A Strong Electroweak Sector at Future $\mu^+\mu^-$ Colliders

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Abstract. We discuss the prospects for detecting at a muon collider the massive new vector resonances $V$ and light pseudo-Nambu-Goldstone bosons $P$ of a typical strongly interacting electroweak sector (as represented by the BESS model). Expected sensitivities to $V$'s at a high energy collider are evaluated and the excellent prospects for discovering $P$'s via scanning at a low energy collider are delineated.

INTRODUCTION

In this contribution we consider some aspects of strong electroweak symmetry breaking at future $\mu^+\mu^-$ colliders. We will concentrate on the possibility of detecting new vector resonances and pseudo-Nambu-Goldstone bosons (PNGB) originating from the strong interaction responsible for electroweak symmetry breaking. The importance of technivector and technipion physics at muon colliders was discussed during the strong dynamics working group meetings [1]. This study will be here performed within the framework of the BESS model [2] and of its generalizations [3]. We recall the main features of this model. The BESS model is an effective lagrangian parameterization of the symmetry breaking mechanism, based on a symmetry $G = SU(2)_L \otimes SU(2)_R$ broken down to $SU(2)_{L+R}$. New vector

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particles are introduced as gauge bosons associated with a hidden $H' = SU(2)_V$. The symmetry group of the theory becomes $G' = G \otimes H'$. It breaks down spontaneously to $H_D = SU(2)$, which is the diagonal subgroup of $G'$. This gives rise to six Goldstone bosons. Three are absorbed by the new vector particles while the other three give mass to the standard model (SM) gauge bosons, after the gauging of the subgroup $SU(2)_L \otimes U(1)_Y \subset G$. The parameters of the BESS model are the mass of these new bosons $M_V$, their self coupling $g''$, and a third parameter $b$ whose strength characterizes the direct couplings of the new vectors $V$ to the fermions. However, due to the mixing of the $V$ bosons with $W$ and $Z$, the new particles are coupled to the fermions even when $b = 0$. The parameter $g''$ is expected to be large due to the fact that these new gauge bosons are thought of as bound states from a strongly interacting electroweak sector. By taking the formal $b \to 0$ and $g'' \to \infty$ limits, the new bosons decouple and the SM is recovered. By considering only the limit $M_V \to \infty$ they do not decouple.

The extension of the BESS model we will consider here is obtained by enlarging the original chiral symmetry $SU(2)_L \otimes SU(2)_R$ to the larger group $SU(8)_L \otimes SU(8)_R$ [3]. The main new feature of this extension is the presence of 60 PNGB’s. Their masses come from the breaking of the chiral group provided by the $SU(3) \otimes SU(2)_L \otimes U(1)_Y$ gauge interactions and by Yukawa couplings [4]. We emphasize that most non-minimal models of a strongly interacting electroweak sector will contain PNGB’s, although the number and their exact properties are model dependent.

**BOUNDS ON THE PARAMETER SPACE FOR THE NEW VECTOR BOSONS**

Bounds on the parameter space of the BESS model from existing data have already been studied (see for instance [5]). Future lepton colliders can improve these limits by testing virtual effects of the new vector particles, especially in the annihilation channels $l^+l^- \to W^+W^-$. In fact, the most relevant observable is the differential cross section $d\sigma(l^+l^- \to W_{L,T}^-W_{L,T}^+)/d\cos\theta$, where $\theta$ is the overall center of mass scattering angle and the decays of the $W^+$ and $W^-$ are used to reconstruct the final $W$ polarizations. The analysis is performed by taking 19 bins in the angular region restricted by $|\cos\theta| \leq 0.95$. Since the new vectors strongly couple to the longitudinal $W$, the most relevant process is $l^+l^- \to W_{L,T}^+W_{L,T}^-$. We have studied the channel with one $W$ decaying leptonically and the other one hadronically. In Fig. 1 we present the 90% C.L. contours in the plane $(b, g/g'')$ for $M_V = 1$ TeV. The continuous and the dashed lines correspond to the bound from the combined differential $W_{L,T}^-W_{L,T}^+$ cross sections at a 500 GeV lepton collider with 20 $fb^{-1}$ of integrated luminosity, assuming effective branching ratios $B = 0.1$ and $B = 0.2$, respectively. The first case can correspond to an $e^+e^-$ machine where the loss of luminosity from beamstrahlung is taken into account. The second one to a muon collider or to an electron collider with a low beamstrahlung loss. The regions within
which one can exclude or detect the $V$ at the 90% CL are the external ones.

