A single-layer RNN can approximate stacked and bidirectional RNNs, and topologies in between

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Abstract

To enhance the expressiveness and representational capacity of recurrent neural networks (RNN), a large body of work has emerged exploring stacked architectures with additional topological modifications like shortcut connections or bidirectionality. However, choosing the best network for a particular problem requires a combinatorial search over architectures and their hyperparameters. In this work, we show that a single-layer RNN can perfectly mimic an arbitrarily deep stacked RNN under specific constraints on its weight matrix and a delay between input and output. This obviates the need to manually select hyperparameters like the number of layers. Additionally, we show that weakening weight constraints while keeping the delay gives rise to partial acausality in the single-layer RNN, much like a bidirectional network. Synthetic experiments confirm that the delayed RNN can mimic bidirectional networks in perfectly solving some acausal tasks, outperforming them in others. Finally, we show that in a challenging language processing task, the delayed RNN performs within 0.3% of the accuracy of the bidirectional network while reducing computational costs.

1 Introduction

Recurrent neural networks (RNN) have successfully been used for sequential tasks like language modeling [31], machine translation [32], and speech recognition [5]. They approximate complex, non-linear temporal relationships by maintaining and updating an internal state for every input element. However, they face several challenges while modeling long-term dependencies, motivating work on variant architectures.

Firstly, due to the long credit assignment paths in RNNs, the gradients might vanish or explode [11]. This has led to gated variants like the Long Short-term Memory (LSTM) [18] that can retain information over long timescales. Secondly, it is well known that deeper networks can more efficiently approximate a broader range of functions [10][12]. While RNNs are deep in time, they are limited in the number of non-linearities applied to recent inputs. To increase depth, there has been extensive work on stacking RNNs into multiple layers [9][28]. In vanilla stacked RNNs, each layer applies a non-linearity and passes information to the next layer, while also maintaining a recurrent connection to itself. To effectively propagate gradients across the hierarchy, skip or shortcut connections can be
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Another development is the bidirectional RNN (Bi-RNN) [29, 16]. While RNNs are inherently
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recurrent weights. While this approximation carries some error, this method is capable of yielding
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constraints on the recurrent weights and outputs. This equivalence trivializes the search among stacked
RNN architectures, as equivalent solutions can be learned in a single layer. The second aim of this
paper is to show that a flattened RNN can approximate a Bi-RNN by removing constraints on the
recurrent weights. While this approximation carries some error, this method is capable of yielding
similar solutions at potentially much lower computational cost.

2 Background

Given a sequential input \( \{x_t\}_{t=1..T}, x_t \in \mathbb{R}^d \), a single-layer RNN is defined by:

\[
\hat{h}_t = f \left( \hat{W}_x x_t + \hat{W}_h \hat{h}_{t-1} + \hat{b}_h \right), \quad \hat{y}_t = g \left( \hat{W}_o \hat{h}_t + \hat{b}_o \right),
\]

where \( f(\cdot) \) and \( g(\cdot) \) are element-wise activation function such as \( \tanh \) and softmax, \( \hat{h}_t \in \mathbb{R}^n \) is
the hidden state at timestep \( t \) with \( n \) units, and \( \hat{y}_t \in \mathbb{R}^m \) is the network output. Learned parameters
include input weights \( \hat{W}_x \), recurrent weights \( \hat{W}_h \), bias term \( \hat{b}_h \), output weights \( \hat{W}_o \), and bias term
\( \hat{b}_o \). The initial hidden state is denoted \( \hat{h}_0 \).

Stacked recurrent units are typically used to provide depth in RNNs [9, 28]. Based on [1], a stacked
RNN with \( k \) layers is given by:

\[
\begin{align*}
\hat{h}_t^{(1)} &= f \left( \hat{W}_x^{(1)} x_t + \hat{W}_h^{(1)} \hat{h}_{t-1}^{(1)} + \hat{b}_h^{(1)} \right) \\
\hat{h}_t^{(i)} &= f \left( \hat{W}_x^{(i)} \hat{h}_{t-1}^{(i-1)} + \hat{W}_h^{(i)} \hat{h}_{t-1}^{(i)} + \hat{b}_h^{(i)} \right), \quad i = 2, \ldots, k \\
y_t &= g \left( \hat{W}_o \hat{h}_t^{(k)} + \hat{b}_o \right),
\end{align*}
\]

where the activation function and parameterization follow the single-layer RNN. Separate weights
and bias terms for each layer \( i \) are given by \( \hat{W}_x^{(i)}, \hat{W}_h^{(i)}, \) and \( \hat{b}_h^{(i)} \). The hidden state for this layer at
timestep \( t \) is \( \hat{h}_t^{(i)} \). The stacked RNN has hidden state vectors \( \hat{h}_t^{(1)} \ldots \hat{h}_t^{(k)} \) corresponding to
the \( k \) layers. The hat operator is used for vectors and matrices in the single-layer RNN, while those
without are for the stacked RNN.

3 A stacked RNN is equivalent to a single-layer RNN

The mathematical structure of a stacked RNN is similar to a single-layer RNN with the addition of
between-layer connections that add depth. Here we show that any stacked RNN can be flattened
into a single-layer RNN that produces the exact same sequence of hidden states and outputs, albeit
with some time delay. To illustrate this, we rewrite the parameters of a single-layer RNN using the

\[
\begin{align*}
\hat{h}_t^{(1)} &= f \left( \hat{W}_x^{(1)} x_t + \hat{W}_h^{(1)} \hat{h}_{t-1}^{(1)} + \hat{b}_h^{(1)} \right) \\
\hat{h}_t^{(i)} &= f \left( \hat{W}_x^{(i)} \hat{W}_x^{(i-1)} \hat{h}_{t-1}^{(i)} + \hat{W}_h^{(i)} \hat{h}_{t-1}^{(i-1)} + \hat{b}_h^{(i)} \right), \quad i = 2, \ldots, k \\
y_t &= g \left( \hat{W}_o \hat{h}_t^{(k)} + \hat{b}_o \right),
\end{align*}
\]
weights and bias terms of the stacked RNN from Equations (2)-(4):

\[ \hat{W}_h = \begin{bmatrix} W^{(1)}_h & 0 & \cdots & 0 \\ W^{(2)}_h & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & W^{(k)}_h & 0 \end{bmatrix}, \quad \hat{b}_h = \begin{bmatrix} b^{(1)}_h \\ \vdots \\ b^{(k)}_h \end{bmatrix}, \quad \hat{W}_x = \begin{bmatrix} W^{(1)}_x \\ \vdots \\ 0 \end{bmatrix}, \quad \hat{b}_x = b_o, \]

where \( \hat{W}_x \in \mathbb{R}^{kn \times d} \) are the input weights, \( \hat{W}_h \in \mathbb{R}^{kn \times kn} \) the recurrent weights, \( \hat{b}_h \in \mathbb{R}^{kn} \) the biases, \( \hat{W}_o \in \mathbb{R}^{m \times kn} \) the output weights, and \( \hat{b}_o \in \mathbb{R}^{m} \) the output biases.

