Probing neutrino masses and tri-bimaximality with lepton flavor violation searches

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Abstract

We examine relation between neutrino oscillation parameters and prediction of lepton flavor violation, in light of deviations from tri-bimaximal mixing. Our study shows that upcoming experimental searches for lepton flavor violation process can provide useful implications for neutrino mass spectrum and mixing angles. With simple structure of heavy right-handed neutrino and supersymmetry breaking sectors, the discovery of $\tau \rightarrow \mu \gamma$ decay determines neutrino mass hierarchy if large (order 0.1) reactor angle is established.

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1 Introduction

In the last decades, one of the most striking developments in particle physics beyond the standard model (SM) is the experimental establishment [1] of neutrino masses and the large mixing property, which is quite different from the small mixing in the quark sector. Neutrino oscillation experiments have revealed neutrino mass-squared differences and its flavor mixing angles. Notably, a simple form of mixing matrix, referred to as tri-bimaximal mixing, is well descriptive of the observed mixing structure [2]. Vast numbers of flavor models have been proposed in order to derive the tri-bimaximal mixing [3]; thus, from a theoretical viewpoint, it is one of the most important subjects to realize difference between tri-bimaximal and observed mixing angles. In addition, recent results of the three-flavor global data analysis [4, 5] indicate non-zero $\theta_{13}$; as the best-fit value, not so small one $\sin \theta_{13} \simeq \mathcal{O}(0.1)$ is obtained. Therefore, it seems interesting to examine deviations of neutrino mixing angles from the tri-bimaximal pattern [6], which leads to $\sin^2 \theta_{12} = 1/3$, $\sin^2 \theta_{23} = 1/2$, and $\sin^2 \theta_{13} = 0$, for the coming future precise experiments.

The present knowledge of neutrino parameters (i.e. masses, mixing angles and phases) is not only important for low-energy characters of neutrinos, but also a key ingredient of the origin of flavor structure in the SM fermions. It is thus meaningful to make clear the relation between neutrino parameters and high-energy phenomenologies. Among them, charged lepton flavor violation (LFV) process would give an intriguing clue, since supersymmetry (SUSY) and the seesaw mechanism [7] could enhance LFV as reachable in near future experiments [8]. The seesaw mechanism naturally provides desired neutrino mass scale, and predicted LFV fractions are affected by low-energy neutrino parameters and heavy Majorana masses via SUSY breaking terms [9, 10].

In this Letter, we examine the relation between neutrino parameters and LFV prediction, in light of the tri-bimaximal mixing and the recent precision oscillation data. We use a particular parametrization [11] of the MNS matrix [12], where the tri-bimaximal mixing is taken as its zeroth order approximation, and give a detailed analysis of LFV prediction especially for a simple case of right-handed Majorana mass and SUSY breaking structures. Our study shows that upcoming experimental LFV searches provide useful implications for neutrino mass spectrum and mixing angles. If we possess simple structure of heavy Majorana masses, future discovery of $\tau \rightarrow \mu \gamma$ implies that inverted hierarchy (IH) and quasi-degenerate (QD) neutrino mass spectra are inconsistent with a large reactor angle of order 0.1, though normal hierarchy (NH) is still allowed.

The Letter is organized as follows. In Section 2, we introduce a useful parametrization [11]
of the MNS matrix for examining how the mixing angles deviate from the tri-bimaximal ones. In Sections 3 and 4, relation between neutrino parameters and LFV prediction in a literature of the minimal supersymmetric standard model (MSSM) with the seesaw mechanism is studied. Detailed analysis is given in Section 4 by using the parametrization. Section 5 is devoted to study implications for neutrino parameters with future LFV searches. We summarize our result in Section 6.

2 Neutrino parameters and tri-bimaximal mixing

Current data obtained by neutrino oscillation experiments is consistent with a simple mixing structure called tri-bimaximal mixing [2]. We adopt a particular parametrization proposed in Ref. [11] to describe the MNS matrix by deviations from exact tri-bimaximal mixing angles. It is helpful to systematically analyze the neutrino tri-bimaximality.

In the basis where the charged lepton mass matrix is diagonal, neutrino mass matrix is given by

$$M_\nu = U D_m U^T,$$

where

$$D_m = \text{diag}(m_1, m_2, m_3)$$

with positive neutrino mass eigenvalues $$m_{1,2,3}$$ and $$U$$ is the unitary lepton mixing matrix. Three mixing angles and phases are involved in the matrix $$U$$; using the standard parametrization [13], it can be expressed as

$$U = \begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
  -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
  s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix} P_M,$$

where $$c_{ij} \equiv \cos \theta_{ij}$$, $$s_{ij} \equiv \sin \theta_{ij}$$, and $$\delta$$ is the Dirac CP violating phase. $$P_M$$ stands for the diagonal phase matrix which involves two Majorana phases.

