OPTIMAL DESIGN OF FINITE PRECISION AND INFINITE PRECISION NON-UNIFORM COSINE MODULATED FILTER BANK

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Abstract. This paper investigates the design of non-uniform cosine modulated filter bank (CMFB) with both finite precision coefficients and infinite precision coefficients. The finite precision filter bank has been designed to reduce the computational complexity related to the multiplication operations in the filter bank. Here, non-uniform filter bank (NUFB) is obtained by merging the appropriate filters of an uniform filter bank. An efficient optimization approach is developed for the design of non-uniform CMFB with infinite precision coefficients. A new procedure based on the discrete filled function is then developed to design the filter bank prototype filter with finite precision coefficients. Design examples demonstrate that the designed filter banks with both infinite precision coefficients and finite precision coefficients have low distortion and better performance when compared with other existing methods.

1. Introduction. Filter banks are extensively used in different signal processing and communication applications such as compression of audio, speech, image, video data, transmultiplexing, multi-carrier modulation and adaptive signal processing [23]-[21]. Filter banks decompose the spectrum of a given signal into different subbands where each subband is associated with a specific frequency interval. In many applications such as wireless communications and subband adaptive filtering, a NUFB is preferred [21]-[14].

Different methods exist for the designs of various types of NUFBs. In one approach, two channel filter banks are used as building blocks and a tree structure filter bank is generated for getting non-uniform band splitting [20]. However, the set of decimation integers of the NUFB is constrained by the tree structure.

In the second approach, the filters in the NUFB are obtained by performing the cosine modulation of the prototype filter. This is known as the non-uniform CMFB. For the design of non-uniform CMFBs, more than one prototype filter is designed and all the other filters are obtained by performing the cosine modulation on the prototype filters [1]. However, the total number of filter coefficients required for the

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design increases. In [19], a direct approach is proposed for the design of non-uniform CMFBs. The frequency response of the prototype filter is stretched and modulated. Although both the transfer functional distortion and the aliasing distortion can be significantly reduced, the design algorithm is quite complicated.

In perfect reconstruction (PR) filter banks, the output will be a weighted delayed replica of the input [20]. In case of near perfect reconstruction (NPR) filter banks, a tolerable amount of aliasing and amplitude distortion errors is permitted [19]. The design of NPR CMFB can achieve a set of filters with better frequency selectivities compared to the corresponding PR CMFB. Even though small amount of aliasing and amplitude distortion errors exist, these filter banks are widely used in practice.

Design of filter banks with good frequency response characteristics and reduced implementation complexity is highly desirable in practical applications. Multipliers are the most expensive components for implementing the digital filters in hardware. As such, the multipliers in the filters are represented by signed power-of-two (SPT) terms [16]-[25] to reduce the implementation of multipliers to shifters and adders. In multiplier-less filter banks, the filter coefficients are represented by SPT terms [16] with the multiplications being carried out as additions, subtractions and shifting.

In this paper, we investigate the design of non-uniform CMFB with both infinite precision coefficients and finite precision coefficients. The non-uniform CMFB is designed by combining appropriate filters of the uniform CMFB [14]. First, we aim to design the non-uniform CMFB with minimum distortion and infinite precision coefficients. An efficient procedure is developed for the design of the prototype filter for the non-uniform CMFB. Then, we investigate the design of the filter bank with finite precision to reduce the complexity for the implementation of the multipliers. An optimization approach based on the discrete filled function is developed for the finite precision prototype filter [9]. Design examples show that the optimized infinite precision and finite precision filter banks have significantly lower distortion and lower prototype filter stopband attenuation when compared with other existing methods.

The outline of the paper is given as followed. In Section 2, we discuss the NUFB design problem. The design of NUFB with infinite precision coefficients is investigated in Section 3 while the design of NUFB with finite precision coefficients is developed in Section 4. Design examples are given in Section 5 while the conclusions are drawn in Section 6.

