COMPARING THE WHITE DWARF COOLING SEQUENCES IN 47 Tuc AND NGC 6397

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ABSTRACT

Using deep Hubble Space Telescope imaging, color–magnitude diagrams are constructed for the globular clusters 47 Tuc and NGC 6397. As expected, because of its lower metal abundance, the main sequence of NGC 6397 lies well to the blue of that of 47 Tuc. A comparison of the white dwarf cooling sequences of the two clusters, however, demonstrates that these sequences are indistinguishable over most of their loci—a consequence of the settling out of heavy elements in the dense white dwarf atmosphere and the near equality of their masses. Lower quality data on M4 continues this trend to a third cluster whose metallicity is intermediate between these two. While the path of the white dwarfs in the color–magnitude diagram is nearly identical in 47 Tuc and NGC 6397, the numbers of white dwarfs along the path are not. This results from the relatively rapid relaxation in NGC 6397 compared to 47 Tuc and provides a cautionary note that simply counting objects in star clusters in random locations as a method of testing stellar evolutionary theory is likely dangerous unless dynamical considerations are included.

Key words: globular clusters: individual (47 Tuc, NGC 6397) – Hertzsprung–Russell and C–M diagrams – stars: kinematics and dynamics – stars: Population II – white dwarfs

Online-only material: color figures

1. INTRODUCTION

The majority of white dwarfs (WDs) have no metals in their atmospheres; their spectra indicate the presence of only H and/or He. This is in line with theoretical expectations (Vaucouleurs et al. 1979) where heavy elements in cooler WDs are expected to sink rapidly under the strong gravity of the WD. For most WDs, the star’s location in the color–magnitude diagram (CMD) is not dependent on the metal abundance of its progenitor.

In addition to the locus of the WD sequence in clusters of different metallicity, the number distribution down the cooling sequence is also of interest. This could provide information on a cluster’s initial mass function or on WD physics such as the slow-down in cooling when the WD core undergoes crystallization (van Horn 1968). Additionally, it could be a window to the dynamical processes internal to the cluster. Young WDs have recently changed their masses by about 50%. Hence, they should be in the process of altering their radial distribution and may be possible to observe this.

In this paper, we examine the location and number distribution in detail in the CMDs of WDs in two globular star clusters of very different metallicities: NGC 6397 and 47 Tuc ([Fe/H] = −2.0 and −0.75, respectively; Carretta et al. 2009). Using somewhat lower quality data, we also include M4 in a comparison of the WD loci. The number of WDs down the cooling sequences of 47 Tuc and NGC 6397 are explored, found to be quite different, and then compared with the results of N-body calculations which suggest that internal dynamics have had an important role in controlling these distributions.

2. DATA AND CMDs

NGC 6397 is one of the closest globular clusters (2.6 kpc; see Richer et al. 2008 for a summary) and 47 Tuc is a little less than twice as far from us (4.6 kpc; Woodley et al. 2012). For both clusters, we made use of the superb imaging capabilities of the Advanced Camera for Surveys (ACS) on board the Hubble Space Telescope (HST) to explore their complete stellar populations. For NGC 6397, we already had deep imaging data from previous work (Richer et al. 2006; Hansen et al. 2007; Anderson et al. 2008), based on a large program in HST Cycle 13 (GO-10424, 126 orbits). This was supplemented by a new epoch of observations in 2010 (GO-11633, 9 orbits) in a single filter (F814W) in the same field that was used to proper motion (PM)-clean the earlier data.

For 47 Tuc, our team was awarded 121 orbits in Cycle 17 to image the cluster (GO-11677). The main science goal was to obtain photometry with the ACS F606W and F814W filters that would go deep enough to study the entirety of the WD cooling sequence (Hansen et al. 2013). A detailed discussion of the observations and data reduction can be found in Kalirai et al. (2012).

From these observations, we derived CMDs for the two clusters. In the present paper, for 47 Tuc we use a CMD constructed from stacks of the ACS images in the F606W and F814W filters. Our CMD is largely free of distracting artifacts; with such clean images, the CMD penetrates to F606W > 30, even in a field as crowded as this one. The left panel of Figure 1 displays the 47 Tuc CMD, showing all objects that were at least 2.5σ above the sky in both filters, and whose photometry fell...
Figure 1. CMDs of 47 Tuc and NGC 6397. Left: all objects measured in our 47 Tuc field (at 1.9 half light radii) that are more than 2.5σ above the sky and that pass a modest error cut are included. The main sequence of the Small Magellanic Cloud, which is located in the background of 47 Tuc, lies between the 47 Tuc main sequence and its white dwarf cooling sequence. In addition (middle panel), these data were PM-cleaned so as to include just the cluster stars. The axes in this plot are absolute magnitude and intrinsic color. Right: PM-cleaned CMD of NGC 6397 (at 1.7 half light radii). F606W apparent distance moduli of 13.42 and 12.61 are used for 47 Tuc and NGC 6397, respectively, with their associated reddenings of 0.045 and 0.223.

within an error cut that grew from 0.037 mag at F606W = 20 to 0.15 mag at F606W = 30. In addition, these data were PM-cleaned using archival images taken between 2002 and 2012 (see Richer et al. 2013 for details).

