Spin-based quantum gating with semiconductor quantum dots by bichromatic radiation method

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A potential scheme is proposed for realizing a two-qubit quantum gate in semiconductor quantum dots. Information is encoded in the spin degrees of freedom of one excess conduction electron of each quantum dot. We propose to use two lasers, radiating two neighboring QDs, and tuned to blue detuning with respect to the resonant frequencies of individual excitons. The two-qubit phase gate can be achieved by means of both Pauli-blocking effect and dipole-dipole coupling between intermediate excitonic states.

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Quantum computing with semiconductor quantum dots (QDs) has drawn more and more interest over the past few years. Besides the ease of scalability, semiconductor QD quantum computing is more appealing than other quantum computing schemes due to the existence of the industrial basis for semiconductor processing as well as the promise of being easily integrable.

In the schemes for quantum computing with semiconductor QDs proposed so far, either the excitonic or spin degrees of freedom have been identified as potential qubits. Quantum gating based on excitons, although being strongly restricted by the short decoherence time of the exciton, can be implemented ultrafastly with optical manipulations. In contrast, spin qubits, due to their relatively long decoherence time, allow for longer storage of quantum information. Quantum gates can be carried out by means of Coulomb interaction or by coupling to a cavity mode. When nearest-neighbor coupling plays an important role one has to face a significant overhead for coupling two distant QDs. On the contrary if the QDs are put into a cavity, two distant QDs can interact directly through coupling to the same cavity mode. Nevertheless, the implementation time in such a model is typically quite long due to the large detuning technique adopted for avoiding the cavity decay and the weak cavity-laser-QD coupling.

More and more experimental evidences, based on current state-of-art nanostructure and laser technology, have shown the present capacity to manipulate semiconductor QDs based qubits. The single QD cooled and prepared with one excess conduction band electron only, which is the prerequisite of spin-based QD quantum computing schemes, has already been achieved. Entangled excitonic states have been realized by optical method and the ultrafast spin rotation by laser pulses in a magnetic field has been presented. More recently, the writing and readout operations for the spin states of the single conduction band electron were performed in an n-doped InAs-GaAs QD by nonresonant circularly polarized optical pumping. In the same experiment, a long lifetime of the electron spin was observed. Although no experiment so far has demonstrated a quantum algorithm, the experimental progress outlined above has paved the road to a working quantum computer with semiconductor QDs.

The present work concentrates on an alternative two-qubit proposal for spin qubits of semiconductor QDs, which is inspired by \(^2\). The essential ingredient of our gate proposal is provided by the bichromatic radiation approach widely employed in ion trap quantum computing. As in \(^2\), quantum information in our scheme will be encoded in the spin states of the single excess conduction electron of the QD and the two-qubit quantum gate will be realized by exploiting the biexcitonic shift due to the dipole-dipole interaction between adjacent excitonic states. However, at variance from \(^2\), the two-qubit gate proposed here is implemented by the bichromatic radiation approach. To this aim, we have to modify the original formulation used for trapped ions; this is due to the differences between QDs and atomic ions, i.e., degeneracy of qubit states in QDs in the absence of external field, no metastable levels to use as ancillary states in QDs, no motional degrees of freedom attached to the qubit states and no exactly identical self-assembled QDs.

Consider two lasers, radiating two neighboring QDs, and tuned to blue detuning with respect to the resonant frequencies of individual excitons. If the sum of the two blue detunings equals to the biexcitonic shift, an effective coupling can be generated between \(|1\rangle_{ab}\) and \(|XX\rangle_{ab}\), where \(|1\rangle_k\) and \(|X\rangle_k\) denote a qubit state and the excitonic state of the QD \(k\) respectively (defined later, see Fig. 1). We shall show how to realize a conditional phase gate, based on this effective coupling, by means of properly tailored ultrafast laser pulses. We shall focus on the case of two QDs. Both our method and results can be in principle extended to multi-QD systems.
Let us suppose that each QD contains only one excess conduction-band electron. We employ the spin states \( m_j^e = 1/2 \) and \(-1/2\) of such electron as qubit states \([1]\) and \([0]\) respectively. Excitonic states are introduced as ancillary ones. Besides the Coulomb repulsion, the Pauli-blocking mechanism is essential to our scheme. When we radiate a \( \sigma^- \) polarized light with suitable energy on the QD, due to the Pauli exclusion principle, the exciton states \( |m_j^e = \frac{1}{2}, m_j^g = -\frac{1}{2}\rangle \) in the s-shell will be produced if and only if the excess electron has a spin projection \( 1/2 \). This Pauli-blocking mechanism has been used to experimentally produce entangled excitonic states in [11]. We define \( |0\rangle_\nu = c^\dagger_{\nu, -\frac{1}{2}} |\text{vac}\rangle \), \( |1\rangle_\nu = c^\dagger_{\nu, \frac{1}{2}} |\text{vac}\rangle \), and the excitonic states \( |X\rangle_\nu = c^\dagger_{\nu, -\frac{1}{2}} d^\dagger_{\nu, \frac{1}{2}} |\text{vac}\rangle \), where \( c^\dagger_{\nu, \sigma}(d^\dagger_{\nu, \sigma}) \) is the creation operator for a conduction (valence) band electron (hole) in QD \( \nu \) with spin projection \( \sigma \), and \( |\text{vac}\rangle \) denotes the electron-hole vacuum. The general Hamiltonian of such a system can be found in [6]. Here we only consider a special configuration, radiated by two blue-detuned lasers simultaneously, as shown in Fig. 1. The Hamiltonian of the QD system \( H_0 + H_I \) can be written in unit of \( \hbar \) as