Statistical errors are taken into account and we have assumed a systematic error of 1.5%. The dot-dashed line corresponds to the bound from the total cross section $pp \rightarrow W^\pm, V^\pm \rightarrow W^\pm Z \rightarrow \mu\nu\mu^+\mu^-$ at LHC if no deviation is observed with respect to the SM within the experimental errors. Statistical errors and a systematic error of 5% have been included. In conclusion, for models of new strong interacting vector resonances the measurement of $d\sigma(l^+l^- \rightarrow W^+_{L,T}W^-_{L,T})/d\cos\theta$ gives rather strong bounds, provided one is able to reconstruct the final $W$ polarizations. These bounds become very stringent for increasing energy of the collider [5].

Direct production of the new vector bosons can also be considered. A muon collider of 3 – 4 TeV will enable us to completely explore the strongly interacting electroweak option [6].

Since one must rely on an effective non-renormalizable description, one has to take into account the partial wave unitarity limits from $WW$ scattering. In fact when the mass of the new vectors is in the range 2 – 4 TeV, these bounds come out to be quite restrictive. If we denote by $A(s, t, u)$ the amplitude for the scattering $W^+W^- \rightarrow ZZ$, one gets

$$A(s, t, u) = \left(1 - \frac{3\alpha}{4}\right) \frac{s}{v^2} + \frac{\alpha M_V^2}{4 v^2} \left(\frac{u - s}{t - M_V^2 + iM_V\Gamma_V} + \frac{t - s}{u - M_V^2 + iM_V\Gamma_V}\right)$$

where $\alpha = 192\pi v^2\Gamma_V/M_V^3$ and $v = 246$ GeV.

Projecting the components with definite isospin into the lower partial waves and requiring that the bounds $a_{IJ} \leq 1$ are valid up to energies $\Lambda$, such that $\Lambda/M_V \leq 1.5$, we get the limitations in the plane $(M_V, \Gamma_V)$ given in Fig. 2. The intersection of the three allowed regions gives a general upper bound on the mass $M_V \sim 3$ TeV. The previous considerations can be applied also to the technirho case, which is obtained by taking $\alpha = 2$. In this model the unitarity bound turns out to be $M_{\rho_T} \leq 2$ TeV. In Fig. 3 we translate these limits into restrictions on the parameters of the BESS model, using the relation $\Gamma_V = M_V^3/(48\pi v^4 g''^2)$.

We conclude that the unitarity bounds imply that one or more of the heavy vector resonances should be discovered at the LHC, NLC, a $\sqrt{s} \sim 500$ GeV muon collider, or, for certain, at a 3 – 4 TeV muon collider, unless $g''$ is very large and $b$ is very small so that they are largely decoupled.

**PNGB PRODUCTION AT A MUON COLLIDER IN THE EXTENDED BESS MODEL**

In this section, we consider $s$-channel production of the lightest neutral PNGB $P^0$ at a future $\mu^+\mu^-$ collider. Although we shall employ the specific $P^0$ properties as predicted by the extended BESS model with $SU(8) \otimes SU(8)$ symmetry [3], many of our results apply in general fashion to other models of a strongly interacting electroweak sector. An example of large cross section for the production of an isoscalar and an isovector technipion of 110 GeV was shown by Bhat [7].
FIGURE 1. BESS model 90% C.L. contours in the plane $(b, g/g'')$ for $M_V = 1$ TeV. The continuous and the dashed lines correspond to the bound from the differential $W_{L,T}W_{L,T}$ cross sections at a 500 GeV lepton collider for $B = 0.1$ and $B = 0.2$ respectively. The dot-dashed line corresponds to the bound from the total cross section $pp \rightarrow W^\pm, V^\pm \rightarrow W^\pm Z$ at LHC. The allowed regions are the internal ones.

FIGURE 2. Unitarity bounds in the plane $(M_V, \Gamma_V)$ with $\Lambda/M_V = 1.5$. The dashed line corresponds to the bound from the partial wave $a_{00}$, the dotted one to $a_{20}$, and the continuous one to $a_{11}$. The allowed regions are the internal ones.
FIGURE 3. Unitarity bounds in the plane \((M_V, g/g'')\) with \(\Lambda/M_V = 1.5\). The dashed line corresponds to the bound from the partial wave \(a_{00}\), the dotted one to \(a_{20}\), and the continuous one to \(a_{11}\). The allowed regions are the internal ones.

