CLUSTERING OF THE DIFFUSE INFRARED LIGHT FROM THE COBE DIRBE MAPS. III. POWER SPECTRUM ANALYSIS AND EXCESS ISOTROPIC COMPONENT OF FLUCTUATIONS

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ABSTRACT

The cosmic infrared background (CIB) radiation is the cosmic repository for energy release throughout the history of the universe. The spatial fluctuations of the CIB resulting from galaxy clustering are expected to be at least a few percent on scales of a degree, depending on the luminosity and clustering history of the early universe. Using the all-sky data from the COBE DIRBE instrument at wavelengths 1.25–100 μm, we attempt to measure the CIB fluctuations. In the near-IR, foreground emission is dominated by small-scale structure due to stars in the Galaxy. There we find a strong correlation between the amplitude of the fluctuations and Galactic latitude after removing bright foreground stars. Using data outside the Galactic plane (|b| > 20°) and away from the center (90° < l < 270°), we extrapolate the amplitude of the fluctuations to csc |b| = 0. We find positive intercepts of \(\delta F_{\text{rms}} = 15.5^{+7.0}_{-3.7}, 5.9^{+1.6}_{-3.7}, 2.4^{+0.5}_{-0.5}, \) and \(2.0^{+0.25}_{-0.25}\) nW m\(^{-2}\) sr\(^{-1}\) at 1.25, 22, 3.5, and 4.9 μm, respectively, where the errors are the range of 92% confidence limits. For color subtracted maps between band 1 and 2 we find the isotropic part of the fluctuations at \(7.6^{+1.4}_{-1.4}\) nW m\(^{-2}\) sr\(^{-1}\). Based on detailed numerical and analytic models, this residual is not likely to originate from the Galaxy, our clipping algorithm, or instrumental noise. We demonstrate that the residuals from the fit used in the extrapolation are distributed isotropically and suggest that this extra variance may result from structure in the CIB. We also obtain a positive intercept from a linear combination of maps at 1.25 and 22 μm. For 2° < θ < 15°, a power-spectrum analysis yields firm upper limits of \((θ/5°) \times δF_{\text{rms}}(θ) < 6, 2.5, 0.8,\) and 0.5 nW m\(^{-2}\) sr\(^{-1}\) at 1.25, 22, 3.5, and 4.9 μm, respectively. From 10 to 100 μm, the dominant foregrounds are emission by dust in the solar system and the Galaxy. The upper limits on the CIB fluctuations are below 1 nW m\(^{-2}\) sr\(^{-1}\) there and are lowest (≤0.5 nW m\(^{-2}\) sr\(^{-1}\)) at 25 μm.

Subject headings: cosmology; observations — diffuse radiation — infrared; galaxies

1. INTRODUCTION

Diffuse backgrounds and their structure contain important information about the history of the early universe, when discrete objects either do not exist or are not accessible to telescopic studies. The formation and early evolution of galaxies should have generated radiation redshifted into the infrared bands (e.g., Partridge & Peebles 1967; Bond, Carr, & Hogan 1986). This cosmic infrared background (CIB) may come from the entire history of the universe between the epoch of last scattering, mapped by the microwave background, and the present day.

The COBE Diffuse Infrared Background Experiment (DIRBE) (Boggess et al. 1992; Silverberg et al. 1993; Weiland et al. 1998) mapped the entire sky with 0.3 pixels and 0.7 resolution in 10 bands between 1.25 and 240 μm. The results have been published (Hauser et al. 1998; Kelsall et al. 1998; Arendt et al. 1998; Dwek et al. 1998), based on a zodiacal light model believed to be accurate within a few percent (Kelsall et al. 1998). Their Galactic dust emission model (Arendt et al. 1998) is derived from fits to the DIRBE data and comparison to hydrogen maps and accounts for the variation of dust temperature. Their model for Galactic starlight is derived from external star count models, with no free parameters. The residuals from the modeling were not significantly above the uncertainties except at 140 and 240 μm. The Far-Infrared Absolute Spectrophotometer (FIRAS) instrument on COBE gave similar answers (Fixsen et al. 1998), using three different methods for removing the foregrounds. The agreement is significant because the instruments were calibrated independently. The measured values of the far-IR background are comparable to the total observed Galactic emission at visible and near-IR wavelengths and imply that about one-half of the luminosity of the universe has been obscured by dust and converted into far-IR radiation. Some distant galaxies might be more reddened or obscured by dust than otherwise expected, and the CIB and its fluctuations might be brighter than predicted from visible band galaxy counts.

In this paper we continue our search for a CIB by trying to detect its spatial structure in the DIRBE data. The method is similar to that first suggested by Gunn and later applied in the visible (Shectman 1973, 1974) and UV (Martin & Bowyer 1989). Recently, Vogeley (1997) applied the method to the Hubble Deep Field, constraining the amount of the visible cosmic background, concluding that there is very little visible cosmic background from unresolved sources. They considered arcsecond scales, at which fluctuations are dominated by random parts of individual galaxies (Vogeley 1997) or cluster (Shectman 1974) profiles. We apply the method to degree scales where the dominant contribution comes from galaxy clustering. Our previous results are described by Kashlinsky et al. (1996b, hereafter Paper I) and Kashlinsky, Mather, & Odenwald (1996a, hereafter Paper II).

If we assume that the galaxies producing the CIB are clustered, the amplitude of the resulting CIB two-point correlation function, \(C(θ)\), depends on the rate of the flux emission, \(dF/dz\), and on the galaxy two-point correlation...
function, $\xi(r; z)$. If the latter is known on the linear scales subtended by a given angle, measurements or upper limits on $C(\theta)$ can constrain the levels of the CIB emitted by clustered material. Measuring or limiting the structure of the CIB can be especially valuable in the mid- to far-IR bands, where the foreground emission is very bright, but smooth.

The plan of the paper is as follows. In § 2 we define the quantities used and provide mathematical background. We show that for the measured galaxy correlation function we expect fluctuations of about 5%–10% in the CIB flux on 0.5 scales, almost independently of the particular mechanism of the CIB production. We discuss our search for the CIB structure in the DIRBE data between 1 and 100 $\mu$m. The last two bands of DIRBE—140 and 240 $\mu$m—are not useful for this because of the large instrument noise there. Section 3 deals with the beam profile and the spatial window function for power spectrum analysis. Section 4 discusses the data for $\lambda \leq 5 \mu$m, where the foregrounds are dominated by stellar emission from the Galaxy. We find a significant trend in the amplitude of the measured fluctuations with Galactic latitude. When our star removal algorithm is applied to simulated data, we recover the measured slope of the correlation with Galactic latitude, and extrapolation of the simulated fluctuations to $\csc |b| = 0$ leads to a zero intercept. We derive a formula relating the measured fluctuations to star counts and show that for a plane-parallel Galaxy there should be a simple relationship between the $b$ dependence of the fluctuations and the slope of the star counts function at the Galactic pole. Then we show that for $90^\circ < l < 270^\circ$ and $|b| > 20^\circ$ the measured structure of the Galaxy fits the plane-parallel model. Furthermore, the scatter in the $C(0)$–$\csc |b|$ plot is low enough to allow an extrapolation to $\csc |b| = 0$, with positive intercepts in all four near-IR bands. We perform a power spectrum analysis, but most of the measured structure is due to fluctuating star counts. If the star removal algorithm removes stars fainter than 5 times the confusion noise limit, there are too few pixels left to calculate a power spectrum. Nevertheless, limits are sufficiently low to be of interest in testing theoretical predictions. We test our methods with numerical and analytic models of the Galaxy and possible CIB contributions and find good agreement with the data. Results from the mid- to far-IR bands, where foreground emission structure is dominated by the large-scale gradients from dust emission in the solar system and the Galaxy, are presented in § 5. The upper limits on these fluctuations are $<1$ nW m$^{-2}$ sr$^{-1}$, depending on wavelength, and provide much stronger tests of the predictions of galaxy counts than do direct measurements from inside the zodiacal dust cloud. We summarize our results in § 6.

2. THEORETICAL PRELIMINARIES

Here we provide a general mathematical basis without specific cosmological and galaxy evolution models. We will use the data to constrain the CIB properties in a model-independent way, and then to constrain models of galaxy evolution. We extend our previous work (Paper I; Paper II; Jimenez & Kashlinsky 1999) to show the power spectrum of the fluctuations and to estimate the typical amplitude of the fluctuations in a model-independent way.

We start with definitions. The surface brightness in the CIB per unit wavelength will be denoted as $I_\lambda$, per unit frequency as $I_\nu$, and per logarithmic wavelength interval $F = \lambda I_\lambda = \nu I_\nu$, and we call them all “flux.” The fluctuation in the CIB flux is then $\delta F(x) = F(x) - \langle F \rangle$, where $x$ is the two-dimensional coordinate on the sky and $\langle \ldots \rangle$ denotes ensemble averaging. We also use the two-dimensional Fourier transform, $\delta F(\theta) = (2\pi)^{-2} \int \delta F_x \exp(-i q \cdot \theta) d^2 q$.

If $\delta F(x)$ is a random variable, then it can be described by the moments of its probability distribution function. The first nontrivial moment is the projected two-dimensional correlation function $C(\theta) = \langle \delta F(x + \theta) \delta F(x) \rangle$. The two-dimensional power spectrum is $P_2(q) = \langle |\delta F_q|^2 \rangle$, where the average is performed over all phases. The correlation function and the power spectrum are a pair of two-dimensional Fourier transforms and for isotropically distributed signal are related by

$$\langle \delta F_q \rangle = \frac{1}{2\pi} \int_0^\infty P_2(q) J_0(q\theta) d\theta$$  \hspace{1cm} (1)

and

$$P_2(q) = 2\pi \int_0^\infty C(\theta) J_0(q\theta) d\theta$$ \hspace{1cm} (2)

where $J_n(x)$ is the $n$th order cylindrical Bessel function. If the phases are random, then the distribution of the flux field is Gaussian and the correlation function, or its Fourier transform, the power spectrum, uniquely describe its statistics. In measurements with a finite beam radius $\delta$ the intrinsic power spectrum is multiplied by the window function $W$ of the instrument. Another useful quantity is the mean square fluctuation within a finite beam, or zero-lag correlation signal, which is related to the power spectrum by

$$\langle (\delta F_q)^2 \rangle_q = \frac{1}{2\pi} \int_0^\infty P_2(q) W_{\text{TTH}}(q\delta) q d\theta$$ \hspace{1cm} (3)

For a top-hat beam the window function is $W_{\text{TTH}} = [2J_1(x)/x]^2 = 0.5$ at $x \approx \pi/2$, where $x = q\delta$, and hence the values of $q^{-1}$ correspond to fluctuations on angular scales of diameter $\sim \pi/q$.

The CIB flux and its structure are measured in projection on the celestial sphere and reflect both the three-dimensional clustering pattern of the galaxy distribution and the rate of emission at redshift $z$. We introduce the three-dimensional two-point correlation function of galaxy clustering, $\xi(r)$, and its three-dimensional power spectrum, $P_3(k)$. These are related via three-dimensional Fourier transforms and, assuming isotropy, are related by

$$\xi(r) = \frac{1}{2\pi} \int_0^\infty P_3(k) j_0(kr) k^2 dk$$ \hspace{1cm} (4)

The projected CIB correlation function is related to the underlying two-point correlation function of the galaxy distribution and the rate of the CIB flux emission via the Limber equation,

$$C(\theta) = \int_{z_1}^{z_2} \int_{z_1}^{z_2} dF dz_1 \frac{dF}{dz_1} \xi(r_{12}; z) dz_1 dz_2$$ \hspace{1cm} (5)

where $r_{12}$ is the proper length subtended by the angle $\theta$ and redshifts $z_1, z_2$. In the limit of small angles, $\theta \ll 1$, the
The Limber equation becomes (e.g., Peebles 1980)

\[
C(\theta) = \int_0^\infty \frac{J_0}{2\pi(1+z)^2} \frac{df}{dz} \left[ \frac{\lambda}{1+z} dL + e^{-\theta^2} \right] d\ell \quad \ldotp (6)
\]

For Robertson-Walker metrics the proper separation is given by

\[
\frac{d\ell}{dz} = (1+z)^2 \frac{dx/dz}{\sqrt{1 - kx^2}} u^2 + \frac{x^2(z)\theta^2}{(1+z)^2}
\]

(7)

\[
e^2 \left[ \frac{dt}{dz} \right] u^2 + d^2(z)\theta^2 \quad . \ldotp (8)
\]

Here \(d_*(z)\) is the angular diameter distance, \(x(z) = d_*(z) / (1 + z)\), \(u\) is the integration variable, \(k\) in equation (7) is in units of physical curvature, and \(t\) is the cosmic time.

