Cavity-enhanced field-resolved spectroscopy

Philipp Sulzer 1,2,3,4,8✉, Maximilian Högener 1,2,8✉, Ann-Kathrin Raab1,2,5, Lukas Fürst2,6, Ernst Fill1,6, Daniel Gerz1,2,7, Christina Hofer1,2, Liudmila Voronina1,2 and Ioachim Pupeza1,2✉

Femtosecond enhancement cavities are key to applications including high-sensitivity linear and nonlinear gas spectroscopy, as well as efficient nonlinear optical frequency conversion. Yet, to date, the broadest simultaneously enhanced bandwidths amount to <20% of the central optical frequency. Here, we present an ultrabroadband femtosecond enhancement cavity comprising gold-coated mirrors and a wedged-diamond-plate input coupler, with an average finesse of 55 for optical frequencies below 40 THz and exceeding 40 in the 120–300 THz range. Resonant enhancement of a 50-MHz-repetition-rate offset-free frequency comb spanning 22–40 THz results in a waveform-stable ultrashort circulating pulse with a spectrum supporting a Fourier limit of 1.6 cycles, enabling time-domain electric-field-resolved spectroscopy of molecular samples with temporally separated excitation and molecular response. The contrast between the two is improved by taking advantage of destructive interference at the input coupler. At an effective interaction length with a gas of up to 81 m, this concept promises parts-per-trillion-level ultrabroadband electric-field-resolved linear and nonlinear spectroscopy of impulsively excited molecular vibrations.

The spatial confinement of laser light propagating within passive free-space optical resonators enables multiple interactions of the circulating radiation with atoms, molecules or clusters located in the beam path. This underlies sensitivity enhancement in techniques like cavity-enhanced absorption spectroscopy, cavity-ringdown spectroscopy and noise-immune cavity-enhanced optical heterodyne molecular spectroscopy and cavity-enhanced Vernier spectroscopy. The same principle enables driving (extreme) nonlinear processes, like high-harmonic generation, at multi-megahertz pulse repetition rates, generating broadband frequency combs in the vacuum/extreme ultraviolet.

Broadband passive enhancement in a femtosecond enhancement cavity (fsEC), however, imposes challenging requirements for the mirrors, namely, simultaneous broadband reflectivity and flat spectral phase. Consequently, typical fsECs in the ultraviolet, visible and infrared (IR) spectral regions rarely simultaneously enhance comb bandwidths broader than 10% of the central frequency, with a record value of less than 20% demonstrated in the near-IR region. Furthermore, fsECs have so far been exclusively used in combination with time-integrating detection techniques (such as Fourier-transform infrared (FTIR) spectroscopy), requiring the detection of weak sample responses on top of often orders of magnitude stronger excitation intensities. This challenge to the dynamic range of linear detection becomes particularly severe with the advent of novel, (multi)-Watt-level mid-IR frequency combs.

In this Letter, we introduce ultrabroadband cavity-enhanced field-resolved spectroscopy, combining two cutting-edge photonic technologies to overcome both aforementioned shortcomings. First, employing gold-coated mirrors and reflective input coupling via a wedged diamond plate oriented close to Brewster’s angle, we realized an fsEC with a finesse exceeding 40 over multiple octaves, and demonstrated the simultaneous enhancement of an offset-free frequency comb in the 22–40 THz range. Second, the pulse-to-pulse carrier-envelope-phase (CEP)-stable electric field of this offset-free comb enables time-domain optical-waveform detection via free-space electro-optic sampling (EOS). This leverages the unparalleled dynamic range of linear detection and the excitation-power scalability afforded by EOS in the molecular fingerprint region. The combination of fsEC, impulsive excitation and EOS possesses the additional advantages of temporal separation of the resonantly enhanced sample-specific molecular response from the few-cycle excitation and the potentially strong attenuation of the few-cycle excitation owing to destructive interference at the input coupler (IC).

