ABSTRACT

The exponential model with pairing attenuation (EMPA) is extended and used to analysis the superdeformed rotational bands (SDRB’s) in Thallium nuclei in the mass region $A \sim 190$. The level spins are extracted by fitting the experimental dynamical moment of inertia $J^{(2)}$ to the theoretical version of Harris expansion in even powers of rotational frequency $\hbar \omega$. Using the assigned spins, the parameters of the extended exponential model are adjusted by using a computer simulated search program to fit the transition energies with experimental ones. The best adopted parameters are used to calculate transition energies $E_\gamma(I)$ rotational frequencies $\hbar \omega$, kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia. The variation of $J^{(1)}$ and $J^{(2)}$ versus $\hbar \omega$ are examined. A staggering function depends on the concept of EGOS and finite differences are suggested to investigate and exhibit the $\Delta I=1$ staggering effect in transition energies in the studied SDRB’s. The phenomenon of identical bands is also investigated in Thallium nuclei.

Keywords: Pairing Attenuation, Staggering, Thallium nuclei

1. INTRODUCTION

Since the discovery of the first superdeformed (SD) bands in $^{152}$Dy[1] and $^{199}$Hg[2], a great number of superdeformed rotational bands (SDRB’s) were identified in different mass regions $A \sim 190,150,130,80,60$[3] which have their own characteristic features. These high spin SDRB’s are associated with extremely large quadrupole deformation parameter. The difference between the SDRB’s in various mass regions are manifested through the behavior of the dynamical moment of inertia. The $A \sim 190$ mass region is of special interest, since in this region the SD bands were observed down to quite low spin, Most SDRB’S in $A \sim 190$ region display the same smooth rise in dynamical moment of inertia with increasing rotational frequency [4]. This rise behavior was attributed in the cranked shell model calculations mainly to the successive alignment of both $J^{15/2}$ quasi neutron and $J^{13/2}$quasi proton pairs from high N intruder orbital under rotation in the presence of pairing correlations [5,6]. For all SDRB’S the transition energies are the only quantities to be detected, there are no direct experimental method to determine spins, parities and excitation energies ,this is due to the difficulty of establishing the de excitation of SD band into ground states .Several theoretical procedures for assigning spins were proposed[7,13]. One of the unexpected feature happen in SDRB’s is the existence of identical bands (IB’s) [14,15],that is states in different SD bands having nearly the same $\gamma$ – ray transition energies .This indicate that the rotational frequencies as well as the dynamical moment of inertia are very similar. Many authors tried to understand the IB’s phenomenon [16-19]. The interesting discovery of IB’s was seen also in pairs of normal deformed nuclei [20-22].

Another interesting feature related to the SDRB’s is the measurement of magnetic properties. The branching ratio of M1 transition linking two signature partners SD bands were observed [23-24] and is denoted as $\Delta I=1$
staggering. The crosstalk was originally observed in one direction, then it was established that the crosstalk goes both ways [25]. The \( \Delta I=1 \) staggering effect is also investigated in normal deformed nuclei between the energies of states of the ground and octupole bands [26]. Some SDRB’s in mass region \( A \sim 190 \) show an unexpected \( \Delta I=2 \) staggering effects in the transition energies [27,28]. This is commonly called \( \Delta I=4 \) bifurcation because the states of the band splits into two sequences of transition energies differing by \( 2\hbar \) of angular momentum and the states in each sequence differ by \( 4\hbar \) of angular momentum. The spin values in one sequence are \( I, I+4, I+8, \ldots \) while in the other sequence are \( I+2, I+6, I+10, \ldots \) Several theoretical attempts were done to explain the \( \Delta I=2 \) energy staggering phenomenon [29-34].

