OPERATOR WEAK VALUES AND BLACK HOLE COMPLEMENTARITY

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Abstract. In conventional field theories, the emission of Hawking radiation in the background of a collapsing star requires transplanckian energy fluctuations. These fluctuations are encoded in the weak values of the energy-momentum operator constructed from matrix elements between both -in and -out states. It is argued that taming of these weak values by back-reaction may lead to geometrical backgrounds which are also built from weak values of the gravitational field operators. This leads to different causal histories of the black hole as reconstructed by observers crossing the horizon at different times but reduces, in accordance with the equivalence principle, to the classical description of the collapse for the proper history of the star as recorded by an observer comoving with it. For observers never crossing the horizon, the evaporation would be interpreted within a topologically trivial “achronon geometry” void of horizon and singularity: after the initial ignition of the radiation from pair creation out of the vacuum of the collapsing star of mass

* presented at the Oskar Klein Centenary Symposium (September 1994)
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M, as in the conventional theory, the source of the thermal radiation would shift gradually to the star itself in a time at least of order $4M \ln 2M$. The burning of the star could be consistent with a quantum unitary evolution along the lines suggested by 't Hooft. A provisional formal expression of general black hole complementarity is proposed and its possible relevance for testing features of a theory of quantum gravity is suggested.
1. General Considerations.

Black hole evaporation questions acutely the consistency of quantum physics and general relativity. In the original derivation of black hole radiation\cite{1}, the thermal density matrix describing the radiation comes out as a consequence of tracing pure quantum states over states hidden from an external observer by the event horizon. Disappearance of the black hole would then result in a violation of unitarity within the universe of external observers, as suggested by Hawking\cite{2}. A halt of the evaporation process at the Planck mass could formally save unitarity by correlating the distant radiation to a stable planckian remnant\cite{3}. The latter could then be generated in a huge and probably infinite number of ways. Thus, the end point of the evaporation process would leave us with the problem either to understand why, despite its breakdown at the Planck scale, quantum physics is operative at larger scales, or to enlarge in a consistent way the framework of ordinary quantum field theory to incorporate infinitely degenerate Hilbert spaces to describe this Planck scale.

An alternative approach to the dilemma has been proposed by 't Hooft\cite{4,5}. The Hawking radiation would induce some strong back-reaction on the geometry which would appear, to the external observer, free of singularity and horizons. The emission process would be part and parcel of a fully unitary evolution of the black hole. Susskind and al\cite{6} have suggested to implement this idea by materializing in the planckian vicinity of the event horizon a physical “stretched horizon”, invisible to the free falling observer, where the incoming information is deposited and then burned away. In this way full evaporation would be consistent with unitarity resulting from long time correlations between otherwise thermal quanta. As the very existence of the stretched horizon would be observer dependent, they argue that unitary evolution implies a departure from hitherto admitted physical realism as expressed by the concept of an absolute, observer independent, event. They characterize such a relativity of events as a kind of “complementarity”.

Unitarity is not the only conceptual problem posed by black hole evaporation.
In conventional quantum field theory the emergence of black hole radiance from vacuum fluctuations in the background of a collapsing star requires, because of the unbounded blueshift in the horizons vicinity, frequencies well above the Planck scale. In particular, any typical Hawking quantum detected at future light-like infinity $I^+$ originates at past light-like infinity $I^-$ from transplanckian frequencies highly tuned on distances much smaller than the Planck size and these transplanckian fluctuations constitute a gauge invariant effect\cite{7},\cite{8}. Transplanckian fluctuations on $I^-$ do not pose any problem as long as gravitational non-linearities are not taken into account, but one would expect that their detailed structure should not lead to observable effects on cisplanckian scales. Nevertheless any smearing of these fluctuations above the Planck scale would totally invalidate the conventional derivation of the Hawking radiation.

There is however little doubt that Hawking radiation does occur because it appears to follow from general thermodynamic considerations. The information stored in the event horizon gives credence to the Bekenstein conjecture\cite{9} that the area of the event horizon is a measure of entropy. This entropy must then be, for dimensional reasons, inversely proportional to the Planck constant and this in turn requires that static matter surrounding an eternal black hole would thermalize at a global temperature proportional to $\bar{\hbar}$. Consider indeed the classical Killing identity\cite{10} for a black hole surrounded by static matter. This identity may be viewed as the integrated constraint equation over a static coordinate patch and can be written as\cite{11}

$$\delta M_\infty = \delta H_{\text{matter}} + \kappa \frac{\delta A}{2\pi}$$

(1)

where $\kappa$ is the surface gravity of the hole, $M_\infty$ the total mass at infinity and $\delta H_{\text{matter}}$ is the variation of all non gravitational parameters in the matter hamiltonian outside the horizon. In the thermodynamic limit $\delta H_{\text{matter}}$ is the variation of all interactions at the microscopic level and can be written as $T dS_{\text{matter}} + \sum_\mu \mu_i dN_i$ where $S_{\text{matter}}$ is the matter entropy and the sum is over all matter constituents. Thus, the Bekenstein conjecture also implies that there exists a global equilibrium
temperature $T$ proportional to the surface gravity for a system composed of black hole and matter. But Eq.(1) being a classical equation, this temperature should be proportional to $\hbar$ in order to cancel the $\hbar^{-1}$ in the entropy. This reasoning is corroborated by the estimate of this temperature via euclidean continuation of the metric, either for Green’s functions\cite{12}, for partition functions\cite{13} or for tunneling amplitudes\cite{14}, because euclidean continuation always leads to the required dependence of the temperature on $\hbar$. Moreover these methods all yield the same result, namely

$$T = \frac{1}{8\pi M}$$

(2)

for a black hole of mass $M$ in absence of surrounding matter$^\star$. The same value was also originally obtained for the temperature of the thermal radiation emitted by an incipient black hole using the dynamics of free fields in the curved background of a collapsing star\cite{1}.

The above thermodynamic argument suggests that, possibly up to a multiplicative constant, the result Eq.(2) does not really rely on the mechanism used to derive it but rather on its internal consistency, and the original derivation of the Hawking radiation from a collapsing star is perfectly consistent in absence of gravitational back-reaction. The fact that the thermodynamic argument refers more directly to hypothetical eternal black holes than to incipient ones does not weaken its significance. In an incipient black hole the drop of the centrifugal barrier close to the horizon results there in a thermalization of predominantly high angular momentum quanta at the high local temperatures consistent with the global Hawking temperature Eq.(2). This means that, close to its horizon, incipient black holes tend to behave as eternal ones. Therefore thermodynamics indeed strongly suggests that Hawking radiation is a necessary consequence of collapse.

$^\star$ Note however that there would be a decrease by a factor of two in the Euclidian periodicity if both asymptotically flat spaces connected by the black hole throat were identified. This is the ambiguity by a factor of two in the temperature raised by ’t Hooft\cite{15}
The emergence of transplanckian frequencies is not specific to black-holes; it is rather a general feature of event horizons. There is therefore not \textit{a priori} a relation between the transplanckian problem and the black-hole unitarity problem. However both problems most likely require some understanding of the nature of space-time at the Planck scale and hence one may hope that solving the former problem may shed light on the latter. In this paper we shall motivate and present a conceptual framework in which the taming of transplanckian frequencies can be envisaged. It will be seen that this framework accommodates the approach to the unitarity issue proposed by ’t Hooft. It leads to an enlarged notion of complementarity which can be formulated within the realms of quantum theory using the concept of “weak value” introduced by Aharonov and al.\cite{16}. Clearly however, in absence of any reliable dynamics at the Planck scale, our approach will be qualitative and provisional and the concepts developed here may well turn out to be wrong. But if they contain some elements of truth they may be of some help in unravelling the underlying structure of quantum gravity.

Let us now summarize our approach.

