Dispersion curves for Acoustic Multipole Sources on guided waves in Isotropic Tubular Structure Liquid Metal Sodium Filled

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Abstract: A new analytical method is proposed for the multi-pole sound source of the isotropic tubular structure liquid metal sodium with the symmetry principal axis parallel to the tube axis. For the first time in this paper, a new method is used to simulate the dispersion characteristics of an isotropic tubular structure filled with liquid sodium. By selecting a set of displacement potentials and a cylindrical coordinate system along the tube axis, the multipole sound field solution of the isotropic tubular structure filled with liquid sodium is derived. The sound field of the multi-pole source inside and outside the tube is studied, and the dispersion characteristics in the tube are numerically simulated. The results show that in the isotropic tubular structure in which the symmetrical main axis of the tube excited by the monopole, dipole and quadrupole sources is parallel to the tube axis, the dispersion characteristics of the filled liquid metal sodium vary with the thickness and temperature of the tube and change.

1. The framework model
With the deepening of research on acoustic waves, people have discovered that wave propagation in anisotropic media has become a relatively active research topic. What is more interesting in the study of anisotropic formations is the transversely isotropic medium (TIM). This is mainly because PTL (periodical thin-layer) and EDA (extended-dilatancy anisotropy) models are very important for reservoir rocks, and both are equivalent to transversely isotropic formations under the assumption of long wavelength. With the development of well source technology, the study of acoustic wave propagation in anisotropic media has become an important part of acoustic logging. When the medium outside the well is TIM, the wave field solution can be solved analytically only when the symmetric main axis of the TIM is parallel to the borehole axis. In order to study the wave field characteristics in important situations where the TIM symmetry principal axis is not parallel to the borehole axis, several approximate methods are proposed. Ellefsen, Sinha and Norris proposed a perturbation method to study the influence of weak elastic anisotropy on tube wave velocity [1-5]. However, their perturbation methods cannot be used to study the characteristics of full waves and longitudinal and transverse waves. Zhang et al. [6-9] proposed another perturbation method for processing the TIM full-wave sound field.
with the longitudinal axis perpendicular to the borehole axis. They obtained the zero-order and first-order approximate solutions of the sound field inside and outside the borehole. This method can be used to resolve the wellbore shear wave splitting problem analytically. Zhang et al. [10] studied the frequency wavenumber domain of the multipole sound source perturbation method in transversely isotropic media.

This paper presents an effective analysis method for the multipole sound source in the isotropic tubular structure, which is to fill the liquid metal sodium with the symmetry main axis parallel to the tube axis, although an accurate solution is obtained. The characteristics of each order of the perturbation solution outside the tube when the radial variable $r$ changes are discussed. The sound field inside and outside the borehole excited by the multipole sound source is studied. The dispersion characteristics excited by monopole, dipole and quadrupole sources vary with the physical characteristics of the tube.

2. FORMULATION

Consider a fluid-filled tube surrounded by tubular structure whose symmetric principal axis is parallel to the tube axis. The density and velocity of the fluid inside the tube are $\rho_f$ and $V_f$, respectively, $R_1$ and $R_2$ are the inside radius and the outside radius of the tube. We adopt a cylindrical coordinate system $(r, \theta, z)$ centered at the center of multipole source and oriented along the tube axis. The displacement in the isotropic tubular structure outside the tube satisfies:

$$\mathbf{u} = \nabla \phi + \nabla \times (\nabla \times \phi) = \nabla \times \mathbf{e}_z = \nabla \phi$$

(1)

Where $K_n$ is the second kind of nth-order modified Bessel function, $r_0$ is the multipole source separation, $\psi = \sqrt{k_2^2 - k_3^2}$, $k_2^2 = \rho \omega^2 / (\lambda + 2\mu)$, $k_3^2 = \rho \omega^2 / \mu$, and $B_n, C_n, D_n, E_n, F_n, G_n$ are the weighting coefficients.

