Coulomb drag between one-dimensional conductors

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We have analyzed Coulomb drag between currents of interacting electrons in two parallel one-dimensional conductors of finite length $L$ attached to external reservoirs. For strong coupling, the relative fluctuations of electron density in the conductors acquire energy gap $M$. At energies larger than $\Gamma = \text{const} \times v_\text{w} \exp(-LM/v_\text{w})/L + \Gamma_\text{+,}$ where $\Gamma_\text{+}$ is the impurity scattering rate, and for $L > v_\text{w}/M$, where $v_\text{w}$ is the fluctuation velocity, the gap leads to an “ideal” drag with almost equal currents in the conductors. At low energies the drag is suppressed by coherent instanton tunneling, and the zero-temperature transconductance vanishes, indicating the Fermi liquid behavior.

Current drag between two parallel conductors provides important information about electron correlations both between the conductors and inside them. The dominant drag mechanism is the momentum transfer between electrons in the two conductors by Coulomb scattering, and in the case of Fermi liquids, the usual phase-space arguments applied to this scattering show that the linear drag trans-conductance (and trans-resistance) should vanish as $T^2$ when temperature $T$ goes to zero. Vanishing zero-temperature transconductance can be viewed then as a manifestation of the Fermi-liquid behavior. In this work, we study the low-energy behavior of the current drag between one-dimensional conductors of interacting electrons \(\parallel\), where the near-ideal equality of currents in the two conductors was predicted in the strong-coupling regime. It is shown below, however, that despite this “ideal” drag at large energies, the reservoirs and impurity scattering suppress the drag at low energies making the linear zero-temperature trans-conductance zero. Experimentally, this behavior of the current drag can be studied in coupled and individually contacted one-dimensional AlGaAs/GaAs heterostructures, where the drag has been found recently.

The set-up we consider consists of two identical parallel 1D conductors of finite length $L$ attached to external reservoirs. Our description of the current drag is based on the sin-Gordon model. In the language of this model, when the conductors are infinitely long, strong coupling between the currents in the two conductors is equivalent to opening of the energy gap $M$ in the spectrum of the relative electron density fluctuations. This requires an interacting repulsive Tomonaga-Luttinger liquid (TLL) in the individual conductors. For conductors of finite length $L$, the strong coupling is expected if $M > T_L \equiv v_\text{w}/L$, where $v_\text{w}$ is velocity of the relative electron density fluctuations. Here and below $e = \hbar = 1$. Deviations between the currents in the two conductors in this regime are due to tunneling of instantons of energy $M$. To describe this tunneling we derive below a low-energy model using the Schmid’s duality transformation, and solve it by fermionization. The main prediction that follows from the solution is the existence of a new low-energy regime of coherent multi-instanton tunneling that leads to a complete suppression of the Coulomb drag at small temperatures and voltage differences $V$ across the two conductors. Inclusion of a weak one-electron impurity scattering with small rate $\Gamma_\text{+}$ suppresses the trans-conductance further, increasing the energy range $\Gamma$ of the coherent multi-instanton tunneling as $\Gamma = \text{const} \times \exp(-LM/v_\text{w})v_\text{w}/L + \Gamma_\text{+}$.

Transport through a one-channel wire confined between two reservoirs of spinless electrons and close to a screening gate can be modeled by a 1D system of electrons whose interaction is local and switched off outside the finite length of the wire. Haldane’s bosonization applied to each of the two wires relates their 1D electron densities $\rho_b$ to the appropriate bosonic fields $\phi_b$ as $\rho_b(x,t) = (2k_bT + \partial_x \phi_b(x,t)) \sum \exp\{ik_b(x) + i\phi(x,t)/2\}/2\pi$. Here summation runs over even $n$, and $k_bT, b = 1, 2$, are the Fermi momenta of the two wires. Evolution of each bosonic field $\phi_b(x,t)$ is described by the Lagrangian of an inhomogeneous TLL

$$\mathcal{L}_b = \int dx \frac{1}{2g_b(x)} \left\{ \left( \frac{\partial_t \phi_b(x,t)}{\sqrt{4\pi}} \right)^2 - \left( \frac{\partial_x \phi_b(x,t)}{\sqrt{4\pi}} \right)^2 \right\},$$