It is reasonable to suppose that transplanckian frequencies are tamed to planckian ones by some hitherto unknown Planck scale physics. The thermodynamic argument suggests that detection of Hawking radiation survives the taming process although the dynamics of the thermal emission might be drastically altered. Taming would arise if back-reaction would slowdown the collapse to a halt so as to prevent the blueshifted fluctuations to reach the Planck scale at past light-like infinity. We shall argue, on the basis of an analysis of the energy-momentum tensor of the radiating field in the background of a collapsing star, that such a taming mechanism can be expected when the back-reaction of the radiation on the geometry is taken into account. Energy and momentum of the collapsing star would then be converted into heat by degrees of freedom at the Planck scale, giving rise to thermal emission at the Hawking temperature. The star would then burn and evaporate within a trivial geometrical background displaying neither horizon nor singularity. This mechanism for removing transplanckian frequencies is in line
with the original proposal of t’ Hooft: Hawking radiation would remain in causal contact with the collapsing matter and evaporation can therefore possibly be consistent with unitarity. It is at odds with the conventional description of black hole evaporation where the information is entirely contained in the collapsing star which causally separates from the emission process, leaving a genuine thermal radiation void of any information.

For this scenario to make sense it will be necessary to assume that, at a more fundamental level, the classical description of the collapsing objects results from an average over quantum spreads of planckian constituents. Even so, it apparently contradicts the equivalence principle as the small classical gravitational field at the horizon appears inconsistent with the huge acceleration required to bring the collapse to a halt. This clash can be resolved when the metric field $g_{\mu\nu}(x)$ is promoted to a quantum operator. Indeed, the above-mentioned analysis of the energy-momentum tensor will suggest that, in presence of horizons, the geometrical background $g_{\mu\nu}(x)$ in which matter evolution can be described should not be identified with the expectation value of the corresponding Heisenberg operator $\hat{g}_{\mu\nu}(x)$ in the normalized “in” quantum state $|i\rangle$ and thus

$$g_{\mu\nu}(x) \neq \langle i | \hat{g}_{\mu\nu}(x) | i \rangle.$$  \hspace{1cm} (3)

Rather, the background geometry should be determined, as anticipated by ’t Hooft\textsuperscript{[5]} from both the “in” state $|i\rangle$ and the “out” state. This will lead to different causal histories of the black hole as reconstructed by observers crossing the horizon at different times but will reduce, in accordance with the equivalence principle, to the classical description of the collapse for the proper history of the star as recorded by an observer comoving with it.

To understand this point in qualitative terms, consider a detector sensitive only to cisplanckian effects. We call such a detector an observer. Let us first confine the motion of this observer within the space-time outside the event horizon of a sufficiently massive collapsing star. It will necessarily encounter radiation. The
radiation recorded by such observers can be encoded in some “out”-state. Thus in the space-time available to “external” observers, there exist outgoing states describing a particular set of detectable quanta covering the whole history of the evaporating black hole. This information about a particular decay mode can be added to the characterization of the system by the Schrödinger state of the star before collapse, or equivalently by the corresponding Heisenberg state $|i\rangle$. More precisely we could specify that the system is likely to be be found at sufficiently late times in a state characterized by some typical distribution of Hawking quanta. It may seem at first sight that this added information about the future detection of Hawking radiation is irrelevant for the analysis of the energy momentum tensor and of the metric at intermediate times. This could be an incorrect conclusion for reasons we now explain.

The expectation value $\langle i \mid \hat{A}(t) \mid i \rangle$ of a Heisenberg operator $\hat{A}(t)$ in the normalized quantum state $|i\rangle$ is often expressed as

$$\langle i \mid \hat{A}(t) \mid i \rangle = \sum_{\alpha} P_{\alpha} A_{\alpha} \quad P_{\alpha} = |\langle \alpha \mid i \rangle|^2$$  \hspace{1cm} (4)

where the eigenvectors $|\alpha\rangle$ relative to eigenvalues $A_{\alpha}$ form a complete set of orthonormal states. One interprets then $\langle i \mid \hat{A}(t) \mid i \rangle$ as the average over the probability distribution $P_{\alpha}$ of finding the value $A_{\alpha}$ if exact measurements of a complete set of commuting observables containing $\hat{A}(t)$ are performed at time $t$ on a quantum system “pre-selected” to be in the initial Schrödinger state $|t_1, i\rangle = U(t_1, t_0) |i\rangle$ at time $t_1$. $U(t_1, t_0)$ is the evolution operator to the time $t_1$ from the time $t_0$ where the Schrodinger state is identified with the Heisenberg one.

More information can be gained if the system is also “post-selected” to be found at a later time $t_2$ in a given Schrödinger state $|t_2, f\rangle$[16]. One may then express expectation values $\langle i \mid \hat{A}(t) \mid i \rangle$ as an average of weak values defined for $t_1 < t < t_2$ by

$$A_{\text{weak}}^f \equiv \frac{\langle f \mid \hat{A}(t) \mid i \rangle}{\langle f \mid i \rangle}$$  \hspace{1cm} (5)
where $|f\rangle = U(t_0, t_2)|t_2, f\rangle$. One gets

$$\langle i | \hat{A}(t) | i \rangle = \sum_f f^\dagger P_f A_f^{weak} \quad P_f = |\langle f | i \rangle|^2. \quad (6)$$

Eq.$(6)$ suggests that weak values represent measurable quantities for a pre- and post-selected system. This is indeed the case if a measurement of $\hat{A}(t)$ is performed on the system with sufficient quantum uncertainty to avoid disrupting the evolution of the system. Such “weak” measurements yield not only the real part of $A_f^{weak}$ but also its imaginary part and reconstruct in this way the available history between $t_1$ and $t_2$ for a system pre-selected at $t_1$ and post-selected at $t_2$[16].

Generally, the information gained by post-selection and weak values is relevant only if one post-selects a state describing rare events for the pre-selected state considered, that is if $P_f$ is located in the tail of the distribution probability. However, the situation is different when one considers the Hawking emission process in the classical background of a collapsing star. There, in absence of back-reaction, the energy-momentum tensor of the radiation can be computed exactly in some simplified models. It then appears that post-selected states defined on a space-like surface $\sigma$ arbitrarily close to the union of the event horizon $\mathcal{H}$ and of the future light-like infinity $\mathcal{I}^+$ may yield weak values of the energy-momentum tensor operator $\hat{T}_{\mu\nu}(x)$ very different from its average value. While the latter remains smooth on the scale of the Schwartzschild radius, the former may exhibit in that region oscillations of unbounded amplitudes and a singularity on the event horizon[7]. These features, which are a consequence of the unbounded blueshift experienced in the vicinity of the horizon by the vacuum fluctuations generating the Hawking quanta, persist in generic post-selected states detectable by external observers. The energy content of these fluctuations show up in the weak values of $\hat{T}_{\mu\nu}(x)$ but are averaged out in expectation values. However observers who do cross the horizon detect different post-selected states. These yield weak values of $\hat{T}_{\mu\nu}(x)$ which are smooth as the observer approaches the horizon.
Taming the energy fluctuations will lead rather naturally to a back-reaction picture in which the geometry reconstructed by external observers exhibits neither horizon nor singularity. This geometry would not be determined by the expectation value of the metric but would be obtained from the weak values

$$g_{\mu\nu}(x)^{weak}_{f} \equiv \frac{\langle f | \hat{g}_{\mu\nu}(x) | i \rangle}{\langle f | i \rangle}. \quad (7)$$

