Closed shell effects from the stability and instability of deformed and superdeformed nuclei against cluster decays in the mass regions 130-158 and 180-198

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The stability and/or instability of the deformed and superdeformed nuclei, and Pb parents, coming from three regions of different superdeformations, are studied with respect to the α and heavy cluster decays. The α-decay studies also include the heavier 190–210 Pb nuclei, for reasons of spherical magic shells at Z=82 and N=126. The calculations are made by using the preformed cluster-decay model, and the obtained α-decay half-lives are compared with the available experimental data. Having met with a very good success for the comparisons of α-decay half-lives and in giving the associated known magic or sub-magic closed shell structures of both the parent nuclei and daughter products, the interplay of closed shell effects in the cluster-decay calculations is investigated. The cluster-decay calculations also give the closed shell effects of known spherical magicities, both for the parent and daughter nuclei, and further predict new (deformed) closed shells at Z=72-74 and N=96-104 due to the stability and instability of Hg and Pb parents against cluster decays. Specifically, a new deformed daughter radioactivity is predicted for various cluster decays of 186–189 Hg and 194–195 Pb parents with the best possible measurable cases identified as the 6Be and 13C decays of 176–177 Hg and/or 192Pb parents. The predicted decay half-lives are within the measurable limits of the present experimental methods. The interesting point to note is that the parents with measurable cluster decay rates are normal deformed nuclei at the transition between normal and superdeformation.

I. INTRODUCTION

The α-decay results have been used for identifying the shell closure effects for quite some time now, including even the very weak, sub-shell closures. For example, the Z=64 sub-shell was first noted by observing the systematics of α-decay energies [1], and later by a dip at Z=64 in the measured α-decay reduced widths [2], of a few N=84 isotones in its neighborhood. In the recent past, some of us and collaborators [3–9] have coupled the α-decay studies with the exotic cluster-decay result of the observed spherical closed-shell daughter (208Pb or a neighbouring nucleus), called cluster radioactivity [10–12]. This allowed us to predict two other new spherical closed-shell daughter radioactivities, namely 100Sn and 132Sn daughter radioactivities [4–7], and also a deformed daughter radioactivity at Z=74-76 and N=98-104 [8]. The spherical 100Sn daughter radioactivity has been emphasized also by Poenaru, Greiner and Gherghescu [13], and a couple of, so-far unsuccessful, experimental attempts have also been made to observe it as the ground-state decay of 114Ba nucleus produced in heavy-ion reactions [14,15]. This decay is now believed to belong to an excited compound nucleus decay, studied for 12C decay of 116Ba* [16,17]. Furthermore, the cluster decay studies are also used to point out the shell stabilizing effects of the parent nucleus [3,8,9]. Thus, both the cases of large and small decay rates (equivalently, the small and large decay half-lives) are found important, the large ones referring to closed shell effects of the daughter nucleus and the small ones to closed shell effects of the parent nucleus. In other words, taking a clue from the experiments, in a decay calculation, the presence of a known spherical or deformed daughter should result in a large decay rate (small decay half-life) or alternatively, a large decay rate (small decay half-life) should refer to the existence of a known or un-known (new), spherical or deformed, closed shell for the daughter nucleus.

In the above mentioned calculations, we have so-far investigated the alpha and/or cluster decays of various neutron-deficient and neutron-rich rare-earths 54Xe to 64Gd [4–7] and the even-A deformed and superdeformed 180–193Hg nuclei [8]. In mercury nuclei, the superdeformation begins at the 189Hg isotope, and the axes ratios are ≈1.7:1 [18]. Note that the superdeformation here refers to the observation of (excited) superdeformed band(s) in these nuclei, though their ground-state deformations are not very much different from other neighbouring nuclei. This is illustrated in Fig. 1, where the data for ground-state quadrupole deformation parameter β2 is taken from the calculations of Möller et al. [19], since a similar data from experiments is not available for all the nuclei studied here. On the other hand, a deformed or normal deformed nucleus is one where superdeformed band(s) are not observed and it comes from the well known mass region 150 < A < 190 of deformation. In the above stated nuclei, the closed shell effects of
both the daughter products and the parent nuclei were analyzed. The stability of parent nuclei was also studied for the mass region A=68-82 [3,9], which includes several deformed and superdeformed nuclei (here, both in the ground states). However, there are several other regions of various deformations and superdeformations in the mass regions A=130-158 and 180-198 [18] whose decay characteristics still remain to be probed. The aim of this paper is to make a complete analysis of the decay properties of nuclei in these two mass regions, in order to get a general picture of how the deformed, in particular the superdeformed, nuclei behave against the \( \alpha \) and heavier cluster decays. The superdeformed nuclei are expected to be more instable, though, like for the mass region A=68-82 [3,9], the following analysis does not seem to support this contention. Instead, the superdeformed nuclei are found to be rather poor \( \alpha \) emitters, as compared to their lower mass, normal deformed nuclei. They are, however, shown to be the better emitters than the heavier mass (heavier than superdeformed nuclei), normal deformed nuclei. The same is found true for cluster decay results. Such an unexpected situation is presented by the presence of known and/or un-known (new) closed shell effects of the daughter products. Also, the closed shell effects of either the protons or neutrons, as well as the neutron/proton asymmetry, of the parent nucleus play a role.

