Calculation of the Fatigue Limit of High-Strength Steel Specimens at Different Loading Conditions Based on Inclusion Sizes

Jens Schumacher* and Brigitte Clausen

A calculation method for the fatigue limit of components made of high-strength steels that fail at inclusions is described. The method consists of two parts: first, the size of failure-critical inclusions is determined based on metallographic investigations. Then, the fatigue limit is determined based on the expected inclusion sizes using a fracture mechanics model. Influencing factors such as sample size, heat-treatment conditions, and mean stresses are considered. The model parameters are derived from fatigue test data collected over decades within the framework of several research projects. The dominant influencing factors are the inclusion size and the hardness of the steel matrix. Finally, the model is used to predict the fatigue limit of different specimens made of a bearing steel SAE 52 100. The sizes of failure-critical inclusions and fatigue limits calculated with the model are compared and evaluated on the basis of the inclusion sizes found in the samples’ fracture surface and the experimentally determined fatigue limits.

1. Introduction

In the case of fatigue, component and specimen failure already occur at stress amplitudes below the yield strength of the material. The failure is caused by cracks that initiate at critical defects on the surface or in the volume and subsequently propagate. When a crack is initiated below the surface, inclusions are often the starting points of the crack. This failure mechanism is typical for high-strength steels where failure occurs in the high cycle fatigue (HCF) or very-high cycle fatigue (VCHF) region. In the literature, oxides, nitrides, and sulfides are the most frequently mentioned inclusion types. In general, the fatigue limit of components decreases with increasing inclusion size. According to Murakami, the inclusion type has nearly no influence on the fatigue limit. In contrast, other authors assign different kinds of harmfulness to different inclusion types.

In recent decades, the degree of purity of steels and thus their performance have increased continuously through improvements in the steel manufacturing process. Despite the reduced number of inclusions in the steel and a reduction in their size, they still are the main causes for crack initiation at low cyclic loads, that is, large numbers of cycles to failure, and thus restrict the fatigue limit of steel components. A recognized method of quantifying the proportion of nonmetallic inclusions is the light microscopic examination of metallographic sections to rate the cleanliness of steel batches. How the nonmetallic inclusions are counted and evaluated depending on the chosen standard or evaluation specification. Especially with regard to fatigue properties, it must be noted that the most critical defect always determines the failure behavior, but it is not necessarily found in the metallographic examinations, even if a great effort is made in the metallographic examinations.

Based on experience, the suitability of the respective steel batch for the application can be assessed on the basis of cleanliness. However, a direct quantitative determination of the fatigue limit is not possible on this baseline. Nevertheless, the advantages of metallographic investigations compared with fatigue tests for the characterization of steel batches are significantly lower costs and time saving. This means that for an economic evaluation of the performance of steel a cleanliness evaluation with the current methods should be sought.

The influence of inclusions on the fatigue limit of high-strength steels has been investigated in detail by Murakami. In his investigations, he considers the inclusion area projected perpendicular to the loading direction as a crack of equal size, that is, the stress intensity is assumed identical for an inclusion and a crack under the same external load. The stress intensity factor range \( \Delta K \) at an inclusion can be calculated according to Murakami as follows.

\[
\Delta K = \alpha \cdot \Delta \sigma \cdot \sqrt{\text{area}}
\]  

The stress intensity factor range depends on the stress range \( \Delta \sigma \) and the size of the inclusion \( \sqrt{\text{area}} \). A factor of \( \alpha = 0.5 \) was set.
for inclusions in the volume and $a = 0.65$ for inclusions in contact with the surface. Short cracks behave differently than long ones. For example, the threshold value for cyclic crack propagation depends on the crack length itself. The threshold stress intensity factor range $\Delta K_{th}$ depends, according to Murakami and Yamashita,[13] for an inclusion in the interior of the specimen on the Vickers hardness $H_V$ of the steel and the inclusion size $\sqrt{\text{area}}$ causing failure

$$\Delta K_{th} = 0.00277 \cdot (H_V + 120) \cdot (\sqrt{\text{area}})^{1/3}$$ (2)