The working principle of an fsEC resonantly seeded by a train of CEP-stable pulses (red) is illustrated in Fig. 1a. Provided the comb lines of the driving laser overlap with the fsEC’s resonances, the seeding and circulating pulses constructively interfere inside the cavity, whereas the immediate reflection of the seeding pulses off the IC interferes destructively with a portion of the circulating field transmitted through the IC. In the case of a perfectly impedance- and mode-matched fsEC, this leads to a complete suppression of the reflected field. This changes, however, when a gas with optically active molecular vibrations is introduced in the beam path. The coherent electric field emitted as a consequence of the impulsive vibrational excitation (that is, the molecular response) consists of long-lasting oscillations (green), which, after excitation, lack a matching counterpart to destructively interfere with at the IC. Consequently, the beam reflected off the fsEC contains the enhanced molecular response and suppresses the excitation pulse.

The field-resolved spectroscopy setup, which we used to elucidate these phenomena in the time domain, is sketched in Fig. 1b. A fibre-laser front end generates a frequency comb with controllable repetition rate and offset frequency (Methods). Intrapulse difference-frequency generation (IPDFG) in a 1-mm-thick GaSe crystal inherently generates an offset-free frequency comb with an average power of approximately 0.1 W, which then seeds the fsEC. Figure 1c depicts the fsEC design. Reflective input/output coupling by means of a wedged diamond obviates the need of a partially transmissive input coupling mirror, enabling the exclusive...
Fig. 1 Concepts and experimental implementation. a, Principle of the time-resolved response of an impedance-matched passive fsEC to a seeding train of ultrashort pulses. The fsEC is placed in a chamber containing molecules with IR-active vibrations. Red, exciting pulses; green, re-emitted electric field (molecular signal); black (dashed), IC reflection. b, Experimental setup with the fundamental frequency comb (blue) generated by an Er:fibre oscillator with nonlinear frequency conversion in a highly nonlinear fibre (HNF) and Tm:fibre-based chirped-pulse amplification (green) followed by nonlinear pulse compression in a photonic crystal fibre (PCF). IPDFG in a 1-mm-thick GaSe crystal generates an offset-free frequency comb in the mid-IR spectral region (red), which seeds the fsEC. Additional seeding with the fundamental near-IR (NIR) comb enables the generation of a Hänsch–Couillaud (HC) error signal in the near-IR region (Methods). EOS analyses the fsEC output. f–2f interferometry controls the fundamental comb offset frequency \( f_0 \). c, fsEC design consisting of four gold mirrors and a 500-μm-thick, 1°-wedged diamond plate. d, Schematic of the near-IR (blue) and mid-IR (MIR; red) comb modes versus cavity resonances (grey). The HC lock controls \( f_{\text{rep}} \) to overlap the cavity resonances and the near-IR comb modes (right; passing a 12-nm-wide, 1,550-nm-centre-wavelength bandpass filter), fixing the relative position of the mid-IR resonances and offset-free comb modes (left) in the frequency space. Tuning \( f_0 \) thus, results in a linear change in \( f_{\text{rep}} \), bringing the mid-IR comb lines into resonance with fsEC. e, Intensity at the fsEC transmission port for near-IR (blue) and mid-IR (red) and HC error signal (cyan) as a function of \( f_{\text{rep}} \).

The use of unprotected gold-coated mirrors. It, furthermore, avoids etalon effects caused by spatially overlapping multiple reflections in a parallel-plate IC\(^{29}\). In the following, the terms reflection port and transmission port are used in relation to the fsEC (Fig. 1c) to describe the physical effects at the respective ports. This nomenclature is consistent with the fsEC literature, which mostly uses transmission through one of the cavity mirrors for input coupling (Fig. 1a). In our coupling scheme, the beam transmitted through the IC instead of the one reflected off the IC interferes with the pulses coupled out of the cavity. Input coupling takes place at the upper interface of the wedge (Fig. 1a, inset), whereas the lower interface takes on the role of an output coupler.

