Pseudogap Phenomena of an Ultracold Fermi Gas with a $P$-wave Feshbach Resonance

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Abstract. We investigate pseudogap phenomena in an ultracold Fermi gas with a $P$-wave tunable pairing interaction. Including $P$-wave pairing fluctuations within a strong-coupling $T$-matrix theory, we calculate the single-particle density of states (DOS) above the superfluid transition temperature $T_c$. Starting from the weak-coupling regime, we show that, at first, a dip structure gradually appears to become remarkable in DOS with increasing the strength of the $P$-wave pairing interaction at $T_c$. However, this pseudogap structure again becomes less remarkable to eventually disappear, as one passes through the intermediate coupling regime to enter the strong-coupling region. This non-monotonic interaction dependence of the pseudogap is quite different from the $s$-wave case, where the pseudogap size simply increases with increasing a $s$-wave interaction strength. This difference is found to originate from the momentum dependence of the $P$-wave interaction. We also determine the pseudogap temperature below which the dip structure in DOS appears, and identify the pseudogap regime in the phase diagram of a Fermi gas with respect to the temperature and $P$-wave interaction strength.

1. Introduction

A $P$-wave Feshbach resonance allows us to introduce a tunable $P$-wave pairing interaction to an ultracold Fermi gas. This anisotropic Feshbach resonance has been observed in $^{40}$K[1] and $^6$Li[2] Fermi gases. In addition, the formation of $P$-wave Feshbach molecules has been also reported in these Fermi gases[2, 3]. Although the $P$-wave superfluid has not been obtained in cold Fermi gases, the realization of this unconventional pairing state using the $P$-wave Feshbach pairing mechanism is highly expected. Once we can reach this superfluid state, by adjusting the threshold energy of a $P$-wave Feshbach resonance, we can study $P$-wave superfluid properties from the weak-coupling BCS (Bardeen-Cooper-Schrieffer) regime to the strong-coupling BEC (Bose-Einstein condensation) regime in a unified manner. This kind of study has been done in the $s$-wave case by using a $s$-wave Feshbach resonance. However, since various pairing states (that originate from the $L = 1$ angular momentum of Cooper pairs) are possible in the $P$-wave case, we can expect much richer physics than the $s$-wave case.

In the current stage of research for the $P$-wave superfluid Fermi gas, the most important challenge is to reach the $P$-wave superfluid phase transition temperature $T_c$. In this regard, when one increases the strength of a pairing interaction by adjusting the threshold energy of a $P$-wave Feshbach resonance, $P$-wave pairing fluctuations would become strong[4], which would...
induce various precursor phenomena of the p-wave superfluid state even above $T_c$. In particular, the so-called pseudogap phenomenon, which has been recently discussed in s-wave superfluid Fermi gases[5], is also expected in the p-wave case, where a pseudogap (dip) structure appears in the single-particle density of states (DOS) in the normal state. Since the pseudogap should be remarkable near $T_c$, one may use this phenomenon in order to see to what extent the current experimental situation is close to the p-wave superfluid phase transition.

In this paper, we investigate the pseudogap phenomena in an ultracold Fermi gas with a p-wave pairing interaction. Including p-wave pairing fluctuations within the framework of a strong coupling T-matrix theory, we calculate DOS above $T_c$. From the temperature dependence of the pseudogap in DOS, we determine the pseudogap temperature $T^*$, as well as the pseudogap regime in the phase diagram with respect to the temperature and the interaction strength.

2. Formulation
We consider a single component Fermi gas with a p-wave pairing interaction, described by the BCS-type Hamiltonian,

$$H = \sum \xi_p c_p^\dagger c_p + \frac{1}{2} \sum_{p,p',q} U(p,p') c_p^\dagger c_{p+q}^\dagger c_{p+q} c_{p'-q}.$$  

(1)

Here, $c_p^\dagger$ is the creation operator of a Fermi atom with the kinetic energy $\xi_p = p^2/(2m) - \mu$, measured from the chemical potential $\mu$. $U(p,p') = -U p \cdot p' (U > 0)$ is an assumed p-wave pairing interaction. In this paper, we ignore the detailed Feshbach mechanism, and simply treat $U$ as a tunable parameter. We also ignore effects of a harmonic trap, for simplicity. We will improve these in our future paper.

In cold atom physics, the p-wave interaction is frequently measured in terms of the scattering volume $v$ and the effective range $k_0$. These are related to $U$ as $4\pi v/m = -U/[3 - U \sum_\omega \omega^2/(2\varepsilon_p)]$, and $k_0 = -(4\pi/m^2) \sum_\omega \omega^2/(2\varepsilon_p)$, where $\omega_c$ is a high-energy cutoff. In this paper, we also employ $v$ and $k_0$ to measure the interaction strength.

The single-particle Green’s function has the form $G(p, i\omega_m) = [i\omega_m - \xi_p - \Sigma(p, i\omega_m)]^{-1}$, where $\omega_m$ is the fermion Matsubara frequency. The self-energy correction $\Sigma(p, i\omega_m)$ involves effects of p-wave pairing fluctuations. Within the T-matrix approximation, it is diagrammatically given by Fig. 1. Summing up these diagrams, we obtain $\Sigma(p, i\omega_m) = \frac{2}{\beta} \sum_{q, i\nu_n, i,j} \tilde{\Gamma}_{ij}(p, p, q, i\nu_n) G_0(-p + q, -i\omega_m + i\nu_n)$, where $G_0(p, i\omega_m) = (i\omega_m - \xi_p)^{-1}$ is the free fermion Green’s function. $\tilde{\Gamma}_{ij}(p, p', q, i\nu_n) = (p_i - q_i/2) \hat{\gamma}_{ij}(q, i\nu_n)(p'_j - q_j/2)$ is a $3 \times 3$ matrix particle-particle scattering matrix ($i, j = x, y, z$), where $\hat{\gamma}(q, i\nu_n) = -U/[1 - U1(\Gamma, i\nu_n)]$.

