Study of Resonances in $1 \times 25$ kV AC Traction Systems

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CONTENTS

1. Introduction
2. $1 \times 25$ kV Supply of AC Traction Systems
3. Traction System Harmonic Analysis
4. Analytical Characterization of the Traction System Harmonic Response
5. Application of Harmonic Resonance Location
6. Conclusions
Funding
References

Abstract—AC traction systems are $1 \times 25$ or $2 \times 25$ kV 50-Hz single-phase, non-linear, time-varying loads that can cause power quality problems. One of the main concerns about these systems is voltage distortion, because adjustable-speed drives for trains may inject harmonic currents of frequencies below 2 kHz. Since the presence of parallel resonances in the contact feeder section of the traction circuit worsens the scenario, traction system resonance phenomena should be analyzed to prevent problems. Several works addressed these phenomena, but they only drew weak numerical conclusions based on the frequency scan method. This article studies $1 \times 25$ kV traction system resonances at pantograph terminals and provides more effective analytical expressions to locate them and determine the impact of traction system parameters on them. These expressions are validated from several traction systems in the literature.

1. INTRODUCTION

Although $2 \times 25$ kV traction systems are common in high-speed railways because of their ability to meet high power requirements with lower currents at the contact feeder section, $1 \times 25$ kV traction systems are still operating in traditional and high-speed railways [1–6]. The main concerns about these systems are related to power quality because traction loads are single-phase, non-linear, time-varying loads closely connected to the utility power supply system [7–10]. In particular, special attention must be paid to the presence of harmonics [1, 9–12]. Many distorting sources in traction systems inject harmonics into transformer substations and traction lines, with the main being adjustable-speed drives for trains [1, 7]. Although pulse-width-modulated (PWM) drives with nearly unity power factor and reduced harmonic current content are the most widely used in modern locomotives, the harmonic problem remains important because of the wide range of frequencies of injected currents (especially the 1–2 kHz range close to the converter switching frequency) [2, 7, 10, 11]. The problem is worse in trains equipped with phase-controlled thyristor converters because they consume currents with a low displacement power factor and high frequency content (in particular, frequencies
below 450 Hz) [4, 11–13]. Moreover, the use of converters with advanced control characteristics and new signaling systems requires a higher power quality level to avoid electromagnetic interference in power and signaling devices [1, 7], which can strongly affect system safety and reliability [11].

The harmonic problem is worsened with the presence of resonances in the contact feeder section of the traction circuit because they can increase harmonic voltage distortion. Thus, many works have addressed the resonance problem at traction system pantograph terminals experimentally and numerically [1–3, 7, 10, 14–16]. In [2, 3, 10, 15], resonance is analyzed from measurements, and in [1, 2, 7, 10, 14–16], it is studied from simulations considering line distributed models to determine the influence of the position of the train on the track. These works only report examples about traction system resonance frequencies obtained by the frequency scan method but do not analyze resonances in depth or provide analytical expressions for their location. These studies concluded that resonances are in the range of 1 to 20 kHz, and some more attention was paid to the lowest resonance (close to the 1–2 kHz switching frequency of PWM drive converters) by briefly analyzing the influence of system parameters on it. From the frequency scan study, most works deduced that the lowest resonance mainly depends on the substation reactance and the capacitance between the contact wire and the ground rather than on the train position on the track. However, none points out or analyzes the limitation of this assertion [2, 4, 7, 14, 15]. Thus, the main challenge is to provide analytical expressions that are more accurate and useful than frequency scan plots in understanding and predicting resonance phenomena. To reduce harmonic currents injected into networks and mitigate resonances, active and hybrid systems can be used in locomotives and RLC and broadband passive filters in traction systems. These filters are often linked with the fundamental reactive power compensation goal. Because several different types of locomotives run simultaneously on the same traction section, placing passive filters at the 25 kV side of the traction substation is usually the most effective and economical mitigation technique [3, 8, 11, 13, 15–17]. Most previous works study unfiltered traction systems to determine the extent of the harmonic problem and subsequently analyze how filters alleviate the resonance problem and plan their connection.

