The $W$ boson Mass and Muon $g - 2$: Hadronic Uncertainties or New Physics?

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There are now two single measurements of precision observables that have major anomalies in the Standard Model: the recent CDF measurement of the $W$ mass shows a $7\sigma$ deviation and the Muon $g - 2$ experiment at FNAL confirmed a long-standing anomaly, implying a $4.2\sigma$ deviation. Doubts regarding new physics interpretations of these anomalies could stem from uncertainties in the common hadronic contributions. We demonstrate that the two anomalies pull the hadronic contributions in opposite directions by performing electroweak fits in which the hadronic contribution was allowed to float. The fits show that including the $g - 2$ measurement worsens the tension with the CDF measurement and conversely that adjustments that alleviate the CDF tension worsen the $g - 2$ tension beyond $5\sigma$. This means that if we adopt the CDF $W$ mass measurement, the case for new physics in either the $W$ mass or muon $g - 2$ is inescapable regardless of the size of the SM hadronic contributions. Lastly, we demonstrate that a mixed scalar leptoquark extension of the Standard Model could explain both anomalies simultaneously.

INTRODUCTION

The CDF collaboration at Fermilab recently reported the world’s most precise direct measurement of the $W$ boson mass, $M_W^{CDF} = 80.4335 \pm 0.0094 \text{ GeV}$ [1], based on 8.8/fb of data collected between 2002-2011. This deviates from the Standard Model (SM) prediction by about $7\sigma$. The recent FNAL E989 measurement of the muon’s anomalous magnetic moment ($a_{\mu}$) furthermore implies a new world average that is in $4.2\sigma$ tension [2] with the SM prediction from the Muon $g - 2$ Theory Initiative [3]. \!

Whilst the Fermilab $g - 2$ measurement was in agreement with the previous BNL E821 measurement [30], as shown in fig. 1 there appears to be tension between the new CDF measurement and previous measurements, including the previous CDF measurement with only 2.2/fb of data [31]. Updates to systematic uncertainties shift the previous measurement by 13.5 MeV, however, such that the CDF measurements are self-consistent. In the Supplemental Material we find a reduced chi-squared from a combination of $N = 7$ measurements of about $\chi^2/(N - 1) \approx 3$ and a tension of about $2.5\sigma$. Nevertheless, we show that these two measurements could point towards physics beyond the SM with a common origin and, under reasonable assumptions, that the new CDF $W$ mass measurement pulls common hadronic contributions in a direction that significantly strengthens the case for new physics in muon $g - 2$.

We now turn to the SM predictions for the $W$ mass and muon $g - 2$. Muon decay can be used to predict $M_W$ in the SM from the more precisely measured inputs, $G_\mu$, $\alpha$, and $M_Z$.

\begin{equation}
M_W^2 = M_Z^2 \left\{ \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi \alpha}{\sqrt{2} G_\mu M_Z^2} (1 + \Delta r)} \right\}. \quad (1)
\end{equation}

The loop corrections are contained in $\Delta r$: full one-loop contributions were first calculated in Refs. [40, 41], and the complete two-loop contributions are now avail-
able [42–59]. These have been augmented with leading three-loop and leading four-loop corrections [60–69]. The state-of-the-art on-shell (OS) calculation of \( M_W \) in the SM [39] updated with recent data gives 80.356 GeV [70], whereas the \( \overline{\text{MS}} \) scheme [71] result is about 6 MeV smaller when evaluated with the same input data. Direct estimates of the missing higher order corrections were a little smaller (4 MeV for OS and 3 MeV for \( \overline{\text{MS}} \)).

The predictions also suffer from parametric uncertainties, with the largest uncertainties coming from \( m_t \) and may be around 9 MeV [71], and depend on estimates of the hadronic contributions to the running of the fine structure constant, \( \Delta \alpha_{\text{had}} = \Delta \alpha_{\text{had}}^{(5)}(M_Z^2) \), defined at the scale \( M_Z \) for five quark flavors. This is constrained by electroweak (EW) data and by measurements of the \( e^+e^- \to \text{hadrons} \) cross section (\( \sigma_{\text{had}} \)) through the principal value of the integral [72]

