Comparative Synthesis: Learning Near-Optimal Network Designs by Query

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When managing wide-area networks, network architects must decide how to balance multiple conflicting metrics, and ensure fair allocations to competing traffic while prioritizing critical traffic. The state of practice poses challenges since architects must precisely encode their intent into formal optimization models using abstract notions such as utility functions, and ad-hoc manually tuned knobs. In this paper, we present the first effort to synthesize optimal network designs with indeterminate objectives using an interactive program-synthesis-based approach. We make three contributions. First, we present comparative synthesis, an interactive synthesis framework which produces near-optimal programs (network designs) through two kinds of queries (Propose and Compare), without an objective explicitly given. Second, we develop the first learning algorithm for comparative synthesis in which a voting-guided learner picks the most informative query in each iteration. We present theoretical analysis of the convergence rate of the algorithm. Third, we implemented NET10Q, a system based on our approach, and demonstrate its effectiveness on four real-world network case studies using black-box oracles and simulation experiments, as well as a pilot user study comprising network researchers and practitioners. Both theoretical and experimental results show the promise of our approach.

CCS Concepts:
• Software and its engineering → Automatic programming;
• Networks → Traffic engineering algorithms;
• Theory of computation → Active learning;
• Human-centered computing → Interactive systems and tools.

Additional Key Words and Phrases: Program Synthesis, Traffic Engineering, Optimization, Query, User Interaction

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1 INTRODUCTION

Synthesizing wide-area computer network designs typically involves solving multi-objective optimization problems. For instance, consider the task of managing the traffic of a wide-area network — deciding the best routes and allocating bandwidth for them — the architect must consider myriad

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considerations. She must choose from different routing approaches — e.g., shortest path routing [Fortz and Thorup 2000], and routing along pre-specified paths [Hong et al. 2013; Liu et al. 2014]. The traffic may correspond to different classes of applications — e.g., latency-sensitive applications such as Web search and video conferencing, and elastic applications such as video streaming, and file transfer applications [Hong et al. 2013; Jain et al. 2013; Kumar et al. 2015]. The architect may need to decide how much traffic to admit for each class of applications. It is desirable to make decisions that can ensure high throughput, low latency, and fairness across different applications, yet not all these goals may be simultaneously achievable. Likewise, a network must not only perform acceptably under normal conditions, but also under failures — however, providing guaranteed performance under failures may require sacrificing normal performance [Jiang et al. 2020; Liu et al. 2014].

Traffic engineering formulates network design problems as optimization problems [Fortz and Thorup 2000; Jain et al. 2013; Jiang et al. 2020; Kumar et al. 2015; Wang et al. 2010], e.g., minimizing a weighted sum of link utilization and latency subject to constraints. In this context, architect must provide the objectives as well-defined mathematical functions (which we henceforth refer to as target functions), which is a challenging task in the first place. Even the simplest target functions may involve several knobs to capture the relative importance of different criteria (e.g., throughput, latency, and fairness, performance under normal conditions vs. failures). These knobs must be manually tuned by the architect in a “trial and error” fashion to result in a desired design. Further, many optimization problems (e.g., [Kumar et al. 2015]) require architects to use abstract functions that capture the utility an application sees if a given design is deployed. Utility functions are often non-linear (e.g., logarithmic) and may involve weights, which are not intuitive for a designer to specify in practice [Srikant 2004]. Finally, objectives are often chosen in a manner to ensure tractability, rather than necessarily reflecting the true intent of the architect.

Through the lens of programming languages, this problem can be viewed as an instance of specification mining [Ammons et al. 2002], a long-standing problem in the formal methods community which recognizes that a precise specification may not always be available. Beyond the context of network design, the challenge of indeterminate objectives is also commonly seen in many quantitative synthesis problems beyond the networking domain. For example, the default ranker for the FlashFill synthesizer is manually designed and highly tuned by experts [Gulwani et al. 2019]. In quantitative syntax-guided synthesis (qSyGuS) [Hu and D’Antoni 2018], the objective should be provided as a weighted grammar, which is nontrivial for average programmers.

As our first step toward addressing this fundamental problem for quantitative program synthesis, this paper proposes Comparative Synthesis, an interactive approach based on the key insight that when a user has difficulty in providing a concrete objective, it is relatively easy and natural to give preferences between pairs of concrete candidates. The approach may be viewed as a new variant of programming-by-example (PBE), where preference pairs are used as “examples” instead of input-output pairs in traditional PBE systems. In this paper, we focus on the networking domain and illustrate how comparative synthesis enables learning near-optimal network designs with indeterminate objectives, and briefly discuss the application to other quantitative synthesis problems as future research directions. The main contributions of the paper include:

- **A novel user-interaction paradigm (§3).** We present a rigorous formulation of an interactive synthesis framework which we refer to as comparative synthesis. As Fig 1 shows, the framework consists of two major components: a comparative learner and a teacher (a user or a black-box oracle). The learner takes as input a clearly defined qualitative synthesis problem (including a parameterized program and a specification), a metric group and a target function space, and is tasked to find a near-optimal program w.r.t. the teacher’s quantitative intent through two kinds of queries — Propose and Compare.
Comparative Synthesis: Learning Near-Optimal Network Designs by Query

The notion of comparative synthesis stems from our position paper [Wang et al. 2019]. The preliminary work lacks formal foundation and query selection guidance, and may involve impractically many rounds of user interaction (see §2). In contrast, the formalism of our framework enables the design and analysis of learning algorithms that strive to minimize the number of queries, and are amenable for real user interaction.

- **The first algorithm for comparative synthesis** (§4). We develop the first, voting-guided learning algorithm for comparative synthesis, which provides a provable guarantee on the quality of the found program. The key insight behind the algorithm is that objective learning and program search are mutually beneficial and should be done in tandem. The idea of the algorithm is to search over a special, unified search space we call *Pareto candidate set*, and to pick the most informative query in each iteration using a voting-guided estimation.

  We analyze the convergence of voting-guided algorithm, i.e., how fast the solution approaches the real optimal as more queries are made. We prove that the algorithm guarantees the median quality of solutions to converge logarithmically to the optimal. When the target function space is sortable, which covers a commonly seen class of problems, a better convergence rate can be achieved — the median quality of solutions converges linearly to the optimal.

- **Evaluations on network case studies and pilot user study** (§5). We developed Net10Q, an *interactive network optimization system* based on our approach. We evaluated Net10Q on four real-world scenarios using oracle-based evaluation. As a first-of-its-kind system, Net10Q does not have any similar systems to compare with. Therefore, we developed a variant of Net10Q as the baseline system. Experiments show that Net10Q only makes half or less queries than the baseline system Net10Q-NoPrune to achieve the same solution quality, and robustly produces high-quality solutions with inconsistent teachers. We conducted a pilot study with Net10Q among networking researchers and practitioners. Our study shows that user policies are diverse, and Net10Q is effective in finding allocations meeting the diverse policy goals in an interactive fashion.

2 MOTIVATION

In this section, we present background on network design, how it may be formulated as a program synthesis problem, and discuss challenges that we propose to tackle.

**Network design background.** In designing Wide-Area Networks (WANs), Internet Service Providers (ISPs) and cloud providers must decide how to provision their networks, and route traffic so their traffic engineering goals are met. Typically WANs carry multiple classes of traffic (e.g., higher priority latency sensitive traffic, and lower priority elastic traffic). Traffic is usually specified
as a matrix with cell \((i, j)\) indicating the total traffic which enters the network at router \(i\) and that exits the network at router \(j\). We refer to each pair \((i, j)\) as a flow, or a source-destination pair. It is typical to pre-decide a set of tunnels (paths) for each flow, with traffic split across these tunnels in a manner decided by the architect, though traffic may also be routed along a routing algorithm that determines shortest paths (§5.1).

Given constraints on link capacities, it may not be feasible to meet the requirements of all traffic of all flows. An architect must decide how to allocate bandwidth to different flows of different classes and how to route traffic (split each flow’s traffic across its paths) so desired objectives are met. In doing so, an architect must reconcile multiple metrics including throughput, latency, and link utilizations [Hong et al. 2013; Jain et al. 2013; Kumar et al. 2018; Subramanian et al. 2020], ensure fairness across flows [Danna et al. 2012; Kumar et al. 2015; Srikant 2004], and consider performance under failures [Chang et al. 2017; Jiang et al. 2020; Liu et al. 2014; Wang et al. 2010].

**Network design as program synthesis problems.** Consider a variant of the classical multi-commodity flow problem used in Microsoft’s Software Defined Networking Controller SWAN [Hong et al. 2013], which we refer to as MCF. MCF allocates traffic to tunnels optimally trading off the total throughput seen by all flows with the weighted average flow latency [Hong et al. 2013]. We consider a single class (see §5.1 for multiple classes).

Fig 2 shows how the demand-capacity constraints may be described as a sketch-based synthesis problem, in which the programmer specifies a sketch — a program that contains unknowns to be solved for, and assertions to constrain the choice of unknowns. The Topology struct represents the network topology (we use the Abilene topology [Knight et al. 2011] with 11 nodes, 14 links and 220 flows as a running example). The allocate function should determine the bandwidth allocation (denoted by ??), which is missing and should be generated by the synthesizer. The function also serves as a test harness and checks that the synthesized allocation is valid, satisfying all demand and capacity constraints (lines 12–13). Finally, the main function takes the generated allocation, and computes and returns the total throughput and weighted latency.

Now Fig 2 has encoded all hard constraints and represented a qualitative synthesis problem, which can be solved by Sketch [Solar-Lezama et al. 2006] easily. The bandwidth allocations generated by the synthesizer (the values of bw) is just a network design solving the MCF problem. However, there are many different ways to fill the ??, corresponding to many different ways of assigning paths and leading to different throughput-latency combinations as computed in main. Which solution is the most desirable one? Traditionally, the architect has to explicitly provide a target function which maps each possible solution to a numerical value indicating the preference. Given a well-specified target function, the bandwidth allocation problem becomes a quantitative synthesis problem and can be solved using existing techniques from both synthesis and optimization communities. E.g., in Fig 2, one can explicitly add a target function \(O_{real}\) and use the minimize construct (cf. Sketch manual [Solar-Lezama 2016]) to find the optimal solution.