Figure 1d demonstrates how locking the fundamental near-IR comb to the cavity creates a linear link between the independent degrees of freedom \( f_{\text{rep}} \) and \( f_0 \) of the oscillator. Adjusting \( f_{\text{rep}} \) maintains the lock in the near-IR regime, and recording the spectra transmitted through the fsEC allows for scanning over the resonances and thus gaining information about finesse and dispersion of the fsEC (Fig. 2). The gold and diamond optics exhibit low dispersion across a broad bandwidth, allowing for the simultaneous enhancement of more than three octaves, provided that the seeding comb modes can be overlapped with the cavity resonances by tuning both \( f_{\text{rep}} \) and \( f_0 \) (Fig. 2a, left). Our IPDFG comb, however, has zero offset frequency, which inherently cannot match the optimum offset frequency (OOF\(^{30}\)) required by the empty cavity (Methods). We overcome this obstacle by filling the cavity with a gas (here humidity-controlled air) and thus shifting the fsEC’s OOF and introducing minimum dispersion in the 8–15 THz range. This procedure enables the enhancement of a nearly octave-spanning spectrum (theory, Fig. 2a (grey areas, right); experiment, Fig. 2b). For a proof-of-principle spectroscopic measurement, we additionally introduced 1 ppm methanol into the chamber as an IR-active molecular sample.

We characterized the fsEC by recording the mid-IR spectra using a commercial FTIR spectrometer at the fsEC transmission port as \( f_0 \) is tuned (Methods). Figure 2b shows the power spectral density (PSD) enhancement as a function of the resulting change in \( f_{\text{rep}} \) determined from these data. The simultaneous enhancement over the 22–40 THz band agrees well with the numerical model (the red dashed line tracks the position of maximum enhancement; Methods). The fsEC finesse was also obtained from the measured spectra and is depicted together with matching theoretical
predictions for the empty and (humidity-controlled) air-filled chamber and the input spectra (Fig. 2c).

We analysed the waveforms reflected by the fsEC (with locking parameters enabling maximum enhancement) using EOS. They consist of the excitation pulse, followed by the much weaker molecular response (Fig. 3). The black line represents data recorded when the fsEC was tuned in the off-resonance condition (Fig. 3a, left inset). Owing to the high transmissivity of the IC, this beam corresponds to a single pass of almost the full seeding intensity through approximately 6.5 m of the gas-filled chamber. The red dataset (Fig. 3a, right inset) is measured with the comb locked to the ultrabroadband working point described above, and features a reduced maximum amplitude. This attenuation (Fig. 3b) is due to the destructive interference between the output-coupled intracavity beam and the portion of the seed transmitted through the IC (Fig. 1a). Due to the imperfect transverse mode and impedance matching, the reduction in amplitude in the main pulse amounts to only 38%, which, nevertheless, corresponds to an intensity suppression of more than 60%. The molecular response generated inside the fsEC leaks out without destructive interference (Fig. 3c). The improved ratio between the long-lasting oscillations rich in spectroscopic information and the comparatively strong excitation benefits power scaling and relaxes the demands on the dynamic range of the detection scheme. Note that interferometric suppression of the excitation profits both time-resolved spectroscopy where it adds to the advantages of temporal filtering and time-integrating detection schemes.

Further analysis of these traces was performed in the frequency domain. Figure 4a shows the amplitude spectra corresponding to a single pass through the chamber (path length, 6.5 m; black) and to pulses recorded in the on-resonance condition (red). The latter is interference between the single pass and output-coupled portion of the circulating field. The availability of amplitude and phase information through EOS detection readily allows for the reconstruction of the intracavity pulses by forming the difference between the single-pass and on-resonance signals (blue). The numerical prediction for the intracavity spectrum is depicted as a dashed grey line, and the discrepancy between theory and experiment is attributed to imperfect mode matching (Methods). The spectrum of the single-pass EOS signal (Fig. 4a, inset) shows only weak resonant features connected to molecular vibrations. The on-resonance signals, on the other hand, feature these narrowband resonances stronger due to the increased effective path length. Sampling the electric field in the time domain enables the temporal filtering of the recorded signals, isolating molecular emission from the excitation16 (Fig. 4b). This step reveals a plethora of emission peaks at frequencies coinciding with the HITRAN database17 absorption data for water (14% humidity), CO2 (360 ppm) and methanol (1 ppm), as expected from the calculations (Methods). The dominant peak from the molecular re-emission of methanol close to 30 THz is enhanced by a factor of 6 compared with the single-pass data. This value is compatible with PSD enhancement (Fig. 2), taking into account the imperfect transverse mode matching (estimated to be 40%; Methods).

