Supernovae-induced accretion and star formation in the inner kiloparsec of a gaseous disc

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ABSTRACT

We consider the effects of supernovae (SNe) on accretion and star formation in a massive gaseous disc in a large primeval galaxy. The gaseous disc we envisage, roughly 1 kpc in size with \( \gtrsim 10^8 \, M_\odot \) of gas, could have formed as a result of galaxy mergers where tidal interactions removed angular momentum from gas at larger radius and thereby concentrated it within the central \( \sim 1 \) kpc region. We find that SNe lead to accretion in the disc at a rate of roughly 0.1–1 \( M_\odot \) yr\(^{-1}\) and induce star formation at a rate of \( \sim 10–100 \, M_\odot \) per year which contributes to the formation of a bulge; a part of the stellar velocity dispersion is due to SN shell speed from which stars are formed and a part due to the repeated action of stochastic gravitational field of SNe remnant network on stars. The rate of SN in the inner kpc is shown to be self-regulating, and it cycles through phases of low and high activity. The SN-assisted accretion transports gas from about 1 kpc to within a few pc of the centre. If this accretion were to continue down to the central black hole then the resulting ratio of black hole mass to the stellar mass in the bulge would be of the order of \( \sim 10^{-2}–10^{-3} \), in line with the observed Magorrian relation.

Key words: accretion, accretion discs – methods: analytical – stars: formation – supernovae: general – galaxies: bulges.

1 INTRODUCTION

The CO observations of ultra-luminous infrared galaxies (ULIGals) find the gas mass in the inner regions of the galaxy to be about \( 5 \times 10^9 \, M_\odot \), and the average particle density to be of the order of \( 10^3 \) cm\(^{-3}\) and the kinetic temperature of molecular gas \( \sim 50–100 \, K \) (e.g. Downes & Solomon 1998; see Sanders & Mirabel 1996, for a review). The gas in the central \( \sim \)kpc region of the galaxy is likely to have come from distances of the order of 10 kpc when it lost some of its angular momentum due to gravitational tidal torques (Barnes & Hernquist 1992, and references therein; see Barnes 2002, for a more recent numerical simulation).

The inner kiloparsec region of most young massive galaxies is likely composed of a gaseous disc with a mass of several hundred million solar masses, ULIGal being at the extreme end of the mass distribution. This gaseous disc is expected to host star formation at a large rate. Some of these stars will explode and give rise to shock waves in the gaseous disc which will spawn both more star formation and accretion of gas towards the centre of the galaxy. We consider these processes analytically in some detail in this paper, paying special attention to the effect they might have on the evolution of the central parts of the galaxy and on the growth of a central black hole.

There exists a large body of work on the subject of galaxy mergers, star formation and black hole growth, e.g. Sanders et al. (1988), Kauffmann & Haehnelt (2000), Kawakatu & Umemura (2002), Granato et al. (2004), Croton et al. (2006), Kauffmann & Heckman (2009), Chen et al. (2009) (see Kormendy & Kennicutt 2004, for a review), and sophisticated numerical simulations, e.g. Barnes & Hernquist (1991, 1996), Mihos & Hernquist (1996), Di Matteo, Springel & Hernquist (2005), Springel, Di Matteo & Hernquist (2005), Hopkins et al. (2005), Hopkins & Hernquist (2009). What is different in this paper is that we try to capture some of the basic properties of this complex system using analytic results for supernova (SN) remnant evolution and other simple physical scalings, which are hard to capture in numerical simulations due to the large ratio of galaxy size and SN shell radius.

The physical system we consider is described in Section 2 along with the effect SNe have on accretion. Bulge formation as a product of SN-induced star formation in the gaseous disc is discussed in Section 3. The main conclusions and uncertainties of this study can be found in Section 4.

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2 SUPERNova-INDUCED ACCRETion

IN A GAS DISC

Numerical simulations of gaseous discs (e.g. Wada & Norman 2001) find the medium to be multiphase and highly filamentary as a result of star formation and stellar explosion. For the analytical calculations in this work, where our primary interest is in average disc properties, we consider a simplified disc structure that ignores its filamentary density structure; accordingly, the gas distribution is taken to be a smooth function of distance from the centre. Many of the results reported in this work, as we shall see, have a weak dependence of the interstellar medium (ISM) density and therefore the error introduced by the assumption of smooth density field in the disc should not be large.

We consider a disc, roughly 1 kpc in radius, consisting of \( \gtrsim 10^{10} \text{M}_\odot \) in gas that came from larger radius (approximately tens of kpc) due to, e.g., tidal interaction with another galaxy. The mean gas density in the disc at 1 kpc is \( \sim 10^3 \text{ cm}^{-3} \). Stars form and die in the disc at a certain rate; we take the SN rate in the disc to be \( dN_{\text{SN}}(r)/dr \) per unit area per year. The interaction between the remnant of an SN with the gaseous disc carves out a cavity in the disc. The gas swept up by an SN is compressed into a thin shell during the snowplough phase, which starts when the thermal cooling time-scale of the shocked gas is less than the age of the remnant. This means that a certain amount of gas is pushed closer to the Galactic Centre by the SN shock wave and, of course, a certain amount is pushed outwards. However, unless gas pushed inwards by the SN shock loses angular momentum it will be pushed back out to a larger radius when the shock weakens. We shall estimate later in this section the loss of angular momentum for gas pushed closer to the Galactic Centre by the SN, in order to determine if it is sufficient to keep the swept-up gas at a smaller radius. However, first, we estimate the amount of gas swept up and pushed to smaller radius by an SN.

