_\nu = \begin{pmatrix} a & d & d^* \\ d & c & b \\ d^* & b & c^* \end{pmatrix}, \]  

(4)

where $a, b$ are real whereas $c, d$ are complex a priori. It is necessary that both $c \neq 0$, $d \neq 0$, and $\text{Im}(d^2c^*) \neq 0$, to ensure $\theta_{13} \neq 0$ [9, b] because a rephasing transformation can turn $M_\nu$ to a matrix invariant under the simpler (unitary) $\mu\tau$ interchange symmetry.

One can show that a matrix of the form (4) can be always diagonalized by a matrix of the form [9]

\[ U_0 = \begin{pmatrix} u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \\ w_1^* & w_2^* & w_3^* \end{pmatrix}, \]  

(5)

where $u_i$ are real and conventionally positive. Application of complex conjugation on $M_\nu$ and $U_0$ shows that the diagonalization of (4),

\[ U_0^T M_\nu U_0 = \text{diag}(\pm m_i), \]  

(6)

already leads to real diagonal entries, so that the Majorana phases are trivial, i.e., either 1 or $i$. Therefore, we can write for the complete diagonalization matrix,

\[ U_\nu = U_\nu^{(0)} K_\nu, \]  

(7)

where $U_\nu^{(0)}$ has the form (5) and $K_\nu$ is diagonal and contains the Majorana phases $(K_\nu)_{ii} = 1$ or $i$. We denote the different possibilities by

\[ \text{diagonal of } K_\nu^2 \sim (++, (--), (+-), (+-)) \text{ or } (+-), \]  

(8)
which correspond to the CP parities of $\nu_{iL}$ assuming $\text{CP}^{\mu\tau}$.

It is easy to see that $U_0$ obeys

$$|(U_0)_{\mu j}| = |(U_0)_{\tau j}|, \quad \text{for } j = 1, 2, 3.$$  \hspace{1cm} (9)

The equality for $j = 3$ signals maximal $\theta_{23}$. The equality for $j = 1, 2$, easily seen in the standard parametrization, leads to \cite{9}

$$\sin \theta_{13} \sin \delta_{\text{CP}} = 0.$$  \hspace{1cm} (10)

This signals maximal $\delta_{\text{CP}}$ since $\theta_{13} \neq 0$.

### III. NEUTRINO-LESS DOUBLE BETA DECAY IN THEORIES WITH $\text{CP}^{\mu\tau}$

For Majorana neutrinos, there is a nonzero probability of neutrino-less double beta decay to occur. The rate depends on the square of the modulus of

$$m_{ee} \equiv \sum_i m_i U_{ei}^2.$$  \hspace{1cm} (11)

In general, this quantity depends on the Dirac CP phase (depending on the convention) and Majorana CP phases. For the theory invariant under $G_{\nu} = \mathbb{Z}_2^{\text{CP}}$ and $G_{l} \subset U(1)_{\mu-\tau}$, $\delta_{\text{CP}} = \pm \pi/2$, only a discrete choice of possibilities for the Majorana phases remain. We obtain

$$m_{ee} = \sum_i m'_i U_{ei}^{(0)}^2,$$  \hspace{1cm} (12)

where $U_{ei}^{(0)}$ are real positive quantities fixed by $\theta_{12}, \theta_{13}$, cf. (7), and $m'_i = \pm m_i$ are the light neutrino masses with its CP parities.

In Fig. 1 we show the discrete possibilities for $|m_{ee}|$ as a function of the lightest neutrino mass $m_0$ ($m_1$ for NH and $m_3$ for IH). We vary $\Delta m^2_{21}, \Delta m^2_{31}, \theta_{12}, \theta_{13}$ within their 3-$\sigma$ allowed values \cite{2} ($\theta_{23} = \pi/4$ is fixed from symmetry). We can see that some CP parities can be distinguished if independent information on the mass hierarchy and sufficiently precise information of the absolute mass scale is known. Specially for IH, we can distinguish between $(+++)/(+−)$ CP parities for $\nu_L$ and $(-−+)/(+−)$. For NH, some cases can be distinguished for some ranges of the absolute mass scale. For example the $\tilde{S}_4 \ (A_4 \rtimes \mathbb{Z}_2^{\text{CP}})$ model of Ref. [11, b] lies in the lower (NH) yellow $(-++)$ band. With enough precision, even in the quasi-degenerate spectrum we can distinguish between $(+++)/(+−)$ and $(-−+)/(+−)$ CP parities. Notice that some bands would completely overlap in the $m_0 \to 0$ limit. Regions similar to the ones we show here can be seen, in the general
phenomenological analysis of Ref. [17] (see its Fig. 2 with dashed curves denoted as (±±)), but without the underlying symmetry discussion. Note that this predictions for neutrinoless double beta decay is the same as for the strictly CP conserving case at low energies but in our case the Dirac CP phase is maximal instead of being 0 or π, a fact that can be distinguished in future oscillation experiments.

![Diagram](image)

**FIG. 1:** $|m_{ee}|$ as a function of the lightest mass $m_0$ ($m_1$ for NH and $m_3$ for IH) for CP parities $K_{\mu\mu}^2$ of the light neutrinos $\nu_{iL}$: (+ + +) (green), (− + +) (yellow), (+ − +) (blue) and (+ + −) (red). Darker colors denotes NH and lighter colors denotes IH. For the latter, light blue and yellow (light red and green) are largely overlapped. We use the 3-σ allowed ranges for $\Delta m^2_{21}$, $\Delta m^2_{31}$, $\theta_{12}$, $\theta_{13}$ of Ref. [2]. The vertical dashed lines shows the current bound coming from the cosmological data on $\sum m_i$; cf. (13).

Also shown in Fig. 1 are the cosmological bounds for $m_0$,

$$
\text{NH : } m_0 = m_1 < 0.0716 \text{ eV},
\text{IH : } m_0 = m_3 < 0.0665 \text{ eV}.
$$

(13)

These values are obtained from the cosmological bound of $\sum m_i < 0.23$ at 95% C.L. reported by the Planck collaboration [18] when 3-σ range of $\Delta m^2_{21}$ and $\Delta m^2_{31}$ are considered.

**IV. LEPTOGENESIS**

Neutrino mass mechanisms are widely considered to have a connection to the origin of matter via leptogenesis [19]. In this section, we discuss this in the class of models we are discussing
here. The first consideration of leptogenesis with $\text{CP}^{\mu\tau}$ symmetry was made in [9, b]. The authors concluded that leptogenesis is not possible because $\text{CP}^{\mu\tau}$ invariance of the neutrino sector ensured that all elements $(\lambda\lambda^\dagger)_{ij}^2$ were real leading to vanishing CP asymmetry, with $\lambda$ being the $N_R$ Yukawa coupling in the basis where the RHNs are mass eigenstates. Such a conclusion, however, is only valid for the case where the heavy neutrinos are hierarchical and charged lepton flavor effects are unimportant (the so-called one-flavor approximation), i.e., for $T \sim M_1 \gtrsim 10^{12}\text{GeV}$, where $M_1$ is the mass of the lightest right-handed neutrino. Below that temperature, the tau lepton enters into thermal equilibrium due to its Yukawa interaction with $\tau_R$ and flavor effects must be considered (the so-called flavored leptogenesis [20]). We will see that successful leptogenesis is possible even with $\text{CP}^{\mu\tau}$ symmetry in the intermediate range $10^9\text{GeV} \lesssim M_1 \lesssim 10^{12}\text{GeV}$ if flavor effects are taken into account. Therefore, we do not need small $\text{CP}^{\mu\tau}$ breaking for successful leptogenesis as in Ref. [15, b]. Surprisingly, $\text{CP}^{\mu\tau}$ symmetry seems to preclude successful leptogenesis for $M_1 \lesssim 10^9\text{GeV}$ for hierarchical heavy right-handed neutrinos because both $\tau$ and $\mu$ flavors are in thermal equilibrium; see Sec. IV A. This result holds even if the resonant enhancement of CP asymmetries due to quasi-degenerate heavy right-handed neutrinos are considered; see Sec. IV B.

To prove our assertion, let us first review the consequences of $\text{CP}^{\mu\tau}$ on the quantities relevant for leptogenesis. It is clear from the form of $U_\nu$ in (7) that $\text{CP}^{\mu\tau}$ implies the CP property

$$X U^*_{\nu} = U_{\nu} K^2_{\nu} \quad \text{or} \quad U^*_{\nu} = X \dagger U_{\nu} K^2_{\nu}. \quad (14)$$

This can be also generically inferred from the relation (2). As can be checked explicitly in the CP-basis, $K^2_{\nu}$ corresponds to the CP parities of $\nu_i L$ considering $\text{CP}^{\mu\tau}$ is conserved in the neutrino sector. A similar relation is also valid for $U_R$, the matrix that diagonalizes $M_R$:

$$U^*_R = X U_R K^2_{R} \quad \text{and} \quad U_R = U_R^{(0)} K_R. \quad (15)$$

Note that the previous relation assumes $M_R$ is in the symmetry basis. We also assume the charged lepton mass matrix (squared) is diagonal (flavor basis) so that the PMNS matrix is $U = U_\nu$.

Let us write the type-I seesaw Lagrangian in the form

$$-\mathcal{L} = y_\alpha \bar{L}_\alpha H l_{\alpha R} + \bar{N}_{iR}^{\lambda} \lambda^{\alpha} \tilde{H} \dagger L_\alpha + M_i \bar{N}_{iR} N_i^c, \quad (16)$$

where the sum of repeated indices is implicit. In this basis, the CP asymmetries depend only on $\lambda$ and the heavy masses $M_i$.

