ERRATA

STOCHASTIC CALCULUS OVER SYMMETRIC MARKOV PROCESSES WITHOUT TIME REVERSAL

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1. Errata. The sentence “In view of Theorem 2.2 in [22], . . . for q.e. x ∈ E.” at page 1538 should be eliminated. The definitions of \( \hat{M}_d \), \( \hat{M}_j \) and \( \hat{M}_\kappa \) are corrected to be like \( \hat{M}_d := \{ M \in \hat{M} \mid \langle M, N \rangle \equiv 0 \text{ for } N \in \hat{M}^c \} \).

The statements of Theorem 2.1 and Corollary 2.3 in [3] are incorrect, which come from the error in [1] (see [2]). The corrected statement of Theorem 2.1 in [3] can be found below. Its proof can be obtained in the same way as in [3]. The class \( \hat{J} \) introduced in [3] is unnecessary for the corrected statement.

THEOREM 1.1 (Corrected statement of Theorem 2.1 in [3]). There exists a one-to-one correspondence between \( \hat{J}/\sim \) and \( \mathcal{M}_{\text{loc}}^{d,\llbracket0,\xi\rrbracket} \) which is characterized by the relation that for \( \phi \in \hat{J} \) (resp., \( M \in \mathcal{M}_{\text{loc}}^{d,\llbracket0,\xi\rrbracket} \)), there exists \( M \in \mathcal{M}_{\text{loc}}^{d,\llbracket0,\xi\rrbracket} \) (resp., \( \phi \in \hat{J} \)) such that \( \Delta M_t = \phi(X_t - x, X_t) \), \( t \in [0, \xi[, \mathbb{P}_x \)-a.s. for q.e. \( x \in E \). Moreover, we have \( \langle M \rangle_t = \int_0^t \int_E \phi^2(x, y)N(X_s, dy) dH_s \) for all \( t \in [0, \infty[ \) \( \mathbb{P}_x \)-a.s. for q.e. \( x \in E \).

We define subclasses of \( \mathcal{M}_{\text{loc}}^{d,\llbracket0,\xi\rrbracket} \) as follows:

\[
\mathcal{M}_{\text{loc}}^{j,\llbracket0,\xi\rrbracket} := \{ M \in \mathcal{M}_{\text{loc}}^{d,\llbracket0,\xi\rrbracket} \mid \phi(\cdot, \partial) = 0, \kappa\text{-a.e. on } E \},
\]

\[
\mathcal{M}_{\text{loc}}^{k,\llbracket0,\xi\rrbracket} := \{ M \in \mathcal{M}_{\text{loc}}^{d,\llbracket0,\xi\rrbracket} \mid \phi = 0, J\text{-a.e. on } E \times E \}.
\]

Then we have that \( M \in \mathcal{M}_{\text{loc}}^{j,\llbracket0,\xi\rrbracket} \), \( N \in \mathcal{M}_{\text{loc}}^{k,\llbracket0,\xi\rrbracket} \) imply \( \langle M, N \rangle \equiv 0 \mathbb{P}_x\)-a.s. for q.e. \( x \in E \), and every \( M \in \mathcal{M}_{\text{loc}}^{\llbracket0,\xi\rrbracket} \) is decomposed to \( M = M^c + M^j + M^k \), where \( M^c \in \mathcal{M}_{\text{loc}}^{c,\llbracket0,\xi\rrbracket} \), \( M^j \in \mathcal{M}_{\text{loc}}^{j,\llbracket0,\xi\rrbracket} \), \( M^k \in \mathcal{M}_{\text{loc}}^{k,\llbracket0,\xi\rrbracket} \) have the properties \( \langle M^c, M^j \rangle \equiv 0 \mathbb{P}_x\)-a.s. for q.e. \( x \in E \).

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\[ \langle M^j, M^\kappa \rangle \equiv \langle M^\kappa, M^c \rangle \equiv 0 \] in view of Theorem 1.1. The statement of Remark 2.3 in [3] is changed to be the following:

**Remark 1.1.** For each \( i = c, d, j, \kappa \), we let

\[ \mathcal{M}^{i,+}_{\text{loc}} := \{ M \mid \text{there exists } \{ G_n \} \in \Theta \text{ and } M^{(n)} \in \mathcal{M}^i \text{ such that} \]

\[ M_t = M_t^{(n)} \text{ for all } t \leq \tau_{G_n} \text{ and } n \in \mathbb{N}, \mathbb{P}_x \text{-a.s. for q.e. } x \in E \}. \]

Then \( \mathcal{M}^{i,\lfloor 0, \zeta \rfloor}_{\text{loc}} = \mathcal{M}^{i,+}_{\text{loc}} (i = c, d, j, \kappa) \). More strongly, we have \( \mathcal{M}^{c,\lfloor 0, \zeta \rfloor}_{\text{loc}} = \mathcal{M}^c_{\text{loc}} \).

All the statements of Corollary 4.1 and 4.3, Definition 4.3 and Theorem 4.1 in [3], the generalized Fukushima decomposition (Theorem 4.2 in [3]), the generalized Itô formula (Theorem 4.3 in [3]) and their corollaries (Corollaries 4.4 and 4.5 in [3]) hold only for \( t \in [0, \zeta] \mathbb{P}_x \text{-a.s. for q.e. } x \in E \). The proofs of them can be done in the same way as in [3]. The classes \( \mathcal{F}^\dagger_{\text{loc}} \) and \( \mathcal{F}^\flat_{\text{loc}} \) introduced in [3] are unnecessary for the corrected statements. We only expose the corrected statement of generalized Fukushima decomposition below for completeness.

**Theorem 1.2** (Corrected statement of Theorem 4.2 in [3]). For \( u \in \mathcal{F}^\dagger_{\text{loc}} \), the additive functional \( A^u \) defined by \( A^u_t := u(X_t) - u(X_0) \) can be decomposed as

\[ A^u = M^u + N^u, \quad M^u \in \mathcal{M}^{\lfloor 0, \zeta \rfloor}_{\text{loc}}, \quad N^u \in \mathcal{N}_{c,\text{loc}} \]

in the sense that \( A^u_t = M^u_t + N^u_t, t \in [0, \zeta], \mathbb{P}_x \text{-a.s. for q.e. } x \in E \). Such a decomposition is unique up to the equivalence of additive functionals on \( \lfloor 0, \zeta \rfloor \) (or of local additive functionals).

**References**

[1] **Chen, Z. Q., Fitzsimmons, P. J., Kuwae, K. and Zhang, T. S.** (2008). Stochastic calculus for symmetric Markov processes. *Ann. Probab.* 36 931–970. MR2408579

[2] **Chen, Z. Q., Fitzsimmons, P. J., Kuwae, K. and Zhang, T. S.** (2012). Errata for “Stochastic calculus for symmetric Markov processes.” *Ann. Probab.* 36 (2008) 931–970] *Ann. Probab.* 40 1375–1376.

[3] **Kuwae, K.** (2010). Stochastic calculus over symmetric Markov processes without time reversal. *Ann. Probab.* 38 1532–1569. MR2663636