\[
H_0 = \Delta |XX\rangle \langle XX| + \omega_a |X\rangle \langle X| \otimes \hat{I}_b + \omega_b \hat{I}_a \otimes |X\rangle b \langle X| \tag{1}
\]

and

\[
H_I = \frac{1}{2} \left[ \Omega_a(t) \left( e^{i\omega_{L1}t} + e^{i\omega_{L2}t} \right) |1\rangle_a \langle X| \otimes \hat{I}_b + \Omega_b(t) \left( e^{i\omega_{L1}t} + e^{i\omega_{L2}t} \right) \hat{I}_a \otimes |1\rangle_b \langle X| + h.c. \right] \tag{2}
\]

where \( \Omega_k(t) \) \((k = a \text{ and } b)\) denotes the couplings of lasers with QDs \( a \text{ and } b \), respectively. For simplicity, we assume here that the coupling strength \( \Omega_k(t) \) is identical for each QD irradiated by different laser beams. But this assumption is not essential to the following deduction. \( \hat{I}_k \) is the identity operator with respect to QD \( k \), and \( \omega_k \) is the resonant energy of a single exciton produced in individual QDs, and \( \Delta \) is the biexciton shift due to Coulomb repulsion. \( \omega_{Ln} \) \((n = 1 \text{ or } 2)\) is the laser frequency applied on QDs \( a \text{ and } b \), and \( h.c. \) means hermitian conjugate. The Pauli blocking is reflected by the absence of the transition from \([0]\) to \([X]\). In the rotating frame with respect to \( H_0 \), we have

\[
H' = \frac{\Omega_a(t)}{2} \left( e^{i\omega_{L1}t} + e^{i\omega_{L2}t} \right) e^{-i\omega_at} |1\rangle_a \langle X| \otimes (|X\rangle \langle X| e^{-i\Delta t} + |1\rangle \langle 1| + |0\rangle \langle 0|) b
\]

\[
+ \frac{\Omega_b(t)}{2} \left( e^{i\omega_{L1}t} + e^{i\omega_{L2}t} \right) e^{-i\omega bt} (|X\rangle \langle X| e^{-i\Delta t} + |1\rangle \langle 1| + |0\rangle \langle 0|) a \otimes |1\rangle_b \langle X| + h.c.
\]

\[
\tag{3}
\]

Since the two QDs are irradiated by two lasers simultaneously, there should be four detunings. We define \( \delta_a = \omega_{L1} - \omega_a, \delta'_a = \omega_{L2} - \omega_a, \delta_b = \omega_{L1} - \omega_b, \text{ and } \delta'_b = \omega_{L2} - \omega_b \) with \( \delta_a + \delta'_a = \delta_b + \delta'_b = \Delta \) to achieve the two-photon resonance. If we adjust the two lasers to satisfy \( \Delta \to 2 \ll \min(\delta_a, \delta_b, \delta'_a, \delta'_b) \), then there would be no actual excitation in the intermediate states \([1X]\) \(_{ab}\) and \([X1]\) \(_{ab}\). We thus reach an effective Hamiltonian

\[
H_{eff} = \frac{\tilde{\Omega}(t)}{2} (|XX\rangle \langle 11| + h.c.)_{ab} \tag{4}
\]

with

\[
\frac{\tilde{\Omega}(t)}{2} = \frac{1}{2} \frac{\Omega_a(t)\Omega_b(t)(1/\delta_a + 1/\delta'_a + 1/\delta_b + 1/\delta'_b)}{\Omega_a(t)\Omega_b(t) \left[ 1/\delta_a + 1/(\Delta + \delta - \delta_a) + 1/(\delta_a - \Delta) + 1/(\Delta - \delta_a) \right]}
\]

\[
\tag{5}
\]

where \( \delta = \omega_b - \omega_a \) is the resonance frequency difference between the two QDs. Based on Eq. (4), returning to the Schrödinger representation, we have the time evolution as follows

\[
|11\rangle \rightarrow \cos \left[ \frac{1}{2} \int_0^T \tilde{\Omega}(t) dt \right] |11\rangle - ie^{i(\omega_a + \omega_b + \Delta)t} \sin \left[ \frac{1}{2} \int_0^T \tilde{\Omega}(t) dt \right] |XX\rangle
\]

\[
\tag{6}
\]

and a similar equation for \([XX]\).

