Learning Obstacles among Indonesian Eighth Graders on Ratio and Proportion

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Abstract. Many studies reported that the students lack understanding on the topic of ratio and proportion. This topic basically becomes a cornerstone for understanding the other mathematics topics, developing mathematical skills, and helping the students to solve real-life problems, but many researchers have documented the obstacles regarding this topic. For such a reason, this study intends to investigate kinds of learning obstacles on the topic of ratio and proportion that can emerge due to some causes. It was part of didactical design research which was conducted to eighth graders who had already been taught about its topic. The data were collected from students’ answer and interview in solving ratio and proportion problems. The data analysis reveals that the obstacles are classified into ontogenic obstacle, didactical obstacle, and epistemological obstacle. It can be identified from students’ understanding or conception about ratio and proportion which was caused by students’ prior knowledge, didactical practices, or the lack of contexts. These findings are expected to overcome learning obstacles in teaching and learning of ratio and proportion.

1. Introduction
The success of mathematics teaching and learning cannot be separated from successful teachers in overcoming or minimizing the emergence of learning obstacle. The learning obstacle is such a barrier for students to learn mathematics, both the material being learned and they will learn later on. This becomes one of problems that needs to be considered by the teachers in developing didactical and pedagogical anticipation as the relationship between the teacher and materials, besides didactical and pedagogical relationship [1].

The emergence of learning obstacle is basically due to the students’ prior knowledge, the ways of teaching, the textbooks used, as well as the limited of students’ understanding in certain contexts. It can be classified as ontogenic obstacle, didactical obstacle, and epistemological obstacle. Ontogenic obstacle is essentially related to the students’ limitation at the time of their development or their mental readiness in learning [2]. This obstacle deals with psychological causes such as prior knowledge, motivation, experience, and abilities [5] which can actually hinder what the students should have or develop. Those are the factors of ontogenic obstacle and prior knowledge is the most important factor that affects the effectiveness of learning. In other words, this obstacle can emerge when there is discrepancy between students’ prior knowledge and the new ones due to the lack of basic concept or inaccurate, unadapted, and incomplete prior knowledge [3, 4, 2, 5]. It has to be noticed by teachers because mathematics teaching and learning should encourage the students’ experiences or interactions
in such a way that it can strengthen their mentality to construct their new experiences or knowledge easily [6, 7]. The second one is didactical obstacle which can emerge from didactical practices in the class. It can be identified as the results of the methods or approaches used by the teacher [8]. It becomes a reflection process that the students’ understandings also depend on the quality of teachers’ instruction. It indicates the importance of finding the appropriate methods, a good understanding of materials and instructions. The other kind of obstacle is the obstacle in gaining new knowledge, called epistemological obstacle. This obstacle emerges due to the students’ limitation in the contexts [9]. The students are not facilitated with variation of contexts for developing their experiences and understanding, while it is important and should be provided in mathematics teaching and learning [10] so that it helps them to construct new understanding.

Regarding the importance of investigating learning obstacle in mathematics teaching and learning, one of the topics that shows the existence of learning obstacle is ratio and proportion. This topic is listed in curriculum in Indonesia [11] and becomes a cornerstone for understanding the other mathematical topics [12, 13]. In addition, the students’ understanding will lead them to develop mathematical skills [14] and help them to solve real-life problems [15, 16], consequently they have to master it. They need to understand the definition of ratio, its related concepts, characteristics in a concept, its representation, the relationship between ratio and proportion, until the extension of the concept, such as direct and inverse proportion. Ratio is a comparison between quantities and it has a multiplicative relationship between those quantities [17, 18] so that the significance of quantity as well as the difference between multiplicative and additive relationship needs to be emphasized. The next rule is based on the notation of ratio which is written as $a:b$ or $\frac{a}{b}$ where $b \neq 0$ [13]. It indicates that the understanding of fraction concept is required. Their understanding about fraction and ratio will be a prerequisite for understanding proportion concept. An equivalent fraction is used in equivalent ratio which denotes a proportion. It leads to the strategy of solving problems, called cross-multiplication [17] and they need to understand how the cross product works in the notation of proportion, i.e. “if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$, where $a, b, c,$ and $d$ are positive integers” [19, 18]. Furthermore, the students have to understand two kinds of proportional relationships, namely direct and inverse proportion. Those understandings are going to help them to solve the problems.

