Baryon Spectra in Deformed Oscillator Quark Model

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Abstract

We study theoretically the baryon spectra in terms of a deformed oscillator quark (DOQ) model. This model is motivated by confinement of quarks and chiral symmetry breaking, which are the most important non-perturbative phenomena of QCD. The minimization of the DOQ Hamiltonian with respect to the deformation for each principal quantum number results in deformations for the intrinsic states of excited baryonic states. We find that the resulting baryon spectra agree very well with the existing experimental data. The spatial deformation of the baryonic excited states carry useful information on the quark confinement and provide a clue to understand the confining mechanism.

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It is very important to understand the structure of hadrons for the full understanding of the quark nuclear physics, which is governed by the QCD dynamics. QCD is a non-abelian gauge theory and exhibits many interesting non-perturbative phenomena. Although quarks and gluons are the fundamental degrees of freedom in the QCD lagrangian, they are confined in hadrons. At the same time, chiral symmetry is spontaneously broken in the hadronic phase, which provides the constituent masses to the quarks inside hadrons. These non-perturbative properties in turn play a crucial role in hadron structures and their interactions.

The direct theoretical study of QCD for the baryon spectra including excited baryons in terms of lattice QCD seems difficult at present and therefore we want to learn the physics of strong interaction from the nature. In this respect, there was an interesting suggestion made a decade ago in the baryon spectra [1]. The observation of the nucleon, delta and lambda excitation spectra motivated a theoretical description, where the ground state baryons such as the nucleon are spherical, but the excited baryons such as the Roper are deformed. The use of the SU(3) version of the interacting boson model, in which the Roper is identified as the band head of the rotational band, provides spectra in good agreement with experiment. The same observation was made by Bhaduri and his collaborators [2].

These works were performed almost decade ago. There is also an elaborated model as the non-relativistic quark model with various perturbative terms of Isgur and Karl [3]. The baryons as well as the meson spectra are discussed also in terms of the group theoretical model by Iachello and his collaborators [4]. Since the experimental facilities such as CEBAF, the GeV photon facility at SPring-8 and the storage ring LISS project at Indiana will provide variety of detailed data on the hadron spectra, we believe it important to point out the interesting regularity seen clearly in the baryon spectra [5]. The theoretical view on the non-perturbative phenomena are also advanced very much in these days due to the detailed study of the dual Ginzburg-Landau (DGL) theory [6] and the lattice gauge theory on the confinement and the chiral symmetry breaking mechanisms [7].

Hence, the purpose of this paper is to introduce a model for baryons with a spatially deformed shape to describe the confinement for quarks with the constituent quark mass due to chiral symmetry breaking. The spatial deformation is implemented by a simple deformed harmonic oscillator potential, and therefore the model is referred to as the
deformed oscillator quark (DOQ) model. The predictions of the DOQ model is then compared with existing experimental data.

To begin with, we discuss a possible scenario of connection of the DOQ model with QCD. The QCD lagrangian consists of quarks and gluons, which is written as

\[
L = \bar{\psi} (i\gamma_\mu \partial^\mu + e\gamma_\mu A^\mu - m) \psi - \frac{1}{4} \text{tr} G_{\mu\nu}G^{\mu\nu}.
\]

Here, \( \psi \) denotes the quark field with current masses \( m \) and \( A^\mu \) are the gluon fields. QCD provides confinement of quarks and chiral symmetry breaking as the most important non-perturbative phenomena for strong interactions. Although the explicit procedure is not yet known, we may construct the baryonic states using the concept of the mean field approximation on gluon fields, which leads to the baryon lagrangian for quarks as

\[
L = \bar{\psi} \left( i\gamma_\mu \partial^\mu - M(\bar{A}) - \phi(\bar{A}) \right) \psi + \text{(higher order)}.
\]

Here, the masses \( M(\bar{A}) \) of constituent quarks are acquired by spontaneous chiral symmetry breaking. The mean field, \( \bar{A} \), should be provided by knowing the quark configuration and its form may be highly complicated. The confinement of quarks is expressed in terms of \( \phi(\bar{A}) \), which is considered here as a scalar like function. The constituent quark masses and the confinement potential are functions of the mean fields of the gluon fields, \( \bar{A} \), which are at the same time dependent on the quark wave function, \( \psi \). The higher order terms may contain, for example, Goldstone boson exchange terms, perturbative gluon exchange terms and so on. We may be able to formulate this scenario in terms of the dual Ginzburg-Landau theory, which describes both confinement and chiral symmetry breaking through the QCD monopole fields and their condensation in the QCD vacuum.

