Parametric system identification of catamaran for improving controller design

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Abstract. This paper presents an estimation of simplified dynamic model for only surge- and yaw- motions of catamaran by using system identification (SI) techniques to determine associated unknown parameters. These methods will enhance the performance of designing processes for the motion control system of Unmanned Surface Vehicle (USV). The simulation results demonstrate an effective way to solve for damping forces and to determine added masses by applying least-square and AutoRegressive Exogenous (ARX) methods. Both methods are then evaluated according to estimated parametric errors from the vehicle’s dynamic model. The ARX method, which yields better estimated accuracy, can then be applied to identify unknown parameters as well as to help improving a controller design of a real unmanned catamaran.

1. Introduction

Periodical river/canal surveys are one of the most important operations for agricultural and water-resource management in Thailand. However, to survey vast area of river/canal using small number of official personels, enormous budget as well as lengthy survey period are needed. An Unmanned Surface Vehicle (USV) could be an instrument to assist in collecting survey data. So, autonomous control system design as well as intelligent system of USV are important for vast field operation in different environments. However, effectiveness of autonomous control system is depended on either accuracy of USV dynamic parameters or robustness of controller. In this research work, estimation of parameters in USV dynamic equations is examined to increase the accuracy of the controller design.

Examples of the implementation of Unmanned Surface Vehicle (USV) are in several research. Zhi Li and Ralf Bachmayer [1] developed autonomous way-point control system of catamaran motion for testing unknown variables of a catamaran model using Least-Square method. Consequently, a comparisons of the measured and estimated values are shown for the technical parameters. Moreover, researcher Jeong-Hong Park, Hyung-Won Shim and team [2] demonstrated the technique for predicting parameters in dynamic model using Linear-Quadratic system-identification method. By applying specified force to motor propellers, the parameters of vehicle could be estimated with good accuracy.

Moreover, the parameter estimations are implemented in several application for enhancing accuracy of the systems. For example, Huaiyu Wu, Dong Sun and Zhaoying Zhou used a small control board for performing flight control in Micro Air Vehicle (MAV) [3]. By using a multi-channel remote control, the correlation of the vertical and horizontal flight of the MAV could be recorded. In this study, three inputs types are: 1) four inputs (throttle, rudder, aileron, elevator) 2) two inputs (aileron, elevator) and 3) one
input (aileron) that are used for identifying variables of MAV dynamic equation with the Autoregressive exogenous (ARX) method in MATLAB Toolbox. Characteristic equation is used in modeling and analyzing of Pitch-Control with MATLAB Toolbox. Moreover, from China Chongqing University by Yupeng Yuan, Shan Liang and Min Gao [4], also applied ARX method to a microwave heating process in vast industries. This microwave heating process, a closed system, requires a precision control and high safety. Parameters of microwave heating can be calculated using main input variable is the time delay. Experimental data was analyzed for unknown parameters base on the ARX model technique. In addition, Bito Irie from University of Texas at Arlington [5], studied the input/output characteristics relationship between measured dynamic tire loads and road profiles of the tractor (Tractor-semitrailer) in the vehicle model, which approach to the system identification by polynomial. These state-space black-box modeling are formulated and compared with the measured values. The parameters are estimated with ARX, ARMAX based on estimation to obtain data for simulations and compared for the best techniques. It is observed that the ARMAX technique as well as the most accurate method for the tractor model.

This paper presents an importance of predicting and identify unknown parameters of the motion equations unmanned surface vehicles (USV). This research are organized as follows. Section 2 describes kinematic and dynamics equations of USV. The numerical parameter estimation methods both LSM and ARX are presented in Section 3. Section 4 analyzes dynamic system based on magnitude as function of frequency in order to select proper frequency input range. Last, USV parameter estimation using both LSM and ARX methods are demonstrated and compared in Section 5.

2. Kinematic and dynamic equations of USV
For analyzing USV motion in simulation, kinematics and dynamics models, particularly for catamaran, are described [6-9]. Generally, USV motion consists of 6 Degrees of Freedoms (DOFs): 3 positions and 3 rotations. Corresponding parameters of vehicle motion and Earth- and Body-fixed coordinate frames are shown in figure 1.