Recent progress in neutrino experiments greatly increase data for neutrino masses and mixing angles. The updated result of the three-flavor global data analysis [4] indicates the following best-fit values with $$3\sigma$$ intervals of three (solar, atmospheric, reactor) mixing angles and two (solar, atmospheric) mass squared differences:

$$\sin^2 \theta_{12} = 0.304^{+0.066}_{-0.054}, \quad \sin^2 \theta_{23} = 0.50^{+0.17}_{-0.14}, \quad \sin^2 \theta_{13} = 0.010 \,(\leq 0.056),$$

$$\Delta m^2_{\text{sol}} \equiv m_2^2 - m_1^2 = (7.65^{+0.69}_{-0.60}) \times 10^{-5} \text{ eV}^2,$$

$$\Delta m^2_{\text{atm}} \equiv |m_3^2 - m_1^2| = (2.40^{+0.35}_{-0.33}) \times 10^{-3} \text{ eV}^2.$$ (2.2)

The Dirac phase $$\delta$$ has not yet been constrained by the experimental data.

The flavor structure could be determined by profound principles such as flavor symmetry in a high-energy regime. Although the origin of the flavor is unrevealed, the current neutrino
mixing angles are known to be consistent with a simple mixing matrix $U_{TB}$,

$$U_{TB} = (R_{TB})_{23}(R_{TB})_{12} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & \sqrt{3} \\ 1 & -\sqrt{2} & \sqrt{3} \end{pmatrix}.$$  

(2.4)

where two (non-diagonal) rotational matrices are given by

$$(R_{TB})_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \quad (R_{TB})_{12} = \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{3} & 2/\sqrt{6} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (2.5)$$

Although the matrix $U_{TB}$ approximately describes the MNS matrix (2.1), it is an interesting subject to investigate the difference between them in reality. For this purpose, the following unitary matrix is useful to parametrize the MNS mixing structure:

$$U = (R_{TB})_{23} \begin{pmatrix} c_w c_z & c_w s_z e^{-i\delta} \\ -s_w c_y - c_x s_y s_z e^{i\delta} & c_x c_y - s_x s_y s_z e^{i\delta} \\ s_x s_y - c_x c_y s_z e^{i\delta} & -c_x s_y - s_x c_y s_z e^{i\delta} \end{pmatrix} (R_{TB})_{12} P_M, \quad (2.6)$$

where $c_w$ and $s_w$ denote $\cos \epsilon_w$ and $\sin \epsilon_w$ ($w = x, y, z$), respectively. In the limit where $\epsilon_{x,y,z} \to 0$, $U$ in (2.6) goes back to $U_{TB}$ except $P_M$. With this parametrization, experimental data (2.2) indicates

$$-0.092 \leq \epsilon_x \leq 0.038, \quad -0.14 \leq \epsilon_y \leq 0.17, \quad |\epsilon_z| \leq 0.239, \quad (2.7)$$

in its $3\sigma$ ranges. Since the deviation parameters are much suppressed than $O(1)$, expansions around $\epsilon_{x,y,z} = 0$ could give a good approximation to physical quantities related to lepton mixing angles.

3 LFV in MSSM with type-I seesaw

Let us study LFV prediction in MSSM with heavy right-handed Majorana neutrinos for the type-I seesaw mechanism [7]. The relevant part of the MSSM superpotential is given by

$$W_{\text{lepton}} = L_i (Y_e)_{ij} \bar{e}_j H_d + L_i (Y_\nu)_{ij} \bar{\nu}_j H_u + \frac{1}{2} \bar{\nu}_i (M_R)_{ij} \nu_j, \quad (3.1)$$

where $Y_e$ and $Y_\nu$ are charged lepton and neutrino Yukawa matrices. Superfields $L_i$, $\bar{e}_i$, $\bar{\nu}_i$ ($i = 1, 2, 3$) and $H_u(d)$ include lepton doublets, charged leptons, right-handed neutrinos and up

\footnote{One can easily find that $\epsilon_z = \theta_{13}$ from (2.1) and (2.6), owing to $\theta_{13} = 0$ in the tri-bimaximal limit.}
(down)-type Higgs doublet, respectively. $M_R$ gives Majorana mass matrix of the right-handed neutrinos, whose scale is assumed to be much larger than the electroweak scale ($\sim 10^2$ GeV). It is also noted that scale of $M_R$ is required to be smaller than the grand unified theory (GUT) scale in order to reproduce known solar and atmospheric neutrino mass scales unless $Y_\nu$ is much larger than $O(1)$. Without loss of generality, we take a basis where $M_R$ is diagonal.

