Meyerovitch, Tom; Yadin, Ariel
Harmonic functions of linear growth on solvable groups. (English) Zbl 1362.43006
Isr. J. Math. 216, No. 1, 149-180 (2016).

The authors prove the following conjecture (the converse direction of Klein’s theorem) for solvable groups:
Let $G$ be a finitely generated group, and let $\mu$ be a symmetric probability measure on $G$, with finite support that generates $G$. Let $HF_k(G, \mu)$ denote the space of $\mu$-harmonic functions on $G$ whose growth is bounded by a degree $k$ polynomial. Then the following are equivalent: (1) $G$ is virtually nilpotent. (2) $G$ has polynomial growth. (3) $\dim HF_k(G, \mu) < \infty$ for all $k$. (4) There exists $k \geq 1$ such that $\dim HF_k(G, \mu) < \infty$. The investigation is motivated by Kleiner’s proof for Gromov’s theorem on groups of polynomial growth.

Reviewer: Bolis Basit (Clayton)

MSC:
43A70 Analysis on specific locally compact and other abelian groups
60B15 Probability measures on groups or semigroups, Fourier transforms, factorization

Full Text: DOI arXiv

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