Abstract. An overview is given of the potential for neutrino physics studies through parasitic use of the intense high energy neutrino beams that would be produced at future many-TeV muon colliders. Neutrino experiments clearly cannot compete with the collider physics. Except at the very highest energy muon colliders, the main thrust of the neutrino physics program would be to improve on the measurements from preceding neutrino experiments at lower energy muon colliders, particularly in the fields of B physics, quark mixing and CP violation. Muon colliders at the 10 TeV energy scale might already produce of order $10^8$ B hadrons per year in a favorable and unique enough experimental environment to have some analytical capabilities beyond any of the currently operating or proposed B factories. The most important of the quark mixing measurements at these energies might well be the improved measurements of the important CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$ and, possibly, the first measurements of $|V_{td}|$ in the process of flavor changing neutral current interactions involving a top quark loop. Muon colliders at the highest center-of-mass energies that have been conjectured, 100–1000 TeV, would produce neutrino beams for neutrino-nucleon interaction experiments with maximum center-of-mass energies from 300–1000 GeV. Such energies are close to, or beyond, the discovery reach of all colliders before the turn-on of the LHC. In particular, they are comparable to the 314 GeV center-of-mass energy for electron-proton scattering at the currently operating HERA collider and so HERA provides a convenient benchmark for the physics potential. It is shown that these ultimate terrestrial neutrino experiments, should they eventually come to pass, would have several orders of magnitude more luminosity than HERA. This would potentially open up the possibility for high statistics studies of any exotic particles, such as leptoquarks, that might have been previously discovered at these energy scales.
I INTRODUCTION: THE ROLE OF MIGHTY MURINEs

The dominant motivation for high energy muon colliders (HEMCs) is unquestionably to explore elementary particle physics at many-TeV energy scales. For the sake of completeness, however, this paper instead discusses what would be the most promising subsidiary fixed target physics program, namely, the parasitic use of the free and profuse neutrino beams at HEMCs to provide complementary precision studies of high energy physics (HEP) at lower energies. Perhaps, this might complement collider studies in fostering new and helpful insights into the properties of elementary particles.

Neutrino interactions have unique potential for precision HEP studies because they only participate in the weak interaction. Today’s neutrino beams from pion decays lack the intensity to fully exploit this potential but future MUon RIing Neutrino Experiments (MURINEs), using neutrino beams from the decays of muons in a muon collider or other storage ring, hold the promise of neutrino beams that are several orders of magnitude more intense than today’s beams (1). The first MURINEs may well be muon storage rings dedicated to neutrino production (2) ("neutrino factories") while the collider rings of any first generation muon colliders will also make excellent MURINEs. The topic of this paper is MURINEs at very high energies and these will be referred to as “Mighty MURINEs”.

At a minimum, Mighty MURINEs will improve on previous MURINEs in providing much useful bread-and-butter precision HEP to feed the hungry masses of HEP experimentalists with aversions to collider mega-experiments. More interestingly, there are a couple of plausible scenarios under which they might do much more; namely (i) if quark mixing and/or B physics offer more than predicted by our naive prejudices as parameterized in the standard model (SM) of elementary particles, and (ii) if leptoquarks or other exotica begin to emerge at or below the 100 GeV energy scale.

The following section surveys the experimental conditions and parameters that might be found at Mighty MURINEs. This is followed by an overview of the potential physics analyses and by three sections going into more detail on the most interesting topics: one each on exploiting Mighty MURINEs as B factories, on the possibilities for quark mixing studies and on the potential for heavy particle production up to the 100 GeV scale.

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2) this is a play on words alluding to the venerable cartoon character Mighty Mouse through the dictionary definition of “murine” as “to do with mice”.

2
II EXPERIMENTAL OVERVIEW

A The Neutrino Beams

Neutrinos are emitted from the decay of muons in the collider ring:

\[
\begin{align*}
\mu^- & \rightarrow \nu_\mu + \bar{\nu}_e + e^-, \\
\mu^+ & \rightarrow \bar{\nu}_\mu + \nu_e + e^+.
\end{align*}
\]

(1)

As is illustrated in figure 1, the thin pencil beams of neutrinos for experiments will be produced from the most suitable long straight sections in the collider ring or, possibly, in the accelerating rings. These will be referred to as the production straight sections. The divergence of the neutrino beam is typically dominated by the decay opening angles of the neutrinos rather than the divergence of the parent muon beam. Relativistic kinematics boosts the forward hemisphere in the muon rest frame into a narrow cone in the laboratory frame with a characteristic opening half-angle, \(\theta_\nu\), given in obvious notation by

\[
\theta_\nu \sin \theta_\nu = 1/\gamma_\mu = \frac{m_\mu c^2}{E_\mu} \approx \frac{10^{-4}}{E_\mu [\text{TeV}]}.
\]

(2)

For the example of 5 TeV muons, the neutrino beam will have an opening half-angle of approximately 0.02 mrad.

The large muon currents and tight collimation of the neutrinos results in such intense neutrino beams that potential radiation hazards (1; 3) are a serious design issue for the neutrino beam-line and even for the less intense neutrino fluxes emanating from the rest of the collider ring.
B Luminosities at Neutrino Experiments

For a cylindrical experimental target extending out from the beam center to an angle $\theta_\mu = 1/\gamma_\mu$, the luminosity, $L$, is proportional to the product of the mass depth of the target, $l$, and the number of muon decays per second in the beam production straight section, according to:

$$L [\text{cm}^{-2}.\text{s}^{-1}] = N_{\text{Avo}} \times f_{ss} \times n_\mu [\text{s}^{-1}] \times l [\text{g.cm}^{-2}],$$

where $f_{ss}$ is the fraction of the collider ring circumference occupied by the production straight section, $n_\mu$ is the rate at which each sign of muons is injected into the collider ring (assuming they all circulate until decay rather than being eventually extracted and dumped) and the appropriate units are given in square brackets in this equation and all later equations in this paper. The proportionality constant is Avagadro’s number, $N_{\text{Avo}} = 6.022 \times 10^{23}$, since exactly one neutrino per muon is emitted on average into the boosted forward hemisphere, i.e. each muon decay produces two neutrinos and half of them travel forwards in the muon rest frame.