The nuclei that have superdeformations identical to those of \( ^{190-194} \text{Hg} \) nuclei are the \( ^{191} \text{Au}, \text{81}^{191-195} \text{Tl}, \text{82}^{192-196,198} \text{Pb} \) and \( ^{197} \text{Bi} \) nuclei. Then, there are several rare-earths, from \( ^{62} \text{Sm} \) to \( ^{66} \text{Nd} \), which have even more strongly superdeformed shapes with axes ratios \( \approx 2:1 \). Also, some other rare-earths, the \( ^{57} \text{La} \) to \( ^{60} \text{Nd} \), have superdeformed species with axes ratios \( \approx 1.5:1 \). In this paper, we choose to work specifically with both the odd- and even-A \( ^{60} \text{Nd}, ^{64} \text{Gd} \) and \( ^{82} \text{Pb} \) parents, which comprise the three regions of different superdeformations mentioned above, along with some normal deformed nuclei. Note that \( ^{154-158} \text{Gd} \) is known \( \alpha \)-stable nuclei, but are found to be of interest from the point of view of heavy-cluster instabilities and the associated closed shell effects (Section III.B.2). We have also included in our analysis here, the already studied \([8]\) mercury nuclei, extended to both the odd- and even-A \( ^{176-194} \text{Hg} \), and the heavier \( ^{199-210} \text{Pb} \) nuclei where some experimental data for \( \alpha \)-decays are available. Thus, the cases of both the normal deformed and superdeformed nuclei, and the spherical closed shell nuclei at and around \( Z=82, \text{N}=126 \), are covered in our study. Figure 1 shows that all the superdeformed nuclei chosen here come from the transition (both lighter and heavier) regions of known deformed nuclei in the mass region \( 150 < A < 190 \).

The paper is organised as follows. The calculations are made by using the preformed cluster-decay model (PCM) of Gupta and collaborators [20-23] whose brief outline is presented in section II. Section III deals with the calculations and results obtained from this study. A summary of our results and conclusions are presented in section IV.

## II. THE PREFORMED CLUSTER-DECAY MODEL

The preformed cluster-decay model (PCM) is a well established method for cluster decay studies. We refer the reader to original papers [20–22] or the reviews in Refs. [12,23] for complete details on the model. In the PCM, the decay constant \( \lambda \) (or, inversely, the decay half-life time \( T_{1/2} \)) is the product of the cluster preformation probability \( P_0 \), the barrier impinging frequency \( \nu_0 \), and the barrier penetration probability \( P \),

\[
\lambda = \frac{\ln 2}{T_{1/2}} = P_0 \nu_0 P. \tag{1}
\]

For calculating \( P_0 \) and \( P \), the authors introduced, respectively, the dynamical collective coordinate of mass asymmetry \( \eta=(A_1-A_2)/A \), with \( A=A_1+A_2 \), and relative separation \( R \) between the two fragments, via the stationary Schrödinger equation

\[
H(\eta,R)\psi_{\eta}(\eta,R) = E_n \psi_{\eta}(\eta,R). \tag{2}
\]

The potential part of the Hamiltonian in this equation is defined by

\[
V(\eta,R) = \sum_{i=1}^{2} B_i(A_i,Z_i) + \frac{Z_i Z_2 e^2}{R} + V_p, \tag{3}
\]

given as the sum of the experimental binding energies [24] and the Coulomb and nuclear proximity [25] potentials. The fragmentation potential \( V(\eta) \) and the scattering potential \( V(R) \) are obtained from Eq. (3), respectively, for fixed \( R \) and \( \eta \). The \( R \) is fixed at the touching configuration, \( R=C_0=C_1+C_2 \), the \( C_i \) being the Süssmann central radii \( C_i = R_i - 1/R_i \) (in fm) with \( R_i \) as the equivalent spherical radii \( R_i = 1.28 A_i^{1/3} - 0.76 + 0.8 A_i^{-1/3} \) fm; and \( \eta \) is fixed by the emitted cluster. The charges \( Z_i \) in (3) are fixed by minimizing the potential (without \( V_p \)) in the charge asymmetry coordinate \( \eta_Z=(Z_1-Z_2)/Z \), with \( Z=Z_1+Z_2 \).

