WMAP5 Observationnal Constraints on Braneworld New Inflation Model

R. Zarrouki\textsuperscript{1}, Z. Sakhi\textsuperscript{2,3} and M. Bennai\textsuperscript{1,3}\textsuperscript{*}

\textsuperscript{1}L.P.M.C, Faculté des Sciences Ben M’zik, B.P. 7955, Université Hassan II-Mohammedia, Casablanca, Maroc.
\textsuperscript{2}LISRI, Faculté des Sciences Ben M’Sik, Université Hassan II-Mohammedia, Casablanca, Maroc,
\textsuperscript{3}Groupement National de Physique des Hautes Energies, Focal point, LabUFR-PHE, Rabat, Morocco.

October 27, 2009

Abstract

We study a new inflation potential in the framework of the Randall-Sundrum type 2 Braneworld model. Using the technic developped in\cite{1}, we consider both an monomial and a new inflation potentials and apply the Slow-Roll approximation in high energy limit, to derive analytical expression of relevant perturabtion spectrum. We show that for some values of the parameter $n$ of the potential ($V(\phi) = V_0 - \frac{1}{2}m^2\phi^2 + \frac{\alpha}{2^n}\phi^{2n}$) we obtain an perturbation spectrum wich present a good agreement with recent WMAP5 observations.

Keywords: RS Braneworld, New inflation potential, Perturbation Spectrum, WMAP5.

PACS numbers: 98.80. Cq

Contents

1 Introduction

2 Slow-roll Braneworld inflation
   2.1 Randall-Sundrum model
   2.2 Slow-Roll approximation and perturbation spectrum on brane

3 Perturbation spectrum in Braneworld New inflation
   3.1 Monomial potential
   3.2 New inflation model

4 Conclusion

\textsuperscript{*}E-mail adress: m.bennai@univh2m.ac.ma, bennai_idrissi@yahoo.fr
1 Introduction

Recently Braneworld scenario\cite{2,3,4} has become a central paradigm of modern inflationary cosmology. Standard inflation has been mainly studied and was early confirmed by observations\cite{5}. Brane inflation is proposed to solve important cosmological problems like as dark energy\cite{6}, tachyonic inflation\cite{7} or Black Holes systems\cite{8,9}. Others motivations are observations on accelerating universe\cite{10} as well as results on interpretations of these phenomena in terms of scalar field dynamics. Generally, scalar fields naturally arise in various particle physic theories including string/M theory and expected to play a fundamental role in inflation\cite{11,12}.

In Randall-Sundrum model\cite{13} which is one of the most studied models, our four-dimensional universe is considered as a 3-brane embedded in five-dimensional anti-de Sitter space-time(AdS5), while gravity can be propagated in the bulk. The most simplest inflationary models studied in the context of Randall-Sundrum scenario is the chaotic inflation\cite{15}, but in relation with recent WMAP observations\cite{16,17,18}, more generalized models must be studied.

In this work, we are interested on a new inflationary model in the framework of Randall-Sundrum Braneworld inflation in relation with recent WMAP5\cite{18} for both monomial and New inflation potentials.

We first start in section 2, by recalling the foundations of the Braneworld inflation precisely the modified Friedmann eqs, and various inflationary perturbation spectrum parameters. In the section 3 we present our results for both Monomial and New inflation models. We have applied here the Slow-roll approximation in the high energy limit to drive various perturbations parameters spectrum for these models. We show that for some values of the parameter n of the potential \( V(\phi) = V_0 - \frac{1}{2}m^2\phi^2 + \frac{\alpha}{n}\phi^{2n} \), we obtain an perturbation spectrum wich present a good agreement with recent WMAP5 observations. A conclusion and a perspective of this work are given in the last section.

