Self-consistent simulation of an electron beam for a new autoresonant x-ray generator based on $TE_{102}$ rectangular mode

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Abstract. The space cyclotron autoresonance interaction of an electron beam with microwaves of $TE_{102}$ rectangular mode is simulated. It is shown that in these conditions the beam electrons can achieve energies which are sufficient to generate hard x-rays. The physical model consists of a rectangular cavity fed by a magnetron oscillator through a waveguide with a ferrite isolator, an iris window and a system of dc current coils which generates an axially symmetric magnetic field. The 3D magnetic field profile is that which maintains the electron beam in the space autoresonance regime. To simulate the beam dynamics, a full self-consistent electromagnetic particle-in-cell code is developed. It is shown that the injected 12keV electron beam of 0.5A current is accelerated to energy of 225keV at a distance of an order of 17cm by 2.45GHz standing microwave field with amplitude of 14kV/cm.

1. Introduction

X-ray radiation of traditional sources, where the electrons are accelerated by an electrostatic field, runs into the range of 50-150keV. The upper limit of the x-ray energy is determined by electric isolation problems. However, among the existing methods for producing x-rays, of great importance for our research are those that are based on the electron cyclotron phenomenon. A cyclic electron accelerator as an x-ray source is proposed in [1]. A compact x-ray source based on a mirror trap where electron heating is realized on the fundamental ECR frequency and its harmonics is described in [2]. Electron acceleration by microwaves of $TE_{111}$ mode in a magnetostatic field under the electron cyclotron resonance conditions is studied theoretically [3]. An ECR acceleration of electrons in $TE_{101}$ rectangular resonant cavity in the presence of a uniform magnetic field is simulated [4]. The results of this study make it possible to fabricate and test an x-ray source where the electrons are accelerated in spiral orbits in the cavity midplane [5].

It is worth noting that in the above mentioned accelerators the electron cyclotron frequency can be maintained equal to the microwave frequency within a very limited number of cyclotron periods, which severely constrains the maximum operating energy. Theoretically speaking, it is possible to circumvent the problem by increasing a homogeneous magnetic field at the time with a rate that compensates the relativistic electron mass rise. In such a manner the resonance can be maintained and large electron energies can be achieved [5,6]. Recently, we have analyzed the cyclotron resonance speeding of electrons in the combined inhomogeneous magnetostatic and standing microwave fields both through numerical experiment and analytical analysis [7–9].
The 3D inhomogeneous magnetic field profile is determined by the necessity for sustaining the ECR conditions along all electron trajectories. This type of electron cyclotron resonance is named spatial autoresonance acceleration (SARA). An X ray source based on this concept has been certified recently [10]. In the present paper, a physical scheme of an autoresonant accelerator system based on $TE_{102}$ rectangular cavity is presented. This device can be classified as compact due to its characteristic dimensions of about 30cm. The autoresonant interaction between 0.5A electron beam and 2.45GHz standing microwave field of 14kV/cm of amplitude is studied. Taking into consideration that the problem is relativistic, a full electromagnetic particle-in-cell (PIC) code [12] is developed.

2. Theoretical formalism
The physical scheme of the autoresonant accelerator which can produce x-rays is shown in Figure 1. A magnetron emits 2.45GHz microwaves into $TE_{102}$ rectangular cavity through a waveguide with a ferrite isolator and an iris window. The field amplitude in the cavity is $E_{0l} = 14kV/cm$. An axially symmetric magnetostatic field can be formed by a system of DC current coils or permanent magnetic rings. An electron gun injects the electron beam into the cavity along the magnetic field axis. The magnetic field parameters are those which maintain the electron beam in the space cyclotron autoresonance regime [7, 8]. To produce an x-ray flux, a molybdenum target is placed at the acceleration zone end. A linear polarized $TE_{102}$ electromagnetic field is excited in the cavity.