\[
\Delta \alpha_{\text{had}} = \frac{M_Z^2}{4\pi^2\alpha} \int_{m_{\pi^0}}^{\infty} \frac{ds}{M_Z^2 - s} \sigma_{\text{had}}(\sqrt{s}),
\]

where \( m_{\pi^0} \) is the neutral pion mass. The parametric uncertainties may be estimated through global EW fits. For example, two recent global fits without any direct measurements of the \( W \) boson mass predict 80.354 ± 0.007 GeV [37] and 80.3591 ± 0.0052 GeV [73] in the OS scheme. Lastly, the CDF collaboration quote 80.357 ± 0.006 GeV [1]. While the precise central values and uncertainty estimates vary a little, all of these predictions differ from the new CDF measurement by about \( 7\sigma \).

Turning to muon \( g - 2 \), the SM prediction for \( a_\mu \) includes hadronic vacuum polarization (HVP) and hadronic light-by-light (HLbL) contributions in addition to the QED and EW contributions that can be calculated perturbatively from first principles [3]. Although HVP is not the main contribution for \( a_\mu \), it suffers from the largest uncertainty and it is hard to pin down its size. Two approaches are commonly used to extract the contributions from HVP. First, a traditional data driven method in which the HVP contributions are determined from measurements of \( \sigma_{\text{had}} \) using the relationship [74]

\[
a_\mu^{\text{HVP}} = \frac{m_\mu^2}{12\pi^3} \int_{m_{\pi^0}}^{\infty} \frac{ds}{s} K(s) \sigma_{\text{had}}(\sqrt{s}),
\]

where \( m_\mu \) and \( m_{\pi^0} \) are the muon and neutral pion masses, respectively, and \( K(s) \) is the kernel function as shown in Refs. [74, 75]. This approach results in \( a_\mu^{\text{HVP}} = 693.1(4.0) \times 10^{-10} \) with an uncertainty of less than 0.6\% [8–10, 12, 13, 76]. The second approach uses

\[\Delta M_W = 9.4 \text{ MeV} \text{ is fixed when obtaining the \( \pm 1\sigma \) region.}\]

lattice QCD calculations. The recent leading-order lattice QCD calculations for HVP from the BMW collaboration significantly reduced the uncertainties and resulted in \( a_\mu^{\text{HVP}} = 707.7(5.5) \times 10^{-10} \) [77]. This, however, shows tension with the \( \sigma_{\text{had}} \) measurements method.

The \( M_W \) and muon \( g - 2 \) calculations are in fact connected by the fact that both \( \Delta \alpha_{\text{had}} \) and the HVP contributions can be extracted from the hadronic cross section, \( \sigma_{\text{had}}(\sqrt{s}) \), through eqs. (2) and (3). We assume that the energy dependence of this cross-section, \( g(\sqrt{s}) \), is reliably known for \( \sqrt{s} \geq m_{\pi^0} [9, 12] \), but that the overall scale, \( \sigma_{\text{had}} \), may be adjusted,

\[
\sigma_{\text{had}}(\sqrt{s}) = \sigma_{\text{had}} g(\sqrt{s}).
\]

This is similar to scenario (3) in Ref. [72]. Using eqs. (2) and (4) we may trade \( \sigma_{\text{had}} \) for \( \Delta \alpha_{\text{had}} \) giving \( \Delta \alpha_{\text{had}} \propto \sigma_{\text{had}} \). The HVP contributions depend on \( \Delta \alpha_{\text{had}} \) and conversely estimates of the HVP contributions from either hadronic cross-sections or lattice QCD constrain \( \Delta \alpha_{\text{had}} \). Further details of the transformation between \( \Delta \alpha_{\text{had}} \) and \( \sigma_{\text{had}} \) are provided in the Supplemental Material. Thus we can transfer constraints on \( \Delta \alpha_{\text{had}} \) from measurements of \( M_W \) to constraints on the HVP contributions to muon \( g - 2 \) and vice-versa [78–80] through global EW fits. Motivated by the above, we study how the new \( M_W \) measurement from CDF impacts estimates of muon \( g - 2 \) in global EW fits, and then show that a scalar leptoquark model could provide a simultaneous explanation of both muon \( g - 2 \) and the \( W \) mass anomalies.

