A REVIEW ON TROPICAL CYCLONES

Indrajit Ghosh¹ Sukhen Das² Nabajit Chakravarty³

¹Dept. of Physical and Material Sciences, College of Engineering and Management Kolaghat, East Medinipur, West Bengal, India.

²Dept. of Physics, Jadavpur University, Kolkata, West Bengal, India.

³Positional Astronomy Centre, India Meteorological Department, Ministry of Earth Science, Plot No. 8, Block-AQ, Sector-V, Salt Lake City, Kolkata, West Bengal, India.

³Department of Applied Optics & Photonics, University of Calcutta, JD-2, Sector-III, Salt Lake, Kolkata, West Bengal, India.

¹indraghosh06@gmail.com, ²sukhendas29@gmail.com, ³nabajit_c@yahoo.com

Corresponding author: Nabajit Chakravarty

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Abstract

In this review, we have discussed the important recent theoretical research works on tropical cyclone dynamics. For mitigation of the devastating effect of tropical cyclones on coastal human civilization more and more advanced forecasting techniques are evolving nowadays with the increase in the frequency of generation of tropical cyclones. Thus it is of utmost necessity to understand the anatomy and physiology of the dynamics of tropical cyclones. So researchers explain the cyclonic system from a different point of view and that is highlighted in this review. So this review illustrates, in brief, some important developed models.

Keywords: tropical cyclone, cyclostrophic flow, thermal wind, gale wind, wind gusts, storm surge, bathymetry, barotropic wind, baroclinic atmosphere, gradient wind, potential temperature.

I. Introduction

I.i. Background

One of the most devastating natural calamities affecting human civilization is tropical cyclones that generate over the ocean bed due to the satisfaction of a number of critical parameters, gain potential from the infinite energy source of the ocean and die out in the land. The life cycle of a tropical cyclone is mainly classified into 4 stages: - a) formation stage b) intensification stage c) mature stage and d) decay stage respectively. Among these four stages the formation or genesis stage of a tropical
cyclone, the intensification and mature stage (cyclone stage) are most important in the perspective of cyclone dynamics.

Tropical cyclones are characterized by intense, cyclostrophically rotating low vortices weather systems that form over the tropical oceans. The wind speed ranges from 17 m/s (60 km/hr, 32 Kn) to greater than 33 m/s (120 km/hr, 64 Kn). Tropical cyclones are classified accordingly taking their site of occurrence and intensities as parameters. They have different names in different parts of the world. They are called Hurricanes over the Atlantic Ocean, the East Pacific Ocean, and the Caribbean sea. They are named popularly as cyclones in India and as Typhoons over the Western North Pacific Ocean. India Meteorological Department (IMD) mainly monitors the cyclones that are formed over the North Indian Ocean. Cyclones are classified into 7 categories according to the intensity as: i) depression (speed ranges 16 – 28 Kn or 32 – 51 Km/hr), ii) deep depression (speed ranges 29 – 34 Kn or 50 – 63 Km/hr), iii) cyclonic storm (speed ranges 35 – 48 Kn or 64 – 87 Km/hr), iv) severe cyclonic storm (speed ranges 49 – 64 Kn or 88 – 118 Km/hr), v) very severe cyclonic storm (speed ranges 65 – 90 Kn or 119 – 166 Km/hr), vi) extremely severe cyclonic storm (speed ranges 91 – 120 Kn or 167 – 221 Km/hr), vii) super cyclonic storm (speed ≥ 120 Kn or 220 Km/hr).

Giaiotti and Stel (2006) proposed the Rankine vortex model that explains tropical cyclone vorticity. Vorticity is defined as a microscopic rotational parameter of turbulence while circulation is a macroscopic rotational parameter of turbulence. Thus vorticity is a local rotational parameter of cyclone turbulence and has a profound impact on the structural characteristics of a cyclone. Important works like ‘Convective forcing in the Inter-Tropical Convergence Zone of the Eastern Pacific’ by Raga and Raymond (2003); ‘The Mechanics of Gross Moist Stability’ by Raymond et al. (2009) elucidate some important critical factors behind the genesis of tropical cyclones.

The important critical factors are as follows: -

i) Moderately warm ocean waters having a critical temperature of about 26°C.

ii) The location of the cyclone ought to be around 5° latitude from the equator.

iii) Presence of sufficiently low vertical wind shear.

iv) Presence of sufficient ambient moisture in the mid-troposphere zone of the atmosphere.

v) Unsteady or wobbly conditions prevailing in the atmosphere.

vi) Prevailing antecedent interruptions or disruptions in the atmosphere.

A compact literature survey signifies that a lot of researches have been done on the above-mentioned six parameters.

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Global warming and climate change are the most significant factors behind the rise of ocean temperatures in the past two decades. Subsequently, the rise in ocean temperatures has conversely helped in the increase in the frequency of development of cyclones in the tropical oceans. For the past 30 years, the increase in intensity and frequency of tropical cyclones and the subsequent devastation effect on our civilization was described by Emanuel (2005). A detailed analysis of tropical cyclone data was done by Wang (2016). The highest frequency of tropical cyclones in number occurs in tropical and equatorial regions. Studying the dynamics of tropical cyclones means studying the nature and solution of the governing equations of fluid dynamics applicable for the dynamics of a field point within the representative fluid parcel of a tropical cyclone. Due to the extreme complexity in the process of solving the governing equations of fluid dynamics analytically or semi-analytically applicable for tropical cyclone dynamics (starting from the depression to the dissipation stage of a tropical cyclone), the general procedure used thoroughly worldwide is the numerical technique known as computational fluid dynamics (CFD). To study in detail the impact of Typhoon hazards in the coastal regions of China the numerical simulation approach in three-dimension made by Liu et al. (2019), bears a strong significance. By using pseudo-random fluctuations the boundary conditions of the simulations are provided. The pseudo-random fluctuations include the present meteorological pattern of the system. A number of meteorological simulations were done for artificial cyclones. The results helped immensely in predicting the track and intensity of future typhoons. A detailed description of hurricane dynamics, its internal structure, basic guiding equations, boundary conditions, vortex formation etc. were discussed by Wang and Xu (2010). Accordingly, the hurricane has the core innermost region known as the eye which is a cylindrical region having the cyclonic cylinder axis as its central axis. The eye region of the cyclone is relatively calm. The cyclone eye has been followed by the wall cloud region where the turbulence speed of the cyclone wind field starts to increase and becomes maximum at the wall cloud periphery. Then, the turbulence speed of the cyclone wind field decreases. The recent effective research progress on tropical cyclone dynamics was summed up by Wang and Wu (2004). Asymmetric and symmetric are two important models of mature tropical cyclones. Different phenomena like wind gusts, a gale wind, spiral rain bands, concentric eyewall cycle, geostrophic wind, squall lines, thermal wind, cyclostrophic wind, geopotential and potential temperature, barotropic and baroclinic atmospheres, circulation, vorticity, anomaly temperature, turbulence etc. are important terminologies linked with tropical cyclone dynamics. We shall discuss these terminologies in detail in a later section. Circulation and vorticity are respectively the macroscopic and microscopic rotational parameters linked with the constitutional property of tropical cyclones. Important recent work like, ‘When will cyclogenesis commence given a favourable environment?’ by Levina and Montgomery (2013) describes the censorious components for tropical cyclone evolution and augmentation. The cyclone wind field near core structure from Atlantic and Eastern pacific storms (1977-2001) are examined in detail by Mallen et al. (2005). Utilizing flight level observations combined from a huge number of radial
flight segments they arrived at a conclusion that the eye region of a tropical cyclone is
mainly characterized by comparatively weak cross-radial wind deterioration, although
there is a presence of strong vorticity at the core. In simple words, the axial region of
cyclonic turbulence is relatively calm compared to the periphery. In this scenario, we
must also concentrate on some earlier works on tropical cyclone dynamics. Mandal
(1986) propounded an important semi-empirical formula relating anomaly
temperature with the tropical cyclone storm intensity. Estoque (1962) showed that
the gradient wind balance and hydro-static balance linked with a fluid parcel in a
tropical cyclone can be employed to estimate the upright and spiral dynamics
connected with a specified cross radial and temperature distribution of a tropical
cyclone. Kilroy et al. (2014) developed an important model for tropical cyclone
augmentation. They stressed the increase of cross radial velocity towards the core of a
tropical cyclone boundary layer as an important contributing factor behind tropical
cyclone stabilization. A non-linear empirical relationship between Atlantic tropical
cyclone uninterrupted velocity and the subsequent sea-level temperature was
constructed by De-Maria and Kaplan (1994a). By numerical SSP (Statistical Software
Package) analyses a semi-empirical simple linear relationship between the maximum
speed of tropical cyclones and the corresponding sea surface temperature over the
Eastern-North Pacific Ocean was derived by Whitney and Hobgood (1997). Crinivec
et al. (2015) also worked on the same topic. Arora and Dash (2016) explained by
the enthalpy exchange system that the air-sea surface temperature variation during low-
pressure formation or cyclogenesis acts as a fuelling factor affecting the maximum
tangential speed of tropical cyclones.