In the symmetry basis, $\lambda_{\text{sym}}$ obeys

$$X \dagger \lambda_{\text{sym}} X = \lambda_{\text{sym}}^* . \quad (17)$$
In the basis of (16), we have

$$\lambda = U_R^\dagger \lambda_{\text{sym}},$$

(18)

and it obeys

$$\lambda^* = K_R^2 \lambda X.$$  

(19)

### A. Hierarchical heavy neutrinos

We can see the consequences of CP$^{\mu\tau}$ on leptogenesis for the case where the right-handed neutrinos $N_{iR}$ have hierarchical masses and only the decay of lightest state $N_1$ is relevant for leptogenesis. Our discussion, however, apply also to cases where the hierarchy is mild. In our notation, the flavored CP asymmetries for the decay $N_1 \to l_\alpha + \phi, \alpha = e, \mu, \tau$, read (see e.g. [19])

$$\epsilon_\alpha = \frac{1}{8\pi(\lambda\lambda^\dagger)_{11}} \sum_{j \neq 1} \left\{ \text{Im} \left[ (\lambda\lambda^\dagger)_{j1} \lambda_{j\alpha} \lambda_{1\alpha}^* \right] g(x_j) + \text{Im} \left[ (\lambda\lambda^\dagger)_{1j} \lambda_{j\alpha} \lambda_{1\alpha}^* \right] \frac{1}{1 - x_j} \right\},$$

(20)

where $x_j \equiv M_j^2/M_1^2$ and

$$g(x) \equiv \sqrt{x} \left[ \frac{1}{1 - x} + 1 - (1 + x) \ln \left( \frac{1 + x}{x} \right) \right] \equiv \frac{\sqrt{x}}{1 - x} + f(x),$$

(21)

where $f(x)$ is the vertex function. The part proportional to $f(x)$ corresponds to the one-loop vertex contribution while the rest corresponds to the self-energy contribution for $N_R$. We are assuming that $N_{iR}$ masses are hierarchical, i.e., $M_3 - M_1 > M_2 - M_1 \gg \Gamma_1$. We comment on the possibility of resonant enhancement in Sec. IV B.

Now if we apply the symmetry properties (19) of $\lambda$ in (20), we conclude that

$$\epsilon_e = 0, \quad \epsilon_\mu = -\epsilon_\tau.$$  

(22)

For example, note that $\lambda_{j\mu}^* = K_{Rj}^2 \lambda_{j\tau}$ and $K_{Rj}^4 = 1$ for all $j$. The CP$^{\mu\tau}$ symmetry also relates the $\mu$ and $\tau$ washout parameters as

$$\tilde{m}_\mu = \tilde{m}_\tau,$$

(23)

where

$$\tilde{m}_\alpha \equiv \frac{|\lambda_{1\alpha}|^2 v^2}{M_1},$$

(24)
and \( v = 174 \text{GeV} \) in the SM; they quantify the strength of \( N_1 \) decay and also its inverse decays into \( L_\alpha \). Therefore, it is clear that the CP asymmetries for the \( N_1 \) decaying into all flavors,

\[
\epsilon^{(1)} = \epsilon_e + \epsilon_\mu + \epsilon_\tau,
\]

is vanishing and leptogenesis at the high scale \( T \sim M_1 \gtrsim 10^{12} \text{GeV} \) can not proceed.

When \( M_1 \lesssim 10^{12} \text{GeV} \), the tau Yukawa interactions enter in equilibrium (also the muon flavor below \( 10^9 \text{GeV} \)) and distinct leptonic flavors may contribute differently to leptogenesis. In this case, the residual baryon asymmetry can be written as \([19, 20]\)

\[
Y_{\Delta B} \simeq \frac{12}{37} Y_{N_1}^{\text{eq}} \sum_\alpha \epsilon_\alpha \eta_\alpha,
\]

where the sum over \( \alpha \) is performed only over the flavors that can be resolved by interactions at the period of leptogenesis (one, two or three flavors). The quantity \( Y_{N_1}^{\text{eq}} \) is the thermal density of \( N_1 \) per total entropy density and is given by \( Y_{N_1}^{\text{eq}} = \frac{135\epsilon(3)}{4\pi^2 \alpha} \approx 3.9 \times 10^{-3} \), where the last numerical value is for the SM degrees of freedom below the \( N_1 \) mass \( (\alpha = 106.75) \). The factor 12/37 corresponds to the reduction of asymmetry in \( \Delta_\alpha = B/3 - L_\alpha \) to \( B - L \) in the SM due to spharelons\(^1\).

When \( 10^9 \lesssim M_1 \lesssim 10^{12} \text{GeV} \) only the \( \tau \) Yukawa interactions are in equilibrium and then only the \( \tau \) flavor and its orthogonal combination are resolved by interactions. In this case, the asymmetry in (26) can be approximated by

\[
Y_B \simeq \frac{12}{37} Y_{N_1}^{\text{eq}} \left[ \epsilon_2 \eta(\frac{417}{589} \tilde{m}_2) + \epsilon_\tau \eta(\frac{390}{589} \tilde{m}_\tau) \right],
\]

where \( \epsilon_2 = \epsilon_e + \epsilon_\mu \), \( \tilde{m}_2 = \tilde{m}_e + \tilde{m}_\mu \), and

\[
\eta(\tilde{m}_\alpha) \simeq \left( \frac{\tilde{m}_\alpha}{2.1 m_*} \right)^{-1} + \left( \frac{m_* / 2}{\tilde{m}_\alpha} \right)^{-1.16}
\]

(28)

The mass \( m_* \equiv \frac{16\pi^2 v^2}{3M_{\text{pl}}} \sqrt{\frac{2\pi}{\alpha}} \approx 1 \text{meV} \) quantifies the expansion rate of the Universe. The factors 417/589 and 390/589 correspond to the diagonal entries of the \( A \) matrix and quantifies the effects of flavor in the washout processes when changing from the asymmetry in lepton doublets to asymmetries in \( \Delta_\alpha \), see e.g. \([19]\). We can see that the properties (22) of CP\( \mu\tau \) leads to a partial cancellation of the baryon asymmetry in (27) but it is nonzero because the \( \tau \) flavor and its orthogonal combination are washed out differently. The question is then quantitative. We show some cases leading to successful leptogenesis in Sec. IV C.

\(^1\) For the case of two Higgs doublets, this factor is 10/31 but numerically very close.
For $M_1 \lesssim 10^9 \text{GeV}$, the $\mu$ Yukawa interactions are also fast enough so that the three flavors can be resolved. For such a low scale, the CP asymmetries are usually too small to lead to a successful leptogenesis. In the $\text{CP}^{\mu \tau}$ symmetric case, the baryon asymmetry is in fact vanishing. With the three flavors resolved, the baryon asymmetry can be approximated by

$$Y_B \simeq \frac{12}{37} \times Y_{N_1}^{\text{eq}} \times \left[ e_\eta \left( \frac{151}{179} \tilde{m}_e \right) + e_\mu \eta \left( \frac{344}{537} \tilde{m}_\mu \right) + e_\tau \eta \left( \frac{344}{537} \tilde{m}_\tau \right) \right].$$ (29)

Due to the properties (22) and (23), the baryon asymmetry vanishes within this analytic approximation. Note that this is true even for mild hierarchies for $M_i$ and the leptogenesis scale cannot be lowered by tuning the values of the masses.

Therefore, as long as $\text{CP}^{\mu \tau}$ symmetry is valid at the leptogenesis scale, the only temperature range for which leptogenesis might be viable for hierarchical $N_{iR}$ is the intermediate scale $T \sim M_1$ where

$$10^9 \text{GeV} \lesssim M_1 \lesssim 10^{12} \text{GeV}.\quad (30)$$

It is worth emphasizing that CP violation in our case comes from maximal Dirac CP phase of the low-energy sector thereby giving a symmetry setting for some scenarios of leptogenesis driven by low-scale CP violation [4]. All these properties follow from the $G_l$ conservation in the charged lepton sector and $\text{CP}^{\mu \tau}$ conservation of the neutrino sector; see Sec. V.

### B. Resonant leptogenesis

For the usual type-I seesaw scenario, the CP asymmetry produced by $N_1$ decay usually decreases as we lower the mass of $N_1$ since the Yukawa couplings decrease and also the washout effects get stronger. For $M_1 \ll 10^9 \text{GeV}$, successful leptogenesis is not possible for hierarchical $N_{iR}$. However, when some of the masses, say $M_1$ and $M_2$, are quasi-degenerate, it is possible to resonantly enhance the CP asymmetry leading to the resonant leptogenesis scenario [21]. In fact, (20) is singular in that limit because perturbation theory breaks down. We can regulate such a behavior by resummation methods [21]. We will see in the following that $\text{CP}^{\mu \tau}$ still leads to (22) and it largely suppresses the CP asymmetries if $\mu$ and $\tau$ flavors have equal washout strengths.

Suppose $M_3 \gg M_2 \approx M_1$ and also the resonant condition

$$M_2 - M_1 \sim \Gamma_{1,2} \ll M_{1,2}. \quad (31)$$

The resummed flavored CP asymmetry for $N_1 \to L_\alpha + \phi$, neglecting $M_3$ and vertex contributions,
can be approximated by [21] (see also [22])

$$
\epsilon^{(1)}_{\alpha} \approx f^{12}_{\text{reg}} \frac{\text{Im}[(\lambda \lambda^\dagger)_{21} \lambda^\dagger_{1\alpha} \lambda_{2\alpha}] + \frac{M_1}{M_2} \text{Im}[(\lambda \lambda^\dagger)_{12} \lambda^\dagger_{1\alpha} \lambda_{2\alpha}]}{(\lambda \lambda^\dagger)_{11}(\lambda \lambda^\dagger)_{11}},$$

(32)

where

$$f^{12}_{\text{reg}} \equiv \frac{(M_1^2 - M_2^2)M_1 \Gamma_2^{(0)}}{(M_1^2 - M_2^2)^2 + (M_1 \Gamma_2^{(0)})^2}.$$  

(33)

One can see that (32) is a regulated version of (20), neglecting the contribution of $f(x)$ (vertex) and regulating the function $\sqrt{x}/(1 - x)$ by $f^{12}_{\text{reg}}$. See [22] for a discussion about other regulator functions used in the literature. The $N_2$ decay is also resonantly enhanced as

$$\epsilon^{(2)}_{\alpha} \approx \epsilon^{(1)}_{\alpha}.$$  

(34)

Thus with appropriate $\lambda$ we can have an enhanced CP asymmetry of order one compared to $\epsilon \sim 10^{-6}$ required for successful leptogenesis in the conventional case.