More precisely, expressing as in Eq.(6) expectation values in terms of weak values

$$\langle i | \hat{g}_{\mu\nu}(x) | i \rangle = \sum_{f} P_{f} \frac{\langle f | \hat{g}_{\mu\nu}(x) | i \rangle}{\langle f | i \rangle} \quad P_{f} = |\langle f | i \rangle|^{2} \quad (8)$$

we are lead to the assumption that, because of the sensitivity of the horizon to back-reaction effects the right hand side is not dominated by a single classical geometry but by a set of distinct, possibly complex, geometries depending on the post-selected state. A distinct geometrical background would then be defined by a restricted average $< g_{\mu\nu}(x) >_{\{\alpha\}}$ over weak values for post-selected states $f_{\{\alpha\}}$ in Eq.(8) resulting from keeping fixed some macroscopic parameters $\{\alpha\}$, namely

$$< g_{\mu\nu}(x) >_{\{\alpha\}} = \sum_{f_{\{\alpha\}},(\alpha)} P_{f_{(\alpha)}} \frac{\langle f_{(\alpha)} | g_{\mu\nu}(x) | i \rangle}{\langle f_{(\alpha)} | i \rangle} \quad P_{f_{(\alpha)}} = \frac{|\langle f_{(\alpha)} | i \rangle|^{2}}{\sum_{f_{\{\alpha'\}}} |\langle f_{\{\alpha'\}} | i \rangle|^{2}} \quad (9)$$

In this way the the fully evaporated black hole in tamed vacuum would be characterized by all the post-selected Hawking radiation states $|f\rangle$ on $I^{+}$ which would then constitute a Cauchy surface in the future of $x$; the resulting geometry connecting $I^{-}$ to $I^{+}$ would be free from the black hole horizon and singularity and would serve as a background for reconstructing its history. Geometries corresponding to no detectable outgoing Hawking quanta would require a different future Cauchy surface and a different post-selection. They would uncover the classically singular background of the collapsing star with its original mass $M$, and the proper history recorded by the comoving observer which detects essentially no
radiation would then agree with the classical predictions in accordance with the equivalence principle. Post-selections corresponding to observers in free fall after recording a partial evaporation would generate a geometrical background accommodating a black hole with mass \( m \) smaller than \( M \). The role of \( \{\alpha\} \) is played here by the final mass \( 0 \leq m \leq M \) of the star and by the geometry of the future Cauchy surface which should be self-consistently dynamically determined from the final mass \( m \).

In the following sections we shall discuss these ideas within simplified models by confronting the problems met in the conventional derivation of the Hawking radiation based on local field theory. In section 2, we analyse, in the vicinity of the horizon, the discrepancy between weak and expectation values of the energy-momentum tensor in the classical background of a collapsing star. This analysis is based on the work of Massar and Parentani\(^7\) who generalize the concept of weak values. They introduce the notion of partial post-selection on a subspace of the full Hilbert space to cope with the hiding of information by event horizons. We shall present the considerations relevant for our analysis in a comprehensive manner and we shall refer the reader interested in further development on partial post-selection to reference \([7]\). These results are used in section 3 to motivate the proposed taming mechanism and its obvious implications for the unitarity issue. We discuss its consistency with Lorentz invariance and causality. The general formal expression Eq.\((9)\) is proposed as a provisional expression of black hole complementarity and its possible relevance for uncovering features of a theory of quantum gravity is suggested.

2. Weak and Expectation Values in the Background of a Collapsing Shell.

Let us first review the Hawking emission process for a classical collapsing spherically symmetric shell of mass \( M \). Instead of specifying the energy momentum of the shell and deducing from it its trajectory, we shall take, as in references
([17],[18],[8]) the latter as the input of the analysis. Inside the shell, Minkowski space can be described by the coordinate system

\[ ds^2 = dU \, dV - r^2 d\Omega^2 \]

\[ U = \tau - r \quad V = \tau + r, \]

for \( r < r_s(\tau) \) where \( r_s(\tau) \) is the shell radius at time \( \tau \). The input parameter will be taken to be \( w_s \), the radial inwards velocity of the shell with respect to an observer at rest inside the star, namely

\[ w = -\frac{dr_s(\tau)}{d\tau}. \]  

Outside the shell and outside the event horizon we parametrize space-time by tortoise coordinates

\[ ds^2 = g_{00}(r) du dv - r^2 d\Omega^2 \quad g_{00}(r) \equiv \left(1 - \frac{2M}{r}\right) \]

\[ u = t - r^* \quad v = t + r^* \]

\[ dr = (1 - \frac{2M}{r}) dr^*. \]

where \( r \), understood as a function of \( v - u \), is the “radius” which measures the invariant surface \( 4\pi r^2 \) of a sphere and is continuous across the shell. The entire space-time outside the event horizon can be parametrized by a single \((u,v)\) coordinate system by performing inside the shell a coordinate transformation \( U(u), V(v) \). Continuity of \( ds^2 \) and \( r \) across the shell imply on the shell

\[ \frac{dU}{du} \frac{dV}{dv} = g_{00}(r_s) \]  

\[ dV - dU = g_{00}(r_s) (dv - du) \]

An outgoing photon emitted from inside the star is redshifted on \( \mathcal{I}^+ \) at \( v = +\infty \).  

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by a factor $dU/du$. From Eqs (13) and (14),

$$
\frac{dU}{d\nu} = \left[ -w + \sqrt{w^2 + (1 - w^2)g_{00}(r_s)} \right] \frac{1}{1 - w} 
$$

(15)

where $r_s$ is the radius of the shell intersected by the outgoing ray at the value of $u$ considered. For a shell at rest, one gets

$$
\frac{dU}{d\nu} = g_{00}^{1/2}(r_s).
$$

(16)

For a collapsing star, in the vicinity of the horizon where $g_{00}^{1/2}(r_s) \to 0$ and $w \to \alpha$ where $\alpha$ is a constant such that $0 < \alpha \leq 1$, we get the asymptotic value

$$
\frac{dU}{d\nu} = g_{00}(r_s) \frac{1 + \alpha}{2\alpha}.
$$

(17)

Eqs.(16) and (17) show clearly the enhanced redshift of a photon emitted from inside the star due to the motion of the shell.

Let us first examine the s-wave contribution from a free scalar field to the radiation. Neglecting the residual potential barrier, the Heisenberg scalar field operator rescaled by $r$ obeys then

$$
\partial_u \partial_v \Phi = 0.
$$

(18)

It is expanded into a complete set of solutions $\phi_k$, that is

$$
\phi_k = f_k(u) + g_k(v)
$$

(19)

such that

$$
(\phi_j|\phi_i) \equiv i \int_{\Sigma} \left[ \phi_j^* \leftrightarrow \partial_v \phi_i dv - \phi_j^* \leftrightarrow \partial_u \phi_i du \right] = \delta(i - j)
$$

(20)

where $\Sigma$ is an arbitrary Cauchy surface which does not cross the horizon and the $\phi_k$ vanish at $r = 0$. 

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Moving back in time, positive frequency plane waves solutions defined on \( I^+ \) reflect on a timelike curve \( V(v) - U(u) = 2r = 0 \) and propagate to \( I^- \) where they span only the domain \( v < 0 \) (the zero of \( v \) is chosen here to coincide on \( I^- \) with the last rays arriving on \( I^+ \)). Hence wave-packets build out of positive frequencies in the retarded time \( u \) on \( I^+ \) require frequencies of both signs in the advanced time \( v \) on \( I^- \). To build a complete set of solutions of the field equations with positive frequencies on \( I^- \) one needs waves having support on \( v > 0 \) and propagating towards the horizon. The Heisenberg vacuum \( |i\rangle \) annihilated by destruction operators associated with positive frequencies waves on \( I^- \) is then a superposition of pairs formed from outgoing quanta on \( I^+ \) correlated to partner states described by waves propagating towards the horizon \( \mathcal{H} \). While the outgoing quanta are real particles, this is not true for the partners quanta which are associated to waves with no definite frequency sign and should be interpreted as vacuum fluctuations. Taking a trace of \( |i\rangle \langle i| \) over the latter states, one gets a density matrix describing at sufficiently large retarded times \( u \) a thermal flux of outgoing particles on \( I^+ \) at the Hawking temperature Eq.(2). For definiteness we exhibit here the various waves in the case of a light-like shell \( (w = 1) \). Denoting respectively by \( | - \omega^{\text{out}} \rangle \) and \( | + \omega^{\text{out}} \rangle \) the outgoing plane waves and their partner we have[8]

