I. INTRODUCTION

Quantum devices allowing coherent states are in focus of intense experimental and theoretical studies, in particular in connection with the problem of quantum computation. A promising example of such systems is a set of superconducting circuits connected by nanometer size contacts. Phase coherence between two superconductors is maintained if electron (Cooper pair) exchange between them is allowed. A quantum state resulting from such coupling is a coherent superposition of states with different number of Cooper pairs in the linked superconductors. As is well known, transfer of Cooper pairs is a direct manifestation of such states a non-dissipative current depending on the phase difference between the leads can be measured.

II. MODEL SYSTEM

First, to demonstrate this phenomenon let us consider a superconducting grain initially in a state $|n\rangle$ with a fixed number of Cooper pairs $n$ which is brought into contact with a bulk lead and then removed from the latter. Since the Josephson coupling decays strongly with the distance, the grain finally becomes isolated in a state, $|\psi\rangle = \sum_n c_n e^{-i\phi_n} |n\rangle$. The phases $\phi_n$ will then depend on the superconducting phase of the bulk superconductor. Below we will refer to such a state as a Josephson Hybrid (JH). If a grain initially in a hybrid state passes by the lead it acquires a charge which depends on the coefficients $c_n e^{-i\phi_n}$, most importantly on the phases $\phi_n$, as well as on the phase on the lead.

Now let us consider a set of leads and allow the grain to pass the leads sequentially. Since between the leads the grain is decoupled from the environment, the coefficients $c_n$ will remain constant while the phases $\phi_n$ evolve according to $\dot{\phi} = \hbar^{-1} (\langle n | H_c | n \rangle$ where $H_c$ is the Hamiltonian of an isolated grain. Each interaction with a lead causes a phase-dependent charge transfer. The resulting state as well as the total charge transfer is determined by the whole “history” of the grain’s motion.

An important issue is that the JH can be created only if the energy differences between states with different numbers of Cooper pairs are smaller than the Josephson energy $E_J$. This can be achieved even if $E_J$ tends to zero by gate electrodes which induce charges on the grain. If the gate voltage is properly chosen then the states with two subsequent numbers of Cooper pairs on the grain become equal. In other words, the Coulomb blockade of tunneling is lifted, as in the case of a single-electron transistor. As a result, the JH consists of only two states with subsequent numbers of Cooper pairs.

As the grain is removed from the lead the problem of how long the “memory” of a proximity between two superconductors persists arises. This question was first addressed by Leggett in Ref. [7] who considered the motion of a superconducting grain between superconducting...
leads. There it was assumed that the grain motion in the contact regions is slow enough to reach a local thermal equilibrium between the grain and the nearest lead. As sources of destruction of such a memory he discussed both external random fields and the so-called internal mutual dephasing due to relaxation to the equilibrium during subsequent contacts to the leads. In contrast to that approach, our system is intrinsically non-adiabatic in the contact regions. Therefore the equilibrium is not reached during the contacts, the system evolves according to the laws of quantum mechanics, and intrinsic dynamics does not occur.

Another important difference is that we consider the case of strong Coulomb blockade, when the charging energy \( E_C \) is much larger than the Josephson coupling energy \( E_J \). As a result, the hybrid state is constituted of only two charge states differing by one Cooper pair as mentioned above. In this way we avoid the decoherence due to quantum beats between many quantum states discussed in Ref. [10].

### III. CALCULATIONS

For concrete calculations let us consider charge transfer between two bulk leads due to repeated alternate contacts of the grain with those leads when the grain moves periodically in time as shown in Fig. 1. A similar system was recently realized experimentally [10]. If we prepare the system in a state such that after one period its state differs from the previous one only by a phase factor, then the charge transfer is periodic in time. Its magnitude as well as the direction are determined by the phase difference between the leads, as well as by the phase acquired in course of free motion. This is a new mechanism for Josephson charge transfer due to Cooper pair shuttling which is a direct manifestation of coherent transport.