We take now the simplest possible form for the description of baryons in terms of constituent quarks, which realizes the above scenario. We assume that the constituent quark masses are large enough to take the non-relativistic quark model and the confinement potential is expressed in terms of a deformed harmonic oscillator potential. The hamiltonian is then written as

\[
H = \sum_{i=1}^{3} \left[ \frac{p^2_i}{2M_i} + \frac{1}{2}M_i(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) \right]
\]

The oscillator parameters are assumed to satisfy the volume conservation condition, \( \omega_x \omega_y \omega_z = \omega_0^3 \) to minimize the number of parameters. We may relax this condition as
our knowledge on confinement becomes more clear. Hence, there exist two free parameters in the DOQ model, which are fixed by the nucleon and the Roper masses. The system of the deformed oscillator model is worked out for nuclei by Bohr and Mottelson in detail and documented in their book \[10\]. The quarks acquire the constituent masses due to chiral symmetry breaking, which justifies the use of the non-relativistic form of the kinetic energy. For simplicity, we do not include more terms as the pseudo-scalar degrees of freedom as naturally arisen as the Goldstone bosons of chiral symmetry breaking \[7\]. We solve three valence quark systems with the removal of the center-of-mass coordinate, where the intrinsic energy is expressed simply as

\[
E_{\text{int}}(N_x, N_y, N_z) = \omega_x(N_x + 1) + \omega_y(N_y + 1) + \omega_z(N_z + 1),
\]

where \(N_i = n_i^\lambda + n_i^\rho\) are the sum of the oscillator quanta of the intrinsic motion in the Jaccobi coordinates, \(\lambda\) and \(\rho\). We then take the variation of the intrinsic energy (4) with respect to deformation \((\omega_x, \omega_y, \omega_z)\) for each principal quantum number, \(N = N_x + N_y + N_z\). The results of this variation is summarized in Table 1. The lowest three states are \(N = 0, 1\) and \(2\). Their shapes are spherical for \(N = 0\), prolately deformed with the ratio of 2 to 1 for \(N = 1\) and prolately deformed with the ratio of 3 to 1 for \(N = 2\). It is then natural to identify the \(N = 0\) state with the nucleon, the \(N = 1\) state with the negative parity states around 1520 MeV and the \(N = 2\) state with the positive parity excited states.

We have to perform the angular momentum projection when the intrinsic state is deformed. This projection can be done by using the Hill-Wheeler projection method \[11\]. The excitation energy of a state of orbital angular momentum \(L\) is written as

\[
E(N, L) = E_{\text{int}}(N) + \frac{\hbar^2}{2I} \left[ L(L + 1) - \langle L^2 \rangle \right]
\]

\(E_{\text{int}}\) is obtained by the minimization condition for each \(N\). The moment of inertia \(I\) is calculated using the cranking formula \[10\]. \(\langle L^2 \rangle\) is the average angular momentum of the intrinsic state. These quantities are tabulated in Table 1.