2. **Non-uniform CMFB.** Consider a uniform CMFB with the prototype filter \( h = [h(0), \ldots, h(N-1)]^T \) and the transfer function \( H(z) \),

\[
H(z) = \sum_{n=0}^{N-1} h(n)z^{-n}
\]

where \( N \) denotes the filter length. For an \( M \)-channel uniform CMFB, the impulse responses of the analysis and synthesis filters, \( h_k \) and \( g_k \), \( 0 \leq k \leq M - 1 \), are obtained from \( h \) as

\[
h_k(n) = 2h(n) \cos \left( \left( 2k + 1 \right) \frac{\pi}{2M} n - \frac{\pi}{2} \right) + (-1)^k \frac{\pi}{4}
\]

\[
g_k(n) = 2h(n) \cos \left( \left( 2k + 1 \right) \frac{\pi}{2M} n - \frac{\pi}{2} \right) - (-1)^k \frac{\pi}{4}
\]
where 0 \leq n \leq N - 1. The uniform CMFB with the analysis and synthesis filters $H_k(z)$ and $G_k(z)$, 0 \leq k \leq M - 1, and the decimation factor $M$ is depicted as in Fig. 2.

We now consider the design of a non-uniform CMFB with $\bar{M}$ subbands, see Fig. 2. For 0 \leq i \leq \bar{M} - 1, the $i^{th}$ analysis filter for the non-uniform CMFB is obtained by merging $l_i$ filters, $l_i \geq 1$, of the uniform CMFB,

$$\bar{H}_i(z) = \sum_{k=n_i}^{n_i + l_i - 1} H_k(z), \quad n_0 = 0, \quad n_i = n_{i-1} + l_{i-1}. \quad (2)$$

We have

$$l_0 + l_1 + \ldots + l_{\bar{M}-1} = M. \quad (3)$$

The decimation factor $M_i$ for the $i^{th}$ subband is given by

$$M_i = \frac{M}{l_i}, \quad 0 \leq i \leq \bar{M} - 1. \quad (4)$$

The synthesis filter $\bar{F}_i(z)$ for the non-uniform CMFB is obtained similarly as

$$\bar{F}_i(z) = \frac{1}{l_i} \sum_{k=n_i}^{n_i + l_i - 1} F_k(z), \quad 0 \leq i \leq \bar{M} - 1. \quad (5)$$

3. **Infinite precision optimization problem.** The prototype filter $h$ is designed for the non-uniform CMFB with infinite precision coefficients. Here, PR condition is relaxed by allowing a small amount of distortion. We aim to minimize the amplitude distortion of the $\bar{M}$ subbands non-uniform CMFB using the minimax criterion over the frequency range $[0, \pi]$ as in [22],

$$\min_{h} \max_{0 \leq \omega \leq \pi} \sum_{i=0}^{\bar{M}-1} \left| \bar{H}_i(e^{j\omega}) \right|^2 - 1 \quad (6)$$
subject to constraints on the stopband level of the prototype filter,

\[ |H_0(e^{j\omega}) - H_d(e^{j\omega})| \leq \epsilon_s, \ \pi/M \leq \omega \leq \pi \]  

(7)

where \( H_d(e^{j\omega}) \) is the desired response and \( \epsilon_s \) is a small acceptable distortion.

The optimization problem with the objective function defined in (6) and the constraint defined in (7) can be formulated as

\[
\begin{align*}
\min_h & \quad \max_{0 \leq \omega \leq \pi} \left( \sum_{i=0}^{M-1} |\tilde{H}_i(e^{j\omega})|^2 - 1 \right) \\
\text{subject to} & \quad |H_0(e^{j\omega}) - H_d(e^{j\omega})| \leq \epsilon_s, \ \pi/M \leq \omega \leq \pi \\
& \quad \tilde{H}_i(z) = \sum_{k=n_i}^{n_i+l_i-1} H_k(z), \ 0 \leq i \leq \tilde{M} - 1.
\end{align*}
\]  

(8)

The problem defined in (8) is a nonlinear optimization problem with infinite number of constraints. We now develop an efficient algorithm for solving the problem defined in (8). The optimization problem is transformed into an unconstrained optimization problem using a constraint transcription method. The continuous inequality constraint defined in (7) can be expressed equivalently as the following equality constraint

\[ G(h) = 0 \]  

(9)

where

\[ G(h) = \int_{\pi/M}^{\pi} \max \{ g(h,\omega), 0 \} \, d\omega \]  

