Quantum Sci. Technol. 3 (2018) 015001
https://doi.org/10.1088/2058-9565/aa8e15

PAPER

Evanescent-wave Johnson noise in small devices

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Keywords: noise, decoherence, quantum computing, quantum information, Johnson noise

Abstract
In many quantum computer architectures, the qubits are in close proximity to metallic device elements. The fluctuating currents in the metal give rise to noisy electromagnetic fields that leak out into the surrounding region. These fields are known as evanescent-wave Johnson noise. The noise can decohere the qubits. We review and update the general theory of this effect for charge qubits subject to electric noise and for spin and magnetic qubits subject to magnetic noise. A mapping of the quantum-mechanical problem onto a problem in classical electrodynamics simplifies the calculations. The focus is on relatively simple geometries in which analytical calculations can be done. Results are presented for the local noise spectral density in the vicinity of cylindrical conductors such as small antennae, noise from objects that can be treated as dipoles, and noise correlation functions for several geometries. We summarize the current state of the comparison of theory with experimental results on decoherence times of qubits. Emphasis is placed on qualitative understanding of the basic concepts and phenomena.

1. Introduction
The prospect of quantum computing has inspired many designs for the manipulation of small coherent quantum systems—qubits. Qubits are often located very near electrodes that contain many mobile charges and spins. The thermal and quantum motion of these charges and spins creates random electromagnetic fields that can decohere the qubits, an effect strenuously to be avoided. This noise is a species of Johnson noise. Johnson discovered this noise in 1927 in the course of a research program to improve the performance of amplifiers [1]. Nyquist soon explained it theoretically using ingenious applications of equilibrium thermodynamics to thought experiments [2]. When the general relation of fluctuation and dissipation was discovered by Callen and Welton in 1951, they regarded their fluctuation–dissipation theorem (FDT) as a ‘generalized Nyquist relation’ [3]. The later, more general, theory of linear response of Kubo developed out of the FDT [4]. This is an interesting example of important and general basic science coming from research on very specific technological issues.

The Nyquist formula is

\[ \langle V^2 \rangle_\omega = 2k_B T R(\omega), \]  

(1.1)

where

\[ \langle V^2 \rangle_\omega = \int_{-\infty}^{+\infty} dt \ e^{i\omega t} \langle V(0) V(t) \rangle. \]  

(1.2)

Here \( V \) is the voltage drop between the ends of a resistor with a possibly frequency-dependent resistance \( R \). The angle brackets are an average over the stationary random process that \( V \) represents. The rms voltage noise \( \sqrt{\langle V^2 \rangle_\omega} \) is the quantity usually quoted (in units of volts per root Hertz), since it is often practical to measure the drop with a bandpass filter in a frequency range where \( R \) is more or less constant. Johnson himself verified that this formula holds independent of the shape, size, or constitution of the resistor. These days, equation (1.1) is recognized as the high-temperature limit of the more general formula

\[ \langle V^2 \rangle_\omega = \hbar \omega \coth (\hbar \omega / 2k_B T) R(\omega) \]  

(1.3)
that follows from the quantum-mechanical version of the FDT. For applications to qubits we need a generalization of the Nyquist form of the FDT, which gives the voltage drop between two points in a resistor. In particular, we need a theory that works between any two points irrespective of whether they are on a resistor; we would also like to understand the connection between the Nyquist relation with that other famous kind of thermal electromagnetic field—blackbody radiation. Quantum field theory gives the needed generalization. The main difficulty is to formulate finite-temperature quantum electrodynamics in such a way that the only inputs required are the macroscopic electric and magnetic response functions $\varepsilon(\vec{r}, \omega)$ and $\mu(\vec{r}, \omega)$. The outputs of the theory are the noise spectral densities, which are the field fluctuations at a single spatial point (sufficient to calculate the decoherence of point qubits), and the noise correlation functions which give the fluctuations at spatially separated points (required to calculate the decoherence of extended qubits). We will give precise definitions of these quantities below. The formalism required to do this was constructed in the 1950s by Lifshitz [5] and Rylov [6, 7] and the theory was further developed by Agarwal [8]. These authors built on earlier work of Casimir [9]. An accessible treatment is given by Lifshitz and Pitaevskii [10]. There is a fairly large literature on the application of this formalism to heat transfer and friction in small devices which has been reviewed by Volokitin and Persson [11].

Before proceeding with the development of the formalism, we first give a qualitative picture of how we expect noise to leak out of metallic device elements, taking the lead from a paper of Pendry [12]. Consider a piece of metal surrounded by an insulator. For the sake of argument, let us specify that the metal is hotter than its environment. The Stefan–Boltzmann formula tells us that the total EM power radiated depends only on the surface area and the temperature of the object, not on its conductivity. The radiation is the result of photons thermally generated in the metal leaking out through the surface. The metal has a dielectric function $\varepsilon(\omega) = 1 + 4\pi\sigma/\omega$, where the conductivity $\sigma$ nearly always satisfies $\sigma/\omega \gg 1$ (and this is true for all frequencies considered in this paper). $|\varepsilon|$ is much greater than unity, so the speed of light (to the extent that it can be defined for the highly overdamped modes of the metal) is small relative to the surrounding insulator. This immediately implies that the photon density of states and the equilibrium density depends on $\sigma$. This presents a paradox, since the radiated power is independent of $\sigma$. This paradox is resolved by the realization that a high photon density of states is always accompanied by a high probability of internal reflection of the photon [13, 14]. The cancellation of these effects gives the universal coefficient of blackbody radiation. However, internal reflection is always accompanied by an evanescent wave (figure 1). This in turn implies that there will be strong Johnson noise near a metallic surface for any material having $|\varepsilon| \gg 1$. This is called evanescent-wave Johnson noise (EWN). This physical picture tells us that the proper treatment of boundary conditions will be very important. This in turn implies that for ordinary, non-magnetically active metals, the behavior of electric noise is quite different from magnetic noise, since magnetic fields can penetrate those materials much more easily.

It is very important to distinguish between EWN and the more commonly discussed circuit Johnson noise (CJN). If we consider two separate metallic elements in a small device, usually the path of least resistance between them runs through the external circuit. Thus CJN is a physical effect that involves two or more device elements that convey information about the external circuit to the qubit. EWN, in contrast, is an effect that occurs even without the external circuit, and fundamentally arises from individual device elements. CJN and EWN thus come from different physical sources. For the most part, they can be calculated separately and they are basically additive. EWN is also distinct from charge noise and any other noise that comes from localized defects. The noise sources of EWN need not be macroscopic, but their dimensions must be large enough that a dielectric function can be defined by the usual coarse-graining procedure of classical electrodynamics. Atomic-scale defects must be treated by different methods.

![Figure 1. Cartoon depicting evanescent-wave Johnson noise near the surface of a conductor.](image-url)
The implications of Johnson noise for decoherence of atomic qubits were first discussed by Henkel and collaborators [15, 16], in the context of heating of trapped ions by the walls of the trap. The local noise spectral densities for both electric and magnetic fields relevant to the situation of point qubits near a conducting half-space were calculated and loss and decoherence rates were extracted. These predictions were quantitatively verified in experiments that measured losses from magneto-optical traps [17]. The lifetimes in the experiments are of order 10 s and the distances from the walls 10 to 100 μm. At about the same time, other qubit applications were discussed by Sidles et al [18]. In semiconductor and some other solid-state implementations of quantum computing, the distance scales are much less than in the atom experiments and this suggests that the effects of Johnson noise could be appreciable for those systems [19–24]. Indeed, a recent experiment with a diamond film containing NV centers on a silver substrate demonstrated decoherence of qubits due to EWJN in a very direct and quantitative fashion [25].

Charge quantum dot qubits displayed lifetimes in the range of $T_\text{i} \sim 10$ ns, which was shorter than expected based on decoherence mechanisms such as coupling to phonons [26–29]. This spurred theoretical work on CJN for double quantum dots [30], and even though it appears that it cannot be the main mechanism in this instance, the effects are still appreciable.

There has been a small amount of work on the very interesting topic of noise from micromagnets implanted in semiconductors [13, 14]. However, in this paper we shall deal only with non-magnetic materials, so the magnetic permeability $\mu = 1$ everywhere. In this paper, we focus exclusively on EWJN. We cover only analytic calculations and physical considerations. Numerical calculations on realistic devices are not included. To our knowledge, no such calculations exist at present, though the calculations in [13] represent a start in this direction. It would be a major development project to modify existing FEM software to compute EWJN, since the number of coupled equations is large. However, the formal developments in section 2.2 may suggest ways to simplify the problem.

The literature at present only contains analytic results for the half-space, single film [31], and two-film geometries. In the next section we outline the basic formalism of EWJN. Section 2 describes how to apply the results to compute lifetimes of qubits. Section 3 gives the applications to electric field noise and decoherence of charge qubits. Section 4 is a parallel discussion for magnetic field noise and spin qubits. Section 5 gives the current situation with regard to comparison of theory and experiment. Section 6 gives a summary and describes the implications for future qubit designs. The overall structure of the paper is meant to reflect the logical development of the subject, with reasonably complete derivations of the main results. If the reader’s main concern is just with new results, then these are to be found as follows.

- We provide explicit results for the noise spectral density that determines the decoherence of point qubits for new geometries. We find that for the conducting cylinder and electrode, the spectral densities exhibit anisotropies. These anisotropies can serve as a sharp criterion for the presence of EWJN. They can also be exploited to substantially increase $T_1$ and $T_2$ by suitable qubit orientation. These results are found in sections 4.1–4.3 and 5.1–5.3.
- We compute noise correlation functions that are needed for the determination of decoherence times for extended qubits. These results are found in sections 4.1–4.3 and 5.1–5.3.
- We describe in detail how to apply these noise calculations to compute relaxation and decoherence times for qubits in the noise field. These results are found in sections 3.1 and 3.2.

Overall, the comparisons with present experimental results indicate that EWJN is not the dominant relaxation mechanism for many charge qubit implementations. On the other hand, the observed relaxation time for certain spin qubits can be explained by the calculations we present here.

2. General formalism

2.1. Photon green’s functions

We consider a system at temperature $T$ and regard the dielectric function $\varepsilon(\vec{r}, \omega)$ as given. As stated above, $\mu = 1$ everywhere so $\vec{B} = \vec{H}$. We will work in the realm of macroscopic electrodynamics, i.e., all quantities are averaged over distances of order $a$, where $a$ is an interatomic distance. This excludes a large class of physical situations that can be important in qubit devices, namely those in which the noise sources are few in number or otherwise cannot be considered as members of a continuum. The results in this paper do not apply to such situations.
Our derivation in this section follows [10]. We present it here to introduce the concepts and to establish notation. We shall work in the temporal gauge where the scalar potential \( \phi = 0 \). The retarded photon Green’s function is
\[
iG_{ij}(\vec{r}, t; \vec{r}', t') = \Theta(t - t') (\hat{A}_i(\vec{r}, t)\hat{A}_j(\vec{r}', t') - \hat{A}_i(\vec{r}', t')\hat{A}_j(\vec{r}, t)).
\] (2.1)

Here \( \Theta(x) = 1 \) if \( x > 0 \) and \( \Theta(x) = 0 \) if \( x < 0 \). \( i, j \) run over \( x, y, z \). The angle brackets represent a thermal ensemble average. The \( \hat{A} \) are photon operators for the vector potential in the interaction picture. We define
\[
G_{ij}(\vec{r}, \vec{r}', \omega) = \int_{-\infty}^{\infty} dt \ e^{i\omega t} G_{ij}(\vec{r}, t; \vec{r}', 0)
\] (2.2)
and this function satisfies an Onsager relation:
\[
G_{ij}(\vec{r}', \vec{r}, \omega) = G_{ji}(\vec{r}', \vec{r}, \omega).
\] (2.3)

Fortunately, we will not need to consider the operator properties of \( \hat{A} \) in detail. Instead, we will derive a differential equation for \( G \). In the presence of a classical current \( J \), \( H' \) is the perturbation to the free-space Maxwell Hamiltonian with
\[
H' = -\frac{1}{c} \int J_i(\vec{r}, t) A_i \, d^3r,
\] (2.4)
with a summation convention over Cartesian indices. The expectation value of \( \hat{A}_i \) is given by the Kubo formula
\[
\langle \hat{A}_i(\vec{r}, \omega) \rangle = -\frac{1}{\hbar c} \int G_{ij}(\vec{r}, \vec{r}', \omega) J_j(\vec{r}', \omega) \, d^3r'.
\] (2.5)

In the following, angle brackets and the argument \( \omega \) will often be omitted.

In many situations electronic length scales such as the mean free path \( \ell \) is much smaller than all the other lengths in the problem and consequently there is a local relation between the electric displacement and the electric field:
\[
D(\vec{r}) = \varepsilon(\vec{r}) \vec{E}(\vec{r}).
\] Then Maxwell’s equation is
\[
\nabla \times \vec{B} = \frac{4\pi}{c} j - i \frac{\omega}{c} \varepsilon(\vec{r}) \vec{E},
\] (2.6)
and the fields are given in this gauge by
\[
\vec{B} = \nabla \times \vec{A} \quad \text{and} \quad \vec{E} = i \frac{\omega}{c} \vec{A}.
\] (2.7)

Thus we have that
\[
-\nabla^2 \vec{A} + \nabla (\nabla \cdot \vec{A}) - \frac{\omega^2 \varepsilon(\vec{r})}{c^2} \vec{A} = \frac{4\pi}{c} j
\] (2.8)
which in index notation with the summation convention is
\[
-\delta_{ij} \left( \nabla^2 + \frac{\omega^2 \varepsilon(\vec{r})}{c^2} \right) + \partial_i \partial_j \right] A_i = \frac{4\pi}{c} j_i.
\] (2.9)

Since this is true for any \( J_i \), it implies that
\[
\left[ -\delta_{ij} \left( \nabla^2 + \frac{\omega^2 \varepsilon(\vec{r})}{c^2} \right) + \partial_i \partial_j \right] G_{jk}(\vec{r}, \vec{r}') = -4\pi \hbar \delta^3(\vec{r} - \vec{r}') \delta_{jk}.
\] (2.10)

The differential operators act on \( \vec{r} \), not \( \vec{r}' \). For a fixed \( \vec{r}' \) (source position), this is an inhomogeneous partial differential equation when \( j = k \) and a homogeneous partial differential equation when \( j \neq k \) for the functions \( G_{ik} \). Tangential \( \vec{E} \), normal \( D = \varepsilon \vec{E} \) and \( \vec{B} = H \) are continuous at the boundary between different media. We have that \( E_i(\vec{r}) \sim (\omega/c) G_{ij}(\vec{r}, \vec{r}') \) and \( B_i(\vec{r}) \sim \varepsilon(\vec{r}) \partial_i G_{ij}(\vec{r}, \vec{r}') \). Hence the boundary conditions at a surface with a discontinuity in \( \varepsilon(\vec{r}) \) with normal vector \( \vec{n} \) are:
\[
\varepsilon_{ik} n_j G_{jm} \quad \text{continuous for all } k, m,
\]
\[
\varepsilon n_j G_{im} \quad \text{continuous for all } m,
\]
\[
\varepsilon_{ik} \partial_j G_{jm} \quad \text{continuous for all } k, m.
\]
Now assume that we can solve these differential equations and have the response function $G$. Then an application of the FD theorem yields

$$
\int e^{i\omega(t-t')} \langle A_i(\vec{r}, t)A_j(\vec{r}', t') \rangle \, dt \, dt' = \langle A_i(\vec{r})A_j(\vec{r}') \rangle \omega
$$

As we will see below, the relaxation of a charge qubit with level separation $\omega$ in the neighborhood of $\vec{r}$ and $\vec{r}'$ will be determined by a correlation function of the type

$$
\langle E_i(\vec{r})E_j(\vec{r}') \rangle = -\frac{\omega^2}{c^2} \coth \left( \frac{\hbar \omega}{2k_B T} \right) \text{Im} \, G_{ij}(\vec{r}, \vec{r}', \omega).
$$

We shall also have occasion to refer to the mixed correlation function. Thus the single function $G$ yields all the field correlations in the system. This method is general for a closed system. The system is completely specified once the dielectric function $\epsilon(\vec{r}, \omega)$ is given everywhere in space.