In principle, the two coordinates are coupled, but in view of the defining equation (1), the Schrödinger equation (2) is solved in the decoupled approximation of \( \eta \) and \( R \)-motions. Only the ground-state (\( n=0 \)) solution is relevant for the cluster decay to occur in the ground-state of the daughter nucleus. Then, for \( \eta \) motion, the properly normalized fractional cluster preformation probability is

\[
P_{0\eta}(A_2) = |\psi(\eta)|^2 \sqrt{B_{\eta\eta}(\eta)} \frac{2}{A}, \tag{4}
\]

with \( B_{\eta\eta} \) taken as the classical hydrodynamical mass of Kröger and Scheid [26]. For the \( R \)-motion, we use the
WKB approximation for calculating the penetrability $P$. In PCM, the penetration is considered to begin at $R = R_0 = C_1$ and end at $V(R_0) = Q$-value of the decay.

Finally, the impinging frequency $v_0$ in the PCM is defined by considering that the total kinetic energy, shared between the two fragments, is the positive $Q$-value. Then,

$$v_0 = \frac{\text{velocity}}{R_0} = \frac{\sqrt{2Q/mA_2}}{R_0}. \quad (5)$$

Here $R_0$ is the equivalent spherical radius of the parent nucleus and $mA_2$ is the mass of emitted cluster.

### III. CALCULATIONS

In this section, we present our calculations first for $\alpha$-decay, compared with the experimental data, wherever available. Then, we analyze the cluster-decay calculations with a view to look for the role of known magic shells in both the daughter and parent nuclei, and the possible new closed-shell daughter products presenting the signatures of a new radioactivity, if any. The cluster-decay calculations are presented separately for each set of nuclei.

Figures 2 and 3 show the fragmentation potentials for Nd and Pb nuclei, as the representatives of the two mass regions ($A = 130-158$ and $180-198$) studied here. The experimental binding energies used are from the 1995 tables of Audi and Wapstra [24]. We notice that in each case, the potential energy minima occur at $^4\text{He}$ and other $N = Z$, $\alpha$ nuclei, as well as at $N \neq Z$, non-$\alpha$ nuclei for all the parents in the heavier mass region ($A = 180-198$) and for only the heavier parents in the lighter mass region $A = 130-158$. This means that the $\alpha$-nuclei decay products are energetically more favoured for the lighter isotopes of Nd and Gd parents in the lighter mass region $A = 130-158$, and the non-$\alpha$ decay products become equally favourable for all the $\text{Hg}$ and Pb parents in the heavier mass region $A = 180-198$ and for the heavier isotopes of Nd and Gd parents in $A = 130-158$ mass region. We are interested only in the potential energy minima because the preformation factors $P_0$ for nuclei at the minima are the largest, compared to their neighbours, as is depicted in Fig. 4 for Nd and Gd and in Fig. 5 for Hg and Pb parents, where, for some clusters belonging to the minima in the fragmentation potentials, the (negative) logarithm of the preformation probability $P_0$ is plotted as a function of the mass number of the parents. We notice that in all cases, like in our earlier calculations [3-9], the preformation factor is largest for $^4\text{He}$ and it goes on decreasing as the size of the cluster increases. Another point of interest to note in these figures is the change in the penetrability of clusters for the heavier parents (see the dashed parts of lines). For example, in both the Figs. 4 and 5, the cluster $^{16}\text{O}$ for $^{144-154}\text{Gd}$ and $^{176-193}\text{Hg}$ nuclei changes to $^{16}\text{C}$ for heavier $^{155-158}\text{Gd}$ and $^{194}\text{Hg}$ nuclei. However, then the Q-value (for $^{16}\text{C}$ cluster combination) is so small (see Fig. 6) that the penetrability is almost negligible (and hence of not much interest to include such clusters any further in our analysis). Figure 6 also reveals that the Q-value is negative (or nearly zero) for $\alpha$-decay of $^{146}\text{Gd}$, and for both the $\alpha$ and $^8\text{Be}$ decays of Gd nuclei heavier than $^{154}\text{Gd}$. The fact that the penetrabilities $P$ are small (−log$P$ large) for smaller Q-values, is evident from Figs. 7 and 8, which give, similar to Figs. 4 and 5, the results of our calculation for the barrier penetrability $P$. We further notice in Fig. 7 that the penetrability $P$ is in general small (large −log$P$) for non-$\alpha$ clusters in the light mass region $A = 130-158$. The combined effect of the preformation probability $P_0$ and penetrability $P$ gives the measurable decay half-life time $T_{1/2}$, since the impinging frequency $v_0$ is almost constant. The resulting $T_{1/2}$ are presented in Figs. 9 and 10, where their logarithms are plotted with respect to the mass number of the parent nuclei. The structural information obtained from these calculations for each set of parents is discussed separately in the following sub-sections. Note, however, in Fig. 9(b) that $^{154}\text{Gd}$ is almost stable against $\alpha$ and $^8\text{Be}$ decays (large $T_{1/2}$-values), but could be of interest for other heavier cluster decays, as is discussed in section III.B.2.

