Quantized Adiabatic Charge Transport in a Carbon Nanotube

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The coupling of a metallic Carbon nanotube to a surface acoustic wave (SAW) is proposed as a vehicle to realize quantized adiabatic charge transport in a Luttinger liquid system. We demonstrate that electron backscattering by a periodic SAW potential, which results in miniband formation, can be achieved at energies near the Fermi level. Electron interaction, treated in a Luttinger liquid framework, is shown to enhance minigaps and thereby improve current quantization. Quantized SAW induced current, as a function of electron density, changes sign at half-filling.

The mechanism of quantized adiabatic transport, as first conceived by Thouless\textsuperscript{1}, involves a one-dimensional (1D) electron system in a periodic potential that, via backscattering, opens a gap in the electron spectrum. If the potential varies slowly and periodically in time in such a way that the Fermi level lies within a gap of the instantaneous Hamiltonian, then an integer charge $me$ is transported across the system during a single period. This results in a quantized current $j = mef$, where $f$ is the frequency of the external field. If realized experimentally, such a device would present an important application as a current standard.

Electron properties of real 1D conductors, such as nanotubes or quantum wires, are dominated by electron interactions. However, apart from general statements about robustness of the quantization, the effect of interactions on quantized transport has not been explored. In this article we establish the possibility to realize this regime in metallic nanotubes, the purest 1D conductors currently available. We develop a theory that takes full account of electron interactions in this system in the Luttinger liquid framework.

Although the quantized adiabatic transport mechanism is compelling in its simplicity, it has proven difficult to realize experimentally: the goal is to find such a combination of a host 1D system and a sliding external perturbation to engineer a miniband spectrum with minigaps sufficiently large that disorder, thermal excitations, and finite size effects do not compromise the integrity of the quantization. Recently, a surface acoustic wave (SAW) was used to achieve quantized current in a split gate point contact\textsuperscript{10}. The SAW field can be strong enough to induce a bulk gap, and the SAW wavenumber can be chosen to match $2pr$ to pin electrons. Among the existing 1D systems, one possibility is to use quantum wires which can be coupled easily to the SAW. However, since the densities for which adiabatic transport is most pronounced correspond to a few electrons per SAW spatial period (realistically, ca. a few microns), one would need wires with low electron 1D density of around $10^4 \text{cm}^{-1}$. The densities currently available in such systems are at least an order of magnitude higher\textsuperscript{11}.

In this letter we argue that a surface acoustic wave (SAW) coupled to a semi-metallic carbon nanotube presents an ideal system in which quantized transport can be realized. The experimental arrangement is illustrated in Fig 1. A nanotube is placed between two metallic contacts on the surface of a piezoelectric crystal, with a gate electrode nearby to allow adjustment of the Fermi level in the tube. In a piezoelectric substrate the SAW is accompanied by a wave of electrostatic potential that can have an amplitude up to few Volts\textsuperscript{13}. The potential decays both into the free space and into the substrate to a depth comparable to the wavelength $\lambda_{\text{SAW}}$. We assume that the tube is suspended at a height $\ll \lambda_{\text{SAW}}$ above the substrate, so that there is no direct mechanical coupling and only the free space component of a SAW potential matters. When a SAW is launched from a transducer (such as an inter-digitated electrode array) it’s electric field penetrates the tube and electron diffraction on the sliding SAW potential results in miniband formation. By positioning the Fermi level within the energy gap, the conditions for current quantization are fulfilled.

FIG. 1. The low energy spectrum of a metallic Carbon nanotube (broken line) acquires a minigap (solid line) in the presence of a symmetry breaking perturbation. The backscattering transitions induced by the SAW potential are shown. Inset: proposed experimental arrangement consisting of a nanotube suspended between contacts, with a gate on the side, and a SAW source.

High electron velocity in nanotubes, $v \approx 8 \times 10^7 \text{cm/s}$, makes it possible to create large minigaps. The expected minigap size can be estimated in view of the observation of resonant states formed as standing waves in a finite sample. The resonances were found to be spaced by $\hbar v/L$, where $L$ the sample length. This gives $E_{\text{gap}} = 0.6 \text{meV}$ for $L = 3 \mu \text{m}$, which implies that a peri-
odic SAW potential with $\lambda_{\text{SAW}}$ of several microns is sufficient to form large minigaps. A period of the SAW-induced grating down to $200 - 300\text{nm}$ can be realized, so that minigaps as large as $10\text{meV}$ are expected. For comparison, the same periodic perturbation acting on a GaAs 1D channel will induce minigaps of an order of magnitude smaller because of a smaller $v_F \approx 10^7\text{cm/s}$. A further advantage of metallic nanotube system results from its semi-metallic spectrum in which two pairs of oppositely moving spin degenerate states intersect exactly at the Fermi level (at half-filling). Thus despite the fact that the SAW wavelength is always much larger than the inter-atomic distance $a_0$ (ca. 10), backscattering and the Umklapp interactions are small scaling as $1/N = 1/20$. Therefore, taking into account the presence of a symmetry breaking perturbation $\Delta$, and neglecting both backscattering and Umklapp processes, the low energy states of the nanotube system (in the vicinity of the band-crossing $p = \pm k_0$) are described by the Dirac Hamiltonian