In conclusion, we have demonstrated the passive enhancement of nearly octave-spanning, waveform-stable few-cycle mid-IR pulses in a low-dispersion fsEC, and quantitative cavity-enhanced field-resolved vibrational spectroscopy of gases. Our setup features, to the best of our knowledge, the broadest simultaneously enhanced frequency comb demonstrated so far, as well as the first fsEC for a frequency comb covering wavelengths beyond 4 µm, including the difficult-to-access region above 10 µm.

The enhancement of waveform-stable pulses permitted the detection of optical waveforms carrying a cavity-enhanced molecular response via EOS for the first time. We directly observed the time-dependent interference at the IC, showing an improvement of
the contrast between the cavity-enhanced molecular response and the ultrashort excitation reflected by the fsEC. This effect relaxes the dynamic range requirements for linear detection via EOS and potentially grants access to the leading higher-amplitude part of the molecular response to an ultrashort excitation in the presence of excessive excitation intensity noise. Combined with recent improvements in EOS sensitivity in the mid-IR spectral region, broadband spectroscopy of mixed gaseous samples with detection limits in the lower parts-per-trillion range (Methods) come into reach.

This extension of fsEC technology, providing optical-field enhancement of few-cycle pulses at high repetition rates, to the long-wave mid-IR range promises novel applications. These include—but are not limited to—the high-precision investigation of nonlinear light–matter interactions, aided by the tunable CEP, and their exploitation for frequency conversion as well as the exploration of low-energy excitations in non-equilibrium condensed-matter systems.

Online content

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Fig. 4 | Spectra of EOS traces measured at the fsEC reflection port. a, Spectral amplitudes computed from the EOS traces of the excitation pulse and molecular response (sketch). The red line indicates the reflection off the fsEC with the seeding comb locked to a setting that enables the maximum power enhancement around 30 THz. The black line indicates propagation through the gas-filled chamber and cavity IC with the fsEC in the off-resonance condition (solid grey, corresponding simulation). The blue line indicates the difference between the single pass and fsEC reflection, which is proportional to the circulating spectrum (dashed grey, corresponding simulation assuming perfect mode matching; dotted grey, simulation accounting for the mode-matching factor; Methods). The green areas mark the spectral region in which the fsEC enhancement vanishes (due to water-vapour and CO₂ resonances). The inset shows the data normalized to the expected spectral amplitude at 31 THz assuming negligible absorption by methanol and illustrates the 12-fold increase in the relative dip depth (unaffected by imperfect mode matching), in agreement with the numerical model (dashed lines). b, Spectral amplitudes of the EOS traces with the excitation removed by a soft window (sketch in inset and Methods), showing a sixfold improvement owing to limited mode matching. The grey lines indicate numerical predictions for both single pass (solid) and circulating (dotted; includes mode matching) molecular re-emissions (Methods). The acquisition time of the measurements is 40 s and the frequency resolution is 2.5 GHz.
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Methods

The laser system including mid-IR generation and EOS is based on a commercial Er: fibre laser (Menlo C-Fiber, gain = 100 MHz). Pulse picking reduces the repetition rate to 50 MHz and a nonlinear Raman fibre shifts its spectrum to a wavelength of 1,965 nm. Employing these seeding pulses, a chirped-pulse-amplification system with two Ti: sapphire amplifiers yields an output power of up to 40 W at a pulse duration of 250 fs. In two separate branches, photonic crystal fibres are employed for nonlinear self-compression at a centre wavelength of 2 µm (ref. 12). The 30 fs output drives the IPDFF and the 13 fs output is used for temporal gating in EOS. For EOS, the mid-IR pulses from the fsEC reflection port are spatially and temporally overlapped with the near-IR gate pulses in a 541-µm-thick GeSe crystal. The second-order nonlinearity of this crystal maps the electric field of the mid-IR pulses into a polarization change of the near-IR beam. The electro-optic signal is recorded using balanced detection and by scanning the gate-pulse delay with a mechanical stage13.