The comoving volume occupied by a unit solid angle in the redshift interval \(dz\) in \(dV/dz = (1+z)x^2(z)c\, dt/dz\), and the rate of received power per unit wavelength and collecting area in band \(\lambda\) from each galaxy with absolute bolometric luminosity \(L\) at redshift \(z\) is \([L/4\pi x^2(1+z)^3]) f_\lambda(\lambda/(1+z); z)\). Here \(f_\lambda d\lambda\) is the fraction of the total light emitted in the wavelength interval \([\lambda; \lambda + d\lambda]\), and the extra factor of \((1+z)\) in the denominator accounts for the fact that the flux received in band \(\lambda\) comes from a redshifted galaxy. The contribution to the total CIB flux from the redshift interval \(dz\) is given by

\[
\frac{df}{dz} = \frac{R_h}{4\pi(1+z)^2} \frac{dH_0}{dz} \sum_i \mathcal{L}_i(z) \left[ \frac{\lambda}{1+z} + z \right], \quad (9)
\]

where the sum is taken over all galaxy populations contributing flux in the observer rest-frame band at \(\lambda\), and \(f_\lambda\) characterizes the spectral energy distribution (SED) of galaxy population \(i\). Here \(R_h = cH_0^{-1}\) and \(\mathcal{L}_i(z) = \int \Phi(L; z) L\, dL\) is the comoving luminosity density from galaxies with the luminosity function \(\Phi(L; z)\) at epoch \(z\).

It is illustrative to study the redshift dependence of the flux rate production, equation (9). At small redshifts the factor \((1+z)^{-2} dt/dz\) varies little with \(z\), and the rate of flux production is governed by the comoving bolometric luminosity density \(\mathcal{L}(z)\) and the SED of the galaxy emission \(f_\lambda\). If the luminosity evolution at these redshifts is small, the rate of flux production is governed by the SED shape. If \(f_\lambda(z)\) increases toward shorter wavelengths, then \(df/dz\) increases with \(z\). For \(f_\lambda(\lambda) = \text{const.}\) and no luminosity evolution, the rate is roughly constant with \(z\). In addition, there is observational evidence for an increase of \(\mathcal{L}(z)\) out to \(z \approx 1\) in the galaxy rest-frame UV to near-IR (1 μm) bands (Lilly et al. 1996). At sufficiently high redshifts, the evolution in equation (9) would be offset by the factor \((1+z)^{-2} dt/dz\), so that the rate of production would be cut off at sufficiently large \(z\). This factor is responsible for resolving Olbers’ paradox even for a flat SED.

The SED for rest-frame \(\lambda < 10\mu\text{m}\) is dominated by stellar emission, with a peak at visible wavelengths and a decrease for \(\lambda > 0.7\mu\text{m}\). Consequently, most of the predicted J-band CIB comes from redshifts \(z \sim 0.3−1\), which shifts the visible emission of normal stellar populations into the J band (cf. Yoshii & Takahara 1988). In the M band at 5 μm, most of the predicted CIB comes from \(z > 1−2\). At \(\lambda > 10\mu\text{m}\), the emission is dominated by galactic dust and the situation is reversed, so \(f_\lambda\) increases with wavelength roughly as \(\lambda^2\) with \(\alpha \sim 1.5\). Hence, the dusty starburst galaxies observed by IRAS at low redshifts should make the dominant contribution to the 10 μm CIB. In the far-IR, the measured CIB found by DIRBE can have large contributions from high redshifts.

Measurements of the correlation function \(C(\theta) = \langle \delta F(x + \theta) \delta F(x) \rangle\) have an important advantage over direct determinations of the CIB spatial power spectrum, because they are immune to the discontinuities in the maps created by point source removal. (Because, in practice, \(C(\theta)\) is evaluated from masked data, the uncertainties of the \(C(\theta)\) points are themselves correlated.) However, interpreting \(C(\theta)\) in terms of \(df/dz\) and \(\xi(\theta)\) can be cumbersome because the right-hand side of equation (6) contains a double integral and the three-dimensional correlation function is not always positive. By definition, \(\xi(\theta)\) must be negative on large scales so that \(\int_0^\infty \xi(\theta) r^2 \, dr = 0\).

A simpler method is to work with the Fourier transform of the correlation function, \(P_\delta(q)\), which contains the same information as \(C(\theta)\) and is easier to interpret. As for \(C(\theta)\), the effects of the mask will produce correlations of the power spectrum points with each other. Also, the measured power spectrum is the convolution of the Fourier transform of the mask with the true power. Although the uncertainties in \(C(\theta)\) and \(P_\delta(q)\) can be evaluated from theory, we evaluated them from the data by comparing multiple measurements.

The Limber equation (6) can be rewritten directly in terms of the power spectra, substituting equation (4) into equation (6) and using \(\int \int \int \cdots d^3 d\ell = \pi J_0(x)\) to obtain

\[
C(\theta) = \frac{1}{2\pi} \int_0^\infty d\ell \left( \frac{df}{dz} \right)^2 \frac{1}{c\, dt/d\ell} \int_0^\infty P_\delta(k; z) J_0(kd_\lambda \theta) \, dk \quad . \ldotp (10)
\]

Substituting this expression into equation (2) and using the orthogonality relations for the Bessel functions,

\[
\int_0^\infty J_0(x) J_0 (y) dx = \pi \delta (x - y),
\]

leads to

\[
P_\delta(q) = \frac{c^2}{2\pi} \frac{d^2}{dz^2} \left[ \frac{1}{c (1+z)^2} (1+z)^2 \right] P_\delta[qd_\lambda^{-1}(z); z] \ldotp (11)
\]

Equation (11) involves only one integration and the kernel is always positive. Hence it may be easier to derive cosmologically interesting quantities such as \(P\) and \(\xi(\theta)\) or \(P_\delta(k)\). Determining \(P_\delta(q)\) directly from the data is difficult owing to the masking created by point source removal, and it seems best to compute \(C(\theta)\) and then derive \(P_\delta(q)\) from it.

Figure 1 (left panel) plots the argument, \(q(1+z)/c(z)\), of \(P_\delta\) in the right-hand side of equation (11) for the largest wavenumbers (smallest scales) probed by DIRBE at various redshifts \(z\). The dotted line corresponds to \(\Omega = 1, \Omega_\lambda = 0\); the solid line to \(\Omega = 0.1, \Omega_\lambda = 0\), and the dashed line to \(\Omega = 0.1, \Omega_\lambda = 0.9\), where \(\Omega = 3H_0^2(1-\Omega)\) denotes the contribution of the cosmological constant. For redshifts contributing most of the CIB flux (\(z > 0.1−0.2\); cf. Paper I), the DIRBE instrument probes scales that are in the quasi-linear or linear regime and that thus can be approximated as having evolved at the same rate with time. Thus for DIRBE scales, one can write \(P_\delta(k; z) \approx P_\delta(k; 0) \Psi^2(z)\) (e.g., Peebles 1980), where \(\Psi^2(z)\) accounts for the evolution of the clustering and is normalized to \(\Psi^2(0) = 1\). Then equation (11) can be rewritten in a more compact way:

\[
q^2 P_\delta(q) = \int_0^\infty \left( \frac{df}{dz} \right)^2 \frac{\Psi^2(z)}{H_0 dt/dz} \Delta^2 \left[ \frac{q}{d_\lambda^{-1}(z)} \right] dz \ldotp (12)
\]
with

\[ H_0 \frac{dt}{dz} = \frac{1}{(1+z)^2 \sqrt{1 + \Omega_L + \Omega_m (1+z)^2 - 1}}. \]  

(13)

The left-hand side of equation (12) is the same order of magnitude as the mean square fluctuation (eq. [3]), and we have defined

\[ \Delta^2(k) \equiv R_H^{-1} k^2 P_\delta(k; 0). \]  

(14)

The quantity \( \Delta(k) \) is roughly the fluctuation over the line-of-sight cylinder of length \( R_H \) and diameter \( k^{-1} \). For relevant scales and spectra of density fluctuations, \( \Delta(k) \) increases with \( k \). Hence, the CIB fluctuation on a scale of \( q^{-1} \) is \( \sim [q^2 P_2(q)]^{1/2} \approx F \Delta(q/R_H) \).

The present-day power spectrum of galaxy clustering has been measured on scales corresponding to at least the smallest angular scales probed by DIRBE. The most accurate measurement comes from the Automatic Plate Measuring Facility (APM) survey data on the projected angular correlation function (Maddox et al. 1990). Baugh & Efstathiou (1993) deprojected the APM data to obtain the underlying power spectrum of galaxy clustering, \( P_\delta(k) \). Kashlinsky (1998) used the current large-scale data, and the abundance of objects at high redshifts, to reconstruct the pre-galactic density field over 6 orders of magnitude in mass. His result requires significant fluctuation power on small scales, an early epoch of galaxy formation, and high levels of the CIB and its fluctuations. Figure 1 (right panel) plots the data from the Baugh & Efstathiou (1993) deprojection. On small scales (large \( k \)), \( \Delta^2(k) \propto k^{0.7} \), so the integrand in equation (12) behaves as \( \propto z^{-0.7} \) with an integrable singularity at \( z \to 0 \). While a nonnegligible part of the clustering part of the CIB fluctuations comes from nearby galaxies, much of it arises from galaxies at \( z > 1 \).

In Papers I and II we estimated \( \sim 5\%\mathbf{-}10\% \) CIB fluctuations on a scale of 0.5. One can also see this in a more intuitive way from equation (12). Figure 1 (left panel) plots \( q(1+z)/d(z) \), the largest wavenumber that enters on the right-hand side of equation (12), for \( q^{-1} = 0.5 \). In the Friedman-Robertson-Walker universe, \( q/d(z) \) reaches a minimum at \( z \sim 1-3 \), and its value at the minimum depends weakly on cosmological parameters. Thus equation (12) can be rewritten as an inequality:

\[ q^2 P_2(q) \geq \Delta^2 \left\{ \min \left[ \frac{q}{d(z)} \right] \right\} \int_0^\infty \left( \frac{dF}{dz} \right)^2 \frac{\Psi^2(z)}{H_0^2 (dz/dz)} dz. \]  

(15)

On linear and quasi-linear scales over the range of redshifts that contribute most to the CIB, the quantity \( \Psi^2(z)/[H_0^2 (dz/dz)] \) depends weakly on \( z \) (e.g., on linear scales \( \Psi^2 = (1+z)^{-2} \) if \( \Omega = 1 \)). The integral is of the same order of magnitude as \( F^2 \) because the term \( dF/dz \) is a peaked function with a full-width at half-maximum of order unity. Hence, the relative fluctuation in the CIB is \( \sim \Delta \min \{ q/[d(z)] \} \). Figure 1 (right panel) plots \( \Delta(k) \) versus \( k \) using the power spectrum of APM galaxies from Baugh & Efstathiou (1993); it is an increasing function of \( k \). At the wavenumber of the minimum in Figure 1 (left panel), the CIB fluctuation can be as large as \( \sim 5\%\mathbf{-}10\% \). In models where the bulk of the CIB comes from higher redshifts, this number may be somewhat smaller. Because this method measures a two-point process, the results constrain a measure related to the mean square of the CIB emission rate, i.e., \( \sim \frac{1}{dz^2} dF/dz dz \).

