Glitch Behavior of Pulsars and Contribution from Neutron Star Crust

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Abstract

Pulsars are highly magnetized rotating neutron stars (NSs) with a very stable rotation speed. Irrespective of their stable rotation rate, many pulsars have been observed to feature a sudden jump in the spin frequency, known as a pulsar glitch. The glitch phenomena are considered to be an exhibit of superfluidity of neutron matter inside the NS’s crustal region. The magnitude of such a rapid change in rotation rate relative to the stable rotation frequency can quantify the ratio of the moment of inertia (MoI) of the crustal region to the total MoI of the star, also called the fractional moment of inertia (FMI). In this paper, we have calculated the FMI for different masses of a star using six different representative unified equations of state constructed under a relativistic mean field framework. We have performed an event-wise comparison of the FMI obtained from data with that from theoretically calculated values with and without considering the entrainment effect. It is found that larger glitches cannot be explained by the crustal FMI alone, even without entrainment.

Key words: equation of state – stars: interiors – stars: neutron – stars: rotation

1. Introduction

Pulsars are rotating magnetized neutron stars (NSs) with a magnetic field strength of about 10\(^{12}\) G. Most pulsars have very stable rotational periods ranging from milliseconds to a few seconds owing to their compact size and large mass. However, some pulsars do show rotational irregularities, such as sudden jumps in the spin frequency, known as pulsar glitches (Radhakrishnan & Manchester 1969). Over the last five decades, 529 glitches have been reported in radio pulsars (Lyne 1992, 1996; Shemar & Lyne 1996; Lyne et al. 2000; Wang et al. 2000; Krawczyk et al. 2003; Espinoza et al. 2011; Yu et al. 2013; Fuentes et al. 2017). The reported fractional spin-up during a glitch\(^5\) ranges from 10\(^{-9}\) to 33 \times 10\(^{-6}\) (Manchester & Hobbs 2011). Pulsars such as PSR B0833−45, B1046−58, B1338−62, and B1737−30 show glitches with fractional spin-up varying over three orders of magnitude. The glitch events can also be followed by exponential recovery of rotation rate (Yu et al. 2013). It is believed that the interior superfluid is responsible for the glitches, where the excess angular momentum of the pinned component of the superfluid is transferred to the crust at a critical lag between the differential rotation of the NS’s superfluid and non-superfluid component (Alpar et al. 1984a, 1984b, 1985). Observations of pulsar glitches and their recovery provide an important probe of the composition and structure of NSs (Haskell & Melatos 2015) in terms of the depth from the surface of the star where glitches originate. Glitches can also help in understanding how the superfluid component of the star is coupled with its observable crust and whether its core participates in the glitch phenomenon. We aim to investigate this by comparing theoretical calculations with reported glitch measurements.

One way to achieve this goal is to place constraints on moment of inertia (MoI) of different parts of the NS participating in the glitch event. There exist distinct density regions inside the star, defined by the local density, which change from \(\sim 10^3\) g cm\(^{-3}\) to \(\sim 10^{14}\) g cm\(^{-3}\) from the surface of the NS to its center. The outermost layer (outer crust) consists of fully ionized nuclei arranged in a BCC lattice structure embedded in a degenerate electron gas. With increasing density nuclei become more and more neutron-rich (Baym et al. 1971b; Rüster et al. 2006; Nandi & Bandyopadhyay 2011). At a density of \(\sim 10^{13}\) g cm\(^{-3}\), the neutron drip point is reached and the inner crust begins. The inner crust is composed of neutron-rich nuclei arranged in a lattice and immersed in a free electron gas, as well as a gas of dripped neutrons (Baym et al. 1971a; Neggele & Vautherin 1973; Haensel 2001; Nandi et al. 2011), which are expected to be superfluid (Baldo et al. 2005; Sedrakian & Clark 2006; Chamel & Haensel 2008). Beyond the inner crust, the core of the star may consist of superfluid neutrons, superconducting protons, and other exotic matter (Ginzburg & Kirzhnits 1964; Sauls 1989; Baldo et al. 2005; Page & Reddy 2006; Sedrakian & Clark 2006; Chamel & Haensel 2008).

