Comment on cond-mat/0107371: “Dynamical exponents of an even-parity-conserving contact process with diffusion”

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In cond-mat/0107371, de Mendonça proposes that diffusion can change the universality class of a parity-conserving reaction-diffusion process. In this comment we suggest that this cannot happen, due to symmetry arguments. We also present numerical results from lattice simulations which support these arguments, and mention a previous result supporting this conclusion.

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Although there are a large variety of nonequilibrium reaction/reaction-diffusion models which present a transition from an active to an absorbing state, their critical behaviors fall within a small number of universality classes. The simplest one, which can be viewed as the nonequilibrium counterpart of the Ising universality class, is the contact process (CP), or directed percolation (DP) universality class [1, 2, 3]. Another well-established class is the parity-conserving (PC) universality class [4, 5]. More recently, binary spreading (BS) processes (with or without parity conservation), have appeared as representative of a new universality class [6, 7, 8].

It appears that the nature of the creation and annihilation processes is fundamental on determining to which universality class a given process might belong, that is, the number of particles necessary for a reaction to occur seems to play a key role. One way to assess the importance of such terms is via numerical simulations or, whenever possible, by studying the analytical properties of the equations governing the dynamical process.

For any Markovian process, one can write a master equation describing how the probability \(P(\{a\}, t)\) of a given configuration \(\{a\}\) evolves in time [9]. A technique first introduced by Doi [10] allows us to derive a field-theoretic representation for the master equation,
and then to handle fluctuations systematically. Moreover, with the field-theoretic action in hand, one can write down the Langevin equation which governs the stochastic evolution of the field operators, with no ambiguity on the definition of the noise [6].

Even though the noise is always multiplicative in absorbing state phase transitions, it can have different functional forms, which can change entirely the long-term behavior of the system being described. For instance, when two particles are necessary for annihilation to occur, the noise in the Langevin equation is, counter-intuitively, purely imaginary (as in representatives of the PC class). On the other hand, when both creation and annihilation require a pair of particles, there is a competition between “real” and “imaginary” noise (as in representatives of the BS class) [7].

| Universality class | creation | annihilation |
|--------------------|----------|--------------|
| DP                 | unary    | unary        |
| PC                 | unary    | binary       |
| PCPD               | binary   | binary       |

TABLE I: Nature of creation/annihilation processes within each universality class, that is, the number of particles needed for a reaction to occur.

The CP(M) process, invented by Inui and Tretyakov [12], is a parity-conserving dynamical process that can be summarized as follows

\[
\begin{align*}
mA & \rightarrow \emptyset \\
A & \rightarrow (m+1)A.
\end{align*}
\]

It can be interpreted as a process where a particle generates \( m \) others with probability \( p \), while \( m \) particles, upon contact, are annihilated with probability \( 1 - p \). An isolated cluster of \( (m - 1) \) particles cannot diffuse or annihilate. It has been shown by the authors that CP(2), starting with an even number of particles, belongs to the PC universality class [12].

Recently, Mendonça [13] studied the CP(2) process with diffusion of solitary particles. On the basis of finite size scaling of the exact numerical diagonalization of the evolution
operator, he reports that diffusion changes the universality class from PC to DP. Numerical estimates for $z$, the dynamic exponent governing spatial fluctuations of the cluster of alive sites, seem to converge to $z_{DP}$.

At a first glance, the claim that the addition of diffusion might alter the dynamical critical behavior of the CP(2) process does not seem plausible, since it does not change the symmetry of the problem: the number of particles is still conserved modulo 2 and the absorbing state is still unique and devoid of particles. Moreover, as in the analysis of simpler models [14], it can be seen that a coarse-grained description of non-diffusive lattice models generally includes a diffusive term, and even without coarse-graining, an effective diffusion is already present in CP(2), for instance, in the sequence $A\emptyset \emptyset \rightarrow AAA \rightarrow \emptyset \emptyset A$.

To verify this, we have developed a probabilistic cellular automaton for the 1d CP(2)d (with diffusion rate $d = 0.05$) with even and odd sites constituting sub-lattices $A$ and $B$ [15]. Starting from a fully occupied lattice of 8000 sites, we have measured the density of active sites $\rho(t)$, which should decay as $\rho(t) \propto t^{-\delta}$ at criticality. From the data we find $\delta \sim 0.286(5)$.

We have also performed dynamical simulations at the critical point, starting from a single pair of particles and measuring the accumulated number of active sites, $M(t) \propto t^{\eta+1}$, from which we find $\eta \sim 0.0(5)$. These results suggest that diffusion does no alter the critical behavior of CP(2), and only confirm what has already been obtained by Zhong and ben-Avraham, who studied a branching-annihilating random walk with finite annihilation rate [16]. This system is equivalent to the CP(2)d with high diffusion probability, and thus should have the same critical behavior. We point out to cond-mat/0207720 for recent simulations of the diffusive pair contact process [3, 4], a lattice model which (possibly) has a diffusion dependent universality class.

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FIGURE CAPTIONS

FIG. 1. Density of active sites $\rho(t)$ as a function of time (in Monte-Carlo steps). Results are shown for the diffusionless CP(2) (upper) and CP(2)d with $d=0.05$ (lower).

FIG. 2. Cumulative number of active sites, $M(t)$ as a function of time (in Monte Carlo steps) for the CP(2) (upper) and CP(2)d (lower) with $d=0.05$. 
FIGURES

FIG. 1:

FIG. 2: