Teleporting Grin of a Quantum Chesire Cat without cat

Debmalya Das$^1$ and Arun Kumar Pati$^1$

$^1$Quantum Information and Computation Group, Harish-Chandra Research Institute, HBNI, Chhatnag Road, Jhunsi, Allahabad 211 019, India

Quantum Chesire Cat is a counterintuitive phenomenon that provides a new window into the nature of the quantum systems in relation to multiple degrees of freedom associated with a single physical entity. Under suitable pre and postselections, a photon (the cat) can be decoupled from its circular polarization (its grin). In this paper, we explore whether the grin without the cat can be teleported to a distant location. This will be a totally disembodied teleportation protocol. Based on the original Quantum Chesire Cat setup, we design a protocol where the circular polarization is successfully teleported between two spatially separated parties even though the photon is not physically present with them. The process raises questions in our understanding about properties of quantum system. In particular it shows that question like “whose polarization is it” can prove to be vacuous in such scenario.

I. INTRODUCTION

In the standard measurement scenario of quantum mechanics, the state of a system is disturbed irreversibly during the process of measurement. The outcome of the measurement of an observable is indeterministic and the original state collapses into one of the eigenstates of the measured observable. This allows us to gain information about the expectation value of the observable by repeating the measurement over an ensemble of the states. Weak measurement, on the other hand seeks to gain limited information from the quantum system while causing minimal perturbation to the same. As opposed to a projective measurement, this kind of measurement can be achieved by effecting a weak coupling between the system and the measurement device.

In 1988, Aharonov et. al. proposed the concept of the weak value $\langle \Psi^f | \Psi^i \rangle$ which is claimed to be the value of an observable $A$, for an ensemble which is initially prepared in the state $|\Psi^i\rangle$ and finally postselected in the state $|\Psi^f\rangle$. The weak value of a observable $A$ is obtained in the following way. The system, in the initial state $|\Psi^i\rangle$, is weakly coupled to a suitable measurement apparatus or meter, thus causing the weak measurement of the observable $A$. This is then followed by the projective measurement of a second observable $B$ that is incompatible with $A$. In the final step, one of the eigenstates, $|\Psi^f\rangle$, of the measured observable $B$, is postselected. For all successful postselections of the state $|\Psi^f\rangle$, the meter readings corresponding to the weak measurements of $A$ are taken into consideration while the others are discarded. The shift in the meter readings, on an average, for all such postselected systems is $A_w$ which is known as the weak value of $A$ and given by

$$A_w = \frac{\langle \Psi^f | A | \Psi^i \rangle}{\langle \Psi^f | \Psi^i \rangle}. \quad (1)$$

Clearly, this is a strange value of the observable $A$ that the system reveals between the pre-selection $|\Psi^i\rangle$ and the post-selection $|\Psi^f\rangle$. In other words, it can be viewed as a property of the system, which the projective measurement fails to capture. Some of the fascinating aspects of the weak value are that it can have anomalous values that lie outside the eigenvalue spectrum $|1,2 \rangle$ and can even be complex $|3 \rangle$.

Although the weak value has been measured experimentally in several quantum systems $|4,8 \rangle$, its meaning has ever been a subject of numerous discussions and controversies $|2,3,9,11 \rangle$. It has been used effectively in signal amplification and in providing explanations for the spin Hall effect, the three-box paradox and Hardy’s paradox $|12,14 \rangle$. It has also been employed to measure the wavefunction of a single photon $|6 \rangle$ and to measure the expectation value of non-Hermitian operators $|15,16 \rangle$. Quantum Chesire Cat is a theoretical scheme developed in Ref. $|17 \rangle$ to challenge some of the pre-conceived notions about the nature of a quantum system. It is based on interferometry and asks whether an intrinsic property, attributed to a system can exist in isolation to the system itself.

In the analysis given in Ref. $|17 \rangle$, the two properties in question are the position of the photon and its polarization. Going by the experience in the classical world, it would seem that a property, like the polarization of the photon, can only exist in a region where the photon actually passes through. In other words, the polarization cannot have an existence independent of the photon itself. The counterintuitiveness of Quantum Chesire Cat lies in the fact that the photon is detected in one region of the interferometer while its polarization is detected in a mutually exclusive region. In the next section, we recapitulate this phenomenon more rigorously.

The topic of Quantum Chesire Cat has recently drawn a great deal of attention from a large number of researchers working in quantum information and foundations and has led to a great number of debate and discussions $|18,22,23,27 \rangle$. In Ref. $|19 \rangle$ it was argued that the original formulation of Ref. $|17 \rangle$ is an incomplete one as it decouples only one component of the polarization from the position. The former comes up with an alternative interferometric setup that seeks to decouple all the components of polarization degree of freedom from the path degree of freedom. It was also shown in Ref. $|20 \rangle$ that more than two degrees of freedom can be separated in
a similar way, a phenomenon called Twin Chesire Cats. The
three-box paradox, in which a single particle appears
with certainty in two disjoint locations, under the context
of postselection has also been analyzed using Quantum
Chesire Cat \[21\]. Recently, in Ref.\[24\], the effect has
been studied in the presence of decoherence. The
phenomenon has also been observed experimentally using
perfect silicon crystal interferometer that separates neu-
trons from their magnetic moments under suitable pre
and postselection \[25\]. Other experimental realization of
the Quantum Chesire Cat can be found in Ref.\[20\].

In this paper we ask ‘Can we teleport a property with-
out an object?’ To answer the question, we consider the
possibility of using the grin of the Quantum Chesire Cat
for teleportation without the cat. Using a photon inter-
ferometer, we isolate the circular polarization of the pho-
ton. We send it to a party, who teleports it to another
spatially separated party using a shared entangled state,
local operation and classical communication (LOCC). We
demonstrate that although the first party has no photon
and no knowledge of the input polarization state, a pro-
tocol can be designed where it can be successfully tele-
ported to a distant location. At various points, the state
of the photon polarization can be checked using weak
measurements. This can confirm the successful telepor-
tation of the grin. Using realizations gained from the
thought experiment we discuss the implications it has
regarding associating a property of an entity with itself,
in the presence of multiple quantum systems.

The paper is organized as follows. Section II is a re-
capitulation of the Quantum Chesire Cat effect. In Sec-
tion III, we present our protocol for teleportation of the
grin of the Chesire Cat. Section IV deals with some new
implications the protocol has towards understanding the
nature of physical property of the quantum system. We
finally conclude with some discussions in Section V.

II. THE QUANTUM CHESIRE CAT

The phenomenon of Quantum Chesire Cat can be re-
alized by a scheme that is based on a Mach Zehnder in-
terferometer, first presented in Ref.\[17\]. A source sends
a linearly polarized single photon towards a 50:50 beam-
splitter \(BS_1\) that channels the photon into a left and
right path. Let \(|L\rangle\) and \(|R\rangle\) denote two orthogonal states
representing the two possible paths taken by the photon,
the left and the right arm, respectively. If the photon
is initially in the horizontal polarization state \(|H\rangle\), the
photon after passing through the beam-splitter \(BS_1\) can be prepared in the state
\[
|\Psi\rangle = \frac{1}{\sqrt{2}} (i |L\rangle + |R\rangle) |H\rangle,
\]  
where the relative phase factor \(i\) is picked up by the
photon traveling through the left arm due to the reflection
by the beam splitter. The postselection block, con-
ducting the process of projective measurement and even-