Because Avagadro’s number is so large, the luminosities at Mighty MURINEs will be enormous compared to those at collider experiments. The luminosities for a reasonable scenario using the workshop’s straw-man parameter sets (4) are given in table 1 along with, for comparison, the final design goal luminosity for the HERA ep collider. It can be seen that, roughly speaking, Mighty MURINEs might achieve of order a million times the luminosity of HERA. An “accelerator year’s” running – $10^7$ seconds – at the 50 TeV MURINE’s luminosity of $2 \times 10^{37}$ cm$^{-2}$.s$^{-1}$ would amount to an impressive integrated luminosity of 200 inverse attobarns per year while the even bigger straw-man luminosity at 5 TeV, $1 \times 10^{39}$ cm$^{-2}$.s$^{-1}$, requires a luminosity prefix that is even less familiar to the HEP community: 10 inverse zeptobarns per year.

C Center-of-Mass Energies for the Neutrino-Nucleon Interactions

It will be seen in section VI that HERA provides a useful comparison for some of the physics capabilities of Mighty MURINEs, particularly since the maximum center-of-mass energies, $E_{\text{CoM}}$, at the highest energy HEMCs might even be comparable to those at the HERA collider. The electron-proton $E_{\text{CoM}}$ at the collider is given by relativistic kinematics as

$$E_{\text{CoM}}^{\text{HERA}} = 2 \sqrt{E_p E_e},$$

which is 314 GeV for the proton and electron energies of the year 2000 upgrade to HERA, $E_p = 820$ GeV and $E_e = 30$ GeV. For comparison, the MURINE’s $E_{\text{CoM}}$ is

$$E_{\text{CoM}}^{\text{MURINE}} = \sqrt{2E_\nu M_p c^2 + (M_p c^2)^2},$$

where $E_\nu$ is the neutrino energy.
where the proton mass corresponds to $M_p c^2 = 0.938 \text{ GeV}$. The neutrino energy can range right up to the muon beam energy, $E_{\nu}^{\text{max}} = E_{\mu}$, and the energy spectrum seen by the detector is relatively hard (5), with an average neutrino energy within the $1/\gamma_{\mu}$ cone that is 49% of the muon beam energy. The comparative center-of-mass energies for Mighty MURINEs and HERA are summarized in table 1.

**TABLE 1.** Energy, luminosity and event rates for Mighty MURINEs at the 10 TeV and 100 TeV CoM muon colliders given in the HEMC’99 straw-man parameter sets. The center-of-mass energy, $E_{\text{CoM}}$, is given for the neutrino-nucleon system. The energy and luminosity at HERA are provided for comparison. The event rates assume fractional straight section lengths of $f_{ss} = 0.02, 0.01$ for the 5+5 TeV and 50+50 TeV parameter sets, respectively, and a detector mass-per-unit-area of $l = 1000 \text{ g.cm}^{-2}$ that intercepts the neutrino beam out to an angle $\theta_{\mu} = 1/\gamma_{\mu}$ subtended at the beam production straight section.

| Facility       | $E_{\text{CoM}}$ | Luminosity, $\mathcal{L}$ | events/year |
|----------------|------------------|---------------------------|-------------|
| 5 TeV MURINE   | 0 to 97 GeV      | $1 \times 10^{39} \text{ cm}^{-2}\text{.s}^{-1}$ | $1.7 \times 10^{14}$ |
| 50 TeV MURINE  | 0 to 306 GeV     | $2 \times 10^{37} \text{ cm}^{-2}\text{.s}^{-1}$ | $4 \times 10^{10}$ |
| HERA (2000 upgrade) | 314 GeV          | $7 \times 10^{33} \text{ cm}^{-2}\text{.s}^{-1}$ | N.A. |

### D Cross Sections and Event Rates

The event rate in the neutrino detector is a product of the luminosity given in table 1 and the neutrino-nucleon scattering cross-section, which we now discuss.

The predominant interactions of neutrinos and anti-neutrinos at all energies above a few GeV are charged current (CC) and neutral current (NC) deep inelastic scattering (DIS) off nucleons ($N_i$, i.e. protons and neutrons) with the production of several hadrons ($X$):

\begin{align*}
\nu(\overline{\nu}) + N &\rightarrow \nu(\overline{\nu}) + X \quad (NC) \\
\nu + N &\rightarrow l^- + X \quad (\nu - CC) \\
\overline{\nu} + N &\rightarrow l^+ + X \quad (\overline{\nu} - CC),
\end{align*}

(6)

where the charged lepton, $l$, is an electron if the neutrino is an electron neutrino and a muon for muon neutrinos. The cross-sections for these processes are approximately proportional to the neutrino energy, $E_{\nu}$, with numerical values of (6):

\[
\sigma_{\nu N} \text{ for } \begin{pmatrix} \nu - CC \\ \nu - NC \\ \overline{\nu} - CC \\ \overline{\nu} - NC \end{pmatrix} \approx \begin{pmatrix} 0.72 \\ 0.23 \\ 0.38 \\ 0.13 \end{pmatrix} \times 10^{-35} \text{ cm}^2 \times E_{\nu}[\text{TeV}].
\]

(7)

The number of events in the detector is easily seen to be given by:
$$N_{\text{events}} = \mathcal{L}[\text{cm}^{-2}\cdot\text{s}^{-1}] \times 0.73 \times 10^{-35} \times 0.49 \times E_\mu[\text{TeV}] \times T[\text{s}], \quad (8)$$

where $T$ is the running time, $0.49 \times E_\mu$ is the average neutrino beam energy into the detector (5) and $0.73 \times 10^{-35}$ is the total cross-section-divided-by-energy that is obtained from equation 7 after summing over the NC and CC interactions and averaging over neutrinos and anti-neutrinos.

The final column of table 1 shows the impressive event sample sizes predicted from equation 8: up to of order $10^{11}$ events per year in a reasonably sized neutrino target.