$$H = \int dx \sum_{\alpha=1}^4 \psi_\alpha [-\hbar v_F \sigma_3 \partial_x + \Delta] \psi_\alpha + \frac{1}{2} \sum_q \hat{\rho}_q V(q) \hat{\rho}_{-q}$$  \hspace{1cm} (1)$$

where $\psi = \psi^\dagger \sigma_1$. Here Pauli matrices $\sigma_{1,2}$ operate in the two-component Dirac operator space $\psi_\alpha = (\psi_1, \psi_2)_\alpha$ with pseudospin components corresponding to the right and left moving states, and $\hat{\rho}(x) = \sum_\alpha \psi_{\alpha}^\dagger(x) \psi_{\alpha}(x)$ is charge density operator. The second term in (1) describes the left/right mixing and yields a gap in the spectrum. Different mixing mechanisms lead to different values of $\Delta$. For example, a parallel magnetic field produces $\Delta = \hbar v_F / R$, where $R$ is the nanotube radius and $\phi = \Phi / \Phi_0$ is the magnetic flux through nanotube cross-section (measured in units of the flux quantum $\Phi_0 = \hbar c / e$).

The harmonically varying electrostatic potential of the SAW decays exponentially in the direction normal to the surface: $A e^{-k_z x} \sin k(x - ut)$, where $u$ is SAW velocity. Since the wavelength $\lambda_{\text{SAW}} = 2\pi / k$ is much larger than the tube diameter $2R$, one can ignore the potential variation $e^{-k_z x}$ over the tube cross-section. For example, for $\lambda_{\text{SAW}} = 1\mu m$ and $R = 1\mu m$, the potential variation is less than 1%. Therefore, it is sufficient to take into account just the parallel component of the SAW electric field. Since the SAW velocity is small, $u \ll v$, the spectrum can, therefore, be obtained within a stationary approximation.

To simplify our analysis, let us first consider the non-interacting system. In the stationary approximation, the single-particle spectrum of each degenerate ‘flavor’ can be obtained from the perturbed 1D Dirac system,

$$\epsilon \psi(x) = (-i\hbar v_F \sigma_3 + \Delta \sigma_1 + A \sin k x) \psi(x). \hspace{1cm} (2)$$

Here the selection rule described earlier is manifest: for $\Delta = 0$, Eq. (2) separates into two independent equations for right and left moving particles. The SAW affects only the phase of the wavefunction. For $\Delta \neq 0$, the backscattering effect of the SAW potential is restored, and minigaps are induced in the spectrum.

To explore the miniband structure of the Dirac system it is convenient to implement a gauge transformation, $\psi(x) = e^{i \lambda \sigma_3 \cos k x} \psi'(x)$, where $\lambda = 2A / \hbar v_F$, and

$$\epsilon \psi'(x) = (-i\hbar v_F \sigma_3 \partial_x + \Delta e^{-i \lambda \sigma_3 \cos k x} \sigma_1) \psi'(x) \hspace{1cm} (3)$$

The periodic system is characterized by Bloch states $\psi_p(x) = u_p(x) e^{ipx}$ with quasimomentum $p$ taking values in the Brillouin zone defined by the SAW period, $-k/2 < p < k/2$. The corresponding energy spectrum can easily be obtained numerically (Fig. 3), by integrating the system of first-order differential equations (3) over the SAW spatial period $0 < x < 2\pi / k$. The spectrum has an electron-hole symmetry, $\epsilon \rightarrow -\epsilon$, characteristic of a Dirac system.

The problem (3) can also be solved analytically for $\Delta \ll \hbar v_F$ by treating the second term of Eq. (3) as a perturbation [13]. Separated into Fourier components,
\[ e^{-i\lambda \cos kx} = \sum_{m=\pm\infty}^\infty (-i)^m J_m(\lambda) e^{-imkx}, \]

where \( J_m(\lambda) \) are Bessel functions, each harmonic of the perturbation \( \Phi \) mixes right and left modes with \( p - p' = mk \). When these states are in resonance (i.e., when \( p = -p' = mk/2, \varepsilon'_m = mkv/2 \)), the spectrum can be found by standard two-wave matching. This gives energy gaps

\[ \Delta_m = 2\Delta |J_m(2A/hkv)| \]

which are oscillatory functions of the SAW amplitude \( A \), with zeros at the nodes of Bessel functions. In particular, for \( A \ll hkv \), \( \Delta_m \approx 2\Delta(A/hkv)^{|m|/m!} \).

