Thermal correction to resistivity of 2D electron (hole) gas in low-temperature measurements at $B=0$

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We calculate the zero magnetic field resistivity, taking into account the degeneracy of the 2D electron (hole) gas and the thermal correction due to the combined Peltier and Seebeck effects. The resistivity is found to be universal function of temperature, expressed in units of $\frac{k}{(kT)^2}$. PACS numbers: 73.40.Qv, 71.30+h, 73.20.Fz

Recently, a great deal of interest has been focussed on the anomalous behaviour of a wide variety of low-density 2D electron and hole systems, whose resistivity unexpectedly decreases as the temperature is lowered, exhibiting a behaviour generally associated with metals, rather than insulators. In particular, it has been found that, below some critical 2D electron density $n_\text{c}$, cooling causes an increase in resistivity, whereas at $n_\text{c} > n_\text{c}$ the resistivity decreases. Although numerous theories have been put forward to account for this effect, the origin of this metallic behaviour is still the subject of a heated debate.

In the present paper, we report on a study of low-temperature transport in 2D electron gas at zero magnetic field, taking into account both the electron degeneracy and the Peltier-effect-induced correction to resistivity. It is well known that ohmic measurements are carried out at low current density in order to prevent heating. Usually, only the Joule heat is considered to be important. In contrast to the Joule heat, the Peltier and Thomson effects are linear in current. As shown in [6], [7], the Peltier effect influences ohmic measurements and results in a correction to a measured resistivity. When current is flowing, one of the sample contacts is heated, and the other cooled, because of the Peltier effect. The established temperature gradient is proportional to the current. The Thomson heat is then proportional to squared current and can therefore be neglected. Then, the voltage drop across the circuit includes the thermoelectromotive force induced by the Peltier effect, which is linear in current. Finally, there exists a thermal correction $\Delta \rho$, to the ohmic resistivity, $\rho$, of the sample. As was demonstrated in [7] for degenerate electrons, $\Delta \rho / \rho \approx (kT/\mu)^2$, where $\mu$ is the Fermi energy. Hence, the above correction may be comparable with the ohmic resistance of a sample when $kT \sim \mu$. We further discuss the features of thermal correction within 2D electron-density-modulated low-temperature ohmic measurements.

Let us consider for clarity a 2DEG sample and dc current flowing in it. The 2DEG structure (MOS, quantum well, etc.) is arbitrary, electrons are assumed to occupy the first quantum-well subband with isotropic energy spectrum $\varepsilon(k) = \frac{p^2}{2m}$. Here, $m$ is the electron effective mass, $p = \hbar k$ is the electron quasi-momentum, and $k$ is the wave vector. The sample is connected (see Fig. insert) by means of two identical leads to the current source. Both contacts are assumed to be ohmic. The voltage is measured between the open ends (“e” and “d”) kept at the temperature of the external thermal reservoir. The sample is placed in a sample chamber (not shown) with mean temperature $T_0$.

According to our basic assumption, the contacts (“a” and “b”) may have different respective temperatures $T_a$ and $T_b$. With the temperature gradient term included, the current density $j$ and the energy flux density $q$ are given by

$$j = \sigma (E - \alpha \nabla T),$$

$$q = (\alpha T - \zeta/e) j - \kappa \nabla T,$$

Here, $E = \nabla \zeta/e$ is the electric field, and $\zeta = \mu - e \varphi$ is the electrochemical potential. Then, $\sigma = Ne^2/\hbar m$ is the conductivity, $N$ is the 2D electron concentration, $\tau$ is the momentum relaxation time, $\kappa$ is the thermal conductivity, and $\alpha$ is the 2DEG thermopower.

It is well known that the Peltier heat is generated by current flowing across the interface between two different conductors. At the contact (“a” in Fig. insert), the temperature $T_a$, electrochemical potential $\zeta$, normal components of the current $I = jd$, and energy flux $qd$ are continuous. Here, $d$ is the sample width. Then, there exists a difference of thermopowers $\Delta \alpha = \alpha_{me} - \alpha$, where $\alpha_{me}$ is the thermopower of the metal lead. For $\Delta \alpha > 0$ and the current direction depicted in Fig. contact “a” is heated, and contact “b” is cooled. Thus, the contacts are at different temperatures, and $T_a - T_b = \Delta T > 0$.

