Mini–review on Monte Carlo programs for Bhabha scattering

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We review the status of Monte Carlo generators presently used for simulations of the large–angle Bhabha process at electron–positron (e+e−) accelerators of moderately high energy (flavour factories), operating at centre–of–mass energies between about 1 GeV and 10 GeV. It is shown how the theoretical accuracy reached by present Bhabha programs for physics at flavour factories is at the level of 0.1% and, therefore, comparable with that reached about a decade ago for luminosity monitoring through small–angle Bhabha scattering at LEP.

1. INTRODUCTION

At modern electron–positron (e+e−) accelerators built for a deeper and deeper understanding of the fundamental interactions between elementary particles, the machine (integrated) luminosity can be derived precisely by means of the relation \( \int L \, dt = N_{\text{obs}} / \sigma_{\text{th}} \), where \( N_{\text{obs}} \) and \( \sigma_{\text{th}} \) are the number of events and the theoretical cross section of a given reference process, respectively. Because of the latter relation, the reference process must be a reaction with clean topology, high statistics and calculable with high theoretical accuracy, to maintain small the total luminosity error given by the sum in quadrature of the relative experimental and theoretical uncertainty.

For example, at high–energy accelerators LEP/SLC running in the ’90s around the Z pole to perform precision tests of the Standard Model, the process of e+e− production (Bhabha scattering [1]), with the final–state leptons detected at small scattering angles, was used because dominated by the electromagnetic interaction and, therefore, calculable in perturbation theory, at least in principle, with very high theoretical accuracy. A total (experimental plus theoretical) precision of \( \sim 0.05 \pm 0.1\% \) was achieved at the end of LEP/SLC operation [2,3,4], thanks to the work of different theoretical groups and the excellent performances of precision luminometers.

At presently running e+e− colliders of moderately high energy (between about 1 GeV and 10 GeV), such as the Φ–factories DAΦNE and VEPP–2M, the charm–factories CESR and BEPC, as well as the B–factories KEK–B and PEP–II (globally denoted as flavour factories), the normalization reaction primarily used is the large–angle Bhabha process [5]. Actually, at all flavour factories, the final–state leptons are detected at wide scattering angles, within a typical central acceptance region of \( \sim 30^\circ \sim 150^\circ \), because of the absence of dedicated luminosity counters, for example, at small scattering angles. A precision luminosity measurement at flavour factories is of utmost importance to perform accurate measurements of the e+e− → hadrons cross section, which is, in turn, a key ingredient in high–precision calculations of the running of αQED and lepton \( g–2 \).

It is worth noting that high theoretical accuracy and, in particular, comparison with pre-
cise data require the development of sophisticated Monte Carlo (MC) programs, including radiative corrections with a relative precision at the per mille level. In this review, only the large–angle Bhabha generators used at present $e^+e^-$ colliders will be considered. A description of the small–angle Bhabha programs available in the literature and used at LEP can be found in [6].

2. LARGE–ANGLE BHABHA GENERATORS

Since pure weak corrections are completely irrelevant in the energy range explored by flavour factories, the dominant part of radiative corrections, i.e. leading–log QED contributions, can be kept under control by means of different theoretical methods, such as QED Structure Functions (SF) [7] and Yennie–Frautschii–Suura (YFS) exponentiation [8], as done in many applications to precision physics at LEP [3]. For instance, in the generator BabaYaga v3.5 [9,10], a pure QED Parton Shower (PS) is implemented, through a MC solution of the QED DGLAP equation satisfied by the electron SF in the non–singlet approximation. Once the electron SF is numerically derived, the QED corrected cross section can be simply obtained by dressing the hard–scattering cross section of the process under consideration with a SF for any charged leg. A remarkable advantage of this universal approach is that the four–momenta of the final–states products (charged particles and photons) can be exclusively generated through the shower cascade, as in the case of the QCD PS programs for the momenta of quarks and gluons.

A comparison between BabaYaga predictions and data published by the KLOE Collaboration for the measured energy distribution of the $e^+e^-$ pairs produced in the large–angle Bhabha process at DAΦNE is reported, from [11], in Fig. 1 showing a nice agreement between data and theory. In spite of that, it is clear that an obvious drawback of a pure PS generator like BabaYaga v3.5 is that it lacks the effect of $O(\alpha)$ non–logarithmic contributions and, therefore, the corresponding theoretical accuracy is limited to that of a leading–log (LL) approximation. However, the physical precision of the formulation underlying such a program can be substantially improved from about a per cent accuracy down to the per mille level by performing a matching of the exact next–to–leading (NLO) corrections with the multiple photon effects taken into account by the PS. This matching procedure, which is described in detail in [12], allows to obtain an $O(\alpha)$ corrected cross section that coincides, by construction, with the exact NLO result, avoiding double counting and preserving resummation of leading contributions. This theoretical formulation is on the grounds of an improved version of BabaYaga v3.5, denoted as BabaYaga@NLO, having a precision tag of 0.1%. Needless to say, thanks to the matching algorithm, the theoretical error of BabaYaga@NLO is shifted to $O(\alpha^2)$ contributions of the next–to–next–to–leading (NNLO) perturbative expansion.