2 Slow-roll Braneworld inflation

2.1 Randall-Sundrum model

We start this section by recalling briefly some fundamentals of Randall-Sundrum type II Braneworld model\cite{13}. In this model, our universe is supposed living in a brane embedded in an Anti-de Sitter (AdS) five-dimensional bulk spacetime. One of the most relevant consequences of this model is the modification of the Friedmann equation for energy density of the order of the brane tension, and also the appearance of an additional term, usually considered as dark radiation term. In the case where the dark radiation term is neglected, the gravitationaln Einstein eqs, leads to the modified Friedmann equation on the brane as\cite{15}

\[
H^2 = \frac{8\pi}{3M_{pl}^2}\rho \left[ 1 + \frac{\rho}{2\lambda} \right]
\]

with \( \lambda \) is the brane tension, \( H \) is the Hubble parameter and \( M_{pl} \) is th Planck mass. It’s clear that the crucial correction to standard inflation is given by the density quadratic term \( \rho^2 \). Brane effect is then carried here by the deviation factor \( \rho/2\lambda \), with respect to unity. This deviation has the effect of modifying the dynamics of the universe for density \( \rho \gtrsim \lambda \). Note also that in the limit \( \lambda \to \infty \), we recover standard four-dimensional standard inflation results. In inflationary theory, the energy density \( \rho \), and pressure \( p \), are expressed in term of inflaton potential \( V(\phi) \) as \( \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \) and \( p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \), where \( \phi \) is the inflaton field.
Inflation theory, the scalar potential $V(\phi)$, depending on the scalar field $\phi$, play a fundamental role and represent the initial vacuum energy responsible of inflation. Along with these equation, one also has a second inflation *Klein-Gordon* equation governing the dynamic of the scalar field $\phi$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$  \hspace{1cm} (2)

This is a second order evolution equation which follows from conservation condition of energy-momentum tensor $T_{\mu \nu}$. To calculate some physical quantities as scale factor or perturbation spectrum, one has to solve equations(1,2) for some specific potentials $V(\phi)$. To do so, the Slow-Roll approximation was introduced and applied by many authors to drive inflation perturbation spectrum[14].

### 2.2 Slow-Roll approximation and perturbation spectrum on brane

Inflationary dynamics requires that inflaton field $\phi$ driving inflation moves away from the false vacuum and slowly rolls down to the minimum of its effective potential $V(\phi)$[23]. In this scenario, the initial value $\phi_i = \phi(t_i)$ of the inflaton field and the Hubble parameter $H$ are supposed large and the scale factor $a(t)$ of the universe growth rapidly. Applying the slow roll approximation, $\dot{\phi}^2 \ll V$ and $\ddot{\phi} \ll V'$, to brane field equations(1,2), we obtain:

$$H^2 \simeq \frac{8\pi V}{3M^4} \left( 1 + \frac{V}{2\lambda} \right), \quad \dot{\phi} \simeq -\frac{V'}{3H}$$  \hspace{1cm} (3)

Note that slow roll approximation puts a constraint on the slope and the curvature of the potential. This is clearly seen from the field expressions of $\epsilon$ and $\eta$ parameters given by[15],

$$\epsilon = \frac{H}{H^2} \equiv \frac{M^2}{4\pi} \left( \frac{V'}{V} \right)^2 \left[ \frac{\lambda(\lambda + V)}{2\lambda + V} \right]$$  \hspace{1cm} (4)

$$\eta = \frac{\dot{V}}{3H^2} \equiv \frac{M^2}{4\pi} \left( \frac{V''}{V} \right) \left[ \frac{\lambda}{2\lambda + V} \right].$$  \hspace{1cm} (5)

Slow-roll approximation takes place if these parameters are such that $\max\{\epsilon, |\eta|\} \ll 1$ and inflationary phase ends when $\epsilon$ and $|\eta|$ are equal to one. Other inflationary important quantity is the number $N_e$ of e-folding which, in slow roll approximation, reads as

$$N_e \simeq \frac{8\pi}{M^4} \int_{\phi_i}^{\phi_f} \frac{V}{V'} \left( 1 + \frac{V}{2\lambda} \right) d\phi.$$  \hspace{1cm} (6)

where $\phi_i$ and $\phi_f$ stand for initial and final value of inflaton.

Before proceeding, it is interesting to comment low and high energy limits of these parameters. Note that at low energies where $V \ll \lambda$, the slow-roll parameters take the standard form. At high energies $V \gg \lambda$, the extra contribution to the Hubble expansion dominates. The number of e-folding in this case becomes $N_e \simeq -\frac{12\pi}{\lambda M^4} \int_{\phi_i}^{\phi_f} \frac{V}{V'} d\phi$.