**Why synthesis with indeterminate objectives?** Unfortunately, in practice, it is hard for network architects to precisely express their true intentions using target functions. For example, to capture the intuition that once the throughput (resp. latency) reaches a certain level, the marginal benefit (resp. penalty) may be smaller (resp. larger), an architect may need to use a target function like below:

\[
O_{real}(\text{throughput, latency}) \overset{\text{def}}{=} 2 \cdot \text{throughput} - 9 \cdot \text{latency} - \max(\text{throughput} - 350, 0) - 10 \cdot \max(\text{latency} - 28, 0)
\]  

We will use network design and network program interchangeably in the paper, as network design can always be extracted from the synthesized network program.

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Comparative Synthesis: Learning Near-Optimal Network Designs by Query

1 struct Topology {
2   int n_nodes; int n_links; int n_flows;
3   bit[n_nodes][n_nodes] links;
4   /* every link has a capacity and a weight, every flow consists of multiple links and has a demand */
5   float[n_links] capacity; int[n_links] weight;
6   bit[n_links][n_flows] in_f; float[n_flows] demand; ...
7 }
8 Topology abilene = new Topology(n_nodes=11, n_links=220, ...);
9 float[] allocate(Topology T) {
10   float[] bw = ???; // generate bandwidth allocation
11   assert that every flow’s demand is satisfied and every link’s bandwidth is not exceeded */
12   assert \[ \forall i \exists j \left( \sum_i \left( \sum_j \text{in}_{f[j][i]} \right) \times \text{bw}[i] : 0 \right) \leq T\.capacity[j] ;
13   return bw; }
14 float[] main() {
15   float[] alloc = allocate(abilene);
16   /* compute the throughput and weighted latency */
17   float throughput = \[ \sum_i \text{alloc}[i] , latency = \left[ \sum_i \left( \sum_j \text{in}_{f[j][i]} \times \text{weight}[j] : 0 \right) \right] ;
18   return \{ throughput, latency \}; }

Fig. 2. MCF allocation encoded as a program sketch.

The intuition behind the network design choice is allocating resources based on the incremental value of additional allocation. The network architect wants to make sure the throughput is no less than a certain level hence giving higher reward to throughput when throughput is below the threshold. The network architect cares less about further improvements in throughput hence giving smaller reward to the portion of throughput that is beyond the threshold.

More generally, the marginal reward in obtaining a higher bandwidth allocation is smaller capturing which may require a target function of the form

$$O(\text{throughput}, \text{latency}) = 1 \cdot \log_2(\text{throughput}/tmax + 1) + 5 \cdot \log_2(lmin/\text{latency} + 1)$$

where \(tmax\) is the maximum possible throughput and \(lmin\) is the minimum possible latency. Expressing such abstract target functions is not trivial, let alone the parameters associated with the functions. We present several other examples in §5.1.

**Naïve objective synthesis is not enough.** A preliminary effort [Wang et al. 2019] argued for synthesizing target functions by having the learner (synthesizer) iteratively query the teacher (user) on its preference between two concrete network designs. In each iteration, any pair of designs may be compared as long as there exist two target functions that (i) disagree on how they rate these designs, and (ii) both satisfy preferences expressed by the teacher in prior iterations. The process continues until no disagreeing target functions are found. However, this work only focuses on objective learning and does not explicitly consider design synthesis. Moreover, it does not address how to generate good preference pairs to minimize queries. As most network design problems are in large search space (and mostly, in real-valued space), the target function can be refined with excessive number of times or even ad infinitum, and it is unclear when the objective function is
precise enough to stop. These limitations make this naïve approach not amenable for real user interaction. Fig 3 shows the performance of a design optimal for a target function synthesized using this procedure if it were terminated after a given number of queries. The resulting designs achieve a reward only 60% of the true optimal design under the ground truth (Eq 2.1), and there is hardly any performance improvement in the first 100 queries.

3 COMPARATIVE SYNTHESIS, FORMALLY

In this section, we provide a formal foundation for the comparative synthesis framework, based on which we design and analyze learning algorithms. The key novelty of our framework compared to past work on quantitative synthesis [Bornholt et al. 2016; Chaudhuri et al. 2014; Hu and D’Antoni 2018; Schkufta et al. 2013, 2014; Černý and Henzinger 2011] is that comparative synthesis does not require the user to explicitly specify the objective. Instead, we approach synthesis via interaction through comparative queries where queries simply involve the users comparing two alternative programs and indicating which is more preferable. Since a user will only be willing to answer a small number of queries and may choose to stop at any point of the interactions, finding a perfect quantitative objective can be unrealistic. Therefore, our goal is to generate a near-optimal program within a budgeted number of queries. As the real ground truth optimal is not accessible, we also introduce a natural notion, called quality of solution, to estimate how close a solution is to optimal.

Roadmap. In §3.1, we formally define quantitative synthesis, a necessary first step for us to formally treat comparative synthesis. Rather than restrict quantitative synthesis to objectives which are closed-form mathematical functions, we formulate quantitative synthesis more generally as we motivate and discuss in §3.1. We formally define comparative queries, and solution quality in §3.2. We conclude with a formal definition of comparative synthesis in §3.3.

3.1 Quantitative Synthesis with Metric Ranking

In this section, we present a formal definition of quantitative synthesis, as a first step towards defining comparative synthesis more precisely. Quantitative synthesis may be viewed as a goal of synthesizing a program that meets a set of “hard constraints”, while performing well on a quantitative objective. Rather than restrict quantitative synthesis to objectives which are closed-form mathematical functions, we formulate quantitative synthesis when given a ranking over all possible programs (which we refer to as a metric ranking). This more general formulation is motivated by the fact that we wish to capture a rich set of user policies in terms of relative preferences across programs, and not restrict the user to objectives that are closed-form mathematical functions.

We start by defining qualitative synthesis (which captures the “hard constraints” that any acceptable program must meet), and then discuss quantitative synthesis with metric ranking.

Definition 3.1 (Qualitative synthesis problem). A qualitative synthesis problem is represented as a tuple \((P, C, \Phi)\) where \(P\) is a parameterized program, \(C\) is the space of parameters for \(P\), and \(\Phi\) is a
verification condition with a single free variable ranging among C. The synthesis problem is to find a value \( \text{ctr} \in C \) such that \( \Phi(\text{ctr}) \) is valid.

**Example 3.2.** Our running example can be formally described as a qualitative synthesis problem \( \mathcal{L}_{\text{MCF}} = (P_{\text{abilene}}, \mathbb{R}^{220}, \Phi_{\text{abilene}}) \), where \( P_{\text{abilene}} \) is the program sketch presented in Fig 2, \( \mathbb{R}^{220} \) is the search space of unknown hole (line 10) which includes 220 bandwidth values of the Abilene network, \( \Phi_{\text{abilene}} \) is the verification condition, taking a candidate solution \( c \in \mathbb{R}^{220} \) as input and checking whether \( P_{\text{abilene}}[c] \) satisfies all assertions in Fig 2. Any solution that satisfies assertions in Fig 2 is a feasible program to the qualitative synthesis problem \( \mathcal{L}_{\text{MCF}} \).

While a qualitative synthesis problem captures all hard constraints, there are potentially infinitely many solutions. Which one is the most desirable? Quantitative synthesis concerns itself with this question and extends a qualitative synthesis problem with a quantitative goal, which is evaluated using a metric group, as defined below.

**Definition 3.3 (Metric).** Given a parameterized program \( P[c] \) where \( c \) ranges from a search space \( C \), a metric with respect to \( P \) is a computable function \( m_P : C \rightarrow \mathbb{R} \). In other words, \( m_P \) takes as input a concrete program in the search space and computes a real value.

**Definition 3.4 (Metric group).** Given a parameterized program \( P \), a \( d \)-dimensional metric group \( M \) w.r.t. \( P \) is a sequence of \( d \) metrics w.r.t. \( P \). We write \( M_i \) for the \( i \)-th metric in the group and \( M(c) \) for the value vector \( (M_1(c), \ldots, M_d(c)) \).

**Example 3.5.** A metric can be computed from the syntactical aspects of the program. For example, a metric \( size_P(c) \) can be defined as the size of the parse tree for \( P[c] \).

**Example 3.6.** A metric can simply be the value of a variable on a particular input (or with no input). In our running example in Fig 2, the two variables throughput and latency of the main function can be used to define two metrics. As the latency as a metric is not beneficial, i.e., smaller latency is better, we can simply use its inverse latency as a beneficial metric. The two metrics form a metric group \( M_{\text{MCF}} \) = \( (\text{throughput}, \text{-latency}) \).

Now given a metric group, the quantitative intent of a user can be captured either syntactically (using a target function) or semantically (using a metric ranking). We formally define them below and discuss their relationship.

**Definition 3.7 (Target function).** Given a metric group \( M \), a target function with respect to \( M \) is a computable function \( \mathbb{R}^{\left| M \right|} \rightarrow \mathbb{R} \).

**Definition 3.8 (Metric ranking).** Given a \( d \)-dimensional metric group \( M \), a metric ranking for \( M \) is a total preorder \( \preceq_M \) over \( \mathbb{R}^d \). In other words, \( \preceq_M \) satisfies the following properties: for any \( u, v, w \in \mathbb{R}^d \), if \( u \preceq_M v \) and \( v \preceq_M w \), then \( u \preceq_M w \) (transitivity); for any two vectors \( u, v \in \mathbb{R}^d \), if \( u \preceq_M v \) or \( v \preceq_M u \) (connexivity).

We write \( u \approx_M v \) if \( u \preceq_M v \) and \( v \preceq_M u \). In this paper, we also flexibly write \( u \succeq_M v, u \prec_M v, u >_M v \) and \( u >_M v \) with the expected meaning. Moreover, we also abuse \( \preceq_M \) and other derived symbols we just described as relations between programs: when the metric group \( M \) is clear from the context, for any two program parameters \( c_1, c_2 \in \text{dom}(M) \), we write \( c_1 \preceq_M c_2 \) to indicate that \( M(c_1) \preceq_M M(c_2) \).