The far right panel of Figure 1 displays the PM-cleaned CMD for NGC 6397. Because we will want to compare the clusters directly, the CMDs have been plotted as absolute magnitude and intrinsic color in the two right-most panels. To accomplish this, we begin with literature values for true distance moduli, reddening, and the ratio of total-to-selective absorption RV (taken as 3.1 (Cardelli et al. 1989) for both clusters). The absolute distance moduli for 47 Tuc and NGC 6397, respectively, are 13.30 ± 0.08 and 12.07 ± 0.06 with reddening values in (B − V) of 0.04 ± 0.01 and 0.18 ± 0.02. This 47 Tuc distance was obtained from a mean of those discussed in Woodley et al. (2012), excluding their WD derived distance (in order to avoid bias later when we compare the cooling sequences of the cluster WDs), and the NGC 6397 values are from Richer et al. (2008), again excluding WD derived values.

We now need to convert these to F606W and F814W and correctly apply the extinction in our filters. Since we are interested mainly in the WDs in this contribution, we determine the extinction corrections for stars with average WD temperatures. For this we use the spectral model grid from Tremblay et al. (2011) to determine a model spectrum for a 10,000 K WD that falls near the center of our WD cooling sequences in these clusters. We then use the reddening curve from Fitzpatrick (1999) to calculate the magnitudes of this object in our filters with and without interstellar reddening applied. We use the differences of these magnitudes to calculate the appropriate extinction in each filter and correct our CMDs to an unreddened frame. We then use true distance moduli to adjust each CMD so that it lies in an extinction- and distance-corrected reference frame.

3. MAIN SEQUENCE AND WHITE DWARF LOCI

It is not immediately obvious from Figure 1, but the main sequences (MSs) of the two clusters have rather different colors. This becomes evident when we overplot the two CMDs, as in Figure 2, where the CMD of NGC 6397 was plotted in red and that of 47 Tuc was overplotted in black. At the brightness level of the hot WDs, the MSs differ in color index by about half a magnitude. This is of course due to the large difference between the metallicities of the two clusters; it has been recognized for many decades that MS color is controlled by the amount of line blanketing that metals produce in a stellar atmosphere (e.g., Sandage & Walker 1966).

The difficulty in using MS turnoffs for age determination becomes obvious in this diagram; the large difference between turnover locations (which eventually determines the age) could be
entire CMDs of NGC 6397 (red) overplotted with that of 47 Tuc (black) in absolute magnitude and intrinsic color. The MSs of the clusters do not align while the WD cooling sequences do. (A color version of this figure is available in the online journal.)

due either to a metallicity and/or an age difference. By contrast, the expectation is that the WD cooling sequences of the clusters should align, independent of the metallicity of the progenitor. The WDs should have no metals in their atmospheres (Fontaine & Michaud 1979). These sink rapidly under the strong gravity of the WD unless the star is very hot and can radiatively levitate atoms (Holberg et al. 1994; Chayer et al. 1995), or unless the atmosphere is continually being polluted from circumstellar material (e.g., Farihi et al. 2009). Hence, their spectra and location in the CMD do not explicitly depend on the cluster metal abundance—except for a possible WD mass dependence on metallicity through the initial-final mass relation (Kalirai et al. 2008, 2009). This point was discussed in detail in Woodley et al. (2012), but there is no evidence for a range in the masses of, for example, globular cluster WDs with metallicity at the top of the cooling sequence (Moehler et al. 2004; Kalirai et al. 2009), although the data here are not yet conclusive.

Over the full range of the WD cooling sequences, the overlap between the clusters is remarkable (see Hansen et al. 2013 Figure S5 for an earlier version of this diagram) and we demonstrate below that this result is very robust to modest changes in distance modulus and reddening.