Many models have calculated the expected CIB over the 1–100 \( \mu m \) range (e.g., Partridge & Peebles 1967; Stecker, Puget, & Fazio 1977; Bond et al. 1986; Fall, Charlot, & Pei 1996; Wang 1991; Beichman & Helou 1991; Franceschini et al. 1991; Cole et al. 1992; Malkan & Stecker 1998; Jimenez & Kashlinsky 1999; Dwek et al. 1998). The models are normalized to galaxy counts and predict a typical flux of \( F \sim 10 \text{ nW m}^{-2} \text{ sr}^{-1} \). Deep \( K \)-band counts of galaxies (e.g., Cowie et al. 1994; Djorgovski et al. 1995) or at 12–100 \( \mu m \) (Hacking & Soifer 1991) suggest minimal fluxes of at least a few nW m\(^{-2}\) sr\(^{-1}\). Therefore the CIB fluctuations on 0.5 scales may be \( \sim 1 \text{ nW m}^{-2} \text{ sr}^{-1} \).

The APM measurements of the galaxy correlation function in the blue band might not apply to the infrared. However, the \( r \)-band Palomar survey (Picard 1991), and the \( R \)-band Las Campanas survey (Shectman et al. 1996; Lin et al. 1996) give results identical to the APM survey. On the other hand, for \( IRAS \) galaxies on small (nonlinear) scales the correlation function has a lower coherence length. Saunders et al. (1992) show that on small scales \( IRAS \) galaxies at 60 \( \mu m \) cluster with \( r_{\delta,IRAS} = (r/r_{\delta,IRAS})^{-1} \) where \( r_{\delta,IRAS} \approx 4 \text{ h}^{-1} \text{ Mpc} \) and \( \gamma_{IRAS} \approx 0.6 \), as opposed to \( r_{\delta} = 5.5 \text{ h}^{-1} \text{ Mpc} \) and \( \gamma \approx 0.7 \) for the \( AP \) galaxies (Moore et al. 1994). The slight difference could be due a tendency of the \( IRAS \) dusty starburst galaxies to avoid the central regions.
of rich clusters of galaxies. However, on larger (linear) scales where the galaxy clustering pattern presumably traces the pregalactic density field, IRAS galaxies as measured by the QDOT counts-in-cells analysis (Saunders et al. 1991) are consistent with the APM galaxies power spectrum. These linear to quasi-linear scales are relevant for the DIRBE beam size, so we use the APM numbers for all infrared wavelengths. In surveys with smaller beams, the differences will be more pronounced, but the differences between the IRAS and APM correlation functions do not lead to appreciably smaller mid- to far-IR CIB fluctuations.

In addition to the clustering term, there is a white-noise component in the correlation signal due to individual galaxies (e.g., Peebles 1980). Its amplitude at zero lag is

$$C_{\text{WN}}(0) = \frac{\theta_{\text{beam}}^{-2}}{16\pi^{2}} \int \left( \Phi(L; z) L^{2} dL \right) \frac{dH_{0} dt}{dz} \times \left[ \frac{\lambda f_{0}}{1 + z} \right]^{2} dz. \quad (16)$$

Like white noise from discrete stars, the white noise from galaxies is dominated by the nearest objects, unlike the clustered component (eq. [6]), because of the presence of $x^2(z)$ in the denominator of the integrand in equation (16). Because the galaxies are almost undetectable against the star fluctuations, the galaxy white noise is also negligible.

3. DIRBE DATA AND BEAM PROFILE

The COBE DIRBE instrument provided an all-sky 41 week survey with a 10 band photometer (Boggess et al. 1992). The DIRBE bolometer measurements at 140 and 240 $\mu$m are too noisy for our purposes. The remaining eight bands are centered on wavelengths 1.25, 2.2, 3.5, 4.9, 12, 25, 60, and 100 $\mu$m and are labeled bands 1, 2, 3, and 4, respectively. We obtained the 41 weekly averaged DIRBE maps from the National Space Science Data Center (NSSDC), subtracted the zodiacal light model developed by the DIRBE team (Kelsall et al. 1998; Weiland et al. 1998), and averaged the weeks together.

The maps are stored in a cube format and pixelized into six faces of 256$^2$ approximately square pixels $\sim 0.3$ on a side. For a finite beam the ideal map is convolved with the beam window function. For a circular top-hat beam similar to DIRBE’s square top hat, the Fourier transform of the window function is $W(x) = [2J_1(x)/x]^2$, where $J_1(x)$ is the first-order cylindrical Bessel function. The measured power spectrum is then the product of the underlying power spectrum and the beam window function, i.e., $P_{\text{measured}}(q) = P_{2}(q)W(q\theta)$, where $\theta$ reflects the beam size. On large scales (small $q$), $W(q) \sim 1$. Throughout the rest of the paper, $P(q)$ will refer to the two-dimensional power spectrum of the diffuse emission computed from DIRBE maps after deconvolution from the beam profile, i.e., $P(q) = P_{\text{measured}}(q)/W(q\theta)$.

We determined the effective $W$ and the effective beam size from maps of the beam response function archived at the NSSDC with $181 \times 225$ 0’0065 square pixels, which we embedded in 256$^2$, 512$^2$, and 1024$^2$ pixel arrays. The results for band 1 are shown in Figure 2, and are similar for all eight bands. The solid line shows a top-hat beam profile with $\theta = 0.4$, which is slightly lower than the value of $\theta = 0.45$ adopted in Paper I. From eye fits we adopted $\theta = 0.4$ in band 1, $0.37$ in bands 2 and 3, and $0.35$ in bands 4–8. The combined effects of beam smearing from the motion of the beam during sampling, pixelization, and pointing error would increase the effective beam size by about 10%, and agree better with Paper I. The beam response function drops below 10% at $q^{-1} < 0.15$ and the pixelization prevents measurement below $q^{-1} = 0.1$.

4. NEAR-IR ANALYSIS

4.1. Foregrounds and Point Source Removal

Foreground emission from the Galaxy and the solar system is the main problem in unveiling the expected CIB. At wavelengths less than 10 $\mu$m, the dominant foreground after removing the zodiacal light model is emission from stars in our Galaxy. Using the SAO Bright Star Catalog magnitudes and colors, we simulated $J$-band maps. We found that aside from the large-scale shape of the galaxy and a few star clusters, this foreground has an almost uncorrelated spatial distribution. At 12 and 25 $\mu$m (DIRBE bands 5 and 6) the zodiacal dust is so bright that residuals from the model subtraction dominate the map structure outside the Galactic plane. At wavelengths of 60 and 100 $\mu$m (DIRBE bands 7 and 8) most foreground residual fluctuations come from cirrus dust clouds in the Galaxy, and stars contribute little.

The measured fluctuations contributed by point sources can be reduced substantially by identifying and removing them down to near the confusion noise limit. We used the point source finding routine developed by the DIRBE team and adopted in Paper I and called it the “clipping” algorithm. The data from a selected region (patch) are first used to construct a smoothed model of the sky background emission for the whole patch, as follows. Surrounding each pixel, a window of size $f_{\text{size}} \times f_{\text{size}}$ is searched for the minimal flux value. We use the minimal value rather that the median because the stellar brightness distribution is highly skewed.

The map of this minimal (or lower envelope) flux is then subtracted from the sky map, and the standard deviation $\sigma_{\text{m}}$ is calculated. Then, each of the pixels with flux above $N_{\text{cut}} \sigma_{\text{m}}$ is masked out along with

![Fig. 2.—Window function and beam profile for DIRBE band 1 beam. Crosses show embedding in a 256$^2$ pixel field, asterisks 512$^2$ pixels, and diamonds 1024$^2$ pixels. Solid line is a top-hat profile with a beam radius of $\theta = 0.4$.](image-url)
the 8 adjacent pixels, about twice the DIRBE beam area. This process of fitting a polynomial, identifying bright pixels, and masking them out, is iterated about 5 times, until no more fluxes above \( N_{\text{cut}} \sigma_0 \) are found. In the near-IR no noticeable improvement is reached above \( n_{\text{fit}} = 2 \). We discuss the dependence on \( n_{\text{fit}} \) in § 4.5; for the results presented below we used \( n_{\text{fit}} = 5 \). The polynomial fit was used only for point source masking, no gradients were subtracted from the maps analyzed in this (near-IR) section, and the results at high Galactic latitude (\(|b| > 20^\circ\)) were independent of this choice.

For the near-IR, where foreground stars are the dominant source of fluctuations, there should be a clear correlation between the fluxes at different wavelengths. Furthermore, because most of the flux and fluctuations come from K and M stars, and the DIRBE pixels are large enough to include many stars, the dispersion in the color diagrams for the four near-IR bands should be small. In Paper I we used these properties to search for CIB fluctuations that have a different color because of redshifts or a different source spectrum. We construct a linear combination of maps at two different bands:

\[
\Delta_{12} = \delta F_1 - \beta_{12} \delta F_2 .
\]  

Note that this \( \Delta \) is unrelated to the \( \Delta \) of equation (12). The variance of this map, \( C_b(0) = \langle \Delta_{12}^2 \rangle \), is minimized for

\[
\beta_{12} = \frac{\langle \delta F_1 \delta F_2 \rangle}{\langle \delta F_2^2 \rangle} .
\]  

Because the dispersion in the color index \( \beta = \delta F_1/\delta F_2 \) is small, i.e., \( \sigma_\beta = (\langle \beta^2 \rangle - \langle \beta \rangle^2)^{1/2} \ll \beta_{12}^2 \) with the average taken over all the pixels in the patch, the foreground contribution to \( C(0) \) will be reduced by a factor \( \sim (\beta_{12}/\sigma_\beta)^2 \gg 1 \). If the fluxes in adjacent bands do not correlate, the emission in the two bands comes from different sources or from measurement errors (noise).

Since most of the predicted CIB comes from galaxies at \( z > 0.2 \) with typical stellar populations, its color should differ from that of the foreground stars, and the color subtracted maps should retain some of the CIB structure. If most of the CIB comes from high redshifts so that

\[
\beta_{\text{CIB}} = \langle \delta F_{1,\text{CIB}} \delta F_{2,\text{CIB}} \rangle / \langle \delta F_{2,\text{CIB}}^2 \rangle \leq 2 \beta_{12} ,
\]  

the CIB fluctuations in the color subtracted maps will be larger than in the single band maps. We used equation (18) to make all-sky color-subtracted maps for all the adjacent band pairs, in order to search for a coordinate-independent part of the fluctuations. This method can be generalized to a multicolor subtraction method, e.g., minimize \( \Delta_{123} = \delta F_1 - \beta \delta F_2 - \alpha \delta F_3 \) with respect to \( \alpha, \beta \). We applied this to the DIRBE maps, but without significantly different results.

### 4.2. All-Sky Variance Analysis: \( C(0) \)

We divided the sky into 384 patches, each with 32 × 32 pixels, and clipped each patch individually using the above procedure, with \( N_{\text{cut}} = 7, 5, 3.5 \) and 3. Because the foreground emission at these wavelengths is dominated by point sources, very few pixels are left for \( N_{\text{cut}} < 3 \). We performed the same analysis on 96 patches of 64 × 64 pixels, with similar results.

Our star finding algorithm cuts deeper into the distribution than a simple interpretation might suggest. Each bright star is observed in about 5 pixels, depending on the position of the star. The noise distribution of these pixels in the absence of the bright star still has a non-Gaussian distribution with a nonzero mean. Therefore a star near the clipping threshold will be identified in the pixel where the background noise fluctuations are greatest, rather than at the true star location. Hence, the clipping algorithm finds stars about 1 \( \sigma \) fainter than \( N_{\text{cut}} \), as we confirmed with simulations.

After clipping the 384 patches, we computed the color indices according to equation (18), the single band \( C(0) \), and the color-subtracted \( C_b(0) \) for each patch. Figure 3 shows histograms of the numbers of the remaining pixels after clipping. In the near-IR bands, about 350–450 pixels out of 1024, or about uncorrelated 75–90 beams per patch, remain for \( N_{\text{cut}} = 3.5, 3 \), making the intrinsic uncertainty of \( C(0) \) less than 15%.

In each patch there is a clear correlation between the fluxes in the adjacent bands. Paper I gave flux correlation plots for selected patches and an earlier model of the zodiacal light. The current data give similar results. For bands 1–4 all the patches have correlation coefficient \( R > 0.9 \) between all the pairs of bands. The color indices have very small dispersion, \( \sigma_\beta/\beta < 10% \). Because the dispersion in \( \beta \) is so small, and most of the predicted CIB emission at these wavelengths comes from redshifts \( z > 0.1 \), the CIB fluctuations should not be removed by the color subtraction.