Intuitively, one can see from Eq. (5) that each layer in the stacked RNN is converted into a group of units in the flattened RNN. The block bidiagonal structure of the recurrent weight matrix \( \hat{W}_h \) makes the hidden state act as a buffer, where each group of units only receives input from itself and the previous group. Information processed through this buffering mechanism eventually arrives at the output after \( k-1 \) timesteps. It is important to note that the flattened RNN performs the same computations as the stacked version by trading depth in layers for depth in time.

Next, we define the following notation: for a vector \( v \in \mathbb{R}^{kn} \) with \( k \) blocks, the subvector \( v^{(i)} \in \mathbb{R}^{n} \) refers to its \( i \)th block following the partition from Equation (5). We now prove that a single-layer RNN parametrized by Eq. (5)-(6) is exactly equivalent to the stacked RNN in Eqs. (2)-(4). The proof easily extends to more complex recurrent cells such as LSTMs and GRUs.

**Theorem 1.** Given an input sequence \( \{x_t\}_{t=1..T} \) and a stacked RNN with \( k \) layers defined by Equations (2)-(4) with activation functions \( f(\cdot) \) and \( g(\cdot) \), and initial states \( \{h_0^{(i)}\}_{i=1..k} \), the single-layer RNN defined by Equations (5)-(6) and initialized with \( \hat{h}_0 \) such that \( \hat{h}_{i-1}^{(i)} = h_0^{(i)}, \forall i = 1 \ldots k \), produces the same output sequence but delayed by \( k-1 \) timesteps, i.e., \( \hat{y}_{t+k-1} = y_t \) for all \( t = 1 \ldots T \). Further, the sequence of hidden states at each layer \( i \) are equivalent with delay \( i-1 \), i.e., \( \hat{h}_{t+i-1}^{(i)} = h_t^{(i)} \) for all \( 1 \leq i \leq k \) and \( t \geq 1 \).

**Proof.** The proof is included in Section 1 of the supplementary material.

Theorem 1 makes an assumption that \( \hat{h}_0 \) in the single-layer RNN can be initialized such that it achieves \( \hat{h}_{i-1}^{(i)} = h_0^{(i)} \) for all blocks. Lemma 1 below implies that initialization for the flattened RNN can always be computed from the stacked RNN. The intuition behind it is that we can compute recursively from \( \hat{h}_{i-1}^{(i)} = h_0^{(i)} \) to \( h_0^{(i)} \) for block \( i \), while “inverting” the activation function effect in the process. All commonly used activation functions are surjective, thus it is enough to know the right-inverse of the activation function \( f(\cdot) \) (see proof of Lemma). For example, when \( f(\cdot) \) is the ReLU, the right-inverse is the identity function \( r(d) = d \).

**Lemma 1.** Let \( f: \mathbb{R} \rightarrow D \) be a surjective activation function that maps elements in \( \mathbb{R} \) to elements in interval \( D \). Also, let \( h_0^{(i)} \in D^n \) for \( i = 1 \ldots k \) be the hidden state initialization for a stacked RNN with \( k \) layers as defined in (2)-(3). Then, there exists an initial hidden state vector \( \hat{h}_0 \in \mathbb{R}^{kn} \) for a single-layer network in Equation (5) such that \( \hat{h}_{i-1}^{(i)} = h_0^{(i)} \) for all \( i = 1 \ldots k \).

**Proof.** The proof is included in Section 2 of the supplementary material.
A stacked RNN is equivalent to a single-layer RNN under the given weight constraints. The flattened RNN produces the same representations as the stacked network, albeit with a time delay. (a) Stacked RNN with \( k = 4 \) layers where connections show the different weight parameters. (b) Weights of the flattened RNN that are equivalent to connections in the stacked RNN.

### 3.1 The weight matrices

Suppose one takes a flattened RNN as in Eq. (5) and adds non-zero elements to regions not populated by weights from the equivalent stacked RNN. These non-zero weights do not correspond to existing connections in the stacked RNN. So what do they correspond to?

To explore this question we illustrate a 4-layer stacked RNN in Figure 1(a). Here solid arrows show the standard stacked RNN connections. The flattened RNN weight matrices \( \hat{W}_h, \hat{W}_x, \) and \( \hat{W}_o \) are shown in Figure 1(b), where the color of each block matches the corresponding arrow in Figure 1(a). Blocks on the main diagonal of \( \hat{W}_h \) connect groups of units to themselves recurrently, while blocks on the subdiagonal correspond to connections between layers in the stacked RNN. More generally, block \((i, j)\) in \( \hat{W}_h \) corresponds to a connection from \( h_{j}^{(j)} \) to \( h_{i}^{(i)} \) in the stacked RNN. Thus, blocks in the lower triangle (i.e. \( i > j + 1 \)) correspond to connections that point backwards in time, and from a lower layer to a higher layer. For example, the orange block \((3, 1)\) in Figure 1(b) (and the dashed orange lines in Figure 1(a)) connects layer 1 at time \( t \) to layer 3 at time \( t - 1 \). In a later section we will test whether this type of connection can be exploited to mimic some aspects of a Bi-RNN. Conversely, blocks in the upper triangle (i.e. \( j > i \)) point forward in time and from a higher layer to a lower layer. For example, the red block \((3, 4)\) in Figure 1(b) (and the dashed red lines in Figure 1(a)) connects layer 4 at time \( t \) to layer 3 at time \( t + 2 \).

Which possible connections in a stacked RNN cannot be represented in the flattened RNN? First, the flattened RNN cannot emulate connections that go backwards in time from a higher layer to a lower layer. However, such connections introduce loops and thus are also impossible in stacked RNNs. Second, the flattened RNN cannot emulate connections that go forward in time from a lower layer to a higher layer. However, such connections would merely serve as “shortcuts”, and don’t facilitate additional non-linear transformations to the input.

Thus we see that adding weights to empty regions in the flattened RNN can mimic the behavior of many stacked recurrent architectures that have previously been proposed. Among others, it can approximate the IndRNN [20], td-RNN [33], skip-connections [14], and all-to-all layer networks [13]. Simply removing the constraints on \( \hat{W}_h \) during training will enable a single-layer RNN to learn the necessary stacked architecture. However, unlike an ordinary RNN, this requires the output to be delayed based on the desired stacking depth. Further, while the single-layer network has the same total number of units as the corresponding stacked RNN, relaxing constraints on \( \hat{W}_h \) would mean that the single-layer would have many more parameters.