\[
| - \omega^{\text{out}} \rangle = \frac{1}{\sqrt{4\pi\omega}} \left[ \exp(-i\omega u) - \Theta(-v) \exp(i4M\omega \ln \frac{-v}{A}) \right] \tag{21}
\]

\[
| + \omega^{\text{out}} \rangle = \frac{1}{\sqrt{4\pi\omega}} \Theta(v) \exp(-i4M\omega \ln \frac{v}{A}) \tag{22}
\]

where the frequencies \( \omega \) span the positive real axis. A convenient complete set of positive frequency in-modes on \( I^- \) is, taking \( u \to \infty \),

\[
| \pm \omega^{\text{in}} \rangle = \frac{1}{\sqrt{8\pi\omega \sinh(\omega 4\pi M)}} \Theta(v) \exp(\mp i4M\omega \ln \frac{v}{A}) \exp(\pm i\omega 2\pi M)
\]

\[+ \frac{1}{\sqrt{8\pi\omega \sinh(\omega 4\pi M)}} \left[ \Theta(-v) \exp(\mp i4M\omega \ln \frac{-v}{A}) \exp(\mp i\omega 2\pi M) - \exp(\pm i\omega u) \right]. \tag{23}
\]

Defining the destruction operators \( a_{\pm\omega}^{\text{in}} \) and \( a_{\pm\omega}^{\text{out}} \) associated to the modes Eqs.(21),(22)
and (23), we have
\[
\Phi(u, v) = \int_0^\infty d\omega \left[ -\omega a_{-\omega} \right] + \int_0^\infty d\omega \left[ +\omega a_{+\omega} \right] \tag{24}
\]
or equivalently
\[
\Phi(u, v) = \int_0^\infty d\omega \left[ -\omega a_{-\omega} \right] + \int_0^\infty d\omega \left[ +\omega a_{+\omega} \right] \tag{25}
\]

The Bogoliubov transformation relating in- and out-operators take the form
\[
a_{\omega}^\text{in} = \alpha_{\omega} a_{\omega}^\text{out} - \beta_{\omega} a_{-\omega}^\dagger \]
\[
a_{-\omega}^\text{in} = \alpha_{\omega} a_{-\omega}^\text{out} - \beta_{\omega} a_{+\omega}^\dagger \tag{26}
\]
where
\[
\alpha_{\omega} = \frac{\exp(\omega 2\pi M)}{\sqrt{2 \sinh(\omega 4\pi M)}} \quad \beta_{\omega} = \frac{\exp(-\omega 2\pi M)}{\sqrt{2 \sinh(\omega 4\pi M)}} \tag{27}
\]

From Eqs.(26) and (27) one easily verifies Eq.(2).

The transplanckian problem arises because Hawking quanta arriving at late time \( u_t \) on \( \mathcal{T}^+ \) are build out of vacuum fluctuations with very high frequencies on \( \mathcal{T}^- \) in a the Lorentz frame fixed by the spherical symmetry of the shell. We represent a typical quantum detected at the retarded time \( u_t \) on \( \mathcal{T}^+ \) by a wave-packet centred, in accordance with Eq.(2) around a positive frequency \( \omega = O(1/2M) \) and extending over a distance of the order of its wavelenght \( 2M \). Its complex frequency spectrum \( f(\omega) \) has the form
\[
f(\omega) = |f(\omega)| \exp(-i\omega u_t) \tag{28}
\]
where \(|f(\omega)|\) is spread over a range comparable to its central frequency. When this wave packet moves back in time with the velocity of light, it is blueshifted
by $du/dU$ given in Eq.(17). In the vicinity of the horizon, one may choose the arbitrary constant defining $r^*$ to write, when $1 - 2M/r < 1$,

$$1 - \frac{2M}{r} \simeq \exp \left( \frac{v - u}{4M} \right).$$

(29)

Hence at late retarded times

$$g_{00}(r_s) = \exp \left( \frac{v_\infty - u}{4M} \right),$$

(30)

where $v_\infty$ is the asymptotic coordinate of the collapsing shell on the horizon. Labeling by $\tilde{\omega}$ the central frequency of the packet traced back to $I^-$ one thus gets from Eqs.(17) and (30)

$$\frac{\tilde{\omega}}{\omega} \simeq g_{00}^{-1}(r_s) \simeq -\frac{4M}{U(u_l)}$$

(31)

where the zero of $U$ is chosen on the horizon. One may verify Eq.(31) from the explicit expression of the wave Eq.(23) which yields a local frequency $\tilde{\omega} = 4M\omega/(-v)$; for $w = 1 v = V$ and the relevant waves satisfy $U(u_l) - V = U(u_l) - v = 0$. The increase in frequency from $\omega$ to $\tilde{\omega}$ is accompanied by a localisation within a wave-length $\tilde{\omega}^{-1}$. The wave-packet propagating backwards in time reflects on $r = 0$ and is correlated on $I^-$ to a partner centred at $v > 0$. This correlation arises because a plane wave of frequency $\tilde{\omega}$ extends on both sides of $v = 0$. The partner with $v > 0$ can then be similarly depicted as a wave packet with the same frequency $\tilde{\omega}$ as the ancestor of the Hawking quantum but the separation between the two packets is on $I^-$ comparable to their spread$^7$,$^8$.

Although in absence of back-reaction the Planck scale does not enter the problem, it is useful for the forthcoming discussion of section 3, to characterize the different time scales for Hawking emission in the space-time background used here with respect to this scale. We first note that Hawking emission starts about a retarded time $u_0$ (and a corresponding Schwartzschild time $t_0$) when the asymptotic value Eq.(30) becomes accurate, that is when $v$ on the star surface can be well
approximated by the constant $v_\infty$. This happens when $\delta \equiv r - 2M$ is still of order $2M$ and we label the corresponding coordinate separation by $\delta_0$. Thus

$$\delta_0 = O(2M).$$

A typical Hawking quantum requires, from Eqs. (30) and (31) transplanckian vacuum frequencies $\tilde{\omega} > 1$ inside the shell and on $\mathcal{I}^-$ after retarded and Schwartzschild times $u_1$ and $t_1$ such that

$$t_1 - t_0 = \frac{u_1 - u_0}{2} = O(2M \ln 2M).$$

while for a static observer just outside the shell the frequencies remain cisplanckian up to times such that $\omega/\sqrt{g_{00}(r_s)} \simeq 1/2M\sqrt{g_{00}(r_s)}$ becomes of order unity, that is

$$t_2 - t_0 = \frac{u_2 - u_0}{2} = O(4M \ln 2M).$$

The corresponding location of the centre of the packets in the vicinity of the shell are, from Eq. (30)

$$\delta_1 = O(1); \quad \delta_2 = O(1/2M).$$

Note that, at fixed time $t$, $\delta_1$ measures a planckian distance to the horizon inside the star while $\delta_2$ measures a planckian distance to the horizon outside the star, as follows from the metric Eq.(12). At time $t_2$, the local temperature $T/g_{00}^{(1/2)}(r_s)$ reaches, from Eq.(35), the Planck temperature just outside the shell.