### E High Performance Neutrino Detectors for Mighty MURINEs

The unprecedented event samples in small targets at MURINEs will undoubtedly also spark a revolution in neutrino detector design and performance, both to cope with event rates and to fully exploit the physics potential of the beams. An example of a novel general purpose neutrino detector that has been proposed previously (7) for MURINEs is shown in figure 2. The neutrino target is the cylinder at mid-height on the left hand side of the figure. It comprises a stack of equally-spaced CCD tracking planes, oriented perpendicular to the beam and with spacings of order 1 mm, that provides vertex tagging for events with hadrons containing charm or bottom quarks.

The general detector design of figure 2 should remain appropriate for Mighty MURINEs although it would likely be elongated to cope with the more boosted events, including perhaps lengthening the target to several meters to increase the target mass-per-unit-area and, correspondingly, the event rate.

At these higher energies, the target mass-per-unit-area could be increased still further by interspersing thin tungsten disks with the CCD planes. There are two reasons why such a dense, high-Z target becomes more practical than at lower energies: (i) multiple coulomb scattering becomes less important at higher energies, so the tracking resolution is degraded less, and (ii) the narrower pencil beam for Mighty MURINEs allows the disks to have smaller radii than at lower energies – smaller than the characteristic Moliere radius for electromagnetic showers – so it is speculated that electromagnetic showers will not develop to excessively pollute the events, despite the large number of radiation lengths along the axis of the target. (This assumption needs to be checked in more detailed follow-up studies.)

A specific scenario for the neutrino target that gives the 1000 g cm$^{-2}$ target mass assumed in table 1 is as follows: a 4 meter long target containing 4000 millimeter-long tracking subunits, where each tracking subunit contains a thin tungsten disk of thickness 118 microns (0.227 g cm$^{-2}$) in front of a 100 micron thick CCD pixel detector (0.023 g cm$^{-2}$). Each tungsten disk could have a radius of 2 cm to match the beam radius at approximately 1 km from production for a 5 TeV muon beam.
(as predicted from equation 2). The CCD detectors can be wider than the beam radius to also track particles moving outside the radial extent of the neutrino beam.

The vertex tagging performance of the target in figure 2 is expected (7) to be better than any other existing or planned high-rate detector for heavy quark physics, and should continue to improve with beam energy. Mighty MURINEs should attain close to 100 percent efficiency for both c and b (easier) vertex tagging in the target (excepting all-neutral decay modes, of course) since the average boosted lifetime for TeV-scale charm and beauty hadrons – of order 10 cm – would span many planes of CCD’s. The extremely favorable geometry for vertexing is illustrated in figure 3, where it is also compared with the vertexing geometry at a collider detector.

As with lower energy MURINEs, the detector backing the neutrino target should faithfully reconstruct both CC and NC event kinematics. The lower energy MURINEs provide essentially full particle identification through the muon toroids and dE/dx plus cherenkov radiation in the TPC. The particle ID for long-lived charged hadrons would become more difficult at higher energies although, speculatively, the cherenkov radiation in an elongated TPC might still give effective PID for particle energies up to a couple of hundred GeV – this requires further study. To somewhat compensate, the trajectories of most photons should be very well measured when they convert in the stack of tungsten disks that comprise most of the target mass. This will be particularly helpful for the reconstruction of neutral pions.

III MURINES: A NEW REALM FOR NEUTRINO PHYSICS

This subsection gives a brief non-technical overview of the high rate neutrino physics topics expected for MURINEs in general. Topics that might be expanded on with the higher energies of Mighty MURINEs are pointed out in preparation for more detailed discussion in the following sections.

A Neutrino Interactions with Quarks

Put simply, the DIS interactions of equation 6 involve a simple projectile (the neutrino), interesting interactions (the CC and NC weak interactions) and a complicated target (the nucleon). The bulk of the physics interest lies in the interactions with the quark constituents rather than in the properties of the neutrinos themselves. The complementary analyses that study the potential oscillations of neutrino flavors tend to be more the domain of lower energy MURINEs, at muon energies of order 100 GeV or below, and won’t be discussed further here.

By the TeV-energy scale and above, the CC and NC interactions of equation 6 have become very well described as being the quasi-elastic (elastic) scattering of neutrinos off one of the many quarks (and anti-quarks), q, inside the nucleon:
FIGURE 2. Schematic example of a general purpose neutrino detector, reproduced from reference (7). A human figure in the lower left corner illustrates its size. The neutrino target is the small horizontal cylinder at mid-height on the right hand side of the detector. Its radial extent corresponds roughly to the radial spread of the neutrino pencil beam, which is incident from the right hand side. Further details are given in the text.

\[ \nu(\overline{\nu}) + q \rightarrow \nu(\overline{\nu}) + q \]  
\[ \nu + q^{(-)} \rightarrow l^{-} + q^{(+)} \]  
\[ \overline{\nu} + q^{(+)} \rightarrow l^{+} + q^{(-)} \]

(9)  
(10)  
(11)

The CC and NC interactions are mediated through the exchange of a virtual W or Z boson, respectively. All quarks – up quarks (u), down quarks (d) and the smaller “seas” of the progressively heavier strange (s), charm (c) and even beauty (b) quarks – participate in NC scattering interactions of both neutrinos and anti-neutrinos. In contrast, charge conservation specifies the charge sign of the quarks participating in the CC processes as indicated by the labels: \( q^{(-)} \in d, s, b, \overline{u}, \overline{c} \) and \( q^{(+)} \in u, c, d, s, b \).

The hadrons seen in the detector are produced by the “hadronization” of the final state struck quark at the nuclear distance scale.

B The Intrinsic Richness of Neutrino Physics

Experimentally, the interaction type of almost all events can be distinguished with little ambiguity by the charge of the final state lepton (and its flavor: i.e. electron or muon): neutral (an unseen neutrino), negative or positive for the 3 respective processes in equation 6.
FIGURE 3. Conceptual illustration of the vertex tagging superiority at MURINEs over that with collider experiment geometries. MURINEs could have a vertex plane of CCD pixel detectors every millimeter. For comparison, the VXD3 vertexing detector at the SLD (8) experiment, which is universally regarded as the best existing vertex detector in a collider experiment, has its two innermost CCD tracking planes at 2.8 cm and 3.8 cm from the interaction point. A cartoon of a 1-prong $D^+$ decay has been drawn to illustrate the advantages of closely spaced vertex detectors. For clarity of illustration, the kink deflection angle has been drawn much larger than would be typical. The 2 cm distance to decay for the $D^+$ charmed meson corresponds to the average boosted lifetime for a 120 GeV $D^+$. Most charm and beauty hadrons at a Mighty MURINE, having much higher energies than this, will travel even further and hence traverse even more planes of tracking before decaying.

