Thermal $\Upsilon(1s)$ and $\chi_{b1}$ suppression in $\sqrt{s_{NN}} = 2.76$ TeV Pb-Pb collisions at the LHC

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I compute the thermal suppression of the $\Upsilon(1s)$ and $\chi_{b1}$ states in $\sqrt{s_{NN}} = 2.76$ TeV Pb-Pb collisions. Using the suppression of each of these states I estimate the total $R_{AA}$ for the $\Upsilon(1s)$ state as a function of centrality, rapidity, and transverse momentum. I find less suppression of the $\chi_{b1}$ state than would be traditionally assumed; however, my final results for the total $\Upsilon(1s)$ suppression are in good agreement with recent preliminary CMS data.

The behavior of nuclear matter at extreme temperatures is now being studied with the highest collision energies ever achieved using the Relativistic Heavy Ion Collider at Brookhaven National Laboratory and the Large Hadron Collider at CERN. For RHIC $\sqrt{s_{NN}} = 200$ GeV Au-Au collisions, initial central temperatures of $T_0 \sim 350$ MeV were generated. For current LHC $\sqrt{s_{NN}} = 2.76$ TeV collisions one obtains $T_0 \sim 500 - 600$ MeV [1] and for upcoming full energy runs with $\sqrt{s_{NN}} = 5.5$ TeV one expects $T_0 \sim 700 - 800$ MeV. At such high temperatures, one expects to generate a quark-gluon plasma (QGP) in which the formation of quark bound states is suppressed in favor of a deconfined plasma of quarks and gluons.

Suppression of quark bound states follows from the fact that in the QGP one has Debye screening of color charge [2]. Heavy quarkonium has received the most theoretical attention, since heavy quark states are dominated by short distance physics and can be treated using heavy quark effective theory. Based on such effective theories of QCD, non-relativistic quarkonium states can be reliably described. Their binding energies are much smaller than the quark mass $m_Q \gg \Lambda_{QCD}$ ($Q = c, b$), and their sizes are much larger than $1/m_Q$. At zero temperature, since the velocity of the quarks in the bound state is small ($v \ll c$), quarkonium can be understood in terms of non-relativistic potential models such as the Cornell potential which can be derived directly from QCD using effective field theory [2].

Using such non-relativistic potential models, studies of quarkonium spectral functions and meson current correlators have been performed [2]. The results have been compared to first-principles lattice QCD calculations [3] which rely on the maximum entropy method [4]. In recent years, however, there has been an important theoretical development, namely the first-principles calculation of the thermal widths of heavy quarkonium states which emerge from imaginary-valued contributions to the heavy quark potential. The first calculation of the leading-order perturbative imaginary part of the potential due to gluonic Landau damping was performed by Laine et al. [5]. Since then an additional imaginary-valued contribution to the potential coming from singlet to octet transitions has also been identified [8]. The consequences of such imaginary parts on heavy quarkonium spectral functions [9], perturbative thermal widths [7][9], and quarkonium suppression in a T-matrix approach [10] have recently been studied; however, these studies were restricted to the case of an isotropic thermal plasma, which is only the case if one assumes ideal hydrodynamical evolution.

The calculation of the heavy quark potential has since been extended to the case of a plasma with finite momentum-space anisotropy for both the real [11] and imaginary [12] parts. Additionally, the impact of the imaginary part of the potential on the thermal widths of the states in both isotropic and anisotropic plasmas was recently studied [13]. The consideration of momentum-space anisotropic plasmas is necessary since, for any finite shear viscosity, the quark-gluon plasma possesses local momentum-space anisotropies [11][15]. Depending on the magnitude of the shear viscosity, these momentum-space anisotropies can persist for a long time and can be quite large, particularly at early times or near the edges of the plasma.

This has motivated the development of a new dynamical formalism called “anisotropic hydrodynamics” (aHYDRO) which extends traditional viscous hydrodynamical treatments to cases in which the local momentum-space anisotropy of the plasma can be large [14][15]. In this Letter I present first results of the combination of the aHYDRO temporal evolution of Ref. [15] together with results obtained in Ref. [13] for the real and imaginary parts of the binding energy. Using this framework I compute the suppression of the $\Upsilon(1s)$ and $\chi_{b1}$ mesons as a function of centrality, rapidity, and transverse momentum.