The inflationary spectrum perturbations is produced by quantum fluctuations of fields around their homogeneous background values. Thus the scalar amplitude $A^2_s$ of density perturbation, evaluated by neglecting back-reaction due to metric fluctuation in fifth dimension, is given by[15]

$$A^2_s \simeq \frac{512\pi}{75M^4} \left| \frac{V^3}{V''^2} \left[ \frac{2\lambda + V}{2\lambda} \right]^3 \right|_{k=aH},$$  \hspace{1cm} (7)

Note that for a given positive potential, the $A^2_s$ amplitude is increased in comparison with the standard result. In high energy limit this quantity behaves as

$$A^2_s \simeq \frac{64\pi}{75\lambda^3 M^4} \frac{V^6}{V''}.$$  \hspace{1cm} (8)
On the other hand, using eqs (4,5), one can compute the perturbation scale-dependence described by
the spectral index  \( n_s \equiv 1 + d (\ln A_s^2) / d (\ln k) \) and find

\[
    n_s - 1 \simeq 2\eta - 6\epsilon ,
\]

(9)

Note that at high energies \( \lambda / V \), the slow-roll parameters are both suppressed; and the spectral index is
driven towards the Harrison-Zel’dovich spectrum; \( n_s \to 1 \) as \( V / \lambda \to \infty \).

In what follows, we shall apply the above Braneworld formalism by singling out two specific kinds of inflaton potentials. These are the monomial and a new inflation potentials recently studied by Boyanovsky et al. [21] in standard inflation.

3 Perturbation spectrum in Braneworld New inflation

To begin, recall that chaotic inflationary model, which was first introduced by Linde [19], has been
reconsidered recently by several authors in the context of Braneworld scenario [15, 20]. In the present
work, we are interested by others types of potential inflation. The authors of [21] have shown that
combining the WMAP data with the slow roll expansion constraints the inflaton potential to have the
form

\[
    V(\phi) = N_e M^4 w(\chi)
\]

(10)

\( M \) is the inflation energy scale determined by the amplitude of the scalar adiabatic fluctuations\[7\] to be
\( M \sim 0.00319 \ M_{Pl} = 0.77 \times 10^{16} \text{GeV} \), where a dimensionless rescaled field variable is introduced

\[
    \chi = \frac{\phi}{\sqrt{N_e} M_{Pl}},
\]

(11)

here \( N_e \) is the number of efolds.

In this new notation, the Slow-Roll parameters become

\[
    \epsilon = \frac{\lambda}{4\pi N_e^2 M^4} \left( \frac{w'(\chi)^2}{w(\chi)^3} \right)
\]

(12)

\[
    \eta = \frac{\lambda}{4\pi N_e^2 M^4} \left( \frac{w''(\chi)}{w(\chi)^2} \right)
\]

(13)

where the prime stands for derivative with respect to \( \chi \) : \( w' = \frac{dw}{d\chi} \) and \( w'' = \frac{d^2w}{d\chi^2} \).

The perturbations parameters are now expressed in term of the new variable as

\[
    N_e = \frac{1}{4\pi M^4} \int_{\chi_e}^{\chi_{end}} \frac{w(\chi)^2}{w'(\chi)} d\chi
\]

(14)

where \( \chi_e \) is the value of \( \chi \) corresponding to \( N_e \) e-folds before the end of inflation, and \( \chi_{end} \) is the value of \( \chi \) at the end of inflation.