It is seen that equations 9 through 11 probe 3 different weightings of the quark flavors inside a nucleon, through weak interactions involving both the W and Z. For comparison, only a single and complementary weighting is probed by the best competitive process – the photon exchange interactions of charged lepton scattering experiments at HERA and fixed target facilities. Much of the uniqueness and richness of neutrino scattering physics derives from this variety of interaction processes.
C Physics Topics at MURINEs

Mighty MURINEs will extend and improve on the already considerable range of unique topics that will have been explored at lower energy MURINEs. The interested reader is referred to reference (5) for more details. Here we list those topics that will have already been well addressed in earlier MURINEs and comment on any added potential that might be available using the higher energies at Mighty MURINEs.

Probing Nucleon Structure

The redundant probes of the proton’s and neutron’s internal structure should provide some of the most precise measurements and tests of perturbative QCD - the theory of the strong interaction - and will also be invaluable input for many analyses at pp colliders. Neutrinos are also intrinsically 100% longitudinally polarized, so experiments with polarized targets could additionally map out the spin structure of the nucleon. Some of these analyses might obtain modest benefits from the higher energies and statistics at Mighty MURINEs.

Precision Electroweak Measurements

Besides using W and Z exchange as nuclear probes, the interactions themselves provide important precision tests of the standard model of elementary particles. Two measurements of total interaction cross sections will provide determinations of the fundamental weak mixing angle parameter of the electroweak theory, \( \sin^2 \theta_W \), from (i) the ratio of NC to CC total cross sections and (ii) the absolute cross section for the rarer process of neutrino-electron scattering, which is 3 orders of magnitude less common than neutrino-nucleon scattering. In both cases, the fractional uncertainties in \( \sin^2 \theta_W \) might approach the \( 10^{-4} \) level, which would be complementary and competitive to the best related measurements in collider experiments.

The first of the two measurements will already be systematically limited at MURINEs (5) so large gains should not be expected at Mighty MURINEs. The situation is not so clear for the electron scattering process, where the higher event statistics could still be beneficial. Speculatively, these higher statistics might also allow the use of liquid hydrogen targets with improved experimental capabilities.

Charm and Beauty Factories

MURINEs of all energies will be excellent charm factories, with of order 1% to 10% of the events containing a charmed hadron, depending on the beam energy. TeV-scale MURINEs and above will also produce enough B hadrons to be considered as beauty factories and Mighty MURINES might be very impressive B factories, as will be discussed in section IV.
Quark Mixing Studies

There is much additional interest in experimentally partitioning the CC event sample to obtain the partial cross sections for the various possible quark flavor transition combinations represented by the $q^{(-)}$ and $q^{(+) \text{ symbols}}$ in equations 10 and 11. MURINEs, in general, should have the quark-tagging capability to separate the various quark flavor contributions, as was discussed in the preceding section. Mighty MURINES, with their extra capability for producing heavy final-state quarks, could make great strides beyond previous MURINEs for these “quark mixing” studies, as will be expanded on in section V.

Rare and Exotic Processes

The higher statistics and, particularly, energies available at Mighty MURINEs would clearly expand the scope for studies of rare processes and searches for exotic processes. This will be covered in section VI.

IV MIGHTY MURINES AS B FACTORIES

The charged current production of b quarks off the light quarks in the nucleon is heavily suppressed due to small off-diagonal CKM matrix elements. However, the fraction of neutrino-induced events containing B hadrons rises rapidly with energy (5) due to the decreasing threshold suppression for two higher-order processes involving gluons in the initial state:

1. $b \bar{b}$ pair production in neutral current interactions:

$$\nu N \rightarrow \nu b \bar{b} X.$$  \hspace{1cm} (12)

2. charged current production of $c \bar{b}$ and $b \bar{c}$ from the charged current interactions of neutrinos or anti-neutrinos:

$$\nu N \rightarrow l^-b \bar{c} X$$ \hspace{1cm} (13)

and

$$\bar{\nu} N \rightarrow l^+b \bar{c} X,$$ \hspace{1cm} (14)

respectively.

Preliminary estimates (9; 5) for the fraction of events from each of these processes are tabulated versus neutrino energy in table 2. The second of the two processes is seen to be less common than the first. To compensate, it provides an extremely pure and efficient tag to distinguish between $b$ and anti-$b$ quark production.
TABLE 2. Fraction of events (9) producing B’s in the final states \( b\bar{b} \), \( b\bar{c} \) or \( c\bar{b} \), from neutrinos and anti-neutrinos of energies 1 TeV and 10 TeV. Estimates are preliminary.

| \( \nu \) or \( \bar{\nu} \) | \( E_\nu \) | final state | fraction |
|--------------------------|----------|-------------|----------|
| \( \nu \)                | 1 TeV    | \( b\bar{b} \) | \( 6 \times 10^{-4} \) |
| \( \bar{\nu} \)          | 1 TeV    | \( b\bar{b} \) | \( 6 \times 10^{-4} \) |
| \( \nu \)                | 1 TeV    | \( c\bar{b} \) | \( 2 \times 10^{-5} \) |
| \( \bar{\nu} \)          | 1 TeV    | \( b\bar{c} \) | \( 2 \times 10^{-5} \) |
| \( \nu \)                | 10 TeV   | \( b\bar{b} \) | \( 4 \times 10^{-3} \) |
| \( \bar{\nu} \)          | 10 TeV   | \( b\bar{b} \) | \( 4 \times 10^{-3} \) |
| \( \nu \)                | 10 TeV   | \( c\bar{b} \) | \( 8 \times 10^{-3} \) |
| \( \bar{\nu} \)          | 10 TeV   | \( b\bar{c} \) | \( 6 \times 10^{-5} \) |

production is always accompanied by a positive primary lepton (from anti-neutrino interactions) and anti-b production by a negative primary lepton (from neutrino interactions). This will be very helpful for studies of oscillations of \( B_0 \)’s and \( B_S \)’s.

