Final State Interaction in Exclusive \((e, e'NN)\) Reactions

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Contributions of nucleon-nucleon (NN) correlations, meson exchange currents and the residual final state interactions (FSI) on exclusive two-nucleon knock-out reactions induced by electron scattering are investigated. All contributions are derived from the same realistic meson exchange model for the NN interaction. Effects of correlations and FSI are determined in a consistent way by solving the NN scattering equation, the Bethe-Goldstone equation, for two nucleons in nuclear matter. One finds that the FSI re-scattering terms are non-negligible even if the two nucleons are emitted back to back.

I. INTRODUCTION

Exclusive \((e, e'NN)\) reactions, as well as photoinduced two nucleon knock-out experiments \((\gamma, NN)\) experiments, are often considered as very powerful tools to explore correlations in the nuclear many-body wave function, which are beyond the mean field or Hartree-Fock approximation. Using a simple picture for such reactions the real or virtual photon of inelastic electron scattering is absorbed by a pair of nucleons. This pair of nucleons is knocked out from the target nucleus while the residual (A-2) nucleons can be considered as spectators which form the ground state or another well defined bound state of the daughter nucleus. Due to the progress in accelerator and detector technology such triple coincidence experiments with sufficient resolution to identify specific states of the daughter nucleus have become possible and first results have been reported in the literature [1–4].

If the mean field approximation, in which the nucleons move independent from each other in the Hartree-Fock field, would be valid for the nuclear wave function, such processes should be strongly suppressed. Therefore the analysis of these reactions may provide detailed information about the correlations between the two nucleons which absorb the photon. Various modern models for the nucleon-nucleon (NN) interaction, which all yield an excellent fit to the NN scattering data below the threshold for pion production [5–7], provide different predictions for the short-range and tensor correlations [8,9]. Therefore one hopes that the analysis of exclusive two-nucleon knock-out experiments will provide additional constraints on the models for the NN interaction.

However, correlations in the ground state wave function of the target nucleus is not the only contribution to the cross section of photoinduced two-nucleon knock-out. One may also think about processes in which the photon is absorbed by one nucleon, which propagates off-shell and then shares the absorbed energy and momentum by scattering on a second nucleon. This process of a final state interaction (FSI) might be as important as the effect of correlations in the initial state. In fact, if we describe the absorption of a photon by a correlated pair of nucleons within Brueckner theory, such processes are represented by the diagrams (a) and (b) in Fig. 1. The processes of the FSI, which we just discussed, can be represented by the diagrams displayed in (c) and (d) of the same Fig. 1. Therefore these effects of correlations and FSI could be interpreted as just two different time orderings of NN interaction and photon absorption.

Of course there are some significant differences between these two contributions. The correlation effect can be described in terms of a Brueckner G-matrix, which is evaluated at a starting energy of two bound nucleons. Therefore the resulting G-matrix is real and the corresponding correlated wave function is bound to heal to the uncorrelated wave function at medium relative distances. The FSI on the other hand must be described in terms of a solution of the Bethe-Goldstone equation at a starting energy for the two nucleons above the Fermi energy. The G-matrix for these starting energies becomes complex and therefore we denote it by \(T\). This implies that the wave function of the two outgoing nucleons is a scattering wave function which does not show the healing property. Nevertheless, the symmetry of the contributions displayed in (a) - (d) of Fig. 1 suggests that one should try to treat them on the same level of accuracy. This is in fact the main aim of the study presented in this manuscript.
It should be mentioned that there is another important effect of FSI, which is usually considered in terms of an optical potential for the outgoing nucleons [10]. This mean field contribution, which accounts for the fact that the outgoing nucleons will be slowed down, redirected or absorbed in the field of the residual nucleus, must be be considered in addition to the FSI contribution discussed above.

Furthermore it is important to keep in mind that there are additional mechanisms which also contribute to the cross section of two-nucleon knock-out. Beside the contributions due to the pair correlations in the ground state wave function of the target nucleons and the FSI one must also consider the effects of the two-body terms in the operator for the electromagnetic current. These two-body terms include meson exchange currents (MEC) which can be derived from the commutator of the charge density with the nuclear Hamiltonian to obey the continuity equation. The meson exchange current contributions should be evaluated in a way consistent with the NN interaction used to determine FSI and correlation effects.