Theoretically, the MoI of the crust and core components can be estimated by solving the structure equations with a given equation of state (EoS). In the study of pulsar glitches, mostly non-relativistic EoSs are used (Andersson et al. 2012; Chamel 2013; Ho et al. 2015; Delsate et al. 2016; Li et al. 2016; Pizzochero et al. 2017). On the other hand, EoSs obtained from a relativistic mean field (RMF) model that gives a causal description at all densities of the NS have been widely used in the literature to study various properties of nuclear matter as well as NSs (Glendenning 2000; Dutra et al. 2014; Oertel et al. 2017). Only Piekarewicz et al. (2014) employed RMF EoSs for the NS core to study glitch phenomena. However, for the inner crust they used a polytropic EoS that interpolates between the neutron-drip density and the crust–core transition density.

\(^{5}\) The fractional spin-up is defined as the ratio of increase in rotation rate \(\delta \nu\) to rotation rate \(\nu\) at the time of the glitch.
Since the inner crust contributes more than 99% to the crustal MoI, it is important to have a realistic EoS for it. In the literature, the inner crust and the core are often treated separately and the corresponding EoSs are matched “by hand” at the crust–core boundary, which is also chosen arbitrarily (Glendenning 2000; Read et al. 2009; Fattoyev et al. 2018). The lack of precision in this matching processes can lead to significant differences in the estimation of the MoI of different components (Fortin et al. 2016). Therefore, it is necessary to use a unified EoS, where the EoSs of both the inner crust and the core are calculated from the same microscopic theory and, as a consequence, the crust–core boundary is automatically determined. In this paper, we calculate the MoI using unified EoSs derived from different variations of the RMF model.

A pinned neutron superfluid provides an angular momentum reservoir as its rotation rate is determined by the areal vortex density, which is constant as long as it is pinned to the crust. At the same time, the crust continuously slows down due to loss of its angular momentum in the particle wind and electromagnetic radiation. At a critical lag in this differentially rotating two-component system, superfluid vortices get unpinned, dumping a large amount of angular momentum to the crust, which is observed as a spin-up in the crustal rotation rate, usually inferred by timing the radio pulse (Alpar et al. 1984a, 1985). This implies that the fractional spin-up provides a probe of the extent of angular momentum transfer and hence the MoI of the crustal pinned superfluid. The ratio of the MoI of the crustal pinned superfluid to that of the rest of the star, referred to as the fractional moment of inertia (FMI), can be related to the observed fractional spin-up, allowing a comparison of theoretical estimates with those from observations (Link et al. 1992, 1999; Eya et al. 2017). In this context, it is common practice in the literature (Link et al. 1999; Andersson et al. 2012; Chamel 2013; Ho et al. 2015; Eya et al. 2017) to define a quantity called the activity parameter, which is essentially the average of all glitches observed in a time window for a given pulsar. This parameter is then used to estimate the FMI for different pulsars. However, as we are interested in investigating how far the FMI of the crust can explain the observed glitches, it seems more appropriate to use individual glitches instead of an average. Based on a calculation of the FMI in eight individual Vela pulsar glitches (Alpar et al. 1993), Datta & Alpar (1993) ruled out one out of 19 EoSs for this pulsar. With a better constraint on the maximum mass of an NS for different EoSs and, 25 years on, a much larger glitch database, this question is worth a detailed re-examination. Therefore, we consider each individual glitch cataloged so far (Espinoza et al. 2011) to estimate the FMI.

In this paper, we apply a unified treatment of the EoS, obtained from a variety of RMF models, to estimate the fraction of the stellar MoI of the crust and compare it to that inferred from reported observations of pulsar glitches. In Section 2, the construction of the EoS is described, followed by estimates of the relevant MoI using the structure equations in Section 3. We connect these estimates to observables in Section 4 and present our results in Section 5. A discussion of these results and our conclusions are presented in Section 6.