SUSY breaking should be incorporated in realistic models, since it is not exact symmetry in Nature. We thus introduce SUSY breaking terms, which in general could be new sources of the flavor violation. Although several breaking scenarios have been proposed, one of the most economical and predictive ansatz is to assume the universal form of soft terms in a high-energy regime. In this case, universal SUSY breaking parameters are listed as scalar mass $m_0$, trilinear coupling $A_0$, gaugino mass $M_{1/2}$ and the Higgs bilinear coupling. We refer to these SUSY breaking parameters as their GUT scale values.

The flavor violation in the supersymmetric sector is transmitted to SUSY breaking sector thorough renormalization group (RG) evolution between GUT and heavy Majorana mass scales. At a low-energy regime, one-loop diagrams with SUSY particles give leading corrections to the LFV processes; the branching fractions are approximately written as

$$B(\ell_j \to \ell_i + \gamma) \simeq \frac{\alpha^3}{G_F^2 m_S^8} \left[ \frac{3m_0^2 + A_0^2}{8\pi^2 v_H^2 \sin^2 \beta} \right]^2 \tan^2 \beta |B_{ij}|^2,$$  \hspace{1cm} (3.2)

where $\alpha$ and $G_F$ are the fine-structure and the Fermi coupling constants, $v_H \simeq 174$ GeV, and $\beta$ parametrizes the ratio between vacuum expectation values of the two Higgs scalars in $H_u,d$. The mass parameter $m_S$ is a typical mass scale of SUSY particles. Note that flavor indices only appear in $B_{ij}$ as

$$B_{ij} = v_H^2 \sin^2 \beta \sum_{k=1}^{3} (Y_\nu)_{ik}(Y_\nu^\dagger)_{kj} \ln \frac{M_G}{M_{Rk}},$$  \hspace{1cm} (3.3)

where $M_{Rk}$ denotes the $k$-th eigenvalue of $M_R$, and $M_G \simeq 10^{16}$ GeV is the GUT scale. Thus the flavor dependence in LFV branching fractions is completely involved in (3.3), that is SUSY breaking parameters do not affect the flavor structure in the approximate formula of LFV prediction (3.2).

Although it is generally difficult to reconstruct the combination of neutrino Yukawa matrix in (3.3) from low-energy data, if heavy Majorana neutrinos have an approximately degenerate mass $M_U$ and there are no large CP phases except the Dirac phase $\delta$ in the neutrino sector,
the branching fractions (3.2) are tightly connected to neutrino parameters as

\[ B(\ell_j \rightarrow \ell_i + \gamma) \simeq \frac{\alpha^3}{G_F m_S^2} \left[ \frac{3m_0^2 + A^2_0}{8\pi^2 v^2_H \sin^2 \beta} \right]^2 \tan^2 \beta M^2_U \left( \ln \frac{M_G}{M_U} \right)^2 |b_{ij}|^2, \]  

(3.4)

\[ b_{ij} = \sum_{k=1}^{3} m_k (U^*)_{ik} (U^T)_{kj}. \]  

(3.5)

Note that \( P_M \) in (2.6) disappears in the factor \( b_{ij} \). Here the low-energy neutrino parameters, namely their masses, mixing angles and the Dirac phase, are completely involved in (3.5); thus the LFV branching fractions depend on SUSY parameters in a flavor independent manner. We will discuss more general cases with non-degenerate Majorana masses in the end of the next section.

4 LFV prediction around tri-bimaximal mixing

In this section, we discuss the relation between neutrino parameters and prediction of LFV processes. The parametrization (2.6) allows us to handle deviations from tri-bimaximal mixing in a systematic way. In a definite framework for high-energy theory, we analyze how LFV prediction depends on small deviation parameters with current neutrino oscillation data.

Analytical results Applying the parametrization (2.6) to describe the LFV prediction, \( b_{ij} \) is explicitly written as

\[ b_{12} = \frac{m_{12}}{6\sqrt{2}} c_x (c_y - s_y) (2\sqrt{2} c_{2x} + s_{2x}) + \frac{e^{i\delta}}{6\sqrt{2}} c_z s_x (c_y + s_y) \left[ 3m_{123} + m_{12}(c_{2x} - 2\sqrt{2}s_{2x}) \right], \]

\[ b_{13} = -\frac{m_{12}}{6\sqrt{2}} c_z (c_y + s_y) (2\sqrt{2} c_{2x} + s_{2x}) + \frac{e^{i\delta}}{6\sqrt{2}} c_z s_x (c_y - s_y) \left[ 3m_{123} + m_{12}(c_{2x} - 2\sqrt{2}s_{2x}) \right], \]

\[ b_{23} = \frac{m_{123}}{4} c_z^2 (c_y^2 - s_y^2) - \frac{m_{12}}{12} (1 + s_z^2) (c_y^2 - s_y^2) (c_{2x} - 2\sqrt{2}s_{2x}) \]

\[ - \frac{e^{i\delta}}{12} m_{12} s_x \left[ (c_y - s_y)^2 - (c_y + s_y)^2 e^{-2i\delta} \right] (2\sqrt{2} c_{2x} + s_{2x}), \]  

