Study on Discrete Railway Bulk Transportation Option Based on Game Theory

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Abstract. Railway transportation is an important part of China's transportation system, and its freight price level has a direct impact on the national economy and social development. In this paper, taking railway freight pricing as the research object, based on pricing theories and theorems, the option theory is introduced into the railway freight pricing problem, a Stackelberg game model of railway freight option is established, and an optimization algorithm is proposed to solve the model. It is helpful for railway transportation enterprises to participate in the market competition of railway freight rate system, which provides the theoretical basis and technical support for the market-oriented reform of railway freight rate.

1. Introduction

In the development of China's transport, railway transport has always occupied an important position. In June 2013, China Railway Corporation began to organize a series of reform measures to make railway freight transport more suitable for the market. With the market as the leading force, China Railway Corporation began to adjust railway freight rates, promote the better development of railway freight transport, and push the railway freight transport to the market in an all-round way. Railway freight transport has become the focus of scholars' discussion. The Brownian movement was first proposed by LOUIS BACHELIER in 1900. Kavussanos [1] this paper discusses the commodity trading and economic spillover effect between freight derivatives, find the commodity market rate of return volatility, will affect the freight derivatives market through information dissemination mechanism, Adland [2] found that in addition to market conditions and the choice of transport route lessees and cargo owners' personal choices also have a greater impact on the level of freight options. These studies provide theoretical support for the detailed derivation of freight option pricing formulas. Kou[3] carries out detailed discussion on relevant parameters of derivatives. Kavussanos[4] USES Panamax index to analyze earnings and volatility in two market trading activities, spot freight and forward freight. Adland[5] used the non-parametric Markov diffusion model to study the volatility of freight derivatives in the crude oil transportation market, and found that the spot freight level in the crude oil transportation market could be described by a nonlinear stochastic model. Li Guanghui [6] studied the fluctuation rules of China's coastal dry bulk transport market and found that it was necessary for enterprises to predict the future freight price trend through the historical fluctuation rules. In terms of railway freight transport, Feng Fenling [7] made a preliminary discussion on railway freight option.

On this basis, this paper finds out the execution process of railway bulk freight option, and USES a multi-stage Stackelberg dynamic game to analyze the relationship between railway transportation enterprise and customer, and builds a discrete railway bulk freight option pricing model. Make decision-
making recommendations for customers and railway transportation companies respectively. To achieve a win-win situation for railway transportation companies and customers.

2. Research on Discrete Railway Bulk Freight Option Model

2.1. Problem description

At present, in China's railway freight transport, except for the goods subject to special transport, most of the railway freight transport is gradually realizing marketization, and the railway freight transport activities have options market and spot market. The option market coexists with the spot market, and there is a game between railway transport enterprises and option customers. After the introduction of the option mechanism, the railway transport enterprise needs to infer the optimal action of the option customer, so as to establish a reasonable option fee $W_0$ for the unit capacity of the railway freight option and an option strike price $W_1$ for the unit capacity of the railway transport enterprise. Option customers, on the other hand, decide the amount of subscription and execution of option according to the published pricing ($W_0$ and $W_1$) and unit capacity price of spot market at $t_0$ time. In fact, their decision-making process is in a two-stage Stackelberg dynamic game process. The option trading process is shown in the figure below:

![Option trading process](image)

Figure 1 Option trading process

The behavior of the two stages in the figure is simulated by Stackelberg model, in which the railway transport manufacturer is the leader of the game. In the first stage, the railway transport enterprise gives a set of option parameters, according to which the option customer determines its optimal capacity option order quantity. Assuming information symmetry, railway transport enterprises can infer the order quantity of option customers' capacity based on the information obtained, and make the best decision accordingly. In the second stage, customers can directly purchase transport services from the spot market from the railway transport production enterprise, and determine the amount of exercise of the option and purchase of the capacity in the spot market according to the price in the spot market and the strike price of the capacity option, and the railway transport enterprise can also sell its excess capacity in the spot market.

2.2. Symbol definition

To facilitate representation, you need to define the symbols that will appear in the model before you build the model.

- $w_0$: Option fee per unit capacity of a rail freight option.
- $w_1$: option strike price per unit capacity set by the railway transport enterprise.
- $s_0$: Capacity price in spot market. If $s_0$ obeys a certain distribution, let its probability density function be $h(s_0)$ and its distribution function be $H(s_0)$. 
$q_0$: The number of options exercised by the customer.

$Q$: According to the situation of the existing market, the railway transport enterprise speculates the possible purchase amount of all option customers. The size of $Q$ is related to the strike price and option cost, which meets the price demand supply theory. When the strike price and option cost are higher, the demand will be lower.

