Student’s conceptions and geometry problem-solving of the distance in cube

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Abstract. The purpose of this study is to explore student's conception in solving the distance measures problem on the cube. The subject of this study was a male student of intermediate abilities who was selected for certain considerations on three problems encountered in the process of completion: verbal, spatial, and concept application problems. Overall, student indicates a suitability between the distance concept definition and his conception. Student’s conception about the distance between a point and a line is a perpendicular distance but not always the distance between points with one other point passed by the line; the distance between a point and a plane is a perpendicular distance between point and point on altitude of plane if the plane is an equilateral triangle; while the conception of the distance between a line and a plane is a perpendicular distance between the point and a line, where the point is result of spatial transformation of a line, and the line is the representation of a plane. The student has been an error in solving the problem, and this is an example case of the stages in Van-Hiele's theory are not a strict sequence. The errors are the result of unfounded conceptions of knowledge and experience and do not did the further verify. Learning with investigative and observation activities is an approach to problem-solving especially on geometry.

1. Introduction

Geometry is one of the branches of mathematics and one of the important mathematics learning topics [1, 2, 3, 4, 5]. Geometry is contained in the curriculum of mathematics education in Indonesia from basic to the high-level school. At the intermediate level, geometry studies the relationship between points, lines, and planes [6]. The relationship means is the distance and how many degree of the angle of two geometry elements in a wake of space in the three-dimensional topic. This study only explores the distance on geometry topics. Geometry distance in the solid figure is a continuation of studying geometry on the plane so that its concept is a geometry prior knowledge student [6]. The continuation means that there is a continuity between initial knowledge as a basic concept and a new mathematical concept. It is tended to be the cause of not appropriate with the actual concept base. Such a condition is known as misconception [5, 7, 8].

Moreover, this study would observe regarding be possibility misconception experienced. Misconceptions are the result that arises from conceptual disagreements with students' perceptions [5,
An example of misconception on geometry topic occurring in qualitative descriptive studies [9] is about the distance of two parallel lines and the distance of two parallel planes. Some misconception criteria include incomplete symbols, imprecise in applying concepts, and interpretations that do not correspond to actual conditions [5, 8]. The misconception is a mistake in conception. Conception is a subjective perception of related concepts and processes, intuition of concepts, knowing how, as well as knowledge of mental constructions [7, 8]. Further, conception involves structural thinking constructed from concrete to abstract concepts [4], or in other words, the structural relationship between concept definitions [8, 10]. Conception establishes a stable and permanent understanding and makes understanding last all the time [10, 11]. Conceptions that become knowledge acquired through learning activities, therefore there is a systematic linkage to learning approaches, quality of learning, and the quality of conceptual understanding [4, 5, 11, 12, 13].

Through his conception, students try to organize concepts and information obtained in the learning process. This arrangement is one of the basic methods that contribute to gain insight into the interconnectedness of concepts [4, 12]. The arrangement of mathematical concepts plays a role because the corresponding constructs will have an impact on improving mathematical abilities. Thus, through the process of accommodation, assimilation, and equilibrium the knowledge structure will become increasingly complex, and the structure of knowledge formed will provide support for in-depth understanding [4, 14]. Thus, conceptions and understandings are interconnected [10, 11], as understanding develops and gains the power of conception.

Associated with the topic on geometry, the conception benefits required students in articulating the relationship between image visualization, verbal information, and concept applications [15]. On the other hand, learning the three-dimensional topic in high school encourages students to solve the geometry measurements problem. Problems are constraints facing a person so as not to immediately find a solution [16]. The problem that students must solve in this research is the test of distance on the cube. This research further examines how students answer, about the geometry of the cube, such as: the distance between two points, the distance between the point and the line, the distance between the point and the plane, the distance between two lines, the distance between the line and the plane, and the distance between the two planes, which is a matter of measuring geometry in solid figures [6]. The geometry problem in solid figures relates to spatial intuition [1, 2, 9]. Therefore, it is not possible that these things become some of the factors causing geometry learning to be not easy, because [1, 5, 8, 9] states that sometimes students still show particular difficulty, especially on the achievement of geometry topics, although the paradigm of learning today is an active and constructive process.

Constructing a geometry problem-solving is one of the contributions to student conceptions. Conception will be the rational basis between the known information in geometry problem and the problem-solving found, where the process occurs throughout the problem-solving activity. The search for the role of conception in the process of solving a geometry problem is related to the purpose of this study which is exploring the students' conception of distance in solving geometry measure problem. In addition, this study analyzes the important factors that have a major effect on problem-solving so the geometry measure is not found incorrect solution completion.

