Correspondence between the contracted BTZ solution of cosmological topological massive gravity and two-dimensional Galilean conformal algebra

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Abstract
We show that a BTZ black hole solution of cosmological topological massive gravity has a hidden conformal symmetry. In this regard, we consider the wave equation of a massless scalar field propagating in BTZ spacetime and find that the wave equation could be written in terms of the $SL(2, \mathbb{R})$ quadratic Casimir. From the conformal coordinates, the temperatures of the dual conformal field theories (CFTs) could be read directly. Moreover, we compute the microscopic entropy of the dual CFT by the Cardy formula and find a perfect match to the Bekenstein–Hawking entropy of a BTZ black hole. Then, we consider Galilean conformal algebras (GCA), which arises as a contraction of relativistic conformal algebras ($x \rightarrow \epsilon x$, $t \rightarrow t$, $\epsilon \rightarrow 0$). We show that there is a correspondence between GCA$_2$ on the boundary and contracted BTZ in the bulk. For this purpose we obtain the central charges and temperatures of GCA$_2$. Then, we compute the microscopic entropy of the GCA$_2$ by the Cardy formula and find a perfect match to the Bekenstein–Hawking entropy of a BTZ black hole in a non-relativistic limit. The absorption cross section of a near-region scalar field also matches the microscopic absorption cross section of the dual GCA$_2$. So we find further evidence that shows correspondence between a contracted BTZ black hole and two-dimensional GCA.

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1. Introduction

Recently, there has been some interest in extending the AdS/conformal field theory (CFT) correspondence to non-relativistic field theories [1, 2]. The Kaluza–Klein-type framework for non-relativistic symmetries, used in [1, 2], is basically identical to the one introduced in [3] (see also [4]). The study of a different non-relativistic limit was initiated in [5].
where the non-relativistic conformal symmetry was obtained by a parametric contraction of the relativistic conformal group. Galilean conformal algebra (GCA) arises as a contraction of relativistic conformal algebras [5, 6], where in 3 + 1 spacetime dimensions the Galilean conformal group is a 15-parameter group which contains the ten-parameter Galilean subgroup. An infinite dimensional Galilean conformal group has been reported in [6], the generators of this group are

\[ L^n = -(n+1)t^n x_i \partial_i - t^{n+1} \partial_t, \quad M^n_i = t^{n+1} \partial_i, \quad J^n_{ij} = -t^n (x_i \partial_j - x_j \partial_i) \]

for an arbitrary integer \( n \), where \( i \) and \( j \) are specified by the spatial directions. There is a finite-dimensional subgroup of the infinite-dimensional Galilean conformal group which is generated by \((J^0_{ij}, L^{\pm 1}, M^{\pm 1}_i, M^0_i)\). These generators are obtained by contraction \((t \rightarrow t, x_i \rightarrow \epsilon x_i, \epsilon \rightarrow 0, v_i \sim \epsilon)\) of the relativistic conformal generators. Recently, the authors of [7] (see also [8]) have shown that the GCA2 is the asymptotic symmetry of cosmological topological massive gravity (CTMG) in the non-relativistic limit. They have obtained the central charges of GCA2, and also a non-relativistic generalization of the Cardy formula. In this paper, we obtain a similar result by another method. But our aim in this paper is more than this. We show that the BTZ black hole solution of CTMG has a hidden conformal symmetry, not only in the relativistic case, but also in the non-relativistic case. We show that for a massless scalar field, in non-relativistic CTMG there exists a finite Galilean conformal symmetry acting on the solution space. According to our knowledge, this is the first paper that studies the hidden conformal symmetry for a black hole in the non-relativistic case.

