Invited Comment

How many principles does it take to change a light bulb...into a laser?

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Abstract

Quantum optics did not, and could not, flourish without the laser. The present paper is not about the principles of laser construction, still less a history of how the laser was invented. Rather, it addresses the question: what are the fundamental features that distinguish laser light from thermal light? The obvious answer, 'laser light is coherent', is, I argue, so vague that it must be put aside at the start, albeit to revisit later. A more specific, quantum theoretic, version, 'laser light is in a coherent state', is simply wrong in this context: both laser light and thermal light can equally well be described by coherent states, with amplitudes that vary stochastically in space. Instead, my answer to the titular question is that four principles are needed: high directionality, monochromaticity, high brightness, and stable intensity. Combining the first three of these principles suffices to show, in a quantitative way—involving, indeed, very large dimensionless quantities (up to ~10^5)—that a laser must be constructed very differently from a light bulb. This quantitative analysis is quite simple, and is easily relatable to 'coherence', yet is not to be found in any textbooks on quantum optics to my knowledge. The fourth principle is the most subtle and, perhaps surprisingly, is the only one related to coherent states in the quantum optics sense: it implies that the description in terms of coherent states is the only simple description of a laser beam. Interestingly, this leads to the (not, as it turns out, entirely new) prediction that narrowly filtered laser beams are indistinguishable from similarly filtered thermal beams. I hope that other educators find this material useful; it may contain surprises even for researchers who have been in the field longer than I have.

Keywords: lasers, quantum optics, light source

1. Introduction

The importance of the laser to quantum optics, and to optical sciences and technologies more generally, can hardly be overstated. But what makes laser light special? More specifically, what differentiates it from thermal radiation, which was mankind’s only source of illumination for most of history? A one-word answer that springs to the mind of many is 'coherence'. Indeed, wikipedia [1] states

A laser differs from other sources of light in that it emits light coherently.

For those with a quantum optics education, a more technical answer may spring to mind: laser light is in a coherent state. This idea has long [2] been attributed to Glauber, although what he actually said in 1963 [3] (when the term 'laser' had yet to supplant 'optical maser') is much more nuanced:

The density operator which represents an actual maser beam is not yet known. It is clear that such a beam cannot be represented by ... [a] coherent state ...unless the phase and amplitude stability of the device is perfect. On the other hand, a maser beam is not at all likely...
to be described by ...[an] incoherent classical model .... More plausible models for a steady maser beam are much closer in behavior to the ideal coherent states.

In this pedagogical paper I maintain that the word ‘coherent’ has too many meanings to be a useful answer in itself, while the idea that a laser is in a coherent state is simply wrong in the context of trying to understand how laser light differs from thermal light. Instead, I will argue that there are four key features which distinguish laser light from thermal light.

This paper is based upon material I teach to advanced undergraduate students as a small part of an introductory course on quantum electrodynamics. In class I ask them the question ‘what makes laser light special?’ and invariably educe with little difficulty three of the four key features I have in mind:

(i) High directionality;
(ii) Monochromaticity;
(iii) High brightness.

Pleasingly, these three are also easily drawn out from the following statement on the official Year of Light home page [4].

A laser is an optical amplifier—a device that strengthens light waves. Some lasers have a well-directed, very bright beam with a very specific color; others emphasize different properties, such as extremely short pulses. The key feature is that the amplification makes light that is very well defined and reproducible, unlike ordinary light sources such as the Sun or a lamp.

Ignoring the clause covering lasers that are not CW, my first three key features of laser light are clearly stated here (albeit in a different order), as emphasized in italic font by me. I have also emphasized in the above quote ‘well defined’ because, as we will see, there is a property of laser light that is well-defined, and not covered by the above three features:

(iv) Stable intensity.

The above four criteria were already formulated as ‘four quantitative conditions that the output of a device must satisfy in order for the device to be considered a laser’ by me in 1997 [5]. What is new in the present paper is two-fold. First, I give a detailed and quantitative examination of how light from a light bulb fails to satisfy these criteria, and how, even using spatial and frequency filtering, it would be wildly impractical to obtain a beam satisfying the first three criteria, and impossible to create one satisfying all four. Second, I discuss why that which Glauber implied was unknown (in 1963)—an explicit expression for the density operator which represents an actual laser beam, written in a way comparable to that for the density operator for thermal radiation—is still unknown today. That is not to say that the state of a laser beam cannot be described; the quantum properties of a laser beam have been understood since the later 1960s (see the textbooks [6, 7] and references therein). But this is not the same as an explicit equation for the multimode density operator, as can be given for the case of thermal light. Interestingly, this difference has experimental consequences for the intensity fluctuations of frequency-filtered laser light, as I will discuss.

The remainder of this paper is organized as follows. I begin in section 2 by introducing some notation and identities for single-mode mixed states that will be useful subsequently. Then, in section 3, I consider the multimode mixed state describing emission from a light bulb. This leads to the discussion, in section 4, as to what could or could not be done to light-bulb light to give it the above four properties of laser light. In section 4.1 I consider collimation to achieve directionality, and calculate how hot the light-bulb filament would need to be to give a beam of the same power as a laser beam. In section 4.2 I consider spectral filtering to achieve (near) monochromaticity, and again calculate the required light-bulb temperature to reproduce a beam with the power and spectral properties of a laser. In section 4.3 I show how it is the beam brightness that results in the astronomically high temperatures calculated in the preceding sections, and also calculate what fraction of the radiated power would survive the required spatial and frequency filtering. The staggeringly small answer gives, I think, the best feel for how special laser light is. In section 4.4 I explain how, even if all of this were possible, it would still not give a beam satisfying the fourth criterion. While the difference is simple to state in the time (or longitudinal position) domain, the contrast is even more marked in the frequency (or wavenumber) domain: there is a simple expression for the quantum state of thermal light in terms of frequency components, but no such simple expression for the state of laser light. This last topic—the quantum state of a laser beam—does require something beyond undergraduate-level mathematics to understand, and so most of the analysis is presented in the appendix. I conclude in section 5 with a reconsideration of the notion of ‘coherence’, and a return to the Glauber quote from the opening paragraph.