![Figure 1. Vehicle dynamic models based on earth-fixed and body-fixed coordinate frames.](image)

2.1. USV kinematic model
The variables can be described [6] by the body-fixed frame and the earth-fixed frame in terms of distance, velocity for linear motions, and angular motions of the vehicle, which can be summarized as shown on Table 1.
Table 1. Notations used for kinematic models of unmanned surface vehicles. [6]

| DOF | Motion Description     | Positions & Euler angles | Linear & Angular Velocities |
|-----|------------------------|--------------------------|----------------------------|
| 1   | Motion in x-direction (surge) | x                         | u                          |
| 2   | Motion in y-direction (sway) | y                         | v                          |
| 3   | Motion in z-direction (heave) | z                         | w                          |
| 4   | Rotation about x-axis (roll) | φ                         | p                          |
| 5   | Rotation about y-axis (pitch) | θ                        | q                          |
| 6   | Rotation about z-axis (yaw)  | ψ                         | r                          |

Combining these kinematic variables in Earth- and Body-fixed frames into two sets of column vectors, the previous parameters can be expressed in equation (1).

$$\eta = \begin{bmatrix} \eta_1^T, \eta_2^T \end{bmatrix}; \quad v = \begin{bmatrix} v_1^T, v_2^T \end{bmatrix}$$  \hspace{1cm} (1)

Linear-velocity vector ($v_1$) with respect to vehicle’s center of mass (CM) can be transformed to linear-velocity vector in the Earth-fixed frame ($\eta_1$) using a Jacobian matrix ($J_1(\eta_2)$) in equation (2)-(3), which is a function of Euler angles.

$$\dot{\eta}_1 = J_1(\eta_2) \cdot v_1,$$ \hspace{1cm} (2)

where

$$J_1(\eta_2) = \begin{bmatrix} c\psi c\theta & -s\psi c\phi + c\psi s\theta & s\psi s\phi + c\psi c\phi\theta \\ s\psi c\theta & c\psi c\phi + s\phi s\theta & -c\psi s\phi + s\theta s\psi c\phi \\ -s\phi & c\theta s\phi & c\theta c\phi \end{bmatrix}$$ \hspace{1cm} (3)

Similarly, a Jacobian matrix ($J_2(\eta_2)$) can transform the angular-velocity vector ($v_2$) at vehicles’ center of mass (CM) to the angular-velocity vector ($\eta_2$) in the Earth-fixed frame, as shown in equation (4).

$$\dot{\eta}_2 = J_2(\eta_2) \cdot v_2,$$ \hspace{1cm} (4)

where the Jacobian matrix ($J_2(\eta_2)$) is

$$J_2(\eta_2) = \begin{bmatrix} 1 & s\phi \theta & c\phi \theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi / c\theta & c\phi / c\theta \end{bmatrix}$$ \hspace{1cm} (5)

Integrating equation (3) and (5), the 6-DOF kinematics equation of vehicle can be expressed as the following:

$$\eta = J(\eta) v$$ \hspace{1cm} (6)

$$\begin{bmatrix} \eta_1^T \\ \eta_2^T \end{bmatrix} = \begin{bmatrix} J_1(\eta_2) & 0_{3 \times 3} \\ 0_{3 \times 3} & J_2(\eta_2) \end{bmatrix} \begin{bmatrix} v_1^T \\ v_2^T \end{bmatrix}$$ \hspace{1cm} (7)

2.2. USV dynamics model [6]

To identify USV parameters using dynamics simulation, the motions of the surface vessel are assumed to be restricted only in the horizontal plane without roll motion (or $p = 0$) and pitch motion (or $q = 0$). Thus, there are translation motions in surge (along x-axis) and sway (along y-axis), while heave motion along z-axis (or $w = 0$) is negligible small. As a result, 3-DOF dynamic equations of USV in the body-fixed frame, which are derived from the Newton second law, are written in equation (8).
\[ M \dot{v} + C(v) v + D(v) v = \tau \]  

where \( M \) matrix is a combination of the rigid-body mass and inertia (\( M_{RB} \)) and hydrodynamics forces and moments (\( M_A \)) and \( m \) is mass the vehicle, \( x_G \) is center of gravity in x-direction, \( I_z \) is moment of inertia in about z-axis. Or, \( M = M_{RB} + M_A \) are expressed in equation (9).

\[
M_{RB} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & mx_G \\ 0 & mx_G & I_z \end{bmatrix}; M_A = \begin{bmatrix} -X_u & 0 & 0 \\ 0 & -Y_v & -Y_r \\ 0 & -Y_v & -N_r \end{bmatrix}
\]

Similarly, coriolis and centripetal matrix (\( C = C_{RB} + C_A \)) denote \( X_u, Y_v, Y_r, N_r \) by reference [6] including from rigid-body motion (\( C_{RB} \)) and hydrodynamic force and moments, created by surrounding fluid, (\( C_A \)) can be written in equation (10).