ELECTROWEAK FITS OF THE W MASS AND MUON \( g - 2 \)

We first investigated the impact of the \( W \) mass on the allowed values of \( \Delta \alpha_{\text{had}} \) by performing EW fits using \texttt{Gfitter} [37, 81–84] with data shown in table S1 in which \( m_s, m_t, M_Z, \alpha_s \) and \( \Delta \alpha_{\text{had}} \) were allowed to float. The Fermi constant \( G_F = 1.1663787 \times 10^{-5} \text{ GeV}^{-2} \) and the fine-structure constant \( \alpha = 1/137.035999074 \) [85] were fixed in our calculation. Although \( \Delta \alpha_{\text{had}} \) is not a free parameter of the SM as it is in principle calculable, it isn’t precisely known and so we allowed it to float, following the approach used in Ref. [72]. We found the allowed \( \Delta \alpha_{\text{had}} \) when assuming specific \( W \) masses between 80.3 GeV and 80.5 GeV; the results form the diagonal red band in fig. 2. The previous world average (PDG 2021) and current CDF measurement (CDF 2022) of the \( W \) mass are shown by blue and green vertical bands, respectively, and the corresponding best-fit \( \Delta \alpha_{\text{had}} \) are indicated by blue and green dashed horizontal lines, respectively. From the intersection of regions allowed by CDF 2022
had \times 10^4

\begin{align*}
\text{SM} &\pm 1-\sigma \\
\text{PDG 2021} &\\
\text{CDF 2022} &\\
\text{e^+e^-} &\\
\text{BMWc} &\\
\end{align*}

\begin{align*}
\Delta \alpha(5) &\\
\end{align*}

Figure 2. The allowed values of $\Delta \alpha_{\text{had}}$ assuming specific values of the W boson mass in the SM found from EW global fits. The solid line indicates the central value from the fit without any input for $\Delta \alpha_{\text{had}}$, while the red band shows the $1\sigma$ region. The current world average (PDG 2021) and new measurement (CDF 2022) for $M_W$ are indicated by vertical bands. We indicate the favored $\Delta \alpha_{\text{had}}$ from BMWc lattice calculations (gray), $e^+e^-$ cross section measurements (magenta), our fit using $M_W$ from PDG 2021 (blue) and our fit using $M_W$ from CDF 2022 (green).

SCALAR LEPTOQUARK MODEL

Even without light new physics, sizable BSM contributions to muon $g - 2$ can be obtained by an operator that gives an internal chirality flip in the one-loop muon $g - 2$ corrections (see e.g. Refs. [86, 87] for a review). On the other hand, BSM contributions to the W mass can be obtained when there are large corrections to the oblique parameter $T$ [88]. We show that a scalar leptoquark model can satisfy both of these criteria and provide a simultaneous explanation of both muon $g - 2$ and the W mass anomalies. We anticipate other possibilities, including composite models with non-standard Higgs bosons [89].

Scalar leptoquarks (LQs) (see Ref. [90] for a review), or more specifically the scalar leptoquarks referred to as $S_1 (3, 1, 1/3)$ and $R_2 (3, 2, 7/6)$ in Ref. [91–93], are well known to provide the chirality flip needed to give a large contribution to $a_\mu$ [94], and have also been proposed for a simultaneous explanation of the flavour anomalies [95]. Furthermore due to the mass splitting between its physical states the SU(2) doublet $R_2$ is capable of making a considerable contribution to the W mass. However we find that the mass splitting from a conventional Higgs portal interaction cannot generate corrections big enough to reach the CDF measurement, unless the interaction $\lambda_{HH} R_2 R_2^* H^H$ is non-perturbative. We thus analyze the plausibility of situations in which one-loop contributions to the anomalous muon magnetic moment and W mass corrections are created via the mixing of two scalar LQs through the Higgs portal. For simplicity, we consider the $S_1 & S_3 (\mathbf{3}, \mathbf{3}, 1/3)$ scenario,

$$\mathcal{L}_{S_1 & S_3} = \mathcal{L}_{\text{mix}} + \mathcal{L}_{\text{LQ}},$$

where the first term is responsible for the mixing of the two LQs, and the second specifies the interaction between...
Table I. SM predictions from EW fits for $\Delta \alpha_{\text{had}}$ and $M_W$, and the differences with respect to measurements of muon $g-2$ and the $W$ mass, $\delta_{\mu}$ and $\delta M_W \equiv M_W^{\text{CDF}} - M_W$. The input data for $\Delta \alpha_{\text{had}}$ and $M_W$ are listed in first two rows for each case.