**Why metric ranking?** Target function and metric ranking are closely related, but metric ranking is a more general and unique representation for quantitative intent, and can capture a richer set of user policies in terms of which of two feasible programs is preferable. First, every target function...
implicitly determines a metric ranking (see Def 3.9 below). Second, multiple target functions may have the same metric ranking. For example, any target functions $O$ and $O'$ such that $O'(v) = 2 \cdot O(v)$ for any $v \in \mathbb{R}^{|M|}$ have the same metric ranking. Third, some metric ranking does not correspond to any target function, e.g., one can define a metric ranking $\leq$ between integer metric values such that $n_1 \leq n_2$ if and only if the $n_1$-th digit of $\Omega$ is less than or equal to the $n_2$-th digit of $\Omega$, where $\Omega$ is a Chaitin’s constant [Chaitin 1975] representing the probability that a randomly generated program halts. To this end, we define quantitative synthesis problem below using metric ranking.

**Definition 3.9.** Given a target function $O$ w.r.t. a $d$-dimensional metric group $M$, the corresponding metric ranking $\leq_O \subseteq \mathbb{R}^d \times \mathbb{R}^d$ is defined as follows: for any two program parameters $c_1, c_2 \in \text{dom}(M)$, $c_1 \leq_O c_2$ if and only if $O(M(c_1)) \leq O(M(c_2))$. It can be easily verified that $\leq_O$ is indeed a metric ranking.

**Definition 3.10 (Quantitative synthesis problem).** A quantitative synthesis problem is represented as a tuple $\mathcal{Q} = (P, C, F, M, \leq_M)$ where $(P, C, F)$ forms a qualitative synthesis problem $\mathcal{Q}_{qual}$. $M$ is a metric group w.r.t. $P$ and $\leq_M$ is a metric ranking for $M$. The synthesis problem is to find a solution $s_t$ to $\mathcal{Q}_{qual}$ such that for any other solution $s_t'$, $s_t' \preceq_M s_t$.

**Example 3.11.** With the metric group $M_{MCF}$ defined in Example 3.6, the function $O_{real}$ defined in Equation 2.1 is a 2-dimensional target function with the corresponding metric ranking $\leq_{real}$. Then the qualitative synthesis problem $\mathcal{Q}_{MCF} = (P_{abilene}, \mathbb{R}^{220}, F_{abilene})$ can be extended to a quantitative synthesis problem $\mathcal{Q}_{MCF} = (P_{abilene}, \mathbb{R}^{220}, F_{abilene}, M_{MCF}, \leq_{real})$.

### 3.2 Interaction through Comparative Queries

Quantitative synthesis problem as defined in Def 3.10 expects a metric ranking explicitly or implicitly (e.g., through a target function). Comparative synthesis is more challenging as it seeks to synthesize a program that is near-optimal in terms of the objective, but without being explicitly given the objective. To achieve the goal, our comparative synthesis framework is interactive between a learner and a teacher (see Fig 1). As the teacher may choose to stop at any point of the interactions, our comparative synthesis framework maintains the best candidate solution found through the synthesis process and recommends the best solution confirmed by the teacher when terminated.

As the quantitative objective is assumed to be very complex and not directly accessible from the teacher, the comparative learner can only make several types of queries to the teacher, whose responses provide indirect access to the specifications. The query types serve as an interface between the learner and the teacher, and different query types lead to different synthesis power (e.g., see the query types discussed in [Jha and Seshia 2017]). What makes our framework special is that the learner can make queries about the metric ranking — queries that compare two program (based on their corresponding metric value vectors).

Let us fix a parameterized program $P$ and a metric group $M$. The learner and the teacher interact using two types of queries:

- **Compare($c_1$, $c_2$) query:** The learner provides two concrete programs $P[c_1]$ and $P[c_2]$, and asks “Which one is better under the target metric ranking $\leq_M$?” The teacher responds with $<$ or $>$ if one is strictly better than the other, or $=\text{ if } P[c_1] \text{ and } P[c_2]$ are considered equally good.

- **Propose($c$) query:** The learner proposes a candidate program $P[c]$ and asks “Is $P[c]$ better than the running best candidate $r_{best}$?” If the teacher finds that $P[c]$ is not better than the running
best, she can respond with ⊥. Otherwise, the teacher considers that \( P[c] \) is better and responds with \( \top \); in that case, the running best will be updated to \( P[c] \).

Now in comparative synthesis, the specification (a metric ranking \( R \)) is hidden to the learner. Instead, the learner can approximate/guess the specifications by making queries to the teacher. Ideally, the teacher should be perfect, i.e., the responses she makes to queries are always consistent and satisfactory. Formally,

**Definition 3.12 (Perfect teacher).** A teacher \( T \) is perfect if there exists a metric ranking \( \leq_M \) such that: 1) for any query \( \text{COMPARE}(v_1, v_2) \), the response is “<” if \( v_1 \leq_M v_2 \) and \( v_2 \not\leq_M v_1 \), or “>” if \( v_1 \not\leq_M v_2 \) and \( v_2 \leq_M v_1 \), or “=” if \( v_1 \leq_M v_2 \) and \( v_2 \leq_M v_1 \); 2) for any query \( \text{PROPOSE}(c) \) with the current running best \( r_{\text{best}} \), the response is “\( \top \)” if \( c >_M r_{\text{best}} \); or “\( \bot \)” otherwise.

We denote the perfect teacher w.r.t. \( \leq_M \) as \( T_{\leq_M} \). We also denote the metric ranking \( \leq_M \) represented by a perfect teacher \( T \) as \( \leq_T \). A perfect teacher guarantees that an optimal solution exists among all candidates. For now, let us assume that the teacher is perfect, i.e., consistent and able to answer all queries; but in the real world, a human teacher may be inconsistent and responds incorrectly. We do consider imperfect teachers in our evaluation (see §5.3).

**Why budgeted number of queries?** Ideally, the goal of the learner is to find an objective target (in the form of target function or metric ranking) that matches the teacher’s mind and the corresponding optimal program that optimizes the objective. However, finding the target function can be impossible as the objective target may have no closed-form representation and not in the target function space. As the teacher is free to terminate the synthesis process at any point, pinpointing the target function in a potentially infinitely large space can also be impossible, even if the target function is in the target function space.

Therefore, the goal of the learner is to *spend a budgeted number of queries and to produce a near-optimal program from the perspective of the teacher*. Note that because the learner may use a conjectured objective to guide the search process, finding a perfect target function is not a goal. This is also a key insight for our algorithm design (cf. §4).

Now to determine how close a solution is to the ground truth optimal, we introduce a natural notion called *quality of solution* which is intuitively the “relative rank” of the solution among all solutions. E.g., a solution of quality 0.9 is better than or equal to 90% of possible solutions. From a probability theory point of view, the quality is just the cumulative distribution function (CDF). Below we formally define the quality of solutions.

**Definition 3.13 (Quality of solution).** Let \( \mathcal{D} = (\mathcal{P}, C, \Phi, M, \leq_M) \) be a quantitative synthesis problem and let \( \text{ctr} \) be a solution to \( \mathcal{D}^{\text{qual}} \). The quality of \( \text{ctr} \) is defined as

\[
\text{Quality}_\mathcal{D}(\text{ctr}) \overset{\text{def}}{=} P(\text{ctr} \geq_T X^{M}_\mathcal{D})
\]

where \( X^{M}_\mathcal{D} \) is a variable randomly sampled from the uniform distribution for Solutions( \( \mathcal{D}^{\text{qual}} \)).

In particular, when \( \text{Quality}_\mathcal{D}(c) = 1 \), \( c \) is better than or equal to all other possible solutions, i.e., is the optimal solution under the teacher’s preference. Note that computing the exact quality can be very expensive, if not impossible. However we can estimate the quality by sampling, as we do in evaluation (see §5.3).

**Example 3.14.** The quantitative synthesis problem \( \mathcal{D}_{\text{MCF}} \) in Example 3.11 involves a metric ranking \( \leq_{\text{real}} \). Let \( T_{\text{real}} \) be a perfect teacher w.r.t. \( \leq_{\text{real}} \). Table 1 illustrates how a voting-guided learning algorithm (which we present later in §4) serves as the learner and learns a near-optimal solution to \( \mathcal{D}_{\text{MCF}} \) through queries to \( \leq_{\text{real}} \). In the first iteration, the learner solves the synthesis problem in Fig 2 and gets a first mediocre allocation \( P_0 \) and presents it to the teacher, using query

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The teacher accepts the proposal as this is the first running best. In the sixth iteration, the learner presents two programs $P_6$ and $P_7$ to the teacher and asks her to compare them. Based on the feedback that the teacher prefers $P_6$ to $P_7$, the learner proposes $P_8$ which is confirmed by the teacher to be the best program so far. After seven queries, the running best is already very close to the optimal under the real objective (Quality of this solution has already achieved 97.8%). If the teacher wishes to answer more queries, the solution quality can be further improved.

In Example 3.14, a perfect teacher $T_{real}$ is queried. In practice, the teacher is usually a network architect who has a sense of which metrics she cares and how these metrics compete. The network architect answers comparative queries based on her own criterion. For example, when comparing $P_6$ and $P_7$, an architect may prefer higher throughput and does not mind sacrificing latency a bit, then she may choose $P_6$. Another architect may have a strict latency threshold in mind and end up picking $P_7$.

### 3.3 The Comparative Synthesis Problem

Given the approximation nature of query-based interaction and the quality of solution defined above, the learner is tasked to solve what we call *comparative synthesis problem*, which is formally defined below.

**Definition 3.15 (Comparative synthesis problem).** A comparative synthesis problem is represented as a tuple $\mathcal{C} = (\mathcal{P}, C, \Phi, M, T)$ where $\mathcal{P}$ is a parameterized program, $C$ is the space of parameters for $\mathcal{P}$, $\Phi$ is a verification condition for $\mathcal{P}$, $M$ is a metric group w.r.t. $\mathcal{P}$ and $\mathcal{T}$ is a perfect teacher, such that $(\mathcal{P}, C, \Phi, M, \leq_T)$ forms a quantitative synthesis problem, which is denoted as $\mathcal{Q}$. The synthesis problem is to find, by making a sequence of $\text{COMPARE}$ and $\text{PROPOSE}$ queries to the teacher $\mathcal{T}$, a near-optimal solution ctr to $\mathcal{Q}$ with a provable guarantee on Quality $\mathcal{Q}(\text{ctr})$.

