The current state of cosmology is easy to summarize: a very successful standard model – the hot big-bang cosmology – that accounts for the evolution of the Universe from $10^{-2}$ sec until the present; bold ideas based upon early-Universe physics – foremost among them inflation and cold dark matter – that can extend the standard cosmology to times as early as $10^{-32}$ sec and address the most pressing questions; and a flood of observations – from determinations of the Hubble constant to measurements of CBR anisotropy – that are testing inflation and cold dark matter.

1 Introduction

The value of the Hubble constant has changed by about a factor of ten since Edwin Hubble’s pioneering measurements. The context in which we view the Universe has changed just as profoundly. Until 1964 cosmology was mostly concerned with cosmography; the spirit of this period was perhaps best captured by Sandage, “the quest for two numbers ($H_0$ and $q_0$).” The discovery of the Cosmic Background Radiation led to the establishment of a physical foundation for the expanding Universe – the hot big-bang cosmology. The 1970s saw this model become firmly established as the standard cosmology. In the 1980s cosmologists began trying to extend the standard cosmology by rooting it in fundamental physics. Inflation is a potential first step in this program. Today, a host of cosmological observations are testing inflation and its cold dark matter theory of structure formation. Although there is not agreement on how inflation is faring, most would agree that inflation will soon be tested decisively.

2 Foundations

The hot big-bang cosmology is a remarkable achievement. It provides a reliable accounting of the Universe from around $10^{-2}$ sec until the present, some 10 Gyr to 15 Gyr later. It, together with the standard model of particle physics and speculations about the unification
Figure 1: Summary of CBR anisotropy measurements. Plotted are the squares of the measured multipole amplitudes ($C_l = \langle |a_{lm}|^2 \rangle$) versus multipole number $l$. The relative temperature difference on angular scale $\theta$ is given roughly by $\sqrt{l(l+1)C_l/2\pi}$ with $l \sim 200^\circ/\theta$. The theoretical curves are standard CDM (upper curve) and CDM with $n = 0.7$ (from Ref. [2]).

of the fundamental forces and particles, provides a firm foundation for the sensible discussion of earlier times.

The standard cosmology rests on four observational pillars:

- The expansion of the Universe. The redshifts and distances of thousands of galaxies have been measured and are in accord with Hubble’s Law, $z = H_0d$, a prediction of big-bang models for $z \ll 1$.

- The Cosmic Background Radiation (CBR). The CBR is the most precise black body known – deviations from the Planck law are smaller than 0.03% of the maximum intensity. Its temperature has been measured to four significant figures: $T_0 = 2.728 \pm 0.002$ K [1]. The only plausible origin is the hot, dense plasma that existed in the Universe at times earlier than $10^{13}$ sec (epoch of last scattering and recombination).

- Temperature fluctuations in the CBR. Temperature differences of order $30\mu$K between directions on the sky separated by angles from less than one degree to ninety degrees
Figure 2: Big-bang production of the light elements; widths of the curves show the two-sigma theoretical uncertainty. The primeval abundances of D, $^3$He, $^4$He and $^7$Li can be explained if the baryon density is between $1.5 \times 10^{-31}$ g cm$^{-3}$ and $4.5 \times 10^{-31}$ g cm$^{-3}$ ($\Omega_B h^2 = 0.008 \pm 0.024$).

have been measured by more than ten different experiments [2] (Fig. 1). They establish the existence of density inhomogeneities at the same level, $\delta \rho/\rho \sim \delta T/T \sim 10^{-5}$, on length scales $\lambda \sim 100 h^{-1}$ Mpc ($\theta/\text{deg}) \sim 30 h^{-1}$ Mpc $- 10^3 h^{-1}$ Mpc. Density perturbations of this amplitude, when amplified by the attractive action of gravity over the age of the Universe, are sufficient to explain the structure seen today.

- Primeval abundance pattern of D, $^3$He, $^4$He and $^7$Li. These light nuclei were produced a few seconds after the bang; the predicted abundance pattern is consistent that seen in primitive samples of the cosmos – provided that the present baryon density is between $1.5 \times 10^{-31}$ g cm$^{-3}$ and $4.5 \times 10^{-31}$ g cm$^{-3}$. This corresponds to a fraction of critical density $\Omega_B h^2 = 0.008 - 0.024$ [3] (Fig. 2). Nucleosynthesis is the earliest test of the hot big bang and provides the best determination of the density of ordinary matter.