$m$: The probability that the railway transport enterprise will find the corresponding buyer of the surplus capacity in the spot market. $M$ is related to the railway transport demand in the spot market. The greater the demand, the greater the $m$. ($0 \leq m \leq 1$)

c: The railway transport enterprise provides the fixed cost of unit capacity.

$b_1$: Long-term preparation cost of railway transportation enterprises.

$b_2$: Short term preparation cost of railway transportation enterprise.

$q_i$: Refers to the volume of freight carried by a certain mode of transport.

$M$: The sum of the freight volumes for all modes of transport.

2.3. Research hypothesis

Hypothesis 1: A railway transport enterprise is an economic man, and it will make a rational choice to maximize its own profits under a given market environment.

Hypothesis 2: The price of railway freight capacity in the freight market is determined entirely by market economic conditions and is not affected by railway freight companies and option customers. In other words, with the improvement of market economic conditions, the price of railway freight capacity in the freight market will increase.

Hypothesis 3: The long-term preparation cost per unit capacity of the railway freight enterprise is less than the short-term production cost, that is, $b_1 \leq b_2$, and the railway transport enterprise chooses to sell the remaining capacity on the spot market only if $s_0 > b_2$, so as to obtain profits.

Hypothesis 4: The spot market capacity price is normally distributed.

2.4. Model

When developing option strategies, railway transport enterprises first judge the possible option purchase amount of all option customers in the market according to their own grasp of the market, and then judge the inferred option purchase amount. Develop an optimal option strategy $(w_0^*, w_1^*)$ to maximize its own returns.

Upper function:

According to the description of the model, in the course of option trading between railway transport enterprises and option customers on the market, railway transport enterprises will allocate vehicles, organize production and organize transportation according to the option purchase amount of option customers. In the option maturity date on the same day, when the agreement when the customer’s order execution amount is less than its options, the railway transport enterprise will its remaining capacity on the spot market for sale, because the model assumes that the market is uncertain, therefore assume that the railway freight enterprise on the spot market for the rest of the capacity to find the probability of the corresponding customer is $m$, $(0 \leq m \leq 1)$. The cost of the capacity of railway transport enterprises can be divided into long-term preparation cost $b_1 b_2$ and short-term preparation cost, cost of long-term preparation for railway transport enterprises to meet all customer capacity options quantity of production cost, it will cost you a short-term production costs for the railway transport enterprise will its excess capacity on the spot market sales drop, and achieve the final cost required for transportation. According to the above description, the profit function of railway transport enterprises can be expressed as:

$$\Pi = w_0 Q + q_0 (w_1 - b_1) + m (K - q_0) (s_0 - b_2) - c K$$

(1)

The first item on the right of the formula represents the option fee paid by the option customer to the railway transportation enterprise, and the second item represents the income brought to the railway transportation enterprise by all option customers exercising the option. The third represents the profits
brought by railway transportation enterprises to sell their surplus capacity on the spot market. The fourth represents fixed investment in railway transport production.

On the expiration date, option customers will determine the amount of options to be exercised based on the spot market capacity price. When the spot market capacity price is lower than the option strike price, the client will give up the option contract and choose to buy the capacity in the spot market. When the spot market capacity price is higher than the option strike price, the client chooses to execute the option contract. Therefore, the option exercise of the client can be expressed as:

$$q_0 = \begin{cases} Q, s_0 \geq w_1 \\ 0, 0 < s_0 < w_1 \end{cases}$$  \hspace{1cm} (2)$$

Lianli (1) and (2) can obtain the profit function of railway transportation enterprises:

$$
\begin{align*}
\mathcal{L}(q, s_0) &= s_0 - q + 10 \cdot \max\{0, s_0 - q - K\} \\
n_{q, s_0} &= \begin{cases} \int_{s_0}^{w_1} \max\{0, s_0 - q(x) - K\} ds_0 & q \leq \frac{w_1 - s_0}{10} \\ \int_{s_0}^{w_1} \max\{0, s_0 - q(x) - K\} ds_0 & q > \frac{w_1 - s_0}{10} \end{cases}
\end{align*}
$$

The lower function can be changed into:
\[
\min G(q_i, e) = \sum_{i=0}^{n} \int_{0}^{\infty} f_i(x)dx + \int_{0}^{\infty} L(x)dx
\]  

(9)

Since there is surplus capacity in the sum of the modes of transport, it is equivalent to adding a mode of transport. Can be expressed as:

\[
\min G(q_i) = \sum_{i=0}^{n+1} \int_{0}^{\infty} f_i(x)dx
\]  

(10)

The cost of transportation can be expressed as a function:

\[
f_i(q_i) = a(q_i)^y + V_i
\]  

(11)