The transplanckian vacuum fluctuations give rise to corresponding transplanckian weak-values of the energy-momentum tensor for post-selected states describing Hawking quanta and these unbounded fluctuations are responsible for the discrepancy between average and weak values in the vicinity of the horizon. In general, post-selected states are defined by eigenstates of a complete set of commuting observables on some Cauchy surface $\Sigma_f$ laying in the future of the space-time points.
\{x\}$ where weak values are computed. We shall take for $\Sigma_f$ a space-like surface infinitesimally close to the union of the future light-like infinity $I^+$ and of the event horizon $\mathcal{H}$. We shall consider the subset of post-selected states which can be obtained by measuring devices sensitive only to cisplanckian frequencies. This can be achieved in the following way. We write the full Hilbert space $H$ of post-selected states as

$$H = H_\mathcal{H} + H_{I^+}$$

where $H_\mathcal{H}$ and $H_{I^+}$ are engendered respectively by the operators acting in $\mathcal{H}$ and in $I^+$. Consider a post-selection which would be defined only in the Hilbert space $H_{I^+}$ and let $|P_{1\alpha}\rangle$ be such a state. This state is EPR correlated to a state $|P_{2\beta}\rangle$ which is, up to a normalization constant, equal to $\langle P_{1\alpha}|i \rangle$. The corresponding post-selected state is, up to normalization, $|P_{1\alpha}\rangle\langle P_{1\alpha}|i \rangle$ and we shall call it a partially post-selected state. Thus the weak value of the renormalized energy-momentum tensor operator $\hat{T}_{\mu\nu}$ of the scalar field $\Phi$, corresponding to this partially post-selected state, is according to Eq.(5)

$$T_{\mu\nu}(x)_{\text{weak}}^{P_{1\alpha}} \equiv \frac{\langle i| P_{1\alpha}\rangle \langle P_{1\alpha}| \hat{T}_{\mu\nu}(x)|i \rangle}{\langle i| P_{1\alpha}\rangle \langle P_{1\alpha}| i \rangle}. \quad (37)$$

Note that one may insert in Eq.(37) the projection operator $I_{H\mathcal{H}}$ on the Hilbert space $H_\mathcal{H}$ and define a post-selection density matrix $\Pi_S \equiv I_{H\mathcal{H}} |P_{1\alpha}\rangle \langle P_{1\alpha}|$. In this way, Eq.(37) can be rewritten in the formulation of partial post-selection given in reference [7], namely

$$T_{\mu\nu}(x)_{\text{weak}}^{P_{1\alpha}} \equiv \frac{\langle i| \Pi_S \hat{T}_{\mu\nu}(x)|i \rangle}{\langle i| \Pi_S |i \rangle}. \quad (38)$$

The probability of finding $|P_{1\alpha}\rangle$ by tracing over $H_\mathcal{H}$ is

$$p_\alpha = \langle i| P_{1\alpha}\rangle \langle P_{1\alpha}| i \rangle \quad (39)$$

and it is easy to check that it is equal to the probability of finding simultaneously
$|P_{1\alpha}\rangle$ and its EPR-correlated state in $|i\rangle$. We have

$$\sum_\alpha p_\alpha T_{\mu\nu}(x)|P_{1\alpha}\rangle \rightarrow \sum_\alpha \langle i| I_{H_+} \hat{T}_{\mu\nu}(x) |i\rangle = \langle i| \hat{T}_{\mu\nu}(x) |i\rangle.$$  \hfill (40)

Eq.(40) expresses that summing weak values over all partially post-selected states on $I^+$ yields back again the expectation value of $\hat{T}_{\mu\nu}(x)$. Each of the partially post-selected states $|P_{1\alpha}\rangle$ can be interpreted as a state which could have been detected by observers whose motion is confined to the space-time region bounded by the event horizon. We shall say that these are the post-selected states available to external observers: they contain the maximal amount of information accessible to these observers that can be added to the one contained in the state $|i\rangle$.

To compare the weak values Eq.(37) with the expectation value of $\hat{T}_{\mu\nu}$ we write

$$\hat{T}_{\mu\nu}(x) = :\hat{T}_{\mu\nu}(x) : + T_{\mu\nu}(x)$$ \hfill (41)

where $:\hat{T}_{\mu\nu}(x) :$ stands for the normal ordered operator in the Heisenberg state $|i\rangle$; hence the c-number $T_{\mu\nu}(x)$ is

$$T_{\mu\nu}(x) = \langle i| \hat{T}_{\mu\nu}(x) |i\rangle.$$ \hfill (42)

Outside the star, $T_{\mu\nu}(x)$ can be computed exactly in the 2-dimensional approximation corresponding to the reduced field equation Eq.(18). In that approximation the 2-dimensional energy-momentum tensor operator $\hat{T}_{\mu\nu}^{(2)}(x) = \hat{T}_{\mu\nu}(x)4\pi r^2$. One has

$$:\hat{T}_{\mu\nu}^{(2)}(x) : = : \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} g_{\mu\nu}(g^{\sigma\tau} \partial_\sigma \Phi \partial_\tau \Phi) :.$$ \hfill (43)

The expectation value of $\hat{T}_{\mu\nu}^{(2)}(x)$ follows from the trace anomaly

$$T_{uv}^{(2)}(x) = -\frac{1}{12\pi} \partial_u \partial_v \rho,$$ \hfill (44)

where $\exp(2\rho)$ is the conformal factor in the $(u, v)$ coordinate system which coin-
cides with $g_{00}(r)$ outside the star, and from the conservation law

$$\hat{T}_{\nu;\mu}^\mu(x) = 0.$$  \hfill (45)

One gets outside the shell

$$4\pi r^2 T_{uu}(x) = \langle i | \hat{T}_{uu}^{(2)}(x) | i \rangle = \frac{1}{12\pi} \left[ -\frac{M}{2r^3} (1 - \frac{2M}{r}) - \frac{M^2}{4r^4} \right] + t_u(u) \hfill (46)$$

$$4\pi r^2 T_{vv}(x) = \langle i | \hat{T}_{vv}^{(2)}(x) | i \rangle = \frac{1}{12\pi} \left[ -\frac{M}{2r^3} (1 - \frac{2M}{r}) - \frac{M^2}{4r^4} \right] + t_v(v) \hfill (47)$$

$$4\pi r^2 T_{uv}(x) = \langle i | \hat{T}_{uv}^{(2)}(x) | i \rangle = -\frac{1}{12\pi} \frac{M}{2r^3} (1 - \frac{2M}{r}). \hfill (48)$$

The last terms in Eqs.(46) and (47) are determined from boundary conditions. The vanishing of $T_{\mu\nu}$ on $\mathcal{I}^-$ gives $t_v = 0$ and the regularity of $T_{\mu\nu}$ on the horizon in an inertial frame requires that $T_{uu}$ vanishes when $u \to \infty$ as $O(1 - 2M/r)^2$ or equivalently

$$t_u = \frac{\pi}{12} \frac{1}{(8\pi M)^2}. \hfill (49)$$

This term represents from Eq.(46) the outgoing energy flux as $r \to \infty$ and indeed coincides with the thermal flux of the Hawking radiation.

These values of $t_u$ and $t_v$ characterize the boundary conditions of the “Unruh vacuum” in contradistinction with the values $t_u = t_v = 0$. The latter define the “Boulware vacuum” where $T_{\mu\nu}$ vanishes both on $\mathcal{I}^-$ and $\mathcal{I}^+$. As follows from Eq.(46) the energy density then diverges negatively in an inertial frame as one approaches the horizon. The Boulware vacuum mimics the vacuum energy that would be provoked by a static shell sitting on the horizon.

