Coherence-powered work exchanges between a solid-state qubit and light fields

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How does quantum coherence impact energy exchanges between quantum systems? This key question of quantum thermodynamics is also of prime importance for the energy management of emerging technologies based on quantum coherence. Pioneering theoretical frameworks have been proposed to describe the role of coherence in the energetic exchanges between a qubit and the electromagnetic field. Here, we experimentally study the work transferred during the spontaneous emission of a solid-state qubit into a reservoir of modes of the electromagnetic field, a step that energetically corresponds to the charging of a quantum battery. We show that the amount of transferred work is proportional to the initial quantum coherence of the qubit, and is reduced at higher temperatures. In a second step, we study the discharge of the battery and its energy transfer to a classical- i.e.- laser field using homodyne-type measurements. Our research shows that the amount of energy and work transferred to the laser field is controlled by the relative classical optical phase between the two fields, the quantum purity of the charged battery field as theoretically predicted, as well as long-term fluctuations in the qubit solid-state environment. Our study lays the groundwork for the energetics of quantum light generation and optical quantum interferences - two key processes that are at the core of most light-based quantum technologies.

Introduction Quantum thermodynamics aims to extend the concepts and laws of thermodynamics into the quantum realm \cite{1–4} with the study of quantum coherence as an energetic resource for primary motivation \cite{5–9}. With the rise of quantum technologies, it appears that these investigations are relevant to mitigate the energetic footprint of quantum information processing \cite{10–18}, hence the energy consumption of future quantum computers \cite{19}. In this realm, quantum batteries are key devices for energy management, that allow to store and retrieve work on demand. Quantum coherence has been predicted to impact their charging speed, ability for work extraction, and their charging power \cite{20–24}. The physics of quantum batteries can fruitfully be described in the fully quantum regime, where energy flows between two coupled quantum systems, \(A\) and \(B\), that are otherwise isolated \cite{25–29}. This captures the charging of a quantum battery by a work provider, and reciprocally, its discharging into a work receiver \cite{30–32}. The energy change of each system satisfies \(−\Delta E_A = \Delta E_B\) and the work \(W_{A,B}\) exerted by \(A\) on \(B\) is defined as the fraction of energy transfer stemming from an effective unitary interaction between the two systems. The amount of work exchanged is limited by quantum correlations between the two systems so that \(\Delta E_B = W_{A,B} + Q_{A,B}\) where \(Q_{A,B}\) represents the quantum correlation energy or “heat” \cite{25–29}. The work transfer efficiency \(\eta_{A,B} = W_{A,B}/|\Delta E_A|\) reaches \(\eta_{A,B} = 1\) only when one of the systems behaves as a perfect work source. A qubit coupled to a reservoir of electromagnetic modes \cite{33, 34} is an interesting platform to explore the impact of coherence on energetic transfers in this fully quantum regime \cite{35}. It is also a key system for quantum technologies - where atom-light interaction is central to quantum light generation and qubit manipulation \cite{29, 30}. It was shown in Refs. \cite{29, 30, 36} that the work exchanged between a qubit and the field is equal to the energy change in the coherent component of the field, making work an observable. When a coherent and intense field drives the qubit, both systems remain non-entangled and only exchange work: the qubit purity is unaltered, an ideal regime to implement single qubit gates \cite{12, 13, 29}. Conversely, when the qubit is coherently excited, the energy it spontaneously releases contains a work component proportional to the initial quantum coherence of the qubit, representing at most 50 \% of the energy transferred \cite{30}. Here, we study the energy exchanges between a solid-state qubit and light fields - following a two-step protocol sketched in Fig. 1(a), that features the charge and the discharge of a quantum battery. The qubit is a semiconductor quantum dot inserted in an optical microcavity - a system that years of technological developments have brought close to the ideal “one-dimensional atom” at low temperature \cite{37, 38}. In the first (charging) step, the qubit provides work to a reservoir of empty electromagnetic modes (initially uncharged quantum battery). We show that the efficiency of the work transfer corresponds to the visibility of fringes recorded in a self-homodyne measurement of the emitted light field, and we observe a work transfer close to the ideal value. We explore the effect of decoherence by in-
FIG. 1. Protocol to study energy exchanges between a qubit and light fields. (a) Schematic of the protocol. A classical drive generates coherence in a two-level system (qubit). In turn, the qubit exerts work on an empty quantum battery (electromagnetic modes) via spontaneous emission, charging the quantum battery (step 1). We discharge the quantum battery through homodyne-type interference with a coherent field (classical receiver) at a 50:50 beam splitter (step 2). (b) Experimental implementation: a resonant pulsed laser excites the QD-cavity device. The emitted photonic field is filtered from the laser drive in crossed-polarization at a PBS. In configuration (1) with an additional flip mirror (FM), the battery field is split into two paths at a beam splitter (BS2) and one arm is delayed such that two subsequently emitted photonic fields interfere at a 50:50 beam splitter (BS1). In configuration (2) the classical receiver is derived from the same drive laser, shaped using a Fabry-Perot etalon (FP) and reflected off a mirror (M) in the same cryostation as the QD device to minimize the impact of mechanical vibrations. The battery and classical receiver fields interfere at BS1. For both steps, the output intensities are recorded using two superconducting nanowire single-photon detectors (SNSPDs), D1 and D2.

Increasing the qubit temperature and observe reduced energetic performances. In the second step, we discharge the battery into a coherent field by making the emitted light and the receiving laser field interfere on a balanced beam splitter. We demonstrate that the visibility of the interference gives direct access to the work transfer, that is shown to be limited by the relative coherence of both fields.

Work transfer during spontaneous emission: Quantum Battery Charging. In the absence of decoherence [30], the qubit, resonantly excited by the laser drive, is brought to the pure quantum superposition state \( |\Psi_q(0)\rangle = \cos(\theta/2) |g\rangle + \sin(\theta/2)e^{i\phi} |e\rangle \), where \(|g\rangle\) and \(|e\rangle\) are the qubit ground and excited states separated by an energy \( \hbar \omega_0 \). \( \theta, \phi \in [0, \pi] \) are the pulse area and the classical phase of the driving laser pulse respectively. The total initial energy brought to the qubit by the laser drive is \( \mathcal{E}_q(0) = \hbar \omega_0 \sin^2(\theta/2) \). At the end of the spontaneous emission process, this energy is entirely transferred to the quantum battery field, which gained an energy satisfying \( \mathcal{E}_b = \mathcal{E}_q(0) = \mathcal{E}_q(0) \). The emitted field state is pure and reads \(|\Psi_b\rangle = \cos(\theta/2) |0\rangle + \sin(\theta/2) |1\rangle \), where \(|0\rangle\) and \(|1\rangle\) are the photon number states in the battery optical modes. The work provided by the qubit to the battery corresponds to the coherent part of the emitted field energy [29, 30, 36] and could be used to coherently drive another qubit, e.g. perform work on it [29]. It reads \( W_{q,b} = \hbar \omega_0 s^2 \), where \( s \) is the initial coherence of the qubit, \( s = \cos(\theta/2) \). The amount of work is thus maximal at \( \theta = \pi/2 \). This maximizes the complex amplitude of the battery field, hence the precision of its classical phase in the quadrature space [30]. The charging efficiency is given by: \( \eta_{q,b} = W_{q,b}/\mathcal{E}_b = \cos^2(\theta/2) \). Reciprocally, the heat exchange corresponds to the incoherent component of the battery field and is maximal for \( \theta = \pi \). Our qubit, an InGaAs quantum dot (QD) coupled to a micropillar cavity [39], is subject to various sources of decoherence such as charge noise [40], and phonon coupling [41] that alter both the excitation and the spontaneous emission processes. Since the energetics of spontaneous emission for a qubit subject to various baths is beyond currently accessible theoretical frameworks, we adopt a simple approach where we attribute all of the decoherence to the excitation process and the work still corresponds to the coherent part of the emitted light field, now described by the density matrix \( \hat{\rho}_b = \cos^2(\theta/2)\hat{\rho}_0 + \sin^2(\theta/2)\hat{\rho}_1 + \cos(\theta/2) \sin(\theta/2)(\hat{\rho}_{01} + \hat{\rho}_{10}) \) (see Suppl.) where the subscripts 0 and 1 stand for the vacuum or one photon part of the field. \( M_s = \text{Tr}[\hat{\rho}_s^2] \) is the single photon indistinguishability or purity of the single photon component in the temporal domain [42]. The reduction of quantum coherence between the vacuum and the one-photon component is captured by \( C = \text{Tr}[\rho_{01}\rho_{10}] \), and satisfies \( C \leq \sqrt{M_s} \). The work provided by the qubit to the quantum battery now reads: \( W_{q,b} = \hbar \omega_0 C \cos^2(\theta/2) \sin^2(\theta/2) \) (see Suppl.).

