Flavour puzzle or
Why Neutrinos Are Different?

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The Flavour Puzzle In A Nutshell

✓ Why three families in the SM?
  - Hierarchical masses + small mixing angles

✓ Why massive neutrinos?
  - Tiny masses + two large mixing angles

✓ Why very suppressed FCNC?
  - Strong limits on a TeV scale extension of the SM

Proposed solution:
  A model of family replication in 6D
3 Families In 4D From 1 Family In 6D

- Our 3D World is a core of Abrikosov-Nielsen-Olesen vortex:
  \[ U_g(1) \text{ gauge field } A + \text{scalar } \Phi \]

- There is only single vector-like fermionic generation in 6D

- Chiral fermionic zero modes are trapped in the core due to specific interaction with the \( A \) and \( \Phi \). Specific choice of \( U_g(1) \) fermionic gauge charges \( \Rightarrow \)
  
  Number of zero modes = 3

- Zero modes \( \iff \) 4D fermionic families
### Field Content

| Fields | Profiles | Charges | Representations |
|--------|----------|---------|-----------------|
| scalar $\Phi$ | $F(r)e^{i\phi}$  
$F(0) = 0$, $F(\infty) = v$ | +1 | $U_g(1)$ $U_Y(1)$ SU$_W$(2) SU$_C$(3) |
| vector $A_\phi$ | $A(r)/e$  
$A(0) = 0$, $A(\infty) = 1$ | 0 | 0 0 0 0 |
| scalar $X$ | $X(r)$  
$X(0) = v_X$, $X(\infty) = 0$ | +1 | 0 1 1 |
| scalar $H$ | $H(r)$  
$H_i(0) = \delta_{2i}v_H$, $H_i(\infty) = 0$ | -1 | +1/2 2 1 |
| fermion $Q$ | 3 L zero modes  
axial (3, 0) | +1/6 | 2 3 |
| fermion $U$ | 3 R zero modes  
axial (0, 3) | +2/3 | 1 3 |
| fermion $D$ | 3 R zero modes  
axial (0, 3) | -1/3 | 1 3 |
| fermion $L$ | 3 L zero modes  
axial (3, 0) | -1/2 | 2 1 |
| fermion $E$ | 3 R zero modes  
axial (0, 3) | -1 | 1 1 |
| fermion $N$ | Kaluza-Klein spectrum | 0 | 0 1 1 |

**Why Neutrinos Are Different?**

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Hierarchical Dirac Masses

- 3 zero modes have different shapes, and different angular momenta \( n = 0, 1, 2 \)

\[
\hat{J} \psi_n \equiv - \left( i \partial_\varphi + \frac{1 + \Gamma_7}{2} \right) \psi_n = n \psi_n
\]

\[
\psi_n(r \to 0) \sim r^n
\]

- \( m_{nm} \propto \int_0^{2\pi} d\varphi \int_0^R dr \bar{\psi}_n \psi_m HX (or \Phi) \sim \sigma^{2n(-1)} \delta_{nm(\pm 1)} \)

- \( \sigma \) depends on the parameters of the model. Hierarchy arises at \( \sigma \sim 0.1 \)

\[
m_2 : m_1 : m_0 \sim \sigma^4 : \sigma^2 : 1 \sim 10^{-4} : 10^{-2} : 1
\]

\[
U_{CKM} \sim \begin{pmatrix}
1 & \sigma & \sigma^4 \\
\sigma & 1 & \sigma \\
\sigma^2 & \sigma & 1
\end{pmatrix}
\]

Generation number \( \iff \) Angular momentum

✓ The scheme is very constrained, as the profiles are dictated by the equations
Neutrinos masses. Why is it different?

- N -- additional neutral spinor
  - Free propagating in the extra dim (up to dist. $R \sim (10 \div 100 \text{TeV})^{-1}$).
  - Majorano-like 6D mass term
    \[ \frac{M}{2} \bar{N}^c N + \text{h.c.} \]
  - Kaluza-Klein tower in 4D (no zero mode)
  - Effective 6D couplings with leptons allowed by symmetries
    \[
    \sum_{S_+} \bar{H} S_+ \bar{L} \frac{1 + \Gamma_7}{2} N + \sum_{S_-} \bar{H} S_- \bar{L} \frac{1 - \Gamma_7}{2} N + \text{h.c.}
    \]

Non-zero windings $\Rightarrow$ more composite structure of the mass matrix

- 4D Majorano neutrinos masses are generated by See-saw mechanism
**Neutrinos:**

\[
m_{mn}^\nu \sim \int_0^{2\pi} d\varphi \int_0^R dr F(r, \varphi) \left[ c^L \propto LL \right]
\]

\[
\sim \int_0^{2\pi} d\varphi e^{i(4-n-m+\ldots)\varphi} \sim \delta_{4+\ldots,m+n}
\]

\[
\sim \begin{pmatrix}
\ddots & 1 \\
\sigma^2 & \ddots \\
1 & \ddots & \ddots
\end{pmatrix}
\]

\[
U_v^\dagger m_v U_v^* \sim \text{diag}(-m, m, m\sigma^2)
\]

\[
U_v \sim \begin{pmatrix}
1/\sqrt{2} & 1/\sqrt{2} & \sigma \\
\sigma & \sigma & 1 \\
-1/\sqrt{2} & 1/\sqrt{2} & \sigma
\end{pmatrix}
\]