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
\text{Input} & M_W & \Delta \alpha_{\text{had}} & \text{Indirect} & M_W & \Delta \alpha_{\text{had}} & \text{Indirect} & M_W & \Delta \alpha_{\text{had}} & \text{Indirect} \\
\hline
\text{BMW} & 280.9(1.4) & 275.9(1.1) & 274.4(1.4) & 80.379(12) & 80.379(12) & 80.379(12) & 80.4335(94) & 80.4335(94) & 80.4335(94) \\
\text{PDG} & 281.8(1.5) & 276.1(1.1) & - & 18.32/15 & 16.01/15 & 15.89/14 & 79(11) & 73(11) & 67(12) \\
\text{CDF} & 281.8(1.5) & 276.1(1.1) & - & 23.41/16 & 18.74/16 & 17.59/15 & 74.51/16 & 62.58/16 & 47.19/15 \\
\hline
\end{array}
\]

By construction, the LQ mixing is mediated by the Yukawa couplings of the $LQ$ fields to the $S_1$ and $S_3$ scalar fields, namely $y_R^{ij} S_1^i S_1^j$ and $y_L^{ij} S_3^i S_3^j$, which generate mass eigenstates,

\[
m_{S_{\pm}}^2 = \frac{m_{S_1}^2 + m_{S_3}^2}{2} \pm \frac{1}{2} \sqrt{(m_{S_1}^2 - m_{S_3}^2)^2 + 4\delta^2} \tag{8}
\]

where $\delta \equiv \lambda v^2/2$. We also define $\Delta m \equiv m_{S_+} - m_{S_-}$ as the difference between mass parameters of the gauge eigenstates $S_1$ and $S_3$. We see that $\delta$ is the induced splitting when $\Delta m = 0$. This mass splitting can generate a non-vanishing oblique correction to the $T$ parameter at one-loop [96],

\[
T = \frac{3}{4\pi^2 s_W^2} \frac{1}{M_W^2} \left[ F \left(m_{S_+}, m_{S_-}\right) \cos^2 \phi + F \left(m_{S_+}, m_{S_-}\right) \sin^2 \phi \right] \tag{9}
\]

with

\[
F \left(m_1, m_2\right) = m_1^2 + m_2^2 - \frac{2m_1^2m_2^2}{m_1^2 - m_2^2} \log \left(\frac{m_1^2}{m_2^2}\right) \tag{10}
\]

where $\phi$ is the mixing angle determined via $\tan 2\phi = 2\delta/(m_{S_+}^2 - m_{S_-}^2)$. The shift in $M_W$ from the SM prediction can be related to the oblique $T$ parameter via,

\[
\Delta M_W^2 \equiv M_W^2 \left|_{\text{BSM}} - M_W^2 \right|_{\text{SM}} = \frac{\alpha c_W^4 m_W^2}{c_W^2 - s_W^2} T \tag{11}
\]

where $c_W$ and $s_W$ are the cosine and sine of the Weinberg angle. There are, furthermore, contributions from $S$ and $U$ that are subdominant in our LQ model. We determine the $T$ that is required to explain the CDF 2022 measurement from our EW global fits and use that in combination with eq. (9) to test if LQ scenarios can explain this data. We checked analytically and numerically that our calculation obeys decoupling, with the additional BSM contributions approaching zero in the limit of large LQ masses. We cross-checked eq. (9) with a full one-loop calculation of the $T$ parameter using \textsc{Sarah} 4.14.3 [97], \textsc{FeynArts} 3.11 [98], \textsc{FormCalc} 9.9 [99] and \textsc{LoopTools} 2.16 [100], finding good agreement with the results using just eq. (9). With the same setup we also verified that the combined contributions from $S$ and $U$ to $M_W$ are small and do not impact significantly on our results.