(10)

and

\[ g(h,\omega) = |H_0(e^{j\omega}) - H_d(e^{j\omega})| - \epsilon_s. \]  

(11)

The concept of the penalty function is used to append the equality constraint defined in (9) to the cost function defined in (6), forming an unconstrained optimization problem as follow:

\[ \min_h f(h) \]  

(12)

where

\[ f(h) = \max_{0 \leq \omega \leq \pi} \left( \sum_{i=0}^{\tilde{M}-1} |\tilde{H}_i(e^{j\omega})|^2 - 1 \right) + \lambda G(h) \]  

(13)

and \( \lambda \) is a weighting factor for the penalty function.

As the cost function \( f(h) \) in (12) is nonlinear and nonconvex, it may have many local minima. As such, the cost function defined in (12) is solved with multiple initial solutions. Here, we use the Kaiser window and the method developed in [13] as an initial solution and for the purpose of comparison. For an initial stopband level \( A_s \) for the prototype filter, the shaping parameter \( \beta \) for the Kaiser window [13] can be obtained as

\[
\beta = \begin{cases} 
0, & \text{if } A_s \leq 21 \\
0.5842(A_s - 21)^{0.4} + 0.07886(A_s - 21), & \text{if } 21 \leq A_s < 50 \\
0.1102(A_s - 8.7), & \text{if } A_s \geq 50.
\end{cases}
\]  

(14)

We observe that the Kaiser window is very sensitive with respect to the parameter \( \beta \). Thus, we propose a two-stage optimization procedure which includes a combination of a one dimensional search for \( \beta \) and the search for the optimal prototype filter coefficients as follows.

Procedure 3.1. Combined search for \( \beta \) and the prototype filter coefficients.
Step 1. Initialize the level $A_s$ related to the desired stopband level of the prototype filter. Obtain the initial $\beta_0$ according to (14). The value $\beta$ is searched in the range $[\beta_0, \beta_0 + a]$, where $a$ is a positive constant with a step size $s$, $s > 0$. Set $\beta = \beta_0$.

Step 2. For a fixed $\beta$, obtain the prototype filter coefficient vector $h_\beta$ using the Kaiser window by the method developed in [13].

Step 3. Obtain the optimum prototype filter coefficient vector $h^*_\beta$ by optimizing the problem (12) using the initial $h_\beta$ and a gradient optimization method. If the constraints in (7) are not satisfied, then increase $\lambda$ and go to the beginning of Step 3. Otherwise, go to Step 4.

Step 4. If $\beta + s > \beta_0 + a$, then go to the Step 5. Otherwise, set $\beta := \beta + s$ and return to the beginning of Step 2.

Step 5. The optimal prototype filter coefficient vector is obtained as $h_{\text{opt}} = \arg\min_{h_\beta} f(h^*_\beta)$. (15)

Output $h_{\text{opt}}$.

In the following, we investigate the design of the non-uniform CMFB with finite precision coefficients to reduce the complexity associated with the implementation of the filter bank.

4. Finite precision optimization problem. Consider the design of finite precision prototype filter for the non-uniform CMFB with a constraint on the maximum allowable SPT terms. As the maximum magnitude for the optimal infinite precision prototype filter might not be around 1, we propose to shift the finite precision coefficients by $2^{B_1}$ where $2^{B_1}$ is the closest power-of-two to the maximum magnitude of the optimal infinite precision coefficient,

$$B_1 = \arg\min_k 2^k - \max_{0 \leq n \leq N-1} |h_{\text{opt}}(n)|.$$

For $0 \leq n \leq N - 1$, the prototype filter coefficient $h(n)$ with $B$ bits can be expressed as

$$h(n) = \sum_{i=1}^{B} s_{n,i}2^{-i+B_1}$$

where $s_{n,i} \in \{-1, 0, 1\}$. Denote by $Q$ the total allowable number of SPT terms, the coefficients $s_{n,i}$ are constrained by

$$\sum_{n=0}^{N-1} \sum_{i=1}^{B} |s_{n,i}| \leq Q.$$ (17)

