Over-the-Air Computation via Cloud Radio Access Networks

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Abstract—Over-the-air computation (AirComp) has recently been recognized as a promising scheme for a fusion center to achieve fast distributed data aggregation in wireless networks via exploiting the superposition property of multiple-access channels. Since it is challenging to provide reliable data aggregation for a large number of devices using AirComp, in this paper, we propose to enable AirComp via the cloud radio access network (Cloud-RAN) architecture, where a large number of antennas are deployed at separate sites called remote radio heads (RRHs). However, the potential densification gain provided by Cloud-RAN is generally bottlenecked by the limited capacity of the fronthaul links connecting the RRHs and the fusion center. To this end, we formulate a joint design problem for AirComp transceivers and quantization bits allocation and propose an efficient algorithm to tackle this problem. Our numerical results shows the advantages of the proposed architecture compared with the state-of-the-art solutions.

I. INTRODUCTION

It is expected that a huge number of Internet of things (IoT) devices will be connected to wireless networks to boost the proliferation of intelligent services in our daily life [1], [2], [3]. To realize this promising vision, one key challenge is the urgent need of ultra-fast wireless data aggregation, which pervades a wide range of applications, including massive machine type communication [4] and on-device federated machine learning [5]. As a result, we need to use wireless communication technology to quickly collect and process data distributed across a massive number of devices. However, it is difficult to use the conventional interference-avoiding channel access schemes to aggregate massive distributed data while achieving high spectrum utilization efficiency and low network latency [6], [7]. To overcome this challenge, a promising solution called over-the-air computation (AirComp) has recently emerged as a multi-access scheme that exploits the superposition property of multiple-access channels [8], [9], [10], [11].

There have been extensive research works on investigating the different network architectures for AirComp [6], [7], [8], [12], [13]. In particular, a single-antenna AirComp system was developed in [7] to minimize the computation error by jointly optimizing the transmit power at devices and a signal scaling factor at the base station (BS) by using channel-inversion power control method. For multiple-input multiple-output (MIMO) AirComp systems, zero-forcing precoding was designed at the transmitter and the multi-antenna server attempts to apply receive beamforming, called aggregation beamforming, to achieve simultaneous magnitude alignment of spatially multiplexed multiuser signals to receive parallel functional streams [14]. Further, intelligent reflecting surface (IRS) aided AirComp system was developed in [15], [16], [17], [18] to build controllable wireless environments, thereby boosting the received signal power significantly by optimizing the phase shifts matrix at IRS.

Despite of the previous research, alleviating the performance deterioration due to channel fading is still a great challenge for the existing AirComp systems [14]. The conventional massive MIMO AirComp system can not receive reliable signals transmitted by the devices which are far away from the BS due to the severe path loss. Hence, we propose to deploy Cloud-RAN architecture to support AirComp, in which the devices upload local data to the baseband unit (BBU) through distributed RRHs and the signal is transmitted through fronthaul links between RRHs and BBU [19]. Relying on a large number of RRHs geographically spread out over a region densely, Cloud-RAN facilitates scaling and increasing the baseband processing density and reducing path loss of the channels between the RRHs and devices. Furthermore, it also achieves a high diversity gain against channel fading by exploiting the independent fading of their signals to aggregate data more accurately in AirComp systems. However, in practice, the capacity of fronthaul links connecting the RRHs and the BBU in Cloud-RAN is limited, which limits the performance gain provided by dense RRHs [20], [21], [22].

To tackle the challenge due to limited fronthaul capacity, we formulate an optimization problem of joint devices’ transmit beamforming and RRHs’ quantization bits allocation and BBU’s receive beamforming design to minimize the mean square error (MSE) of AirComp. We propose an efficient solution to this complicate optimization problem. Specifically, the BBU jointly optimizes the transceivers and quantization bits allocation to minimize the MSE based on the alternating optimization technique. Furthermore, numerical results demonstrate that our proposed solution enjoys the near optimal performance, and Cloud-RAN architecture for AirComp outperforms the conventional massive MIMO architecture.

