Description Theory, LTAGs and Underspecified Semantics*

Reinhard Muskens
Department of Linguistics
Tilburg University
P.O. Box 90153
5000 LE Tilburg, The Netherlands
r.a.muskens@kub.nl

Emiel Krahmer
IPO
Eindhoven University of Technology
P.O. Box 513
5600 MB Eindhoven, The Netherlands
krahmer@ipo.tue.nl

An attractive way to model the relation between an underspecified syntactic representation and its completion is to let the underspecified representation correspond to a logical description and the completions to the models of that description. This approach, which underlies the Description Theory of (Marcus et al. 1983) has been integrated in (Vijay-Shanker 1992) with a pure unification approach to Lexicalized Tree-Adjoining Grammars (Joshi et al. 1975, Schabes 1983) has been integrated in (Vijay-Shanker 1990). We generalize Description Theory by integrating semantic information, that is, we propose to tackle both syntactic and semantic underspecification using descriptions.1 Our focus will be on underspecification of scope. We use a generalized version of LTAG, to which we shall refer as LFTAG. Although trees in LFTAG have surface strings at their leaves and are in fact very close to ordinary surface trees, there is also a strong connection with the Logical Forms (LFs) of (May 1977). We associate logical interpretations with these LFs using a technique of interrealising the logical binding mechanism (Muskens 1996). The net result is that we obtain a Description Theory-like grammar in which the descriptions underspecify semantics. Since everything is framed in classical logic it is easily possible to reason with these descriptions.

1 Syntactic Composition

Descriptions in our theory model three kinds of information. First, there are input descriptions, which vary per sentence. For example, for sentence (1) we have (2) as an input description. It says that there are two lexical nodes,' labeled John and walks respectively; that the first of these precedes the second; and that these two lexical nodes are all that were encountered. Secondly, there is a lexicon which includes semantic information. The entries for John and walks are given in (3) and (4).

(1) John walks.

(2) \exists n_1 n_2 (\mathit{lex}(n_1) \land \mathit{lex}(n_2) \land n_1 < n_2 \land \mathit{lab}(n_1, \text{john}) \land \mathit{lab}(n_2, \text{walks}) \land \forall n ((\mathit{lex}(n) \rightarrow (n = n_1 \lor n = n_2)))

(3) \forall n_1 (\mathit{lab}(n_1, \text{john}) \rightarrow \exists n_3 (\mathit{lab}(n_3, \text{np}) \land n_3 < n_1 \land a^+(n_3) = n_1 \land \sigma(n_3) = \text{John} \land \forall n (\sigma(n) = n_1 \rightarrow (n = n_3 \lor n = n_2))) \land \forall n (\sigma(n) = n_1 \rightarrow n = n_3))

(4) \forall n_2 (\mathit{lab}(n_2, \text{walks}) \rightarrow \exists n_1 n_3 n_4 n_5 n_6 (\mathit{lab}(n_1, \text{st}) \land \mathit{lab}(n_3, \text{np}) \land \mathit{lab}(n_5, \text{vp}) \land \mathit{lab}(n_6, \text{vp}) \land n_2 < n_1 \land n_1 < n_3 \land n_3 < n_4 \land n_4 < n_5 \land n_5 < n_6 \land a^- (n_6) = n_2 \land \forall n (a^+(n) = n_2 \rightarrow (n = n_4 \lor n = n_5 \lor n = n_6)) \land a^-(n_6) = n_2 \land \forall n (\sigma(n) = n_2 \rightarrow (n = n_3 \lor n = n_4 \lor n = n_5 \lor n = n_6)) \land \sigma(n_1) = \sigma(n_6) (\sigma(n_5) = \text{vp.walks}))

The function symbol \(a^+\) used in these descriptions positively anchors nodes to lexical nodes, \(a^-\) negatively anchors nodes and \(\sigma\) gives a node its semantic value. Since descriptions are unwieldy we partially abbreviate them with the help of pictures:

\[
\begin{array}{c}
\text{np}_9 \\
\text{vp}_6 \\
\text{vp}_7 \\
\end{array}
\]

\[
\begin{array}{c}
\text{john}_1 \\
\text{walks}_2 \\
\end{array}
\]

Here uninterrupted lines represent immediate dominance (\(<\)) and dotted lines represent dominance (\(<^*\)), as usual. Additionally we mark positive and

\[\text{112}\]
negative anchoring in the following way. If a description contains the information that a certain nonlexical node is positively (negatively) anchored, the term referring to that node gets a plus (minus) sign. But pluses and minuses cancel and terms that would get a ± by the previous rule will be left unmarked. Terms marked with a plus (minus) sign are to be compared with the bottom (top) parts of Vijay-Shanker’s ‘quasi-nodes’ in Vijay-Shanker (1992).