The discrete railway bulk cargo option pricing model can be obtained as follows:

\[
\max \left\{ \int_{0}^{\infty} \left[ w_0 Q + mK(s_0 - b_2) - cK \right] h(s_0)ds_0 \right. \\
+ \left. \int_{0}^{\infty} \left[ w_0 Q + Q(w_1 - b_1) + mK(\bar{Q} - s_0 - b_2) - cK \right] h(s_0)ds_0 \right\}
\]  

\[
\min \left\{ \sum_{i=0}^{n+1} \int_{0}^{\infty} f_i(x)dx \right. \\
\left. \begin{array}{l}
0 \leq q_0 \leq Q \\
0 < m < 1 \\
w_0 \leq s_0 \\
w_1 \leq s_0 \\
\text{ST} b_1 + b_2 \leq w_0 + w_1 \\
p_{\max} \leq 1.2 p \\
q_i \geq 0 \\
\sum_{i=0}^{n+1} q_i = M
\end{array} \right\}
\]  

Including 1 constraint expression option purchases cannot exceed total market capacity, 2 will said the railway transport enterprise surplus capacity constraints in the spot market to find the probability and the demand of buyers were positively correlated, corresponding constraints option price 3 and 4 said the strike price is higher than the spot market price is meaningless, 5constraint condition is to ensure that the transport enterprises are profitable, 6 said the current rate of floating policy constraints do not allow the freight rate floats over 20%, the constraint conditions of 7 in order to guarantee a certain mode of transportation of freight must be meaningful, constraint condition and 8 for any mode of transport of freight.

The optimal solution of the transportation enterprise and the minimum transportation cost of the option customer can be obtained through the optimization of the above functions.

2.5. Model solving

The sensitivity analysis algorithm is used to solve the model to find the specific formula of the lower function and substitute the lower function into the upper function to find the optimal solution.

Railway freight rate is the influencing factor. Other factors can be ignored. Then there is inequality:

\[
f \left( q_i^* (p_i), p_i, q_i - q_i^* (p_i) \right) = 0
\]  

(12)

In the equation:

\[
q_i^* = \left( q_i^{i^*} , q_i^{2^*} , \cdots , q_i^{n^*} \right) \right; \\
q_i = \left( q_i^1 , q_i^2 , \cdots , q_i^n \right)^T; \\
q_i^* = \left( q_i^{i*} , q_i^{2*} , \cdots , q_i^{n*} \right)^T;
\]
\[ f(q^*_i) = \left( f(q^*_1), f(q^*_2), \ldots, f(q^*_n) \right) \]

A necessary condition for the existence of a unique solution:
\[ f(q^*_i(p_i), p_i) - \mu = 0 \]  \hspace{1cm} (13)
\[ M = \nabla^2 f(q^*_i(p_i)) \]  \hspace{1cm} (14)
\[ \nabla^2 f(p_i)(p_i) = J^{-1}(p_i)[\ldots - J_R(\nabla f(p_i))] \]  \hspace{1cm} (15)

The initial value of a given rail freight pricing. When the freight pricing of other modes of transport is guaranteed to remain unchanged, the formula is expanded by using the Jacobian matrix using sensitivity analysis.

\[ q_i(p_i) = q_i^*(p_i(0)) + \frac{\partial q_i}{\partial p_2}(p_i - p_i(0)) \]  \hspace{1cm} (16)

Specific steps based on sensitivity algorithm:

Step1: Initialization. First, take a freight rate in the railway, and make the number of iterations t=0.
Step2: The freight rate is substituted into the lower level programming model. MATLAB is used to solve the freight volume under various modes of transport.
Step3: The required function is obtained using the Jacobian matrix using sensitivity analysis.
Step4: Plug it into the upper function to get the new rate.
Step5: Perform a convergent judgment. If \[ |p_i^{(k+1)} - p_i^{(k)}| \leq \delta \] , then the algorithm terminates.

Thus we can analyze the optimal option fee and the optimal strike price of the unit capacity of the railway freight option formulated by the railway transport enterprise under the co-existence of the spot market and the option market.

