The $\tau$ forward-backward asymmetry within Grand Unified Theories

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Abstract

We will study some possible non universal corrections that could affect single measurements of the Weinberg angle $\sin^2 \theta_W$ (leaving the full average essentially unchanged). We will concentrate on the class of the models that introduces a new light gauge boson and an extended Higgs sector, but we restrict the Higgs charges choice in order to automatically achieve zero mixing at the tree level between the $Z_0$ and the $Z'$. This will more naturally keep small the universal effects with a relatively light $Z'$, but it will imply only vector-like coupling between this boson and the standard fermions. A SO(10) gauge boson which distinguishes between families can have such properties and could explain a deviation in the $A^\tau_{FB}$ without affecting the $\tau$ polarization measurements at LEP.

1 Introduction

The analysis of the electroweak physics at the $Z_0$ peak has confirmed the agreement between the Standard Model (SM) and the experiments beyond a trivial level of accuracy [1]. One loop pure electroweak corrections, involving the top quark and the Higgs boson sector have been tested. The top mass measured by CDF/D0 strengthen this success. The experimental result for the Weinberg angle is obtained taking the average of all the available precise measurements. The global average between very different type of measurements (both hadronic and leptonic) is justified by the fact that any new physics [2] contributing to the vacuum polarization of the weak bosons (the most likely place where to look for new effects) will affect in a universal way all the precise measurements [1]. Nevertheless this does not exclude the possibility of one

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single measurement being affected while leaving the whole average essentially unchanged.

In the following we discuss how an additional gauge boson [3] could change only one (or some) of the forward-backward asymmetries and we try to embed it in a Grand Unified scenario.

2 Electroweak precise measurements: the $\tau$ forward-backward asymmetry

The forward-backward asymmetry of the $\tau$ lepton deviates from the Standard Model of about $2 \sigma$. On the contrary the $\tau$ polarization measurements agree with the SM; an anomalous coupling between the $Z_0$ and the $\tau$ cannot explain their discrepancy. But if we add to the SM scattering amplitude

$$e^+ + e^- \rightarrow Z_0 \rightarrow \tau^+ + \tau^-$$

(1)

a new contribution

$$e^+ + e^- \rightarrow X \rightarrow \tau^+ + \tau^-$$

(2)

which is imaginary and thus it interferes with the SM amplitude induced by the $Z_0$ production, we could affect the $A_{FB}^\tau$ keeping negligible effects to all the other precise measurements. For instance, let us assume that this $X$ is a new $Z'$. In a physically interesting class of models described in the next section the mixing between the $Z_0$ and the $Z'$ can be very small, so we will consider it to be negligible. In these models the coupling of this new boson with the standard fermions can only be vector-like (see next section). We fix the interaction between the $\tau$ and the boson to be

$$g_\tau Z'_\mu \bar{\tau} \gamma^\mu \tau.$$  

(3)

This will define the coupling constant $g_\tau$. Analogously we define $g_e, g_\mu, \ldots$ the couplings with the electron, the muon etc. This $Z'$ will add to the above scattering amplitude a term ($p^2 = M_Z^2$)

$$g_e g_\tau \bar{e} \gamma^\mu \ell_\mu \frac{1}{M_Z^2 - M_{Z'}^2 + iM_Z \Gamma_{Z'}}$$

(4)

whose imaginary part is ($\Gamma_{Z'} << M_Z - M_{Z'}$)

$$g_e g_\tau \bar{e} \gamma^\mu \ell_\mu \tau \times \frac{iM_Z \Gamma_{Z'}}{(M_Z^2 - M_{Z'})^2} = i g_e g_\tau \bar{e} \gamma^\mu \ell_\mu \tau \times \frac{C}{M_Z^2}$$

(5)

\footnote{The effects of the mixing of a $Z'$ with a $Z_0$ are extensively discussed in the literature.}
Table 1
Effects to the electroweak asymmetries.