Locking the Er: fibre laser to the fsEC. The polarization discrimination introduced by the wedged IC lends itself to the implementation of the Hänisch–Couillaud locking scheme14. Its error signal acts on the fundamental frequency comb’s repetition rate via actuators provided by the manufacturer. The seeding frequency comb system also provides independent control over f0.

Parameters of empty fsEC. The 6-m-long fsEC (Fig. 1c) comprises two plane mirrors, two 1,000-mm-radii-of-curvature spherical mirrors placed 1,064 mm apart and wedge-diamond IC (wedge angle, 1°; thickness, 300 µm). The mid-IR seeding beam impinges on the IC at an angle of incidence (56.8°) offset from Brewster’s angle (67.2°), with a measured reflectivity of R0 = 4.3% (Fig. 1c, inset). After traversing the fsEC, the beam is reflected at the first surface of the IC into the fsEC transmission port (R1 = 2.7%) and at the second surface into the fsEC reflection port (R2 = 4.0%). Absorption (0.7%) and Fresnel (R0 + R2) losses in the diamond wedge and 1.0% loss at each gold mirror amount to a total round-trip loss of the empty fsEC of 11.0% (that is, the round-trip transmission provided by the manufacturer (Diamond Materials). Diamond was chosen as the index (1.0–100.0 μm) and absorption (2.5–15.0 µm) data of the diamond IC, respectively. However, a resonator with a vanishing total geometrical phase and slippage (1–0.004) and absorption losses on the IC, round-trip phase ϕ0, seeding-pulse-train repetition period τ0, and CEP slippage Δϕ. The values for R0, R1, τ0, ϕ0, and were obtained as described in the ‘Parameters of empty fsEC section’. For τ0, ϕ0, and, we additionally considered the humid-air complex refractive index, assembled from the complex line profiles with Lorentz parameters from HITRAN1,2, for water vapour (14% relative humidity), CO2 (360.0 ppm) and CH4 (1.8 ppm) at standard temperature and pressure, as well as the Gouy phase shifts for the GH00 eigenmode (determined by the fsEC geometry; ‘Parameters of empty fsEC section’). Water-vapour and CO2 concentrations were chosen for best agreement with the experimental data. The pulse-train parameters τ0, and Δϕ are related to the comb parameters f0 and f0, and were optimized for best enhancement around the optical frequencies marked with the red star symbols in Fig. 2a. The PSD enhancement e(ν) for f0 = Δϕ = 0 was computed versus ν and f0, and the position of maximum enhancement for each value of ν was determined and depicted as a red line for comparison with the experimental data (Fig. 2b). Figure 2c shows the simulated finesse: F ≡ 1/2πτ0 IC(ν). Both datasets were calculated with a frequency resolution of 342 MHz.

Finesse and PSD enhancement measurement. The PSD at the fsEC transmission port, S0(v), was measured using an FTIR spectrometer while the near-IR fundamental comb was locked to the cavity and its f0 was tuned, leading to a change in f0 of both fundamental near-IR and mid-IR combs. In turn, the repetition rate f0 was computed versus ν and f0, and the position of maximum enhancement for each value of ν was determined and depicted as a red line for comparison with the experimental data (Fig. 2b). Figure 2c shows the simulated finesse: F ≡ 1/2πτ0 IC(ν). Both datasets were calculated with a frequency resolution of 342 MHz.

