Further evidence that the transition of 4D dynamical triangulation is 1st order.

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Abstract

We confirm recent claims that, contrary to what was generally believed, the phase transition of the dynamical triangulation model of four-dimensional quantum gravity is of first order. We have looked at this at a volume of 64,000 four-simplices, where the evidence in the form of a double peak histogram of the action is quite clear.
1 Introduction

Dynamical triangulation is a relatively recent approach to the problem of formulating a theory of four-dimensional quantum gravity [1, 2, 3]. Although at the moment only a euclidean version of the dynamical triangulation model exists, many interesting results have been obtained in this version.

In dynamical triangulation, the path integral over metrics is replaced by a sum over ways to glue together a number $N_4$ of four-dimensional simplices in all possible ways that have a certain fixed topology that is usually taken to be the sphere $S^4$. In this procedure, the continuum euclidean action

$$S = \frac{1}{16\pi G} \int d^4 x \sqrt{g} (2\Lambda - R)$$

(1)

can be rewritten in the discrete form

$$S = -\kappa_2 N_2 + \kappa_4 N_4,$$

(2)

where the $N_i$ are the number of $i$-simplices in the triangulation. For later reference, we can use the geometrical relation between these $N_i$

$$N_0 = \frac{N_2}{2} - N_4 + 2,$$

(3)

to write the discrete action (2) in terms of the number of vertices $N_0$

$$S = -2\kappa_2 N_0 + (\kappa_4 - 2\kappa_2) N_4 + 4\kappa_2.$$

(4)

This system turns out to have a phase transition at a critical $\kappa_2$ value $\kappa_2^c$ that depends somewhat on the volume $N_4$ and for large volumes converges to a value that was recently measured as $\kappa_2^c = 1.336(6)$ [4]. Various evidence is present that indicates that the transition is a continuous one [4, 5]. Recently, however, data have been presented [6] that indicate that the phase transition is a first order transition. This point of view was already expressed in [3] where some hysteresis was observed, but this was retracted in [7].

2 Simulations

We have simulated the system at volumes of 32,000 and 64,000 simplices at several values of $\kappa_2$ close to the phase transition. There is no known
method to use ergodic moves that keep the volume constant and probably no such method can exist \[8\]. It is known that such a method cannot exist for manifolds that are unrecognizable and also that some 4-manifolds indeed are unrecognizable, although for the 4-sphere used in our simulations this is not known. It would be quite surprising if there turned out to be a set of local moves whose ergodicity depends on the topology of the manifold, but for non-local moves such as baby universe surgery \[4\] this is easier to imagine.

To make sure the moves in the simulations are ergodic, we therefore have to allow fluctuations of the number of simplices \(N_4\). This is done by the usual method of allowing the number of simplices to vary, but at the same time adding a quadratic term to the action to keep this number close to some desired value \(V\). The action then becomes

\[
S = -\kappa_2 N_2 + \kappa_4 N_4 + \gamma (N_4 - V)^2, \tag{5}
\]

where \(\gamma\) is a parameter that controls the volume fluctuations and that we set to \(5 \cdot 10^{-4}\). Because a constant in the action is irrelevant, it is possible to eliminate either \(\kappa_4\) or \(V\) from the action, but we will not do so. In the simulations, the magnitude of the fluctuations in \(N_4\) is

\[
\delta N_4 = \sqrt{\langle N_4^2 \rangle - \langle N_4 \rangle^2} = \sqrt{1/2\gamma} \approx 30. \tag{6}
\]

Note also that the modification of the action in (5) only depends on \(N_4\). Therefore, the relative weights of configurations at a particular value of \(N_4\) does not change with respect to the original action.

The fluctuations of \(N_4\) introduce some extra fluctuations in the values of \(N_0\) that we are going to measure. Because our parameter \(\kappa_2\) couples to the number of triangles \(N_2\), one would at first sight consider measuring \(N_2\). The reason we use \(N_0\) is that it suffers much less from these extra fluctuations. This can be seen as follows. Because of the relation (3), the ratio of the fluctuations \(\delta N_2/\delta N_0\) at fixed \(N_4\) equals 2. However, the ratio of the \(N_i\) fluctuations due to the volume fluctuations at fixed energy density (that is fixed \(N_i/N_4\)) is \(\delta N_2/\delta N_0 = N_2/N_0 \geq 10\), the exact ratio depending on \(\kappa_2\).