To the third and final kind of descriptions belong axioms which say that < ≤ and ≤ behave like immediate dominance, dominance and precedence in trees (A1 - A10, see also e.g., Cornell 1994, Backofen et al. 1995:9) combined with other general information, such as the statements that labeling is functional (A11), and that different label names denote different labels (A12). A13 and A14 say that all nodes must be positively anchored to lexical nodes and that all lexical nodes are positively anchored to themselves. The axioms for negative anchoring (A15 and A16) are similar, but allow the root r to be negatively anchored to itself.

A1  ∀k [r < ≤ k v r = k]
A2  ∀k ¬k ≤ k
A3  ∀k1 k2 k3 [[k1 ≤ k2 ∧ k2 ≤ k3] → k1 ≤ k3]
A4  ∀k ¬k ≤ k
A5  ∀k1 k2 k3 [[k1 ≤ k2 ∧ k2 ≤ k3] → k1 ≤ k3]
A6  ∀k1 k2 k3 k4 k5 [k1 ≤ k2 ∧ k2 ≤ k1 ∧ k1 ≤ k+ k2 V k2 ≤ k3 ∧ k3 ≤ k1 ∧ k1 ≤ k+ k3 V k3 ≤ k1 ∧ k1 ≤ k+ k3]
A7  ∀k1 k2 k3 [k1 ≤ k2 ∧ k2 ≤ k1 → k2 ≤ k2]
A8  ∀k1 k2 k3 [k1 ≤ k2 ∧ k2 ≤ k1 → k3 ≤ k2]
A9  ∀k1 k2 [k1 < k2 → k1 <+ k2]
A10 ∀k1 k2 k3 ¬[k1 < k2 ∧ k1 <+ k2 ∧ k2 <+ k3]
A11 ∀kVk1 k2 [[lab(k, k1) ∧ lab(k, k2)] → k1 = k2]
A12 l1 ≠ l2, if l1 and l2 are distinct label names
A13 ∀k lex(α°(k))
A14 ∀k [lex(k) → α°(k) = k]
A15 ∀k [k = r v lex(α°(k))]
A16 ∀k [[lex(k) v k = r] → α°(k) = k]

Together with this extra information (2), (3) and (4) conspire to determine a single model. Only n1 and n2 are lexical nodes. All nodes must be positively anchored to a lexical node. The set of nodes positively anchored to n1 is {n1, n3} and the set positively anchored to n2 is {n2, n4, n7}. So the remaining n5 and n6 must corefer with one of the constants mentioned, the only possibility being that n5 = n3 and that n6 = n7. The reader will note that in the resulting model σ(n4) = walk John. The general procedure for finding out which models satisfy a given description is to identify positively marked terms with negatively marked ones in a one-to-one fashion. The term r, denoting the root, counts as negatively marked.

In the given example only one tree was described, but this is indeed an exceptional situation. It is far more common that a multiplicity of trees satisfy a given description. This kind of underspecification enabled (Marcus et al. 1983) to define a parser which does not only work in a strict left-right fashion but is also incremental in the sense that at no point during a parse information need be destroyed. A necessary condition for this form of underspecification is that there are structures which can be described. In the context of semantic scope differences it therefore is natural to turn to (May 1977)'s Logical Forms, as these are the kind of models required. In fact we use a variant of May’s trees which is very close to ordinary surface structure: although we will allow NPs to be raised, the syntactic material of such NPs will in fact remain in situ. But while the only syntac-
The basic idea here is that the long-distance tic effect of raising will be the creation of an extra node and Logical Forms will have their corresponding surface structures as subtraces, the 'movement' has an important effect on semantic interpretation. Consider example (5).

(5) Every man loves a woman.

We have depicted its five lexical items in fig. 1. With two exceptions they pretty much conform to expectation. The exceptions are that each determiner main of extended locality of a determiner. In each case the semantics of the higher S will be composed out of the semantics of the lower S and the semantics of the NP, the semantic composition rule being quantifying-in.4 The two exceptions they pretty much conform to May's theory. There is also an obvious connection with the (single) 19 possibility, as it can be the case that \( n_2 = n_7 \).