We introduce a one-to-one transformation for the prototype filter coefficients $h(n)$, $0 \leq n \leq N - 1$, from the discrete domain to the coefficient $x(n)$ in the integer domain as

$$x(n) = 2^{B-B_1} h(n).$$

Then, $x(n) \in \mathcal{X} = \{-2^B, \ldots, 2^B - 1\}$. The coefficient $x(n)$ can be expressed as

$$x(n) = \sum_{i=1}^{B} s_{n,i}2^{B-i}, \quad s_{n,i} \in \{-1, 0, 1\}.$$ (19)

The design of the prototype filter with discrete coefficients can be formulated as
\[
\begin{align*}
\min_{x \in \mathcal{X}} \max_{0 \leq \omega \leq \pi} & \sum_{i=0}^{M-1} |\bar{H}_i(e^{j\omega})|^2 - 1 \\
|H_0(e^{j\omega}) - H_d(e^{j\omega})| & \leq \epsilon_d, \quad \pi/M \leq \omega \leq \pi \\
\bar{H}_i(z) & = \sum_{k=n_i}^{n_i+I-1} H_k(z), \quad 0 \leq i \leq M-1 \\
x(n) & \in \mathcal{X}, \quad \sum_{n=0}^{N-1} \sum_{i=1}^B |s_{n,i}| \leq Q
\end{align*}
\]

where \(\epsilon_d\) is the stopband specification for the discrete prototype filter. We define \(f(x)\) similar to (13) where \(g(x, \omega) = |H_0(e^{j\omega}) - H_d(e^{j\omega})| - \epsilon_d\). (21)

The optimization problem (20) can be reformulated as

\[
\begin{align*}
\min_{x \in \mathcal{X}} f(x) \\
x(n) & \in \mathcal{X}, \quad \sum_{n=0}^{N-1} \sum_{i=1}^B |s_{n,i}| \leq Q.
\end{align*}
\]

(22)

Denote by \(P(x)\) the number of power-of-two terms required for an integer \(x \in [-2^B + 1, 2^B - 1]\). We now state a simple procedure to calculate \(P(x)\) for a given \(x\) by using the closest power-of-two term as follows:

- Step 1. Initialize \(P(x) = 0\).
- Step 2. Search for a power-of-two term \(\zeta\),

\[
\zeta \in B = \{-2^{B-1}, \ldots, -2^0, 2^0, \ldots, 2^{B-1}\},
\]

closest to \(x\)

\[
|x - \zeta| = \min_{\eta \in B} |x - \eta|.
\]

(24)

Set \(P(x) := P(x) + 1\) and \(x := x - \zeta\). If \(x \neq 0\), return to the beginning of Step 2. Otherwise, stop the procedure and output \(P(x)\).

\(\square\)

The value \(P(x)\) can be pre-calculated for all integer \(x \in \mathcal{X}\). The total number of power-of-two terms required to implement a coefficient vector \(x\) can be obtained as

\[
P(x) = \sum_{n=0}^{N-1} \sum_{i=1}^B |s_{n,i}| = \sum_{n=0}^{N-1} P(x(n)).
\]

(25)

Thus, the constraint in (22) is equivalent to \(P(x) \leq Q\). The problem (22) can be transformed to an unconstrained optimization problem as

\[
\min_{x \in \mathcal{X}} F(x) = f(x) + (M - f(x))g(P(x))
\]

(26)

where \(M\) is a large number and

\[
g(P(x)) = \begin{cases} 
0, & \text{if } P(x) \leq Q \\
1, & \text{otherwise.}
\end{cases}
\]

(27)
We now discuss a two-stage procedure to solve the discrete optimization problem (26). In the first stage, a local search is implemented for the problem (26). We have the following definition:

**Definition IV.1.** For any \( x \in \mathcal{X}^N \), the neighborhood \( \mathcal{N}(x) \) of \( x \) is defined by

\[
\mathcal{N}(x) = \{ x \pm e_i, \ 0 \leq i \leq N - 1 \}
\]  

(28)

where \( e_i \) is a unit vector with the \( i^{th} \) component equal to 1 and all other components are 0.