The fact is the students still lack understanding. Many studies reveal that teaching and learning of ratio and proportion just force them to apply procedural methods rather than to develop their mathematical concept and skills [20, 21, 22]. The students tend to forget the purpose of learning the concept so that they do not use it to solve the problems [23, 24, 25, 16, 21]. They also cannot distinguish between direct and inverse proportion, as well as lack of experiences in exploring the contexts. This condition certainly shows that so many learning obstacles on the topic of ratio and proportion. It eventually becomes the obstacle in learning the other mathematical concepts.

Based on the results above, this study intends to investigate learning obstacles which are identified. It explains crucial aspects in each obstacle so that it is expected to be guidance for teachers to develop didactical design and to overcome the emergence of learning obstacle on ratio and proportion.

2. Method

The study was conducted based on interpretive paradigm which was part of Didactical Design Research. This paradigm concerns the problems of didactical design. It carries out the effort to probe and to understand phenomenon occurred [26, 27] as a contribution to give a better solution, for instance, the phenomenon of learning obstacles. This identification is based on previous didactical design. The understanding about learning obstacle is followed up as a critical paradigm. It means that this study is useful to create learning innovations.

There were 72 students of grade 8 in one of junior high schools in Bandung, Indonesia, who had learned ratio and proportion. They had to solve six problems about ratio and proportion (Fig.1). The problems are basically designed to investigate students’ understanding about ratio and proportion concept (including direct and inverse proportion) as well as its application; and also applying the
notation, rules, or formula. Problem 1 and 6 lead the students to compare ratios and to realize the existence of nonproportional relationships. Problem 2 and 3 are related to equivalent ratios (proportional situations) which lead the students to use the concept of proportion, but Problem 3 invites them to prove the condition. Problem 4 emphasizes the contexts that cannot be represented by the notation of proportion (direct proportion), even though the problem resembles proportion problems. Furthermore, problem 5 combines the concept of direct and inverse proportion which is applied in scale drawing and speed. Based on the results of students’ answer, some of them were selected for an in-depth interview, and then the results were analyzed qualitatively. The data shows about their strategies in solving problems, the ways of thinking, the application of the concepts, and learning obstacles.

3. Result and Discussion

Based on the results of students’ answer on the test given and interview, the researchers found three kinds of learning obstacle, called ontogenic obstacle, didactical obstacle, and epistemological obstacle. The factors of epistemological obstacle have been investigated in the previous study [28], however this study finds out their strategies in solving ratio and proportion problems (Fig. 1) can also be identified as ontogenic obstacle and didactical obstacle.

![Figure 1. The problems about ratio and proportion concept.](image)

**Figure 1.** The problems about ratio and proportion concept.

**Figure 2.** Photograph of student’s answer in solving Problem 1.

3.1. Ontogenic Obstacle

The students’ answers show ontogenic obstacle when there is discrepancy between the ways of thinking and the material. The students are forced to think of something, while their prior knowledge is still insufficient. Hence, their conception is incomplete. They do not understand because they miss the important things from definition. They should know about quantity and the characteristics of ratio and proportion, but they do not get it. It can be depicted from the strategy in solving Problem 1 (Fig. 2). The student realizes the problem is about ratio, s/he uses the notation of ratio, even though s/he does not understand why s/he has to do it. This answer has ever been classified as epistemological obstacle [28], but it can be an ontogenic obstacle because most of the students just know about the notation without understanding the meaning. It might be due to the lack of knowledge to achieve a complete understanding of the concept.
Ontogenic obstacle is also identified when the students solve Problem 2. All of them do not understand how to represent the problem using proportion concept. Some students are used to solve such problem using calculation manually without noticing related concept. Some know that ratio can be written as fraction like what they do in solving the first problem (Fig. 3), but they have no idea when they have to apply it in solving Problem 2. They basically do not understand how equivalent fraction is related to proportion concept.