2. Single mode mixed states

Two types of single-mode mixed state will play a role in the later sections of this paper. The first is a thermal state. For a harmonic oscillator, such as a single mode of the EM-field, in thermal equilibrium at temperature T, the mean number of excitations (photons) is

$$\bar{n}_T(\omega) = \frac{1}{\exp(\hbar \omega / k_B T) - 1}. \quad (1)$$

In terms of this, the state matrix, also known as a density operator, for a single-mode thermal (SMT) state can be written as

$$\rho_T(\bar{n}) = \frac{1}{1 + \bar{n}} \sum_{n=0}^{\infty} \left( \frac{\bar{n}}{\bar{n} + 1} \right)^n |n\rangle \langle n|. \quad (2)$$

The photon-number variance of the SMT state is $\bar{n}^2 + \bar{n}$. That is, the standard deviation in $n$ is always larger than the mean. The SMT state can also be expressed in terms of coherent
states as

$$\rho_{\text{SM}}(n) = \frac{1}{\pi n^2} \int d^2 \alpha \exp\left(-|\alpha|^2/n\right)|\alpha\rangle\langle\alpha|.$$  

The second type of single-mode mixed state is what I will call a single-mode laser (SML) state. This is meant to represent the state of the cavity mode for a laser in steady state. An ideal laser, far above threshold, will have a near-Poissonian photon number distribution, the same as a coherent state \([5-8]\). But the phase of the laser, relative to any other optical-frequency clock\(^1\) will be completely unpredictable. Even if the phase \(\phi\) were established at some time, it would quickly (on a human time-scale; often slowly on an atomic physics time-scale) become undefined, through a process of phase diffusion. The simplest model for this, yielding a laser with a Lorentzian spectrum, is a constant-diffusion Fokker–Planck equation for the phase probability distribution \([5, 6]\):

$$\frac{\partial}{\partial t} p(\phi) = \frac{1}{2} \left(\frac{\partial^2}{\partial \phi^2}\right) p(\phi),$$

where \(\Gamma\) is the linewidth, the reciprocal of its coherence time.

Thus if a SMT state is described by the mixture (3), a SML state is described by the mixture

$$\rho_{\text{laser}}(\mu) = \frac{1}{2\pi} \int d\phi \sqrt{\mu e^{i\phi}} \sqrt{\nu e^{i\nu}}.$$  

Here \(\sqrt{\nu e^{i\nu}}\) is a coherent state, with a mean photon number of \(\mu\). Equivalently, the SML can be written

$$\rho_{\text{laser}}(\mu) = \sum_{n=0}^{\infty} e^{-\mu} \frac{\mu^n}{n!} |n\rangle\langle n|.$$  

There is no more reason to say that a SML is ‘really’ in a coherent state than that a SMT state is. The crucial difference between them is that in a SML the standard deviation in the photon number is much less than the mean \(\mu\), as long as the latter is large (as it will typically be).

### 3. Light-bulb light

Light-bulb light is the sort of light that comes out an old-fashioned (incandescent) light bulb. If we use \(\kappa\) as an index for the modes—radiating in all directions, with all frequencies, and with both polarizations—populated by light from the light bulb, the total state of these modes will simply be a tensor product over all the modes:

$$\rho = \bigotimes_k \rho_{\text{ph}}(\bar{n}_{\text{ph}}(\omega_k)).$$

Here \(\bar{n}_{\text{ph}}(\omega_k)\) is as defined in equation (1), and \(\rho_{\text{ph}}\) is the SMT state of equation (2). This tensor-product form follows simply from the fact that the energy is additive over all these modes (each one is a harmonic oscillator).

\(^1\) See \([9]\) for arguments as to why a laser is as much a clock as anything can be at optical frequencies.

The Stefan–Boltzmann law says that a blackbody source of area \(A\), at temperature \(T\), has a radiative power of

$$P_{\text{total}} = \frac{\pi^2}{60} A (k_B T)^4.$$  

Wien’s displacement law says that \(\lambda_{\text{max}}\), the wavelength of maximum spectral power, is given by

$$\lambda_{\text{max}} = \frac{2\pi h c}{x k_B T} \approx 4.965.$$  

Combining these, one finds

$$A \approx 2.37 c^{-2} h^{-1} \lambda_{\text{max}}^4.$$  

We can use this to calculate the effective area of the radiating element of a light bulb, using the fact that the spectrum of a typical light bulb has its peak power in the infra-red, with \(\lambda_{\text{max}} \approx 1 \mu\text{m} \) micron. For a 60 W bulb this gives \(A \approx 15 (\text{mm})^2\), which seems reasonable. This will be used in section 4.3.

The rest of this paper is devoted to teasing out how the thermal light described here is different from laser light.

### 4. Changing light bulb light into laser light

#### 4.1. Collimated polarized thermal light

The first obvious property of laser light which contrasts with thermal light is that it is not isotropic. Rather, it propagates in a single direction, and is usually polarized. These properties could, in principle, be produced from a thermal source (a light bulb) by collimating it—that is, passing it through a series of finite apertures and lenses—and then passing it through a polarizer. This will lead, in the ideal limit, to a field with modes described by a single parameter, the ‘wave-number’ \(k > 0\) in the direction of propagation. Since the collimation does nothing but exclude some modes, the state of the collimated polarized field is still a tensor product of SMT states:

$$\rho = \bigotimes_k \rho_{\text{SM}}^k(\bar{n}_{\text{SM}}(c k)).$$

Because the light is collimated, the energy density (which scales as \((k_B T)^4/(hc)^3\)) for the field inside a black-body oven is not a sensible quantity to consider. The energy per unit volume can be changed simply by expanding or focussing the collimated beam. Rather, we should calculate the energy per unit length in the direction of propagation. Introducing a normalization length \(L\), this is given by

$$\left\langle \hat{H} \right\rangle/L = \frac{1}{L} \sum_k \hbar c \bar{n}_{\text{SM}}(c k) = \frac{\hbar c}{2\pi} \sum_k (\Delta k) k \bar{n}_{\text{SM}}(c k),$$

where \(\Delta k = 2\pi/L\) is the separation of the modes in \(k\)-space. In the limit \(L \to \infty\) this separation becomes infinitesimal and the sum can be converted to an integral, \(\int_0^\infty dk\). The result is \(\pi (k_B T)^2/(12hc)\).
Since the light is propagating in one direction, it is natural to convert this result into the power (energy per unit time) in the collimated beam:

\[
P_{\text{coll}} = \frac{\langle \hat{H} \rangle c}{L} = \frac{\pi \left( k_B T \right)^2}{12 \hbar}. \tag{13}
\]

This can now be easily compared to the power of the output of a laser, which is typically of order 100 milliW. Solving for the temperature, one finds \( T' = 4.6 \times 10^3 \) K. Here we see the reason for using \( T' \) rather than \( T \): the required temperature is very different from that of an actual light bulb, \( T \approx 3000 \) K. Indeed, to replicate the laser power, our ‘light bulb’ would have to be much hotter than the surface of the Sun, and indeed hotter than the surface of the hottest known stars (newly formed white dwarfs) [10].