Whilst the mass splitting impacts the $W$ mass, the mixing impacts muon $g - 2$. Indeed, the mixing between interaction eigenstates allows the physical mass eigenstates to have both left- and right-handed couplings to muons and induces chirality flipping enhancements to muons $g - 2$ [96],

\[
\delta_{\mu} = -\frac{3m_{\mu}^2 m_t}{16\pi^2 m_{\mu}} \sin 2\phi \frac{y_L y_R}{m_{S_+}^2 - m_{S_-}^2} \left[ G \left(x_t^+\right) - G \left(x_t^-\right) \right] \tag{12}
\]

where $x_t^{\pm} = m_t^2/m_{S_{\pm}}^2$, the loop function is $G(x) = \frac{1}{x^2 - 1}$.
Δm eigenstates, it is possible to find explanations of both anomalies that predict the $W$-boson mass and muon $g - 2$ in agreement with measurements. The mixing coupling is set to be 1 and $m_{S_1} = 2$ TeV.

\[ \frac{1}{3}g_S(x) - g_F(x) \] with

\[ g_S(x) = \frac{1}{x - 1} - \frac{\log x}{(x - 1)^2} \]
\[ g_F(x) = \frac{x - 3}{2(x - 1)^2} + \frac{\log x}{(x - 1)^3} \]

and we simplify our notation by letting $y_L \equiv y_{L}^{b\mu}$ and $y_R \equiv y_{R}^{b\mu}$. As shown in eq. (12), the sign of $\delta a_\mu$ depends on the sign of $\sin 2\phi$, which in turn depends on the sign of the mass splitting, $\Delta m$. There is a discontinuity in $\tan 2\phi$ at $\Delta m = 0$ as it depends on whether we approach $\Delta m > 0$ or from $\Delta m < 0$. Once we fix $\Delta m > 0$, the corresponding $\sin 2\phi$ is negative definite, resulting in a positive $\delta a_\mu$.

Note that in this case there is a cancellation between the contribution of the lighter and heavier mass eigenstates, which reduces the effect of the very large chirality flipping enhancement $m_t/m_\mu$ somewhat. If we were to consider couplings between $S_1$ state and left-handed muons as well, the contributions would be considerably enhanced, so this would simply make it easier to explain $a_\mu$ while having little or no impact on the $W$ mass prediction.

Since the dominant BSM contribution to $a_\mu$ in eq. (12) is proportional to $y_L y_R$ and the $W$ mass corrections can be controlled by the mass splitting $\Delta m$ of the gauge eigenstates, it is possible to find explanations of both $a_\mu$ and the 2022 CDF measurement of $M_W$ by varying $y_L y_R$ and $\Delta m$. In fig. 3 we show regions in the $\Delta m - \sqrt{y_L y_R}$ plane that explain both measurements, where we have fixed the LQ mass to 2 TeV, a little above the LHC limit, and we have also fixed $\lambda = 1$ which affects the mixing.

LQ couplings of around one can explain the $a_\mu$ measurement within 1σ when we use the SM prediction from the theory white paper, where $e^+e^-$ data is used for $a_\mu^{\text{HVP}}$. Explaining the SM prediction from the BMW collaboration requires even smaller couplings, though in this case the tension with the SM is anyway less than 2σ. Using $e^+e^-$ data to also fix $\Delta m_{\text{had}}$ means there then remains an additional deviation between the SM $M_W$ and $a_\mu$ anomalies can be achieved in the region where the yellow BMW band in fig. 3 can extend to $\Delta m \approx 0$, but to within 1σ a small non-zero $\Delta m$ is required.

CONCLUSIONS

There are two major outstanding anomalies in the SM: a recently announced anomaly in the $W$ boson mass measured by CDF and a longstanding anomaly in muon $g - 2$. We show that because the anomalies pull the hadronic contributions in opposite directions, the new CDF $W$ mass measurement indirectly increases the deviation in muon $g - 2$. Indeed, combining $e^+e^-$ data and the new CDF measurement we find that the SM prediction for muon $g - 2$ deviates from the world average by more than 4σ. Regardless of the method used to estimate the hadronic contributions to muon $g - 2$ and $M_W$, we found that the tension between one of the measurements and the SM prediction is always more than 4σ. We then considered a scalar leptoquark extension of the SM and showed that it could simultaneously explain both anomalies. This shows that relatively simple new physics explanations of both anomalies exist, demonstrating that it is plausible that the anomalies are signs of new physics. The results point to new physics that has large chirality flipping enhancements in the one-loop diagrams for muon $g - 2$ and significant BSM contributions to the oblique $T$ parameter that can be given through custodial symmetry violation.