Now, since the Yukawa structure in (32) is the same as in the hierarchical case (20), the consequences of CP$^{\mu \tau}$ are the same: the flavored CP asymmetries $\epsilon^{(1)}_{\alpha}, \epsilon^{(2)}_{\alpha}$ obey (22). Therefore, if the effects of washout for $\mu$ and $\tau$ flavors are the same, the CP asymmetries for $\mu$ and $\tau$ will cancel each other precluding leptogenesis even when $M_1 \sim M_2 \lesssim 10^9\text{GeV}$. This would be the case in the analytic approximation (29) arising from the classical Boltzmann equation solutions. However, to properly quantify the baryon asymmetry, including washout effects, a full flavored and quantum description is necessary and we will not address it here. Moreover, when the three right-handed neutrinos are quasi-degenerate, a more complicated expression holds for the CP asymmetries [22] and it is not clear if the properties (22) will still hold.

C. Quantitative analysis and $N_3$ decoupled case

To assess quantitatively if leptogenesis can be successful with $G_F = G_t \times G_\nu$ symmetry, we can use the Casas-Ibarra parametrization that uses a complex orthogonal matrix $R$:

$$R = \hat{M}_R^{-1/2}(\lambda v)U_{\nu}M_{\nu}^{-1/2},$$

(35)

where the hatted matrices correspond to the diagonalized matrices and $\lambda$ is in the basis (16).

We can see that the CP$^{\mu \tau}$ symmetry implies

$$R^* = K_R^2 R K_{\nu}^2.$$  

(36)
This means that there is no CP violating effect coming from $R$ when there is $\text{CP}^{\mu\tau}$ symmetry. A similar result was found for usual CP symmetry in [4]. CP invariance in $R$ is more apparent if we eliminate the potential purely imaginary $i$ factors as in

$$R = K_R^* R^{(0)} K_\nu.$$

(37)

where $R^{(0)}$ is a real matrix, as can be seen from the properties of $R$. Therefore, $R^{(0)}$ obeys

$$R^{(0)\dagger} K_R^2 R^{(0)} = K_\nu^2, \quad R^{(0)} K_R^2 R^{(0)\dagger} = K_R^2.$$

(38)

This is just the defining relation for a real orthogonal matrix when $K_R^2 = K_\nu^2 = 1$ or a real hyperbolic2 $R^{(0)}$ in $O(2,1)$, when $K_R^2 = K_\nu^2 = \text{diag}(-1,1,1)$ or any independently permuted diagonal entries for $K_R^2$ or $K_\nu^2$. There is no other possibility and we conclude that the CP parities of $\nu_{iL}$ ($N_{iR}$) are either all equal or only one is different.

When $M_i$ are hierarchical, the flavored CP asymmetries in (20) can be approximated to [4, 19]

$$\epsilon_\alpha = - \frac{3 M_1}{16 \pi v^2} \frac{\text{Im}\{\sum_{i,j} \sqrt{m_i m_j} m_j R_{ij} U_{\alpha i}^* U_{\alpha j}\}}{\sum_j m_j |R_{ij}|^2},$$

(39)

where $M_1 \ll M_2, M_3$ is assumed. One can check (25) also in this form from the properties for $R$ and $U_{\alpha j}$ in Eqs. (14) and (37). Hence we only need $\epsilon_\tau$.

If we eliminate the CP parities $K_\nu, K_R$, we obtain

$$\epsilon_\tau = - \frac{3 M'_1}{16 \pi v^2} \frac{\sum_{i,j} \sqrt{m_i m_j} m_j' R_{ij}^{(0)} U_{\tau i}^{(0)*} U_{\tau j}^{(0)}}{\sum_j m_j (R_{ij}^{(0)})^2},$$

(40)

where $M'_1 = (K_R^2)_{11} M_1 \equiv \pm M_1$ and $m'_j \equiv (K_\nu^2)_{jj} m_j = \pm m_j$ are the masses including the CP parities. We can simplify further as

$$\epsilon_\tau = \frac{3 M'_1}{16 \pi v^2 \tilde{m}} \frac{J_{\text{CP}}}{|U_{e1} U_{e2} U_{e3}|} \left\{ B_{12} R_{11}^{(0)} R_{12}^{(0)} - B_{13} R_{11}^{(0)} R_{13}^{(0)} + B_{23} R_{12}^{(0)} R_{13}^{(0)} \right\},$$

(41)

where

$$B_{ij} \equiv \sqrt{m_i m_j} (m'_j - m'_i) |U_{ek}|,$$

$$\tilde{m} \equiv \sum_\alpha \tilde{m}_\alpha = \sum_j m_j |R_{1j}|^2 = \sum_\alpha \sum_{ij} \sqrt{m_i m_j} R_{1i}^{(0)} R_{1j}^{(0)} \text{Re}(U_{\alpha i}^{(0)*} U_{\alpha j}^{(0)}),$$

(42)

with $(ijk) = (123)$ or permutations and $J_{\text{CP}}$ is the Jarlskog invariant

$$J_{\text{CP}} \equiv \text{Im}[U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^*].$$

(43)

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2 Lorentz transformations in 2+1 dimensions.
To obtain (41), we have multiplied and divided by $U_{11}^{(0)}U_{12}^{(0)}U_{13}^{(0)} = |U_{11}U_{12}U_{13}|$ and included the appropriate factors inside the imaginary part. Notice that we are assuming CP$^{\mu\tau}$ and (5). We also used the fact that the Jarlskog invariant can be written in terms of different entries of $U$.

In the standard parametrization, the Jarlskog invariant is

$$J_{CP} = (s_{13}^2 c_{13}^2)(s_{12}^2 c_{12})(s_{23}^2 c_{23}) \sin \delta_{CP}.$$  \hspace{1cm} (44)

Therefore, in the CP$^{\mu\tau}$ symmetric case, we obtain

$$\frac{J_{CP}}{|U_{e1}U_{e2}U_{e3}|} = \pm \frac{1}{2},$$

for $\delta_{CP} = \pm \pi/2$, respectively [9, b]. We can see from (41) that $\epsilon_\tau$ depends only on the low-energy CP violation coming from $J_{CP}$. Other than that, $\epsilon_\tau$ only depends on the three $R_{1i}^{(0)}$, on the absolute neutrino scale and the discrete choice of $\nu_{iL}$ CP parities.

We can finally use $Y_B$ in (27), $\epsilon_\tau$ in (41) and $\tilde{m}_a$ in (42) to calculate the baryon asymmetry produced by leptogenesis using the Casas-Ibarra parametrization. To simplify the numerical study even further, we employ the approximation where $M_3 \gg M_1, M_2$ and $N_{3R}$ decouples. In that case, the $R$ matrix can be written as [23]

$$\begin{cases}
\text{NH: } R = \begin{pmatrix} 0 & * & * \\ 0 & * & * \\ 1 & 0 & 0 \end{pmatrix}, & m_1 \to 0, \\
\text{IH: } R = \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & 1 \end{pmatrix}, & m_3 \to 0.
\end{cases}$$

Then we can denote the different cases of CP parities for $N_{iR}$ and $\nu_{iL}$ as in Table I. In the decoupling limit, when $R$ is not real, we only have the cases [cf. (38)]

$$\begin{align*}
\text{NH: } & (31), (12), (13), (21), (23) \\
\text{IH: } & (33), (11), (12), (21), (22).
\end{align*}$$

\hspace{1cm} (48)
Note that, differently from the strength of double beta decay, leptogenesis also depends on the CP parities of the heavy right-handed neutrinos.

We show our results for leptogenesis induced by hierarchical $N_{iR}$ and decoupled $N_{3R}$ in Figs. 2 and 3. We use the maximum possible value for $M_1$ within flavored leptogenesis with $\tau$-flavor in equilibrium: $M_1 = 10^{12}$GeV. Given the parametrization in (39) ($M_1$ only appears linearly in the prefactor), lowering $M_1$ leads to proportional lowering of $\epsilon_\tau$ and also $|Y_B|$. Plots with smaller $M_1$ can be obtained by scaling down the lines proportionally. Note that $\theta_{23} = 45^\circ$ (and $\delta_{CP} = \pm \pi/2$) is fixed from symmetry and this makes the curves of $|Y_B|$ smoother, with less possibility of cancellations.

Let us begin with Fig. 2, left. We treat the case where all CP parities are equal for light and heavy neutrinos, i.e., cases (00)-NH and (00)-IH, and the figure shows the ratio of the baryon asymmetry of the model over its experimental value, $Y_B/Y_{B_{\text{exp}}}$, in terms of $R_{12}$. Since the third $N_{3R}$ decouples, the same plots also applies to the case where the CP parity of $N_{3R}$ is different from the rest, i.e., $K_R = \text{diag}(1, 1, i)$. The property in (38) requires that we are only left with the cases (31)-NH [same as (00)-NH] and (33)-IH [same as (00)-IH]. Thus successful leptogenesis can happen for normal hierarchy [(00)-NH and (31)-NH] but not for the inverted hierarchy [(00)-IH and (33)-IH]. For normal hierarchy, we can read from the plot that the scale of $M_1$ can be lowered at most by a factor $|Y_B|_{\text{max}}/Y_{B_{\text{exp}}} = 15.3$ and we need $0.65 \times 10^{11} \lesssim M_1 \lesssim 10^{12}$GeV. A similar situation of leptogenesis induced solely by $\delta_{CP}$ was also considered in Ref. [4]. Here we furnish a symmetry justification for that case.

In Fig. 2, right, the remaining cases for NH are considered, i.e., (12)/(23) and (13)/(22). We show the ratio $|Y_B|/Y_{B_{\text{exp}}}$ in terms of $\xi$, which parametrizes the nonzero $R_{1i}$. The cases (12) and (23) [(13) and (22)] are represented by the same blue (green) curve. We can see that the cases (13)-NH and (22)-NH do not lead to successful leptogenesis. For (12)-NH and (23)-NH, successful leptogenesis is also possible for $0.5 \times 10^{11} \lesssim M_1 \lesssim 10^{12}$GeV ($|Y_B|_{\text{max}}/Y_{B_{\text{exp}}} = 20.2$).