### 2.2. Physical analogy

Physical intuition for the meaning of $G_{\parallel}(\vec{r}, \vec{r}', \omega)$, and a practical calculation method, may be obtained by noting the similarity of equations (2.9) and (2.10). Place a fictitious point electric dipole $\vec{p}$ at the point $\vec{r}'$. The current is

$$
\vec{I}^{(f)}(\vec{r}) = -i\omega \vec{p} \delta^3(\vec{r} - \vec{r}').
$$

and the electric field is given by

$$
\left[ \partial_i \partial_i - \delta_{ij} \nabla^2 - \delta_{ij} \frac{\omega^2 \epsilon(\vec{r})}{c^2} \right] E^{(f)}_j(\vec{r}) = \frac{4\pi \omega^2}{c^2} \delta^3(\vec{r} - \vec{r}').
$$

On the other hand, multiplying equation (2.10) by $p_k$ and summing over $k$ we find:

$$
\left[ \partial_i \partial_i - \delta_{ij} \nabla^2 - \delta_{ij} \frac{\omega^2 \epsilon(\vec{r})}{c^2} \right] G_{ik}(\omega; \vec{r}, \vec{r}') p_k = -4\pi \hbar p_j \delta^3(\vec{r} - \vec{r}').
$$

Comparison of equations (2.17) and (2.18) says that

$$
G_{ik}(\omega; \vec{r}, \vec{r}') p_k = -\frac{\hbar c^2}{\omega^2} E^{(f)}_l.
$$

and, using equation (2.14)

$$
\langle E_i(\vec{r})E_j(\vec{r}') \rangle \omega = \frac{\hbar \omega}{2k_B T} \text{Im} \, E^{(f)}_l
$$

Hence if we wish to find (say) $G_{xy}$ we solve the fictitious classical problem of an oscillating dipole $\vec{p} = (0, P_y, 0)$ at the point $\vec{r}'$ and compute $E^{(f)}_x$ at the point $\vec{r}$. Then

$$
G_{xy}(\vec{r}, \vec{r}') = -\frac{\hbar c^2}{\omega^2} E^{(f)}_x / P_y.
$$

We can compute all 9 components of $G$ in this way.

There is a similar analogy for magnetic fluctuations. The current of a point magnetic dipole $\vec{m}$ at $\vec{r}'$ may be written as

$$
J^{(f)}_i(\vec{r}) = \epsilon \epsilon_{ijk} \partial_j \delta^3(\vec{r} - \vec{r}') \, m_k.
$$
which creates a fictitious magnetic field \( B^{(f)}(\vec{r}, \vec{r}') \) at \( \vec{r} \). It is related to \( G \) by
\[
\frac{1}{\hbar} \epsilon_{ijkl} e^{\text{lin}} m_n \partial_i \partial_j G^f_{jk}(\vec{r}, \vec{r}') = B^{(f)}_k(\vec{r}, \vec{r}').
\] (2.23)

Hence if we wish to find the magnetic correlations, we first solve the fictitious classical problem of the magnetic field \( B^{(f)}(\vec{r}, \vec{r}') \) at the point \( \vec{r} \) resulting from an oscillating point magnetic dipole \( \vec{m} \) at the point \( \vec{r}' \). For example, to find the physical magnetic field noise spectral density we place a point magnetic dipole \( \vec{m} \) in the \( j \)th direction at \( \vec{r}' \), compute \( B^{(f)}_j(\vec{r}, \vec{r}') \), and then
\[
\langle B_j(\vec{r}) B_k(\vec{r}') \rangle = \frac{\hbar}{m_j} \coth(h\omega/2k_B T) \text{Im} B^{(f)}_j(\vec{r}, \vec{r}').
\] (2.24)

The Maxwell equations relate \( \vec{E} \) at even orders in \( \omega \) with \( \vec{B} \) at odd orders and vice versa, so the theory has two uncoupled sectors. This is the reason that we need the two separate analogies represented by equations (2.19) and (2.23).

In the fictitious problem, the equations satisfied by the fields in the vacuum are \( \nabla^2 E^{(f)} = 0, \nabla \cdot B^{(f)} = 0, \nabla \cdot E^{(f)} = 4\pi \rho/\varepsilon_0 \), \( \nabla \cdot B^{(f)} = 4\pi \rho/\varepsilon_0 \), and in the metal we have \( \nabla^2 E^{(f)} + 2\delta^2 E^{(f)} = 0, \nabla^2 B^{(f)} + 2\delta^2 B^{(f)} = 0, \nabla \cdot E^{(f)} = 0, \nabla \cdot B^{(f)} = 4\pi \rho/\varepsilon_0 \), in the quasistatic case. The boundary conditions are that the tangential component of \( E^{(f)} \) and \( B^{(f)} \) are continuous at the interface of dielectric and metal, while the normal component \( E^{(f)}_n \) of \( E^{(f)} \) satisfies \( 4\pi i \sigma/\omega E^{(f)}_n(m) = \varepsilon_d E^{(f)}_n(d) \), where \( E^{(f)}_n(m) \), \( E^{(f)}_n(d) \) is the normal component of \( E \) in the metal (respectively, the dielectric) as the surface is approached. \( \sigma \) is the DC conductivity of the metal. \( \varepsilon_d \) is the dielectric constant in the dielectric material. These results show that the quantum and thermal fluctuations of the electromagnetic field can be computed solely using the methods of classical electrodynamics. Furthermore, they suggest that it may be possible to modify standard FEM software tools to compute EWJN.

The dipole analogy makes clear at a formal level that the physics of EWJN is closely related both to the image charge arguments and to van der Waals forces. The currents induced by the fictitious dipole can be thought of as a distributed fluctuating image charge, much as we think of the induced currents in the van der Waals attraction.

### 2.3. Quasistatic approximation

The subject of this paper is the random electric and magnetic fields that decohere qubits in the neighborhood of small metallic objects. The characteristic frequencies for the decoherence rarely exceed a few GHz, so we restrict our attention to frequencies at or below this range. For this reason we employ the quasistatic approximation from the start, setting the vacuum wavevector \( k = \omega/c = 0 \). In the interior of a metal object with conductivity \( \sigma \) the characteristic length scale of the fields is the skin depth \( \delta = c/\sqrt{2\pi \sigma \omega} \). The inverse skin depth \( \delta^{-1} \) is proportional to \( (\sigma/\omega) k >> k \) and it is retained in the theory. For example, the term \( \omega^2 \varepsilon(\vec{r})/c^2 \) in equation (2.9) can be neglected when \( \vec{r} \) is in the dielectric or vacuum where \( \varepsilon \sim 1 \) but not when \( \vec{r} \) is in the metal. In this approximation, radiation fields are neglected. We assume that the Drude model is a good approximation for the metals in question, and that \( \omega \ll 1/\tau \), where \( \tau \) is the relaxation time of electrons in the metals. The dielectric function is always approximated as \( \varepsilon = 4\pi i \sigma/\omega \).

### 2.4. Nonlocal effects

This paper focuses on the cases where local response is valid. Roughly speaking, this is when the distance of \( \vec{r} \) and \( \vec{r}' \) from the nearest metal surface is greater than the electron mean free path in the metal. However, when the distance to the metal tends to zero, the local expressions for noise strengths diverge, which is clearly unphysical. For completeness, we briefly outline how to include nonlocality in the theory. Generally \( \vec{D}(\vec{r}) \), the electric displacement, depends on \( \vec{E}(\vec{r}') \) according to \( D_i(\vec{r}, t) = \int d\vec{r}' \varepsilon_i(\vec{r} - \vec{r}', t - t') \vec{E}_j(\vec{r}', t') \) and when Fourier transformed this becomes \( D_i(\vec{k}, \omega) = \varepsilon_i(\vec{k}, \omega) \vec{E}_j(\vec{k}, \omega) \). Equation (2.10) becomes
\[
\left( -\delta_{ij} \nabla^2 + \partial_i \partial_j \right) G_{\delta k}(\vec{r}, \vec{r}') - \delta_{ij} \frac{\omega^2}{c^2} \int d\vec{r}'' \varepsilon_{im}(\vec{r}, \vec{r}'') G_{mk}(\vec{r}'', \vec{r}')
\] (2.25)
\[
= -4\pi \hbar \delta^3(\vec{r} - \vec{r}') \delta_{ij}.
\] (2.26)

Use of this equation with an appropriate choice for \( \varepsilon(\vec{r}, \vec{r}') \) cures the unphysical divergence at small distances. In practice, to date only the problems of a conducting half-space and conducting films have been treated using the nonlocal formalism [23, 24, 31].
3. Application to qubits

In this section we treat relaxation and dephasing of qubits. Relaxation is due to absorption or spontaneous emission of a photon, the latter being thought of as an electromagnetic mode of a metallic object. Dephasing is due to modulation of the energy level separation of a qubit by random electromagnetic fields.

3.1. Relaxation

A qubit system in a noisy environment is described by a Hamiltonian \( H = H_q + H_{\text{t}}(t) \) where \( H_q \) admits two eigenstates \(|0\rangle, |1\rangle\) such that \( H_q |i\rangle = \epsilon_i |i\rangle \). The relaxation rate for such a qubit in the presence of EWJN is given by the Golden Rule-type formula

\[
\frac{1}{T_1} = \frac{1}{\hbar^2} \int_{-\infty}^{\infty} \langle 0 | H_{\text{t}}(t) | 1 \rangle \langle 1 | H_q(0) | 0 \rangle e^{-i\omega t} dt. \tag{3.1}
\]

Consider a qubit with charge, mass, and g-factor \( e, m, g \) respectively placed in a time dependent electromagnetic field described by \( \vec{A}(r, t) \). The full Hamiltonian is

\[
H = \frac{1}{2m} \left( \vec{\Pi} - \frac{e}{c} \vec{A} \right)^2 + V(\vec{r}) - \frac{eg}{2m} \vec{B} \cdot \vec{S}, \tag{3.2}
\]

where \( \vec{\Pi} = -i\hbar \nabla \). Here we will restrict ourselves to \( \mathcal{O}(e) \) so the Hamiltonian can be written

\[
H = \frac{\vec{\Pi}^2}{2m} + V(\vec{r}) - \frac{e}{2mc} (\vec{\Pi} \cdot \vec{A} + \vec{\Pi} \cdot \vec{\Pi}) - \frac{eg}{2m} \vec{B} \cdot \vec{S}. \tag{3.3}
\]

Imposing the gauge condition \( \phi = 0 \) we find a Hamiltonian readily treated in the interaction picture. The charge distribution generating the noise is contained in the metal, so at a nearby qubit we have \( \nabla \cdot E = \nabla^2 \phi + \frac{1}{c^2} \partial_t (\nabla \cdot \vec{A}) = 0 \). For finite frequency noise, this implies \( \nabla \cdot \vec{A} = 0 \), and thus \( [\vec{\Pi}, \vec{A}(\vec{r})] = -i\hbar \nabla \cdot \vec{A}(\vec{r}) = 0 \). The time dependence of the system is entirely due to the electromagnetic noise and the static Hamiltonian \( H_q = \frac{\vec{\Pi}^2}{2m} + V(\vec{r}) \). We are left with

\[
H = H_q + H_{\text{t}}(t), \tag{3.4}
\]

\[
H_{\text{t}}(t) = -\frac{e}{mc} \vec{A}(r, t) \cdot \vec{\Pi} - \frac{eg}{2mc} \vec{B}(r, t) \cdot \vec{S}. \tag{3.5}
\]

Our interaction Hamiltonian can be written as a spatial Taylor series as follows

\[
H_{\text{t}}(t) = -\frac{e}{mc} \sum [A_i(r, t) + (\nabla_j A_i(r, t))_{r_j=0} + ...] \vec{\Pi}_i \frac{-eg}{2mc} B_i S_i. \tag{3.6}
\]

This allows us to treat the relevant matrix elements term by term in multipole moments, as described in [32].

Truncating the series at second order and evaluating the off-diagonal matrix elements gives us

\[
\frac{1}{T_1^\text{rel}} = \frac{\hbar^2}{2m} \left( \langle p_i \rangle \langle p_i^\ast \rangle \langle E_i(\vec{r}) E_i(\vec{r}) \rangle_\omega \right); \quad \frac{1}{T_1^\text{rel}} = \frac{\hbar^2}{2m} \left( \langle m_i \rangle \langle m_i^\ast \rangle \langle B_i(\vec{r}) B_i(\vec{r}) \rangle_\omega \right), \tag{3.7}
\]

\[
\frac{1}{T_1^\text{cross}} = \frac{\hbar^2}{2m} \left( \langle p_i \rangle \langle m_k \rangle \langle E_i(\vec{r}) B_k(\vec{r}) \rangle_\omega + \langle m_k \rangle \langle p_i \rangle \langle B_k(\vec{r}) E_i(\vec{r}) \rangle_\omega \right). \tag{3.8}
\]

Above we set \( \hbar \omega = \epsilon_1 - \epsilon_0 \) via equation (3.1). For brevity we also use \( \langle x \rangle \equiv \langle 0 | x | 1 \rangle \) and \( \langle F_i(t) F_j(0) \rangle_\omega = \langle F_i F_j \rangle_\omega \). Here we only include dipole contributions; higher order multipole moments and more details of the calculation are treated in the appendix.

In the case of the spin qubit the states \(|0\rangle, |1\rangle\) are up and down states of the spin part of the wavefunction. Hence \(|0\rangle = |\psi_0 \rangle \otimes |0\rangle\) where \(|\psi_0 \rangle\) is the orbital part of the wavefunction which is common to both states of the spin qubit. Immediately we see that all the spatial operator matrix elements \( \langle p_i \rangle = \langle q_i \rangle = \langle l_i \rangle = 0 \). Hence the above expression simplifies to

\[
\frac{1}{T_1} = \frac{\hbar^2}{2m} \left( \langle S_z \rangle \langle S_z^\ast \rangle \langle B_z(\vec{r}) B_z(\vec{r}) \rangle_\omega + \langle B_z(\vec{r}) B_z(\vec{r}) \rangle_\omega \right). \tag{3.9}
\]

Spatially localized qubits are defined as those whose spatial extent is much less than the typical wavelength of the noise field. For them, only the local \( \langle \vec{r} = \vec{r}' \rangle \) noise correlations are needed. For example, a local spin qubit whose up and down states are eigenstates of \( S_z \), has a relaxation time given by

\[
\frac{1}{T_1} = \frac{\hbar^2}{4m} \left( \langle B_z(\vec{r}) B_z(\vec{r}) \rangle_\omega + \langle B_z(\vec{r}) B_z(\vec{r}) \rangle_\omega \right). \tag{3.10}
\]
3.2. Dephasing

Qubit relaxation is due to the off-diagonal matrix elements $\langle 0 \mid H_{\text{dd}} \mid 1 \rangle$ and $\langle 1 \mid H_{\text{dd}} \mid 0 \rangle$ of the noise Hamiltonian. The diagonal elements $\langle 0 \mid H_{\text{dd}} \mid 0 \rangle$ and $\langle 1 \mid H_{\text{dd}} \mid 1 \rangle$ produce dephasing. If the initial state is $(1/\sqrt{2})(\mid 0 \rangle \pm \mid 1 \rangle)$, and the state at time $t$ is $(1/\sqrt{2})(\mid 0 \rangle \pm e^{i\phi(t)}\mid 1 \rangle)$ then $\phi$ is random after a time $T_2$. The basic formulas for $T_2$ are as follows. We have

$$\frac{1}{T_2} = \frac{1}{2T_0} + \frac{1}{T_0},$$

(3.11)

where $T_0$ is the dephasing time. For a Johnson-type noise mechanism, the Gaussian approximation for $T_0$ should be very accurate, since many modes of the metal contribute to the noise. $T_0$ is then calculated in the following way. Again let the applied field be in the $\mathbf{t}$th direction. The initial condition is $\phi(t = 0) = 1$. We then repeatedly measure $X = \mid 0 \rangle \langle 0 \rangle + \mid 1 \rangle \langle 1 \rangle$, average to get $X(t)$ and the function $\Gamma(t)$ is defined by

$$X(t) = \exp\{-\Gamma(t)\}X(0)\cos\omega t.$$

(3.12)

and the Gaussian result for $\Gamma(t)$ is

$$\Gamma(t) = \frac{t^2}{2} \int_{-\infty}^{\infty} d\omega \frac{\sin^2(\omega t/2)}{(\omega t)^2},$$

(3.13)

with

$$S(\omega) = \frac{1}{\hbar^2} \int_{-\infty}^{\infty} dt \{\langle 1 \mid H_n(t) \mid 1 \rangle - \langle 0 \mid H_n(t) \langle 0 \rangle \}
\times \{\langle 1 \mid H_n(t) \rangle - \langle 0 \mid H_n(t) \rangle e^{-i\omega t}\}.$$