Other perturbation quantities are also calculated in term of \( w(\chi) \) as

\[
    A_s^2 = \frac{64\pi N_e^2 M_{Pl}^{16}}{75\lambda^2 M^4} \left( \frac{w(\chi)}{w'(\chi)^2} \right).
\]

(15)

The spectral and running index are now respectively written in the following form

\[
    n_s - 1 = \frac{\lambda}{4\pi N_e^2 M^4} \left( \frac{2 w''(\chi)}{w(\chi)^2} - 6 \frac{w'(\chi)^2}{w(\chi)^3} \right),
\]

(16)

\[
    \frac{dn_s}{d\ln k} = -\frac{\lambda^2}{8\pi^2 M^8 N_e^4} \left( -8w''(\chi) w'(\chi)^2 \frac{w'(\chi)^4}{w(\chi)^5} + 9 \frac{w'(\chi)^4}{w(\chi)^5} + \frac{w'(\chi) w''(\chi)}{w(\chi)^4} \right).
\]

(17)
Finally, the ratio of tensor to scalar perturbations \( r \) reads

\[
r = \frac{6\lambda}{\pi N^2 M^4} \left( \frac{w'(\chi)^2}{w(\chi)^3} \right).
\]  

In what follows we will determine all this inflationary perturbation spectrum parameters at \( \chi = \chi_c \), for a monomial and a new inflation potential and compare our results with recent WMAP experimental data in the later case.

### 3.1 Monomial potential

Let us begin by a monomial potential which generalize the chaotic one. Chaotic inflation was mainly studied in the context of standard\[19\], Brane\[15, 20\] and recently Chaplygin inflation on the brane \[22\]. Here, we consider a general potential of the form

\[
V(\phi) = \frac{\alpha}{2n^2} \phi^{2n},
\]  

where \( \alpha \) and \( n \) are constants.

In term of \( \chi \), we get

\[
w(\chi) = \frac{\chi^{2n}}{2n},
\]  

where we have used

\[
M^4 = \alpha N^{n-1} M^2_{pl}
\]  

Thus, slow-roll parameters are represented by

\[
\varepsilon = \frac{2n}{N^2 M^4} \left( \frac{1}{\chi^{2n+2}_c} \right),
\]  

\[
\eta = \frac{\lambda n^2 (2n-1)}{N^2 M^4} \left( \frac{1}{\chi^{2n+2}_c} \right)
\]  

and scalar spectral index \( n \) and the ratio \( r \) are respectively expressed as

\[
n_s - 1 = -\frac{2n}{N^2 M^4} (4n + 1) \left( \frac{1}{\chi^{2n+2}_c} \right),
\]  

\[
r = 48\frac{\lambda n^3}{N^2 M^4} \left( \frac{1}{\chi^{2n+2}_c} \right)
\]  

Finally the running index takes the following expression

\[
\frac{dn_s}{d\ln k} = -\frac{4\lambda^2}{\pi^2 M^2 N^4} \left( 4n^2 + 5n + n^5 \right)
\]  

Inflation ends at \( \chi_{end} = 0 \), thus the value of the dimensionless field \( \chi_c \) before the end of inflation is

\[
\chi_c^{2n+2} = \frac{\lambda n^2 (2n + 2)}{N^2 M^4}
\]  

Winding this result, various perturbations parameters are obtained in term of potential parameter \( n \) and e-fold number \( N_e \)

\[
\varepsilon = \frac{n}{N_e (n + 1)}, \quad \eta = \frac{2n - 1}{2N_e (n + 1)},
\]  

\[
n_s - 1 = -\frac{4n + 1}{N_e (n + 1)}, \quad r = 24\frac{n}{N_e (n + 1)},
\]  

\[
\frac{dn_s}{d\ln k} = -\frac{(4n^2 + 5n + 1)}{N_e^2 (n + 1)^2}
\]  

\[
\frac{d^2 n_s}{d\ln k^2} = \frac{(4n^2 + 5n + 1)}{N_e^2 (n + 1)^2}
\]
It will be interesting to study the variation of these perturbation parameters as function of potential parameter $n$, and compare the results to recent WMAP5 observations. In the following, we do this for a more generalized new inflation potential.

### 3.2 New inflation model

Consider now a new inflation potential of the form[1]

$$w(\chi) = w_0 - \frac{1}{2} \chi^2 + \frac{g}{2n} \chi^{2n}$$  \hspace{1cm} (31)

where $w_0$ and the coupling $g$ are dimensionless.