#### A. The $\alpha$-decay results

We have noted above that the preformation factor $P_0$ is largest for $^4\text{He}$. This is of the order of $10^{-5}$ − $10^{-8}$ for all superdeformed $^{133-137}\text{Nd}$, superdeformed and some heavier mass, normal deformed $^{144-154}\text{Gd}$ nuclei (superdeformation in Gd nuclei stops at $^{150}\text{Gd}$, and Gd nuclei beyond $^{154}\text{Gd}$ are $\alpha$-stable, $Q_\alpha < 0$; Q-value is small but positive for $\alpha$-decay of $^{154}\text{Gd}$, though experimentally it is a known $\alpha$-stable nucleus), all superdeformed and some lighter mass (mainly odd-A), normal deformed $^{183-184}\text{Hg}$ (here superdeformation begins at $^{189}\text{Hg}$) and all superdeformed $^{192-198}\text{Pb}$ nuclei. However, $P_0$ is much larger, $∼ 10^{-3}$, for almost all lighter mass, normal deformed $^{176-188}\text{Hg}$ and $^{182-191}\text{Pb}$ nuclei. This suggests that superdeformed nuclei are the poorer $\alpha$-emitters, as compared to their light mass, normal deformed species. Interesting enough, the same result is born out in the calculated $\alpha$-decay half-lives $T_{1/2}$, plotted in Fig. 11 as $\log_{10} T_{1/2}$ versus parent mass number, and compared with the available experimental data (taken from Refs. [27,28]). We have also included here (see inset, Fig. 11) the other heavier isotopes of Pb in order to include the lone experimental data for $^{210}\text{Pb}$, in the neighborhood of doubly magic $^{208}\text{Pb}$ nucleus. The Fig. 11 presents not only the interesting comparison of calculated results with experiments, but also interesting shell structure effects of both the parent nuclei and their daughter products which are mostly known but not yet observed experimentally via $\alpha$-decay studies (the $\alpha$-decay experimental data are...
not yet complete). We have also compared our results and the available experimental data with another calculation due to the generalized liquid drop model (GLDM) [27] in Table 1. We notice that the PCM and the GLDM calculations give identical results, both within one order of magnitude of the experimental data.

The following results are evident from Fig. 11: (i) The PC calculations compare nicely with experiments, showing even the (small) odd-even effects in both the light mass, normal deformed 176–188Hg and 182–191Pb nuclei. The odd-even effects seem to become weaker in all the five even effects as clearly as are given by the calculations. 210

(ii) The light mass, normal deformed showing even the (small) odd-even effects in both the calculations give identical results, both within one order

We have seen above, and is also known from our earlier calculations [4–7,9], that as the N:Z ratio of parent nuclei increases, the N≠Z, non-α nuclei cluster emissions become equally, or even more, probable as compared to N=Z, α nuclei cluster emissions (see, e.g., the crossing over of the curves for 12C and 14C clusters in Fig. 4(b) for heavier Gd parents or in Fig. 5(a) for Hg parents; P_0 becomes larger for 14C, as compared to that for 12C). Hence, the α-nucleus cluster emission effects must be more prevalent for parents in the low mass region A=130–158 and the non-α nucleus cluster emissions for parents in the heavier mass region A=180–198. We know that all the radioactive exotic cluster-decays from the parent masses A>222 consist of only non-α nuclei clusters [12], such as 14C, 18,20O, 23F, 22,24,26Ne, 28,30Mg and 32,34Si. Secondly, it is interesting to note that, though the cluster preformation factors P_0 are of similar orders for both the chosen regions of parent nuclei (compare Figs. 4 and 5), the penetrabilities P are much smaller (larger −log_{10}P) for parents in the lighter mass region A=130–158 (compare Figs. 7 and 8). This means that, like for α-decays, the cluster-decay rates for parents in the lighter mass region are also expected to be smaller (larger cluster-decay half-lives) than for parents in the heavier mass region. In other words, the parents in lighter mass region A=130–158 are likely to be more stable against cluster decays, than the parents in heavier mass region A=180–198. We discuss these results in the following for each set of parent nuclei separately.