2. OUTLINE OF EXPONENTIAL MODEL WITH PAIRING ATTENUATION

The pairing energy \( \Delta \) depends on the angular momentum \( I \) by the relation [35]

\[
\Delta (I) = \Delta_0 \left( 1 - \frac{I}{I_C} \right)^{\frac{1}{2}}
\]

(1)

where \( \Delta_0 \) is the energy pairing gap and \( I_C \) is the critical spin value at which pairing correlations disappear completely (\( I_C = I \)). That is the pairing gap vanishes. Using the cranked model, it was found [36] that the nuclear moment of inertia \( J \) depends exponentially on pairing correlation parameter \( \Delta \)

\[
J(\Delta) = J_0 \exp (-\Delta)
\]

(2)

where the parameter \( J_0 \) can be determined from the (ln \( J \)) versus \( \Delta \) plot. Inserting the moment of inertia \( J(\Delta) \) in the \( I(I+1) \) rule of excitation energy

\[
E(I) = \frac{\hbar^2}{2J} I(I+1)
\]

(3)

yield the exponential model with pairing attenuation [37]

\[
E(I) = A I(I+1) \exp [\Delta_0 \left( 1 - \frac{I}{I_C} \right)^{\frac{1}{2}}]
\]

(4)

with \( A \) for SD bands. The \( \gamma \) – ray transition energy \( E_\gamma \) are the only quantity experimental measured, therefore, we can write \( E_\gamma \) between two levels differentiates by two units of angular momenta as

\[
E_\gamma(I) = E(I) - E(I-2)
\]

\[
= A \left\{ I(I+1) \exp [\Delta_0 (1-I/I_C)^{1/2}] - (I-1)(I-2) \exp [\Delta_0(1-(I-2)/I_C)^{1/2}] \right\}
\]

(5)

Equation (5) can be used to fit the transition energies of SDRB’s for fixed \( I_C \) and keeping \( A \) and \( \Delta_0 \) as a free parameter.

3. SPIN ASSIGNMENT FOR SDRB’s

The determination of spins and parity of states in all SD bands is one of most difficult and unsolved problems in the study of superdeformation. This is due to the difficulty of establishing the de excitation of a SD band into known yrast state. Several theoretical approaches [3] were attempted to assign the spins of the SD bands. Most of procedures begin with the Harris model [39]. The two parameter Harris formula for excitation energy it given by

\[
E(\omega) = \frac{1}{2} \alpha \omega^2 + \frac{3}{2} \beta \omega^4
\]

(6)

The dynamic moment of inertia \( J^{(2)} \) for the Harris formula is

\[
J^{(2)} = \frac{1}{\omega} \left( \frac{dE}{d\omega} \right)
\]

(7)

\[
J^{(2)} = \alpha + 3\beta \omega^2
\]

(8)

where \( \alpha \) corresponds to the bandhead moment of inertia. Integrating equation (8) yield the level spin \( I \)

\[
\hbar I = \int J^{(2)} d\omega = \alpha \omega + 3\beta \omega^3 + i
\]

(9)

where the intrinsic alignment \( i \) appears as a constant of integration and \( I = \sqrt{I(I+1)} \) is the intermediate nuclear spin.

The parameters \( \alpha \) and \( \beta \) are obtained from the fitting procedure of \( J^{(2)} \) with experimental data.

The kinematic moment of inertia \( J^{(1)} \) for Harris formula is
\[ (J^{(2)}/\hbar^2) = h(1/\omega) = \alpha + \beta \omega^2 \]  
(10)

Eliminating \( \omega \) from the two equations (9) and (10) we get a cubic equation for the energy of the rotational band levels.

\[ e^3 + 2e^2 + (1 + 36d)e - 4(1 + 27d) = 0 \]  
(11)

where \( e = (4\beta/\alpha^2) \), \( d = (\beta/2\alpha^3) I(I+1) \)

Equation (11) give the excitation energy \( E \) as a function of spin. Putting \( \gamma = 2d \) we have

\[ E(I) = (h^2/2\alpha)(I(I+1) \left[ 1 - \gamma + 4\gamma^2 - 24\gamma^3 + \ldots \right] \]  
(12)

The rotational frequency \( h\omega \) , the dynamic \( J^{(2)} \) and kinematic \( J^{(1)} \) moments of inertia are related to the transition energy \( E(I) \) by the relations

\[ h\omega = (1/4)[E(I+2→I) + E(I→I-2)] \quad (\text{MeV}) \]  
(13)

\[ J^{(2)} = 4[I E(I+2→I) - E(I→I-2)] / (h^2 \text{ MeV}^{-1}) \]  
(14)

\[ J^{(1)} = (2I-1) / E(I→I-2) \quad (h^2 \text{ MeV}^{-1}) \]  
(15)