![Graph of electron energy spectrum vs. SAW amplitude](image)

**FIG. 2.** Electron energy spectrum of Eq. (2) vs. the SAW field strength \( A \), scaled by \( \epsilon_0 = k_v \). Minigaps oscillate as a function of \( A \), in agreement with the perturbation theory (3), vanishing at values close to the roots of Bessel functions.

Electrons in the half-filled (undoped) system represent a solid state analogue of the Dirac vacuum: under the SAW perturbation, the many-body state carries neither charge nor current. For a weak SAW potential this follows from adiabatic continuity: quantized transport takes place when the chemical potential \( \mu \) falls in one of the minigaps. The value of the quantized current will remain the same (with a whole range of values of \( \mu \) and \( A \) that stay within a gap. Since the spectral gap at the band center is adiabatically connected to the minigap at \( A = 0 \) (induced by the symmetry breaking perturbation \( \Delta \)), it is evident that at half-filling the current is zero. Similarly, for \( m \) fully occupied minibands taking into account the four-fold valley and spin degeneracy, the electron density (counted from that at \( \mu = 0 \)) is \( \delta n = 4m k/2\pi \). This results in a current \( j = e\delta n \). Identifying \( uk/2\pi \) with the SAW frequency \( f \), we obtain quantized current \( j = 4mf \). The dependence of the energy gaps on \( A \), shown in Fig. 3, describes the width of the plateaus of quantized current.

To complete our analysis it remains only to explore the integrity of the current quantization in the presence of electron interactions. To undertake this program it is convenient to first bosonize the Hamiltonian (1) setting \( \psi_j(x) \propto \exp(i\sqrt{\pi}\phi_j(x)) \). Introducing the linear combination of bosonic fields

\[
\begin{pmatrix}
\Phi_0 \\
\Phi_1 \\
\Phi_2 \\
\Phi_3
\end{pmatrix} = \frac{1}{2} \begin{pmatrix}
1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 \\
1 & -1 & -1 & 1 \\
1 & 1 & 1 & 1
\end{pmatrix} \begin{pmatrix}
\phi_1 \\
\phi_2 \\
\phi_3 \\
\phi_4
\end{pmatrix},
\]

the part of the Hamiltonian (1) without the mass term \( \Delta \bar{\psi} \bar{\psi} \) is diagonalized. Setting \( V_{ext}(x) = V_\delta + A\sin kx \), where \( V_\delta \) represents the external gate potential, and \( K(q) = 1 + 4V_0(q)/\pi h_v \), the corresponding Lagrangian

\[ \mathcal{L}_0 = \frac{1}{2} \sum_\alpha \left[ \partial_\tau \Phi_\alpha(q) \partial_\tau \Phi_\alpha(-q) + K(q)q^2 \Phi_\alpha(q)\Phi_\alpha(-q) \right] + \int dx \left[ \frac{1}{2} \sum_{\alpha=1}^3 (\partial_\tau \Phi_\alpha)^2 + \frac{2}{\sqrt{\pi}h_v} V_{ext}(x) \Phi_\alpha(x) \right] \]

describes the dynamics of one charged and three neutral modes. Restoring the mass term perturbation, the total Lagrangian is given by \( \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\Delta \), where

\[ \mathcal{L}_\Delta = -2\Delta \int dx \sum_{\alpha=1}^4 \cos(\sqrt{\pi}\phi_\alpha). \]

Applied to \( \mathcal{L} \), the conventional RG approach demonstrates that the perturbation \( \mathcal{L}_\Delta \) is relevant and grows. Depending on the density, controlled by \( V_\delta \), the resulting state can be gapped with a finite correlation length, or gapless.

Let us first focus on the influence of electron interactions on the energy gap at the band center considering the system at half-filling and in the absence of the SAW (i.e. \( V_{ext} = 0 \)). Technically, this involves estimating the energy of a soliton field configuration \( \phi_j \) (with any flavor \( j \)). A variational analysis which takes into account the renormalization due to the three neutral modes, obtains

\[ E_{\text{gap}} \approx K^{1/2} E_0^{1/5} \Delta^{4/5}, \]

where \( E_0 = \hbar \nu/d \), substantially larger than the non-interacting result, \( \Delta \).