\[
|\Psi_f\rangle = \frac{1}{\sqrt{2}} (|L\rangle |H\rangle + |R\rangle |V\rangle),
\]  
where \(|V\rangle\) refers to the vertical polarization state orthog-
onal to the initial polarization state \(|H\rangle\). The PBS flips
the polarization of the photon from \(|H\rangle\) to \(|V\rangle\) and vice-
versa. The phase-shifter (PS) adds a phase factor of \(i\) to
the beam. The beam-splitter \(BS_2\) is such that when a photon in the state \(\frac{1}{\sqrt{2}} (|L\rangle + i |R\rangle)\) is incident on it, the
detector \(D_2\) never clicks. In other words, in such cases,
the photon always emerges towards the PBS. The PBS
is chosen such that it always transmits the horizontal
polarization \(|H\rangle\) and always reflects the vertical polar-
ization \(|V\rangle\). The above arrangement thus ensures that
only a state that is given by \(|\Psi_f\rangle\), before it enters the
postselection block, corresponds to the click of detector
\(D_1\). Any clicking of the detectors \(D_2\) or \(D_3\) implies a dif-
ferent state entering the postselection block. Therefore,
selecting the clicks of the detector \(D_1\) alone and discard-
ing all the others leads to the postselection onto the state
\(|\Psi_f\rangle\).
Define a circular polarization basis as
\[
|+\rangle = \frac{1}{\sqrt{2}}(|H\rangle + i|V\rangle),
\]
\[
|\rangle = \frac{1}{\sqrt{2}}(|H\rangle - i|V\rangle).
\]

Also consider the operator
\[
\sigma_z = |+\rangle \langle +| - |\rangle \langle |\rangle.
\]

Suppose we want to know which arm a photon, prepared in the state $|\Psi\rangle$ and was ultimately postselected in the state $|\Psi_f\rangle$, passed through. This can be effected by performing weak measurements of the observables $\Pi_L = |L\rangle \langle L|$ and $\Pi_R = |R\rangle \langle R|$ by placing weak detectors in the two arms. Similarly, the polarizations can be detected in the left and the right arms by respectively performing weak measurements of the following operators
\[
\sigma^L_z = \Pi_L \otimes \sigma_z,
\]
\[
\sigma^R_z = \Pi_R \otimes \sigma_z.
\]

The weak values of the photon positions are measured to be
\[
(\Pi_L)^w = 1 \text{ and } (\Pi_R)^w = 0
\]
which implies that the photon in question has traveled through the left arm. The measured weak values of the polarization positions, on the other hand, turn out to be
\[
(\sigma^L_z)^w = 0 \text{ and } (\sigma^R_z)^w = 1.
\]

Equations \((7)\) and \((8)\) together reveal that the photon traveled through the left arm but its circular polarization traveled through the right arm. This means the two degrees of freedom of a single entity can, in fact, be decoupled. That is, a property of a quantum system can exist independent of its existence in that region.

**III. TELEPORTATION USING THE DECOUPLED POLARIZATION**

In quantum teleportation, one can recreate the quantum state at a distant location using entanglement, local operation and classical communication. However, the particle is present at one end of the shared entangled state, with the teleporter, where the Bell-state measurement is carried out. \textit{Whereas here we will discuss the teleportation of the photon polarization, while the photon itself is at a different place.} Consider four parties Alice, Bob, Charlie and Dave, who are all spatially separated from each other. The setup is primarily based on the already discussed Mach-Zehnder interferometer arrangement of the Quantum Chesire Cat. Charlie prepares the initial state and Dave performs the postselection. Alice and Bob are situated on an arm of the interferometer through which the disembodied polarization state traveled, conditioned to the appropriate postselection by Dave. We would like to test whether it is possible for Alice to teleport the polarization of a photon to Bob, while the photon is not physically present with her. The initial linear polarization is taken to be in an arbitrary direction
\[
|\psi\rangle = \alpha |H\rangle + \beta |V\rangle.
\]

Let the initial state $|\Psi\rangle$, prepared by Charlie, using $BS_1$ be
\[
|\Psi\rangle = \frac{1}{\sqrt{2}}(i|L\rangle + |R\rangle) |\psi\rangle.
\]

He sends the photon and its polarization to Dave who is in possession of the postselection block, consisting of the HWP, PS, $BS_2$, PBS and the detectors $D_1$, $D_2$ and $D_3$. It is Dave who postselects the state $|\Psi_f\rangle$, given by
\[
|\Psi_f\rangle = \frac{1}{\sqrt{2}}(|L\rangle |\psi\rangle + |R\rangle |\psi^\perp\rangle),
\]
where $|\psi^\perp\rangle$ represents the polarization state, orthogonal to the initial linear polarization $|\psi\rangle$. Thus,

$$|\psi^\perp\rangle = \beta^* |H\rangle - \alpha^* |V\rangle.$$  \hspace{1cm} (12)