It is seen that the dynamic moment of inertia \( J^{(2)} \) can be extracted from the energy difference between two consecutive transition energies in the band ,it does not depend on the spin while the kinematic moment of inertia \( J^{(1)} \) depend on the spin proposition . The two moments of inertia \( J^{(2)} \), \( J^{(1)} \) are related as follows.

\[ J^{(2)}/\hbar^2 = (1/\hbar)(dI/d\omega) = (1/\hbar^2)[J^{(1)}+(dJ^{(1)}/d\omega)] \]  
(16)

3. Analysis of energy staggering in SDRB’S

The \( \Delta I=1 \) staggering phenomenon, several pairs of signature partners SDRB’s exhibit a \( \Delta I=1 \) energy staggering [17]. these signature partners show signature splitting with large amplitude.

To exhibit the \( \Delta I=1 \) energy staggering, we consider EGOS staggering function. It represents the gamma transition energy over the spin

\[ \text{EGOS}(I) = E_\gamma(I) / 2I \]  
(17)

with \( E_\gamma(I) = E_\gamma(I ) - E_\gamma(I - 2) \)

when EGOS is plotted against \( 2I \) for pure rotator E (I) = A I (I+1) it represents a straight line parallel to the spin axis.

4. NUMERICAL RESULTS AND DISCUSSION

Our selected data set includes twelve super deformed rotational bands (SDRB’s) for Thallium nuclei The \( \gamma \)-ray transition energies in a band is assumed as the difference in energies between two levels separated by two units of angular momentum .The experimental transition energies of our SDRB’s are taken from Ref [3].The level spins of each band are determined by fitting the dynamic moments of inertia for each rotational frequency \( h\omega \) extracted from experimental transition energies with the Harris two parameter \( \alpha \) and \( \beta \) equation (10) in order to obtain a minimum root mean square deviation by the common definition of the \( \chi(J^{(2)}) \)

\[ \chi(J^{(2)}) = \frac{1}{N} \sum_{i=1}^{N} \frac{(E_{\gamma}^{exp}(oi) - E_{\gamma}^{cal}(oi))^2}{\delta E_{\gamma}^{exp}(oi)} \]  

where \( N \) is the total number of the experimental points entering into the fitting procedure and \( \delta J^{(2)exp} (oi) \) is the experimental errors in \( J^{(2)} \).

A computer simulated search program is used in fitting procedure. The calculated assigned bandhead spins and the best adopted Harris parameters \( \alpha \) and \( \beta \) are listed in Table (1).

After knowing the level spins for each band, another fitting procedure has been performed to adjust the two parameters \( \Delta \) and \( A \) of the exponential model with pairing attenuation by fitting the experimental transition energies with the corresponding theoretical one to minimize \( \chi(E_\gamma) \)

\[ \chi(E_\gamma) = \frac{1}{N} \sum_{i=1}^{N} \frac{(E_\gamma^{exp}(oi) - E_\gamma^{cal}(oi))^2}{\delta E_\gamma^{exp}(oi)} \]  

where \( \delta E_\gamma^{exp} (oi) \) is the experimental error in transition energy \( E_\gamma (oi) \).

Table (1) show the adopted best model parameters \( \Delta \), \( A \) and the rms deviation \( \chi (E_\gamma) \). The experimental lowest transition energy \( E_\gamma \) (I_0 +2→I_0). For each (SD) band is also indicated in Table (1). The calculated transition energy \( E_\gamma^{cal} (I) \) are compared to experimental...
ones $E^{\text{exp}}(\mathcal{I})$, and the results are shown in Table (2). We see that a very good agreement between calculation and experiment is obtained, which give good support to the proposed exponential model with pairing attenuation.

The moments of inertia $J^{(1)}$ and $J^{(2)}$ for the three pairs of identical bands (IB's) in $^{193,194,195}\text{Tl}$ are shown in Figure (1), they display similarities in the frequency range $0.10 \text{ MeV} < \hbar \omega < 0.35 \text{ MeV}$ for the pair $[^{193}\text{Tl} \,(SD1),^{194}\text{Tl} \,(SD3)]$, and $0.10 \text{ MeV} < \hbar \omega < 0.40 \text{ MeV}$ for the two pairs $[^{193}\text{Tl} \,(SD1),^{195}\text{Tl} \,(SD1),^{193}\text{Tl} \,(SD2),^{195}\text{Tl} \,(SD2)]$.