The QD is placed in a cryostat at 5-20 K and is resonantly driven by a pulsed Ti:Sapphire laser at a 81 MHz repetition rate (see Fig. 1(b)). The emitted photonic field is separated from the laser drive in a cross-polarization configuration. Black symbols in Fig. 2(b) correspond to the normalized intensity \( \mu_b \) of the battery field as a function of the pulse area \( \theta \). We observe the onset of Rabi oscillations attesting the coherent control over the qubit, i.e. the ability to generate arbitrary quantum superpositions of the qubit ground and excited state. Here, we assume a near unity occupation of the qubit excited state at the power \( P_e \) corresponding to the highest intensity so that the pulse area is \( \theta = 2 \arcsin(\sqrt{P_f/P_e}) \). This experimental curve then corresponds to the normalized energy
of the battery field $E_b/(\hbar \omega_0) = \eta_b = \sin^2(\theta/2)$ (black line). The coherent component of the battery field is measured by performing a self-homodyne measurement. Two battery fields are generated by two successive excitations of the qubit (12 ns apart) and are temporally overlapped at the two inputs of a beam splitter [43]. The counts of the two detectors at the outputs show anticorrelated intensities as a function of the relative phase between the two input fields, characterized by an interference visibility $v$ that is a lower bound to the work transfer efficiency, $v \leq \eta_{q,b} = C \cos^2(\theta/2)$ (see Suppl.). Fig. 2(a) shows the measured visibility of the interference fringes $v$ as a function of $\theta$. We fit the data including a small correction for imperfect suppression of the classical drive laser. At 5 K, the experimentally measured efficiency decreases continuously as $\theta$ increases, as theoretically expected. The measured behaviour is remarkably close to the ideal case of an isolated qubit, with $C(5K) = 0.975 \pm 0.007$. Combining this visibility measurement with the measurement of the field energy $E_b$ (see Fig. 2(b)), we deduce a lower bound to the amount of work transferred into the quantum battery: $W_{q,b} \geq E_b \times v$. Fig. 2(b) shows the experimental normalized values of $E_b \times v$ as a function of $\theta$ (open symbols) as well as the corresponding heat $Q_{q,b} = E_b \times (1 - v)$ (filled symbols) and the corresponding theoretical predictions (lines). At low $\theta$, most of the energy transferred from the qubit to the electromagnetic field corresponds to work [30]. Such behavior can be understood considering that, in the low excitation regime, the radiated field comes from the qubit dipole: no entanglement takes place between the two systems, and the emitted field is remarkably close to a coherent field. Conversely, light-matter entanglement emerges during the spontaneous emission process when a qubit population is created for increasing $\theta$. Quantum correlations then reduce the amount of work eventually transferred to the electromagnetic field, reaching the situation where all the energy is transferred in the form of heat for $\theta = \pi$. Remarkably, this is when the qubit acts as a deterministic single-photon source—a key device for discrete variable optical quantum technologies.

The work transfer is maximal for $\theta = \pi/2$, where our observations demonstrate an equipartition of work and heat at 5 K, as expected for the decoherence free situation. This reflects the efficient suppression of decoherence at 5 K owing to the acceleration of spontaneous emission provided by the coupling to the cavity. This is consistent with the generation of single photons with near-unity indistinguishability with similar devices [39].

We explore the effect of decoherence by increasing the QD temperature to 20 K. Coupling to acoustic phonons affects both the coherent control of the quantum dot with the classical drive [44] as well as the single photon indistinguishability [45] which is decreased from $M_s(5K) = (92.6 \pm 0.1)\%$ to $M_s(20K) = (58.0 \pm 1.0)\%$ at $\theta = \pi$. The self-homodyne measurement on the battery field still reveals single photon interference, evidencing a work transfer between the qubit and the quantum battery. However, the work transfer efficiency is reduced to $C(20K) = 0.594 \pm 0.007$. The lower bound to the work transferred is still maximum for $\theta = \pi/2$ but the maximum value is reduced to $W_{q,b} = (15.8 \pm 0.6)\%$. Note that the relation $C \leq \sqrt{M_s}$ is verified in our experimental observations where we independently measured $M_s = (92.6 \pm 0.1)\%$ at 5 K, and $M_s = (58.0 \pm 1.0)\%$ at 20 K.

**Energy exchanges in a quantum interference - Quantum Battery Discharging.** In the second step, we discharge the quantum battery into a propagating coherent field (step 2 in Fig. 1) As mentioned above, coherent fields are proper resources to drive e.g. qubits and implement quantum gates, making them convenient work receivers. To do so, we interfere both fields on a 50:50 beam split-
ter – a standard configuration to obtain effective light-light interaction in optical quantum technologies. A full battery’s discharge corresponds to a complete extinction of the battery field after the beam splitter. When the battery’s input state is pure, this situation is obtained if both fields have the same input energies and complex amplitudes. This only happens if the battery field itself is a coherent field, i.e. in the limit $\theta \to 0$.

In what follows, we keep the condition of equal input energies $E_{b}^{in} = E_{c}^{in} = \hbar \omega_{0} \sin^{2}(\theta/2)$, such that the energy transferred to the receiver reads $\Delta E_{c} = E_{b}^{in} v$ where $v$ is the interference visibility. $v = 1$ signals a complete discharge. When the battery field is not pure, the energy transfer is lowered by a factor $C_{b,c}$ which captures the reduction of the overall coherence between the two fields. The visibility becomes $v = C_{b,c} \cos(\theta/2)$ and $C_{b,c} \leq \sqrt{M_{b,c}}$ (see Suppl.), where $M_{b,c}$ is the mean wave packet overlap between the battery and the receiver.