On the other hand, we may write, using the Bogoliubov transformations $\hat{T}_{\mu\nu}(x)$ : acting on $|i\rangle$

$$: \hat{T}_{\mu\nu}(x) : |i\rangle = :: \hat{T}_{\mu\nu}(x) :: |i\rangle + \Delta T_{\mu\nu}(x) |i\rangle$$ \hfill (50)

where :: $\hat{T}_{\mu\nu}(x)$ :: contains only creation operators on out-states and $\Delta T_{\mu\nu}(x)$ is a
c-number. Thus
\[
\Delta T_{\mu\nu}(x) = \frac{\langle f_0| : \hat{T}_{\mu\nu}(x) : |i\rangle}{\langle f_0| i\rangle},
\]  
where $|f_0\rangle$ labels the out-vacuum. The weak value Eq.(37) of the energy-momentum tensor for the partially post-selected state $|P_{1\alpha}\rangle$ can be written as
\[
T_{\mu\nu}(x)_{P_{1\alpha}}^{\text{weak}} = \frac{\langle i| P_{1\alpha}\rangle \langle P_{1\alpha}| : \hat{T}_{\mu\nu}(x) :: |i\rangle}{\langle i| P_{1\alpha}\rangle \langle P_{1\alpha}| i\rangle} + \Delta T_{\mu\nu}(x) + T_{\mu\nu}(x).
\]  
Computing $\Delta T_{\mu\nu}(x)$ on $I^+$ in the asymptotic region $u > u_0$, we get from Eqs.(43), (25) and (23)*, using the Bogoliubov transformation Eq.(26),
\[
4\pi r^2 \Delta T_{\mu\nu} = - \int_0^\infty d\omega \frac{2\beta(\omega)}{\alpha(\omega)} \partial_\mu| - \omega in)\partial_\nu| - \omega in)
\]
or explicitly
\[
4\pi r^2 \Delta T_{uu} = - \frac{1}{2\pi} \int_0^\infty d\omega \frac{\omega}{e^{\omega 8\pi M} - 1} = - \frac{\pi}{12} \frac{1}{(8\pi M)^2}
\]
\[
4\pi r^2 \Delta T_{vv} = - \frac{1}{2\pi} \frac{1}{(8\pi M)^2} \frac{16M^2}{(v - i\epsilon)^2}
\]
\[
4\pi r^2 \Delta T_{uv} = 0.
\]
Going to $I^+$ we get from Eqs.(40) and (52)
\[
\lim_{v \to +\infty} \sum_\alpha p_\alpha \frac{\langle i| P_{1\alpha}\rangle \langle P_{1\alpha}| : \hat{T}_{\mu\nu}(x) :: |i\rangle}{\langle i| P_{1\alpha}\rangle \langle P_{1\alpha}| i\rangle} = \frac{\pi}{12} \frac{1}{(8\pi M)^2}\delta_{\mu\nu}
\]  
where the sum extends over all partially post-selected states on $I^+$. This is consistent with the interpretation that
\[
\langle i| P_{1\alpha}\rangle \langle P_{1\alpha}| : \hat{T}_{\mu\nu}(x) :: |i\rangle / \langle i| P_{1\alpha}\rangle \langle P_{1\alpha}| i\rangle
\]

* We use here the waves $|\pm \omega n\rangle$ for the light-like collapse $w = 1$. It is easily verified that in the asymptotic region the results Eqs.(54),(55) and (56) are valid for the general collapse $0 < w \leq 1$. 

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represents the energy density contained in the post-selected Hawking quanta forming the state $|P_{\alpha}\rangle$: Eq. (57) tells that the average energy density carried by the post selected Hawking quanta is equal to that of the Hawking flux on $\mathcal{I}^+$. More generally Eq. (52) describes these quanta as if they were travelling on a vacuum whose energy density is $T_{\mu\nu} + \Delta T_{\mu\nu}$. Outside the shell $v > 4M$ and $\Delta T_{vv}$ becomes negligible compared to $\Delta T_{uu}$ as $v$ increases. Comparing Eqs. (54), (55) and (56) to Eqs. (46), (47), (48) and (49) we see that the term $t_u$ has cancelled out in the new vacuum and that outside the shell the post-selected quanta travel thus on a vacuum whose energy density is essentially the one due to a Boulware vacuum characterized by $t_u = t_v = 0$.

Thus, by post-selection we switch from a description where the energy of the Hawking radiation appears as a property of the Unruh vacuum to a description in which excited particles building the Hawking flux travel on a Boulware vacuum. The latter has a negative energy density which diverges on the horizon in an inertial frame. Moving backward in time this divergence reflects back on $r = 0$ up to $\mathcal{I}^-$, as seen from Eq. (55). In fact $\Delta T_{vv}$ is symmetric with respect to $v = 0$ and the vanishing of the total minkowskian energy on $\mathcal{I}^-$ is ensured from Eq. (55) by a positive energy singularity carried by the last rays [7]. The last rays may be viewed as causally generating the horizon and thus the Boulware vacuum energy density is in that sense causally related to energy densities lay down on $\mathcal{I}^-$. The positive energy singularity carried by the last rays accordingly generates a singularity on the horizon itself, and this constitutes a general feature of the Boulware vacuum [20]. The equivalence on the average between the description in terms of excited quanta on top of a Boulware vacuum causally generated by the singular last rays and the description in terms of the Unruh vacuum, is ensured by Eq. (40). This equivalence is however the result of a very delicate interference process. Indeed, the weak energy-momentum carried by the post-selected quanta (the first term in Eq. (52)) in the vicinity of the last rays and of the horizon exhibits oscillations with divergent amplitudes and frequencies [7],[8]. This is a consequence of the unbounded differential blueshift which distort the small oscillations inherent
to wave packets built on $I^+$ from positive frequencies only. On the contrary, the causal Boulware vacuum energy diverges monotonically in the vicinity of the last rays and of the event horizon. Therefore, even for generic partially post-selected states on $I^+$ expectation and weak values of $\hat{T}_{\mu\nu}(x)$ will behave very differently and the unbounded fluctuations of $T_{\mu\nu}(x)^{weak}_{P_{\alpha}}$ will survive such generic post-selection.

The post-selected states considered above contain the maximal amount of information available for external observers. However an observer comoving with the star will detect practically no quanta and cannot gain information from post-selection. In other words, the partial post selected states available to comoving observers is $I_{H^+}|i\rangle = |i\rangle$ and the corresponding weak value

$$T_{\mu\nu}(x)^{weak} = \langle i|\hat{T}_{\mu\nu}(x)|i\rangle$$  \hspace{1cm} (58)

then simply coincides with the expectation value. Observers who cross the horizon at later advance time $v_c$ than the shell stop detecting radiation after some critical retarded time $u_c(v_c)$. They can be similarly be characterised by some post-selected states containing the maximal information available to them. We surmise that these would lead, as in Eq.(58), to a regular weak value for the energy momentum tensor on the horizon, but we have not carried out an explicit evaluation for such “intermediate” cases. Of course the fact that the weak energy can be either regular or singular on the horizon, depending on the information available to observers with different state of motion, does not lead at this stage to any paradox but simply emphasizes that detectable quanta on $I^+$ require violent energy-momentum fluctuations in the vicinity of the horizon and back to $I^-$. 

This conclusion is not significantly altered when higher angular momentum are included. The above description remains essentially valid for all angular momenta except that most of the high angular momentum modes are reflected back outside the star towards the horizon. Transplanckian fluctuations exist for all angular momenta but outside the shell the distribution of Hawking quanta is the result of a balance of outgoing and reflected waves. This brings them nearly in
thermal equilibrium close to the horizon outside the star with a local blueshifted temperature

\[ T_{\text{loc}} = \frac{1}{8\pi M} \left(1 - \frac{2M}{r}\right)^{-1/2} \]  

of the order of the blueshifted frequency there. Thus, the inclusion of higher angular momentum for free fields does not change the transplanckian character of the production process. Rather, it imbeds, outside the shell, the s-wave transplanckian frequencies in a transplanckian thermal bath Eq.(59) which would be unaffected by interactions due do asymptotically free interacting renormalizable field theory mixing different angular momenta. Conversely, the existence of such a local temperature, given the potential barrier, is sufficient to generate the Hawking radiation at infinity\(^6\),\(8\).