Our conditional phase gate is based on Eq. (6), which can lead to a universal quantum computing along with single qubit operations. Since our computational subspace is spanned by \([0]\) \(_{ab}\) and \([1]\) \(_{ab}\), an evolution with \( \int_0^T \tilde{\Omega}(t) dt = 2\pi \) yields \([11]\) \(_{ab}\) \( \rightarrow -[11]\) \(_{ab}\), but no change in \([10]\) \(_{ab}\), \([01]\) \(_{ab}\) and \([00]\) \(_{ab}\). This is a typical conditional phase gate. During the gating, however, the excitonic states are actually excited. As a result, our gating time must be shorter than
the decoherence time of the exciton. Let us suppose that the QDs are made of III-V semiconductor materials. The biexcitonic shift $\Delta$ between adjacent excitons, corresponding to the inter-dot distance of 10 nm in the presence of an in-plane electric field $F = 75kV/cm$, is about $\Delta = 4$ meV. We assume $\delta = 1$ meV and the laser pulses to be Gaussian where $\Omega_k(t) = \Omega_k e^{-t^2/\tau^2}$ with $\tau$ the pulse duration and $\Omega_0 \approx \Omega_b = \Omega_0$. Then we have to satisfy the relation

$$\frac{1}{2} \Omega^2(0) \left[ 1/\delta_a + 1/(5 - \delta_a) + 1/(\delta_a - 1) + 1/(4 - \delta_a) \right] \int_{-t/2}^{t/2} e^{-t^2/\tau^2} dt = \pi$$

where the implementation time of the conditional phase gate $T$ should be shorter than the dephasing time of the excitons, which is of the order of 1 ns. To analyze the constraints given by the short dephasing time of the excitons and the virtual excitation of the intermediate states, let us define $R = \Omega_0/2\delta_{min}$ with $\delta_{min} = \min\{\delta_a, \delta_b, \delta'_a, \delta'_b\}$. The numerical results in Fig. 2 demonstrate that when $\delta_a = 2.5$ meV, we have shortest gating times. So in what follows, for simplicity, we shall only consider this optimal case. If we consider $R = 1/2$, which was adopted in Ref. 17 for building entangled states of trapped ions based on the proposals of Refs. 13, then $\tau \approx 1.0$ ps. But this R is too big to carry out our scheme with high fidelity. To avoid the excitation in the intermediate states defined as $\bar{n} = 2R^2$ though, we have to restrict $R$ to be smaller than 1/7, i.e., $\bar{n} < 5\%$. Thus to realize a gating with such a high fidelity, we would have $\tau \approx 0.5$ ps.

As mentioned above, our scheme is rooted in Ref. 6. Besides using the same qubits, both the schemes perform the quantum gating by means of the biexcitonic shift and Pauli blocking. The important difference is that, instead of an adiabatic process for accumulating the conditional phase factor, our conditional phase gate is based on the resonant transition between $|11\rangle$ and $|XX\rangle$. This can be achieved by one-step implementation, which much simplifies the operations in the original proposal of the bichromatic radiation. Nevertheless, due both to detunings to the individual excitation of the exciton and to the second-order process (Eq. (5)) employed in our scheme, the coupling of the lasers to the QDs cannot be large. As a result, the implementation time our gating takes is of the same order of in Ref. 6. On the other hand, since the bichromatic radiation approach has been proven experimentally in Ref. 17 in atomic physics to be an efficient and reliable way of entangling states with high fidelity, as long as $\Omega_b/2 \leq \pi \min\{\delta_a, \delta_b, \delta'_a, \delta'_b\}$ is satisfied, we are optimistic for the possibility to achieve a conditional phase gate with the fidelity higher than 95%.

Besides the two-qubit gate, single-qubit operation is necessary for a universal quantum computing scheme. As done in Refs. 2, 3, based on the exact knowledge of specific QDs, we assume the individually addressing of the QDs is available with laser beams by using energy selective schemes rooted in the characteristic size fluctuations of self-assembled QDs combined with near field technique. By employing resonant Raman coupling between $|0\rangle$ and $|1\rangle$ under the radiation of two lasers with different polarizations and suitable frequencies, single-qubit rotation can be readily carried out within the order of ps. However we noticed that the light hole $|m_h^j = 1/2\rangle$ is an excited state in III-V semiconductor materials, whose decoherence is not advantageous to our single-qubit gating. To avoid this problem, we can choose II-V semiconductor QDs, in which the light hole state is energetically favoured.

