PARTIAL DYNAMICAL SYMMETRY IN A FERMIONIC MANY-BODY SYSTEM

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The concept of partial symmetry is introduced for an interacting fermion system. The associated Hamiltonians are shown to be closely related to a realistic nuclear quadrupole-quadrupole interaction. An application to $^{12}$C is presented.

1 Introduction

The fundamental concept underlying algebraic theories in quantum physics is that of an exact or dynamical symmetry. Realistic quantum systems, however, often require the associated symmetry to be broken in order to allow for a proper description of some observed basic features. Partial dynamical symmetry (PDS) describes an intermediate situation in which some eigenstates exhibit a symmetry which the associated Hamiltonian does not share. The objective of this approach is to remove undesired constraints from the theory while preserving the useful aspects of a dynamical symmetry, such as solvability, for a subset of eigenstates. Here we present an example of a PDS in an interacting fermion system. In the symplectic shell model of nuclei, we introduce PDS Hamiltonians which are closely related to the nuclear quadrupole-quadrupole interaction. An application to $^{12}$C is discussed.

2 PDS Hamiltonians and quadrupole-quadrupole interaction

The quadrupole-quadrupole interaction is an important ingredient in models that aim at reproducing quadrupole collective properties of nuclei. A model which is able to fully accommodate the action of the collective quadrupole operator, $Q_{2m} = \sqrt{16\pi/5} \sum_s r_s^2 Y_{2m}(\hat{r}_s)$, is the symplectic shell model (SSM), an algebraic scheme which respects the Pauli principle. In the SSM, this operator takes the form $Q_{2m} = \sqrt{3}(\hat{C}_{2m}^{(11)} + \hat{A}_{2m}^{(20)} + \hat{B}_{2m}^{(02)})$, where $\hat{A}_{lm}^{(20)}$, $\hat{B}_{lm}^{(02)}$, and $\hat{C}_{lm}^{(11)}$ are symplectic generators with good SU(3) $^\text{superscript} (\lambda, \mu)$ and SO(3) $^\text{subscript} l, m$ tensorial properties. The $\hat{A}_{lm}^{(20)}$, $\hat{B}_{lm}^{(02)}$, $l = 0$ or 2, create (annihilate) $2\hbar \omega$ excitations in the system. The $\hat{C}_{lm}^{(11)}$, $l = 1$ or 2, generate a SU(3) subgroup and act only within one harmonic oscillator (h.o.) shell ($\sqrt{3}\hat{C}_{2m}^{(11)} = Q_E^{E}$, the quadrupole operator of Elliott, which does not couple
different h.o. shells\(\hat{C}_{1m}^{(11)} = \hat{L}_m\), the angular momentum operator). The symplectic basis is generated by repeated application of \(\hat{A}^{(20)}\) to a 0\(\hbar\omega\) shell model configuration, labeled by its Elliott quantum numbers \((\lambda_\sigma, \mu_\sigma)\). The resulting \(N\hbar\omega\) excited states \((N=0,2,\ldots)\) are coupled to good SU(3)⊃SO(3) symmetry \((\lambda, \mu)\), where \(\kappa\) enumerates multiple occurrences of \(L\) in the SU(3) irrep \((\lambda, \mu)\). This labeling scheme defines a dynamical symmetry basis.

The quadrupole-quadrupole interaction connects h.o. states differing in energy by \(\pm 2\hbar\omega\), and \(\pm 4\hbar\omega\), and may be written as

\[
Q_2 \cdot Q_2 = 9\hat{C}_{SU3} - 3\hat{C}_{Sp6} + \hat{H}_0^2 - 2\hat{L}_0 - 3\hat{L}_2 - 6\hat{A}_0\hat{B}_0 + \{\text{terms coupling different h.o. shells}\},
\]

where \(\hat{C}_{SU3}\) and \(\hat{C}_{Sp6}\) are Casimir invariants of SU(3) and Sp(6,R). These operators, as well as \(\hat{H}_0\) and \(\hat{L}_2\), are diagonal in the dynamical symmetry basis. Unlike the Elliott quadrupole-quadrupole interaction, \(Q_2 \cdot Q_2\) breaks SU(3) symmetry within each h.o. shell since \(\hat{A}_0\hat{B}_0\) mixes different SU(3) irreps.

To study the action of \(Q_2 \cdot Q_2\) within such a shell, we consider the Hamiltonians

\[
H(\beta_0, \beta_2) = \beta_0 \hat{A}_0\hat{B}_0 + \beta_2 \hat{A}_2 \cdot \hat{B}_2
\]

\[
= \frac{\beta_2}{18}(9\hat{C}_{SU3} - 9\hat{C}_{Sp6} + 3\hat{H}_0^2 - 36\hat{H}_0) + (\beta_0 - \beta_2)\hat{A}_0\hat{B}_0.
\]

For \(\beta_0 = \beta_2\), one recovers the dynamical symmetry, and for \(\beta_0 = 12, \beta_2 = 18\), one obtains \(Q_2 \cdot Q_2 = H(\beta_0 = 12, \beta_2 = 18) + \text{const}(N) - 3\hat{L}_2 + \text{terms coupling different shells},\) where \(\text{const}(N)\) is constant for a given h.o. \(N\hbar\omega\) excitation.