1. Nd parents

First of all we look at the calculations in Figs. 4(a) and 7(a), respectively, for the preformation probability P_0 and penetrability P for Nd parents. We notice that -log_{10}P_0 for the lightest two clusters 8Be and 12C are structure-less (remains almost constant), but for heavier clusters develop into maxima (minima for P_0) each at the even-A parents 134Nd and 136Nd which grow as the size of the cluster increases. On the other hand, for the penetrability, -log_{10}P is structure-less for all clusters, except for a steep rise for 8Be decay and small maximum (minimum for P) at 134Nd (or a minimum at 135Nd) for its 14C decay. The fact that these maxima and minima are simply the result of an odd-even effect in Nd nuclei is evident from the almost constant cluster decay half-lives T_1/2 in Fig. 9(a). The notable exception is again for 8Be decay, where the decay half-life is an ever increasing function of parent mass, with T_1/2 > 10^{100}(s) and hence stable against such a decay. The interesting result is that some heavy clusters, like 30,32Si, have decay half-lives of the same order as for the light clusters like 12C and 16O. These are apparently due to, say, the neighbouring Z=50 magic shell in 12_{19}Te daughter, or the mid-shell effects of the known neutron magic shells, in 16O decay of 135Nd parent. The decay half-lives are, however, large ~10^{50} (s). In other words, the Nd parents are as stable, rather more stable, against cluster decays as they are against

B. The heavy cluster-decays and closed-shell effects

We have seen above, and is also known from our earlier calculations [4–7,9], that as the N:Z ratio of parent
α-decays.

2. Gd parents

For Gd parents, the P₀ in Fig. 4(b) seem to behave smoothly, except for a small minimum at 148Gd parent, which turn into a minimum at 145Gd and a maximum at 144Gd parent for the heavier clusters. Also, the division between the superdeformed and normal deformed nuclei is evident at 152Gd where P₀ increases suddenly (−log₁₀P₀ decreases) for all the clusters and stays nearly independent of the mass of normal deformed parents (except for small oscillations, the odd-even effect). On the other hand, the P in Fig. 7(b) show the maxima, minima structure for different light clusters at any one of the parent nuclei 146−152Gd. The heavier clusters are all peaked around 146Gd. These results combine to give four significant minima for T₁/2 in Fig. 9(b): one at 150Gd for 8Be decay, another at 152Gd for 12C decay, the third one at 154Gd for 14C decay and finally the fourth one at 156Gd for 18O decay. All these minima refer to N=82 magicity of the respective daughters 5012Nd, 58140Ce, 58140Ce and 56188Ba. In other words, these are the only four isotopes of Gd (152−156Gd) which are prone to heavier cluster decays, though the predicted decay half-lives are large ~10^28−10^32(s). Note that one of these parents (150Gd) is a superdeformed nucleus whereas the other three heavier ones (152,154,156Gd) are normal deformed nuclei. Also, of these four, 8Be and 12C decays of 150Gd and 152Gd, respectively, are more probable (smaller T₁/2). Then, there are some maxima appearing in Fig. 9(b), mainly at 146Gd, which refer to the stability of this parent nucleus against the cluster decays due to its N=82 shell closure. Note that 146Gd is already stable against 8Be decay (Q < 0). Thus, the structure effects of both the parent(s) and daughters come into play in the cluster-decay properties of Gd nuclei, but the predicted cluster decay half-lives are beyond the limits of the present measurements, which go only up to ~10^28(s) [12].

3. Hg parents

In this sub-section, we discuss the results of our calculations for normal deformed 176−188Hg and superdeformed 189−194Hg nuclei, presented in Figs. 5(a), 8(a) and 10(a) for P₀, P and T₁/2, respectively. First of all, we notice a number of minima and maxima in Fig. 5(a) for P₀, which correspond to the odd-even effects of the parents. Then, the P₀ increases suddenly (−log₁₀P₀ decreases) near the transition point of deformed to superdeformed region where it has the largest value for almost all the cluster preformations in 188Hg, the normal deformed nucleus at the transition. As we shall see below for the T₁/2 calculations, this result corresponds exactly to the one observed for the α-decay half-lives of Gd isotopes in Fig. 11, i.e. of the change of shape in going from a maximum (peaking) to the minimum (valley). On the other hand, the P are nearly smooth functions of the parent mass, except for a noticeable minimum (enhanced penetrability) at 185Hg, and/or for some clusters at 189Hg, in Fig. 8(a). The resulting T₁/2 in Fig. 10(a) show interesting maxima and minima, like for P₀, for normal deformed 176−185Hg nuclei, referring to the larger stability of the even parents (at maxima) relative to the odd parents (at minima). Then, a (broad) minimum or valley of instability (smaller values next to a maximum in T₁/2) occurs for the normal deformed and superdeformed transitional nuclei 186−190Hg.