\[
C_{RB}(v) = \begin{bmatrix} 0 & 0 & -m(x_G r + v) \\ 0 & 0 & mu \\ m(x_G r + v) & -mu & 0 \end{bmatrix}; C_A(v) = \begin{bmatrix} 0 & 0 & Y_v + Y_r r \\ 0 & 0 & -X_u u \\ -Y_v - Y_r r & X_u u & 0 \end{bmatrix}
\]

The hydrodynamic damping matrix (\( D \)) exhibits uncoupled between surge-motion and sway- and yaw-motion, damping forces and moment (neglect heave, roll, pitch) [6] formulated in equation (11).

\[
D(v) = \begin{bmatrix} -X_u & 0 & 0 \\ 0 & -Y_v & -Y_r \\ 0 & -N_v & -N_r \end{bmatrix}
\]

Lastly, external forces and moments acting on USV, denoted by \( \tau = [X Y N]^T \), can be described by a multiplication of thruster-configuration matrix (\( B \)) and controlled input-force vector (\( u = [T_L T_R]^T \)), as shown in equation (12).

\[
\tau = Bu
\]

According to Figure 2, USV has two thrusters, fixed at the stern, therefore a forward force \( X = T_L + T_R \) is a sum of two controlled input-forces and a sway force \( Y \) is zero and yaw moment \( N = T_L X_L - T_R X_R \) around the center of mass (CM) or the body-fixed reference frame. \( X_L \) and \( X_R \) are distances along y-axis from the CM of USV to left- and right-thrusters, respectively. Thus, the thruster-configuration matrix can be written as in equation (13). [10]

\[
B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ X_L & X_R \end{bmatrix}, u = \begin{bmatrix} T_L \\ T_R \\ X_L \end{bmatrix}
\]

**Figure 2.** Earth-fixed and body-fixed coordinate frames of USV with two fixed thrusters at stern.
Nevertheless, the sway motion is comparable smaller than linear surge-motion and angular yaw-motion, thus USV’s dynamic equation can be further simplified into 2-DOF dynamic equation in surge-and yaw-directions, as written in Equation (14).

\[
\begin{align*}
\dot{u} &= \left(\frac{X_u}{m-X_u}\right)u + \left(\frac{1}{m-X_u}\right)T_L + \left(\frac{1}{m-X_u}\right)T_R \\
\dot{r} &= \left(\frac{N_r}{I_{zz}-N_r}\right)r + \left(\frac{X_L}{I_{zz}-N_r}\right)T_L - \left(\frac{X_R}{I_{zz}-N_r}\right)T_R
\end{align*}
\]  

(14)

3. Method Identification

This section briefly explains two system-identification techniques for estimating unknown parameters of USV dynamic system. The Least-Square Method (LSM) and AutoRegressive eXogenous (ARX) methods described [11] are based on an input-output description of the dynamic system. The simplified USV model in equation (14) can be reformatted into a state-space form for surge- and yaw-motions in equation (15) and (16), respectively.

\[
Y_1 = [0X_1] = \left[\frac{X_u}{m-X_u} \right] \left[\frac{1}{m-X_u} \right] \left[\frac{1}{m-X_u}\right] \begin{bmatrix} u \cr T_L \cr T_R \end{bmatrix} + e_1
\]  

(15)

\[
Y_2 = [0X_2] = \left[\frac{N_r}{I_{zz}-N_r} \right] \left[\frac{X_L}{I_{zz}-N_r} \right] \left[\frac{X_R}{I_{zz}-N_r}\right] \begin{bmatrix} u \cr T_L \cr T_R \end{bmatrix} + e_2
\]  

(16)

where \(e_1\) and \(e_2\) are random external disturbances.

3.1. Least-Square Method (LSM) [11]

The Least-Squares Method (LSM) could be used to estimate unknown parameters in the dynamic system by minimizing a weighted least-square objective function \(J\), defined as in equation (17).

\[
J = e^\top We = (Y - \hat{Y})^\top W(Y - \hat{Y})
\]  

(17)

where a prediction error \(e\) is defined as: \(e = Y - 0X\). By minimizing the cost function \(J\), optimal values of unknown parameters can be computed from equation (18).

\[
\hat{\theta} = (X^\top WX)^{-1}X^\top WY
\]  

(18)

Moreover, these estimated parameters \(\hat{\theta}\) could be employed to evaluate the prediction error as well as to adjust the weight so that \(e\) can be further reduced.