In this empiric formula, $\Delta K_{th}$ is expressed in MPa $\sqrt{\text{m}}$ and $\sqrt{\text{area}}$ in $\mu$m. To calculate the fatigue limit of steel specimens, Murakami combined Equation (1) and (2). The relationship between the inclusion size area in the specimen volume, the steel hardness $H_V$, and the fatigue limit $\sigma_W$ is shown in Equation (3).

$$\sigma_W = 1.56 \cdot (H_V + 120) \cdot (\sqrt{\text{area}})^{1/2}$$ (3)

With increasing inclusion size area, the fatigue limit of the steel, in which this inclusion is located, decreases. Consequently, the size of the largest inclusion in the highly stressed volume that varies from specimen to specimen within a batch leads to scatter of the fatigue limit of that batch. However, Murakami does not consider the influence of the spatial arrangement of the inclusions and the inclusion properties, such as the mechanical properties, the connection to the steel matrix, or the inclusion geometry.

Murakami determined his threshold formula on the basis of samples with small artificial defects in the surface. Schumacher et al.[16] used the same approach as Murakami to determine the threshold value. However, information from over 600 fatigue specimens from different steels, more precisely case-hardening steels, quenched and tempered steels, and roll-bearing steels, specimens from different steels, more precisely case-hardening steels, quenched and tempered steels, and roll-bearing steels, was used as basis for data. The determination relationship is shown in Equation (4).

$$\Delta K_{th} = 0.00290 \cdot (H_V + 95) \cdot (\sqrt{\text{area}})^{0.384}$$ (4)

Oxides, sulfides, nitrides, and oxides were found in the fracture surfaces in this investigation. In most cases, their distances from the sample surface clearly exceeded the only size $\sqrt{\text{area}}$. An influence of the nature of the various failure-causing inclusions on the threshold value was not found. The hardness of the steel and the size of the defects are hence considered as the dominant influencing factors.

Besides the inclusion size, the loading condition has a big influence on the fatigue behavior of specimens and parts. Mean stresses can increase or decrease the fatigue limit depending on whether they are compressive or tensile. Multiaxial stress conditions are present or can occur in notches with uniaxial loading. Furthermore, multiaxial residual stresses caused by specimens production or heat treatment may exist in the highly stressed volume and have to be considered in fatigue loading evaluation. Fatigue criteria are used to compare arbitrary loading conditions of parts with fatigue limits determined uniaxially and without mean stresses. From a complex loading condition, an equivalent stress amplitude $\sigma_{eq}$ has to be calculated with a fatigue criterion, which can be compared with the uniaxial determined fatigue limits. When determining the threshold value in Equation (4), Schumacher et al.[16] applied the fatigue criteria of Bomas,[4] Crossland,[17] Dang Van,[18] and Sines[19] to take residual stresses and the loading condition into account. It was found that all four fatigue criteria describe the influencing factors with similar accuracy. The slightly best fit was achieved with the Bomas fatigue criterion (Equation (5)).

$$\sigma_{eq} = 2 \cdot (\tau_{\max} + \sigma_m \cdot p_m)$$ (5)

In the fatigue criterion of Bomas, the equivalent stress amplitude $\sigma_{eq}$ is a linear combination of the maximum shear stress amplitude $\tau_{\max}$ and the mean hydrostatic pressure $p_m$. Mean and residual stresses are included in the calculation of the equivalent stress amplitude via the mean hydrostatic stress. The parameter $\sigma_m$ depends on the material and is usually determined experimentally. According to Schumacher et al.,[16] the parameter $\sigma_m$ is calculated for each individual steel batch depending on its local hardness

$$\sigma_m = 0.00192 \cdot H_V - 0.575$$ (6)

The influence of inclusion size on fatigue limit, as presented by Murakami[7] and Schumacher et al.,[16] shows that predicting the maximum inclusion size in the specimens is a critical factor in fatigue limit estimation.