This prompts us to define a new basis for circular polarization,

$$|\psi_+\rangle = \frac{1}{\sqrt{2}}(|H\rangle + i|V\rangle),$$

$$|\psi_-\rangle = \frac{1}{\sqrt{2}}(|H\rangle - i|V\rangle),$$  \hspace{1cm} (13)

with $|\psi_+\rangle$ and $|\psi_-\rangle$ being the eigenstates of the operator

$$\sigma_{\psi_z} = |\psi_+\rangle \langle \psi_+| - |\psi_-\rangle \langle \psi_-|.$$  \hspace{1cm} (14)

Clearly, $\sigma_{\psi_z}$ is related to $\sigma_z$ by

$$\sigma_{\psi_z} = U^\dagger \sigma_z U,$$  \hspace{1cm} (15)

where $U$ is a unitary operator. The measurement of the polarization state of the photon is carried out by measuring the operator $\sigma_{\psi_z}$. More specifically, the weak measurements of $\sigma_{\psi_z}^L$ and $\sigma_{\psi_z}^R$ are carried out in the left and right arms of the interferometer by Charlie to find out which way the polarization went without disturbing the state. Here, $\sigma_{\psi_z}^L$ and $\sigma_{\psi_z}^R$ are, respectively, defined as

$$\sigma_{\psi_z}^L = \Pi_L \otimes \sigma_{\psi_z},$$

$$\sigma_{\psi_z}^R = \Pi_R \otimes \sigma_{\psi_z}.$$  \hspace{1cm} (16)

It is noteworthy that Charlie, and not Alice or Bob, conducts this measurement of photon polarization, since it is impossible for Alice or Bob to know the initial polarization state unless this information is shared by Charlie. Hence, Alice and Bob are unaware of the basis required for defining the operator $\sigma_{\psi_z}$. They can always gain information about the polarization state using projective measurement, but that will amount to disturbing the system and jeopardizing the whole process of teleportation of the disembodied polarization.

Now, consider the Bell states in the $\{|H\rangle, |V\rangle\}$ basis as, given by

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle),$$

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}(|HH\rangle - |VV\rangle),$$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|HV\rangle + |VH\rangle),$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|HV\rangle - |VH\rangle).$$  \hspace{1cm} (17)

From the previous discussion we know that, for the postselected states, the photon travels through the left arm while the circular polarization goes via the right arm. As discussed earlier, the two parties Alice and Bob, who are spatially separated, occupy two positions in the right arm. Alice receives the polarization state $|\psi\rangle$, unknown to her, and is required to communicate it to Bob. This can be achieved in the following way. Alice and Bob mutually share a singlet polarization state $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|HV\rangle - |VH\rangle)$ between each other. This state is appropriate for teleportation, as it remains invariant up to a phase with the change is basis. This is important because for Alice and Bob the pure state $|\psi\rangle$ they would like to teleport is unknown. Thus the basis defining the operator $\sigma_{\psi_z}$ is also unknown to them. Consequently, Alice and Bob are free to define their mutually shared polarization state in the $\{|H\rangle, |V\rangle\}$ basis due to their ignorance of the state of the polarization to be teleported.

On the joint state consisting of the polarization state, sent by Charlie, and the shared singlet state, Alice performs a local joint Bell measurement and projects her polarization state into one of the states given by Equation (17). For example, if the outcome of Alice is $|\psi^+\rangle$, then the total state consisting of the input state and the joint state she shares with Bob now becomes $|\psi^+\rangle (-U_z |\psi\rangle)$ with

$$U_z = |H\rangle \langle H| - |V\rangle \langle V|.$$  \hspace{1cm} (18)

Alice then classically communicates her outcome to Bob. To reproduce the state $|\psi\rangle$, at his end, Bob applies the $U_z$ operator locally and achieves the same state $\psi$ up to a minus sign. Similarly, depending upon Alice’s outcomes being $|\phi^+\rangle$, $|\phi^-\rangle$ or $|\psi^-\rangle$, which are classically communicated to Bob, he subsequently applies one of the local operators $U_\phi = -i |H\rangle \langle V| + i |V\rangle \langle H|$, $U_z = |H\rangle \langle V| + |V\rangle \langle H|$ or $I$ on the state he shares with Alice. In doing so the polarization state $|\psi\rangle$ is reproduced at Bob’s end and is then sent to Dave who proceeds with the strong measurement and postselection as described before.

