Engineering W-type steady states for three atoms via dissipation in an optical cavity

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We propose a scheme for dissipative preparation of W-type entangled steady-states of three atoms trapped in an optical cavity. The scheme is based on the competition between the decay processes into and out of the target state. By suitable choice of system parameters, we resolve the whole evolution process and employ the effective operator formalism to engineer four independent decay processes, so that the target state becomes the stationary state of the quantum system. The scheme requires neither the preparation of definite initial states nor precise control of system parameters and preparation time.

PACS numbers: 03.67.Bg, 42.50.Dv, 42.50.Pq

I. INTRODUCTION

Entanglement of multiple particles is not only an essential ingredient for a test of quantum nonlocality, but also a key resource for implementation of quantum information processing (QIP) [1–3]. Preparing entangled states faithfully and reliably has been one of the main tasks in quantum computation [4, 5]. To achieve this, one of the main obstacles is decoherence induced by the environment. Recently, many strategies using decoherence as a resource have been developed in quantum computation and entanglement engineering [6–16]. Schemes based on dissipative preparation require neither the preparation of definite initial states nor precise control of system parameters and preparation time. Particularly, Kastoryano and Reiter et al. [6–8] proposed a scheme to produce maximally entangled states for two atoms trapped in an optical cavity via engineering the decay process. Busch et al. [9] showed that two atoms in an optical cavity can be cooled to a maximally entangled state by employing level shifts induced by laser fields. The distinct feature of these schemes [6–9] is the linear scaling of the fidelity with the cooperativity as compared to square root scaling of the fidelity for the schemes based on unitary dynamics. The idea of Refs. [6, 8] has been applied to dissipative preparation of maximally entangled states for two atoms trapped in two coupled cavities [10]. However, most of the previous theoretical schemes [6–13] and experiments [17] concentrate on the preparation of entangled states of two atoms. To our knowledge, there is no experimental report for dissipative preparation of multipartite entangled states in cavity QED.

One of the most important multipartite entangled state is the W-type state which has been shown to have valuable applications in QIP such as quantum teleportation [18], quantum dense coding [19], quantum cloning machine [20], etc. Recently, three-qubit W states have been achieved in optical systems [21], ion traps [22], and superconducting phase qubits [23]. Numerous schemes have been proposed for generation of such states in cavity QED via unitary dynamics [24–28]. However, a W state has not been realized experimentally in cavity QED. The fidelity of schemes based on cavity QED will suffer errors coming from spontaneous emission and cavity decay, but these two error sources can not be decreased at the same time in unitary dynamics [6].

In this paper, we propose a scheme for the dissipative preparation of W-type steady-state (the target state) of three atoms in an optical cavity. The scheme is based on the competition between the decays into and out of the target state. Each laser field, assisted by the dissipative cavity mode and atomic spontaneous emission, induces a collective atomic decay process independently. The total decay rate between any pair of collective atomic states is the sum of those associated with the four engineered decay processes. By suitable choice of system parameters, the rate of decay into the target state is much larger than that out of the target state so that the system finally approaches the target state no matter what the initial state is. Numerical results show that the W-type steady entanglement can be obtained with fidelity as high as 90%, despite of the cooperativity parameter C as low as 75, where $C = g^2 / \kappa \gamma$.

II. ENGINEERING W-TYPE STEADY STATE

As shown in FIG. I three Λ-type atoms are trapped in a single-mode cavity. Each atom has two ground states $|0\rangle$ and $|1\rangle$ and an excited state $|2\rangle$. The cavity mode is coupled to the $|1\rangle \leftrightarrow |2\rangle$ transition resonantly. Two off-resonance optical lasers, each with detuning $\Delta_k$ and Rabi frequency $\Omega_k$, drive the transition $|0\rangle \leftrightarrow |2\rangle (k = 1, 2)$. The $|1\rangle \leftrightarrow |2\rangle$ transition is driven by two other different lasers, each with detuning $\Delta_k$ and Rabi frequency $\Omega_k (k = 3, 4)$ respectively.