Different methods are reported in the literature to predict the size of failure-critical inclusions in fatigue specimens based on metallographic investigations.[10,20–23] Two different approaches are pursued to determine the inclusion sizes in cross sections. On the one hand, for several cross sections, the largest inclusion is determined[10,20,22] and on the other hand, all inclusions in a cross section, whose size exceeds a threshold value, are recorded.[10,21–23]

Thumser et al.[20] compared the sizes of failure causing inclusions in fracture surfaces of fatigue specimens and the biggest inclusion sizes found in cross sections. The inclusions found in cross sections are much smaller than those found in the fracture surfaces. An approach for the prediction of the inclusion size in the fatigue specimens based on the metallographic data is not shown. Anderson, Maré and Rootzén,[22] as well as Shi et al.[21] fit various distribution functions to the metallographic data base to predict the failure-critical inclusion sizes. However, no comparison was made with actually occurring failure-critical inclusions. Zhang et al.[10] compared predicted inclusion sizes with inclusion sizes determined experimentally. Based on the largest inclusions found in cross sections, the actual inclusion size in the steel was overestimated and based on the metallographically determined inclusion sizes exceeding a threshold value, the actual inclusion size was underestimated. The authors expect a suitable inclusion size prediction by combining both approaches.

Melander et al.[2] and Meurling et al.[24] used inclusion sizes measured at cross sections to predict the fatigue limit of steels. They combined the inclusion size prediction directly with a fracture mechanical approach to predict the fatigue limit of the specimens. For this reason, no predicted inclusion sizes are available as interim results. Schumacher et al.[23] predicted the fatigue
Limit of specimens failed at manganese sulfides based on metallographic investigations. Again, the expected inclusion sizes were not available as interim results. The experimental investigation involved small bending samples, whose test volume was less than 1 mm³; so a transferability of this model to significantly larger samples cannot be assumed.

The standard ASTM E 2283 is used to evaluate the cleanliness of steels. [12] Within this standard, the biggest inclusion of each cross section is determined and a Gumbel distribution is fit to the inclusion sizes. This approach is not used to determine inclusion sizes in samples but to determine the expected inclusion sizes in a cross section of 150 000 mm². A standard to predict tension sizes in samples but to determine the expected inclusion sizes. This approach is not used to determine inclusion sizes and used as a basis for the inclusion size prediction. The expected inclusion size serves as the parameter, an estimation order and for each inclusion i the cumulative probability $P_i$ is estimated with the following equation.

$$P_i = i/(n_{cs} + 1)$$

The mathematical description of the size of the largest inclusion in the microsection is based on a Gumbel distribution $F_{Gumbel}$ (Equation (9)), as well as a Fréchet distribution $F_{Fréchet}$ (Equation (10)).

$$F_{Gumbel}(d) = \exp\left(-\exp\left(-\frac{(d - \lambda)}{\delta}\right)\right)$$

$$F_{Fréchet}(d) = 2^{-\left(d/d_0\right)^\delta}$$

The Gumbel distribution is used in ASTM E 2283 standard to describe the size distribution of inclusions. The parameters $\lambda$ and $\delta$ in Equation (9) describe the size or the scatter of the determined diameter. Zierbart and Heckel, [25] Huster, [26] and Bomas, Linkewitz, and Mayr [27] assume that the size of the largest inclusion in a sample obeys a two-parameter Fréchet distribution. Hence, this distribution is used in this investigation as well. The parameter $d_0$ is the median of the Fréchet distribution and the parameter $c$ describes the scatter of the inclusion size in Equation (10).

Based on probability theory considerations and with knowledge of the parameters of Equation (9) for the Gumbel distribution and Equation (10) for the Fréchet distribution, the size of the largest inclusion $d_{0,V}$ in a specimen with the highly stressed volume $V$ can be calculated. The size distribution $F_{specimen}$ of the largest inclusion in a specimen depends on the size distribution determined at the cross section $F_{microsection}$ and the ratio of the specimen’s highly stressed volume $V$ and the reference volume $V_0$.

$$F_{specimen} = (F_{microsection})^{\frac{d}{V_0}}$$

$F_{microsection}$ can either be $F_{Gumbel}$ or $F_{Fréchet}$. To determine the median of the largest inclusion $d_{0,V}$ in a specimen, the function $F_{specimen}$ is set equal to 0.5 and solved to $d$. The resulting formula for the Gumbel distribution and the Fréchet distribution are shown in Equation (12) and (13), respectively.