We experimentally study the discharge both at 5 K and 20 K (configuration (2) in Fig. 1(b)). To ensure overall coherence between the fields, we derive the receiver from the same laser that is used to drive the qubit, and shape it to match the temporal wave packet of the battery field (see Fig. S.2). The visibility is plotted as a function of pulse area $\theta$ in Fig. 3(a). As theoretically expected, an increasing visibility is observed when reducing $\theta$, both at 5 K and 20 K, evidencing a discharge process of increasing efficiency. Our observations are well reproduced by $v = \Delta E_{c}/E_{b}^{(in)} = \cos(\theta/2) C_{b,c}$ for $C_{b,c}(5K) = (36.3 \pm 0.4)\%$ and $C_{b,c}(20K) = (27.2 \pm 0.4)\%$. Note that the upper bound for the energy transfer, $C_{b,c} \leq \sqrt{M_{b,c}}$, is not reached experimentally: our measurements yield $M_{b,c} = (48.9 \pm 0.3)\%$ at 5 K and $M_{b,c} = (32.3 \pm 0.7)\%$ at 20 K. This indicates that blinking of the quantum dot charge state may not be properly captured by the extracted mean wave packet overlap.

We now analyze the energy transferred from the battery to the receiver $\Delta E_{c}$ as the sum of a work $W_{b,c}$ and a heat $Q_{b,c}$ contribution. The former (resp. the latter) corresponds to the increase of the coherent (resp. incoherent) component of the receiver after crossing the beam splitter (see Suppl.). The heat can arise only from the battery field so that $Q_{b,c} = Q_{q,b}/2$. Conversely, the work contribution reads $W_{b,c} = E_{b}^{in} v - Q_{q,b}/2$. Interestingly, it is shown to be upper bounded by the work transferred during the charge, $W_{q,b} \geq W_{b,c}$ (see Suppl.): the battery cannot provide more work than it initially received.

Fig. 3(b) presents the work, heat and total energy transfers in the ideal situation of a pure battery state. Most of the energy is transferred in the form of work in the limit where $\theta \to 0$, i.e. when the state of the battery field is the closest to a coherent field and the discharge is complete. $\Delta E_{c}$ is maximum for $\pi/2 < \theta < \pi$, a behavior that results from a trade-off between the maximization of the battery field amplitude reached at $\theta = \pi/2$, and the normalization condition imposed on the receiver, whose coherence continuously increases with $\theta$. For high values of $\theta$, the work becomes negative, signalling an undesired flow from the receiver to the battery. Finally at $\theta = \pi$, no energy is transferred to the receiver as heat and work exchanges perfectly cancel out.

Fig. 3(c) shows the measured work, heat and energy transfers deduced from the visibility measured in FIG. 3. Energy exchanges in a quantum interference - Quantum Battery Discharging. (a) Measured visibility $v$ as a function of $\theta$ when interfering the quantum battery with the classical receiver (blue 5 K, red 20 K). (b) Theoretical energy, work and heat exchanged between the quantum battery and the classical receiver in the ideal case. (c) Measured energy discharged from the quantum battery into the classical receiver, $\Delta E_{c}$ as a function of $\theta$, heat $Q_{b,c}$ deduced from Fig. 2(b) and work transferred $W_{b,c}$ (blue 5 K, red 20 K). Lines correspond to experimental data fit with deduced parameters $C_{b,c}$ and $C$. At each temperature.
Fig. 3(a) and the measurement of the heat received by the battery from Fig. 2(b). While the battery charging at 5 K was close to the ideal case, this second step is now further from the ideal situation, with significantly reduced maximum energy exchanged, and positive work flows for smaller $\theta$ range. The situation is even worse at 20 K, where the work flows the opposite direction for most of the $\theta$ range. This is a result of a strongly reduced mode overlap between the receiver and the battery field limited by long-term fluctuations in the qubit environment.

**Conclusion** We have provided theoretical and experimental tools to understand the energy exchanged between a two-level system and the electromagnetic field as well as between two light fields. Both interactions constitute key building blocks for a multitude of quantum technologies from atom-based quantum memories, linear optical gates to Bell state measurements among others. Our study reveals how both classical and quantum coherence impacts energetic transfers and can be accessed by homodyne measurements. We have shown that a maximum work transfer efficiency is obtained during spontaneous emission at the onset of the qubit population inversion, validating the picture that work transfer is reduced by the light-matter entanglement. This work has then been successfully transferred to a coherent light field through a quantum interference. The maturity of the quantum dot-cavity system allowed us to study these various phenomena in a variety of situations by controlling the degree of decoherence exerted on the qubit. The present work hence carries the seeds of an energetic investigation of realistic processes at the core of optical quantum technologies. We hope it will stimulate fundamental research in the area.

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Data and materials availability: All data acquired and used in this work is property of the Centre for Nanoscience and Nanotechnology and is available upon reasonable request.

Supplementary information Supplementary Information is available for this paper.

**METHODS**

Device and setup The qubit is a neutral InGaAs quantum dot (QD) embedded in an electrically contacted micropillar $\lambda$-cavity with 14 (28) GaAs/AlAs Bragg pairs in the top (bottom) mirror, which has previously been used as a bright single-photon source [39, 46]. The device is placed in a cryostation (Cryostation s200, Montana Instruments), operating at either 5 K or 20 K, see Fig. 1(b). Note that at 20 K, an application of an electrical bias on the QD structure allows us to maintain the qubit-cavity resonance through Stark shift and keep all other parameters of the system mostly unchanged with respect to the measurements at 5 K [45]. The qubit is resonantly excited using a Ti:Sapphire laser which generates 3 ps pulses centred at 925 nm at a 81 MHz repetition rate. The pulse duration is increased to 7 ps with a 4f-shaping line to optimize the mode overlap with the cavity. The drive laser is aligned along the polarization axis of the cavity using a quarter wave plate (QWP) and half wave plate (HWP). An objective lens ($L_1$) inside the cryostation focuses the drive laser onto the micropillar cavity. The emitted single-photon field (charged quantum battery) is separated from the drive laser via cross-polarization at a polarizing beam splitter (PBS).

A flip mirror (FM) in the setup in Fig. 1(b), allows switching between two measurement configurations. In the first configuration (1) with the flip mirror in, we probe the work transfer between qubit and quantum battery $W_{q,b}$. We measure this work transfer by interfering the charged quantum battery with a copy of itself (a subsequently emitted single-photon field) by inserting a second beam splitter (BS2) in the charged battery path. The two fields interfere at a 50:50 beam splitter (BS1) and two SNSPDs (D1, D2; Single Quantum Eos) at the outputs of BS1 record the photon intensities with 100 ms resolution over 20 min for each prepared qubit state $\theta$.

With the second configuration (2) in Fig. 1(b) we measure the amount of work transfer between the charged battery and a classical receiver (a coherent photon field). The quality $Q$ limits the amount of work transport between the charged quantum battery and the classical receiver. The input/output fields overlap is $\lambda_{in} = 1 - \sin^2(\theta)$. The mode overlap between the receiver and the battery field is limited by long-term fluctuations in the qubit environment.
field derived from the classical drive). To minimize the effect of the vibrations in our closed cycle cryostation, the classical receiver is sent into the same cryostation, and focused by an objective lens ($L_1$) on a mirror. The reflected coherent field is subsequently sent through a Fabry-Pérot etalon (FP) where it is shaped to match the temporal profile of the emitted photonic field, before arriving at the 50:50 beam splitter BS1. At BS1 the coherent field interferes with the charged quantum battery and we record the resulting photon intensities with 100 ms resolution with two SNSPDs for 20 min per prepared qubit state $\theta$.