The standard cosmology is successful in spite of our ignorance of the basic geometry of the Universe – age, size, and curvature – which hinge upon accurate measurements of the
Hubble constant and energy content of the Universe (fraction of critical density in matter, radiation, vacuum energy, and so on). The expansion age, which is related to $H_0^{-1}$ and the energy content of the Universe, is an important consistency check – it should be larger than the age of any object in the Universe. The curvature radius of the Universe is related to $H_0$ and $\Omega_0$: $R_{\text{curv}} = H_0^{-1} / \sqrt{|\Omega_0 - 1|}$.

Note, the deceleration parameter is related to energy content of the Universe, $q_0 = \frac{1}{2}(\Omega_0 + 3 \sum_i w_i \Omega_i)$, where $\Omega_0$ is the total energy density divided the critical energy density, $\Omega_i$ is the fraction of critical density in component $i$ and $w_i$ is the ratio of the pressure contributed by component $i$ to its energy density. For a universe filled with nonrelativistic matter, $q_0 = \frac{1}{2} \Omega_0$; for a universe with nonrelativistic matter + vacuum energy (cosmological constant, $w_\Lambda = -1$), $q_0 = \frac{1}{2} \Omega_0 - \frac{3}{2} \Omega_\Lambda$.

3 Aspirations

The hot big-bang model provides a firm physical basis for the expanding Universe, but it leaves important questions unanswered.

- **Quantity and composition of dark matter.** Most of the matter in the Universe is dark and of unknown composition [4]. The peculiar velocities of the Milky Way and other galaxies indicate that $\Omega_{\text{Matter}}$ is at least 0.3, perhaps as large as unity [5]. Luminous matter accounts for less mass density that the lower limit to the baryon density from nucleosynthesis ($\Omega_{\text{Lum}} \simeq 0.003h^{-1} < 0.008h^{-2} < \Omega_B$), and the upper limit to the baryon density from nucleosynthesis is less than 0.3 ($\Omega_B < 0.024h^{-2} < 0.3$). This defines the two dark-matter problems central to cosmology (Fig. 3). What is the nature of the dark baryons? What is the nature of the nonbaryonic dark matter?

- **Formation of large-scale structure.** Gravitational amplification of small primeval density inhomogeneities provides the basic framework for understanding structure formation, but important questions remain. What is the origin of these perturbations? What are the details of structure formation? The latter is clearly tied to the dark-matter question.

- **Origin of matter-antimatter asymmetry.** During the earliest moments ($t \lesssim 10^{-6}\text{ sec}$), when temperatures exceeded the rest-mass energy of nucleons, matter and antimatter existed in almost equal amounts (thermal pair production made nucleons and antinucleons as abundant as photons); today there is no antimatter and relatively little matter (one atom for every billion photons). For this to be so, there must have been a slight excess of matter over antimatter during the earliest moments: about one extra nucleon per billion nucleons and antinucleons, for a net baryon number per photon of about $10^{-9}$. What is the origin of this small baryon number?

- **Origin of smoothness and flatness.** Why in the large is the Universe so smooth (as evidenced by the CBR)? The generic cosmological solutions to Einstein’s equations
Figure 3: Determinations of the matter density. The lowest band is luminous matter, in the form of bright stars and associated material; the middle band is the big-bang nucleosynthesis determination of the density of baryons; the upper band is the estimate of $\Omega_{\text{Matter}}$ based upon the peculiar velocities of galaxies. The gaps between the bands illustrate the two dark matter problems: most of the ordinary matter is dark and most of the matter is nonbaryonic.

are not smooth; further, microphysical processes could not have smoothed things out because the distance a light signal can travel at early times covers only a small fraction of the Universe we can see. Why was the Universe so flat in the beginning? Had it not been exceedingly flat, it would have long ago recollapsed or gone into free expansion, resulting in a CBR temperature of much less than 3 K.

- The beginning. What launched the expansion? What is the origin of the entropy (i.e., CBR)? What was the big bang? Is there a before the big bang? Were there other bangs? Are there more spatial dimensions to be discovered?