Let us consider an SN going off in the disc at radius \( r \). The number density of particles in the disc at this radius is \( n(r) \), the vertical scaleheight is \( H(r) \) and the mean rotation speed of gas on its circular orbit is \( V_{\text{orb}}(r) \). During the adiabatic expansion of the SN shell – the Sedov–Taylor phase – the radius, speed and temperature of the shock front are given by

\[
R_s(t) = 3.4 t_4^{-2/5} n_3^{-1/5} E_{51}^{2/5} \text{ pc},
\]

\[
V_s(t) = 123 t_4^{-3/5} n_3^{-1/5} E_{51}^{2/5} \text{ km s}^{-1},
\]

\[
T(t) = 2.1 \times 10^4 t_4^{-6/5} n_3^{-2/5} E_{51}^{2/5} \text{ K},
\]

where \( t_4 = t/10^4 \text{ yr}, E_{51} = E/10^{51} \text{ erg} \) and \( n_1 = n/10^4 \text{ cm}^{-3} \) is the particle number density of gas in the disc at radius \( r \). The Sedov–Taylor phase ends when the radiative cooling time \( t_{\text{cool}} = 0.67 KT/(\pi n\alpha) \) is equal to the dynamical time; \( \Lambda \approx 10^{-16} T^{-1} \text{ erg cm}^{-3} \text{ s}^{-1} \) (Blondin et al. 1998). The radiative phase begins at time

\[
t_{\text{snow}} \approx 723 n_3^{-9/17} E_{51}^{11/17} \text{ yr}.
\]

Subsequently, during the snowplough phase (\( t > t_{\text{snow}} \)), the evolution is described by (cf. Chevalier 1974; equation 26)

\[
R_s(t) \approx 0.8(t/t_{\text{snow}})^{0.31} n_3^{-7/17} E_{51}^{2/17} \text{ pc},
\]

\[
V_s(t) \approx 595(t/t_{\text{snow}})^{-0.69} n_3^{2/17} E_{51}^{11/17} \text{ km s}^{-1},
\]

\[
M_s(t) \approx 133(t/t_{\text{snow}})^{0.03} n_3^{-4/17} E_{51}^{15/17} \text{ M}_\odot,
\]

where \( M_s \) is the mass of gas swept up by the SN remnant. These equations are valid only as long as the shell radius is less than the vertical scaleheight \( H \):

\[
H = \frac{2^{1/2} (C_s^2 + V_s^2)^{1/2}}{\Omega} = \frac{2^{1/2} r (C_s^2 + V_s^2)^{1/2}}{V_{\text{orb}}},
\]

\[
\sim (500 \text{ pc}) r_3^{3/2} \left( \frac{V_s}{10 \text{ km s}^{-1}} \right) \left( \frac{M_s}{\text{10^5 M}_\odot} \right)^{-1/2},
\]

where \( r_3 = r/10^3 \text{ pc}, C_s \sim 1 \text{ km s}^{-1} \) is the sound speed, \( V_s \) is the rms turbulence velocity in the gaseous disc (produced by SN explosions and winds from early-type stars) and \( M_s \) is the total mass enclosed within radius \( r \).

The Toomre \( Q \) parameter for the gravitational instability of the gaseous disc is

\[
Q = \frac{2 (C_s^2 + V_s^2)^{1/2} \Omega}{\pi G \Sigma} \sim 1 V_{\text{s},0}M_{\text{s}}^{-1/2} r_3^{-1/2},
\]

where \( \Sigma \) is the mass density per unit area, \( V_{\text{s},0} \equiv V_s/10^6 \text{ cm s}^{-1} \) and \( M_s = M_s/10^5 \text{ M}_\odot \). We see from the above equation that the disc is gravitationally unstable, and will support an ongoing star formation activity.