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Supplementary Material

I. Description of the quantum battery field

I.1 General formalism

In this section we provide a description of the battery field state in realistic conditions where decoherence perturbs the spontaneous emission mechanism. The photonic density operator in the pulse-mode formalism [47] of a single propagating mode \( \hat{a}_b(t) \) containing at most one photon reads:

\[
\hat{\rho}_b = p_0 |0\rangle \langle 0| + p_1 \int dt dt' \xi(t, t') \hat{a}_b^\dagger(t) |0\rangle \langle 0| \hat{a}_b(t') + \sqrt{p_0 p_1} \int dt \zeta(t) \hat{a}_b^\dagger(t) |0\rangle \langle 0| + \text{h.c.}
\]

(S.1)

where \( \xi(t, t') = \xi^\ast(t', t) \) is a Hermitian function describing the temporal shape and coherence of the single photon and \( \zeta(t) \) is the complex amplitude describing the time dynamics of the photon-number coherence. The photon-number probabilities \( p_0 \) and \( p_1 \) satisfy \( p_0 + p_1 = 1 \) and so \( \text{Tr}[\hat{\rho}_b] = 1 \) implies that \( \int \xi(t, t') dt = 1 \). The total purity of this photonic state is

\[
P = \text{Tr}[\hat{\rho}_b^2] = p_0^2 + p_1^2 M_s + 2 p_0 p_1 \mathcal{C},
\]

(S.2)

where

\[
M_s = \text{Tr}[\hat{\rho}_s^2] = \int dt dt' |\xi(t, t')|^2
\]

(S.3)

is the single-photon indistinguishability, or purity in the temporal domain, and

\[
\mathcal{C} = \text{Tr}[\hat{\rho}_{01} \hat{\rho}_{10}] = \int dt |\zeta(t)|^2
\]

(S.4)

is the number purity of the coherence between the single photon and the vacuum.

I.2. Relationship between the purity in the number basis and in the temporal domain

In Ref. [43], we discussed the self-homodyne measurement of two fields emitted by a quantum dot and introduced a formalism separating the coherence of the field in the photon number basis and in the time/frequency domain (indistinguishability). In this section, we present a more rigorous approach allowing to discuss both coherences.

In general, the mixed photonic state can be described by a classical probability distribution \( \hat{\rho} = \sum_k q_k |\psi_k\rangle \langle \psi_k| \) of pure photonic states \( |\psi_k\rangle = p_{0,k} |0\rangle + p_{1,k} \int dt f_k(t) \hat{a}_1^\dagger(t) |0\rangle \) each occurring with the probability \( q_k \). The single photon composing each state is described by a complex amplitude \( f_k(t) \) normalized such that \( \int dt |f_k(t)|^2 = 1 \). In this notation, the total photon number probabilities are given by the weighted summations \( p_0 = \sum_k q_k p_{0,k} \) and \( p_1 = \sum_k q_k p_{1,k} \). By expanding out \( \hat{\rho} \) in terms of \( |\psi_k\rangle \), we can also see that the single-photon trace purity \( M_s \) and number purity \( \mathcal{C} \) can be written in terms of \( f_k(t) \) as:

\[
M_s = \frac{1}{p_1^2} \sum_{k,k'} q_k q_{k'} p_{1,k} p_{1,k'} |\hat{c}_{k,k'}|^2
\]

(S.5)

\[
\mathcal{C} = \frac{1}{p_0 p_1} \sum_{k,k'} q_k q_{k'} \sqrt{p_{0,k} p_{0,k'}} p_{1,k} p_{1,k'} |\hat{c}_{k,k'}|^2
\]

where \( \hat{c}_{k,k'} = \int dt f_k(t) f_{k'}^\ast(t) \) is the complex amplitude overlap. Note that \( \hat{c}_{k,k'} = \hat{c}_{k',k}^\ast \) and so \( \mathcal{C} \) is real. Using a Cauchy-Schwarz inequality, we can now see that

\[
\mathcal{C}^2 \leq \frac{1}{p_0 p_1} \left( \sum_{k,k'} q_k q_{k'} p_{0,k} p_{0,k'} \right) \left( \sum_{k,k'} q_k q_{k'} p_{1,k} p_{1,k'} |\hat{c}_{k,k'}|^2 \right)
\]

(S.6)

\[
= \frac{1}{p_1^2} \sum_{k,k'} q_k q_{k'} p_{1,k} p_{1,k'} |\hat{c}_{k,k'}|^2 = M_s
\]
Hence, a vanishing temporal purity will prohibit coherence in the number basis, but number coherence can vanish even if there is high temporal purity. This inequality defines a useful figure of merit \( \lambda^2 = C/\sqrt{M_s} \), where \( 0 \leq \lambda \leq 1 \) quantifies the amount of decoherence in the photon-number basis which cannot be attributed to temporal decoherence of the involved single photon. Note that if \( M_s = 1 \), \( \lambda = |\langle 0|\hat{\rho}_q(0)|1 \rangle|/\sqrt{p_0 p_1} \) describes the reduction in number coherence of the initial qubit state, otherwise it accounts for both the degradation of the initial state preparation and the excess decoherence occurring during emission that is not directly associated with the reduction in single-photon indistinguishability.

### II. Relating energetic and optical quantities

In this section, we theoretically propose a framework to experimentally access work transfer mechanisms. We demonstrate that homodyne measurements, where two optical fields interfere on a balanced beam splitter (see Fig. 1), give direct access to the work exchanged between two fields. For two input fields impinging a balanced beam splitter, the transformation is given by:

\[
\begin{pmatrix}
\hat{a}_3(t) \\
\hat{a}_4(t)
\end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{i\phi} \\
-1 & 1 \end{pmatrix}
\begin{pmatrix}
\hat{a}_1(t) \\
\hat{a}_2(t)
\end{pmatrix}
\]

(S.7)

The visibility of interference is defined as:

\[
v = \frac{\mu_3 - \mu_4}{\mu_3 + \mu_4}
\]

(S.8)

where \( \mu_j = \int \langle \hat{a}_j(t)^\dagger \hat{a}_j(t) \rangle dt \) is the average photon number detected in mode \( j \) at the output of the beam splitter.

In what follows, we successively study the case of a self-homodyne experiment, where the input fields are copies of the battery field of Eq. (S.1), i.e. \( a_1(t) = a_{b1}(t) \), and \( a_2(t) = a_{b2}(t) \), and the case of a standard homodyne experiment where the input fields are the quantum battery (b) and the classical receiver (c), i.e. \( a_1(t) = a_c^{(in)}(t) \), \( a_2(t) = a_b^{(in)}(t) \), \( a_3(t) = a_c^{(out)}(t) \), \( a_4(t) = a_b^{(out)}(t) \).

#### II.1. Measuring the efficiency of battery charging with self-homodyne interference

In Refs. [29, 30], it has been demonstrated that when the qubit-field system is isolated, the total work provided by the qubit to the field’s mode (quantum battery) can be extracted from the final field’s state:

\[
W_{q,b} = \hbar \omega_0 \int dt |\langle \hat{a}_b(t) \rangle|^2
\]

(S.9)

We extended the definition above to the experimental situation where the joint qubit-field state may not be perfectly pure. By evaluating Eq. (S.9) on the general state in Eq. (S.1) with \( p_0 = \cos^2(\theta/2) \), and \( p_1 = \sin^2(\theta/2) \), we find

\[
W_{q,b} = \hbar \omega_0 \cos^2(\theta/2) \sin^2(\theta/2) \int dt |\langle \zeta(t) \rangle|^2.
\]

Now using Eq. (S.4), we find the expression for the work presented in the main text, \( W_{q,b} = \hbar \omega_0 \cos^2(\theta/2) \sin^2(\theta/2)C \).