In ref.[1] the authors used this model in standard inflation and they have shown that for lower values of $n$ the results reproduce observation. In the present work, we reproduce a new results for all known inflation spectrum parameters, but in the context of Randall-Sundrum Branelworld model.

New inflation model described by the dimensionless potential given by eq.(31) have a minimum at $\chi_0$ which is the solution to the following conditions

$$w'(\chi_0) = w(\chi_0) = 0$$  \hspace{1cm} (32)

These conditions yield

$$g = \frac{1}{\chi_0^{2n-2}}, \quad w_0 = \frac{\chi_0^2}{2n} (n - 1)$$  \hspace{1cm} (33)

As using the previous results the equation (31) becomes

$$w(\chi) = \frac{(n - 1)}{2n} \chi_0^2 - \frac{\chi^2}{2} + \frac{\chi_0^{2-2n}}{2n} \chi^{2n}$$  \hspace{1cm} (34)

$\chi_0$ determines the scale of symmetry breaking $\phi_0$ of the inflaton potential upon the rescaling eq.(11), namely

$$\phi_0 = \sqrt{N_e} M_p \chi_0$$  \hspace{1cm} (35)

It is convenient to introduce the dimensionless variable

$$x = \frac{\chi}{\chi_0}$$  \hspace{1cm} (36)

Then, from eq.(36), the potential of inflation model eq.(34) takes the form

$$w(x) = \frac{\chi_0^2}{2n} \left[ n \left( 1 - x^2 \right) + x^{2n} - 1 \right], \quad \text{broken symmetry}$$  \hspace{1cm} (37)

Inflation ends when the inflaton field arrives at the minimum of the potential. As shown by Linde[23], the symmetry is broken for non vanishing minimum of the potential. Thus, for our new inflation model eq.(34) inflation ends for

$$\chi_{\text{end}} = \chi_0$$  \hspace{1cm} (38)

According to the new variable $x$, the condition eq.(14) becomes

$$1 = \frac{\pi N_e M^4 \chi_0^4}{\lambda n^2} I_n (X)$$  \hspace{1cm} (39)

where

$$I_n (X) = \int_X^1 \left( \frac{n \left( 1 - x^2 \right) + x^{2n} - 1}{1 - x^{2n-2}} \right)^2 \frac{dx}{x}$$  \hspace{1cm} (40)

and

$$X = \frac{\chi}{\chi_0}$$  \hspace{1cm} (41)
For small field and $X \to 1^-$ the integral $I_n(X)$ obviously vanishes and by expanding the potential (eq.34) near the minimum $\chi_0$ ($n > 1$)

$$w(\chi) \approx \frac{(2n-2)(\chi-\chi_0)^2}{2}$$

(42)

The expression approached of the potential (eq.42) allows us to recover the expression of the monomial potential (eq.20) for $n = 1$ by simple shift

$$\chi \to \sqrt{(2n-2)}(\chi-\chi_0); \quad (n > 1)$$

(43)

So, we can determine all inflationary perturbation spectrum parameters near the minimum $X = 1$. Therefore for $X \sim 1$ the quadratic monomial is an excellent approximation to the family of higher degree potentials.

The slow-roll parameters become in term of the variable $X$

$$\varepsilon = \frac{2nI_n(X)}{N_e} \frac{(-X + X^{2n-1})^2}{(n (1 - X^2) + X^{2n - 1})^3}$$

(44)

$$\eta = \frac{I_n(X)}{N_e} \frac{(-1 + (2n - 1) X^{2n-2})}{(n (1 - X^2) + X^{2n - 1})^2}$$

(45)

In the following, we study the variation of these parameters as function of $X$, by numerical calculations for $N_e = 50$.

Figure 1: $\varepsilon$ vs $X$ for new inflation potential $(n = 2, 3, 4)$. 


Figure 2: $\eta$ vs $X$ for new inflation potentialor ($n = 2, 3, 4$).

The figures 1 and 2 show the increasing behavior of the two functions $\varepsilon$ and $\eta$ for small values for $X$ and for large $X$, the both functions become constant. We can remark again that a large domain of variation of $X$ verifies the conditions of inflation since during inflation we have $\varepsilon \ll 1$ and $|\eta| \ll 1$.