OOF. Because the seeding comb obtained by IPDFF is intrinsically offset free, the fsEC must be engineered for an OOFS to zero to enable simultaneous broadband enhancement. The OOFS is determined by the round-trip spectral phase of the resonator, which has contributions from the optics itself and the geometrical phases acquired during propagation. Our optics have negligible dispersion (‘Parameters of empty fsEC section’) and only a very small CEP shift (four gold-coated mirrors without a protective layer, −0.2°; 300-µm-thick diamond, 2.5°; determined for the measured seed spectrum described above). The total round-trip CEP shift is, therefore, nearly exclusively determined by the geometrical (Gouy) phases ϕ = (n + m + 1) ∑n argn f + argn f, where n and m are the transverse-mode numbers of the excited Hermite–Gaussian cavity eigenmode, and gama, and, are the complex beam parameters after the first and before the second optical element for the ith free-space propagation inside the resonator, respectively. However, a resonator with a vanishing total geometrical phase and optical elements that do not induce a CEP shift is inherently unstable for the fundamental transverse mode: due to the (n + m + 1) prefactor, the geometrical phase equals to zero for all the transverse modes, that is, all the modes would be simultaneously resonant. For a bowtie cavity, it is well known that this situation is only possible at the resonator’s stability edge15. To address this problem, the resonator can, in principle, be operated with a higher-order transverse mode (at the expense of achievable enhancement due to the reduced spatial overlap with the seed), or an additional optical element with a larger CEP shift can be introduced (in general, at the expense of a decrease in bandwidth due to increased dispersion). Here we chose the latter option, and filtered the chamber with humid air (CEP shift of 132.2° for 14% humidity and 360 ppm CO2) to permit a round-trip CEP shift of zero and broadband operation in a stable resonator with the fundamental Hermite–Gaussian transverse eigenmode. The humidity was controlled by filling the chamber with nitrogen and mixing in humid air until broadband operation was achieved. The relative humidity was measured with a consumer-electronics sensor (16.0 ± 3.8%) and was found to be in line with the value expected from the numerical model (14.0%).
of the strong molecular absorptions due to water vapour and CO₂ (20–23 and 37–40 THz), we found an underestimation, by at most 15%.

Path-length enhancement. The path-length enhancement \( \eta \) relates to the PSD enhancement \( \epsilon \) and input/output coupling ratios of the fsEC as follows: for an absorber in the cavity, the on-resonance field enhancement \( \sqrt{\epsilon} \) is equal to \( \sqrt{\eta} \sqrt{\eta_{\text{occ}}} \). The total cavity transmission coefficient is obtained as the product of field enhancement \( \sqrt{\epsilon} \) and output-coupler transmission coefficient \( \sqrt{\eta_{\text{occ}}} \). For small \( n \), \( \eta_{\text{occ}} \approx 1 - 0.5H_{\text{in}}/\eta \sqrt{\eta_{\text{total}}} \). Comparing with the corresponding single-pass value \( \eta_{\text{sp}} \), we find \( \eta_{\text{total}} \approx \eta_{\text{sp}} \) yields the path-length enhancement due to an absorber, that is, \( \eta \approx \eta_{\text{sp}} \). For a lossless cavity, \( \eta_{\text{total}} \approx 1 \), which yields the well-known formula \( \eta_{\text{sp}} = \sqrt{\eta_{\text{total}}} \) (for a travelling-wave cavity as in our case; for a standing-wave cavity, it is \( \eta_{\text{sp}}^2 \)). In our case, a substantial part of the seeding energy is dissipated in the cavity optics. Parameters of empty fsEC section) such that here the general formula for amplitude-change enhancement has to be invoked: \( \eta = \eta_{\text{sp}} \sqrt{\epsilon_{\text{in}} \sqrt{\eta_{\text{total}}} = \eta_{\text{sp}} \sqrt{\epsilon_{\text{in}}} \sqrt{\epsilon_{\text{out}}} \). For \( \epsilon_{\text{in}} = 13.6 \) (measured in the on-resonance condition at 30 THz; Fig. 2b), the expected path-length enhancement for detection via the fsEC reflection port \( (\eta_{\text{EC}} = \eta_{\text{sp}} = 2.7\%) \) would yield a slightly lower path-length enhancement of 10.8 instead of 13.1.