Figure 4 plots the histograms of \( \beta \) for \( N_{\text{cut}} = 3 \). The maps for the band pair [4, 5] have color index \( \beta \approx 0 \) because the dust in the solar system is not strongly correlated with the stars seen in band 4. Between bands 1 and 2, most of the patches have a color index of \( \beta_1 \approx 2 \), typical of K–M giants. In band pairs [2, 3] and especially [3, 4] the spread in \( \beta \) is substantial, suggesting that star fluctuations are not the only source, or that their color is different in different regions. The range of color indices for the band differences [2–3] and [3–4] is reduced for \(|b| > 20^\circ\), but nevertheless remains much wider than for the [1–2] maps. Most of the map peaks are due to stars, as we found them in simulated maps derived from the SAO Bright Star Catalog, using wavelength extrapolations appropriate to the tabulated spectral types.
The lowest limits on $C(0)$ for single bands are similar to those in Paper II, and for the color subtracted bands they are not very different from Paper I. However, we can now address the dependence of $C(0)$ on Galactic coordinates. Figure 5 shows a strong correlation of $[C(0)]^{1/2}$ with $\csc |b|$. Figure 6 plots $[C(0)]^{1/2}$ versus $\csc |b|$ for the color subtracted maps [1–2], [2–3], and [3–4]. Although the color subtracted $C(0)$ is a factor of $\sim 10$ lower than for single bands, the residual fluctuations still depend strongly on Galactic latitude.

4.3. Near-IR Fluctuations from Galactic Stars

We would now like to extrapolate the data to estimate the extragalactic contribution to the fluctuations. We use analytic and numerical models to support a power-law dependence of the stellar contribution to $C(0)$ on $\csc |b|$. We also show how to extrapolate the fluctuations due to Galactic stars to $\csc |b| = 0$ and that we expect no isotropic residual for $C(0)$ due to stars.

We assume that Galactic stars are distributed with a white noise power spectrum; i.e., they are spatially uncorrelated except for large-scale distribution functions such as those in the Bahcall & Soneira (1980) and Wainscoat et al. (1992) models. Thus $C(0)$ in the direction $b$ is given by the following (Galactic) version of the Limber equation:

$$\sigma^2(x) = \sum_i \int \frac{L_i}{4\pi r^2} \frac{L_2}{4\pi r_2^2} dP_{12} = \sum_i \frac{L_i^2}{i^2 4\pi} \int \int_{R_i} n_i(r; x) \frac{1}{r^2} \, dr. \quad (20)$$

Here the sum is taken over stars of type $i$ with intrinsic luminosity $L_i$, $\omega$ is the pixel solid angle, $x = \csc |b|$, $r$ is the distance to each star, and $n_i$ is the number density of stars at that distance in the direction of $b$. The distance to the star is related to the height, $Z$, above the Galactic plane by $r = Z \csc |b|$. In our clipping algorithm, we remove sources exceeding the flux limit of $N_{\text{cut}}$ (standard background polynomial fit). This standard deviation is approximately $\sigma$ from equation (20) above, if the integration limits are chosen to match the threshold for star detection.

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**Fig. 4.** Histograms of color indices $\beta$ from eq. (18) for 384 patches. Thin solid lines are all-sky; thick lines are patches with $|b| > 20^\circ$.

**Fig. 5.** $[C(0)]^{1/2}$ vs. $\csc |b|$ for bands 1–4 and $N_{\text{cut}} = 3.5$

**Fig. 6.** Same as Fig. 5 but for color subtracted maps
and clipping. We find a very good correlation between the deviation of the fit and the value of the residual fluctuations in the patch; if all the histograms had the same non-Gaussian shape the two should be proportional. Hence, the lower limit radius can be approximated to depend on $\sigma$ itself via

$$R_i = \left[ L_i/(4\pi N_{\text{cut}} \sigma) \right]^{1/2}.$$  \hfill (21)

Equations (20) and (21) then allow us to determine $\sigma(b)$ for a given distribution of stars.

We now derive the latitude dependence implied by these equations. As an example, we use a plane parallel exponential model for stars in the Galaxy: $n_l = n_{0,l} \exp \left(-|r|/h_i |x|\right)$. This should be valid if our clipping radius $R$ is small compared to the exponential scale length for the Galactic disk $\alpha^{-1} \approx 3-4$ kpc and is in rough agreement with the disk in hydrostatic equilibrium with the Galactic gravity field (Mihalas & Binney 1981). Now equation (20) becomes

$$\sigma^2(x) = \sum_i R_i^{-1} L_i^2 n_{0,i} \omega E_i \left( \frac{R_i}{h_i |x|} \right),$$  \hfill (22)

where $E_i(u) \equiv \int^\infty_0 t^{-n} \exp (-ut)dt$. We express equation (22) in terms of the total number of stars in the $dR_i$ interval:

$$\frac{dN_i}{dR_i} = n_{0,i} \omega R_i^2 \exp \left(-\frac{R_i}{h_i |x|}\right).$$  \hfill (23)

Then the equation for $\sigma(x)$ becomes

$$1 = N_{\text{cut}}^2 \sum_i \left[ \frac{dN_i}{dR_i} \right] \left. \exp \left(q E_i(q) \right) \right|_{q=R_i/h_i |x|}.$$  \hfill (24)

Consider first the limit when $R_i \ll h_i$. In this case $q < 1$ and the term in square brackets near unity. The number of clipped stars per beam width then becomes

$$N_x = \omega \sum_i \int_0^{R_i} n(r; x)r^2 dr \approx 3^{-1} \sum_i \left( R_i dN_i/dR_i \right),$$

leading to $N_x = N_{\text{cut}}^{-2}/3$. This is much smaller than we find in the data. Our clipping algorithm removes 9 pixels per star and cuts deeper than $N_{\text{cut}} = 3.5$ would indicate, leaving only about 40% of the pixels. This proves that $q > 1$, i.e., bright stars are seen out to the scale height of the disk and beyond.

We now consider the variations in star counts with magnitude in a plane-parallel Galaxy. The differential counts in the magnitude interval $dm$ are given by $dN/dm = \omega [dn/m/dm] n(r_m)r_m^2$, where $r_m \propto 10^{0.2m}$ is the radial distance to the star of apparent magnitude $m$. In the limit of $r_m < h_i$ or $n \approx $ const, the counts converge to the uniform distribution limit, $dN/dm \propto 10^{0.6m}$. In a plane-parallel Galaxy in which the radial structure can be neglected, the number density of stars depends only on $|Z| \propto 10^{0.2m} \sin |b|$, and the differential counts in the direction $x = \csc |b|$ can be related to those at the Galactic pole by

$$\frac{dN}{dm} \bigg|_x = x^4 \frac{dN(m-5 \log_{10} x)}{dm} \bigg|_{\text{Pole}}.$$  \hfill (25)

Hence, once the counts at the pole are measured, we can evaluate the expected number counts in any direction $b$ and then compute the fluctuations in the flux they produce via

$$\sigma^2(x) \propto \int_{-m}^{\infty} 10^{-0.8m} \frac{dN}{dm} \bigg|_x dm.$$  \hfill (26)

This formulation has two advantages: (1) the data on star counts in the polar regions can be used to evaluate the dependence of star fluctuations on $\csc |b|$; and (2) the only assumption is plane parallelism. We tested this assumption with both models and observations.

We now determine $dN/dm$ from the DIRBE data and compare it with prior observations. Figure 7 shows a band 2 histogram of $\log_{10}(F - \langle F \rangle)$ for a 128 pixel square (38.4$\times$38.4) region at the North Galactic Pole (NGP). This logarithmic form is useful for displaying both large and small deviations, and we have normalized it to the equivalent number of stars per magnitude per square degree. The equivalent $K$ magnitude is plotted on the top horizontal axis. With the DIRBE beam of $1.42 \times 10^{-4}$ sr (4.5 pixels), $F = 1$ nW m$^{-2}$ sr$^{-1}$ corresponds to $m_K = 9.45$. Because the CIB level and the smooth foreground contributions from dust and faint stars are not known in advance, we must subtract the mean of this confusion noise before seeking to measure brightnesses of individual bright stars. We define $\langle F \rangle$ as the mean flux of the map clipped with $N_{\text{cut}} = 3.5$; in this case it is 66 nW m$^{-2}$ sr$^{-1}$ or $m_K = 5$, near the turnover of the distribution. The total number of pixels with positive values of the flux after the subtraction is 12,359; the remaining 24.6% of the pixels have negative fluxes. We have plotted both positive and negative deviations to illustrate the asymmetry of the histogram far from the peak, as well as its symmetry near the peak. Negative deviations are statistical fluctuations, while very bright pixels are individual cataloged stars. For the clipped map at $N_{\text{cut}} = 3.5$, a total of 6,380 pixels remain. Out of these, 2,982 pixels (47%) have positive fluxes, showing the symmetry of the flux distribution for the clipped map.

Figure 7 also shows a Gaussian distribution with the variance $\sigma \approx 10$ nW m$^{-2}$ sr$^{-1}$ computed with $N_{\text{cut}} = 3.5$, and for $\sigma = 9$ nW m$^{-2}$ sr$^{-1}$. The difference between these two Gaussians is not large on this plot, showing the difficulty of direct detection of a subpopulation of Gaussian fluctuations with dispersion of 5 nW m$^{-2}$ sr$^{-1}$.

The NGP star counts were observed directly by Elias (1978). We show his data at $K = 1, 2.5, 3.25$ and $8$ with $1^{1/2}$ error bars and our binning of his data. Further NGP data were obtained by the 2MASS survey in the $K_s$-band, almost identical to the DIRBE band 2, and were kindly provided to us by T. Jarrett (1998, private communication). The cumulative counts from these measurements were shown in Figure 1 of Beichman (1996) out to $K_i > 15$, who found that they follow $dN/dm \propto 10^{0.5m}$ (cf. his Table 4). Actual 2MASS star counts from a region of 5 deg$^2$ centered on the NGP are plotted in Figure 7. The agreement between the DIRBE counts, the Elias (1978) and Jarrett (1998, private communication) data, and the $B = 0.3$ extrapolation is excellent over 15 mag, or six decades in flux. South Galactic Pole counts from Minezaki et al. (1998) are also shown and confirm the slope as well as the north-south symmetry of the Galaxy. At $m_K < 1.5$, the counts tend to the slope of $B = 0.6$ coming from stars much closer than the scale height; if $B$ were less than 0.6, the integrated star brightness would diverge at the bright end.

The star counts agree with model predictions. Both Beichman (1996) in his Figure 1 and Minezaki et al. (1998) in their Figure 1 show that the counts are fitted well by extensions of either the Bahcall & Soneira (1980) or Wainscoat et al. (1992) models. An eyeball fit to their data gives $B = 0.3-0.32$ at $K = 11$. The Wainscoat et al. model at
K = 11 shown in Figure 1 of Minezaki et al. (1998) gives log \((dN/dm) \approx 1.35\), whereas continuation of the solid line in our Figure 7 to \(K = 11\) gives log \((dN/dm) = 1.3\) if \(B = 0.3\) and 1.4 if \(B = 0.33\). The agreement between the two slopes and normalizations is thus very good. Even the large-beam DIRBE instrument sees far beyond the scale height of the bright \(K\)-band stars.

We can now estimate the star brightness fluctuations. We begin with a power law for star counts in a plane-parallel Galaxy, \(dN/dm|_{\text{Pole}} \propto 10^{Bm}\). Then the fluctuation due to stars fainter than \(m\) at the Galactic pole becomes

\[
\sigma^2 \bigg|_{\text{Pole}} \propto \int_m^{\infty} 10^{-0.8m' + Bm'} dm'.
\]

(27)

We now derive the latitude scaling law implied by these assumptions. In our clipping method the lower limit on \(m\) in the integral is given by \(10^{-0.4m} \propto N_{\text{cut}} \sigma\). Combining this with equation (25) we find that the total star flux at latitude \(b\) is given by the power law

\[
\sigma \propto (\csc b |b|)^{1/2(\beta - 2)}.
\]

(28)

A uniform, infinite star cloud has \(B = 0.6\), so there is no latitude dependence in that case. If \(B = 0.4\), then we expect \(\sigma \propto \csc b |b|\), while if \(B = 0.3\) then \(\sigma \propto \csc^2 |b|\). The dependence of \(C(0)\) on \(\csc b |b|\) can also be inverted via equations (25) and (26) to give a model for \(dN/dm\). There is a unique relation between the two functions, for a plane-parallel Galaxy and our method of bright star extraction.