I ask students to perform the above calculation, and then ask them in class what conclusions they draw from this. The answer is usually readily forthcoming: that a half-million-degree light bulb is not what is actually hiding inside a typical laser, so a laser must be different from a thermal source.

### 4.2. Monochromatic collimated polarized thermal light

As well as being unidirectional and polarized, laser light is close to monochromatic. That is, almost all of the power is in a narrow frequency band. This property could be achieved from collimated polarized thermal light by passing it through filters of increasingly narrow frequency resolution (such as a refracting prism, then a diffraction grating, then a series of Fabry–Perot etalons). The overall transmission can be described by a filter function \( 0 \leq f(\omega) \leq 1 \). To mimic the frequency spread of a typical laser, we can take the filter function to be Lorentzian, with

\[
f(\omega) = \frac{(\Gamma/2)^2}{(\Gamma/2)^2 + (\omega - \omega_0)^2}, \tag{14}
\]

where \( \omega_0 \) is the mean frequency and \( \Gamma \ll \omega_0 \) is the full width at half maximum height linewidth. Note that for a laser, this \( \Gamma \) is the phase diffusion rate mentioned in section 2. Since this filtering is a passive process, the state of each mode \( k \) remains a SMT state, just with a modified mean occupation number

\[
\bar{n}_f(\omega) = \bar{n}_R(\omega)f(\omega) \approx \nu f(\omega), \tag{15}
\]

where \( \nu \equiv \bar{n}_R(\omega_0) \). Here the approximation holds in the frequency band of interest—around \( \omega_0 \)—although not for very low (radio) frequencies \( \lesssim \Gamma \). In the optical band, the state of the filtered light can thus be written as

\[
\rho = \bigotimes_k \rho_R^{\text{filt}}(\nu f(\omega)). \tag{16}
\]

The power in this filtered collimated polarized thermal light is

\[
\left\langle \hat{H} \right\rangle c = \frac{P_{\text{coll}}}{\frac{L}{c} \sum_k k \hbar \bar{n}_f(ck)}. \tag{17}
\]

In the limit \( \Gamma \ll \omega_0 \) this can be evaluated as

\[
P_{\text{filt}} = \nu \hbar \omega_0 \Gamma /4. \tag{18}
\]

For high temperatures (\( k_B T \gg \hbar \omega_0 \)), which, unsurprisingly, will be the relevant limit) the expression for \( \nu = \bar{n}_R(\omega_0) \) simplifies to give

\[
P_{\text{filt}} = k_B T'' \Gamma /4. \tag{19}
\]

I use \( T'' \) to emphasize that this is a different temperature both from an actual light-bulb temperature \( T \) and from that required in the preceding section, when considering only collimation, \( T' \). Let us compare with a laser of moderate quality, with an output power of order 100 milliW (as above) and a linewidth \( \Gamma \) of order \( 10^3 \) s\(^{-1}\) (1.6 MHz). This time, the required temperature is \( T'' = 2.9 \times 10^{15} \) K. This is far higher than any temperature that has been produced on the Earth, and corresponds to that of the Universe when it was less than \( 10^{-12} \) s old. This time the message is even more emphatic: a laser is profoundly different from a thermal source.

### 4.3. Bright Monochromatic collimated polarized thermal light

The reason such astronomically high temperatures are required for a hypothetical thermal source behind a laser beam is that so much of the original radiation would have to be discarded to obtain a beam with the desired properties. One can learn more by examining just how much light must be thrown away.

First, consider the collimation process. Using equations (8), (9), and (13), one can show that, by numerical coincidence, the proportion of power not discarded in the necessary collimation process is extremely well approximated by a simple ratio:

\[
\frac{P_{\text{coll}}}{P_{\text{total}}} \approx \frac{\lambda'_\max^2}{\lambda''_\max} \frac{\Gamma}{A}. \tag{20}
\]

Here \( \lambda'_\max \) is the peak wavelength corresponding to temperature \( T' \) defined in equation (13). This scaling is easily understood from elementary transverse coherence theory. If one considers a large sphere of radius \( R \), concentric with a source of size \( r \) that produces light of wavelength \( \lambda \), the coherence length on the surface of the large sphere scales as \( R \lambda / r \) [11]. Hence the number of transversely coherent modes in the far field scales as \( (\lambda / r)^2 \), so discarding all but one gives the type of ratio appearing in equation (20). For the parameters used in section 4.1, and using \( A = 15 \) (mm\(^2\)) from section 3, equation (20) evaluates to \( 2.6 \times 10^{-12} \).

Next, consider adding filtering. This time, we will ignore factors of order unity, for simplicity. Then, using equations (8), (9), and (18), we find

\[
\frac{P_{\text{filt}}}{P_{\text{total}}} \sim \frac{\lambda''_\max^2}{\lambda'_\max^2} \frac{\Gamma}{A} \frac{\omega''_\max}{\omega'_0}. \tag{21}
\]

Here \( \lambda''_\max \) and \( \omega''_\max \) refer to the temperature \( T'' \) defined in equation (18). For the parameters in section 4.2, the first (collimation) factor evaluates to something far smaller even than that in the preceding paragraph, this time of order \( 10^{-31} \), while the second (frequency filtering) factor evaluates to \( \sim 10^{-20} \). Thus equation (21) evaluates to \( \sim 10^{-51} \), a dimensionless number whose reciprocal could truly be described as astronomical.
Equation (21) involves the parameter $\omega''_{\text{max}}$ (or $\lambda''_{\text{max}}$) which has no simple interpretation in terms of the properties of the laser beam, unlike $\omega_0 = 2\pi c/\lambda_0$, the actual central frequency of the laser. However, using the (appropriate) high-temperature limit $k_B T = \nu/\omega_0$, one can rewrite equation (21) as

$$\frac{P_{\text{eff}}}{P_{\text{total}}} \sim \frac{2}{A \omega_0 \nu^3}. \quad (22)$$