(3.14)

Evidently we need the diagonal matrix elements of the time-dependent part of the Hamiltonian from equation (3.5). Defining moments $p_i = c_i t$ and $m_i = \frac{c_i}{2mc}(l_i + gS_i)$,

$$\langle 1 \mid H_n(t) \mid 1 \rangle - \langle 0 \mid H_n(t) \rangle = -B_i(t)(\langle m_i \rangle - \langle m_i \rangle_0) + E_i(t)(\langle p_i \rangle - \langle p_i \rangle_0).$$

To keep things short let $\Delta x = (\langle 1 \mid x \rangle - \langle 0 \mid x \rangle)$ for any operator $x$. The integral kernel becomes

$$S(\omega) = \frac{1}{\hbar^2} \int_{-\infty}^{\infty} dt \{\langle B_i B_i \rangle \omega \Delta m_i \Delta m_j - \langle B_i E_i \rangle \omega \Delta m_i \Delta p_j - \langle E_i B_i \rangle \omega \Delta p_i \Delta m_j + \langle E_i E_i \rangle \omega \Delta p_i \Delta p_j\}.$$

(3.15)

In order to make use of equation (3.13) we need to make some mild assumptions on the frequency dependence of the noise spectral density terms. We can write

$$\Gamma(t) = \frac{t^2}{2} \int_{0}^{1/T} d\omega \frac{f(\omega)}{\omega} \coth\left(\frac{\hbar\omega}{2k_B T}\right) \frac{\sin^2(\omega t/2)}{(\omega t)^2}.$$

(3.16)

Again, $f(\omega)$ contains all the information about conductivity, qubit position, device geometry, etc, but it depends weakly on frequency at low frequency, and here we will take it to be independent of frequency $f(\omega) = f_0$ until it falls rapidly to zero at $\omega = 1/T$, where $T$ is the electron relaxation time. We note first that at very short times ($t \ll T, \hbar/k_B T$) we always get $\Gamma(t) \sim t^2/t_0^2$ (Gaussian decay), where

$$t_0^2 = \frac{4\pi^2}{f_0^2} \tanh\left(\frac{\hbar}{2k_B T}\right).$$

As a result, Gaussian decay is only observed when the noise is quasi-static. Exponential decay at longer times is the most important from the standpoint of EWJN. This is where $t \gg T$ and $t \gg \hbar/k_B T$ and then we can write

$$\Gamma(t) = 4f_0 \int_{0}^{1/2T} dx \coth\left(\frac{h}{k_B T}\right) \frac{\sin^2 x}{x} \approx \frac{2\pi f_0 k_B T t}{\hbar}. $$

The agreement of these limiting cases with the exact integral is shown in figure 2. Hence, at any experimentally accessible temperature

$$\frac{1}{T_0} = \frac{2\pi f_0 k_B T}{\hbar}. $$

(3.17)
We see that only off diagonal elements of the multipole moments determine \( T_1 \) and all of the matrix elements come into the determination of \( T_2 \). If the expectation values of the multipole moments are not significantly different between the ground and excited qubit states \( T_1 \) will be small and \( T_2 \) will generally be of the same order of magnitude, which distinguishes EWJN from many other noise mechanisms.

In many experiments, it appears that the noise spectrum has two components, a ‘1/f’ component that dominates at low frequencies, and a white component that is bigger at high frequencies. Using the qubit as a spectrometer \[33\] it has been shown that this happens both in GaAs devices \[34\] and in Si devices \[35\]. Echo techniques can mitigate the low-frequency noise but not the more pernicious white part. \( T_2^{\text{echo}} \), the decoherence time after echoing, can serve as a diagnostic for EWJN in this situation. The experiment of \[34\] is particularly interesting in this regard, since it shows that the white component of the noise has a strong temperature dependence which the \( 1/f \) part is largely temperature (\( T \)) independent, strongly suggesting different origins for the two types of noise. However, \( T_2^{\text{echo}} \) was proportional to \( T^{-2} \), while equation (3.17) would predict a \( T^{-1} \) behavior.

### 4. Electric noise

The noise spectral density \( \langle E_i(\vec{r})E_j(\vec{r'})\rangle_{\omega} \) generally involves four length scales: \( |\vec{r} - \vec{r'}| \), the distance over which the correlations are to be measured; \( d \), the distance from the qubit to the conducting object(s); \( \delta = \epsilon / \sqrt{2\pi \sigma\omega} \), the skin depth in the conductor(s); and \( L \), the linear size of the conducting object(s). In practice \( L \) may be larger or smaller than \( \delta \) and we give results for both cases where possible. In most cases, the size of the qubit is small, which means that usually the case \( \vec{r} \approx \vec{r'} \) is of interest, and \( |\vec{r} - \vec{r'}| \) is the smallest length in the problem. However, qubits can also be extended objects, so we will give formulas as a function of \( \vec{r} - \vec{r'} \) where possible. As stated above, the vacuum wavelength is always taken to be infinite. The simple geometries treated in this paper are shown in figure 3.

We will focus first on some limiting cases in which at least one of the other three lengths is very different from the two others.

#### 4.1. Half space

There is a semi-infinite slab of metal with conductivity \( \sigma \) occupying the half-space \( z < 0 \). We are interested in the electric noise in a dielectric with dielectric constant \( \epsilon \) at \( z < 0 \).

##### 4.1.1. Point qubit

We first focus on some simple methods to compute \( G_{ij}(\vec{r}, \vec{r'} = \vec{r}) = \delta_{ij} G_{ij}(\vec{r}, \vec{r'} = \vec{r}) \), which is sufficient for the calculation of the decoherence of a point qubit. This case lends itself to some simple approximations that are physically illuminating. The results in the subsection are well-known \[7, 36, 37\] and are included here for completeness and to establish notation.

- **Image regime.** To understand this problem physically, we first outline the solution when \( d \ll \delta \), since the problem is then essentially elementary. The greater part of the electric field is concentrated within a sphere of...
radius of order $d$ of the dipole. This implies that inside the metal we have that $\nabla^2 \tilde{E} = (-2i/\delta^3) \tilde{E} \approx 0$, since the skin depth $\delta$ may be taken to be large. The problem now reduces to the image problem for a static point charge in a medium with dielectric constant $\varepsilon_d$ located at a distance $d$ from a half space with dielectric constant $\varepsilon_m \approx 4\pi\sigma/\omega$. For $z > 0$ we have the equations $\nabla \cdot \tilde{E} = 4\pi\rho = 4\pi\delta^3(\tilde{r} - \tilde{r}^\prime)$ and $\nabla \times \tilde{E} = 0$. For $z < 0$, we have $\nabla \cdot \tilde{E} = 0$ and $\nabla \times \tilde{E} = 0$. At the interface we have $\varepsilon_d E_z(z = 0_+) = \varepsilon_m E_z(z = 0_-)$ and $E_{x,y}(z = 0_+) = E_{x,y}(z = 0_-)$. This is the textbook image problem. Hence the solution for $z > 0$ is given by

$$E = -\nabla \Phi, \quad \Phi(x, y, z) = q/|\tilde{r} - \tilde{r}^\prime| + q'/|\tilde{r} - \tilde{r}''|$$

and for $z < 0$ by $\Phi_2(x, y, z) = q''/|\tilde{r} - \tilde{r}^\prime|$. Here $q' = -q[(\varepsilon_m - \varepsilon_d)/(\varepsilon_m + \varepsilon_d)]$ and $q'' = q[(2\varepsilon_m)/(\varepsilon_m + \varepsilon_d)]$. This satisfies the differential equations and the boundary conditions. Hence the textbook image solution carries over to this case.

We will calculate the $\langle E_x(\tilde{r}) E_x(\tilde{r}^\prime) \rangle_\omega$ correlation function first, so we place a fictitious dipole $\tilde{p} = p\hat{x}$ at $\tilde{r}^\prime = (0, 0, d)$. Then we need the induced field at $\tilde{r}$. It is produced by the image dipole $\tilde{p}'$ at $\tilde{r}''$:

$$p' = -p \frac{\varepsilon_m - \varepsilon_d}{\varepsilon_m + \varepsilon_d} \approx -p \left(1 + \frac{i\omega \varepsilon_d}{2\pi \sigma}\right)$$

and the field from this charge is

$$E^{(f)}_{x}(\tilde{r}) = p' \frac{3(\tilde{r} - \tilde{r}^\prime)x(\tilde{r} - \tilde{r}'')_x - |\tilde{r} - \tilde{r}''|^2}{|\tilde{r} - \tilde{r}''|^3} = p \left(1 + \frac{i\omega \varepsilon_d}{2\pi \sigma}\right) \frac{1}{(2d)^3}$$

so

$$G_{xx}(\tilde{r}, \tilde{r}^\prime, \omega) = -\frac{\hbar^2}{\omega^2} \left(1 + \frac{i\omega \varepsilon_d}{2\pi \sigma}\right) \frac{1}{(2d)^3}$$

and using equation (2.19) we find at the position $\tilde{r} = \tilde{r}''$ of the qubit that the physical local noise spectral density is

$$\langle E_x(\tilde{r}) E_x(\tilde{r}) \rangle_\omega = \hbar \omega \varepsilon_d \coth \left(\frac{\hbar \omega}{2k_B T}\right)$$

which at low temperatures $k_B T \ll \hbar \omega$ reduces to

$$\langle E_x(\tilde{r}) E_x(\tilde{r}) \rangle_\omega = \frac{\hbar \omega \varepsilon_d}{16\pi\sigma d^3}$$

and at high temperatures $k_B T \gg \hbar \omega$ to

$$\langle E_x(\tilde{r}) E_x(\tilde{r}) \rangle_\omega = \frac{k_B T \varepsilon_d}{8\pi\sigma d^3}$$

Of course cylindrical symmetry implies that $\langle E_x(\tilde{r}) E_x(\tilde{r}) \rangle_\omega = \langle E_y(\tilde{r}) E_y(\tilde{r}) \rangle_\omega$.

It is important to note that the electric noise is inversely proportional to $\sigma$. For really good metals, the screening is complete and there is no dissipation and therefore no fluctuations in the field. It is a general result that the result for $E^{(f)}$ depends only on the ratio of dielectric constants in the two media, that is, on $(4\pi\sigma/\omega)/\varepsilon_d$. This follows immediately from inspection of the boundary condition, which is the only place that $\varepsilon_d$ enters the calculation. The $d^{-3}$ dependence follows immediately from the physical analogy to the image problem.

Now we will do the $\langle E_x(\tilde{r}) E_x(\tilde{r}^\prime) \rangle_\omega$ correlation function, so we place a dipole $\tilde{p} = p\hat{x}$ at $\tilde{r}^\prime = (0, 0, d)$. Then we need the induced field at $\tilde{r}$. The calculation proceeds as for the $x$ direction except for a change in sign of
the fictitious image dipole $\tilde{p}^z$ at $\vec{r}'' = (0, 0, -d)$ with the result that

$$\langle E_x(\vec{r}') E_x(\vec{r}') \rangle_\omega = \frac{\hbar \omega \delta}{8 \pi \sigma d^3} \coth \left( \frac{\hbar \omega}{2 k_B T} \right)$$

(4.7)

which is greater than $\langle E_x(\vec{r}') E_x(\vec{r}') \rangle_\omega$ by a factor of 2. This anisotropy is quite significant for detailed exploration of the theory by experiment.

b. Induction regime. This regime is characterized by the opposite limit $d \gg \delta$. The qubit is far away from the interface on the length scale of the penetration depth. The image problem does not carry over directly since the electric field in the metal satisfies $\nabla^2 E^{(f)} = (-2i/\delta^2) E^{(f)}$ in the metal and $\delta^2$ cannot be neglected, as it was in the image regime. However, we may now use the fact that the field penetrates only a short distance into the metal, and this allows us to develop a perturbation series in $\omega$ for the complex amplitudes $E^{(f)}$, $B^{(f)}$ in the frequency domain. At order $\omega^0$ we have an electric field $E^{(f)}$ but $B^{(f)}$ vanishes. $E^{(f)}$ is the static field from the previous image calculation that is normal to the interface. At order $\omega^1$ there is a magnetic field that corresponds to the static electric field according to the equation $\nabla \times \vec{B} = -i \omega \vec{E}$. To compute $B^{(f)}$ at this order we again put a dipole $\tilde{p} = p\vec{x}$ at $\vec{r}' = (0, 0, d)$ together with its image dipole $-p\vec{x}$ at $\vec{r}'' = (0, 0, -d)$. This corresponds to a current $\vec{j}(\vec{r}) = p(\partial \delta^3(\vec{r} - \vec{r}')/\partial x) - p(\partial \delta^3(\vec{r} - \vec{r}'')/\partial x)$. Computing the magnetic field due to this current we have:

$$B^{(f)}_y(z = 0) = -\frac{2i p \rho \omega}{c (\rho^2 + d^2)^{1/2}}$$

and $B^{(f)}_z(z = 0) = B^{(f)}_x(z = 0) = 0$, (4.8)

correct to order $\omega$. $B^{(f)}_y$ is continuous at the interface and $\nabla^2 B^{(f)} = -2i \delta^2 B^{(f)}$ for $z < 0$. The crucial point is that since $\delta^2$ is large we may neglect the $x$ and $y$ derivatives in both $B^{(f)}_x$ and $E^{(f)}_x$ for $z < 0$ and we have that

$$B^{(f)}_y = -\frac{2i p \rho \omega}{c (\rho^2 + d^2)^{1/2}} \exp [(1 - i)z/\delta].$$

(4.9)

Since $\nabla \times \vec{E} = i \omega \vec{B}/c$ for $z < 0$, consistency requires that

$$\frac{\partial E^{(f)}_z}{\partial z} = (1 - i) \delta^{-1} E^{(f)}_x(z) = i \omega B^{(f)}_y(z)/c$$

(4.10)

at order $\omega^2$. Solving these equations gives

$$E^{(f)}_x(z = 0) = \frac{(1 + i) p d \delta \omega^2}{c (\rho^2 + d^2)^{1/2}}.$$  

(4.11)

$E^{(f)}_x$ is continuous at the interface so we also get a correction to the field for $z > 0$ at order $\omega^2$.

For $z > 0$ the field components satisfy the Laplace equation $\nabla^2 E^{(f)} = 0$, so we can get the field everywhere by applying Green’s theorem to the components of $E^{(f)}$. Using equations (2.19) and (2.20) we find

$$\langle E_x(\vec{r}) E_x(\vec{r}) \rangle_\omega = \langle E_x(\vec{r}) E_x(\vec{r}) \rangle_\omega = \frac{\hbar \omega}{8 \pi \sigma d^3} \coth \left( \frac{\hbar \omega}{2 k_B T} \right)$$

$$\approx \begin{cases} \frac{\hbar \omega}{8 \pi \sigma d^3} & \text{for } k_B T \ll \hbar \omega, \\ \frac{\hbar \omega}{4 \pi \sigma d^3} & \text{for } k_B T \gg \hbar \omega. \end{cases}$$

(4.12)

Since $\delta \sim \frac{1}{\sqrt{\omega/\sigma}}$, in the classical limit we have that the noise is proportional to $\sqrt{\omega/\sigma}$, an interesting contrast to the $\omega/\sigma$ dependence in the image regime.

For the $z-z$ correlation function the derivation is only slightly different. We now put a dipole $\tilde{p} = p\vec{z}$ at $\vec{r}' = (0, 0, d)$. $\vec{j}(\vec{r}) = p(\partial \delta^3(\vec{r} - \vec{r}')/\partial z) + p(\partial \delta^3(\vec{r} - \vec{r}'')/\partial z)$. The result for the noise spectral density is

$$\langle E_x(\vec{r}, \omega) E_x(\vec{r}, \omega) \rangle_\omega = \frac{\hbar \omega}{8 \pi \sigma d^3} \coth \left( \frac{\hbar \omega}{2 k_B T} \right).$$

(4.13)

This is the same as equation (4.12), so the noise becomes isotropic at large distances from a metal surface.

c. Summary of approximate results for the point qubit. The two regimes are distinguished by the relative magnitudes of $d$ and $\delta$—the distance of the source from the half space and the skin depth. The following physical considerations serve as the basis for understanding electric field noise in small devices.