Similarly, the SAW-induced minigaps (3) are also enhanced by interaction. Considering the regime \( \Delta \ll \hbar k_v \), this enhancement is most straightforwardly demonstrated by mapping the SAW-induced gap onto the gap at the band center. This is achieved by a variable shift,

\[ \phi_j \to \phi_j - (\sqrt{\pi}h_v)^{-1} \int_0^x \tilde{K}^{-1} V_{ext}(x') dx', \]

eliminating the term linear in \( \Phi_0 \) from Eq. (3). [The operator in (4) is diagonal in Fourier representation, \( \tilde{K} = K(q) \).] At the same time the mass term (1) is transformed as
\[ \mathcal{L}_\Delta = -\Delta \int dx \sum_{f=1}^{4} e^{i(\sqrt{\pi} \phi_f + \tilde{\lambda} \cos kx - 2\bar{V}_g x)} + \text{c.c.}, \tag{10} \]

where \( \tilde{\lambda} = 2A/K\hbar v_f, \bar{V}_g = V_f/K\hbar v_f \), and \( K = 1 + \frac{\pi}{2} V(k) \). (Indeed, the shift (8) is nothing but the bosonization representation of the gauge transformation used to solve the free fermion Dirac equation (3).) Now, by analogy with the treatment of Eq. (3) above, one can expand \( \mathcal{L}_\Delta \) in Fourier components \( m \). The density corresponding to \( m \) filled minibands can be chosen by setting \( \bar{V}_g = mk/2 \). In this case, all terms in the Fourier series \( \mathcal{L}_m \) with \( m \neq 2\bar{V}_g/k \) give rise to expressions with oscillatory spatial dependence. Discarding these non-resonant terms one arrives at an expression of the form (8) with \( \Delta \) replaced by \( \Delta J_m(\tilde{\lambda}) \). Being now formally equivalent to the problem at half-filling considered above, we deduce that the interaction brings about a renormalization of the minigap such that

\[ E_{\text{gap}}^{(m)} \approx K^{1/2} E_0^{1/5} |\Delta J_m(2A/K\hbar v_f)|^{4/5}. \tag{11} \]

Several features of this result are worth noting: the general form of the energy gap dependence on the SAW amplitude, with nodes at the roots of Bessel functions, is unaffected by electron interaction. The magnitude of the minigap is enhanced by ca. \( K^{1/2} (E_0/\Delta)^{1/5} \) as compared to the non-interacting case. The rescaling of the SAW amplitude \( A \) and of \( \bar{V}_g \) by \( K \), manifest in Eq. (11), describes the effect of screening due to the 1D electron system. For the substrate dielectric constant \( \epsilon = 12 \) (which corresponds to GaAs) we estimate the screening factor as \( K \approx 15 \).

To complete our discussion, let us comment on the feasibility of the experiment (Fig. 1 inset). Maximal values of the SAW induced minigaps in Fig. 3 are close to \( \Delta \), one half of the value of the central gap. If a longitudinal magnetic field is used to open the central gap, then for a single-walled nanotube with a diameter 1.6 nm (such as that grown by Ref. [23]), and a field \( B = 16 \text{T} \), one finds \( \Delta \approx 5 \text{ meV} \). Applied to the spectra in Fig. 3 where \( \Delta = 0.4\epsilon_0 \) (i.e. \( \epsilon_0 = 12 \text{ meV} \)), it corresponds to a SAW wavelength of \( \lambda_{\text{SAW}} \approx 0.25 \mu \text{m} \), frequency \( f = 13 \text{ GHz} \), and quantized current of around 8 nA. In order to reach a maximum value of the principal SAW induced minigap shown in Fig. 3 the SAW potential should be around \( A = 10 \text{ meV} \). This value obtained in the single electron approximation should be corrected by the factor \( K \approx 15 \) to account for the screening in the 1D system. Thus a SAW potential of around several hundred meV may be required. These values do not present a problem even when a weak piezoelectric such as GaAs is used (11). Moreover, for the experiments with nanotubes one can use a much stronger piezoelectric such as LiNbO3 as a substrate, which will make SAW potential in the eV range available. A strong piezoelectricity of the substrate will also facilitate generation of the high frequency SAW required for the proposed experiment (in LiNbO3, the SAW frequencies of ca. 17 GHz have been reported [24]). One could also use the “zig-zag” nanotubes in which the central gap opens (13) due to the curvature of the carbon sheet. In this case the gap is predicted (13) to be in a range up to 20 meV for a tube diameter of 1.6 nm, and the magnetic field is not necessary. Thus a SAW induced minigap as large as 10 meV could be obtained in this case.

To summarize, we have considered a metallic carbon nanotube in the field of a slowly moving periodic potential. If the nanotube is subjected to a further perturbation that mixes right and left moving states, the coupling between the electrons and the SAW potential acts as a charge pump conveying electrons along the tube. An estimate of the miniband spectrum induced by electron diffraction on the sliding potential revealed that minigaps of ca. 10 meV are viable. We therefore conclude that the carbon nanotube combined with the SAW provides a promising system in which quantized adiabatic charge transport can be observed. As demonstrated above, the energy gaps, that can be detected experimentally through quantization plateau widths, are sensitive to the character of electron interactions. Thus, quantized transport in this strongly interacting system can be viewed as a novel probe of the Luttinger liquid physics.

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