The rest of this paper is organized as follows. In Section II, we describe our system model of Cloud-RAN architecture for AirComp and formulate the MSE minimization problem. In Section III, we elaborate on our proposed optimization approach to the formulated problem. Section IV provides the simulation results to verify the effectiveness of Cloud-RAN architecture for AirComp. Finally, we conclude our work in

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can be written as $g(t)$ and varies from one to another. The received parameter at RRH $i$ is used in the case with quasi-static flat-fading channels, where the channel writing $g$ is based on the BBU aims to recover the variable which can be given in the transmit symbols are normalized to have unit variance.

denotes the transmitter scalar of device $i$, the signal received at the $m$th antenna of RRH $i$, where $x_k$ is the pre-processing function of device $k$. We denote $x_k(t) := \phi_k(\theta_k(t))$ as the transmit symbols. In this work, we assume that transmit symbols are normalized to have unit variance.

In order to estimate the target function at time slot $t$, the BBU aims to recover the variable which can be given in the form as $g(t) := \sum_{k \in N_W} x_k(t)$.

To simplify the notation, we omit the time slot index by writing $g$ and $x_k$ instead of $g(t)$ and $x_k(t)$. Then we consider the case with quasi-static flat-fading channels, where the channel conditions remain unchanged in a certain time slot but may vary from one to another. The received parameter at RRH $i$ can be written as

$$y_i = \sum_{k \in N_W} h_{i,k} b_k x_k + z_i, \quad i \in N_A$$

where $y_i = [y_{i,1}, \ldots, y_{i,M}]^T$ with $y_{i,m}, 1 \leq m \leq M$ denoting the signal received at the $m$-th antenna of RRH $i$, and $b_k$ denotes the transmitter scalar of device $k$, $h_{i,k} \in \mathbb{C}^{M \times 1}$ denotes the communication channel vector from the device $k$ to the RRH $i$ and the interference channel vector between the radar transmitter and the BS receiver, and $z_i \sim \mathcal{CN}(0, \sigma_z^2 I)$ denotes the additive white Gaussian noise at RRH $i$. We assume that $\{z_i\}_{i \in N_A}$ are independent over $i$.

Furthermore, in practice, each device $k \in N_W$ is constrained by a power budget $P_k$, i.e., the transmit power constraints of each device can be written as

$$\mathbb{E}(|b_k x_k|^2) = |b_k|^2 \leq P_k.$$  

Each multi-antenna RRH quantifies and forwards the received baseband symbol $y_i$ to the BBU via the limited-capacity fronthaul link. Each RRH first demodulates the signal received from each antenna to the baseband, and then conducts a scalar quantization over each output in parallel, and finally forwards the quantized bits to the BBU via the fronthaul link. Specifically, we apply the uniform quantization to each element of compressed signal $\hat{y}_i = [\hat{y}_{i,1}, \ldots, \hat{y}_{i,L}]^T$ at RRH $i$ via separate I/Q scalar quantization and the baseband quantized signal can be given as

$$\hat{y}_i = y_i + q_i = \sum_{k \in N_W} h_{i,k} b_k x_k + z_i + q_i, \quad i \in N_A$$

where $q_i = [q_{i,1}, \ldots, q_{i,M}]^T$ with $q_{i,m}, 1 \leq m \leq M$, modeling the quantization distortion for the $y_{i,m}$ as being independent of $y_{i,m}$ and distributed as $q_{i,m} \sim \mathcal{CN}(0, \omega_i)$. Let $C_{i,m}$ denote the number of bits that RRH $i$ uses to quantize the I-branch or Q-branch of $y_{i,m}$. According to the results of [21], the quantization noise level $\omega_{i,m}$ due to $q_{i,m}$ for uniform quantization can be written as

$$\omega_{i,m} = 3 \left( \sum_{k \in N_W} |e_m^T h_{i,k}|^2 |b_k|^2 + \sigma_z^2 \right)^{2-2C_{i,m}},$$

where $e_m$ denotes the unit vector whose $m$-th entry is 1.