### 3.2 The delay in the output

Flattening a stacked RNN introduces a delay of \( k - 1 \) timesteps between the input at timestep \( t \) and its respective output at timestep \( t + k - 1 \). This delay plays the role of the \( k \) non-linear transformations.
that the stacked RNN computes for each timepoint. That is, non-linearities are achieved in the temporal direction instead of across layers. The idea of increasing the number of temporal non-linearities has been previously explored as micro-steps \[ \text{max}(1-\Delta t, 1+\Delta t) \], where additional timesteps are inserted between each pair of elements in both the input and output sequences. For a sequence of length \( T \), the computational effort of micro-step models grows with the delay \( d \) proportionally to \( O(dT) \). In contrast, temporal depth in a flattened RNN is obtained by applying the delay as in Theorem 1. This allows the model to maintain \( k \) non-linear transformations between input and output, but to have a computational complexity that only grows proportionally to \( O(d + T) \).

### 3.3 Approximating bidirectional RNNs

When sparsity constraints on the weight matrices are removed, a flattened RNN gains the ability to peek at future inputs. A similar idea was used in the past as a baseline for bidirectional recurrent neural networks (Bi-RNNs) \[29, 10\]. These papers showed that Bi-RNNs were superior to delayed RNNs for relatively simple problems, but it is not clear that this comparison holds true for problems that require more non-linear solutions. If a recurrent network can compute the output for time \( t \) by exploiting future input elements, what conditions are necessary to approximate its Bi-RNN counterpart? Moreover, can the delayed-output RNN obtain the same results? And, given these conditions, is there a benefit to use the delayed-output RNN instead of the Bi-RNN?

To answer these questions we first focus on the functional richness and the influence of each element in the sequence. Figure 2 shows the number of non-linear transformations that each network can apply to any input element before computing the output at timestep \( t_0 \). The generic RNN processes only past inputs (\( t \leq t_0 \)), and the number of non-linearities decreases for inputs closer to timestep \( t_0 \), reaching 1 at \( t = t_0 \). The Bi-RNN has identical behavior for causal inputs but is augmented symmetrically for acausal inputs by the inclusion of a backward RNN. In contrast, the delayed-output RNN (d-RNN) has a similar behavior for the causal inputs but with a higher number of non-linearities. This trend continues for the first \( d \) acausal inputs with a decreasing number of non-linearities until the number reaches zero at \( t = t_0 + d + 1 \).

In order for a d-RNN to have at least as many non-linearities as a Bi-RNN for every element in a sequence, it needs a delay that is twice the sequence length. However, long delays have the disadvantage of increasing the memory requirements on the network. On the other hand, a d-RNN can beat a Bi-RNN when the needed acausal information is limited to only a few future elements or when the non-linear influence of these nearby inputs on the learned function is higher than the remaining ones in the sequence.

Interestingly, using the d-RNN with low delay values to approximate a Bi-RNN has additional benefits. One advantage is in computational cost: for a sequence of length \( T \), the cost to compute a forward-pass for the d-RNN is \( T + d \), while for the Bi-RNN the cost is \( 2T \). Thus, in practice it is possible to trade-off small performance degradations for computationally cheaper networks. Beyond the computational costs, d-RNNs can also be used in applications where it is critical to output values in (near) realtime \([6, 17]\).
4 Experiments

We test the capabilities of the d-RNN in three experiments designed to shed more light on the relationships between d-RNNs, RNNs, and Bi-RNNs. For this purpose, the RNN implementation is switched to LSTMs, which avoid vanishing gradients and are better able to retain information across delays. The delayed LSTM networks are denoted as d-LSTMs. To train each d-LSTM, the input sequences are padded at the end with zero-vectors and loss is computed by ignoring the first “delay” timesteps. All models are trained using the Adam optimization algorithm [19] with learning rate $1e^{-3}$, $\beta_1 = 0.9$, and $\beta_2 = 0.999$. During training, the gradients are clipped at 1.0 to avoid explosions. Experiments were implemented using PyTorch 1.1.0 [25].

4.1 Experiment 1: sequence reversal

First, we propose a simple test to illustrate how the d-LSTM can interpolate between a regular LSTM and Bi-LSTM. In this test we require the recurrent architectures to output a sequence in reverse order while reading it, i.e. $y_t = x_{T-t+1}$ for $t = 1, \ldots, T$. Solving this task perfectly is only possible when a network has acausal access to the sequence. Moreover, depending on how many acausal elements a network accesses, it is possible to analytically calculate the expected maximum performance that the network can achieve. Given a sequence of length $T$ with elements from a vocabulary $\{1, \ldots, V\}$, a causal network such as the regular LSTM can output the second half of the elements correctly and guess those in the first half with probability $1/V$. When a network has access to $d$ acausal elements it can start outputting correct elements before reaching the halfway point, and can achieve an expected true positive rate (TPR) of $\frac{d}{2}(1 + \frac{1}{V}) + \left\lfloor \frac{d+1}{2} \right\rfloor \frac{1 - \frac{1}{V}}{2}$. We generate data sequences of length $T = 20$ by uniformly sampling integer values between 1 and $V = 4$. The training set consists of 10,000 sequences, the validation set 2,000, and test set 2,000. Output sequences are the input sequences reversed. Values in the input sequences are fed as one-hot vector representations. All networks output via a linear layer with a softmax function that converts to a vector of $V$ probabilities to which cross-entropy loss is applied. The LSTM and d-LSTM networks have 100 hidden units, while the Bi-LSTM has 70 in each direction in order to keep the total number of parameters constant. We use batches of 100 sequences and train for 1,000 epochs with early stopping after 10 epochs and $\Delta = 1e^{-3}$.

Figure 3 shows accuracy on this task as a function of the applied delay. The LSTM network does not use acausal information and is unable to reverse more than half of the input sequence. Conversely, the Bi-LSTM has full access to every element in the sequence, and can perfectly solve the task. For the d-LSTM network, performance increases as we increase the delay in the output, reaching the same level as the Bi-LSTM once the network has access to the entire sequence before being required to produce any output (delay 19). This experiment shows that, for a simple linear task, the d-LSTM can “interpolate” between LSTM and Bi-LSTM by choosing a delay that ranges from zero to the length of the input sequence.
We generate datasets with different combinations of $\gamma$ with different filters $w$. The first experiment showed how a d-LSTM with sufficient delay can mimic a Bi-LSTM. In the next experiment, we aim at comparing how well d-LSTM, LSTM, and Bi-LSTM networks approximate functions with varying degrees of non-linearity and acausality.