Finally, Fig. 3 shows the remaining cases for IH: (11)/(12) and (12)/(22). We show again the ratio $|Y_B|/Y_{B_{\text{exp}}}$ in terms of $\xi$, which parametrizes the nonzero $R_{1i}$. In all cases leptogenesis is possible for slightly different ranges for $M_1$. For (11)/(21), we need $0.44 \times 10^{11} \lesssim M_1 \lesssim 10^{12}$GeV ($|Y_B|_{\text{max}}/Y_{B_{\text{exp}}} = 22.8$). For (12)/(22), $2.3 \times 10^{11} \lesssim M_1 \lesssim 10^{12}$GeV ($|Y_B|_{\text{max}}/Y_{B_{\text{exp}}} = 4.4$). If we assume negative $\delta_{CP}$, preferred from global fits [1, 2], then the range for case (11)/(21) shrinks almost to the single value $M_1 \approx 10^{12}$GeV because the right portion of the curve leads to anti-matter dominance instead of matter dominance; see figure.

We conclude that successful leptogenesis is not possible for the cases (00)-IH, (33)-IH, (13)-NH and (22)-NH. Therefore, for IH, successful leptogenesis requires that the CP parity of $\nu_{1L}$ or $\nu_{2L}$
FIG. 2: Left: ratio of $|Y_B|$ over $Y_{B\text{exp}} = 8.75 \times 10^{-11}$ as a function of $R_{12}$ for $M_1 = 10^{12}$GeV in the $N_{3R}$ decoupling limit; the blue curve corresponds to both (00)-NH and (31)-NH, with $R_{11} = 0$, $|R_{12}|^2 + |R_{13}|^2 = 1$ and $R_{13} > 0$, while the green curve corresponds to both (00)-IH and (33)-IH, with $R_{13} = 0$, $|R_{11}|^2 + |R_{12}|^2 = 1$ and $R_{11} > 0$. Right: ratio of $|Y_B|$ over $Y_{B\text{exp}}$, for $M_1 = 10^{12}$GeV and in the $N_{3R}$ decoupled limit, as a function of $\xi$ in $R_{1i} = (0, \cosh \xi, -i \sinh \xi)$ for (12)-NH (blue) and $R_{1i} = (0, -i \sinh \xi, \cosh \xi)$ for (13)-NH (green); the blue (green) curve also describes the case (23)-NH [(22)-NH], with $R_{12}, R_{13}$ exchanged and $\xi \to -\xi$. We use the best-fit values of Ref. [2] for $\theta_{12}, \theta_{13}$ and the squared mass differences. The solid curves correspond to $Y_B > 0$ for $\delta_{\text{CP}} = -90^\circ$ (preferred, cf. [1, 2]) while the dashed curves correspond to $Y_B > 0$ for $\delta_{\text{CP}} = 90^\circ$.

be different of the rest. On the other hand, the cases (00)-IH and (33)-IH correspond to the largest value of $|m_{ee}|$ in Fig. 1. If this value of $|m_{ee}|$ were measured in future experiments, then CP\textsuperscript{\mu\tau} symmetric leptogenesis with hierarchical right-handed neutrinos and decoupled $N_{3R}$ is excluded as the origin of the present baryon asymmetry of the Universe.

V. SYMMETRY CHOICE AND PROPERTIES

We now turn to a theoretical discussion of CP\textsuperscript{\mu\tau} symmetry and follow it up in the subsequent section with a model realization. As already noted, a much pursued idea in the neutrino literature is that flavor symmetries may be behind the structure of masses and mixing angles of the leptons [5]. A very predictive setting consists of assuming that the charged lepton sector and neutrino sectors are invariant under different groups $G_l$ and $G_\nu$, respectively. These groups are then part of a larger group $G_F$ that may be entirely or partially valid at higher energies (the latter if some factor appears accidentally). A less ambitious variations of the above idea is (i) to allow more free parameters by requiring less symmetry for $G_\nu$ or $G_l$ or (ii) including generalized CP (GCP) symmetries as part
of the flavor group. Here we pursue a direction where we identify a minimal setting with $G_l$ being abelian and $G_\nu$ being a GCP transformation. We find that we are largely restricted to $\mathbb{CP}^{\mu\tau}$ for $G_\nu$.

To discuss our strategy, we assume Majorana neutrinos, with the leptonic Lagrangian below EWSB in the flavor basis to be

$$-\mathcal{L} = m_\alpha \bar{l}_\alpha L + \nu_\alpha^c L (M_\nu)_{\alpha\beta} \nu_\beta L + h.c., \quad (49)$$

where the implicit sum over $\alpha = e, \mu, \tau$ is understood. Note that in the flavor basis, the interaction with $W$ gauge bosons is diagonal, $W_\mu \bar{l}_\alpha \gamma^\mu \nu_\alpha L$, and the PMNS matrix comes from the diagonalization of $M_\nu$.

It is clear that the charged lepton part of (49) is invariant under three separate family lepton numbers $L_e, L_\mu, L_\tau$, that should be broken in the neutrino part. Although these symmetries are automatically present whenever we diagonalize the charged lepton mass matrix [24], we assume some subgroup of it, $G_l$, is a symmetry of the theory at higher scales for the charged lepton sector (we allow for the fact that it may be accidental). Since charged leptons and left-handed neutrinos come from the same leptonic doublet $L_\alpha$ above the EW scale, the group $G_\nu$ should also act on the same space. Let us look for the minimal $G_l$ and $G_\nu$ where the former is abelian and the latter is a GCP.
We assume $G_l$ has a generic element acting on $L_\alpha = (l_{\alpha L}, \nu_{\alpha L})$ of the form (more generic forms are considered in appendix A)

$$G_l : \quad T = \begin{pmatrix} 1 & e^{i\theta} \\ e^{-i\theta} & 1 \end{pmatrix}. \quad (50)$$

For the moment, $G_l$ can be a continuous $U(1)$ group (which can therefore be the group $U(1)_{\mu-\tau}$ of $L_\mu - L_\tau$) or a discrete abelian group $\mathbb{Z}_n$, with $n \geq 3$ to avoid degenerate $T$. We are in the basis where $T_L = T_{\mu R} = T$ act all in the same way on left-handed doublets and right-handed singlets but they can be in different irreducible representations (irreps) if $T$ is embedded in a larger group. In this case, $G_l$ will refer to the group acting on the left-handed doublets $L_\alpha$.

Next, we assume the symmetry of the neutrino sector of (49), $G_\nu$, is composed of a generalized CP (GCP) symmetry [25] of the form

$$G_\nu : \quad L(x) \rightarrow XL^CP(\hat{x}), \quad (51)$$

where $L^{CP} = -iCL^*$ is the usual CP transformation and $X$ is a generic $3 \times 3$ unitary and symmetric matrix acting in the space of three families; $\hat{x}$ is the space inversion of $x$. Symmetric $X$ guarantee that the application of (51) two times, leads to the identity. Note that a global rephasing is unimportant for $X$.

Now we demand that $G_l$ and $G_\nu$ close as a group acting on $L_\alpha$. If $G_{l,\nu}$ were unitary and we demanded that the product of its generators be finite, we would obtain von Dyck groups that were extensively studied in this context [26]. Instead, (51) is a GCP symmetry and we should demand that the following composition of $G_\nu$ and $G_l$ induce an automorphism [11, d]:

$$XT^*X^\dagger = T' \in G_l. \quad (52)$$

where $T, T'$ are elements of the same group. This equation can be rewritten as

$$X = T'XT^T \in G_l. \quad (53)$$

This equation and the previous one are not restricted to diagonal $T$ but are valid for any unitary $T$ in any basis.

If $G_l = U(1)$, irrespective of the form in Eq.(50), there are only two possible automorphisms:

$$\begin{align*}
(i) \quad & T' = T^{-1} \\
(ii) \quad & T' = T.
\end{align*} \quad (54)$$

---

3 For a different approach based on $\mathbb{Z}_2 \times \mathbb{Z}_2$, see [27].
These are also automorphisms for all subgroups $\mathbb{Z}_n$ and, in particular, for $n = 3, 4$, they are the only ones. For general $\mathbb{Z}_n$, with $n \neq 3, 4$, additional automorphisms $T' = T^k$ are possible but not with the form (50). For these automorphisms, (53) and the form of $T$ in (50) leads to

$$\begin{align*}
(i) & \quad X = \begin{pmatrix} 1 & \vspace{1em} \\
1 & \vspace{1em} \\
1 & \end{pmatrix} \quad \text{or} \quad (ii) & \quad X = \begin{pmatrix} 1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{pmatrix}, \\
\end{align*} \tag{55}$$

after rephasing some fields appropriately. The first case is just usual CP transformation and we can see that the charged lepton part of (49) is automatically invariant under such a transformation, thus leading to CP invariance in the whole theory. This symmetry prevents CP violation in the leptonic sector and hence we consider it no further. Instead we focus on the second case which we will denote as $\mathbb{CP}^{\mu \tau}$ and it is a well-known GCP symmetry in the literature called $\mu \tau$-reflection symmetry [9]. CP breaking arises in this setting because of the clash between the neutrino part and the charged lepton part in (49): the former is invariant under $\mathbb{CP}^{\mu \tau}$ while the latter is invariant under the usual CP (after rephasing). What distinguishes our work from the previous ones is that in previous works on $\mathbb{CP}^{\mu \tau}$, neither the symmetry $G_l$ was identified nor its relation with $G_\nu$ was stressed as we do here. Also, in later approaches using GCP symmetry with finite flavor symmetries, much more complicated automorphism structures (compared to ours) needed to be studied for some groups [11, 12].

In fact, this settings is much more general: the two forms for $X$ in (55) are unique for any diagonal $T$ and the form for $T$ in (50) is also unique for $G_l = U(1)$ or $G_l = \mathbb{Z}_n$ with prime $n$ or $n = 4, 6$. The uniqueness is up to simultaneous permutations of rows and columns that leaves $T$ diagonal. This result is proved in appendix A, where we also show the first different form for $T$ – it occurs for $\mathbb{Z}_8$.