For the electron motion analysis, the Cartesian coordinates, whose origin is set in the center of the cavity lateral side (Figure 1(a)), are applied. The z-axis coincides with the cavity axis along which the electron beam is injected; the other axes are shown in Figure 4. In these coordinates, the width, height and length of the cavity dimensions are $L_x=7.74cm$, $L_y=3.82cm$ and $L_z=20cm$, respectively.

The magnetostatic field is generated by three axisymmetric coils using the same parameters given in [7]. This profile is shown in Figure 1(b). The self-consistent simulation of this system is studied by using the particle in cell method whose algorithm is presented in Figure 2 through a computational time cycle which involves the following steps:

(i) calculation of the current densities in the mesh points on the basis of the superparticle (SP) position and velocity data,
Computation of the fields on the mesh points,
(iii) calculation of the forces exerted on the SPs,
(iv) calculation of new positions and velocities of the SPs through integration of their equations of motion.

In our simulations, the current density is found using the charge conservative method proposed by Umeda [11] and the beam electric and magnetic fields on the mesh points are found using the Yee method [13]. It is worth noting that the $TE_{10}$ microwaves that propagate along the waveguide and are incident on the iris window excite in the cavity a linearly polarized $TE_{102}$ mode which is calculated using the method described in [14]. The electric and magnetic field on the SP positions, $\vec{E}_P$ and $\vec{B}_P$, are calculated by a trilinear interpolation of the $\vec{E}$ and $\vec{B}$ mesh node data. To obtain the total magnetic field, fields produced by the current coils and by the beam are summed up. The SP dynamics is described by the relativistic Newton-Lorentz equation which is solved numerically for all the SPs using the Boris leapfrog procedure [12]. In order to accelerate the calculation process and reach faster the microwave steady state regime in the cavity, the Hanning method is applied [15]. To avoid non-physical actions of the microwaves, we use the Uniaxial Perfectly Matched layer (UPML) technique [16]. The phase shift between the particle-velocity vector and the right-hand electric component of the microwave field, represented by $\varphi$, is calculated indirectly beginning from the phase of $TE_{102}$ electric field component. The simulation time step $\Delta t$ is taken according to the Courant stability condition [16].

3. Results and discussion
The numerical simulations on a spatial autoresonance interaction of an electron beam and microwaves are realized under the above-mentioned physical parameters. The electron beam of 0.5A is injected at the initial energy of 12keV through an orifice of 0.2cm into the cavity along its axis which taken as z-axis. The total length of 32.34cm, for which the simulations are fulfilled, is represented by the waveguide and the cavity. To excite a 14kV/cm field in the cavity, an input power is to be equal to 2.4MW in a continuous mode. Such a high level of the microwave power makes the system operate in a pulse regime. For example, the 2.4MW pulses of 1$\mu$s duration at 25Hz repetition frequency results in an acceptable consumed power of 60W. The electrons beam also needs to be injected in a pulsed regime at the same frequency. In these conditions the beam length will be of the order of 0.5m.

To execute the simulations during an appropriate time lapse the beam electrons which are injected during a microwave period are divided into $8.49 \times 10^3$ superparticles. The calculations are fulfilled on a rectangular mesh of $\Delta x = \Delta y = \Delta z = 0.07 cm$ and using a time step...
$\Delta t = 1.315 \text{ ps}$. The simulations are considered finished when the electrons impinge on a non-magnetic metal target whose longitudinal position is $z=17.5\text{cm}$. The dimensions of the target are $LT_x = 0.28\text{cm}$, $LT_y = 1.2\text{cm}$ and $LT_z = 1.0\text{cm}$.

The steady electric field distribution in the $y = 0$ cavity plane without the electron beam is shown in Figure 3(a). One can see the $TE_{102}$ electric field image with distortions in the vicinity of the iris microwave aperture (the black line) and above the metal target (yellow spot) whose $z$-coordinate is 17.5cm. The first one is irrelevant because the beam does not appear there. Figure 3(b) evidences that the electric field strength is increased significantly at the target surface. But this distorted electric field does not prevent the beam from bombarding the target because it pointed perpendicularly to the target surface.