The above noted stability of even-A 176−185Hg nuclei point to the closed shell effects of these parents at Z=80 (in the neighborhood of magic Z=82) coupled with a magic or semi-magic nature of their neutron shells with N=96,98,100,102 and 104. On the other hand, the instability of 186−190Hg parents, against various clusters, reflect the closed shell effects of the daughter nuclei, like 178−182Os, 174−178W, ... etc., referring to Z=76,74,... and N=106,104,102,... closed or near-closed shells. Coupling the results of both the stability and instability in this region, with the fact that 8Be and 12C are shown as the most probable cluster decays (smallest T₁/2-values), the Z=76 or 74 and N=96-104 seem to point to the major (deformed) closed shells. The same result was obtained in our earlier studies [4,8] and supports the structure calculations of other authors [9,30]. Also, Fig. 10(a) shows that the best measurable 8Be and 12C decays come from 176,177Hg, with T₁/2 ~10^18(s) and T₁/2 ~10^22(s), which are well within the limits of present day experiments.

4. Pb parents

The calculations for Pb parents are presented in Figs. 5(b), 8(b) and 10(b), respectively, for P₀, P and T₁/2. The P₀ are almost constant, except for a small enhancement (minimum −log₁₀P₀) in the case of 190Pb parent for all cluster configurations. The same is true for the P, except that the enhancement is now for the 193Pb parent. The net result is a shallow minimum in T₁/2 for heavier clusters, like 20O and 24Ne, from 194Pb or 195Pb parents, which means the shell stabilizing effects of the daughters 174,175W and 170,171Hf. This means supplementing the above noted results for Hg isotopes that the major closed shells could even occur at Z=72,74, N=98-100. The minimum decay half-lives are for the 8Be and 12C decays of 192Pb, both ~10^17, which are still not within the reach of experiments.
We have made a systematic study of the $\alpha$- and heavy-cluster decays of nuclei with masses $A=130-158$ and $180-198$, comprising three regions of superdeformations to different orders. Specifically, the $\text{135}^{\text{Nd}}, \text{144}^{\text{Gd}}, \text{176}^{\text{Hg}}$ and $\text{192}^{\text{Pb}}$ nuclei are studied, which also include the normal deformed nuclei on both the lighter and heavier sides of the superdeformed nuclei. Furthermore, for $\alpha$-decay of Pb nuclei, we have also considered the very heavy isotopes up to $\text{210}^{\text{Pb}}$ which means including also the spherical closed shell effects of $\text{208}^{\text{Pb}}$ with doubly magic $Z=82$ and $N=126$. The main idea of this work is to look for new (spherical or deformed) closed shells via cluster decay studies or, in other words, the possible signatures of any new cluster radioactivity in the superdeformed nuclei. A parent nucleus is stable against $\alpha$ and other cluster-decays, if the decay half-lives are large. On the other hand if the decay half-life is small (measurable or close to measurable) then, in view of the so-far observed radioactive decays, it must refer to the closed shell effects of the daughter product(s). Based on such an analysis for both the $\alpha$ and heavier cluster decays, we have obtained the following results from the chosen nuclear mass regions.

The superdeformed nuclei are better $\alpha$ emitters, as compared to their heavier mass, normal deformed isotopes, though both of these are poorer $\alpha$ emitters than their lighter mass, normal deformed species. The closed shell effects of the parents (for $\text{140}^{\text{Gd}}$ and $\text{208}^{\text{Pb}}$ parents), at the spherical sub-magic $Z=64$, magic $N=82$ and doubly magic $Z=82$, $N=126$, are evident in terms of the maxima or peakings of the $\alpha$-decay half-lives, whereas the same for daughter nuclei $\text{130}^{\text{Ce}}$ and $\text{144}^{\text{Sm}}$ (for $\text{134}^{\text{Nd}}$ and $\text{148}^{\text{Gd}}$ parents, respectively) are given as minima or valleys in $\alpha$-decay half-lives due to the mid-shell effect of the magic $Z=50$ and sub-magic $Z=64$, and the $N=82$ magic shell. These are the known shell closure effects, which apparently are nicely reproduced in the PCM calculations.