Note that $q_{i,m}$’s are independent over $m$ due to independent scalar quantization for each element of $y_n$, and also over $i$ due to independent processing at different RRHs. Thus the covariance matrix of $q_i$ is a function of $b_k, k \in N_W$ as well as $C_{i,m}, i \in N_A$, which is given by

$$q_i = \text{diag}(\omega_{i,1}, \ldots, \omega_{i,M})$$

Then, each RRH forwards the quantized bits to the BBU via the fronthaul link. The transmission rate of RRH $i$’s fronthaul link is expressed as

$$T_i = 2B \sum_{m=1}^M C_{i,m}, \quad i \in N_A$$

where $B$ is the channel bandwidth.

Based on the received quantized signals $\hat{y}_i, \forall i$, the BBU estimates the target function $g$ in (2).

As illustrated in Fig. 1 we consider an AirComp task performed on a multi-antenna Cloud-RAN system which consists of $N_W$ single-antenna devices, $N_A$ multi-antenna RRHs and one BBU. Each RRH is equipped with $M$ antennas. Each device sends their local data to the BBU through $N_A$ multi-antenna RRHs, while the RRHs transmit the information to the BBU via a fronthaul link which is modeled as a digital link of capacity $C$ bits/sample. We define the sets $N_W = \{1, 2, \ldots, N_W\}$ and $N_A = \{1, 2, \ldots, N_A\}$ for the indices of devices and RRHs, respectively.

For a specific time slot $t$, we define the data aggregated at device $k$ as $\theta_k(t) \in \mathbb{C}$ and $\sum_{i=1}^N g(t)$ to each element of compressed signal $y_i$. Then the target function for aggregating local updates at the BBU can be written as

$$f(t) = \Phi \left( \sum_{k \in N_W} \phi_k(\theta_k(t)) \right),$$

where $\Phi(\cdot)$ is the post-processing function at BBU, and $\phi_k$ is the pre-processing function of device $k$. Let

$$t \sim \mathcal{CN}(0, \omega_i).$$

Fig. 1. Cloud-RAN system
define a vector $\hat{y} = [\hat{y}_1^T, \hat{y}_2^T, \ldots, \hat{y}_{N_W}^T]$ which is stacked by the quantized signal at each RRH, and denote $h_k = [h_{1,k}^T, h_{2,k}^T, \ldots, h_{N_{RA,k}}^T]^T$ as the channel vector stacked by the channel vector from device $k$ to RRHs. Then, vector $\hat{y}$ can be written as

$$\hat{y} = \sum_{k\in N_W} h_k b_k x_k + z + q,$$

where we define $z = [z_1^T, z_2^T, \ldots, z_{N_{RA_A}}]^T \sim \mathcal{CN}(0, \sigma^2 I)$ and $q = [q_1^T, q_2^T, \ldots, q_{N_{RA_A}}]^T \sim \mathcal{CN}(0, \Omega)$ with $\Omega = \text{diag}(\{1, \ldots, q_{N_{RA_A}}\})$.

By assuming that BBU performs a linear estimation of the target parameter $g$ from $\hat{y}$, the estimation of $g$ at the BBU can be given by

$$\hat{g} = m^H \hat{y} = m^H \sum_{k\in N_W} h_k b_k x_k + m^H(z + q),$$

where $m \in \mathcal{CN}_{N_A \times 1}$ is the receiver beamforming vector. Then each element of the target vector $g$ can be obtained at the BBU through $\hat{g}$. The distortion of $\hat{g}$ with respect to the target value $g$ can be measured by the mean-squared-error (MSE) which is given as

$$\text{MSE}(\hat{g}, g) = \mathbb{E}((\hat{g} - g)^2) = \sum_{k\in N_W} |m^H h_k b_k - 1|^2 + m^H(\sigma^2 I + \Omega)m.$$  \hspace{1cm} (11)