The value of \(N_{\text{cut}}\) enters indirectly, since it determines the magnitude \(m\) of the cutoff of detected stars and hence the magnitude at which \(B\) is important. Another measure of the star distribution function is the number of clipped stars, \(N_{\geq}\). In the power law case we have a very simple result:

\[
N_{\geq} = \frac{0.8 - B}{B N_{\text{cut}}^2} = \frac{2p + 1}{3 N_{\text{cut}}^2},
\]

(29)

where \(p = (1.2/B) - 2\) is the logarithmic slope in equation (28). The dependence of \(N_{\geq} \propto N_{\text{cut}}^2\) agrees with our results from DIRBE. Evaluating equation (29) for \(B = 0.6\) (or \(r_m < h\)) we find \(N_{\geq} = 1/3 N_{\text{cut}}^2\) in agreement with the exponential distribution of equation (24) with \(h = \infty\), while for \(B = 0.3\) we find \(N_{\geq} = 5/3 N_{\text{cut}}^2\). The fraction of pixels clipped in the near-IR DIRBE maps at \(N_{\text{cut}} = 3.5\) is \(\sim 60\%\). Because the clipping algorithm uses a mask with about twice the beam area, this corresponds to \(N_{\geq} = 0.35\) so the effective \(N_{\text{cut}} = 2.2 - 2.4\) for \(p = 2.0 - 2.5\). The effective \(N_{\text{cut}}\) is less than the input value of 3.5 because each star is observed in many pixels, each with its own noise.

Similarly, equations (25) and (26) allow us to evaluate the expected variation of the residual fluctuation with \(N_{\text{cut}}\), \(\sigma \propto N_{\text{cut}}^{-1} + 0.8/B\). In the limit of a spatially uniform distribution, \(B = 0.6\), and \(\sigma \propto N_{\text{cut}}^{1/3}\). For \(B = 0.3\) it follows that \(\sigma \propto N_{\text{cut}}^{1/7}\), and reducing \(N_{\text{cut}}\) from 7 to 3.5 should decrease \(C(0)\) by a factor of 10, in agreement with the DIRBE data.

To summarize this section, we have shown that, for \(dN/dm \propto 10^{Bm}\) with \(B = 0.3\) out to \(K < 15\), and a plane-parallel Galaxy, we should recover a unique dependence for Galactic star fluctuations, \([C(0)]^{1/2} \propto (\csc b |b|)^2\). This value of \(B\) is consistent with the \(N_{\text{cut}}\) dependence of the fraction of clipped pixels and the amplitude \(\sigma\) of the remaining fluctuations in the DIRBE data.
Figures 8 and 9. The dependence on longitude is significant data with 90°-tic longitudes between 90° and 270° and high latitudes there the Galaxy. We divided the sample of 384 patches into four mental parts of the signal, minimizing for example, with a subtracted [2

We tried several fitting functions for the 127 patches for which |b| > 20°, 90° < l < 270°:

\[ \sqrt{C_{\text{fit}}(0)} = a + A(csc \ |b|)^p, \]  
where \( a \) presumably contains the cosmological and instrumental parts of the signal,

\[ C_{\text{fit}}(0) = a^2 + A^2(csc \ |b|)^{2p}, \]  

\[ \sqrt{C_{\text{fit}}(0)} = a + \frac{1}{2} \sum_{i=1}^{7} A_i(csc \ |b|)^i, \]  

and

\[ C_{\text{fit}}(0) = a \exp(A \ csc \ |b|), \]  

minimizing, for example, \((\langle [C(0) - C_{\text{fit}}(0)]^2 \rangle)^{1/2} \) with respect to \( a, A, p \). The fits for equation (31) are shown in Figures 10 and 11 and in Table 1. Except for the color-subtracted [2 - 3] maps, all fits give positive values of \( a \). The color-subtracted maps [2 - 3] and [3 - 4] have large scatter in the color indices at which the variance \( C(0) \) is minimized, and do not allow for a robust determination of \( a \). Note that \( p > 1 \) as expected from the model. The star and CIB fluctuations add in quadrature, as we demonstrated by simulation, so functional forms such as equation (31) are better justified than the other fits, although all give consistent results. The last row in Table 1 summarizes our limits on \( a \) with errors corresponding to the extreme range from 92% confidence levels from all the fits, around our preferred central value from equation (31).

We estimated the statistical uncertainty as follows. We define a relative variance \( \sigma^2_a \equiv \min \langle ([\text{data} - \text{fit}]/\text{data})^2 \rangle \) and a normalized \( \chi^2_a(a, A, p) \equiv \langle ([\text{data} - \text{fit}]/\text{data})^2 \rangle/\sigma^2_a \). Figure 12 plots deviation histograms versus \( \delta_{\text{fit}} \equiv (C(0) - a^2 - A^2x^2y)/[C(0)]^{1/2} \). For purely Gaussian deviations the histograms would be straight lines of slope \(-\frac{1}{2} \).

We plot contours for \( \Delta \chi^2 = 7 \), which for 3 parameters \((a, p, A)\) corresponds to a confidence level of 92%. Figure 13 shows thus determined confidence contours projected onto the \((a, p)\) plane for bands 1 to 4, according to equation (31). The uncertainties shown in Table 1 correspond to the largest span of \( a \) in the panels in Figure 12; for any given value of \( p \) the corresponding uncertainty levels are reduced significantly. This is the reason for the smaller formal uncertainties on \( a \) when an exponential fit—equation (33)—is assumed. Excluding from the analysis the patches that lie close to the Ecliptic plane further reduces the scatter in Figures 10 and 11, but the contours are almost identical to those in Figure 13. Likewise, keeping only the patches at 120° < l < 240° produces the same result. The results for \( N_{\text{cut}} = 7 \) and 5 are consistent but with a larger uncertainty, whereas for \( N_{\text{cut}} = 3 \) the results are close to those shown in Table 1. The values for \( p \) are in good agreement with the expectations from the star counts. For band 2, the maximal spread is 1.5 ≤ p ≤ 2.6, which corresponds to 0.26 ≤ B ≤ 0.34 in good agreement with Figure 7.

To summarize, we have justified simple power law star count models and a plane parallel model of Galactic fluctuations by comparing the predictions to the measurements. The model applies only far from the Galactic center and Galactic plane. There is a statistically significant residual term in the DIRBE data after extrapolation to zero csc \(|b|\), with consistent values for four different fitting functions and for different \( N_{\text{cut}} \). Plate 1 of our Paper II shows visually that there is no obvious structure in the selected anticenter \(|b| > 20°\) data set. We checked the validity of our simple models by studying the residuals of the fits. Figure 14 shows the residuals for equation (31) for each patch fluctuations, \([C(0)]^{1/2}\) in band 2. There is no apparent correlation with Galactic longitude or latitude and the residuals are similarly independent of the ecliptic coordinates.

To consider whether the instrument itself or the zodiacal light could be a source for this residual, we review the estimates in Paper I. There we constructed maps of the differences between different weeks of observation, and found noise levels \([C(0)]^{1/2} \sim 1.5, 0.3, 0.1\ nW\ m^{-2}\ sr^{-1}\) in bands 1 through 3 for \( N_{\text{cut}} = 3.5 \). These are much smaller than \( a \) in Table 1, and it is unlikely that \( a \) is due to noise. Similar arguments apply to errors in the zodiacal light modeling, which varies from week to week, and would contribute to the measurement error calculation. In any case, in the near-IR bands the contribution of the zodiacal light is small. It should be largest at 12 and 25 \( \mu \)m, where the residual fluctuations are below \( 1\ nW\ m^{-2}\ sr^{-1} \) (Paper II and § 5 of this paper), or <1% of the total foreground. Therefore, the near-IR zodiacal modeling errors should be quite negligible compared to \( a \). Furthermore, zodiacal light has a sharp cusp near the ecliptic plane, and zodiacal model errors would reflect this spatial dependence. We tried excluding patches in or near the ecliptic plane and found no change in \( a, p \). The spatial correlation function that we find for \( N_{\text{cut}} = 3.5 \) (cf. Fig. 19) is significantly different from that expected for zodiacal light. The zodiacal light is smooth except near the ecliptic plane, where dust resonances and asteroid family debris are found.

We conclude that the single band plots all indicate a positive and approximately isotropic residual term that is unlikely to be produced by either instrumental noise or errors in the zodiacal modeling. Since independent contributions add linearly to the combined variance, such a component would contribute only \sim 10%\ to the total dispersion of the confusion noise at the faint end of the flux distribution plotted in Figure 7, and would not be detectable there. The figure shows two Gaussians differing in dispersion by 10%, and they both seem reasonable fits.

4.5. Analytic and Numerical Modeling

Since the measured residual \( a \) cannot be explained by known errors, we must investigate the stellar foreground fluctuations more carefully. To do so, we simulated the confusion noise process both analytically and numerically.

Our analytical approach assumes a power law for \( dN/dm \), with a Fourier method to simulate the histogram of Figure 7. Let \( \mathcal{P}(F) \) be the probability distribution function to find a single star in the line of sight with flux \( F \). The probability
distribution function and its characteristic function $G(f)$ are a pair of one dimensional Fourier transforms, $G(f) = \int \mathcal{P}(F) \exp(\imath f F) dF$. The probability distribution function of finding two stars is the convolution of $\mathcal{P}$ with itself and the characteristic function for it is $G^2(f)$. Similarly, for $n$ stars the characteristic function is $G^n(f)$. For sufficiently large $n$ the characteristic function tends to a Gaussian, which can be seen from expanding $G(f)^n \approx 1 - \frac{f^2}{2} \mathcal{P}(F)$, (Chandrasekhar 1954). For a Poisson distribution with $m$ stars per pixel on the average, the probability to find $n$ stars in a given pixel is $\mathcal{P}(n) = m^n \exp(-m)/n!$. Then the complex Fourier transform of the Poisson distribution of stars with an average of $m$ stars per pixel is $\exp[m(G - 1)]$. For many stars and sufficiently small $f$ or large fluctuations $G(f) \approx 1 - \frac{f^2}{2} \mathcal{P}(F)^2$, and the previous expression converges to a Gaussian distribution whose width $\propto m^{-1/2}$. This prescription was implemented numerically assuming $\mathcal{P}(F) \propto 10^{-2.5B \log_{10} F}$ with $B = 0.3$. Convolution of the predicted star histogram with Gaussian measurement noise and Gaussian cosmic background fluctuations can be included by multiplying $G(f)$ by a Gaussian. The effective beam size sets the maximum value of the apparent $dN/dm$ in the confusion noise region. We are able to reproduce Figure 7 very well, including the negative fluctuations and the transition from the confusion noise region to direct star detection. The result is robust in that the effective value of $B$ measured at the bright end of the distribution is not altered by subtracting the mean value, and clipping with various $N_{\text{cut}}$ has no effect on the value of $B$ at the bright end. The apparent number of bright stars is about 10% larger than the input value, owing to the bias introduced by undetected stars of medium brightness near the detected bright stars.