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Supplemental Material: The W boson Mass and Muon g − 2: Hadronic Uncertainties or New Physics?

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In this supplemental material, we present the data in our global fits (table S1), our procedure of our simple combination of $M_W$ and the transformation between $\Delta \alpha_{\text{had}}$ and $\alpha^\mu_{\text{HVP}}$.

| Parameter          | Measured value | Ref. |
|--------------------|----------------|------|
| $M_W$ [GeV]        | 80.379(12)     | [85] |
| $M_W$ [GeV]        | 80.4335(94)    | [1]  |
| $\Delta \alpha_{\text{had}}(M_Z^2)$ | See text     |
| $m_b$ [GeV]        | 125.25(17)     | [85] |
| $m_t$ [GeV]$^a$    | 172.76(58)     | [85] |
| $\alpha_s(M_Z)$    | 0.1179(9)      | [85] |
| $\Gamma_W$ [GeV]   | 2.085(42)      | [85] |
| $\Gamma_Z$ [GeV]   | 2.4952(23)     | [101]|
| $M_Z$ [GeV]        | 91.1875(21)    | [101]|
| $A_{\text{FB}}^{0,b}$ | 0.0992(16)   | [101]|
| $A_{\text{FB}}^{0,c}$ | 0.0707(35)   | [101]|
| $A_{\text{FB}}^{0,t}$ | 0.0171      | [101]|
| $A_b$              | 0.923(20)      | [101]|
| $A_c$              | 0.670(27)      | [101]|
| $A_t(SLD)$         | 0.1513(21)     | [101]|
| $A_t(\text{LEP})$ | 0.1465(33)     | [101]|
| $R_0^b$            | 0.21629(66)    | [101]|
| $R_0^c$            | 0.1721(30)     | [101]|
| $R_0^t$            | 20.767(25)     | [101]|
| $\sigma_0^h$ [nb] | 41.540(37)     | [101]|
| $\sin^2 \theta_{\text{eff}}(Q_{\text{FB}})$ | 0.2324(12)   | [101]|
| $\sin^2 \theta_{\text{eff}}(\text{Teva})$ | 0.23148(33)  | [102]|
| $\mu_c$ [GeV]     | 1.27(2)        | [85] |
| $\mu_b$ [GeV]     | 4.18^{+3}_{-2} | [85] |

$^a$ 0.5 GeV theoretical uncertainty is included.

Table S1. The measurements included in the global EW fit. Correlations among $(M_Z, \Gamma_Z, \sigma_0^h, R_0^b, A_{\text{FB}}^{0,c}, A_{\text{FB}}^{0,b}, A_c, R_0^t, R_0^c)$ are also taken into account [101].

**SIMPLE COMBINATION OF W MASS MEASUREMENTS**

We compute a weighted average of $N$ measurements following a standard procedure (see e.g., Ref. [38]),

$$\bar{x} \pm \Delta x = \frac{\sum_{i=1}^{N} w_i x_i}{\sum_{i=1}^{N} w_i} \pm \left(\sum_{i=1}^{N} w_i^{-1}\right)^{-1/2}$$  \hspace{1cm} (S.1)
where \(w_i = 1/\sigma_i^2\). We include the seven measurements — LEP [32], LHCb [33], ATLAS [34], D0 [35] 92-95 (95/pb) and 02-09 (4.3/fb), CDF [1, 31, 36] 88-95 (107/pb) and 02-11 (9.1/fb) — avoiding double-counting the CDF data. This resulted in our simple combination

\[
M_W = 80.411 \pm 0.007 \text{ GeV.} \tag{S.2}
\]

This combination remains about 6\(\sigma\) away from the SM. The chi-squared associated with this estimate,

\[
\chi^2 = \sum_{i=1}^{N} w_i (x_i - \bar{x})^2 , \tag{S.3}
\]

was 17.7 with \(N - 1 = 6\) degrees of freedom. The associated significance was 2.5\(\sigma\), found from

\[
p = 1 - F_{\chi^2}(\chi^2) \quad \text{and} \quad Z = \Phi^{-1}(1 - p) \tag{S.4}
\]

where \(F_{\chi^2}\) is the chi-squared cumulative density function with \(n\) degrees of freedom and \(\Phi\) is the standard normal cumulative density function. This result depends on the number and choices of measurements combined. The code to reproduce these calculations is available at ☰.