Despite initial appearances, this, like equation (21), is independent of $\omega_0$. To analyse the contribution of each term we must choose a value of $\lambda_0$. Let us take $\lambda_0 = 1 \mu m$, the same as $\lambda_{\text{max}}$ for the original (realistic) light bulb of section 3, and a pretty representative figure for lasers. The first fraction in equation (22) is the geometric factor one might naively expect from collimation, as per the argument in the first paragraph of this section. For the above parameters, it evaluates to $\sim 10^{-7}$. The second fraction is a factor one might naively expect from filtering. For the above parameters, it evaluates to $\sim 10^{-8}$. Thus, the greatest contribution to the overall ‘efficiency’ of $\sim 10^{-51}$ is from the third fraction. For the above parameters, $\nu \sim 10^{12}$, so that $\nu^{-3} \sim 10^{-36}$, as required. The reason this factor appears is that, in order to achieve the actual brightness of a laser beam, it is necessary to start with an extremely hot source, so that $\lambda''_{\text{max}} < \lambda_0$ and $\omega''_{\text{max}} \gg \omega_0$.

One might object that equation (22) contains the parameter $\nu$ which has not been given any simple interpretation. But in fact it does have one. From this expression

$$\nu = \frac{P}{h\omega_0 \Gamma} \frac{4}{A} \quad (23)$$

one can see that $\nu$ is the number of photons per unit time, multiplied by the coherence time of the beam (ignoring constants of order unity). That is, $\nu$ is roughly the number of photons that come out coherent with one another, before the phase of the beam randomly shifts to some other value. Of course strictly it makes no sense to talk of the phase of a beam made of single photons, or containing a fixed number of photons, but hopefully the preceding sentence conveys meaning intuitively. Thus we see that $\nu$ is a natural dimensionless way to quantify the intensity or brightness of the beam, just as $\Gamma/\omega_0$ is the natural way to quantify its monochromaticity. And it is the fact that $\nu \gg 1$ for typical laser parameters that makes it astronomically impractical to have a very poorly defined photon number, while the latter can have a very well-defined photon number.

In section 4.3, I talked about the coherence time in terms of the random variation of the phase. In fact, a filtered collimated polarized thermal beam would have huge intensity fluctuations as well as phase fluctuations. They would both occur on the time scale of the coherence time, $\Gamma^{-1}$, and in fact both would contribute equally to the decay of coherence. For those familiar with stochastic calculus, and quantum optics, it is possible to be more specific. The state (16) is equivalent to a spatially stochastic coherent state. That is, it is an ensemble where each member is an eigenstate of all the annihilation operators $\{a_k\}$, with eigenvalues $\{\alpha_k\}$ (coherent-state amplitudes) that differ in different member of the ensemble. Specifically, a randomly drawn member of the coherent-state ensemble representing (16) can be generated by choosing each $\alpha_k$ as an independent random variable from the probability distribution $[\pi\hbar^2/(\omega_k)]^{-1} \exp[-|\alpha_k|^2/\hbar^2(\omega_k)] d^2\alpha_k$. An alternative way of expressing this is by converting (via the Fourier transform) from $k$-space to $x$-space (longitudinal position), yielding, for any given ensemble member, a coherent-state amplitude $\alpha(x)$. In contrast to $k$-space, the different $\alpha(x)$ coherent-state amplitudes are not independent random variables. Rather, as stated above, they are correlated in both amplitude and phase on a time scale of $\Gamma^{-1}$. Specifically, a randomly drawn $\alpha(x)$, for all $x$, can be generated with the correct statistics by this equation:

$$\alpha(x) = e^{-ik_0x} e^{\int_{-\infty}^{\infty} (\Gamma/2)(t-t')^2 \zeta(t')} dt \int_{-\infty}^{\infty} (\Gamma/2)(t-t')^2 \zeta(t'). \quad (24)$$

Here $\zeta(t)$ is a complex white noise process [15], satisfying $E[\{\zeta(t)\}^n(t')] = \delta(t-t')$, with all other first and second-order moments vanishing. The coherent-state amplitude $\alpha(x)$ is normalized so that $E[|\alpha(x)|^2]$ is the photon flux (mean number of photons per unit time). This evaluates, as it should, to $\nu \Gamma/4$. The direction of propagation has been taken to be in the negative $x$ direction; a particular member of the ensemble is not a stationary state, but rather changes in time by propagation at the speed of light so that $\alpha(x) = \alpha_{x+y}(x + ct)$. The quantum state $\rho$ of the whole ensemble is of course stationary, because the statistics of equation (24) are stationary (invariant under displacements of $x$).

A laser beam can also be described as an ensemble of coherent states, which can again be thought of as a state with a stochastically varying coherent-state amplitude (varying in space at any particular time, or in time as it passes by any particular point in space). By contrast with a thermal-derived beam, however, a laser beam has an essentially fixed intensity. Ideally it has only Poissonian fluctuations in the number of photons in any given time interval. Specifically, in the limit where the state of the laser cavity mode is given by equation (6), and its phase fluctuations are described by equation (4), the laser beam has a coherent-state amplitude that varies stochastically with position $x$ as

$$\alpha(x) = e^{-ik_0x} \sqrt{\frac{\nu \Gamma}{4}} \exp \left[ i \int_{-\infty}^{\infty} dt \sqrt{\Gamma} \xi(t) \right]. \quad (25)$$

Here $\xi(t)$ is a real white noise process, satisfying $E[\xi(t)\xi(t')] = \delta(t-t')$. That is, the amplitude (by which
I mean the modulus of the coherent-state amplitude) is constant, but the phase is stochastic. Although this is very different from the stochasticity in equation (24), they have the same power spectrum

\[ S(\omega) = \int_{-\infty}^{\infty} dt \ e^{i\omega t} E[\alpha^d(x + c t)\alpha(x)] = \nu f(\omega), \tag{26} \]

which, as defined here, is the photon flux per unit frequency, a dimensionless quantity. Recall that the filter function \( f(\omega) \) as defined in equation (14) is dimensionless, as is \( \nu \gg 1 \), the number of photons per coherence time.