Combining the numbers in tables 1 and 2 predicts event rates of perhaps \( 10^8 \) to \( 10^9 \) B’s per year at Mighty MURINEs. This is intermediate between the expectations of the \( e^+e^- \) B factory experiments (\( \sim 10^7 \) events/year) and the hadron B factories, HERA-B, BTeV and LHC-B (up to \( \sim 10^{11} \) events/year, with up to a few times \( 10^9 \) events tagged for analysis). As already mentioned, however, the vertexing capabilities and other experimental conditions at Mighty MURINES should be superior in some aspects to those at the \( e^+e^- \) B factories and vastly superior to the very difficult experimental conditions at the hadronic B factories.

Three speculative examples of B analyses that would benefit from the unique experimental conditions at Mighty MURINEs are:

- the superior vertexing capabilities should be ideal for studying the expected fast oscillations of \( B_s \)’s, perhaps following up on previous B factories with more precise measurements of the oscillation frequency and greater sensitivity to any asymmetry in the \( B_s \) and \( \overline{B}_s \) decay rates

- some studies of the B baryons, \( \Lambda_b, \Xi_b^- \) and \( \Xi^0_b \), which are not produced in \( e^+e^- \) B factories, may also plausibly be best performed at a Mighty MURINE

- it might have a chance (10) to measure the branching ratio for the all-neutral rare decay \( B_d \rightarrow \pi^0\pi^0 \), which is expected to be of order \( 10^{-6} \). This would provide an estimate for the otherwise problematic “penguin-diagram pollution” in the analogous charged pion decay \( B_d \rightarrow \pi^+\pi^- \), and this could go some way to resurrecting the charged decay mode as one of the central CKM processes at B factories. However, observing the neutral decay mode does not look feasible at any future B factories other than Mighty MURINEs.

The final process deserves some further explanation since the decay itself doesn’t provide a vertex. However, the close to 100% vertex reconstruction efficiency could
instead act as a veto to reduce the backgrounds from the pair-produced B’s in neutral current interactions. The signature for the signal process would be a neutral current event with (i) a single vertex from the other B, (ii) 4 converted gammas reconstructing to 2 high energy $\pi^0$’s that, in turn, reconstruct to the $B_d$ mass and (iii) no suspicion of another B or charm vertex. Hence, the analysis – although admittedly still exceedingly difficult – would benefit from both the exceptional vertexing and neutral pion reconstruction at Mighty MURINEs.

Therefore, to summarize this section, the initial expectation is that Mighty MURINEs should be able to do follow-up precision studies in at least some of the most difficult areas of B physics, even after the other B factories have run.

V QUARK MIXING: MEASUREMENTS BEYOND THE B FACTORIES

This section enlarges on the theoretical interest in measurements of quark mixing at MURINEs and also provides detail on the central role that Mighty MURINEs could assume if they reached sufficient energies to begin producing top quarks.

A Theoretical Interest in the CKM Matrix

Quark mixing is one of the least understood and most intriguing parts of elementary particle physics, and the confinement of quarks inside hadrons also makes it one of the hardest areas to study. The CC weak interaction for quarks differs from this interaction for leptons by mixing quarks from different families, i.e. any positively charged quark, $q^{(+)} \in u, c, t$, has some probability of being converted into any of its negatively charged counterparts, $q^{(-)} \in d, s, b$, and vice versa, rather than being uniquely associated with its same-family counterpart (i.e. $d \leftrightarrow u$, $s \leftrightarrow c$ and $b \leftrightarrow t$). This feature is accommodated in the standard model of elementary particle physics through the unitary 3-by-3 Cabbibo-Kobayashi-Maskawa (CKM) matrix, $V_{ij}$, that connects the positively charged $q_i^{(+)}$’s with the three $q_j^{(-)}$’s. (The corresponding matrix for leptons is trivially the 3-by-3 identity matrix.)

Apart from verifying that the SM description for quark mixing is indeed correct, the CKM matrix has additional interest through its hypothesized association with CP violation: the puzzling phenomenon that some particle properties, such as decay rates, have been found to differ slightly from those of the corresponding anti-particles. CP violation could also have cosmological implications; it has been invoked as one possible explanation for the comparative scarcity of anti-matter in the universe. The presence of a complex phase in the CKM matrix is the largely untested standard model explanation/parameterization for CP violation.

It is a testament to the perceived importance of the CKM matrix and CP violation that much of today’s experimental HEP effort is devoted such studies, including B and phi factory colliders, LHC-B, HERA-B, B-TeV, K-TeV and many others.
Neutrino-nucleon scattering has impressive potential to augment these studies but, until the arrival of MURINEs, it will be held back by inadequate beam intensities.

**B Quark Mixing Studies at MURINEs**

The new neutrino studies at MURINEs will be much cleaner theoretically than most of the other experimental processes and will offer much complementary information.

Figure 4 is the Feynman diagram for the basic scattering process, showing that the CKM matrix element $V_{qq'}$ participates as an amplitude in the W-quark coupling. As a fundamental difference between $\nu N$ DIS and all other types of CKM measurements, the scattering process involves the interaction of an external W boson probing the quarks inside a nucleon rather than an internal W interaction inside a hadron that, e.g., initiates a B decay. (In principle, the HERA ep collider could also do measurements involving an external W exchange, but these turn out not to be feasible in practice (11).) This is a substantial theoretical advantage for neutrino scattering because the “asymptotic freedom” property of QCD predicts quasi-free quarks with reduced influence from their hadronic environment at the higher 4-momentum-transfer (Q) scales available with an external W exchange.