2. Equation of State

It has been shown (Fortin et al. 2016) that for an unambiguous calculation of NS properties (especially radius and crust thickness) one needs to employ a unified EoS, i.e., the EoS of crust and core should be obtained within the same many-body theory. As glitches are supposed to be very sensitive to the thickness of the crust, we employ only unified EoSs here. We construct the EoSs of the inner crust and core adopting the RMF approach, where the interaction between nucleons is described by the exchange of $\sigma$, $\omega$, and $\rho$ mesons. We choose EoSs that represent different variations of the RMF model and are also used widely in the literature (Dutra et al. 2014). In particular, we use the parameter sets NL3 (Lalazissis et al. 1997) and GM1 (Glendenning 2000), which contain nonlinear self-interaction of $\sigma$ mesons, TM1 (Sugahara & Toki 1994), which has self-interacting terms for both $\sigma$ and $\omega$ mesons, NL3$\omega\rho$ (Horowitz & Piekarwiecz 2001), which contains self-interaction of $\sigma$ mesons and coupling between $\omega$ and $\rho$ mesons, and DDME2 (Lalazissis et al. 2005) and BHB$\Lambda\phi$ (Banik et al. 2014), where the coupling parameters are considered to be density dependent. The EoSs of the inner crust along with the crust–core transition density for all parameter sets excluding BHB$\Lambda\phi$ are taken from Grill et al. (2014), whereas the EoSs of that core, which contains neutrons, protons, electrons, and muons, are generated by the standard procedure (Glendenning 2000; Dutra et al. 2014). As the EoSs of both the crust and the core are described by the same parameter set, they can be joined smoothly at the crust–core boundary without any jump in pressure and density. For the EoS of the outer crust, we choose the DH EoS (Haensel et al. 2007). The choice of outer crust does not have any significant impact on the observables as most of it is determined from experimentally measured nuclear masses. The unified BHB$\Lambda\phi$ EoS is obtained following Banik et al. (2014). Apart from nucleons and leptons, the core EoS of BHB$\Lambda\phi$ also includes $\Lambda$ hyperons, which interact among themselves via the exchange of $\phi$ mesons. All six EoSs used here give maximum NS masses ($M_{\text{max}}$) of more than 2$M_\odot$ and are therefore compatible with the constraint $M_{\text{max}} = 2.01 \pm 0.04M_\odot$ obtained from observation (Antoniadis et al. 2013). We use these EoSs to estimate the MoI relevant for the present work.

3. Structure

The equilibrium structure of a spherically symmetric, non-rotating NS is calculated from solutions of the Tolman–Oppenheimer–Volkoff (TOV) equations:

$$\frac{dP(r)}{dr} = - \frac{[\varepsilon(r) + P(r)][M(r) + 4\pi r^3 P(r)]}{r[2M(r)]}$$

(1)

$$\frac{d\nu(r)}{dr} = - \frac{1}{\varepsilon(r) + P(r)} \frac{dP(r)}{dr}$$

(2)

$$\frac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r).$$

(3)

Here, $\varepsilon(r)$, $P(r)$, $M(r)$, and $\nu(r)$ represent the radial profiles for energy density, pressure, enclosed mass, and the metric potential respectively. Complemented with an EoS, the structure equations (1)–(3) are solved numerically to generate the profiles for those quantities, and also the total mass and radius of the star for a given value of central energy density. Assuming the star is rotating uniformly and the angular velocity ($\Omega$) is sufficiently slow ($\Omega^2 R^3 \ll M$) compared to its Kepler limit, the MoI of a star can be calculated in the slow-rotation approximation using Hartle’s prescription (Hartle & Thorne 1968). The metric of a slowly, uniformly rotating star

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can be expressed at the first order of spin frequency $\Omega$ as
\[
d s^2 = -e^{2\nu(r)}d\tau^2 + e^{2\lambda(r)}dr^2 - 2\omega(r)r^2 \sin^2 \theta d\phi dt + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2,
\]
where $\omega$ is the frame-dragging frequency, which satisfies the following differential equation:
\[
d\left( r^2 j(r) \frac{d\omega(r)}{dr} \right) + 4r^3 \frac{dj(r)}{dr} \omega(r) = 0,
\]
where $\omega(r) = \Omega - \omega(r)$ and $j(r)$ is defined as
\[
j(r) = e^{-\nu(r)+\lambda(r)} = e^{-\nu(r)} \sqrt{1 - 2GM(r)/r}, \quad r \leq R.
\]