It is a tough work to obtain the analytical result of the present system for the reason that there is no interaction picture in which the system Hamiltonian becomes time independent. When the classical fields are sufficiently weak, the condition $\Omega_k \Omega_l (1/\Delta_k + 1/\Delta_l)/2 \ll |\Delta_k - \Delta_l|$
(k, l = 1, 2, 3, 4; k ≠ l) is satisfied. We can neglect the Raman transition between two any classical fields. In this case, each laser field, together with the dissipative cavity mode and atomic spontaneous emission, induces a collective atomic decay process independently. The whole dissipative process is the incoherent combination of the four independent decay processes.

Under the rotating wave approximation, the Hamiltonian associated with the kth independent decay process in the interaction picture reads \( H^{(k)} = H_0^{(k)} + V_+^{(k)} + V_-^{(k)} \), where

\[
H_0^{(k)} = \Delta_k a^\dagger a + \sum_{m=1}^{3} \Delta_k \langle 2 |m \rangle \langle 2 |m \rangle,
\]

\[
V_+^{(k)} = \frac{\Omega_k}{3} \sum_{m=1}^{3} \langle 2 |m \rangle \langle 0 |m \rangle, \quad (k = 1, 2)
\]

\[
V_-^{(k)} = \frac{\Omega_k}{3} \sum_{m=1}^{3} \langle 2 |m \rangle \langle 1 |m \rangle, \quad (k = 3, 4)
\]

\( V_-^{(k)} \) is the annihilation operator for the cavity mode, and \( g \) is the atom-cavity coupling constant.

In the following, we assume that the system-environment interaction is Markovian such that the evolution of the density matrix \( \rho \) can be described by a master equation of Lindblad form

\[
\dot{\rho} = i \{ \rho, H^{(k)} \} + \sum_{x} \left\{ L_x \rho (L_x)^\dagger - \frac{1}{2} \left[ (L_x)^\dagger L_x \rho + \rho (L_x)^\dagger L_x \right] \right\},
\]

where the Lindblad operators \( L_x \) represent various decay sources. In the atom-cavity system, two decay sources will inherently be present: spontaneous emission of the excited state to the ground states \( |0 \rangle \) and \( |1 \rangle \) with decay rates \( \gamma_{0,m} \) and \( \gamma_{1,m} \) (\( m = 1, 2, 3 \)), and cavity leakage at a decay rate \( \kappa \). Seven Lindblad operators associated with dissipation can be expressed as \( L_k = \sqrt{\kappa} a, L_{\gamma,0,m} = \sqrt{\gamma_{0,m}} \langle 0 |m \rangle \langle m |0 \rangle, L_{\gamma,1,m} = \sqrt{\gamma_{1,m}} \langle 1 |m \rangle \langle m |1 \rangle \). We assume \( \gamma_{0,m=1,2,3} = \gamma_{1,m=1,2,3} = \gamma/2 \) for simplicity.

According to Ref. [3], based on the competition between the unitary dynamics induced by the classical fields and the dissipation dynamics, the time evolution of the ground subspace is much slower than that of the excited subspace when the excited states are not initially populated under the condition of weak classical fields. We can adiabatically eliminate the excited cavity field modes and excited states of the atoms. The system dynamics will be reduced to the ground subspace in a strongly dissipative environment. To the second order in perturbation theory, the dynamics of the system is governed by an effective master equation in Lindblad form [6–8]

\[
\dot{\rho} = i \{ \rho, H_{\text{eff},k} \} + \sum_{x} \left\{ L_x^{\text{eff},k} \rho (L_x^{\text{eff},k})^\dagger - \frac{1}{2} \left[ (L_x^{\text{eff},k})^\dagger L_x^{\text{eff},k} \rho + \rho (L_x^{\text{eff},k})^\dagger L_x^{\text{eff},k} \right] \right\}.
\]

It contains the effective Hamiltonian \( H_{\text{eff},k} \) and the effective Lindblad operator \( L_x^{\text{eff},k} \) (\( L_x \) represents seven Lindblad operators defined previously)

\[
H_{\text{eff},k} = -\frac{1}{2} V_x \left( H_{\text{NH},k}^{-1} + (H_{\text{NH},k}^{-1})^\dagger \right) V_x, \quad (6)
\]

\[
L_x^{\text{eff},k} = L_x H_{\text{NH},k}^{-1} V_x, \quad (7)
\]

where \( H_{\text{NH},k}^{-1} \) is the inverse of the non-Hermitian Hamiltonian \( H_{\text{NH},k} = H_0^{(k)} - \frac{\kappa}{3} \sum_x (L_x)^\dagger L_x \).