Drawing inspiration from [29], we require each recurrent network to learn the function $y_t = \sin(\gamma \sum_{j=-c+1}^a w_j + x_{t+j})$. The parameter $\gamma$ scales the argument of the sine function and thus controls the degree of non-linearity in the function: for small $\gamma$ the function is roughly linear, while for large $\gamma$ the function is highly non-linear. Scalars $a \geq 0$ (acausal) and $c \geq 0$ (causal) control the length of the causal and acausal portions of the linear filter $w$ that is applied to the input $x$.

We generate datasets with different combinations of $\gamma \in [0.1, \ldots, 5.0]$ and $a \in [0, \ldots, 10]$, choosing $c$ such that $a + c = 20$. For each combination, we generate a filter $w$ with 20 elements drawn uniformly in $[0.0, 1.0)$, and random input sequences with $T = 50$ elements drawn from a uniform distribution $[0.0, 1.0)$. In total, there are 10,000 generated sequences for training, 2,000 for validation, and 2,000 for testing with each set of parameter values. The output is computed following the previous formula and with zero padding for the borders. We generate 5 repetitions of each dataset with different filters $w$ and inputs $x$.

We train LSTM, d-LSTM with delays 5 and 10, and Bi-LSTM networks to minimize mean squared error (MSE). The LSTM and d-LSTM have 100 hidden units and the Bi-LSTM has 70 per network, matching the numbers of parameters. A linear layer after the recurrent layer outputs a single value error (MSE). The LSTM and d-LSTM have 100 hidden units and the Bi-LSTM has 70 per network, matching the numbers of parameters. A linear layer after the recurrent layer outputs a single value error (MSE). The LSTM and d-LSTM have 100 hidden units and the Bi-LSTM has 70 per network, matching the numbers of parameters. A linear layer after the recurrent layer outputs a single value error (MSE).

Figure 4 shows the average test MSE for each model as a function of $(\gamma, a)$ value.

Figure 4 shows the average test MSE for each model as a function of $(\gamma, a)$ value. LSTM performance (Fig. 4(a)) is poor everywhere except where the filter is purely causal. Surprisingly, the network performs quite well even when the amount of non-linearity $(\gamma)$ is quite high. The reason for this seems to be that temporal depth enables the LSTM to approximate this function well. Bi-LSTM performance (Fig. 4(b)) follows a similar trend for the causal case ($a = 0$) as the forward LSTM, but also has good performance for acausal filters ($a > 0$) when the function is nearly linear ($\gamma$ is small). As the non-linearity of the function increases, however, Bi-LSTM performance suffers. This occurs because the Bi-LSTM needs to approximate a highly non-linear function with a linear combination of its forward and backward outputs, which cannot be done with small error. Improving performance would require stacked Bi-LSTM layers.

In contrast, d-LSTM networks have excellent performance for both non-linear and acausal functions. The d-LSTM with delay 5 (Fig. 4(c)) shows a clear switch in performance from acausality $a = 5$ to 6. This perfectly matches by the limit of acausal elements that the network has access to. For the d-LSTM with delay 10 (Fig. 4(d)), the network performs well for acausality values $a$ up to 10.

An interesting outcome of this experiment is the better performance observed for the d-LSTM over the Bi-LSTM. This shows that the d-LSTM can be a better fit than a Bi-LSTM for the right task. Furthermore, the d-LSTM network seems to approximate the functionality of a stacked Bi-LSTM by approximating highly non-linear functions. In practice, this could be a great benefit for applications.
The models are composed of two subnetworks at character-level and word-level. Best bidirectional network and best forward-only network are marked in bold for each language.

| Lang. | Char-level network | Word-level network | Validation Accuracy | Test Accuracy |
|-------|--------------------|--------------------|---------------------|---------------|
| DE    | LSTM               | LSTM               | 92.05 ± 0.16        | 91.58 ± 0.11  |
|       | d-LSTM delay=1     | d-LSTM delay=1     | 93.48 ± 0.31        | **92.87 ± 0.24** |
|       | d-LSTM delay=1     | Bi-LSTM            | 93.93 ± 0.06        | **93.39 ± 0.18** |
|       | Bi-LSTM            | Bi-LSTM            | 93.88 ± 0.13        | 93.15 ± 0.08  |
| EN    | LSTM               | LSTM               | 92.05 ± 0.13        | 92.14 ± 0.10  |
|       | d-LSTM delay=1     | d-LSTM delay=1     | 94.57 ± 0.08        | **94.57 ± 0.14** |
|       | d-LSTM delay=1     | Bi-LSTM            | 94.94 ± 0.07        | **94.95 ± 0.06** |
|       | Bi-LSTM            | Bi-LSTM            | 94.85 ± 0.05        | 94.84 ± 0.08  |
| FR    | LSTM               | LSTM               | 96.67 ± 0.07        | 96.10 ± 0.11  |
|       | d-LSTM with delay=1| d-LSTM with delay=1| 97.49 ± 0.04        | **97.04 ± 0.13** |
|       | d-LSTM with delay=1| Bi-LSTM            | 97.67 ± 0.07        | **97.23 ± 0.12** |
|       | Bi-LSTM            | Bi-LSTM            | 97.63 ± 0.06        | 97.22 ± 0.11  |

where there is no need to treat the whole sequence. Moreover, this could be impossible in other cases, such as streamed data. In such cases, the d-LSTM would shine over bidirectional architectures. On the other hand, we expect the Bi-LSTM to perform better when the acausality needs for the task are longer than the delay, i.e., $a > d$.

4.3 Experiment 3: real-world part-of-speech tagging

In the previous experiments, we show that d-LSTM is capable of approximating and even outperforming a Bi-LSTM in some cases. In practice, however, the elements in a sequence may have different forward and backward relations. This poses a challenge for delayed networks that are constrained to a specific delay. If the delay is too low, it may not be enough for some long dependencies between elements. If it is too high, the network may forget information and require higher capacity (and maybe training data). This is prevalent in several NLP tasks. Therefore, we test on the Part-of-Speech (POS) tagging task where Bi-LSTMs achieve state-of-the-art performance [26, 21, 7]. The task involves processing a variable length sequence to predict a POS tag (e.g. Noun, Verb) per word, using the Universal Dependencies (UD) [22] dataset. More details can be found in the supplementary material.

The dual Bi-LSTM architecture proposed by Plank et al. [26] is followed to test the approximation capacity of the d-LSTMs. In this model, a word is encoded using a combination of word embeddings and character-level encoding. The encoded word is fed to a Bi-LSTM followed by a linear layer with softmax to produce POS tags. The character-level encoding is produced by first computing the embedding of each character and then feeding it to a Bi-LSTM. The last hidden state in each direction is concatenated with the word embedding to form the character-level encoding.