MODEL POTENTIAL: The phase-space distribution of gluons in the local rest frame is assumed to be given by $f(x,p) = f_{iso} \left( \sqrt{p^2 + \xi(p \cdot n)^2}/p_{hard} \right)$ where $p_{hard}$ is a scale which specifies the typical momentum of the particles and can be identified with the temperature in an isotropic plasma ($\xi = 0$) [16]. In general, the parameter $\xi$ measures the degree of anisotropy of the plasma via $\xi = \frac{1}{3} (p_z^2 / p_{t}^2) - 1$ where $p_z$ and $p_t$ are the partonic longitudinal and transverse momenta in the local rest frame, respectively.

The perturbative heavy quark potential as function of $\xi$ has been evaluated previously and has both real [11]
and imaginary contributions \[12\]. For \( N_c = 3 \) and \( N_f = 2 \) the real part of the resulting potential can be well-approximated by \( \Re[V_{\text{pert}}] = -\alpha \exp(-\mu r)/r \) with

\[
\left(\frac{\mu}{m_D}\right)^{-4} = 1 + \xi \left(1 + \sqrt{2}(1 + \xi)^2(\cos(2\theta) - 1)/(2 + \xi)^{5/2}\right),
\]

where \( \alpha = 4\alpha_s/3 \), \( m_D^2 = (1.4)^2 16\pi\alpha_s \rho_{\text{hard}}^2/3 \) is the isotropic Debye mass, and \( \theta \) is the angle with respect to the beamline. The factor of \((1.4)^2\) accounts for higher-order corrections to the isotropic Debye mass \[17\].

However, for describing finite-temperature states which can have large radii compared to \( \Lambda_{\text{QCD}}^{-1} \), one must supplement the perturbative short range contribution above by a long range contribution. Following previous work \[11\], I generalize the Karsch-Mehr-Satz potential \[2\] by including the anisotropic mass scale \( \mu \) given in Eq. \[1\] in place of the isotropic Debye mass and adding the entropy contribution necessary to obtain the internal energy of the states. Such a construction agrees well with lattice measurements of the real part of the heavy quark potential \[17\]. The resulting model for the real part of the heavy quark potential is

\[
\Re[V] = -\frac{\alpha}{r} (1 + \mu r) \exp(-\mu r) + \frac{2\pi}{\mu} \left[1 - \exp(-\mu r)\right] - \sigma r \exp(-\mu r) - \frac{0.8\sigma}{m_Q^2 r},
\]

where the last term is a temperature- and spin-independent quark mass correction \[13\] and \( \sigma = 0.223 \) GeV is the string tension. Here I ignore the effect of the running of \( \alpha_s \) and fix \( \alpha = 0.385 \) to match zero temperature binding energy data for heavy quark states \[11\].

For the imaginary part of the model potential I use the imaginary part of the perturbative heavy quark potential which has been calculated to linear order in \( \xi \)

\[
\Im[V_{\text{pert}}] = -\alpha \rho_{\text{hard}} \left\{ \phi(\hat{r}) - \xi \left[ \psi_1(\hat{r}, \theta) + \psi_2(\hat{r}, \theta) \right] \right\},
\]

where \( \hat{r} = m_D T \) and \( \phi, \psi_1, \psi_2 \) are defined in Ref. \[12\].

The full model potential is given by \( V = \Re[V] + i\Im[V] \) and can be used in the Schrödinger equation. To solve the resulting Schrödinger equation I use the finite difference time domain method \[19\] extended to the case of a complex-valued potential \[13\]. For the temperature and anisotropy dependence of the resulting real and imaginary parts of the binding energies for the \( \Upsilon(1s) \) and \( \chi_{b1} \) states, I refer the reader to Ref. \[13\]. For a point of reference, in an isotropic plasma the medium-induced width of the \( \Upsilon(1s) \) is approximately \( \Im[E_{\text{bind}}] \sim 0.211 T \).

INITIAL CONDITIONS AND DYNAMICS: Solution of the Schrödinger equation gives the real and imaginary parts of the binding energy of the states. The imaginary part defines the instantaneous width of the state \( \Im[E_{\text{bind}}(\rho_{\text{hard}}, \xi)] \equiv -\Gamma_T(\rho_{\text{hard}}, \xi)/2 \). However, one must account for the complete dissociation of the states when \( \Re[E_{\text{bind}}] < 0 \). I implement this by assigning a large width of 10 GeV \( \sim (0.02 \text{ fm}/c)^{-1} \) to states when \( \Re[E_{\text{bind}}] < 0 \). The final results are insensitive to the value of this rate, as long as it is taken to be greater than 0.5 GeV. Given the binding energy data, I evolve the system using the non-boost invariant HYDRO equations of Ref. \[13\]. Using the output I can compute the temporal evolution of the thermal width which gives \( \Gamma_T(\tau) \).