Our model Hamiltonian as a function of the grain position \( x \) can be expressed as a sum of the electrostatic part

\[
\hat{H}_C = \left[ e^2/2C(x) \right] [2\hat{n} + Q(x)/e]
\]

(1)

and Josephson coupling with the left (\( L \)) and right (\( R \)) leads,

\[
\hat{H}_J = - \sum_{s=L,R} E^s_J(x) \cos(\Phi_s - \tilde{\Phi}) .
\]

(2)

Here \( \hat{n} \) is the operator of the Cooper pair number on the grain, \( Q(x) \) is the charge induced on the grain by the gate, \( \Phi_s \) are the order parameter phase of the \( s \)-th lead while \( \tilde{\Phi} \) is the order parameter phase operator of the grain. The operators \( \hat{n} \) and \( \Phi \) obey the commutation relation \([\hat{n}, \Phi] = i\) as far as we restrict ourselves by the ground state. To make that possible we require the typical frequency of the grain oscillations to be much less than \( \Delta/\hbar \). In the Eqs. (1) and (2), \( C(x) \) is the mutual capacitance, while \( E^s_J(x) \) is the Josephson coupling energy. Only the states with even number of electrons on the grain are taken into account. As it is known [8], that requires the inequality \( \Delta \geq e^2/C \).

![FIG. 1. Schematic of the specific system described in the text. A superconducting grain executes periodic motion between two superconducting bulk leads. The presence of the gate ensures that the Coulomb blockade is lifted during the contacts between the grain and superconductor which allows for the grain to be in the Josephson hybrid state. As the grain moves between the leads Cooper pairs are shuttled between them creating a DC-current through the structure.](image-url)

The energy of the Josephson coupling between the grain and the electrodes depends strongly on position and is given by \( E^L_R(x) = E_0 \exp(-|\delta x_{L,R}|/l) \) where \( \delta x_{L,R} \) is the distance between the grain and the right/left contact and \( l \) is the characteristic tunneling length.

The dynamics of the system are governed by the evolution operator

\[
\hat{U}(t_2; t_1) = \hat{T} \exp(-i\hbar^{-1} \int_{t_1}^{t_2} dt' \hat{H}(x(t'))) .
\]

(3)

For periodic grain motion \( x(t + T) = x(t) \) it is convenient to consider the periodic eigenstates \( |\alpha\rangle \) at \( x_A = x(t_A) \) (cf. with Fig. 2)

\[
\hat{U}(t_A + T; t_A) |\alpha\rangle = e^{-i\lambda_\alpha} |\alpha\rangle .
\]

(4)

The Liouville equation for the statistical operator \( \hat{\rho}(t) \) is chosen in the form

\[
d\hat{\rho}(t)/dt = -i\hbar^{-1} [\hat{H}(t), \hat{\rho}(t)] - \nu(t) [\hat{\rho}(t) - \hat{\rho}_0(t)] .
\]

(5)

The last item in the right-hand side of Eq. (5) allows for the relaxation to the adiabatic equilibrium distribution,

\[
\hat{\rho}_0 = Z^{-1} \exp\left( -\beta \hat{H}(x(t)) \right) .
\]
Here $\beta$ is the inverse temperature while $Z = \text{Tr} \exp \left( -\beta \hat{H} [x(t)] \right)$. We specify the relaxation as $\nu(t) = \nu_0 \exp \left( -|\delta x(t)|/l \right)$. One needs to introduce the relaxation in order to "forget" the initial conditions to Eq. (3). Physically, the relaxation is caused by incoherent charge transfer, such as quasiparticle tunneling. In the following we will assume that the relaxation is infinitely slow. Then it does not enter the results explicitly. However, it determines the populations through the balance equation (6). The latter has the simplest form in the representation of the evolution operator, Eq. (6). In this case $\hat{\rho}$ is diagonal in the $|\alpha\rangle$ representation

$$
\rho_{\alpha\alpha} = \nu(t) \langle \alpha | \hat{U}(T; t) \hat{\rho}(t) \hat{U}(T; t) | \alpha \rangle / T \tag{6}
$$

where $A(t) = T^{-1} \int_0^T dt A(t)$ is the time average over the period $T$ of the grain’s motion. Equation (6) can be directly obtained as a time average of Eq. (5).