To explore the location of the WD loci in more detail, in Figure 3, we include M4, which is the only other globular cluster with data deep enough to see a major component of the cooling sequence. Its metallicity ([Fe/H] = −1.20; Drake et al. 1994) lies between that of 47 Tuc and NGC 6397. The data for this cluster were downloaded from the HST archives. The filters used here were F606W and F775W, so we were required to transform the color to (F606W − F814W). Using a cooling model from Fontaine et al. (2001) and spectral models from Tremblay et al. (2011), we found a relation between temperature and reddened (F606W − F775W) color (using $E(B − V) = 0.35$; Richer et al. 1997 and $R_V = 3.76$; Kaluzny et al. 2013) for WDs of 0.53 $M_\odot$. We then calculated (F606W − F814W) for the same temperatures. This allows us to determine an expected difference between these two colors as a function of reddening (F606W − F775W). This color dependent transform is then applied to our original photometry to generate a transformed (F606W − F814W) CMD. These M4 data are of poorer quality and the scatter at the faint end is much larger. Using the true distance modulus from Kaluzny et al. (2013) of 11.34, we plot the M4 stars in Figure 3 with smaller points, so all cluster sequences are readily visible. We collect the major properties of the clusters we have been discussing in Table 1.

We compared a bright portion of the WD cooling sequences between $M_{606W} = 11.5$–15.0 for all three clusters in order to assess whether or not their slopes and intercepts were consistent. This region avoids the brightest WDs, whose insulating hydrogen layers are not fully degenerate, as well as the fainter WDs where the photometry becomes less certain. We used a maximum likelihood method to fit a line to each WD sequence. Our fitting method also varies distance and reddening. At the end, we marginalize these parameters out with priors that reflect a 20% uncertainty in reddening/extinction and a 0.1 magnitude uncertainty in true distance modulus. The result is a likelihood surface for only the two model parameters (slope and intercept) for each cluster (Figure 4). These contours reflect the inherent scatter in the photometry, the photometric errors, and uncertainties in distance and extinction. The 1$\sigma$ contours overlap for all three clusters, demonstrating the near equality of the loci of the WD cooling sequences for clusters covering almost the entire range of metallicity in the globular cluster system.

Turning now to the full WD cooling sequences, we note that these are well populated in all three clusters and show the theoretically predicted turn to bluer colors at faint magnitudes (Bergeron et al. 1995; Hansen 1999; Saumon & Jacobson 1999). This is due to collision-induced absorption which, in the cool dense atmosphere of such a WD, allows H$_2$ to form with its strong infrared opacity. This forces the radiation out in the bluer spectral regions so that the star gets bluer even though its temperature is getting cooler.
The location of the truncation of the WD cooling sequence is a potentially sensitive age indicator as it is the limit to which the bulk of the WDs could have cooled over the lifetime of the cluster. Older clusters are thus expected to exhibit fainter truncation magnitudes, as is seen for NGC 6397 compared to 47 Tuc. No strong conclusions can be made for M4 considering the large scatter in the data and the lack of incompleteness corrections in this plot. A detailed comparison employing incompleteness corrections for these very faint stars and a full modeling of the WD cooling sequences for 47 Tuc and NGC 6397 have been carried out (Hansen et al. 2013). NGC 6397 is found to be 2 Gyr older than 47 Tuc.

Table 2

| Cluster | WD < 0.5 Gyr | 0.5–2 Gyr | 2–5 Gyr |
|---------|--------------|-----------|---------|
| 47 Tuc  | 0.047(0.003) | 0.071(0.004) | 0.114(0.004) |
| N6397   | 0.063(0.013) | 0.278(0.026) | 0.714(0.039) |

Model

- $t/t_{ch} = 2.5$: 0.038(0.008) 0.086(0.011) 0.180(0.021)
- $t/t_{ch} = 35$: 0.062(0.012) 0.248(0.029) 0.564(0.022)

4. COMPARING NUMBERS OF WDs

4.1. Using Cooling Model Ages

The number of WDs as a function of magnitude (the luminosity function) contains information about the cluster age, its initial-mass function, WD cooling physics, and cluster dynamics. This motivates a detailed comparison of the number distribution with magnitude of WDs in 47 Tuc and NGC 6397.

To explore this quantitatively, we analyze the number of WDs of various ages down the cooling sequence. To accomplish this, we compare the completeness-corrected counts of WDs in various magnitude ranges (that is, cooling ages) to MS stars with masses similar to the WDs ($0.5–0.6 M_\odot$) in the two clusters. This mass was chosen to match that of the WDs so that any differences could be most easily interpreted. The ages of the WDs were obtained from the cooling models of Fontaine et al. (2001). The data are contained in Table 2. Statistical errors based on the numbers of stars counted are included in the brackets beside each entry and the age of the WD sample is indicated.