In addition, there are contributions to the two-body current which are of a different origin. Here we, mention the contribution due to the excitation of intermediate ∆ isobar excitations [13]. The contribution of this isobar current (IC) is not constrained by a continuity equation like the MEC and therefore depends on parameters like the meson-nucleon ∆ coupling constants and the propagator of the ∆ in the nuclear medium.

In the present work we want to evaluate the nuclear matrix elements for the absorption of a photon by a correlated pair of nucleons determining the effects of correlations, MEC and FSI in a consistent way from a realistic NN interaction [14]. As a first step we investigate the absorption of photons by a pair of nucleons in nuclear matter. For that purpose the technique, which has been described in [15], accounting for correlations, MEC and IC effects, is extended to allow the consideration of FSI as well. The investigation of these various effects shall provide a comparison of the relative importance of these different mechanisms and their interference under various kinematical conditions.

It is of course a serious disadvantage of such a study in nuclear matter that it does not provide results for a cross section which can directly be compared with experimental data produced for a specific target nucleus. In particular, it is impossible to take advantage of the fact that reactions leading to specific final states of the residual nucleus can be selective for one of the two-nucleon knock-out mechanisms discussed above. Such a feature has been observed in theoretical studies of \((e,e'pp)\) an \((e,e'pn)\) reactions on \(^{16}\text{O}\) [14,15]. On the other hand, however, a study of nuclear matter shall exhibit general features which are independent on the specific target nucleus considered and the corresponding long-range or low-energy correlations. In particular we would like to explore the importance of the FSI effects as compared to correlation and MEC effects in \(pp\) and \(pn\) knock-out, if the two nucleons are emitted back to back or in more parallel directions.

After this introduction we will briefly present the extension of the approach described in [15] to include the effects of final state interaction between the two ejected nucleons. The discussion of results is presented in section 3 and the final section 4 contains some concluding remarks.
II. FROM THE NUCLEAR CURRENT TO THE CROSS SECTION

As it has already often been described in the literature \cite{10,18,19} the differential cross section for the exclusive \((e, e'NN)\) reaction can be written

\[
\frac{d^3\sigma}{dE_i d\Omega_1 dE_2 d\Omega_2 dE'_2 d\Omega'_2} = \frac{1}{4} \frac{1}{(2\pi)^9} \tilde{p}_1 \tilde{p}_2 \tilde{E}_1 \tilde{E}_2 \sigma_{\text{Mott}} \left\{ v_C W_L + v_T W_T + v_S W_{TT} + v_I W_{LT} \right\} 
\times (2\pi) \delta(E_f - E_i)
\]

(1)

where the nuclear structure functions \(W_i\) \((i = L, T, TT, LT)\) contain the matrix elements of the nuclear current operator which consists of contributions for the different absorption processes of the virtual photon carrying momentum \(\tilde{q}\) and energy \(\omega\). Here, the kinematical variables are the energies \(\tilde{E}_1, \tilde{E}_2\) and the final momenta \(\tilde{p}_1, \tilde{p}_2\) of the two ejected nucleons.

These final nucleon momenta, \(\tilde{p}_1, \tilde{p}_2\), are different from the momenta of the two nucleons after absorption of the photon inside the target, which we denote by \(p'_i\) because of the mean field effect of the final state interaction. This mean field contains the attraction of the outgoing nucleons by the nuclear single-particle potential and essentially reduces the momentum of the outgoing nucleon. In infinite nuclear matter this retardation can simply be parameterised through an effective mass \(m^*\) and we obtain

\[
\tilde{E}_i = \frac{\tilde{p}_i^2}{2m} = \frac{p_i'^2}{2m^*} + U.
\]

(2)