Permutations of the above structure can be discarded for phenomenological reasons as follows. If we adopt $T$ with nontrivial entries in (11)-(22) [or (11)-(33)], the structure of $X$ would also be interchanged and we obtain the relations $|U_{e3}| = |U_{\mu 3}|$ (or $|U_{e3}| = |U_{\tau 3}|$), which leads (respectively) to

$$\begin{align*}
\mathbb{CP}^{e\mu} : & \quad \tan \theta_{13} = \sin \theta_{23}, \\
\mathbb{CP}^{e\tau} : & \quad \tan \theta_{13} = \cos \theta_{23}. \tag{56}
\end{align*}$$

These relations are completely excluded because of small $\theta_{13}$.

At last, we point out a remarkable property of the symmetries $G_l$ generated by $T$ and $G_\nu = \mathbb{Z}_2^{\mathbb{CP}}$
generated by \( \text{CP}^{\mu\tau} \): the two groups commute.\(^4\) Therefore, our minimal flavor group, including GCP, can be just \( G_F = G_l \times G_\nu \).\(^5\) Generically, when \( G_F \) is a subgroup of \( U(3) \), \( G_l \sim \mathbb{Z}_n \) and \( G_\nu \sim \mathbb{Z}_2 \times \mathbb{Z}_2 \) (or subgroup), their commutation is impossible because all mixing angles are nonzero. For that reason, the whole group containing \( G_l \) and \( G_\nu \) tends to be a large nonabelian group. For example, the minimal group that leads to TBM is \( S_4 \)\(^{[28]} \) of order 24. To fix at least the nonzero \( \theta_{13} \), it must be much larger of order 150 or more\(^{[29]} \).

The commutation of \( G_l \) and \( G_\nu \) seems to have another remarkable feature, i.e., the *vev alignment problem*\(^6\) often encountered in flavor symmetry model building – can be naturally avoided in the scalar sector (without supersymmetry) as our examples below show. The solution is simply that \( G_l \) (\( G_\nu \)) can be broken in the neutrino sector (charged lepton sector) preserving \( G_\nu \) (\( G_l \)) by using \( G_\nu \)-invariant (\( G_l \)-invariant) fields with \( G_l \) (\( G_\nu \)) charge. Hence, only complete invariants of both \( G_l \) and \( G_\nu \) interact in the potential. Thus to avoid the contamination of \( G_l \)-breaking effects in the neutrino sector, we just need to avoid the coupling of \( G_l \) breaking scalars to neutrino fields (be it by additional symmetries). The same is valid for the charged lepton sector.\(^7\)

### VI. MODEL

The main challenge in model building with \( \text{CP}^{\mu\tau} \), is to keep it unbroken in the neutrino sector while breaking it sufficiently in the charged lepton sector (keeping \( G_l \)) to generate \( \mu-\tau \) mass splitting. We have found several ways to meet this challenge. Although our general setting can be implemented in many different ways, some distinction is possible on how \( G_l \) appears and how \( G_\nu \) (GCP) is broken in the charged lepton sector. The different possibilities depend on how \( G_l \) appears, i.e., either

- \( G_l \) comes from a symmetry of the whole theory \( G_F \) at high scales; or
- \( G_l \) appears accidentally.

\(^4\) This property is more transparent in the basis where \( G_l \), in the continuous case, is represented by \( SO(2) \) rather than \( U(1) \) and \( \text{CP}^{\mu\tau} \) is represented by usual CP which commutes; see appendix B.

\(^5\) Obviously \( \text{CP}^{\mu\tau} \) may not commute with other symmetries such as the SM gauge group.

\(^6\) This name is not entirely appropriate in our context (we use one-dimensional irreps, see also appendix B) and we specifically refer here to the possibility of different symmetry breaking scalars interacting through the potential.

\(^7\) To see the advantage of our discussion relative to other flavor groups, we can compare our setting with those based on \( A_4 = (\mathbb{Z}_2 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_3 \) group. We can take \( G_l \simeq \mathbb{Z}_3 \) and \( G_\nu \simeq \mathbb{Z}_2 \) and note that they do not commute. In this case, \( G_\nu \)-invariant fields with \( G_l \) charge exist: take the \( 1' \) or \( 1'' \) singlets (in actual models, additional flavons are necessary to partly break \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) of \( A_4 \)). However, there is no irrep with \( G_\nu \) charge but without \( G_l \) charge in \( A_4 \). In actual \( A_4 \) models, usually triplets \( 3 \) with specifically aligned vevs are used to achieve the breaking \( G_F \to G_l \) in the charged lepton sector (and also in the neutrino sector, hence the alignment problem).
In Sec. V, we saw that the largest group for abelian $G_l$ is $G_l = U(1)_{\mu - \tau}$, which is the continuous symmetry of the combination $L_{\mu} - L_{\tau}$. Variations on this respect involve gauging $U(1)_{\mu - \tau}$ or considering only a $\mathbb{Z}_n$ subgroup of it. The latter would allow embedding our $G_l \times G_{\nu}$ into a larger nonabelian discrete group. Either way, we use the nomenclature of $U(1)_{\mu - \tau}$ to describe our models and only make some comments on variants.

Furthermore, our setting requires that only $G_l$ be broken in the neutrino sector and only $G_{\nu}$ be broken in the charged lepton sector – the conservation of $G_l$ and $G_{\nu}$ in the complementary sectors is what leads to predictions. That is achieved through the vacuum expectation value of scalars that we call as $l$-flavons and $\nu$-flavons. They have the following properties:

- $l$-flavons: all conserve $G_l$ but some need to break $G_{\nu}$. Best candidate is a $G_l$ invariant CP$^{\mu \tau}$ odd scalar (we denote it as $\sigma$).
- $\nu$-flavons: all conserve $G_{\nu}$ but some need to break $G_l$. Best candidates are scalars carrying $G_l$ charge but CP$^{\mu \tau}$ even ($G_{\nu}$-invariant); we denote them as $\eta$’s.

Since the alignment problem in the scalar potential can be avoided, we just need to prevent $l$-flavons ($\nu$-flavons) to couple to the neutrino sector (charged lepton sector). Often that can be achieved by additional symmetries.

One remark with respect to additional symmetries of flavons is in order. For the above setting, it is simpler if flavons do not carry other additive quantum numbers other than those of $G_l$ or $G_{\nu}$. For example, let us consider a $\nu$-flavon $\eta_2$ carrying $L_{\mu} - L_{\tau} = 2$ ($G_l$) so that it couples with $N_3$ as $N_{3R}^{\sigma} \eta_2$. If $\eta_2$ carries no other quantum number, we can define its CP$^{\mu \tau}$ transformation as

$$\text{CP}^{\mu \tau}: \quad \eta_2(x) \rightarrow \eta_2(\hat{x}), \quad (57)$$

i.e., $\eta_2$ is composed of two CP even real scalars. However, if $\eta_2$ also carries $B - L = -2$, and $N_{iR}$ carries $B - L = -1$, then its $N_3$ coupling transform as

$$\text{CP}^{\mu \tau}: \quad N_{3R} N_{3R} \eta_2 \rightarrow N_{2R}^{c\sigma} N_{2R}^{c\sigma} \eta_2, \quad (58)$$

which maps a $B - L$ invariant term to a $B - L$ violating term. In this case, consistency with CP$^{\mu \tau}$ requires the existence of another field $\eta_{-2}$ with charges $L_{\mu} - L_{\tau} = -2$ and $B - L = -2$. The transformation property now would be

$$\text{CP}^{\mu \tau}: \quad \eta_2(x) \rightarrow \eta_{-2}^*(\hat{x}). \quad (59)$$

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8 This possibility is raised for general discrete nonabelian symmetries in [11, d] but no model application was discussed.
This corrects the transformation properties for (58) but allow $\text{CP}^{\mu\tau}$ breaking if $|\langle \eta_2 \rangle| \neq |\langle \eta_{-2} \rangle|$.

Therefore, in our setting, we require that $\nu$-flavons carry no other additive quantum number and hence a continuous $B - L$ symmetry cannot be implemented.

The exception to the above feature is when $\nu$-flavons carry only a $\mathbb{Z}_2$ quantum number. In this case, since the representation is real, (57) can be maintained. This means that a $\mathbb{Z}_4$ subgroup of $U(1)_{B-L}$, acting as

$$\mathbb{Z}_4^L: \text{leptons } \sim i, \quad \nu\text{-flavons } \sim -1,$$

(60)
can still be implemented as a symmetry.

At last, we assume leptogenesis is successful in our setting and we will be seeking high scale $(M_R \gtrsim 10^{11}\text{GeV})$ type-I seesaw implementations.

### A. Multi-Higgs implementation

The model below illustrates the general aspects of our setting. In this case, $G_l$ will be accidental and $G_\nu$ will be broken at a high scale and transmitted to the charged lepton sector to generate the $\mu\tau$ mass splitting. The symmetries at the high scale will be $U(1)_{\mu-\tau} \times \mathbb{Z}_2^\text{CP}$, a gauged $U(1)_{\mu-\tau}$ (which is not exactly $G_l$ at the low scale) and global $\text{CP}^{\mu\tau}$. Another implementation where $G_F = G_l \times G_\nu$ is a symmetry of the high scale theory is given in appendix C.

All lepton fields transform alike under $U(1)_{\mu-\tau}$, with $L_{\mu} - L_{\tau}$ charges

$$L_i \sim l_i \sim N_i \sim (0, 1, -1),$$

(61)

where $L_i, l_i \equiv l_iR, N_i \equiv N_iR$ (here we use $l_i, l_i$ instead of $L_\alpha, l_\alpha$) are the three families of lepton doublets, lepton singlets and right-handed neutrino singlets, respectively. The $\text{CP}^{\mu\tau}$ symmetry also acts similarly for all of the three type of fields, as (51) with the second $X$ in (55), and should swap the second with the third family fields. Note that this GCP symmetry commutes with $U(1)_{\mu-\tau}$ and it does not reverse its charges. The SM group charges, however, are reversed by this GCP symmetry.