On the other hand, observations combining WMAP5, BAO(Baryon Acoustic Oscillations) and SN(Type Ia supernovae) data\cite{18}, yields

$$n_s = 0.960 \pm 0.014 \quad (95\% \text{ CL}) \quad (46)$$

$$r < 0.20 \quad (95\% \text{ CL}) \quad (47)$$

$$-0.0728 < \frac{dn_s}{d\ln k} < 0.0087 \quad (95\% \text{ CL}) \quad (48)$$

In our variable $X$, the spectral index becomes

$$n_s - 1 = \frac{2J_n(X)}{N_n(n(1-X^2)+X^{2n}-1)^2} \left[ -6n \frac{(-X+X^{2n-1})^2}{(n(1-X^2)+X^{2n}-1)} - 1 + (2n-1)X^{2n-2} \right] \quad (49)$$

In figure 3, we plot this parameter as function of $X$. So, we can remark that the window of consistency with the WMAP5 + BAO+SN data narrows for growing $n$. Thus we obtain a good potential expression for small $n < 3$ and large $X$. 
Other spectrum parameter is the ratio $r$ presented by

$$r = 48 \frac{n I_n (X)}{N_\epsilon} \frac{(-X + X^{2n-1})^2}{(n(1 - X^2) + X^{2n} - 1)^3} \tag{50}$$

Figure 4: $r$ vs $X$ for new inflation potential ($n = 2, 3, 4$). Horizontal dotted line corresponds to the upper limit $r = 0.2$ (95%CL) from WMAP5,BAO and SN.

This figure shows the same behavior as figure 1 since $r = 24\epsilon$. Note that the observational result is reproduced for small values of $X$ where the three curves are almost confounded.
Figure 5: $r$ vs $n_s$ for new inflation potential ($n = 2, 3, 4$).

To confront simultaneously the observables $r$ and $n_s$ with observation, it will be interesting to study the relative variation of these parameters. In figure 5 we have plotted $r$ vs $n_s$. The black dot corresponds to chaotic potential $\frac{a\phi^2}{2} (X = 1, n_s = 0.95$ and $r = 0.24)$. Values below the black dot correspond to $X < 1$ which constitutes the region where the observation results are recovered. For values corresponding to $X > 1$, the parameter values become in disagreement with observation notably for $r$.

We have also calculated the running index $\frac{dn_s}{d\ln k}$ which is given by

$$
\frac{dn_s}{d\ln k} = -\frac{I_n (X) \phi^2}{8n^4 N_e^2} \left( -\frac{8v''(X) v'(X)^2}{v(X)^5} + 9 \frac{v'(X)^4}{v(X)^6} + \frac{v'(X) v'''(X)}{v(X)^4} \right)
$$

(51)

where

$$
v(X) = \frac{[n (1 - X^2) + X^{2n} - 1]}{2n}
$$

(52)

$$
v'(X) = \frac{\partial v(X)}{\partial X}
$$

We observe, in figure 6, that for any values of $X$ the experimental data is verified. Thus all members of new inflation potential family predict a small and negative running
Figure 6: \( \frac{dn_s}{d \ln k} \) (Running of the scalar index) vs \( X \) for \( n = 2, 3, 4 \) for new inflation potential

For the some reasons as above, we plot in the last figure, the variation of \( \frac{dn_s}{d \ln k} \) vs \( n_s \).

Figure 7: \( \frac{dn_s}{d \ln k} \) vs \( n_s \) for new inflation potential \( (n = 2, 3, 4) \).

We note that for \( n_s \) experimental data \( (0.9392 < n_s < 0.9986) \), all values of \( \frac{dn_s}{d \ln k} \) are conforme with observation for any \( n \). As before, the black dot corresponds to chaotic potential \( \frac{\alpha \phi^2}{2} \) \( (X = 1, n_s = 0.95 \text{ and } \frac{dn_s}{d \ln k} = -0.0010) \).

Thus, a good values of \( X \) which are in agreement within observation correspond to \( X \prec 1 \).