| $\delta A^\tau_{FB}$ | $\delta A^\tau_{LR}^{slac}$ | $\delta A^\tau_{pol}$ | $\delta A^\tau_{pol}$ |
|----------------------|-----------------------------|------------------------|------------------------|
| $1.25 \, g_e \, g_\tau \, C$ | $0.$ | $0.12 \, g_e \, g_\tau \, C$ | $0.12 \, g_e \, g_\tau \, C$ |

where for simplicity we have called $C$ the factor that includes the mass and the width of the new boson. This imaginary part will affect the precise measurements of the $\tau$ lepton. Computing the interference between the above matrix element (5) and the SM amplitude (1) we obtain that the correction to the total cross section $\tau^+ \tau^-$ is proportional to the product of the vector couplings $g_V^e$ and $g_V^\tau$ of the $Z_0$ with the electron and the $\tau$. These parameters are small and the effect is suppressed and negligible. Analogously the polarization measurement of the $\tau$ gets corrections which are proportional to $g_V^e$ or $g_V^\tau$, and thus are very small. On the contrary the forward backward asymmetry of the $\tau$ gets a correction which is proportional to $g_A^e$ and $g_A^\tau$ and therefore is numerically sizable. This argument is true only because we have assumed our new gauge boson to be vector-like (hereafter we will call vector-like a gauge boson with purely vector-like interaction with the standard fermions, see also the next section). In table (1) we show how the measurements of the Weinberg angle involving the $\tau$ lepton are affected by the interference between the amplitude (5) and the SM matrix element (1).

We have seen that an imaginary amplitude such as in (5) can only affect the $A^\tau_{FB}$; now we investigate in some details the specific case of a Grand Unified boson as described in the last section. This boson (see last section) is coupled at tree level only with the third generation of fermions. If the coupling with the $\tau$ is $g_\tau = g$ then from the table (3) we have $g_b = g_t = -1/3g$ and $g_\nu_\tau = g$. Even if the tree level coupling of this boson with the electron is zero, at one loop a small coupling can arise. For instance the two-point Green function $Z' - \gamma$ is generated by the one loop fermionic corrections. The imaginary part of this function gives an amplitude similar to the (5)

$$e_\gamma g \bar{e}_\mu \gamma^\mu \tau \times \frac{i A_\gamma Z'}{M_Z^2(M_Z^2 - M_{Z'}^2)}$$

where $e_\gamma$ is the electromagnetic coupling, $1/M_Z^2$ and $1/(M_Z^2 - M_{Z'}^2)$ come from the photon and the $Z'$ propagators; to compute the constant $A_\gamma Z'$ we have to evaluate the imaginary part of the fermionic loop which mixes the photon and the $Z'$ propagators. The only fermions contributing to the imaginary part in this loop are the bottom and the $\tau$ since the top quark is above the threshold. The couplings have been given previously. The only measurement affected would be the $A^\tau_{FB}$; the correction can be computed from table (1) replacing

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3 A small electron coupling could also arise through a two-point function $Z' - Z''$ where $Z''$ is a vector-like boson much heavier than the $Z'$ and coupled to the electron.
Table 2
The $\tau$ forward-backward asymmetry compared with the SM value.

| EXP.   | DATA     | LEP AVERAGE | STANDARD MODEL |
|--------|----------|-------------|----------------|
| ALEPH  | 0.0196 ± 0.0028 | 0.0201 ± 0.0018 | 0.0160          |
| DELPHI | 0.0223 ± 0.0039 | 0.0201 ± 0.0018 | 0.0160          |
| L3     | 0.0233 ± 0.0049 | 0.0201 ± 0.0018 | 0.0160          |
| OPAL   | 0.0178 ± 0.0034 | 0.0201 ± 0.0018 | 0.0160          |

the factor $g_eC$ in (3) with the proper one in (3); calculating the loop $A_{\gamma Z'}$ we obtain

$$\delta A_{\tau}^{FB} = 0.0002 \times \frac{g^2}{e^2} \times \frac{M_Z^2}{M_Z^2 - M_{Z'}^2}$$

(7)