### 3 Results

Two time histories of \(N_0\) are plotted in figures 1 and 2. The horizontal units are 100 sweeps, where we define a sweep as \(N_4\) accepted moves. The quantity
$N_0$ (the number of vertices in the configuration) is at fixed $N_4$ and up to a constant directly proportional to the action. We can see this from equation (4). Several other time histories that were made show the same effect: there are two states, one at high $N_0$ and the other at low $N_0$. The system stays in one of these states for a long time and occasionally the system flips from one state to the other. This is a good indication that there are two separate minima in the free energy, creating a first order phase transition.

A histogram of observed $N_0$ values is plotted in figure 3. The size of the bins is 5, while $N_0$ values were taken each 10 sweeps. Because of the fluctuations in $N_4$ explained above, the $N_0$ values are slightly more spread out than what would be the case if $N_4$ were really fixed. As the fluctuations in $N_4$ are about 30 and $N_0/N_4 \approx 1/6$, these extra fluctuations in $N_0$ are about 5, that is the size of one bin.

The double peak structure in figure 3 is clear. It should be obvious from the limited time histories in figures 1 and 2 that the relative strengths of the
peaks in the histogram are not very significant. To find the actual relative contribution of the two kinds of configurations to the partition function would require many more flips between the states, which would take inordinate amounts of computer time.

The fits shown in the figure are fits to a double gaussian

$$C_1 \exp\left(\frac{-(N_0 - \mu_1)^2}{2\sigma_1}\right) + C_2 \exp\left(\frac{-(N_0 - \mu_2)^2}{2\sigma_2}\right). \quad (7)$$

The two states with large and small $N_0$ are directly related to the crumpled or elongated structure of the configuration. This structure is quantified using the average distance between any two simplices in a particular configuration. Figure 4 shows the correlation of the two quantities. There seem to be two separated areas, one where $N_0$ and the average distance are small, and one where both are large.

Because we fix $\kappa_4$ in the action (3) at some reasonable but arbitrarily
chosen value in our simulations, the actual average volume is not exactly the 64,000 we mention, but 63,912. This also means that the transition occurs at a slightly lower value of $\kappa_2$ than if the average volume were exactly 64,000.

For comparison, we have also plotted a histogram of $N_0$ values at the smaller volume of $N_4 = 32,000$. In this case the actual average volume was $N_4 = 31,911$. Only at $\kappa_2 = 1.258$ can we see a double peak structure, and even that one is quite weak. This shows that the effect grows with the volume. Also, the distance between the peaks grows roughly linearly with the volume, from 160 at $N_4 = 32,000$ and $\kappa_2 = 1.258$ to 366 and 301 at the two $\kappa_2$ values shown at $N_4 = 64,000$. In other words, if we take an intensive quantity like $N_0/N_4$ the distance between the peaks stays constant. If the transition was second order and the double peak structure a finite volume effect, this distance would shrink with the volume.

Figure 3: Histogram of $N_0$ values at $N_4 = 64,000$. 


4 Finite size scaling

Using the Monte Carlo runs we made at various values of $\kappa^2$, we can calculate the susceptibility

$$\chi = \frac{1}{N^4} \left( \langle N_0^2 \rangle - \langle N_0 \rangle^2 \right),$$

as a function of $\kappa$. To cover intermediate values of $\kappa$, we use the Ferrenberg-Swendsen reweighting procedure [9, 10]. The results are plotted in figure 6. The errors (represented by the dotted curves) were generated using the jackknife method. The large errors at the end of the curves arise because the reweighting procedure cannot extrapolate well far from the actual $\kappa$ values that were used in the simulations. The maxima occur at $\kappa = 1.257(1)$ and $\kappa = 1.280(1)$ respectively.