We add two more Higgs doublets $\phi_{\pm 2}$ with $U(1)_{\mu-\tau}$ charge $\pm 2$ in addition to the SM doublet $\phi_0$. The Lagrangian for the charged lepton sector is

$$-\mathcal{L}^l = y_0 \bar{L}_1 \phi_0 l_1 + y_2 \bar{L}_2 \phi_2 l_3 + y_{-2} \bar{L}_3 \phi_{-2} l_2.$$

(62)

We prevent $\phi_0$ from coupling to $\bar{L}_2 l_2$ and $\bar{L}_3 l_3$ by assigning

$$\mathbb{Z}_2 : \quad l_2, l_3, \phi_{\pm 2} \text{ are odd}. $$

(63)
Such a symmetry also leads to the accidental symmetry

\[ G_l : \quad L_2 \sim l_3 \sim e^{i\theta}, \quad L_3 \sim l_2 \sim e^{-i\theta}. \]  \tag{64}

The Higgs doublets are invariant under this symmetry and so it leaves the symmetry invariant even after EWSB. It is this symmetry that will correspond to \( G_l \) at low energies and will differ from our original \( U(1)_{\mu-\tau} \) only for \( l_R \). The \( CP^{\mu\tau} \) acts in the same form for \( G_l \) as it does for \( U(1)_{\mu-\tau} \).

The \( CP^{\mu\tau} \) symmetry acts on the doublets as

\[ \phi_0 \to \phi_0^* , \quad \phi_2 \to \phi_{-2}^*. \]  \tag{65}

This implies \( y_0 \) is real and \( y_{-2}^* = y_2 \).

If we write

\[ \langle \phi_{-2}^{(0)} \rangle = v_{-2} \quad \text{and} \quad \langle \phi_2^{(0)} \rangle = v_2 , \]  \tag{66}

the \( CP^{\mu\tau} \) breaking will come from

\[ |v_{-2}| \gg |v_2| , \]  \tag{67}

which induces the \( \mu\tau \) mass splitting

\[ m_\mu = |y_2 v_2| \ll m_\tau = |y_{-2} v_{-2}| . \]  \tag{68}

Note that prior to EWSB \( CP^{\mu\tau} \) renders \( \mu\tau \) flavors indistinguishable and the \( |v_{-2}| \ll |v_2| \) leads physically to the same situation. The \( CP^{\mu\tau} \) breaking in (67) will be induced by a large vev of a CP odd scalar \( \sigma \) in the potential [30].

The Higgs potential is

\[ V_2 = \mu_2 (|\phi_2|^2 + |\phi_{-2}|^2) + \mu_0 |\phi_0|^2 , \]

\[ V_4 = \frac{1}{2} \lambda_0 |\phi_0|^4 + \frac{1}{2} \lambda_1 (|\phi_2|^2 + |\phi_{-2}|^2)^2 + \lambda_2 |\phi_2|^2 |\phi_{-2}|^2 
+ \lambda_{22} (\phi_0^\dagger \phi_2 \phi_0 \phi_{-2} + h.c.) + \lambda_{02} |\phi_0|^2 (|\phi_2|^2 + |\phi_{-2}|^2) 
+ \lambda'_{02} (|\phi_0^\dagger \phi_2|^2 + |\phi_0 \phi_{-2}|^2) , \]  \tag{69}

\[ \delta V = \mu_\sigma \sigma (|\phi_2|^2 - |\phi_{-2}|^2) + (\lambda_{-4} \phi_2^\dagger \phi_{-2} \eta_2^2 + h.c. ) , \]

where \( \sigma \) is a CP odd scalar and \( \eta_2 \) is a CP-even scalar with \( U(1)_{\mu-\tau} \) charge 2 and will couple to \( N_2^2, N_3^2 \). We have omitted a term similar to the \( \lambda_2 \)-term because only neutral vevs are sought and they are not relevant to the discussion below. We could also replace \( U(1)_{\mu-\tau} \) by \( Z_8 \) by adding the terms \( (\phi_2^\dagger \phi_{-2})^2 \).
After $\sigma$ and $\eta_2$ acquire vevs at the high scale, we get from $\delta V$ and $V_2$ an effective quadratic term for $\phi_{\pm 2}$,

$$V_{\text{eff}} = M_2^2 |\phi_2|^2 + M_{-2}^2 |\phi_{-2}|^2 + M_{22}^2 \phi_2^\dagger \phi_{-2} + h.c.,$$  \hfill (70)

where

$$M_2^2 = \mu_2 + \mu_\sigma \langle \sigma \rangle, \quad M_{-2}^2 = \mu_2 - \mu_\sigma \langle \sigma \rangle, \quad M_{22}^2 = \lambda_{-4} \langle \eta_2 \rangle^2.$$  \hfill (71)

Irrespective of the phases of $\lambda_{-4}, \langle \eta_2 \rangle$, we apply rephasing transformations so that $M_{22}^2$ is real and negative.

Now we adjust $\langle \sigma \rangle$ so that $|M_2^2| \approx \varepsilon^{-1}|M_{22}^2| \approx \varepsilon^{-2}|M_{-2}^2| \sim v_{\text{ew}}$. The phases of the vevs are trivial in the minimum when $\lambda_{22} < 0$. This leads to a high scale mass matrix for $(\phi_{-2}, \phi_2)$ of the form:

$$M_2^2 = M_2^2 \begin{pmatrix} \varepsilon^2 & \sim \varepsilon \\ \sim \varepsilon & 1 \end{pmatrix}$$  \hfill (72)

The two approximate eigenvectors of this matrix are: $H' \approx \phi_2 + \varepsilon \phi_{-2}$ and $h_0 \approx \phi_{-2} - \varepsilon \phi_2$. By fine tuning we keep $\varepsilon \sim \mu_\tau / m_\tau$ and $H'$ as superheavy whereas $h_0$ mass is negative and weak scale. Then below the scale of $\langle \eta \rangle$ and $\langle \sigma \rangle$, the effective charged lepton Yukawa couplings in (C5) look like:

$$-L_{\text{eff}}^I \simeq y_0 \bar{L}_1 \phi_0 l_1 + y_2 \varepsilon \bar{L}_2 h_0 l_3 + y_2^* \bar{L}_3 h_0 l_2.$$  \hfill (73)

After a $90^\circ$ rotation of the right-handed charged leptons, this gives $m_\tau = |y_2^* \langle h_0^{(0)} \rangle|$ and $m_\mu = |y_2 \varepsilon \langle h_0^{(0)} \rangle|$ as desired for a realistic theory.

For the neutrino sector we add three singlet scalars $\eta_k, k = 0, 1, 2$, with $U(1)_{\mu-\tau}$ charge $k$; $\eta_0$ is a real scalar. When they acquire vevs (for $k \neq 0$), they break $U(1)_{\mu-\tau}$ without breaking $\text{CP}^{\mu\tau}$, as discussed previously, and they transform trivially under $\text{CP}^{\mu\tau}$:

$$\text{CP}^{\mu\tau} : \eta_k(x) \rightarrow \eta_k(\hat{x}),$$  \hfill (74)

where $\hat{x} = (x_0, -x)$ for $x = (x_0, x)$. We also assume the symmetry $\mathbb{Z}_4^L$ in (60) where $\eta_k \sim -1$.

The Lagrangian for $N$,

$$-\mathcal{L} \supset \frac{1}{2} k_1 \tilde{N}_1 N_1^c \eta_0 + k_{23} \tilde{N}_2 N_3^c \eta_0 + \frac{1}{2} k_2 \tilde{N}_2 N_2^c \eta_2 + \frac{1}{2} k_3 \tilde{N}_3 N_3^c \eta_2^* + \frac{1}{2} k_{12} \tilde{N}_1 N_2^c \eta_1 + k_{13} \tilde{N}_1 N_3^c \eta_1^*,$$  \hfill (75)
gives rise to $M_R$ in the $\text{CP}^{\mu\tau}$ symmetric form (4) after $\eta_k$ acquire generic vevs. GCP symmetry imposes real $k_1$, real $k_{23}$, $k_3 = k_2^*$, $k_{13} = k_{12}^*$. Given the necessary structure (4) and the requirement for $\theta_{13} \neq 0$, we indeed need both fields $\eta_{1,2}$. Note that $\mathbb{Z}_4^L$ prevents $\sigma$ from coupling to $N_iR$.

It can be seen that $\text{CP}^{\mu\tau}$ symmetric $M_R$ also leads to a $\text{CP}^{\mu\tau}$ symmetric $M_{R}^{-1}$. Such a structure is maintained from the neutrino Dirac mass matrix $M_D$ coming from

$$- \mathcal{L} \supset f_0 \bar{N}_1 \phi_0^* L_1 + f_2 \bar{N}_2 \phi_0^* L_2 + f_3 \bar{N}_2 \phi_0^* L_3, \quad (76)$$

where $\phi_0$ is the same Higgs doublet that couples to electrons and quarks. The reality of $f_0$ and $f_3 = f_2^*$ follow from $\text{CP}^{\mu\tau}$ and we obtain

$$M_D = \begin{pmatrix} x_\nu & z_\nu \\ z_\nu & z_\nu^* \end{pmatrix}, \quad (77)$$

The neutrino mass matrix given by the seesaw formula [31]

$$M_\nu = -M_D^T M_R^{-1} M_D, \quad (78)$$

is $\text{CP}^{\mu\tau}$ invariant and has the form (4) as advertised.

The leptogenesis aspects studied in Sec. IV has to be adapted in this case because $v = 174 \text{GeV}$ has to be replaced by $v_u = v \sin \beta$. The plots shown in Figs. 2 and 3 apply now for $M_1/\sin^2 \beta = 10^{12} \text{GeV}$ and limits for the $M_1$ window changes accordingly.