As discussed before as the second critical parameter behind tropical cyclone genesis
and intensification, is the favourable location of the site of occurrence of the cyclone
at $5^\circ$ a latitude from the equator. The warm ocean waters of the critical temperature
$26^\circ$C are most favourable for cyclonic depression genesis. The formed depression
then generally circulates towards the northwest owing to the rotation of the earth
about its axis from west to east. In general, for places where there is a resultant finite
angular velocity vector, the probability for the sustainability of tropical cyclones is
higher there (Emanuel 2011). Choi et al. (2015) studied in detail the maximum
intensity and location site (location in terms of latitude and longitude) for cyclones at
the western-north Pacific ocean for a considerable period. They used statistical time
series analysis with the data and came up with important conclusions.

The third important critical factor for tropical cyclone development and
intensification as stated before is the existence of low vertical wind shear especially
in cyclone core. The difference in wind velocity and direction at two different
altitudes or geopotential heights of the atmosphere is defined by vertical wind shear.
The important physics behind the thermodynamics of tropical cyclone formation is
the Carnot Cycle developed by extraction of heat and moisture from the sea surface
(having ambient temperature of around $26^\circ$C which is ideal for cyclone formation) by
adjacent air and then rising to high altitude with this moisture content in low vertical

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shear environment without changing of the significant magnitude of velocity and direction. Accordingly, when this moisture-containing air comes in contact with cooler temperatures at high altitudes the moisture content condenses and releases latent heat in the upper troposphere. This latent heat gets liberated in the form of conversion to potential energy. This huge potential energy release in the upper troposphere works as a significant fuelling factor behind the intensity development of tropical cyclones and its consequent devastation to civilization. So, this entire phenomenon is highly catalysed in a low vertical shear environment. It’s often being said that a tropical cyclone is preceded by a calm environment. The fourth factor behind tropical cyclone formation and intensification is the presence of ambient moisture in the atmosphere. Zhang et al. (2013) underwent a detailed study into this parameter of low vertical shear in the inner core region of a tropical cyclone. They described in detail the thermodynamic Carnot cycle linked with the transport of moisture from sea level to the generation of cyclone potential energy through shear. This cyclone potential energy also called destructive potential energy is related to tropical cyclone intensity. Powell and Reinhold (2007) defined a parameter named Cyclone Integrated Kinetic Energy (IKE) and developed a mathematical relationship between cyclone destructive potential and integrated kinetic energy. The strength of the wind field or the total destruction field of a cyclone comes as an estimation from its Integrated Kinetic Energy (IKE). The quick natural boosting process and subsequent programming of hurricanes were studied in detail by Reasor et al.(2000). They came up with the conclusion that the augmentation process was deeply characterized by brine layer vertical shear. They also studied in detail the internal structure of storms and concluded that the variation of storm intensity during the time of occurrence of the cyclone has a deep complex interaction with vertical wind shear. Literature review suggests that due to the presence of low vertical shear in the axis portion of the tropical cyclone geometrical structure (considering the cyclone geometrical structure to be a rotating hollow cylinder analogue), the eye portion (core-region) of the cyclone is calm. The effect of vortex Rossby waves in the stabilization of eyewall of cyclone has been discussed by Zhang et al. (2009). Bretherton et al. (2004) developed a non-linear relationship between moisture of tropical oceans (originated from warm ocean waters) and subsequent precipitation in connection with tropical cyclones. The fifth critical factor for tropical cyclone genesis and intensification is the presence of pre-existing atmospheric disturbances in the cyclone site that acts as a positive catalyst of the genesis of a strong tropical cyclone from weak depression. The warm ocean waters of tropical regions produce moisture which in turn gets adhered to the adjacent air to the sea bed. The moisture mixed warm air adjacent to sea bed gets lifted upward by convection process due to the presence of cold heavy air in higher altitude. In this process the position of cold heavy air of higher altitude is occupied by the transport of warm moisture containing hot air adjacent to warm sea water surface, consequently the cold heavy air of higher altitude are transported down. These process is called convective cycle where the atmosphere acts as the source and the sea as the infinite energy reservoir (Emanuel 1991). This process is idealized in presence of low vertical shear (Smith 2006) which
gives a reviewed detailed description of the present mathematical perceptions and theories on tropical cyclone genesis, impact, structural dynamics, subsequent physics etc. The early prediction and forecasting of tropical cyclones is the main scope of research lately. Tropical cyclones are not common in regions of undulating landscapes or of higher altitudes. The higher rate of decaying of the intensity of tropical cyclones in undulating land-forms compared to flat land-forms was shown analytically by Lala et al. (2014). In lower latitudes, the dynamics of frictional boundary layer of atmosphere plays an important role in the concentration and stabilization of tropical cyclones. This was chalked out by Kilroy et al. (2017). Raymond et al. (2009) defined an important critical parameter for tropical cyclone development from depressions called the normalized gross moist stability and estimated a critical value of cyclone radius from it. Accordingly it is defined as the ratio of the net emergence of moist entropy or moist potential energy produced from thermodynamic Carnot cycle to the total moist energy of the system produced from the atmospheric convection process. In this regard the important analytic performance on total cyclone dynamics of Ghosh and Chakravarty (2018) deserves special mention. They solved the dynamical Navier- Stoke's equation in cylindrical polar coordinate system applicable to tropical cyclones with the WKB approximation and hence developed a critical stability parameter for tropical cyclone development. Williams et al. (2013) did study the boundary layer pumping of tropical cyclones and obtained shock like structures when they plotted the boundary layer component velocities with respect to the radial distance from the core of the storm. De-maria and Kaplan (1994b, 1999) developed a model for estimating the maximum potential intensity parameter of tropical cyclones occurring over the Atlantic and East North Pacific Ocean. Kieu (2004a, 2004b) also prepared a numerical model for tropical cyclone dynamics. The dissipation energy plays an important role in the devastation aspect of tropical cyclone to human civilization. For the determination of maximum intensity, frictional gratification energy for storm surge they used numerical simulation on the model produced by numerical analysis of the guiding equations of tropical cyclone dynamics. This dissipation energy has also been a major important concern in the work of Emanuel (2007) who showed the importance of environmental parameters in the concentration of tropical cyclones. In the field of atmospheric physics, dynamic meteorology, fluid mechanics, earth sciences, hydrology etc. tropical cyclone dynamics constitutes a major portion of nowadays research.