(4.1)

where \( c_{2x} = \cos 2\epsilon_x \) and \( s_{2x} = \sin 2\epsilon_x \). Note that neutrino masses appear just as particular combinations

\[ m_{12} = m_2 - m_1, \quad m_{123} = 2m_3 - m_2 - m_1. \]  

\[ \text{One can also discuss relation between neutrino parameters and LFV prediction with other seesaw mechanisms than the conventional type-I [14]. For instance, in the type-II seesaw scenario [15], neutrino masses in } b_{ij} \text{ are replaced by those squared, and mixing parameter dependence of LFV prediction can be studied as our analysis.} \]
It is also noted that \( b_{12}(\epsilon_y, \epsilon_z; \delta) = -b_{13}(-\epsilon_y, -\epsilon_z; \delta) = -b_{13}(\epsilon_y, \epsilon_z; \delta \pm \pi) \) holds between \( b_{12} \) and \( b_{13} \); the relation is just the \( \mu-\tau \) symmetric property in the tri-bimaximal limit.

As mentioned in Section 2, the observed values of lepton mixing angles are consistent with the tri-bimaximal mixing pattern. Focusing on the LFV prediction with the tri-bimaximal limit \((\epsilon_x, \epsilon_y, \epsilon_z) \to (0, 0, 0)\) in (4.1), we obtain

\[
\mathcal{B}(\mu \to e\gamma) \propto |b_{12}|^2 \to \left(\frac{m_{12}}{3}\right)^2 = \frac{m_{12}^2}{9},
\]

\[
\mathcal{B}(\tau \to e\gamma) \propto |b_{13}|^2 \to \left(\frac{m_{12}}{3}\right)^2 = \frac{m_{12}^2}{9},
\]

\[
\mathcal{B}(\tau \to \mu\gamma) \propto |b_{23}|^2 \to \frac{1}{16} \left(m_{123} - m_{12}\right)^2 = \frac{m_{123}^2}{16} - \frac{m_{123}m_{12}}{24} + \frac{m_{12}^2}{144},
\]

where only \( |b_{23}|^2 \) depends on \( m_{123} \) and \( |b_{12}|^2 = |b_{13}|^2 \) holds.

Experimental values of solar and atmospheric neutrino mass differences indicate that \( m_{123} \) is much larger than \( m_{12} \). The ratio \( \hat{m} \equiv m_{12}/m_{123} \) depends on the neutrino mass spectrum. For NH of neutrino masses \((m_1 < m_2 < m_3)\), we obtain

\[
\hat{m} = \frac{\sqrt{\Delta m_{\text{sol}}^2 + m_1^2 - m_1}}{2 \sqrt{\Delta m_{\text{atm}}^2 + m_1^2 - \Delta m_{\text{sol}}^2}} \to \frac{1}{2} \frac{\sqrt{\Delta m_{\text{sol}}^2}}{\Delta m_{\text{atm}}^2},
\]

while for the IH case \((m_3 < m_1 < m_2)\), we have

\[
\hat{m} = \frac{\sqrt{\Delta m_{\text{atm}}^2 + \Delta m_{\text{sol}}^2 + m_3^2 - \sqrt{\Delta m_{\text{atm}}^2 + m_3^2}}}{2 m_3 - \sqrt{\Delta m_{\text{atm}}^2 + \Delta m_{\text{sol}}^2 + m_3^2 - \sqrt{\Delta m_{\text{atm}}^2 + m_3^2}}} \to -\frac{1}{4} \frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2}.
\]

The last expressions in (4.4) and (4.5) imply the massless limit of the lightest neutrino, where \( \Delta m_{\text{sol}}^2/\Delta m_{\text{atm}}^2 \simeq 0.04 \) with the present data. We plot in Fig. 1 the ratio \( \hat{m} \) as the function of the lightest neutrino mass eigenvalue \( m_{\text{ref}} \). It shows that the ratio becomes relatively large (small) if the neutrino mass spectrum is NH (IH). With being large value of \( m_{\text{ref}} \), namely QD mass spectrum limit, \( \hat{m} \) takes similar values for normal and inverse ordering cases.

