Dependence of Lamé transformation for concrete pipeline design calculation

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Abstract. Determination of concrete strength and wall thickness for low pressure concrete pipe underground pipeline by possible way to identifications of the known dependencies by entering the correlation “wall diameter/wall thickness” and transformation of the Lamé formula. The conversion of the known dependencies, allowing to determine the normal hoop stress from the external equivalent load and internal pressure according to the Lamé dependence, allowed us to obtain new transformed dependencies for calculating the stresses in pipes under the combined action of internal pressure and external (reduced) load. Using the obtained equations allows you to determine the range of application of concrete (without metal reinforcement) pipes and select the concrete composition for the pipe production. It was determined in this study the stresses in the walls of concrete pipes under the combined action of an external (compressing pipe) load (the pipeline depth – the magnitude of the working fluid pressure) and internal pressure from the fluid transported under pressure.

1. Introduction
For gravity-pressure irrigation and drainage systems, reinforced concrete and concrete unreinforced pipes are used [1, 2]. Concrete pipes are about 2 times cheaper than reinforced concrete ones. They are more durable in the presence of contact with saline groundwater, because they do not contain corrosion-resistant reinforcement. However, in some (severe) laying conditions, their strength is insufficient [3, 4].

The research objective is to determine the stresses in the walls of concrete pipes under the combined action of an external (compressing pipe) load and internal pressure from the fluid transported under pressure. An analysis of the nature and magnitude of the stresses is necessary to select the material of the pipe walls, as well as to establish a possible combination of the maximum external loads (the pipeline depth – the magnitude of the working fluid pressure).

2. Bulk of research
Loads on underground pipelines can be divided into two groups [1, 5, 6]:
1. Pressure of overlying soil, passing vehicles (external load) – \( P \);
2. Pressure of the transported fluid (internal load) – \( q \).

The forces acting in the pipe section are given in Figure 1.
The magnitude of the reduced external load $P$ is determined by the dependence [1]

$$P = 0.325 \cdot H \cdot \gamma \cdot (1 - 0.06H) \cdot (d + 0.8).$$  \hspace{1cm} (1)

For averaged soil conditions at $\gamma = 1.8 \cdot T / M 3$, 

$$P = 0.585 \cdot H \cdot (1 - 0.06H) \cdot (d + 0.8).$$  \hspace{1cm} (2)

where $H$ – pipeline depth, counting from the top of the pipe, m; $d$ – pipe diameter, m.

The load $P$ is usually given (in normative documents) in t/linear meters. Under the action of $P$, bending moments arise in the pipe section (figure 2, a), reaching a maximum value along the line of application $P$ at the inner surface of the pipe (points A and A$_1$). At these points, normal hoop stresses $\delta_p$ have maximum values and are determined by the dependence [7-10]:

$$\sigma_p = \frac{1.1 \cdot P \cdot r_c}{b \cdot c^2}. \hspace{1cm} (3)$$

where $P$ – reduced load; $r_c$ – average radius of the pipe; $c$ – wall thickness.

Note: since $P$ is given in t/linear meters in the norms, the values of $P$ given in the norms must be converted into kg/cm. From the internal pressure $q$ (kg/cm$^2$), the normal stresses $\delta_q$ are uniformly distributed over the entire section section, reaching a maximum near the inner surface, their value is determined by the classical Lamé formula [6, 10-13]:

$$\delta_q = q \frac{r^2_q + r^2_q}{r^2_{r1} - r^2_{r2}}, \hspace{1cm} (4)$$

where $r_{r1}$ and $r_{r2}$ are the inner and outer radius of the pipe, cm; $q$ – internal pressure, kg/cm$^2$.

For practical calculations of the bearing capacity of the pipe, it is necessary to solve several problems:

1. Direct task. Given the loads ($P$ and $q$) and the geometric size of the pipe ($g$ and $s$), determine the magnitude of the normal stresses in zone A (figure 2);

2. Inverse problem. For given loads ($P$ and $q$) and material strength ($R_{bt}$), calculate the required value of "C" – the pipe wall thickness.

