Magnetic moment for the negative parity $\Lambda \rightarrow \Sigma^0$ transition in light cone QCD sum rules

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Abstract

The magnetic moment of the $\Lambda \rightarrow \Sigma^0$ transition between negative parity, baryons is calculated in framework of the QCD sum rules approach, using the general form of the interpolating currents. The pollution arising from the positive–to–positive, and positive to negative parity baryons are eliminated by constructing the sum rules for different Lorentz structures. Nonzero value of the considered magnetic moment can be attributed to the violation of the $SU(3)$ symmetry.

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1 Introduction

Magnetic moment of baryons is one of the most important quantities in investigation of their electromagnetic structure, and can provide essential information about the dynamics of the strong interaction at low energies. The magnetic moments of the octet baryons have already been calculated in various theoretical approaches. These calculations have the privilege that they can be checked against the available precise experimental data. The study of the $\Lambda \rightarrow \Sigma^0$ transition magnetic moment can play a critical role in investigation of the properties of the octet baryons.

In recent years the study of the negative parity baryons have become of the most promising direction on connection with the experiments conducted and planned at Jefferson laboratory [1], and Mainz Microtron facility (MAMI) [2, 3]. The magnetic moments of $N*$ are planned to be measured at MAMI [3, 4]. In the present work we calculate the transition magnetic moment between the negative parity $\Lambda^*$ and $\Sigma^{0*}$ baryons within the QCD sum rules method (LCSR) (here and in future discussions, we denote the negative parity baryons as $B^*$). This method is based on operator product expansion (OPE) near light cone. The OPE is performed over the twist of the operators rather than dimension, as is the case in the traditional QCD sum rules method. In this version all nonperturbative dynamics is encoded in light cone distribution amplitudes. These amplitudes appear when the matrix elements of the nonlocal operators are sandwiched between the vacuum and one–particle states (about the details of the LCSR see [5]). The magnetic moment of the $\Lambda \rightarrow \Sigma^0$ transition has already been calculated in framework of the traditional QCD sum rules [6], the external field method in the traditional QCD sum rules [7], and in the light cone version of the QCD sum rules method [8]. Note that the magnetic moments of the negative parity octet baryons, $J^{P} = 3^-/2$ heavy baryons, as well as diagonal and transition magnetic moments of negative parity heavy baryons are calculated within the same framework in [9], [10] and [11], respectively.

The work is arranged as follows. In section 2 the LCSR for the magnetic moment of the $\Lambda^* \rightarrow \Sigma^{0*}$ transition is derived. In section 3 we numerically analyze these LCSR obtain for the transition magnetic moment. This section also contains concluding remarks.

2 Light cone QCD sum rules for the magnetic moment of the $\Lambda^* \rightarrow \Sigma^{0*}$ transition

In order to obtain the light cone sum rules for the magnetic moment of the $\Lambda^* \rightarrow \Sigma^{0*}$ transition the following time ordered correlation function in the vacuum in presence of the external magnetic field is considered,

$$\Pi = i \int d^4 x e^{ipx} \langle 0 | T \{ \eta_{\Sigma^0}(x) \bar{\eta}_{\Lambda}(0) \} | 0 \rangle_\gamma ,$$

where $\eta_B$ is the interpolating current of the corresponding baryon. Firstly, on the phenomenological side the calculation is carried out by saturating a tower of hadronic intermediate states carrying the same quantum numbers as the interpolating current. Secondly, on the QCD side it is described in terms of quarks and gluons. The QCD sum rules is
constructed by matching these two representations. The interpolating currents needed in
calculation of the correlation function are constructed from the quark fields with the same
quantum number as the interpolating currents, and isolating the ground state
contribution we get,

\[ \eta_\Lambda = 2 \sqrt{\frac{1}{2} \varepsilon^{abc}} \left\{ 2(u^{\alpha T} C d^b) \gamma_5 s^c + (u^{\alpha T} C s^b) \gamma_5 d^c - (d^{\alpha T} C s^b) \gamma_5 u^c + 2\beta (u^{\alpha T} C \gamma_5 d^b) s^c \\
+ \beta (u^{\alpha T} C \gamma_5 s^b) d^c - \beta (d^{\alpha T} C \gamma_5 s^b) u^c \right\}, \]

\[ \eta_{\Sigma^0} = \sqrt{2} \varepsilon^{abc} \left\{ (u^{\alpha T} C s^b) \gamma_5 d^c + (d^{\alpha T} C s^b) \gamma_5 u^c + \beta (u^{\alpha T} C \gamma_5 s^b) d^c + \beta (d^{\alpha T} C \gamma_5 s^b) u^c \right\}. \]

(2)

where \( a, b, c \) are the color indices, \( C \) is the charge conjugation operator, superscript \( T \)
denotes the transpose operator, and \( \beta \) is the arbitrary parameter with \( \beta = -1 \) corresponding
to the Ioffe current.