For general \(\beta_0 \neq \beta_2\), \(H(\beta_0, \beta_2)\) exhibits partial SU(3) symmetry: The Hamiltonian is not SU(3) invariant, yet it possesses a subset of 'special' states which respect the symmetry: All \(0\hbar\omega\) states are unmixed and span the entire \((\lambda_\sigma, \mu_\sigma)\) irrep. Moreover, among the excited configurations \((N > 0)\), one finds additional states with good SU(3) symmetry. Unlike the \(0\hbar\omega\) states, however, they span only part of the corresponding SU(3) irreps. There are other states at each excited level which do not preserve the symmetry and therefore contain a mixture of irreps. The partial SU(3) symmetry of \(H(\beta_0, \beta_2)\) is converted into partial dynamical symmetry by adding to it SO(3) rotation terms which lead to \(L(L+1)\)-type splitting but do not affect the wave functions. The solvable states then form rotational bands and since their wave functions are known, one can evaluate their energies and the E2 rates between them analytically\(\dagger\).

3 Application to \(^{12}\text{C}\)

To illustrate that the PDS Hamiltonians introduced here are physically relevant, we present an application to \(^{12}\text{C}\). In Fig. 1, we compare the energy...
Figure 1. Energy spectra for $^{12}$C. $K=0$ indicates the ground band in all three parts of the figure. In addition, resonance bands dominated by $2\hbar\omega\ (K=2_1, 0_2, 1_1, 0_3)$, $4\hbar\omega\ (K=4_1)$, and $6\hbar\omega\ excitation\ (K=6_1)$ are shown for the two calculations. The angular momenta of the states in the rotational bands are $L=0, 2, 4, \ldots$ for $K=0$ and $L=K, K+1, K+2, \ldots$ otherwise.

spectra of $H_{PDS} = h(N) + \xi H_0(\beta_0 = 12, \beta_2 = 18) + \gamma_2 \hat{L}^2 + \gamma_4 \hat{L}^4$ and $H_{Q\cdot Q} = H_0 - \chi Q_2 \cdot Q_2 + d_2 \hat{L}^2 + d_4 \hat{L}^4$, where $h(N)$ is constant for a given $N\hbar\omega$ excitation. $H_{PDS}$ has families of pure SU(3) eigenstates which can be organized into rotational bands; they are indicated in the figure.

Although the PDS Hamiltonian cannot account for intershell correlations, it is able to reproduce various features of the quadrupole-quadrupole interaction, as can be seen in Fig. 2 where the structure of selected PDS eigenstates is compared to that of the corresponding $Q_2 \cdot Q_2$ eigenstates: PDS eigenfunctions do not contain admixtures from different $N\hbar\omega$ configurations, but belong entirely to one level of excitation. We find that, for reasonable interaction parameters, the $N\hbar\omega$ level to which a particular PDS band belongs is also dominant in the corresponding band of exact $Q_2 \cdot Q_2$ eigenstates. Moreover, within this dominant excitation, eigenstates of both Hamiltonians have similar SU(3) distributions. Structural differences, nevertheless, do arise and are reflected in the very sensitive interband transition rates. Overall, however, we may conclude that PDS eigenstates approximately reproduce the structure of the exact $Q_2 \cdot Q_2$ eigenstates, for both ground and the resonance bands.

4 Summary

The notion of partial dynamical symmetries extends the familiar concepts of exact and dynamical symmetries. It is applicable when a subset of states exhibit a symmetry which does not arise from invariance properties of the relevant Hamiltonian. Recent studies, including the one presented here, show that partial symmetries are indeed realized in various quantum systems.
Figure 2. Decompositions for calculated $L^\pi = 2^+$ states of $^{12}$C. Individual contributions from the relevant SU(3) irreps at the $0\hbar\omega$ and $2\hbar\omega$ levels are shown for both the $8\hbar\omega Q_2^\ast Q_2$ calculation and the PDS calculation. In addition, the total strengths contributed by the $N\hbar\omega$ excitations for $N > 2$ are given for the $Q_2^\ast Q_2$ case.

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References

1. A. Leviatan, Phys. Rev. Lett. 77, 818 (1996); A. Leviatan and I. Sinai, Phys. Rev. C 60, 61301 (1999); A. Leviatan and J.N. Ginocchio, Phys. Rev. C 61, 24305 (2000).
2. J. Escher and A. Leviatan, Phys. Rev. Lett. 84, 1866 (2000).
3. G. Rosensteel and D.J. Rowe, Phys. Rev. Lett. 38, 10 (1977); Ann. Phys. 126, 343 (1980); D.J. Rowe, Rep. Prog. Phys. 48, 1419 (1985).
4. J.P. Elliott, Proc. Roy. Soc. A 245, 128 (1958); 245, 562 (1958).
5. J. Escher and A. Leviatan, to be submitted.