Recent investigation on the holographic dual descriptions for the black holes has achieved substantial success. According to the Kerr/CFT correspondence [9], the microscopic entropy of a four-dimensional extremal Kerr black hole was calculated by studying the dual chiral CFT associated with the diffeomorphisms of a near-horizon geometry of the Kerr black hole. Subsequently, this work was extended to the case of near-extreme black holes [10]. The main progress is made essentially on the extremal and near-extremal limits in which the black hole near-horizon geometries consist a certain AdS structure and the central charges of dual CFT can be obtained by analyzing the asymptotic symmetry following the method in [11] or by calculating the boundary stress tensor of the 2D effective action [12]. Recently, Castro et al [13] have provided evidence that the physics of Kerr black holes might be captured by a CFT. The authors have discussed that the existence of conformal invariance in a near-horizon geometry is not a necessary condition, instead the existence of a local conformal invariance in the solution space of the wave equation for the propagating field is sufficient to ensure a dual CFT description (see also [14]). The scalar Laplacian in the low frequency limit could be written as the \( SL(2, R) \) quadratic Casimir, showing hidden \( SL(2, R) \times SL(2, R) \) symmetries. In the microscopic description, using the Cardy formula for the microscopic degeneracy, they reobtain the Bekenstein–Hawking entropy of the black hole.

In this paper, we investigate the massless scalar wave equation in the background of a BTZ black hole solution of CTMG and show that the wave equation can be written in terms of \( SL(2, R) \) Casimir invariants. From the conformal coordinates introduced in [9], we read the temperature of the dual CFT. The microscopic counting supports this holographic picture. Then, we consider the non-relativistic limit of both sides of this correspondence, i.e. the non-relativistic limit of 2D CFT which give us GCA2 from one side, and a non-relativistic limit of CTMG which give us contracted BTZ from another side. For this purpose we obtain the central charges and temperatures of GCA2. Then, we show that the radial part of the Klein–Gordon (KG) equation in the background of a contracted BTZ black hole, where \((j \rightarrow \epsilon j, \varphi \rightarrow \epsilon \varphi)\), can be given by the non-relativistic limit of a quadratic Casimir of \( SL(2, R) \). We could read the GCA2 temperatures from the correspondence of the radial part of a non-relativistic KG equation and Casimir of GCA2. Then, we compute the microscopic entropy of the GCA2 by the Cardy formula and find a perfect match to the Bekenstein–Hawking entropy of a contracted
BTZ black hole. In section 4, we compute the absorption cross section of a near-region scalar field and find a perfect match to the microscopic cross section in dual GCA$_2$. These results support the idea of a correspondence between a contracted BTZ black hole and dual GCA in two dimensions.

2. Massless scalar field in the background of CTMG

In this section, we introduced the idea of the hidden conformal symmetry into the CTMG, and obtain the Virasoro algebras as a local symmetry of massless scalar fields propagating in the BTZ black hole background. The existence of the two-dimensional CFT behind the asymptotically AdS$_3$ spacetime, including the BTZ black hole solutions, was already pointed out by using Brown–Henneaux’s method, see [11] for the case without the gravitational Chern–Simons term, and [15] for the case with the Chern–Simons term.

We show that for a massless scalar field $\Phi_1$ propagating in the background of CTMG, there exists an $SL(2, R)_L \times SL(2, R)_R$ conformal symmetry acting on the solution space. The BTZ spacetime is given by the line element [7]

$$ds^2 = \left(-f(r) + \frac{16G^2j^2}{r^2}\right)dt^2 + \frac{dr^2}{f(r)} + r^2 d\phi^2 + 8Gj dt d\phi,$$

(1)

where

$$f(r) = \left(\frac{r^2}{l^2} - 8GM + \frac{16G^2j^2}{r^2}\right)$$

(2)

which is the solution of the Einstein equation. The event horizons of the spacetime are given by the singularities of the metric function which are the real roots of $r^2 f(r) = 0$. In the above metric, $G$, $j$, $M$ and $-\frac{l^2}{2}$ are the gravitational constant, rotational parameter, mass of a black hole and cosmological constant, respectively. But their definitions in CTMG are

$$M = m + \frac{1}{\mu}j, \quad J = j + \frac{1}{\mu}m,$$

(3)

where $\frac{1}{\mu}$ is the coupling constant of a gravitational Chern–Simons term. Now we consider a bulk massless scalar field $\Phi$ propagating in the background of (1). The KG equation

$$\Box \Phi = \frac{1}{\sqrt{-g}} g^{\mu \nu} \partial_{\mu} \Phi \partial_{\nu} \Phi = 0$$