$$d_{0,V} = \lambda - \delta \cdot \ln(V_0/V) \cdot \ln(0.5)$$

$$d_{0,V} = d_0 \cdot (V/V_0)^{1/c}$$
A fracture mechanical approach is used to predict the fatigue limit of specimens based on the expected inclusion sizes. The inclusions are regarded as a crack with the same size as the inclusion in a plane perpendicular to the loading direction, as described by Murakami.[7] The inclusion size $d_{0,V}$ must be converted into the size $\sqrt{\text{area}}$ for this purpose.

$$\sqrt{\text{area}} = d_{0,V} \cdot \frac{\sqrt{R}}{2} \quad (16)$$

The stress intensity factor range $\Delta K$ at an inclusion is calculated by Equation (1). Assuming that the failure-critical inclusions are located inside the samples and are not in contact to or near the surface, a value of 0.5 is used for the $\alpha$ parameter. The threshold value $\Delta K_{th}$ for uniaxial mean stress free loading is calculated according to Equation (4). Combining Equation (1) and (4) leads to the fatigue limit $\sigma_F$ of the specimen.

$$\sigma_F(\sigma_m = 0) = (1.64 \cdot (H_V + 95))/(\sqrt{\text{area}}^{0.116}) \quad (17)$$

In the case that multiaxial stresses or mean stresses are to be considered, the equivalent stress amplitude $\sigma_{ae}$ must be used instead of the stress amplitude $\sigma_a$. This is calculated according to the Bomas[4] fatigue criterion with Equation (5), whereby the parameter $\alpha_m$ is determined by Equation (6).

$$\sigma_{ae} = 2 \cdot (\tau_{a,max} + \alpha_m \cdot \rho_m) = \sigma_F \quad (18)$$

In summary, with the knowledge of the failure-critical inclusion size and the hardness of the steel, the fatigue limit of a specimen can be calculated.

### 3. Experimental Section

For the experimental investigations, the roller-bearing steel 100Cr6 (SAE 52 100) was used. Three types of fatigue samples were taken from bars in such a way that their axes of symmetry and thus their highly stressed volume were at half the radius of the bar material. Their geometry is shown in Figure 1. The highly stressed volumes of the medium and large specimen were 229 and 3483 mm³, whereby only the cylindrical test area of the specimens was considered. For the small specimen, the highly stressed volume was 10.3 mm³, whereby the areas of the specimen that experienced at least 90% of the maximum stress were taken into account. An overview of how the specimens were heat treated and fatigue tested is shown in Figure 1. All specimen types were austenitized at 850 °C for 45 min. After oil quenching of the small and medium specimens or gas quenching of the large specimens, the specimens were tempered at 180 °C for 2 h. A part of the medium-sized specimens was tempered at 240 °C for 2 h. In the final postprocessing, the clamping area was reworked to minimize unwanted superimposed bending loads. In addition, the test area of the samples was reworked to get a smooth compressive residual stress-affected surface to create inclusion failure in the volume instead of surface fatigue failure. Postprocessing of the medium-sized specimens was conducted by grinding and of the small and large specimens by precision turning.

All specimen variants were tested in the area of fatigue limit at a stress ratio of $R = -1$. In addition, the small fatigue specimens and both heat-treatment variants of the middle-sized specimens were tested at a stress ratio of $R = 0$. The fatigue tests were conducted up to a limit of ten million load cycles. Specimens that reached this number of cycles without failure were tested again at a higher stress amplitude until failure, to be able to identify the

![Figure 1](image-url)
weak point, which initiated no failure at the run-out stress amplitude in these cases. The testing of the small specimens was force controlled on a high-frequency pulsator (type Amsler 2HFP) at 98.0 Hz ± 3.1 Hz. The middle-sized specimens were tested force controlled on resonance testing machines (type Rumul, Russenberger Prüfmaschinen AG) at 79.4 Hz ± 1.5 Hz and the large fatigue specimens were also tested force controlled on a horizontal pulsator (type Schenck PHT) at 33.7 Hz ± 0.5 Hz.