This could be sizable if the $Z'$ is not too far from the $Z_0$ (the limits for such a boson essentially coupled only to the third family and decaying mainly in $\tau$ or invisible/dark matter are weak). All the four LEP experiments give an excess in the $A_{\tau}^{FB}$ (see table (2)), but the average is only $2\sigma$ away from the SM. Before concluding this section, it is worthwhile to note again that, differently from the $\gamma - Z'$, a $Z_0 - Z'$ mixing cannot explain the discrepancy between $A_{\tau}^{FB}$ and $A^{FB}$: the effect would be the same in both measurements, since it could be reabsorbed into a redefinition of the $\tau$ coupling with the $Z_0$; to compute such a correction we should evaluate the real part of the fermionic loop, which depends on the spectrum (and its several free parameters) of the theory above the $Z_0$ mass (remembering that the $Z_0$ is purely imaginary and the photon is real at the $Z_0$ peak, the phase of the fermionic loop must be chosen accordingly).

3 Adding a new gauge boson

Within the SM the $Z_0$ and $W$ mass are related at tree level by a well known relation

$$M_Z^2 = M_W^2 \frac{g_1^2 + g_2^2}{g_2^2}.$$  

(8)

This give a prediction of the $W$ mass once the $Z_0$ mass and the weak coupling are known, if we use a minimal representation of Higgs sector: i.e. one (or more) electroweak doublet.

If we add an extra massive gauge boson to our model the Higgs sector must be enlarged, since the longitudinal polarization of the new massive boson must
come from the would-be goldstone boson of an additional scalar representation. A new Higgs must acquire a vev, which must carry a non zero charge with respect the new gauge boson. We briefly discuss two possible scenarios:

3.1 non minimal low energy Higgs sector

at least one vev carries charges both of SU(2)×U(1) and U′(1). As a consequence this vev will introduce a mixing between the Z₀ and the Z′ proportional to

\[ v^2 Z_0 Z'. \]  

(9)

The physical mixing between the Z₀ and the Z′ can be small if we add another vev \( v' \gg v, v_{\text{SM}} \) neutral with respect the SM gauge generators but with a \( Z' \) charge: the new boson becomes very heavy and the mixing negligible. In this case the relation (8) is a consequence of a particular choice of the Higgs potential, where two Higgs acquire vev’s of different order of magnitude. Since these two fields share the U(1) interaction, one has probably to introduce a mechanism to explain this hierarchy.

3.2 minimal low energy Higgs sector: a vector-like gauge boson

Another possible scenario arises if we just add a minimal extension of the Higgs sector of the low energy effective lagrangian. In such a case we add only one Higgs scalar which is a SM singlet and coupled only to the \( Z' \), while the usual standard model Higgs doublet is not coupled to the \( Z' \). Only this charge choice of the scalars guarantees that no mixing arises at tree level between the \( Z_0 \) and the \( Z' \). As for the SM the tree level relation (8) is simply a consequence of the Higgs charge choice, regardless the shape of the Higgs potential and its arbitrary parameters. Differently from the first case, now the new gauge boson can be more easely much closer to the weak scale, since no assumption on the Higgs potential and no hierarchy is required.

The models with a minimal extension of the Higgs sector of the low energy effective lagrangian have an important property in addition to the tree level equation (8). The additional gauge boson can only have a vector-like interaction with the dirac fermions of the standard model: each standard fermion (having the standard charge assignments with respect SU(2)×U(1)) can get mass only through the SU(2) Higgs doublet. Remembering that this does not carry \( Z' \) charge (see above), a gauge invariant Yukawa interaction between the dirac fermion and the Higgs doublet is allowed only if the left and right
handed component of the dirac fermion transform with the same charge with respect $U'(1)$ gauge transformation. In other words the coupling between this $U'(1)$ and the standard fermions is vector-like.

In this work we have concentrated on the latter class of minimal models where the $(8)$ is a direct consequence of the Higgs representation choice, regardless the Higgs potential shape of the low energy effective lagrangian; in the last section we will comment on the consequences of the embedding such a gauge boson in a Grand Unified model.