The CKM measurements at MURINEs will be complementary to, say, the CKM measurements at B factories in that the measurements are of the magnitudes of individual CKM matrix elements rather than of interference terms involving pairs of elements. As can be inferred from figure 4, this arises because the differential cross-sections, $\frac{d\sigma}{dx}(q_i \rightarrow q_j)$, for the quark transitions are proportional to the absolute squares of the CKM elements:

$$\frac{d\sigma}{dx}(d \rightarrow c) \propto x d(x)|V_{cd}|^2 \times T(m_c, x)$$

(15)
\[ \frac{d\sigma}{dx}(s \to c) \propto x s(x) |V_{cs}|^2 \times T(m_c, x) \quad (16) \]
\[ \frac{d\sigma}{dx}(u \to b) \propto x u(x) |V_{ub}|^2 \times T(m_b, x) \quad (17) \]
\[ \frac{d\sigma}{dx}(c \to b) \propto x c(x) |V_{cb}|^2 \times T(m_b, x) \quad (18) \]
\[ \frac{d\sigma}{dx}(d \to t) \propto x d(x) |V_{td}|^2 \times T(m_t, x) \quad (19) \]
\[ \frac{d\sigma}{dx}(s \to t) \propto x s(x) |V_{ts}|^2 \times T(m_t, x) \quad (20) \]
\[ \frac{d\sigma}{dx}(b \to t) \propto x b(x) |V_{tb}|^2 \times T(m_t, x) \quad (21) \]

where the Bjorken scaling variable, \( x \), with \( 0 < x < 1 \), is a relativistically invariant quantity that can be reconstructed for each event and, roughly speaking, measures the fraction of the nucleon’s 4-momentum carried by the struck quark. The respective initial-state quark densities as functions of Bjorken \( x \) have been labeled \( d(x), s(x), u(x), c(x) \) and \( b(x) \), and the \( T(m_q, x) \)’s are threshold suppression factors due to the masses, \( m_q \), of the final-state quarks.

The \( T(m_q, x) \) mass suppression factors are zero or much less than unity for all \( x \) below neutrino energies that can readily supply enough CoM energy to produce the massive final state quarks. From equation 5, the \( T(m_q, x) \)’s will asymptotically approach unity only for muon beam energies such that:

\[ m_q^2 \ll 2M_p E_\mu/c^2 + M_p^2. \quad (22) \]

This places the following lower bounds on beam energies for the efficient production of charm, beauty and top quarks, respectively:

\[ m_c \sim 1.3 - 1.7 \text{ GeV}/c^2 \quad \Rightarrow \quad E_\mu \gg 1 \text{ GeV} \quad (23) \]
\[ m_b \sim 5 \text{ GeV}/c^2 \quad \Rightarrow \quad E_\mu \gg 13 \text{ GeV} \]
\[ m_t \sim 175 \text{ GeV}/c^2 \quad \Rightarrow \quad E_\mu \gg 16 \text{ TeV}. \]

The extraction of the CKM matrix elements from the MURINEs’ experimental data will be analogous to, but vastly superior to, current neutrino measurements of \( |V_{cd}| \), the only CKM matrix element that is currently best measured in neutrino-nucleon scattering (12). The experimentally determined event counts and kinematic distributions of the quark-tagged event samples provide measurements of the differential distributions for each of the final state quarks. The differential cross-sections, \( \frac{d\sigma}{dx}(q_i \to q_j) \), and CKM matrix elements, \( |V_{ij}| \), are derived from equations 15 through 21 using some auxiliary knowledge of the quark \( x \)-distributions within the nucleons and also a model for the mass threshold suppression terms, \( T(m_q, x) \). In practice, this information should be obtainable largely from the data samples themselves: from CC and NC structure function measurements and the observed kinematic dependences in the heavy quark event sample.
C Expected Measurement Precisions at MURINEs

Today’s measurements of $|V_{cd}|$ in $\nu N$ scattering are already the most precise in any process, despite the coarse instrumentation of the neutrino detectors and the consequent requirement to use the semi-muonic subsample of charm decays for final state charm tagging. Even the lowest energy MURINEs under consideration (dedicated neutrino factories with $E_\mu \simeq 10$ GeV and up) will provide an opportunity to extend to unique and precise measurements of the elements $|V_{cd}|$ and probably $|V_{cs}|$, now using vertex tagging of charm and with much improved knowledge of the quark distributions.

Further measurements of the more theoretically interesting elements $|V_{ub}|$ and $|V_{cb}|$ will become available at MURINEs with muon energies of around 100 GeV or above, which can provide high enough neutrino energies for $B$ production. The $B$-production analyses at these higher energy MURINEs should be experimentally rather similar to the charm analyses but would have vastly greater theoretical interest.

Both $|V_{ub}|$ and $|V_{cb}|$ determine the lengths of sides of the “unitarity triangle” that is predicted to exist if the CKM matrix is indeed unitary (13). The main goal of today’s $B$ factories is to measure the interior angles of this triangle to confirm that it is indeed a triangle, and the complementary input from a MURINE will be an enormous help in this verification process. In particular, the predicted (7; 5) 1-2% accuracy in $|V_{ub}|^2$ is several times better than predicted accuracies in any future measurements of other processes, and will obviously provide a very strong constraint on the unitarity triangle.

Predicted experimental accuracies for a 500 GeV MURINE are summarized in table 3. These measurements would likely be improved still further with the higher event statistics and cleaner theoretical analysis available at a Mighty MURINE.

D Possible Measurements of $V_{td}$ in Flavor Changing Neutral Current Interactions

The increased event statistics and neutrino energies at Mighty MURINEs might even allow the measurement of the further matrix element $|V_{td}|$ through the flavor changing neutral current (FCNC) interaction of figure 5.