2. Research Methodology

This single case study research explores students' conceptions of the distance in solving geometry measures problems. Somethings as a problem in solving the concept of geometry are: (a) verbal problems concerning what is known and what is asked in geometry problem [5, 15]; (b) spatial problems related to spatial orientation, spatial visualization and spatial transformations [1, 2, 5, 6, 15]; (c) concept application problems related to calculations and used concepts in determining of distance [6, 8, 9, 15]. The focus of this study is the conception of concepts related to concept application problems, not about spatial and verbal issues. Usually, low ability students are someone with complicated difficulties, and spatial problems are often experienced by female students [1]. Therefore, there are considerations for the subject to be sampled not in the verbal and spatial problem, so that from some male students in a class, a male student of moderate cognitive abilities is selected.
The geometry test is to determine the shortest distance on the cube, such as: the distance between point $A$ and point $G$, the distance between point $D$ and line $AC$, and the distance between plane $ACH$ and $BEG$. Based on student answer and interrelation of its solution, student's conception would be observed just three problems of geometry distance in the discussion. Subsequently, subjects were interviewed about the concepts related to distance, so that the subject description obtained in the interview section related to the completion of the test process. Test and examples of interview questions are included in the appendix.

3. Result and Discussion
The test answers from the subject were preliminary data of research and obtained 3 of 6 questions solved with the correct answer. Here is a look at how the conception of a student in solving the distance of geometry problems.

3.1. The distance between a point and a line
The question of the distance between the point $D$ to line $AC$ on cube $ABCD EFGH$ has solved the student correctly. Here's a picture of the solution he is made.

![Figure 1](image)

The picture on above shows that student is able to know how to solve it by taking an initial step of his conception on square properties. In the interview section, a student reveals the reason for determining the distance between a point and a line. Here is the summary:

Researcher: *Why did you choose that distance?*
Student: *Because of the distance between the point and the line is the distance from the point and perpendicular to the line. The diagonal side of $AC$ and $BD$ definitely intersect in the middle and these properties are square. So the distance is the distance from point $D$ to the diagonal cut point. In addition, the triangle $ACD$ is an equilateral triangle so the distance is from point $D$ to the midpoint of the $AC$ line.*

Researcher: *Is it possible to distance it to another, eg distance from point $A$ to point $D$.*
Student: *No, this is the point distance with the line, not from point to point.*

The Student's answers are based on the conceptual reasoning of geometry concepts that have been accepted in his learning experiences. In accordance with [17] that conception arises from experience and as experience. In this section, there is no indication of misconceptions about the problem between two elements of geometry property. The student recognizes the relationship of the distance concept to the concept of square and triangle. In Van Hiele theory, it is able to recognize the interconnectedness of concepts and know how to apply the property of geometry at the abstraction level [12, 15, 16]. The one of Van Hiele theory describing how a student thinks when involved with geometry problems [4, 12, 15, 16]. According to the answer above, also seen that student is capable of spatial visualization so he did not show a spatial problem and be able to connect some necessary concepts to solve the problem.
The explanations based on square and triangle concepts are justifications for his reasoning. Therefore, this corresponds a quote which states that solving a problem is often done by looking back on the previous experience with the note that student was familiar with concepts, rules, and schemes in finding solutions [4, 12, 18]. The conception is based on knowledge and experience [13, 17]. Furthermore, this fact is also in accordance with the claim [7] which states that in order to acquire certain mathematical knowledge, the subject may not have a unique conception, but a set of conceptions that may be combined and dependent on the situation involved.

In addition, the student's conception of the distance between point and line can't be represented by point $A$ to point $D$ even if one of the points is bypassed by a line. In other words, the geometry spacing, in this case, can't be represented by the distance between the points with a point that the line passes because the distance represented by the two points is not perpendicular to the line. So that student's conception of the distance concept between point and line is distance between two geometry elements perpendicularly and can't always be represented by the distance between two points even though one other point is bypassed the line. In this case, he knows about the distance concept but has not yet touched on the nearest distance as a result of perpendicular concept. Thus, it is true that knowledge and conception will affect his ability to solve geometry problems [16].

3.2. The distance between point $D$ and plane $BEG$

There are three numbers of geometry problems that are done haphazardly. One of them is about the distance between point $D$ and plane $BEG$.