Specifically, the Coromandel Coast of India gets maximum affected from cyclones generated over Bay of Bengal (To be more precise the Paradip Island, Coastal regions of Odisha, Andhra Pradesh and Tamil Nadu alongside with some Southern districts of West Bengal also). After the striking of the fatal 1999 Odisha super cyclone on Paradip island and coastal regions of Odisha a large number of destructive cyclones crushed different portions of Coromandel Coast from time to time. Some of them generated in North Indian Ocean and Bay of Bengal are respectively, Cyclone Aila (May 2009), Giri (2010), Laila (2010), Nargis (2008), Phailin (2013), Nilam (2012), Helen (2013), Hudhud (2014), Vardah (2016), Titli (2018), Gaja (2018) etc. The quantum of destruction to both lives and properties was maximum inflicted by Nargis

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and Super Cyclone 05B. Now if we move to other cyclone-prone areas of the world like North Atlantic and Pacific Ocean then category 5 storms like Emily and Katrina (2005), Hurricane Patricia (2015), Typhoon Haiyan (2013) etc. deserves attention. The main dynamical equations guiding cyclone dynamics are basically the core equations applicable for the dynamics of a fluid parcel in fluid dynamics and thermodynamics viz:- the Navier Stoke's equation, the momentum conservation equation, the equation of continuity in fluid dynamics and the equation of state. The exact analytic algebraic solution of the Navier Stoke's equation is impossible without finite approximation due to the presence of non-linear terms on it. That's why researchers resort to numerical solution with necessary stipulated boundary conditions called computational fluid dynamics (CFD). The description of tropical cyclone dynamics through analytic solution of the corresponding equations viz:- the Navier- Stokes' equation, the momentum conservation equation, the equation of state and the equation of continuity in thermodynamics is not plentiful. Kieu (2008) developed an analytic model for describing the tropical cyclone dynamics by considering the axisymmetric nature of the tropical cyclones, retaining all the non-linear terms in the Navier stoke's equation and considering the vertical velocity to be directly proportional to Joule Coefficient of heating. He also developed a numerical model with necessary approximations on tropical cyclone dynamics. In gross, for the purpose of our research simplicity and also to reduce the complexity in the dynamics of tropical cyclones we divide the life-cycle of tropical cyclones in four stages, viz:- the genesis stage or the formation stage of a stable strong tropical cyclone from a mere environmental depression, with absence of closed isobars at the boundary surface, the tropical depression stage which is also marked as the second stage in the life cycle of tropical cyclones, the tropical storm or hurricane stage at which the rate of devastation rendered by tropical cyclones reaches its peak and lastly the decay stage where the cyclone intensity decays with time. Significantly analytic research on all these four stages to describe the basic physics and hence proper validation of tropical cyclone dynamics has been a major point of concern of the meteorologists fraternity all over the World. The annual and seasonal variability in tropical cyclone genesis and its nature in the Atlantic, Indian, Eastern North Pacific etc. tropical oceans and water bodies and basins has been attributed to both thermodynamic and dynamical factors. That sea surface temperature of 26°C is ambient and effective for tropical cyclone genesis and formation and hence intensification was first indicated by Palmen (1948). Riehl (1948) explained the role of vertical shear of the horizontal cyclone wind in the action of inhibiting the process of tropical storm formation. In simple words he has shown that strong vertical shear of horizontal wind acted as a negative catalyst in the process of cyclone genesis. Gray (1998) defined a parameter for cyclone genesis. This parameter evolves from the necessary thermodynamic and dynamic terms. The behaviour change of hurricanes with altitude or geopotential height was studied in details by Koteswaram (1967). After a detailed analysis he arrived at a conclusion that there is an approximate height of 15Km from sea level after which the extension of warm core of hurricanes changes to cold core. Hawkins and Imbembo (1976) studied in details the features of small intense hurricane Inez.
La seur and Hawkins (1963) explained the detailed analysis of hurricane Cleo (1958). By analysing the details of research flight missions into Hurricane Daisy (1958), Riehl and Malkus (1961) described in details some important aspects like heat budget, moisture, kinetic energy, momentum fluxes etc. In a similar fashion Hawkins and Rubsam (1968) described the detailed features of hurricane Hilda (1964). Hilda was a fairly symmetrical storm. The strongest horizontal pressure gradient was obtained in the eye wall at a pressure level below 550 mb. Frank (1977) has shown that the mean two dimensional temperature anomalies persist thoroughly through the troposphere. According to his work the variation of temperature with radius is negligible in the boundary layer despite decrease of pressure with maximum pressure occurring at 250 mb. Anomaly temperature being defined as the variation of temperature at a certain altitude from the standard reference level due to the atmospheric perturbation of cyclone. In connection to cyclone genesis Macbride (1995) developed an anomaly temperature parameter and showed its well correlation with global tropical cyclone formation region. Further details study on low vertical shear on Atlantic tropical cyclone genesis was carried out by Gray (1984). Shaipro and Goldenberg (1998) showed the frequency of tropical cyclones with the increase of sea surface temperature over Atlantic ocean. Global warming in recent years has also been a major concern of the research for meteorological scientists in the past two decades. Chou and Nelin (2004) developed a proportional relationship between tropical cyclone precipitation and global warming. An important statistical relationship between wind speed (maximum potential intensity of wind) and precipitation during occurrence of tropical cyclones was developed by Back and Bretherton (2005). Back and Bretherton (2009) worked for developing the relationship between tropical cyclone convergence, boundary layer of wind flow and the temperature gradient, which varies radially on the surface. They developed another lucid model involving the climatology dynamics and vertical flow pattern of tropical cyclones. Every aspect of geophysical fluid dynamics like assemblage transmission, boundary layer theory, thermodynamic cycles, surface wave dynamics, currents of the upper part of the ocean, barotropic and baroclinic instability, Rossby waves, sea surface wind water interaction etc. are amalgamated in tropical cyclone dynamics. So necessary scope of research in this entire fraternity of tropical cyclones is immense. However the main barrier in theoretical analysis of tropical cyclone dynamics is the extreme critical nature of the analytic solution of the Navier-Stoke’s equation of fluid dynamics which prevents us to obtain the elaborate analytic insight of the mechanical, statistical, dynamical and thermodynamic processes involved in tropical cyclone. In this connection the solution of cross radial velocity of cyclone profile by Ghosh and Chakravarty (2017) deserves mention. Ghosh et al. (2020) also correlated astrophysical parameters viz. stellar scintillation and occultation in the turbulence mechanism of tropical cyclones. Especially the evolution of tropical cyclones from the infant genesis stage is characterized by a strong vigorous growth having suitable environmental parameters aligned in the process before arriving a quasi-stationary mature stage. This is a critical stage of transition period in the life cycle of tropical cyclones and this genesis stage of stable tropical cyclone needs to be