We extract \( C \) from the visibility of the self-homodyne interference: two copies of the quantum battery are incident on a balanced beam splitter. The two input states are identical except for a relative phase \( \phi \) on the number coherence between them which is due to the difference in path length. The numerator of Eq. (S.8) is given by:

\[
2\text{Re} \left[ \int dt e^{i\phi} \langle \hat{a}_{b1}^\dagger(t) \hat{a}_{b2}(t) \rangle \right] = 2\text{Re} \left[ \int dt e^{i\phi} \langle \hat{a}_{b1}(t) \rangle \langle \hat{a}_{b2}(t) \rangle \right] = 2\cos(\phi) \int dt |\langle \hat{a}_b(t) \rangle|^2
\]

(S.10)

where the first equality comes from the fact that the states of modes b1 and b2 are uncorrelated, the second from the fact that the states are identical, i.e. \( \langle \hat{a}_{b1} \rangle = \langle \hat{a}_{b2} \rangle = \langle \hat{a}_b \rangle \).

Since the two fields carry the same number of photons, the denominator of Eq. (S.8) is simply \( 2\mu_b = 2E_b/(\hbar \omega_0) \).

Putting all together we get:

\[
v = \cos(\phi) \frac{\int dt |\langle \hat{a}_b(t) \rangle|^2}{\mu_b} = \cos(\phi) \frac{W_{q,b}}{E_b}
\]

(S.11)
The visibility of the interference varies with the relative phase between the two inputs, \( \phi \), and reaches maximum constructive (destructive) interference when \( \phi = 0 (\pi) \).

II.2. Discharging the quantum battery through homodyne interference

We discharge the quantum battery via interference with a coherent field, which acts as a classical receiver. The visibility of this classical-homodyne measurement allows us to quantify the energy and the work transfer in the discharge process. The quantum battery (b) field enters the balanced beam splitter in channel 2, with \( \langle \hat{a}_{b}^{(in)}(t) \rangle = \sin (\theta/2) \cos (\theta/2) \zeta (t) \). The classical receiver (c) field, \( |\beta\rangle \), enters the beam splitter in channel 1, with \( \langle \hat{a}_{c}^{(in)}(t) \rangle = \beta(t) \).

The final energy of the classical receiver field, reads:

\[
E_c^{\text{(out)}} = \frac{1}{2} \left[ E_c^{\text{(in)}} + E_b^{\text{(in)}} \right] + \hbar \omega_b \Re \left[ \int dt \langle \hat{a}_{b}^{(in)}(t) \rangle \langle \hat{a}_{b}^{(in)}(t) \rangle^* \right] = \frac{\hbar \omega_0}{2} \left[ \int dt |\beta(t)|^2 + \sin^2 (\theta/2) \right] + \hbar \omega_0 \cos (\theta/2) \sin (\theta/2) \Re \left[ \int dt \zeta(t) \beta(t)^* \right]
\]  

(S.12)

The efficiency of the discharge process reads \( G = E_c^{\text{(out)}} / \left[ \hbar \omega_0 \int dt |\beta(t)|^2 + \hbar \omega_0 \sin^2 (\theta/2) \right] \), replacing the numerator with the expression above, we find that \( G \) is maximal when \( \int dt |\beta(t)|^2 = \sin^2 (\theta/2) \), i.e. \( \beta(t) = \sin (\theta/2) \sqrt{\zeta(t,t)} e^{-i\phi(t)} \), namely the two fields are matched in intensity. This condition is met in the experiment presented in the main and it corresponds to an interference visibility reading:

\[
v = \frac{1}{\mu_b} \Re \left[ \int dt \langle \hat{a}_{c}^{(in)}(t) \rangle \langle \hat{a}_{b}^{(in)}(t) \rangle^* \right] = \cos (\theta/2) \Re \left[ \int dt \zeta(t) \sqrt{\zeta(t,t)} e^{i\phi(t)} \right] = \cos (\theta/2) C_{b,c}
\]  

(S.13)

where, in the last equality, we defined the quantity \( C_{b,c} = \Re \left[ \int dt \zeta(t) \sqrt{\zeta(t,t)} e^{i\phi(t)} \right] \) accounting for both the classical and the quantum coherence of the process. It is clear that the visibility and hence the relative efficiency \( G = (1+v)/2 \) are maximal when \( C_{b,c} = 1 \). The energy change of the two fields hence reads:

\[
\Delta E_c = \hbar \omega_0 \cos (\theta/2) \sin^2 (\theta/2) C_{b,c} = -\Delta E_b
\]  

(S.14)

We can split this energy change into work and heat energy terms:

\[
\Delta E_c = W_{b,c} + Q_{b,c}
\]  

(S.15)

Using the analysis introduced in Refs. [25–29], we find the work:

\[
W_{b,c} = \hbar \omega_0 \left( \int dt |\hat{a}_{b}^{(out)}(t)|^2 - \int dt |\hat{a}_{b}^{(in)}(t)|^2 \right) = \hbar \omega_0 \Re \left[ \int dt \langle \hat{a}_{c}^{(in)}(t) \rangle \langle \hat{a}_{b}^{(in)}(t) \rangle^* \right] + \frac{\hbar \omega_0}{2} \left( \int dt |\hat{a}_{b}^{(in)}(t)|^2 - \int dt |\hat{a}_{b}^{(in)}(t)|^2 \right)
\]  

(S.16)

It can be easily verified that for any choice of \( \langle \hat{a}_{c}^{(in)}(t) \rangle \) we find \( W_{b,c} \leq W_{q,b} \). Subtracting the work from the total energy change we find the heat:

\[
Q_{b,c} = \hbar \omega_0 \sin^2 (\theta/2) (1 - \cos^2 (\theta/2) C) / 2 = \frac{E_{b,c}^{\text{(in)}} - \hbar \omega_0 \int dt |\hat{a}_{b}^{(in)}(t)|^2}{2} = Q_{c,b}/2
\]  

(S.17)

II.3. Upper bound to \( C_{b,c} \)

Similar to the self-homodyne interference, we derive a relationship between \( C_{b,c} \) and the mean wave packet overlap between the battery field and the coherent field \( M_{b,c} \). In terms of the amplitudes \( f_k \) composing \( \zeta \), we have \( C_{b,c} = \).
where $g = \int dt \text{Re} [\beta^*(t) f_k(t)] / \sqrt{p_c}$ is the real-valued normalized amplitude overlap of the coherent field with the $k$th pure state composing $\hat{\rho}$, and $\beta(t) = \langle \hat{a}_c(t) \rangle$ is the complex amplitude of the coherent state. Similarly, we can write the mean wave packet overlap in terms of $f_k$ as $M_{b,c} = (1/p_1) \sum_k q_k p_{1,k} M_k$, where $M_k = \left| \int dt \beta^*(t) f_k(t) \right|^2 / \mu_c$ is the mean wave packet overlap between the coherent state and the $k$th pure state composing $\hat{\rho}$. Using a Cauchy-Schwarz inequality, we can again see that

$$
|C_{b,c}|^2 \leq \frac{1}{p_0 p_1} \left( \sum_k q_k p_{0,k} \right) \left( \sum_k q_k p_{1,k} |c_k|^2 \right) = \frac{1}{p_1} \sum_k q_k p_{1,k} |c_k|^2 \leq M_{b,c},
$$

(S.18)

where the last inequality arises from the fact that $|c_k|^2 = |\text{Re} \left[ \int dt \beta^*(t)f_k(t)/\sqrt{\mu_c} \right]|^2 \leq \left| \int dt \beta^*(t)f_k(t) \right|^2 / \mu_c = M_k$.