We note, however, that a true combination should include careful consideration of correlated systematic errors and expert judgment about unstated or underestimated errors and systematics. For example, if the reduced chi-squared indicates discrepant measurements, the PDG [38] may inflate the estimated errors, though this does not impact the central value, or decline to combine them (for further discussion see e.g., Ref. [103–106]). In our case, the PDG prescription would inflate the error by about two, if the measurements were combined. This would reduce the discrepancy with the SM to about 3\(\sigma\).

## THE RELATIONSHIP BETWEEN \(\Delta \alpha_{\text{had}}\) AND \(\alpha^\mu_{\text{HVP}}\)

Since both \(\Delta \alpha_{\text{had}}\) and \(\alpha^\mu_{\text{HVP}}\) can be extracted from \(\sigma_{\text{had}}\) measurements, changes in \(\sigma_{\text{had}}\) affect the transformation between \(\Delta \alpha_{\text{had}}\) and \(\alpha^\mu_{\text{HVP}}\). On the one hand, one can directly use the experimental data from \(\sigma_{\text{had}}\) measurements (the \(e^+e^-\) data) to derive \(\Delta \alpha_{\text{had}}\) and \(\alpha^\mu_{\text{HVP}}\) with the data-driven method as shown in eqs. (2) and (3). On the other hand, we can extract \(\Delta \alpha_{\text{had}}\) from EW fits or from estimates of the HVP contributions from the BMW lattice calculation and use that to indirectly indicate possible changes in \(\sigma_{\text{had}}\) compared with the experimental measurements. For the latter, we must make assumptions about the energy dependence of \(\sigma_{\text{had}}\) and the energy range in which it changes.

As pointed out in Ref. [72], we could modify \(\sigma_{\text{had}}\) only in the energy ranges:

\[
m_{\pi_0} \leq \sqrt{s} \leq 1.937 \text{ GeV,} \tag{S.5}
\]

\[
m_{\pi_0} \leq \sqrt{s} \leq 11.199 \text{ GeV or} \tag{S.6}
\]

\[
m_{\pi_0} \leq \sqrt{s} \leq \infty , \tag{S.7}
\]

that is, at low energies, at any moderate energies, or across the entire energy range. The hadronic cross section is unchanged above these thresholds. We previously only considered the latter possibility eq. (S.7); we now consider eqs. (S.5) and (S.6) and reconsider the relationship between \(\Delta \alpha_{\text{had}}\) and \(\alpha^\mu_{\text{HVP}}\), i.e. we follow the procedure introduced in Ref. [72]. We find that using eq. (S.7) and transforming \(\Delta \alpha_{\text{had}}\) to \(\alpha^\mu_{\text{HVP}}\) (\(\alpha^\mu_{\text{HVP}}\) to \(\Delta \alpha_{\text{had}}\)) results in the most conservative (aggressive) deviation from the \(e^+e^-\) data. However, the low energy range projection eq. (S.5) shows the opposite behaviour. We assume that \(\sigma_{\text{had}}\) changes by an overall factor over the range \(m_{\pi_0}\) to infinity, eq. (S.7). This is scenario (3) in Ref. [72]. Based on this assumption, we derive \(\alpha^\mu_{\text{HVP}}\) from \(\Delta \alpha_{\text{had}}\) (and vice-versa) with a naive and uniform scaling of the cross-section from the \(e^+e^-\) data [3, 9]. In this way, we obtain alternative predictions for \(\alpha^\mu_{\text{HVP}}\) that correspond to the \(M_W\) measurement under the assumption that no new physics affects the EW fits.