(4)

can be simplified by assuming the following form of the scalar field:

$$\Phi(t, r, \theta, \phi) = \exp(-im\phi + i\omega t)S(\theta)R(r),$$

(5)

and for $l = 1$ is reduced to the following equation:

$$\left(\Delta u \partial_u \right) + \frac{1}{4} \left[ \frac{(2\omega r - m r_+)^2}{(u - u_+)(u_+ - u_2)} - \frac{(2\omega r - m r_-)^2}{(u - u_-)(u_2 - u_-)} \right] R(u) = 0,$$

(6)

where

$$\Delta = uf(u), \quad u = r^2, \quad u_{\pm} = r_{\pm}^2,$$

(7)

and

$$r_{\pm} = \sqrt{2G(m + j)} \pm \sqrt{2G(m - j)}.$$  

(8)

We can show that this equation can be reproduced by the introduction of conformal coordinates. We introduce the conformal coordinates [9]

$$\omega^* = \frac{u - u_+}{u - u_-} \exp(2\pi T_R \phi + 2n_R t)$$

(9)
\[ \omega^- = \sqrt{\frac{u - u_+}{u - u_-}} \exp(2\pi T_L \varphi + 2n_L t) \]  
\[ y = \sqrt{\frac{u_+ - u_-}{u - u_-}} \exp(\pi (T_R + T_L) \varphi + (n_R + n_L)t). \]

We define left- and right-moving vectors by
\[ H_1 = \partial \]  
\[ H_0 = (\omega^+ \partial_+ + \frac{1}{2} y \partial_y), \]
\[ H_{-1} = ((\omega^+)^2 \partial_+ + \omega^+ y \partial_y - y^2 \partial_-) \]
\[ \overline{H}_1 = \partial_- \]
\[ \overline{H}_0 = (\omega^- \partial_- + \frac{1}{2} y \partial_y) \]
\[ \overline{H}_{-1} = ((\omega^-)^2 \partial_- + \omega^- y \partial_y - y^2 \partial_+) \]
which each satisfy the \( SL(2, \mathbb{R}) \) algebra
\[ [H_0, H_{\pm 1}] = \mp H_{\pm 1}, \quad [H_{-1}, H_1] = -2H_0 \]
\[ \overline{H}_0, \overline{H}_{\pm 1} = \mp \overline{H}_{\pm 1}, \quad [\overline{H}_{-1}, \overline{H}_1] = -2\overline{H}_0. \]

The quadratic Casimir is
\[ H_2^2 = \partial_y(\Delta \partial_y) = \frac{u_+ - u_-}{u - u_+} \left( \frac{T_L + T_R}{4A} \partial_\varphi - \frac{n_L + n_R}{4\pi A} \partial_\varphi \right)^2 + \frac{u_+ - u_-}{u - u_-} \left( \frac{T_L - T_R}{4A} \partial_\varphi - \frac{n_L - n_R}{4\pi A} \partial_\varphi \right)^2 A = T_L n_R - T_R n_L. \]

The crucial observation is that this Casimir, when written in terms of \( \varphi, t, u \)
\[ T_L = \frac{r_+ + r_-}{2\pi} \quad T_R = \frac{r_+ - r_-}{2\pi} \]
\[ n_L = \frac{r_+ + r_-}{2} \quad n_R = \frac{r_+ - r_-}{2}. \]

The microscopic entropy of the dual CFT can be computed by the Cardy formula which matches with the black hole Bekenstein–Hawking entropy
\[ S_{\text{CFT}} = \frac{\pi^2}{3} (c_L T_L + c_R T_R). \]
The central charges of CTMG are
\[ c_L = \frac{3}{2G} \left( 1 + \frac{1}{\mu} \right), \quad c_R = \frac{3}{2G} \left( 1 - \frac{1}{\mu} \right). \] (24)

From the central charges (24) and temperature (22) we have
\[ S_{\text{CFT}} = \frac{\pi r_+}{2G} + \frac{1}{\mu} \frac{\pi r_-}{2G}, \] (25)
which agrees precisely with the gravity result. The contribution to the entropy that is due to the gravitational Chern–Simons (last term in equation (25)) was first obtained by Solodukhin [16]. It is curiously proportional to the area of the inner horizon rather than that of the outer horizon.