In the extended BESS model, the PNGB mass derives both from gauge contributions and from the effective low-energy Yukawa interactions between the PNGB’s and ordinary fermions [4]. The lightest neutral PNGB’s are the following combinations of the isosinglet and isotriplet components: \(P^0 = (\tilde{\pi}_3 - \pi_D)/\sqrt{2}\), \(P^{0'} = (\tilde{\pi}_3 + \pi_D)/\sqrt{2}\). The \(P^0\) boson couples to the \(T_3 = -1/2\) component of the fermion doublet while \(P^{0'}\) couples to the \(T_3 = 1/2\) component. It is the \(P^0\) upon which we focus. The expressions for the \(P^0\) and \(P^{0'}\) masses are [4]

\[
m^2_{P^0} = \frac{2\Lambda^2}{\pi^2 v^2} m^2_b, \quad m^2_{P^{0'}} = \frac{2\Lambda^2}{\pi^2 v^2} m^2_t
\]

where \(\Lambda\) is an UV cut-off, situated in the TeV region. The first result above can be written as \(m_{P^0} \sim 8\text{ GeV} \times \Lambda(\text{TeV})\). Thus, not only does the \(P^0\) have the \(\mu^+\mu^-\) coupling needed for \(s\)-channel production at a muon collider, but also, in this model, \(m_{P^0}\) should be relatively small, \(\lesssim 80\text{ GeV}\) for \(\Lambda \lesssim 10\text{ TeV}\) (as expected in the present model).

The \(P^0\) Yukawa couplings to fermions are [4]

\[
\mathcal{L}_Y = -i\lambda_b \bar{b} \gamma_5 b P^0 - i\lambda_\tau \bar{\tau} \gamma_5 \tau P^0 - i\lambda_{\mu} \bar{\mu} \gamma_5 \mu P^0
\]

with

\[
\lambda_b = \sqrt{\frac{2 m_b}{3 v}}, \quad \lambda_\tau = -\sqrt{6\frac{m_\tau}{v}}, \quad \lambda_{\mu} = -\sqrt{6\frac{m_\mu}{v}}
\]
For the $P^0$, the $\gamma\gamma$ and gluon-gluon channels are also important; the corresponding couplings are generated by the ABJ anomaly. Clearly these couplings are model dependent and, as an example, we borrow them from technicolor theories [8]. We list here all the partial widths relevant for our analysis:

$$\Gamma(P^0 \to \bar{f}f) = C \frac{m_{P^0}}{8\pi} \lambda_f^2 (1 - \frac{4m_f^2}{m_{P^0}^2})^{1/2}$$

$$\Gamma(P^0 \to gg) = \frac{\alpha_s^2}{48\pi^3 v^2} N_{TC} m_{P^0}^3$$

$$\Gamma(P^0 \to \gamma\gamma) = \frac{2\alpha^2}{27\pi^3 v^2} N_{TC} m_{P^0}^3$$

(5)

where $C = 1(3)$ for leptons (down-type quarks) and $N_{TC}$ is the number of technicolors. The corresponding branching ratios are shown in Fig. 4.

There are presently no definitive limits on the mass of the $P^0$. Potentially useful production modes arise through its ABJ anomaly coupling to pairs of electroweak gauge bosons [9,10]. At LEP the dominant production mode is $Z \to \gamma P^0$. The limit of [10], obtained by requiring a $Z \to \gamma \varphi$ decay width of $2 \times 10^{-6}$ GeV in order to make the $\varphi$ visible in a sample of $10^7$ $Z$ bosons, can be rescaled to the case of the $P^0$. We find that for $N_{TC} \leq 9$ there is no limit on $m_{P^0}$, while, for instance, for $N_{TC} = 10$ $m_{P^0} \geq 12$ GeV is required.