The character-level Bi-LSTM has 100 units in each direction and the LSTM/d-LSTMs have 200 units to generate encodings of the same size. For the word-level subnetwork, the hidden state is of size 188 for the Bi-LSTM, and 300 units for the LSTM/d-LSTM to match the number of parameters. The networks are trained for 20 epochs with cross-entropy loss. We train combinations of networks with delays 0 (LSTM), 1, 3, and 5 for the character-level subnetwork, and delays 0 through 4 for the word-level. Each network has 5 repeats with random initialization.

Results are presented in Table 1. For brevity, we include a subset of the combinations for each language (the complete table can be found in the supplementary material). For the character-level model, LSTMs without delay yield reduced performance. However, replacing only the character-level Bi-LSTM with a LSTM does not affect the performance (supplementary material). This suggests that only the word-level subnetwork benefits from acausal elements in the sentence. Interestingly, using a d-LSTM with delay 1 for the character-level network achieves a small improvement over the double-bidirectional model in English and German. Replacing the word-level Bi-LSTM
with an LSTM decreases performance significantly. However, using a mere d-LSTM with delay 1 improves performance to within 0.3% of the original Bi-LSTM model.

5 Conclusions

In this paper we show that stacked RNNs, which are frequently used to increase depth and representational complexity for sequence problems, can always be flattened into a single-layer RNN with delayed output. Relaxing constraints imposed by the flattening process allows this delayed RNN to look at future as well as past elements, making it possible to approximate bidirectional RNNs but at reduced computational cost. Although delayed-output RNNs have been touched upon previously, this paper reinforces the idea that the simple action of introducing a delay can have significant impact on the capabilities and performance of RNNs.

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A Theorem 1 proof

Proof. We prove Theorem 1 by induction on the sequence length $t$. First, we show that for $t = 1$ the stacked RNN and the flattened single-layer RNN are equivalent. Namely, for $t = 1$ we show that the outputs and the hidden states are the same, i.e., $\hat{y}_k = y_1$ and $\hat{h}_i^{(i)} = h_1^{(i)}$, respectively. Without loss of generality, we have for any $j$ in $1 \ldots k$ the following:

$$\hat{h}_i^{(i)} = f^{(i)} \left( W_x x_i + \hat{W}_h \hat{h}_{i-1} + \hat{b}_h \right)$$

$$= f \left( W_x^{(i)} x_i + W_h^{(i)} \hat{h}_{i-1}^{(i-1)} + W_h^{(i)} \hat{h}_{i-1}^{(i)} + b_h^{(i)} \right)$$

$$= f \left( 0 + W_h^{(i)} \hat{h}_{i-1}^{(i-1)} + W_h^{(i)} h_0^{(i)} + b_h^{(i)} \right)$$

$$= f \left( W_h^{(i)} \hat{h}_{i-1}^{(i-2)} + W_h^{(i)} h_0^{(i-1)} + b_h^{(i-1)} \right) + W_h^{(i)} h_0^{(i)} + b_h^{(i)}$$

$$\ldots$$

$$= f \left( W_h^{(i)} \ldots f \left( f \left( W_h^{(2)} f \left( W_h^{(1)} h_0^{(1)} + b_h^{(1)} \right) + W_h^{(2)} h_0^{(2)} + b_h^{(2)} \right) \ldots + W_h^{(j)} h_0^{(j)} + b_h^{(j)} \right) \ldots + W_h^{(i)} h_0^{(i)} + b_h^{(i)} \right)$$

$$= f \left( W_h^{(i)} \ldots f \left( W_h^{(i)} h_0^{(i)} + b_h^{(i)} \right) \ldots + W_h^{(i)} h_0^{(i)} + b_h^{(i)} \right)$$

$$\ldots$$

$$= f \left( W_h^{(i)} h_0^{(i-1)} + W_h^{(i)} h_0^{(i)} + b_h^{(i)} \right) \ldots + W_h^{(i)} h_0^{(i)} + b_h^{(i)}$$

$$\ldots$$

$$= f \left( W_h^{(i)} h_0^{(i-1)} + W_h^{(i)} h_0^{(i)} + b_h^{(i)} \right)$$

$$= h_1^{(i)}$$

where we used the initialization assumption $\hat{h}_i^{(i)} = h_0^{(i)}$ for all $i = 1 \ldots k$, and the definition of the hidden state in Eq. (1) for $j - 1$ blocks, in the previous steps. In particular, we have for $j = k$,

$$\hat{h}_k^{(k)} = h_1^{(k)}.$$

Plugging this result and the definition of the output weights and biases in Equation (6) into Equation (1) for the computing the output, we obtain

$$\hat{y}_k = g \left( W_\alpha \hat{h}_k + \hat{b}_\alpha \right) = g \left( W_\alpha \hat{h}_k^{(k)} + b_\alpha \right) = g \left( W_\alpha h_1^{(k)} + b_\alpha \right) = y_1. \quad (A.7)$$

Which concludes the basis of the induction.

Next, we assume that $\hat{h}_i^{(i)} = h_i^{(i)}$ for all $1 \leq i \leq k$ and $t \leq T - 1$, and prove that it holds for the hidden states for all layers when $t = T$: $\hat{h}_i^{(i)} = h_i^{(i)}$, $\forall 1 \leq i \leq k$. Without loss of generality, we
have for the hidden state $\hat{h}_{T+i-1}^{(i)}$ in single-layer RNN that,

$$
\hat{h}_{T+i-1}^{(i)} = f^{(i)} \left( W_x x_{T+i-1} + W_h \hat{h}_{T+i-2} + b_h \right)
= f \left( W_x^{(i)} x_{T+i-1} + W_x^{(i)} \hat{h}_{T+i-2}^{(i-1)} + W_h^{(i)} \hat{h}_{T+i-2}^{(i)} + b_h^{(i)} \right)
= f \left( 0 + W_x^{(i)} \hat{h}_{T+i-2}^{(i-1)} + W_h^{(i)} \hat{h}_{T+i-2}^{(i)} + b_h^{(i)} \right)
= f \left( W_x^{(i)} \cdot f \left( W_x^{(i-1)} x_{T+i-2} + W_x^{(i-1)} \hat{h}_{T+i-3}^{(i-2)} + W_h^{(i-1)} \hat{h}_{T+i-3}^{(i-1)} + b_h^{(i-1)} \right)
+ W_h^{(i)} \hat{h}_{T+i-2}^{(i)} + b_h^{(i)} \right)
= f \left( W_x^{(i)} \cdot f \left( W_x^{(i-1)} \hat{h}_{T+i-3}^{(i-2)} + W_h^{(i-1)} \hat{h}_{T+i-3}^{(i-1)} + b_h^{(i-1)} \right) + W_h^{(i)} \hat{h}_{T+i-2}^{(i)} + b_h^{(i)} \right)
\ldots
= f \left( W_x^{(i)} \ldots f \left( W_x^{(j)} \ldots \right. \right.