3. GCA in two dimensions

GCA in two dimensions can be obtained from contracting conformal algebra in two dimensions [6]. In two dimensions the non-trivial generators are given by
\[ L_n = -(n + 1)\epsilon x \partial_x - \epsilon^{n+1} \partial_t, \quad M_n = \epsilon^{n+1} \partial_x. \] (26)

The above generators obey the following commutation relations, where we define the GCAs:
\[ [L_m, L_n] = (m - n)L_{m+n}, \]
\[ [L_m, M_n] = (m - n)M_{m+n}, \]
\[ [M_n, M_m] = 0. \] (27)

2D conformal algebra at the quantum level is described by two forms of Virasoro algebra:
\[ [\mathcal{L}_m, \mathcal{L}_n] = (m - n)\mathcal{L}_{m+n} + \frac{c_R}{12} m(m^2 - 1)\delta_{m,n,0} \]
\[ [\mathcal{L}_m, \mathcal{L}_n] = (m - n)\mathcal{L}_{m+n} + \frac{c_L}{12} m(m^2 - 1)\delta_{m,n,0}. \] (28)

From these, one obtains a centrally extended 2D GCA
\[ [L_m, L_n] = (m - n)L_{m+n} + c_1 m(m^2 - 1)\delta_{m,n,0}, \]
\[ [L_m, M_n] = (m - n)M_{m+n} + c_2 m(m^2 - 1)\delta_{m,n,0}, \]
\[ [M_n, M_m] = 0. \] (29)

GCA central charges \( C_1 \) and \( C_2 \) are defined in terms of CFT central charges [7]:
\[ C_1 = \lim_{\epsilon \to 0} \frac{c_L + c_R}{12} = \frac{1}{4G} C_2 = \lim_{\epsilon \to 0} \left( \epsilon \frac{c_L - c_R}{12} \right) = \frac{1}{4G \mu}. \] (30)

CFT entropy (23) with the limit (\( \epsilon \to 0 \)) is converted to Galilean conformal entropy:
\[ S_{\text{GCA}} = \lim_{\epsilon \to 0} \left( \frac{\pi^2}{3} \left[ 6C_1(T_L + T_R) + 6C_2 \left( \frac{T_L - T_R}{\epsilon} \right) \right] \right). \] (31)

We define Galilean conformal temperatures as follows:
\[ T_1 = \lim_{\epsilon \to 0} 6(T_L + T_R)T_2 = \lim_{\epsilon \to 0} 6 \frac{T_L - T_R}{\epsilon}. \] (32)

The GCA entropy is
\[ S_{\text{GCA}} = \frac{\pi^2}{3} (C_1 T_1 + C_2 T_2). \] (33)
To make the GCA entropy finite, $T_L + T_R \sim O(1)$ and $T_L - T_R \sim O(\epsilon)$; as a result $r_+ \sim O(1)$ and $r_- \sim O(\epsilon)$. These results appear in the limits of $j \rightarrow \epsilon j$ and $m \rightarrow m$ that correspond with the result of [7]. Finally, from equation (22) and the above discussion we have introduced finite $n_1$ and $n_2$ (in non-relativistic limit) in terms of $n_L$ and $n_R$:

$$n_1 = \frac{n_L + n_R}{\epsilon}, \quad n_2 = n_L - n_R.$$  \hspace{1cm} (34)