The bandhead moments of inertia which depends on the intrinsic structure of the rotational band for these three pairs of IB's are nearly identical.

From the analysis of $J^{(1)}$ and $J^{(2)}$ moments of inertia we observed that the lowest values of $J^{(2)}$ for the yrast superdeformed bands in Tl isotopes exhibit a zigzag moments of inertia staggering. Table (3) and Figure (2) shows the calculated and the experimental lowest values of $J^{(2)}$ versus the neutron number of the yrast SD bands in $^{191-195}\text{Tl}$ nuclei.

The main results of present paper are the observation of a $\Delta l=1$ energy staggering effect in five pairs of signature partners in odd A and odd-odd $^{192,194}\text{Tl}$ nuclei namely:

$^{191}\text{Tl} \,(SD1, SD2),^{192}\text{Tl} \,(SD1, SD2),^{193}\text{Tl} \,(SD1, SD2),^{194}\text{Tl} \,(SD1, SD2)$, and $^{195}\text{Tl} \,(SD1, SD2)$. We investigated this Effect by calculating the staggering function EGOS equation (17). The results are plotted versus nuclear spin $I$ in Figure (3). Zigzag pattern with large amplitude is shown. The numerical values are listed in Table (4).

### Table (1) The estimated bandhead spin $I_0$ and the adopted best model parameters $\alpha$, $\beta$, $\Delta$, $A$ obtained from the fitting produce for studied SDRB'S in Tl the nuclei. The experimental lowest transition energy $E_\gamma (I_0+2\rightarrow I_0)$ for each (SD) band is also given. The rms deviation $\chi$ is indicated.

| SD band      | $\beta \text{(h}^2\text{MeV}^{-1})$ | $\Delta(10^{-1})$ | $A(\text{Kev})$ | $\chi$ | $E_\gamma (^\text{exp})$ (Kev) |
|--------------|-----------------------------------|-------------------|-----------------|-------|-------------------------------|
| $^{191}\text{Tl} \,(SD1)$ | 92.7007                           | 3.3009            | 3.9958          | 0.754 | 276.5                         |
| $^{191}\text{Tl} \,(SD2)$ | 92.1013                           | 5.2868            | 3.5564          | 0.338 | 296.3                         |
| $^{192}\text{Tl} \,(SD1)$ | 102.7981                          | 3.77642           | 3.9097          | 1.100 | 283.0                         |
| $^{192}\text{Tl} \,(SD2)$ | 103.5797                          | 3.12090           | 4.1003          | 1.495 | 337.5                         |
| $^{193}\text{Tl} \,(SD1)$ | 96.4098                           | 3.17970           | 3.9123          | 0.693 | 206.6                         |
| $^{193}\text{Tl} \,(SD2)$ | 96.4333                           | 2.6883            | 4.0853          | 0.491 | 227.3                         |
| $^{194}\text{Tl} \,(SD1)$ | 96.9760                           | 3.9958            | 3.9097          | 1.100 | 283.0                         |
| $^{194}\text{Tl} \,(SD2)$ | 99.9000                           | 2.15538           | 4.1178          | 0.510 | 209.3                         |
| $^{194}\text{Tl} \,(SD3)$ | 90.7109                           | 2.88292           | 4.0276          | 0.533 | 240.5                         |
| $^{194}\text{Tl} \,(SD5)$ | 100.4390                          | 1.91747           | 4.1322          | 0.454 | 187.9                         |
| $^{195}\text{Tl} \,(SD1)$ | 95.5000                           | 2.8735            | 4.0470          | 0.360 | 146.2                         |
| $^{195}\text{Tl} \,(SD2)$ | 95.0010                           | 6.5               | 3.8300          | 1.414 | 167.5                         |
Table (2). The calculated transition energy $E_\gamma (I)$ for our calculated SDRB’s and comparison with experimental data. The model parameters and the band head spins are listed in Table (1)