Knowing the eigenstates (4) one can easily calculate the electric current

$$
\bar{I} = 2e f \sum \rho_{\alpha\alpha} I_{\alpha}, \quad f = T^{-1} \tag{7}
$$

where $I_{\alpha}$ is the charge carried through the cross section $x_A$ (cf. with Fig. 2) by the state $|\alpha\rangle$ during one cycle and is given by the general relation

$$
I_{\alpha} \equiv \langle \alpha | \hat{n} - \hat{U}^{\dagger}(t_B; t_A) \hat{n} \hat{U}(t_B; t_A) | \alpha \rangle = \partial \lambda_\alpha / \partial \Phi L. \tag{8}
$$

The dynamics of the system under consideration is most simple if the electrostatic energy cost per tunneling Cooper pair, $E_C \equiv (2e)^2/2C$, is larger than the Josephson coupling energy, $E_J$. Then the tunneling transport can be blocked by Coulomb effects, and resonant tunneling of Cooper pairs is possible only at very specific values of the induced charge $Q$. In particular, when $Q = Q_n = -(2n+1)e$ only two states with the number of excess Cooper pairs at the grain which differs by one are possible. This leaves us inside a two-state Hilbert space which can be characterized by the Cooper pair number at the grain, $|n\rangle$ and $|n+1\rangle$. If at that instant the Josephson coupling is “on”, a coherent hybrid, $c_1 |n\rangle + c_2 e^{i\vartheta} |n+1\rangle$, is formed. Such a situation takes place each time the grain passes the lead. Consequently, the state evolution can be decoupled into the “scattering” regions where the grain is in touch with a lead and “free motion” regions where the grain is decoupled from the leads and its state evolves as that of a non-interacting system. The grain’s trajectory is schematically shown in Fig. 2. For such motion the time evolution operator $\hat{U}$ has the form of a $2 \times 2$ matrix which can be factored into four parts pertaining to the two scattering events and the evolution of the state between the leads i.e.

$$
\hat{U}(t_A + T; t_A) = \hat{U}_- \hat{S}_R \hat{U}_+ \hat{S}_L. \tag{9}
$$

Here

$$
\hat{U}_\pm = \exp (i \chi_\pm \sigma_3), \chi_\pm = (1/h) \int dt \delta E_L[x(t)] \tag{10}
$$

describe the “free” evolution along the upper, right moving, and the lower, left moving, paths respectively. The quantity $\delta E_L(x)$ is the difference in electrostatic energy for the two charge states in the hybrid while $\sigma_3$ is the Pauli matrix. The scatterings with the leads are given by the matrices

$$
\hat{S}_s = \exp [i \partial_\varphi (\sigma_1 \cos \Phi_s + \sigma_2 \sin \Phi_s) ]. \tag{11}
$$

Here $\vartheta_s$ are dimensionless contact times with the leads $s = R, L$ defined as $\vartheta_s = h^{-1} \int dt \delta E_s(x(t))$. In the following we will assume for simplicity that $\vartheta_R = \vartheta_S = \vartheta$. Since the exponential rapidly decays when the grain is far from the lead the integral can be decoupled into the integrals over the scattering regions and the integrals over the regions of the free motion. Inserting Eq. (3) for the evolution operator into the expression (5) for the electric current, one finds the relationship between the dc current, $\bar{I}$ and the eigenvalues of the evolution operator,

$$
\bar{I} = 2ef \sum \rho_{\alpha\alpha} I_{\alpha} = \epsilon f (\partial \lambda_1 / \partial \Phi R)(\partial \lambda_1 / \partial \vartheta). \tag{12}
$$

This expression is non-trivial because it expresses the current in a non-equilibrium state in an equilibrium fashion.

The eigenvalues for our two-state system can be expressed as $\exp (-i\lambda_\alpha)$, where $\lambda_1 = -\lambda_2 \equiv \lambda$ is given by the equation

$$
\cos \lambda = \cos^2 \vartheta \cos \chi - \sin^2 \vartheta \cos \Phi. \tag{13}
$$

Here we have defined $\chi = \chi_+ - \chi_-$ and $\Phi = (\Phi_R - \Phi_L) + (\chi_+ - \chi_-)$. The dc current for the case $\vartheta_L = \vartheta_R \equiv \vartheta$ is

$$
\bar{I} = 2ef \cos \vartheta \sin^3 \vartheta \sin \Phi (\cos \chi + \cos \Phi) \left( 1 - (\cos^2 \vartheta \cos \chi - \sin^2 \vartheta \cos \Phi)^2 \right). \tag{14}
$$
IV. DISCUSSION AND CONCLUSIONS