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addressed properly by strong scientific exploration and physical reasoning. The geometrical structure of a tropical cyclone and its alignment from the axis also affects its devastation pattern on civilization.

I.ii. Main Research Objective

The main research objectives behind tropical cyclone dynamics are:

- Whether a system of analytic solution to the non-linear Navier Stoke’s equation in cylindrical polar co-ordinate system exists for each component velocities to generate analytic solution model?
- By solving the basic equations in fluid dynamics can the genesis phenomenon and stability phenomenon in connection to tropical cyclones be expressed in a more scientific and effective way?
- By solving the Navier- Stoke's equation for the velocity components of a representative random fluid parcel in tropical cyclone can we estimate a mathematical stability parameter for the generation of tropical cyclones?
- Can a model be generated involving the anomaly temperature of tropical cyclones that can also aid in better forecasting of tropical cyclone tracks and will hence help to determine that which depression after its genesis on ocean bed ultimately converges to a mature tropical cyclone?
- Can the four stages of life cycle of a tropical storm viz: the genesis, tropical storm, hurricane stage and decay stage be uniformly counterfeited both by analytic modelling and hence valediction with real cyclone data?
- Does analytic model satisfying cyclone dynamics (physically viewed) with synthetic data assimilation satisfy actual cyclone data?

II. Review of the existent theories

The basic guiding equations for the dynamics of tropical cyclones are the following system of fluid equations:

II.i. Navier Stoke’s equation

For Newtonian, in-compressible fluid in three dimension the Navier- Stoke's equation is given by,

\[ \frac{dU}{dt} = -\frac{1}{\rho} \nabla p + \mathbf{g} + \nu \nabla^2 \mathbf{U} \]  \hspace{1cm} (1)

Where, \( \frac{1}{\rho} \nabla p \) is the pressure gradient force/ unit mass, \( \nabla \) being the gradient operator.

\( \mathbf{g} \) is the gravitational force/ unit mass.

\( \nu \nabla^2 \mathbf{U} \) is the viscous force per unit mass.

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\( \nu \) is the viscosity coefficient per unit mass. Now,

\[
\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \vec{U} \cdot \vec{\nabla}T
\]  

(2)

\( \frac{\partial T}{\partial t} \) is the local temperature change in absolute scale.

\( \frac{DT}{Dt} \) is the global temperature change rate with respect to time.

Now,

\[
\frac{\partial T}{\partial t} = \frac{DT}{Dt} - \vec{U} \cdot \vec{\nabla}T
\]

(3)

Now,

\[
\vec{U} = iu + jv + kw
\]

(4)

Where, \( \vec{U} \) is the 3 \( \times \) velocity vector in any concerned point within a fluid parcel of the cyclone. \( u, v \) and \( w \) are the respective projections or components of \( \vec{U} \) along the \( X, Y \) and \( Z \) axes respectively. The term \( -\vec{U} \cdot \vec{\nabla}T \) in equation (3) is called the temperature advection term. This is a complicated non-linear term which has profound physical significance. This signifies the local temperature change due to wind flow in a certain region. Thus we can conclude that if wind flows by diffusion to maintain temperature equilibrium from a cold region to a warm region then \( -\vec{U} \cdot \vec{\nabla}T \) will be negative and it is called negative convection and obviously this reduces resultant temperature advection. Similarly, if we consider the flow of wind from warm region to a cold region then the phenomenon is called positive convection and this enhances resultant temperature advection, (Holton 1972).

If we consider the rotation of the earth from west to east with angular velocity \( \vec{\omega} \) then the Navier Stokes equation further complicates to :-

\[
\frac{Du}{Dt} = -2\vec{\omega} \times \vec{U} - \frac{1}{\rho} \vec{\nabla} p + g + F_r
\]

(5)

Here, \( -2\vec{\omega} \times \vec{U} \) is the Coriolis force and \( F_r \) is the frictional force.

For considering the cyclone dynamics with a rotating cylinder analogue we convert the Navier Stokes equation in fixed stationary frame (3) to cylindrical coordinate system having parameters \( r, \theta \) and \( z \). The \( r \)-component equation is given by :-

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The component equation is given by:

\[
\rho \left( \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_r u_{\theta}}{r} - \frac{u_{\theta}^2}{r} + u_z \frac{\partial u_r}{\partial z} \right) = -\frac{\partial p}{\partial r} + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_r}{\partial r} \right) - \frac{2}{r^2} \frac{\partial u_{\theta}}{\partial \theta} + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_r}{\partial z^2} \right] + \rho g_z \] (6)

The \( \theta \) component equation is given by:

\[
\rho \left( \frac{\partial u_{\theta}}{\partial t} + u_r \frac{\partial u_{\theta}}{\partial r} + \frac{u_r u_{\theta}}{r} + \frac{u_{\theta} u_z}{r} + u_z \frac{\partial u_{\theta}}{\partial z} \right) = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_{\theta}}{\partial r} \right) - \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} + \frac{1}{r^2} \frac{\partial^2 u_{\theta}}{\partial \theta^2} + \frac{\partial^2 u_{\theta}}{\partial z^2} \right] + \rho g_z \] (7)

The \( z \) component equation is given by:

\[
\rho \left( \frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_r u_z}{r} + u_z \frac{\partial u_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} \right] + \rho g_z \] (8)

Here the suffix indices \( r, \theta \) and \( z \) to a certain quantity are the respective components of that vector quantity along radial (\( r \)), cross radial (\( \theta \)) and vertical (\( z \)) direction respectively. We have mainly focused in the solution of equation (7) (the \( \theta \) - component or the cross-radial component of the Navier-Stoke's equation) with axial symmetry and retaining the non-linear terms. This is mainly due to the fact that the turbulence of a tropical cyclone is mainly estimated in terms of its cross-radial or tangential velocity(\( u_{\theta} \)).