| $^{194}$TI (SD1) | $^{195}$TI (SD2) | $^{192}$TI (SD1) | $^{192}$TI (SD2) |
|------------------|------------------|------------------|------------------|
| $I(h\ell)$ | $E_\gamma (I)$ (KeV) | $I(h\ell)$ | $E_\gamma (I)$ (KeV) | $I(h\ell)$ | $E_\gamma (I)$ (KeV) | $I(h\ell)$ | $E_\gamma (I)$ (KeV) |
| EXP | CAL | EXP | CAL | EXP | CAL | EXP | CAL |
| 13.5 | 276.5 | 278.687 | 14.5 | 296.3 | 294.725 | 13 | 283 | 282.121 | 16 | 337.5 | 332.139 |
| 15.5 | 317.7 | 319.431 | 16.5 | 337.2 | 336.487 | 15 | 320.8 | 320.898 | 18 | 374.9 | 372.608 |
| 17.5 | 359 | 359.574 | 18.5 | 377.8 | 377.249 | 17 | 359 | 359.928 | 20 | 413.4 | 412.489 |
| 19.5 | 398.8 | 399.100 | 20.5 | 416.9 | 416.995 | 19 | 397.8 | 397.599 | 22 | 451.1 | 451.767 |
| 21.5 | 438.3 | 437.993 | 22.5 | 455.7 | 455.704 | 21 | 437.1 | 437.433 | 24 | 489.6 | 490.427 |
| 23.5 | 476.8 | 476.238 | 24.5 | 492.8 | 493.354 | 23 | 476.1 | 476.253 | 26 | 527.4 | 528.450 |
| 25.5 | 514.5 | 513.815 | 26.5 | 529.6 | 529.921 | 25 | 515.2 | 515.521 | 28 | 565.5 | 565.816 |
| 27.5 | 551.4 | 550.706 | 28.5 | 566.1 | 565.380 | 27 | 554.4 | 553.840 | 30 | 603.1 | 602.505 |
| 29.5 | 587.5 | 586.888 | 30.5 | 600.1 | 599.701 | 29 | 593 | 592.606 | 32 | 640.9 | 638.491 |
| 31.5 | 621.8 | 622.336 | 32.5 | 633.4 | 632.852 | 31 | 632 | 631.352 | 34 | 677.6 | 673.546 |
| 33.5 | 656.3 | 657.026 | 34.5 | 665.9 | 664.75 | 33 | 670 | 671 | 35 | 715.3 | 708.983 |
| 35.5 | 689.8 | 691.19 | 36.5 | 697.5 | 695.43 | 35 | 707 | 706 |

| $^{190}$TI(SD1) | $^{190}$TI(SD2) | $^{194}$TI(SD1) | $^{194}$TI(SD2) |
|------------------|------------------|------------------|------------------|
| $I(h\ell)$ | $E_\gamma (I)$ (KeV) | $I(h\ell)$ | $E_\gamma (I)$ (KeV) | $I(h\ell)$ | $E_\gamma (I)$ (KeV) | $I(h\ell)$ | $E_\gamma (I)$ (KeV) |
| EXP | CAL | EXP | CAL | EXP | CAL | EXP | CAL |
| 10.5 | 206.6 | 209.066 | 11.5 | 227.3 | 229.038 | 14 | 268 | 269.062 | 11 | 209.3 | 210.251 |
| 12.5 | 247.3 | 249.341 | 13.5 | 267.9 | 269.263 | 16 | 307 | 307.569 | 13 | 248.4 | 249.253 |
| 14.5 | 287.7 | 289.077 | 15.5 | 308.2 | 309.028 | 18 | 345.1 | 345.701 | 15 | 287.5 | 287.901 |
| 16.5 | 327.4 | 328.262 | 17.5 | 348 | 348.321 | 20 | 384.2 | 383.448 | 17 | 326 | 326.187 |
| 18.5 | 366.4 | 366.888 | 19.5 | 387 | 387.129 | 22 | 421 | 420.799 | 19 | 364.4 | 364.1 |
| 20.5 | 405.3 | 404.931 | 21.5 | 425.4 | 425.441 | 24 | 457 | 457.743 | 21 | 401.7 | 401.63 |
| 22.5 | 442.9 | 442.385 | 23.5 | 463.7 | 463.242 | 26 | 494.9 | 494.266 | 23 | 439.3 | 438.766 |
| 24.5 | 479.7 | 479.231 | 25.5 | 501.1 | 500.518 | 28 | 530.9 | 530.355 | 25 | 475.9 | 475.496 |
| 26.5 | 516.1 | 515.450 | 27.5 | 537.5 | 537.250 | 30 | 567 | 565.993 | 27 | 512 | 511.807 |
| 28.5 | 551.6 | 551.023 | 29.5 | 573.4 | 573.421 | 32 | 601.2 | 601.849 | 29 | 548 | 547.684 |
| 30.5 | 586.5 | 585.930 | 31.5 | 608.8 | 609.010 | 34 | 634.9 | 635.849 | 31 | 583.5 | 583.110 |
| 32.5 | 620.3 | 620.145 | 33.5 | 643.8 | 643.997 | 36 | 669.8 | 670.675 | 33 | 617.5 | 618.070 |