This process is analogous to the predicted rare $B$ decay $B \rightarrow X_d \nu \bar{\nu}$, shown in figure 6, where $X_d$ represents inclusive production of hadrons containing a down quark. As related measurements, the very rare kaon decay processes $K^- \rightarrow \pi^- \nu \bar{\nu}$ and $K^0_L \rightarrow \pi^0 \nu \bar{\nu}$ proceed through diagrams equivalent to 6 except with the incoming $b$ quark replaced by an $s$ quark and, correspondingly, $V_{tb}$ replaced by $V_{ts}$. Therefore, the charged kaon decay has the potential to measure $|V_{ts}^*V_{td}|$ and its neutral counterpart actually measures the imaginary part of this quantity, $\text{Im}(V_{ts}^*V_{td})$, due to $K^0 - \bar{K^0}$ interference. One event of the decay $K^- \rightarrow \pi^- \nu \bar{\nu}$ has been seen so
TABLE 3. Predicted CKM measurements for lower energy MURINES, reproduced from reference (14). The first row for each element, in bold face, is the absolute square of that matrix element, which is proportional to the experimental event rate – see equations 15 through 21. The second row for each element gives current percentage uncertainties in the absolute squares, without applying unitarity constraints, and speculative projections of the uncertainties after analyses from a 500 GeV MURINE. The measurements of \(|V_{cd}|^2\) and \(|V_{cs}|^2\) might be comparably good even for a 50 GeV MURINE but \(|V_{ub}|^2\) and \(|V_{cb}|^2\) would not be measured.

|     | d     | s     | b     |
|-----|-------|-------|-------|
| u   | 0.95  | 0.05  | 0.0001|
|     | ±0.1% | ±1.6% | ±50%  → 1.2% |
| c   | 0.05  | 0.95  | 0.002 |
|     | ±15%  → 0.2-0.5% | ±35%  → ~1% | ±15%  → 3.5% |
| t   | 0.0001| 0.001 | 1.0   |
|     | ±25%  | ±40%  | ±30%  |

FIGURE 5. The Feynman diagram for B production through a flavor changing neutral current interaction involving a top quark loop.

far (15), consistent with its predicted tiny branching ratio of \((8.2 ± 3.2) \times 10^{-11}\), and the even rarer neutral decay process has yet to be observed.

The search for the B decay signal looks to be extremely challenging even at future B factories, so it is unlikely to yield an accurate measurement of \(|V_{td}|\). Therefore, a neutrino measurement of this quantity, at or below the 10% level, might still be valuable even after the B factories have run, augmenting the complementary measurements, perhaps eventually with comparable accuracy (16), of \(|V_{ts}^* V_{td}|\) and
Im($V_{ts}^* V_{td}$) expected in future generations of rare kaon decay experiments and the precise measurement of the ratio $|V_{ts}|/|V_{td}|$ that is to be eventually expected from $B_d$ and $B_s$ oscillations.

For neutrino energies well above the $B$ production threshold, the process of figure 5 will occur at the level (5) of order $10^{-8}$ of the total neutrino-induced event sample unless some exotic physics process intervenes to increase the production rate. The signature is production of single $B^-$ mesons from the valence $d$ quarks at high $x$, whose rate should be directly proportional to the product of $|V_{td}|$ and the known valence quark density in nucleons.

The main background will come from $b$–anti-$b$ production where the partner $B$ meson containing the $b$ anti-quark has escaped detection from either its primary decay vertex or through the decay vertex of its daughter charmed meson. The background process is easily separable from the signal on a statistical basis because it is symmetric in $B^-$ versus $B^+$ mesons. However, the raw production rate (5) is roughly five orders of magnitude above the signal so the statistical viability of the analysis would require the raw background to be reduced by perhaps 4 to 5 orders of magnitude. This can be contemplated only because of (1) the very different event kinematics, with almost all of the background events at low Bjorken $x$ and the signal mostly at high $x$, and (2) the unprecedented veto power for $B$ decays that is expected in the vertexing detectors at MURINEs. Even so, a raw signal event sample of thousands of events might be needed for a 10% measurement of $|V_{td}|$ given the statistical dilution that background processes might entail. This would require several years running for the experimental parameters of table 1.
E CKM Measurements from the Production of Top Quarks at the Highest Energy Mighty MURINEs

Aside from top production in loops, a daunting leap of 3 orders of magnitude in beam energy would be required to move from the CKM elements involving B production to those involving top production, as is seen from comparing the second and third rows of equation 24. Uniquely precise direct measurements of $|V_{td}|^2$ and $|V_{ts}|^2$ and, possibly, $|V_{tb}|^2$ from the production of top quarks will become available if and when muon colliders eventually reach the 100 TeV CoM energy scale. (Note that muon collider energies even up to 1000 TeV, i.e. 1 PeV, have been speculated, using muon acceleration in linacs (17).)

Such impressive machines are prospects for the far distant future, and would be intended to zero in on a coherent understanding of the elementary building blocks of our universe. It should be stated that a major sea change from current theoretical prejudices would be required if the CKM matrix and its information on CP violation was to become central to the construction or verification of such a “theory of everything”. Disregarding the current prejudices, top production at these highest energy Mighty MURINEs would move the experimental probing of the CKM matrix to a level of accuracy that appears to be inaccessible to any other type of experiment.

As will be explained further in the following section, a simple scaling from top production estimates calculated for HERA (18) predicts of order $10^5$ top quark events for 1 inverse zeptobarn of integrated luminosity at muon energies slightly above 50 TeV. (A more accurate and detailed calculation is obviously required!)

Experimentally, top production should be relatively easy to tag with very high efficiency and purity. The two signatures are:

\[ \nu_\mu N \rightarrow \mu^- (2\text{ jets})(b\text{ jet}) \]  
\[ \nu_\mu N \rightarrow \mu^- l^+ \nu (b\text{ jet}) , \]

with 68% and 32% BR’s, respectively. Because of the large top mass, the final state jets can each have large acoplanarities, and the rarity of backgrounds with b quarks makes both signatures particularly distinctive. Additionally, in the first case the 2 other jets will reconstruct to the W mass while the presence of a second high-$p_t$, high energy lepton and large missing $p_t$ from the neutrino will make the second signature even more striking.

No attempt will be made to even guess at the measurement accuracy. As a general comment, the beam energy will never be very far above the threshold for top quark production, so the feasibility and accuracy of the measurements would depend more strongly on the muon beam energy than the beam intensity. In almost all cases, the measurements of the CKM matrix elements involving top should be statistically limited because of the relatively small statistics (except at PeV-scale colliders!), their distinctive experimental signature and the accurately predictable threshold behavior. The sequentially decreasing populations at high $x$ of the progressively
FIGURE 7. Conceptual illustration of A) the relatively soft electromagnetic interactions, involving the exchange of a photon, that dominate the event sample at HERA, and B) the much harder weak interactions that will occur in Mighty MURINEs and that exist on the “hard scattering tail” at HERA.

