Analytical Calculation of Large Deflection of Annular Thin Plate of Damping Valve

Yijie Chen¹, Wei Nie¹², Xiaodong Gao¹, Baoqiang Li¹, Yiqiang Wan¹
¹China North Vehicle Research Institute, Beijing
²School of Mechanical Engineering, Beijing Institute of Technology, Beijing

chenyijie1206@163.com

Abstract: Aiming at the phenomenon of bending deformation caused by the pressure difference of the throttle plate in the working process of the damping valve, the analytical calculation of the large deflection of the annular thin plate was carried out. On the basis of extracting the physical model of the thin plate, the boundary conditions and initial conditions of the annular thin plate were set by using the Qian's perturbation method, and the nonlinear large deflection deformation analytical formula was derived; Through the finite element modeling, the accurate numerical solution of thin plate deformation was obtained, and the key parameters of the analytical formula were corrected by the least squares method. Through verification, the modified analytical formula had the characteristics of easy calculation and high calculation accuracy, and the influence law of the large deflection of the throttle valve was studied, which provided the necessary theoretical support for the accurate design of the damping valve performance.

1. Introduction

In the vehicle suspension damping device, there are many ways to generate the damping force, and the use of the throttle plate deformation slit to throttle the oil is widely used in engineering practice. After the valve is opened, the throttle plate is mainly in a large deflection state. However, in the existing design method, the small deflection deformation based on the classical loading method is mostly solved, and such an approximation method causes a large error. It is difficult to make the actual damping force of the damper correspond to the theoretical value, which does not meet the design requirements well⁴⁻⁵.

Due to the complicated boundary conditions of the large deflection of the annular thin plate, the research results are few, so there is no mature theory applicable to engineering practice. Based on this situation, this paper proposes a derivation and systematic study on the large deflection of the annular throttle valve using the curve fitting method combining the Qian's perturbation method and the finite element analysis.

2. Physical model of throttle plate

Figure 1 is a schematic view showing the assembly of the throttle plate of the vibration damping device, the throttle valve piece is installed in the end of the piston. During the reciprocating motion of the piston, due to the different pressures of the upper and lower chambers, the pressure difference generated causes the outer edge of the valve plate to deform, and the oil will flow through the deformed annular gap, thereby generating a damping force to attenuate the vibration of the vehicle body. The throttle plate is separately extracted for simplification of the model, as shown in figure 2, in which the inner ring of the annular valve is fully constrained and the outer ring is in a freely deformed state. In the figure, q is the
uniform load, \( h \) is the thickness of the valve, \( r_a \) is the inner radius of the valve, and \( r_b \) is the outer radius of the valve.

3. Derivation of large deflection deformation of throttle plate

3.1 Derivation of deflection deformation based on perturbation method

The Von Kanman axisymmetric thin plate equation is transformed into the following form\(^{[3-4]}\):

\[
\begin{aligned}
y^2 \frac{d^2 \varphi(y)}{dy^2} &= \varphi(y) S(y) + y^2 \psi(y) \\
y^2 \frac{d^2 S(y)}{dy^2} &= -\frac{1}{2} \varphi^2(y)
\end{aligned}
\]  
(1)

Introduce the following dimensionless parameter:

\[
\begin{aligned}
y &= \frac{r^2}{r_b^2}; \quad W = \sqrt{3(1-u^2)} \frac{W}{h}; \quad \varphi(y) = y \frac{dW}{dy}; \quad \alpha = \left( \frac{r_a}{r_b} \right)^2 \\
S(y) &= 3(1-u^2) \frac{r_b^3 N_y y}{Eh^3}; \quad \psi(y) = \frac{3r_b^2 (1-u^2) \sqrt{3(1-u^2)}}{4\pi Eh^3} R;
\end{aligned}
\]  
(2)

Where \( w \) is the deformation of the outer edge of the annular valve piece, \( u \) is the Poisson's ratio, \( E \) is the elastic modulus, \( N_y \) is the radial film tension, and \( R \) is the combined external load. In the case of uniform load, there is:

\[
R = \int_{r_a}^{r_b} q(r) 2\pi rdr = r_b^2 \pi \int_{a}^{b} qdy
\]  
(3)

Bring equation (3) into (2) and organize it:

\[
\psi(y) = \frac{3r_b^2 (1-u^2) \sqrt{3(1-u^2)}}{4Eh^3} q \left( 1 - \frac{\alpha}{y} \right) = Q \left( 1 - \frac{\alpha}{y} \right)
\]  
(4)