For the statistical evaluation of the fatigue limits, the experimental fracture probabilities \( P_{\text{exp}}(S_a) \) were determined for each stress level \( S_a \) using Equation (19). It was calculated from the ratio of the number of fractured samples \( n_a(S_a) \) to the number of tested samples \( n(S_a) \) at a stress level \( S_a \).

\[
P_{\text{exp}}(S_a) = \frac{n_a(S_a)}{n(S_a)}
\]  

(19)

The mathematical description of the scattering fatigue limits was based on the assumption that they obey a two-parameter Weibull distribution (Equation (20)). The parameters of the distribution function of the fatigue limit \( F_{\text{fatigue limit}} \) were determined using the maximum likelihood method. The location parameter \( S_F \) of the Weibull distribution corresponds to the median of the distribution and thus the fatigue limit of the specimen for a fracture probability \( P_f = 0.5 \), whereas the shape parameter \( m \) describes the scatter of the distribution.

\[
F_{\text{fatigue limit}}(S_a) = P_f(S_a) = 1 - 2^{-\left(\frac{S_a}{S_F}\right)^m}
\]  

(20)

All fracture surfaces were investigated with scanning electron microscopy to identify the crack initiation site. In case of inclusion failure, the type, surface distance, and size of the inclusion were determined. To describe the size distribution of the crack-initiating inclusions, a two-parametric Fréchet distribution is used.

\[
F_{\text{Inclusions size}}(\sqrt{\text{area}}) = 2^{-\left(\frac{\text{area}}{\text{area}_0}\right)^{-\alpha}}
\]  

(21)

In Equation (21), \( \sqrt{\text{area}_0} \) is the location parameter and \( \alpha_0 \) the scale parameter. To adjust the parameters to the measured inclusion sizes, the \( n \) inclusions were ordered by size from \( i = 1 \) to \( n \). Equation (22) is used to assign them an experimental exceedance probability \( P_{\text{ex}} \).

\[
P_{\text{ex}} = \frac{i}{n + 1}
\]  

(22)

In total, 25 longitudinal cross sections were taken from the semifinished bars at half the radius to analyze the cleanliness of steel. In addition, longitudinal and cross sections of the fatigue specimens in the final condition were metallographically investigated to characterize the microstructure.

Residual stresses in the fatigue specimens in axial and circumferential direction were measured by X-ray diffraction using a type F diffractometer from the manufacturer Siemens. The measurement was carried out according to the \( \sin^2\psi \) method on the \{211\} lattice planes of the ferritic phases with Cr K\(_\alpha\) radiation. The accelerating voltage was 33 kV and the beam current 40 mA. The aperture had a diameter of 0.5 mm. A total of 11 \( \psi \) angles were measured in the range from \(-45\) to \(45^\circ\). For the evaluation, a modulus of elasticity of 220 GPa and a Poisson’s ratio of 0.28 were assumed. Depth profiles were produced by stepwise electrolytic removal of material layers followed by a stress measurement at the new surface. Though the residual stress state of the specimens was influenced by the removal, the measured residual stresses were not corrected mathematically as the thickness of the removed layer was very low compared with the diameter of the samples.

4. Results

4.1. Experimental Investigation of the Fatigue Specimen

After heat treatment, the samples show a martensitic microstructure. The medium-sized specimens tempered at 240 °C have a hardness of 705 HV, as shown in Table 1. After tempering at 180 °C, the medium and large specimens reached hardness of about 768 HV and the small samples about 779 HV. The grinding of the medium samples leads to compressive residual stresses at the surface in axial and in circumferential direction in the range from \(-331\) to \(-470\) MPa. At a depth of about 10 \( \mu \)m, the residual compressive stresses have almost completely disappeared. The precision-turned small and large specimens showed higher compressive residual stresses at the surface of the specimens compared with the medium-sized specimens. They reached up to \(-821\) MPa in case of the small specimens and the large specimens reached up to \(-1036\) MPa. Residual compressive stresses are measurable in depths of up to 90 or 60 \( \mu \)m, respectively.