Rewriting (7) taking \( u_r = u; u_{\theta} = v; u_z = w \) and introducing the axial symmetry we get,

\[
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + \frac{v}{r} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial r} + \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v}{\partial r} \right) - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2} \right] + \rho g_z \] (9)

II.ii. Equation of continuity in fluid mechanics

The equation of continuity for a fluid parcel in fluid mechanics is given by,

\[
\frac{D\rho}{Dt} + \rho \vec{V} \cdot \vec{u} = 0 \] (10)

Where, \( \frac{D\rho}{Dt} \) is the global rate of change of density of the fluid parcel with respect to time. For an incompressible fluid the vector field within the fluid is solenoidal and the equation of continuity (10) reduces to \( \frac{D\rho}{Dt} = 0 \) and hence \( \rho \) is constant.

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Thus the fluid density is constant for an in-compressible fluid.

Hence for an in-compressible fluid in the cylindrical polar coordinate system the equation of continuity becomes, in terms of \( r, \theta \) and \( z \) as

\[
\frac{\partial}{\partial r} (\rho wr) + \frac{\partial}{\partial z} (\rho wr) = 0
\]

(11)

II.iii. The equation of state

If we consider a certain reference point within a fluid parcel then its thermodynamic state is represented in terms of three parameters viz.: the pressure, absolute temperature and density. These three variables for an ideal gas is related by the equation of state given as,

\[
P = \rho RT
\]

(12)

Where, \( P \) is the pressure at the point under consideration.

\( R \) being the universal gas constant which is numerically given as \( 287 \text{ Jkg}^{-1} \text{ K}^{-1} \)

\( T \) is the absolute temperature.

Now, actually for a fixed volume ' \( V \) ' of an ideal gas of pressure ' \( P \)' and absolute temperature ' \( T \)', the equation of state becomes as,

\[
P V = nRT
\]

(13)

[n is the number of moles under consideration]

Now the equation (13) reduces to

\[
P = \rho RT
\]

(14)

Where \( \rho = \frac{n}{V} \) is the molar density.

II.iv. The equation of hydro-static balance

The equation of hydro-static balance basically signifies the decrease of pressure with respect to the increase in height of the field point (i.e. the increase of geopotential) from the sea surface upto tropopause in atmosphere. It is given by,

\[
\frac{\partial p}{\partial z} = -g \rho
\]

(15)

Where \( \frac{\partial p}{\partial z} \) is the pressure gradient with respect to height.

II.v. The thermodynamic equation

The thermodynamic equation is nothing but the mathematical combination of first and second laws of thermodynamics. The first law of thermodynamics gives us for any isolated closed system, \( dQ = du + dw \)

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Where, \( dQ \) is the heat produced in the process.

\( du \) is the change in internal energy of the system.

\( dw \) is the external work done.

From the definition of entropy, \( dS = \frac{dQ}{T} \)

Thus combining first law and definition of entropy we get the second law of thermodynamics as, \( du = Tds - pdv \)

Thus ultimately we get as,

\[
C_p \frac{dT}{dt} - \frac{1}{\rho} \frac{dP}{dt} = J \tag{16}
\]

Where \( C_p \) is the molar specific heat at constant pressure, \( J \) is the mechanical or Joule equivalent of heat.

If we apply Reynold's average to the right hand side of the equations (5), (10), (14) and (16) will contain some eddy components viz: \((u', v'), (u', w'), (u', T')\) that requires parameterization. Giving the forcing terms \( F \) and \( J \) the parameterization for the eddy terms, the above system is closed with six unknowns \((u, v, w, p, \rho, T)\) which can be solved. The intense challenging research task in imbibing the dynamics of tropical cyclone is the parameterization of eddy forcing terms with applicable boundary conditions and the non-linearity of the operator present in the right hand side of equation (5). Since for the development of a stable cyclonic vortex the tangential or cross radial velocity \((v)\) is always some magnitude higher than the radial velocity \((u)\) and the vertical component velocity \((w)\) the solutions of equations (5), (12), (15) and (16) are often expressed in a non-dimensional ratio of \(u\) and \(v\). Due to azimuthal and axisymmetric nature of tropical cyclones we consider the generic geometrical structure associated with a stable tropical cyclone is analogous to a rotating cylinder. Thus for necessary mathematical operations for the sake of development of the tropical cyclone dynamical model under mesoscale analysis equations (5), (10), (14) and (16) are converted to (9), (11), (14) and (16) respectively. No change in equations (14) and (16) occurs since the azimuthal and axisymmetric assumption parameters remain changed in this equation.

II.vi. Balanced dynamics

The gradient wind approximation is among the non-linear earlier most model for describing the tropical cyclone dynamics. The gradient wind approximation basically establishes a three way relationship between centrifugal acceleration, the pressure gradient force and the Coriolis force per unit mass. This approximation does not allow us to consider the interaction between secondary circulation and primary

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circulation. Since the secondary circulation is the resultant of the effects produced by the convergent inflow of the planetary boundary layer (PBL), vertical motion in the inner core region and the divergent outflow aloft, so it’s definitely worthy to consider its interaction with primary circulation to describe the complicated tropical cyclone dynamics. Due to this drawback the quasi-balanced model came to the forefront for describing the structural geometry of tropical cyclones. In the description of secondary circulation from adiabatic and frictional processes, Shaipro and Willoughby (1982) proposed slow evolution of axisymmetric vortex (Sundquist 1970, Hack and Schubert 1986). The essentiality of the quasi balanced model is based on the following equations:-

**II.vii. The Saywer-Elliassen equation**

This equation denotes a fundamental combination of hydro-static approximation (Yanai 1964), the thermal wind equation and the gradient wind approximation. It is given by,

$$\frac{\partial}{\partial r} \left( \frac{A}{r} \frac{\partial (r\psi)}{\partial r} + B \frac{\partial \psi}{\partial z} \right) + \frac{\partial}{\partial z} \left( B \frac{\partial (r\psi)}{r \partial r} + C \frac{\partial \psi}{\partial z} \right) = \frac{g}{\theta_0} \frac{\partial f}{\partial r} + \frac{\partial (\eta F_r)}{\partial z}$$

(17)

\(\psi\) being the stream function for cross-radial circulation. In azimuthal, axisymmetric symmetry of tropical cyclone development the stream function is defined by,

$$\frac{\partial \psi}{\partial z} = \rho w r \frac{\partial \psi}{\partial r} = -\rho wr$$

(18)

\(\psi = f(r, z)\) and it is basically a perfect differential of \(r, z\).

\(\eta\) is called the absolute vorticity. \(\theta_0\) is the potential temperature at reference level of 100 hPa.

The coefficients \(A, B\) and \(C\) are given by, \(A = \frac{g}{\rho \theta_0} \frac{\partial \theta}{\partial z}\) which actually depicts the static stability. \(B = -\frac{\rho}{\rho \theta_0} \frac{\partial \theta}{\partial r}\) represents the baroclinic stability.

$$C = \frac{f^2}{4\pi \rho} \frac{\partial}{\partial z} \left[ \frac{2\nu + r^2}{f} \right]$$

is the inertial stability.

The horizontal momentum equation for the cross radial wind,

$$\frac{\partial v}{\partial t} = \left( \frac{u}{\partial r} + \frac{w}{\partial z} + \frac{uv}{r} \right) - f u$$

(19)

\(\frac{\partial u}{\partial t}\) is the horizontal component of force per unit mass.