The heights of the susceptibility peaks allow us to calculate some of the finite size scaling exponents of the phase transition. Some caution should be exercised when interpreting the results, because only two volumes were
available. Simulating to the same accuracy at another large volume would take too long, while smaller volumes showed no sign of a first order transition and therefore are apparently too small to give meaningful large volume behaviour.

We will first look at the susceptibility exponent $\Delta$ defined by

$$\chi_{\text{max}} \propto N_4^\Delta.$$  \hspace{1cm} (9)

In a regular spin system with a second order transition, $\Delta$ would be related to the susceptibility exponent $\gamma$, the correlation length exponent $\nu$ and the dimension $d$ by the relation $\Delta = \gamma/\nu d$, but whether such a relation also holds in simplicial quantum gravity is not clear. In particular, one may wonder what the dimension of the system is. In [11] a scaling dimension $d_s$ was defined that turned out to be compatible with 4, but we will not attempt to use that in the present discussion.

The susceptibility exponent $\Delta$ following from the data of figure 6 is 0.81(4). This is not compatible with an exponent of 1 expected at a first
order phase transition. It is however much larger than the value of 0.259(7) obtained at volumes up to 8,000 in [5]. Apparently the number goes up with the volume and may very well approach 1 at even larger volumes. One should also note that the number of 0.81(4) was obtained by taking the absolute values of the peaks, and not their height relative to the “background” susceptibility (arising from the fluctuations within one free energy minimum) that does not increase with the volume. Doing this would result in a higher exponent, but we cannot simply subtract the value observed away from the transition, because the background susceptibility is so different in both phases.

Another exponent to consider is the one that describes the width of the susceptibility peak

$$\delta \kappa_2 \propto N_4^{-\Gamma}.$$  \hspace{1cm} (10)

As the width of the peak, we arbitrarily take the width at 75% of the peak height. Taking the usual definition of the width at half the maximum height would take us too far into the left hand tail of the curve at $N_4 = 32,000$. The
exponent $\Gamma$ we get from the data in figure 3 is $1.24(18)$. This is compatible with the value of 1 expected at a first order transition. In this case, the background susceptibility makes the exponent come out higher than its actual value in the infinite volume limit.

The finite size scaling exponent that governs the change of the apparent critical value $\kappa_2^c(N_4)$ with the volume cannot be determined from only two volumes.

5 Discussion

We have seen that at a volume of $N_4 = 64,000$ the time history flipping and double peak structure in the action are very clear indicating a first order transition. We calculated some finite size scaling exponents, but due to the small number of volumes they cannot be interpreted as the definitive values.

It has been suggested [6] that constraining the volume in the simulations may create an artificial potential barrier, causing the flipping between the elongated and the crumpled state. This seems unlikely to me, because of the following. There cannot be an energy barrier between the states, because intermediate states by definition have intermediate energy. And a barrier in the volume, in the sense that one needs a higher volume to get from one state to the other, simply means that at the value of $N_4$ under consideration the number of intermediate states is small, meaning that the first order transition is genuine.

If the process is ergodic in practice, all states at fixed $N_4$ will be visited with frequency proportional to their respective Boltzmann weights. In this case there is obviously no problem and the double peak structure is real. If the double peak structure is caused by a non-ergodicity of the moves, this can only be because many of the intermediate states (at the gap between the peaks) do exist, but are unreachable. This seems unlikely, because such unreachable states have not been observed [12, 13] and if they exist in significant numbers one would expect them to have some extremal feature and not be in the middle of the $N_2$ distribution.

As has been pointed out earlier [11], the system may scale even away from the phase transition, making a continuous transition not vital for continuum behaviour of the theory. As an example, in the case of $\mathbb{Z}_N$ gauge theory [14, 15, 16] the system has infinite correlation length in a whole region of values of the coupling constant (the Coulomb phase). A different scenario
can be found in SU(2) gauge-Higgs theory (see e.g. [17, 18, 19] and references therein). This system has a first order transition, but decreasing the lattice spacing increases the correlation length in lattice units in such a way that the system still scales.

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