![Figure 3](image)

**Figure 3.** Microwave electric field before the electron beam injection: (a) in the longitudinal $y = 0$ plane and (b) in the transverse $z = 17.5\text{cm}$ plane.

Figure 4 shows some snapshot of electrons beam evolution (red dots) from the injection point up to an impact on the target. Close to the injection point, the electrons move along an almost straight line due to low magnitudes of the microwave field in this region (see Figure 4(a)). In the longitudinal magnetostatic field the electrons move in a counterclockwise direction around the magnetic field axis. In Figure 4(b), this motion is indicated by the black arrows. In the position $z=10\text{cm}$ the microwave field changes its phase to $\pi$. However, after some microwave periods the electrons regain their acceleration phases because of high frequency of the microwaves. Figure 4(d) shows the moment of the impact of electrons on the target.

![Figure 4](image)

**Figure 4.** Some snapshot of electrons beam evolution simulation from its injection up to impact with the target.
Figure 5 shows that the self-consistent electric field on the beam surface field in the plane \( z = 10 \text{cm} \) is as high as 1.6kV/cm and thus cannot be ignored although this value remains significantly smaller than the microwave amplitude of 14kV/cm.

![Figure 5. Transversal self consistent electric field strength in \( z = 10 \text{cm} \) plane.](image)

The evolution of the phase shift between the particle velocity and the circular right-hand electric component of microwaves along the beam trajectory is shown in Figure 6(a). One can see that the electrons are injected with arbitrary phases with respect to the microwave field phase. Then, in the process of the interaction with the microwave field some electrons are accelerated, with some of them losing their energy, which results in a phase-focalization effect. The phase-focalization process is completed after 3cm run when the phase shift is observed practically equal to the exact cyclotron resonance value of \( \pi \).

![Figure 6. Phase shift between the right-hand circular polarized component of the electric microwave field and the transversal velocity of electrons (a) and the evolution of the beam energy (b) along the beam trajectory.](image)

The rate of beam energy change as the beam moves into the increasing magnetostatic field is presented in Figure 6(b). The energy stop rising in the zone of \( z = 10 \text{cm} \) is due to the change of the microwave polarization vector to the opposite sense. In spite of the polarization direction jump all the electrons remain in the acceleration band because the electric field in this zone is very low at a local decrease of the magnetostatic field (see Figure 1). The latter provokes reduction of the cyclotron frequency magnitude. The evolutions of the transversal and longitudinal velocity components are shown in Figure 7.

One can see that in the zone 12cm<\( z < 14 \text{cm} \) the longitudinal electron velocities have their minimum values that are accompanied by an increase of the local density that causes rise of the self-consistent electric field and the dispersion of the energy in this zone (see Figure 6). Finally,
the electrons impact on the target at a position of \( z = 17 \text{cm} \) with an energy of 225keV. In this position, the longitudinal velocity is so small that during the period of the cyclotron rotation the beam displaces in the longitudinal direction at a distance of 4-6mm which is smaller than the dimension of the target along \( z \) axis whose area is \( 1.2 \times 1.0 \text{cm}^2 \). The transversal velocity and energy attain their maxima magnitudes in the impact position that guarantee the production of hard 170keV.

Figure 7. Evolution of the longitudinal (red dots) and transversal (blue dots) components of electron velocities along the beam trajectory.

4. Conclusions
The realized numerical experiment shows that the studied system based on a \( TE_{102} \) rectangular cavity can create the conditions for space cyclotron autoresonance acceleration of electron beams of order of 0.5A to energies of hundred electron-volts which can be used for an x-ray production. For the simulations, the full 3D electromagnetic software is developed. To enhance the effectiveness of the studied system in reference to a microwave power harnessing, an optimization of waveguide-cavity coupling system is scheduled.

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