Future limits from the Tevatron and LHC have also been considered [10]. In the single production mode, the best hope of finding the $P^0$ at hadron colliders is via the anomalous decay $P^0 \to \gamma\gamma$. The signal in this channel is similar to

**FIGURE 4.** Branching ratios for $P^0$ decay into $\mu^+\mu^-$, $\tau^+\tau^-$, $b\bar{b}$, $\gamma\gamma$, and $gg$. 
that of a Standard Model Higgs boson of the same mass, given the comparable branching ratio illustrated in Fig. 4. However, for the range of $P^0$ masses we are considering the signal will be hard to see since the $\gamma\gamma$ continuum background is very large at low mass. Another possibility would be to produce pairs of PNGB’s, as for instance, in the resonant production $pp \rightarrow V^\pm \rightarrow P^\pm P^0 + X$ [11], where $V$ is the vector resonance discussed in the Introduction. However, the discovery of the PNGB’s via $\bar{t}bb$ or $\bar{t}bgg$ decays, needs a careful evaluation of backgrounds in the LHC environment. One could also consider the process $pp \rightarrow gg \rightarrow P^0 P^0$, mediated by the anomalous $ggP^0$ vertex, which could be detected by looking for equal mass pairs. Again, backgrounds will be large. Thus, as far as we know, reliable bounds will not be obtained at hadron colliders.

![Figure 5](image_url)

**FIGURE 5.** $\Gamma_{\text{tot}}$ for the $P^0$ as a function of $m_{P^0}$.

Thus, it is clearly of great importance to find a means for discovering or eliminating a $P^0$ with mass between, roughly, 10 GeV and 100 GeV. For much of this mass range a muon collider would be the ideal probe. First, we note that the $P^0$ has a sizeable $\mu^+\mu^-$ coupling [eq. (4)]. Second, the muon collider is unique in its ability to achieve the very narrow Gaussian spread, $\sigma_{\sqrt{s}}$, in $\sqrt{s}$ necessary to achieve a large $P^0$ cross section given the very narrow width of the $P^0$ (as plotted in Fig. 5). One can achieve $R = 0.003\%$ beam energy resolution with reasonable luminosity at the muon collider, leading to

$$\sigma_{\sqrt{s}} \sim 1 \text{ MeV} \left( \frac{R}{0.003\%} \right) \left( \frac{\sqrt{s}}{50 \text{ GeV}} \right);$$

in addition, the beam energy can be very precisely tuned ($\Delta E_{\text{beam}} \sim 10^{-5}E_{\text{beam}}$ is ‘easy’; $10^{-6}$ is achievable) as crucial for scanning for a very narrow resonance.
To quantitatively assess the ability of the muon collider to discover the $P^0$ we have proceeded as follows. We compute the $P^0$ cross section by integrating over the resonance using a $\sqrt{s}$ distribution given by a Gaussian of width $\sigma_{\sqrt{s}}$ (using $R = 0.003\%$) modified by bremsstrahlung photon emission. (Beamstrahlung is negligible at a muon collider.) See Ref. [12] for more details. We separate $\tau^+\tau^-$, $b\bar{b}$, $c\bar{c}$ and $q\bar{q}$, $gg$ final states by using topological and $\tau$ tagging with efficiencies and mistagging probabilities as estimated by B. King [13]: $\epsilon_{bb} = 0.55$, $\epsilon_{cc} = 0.38$, $\epsilon_{bc} = 0.18$, $\epsilon_{cb} = 0.03$, $\epsilon_{qb} = \epsilon_{gq} = 0.03$, $\epsilon_{qc} = \epsilon_{gc} = 0.32$, $\epsilon_{\tau\tau} = 0.8$, $\epsilon_{\tau b} = \epsilon_{\tau c} = \epsilon_{\tau q} = 0$, where, for example, $\epsilon_{bb}$ ($\epsilon_{bc}$) is the probability that a $b$-quark jet is tagged as a $b$ ($c$). Only events in which the jets or $\tau$’s have $|\cos \theta| < 0.94$ (corresponding to a nose cone of $20^\circ$) are considered. A jet final state is deemed to be: $b\bar{b}$ if one or more jets is tagged as a $b$; $c\bar{c}$ if no jet is tagged as a $b$, but one or more jets is tagged as a $c$; and $q\bar{q}$, $gg$ if neither jet is tagged as a $b$ or a $c$.