Charlie can install two detectors, one weakly measuring $\Pi_L$ and the other weakly measuring $\sigma_{\psi_z}^R$ on the left and right arm of the interferometer, respectively, to check whether the photon is traveling through the left arm and the circular polarization is traveling through the right arm, as before, using the corresponding weak values, for all successful postselections of $|\Psi_f\rangle$. The results are same as obtained earlier. Thus,

$$(\Pi_L)^w = 1 \text{ and } (\Pi_R)^w = 0$$  \hspace{1cm} (19)

Also,

$$(\sigma_{\psi_z}^L)^w = 0 \text{ and } (\sigma_{\psi_z}^R)^w = 1.$$  \hspace{1cm} (20)

which means that Charlie has checked that the circular polarization has been sent through the right arm, and the photon through the left arm, for all cases in which the state $|\Psi_f\rangle$ is postselected in future. This circular polarization is teleported by Alice to Bob, while the photon itself is in the left arm. Dave can also install weak detectors and double check whether Bob actually recreated the input polarization. It is, however, imperative that Alice and Bob do not perform any measurement, strong
or weak, to gain knowledge about the state or to check
the success of teleportation. This is because, strong mea-
surement will rupture the whole process by disturbing the
system and weak measurements need to be performed in
a specific basis, unknown to both Alice and Bob.

IV. WHOSE GRIN MAKES IT TO THE END?

We have shown that the circular polarization of a pho-
ton can be teleported while the photon itself is at some
other location. In this section we throw light on another
curious aspect pertaining to the process. Notice that
when the polarization arrives at Alice’s port, true to the
spirit of teleportation, she does not physically transport
the polarization to Bob’s port. Instead, the shared EPR
state between Alice and Bob is converted to the input
state at Bob’s end by virtue of the operations done by
Alice and Bob. It is this polarization state, that Bob
sends to Dave for combination with the spatial degree of
freedom, traveling through the other arm. This begs us
to ask the question, if the grin of the cat ends its jour-
ney at Alice’s port, whose grin recombines with the cat
at Dave’s location? It would seem that this grin belongs
to one of the polarization degrees of freedom of the EPR
state, Bob’s subsystem of the polarization singlet state
in our case. In other words we have exchanged the grin
of two Quantum Chesire Cats.

There are more counterintuitive aspects to this process.
The grin or the polarization derived from the original in-
put state ends its journey with Alice. Alice thus has at
her disposal the polarization from the input state and the
spatial and polarization degrees of freedom from the
shared EPR state. On the other hand, Bob is left with
the spatial degree of freedom obtained from the EPR
state. Thus, at the end of the entire process, Alice has
a polarization without the photon while Bob has a photon
without a polarization. This situation is a more perma-
nent decoupling of the cat and its grin as opposed to the
usual Quantum Chesire Cat scenario in which the grin
recombines with the cat at Dave’s port.

It must be remembered that all the anomalous effects
discussed so far, starting from the separation of the pho-
ton and its polarization, the teleportation of the polar-
ization alone to the permanent decoupling of the photon
and the polarization, are all in the context of a success-
ful postselection. For all other outcomes the process pro-
ceeds as expected without the separation of the cat and
its grin.

V. CONCLUSIONS

To summarize, using the original Quantum Chesire Cat
setup we have used the isolated photon polarization
to perform teleportation even when the photon is not
present. It has been shown that our protocol does not
require a knowledge of the polarization state for the
teleporter. But it is also revealed that the parties partic-
ipating in the teleportation must remain ignorant of the
original state of the photon polarization if the informa-
tion if not shared with them a priori. We have also hinted
at the counterintuitiveness of different photons exchang-
ing their polarizations.