In general, one can easily solve Eq.1, and then find $\Delta T$ for an arbitrary circuit cooling. Since the electron-phonon coupling is weak below $\sim 1K$, the heat conduction from 2DEG to mixing chamber could predominately occur through the ohmic contacts of the sample and the leads connected to them. However, the experimental observations in Ref. demonstrate that the electron gas is, in fact, the dominant thermal resistance in this problem. Actually, the cooling of 2D electrons with respect to bath is provided by thermal conductivity found to follow Wiedemann-Franz law. Accordingly, we neglect further the contact related cooling of 2DEG electron gas. Then, for actual $I \rightarrow 0$ case we will omit the Joule heating. Let us suppose for a moment that the cooling conditions are adiabatic, with the 2D electron gas thermally insulated...
from the environment. These assumptions will be later justified for an actual 2DEG system. We emphasize that under the above conditions, the sample is not heated. Indeed, at small currents, \( T_a \approx T_b \approx T_0 \). Hence, the amount of the Peltier heat, \( Q_a = I \Delta \alpha T_0 \), evolved at contact “a” and that absorbed at contact “b” are equal. If it is recalled that the energy flux is continuous at each contact, the difference of the contact temperatures is given by \( \Delta T = I \Delta \alpha T_0 / \pi d \), where \( \pi d \) is the sample length. As expected, \( \Delta T \) is linear in current.

As shown in Ref. [7], standard ohmic measurements always result in a thermal correction to the resistance measured. Using Eq. (3), we find for the voltage drop \( U \) between ends “c” and “d”

\[
U = R I + \int_c^d \alpha dT,
\]  

where \( R \) is the total ohmic resistance of the circuit. The second term in Eq. (3) coincides with the expression for the conventional thermoelectromotive force, \( \varepsilon_T \), under zero current conditions. Let us assume that the temperature gradient is small. In this case \( \sigma, \alpha, \varpi \) can be considered constant. The thermoelectromotive force is then given by \( \varepsilon_T = \Delta \alpha \Delta T \). Since \( \Delta T \sim I \), there always exists a thermal correction to the ohmic resistance \( \Delta R = \varepsilon_T / I \). Finally, the total resistivity of the 2DEG-sample is given by

\[
\rho^{\text{tot}} = \rho \left( 1 + \frac{\alpha^2}{T} \right),
\]

where \( \rho = 1 / \sigma \) is the ohmic resistivity of the sample. In Eq. (3) we take into account that for the actual case of metal leads \( \Delta \alpha \simeq -\alpha \). Then, for the low-temperature case in question we omit the phonon-related contribution to thermal conductivity. Therefore, \( \varpi = LT\sigma \), where \( L = \frac{\pi^2 k_e^2}{3\mu} \) is the Lorentz number. It is noteworthy that the above result is valid for a 2D hole gas as well.

Using the conventional Gibbs statistics and the energy spectrum specified above, the 2DEG concentration \( N = -\frac{\Omega}{\mu k_e^2} T \) yields

\[
N = N_0 \xi F_0 (1 / \xi)
\]

where \( \Omega = -kT \sum_k \ln(1 + \exp(\frac{\epsilon_k}{kT})) \) is the thermodynamic potential of the 2D electron gas, and \( \xi = kT / \mu \) is dimensionless temperature. Then, \( N_0 = \frac{\Omega}{\mu k_e^2} \) is the density of strongly degenerate 2DEG, and \( F_n(z) = \int_0^\infty x^n(1 + \exp(x - z))^{-1} dx \) is the Fermi integral. In Fig. 3, we plot the temperature dependence of the dimensionless concentration \( n = N / N_0 \) given by Eq. (4). In the classical Maxwell-Boltzman limit (\( \xi < 0, |\xi| \ll 1 \)), the 2D electron density is thermally activated, and, therefore, \( n = |\xi| \exp(-1 / |\xi|) \). In the case of strongly degenerate electrons (\( \xi \ll 1 \)), we obtain \( n = 1 + \xi \exp(-1 / \xi) \). Then, at elevated temperatures \( \xi \gg 1 \), the dependence of the 2DEG concentration \( n = 1 / 2 + \xi \ln 2 \) becomes linear in temperature. It is noteworthy that the 2D electron concentration \( N \) may exceed the zero-temperature value \( N_0 \).