For a fixed \( Q \), the quantization method [16] is employed to obtain the quantized solution \( x_q \) from the optimal infinite precision solution \( h_{\text{opt}} \) with a fixed \( Q \) on the number of power-of-two terms. The local search method aims to improve the quantized solution \( x_q \) by searching for a local minimizer \( x_{\ell, \text{opt}} \) around the neighborhood of the quantized solution. The local optimal solution is updated once a better solution is obtained and stops when no improvement is achieved. The local search method can be summarized as:

**Procedure 4.1. Local optimal solution**

- Step 1. Obtain the quantized solution \( x_q \) as in [16]. Initialize \( x_{\ell, \text{opt}} = x_q \).
- Step 2. Check if \( x_{\ell, \text{opt}} \) is a local minimizer of \( F(x) \). This can be done by finding

\[
F_{\text{min}} = \min_{0 \leq i \leq N - 1} F(x_{\ell, \text{opt}} \pm e_i).
\]

- Step 3. If \( F_{\text{min}} \geq F(x_{\ell, \text{opt}}) \) then stop the procedure and \( x_{\ell, \text{opt}} \) is a local minimizer of \( F(x) \). Otherwise, obtain a vector \( x \) in \( \mathcal{N}(x_{\ell, \text{opt}}) \) such that \( F(x) = F_{\text{min}} \). Assign \( x_{\ell, \text{opt}} := x \) and return to Step 2.

In the second stage, we aim to obtain another solution with a lower objective function than the current local optimal solution \( x^* \). As such, we employ an auxiliary function as in [18] based on the current local minimum solution \( x^* \). By applying the search in Stage 1 to the auxiliary function, we can escape from the current local optimal solution to another solution with a lower objective function. The auxiliary function can be given as:

\[
A(x, x^*) = \frac{w((F(x) - F(x^*))(1 + \|x - x^*\|))}{1 + \|x - x^*\|}
\]

(29)

where \( w(t) = t \) if \( t \leq 0 \) and 1 otherwise. We have the following property:

**Property 4.1.** If there exists a local minimizer of \( x_1^* \) such that \( F(x_1^*) < F(x^*) \), then \( x_1^* \) is also a local minimizer of \( A(x, x^*) \).

This follows from the definition of the auxiliary function in (29) and the fact that \( A(x, x^*) > 0 \) for \( F(x) > F(x^*) \). The global optimal solution for the discrete optimization can be obtained as follow.

**Procedure 4.2. Global optimal solution**

- Step 1. Initialize \( x^* \) as the local optimal solution \( x^* := x_{\ell, \text{opt}} \).
- Step 2. For all \( 0 \leq i \leq N - 1 \), apply the local search in Stage 1 to \( A(x, x^*) \) around the neighborhood of \( x = x^* + e_i \) and \( x = x^* - e_i \) to obtain the local minimizers \( x_{i,+} \) and \( x_{i,-} \).
• Step 3. Obtain
\[
F_{\text{min}} = \min_{0 \leq i \leq N-1} \{F(x_{i+}^*), F(x_{i-}^*)\}. \tag{30}
\]
If \(F_{\text{min}} \geq F(x^*)\) then go to Step 4. Otherwise, obtain an index \(i_0\) such that \(\min\{F(x_{i_0+}^*), F(x_{i_0-}^*)\} = F_{\text{min}}\). For the case \(F(x_{i_0+}^*) = F_{\text{min}}\) then set \(x^* = x^* + e_{i_0}\), otherwise set \(x^* = x^* - e_{i_0}\). Return to Step 2.

• Step 4. Output the optimal solution \(x^*\) and stop the procedure. \(\square\)

In [18], [9], Procedure 4.2 stops when no improvement in the objective function is obtained. As the number of prototype filter coefficients is large, this will result in a large number of iterations and a large computational complexity. In order to reduce the computational complexity, we restrict the local search in Step 2 for \(A(x, x^*)\) in the region
\[
R = \{x : \|x_i - x_i^*\| \leq r, \ 0 \leq i \leq N - 1\} \tag{31}
\]
where \(r\) is a pre-chosen positive value if there is no improvement recorded for \(F(x^*)\).

5. Design examples. In the following, we investigate the design of the non-uniform CMFB with both infinite precision coefficients and finite precision coefficients.