The image regime of small $d/\delta$ is fairly easily understood. In the fictitious problem, the electric field penetrates the metal in the same way it does in the textbook case of two dielectrics of strongly different dielectric
constants. The field is strongly screened at the surfaces so that the field lines bend sharply at the interface. This field dissipates energy at the usual rate \( \sim \sigma |\overrightarrow{E}|^2 \) per unit volume in the fictitious problem, and the physical fluctuations are also proportional to this. However, the ‘impedance mismatch’ dominates to the extent that \( |\overrightarrow{E}| \sim 1/\sigma \) in the metal overall and the noise spectral density at a given frequency is proportional to \( 1/\sigma \). The noise is stronger for poor conductors since the field penetrates further. Once the dependence on the conductivity has been determined, the \( 1/d^8 \) spatial dependence follows by dimensional analysis or noting that the fictitious field is produced by an image dipole.

The induction regime of large \( d/\delta \) is somewhat different. The electric field outside the metal is, again, essentially normal to the interface. This induces a magnetic field parallel to the interface which penetrates only a distance \( \delta \) into the metal. This in turn induces an orthogonal electric field parallel to the surface that dissipates energy. The volume in which the energy is dissipated is of thickness \( \delta \), so the dissipation is proportional to \( \delta \). Thus the image result is reduced by the factor \( \delta/d \), and the noise spectral density is proportional to \( 1/d^2 \sqrt{\sigma} \).

### 4.1.2. Extended qubits

For extended qubits, we need the full \( \overrightarrow{r} \) and \( \overrightarrow{r} \) dependence of \( G \). We compute using a method that will be used repeatedly in what follows. Details are given in the appendix, along with explicit forms for the components of the noise tensors. We place a fictitious dipole \( \overrightarrow{p} = \overrightarrow{p}^i = (0, 0, d) \) and find the induced field

\[
\overrightarrow{E}^{(\text{ind})}(\overrightarrow{r}) = -\frac{\hbar}{2\pi} \int d^3q \left( -i q_x - i q_y, q \right) e^{-i \overrightarrow{q} \cdot \overrightarrow{r}} \times \frac{1 - (\varepsilon_m/\varepsilon_d)q/\alpha}{1 + (\varepsilon_m/\varepsilon_d)q/\alpha} \hat{e}_x e^{-iq_z}.
\]

(4.14)

for \( z > 0 \), and the corresponding electric noise is given by equation (2.20):

\[
\langle \overrightarrow{E}(\overrightarrow{r} = (\overrightarrow{p}, z)) E_{r}\rangle_{\omega} = -\frac{\hbar}{2\pi} \coth \frac{\hbar \omega}{2\kappa B T} \times \text{Im} \int d^3q \left( -i q_x - i q_y, q \right) e^{-i \overrightarrow{q} \cdot \overrightarrow{r}} \times \frac{1 - (\varepsilon_m/\varepsilon_d)q/\alpha}{1 + (\varepsilon_m/\varepsilon_d)q/\alpha} \hat{e}_x e^{-iq_z}.
\]

(4.15)

The diagonal is complicated, but it can be evaluated numerically and it simplifies in the limits of large and small \( d \). When \( d \ll \delta, \alpha \approx q \) and we find for the physical noise

\[
\langle \overrightarrow{E}(\overrightarrow{r}) E_{r}\rangle_{\omega} \approx -\frac{\hbar \omega \delta}{2\pi \sigma} \coth \frac{\hbar \omega}{2\kappa B T} \frac{d + z}{[(d + z)^2 + \rho^2]^{3/2}}.
\]

(4.16)

The diagonal component of this equation reduces to equation (4.7) when \( \overrightarrow{r} = \overrightarrow{r}' = (0, 0, d) \), satisfying an important check. This case has the unusual feature of anticorrelations in \( E_x \) for large lateral separations of \( \overrightarrow{r} - \overrightarrow{r}' \). \( \rho > \sqrt{\delta}(d + z) \). This implies that in the appropriate geometry there can be cancellations in the integral that determines qubit decoherence. This can be incorporated as a design feature.

For \( d \gg \delta \) (but still \( d \ll \delta \omega/\sigma \)) we have \( \alpha \approx (1 - i)\delta^{-1} \) and the physical noise correlation function is

\[
\langle \overrightarrow{E}(\overrightarrow{r}) E_{r}\rangle_{\omega} = -\frac{\hbar \omega \delta}{2\pi \sigma} \coth \frac{\hbar \omega}{2\kappa B T} \frac{1}{[(d + z)^2 + \rho^2]^{3/2}}.
\]

(4.17)

Again, it can be verified that the diagonal component of this equation reduces to equation (4.13) when \( \overrightarrow{r} = \overrightarrow{r}' = (0, 0, d) \). The situation for \( \langle \overrightarrow{E}(\overrightarrow{r}) E_{r}\rangle_{\omega} \) is somewhat more complicated because of the lack of cylindrical symmetry. However, the method of the previous section does not depend on the symmetry and it can still be used. We now use \( \overrightarrow{p} = \overrightarrow{p}^i \). This leads to a fictitious induced electric field for \( z > 0 \):

\[
\overrightarrow{E}^{(\text{ind})}(\overrightarrow{r}) = -\frac{\hbar}{2\pi} \frac{\partial}{\partial x} \int d^3q \left[ \frac{1}{q} e^{-i q(d + z)} \frac{1 - (\varepsilon_m/\varepsilon_d)q/\alpha}{1 + (\varepsilon_m/\varepsilon_d)q/\alpha} \hat{e}_x e^{i q x + i q y} \right]
\]

and the physical noise correlation is

\[
\langle \overrightarrow{E}(\overrightarrow{r}) E_{x}\rangle_{\omega} = -\frac{\hbar}{2\pi} \frac{\partial}{\partial x} \int d^3q \left[ \text{Im} \frac{1 - (\varepsilon_m/\varepsilon_d)q/\alpha}{1 + (\varepsilon_m/\varepsilon_d)q/\alpha} \hat{e}_x e^{i q x + i q y} \coth \frac{\hbar \omega}{2\kappa B T} \right].
\]

(4.19)

For \( d \ll \delta \) we can calculate the diagonal element of the physical noise spectral density and show it is in agreement with equation (4.4). For \( d \gg \delta \) (but still \( d \ll \delta \omega/\sigma \)) we have

\[
\langle \overrightarrow{E}(\overrightarrow{r}) E_{x}\rangle_{\omega} = \frac{\hbar}{2\pi} \frac{\delta}{\sigma \delta} \left[ \frac{1}{\rho^2} \left( 1 - \frac{d + z}{[(d + z)^2 + \rho^2]^{3/2}} \right) \coth \frac{\hbar \omega}{2\kappa B T} \right].
\]

(4.20)

and the various components of the tensor may be calculated from this expression.
It is difficult to give detailed physical interpretations of the expressions for nonlocal correlations, other than to point out the dependence on the components of $\mathbf{r} - \mathbf{r}'$ has takes the form of inverse power laws consistent with the picture of a fluctuation-induced force.

4.1.3. Between induction and image regimes

In general equation (4.15) cannot be simplified, but in both the image and induction regime we can find analytic results (equations (4.7) and (4.13)). Using these two results, we can interpolate a function to compute correlation functions for qubit geometries that do not fall into either of the extremal cases treated here. For two functions $f_1$ and $f_2$ we define a family of interpolated functions

$$f_{\text{int}}(p) = (f_1^p + f_2^p)^{1/2}$$

and search for the $p \in \mathbb{R}$ that optimizes the interpolated function’s agreement with the extended qubit noise spectral density. The interpolated function is plotted alongside numerical results for equation (4.15) in figure 4.

4.2. Conducting cylinder

We consider an infinite conducting circular cylinder (conductivity $\sigma$ and radius $a$) with its axis along the $z$-direction. There is a qubit at the point $\mathbf{r}' = (d, 0, 0)$. We wish to compute $\langle B_i(\mathbf{r}') B_i(\mathbf{r}'') \rangle$ with $i = x, y, z$. We are particularly interested in the anisotropy of relaxation times, which depend on the ratios of this correlation function for different values of $i$. The most common case is when the skin depth $\delta \gg a$. We will also be mainly interested in thin wires also in the sense that $d \ll a$. This means that the fictitious applied field is slowly varying over the cylinder. The problem reduces to a computation of the electric polarizability.

The problem of the magnetic polarizability of a conducting cylinder in a uniform field is a standard one [38]. We modify the solution to obtain the electric polarizability $\beta_i$, defined by $P_i = \pi a^2 \beta_i E_i$, where $P_i$ is the electric dipole moment per unit length in direction $i$. We find

$$\beta_i = \frac{1}{2\pi} \frac{4\pi \sigma / \omega - C}{4\pi \sigma / \omega + C}$$

with

$$C = -1 + \frac{ka J_0(ka)}{J_1(ka)}$$

and $k = (1 + i) / \delta$.

Again, the most interesting case (and the easiest one to calculate) is when $\delta \gg a$, so $|ka| \ll 1$ and

$$\frac{ka J_0(ka)}{J_1(ka)} \approx ka \frac{1}{ka / 2} = 2,$$

and then we find

$$\text{Im} \beta_i = \frac{\omega}{\pi \sigma}.$$
When \(d \gg a\) we can integrate along the z-axis assuming uniform applied field. We find

\[
\langle E_x(\vec{r}') E_x(\vec{r}'') \rangle = \frac{123\omega\hbar^2}{256\sigma d^5} \coth \left( \frac{\hbar \omega}{2k_B T} \right)
\]  \hspace{1cm} (4.26)

and

\[
\langle E_y(\vec{r}') E_y(\vec{r}'') \rangle = \frac{3\omega\hbar a^2}{32\sigma d^5} \coth \left( \frac{\hbar \omega}{2k_B T} \right).
\]  \hspace{1cm} (4.27)

We may calculate the noise correlation for the z-direction in the same way. However, end effects are likely to be very important for this case. We present the result as a conjecture to be investigated in further work:

\[
\langle E_z(\vec{r}') E_z(\vec{r}'') \rangle = \frac{27\pi\hbar a^4}{2048d^6} \coth \left( \frac{\hbar \omega}{2k_B T} \right).
\]  \hspace{1cm} (4.28)

The infinite cylinder geometry differs qualitatively from the other cases considered in that the dimension of the source object in the z-direction is always long compared with \(d\), while the most interesting case in physical devices is where the radius \(a\) and the qubit separation \(d\) are short compared with \(d\). This results in an anisotropy which is not merely a dimensionless geometrical factor but which also depends on \(d\) and therefore on the frequency.

### 4.3. Distant object

We now treat the electrical noise of a metallic object far away from the qubit (\(d \gg L\)). We consider a fictitious point dipole \(\vec{p}\) at \(\vec{r}'\), the metallic object approximated by a sphere at the origin and an observation point \(\vec{r}\).

Equation (2.14) gives the correlation function:

\[
\langle E_\vec{p}(\vec{r}') E_\vec{k}(\vec{r}'') \rangle = \hbar \ coth \left( \frac{\hbar \omega}{2k_B T} \right) \times \text{Im} \left[ \alpha(\omega) \right] \frac{9x_i x_i' r'_i + \delta_{ik} r^2 + 3x_i x_i' r_i r_i' - 3x_i' x_i r_i r_i'}{r^5 r'^5},
\]  \hspace{1cm} (4.29)

where now \(\vec{E}\) is the physical fluctuating field. The local noise at \(\vec{r}\) is

\[
\langle E_\vec{p}(\vec{r}') E_\vec{k}(\vec{r}'') = \vec{p} \rangle = \hbar \ coth \left( \frac{\hbar \omega}{2k_B T} \right) \text{Im} \left[ \alpha(\omega) \right] \frac{3x_i x_i + \delta_{ik} r^2}{r^6}.
\]  \hspace{1cm} (4.30)

The \(r^{-6}\) dependence is familiar from the van der Waals force, which has a similar physical origin.

The anisotropy in lifetimes of a qubit in the presence of a spherical electrode is independent of the value of \(\alpha\). If the qubit is located at \(\vec{r} = r \hat{z}\), then

\[
\langle E_x(\vec{r}') E_x(\vec{r}'') \rangle = \langle E_y(\vec{r}') E_y(\vec{r}'') \rangle = \hbar \ coth \left( \frac{\hbar \omega}{2k_B T} \right) \frac{\text{Im} \left[ \alpha(\omega) \right]}{r^6},
\]  \hspace{1cm} (4.31)

\[
\langle E_z(\vec{r}') E_z(\vec{r}'') \rangle = 4\hbar \ coth \left( \frac{\hbar \omega}{2k_B T} \right) \frac{\text{Im} \left[ \alpha(\omega) \right]}{r^6}.
\]  \hspace{1cm} (4.32)

The anisotropy

\[
\langle E_z(\vec{r}') E_z(\vec{r}'') \rangle = 4\langle E_x(\vec{r}') E_x(\vec{r}'') \rangle
\]  \hspace{1cm} (4.33)

is stronger than in the half-space case. Thus the problem of noise from a distant metallic object reduces to a calculation of \(\text{Im} \left[ \alpha(\omega) \right]\), the dissipative part of the polarizability of the electrode. To get \(\alpha\), we need to calculate the change in the charge density of the electrode due to a distant oscillating dipole, and the electric field that results from this charge. We do this now in two limits.

**a. Image regime.** We first consider a metallic sphere of radius \(a\) with \(\delta \gg a\). Once again the fictitious problem is mathematically identical with that of a dielectric sphere in a static field, so we may simply transcribe the textbook formulas for the polarizability:

\[
\alpha = \frac{\varepsilon_m / \varepsilon_d - 1}{\varepsilon_m / \varepsilon_d + 2} \approx \left( 1 + \frac{3i \omega \varepsilon_d}{4\pi \sigma} \right) a^3.
\]  \hspace{1cm} (4.34)

Hence

\[
\langle E_i(\vec{r}') E_k(\vec{r}'') \rangle = \frac{3i \omega \varepsilon_d a^3 x_i x_k + \delta_{ik} r^2}{4\pi \sigma} \coth \left( \frac{\hbar \omega}{2k_B T} \right).
\]  \hspace{1cm} (4.35)

For a metallic ellipsoid with radii \(a_x, a_y, a_z\) in the \(x, y, z\) directions the coordinate system is aligned with the axes of the ellipsoid and the polarizability tensor satisfies \(\alpha_{ij} = \delta_{ij} \alpha_{ii}\) with
\[
\alpha_{ij} = \frac{1}{3} \frac{\varepsilon_m / \varepsilon_d - 1}{1 + (\varepsilon_m / \varepsilon_d - 1) n_i} a_x a_y a_z \approx \left(1 + \frac{i\omega \varepsilon_d}{12n^2 \pi \sigma}\right) a_x a_y a_z. \tag{4.36}
\]

The depolarizing factors \(n_x, n_y, n_z\) are positive and satisfy \(n_x + n_y + n_z = 1\) and \(n_i\) are decreasing functions of \(a_i\). In particular, if \(a_x < a_y < a_z\), then \(n_x > n_y > n_z\). The connection between the \(n_i\) and the \(a_i\) involves elliptic integrals. Exact expressions and tables may be found in [39]. Using equation (B.23) we have

\[
\langle E_i(\vec{r}) E_k(\vec{r}') \rangle = \hbar \coth \left(\frac{\hbar \omega}{2k_B T}\right) \text{Im} (\alpha_{ij}) f_i(\vec{r}) f_j(\vec{r}')
= \frac{\hbar \omega \varepsilon_d V}{16\pi^3 \sigma} \coth \left(\frac{\hbar \omega}{2k_B T}\right) \times \frac{1}{n_j} \frac{3x_i'^2 x_j' - \delta_{ij} r'^2}{r'^5} \frac{3x_i x_j - \delta_{ij} r^2}{r^5}, \tag{4.37}
\]
a distance \(r\) from the center of the ellipsoid of volume \(V\). To understand the physics of this formula, think of a qubit at \(\vec{r} = r\hat{z}\) with the origin of coordinates at the center of the ellipsoid. Then the off-diagonal components of the noise tensor vanish and the formula exhibits the anisotropy mentioned above. This expression confirms the intuition that the noise should be stronger in the directions where the axis is longer, since the polarizability is greater.

b. Induction regime. Again we first consider a metallic sphere of radius \(a\). We find

\[
\alpha = \frac{3(1 + i)\omega \sigma^4}{8\pi \sigma} \tag{4.38}
\]
for the polarizability in the induction regime. Using equation (4.30), we have that if the qubit is located at \(\vec{r} = r\hat{z}\), then

\[
\langle E_x(\vec{r} = r\hat{z}) E_x(\vec{r} = r\hat{z}) \rangle = \langle E_y(\vec{r} = r\hat{z}) E_y(\vec{r} = r\hat{z}) \rangle = \frac{3\hbar \omega \sigma^4}{8\pi \sigma \epsilon \delta^4} \coth \left(\frac{\hbar \omega}{2k_B T}\right), \tag{4.39}
\]

\[
\langle E_z(\vec{r} = r\hat{z}) E_z(\vec{r} = r\hat{z}) \rangle = \frac{3\hbar \omega \sigma^4}{2\pi \sigma \epsilon \delta^4} \coth \left(\frac{\hbar \omega}{2k_B T}\right). \tag{4.40}
\]

c. General result. The problem of the polarization of a metallic sphere is exactly solvable for all \(\delta / \sigma\) but it is not trivial. The method may be found in [40], and it is discussed in [41], but seems not to have been solved prior to 2008.