The condition above shows that the students’ prior knowledge about fraction is not conveyed properly in teaching and learning of ratio and proportion. They are not directed to see the relationship between fraction and proportion concept. They are just given the notation, the formula, the examples, whereas the understanding of ratio should lead them to find the value of the other ratio regarding the proportion concept [18]. Furthermore, it affects their strategy in solving Problem 3, as shown in Figure 5. It shows that the student does not have an effective strategy so that s/he just guesses the distance and fuel, looks for numbers which close to 267 as the distance and 21 as litres of fuel. The student does not know how and when the proportion concept can be used, moreover s/he does not realize whether it is a proportion problem.

The result of students’ answers indicates that there is a gap for the ways of thinking regarding the introduction of quantity and multiplicative relationship. The introduction can actually emphasize “what are we comparing?” Ratio is almost used when comparing quantities, such as distance, time, temperature, length, height, speed [12, 29]. It is an illustration or description of an object and its characteristic including the unit of measurement [30]. The students also have to know about distinguishing whether a condition is ratio, and the concept involved. If they understand the fraction concept and its application, they will realize a condition where zero is only for numerator [13], as the example. Those are the aspects why they do not achieve the understanding of ratio and proportion concept.

3.2. Didactical Obstacle
Didactical obstacle is identified from students’ answers which emphasize procedural methods without understanding. When the students solve Problem 4, they confidently assume that it is about direct proportion. They just remember that if they find a problem with three information and one is missing, they can solve it using the ways as shown in Figure 6. They recognize it as a problem they usually get
in the class. It also proves that a leap happening is based on the ways of teaching in the class as shown in the following interview.

**Researcher**: what did you do here? Where did you find 2400?
**Student**: $800 \times 3$.

**Researcher**: why did you multiply it by three?
**Student**: because the sister is here (200) and here is younger sister (800). It means that $600 \div 3$.

**Researcher**: oh I see. So, it becomes $800 \times 3$?
**Student**: yes, that’s why the answer is 2400.

**Researcher**: but why did you have to multiply it?
**Student**: Hmmm... because of the examples I used to get.

The interview above reveals that the procedure the student uses as the things s/he always get in the class when s/he learns about proportion, so s/he relies on the examples. It denotes that the teacher does not tell the students about when it can apply in a problem. This condition is related to the contexts whether it is ratio or non-ratio. Even though 58.33% students solve Problem 4 correctly, it does not guarantee they can distinguish between ratio and non-ratio problem. It can be detected from their strategies in solving all problems given.

![Figure 6](image1.png)

**Figure 6.** Photograph of student’s answer in solving Problem 4.

![Figure 7](image2.png)

**Figure 7.** Photograph of student’s answer in solving Problem 5

![Figure 8](image3.png)

**Figure 8.** Photograph of illustration about scale drawing formula

Didactical obstacle is also depicted when they solve Problem 5. The previous study already shows the strategy in Figure 7 as epistemological obstacle [28], but it can actually be classified as didactical obstacle. The student creates an illustration which is ever given in the class to remember scale drawing formula (Fig. 8), but the illustration is wrong and s/he has no idea at all about what the scale is. Even though the student realizes the topic of the problem, s/he does not understand the concept so that the illustration is useless when s/he forgets where s/he has to put the conditions. In fact, the way the student uses proportion there solely just for getting the answer. It does not mean proportion. The student just places the quantity randomly.
The results above illustrate the learning conditions that occur in the class. The students explain their strategies used based on what they have got in the class, based on the way the teacher teaches and emphasizes the things that these things are enough to remember. The teacher does not emphasize on using the rules or formulas correctly, what needs to be understood in using them, what concepts are involved, as well as when and where a procedure, rule, or formula can be applied. It also proves that the teacher teaches mathematics just for making the students do mathematical calculations and to stick to the rule, rather than educating them to construct their knowledge and develop their mathematical skills [31, 32, 33, 34, 35]. This condition is one of the implications of the emergence of didactical obstacle. The teachers should facilitate them to find formulas or rules so that the students do not rely on memorization of formulas, but on constructing conceptual understanding. The teacher should also understand how to provide the ways of thinking properly so that the students understand the concept completely.