Firstly we shall calculate the phenomenological part of the correlation function given in
Eq. (1). Saturating the interpolating current with the intermediate hadronic states having
the same quantum number as the interpolating currents, and isolating the ground state
contributions we get,

\[ \Pi = \left\langle 0 | \eta_\Sigma | \Sigma^0(p_2) \right\rangle \frac{\left\langle \Sigma^0(p_2) \gamma(q) | \Lambda(p_1) \right\rangle}{p_2^2 - m_{\Sigma^0}^2} \frac{\left\langle \Lambda(p_1) | \bar{\eta}_\Lambda | 0 \right\rangle}{p_1^2 - m_\Lambda^2} + \left\langle 0 | \eta_{\Sigma^0} | \Sigma^0(p_2) \right\rangle \frac{\left\langle \Sigma^0(p_2) \gamma(q) | \Lambda^*(p_1) \right\rangle}{p_2^2 - m_{\Sigma^0}^2} \frac{\left\langle \Lambda^*(p_1) | \bar{\eta}_\Lambda | 0 \right\rangle}{p_1^2 - m_\Lambda^2} + \left\langle 0 | \eta_\Sigma | \Sigma^0(p_2) \right\rangle \frac{\left\langle \Sigma^0(p_2) \gamma(q) | \Lambda^*(p_1) \right\rangle}{p_2^2 - m_{\Sigma^0}^2} \frac{\left\langle \Lambda^*(p_1) | \bar{\eta}_\Lambda | 0 \right\rangle}{p_1^2 - m_\Lambda^2} + \left\langle 0 | \eta_{\Sigma^0} | \Sigma^0(p_2) \right\rangle \frac{\left\langle \Sigma^0(p_2) \gamma(q) | \Lambda^*(p_1) \right\rangle}{p_2^2 - m_{\Sigma^0}^2} \frac{\left\langle \Lambda^*(p_1) | \bar{\eta}_\Lambda | 0 \right\rangle}{p_1^2 - m_\Lambda^2}, \]

(3)

where superscript \( * \) means it is a negative parity baryon. The matrix elements in Eq. (3)
are determined in the following way:

\[ \left\langle 0 | \eta | B(p) \right\rangle = \lambda_B u(p), \]

\[ \left\langle 0 | \eta | B^*(p) \right\rangle = \lambda_{B^*} \gamma_5 u(p), \]

\[ \left\langle B_2(p_2) \gamma(q) | B_1(p_1) \right\rangle = e\varepsilon^{\mu}(p_2) \left[ f_1^\mu \gamma_\mu - i \frac{\sigma_{\mu\nu} q^\nu}{m_{B_1} + m_{B_2}} f_2 \right] u(p_1), \]

\[ \left\langle B_2^*(p_2) \gamma(q) | B_1^*(p_1) \right\rangle = e\varepsilon^{\mu}(p_2) \left[ f_1^* \gamma_\mu - i \frac{\sigma_{\mu\nu} q^\nu}{m_{B_1} + m_{B_2}} f_2^* \right] u(p_1), \]

\[ \left\langle B_2^*(p_2) \gamma(q) | B_1(p_1) \right\rangle = e\varepsilon^{\mu}(p_2) \left[ f_1^T \gamma_\mu - i \frac{\sigma_{\mu\nu} q^\nu}{m_{B_1} + m_{B_2}} f_2^T \right] \gamma_5 u(p_1). \]

(4)

Substituting these matrix elements into Eq. (3), and performing summation over the spins
of the baryons we get,

\[ A' (\not p_2 + m_{\Sigma^0}) \not (\not p_1 + m_\Lambda) + B' (\not p_2 - m_{\Sigma^0}) \not (\not p_1 - m_{\Lambda^*}) + C' (\not p_2 - m_{\Sigma^0}) \not (\not p_1 + m_\Lambda) + D' (\not p_2 + m_{\Sigma^0}) \not (\not p_1 - m_{\Lambda^*}) + \cdots, \]