The polarizability \(\alpha\) for the sphere of radius \(a\) is given by

\[
a^{-3} \alpha = \frac{1}{2} \frac{\kappa^2 a}{j_0 + (1 + 2\varepsilon)} \frac{j''}{j_0}, \tag{4.41}
\]

The symbols are defined as \(\kappa = (1 + i) / \delta\), \(j_0 = (1/\kappa a) \sin \kappa a\), \(j'' = (1/\kappa a) \cos \kappa a - (1/\kappa a^2) \sin \kappa a\).

To obtain the first correction in the case \(\delta \gg a\) we expand to first order in \(\omega / \sigma\) and \(a / \delta\), (always assuming \(\omega / \sigma \ll a / \delta\)) and find

\[
j_0 \approx 1, \quad j'' \approx -\frac{1}{3} \kappa^2 a \tag{4.42}
\]

and we have

\[
\alpha = a^3 \left(1 + \frac{3\omega}{4\pi \sigma}\right), \tag{4.43}
\]
in agreement with equation (4.34) for the dissipative part. Note that the term that is zeroth-order in \(\omega\) gives a polarizability \(\alpha = a^3\), which is the proper static limit given in many textbooks.

When \(\delta \ll a\), then

\[
j_0 \approx \frac{i}{2\kappa a} e^{-it + i\omega / \delta}, \quad j'' \approx \frac{1}{2a} e^{-it + i\omega / \delta}
\]

and

\[
\alpha \approx a^3 \left(1 + \frac{3a(1 + i)\omega}{8\pi \sigma \delta}\right). \tag{4.44}
\]

As \(\omega\) increases, we find that \(\text{Im} \alpha\) increases, so it is a monotonic function of \(\omega\). The results in this section are easily understood physically, since they correspond to the dissipative part of the van der Waals interaction. This gives the familiar \(r^{-6}\) dependence to the fluctuation fields.
4.4. Multiple objects

Real devices tend to have complex geometries with multiple metallic device elements. A modern spin qubit experiment may involve a back gate or an accumulation gate having a layer or half-space shape. There may be up to tens of finger gates for lateral or voltage control that are approximately cylindrical. Clearly a numerical approach is indicated for these cases, which is beyond the scope of this paper. We therefore limit ourselves to a few remarks.

In many cases, it may be reasonable to regard different metallic elements as noise sources that are statistically independent. If this assumption holds, then

$$\langle E_i(\mathbf{r}) E_j(\mathbf{r}') \rangle_\omega = \sum_{i=1}^{N} \langle E_i(\mathbf{r}) E_j(\mathbf{r}') \rangle_\omega^0,$$

(4.45)

where the \((s)\) indexes the sources, of which there are \(N\) total. The various noise sources add incoherently.

The physical analogy of section 2 shows that this assumption cannot be strictly correct. The various device elements are in fact all driven by a single fictitious dipole and they are therefore in phase. However, unless the qubit occupies a position of high symmetry with regard to at least one pair of metallic objects. This can occur: it is common to place qubits near the tips of opposing finger gates. However, in most other cases the symmetry is low and equation (4.45) can be used.

4.5. Sharp points

A serious concern for qubit decoherence is the geometrical enhancement of noise in the neighborhood of surface asperities of conductors. The question is whether the well-known divergence of local field strengths at such structures carries over to noise. This is not a particularly pressing issue for for semiconductor qubits where gate features are only defined on length scales of 10 nm or longer. But one may also consider tunneling devices closer to scanning tunneling microscopes with much sharper tips. However, it can be seen fairly simply that electric noise is not greatly enhanced by asperities in the case that \(\delta\) is greater than the size of the surface feature (the usual case). We imagine a spherical geometry with a sharp point added on top, and a qubit near the point (see figure 5). Qualitatively, the quasistatic electric field lines will gather at the point, giving the familiar lightning-rod effect. However, these lines are outside the object and they do not produce the dissipation that is associated with field fluctuations and noise. Inside, the magnitude of the field is reduced by a factor \(p\).

The z component of the fictitious electric field at the point \(\mathbf{r} = (0, 0, -d)\) is proportional to

$$E_z^{(f)}(\mathbf{r}) \sim \int_0^p d'r' \frac{\sigma(\mathbf{r}') (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} \sim \int_0^p \sin \alpha \ r' d'r' \frac{(r')^{1+\nu}(-d - r' \cos \alpha)}{(r')^2 + d^2 + 2d r' \cos \alpha}^{3/2},$$

(4.46)

where \(p\) is an upper cutoff on the size of the cone. We are only interested in the small \(d\) behavior, which follows from simple scaling arguments as \(E_z^{(f)} \sim d^{-\nu+1/2}\) and this carries over to the physical field fluctuations \(\langle E_i(\mathbf{r}) E_j(\mathbf{r}) \rangle_\omega \sim d^{-\nu+1/2}\). So the divergence of the fields as the point is approached along the surface does not carry over to the noise in the immediate region near the tip but outside the conductor.
4.6. Charge qubits

To understand qubit decoherence in the presence of noise, the frequency dependence of the noise is of paramount importance. To this end, write the noise spectral density $\langle E_i(\vec{r}) E_j(\vec{r'}) \rangle$ from EWJN as

$$\langle E_i(\vec{r}) E_j(\vec{r'}) \rangle = f(\omega) \omega \coth \left( \frac{\hbar \omega}{2k_B T} \right)$$

where all spatial and device geometry information is contained in $f(\omega)$. For EWJN, $f(\omega) \rightarrow f_0$, a constant as $\omega \rightarrow 0$, $f_0$ sets the overall scale of the noise strength. In addition there is a high-frequency cutoff $1/\tau$ at the relaxation time for the conduction electrons in the metal. Thus $f(\omega) \rightarrow 0$ when $\omega \gg 1/\tau$. Physically, the factor of $\omega$ comes from the connection of noise to dissipation. Photons are non-interacting bosons—hence the cotangent factor. This sort of noise is white, or at least white-ish. This means that echo techniques are not likely to be very useful for extending qubit lifetimes when EWJN is the dominant source of decoherence. This noise is essentially the same as that of the well-known spin-boson model and the results are well known, so we only briefly summarize results here and give no derivations.

There are three frequency regime for the spectral density. 1. When $0 \leq \omega < 2k_B T/\hbar$, then $\langle E_i(\vec{r}) E_j(\vec{r'}) \rangle = 2k_B T f_0 / \hbar$. 2. When $2k_B T / \hbar < \omega < 1/\tau$ we have $\langle E_i(\vec{r}) E_j(\vec{r'}) \rangle = f(\omega) \omega$, where typically the frequency dependence of $f(\omega)$ is weak. 3. When $\omega > 1/\tau$, then the frequency dependence is material-dependent but we may usually assume that the noise is cut off.

In regime 1, the fluctuations are thermal. Regime 2 is the quantum regime and the linear spectrum is referred to as ‘ohmic’. Regime 3 is above the high-frequency cutoff, whose presence is implicit in this paper. The symbol $\sigma$ denotes the DC conductivity; however, no equation in which it appears can be used at frequencies greater than $1/\tau$. This frequency range is generally in the infrared for metals.

The qubit energy level separation is $\hbar \omega_0$ and $\omega_0$ may be in either regime 1 or regime 2, depending on the implementation. No existing implementation operates in Regime 3.

5. Magnetic noise

5.1. Half space

For magnetic noise the image method is not useful, so we proceed directly to general results for extended qubits. Again, we are interested in a metal with conductivity $\sigma$ that occupies the half space $z < 0$. The equations satisfied by the fields are the same as for the electric case. The only difference for magnetic fields is that $\vec{B}$, unlike $\vec{E}$, is continuous at the interface, since we dealing with non-magnetic materials. The derivations for magnetic noise are similar to those for electric noise so we mainly give results.

For this problem we place a fictitious magnetic dipole moment $\vec{m}$ at the point $\vec{r'} = (0, 0, d)$ and the physical noise spectral density is...
The interpolated function is plotted alongside numerical results for equation (5.1) in figure 6. For a point qubit \( \vec{r} = \vec{r}' \) we have

\[
\langle B_x(\vec{r}) B_x(\vec{r}') \rangle_\omega = \frac{\hbar}{4\pi\delta^2} \coth \frac{\hbar \omega}{2k_B T} \operatorname{Im} \int \frac{d^2q}{q^2} \frac{1}{q_1} e^{-q_1 x + i q_1 y}
\]

\[
= -\frac{\hbar}{4\pi\delta^2} \coth \frac{\hbar \omega}{2k_B T} \nabla \int \frac{d^2q}{q} \frac{1}{q_1} e^{-q_1 x + i q_1 y}.
\]

(5.1)

The interpolated function is plotted alongside numerical results for equation (5.1) in figure 6. For a point qubit \( \vec{r} = \vec{r}' \) we have

\[
\langle B_x(\vec{r}) B_x(\vec{r}') \rangle_\omega = \frac{\hbar}{4\delta^2} \coth \frac{\hbar \omega}{2k_B T}.
\]

(5.2)

And in the high \( T \) limit this reduces to

\[
\langle B_x(\vec{r}) B_x(\vec{r}') \rangle_\omega = \frac{k_B T}{2\omega d^2}.
\]

(5.3)

The \( 1/d \) dependence comes from integrating the \( 1/r^3 \) behavior of the dipole interaction over the half space. This is in contrast the the \( 1/d^3 \) dependence of the electric field noise. The difference is due to the fact that metals screen electric, but not magnetic, fields. This relatively slow spatial decays is the main reason that magnetic noise is likely to be of more practical importance in devices. For \( d \gg \delta \) we have

\[
\frac{1 - q/\alpha}{1 + q/\alpha} \approx (1 + q\delta + iq\delta),
\]

(5.4)

and

\[
\langle B_x(\vec{r}) B_x(\vec{r}') \rangle_\omega = -\frac{\hbar}{2\delta^2} \coth \frac{\hbar \omega}{2k_B T} \nabla \int d^2q \frac{1}{q_1} \frac{1 - q/\alpha}{1 + q/\alpha} e^{iq_1 x + i q_1 y}.
\]

For a point qubit only the diagonal component is nonzero:

\[
\langle B_x(\vec{r}) B_x(\vec{r}') \rangle_\omega = \frac{3\hbar}{8\delta^3} \coth \frac{\hbar \omega}{2k_B T}.
\]

(5.5)

The \( x \)-component is more complicated because of the lack of cylindrical symmetry, but the essential procedure is the same. We find

\[
\langle B_x(\vec{r}) B_x(\vec{r}') \rangle_\omega = \frac{\hbar}{2\pi} \coth \frac{\hbar \omega}{2k_B T} \partial_x \operatorname{Im} \int d^2q \frac{1}{q_1} \frac{1 - q/\alpha}{1 + q/\alpha} e^{iq_1 x + i q_1 y}.
\]

For \( d \ll \delta \) this is

\[
\langle B_x(\vec{r}) B_x(\vec{r}') \rangle_\omega = -\frac{\hbar}{2\delta^2} \coth \frac{\hbar \omega}{2k_B T} \partial_x \int_0^\infty dq \frac{1}{q} e^{-q_1 x + i q_1 y} J_0(q\rho).
\]

At \( \vec{r} = \vec{r}' \) we have that only the diagonal component is non-vanishing and

\[
\langle B_x(\vec{r}') B_x(\vec{r}') \rangle_\omega = \frac{\hbar}{8d^2} \coth \frac{\hbar \omega}{2k_B T}.
\]

In the high \( T \) limit this reduces to

\[
\langle B_x(\vec{r}') B_x(\vec{r}') \rangle_\omega = \frac{k_B T}{4\omega d^2}.
\]

For \( d \gg \delta \) we get

\[
\langle B_x(\vec{r}) B_x(\vec{r}') \rangle_\omega = -\frac{\hbar}{\delta^2} \coth \frac{\hbar \omega}{2k_B T} \partial_x \partial_z \frac{1}{(d + z)^2 + \rho^2} \frac{1}{T^2}.
\]

For \( \vec{r} = \vec{r}' \) only the diagonal component is nonzero:

\[
\langle B_x(\vec{r}') B_x(\vec{r}') \rangle_\omega = \frac{3\hbar}{16\pi d^3} \coth \frac{\hbar \omega}{2k_B T}.
\]

In the high \( T \) limit this becomes

\[
\langle B_x(\vec{r}') B_x(\vec{r}') \rangle_\omega = \frac{3\hbar}{8\pi d^3}.
\]

Overall, the most notable difference between electric noise and magnetic noise is that electric noise is screened by a metal and magnetic noise is not. This accounts for the \( 1/d \) dependence in the \( d \ll \delta \) case for magnetic noise. This relatively slow decline in strength suggests that magnetic noise is, somewhat counter to intuition, more likely to be important in small devices. This will be confirmed in section 6 where comparisons to experiment are presented.
5.2. Cylinder

In this section, we consider a infinitely long conducting circular cylinder as a source of EJW. The cylinder has conductivity $\sigma$ and radius $a$ and its axis is along the $z$-direction. This geometry is important, since cylindrical microwave antennas are used for single qubit rotations. There is a qubit at the point $\vec{r}' = (d, 0, 0)$. We wish to compute $\langle B_i(\vec{r}') B_i(\vec{r}) \rangle$ with $i = x, y, z$. We are particularly interested in the anisotropy of relaxation times, which depend on the ratios of this correlation function for different values of $i$. The most common case is when the skin depth $\delta \gg a$. We will also be mainly interested in thin wires also in the sense that $d \ll a$.

We need the solution to the problem of the magnetic polarizability of a conducting cylinder in a uniform field. This is given by [38]. The polarizabilities $\alpha_i$ are defined by the formulas

$$
M_i = \pi a^2 \alpha_i B_i,
$$

where $M_i$ is the magnetic moment per unit length in direction $i$. Here

$$
\alpha_x = \alpha_y = -\frac{1}{2\pi} \left[ 1 - \frac{2}{ka} \frac{J_1(ka)}{J_0(ka)} \right],
$$

$$
\alpha_z = -\frac{1}{4\pi} \left[ 1 - \frac{2}{ka} \frac{J_1(ka)}{J_0(ka)} \right],
$$

with $k = (1 + i) / \delta$. We will mainly need the imaginary part in the limit where $\delta \gg a$, which is

$$
\text{Im } \alpha_x = \text{Im } \alpha_y = \frac{a^2}{8\pi \delta^2},
$$

$$
\text{Im } \alpha_z = \frac{a^2}{16\pi \delta^2}.
$$

According to the usual prescription, we find

$$
\langle B_x(\vec{r}') B_x(\vec{r}) \rangle = \frac{27\pi}{256d^4} \text{Im} \left[ \frac{2}{ka} \frac{J_1(ka)}{J_0(ka)} \right] \coth \left( \frac{\hbar \omega}{2k_B T} \right),
$$

valid for any value of $\delta/a$.