5.1. Non-uniform CMFB with infinite precision coefficients. Case 1. Consider the design of a 5-channel non-uniform CMFB with infinite precision coefficients and the decimation factors \((4, 4, 8, 8, 4)\), e.g. \(M_1 = M_2 = 4, M_3 = M_4 = 8\) and \(M_5 = 4\). The non-uniform CMFB is obtained by merging the filters of an 8-channel uniform CMFB with \(l_1 = l_2 = 2, l_3 = l_4 = 1\) and \(l_5 = 2\). The length of the prototype filter is 154.

Procedure III.1 is employed to obtain the optimum of the problem (12). Table 1 shows the results for (i) weighted Chebyshev method discussed in [12]; (ii) WCLS approach discussed in [12]; (iii) the window method discussed in [13] with \(A_s = 65\); (iv) the optimization problem (12) with the window method [13] and \(A_s = 65\) as the initial subject to a constraint of \(-65\) dB imposing on the stopband of the prototype filter; and (v) our proposed method as in Procedure 3.1 with a constraint of \(-65\) dB imposing on the stopband of the prototype filter. As it can be seen from the table, the optimization of the problem (12) significantly reduces the amplitude distortion of the NUFB and the stopband attenuation of the prototype filter over the initial solution obtained using the window method [13]. The amplitude distortion is reduced further by searching for an optimal \(\beta\) of the Kaiser window as in Procedure 3.1. More specifically, the amplitude distortion of the optimal filter bank is 0.00048 and the stopband attenuation of the prototype filter is \(-78.23\) dB, while the amplitude distortion for the WCLS approach discussed in [12] is much larger 0.0029 and the stopband attenuation of the prototype filter is \(-61.49\) dB.

Figs. 3 and 4 show, respectively, the magnitude response and the amplitude distortion for the optimal non-uniform CMFB obtained by our proposed method using Procedure 3.1. The NUFB has a low amplitude distortion with a maximum value of 0.00048.

Case 2. We consider the design of a 4-channel non-uniform CMFB with the decimation factors \((8, 8, 4, 2)\) and \(M_1 = M_2 = 8, M_3 = 4, M_4 = 2\). Here, we have \(l_1 = l_2 = 1, l_3 = 2\) and \(l_4 = 4\).

Table 2 shows the results with the prototype filter length chosen as 154 and 198. For \(L = 154\), the table shows the result using the weighted Chebyshev method [12],...
the WCLS method [12] and our proposed method with \( A_s = 65 \). Our proposed method results in a much lower maximum amplitude distortion 0.00061 compared to 0.0028 as in [12] and a much lower stopband attenuation of the prototype filter compared to [12].

For \( L = 198 \), the table shows the method discussed in [13] reported in [12], our proposed method with the stopband constraint \(-80\) dB imposed in the prototype filter and our proposed method with the stopband constraint \(-90\) dB imposed in the prototype filter. The maximum amplitude distortion is significantly reduced by using our proposed method while maintaining approximately the same stopband attenuation for the prototype filter.

Figs. 5 and 6 show the magnitude response and the amplitude distortion of the optimal non-uniform CMFB using our proposed method with a specification of \(-85\) dB imposed on the stopband of the prototype filter. The maximum amplitude distortion error is 0.0011, which is much smaller than the result obtained in [13] with a lower maximum stopband error for the prototype filter.

Figs. 7 and 8 show the result for the optimal filter when the specification of \(-90\) dB is imposed on the stopband of the prototype filter. The maximum amplitude distortion error is lightly increased to 0.0012 but with a lower stopband level for the prototype filter when compared with the previous case.

### 5.2. Non-uniform CMFB with finite precision coefficients

We now consider the design of a 4-channel non-uniform CMFB with discrete coefficients and the decimation factor \((8, 8, 4, 2)\), e.g. \( M_1 = M_2 = 8, M_3 = 4 \) and \( M_4 = 2 \) with \( l_1 = l_2 = 1, l_3 = 2 \) and \( l_4 = 4 \). The number of bits is \( B = 14 \) with \( B_1 = 3 \).