Ignoring density gradients in the disc (\( R_s \ll r \) and \( R_s < H \)), the expansion of an SN shell is nearly spherically symmetric until the Coriolis or centrifugal force per unit mass becomes of the order of the deceleration of SN remnant. This occurs when \( V_s \Omega_{\text{orb}} \approx d V_s/dt \), or \( t \approx 0.69/\Omega_{\text{orb}} \), and defines a characteristic time when the shell is no longer spherical

\[
t_{\text{shear}} \sim 7 \times 10^5 r_3^{-3/5} V_{\text{s},0}^{-2} \text{ yr},
\]

where \( V_{\text{s},0} = V_s/100 \text{ km s}^{-1} \). In fact, at this time the magnitude of the shear velocity across the shell – \( R_s \Omega_{\text{orb}}/d \) in \( r \) – is of the order of the speed of the SN remnant. The SN shell is thus dispersed by the shear flow in the disc and mixed with the ambient ISM on this timescale. The radius, velocity and mass of the remnant at this time are

\[
R_s(t_{\text{shear}}) \sim 14 n_3^{-0.25} E_{51}^{0.22} r_3^{3/5} V_{\text{s},0}^{-0.31} \text{ pc},
\]

\[
V_s(t_{\text{shear}}) \sim 1.1 n_3^{-0.25} E_{51}^{0.22} V_{\text{s},0}^{0.69} r_3^{-0.69} \text{ km s}^{-1},
\]

\[
M_s(t_{\text{shear}}) \sim 7 \times 10^5 r_3^{0.03} V_{\text{s},0}^{-0.93} r_3^{0.25} E_{51}^{0.66} \text{ M}_\odot.
\]

Note that \( R_s(t_{\text{shear}}) \lesssim H \), and therefore the SN shell is confined within the disc unless \( r \lesssim 10 \text{ pc} \). Since the shell velocity is small compared with the orbital speed when \( R_s \sim H \text{ pc} \), the swept-up gas in the disc cannot escape the galactic potential.

During the snowplough phase, the swept-up gas is compressed into a thin shell, forming a hollow sphere. Therefore, half of the swept-up ISM gas, of mass \( M_s/2 \), is pushed closer to the centre of the galaxy by a distance \( \sim R_s/2 \). For an ensemble of SNe going off in the disc the net amount of gas that is accreted at the Galactic Centre by the SN is strongly dependent on the SN rate as a function of distance from the Galactic Centre (\( r \)) and is given by

\[
M_{\text{acc}} \sim R_s \frac{d}{dr} \left( \frac{M_s(r)}{2 - \pi n_r^2 r dN_{\text{SN}}}{dA} \right) \sim M_s (R_s/r)^2 f_{\text{SN}}(r),
\]

where \( f_{\text{SN}}(r) \) is the cumulative SN rate within the radius \( r \). We assumed that \( (M_s, R_s, f_{\text{SN}}) \) is an increasing function of \( r \) in deriving the second part of the above equation; otherwise, SNe would lead to a net outflow of gas in the disc to larger distances. Since \( M_s \sim R_s \approx n_1^{0.01} \) during the snowplough phase (combine equations 5 and 7, and substituting for \( t_{\text{snow}} \) from equation 4) as long as \( f_{\text{SN}}(r) \)
increases with distance faster than \( r^1 \) there is a net mass accretion even when SN remnants are spherically symmetric. For a Mestel disc the mass enclosed inside radius \( r \) increases linearly with \( r \), and in that case there is no net accretion – for spherical SNe – if the rate of stellar explosions is linearly proportional to gas mass. We shall see below that the condition on \( f_{SN}(r) \) is relaxed when we consider the distortion of SNe remnants by Coriolis and centrifugal forces. Equation (14) for accretion rate is also modified when SNe shells undergo collision before \( t_{\text{coll}} \); this is discussed below. However, in any case we need to make sure that gas pushed inwards by an SN loses specific angular momentum; otherwise, it would be pushed back out when the shock becomes sufficiently weak.

It should be noted that half of an SN shell has negative angular momentum (as seen by an observer at the centre of the explosion comoving with the disc), and the other half, with prograde velocity field, has positive angular momentum. The magnitude of the total positive/negative angular momentum grows during the adiabatic expansion as \( |L_s| \approx M_s V_s r/2 \propto 1 / V_s \) (the second part of this relation follows from energy conservation during the adiabatic expansion phase). However, the angular momentum does not increase much during the snowplough phase when \( M_s V_s \) is approximately constant. The magnitude of the negative angular momentum carried inwards by the lower half of an SN shell \( (|L_s|) \) is quite large and that can lead – as we shall see shortly – to an accretion rate in a gas-rich disc (like ULIGal) of the order of a few solar masses per year.

It can be shown that, for a freely expanding shell, particles on half of the shell with negative angular momentum – that have retrograde velocity as seen by a comoving observer at the centre of explosion – will on average descend a distance of \( \Delta t = (8 V_{\text{orb}} / \pi V_s - 1) R_s^2 / 4 \Delta \) in the radial direction (as a result of Coriolis and centrifugal forces in the comoving frame), as long as \( V_s \gtrsim V_{\text{orb}} \Delta \sim \Delta \). Particles on the other half of the shell with positive angular momentum, or prograde velocity, will move outwards the same distance. Thus, the shell is continuously distorted with time as a result of tidal stretching and Coriolis force.