### 3.3. Epistemological Obstacle

The explanations related to factors influencing the emergence of epistemological obstacle on the topic of ratio and proportion basically have been analyzed in the previous studies [28], but it only focuses on some problems. A broader perspective is carried out in this study which describes the epistemological obstacle of all problems. It is depicted when the students are only limited to understanding the context of a certain problem. They usually suddenly do not understand what concepts they should use when they find different contexts or types of questions, even though it actually uses the concepts they have got in the class. This condition is shown when they solve Problem 1, 3, 5, and 6.

Their strategies for Problem 1 are classified as epistemological obstacle because the students never get the problems like proving the quantity of a group with the other group [28]. They never realize the meaning of quantity from the point of view of the "part" which indicates a multiplicative relationship, not additive one. It makes their answers focus solely on additive situation and tell that girls on team A are just the same with team B, as shown in Figure 2.

The answer for the previous one in Problem 3 (Fig. 5) can also reflect an epistemological obstacle because almost all students have no idea how to solve it using proportion concept. They often deal with the context to find a quantity from the other ratios as a ratio concept [18], but the demand to prove a condition is very rarely given in the class. The students cannot explore their understanding further because they do not know the basic concept. Thus, they just rely on understanding of routine problems in the class.

![Figure 9. Photograph of student’s answer in solving Problem 6 using addition.](image1.png)

The other conditions are detected when the students solve Problem 5. Their strategies are dominated by working on numbers and formulas without understanding, as shown in Figure 7. In fact, the students just know the problems in accordance with the topic, for instance, if the teacher is introducing them about direct proportion, then the examples will be full about it in the same type, as well as the topic of inverse proportion [28]. They are often not facilitated to realize and prove the differences directly from the contexts, especially faced with a problem that combines both.

The same situation also occurs when the students solve Problem 6. There are no students who answer this problem correctly because they do not realize that the enlargement uses the concept of ratio and
proportion [28]. Figure 9 and 10 show that all of them do not understand the meaning of enlargement. Some students consider it as additive situation, as shown in Figure 9 and some try to relate it to the concept of area, and then look for the enlargement of photo (Fig. 10). It shows that ratio and proportion problems that students have so far have not varied. The concept is not yet fully understood so that these simple problems are difficult for them.

Based on the identification of learning obstacles on the topic of ratio and proportion above, it is expected to be knowledge for teachers to anticipate the emergence of learning obstacles. In addition, this understanding can be guidance to how the topic of ratio and proportion should be taught, what aspects have been lost or missed, and what mathematical skills can be developed [36, 28, 37]. When the teachers are able to understand it, then the quality of learning can be better and make them as professional educators.

4. Conclusion
The students cannot solve the problems related ratio and proportion because the topic was learned when students do not have sufficient prior knowledge, which is classified as ontogenic obstacle. There are important things which are overlooked and there is a connection among the concepts neglected. This condition can also happen due to didactical practices in the class so that the students do not develop their conceptual understanding and lack experience in applying concepts. It is eventually classified as didactical and epistemological obstacle. Therefore, for a successful teaching and learning of ratio and proportion in the future, teachers must provide the instructions or didactical design to avoid those kinds of obstacle. Of course, it is not using a lecture method, but more interactive and independent learning. It can be carried out by understanding learning obstacles identified. This understanding also intends that the teachers need to master the material they want to teach, to understand mathematical skills that need to be developed, so they create an effective teaching and learning of ratio and proportion.

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