f \left( W_x^{(2)} f \left( W_x^{(1)} x_T + W_h^{(1)} \hat{h}_{T-1}^{(1)} + b_h^{(1)} \right) + W_h^{(2)} \hat{h}_{T-1}^{(2)} + b_h^{(2)} \right)
+ \ldots + W_h^{(j)} \hat{h}_{T+j-2}^{(j)} + b_h^{(j)} \right) + W_h^{(i)} \hat{h}_{T+i-2}^{(i)} + b_h^{(i)}
\ldots
= f \left( W_x^{(i)} \ldots f \left( W_x^{(j)} h_{T-1}^{(j)} + W_h^{(j)} h_{T-1}^{(j)} + b_h^{(j)} \right) \ldots + W_h^{(i)} h_{T-1}^{(i)} + b_h^{(i)} \right)
\ldots
= f \left( W_x^{(i)} h_{T}^{(i-1)} + W_h^{(i)} h_{T}^{(i)} + b_h^{(i)} \right)
= h_{T}^{(i)}
$$

From the inductive assumption, we have that $\hat{h}_{T+j-2}^{(j)} = h_{T-1}^{(j)}$ for all $1 \leq j \leq k$, then it follows that

$$
\hat{h}_{T+i-1}^{(i)} = f \left( W_x^{(i)} \ldots f \left( W_x^{(j)} \ldots \right. \right.

f \left( W_x^{(2)} f \left( W_x^{(1)} x_T + W_h^{(1)} h_{T-1}^{(1)} + b_h^{(1)} \right) + W_h^{(2)} h_{T-1}^{(2)} + b_h^{(2)} \right)
+ \ldots + W_h^{(j)} h_{T-1}^{(j)} + b_h^{(j)} \right) + W_h^{(i)} h_{T-1}^{(i)} + b_h^{(i)}
\ldots
= f \left( W_x^{(i)} \ldots f \left( W_x^{(j)} h_{T-1}^{(j)} + W_h^{(j)} h_{T-1}^{(j)} + b_h^{(j)} \right) \ldots + W_h^{(i)} h_{T-1}^{(i)} + b_h^{(i)} \right)
\ldots
= f \left( W_x^{(i)} h_{T}^{(i-1)} + W_h^{(i)} h_{T}^{(i)} + b_h^{(i)} \right)
= h_{T}^{(i)}
$$

where we used the definition of the hidden states in Equations (2)-(3). In particular, we have for $i = k$ that $\hat{h}_{T+k-1}^{(k)} = h_{T}^{(k)}$.

Now, we show that $\hat{y}_{T+k-1} = y_T$. By the definition of the output weights and biases in Equation (6), and by the fact that $\hat{h}_{T+k-1}^{(k)} = h_{T}^{(k)}$, we obtain

$$
\hat{y}_{T+k-1} = g \left( W_o \hat{h}_{T+k-1} + b_o \right) = g \left( W_o h_{T+k-1}^{(k)} + b_o \right) = g \left( W_o h_{T}^{(k)} + b_o \right) = y_T,
$$

which completes the proof.

### B Lemma 1 proof

We show next that there exists an initialization vector that allows us to initialize the single-layer RNN as defined in Theorem 1.
Proof of Lemma 1. From the surjective definition of the activation function $f(\cdot)$, we know that the function $f(\cdot)$ is right-invertible. Namely, there is a function $r : D \to \mathbb{R}$ such that for any $d \in D$, $r(\cdot)$ satisfies $f(r(d)) = d$. First, we note that for $i = 1$, we have $\hat{h}_0^{(1)} = h_0^{(1)}$. When $i = 2$, we have
\[
\hat{h}_0^{(2)} = \hat{h}_1^{(2)} = f\left( W^{(2)}h_0^{(1)} + W^{(2)}h_0^{(2)} + b_h^{(2)} \right). \tag{B.8}
\]
From (B.8) and the right-invertible function $r(\cdot)$ satisfies $h_0^{(2)} = f\left( r(h_0^{(2)}) \right)$, we obtain
\[
\hat{h}_0^{(2)} = W_h^{(2)} r(h_0^{(2)}) + b_h^{(2)} \implies \hat{h}_0^{(2)} = W_h^{(2)} r(h_0^{(2)}) - W_x^{(2)} h_0^{(1)} - b_h^{(2)}, \tag{B.9}
\]
where $A^\dagger$ is the pseudoinverse of matrix $A$.

We assume that we obtained the initializations for $i - 1$ and compute the initialization for block $i$. In general, for block $i$ we have
\[
h_0^{(i)} = \hat{h}_0^{(i)} = f\left( W_x^{(i)} \hat{h}_{i-1}^{(i-1)} + W_h^{(i)} \hat{h}_{i-2}^{(i)} + b_h^{(i)} \right)
\]
We can plug in the initialization and the intermediate computed hidden states for block $i - 1$ to obtain
\[
\hat{h}_{i-2}^{(i)} = W_h^{(i)} r(h_0^{(i)}) - W_x^{(i)} \hat{h}_{i-1}^{(i-1)} - b_h^{(i)}.
\]
We continue to reapply the recursive formula one step at a time until we reach the last step before the initialization $\hat{h}_0^{(i)}$:
\[
\hat{h}_{i-j}^{(i)} = W_h^{(i)} r(\hat{h}_{i-j+1}^{(i)}) - W_x^{(i)} \hat{h}_{i-j+1}^{(i-1)} - b_h^{(i)}
\]
\[
\vdots
\]
\[
\hat{h}_1^{(i)} = f\left( W_x^{(i)} \hat{h}_1^{(i-1)} + W_h^{(i)} \hat{h}_0^{(i)} + b_h^{(i)} \right)
\implies \hat{h}_0^{(i)} = W_h^{(i)} r(\hat{h}_0^{(i)}) - W_x^{(i)} \hat{h}_1^{(i-1)} - b_h^{(i)}, \tag{B.10}
\]
Following these steps from $h_0^{(i)}$ to obtain $\hat{h}_0^{(i)}$, we constructed the initialization of the single-layer RNN to accurately mimic the initialization of the stacked RNN. This completes the proof of Lemma 1.

C Weight constraints and connections in flattened RNN

Figure 5 shows the weight constraints imposed to achieve equivalence between the stacked and single-layer RNNs, and a visualization of the weights as connections in the flattened RNN.
which is trained with the network. Word embeddings (64 dimensions). We build our own alphabets based on the most frequent 100 characters in the vocabularies. All the networks have a 100-dimensional character-level embedding, which is trained with the network.

Table 2: Parts-of-Speech results for German. The table shows all possible combinations of delays or bidirectional LSTM networks. The best forward-only network is marked in bold.

| Character-level network | Word-level network | Validation Accuracy | Test Accuracy |
|-------------------------|--------------------|---------------------|--------------|
| Bi-LSTM                 | Bi-LSTM            | 93.88 ± 0.13        | 93.15 ± 0.08 |
| Bi-LSTM                 | LSTM               | 92.00 ± 0.16        | 91.50 ± 0.05 |
| Bi-LSTM                 | d-LSTM with delay=1| 93.32 ± 0.23        | 92.81 ± 0.14 |
| Bi-LSTM                 | d-LSTM with delay=2| 93.15 ± 0.06        | 92.67 ± 0.08 |
| Bi-LSTM                 | d-LSTM with delay=3| 92.82 ± 0.14        | 92.25 ± 0.16 |
| Bi-LSTM                 | d-LSTM with delay=4| 92.41 ± 0.12        | 91.95 ± 0.17 |
| Bi-LSTM                 | d-LSTM with delay=5| 91.80 ± 0.11        | 91.57 ± 0.20 |
| LSTM                    | Bi-LSTM            | 93.96 ± 0.12        | 93.43 ± 0.07 |
| LSTM                    | LSTM               | 92.05 ± 0.16        | 91.58 ± 0.11 |
| LSTM                    | d-LSTM with delay=1| 93.46 ± 0.16        | 92.71 ± 0.11 |
| LSTM                    | d-LSTM with delay=2| 93.13 ± 0.10        | 92.61 ± 0.26 |
| LSTM                    | d-LSTM with delay=3| 92.91 ± 0.13        | 92.38 ± 0.15 |
| LSTM                    | d-LSTM with delay=4| 92.56 ± 0.17        | 92.06 ± 0.19 |
| d-LSTM with delay=1     | Bi-LSTM            | 93.93 ± 0.06        | 93.39 ± 0.18 |
| d-LSTM with delay=1     | LSTM               | 92.04 ± 0.11        | 91.58 ± 0.14 |
| d-LSTM with delay=1     | d-LSTM with delay=1| 93.48 ± 0.31        | 92.87 ± 0.24 |
| d-LSTM with delay=2     | d-LSTM with delay=2| 93.11 ± 0.18        | 92.54 ± 0.08 |
| d-LSTM with delay=3     | d-LSTM with delay=3| 92.85 ± 0.14        | 92.28 ± 0.19 |
| d-LSTM with delay=4     | d-LSTM with delay=4| 92.50 ± 0.12        | 92.11 ± 0.19 |
| d-LSTM with delay=3     | Bi-LSTM            | 94.00 ± 0.17        | 93.32 ± 0.18 |
| d-LSTM with delay=3     | LSTM               | 92.10 ± 0.24        | 91.61 ± 0.18 |
| d-LSTM with delay=3     | d-LSTM with delay=1| 93.29 ± 0.09        | 92.68 ± 0.09 |
| d-LSTM with delay=3     | d-LSTM with delay=2| 93.09 ± 0.21        | 92.59 ± 0.16 |
| d-LSTM with delay=3     | d-LSTM with delay=3| 92.86 ± 0.24        | 92.42 ± 0.16 |
| d-LSTM with delay=3     | d-LSTM with delay=4| 92.53 ± 0.17        | 92.08 ± 0.18 |
| d-LSTM with delay=5     | Bi-LSTM            | 93.88 ± 0.17        | 93.27 ± 0.06 |
| d-LSTM with delay=5     | LSTM               | 91.88 ± 0.18        | 91.54 ± 0.11 |
| d-LSTM with delay=5     | d-LSTM with delay=1| 93.31 ± 0.14        | 92.74 ± 0.10 |
| d-LSTM with delay=5     | d-LSTM with delay=2| 93.17 ± 0.13        | 92.57 ± 0.17 |
| d-LSTM with delay=5     | d-LSTM with delay=3| 92.84 ± 0.19        | 92.25 ± 0.10 |
| d-LSTM with delay=5     | d-LSTM with delay=4| 92.50 ± 0.22        | 91.96 ± 0.19 |

D Additional plots for error maps

Figure 6 presents the standard deviation diagrams for the error maps in Figure 4.

E Part-of-speech: additional details and results

In this section, we include more details about the dataset and the results of all the combinations for the Parts-Of-Speech experiment. We used treebanks from Universal Dependencies (UD) version 2.3. We selected the English EWT treebank [30, 1] (254,854 words), French GSD treebank [2] (411,465 words), and German GSD treebank [3] (297,836 words) based on the quality assigned by the UD authors. We follow the partitioning onto training, validation and test datasets as pre-defined in UD. All treebanks use the same POS tag set containing 17 tags. We use the Polyglot project [4] word embeddings (64 dimensions). We build our own alphabets based on the most frequent 100 characters in the vocabularies. All the networks have a 100-dimensional character-level embedding, which is trained with the network.

Results for German, English, and French can be found in Tables 2, 3, and 4 respectively. The best result that does not use a bidirectional network is marked in bold for each language.
Figure 6: Error maps presented in Figure 4 (left column) together with their standard deviation

(a) LSTM

(b) Bi-LSTM

(c) d-LSTM with delay=5

(d) d-LSTM with delay=10
Table 3: Parts-of-Speech results for English. The table shows all possible combinations of delays or bidirectional LSTM networks. The best forward-only network is marked in bold.