This is an ambitious list. However, the study of the unification of the forces of Nature and the application of these ideas to cosmology has allowed these questions to be addressed, and many of us believe that answers will be found in the physics of the early Universe. Over the past fifteen years a number of important ideas have been put forth – baryogenesis,
topological defects (cosmic strings, monopoles, textures, and domain walls), particle dark
matter, baryogenesis, and inflation. I will focus on inflation – it is the most expansive,
addressing almost all the questions mentioned above – and is ripe for testing.

4 Inflation and Cold Dark Matter

Inflation \[^{7}\] holds that very early on (perhaps around \(10^{-34}\) sec) the Universe underwent
a burst of exponential expansion driven by the energy of a scalar field displaced from the
minimum of its potential-energy curve. (There are many candidates for the scalar field that
drives inflation; all involve new fields associated with physics beyond the standard model of
particle physics.) During this growth spurt, the Universe expanded by a larger factor than
it has since. When the scalar field evolved to the minimum of its potential, its energy was
released into a thermal bath of particles. This entropy is still with us today as the Cosmic
Background Radiation.

The tremendous growth in size during inflation explains the large-scale flatness and
smoothness of the Universe: After inflation, a very tiny patch of the pre-inflationary Uni-
verse, which would necessarily appear flat and smooth, becomes large enough to encompass
all that we see today and more. Since spatial curvature and \(\Omega_0\) are related, inflation predicts
a critical density Universe.\[^{1}\]

The most stunning prediction of inflation is the linking of large-scale structure in the
Universe to quantum fluctuations on microscopic scales \[^{9}\] (\(\ll 10^{-16}\) cm): The wavelengths
of quantum fluctuations in the scalar field that drives inflation are stretched to astrophys-
ical size by the expansion that occurs during inflation. The continual creation of quantum
fluctuations and expansion leads to fluctuations on all length scales; they develop into den-
sity perturbations when the vacuum energy is converted into radiation. The spectrum is
approximately scale invariant, that is, fluctuations in the gravitational potential that are
independent of length scale. The overall normalization of the spectrum is dependent upon
the shape of the scalar potential, and achieving fluctuations of the correct size to produce
the observed structure in the Universe places an important constraint on it.

An inflationary model must incorporate two other pieces of early-Universe physics: baryo-
genesis \[^{10}\] and particle dark matter \[^{11}\]. Since the massive entropy released at the end
of inflation exponentially dilutes any asymmetry that might have existed between matter
and antimatter, an explanation for the matter – antimatter asymmetry must be provided.
Baryogenesis is an attractive one. It holds that particle interactions that do not conserve
baryon-number and do not respect \(C\) and \(CP\) (matter-antimatter) symmetry occurred out-
of-thermal-equilibrium and gave rise to the small excess of matter over antimatter needed
to ensure the existence of matter today. Details of baryogenesis remain to be worked out
and tested – did baryogenesis occur at modest temperatures \(T \sim 200\) GeV and involve the
baryon-number violation that exists in the standard model or did it occur at much higher

\[^{1}\]Recently, it has been shown that inflation can accommodate \(\Omega_0 < 1\), but at the expense of tuning
precisely the amount of inflation \[^{8}\].
temperatures and involve grand unification physics.

Particle dark matter is necessary since inflation predicts that the Universe is at the critical density and baryons can contribute at most 10% of that. While the standard model of particle physics does not provide a particle dark matter candidate, many theories that attempt to unify the forces and particles predict the existence of new, long-lived particles whose abundance today is sufficient to provide the critical mass density. The three most promising candidates are: a neutrino of mass around \(30\,\text{eV}\); a neutralino of mass between \(10\,\text{GeV}\) and \(500\,\text{GeV}\) [12]; and an axion of mass between \(10^{-6}\,\text{eV}\) and \(10^{-4}\,\text{eV}\) [13].

Inflation addresses essentially all the previously mentioned questions, including the nature of the big bang itself. As Linde [14] has emphasized, if inflation occurred, it has occurred time and time again (eternally to use Linde’s words). What we refer to as the big bang is simply the beginning of our inflationary bubble, one of an infinite number that have been spawned and will continue to be spawned ad infinitum. From the inflationary view, there is no need for a beginning.

