BRANS-DICKE THEORY AND PBH IN EARLY MATTER-DOMINATED ERA

B.Nayak\textsuperscript{1} and L.P.Singh\textsuperscript{2}
Department of Physics, Utkal University, Bhubaneswar-751004, India
\textsuperscript{1} bibeka@iopb.res.in
\textsuperscript{2} lambodar\_uu@yahoo.co.in

Abstract

We have shown that PBHs can be formed in the matter-dominated era if gravity is described by Brans-Dicke theory. Considering an early matter-dominated era in between inflation and reheating, we found that the PBHs which are formed during this era lives longer than the standard case where PBHs are formed in early radiation-dominated era. Thus in comparison with standard scenario more number of PBHs are existing today. Again the constraint on PBH formation becomes stronger than the standard case indicating a significant enhancement in the formation of PBH during the matter-dominated era.

PACS numbers : 98.80.-k, 97.60.Lf
Key words : Brans-Dicke theory, primordial black holes, early matter-dominated era, density fluctuation.

1 Introduction

Assuming Gravitational constant($G$) as a time independent quantity Einstein proposed General Theory of Relativity(GTR)[1] in 1916. As the extensions of GTR, many scalar tensor theories have been developed by considering $G$ as a time dependent quantity. Among them Brans-Dicke(BD) theory [2] is the simplest one. In BD theory the gravitational constant is set by the inverse of a time dependent scalar field which couples to gravity with coupling parameter $\omega$. GTR can be recovered from BD theory in the limit $\omega \rightarrow \infty$ [3]. BD theory has an important property that it gives simple expanding solutions[4] for scalar field $\phi(t)$ and scale factor $a(t)$ which
are compatible with solar system observations[5]. Again BD theory is successful in explaining many cosmological phenomena such as inflation[6], early and late time behaviour of the Universe[7], cosmic acceleration and structure formation[8], cosmic acceleration and coincidence problem[9] etc.

It was first predicted by Zeldovich and Novikov[10] in 1967 and later by Hawking[11] in 1971 that black holes could be formed in the early Universe which are known as Primordial Black Holes (PBHs). PBHs may be formed as a result of density fluctuation[12], inflation[13], phase transition[14], bubble collision[15] etc. These black holes are of special interest because these are the only black holes whose masses could be small enough to have evaporated by the present epoch as a result of quantum emission[16]. Again PBHs could act as seeds for structure formation[17] and could also form a significant component of dark matter[18].

From standard Cosmology[19], we know that the Universe is radiation-dominated just after inflation and it becomes matter-dominated much later. So PBHs were expected to be only formed in radiation-dominated era. But the detailed analysis[20] of the probability of the PBH formation at the matter-dominated era shows it’s strong enhancement as compared to the case of radiation-dominated era. Combining both the ideas, it has been conjectured that there may be an early matter-dominated era[21] in between the end of the inflation and reheating where PBH could be formed. It is, therefore, an open and interesting problem to investigate PBH formation and evolution in matter-dominated era within the context of BD theory, although a number of similar studies have been done in radiation-dominated era[22-24]. In this work, we have undertaken the analysis in BD theory and show that PBHs can indeed be formed in early matter-dominated era. We have also studied how it affects constraints and evaporation time of PBHs.

2 PBH in matter-dominated era

The gravitational field equations for a spatially flat FRW Universe with scale factor 'a' described by BD Theory are

\[ \frac{\ddot{a}}{a^2} + \frac{\dot{a}}{a} \frac{\dot{\Phi}}{\Phi} - \frac{\omega}{6} \frac{\dot{\Phi}^2}{\Phi^2} = \frac{8\pi \rho}{3\Phi} \]  

(1)

\[ 2 \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + 2 \frac{\dot{a}}{a} \frac{\dot{\Phi}}{\Phi} + \frac{\omega}{2} \frac{\dot{\Phi}^2}{\Phi^2} + \frac{\ddot{\Phi}}{\Phi} = -\frac{8\pi p}{\Phi} . \]  

(2)

The wave equation for BD scalar field is

\[ \frac{\ddot{\Phi}}{8\pi} + 3 \frac{\dot{a}}{a} \frac{\dot{\Phi}}{8\pi} = \frac{\rho - 3p}{2\omega + 3} . \]  

(3)
Using above three equations and the perfect fluid equation of state $p = \gamma \rho$, energy conservation equation can be written as

$$\dot{\rho} + 3 \frac{\dot{a}}{a} (\gamma + 1) \rho = 0$$  \hspace{1cm} (4)$$

For matter-dominated era $p = 0$ leading to $\gamma = 0$.