### 4. Massless scalar field in the background of non-relativistic CTMG

In this section, we will show that for the massless scalar field $\Phi$, in non-relativistic CTMG there exists a finite Galilean conformal symmetry acting on the solution space. We consider a bulk massless scalar field $\Phi$ propagating in the background of (1). The KG equation (6) in a non-relativistic limit ($j \rightarrow \epsilon j$, $\varphi \rightarrow \epsilon \varphi$) reduces to

$$\partial_u (\Delta \partial_u) - \frac{1}{4} \left[ \frac{r_+^2}{(u - u_+)(u - u_-)} \delta^2 \left( -2r'_+ r'_- \left( \frac{1}{u - u_+} - \frac{1}{u - u_-} \right) \right) \partial_t \partial_u \right] \Phi = 0,$$

$$\delta^2 \Phi = 0,$$  \hspace{1cm} (35)

where $r'_+ = 2\sqrt{2}Gm$ and $r'_- = \sqrt{\frac{6G}{m}}j$. We can show that the above equation can be reproduced by the conformal coordinate (9), (10), (11) in the non-relativistic limit. $H^2$ is the Casimir of $\mathbb{SL}(2, R)_L \times \mathbb{SL}(2, R)_R$ so

$$[H^2, H_{0 \pm 1}] = 0 \quad [H^2, \bar{H}_{0 \pm 1}] = 0.$$  \hspace{1cm} (36)

Since, 2D Galilean conformal generators are created by the mixing of 2D conformal generators in the non-relativistic limit, from equation (36) it can be shown that the Casimir of the conformal group is the same as that of the Galilean conformal group in the non-relativistic limit. In another term, since

$$H^2 = \lim_{\epsilon \rightarrow 0} H^2, \quad L_{0 \pm 1} = \lim_{\epsilon \rightarrow 0}(H_{0 \pm 1} + \bar{H}_{0 \pm 1}), \quad M_{0 \pm 1} = \lim_{\epsilon \rightarrow 0} \left( \frac{H_{0 \pm 1} + \bar{H}_{0 \pm 1}}{\epsilon} \right)$$  \hspace{1cm} (37)

we have

$$\lim_{\epsilon \rightarrow 0}[H^2, H_{0 \pm 1} + \bar{H}_{0 \pm 1}] = 0 \quad \lim_{\epsilon \rightarrow 0} \left[ H^2, \frac{H_{0 \pm 1} + \bar{H}_{0 \pm 1}}{\epsilon} \right] = 0$$  \hspace{1cm} (38)

So, the Casimir operator (21) in the non-relativistic limit is the Casimir of GCA ($H^2$ is the non-relativistic limit of $H^2$). Finally, the Casimir operator of GCA is

$$H^2 = \partial_u (\Delta \partial_u) - \frac{u_+ - u_-}{u - u_-} \left( \frac{T_1}{\pi A^2} \right)^2 \delta^2 + \frac{2(u_+ - u_-)}{\pi A^2} \left( \frac{T_1 n_1}{u - u_+} - \frac{T_2 n_2}{u - u_-} \right) \partial_t \partial_u \phi$$

$$+ \left( \frac{1}{\epsilon^2} \frac{u_+ - u_-}{r - r_-} \frac{n_2^2}{\pi^2 A^2} - \frac{u_+ - u_-}{r - r_+} \frac{n_1^2}{\pi^2 A^2} \right) \delta^2 A' = T_1 n_2 - T_2 n_1.$$  \hspace{1cm} (39)
The above equation is reduced to the radial equation (35), with the identifications

\[ T_1 = \frac{6r'}{\pi}, \quad T_2 = \frac{6r'}{\pi}, \quad n_1 = r', n_2 = r'. \]  

(40)

The microscopic entropy of the dual GCA can be computed by the non-relativistic Cardy formula (33). From the central charges (30) and temperatures (40) we have

\[ S_{\text{GCA}} = \pi \left( \sqrt{\frac{2m}{G}} + \frac{j}{\mu} \sqrt{\frac{1}{2Gm}} \right) = S_{\text{BH}}, \]  