### 4 VOTING-GUIDED LEARNING ALGORITHM

In this section, we focus on the learner side of the framework and propose a voting-guided learning algorithm that can play the role of the comparative learner and solve the comparative synthesis problem. Below we propose a novel search space combining the program search and objective learning, then present an estimation of query informativeness, based on which our voting-guided algorithm is designed. We discuss the convergence of the algorithm at the end.

#### 4.1 A Unified Search Space

A syntactical and natural means to describing quantitative specification is *target function* (in contrast to the semantically defined metric ranking in Def 3.8). Now to solve a comparative synthesis problem efficiently, an explicit task of the learner is *program search*: the goal is to minimize human interaction (i.e., the number of queries) and maximize the quality of the solution (see Def 3.13) proposed through $\text{PROPOSE}$ queries. Another implicit task of the learner is *objective learning*: to steer program search...
faster to the optimal and minimize the query count, the learner should conjecture target functions that fit the teacher-provided preferences, and use them to determine which programs are more likely to be optimal. Note that the conjectured target function need not (and sometimes cannot) be perfect — the goal is just to approximate the teacher’s metric ranking $\leq_T$.

Our key insight is that the two tasks are inherently tangled and better be done together. On one hand, the quantitative synthesis task needs to be guided by an appropriate objective; otherwise the search is blind and unlikely to steer to those candidates satisfying the user. On the other hand, learning a perfect target function can be extremely expensive (if not impossible — see the “why metric ranking” discussion in §3) and unnecessary — even an inaccurate target function may guide the program search. We first define the target function space.

**Definition 4.1 (Target function space).** A target function space $O$ is a set of target functions with respect to a $d$-dimensional metric group $M$ such that for any metric ranking $\leq_M \subset \mathbb{R}^d \times \mathbb{R}^d$ and any finite subset $S \subset \mathbb{R}^d$, there exists a target function $O \in O$ such that for any $u, v \in S, u \leq_M v$ if and only if $O(u) \leq O(v)$.

**Example 4.2.** The class of conditional linear integer arithmetic (CLIA) functions forms a target function space. A CLIA target function, intuitively, uses linear conditions over metrics to divide the domain into multiple regions, and defines in each region as a linear combination of metric values. Formally, for any $d$-dimensional metric group $M$, a target function space $O^{d}_{\text{CLIA}}$ can be defined as the class of expressions derived from the nonterminal $T$ of the following grammar:

$$
T ::= E \mid \text{if } B \text{ then } T \text{ else } T \\
B ::= E \geq 0 \mid B \land B \mid B \lor B \\
E ::= v_1 \mid \ldots \mid v_d \mid c \mid E + E \mid E - E
$$

where $c \in \mathbb{Z}$ is a constant integer, and $v_i$ is the $i$-th value of the metric vector. It is not hard to see that $O^{d}_{\text{CLIA}}$ is indeed a target function space, because with arbitrarily many conditionals, one function can be constructed to fit any finite subset of any metric ranking.

While the $O^{d}_{\text{CLIA}}$ space above is general enough for any comparative synthesis problem, for many concrete metric groups, smaller, domain-specific templates usually exist. The algorithm can search more efficiently by starting with the most specific template and falling back to more general templates if necessary. Below shows a domain-specific target function space used for our running example.

**Example 4.3.** For our running example, a commonly used function to quantify this trade-off is the multi-commodity flow functions used in software-driven WAN [Hong et al. 2013]. The $O_{\text{real}}$ function (Equation 2.1) in our running example is an instance of the generalized, two-segment MCF function space, which can be described in the following form:

$$
O(\text{throughput, latency}) \overset{\text{def}}{=} \text{throughput} \ast ?? - \max(\text{throughput} - ??, 0) \ast ?? \\
- \text{latency} \ast ?? - \max(\text{latency} - ??, 0) \ast ??
$$

where ?? can be arbitrary weights or thresholds. Note that the two-segment template is insufficient to characterize an arbitrary finite metric ranking. In that case, the template may be extended to more segments. We call this layered target function space $O_{\text{MCF}}$.

Now as the learner’s task is to search two spaces — one for programs and one for target functions — we merge the two tasks into a single one, searching over a unified search space which we call Pareto candidate set:
Definition 4.4 (Pareto candidate set). Let $C = (P, C, \Phi, M, T)$ be a comparative synthesis problem and $O$ be a target function space w.r.t. $M$. A Pareto candidate set (PCS) with respect to $C$ and $O$ is a finite partial mapping $\mathcal{G} : O \to C$ from a space of target functions $O$ to a space of program parameters $C$, such that for any $O \in O$, $\mathcal{G}(O)$ is the effectively optimal solution under target function $O$, i.e., a solution $c \in C$ such that $\mathcal{G}(O) \leq_O c$, if exists, cannot be effectively found. Specifically, for any other $O' \in O$, $\mathcal{G}(O') \leq_O \mathcal{G}(O)$.

Intuitively, a Pareto candidate set (PCS) $\mathcal{G}$ maintains a set of candidate target functions, a set of candidate programs, and a mapping between the two sets, and guarantees that every candidate target function $O$ is mapped to the best candidate program under $O$.

4.2 Query Informativeness

Now with the unified search space — PCS in Def 4.4 — comparative synthesis becomes a game between the learner and the teacher: a PCS $\mathcal{G}$ is maintained as the current search space; and in each iteration, the learner makes a query and the teacher gives her response, based on which $\mathcal{G}$ is shrunken. The learner’s goal should be, in each iteration, to pick the most informative query in the sense that it can reduce the size of $\mathcal{G}$ as fast as possible. The key question is how to evaluate the informativeness of a query.

In this paper, we develop a greedy strategy which evaluates the informativeness by computing how many candidate programs from $\mathcal{G}$ can be removed immediately with the teacher’s response. As the teacher’s response can be arbitrary, our evaluation considers all possible responses and take the minimum number among all cases. The formulation is shown in Fig 4 and explained below.

- **Compare query:** For Compare($c_1, c_2$), recall the teacher may prefer $P[c_1] \to P[c_2]$, or vice versa, or consider the two programs equally good (corresponding to the three responses: $<, >$ and $=\,$). In each case, we can remove all the candidate target functions that have a different relative ordering of $P[c_1]$ and $P[c_2]$ than the teacher’s preference. We denote the number of candidates that can be removed when the teacher prefers $P[c_1]$ (resp. $P[c_2]$) as $|c_1 \leq_O c_2|$ (resp. $|c_1 \geq_O c_2|$). Further let $|c_2 \neq_O c_1|$ denote the candidates that can be removed if the user indicates both programs are equally good. The overall informativeness is just the minimum of the three cases.

- **Propose query:** For Propose($c$), recall the teacher may confirm that $P[c]$ is indeed better than the running best $P[r\_best]$ (response $\top$), or keep the current running best (response $\bot$). Like the compare query, in each case, we can eliminate all candidate target functions that do not satisfy this relative preference between $P[c]$ and $P[r\_best]$ expressed by the user. However, in the case that the user prefers $P[c]$, we can additionally remove all candidate target functions for which $P[c]$ is already the best choice, i.e., more queries are not needed for further improvement — we use $|NewBest(c)|$ in Fig 4 to denote the total number of eliminated candidate programs in this case. The overall informativeness is just the minimum of the two cases.

$^4$Note that in general, the best candidate program under $O$ is not necessarily unique. To break the tie and make $\mathcal{G}$ uniquely determined by the component sets of target functions and programs, when two candidate programs $c_1$ and $c_2$ both get the highest reward under $O$, we assume $\mathcal{G}(O)$ is $c_1$ if $M(c_1)$ is smaller than $M(c_2)$ in lexicographical order, or $c_2$ otherwise.
**Example 4.5.** Table 2 shows a PCS $\mathcal{G}_{\text{ex}}$ that consists of 5 candidate target functions, namely $O_i$ for $0 \leq i \leq 4$, and 3 candidate programs, namely $P_0, P_1$ and $P_2$. The rankings of all candidate programs and the current running best $r_{\text{best}}$ under target functions are also shown in Table 2. The informativeness of COMPARE and PROPOSE queries for PCS $\mathcal{G}_{\text{ex}}$ is presented in Tables 3 and 4, respectively. Take COMPARE($P_1, P_2$) as an example, $[P_1 \preceq O P_2]$ is 2 since all candidate target function except $O_3$ believes $P_1 \preceq O P_2$ and removing these four target functions essentially removes $P_0$ and $P_2$ from PCS $\mathcal{G}_{\text{ex}}$. As per Fig 4, $[P_1 \succeq O P_2]$ and $[P_1 \prec O P_2]$ are also 2. The informativeness of COMPARE($P_1, P_2$) is therefore 2, the minimum of the three cases above, which means at least 2 candidate programs will be removed from $\mathcal{G}_{\text{ex}}$ no matter which the user prefers. Consider PROPOSE($P_2$) as another example, $[\text{NewBest}(P_2)]$ is 2, since $O_2$ prefers $r_{\text{best}}$ over $P_2$ and $P_2$ is the best choice under $O_0$, $O_1$ and $O_4$. Candidate program $P_1$ and $P_2$ will be removed from $\mathcal{G}_{\text{ex}}$ as their target functions either do not satisfy user preference or cannot improve the running best further. Given PCS $\mathcal{G}_{\text{ex}}$, both PROPOSE($P_2$) and COMPARE($P_1, P_2$) share highest informativeness, which is 2. In this case, PROPOSE($P_2$) will be presented to the user.

### 4.3 The Algorithm

Our learning algorithm is almost straightforward: for each iteration, compute the informativeness of every possible query and make the most informative query. The remaining issue is that it is not realistic to keep a PCS that contains all possible candidates, because the number of candidates is usually very large, if not infinite. For example, $\mathcal{L}_{\text{MCF}}$ in Example 3.2 has infinitely many Pareto optimal solutions, ranging in the continuous spectrum from maximizing throughput to minimizing latency. To this end, our voting-guided algorithm maintains a moderate-sized PCS by sampling from the search space, from which queries are generated and selected based on their informativeness.