The acoustical potential in a tube for an nth multipole source can be written the same form as that in the isotropic medium [7-14]

$$\phi_n(r, \theta, k_z, \omega) = -\frac{V_n(\omega)}{2\pi} \left( \frac{r_0}{n!} \right)^n \left[ B_n K_n(v(r)) + C_n I_n(v(r)) \right] \cos(\theta - \theta_0)$$

$$\chi_n(r, \theta, k_z, \omega) = -\frac{V_n(\omega)}{2\pi} \left( \frac{r_0}{n!} \right)^n \left[ D_n K_n(v(r)) + E_n I_n(v(r)) \right] \sin(\theta - \theta_0)$$

$$\psi_n(r, \theta, k_z, \omega) = -\frac{V_n(\omega)}{2\pi} \left( \frac{r_0}{n!} \right)^n \left[ F_n K_n(v(r)) + G_n I_n(v(r)) \right] \cos(\theta - \theta_0)$$

(2)

In the Cartesian coordinate system $(x, y, z)$, the displacement $\mathbf{u}$ in the isotropic tubular structure outside the tube satisfies:

$$\mathbf{u} = \nabla \phi + \nabla \times (\nabla \times \phi) = \nabla \times \mathbf{e}_z$$

(1)
\[
\phi_i^j(\omega, k) = -\frac{1}{4\pi n!\left(\frac{v_0}{2}\right)^4} \left[ e_i R_i(\nu r) \cos(n(\theta-\theta_i)) + A_i I_i(\nu r) \cos(n(\theta-\theta_i)) \right]
\]  

(3)

Where, \(V_f\) is the acoustic velocity of the fluid inside the tube, \(\varepsilon_n = 2\) \(\delta_{n0}\) is Neumann’s factor, \(r_0\) is the multipole source separation, \(I_n\) is the first kind of the nth-order modified Bessel function, and \(A_n\) is the reflection coefficient which can be determined by the boundary conditions at tube wall.

The boundary conditions at \(r = R_\circ\) are [6-8, 10-12]

\[
U^I = U^II, \\
-P^I = \tau^I, \\
0 = \tau^II, \\
0 = \tau^I_{r\theta}.
\]  

(4)

The boundary conditions at \(r = R_\circ\) are [6-8, 10-12]

\[
\tau^I_r = 0, \\
\tau^II_r = 0, \\
\tau^I_{r\theta} = 0.
\]  

(5)

Where the superscripts I and II represent the media inside and outside the tube. Substituting the fields inside and outside the tube into Eqs. (4) and (5), yields a linear equation group about \(A_{n1}, B_{n1}, C_{n1}, D_{n1}, E_{n1}, F_{n1}, G_{n1}\) and \(H_{n1}\). By this linear equation, we can calculate all unknown coefficients inside and outside the tube. Then, it is easy to analyze the acoustic field characteristics by the numerical simulation.

According to the linear equations of Eqs. (4) and (5):

\[
\begin{bmatrix}
    a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} \\
    a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} \\
    a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} \\
    a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & a_{47} \\
    a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & a_{57} \\
    a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & a_{67} \\
    a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77}
\end{bmatrix} \begin{bmatrix}
    A_n \\
    B_n \\
    C_n \\
    D_n \\
    E_n \\
    F_n \\
    G_n
\end{bmatrix} = \begin{bmatrix}
    b_1 \\
    b_2 \\
    0 \\
    0 \\
    0 \\
    0 \\
    0
\end{bmatrix}
\]

(6)

Where \(a_{ij}(i, j = 1, 2, 3, 4, 5, 6, 7), b_1\) and \(b_2\) are been canceled, because limit of the paper.