Coupling the intrinsic spin \(S\) of three quarks with the orbital angular momentum \(L\), take for example \(S = 1/2\), we construct mass spectra for nucleon excited states as shown in Fig. 1. On the left hand side we show the theoretical results: one for the positive parity states of \(L = 0, 2, 4, \cdots\) and the other for the negative parity states of \(L = 1, 3, \cdots\). In the theoretical side, two spin states \(J = L \pm 1/2\) degenerate when spin-orbit coupling
is ignored, as experimental data suggest. On the right hand side, experimental masses of well observed nucleon excited states with four stars are shown [5]. One exception is the \(5/2^-\) state of \(D_{15}(2200)\) with two stars. This state is very likely to form a spin doublet with \(G_{17}(2190)\). We do not list all the states but those which are well identified with the \(^2S^+\) representations \((S = 1/2)\) of the spin-flavor group, and are well compared with the DOQ model predictions. The mass formula (5) contains two parameters; one is the constant energy which determines the absolute value of the ground state energy and the other is the oscillator parameter \(\omega_0\). Hence Eq. (5) is essentially the formula with one parameter \(\omega_0\). The oscillator parameter \(\omega_0\) is determined here by the average mass splitting between the first excited states of \(1/2^+\) (Roper like states) and the corresponding ground states for the flavor SU(3) baryons (for details of SU(3) baryons, see the discussion below). The resulting value is \(\omega_0 = 607\) MeV.

Considering the simplicity of the DOQ model, it is remarkable that many observed states fit very well to the theoretical rotational band. In particular, the first \(1/2^+\) excited state, the Roper resonance, appears near the first \(1/2^-\) excited states. In the DOQ model, the energy subtraction due to the angular momentum fluctuation, \(\hbar^2/2I\langle L^2 \rangle\), is significant. The sum of the rotational energy and the energy subtraction results in the theoretical Roper state very close to the observed one. If we look at the spectra in more detail, theoretical masses of \(1/2^-\) states appear lower than experiments. Shifting up the theoretical masses by a constant amount, the agreement would become better.

If the fundamental structure of the excitation spectrum is produced by gluon dynamics through confinement and chiral symmetry breaking, we should see a similar pattern in other members of the spin-flavor group. For this purpose, we tabulate in Table 2 well observed SU(3) baryon excited states with three or four stars up to about 2 GeV. In Figs. 2, the masses of the excited states \((M^*)\) measured from the corresponding ground state \((M)\) are shown as compared with theoretical predictions with the oscillator parameter \(\omega_0 = 607\) MeV as before. Figs. 2a and 2b are for positive and negative parity states. There we have shown those states which seem to be well identified with the rotational band of the DOQ model. They are the representations which include the ground state: \(^2S^+\) for \(N, \Lambda\) and \(\Sigma\), and \(^4^{10}\) for \(\Sigma\) and \(\Delta\). For correspondence, we also show \(^210\) of negative parity \(\Delta\). Here the notation is for the spin-flavor group, \(^{2S+1}D\), where \(S\) is the total spin.
of the three quarks and \( D \) the representations of SU(3). States are then specified by the
coupling of \( S \) and \( L \). Now we explain the identification of various states.

**Ground states**

Because the three quarks are in the lowest \( S \)-state and form the symmetric spatial wave
function, the possible spin-flavor states are \( ^2S \) and \( ^4S \). They form the \( 56^+ \) dimensional
representation. We do not discuss the mass differences within this multiplet, but rather
the masses of the ground states are fixed as inputs on which excited states are constructed.
Although multiplets such as \( N(^4S), \Lambda(^2S), \Sigma(^4S), \Delta(^2S) \) are not allowed for the ground
states (this is shown in Table 2 by the symbol \( -- \)), we keep those columns when the
corresponding states exist in excited states.