where the coordinate $x$ was scaled by inverse velocity. The inhomogeneous interaction constant of the TLL model, $g(x) = 1 + (g-1)\varphi(x)$, $\varphi(x) = \theta(x)\theta(L-x)$, corresponds to non-interacting electrons outside the wire, $g(x) = 1$, and takes on some interacting value $g(x) = g < 1$ inside it. The fields $\phi_{\pm} \equiv (\phi_1 \pm \phi_2)/\sqrt{2}$ related to the fluctuations of the total and relative electron densities in the wires have the Lagrangians $\mathcal{L}_\pm$ of the same form, and the total Lagrangian is $\mathcal{L} = \sum_b \mathcal{L}_b - \sum_{b=1,2} \mathcal{L}_b$. Inter-wire interaction adds another part to the total Lagrangian, $\mathcal{L}_{\text{int}} = -\int_0^L dx dy U(x-y) \rho_1(x) \rho_2(y)$, where $U(x-y)$ is short-ranged on the scale of the conductors’ length $L$. Substituting bosonic expression for the densities into $\mathcal{L}_{\text{int}}$ we write the total Lagrangian as
\[ \mathcal{L} = \int dx \sum_{\pm} \left( \frac{\partial_t \phi_{\pm}(t,x)}{g(x)\sqrt{8\pi}} \right)^2 - \frac{\pi + \varphi(x)U_0 g}{\pi} \left( \frac{\partial_t \phi_{\pm}(t,x)}{g(x)\sqrt{8\pi}} \right)^2 - \frac{E_F^2 U_i}{2\pi^2} \varphi(x) \cos(2\Delta k_F x + \sqrt{2}\varphi_{-}(t,x)) , \] (1)

where \( U_{0,1}/(2\pi) \) are constants that for weak potential \( U(x) \) are equal to the amplitudes of the forward and backward scattering, and \( \Delta k_F = k_1 - k_2 \). Equation (1) means that the forward scattering modifies the constants \( g_+ \) of the TLL interactions and velocities of the modes: \( g_+ = g/\sqrt{1+gU_0/\pi} \) (i.e., \( g_+ < g_- \)), and \( v_+ = \sqrt{1+gU_0/\pi} \). The inter-wire interaction also leads to backscattering of amplitude \( E_F^2 U_i/2\pi^2 \) in the “−” mode, making evolution of this mode subject to the sin-Gordon Lagrangian. For the zero chemical potential \( \mu \equiv v_\pm \Delta k_F = 0 \), a renormalization-group analysis [1] of the uniform sin-Gordon model at energies larger than \( T_L \) leads to the renormalized values of the parameters in (1) for the − mode. When \( g_- > 1 \), the backscattering amplitude goes to zero if its initial value is sufficiently small, \( g_- - 1 > |U_1|v_-/2\pi \). Otherwise, the backscattering amplitude increases and becomes of order 1, while \( g_- \) renormalizes to 1/2 as the energy cut-off scales down to the gap \( M_0 \) opening in the − mode and evaluated as \( M_0 \approx E_F|U_1|v_-/4\pi^2(1-g_-) \) for \( 1 - g_- \gg |U_1|v_-/4\pi^2 \). A finite \( \mu \) does not change this behavior unless \( \mu > M_0 \). Since \( U_0 > |U_1| \) for realistic interactions, if \( g = 1 \), this scaling always brings the “−” mode into the gapless TLL regime. Therefore, existence of the *interacting repulsive* TLL’s in the individual wires with \( g < 1 \) is a prerequisite for the opening of the gap in the energy spectrum of the relative density excitations and associated strong Coulomb drag. Assuming the gap, we construct below an effective low-energy model for electron transport taking into account impurity scattering inside the wires which introduces the term

\[ \mathcal{L}_{\text{imp}} = -\frac{2E_F}{\pi} \int_0^L dx \sum_j \left[ A_j(x)e^{i\phi_j(x,t)} + h.c. \right] , \] (2)

to the total Lagrangian. Here \( A_{1,2}(x) \) are the amplitudes of weak one-electron backscattering in the two wires.