III. Experimental methods

III.1. Battery field indistinguishability

To measure $C$ and $M_k$ for the battery field, two sequentially generated copies of the state interfere at a 50:50 beam splitter. We monitor the single-photon intensity at the two outputs of the beam splitter to observe interference fringes, and also simultaneously monitor the two-photon coincidences where we observe bunching due to Hong-Ou-Mandel interference [43, 48, 49]. To obtain the single-photon indistinguishability $M_s$, we first measure the mean wave packet overlap between two subsequently emitted battery fields, $M$ [48]. The mean wave packet overlap is defined by $M = (1/\mu_b^2) \int dt dt' |G^{(1)}(t,t')|^2$, where $\mu_b = \int dt I_b(t) = \sum_i n_i$ is the average photon number, $I_b(t) = \langle \hat{a}_b^\dagger(t) \hat{a}_b(t) \rangle$ is the wave packet temporal envelope, and $G^{(1)}(t,t') = \langle \hat{a}_b^\dagger(t') \hat{a}_b(t) \rangle$ is the first-order (amplitude) correlation function. Experimentally, the integrated coincident counts after HOM interference binned with respect to the detection delay $\tau$ produce histograms proportional to $\int dt G_{\text{HOM}}^{(2)}(t,t+\tau)$, where $2G_{\text{HOM}}^{(2)}(t,t') = I_b(t)I_b(t') + G^{(2)}(t,t') - |G^{(1)}(t,t')|^2$ is the phase-averaged intensity correlation after HOM interference and $G^{(2)}(t,t') = \langle \hat{a}_b^\dagger(t) \hat{a}_b^\dagger(t') \hat{a}_b(t') \hat{a}_b(t) \rangle$ is the intensity autocorrelation measured using a Hanbury Brown–Twiss setup.

The required normalization $\mu_b^2$ for $M$ is obtained by comparing the coincident counts for when the inputs are co-polarized to when they are cross-polarized. In this latter case, we have $G^{(1)} = 0$. Then, if the fully integrated and normalized intensity correlation $g^{(2)} = (1/\mu_b^2) \int dt dt' G^{(2)}(t,t')$ is very small, the HOM histograms allow us to quantify $M$ via the HOM visibility defined as:

$$
V_{\text{HOM}} = \frac{g_{\text{HOM},\|}^{(2)} - g_{\text{HOM},\perp}^{(2)}}{g_{\text{HOM},\perp}^{(2)}},
$$

(S.19)

where $g_{\text{HOM},\|}^{(2)} = A_{\|} / |\mathbf{A}_{\|}|$ and $g_{\text{HOM},\perp}^{(2)} = A_{\perp} / |\mathbf{A}_{\perp}|$ are the central peak areas (A$_{\|}$) in co- (||) and cross- (⊥) input polarization configurations normalized by the average peak area of the histogram peaks arising from uncorrelated counts (|\mathbf{A}_{\|}| or |\mathbf{A}_{\perp}|). Note that, due to the interferometer delay needed to interfere subsequently generated states, the first side peak of the || case is partially suppressed due to antibunching and hence is excluded from the normalization. In addition, Eq. (S.19) is an accurate measurement of $M$ only when there is not much first-order coherence in the number basis so that $|\mathbf{A}_{\|}\rangle$ is not suppressed by possible interference fringes. Hence, this measurement approach is accurate for pulse areas $\theta \approx \tau$; however, $M$ is expected to take similar values for all $\theta$.

If the mode contains no more than one photon, then $G^{(1)}(t,t') = \mu_b^2 \xi(t,t')$ and so $V_{\text{HOM}} = M = M_s$. For small nonzero $g^{(2)}$, both $A_{\|}$ and $A_{\perp}$ are increased equally, which increases the denominator in Eq. (S.19) and causes $V_{\text{HOM}}$ to underestimate $M$. This small underestimate of $M$ can be corrected by taking $M = V_{\text{HOM}}(1 + g^{(2)})$. Note that this measurement approach and subsequent $g^{(2)}$ correction are different than those used in Ref. [48], where the visibility was defined as $V_{\text{HOM}} = 1 - 2g_{\text{HOM},\|}^{(2)}$, which implies $M = V_{\text{HOM}} + g^{(2)}$. Both approaches should predict identical values of $M$. Furthermore, if $g^{(2)}$ is small but nonzero and one of the two photons is approximately distinguishable from the emitted single photon state [48], then $M$ is related to $M_s$ via $M \simeq M_s(1 - g^{(2)})$. Hence, in
**FIG. S.1. Performance of Single-Photon Source** (a) Auto-correlation measurement of the single-photon wave packets generated by the single-photon source with $\theta = \pi$ at 5 K. The resulting coincidence histogram allows for extraction of the single-photon purity $g^{(2)}$. (b) The coincidence histogram obtained from Hong-Ou-Mandel interference measurements $g_{\text{HOM}}^{(2)}$ for $\theta = \pi$ at 5 K. From Eq. 24 and the single-photon purity extracted from (a) we can deduce the mean wave packet overlap, or single-photon indistinguishability, $M_s$. (c) and (d) the same as (a) and (b) but at 20 K. Note that the small peaks observed in the coincidence histograms at non-integer multiples of the pulse separation time of 12 ns are due to electronic reflections in our measurement setup and can be neglected.

Our experiments, $M_s$ is accurately estimated by

$$M_s \simeq V_{\text{HOM}} \frac{1 + g^{(2)}}{1 - g^{(2)}}.$$  \hspace{1cm} (S.20)

Fig. S.1 shows the results of these second-order intensity correlation measurements with Fig. S.1(a) and Fig. S.1(b) (Fig. S.1(c) and Fig. S.1(d)) corresponding to the auto-correlation $g^{(2)}$, and the time integrated HOM measurement $g_{\text{HOM}}^{(2)}$ at 5 K (20 K), all taken at $\theta = \pi$. We extract similar auto-correlation values at 5 K and 20 K: $g^{(2)}$ of (2.84 ± 0.08)% and (2.28 ± 0.08)% respectively. However, the indistinguishability between two subsequently emitted single-photon wave packets is reduced at the higher temperature. From the indistinguishability measurements we deduce a single-photon indistinguishability of $M_s = (92.6 \pm 0.1)\%$ at 5 K and $M_s = (58.0 \pm 1.0)\%$ at 20 K.

### III.2. Battery and classical receiver fields overlap

To generate temporal mode overlap between the battery field and the classical receiver field we shape the latter with a Fabry-Pérot etalon. By time resolving the emission dynamics of both fields (see Fig. S.2), we qualitatively ensure temporal overlap between the classical receiver (black dashed) and the quantum battery (blue 5 K, red 20 K). To then quantify the mean wave packet overlap between the battery and classical receiver fields, $M_{b,c}$, we perform a HOM experiment with the two fields for $\theta = \pi$. Here it is important to account for the non-negligible classical...
**FIG. S.2.** Temporal profiles quantum battery and classical receiver The classical receiver (dashed black), derived from the same coherent laser field used to drive the quantum dot, is temporally shaped to overlap with the temporal profile of the quantum battery at 5K and 20K (blue and red, solid).