Algorithm 1 illustrates the voting-guided algorithm. The algorithm takes as input a comparative synthesis problem $\mathcal{C}$ and a target function space $O$, and maintains a PCS $\mathcal{G}$ w.r.t. $\mathcal{C}$ and $O$ and set of preferences $R$, both empty initially. In each iteration, the algorithm computes the informativeness of all possible queries that can be made about the current candidates image($\mathcal{G}$), and picks the highest-informativeness query according to the computation presented in Fig 4 (line 7). After the query is made and the response is received, an update subroutine is invoked to update $\mathcal{G}$ and remove all candidates violating the preference (lines 11–15). Moreover, the algorithm also checks at the beginning of every iteration the size of $\mathcal{G}$; if image($\mathcal{G}$) is below a fixed threshold $\text{Thresh}$, the algorithm attempts to sample from the search space and extend $\mathcal{G}$ using a generate-more subroutine. The algorithm terminates and returns the current running best when $\mathcal{G}$ becomes 0 or NQUERY queries have been made, where NQUERY is the number of queries that the teacher promises to answer (line 17). Table 1 shows an example run of this algorithm.

The subroutines involved in the algorithm are shown as Algorithm 2. The update subroutine is straightforward, taking a new preference pair and shrinking $\mathcal{G}$ accordingly. The generate-more subroutine is tasked to expand $\mathcal{G}$ as much as possible within a time limit. Each time, it tries
The voting-guided learning algorithm.

4.4 Convergence

In the rest of the section, we discuss the convergence of the algorithm. Recall that our algorithm only produces quasi-optimal programs as the ground-truth target function is not present. Therefore, the algorithm should be evaluated on the rate of convergence [Gautschi 1997], i.e., how fast the median quality of solutions (see Def. 3.13) approaches 1 as more queries are made. Our first result is that the algorithm guarantees a logarithmic rate of convergence.

**Theorem 4.6.** Given a comparative synthesis problem \( \mathcal{C} \) and a target function space \( O \) as input, if Algorithm 1 terminates after \( n \) queries, the median quality of the output solutions is at least \( 2^{\frac{1}{n+1}} \).

---

\(^5\)The algorithm involves random sampling and results we prove below are for the median quality of output solutions; the proofs can be easily adapted to get similar results for the mean quality of solutions.
input : Two program metric vectors \(m, n\) and their comparison result \(\text{response}\)
modifies: The current metric vector preferences \(R\) and the current PCS \(\mathcal{G}\)
def \textbf{update} \((m, n, \text{response})\):
    if \(\text{response} = (>)\):
        \(R \leftarrow R \land m > n\)
    elif \(\text{response} = (<)\):
        \(R \leftarrow R \land n > m\)
    else:
        \(R \leftarrow R \land m = n\)
    \(\mathcal{G} \leftarrow \mathcal{G} |_{\{O | O=R\}}\)
return : A parameterized program \(\mathcal{P}\), a metric group \(M\), current metric vector preferences \(R\) and current running best \(r_{\text{best}}\)
modifies: The Pareto candidate set \(\mathcal{G}\)
def \textbf{generate-more} \((\mathcal{P}, M, R, r_{\text{best}}, \mathcal{G})\):
    repeat
        \(c \leftarrow \text{SYNPROG}(\mathcal{P}, M)\); // synthesize an arbitrary (Pareto optimal) program
        \(O \leftarrow \text{SYNOBJ}(R \land M(\mathcal{P}[c]) > M(\mathcal{P}[r_{\text{best}}]))\) // synthesize an objective that prefers the new \(c\) over \(r_{\text{best}}\)
        if \(O \neq ⊥\):
            \(c \leftarrow \text{IMPROVE}(O, \mathcal{P}, M, c)\) // this is optional: try to improve \(\mathcal{P}[c]\)
        else:
            \(O \leftarrow \text{SYNOBJ}(R)\) // synthesize an arbitrary objective satisfying \(R\)
            \(c \leftarrow \text{IMPROVE}(O, \mathcal{P}, M, r_{\text{best}})\) // synthesize a best possible program under \(O\), but at least better than \(r_{\text{best}}\)
            \(\mathcal{G} \leftarrow \mathcal{G} \cup (O, c)\)
    until \(\text{timeout}\);
Algorithm 2: The subroutines involved in the voting-guided learning algorithm.

PROOF. Note that every query will discard at least one candidate program from the PCS, regardless of the query type. In other words, the final output \(c\) must be the optimal among at least \((n + 1)\) randomly selected candidates from the uniform distribution. Therefore, the quality of \(c\) is at least the \((n + 1)\)-th order statistic of the uniform distribution, which is a beta distribution Beta\((n + 1, 1)\), whose median is \(2\pi^{1}\).

The proved lower-bound in the theorem above is tight only when each query only removes one candidate from the PCS \(\mathcal{G}\). Unfortunately, the following lemma shows that in general, this scenario is always realizable:

**Theorem 4.7.** The bound in Theorem 4.6 is tight.

The PCS constructed for the following lemma serves as a witness of the bound tightness:

**Lemma 4.8.** Let \(\mathcal{S} = (\mathcal{P}, C, \Phi)\) be a qualitative synthesis problem with infinitely many solutions and \(O\) be a target function space. For any integer \(n > 0\), there exist a PCS \(\mathcal{G} : O \rightarrow C\) and a parameter \(r_{\text{best}} \in C\) such that: (1) \(|\text{image}(\mathcal{G})| = n\); (2) for any \(c_1, c_2 \in \text{image}(\mathcal{G})\), Info(\text{COMPARE}(c_1, c_2)) = 1; (3) for any \(c \in \text{image}(\mathcal{G})\), Info(\text{PROPOSE}(c)) = 1.

**Proof.** As \(C\) has infinitely many solution, we can pick arbitrary \(n\) solutions, say \(c_1, \ldots, c_n\). For each \(1 \leq i \leq n\), one can construct a total order \(\preceq_i\) such that \(c_n \preceq_i \ldots c_{i+1} \preceq_i c_{i-1} \ldots c_1 \preceq_i c_i\). According to the definition of target function space (Def 4.1), there exists a target function \(O_i\) that
fits \( \leq_i \). Now we can construct \( \mathcal{I} \) such that \( \text{dom}(\mathcal{I}) = \{O_1, \ldots, O_n\} \), and \( \mathcal{I}(O_i) = c_i \) for each \( i \). It can be verified \( \mathcal{I} \) is a Pareto candidate set satisfying the required conditions.

\[ \square \]

### 4.5 Better Convergence Rate with Sortability

We have shown that our voting-guided algorithm guarantees a logarithmic rate of convergence in general, but are there scenarios for which the algorithm guarantees faster convergence? We next show that when the target function space is sortable, our algorithm guarantees a faster, linear convergence.

The idea of the proof bears a similarity to the convergence guarantee for many algorithms in traditional convex optimization [Boyd and Vandenberghe 2004]; but the key difference is that the objective is indeterminate for our algorithm. We first introduce the notion of sortable target function space, which makes sure that the candidates in the PCS can be ordered appropriately such that every target function with corresponding candidate \( c \) always prefers its nearer neighbors to farther neighbors.

**Definition 4.9 (Sortability).** A PCS \( \mathcal{I} \) is sortable if there exists a total order \( \prec \) over \( \text{image}(\mathcal{I}) \) such that for any target functions \( O, P, Q \in \text{dom}(\mathcal{I}) \) with \( \mathcal{I}(O) \prec \mathcal{I}(P) \prec \mathcal{I}(Q) \), the following two conditions hold: \( \mathcal{I}(P) \succ O \mathcal{I}(Q) \), and \( \mathcal{I}(P) \succ O \mathcal{I}(O) \). A target function space \( O \) is sortable with respect to a comparative synthesis problem \( \mathcal{C} \) if any PCS \( \mathcal{I} \) w.r.t. \( \mathcal{C} \) and \( O \) is sortable.

The following lemma shows that if a PCS is sortable, one can make a query to cut at least half of the candidates, no matter what the teacher’s response is.

**Lemma 4.10.** If a Pareto candidate set \( \mathcal{I} \) is finite and sortable, then there exists a query whose quality for \( \mathcal{I} \) as computed in Fig 4 is \( \left\lceil \frac{|\text{image}(\mathcal{I})|}{2} \right\rceil \).

**Proof.** Let \( n = |\text{image}(\mathcal{I})| \) and \( m = \left\lfloor \frac{|\text{image}(\mathcal{I})|}{2} \right\rfloor \). As \( \mathcal{I} \) is sortable, by Def 4.9, there exists a total order \( \mathcal{I}(O_1) \prec \cdots \prec \mathcal{I}(O_n) \). Now we claim that \( \text{Info}(\text{COMPARE}(\mathcal{I}(O_m), \mathcal{I}(O_{m+1}))) = m \). By Def 4.9, for any \( 1 \leq i \leq m, \mathcal{I}(O_m) \succ O \mathcal{I}(O_{m+1}) \), and for any \( m+1 \leq j \leq n, \mathcal{I}(O_m) \prec O \mathcal{I}(O_{m+1}) \). Then according to the query quality estimation described in Fig 4, both \#RemNEQ(\mathcal{I}(O_m), \mathcal{I}(O_{m+1})) \) and \#RemEQ(\mathcal{I}(O_{m+1}), \mathcal{I}(O_m)) \) are at least \( m \). Therefore, \( \text{Info}(\text{COMPARE}(\mathcal{I}(O_m), \mathcal{I}(O_{m+1}))) = m \), which is \( \left\lceil \frac{|\text{image}(\mathcal{I})|}{2} \right\rceil \). \( \square \)

With the lower bound of removed candidates guaranteed by Lemma 4.10, our voting-guided synthesis algorithm guarantees to produce a unique best candidate after a logarithmic amount of queries:

**Theorem 4.11.** Given a comparative synthesis problem \( \mathcal{C} \) with metric group \( M \) and a sortable target function space \( O \) w.r.t. \( M \) as input, if Algorithm 1 terminates after \( n \) queries, the median quality of the output solutions is at least \( 1 - \frac{1}{\Omega(1.5^n)} \).