### B. Higgs spectrum

At low energies, the scalar sector of this model acts like a lepton-specific (also called type-X) two Higgs doublet model [32] with the Higgs doublets being $h_0$ and $\phi_0$, except for the Higgs couplings to electrons; cf. (73). Both of the doublets acquire vevs such that $\sqrt{\langle \phi_0 \rangle^2 + \langle h_0 \rangle^2} = v = 174 \text{ GeV}$. The ratio of vevs is given by $\langle h_0 \rangle/\langle \phi_0 \rangle = \tan \beta$ and the mixing between the real neutral Higgs fields is denoted by $\tan \alpha$. The effective Higgs potential in terms of $\phi_0$ and $h_0$ is given by:

$$V(\phi_0, h_0) = -\mu_0^2 |\phi_0|^2 - \mu_h^2 |h_0|^2 + \frac{1}{2} \lambda_0 |\phi_0|^2 + \frac{1}{2} \lambda_1 |h_0|^2 + \lambda_{02} |\phi_0|^2 |h_0|^2 + \lambda_{02}' |\phi_0|^2 |h_0|^2 + \lambda_{22} \varepsilon (\phi_0^\dagger h_0 \phi_0^\dagger h_0 + \text{h.c.}). \quad (79)$$

The spectrum of Higgs states is given by [33]

$$m_A^2 = -4 \lambda_{22} \varepsilon v^2, \quad m_{H^+}^2 = -(\lambda_{02}' + 2 \varepsilon \lambda_{22}) v^2, \quad (80)$$
where \( v = 174 \) GeV (we use a different normalization compared to [33]), while the mass matrix for the CP even states, in the basis \( \sqrt{2}(\text{Re} h_0 - v_{-2}, \text{Re} \phi_0 - v_0) \), is

\[
M_{h,H}^2 = 2 \begin{pmatrix}
\lambda_1 v_{-2}^2 & \lambda_{345} v_0 v_{-2} \\
\lambda_{345} v_0 v_{-2} & \lambda_0 v_0^2
\end{pmatrix},
\]

where \( \lambda_{345} = \lambda_{02} + \lambda'_{02} + 2\varepsilon \lambda_{22} \). We are using \( \langle h_0 \rangle \approx \langle \phi_{-2} \rangle \approx v_{-2} \).

Since our parameter \( \lambda_{22} \) comes from the high energy theory (decoupled \( \phi_2 \)), it can not be arbitrarily large. If we impose it to be perturbative, \( |\lambda_{22}| < 4\pi \) we obtain an upper bound for the pseudoscalar \( A \) as

\[
m_A = 2v \sqrt{\varepsilon |\lambda_{22}|} \lesssim 2v \sqrt{4\pi \frac{m_\mu}{m_\tau}} \approx 300 \text{ GeV},
\]

hence non-decoupling. This is smaller than \( 2m_t \) and \( t \bar{t} \) cannot be produced. Neutral scalars in the 2HDMs are less constrained than the charged higgses (e.g. from flavor observables [34]) and the strongest limits are available for the MSSM (or type-II) [35]. Usually they appear as lower bounds on the heavy masses because the decoupling limit is usually a good description. Very light pseudoscalars of mass below \( O(10\text{GeV}) \) can also have its couplings constrained [35, 36]. Current LHC limits for the different types of 2HDM constrain the various 2HDMs to be close to the alignment limit [37]. Even in this limit, a portion of the parameter space is already excluded. For example, only \( \tan \beta \gtrsim 3 \) is allowed by data (above 200GeV). Also, being an effective 2HDM, the triple Higgs coupling for the interaction \( h^3 \) is different from the SM and can be probed in the future [38].

**VII. SUMMARY**

We have presented a minimal setting where \( G_l \) is conserved in the charged lepton sector and \( G_\nu \) is conserved in the neutrino sector. The largest \( G_l \) can be identified with the combination \( L_\mu - L_\tau \) symmetry and \( G_\nu \) is generated by a generalized CP symmetry, \( \text{CP}^{\mu\tau} \), that combines CP with \( \mu\tau \) exchange. When \( G_l \) is conserved in the charged lepton sector and \( G_\nu \) is conserved in the neutrino sector, we obtain the usual prediction of maximal \( \theta_{23} \) and \( \delta_{\text{CP}} \) with nonzero \( \theta_{13} \). Additionally, Majorana phases are fixed up to discrete choices and they lead to very specific predictions for neutrino-less double beta decay and leptogenesis.

In our setting, the two symmetries \( G_l \) and \( G_\nu \) commute and this feature allows us to naturally avoid the alignment problem in the scalar sector. Additional symmetries can be used to keep the \( G_l \)- and \( G_\nu \)-breaking effects restricted to the neutrino sector and charged lepton sector, respectively.
Additionally, continuous $B-L$ cannot be imposed (hence not gauged) in our setting and only a $\mathbb{Z}_4$ subgroup may be imposed to keep $\text{CP}^{\mu \tau}$ naturally unbroken in the neutrino sector. Our construction also illustrates that generalized CP symmetries based on the *trivial* automorphism of flavor groups – much less considered in the literature – may still lead to interesting model constructions.

For the neutrino-less double beta decay, the discrete choice of Majorana phases (or CP parities) leads to specific strips that can be clearly distinguished in some cases; see Fig. 1. For example, for inverted hierarchy, the case of all equal CP parities or only $\nu_{3L}$ with different CP parity can be distinguished from the rest and can be potentially measured or falsified in the near future. We emphasize that, key predictions of these models are: (i) $\theta_{23} = 45^0$ and $\delta_{CP} = \pi/2$ simultaneously i.e. if experimentally measured values for either of these observables deviate from the above predictions, $\text{CP}^{\mu \tau}$ violating terms will be necessary to keep these ideas viable.

The consequences of $\text{CP}^{\mu \tau}$ for leptogenesis leads to the natural implementation of the purely flavored leptogenesis scenario where the total CP asymmetry due to $N_1$ decay is vanishing. Successful leptogenesis is possible only when flavored leptogenesis is considered and that must take place at the intermediate temperature range of $10^9$–$10^{12}$GeV. Flavored leptogenesis below $10^9$GeV seems to be precluded even if the CP asymmetry is resonantly enhanced by quasi-degenerate $N_{1R}$ and $N_{2R}$ if the $\mu$- and $\tau$-flavors are washed out equally. For effective two heavy and hierarchical right-handed neutrinos the window for successful leptogenesis is even narrower: $5 \times 10^{10}$–$10^{12}$GeV.

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**Appendix A: Uniqueness of $\text{CP}^{\mu \tau}$**

We show here that the GCP defined by $X$ in (55) for the abelian symmetry generated by $T$ in (50) are the only possibilities for any $G_l = U(1)$ or $G_l = \mathbb{Z}_n$, with prime $n$ or $n = 4, 6$. A different possibility arises only for $T$ (the possibilities for $X$ are the same) beginning with $n = 8$. The case of $G_l = U(1)$ was considered in the text. We only need to consider $G_l = \mathbb{Z}_n$. 
To show the assertion, we generalize the form of $T$ from (50) to

$$
T = \begin{pmatrix}
  z_1 \\
  z_2 \\
  z_3
\end{pmatrix},
$$

(A1)

where $z_i$ are complex number of modulus unity. We also keep $\det T = z_1z_2z_3 = 1$ because its nontrivial contribution can be factored out to usual lepton number. Let us also consider more general automorphisms for $\mathbb{Z}_n$ in (53): $T' = T^k$ where $k$ cannot divide $n$. Then the consistency condition (53) can be recast in the following form:

$$
z_i^k z_j = 1 \quad \text{if} \quad X_{ij} \neq 0.
$$

(A2)

Let us take the first row of $X$. Because $X$ is nonsingular, at least one element of the first row has to be nonzero. Suppose two elements are nonzero. If $X_{11} \neq 0$ and $X_{12} \neq 0$, then condition (A2) implies

$$
z_1^{k+1} = z_2^k z_3 = 1,
$$

(A3)

and then $z_1 = z_2$ which is impossible because $T$ is nondegenerate. The same conclusion is reached if any two of the elements of the first row is nonzero. The argument is independent of the row and hence only one element in each row (or column) can be nonzero. Listing all possibilities and selecting only the symmetric matrices, the nonzero entries of $X$ coincides with the positions of the unity in (55), after eliminating similar forms that are related by the simultaneous permutations of rows and columns that keep $T$ diagonal. Rephasing of fields leads to (55). Thus $X$ is restricted to (55) except for permutations.

Now, for the first case of $X = 1$, we reach the conclusion that

$$
z_1^{k+1} = z_2^{k+1} = z_3^{k+1} = 1.
$$

(A4)

This means $T^{k+1} = 1$ and if $T$ is a faithful representation, $k + 1 = 0 \mod n$. Therefore, $k = -1$ is the only possibility.

For the second case of $X$ being (23)-transposition, we have the conditions

$$
z_1^{k+1} = z_2^k z_3 = z_3^k z_2 = 1.
$$

(A5)

This imposes conditions on $z_1$ and also $z_2^{k-1} = z_3^{k-1}$. For prime $n$ the last relation is only possible if $k = 1$: this leads to (50) (we exclude $z_2 = z_3$). The cases $n = 4, 6$ do not lead to different forms because only $k = 1$ or $k = -1$ correspond to automorphisms. The cases so far proves the assertion.
The first different form appears for $\mathbb{Z}_8$ and one example is

$$T = \begin{pmatrix} -1 & \omega_8 \\ \omega_8^2 & \omega_8^3 \end{pmatrix},$$

(A6)

which obeys $XT^*X^{-1} = T^5$; $\omega_8$ denotes $e^{i2\pi/8}$. If we allow $X$ to be nonsymmetric, other possibilities appear such as for $\mathbb{Z}_7$ where $X$ is the cyclic permutation and $T = \text{diag}(\omega_7, \omega_7^2, \omega_7^4)$ (the same that appears for the $T_7$ group).

Appendix B: $G_l \times G_\nu$ in the real basis

We show here how the $\text{CP}^{\mu\tau}$ symmetry of (51) and the $U(1)_{\mu-\tau}$ symmetry of (50) are rewritten in a real basis where $\text{CP}^{\mu\tau}$ is just the usual CP transformation. In this basis, the commutation of $\text{CP}^{\mu\tau}$ and $U(1)_{\mu-\tau}$ is transparent and it also shows how the combination $U(1)_{\mu-\tau} \times \mathbb{Z}_2^{\text{CP}}$ leads effectively to a two-dimensional representation when the fields are complex, i.e., carrying quantum numbers other than $U(1)_{\mu-\tau} \times \mathbb{Z}_2^{\text{CP}}$.