We will depict its five lexical items in fig. 1. Let it be stressed that the technique discussed here cannot be used to embed predicate logic into (the first-order part of) type theory, with the side-effect that binding can take place on the level of registers. Write

\[
R\delta_1, \ldots, \delta_n \text{ for } \lambda_i. R(V(\delta_1)(i), \ldots, V(\delta_n)(i)),
\]

\[
\forall \varphi \text{ for } \lambda^i \varphi(i),
\]

\[
\varphi \land \psi \text{ for } \lambda_i [\varphi(i) \land \psi(i)],
\]

\[
\varphi \Rightarrow \psi \text{ for } \lambda_i [\varphi(i) \rightarrow \psi(i)],
\]

\[
\text{some } \delta \text{ for } \lambda_i \exists j [\delta(j) \land \varphi(j)],
\]

\[
\text{all } \delta \text{ for } \lambda_i \forall j [\delta(j) \rightarrow \varphi(j)].
\]

We have essentially mimicked the Tarski truth conditions for predicate logic in our object language and in fact it can be proved that, under certain conditions, we can reason with terms generated in this way as if they were the predicate logical formulas they stand proxy for variables and constant registers for constants. However, since registers are simply objects in our models, both variable registers and constant registers can be denoted with variables as well as with constants. Here are some axioms:

\[
\forall_i \forall_w \forall_x [VAR(w) \rightarrow \exists j, i[w]j \land V(w)(j) = x]
\]

\[
\forall_k VAR(u(k))
\]

\[
\forall k_1 k_2 [u(k_1) = u(k_2) \rightarrow k_1 = k_2]
\]

\[
\forall i. V(John)(i) = john,
\]

\[
\forall i. V(Mary)(i) = mary, \ldots
\]

Here \( VAR \) is a predicate which singles out variable registers, \( V \) assigns a value to each register \( v \) in each state \( j \), and \( i[\delta]j \) is an abbreviation of \( \forall w[w \neq \delta \rightarrow V(w)(j) = V(w)(j)] \). \( \lambda^i \) forces states to behave like assignments in an essential way. The function \( u \) assigns variable registers to nodes (A18). Each node is assigned a fresh register (A19). Constant registers have a fixed value (A20). For more information on a strongly related set of axioms see (Muskens 1996).

These axioms essentially allow our logical language to speak about binding and we can now use this expressivity to embed predicate logic into (the first-order part of) type theory, with the side-effect that binding can take place on the level of registers. Write

\[
R\delta_1, \ldots, \delta_n \text{ for } \lambda_i. R(V(\delta_1)(i), \ldots, V(\delta_n)(i)),
\]

\[
\forall \varphi \text{ for } \lambda^i \varphi(i),
\]

\[
\varphi \land \psi \text{ for } \lambda_i [\varphi(i) \land \psi(i)],
\]

\[
\varphi \Rightarrow \psi \text{ for } \lambda_i [\varphi(i) \rightarrow \psi(i)],
\]

\[
\text{some } \delta \text{ for } \lambda_i \exists j [\delta(j) \land \varphi(j)],
\]

\[
\text{all } \delta \text{ for } \lambda_i \forall j [\delta(j) \rightarrow \varphi(j)].
\]

We have essentially mimicked the Tarski truth conditions for predicate logic in our object language and in fact it can be proved that, under certain conditions, we can reason with terms generated in this way as if they were the predicate logical formulas they stand proxy for (see Muskens 1998).

It should be stressed that the technique discussed above can be used to embed any logic with a decent interpretation into classical logic. For example, (Muskens 1996) shows that we can use the same mechanism to embed Discourse Representation Theory (Kamp & Reyle 1993) into classical logic. In a fuller version of this paper we shall also present a version of LTAG based on Discourse Representations.

---

4In this paper only quantification into S is considered, but in a fuller version we shall generalize this to quantification into arbitrary phrasal categories.

---

5The relevant condition is that in each term \( \varphi \) we are using in this way, and each pair \( u(n), u(n') \) occurring in \( \varphi \), with \( n \) and \( n' \) syntactically different, we must be justified to assume \( n \neq n' \). In the application discussed below this condition is met automatically.
$\sigma(r) = \text{all } u_{n_5}[\text{man } u_{n_5} \Rightarrow \text{some } u_{n_18}[\text{woman } u_{n_18} \& u_{n_5} \text{ loves } u_{n_18}]]$ \\
$\sigma(r) = \text{some } u_{n_18}[\text{woman } u_{n_18} \& \text{all } u_{n_5}[\text{man } u_{n_5} \Rightarrow u_{n_5} \text{ loves } u_{n_18}]]$ \\

Figure 2: A Derivable Disjunction

3 Semantic Composition

We can now integrate semantic equations with the lexical items occurring in fig. 1.