4 Grand Unified Theories

All the discovered gauge bosons have couplings with the fermions which fit in a very nice and simple way with a SU(5) Grand Unified minimal scenario [4]. Therefore a new $Z'$ that can be easily accommodated in the above picture is certainly welcome. The experimental constraints for (light) Grand Unified extra $U(1)$ are in general very compelling. If they have a tree level coupling with all the existent fermions then strong constraints come from CDF/UA2: its direct production would certainly have been detected in the leptonic decay modes. But if its leptonic coupling is suppressed and/or the light quark sector is decoupled, this gauge boson could be very light, even close to the weak scale, probably escaping all the direct searches.

In the following we will discuss how such a gauge boson could be accommodated in a Grand Unified scenario.

4.1 A GUT gauge boson

If both the left handed up quark and the charge conjugated of the right handed up quark belong to the same irreducible representation (of a Grand unified group) then a vector-like gauge boson has opposite couplings with the above states of the same representation: its generator cannot commute for instance with a SU(5) embedding the SM. Equally, if we require leptophobia the gauge boson cannot commute with this group since the electron components (left and right handed) and the quarks are contained in the same irreducible representations of the unified group. Conversely the generator of this gauge boson must commute with SU(3) $\times$ SU(2) $\times$ U(1): it cannot carry color (one of the reasons is because we want it to have a small but non zero coupling with the electron, for colored gauge bosons see after), we do not want it to carry SU(2) $\times$ U(1) charges, it must not have neither electromagnetic charge nor weak isospin charge, otherwise it would be or a charged current or at least a
member of a SU(2) multiplet with a charged boson (the splitting within this
SU(2) multiplet would be bounded by the electroweak precise measurements,
e.g. $\varepsilon_1$). Since the generator of the new boson commutes with SU(2), the left-
headed components of the top and bottom quarks must have the same charge
$g_q$. From the vector-like nature of the new boson we obtain that also their
righthanded components have charge $g_q$. For the same reasons all the leptons
have the same charge $g_l$. The anomaly cancellation (more precisely the square
of the hypercharge times the new generator) will force us to fix the charges
as in table (3) (a righthanded neutrino is also required). This charge choice
corresponds to a linear combination of the two $U(1) \times U(1)'$ in SO(10). It
is manifest that this SO(10) cannot be the standard one containing the SM
gauge bosons, otherwise this new boson would be coupled to the electron, and
certainly (if not too heavy) it would have been seen at CDF/D0. Therefore it
has to belong to an additional SO(10)$_{NEW}$: the standard SO(10) is coupled to
all the three fermion families while the additional SO(10)$_{NEW}$ is not coupled
with the two lightest families.

For example, one could be attracted by the idea that some family generation
of gauge bosons could exist as well as the generations of fermion families.

If the existence of fermion family copies is not understood in the context of a
simple Grand Unified model one could suspect that the answer to this problem
is beyond the unification scale and the mechanism (generating the families)
could not distinguish between fermion and gauge boson but could simply give
some copies of the same gauge lagrangian

$$L = L_1 + L_2 + L_3 + ...$$

1, 2, 3, ... = indices of generations  \(10\)

The gauge bosons would have a family index exactly as the fermions have.
To reduce the number of non-abelian groups to the SM SU(3) \(\times\) SU(2), we
need representations obtained from the tensorial products: for instance, any
Higgs field $\phi^{\alpha, \beta}$ belonging to the $(8_i, 8_j)$ of SU(3)$_i \times$ SU(3)$_j$
could acquire a
vev $\phi^{\alpha, \beta} = v \delta^{\alpha, \beta}_{\text{kron}}$. This will break SU(3)$_i \times$ SU(3)$_j \Rightarrow$ SU(3)$_{i+j}$. The same
breaking can occur for the SU(2).