**B. Problem Formulation**

In this paper, our objective is to minimize $\text{MSE}$ that quantifies the distortion after the decoding process at the BBU by optimizing the devices’ transmit beamforming $\{b_k\}, \forall k$, the receive beamforming vector at the BBU $m$, as well as the quantization bits allocation at each RRH $\{C_{i,m}\}, \forall i, m$. Specifically, the formulated optimization problem can be expressed as:

$$\begin{align*}
\text{minimize} & \quad \text{MSE}(\hat{g}, g) \hspace{1cm} (12) \\
\text{Subject to} & \quad |b_k|^2 \leq P_k, \forall k \hspace{1cm} (13) \\
& \quad 2B \sum_{m=1}^{M} C_{i,m} \leq T_i, \forall i, m \hspace{1cm} (14) \\
& \quad C_{i,m} \in \mathbb{N}^+, \forall i, m. \hspace{1cm} (15)
\end{align*}$$

It can be observed that the optimization problem (12) is a non-convex problem since all the optimization variables are coupled in the objective function, and the quantization bits at each RRH $\{C_{i,m}\}, \forall i, m$ are discontinuous variables. In Section III, we shall leverage the alternating optimization method to solve this problem.

**III. OPTIMIZATION FRAMEWORK**

In this section, we propose to solve problem (12) by utilizing the alternating optimization approach. Specifically, the receive beamforming vector $m$, the devices’ transmit beamforming $\{b_k\}, \forall k$ and the quantization bits allocation $\{C_{i,m}\}, \forall i, m$ are optimized in an alternative manner until the algorithm converges.

**Algorithm 1:** Overall Algorithm for Solving Problem (12)

1. Initialize: Set $C_{i,m}^{(0)} = [T_i/(2BM)], \forall i, m,$ and $i = 0$.
2. repeat
3. \hspace{0.5cm} $i = i + 1$;
4. \hspace{0.5cm} Solve sub-problem (16) and obtain $\{b_k^{(i)}\}$ by using interior-point method with $C_{i,m}^{(i)} = C_{i,m}^{(i-1)}$, $\forall i, m$;
5. \hspace{0.5cm} Substitute $\{b_k^{(i)}\}$ into (20) to obtain $m^{(i)}$;
6. Obtain continuous $\{C_{i,m}^{(i)}\}$ by solving sub-problem (21) with interior-point method.
7. Apply bisection method to $\{C_{i,m}^{(i)}\}$;
8. Update the solution of problem (12).
9. until

$$\text{MSE}^{(i)} - \text{MSE}^{(i-1)} \leq \epsilon_2,$$ where $\text{MSE}^{(i)}$ is the objective value of problem (12) achieved by $m^{(i)}$, $\{b_k^{(i)}\}$ and $\{C_{i,m}^{(i)}\}$, and $\epsilon_2$ is the arithmetic accuracy of the overall algorithm.

**A. Optimizing transmit beamforming and Receive Beamforming**

We firstly fix the quantization bits allocation in problem (12) to optimize transmit power control and receive beamforming vector by solving the following problem:

$$\begin{align*}
\text{minimize} & \quad \text{MSE}(\hat{g}, g) \hspace{1cm} (16) \\
\text{Subject to} & \quad |b_k|^2 \leq P_k, \forall k. \hspace{1cm} (17)
\end{align*}$$

Problem (16) is still a non-convex problem since transmit beamforming $\{b_k\}, \forall k$ and receive beamforming vector $m$ are coupled. However, either fix receive beamforming vector $m$ or fix transmit power constraints $\{b_k\}$ can reduce the problem to be convex. Hence, with fixed $m$, we efficiently solve the following problem by applying interior-point method (23):

$$\begin{align*}
\text{minimize} & \quad \text{MSE}(\hat{g}, g) \hspace{1cm} (18) \\
\text{Subject to} & \quad |b_k|^2 \leq P_k, \forall k. \hspace{1cm} (19)
\end{align*}$$