It will be convenient to work in the Fourier transformed basis of the atomic ground states: \( \{ |000\rangle, |S_{1,j}\rangle, |S_{2,j}\rangle, |111\rangle \} \) \((j = 1, 2, 3)\), where

\[
|S_{1,j}\rangle = \frac{1}{\sqrt{3}} \left( e^{i \frac{2\pi}{3}} |100\rangle + e^{i \frac{4\pi}{3}} |010\rangle + |001\rangle \right),
\]

\[
|S_{2,j}\rangle = \frac{1}{\sqrt{3}} \left( e^{i \frac{2\pi}{3}} |110\rangle + e^{i \frac{4\pi}{3}} |101\rangle + |011\rangle \right),
\]

\[
|S_{1,3}\rangle = \frac{1}{\sqrt{3}} \left( (|100\rangle + |010\rangle + |001\rangle) \right) \text{ is the desired W-type entangled state.}
\]

Applying Eq. [6] and [7] to each decay process, we derive the corresponding effective Hamiltonian and effective Lindblad operators due to the competition between unitary and dissipative dynamics (see Appendix A). We insert these effective operators back into Eq. [5] and use the projection operator approach for the density operator (i.e. \( \langle y | \rho | y \rangle \)). We note that \( \langle y | \rho | y \rangle \) contains only diagonal terms, which indicates there are no coherence between the states. Hence, we can reduce the effective master equation Eq. [5] to the rate equation for the population of states only. We also notice that the effective Hamiltonian does not induce any transition between the transformed basis states. Therefore, the transitions between these basis states are caused by collective decays.
TABLE I. The decay rates \( \mu_{\text{eff},|y\rangle,|S_{1,3}\rangle} \) and \( \mu_{\text{eff},|S_{1,3}\rangle,|y\rangle} \) correspond to the effective decay channels from \( |y\rangle \) to \( |S_{1,3}\rangle \) and from \( |S_{1,3}\rangle \) to \( |y\rangle \) respectively by combining four independent decay processes in Eq. (9).

| \(|y\rangle\) | \( \mu_{\text{eff},|y\rangle,|S_{1,3}\rangle} \) | \( \mu_{\text{eff},|S_{1,3}\rangle,|y\rangle} \) |
|---|---|---|
| \(|000\rangle\) | \[ \sum_{k=1}^{3} \left( \frac{\gamma_{1} \Omega_{k}}{\sqrt{3} R_{1,k}} \right)^2 + \frac{3}{2} \frac{\gamma_{3} \Omega_{k}}{3 \sqrt{6} R_{1,k}} \] | \[ \sum_{k=1}^{3} \frac{3}{3 \sqrt{6} R_{1,k}} \] |
| \(|S_{1,2}\rangle, |S_{1,3}\rangle\) | \[ \sum_{k=1}^{3} \left( \frac{\gamma_{1} \Omega_{k}}{\sqrt{2} R_{1,k}} - \frac{i \gamma_{1} \Omega_{k}}{6 \sqrt{2} R_{1,k}} \right)^2 + \frac{3}{2} \frac{\gamma_{3} \Omega_{k}}{9 \sqrt{2} R_{1,k}} \] | \[ \sum_{k=1}^{3} \frac{3}{9 \sqrt{2} R_{1,k}} \] |
| \(|S_{2,1}\rangle, |S_{2,2}\rangle\) | \[ \sum_{k=1}^{3} \left( \frac{\gamma_{1} \Omega_{k}}{\sqrt{2} R_{2,k}} \right)^2 + \frac{3}{2} \frac{\gamma_{3} \Omega_{k}}{9 \sqrt{2} R_{2,k}} \] | \[ \sum_{k=1}^{3} \frac{3}{9 \sqrt{2} R_{2,k}} \] |
| \(|S_{2,3}\rangle\) | \[ \sum_{k=1}^{3} \left( \frac{\gamma_{1} \Omega_{k}}{\sqrt{2} R_{2,k}} \right)^2 + \frac{3}{2} \frac{\gamma_{3} \Omega_{k}}{9 \sqrt{2} R_{2,k}} \] | \[ \sum_{k=1}^{3} \left( \frac{3}{9 \sqrt{2} R_{2,k}} + \frac{3}{3 \sqrt{6} R_{1,k}} \right)^2 \] |