The fact that the stochastic equation (24) is equivalent to a very simple explicit expression, equation (16), for the quantum state \( \rho \) of thermal-derived beam in terms of single-mode thermal states tempts one to assume the analogous relation for a laser beam. That is, for physicists sufficiently well read to know that a laser cavity mode is not in a coherent state of fixed phase, but rather in a state like that of equation (5), nothing is more obvious than the following guess:

\[ p_{\text{laser}} = \bigotimes_k p^k_{\text{laser}}(\nu f(ck)) \tag{27} \]

for the quantum state of a laser in \( k \)-space. If this were true then further filtering within the laser spectrum would produce a state of the same form, just with a narrower spectral function \( f(\omega) \), as is the case for the thermal-derived beam (16).

I succumbed to the above temptation many years ago, and only the finalization of the current paper in the last week before the submission deadline (16th September 2015), has taught me that equation (27) is false. Not only is equation (27) not equivalent to equation (25); it does not describe a CW beam at all. That is, it does not describe a beam of indefinite length, such that the statistical properties of the beam are independent of its length and invariant under translations along it. Moreover, a laser beam does not have the property that filtering within the laser spectrum would produce a state of the same form. Rather, this extra filtering would produce a state with a narrower spectrum but with extra intensity noise. In particular, in the limit of a very narrow filter, the state produced would have exactly the same properties as the thermal-derived beam (16). That is, it would have huge intensity fluctuations, with standard deviation the same size as the mean. The proofs of all these non-obvious statements are given in the (late written) appendix.

One might wonder whether the above difference between thermal-derived light and laser light is a big enough deal to list a stable intensity as the final key feature of laser light. After all, if a laser has large (\( \sim \pi \)) phase fluctuations on a time scale \( \Gamma^{-1} \), why would having large (of order the mean) intensity fluctuations on the same time scale make the light qualitatively worse? For the purposes of cutting through a thin sheet of metal with an ‘industrial strength’ laser, intensity fluctuations would presumably make no difference. The characteristic time of the cutting process is (I presume) much greater than \( \Gamma^{-1} \), so the intensity fluctuations would average out. But in this example even the narrow linewidth of a laser is perhaps not relevant; only the high photon flux and single transverse mode necessary for tight focussing would be.

However the situation is certainly different for scientific applications such as atomic physics experiments. There the system (atoms) typically have a coherence time much shorter than \( \Gamma^{-1} \). The atomic coherence time upper bounds the effective time of a single ‘shot’ of the experiment (e.g. a single photon emission from an atom). Thus the laser phase (and intensity) is effectively constant over a single shot. Moreover, because atomic processes, in continuous-wave experiments, are insensitive to the absolute phase of light, effectively the same experiment is performed in each independent shot. But to gather good statistics, such an experiment is typically performed over a total time of order \( \Gamma^{-1} \) or longer. Thus if the intensity of the beam were to be fluctuating over that time-scale, this would add a great deal of noise to the process being investigated, because atomic processes are certainly not independent of the intensity of light. The stable intensity of a laser enables the experimenter to perform essentially the same deterministic operation on the atoms in every shot.

Equation (25) implies that the photon statistics of the beam are Poissonian. This means that in any time interval, the uncertainty in the number of photons in the beam is equal to the mean number: \( \delta N = \sqrt{\bar{N}} \). Of course this is not exactly true for real systems, and for very long time scales (very low frequencies) there will always be fluctuations due to technical noise. But from arbitrarily short times up to times much greater than the coherence time \( \Gamma^{-1} \), a good laser will have \( \delta N = O(\sqrt{\bar{N}}) \), which is much smaller than the mean if the latter is large. Even over longer times, where technical noise dominates, it is still the case that \( \delta N \ll \bar{N} \). This is to be contrasted with a collimated and filtered thermal source. In that case, if one looks at a time interval of order \( \Gamma^{-1} \) or shorter, one would see the super-Poissonian statistics of a SMT state (2), in which \( \delta N \gg \bar{N} \) — fluctuations that are not small relative to the mean.

5. Conclusion

We have seen that there are four key differences between laser light (in the CW regime for simplicity) and light-bulb light. To be quantitative, we can consider typical devices as above: a laser with power of 100 mW, a linewidth \( \Gamma \) of \( 10^7 \) s\(^{-1} \), and a wavelength \( \lambda_0 \) of 1 \( \mu \)m; and a light bulb with filament area \( A = 15 \) mm\(^2 \), and a peak-spectral wavelength \( \lambda_{\text{max}} \) of 1 \( \mu \)m. Thus:

(i) Laser light is polarized and has a single transverse mode; light-bulb light is unpolarized and is emitted into something like \( A/\lambda_{\text{max}}^2 \sim 10^7 \) transverse modes.
(ii) Laser light is monochromatic, with $\Gamma/\omega_0 \sim 10^{-8}$; light-
bull light is broad-spectrum with $\delta \omega \sim \omega_{\text{max}}$.

(iii) Laser light is intense, with $\nu \sim 10^{12}$ photons per
cohere time; light-bull light has
\[ \bar{n}_\text{b} \approx 1/\chi \approx 0.2 \text{ photons per spatio-temporal}
\text{mode at spectral peak.} \]

(iv) Laser light has a stable intensity, with photocount
uncertainty $\delta N \ll \bar{N}$ over time intervals long enough
that $\bar{N} \gg 1$; light-bull light, if collimated and filtered,
would have a photocount uncertainty $\delta N$ larger than $\bar{N}$
for time intervals $\leq \Gamma^{-1}$.

None of the above principles state the difference in terms
of laser light being coherent, but all of them can be regarded
as aspects of coherence. The first two are purely classical
aspects of coherence: (i) complete transverse coherence; (ii)
very high longitudinal coherence. The third is quantum, in
that it involves photon number per coherence time, a quantity
that is not defined classically. Stated loosely as the require-
mong of having ‘many photons coherent with one another’,
it is clearly an aspect of coherence. The fact that $\nu \gg 1$ is the
reason it is possible to interfere light from two independent
lasers, as in [12], and see an interference pattern emerge in a
time $\ll \Gamma^{-1}$—before the phase difference between the lasers
diffuses to a different random value—. The fourth principle is
also quantum, and could be argued to be ‘more quantum’ in
that it requires considering photon-number fluctuations as
well as the mean. From his first paper on the topic, quoted
above, Glauber regards any light from a thermal source as
incoherent, even allowing for ‘collimated, completely inco-
herent beams’ and ‘incoherent beams of exceedingly narrow
bandwidth’ [3]. For Glauber it is (near) Poissonian statistics
that distinguishes a coherent beam from an incoherent one.