![Figure 2. Geometry distance completion between a points and a plane](image)

In the above figure, the initial step of the settlement can be declared true because he answer that the closest distance between the point and the plane is the distance from the point to a point passed by the high line of the equilateral triangle $BEG$, if the area is an equilateral triangle. However, in Figure 2 (b), students state that $BK$ is half of $BM$. That is, students indirectly state that the distance of point $D$ and plane $BEG$ is equal to the distance from point $D$ to the midpoint of $BM$. In accordance with the following interview quotes.

**Researcher**: Which distance is meant?
**Student**: The plane $BEG$ is an equilateral triangle and $BM$ is the line of height. The line divides the triangle into two equal parts. Then, the distance is from point to point that is located in the middle of the triangle's high line.

**Researcher**: Why?
**Student**: I assume that the distance is the perpendicular distance from the point to the plane.

Actually, the distance between point $D$ with the midpoint of triangle $BEG$ is not the closest distance and perpendicular from a point to a plane. This is a misconception about the concept of distance and triangle concept in solving of geometry measure problem. One of ability for students in the abstraction stage in Van Hiele theory is to combine forms and properties to give exact definition and informal arguments [16], but in this case, the student is less precise in defining the distance and the
perpendicular position between the two geometry elements. Here, there is a fallacy of way students finish because it is not based on their geometry understanding of the knowledge and learning experience. While [13, 17] have affirmed that conception arises from knowledge and experience. In this case, the student determines the geometry distance just based on assumptions without examination or further clarification. While [19] states that conception is a way of knowing a conceptual object based on observation. This is the cause of failure due to the limitations of the subject in viewing perpendicularly between a point and a plane. In accordance with the assertion [19], that conception is a theoretical model that explains the ability with the limitations of the subject. In the preceding section, student’s conception of geometry distances is the perpendicular distance between two geometry elements, so student’s limitation showed is combining geometry concept to provide the distance’s definition and perpendicular position between two geometry elements. Assumptions are statements based on subjective perceptions. Conception is an explanatory model of a conceptual object [13, 17, 19]. Thus, in this case, student’s conception of the distance between point \( D \) and plane \( BEG \) is the perpendicular distance from point \( D \) to the midpoint of altitude triangle \( BEG \) when the triangle is an equilateral.

3.3. The distance between line AC and plane BEG
There is an ambiguity when student complete test about the distance between line \( AC \) and plane \( BEG \) on cube \( ABCD \) \( EFGH \). The following is presented in figure 3.

![Figure 3. Geometry distance completion between line and plane](image)

In the picture above, he makes a right triangle \( BQR \) with point \( Q \) is the middle of \( AC \) and point \( R \) is midpoint line \( EG \). Spatial visualization helps him to transform triangle \( BEG \) (Figure 3.a) to be hypotenuse side at right triangle \( BQR \) (Figure 3.b). Creating a spatial transformation is the right step for completion. When he interprets the transformation of three-dimensional (3D) to two-dimensional (2D) term, he compares the 3D object and manipulates different representational models of the three-dimensional object, this corresponds to [20]. Furthermore, the construction of 3D transformation term on 2D media can be a key factor [2, 14, 21], and utilization of visual images facilitates student’s understanding for specific situations and discovery of problem solutions [1, 4, 12, 18].

Creating spatial transformation is student’s ability to combine forms (in spatial spaces) and their properties that are related to other forms (from 3D to 2D) so that these abilities are in abstraction stage of Van Hiele theory. But, after the spatial transformation term (triangle \( BQR \)) is made correctly, the student fails to identify triangle and perpendicular properties in the analysis stage on Van Hiele theory. Thus, he is able to get through the abstraction stage, but he is mistaken at the analytical stage, and this is a special case. The abstraction stage in Van Hiele theory is higher than the analytical stage [5, 15], so this case becomes one such example in [4] which states that the stage of development in Van Hiele theory is not a strict sequence stage.

In figure 3 (b), the student knows the length of \( BQ, RQ, \) and \( BR \) so it is based on the length of sides. But the line drawn through point \( Q \) is exactly perpendicular in the middle of line \( BR \). Its means, distance between line \( AC \) and plane \( BEG \) is the length of a perpendicular line. In this case, the student’s conception of the distance is consistent with distance’s definition and concept at the previous
completion. Triangle \( \triangle BQR \) is a right triangle at point \( R \), so that the line was drawn through point \( Q \) and perpendicular to line \( BR \) will not cut line \( BR \) exactly in the middle. In addition, there is an ambiguity. In one point of view, he knows the length of the triangle sides. While at another point of view, he assumes that the line was drawn at point \( Q \) will be perpendicular to the midpoint of line \( BR \). This assumption leads him wrongly so that the process and results in completing the test become less precise.