The solution to the direct problem. According to the principle of independence of the action of forces, the total voltage of $\delta_i$ at point A is composed of the stresses from the external load ($\delta_p$) and internal pressure ($\delta_q$):
\[ \sigma_A = \frac{1.1 \cdot P \cdot r_c}{b \cdot c^2} + q \frac{r_n^2 + r_b^2}{r_n^2 - r_b^2}. \]  

(5)

This problem is solved quite simply. The solution of the inverse problem is associated with significant difficulties, since each value of \( r \) contains the value "C" in a hidden form (Figure 1): 

\[ r_c = r_b + \frac{c}{2} \quad \text{and} \quad r_n = r_b + c, \]

to highlight "C" the equation takes the form:

\[ \sigma_A = \frac{1.1 \cdot P \cdot (r_b + \frac{c}{2})}{c^2} + q \frac{(r_b + c)^2 + r_b^2}{(r_b + c)^2 - r_b^2}, \]

(6)

Figure 2. Main piping loads:

a) external reduced load \( P \),
b) diagram of moments from load \( P \),
c) internal pressure \( q \),
d) diagram of circumferential stresses from pressure \( q \).

The quadratic equation should be solved to open the brackets for the definition of “C” with known \((\delta_A, r_n, P \text{ and } q)\).

To simplify the dependence (6) based on the analysis of the geometry of the pipe revealed that in the range of diameters of concrete pipes:

100 to 1000 mm. The relation \( n=d/c \) remains practically unchanged with some deviations. The introduction of the relation \( n=d/c \) in formulas (3) and (4) allows these formulas to lead to the form:

1. Klein formula [7]:

\[ \sigma_A = \frac{1.1 \cdot P \cdot (d + \frac{d}{n})}{100.2 \cdot \frac{d^2}{n^2}} = 0.0055 \cdot P \cdot (d + \frac{d}{n}) \cdot \frac{n^2}{d^2} = 0.0055 \cdot P \cdot \left( \frac{d + nd}{n} \right) \cdot \frac{n^2}{d^2} = \]

\[ 0.0055 \cdot P \cdot \left( \frac{dn^2 + n^3d}{n^2d} \right) = 0.0055 \cdot P \cdot \left( \frac{dn^2(1 + n)}{nd^2} \right) = 0.0055 \cdot P \cdot \left( \frac{dn^2(1 + n)}{nd^2} \right) = \]

\[ 0.0055 \cdot P \cdot \frac{n}{d} (1 + n) = 0.0055 \cdot n(1 + n) \cdot \frac{P}{d}. \]

We introduce the coefficient \( A = 0.0055n(1 + n) \), then:

\[ \sigma_p = A \frac{P}{d}. \]

(7)
We introduce the relation \( n = \frac{d}{c} \), similarly to the previous one, into the Lamé formula. If \( r_\xi = \frac{d}{2}, \quad r_\eta = \frac{d}{2} + c, \quad d = nc \), then the formula (4) takes the form:

\[
\sigma_\eta = q \frac{0.5d^2 + dc + c^2}{dc + c^2} = q \frac{0.25d^2 + dc + c^2 - 0.25d^2}{dc + c^2} = q \frac{0.5n^2c^2 + nc^2 + c^2}{nc^2 + c^2} = q \frac{c^2(0.5n^2 + n + 1)}{c^2(n + 1)} = q \frac{n^2 + 2n + 2}{2n + 2} = q \left[ \frac{n^2}{2n + 2} + 1 \right].
\]

We introduce the coefficient \( B = \left[ \frac{n^2}{2n + 2} + 1 \right] \), then \( \sigma_\eta = q \cdot B \). (8)

A.G. Vandolovsky and B.N. Younis proposed to present the coefficients A and B in tabular form. Based on our statistical analysis, it was established that for concrete pipes, the thickness varies with walls (c) in the ratio \( d/c \) remains in the range of values 6-9 (Table 1), and in this interval the coefficient value \( A = 0.0055 n(1 + n) \) can be replaced with the expression:

\[
A = 0.007618 \cdot n^{1.9}. \tag{9}
\]

The analytical values of A and given in table 2 and according to formula 9 (Figure 3) show that the maximum deviations do not exceed 0.3%. Similar to the previous one in the converted Lamé formula:

\[
\sigma_\eta = q \left[ \frac{n^2}{2n + 2} + 1 \right].
\]

Coefficient \( B = \left[ \frac{n^2}{2n + 2} + 1 \right] \), which is in the previously specified range of values can be represented by an approximate formula: \( B = 0.078 \cdot n^{0.85} \). (10)

Table 3 gives the exact values of B calculated by the Lamé formula (2) and the values calculated by the transformed formula (10) (Figure 3). Deviations do not exceed 0.6%. The studies performed allowed us to obtain new transformed dependences on the external compressive force \( P \) and internal fluid pressure \( q \), given in table 4, which are necessary for the calculation of pipelines.