**Duality Transformation.** An effective model for energies lower than some cut-off \( D' \) specified below can be derived from the expression for the partition function \( Z \) associated with the combined Lagrangians (1) and (2) following Schmid [3]. Without impurities, the − and + modes are decoupled. Integrating out \( \phi_{-} \) in the reservoirs, we see that the “−” part of \( Z \) describes rare tunneling of the massive − mode between neighboring degenerate vacua characterized by the quantized values of \( \sqrt{2}\varphi_{-}(\tau,x) + 2\mu x = 2m \), where \( m \) is integer. Variation of \( m \) by ±1 corresponds to tunneling of an instanton (anti-instanton) through the wire. The tunneling amplitude has been found as \( Pe^{-M/T_L} \), with the energy gap \( M = \sqrt{M_0^2 - \mu^2} = M_0 \sin \varpi \), from evaluation [3] of the instanton mass \( M_0 \) and by mapping onto a free fermionic model [2]. The instanton calculation [3] also gives the prefactor \( P = C \times \sqrt{D}(\sin^3 \varpi M_0 T_L)/\tau \) up to a constant \( C \) of order 1. A high-energy cut-off \( D' \) of the long-time asymptotics of the instanton-instanton interaction \( F(\tau) = \ln \{ \sqrt{\tau^2 + 1}/D' \} \) created by the reservoirs varies with \( \mu \) from \( D' \approx (M_0/\mu)T_L \) for \( \mu = 0 \), to \( D' \approx (M_0/\mu)T_L \) for \( \mu > T_L \).

Weak impurity pinning does not change the form of instantons or their dynamics as long as \( E_F \int dx |A_{1,2}(x)| \ll M \). However, it couples the ± modes, since the impurity Lagrangian (3) restricted to the \( m \)th vacuum of the − mode creates a scattering potential for the + mode

\[ \mathcal{L}_{\text{imp}}' = -\frac{2E_F(-1)^m}{\pi} \int_0^L dx \left[ e^{i\phi_{+}(x,\tau)/\sqrt{2}} \sum_j A_j(x)e^{i(-1)^j\mu x} + h.c. \right] . \] (3)

The sign \((-1)^m\) of this potential varies as vacuum is switched from \( m \) to \( m \pm 1 \) by instanton tunneling. For energies smaller than \( T_L'' = v_+^2/L \), the Lagrangian (3) is equivalent to that of an effective point-like scatterer, \((-1)^m(2V_{\text{imp}}T_L^2/\pi)\cos(\phi_{+}(\tau,0)/\sqrt{2})\), in the uniform non-interacting (\( g = 1 \)) TLL, where the amplitude \( V_{\text{imp}} \) is proportional to \( A_{1,2} \) in the lowest order [2]: \( V_{\text{imp}} \approx (E_F/T_L^2)^{1-g_-/2} \int dx \left( \sum_j A_j(x)e^{i(-1)^j\mu x} \right) \). Variation of the amplitude sign in Eq. (3) can be accounted for by an auxiliary pseudospin variable with the corresponding Pauli matrix \( \sigma_3 \). The energy splitting becomes an operator \( \sigma_3(2V_{\text{imp}}T_L^2/\pi)\cos(\phi_{+}(\tau,x)/\sqrt{2}) \) acting on the pseudospin, and every (anti-)instanton tunneling reverses the \( \sigma_3 \) values with the Pauli matrix \( \sigma_1 \). The partition function can then be written as

\[ Z \propto \sum_{N=0}^\infty \sum_{a_i=\pm} \int D\phi_+ e^{-S_0[\phi_+]} \frac{N!}{N!} \times T_{\sigma_3}[T \left\{ \int \left( \prod_{i=1}^N d\tau_i Pe^{\frac{N T_L}{4} \sigma_1(\tau_i)} \right) \right. \] (4)

\[ \times \left. e^{-\sum_{i,j} a_{i,j} F(\tau_i,\tau_j) + \frac{T_L^2}{8} V_{\text{imp}} \int d\tau \sigma_3(\tau) \cos(\phi_{+}(\tau,0)/\sqrt{2})} \right\} \].