When $a \ll \delta$ we expand the Bessel functions for small argument and find

$$
B_x^{(\text{ind})} = \frac{\pi a^2 \alpha_x m}{d^3} \frac{27\pi}{128},
$$

$$
\langle B_x(\vec{r}') B_x(\vec{r}) \rangle = \frac{27\pi}{2048 \delta^4} \coth \left( \frac{\hbar \omega}{2k_B T} \right).
$$

The same computation can be performed for the $x$ and $y$ directions. The results for $i = j = x$ are

$$
\langle B_x(\vec{r}') B_x(\vec{r}) \rangle = \frac{123\pi}{256d^4} \text{Im} \left[ \frac{2}{ka} \frac{J_1(ka)}{J_0(ka)} \right] \coth \left( \frac{\hbar \omega}{2k_B T} \right),
$$

valid for any value of $\delta/a$ and for $a \ll \delta$ we have

$$
\langle B_x(\vec{r}') B_x(\vec{r}) \rangle = \frac{123\pi}{1024d^4} \coth \left( \frac{\hbar \omega}{2k_B T} \right),
$$

while for $i = j = y$

$$
\langle B_y(\vec{r}') B_y(\vec{r}) \rangle = \frac{3\pi}{32d^4} \text{Im} \left[ \frac{2}{ka} \frac{J_1(ka)}{J_0(ka)} \right] \coth \left( \frac{\hbar \omega}{2k_B T} \right),
$$

valid for any value of $\delta/a$ and for $a \gg \delta$

$$
B_y^{(\text{ind})} = \frac{\pi a^2 \alpha_y m}{d^3} \frac{3\pi}{16},
$$

$$
\langle B_y(\vec{r}') B_y(\vec{r}) \rangle = \frac{3\pi}{128d^4} \coth \left( \frac{\hbar \omega}{2k_B T} \right).
$$

These considerations lead to very substantial anisotropy in the correlation functions and in the relaxation times. We have that for $d \gg a$.
\[ \langle B_x(\mathbf{r}) B_x(\mathbf{r'}) \rangle : \langle B_y(\mathbf{r}) B_y(\mathbf{r'}) \rangle : \langle B_z(\mathbf{r}) B_z(\mathbf{r'}) \rangle = 82 : 16 : 9. \] (5.10)

The cylindrical geometry occurs when wires or antennas are close to the qubit. The anisotropy can serve as a signature of noise originating from such a structure. The pattern of the anisotropy with the \( z \)-z correlations exceeding the \( x \)-x correlations is not difficult to understand. The longer dimension corresponds naturally to greater polarizability and therefore to stronger noise.

5.3. Distant object

We now treat the magnetic noise of a metallic object whose maximum linear dimension is short compared with the distance to the qubit: \( d \gg L \). We consider a fictitious point magnetic dipole \( \mathbf{m} \) at \( \mathbf{r'} \) and a magnetically polarizable metallic object at the origin. The observation point is \( \mathbf{r} \). Since \( L \) is small, we may take the field \( B \) at the object due to the test dipole to be uniform over the object. If we assume that the electrode is spherical and its dielectric function is isotropic then the magnetic polarizability can be written as

\[ \alpha = \frac{\hbar \omega}{2 k_B T} \left[ 3 x_i x_i - \delta_{ij} r^2 \right] \frac{\text{Im} \beta}{r^3}. \]

This manifestly satisfies the Onsager relation

\[ G_{\beta}(\omega; \mathbf{r}, \mathbf{r'}) = G_{\beta}(\omega; \mathbf{r'}, \mathbf{r}). \] (5.11)

The local noise at \( \mathbf{r} = \mathbf{r'} \) is

\[ \langle B_i(\mathbf{r}) B_i(\mathbf{r'} = \mathbf{r}) \rangle = \hbar \coth \left( \frac{\hbar \omega}{2 k_B T} \right) \text{Im} \beta \frac{3 x_i x_i + \delta_{ij} r^2}{r^3}. \] (5.12)

Thus the problem reduces to a calculation of \( \text{Im} \beta(\omega) \), the dissipative part of the polarizability of the electrode. For a spherical electrode with radius \( a \) and conductivity \( \sigma \), we have that [38]

\[ \text{Im} \beta = -\frac{3 a^2 \delta}{4} \left[ 1 - \frac{a \sin (2a/\delta) + \sin (2a/\delta)}{\cosh (2a/\delta) - \cos (2a/\delta)} \right]. \] (5.13)

which reduces when \( \delta \gg a \) to

\[ \text{Im} \beta = \frac{a^2}{15 \delta^2}. \] (5.14)

and when \( \delta \ll a \) to

\[ \text{Im} \beta = \frac{3 a^2 \delta}{4}. \] (5.15)

Notice that the anisotropy in lifetimes of a qubit in the presence of a spherical electrode is independent of \( \beta \). If the qubit is located at \( \mathbf{r} = r\hat{z} \), then

\[ \langle B_x(\mathbf{r}) B_x(\mathbf{r'} = \mathbf{r}) \rangle = \hbar \coth \left( \frac{\hbar \omega}{2 k_B T} \right) \text{Im} \beta(\omega) \frac{1}{r^6}. \] (5.16)

\[ \langle B_y(\mathbf{r}) B_y(\mathbf{r'} = \mathbf{r}) \rangle = 4 \hbar \coth \left( \frac{\hbar \omega}{2 k_B T} \right) \text{Im} \beta(\omega) \frac{1}{r^6}. \] (5.17)

The \( r^{-6} \) dependence is familiar from the van der Waals force, which has a similar physical origin. The anisotropy

\[ \langle B_z(\mathbf{r}) B_z(\mathbf{r}) \rangle = 4 \langle B_y(\mathbf{r}) B_y(\mathbf{r}) \rangle \] (5.18)

is stronger than in the half-space case.

6. Comparison with experiment

In this section we provide some numerical estimates for the noise strength and the resulting qubit relaxation times, which will allow us to evaluate the relevance of EWJN for current experiments. We shall focus on the half-space geometry, since this case is the important one for existing devices; the greatest masses of metal in semiconductor qubit systems are usually in global gates.

6.1. Charge qubits

The noise spectral energy density is of some interest. Taking \( \omega = 10^9 \text{ s}^{-1} \), \( \sigma = 10^{17} \text{ s}^{-1} \), we get

\[ \delta = \sqrt{2 \pi \sigma \omega} = 12 \times 10^{-4} \text{ cm} = 12 \mu \] and we will only consider the regime \( d \ll \delta \). The vacuum wavelength \( \lambda = 60 \text{ cm} \) is the longest length in the problem and plays no role in our quasistatic regime. At a distance \( d \) from
a half space we find and $T = 1 \text{ K}$ and $\varepsilon_d = 10$ we have:

$$\langle E_x(d)E_x(d)\rangle_\omega \approx \frac{k_B T}{8\pi \sigma d^2} = 9 \times 10^{-22} \text{ erg/cm}^3 \text{s}. \quad (6.1)$$

This noise will relax qubits. In figure 7 we give numerical estimates for $T_1$ of a charge qubit in a half-space geometry. The curves are plotted using equations (3.7) and (4.7) assuming a point qubit. Each curve represents $T_1(d)$ for various values of the distance $d$ from the half space and the dot separation $L$, the latter being listed in the inset. We have assumed $\omega = 10^6 \text{ s}^{-1}$, $\sigma = 10^{16} \text{ s}^{-1}$, $T = 0.1 \text{ K}$. Indicated on the figure are experimental values for $T_1$ and the predictions our model makes based on estimates of the particular experiment’s qubit and surrounding geometry. The measured values are an order of magnitude or two smaller than the predictions made by our model, indicating that EWJN is probably not the dominant mechanism behind qubit relaxation in these experiments. However, the estimates here are made with very limited knowledge of the particular experimental values of $d$ and $L$, which are normally not very accurately determined. Since $T_1 \propto d^2/L^2$ a factor of 2 could account for an order of magnitude correction. A further serious source of uncertainty is that $\sigma$ is not measured and generally is poorly known. If $\sigma$ is too large, the mean free path is the electrons in the metal may become comparable to the gate dimensions, invalidating the local electrodynamics used in this paper. These considerations taken together mean that it is difficult to give a clear evaluation of the role of EWJN in charge qubit experiments. In any case, it seems safe to say that even rather minor improvements in other decoherence mechanisms would make the EWJN mechanism competitive with the others.

6.2. Spin qubits

We can now repeat the numerical estimates for the noise strength and the resulting qubit relaxation times for magnetic noise and spin qubits.

The noise energy density is again of some interest. With $\omega = 10^6 \text{ s}^{-1}$, $\sigma = 10^{12} \text{ s}^{-1}$, $T = 1 \text{ K}$, $d = 50 \text{ nm}$ from a half space we have:

$$\langle B_x B_y \rangle_\omega \approx \frac{\pi k_B T \sigma}{d c^2} = 3.0 \times 10^{-15} \text{ erg/cm}^3 \text{s}. \quad (6.2)$$

This noise will relax qubits. In figure 8 we give numerical estimates for $T_1$ of a spin qubit in a half-space geometry. The curves are plotted using equation (3.7) assuming a point qubit. Each curve represents $T_1(d)$ for various values of the distance $d$ from the half space for a fixed $B$ field, which enters $T_1$ via $\hbar \omega = g\mu_B B$ with $g = 2$. We have assumed $\sigma = 10^{16} \text{ s}^{-1}$, $T = 0.1 \text{ K}$. Indicated on the figure are experimental values for $T_1$ and the predictions our model makes based on estimates of the particular experiment’s qubit and surrounding geometry. The geometries are somewhat better determined in these experiments, meaning that that the main source of uncertainty is in the conductivity $\sigma$, which may differ from our assumption by an order of magnitude.

The experimental values shown in figure 7 are in small devices characterized by linear dimensions of order 100 nm, but we note that in certain MOS devices the relevant distances can be closer to 10 nm [45, 46].
Other qubit architectures such as atom traps, ion traps, or superconductors, are generally considerably larger. This makes it unlikely that EWJN plays a large role in the decoherence of these devices, since the power-law falloffs reduce the noise strength at the qubit positions. This could change as these devices are miniaturized \[47, 48\].

### 7. Conclusions

Qubits with long relaxation times are necessary for quantum computation. Most such devices are controlled electrically. This creates a control—isolation dilemma: connections from the outside world are what make the devices useful, but they are also sources of decoherence. In particular, one may wish to place charge or spin qubits close to metallic device elements used to confine or control the qubits. However, the fluctuating currents and charges in metals give rise to noise that leaks out of the metal into the surrounding region, decohering the qubits. This is standard physics, (though not often treated in textbooks) and results for the noise spectral densities near a half plane are well known. However, results for the more complicated geometries of real devices have not been available at all. The results presented above represent a first step in the direction of repairing this situation.

Most importantly, we have given a streamlined method for the calculation of both noise spectral densities and noise correlation functions. We have presented new results for the spectral density of cylinders and distant objects, and for the noise correlation functions for half spaces and distant objects. The new method also enables us to give more qualitative, but still useful, discussions of issues such as asperities on metal surfaces.

Numerical estimates of the effect of EWJN on qubits indicates that it is probably not a dominant effect on the current generation of charge qubit devices. For spin qubits the situation is different. Experiments in which the gates are close to the qubits may already be showing the effects of EWJN.

### Acknowledgments

We have benefited from discussions with Amrit Poudel, Luke Langsjoen, Andrea Morello, and John Nichol. This work was supported by the ARO under award No. W911NF-12-0607.

### Appendix A. Multipole moments in $T_1$ and $T_2$

The expressions in equations (3.7) and (3.15) can be generalized to higher order multipole moments by keeping more terms in the Taylor expansion of the electromagnetic potentials. Define the electric moments $q_i = e\eta_i$, $q_0 = e\eta r$, and magnetic dipole $m_i = \frac{\epsilon}{2mc}((F \times \vec{\Pi})_i + gS_i)$. We then have

![Figure 8. Spin qubit relaxation time $T_1$ as a function of the distance $d$ from the qubit to a planar metal gate for various values of the external magnetic field, as listed in the inset. The experimental data are taken from [42-44]. The theoretical predictions are indicated by solid squares. The conductivities are roughly estimated as $\sigma = 10^{16}$ s$^{-1}$. The temperature is $T = 0.1$ K.](image)
We can find the quadrupole contribution by expanding equation (3.5) and keeping track of all first derivative terms giving us

\[
\langle 0 | H_\alpha(t) | 1 \rangle = - \frac{e}{mc} \langle A_i(0, t) | 0 \rangle \Pi_i + \frac{eg}{2mc} B_i \langle 0 | S_i | 1 \rangle \\
+ \langle 0 | \left( \frac{1}{2} [\nabla_j A_i(t, r_j) \gamma_{ij} \Pi_j + r_j \Pi_i] \right) | 1 \rangle - \frac{e}{2mc} \langle \nabla \times \tilde{A}(kr_i) \gamma_{ij} r_j \Pi_i | 1 \rangle \\
- \frac{e}{2mc} \langle \nabla \times \tilde{A}(kr_i) \gamma_{ij} r_j | 1 \rangle - \frac{eg}{2mc} B_i \langle 0 | S_i | 1 \rangle \\
= \frac{i}{e} \left( 0 | P_i | 1 \right) A_i(0, t) - \frac{e}{mc} \langle 0 | \frac{1}{2} [\nabla_j A_i] \gamma_{ij} r_j \Pi_j | 1 \rangle - \frac{eg}{2mc} B_i \langle 0 | S_i | 1 \rangle \\
- B_i (0, t) \langle 0 | m_i | 1 \rangle.
\]

We have employed the vector identity

\[
[\nabla]A_i(t, r) = \frac{1}{2} \left[ 0 | \nabla_j A_i(t, r) \gamma_{ij} r_j \Pi_i | 1 \right] - \frac{1}{2} \epsilon_{ij} (\nabla \times \tilde{A}(kr_i) \gamma_{ij} r_j \Pi_i, t).
\]

Now we can work out an expression for \( T_1 \) using equation (3.1)

\[
\frac{1}{T_1} = \frac{1}{\hbar^2} \left[ \langle p_i \rangle \langle p_i \rangle^* \langle E_i E_i \rangle_{\omega} + \frac{1}{2} \langle p_i \rangle \langle q_{l m} \rangle^* \langle E_i \nabla_m E_i \rangle_{\omega} + \frac{1}{2} \langle q_{l j} \rangle \langle p_i \rangle (\nabla E_i E_i)_{\omega} \\
+ \langle p_i \rangle \langle m_{n} \rangle^* \langle E_i B_{n} \rangle_{\omega} + \langle m_{k} \rangle \langle p_i \rangle^* \langle B_k E_i \rangle_{\omega} + \langle m_{k} \rangle \langle m_{n} \rangle^* \langle B_k B_{n} \rangle_{\omega} \\
- \frac{1}{2} \langle q_{l j} \rangle \langle m_{l} \rangle^* \langle \nabla E_i B_{l} \rangle_{\omega} + \frac{1}{2} \langle m_{k} \rangle \langle q_{l m} \rangle^* \langle B_k \nabla_m E_i \rangle_{\omega} + \frac{1}{4} \langle q_{l j} \rangle \langle q_{l m} \rangle^* \langle \nabla E_i \nabla_m E_i \rangle_{\omega} \right].
\]

A naive application of the analysis from the preceding calculation would indicate that \( E \)-field noise will not contribute to diagonal elements of \( H_\alpha(t) \), but this is due to the incomplete application of the gauge condition \( \phi = 0 \). If we begin with the gauge-invariant Schrödinger equation with an arbitrary scalar potential \( \phi(r, t) \) and vector potential obeying \( \nabla \cdot A = 0 \) and eliminate the residual gauge freedom via \( \tilde{A}(r, t) = A(r, t) + \nabla \alpha(r, t), \phi'(r, t) = \phi(r, t) - \alpha(r, t) = 0 \) and \( \alpha' = e^{-i\omega/t} \psi'(r, t) \) we find that the wave function obeys

\[
\frac{i}{\hbar} \dot{\psi} = (e^{i\omega/\hbar} H' - i\omega/\hbar - e\alpha) \psi.
\]

The Hamiltonian \( H' \) in the gauge with no scalar potential is complemented by the gauge fixing term that retains the electric field contribution in the equations of motion. The operator we need in equation (3.14) can be expanded in Taylor series as

\[
H_\alpha(t) = -\frac{e}{mc} \langle A_i(0, t) \rangle + \langle \nabla_j A_i(t, r_j) \gamma_{ij} \Pi_j + ... \rangle \Pi_i \\
- \frac{eg}{2mc} B_i \langle 0 | S_i | 1 \rangle - e\alpha(0, t) + \nabla \alpha(0, t) r_j + ...
\]

Now turning to the relevant matrix elements of equation (3.5) with the gauge term (equivalently, the scalar potential), we begin by treating the vector potential terms

\[
= -\frac{e}{mc} \langle 1 | A_i(0, t) \Pi_j | 1 \rangle = \frac{i}{\hbar c} A_i(0, t) \langle 1 | p_i H_\alpha | 1 \rangle - \langle 1 | H_\alpha p_i | 1 \rangle = \frac{i}{\hbar c} A_i (\epsilon_i - \epsilon_i) \langle 1 | p_i | 1 \rangle = 0,
\]

\[
= -\frac{e}{mc} \langle 1 | \nabla_j A_i \gamma_{ij} r_j \Pi_j | 1 \rangle = -\frac{e}{mc} \left[ \frac{1}{2} \langle \nabla_j A_i | 1 | r_j \Pi_j + r_j \Pi_i | 1 \rangle + \frac{1}{2} B_i \langle 1 | d_i | 1 \rangle \right]
\]

\[
= -\frac{e}{mc} \left( \frac{i}{2} \nabla_j A_i \delta_{ij} - \frac{e}{2mc} B_i \langle 1 | d_i | 1 \rangle \right) = -\frac{e}{2mc} B_i \langle 1 | d_i | 1 \rangle.
\]

The last equality follows from \( \nabla \cdot \tilde{A} = 0 \). We have \( E_i(r, t) = -\nabla \alpha(r, t) \). Using the same methods we obtain an expression for the integral kernel \( S(\omega) \) for \( T_2 \) to quadrupole order.
Appendix B. Spectral density tensors

Here we include the details and off-diagonal components of the noise spectral density tensor for the simple geometries treated in the main body of the paper.