According to the linear equations can be obtained:

\[
A_n = \frac{2\pi j A_{n1}}{\partial A_n / \partial z_{k_0}^c}, B_n = \frac{2\pi j A_{n1}}{\partial A_n / \partial \omega^c}, C_n = \frac{2\pi j A_{n1}}{\partial A_n / \partial k_c^c}, D_n = \frac{2\pi j A_{n1}}{\partial A_n / \partial \nu^c},
\]

\[
E_n = \frac{2\pi j A_{n1}}{\partial A_n / \partial k_z^c}, F_n = \frac{2\pi j A_{n1}}{\partial A_n / \partial \omega^c}, G_n = \frac{2\pi j A_{n1}}{\partial A_n / \partial \nu^c}
\]

and:
\[
\Delta_n = \begin{pmatrix}
 a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} \\
 a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} \\
 a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} \\
 a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & a_{47} \\
 a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & a_{57} \\
 a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & a_{67} \\
 a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77}
\end{pmatrix}
\]

\[
\Delta_{1n} = \begin{pmatrix}
 b_1 & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} \\
 b_2 & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} \\
 0 & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} \\
 0 & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & a_{47} \\
 0 & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & a_{57} \\
 0 & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & a_{67} \\
 0 & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77}
\end{pmatrix}
\]

\[
\Delta_{2n} = \begin{pmatrix}
 a_{11} & b_1 & a_{13} & a_{14} & a_{15} & a_{16} & a_{17} \\
 a_{21} & b_2 & a_{23} & a_{24} & a_{25} & a_{26} & a_{27} \\
 0 & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} \\
 0 & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & a_{47} \\
 0 & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & a_{57} \\
 0 & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & a_{67} \\
 0 & a_{71} & 0 & a_{73} & a_{74} & a_{75} & a_{76}
\end{pmatrix}
\]

\[
\Delta_{3n} = \begin{pmatrix}
 a_{11} & a_{12} & b_1 & a_{14} & a_{15} & a_{16} & a_{17} \\
 a_{21} & a_{22} & b_2 & a_{24} & a_{25} & a_{26} & a_{27} \\
 0 & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} \\
 0 & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & a_{47} \\
 0 & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} & a_{57} \\
 0 & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} & a_{67} \\
 0 & a_{71} & a_{72} & 0 & a_{74} & a_{75} & a_{76}
\end{pmatrix}
\]

\[
\Delta_{4n} = \begin{pmatrix}
 a_{11} & a_{12} & a_{13} & b_1 & a_{15} & a_{16} & a_{17} \\
 a_{21} & a_{22} & a_{23} & b_2 & a_{25} & a_{26} & a_{27} \\
 a_{31} & a_{32} & a_{33} & 0 & a_{35} & a_{36} & a_{37} \\
 a_{41} & a_{42} & a_{43} & 0 & a_{45} & a_{46} & a_{47} \\
 a_{51} & a_{52} & a_{53} & 0 & a_{55} & a_{56} & a_{57} \\
 a_{61} & a_{62} & a_{63} & 0 & a_{65} & a_{66} & a_{67} \\
 a_{71} & a_{72} & a_{73} & 0 & a_{75} & a_{76} & a_{77}
\end{pmatrix}
\]

\[
\Delta_{5n} = \begin{pmatrix}
 a_{11} & a_{12} & a_{13} & a_{14} & b_1 & a_{16} & a_{17} \\
 a_{21} & a_{22} & a_{23} & a_{24} & b_2 & a_{26} & a_{27} \\
 a_{31} & a_{32} & a_{33} & a_{34} & 0 & a_{36} & a_{37} \\
 a_{41} & a_{42} & a_{43} & a_{44} & 0 & a_{46} & a_{47} \\
 a_{51} & a_{52} & a_{53} & a_{54} & 0 & a_{56} & a_{57} \\
 a_{61} & a_{62} & a_{63} & a_{64} & 0 & a_{66} & a_{67} \\
 a_{71} & a_{72} & a_{73} & a_{74} & 0 & a_{76} & a_{77}
\end{pmatrix}
\]
We can get the dispersion curve of the guided wave. This method is used to study the guided waves excited by monopole, dipole and quadrupole in the isotropic tubular structure with the main axis parallel to the tube axis.

\[ \Delta \psi_n = 0. \]

We can get the dispersion curve of the guided wave. This method is used to study the guided waves excited by monopole, dipole and quadrupole in the isotropic tubular structure with the main axis parallel to the tube axis.