Table 3 shows the result for the non-uniform CMFB with discrete coefficients and the filter length \( N = 154 \). The table shows (i) the infinite precision solution with \( A_s = 90 \); (ii) the optimal solution obtained by our proposed method with \( Q = 250 \); (iii) the optimal solution obtained by our proposed method with \( Q = 260 \); and (iv) the solution obtained using the GA method discussed in [12]. For the optimal solution, the table also shows (i) the initial quantized solution for a fixed number of SPT terms \( Q \); (ii) the local optimal solution obtained using Procedure 4.1; and (iii) the global discrete solution obtained using Procedure 4.2.

The discrete optimization problem (26) is optimized with a constraint \( \epsilon_d = -58\) dB imposed on the stopband of the prototype filter. The value \( \epsilon_d = -58\) dB is chosen so that a fair comparison can be made with the results in [12]. It can be seen from the table that the quantized solution, the local optimal solution and the global discrete solution all satisfy the constraint with the stopband attenuation lower than \(-58\) dB. For \( Q = 250 \), the amplitude distortion significantly reduces for the local optimal solution when compared with the quantized solution. In fact, the amplitude distortion for the local optimal solution, 0.000696, is much smaller than 0.0018 for the quantized solution. The amplitude distortion for the optimal solution reduces further to 0.000318 when compared with the local optimal solution.

Similarly, for \( Q = 260 \), the amplitude distortion of the optimal solution, 0.000312, is significantly lower than the quantized solution, 0.0019. In addition, the optimal solution significantly improves the solution discussed in [12] based on the GA method. For \( Q = 260 \), the discrete optimal solution has the amplitude distortion of 0.000318 and the stopband attenuation of \(-66.52\) dB compared to the method discussed in [12] with the amplitude distortion of 0.0058 and the stopband attenuation of \(-56.25\) dB.
Table 4 shows the result for the non-uniform CMFB with finite precision coefficients when the number of prototype filter coefficients increases to \( N = 198 \). The value of \( Q \) is chosen as 290 and 300. The constraint on the stopband attenuation \( \epsilon_d = -75 \) dB is chosen approximately the same as the stopband level of the quantized solution. From the table, it can be seen that the local optimal solution significantly improves the quantized solution. For example, for \( Q = 290 \), the amplitude distortion for the local optimal solution is 0.000715 compared to 0.0013 for the quantized solution. The amplitude distortion for the global optimal solution reduces further from the local optimal solution to 0.000498.

Finally, the optimal discrete solution significantly improves over the solution discussed in [12]. For \( Q = 290 \), the optimal solution has the amplitude distortion of 0.000498 and the stopband attenuation of \(-75.30\) dB compared to the solution discussed in [12] with the amplitude distortion of 0.003 and the stopband attenuation of \(-62.3\) dB.

Figs. 9 and 10 show the magnitude response and the amplitude distortion for the optimal non-uniform CMFB with finite precision coefficients. The length of the prototype filter is 198 with \( Q = 300 \). The amplitude distortion for the NUFB is low with the maximum value of 0.000505 and a maximum stopband attenuation of \(-75.10\) dB for the prototype filter.

6. Conclusions. This paper investigates the design of non-uniform CMFB with both infinite precision coefficients and finite precision coefficients. An efficient optimization approach is developed for the design of non-uniform CMFB with infinite precision coefficient. An effective optimization procedure based on the discrete filled function is then developed to design the prototype filter with finite precision coefficients. Design examples show that the designed infinite precision and finite precision filter banks have lower distortion and better performance compared with other existing methods.

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Table 1. Non-uniform (4,4,8,8,4) CMFB with infinite precision coefficients and $N = 154$

| Methods                                                                 | Amplitude distortion | Stopband attenuation |
|------------------------------------------------------------------------|----------------------|----------------------|
| Weighted Chebyshev in [12]                                            | 0.0042               | −60.65 dB            |
| WCLS approach in [12]                                                 | 0.0029               | −61.49 dB            |
| Window method [13] with $A_s = 65$                                     | 0.0067               | −69.85 dB            |
| Window method [13] with $A_s = 65$ as the initial to (12) with a constraint of −65 dB for prototype filter stopband | 0.0014               | −65.00 dB            |
| Proposed method with $A_s = 65$ and a constraint of −65 dB for prototype filter stopband | 0.00048              | −78.23 dB            |

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Figure 3. Magnitude response for the 5-channel non-uniform CMFB with decimation factor (4,4,8,8,4) and infinite precision coefficients

Figure 4. Amplitude distortion for the 5-channel non-uniform CMFB with decimation factor (4,4,8,8,4) and infinite precision coefficients
Figure 5. Magnitude response for the 4-channel non-uniform CMFB with decimation factor (8,8,4,2) and infinite precision coefficients for $-85$ dB restriction in the prototype filter stopband.