In \cite{13} we investigated the influence of correlations (diagrams (a) and (b) of Figure 1), meson exchange currents (MEC, \cite{11,12}) arising from pions and \(\rho\) mesons, and isobaric currents (IC, \cite{13}) (diagram (e) of Figure 1) on the structure functions of \((e, e'pp)\) and \((e, e'pn)\) reactions. As an example we recall the expression for the matrix elements entering the structure functions from the correlation effect displayed in Figure 1(a)

\[
\int d\tilde{p}_a \langle \tilde{p}_a' | \tilde{J}_N | \tilde{p}_a \rangle S_2(\tilde{p}_a, \tilde{p}_a') \langle \tilde{p}_a', \tilde{p}_a' | G | \tilde{p}_1 \tilde{p}_2 \rangle.
\]

(3)

Here, \(\tilde{J}_N\) is the current operator for the absorption of the virtual photon with momentum \(\tilde{q}\) and energy \(\omega\) on a single nucleon \(\tilde{a}\). The matrix element of \(G\) is calculated for the starting energy of two bound nucleons with momenta \(\tilde{p}_1\) and \(\tilde{p}_2\). Also the two-particle propagator \(S_2\) contains the energy of these bound states and a Pauli operator which guarantees that the momenta \(\tilde{p}_a\) and \(\tilde{p}_a'\) are larger than the Fermi momentum. All this ensures that the dynamical correlation function, which is described in terms of \(S_2\) and \(G\), exhibits the so-called healing property.

In close analogy to this correlation term, we construct the single-particle current contribution with a re-scattering process taking place after the absorption of a virtual photon on a single bound nucleon. In order to give more insight in the evaluation of the corresponding structure functions entering the cross section in eq.(1), we take diagram (c) of Figure 1 as an example.

The matrix element for the absorption process displayed in Figure 1(c) for initial momenta \(\tilde{p}_1\) and \(\tilde{p}_2\) and final momenta \(\tilde{p}_1'\) and \(\tilde{p}_2'\) of the nucleon-nucleon pair reads

\[
\int d\tilde{p}_c \langle \tilde{p}_1' \tilde{p}_2' | T | \tilde{p}_c \tilde{p}_2 \rangle S_1(\tilde{p}_c) \langle \tilde{p}_c | \tilde{J}_N | \tilde{p}_1 \rangle.
\]

(4)

The one-nucleon propagator \(S_1(\tilde{p}_c)\) describes the propagation of the nucleon after absorption of the energy and momentum of the photon. It also contains a Pauli operator for the momentum of the intermediate nucleon \(\tilde{p}_c\). Finally, the re-scattering after the photon absorption is described via the \(T\) matrix element \(\langle \tilde{p}_1' \tilde{p}_2' | T | \tilde{p}_c \tilde{p}_2 \rangle\). The \(T\) matrix entering this calculation of the re-scattering process is derived from the solution of the Bethe-Goldstone equation for starting energies which lie above the threshold of twice the Fermi energy. In contrast to the case of eq.(3) this leads to complex \(T\) matrix elements.
III. RESULTS AND DISCUSSION

The calculations are performed in nuclear matter at saturation density \(k_F = 1.35 \text{ fm}^{-1}\). The coupling constants for the \(\pi\) exchange are \(f_{\pi NN} = 1.005\), \(f_{\pi N\Delta} = 2f_{\pi NN}\), and \(f_{\gamma N\Delta} = 0.12\). For the \(\rho\) exchange, the coupling constants were chosen to be \(g_{\rho}^2/4\pi = 0.86\), \(\kappa_{\rho} = 6.1\). The cutoff masses for the \(\pi\)-nucleon and \(\rho\)-nucleon form factors were fixed at \(\Lambda_\pi = 1.3\ \text{GeV}\) and \(\Lambda_\rho = 1.95\ \text{GeV}\) respectively. The \(\pi\)-nucleon and the \(\rho\)-nucleon vertices are compatible with the \(\pi\) and the \(\rho\) exchange part of the One-Boson-Exchange potential BONN A \([14]\) we used to determine the \(G\) and the \(T\) matrix for calculating the effects of NN correlations and re-scattering processes, respectively. The same potential has also been used to determine the single-particle energies for the nucleons in the nuclear medium which were parametrised according to \([8]\) by \(m^* = 623\ \text{MeV}\) and \(U = -86.8\ \text{MeV}\).