It is worth returning to the opening of the quote from
Glauber in section 1, this time in more complete form:

The density operator which represents an
actual maser beam is not yet known. It is clear
that such a beam cannot be represented by a
product of individual coherent states,
$\bigotimes_k |\alpha_k\rangle \langle \alpha_k|$, unless the phase and amplitude
stability of the device is perfect. On the other
hand, a maser beam is not at all likely to be
described by ...[an] incoherent classical model
... More plausible models for a steady maser
beam are much closer in behavior to the ideal
coherent states.

The second sentence, in which $k$ has the same meaning
as in the rest of this paper, is undoubtedly true. But while perfect
amplitude stability (that is, a Poissonian distribution of pho-
ton number with a time-constant mean) is a harmless idealiz-
ation, perfect phase stability is not. It corresponds to
assuming a value of zero for the linewidth $\Gamma$. But the fact that
a laser (or maser) has a minimum value for $\Gamma$ set by quantum
fluctuations [13] is arguably the most fundamental result of
laser theory, discovered by Schawlow and Townes five years
before Glauber’s paper [14]. This implies that a CW laser
beam is not in a coherent state (or any other pure state), so one
might wonder whether Glauber’s terminological contrast of
something ‘much closer ...to the ideal coherent states’ with an
‘incoherent classical model’ holds up to scrutiny.

By an ‘incoherent classical model’ Glauber means what
he called earlier an ‘incoherent light beam’, defined (using my
notation) as $\rho = \bigotimes_k \rho^{\text{in}}_k (\nu_k)$. This is exactly the same as
equation (16), but allowing for an arbitrary distribution of
mean occupation numbers $\nu_k$ rather than my $\nu_k = \nu_f (ck)$. This
class of states is, in fact (see appendix A.5), the only
class that are mixtures of coherent states, that describe CW
beams, and that can be written as a tensor product over differ-
ent $k$-modes. A single realization of a CW laser beam can
certainly be formally written as a tensor product of coherent
states over different $k$-modes, as in Glauber’s expression
$\bigotimes_k |\alpha_k\rangle \langle \alpha_k|$, simply by writing the Fourier transform of the
stochastically varying in space coherent-state amplitude $\alpha(x)
$ of equation (25). However (see appendix A.1), there seems no
to way to evaluate the ensemble average to obtain the quantum
state $\rho$ for a laser—one has to make do with the infinite
ensemble of coherent-state realizations itself. Thus, even
though the idea that laser light is in a coherent state, wrongly
attributed to Glauber, is indeed wrong, Glauber’s distinction
—between incoherent light, for which we can easily write an
explicit expression for $\rho$ without using the notion of coherent
states, and a ‘steady maser beam’, for which there is no good
option but to use coherent states in our description—is still a
relevant one.

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Appendix. Quantum states of CW beams

A.1. The quantum state of a laser beam

The simple quantum theory of a laser, pumped far above
threshold, predicts a state of the cavity mode with the same
(Poissonian) number statistics as a coherent state [5–8, 13]. It
can be modelled as a coherent state with fixed amplitude $|\alpha|$ and a phase $\phi$ undergoing diffusion, as per equation (4).

Using stochastic calculus [15], we can write the phase as a temporally stochastic variable

$$\phi(t) = \sqrt{\Gamma} W(t),$$

(A.1)

where $W(t)$ is a Wiener process, obeying $W(t) = \xi(t)$ with initial condition $W(0) = \phi(0)/\sqrt{\Gamma}$. That is, the state of the cavity mode can be modelled as a single-mode coherent state that varies stochastically in time,

$$|\sqrt{\mu} e^{i\phi(t)} W(t)\rangle.$$  

(A.2)

Passive optics transforms coherent states into coherent states. Thus with the standard cavity output coupling, corresponding to an intensity decay rate for a ‘cold’ (unpumped) cavity of $\kappa$, the output beam can also be described by a tensor product of stochastic coherent states:

$$|\psi\rangle = \bigotimes_j |\sqrt{\mu} e^{i\phi(t)} W(t)\rangle \bigotimes_j |\sqrt{\kappa}\Delta t/c\rangle^T.$$  

(A.3)

Here the $T$ superscript indicates that I am using ‘temporal modes’ of duration $\Delta t = t_{j+1} - t_j$ so that $\mu\kappa\Delta t$ is the mean number of photons in each such mode. This duration is chosen so that $\omega_0^{-1} \ll \Delta t \ll \Gamma^{-1}$, so that it is infinitesimal on the scale of the stochastic evolution but long enough that we can still use the standard (rotating-wave) treatment for the coupling of the cavity mode to the output modes. The time variable here refers to the time at which each bit of the output field is generated by this coupling, after which it propagates away at the speed of light. Thus we can equally well (in fact, with greater clarity) describe the entire output field by noting that each output mode is a beam of stochastic spatial variation in its phase:

$$|\psi\rangle = \bigotimes_j \left| \sqrt{\mu} e^{i\phi(t)} W(t) e^{|\kappa\Delta t/c| T} \right\rangle.$$  

(A.4)

Here, for mathematical simplicity in converting from time to space, $x = 0$ is the point farthest from the laser, and $x = L$ the point at the laser’s output mirror.

The next step is to convert to $k$-space, the reciprocal variable to $x$. Since we have a beam of length $L$, $k$ is a discrete variable, with $k_{j+1} - k_j = \Delta k = 2\pi/L$ as in section 4.1. A tensor product of coherent states remains so under any change of mode-basis, and here we have:

$$|\psi\rangle = \bigotimes_j \left| \sum_{l} \Delta x \sqrt{\mu} e^{-i\phi(T - \Delta t L)} e^{i\Delta k/l} \right\rangle^K.$$  

(A.5)

Taking the limit $\Delta x \to 0$, this becomes, with $k = (2\pi/L)l$ implicit

$$|\psi\rangle = \bigotimes_l \left| \frac{1}{L} \int_0^L \mathrm{d}x \ e^{ikx} e^{i\phi(T - \Delta t L)} \right\rangle^K,$$  

(A.6)

where I have defined

$$u(t) = \sqrt{\mu \kappa / c} e^{-i\phi(t)/\sqrt{\Gamma}}.$$  

(A.7)

Note the equivalence to equation (25); prior to this point the calculation has been in a frame rotating at the laser frequency $\omega_0$, but in equation (A.7) I have changed to a non-rotating frame by explicitly introducing the oscillation frequency.