Here \( S_0[\phi_+] = \int_0^\beta d\tau \mathcal{L}_+ \) with \( g_+ = 1 \) is the free TLL Euclidean action, and \( T \) denotes time-ordering. All \( \tau \)-integrals run from 0 to inverse temperature \( \beta \), and \( \sum_j a_j = 0 \). To have all \( \sigma_{1,3} \) matrices time-ordered in
\[ Z \propto T \int \mathcal{D}\phi_+ D\phi_- e^{-S_0[\phi_+] - S_0[\phi_-] + S'[\phi_1, \phi_2, \theta]} \] (5)

The action in (5) is similar to a model of point scatterer with internal degrees of freedom in the two-component TLL. This “dual” model (5) is equivalent to the initial one (4) at low energies. Its imaginary time evolution is described \[ \mathcal{H}_L \] with a gauge transformation \[ \mathcal{H}_{\phi} \] and antisymmetrical tensor \[ e^{123} = 1 \]. Point-like nature of the interaction in (5), and appropriate time dependence of the \[ \theta_\tau \] and \[ \phi_\tau \] correlators, allow us to fermionize them introducing the operators:

\[ \psi_\pm(0) = -\sqrt{\frac{\gamma}{2\pi}} \xi_\pm e^{i\varphi_{\pm}(0)/\sqrt{2}}, \quad \psi_\mp(0) = \sqrt{\frac{\gamma}{2\pi}} \xi_\mp e^{i\varphi_{\mp}(0)/\sqrt{2}}. \]

Here \( \psi_\pm(0) \) are the \( x = 0 \) values of the fermionic fields attributed to fluctuations of the total and relative densities. These fields have linear dispersion relations “inherited” from the bosonic fields, with the cut-offs \( T_L \) and \( D' \), respectively. Substitution of these fields into the Hamiltonian of the dual model (5) makes it free-electron-like with the interaction reduced to tunneling between the \( \psi_\pm \) fermions and the Majorana fermion \( \xi_2 \equiv \xi \).

Application of voltages \( V_{1,2} \) across the first and second wires can be described (4) with a gauge transformation \( \phi_{1,2} \to \phi_{1,2} - V_{1,2}t \) in the real-time Lagrangian (5). It is equivalent to transformation \( \phi_{\pm} \to \phi_{1,2} - \sqrt{2}V_{\pm}t \), \( \phi_{\pm} \equiv V_{1,2}/2 \), in the arguments of the cos-terms in the Lagrangians (4) and (5). Since each instanton tunneling in the \( - \) mode transfers charge \( \Delta\phi_\tau/\sqrt{2\pi} = 1 \) and energy \(-V\), this transformation of \( \phi_{\pm} \) causes a shift \( \theta_{\pm}/\sqrt{2} \to \theta_{\pm}/\sqrt{2} + V\tau \) of the cos-term in the real-time form of the action (5). Assuming that both voltages are sufficiently small, \( |V_{1,2}| < T_L < M \), we neglect their effect on the other parameters. The real-time Lagrangian associated with the fermionized Hamiltonian is:

\[ \mathcal{L}_F = i\xi_1 \partial_\tau \xi(t) + i \frac{\gamma}{4} \int dx \psi_\pm^\dagger(x, t) (\partial_x + \partial_{\tau}) \psi_\mp(x, t) 
- \sqrt{\frac{\gamma}{2\pi}} \psi_\pm^\dagger(0, t) \xi(t)e^{i\omega_f t} + h.c. \] (6)