It is clear that the charged lepton part of (49) breaks the $\text{CP}^{\mu\tau}$ symmetry strongly as $m_\tau/m_\mu \sim y_\tau/y_\mu \sim 17$ (if $l_\alpha$ transform similarly to $L_\alpha$ and $H$ transforms as usual). This breaking can be analyzed in a different basis. Since the matrix $X$ in $\text{CP}^{\mu\tau}$ is symmetric, there is a change of basis where such a basis change is

$$\begin{pmatrix} L_\mu \\ L_\tau \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} \begin{pmatrix} L'_\mu \\ L'_\tau \end{pmatrix}.$$  

(B1)

For the right-handed singlets $l_i$ we apply the same transformations. The CP transformation in the new basis will be just the usual

$$L'_i \to (-iC)L'^{\tau^*}_i,$$

(B2)

and similarly for $l_i$.

The Yukawa coefficients in $\bar{L}_i Y_{ij} H l_j$ in the new basis will be just

$$Y = \begin{pmatrix} y_e \\ y_\mu \\ y_\tau \end{pmatrix} \to Y' = \begin{pmatrix} y_e \\ \bar{y} \\ -i\Delta y/2 \\ i\Delta y/2 \\ \bar{y} \end{pmatrix},$$

(B3)
where \( \bar{y} \equiv (y_\tau + y_\mu)/2 \) and \( \Delta y \equiv y_\tau - y_\mu \). One can see that if \( y_\tau \neq y_\mu^* \), CP is violated because the phases of \( \bar{y} \) and \( i\Delta y \) can not be simultaneously removed [keeping (B4)] while in this basis \( M_\nu \) should be a real matrix. For example, if \( y_\mu, \tau \) are real CP is violated by \( i\Delta y \). The latter term is however still invariant by the following \( SO(2) \) without being proportional to the identity:

\[
\begin{pmatrix}
    L_\mu \\
    L_\tau
\end{pmatrix}
\rightarrow
\begin{pmatrix}
    \cos \theta & \sin \theta \\
    -\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
    L_\mu \\
    L_\tau
\end{pmatrix},
\]  
(B4)

In this basis it is clear that the CP transformation (B2) commutes with the \( SO(2) \) transformation in (B4).

In this basis it is also clear \( \text{U}(1)_{\mu-\tau} \times \mathbb{Z}_2^\mathbb{C} \) have the irreducible representations shown in table II, where \( (q, \pm) \) denotes charge \( q \) for \( \text{U}(1)_{\mu-\tau} \) and CP parities \( \pm \) while \( * \) denotes a complex field transforming as \( \phi \rightarrow \phi^* \) in the real basis or \( \phi_q \rightarrow \phi_q^* - q \) in the \( \text{U}(1)_{\mu-\tau} \) diagonal basis.

**Appendix C: Single Higgs implementation**

In this implementation, the symmetry at the high scale is \( G_F = G_l \times G_\nu \) where \( G_l = \text{U}(1)_{\mu-\tau} \) (gauged) and \( G_\nu = \mathbb{Z}_2^{\mathbb{C}} \). At low energy, right above the electroweak scale, we effectively have the SM with one Higgs doublet.

The neutrino sector is the same as in the multi-Higgs model of Sec. VI A, with additional simplification by eliminating \( \eta_0 \) and the symmetry \( \mathbb{Z}_4^L \). If we replace \( \text{U}(1)_{\mu-\tau} \) by \( \mathbb{Z}_3 \), we can simplify further by identifying \( \eta_1 = \eta_2^* \), and we are left with only one \( \nu \)-flavon.

The charged lepton sector needs to be modified. We still assume CP\(^{\mu\tau} \) is spontaneously broken by a vev of a CP odd scalar, which now we rename as \( \sigma_- \). We also need a CP even scalar \( \sigma_+ \). To confine the CP breaking to the charged lepton sector, we introduce a \( \mathbb{Z}_2 \) symmetry for which

\[
\mathbb{Z}_2 : \sigma_\pm, l_{iR} \text{ are odd},
\]  
(C1)
and the rest are even. Both $\sigma_{\pm}$ are invariant under $U(1)_{\mu-\tau}$. We can write an effective Lagrangian as

$$-L_{\text{eff}} = \frac{\sigma_e}{\Lambda_{\text{CP}}} \bar{L}_e H l_e + \frac{\sigma_\mu}{\Lambda_{\text{CP}}} \bar{L}_\mu H l_\mu + \frac{\sigma_\tau}{\Lambda_{\text{CP}}} \bar{L}_\tau H l_\tau + \text{h.c.}. \quad (C2)$$

where $\sigma_\alpha$, $\alpha = e, \mu, \tau$ are some complex linear combinations of $\sigma_{\pm}$. GCP invariance requires

$$\sigma_e = a_e \sigma_+ + ib_e \sigma_-,$$

$$\sigma_\mu = a_\mu \sigma_+ + ib_\mu \sigma_-,$$

$$\sigma_\tau = a_\tau \sigma_+ + ib_\tau \sigma_-,$$  

(C3)

where $a_e, b_e$ are real coefficients and $a_\tau = a_\mu^*, b_\tau = b_\mu^*$ are generally complex. The $\mu\tau$ mass splitting is generated from

$$\frac{m_\tau^2 - m_\mu^2}{v^2} = \frac{1}{\Lambda_{\text{CP}}^2} \left[ |a_\mu^* u_+ + ib_\mu u_-|^2 - |a_\mu u_+ + ib_\mu u_-|^2 \right] = \frac{u_+ u_-}{\Lambda_{\text{CP}}^2} 4 \text{Im}(a_\mu^* b_\mu), \quad (C4)$$

where $\langle \sigma_+ \rangle = u_+$ and $\langle \sigma_- \rangle = u_-$. We can see that CP breaking, and hence $\mu\tau$ mass splitting, requires both $u_\pm$ to be nonzero.

One example for a UV completion of (C2) can be achieved by introducing three heavy vector-like charged lepton fields $E_iL$ and $E_iR$, the latter with the same SM quantum number of $l_iR$. They are charged under $U(1)_{\mu-\tau}$ just like the rest of the leptons as (61) but they are even under the additional $Z_2$ symmetry of (C1). The Lagrangian is then

$$-L_i = y'_1 \bar{L}_1 H E_{1R} + y'_2 \bar{L}_2 H E_{2R} + y'_3 \bar{L}_3 H E_{3R} + M_{E_i} E_iL E_{iR} + \sigma_i \bar{E}_iL l_i, \quad (C5)$$

where $y'_3 = y'_2^*$ and $\sigma_i$ are some linear combinations of $\sigma_{\pm}$ just like (C3); $M_{E_1}$ is real from GCP and $M_{E_3} = M_{E_2}$ can be taken real by convention. We obtain (C2) for the charged leptons after integrating out the heavy leptons $E_i$, with the identification

$$\frac{\sigma_e}{\Lambda_{\text{CP}}} = -\frac{y'_1}{M_{E_1}} \sigma_1,$$

$$\frac{\sigma_\mu}{\Lambda_{\text{CP}}} = -\frac{y'_2}{M_{E_2}} \sigma_2,$$

$$\frac{\sigma_\tau}{\Lambda_{\text{CP}}} = -\frac{y'_3}{M_{E_3}} \sigma_3.$$  

(C6)

In particular, the electron Yukawa is naturally small for $M_{E_1} \gg M_{E_2}$.

We should mention that $U(1)_{\mu-\tau}$ breaking would be induced in the charged lepton sector by
the additional couplings between $E_i$ and $\eta_k$ as
\begin{equation}
-L^l \propto \mu'_{12} \bar{E}_{1L} E_{2R} \eta_1^* + \mu'_{13} \bar{E}_{1L} E_{3R} \eta_1
+ \mu'_{21} \bar{E}_{2L} E_{1R} \eta_1 + \mu'_{31} \bar{E}_{1L} E_{3R} \eta_1^*
+ \mu'_{23} \bar{E}_{2L} E_{3R} \eta_2 + \mu'_{32} \bar{E}_{3L} E_{2R} \eta_2^* + h.c.,
\end{equation}
where $\mu'_{32} = \mu'_{23}$, $\mu'_{13} = \mu'_{12}$, $\mu'_{31} = \mu'_{21}$. However, we can assume that $U(1)_{\mu-\tau}$ breaking scale is much smaller than the bare mass terms for $E_i$ as
\begin{equation}
|\langle \eta_{1,2} \rangle| \ll M_{E_2} \ll M_{E_1}.
\end{equation}
In this case, the $U(1)_{\mu-\tau}$ breaking effects can be neglected and (C2) is effectively obtained after $E_i$ are integrated out. Since $\langle \eta_k \rangle$ are related to $N_R$ masses, more specifically to the generation of $\theta_{12}, \theta_{13}$ and $N_2, N_3$ mass splitting, (C8) means that $N_R$ mass scale is much smaller than the $E_i$ scale. An alternative way of avoiding $U(1)_{\mu-\tau}$ breaking in the charged lepton sector would be to use $Z_{L}^4$.

As for the scale of $\langle \sigma_{\pm} \rangle$, we should have $\langle \sigma_{\pm} \rangle/M_{E_2} \gtrsim 10^{-2}$ for an order one $y'_3$ coupling in (C6), and it can lie below or above the $U(1)_{\mu-\tau}$ breaking scale. Anyhow, $\sigma_{\pm}$ does not couple to $N_R$ at renormalizable level due to the $Z_2$ symmetry and CP breaking is not induced at leading order to the neutrino sector since $\eta_k$ only couple to CP even combinations $\sigma_{1,2}^+$ and $\sigma_{1,2}^-$. We assume, however, that all $\langle \eta_k \rangle, \langle \sigma_{\pm} \rangle$, are greater than the scale where leptogenesis takes place, typically $10^{11}$GeV in our case, so that CP breaking in the charged lepton sector can be manifest.

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