From the framework in [18], the present article provides analytical expressions for locating all the resonance frequencies observed from the traction load at pantograph terminals in unfiltered 1 × 25 kV railway power systems. These expressions make it possible to investigate 1 × 25 kV railway power system resonances in more detail than frequency scan plots. Moreover, the influence of traction system parameters and train position on resonance is determined. The expressions are validated with several traction systems in the literature where resonances are numerically and experimentally located.

2. 1 × 25 KV SUPPLY OF AC TRACTION SYSTEMS

Many electrified traction systems operate on 1 × 25 kV and 50 Hz. In these systems, traction loads are supplied by a single-phase overhead contact line distributed in different sections along the line. These sections, usually of 30- or 40-km lengths (D) [3, 5, 12, 14–16], are connected through a transformer to the main power network (Figure 1(a)). For steady-state studies, railway traction systems are modeled with their equivalent circuit, which is formed by

- the power system: characterized from its open-circuit voltage \( U_O \) and short-circuit power \( S_S \) at the point of coupling;
3. TRACTION SYSTEM HARMONIC ANALYSIS

The harmonic behavior of the passive set “observed” from the traction load is studied to locate resonances. As can be seen in Figure 1(b), this set is formed by the impedances (or admittances) of the power system, the railroad substation transformer, the passive filters, and the contact feeder section [3, 13, 15–17]:

- power system admittance: the impedance of the power supply and the short-circuit impedance of the three-phase transformer feeding the traction system:
  \[
  Y_{Sk} = Z_{Sk}^{-1} \approx \frac{1}{j k X_S} \left( j k \frac{U_o^2}{S_S} \frac{U_{N2}}{U_{N1}} \right)^2 \]

- railroad substation transformer admittance: represents the short-circuit impedance of the transformer feeding the contact feeder section:
  \[
  Y_{TRk} = Z_{TRk}^{-1} \approx \frac{1}{j k X_{TR}} \left( j k \frac{U_o^2}{S_N} \right)^{-1} \]

- RLC and broadband filter admittances: represent the per-unit length longitudinal and transversal impedances of the catenary lines:
  \[
  Y_{Lk} = Z_{Lk}^{-1} = \frac{1}{R_L + j k X_L}, \quad Y_{Ck} = Z_{Ck}^{-1} = j \frac{k}{X_C} \]

The resistances of the power system and substation transformer impedances (i.e., \( R_S \) and \( R_{TR} \), respectively) are
Neglected in this study because it is well-known that they damp the system harmonic response but do not affect resonance location significantly [18].

By considering point \( N \) in Figure 1(b) as the reference bus, the harmonic behavior of the system can be characterized by the admittance matrix

\[
\begin{bmatrix}
Y_{1k} \\
\vdots \\
Y_{5k}
\end{bmatrix} = \begin{bmatrix}
Z_{11k} & \cdots & Z_{51k} \\
\vdots & \ddots & \vdots \\
Z_{15k} & \cdots & Z_{55k}
\end{bmatrix} \begin{bmatrix}
0 \\
\vdots \\
L_k
\end{bmatrix}
\]

\[
= \begin{bmatrix}
Y_{2k} & 0 & 0 & -Y_{24k} & Y_{25k} \\
0 & Y_{3k} & 0 & -Y_{34k} & 0 \\
0 & 0 & Y_{4k} & -Y_{45k} & 0 \\
0 & -Y_{5k} & 0 & -Y_{54k}/d & Y_{55k}
\end{bmatrix}^{-1} \begin{bmatrix}
0 \\
\vdots \\
0
\end{bmatrix}
\]

where

\[
Y_{2k} = Y_{sk} + dY_{ck} + \frac{(D - d)Y_{ck}}{2} + Y_{Fk} + Y_{Fhk},
\]

\[
Y_{3k} = Y_{sk} + Y_{TRk},
\]

\[
Y_{4k} = Y_{TRk} + \frac{dY_{ck}}{2} + \frac{Y_{Lk}}{d} + Y_{Fk} + Y_{Fhk}
\]