**Proof.** Note that Algorithm 1 generates candidates for the Pareto candidate set \( \mathcal{I} \) (through the generate-more subroutine) through random sampling. Therefore, if a query cuts the size of current candidate pool (\( \mathcal{I} \) and the running best) by a ratio of \( r \), the search space (those candidates satisfying all preferences in \( R \)) is cut by an equal or higher ratio in that iteration (extra candidates may be discarded by generate-more, before the query). Now as \( \mathcal{I} \) is sortable, by Lemma 4.10, after the highest-informativeness query, the number of candidates remaining in \( \mathcal{I} \) is at most \( \left\lceil \frac{|\text{image}(\mathcal{I})|}{2} \right\rceil \).
In other words, the query reduces the size of \( \mathcal{G} \) by a ratio of at least \( \frac{2}{3} \) (when \( |\text{image}(\mathcal{G})| = 2 \), the total number of candidates including the running best, reduces from 3 to 2), except for the last query. Therefore, the output is the best among \( O(1.5^n) \) randomly selected candidates, which is Beta\((1.5^n, 1)\)-distributed. Hence by Def 3.13, the median of the quality of the output is \( \frac{1}{2(1+m)} \), which is asymptotically equivalent to \( 1 - \frac{1}{\Omega(1.5^n)} \). □

A common class of problems with sortability. We now show a common class of comparative synthesis problems for which the sortability can be obtained: when the comparative synthesis problem is convex and the target function space is concave with two metrics. The class of problems is commonly seen in practice, covering half of optimization scenarios studied in §5. Intuitively, it captures the assumption that there are two competing metrics (e.g., throughput and latency) such that for each metric continued improvement leads to diminishing marginal utility (e.g., increasing throughput from 1Gbps to 2Gbps is more favorable than increasing throughput from 2Gbps to 3Gbps).

We first formally define the convexity of the comparative synthesis problem and the concavity of the target function space, and build the main convergence result by proving the sortability.

Definition 4.12 (Convexity of comparative synthesis problem). A comparative synthesis problem \( \mathcal{C} \) with metric group \( M \) is convex if for any two solutions \( c_1, c_2 \) to \( \mathcal{C}^{\text{qual}} \) and any \( \alpha \in [0, 1] \), a solution \( c_3 \) to \( \mathcal{C}^{\text{qual}} \) can be effectively found such that \( M(c_3) \geq \alpha \cdot M(c_1) + (1 - \alpha) \cdot M(c_2) \).

Definition 4.13 (Concavity of target function space). Let \( O \) be a target function space w.r.t a \( d \)-dimensional metric group \( M \). \( O \) is concave if for any \( O \in O \), for any \( v_1, v_2 \in \mathbb{R}^d \) and any \( \alpha \in [0, 1] \), \( O(\alpha \cdot v_1 + (1 - \alpha) \cdot v_2) \geq \max(O(v_1), O(v_2)) \).

Example 4.14. Our running example meet both the convexity and concavity conditions.

First, the comparative synthesis problem \( \mathcal{C}_{\text{MCF}} \) in Example 3.14 is convex. As shown in Fig 2, both throughput and latency are weighted sum of allocations to every link. Therefore given any two solutions \( c_1 \) and \( c_2 \), their convex combination is still feasible, and the metric vector is also the corresponding convex combination of \( M(c_1) \) and \( M(c_2) \).

Second, the target function space \( O_{\text{MCF}} \) in Example 4.3 is concave. It is not hard to verify that both the weights of throughput and latency decrease when their values are good enough and exceed a threshold.

Theorem 4.15. Let \( \mathcal{C} \) be a convex comparative synthesis problem with a 2-dimensional metric group \( M \) and \( O \) be a concave target function space w.r.t. \( M \), then \( O \) is sortable w.r.t. \( \mathcal{C}^{\text{qual}} \).

Proof. We shall show the sortability of any Pareto candidate set \( \mathcal{G} \) w.r.t. \( \mathcal{C}^{\text{qual}} \) and \( O \). We claim that the lexicographic order \( \langle_{\text{lex}} \rangle \) over \( \mathbb{R}^2 \) (i.e., \( (a_1, a_2) \prec_{\text{lex}} (b_1, b_2) \) if and only if \( a_1 < b_1 \) or \( a_1 = b_1 \land a_2 < b_2 \)) witnesses the sortability. Per Def 4.9, for any target functions \( O, P, Q \in \text{dom}(\mathcal{G}) \) such that \( \mathcal{G}(O) \prec_{\text{lex}} \mathcal{G}(P) \prec_{\text{lex}} \mathcal{G}(Q) \), we shall show \( \mathcal{G}(P) >_Q \mathcal{G}(O) \) below. It can be similarly proved that \( \mathcal{G}(P) >_Q \mathcal{G}(O) \).

Let \( M(\mathcal{G}(O)) = (o_1, o_2), M(\mathcal{G}(P)) = (p_1, p_2), \) and \( M(\mathcal{G}(Q)) = (q_1, q_2) \). Note that by Def 4.4, each of \( \mathcal{G}(O) \), \( \mathcal{G}(P) \) and \( \mathcal{G}(Q) \) is optimal under a distinct target function, therefore \( M(\mathcal{G}(O)) \), \( M(\mathcal{G}(P)) \), and \( M(\mathcal{G}(Q)) \) are pairwise incomparable, i.e., \( \{o_1, p_1, q_1\} \) and \( \{o_2, p_2, q_2\} \) are all distinct values. Due to the lexicographic order \( \langle_{\text{lex}} \rangle \), we have \( o_1 < p_1 < q_1 \) and \( o_2 > p_2 > q_2 \). Now by Def 4.12, one can
Table 5. Summary of topologies.

| Topology | #nodes | #links | B4 | CWIX | BTNorthAmerica | Tinet | Deltacom | Ion |
|----------|--------|--------|----|------|----------------|-------|----------|-----|
| Abilene  | 11     | 14     | 12 | 21   | 36             | 48    | 103      | 114 |

Table 6. Summary of optimization scenarios.

| Scenario | Metric group | Target function space | Sortable? |
|----------|--------------|-----------------------|-----------|
| MCF      | (throughput, -latency) | $\text{throughput} \ast ?? - \max(\text{throughput} - ??, 0) \ast ??$ -latency $\ast ?? - \max(\text{latency} - ??, 0) \ast ??$ | Yes |
| BW       | ($a_{g_k}$; average allocation to the flows in the k-th class) | $\sum_{1 \leq k \leq K} w_k \log(a_{g_k})$ ($w_k > 0$) | No |
| NF       | ($z_n_i$, $z_f_i$; guaranteed fraction of the traffic demand of group i under normal conditions and failures respectively) | $\sum_i w_{n_i} \ast z_{n_i} + w_{f_i} \ast z_{f_i}$ ($w_{n_i}, w_{f_i} > 0$) | No |
| OSPF     | (−latency, −utilization) | utilization | utilization >?? utilization >?? | Yes |

effectively find a solution $c$ such that

$$M(c) \geq \left(\frac{q_1 - p_1 \cdot o_2 + (p_1 - o_1) \cdot q_2}{q_1 - o_1}, \frac{q_1 - p_1 \cdot o_2 + (p_1 - o_1) \cdot q_2}{q_1 - o_1}\right) = \left(p_1, \frac{q_1 - p_1 \cdot o_2 + (p_1 - o_1) \cdot q_2}{q_1 - o_1}\right)$$

Then by Def 4.4, $\mathcal{G}(P)$ is at least as good as $c$ and $p_2 \geq \frac{(q_1 - p_1) \cdot o_2 + (p_1 - o_1) \cdot q_2}{q_1 - o_1}$. Finally, by Def 4.13, we have $O((p_1, p_2)) \geq O((p_1, \frac{(q_1 - p_1) \cdot o_2 + (p_1 - o_1) \cdot q_2}{q_1 - o_1})) \geq O((q_1, q_2))$. In other words, $\mathcal{G}(P) >_O \mathcal{G}(Q)$. \hfill $\square$

5 EVALUATION

We have prototyped the comparative synthesis framework and the voting-guided learning algorithm as Net10Q — an interactive system that produces near-optimal network design by asking 10 questions to the user — through which we evaluate the effectiveness and efficiency of our approach. We selected four real-world network design scenarios and conducted experiments with both oracles and human users. Our evaluations were conducted on seven real-world, large-scale internet backbone topologies obtained from [Jain et al. 2013; Knight et al. 2011] (sizes summarized in Table 5). Note that the size of our largest topologies, namely Deltacom and Ion, are already beyond the ones typically considered in the traffic engineering community.

5.1 Network Optimization Problems

We summarize the four optimization scenarios in Table 6, including their metric groups, target function spaces and sortability. These scenarios are all well known for the networking community. The metric groups and target function spaces are all from the literature. It is also noteworthy that existing approaches require network architects to stringently follow the scenario-specific target function spaces and hand tune target functions carefully. In contrast, our approach does not require such hand tuning. While the learner uses the templates shown in Table 6, we evaluate our approach in settings with perfect teachers which answers queries based on a ground truth target function, as well as teachers that are imperfect. Finally, we conduct a user study where the teacher (user) is free to choose an arbitrary policy. We present some details about the scenarios below.
Balancing throughput and latency (MCF). This is our running example based on [Hong et al. 2013] described throughout the paper. This bandwidth allocation problem focuses on a single traffic class and considers balancing the throughput and latency in the network.

Utility maximization with multiple traffic classes (BW). A well-studied optimization problem is maximizing utility when allocating bandwidth to traffic of different classes [Ghosh et al. 2013; Kumar et al. 2015; Srikant 2004]. Many applications such as file transfer have concave utility functions which indicate that as more traffic is received, the marginal utility in obtaining a higher allocation is smaller. A common concave utility function which is widely used is a logarithmic utility function, where a flow that receives a bandwidth allocation of $x$ gets a utility of $\log x$. Consider \( N \) flows, and \( K \) classes. Each flow belongs to one of the classes with \( F^k \) denoting the set of flows belonging to class \( k \). The weight of class \( k \) is denoted by \( w_k \) and is a knob manually tuned today to control the priority of the class, which we treat as an unknown in our framework.