3.2. AutoRegressive Exogenous (ARX) Method [11]

The AutoRegressive Exogenous (ARX) method is a one of the effective system-identification technique. The ARX method can be applied with discrete-time model, expressed in a linear difference equation with external disturbance \(e(t)\), as show in equation (19).

\[
y(t) + a_1y(t-1) + \ldots + a_{n_y}y(t-n_y) = b_1u(t-1) + \ldots + b_{n_u}u(t-n_u) + e(t)
\]  

(19)

where \(y(t-i)\) and \(u(t-i)\) are delayed output and input by i-time step, respectively. Reformulating this difference equation in a form of a backward shift operator \((q^{-1})\) or \(q^{-1}u(t) = u(t-1)\), the discrete-time model can be described as a ratio of polynomial functions, \(A(q)\) and \(B(q)\), in equation (20).

\[
y(t) = \frac{B(q)}{A(q)}u(t) = G(q)u(t)
\]  

(20)
Polynomial functions, $A(q)$ and $B(q)$, are correspondingly composed of finite constant coefficients of output and input, which are truncated up to $n_a$ and $n_b$ terms. In ARX method, $n_k$ indicates an input-output delay, $n_a$ are order of $A(q)$ polynomial, related to number of poles and $n_b$ are order of $B(q)$ polynomial, related to number of zeroes plus one. These coefficients, $a_i$ and $b_i$, are unknown parameters of the specified model structure in a form of linear difference equation.

$$A(q) = \sum_{k=0}^{n_a} a_k q^{-k} = 1 + a_1 q^{-1} + a_2 q^{-2} + \ldots + a_{n_a} q^{-n_a}$$

$$B(q) = \sum_{k=0}^{n_b} b_k q^{-k} = b_1 q^{-1} + b_2 q^{-2} + \ldots + b_{n_b} q^{-n_b}$$

Therefore, the model structure can be rewritten as the linear regression below.

$$y(t, v) = \Theta(t)^T \mathbf{v}$$

The unknown discrete parameters, which are lumped as a vector: $\mathbf{v} = [a_1, a_2, \ldots, a_{n_a}, b_1, b_2, \ldots, b_{n_b}]^T$, can be estimated form input-output relation using the LSM. The output vector is denoted as $y(t) = [y(\text{max}(n_a, n_b)), \ldots, y(N)]^T$, where $N \gg \text{max}(n_a, n_b)$ and a matrix $\phi(t)^T$ is a regression matrix for the case with $n_a \geq n_b$, as shown in equation (24).

$$\phi = \begin{bmatrix}
-y(n_a - 1) & \cdots & -y(0) & u(n_a - 1) & \cdots & u(n_a - n_b) \\
-y(n_a) & -y(1) & u(n_a) & \cdots & u(n_a - n_b + 1) \\
\vdots & & \ddots & & \vdots & \\
-y(N - 1) & \cdots & -y(N - n_b) & u(N - 1) & \cdots & u(N - n_b)
\end{bmatrix}$$

For the case with $n_b \geq n_a$, the $n_a$ can be replaced $n_b$ and vice versa. As a result, the ARX method can estimate these unknown parameters, as expressed in Equation (25).

$$\hat{\mathbf{v}} = (\phi^T \phi)^{-1} \phi^T \hat{y}$$

4. Analytical Frequency Range

In order to perform system-identification technique, two sinusoidal inputs or thrust forces with different and varying frequencies are imposed on the USV dynamics models in equation (14), thus system resonance or natural frequency must be recognized beforehand. Using Bode diagram, natural frequencies can be determined from the dynamic equation in state-space form, shown in Equation (26).

$$X = AX + BU$$

$$Y = CX$$

where $X = [u \; r]^T$ is a state vector and $U = [T_L \; T_R]^T$ is an input force vector, consisting of input1 ($T_L$) and input2 ($T_R$). The magnitude and phase responses for states ($u, r$) are plotted as function of varying frequency, as demonstrated in Figure 3 and Figure 4. There is no natural frequency for these two first-order systems, there are only corner frequencies at $\omega_{n,1} = 0.0115$ rad/sec and $\omega_{n,2} = 1.1049$ rad/sec. The frequency range of the forcing sinusoidal functions from Bode diagram can be guideline for selecting forcing frequency of the system-identification analysis in next section.
5. Performance Analysis of System identification

The dynamic model of the USV, defined in equation (14), is employed as a performance evaluation between LSM and ARX system-identification techniques. Rewriting the USV system in Equation (27) and (28) can lumped USV parameters together in a form of \( \alpha \)'s and \( \beta \)'s.

\[
\begin{align*}
    u &= \alpha_1 u + \alpha_2 T_L + \alpha_3 T_R \\
    r &= \beta_1 r + \beta_2 T_L + \beta_3 T_R
\end{align*}
\]

The estimated USV parameters using the LSM and ARX methods are validated against known USV parameters such that parametric estimation errors can be considered as function of input frequency and response time. The coupled sinusoidal thrust forces with varying frequency \((\omega = 2\pi f)\) are specified as inputs in MATLAB/Simulink. Estimated parameters from dynamic response in time-domain are then compared with known parameters.