**FIG. S.3.** Mean wave packet overlap between classical coherent field and single-photon field (a) The coincidence histograms of time integrated Hong-Ou-Mandel interference measurements between the single-photon field (generated with pulse area $\theta = \pi$) and the classical coherent field at 5 K, measured in co- ($g_{\text{HOM},\parallel}^{(2)}$) and cross- ($g_{\text{HOM},\perp}^{(2)}$) polarization configuration. (b) Same as (a) but at 20 K. Using Eq. 25 we obtain mean wave packet overlaps of $M_{b,c} = (48.9 \pm 0.3)\%$ at 5 K and $M_{b,c} = (32.3 \pm 0.7)\%$ at 20 K.

Intensity correlation $g^{(2)}_c = 1$ of the coherent state input. The coincidence counts can arise in three ways. (1) A photon from each input do not bunch when leaving the beam splitter, leading to a contribution of $2\mu_b\mu_c(1-M_{b,c})$. The factor of two in this term arises from the two ways to obtain a coincidence count (both photons reflected or both photons transmitted). (2) Two photons arrive from the coherent state when the battery field is vacuum (or lost), leading to an additional contribution $\mu_c^2 g_c^{(2)} = \mu^2$ to the coincidence counts. (3) Two photons arrive from the battery field input, leading to a small contribution $\mu_b^2 g_b^{(2)}$ to the coincidence counts. If we now assume that $\theta$ is near $\pi$-pulse, the average uncorrelated histogram peak area $A_{r>0,\parallel}$ is given by $2\mu_c\mu_b$. Thus, for the co-polarized case we have $g_{\text{HOM},\parallel}^{(2)} = 1 - M_{b,c} + \bar{g}^{(2)}$, where $\bar{g}^{(2)} = (\mu_c/\mu_b)g_c^{(2)}/2 + (\mu_b/\mu_c)g_b^{(2)}/2$ is the weighted average input intensity correlation. The intensity correlation after interference of inputs in cross-polarization is then simply obtained by $g_{\text{HOM},\perp}^{(2)} = 1 + \bar{g}^{(2)}$. Hence, $M_{b,c}$ is measured by

$$M_{b,c} = \frac{g_{\text{HOM},\perp}^{(2)} - g_{\text{HOM},\parallel}^{(2)}}{g_{\text{HOM},\perp}^{(2)}} \left( 1 + \bar{g}^{(2)} \right). \quad (S.21)$$

When the inputs are balanced ($\mu_c = \mu_b$), we have $1 + \bar{g}^{(2)} = 1 + (g^{(2)} + 1)/2$. If the battery field’s $g^{(2)} \ll 1$, the correction factor becomes $3/2$.

Fig. S.3 shows the coincidence histograms of HOM measurements with the quantum battery and the classical receiver field at 5 K and 20 K, Fig. S.3(a) and Fig. S.3(b), respectively. We perform the measurements with the two fields cross-polarized ($g_{\text{HOM},\perp}^{(2)}$, grey), and co-polarized ($g_{\text{HOM},\parallel}^{(2)}$, blue and red). From the coincidence histograms we extract a mean wave packet overlap of $M_{b,c} = (48.9 \pm 0.3)\%$ at 5 K and $M_{b,c} = (32.3 \pm 0.7)\%$ at 20 K.
To determine both $C$ and $C_{b,c}$, homodyne measurements are performed where we measure the difference in count rate at each detector as the interferometer phase evolves. We detail here the protocol for the self-homodyne measurement (step 1). The same protocol applies for the battery discharge with some small adjustments as indicated later on. The maximum visibility of these interference fringes $v$ give the integrated first-order coherence $c^{(1)}$ [43, 49] of the battery field, where $v \equiv c^{(1)} = \langle 1 / \mu_b \rangle \int dt \vert \langle \hat{a}_b(t) \rangle \vert^2$. If the mode contains no more than one photon, then $\langle \hat{a}_b(t) \rangle = \sqrt{\mu_b \mu_1} \zeta(t)$ and $\mu_b = p_1$. Thus, $C$ can be determined from $c^{(1)} = p_0 C$.

Experimentally, we interfere two fields which are matched in intensity, polarization, and time-of-arrival at a 50:50 beam splitter. We monitor the intensity at the two outputs of the beam splitter using two superconducting nanowire single photon detectors (SNSPDs, Single Quantum Eos), which register the single-photon counts as a function of time, with 100 ms resolution. When interfering two fields that show photon-number coherence, the count rates at the two outputs of the beam splitter are found to be anti-correlated. If the two fields are in (out of) phase then we see maximum constructive (destructive) interference in output 1, corresponding to a maximum (minimum) count rate in detector 1, respectively. In order to determine the maximum interference visibility we record the intensities measured by the two detectors for approximately 20 minutes whilst we let the phase between the two signals freely evolve. (b) The visibility $V$ as a function of time obtained from Eq. S.8 corresponding to the data shown in (a). (c) Histogram of visibility time trace presented in (b). Dashed vertical lines indicate the extracted maximum visibility $v$. This data set was taken at $\theta = 0.55\pi$.

### III.3. Measuring $C$ and $C_{b,c}$

If the wave packet has a small probability of containing more than one photon, then $v \equiv c^{(1)}$ can overestimate $C$. To correct for this effect, we use the approach detailed in Ref. [48] and consider that a small amount of noise is added to the ideal state in Eq. S.1 by a beam splitter interaction. We then decompose the battery field amplitude $\langle \hat{a}_b(t) \rangle$ into a contribution from the desired quantum state (subscript s) and the additional noise (subscript n) $\langle \hat{a}_b(t) \rangle = \cos(\vartheta) \langle \hat{a}_s(t) \rangle + \sin(\vartheta) \langle \hat{a}_n(t) \rangle$, where $\vartheta$ is a noise parameter governing the amount of added noise. Then, the integrated first-order coherence becomes

$$\mu_b c^{(1)} = p_1 p_0 C \cos^2(\vartheta) + 2 \sqrt{p_1 p_0} c_n^{(1)} \cos(\vartheta) \sin(\vartheta) + \mu_n c_n^{(1)} \sin^2(\vartheta), \quad (S.22)$$
where $\mu_b = \mu_s + \mu_n = p_1 + \mu_n$, $\epsilon^{(1)}_{s,n} = (1/\sqrt{\mu_s\mu_n}) \int dt \text{Re} [\langle \hat{a}_s(t) \rangle \langle \hat{a}_n(t) \rangle]$ quantifies the first-order coherence between the noise and the quantum state, and $\xi^{(1)}_n = (1/\mu_n) \int dt |\langle \hat{a}_n(t) \rangle|^2$ quantifies the first-order coherence of the classical noise itself. In our experiments, the noise arises from reflected unfiltered laser from the fast qubit excitation pulse, which is temporally separate from the light emitted into the battery field by the qubit. Hence, the noise is not coherent with the quantum state $\epsilon^{(1)}_{s,n} = 0$ but is itself coherent by definition $\xi^{(1)}_n = 1$. In this notation, $g^{(2)} \ll 1$ can be similarly written as in the supplementary of Ref. [48]

$$
\mu_b^2 g^{(2)} = 2(1 + M_{s,n})p_1\mu_n \cos^2(\theta) \sin^2(\theta) + \mu_n^2 \sin^4(\theta)
$$