Performance with and without failures (NF). Resilient routing mechanisms guarantee the network does not experience congestion on failures [Bogle et al. 2019; Chang et al. 2017; Jiang et al. 2020; Liu et al. 2014; Wang et al. 2010] by solving optimization problems that conservatively allocate bandwidth to flows while planning for a desired set of failures. We consider the model used in [Liu et al. 2014] to determine how to allocating bandwidth to flows while being robust to single link failure scenarios. We consider an objective with (unknown) knobs \( w_{ni} \) and \( w_{rf} \) that trade off performance under normal conditions and failures tuned differently for each group of flows \( i \).

Balancing latency and link utilization (OSPF). Open Shortest Path First (OSPF) is a widely used link-state routing protocol for intra-domain internet and the traffic flows are routed on shortest paths [Fortz and Thorup 2000]. A variant of OSPF routing protocol assigns a weight to each link in the network topology and traffic is sent on paths with the shortest weight and equally split if multiple shortest paths with same weight exist. By configuring the link weights, network architect can tune the traffic routes to meet network demands and optimize the network on different metrics [Fortz and Thorup 2000]. We consider a version of the OSPF problem where link weights must be tuned to ensure link utilizations are small while still ensuring low latency paths [Gvozdiev et al. 2018]. We generalize [Gvozdiev et al. 2018] to allow for more expressive policies that mirror typical network operator intent through conditionals. Intuitively, when utilization is higher than a threshold, it becomes the primary metric to optimize, and when lower than a threshold, minimizing latency is the primary goal. In between the thresholds, both latency and utilization are important, and can be scaled in a manner chosen by the network architect. We treat the thresholds and the scale factors as unknowns in the objective. Policies with such conditionals are not as naturally captured in network optimization models.

5.2 Implementation

What users compare. Net10Q presents to the user only aggregate metrics for the two designs being compared. For each design scenario, the presented metrics are the metric groups indicated in Table 6. For example, in the MCF scenario, users are asked to compare tuples of the form \((\text{thrpt} = t_1, \text{ltncy} = l_1)\) and \((\text{thrpt} = t_2, \text{ltncy} = l_2)\) where each tuple represents the sum of throughput, and the sum of latency across flows associated with the designs being compared. Table 1 shows several concrete examples of queries that are presented by Net10Q for the MCF scenario.

Pre-computed candidates. Note that in Net10Q, once the scenario and the topology are fixed, we can pre-compute a large pool of objective-program pairs, from which the PCS is generated. For each scenario-topology combination, we used the templates shown in Table 6 to generate a pool of random target functions. Then for all scenarios except for OSPF, we generate their corresponding optimal allocations using Gurobi [Gurobi Optimization 2020], a state-of-the-art solver for linear
and mixed-integer programming problems. For OSPF, as we are not aware of any existing tools that can symbolically solve the optimization problem, we used traditional synthesis approaches (cf. Fig 2) to generate numerous feasible link weight assignments. The pre-computed target functions and allocations are paired to form a large PCS serving as the candidate pool. We provide details of pool creation in 5.3.

Inconsistency handling. When the teacher is an imperfect oracle or a human user, inconsistent answers may potentially result in the algorithm unable to determine objectives that meet all user preferences. To ensure NET10Q robust to an imperfect oracle, inspired by the ensemble methods [Dietterich 2000], we implemented NET10Q as a multi-threaded application where a primary thread accepts all inputs and the backup threads run the same algorithm but randomly discard some user inputs. In case no objective could satisfy all user preferences, a backup thread with the largest satisfiable subset of user inputs would take over.

5.3 Oracle-Based Evaluation

We used NET10Q to solve all scenario-topology combinations described above, through interaction with (both perfect and imperfect) oracles who answer queries based on their internal objectives. As a first-of-its-kind system, NET10Q does not have any similar systems to compare with. Therefore, we developed a variant of NET10Q which adopts a simple but aggressive strategy: repeatedly proposing optimal candidates generated from randomly picked target functions. We call this baseline algorithm NET10Q-NoPrune, as the teacher’s preference is not used to prune extra candidates from the search space. As a solution’s real quality (per Def 3.13) is not practically computable, we approximate its quality using its rank in our pre-computed candidate pool. Moreover, as NET10Q involves random sampling, we ran each synthesis task 301 times and reported the median of the (approximated) solution quality achieved after every query.

Evaluation on perfect teacher. We built an oracle to play the role of a perfect teacher who answers all queries correctly based on a ground truth objective. For each scenario, we as experts manually crafted a target function that fits the template and reflects practical domain knowledge.

We presents the performance of NET10Q and NET10Q-NoPrune on solving four network optimization problems (cf. Table 6) on seven different topologies (cf. Table 5). Our key observation is that NET10Q performed constantly better than NET10Q-NoPrune in every scenario-topology combination. In the interest of space, we collected the quality of solutions achieved over all seven topologies for each optimization scenario and presented the median (shown as dots) and the range from max to min (shown as bars). As Fig 5 shows, our voting-guided algorithm is very effective. NET10Q always only needs 5 or fewer queries to obtain a solution quality achieved by NET10Q-NoPrune in 10 queries. We note that although the all-topology range for NET10Q sometimes overlaps with the corresponding range for NET10Q-NoPrune (primarily for the NF scenario), NET10Q still outperformed NET10Q-NoPrune for every topology. Further, in all the cases where we could compute the optimal under the ground truth objective, we confirmed that programs recommended by NET10Q achieved at least 99% of the optimal.

Evaluation on imperfect teacher. We also adapt the oracle to simulate imperfect teachers whose responses are potentially inconsistent, based on an error model described below. When an allocation candidate is presented, the imperfect oracle assigns a random reward that is sampled from a normal distribution, whose expectation is the true reward under corresponding ground truth objective.

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6The quality is computed among Pareto optimal solutions only. In other words, the solution quality as per Definition 3.13 should be higher than what we report here.

7The ground truth does not have to match the template; see §5.4 for human teacher who is oblivious to the template.
Comparative Synthesis: Learning Near-Optimal Network Designs by Query

Fig. 5. Comparing Net10Q and Net10Q-NoPrune with perfect oracle (across all seven topologies). Curves to the left are better. (More detailed, per-topology results for NF is available in Appendix ??)

Fig. 6. Performance of Net10Q with imperfect oracle (BW on CWIX).
The standard deviation is $p\%$ of the distance between average reward and optimal reward under ground truth objective.

Fig 6 shows our experimental results with imperfect teacher on BW with the CWIX topology. We also see similar trend in results of other scenarios. Fig 6a compares Net10Q with Net10Q-NoPrune under the inconsistency level $p = 10$. Net10Q continues to outperform Net10Q-NoPrune. Fig 6b presents the sensitivity of Net10Q on the inconsistency level ($p = 0, 5, 10, 20$). Although the solution quality degrades with higher inconsistency $p$, Net10Q achieves relatively high solution quality even when $p$ is as high as 20. The results show that Net10Q tends to be able to handle moderate feedback inconsistency from an imperfect teacher, although investigating ways to achieve even higher robustness is an interesting area for future work.

Runtime and Scaling. We first discuss the online query time experienced by users. For every synthesis task mentioned above, and across all topologies, the average running time spent by Net10Q for each interactive user query is less than 0.15 seconds. The approach scales well with topology size since a pool of objective-program pairs is created offline. When creating a pool, the solving time for a single optimization problem is under a second for most topologies on all scenarios on a 2.6 GHz 6-Core Intel Core i7 laptop with 16 GB memory, and we used a pool size of 1000 objectives. The only exception was the NF on the two largest topologies, Deltacom and Ion, which took 11.8 and 15.5 seconds respectively, and we used a smaller pool size in these cases to limit the pool generation time. Note that the pool creation occurs offline. Further, it involves solving multiple distinct optimization problems, and is trivially parallelizable.

To examine sensitivity to pool size, we first generated 5000 objective-program pairs and then randomly sampled a given number of objective-program pairs to form a candidate pool. Evaluating on candidate pools of size ranging from 10 to 5000, we found that pools with 300 objective-program pairs are sufficient for Net10Q to achieve over 99% optimal after 10 iterations.

In the interest of space, we defer additional experimental results to supplemental material.

5.4 Pilot User Study

We next report on a small-scale study involving 17 users. The primary goal of the user study is to evaluate Net10Q when the real objective is arbitrarily chosen by the user, and even the actual shape is unknown to Net10Q. This is in contrast to the oracle experiments where the ground truth objectives are drawn from a template (with only the parameters unknown to Net10Q). Further, like with imperfect oracles, users may not always correctly express relative preferences.

The user study was conducted online using an IRB-approved protocol. Participants were recruited with a minimal qualifying requirement being they have taken a university course in computer
networking. Figs 7a and 7b show the background of users. 88% of them are computer networking researchers or practitioners. 53% of users have more than 2 years of experience managing networks.

Our user study used an earlier version of Algorithm 1 implemented as an online web application. Specifically, the PCSes were generated on the fly, rather than pre-generated. To ensure responsiveness, the deployed algorithm set the threshold $T/hr/thrs$ = 2. We note that the cloud application for our user study was developed and tested over multiple months, and in parallel to refinements we developed to the algorithm. We were conservative in deploying the latest version given the need for a robust user-facing system, and to ensure all participants saw the same version of the algorithm.

The study focused on the BW scenario (with four classes of traffic) and the Abilene topology. The user was free to choose any policy on how bandwidth allocations were to be made, and answer queries based on their policy. In each iteration, the user was asked to choose between two different bandwidth allocations generated by Net10Q. The user could either pick which allocation was better, or indicate it was too hard to call if the decisions were close. The study terminated after the user answered 10 questions, or when the user indicated she was ready to terminate the study.

In the post-study questionnaire, users were asked to characterize their policy by choosing how important it was to achieve each of three criteria below: (i) **Balance**, indicating allocation across classes is balanced; (ii) **Priority**, indicating how important it is to achieve a solution with more allocation to a higher priority class if a lower priority class does poorly; and (iii) **No-Starvation**, indicating how important it is to ensure lower priority classes get at least some allocation. Fig 7c presents a breakdown of user policies. 70.6% of users indicated **Balance** were somewhat or very important. 82.3% of users indicated **Priority** were somewhat or very important, while 94.1% of users indicated **No-Starvation** was somewhat or very important. The results were consistent with the qualitative description each user provided regarding his/her policy. Overall, almost all users were seeking to achieve **Balance** and **Priority** avoiding the extremes (starvation) - however, they differed considerably in terms of where they lay in the spectrum based on their qualitative comments.