$\sigma(n_3) = u_{n_5}$ \\
$\sigma(n_1) = \text{all } u_{n_5}[\sigma(n_9)(u_{n_5}) \Rightarrow \sigma(n_2)]$ \\
$\sigma(n_{10}) = \lambda u.v \text{ loves } \sigma(n_{13})$ \\
$\sigma(n_7) = \sigma(n_9)(\sigma(n_8))$ \\
$\sigma(n_{16}) = u_{n_{18}}$ \\
$\sigma(n_{14}) = \text{some } u_{n_18}[\sigma(n_{10})(u_{n_{18}}) \& \sigma(n_{15})]$ \\
$\sigma(n_{21}) = \lambda u.\text{man } u$ \\
$\sigma(n_{22}) = \lambda u.\text{woman } u$

The first two equations derive from the lexical item for every, the third and fourth from loves, the fifth and sixth from a, and the last two from the common nouns. Note that in the translation of every, \( n_3 \) only gets a referent as its translation (namely \( u(n_5) \)), which for readability we write as \( u_3 \), while the real action is taking place upstairs. A similar remark holds for the other determiner.

As we have seen earlier, in any model of the relevant descriptions \( n_8 = n_{21}, n_{19} = n_{22}, n_9 = n_{10}, n_8 = n_{13}, \text{ and } n_{13} = n_{16} \) hold. From this it follows that

$\sigma(n_7) = u_{n_5} \text{ loves } u_{n_{18}}$ \\
$\sigma(n_1) = \text{all } u_{n_5}[\text{man } u_{n_5} \Rightarrow \sigma(n_2)]$ \\
$\sigma(n_{14}) = \text{some } u_{n_18}[\text{woman } u_{n_{18}} \& \sigma(n_{15})]$ \\

The relevant constraints further imply that either \( n_2 = n_{14} \text{ and } n_{18} = n_7 \text{ or, alternatively, that } n_{18} = n_{13} \text{ and } n_2 = n_{17} \). For the moment let us assume the second possibility. Since \( u_{n_5} \text{ loves } u_{n_{18}} \) is a closed term (\( u \) is a function constant and \( n_5 \) and \( n_{18} \) are constants that witness existential quantifiers in the input description of (5)), the assumption that \( n_2 = n_{17} \) allows us to conclude that

$\sigma(n_1) = \text{all } u_{n_5}[\text{man } u_{n_5} \Rightarrow u_{n_5} \text{ loves } u_{n_{18}}]$ \\

Note that this is the point where we have made essential use of our internalisation of binding: had we used ordinary variables instead of our register-denoting terms, the substitution would not have been possible.

Continuing our reasoning, we see that under the given assumption the root node \( r (=n_{14} \text{ in this case}) \) will be assigned the \( \exists v \) reading of the sentence. Without assumptions the disjunction in fig. 2 is derivable.

We conclude that the leading idea behind Marcus' Description Theory allows us to underspecify semantic information much in the same way as syntactic information is underspecified in this theory. The price is that we must accept that different semantic readings correspond to different structures, as the method only allows underspecification of the latter.

References

Backofen, R., J. Rogers and K. Vijay-Shanker. 1995. A First-Order Axiomatization of the Theory of Finite Trees. *Journal of Logic, Language and Information* 4:5-39.

Cooper, R. 1983. Quantification and Syntactic Theory. Reidel. Dordrecht.

Cornell, T. 1994. On Determining the Consistency of Partial Descriptions of Trees. *Proceedings of ACL 94*. 163-170.

Joshi, A., L. Levy and M. Takahashi. 1975. Tree Adjunct Grammars. *Journal of the Computer and System Sciences* 10: 135-163.

Kamp H. and U. Reyle. 1993. From Discourse to Logic. Kluwer, Dordrecht.

Marcus, M., D. Hindle and M. Fleck. 1983. D-theory: Talking about Talking about Trees. *Proceedings of the 21st ACL*. 129-136.

May, R. 1977. *The Grammar of Quantification*, PhD thesis, MIT, Cambridge.

Muskens, R. 1995. Order-Independence and Underspecification. In J. Groenendijk, editor, *Ellipsis, Underspecification, Events and More in Dynamic Semantics*. DYANA Deliverable R.2.2.C, 1995.

Muskens, R. 1996. Combining Montague Semantics and Discourse Representation. *Linguistics and Philosophy* 19: 143–216.

Muskens, R. 1998. Underspecified Semantics. Manuscript, Tilburg University.

Schabes, Y. 1990. *Mathematical and Computational Aspects of Lexicalized Grammars*, Ph.D. thesis, University of Pennsylvania.

Vijay-Shanker, K. 1992. Using Descriptions of Trees in a Tree Adjoining Grammar. *Computational Linguistics* 18:481-518.