3. The example analysis
The spot market price of rail capacity on the day before the option is exercised is normally distributed. The long-term preparation cost of railway transportation enterprises \( b_1 = 120 \). Short-term preparation cost of railway transportation enterprises \( b_2 = 150 \). Fixed cost of unit capacity provided by railway transport enterprises \( c = 30 \). In practical problems, there is a certain relationship between the option fee per unit capacity of railway freight option and the option strike price per unit capacity set by railway transport enterprises.

\[ f_1(q_1) = 3q_1^{0.3} + 0.47p_1 + 348.34 \]
\[ f_3(q_2) = 3q_2^{0.3} + 479 \]

The maximum quantity demanded of the goods is 100 tons. Assuming that the relation of the demand function is \( M = 100 - 0.1p_i \), the inverse function is. Then, there are three modes of transportation \( D^{-1}(Q) = 1000 - 10M \), and the function of the third mode is:

Lower function:
\[ \min G(q_i) = \int_0^\infty (3q_1^{0.3} + 0.47p_1 + 348.34) dq_1 + \int_0^\infty (3q_2^{0.3} + 479) dq_2 + \int_0^\infty 10q_3 dq_3 \]
\[ f_3(q_3) = D^{-1}(x) = 1000 - 10(100 - q_3) = 10q_3 \]
\[ q_1 + q_2 + q_3 = 100 \]

The current freight rate is 558, which is also the initial value. Then substitute it in to get:
\[ q_1^{(0)} = 5.2718, q_2^{(0)} = 20.8003, q_3^{(0)} = 73.9278 \]
\[ f(q^{(0)}, p) = \begin{bmatrix} 3q_1^{(0)0.3} + 0.47p_1 + 348.34 \\ 3q_2^{(0)0.3} + 479 \\ 10q_3^{(0)} \end{bmatrix} \]
\[ q_1^{(0)} + q_2^{(0)} + q_3^{(0)} = 100 \]

Get \( J_y(p_1) \) and \( J_y(p_2) \):

\[
J_y(p) = \begin{pmatrix}
3q_1^{(0)0.3} & 0 & 0 & -1 \\
0 & 3q_2^{(0)0.3} & 0 & -1 \\
0 & 0 & 10 & -1 \\
1 & 1 & 1 & 0
\end{pmatrix}
\]

\[
J_p(p) = \begin{pmatrix}
0.47 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\]

Formula is used to develop the relationship between freight volume and freight rate:

\[
\begin{pmatrix}
q_1^{(0)} \\
q_2^{(0)} \\
q_3^{(0)} 
\end{pmatrix} = \begin{pmatrix}
5.2718 \\
20.8003 \\
73.9278
\end{pmatrix} + \begin{pmatrix}
-0.0368 & 0 & 0 \\
0.0165 & 0 & 0 \\
0.0197 & 0 & 0
\end{pmatrix} \begin{pmatrix}
p_1^{(0)} \\
p_2^{(0)} \\
p_3^{(0)}
\end{pmatrix}
\]

The function obtained is:

\[
q_1^{(0)}(p_1^{(0)}) = 5.2718 - 0.0368 \times (p_1^{(0)} - 558)
\]

Put the function into the upper layer function to find out the optimal pricing strategy of railway transportation enterprises.

By substituting into the upper function, the data of railway freight volume and freight rate can be obtained by substitution once and again as shown in the following table.

| Number of substitution | Railway freight rate (Yuan/ton) | Railway freight volume (ton) | Railway revenue (Yuan) |
|------------------------|---------------------------------|-----------------------------|------------------------|
| 0                      | 558                             | 5.2718                      | 2941.6644              |
| 1                      | 347.2564                        | 13.259                      | 4604.3012              |
| 2                      | 349.1764                        | 13.1863                     | 4604.3498              |
| 3                      | 349.2561                        | 13.1832                     | 4604.3457              |

Figure 2  Freight volume freight rate relation
Figure 3 Revenue tariff relationship

The option fee per unit capacity and the strike price per unit capacity set by the railway transport company increase with the increase of the probability that the railway transport company sells surplus capacity to customers in the spot market. To sum up, the optimal freight rate is 347.1746 (yuan/ton), and the maximum profit will be obtained when the freight rate decreases by 37.78%, and the reduction of the price can also attract freight volume.

4. conclusionS

Combined with the characteristics of my country’s railway bulk cargo transportation. Take advantage of the similarities between the rail bulk freight agreement market and the financial option market. According to the related research results of commodity supply chain, the concept of railway capacity option is proposed. Designed the transaction process of railway capacity options in which the option market and the spot market coexist. And based on this process, a Sackelberg game model between railway transportation companies and option customers is constructed to establish the relationship between option costs and exercise prices. The two-variable optimization problem with constraints is simplified to a single-variable optimization problem with constraints. Take a railway bureau as an example to extract real data. Obtain the Sackelberg game model of railway transportation companies and option customers to formulate reasonable price strategies. Put forward strategies for the future operation of railway bulk freight options.

This article only discusses and studies the situation that the spot market capacity price obeys the normal distribution. In actual situations, the spot market capacity price may also obey other distributions. Therefore, we study the spot market capacity price distribution under other more typical conditions and the corresponding optimal pricing strategy. The formulation and further improvement of the applicability of the model is one of the important aspects of future research.

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