| Character-level network | Word-level network | Validation Accuracy | Test Accuracy |
|-------------------------|--------------------|---------------------|---------------|
| Bi-LSTM                 | Bi-LSTM            | 94.85 ± 0.05        | 94.84 ± 0.08  |
| Bi-LSTM                 | LSTM               | 91.90 ± 0.12        | 92.05 ± 0.09  |
| Bi-LSTM                 | d-LSTM with delay=1| 94.47 ± 0.06        | 94.41 ± 0.05  |
| Bi-LSTM                 | d-LSTM with delay=2| 94.17 ± 0.13        | 94.14 ± 0.10  |
| Bi-LSTM                 | d-LSTM with delay=3| 93.70 ± 0.07        | 93.87 ± 0.07  |
| Bi-LSTM                 | d-LSTM with delay=4| 93.11 ± 0.14        | 93.26 ± 0.08  |
| Bi-LSTM                 | d-LSTM with delay=5| 92.54 ± 0.16        | 92.70 ± 0.10  |
| LSTM                    | Bi-LSTM            | 95.03 ± 0.14        | 94.99 ± 0.15  |
| LSTM                    | LSTM               | 92.05 ± 0.13        | 92.14 ± 0.10  |
| LSTM                    | d-LSTM with delay=1| 94.53 ± 0.08        | 94.58 ± 0.11  |
| LSTM                    | d-LSTM with delay=2| 94.29 ± 0.05        | 94.28 ± 0.05  |
| LSTM                    | d-LSTM with delay=3| 93.81 ± 0.11        | 93.85 ± 0.12  |
| LSTM                    | d-LSTM with delay=4| 93.39 ± 0.12        | 93.55 ± 0.10  |
| d-LSTM with delay=1     | Bi-LSTM            | 94.94 ± 0.07        | 94.95 ± 0.06  |
| d-LSTM with delay=1     | LSTM               | 91.96 ± 0.16        | 92.09 ± 0.10  |
| d-LSTM with delay=1     | d-LSTM with delay=1| 94.57 ± 0.08        | **94.57 ± 0.14** |
| d-LSTM with delay=1     | d-LSTM with delay=2| 94.29 ± 0.12        | 94.37 ± 0.08  |
| d-LSTM with delay=1     | d-LSTM with delay=3| 93.86 ± 0.05        | 93.84 ± 0.10  |
| d-LSTM with delay=1     | d-LSTM with delay=4| 93.35 ± 0.10        | 93.56 ± 0.13  |
| d-LSTM with delay=3     | Bi-LSTM            | 94.98 ± 0.09        | 94.91 ± 0.10  |
| d-LSTM with delay=3     | LSTM               | 91.96 ± 0.08        | 92.08 ± 0.10  |
| d-LSTM with delay=3     | d-LSTM with delay=1| 94.47 ± 0.03        | 94.51 ± 0.10  |
| d-LSTM with delay=3     | d-LSTM with delay=2| 94.21 ± 0.05        | 94.18 ± 0.03  |
| d-LSTM with delay=3     | d-LSTM with delay=3| 93.80 ± 0.13        | 93.88 ± 0.13  |
| d-LSTM with delay=3     | d-LSTM with delay=4| 93.23 ± 0.13        | 93.38 ± 0.11  |
| d-LSTM with delay=5     | Bi-LSTM            | 94.90 ± 0.07        | 94.87 ± 0.09  |
| d-LSTM with delay=5     | LSTM               | 91.84 ± 0.11        | 91.98 ± 0.20  |
| d-LSTM with delay=5     | d-LSTM with delay=1| 94.36 ± 0.09        | 94.44 ± 0.08  |
| d-LSTM with delay=5     | d-LSTM with delay=2| 94.05 ± 0.07        | 94.19 ± 0.05  |
| d-LSTM with delay=5     | d-LSTM with delay=3| 93.61 ± 0.07        | 93.76 ± 0.05  |
| d-LSTM with delay=5     | d-LSTM with delay=4| 93.14 ± 0.04        | 93.27 ± 0.12  |
Table 4: Parts-of-Speech results for French. The table shows all possible combinations of delays or bidirectional LSTM networks. The best forward-only network is marked in bold.

| Character-level network | Word-level network | Validation Accuracy | Test Accuracy |
|-------------------------|--------------------|---------------------|---------------|
| Bi-LSTM                 | Bi-LSTM            | 97.63 ± 0.06        | 97.22 ± 0.11  |
| Bi-LSTM                 | LSTM               | 96.67 ± 0.05        | 96.15 ± 0.17  |
| Bi-LSTM                 | d-LSTM with delay=1| 97.48 ± 0.02        | 96.98 ± 0.05  |
| Bi-LSTM                 | d-LSTM with delay=2| 97.41 ± 0.02        | 96.91 ± 0.12  |
| Bi-LSTM                 | d-LSTM with delay=3| 97.31 ± 0.05        | 96.84 ± 0.09  |
| Bi-LSTM                 | d-LSTM with delay=4| 97.12 ± 0.05        | 96.61 ± 0.06  |
| Bi-LSTM                 | d-LSTM with delay=5| 96.88 ± 0.10        | 96.20 ± 0.14  |
| LSTM                    | Bi-LSTM            | 97.70 ± 0.07        | 97.19 ± 0.09  |
| LSTM                    | LSTM               | 96.67 ± 0.07        | 96.10 ± 0.11  |
| LSTM                    | d-LSTM with delay=1| 97.49 ± 0.07        | 97.03 ± 0.07  |
| LSTM                    | d-LSTM with delay=2| 97.49 ± 0.05        | 97.00 ± 0.06  |
| LSTM                    | d-LSTM with delay=3| 97.34 ± 0.04        | 96.89 ± 0.09  |
| LSTM                    | d-LSTM with delay=4| 97.16 ± 0.06        | 96.66 ± 0.15  |
| d-LSTM with delay=1     | Bi-LSTM            | 97.67 ± 0.07        | 97.23 ± 0.12  |
| d-LSTM with delay=1     | LSTM               | 96.66 ± 0.06        | 95.97 ± 0.07  |
| d-LSTM with delay=1     | d-LSTM with delay=1| 97.49 ± 0.04        | 97.04 ± 0.13  |
| d-LSTM with delay=1     | d-LSTM with delay=2| 97.43 ± 0.05        | 96.98 ± 0.05  |
| d-LSTM with delay=1     | d-LSTM with delay=3| 97.36 ± 0.08        | 96.80 ± 0.10  |
| d-LSTM with delay=1     | d-LSTM with delay=4| 97.22 ± 0.06        | 96.57 ± 0.10  |
| d-LSTM with delay=3     | Bi-LSTM            | 97.67 ± 0.08        | 97.21 ± 0.08  |
| d-LSTM with delay=3     | LSTM               | 96.67 ± 0.07        | 95.98 ± 0.14  |
| d-LSTM with delay=3     | d-LSTM with delay=1| 97.52 ± 0.04        | 97.02 ± 0.09  |
| d-LSTM with delay=3     | d-LSTM with delay=2| 97.44 ± 0.02        | 96.97 ± 0.12  |
| d-LSTM with delay=3     | d-LSTM with delay=3| 97.28 ± 0.04        | 96.74 ± 0.07  |
| d-LSTM with delay=3     | d-LSTM with delay=4| 97.13 ± 0.05        | 96.57 ± 0.09  |
| d-LSTM with delay=5     | Bi-LSTM            | 97.61 ± 0.03        | 97.12 ± 0.06  |
| d-LSTM with delay=5     | LSTM               | 96.64 ± 0.06        | 96.08 ± 0.08  |
| d-LSTM with delay=5     | d-LSTM with delay=1| 97.46 ± 0.02        | 96.96 ± 0.13  |
| d-LSTM with delay=5     | d-LSTM with delay=2| 97.41 ± 0.06        | 96.87 ± 0.06  |
| d-LSTM with delay=5     | d-LSTM with delay=3| 97.36 ± 0.05        | 96.82 ± 0.07  |
| d-LSTM with delay=5     | d-LSTM with delay=4| 97.15 ± 0.05        | 96.51 ± 0.07  |