Part of the lower half of the shell (lying closer to the Galactic Centre) with transverse velocity component in the direction of the orbital velocity has a positive angular momentum with respect to the centre of explosion, and the part with transverse velocity opposite to the orbital motion has negative specific angular momentum. The net amount of negative angular momentum carried by the distorted lower half of the shell, during the snowplough phase, as seen by an observer comoving with the centre of explosion, is

\[
\delta L \approx \Delta \sigma(L_s / R_s) \approx M_s R_s V_{\text{orb}} / \pi. \quad (15)
\]

When the lower half of the shell is mixed – due to shear stretching or collision with other shells – the net mean specific angular momentum of the mixed fluid is smaller by \( \sim 2 R_s V_{\text{orb}} / \pi \) than a particle at the centre of the explosion orbiting the galaxy. This is a sufficient amount of negative angular momentum to keep the mixed lower half of shell moving on a circular orbit at a smaller radius of \( (r - R_s / 2) \). Thus, the constraint discussed earlier on \( f_{SN} \) for inward mass accretion is relaxed because there is a net outward transport of positive angular momentum associated with each SN shell.

The rate of angular momentum transported by an ensemble of SNe in the disc is

\[
L \sim \delta L (2 \pi R_s) \frac{df_{SN}}{dA} \sim \frac{2 M_s}{\pi} R_s V_{\text{orb}} (R_s / r) f_{SN}. \quad (16)
\]

and the accretion rate resulting from this outward angular momentum transport is

\[
M_{\text{acc}} \approx \frac{\delta L}{r V_{\text{orb}}} \approx \frac{2 M_s}{\pi} R_s V_{\text{orb}} (R_s / r)^2 f_{SN}. \quad (17)
\]

which is similar in magnitude to that given by equation (14), i.e. SNe transport mass inwards for a larger class of functions for \( f_{SN}(r) \) than suggested by the discussion following equation (14).

The accretion rate depends on the shell radius \( (R_s) \) at the time when SN shells collide with each other or when the shell is dispersed and mixed due to shear flow in the disc. We consider both of these cases below.

Given an SN rate of \( f_{SN} \) per year within radius \( r \) of the disc, the mean separation between SNe that occurred within time \( t \) is

\[
d_{SN} \approx 1.8 \frac{r}{1 \text{kpc}} \left( \frac{t}{1 \text{yr}} \right)^{-1/2} f_{SN}^{1/2} \text{kpc}. \quad (18)
\]

SNe shells collide when their size is of the order of the mean separation, i.e. \( R_s(t) \approx d_{SN} \). The mean collision time, \( t_{\text{coll}} \), is estimated using equation (5), and is given by

\[
t_{\text{coll}} \approx 1.7 \times 10^5 n_3^{3/2} f_{SN}^{0.62} E_51^{0.27} R_3^{1.23} \text{yr}. \quad (19)
\]

The ratio of the collision time and the time it takes for an SN shell to be dispersed due to shear velocity in the disc is

\[
\frac{t_{\text{shear}}}{t_{\text{coll}}} \approx 40 n_3^{-0.31} f_{SN}^{0.62} R_3^{0.23} V_{\text{orb}}^{-1} f_{SN}^{0.27} E_51^{1.1}. \quad (20)
\]

The accretion rate when \( t_{\text{shear}} / t_{\text{coll}} \lesssim 1 \) is given by (using equations 11, 13 and 17)

\[
M_{\text{acc}} \sim M_s (t_{\text{shear}}) f_{SN} \left( \frac{R(t_{\text{shear}})}{r} \right)^2 \\sim 80 f_{SN} n_3^{-0.25} R_3^{-0.45} V_{\text{orb}}^{-1} f_{SN}^{1.27} E_51 M_\odot \text{yr}^{-1}. \quad (21)
\]

The condition for a remnant not to collide with others before \( t_{\text{shear}} \) places a limit on the SN rate of

\[
f_{SN} \lesssim 3 \times 10^{-3} n_3^{1/2} V_{\text{orb}}^{1.44} E_51^{0.44} V_{\text{orb}}^{1.1} \approx f_{SN}^{0.27}. \quad (22)
\]

The second part of equation (21) is valid only when the SN rate is less than the rate given in equation (22); the accretion rate corresponding to this limiting SN rate is

\[
M_{\text{acc}} \sim 0.3 n_3^{0.25} f_{SN}^{0.66} E_51^{0.08} V_{\text{orb}}^{0.05} M_\odot \text{yr}^{-1}. \quad (23)
\]

The accretion rate has a very weak dependence on \( f_{SN} \) when the SN rate is larger than the rate given in equation (22), i.e. when SN shells collide before \( t_{\text{shear}} \). The reason is that the SN shell radius at the time of shell collision \( (t_{\text{coll}} \propto f_{SN}^{0.62}) \) is \( R_s \propto f_{SN}^{0.19} \) and the shell mass is \( M_s \propto R_s^2 \propto f_{SN}^{0.57} \); therefore, the accretion rate \( M_{\text{acc}} \sim M_s (R_s / r)^2 f_{SN} \propto f_{SN}^{0.9} \). For the case of \( t_{\text{shear}} / t_{\text{coll}} > 1 \), the accretion rate is given by

\[
M_{\text{acc}} \approx 0.3 n_3^{0.25} f_{SN}^{0.05} E_51^{0.14} M_\odot \text{yr}^{-1}. \quad (24)
\]