A) electromagnetic interaction at HERA

B) weak interaction at MURINE

Heavier initial state quarks – d, s and b – should compensate or over-compensate the trend for the higher couplings to the top quark in the respective measurements of $|V_{td}|$, $|V_{ts}|$ and $|V_{tb}|$. Whether the first, the first two or all three matrix could be measured would presumably also depend strongly on the beam energy.

CKM measurements involving top would extend CKM studies beyond the paradigm of the unitarity triangle that is connected to B factory studies. For example, the conventional unitarity triangle is formed from the dot product of columns 1 and 3 of the CKM matrix (13). Measurements of the CKM elements involving the top quark would also provide enough experimental input to test the corresponding triangle involving rows 1 and 3 of the matrix, which is of comparable theoretical value in exploring CP violation. The analysis of experimental results would more likely be couched in more general theoretical terms, including unitarity tests and global fits to the 4 parameters – three magnitudes and a phase – that characterize the unitary 3-by-3 CKM matrix. Consistency of these fits would probe the SM hypothesis at a level that would not be possible without Mighty MURINEs.
VI HEAVY PARTICLE PRODUCTION – MIGHTY MURINES VERSUS HERA

The HERA ep collider is a convenient reference point for assessing the physics potential of Mighty MURINEs at the highest energy scales. As indicated in figure 7, MURINEs have the same weak interactions as HERA while avoiding the predominantly soft electromagnetic interactions that dominate the HERA event trigger rates but are less interesting because lower energy transfers probe physics at relatively lower energy scales. The event samples at MURINEs will correspond to the weak interactions in the “hard scattering tail” of the HERA event sample.

| Particle/Process       | HERA (1 inverse femtobarn) | Mighty MURINE (1 inverse zeptobarn) |
|------------------------|----------------------------|--------------------------------------|
| b quark: c → b         | O(10) events               | O\(10^6\) events                     |
| b quark: u → b         | O(1) event                 | O\(10^5\) events                     |
| top quark              | O(1) event                 | O\(10^5\) events                     |
| W, Z bosons            | tens of events             | O\(10^5\) events                     |
| 120 GeV SM Higgs       | O(1) event                 | O\(10^5\) events                     |
| exotica with small \(\sigma\) | luminosity limited       | luminosity OK!                       |

The HERA event samples involving weak interactions can be compared with MURINEs at energies where the high energy tail of the neutrino beam is comparable to the 314 GeV HERA center-of-mass energy. The maximum neutrino energy is the muon beam energy, which, according to equation 5, equals the HERA CoM energy for \(E_\mu = 53\) TeV. At this energy or slightly above, very rough estimates of the event rates of similar or identical processes can be simply transcribed from HERA calculations after scaling by the \(10^5\) luminosity ratio shown in table 1. Such a comparison is shown in table 4. A range of MURINE energies has been given, in deference to the very approximate nature of the comparison. At the low end \((E_\mu = 50\) TeV\), the MURINE event rates will probably be lower than the estimate, and the rates will normally be higher at the high end \((E_\mu = 100\) TeV\).

The standard model physics processes involving weak interactions are the same in all cases except for the production of W and Z bosons, where HERA has the advantage due to processes involving photon exchange. The SM Higgs has not been found at the time of writing, so the 120 GeV mass is an example only.

The first three processes in table 4 have already been discussed in the preceding section. To be realistic, at the event rates shown it is very doubtful that W, Z and SM Higgs production could contribute anything useful beyond collider studies, despite the the astounding neutrino beam parameters and superior event reconstruction. Beyond this, possible exotic processes at the 100 GeV scale or
below provide the only substantial potential for exciting discoveries. This motivation could become much stronger in the near future if, for example, one of the current leptoquark searches at HERA returned a discovery. It is noted that the leptoquarks produced at a Mighty MURINE might well be different – coupling to neutrinos rather than electrons – and so studies at MURINEs could potentially be complementary to those at a future ep collider with a higher \( E_{\text{CoM}} \) than HERA.

### VII SUMMARY

The Mighty MURINE neutrino experiments that would come almost for free at any future many-TeV muon collider could improve on the pioneering advances from the previous MURINEs that would have existed at lower energy muon colliders. The most important improvements might well be on the unique and important measurements from previous lower energy MURINEs of \( |V_{ub}| \) and \( |V_{cb}| \), perhaps pushing the accuracy of both measurements below 1%. With total event statistics of a few times \( 10^{11} \) events, the rare production of B’s through flavor changing neutral current interactions off valence d quarks might provide one of the best indirect determinations of \( |V_{td}| \). More common channels for B production, particularly through neutral current interactions, might also provide some capabilities as a B factory with novel experimental strengths.

Upon crossing the threshold for top production, the even more interesting elements \( |V_{td}| \), \( |V_{ts}| \), and \( |V_{tb}| \) could become successively available to uniquely precise measurements at the highest energy Mighty MURINEs. The addition of any or all of these three precise measurements would clearly advance our knowledge of the CKM matrix to a level where small perturbations from the Standard Model scenario could be searched for and, if found, could be studied. MURINEs would then truly play the central role in determining the CKM matrix parameters, with the best measurements of the magnitudes of perhaps seven of the nine elements (all but the two elements that are currently best measured: \( |V_{ud}| \) and \( |V_{us}| \)) to add to the phase information from various other experimental processes.

If muon colliders ever reach the 100 TeV center of mass energy scale then their neutrino experiments will attain a center of mass energy reach comparable to the existing HERA ep collider, but at a luminosity that might be perhaps 5 orders of magnitude higher. HERA then becomes a convenient reference point for assessing their physics capabilities. Despite the promise of impressive luminosities, none of the standard model processes other than the CKM matrix appear to offer the chance of competitive physics potential to studies of the same processes at colliders. Therefore, only i) an enlarged theoretical importance for the CKM matrix or ii) the discovery, then or beforehand, of an exotic process that is accessible to Mighty MURINEs, would give Mighty MURINEs a chance for physics analyses of a comparable importance to those at the colliders. Leptoquarks that couple to neutrinos are the obvious candidate for such a new process.
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