**Even parity excited states**

\( N = 2 \) excited states yield the rotational band of \( L = 0, 2, 4, \cdots \). Although, the spatial
wave function can be either symmetric or mixed symmetric, we simply try first to put
excited states into the spatially symmetric states. Then if there remain some, they will be
identified with other allowed multiplets such as \( ^4S \ (S = 3/2) \). It is interesting that many
of the well observed excited states of the nucleon (\( N \)) fit into the rotational band of \( ^2S \).
One exception is \( P_{11}(1710) \) which would be identified with \( ^4S \). For the lambda (\( \Lambda \)) sector,
again, more than half of the well observed states are identified with the \( ^2S \) rotational
band. The \( ^4S \ \Lambda \) states have larger masses than those of the \( ^2S \) multiplets. There should
be spin correlations, which provide the difference between these multiplets. For \( \Sigma \) states,
in addition to the \( ^2S \) rotational band, there seems to be a strong evidence that there is a
rotational band of the decouplet \( ^4S \). In order to emphasize this, we have also listed the
one or two star states, \( P_{13}(1840), F_{15}(2070) \) and \( P_{13}(2080) \). In particular, it is interesting
that the one star state \( P_{13}(1840) \) is likely to be identified with the Roper state of the
decouplet \( \Sigma \). In the delta (\( \Delta \)) sector, all the excited states up to \( L = 2 \) are observed and
fit into the \( ^4S \) rotational band.

**Odd parity excited states**

\( N = 1 \) excited states yield the rotational band of \( L = 1, 3, 5, \cdots \). For \( L = 1 \), the spatial
wave function is mixed symmetric because the symmetric one is for the spurious center
of mass motion, and therefore, the allowed spin-flavor states are \( ^2S, ^4S, ^2P \) and \( ^2P \). They
form the \( 70^- \) dimensional representation. For the nucleon sector, five excited states are
clearly observed which may fit into $^2S^2$ and $^4S^2$ of $L = 1$. As in the case of positive parity $\Lambda(\Lambda(4S))$, the masses of negative parity $N(4S)$ are larger than those of the $^2S^2$ multiplets. Again the difference between these multiplets have to be reproduced with spin correlations. For the lambda sector, we have identified the observed seven states with $^2S^2$, $^4S^2$ and $^2P^2$. The $\Lambda(1405)$ and $\Lambda(1520)$ are identified with $^2P^2$, but are out of the systematics of the DOQ model, as they are much lighter than the model prediction. As discussed many times, these resonances, particularly $\Lambda(1405)$ would have a large $KN$ component rather than the single particle quark state [12]. For the $\Sigma$ sector, we have identified three states with $^2S^2$ and $^4S^2$ of $L = 1$. However, we can not exclude other possibilities, for instance, that they would belong to the same multiplet of $^4S^2$. For the delta sector, two states are identified with $^2P^2$ of $L = 1$. There are several well known states for $\Xi$, but we do not list them here. There are three states that we do not list in the table (below 2 GeV): $\Delta S_{31}(1900)$, $\Delta D_{35}(1930)$ and $\Sigma D_{13}(1940)$. It seems to be difficult to put them in the systematics of the DOQ model.

In the present study of excited baryons, we emphasize that most of the well observed states fit well to the predictions of the DOQ model. Furthermore, the rotation like structure seems to be rather universal in flavor SU(3), and therefore, it is strongly implied that the dynamics of excited baryons are dominantly flavor independent as provided by quark-gluon interactions.

In conclusion, we have studied the baryon spectra using the non-relativistic deformed oscillator quark model. We find that excited states of various spin-flavor multiplet are deformed. The comparison of the spectrum of the DOQ model with experimental data is very good. In order to establish the underlying mechanism of the baryon spectra, we have to get more information experimentally as the higher spin states and gamma transitions. Theoretically we have to calculate the gamma transitions to distinguish various models, which are being studied.

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References
[1] H. Toki, J. Dey and M. Dey, Phys. Lett. B133 (1983) 20.

[2] R.K. Bhaduri and J.C. Waddington, Phys. Rev. D30 (1984) 152.

[3] N. Isgur and G. Karl, Phys. Rev. D18 (1978) 4187.

[4] R. Bijker, F. Iachello and A. Leviatan, Annals of Phys. 236 (1994) 69.

[5] R.M. Barnett et al. (Particle Data Group), Phys. Rev. D54 (1996) 1.

[6] H. Suganuma, S. Sasaki and H. Toki, Nucl. Phys. B435 (1995) 207; H. Toki, H. Suganuma and S. Sasaki, Nucl. Phys. A577 (1994) 353c

[7] S. Kitahara, Y. Matsuyanagi and T. Suzuki, Prog. Theor. Phys. 93 (1995) 1; O. Miyamura, Nucl. Phys. B42 (1995) 538c

[8] T. Chen and L. Li, ”Gauge Theory of Elementary Particle Physics”, Oxford Science Pub. (1986).