Let $\tilde{b} = [\tilde{b}_1, \ldots, \tilde{b}_k]^T$ denote the solution to problem (18). Note that given a certain transmit beamforming, finding the optimal receive beamforming vector $m$ becomes a quadratic optimization problem without any constraint. The closed-form solution is given as

$$\tilde{m} = \left(\sum_{k\in N_W} |\tilde{b}_k|^2 h_k h_k^T + \sigma^2 I + \Omega\right)^{-1} \sum_{k\in N_W} \tilde{b}_k h_k.$$  \hspace{1cm} (20)
B. Optimizing Quantization Bits Allocation

In this subsection, we fix transmit beamforming and receive beamforming vector and optimize quantization bits allocation by solving the following problem:

\begin{align}
\text{minimize} & \quad \text{MSE}(\hat{g}, g)' \quad (21) \\
\text{Subject to} & \quad 2B \sum_{m=1}^{M} C_{i,m} \leq \bar{T}_i, \quad \forall i, m \quad (22) \\
& \quad C_{i,m} \in \mathbb{N}^+, \forall i, m \quad (23)
\end{align}

where \( \text{MSE}(\hat{g}, g)' \) is obtained by substituting \( \bar{b} \) and \( \bar{m} \) into (11), and can be written as

\begin{align}
\text{MSE}(\hat{g}, g)' = \sum_{k \in \mathcal{N}_W} |\bar{m}_k^H \hat{b}_k - 1|^2 + \bar{m}_k^H (\sigma_z^2 \mathbf{I} + \bar{\Omega}) \bar{m}_k. \quad (24)
\end{align}

where \( \bar{\Omega} \) is obtained by substituting \( \{\hat{b}_k\} \) into \( \mathbf{6} \).

Problem (21) is challenging to be solved due to the integer constraints for quantization bits \( C_{i,m} \). Note that if quantization bits allocation \( C_{i,m}, \forall i, m \) is assumed to be continuous, the quantization noise power \( \omega_{i,m} \) will turn to be a continuous function. In the following, we first solve the relaxation of problem (21) without integer constraints which we denote as problem (21)'. Then we propose an efficient algorithm to obtain a set of integer solutions for all \( \{C_{i,m}\}, \forall i, m \) based on the solution of problem (21)'. Firstly, we have

\begin{align}
m^H \bar{\Omega} m = & \sum_{i=1}^{N_A} \sum_{m=1}^{M} \omega_{i,m} |\bar{m}_{i-1} M + m|^2 \\
= & \sum_{i=1}^{N_A} \sum_{m=1}^{M} \xi_{i,m} 2^{-2C_{i,m}}, \quad (25)
\end{align}

where \( m_j \) denotes the \( j \)-th element of \( \bar{m} \), \( 1 \leq j \leq N_A \times M \) and

\begin{align}
\xi_{i,m} = 3 |\bar{m}_{i-1} M + m|^2 \left( \sum_{j=1}^{N_W} b_j |h_{i,j}|^2 + \sigma_z^2 \right), \quad \forall i, m \quad (26)
\end{align}

Note that \( \xi_{i,m} \) can be interpreted as the effective quantization noise power due to the quantized dimension at RRH \( i \). Further, it is worth noting that both the first term of \( \text{MSE}(\hat{g}, g) \) i.e. \( \sum_{k \in \mathcal{N}_W} |\bar{m}_k^H \hat{b}_k - 1|^2 + \bar{m}_k^H (\sigma_z^2 \mathbf{I} + \bar{\Omega}) \bar{m}_k \) remains constant in this alternating optimization step, we can reformulate problem (21)' as the following optimization problem

\begin{align}
\text{minimize} & \quad \sum_{i=1}^{N_A} \sum_{m=1}^{M} \xi_{i,m} 2^{-2C_{i,m}} \quad (27) \\
\text{Subject to} & \quad 2B \sum_{m=1}^{M} C_{i,m} \leq \bar{T}_i, \quad \forall i, m \quad (28)
\end{align}