The p-wave pair correlation function $\Pi_{ij}$ is given by

$$\Pi_{ij}(q, i\nu_n) = \sum_p p_i p_j \frac{1 - f(\xi_{p+q}/2) - f(\xi_{-p+q}/2)}{\xi_{p+q}/2 + \xi_{-p+q}/2 + i\nu_n}.$$  

(2)
The superfluid transition temperature $T_c$ is conveniently determined by the Thouless criterion, 
\[ 1 = \frac{U}{3} \sum_{\mathbf{p}} \frac{p^2}{2} \tanh \left( \frac{\varepsilon_p - \mu}{2T} \right). \] 
(3)

Since the chemical potential $\mu$ in Eq. (3) is known to remarkably deviate from the Fermi energy $\varepsilon_F$ in the strong-coupling regime, we need to solve the $T_c$-equation (3), together with the equation for the number $N$ of Fermi atoms, 
\[ N = T \sum_{\mathbf{p}, i \omega_m} G(\mathbf{p}, i \omega_m) e^{i \omega_m \delta}, \] 
(4)

and self-consistently determine $T_c$ and $\mu$ for a given interaction strength. We show the calculated $T_c$ and $\mu$ in Fig. 2(a) and (b), respectively. We briefly note that the pairing interaction becomes strong when $1/(vp^2_F)$ becomes large in Fig. 2.

Once $T_c$ is obtained, we next solve the number equation (4) to determine $\mu$ above $T_c$. Using the calculated $\mu(T \geq T_c)$ shown in Fig. 2(c), we calculate DOS $\rho(\omega)$ given by
\[ \rho(\omega) = -\frac{1}{\pi} \sum_{\mathbf{p}} \text{Im} G(\mathbf{p}, i \omega_m \rightarrow \omega + i \delta). \] 
(5)

3. $P$-wave Pseudogap Phenomenon in DOS

Figure 3(a) shows DOS at $T_c$. Starting from the weak-coupling regime ($1/(vp_F^2) = -14.0$), a dip structure around $\omega = 0$ at first gradually becomes remarkable with increasing the interaction strength. However, when the interaction strength exceeds a certain value, the pseudogap start shrinking to eventually disappear, although the pairing interaction becomes strong. (See the results for $1/(vp_F^2) = -8.0 \sim 0.0$ in panel (a).) In the strong-coupling regime where $\mu < 0$ ($1/(vp_F^2) > 0$), DOS gradually reduces to that for a free Fermi gas with a negative chemical potential ($\rho(\omega) \propto \sqrt{\omega + |\mu|}[5]$).

The non-monotonic behavior of the pseudogap seen in Fig. 2(a) is due to the momentum dependence of the $p$-wave interaction, $-U \mathbf{p} \cdot \mathbf{p}'$. In the weak-coupling BCS theory, the momentum region near the Fermi surface is known to be crucial for the Cooper pairing. Since the present pseudogap phenomenon purely comes from strong pairing fluctuations, the momentum region around $\tilde{p}_F = \sqrt{2m\mu}$ ($\mu > 0$) is expected to be also crucial for this precursor phenomenon. When we roughly evaluate the interaction strength around $\tilde{p}_F$, one finds
\[ -U \mathbf{p} \cdot \mathbf{p}' \sim -2Um\mu, \] 
(6)
Figure 3. (a) Calculated DOS $\rho(\omega)$ at $T_c$. (b) Temperature dependence of the pseudogap in DOS when $1/(vp^3_f) = -8.0$. We have offset the results in this panel. (c) Pseudogap temperature $T^*$ (dashed line). The solid line shows $T_c$.

which indicates that there is a competition between the increase of $U$ and the decrease of $\mu$ (See Fig.2(b))., as one approaches the unitarity limit ($1/(vp^3_f) = 0$). In the weak coupling region, Eq.(6) increases with increasing the interaction strength $U$. However, near the unitarity limit, Eq. (6) decreases because $\mu$ approaches zero, as shown in Fig.2(b). Since the $s$-wave interaction $-U$ does not depend on the momentum, the interaction strength at $\tilde{p_F}$ is simply dominated by the interaction strength $U$, leading to the monotonic increase of the pseudogap size[5].

Figure 3(b) shows the temperature dependence of DOS. As expected, the pseudogap structure becomes obscure with increasing the temperature, because of the weakening of $p$-wave pairing fluctuations. Determining the pseudogap temperature $T^*$ as the temperature at which the dip structure disappears in DOS, we obtain Fig.3(c). The region surrounded by $T^*$ and $T_c$ in this figure is the $p$-wave pseudogap regime of cold Fermi gases. Since the maximum $T^*$ is much higher than $T_c$, the pseudogap regime is expected to be experimentally more accessible than the superfluid phase.

4. Summary
To summarize, we have discussed pseudogap phenomena in an ultracold Fermi gas with a $p$-wave pairing interaction. Using the $T$-matrix theory, we have calculated DOS above $T_c$. We showed that the interaction dependence of the $p$-wave pseudogap in DOS is quite different from the $s$-wave case, because of the momentum dependence of the $p$-wave interaction. From the temperature dependence of the pseudogap in DOS, we determined the pseudogap temperature $T^*$, as well as the pseudogap regime in the temperature-interaction phase diagram. Since the measurement of single-particle excitations has recently become possible by the photoemission-type experiment in cold Fermi gases[6], the observation of the pseudogap by strong $p$-wave pairing fluctuations would be useful for the study to realize $p$-wave superfluid Fermi gases.

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