We obtain the effective decay rate

\[
\mu_{\text{eff},|y\rangle,|z\rangle} = \sum_{k=1}^{4} \langle k | \mu_{\text{eff},|y\rangle,|z\rangle | k \rangle
\]

(9)

from the basis state \(|y\rangle\) to \(|z\rangle\) by combining four decay processes. To assure that the state \(|S_{1,3}\rangle\) becomes the stationary state of the atom-cavity system, we need to choose detuning \( \Delta_{k} \) such that the transition out of \(|S_{1,3}\rangle\) can be strongly suppressed, while almost all the population of the undesired states are driven into the target state by collective decay. Then we keep the terms with respect to \(|S_{1,3}\rangle\). These terms are summarized in Table 1, where \( \Delta_{k} = \Delta_{k} - \frac{g^2}{\gamma} \), \( \tilde{\delta}_{k} = \Delta_{k} - \frac{g^2}{\gamma} \), \( R_{n,k} = \tilde{\Delta}_{k} \tilde{\delta}_{k} - n g^2 \). In particular, \(|111\rangle\) cannot be driven into the target state directly. Effective decay from \(|111\rangle\) to \(|S_{1,3}\rangle\) is mediated by \(|S_{2,1}\rangle, |S_{2,2}\rangle, |S_{2,3}\rangle\). Table 2 shows the decay rates between \(|111\rangle\) and \(|S_{2,1}\rangle, |S_{2,2}\rangle, |S_{2,3}\rangle\). Under the condition \( g \gg \Omega, \kappa, \gamma \), if we set \( \Delta_1 = 0, \Delta_2 = g, \Delta_3 = \sqrt{3} g, \Delta_4 = \sqrt{2} g \), then the conditions \( \mu_{\text{eff},|y\rangle,|S_{1,3}\rangle} \gg \mu_{\text{eff},|S_{1,3}\rangle,|y\rangle} \) and \( \mu_{\text{eff},|y\rangle,|111\rangle} \sim \mu_{\text{eff},|111\rangle,|y\rangle} \) are satisfied. Thus, we can obtain state \(|S_{1,3}\rangle\) with high fidelity starting from random initial states, where the stationary state fidelity \( F = \langle S_{1,3}|\rho|S_{1,3}\rangle \) is \( P_{1,3} \).

In order to evaluate the performance of the scheme, we insert the decay rates obtained from the effective opera-

TABLE II. The decay rates \( \mu_{\text{eff},|y\rangle,|111\rangle} \) and \( \mu_{\text{eff},|111\rangle,|y\rangle} \) correspond to the effective decay channels from \(|y\rangle\) to \(|111\rangle\) and from \(|111\rangle\) to \(|y\rangle\) respectively by combining four independent decay processes in Eq. (9).

| \(|y\rangle\) | \( \mu_{\text{eff},|y\rangle,|111\rangle} \) | \( \mu_{\text{eff},|111\rangle,|y\rangle} \) |
|---|---|---|
| \(|S_{2,1}\rangle, |S_{2,2}\rangle\) | \[ \sum_{k=3}^{3} \frac{3 \sqrt{3} \gamma_{1} \Omega_{k}}{3 \sqrt{6} R_{1,k}} \] | \[ \sum_{k=3}^{3} \frac{3 \sqrt{3} \gamma_{1} \Omega_{k}}{3 \sqrt{6} R_{1,k}} \] |
| \(|S_{2,3}\rangle\) | \[ \sum_{k=3}^{3} \frac{3 \sqrt{3} \gamma_{1} \Omega_{k}}{3 \sqrt{6} R_{1,k}} \] | \[ \sum_{k=3}^{3} \left( \frac{3 \sqrt{3} \gamma_{1} \Omega_{k}}{3 \sqrt{6} R_{1,k}} + \frac{3}{9 \sqrt{2} R_{2,k}} \right)^2 \] |