### Table 1. Three groups of the parameters.

| Note | densities of the fluid sodium | Velocity of the fluid sodium |
|------|-------------------------------|-----------------------------|
| 1(Temperature=900℃) | 750 kg/m³ | 2097 m/s |
| 2(Temperature=950℃) | 740 kg/m³ | 2070 m/s |
| 2(Temperature=1000℃) | 730 kg/m³ | 2043 m/s |

| densities of the isotropic tubular structure | the shear wave velocity | P-wave velocity |
|---------------------------------------------|------------------------|----------------|
| 7930 kg/m³ | 4000 m/s | 5758 m/s |
| 7930 kg/m³ | 4000 m/s | 5758 m/s |
| 7930 kg/m³ | 4000 m/s | 5758 m/s |

### 3. Numerical Simulation

In numerical simulation, the multipole source separation radius is taken to be 0.01 m, inside of the tube radius \( R_1 \) is taken to be 4 m, outside of the tube radius \( R_2 \) are taken to be 4.01 m, 4.02 m, 4.03 m, 4.04 m and 4.05 m, respectively. Three groups of the parameters are given in Table 1.

Figure 2. Dispersion curves of monopole Stoneley waves (Thickness of tube are 1 cm, 2 cm, 3 cm, 4 cm and 5 cm, Temperature=900℃).
Figures 2 and 3 show the dispersion curves of the Stoneley wave monopole guided wave. The five solid lines in Figure 2 represent the exact solution results of the isotropic tubular structure in parameter group 1. The tube radius is 4.01m, 4.02m, 4.03m, 4.04m, and 4.05m, that is, the tube wall thickness is 1cm, 2cm, 3cm, 4cm and 5cm. The study found that under the excitation of a monopole source, the dispersion characteristics of the isotropic tubular structure filled with liquid metallic sodium change with the thickness of the tube wall. This difference only appears in their velocity amplitude. The greater the thickness of the tube wall, the greater the speed in the low frequency range. The three solid lines representing the temperature results in Figure 3 are respectively taken as 900°C, 950°C and 1000°C. The results show that the dispersion characteristics of the isotropic tubular structure filled with liquid metal sodium change with temperature under the excitation of a monopole source, but the dispersion characteristics change little at 900°C, 950°C and 1000°C.

The symmetrical axis of the dipole source and the quadrupole source excitation is parallel to the tube axis in the tube. The dispersion characteristics of the isotropic tubular structure at low frequencies are similar to that of the monopole source, but it is greater than that of the monopole source at low frequencies (Figure 4).

Figure 4. Dispersion curves excited by monopole source, dipole source and quadrupole source (Temperature=900°C, Thickness of tube=2cm).

4. SUMMARY

In summary, this article uses this method to study the guided waves excited by monopole, dipole and quadrupole sources in a fluid-filled sodium tube in an isotropic tubular structure with a principal axis parallel to the tube axis. The study found that under the excitation of multi-pole sources, in the isotropic tubular structure filled with liquid metal sodium, the symmetrical axis is parallel to the dispersion characteristics of the tube axis in the tube. The results show that this method is a feasible method and is suitable for complex anisotropic tubular structures in acoustic multipole sources.

Acknowledgments

This research was financially supported by Topic Foundation of Changchun Institute of Technology (grant number: 320200040), Young People Foundation of Changchun Institute of Technology (grant number: 320200033) and by the Project of Education Department of Jilin Province College students venture project (Grant No.202011437017).
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