Now the overall phase of $u(t)$ is uniformly random, so any explicit expression for the quantum state of the laser beam would have no coherences between states with different numbers of photons. Thus to try to obtain an explicit expression, like equation (16), we should change to the number basis, and consider the projector

$$|\psi\rangle \langle \psi| = \bigotimes_l \sum_{n, n'} |n\rangle^n \langle n'| \exp\left[-i\sum_{l} \hat{a}_l(k) \hat{a}^\dagger_l(k) \right] \frac{1}{\sqrt{n!}} \frac{1}{\sqrt{n'+1}}.$$  

(A.8)

where

$$\hat{a}_l(k) = \frac{1}{\sqrt{L}} \int_0^L \mathrm{d}x \ e^{ikx} u(x/c).$$  

(A.9)

Now the quantum state $\rho$ is an ensemble average over all possible realizations: $\rho = E_W[|\psi\rangle \langle \psi|]$. To determine this it would be necessary to evaluate ensemble averages of products of $\hat{a}_l(k)$ and $\hat{a}^\dagger_l(k)$ raised to arbitrary non-negative integer powers, involving simultaneously all the possible values of $k = 2\pi L$. There is one simplification: as noted above, the overall phase of $u(t)$ is uniformly random, since $\sqrt{\Gamma} W(0) = \phi(0)$ is uniformly random, and independent of $W(t) - W(0)$. Thus the ensemble average will be zero unless we can pair every $\hat{a}_l(k)$ with a $\hat{a}^\dagger_l(k')$. As the simplest example

$$E[\hat{a}_l(k) \hat{a}^\dagger_{l'}(k')] = \frac{\mu \kappa}{c} \left( \Gamma / 2c^2 \right) \left( k - k_0 \right)^2,$$  

(A.10)

$$\Gamma / (2c^2) + (k - k_0)^2, = \nu f(\kappa k),$$  

(A.11)

where $f$ is the Lorentzian filter function in equation (14) and $\nu = \kappa \mu (4/\Gamma)$ is the number of photons per coherence time, as in the body of this paper. The agrees with equation (26), as expected from the Wiener–Khintchin theorem [15].

However, the calculation of the expectation value of more general products seems overwhelming. Preliminary explorations suggests that the number of (non-stochastic) integrals to be evaluated in the expectation value of a given term grows exponentially with the sum of the powers of the $\hat{a}$ and $\hat{a}^\dagger$s in that term. There are terms with all possible powers, and, more to the point, a laser beam is a bright beam, with of order $\nu = 10^3$ photons on average in each $k$-mode. Thus no small-amplitude approximation, limiting the size of products that need be considered, is possible.

Perhaps there is a clever trick that would enable $\rho = E_W[|\psi\rangle \langle \psi|]$ to be evaluated, but at the present the situation is the same as it was in 1963 [3] with regard to the lack of an explicit expression for the quantum state of a laser beam.

## A.2. The quantum state of a thermal-derived beam

In this subsection I show that, despite starting with the same type of equation (a stochastically spatially varying coherent
state) it is possible to derive an exact expression for the quantum state of a beam if the stochastic coherent-state amplitude is as given in equation (24). That is

\[ u(t) = e^{-i\omega t} \sqrt{\nu/c} \int_{-\infty}^{\infty} ds \left( \frac{\Gamma}{2} \right)^{1/2} (t-s) \zeta(s), \quad (A.12) \]

which is a Gaussian random variable (at each \( t \)). Its Fourier transform

\[ \tilde{u}_k(t) = \frac{1}{\sqrt{L}} \int_0^L dx \ e^{ikx} u(x/c), \quad (A.13) \]

is also a set (indexed by \( k = 2\pi l/L \)) of Gaussian random variables, with mean zero, each with variance given by equation (A.10). In addition, in the limit of large \( L \), these variables are independent:

\[ E[\tilde{u}_k(t)\tilde{u}_k^*(t')] = \delta_{k,k'} \nu f(k). \quad (A.14) \]

Thus each \( k \)-mode is an independent Gaussian mixture of coherent states with uniformly random phase, and mean photon number \( \nu f(k) \). That is, the quantum state of the beam is that of a collimated, polarized, filtered thermal beam (16).

A.3. The non-convergence of the periodogram to the spectrum, and filtering

The result (16) could be derived in the preceding section because of the exactly Gaussian statistics of the coherent field, in either \( x \)- or \( k \)-space. However, Gaussian statistics in the \( k \)-space are actually generic in the following sense [16]: if \( u(t) \) is a stationary ergodic process, then the limit as \( L \to \infty \) of \( \tilde{u}_k(t) \) is a complex Gaussian random variable of uniformly random phase, whose statistics are thus completely defined by its second moment \( \nu \equiv \lim_{L \to \infty} E[|\tilde{u}_k(t)|^2] \).

This result means that, contrary to what may be found in at least one textbook on stochastic methods widely used by physicists [17], the periodogram does not converge to the spectrum, even for ergodic systems. The periodogram is a common way to approximate the spectrum of a stationary stochastic process, by taking a long time series of the process, Fourier transforming it, and squaring its modulus. But even in the limit of an infinite time series this does not yield the spectrum:

\[ \lim_{L \to \infty} \nu \tilde{u}_k(t)^2 = S(ck) \equiv \lim_{L \to \infty} E[|\tilde{u}_k(t)|^2]. \quad (A.15) \]

Rather, since \( \tilde{u}_k(t) \) is, asymptotically, Gaussian, the periodogram actually has an exponential distribution, with a mean, and standard deviation, equal to the spectrum.

A consequence of the above is that if one were to filter out a single frequency component from any indefinitely long sample of a stationary stochastic coherent optical field, the result would have the same statistics as a SMT state. From

\[ \text{To obtain this it is necessary to define } u(t) \text{ to have period } L, \text{ since } k = 2\pi l/L, l \in \mathbb{Z}. \text{ This can be done for a stochastic process by selecting, from the ensemble of all possible realizations, the subensemble (of measure zero, but still continuously infinite) with } u(L/c) = u(0). \text{ As long as } L \gg c/\Gamma, \text{ the local statistical correlations of } u(t) \text{ everywhere in the interval } [0, L/c] \text{ are almost unchanged.} \]

shot to shot the measured intensity would differ, according to an exponential distribution. Of course in reality there is no way to measure a single frequency component from an infinitely long field. However, my presumption is that the above results would hold approximately for a sufficiently narrow filter, with width \( \delta \omega \) much smaller than the spectral width of the source. In this case, the intensity fluctuations ‘from shot to shot’ in the filtered field would occur on a time scale of order \( (\delta \omega)^{-1} \).