After all the non abelian groups are broken into the SM ones, the low energy
model would have the SM gauge bosons but in addition one could have an
abelian gauge boson (remained unbroken at the unification scale) which is
coupled only to the third (and to an extra heavy vectorial family, see below). We
would have three generation of fermions at the weak scale plus another heavier
vectorial family in order to satisfy the orthogonality condition $\text{Tr} T'_{SM} T' = 0$,
with $T'_{SM}$ any SM gauge boson generator and $T'$ the boson in table (3). Other
light dark generations not coupled with the standard boson but coupled with
the $Z'$ obviously cannot be ruled out. The overall normalization of the effective
low energy coupling in table (3) is a free parameter, and it also depends on the
Table 3
Couplings of the vector-like gauge boson to the 16 of SO(10)\textsubscript{NEW}.

|   | t | b | \( \tau \) | \( \nu_e \) |
|---|---|---|---|---|
|   | \(-\frac{1}{3}\) | \(-\frac{1}{3}\) | 1 | 1 |

total number of families coupled to the boson and the details of the symmetry breaking at the GUT scale. Finally, we point out that its vector-like nature will easily allow an electroweak breaking which automatically preserves the tree level relation between the \(Z_0\) and the \(W^\pm\) masses. It is clear from table (3) that it is possible to give mass to the third generation with a Higgs neutral under the \(Z'\), in a GUT scenario it could be the doublet in the 10 of SO(10), whose charge with respect the generator of table (3) is zero. On the contrary the \(Z'\) can get mass from the SM singlet in the 16 of SO(10)\textsubscript{NEW}. No Higgs that is coupled both to the \(Z_0\) and the \(Z'\) acquire a vev and the mixing between the two gauge bosons is zero at the tree level. At one loop a small mixing can arise between the two gauge bosons. Similarly, the radiative corrections could introduce a higher order operator

\[
H_2H_0b_Ls_R
\]

that can explain a small mixing between the third generation and the second one. Since both the strange quark \(s_R\) and the Higgs doublet \(H_2\) are neutral under \(Z'\), the additional Higgs vev \(H_0\) is necessary to build a gauge invariant mixing in the (11). It carries only a \(Z'\) charge and this will avoid a tree level \(Z_0 - Z'\) mixing.

5 Conclusions

After the success of the precision tests of the universal corrections to the observables at the \(Z_0\) peak, we have tried to look for possible signals of new physics in single measurements of the \(\sin^2\theta_w\). Given that the electroweak relation

\[
M_Z^2 = \frac{g_1^2 + g_2^2}{g_2^2}M_W^2
\]

is remarkably true, we have studied the special class of models in which, as for the SM, the above relation is a direct consequence of the Higgs representation choice, with no assumption on the free parameters of the Higgs potential. These models demand that the new \(Z'\) has a vectorial interaction with the standard fermions. Thus deviations in the forward-backward asymmetries could signal the existence of such bosons.
We have also discussed this $Z'$ in the context of Grand Unified Theories. If we demand that the $Z'$ has a suppressed coupling with the electron and its tree level mixing with the $Z_0$ is automatically and naturally zero (see above), we are practically forced to introduce a new generation of gauge bosons.

This could belong to the adjoint of SO(10). This group contains a vector-like gauge boson, which is a linear combination of the $U(1) \times U'(1)$ commutating with $SU(3) \times SU(2)$. If this gauge boson is coupled with only the third generation of fermions it could escape all the direct searches even if it is light; it will not mix with the $Z_0$, but it could have a residual and interesting effect to the $\tau$ forward-backward asymmetry (it will also give a very small shift to the bottom forward-backward asymmetry towards the global average). At present, all the four LEP experiment give an excess in the $A_{FB}^\tau$, but the average is still at the level of only 2$\sigma$. Since the technique to extract this data is different from the $A_{FB}^\mu$ and $A_{FB}^e$, one could also suspect that the systematic error in this measurement is underestimated. Before concluding, we also point out that the existence of new generations of gauge bosons implies additional massive gluons which could have an interesting role in explaining the CDF excess in the jet high $p_t$ distribution [5].

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