(41)

which agrees precisely with the gravity result (in non-relativistic limit) presented in [7]. As we have mentioned in the introduction, the authors of [7] have studied the Galilean non-relativistic limit of the dual field theory of the BTZ black hole in CTMG. From the Galilean limit of Virasoro algebra and the Galilean limit of the BTZ black hole, they obtain the result that the entropy of the Galilean limit of the BTZ black hole is consistent with the entropy calculated using the Cardy formula from the Galilean limit of the Virasoro algebra. This agreement between our result and the result of [7] is interesting, and shows that the non-relativistic version of the BTZ solution of CTMG really has hidden conformal symmetry in the near region of the black hole. In the next section, by obtaining the absorption cross section we present final evidence of the existence of this hidden symmetry.

5. Absorption cross section

In this section, we give a brief review for scattering of the scalar field \( \Phi^1 \) which propagates in the (contracted) BTZ background [17]. We calculate the absorption cross section from gravity side and match the result with the 2D (GCA) CFT cross section. A two-point function of conformal invariant fields is given by [10, 18]

\[ G(t^+, t^-) \sim (-1)^{h_R + h_L} \left( \frac{\pi T_L}{\sinh(\pi T_L t^+)} \right)^{2h_L} \left( \frac{\pi T_R}{\sinh(\pi T_R t^-)} \right)^{2h_R}, \]  

(42)

where \( t^\pm \) are the coordinates of the 2D CFT and \( (h_R, h_L) \) are eigenvalues of \( L_0 \) and \( \bar{L}_0 \), respectively. The absorption cross section in terms of frequency and temperature, from Fermi’s golden rule [10, 18], can be read as

\[ P_{\text{abs}} \sim \int dt^+ dt^- e^{-i\omega_R t^- - i\omega_L t^+} [G(t^+ - i\delta, t^- - i\delta) - G(t^+ + i\delta, t^- + i\delta)]. \]  

(43)

Using the integral

\[ \int dx e^{-i\omega x} (-1)^\Delta \left( \frac{\pi T}{\sinh(\pi T(x \pm i\delta))} \right)^{2\Delta} = \frac{(2\pi T)^{2\Delta-1}}{\Gamma(2\Delta)} e^{\pm \omega/2T} \left| \Gamma \left( \Delta + i \frac{\omega}{2\pi T} \right) \right|^2, \]  

(44)

the absorption cross section becomes

\[ P_{\text{abs}} \sim T_L^{2h_L-1} T_R^{2h_R-1} \sinh \left( \frac{\omega_L}{2T_L} + \frac{\omega_R}{2T_R} \right) \left| \Gamma \left( h_L + i \frac{\omega_L}{2\pi T} \right) \right|^2 \left| \Gamma \left( h_R + i \frac{\omega_R}{2\pi T} \right) \right|^2. \]  

(45)

From the above method, the non-relativistic limit of the absorption cross section can be computed. The non-relativistic limit of a two-point function is given by

\[ \lim_{\epsilon \rightarrow 0} G \sim (-1)^\Delta \left( \frac{T_1}{12 \sinh \left( \frac{\pi T_1}{12} (t \pm i\delta) \right)} \right)^{2\Delta} \exp \left( \frac{2T_2}{T_1} \xi \right). \]  