In particular, by passing an ideal laser beam through an optical frequency filter with \( (\delta \omega) \ll \Gamma \), the intensity fluctuations would be greatly increased, from near-Poissonian, with second-order intensity autocorrelation function \( g^{(2)}(\tau) \approx 1 \) for all \( \tau \), to massively super-Poissonian, with \( g^{(2)}(\tau) \) approaching the thermal value of 2 for \( \tau \ll (\delta \omega)^{-1} \). A real laser may have more complicated stochastic phase dynamics than simple diffusion at rate \( \Gamma \). It may suffer from ‘technical noise’ such as frequency jitter, equivalent to a narrowband (\( \Gamma \)) laser whose central frequency wanders around, on a much slower time scale, in a band \( \Delta \omega \gg \Gamma \). In this case, as the bandwidth \( \delta \omega \) of the filter cavity is smoothly reduced I would expect the intensity fluctuations to vary as:

- near shot-noise, \( g^{(2)} \approx 1 \), for \( \delta \omega \gg \Delta \omega \),
- well above shot noise, \( g^{(2)} > 1 \) for \( \delta \omega \sim \Delta \omega \),
- enormous fluctuations (off for most of the time, with bursts of transmission), \( g^{(2)} \gg 1 \), for \( \Gamma \ll \delta \omega \ll \Delta \omega \),
- thermal-like fluctuations, \( g^{(2)} \approx 2 \), for \( \delta \omega \ll \Gamma \).

A.4. Not the quantum state of a laser beam

As stated in the main text, equation (27), here reproduced:

\[ \bigotimes_k \hat{b}_{\text{laser}}^k (ov (ck)), \quad (A.16) \]

is not an approximation to, or idealization of, the state of a CW laser beam. In fact it does not describe any any CW beam (that is, a beam of indefinite length with stationary ergodic statistics). From the preceding subsection, this is simple to see. Since a SML state (5) is a mixture of coherent states, so is equation (A.16). In the spatial mode representation, it is still a mixture of states where each mode is in a coherent state. That is, it can be represented by a stochastically varying (in space) coherent-state amplitude. But by the above theorem from [16], such a stochastic amplitude, if indefinitely extendible and stationary and ergodic (as a CW beam would be) must have an exponential intensity distribution at each frequency, as in a thermal beam.

If not a laser beam, what then, does equation (A.16) describe? Consider an individual sample, a stochastically varying coherent-state amplitude. This will be a superposition, with random phases, of different spatial frequencies \( k \) with deterministic amplitudes \( \sqrt{ov (ck)} \), creating an irregular pulse of length \( L \). In particular, the largest contribution to the pulse is always the \( k = k_0 \) mode. If we now increase the value of \( L \) to \( L' \), the nature of a randomly drawn pulse will change. Its largest contribution will now still be the \( k = k_0 \) mode, but this corresponds to a constant (in the rotating frame) pulse of length \( L' \). Thus the state (A.16) does not describe a coherent
field which is a stationary stochastic process; it is rather a probabilistic mixture of pulses, each with a shape that depends on the length $L$. That, at least, is my best current understanding.

### A.5. An apparent paradox

For CW beams with a coherent-state description, the results of [16] prove not only that $|\tilde{u}_k(k)|^2$ is, asymptotically, exponentially distributed with some mean photon number $\nu_k$ (so that each $k$-mode individually is in a SMT state), but that

$$\lim_{L \to \infty} E[\tilde{u}_k(k)\tilde{u}_k'(k')] = \delta_{k,k'} \nu_k,$$

(A.17)

This seems to yield the paradoxical conclusion that all such CW beams are what Glauber called incoherent, with each $k$-mode independently ‘prepared’ in some SMT state. In particular, since a laser beam can have the same spectrum as a thermal-derived beam, it would seem that it would have to be identical in all ways to a thermal-derived beam, if it is truly CW.

The following may point the way to a resolution. For thermal-derived beams, with enforced periodicity, the $\tilde{u}_{k,k'}$ on the right-hand-side of equation (A.17) holds for any $L$, as in equation (A.14). But for more general stochastic coherent fields it appears only in the asymptotic limit. For finite but large $L$, there are leading-order corrections scaling as $1/L$. The relevance of such terms can be illustrated as follows. If one does not enforce periodicity then, from either the thermal model (A.12) or the phase-diffusing laser model (A.7), a relatively simple calculation yields the leading order correction

$$E[\tilde{u}_k(k)\tilde{u}_k'(k')] = \delta_{k,k'} \nu_k [1 - (k' - k_0)(k - k_0)(2\pi/c)^2].$$

(A.18)

This might seem like an unimportant correction since we are always interested in the large $L$ limit. But it must be remembered that the number of $k$-modes that are significantly populated is of order $L/(2\pi c) = L'/(2\pi c)$. Thus the small correction for each pair of modes sums, over all modes, to something non-negligible. This is how it is possible to obtain, when taking the Fourier transform back to $x$-space, a field $u(x/c)$ with only local correlations, and no artificial high correlations between $u$ for $x < c/L'$ and $u$ for $x > L - c/L'$.

The above correction has nothing to do with non-Gaussianity; as stated above, it appears in the thermal case for a non-period field. But it does illustrate how the theory stating that the periodograms $\tilde{u}_k(k)$ for different $k$ are asymptotically pairwise independent Gaussians could be compatible with the field $u$ itself being strongly non-Gaussian, as in the phase diffusion model (A.7). If the independence between different $k$ were exact, with no corrections scaling as $1/L$, then the coherent-state amplitudes in the different frequency modes would be strictly statistically independent, and the only allowed CW beams would be Glauber’s ‘incoherent light beams’ with $\rho = \bigotimes_k \rho_k(\nu_k)$.

**Note added in proof.** Subsequent to acceptance it was brought to my attention that the ‘prediction’ I made in section 4.4 (see also Appendix A.3), that a laser filtered well within its linewidth would be indistinguishable from a thermal beam, is by no means new. In essence it was proven by Armstrong in 1966 [18], although he found the thermal negative-exponential intensity distribution only numerically, not analytically. Moreover, an experiment seeing an increase in $g^{(2)}(0)$ towards 2 as the filter was narrowed, was performed in 1992 [19]. See also [20].

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