The readout of the final state may be performed again via Pauli-blocking schemes: only if the final state is $|1\rangle$, an exciton may be induced by a $\sigma^-$ polarized laser pulse of suitable frequency. Therefore, a $\sigma^-$ polarized photon will be created after the exciton decays. By detecting this photon, we shall know whether the QD spin state is in $|1\rangle$ or $|0\rangle$. Since in the readout stage the spins are in product states, we only need to consider the lifetime (i.e., $T_1$) of the spin state. The lifetime of $|1\rangle$ is of the order of $\mu$s. So we can repeat this laser pulse excitation for thousands of times, which is very similar to the electronical shelving amplification used in ion trap experiments. Although the detection efficiency in our scheme would be somewhat lower than that in a microcavity due to the finite angle coverage of the detector, the information amplification mentioned above can guarantee our readout to be correct and effective.

However, like previous proposals, our conditional phase gate is based on a nearest-neighbor coupling, which need significant overhead for coupling two distant qubits, and like in Refs. 2, 3, the external electric field is necessary in our scheme to enlarge the biexcitonic shift $\Delta$. On the contrary, in our other scheme, the qubits based on the QDs embedded in a high-Q single-mode cavity enjoy an effective coupling between two non-neighboring QDs through coupling to the same cavity mode and also need no external field. Nevertheless, in the present scheme, the conditional two-qubit gate can be carried out more quickly than in Ref. 6, which is of great importance in view of decoherence. The scheme in Ref. 6 is strongly restricted by the number of the QDs available in a cavity. It is still experimentally challenging to have few QDs in a high-Q cavity with desired couplings. On the contrary, the present scheme is more easily scalable.

The quantum gate based on our scheme can be carried out with high fidelity. The decoherence time of the spin state of the conduction band electron can be of the order of $\mu$s, which is much longer than the decoherence time of the exciton and thereby will not affect our gating. If we neglect in our discussion any errors due to incorrect or inappropriate operations, potential error sources for our scheme are from (1) actual excitation of the intermediate
states and the spontaneous emission from excitonic states; (2) small admixture of heavy hole component to the light hole wavefunction due to the interaction between the hole bands in actual QDs; (3) possible spectral diffusion due to strong built-in fields and many-body effect; (4) the Förster process happening in the nearest-neighbor coupled QDs. As discussed above, error (1) can be highly suppressed by reasonable laser-QD coupling and implementation time. Error (2) yields slight violation of the Pauli blocking, i.e., a partial excitation of the excitonic state \(|m_J = 1/2, m_I = -3/2\) produced in each radiation with the \(\sigma^-\) polarization when the spin projection of the only excess conduction electron is \(-\frac{1}{2}\). But this can be avoided in our single-qubit gate by using in-plane directed laser pulses. Because of the restriction from symmetry, the induction of the heavy hole part is prohibited in the mixed wave function for any radiation along the growth direction. In the implementation of our two-qubit phase gate instead, in order to avoid decoherence related to light-heavy hole mixing, it is possible to resort to adiabatic techniques. Error (3) would result in random level shift and cross biexcitons. Fortunately, in the low temperature as considered in this work, it happens on the timescale of seconds, much longer than our gating time. So we can neglect it. Error (4) will be also greatly suppressed because of the energy spectrum natural mismatch between different QDs.

In conclusion, a bichromatic radiation scheme for implementing two-qubit phase gate with semiconductor QDs has been proposed. Our scheme can be considered as a combination of the Pauli-blocking spin-based quantum gating and the bichromatic radiation approach. In principle, both the method and the results in this work can be extended to multi-QD case. Since experimentally the mechanism of both the Pauli-blocking and the bichromatic radiation approach has been tested and simple manipulation of spin qubits in semiconductor QDs has been demonstrated, we believe our scheme to be feasible in the near future.

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FIG. 1: Two neighboring QDs \( a \) and \( b \), radiated by two \( \sigma^- \) polarized lasers, where the arrows represent the laser radiation. (A) The Pauli blocking is reflected by the absence of transition between \( |0\rangle \) and \( |X\rangle \). (B) The two-photon process for transition between \( |11\rangle \) and \( |XX\rangle \) is composed of two blue detunings with respect to QDs \( a \) and \( b \), respectively, where \( |1X\rangle \) and \( |X1\rangle \) are non-populated intermediate states.

FIG. 2: The relation between \( R = \Omega_0/2\delta_{\min} \) and \( \tau \) in the implementation of a conditional phase gate, where the solid, dashed and dotted curves represent the cases of \( \delta_a = 1.5, 2.5, \) and \( 3.0 \) meV respectively.