3. Weak values and Complementarity.

As discussed in section 1, in presence of gravitational non-linearities, frequencies on \( I^{-} \) are expected to be tamed to the Planck scale by some hitherto unknown mechanism without altering significantly the emitted radiation on \( I^{+} \). In the shell model considered in the preceding section, transplanckian energies arose from quanta travelling at distances closer than the Planck size from the horizon. Within this toy model, these energies could be tamed to Planck energies if the planckian forces would bring the collapse to a halt at a \( \delta_2 = O(1/2M) \), or equivalently at a Planck distance of its Schwarzschild radius. This would constitute a dynamical taming of the “hot” Boulware vacuum which emerged in section 2 from the description in terms of the weak energy-momentum tensor from the post-selected states available to external observers. Such an assumption is rather natural if the background metric would be generated from weak values of the gravitational field operators and not from their expectation values; these weak values should be sensitive to the weak values of the energy-momentum source terms because, at least in the linear approximation, weak values satisfy the same Heisenberg equations of motion as the expectation values.
The assumption that, in presence of horizons, background are built from weak values and hence dependent on post-selection will be discussed below. We first examine how to give some substance to the toy model if quantum gravity would indeed generate a background describing a stopped collapse*. At the time \( t_1 \) given by Eq.(33) transplanckian fluctuations appear inside the collapsing shell and at the time \( t_2 \) given by Eq.(34) they reach the Planck scale for an observer sitting outside the shell at a fixed planckian distance from the horizon, that is at \( \delta_2 = O(1/2M) \), in a heat bath at the Planck temperature. Thus the halting must take a time \( t_2' - t_1 \) comparable or larger than \( t_2 - t_1 = O(2M \ln 2M) \). During this time span the emission process must be gradually altered and the original mechanism be completely lost after the time \( t_2' \) when the background geometry becomes static. For the radiation at \( I^+ \) to survive the halt of collapse, the local temperature in the neighbourhood of the shell must still be of the order of the Planck temperature. But such a thermal bath in the static geometry can only occur if the halting of the collapse has heated the collapsing matter, thereby providing a source for the radiation, and such a heating is indeed expected from energy conservation in a way similar to the heating of an object maintained by external forces at a fixed radius from the horizon of a black hole horizon. The role of the external force should be here played by the unknown planckian forces responsible for the stopping. However such a process requires quantum transitions between the constituents of the shell and the planckian thermal bath and cannot be accounted for by the toy model.

We therefore have to trade the classical shell and its surrounding vacuum for a quantum object whose average classical configuration tends towards a static one in the planckian neighbourhood of its Schwartzschild radius. Such a picture seems to contradict Eqs.(46),(47) and (48) which lead to zero flux for a static classical trajectory. However, these equations rely on the conservation law Eq.(45) which should now be modified to take into account the source provided by planckian

* In a preliminary version[21], this scenario was suggested but the conceptual framework discussed here was only sketched.
constituents of the quantum shell. Phenomenologically one may write

\[ T_{\nu,\mu}^\mu (x) = J_\nu (x) \]  \hspace{1cm} (60)

where \( J_\nu (x) \) represents the source of the radiation arising from these constituents averaged over a Planck distance. In absence of a theory of quantum gravity one may view this source term as an external one and a possible term is, in the global \((u, v)\) system,

\[ J_u (u, v) = 2 e^{-2\rho} k_u (u) \delta [v - v_s (u)] \]  \hspace{1cm} (61)

\[ J_v (u, v) = 2 e^{-2\rho} k_v (v) \delta [u - u_s (v)] \]  \hspace{1cm} (62)

where \( v_s (u) \) and \( u_s (v) \) are parametrizations of the shell averaged trajectory. The functions \( k_u (u) \) and \( k_v (v) \) measure the radiation due the shell. Instead of Eq.(46) and (47) we get indeed outside the shell, adding the source term to the trace anomaly,

\[ 4\pi r^2 T_{uu} = \frac{1}{12\pi} \left[ \frac{M}{2r^3} (1 - \frac{2M}{r}) - \frac{M^2}{4r^4} \right] + t_u (u) + \Theta [F(u, v)] k_u (u) \]

\[ 4\pi r^2 T_{vv} = \frac{1}{12\pi} \left[ -\frac{M}{2r^3} (1 - \frac{2M}{r}) - \frac{M^2}{4r^4} \right] + t_v (v) - \Theta [F(u, v)] k_v (v) \]  \hspace{1cm} (63)

where \( \Theta [F(u, v)] \) is a function which goes through zero on the shell and is positive outside. The energy flux across a sphere of radius \( r \) at a retarded time \( u \) becomes in general

\[ T_{uu}^{(2)} - T_{vv}^{(2)} = t_u (u) + k_u (u). \]  \hspace{1cm} (64)

For for \( u \approx u_0 \) it is all contained in \( t_u (u) \) and gradually shifts to the source term \( k_u (u) \) as the shell slows down and stops. One may verify that in contradistinction with \( t_u (u) \), \( k_u (u) \) has conventional tensor transformation under conformal reparametrizations.

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The quantum shell represents only an outer region of finite thickness of a realistic star. Consider a structureless classical shell of finite thickness $\Delta L = \xi 2M$ at rest before collapse in the asymptotically flat space time region. The shell may be viewed as part of a larger star of mass $M$; $\xi$ is smaller than 1 but still large compared to $1/M$. At the classical level, the collapse of the shell would induce a Lorentz contraction to a coordinate separation $\Delta r$

$$\Delta r = \left(1 - \frac{2M}{r}\right)^{\frac{1}{2}} (1 - v^2)^{\frac{1}{2}} \Delta L$$ \hspace{1cm} (65)

where $v$ is the velocity of the shell in the local static frame and hence

$$(1 - v^2) = C \left(1 - \frac{2M}{r}\right)$$ \hspace{1cm} (66)

where $C$ is of order 1 and in fact equal to unity for a shell initially at rest at $r = \infty$. Thus for $t < t_0$, $\delta r \simeq \xi \delta$, meaning that the size of the shell $\xi \sqrt{(2M\delta)}$ shrinks to the Planck size between $t_1$ and $t_2$. To get thermal equilibrium at the local Planck temperature, the shell should stick to that size. For the classical shell this could occur neither if the collapse went on (in which case its size would become too small) nor if it would be brought to a halt (in which case the size would become too big). But a quantum object composed of planckian constituents would interact with the local Hawking excitations at that momentum scale. This interaction could localize these constituents within a planckian distance of the horizon, depending on the fundamental structure of matter\[22\], and the whole shell could then remain localized on such size.

Thus we envisage that the source of Hawking radiation shifts gradually from the vacuum fluctuations in the metric of a collapsing star to that of a configuration which appears, on the average, static on time scales short compared to $M^3$ that is to the evaporation time. This means that the background geometry has a trivial topology. Such structures where the star sticks within a Planck size of its Schwartzschild radius have been introduced previously and labeled as “achronons”\[14\].
For a realistic star, the process of terminating a collapse into an achronon heated at the Planck temperature raises a causality issue. The stopping of the star is a nonlocal process and halting the perturbations around spherical symmetry requires correlations to be established over the sphere in a time comparable to the stopping time which is of the order of $4M \ln 2M$ or larger. These correlations cannot be set in faster than the time necessary for a light ray emitted during the collapse to reach the opposite point of the sphere. The shortest travel time for the signal is achieved, when $\delta$ becomes of order 1, by sending it *outwards* to minimize time dilation effects. Outgoing (and ingoing) waves travelling radially in the neighbourhood of the horizon take a time lapse $\Delta t$ to reach (to come from) a distance comparable to the Schwartzschild radius of order

$$\Delta t = 2M \ln \frac{\delta_0}{\delta(t)}$$

(67)

where $t$ labels the emission (arrival) time. Neglecting the time of order $M$ needed to travel on the outer sphere at $\delta = O(2M)$, we see that the travel time is of order $\gamma M \ln 2M$ where $\gamma$ is a number between 4 and 8, depending on the time of emission of the signal during the collapse after the onset of Hawking radiation. This seems to indicate that there is hope that such correlations can be established⋆.