For \( M_{\text{acc}} \) to be independent of \( r \) the density scale should be \( f_{SN}(r) \sim r^{-0.2} \). However, for a non-equilibrium situation in the early phases of galaxy formation and frequent mergers this equilibrium density scaling is not applicable. When a large quantity of gas is deposited within a few kpc following a merger event or a tidal encounter, the high rate of star formation and SNe at \( r \sim 1 \) kpc would cause a rapid rate of accretion of gas to smaller radii, \( M_{\text{acc}} \sim M_s (t_{\text{shear}}) f_{SN} \sim 10^7 M_\odot \text{yr}^{-1} \) after a lag of \( 10^7 \) yr, which will continue until star formation and SN explosions at smaller radii start to inhibit this large accretion rate, the subsequent accretion rate would settle down to the value given by equation (24).
Note that the cumulative effect of SNe is to create a random velocity field in the disc, but that does not automatically ensure accretion. We must have an outward angular momentum transport in order for accretion to proceed. An interesting example is that of convective instabilities in a disc; Ryu & Goodman (1992) have shown that disc convection transports angular momentum inwards, and therefore the turbulent velocity field associated with it does not lead to any accretion. Similarly, the random velocity field in a disc is large when the SN rate is high and yet because shells collide before they are significantly deformed the outward transport of angular momentum increases very weakly with \( f_{SN} \) (equation 24).

The accretion rate in terms of an effective \( \alpha \)-viscosity, \( \dot{\nu} \equiv \alpha \cdot \nabla (t_{coll}) \)
\( \nabla (t_{coll}) \) in the limit that \( f_{SN} > f_{SN}^{coll} \) is given by
\[
\dot{M}_{\text{acc}} \sim 2\pi n_{\rho} H \nu \sim 1.2 \alpha (H / 0.1 r) n_{\rho}^{0.59} f_{SN}^{0.24} E_{51}^{0.54} n_{\rho}^{0.53} M_{\odot} \text{yr}^{-1}.
\]
(25)

Comparing this with equation (24) we see that \( \alpha \sim 1 \) for \( f_{SN} \sim f_{SN}^{coll} \) (given by equation 22), and \( \alpha \) is smaller for a larger SN rate, although the effective \( \alpha \) is dependent on \( r \).

If SNe remnants punch through the disc in the vertical direction, but are still confined by the galactic potential, then some fraction of gas leaving the disc will eventually fall back on to the disc at a smaller radius and contribute to the net accretion rate. An SN is confined to the galaxy provided that
\[
R_{e}(\min(t_{coll}, t_{\text{shell}})) < H \quad \text{or} \quad V_{s}(R_{e} = H) < V_{\text{coll}}.
\]
(26)

If many SNe shells collide and coalesce before they are dispersed they would form super-shells and if their velocity is sufficiently high they can escape the galactic potential. Otherwise, these super-shells would also contribute to transporting gas to smaller radius.

### 3 Star and Bulge Formation and Growth of a Central Black Hole

The possibility of SN-induced star formation has been discussed in numerous contexts, from the early universe to the present-day Milky Way (e.g., Woodward 1976; Bedogni & Woodward 1990; Yamada & Nishi 1998; Mackey, Bromm & Hernquist 2003; Bratsolis, Kontizas & Bellas-Velidis 2004; Johnson & Bromm 2006; Joung & Mac Low 2006; Leão et al. 2009; Nagakura, Hosokawa & Omukai 2009; Sakuma & Susa 2009). Here we consider how this process may compete with the fuelling of black holes and contribute to the formation of galactic bulges.

For a Miller–Scalo initial mass function (IMF) for stars (Scalo 1986):
\[
\frac{dN}{dM} = 4.5N_{c} M^{-1.3} \left( \frac{M}{0.01 M_{\odot}} \right)^{-0.5} \left( \frac{M}{0.05 M_{\odot}} \right)^{0.25} \left( \frac{M}{0.5 M_{\odot}} \right)^{-0.15} \left( \frac{M}{5 M_{\odot}} \right)^{0.05} \left( \frac{M}{50 M_{\odot}} \right)^{-1}.
\]
(27)

with \( \alpha_{1} = 0.3, \alpha_{2} = 1.8 \pm 0.5, \alpha_{3} = 2.3 \pm 0.7 \) (the parameters are taken from Kroupa 2001), and \( N_{c} = \int dM \frac{dN}{dM} / \dot{M} \). The mass fraction in high-mass stars \( (M \gtrsim 8 M_{\odot}) \), capable of SN explosion, to the total star mass is about 0.3, and the fraction by number is about 4 \times 10^{-3}. Thus, the expected SN rate is 0.3 yr^{-1} for the Miller–Scalo IMF and a star formation rate (SFR) of 10 M_{\odot} yr^{-1}, which is of the order observed in ULIGals and needed for forming galactic bulges in L_{\odot} galaxies.