Barrow and Carr\[22\] have found that for matter-dominated era, the solutions of above equations are

$$a(t) \propto t^{\frac{2}{3}n}$$  
$$G(t) = G_0 \left( \frac{a}{a_0} \right)^n$$  \hspace{1cm} (5)$$

where $t_0$ is the present time, $G_0$ is the present value of $G$ and $n = \frac{2}{3+3\omega}$. Observations\[25\] give $|\omega| \geq 10^4$. Taking $|\omega| = 10^4$, we found $n \approx -0.00007$.

Integrating equation(4), one gets

$$\rho = \rho_0 \left( \frac{a}{a_0} \right)^{-3}$$  \hspace{1cm} (6)$$

which in conjunction with equation(5) leads to

$$\rho = \rho_0 \left( \frac{t_0}{t} \right)^{2-n}$$  \hspace{1cm} (7)$$

Now we are going to discuss about the PBH formation in matter-dominated era\[20\]. We consider that the density fluctuation is responsible for forming PBHs\[20\] in matter dominated era. This density fluctuation grows to a sufficiently homogeneous and isotropic configuration which separates itself from cosmological expansion and contracts within it’s gravitational radius.

Let $t_1$ be the time when contraction starts

- $r_1$ be the size of the configuration at time $t_1$
- $S$ be the deviation of configuration from the spherical form at time $t_1$
- $u$ be the inhomogeneity of the density distribution inside the configuration i.e. $u \sim \frac{\delta \rho}{\rho}$

and $\rho_1$ be the mean cosmological density at time $t_1$.

So

$$\rho_1 = \rho_0 \left( \frac{t_0}{t} \right)^{2-n}$$  \hspace{1cm} (8)$$

The mean density of the primordial black holes formed as the result of contraction is

$$\rho_{BH} \sim \frac{M}{r_1^3} \sim \frac{\rho_1}{x^3}$$  \hspace{1cm} (9)$$
where \( x = \frac{r_g}{r_1} = 2\rho_1 G t_1^2 \).

Here \( r_g = 2GM \) is the gravitational radius of considered configuration.

So

\[
\rho_{BH} = \frac{(t_{BH})^{3n}}{8\rho_0^2 G_0 t_0^4 t_1^{2+2n}}.
\]

(10)

The maximal density which may be reached in the contraction of non-spherical configuration is given by

\[
\rho_{max} = \frac{\rho_1}{S^3}.
\]

(11)

In order to form the black hole the configuration should be nearly spherically symmetric. i.e.

\[
S \leq x < 1.
\]

(12)

The upperbound \( x < 1 \), in conjunction with the negative value of \( n \) gives

\[
t_{BH} < \left( \frac{1}{2\rho_0 G_0 t_0^2} \right)^{\frac{1}{|n|}} t_1.
\]

(13)

Since \( t_1 < t_{BH} \), we can write \( t_1 \leq (t_{BH})^{1-\epsilon} \) where \( \epsilon \) is a very small quantity. Thus equation(13) gives

\[
t_{BH} < \left( \frac{1}{2\rho_0 G_0 t_0^2} \right)^{\frac{1}{|n|}} \epsilon.
\]

(14)

But \( \frac{1}{|n|} > 10^4 \), \( \frac{1}{\epsilon} > 1 \) and \( \frac{1}{2\rho_0 G_0 t_0^2} \approx 3.74 \).

So \( \left( \frac{1}{2\rho_0 G_0 t_0^2} \right)^{\frac{1}{|n|}} > (3.74)^{10^4} \gg t_0 \).

The sufficient condition for the PBH formation imposes constraint on the inhomogeneity of the density distribution of the configuration at time \( t_{BH} \) in the form

\[
\frac{\delta \rho_{BH}}{\rho_{BH}} < 1
\]

(15)

which is also satisfied in our case.

We thus arrive at the conclusion that PBHs can be formed in the matter-dominated era in the BD theory.