\[
Y_{5k} = \frac{dY_{ck}}{2} + \frac{Y_{Lk}}{d} + \frac{(D - d)Y_{ck}}{2} + Y_{2k},
\]

\[
Y_{24k} = \frac{dY_{ck}}{2} + Y_{Fk} + Y_{Fhk}
\]

\[
Y_{25k} = \frac{dY_{ck}}{2} + \frac{(D - d)Y_{ck}}{2} + Y_{2k},
\]

\[
Y_{2k} = \frac{(Y_{Lk}Y_{ck}/2)}{(D - d) + (D - d)Y_{ck}/2}
\]

and \( d \) is the train position along the contact feeder section. From Eq. (5), the equivalent harmonic impedance, which relates the \( k \)th harmonic current and voltage at the pantograph node (i.e., at node 5 in Figure 1(b)), can be obtained as follows:

\[
V_{sk} - V_{2k} = (Z_{55k} + Z_{22k} - 2Z_{25k}) L_k = Z_{Eqk} L_k.
\]

The analysis of this impedance in a frequency range makes it possible to locate the resonances observed from the traction load. As an example, Figure 2 shows the frequency response of the unfiltered and filtered system equivalent impedance numerically obtained from Eqs. (5) and (7) considering the train position \( d = D/2 \) with \( D = 30 \) km and the impedance values \( X_i = 0.6944 \Omega \) (\( S_N = 900 \text{ MVA} \)), \( X_{TR} = 6.25 \Omega \) (\( S_N = 12 \text{ MVA} \) and \( e_{cc} = 12\% \)), \( R_L = 0.0232 \Omega/\text{km} \), \( X_L = 0.0625 \Omega/\text{km} \), and \( X_C = 1.25 \cdot 10^3 \Omega \cdot \text{km} \). Track length and impedance values are obtained from the usual traction system parameters in Table 1. Two cases are considered in the filtered system: (1) an RLC filter tuned at the third harmonic and (2) a combination of an RLC filter and a broadband filter tuned at the third and fifth harmonics, respectively. The unfiltered system impedance in Figure 2 (solid gray line) has the typical parallel resonances in the literature, and only the first at \( k_p,1 \approx 23 \) (i.e., \( f_{p,1} \approx 1.2 \text{ kHz} \)) can be problematic due to its proximity to the switching frequency of train converters [2, 3, 7, 10, 11, 14, 15]. The filters allow shifting the parallel resonances to higher frequencies and damping their magnitude, thereby reducing the harmonic problem.

The unfiltered system impedance is analytically determined, and simple expressions to locate resonances are obtained in the following section. This makes it possible to predict the harmonic problem and design shunt filters to avoid it.

4. ANALYTICAL CHARACTERIZATION OF THE TRACTION SYSTEM HARMONIC RESPONSE

4.1. Traction System Harmonic Impedance

Impedance \( Z_{Eqk} \) is obtained from Eqs. (5) and (7) without considering the filters and is normalized with respect to the substation transformer reactance to reduce the number of variables in the study:

\[
Z_{Eqk, N} = \frac{Z_{Eqk}}{X_{TR}}.
\]
\[ X_{TR}^{-1} \left( \frac{Y_{ck}(D-d)(4Y_{lk} + (D-d)^2 Y_{ck})}{2(2Y_{lk} + (D-d)^2 Y_{ck})} \right) + \frac{2Y_{pk} Y_{lk} + d \cdot Y_{ck} (d \cdot Y_{pk} + Y_{lk})}{2(d \cdot Y_{pk} + Y_{lk})} \), \quad (8) \]

where

\[ Y_{pk} = \frac{Y_{sk} Y_{TRK}}{Y_{sk} + 2Y_{TRK}} + \frac{d}{2} Y_{ck}. \quad (9) \]

Expression (8) can also be obtained directly by simple inspection of the circuit in Figure 1(b).

To validate the analytical expression of \( Z_{Eqk, N} \) in Eq. (8), Figure 2 compares its frequency response (shown by a dashed black line) with that calculated in Section 3. It is verified that the accuracy obtained can be extended to any value of the system parameters in Table 1.