Now we can compute the MoI of the star by solving Equation (5) augmented with the TOV equations:
\[
I_{\text{total}} = \frac{8\pi}{3} \int_0^R dr r^4 \frac{(\varepsilon(r) + P(r))}{\sqrt{1 - 2GM(r)/r}} \frac{\omega(r)}{\Omega} e^{-\nu(r)},
\]
where $R_c$ denotes the crust-core boundary. Note that $I_{\text{crust}}$ consists of both the non-superfluid and the pinned superfluid component of the MoI of the crust. Thus, $I_{\text{crust}}$ cannot be directly related to the FMI using the measurements of observed fractional spin-up during a glitch. We provide a prescription for comparing this theoretical estimate with observations in the next section.

4. Interpreting the Data

The pulsed emission in the radio waveband is a direct measure of the rotation of the crust of the pulsar, which can be very precisely modeled using the pulsar timing technique. In pulsar timing, predictions of the pulse times-of-arrival (TOAs) from a rotational model of the star are compared with observed TOAs. The difference between the observed and predicted TOAs are called timing residuals, which are minimized in a least-squares sense to improve the parameters of the rotational model, thus precisely estimating the time evolution of the rotation of the pulsar. The pre- and post-glitch models show a sudden jump in the rotation period $\nu$. The fractional change in $\nu$ is therefore available from observations.

If we assume that the spin-up is due to the transfer of angular momentum from a superfluid, which is not co-rotating with the crust, then the fractional MoI of this superfluid to the MoI of the rest of the star for each glitch is given by (Eya et al. 2017)
\[
\frac{I_{\text{crsf}}}{I_{\text{rest}}} = -\frac{1}{\nu_c} \frac{\Delta \nu_i}{t_i}
\]
where $I_{\text{crsf}}$, $I_{\text{rest}}$, $\Delta \nu_i$, $\nu_c$, and $t_i$ are the MoI of the pinned superfluid (not co-rotating with the crust), the MoI of the rest of the star (assumed to be co-rotating with the crust), the spin-up at the $i$th glitch, the mean rotational spin-down rate, and the time elapsed before the $i$th glitch since the preceding glitch, respectively.

\[\text{Figure 1. Mass–radius diagram for NL3 (sky blue), NL3wp (red), TM1 (green), GM1 (orange), DDME2 (dark blue) and BHBAX (gray) EoS. The blue dotted line represents the mass constraint of 2.01 \pm 0.04 M}_\odot \text{ from observations (Antoniadis et al. 2013).}\]

The above equation can be simplified to
\[
\frac{I_{\text{crsf}}}{I_{\text{rest}}} = 2\tau_c \frac{\Delta \nu}{\nu_i} \frac{1}{t_i},
\]
where $\tau_c = -\nu(2\nu_i)$ is the characteristic age of the pulsar. Since $t_i$ is the time preceding the last glitch, one cannot calculate the $I_{\text{crsf}}/I_{\text{rest}}$ for the first glitch. We want to connect this $I_{\text{crsf}}/I_{\text{rest}}$ with $I_{\text{crust}}/I_{\text{total}}$. Assuming no contribution in the angular momentum transfer from the core superfluid, we have
\[
\frac{I_{\text{crust}}}{I_{\text{total}}} = \frac{I_{\text{crust}}}{I_{\text{crsf}}} \frac{I_{\text{crsf}}}{I_{\text{rest}}} = \frac{1}{1 + I_{\text{crsf}}/I_{\text{rest}}},
\]
where $I_{\text{crsf}}$ is the non-superfluid (and hence co-rotating with the crust) component of the crustal MoI. Now, since $I_{\text{rest}} \gg I_{\text{crsf}}$, we have $I_{\text{crust}}/I_{\text{total}} > I_{\text{crsf}}/I_{\text{rest}}$ and we can write using Equation (10)
\[
\frac{I_{\text{crust}}}{I_{\text{total}}} > 2\tau_c \frac{1}{\nu_i} \left( \frac{\Delta \nu}{\nu} \right)^i.
\]
This connects our theoretical estimate of the FMI with that inferred from observations and this equation is used in the next section to produce histograms to estimate $I_{\text{crust}}/I_{\text{total}}$.