For all presented structure functions, the MEC contributions contain the seagull and meson in-flight currents for the exchange of a \(\pi\) meson as well as the seagull, pair and in-flight currents for the \(\rho\) meson. As we reported already in \([14]\), MEC contributions like the photon absorption on a pair of \(\pi\) and \(\rho\) mesons were found to give negligible contributions in all kinematical setups we investigated.

Our earlier investigations \([15]\) showed that the so-called 'super parallel' kinematical setup \([4]\) is very appropriate to investigate effects of nucleon-nucleon correlations. Therefore we will first present results of calculations including the re-scattering processes for this kinematical situation. For that purpose we chose once again the setup in which one of the two ejected nucleons moves in direction of the momentum of the absorbed photon and the other antiparallel to this direction. In Figures 2 and 3 we show the longitudinal and transverse structure functions for the knock-out of a proton-proton and a proton-neutron pair, respectively. In both cases, the virtual photon carries an energy of \(\omega = 215\ \text{MeV}\).
FIG. 3. Longitudinal (upper part) and transverse structure functions (lower part) for the knock-out of a proton-neutron pair in a 'super parallel' kinematical situation with angles $\theta'_p = 0^\circ$ and $\theta'_n = 180^\circ$ of the proton and the neutron with respect to the direction of the photon momentum. The left part of the figure assumes final kinetic energies $T_p = 156 \text{ MeV}$ and $T_n = 33 \text{ MeV}$ of the two protons while in the right part the final kinetic energies are $T_p = 116 \text{ MeV}$ and $T_n = 73 \text{ MeV}$. The photon energy was chosen to be $\omega = 215 \text{ MeV}$ in all cases. Together with the total structure functions (solid line) the contributions arising from correlations (dashed line), FSI (solid line with crosses), MEC (solid line with squares for the $\pi$ exchange, solid lines with circles for the $\rho$ exchange), and IC (dot-dashed line) are shown. Please note the different scales in the various parts of the figure. All structure functions have been multiplied by a common factor $10^{10}$.

For the structure functions of the $(e, e' pp)$ reaction displayed in Figure 2 we find non-negligible contributions from the FSI re-scattering term. This is true in particular in the case of a very asymmetric splitting of the available energy to the kinetic energy of the outgoing protons ($T_{p,1} = 156 \text{ MeV}$ and $T_{p,2} = 33 \text{ MeV}$), which is presented in the left part of the Figure. The contribution from FSI re-scattering is almost as large as the correlation effects. Both are significantly stronger than the contribution of the isobar current. Correlation and FSI effects add up in a rather coherent way to the total cross section.

The values for the structure functions are reduced by almost one order of magnitude if one considers the more symmetric distribution of the available energy among the two emitted protons, which is considered in the right part of the figure ($T_{p,1} = 116 \text{ MeV}$ and $T_{p,2} = 73 \text{ MeV}$). This shows that correlations as well as FSI effects are not very efficient in redistributing energy and momentum from the proton emerging in direction parallel to the momentum transfer $\vec{q}$ to the second proton, which is emitted antiparallel to $\vec{q}$. In this example the isobar current yields the largest contribution to the transverse structure function. It is worth noting that the FSI re-scattering effect is smaller than the correlation effect. This is true in particular for the longitudinal structure function.

In the case of the knock-out of a proton-neutron pair in Figure 3 the effects of correlations and final state interaction are masked to a large extent by the effects of meson exchange currents (MEC). The large MEC contribution is one reason for the fact that the structure functions of proton-neutron knock-out are almost one order of magnitude larger than the corresponding structure functions for proton-proton emission. In the case of the asymmetric energy splitting (large kinetic energy for the proton emitted parallel to $\vec{q}$), however the dominant contribution to the longitudinal structure function originates from correlation effects. The larger correlation contribution in $pn$ knock-out as compared to $pp$ reflects the importance of tensor correlations in the case of the $pn$ pair. In contrast to the proton-proton knock-
out, the influence of the final state interaction is negligible for the more symmetric distribution of the kinetic energies (see right part of Figure 3).

Comparing the absolute values of the contributions arising from re-scattering processes, one finds that these contributions are about twice as large as for the knock-out of a proton-proton pair. This demonstrates that the proton-neutron interaction is in general stronger than the proton-proton interaction. The enhancement of the correlation effect in going from the $pp$ case to the $pn$ case, however, is much stronger.