(46)
where the scaling dimension $\Delta = h_L + h_R$ is the eigenvalue of $L_0$ and the rapidity $\xi = \lim_{\epsilon \to 0} [\epsilon (h_L - h_R)]$ is the eigenvalue of $M_0$. From equations (43), (44), (46) and using the relation
\[
\lim_{x \to k} (1 + f(x))^{g(x)} = \lim_{x \to k} \exp(f(x)g(x)),
\]
where
\[
\lim_{x \to k} f(x) = 0, \quad \lim_{x \to k} g(x) = \infty,
\]
the cross section for 2D GCA is given by
\[
P_{abs} \sim \exp\left(\frac{2T_2}{T_1} \xi \right) |T_{1|}^{\Delta - 1} \sinh\left(\frac{6\omega_1}{T_1}\right) \left| \Gamma\left(\Delta + i \frac{6\omega_1}{\pi T_1}\right) \right|^2,
\]
where $\omega_1 = \omega_L + \omega_R$. We can study the absorption cross section of BTZ (contracted BTZ), from gravity side, in the relativistic (non-relativistic) limit. The results can be verified in agreement with the corresponding results for the operator dual to the scalar field in the 2D CFT (45) (2D GCA (49)). The radial part of the KG equation
\[
(\Box - M^2) \phi = 0
\]
for the massive scalar field $\phi$ is given by
\[
\left[\partial_u (\Delta \partial_u) + \left(\frac{A}{u-u_+} + \frac{B}{u-u_-} + C\right)\right] R(u) = 0,
\]
where
\[
A = \frac{(\omega r_+ -mr_-)^2}{4(u_+ - u_-)}, \quad B = -\frac{(\omega r_- -mr_+)^2}{4(u_+ - u_-)}, \quad C = \frac{M^2}{4}.
\]
The wavefunction satisfying the ingoing boundary condition at the horizon is of the form
\[
R(z) = z^{\alpha}(1-Z)^\beta F(a,b,c;z),
\]
where $z = \frac{u-u_-}{u_+ - u_-}$, and
\[
\alpha = \sqrt{A} \quad \beta = \frac{1}{2}(1 - \sqrt{1 - 4C}) \quad \gamma = \sqrt{-B},
\]
where
\[
c = 1 - 2i\alpha \quad a = \beta + i(\gamma - \alpha) \quad b = \beta - i(\gamma + \alpha).
\]
The asymptotic form can be read out by taking the limit $z \to 1$ and $1-z \to u^{-1}$
\[
R(u) \sim Dr^{-2h} + Er^{-2h},
\]
where
\[
D = \frac{\Gamma(c)\Gamma(2h - 1)}{\Gamma(a)\Gamma(b)} \quad E = \frac{\Gamma(c)\Gamma(1 - 2h)}{\Gamma(c - a)\Gamma(c - b)}
\]
and $h$ is the conformal weight
\[
h = \frac{1}{2}(1 + \sqrt{1 - 4C}).
\]
The absorption cross section is captured by the coefficient $D$:
\[
P_{abs} \sim |D|^{-2} \sim \sinh(2\pi \alpha) |\Gamma(a)|^2 |\Gamma(b)|^2.
\]
To explicitly see that $P_{abs}$ matches with a microscopic gray-body factor of the dual CFT we need to identify the conjugate charges $\delta E_L$ and $\delta E_R$ defined by
\[
\delta S_{BH} = \delta S_{CFT} = \frac{\delta E_L}{T_L} + \frac{\delta E_R}{T_R}.
\]
We have

\[ \delta E_L = \omega L \delta \quad E_R = \omega R, \]

where

\[ \omega_R = \frac{\omega + m}{2(r_+ + r_-)} \omega_L = \frac{\omega - m}{2(r_+ - r_-)}. \]

Hence, the coefficient \( a, b \) can be expressed in terms of parameters \( \omega_L \) and \( \omega_R \)

\[ a = h_R + \frac{i \omega_R}{2\pi T_R} \quad b = h_L + \frac{i \omega_L}{2\pi T_L}, \]

where for the scalar field \( h_L = h_R = h \). Similarly, we have

\[ 2\pi a = \frac{\omega_L}{2T_L} + \frac{\omega_R}{2T_R}. \]

Finally, from equations (59), (63) and (64) the absorption cross section can be expressed as

\[ P_{abs} \sim \frac{\gamma_1}{\Gamma(\omega_L^2 T_L + \omega_R^2 T_R)}, \]

which is the finite temperature absorption cross section for 2D CFT. In following, we consider
the scattering of a non-relativistic limit of BTZ, from gravity side, and match the result with
a 2D GCA cross section (49). The radial part of the KG equation in the non-relativistic limit
can be expressed in term of equation (51), with the identifications

\[ A_n = \frac{(\omega r' - mr')^2}{4u'^2}, \quad B_n = -\frac{(\omega r' - mr')^2}{4u'^2} + \frac{M^2}{4}. \]