In the tri-bimaximal limit, \( \mathcal{B}(\mu \to e\gamma) \) and \( \mathcal{B}(\tau \to e\gamma) \) in (4.3) are much suppressed than \( \mathcal{B}(\tau \to \mu\gamma) \) since they do not involve \( m_{123} \). This implies that these processes are sensitive to deviations from the tri-bimaximal mixing. As argued in Section 2, the deviation parameters in (2.6) are sufficiently small, so that we can use them as expansion parameters in LFV.
branching fractions. Up to $\mathcal{O}(\epsilon_w^2)$, one can obtain the following expressions:

$$|b_{12}|^2 \simeq \frac{1}{16}(\tilde{m}_{12} - m_{123})^2 - \frac{\tilde{m}_{12}}{4}(\tilde{m}_{12} - m_{123})\epsilon_x^2 + \frac{1}{4}(\tilde{m}_{12} - m_{123})^2\epsilon_y^2$$

$$|b_{23}|^2 \simeq \frac{1}{16}(\tilde{m}_{12} - m_{123})^2 - \frac{\tilde{m}_{12}}{4}(\tilde{m}_{12} - m_{123})\epsilon_x^2 + \frac{1}{4}(\tilde{m}_{12} - m_{123})^2\epsilon_y^2$$

where $\tilde{m}_{12} \equiv m_{12}/3$ and $|b_{13}|^2(\epsilon_y, \epsilon_z \cos \delta) = |b_{12}|^2(-\epsilon_y, -\epsilon_z \cos \delta)$.

From the fact that $m_{123}$ is much larger than $m_{12}$, the typical correlation between neutrino parameters and the branching fractions can be understood. In (4.6), the terms which remain in the tri-bimaximal limit are proportional to $m_{12}^2$, and $m_{123}$ always appears with involving the deviation parameters, especially $\epsilon_z$. Thus $|b_{12(13)}|^2$, namely $B(\mu(\tau) \to e\gamma)$, is sensitive to the parameters, while $B(\tau \to \mu\gamma)$ does not have so large dependence on them since $m_{123}^2$ is the dominant term in $|b_{23}|^2$. Moreover, the leading contributions in $|b_{12}|^2$ and $|b_{23}|^2$ can be expressed as

$$|b_{12}|^2 \simeq \frac{m_{123}^2}{9} \left( \tilde{m}^2 + \frac{3}{\sqrt{2}}\tilde{m}\epsilon_z \cos \delta + \frac{9}{8}\epsilon_z^2 \right) + \cdots, \quad |b_{23}|^2 \simeq \frac{m_{123}^2}{16} + \cdots$$
since the ratio $\hat{m}$ is also the small quantity as well as deviation parameters. One can find that $\epsilon_z$ plays an important role to determine $B(\mu(\tau) \rightarrow e\gamma)$ and that $\epsilon_x$ and $\epsilon_y$ are less effective because they only appear as sub-leading corrections. The $\epsilon_z$ dependence is controlled by the neutrino mass spectrum and the Dirac phase through $\hat{m}$ and $\cos \delta$. For example, $|b_{12}|^2$ is minimized at

$$\epsilon_z = \theta_{13} \simeq -\frac{2\sqrt{2}}{3} \hat{m} \cos \delta \simeq -\hat{m} \cos \delta.$$ \tag{4.9}

Note that the value of $\epsilon_z$ in (4.9) has significant distinction between NH and IH by the magnitude of $\hat{m}$. We can check these properties with numerical analysis.

**Numerical searches** We discussed the neutrino parameter dependence of the LFV prediction using the approximate formula in (3.2). To make our study more complete, we proceed to numerical examination of the LFV prediction. Here we take SPS1a [16] for SUSY particle mass spectrum; SUSY breaking parameters are fixed as $(m_0, M_{1/2}, A_0) = (100, 250, -100)$ GeV at the GUT scale and $\tan \beta = 10$. SUSY parameter dependence is mostly flavor blind, and has been greatly studied [8]. If one takes other types of SUSY mass spectrum, following results do not alter as long as the universality of the SUSY breaking is assumed.

Given set of SUSY parameters, we numerically estimate RG evolutions between GUT and electroweak scale taking heavy right-handed neutrinos into account. Above the right-handed neutrino mass scale, which is taken as $M_U = 10^{14}$ GeV in the analysis, the right-handed neutrinos are decoupled with the theory. Two-loop RG equations for gauge and Yukawa couplings, and one-loop ones for the soft SUSY breaking parameters are numerically solved. LFV fractions are estimated with one-loop diagrams in the SUSY particle mass eigenbasis rather than the mass-insertion approximation.