With the success in performing quantum teleportation
with the grin of the Quantum Chesire Cat, it would be
interesting to explore the possibilities of performing other
quantum information processing tasks with the same. We
can speculate that the separation of a quantum system
from its intrinsic property may lead to a greater security
in future quantum communications but it requires fur-
ther exploration. We are also currently expanding upon
the idea of swapping of grins of two Quantum Chesire
Cats. Thus, in the quantum world, property of a quan-
tum system cannot be claimed to be its own. It could be
someone else’s property that a particle owns momentar-
ily.

[1] Y. Aharonov, D. Z. Albert, and L. Vaidman, Phys. Rev.
Lett. 60, 1351-1354 (1988).
[2] I. M. Duck, P. M. Stevenson, and E. C. G. Sudarshan,
Phys. Rev. D, 40, 2112-2117 (1989).
[3] R. Jozsa, Phys. Rev. A, 76, 044103 (2007).
[4] N. W. M. Ritchie, J. G. Story and R. G. Hulet, Phys.
Rev. Lett. 66, 1107-1110 (1991).
[5] G. J. Pryde, J. L. O’Brien, A. G. White, T. C. Ralph
and H. M. Wiseman, Phys. Rev. Lett. 94, 220405, 2005.
[6] J. S. Lundeen, B. Sutherland, A. Patel, C. Stewart and
C. Bamber, Nature 474, 188-191, (2011).
[7] C. Wu, J. Zhang, Y. Xie, B. Ou, W. Wu and P. Chen,
ArXiv e-prints, 1811.06170 quant-ph (2018).
[8] M. Pal, S. Saha, Athira B. S, Dutta Gupta and N.
Ghosh, ArXiv e-prints, 1807.04103 quant-ph (2018).
[9] Y. Aharonov and L. Vaidman, Phys. Rev. A, 41, 11-20,
(1990).
[10] D. Sokolovski, Quanta 2, 5057 (2013).
[11] M. Cormann, M. Remy, B. Kolaric, and Y. Caudano,
Phys. Rev. A 93, 042124 (2016).
[12] O. Hosten and P. Kwiat, Science 319, 787-790 (2008).
[13] Y. Aharonov, S. Popescu and J. Tollaksen, Physics To-
day, 63, 27-32 (2010).
[14] Y. Aharonov and S. Dolev, Springer Berlin Heidelberg,
283-297 (2005).
[15] A. K. Pati, U. Singh and U. Sinha, Phys. Rev. A 92,
052120 (2015).
[16] G. Nirala, S. N. Sahoo, A. K. Pati, and U. Sinha, Phys.
Rev. A 99, 022111 (2019).
[17] Y. Aharonov, S. Popescu, D. Rohrlich and P. Skrzypczyk,
New Journal of Physics, 15, 113015 (2013).
[18] J. Bancal, Nature Physics, 10, 11 (2013).
[19] Y. Guryanova, N. Brunner and S. Popescu, ArXiv e-
prints, 1203.4215 quant-ph (2012).
[20] I. Ibnouhsein and A. Grinbaum, ArXiv e-prints, 1202.4894 quant-ph (2012).
[21] A. Matzkin and A. K. Pan, Journal of Physics A: Mathematical and Theoretical, 46, 315307 (2013).
[22] R. Corrêa, M. F. Santos, C. H. Monken and P. L. Saldanha, New Journal of Physics, 17, 053042 (2015).
[23] Q. Duprey and S. Kanjilal and U. Sinha and D. Home and A. Matzkin, Annals of Physics, 391, 1 (2018).
[24] M. Richter, B. Dziewit and J. Dajka, Advances in Mathematical Physics, 2018, 7060586 (2018).
[25] T. Denkmayr, H. Geppert, S. Sponar, H. Lemmel, A. Matzkin, J. Tollaksen and Y. Hasegawa, Nature Communications, 5, 4492 (2014).
[26] S. Sponar, T. Denkmayr, H. Geppert and Y. Hasegawa, Atoms, 4, 11 (2016).
[27] D. P. Atherton, G. Ranjit, A. A. Geraci and J. D. Weinstein, Opt. Lett., 40, 879 (2015).