B.1. Electric noise

Place a fictitious electric dipole moment \( \vec{p} \) at the point \( \vec{r}' = (0, 0, d) \) in the half-space geometry. The electric field in free space would be

\[
E^{(ed)}_j(\vec{r}) = -\frac{\partial}{\partial x_j} \frac{1}{|\vec{r} - \vec{r}'|} = -p_k \frac{\partial}{\partial x_j} \frac{1}{|\vec{r} - \vec{r}'|}, \tag{B.1}
\]

which satisfies \( \nabla \cdot E^{(ed)} = 4\pi \rho \) with \( \rho = -\vec{p} \cdot \nabla \delta^3(\vec{r} - \vec{r}_0) \).

We will represent this fictitious field by using the identity

\[
\frac{1}{|\vec{r} - \vec{r}'|} = \frac{1}{2\pi} \int \frac{d^3q}{q} \, e^{i\vec{q} \cdot \vec{r} - q|z - d|}, \tag{B.2}
\]

where \( \vec{q} = (q_x, q_y) \) and \( \vec{r} = (x, y) \). Thus

\[
E^{(ed)}_j(\vec{r}) = -\frac{p_k}{2\pi} \int \frac{d^3q}{q} \frac{\partial}{\partial x_j} \frac{\partial}{\partial x_k} \, e^{i\vec{q} \cdot \vec{r} - q|z - d|}. \tag{B.3}
\]

The induced field for \( z > 0 \) is expanded as

\[
E^{(ind)}_j(\vec{r}) = -\frac{p_k}{2\pi} \int d^3q \, f_j(\vec{q}) e^{i\vec{q} \cdot \vec{r} - qz}, \tag{B.4}
\]

and the Maxwell equations imply

\[
\nabla^2 E^{(ind)} = 0, \quad \nabla \cdot E^{(ind)} = 0, \quad z > 0, \tag{B.5}
\]

so

\[
q = \sqrt{q_x^2 + q_y^2}, \quad iq_x f_x + iq_y f_y = qf_z. \tag{B.6}
\]

The induced field for \( z < 0 \) is defined by

\[
E^{(ind)}_j(\vec{r}) = -\frac{p_k}{2\pi} \int d^3q \, g_j(\vec{q}) e^{i\vec{q} \cdot \vec{r} + qz}, \tag{B.7}
\]

and we have

\[
\nabla^2 E^{(ind)} + 2i\delta^{-2} E^{(ind)} = 0, \quad \nabla \cdot E^{(ind)} = 0, \quad z < 0 \tag{B.8}
\]

and \( \text{Re} \alpha > 0 \) and so

\[
\alpha^2 = q_x^2 + q_y^2 - 2i\delta^{-2} = q^2 - 2i\delta^{-2}, \quad iq_x g_x + iq_y g_y = -\alpha g_z. \tag{B.9}
\]

The tangential component of \( \vec{E} \) is continuous but the normal component satisfies \( E_{\text{norm, out}} = \varepsilon E_{\text{norm, in}} \approx (4\pi i\varepsilon/\omega) E_{\text{norm, in}} \) so \( |E_{\text{norm, out}}| \gg |E_{\text{norm, in}}| \). \( \vec{B} \) is continuous. The fictitious dipole \( \vec{p} = p\hat{z} \) produces a field for \( 0 < z < d \).  

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\[ E^{(ed)}_x(\vec{r}) = -\frac{p}{2\pi} \int d^2q \ iq_x e^{iq \cdot \hat{r} \hat{e}(z-d)}, \]
\[ E^{(ed)}_y(\vec{r}) = -\frac{p}{2\pi} \int d^2q \ iq_y e^{iq \cdot \hat{r} \hat{e}(z-d)}, \]
\[ E^{(ed)}_z(\vec{r}) = -\frac{p}{2\pi} \int d^2q \ q e^{iq \cdot \hat{r} \hat{e}(z-d)}, \]

and the induced field is defined by
\[ E^{(ind)}_x = -\frac{p}{2\pi} \int d^2q \ \tilde{f}(\vec{q}) e^{iq \cdot \hat{r} \hat{e} \alpha} \text{ for } z > 0 \]
\[ E^{(ind)}_y = -\frac{p}{2\pi} \int d^2q \ \hat{g}(\vec{q}) e^{iq \cdot \hat{r} \hat{e} \sigma} \text{ for } z < 0. \]

The boundary conditions yield
\[
\begin{align*}
\imath q_x e^{-\imath q d} + f_x &= g_x, \\
\imath q_y e^{-\imath q d} + f_y &= g_y, \\
q e^{-\imath q d} + f_z &= (\varepsilon_m / \varepsilon_d) g_z, \\
\imath q_x f_x + \imath q_y f_y &= q f_z, \\
\imath q_z g_x + \imath q_y g_y &= -\omega g_z.
\end{align*}
\]

The solution is
\[ (f_x, f_y, f_z) = (-\imath q_x, -\imath q_y, q) e^{-\imath q d} \frac{1 - (\varepsilon_m / \varepsilon_d) q / \omega}{1 + (\varepsilon_m / \varepsilon_d) q / \omega}, \tag{B.8} \]

which gives us an integral expression for the induced field and thus (4.15). For \( \vec{p} = \hat{p} \hat{x} \) the dipole produces a field for \( 0 < z < d \)
\[ E^{(ed)}_x(\vec{r}) = \frac{p}{2\pi} \int d^2q \ \frac{q^2}{q} e^{iq \cdot \hat{r} \hat{e}(z-d)}, \]
\[ E^{(ed)}_y(\vec{r}) = \frac{p}{2\pi} \int d^2q \ \frac{q x}{q} e^{iq \cdot \hat{r} \hat{e}(z-d)}, \]
\[ E^{(ed)}_z(\vec{r}) = -\frac{i p}{2\pi} \int d^2q \ \frac{q x}{q} e^{iq \cdot \hat{r} \hat{e}(z-d)}. \]

For \( d \ll \delta \) we find
\[ \text{Im} E^{(ind)}_x(\vec{r}) = -\frac{p}{2\pi} \frac{\omega \varepsilon_d}{\omega} \frac{1}{\nabla} \int d^2q \ \frac{1}{q} e^{-q(x-z)} e^{iq \cdot \hat{r} \hat{e} \sigma}, \]
\[ = -\frac{p}{2\pi} \frac{\omega \varepsilon_d}{\omega} \frac{1}{\nabla} \frac{x}{(z+d)^2 + \rho^2 \varepsilon_d^2}. \]

So, for example,
\[ \text{Im} E^{(ind)}_x(\vec{r}) = -\frac{p}{2\pi} \frac{\omega \varepsilon_d}{\omega} \frac{2x^2 - (z+d)^2 - y^2}{(z+d)^2 + \rho^2 \varepsilon_d^2}. \tag{B.9} \]

In the regime \( d \gg \delta \) we find
\[ \text{Im} E^{(ind)}_x(\vec{r}) = -\frac{p}{2\pi} \frac{\omega \varepsilon_d}{\omega} \frac{1}{\nabla} \int d^2q \ \frac{1}{q} e^{-q(x-z)} e^{iq \cdot \hat{r} \hat{e} \sigma}, \]
\[ = -\frac{p}{2\pi} \frac{\omega \varepsilon_d}{\omega} \frac{1}{\nabla} \left\{ \frac{x}{\rho^2} \frac{1 - d + z}{(z+d)^2 + \rho^2 \varepsilon_d^2} \right\}, \]

which gives us the correlation functions presented in the main text.

For \( \vec{p} = \hat{p} \hat{x}, \vec{r}' = (0, 0, d) \) and \( d \ll \delta \) the matrix elements are:
\[ \langle E_z(\vec{r}) E_z(\vec{r}') \rangle_\omega = \frac{\hbar \omega \varepsilon_d}{2\pi \sigma} \frac{2(d+z)^2 - 2\rho^2}{[(d+z)^2 + \rho^2 \varepsilon_d^2]} \coth \frac{\hbar \omega}{2k_B T}, \tag{B.10} \]
\[ \langle E_z(\vec{r}) E_z(\vec{r}') \rangle_\omega = \frac{3\hbar \omega \varepsilon_d}{2\pi \sigma} \frac{x(d+z)}{[(d+z)^2 + \rho^2 \varepsilon_d^2]} \coth \frac{\hbar \omega}{2k_B T}, \tag{B.11} \]
When \( d \gg \delta \) we have

\[
\langle \hat{E}_x(\vec{r}) \hat{E}_x(\vec{r}') \rangle_\omega = \frac{\hbar \omega_d}{2\pi} \frac{d + z}{(d + z)^2 + \rho^2 \frac{\rho^2}{2}} \coth \frac{\hbar \omega}{2k_B T}, \tag{B.12}
\]

\[
\langle \hat{E}_y(\vec{r}) \hat{E}_y(\vec{r}') \rangle_\omega = \frac{\hbar \omega_d}{2\pi} \frac{y}{(d + z)^2 + \rho^2 \frac{\rho^2}{2}} \coth \frac{\hbar \omega}{2k_B T}, \tag{B.13}
\]

\[
\langle \hat{E}_z(\vec{r}) \hat{E}_z(\vec{r}') \rangle_\omega = \frac{\hbar \omega_d}{2\pi} \frac{x}{(d + z)^2 + \rho^2 \frac{\rho^2}{2}} \coth \frac{\hbar \omega}{2k_B T}. \tag{B.14}
\]

Now we turn to the solution for \( \vec{p} = p\hat{x} \) and \( d \ll \delta \). The matrix elements are

\[
\langle \hat{E}_x(\vec{r}) \hat{E}_x(\vec{r}') \rangle_\omega = \frac{\hbar \omega_d}{2\pi} \frac{2x^2 - (z + d)^2 - y^2}{(z + d)^2 + \rho^2 \frac{\rho^2}{2}} \coth \frac{\hbar \omega}{2k_B T}, \tag{B.16}
\]

\[
\langle \hat{E}_y(\vec{r}) \hat{E}_y(\vec{r}') \rangle_\omega = \frac{3\hbar \omega_d}{2\pi} \frac{xy}{(z + d)^2 + \rho^2 \frac{\rho^2}{2}} \coth \frac{\hbar \omega}{2k_B T}, \tag{B.17}
\]

\[
\langle \hat{E}_z(\vec{r}) \hat{E}_z(\vec{r}') \rangle_\omega = -\frac{3\hbar \omega_d}{2\pi} \frac{x(d + z)}{(z + d)^2 + \rho^2 \frac{\rho^2}{2}} \coth \frac{\hbar \omega}{2k_B T}, \tag{B.18}
\]

and comparison with equation (B.11) shows that the Onsager relation is satisfied.

The components when \( d \gg \delta \) are:

\[
\langle \hat{E}_x(\vec{r}) \hat{E}_x(\vec{r}') \rangle_\omega = \frac{\hbar \omega_d}{2\pi} \frac{d + z}{(d + z)^2 + \rho^2 \frac{\rho^2}{2}} \coth \frac{\hbar \omega}{2k_B T} \times \left\{ \frac{y^2 - x^2}{\rho^2} \left[ 1 - \frac{d + z}{(z + d)^2 + \rho^2 \frac{\rho^2}{2}} \right] + \frac{x^2(d + z)}{\rho^2((z + d)^2 + \rho^2 \frac{\rho^2}{2})} \right\}, \tag{B.19}
\]

\[
\langle \hat{E}_y(\vec{r}) \hat{E}_y(\vec{r}') \rangle_\omega = \frac{3\hbar \omega_d}{2\pi} \frac{xy}{(z + d)^2 + \rho^2 \frac{\rho^2}{2}} \coth \frac{\hbar \omega}{2k_B T} \times \left\{ \frac{x}{\rho^2} \left[ 2 \left[ 1 - \frac{d + z}{(z + d)^2 + \rho^2 \frac{\rho^2}{2}} \right] - \frac{d + z}{((z + d)^2 + \rho^2 \frac{\rho^2}{2})} \right] \right\}, \tag{B.20}
\]

\[
\langle \hat{E}_z(\vec{r}) \hat{E}_z(\vec{r}') \rangle_\omega = \frac{3\hbar \omega_d}{2\pi} \frac{x}{(z + d)^2 + \rho^2 \frac{\rho^2}{2}} \coth \frac{\hbar \omega}{2k_B T}, \tag{B.21}
\]

and comparison with equation (B.14) shows that the Onsager relation is satisfied.

For the distant object geometry since \( L \), the qubit size, is small, we may take the field \( E'' \) at the electrode due to the test dipole to be uniform over the object. It is given by

\[
E'_j(\vec{r} = 0) = p_k \frac{\partial}{\partial k} \frac{1}{|\vec{r}|} = p_k \frac{3x'_j x'_i - \delta_{ij} r^2}{r^5}, \tag{B.22}
\]

where we have defined the dipole function

\[
f_{ij}(\vec{r}) = \frac{3x'_i x'_j - \delta_{ij} r^2}{r^5}. \tag{B.22}
\]

We shall take only the first term in the multipole expansion of the field produced by the object. We will write this dipole as \( \vec{p}^{(el)} \) (‘el’ for ‘electrode’). It can be written as \( p^{(el)}_j(\omega) = \alpha_{jn}(\omega) E'_n(\omega) \). At the observation point \( \vec{r} \) the (again fictitious) field is

\[
E_i(\vec{r}) = p^{(el)}_j \frac{\partial}{\partial k} \frac{1}{|\vec{r}|} = \alpha_{jn} E'_n f_{ij}(\vec{r}) = \alpha_{jn} p^{(el)} f_{ij}'(\vec{r}) f_{ij}(\vec{r}).
\]
This leads directly to

$$
\langle E(\vec{r}) E(\vec{r}') \rangle = \hbar \coth \left( \frac{\hbar \omega}{2k_B T} \right) \text{Im} \left( \alpha_{jn} f_{jk}(\vec{r}') f_{kj}(\vec{r}) \right).
$$

(B.23)

Hence only the polarizability of the object is relevant in the problem. If we assume that the electrode is spherical and its dielectric function is isotropic then $P_{ij}^{(\text{ph})}(\omega) = \alpha(\omega) \delta_{ij} E_n^2(\omega)$ and

$$
E_{ij}^{(\text{ph})}(\vec{r}) = \alpha(\omega) P_{ij} f_{jk}(\vec{r}') f_{kj}(\vec{r})
$$

$$
= \alpha(\omega) P_{ij} \frac{9\varepsilon_0 \varepsilon'_0}{r^4} \vec{r}' \cdot \vec{r}' + \delta_{ij} r^2 r'^2 - 3x'_i x'_j r^2 - 3x'_i x'_j r'^2.
$$

Using equation (2.19), we have

$$
-\frac{\omega^2}{\hbar c^2} G_{ij}(\omega; \vec{r}, \vec{r}') = \alpha(\omega) f_{ij}(\vec{r}) f_{jk}(\vec{r'}).
$$

(B.24)

This manifestly satisfies the Onsager relation

$$
G_{ij}(\omega; \vec{r}, \vec{r}') = G_{ji}(\omega; \vec{r}', \vec{r}').
$$

(B.25)