When the entrainment coupling between the neutron superfluid and the crustal non-superfluid component is taken into consideration, we get (Andersson et al. 2012)
\[
\frac{I_{\text{crust}}}{I_{\text{total}}} > 2\tau_c \frac{\langle m_n^{\text{eff}} \rangle}{m_n} \frac{1}{\nu_i} \frac{\Delta \nu}{\nu} ^i,
\]
where $\langle m_n^{\text{eff}} \rangle/m_n$ is the ratio of the average effective mass of neutrons in the inner crust and bare neutron mass, and has values in the range 4.3–5.1 (Andersson et al. 2012; Chamel 2012; Delsete et al. 2016).

5. Results

We have used six different unified EoSs, NL3, NL3wp, GM1, TM1, DDME2, and BHBAX, as described in Section 2. The mass–radius sequences of these six EoSs are shown in Figure 1. All these EoSs satisfy the current observational
Figure 2. Left: fraction of MoI of the crust compared to the total MoI (expressed as a percentage) for six different EoSs as a function of stellar mass. Right: crust thickness in kilometers as a function of stellar mass.

Figure 3. Distribution of $I_{\text{crust}}/I_{\text{total}}$. The vertical lines in each plot correspond to the fractional moment of inertia for the six different EoSs. The plots from top left to bottom right correspond to masses of 1.0, 1.2, 1.4, 1.6, 1.8, and 2.0 $M_\odot$ respectively. The distribution plotted in yellow corresponds to that without entrainment, whereas the distribution plotted in blue has been constructed taking the entrainment value of $\frac{\omega_0}{m_0} = 4.35$ into account. Both distributions are obtained from 335 glitches.
constraint on the NS maximum mass \( (2.01 \pm 0.04) \, M_\odot \) (Antoniadis et al. 2013). (Note that the color code used to represent the various EoSs in Figure 1 has been uniformly applied in all subsequent diagrams.)

We have calculated the fraction of the MoI of the crust to the total MoI as a function of mass of the stars (in percent) for each of the six EoSs using Equations (7) and (8), and the results are shown in the left panel of Figure 2. The distance of the crust–core boundary from the center of the star \( (R_c) \) and the radius of the star for each EoS are obtained from the solution of the TOV equations. Hence, one can readily calculate the crust thickness as \( \Delta R = R - R_c \). The right panel of Figure 2 shows the crust thickness as a function of stellar mass. It can be clearly seen that the thickness of the crust can be very large \( \sim (1.4 - 1.8) \, \text{km} \) for a lighter star of mass \( \sim 1 \, M_\odot \), whereas for stars of mass \( \sim 2 \, M_\odot \) it can be as small as \( \sim 0.5 \, \text{km} \). From Figure 2, it is clearly evident that the relation of the crust thickness and its FMI with stellar mass is close. If, instead of a unified EoS, a polytropic EoS is used for the inner crust, as was done by Piekarewicz et al. (2014), we find that, depending on the choice of polytropic index, the FMI gets overestimated by 0%–8% for a 1.4 \( M_\odot \) NS for the NL3\(\omega\rho \) EoS, and the error is greater for low-mass stars. Similar results are expected for the other EoSs.

The values of the FMIIs estimated from all the observed glitches cataloged so far (Espinoza et al. 2011) are shown in Figure 3, where the distribution of \( \log_{10} \left( \frac{I_{\text{crust}}}{I_{\text{total}}} \right) \), estimated using Equation (11), is plotted for different assumed masses of the NS (1.0, 1.2, 1.4, 1.6, 1.8, and 2.0 \( M_\odot \), respectively). The error bars given on each bin are Poissonian. The vertical lines are constraints from our theoretical calculations of \( \log_{10} \left( \frac{I_{\text{crust}}}{I_{\text{total}}} \right) \) for the six EoSs considered. We note that the observed \( I_{\text{crust}}/I_{\text{total}} \) always overestimates the superfluid reservoir as it assumes the crust is made entirely of pinned superfluid. Hence, the actual contribution from the superfluid in the crust toward the glitch is smaller than the estimated \( I_{\text{crust}}/I_{\text{total}} \). In all cases, the NL3\(\omega\rho \) model can explain a larger FMI as it has the largest crust (Figure 2). It is evident that there is a significant number of glitch events, that cannot be explained solely by the crustal superfluidity. The numbers of such events (out of 335 glitches) are tabulated in Table 1. The estimates of \( \log_{10} \left( \frac{I_{\text{crust}}}{I_{\text{total}}} \right) \) considering entrainment are also plotted in Figure 3. As expected, the fraction of glitches that cannot be explained entirely by crustal superfluidity alone is larger in the presence of entrainment.