The oscillating dependence of the dc current on the phase difference $\Phi_R - \Phi_L$ is a direct consequence of the fact that the coherent states are controlled by the phase difference $\Phi$, see Eq. (13). In this way the Cooper pair shuttle discussed above provides a way to prepare and control a coherent superposition of quantum states in a two-state system which is referred to as a qubit. If there is no phase difference, $\Phi_L = \Phi_R$, but the grain’s trajectory is asymmetric, $\chi_+ \neq \chi_-$, the current still does not vanish. If the grain’s trajectory embeds some magnetic flux created by external magnetic field with vector-potential $A(r)$, an extra item $\left(2\pi/\Phi_0\right) \oint A(r) \cdot dr$ enters the expression for the phase difference $\Phi$ which must be gauge-invariant. Here $\Phi_0 = \pi hc/e$ is the magnetic flux quantum.

At small contact times, $\vartheta \ll 1$,

$$I \approx 2ef \vartheta^3 \sin \Phi (\cos \chi + \cos \Phi).$$

(15)

The smallness of this current in this case is natural because it is the contact with the leads which creates the coherent state.

To make the system under consideration realistic, one has to design a setup with large decoherence times. Firstly, the device should be carefully screened from electromagnetic perturbations as well as from other time-dependent magnetic fields. Secondly, the gate electrode, as well as the insulating region between the gate and the grain, should contain as little as possible of charged dynamic defects. Time-dependent switching between the states of those defects would produce charge fluctuations coupled to the charge states of the grain. It seems very difficult to estimate theoretically the total dephasing. For the estimate of the dephasing one can take the experimental results obtained by Nakamura et al. [1] who have observed coherent response of a superconducting qubit during the time $\tau_{\phi} = 10^{-9} - 10^{-8}$ s. This is a lower bound for our case because the grain is coupled to the leads only during a small part of the period of its motion. Consequently, the contact time is of the order of $\tau_c \approx T \sqrt{l/d}$ where $d$ is the distance between the leads. It is this quantity that should be compared to the dephasing time, $\tau_{\phi}$, since the rest of the period the grain is decoupled from the leads. One can imagine another implementation of the Cooper pair shuffling where the alternating couplings between the grain and the leads are controlled by time dependent potential barriers rather than by mechanical motion. To conclude, we have demonstrated a possibility to create a coherent quantum superposition between two charge states of a superconducting grain by repeated alternating contacts with two (or more) superconducting leads. This state can be controlled by the phase difference between the leads and monitored by a phase-dependent dc current though the closed circuit including the leads.

---

[1] Y. Nakamura, Yu. A. Pashkin and J. S. Tsai, Nature 398, 786 (1999).
[2] B. D. Josephson, Phys. Lett 1, 251 (1962).
[3] M. T. Tuominen, R. V. Krotkov and M. L. Breuer, Phys. Rev. Lett. 83, 3025 (1999).
[4] H. Park, J. Park, A. K. L. Lim, E. H. Anderson, A. P. Alivsatos, and P. L. McEuen, Nature 407, 57 (2000).
[5] L. Y. Gorelik, A. Isacsson, M. V. Voinova, B. Kasemo, R. I. Shekhter, and M. Jonson, Phys. Rev. Lett. 80, 4526 (1998).
[6] T. Rueckes, K. Kim, E. Joselevich, G. J. Tseng, C.-L. Cheung, C. M. Lieber, Science 289, 94 (2000).
[7] A. J. Leggett Found. Phys. 21, 353 (1991).
[8] K. A. Matveev, M. Gisselblatt, L. I. Glazman, M. Jonson and R. I. Shekhter, Phys. Rev. Lett. 70, 2940 (1993).
[9] J. E. Mooij, T. P. Orlando, L. Levitov, Lin Tian, Caspar H. van der Wal, Seth Lloyd, Science, 285, 1036 (1999).
[10] A. Erbe, C. Weiss, W. Zwerger and R. H. Blick, Phys. Rev. Lett. 87, 096106 (2001).