Assumptions are examples of indications that show a student’s conceptual knowledge was not strong [3]. The student’s assumption is a conception of geometry distance. This condition appropriate the definition was claimed that conception is not only a kind of property or student’s knowledge that is thought to originate from the subject but also a property or student’s knowledge about something at a time in certain situations [7]. Thus, the student’s conception of geometry distance between a line and a plane is the perpendicular distance between a point and a line of the spatial transformation from 3D to 2D. This conception is not contradictory with the definition of distance between two elements of geometry.

### 3.4. The Geometry problem, conception, and learning approach

Some stages in van Hiele theory are visualization, analysis, abstraction, deduction and rigor [5, 15, 1the 6]. While the problem in solving geometry problems such as verbal, spatial, and concept applications [15]. The first step in solving the question of geometry measure is to understand given verbal information. Its verbal information is then made in the form of visualization of the image, and in this case is solid like a cube. After cube is formed, two geometry elements are created so as to create a mental image with the position and distance of the geometry element in spatial space. It's distance such as: point and the line, point and plane, or line and plane. Finding position and distance with the conception of distance’s concept definition, with spatial intuition like spatial visualization, the spatial orientation until to make a spatial transformation term. The conception of distance in spatial space is not a unique conception but there are several combinations of conceptions such as conceptions of perpendicular concept two geometry elements, the oblique projection of the cube image, and still others.

The transformation of abstract spatial space takes place in the mental process. The process of transformation is characterized by creating shapes and attributes that are related to other forms. The process is classified in the abstraction stage on Van Hiele theory [15, 16]. However, after the transformation form is known, the next poses are to identify the shape because there is a spatial orientation between geometry elements so that the measures geometry can be known. It has certain properties that will be useful for calculating geometry distances and solving geometry measures problems. The process of completion does not explain stages in Van Hiele theory hierarchically because the results of this study indicate a subject error in analysis stage but true when in abstraction stage, and this is a problem of concept application. Whereas the analysis stage is lower level than abstraction stage [4]. Therefore, learning and solving geometry problems is necessary for looking back on every solving step. Looking back on the problem-solving step is in line with the process of conception and mathematical investigation in learning and finding a solution. Even, when investigation as a learning approach will be capable of causing cognitive conflict that may shift misconceptions so that reconstruction of cognitive structures is correct [22].

### 4. Conclusion

Based on the results of the study, it is concluded that the conception of the distance’s concept between a points and a lines is a perpendicular distance between two elements of the geometry, but should not always be represented by the distance between two points even though one other point is bypassed the line. Factors that influence in solving geometry problems are knowledge and experience on previous learning concepts, in this case about the triangle and square concept. Meanwhile, the conception of distance between a point and a plane is a perpendicular distance point to point that lies on the altitude of the plane if the plane is an equilateral triangle. The influencing factors in the calculation of
geometry measures distance are student’s ability to combine geometry concepts for defining the distance and perpendicular position of two geometry elements, mastering informal argumentation as a foundation of its geometry understanding. While the conception of distance between a line and a plane is a perpendicular distance between a point and a line from the result of the spatial transformation of two geometry elements in 3D into 2D. The factors affected the completion are student’s analytical abilities of relevant geometry concepts. The student in this research did an error and it indicated that the stages of Van Hiele theory were not a strict sequence. Its errors arise from unconstructed conceptions of knowledge and experience and do not further verify. Learning with investigative and observation activities is a compatible approach to problem-solving especially on geometry.

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Appendix A. Test Items
The $ABCD.EFGH$ cube and the length of edge 4 cm.

1. Calculate the distance between point A and point G.
2. Calculate the distance between point D and line AC.
3. Calculate the distance between point D and plane BEG.
4. Calculate the distance between line AC and line HB.
5. Calculate the distance between line AC and plane BEG.
6. Calculate the distance between ACH and plane BEG.

Appendix B. Interview Items
1. How do you determine the distance?
2. Where is the distance position?
3. Why do you state that the distance is the distance the question is about?
4. How can the distance be the distance referred to in the test?
5. Is there any other distance that also states the distance referred to the test?

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