The calculated cluster-decay half-lives show that the lighter mass region $A=130-158$ is more stable (larger $T_{1/2}^{\alpha}$ values) than the heavier mass region $A=180-198$. The light mass region presents the closed shell effects of known magic $Z=50$ or $N=82$ shells (for $^{16}O$ decay of $\text{135}^{\text{Nd}}$, $^{8}Be$ decay of $^{150}Gd$, $^{12}C$ and $^{15}O$ of decays of $^{152,154}Gd$, respectively, and $^{18}O$ decay of $^{156}Gd$), with the predicted cluster decay half-lives beyond the present measurable limits of the experiments ($T_{1/2}^{\alpha} \sim 10^{23}(s)$ or more). On the other hand, the heavier mass region $A=180-198$ present not only the stability effects of neighbouring $Z=82$ magic shell for even-$A$ $\text{176}^{\text{Hg}}$ parents, but also interesting new possibilities of deformed-daughter cluster radioactivity at $Z=72-76$ and $N=96-104$ for $^{186-190}Hg$ and $^{194,195}Pb$. In other words, new deformed magic shells are likely to occur at $Z=72-76$ and $N=96-104$ for various cluster decays of $^{186-190}Hg$ and $^{194,195}Pb$. The best, observable cases are predicted to be $^{8}Be$ and $^{12}C$ decays of $^{176,177}Hg$ and/or $^{192}Pb$ parents (with $T_{1/2}^{\alpha} \sim 10^{18}$ and $10^{23}$ (s), respectively). The interesting point is that in both the nuclear regions under study, more of these nuclei are normal deformed nuclei rather than superdeformed ones. In fact, they lie at the transition between the normal deformation and superdeformation, but more towards the region of normal deformation. The best possible cases also come from the normal deformed regions. In other words, the region of the change of shape, i.e. the valley(s) in the immediate neighbourhood of a peak, seems to be the criterion for the location of new magicities or for the new cluster radioactivity(ies).

### IV. SUMMARY OF RESULTS AND CONCLUSION

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Figure Captions:

Fig. 1. The variation of ground-state quadrupole deformation parameter $\beta_2$ with mass number for the selected Nd, Gd, Hg and Pb parent nuclei. The data are from the calculations of Möller et. al [19]. The region of nuclei where superdeformed bands are observed is marked in each case.

Fig. 2. The mass fragmentation potentials as a function of the mass number of light fragments, for the superdeformed isotopes of Nd parents. The calculations are made at the touching configuration $R=C_1+C_2=C_t$ and by using experimental binding energies [24]. Only the light fragments (clusters) at minima are marked.

Fig. 3. The same as for Fig. 2, but for both the normal deformed and superdeformed isotopes of Pb parents.

Fig. 4. The logarithms of the cluster preformation probability $P_0$ as a function of the mass number of (a) Nd, and (b) Gd parents, for different clusters. For the same cluster mass number, the dashed line shows $P_0$ if the charge number is changed.

Fig. 5. The same as for Fig. 4, but for (a) Hg, and (b) Pb parents.

Fig. 6. The variation of Q-value with mass number for (a) Gd and (b) Hg parents. The binding energies used are the experimental binding energies [24].

Fig. 7. The same as for Fig. 4, but for the penetrability $P$.

Fig. 8. The same as for Fig. 7, but for (a) Hg, and (b) Pb parents.

Fig. 9. The same as for Fig. 4, but for the logarithms of cluster decay half-life, $\log_{10}T_{1/2}(s)$.

Fig. 10. The same as for Fig. 9, but for (a) Hg, and (b) Pb parents.

Fig. 11. The logarithm of the calculated $\alpha$-decay half-lives, $\log_{10}T^{\alpha}_{1/2}(s)$, as a function of the parent mass number for various isotopes of Nd, Gd, Hg and Pb nuclei, compared with the experimental data (taken from Refs. [27,28]). The inset shows the same calculation for the heavier isotopes of Pb, where the experimental data for only $^{210}$Pb is known.
Table 1: The logarithms of α-decay half-lives and other characteristic quantities calculated by using the preformed cluster model (PCM) of Gupta and collaborators. The impinging frequency $\nu_0$ is nearly constant, of the order of $10^{21}\text{s}^{-1}$. The PCM calculations are compared with the available GLDM calculations of Royer [27] and the experimental data.