Table (3). The lowest dynamic moment of inertia $J^{(2)}$ for the yrast SDRB’s in $^{191-195}$TI [N is the neutron number]

| SD band | $N$ | $J^{(2)}_{\text{exp}} (\hbar^2 \text{MeV}^{-1})$ | $J^{(2)}_{\text{eat}} (\hbar^2 \text{MeV}^{-1})$ |
|---------|-----|---------------------------------|---------------------------------|
| $^{191}$TI(SD1) | 110 | 97.083 | 98.1739 |
| $^{192}$TI(SD1) | 111 | 105.8201 | 103.1539 |
| $^{193}$TI(SD1) | 112 | 98.2800 | 99.3171 |
| $^{194}$TI(SD1) | 113 | 102.5641 | 103.8732 |
| $^{195}$TI(SD1) | 114 | 95.2380 | 96.6673 |
Table (4). The calculated $\Delta I=1$ staggering function $\text{EGOS}$ ($I$) versus nuclear spin $I$ for the signature partners $^{191}\text{Tl}$ (SD1, SD2), $^{192}\text{Tl}$ (SD1, SD2), $^{193}\text{Tl}$ (SD1, SD2), $^{194}\text{Tl}$ (SD1, SD2), $^{195}\text{Tl}$ (SD1, SD2)