![Graph](https://via.placeholder.com/150)

**FIG. 4.** Longitudinal (above) and transverse structure functions (below) for the knock-out of a proton-proton pair. The angles of the outgoing protons are $\theta_{p,1} = \theta_{p,2} = 30^\circ$ while the final kinetic energies are $T_{p,1} = T_{p,2} = 70$ MeV. The photon energy was chosen to be $\omega = 230$ MeV. The total structure functions (solid line) consist of the one-body current (dashed line for correlations, solid line with crosses for FSI) and the IC contribution (dot-dashed line). All structure functions have been multiplied by a common factor $10^{10}$.

Sometimes the argument has been used that the FSI re-scattering effects should be minimal if the two nucleons are emitted back to back, the case which we have considered up to now. It is evident that FSI effects are very important if the two nucleons are emitted parallel or even form a deuteron as in $(e, e'd)$ experiments. How important is the FSI re-scattering contribution in kinematical setups, which are in between antiparallel and parallel emission? To answer this question we consider a kinematical set up, which we also inspected in our earlier investigations of $[15]$. Here, the two nucleons are ejected in a symmetrical way ($\theta_{p,1/p} = \theta_{p,2/n} = 30^\circ$, on opposite sides of the momentum transfer $\vec{q}$) having the same kinetic energies of 70 MeV. Results for the structure functions of $pp$ and $pn$ knock-out are displayed in Figures 4 and 5, respectively.

In the longitudinal channel of the $(e, e'pp)$ reaction (Figure 4) the contribution from final state interaction is clearly the dominating one. Contributions from correlations are only of minor importance. For the transverse channel, however, both effects yield contributions of similar size, which are furthermore of the same order of magnitude as the isobar contribution.

The absolute strength of re-scattering contribution to $(e, e'pn)$ displayed in Figure 5 is about twice as large as for the corresponding $(e, e'pp)$ case. The cross section for $pn$ emission in this setup, however, is by far dominated by the meson exchange currents which are related to the pion exchange.
FIG. 5. Longitudinal (above) and transverse structure functions (below) for the knock-out of a proton-neutron pair. The angles of the outgoing nucleons are $\theta'_p = \theta'_n = 30^\circ$ while the final kinetic energies are $T_p = T_n = 70\,\text{MeV}$. The photon energy was chosen to be $\omega = 230\,\text{MeV}$. The total structure functions (solid line) consist of the one-body current (dashed line for correlations, solid line with crosses for FSI), the MEC contributions (solid lines with squares and circles) and the IC contribution (dot-dashed line). All structure functions have been multiplied by a common factor $10^{10}$.

IV. SUMMARY AND CONCLUSIONS

The effects of re-scattering processes in the final state interaction of two-nucleon knock-out reactions induced by electron scattering have been investigated. The corresponding matrix elements leading to longitudinal and transverse nuclear structure functions have been calculated for a pair of nucleons in nuclear matter. The contributions from re-scattering are calculated in a way which is consistent with the evaluation of correlation effects and meson exchange current contributions. All contributions are derived from the same realistic meson exchange model of the NN interaction. This allows a systematic study of the relative importance of these effects under various kinematical conditions.

The effects of final state interactions (FSI) are very important if the two nucleons are emitted in directions with small angle in between. As an example we consider the longitudinal structure function for $(e, e'pp)$ with an angle of $60^\circ$, between the momenta of the outgoing protons. This example is completely dominated by the FSI effects. FSI contributions are larger by a roughly a factor of two in $(e, e'pn)$ as compared to $(e, e'pp)$ processes. Since other mechanisms, like correlation effects and meson exchange current contributions, are enhanced by a factor larger than two in $pn$ as compared to $pp$ knock-out, the relative importance of FSI are less pronounced in $(e, e'pn)$ reactions.

FSI effects are non-negligible also in the case of back to back emission of the two nucleons. In this case the FSI effects tend to be smaller than correlation effects. Nevertheless they yield contributions of similar size and therefore should be treated in a way which is consistent with the treatment of correlation effects.

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