From the Table 1, we can clearly conclude that at least 1%–20% of glitch events without entrainment require the angular momentum reservoir of a pinned superfluid located outside the crust, even in the limiting case of the crust being entirely made up of the pinned superfluid. When entrainment is taken into account, this changes to 4%–36%. Therefore, we must consider the contribution of the core in these glitch events.

### 6. Conclusions and Discussions

In this work, a unified treatment of the EoS, using an RMF approach, has been carried out to estimate the fraction of the MoI of the crust compared to the total NS MoI. We have used individual glitches instead of an average, defined via an activity parameter, in our estimation of the FMI. We compared the theoretical estimates with those obtained from observations and conclude that a small percentage of glitches cannot be explained by angular momentum transfer from the pinned superfluid in the crust alone, even without entrainment. The fraction of such glitches ranges from 1% to 20% for different EoSs and assumed NS masses.

In this context, it is important to note that the recent binary NS merger event GW170817 has provided unique insights on the global properties of isolated NSs (Abbott et al. 2017). Several authors have shown that GW170817 sets an upper limit of \( \sim 2.1–2.2 \, M_\odot \) on the maximum mass of an NS (Margalit & Metzger 2017; Shibata et al. 2017; Rezzolla et al. 2018; Ruiz et al. 2018). The radius of a 1.4 \( M_\odot \) star has also been constrained to \( \lesssim 13.5 \, \text{km} \) (Abbott et al. 2018; Most et al. 2018; Nandi & Char 2018). In this paper, we have used both a stiff EoS (NL3) and a moderately soft EoS (BHB\(\lambda\phi \)) for representative purposes. It is evident from Figures 1 and 2 that a stiffer EoS usually leads to a larger radius. This makes the crustal thickness and \( I_{\text{crust}}/I_{\text{total}} \) larger than that of a moderately softer EoS. While a majority of glitches can be explained by transfer of angular momentum from a crustal superfluid for such stiffer EoSs, some glitches remain unexplained. On the other hand, a larger number of glitches are inconsistent with the transfer of angular momentum solely from a crustal superfluid for a moderately softer EoS; this predicts a maximum mass and radius consistent with the upper limit from GW170817 (Bhat & Bandyopadhyay 2018). Therefore, the conclusion that a crustal contribution to the angular momentum transfer alone cannot account for all glitches becomes stronger if the constraints from GW170817 are taken into account.

The number of unexplained glitches is even larger if entrainment is taken into consideration, as expected. It is worth mentioning that a recent study (Watanabe & Pethick 2017) showed that the effect of entrainment is not as significant as estimated earlier (Chamel 2012). Thus, our calculations suggest that the superfluid in the core is also likely to participate, at least in the larger glitches. The rotation of the crustal superfluid is believed to be constrained due to pinning of vortices to the nuclei in the crustal lattice, thus conserving their areal density (Sauls 1989). In order for the superfluid in other parts of the star, particularly the core, to participate in a glitch, a similar constraint is required on this fraction of the superfluid, which is generally believed to co-rotate with the crust and slow down in synchronisation by expelling vortices.