It is easy to demonstrate that the normalized impedance \( Z_{Eqk, N} \) only depends on the following terms:

\[ X_{TR} Y_{sk} \approx \frac{X_{TR}}{j k x_S} = \frac{1}{j k x_S}, \quad X_{TR} Y_{TRK} \approx \frac{X_{TR}}{j k X_{TR}} = \frac{1}{j k}, \]

\[ X_{TR} Y_{lk} \approx \frac{R_L + j k X_L}{X_C} = \frac{1}{r_L + j k x_L}, \quad X_{TR} Y_{ck} \approx \frac{X_{TR}}{j k x_C} = \frac{k}{x_C}, \quad (10) \]

and therefore, the magnitude of the normalized impedance \( Z_{Eqk, N} \) only depends on the harmonic order \( k \), track length \( D \), train position \( d \), and four ratios \((x_S = X_S/X_{TR}, \ r_L = R_L/X_{TR}, \ x_L = X_L/X_{TR}, \) and \( x_C = X_C/X_{TR})\). Table 1 shows typical values of these ratios derived from the traction system parameter data. In the next subsection, simple expressions to locate the resonances of \( Z_{Eqk, N} \) (or \( Z_{Eqk} \)) based on the previous variables are determined.

### 4.2. Analytical Location of Resonance

The resonances of \( Z_{Eqk, N} \) can be analytically located from Eq. (8) by equating to zero its denominator, which can be compacted as follows:

\[ \text{Den}(Z_{Eqk, N}) = k^6 c_3 + k^4 c_2 + k^2 c_1 + c_0, \quad (11) \]

where the coefficients of the equations are

\[ c_3 = x_S^2 D^2 \cdot \left( D^2 - d^2 \right) \left( 1 + 2x_S \right), \]

\[ c_2 = -2x_C x_L D \cdot \left( D^2 - D \cdot d + d^2 \right) + x_L d \cdot \left( D^2 - d^2 \right)^2 \]

\[ + 4x_S x_L x_C D \cdot \left( D^2 - D \cdot d + d^2 \right), \]

\[ c_1 = 4x_C^2 D \cdot \left( 2 + D \cdot x_L \right) + 4x_S \], \quad c_0 = -8x_C^3. \quad (12) \]

In Eq. (12), resistance \( R_L \) (i.e., ratio \( r_L \)) is neglected to reduce the number of variables involved because it is proved that, as expected, it damps the system harmonic response but does not affect resonance location significantly [18]. It is numerically verified that the discriminant of the function in Eq. (11),

\[ \Delta = 18c_3 c_2 c_1 c_0 - 4c_2^2 c_1 - 4c_3 c_1^2 - 27c_2^3 c_0. \quad (13) \]

is greater than zero for the usual track length, all train positions, and the ratio range in Table 1. Thus, the solution of the polynomial function corresponds to three real roots, i.e., Eq. (14), which allow locating the three parallel resonances in Figure 2:

\[ k_{p,1} = \sqrt{(\alpha - \beta - 2\gamma)}, \quad k_{p,2} = \sqrt{\left( \alpha + \beta \left( 1 + i\sqrt{3} \right) + (1 - i\sqrt{3}) \gamma \right)}, \]

\[ k_{p,3} = \sqrt{(\alpha + \beta \left( 1 - i\sqrt{3} \right) + (1 + i\sqrt{3}) \gamma)}, \]

\[ \alpha = -c_2, \quad \beta = c_3, \quad \gamma = \frac{(c_2 - 3c_1 c_1)}{6c_1 C}. \quad (14) \]

where

\[ C = \left( \frac{1}{2} \left( Q + 2c_3^3 - 9c_3 c_2 c_1 + 27c_2 c_0 \right) \right)^{1/3}, \]

\[ Q = \sqrt{(2c_2^2 - 9c_3 c_2 c_1 + 27c_2 c_0)^2 - 4\left( c_2^2 - 3c_1 c_1 \right)^3}. \quad (15) \]