where the rate of impurity scattering is \( \Gamma_+ = 2T_L V_{\text{imp}}/\pi \) and the rate of the instanton tunneling is \( \Gamma_- = 2\pi C^2 \sqrt{T_L M_0} \sin \frac{\alpha}{2} e^{-2M/T_L} \). The currents flowing through the first and second wire can be related as \( J_{1,2} = (J_+ \pm J_-)/\sqrt{2} \) to the currents of the \( \pm \) modes \( J_{\pm} = -\partial_\tau \phi_3/2\pi \). Duality Transformation makes \( J_- \) equal to the tunneling current \( J_- = -i \sqrt{\Gamma_-/2} \psi_\mp^\dagger(0, t) \xi(t)e^{-i\omega_f t} - h.c. \) in the fermionic model (5). Meanwhile, the tunneling current of the plus mode \( J_+ = i \sqrt{\Gamma_+/2} \psi_\pm^\dagger(0, t) \xi(t)e^{i\omega_f t} - h.c. \) coincides with the backscattered current of the bosonic Lagrangian (4), whose average is simply related to the average direct current (4): \( \langle J_\pm \rangle = V_s/(\sqrt{2}\pi) - \langle J_{\mp} \rangle \).

**Currents.** Since the Lagrangian (5) is Gaussian, both currents can be found with the non-equilibrium Keldysh technique as \( \langle J_\pm \rangle = \left( \frac{\sqrt{2\Gamma_\mp}}{\pi} \right) \langle J(V_{1,2}, \Gamma, \Gamma/T) \rangle \) and \( \langle J_+ \rangle = \left( \frac{\sqrt{2\Gamma_+/\pi}}{\Gamma} \right) \langle J(V_s, \Gamma, \Gamma/T) \rangle \), where

\[ J(V_s, \Gamma, \Gamma/T) = \Gamma \int d\omega f(\omega_f) - f(\omega_f + \sqrt{2\Gamma}) - f(\omega_f) \] (7)

In spite of the coupling between the \( \pm \) modes, each of the currents \( \langle J_\pm \rangle \) is independent of the voltage \( V_s \) applied to the opposite mode. At \( T = 0 \), eq. (6) gives for the currents through the first and the second wire:

\[ \langle J_{1,2} \rangle = \frac{V_s}{2\pi} \left\{ \frac{1}{\pi} \Gamma \arctan \left( \frac{V_s + \Gamma_-}{2\Gamma} \right) \mp \Gamma \arctan \left( \frac{V_s - \Gamma_-}{2\Gamma} \right) \right\} \]

The effect of the Coulomb drag is clearly seen when a voltage \( V_s \) is applied to the first (drive) wire only, \( V_s = V_- = V/2 \). In this case, the average drag current in the second wire \( \langle J_2 \rangle = \frac{V_s}{2\pi} - \frac{1}{\pi} \arctan(V/\sqrt{4}) \) depends only on the total crossover energy \( \Gamma \). The current is suppressed below this energy and approaches a half of the maximum total current \( \sigma_0 V \) above it, where \( \sigma_0 \equiv e^2/2\pi h \) is the conductance of a ballistic 1D conductor. This shows that the Fermi-liquid reservoirs and one-electron impurity scattering have similar effect of suppressing the drag current. However, both mechanisms affect the drive wire current differently, \( \langle J_1 \rangle = \frac{V_s}{2\pi} \left\{ \frac{1}{\pi} \Gamma \arctan(V/\sqrt{4}) \right\} \). Below the crossover, this current reaches maximum \( \sigma_0 V \) in absence of disorder, while the disorder suppresses it. Above the crossover, the current is about a half of the maximum, and the conductance \( \langle J_1 \rangle/V \) takes on the fractional value \( \sigma_0/2 \), which is not affected by the disorder.