**Table 1.** Geometric dimensions of concrete and reinforced concrete pipes.

| 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 400   | 50    | 65    | 55    | 51    | 65    | 8.00  | 6.15  | 7.27  | 7.84  | 6.15  |
| 500   | 60    | 75    | 65    | 66    | 75    | 8.34  | 6.67  | 7.69  | 7.57  | 6.67  |
| 600   | 80    | 85    | 75    | 76    | 95    | 7.50  | 7.05  | 8.00  | 7.90  | 6.31  |
| 800   | 100   | 105   | 95    | 96    | 120   | 8.00  | 7.61  | 8.42  | 8.34  | 6.67  |
| 1000  | 110   | 125   | 115   | 121   | 145   | 9.09  | 8.00  | 8.70  | 8.26  | 6.90  |

1 – Diameter, mm.
2, 6 – Reinforced concrete GOST 6482.0-79.
3, 7 – Concrete GOST 20054-82.
4, 8 – Reinforced concrete I.Q.S 1432/1989.
5, 9 – Concrete I.Q.S 1432/1989.
Table 2. Values of the coefficient A in the interval n = 6 .... 9.

| n  | A = 0.0055(n + 1) | A = 0.007618n^0.9 | Δ A | % |
|----|------------------|-------------------|-----|---|
| 6  | 0.23             | 0.23              | 0.000 | 0 |
| 7  | 0.308            | 0.3073            | 0.001 | 0.3|
| 8  | 0.396            | 0.396             | 0.000 | 0 |
| 9  | 0.495            | 0.4954            | 0.000 | 0 |

Table 3. Values of the coefficient B in the interval n = 5 .... 9.

| n  | B = \left(\frac{n^2}{2n+2} + 1\right) | B = 0.078 \cdot n^{0.85} | Δ B | % |
|----|---------------------------------------|------------------------|-----|---|
| 5  | 3.083                                | 3.064                  | 0.019 | 0.6 |
| 6  | 3.571                                | 3.577                  | 0.006 | 0.2 |
| 7  | 4.062                                | 4.077                  | 0.011 | 0.4 |
| 8  | 4.555                                | 4.567                  | 0.012 | 0.5 |
| 9  | 5.050                                | 5.050                  | 0.000 | 0.0 |

Table 4. Dependencies for determining hoop stresses in concrete pipes.

| Load type                     | Known formula                     | Formula transformation |
|-------------------------------|-----------------------------------|------------------------|
| Internal pressure q           | Lamé formula: \( \delta_q = q \left( r_H^2 + r_B^2 \right) / (r_H - r_B^2) \) | \( \delta_q = q \cdot B \) | \( B = 0.078 \cdot n^{0.85} \) |
| External compression load P   | \( \sigma_P = \frac{1.1 \cdot P \cdot r_c}{b \cdot c^2} \) | \( \delta_q = \frac{A \cdot P}{d} \) | \( A = 0.007618 \cdot n \) |

Note: \( n = d/c \); \( P \) – it is introduced in the form of standard indicators – t/linear meters; \( q \) is the fluid pressure in kg/cm².

Figure 3. Coefficients A and B for the calculation of concrete pipes:
(a) Coefficient A = f(n), (b) Coefficient B = f(n).
3. Conclusion

We used the conversion of the known dependencies which allows determining the normal hoop stress from the external equivalent load and internal pressure according to the Lamé dependence to obtain new transformed dependencies for calculating the stresses in pipes under the combined action of internal pressure and external (reduced) load. The obtained equations can be used to determine the range of application of concrete (without metal reinforcement) pipes and select the concrete composition for the pipe production.

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