![FIG. 2. (Color online) The populations of the target state \(|S_{1,3}\rangle\) (left axis) and the purity of the system (green solid line, right axis) as a function of time for a random initial state. The curves are plotted for a set of optimal parameters \( C = 80, \gamma = 1.5 \kappa, \Omega_1 = \Omega_2 = \Omega, \Omega_4 = 2 \Omega_2 = 1.2 \Omega \). (a) \( \Omega = 0.04 \gamma \). (b) \( \Omega = 0.08 \gamma \). Numerical results in Eq. (10) (red, short dash) correspond with numerical curves obtained from the full master equation (blue, solid line) well.](image)

tors, and get the rate equations

\[
\dot{P}_{y} = \sum_{x \neq y} \left( \mu_{\text{eff},|y\rangle,|z\rangle} P_{x} - \mu_{\text{eff},|z\rangle,|y\rangle} P_{y} \right),
\]

(10)

where \( P_{y} \) is the population of state \(|y\rangle\). Numerical solution in FIG. 2(a) illustrates that we can obtain state

of $\gamma/\kappa$ for different values of $C$ and $\Omega$.

FIG. 3. (Color online) Stationary state fidelity $F$ as a function of $\gamma/\kappa$ for different values of $C$ and $\Omega$.

$|S_{1,3}\rangle$ with high fidelity above 91% and the purity $\eta$ above 82%, where $\eta = Tr(\rho^2)$, in a time 6000/g. FIG. 2 also shows numerical results in Eq. (10) correspond well with numerical curves obtained from the full master equation. The discrepancy can be attributed to the fact that we neglect the Raman transition between any two classical fields.

Now let’s consider how the Rabi frequency $\Omega$ affects the fidelity of the steady state and the convergence speed of our scheme. The convergence speed is primarily governed by the magnitudes of $\Omega$. From FIG. 2 we notice that the convergence speed for the bigger $\Omega$ is about four times larger than that for the smaller one. This is because that the decay rate is proportional to the square of the Rabi frequency. However, the Rabi frequency should not exceed a certain amount, otherwise the condition of weak classical fields will break down, and the Raman transitions between any two classical fields and the populations of the excited states should be taken into consideration. Thus, as long as the conditions $\Omega^2(1/\Delta_k + 1/\Delta_l)/2 < |\Delta_k - \Delta_l|$ and $g \gg \Omega$ are satisfied, an appropriate increase of $\Omega$ can speed up the convergence greatly, but it will decrease the fidelity slightly.

FIG. 3 shows the influence of $\gamma/\kappa$ on the fidelity $F$ of the stationary state for different values of $C$ and $\Omega$. We find that our scheme works best when $\gamma \in [0.8\kappa, 1.8\kappa]$. We set $\gamma = 1.5\kappa$ in this paper. FIG. 4 shows the stationary state fidelity $F$ as a function of the cooperativity parameter $C$ for $\Omega = 0.04g$. Then we carry out the curve fitting for the numerical results of Eq. (10) with the least square method, and obtain the error scaling as $1 - F \propto C^{-1}$ which is similar with the scaling for schemes of Refs. [29, 31]. From the inset of FIG. 4 we find out the actual constants for maximizing the fidelity as follows

$$1 - F \approx 7.2C^{-1}. \quad (11)$$

Fidelity $F$ sees a dramatic increase as the cooperativity parameter $C$ augments in FIG. 4. The fidelity over 90% is attainable under a cooperativity parameter $C$ as low as 75.

Nowadays the experimental parameters ($g$, $\gamma/2$, $\kappa/2$) /$2\pi \approx (34, 2.5, 4.1)$MHz and $C \approx 28$ are achievable [29, 31]. Then the W-type steady states with the fidelity above 75% can be obtained, roughly in a time 5000/g $\approx 23\mu$s. Compared to schemes based on unitary dynamics in cavity QED whose optimal result is $1 - F \propto C^{-1/2}$ [32], the linear scaling of $F$ in the present scheme has an improvement on the cooperativity parameter. Therefore, the proposed scheme is very promising to be realized based on the present QED techniques, and the idea can also be generalized to other systems.