### Table 1

| EoS          | \( m_1 \) | \( m_2 \) | \( m_3 \) | \( m_4 \) | \( m_5 \) | \( m_6 \) | \( m_7 \) | \( m_8 \) | \( m_9 \) |
|--------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| NL3          | 4(16)    | 5(22)    | 8(31)    | 8(39)    | 11(71)   | 13(60)   |          |          |          |
| NL3\(\omega\rho \) | 4(12)    | 5(17)    | 5(25)    | 8(31)    | 8(37)    | 11(47)   |          |          |          |
| DDME2        | 5(17)    | 7(28)    | 8(34)    | 10(43)   | 13(56)   | 16(66)   |          |          |          |
| TM1          | 4(16)    | 5(23)    | 8(32)    | 10(43)   | 13(56)   | 18(70)   |          |          |          |
| GM1          | 5(25)    | 8(33)    | 10(43)   | 13(56)   | 17(68)   | 28(80)   |          |          |          |
| BHB\(\lambda\phi \) | 14(61)   | 21(71)   | 31(81)   | 37(94)   | 53(106)  | 68(121)  |          |          |          |

Note. The quantities in parenthesis show glitch events when entrainment \( (\frac{I_{\text{crust}}}{I_{\text{total}}} = 4.35) \) is taken into account for six different assumed NS masses (columns) and six different EoSs (rows).
Our analysis also confirms the presence of an extra angular momentum reservoir along with the NS crust.

For a glitch to occur in the star, it is essential to pin superfluid vortices to some structures that can co-rotate with the stellar crust. One such possibility is the presence of mixed states at a certain depth of the stellar core. This region is marked by the presence of confined (hadronic) and de-confined (quark) matter co-existing in equilibrium. The energetics of this region force the non-dominating component to form crystal structures of various shapes evolving with density. They could form a zone of an extra angular momentum reservoir (Glendenning 2000). A similar kind of crystalline structure can also form due to a mixed phase of kaon condensates inside the nuclear matter as a first-order phase transition (Glendenning & Schaffner-Bielich 1999). Another method of pinning could be between Abrikosov fluxoids along the magnetic moment due to the presence of a paired proton superconductor (Sauls 1989; Ho et al. 2017) at the core of the NS with Onsager–Feynman vortices along the rotation axis (Bhattacharya & Srinivasan 1995). This novel mechanism of interpinning between fluxoids and vortices has been used in the literature to expel the magnetic fluxoids from the core to the crust via the spinning-down of the NS. But it might not help in building up angular momentum, which can be expelled at once to produce the sudden jump in angular velocity, as observed in glitches.

Recently, Gügercinoğlu & Alpar (2014) have proposed a pinning of the superfluid in the outer core by a toroidal field, similar to pinning a crustal superfluid with an FMI of the superfluid associated with this toroidal magnetic field ($I_{\text{tor}}/I_{\text{total}}$) of the order of (0.3−1.2) $\times 10^{-2}$ (Gügercinoğlu 2017). This presents an attractive alternative for the contribution from the superfluid in the outer core mediated by the magnetic field as an extra MoI reservoir for glitches not explained by the crustal superfluid. The large glitches in PSR B2334+61 and J1718−3718 could be explained by this mechanism (Gügercinoğlu & Alpar 2016). It may be noted that a small fraction of the core MoI is needed to explain the larger FMIs in our sample, as expected considering the fraction of the core superfluid associated with this toroidal magnetic field to be much smaller than the total MoI of the core. Thus, this mechanism could explain the glitches in our study where the crustal superfluid is not sufficient.

Interestingly, in pulsars such as PSR B0833−45, B1046−58, B1338−62, and B1737−30, we see both small and large glitches. The glitch size varies by a factor of 258, 160, 236, and 3811 in these pulsars, respectively. Our calculations suggest that in all these pulsars at least one glitch event cannot be explained by the BHBAb EoS, whereas the other EoSs used can explain the glitches from the crustal angular momentum reservoir except for PSR B1046−58. In that case, the glitch that had $I_{\text{crust}}/I_{\text{total}} = 33\%$ cannot be explained by any of the EoSs used, taking even 1.0 $M_\odot$ as the fiducial mass of the NS. Thus, we conclude at least some glitches require participation of the core, while the smaller ones can be caused by the crust alone. This also may provide a probe for the strength of the coupling agent and the angular momentum mechanism from the core to the crust of the star. This work motivates future theoretical calculations to address this issue.

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