Theoretical approaches rather similar, at least in their basic ingredients, to that of BabaYaga@NLO are implemented in the MC programs MCGPJ [13] and BHWISE [14], both used, as BabaYaga@NLO, at flavour factories for simulations of the Bhabha process. The former is a generator realized by a Dubna–Novosibirsk col-

![Figure 1. KLOE large–angle Bhabha data for the energy distribution of the $e^+e^-$ final state. From [11].](image)
Table 1

Relative size of different sources of correction (in per cent) to the large–angle Bhabha scattering cross section for realistic luminosity set up at Φ– and B–factories. From [12].

| set up | a. | b. | c. | d. |
|--------|----|----|----|----|
| δα    | -13.06 | -17.16 | -19.10 | -24.35 |
| δα non–log | -0.39 | -0.66 | -0.41 | -0.70 |
| δHO   | 0.43 | 0.93 | 0.87 | 1.76 |
| δα²L  | 0.04 | 0.09 | 0.06 | 0.11 |
| δVP   | 1.73 | 2.43 | 4.59 | 6.03 |

3. NUMERICAL RESULTS

To get an idea of which corrections are relevant to achieve a per mille precision in simulations of the Bhabha scattering at flavour factories, we show in Tab. 1 the relative effect of various contributions to the large–angle Bhabha cross section, when considering typical selection criteria at Φ– (set up a. and b.) and B–factories (set up c. and d.) (see [12] for details).

From Tab. 1 it can be seen that $\mathcal{O}(\alpha)$ corrections decrease the Bhabha cross section of about 15% at the Φ–factories and of about 20–25% at the B–factories. Within the full set of $\mathcal{O}(\alpha)$ corrections, non–log terms are of the order of 0.5%, almost independently of the centre-of-mass (c.m.) energy, as expected, and with a mild dependence on the angular acceptance cuts, as due to box/interference contributions. The effect of higher–order corrections due to multiple photon emission is about 0.5-1% at the Φ–factories and reaches 1–2% at the B–factories. The contribution of (approximate) $\mathcal{O}(\alpha^2 L)$ corrections is not exceeding the 0.1% level, while the vacuum polarization increases the cross section of about 2% around 1 GeV and of about 5–6% around 10 GeV. Concerning the latter correction, the non–perturbative hadronic contribution to the running of $\alpha$ is included in BabaYaga@NLO both in the lowest–order and one–loop diagrams through the HADR5N routine [16], that returns a data driven error, thus affecting the accuracy of the theoretical calculation. Analogous results about the size of radiative corrections have been obtained recently [17] for the process $e^+ e^- \rightarrow \gamma \gamma$, also of interest for precision luminosity studies at flavour factories. As a whole, these results indicate that both exact $\mathcal{O}(\alpha)$ and higher–order corrections (including vacuum polarization) are necessary for 0.1% theoretical precision.
4. TECHNICAL AND THEORETICAL ACCURACY

4.1. Technical precision: tuned comparisons

A typical procedure followed in the literature for establishing the technical precision of MC generators is to perform tuned comparisons between independent predictions, using the same set of experimental cuts. An example of such tuned comparisons is given in [12], where it is shown that the agreement between the predictions of BabaYaga@NLO and BHWIDE at the Φ–factories is well below 0.1%, and that also the agreement between BabaYaga@NLO and LAB-SPV, which is a benchmark code by our group with a formulation based on collinear SF very similar to MCGPJ, is very good, below the 0.1% level. This level of agreement, together with further considerations about the size of two-loop corrections discussed in the next subsection, is the reason why in the latest publication by KLOE Collaboration about the measurement of the hadronic cross section at DAΦNE [18] the relative uncertainty assigned to theory in the luminosity measurement is now 0.1%, resulting in a total luminosity error of 0.3%.

Similar comparisons have been performed between the results of BabaYaga@NLO and BHWIDE by A. Denig and A. Hafner of BABAR Collaboration, in the presence of realistic selection cuts for luminosity at PEP–II. Their studies show that the two generators agree at the 0.1% level, both for integrated and differential cross sections, in the physical and statistical significant regions. An example of such a comparison, showing the predictions of BHWIDE, BabaYaga@NLO and BabaYaga v3.5 for the electron energy distribution (upper panels), as well as the relative differences between the results of BHWIDE and the two BabaYaga versions (lower panels), is given in Fig. 2.

4.2. Theoretical precision: comparisons with two–loop calculations

In order to assess the physical precision of the generators, the methods typically used are i) to compare with \( \mathcal{O}(\alpha^2) \) calculations, if the latter – as in the case of Bhabha scattering – are available in the literature [19,20,21,22,23,24] ii) to estimate the size of unaccounted higher–order contributions.