We also want to know whether there is any feature of our processing algorithm that could produce a spurious residual fluctuation $a$. For this, we need a numerical simulation of the sky, including both stars and possible cosmic terms. We used the DIRBE star model of Arendt et al. (1998), who implemented the 96 component star population model of Wainscoat et al. (1992), including spatial distribution models for the disk, bulge, halo, ring and arm populations. These were evaluated using the $K$-band $dN/dm$ for the central pixel of each patch, and a two-dimensional uniform random number generator, and the stars out to $K = 20$ were placed at the centers of the pixels in the map. The flux from each star was then distributed among 9 pixels according to the measured or assumed beam shape, using delta-function, uniform, and Gaussian beam profiles. If the...
Fig. 9.—Same as Fig. 8, but for color-subtracted maps

Fig. 10.—$[\text{C}(\text{O})]^{1/2}$ vs. $b$ for $J$, $K$, $L$, and $M$ bands for $90^\circ < l < 270^\circ$. Solid lines are fits of eq. (31) using data for $|b| > 20^\circ$. 
correct beam profile is used, the simulated maps processed with our algorithms match the DIRBE histograms in each patch very well.

We also simulated CIB fluctuations, starting with Gaussian fluctuations in Fourier space with power spectrum $P(k) = k^{-1.3}$, as expected on the smallest DIRBE scales, and then multiplied by the top-hat beam window function determined in § 3. The resultant field was then Fourier transformed to real space and normalized to the modeled cosmic variance, $\sigma_{\text{sky}}$. We added these maps to the simulated star fluctuation maps and examined the results.

As was discussed in § 4.2, our clipping algorithm clips effectively about $\sim 1$ $\sigma$ below the nominal value of $N_{\text{cut}}$, because each star is seen in multiple pixels, each with its own noise fluctuations. Furthermore, according to equation (29), there should be no variation with $\csc b$ in the number of stars clipped in each patch if the star counts follow the same $B$ over the relevant range of magnitudes. To test this

\begin{table}
\begin{tabular}{cccccccc}
\hline
Variable & Band 1 & Band 2 & Band 3 & Band 4 & Band 1 & Band 2 & Band 3 & Band 4 \\
& (1.25 $\mu m$) & (2.2 $\mu m$) & (3.5 $\mu m$) & (4.9 $\mu m$) & 1 & 1 & 1 & \\
\hline
$\sqrt{C(0)} = a + A x^p$ & & & & & & & & \\
$a$ & $12.5^{+4.5}_{-3.7}$ & $4.8^{+1.8}_{-1.4}$ & $1.9^{+0.6}_{-0.4}$ & $2.0^{+0.2}_{-0.1}$ & $6.7^{+1.6}_{-1.2}$ & $1.9^{+0.5}_{-0.4}$ & 1.47 \\
$p$ & $2.15^{+0.37}_{-0.41}$ & $2.18^{+0.39}_{-0.41}$ & $2.31^{+0.45}_{-0.43}$ & $2.79^{+0.51}_{-0.58}$ & $1.92^{+0.34}_{-0.34}$ & 1.47 & \\
\hline
$C(0) = a^2 + A^2 x^{2p}$ & & & & & & & & \\
$a$ & $15.5^{+3.7}_{-3.7}$ & $5.9^{+1.6}_{-1.3}$ & $2.4^{+0.5}_{-0.5}$ & $2.0^{+0.3}_{-0.2}$ & $7.6^{+1.2}_{-1.4}$ & $1.49^{+0.34}_{-0.33}$ & 1.23 \\
$p$ & $1.78^{+0.27}_{-0.22}$ & $1.79^{+0.28}_{-0.29}$ & $1.89^{+0.33}_{-0.33}$ & $2.03^{+0.50}_{-0.78}$ & $1.49^{+0.34}_{-0.33}$ & 1.23 & \\
\hline
$\sqrt{C(0)} = a + \sum_{i=1}^n A_i x^i$ & & & & & & & & \\
$a$ & $14.9 \pm 5.4$ & $5.6 \pm 2.2$ & $2.5 \pm 1.2$ & $2.7 \pm 1.2$ & $6.6 \pm 0.8$ & $0.9 \pm 0.9$ & \\
\hline
$\sqrt{C(0)} = a \exp(A x)$ & & & & & & & & \\
$a$ & $9.7 \pm 0.6$ & $3.7 \pm 0.2$ & $1.3 \pm 0.1$ & $1.3 \pm 0.1$ & $5.3 \pm 0.2$ & $3.6 \pm 0.2$ & $1.5 \pm 0.1$ \\
$x = [\ln \sqrt{C(0)} - \ln a] / A$ & & & & & & & & \\
$a$ & $8.8 \pm 1.3$ & $3.3 \pm 0.3$ & $1.1 \pm 0.1$ & $0.8 \pm 0.1$ & $4.8 \pm 0.6$ & $1.8 \pm 0.4$ & $1.2 \pm 0.1$ \\
\hline
Summary & & & & & & & & \\
$a$ & $15.5^{+3.7}_{-3.7}$ & $5.9^{+1.6}_{-1.3}$ & $2.4^{+0.5}_{-0.5}$ & $2.0^{+0.3}_{-0.2}$ & $7.6^{+1.2}_{-1.4}$ & $1.49^{+0.34}_{-0.33}$ & \\
\hline
\end{tabular}
\end{table}

Notes.—The ranges correspond to 92% confidence levels. $x = \csc |b|$. 

Fig. 11.—Same as Fig. 10, but for color-subtracted maps.
we computed the effective \( N_{\text{cut, eff}} = [(5/3)N^{-1}]^{1/2} \) according to equation (29) assuming \( B = 0.3 \). The number of clipped stars, \( N_{clipped} \), is one-half the number of clipped pixels since our mask has twice the beam area. The results are plotted against \( \csc |b| \) in Figure 15 for both the real sky and the model Galaxy. The data show that in the anticenter quadrants outside the Galactic plane, there is no trend with \( \csc |b| \), and the effective values of \( N_{\text{cut}} \) have very small dispersion. For the real sky, the mean in this range is \( N_{\text{cut, eff}} = 2.26 \) and the dispersion is \( \sigma_{\text{cut}} = [\langle N_{\text{cut, eff}}^2 \rangle - \langle N_{\text{cut, eff}} \rangle^2]^{1/2} = 0.06 \). For the model sky, the mean \( N_{\text{cut, eff}} = 2.27 \), and the dispersion is 0.07. The effective \( N_{\text{cut}} \) is independent of the simulated beam properties.

We found a simple way to test the assumption of a plane parallel Galaxy. As equation (25) shows, the quantity \( x^{-3}(dN/dm) \) plotted against \( m - 5 \log_{10} x \) should be independent of \( x = \csc |b| \) and be equal to the counts at the pole. We computed histograms for 14 patches of \( 64 \times 64 \) pixels at \( |b| > 20^\circ \), \( |\beta_{\text{ ecl}}| > 30^\circ \), and \( 90^\circ < l < 270^\circ \). The patches were clipped at \( N_{\text{cut}} = 3.5 \) and the average flux for the remaining pixels of each patch was subtracted from the map. The resultant distribution of \( x^{-3} dN/dm \) is plotted in Figure 16 for \( K \)-band, with Poisson error bars for those points that contain at least 25 pixels (or 5.5 stars). The figure confirms that the stars are distributed in a plane-parallel way for this data set, with \( B \approx 0.3 \). The plot includes lines for the model Galaxy, with error bands. They agree with the observations within the errors except at the bright end, where the model is slightly low.

We simulated skies with a cosmic term of \( \sigma_{\text{sky}} = 5 \, \text{nW m}^{-2} \text{sr}^{-1} \) for the same \( 64 \times 64 \) patches. The observations and the simulated model are plotted in Figure 17, with fluxes measured in units of \( [C(0)]^{1/2} \). If all the fluctuations are due to stars drawn from a power law distribution of the same slope, then all of the patches should follow the same line on this diagram. The DIRBE data and the synthetic Galaxy model look nearly identical, and there is no noticeable difference between \( \sigma_{\text{sky}} = 5 \, \text{nW m}^{-2} \text{sr}^{-1} \) and 0. The DIRBE data and the models could be matched even better.
by adjusting the beam shape for the synthetic maps. The simulated confusion noise does not affect the amplitude and slope of the star counts at $m_L < 5.5$.

Using our simulated models we also tested the scaling of the amplitude of the residual variance at given $N_{\text{cut}}$ as a function of the input isotropic component $\sigma_{\text{sky}}$. This is important because our clipping algorithm might remove both cosmic and star fluctuations in a complicated and possibly nonlinear way. We find that even at $N_{\text{cut}} = 3.5$, the residual variance $C(0)$ is to good accuracy the sum of the variances from stars and the simulated CIB contribution. This shows that equation (31) is better justified than the other choices for extrapolating to $\csc |b| = 0$. The choice of beam shape—Gaussian, delta-function, or uniform—has no effect on the effective $N_{\text{cut}}$, but does lead to a systematic change in the residual $C(0)$, with the largest beam area having the highest variance.

Similarly, changing the size $f_{\text{size}}$ of the lower envelope region used by the point source recognition algorithm has no effect on $N_{\text{cut,eff}}$, but leads to a systematic dependence of $C(0) \propto 1/f_{\text{size}}$. Clearly, the larger the value of $f_{\text{size}}$, the more diligently the algorithm recognizes point sources. We used $f_{\text{size}} = 5$, corresponding to about 5 DIRBE beams.

We can now predict what the Galactic star counts ought to be in order to reproduce the $C(0)$ versus $\csc |b|$ relation found in the previous section. Combining equations (25) and (26) shows that for a plane-parallel model the star contribution to the fluctuation is related to $dN_P/dm$ at the pole via

$$\chi \sigma^2(x) = F_0 \int_{m_L - 5 \log_{10} x}^{\infty} 10^{-0.8y} \frac{dN_P}{dy} \, dy, \quad (34)$$

where $F_0$ is the zero magnitude flux; for band 2 it is 6050 nW m$^{-2}$ sr$^{-1}$ per DIRBE pixel. The lower magnitude for our clipping method is given by $m_L = -2.5 \log_{10} \left[ f_m N_{\text{cut}} \sigma_{\text{sky}}(x)/F_0 \right]$. Here $f_m$ is a factor accounting for the beam and the lower envelope used; based on the
discussion in the previous paragraph we expect \( f_m \sim 0.5\text{--}0.6 \). Differentiating both sides of equation (34) leads to
\[
\frac{dN_p}{dm} = 10^{0.4m} 0.410 \frac{[x \sigma^2(x)]/\partial x}{f_m N_{cut} F_0 \sigma/x^2 \sigma(x)/\partial x}. \tag{35}
\]
Here the right-hand side should be evaluated for \( x \) given by
\[
m(x) = -2.5 \log_{10} \left[ \frac{f_m N_{cut} x^2 \sigma(x)/F_0}{1} \right]. \tag{36}
\]
Equations (35) and (36) form a closed set to determine the star counts required to reproduce the \( C(0)-x \) relation, and the star counts at the Galactic poles. They also show whether the \( \text{csc} |b| \)-independent part of \( C(0) \) can be produced by the observed stars.

We can measure the \( f_m \) factor by normalizing the recovered star counts to the DIRBE data at \( m_K = 4 \), where the confusion noise is negligible, and find \( f_m \approx 0.6 \). The solid line in Figure 7 shows the recovered star counts at the pole from \( C(0) \) given by equation (31) with \( a = 0 \) with parameters \((A, p)\) taken from Table 1. The agreement between the star counts inverted from the observed \( C(0)-x \) relation and the actual data is remarkably good, considering the statistical uncertainty in \( A, a \) and \( p \), and given that all three \( [C(0) \text{ vs. } x, N_{cut} \text{ and } dN_p/dm] \) were determined independently. The dashed line in Figure 7 shows the star counts required to reproduce equation (31) with \( a = 4.8 \text{ nW m}^{-2} \text{ sr}^{-1} \). We can now use the simulated sky maps to test the extrapolations to \( \text{csc} |b| = 0 \). We constructed simulated data for 384 patches of \( 32 \times 32 \) pixels that contained both the K-band Galaxy synthetic model, and a contribution from the CIB with \( \sigma_{sky} \) varying from 0 to 20 nW m\(^{-2} \) sr\(^{-1} \). Processing the simulated maps with the standard algorithm, we find that the model without the CIB term is a good match to the DIRBE data at intermediate latitudes, but is significantly steeper as \( \text{csc} |b| \rightarrow 0 \) at \( |b| > 45^\circ \), and has a zero intercept within the statistical errors. On the other hand the simulated data curve at \( |b| > 45^\circ \) flattens out for positive values of \( \sigma_{sky} \).