We emphasize that the 2D electron density is a mono-nmonic function of temperature (see Fig. 2). Therefore, one might expect that the ohmic resistivity \( \rho(T) \sim 1 / N \) decreases with increasing temperature at constant carrier mobility. We now demonstrate that the total resistivity specified by Eq. (3) can, nevertheless, increase in a certain temperature range owing to the Peltier effect-related correction.

Following the conventional Boltzmann equation formalism, the explicit formulae for the 2DEG thermopower (for the 3D case, see Pisarenko, 1940) can be written as follows

\[
\alpha = -\frac{k}{e} \left[ \frac{2 F_0 (1 / \xi)}{F_0 (1 / \xi) - \frac{1}{\xi}} \right]
\]

Here, we assume, for simplicity, that the electron scattering is characterized by energy-independent momentum relaxation time. For strongly degenerated 2DEG (\( 0 < \xi \ll 1 \)), we obtain the temperature dependence of the thermopower (Fig. 2b) as \( \alpha = -\frac{1}{e} \pi^2 \xi / 3 \left(1 + 3\xi \right) \exp(-1 / \xi) \). At elevated temperatures \( \xi > 1 \) the thermopower first grows with temperature, and then approaches an universal value \( \alpha = -\frac{1}{e} \pi^2 \xi \). In the classical Maxwell-Boltzman limit (\( \xi < 0, |\xi| \ll 1 \)) the thermopower is given by the conventional formulae \( \alpha = -\frac{1}{e} (2 - 1 / \xi) \). Its worth noting that in the two last cases the thermopower is of the order of \( k/e \). Accordingly, the thermal correction to resistivity \( \Delta \rho = \rho \alpha^2 / L \) may be of the order of the 2DEG ohmic resistivity.

In Fig. 4, we plot the temperature dependence of the 2DEG resistivity given by Eq. (3) at different Fermi temperatures \( T_F = \mu / k \). At fixed temperature, the resistivity decreases with increasing 2DEG degeneracy. Then, for a fixed Fermi energy (e.g., \( T_F = 0.5 \) in Fig. 4) the temperature dependence of the resistivity exhibits metallic behaviour at \( T < T_F \), and then becomes insulating (i.e. \( \rho > 0 \) ) at \( T > T_F \). Within the low-temperature metallic region \( \xi \ll 1 \), the 2DEG resistivity can be approximated (see dashed line in Fig. 3) with \( \rho^{\text{tot}} = \rho_0 (1 + \pi^2 \xi^2 / 3) \), where \( \rho_0 = \frac{1}{2 \pi^2} (k_F l)^{-1} \) is the resistivity at \( T \to 0 \), \( k_F = \sqrt{2 m_e / h} \) is the Fermi vector, and \( l = h k_F / m \) is the mean free path. Then, for the high-temperature (\( \xi > 1 \)) insulating region we obtain the asymptote \( \rho^{\text{tot}} = \rho_0 (1 + \alpha^2 / \xi^2) \), shown in Fig. 4 by dotted line. This result is confirmed by recent experiments [1, 2] shown that for temperatures well below the Fermi temperature the metallic region data obey a scaling law where the disordered parameter \( 1 / l \) appears explicitly. These experimental observations [3] therefore rule out interactions, the shape of the potential well spin-orbit effects as possible origins of the metallic behaviour mechanism. Then, according to Ref. [4] the "metallic"
state can not be associated with e-e induced quantum interference effects. We argue that the semi-classical mechanism discussed above may be responsible for observed T-behaviour of 2D resistivity.