III. CONCLUSION

In conclusion, we have proposed a scheme for dissipative preparation of W-type entangled steady states of three A-atoms in a single mode optical cavity by engineering the effective decay processes. The dissipative dynamics induced by the external fields and dissipative cavity mode leads to the competition between decays into and out of the target state. By suitable choice of the parameters, the former can dominate the latter so that the target state is the steady state of the system. We have shown that a W state with a high fidelity can be obtained with presently available cooperativity.

IV. ACKNOWLEDGEMENT

X.Y.C., L.T.S., H.Z.W., and C.M.F. acknowledge support from the National Fundamental Research Program under Grant No. 2012CB921601, National Natural Science Foundation of China under Grant No. 10974028, the Doctoral Foundation of the Ministry of Education of China under Grant No. 20093514110009, and the Natural Science Foundation of Fujian Province under
Appendix A: Derivation of the effective decay process

We now consider the 1st independent decay process induced by the 1st optical laser with Rabi frequencies $\Omega_1$ and detuning $\Delta_1$, driving independently the transition $|0\rangle \leftrightarrow |2\rangle$, as shown in Fig. 1(b). In a rotating frame, the Hamiltonian of this system in the interaction picture reads $H^{(1)} = H_0^{(1)} + V_+^{(1)} + V_-^{(1)}$, where

$$H_0^{(1)} = \Delta_1 a^\dagger a + \sum_{m=1}^{3} \Delta_1 |2\rangle_m \langle 2|$$

$$H_{eff,1} = \text{Re} \left[ \frac{\delta_1 \Omega_1^2}{3 R_{1,1}} \right] |000\rangle \langle 000| + \text{Re} \left[ \frac{\delta_1 \Omega_1^2}{18 R_{2,1}} \right] (|S_{1,1}\rangle \langle S_{1,1}| + |S_{1,2}\rangle \langle S_{1,2}|) + \text{Re} \left[ \frac{\Omega_1^2}{6 \Delta_1} \right] (|S_{1,1}\rangle \langle S_{1,1}| + |S_{1,2}\rangle \langle S_{1,2}|) + \text{Re} \left[ \frac{\Omega_1^2}{9 \Delta_1} \right] (|S_{2,1}\rangle \langle S_{2,1}| + |S_{2,2}\rangle \langle S_{2,2}|) + \text{Re} \left[ \frac{\delta_1 \Omega_1^2}{9 R_{3,1}} \right] |S_{2,3}\rangle \langle S_{2,3}|,$$

$$L_{eff,1}^{0,m} = \frac{\sqrt{6} \Omega_1}{\sqrt{3} R_{1,1}} |S_{1,3}\rangle \langle 000| - \frac{\sqrt{6} \Omega_1}{\sqrt{3} R_{3,1}} |111\rangle \langle S_{2,3}| - \frac{\sqrt{6} \Omega_1}{3 R_{2,1}} (-e^{-i \frac{2 \pi}{3}} |S_{2,1}\rangle \langle S_{1,2}| - e^{i \frac{2 \pi}{3}} |S_{2,2}\rangle \langle S_{1,1}| + 2 |S_{2,3}\rangle \langle S_{1,3}|),$$