We now move on to consider the formation of PBH in early matter-dominated era which is sandwiched between the epochs of inflation and reheating and their characteristics.
3 PBH evaporation

Due to Hawking evaporation, the rate at which the PBH mass decreases is given by

\[ \dot{M}_{\text{evap}} = -4\pi r_{BH}^2 a_H T_{BH}^4 \] (16)

where \( r_{BH} \sim \) black hole radius = 2GM

\( a_H \sim \) black body constant

\( T_{BH} \sim \) Hawking temperature = \( \frac{1}{8\pi GM} \).

Now

\[ \dot{M}_{\text{evap}} = -\frac{a_H}{256\pi^3 G^2 M^2} \] . (17)

But for radiation-dominated era[22], \( G = G_0 \left( \frac{t_e}{t_e} \right)^n \).

If early matter-dominated era exists upto \( t_2 \), then the evaporation equation becomes

\[ \int_{M_i}^{M} M^2 dM = -\frac{\alpha}{t_0^{2n}} \left[ \int_{t_i}^{t_2} t^{2n} dt + \int_{t_2}^{t_e} t_e^{2n} dt + \int_{t_e}^{t} t^{2n} dt \right] \] (18)

where \( \alpha = \frac{a_H}{256\pi^3 G_0^2} \approx \frac{1}{G_0} \)

\( t_e = \) end of the radiation-dominated era

and \( t_2 = \) reheating time .

But for standard scenario, where PBHs are formed in early radiation-dominated era, evaporating equation becomes

\[ \int_{M_i}^{M} M^2 dM = -\frac{\alpha}{t_0^{2n}} \left[ \int_{t_i}^{t_e} t_e^{2n} dt + \int_{t_e}^{t} t^{2n} dt \right] . \] (19)

(In our calculation, we have used \( t_e \approx 10^{11}s \) and \( t_2 \approx 10^{-22}s \).

For different initial time \( t_i \), the solutions of equation (18) and (19) are shown in the Table-1.

It is cleared from Table-1 that if we consider early matter-dominated era, then the PBHs which are just evaporated by present epoch in standard case will live for more \( 7 \times 10^{15}s \) and the PBHs which are formed in between \( 2.3518 \times 10^{-23}s \) and \( 2.3646 \times 10^{-23}s \) are still existing in the nature.

4 Constraints on PBH

For early radiation-dominated era, the probability of a region of mass \( M \) forming a PBH [26] is

\[ \beta_0(M) \approx \delta(M) \exp \left( \frac{-\gamma^2}{2\delta^2(M)} \right) \] (20)
Table 1: Evaporation time of PBHs for different formation time are shown in the table with symbols ($M_i$) - Initial mass of PBH in standard scenario, $M_i$ - Initial mass of PBH formed in early matter-dominated era, $\tau_0$ - Evaporation time of PBH in standard scenario and $\tau$ - Evaporation time of PBH formed in early matter-dominated era.

| $t_i$         | $(M_i)_0$          | $M_i$          | $\tau_0$       | $\tau$       |
|---------------|-------------------|---------------|----------------|-------------|
| $10^{-28}s$   | $1.0011 \times 10^{10}g$ | $1.0074 \times 10^{10}g$ | $3.337 \times 10^4s$ | $3.4 \times 10^4s$ |
| $10^{-27}s$   | $1.0072 \times 10^{11}g$ | $1.0071 \times 10^{11}g$ | $3.337 \times 10^4s$ | $3.399 \times 10^4s$ |
| $10^{-26}s$   | $1.0069 \times 10^{12}g$ | $1.0069 \times 10^{12}g$ | $3.337 \times 10^4s$ | $3.395 \times 10^4s$ |
| $10^{-25}s$   | $1.0067 \times 10^{13}g$ | $1.0067 \times 10^{13}g$ | $3.349 \times 10^4s$ | $3.406 \times 10^4s$ |
| $10^{-24}s$   | $1.0066 \times 10^{14}g$ | $1.0066 \times 10^{14}g$ | $3.342 \times 10^4s$ | $3.398 \times 10^4s$ |
| $2.354 \times 10^{-23}s$ | $2.354 \times 10^{15}g$ | $2.367 \times 10^{15}g$ | $4.349 \times 10^4s$ | $4.42 \times 10^4s$ |
| $2.3646 \times 10^{-23}s$ | $2.367 \times 10^{15}g$ | $2.38 \times 10^{15}g$ | $4.42 \times 10^4s$ | $4.493 \times 10^4s$ |

where the primordial fluctuation have a Gaussian distribution with r.m.s. amplitude $\delta(M)$ which is implicitly given by[27]

$$\delta(M) < 0.2 \left[31 - \log_{10}(\frac{M}{10^{15}g})\right]^{-\frac{1}{2}} . \quad (21)$$