As stressed in the analytical discussion, among the three deviation parameters in (2.7), $\epsilon_z$ is crucial for the prediction of $\mu(\tau) \rightarrow e\gamma$ branching fractions. To see its dependence, we plot the LFV predictions as the functions of $\epsilon_z$ in Fig. 2. The prediction of $B(\tau \rightarrow \mu\gamma)$ is insensitive to size of $\epsilon_z$, by contrast that of $B(\mu(\tau) \rightarrow e\gamma)$ strongly depends on. The branching fraction $B(\mu \rightarrow e\gamma)$ is minimized around $\epsilon_z \simeq -0.1(+0.01)$ for the case with NH (IH and QD) mass spectrum, as shown in (4.9). These results are consistent with the previous argument using the analytical expressions. Note that $B(\mu(\tau) \rightarrow e\gamma)$ is highly suppressed than $B(\tau \rightarrow \mu\gamma)$ around the minima. This is an important point to extract implications for neutrino parameters from future LFV searches, and we discuss the issue in the next section.
Figure 2: Predictions of LFV branching fractions as the functions of $\epsilon_z$ are shown. Black, red and gray plots correspond to $B(\mu \to e\gamma)$, $B(\tau \to \mu\gamma)$ and $B(\tau \to e\gamma)$, respectively. Each figure corresponds to different neutrino mass spectrum: from left to right figures, NH ($m_1 = 10^{-4}$ eV), IH ($m_3 = 10^{-4}$ eV), and QD ($m_1 = 10^{-1}$ eV) mass spectra are taken, respectively. The mass squared differences are fixed to central values in (2.3) and the Dirac phase is taken as $\delta = 0$. $\epsilon_x$ and $\epsilon_y$ are scanned with 3$\sigma$ ranges in (2.7).

**Effects of heavy Majorana mass hierarchies** In general, heavy Majorana masses have non-degeneracy and it affects LFV prediction. In order to incorporate the non-degeneracy of $M_R$, it is convenient to focus on $B_{ij}$ in (3.3) rather than $b_{ij}$ in (3.5). $B_{ij}$ can be generally rewritten as follows [9]:

$$B_{ij} = \sum_{k=1}^{3} (U^*_k D_{\sqrt{M_R}})_{ik} (D_{\sqrt{m}} R^T D_{\sqrt{M_R}})_{kj} \ln \frac{M_G}{M_{Rk}},$$

where $D_{\sqrt{m}} = \text{diag}(\sqrt{m_1}, \sqrt{m_2}, \sqrt{m_3})$, $D_{\sqrt{M_R}} = \text{diag}(\sqrt{M_{R1}}, \sqrt{M_{R2}}, \sqrt{M_{R3}})$ and a complex matrix $R$ satisfies $RR^T = 1$. The additional mixing matrix $R$ appears in $Y_\nu Y_\nu^T$ because the right-handed neutrino mixing is not unphysical. As a result, LFV prediction generally depends on the mixing structure of $R$, as minutely studied in [9].

If the right-handed mixing is not important for low-energy neutrino parameters, namely $R \approx 1$, $B_{ij}$ is simplified. Especially $R \to 1$ leads to

$$B_{ij} \approx \sum_{k=1}^{3} M_k (U^*_k)_{ik} (U^T)_kj, \quad M_i \equiv m_i M_{Ri} \ln \frac{M_G}{M_{Ri}}. \quad (4.11)$$

It is obvious that $B_{ij}$ has a similar form to $b_{ij}$, where low-energy neutrino mass $m_i$ in (3.5) is replaced by a combination of light and heavy neutrino masses. Hence, explicit expression of
$B_{ij}$ is easily obtained by (4.1). In this case, LFV prediction depends on both neutrino masses through $\mathcal{M}_{12} = \mathcal{M}_2 - \mathcal{M}_1$ and $\mathcal{M}_{123} = 2\mathcal{M}_3 - \mathcal{M}_2 - \mathcal{M}_1$. In particular, the new mass ratio $\hat{\mathcal{M}} = \mathcal{M}_{12}/\mathcal{M}_{123}$ is essential to determine the neutrino parameter dependence. With typical mass hierarchies of $M_R$, LFV prediction has examined in Ref. [17]. Corrections to $R = 1$ also affect LFV prediction when right-handed neutrinos have large non-degeneracy. For example, if $R$ matrix in (4.10) contains a small mixing angle $\kappa_{ij}$, then $\mu \rightarrow e\gamma$ prediction is modified with including the counterpart of (4.8); up to $O(\kappa_{ij}^2)$ contribution except for $O(\kappa_{ij} \cdot \epsilon_{x,y,z})$ terms, the prediction is explicitly written as follows:

$$|B_{12}|^2 \simeq \frac{\hat{\mathcal{M}}_{123}^2}{9} \left[ \hat{\mathcal{M}}^2 + \frac{3}{\sqrt{2}} \hat{\mathcal{M}} \epsilon_z \cos \delta + \frac{9}{8} \epsilon_z^2 + \frac{\sqrt{2} \Delta_{12} + \sqrt{6} \Delta_{23} + 2\sqrt{3} \Delta_{13}}{\hat{\mathcal{M}}_{123}} + \cdots \right],$$

(4.12)

where we use the notation

$$R = \begin{pmatrix} 1 & \kappa_{12} & \kappa_{13} \\ -\kappa_{12} & 1 & \kappa_{23} \\ -\kappa_{13} & -\kappa_{23} & 1 \end{pmatrix}, \quad \Delta_{ij} \equiv \kappa_{ij} \sqrt{m_i m_j} \left( M_{Rj} \log \frac{M_G}{M_{Rj}} - M_{Ri} \log \frac{M_G}{M_{Ri}} \right).$$

(4.13)

The contribution due to $R \neq 1$ strongly depends on heavy Majorana mass hierarchy. One can similarly see that the leading contribution from $\kappa_{ij}$ appears in $\Delta_{ij}$ for the $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ predictions.