Concerning point i) and considering, for definiteness, the generator BabaYaga@NLO, the strategy consists in deriving from the general formulation the \( \mathcal{O}(\alpha^2) \) cross section, which can be cast in the following form

\[
\sigma^\alpha = \sigma_{SV}^\alpha + \sigma_{SV,H}^\alpha + \sigma_{HH}^\alpha
\]

(1)

where, in principle, each of the above \( \mathcal{O}(\alpha^2) \) contributions is affected by an uncertainty, to be properly estimated. In Eq. (1), the first contribution is the cross section including \( \mathcal{O}(\alpha^2) \) soft plus virtual corrections, whose uncertainty can be evaluated by comparison with the available NNLO calculations. The \( \sigma_{SV}^\alpha \) of BabaYaga@NLO has been compared, in particular, with the calculation by Penin [21], who computed the complete set of two–loop virtual photonic corrections.
in the limit $Q^2 \gg m_e^2$ supplemented by real soft-photon radiation up to non–logarithmic accuracy, and the calculations by Bonciani et al. [22], who computed two–loop fermionic corrections (in the one–family approximation) with finite mass terms and the addition of soft bremsstrahlung and real pair contributions.

The results of such comparisons are shown in Fig. 3 and in Fig. 4 for set up a. at the Φ–factories. In Fig. 3, $\delta \sigma$ is the difference between $\sigma_{SV}$ of BabaYaga@NLO and the cross sections of the two $O(\alpha^2)$ calculations, denoted as photonic (Penin) and $N_F = 1$ (Bonciani et al.), as a function of the logarithm of the infrared regulator $\epsilon$. It can be seen that the differences are given by flat functions, demonstrating that such differences are infrared–safe, as expected, as a consequence of the universality and factorization properties of the infrared divergences. In Fig. 4, $\delta \sigma$ is shown as a function of the logarithm of a fictitious electron mass and for a fixed value of $\epsilon = 10^{-5}$. Since the difference with the calculation by Penin is given by a straight line, this indicates that the two–loop soft plus virtual photonic corrections missing in BabaYaga@NLO are $O(\alpha^2 L)$ not infrared–enhanced contributions. On the other hand, the difference with the calculation by Bonciani et al. shows that the fermionic two–loop effects missing in BabaYaga@NLO are dominated by $O(\alpha^2 L^2)$ contributions. It is important to emphasize that, as shown in detail in [12], the sum of the differences with the two $O(\alpha^2)$ calculations does not exceed the $1.5 \times 10^{-4}$ level, for all the considered set up at Φ– and Β–factories. The second term in Eq. (1) is the cross section containing the one–loop corrections to single hard bremsstrahlung and its uncertainty can be estimated by relying on partial results existing in the literature. Actually, the exact perturbative expression of $\sigma_{SV,H}$ is not available yet for full $s + t$ Bhabha scattering, but, using the results valid for small–angle Bhabha scattering [25] and large–angle $s$–channel processes [26], the relative uncertainty of BabaYaga@NLO in the calculation of $\sigma_{SV,H}$ can be safely estimated at the level of 0.05%. The third contribution in Eq. (1) is the double hard bremsstrahlung cross section, whose uncertainty can be evaluated by comparison with the exact $e^+ e^- \to e^+ e^- \gamma \gamma$ cross section. As shown in [12], the differences registered between $\sigma_{HH}$ as in BabaYaga@NLO and the exact calculation are really negligible, at the $10^{-5}$ level.

Summing all the results for the various sources of uncertainty, it turns out that the total theoretical error in BabaYaga@NLO is $\sim 0.1\%$, when also including the uncertainty due to the running of $\alpha$ as returned by the HADR5N routine and the
contribution, at a few 0.01% level, of light pairs radiation, still missing in BabaYaga@NLO.

5. CONCLUSIONS

During the last few years, there has been a significant progress in reducing the theoretical uncertainty in Bhabha generators used at presently running $e^+e^-$ colliders down to 0.1%. Exact $O(\alpha)$ and multiple photon corrections are necessary ingredients to achieve such a precision. These corrections are implemented in three generators (BabaYaga@NLO, BHWIDE and MCGPJ) for the large–angle Bhabha process, which agree within $\sim 0.1\%$ for integrated cross sections and $\sim 1\%$ (or better) for differential distributions [12, 13].

NNLO QED calculations are essential to establish the theoretical accuracy of existing generators and, if necessary, to improve it below 0.1%. In particular, the one–loop corrections to single hard bremsstrahlung should be calculated for full Bhabha scattering, to get a better control of the theoretical precision.

For next generation $e^+e^-$ accelerators (ILC/GigaZ), if a $10^{-4}$ accuracy is assumed, present MC Bhabha programs need to be improved by the inclusion of weak and two–loop QED corrections, as well as beamstrahlung and new, more precise $\Delta e_{\text{had}}^{(5)}$ parameterizations.

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