**Results.** Fig 8a summarizes how well the recommended allocations generated by Net10Q met user policy goals. 82% of the users indicated that the final recommendation is consistent, or somewhat consistent with their policies. The remaining 18% of the users took the study before we explicitly added the objective question to ask users to rate how well the recommended policy met their goals. However, the qualitative feedback provided by these users indicated Net10Q produced allocations consistent with user goals. For instance, one expert user said: "The study was well done in my opinion. It put the engineer/architect in a position to make a qualified decision to try and chose the most reasonable outcome."

Fig 8b shows that 94% of users indicated the response time with Net10Q was usually acceptable, while 6% indicated the time was sometimes acceptable. Fig 9 shows a cumulative distribution across
users of the average \textsc{Net10Q} time (i.e., the average time taken by \textsc{Net10Q} between receiving the user’s choice and presenting the next set of allocations). For comparison, the figure also shows a distribution of the average user think time (i.e., the average time taken by a user between when \textsc{Net10Q} presents the options and when the user submits her/his choice). The time taken by \textsc{Net10Q} was hardly a second, and much smaller than the user think time which varied from 8 to 12 seconds, indicating \textsc{Net10Q} can be used interactively.

Across all users, \textsc{Net10Q} was always able to find a satisfiable objective that met all of the user’s preferences (it never needed to invoke the fallback approach (§5.2) of only considering a subset of user preferences). This indicates users express their preferences in a relatively consistent fashion in practice. Inconsistent responses may still allow for satisfiable objectives, however we are unable to characterize this in the absence of the exact ground truth objective.

Overall, the results show the promise of a comparative synthesis approach even when dealing with complex user chosen objectives of unknown shape. We believe there is potential for further improvements with all the optimizations in Algorithm 1, and other future enhancements.

6 RELATED WORK

Network verification/synthesis. As we discussed in §2, the naïve approach to comparative synthesis proposed in our previous work [Wang et al. 2019] is preliminary and may involve prohibitively many queries. This inefficiency motivates our generalized comparative synthesis framework and the unified search space. We observed that a precise target function is not required to synthesize a satisfying program. Based on this, we generalize and formalize the framework, design the first synthesis algorithm that combines objective learning and program search together with the explicit goal of minimizing queries, present formal convergence results, and conduct extensive evaluations including a user study. In addition, our previous work [Wang et al. 2019] do not consider the feasibility of the network designs which presented to users. In other words, their system does not take the network hard constraints into consideration and the generated network design may not be realizable in the network. Given this, the synthesized target function may not be the best choice on a specific network.

Much recent work applies program languages techniques to networking. Several works focus on synthesizing forwarding tables or router configurations based on predefined rules [El-Hassany et al. 2017, 2018; Rzyzhk et al. 2017; Saha et al. 2015; Soulé et al. 2014; Subramanian et al. 2017; Yuan et al. 2015], or synthesizing provably-correct network updates [McClurg et al. 2016, 2015]. Much research focuses on verifying network configurations and dataplanes [Beckett et al. 2019; Steffen et al. 2020; Subramanian et al. 2020], and does not consider synthesis. Recent works mine network specification from configurations [Birkner et al. 2020], generate code for programmable switches from program sketches [Gao et al. 2019; Sivaraman et al. 2016], or focus on generating network classification programs from raw network traces [Shi et al. 2021]. In contrast to these works, we focus on synthesizing network designs to meet quantitative objectives, with the objectives themselves not fully specified.

Optimal synthesis. There is a rich literature on synthesizing optimal programs with respect to a fixed or user-provided quantitative objective. Some of these techniques aims to solve optimal syntax-guided synthesis problems by minimizing given cost functions [Bornholt et al. 2016; Hu and D’Antoni 2018]. Other approaches either generate optimal parameter holes in a program through probabilistic reasoning [Chaudhuri et al. 2014] or solve SMT-based optimization problems [Li et al. 2014], under specific target functions. In example-based synthesis, the examples as a specification can be insufficient or incompatible. Hence quantitative objectives can be used to determine to which extent a program satisfies the specification or whether some extra properties hold. Gulwani et al. [2019] and Drosos et al. [2020] defined the problem of quantitative PBE (qPBE) for synthesizing
string-manipulating programs that satisfy input-output examples as well as minimizing a given quantitative cost function. Our work is different from all optimal synthesis work mentioned above as in our setting, the objective is unknown and automatically learnt/approximated from queries.

**Human interaction.** Many novel human interaction techniques have been developed for synthesizing string-manipulating programs. A line of work focuses on proposing user interaction models to help resolve ambiguity in the examples [Mayer et al. 2015] and/or accelerate program synthesis [Drachsler-Cohen et al. 2017; Peleg et al. 2018]. Using interactive approaches to solve multiobjective optimization problems has been studied by the optimization community for decades (as surveyed by Miettinen et al. [2008]). Morpheus [Wang et al. 2009] is a routing control platform that allows users to flexibly specify their policy preferences. Morpheus requires pairwise comparisons on relative weights of metrics as input, which can be viewed as a special form of target functions. Our novelty on the interaction method is to proactively ask comparative queries on concrete network designs, with the aim of minimizing the number of queries and maximizing the desirability of the found solution. The comparison of concrete candidates is easier than asking the user to provide rank scale, marginal rates of substitution or aspiration level, which is done by most existing approaches. The objectives we target to learn for network design also involve guard conditions, which is beyond what most existing methods can handle.

**Oracle-guided synthesis.** The learner-teacher interaction paradigm we use in the paper has been studied in the context of programming-by-example (PBE), aiming at minimizing the sequence of queries. Jha et al. [2010] presented an oracle-based learning approach to automatic synthesis of bit-manipulating programs and malware deobfuscation over a given set of components. Their synthesizer generates inputs that distinguishes between candidate programs and then queries to the oracle for corresponding outputs. Ji et al. [2020] followed up and studied how to minimize the sequence of queries. This line of work allows input-output queries only (“what is the output for this input?”) to distinguish different programs. If two programs are distinguishable, they consider them equivalent or a ranking function is given. In invariant synthesis, Garg et al. [2014] followed this paradigm and synthesized inductive invariants by checking hypotheses (equivalent to Propose queries in our setting) with the teacher. Jha and Seshia [2017] proposed a theoretical framework, called oracle-guided inductive synthesis (OGIS) for inductive synthesis. The framework OGIS captures a class of synthesis techniques that operate via a set of queries to an oracle. Our comparative synthesis can be viewed as a new instantiation of the OGIS framework.

**Active learning.** Our algorithm for comparative synthesis has parallels to active learning [Angluin 2004; Settles 2012] in the machine learning community, which interactively queries a user to label data in settings where labeling is expensive. Query-by-committee (QBC) [Seung et al. 1992] is a general query strategy framework that chooses the most informative query based on the disagreements among a committee of models. How to construct the committee space and how to measure the disagreements among committee members are questions must be answered when instantiating the QBC framework. In contrast, we interactively query users to learn objectives using a carefully designed search space PCS and propose ways to estimate query informativeness specific to our setting.

7 **COMPARATIVE SYNTHESIS BEYOND NETWORK DESIGN**

Our experiments show that comparative synthesis is an effective approach for the network domain. We are encouraged by the results and believe that comparative synthesis can be applied to more quantitative synthesis domains. We discuss two research directions below.

**Syntax-based quantitative synthesis.** A class of quantitative synthesis work [Bornholt et al. 2016; Hu and D’Antoni 2018] determines the “quality” or preference of a program purely based on its syntactic representation, e.g., program size or number of operators. Comparative synthesis may
help this class of work learn user’s preference by asking comparative queries in lieu of weighted tree grammars, sketch ordering or cost functions.

One challenge for this application is that domain-specific target function templates are hardly available. How to ensure good search quality with a general target function template is an interesting future direction. Another interesting observation is that evaluating syntactic metrics usually does not require a full program — one can calculate the expression depth or program length even with a partial program. This feature allows us to save efforts for generating full program candidate and also reduce the burden to the user for comparing two concrete programs with all the details.

**Example-based quantitative synthesis.** Programming-by-Example (PBE) has been the common technique behind many synthesizers as input-output examples are a natural means to specify the behavior of the desired program and can be easily provided by non-programming users in many domains. However, as examples are inherently underspecification, PBE usually requires a carefully crafted ranking function provided by experts to choose programs most likely matching user’s intent [Gulwani et al. 2019]. Comparative synthesis may be applied to interact with users, learn custom ranking functions for them and suggest programs they tend to desire.

The major challenge here is that average users of a PBE system (e.g., FlashFill) typically are not capable of comprehending synthesized programs, not to mention comparing a pair of them and picking a preferred one. Supporting more types of queries will make this approach more flexible and practical. For example, the synthesizer may take the pair of programs to be compared, run a solver to obtain a distinguishing input, i.e., an single input for which the two programs disagree on what the output is, present it to the user and ask which output is correct (or just explicitly ask for the expected output).

Responses from end users are also more error-prone than those from domain experts such as network architects. While we show simple strategies are already able to handle moderately noisy oracles/users for network design, more principled approaches for robustness may be required for other domains. Identifying the minimal unsatisfiable core could be helpful in finding inconsistent inputs. Rather than presenting the conflicts to the user for a resolution, one could also explore ways to achieve higher robustness which involve less user interactions.

## 8 CONCLUSIONS

In this paper, we have presented comparative synthesis for learning near-optimal programs with indeterminate objectives, and applied it to network design. First, we have developed a formal framework for comparative synthesis through queries with users. Second, we have developed the first learning algorithm for our framework that combines program search and objective learning, and seeks to achieve high solution quality with relatively few queries. We proved that the algorithm guarantees the median quality of solutions converges logarithmically to the optimal, or even linearly when the target function space is sortable (a property satisfied by two of our case studies). Third, we have developed Net10Q, a system implementing our approach. Experiments show that Net10Q only makes half or less queries than the baseline system to achieve the same solution quality, and robustly produces high-quality solutions with inconsistent teachers. A pilot user study with network practitioners and researchers shows Net10Q is effective in finding allocations that meet diverse user policy goals in an interactive fashion. Overall, the results show the promise of our framework, which we believe can help in domains beyond networking in the future.

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