5.1. Parametric estimation error by LSM

The Least-Square Method (LSM) is a numerical analysis technique, commonly used in the system identification. According to figure 5, estimated surge parameter \( (\alpha_1) \) and yaw parameter \( (\beta_1) \) improve over increasing response time, but these two parameters are insensitive to varying input frequencies. DC gains of the USV transfer function \((\alpha_2, \alpha_3)\) and \((\beta_2, \beta_3)\) are depended on both response time and input frequency, moreover estimated parameters from LSM can be improved by using low input-frequency. With appropriate input-frequency, parametric estimation errors decrease or converge to zero with time.
Figure 5. Parametric estimation errors using LSM as function of response time and varying input frequency.

5.2. Parametric estimation error by ARX

To apply ARX method, dynamic continuous-time model must be converted to discrete-time model in the z-domain, described by $a_i$ and $b_i$ coefficients in equation (21) to (22). The AutoRegressive eXogenous (ARX) is simple and the most efficient numerical technique, which can be utilized with high-order polynomial model. Similar to section 5.1, results of estimated parameters in this section demonstrate as function of input-frequency and time, as shown in Figure 6. It can be observed that estimation errors of poles and DC gains of surge-and yaw-motions using the ARX technique are small and unchanged. Accuracy of the ARX approach is much better than that of the LSM technique.

Figure 6. Parametric estimation errors using ARX as function of response time and varying input frequency.
5.3. Comparison of parametric estimation error between LSM and ARX
Sources of error from both LSM and ARX methods come from numerical time-integration of dynamic differential system. Computing pseudo inverse numerically in equation [18] is another source of error for LSM. In the ARX method, delayed time (nk) of discrete input structure and transforming discrete-time to continuous-time model contribute to estimation error of system lumped parameters of transfer function. Using LSM, estimation error decreases as integration time increases, because large input-output responses with appropriate frequency excitation can discover more accurate system parameters.

This section presents estimation of surge- and yaw-parameter error (using both LSM and ARX techniques). The accuracy comparison of parametric estimation error are summarized as minimum and maximum errors, as shown in table 2. The results show that the ARX approach yields much better accuracy of parameter estimation that of LSM for most lumped parameters.

Table 2. Maximum and minimum parametric estimation errors between reference parameters and estimated parameters from both system identification techniques

| Axis       | Parameter (unit)   | LSM         | ARX         |
|------------|-------------------|-------------|-------------|
|            |                   | Min error   | Max error   | Min error   | Max error   |
| Surge Motion | $\alpha_1$ ($l$/sec) | -1.879711   | -0.058874   | -0.988450   | 0.95789x10^{-11} |
|            | $\alpha_2$ ($l$/kg) | -1.114721   | -0.0211549  | 0.003565    | 0.0043563   |
|            | $\alpha_3$ (1/kg) | 0.007522    | 0.859333    | 0.005186    | 0.006000    |
|            | $\beta_1$ (1/sec) | -2.604681   | -1.083689   | 0.11237x10^6 | 0.11244x10^6 |
| Yaw Motion  | $\beta_2$ (1/kg · m²) | -0.508471   | 1.116051    | 0.694235    | 0.696756    |
|            | $\beta_3$ (1/kg · m²) | -0.773391   | 0.528940    | 0.710399    | 0.710399    |

6. Conclusion
This paper presents system identification (SI) methods: Least-Square Method (LSM) and AutoRegressive eXogenous (ARX), based on simplified dynamic USV model with two fixed input-thrusters input-thrusters ($u = [T_s, T_d]^T$). The appropriated frequency range is also investigated by using Bode diagrams in order to choose input frequency. Then, the numerical methods: 1) Least-square method (LSM) and 2) AutoRegressive eXogenous (ARX) techniques are implemented to estimate lumped parameters of the USV model. Results reveal that parametric estimation error from LSM can be improved with large response time data with proper input frequency excitation. On the other hand, the ARX approach provide faster parameter convergence to true parameter values using only short interval of input-output response time for this simplified dynamic system, since the accuracy of parameter estimation in discrete-time domain of ARX method is better than that of LSM.

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