We also tried fitting the simulated sky fluctuations to the observations, using \( C(0) = A_m \sigma^2(\sigma_{sky}) + a_m \), and found the effective gain and offset \( A_m, a_m \). This approach has the advantage of including all that is known about the geometrical shape of the Galaxy, since the simulated maps use the Wainscoat et al. (1992) shapes for the disk, bulge, halo, ring and arm. If the model includes a cosmic term of the correct amplitude, then we should find \( A = 1, a_m = 0 \). We find that for a model \( \sigma_{sky} = 0 \) the value of \( A_m = 1.4 \pm 0.3 \), consistent with unity, and the numbers for \( A_m \) for positive values of \( \sigma_{sky} \) are similar. We could achieve \( A = 1 \) by better modeling of the DIRBE beam size, since we have already shown that relationship.

We also confirm the values for the residual fluctuations \( a\) in Table 1. If the modeled sky has \( \sigma_{sky} = 0 \), we find \( (a_m)^{1/2} = 7.3^{+2.0}_{-1.8} \text{ nW m}^{-2} \text{ sr}^{-1} \) in agreement with Table 1. Conversely, if we choose a model \( \sigma_{sky} \) similar to a in Table 1, then \( a_m \) should be consistent with zero. We find \( a_m = 0 \) with the 92\% confidence level for 2.8 nW m\(^{-2} \) sr\(^{-1} < \sigma_{sky} < 8.3 \) nW m\(^{-2} \) sr\(^{-1} \) with the central value lying at \( \sigma_{sky} = 5.2 \text{ nW m}^{-2} \text{ sr}^{-1} \).

We can imagine only one possible feature in the DIRBE data processing that could lead to a false conclusion about the latitude dependence. We have shown that for a range of fixed beam size and shape, the sky models all predict a dependence of star fluctuations on latitude that extrapolates to zero fluctuations at zero \( \text{csc} \mid |b| \). There is however some possibility that the effective beam size might depend on the line of sight and hence possibly on Galactic latitude. To explain the measurements, we would have to find an effect that systematically changes the effective beam area by an amount of the order of 30\%, comparable to the fraction of the fluctuations at the Galactic pole that seem to be of cosmic origin.

The DIRBE team measured the actual beam size very carefully from transits of bright stars through the beam, and did not find a significant dependence of beam profile on time or direction. However, as noted elsewhere, the effective beam profile for measuring fluctuations depends on several other effects. The effective beam area is approximately \( \Omega = 2\pi (\theta_{beam}^2/2 + \theta_{pixels}^2 + \theta_{pointing}^2) \), where \( \theta_{beam} \) is the measured beam radius, \( \theta_{pixels} \) is the rms radius of the pixels, and \( \theta_{pointing} \) is the rms (vector) pointing error. The rms pixel size is \( d_{pix}^2(\theta_{pointing}) \), and increases by a factor of \((2/\sqrt{3})^{1/2} = 1.075 \) to 0.142. Ignoring pointing error, this increases the effective beam area by 3.2\%, a negligible amount in this context. The measured DIRBE pointing error is 1.5\% (1\sigma), and increases the effective beam area by a fixed 3.9\%. If, however, the pointing error were much larger than indicated by the statistics of the residuals from the pointing solution fits, and in addition were strongly dependent on Galactic latitude, the effect could be important for us. The pointing solution was the subject of extraordinary scrutiny by the COBE team, and such errors would have been noticed. We conclude that the latitude fitting method is not subject to errors due to changes of the effective beam size with latitude.

### 4.6. Power Spectrum

Although the dominant spatial structure of the near IR maps is simply the white noise of stars, it is interesting to see whether large-scale averages could reveal a CIB component. We computed both power spectra and angular correlation functions, and the results are shown in Figures 18 and 19. We describe the power spectra first.

We used 96 patches of 64 \times 64 pixels (or \( 19^\circ \times 19^\circ \)), and computed spatial power spectra without star clipping, and with \( N_{cut} = 7 \) and 5, leaving over 90\% and 80\% of the pixels remaining at high latitudes. Smaller \( N_{cut} \) left too few pixels for reliable power spectra determination, and showed significant effects of the masking. We computed power spectra for single bands and for the color-subtracted maps, \( \delta \) = \( -\beta \delta_s \), with \( \beta \) evaluated for each patch according to equation (18). After each power spectrum was calculated, it was divided by the beam window function discussed in \( \S 3 \) to take out the instrument signature. The fact that the power spectra are approximately flat confirms that stars are the dominant sources of fluctuations for \( N_{cut} = 5 \), and that the effective window function, accounting for pixelization, map distortion, beam smearing, and pointing errors, is correct.

The single band power spectra also show large-scale gradients produced by the Galactic structure. The amplitude of the power spectrum decreases with \( N_{cut} \), but the overall shape does not change appreciably, indicating that the
beam mask effects are negligible for these values of $N_{\text{cut}}$. The amplitudes of the power spectrum for all angular scales are too large, and the shape too wrong, to allow for detection of the CIB structure. However, for some patches at $N_{\text{cut}} = 5$ the spectrum is close to that expected from Table 1. For $N_{\text{cut}}$ of 5 or more, we can not extrapolate these power spectra to $\gamma \sim 0$ for most of the scales probed by $P(q)$, and for smaller $N_{\text{cut}}$ there are too few pixels left.

Figure 18 shows the power spectrum for patch No. 7, at Galactic $(l, b) = (115^\circ, 61^\circ)$, and ecliptic $(b_{\text{Ecl}}, \lambda_{\text{Ecl}}) = (56^\circ, 163^\circ)$. In the single bands, the amplitude of the power spectrum decreases strongly after point source removal, but the shape remains approximately the same. The solid line in each panel shows the CIB signal according to Table 1, assuming that the CIB power spectrum has $P \sim q^{-1.3}$. This is a valid approximation for scales below one degree, but on large scales the power index of the CIB power spectrum may be different. The power spectrum of the foreground exceeds the estimated CIB by only a modest factor, particularly in band 4. An instrument with a smaller beam might detect these fluctuations directly.

Color subtraction significantly reduces the foreground structure, as shown in Figure 18. Without clipping, we find a reduction by a factor of $\sim 30$ between adjacent bands, although the spectra are still flat from star fluctuation noise. Galactic stars are dominant in band 4 and zodiacal and cirrus emission are dominant in band 5, so these bands are not strongly correlated. For this patch, color subtraction of bands 4 and 5 shows structure dominated by cirrus dust, which has $P \sim k^n$ with a steep $n = -2$ to $-3$; see §5. The solid line in the lower panels shows the estimated small-scale CIB between band 1 and band 2 with logarithmic slope of $-1.3$ and $C(0)$ taken from Table 1. In bands $[2-3]$, we plot $[C(0)]^{1/2} = 2$ nW m$^{-2}$ sr$^{-1}$.

For lower values of $N_{\text{cut}}$ and $q^{-1} > 1^\circ$, the amplitude of $P(q)$ is comparable to that expected from the CIB according to Table 1. Furthermore, the slope of the power spectrum for this patch flattens significantly in the color-subtracted

![Figure 18](image1.png)

**Fig. 18.** Near IR power spectra for Patch 7, well above the Galactic and Ecliptic planes. Crosses are before point source removal, asterisks show and diamonds show $P(q)$ expected if the CIB power spectrum scales as $q^{-1.3}$ for $a$ in Table 1.

![Figure 19](image2.png)

**Fig. 19.** Angular correlation function evaluated for patch 7 for $N_{\text{cut}} = 3.5$. The zero-lag value is plotted at $\theta = 0.15$, and numbers below $0.1$ nW m$^{-2}$ sr$^{-1}$ are not shown. The filled circle shows $a$ from Table 1, normalizing a CIB correlation function with $C(\theta) \propto \theta^{-6.7}$. Diamonds show the absolute value of the correlation function in the negative range.
maps, and at some scales and bands approaches the logarithmic slope expected from the CIB. However, inspection of the power spectra shows that even in the color-subtracted maps, we do not detect a cosmological signal for $N_{\text{cut}} \geq 5$. The spectra are all consistent with the white-noise stellar distribution and large-scale Galactic gradients. To check for a cosmological power spectrum consistent with Table 1 we would have to clip to much lower levels of $N_{\text{cut}}$ where beam masking problems prevent reliable determination of the power spectrum.

For $N_{\text{cut}} < 5$, it is better to compute angular correlation functions, which are unbiased by masking effects, even for $N_{\text{cut}} = 3.5$ where less than 50% of pixels remain. We evaluated $C(\theta) = \langle \delta F(x \pm \theta) \delta F(x) \rangle$, averaging over the remaining pairs of pixels separated by angular distance $\theta$. The points at different $\theta$ are not statistically independent of each other. For $N_{\text{cut}} = 7$ and 5 the correlation function is that of white noise, a delta function at zero lag, with a level consistent with the power spectrum plotted in Figure 18. It remains close to the zero-lag value on scales inside the beam, $\theta < 0.5$, and rapidly drops to very small (positive or negative) numbers on larger scales. For $N_{\text{cut}} = 3.5$ for patches with low values of $C(0)$ the correlation function flattens significantly, which would be consistent with CIB structure at the levels of Table 1. Figure 19 shows the correlation function for Patch 7 at $N_{\text{cut}} = 3.5$, for 10 bins of 1 degree width. The straight line illustrates the slope of a CIB correlation function $\propto \theta^{-0.7}$. The correlation function amplitude and slope are consistent with Table 1 and a CIB interpretation. Large-scale gradients in the star and dust populations are responsible for the positive correlation function at very large angles.

A further comparison between the results in Table 1 and the power spectrum analysis can be made in the following way. For $N_{\text{cut}} = 5$ at each angular scale $q^{-1}$, we evaluate the minimal value of $P(q)$ for all the patches, and plot the value of a typical fluctuation, $[q^2P(q)/2\pi]^{1/2}$, versus $\theta = \pi/q$ in Figure 20. The slope of the data is roughly that of white noise, $[q^2P(q)/2\pi]^{1/2} \propto \theta$, implying that we are still seeing Galactic stars. The upper limits over angular scales of $2^\circ < \theta < 15^\circ$ are

$$\delta F_{\text{rms}}(\theta) \leq A\left(\frac{\theta}{5}\right)^{-1},$$

with $A = 6, 2.5, 0.8, 0.5$ nW m$^{-2}$ sr$^{-1}$ for bands 1–4, respectively. The process of finding an all-sky minimum of $P(q)$ produces a very smooth curve whose uncertainties are pre-

![Fig. 20.—All-sky minimum fluctuation $[q^2P(q)/2\pi]^{1/2}$ vs. $\pi/q$. Shaded areas show the range expected from Table 1, assuming the CIB power spectrum scales as $q^{-1.3}$.](image-url)
dominantly systematic. These are strong constraints on galaxy evolution. The shaded areas represent the power spectra according to Table 1, assuming that the CIB \( P(q) \propto q^{-1.3} \). Table 1 is consistent with the upper limits given by equation (37), except in band 4. In band 4, Table 1 would require a small scale for the turn-over in the CIB power spectrum, implying that much of it comes from high redshifts.

5. RESULTS IN MID- TO FAR-IR BANDS

5.1. Foregrounds and \( C(0) \) Analysis

At wavelengths greater than 10 \( \mu \)m, dust in the solar system and the Galaxy produce most of the foreground emission. These sources are smooth on small scales, so do not necessarily prevent detection of CIB fluctuations. Odenwald, Newmark, & Smoot (1998) detected <100 nearby galaxies in the DIRBE data at wavelengths greater than 10 \( \mu \)m, but with the exception of M31 and the Large and Small Magellanic clouds, the galaxies are unresolved. Because there are few discrete sources in the mid and far-IR DIRBE data, we can clip the DIRBE maps to lower values of \( N_{\text{cut}} \) and keep the same number of pixels as for the near-IR bands.

Clipping to low values of \( N_{\text{cut}} \) would remove some fluctuations in the CIB as well as the foreground. If the CIB zero-lag signal were as high as the upper limit found in the maps, clipping at \( N_{\text{cut}} = 2, 2.5, \) or 3 levels would decrease the real \( C(0) \) by 20%, 10%, and 0.001%, respectively. Given that we find only upper limits, clipping down to \( N_{\text{cut}} = 2 \) is safe but lower levels would require interpretation.