The parameter dependence of the branching fractions is modified from the degenerate heavy Majorana case by mainly given difference between $\hat{m}$ and $\hat{\mathcal{M}}$. Nevertheless, a particular Majorana mass spectrum is taken such as $M_{R3}$ is dominantly heavy, then similar discussion to the degenerate case is possible as long as effects from $R$ in (4.10) is sufficiently small. Further study on effects of $R$ and Majorana masses from our viewpoint is also important and future task.

5 Probing neutrino parameters with LFV searches

Finally, we investigate possible implications for neutrino parameters from future LFV searches with the analysis obtained in the previous section. In the following we concentrate on a limited scenario where $R = 1$ is assumed, and show how future LFV searches give constraints for neutrino mass spectrum and $\theta_{13} = \epsilon_z$.

*Here $\kappa_{ij}$ is assumed to be real. Complex phases of $R$ can bring further modification into LFV prediction [9].

[1] In a class of models where all the leptonic flavor violation originates in the charged lepton sector correspond to $R = 1$ as in Ref. [9]. This is equivalent to the case that neutrino Yukawa and heavy Majorana mass matrices can be taken as simultaneously diagonal. It is notified that such the situation is approximately realized in some $E_6$ and $SO(10)$ grand unified models, called lopsided mass structure [18].
Experimental discovery of lepton rare decay processes $\ell_j \rightarrow \ell_i + \gamma$ is one of smoking gun signals of physics beyond the SM; thus several experiments have been developed to detect LFV processes. The present experimental upper bounds are given at 90% C.L. [19, 20]:

$$B(\mu \rightarrow e\gamma) \leq 1.2 \times 10^{-11}, \quad B(\tau \rightarrow \mu\gamma) \leq 4.5 \times 10^{-8}, \quad B(\tau \rightarrow e\gamma) \leq 1.2 \times 10^{-7}.$$ 

These bounds are to be modified in near future searches. MEG experiment searches $\mu \rightarrow e\gamma$; the bound is expected to reach $B(\mu \rightarrow e\gamma) \leq O(10^{-13} \sim 10^{-14})$ [21]. Future $B$-factories would also greatly reduce the $\tau$ decay upper bounds [22]. In our analysis, we conservatively adopt

$$B(\mu \rightarrow e\gamma) \lesssim 10^{-13}, \quad B(\tau \rightarrow \mu\gamma) \lesssim 10^{-9}, \quad B(\tau \rightarrow e\gamma) \lesssim 10^{-9}, \quad (5.1)$$

as upcoming upper bounds of LFV fractions. Since the bound for $B(\mu \rightarrow e\gamma)$ is most severe, $B(\mu \rightarrow e\gamma)/B(\tau \rightarrow \mu(e)\gamma)$ must be sufficiently suppressed as $10^{-2} \sim 10^{-5(6)}$ in order to observe both $\mu$ and $\tau$ decay processes. Prediction of $B(\tau \rightarrow e\gamma)$ is always suppressed than $B(\tau \rightarrow \mu\gamma)$ in Fig.2 and we focus on $\tau \rightarrow \mu\gamma$ between the tau decay processes in the following.

The LFV fractions depend on $M_R$ and SUSY breaking parameters in a flavor blind way, and on the neutrino parameters in a flavor dependent manner. As seen in the previous section, predictions of $\mu \rightarrow e\gamma$ and $\tau \rightarrow e\gamma$ highly depend on $\epsilon_z$, but $\tau \rightarrow \mu\gamma$ is nearly independent of it. Thus $B(\tau \rightarrow \mu\gamma)$ is mostly determined by $M_R$ and SUSY breaking parameters; namely, the universal dependence in LFV fractions can be read from $B(\tau \rightarrow \mu\gamma)$. Hence we can express $B(\mu \rightarrow e\gamma)$ using $\epsilon_z$ and $B(\tau \rightarrow \mu\gamma)$. Fig.3 shows contour plots of $B(\mu \rightarrow e\gamma)$ as the function of $\epsilon_z$ and $B(\tau \rightarrow \mu\gamma)$ for the cases with NH and IH. In the figure, shaded regions have already been excluded by the current experimental bound for $\mu \rightarrow e\gamma$, and the current and expected bounds for $\tau \rightarrow \mu\gamma$ are shown by the solid and dotted lines, respectively.