The resulting width \( \Gamma_T(\tau) \) implicitly depends on the initial temperature of the system. I vary the assumed plasma shear viscosity to entropy density ratio \( 4\pi\eta/\Sigma = \{1, 2, 3\} \) and for zero impact parameter collisions I use central temperatures of \( T_0 = \{520, 504, 494\} \) MeV, respectively. The central temperature for \( 4\pi\eta/\Sigma = 1 \) is fixed based on the 3+1d viscous hydrodynamical simulation of Schenke et al which reproduces the particle spectra and elliptic flow seen in \( \sqrt{s_{NN}} = 2.76 \text{ TeV Pb-Pb collisions} \[1\]. The central temperatures at \( 4\pi\eta/\Sigma = \{1, 2, 3\} \) were chosen in order to keep \( dN_{\text{ch}}/dy = 1400 \) fixed for different assumed viscosities. For the rapidity dependence of the initial temperature, I use the Gaussian rapidity profile specified in Ref. \[15\]. Finally, I assume an initial anisotropy of \( \xi_0 = 0 \). Since the current HYDRO implementation does not include transverse dynamics, I model the transverse evolution at zero and finite impact parameter as a set of decoupled longitudinally expanding plasmas with initial temperatures given by \( T(\mathbf{x}_\perp, b) = T_0 \left[ N_{\text{part}}(\mathbf{x}_\perp, b)/N_{\text{part}}(0, 0)\right]^{2/3} \), where the participant density is computed using the Glauber model with a Woods-Saxon profile and \( \sigma_{NN} = 62 \text{ mb} \).

At each transverse point I then evolve the system using HYDRO starting from \( \tau_0 = 0.3 \text{ fm}/c \), terminating the evolution at a final time, \( \tau_f \), where \( \tau_0 \) is the local energy density becomes less than that of an \( N_c = 3 \) and \( N_f = 2 \) ideal gas of quark and gluons with a temperature of \( T = 192 \text{ MeV} \). At this temperature plasma screening effects are assumed to decrease rapidly due to the transition to the hadronic phase and the widths of the states will become approximately equal to their vacuum widths.

FORMATION TIME: It is also important to consider the effect of the time-dilated formation time on the suppression of the \( \Upsilon(1s) \) and \( \chi_{b1} \) states \[20\]. The formation time of a state can be estimated by the inverse of its vacuum binding energy. Here I take \( \tau_{\text{form}}^0 = \{0.2, 0.4\} \) fm/c for the \( \Upsilon(1s) \) and \( \chi_{b1} \) states, respectively, in their local rest frame. These formation times are consistent with the states’ respective vacuum binding energies. In the lab frame the formation time depends on the transverse momentum of the state via the gamma factor \( \gamma_{\text{form}}(p_T) = \gamma_{\text{form}}^0 = E_T/\tau_{\text{form}}^0/M \) where \( M \) is the mass of the relevant state. For averaging over transverse momenta I assume that both states have a \( 1/E_T^4 \) spectrum which is consistent with the high-\( p_T \) spectrum measured by CDF \[21\].

THE SUPPRESSION FACTOR: Having obtained the
FIG. 1. Rapidity- and pr-averaged $R_{AA}$ for $\Upsilon(1s)$ and $\chi_b1$ as a function of $N_{\text{part}}$ using $4\pi\eta/S = 1$.

FIG. 2. Rapidity- and pr-averaged $R_{AA}^{\text{full}}$ for the $\Upsilon(1s)$ as a function of $N_{\text{part}}$ for $4\pi\eta/S \in \{1, 2, 3\}$.

FIG. 3. Centrality- and pr-averaged $R_{AA}^{\text{full}}$ for the $\Upsilon(1s)$ as a function of rapidity for $4\pi\eta/S \in \{1, 2, 3\}$.

FIG. 4. Centrality- and rapidity-averaged $R_{AA}^{\text{full}}$ for the $\Upsilon(1s)$ as a function of rapidity for $4\pi\eta/S \in \{1, 2, 3\}$.