For WDs younger than about 0.5 Gyr, the WD/MS ratio is similar in the two clusters. For WD ages greater than this, the difference between the ratios grows so that for 5 Gyr WDs it is more than six times larger in NGC 6397 than in 47 Tuc.

There are a number of potential reasons for these differences. Age is one possibility. Hansen et al. (2013) have shown that 47 Tuc is younger by about 2 Gyr. The number of WDs will increase with age in a cluster because of stellar evolution, roughly doubling for a factor of two in age. However, the small age difference between 47 Tuc and NGC 6397 is insufficient for this to be the full answer. Other causes for the variations in these ratios may be the binary fraction and the cluster orbit; both possibilities could have an effect on the WD/MS ratio by hiding WDs in the first instance and stellar evaporation in the second case. We also know that there is a metallicity difference that could play a role. However, the biggest difference is likely to be dynamical age. NGC 6397 is post-core-collapse with a half-mass relaxation time ($t_{ch}$) of 0.4 Gyr (Harris 2010), while 47 Tuc is listed in the same reference as non-core-collapse with $t_{ch} \sim 3.5$ Gyr. So the age/$t_{ch} (t/t_{ch})$ values are very different for the two clusters: $\sim 30$ for NGC 6397 and $\sim 3$ for 47 Tuc.

For further insight, we examined a number of N-body star cluster models (see Hurley et al. 2008 and Hurley & Shara 2012 for details of the modeling approach) that included stellar evolution, a Galactic tidal field, binaries, and their formation and destruction. We use these models mainly to investigate trends as opposed to using them to predict actual numbers of stars.

One model in particular is, by design, a good match for NGC 6397. The orbit, binary fraction, and initial compactness were chosen to match the cluster. This model initially consisted of 200,000 stars with 2.5% binaries and, at an age of 11.7 Gyr, has $t/t_{ch} = 35$. By this time, it has lost more than 80% of its stars. By contrast, NGC 6397 currently has about 200,000 stars (Heyl et al. 2012). The ratios of WDs of various ages to MS stars are roughly doubling for a factor of two in age.
stars with masses between 0.5 and 0.6 $M_\odot$ were extracted from this model between 1.2 and 2.2 half-light radii, similar to our observed field in NGC 6397. These ratios, with associated errors based on the numbers of model stars counted, are contained in the final row of Table 2. The increase in the WD/MS ratio here is dramatic, nicely similar to that seen in NGC 6397 and quite different from the behavior observed in 47 Tuc.

Regarding 47 Tuc, we do not have a model that comes even close to being a real representation of this cluster. It currently has about one million stars and likely began life with double this number (Giersz & Heggie 2011). Such a population is still well beyond the capabilities of N-body calculations. In the models we have calculated thus far, clusters with small $t/t_{\text{rh}}$ are generally young. In the same $N = 200,000$ model as above, the cluster has $t/t_{\text{rh}} = 3$ when it is only 4 Gy old. Because of stellar evolution, this would have reduced the total fraction of WDs in the cluster by at least a factor of two, so any comparison with the real cluster would be difficult. In addition, these WDs are more massive than those currently being produced in the real cluster. Our best match to 47 Tuc was a model that began with $N = 100,000$ stars, with quite a different relaxation history from the 200,000 model in that at about 13 Gyr it had $t/t_{\text{rh}} = 2.5$. This model began with 5% primordial binaries. By 13 Gyr, $N$ had been reduced to 79,500 stars, which is a far cry from the million stars still present in 47 Tuc today. For this model cluster, the stellar statistics were extracted between 1.4 and 2.4 half-light radii, similar to the location of our real field in 47 Tuc. These are contained in the second-to-last row of Table 2. Here, the WD/MS variation with age is much more gradual, more in line with the numbers seen in 47 Tuc.

Since the real clusters don’t differ in age by more than 20% (Hansen et al. 2013), and since the loss of stars via evaporation would decrease both the MS and WD numbers, it appears from a comparison with the N-body models that internal dynamical evolution is the dominant contributor to the WD/MS trends observed in these two clusters. The initial mass functions of the two model clusters were identical Kroupa functions (Kroupa et al. 1993) so the variations seen in the models are not due to this quantity. Most of the precursors to the WDs currently seen in our NGC 6397 field were likely born elsewhere (presumably closer to the core) and they migrated into our field after they lost much of their mass upon becoming WDs. Within this scenario, the much shorter half-mass relaxation time in NGC 6397 (0.4 Gyr) versus 3.5 Gyr for 47 Tuc is consistent with the ratios in the two clusters being similar for WDs younger than 0.5 Gyr and growing more rapidly in NGC 6397 relative to 47 Tuc for older WDs.