Background and signal events are analyzed in exactly the same manner. Note, in particular, that even though the $P^0$ does not decay to $c\bar{c}$, some of its $b\bar{b}$ and $gg$ decays will be identified as $c\bar{c}$. In Fig. 6, we plot the integrated luminosity required to achieve $S_i/\sqrt{B_i} = 5$ in a given channel, $i$ (as defined after tagging), taking $\sqrt{s} = m_{P^0}$. We also show the luminosity $L$ needed to achieve $\sum_k S_k/\sqrt{\sum_k B_k} = 5$, where the optimal choice of channels $k$ is determined for each $m_{P^0}$. We observe that very modest $L$ is needed unless $m_{P^0} \sim m_Z$. Of course, if we do not have any information regarding the $P^0$ mass, we must scan for the resonance. To estimate the luminosity required for scanning a given interval so as to either discover or eliminate the $P^0$, we have adopted the following approach. We imagine choosing

![Luminosity for a 5\sigma signal of $P^0$ for the channels: $b\bar{b}$, $\tau^+\tau^-$, $c\bar{c}$, $gg$, and for the optimal combination of these four channels.](image)

**FIGURE 6.** Luminosity for a $5\sigma$ signal of $P^0$ for the channels: $b\bar{b}$, $\tau^+\tau^-$, $c\bar{c}$, $gg$, and for the optimal combination of these four channels.
TABLE 1. Luminosity (in units of 0.01 fb$^{-1}$) required to scan from \(M_{\text{min}} + (m_Z - 90)\) to \(M_{\text{min}} + (m_Z - 90) + 5\) (GeV units) and either discover or eliminate the \(P^0\) at the 3\(\sigma\) level. For scan details, see text.

| \(M_{\text{min}}\) | 11 | 16 | 21 | 26 | 31 | 36 | 41 | 46 | 51 | 56 |
|---------------------|----|----|----|----|----|----|----|----|----|----|
| \(L\)               | 0.028 | 0.051 | 0.079 | 0.10 | 0.13 | 0.18 | 0.23 | 0.29 | 0.40 | 0.55 |
| \(M_{\text{min}}\) | 61 | 66 | 71 | 76 | 81 | 86 | 91 | 96 | 101 | 106 |
| \(L\)               | 0.77 | 1.2 | 2.2 | 5.3 | 17 | 166 | 274 | 52 | 23 | 15 |
| \(M_{\text{min}}\) | 111 | 116 | 121 | 126 | 131 | 136 | 141 | 146 | 151 | 156 |
| \(L\)               | 11 | 9.4 | 8.5 | 8.1 | 8.2 | 8.3 | 8.7 | 8.9 | 9.0 |

\(\sqrt{s}\) values separated by 2\(\sigma\)\(\sqrt{s}\). We assume the worst case scenario in which the resonance sits midway between the two \(\sqrt{s}\) values. The signal and (separately) background rates for these two \(\sqrt{s}\) values are summed together (for the optimal channel combination) and the net \(N_{SD} \equiv (S_1 + S_2)/(B_1 + B_2)^{1/2}\) is computed. We require \(N_{SD} = 3\) to claim a signal. The luminosity required for a successful scan of a given interval is computed assuming that the resonance lies between the last two scan points. This, in combination with the fact that \(\sigma_{\sqrt{s}}\) for \(R = 0.003\%\) is typically a factor of two smaller than \(\Gamma_{\text{tot}}^{P^0}\) (implying that points further away than \(\sigma_{\sqrt{s}}\) from the resonance could be usefully included in establishing a signal) will imply that the integrated luminosities given below are quite conservative. We give in Table 1 the integrated luminosity for a 3\(\sigma\) \(P^0\) discovery after scanning the indicated 5 GeV intervals, assuming \(m_{P^0}\) lies within that interval. If the \(P^0\) is as light as expected in the extended BESS model, then the prospects for discovery by scanning would be excellent. For example, a \(P^0\) lying in the \(\sim 10\) GeV to \(\sim 76\) GeV mass interval can be either discovered or eliminated at the 3\(\sigma\) level with just 0.11 fb$^{-1}$ of total luminosity, distributed in proportion to the (combined) luminosity plotted in Fig. 6. A \(P^0\) with \(m_{P^0} \sim m_Z\) would be much more difficult to discover unless its mass was approximately known. A 3\(\sigma\) scan of the mass interval from \(\sim 106\) GeV to \(161\) GeV would require about 1 fb$^{-1}$ of integrated luminosity.

**CONCLUSION**

We have demonstrated the very substantial suitability, and in many respects superiority, of a muon collider for exploring the full range of physics associated with a strongly-interacting electroweak sector as typified by the extended BESS model. Especially interesting is the potential for discovering any light pseudo-Nambu-Goldstone boson with lepton couplings by scanning. Such bosons are a general feature of models of a strongly interacting electroweak sector and may prove to be quite difficult to detect in any other way.
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