Figure 21 shows histograms of the number of pixels remaining in each patch of 32 \( \times \) 32 pixels after removing point sources with \( N_{\text{cut}} = 2 \). Because of the extended nature of the foreground emission in these bands, we removed large-scale gradients first, using polynomials of order up to 4. Figure 22 shows the variation of the residual \( C(0) \) with \( \text{csc} \mid b \mid \) at \( N_{\text{cut}} = 2 \) for \( 90^\circ < l < 270^\circ \). A significant fraction of the foreground emission still comes from the zodiacal light, even after subtraction of the DIRBE zodiacal light model, and contributes to the large scatter in the plots. Figure 22 also shows the dependence of \( C(0) \) on Galactic latitude for the 111 patches that also have Ecliptic latitude \( \mid \beta_{\text{Ecl}} \mid > 25^\circ \). The scatter is reduced but not enough to extrapolate to \( \text{csc} \mid b \mid = 0 \).

In bands 5 and 6 there is only a weak dependence on \( b \), and the amplitude of the typical fluctuations seen in these bands changes by only \( \sim 50\% \) between \( \text{csc} \mid b \mid = 1 \) and 3, showing approximate isotropy for \( \mid b \mid \geq 20^\circ \). We interpret the lowest values of \( [C(0)]^{1/2} \) as upper limits on the CIB fluctuations. They are slight improvements over Paper II, and are shown in Table 2.

![Fig. 21. Histogram of pixels surviving clipping at \( N_{\text{cut}} = 2 \) in the mid- to far-IR DIRBE bands for 384 patches of 32 \( \times \) 32 pixels. Solid line is band 5, dotted line band 6, dashed line band 7, and dot-dashed line band 8.](image)

![Fig. 22. \( [C(0)]^{1/2} \) vs. \( \text{csc} \mid b \mid \) for bands 5–8 and \( N_{\text{cut}} = 2 \). Top panels are \( 90^\circ < l < 270^\circ \). Lower panels are \( 90^\circ < l < 270^\circ \) and \( \mid \beta_{\text{Ecl}} \mid > 25^\circ \).](image)
In bands 7 and 8 the dependence on Galactic latitude is more prominent. Extrapolation to \( \csc b = 0 \) with equation (30) gives values of \( a \) in agreement with the lowest \([C(0)]^{1/2}\) shown in Table 2, but with significant error bars. Therefore we again interpret the derived \( C(0) \) as upper limits on the CIB fluctuations. The slope of \( C(0) \) with Galactic latitude is consistent with a plane parallel Galaxy distribution, i.e., \([C(0)]^{1/2} \propto \csc b\). The fluctuations are approximately proportional to the total brightness.

The color diagrams at these wavelengths are not as clean as in the near-IR. In many patches no color correlations between the adjacent bands exist, and the color indices show large variations across the sky. Local variations in the parameters of the Galactic and solar system dust (e.g., density and temperature) are expected on large angular scales. No improvement in the CIB limits was achieved with mid-IR color subtraction.

### 5.2. Mid- and Far-IR Power Spectra

We computed power spectra for the same 96 patches of 64 \times 64 pixels used for the near-IR bands. Both single band power spectra and color-subtracted maps show a weak decrease with clipping threshold, a consequence of the extended character of the foreground emission. The reduction of the foreground with the color subtraction is not as large as in the near-IR bands. In order to preserve information about \( P(q) \) at all angular scales, no gradients were removed before power spectrum analysis.

The power spectra have a shape typical of the known cirrus distribution (cf. Gautier et al. 1992). The spectrum \( P(q) \) is steep: \( P(q) \propto q^{-n} \) with \( n \approx 2.5-3 \). This is consistent with little small-scale structure and, hence, no strong dependence in the resultant \( C(0) \) on \( N_{\text{cut}} \). It is also consistent with the power spectrum of cirrus emission measured on arcminute and degree scales (Gautier et al. 1992; Wright 1998). Figure 23 shows power spectra for the same Patch 7 used in the near-IR analysis. It shows the steep power spectrum typical of the cirrus distribution with \( P(q) \propto q^{-3} \) for most of the scales. Note that some of the fluctuations at 12 and 25 \( \mu m \) can be due to errors in modeling the zodiacal light.

As in the near-IR analysis, we evaluated the minimal values of \( P(q) \) for each angular scale \( q^{-1} \) across the entire sky. Figure 23 shows these minimal values of the fluctuation \([q^2P(q)/2\pi]^{-1/2}\) as a function of angular scale \( \pi/q \). The minimal values come close to, but are not as small as, those in Table 2, which were evaluated at \( N_{\text{cut}} = 2 \) after gradient subtraction. Even in the patches with the least fluctuations, the power spectrum is still as steep as \( P \propto q^{-2} \), leaving the fluctuation \([q^2P(q)/2\pi]^{-1/2} \approx \text{const} \). This is indicative of cirrus emission in all areas of the sky. Infrared sky surveys with higher angular resolution should be able to reduce this contribution, and possibly uncover the CIB fluctuations.

### 6. CONCLUSIONS

Figure 24 summarizes our results. At 1–5 \( \mu m \), we find a positive residual fluctuation by extrapolating \( C(0) \) to zero \( \csc b \). Based on detailed numerical and analytic models, this residual is not likely to originate from the Galaxy, our clipping algorithm, or instrumental noise. We conclude that

| Band      | Upper Limit (nW m\(^{-2}\) sr\(^{-1}\)) |
|-----------|-----------------------------------------|
| Band 5 (12 \( \mu m \)) | 1.0 |
| Band 6 (25 \( \mu m \)) | 0.5 |
| Band 7 (60 \( \mu m \)) | 0.8 |
| Band 8 (100 \( \mu m \)) | 1.1 |

FIG. 23.—Upper panels are near-IR power spectra in for Patch 7. Lower panels are all-sky minimum fluctuations \([q^2P(q)/2\pi]^{1/2}\) plotted vs. the scale \( \pi/q \). Cross signs are without clipping, asterisks show \( N_{\text{cut}} = 7 \), and diamonds show \( N_{\text{cut}} = 5 \).
The variance found in this way from individual DIRBE bands 1–4 is plotted with diamonds, with 92% uncertainty levels from Figure 13. The results for color-subtracted maps are plotted with triangles at wavelengths halfway between the bands used. We find a positive residual in the color-subtracted map [1–2] between bands 1 and 2, but not in other color subtracted maps. The color-subtracted map [1–2] has a unique color of $\beta_{12} \approx 2$ with little variation across the sky and the limit measures $\langle 0.5$–0.7 nW m$^{-2}$ sr$^{-1}$ at 25 $\mu$m. These limits are lower than those in Paper II and imply strong constraints on how and when the early galaxies formed and evolved.

On larger scales, $2^\circ < \theta < 15^\circ$, we obtain upper limits on the CIB fluctuations from the all-sky power-spectrum analysis: $(\theta/5^\circ) \times \delta F_{\text{rms}}(\theta) < 6, 2.5, 0.8, 0.5$ nW m$^{-2}$ sr$^{-1}$ in bands 1–4, respectively. These limits, when taken in conjunction with our possible detection of the zero-lag CIB signal, limit the turn-over scale in the spectrum of the primordial density field to not much more than $\sim 100$ $h^{-1}$ Mpc.

We fully recognize the difficulty of finding small fluctuations in the presence of larger fluctuations from foregrounds. While we have found no local explanation for our results, it is still quite possible that the fluctuations are not of cosmic origin, but come from some fault of the instrument, the data processing, or an unexpected feature of the Galactic foreground. The best way to resolve this uncertainty is to get better data, such as from a higher resolution sky survey with exceptionally good attention to flat fielding. This may be possible with satellites like the Space Infrared Telescope Facility (SIRTF) or the proposed Next Generation Sky Survey (NGSS), or even from 2MASS data or rocket data.

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REFERENCES

Arendt, R., et al. 1998, ApJ, 508, 74
Bahcall, J., & Soneira, R. 1980, ApJS, 44, 73
Baugh, C., & Efstathiou, G. 1993, MNRAS, 265, 145
Beichman, C. A. 1996, in Diffuse Infrared Radiation and the IRS, ed. H. Okuda, T. Matsumoto, & T. Roellig (San Francisco: ASP), 82
Beichman, C. A., & Helou, G. 1991, ApJ, 370, L1
Boggess, N., et al. 1992, ApJ, 397, 420
Bond, J. R., Carr, B. J., & Hogan, C. 1986, ApJ, 306, 428
Chandrasekhar, S. 1954, in Noise and Stochastic Processes, ed. N. Wax (New York: Dover), 124
Cowie, L., et al. 1994, ApJ, 434, 114
Djorgovski, G., et al. 1995, ApJ, 438, L13

FIG. 24.—Summary. Diamonds are values for $a$ with 92% uncertainties from Table 1. Triangles are for the color subtracted maps, shown at the mean wavelength for the two bands. The [1–2] limits are the left triangle and [3–4] the right. Segments with arrows are upper limits in bands 4–8.
Dwek, E., et al. 1998, ApJ, 508, 106
Elvis, J. H. 1978, AJ, 83, 791
Fall, S. M., Charlot, S., & Pei, Y. C. 1996, ApJ, 464, L43
Fixsen, D. J., Dwek, E., Mather, J. C., Bennett, C. L., & Shafer, R. A. 1998, ApJ, 508, 123
Franceschini, A., et al. 1991, ApJS, 89, 285
Gautier, N., et al. 1992, AJ, 103, 1313
Hacking, P. B., & Soifer, B. T. 1991, ApJ, 367, L49
Hauser, M., et al. 1998, ApJ, 508, 25
Jimenez, R., & Kashlinsky, A. 1999, ApJ, 511, 16
Kashlinsky, A. 1998, ApJ, 492, 1
Kashlinsky, A., Mather, J., & Odenwald, S. 1996a, ApJ, 473, L9 (Paper II)
Kashlinsky, A., Mather, J., Odenwald, S., & Hauser, M. 1996b, ApJ, 470, 681 (Paper I)
Kelsall, T., et al. 1998, ApJ, 508, 44
Lilly, S., et al. 1996, ApJ, 460, L1
Lin, H., et al. 1996, ApJ, 464, L60
Maddox, S., et al. 1990, MNRAS, 242, 43P
Malkan, M. A., & Stecker, F. 1998, ApJ, 496, 13
Martin, C., & Bowyer, S. 1989, ApJ, 338, 677
Mihalas, D., & Binney, J. 1981, Galactic Astronomy (San Francisco: Freeman)
Minezaki, T., et al. 1998, AJ, 115, 229
Moore, B., et al. 1994, MNRAS, 269, 742
Odenwald, S., Newmark, J., & Smoot, G. 1998, ApJ, 500, 554
Partridge, B., & Peebles, P. J. E. 1967, ApJ, 148, 377
Peebles, P. J. E. 1980, Large Scale Structure of the Universe (Princeton: Princeton Univ. Press)
Picard, A. 1991, ApJ, 368, L7
Saunders, W., et al. 1991, Nature, 349, 32
———. 1992, MNRAS, 258, 134
Shectman, S. 1973, ApJ, 179, 681
———. 1974, ApJ, 188, 233
Shectman, S., et al. 1996, ApJ, 470, 172
Silverberg, R. F., et al. 1995, Proc. SPIE, 190, 180
Stecker, F., Puget, J.-L., & Fazio, G. G. 1977, ApJ, 214, L51
Vogeley, M. 1997, preprint (astro-ph 971209)
Wainscoat, R. J., et al. 1992, ApJS, 83, 111
Wang, B. 1991, ApJ, 374, 465
Weiland, J., et al. 1998, DIRBE Explanatory Supplement (Greenbelt, MD: NASA GSFC)
Wright, E. L. 1998, ApJ, 496, 1
Yoshii, Y., & Takahara, F. 1988, ApJ, 326, 1

\[\text{4 Available at http://www.gsfc.nasa.gov/astro/cobe/dirbe_exsup.html.}\]