Spatial and temporal evolution of the widths I can compute the resulting nuclear suppression factor, $R_{AA}$. Starting from the extracted time-, transverse-coordinate-, and rapidity-dependent instantaneous decay rate, I integrate to obtain $\tilde{\gamma}(x_\perp, p_T, \varsigma, b) \equiv \Theta(\tau_f - \tau_{\text{form}}(p_T)) \int_{\tau_{\text{form}}(p_T), \tau_0}^{\tau_f} d\tau \Gamma_T(\tau, x_\perp, \varsigma, b)$, in proper time [22] where $\varsigma$ is the spatial rapidity. From this I extract $R_{AA}$ via $R_{AA}(x_\perp, p_T, \varsigma, b) = \exp(-\tilde{\gamma}(x_\perp, p_T, \varsigma, b))$. Finally, I average over $x_\perp$ taking into account the transverse dependence of the hard-particle production probability via $\langle R_{AA}(p_T, \varsigma, b) \rangle \equiv \left[ \intdx_\perp T_{AA}(x_\perp) R_{AA}(x_\perp, p_T, \varsigma, b) \right]/\left[ \intdx_\perp T_{AA}(x_\perp) \right]$.

**Feed down:** Since a certain fraction of $\Upsilon(1s)$ states produced in high energy collisions come from the decay of excited states, when computing the total $R_{AA}$ for the $\Upsilon(1s)$ one must also consider the suppression of the excited states which decay or “feed down” to it. In this work I have only computed the $R_{AA}$ for one excited state ($\chi_b1$), so I can only estimate the full feed down effect. I will estimate the full feed down effect by assuming that all excited states have the same $R_{AA}$ as the $\chi_b1$ (shown in Fig. 1). With this assumption the $\Upsilon(1s)$ $R_{AA}$ including feed down can be written as $R_{AA}^{\text{full}} = x R_{AA}^{\text{ground state}} + (1 - x) R_{AA}^{\text{excited states}}$ where $x$ is the percentage of $\Upsilon(1s)$ states which are produced directly. Measurements of bottomonium state feed down in $\sqrt{s} = 1.8$ TeV pp collisions at CDF [23] with a cut $p_T > 8.0$ GeV/c find that the percentage of directly produced $\Upsilon(1s)$ states is $[50.9 \pm 8.2 \text{ (stat)} \pm 9.0 \text{ (syst)}]$%. In all plots shown I use $x = 0.51$.

**RESULTS:** In Fig. 1 I show the $N_{\text{part}}$ dependence of $R_{AA}$ for $\Upsilon(1s)$ and $\chi_b1$. As can be seen from this figure, despite the fact that the initial temperature is not high enough to completely dissociate the $\Upsilon(1s)$, there is still a suppression due to the in-medium decay. At these energies we see a somewhat similar suppression pattern for the $\chi_b1$. This may seem paradoxical since the naive melting temperature for the $\chi_b1$ is $\sim 345$ MeV; however, it is important to consider the finite formation time of the $\chi_b1$ and the transverse dependence of the temperature. In practice, one finds that there are still quite a few $\chi_b1$’s generated from the low temperature regions. After averaging over the $p_T$-spectrum and geometry, the final result is the one shown in the Fig. 1.
In Fig. 2 I show the $N_{\text{part}}$ dependence of $R_{AA}^{\text{full}}$ for three different values of the plasma shear viscosity, $\eta$. The experimental data points shown are preliminary data from the CMS collaboration [24]. In all cases, the statistical error is indicated by the narrow darker (dark red online) error bar while the systematic error is indicated by the broader (purple online) shaded region. In Fig. 3 I show the rapidity dependence of the $\Upsilon(1s) R_{AA}^{\text{full}}$. In Fig. 4 I show the $p_T$ dependence of $R_{AA}^{\text{full}}$ for $4\pi T^2/S = 1$. In this figure I averaged over the $p_T$-bins specified by the experiment, $0 < p_T < 6.5$ GeV, $6.5 < p_T < 10$ GeV, and $10 < p_T < 20$ GeV.

Figures 2, 3 and 4 taken together demonstrate a reasonably good agreement between theory and experiment; however, the $p_T$ dependence of the theoretical result seems to be much flatter than the experimental results. Practically speaking though, based on the limited number of events, it is hard to draw firm conclusions. Looking forward it will be important to include the suppression of the other relevant excited states, e.g. $\Upsilon(2s)$ and $\Upsilon(3s)$, and the effect of transverse expansion in the dynamics. Both of these goals represent work in progress.

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