Problem (27) can be shown to be a convex problem and thus we can solve it via the interior-point method. Let \( \bar{C}_{i,m} \) denote the solution of Problem (27). Considering that the solution of problem (21)' may not satisfy all the integer constraints, inspired by [22], we propose an efficient algorithm to obtain a set of integer solutions. In the following, we round each \( C_{i,m} \), \( \forall i, m \) to its nearby integer as follows.

\begin{align}
\tilde{C}_{i,m} = \begin{cases} \text{floor}(\bar{C}_{i,m}), & \text{if } \bar{C}_{i,m} - \text{floor}(\bar{C}_{i,m}) \leq \tau_i, \\ \text{Ceil}(\bar{C}_{i,m}), & \text{otherwise}, \end{cases} \quad (29)
\end{align}

where \( 0 \leq \tau_i \leq 1, \forall i \).

It is worth noting that we can always find a feasible solution of \( \{\bar{C}_{i,m}\}, \forall i, m \) by simply setting \( \tau_i = 1, \forall i \) in (29). Next, we show how to optimize \( \{\tau_i\}, \forall i \) to find a better feasible solution. Since \( \{\bar{C}_{i,m}\}, \forall i, m \) increases as \( \tau_i \) becomes smaller, the resulting \( \{\bar{C}_{i,m}\}, \forall i, m \) from (29) achieves lower MSE, while the individual quantization bits constraints for RRHs become more difficult to be satisfied. Thus, we propose to utilize the bisection method to find the optimal values of \( \{\bar{C}_{i,m}\}, \forall i, m \) and substitute it into (29) to obtain \( \{\tilde{C}_{i,m}\}, \forall i, m \). The bisection method is specified in the Step 7 of Algorithm 1. The proposed algorithm for solving problem (12) is summarized in Algorithm 1.
In our numerical experiments, we set $d_0$ where $d_\bar{i}$, i.e., assume that all the RRHs have the identical fronthaul capacity, $h$ and $\alpha$ and $N_{RRH}$. Performance Gain of Quantization Bits Allocation at each RRH $A$. Performance Lower Bound: Optimized power control

In this subsection, we compare the performance of AirComp in Cloud-RAN and massive MIMO architecture. To guarantee the fairness of our comparison, we fix the number of total antennas to be deployed. Next, in order to show the advantage of Cloud-RAN system, we study the following two cases: massive MIMO system with all of the antennas being deployed at the BS (which is located at origin of the given region), while for single-antenna Cloud-RAN, the RRHs are randomly located in the given region. Fig. 3 shows that MSE achieved in Cloud-RAN system is lower than the MSE achieved in massive MIMO system when appropriate number of fronthaul capacity is deployed at each RRH. However, if the fronthaul capacity deployed at each RRH is too small, the performance of AirComp in Cloud-RAN is inferior to massive MIMO system due to the large quantization noise. This result indicates that, our proposed architecture for AirComp enjoys significant densification gain in reducing MSE because the RRHs are much closer to the devices compared with conventional massive MIMO architecture.

V. CONCLUSIONS

In this paper, we proposed to leverage the Cloud-RAN architecture to boost the performance for AirComp, thereby achieving accurate and ultra-fast data aggregation. To reduce the path loss of channels and thus provide reliable wireless connectivity to a large number of devices, we developed the Cloud-RAN architecture for AirComp. Then we formulated the optimization problem of joint devices’ transmit beamforming and RRHs’ quantization bits allocation and BBU’s receive beamforming to minimize the MSE of AirComp. By applying our proposed alternating optimization method to solve this problem, the numerical results showed that our proposed approach can obtain the densification gain due to the massive deployment of RRHs compared to the massive MIMO system, and achieve better performance in reducing MSE of AirComp.

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