From equations (54) and (55), we see that

\[ \lim_{\epsilon \to 0} a = a_n = \infty, \quad \lim_{\epsilon \to 0} b = b_n = \infty, \quad \lim_{\epsilon \to 0} (a + b) = \text{finite} \]

so the coefficient \( D \) in the non-relativistic limit is given by

\[ D_n = \frac{\Gamma(c_n) \Gamma(2h - 1)}{\Gamma(a_n + b_n)} \lim_{a \to \infty} \Gamma(a) \lim_{b \to \infty} \Gamma(b) \simeq \Gamma(a_n + b_n). \]

The absorption cross section from gravity side in the non-relativistic limit is captured by the
coefficient \( D_n \):

\[ P_{abs} \sim \frac{\Gamma(\omega_L^2 T_L + \omega_R^2 T_R)}{\Gamma(\omega_L^2 T_L + \omega_R^2 T_R)} \]

To explicitly see that \( P_{abs} \) matches with the microscopic gray-body factor of the dual GCA we
need to identify the conjugate charge \( \delta E_1 \) defined by

\[ \lim_{\epsilon \to 0} \delta S_{BH} = \delta S_{GCA} = \frac{\delta E_1}{T_1}. \]

We have

\[ \delta E_1 = \omega_1 = \lim_{\epsilon \to 0} (\omega_L + \omega_R), \]

where

\[ \omega_1 = \frac{\omega r' - mr'}{2r'_+}. \]

Hence, the coefficients \( a_n, b_n \) can be expressed in term of parameters \( \omega_L \) and \( \omega_R \)

\[ a_n + b_n = \Delta + i \frac{6\omega_1}{\pi T_1}, \]
where for the scalar field $\Delta = 2h$. Similarly, we have
\[
2\pi \alpha_n = \frac{6\omega_1}{T_1}.
\] (72)

Finally from equations (67), (71) and (72), the absorption cross section can be expressed as
\[
(P_{abs} \sim T_1^{2\Delta-1} \sinh \left( \frac{6\omega_1}{T_1} \right) \left| \Gamma \left( \Delta + \frac{1}{\pi T_1} \right) \right|^2)
\] (73)

which is the absorption cross section for 2D GCA.

6. Conclusions

In this paper, at first we showed that there exists a holographic 2D CFT description for a BTZ black hole solution of CTMG. The key ingredient for this is the hidden conformal symmetry in the black hole. We considered the wave equation of a massless scalar field propagating in CTMG spacetime and found that the wave equation could be written in terms of the $SL(2,R)$ quadratic Casimir. We read the temperatures of the dual CFT from conformal coordinates. We recovered the macroscopic entropy from the microscopic counting. Then, we extended this study to the non-relativistic limit, and showed that there exists a correspondence between GCA2 on the boundary and contracted BTZ in the bulk. We have defined Galilean conformal temperatures by equation (32), so we could obtain the finite GCA2 entropy by equation (33). After that we showed that the radial part of the Klein–Gordon (KG) equation in the background of a contracted BTZ black hole, where $(j \rightarrow \epsilon j, \phi \rightarrow \epsilon \phi)$, can be given by the non-relativistic limit of the Casimir $H^2$. We could read the GCA2 temperatures $T_1, T_2$ from the correspondence of the radial part of a non-relativistic KG equation and GCA Casimir $H^2$. The temperatures $T_1, T_2$ given by equation (40) are exactly equal to the previous formula we obtained in (32). Then, we calculated the entropy of a contracted BTZ black hole by using a GCA Cardy formula and non-relativistic temperatures. Finally, we have shown that the absorption cross section of a near-region scalar field matches precisely that of a microscopic dual GCA side.

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