We use the above method to alter the SM prediction for \(a_\mu\) by taking all contributions except for \(\alpha^\mu_{\text{HVP}}\) to be those used in Ref. [3]. We then determine the deviation between the combined 2021 world average and our predictions (\(\delta a_\mu\)), after combining both theoretical and experimental uncertainties. The experimental uncertainty is fixed to \(41 \times 10^{-11}\) [2], but the theoretical uncertainty depends on the way in which \(\alpha^\mu_{\text{HVP}}\) was chosen. Each \(\delta a_\mu\) is indicated on the right-hand side of fig. 2 and listed in table I where we also show how many standard deviations this represents. Finally, we visualize \(\delta a_\mu\) and its tension from table I in fig. S1.
Lastly, to complete our study of the transformation between $\Delta\alpha_{\text{had}}$ and $a_{\mu}^{\text{HVP}}$, we consider case eq. (S.5). Since the BMW collaboration only released the first two bins data ($0 \text{ GeV} < \sqrt{s} \leq 1 \text{ GeV}$ and $1 \text{ GeV} < \sqrt{s} \leq \sqrt{10} \text{ GeV}$) of $\Delta\alpha_{\text{had}}$ [77], case eq. (S.5) may be suggested by BMWc results. Therefore, we include this case in table S2 for readers as a reference. Note we only use the integral breakdown from Ref. [9, 13] which provided smaller uncertainties for $a_{\mu}^{\text{HVP}}$ from the $e^+e^-$ data in their calculations. Hence, the $\delta a_{\mu}$ in table S2 would be reduced if the integral breakdown from other references [8, 12] was used. A significant caveat to keep in mind is that we don’t definitively know whether case eq. (S.5), (S.6) or (S.7) should be preferred.

**Figure S1.** The $a_{\mu}$ from experiment average of FNAL E989 and BNL E821 (purple), SM predictions from BMWc (blue), $e^+e^-$ data (red), EW fits w/o $\Delta\alpha_{\text{had}}$ (orange-dashed), EW fits with $\Delta\alpha_{\text{had}}$ from $e^+e^-$ (red-dashed), EW fits with $\Delta\alpha_{\text{had}}$ from BMWc (blue-dashed). The tensions of $\delta a_{\mu}$ are also shown for the comparison. The upper-left panel is w/o $M_W$ input, the upper-right and bottom panels are with PDG 2021 and CDF 2022 $M_W$ inputs, respectively.

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4 We thank Martin Hoferichter for bringing this to our attention.
| Input   | $M_W$ [GeV] | $\Delta \alpha_{\text{had}}$ | $\Delta \alpha_{\text{had}}^{(5)} (M_Z^2)$ | PDG 2021 | CDF 2022 |
|---------|------------|-------------------------------|------------------------------------|----------|----------|
|         | BMWc      | $e^+e^-$                      | BMWc                               | BMWc     | BMWc     |
|         | Indirect  | Indirect                      | Indirect                           | Indirect | Indirect |
|         | 80.379(12)| 277.4(1.2)                    | 277.4(1.2)                         | 277.4(1.2)| 277.4(1.2)|
|         | 80.379(12)| 276.1(1.1)                    | 276.1(1.1)                         | 276.1(1.1)| 276.1(1.1)|
|         | 80.379(12)| -                             | -                                  | -        | -        |
|         | 80.379(12)| -                             | -                                  | -        | -        |
|         | 80.4335(94)| 277.4(1.2)                    | 277.4(1.2)                         | 277.4(1.2)| 277.4(1.2)|
|         | 80.4335(94)| 276.1(1.1)                    | 276.1(1.1)                         | 276.1(1.1)| 276.1(1.1)|
|         | 80.4335(94)| -                             | -                                  | -        | -        |
|         | 80.4335(94)| -                             | -                                  | -        | -        |
| Fitted  | $\chi^2$/dof | 16.28/15                      | 16.01/15                           | 19.51/16 | 19.51/16 |
|         | $M_W$ [GeV] | 80.355(6)                      | 80.357(6)                          | 80.360(6)| 80.360(6)|
|         | $\Delta \alpha_{\text{had}}^* M^2$ | 277.1(1.2)                      | 275.9(1.1)                          | 276.8(1.1)| 275.6(1.1)|
|         | $\delta \sigma_{\mu} \times 10^{11}$ | 438(396)                         | 438(396)                            | 438(396)| 438(396)|
|         | Tension   | 1.1σ                           | 1.1σ                               | 3.2σ     | 3.2σ     |
|         | $\delta M_W$ [MeV] | 7(11)                         | 7(11)                              | 7(11)    | 7(11)    |
|         | Tension   | 7.2σ                           | 7.0σ                               | 6.7σ     | 6.7σ     |

Table S2. The same as table I but using the low energy projection eq. (S.5) for the transformation between $\Delta \alpha_{\text{had}}$ and $\sigma_{\mu}^{\text{HVP}}$. 