Let us analyze in more detail the cooling conditions, which are known to influence the thermal correction to resistivity. It will be recalled that in the case of adiabatic cooling the electron temperature differs from the bath temperature $T_0$. We now consider the opposite situation of electron cooling due to, for example, finite strength of electron-phonon coupling. Let the phonons are thermalized with respect to bath temperature. Following Ref. [8], below $\sim 0.6K$ in Si MOSFETs the electron-to-phonon thermal exchange is given by $a(T^3 - T_0^3)$, where $a = 2.2 \times 10^{-8}\text{W/K}^3\text{cm}^2$. When $T - T_0 \ll T_0$, the thermal correction to the resistivity $\Delta \rho$ is suppressed by the factor $\beta = \tanh \Lambda$, where $\Lambda = l_0 T_0 \sqrt{3a/4\pi}$ is a dimensionless parameter. Actually, $\beta$ is the ratio of outgoing and internal heat fluxes associated with phonon related thermal leakage and electron heat diffusion, respectively.

When $\lambda \leqslant 1$, the local cooling due to phonons can be neglected and the adiabatic approach is well justified. In the opposite case of intensive cooling ($\lambda \gg 1$), the difference of the contact temperatures $\Delta T$ becomes smaller, and, therefore, the thermal correction to resistivity $\Delta \rho$ vanishes. For $l_0 = 1\text{mm}$, $T_0 = 50\text{mK}$, $\sigma = 2e^2/h = 8 \times 10^{-5}\text{Ohm}^{-1}$ (typical critical region conductance) we obtain $\lambda = 1.2$, and, therefore, $\beta = 0.8$. It worthwhile to notice that Peltier correction to resistivity becomes greater at ultra-low temperatures for short samples, since $\lambda \sim l_0 T_0$. In real experiments both the electron-phonon coupling and the sample-to-bath thermal exchange may be important.

We emphasize that both dc and ac ohmic measurements lead to a thermal correction. The correction is, however, strongly damped at high frequencies because of the thermal inertial effects. As demonstrated in Ref. [8], the above quasi-static approach is valid below some critical frequency $f_{cr} = \chi/l_0^3$. For example, for degenerate electrons the thermal diffusion coefficient $\chi$ is of the order of the diffusion coefficient $D = \frac{\sigma}{\rho} \left( \frac{dN_0}{dp} \right)^{-1}$. Assuming $\sigma = e^2/h$, $l_0 = 1\text{mm}$, for GaAs-based structure we obtain $\chi \sim h/m$, hence $f_{cr} = 1.5KHz$. We suggest that the spectral dependence of the 2D resistivity can be used to estimate the thermal correction.

In conclusion, low-temperature ohmic measurements of a 2D electron (hole) gas at $B = 0$ involve a thermal correction caused by the Peltier effect. The magnitude of thermal correction depends on the 2DEG degeneracy and actual cooling conditions. The resistivity of 2DEG with thermal correction included is found to be universal function of temperature, expressed in units $h/e^2(kp)^{-1}$. This universal behaviour correlates with that found in experiments.

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FIG. 1. Temperature dependence of 2DEG concentration (a) and thermopower (b) given by Eq.(8) and Eq.(9) respectively. Asymptotes: $|\xi| \ll 1$ - dotted line, $\xi \gg 1$ - dashed line. Inserts: the experimental setup (top); position of the Fermi level with respect to the bottom of quantum-well subband (bottom).

FIG. 2. T-dependence of the 2DEG resistivity, given by Eq.(15) for $T_F = 2; 1.75; 1.5; 1.25; 1; 0.75; 0.5; 0.3; 0.25; 0.2; 0.15; 0.1; 0.05; 0.1K$. Asymptotes: $\xi \ll 1$ - dashed line, $\xi > 1$ - dotted line for $T_F = 0.5K$. Insert: density dependence of the 2DEG resistivity within the $T = 0.5-0.9K$ range.
$N/N_0$ vs $kT/\mu$ for classical limit and degenerated 2DEG.