Give the boundary conditions of the physical model of the valve plate in Figure 2:
\[ y = \alpha: \begin{cases} \varphi(y) = 0 \\ S(y) = \frac{2\alpha}{1+u} \frac{dS(y)}{dy} \end{cases} \quad y = 1: \begin{cases} \varphi(y) = \frac{2}{1-u} \frac{d\varphi(y)}{dy} \\ S(y) = 0 \end{cases} \] (5)

Bring into (1) and organize it:

\[ \begin{align*}
  y^2 \frac{d^2 \varphi(y)}{dy^2} &= \varphi(y) S(y) + y^2 Q \left(1 - \frac{\alpha}{y}\right) \\
y^2 \frac{d^2 S(y)}{dy^2} &= -\frac{1}{2} \varphi^2(y)
\end{align*} \quad \alpha < y < 1 \] (6)

Let the dimensionless deflection function \( W(y) \) is:

\[ W(y) = \int_{\alpha}^{1} \frac{1}{\xi} \varphi(\xi) d\xi \] (7)

Then take the dimensionless deflection function \( W_m(y) = \int_{\alpha}^{1} \frac{1}{\xi} \varphi(\xi) d\xi \) at the outer edge \( (y=1) \) of the annular valve plate as the perturbation parameter.

Expand all unknown functions into power series of \( W_m(y) \):

\[ W(y) = \sum_{i=1}^{\infty} W_i(y) W_m^{2i-1} \quad \varphi(y) = \sum_{i=1}^{\infty} \varphi_i(y) W_m^{2i-1} \] (8)

\[ S(y) = \sum_{i=1}^{\infty} S_i(y) W_m^{2i} \quad Q = \sum_{i=1}^{\infty} \alpha_i W_m^{2i-1} \] (9)

Where \( Q \) is the dimensionless parameter of the load strength, \( W_i(y), \varphi_i(y), S_i(y) \) and \( \alpha_i \) are the undetermined coefficients, and \( i \) is the order of the solution.

Under normal circumstances, when \( i=2 \), the accuracy requirement can be satisfied, and equations (8) and (9) are respectively brought into equations (5) and (6) and are arranged:

\[ \begin{align*}
  \varphi_1(\alpha) W_m + \varphi_2(\alpha) W_m^3 &= 0 \\
  S_1(\alpha) W_m^2 + S_2(\alpha) W_m^4 &= \frac{2\alpha}{1+u} \left( S_1'(\alpha) W_m^2 + S_2'(\alpha) W_m^4 \right) \\
  \varphi_1(1) W_m + \varphi_2(1) W_m^3 &= \frac{2}{1-u} \left( \varphi_1'(1) W_m + \varphi_2'(1) W_m^3 \right) \\
  S_1(1) W_m^2 + S_2(1) W_m^4 &= 0
\end{align*} \] (10)

(11)
\[
\begin{align*}
&y^2 \varphi_1''(y) W_m + y^2 \varphi_2''(y) W_m^3 = \varphi_1(y) S_1(y) W_m^3 + \varphi_1(y) S_2(y) W_m^5 + \varphi_2(y) S_1(y) W_m^5 \\
&+ \varphi_2(y) S_2(y) W_m^5 + y^2 \left( \alpha_1 W_m + \alpha_2 W_m^3 \right) \left( 1 - \frac{\alpha}{y} \right) \\
&y^2 S_1''(y) W_m^2 + y^2 S_2''(y) W_m^4 = -\frac{1}{2} \left( \varphi_1''(y) W_m^2 + \varphi_2''(y) W_m^6 + 2 \varphi_1(y) \varphi_2(y) W_m^6 \right)
\end{align*}
\]  

(12)

Compare the coefficients of the similar terms at both ends of the equations (10) to (12) and sort them out:

\[
\begin{align*}
W_m^3: & \quad \varphi_1'(\alpha) = 0, \varphi_1'(1) = \frac{2}{1-u} \varphi_1'(1) \\
W_m^2: & \quad \varphi_1(\alpha) = 0, \varphi_1(1) = \frac{2}{1-u} \varphi_1'(1) \\
W_m: & \quad \varphi_1''(1) = \frac{2}{1-u} \varphi_1''(1)
\end{align*}
\]  

(13)