Figure 3 shows the location of parallel resonances \( k_{p,1}, k_{p,2}, \) and \( k_{p,3} \) as a function of train position \( d \) along the contact feeder section of length \( D = 30 \) km with the ratio range in Table 1. They are calculated from Eq. (14) considering six values of ratio \( x_L \) \((x_L = 0.01, 0.05, 0.09, 0.15, 0.25, \) and 0.35 p.u./km), four values of ratio \( x_C \) \((x_C = 2.0 \cdot 10^4, 1.0 \cdot 10^5, 1.75 \cdot 10^5, \) and 2.5 \cdot 10^5 p.u./km), and a single value of ratio \( x_S \) \((x_S = 0.111 \) p.u.). Similar curves are obtained for the other values of ratio \( x_S \) in the range of Table 1. This ratio has a negligible influence on the resonance because its values, derived from the typically large values of system short-circuit power \( S_S \) (Eq. (1)), are very small [2]. The resonances in the example of Section 3 \((D = 30 \) km, \( d = 15 \) km, \( x_L = 0.111 \) p.u., \( r_L = 0.03712 \) p.u./km, \( x_S = 0.01 \) p.u./km, and \( x_C = 2.0 \cdot 10^4 \) p.u./km) are also shown in Figure 3 to verify the usefulness of Eq. (14) in locating the resonances despite neglecting the longitudinal resistance of the section line. Note that the lower-order harmonics at which the parallel resonances occur are found for higher ratios \( x_L \) and lower ratios \( x_C \). The first parallel resonance \( (k_{p,1}) \), which does not depend on the train position, could be close to the harmonics of the currents injected by train converters \((i.e., \) below the 40th harmonic order) for most values of ratios \( x_L \) and \( x_C \). This is not true for the other two resonances, but \( k_{p,2} \) could also occur below the 40th harmonic order for \( x_L \) greater than 0.1 p.u./km and \( x_C \) smaller than 5.
10^4 \text{p.u.} \cdot \text{km}. The analysis of the effect of feeder section length \( D \) on the location of the first resonance \( k_{p,1} \) from Eq. (14) results in the plot \( k_{p,1} \) versus \( D \) in [7]. It is concluded that the frequency of the resonance is reduced with section line length, but for lengths greater than 40, the influence on resonance location is insignificant. Thus, one design suggestion would be to use the shortest possible sections so that resonance is shifted to higher values.

Although it is easy to locate parallel resonances from the coefficients in Eq. (12) using current software tools, simpler expressions of the polynomial function coefficients can be derived by neglecting the short-circuit reactance \( X_S \), i.e., \( x_S = X_S/X_TR \approx 0 \):

\[
\begin{align*}
   c_3 & \approx x_L^2 D \cdot d^2 \cdot (D - d)^2, \\
   c_2 & \approx -2x_L X_C D \cdot (D^2 - D \cdot d + d^2) + x_L d \cdot (D - d)^2, \\
   c_1 & \approx 4x_C^2 D \cdot (2 + D \cdot x_L), \quad c_0 = -8x_C^3.
\end{align*}
\]

This makes it possible to obtain friendlier expressions (referred to as \( k_{p,i}^{apx} \) with \( i = 1, 2, \) and 3) to locate approximately the harmonics at which parallel resonances occur. To illustrate the goodness of the above approximation, Figure 4 shows the error between \( k_{p,i} \) and \( k_{p,i}^{apx} \) expressions of the three parallel resonances (\( i = 1, 2, \) and 3) as a function of the train position along the contact feeder section of length 30 km and considering six values of ratio \( x_L (x_L = 0.01, 0.05, 0.09, 0.15, 0.25, \) and 0.35 p.u./km), any value of ratio \( x_C \) (the errors are independent of this ratio), and a single value of ratio \( x_S (x_S = 0.111) \). This error is evaluated as

\[
\varepsilon_{i,j}^{apx} = \frac{k_{p,i} - k_{p,i}^{apx}}{k_{p,i}} \quad (i = 1, 2, 3; \quad j = 1).
\]