(5)
where

\[
A' = \frac{\lambda^{0}_S(\beta)\lambda_\Lambda(\beta)}{(m^2_{\Sigma^0} - p^2_2)(m^2_\Lambda - p^1_1)}(f_1 + f_2),
\]
\[
B' = \frac{\lambda^{0}_S(\beta)\lambda_\Lambda*(\beta)}{(m^2_{\Sigma^0*} - p^2_2)(m^2_\Lambda* - p^1_1)}(f^*_1 + f^*_2),
\]
\[
C' = \frac{\lambda^{0}_S(\beta)\lambda_\Lambda(\beta)}{(m^2_{\Sigma^0} - p^2_2)(m^2_\Lambda - p^1_1)}\left[f^T_1 + \frac{m_{\Sigma^0} - m_\Lambda}{m_{\Sigma^0*} + m_\Lambda}f^*_2\right],
\]
\[
D' = -\frac{\lambda^{0}_S(\beta)\lambda_\Lambda*(\beta)}{(m^2_{\Sigma^0} - p^2_2)(m^2_\Lambda* - p^1_1)}\left[f^T_1 + \frac{m_\Lambda* - m_{\Sigma^0}}{m_\Lambda* + m_{\Sigma^0}}f^*_2\right],
\]

(6)

where dots denote rest of the structures other that \(\gamma_\mu\). The \(\Lambda^* \rightarrow \Sigma^0*\) transition magnetic moment in natural units is described by \(f^*_1 + f^*_2\) at the point \(q^2 = 0\). Therefore, in order to determine the magnetic moment of the \(\Lambda^* \rightarrow \Sigma^0*\) transition the four equations in Eq. (5) should be solved.

The result of the calculation for the correlation function from the QCD side can be obtained from the diagonal \(\Sigma^{0*} - \Sigma^{0*}\) transition as follows. It is noted in [13] that the magnetic moment for the \(\Lambda - \Sigma^0\) transition can be determined from the diagonal \(\Sigma^0 - \Sigma^0\) transition by using the relation between the correlation function which is given as (more precisely using the relation between the invariant functions for the different Lorentz structures),

\[
\Pi^\Sigma^0_{\omega}(u+d) - \Pi^\Sigma^0_{\omega}(u+d) = \sqrt{3}\Pi^\Sigma^0_{\omega}\Lambda.
\]

(7)

This relation shows that one can obtain the QCD sum rules for the \(\Lambda^*-\Sigma^{0*}\) transition magnetic moment by making simple substitutions in the result for the diagonal \(\Sigma^{0*} - \Sigma^{0*}\) transition.

The invariant functions for the diagonal \(\Sigma^{0*} - \Sigma^{0*}\) transition are calculated in [9] (see also the Appendix in [14]), and in the same manner with the help of Eq. (7) the same calculation can easily be repeated for the \(\Lambda^*-\Sigma^{0*}\) transition. For this reason, in the present work we do not present the result of the correlation function from the QCD side.

As has already been mentioned, in order to determine the magnetic moment of the negative parity \(\Lambda^* - \Sigma^{0*}\) transition four equations are needed. In constructing these four equations we need four Lorentz structures. In the present work we choose the structures \(p\not\!q, \not\!p, \not\!q, \not\!p\), and denote the corresponding invariant functions as \(\Pi_1, \Pi_2, \Pi_3\) and \(\Pi_4\), respectively.

The sum rules for the \(\Lambda^* - \Sigma^0\) transition is derived by equating the coefficients of the structures \(p\not\!q, \not\!p, \not\!q, \not\!p\) of the correlation from the from the phenomenological and QCD side, and perform double Borel transformation over the variables \(p^2_1 = (p+q)^2\) and \(p^2_2 = p^2\), and then solve the system of algebraic equations. As the result of these steps of calculations we get the following expression for the magnetic moment of the negative parity \(\Lambda - \Sigma^0\) transition,

\[
\mu = \frac{e^{m^2_{\Sigma^0*}/M^2}}{\lambda_\Lambda*\lambda^{0}_S}(m_{\Sigma^0} + m_{\Sigma^0*})(m^2_{\Sigma^0} + 3m^2_{\Sigma^0*}) \left\{ \left[ m_{\Sigma^0}(m_{\Sigma^0} - m_{\Sigma^0*}) - 2m^2_{\Sigma^0*} \right] \Pi^B_1 - 2m_{\Sigma^0}(m_{\Sigma^0} + m_{\Sigma^0*}) \Pi^B_2 - (m_{\Sigma^0} - 3m_{\Sigma^0*}) \Pi^B_3 - m_{\Sigma^0}(m_{\Sigma^0} + m_{\Sigma^0*}) \Pi^B_4 \right\},
\]

(8)
where we have used $M_1^2 - M_2^2 = 2M^2$ and $m_\Lambda \simeq m_{\Sigma^0}$, $m_{\Lambda^*} \simeq m_{\Sigma^0*}$. The residues $\lambda_{\Lambda^*}$ and $\lambda_{\Sigma^0*}$ are calculated in [9].