4 Conclusion

In this work, we have studied a new inflation potential in the framework of Braneworld \textit{Rundall-Sundrum} type 2 model. We have applied Slow-Roll approximation in high energy limit in order to derive analytical expressions for various perturbation spectrum \( (n_s, r \text{ and } \frac{dn_s}{d \ln k}) \). We have considered a monomial and a new inflation potentials to study the behaviours of inflation spectrum for various values of \( n \). We have shown that for some values of the parameter \( n \) of the potential \( (V(\phi) = V_0 - \frac{1}{2}m^2\phi^2 + \frac{\alpha}{2n}\phi^{2n}) \) our results are in a good agreement with recent WMAP5 observations, specially for small fields.
References

[1] D. Boyanovsky, H. J. de Vega, C. M. Ho, and N. G. Sanchez, "New Inflation vs. Chaotic Inflation, higher degree potentials and the Reconstruction Program in light of WMAP3," Phys. Rev. D75, 123504 (2007).

[2] P. Brax, C. Bruck and A. Davis, "Brane World Cosmology," Rept. Prog. Phys. 67(2004)2183-2232.

[3] James E. Lidsey, "Inflation and Braneworlds," Lect. Notes Phys. 646 (2004) 357-379.

[4] P. Brax, C. Bruck, "Cosmology and Brane Worlds," Class. Quant. Grav. 20(2003)R201-R232.

[5] G. Efstathiou (1991), in "Observational Tests of Cosmological Inflation," eds Shanks, T. et al, Kluwer academic.

[6] R. Kallosh and A. Linde, "Dark Energy and Fate of the Universe," JCAP0302(2003)002.

[7] M. Sami, Pravabati Chingangbam and Tabish Qureshi, "Aspects of Tachyonic Inflation with Exponential Potential," Phys. Rev. D66 (2002) 043530.

[8] Adil Belhaj, Pablo Diaz, Mohamed Naciri, Antonio Segui, "On Brane inflation potentials and black hole attractors," Int. J. Mod. Phys. D17(2008)911-920.

[9] Won Tae Kim, John J. Oh, Marie K. Oh, Myung Seok Yoon "Brane-World Black Holes in Randall-Sundrum Model," J. Korean Phys. Soc. 42(2003)13-18.

[10] P. de Bernardis et al., Astrophys. J. 564, 559 (2002).

[11] Nobuyoshi Ohta "Accelerating Cosmologies and Inflation from M/Superstring Theories," Int. J. Mod. Phys. A20 (2005) 1-40.

[12] M. Bennai, H. Chakir and Z. Sakhi, "On Inflation Potentials in Randall-Sundrum Braneworld Model," Electronic Journal of Theoretical Physics 9 (2006) 84–93

[13] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999); Phys. Rev. Lett. 83, 4690 (1999).

[14] Andrew R Liddle, Anthony J Smith "Observational constraints on Braneworld chaotic inflation," Phys. Rev. D68 (2003) 061301.

[15] R. Maartens, D. Wands, B. Basset, and I. Heard, "Chaotic inflation on the brane," Phys. Rev. D 62 (2000)041301.

[16] H. V. Peiris et al. (WMAP collaboration), Ap. J. Suppl.148, 213 (2003);

[17] D. N. Spergel et al., "Wilkinson Microwave Anisotropy Probe (WMAP) three year results: Implications for cosmology," J. Suppl.170, 377 (2007).

[18] E. Komatsu et al., "Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation," Astrophys. J. Suppl.180:330-376,(2009).

[19] A.D. Linde, Phys. Lett. 108B, 389(1982); 114B, 431 (1982); 116B, 335, 340 (1982).

[20] B.C. Paul "Chaotic Inflationary Universe on Brane," Phys. Rev. D68 (2003)127501.

[21] D. Boyanovsky, H. J. de Vega and N. G. Sanchez, Phys. Rev. D73 (2006) 023008.
[22] Ramon Herrera "Chaplygin inflation on the brane," *Phys.Lett.B* 664 (2008) 149-153.

[23] Andrei Linde."Particle Physics and Inflationary Cosmology," *Contemp.Concepts Phys.* 5 (2005) 1-362.
-0.0013
-0.0014
-0.0015
0.0