It must be noted that the maximum error is always below 10%. This error is smaller for lower values of ratio \( x_S \). As an example of the above study, Figure 5(a) compares the results of \( k_{p,1} \) and \( k_{p,1}^{apx} \) as functions of train position \( d \) along the contact feeder section of length 30 km and for \( x_S = 0.111 \) p.u., \( x_C = 2.0 \cdot 10^4 \) p.u. \cdot km, and six values of \( x_L \) (\( x_L = 0.01, 0.05, 0.09, 0.15, 0.25, \) and 0.35 p.u./km). The resonances in the example of Section 3 (\( D = 30 \) km, \( d = 15 \) km, \( x_S = 0.111 \) p.u., \( x_L = 0.003712 \) p.u./km, \( x_L = 0.01 \) p.u./km, and \( x_C = 2.0 \cdot 10^4 \) p.u. \cdot km) are also shown in Figure 5(a) for comparison purposes.
4.3. Study of the First Parallel Resonance

Although the three parallel resonances determined in Eq. (14) and shown in Figure 3 are reported in the literature, only the first (i.e., the \( k_{p,1} \) resonance) is studied because of its proximity to the switching frequency of train converters. This section analyzes the dependence of this resonance on traction system parameters. Moreover, two approximate expressions to locate the resonance are proposed, and their accuracy and limitations are discussed.

By considering the approximations of the longitudinal resistance of the section line and the short-circuit reactance of the power system in the previous section, and the slight dependence of \( k_{p,1} \) (i.e., first resonance) on train position \( d \) (see Figures 4 and 5(a)) \([2, 4, 7, 14, 15]\), this resonance can be approximately located by imposing \( R_L \approx 0 \) and \( X_S \approx 0 \) and setting any value of \( d \) in the admittance expressions in Eq. (6). That is, harmonic \( k_{p,1} \), at which the first resonance occurs, can be roughly determined by neglecting the section line resistance and the power supply reactance in the circuit of Figure 1(b) and placing the train at the beginning of the contact feeder section (i.e., \( d = 0 \) km). As can be seen in the resulting circuit, the expression of the normalized equivalent impedance at the load terminals is

\[
Z_{Eqk, N}^\text{approx2} = \frac{Z_{Eqk}^{\text{approx2}}}{X_{TR}} = \frac{1}{X_{TR}} \frac{X_{Eqk}^{\text{approx2}}}{1} = \frac{1}{X_{TR}} \left( \frac{1}{kX_{TR} - \frac{D \cdot k}{2X_C} + \frac{1}{(kX_L D - \frac{2X_C}{D})}} \right)^{-1}, \quad (18)
\]

and the first resonance of \( Z_{Eqk, N} \) can be approximately located by equating to zero the denominator of Eq. (18), which can be compacted as follows:

\[
\text{Den} \left( Z_{Eqk, N}^{\text{approx2}} \right) = x_L D^3 \cdot k^4 - 2x_C D \cdot (D \cdot x_L + 2) \cdot k^2 + 4x_C^2. \quad (19)
\]

Thus, the first parallel resonance in Eq. (14) \( (k_{p,1}) \) can be approximated by the following root of Eq. (20):

\[
k_{p,1}^{\text{approx2}} = \frac{1}{D} \sqrt{\frac{x_C}{x_L} \left( D \cdot x_L + 2 - \sqrt{D^2 \cdot x_L^2 + 4} \right)}. \quad (20)
\]