Here, few remarks about the calculation of the correlation function from the QCD side are in order. This correlation function contains three different contributions. a) Perturbative part, which corresponds to the case when photon interacts with quarks perturbatively, and all propagators of the free quarks are considered. b) Mixed part which corresponds to the case when photon interacts with quarks perturbatively, and at least one quark propagator is replaced by the corresponding condensates. c) Nonperturbative part. In this case photon interacts with the quarks at long distance. This interaction is described by the matrix element of the nonlocal operators between the vacuum and one–photon states, i.e.,

$$\langle \Gamma(q)|q_i(G_{\mu
u}\Gamma_i)q|0\rangle.$$ 

These matrix elements are parametrized in terms of the photon distribution amplitudes (DAs). The definitions of the above–mentioned matrix elements and photon DAs are presented in [15].

### 3 Numerical results

This section is devoted to the numerical analysis of the sum rules obtained for the $\sigma^0$–$\Lambda$ transition magnetic moment of the negative parity baryons. The values of the input parameters entering to the sum rules are $\langle \bar{u}u\rangle(1\, GeV) = \langle \bar{d}d\rangle(1\, GeV) = -(0.243)^3 GeV^3$, $\langle \bar{s}s\rangle(1\, GeV) = 0.8\langle \bar{u}u\rangle(1\, GeV)$, $m_0^2 = (0.8\pm0.2)\, GeV^2$ [16], $\Lambda = (0.5\pm0.1)\, GeV$ [17], $f_{3\gamma} = -0.039$ [15]. The value of the magnetic susceptibility is determined from the QCD sum rules analysis to have the value $\chi(1\, GeV) = -(2.85\pm0.5)\, GeV^{-2}$ [18], and $m_s(2\, GeV) = (111\pm6)\, MeV$ [19]. Also, the expressions of the photon DAs, which are the main ingredients of the LCSR, are presented in [15].

The sum rules for the transition magnetic moment of the $\Lambda^*–\Sigma^{0*}$ transition contains three auxiliary parameters, namely, the continuum threshold $s_0$, the arbitrary parameter $\beta$ in the interpolating current, the Borel mass parameter $M^2$; and the magnetic moment should be independent of them.

The working region of the Borel mass parameter $M^2$ for the magnetic moment of $\Lambda^*$ and $\Sigma^{0*}$ transition is determined in [9] to have the range $1.6\, GeV^2 \leq M^2 \leq 3.0\, GeV^2$. Since we take $m_\Lambda \simeq m_{\Sigma^0}$ we can also use the same domain in the present analysis.

The second arbitrary parameter of the sum rules is the continuum threshold $s_0$. This parameter is related by the energy of the first state. The energy difference between the first and ground states ranges from 0.3 $GeV$ to 0.8 $GeV$. In our calculations we use the average value $\sqrt{s_0} = (m_{\text{ground}}+0.5)\, GeV$. Finally, in order to determine the domain of the arbitrary parameter $\beta$ that appears in the interpolating current, we consider the dependence of the magnetic moment on $\cos \theta$, where $\beta = \tan \theta$, at several fixed values of $M^2$ and $s_0$ chosen from their respective working regions.

As the result of our detailed numerical study, the magnetic moment of the negative parity $\Lambda^*$ and $\Sigma^{0*}$ transition is found to have the value,

$$\mu_{\Lambda^*\Sigma^{0*}} = (0.2 \pm 0.05)\mu_N,$$
where the error in the result can be attributed to the variation in $M^2$ and $s_0$, as well as the uncertainties in the photon DAs and input parameters.

In the $SU(3)$ limit the $\Lambda^* - \Sigma^0$ transition magnetic moment is equal to zero. The deviation from zero is a measure of the $SU(3)$ symmetry violation, and we see that this violation is about 20%.

In conclusion, the magnetic moment of the $\Lambda - \Sigma^0$ transition for the negative parity baryons is estimated in framework of the LCSR. It is obtained that the violation of $SU(3)$ symmetry leads to nonzero value for the $\Lambda^* - \Sigma^{0*}$ transition magnetic moment.
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