(S.23)

where $M_{s,n} = (1/\mu_s\mu_n) \int dt \int dt' \text{Re} [G^{(1)}_s(t,t')G^{(1)}_n(t',t)] \simeq 0$ is the mean wave packet overlap between the emitted quantum state and the temporally separate classical noise photons. Note that here we used $g^{(2)}_s = 0$ and $g^{(2)}_n = 1$. Defining $\cos^2(\eta) = p_1 \cos^2(\theta)/\mu_b$ and $\sin^2(\eta) = \mu_n \sin^2(\theta)/\mu_b$, we can re-write our expressions in terms of a single parameter:

$$
c^{(1)}(\eta) = p_0 C \cos^2(\eta) + \sin^2(\eta)
$$

$$
g^{(2)}(\eta) = 2 \cos^2(\eta) \sin^2(\eta) + \sin^4(\eta)
$$

(S.24)

Clearly, we have $c^{(1)} = p_0 C$ when $\eta \to 0$ as expected for the ideal case. If nonzero, the lowest-order correction is then given by $\lim_{\eta \to 0} (dc^{(1)}(\eta)/dg^{(2)}(\eta)) = (1 - p_0 C)/2$. Hence we have

$$
p_0 C \simeq \frac{c^{(1)} - g^{(2)}/2}{1 - g^{(2)}/2}.
$$

(S.25)

Note that this formalism allows to account for the residual $g^{(2)}$ in the visibility measurement (blue and red curves in Fig. 2(a) and Fig. 2(b)). Indeed, a small fraction of laser signal in the measured field leads to classical interference that artificially increases the amount of work transferred. This effect contributes all the more as we increase $\theta$ as attested by an increased second-order intensity correlation of the battery field.
### IV. Glossary

**TABLE S.1.** List of terms, symbols, and definitions for quantities related to energetics.

| Name                              | Symbol | Definition or constraint |
|-----------------------------------|--------|--------------------------|
| Qubit initial state              | $Q_q(0)$ | $\cos(\theta/2) |g\rangle + \sin(\theta/2)e^{i\phi} |e\rangle$ |
| Qubit initial energy             | $E_q(0)$ | $\hbar\omega_0 \sin^2(\theta/2)$ |
| Qubit initial coherence          | $s$     | $\cos(\theta/2) \sin(\theta/2)$ |
| Battery mode                     | $\hat{a}_b$ | -                         |
| Battery envelope (intensity profile) | $I_b(t)$ | $\langle \hat{a}_b(t)^\dagger \hat{a}_b(t) \rangle$ |
| Battery initial energy (charged) | $E_b$   | $\hbar\omega_0 \mu_b = E_q(0)$ |
| Battery ideal state              | $|\Psi_b\rangle$ | $\cos(\theta/2) |0\rangle + \sin(\theta/2)e^{i\phi} |1\rangle$ |
| Battery ideal charging work      | $W_{q,b}$ | $\hbar\omega_0 s^2$ |
| Qubit-Battery charging efficiency | $\eta_{q,b}$ | $W_{q,b}/E_b$ |
| Classical receiver mode (coherent state) | $\hat{a}_c$ | -                         |
| Classical receiver amplitude     | $\beta(t)$ | $\langle \hat{a}_c(t) \rangle$ |
| Classical receiver envelope (intensity profile) | $I_c(t)$ | $\langle \hat{a}_c(t)^\dagger \hat{a}_c(t) \rangle = |\beta(t)|^2$ |
| Classical receiver initial energy | $E_c$   | $\hbar\omega_0 \mu_c$ |
| Name | Symbol | Definition or constraint |
|------|--------|-------------------------|
| Photon creation operator (time basis) | \( \hat{a}^\dagger(t) \) | \( [\hat{a}(t), \hat{a}^\dagger(t')] = \delta(t-t') \) |
| Photonic density operator (number basis) | \( \hat{\rho} \) | \( p_0 \hat{\rho}_0 + p_1 \hat{\rho}_1 + \sqrt{p_0 p_1} (\hat{\rho}_{01} + \hat{\rho}_{10}) + \cdots \) |
| Vacuum state | \( \hat{\rho}_0 \) | \( |0\rangle\langle 0| \) |
| Single-photon state | \( \hat{\rho}_1 \) | \( \int dtdt' \xi(t,t') \hat{a}^\dagger(t) |0\rangle \langle 0| \hat{a}(t') \) |
| Single-photon temporal density function | \( \xi(t,t') \) | \( \text{Tr}[\hat{a}^\dagger(t') \hat{a}(t) \hat{\rho}_1] \) |
| Number coherence (between \(|0\rangle\) and \(|1\rangle\)) | \( \hat{\rho}_{01} \) | \( \int dt \zeta(t) \hat{a}^\dagger(t) |0\rangle \langle 0| \hat{a}(t) \) |
| Temporal number coherence amplitude | \( \zeta(t) \) | \( \text{Tr}[\hat{a}(t) \hat{\rho}_0] \) |
| Temporal wavepacket (first-order correlation) | \( G^{(1)}(t,t') \) | \( |\hat{a}^\dagger(t')\hat{a}(t)\rangle \) |
| Temporal envelope (intensity profile) | \( I(t) \) | \( |\hat{a}^\dagger(t)\hat{a}(t)\rangle \) |
| Average photon number | \( \mu \) | \( \int dt I(t) = \sum_n n p_n \) |
| Mean wavepacket overlap | \( M_{i,j} \) | \( \frac{1}{\mu_{i} \mu_{j}} \int \int dt dt' \text{Re}[G^{(1)}_{i}(t,t')G^{(1)}_{j}(t',t)] \) |
| Indistinguishability | \( M \) | \( \frac{1}{\mu \mu_{i} \mu_{j}} \int \int \int \int dt dt' dt'' dt''' \text{Re}[G^{(1)}(t,t')G^{(1)}(t'',t''')] \) |
| Single-photon indistinguishability (trace purity) | \( M_{s} \) | \( \int \int \int \int dt dt' dt'' dt''' \text{Tr}[\hat{\rho}_{i1} \hat{\rho}_{j0}] = \mu \mu_{i} \mu_{j} \) |
| Normalized\(^\dagger\) first-order coherence amplitude overlap | \( c^{(1)}_{i,j} \) | \( \frac{2}{\mu_{i} \mu_{j}} \int dt \text{Re}[\langle \hat{a}_i(t) | \hat{a}_j^\dagger(t) \rangle] \) |
| Number purity | \( C \) | \( \int dt |\zeta(t)|^2 = \text{Tr}[\hat{\rho}_0 \hat{\rho}_1], 0 \leq C \leq \sqrt{M_{s}} \) |
| Number purity parameter | \( \lambda^2 \) | \( C/\sqrt{M_{s}}, 0 \leq \lambda \leq 1 \) |
| Second-order (intensity) correlation | \( G^{(2)}(t,t') \) | \( |\hat{a}^\dagger(t)\hat{a}^\dagger(t')\hat{a}(t')\hat{a}(t)\rangle \) |
| Normalized time-integrated second-order correlation\(^\dagger\) | \( g^{(2)}(\tau) \) | \( \frac{1}{\mu} \int dt G^{(2)}(t,t + \tau) \) |
| Normalized fully integrated second-order correlation\(^\dagger\) | \( g^{(2)} \) | \( \frac{1}{T} \int \int dt dt' G^{(2)}(t,t') = \frac{T}{\mu} \sum_n \binom{n}{2} \) |

\(^\dagger\) Upper-case \( G \) is used for un-normalized correlations, lower-case \( g \) is used for normalized correlations.

\(^\dagger\) An alternative convenient normalization is \( 1/\sqrt{\mu_{i} \mu_{j}} \), which is equivalent when \( \mu_{i} = \mu_{j} \).

\(^\dagger\) Note that \( g^{(2)} \neq g^{(2)}(0) \) in this notation.