The hydro-static balance approximation in a more refined way in terms of the potential temperature (\(\theta\)) is given by,

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The transverse stream function is used to determine the radial and vertical flow component velocities \( u, v \) is given by,

\[
\frac{\partial \phi}{\partial z} = \frac{g \theta}{\theta_0}
\]  

(20)

The quasibalanced secondary circulation can be obtained by solving equation (17) and equation (21) provided we are supplied a diabetic heating distribution \( J \) and frictional distribution \( F \). Thus combining the gradient wind approximation and the solutions obtained from (17) and (21) we can describe the dynamics of a tropical cyclone for a quasibalanced framework and a balanced vortex. The balanced dynamics incorporated work of tropical cyclones associated with potential vorticity (PV) illustrates a different expression of quasi balanced dynamics. In the new mathematical formulation the important relationships are as follows (Davis 1992, Wang and Zhang 2003):- The Charley nonlinear balanced (NLB) equation:-

\[
\nabla^2 \phi = \nabla_h (f \nabla_h \psi') + 2J \left( \frac{\partial \psi'}{\partial x} \right) + \nabla_h \cdot F
\]  

(22)

And the potential vorticity (PV) equation:-

\[
q = \frac{1}{r(z)} \frac{\theta_0}{\theta} \left[ f + \nabla^2 \psi' \right] \frac{\partial^2 \phi}{\partial z^2} + \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \psi'}{\partial x \partial z} - \frac{\partial^2 \psi}{\partial z \partial x} - \frac{\partial^2 \phi}{\partial y \partial z} - \frac{\partial^2 \phi}{\partial y \partial z} - \frac{\partial^2 \psi}{\partial y \partial z}
\]  

(23)

The pseudo vertical height coordinate is given by \( z = \left[ 1 - \frac{\rho}{\rho_0} \frac{R}{C_P} \right] \left( \frac{C_P \theta_0}{g} \right) \) The pseudo density is represented by, \( \rho(z) = \rho_0 \left( \frac{R}{R_0} \right) \frac{C_P}{g} \)

\( \psi' \) is the horizontal stream function.

\( \phi \) is the required geopotential.

We can draw closed contour streamlines for corresponding low pressure zones of tropical cyclones with solutions of \( \phi' \) and \( \phi \). We define the \( q \) value of a tropical cyclone as the volumetric flow rate. Thus for determining 3 dimensional characteristics of a cyclone we need to know its potential vorticity value, given equation of state and other boundary conditions. The NLB equation as stated above is basically an essential equation in the quasi balanced dynamics and consists of following force terms from left to right, viz:- the Coriolis force (since the expression consists of Coriolis parameter \( f = 2 \omega \sin \phi \omega \) being the angular velocity of the earth and \( \phi \) is the longitude of the place under consideration ), the advection term.
(nonlinear) and frictional force term. These represented equations, viz.:-(22) and (23) of the non-linear quasi balanced dynamics has been widely used in number of works like Möler and Montgomery (2000), Möler and Shaipro (2002) for describing the genesis and intensification in tropical cyclone dynamics. It is to be noted that by the quasi balanced dynamics we describe the secondary circulation associated with tropical cyclone dynamics. So before starting anything we have to consider the primary circulation as a consequence of which the secondary circulation gets developed.

II.viii. CISK theory

The conditional instability of the second kind is abbreviated as the CISK theory. It is one of the most primitive but effective theory for describing the dynamics linked with tropical cyclone. The guiding equation of tropical cyclone dynamics i.e. Navier-Stoke's equation consists of the temperature advection term. Due to its non-linearity the solution of the Navier-Stoke's equation becomes too complex in this regard. In the CISK theory quantum mechanics was first introduced in describing dynamics of tropical cyclone. This CISK theory targeted the non-linear explanation of tropical cyclone dynamics. According to this theory non-linear perturbation has been introduced in secondary circulation of a tropical cyclone in the form of a non-linear wave function, \( \psi = \psi(r)e^{\beta t} \) which is basically a function of space and time. Here \( \beta \) is the growth rate of the wave function which varies exponentially with time. The increase of the growth rate depends on how smaller the scale of disturbance of the tropical cyclone is. Thus, for tropical cyclone development the low level convergence associated with a stronger balanced vortex is monitored by Ekman pumping [Charney and Eliason (1964)]. \( \psi \) is the stream function and is a measure of the non-linear perturbation introduced within the solution system. Ooyama (1969) developed similar kind of relationship between the growth rate \( \beta \) and the radius \( r \) of the tropical depressions.

Certain weaknesses also evolve from the CISK theory. Firstly, the practical real data of tropical cyclone growth rate and radius of tropical depressions in the paradigm of tropical cyclone genesis does not support the analytic relationship proposed by the theory. Secondly, the proposed perturbed stream function in this case is not appropriate to be a perfect differential of the radial and vertical distance of the field point within the fluid parcel. Thirdly, no time dependent solution has been obtained in this theory. Thus, formation of quasi balanced vortex for cyclone core does not support appropriate tropical cyclone dynamics.

II.ix. WISHE theory

Wind induced surface heat exchange (WISHE) theory was developed by Emanuel (1986). This theory provides an analytic relationship between the equivalent potential temperature \( (\theta_e) \) and the pressure at the top of the potential boundary layer (PBL). The relationship is given by,

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This relationship has been considered at vertical height, \( z = h \), \( h \) being the top of the boundary layer height which is around \( 20 \text{km} \) (approx.) from sea level. \( T_B \) is temperature at the boundary layer. \( T_{out} \) is the upper level outflow temperature. \( \theta_{ea} \) is the ambient equivalent potential temperature. \( \frac{K}{\rho_0} \) is known as the Exner function. For solving equation (24) Emanuel used another important relationship between \( \theta_e \) and \( \frac{K}{\rho_0} \) which can be derived from the expression of the gradient wind approximation, the relationship is given by,

\[
\ln \theta_e = \ln \theta_{ea} - \frac{C_D}{C_p} \frac{1}{\rho_0} \left( \frac{v^2 + 2r \omega}{2} \right) \tag{25}
\]

Where \( T_0 \) being fixed approximation of \( T_{out} \).

The main outcome of this theory is the dependence of the maximum surface wind speeds on the frictional coefficients in the expression of Navier-Stoke's equation during the tropical cyclone genesis period. Now the destructive potential of a cyclone depends on the formation of a deep vortex. According to this theory a strong cyclone vortex acts as a positive catalyst is sweeping out mixture and heat fluxes from ocean surface within the cyclone core and consequently the tendency of increase in further strength of the vortex develops.