| $^{191}\text{Tl}$ (SD1, SD2) | $^{192}\text{Tl}$ (SD1, SD2) | $^{193}\text{Tl}$ (SD1, SD2) | $^{194}\text{Tl}$ (SD1, SD2) | $^{195}\text{Tl}$ (SD1, SD2) |
|---|---|---|---|---|
| $I(h)$ | $\text{EGOS}$ | $I(h)$ | $\text{EGOS}$ | $I(h)$ | $\text{EGOS}$ | $I(h)$ | $\text{EGOS}$ |
| 14.5 | 1.060 | 16.0 | 0.7029 | 11.5 | 1.7366 | 14.0 | 1.4149 | 8.5 | 2.5223 |
| 15.5 | 1.5939 | 17.0 | 1.6346 | 12.5 | 1.6242 | 15.0 | 1.2557 | 9.5 | 2.0988 |
| 16.5 | 1.0336 | 18.0 | 0.7044 | 13.5 | 1.4757 | 16.0 | 1.2292 | 10.5 | 2.0111 |
| 17.5 | 1.3192 | 19.0 | 1.3679 | 14.5 | 1.3664 | 17.0 | 1.0951 | 11.5 | 1.7207 |
| 18.5 | 0.9554 | 20.0 | 0.6945 | 15.5 | 1.2871 | 18.0 | 1.0841 | 12.5 | 1.5714 |
| 19.5 | 1.1205 | 21.0 | 1.1878 | 16.5 | 1.1656 | 19.0 | 0.9683 | 13.5 | 1.5362 |
| 20.5 | 0.8729 | 22.0 | 0.6515 | 17.5 | 1.1462 | 20.0 | 0.9674 | 14.5 | 1.3884 |
| 21.5 | 0.9766 | 23.0 | 1.0037 | 18.5 | 1.0034 | 21.0 | 0.8658 | 15.5 | 1.2769 |
| 22.5 | 0.7871 | 24.0 | 0.4972 | 19.5 | 1.0381 | 22.0 | 0.8713 | 16.5 | 1.1798 |
| 23.5 | 0.8737 | 25.0 | 0.9403 | 20.5 | 0.8683 | 23.0 | 0.7811 | 17.5 | 1.14 |
| 24.5 | 0.6996 | 26.0 | 0.4277 | 21.5 | 0.9539 | 24.0 | 0.7907 | 18.5 | 1.01 |
| 25.5 | 0.8823 | 27.0 | 0.9237 | 22.5 | 0.753 | 25.0 | 0.7101 | 19.5 | 1.0365 |
| 26.5 | 0.6077 | 28.0 | 0.3299 | 23.5 | 0.8875 | 26.0 | 0.7219 | 20.5 | 0.8646 |
| 27.5 | 0.7558 | 29.0 | 0.9305 | 24.5 | 0.6526 | 27.0 | 0.6496 | 21.5 | 0.9572 |
| 28.5 | 0.5148 | 30.0 | 0.223 | 25.5 | 0.8347 | 28.0 | 0.6624 | 22.5 | 0.7648 |
| 29.5 | 0.729 | 31.0 | 0.9571 | 26.5 | 0.5634 | 29.0 | 0.5975 | 23.5 | 0.896 |
| 30.5 | 0.42 | 32.0 | 0.108 | 27.5 | 0.7927 | 30.0 | 0.6103 | 24.5 | 0.6371 |
| 31.5 | 0.7185 | 33.0 | 1.0008 | 28.5 | 0.4832 | 31.0 | 0.5521 | 25.5 | 0.8486 |
| 32.5 | 0.3235 | 34.0 | -0.0146 | 29.5 | 0.7592 | 32.0 | 0.5641 | 26.5 | 0.5489 |
| 33.5 | 0.7216 | 35.0 | 0.7029 | 30.5 | 0.4101 | 33.0 | 0.5123 | 27.5 | 0.8121 |
| 34.5 | 0.2253 | 36.0 | 1.6346 | 31.5 | 0.7326 | 34.0 | 0.5224 | 28.5 | 0.4535 |
| 35.5 | 0.736 | 37.0 | | 32.5 | 0.3426 | 35.0 | 0.4769 | 29.5 | 0.7844 |
| 36.5 | 0.1253 | 38.0 | | 33.5 | 0.712 | 36.0 | 0.4857 | 30.5 | 0.3732 |
| 37.5 | 0.7601 | 39.0 | | 34.5 | 0.2796 | 37.0 | 0.4453 | 31.5 | 0.7638 |
| 38.5 | 0.0236 | 40.0 | | 35.5 | 0.6961 | 38.0 | 0.4518 | 32.5 | 0.2984 |
| | | | 36.5 | 0.2202 | 33.5 | 0.7492 |
| | | | 37.5 | 0.6862 | 34.5 | 0.2282 |
| | | | 38.5 | 0.162 | 35.5 | 0.7396 |
| | | | 39.5 | 0.678 | 36.5 | 0.1615 |
| | | | 40.5 | 0.108 | 37.5 | 0.7341 |
| | | | 41.5 | 0.6729 | 38.5 | 0.0977 |
| | | | 39.5 | 0.7323 | 40.5 | 0.0362 |
| | | | 41.5 | 0.7338 | | 42.5 | -0.0233 |
Figure (1). The kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moment of inertia verses rotational frequency $\omega$ for the three pairs of identical bands in $^{193,194,195}$Tl.

Figure (2). The lowest dynamic moments of inertia $J^{(2)}$ for the yrst SDRB’S in $^{191-195}$Tl.
5. CONCLUSION

SDRB’s in Tl isotopes with mass number between 191 and 195 have been studied in the framework of exponential model with pairing attenuation. The existence of identical bands and EGOS(I) staggering behaviors are discussed. All the studied SD bands exhibit significant amounts of staggering. We have succeeded in making very good fit \( \gamma \)-ray transition energies \( E_{\gamma} \) by using a simulated search program.

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Figure (3). The \( \Delta I=1 \) staggering parameter EGOS(I) in (KeV) as a function of nuclear spin I for the Five pairs of signature partners in Tl nuclei.
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