[9] W. Buchmueller, Phys. Lett. 112B (1982) 479.

[10] A. Bohr and B. Mottelson, ”Nuclear Structure”, Benjamin Inc. (1975).

[11] R.E. Peierls and J. Yoccoz, Proc. Phys. Soc. London 70 (1957) 381.

[12] E.A. Veit, B.K. Jennings, R.C. Barret and A.W. Thomas, Phys. Lett. B137 (1984) 415; E.A. Veit, B.K. Jennings, A.W. Thomas and R.C. Barret, Phys. Rev. D31 (1985) 1033; A. Mueller-Groeling, K.Holinde and Speth, Nucl. phys. A513 (1990) p557; C.L. Schat, N.N. Scoccola and C. Gobbi, Nucl. Phys. A585 (1995) 627.
Table caption

Table 1: The properties of the deformed oscillator quark (DOQ) model for each principal quantum number $N$. $E_{\text{int}}$ denotes the intrinsic energy in unit of $\hbar\omega_0$ with the ratio of the oscillator parameter, $\omega_x : \omega_y : \omega_z$, which provides the shape of the intrinsic state. When the intrinsic state is deformed as indicated in the column of shape, the rotational states appear with the moment of inertia, $\hbar^2/2I$. $\langle L^2 \rangle$ denotes the average value of the orbital angular momentum for each deformed intrinsic state.

Table 2: Baryon states of $uds$ flavors are categorized to the predictions of the DOQ model. All the details of the identification of the experimentally observed states in this table are described in the text.

Figure caption

Figure 1: The energy spectrum of nucleon resonances for positive and negative parity states in unit of GeV. In the theoretical spectrum, $L$ for each state denotes the orbital angular momentum. The intrinsic spin, $S = 1/2$, is coupled to $L$ to form total spin states, which are denoted in the right hand side of each state. The rotational bands are formed on the first excited positive and the negative parity states. The experimental spectrum is shown in the right hand side of this figure.

Figure 2: The mass spectrum of the positive and the negative parity states for $uds$ baryons in unit of GeV. The masses are measured from the ground states of each spin-flavor quantum number. The theoretical prediction is shown in the left part for each parity.
Energy spectrum of nucleon resonances

Figure 1
Figure 2
| $N$ | $E_{\text{min}}[\hbar \omega_0]$ | $\omega^{-1}_x: \omega^{-1}_y: \omega^{-1}_z$ | shape | $\hbar^2/2I[\hbar \omega_0]$ | $\langle L^2 \rangle$ |
|-----|-------------------------------|--------------------------------|-------|-----------------------------|----------------|
| 0   | 3                             | 1:1:1                         | spherica |                              | 0              |
| 1   | 3.780                         | 1:1:2                         | prolate  | 0.126                       | 3              |
| 2   | 4.327                         | 1:1:3                         | prolate  | 0.072                       | 8              |

Tablê
| Evenparity baryons | N | Λ | Σ | Δ |
|-------------------|---|---|---|---|
| N = 0 (Ground state) | | | | |
| L = 0 | P_{11}(939) | P_{01}(1116) | P_{11}(1189) | P_{13}(1385) |
| L = 2 | F_{15}(1680) | F_{05}(1820) | F_{15}(1915) | F_{15}(2070) * |
| L = 4 | H_{19}(2220) | H_{05}(2350) | | |

| Oddparity baryons | N | Λ | Σ | Δ |
|-------------------|---|---|---|---|
| N = 1 | | | | |
| L = 1 | D_{13}(1520) | D_{15}(1675) | D_{13}(1670) | D_{15}(1775) |
| L = 3 | G_{17}(2190) | G_{19}(2250) | | |
| L = 4 | H_{19}(2220) | H_{05}(2350) | | |

Table 2