And we find

$$
\langle E(\vec{r}) E(\vec{r}') \rangle = \hbar \coth \left( \frac{\hbar \omega}{2k_B T} \right) \text{Im} \left( \alpha(\omega) f_{ij}(\vec{r}') f_{jk}(\vec{r}) \right).
$$

(B.26)

B.2. Magnetic noise

To find the noise tensor in the half space we place a magnetic dipole moment $\vec{m} = m \hat{z}$ at $\vec{r} = (0, 0, d)$ in analogy to the electric field noise calculation. The magnetic field due to this fictitious dipole in free space would be

$$
B_{ij}^{(\text{md})}(\vec{r}) = m_j \frac{\partial}{\partial x_i} \frac{1}{|\vec{r} - \vec{r}'|},
$$

which satisfies $\nabla \times \vec{B}^{(\text{md})} = 4\pi \vec{m}$ and $\vec{J} = \vec{m} \times \nabla \delta^3(\vec{r} - \vec{r}')$. Proceeding analogously to equation (B.1), we have

$$
B_{ij}^{(\text{md})}(\vec{r}) = \frac{m_k}{2\pi} \int \frac{d^3q}{q} \frac{\partial}{\partial x_i} \frac{\partial}{\partial x_j} \frac{1}{q^2 + \gamma q} e^{i \vec{q} \cdot \vec{r}}
$$

(B.28)

where $\vec{q} = (q_x, q_y)$ and $\vec{\rho} = (x, y)$, and

$$
B_{ij}^{(\text{md})}(\vec{r}) = -\frac{m}{2\pi} \int d^3q \left( -i q_x, -i q_y, q \right) e^{-qz} \frac{1 - q / \alpha}{1 + q / \alpha} e^{i q x + i q y - q z} \text{ for } z > 0.
$$

(B.29)

For $d \ll \delta$ we find

$$
\frac{1 - q / \alpha}{1 + q / \alpha} = \frac{\sqrt{q^2 - 2i \delta^2} - q}{\sqrt{q^2 - 2i \delta^2} + q} \approx \left( -\frac{i}{2q^2 \delta^2} \right)
$$

and

$$
B_{ij}^{(\text{md})}(\vec{r}) = \frac{im}{4\pi \delta^2} \int d^3q \frac{1}{q^2} \left( -i q_x, -i q_y, q \right) e^{-q(z + d)} e^{i q x + i q y} \text{ for } z > 0.
$$

(B.30)

For $\vec{m} = m \hat{z}$ and $d \ll \delta$ the components of the noise tensor in the half space are:

$$
\langle B_z(\vec{r}) B_z(\vec{r}') \rangle_\omega = \frac{\hbar}{2\delta^2} \frac{1}{(d + z)^2 + \rho^2 \delta^2 / 2} \coth \left( \frac{\hbar \omega}{2k_B T} \right),
$$

(B.31)

$$
\langle B_x(\vec{r}) B_x(\vec{r}') \rangle_\omega = \frac{\hbar}{2\delta^2} \frac{x}{\rho^2} \left\{ 1 - \frac{(d + z)}{l(d + z)^2 + \rho^2 \delta^2 / 2} \right\} \coth \left( \frac{\hbar \omega}{2k_B T} \right),
$$

(B.32)
\[
\langle B_x(\vec{r}) B_z(\vec{r}') \rangle_\omega = \frac{\hbar}{2\delta^2} \frac{y}{\rho^2} \left( 1 - \frac{(d + z)}{[(d + z)^2 + \rho^2]^2} \right) \coth \frac{\hbar \omega}{2k_B T}.
\]

(B.33)

In the regime where \( d \gg \delta \) we have
\[
\langle B_z(\vec{r}) B_z(\vec{r}') \rangle_\omega = -\hbar \delta \coth \frac{\hbar \omega}{2k_B T} \times \left[ -6(z + d)^2 + 9\rho^2(z + d) \right] \frac{1}{[(d + z)^2 + \rho^2]^2},
\]

(B.34)
\[
\langle B_x(\vec{r}) B_z(\vec{r}') \rangle_\omega = -\hbar \delta \coth \frac{\hbar \omega}{2k_B T} \times \left[ -12 y(z + d)^2 + 3\rho^2 \right] \frac{1}{[(d + z)^2 + \rho^2]^2},
\]

(B.35)
\[
\langle B_y(\vec{r}) B_z(\vec{r}') \rangle_\omega = -\hbar \delta \coth \frac{\hbar \omega}{2k_B T} \times \left[ -12 y(z + d)^2 + 3\rho^2 \right] \frac{1}{[(d + z)^2 + \rho^2]^2}.
\]

(B.36)

Now we turn to \( \vec{m} = m\hat{x} \) and \( d \ll \delta \) where the components are
\[
\langle B_x(\vec{r}) B_x(\vec{r}') \rangle_\omega = \frac{\hbar}{2\delta^2} \coth \frac{\hbar \omega}{2k_B T} \frac{\rho}{\rho^2} \left( \frac{x^2 - y^2}{\left[(d + z)^2 + \rho^2\right]^2} - \frac{x^2}{\rho^2[(d + z)^2 + \rho^2]^2} \right),
\]

(B.37)
\[
\langle B_y(\vec{r}) B_x(\vec{r}') \rangle_\omega = \frac{\hbar}{2\delta^2} \coth \frac{\hbar \omega}{2k_B T} \frac{\rho}{\rho^2} \int_{0}^{\infty} dq \frac{1}{q^2} e^{-q(z+d)} J_0(q\rho)
\]
\[
= -\frac{\hbar}{4\pi \delta^2} \coth \frac{\hbar \omega}{2k_B T} \frac{x}{\rho^2} \left[ \frac{x^2}{\rho^2[(d + z)^2 + \rho^2]^2} - (d + z) \right].
\]

(B.38)

On the other hand when \( d \gg \delta \) we find
\[
\langle B_x(\vec{r}) B_x(\vec{r}') \rangle_\omega = \hbar \delta \coth \frac{\hbar \omega}{2k_B T} \frac{\rho}{\rho^2} \frac{3(d + z)^2 + 3\rho^2 - 15x^2}{[(d + z)^2 + \rho^2]^3},
\]

(B.39)
\[
\langle B_y(\vec{r}) B_x(\vec{r}') \rangle_\omega = -15\hbar \delta \coth \frac{\hbar \omega}{2k_B T} \frac{\rho}{\rho^2} \frac{x y(d + z)}{[(d + z)^2 + \rho^2]^2},
\]
\[
\langle B_z(\vec{r}) B_x(\vec{r}') \rangle_\omega = \hbar \delta \coth \frac{\hbar \omega}{2k_B T} \frac{\rho}{\rho^2} \frac{-12 x (d + z)^2 + 3xz^2}{[(d + z)^2 + \rho^2]^2}.
\]

(B.40)

(B.41)

The distant object geometry results can be obtained by placing a fictitious dipole near a magnetically polarizable electrode. Since \( d \gg L \) we assume the field generated by this electrode is uniform over the qubit and given by
\[
B_x(0) = m_f j_f(\vec{r}),
\]

(B.42)

where again \( f_f(\vec{r}) = (3\gamma_f \mathbf{r} - \mathbf{r}^2 \hat{\gamma}_f)/\mathbf{r}^3 \). We shall take only the first term in the multipole expansion of the field produced by the object, which is completely characterized by its dipole moment \( \vec{m}' \). Assuming linear response yields \( m_f^l = \beta_f B_f^l \), where \( \beta_f \) is the magnetic polarizability of the object. At the observation point \( \vec{r} \) the (again fictitious) field is
\[
B_x(\vec{r}) = m_f^l f_m(\vec{r}) = \beta_{mk} m_f j_f(\vec{r}) f_{mn}(\vec{r}),
\]

and the prescription following equation (2.23) then gives the physical noise function as
\[
\langle B_x(\vec{r}) B_x(\vec{r}') \rangle = \hbar \text{Im} (\beta_{mk}) f_{kj}(\vec{r}) f_{mn}(\vec{r}) \coth \left( \frac{\hbar \omega}{2k_B T} \right).
\]

This leads directly to
\[
\langle B_x(\vec{r}) B_x(\vec{r}') \rangle = \hbar \coth \left( \frac{\hbar \omega}{2k_B T} \right) \text{Im} (\beta_{mk}) f_{kj}(\vec{r}) f_{ij}(\vec{r}).
\]

(B.43)
References

[1] Johnson J B 1928 Thermal agitation of electricity in conductors Phys. Rev. 32 97–109
[2] Nyquist H 1928 Thermal agitation of charge in conductors Phys. Rev. 32 110–3
[3] Callen H B and Welton T A 1951 Irreversibility and generalized noise Phys. Rev. 83 34–40
[4] Kubo R 1966 The fluctuation–dissipation theorem Rep. Prog. Phys. 29 255
[5] Lifshitz E M 1956 The theory of molecular attractive forces between solids Sov. Phys.—JETP 2 73
[6] Byrnes S M 1953 Theory of Electrical Fluctuation and Thermal Radiation (Bedford, MA: Electronics Research Directorate, Air Force Cambridge Research Center, Air Research and Development Command, U.S. Air Force)
[7] Rytov S M, Kravtsov I U and Tatarski V I 1989 Principles of Statistical Radiophysics vol 3 (Berlin: Springer)
[8] Agarwal G S 1975 Quantum electrodynamics in the presence of dielectrics and conductors, i. electromagnetic-field response functions and black-body fluctuations in finite geometries Phys. Rev. A 11 230–42
[9] Casimir H B G 1948 On the attraction between two perfectly conducting plates Indag. Math. 10 261–3
[10] Casimir H B G 1997 Kon. Ned. Akad. Wetensch. Proc. 100N3–4 61
[11] Lifshitz E M and Pitaevskii L P 1980 Statistical Physicals, Part 2 vol 9 (Oxford: Pergamon)
[12] Volokitin A I and Persson B N J 2007 Near-field radiative heat transfer and noncontact friction Rev. Mod. Phys. 79 1291–329
[13] Pendry J B 1999 Radiative exchange of heat between nanostructures J. Phys.: Condens. Matter 11 6621–33
[14] Neumann R and Schreiber L R 2015 Simulation of micro-magnet stray-field dynamics for spin qubit manipulation J. Appl. Phys. 117 193903
[15] Hua A, Joynt R and Culcer D 2015 Do micromagnets expose spin qubits to charge and Johnson noise? Appl. Phys. Lett. 107 172101
[16] Henkel C and Wilkens M 1999 Loss and heating of particles in small and noisy traps Appl. Phys. B 69 379–87
[17] Henkel C and Wilkens M 1999 Heating of trapped atoms near thermal surfaces Europhys. Lett. 47 414–20
[18] Harber D M, McGuirk J M, Obrecht J M and Cornell E A 2003 Thermally induced losses in ultra-cold atoms magnetically trapped near room-temperature surfaces J. Low Temp. Phys. 133 229–38
[19] Sildes J A, Garbin J I, Dougherty W M and Chao S H 2003 The classical and quantum theory of thermal magnetic noise, with applications in spintronics and quantum microscopy Proc. IEEE 91 799–816
[20] Averin D V 1998 Adiabatic quantum computation with cooper pairs Solid State Commun. 105 659–64
[21] Makhlin Y, Schönh G and Shnirman A 2001 Quantum-state engineering with Josephson-junction devices Rev. Mod. Phys. 73 357–400
[22] Dykman M I, Platzman P M and Seddighrad P 2003 Qubits with electrons on liquid helium Phys. Rev. B 67 155402
[23] Huang F and Hu X 2014 Electron spin relaxation due to charge noise Phys. Rev. B 89 195302
[24] Poudel A, Langsjoen L S, Vavilov M G and Joynt R 2013 Relaxation in quantum dots due to evanescent-wave Johnson noise Phys. Rev. B 87 045301
[25] Langsjoen L S, Poudel A, Vavilov M G and Joynt R 2013 Electromagnetic relaxation from evanescent-wave Johnson noise Phys. Rev. A 86 010301
[26] Kolkowitz S et al 2015 Probing Johnson noise and ballistic transport in normal metals with a single-spin qubit Science 347 1129–32
[27] Hayashi T, Fujisawa T, Cheong H D, Jeong Y H and Hirayama Y 2003 Coherent manipulation of electronic states in a double quantum dot Phys. Rev. Lett. 91 226804
[28] Fujisawa T, Hayashi T, Cheong H D, Jeong Y H and Hirayama Y 2004 Rotation and phase-shift operations for a charge qubit in a double quantum dot Proc. 11th Int. Conf. on Modulated Semiconductor Structures; Physica 82 1046–52
[29] Petta J R, Johnson A C, Marcus C M, Hanson M F and Gossard A C 2004 Manipulation of a single charge in a double quantum dot Phys. Rev. Lett. 93 186802
[30] Gorman J, Hasko D G and Williams D A 2005 Charge–qubit operation of an isolated double quantum dot Phys. Rev. Lett. 95 090502
[31] Valente D C B, Mucciolo E R and Wilhelm F K 2010 Decoherence by electromagnetic fluctuations in double-quantum-dot charge qubits Phys. Rev. B 82 125302
[32] Langsjoen L S, Poudel A, Vavilov M G and Joynt R 2014 Electromagnetic fluctuations near thin metallic films Phys. Rev. B 89 115401
[33] Raab R E and Lange O L I 2005 Multipole Theory in Electromagnetism (Oxford: Oxford University Press)
[34] Bylander J, Gustavsson S, Yan F, Yoshitaha F, Harribi K, Fitch G, Cory D G, Nakamura Y, Tsai J–S and Oliver W D 2011 Noise spectroscopy through dynamical decoupling with a superconducting flux qubit Nat. Phys. 7 565–70
[35] Dial O E, Shulman M D, Harvey S P, Bluhm H, Umanovsky V and Yacoby A 2013 Charge noise spectroscopy using coherent exchange oscillations in a single–tritip qubit Phys. Rev. Lett. 110 114604
[36] Muhonen J T et al 2014 Storing quantum information for 30 seconds in a nanoelectronic device Nat. Nanotechnol. 9 986–91
[37] Levin M and Rytov S M 1967 Theory of Equilibrium Thermal Fluctuations in Electrodynamics (Moscow: Nauka) (In Russian)
[38] Jouliain K, Mulet J P, Marqueri F, Carminati R and Greffet J J 2005 Surface electromagnetic waves thermally excited: radiative heat transfer, coherence properties and casimir forces revisited in the near field Surf. Sci. Rep. 57 59–112
[39] Landau L D and Lifshitz E M 1980 Electrodynamics of Continuous Media vol 8 (Oxford: Pergamon)
[40] Osborn J A 1945 Demagnetizing factors of the general ellipsoid Phys. Rev. 67 351–7
[41] Garg A 2008 Conductors in quasistatic electric fields Am. J. Phys. 76 615–20
[42] Garg A 2012 Classical Electromagnetism in a Nastehll (Princeton, NJ: Princeton University Press)
[43] Xiao M, House M G and Jiang H W 2010 Measurement of the spin relaxation time of single electrons in a silicon metal–oxide-semiconductor-based quantum dot Phys. Rev. Lett. 104 096801
[44] Willems van Beveren L H, Witkamp B, Vandervenosh L M K, Elzerman J M, Hanson R and Kouwenhoven L P 2004 Single-shot read-out of an individual electron spin in a quantum dot Nature 430 431–5
[45] Amasha S, Macleak K, Radu I P, Zumbühl D M, Kastner M A, Hanson M P and Gossard A C 2008 Electrical control of spin relaxation in a quantum dot Phys. Rev. Lett. 100 046803
[46] Anders C R, Ferguson A J, Drurak A S and Clark R G 2007 Gate–defined quantum dots in intrinsic silicon Nano Lett. 7 2051–5
[47] Hwang J C Y, Veldhorst M, Hendriks N, Fogarty M A, Huang W, Hudson F E, Morello A and Drurak A S 2016 Impact of g-factors and valleys on spin qubits in a silicon double quantum dot Phys. Rev. B 96 045302
[48] Folman R, Krüger P, Cassetari D, Hessmo B, Maier T and Schmiedmayer J 2000 Controlling cold atoms using nanofabricated surfaces; Atom chips Phys. Rev. Lett. 84 4749–52
[49] Cironne M A, Negretti A, Calarco T, Krüger P and Schmiedmayer J 2005 A simple quantum gate with atom chips Eur. Phys. J. D 35 165–71