If the early stage of the Universe was matter-dominated era, then the constraints on the fraction of the Universe going into PBHs during the matter-dominated era related to $\beta_0(M)$ via the equation[28]

$$\beta(M) = \beta_0(M)\eta \left(\frac{t}{t_{pl}}\right)\left(\frac{t}{t_{pl}}\right)^{\frac{1}{2}} \left(\frac{M}{M_{pl}}\right)^{-\frac{1}{2}} . \quad (22)$$

with $M = \eta \left(\frac{t}{t_{pl}}\right)M_{pl}$.

In our case, we have taken

$$M = \left(\frac{t}{t_0}\right)^n \left(\frac{t}{t_{pl}}\right)M_{pl} . \quad (23)$$

So

$$\beta(M) = \beta_0(M)\left(\frac{t}{t_0}\right)^n\left(\frac{t}{t_{pl}}\right)^{\frac{1}{2}}\left(\frac{M}{M_{pl}}\right)^{-\frac{1}{2}} . \quad (24)$$

For different $M$, $\beta_0(M)$ and $\beta(M)$ are shown in the following Table-2.
| $M$  | $\delta(M) <$ | $\beta_0(M) <$ | $\beta(M) <$ |
|------|--------------|---------------|-------------|
| $10^{10}$g | 0.0333 | $6.364 \times 10^{-24}$ | $6.41 \times 10^{-21}$ |
| $10^{11}$g | 0.0338 | $2.614 \times 10^{-23}$ | $8.324 \times 10^{-21}$ |
| $10^{12}$g | 0.0343 | $1.064 \times 10^{-22}$ | $1.071 \times 10^{-20}$ |
| $10^{13}$g | 0.0348 | $4.325 \times 10^{-22}$ | $1.377 \times 10^{-20}$ |
| $10^{14}$g | 0.0353 | $1.762 \times 10^{-21}$ | $1.768 \times 10^{-20}$ |
| $10^{15}$g | 0.0359 | $7.186 \times 10^{-21}$ | $2.279 \times 10^{-20}$ |

Table 2: The variation of constraints on PBH formation with different initial masses are shown in the table.

From the Table-2, one finds a significant change in $\beta$ values and this deviation of constraint from standard case gradually decreases as we go towards reheating.

But presently evaporating PBHs generate a $\gamma$-ray background whose most of the energy appears at around 100 Mev[29]. This changes the critical density by a factor of $10^{-8}$ which modifies $\delta(M)$ as

$$\delta(M_*) < 0.2 \left[39 - \log_{10}(\frac{M_*}{10^{15}g})\right]^{-\frac{1}{2}}.$$  \hspace{1cm} (25)

Again $M_* \approx 2.367 \times 10^{15}g$.
So $\delta(M_*) < 0.03218$ which gives $\beta_0(M_*) < 1.616 \times 10^{-25}$ and $\beta(M_*) < 3.3323 \times 10^{-25}$. These values are consistent with previous results[30].

5 Conclusions

We have shown that PBHs can be formed in matter-dominated era of cosmic evolution with gravity described by BD theory. For realisation of our result, we consider an early matter-dominated era in between inflation and reheating. We found that the evaporation period of these PBHs are longer than that predicted in the standard scenario where the PBHs are formed in early radiation-dominated era. Thus more number of PBHs are existing today than the standard scenario. Again the constraint on PBH formation becomes more stringent compared with that of standard scenario which provides a strong evidence for the enhancement of PBH formation in matter-dominated era than the radiation-dominated era. It may be recalled that in matter-dominated phase, cosmic matter remains in a nearl y zero pressure state. Absence of an opposing force to gravitational pull, thus, increases the probability of PBH formation in this era. Our work presents an application of BD theory, a
viable alternate theory of gravity, to yet another physical problem in addition to the various ones mentioned in the introduction.

Acknowledgement

We are thankful to Institute of Physics, Bhubaneswar, India, for providing the library and computational facility. B.Nayak would like to thank the Council of Scientific and Industrial Research, Government of India, for the award of JRF, F.No. 09/173(0125)/2007 – EMR – I.