To illustrate the usefulness of the above approximation, Figure 5(b) compares the results from expressions \( k_{p,1} \) (Eq. (14)) and \( k_{p,1}^{\text{approx2}} \) (Eq. (20)), and Figure 6 shows the error \( \epsilon_1^{\text{approx2}} \)
(Eq. (17)) between them as a function of the train position \( d \) along the contact feeder section and for \( D = 30 \text{ km}, x_S = 0.111 \text{ p.u.}, x_L = 2.0 \cdot 10^4 \text{ p.u.} \cdot \text{km}, \) and six values of \( x_L (x_L = 0.01, 0.05, 0.09, 0.15, 0.25, \) and \( 0.35 \text{ p.u.} / \text{km}) \). The resonances in the example of Section 3 \( (D = 30 \text{ km}, d = 15 \text{ km}, x_L = 0.111 \text{ p.u.}, r_L = 0.003712 \text{ p.u.} / \text{km}, x_L = 0.01 \text{ pu/km}, \) and \( x_C = 2.0 \cdot 10^4 \text{ p.u.} \cdot \text{km} ) \) are also shown in Figure 5(b) for comparison purposes. Note that the excellent accuracy of the approximation (with errors below 10%) can be extended to the other values of the traction system parameters in Table 1.

It can be observed that for \( D^2 \cdot x_L^2 << 4 \) (i.e., if the longitudinal reactance \( X_L \) of the catenary is small enough), the \( k_{p,1}^{a p x 2} \) expression in Eq. (20) can be simplified as

\[
k_{p,1} = \frac{1}{D} \frac{x_C}{\sqrt{X_L}}.
\]

This expression can also be deduced by neglecting the section line resistance \( R_L \), the power supply reactance \( X_S \), and the longitudinal reactance \( X_L \) in the circuit of Figure 1(b) and by equating to zero the denominator of the normalized equivalent impedance at the train terminals when the train is located at the beginning of the contact feeder section \( (i.e., d = 0 \text{ km}) \):

\[
Z_{Eqk, N} = Z_{Eqk} = \frac{1}{X_{TR}} \frac{1}{X_{TR}} = \frac{1}{X_{TR}} J \frac{1}{X_{TR}} \times \frac{1}{k X_{TR}} - D \cdot k \frac{1}{X_{C}}^{-1}.
\]

The above approximation is commonly used in traction system studies assuming that the first parallel resonance is independent of the train position along the section line \( [2, 4, 7, 14, 15] \) and is mainly determined by the substation reactance \( X_{TR} \) and the per-unit length capacitive reactance \( X_C \) between the contact wire and the ground \( [2, 7] \). However, to the authors’ best knowledge, its application range has been neither theoretically justified nor limited. To illustrate this assertion, Figure 5(b) compares the results from expressions \( k_{p,1}^{a p x 1} \) (Eq. (14)) and \( k_{p,1}^{a p x 3} \) (Eq. (21)), and Figure 6 shows the error \( \varepsilon_{a p x 3}^{a p x 3} \) (Eq. (17)) between them as a function of the train position \( d \) along the contact feeder section and for \( D = 30 \text{ km}, x_S = 0.111 \text{ p.u.}, x_C = 2.0 \cdot 10^4 \text{ p.u.} \cdot \text{km}, \) and six values of \( x_L (x_L = 0.01, 0.05, 0.09, 0.15, 0.25, \) and \( 0.35 \text{ p.u.} / \text{km}) \). Note that as \( k_{p,1}^{a p x 3} \) is independent of \( X_L \), it is plotted as a single line that moves farther away from \( k_{p,1}^{a p x 1} \) with increasing the longitudinal reactance of the section line \( (i.e., X_L) \). Thus, \( k_{p,1}^{a p x 3} \) gives an acceptable error (close to 10%) only for values \( X_L \) below 0.01 p.u./km. Above this value, the error increases dramatically, invalidating the approximation. This is also true for the other values of the parameters in Table 1.

5. APPLICATION OF HARMONIC RESONANCE LOCATION

The analytical expressions of \( k_{p,1}^{a p x 1} \) (Eq. (14) with coefficients in Eq. (12)), \( k_{p,1}^{a p x 1} \) (Eq. (14) with coefficients in Eq. (16)), \( k_{p,1}^{a p x 2} \) (Eq. (20)), and \( k_{p,1}^{a p x 3} \) (Eq. (21)) are applied to locate the harmonic resonance of three 1 × 25 kV 50 Hz traction power systems in the literature \( [3, 15, 16] \).