With 40% transverse-mode overlap (‘Mode matching’ section), the expected PSD enhancement and thus the path-length enhancement reduces to 13.1 \( \times \) 0.40 = 5.2 (Fig. 2b). In the case of perfect mode matching (for example, spatially filtered seeding model; ‘Mode matching’ section) and impedance matching (for example, custom-manufactured diamond wedge where the transmission port surface is placed under Breugel’s criterion), a cavity with four gold mirrors (loss, \( 1\% \) \( \times \) 2 = 2\%; \( 2\% \) \( \times \) 2 = 4\%) would permit power and path-length enhancement of 24. For the case of lossless mirrors, the 0.7% internal losses in the diamond-wedge IC would be the limiting factor for the path-length enhancement, yielding a value of 143.

Suitability of fsEC for ultrashort pulses. The circulating pulses in the cavity are subject to the dispersion of the fsEC transfer function \( f_{\text{sp}} \) (chosen for the maximum bandwidth). On resonance, the FTIR spectrum measured at the fsEC transmission port (‘Response and PSD enhancement measurement’ section) supports a Fourier limit of 53 fs (1.6 cycles at a carrier frequency of 31 THz). To estimate the effect of fsEC dispersion on pulse duration, we apply the spectral phase of the transfer function to the Fourier-limited spectrum, obtaining a slightly elongated 55 fs pulse.

Evaluation and simulation of EOS traces. To reproduce the single-pass spectrum with the numerical model (Fig. 4, grey line), the excitation of the measured single-pass EOS trace was isolated with a temporal filter (piecewise cosine window, 0- to 1-rise interval from 0.7 to 0.6 ps, 1-to-0 drop interval from 0.9 to 1.0 ps relative to the maximum of the trace). In the frequency domain, this operation results in a smooth version of the single-pass spectrum (without dips due to molecular absorption). Our dissipation of this estimate for the excitation through the fsEC chamber and EOS chamber is described by multiplication of the complex spectrum by \( \epsilon_{\text{in}} \sqrt{\eta_{\text{total}}} \) and \( \epsilon_{\text{out}} \sqrt{\eta_{\text{total}}} \), respectively, where \( \epsilon_{\text{in}} \) is the intracavity medium’s complex refractive index, including 1 ppm methanol (Model of fsEC section), and \( \epsilon_{\text{out}} \) is that for the EOS chamber (35% relative humidity of the ambient air, same CO₂ and CH₄ concentration, and no methanol). The path lengths in the fsEC chamber and EOS chamber are \( \ell_{\text{sp}} = 6.5 \) m and \( \ell = 1.0 \) m, respectively.

The resulting complex spectrum was then transformed back to the time domain, and the excitation was removed with the same soft window as used for the experimental data (cos² window with a 0- to 1-rise interval from 1.5 to 7.7 ps; Fig. 4b, inset). Figure 4b shows the agreement of the complex spectrum of the resulting trace with its measured counterpart. It must be stressed that the peaks in the simulated spectrum cannot be a remainder from the underlying single-pass EOS trace, because the two applied temporal filters (isolating the excitation and removing the molecular re-emission) are mutually exclusive. The EOS signal was recorded at the fsEC reflection port and the fsEC detuned from resonance is a good approximation for a single pass through the chamber because most of the seeding power is transmitted through the IC; inside the cavity, the pulses do not interfere constructively.

Likewise, we obtained the simulated complex spectra corresponding to the output-coupled circulating pulses (Fig. 4a, dashed grey line) as follows: we multiplied the excitation’s complex spectrum by the complex transmissions through the fsEC and EOS chambers (as mentioned above), on-resonance fsEC \( \sqrt{\epsilon_{\text{in}}} \sqrt{\eta_{\text{total}}} \) and EOS chambers \( \sqrt{\epsilon_{\text{out}}} \sqrt{\eta_{\text{total}}} \) (‘Model of fsEC section’). We simulated the output coupling coefficient \( \sqrt{\eta_{\text{occ}}} \), and we corrected for the wedge transmission coefficient included in the single-pass pathway \( (\sqrt{\epsilon_{\text{out}}} \sqrt{\eta_{\text{total}}} \). Figure 4a shows the agreement of the relative dip depths of the simulated versus measured spectra (inset). The ratio of the relative dip depths for enhanced and single-pass methanol signal is 11.5%/0.9% = 12.8, compared with an expected value of (1, 1 × 6 m)/\( \ell = 13.1 \), where \( \eta = 13.1 \) is the path-length enhancement (‘Path-length enhancement’ section).