**Charged fermions:**

\[
m_{mn}^{\text{charged}} \sim \int_0^{2\pi} d\varphi \int_0^R dr F(r, \varphi) \left[ \bar{\psi} \psi \propto \psi^* \psi \right]
\]

\[
\sim \int_0^{2\pi} d\varphi e^{i(n-m+\ldots)\varphi} \sim \delta_{n,m-\ldots}
\]

\[
\sim \begin{pmatrix}
\sigma^4 & \ddots & \\
\sigma^2 & \ddots & \ddots \\
& \ddots & 1
\end{pmatrix}
\]

\[
m_{\text{charged}}^{\text{diag}} \sim \text{diag}(\mu \sigma^4, \mu \sigma^2, \mu)
\]

\[
U_{\text{CKM}} \sim \begin{pmatrix}
1 & \sigma & \sigma^4 \\
\sigma & 1 & \sigma \\
\sigma^2 & \sigma & 1
\end{pmatrix}
\]
Consequences of this structure

- Inverted hierarchy:

\[ \Delta m_{\odot}^2 = \Delta m_{12}^2 \]
\[ \frac{\Delta m_{12}^2}{\Delta m_{13}^2} \sim \sigma^2 \]

- Pseudo-Dirac structure \( \Rightarrow \)

0νββ decay

Partial suppression

\[ |\langle m_{\beta\beta}\rangle| \approx \frac{1}{3} \sqrt{\Delta m_{\odot}^2} \]
Semi-realistic numerical example

\[
\begin{pmatrix}
-50.03 & 0 & 0 \\
0 & 50.79 & 0 \\
0 & 0 & 0.7089
\end{pmatrix}
\]
\[\text{[meV]}, \quad U_{\text{MNS}} = \begin{pmatrix}
0.808 & 0.559 & 0.186 \\
-0.286 & 0.660 & -0.693 \\
-0.514 & 0.502 & 0.696
\end{pmatrix}\]

\[
\Delta m^2_{12} = 7.63 \times 10^{-5} \text{eV}^2 \\
\Delta m^2_{13} = 2.50 \times 10^{-3} \text{eV}^2
\]

\[
\tan^2 \theta_{12} = 0.471 \left( 0.47^{+0.14}_{-0.10} \right) \\
\tan^2 \theta_{23} = 0.997 \left( 0.9^{+1.0}_{-0.4} \right)
\]

\[
\sin^2 \theta_{13} = 3.46 \times 10^{-2} \left( \leq 0.036 \right)
\]

Consequence for $0\nu\beta\beta$ decay

\[
|\langle m_{\beta\beta} \rangle| = \left| \sum_i m_i U^2_{ei} \right| = 17.0 \text{ meV}
\]
Like in the UED, vector bosons can travel in the bulk of space. From the
4D point of view:

1 massless vector boson in 6D = 

1 massless vector boson (zero mode)
+ KK tower of massive vector bosons $M_n \sim \frac{n}{R}$
$\Rightarrow$ FCNC
+ KK tower of massive scalar bosons in 4D
$\Rightarrow$ KK scalar modes do not interact with fermion zero modes
KK vector modes carry angular momentum = family number. In the absence of fermion mixings, family number is an exactly conserved quantity \( \Rightarrow \) processes with \( \Delta G = \Delta J \neq 0 \) are suppressed by mixing.

\[
K_L^0 \rightarrow \mu^\pm e^\mp
\]

\( \Delta G = \Delta J = 0 \)

\( J_{d\bar{s}} = 1 \)

\( J_{\bar{s}} = 1 \)

\( J_{Z'} = 1 \)

\( J_{\mu e} = 1 \)

\( \sim \frac{\kappa^2}{M_{Z'}^2} \)

\( \bar{\mu} \)

\( e \)

\( \mu \rightarrow ee\bar{e} \)

\( \Delta G = \Delta J = 1 \)

\( J_{e\bar{e}} = 0 \)

\( J_{e} = 2 \)

\( \sim \frac{\kappa^2}{M_{Z'}^2} \)

\( \bar{e} \)

\( \checkmark \) \( \kappa = 1 \) for the particular model, but may be \( \ll 1 \) for extensions

Why Neutrinos Are Different?

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Rare processes:

- $\Delta G = 0$: $K_L^0 \to \mu e, K^+ \to \pi^+ \mu^+ e^-$

- $\Delta G = 1$: $\mu \to ee\bar{e}, \mu e$-conversion, $\mu \to e\nu$

- $\Delta G = 2$: mass difference $K_L - K_S$, CP-violation

Bound on $M_{Z'} \gtrsim \kappa \cdot 100 \cdot$ TeV

**NB**: A clear signature of the model would be an observation $K_L^0 \to \mu e$ without observation other FCNC-processes at the same precision level
Search at LHC

- Search for an «ordinary» massive $Z'(W', g', \nu')$
- Search for $pp \rightarrow \mu^+e^- + \ldots$
- Search for $pp \rightarrow \mu^-e^+ + \ldots$ --- one order below due to quark content of protons
- Search for $pp \rightarrow \bar{t} + c + \ldots$ or $pp \rightarrow \bar{b} + s + \ldots$ --- expect a few 1000's events, but must consider background!

LHC thus has the potential (in a specific model) to beat even the very sensitive fixed target $K \rightarrow \mu e$ limit!

Why Neutrinos Are Different?

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Conclusions

- Family replication model in 6D: elegant solution to the flavour puzzle
  - Hierarchical Dirac masses + small mixing angles
  - Neutrinos are different: See-saw + Majorano-like mass for the bulk neutral fermion can fit neutrino data
  - Family/lepton number violating FCNC suppressed by small fermion mixings

- Predictions for neutrinos
  - Inverted hierarchy
  - Reactor angle $\sim 0.1$
  - Partially suppressed neutrinoless $\beta\beta$ decay

- Other predictions
  - $K \rightarrow \mu e$ will show up earlier than other FCNC-processes
  - Massive gauge bosons with mass $\sim$ TeV or higher
  - Search for $pp \rightarrow \mu^+e^-$ at LHC can beat fixed target
  - Constraint on B-E-H boson: should be LIGHT