From the figure, one can realize that future LFV searches give us implications for neutrino parameters. For instance, if near future experiments discover both $B(\mu \rightarrow e\gamma)$ and $B(\tau \rightarrow \mu\gamma)$, then $\epsilon_z$ and neutrino mass spectrum are strongly constrained. In the scenario, on the one hand for NH case $|\theta_{13}|$ is close to 0.1, on the other hands for IH and QD cases such a large value of $\theta_{13}$ is not allowed. It is interesting that the above value of reactor angle for NH is in accord with the best-fit value reported by recent data analyses. Hence, the LFV discovery excludes IH and QD mass spectra when the large $\theta_{13}$ is established in experiments like T2K [23] and Double Chooz [24]. Though in the analysis the Dirac phase is taken as zero, the result is preserved if non-zero value of $\delta$ is incorporated. This is because IH and QD spectra are still inconsistent with the LFV discovery and the large $\theta_{13}$ value, as easily realized from (4.9).

As another scenario, when future LFV searches discover only $B(\tau \rightarrow \mu\gamma)$, the above discussion is still valid. Fig.4 shows the prediction of $B(\tau \rightarrow \mu\gamma)$ as the functions of $\epsilon_z$ for
Figure 3: Contour plots of $\mathcal{B}(\mu \to e\gamma)$ as the functions of $\epsilon_z$ and $\mathcal{B}(\tau \to \mu\gamma)$ for the cases with NH and IH. The other deviation parameters are set to zero, and the neutrino masses are fixed to their central values in (2.3). The Dirac phase is taken as $\delta = 0$. Shaded regions have already been excluded by the current experimental bound for $\mu \to e\gamma$, and the current (expected) bounds for $\tau \to \mu\gamma$ is shown by the solid (dotted) lines.

the cases with NH and IH. All the plotted points satisfy $\mathcal{B}(\mu \to e\gamma) \leq 1.2 \times 10^{-11}$. One can see that NH and IH require different values of $\epsilon_z$ to predict $\mathcal{B}(\tau \to \mu\gamma)$ in future discovery range. However, constraints on $\epsilon_z$ and neutrino mass spectrum are weakened, if $\mathcal{B}(\tau \to \mu\gamma)$ is sufficiently suppressed than the future experimental limit.

6 Conclusion

In this Letter, we have examined relation between neutrino parameters and LFV predictions, in light of the tri-bimaximal mixing and the recent precision data. By using a particular parametrization for the lepton mixing matrix, which is useful to study difference between the MNS and tri-bimaximal mixing matrices, we have explicitly showed that the flavor dependence in LFV predictions is controlled by deviation parameters and neutrino mass differences.

In the setup with universal heavy Majorana masses and soft SUSY breaking parameters, we have found that $\epsilon_z$ and the neutrino mass spectrum are important for predictions of $\mathcal{B}(\mu \to e\gamma)$ and $\mathcal{B}(\tau \to \mu\gamma)$, while $\epsilon_x$ and $\epsilon_y$ are less effective to determine the LFV predictions. The branching fraction $\mathcal{B}(\tau \to \mu\gamma)$ is also shown to be insensitive to neutrino parameters.
Figure 4: Prediction of $B(\tau \to \mu \gamma)$ as the functions of $\epsilon_z$ for the cases with NH (black circles) and IH (gray triangles). All the plotted points satisfy $B(\mu \to e \gamma) \leq 1.2 \times 10^{-11}$. The deviation parameters $\epsilon_x$ and $\epsilon_y$ are scanned with $3\sigma$ ranges (2.7), and the Dirac phase is taken as $\delta = 0$. The current (expected) bounds for $\tau \to \mu \gamma$ is shown by the solid (dotted) lines.

In addition, we have discussed the effects from heavy Majorana mass structure, namely non-degeneracy in right-handed neutrinos and small mixing angles in $R$ matrix.

We have examined and extracted the possible implications for neutrino parameters from upcoming LFV searches. Future discovery of LFV process can give strong constraints on $\theta_{13}$ with respect to the type of neutrino mass hierarchy as long as effects from $R$ in (4.10) is sufficiently small. In particular, $\tau \to \mu \gamma$ discovery excludes IH and QD neutrino mass spectra if $|\theta_{13}| \simeq 0.1$ would be established.

In general, inclusive studies of the precision neutrino parameters and LFV could give us implications for unrevealed issues in the lepton flavor structure, such as the neutrino tri-bimaximality. Further investigations of LFV prediction focusing on effects from the phases and the Majorana mass structure are our future works.

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