There is no standard model of inflation, but there are a set of robust predictions that allow inflation to be tested.

- Flat Universe. Total energy density is equal to the critical density, \(\Omega_0 \equiv \sum_i \Omega_i = 1\). Among the components \(i\) are baryons, slowly moving elementary particles (cold dark matter), radiation (a very minor component today, \(\Omega_{\text{rad}} \sim 10^{-4}\)), and possibly other particle relics or a cosmological constant.

- Approximately scale-invariant spectrum of density perturbations. More precisely, the Fourier components of the primeval density field are drawn from a gaussian distribution with variance given by power spectrum \(P(k) \equiv \langle |\delta_k|^2 \rangle = A k^n\) with \(n \approx 1\) (\(n = 1\) is exact scale invariance), where \(k = 2\pi/\lambda\) is wavenumber and the model-dependent constant \(A\) sets the overall level of inhomogeneity and is related to the form of the inflationary potential.

- Approximately scale-invariant spectrum of gravitational waves. Quantum fluctuations in the space-time metric give rise to relic gravitational waves. The overall amplitude of the spectrum depends upon the scalar potential in a different way than the density perturbations. These relic gravitational waves might be detected directly by laser interferometers that are being built (LIGO, VIRGO, and LISA) or by the CBR anisotropies they produce [15]. If the spectra of both the matter fluctuations and gravity waves can be determined, much could be learned about the inflationary potential [16].

The first two predictions lead to the cold dark matter theory of structure formation.\(^2\)Within the cold dark matter (CDM) theory, there are cosmological quantities that must

\(^2\)As a historical note the more conservative approach of neutrino (hot) dark matter was tried first and found to be wanting [17]: Since neutrinos are light and move very fast they stream out of overdense regions and into underdense regions, smoothing out density inhomogeneities on small scales. Structure forms from the top down: superclusters fragmenting into galaxies – which is inconsistent with observations that indicate that superclusters are just forming today and galaxies formed long ago.
be specified in order to make precise predictions \[20\]. They can be organized into two groups. First are the cosmological parameters: the Hubble constant; the density of ordinary matter; the power-law index $n$ and overall normalization constant $A$ that quantify the density perturbations; and the level of gravitational radiation.\footnote{The level of gravitational radiation is important because density perturbations are normalized by CBR anisotropy and at present it is difficult to separate the contribution of gravity waves to CBR anisotropy from that due to density perturbations \[18\].} (A given model of inflation predicts $A$ and $n$ as well as the level of gravitational radiation; however, there is no standard model of inflation. Conversely, measurements of the above quantities can constrain – and even be used to reconstruct – the scalar potential that drives inflation \[16\].)

The second group specifies the composition of invisible matter in the Universe: radiation, dark matter, and cosmological constant. Radiation refers to relativistic particles: the photons in the CBR, three massless neutrino species (assuming none of the neutrino species has a mass), and possibly other undetected relativistic particles. The level of radiation is crucial since it determines when the growth of structure begins and thereby the shape of the power spectrum of density perturbations today. While the bulk of the dark matter is CDM, there could be other particle relics; for example, a neutrino species of mass $5\, \text{eV}$, which would account for about 20\% of the critical density.

The testing of cold dark matter began more than a decade ago with a default set of parameters (“standard CDM”) characterized by simple choices for both the cosmological and the invisible matter parameters: precisely scale-invariant density perturbations ($n = 1$), $h = 0.5$, $\Omega_B = 0.05$, $\Omega_{\text{CDM}} = 0.95$; no radiation beyond photons and three massless neutrinos; no dark matter beyond CDM; no gravitational waves; and zero cosmological constant. The overall level of the matter inhomogeneity – set by the constant $A$ – was fixed by comparing the predicted level of inhomogeneity today with that seen in the distribution of bright galaxies. Bright galaxies may or may not faithfully trace the distribution of mass. In fact, there is some evidence that bright galaxies are more clustered than mass, by a factor called the bias, $b \simeq 1 - 2$. The distribution of galaxies today only fixes $A$ up to the bias factor $b$.