In the regime linear in \( V_s \), the transport is characterized by the “diagonal” conductance \( GC \) and trans-conductance \( G_T \). To find them we use that \( J(V/T, \Gamma/T) = V_\psi(1/2 + \Gamma/\pi T)/(2\pi T) \equiv G(T/T)V/(2\Gamma) \) for small \( V \). Here \( \psi(x) \) is the derivative of the di-gamma function, and the function \( G \) decreases monotonically from one at zero temperature,
$G = 1 - \frac{1}{3}(\pi T/2\Gamma)^2$, to zero at large temperatures, $G = (\pi T/2T) - 14\zeta(3)(\pi T/2)^2$, where the zeta function $\zeta(3) \approx 1.2$. The linear trans-conductance $G_T = \sigma_0(1 - G)/2$ is a function of $T/\Gamma$, it approaches $1/2$ at low temperatures $T \gg \Gamma$ and vanishes at $T \ll \Gamma$. The diagonal conductance $G_C = \sigma_0[1/2 + G(T/\Gamma)(1 - \Gamma_+)]$ has a fractional large-temperature asymptotics $\sigma_0(1/2 + (\Gamma_+ - \Gamma_)/\pi(4\Gamma))$. The sign of $\Gamma_+ - \Gamma_-$ determines whether the conductance increases or decreases with decreasing temperature. At zero temperature, $G_C = \sigma_0\Gamma_+ - \Gamma_-$ and varies from 0 for $\Gamma_+ \gg \Gamma_+$ to its maximum $\sigma_0$ in absence of disorder.

\begin{equation*}
G_T = \frac{1}{1 - \frac{1}{2}G(T/\Gamma)} \left[1 - \frac{1}{4\Gamma}G(T/\Gamma)\right]^{-1}.
\end{equation*}

In particular, $R_T$ is always enhanced by the scattering at temperatures larger than $\Gamma$ as $R_T = R_T(T/\Gamma_+) + \frac{9}{4} \zeta(3)\Gamma_+ - \Gamma_-$ and remains small below $\Gamma_-$ (see Fig. 1).

The results obtained in this work are valid for the following hierarchy of energies, $V, T \ll T_L \ll M$, and for sufficiently small amplitude of impurity scattering, $V_{\text{imp}} \ll M/T_L(T_L/E_F)^{9/2}$. They show that without impurity scattering, the linear trans-conductance $G_T$ is exponentially close to $\sigma_0/2$ at $\Gamma \ll T \leq T_L$. $G_T$ should remain exponentially close to $\sigma_0/2$ even at larger temperature, $T \geq T_L$, but with deviations from this value now due to the thermal activation of instantons, $G_T - \sigma_0/2 \propto e^{-M/T}$. With further increase of temperature beyond $M$, $G_T$ is expected to deviate from $\sigma_0/2$, decreasing as $1/T$, as can be seen from a solution [13] of a similar problem. When $T_L$ approaches $M$ (e.g., with decreasing conductor length) the width of the peak in the $G_T(T)$ dependence around $T_L$ reduces and this dependence smoothly goes over into the perturbative regime. In this case, similarly to the strong coupling regime, $G_T \propto T^2$ at low temperatures, $T \ll T_L$, but $G_T \propto T^{4/3 - 3} \propto T^{1/2}$ at $T \\geq T_L$. The trans-conductance can even grow with $T$, in particular, $G_T \propto T^{2/3}$ for 1D Fermi liquid conductors above $T_L$. Note, however, that the Fermi liquid description of the drag problem is inconsistent, since it takes into account only the intrawire part of the Coulomb interaction.

To evaluate the minimum length $L_m = v/4M$ necessary for observation of the strong Coulomb drag, we notice that in the optimum situation for observation of drag, the wire parameters: the width $w$, their separation $d$, and the distance $D$ from each wire to its screening gate, are related by the set of inequalities: $w < D < d \approx 1/k_F$. Standard expressions for the electrostatics of inter-wire and intra-wire interactions show that these inequalities provide sufficiently small TLL interaction parameter $g$, and not-too-small backscattering amplitude $U_1$ of the interwire interaction, while keeping the upward renormalization of $g$—caused by the forward scattering $U_0$ to a minimum. We evaluate $L_m \approx 3\mu m$ [17] for $w = 10nm$, $D = 20nm$, $d = 40nm$, $k_F = 30nm$. These parameters are realistic for the present-day GaAs heterostructures and close to satisfying the inequalities.

In conclusion, we have shown that strong Coulomb drag occurs between currents of repulsive TLL in two 1D conductors at temperatures and/or voltage differences above some crossover energy $\Gamma$ and below the energy gap $M$ of the relative density fluctuations. The drag is suppressed below $\Gamma$ and grows to the ideal one at $T, V \rightarrow T_L$.

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