**II.x. Boundary layer model theory**

The most activeness of a tropical cyclone and its interaction to our materialistic World and atmospheric environment take place within the boundary layer i.e. atmosphere extended upto the tropospheric height. Thus considering the \( f \)-plane for a axisymmetric vortex the boundary layer equations for studying tropical cyclone dynamics is given by :-

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( ru^2 \right) + \frac{\partial}{\partial z} (uw) + \frac{v^2}{r} - \frac{ \omega w^2 }{r} + f (v_y - v) = \frac{\partial}{\partial z} \left( \frac{K}{\rho_0} \frac{\partial u}{\partial z} \right) \tag{26}
\]

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 uv \right) + \frac{\partial}{\partial z} (rvw) + f u = \frac{\partial}{\partial z} \left( \frac{K}{\rho_0} \frac{\partial v}{\partial z} \right) \tag{27}
\]

\[
\frac{1}{r} \frac{\partial}{\partial r} (ru\phi) + \frac{\partial}{\partial z} (w\phi) = \frac{\partial}{\partial z} \left( \frac{K}{\rho_0} \frac{\partial \phi}{\partial z} \right) \tag{28}
\]

\[
\frac{\partial}{\partial r} (ru_i) + \frac{\partial}{\partial z} (ri) = 0 \tag{29}
\]
If we consider stationary co-ordinate system \((r, \phi, z)\) in the cylindrical polar co-ordinate system then \((u, v, z)\) are respective components of the velocity vector \((3d)\) at a certain point within the fluid system at a certain instant of time. So, to describe the dynamics of tropical cyclone within the boundary layer these equations need to be solved either analytically or numerically and hence is to be validated with real cyclone data to get the fundamental model of cyclone impact on mankind and civilization. This boundary layer theory was first proposed by Smith and Montgomery (2010).

III. Different important terminologies linked with tropical cyclone dynamics

The atmosphere is considered as an utmost continuum in fluid dynamics. For definition of many atmospheric parameters we have to consider an in-compressible fluid. Hence we start as follows:-

III.i. In-compressible fluid

For a fluid in motion at a certain instant of time within the fluid at a certain point if the \(3d\) velocity has closed flux then it is called in-compressible fluid. Thus, if \(\vec{U}\) represents subsequently the \(3d\) velocity vector within a fluid parcel at a certain instant of time then, \(\nabla \cdot \vec{u} = 0\). Thus for an in-compressible fluid the \(3d\) velocity vector has closed lines of force. Again, utilizing the equation of continuity for an incompressible fluid we obtain that the density is constant of time.

III.ii. Geostrophic approximation

Considering the component equations of the Navier- Stoke's equation as given in equation (5) in the spherical polar co-ordinate system \((\lambda, \phi, z)\) we get,

\[
\frac{Du}{Dt} \frac{uv \tan \phi}{a} + \frac{uw}{a} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + 2\Omega v \sin \phi - 2\Omega w \cos \phi + F_{rx}
\]

\[
\frac{Dv}{Dt} \frac{u \tan \phi}{a} + \frac{uw}{a} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - 2\Omega u \sin \phi + F_{ry}
\]

\[
\frac{ Dw }{ Dt } \frac{ u^2 + v^2 }{ a } = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g + 2\Omega u \cos \phi + F_{rz}
\]

Where \((u, v, w)\) are the respective components of the velocity vector \(\vec{U}\) along the radial, cross- radial and vertical direction respectively. Where 'a': is the earth radius. \((\lambda, \phi, z)\) are respectively the latitude, longitude and vertical distance of the concerned point from the earth's surface.

\(F_{rx}, F_{ry}, F_{rz}\) are the frictional force components per unit mass.

\(\omega\) is the angular velocity of the earth.

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\[ \frac{Du}{Dt} \text{ and } \frac{Dv}{Dt} \] are the global acceleration components along the component perpendicular axes and \[ \frac{1}{\rho} \frac{\partial p}{\partial x}, \frac{1}{\rho} \frac{\partial p}{\partial y}, \frac{1}{\rho} \frac{\partial p}{\partial z} \] are the respective pressure gradient force per unit mass.

\[ 2\Omega \times \vec{U} \] is the Coriolis force term. Where \( 2\omega \sin \phi \) being Coriolis parameter under synoptic scale subsequent latitude analysis. If the Coriolis force and pressure gradient forces balance each other then it is called geostrophic wind approximation. Now, geostrophic wind relationship is only possible in the horizontal field without the introduction of time. The entire velocity field is not considered and hence it is a diagnostic relationship. In geostrophic approximation the tangential term vanishes hence it is written as,

\[ -fv = -\frac{1}{\rho} \frac{\partial p}{\partial x} \quad (33) \]

\[ fu = -\frac{1}{\rho} \frac{\partial p}{\partial y} \quad (34) \]

Now, considering the horizontal velocity field,

\[ \vec{V} = \vec{i}u_g + \vec{j}v_g \quad (35) \]

Where \( u_g \) and \( v_g \) are the respective components of \( \vec{V}_g \) in horizontal plane then the geostrophic approximation gives,

\[ V_g = \vec{k} \times \frac{1}{\rho_f} \nabla p \quad (36) \]

Now, \( f \) is the Coriolis parameter while \( \vec{i}, \vec{j}, \) and \( \vec{k} \) are the unit vectors along the horizontal and vertical plane respectively.

**III.iii. Rossby Number**

It is basically the mathematical dimensionless parameter which signifies the impact of geostrophic approximation to the fluid system. This is defined as the ratio of the inertial force to the Coriolis force. Mathematically it is expressed as

\[ R_o = \frac{U}{L_f} \]

Where \( U \) the characteristic velocity is scale and \( L \) is the characteristic length scale for a concerned fluid parcel.

\[ f = 2\Omega \sin \phi \] is the Coriolis parameter.

Now, since in the definition of Rossby number the denominator term is Coriolis force, so lower the value of Rossby number implies that higher is the impact of geostrophic approximation for the concerned fluid parcel.

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III.iv. Potential temperature

For adiabatic expansion of ideal gas the potential temperature from the first law of thermodynamics is defined as, \[ \theta = T \left( \frac{P_f}{P_i} \right)^{\frac{C_p}{R}} \]

Where \( P_i \) and \( P_f \) are the initial and final pressures of the system.

\( T \) is the absolute temperature and \( C_p \) is the molar specific heat at constant pressure.

Now the temperatures of the atmosphere decrease with height and hence it is dependent on pressure. The variation of temperature with pressure has also got its significance from the definition of potential temperature, the hydro-static balance and the adiabatic lapse rate.

III.v. Adiabatic Lapse Rate

This is basically the mathematical expression of the decrease of temperature with height.

The lapse rate is given by, \[ \frac{dT}{dz} = -\frac{\theta}{C_p} \]

Where \( \frac{dT}{dz} \) is the temperature gradient.

III.vi. Inertial Flow

This is basically the balance between Coriolis force and centrifugal force. Thus what can be concluded is that inertial flow is exactly neutral both to cyclonic or anticyclonic effects in atmosphere.

Mathematically for a certain flow scale from inertial flow,

\[ \frac{V^2}{R} + fV = 0 \]  

(37)

Where \( R \) represents the radius of curvature of a certain fluid parcel, scale having velocity \( V \) and \( f \) is the Coriolis parameter.

Now from momentum conservation equation (30), (31) and (32) to achieve a force balance in equation (37) the horizontal pressure gradient force must vanish. This is only possible if the potential field is constant on a surface of constant pressure.