References

[1] A. Einstein, Sitz. Preuss. Akad. Wiss. Phys. 142, (ξ4) (1917) ; Ann. Phys. 69, 436 (1922).
[2] C. H. Brans and R. H. Dicke, Phys. Rev. 124, 925 (1961).
[3] J. D. Benkestein and A. Meisels, Phys.Rev. D 18, 4378 (1978) ; Astro-phys. J1. 237, 342 (1980).
[4] C. Mathiazhagan and V. B. Johri, Class. Quantum Grav. 1, L29 (1984).
[5] S. Perlmutter et al, Astrophys. J. 517, 565 (1999) ; A. G. Riess et al, Astron. J. 116, 74 (1999) ; P. M. Garnavich et al, Astrophys. J. 509, 74 1998.
[6] D. La and P. J. Steinhardt, Phys. Rev. Lett. 62, 374 (1989).
[7] B. K. Sahoo and L. P. Singh, Mod. Phys. Lett. A 18, 2725 (2003) ; Mod. Phys. Lett. A 17, 2409 (2002).
[8] O. Bertolami and P. J. Martins, Phys. Rev. D 61, 064007 (2001).
[9] B. Nayak and L. P. Singh, arxiv:0803.2930 (Accepted in Mod.Phy.Lett.A);
N. Banerjee and D. Pavon, Phys. Lett. B 647, 477 (2007).
[10] Ya. B. Zeldovich and I. D. Novikov, Sov. Astron. Astrophys. J. 10, 602 (1967).
[11] S. W. Hawking, Mon. Not. R. Astron. Soc. 152, 75 (1971).
[12] B. J. Carr, Astrophys. J. 201, 1 (1975).
[13] M. Y. Kholpov, B. A. Malomed and Ya. B. Zeldovich, Mon. Not. R. Astron. Soc. 215, 575 (1985).
[14] M. Y. Kholpov and A. Polnarev, Phys. Lett. B 97, 383 (1980).
[15] H. Kodma, M. Sasaki and K. Sato, Prog. Theor. Phys. 68, 1979 (1982).
[16] S. W. Hawking, Commun. Math. Phys. 43, 199 (1975).
[17] K. J. Mack, J. P. Ostriker and M. Ricotti, Astrophys. J. 665, 1277 (2007).
[18] D. Blais, C. Kiefer and D. Polarski, Phys. Lett. B 535, 11 (2002) ; D. Blais, T. Bringman, C. Kifer and D. Polarski, Phys. Rev. D 67, 024024 (2003).
A. Barrau, D. Blais, G. Boudoul and D. Polarski, Annalen Phys. **13** 114 (2004)
[19] S. Wienberg, ‘Gravitation and Cosmology’, Wiley, New York, 1972 .
[20] M. Yu Kholpov, ‘Cosmoparticle Physics’, Chapter-4 (page-165) .
[21] B. J. Carr, J. Gilbert and J. Lidsey, Phys. Rev. D **50**, 4853 (1994) .
[22] J. D. Barrow and B. J. Carr, Phys. Rev. D **54**, 6 (1996) .
[23] B. Nayak, L. P. Singh and A. S. Majumdar, arxiv:0902.4553 .
[24] A. S. Majumdar, D. Gangopadhyay and L. P Singh, Mon. Not. R. Astron. Soc. **385**, 1467 (2008) .
[25] B. Bertotti, L. Iess and P. Tortora, Nature **425**, 374 (2004) .
[26] B. J. Carr and S. W. Hawking, Mon. Not. R. Astron. Soc. **248**, 52 (1991) .
[27] B. J. Carr and J. Lidsey, Phys. Rev. D **48**, 2 (1993) .
[28] A. G. Polnarev and M. Yu. Kholpov, Sov. Astron. **26**, 391 (1983) ;
Sov. Phys. Usp. **28**, 213 (1985) .
[29] D. Page and S. W. Hawking, Astrophys. J. **206**, 1 (1976) .
[30] B. J. Carr, Astron. Astrophys. Trans. **5**, 43 (1994) ;
I. D. Novikov et al, Astron. Astrophys. J. **80**, 104 (1979) ;
J. MacGibbon and B. J. Carr, Astrophys. J. **371**, 447 (1991).