An important change occurred with the detection of CBR anisotropy by COBE in 1992 \[21\]. The COBE measurement permitted a precise determination of the amplitude of density perturbations on very large scales, without regard to biasing. (The CBR anisotropy detected by COBE arises mainly from density fluctuations on scales of around $10^3 h^{-1} \text{Mpc}$.) There was a surprise: For standard CDM, the COBE normalization predicts too much power on the scales of clusters and smaller \[19\].

Figure 4 illustrates clearly that this problem simply reflects a poor choice for the standard parameters. It shows that there are many COBE-normalized CDM models that are consistent with measurements of the large-scale structure that exists today (shape of the power spectrum of the galaxy distribution, abundance of clusters, and early formation of structure in the form of damped Lyman-\(\alpha \) clouds; see Ref. \[20\]). Organized into families characterized by their invisible matter content they are: CDM + cosmological constant ($\Lambda$CDM) \[22\], CDM + a small amount of hot dark matter ($\nu$CDM) \[23\], CDM + additional relativistic particles ($\tau$CDM) \[24\], and CDM with standard invisible matter content \[25, 26\].
Figure 4: Acceptable values of the cosmological parameters $n$ and $h$ for CDM models with standard invisible-matter content (CDM), with 20% hot dark matter ($\nu$CDM), with additional relativistic particles (the energy equivalent of 12 massless neutrino species, denoted $\tau$CDM), and with a cosmological constant that accounts for 60% of the critical density ($\Lambda$CDM). The $\tau$CDM models have been truncated at a Hubble constant of 65 km s$^{-1}$ Mpc$^{-1}$ because a larger value would result in a Universe that is younger than 10 Gyr (from Ref. [20]).

There are two additional pieces of data that have significant leverage on CDM models: The Hubble constant/age of the Universe and the cluster baryon fraction. Determinations of the Hubble constant based upon a variety of techniques (Type Ia and II supernovae, IR Tully-Fisher and fundamental plane methods) have converged on a value between 60 km s$^{-1}$ Mpc$^{-1}$ and 80 km s$^{-1}$ Mpc$^{-1}$ [27]. This corresponds to an expansion age of less than 11 Gyr for a flat, matter-dominated model; for $\Lambda$CDM, the expansion age can be significantly higher, as large as 16 Gyr for $\Omega_\Lambda = 0.6$ (Fig. 3). On the other hand, the ages of the oldest globular clusters indicate that the Universe is between 13 Gyr and 17 Gyr old; further, these age determinations, together with the those for the oldest white dwarfs and the long-lived radioactive elements, provide an ironclad case for a Universe that is at least 10 Gyr old [25]. Unless the age of the Universe and the Hubble constant are near the lowest values consistent with current measurements, only $\Lambda$CDM model is viable.

Clusters are large enough that the baryon fraction should reflect its universal value, $\Omega_B/\Omega_{\text{Matter}} = (0.008 - 0.024)h^{-2}/(1 - \Omega_\Lambda)$. Most of the (observed) baryons in clusters are in the hot, intracluster x-ray emitting gas. From x-ray measurements of the flux and
The cross-hatched region is ruled out because $\Omega_{\text{Matter}} < 0.3$. The broken lines indicate the favored range for $H_0$ and for the age of the Universe.

At the moment, the observations point to $\Lambda$CDM as the best fit CDM model \[^{31}\] (Fig. 6). The existence of a cosmological constant raises a fundamental issue – the origin of the implied vacuum energy density. However, one should bear in mind that the case for $\Lambda$CDM hinges upon the Hubble constant and cluster baryon fraction, and neither measurement is completely settled.
Figure 6: Summary of constraints projected onto the $H_0 - \Omega_{\text{Matter}}$ plane: (CBF) comes from combining the BBN limit to the baryon density with x-ray observations of clusters; (PS) arises from the power spectrum; (AGE) is based on age determinations of the Universe; ($H_0$) indicates the range currently favored for the Hubble constant. (Note the constraint $\Omega_\Lambda < 0.7$ has been implicitly taken into account since the $\Omega_\Lambda$ axis extends only to 0.7.) The darkest region indicates the parameters allowed by all constraints (from [32].)