III.vii. Cyclostropic flow

As the name suggests this flow or approximation proposes tropical cyclone genesis.

In cyclostropic flow the pressure gradient force gets balanced by the centrifugal force. For a certain flow scale the cyclostropic approximation is given by,

\[ V = -\left( -R \frac{\partial p}{\partial z} \right) \]

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Where $V$ is the speed of the cyclostropic wind. Now (38) evolves from the horizontal momentum equations and given by,

$$\frac{DV}{Dt} = \frac{\partial p}{\partial t}$$  \hspace{1cm} (39)

$$\frac{V^2}{R} + fV = -\frac{\partial p}{\partial n}$$  \hspace{1cm} (40)

Where $\vec{t}$ is the cross radial direction unit vector and $\vec{n}$ is the radial direction unit vector. $R$ is the radius of curvature of the concerned fluid parcel. ‘$f$ ’ is the Coriolis parameter.

Now to get (38) $fV \approx 0$. Thus for cyclostropic approximation to happen physically, the Coriolis force term should tend to zero.

**III.viii. Gradient wind approximation**

Gradient wind approximation basically illustrates a three way balance between the Coriolis force, centrifugal force and pressure gradient force Thus utilizing (40) the gradient wind velocity is given by,

$$V = \frac{fR}{2} \pm \left( \frac{f^2R^2}{4} + fRV_g \right)^{\frac{1}{2}}$$  \hspace{1cm} (41)

Again rewriting the above equation in the form of the geostropic wind velocity,

$$\frac{V^2}{R} + fV - fV_g = 0$$  \hspace{1cm} (42)

If we divide both sides by $fV$ we get the ratio of geostrophic wind velocity to the gradient wind velocity as,

$$\frac{V_g}{V} = 1 + \frac{V}{fR}$$  \hspace{1cm} (43)

This is an important criterion for cyclone development. For, cyclonic flow the cyclone radius of curvature must be finite and so, $fR > 0$ and thus as a consequence $V_g > V$.

Similarly for anticyclonic flow $fR < 0$ and thus from (40), $V > V_g$.

Thus this is the significance of gradient wind and geostrophic wind. In general for cyclonic flow the gradient wind velocity dominates geostrophic wind velocity.
III.ix  The thermal wind

The thermal wind velocity is basically defined as the difference of geostrophic wind speeds at two different geopotential heights.

For pressure level \( p_1 \) the geostrophic wind velocity is given by,

\[
\mathbf{V}_g(p_1) = \frac{1}{\rho_f} \nabla p_1 \left[ i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} \right].
\]

For pressure level \( p_0 \) the geostrophic wind speed is given by, \( v_g(p_0) = \frac{1}{\rho_f} \nabla p_0 \).

If, \( p_0 < p_1 \), then the thermal wind velocity is given by,

\[
V_T = \frac{1}{\rho_f} [\nabla p_1 - \nabla p_0].
\]

(44)

Again an essential criterion for cyclone genesis is low vertical shear. So, for maintenance of this criterion the thermal wind velocity should be negligible.

Thus for cyclone formation \( V_T \) must be insignificant as far as practicable.

III.x.  Gale wind

Gale winds are a special class of strong devastating winds used to characterize the strength of a tropical cyclone. According to the US National Weather Service winds having speed between 34- 47 Knots (\( 63 – 87 \text{Km/h}, 17.5 – 24.2 \text{m/s} \) or \( 39 – 54 \text{ miles/h} \)) are gale winds. The word 'gale' is originated from the Latin word 'gail' meaning elated or excited etc.

III.xi.  Squall lines

The cumulative locus of multicell or super-cell tropical storms are often characterized by the name squall lines.

III.xii.  Wind gusts

Winds having accelerating tangential speeds generally in a period of time of less than 20 seconds are called wind gusts. Generally such accelerating winds in the form of wind gusts characterize intensity of tropical cyclones.

III.xiii.  Storm surge

Storm surge is the most intense materialistic parameter that characterize with tropical cyclone. It causes the most number of casualties and property damage during the occurrence of a tropical cyclone.

III.xiv.  Bathymetry

The measurement of the depth of water of large water bodies viz. oceans is called bathymetry. This cyclone bathymetry has a huge impact on the devastating effect of

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tropical cyclones on our civilization. For shallow water bodies generally the storm surge is more dangerous than deep water bodies.

III.xv. Barotropic and baroclinic atmospheres

For barotropic atmospheres the atmospheric density is a function of the pressure or geopotential, i.e. \( \rho = \rho(p) \). Thus the isobaric lines i.e. the lows and highs of pressure during a cyclone are also isothermal for a barotropic atmosphere. In a baroclinic atmosphere the atmospheric density is a function of both temperature and pressure. Thus for baroclinic atmosphere \( \rho = \rho(T,p) \)

III.xvi. Circulation

Circulation is an important parameter in the tropical cyclone fraternity. It is the macroscopic rotational aspect of turbulence. For any vector field \( \vec{U} \) the circulation \( \mathcal{C} \) is given by, \( \mathcal{C} = \oint_S \vec{U} \cdot d\vec{l} \). Where \( S \) being the boundary of the closed contour within the vector field \( \vec{U} \) and \( d\vec{l} \) is the concerned line element.

III.xvii. Vorticity

Vorticity is defined as the microscopic rotational parameter for atmospheric turbulence. Mathematically for any vector field \( \vec{U} \) it is written as, \( \vec{\omega} = \nabla \times \vec{U} \). Now, circulation and vorticity constitutes a major part of the atmospheric turbulence connected with tropical cyclone dynamics.

III.xviii. The Boussinesq approximation

The Boussinesq approximation is an important scientific phenomenon that simplifies the tropical cyclone dynamics governing equations within the boundary layer. The basic physics lies in the concept that the density fluctuations within the boundary layer are negligible in the atmosphere. Especially during tropical cyclone genesis within the cyclone core there must be low vertical shear, so further there is emphasis with least change of density with height. Under this approximation the expression of density in the component equations is replaced by a constant density term \( \rho_0 \) which is the mean sea level density. Under this approximations the horizontal momentum component equations, the thermodynamic energy equation and the equation of continuity becomes respectively,
Where $\theta$ is the departure of the potential temperature from its basic value $\theta_0(z)$

### III.xix. WKB approximation

Wentzel, Krammers and Brillouin (WKB) approximation (Griffiths 2005) is a significant profound approach for solving the Schrödinger equation with varying potential in quantum mechanics. This approximation has also been used in the work (Ghosh and Chakravarty, 2018) for solving a particular case of the Navier-Stokes equation in connection to tropical cyclone dynamics.

### IV. Conclusions

This review work gives a detailed idea of the research progress in tropical cyclone till date. It also gives an idea that how much of the main research objectives have been fulfilled so far and how much is yet to achieve to improve our knowledge regarding the entire being of a tropical cyclone and hence how can we achieve more precise forecasting in the coming years. From this work one can surely get the impetus and way to look around the problem of dynamical understanding of tropical cyclone with a new fresh approach keeping the sound substantiated knowledge of all those previous works described in this review.

### Conflict of Interest:

The authors declare that no conflict of interest to report the present study.
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