### 4.2. Using Statistical Ages

In the previous subsection, we relied on models to set the WD cooling timescale and N-body calculations to help interpret the results. Here, the discussion avoids using theoretical models, instead using a statistical approach to estimate cooling ages. The rate of formation of WDs in our 47 Tuc ACS field is $1/0.0095$ WD Gyr$^{-1}$ (Goldsbury et al. 2012). This comes from counting the giant stars in our field, establishing their evolutionary time through theoretical models (giant models from Dotter et al. 2008, not WD models), and assuming that all the giants eventually turn into WDs. The relaxation time in our NGC 6397 field is 0.7 Gyr (Heyl et al. 2012). Note that this is larger than the half-mass relaxation time, as the field is farther from the cluster center. We divide the hot end of the 47 Tuc WD cooling sequence into three sections by age: stars younger than 0.7 Gyr, stars between 0.7 and 2.1 Gyr, and stars from 2.1 to 3.5 Gyr. The number of WDs less than 0.7 Gyr is 0.7 Gyr (Harris 2010), we do not expect significant numbers of stars to arrive or leave this field, on average, in a single relaxation time, so the WD cooling age across each bin should scale as the number of WDs in that bin. Hence, if we proceed down the 47 Tuc WD cooling sequence until we have counted 74 stars, then that will correspond to a cooling time of 0.7 Gyr for the faintest object. Counting down to $3 \times 74$ stars would be at an absolute magnitude corresponding to a cooling time of 2.1 Gyr, with $5 \times 74$ stars at a cooling age of 3.5 Gyr. Here, we assume that the formation rate of WDs has been constant over the last few Gyr. Goldsbury et al. (2012) have shown that this assumption is valid in our 47 Tuc field.

Since the WD cooling rate is not dependent on metallicity, and since the WD cooling sequences align so that there is little if any mass difference, the same absolute magnitude ranges must also correspond to WDs at the same cooling ages in NGC 6397. We now examine what the incompleteness-corrected WD numbers are in these increasingly faint magnitude bins for NGC 6397. The counts are contained in Table 3. In the first NGC 6397 bin (down to an absolute F606W magnitude of 11.63), we count 20 WDs. In the next bin, we expect 40 (the bin is twice as wide in age) and count 49. The Poisson probability of this occurring is 3%. In the final bin, we count 81 where 40 are again expected. The probability of this occurring by chance is $1.8 \times 10^{-8}$.

For NGC 6397 the WD numbers grow as we progress to the fainter bins. Since the two clusters don’t differ in age by a large amount and since evaporative losses would likely be more important in NGC 6397 than in 47 Tuc, it again emerges that the NGC 6397 cooling sequence is being populated not just by stars evolving in the observed field, but by large numbers coming into our field from elsewhere.

### 5. Conclusions

The main result of this paper is a demonstration that WD cooling sequences for globular clusters of very different metallicities align almost perfectly in the CMD. This is in line with theoretical expectations where heavy elements in the dense atmosphere of a WD are expected to sink, yielding atmospheres composed solely of H and/or He. Virtually all knowledge of the original metal abundance of the WD precursor has been lost. This is a strong endorsement of the use of WDs in cosmochronology, where the dependence on metallicity found in other dating techniques (particularly MS turnoffs) are largely eliminated.

| Cluster          | $M_{\text{POOH}} < 11.63$ | 11.63 < $M_{\text{POOH}} < 13.10$ | 13.10 < $M_{\text{POOH}} < 14.15$ |
|------------------|---------------------------|----------------------------------|----------------------------------|
| WD cooling age   | <0.7                      | 0.7–2.1                          | 2.1–3.5                          |
| Number 47 Tuc WDs| 74                        | 148                              | 148                              |
| Number NGC 6397 WDs | 20                      | 49                               | 81                               |

Table 3

WD Numbers at Various Ages
Most of the WDs found in our NGC 6397 field, which is located near two half-mass radii in the cluster, likely formed nearer to the cluster center and have slowly been migrating outward. A red giant with a mass near $0.8\, M_\odot$ located close to the cluster center evolves into a $0.5\, M_\odot$ WD. Two-body relaxation drives the star to a larger radius orbit. Due to the relatively short relaxation time in NGC 6397, over a period of a few Gyr these WDs end up increasing the ratio of WDs to MS stars of the same mass at radii beyond the core. It appears that we are seeing this process in action in our data.

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