Figure 6. Amplitude distortion for the 4-channel non-uniform CMFB with decimation factor (8,8,4,2) and infinite precision coefficients for $-85$ dB restriction in the prototype filter stopband.
Figure 7. Magnitude response for the 4-channel non-uniform CMFB with decimation factor (8,8,4,2) and infinite precision coefficients for $-90$ dB restriction in the prototype filter stopband.

Figure 8. Amplitude distortion for the 4-channel non-uniform CMFB with decimation factor (8,8,4,2) and infinite precision coefficients for $-90$ dB restriction in the prototype filter stopband.
Figure 9. Magnitude response for the 4-channel non-uniform CMFB with decimation factor (8,8,4,2) and finite precision coefficients.

Figure 10. Amplitude distortion for the 4-channel non-uniform CMFB with decimation factor (8,8,4,2) and finite precision coefficients.
Table 2. Non-uniform (8,8,4,2) CMFB with infinite precision coefficients

| \( N \) | Methods | Amplitude dist. | Stopband att. |
|-----|----------|-----------------|---------------|
| 154 | Weighted Chebyshev [12] | 0.0039 | −60.65 dB |
|      | WCLS [12] | 0.0028 | −61.49 dB |
|      | Optimal solution with \( A_s = 65 \) | 0.00061 | −71.44 dB |
| 198 | Method in [13] as quoted in [12] | 0.0025 | −79.65 dB |
|      | Method in [13] with \( A_s = 80 \) | 0.0021 | −89.95 dB |
|      | Proposed method with restriction of −85 dB for prototype filter stopband | 0.0011 | −85.00 dB |
|      | Proposed method with restriction of −90 dB for prototype filter stopband | 0.0012 | −90.01 dB |

Table 3. Non-uniform (8,8,4,2) CMFB with finite precision coefficients and \( N = 154 \)

| SPT | Methods | Amplitude dist. | Stopband att. | Total adders |
|-----|----------|-----------------|---------------|--------------|
| In. precision sol. \( A_s = 90 \) | | 0.0017 | −90.00 dB | - |
| \( Q = 250 \) | \( \epsilon_d = -58 \text{ dB} \) | Quantized solution | 0.0018 | −81.57 dB | 250 |
| | | Local optimal | 0.000696 | −72.63 dB | 250 |
| | | Optimal solution | 0.000318 | −66.52 dB | 250 |
| \( Q = 260 \) | \( \epsilon_d = -58 \text{ dB} \) | Quantized solution | 0.0019 | −81.63 dB | 256 |
| | | Local optimal | 0.000684 | −71.56 dB | 263 |
| | | Optimal solution | 0.000312 | −67.84 dB | 268 |
| | Method in [12] using GA | 0.0058 | −56.25 dB | 266 |

Table 4. Non-uniform (8,8,4,2) CMFB with finite precision coefficients and \( N = 198 \)

| SPT | Methods | Amplitude dist. | Stopband att. | Total adders |
|-----|----------|-----------------|---------------|--------------|
| In. precision sol. \( A_s = 90 \) | | 0.0012 | −90.00 dB | - |
| \( Q = 290 \) | \( \epsilon_d = -75 \text{ dB} \) | Quantized solution | 0.0013 | −77.28 dB | 290 |
| | | Local optimal | 0.000715 | −75.12 dB | 289 |
| | | Optimal solution | 0.000498 | −75.30 dB | 290 |
| \( Q = 300 \) | \( \epsilon_d = -75 \text{ dB} \) | Quantized solution | 0.0014 | −77.14 dB | 300 |
| | | Local optimal | 0.00078 | −76.16 dB | 300 |
| | | Optimal solution | 0.000505 | −75.10 dB | 300 |
| | Method in [12] using GA | 0.003 | −62.30 dB | 315 |