Indeed, this is consistent with the following simple estimate of the SFR in the gaseous disc:
\[
\text{SFR} \sim f_{s} \dot{M}_{\text{disc}} / \dot{H}_{\text{f}}.
\]
(28)

where \( \dot{M}_{\text{disc}} \) is the disc gas mass, \( t_{d} \) is the free-fall time and \( f_{s} \) is the efficiency with which gas is turned into stars on a free-fall time, taken to be of the order of \( f_{s} \sim 0.01 \) (e.g. Krumholz & Tan 2007). Then, for a disc of \( 10^{9} \sim 10^{8} M_{\odot} \), the SFR is about 1–10 M_{\odot} per year, assuming a density of \( 10^{5} \text{cm}^{-3} \).

If a similar fraction \((\sim 1 \text{ per cent}) \) of gas in SNe shells is turned into stars (before shells collide) then the resulting SFR would be \( \sim 140 M_{\odot} n_{\rho}^{0.54} E_{51}^{0.44} f_{SN}^{1.1} 3^{0.4} \text{yr}^{-1} \), and that would result in an SN rate of \( \sim 5 n_{\rho}^{0.54} E_{51}^{0.44} f_{SN}^{0.4} \text{yr}^{-1} \) if the IMF for these ‘daughter’ stars were given by equation (27). However, we show below that formation of massive stars – those capable of SN explosion – is suppressed in SNe remnants when \( f_{SN} \gtrsim 0.5 \).

Star formation in an SN shell is suppressed on large length-scales due to the transverse relative velocity gradient in the remnant. We calculate this length-scale as well as the Jeans length, and estimate the rate of star formation in SNe remnants.

The velocity field in an SN remnant seen by a comoving observer in her neighbourhood is \( \delta V_{s} \sim (V_{s} / R_{s}) \delta r \), where \( \delta r \) is the position vector, tangential to the shock front, pointing from the comoving observer to a point in her neighbourhood. For a gas clump to be able to collapse, the differential velocity across the clump, \( \delta V_{s} \), should be smaller than the gravitational escape speed, i.e. for a clump of size \( \ell, |\delta V_{s}| \sim V_{s}(\ell / R_{s}) \sim (G M_{*} / \ell R_{s}^{2})^{1/2} / 4 \ell^{1/2} \) or
\[
\ell \lesssim \frac{G M_{*} \ell}{4V_{s}^{2}} M_{*} \sim \frac{M_{*}(\ell / 2R_{s})^{2}}{64V_{s}^{2} R_{s}^{2}}.
\]
(29)

Using equations (5)–(7) we find
\[
M_{*} \sim 9 \times 10^{-12} \frac{1}{t_{\text{low}}^{4.9}} n_{\rho}^{4} \frac{n_{\rho}^{n_{51}}}{E_{51}^{2}} M_{\odot}.
\]
(30)

We see from equation (6) that at \( t / t_{\text{low}} \sim 137 n_{51}^{0.17} E_{51}^{-0.9} \), the shell speed is \( V_{s} = 20 \text{ km s}^{-1} \), and at that time \( M_{*} \sim 0.2 n_{51}^{0.42} E_{51}^{2.2} M_{\odot} \); the maximum star mass scales as \( V_{s}^{-7.4} \).

Star formation is disrupted when shells collide. The average speed of SNe shells when they collide, at time \( t_{\text{coll}} \), is \( V_{s} \sim 14 \text{ km s}^{-1} \), \( f_{SN}^{0.4} - 0.8 n_{51}^{0.5} E_{51}^{0.4} \) (obtained from equations 6 and 19). For a small SN rate, \( f_{SN} \lesssim 0.5 \text{ yr}^{-1} \), \( V_{s} \) can drop down to a value where massive stars capable of SN explosion can form before SNe shells collide and star formation is disrupted; however, since lower mass stars form first in SNe remnants they can perhaps significantly suppress the formation of more massive stars. At a higher SN rate, formation of stars more massive than a few solar mass is suppressed; massive stars could still form in the disc after the turbulence generated by shell collisions has subsided and gas has cooled down. This suggests that SNe explosions in a gaseous disc may occur in waves of high and low activity, and during periods of high activity Miller–Scalo IMF is truncated above a few solar mass, and even during periods of lower SN rate the IMF could be more bottom-heavy than the standard IMF.

#### 3.1 The thermal state of the pre-shock gas and the shock-induced stellar IMF

A firm lower limit to the shock front speed, which in turn sets an upper limit to the masses of the stars which may form behind the shock front, is set by the sound speed of the medium upstream of the

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1 Collision between SNe shells disrupts star formation because of turbulence generated in these collisions. Once turbulence dies out – in about a shock-crossing time – star formation can resume provided that the merged shell is not hit again by a high-speed shock.
shock front when SN shock weakens and turns into a sound wave. In order to calculate the sound speed of the upstream gas, we must determine the thermal state of this gas, and we discuss two possible cases in this subsection.