5 Concluding Remarks

At the moment, $\Lambda$CDM best accommodates all the observations, but I believe the evidence is not yet strong enough to abandon the other CDM models. Especially because additional observations will soon be able to decisively distinguish between the different models as well as testing inflation. They include:

- Deceleration parameter. $\Lambda$CDM predicts $q_0 \equiv \frac{1}{2} - \frac{3}{2} \Omega_\Lambda \sim -\frac{1}{2}$, while the other CDM models predict $q_0 = \frac{1}{2}$. Two groups (The Supernova Cosmology Project and The High-z Supernova Team) are hoping to determine $q_0$ to a precision of $\pm 0.2$ by using distant Type Ia supernovae ($z \sim 0.3 - 0.7$) as standard candles. Together, they discovered more than 40 high redshift supernovae last fall and winter and both groups should be announcing results soon.

- Hubble constant. Since the Universe is at least 10 Gyr old, a determination that the Hubble constant is $65 \text{ km s}^{-1} \text{ Mpc}^{-1}$ or greater would rule out all models but $\Lambda$CDM; a
determination that the Hubble constant is greater than $60 \text{ km s}^{-1} \text{ Mpc}^{-1}$ would require nonminimal invisible matter content (e.g., some hot dark matter or extra radiation); a value less than $60 \text{ km s}^{-1} \text{ Mpc}^{-1}$ would make the simplest CDM models viable.

- Cluster baryon fraction. This strongly favors ΛCDM. Further evidence that x-ray measurements have correctly determined the total cluster mass (e.g., from weak gravitational lensing) and baryon mass (e.g., from AXAF) would strengthen the case for ΛCDM. On the other hand, discovery of a systematic effect that lowers the cluster baryon fraction by a factor of two (e.g., underestimation of cluster mass because gas is not supported by thermal pressure alone, or overestimation of cluster gas mass because the gas is clumped) would undermine the case for ΛCDM.

- Evolution of Structure. The study of the Universe at high redshift by HST and Keck will test CDM and distinguish between models. For example, ΛCDM predicts earlier structure formation, while νCDM predicts later structure formation.

- Redshift surveys. Two large redshift surveys are coming on line. The Sloan Digital Sky Survey will gather a million redshifts over a quarter of the sky; the Two-degree Field will gather 250,000 redshifts in hundreds of fields that are two degrees across. Both should be able to discriminate between the different CDM models by better measuring the power spectrum of inhomogeneity and other related quantities.

- Determination of the primeval deuterium abundance. A definitive measurement of the deuterium abundance in a high-redshift hydrogen cloud can be used to determine the primeval deuterium abundance and thereby the baryon density ($\Omega_B h^2$) to a precision of 10% or so. Such a measurement would pin down this important cosmological parameter, sharpen the cluster baryon fraction test, and, when the baryon density is determined from CBR anisotropy, provide a consistency test of the standard cosmology. Results – though not a consensus – for the deuterium abundance in high-redshift hydrogen clouds ($z \sim 2.5 - 4.6$) have been reported [33]; further observations with the Keck and the HST should clarify matters and lead to a 10% determination of the primeval deuterium abundance.

- Laboratory search for particle dark matter. An experiment with sufficient sensitivity to detect axions in the halo of the Milky Way is now taking data [34]; several experiments that can detect neutralinos will start operating soon. In addition, a host of experiments to search for evidence of neutrino mass are underway (e.g., at Los Alamos, CERN, Fermilab, Kamiokande and other laboratories).

- CBR anisotropy in the MAP/COBRAS/SAMBA era. Last, but certainly not least, the high-resolution CBR maps that will be made by these two satellite-borne experiments as well as ground and balloon based experiments will test both inflation and CDM decisively. By measuring the multipole amplitudes out to $l \sim 3000$ they will be able to simultaneously determine $h$, $\Omega_0$, $\Omega_A$, $\Omega_B h^2$, $\Omega_\nu$, $n$, and $T/S$ to good precision (better than 10%) (Fig. 7) [35].
This is an exciting time in cosmology. We have a successful standard model, in inflation a bold and expansive paradigm for extending it, and a flood of observations to test paradigm. Soon we will know if inflation and cold dark matter are to become part of the standard cosmology, and whether or not we are living in the golden age of cosmology.

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