The mechanism considered above to get rid of transplanckian frequencies from planckian effects would keep the source of Hawking radiation in causal contact with the star and remain so during its subsequent evaporation. After a short ignition period during which the collapsing star emits thermal quanta uncorrelated to its structure, the achronon burns away its mass and structure as any hot body does within a topologically trivial background void of horizons and of singularities. Although we have not implemented our description by dynamics, we can see no reason why the evaporation process should not be viewed as a unitary process within this background at least to the extend that background transitions are small.

⋆ it was argued in reference [22] that strings could establish correlations over the horizon in the time spent by a signal emitted at a Planck distance from the horizon, that is for $\gamma = 8$. 28
This is at odds with the usual picture assuming weak back-reactions effects induced by the average energy-momentum tensor of the radiation. In this case, except at the final stage, one could treat in good approximation the emission process adiabatically by fixing the classical Schwarzschild background by the instantaneous mass. The cumulative effect of the varying mass leads however to qualitative effects\textsuperscript{[23],[24]} which we briefly discuss. For a constant mass the global event horizon coincided outside the star with the boundary of trapped surfaces, that is with the apparent horizon. The latter now separates from the former. A hypothetical light ray which would be emitted from inside the star and cross the shell between the two horizons would first recede to smaller radius and diverge again upon reaching the apparent horizon. The evaporation of the black hole is not due to a reduction of the shell mass but results from a negative contribution to the total mass from a cloud concentrated between the event and the apparent horizon. This cloud is then situated at smaller radius than the shell when the latter was in causal contact with the distant observer but is separated by a space-like distance from the shell at the same radius. Thus the march of the classical star towards its final destruction by tidal forces in the vicinity of the classical singularity appears as a causally disconnected history from the evaporation out of a polarisation cloud which fills up with increasing mass (in absolute value) a macroscopic volume of space “outside the star surface” back to $r \simeq 0$.

This sequence of events is fully consistent with a thermal, structureless Hawking radiation encoding no information about the original star. All the information is contained in the collapsing star evolving with its initial mass. This is also consistent with the fact that, as required by the quantum superposition principle, there is no duplication of information between the collapsing star and the Hawking radiation\textsuperscript{[6]}. In fact, the latter argument seem to indicate that the picture emerging from the semi-classical back-reaction anzats is bound to survive quantum corrections and that up to the Planck mass scale, the Hawking radiation of a collapsing star cannot, even in principle, contain any relevant information about the detailed structure of the star. One would then be confronted with the usual
dilemma of either a full evaporation and the concomitant loss of unitarity in the
universe left behind or with the halting of the collapse at the external observers
Planck size, relegating there the huge and presumably infinite degeneracy of a left
over remnant.

The termination of a collapse into an achronon would cure tranplanckian ef-
fects and possibly the unitarity issue, but appears in blatant contradiction with
the equivalence principle as there can be no classical force to generate the huge ac-
celeration needed to halt the collapse in a region where a free-falling object, would,
according to classical general relativity feel, for large collapsing masses, vanishingly
small curvature. A reconciliation was already suggested above from the presumed
dynamical origin of the taming. The tamed metric background in which the black
hole evaporates would stem from to weak values of the energy-momentum tensor;
it would not coincide with the expectation value of $g_{\mu\nu}$ in the Heisenberg state
$|i\rangle$ defined by the star prior to collapse but rather by some average weak value
$< g_{\mu\nu}(x) >_0$. This value would be the result of an averaging over post-selected
states defined by a complete set of commuting observables describing, on a future
space like surface, the Hawking quanta due to the complete evaporation to zero
final mass of the hole. It would describe a geometrical background with a trivial
achronon topology in which the future space-like surface where post-selection of
Hawking quanta is performed is a Cauchy surface. In contradistinction with this
reconstructed history of the collapse by external observers, the proper history of
the star, as recorded by a comoving observer, follows from the back-reaction to a
trivial post-selection corresponding to essentially no detectable Hawking quanta.
This back-reaction should not destroy the classical background and the star would
cross the horizon and end up in the singularity, in accordance with the equivalence
principle. This background we label as $< g_{\mu\nu}(x) >_M$ to recall that the star retains
its full mass $M$. One must then also consider intermediate geometries detected by
observers recording Hawking radiation up to some final mass $m$, $0 < m < M$ and
then collapsing in the background of a star of mass $m$. Thus we could classify
the histories of the star according to classes defined by some classical parameter,
say \( m \), labeling as in Eq.(9) averages of weak values over final states with the corresponding parameters:

\[
< g_{\mu\nu}(x) >_m = \sum_{f_m \in m} P_{f_m} \frac{\langle f_m | \hat{g}_{\mu\nu}(x) | i \rangle}{\langle f_m | i \rangle}; \quad P_{f_m} = \frac{|\langle f_m | i \rangle|^2}{\sum_{f_{m'} \in m} |\langle f_{m'} | i \rangle|^2} \quad (68)
\]

Thus in presence of the classical event horizon an infinity of different background geometries would be generated according to the post-selected events available to a given observer. This form of complementarity is somewhat different from the one proposed in reference [6] where two histories were assumed: the one available to external observers and the proper one. Consistency for observers crossing the horizon after partial evaporation of the hole is then tentatively ensured by arguing that these observers cannot receive cisplanckian signals from the proper motion before reaching the singularity\(^{[25]}\). The present approach based on weak values associate different geometries and hence different histories to detectable differences in the amount of Hawking radiation recordable before crossing the horizon. Histories are relative to the post-selections available to the state of motion of the observer and split upon the latter crossing the horizon\(^*\). In each history, unitarity is expected to be satisfied to the extent that transitions between different background can be neglected. To avoid paradoxes in this enlarged concept of complementarity, interferences between macroscopically distinct histories must be negligible, as is generally the case in ordinary quantum physics for macroscopic systems containing many degrees of freedom. A more dramatic possibility would be that quantum gravity allow fluctuations of the geometry only in the vicinity of a classical background. This alternative would attach to each configuration a well defined meaning to time as the latter is an operational concept only in the vicinity of a classical geometry\(^{[26]}\). Finally we want to stress that the fact that weak values are called upon for selecting backgrounds does not conflict with causality. Causality is not

\(^*\) The observer dependence of effects due to back-reaction in presence of horizons was suggested previously by Gibbons and Hawking\(^{[12]}\).
posed in different terms for weak values than for the more conventional expectation value ansatz: one just selects different matrix elements of the same Heisenberg operator.

Clearly, in absence of a reliable theory of quantum gravity, Eq.(68) has only formal and provisional meaning. In particular, it is not clear how to cope with a possibly complex background metric. Also the very notion of a background geometry can only be made precise if physics admits a description in terms of objects defined on it. Despite and perhaps because of these ambiguities, the possible relevance of our formulation of complementarity is to provide a germ of a possible conceptual framework in which the dynamics of quantum gravity should fit. At present all one can say is that, as mentioned above, the arguments of reference [22] seem to indicate, in a different but related context, that elements of string theories may be relevant in particular for ensuring the Lorentz transformation properties of matter constituents and the correlations over the whole horizon needed to reconstruct the star history by the external observers. In addition the taming of transplanckian frequencies outside the star follows also rather naturally from the spectrum of string theories[8]. Thus string theories may contain some of the ingredients required to realize the above scheme which appears clearly inconsistent with the conventional local field theory whose shortcomings motivated its construction.
Acknowledgements

I am particularly grateful to Y. Aharonov for his help in clarifying the conceptual issues involved in this work. I also thank R. Argurio, R. Brout, A. Casher, S. Massar, R. Parentani and Ph. Spindel for useful discussions.
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