### 3.1.1 High supernova rate

For the first case, we consider a galactic disc with a high SFR, in which the dominant process affecting the thermal state of the gas is photoheating of the gas by massive stars, which eventually explode as SNe. Considering that these stars will each live for roughly $10^7$ yr, the average distance between these massive stars within radius $r$ of the disc, following equation (18), is

$$d_{SN} \approx 0.6 \frac{r}{1 \text{pc}} f_{SN}(r)^{-1/2} \text{pc}.$$  \hspace{1cm} (31)

The number of ionizing photons emitted per second by such massive stars is $\gtrsim 10^{49}$ (e.g. Osterbrock & Ferland 2006). Taking the average gas density to be $\sim 10^3$ cm$^{-3}$, we find the radius of the Strömgren spheres surrounding massive stars (Strömgren 1939) to be of the order of $\sim 1$ pc. Thus, we expect that for an average SN rates of $f_{SN} > 0.4$ yr$^{-1}$ the H II regions surrounding the massive stars in the disc will overlap, and therefore SNe shocks will propagate into such photoionized regions. Largely independent of the metallicity of the gas, the temperature in such a photoionized region will be of the order of $10^4$ K, and the sound speed $\sim 10$ km s$^{-1}$. Therefore, SNe shocks in this case will generally dissipate and turn into pressure waves once they have slowed to speeds of $\sim 10$ km s$^{-1}$. This suggests, following equation (29), that the IMF of stars formed in the material swept-up in SN shocks is likely cut-off at a few solar masses.

### 3.1.2 Low supernova rate

We next consider the case of a lower SFR, corresponding to $f_{SN} \lesssim 0.4$ yr$^{-1}$, for which the distance between massive stars is greater than their average Strömgren radii. In this case, SN shocks will generally propagate into so-called relic H II regions in which gas that was previously photoionized by the progenitor star is recombing and cooling; equations (5) and (6) show that, in general, $R_e \gtrsim 1$ pc (Strömgren radius) when the SN shell speed has dropped to $\sim 10$ km s$^{-1}$. However, even for larger Strömgren radii, the upstream relic H II region gas can in some cases cool to temperatures well below $10^4$ K, which allows for the formation of stars with masses greater than a few solar masses.

The results of a numerical calculation of relic H II region gas temperature ahead of the shock are shown in Fig. 1. This calculation assumes that the density of the gas remains constant at the fiducial value of $n = 10^3$ cm$^{-3}$, and that the gas cools only radiatively through atomic transitions of metals, which is a reasonable assumption for the case we consider here (see Jappsen et al. 2009, for the case of star formation in a more isolated environment). In principle, the radiation emitted by the shocked gas can send a radiative precursor ahead of the shock and heat the upstream gas, however, we neglect this effect as at the shock velocities at which the shocks stall, i.e. $\gtrsim 20$ km s$^{-1}$, radiative precursors do little to ionize or heat the upstream gas (Shull & McKee 1979).

Fig. 1 shows results for four different metallicities: $10^{-1}$, $10^{-2}$, $10^{-3}$ and $10^{-4}$ Z$_\odot$. The metallicity-dependent cooling rate of the gas is taken from Mashchenko, Wadsley & Couchman (2008), who provide a fitting formula to the cooling function calculated by Bromm et al. (2001). The left-hand panel of Fig. 1 shows the temperature of the upstream gas as a function of the time from the death of the central star. The right-hand panel shows the square of the Mach number for the shock front. The SN shock will stall once this ratio approaches unity – at this point the shock will turn into a pressure wave.

For metallicities $\lesssim 10^{-2}$ Z$_\odot$ the SN shock stalls at a time $t_{coll} \sim 10^5$ yr, when the shock velocity is still $\sim 10$ km s$^{-1}$. However, for higher metallicities the shock stalls at later times when $V_s$ has dropped to a smaller value and thus the maximum mass of SN-induced stars can become $M_s \gtrsim 5$ M$_\odot$ for shocks that last for $\gtrsim 2 \times 10^5$ yr (see equation 30). We note also that for the case we consider here of $f_{SN} \lesssim 0.4$, $t_{coll}$ can easily exceed $t_{coll}$, and therefore shell collisions will not interfere with SN-induced formation of these more massive stars at late times.

### 3.2 Stars, bulge and black hole

As shown in the last section, for $f_{SN} \gtrsim 0.4$ yr$^{-1}$, stars forming in SN remnants have peculiar velocities of the order of 10–20 km s$^{-1}$ in radial direction with respect to the centre of the explosion. When viewed from the Galactic Centre the velocities of these newly formed stars in SN remnants would have a random velocity dispersion of $\sim 10$–20 km s$^{-1}$, and therefore these stars would tend to form a bulge at the centre. The velocity dispersion of the newly formed stars would increase with time as these stars are subjected to the stochastic gravitational field of the SN remnant network. Moreover, these stars will suffer some hydrodynamical drag on their way...
of the gaseous disc, and also will be subject to gravitational drag that will modify their velocity dispersion (e.g. Artyomowicz 1994; Nayakshin & Cuadra 2005); the former is likely a small effect due to the small cross-section for star–gas interaction, but the latter can be a significant effect and needs to be included in the calculation to determine the true random velocity distribution of stars formed out of SNe remnants.