| Parent | Q-value (MeV) | $P_0$ | $P$ | $\log_{10}T_{1/2}(s)$ | PCM | GLDM | Expt. |
|--------|---------------|-------|-----|------------------------|-----|------|-------|
| $^{144}\text{Gd}$ | 3.27 | $3.10 \times 10^{-03}$ | $1.38 \times 10^{-28}$ | 10.90 | 9.68 | 9.36 |
| $^{149}\text{Gd}$ | 3.10 | $2.84 \times 10^{-07}$ | $2.84 \times 10^{-29}$ | 13.64 | 11.15 | 11.21 |
| $^{150}\text{Gd}$ | 2.81 | $4.13 \times 10^{-07}$ | $7.13 \times 10^{-31}$ | 15.10 | 14.09 | 13.75 |
| $^{151}\text{Gd}$ | 2.65 | $9.41 \times 10^{-08}$ | $5.92 \times 10^{-32}$ | 16.83 | 15.92 | 15.11 |
| $^{152}\text{Gd}$ | 2.21 | $1.02 \times 10^{-05}$ | $2.78 \times 10^{-36}$ | 19.17 | 22.12 | 21.54 |
| $^{176}\text{Hg}$ | 6.93 | $6.21 \times 10^{-03}$ | $9.63 \times 10^{-18}$ | -2.38 | -1.76 | -1.7 |
| $^{177}\text{Hg}$ | 6.74 | $1.36 \times 10^{-04}$ | $4.69 \times 10^{-18}$ | -0.40 | -1.17 | -0.77 |
| $^{178}\text{Hg}$ | 6.58 | $1.82 \times 10^{-03}$ | $2.39 \times 10^{-18}$ | -1.23 | -0.60 | -0.44 |
| $^{179}\text{Hg}$ | 6.43 | $9.57 \times 10^{-05}$ | $1.22 \times 10^{-18}$ | 0.35 | -0.04 | 0.32 |
| $^{181}\text{Hg}$ | 6.29 | $5.36 \times 10^{-05}$ | $6.04 \times 10^{-19}$ | 0.91 | 0.49 | 1.32 |
| $^{182}\text{Hg}$ | 6.00 | $1.29 \times 10^{-03}$ | $1.23 \times 10^{-19}$ | 0.23 | 1.68 | 1.85 |
| $^{183}\text{Hg}$ | 6.04 | $1.83 \times 10^{-06}$ | $1.62 \times 10^{-19}$ | 2.96 | 1.50 | 1.57 |
| $^{184}\text{Hg}$ | 5.66 | $6.60 \times 10^{-04}$ | $1.49 \times 10^{-20}$ | 1.45 | 3.20 | 3.37 |
| $^{186}\text{Hg}$ | 5.21 | $4.02 \times 10^{-05}$ | $4.49 \times 10^{-22}$ | 4.21 | 5.40 | 5.73 |
| $^{182}\text{Pb}$ | 7.08 | $6.50 \times 10^{-03}$ | $1.57 \times 10^{-17}$ | -2.61 | -1.59 | -1.26 |
| $^{189}\text{Pb}$ | 5.86 | $1.12 \times 10^{-07}$ | $2.57 \times 10^{-20}$ | 4.98 | 3.17 | 4.11 |
| $^{191}\text{Pb}$ | 5.41 | $4.64 \times 10^{-07}$ | $8.29 \times 10^{-22}$ | 5.87 | 5.31 | 5.78 |
| $^{210}\text{Pb}$ | 3.79 | $2.48 \times 10^{-09}$ | $2.63 \times 10^{-30}$ | 16.74 | 16.00 | 16.57 |
Fig. 1 "Closed shell effects from ....." R.K. Gupta et al.

Superdeformed Bands

Data from Moller et al. [19]

Superdeformed Bands

Superdeformed Bands

Superdeformed Bands

Ground-state quadrupole deformation parameter $\beta_2$

Mass number of parent nuclei

Nd

Gd

Hg

Pb
Fig. 2 "Closed shell effects from ....." R.K. Gupta et al.

Audi-Wapstra (1995)

\[ R = C_1 + C_2 \]
Fig. 3 "Closed shell effects from ....." R.K. Gupta et al.
Fig. 4 "Closed shell effects from ....." R.K. Gupta et al.
Fig. 5 "Closed shell effects from ....." R.K. Gupta et al.
Fig. 6 "Closed shell effects from ....." R.K. Gupta et al.
Fig. 7 "Closed shell effects from ....." R.K. Gupta et al.
Fig. 8 "Closed shell effects from ....." R.K. Gupta et al.

(a) Mass number of Hg parents

(b) Mass number of Pb parents

- log_{10} P

16C

4He

8Be

12C

14C

16O

16C

18O

20Ne

20O

22Ne

24Mg

24Ne

26Mg

28Si

30Si

32S

32Si

34Si

36S

38S

42Ar

44Ar

46Ca

48Ca
Fig. 9 "Closed shell effects from ....." R.K. Gupta et al.

(a) Mass number of Nd parents

(b) Mass number of Gd parents

- $^4\text{He}$
- $^8\text{Be}$
- $^{12}\text{C}$
- $^{14}\text{C}$
- $^{16}\text{O}$
- $^{18}\text{O}$
- $^{22}\text{Ne}$
- $^{26}\text{Mg}$
- $^{30}\text{Si}$
- $^{32}\text{Si}$

$\log_{10} T_{1/2} (s)$
Fig. 10 "Closed shell effects from ....." R.K. Gupta et al.

(a)

(b)
Fig. 11 "Closed shell effects from ....." R.K. Gupta et al.