$$L_{eff,1}^{1,m} = \frac{\sqrt{2} \delta_1 \Omega_1}{3 \sqrt{2} R_{1,1}} |000\rangle \langle 000| + \frac{i \sqrt{2} \delta_1 \Omega_1}{18 \sqrt{2} R_{2,1}} (|S_{1,1}\rangle \langle S_{1,1}| + |S_{1,2}\rangle \langle S_{1,2}|) + \frac{i \sqrt{2} \delta_1 \Omega_1}{18 \sqrt{2} R_{2,1}} (|S_{1,1}\rangle \langle S_{1,1}| + |S_{1,2}\rangle \langle S_{1,2}|) + \frac{i \sqrt{2} \delta_1 \Omega_1}{18 \sqrt{2} R_{2,1}} (|S_{1,1}\rangle \langle S_{1,1}| + |S_{1,2}\rangle \langle S_{1,2}|) + \frac{i \sqrt{2} \delta_1 \Omega_1}{9 \sqrt{2} R_{2,1}} (|S_{1,3}\rangle \langle S_{1,3}| + 2 |S_{1,1}\rangle \langle S_{1,1}|) + \frac{i \sqrt{2} \delta_1 \Omega_1}{9 \sqrt{2} R_{2,1}} (|S_{1,3}\rangle \langle S_{1,3}| + 2 |S_{1,1}\rangle \langle S_{1,1}|) + \frac{i \sqrt{2} \delta_1 \Omega_1}{9 \sqrt{2} R_{2,1}} (|S_{1,3}\rangle \langle S_{1,3}| + 2 |S_{1,1}\rangle \langle S_{1,1}|) + \frac{i \sqrt{2} \delta_1 \Omega_1}{9 \sqrt{2} R_{2,1}} (|S_{1,3}\rangle \langle S_{1,3}| + 2 |S_{1,1}\rangle \langle S_{1,1}|),$$

Under the condition of weak classical laser fields, we can adiabatically eliminate the excited cavity field modes and excited states of the atoms when the excited states are not initially populated. Applying Eq. (6) and (7) to our setup, we obtain the effective Hamiltonian and Lindblad operators...
\[ + \left( \frac{\sqrt{\gamma_1 \delta_1}}{18 \sqrt{2 R_{2,1}}} - i \frac{\sqrt{\gamma_1 \Omega_1}}{6 \sqrt{2 \Delta_1}} \right) (e^{i \frac{2m\pi}{3} |S_{2,3}\rangle \langle S_{1,1}| + e^{-i \frac{2m\pi}{3} |S_{2,3}\rangle \langle S_{1,1}|}) \\
- \frac{\sqrt{\gamma_1 \delta_1}}{9 \sqrt{2 R_{2,1}}} (e^{i \frac{2m\pi}{3} |S_{2,2}\rangle \langle S_{1,1}| + e^{-i \frac{2m\pi}{3} |S_{2,2}\rangle \langle S_{1,1}|}) \\
+ \frac{\sqrt{\gamma_1 \delta_1}}{3 \sqrt{6 \Delta_1}} (e^{-i \frac{2m\pi}{9} |111\rangle \langle S_{2,1}| + e^{i \frac{2m\pi}{9} |111\rangle \langle S_{2,1}|} + \frac{\sqrt{\gamma_1 \delta_1 \Omega_1}}{3 \sqrt{6 R_{3,3}}} |111\rangle \langle S_{2,3}|) \] 

\( \frac{V_{+}^{(3)}}{3} = \sum_{m=1}^{3} |2\rangle \langle m| \) (A8)

Applying Eq. (A3) and (A4) to the process, we can obtain the effective Hamiltonian and Lindblad operators of the 2nd decay process in the same way.
+\frac{\sqrt{\gamma_0 \Omega_3}}{9 \sqrt{2 R_{2,3}}} (-e^{-\frac{2(n+1)\pi}{3}} |S_{2,1}\rangle\langle S_{2,2}| + 2|S_{2,2}\rangle\langle S_{2,2}| - e^{\frac{2(n+1)\pi}{3}} |S_{2,3}\rangle\langle S_{2,2}|)
+\frac{\sqrt{\gamma_0 \Omega_3}}{9 \sqrt{2 R_{2,3}}} (-e^{\frac{2(n+1)\pi}{3}} |S_{2,1}\rangle\langle S_{2,3}| - e^{-\frac{2(n+1)\pi}{3}} |S_{2,2}\rangle\langle S_{2,3}| + 2|S_{2,3}\rangle\langle S_{2,3}|)
+\frac{\sqrt{\gamma_0 \Omega_3}}{3 \sqrt{2 R_{3,3}}} |111\rangle\langle 111|.

(A12)

The effective Hamiltonian and Lindblad operators of 4th decay process can be obtained in the same way.

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