Mode matching. The amplitude of the experimentally determined spectrum of the circulating pulses is smaller than the prediction from the numerical model, despite the fact that the measured and simulated finesse, as well as the derived PSD enhancement factor of the fsEC, agree well (Fig. 2). This discrepancy stems from the limited spatial overlap between the seeding transverse mode and cavity eigenmode, as well as the overlap between the beam from the fsEC reflection port and beam of EOS gating pulses. The ratio may be used to quantify the overall mode-matching factor combining seed pulses, fsEC and EOS detection. For this, we computed the squared ratio between simulated and measured intracavity complex spectrum (Fig. 4a). We obtained values between 35% and 70% in the 23–35 THz interval (40° at the position of the methanol feature, that is, 31 THz). In principle, the spatial overlap between the seed and fsEC may be increased further by filtering the seeding mode, for example, with a hollow-core single-mode mid-IR fibre\(^\text{a}\), which would also improve the observed interferometric suppression of excitation. Figure 4b shows the agreement between the simulated output-coupled spectrum (dotted grey), corrected by the smoothed frequency-dependent mode-matching factor, and the measurement, after removing the excitation from both. We attribute the remaining discrepancies of the peak heights (1) to the approximation of the exciting pulses by temporal filtering as described above (‘Evaluation and simulation of EOS traces’ section) and (2) to the uncertainty in the CO₂ and water–vapour concentrations, which were unlike methanol that was prepared with a standardized protocol for low concentrations—not independently calibrated with sufficient precision, but chosen for best agreement with Fig. 2b. Finally, the error on the frequency-resolved enhancement contributes, too.

Limit of detection. The signal-to-noise ratio for the methanol peak at 31 THz (Fig. 4b) is approximately 50, suggesting that the limit of detection for the setup presented here (consisting of seed source, fsEC and detection via EOS) is 20 parts per billion. To estimate the achievable limit of detection for methanol under optimum use of our fsEC for path-length enhancement, we combine the assumptions of (1) an effective path length of 81 m, which is a property of our fsEC (and is independent of the source and detection) with (2) the ability to measure changes in the relative amplitude in a spectral element on the order of 10\(^{-12}\), as recently demonstrated in the EOS of optical frequencies around 30 THz for an averaging time of 25 s and a 4.5 s scanned time window\(^\text{c}\). Under room temperature and ambient bar, the methanol absorption peak at 31 THz has an absorption coefficient of \( \alpha = 21 \) cm\(^{-1}\). With the assumptions (1) and (2) above, this corresponds to an estimated detection limit of 12 parts per trillion (that is, a partial methanol pressure of \( p = 12 \times 10^{-12} \) bar at \( l = 81 \) m effective path length causes a relative spectral amplitude change of \( a/2\pi \times 10^{-11} = 110\%\).
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**Author contributions**

P.S., M.H. and I.P. designed and planned the experiments. P.S. coordinated the experiments. P.S., A.-K.R., L.F. and M.H. performed the experiments and analysed the data. M.H. and A.-K.R. developed the model and performed the simulations. E.F. and L.V. contributed to the experimental concept. D.G. and C.H. provided and operated the amplifier system, mid-IR generation and EOS detection system. P.S., M.H., A.-K.R. and I.P. wrote the manuscript with input from all the other authors. I.P. supervised the project.

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**Competing interests**

The authors declare no competing interests.

**Additional information**

Correspondence and requests for materials should be addressed to Philipp Sulzer, Maximilian Högener or Ioachim Pupeza.

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