The SN-led accretion would also deposit gas in the central parsec region of the galaxy at the rate given by equation (24); at distances smaller than ~1 pc the magneto-rotational viscosity (Balbus & Hawley 1991) is expected to be effective in transporting gas to the black hole at the centre. We note that MRI might also operate at larger radius due to heating and ionization produced by SNe.

Let us assume that a fraction $f_\text{SN}$ of SN remnant mass is converted to stars before shells collide with another shell. Using equations (7) and (19), we estimate the SFR in SN remnants to be

$$M_* \approx (1.5 \times 10^9 M_\odot) f_\text{SN} \frac{0.4}{n_\text{i}} \frac{0.54}{E_{\text{SN}}} \frac{E_{\text{SN}}^{0.4}}{r_3} \frac{1}{1} \text{yr}^{-1}.$$ (32)

Therefore, the ratio of star formation and accretion rates for $f_\text{SN} > 0.5$ yr$^{-1}$ is $\sim 5 \times 10^2 f_\text{SN} \frac{0.3}{n_\text{i}} \frac{E_{\text{SN}}^{0.3}}{r_3}$ (equations 24 and 32). For $f_\text{SN} < 10^{-2}$ and $f_\text{SN} < 1$ yr$^{-1}$ this ratio is of the order of a few hundred. In the case of low SN rate (when the remnant survives until its velocity drops to ~10 km s$^{-1}$ and then turns into a compression wave) the ratio is $3 \times 10^2 f_\text{SN} \frac{0.4}{n_\text{i}} \frac{E_{\text{SN}}^{0.4}}{r_3}$, which is also of the order of a few hundred for $f_\text{SN} < 10^{-2}$; this last expression was obtained by using equations (5) and (7) for $R_\odot$ and $M_*$ corresponding to shell velocity of 10 km s$^{-1}$ (equation 6) to calculate star formation and accretion rate (equation 17).

This ratio of SFR to accretion rate is similar to the reported ratio of bulge and black hole mass in galaxies (Ferrarese & Merritt 2000; Gebhardt et al. 2000), and the scenario we have described offers a plausible physical explanation for this correlation. We note that a number of well-known feedback processes have been left out in the calculations presented in this paper (cf. Cattaneo et al. 2009), and these might significantly modify star formation and accretion rates.

### 4 SUMMARY

We have analysed the effect of SNe occurring within the central kpc region of a gaseous disc on the formation of stars and transport of gas from ~1 kpc to a few pc of the Galactic Centre. The outward transport of angular momentum facilitated by SNe expulsions allows for the inward transport of gas that feeds the central black hole. This is a process that may take place quite generically in any galactic disc hosting star formation, although it may not be the dominant process affecting black hole accretion.

We have shown that associated with the inward transport of gas swept-up by SNe is the shock-induced formation of stars which are born with a random peculiar velocity of ~10 km s$^{-1}$. This velocity dispersion increases with time as a result of the stochastic gravitational field associated with filamentary SN remnants; the stars formed in SN remnants contribute to the stellar population of a central bulge.

Due to the divergent velocity field of an expanding SN shell there is a maximum length-scale for fragmentation of shells or an upper limit to the mass of stars formed in SN remnants; the SN-induced stellar IMF is cut off above a few solar masses and more massive stars can only form if and when an SN shell slows down to ~10 km s$^{-1}$.

We note that numerous observations have suggested connections, such as we have considered in this paper, between star formation, black hole accretion and the formation of a stellar bulge. Heckman et al. (2004) suggest that star formation and black hole accretion rates are correlated, and Chen et al. (2009) report an empirical relation between SN rate and gas accretion rate on the central black hole (see also Xu & Wu 2007). Furthermore, Page et al. (2001) report observations suggesting that central black holes and stellar spheroids form concurrently, and Genzel et al. (2006) describe observations of a galaxy hosting both an accreting black hole and a central stellar bulge, with no evidence of a major merger. Hydrodynamic simulations have demonstrated that SN feedback may produce spherical distributions of stars in dwarf galaxies (Stinson et al. 2009) and in the inner portions of the Galactic bulge (Nakasato & Nomoto 2003). We would like to point out the recent work of Wang et al. (2009) that models SN-induced turbulence as an effective viscosity and describes the evolution of a gaseous disc.

A limitation of this work is that we have ignored radiative feedback effects, which are known to control the steady state accretion rate on to the black hole (e.g. Ostriker et al. 1976; Milosavljevic, Couch & Bromm 2009; Proga, Ostriker & Kurosawa 2008) and probably also affect the formation rate of stars in the central kpc. Ultimately, large-scale simulations resolving the long-term evolution of individual SNe remnants, star formation, the multiphase ISM and the feedback effects of accretion on to a central black hole will be required to more fully elucidate what role SN-induced accretion and star formation play in galaxy formation.

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