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Covariance of stochastic integrals with respect to fractional Brownian motion

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Abstract

We find an explicit expression for the cross-covariance between stochastic integral processes with respect to a \(d\)-dimensional fractional Brownian motion (fBm) \(B_t\) with Hurst parameter \(H > 1/2\), where the integrands are vector fields applied to \(B_t\). It provides, for example, a direct alternative proof of Y. Hu and D. Nualart’s result that the stochastic integral component in the fractional Bessel process decomposition is not itself a fractional Brownian motion.

Keywords: fractional Brownian motion, divergence integral, stochastic integral, fractional Bessel process

2010 MSC: 60G15, 60G18, 60G22, 60H05, 60H07

1. Introduction

Fractional Brownian motion is a family of zero mean stationary Gaussian processes \(B_t = B_t^H\) indexed by \(H \in (0, 1)\) which was mathematically introduced by B.B Mandelbrot and J.W. Van Ness in [1] (cf. [2] as well). It generalizes Brownian motion \((H = 1/2)\) in that \(E B_t^2 = t^{2H}\), and can be used to model various phenomena, in finance as well as in other fields. This is primarily due to the fact that its self-similarity depends on the parameter \(H\), which allows for phenomena exhibiting different kinds of self-similarity to be modeled by fractional Brownian motion with an appropriate \(H\).

Since fractional Brownian motion is not a semimartingale (unless \(H = \frac{1}{2}\)), the ordinary stochastic calculus for semimartingales (such as the Itô integral) does not apply. Instead, there are several approaches for defining a stochastic integral with respect to fractional Brownian motion. The divergence integral is one possible approach, the one discussed in this paper, using the Malliavin divergence operator as the basis for integration, a survey of which can be found in D. Nualart’s book [3]. One other approach for example was developed by Zähle in [4], which involves a pathwise definition of the stochastic integral. This

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