In [3], a harmonic study on the Velesin (Czech Republic) traction system was presented. The voltage and current at the traction substation are measured and compared with Microcap simulation results by programming the equivalent circuit model with the electrical parameters of the traction system. The voltage and current waveforms reveal the presence of resonances in the traction system, which are analyzed from equivalent circuit simulations.

In [15], the performance of a hybrid shunt compensation system connected at one end of a traction feeder section is studied by simulation and experiments to address the resonance phenomena. The study considers typical 1 × 25 kV 50 Hz traction substations with 30-km single-phase contact feeder sections and provides their electrical parameters.

In [16], technical details of several traction systems, such as harmonic distortion, resonance phenomena, and AC filters, were analyzed from their electrical specifications.

Table 2 summarizes the traction system electrical parameter data provided by the above references and the harmonic orders at which the parallel resonances are located. The lack of information on the short-circuit power of the supply networks is not a problem because of its small influence on the resonances (see \( k_{p,1}^{a p x 1} \) errors in Figure 5). Thus, a 700-MVA value is assumed. With these data, a harmonic study on parallel resonance location is performed from the analytical expressions in Section 4. Results are reported in Table 2, together with the results in the references. It must be noted that the results of expressions...
### TABLE 2. $1 \times 25$ kV 50-Hz traction systems in the literature

| Reference | Reactance $X_{TR}$ ($\Omega$) | [3] | [15] | [16] |
|-----------|-------------------------------|-----|-----|-----|
| Substation transformer section | | | | |
| | | | | |
| Contact feeder section | | | | |
| Longitudinal $\pi$-reactance $X_L$ ($\Omega$/km) | 0.4492 | 0.4335 | 0.3142 |
| Longitudinal $\pi$-resistance $R_L$ ($\Omega$/km) | 0.200 | 0.169 | 0.15 |
| Transversal $\pi$-reactance $X_C$ ($\Omega$·km) | 1.55 · 10^5 | 2.89 · 10^5 | 2.12 · 10^5 |
| Track length $D$ | 40 | 30 | 30 |
| Train position $d$ | 20 | 10 | 30 |

| Resonance | $k_{p_i}$, | $k_{p_i}$,apx |
|-----------|------------|---------------|
| First | $\approx 16$ | $\approx 26$ |
| Second | $\approx 49$ | $\approx 90$ |
| Third | $\approx 65.45$ | 599.96 |

$^a$No data are available.

$k_{p_i}$ and $k_{p_i}$,apx ($i = 1, 2$) are in agreement with those in the original works. In the case of [15], the result of $k_{p_2}$,apx does not exactly agree with $k_{p_2}$,[Ref], probably due to differences in the modeling of the section line. These have no effect on the location of the first resonance, but do on the location of the other resonances. Expressions $k_{p_1}$,apx1 and $k_{p_1}$,apx2 give correct results, unlike expression $k_{p_1}$,apx3, the results of which are unacceptable because the longitudinal reactance of the contact feeder section is too large compared to the substation transformer reactance ($x_L > 0.01$ p.u./km in the three examples).

6. CONCLUSIONS

Train converters are non-linear loads that inject harmonic currents at pantograph terminals capable of causing voltage waveform distortion. This problem can be magnified by the parallel resonance of the equivalent impedance observed from the traction load. In unfiltered traction systems, this impedance has three parallel resonances, but only that below 2 kHz can be really dangerous because of its proximity to the frequency of harmonic currents injected by converters. This article contributes to locating the harmonics at which the three parallel resonances occur by providing analytical expressions. Using these expressions and considering system electrical parameters, it is possible to analyze the resonance frequencies in more detail than frequency scan plots. The study of the lowest parallel resonance shows that this resonance hardly depends on train position, allowing the derivation of a simpler expression to locate it. Moreover, if the longitudinal reactance of the section line is 0.01 times smaller than the substation transformer reactance, the previous expression can be further simplified, and the harmonic of the parallel resonance is mainly dependent on the substation reactance, the per-unit length capacitive reactance between the contact wire, and the ground and the contact feeder section length. This approximation is commonly used in the literature without considering its application range. The proposed expressions are validated by analyzing the frequency response of three traction systems in the literature.

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