From equations (7) and (8) there are:

\[
\int_a^L \frac{1}{\xi} \varphi(\xi) d\xi = \int_a^L \frac{1}{\xi} \varphi_1(\xi) d\xi + \int_a^L \frac{1}{\xi} \varphi_2(\xi) d\xi + \left( \int_a^L \frac{1}{\xi} \varphi(\xi) d\xi \right)^2
\]  

(15)

Compare the similar items at both ends:

\[
\int_a^L \frac{1}{\xi} \varphi_1(\xi) d\xi = 1 \quad \int_a^L \frac{1}{\xi} \varphi_2(\xi) d\xi = 0
\]  

(16)

The simultaneous equations (13) to (16) can be used to find the expressions of \( \alpha_1 \), \( \alpha_2 \), \( \varphi_1 \), \( \varphi_2 \) and \( S_1 \) respectively, and bring them into the following equation \([6]\):

\[
Q = \alpha_1 W_m + \alpha_2 W_m^3
\]  

(17)

And finishing:

\[
r_b^4 q = 4Ehw \left[ \frac{\alpha_1 h^2}{3(1-u^2)} + \alpha_2 w^2 \right]
\]  

(18)

The above formula is a large deflection function expression of the second-order annular thin plate under uniform load obtained by the Qian's perturbation method.

3.2 Curve fitting analytical formula

Formulate the linear function expression of equation (18):

\[
w = \frac{r_b^4 q - 4E\alpha_2 hw^3}{16\alpha_1 D}
\]  

(19)
Where $D$ is the bending stiffness of the valve plate.

Write the residual error squared sum expression of the flex function:

$$Q = \sum_{i=1}^{n} (w - w_i)^2 = \sum_{i=1}^{n} \left( \frac{r_i^4 q_i - 4E\alpha_2 h_i w_i^3}{16\alpha_1 D} - w_i \right)^2$$  \hspace{1cm} (20)

Where $i(1 \cdots n)$ is the number of data sets, $h_i$ is the thickness of the valve, $q_i$ is the load value in the different data sets, and $w_i$ is the deflection value obtained from the finite element.

Solve the partial derivatives of the above equations $\alpha_1$ and $\alpha_2$ respectively:

$$\frac{\partial Q}{\partial \alpha_1} = -2\sum_{i=1}^{n} \left[ r_i^4 q_i - 4E\alpha_2 h_i w_i^3 \right] \left[ \frac{r_i^4 q_i - 4E\alpha_2 h_i w_i^3}{16\alpha_1 D} - w_i \right] = 0$$  \hspace{1cm} (21)

$$\frac{\partial Q}{\partial \alpha_2} = -2\sum_{i=1}^{n} \left[ \frac{E h_i w_i^3}{4\alpha_1 D} \left( \frac{r_i^4 q_i - 4E\alpha_1 h_i w_i^3}{16\alpha_1 D} - w_i \right) \right] = 0$$  \hspace{1cm} (22)

For the simultaneous (22) and (23), $\alpha_1 = 48.28, \alpha_2 = 1.03$ can be obtained. The expression of the large deflection function of the annular valve based on curve fitting is:

$$r_i^4 q_i = E h_i w_i \left[ 64.38 \left( \frac{h_i^2}{1-u^2} \right) + 4.14v^2 \right]$$  \hspace{1cm} (23)

### 4 Analytical verification and analysis

Using curve fitting analytical equation (23) and finite element to calculate the large deflection of the outer edge of the throttle plate under different loads, the comparison results are shown in figure 3-4 by Matlab.

**Figure 3.** The contrastive results when slice thickness is 0.4mm

**Figure 4.** The contrastive results when slice thickness is 0.5mm
Figure 5. The contrastive result in different slice E

Figure 6. The contrastive result in different slice thickness

It can be seen from figure 5-6 that with the increase of the elastic modulus, the large deflection of the outer edge of the annular valve plate under the same load is gradually reduced; as the thickness increases, the outer edge of the annular valve plate is greatly deflected, the amount of deformation is reduced.

5. Conclusions
According to the least squares principle fitting, the relevant parameters in the analytical formula are determined. The correctness and accuracy of the fitting analytical formula are verified. The influence of the structural parameters and material properties of the valve on the large deflection is analyzed.

The analytic formula and simulation conclusion of the large deflection of the throttle plate obtained by curve fitting provide the possibility of accurate design of the damping valve, and provide an effective calculation method for the independent research and development of the products.

6. References
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