In the stress vs. number of cycles to failure (SN) diagram in Figure 2, the experimental fatigue test results are shown. The fatigue tests were conducted for all variants in the respective regime of the fatigue limit. As expected, in the range of the fatigue limit, the stress amplitudes at \( R = -1 \) are above the samples tested at \( R = 0 \). The \( R = -1 \) and \( R = 0 \) areas are separated by a gray horizontal line at the 650 MPa stress amplitude level. It can already be seen in the SN diagram that the endurable stress amplitudes increase as the specimen size decreases. The stress amplitudes of the medium specimens tempered at 240 °C are at \( R = -1 \) below those of the specimens tempered at 180 °C. At \( R = 0 \), the stress amplitudes of the two batches are at the same level.

In addition to the lifetime \( N_f \) of the fatigue specimens, the crack initiation site of the specimens, identified in the fracture surface, is displayed in the SN curve. Scanning electron microscopy of the fracture surfaces identified oxides, titanium nitrides (TiN), the surface, and a nondefect in the volume as the crack initiation points. Micrographs taken at these crack initiation points are shown as examples in Figure 3. As mentioned before, the run-out specimens have been tested again at a higher stress amplitude to identify the weak point, which leads to no failure at the run-out stress amplitude. These are also shown at the respective load horizon at which the limit number of ten million load cycles was reached.

As shown in Table 1, at an \( R \) ratio of \(-1\), most specimens failed at globular-shaped oxide inclusions. Only two of the medium-sized specimens tempered at 180 °C failed at the surface. The small specimens showed besides 12 failures at oxides 3 fractures at titanium nitrides and one specimen showed a
Table 1. Highly stressed volume, hardness, residual stresses, fatigue limit, and failure-causing inclusion of the fatigue specimens.

|                             | Small specimen | Medium specimen | Large specimen |
|-----------------------------|----------------|-----------------|----------------|
| Highly stressed volume V [mm³] | 10.3           | 229            | 3483           |
| Tempering                   | 180 °C 2 h     | 180 °C 2 h     | 240 °C 2 h     | 180 °C 2 h     |
| Hardness [HV]               | 779 ± 19       | 768 ± 9        | 705 ± 11       | 768 ± 10       |
| Residual stress at the surface |                |                |                |
| Axial [MPa]                 | −665 ± 148     | −470 ± 29      | −399 ± 17      | −1036 ± 36     |
| Circumferential [MPa]       | −821 ± 62      | −391 ± 40      | −331 ± 49      | −897 ± 20      |
| Influenced depth [μm]       | 90             | 10             | 10             | 60             |
| Stress ratio R              | −1             | 0              | −1             | 0              | −1             |
| Fatigue strength distribution $F_{fatigue}$ strength $S_f$ [MPa] | 888 | 538 | 880 | 532 | 783 | 523 | 725 |
| $m$                         | 28.5           | 6.0            | 42.8           | 33.0           | 32.3           | 23.8           | 20.8           |
| Failure causes and its number |                |                |                |                |
| Oxide                      | 12             | 4              | 14             | 13             | 19             | 14             | 17             |
| TiN                        | 3              | 8              | −              | 6              | −              | 1              |
| Nondefect                  | 1              | −              | −              | −              | −              | −              |
| Surface                    | −              | 11             | 2              | −              | −              | −              |
| Unknown                    | 1              |                |                |                |                |                |
| Oxide size distribution in fracture surface $\sqrt{area_0}$ [μm] | 25.0 | 20.7 | 30.1 | 27.8 | 31.3 | 28.9 | 45.8 |
| $C_0$                      | 3.4            | 1.5            | 5.8            | 8.7            | 6.4            | 3.4            | 1.8            |
| TiN size distribution in fracture surface $\sqrt{area_0}$ [μm] | 9.9 | 9.5 | − | 10.4 | − | − | − |
| $C_0$                      | 13.4           | 9.4            | −              | 7.7            | −              | −              |

Figure 2. Fatigue test result of the small, medium-sized, and large specimens (TiN = titanium nitride).
featureless failure, which means the steel matrix itself was the weak point. In case of loading at $R = 0$, the oxides are again the main failure cause for the medium-sized specimens at both tempering temperatures. Besides the fractures at globular-shaped oxide inclusions, one specimen tempered at 240°C failed at titanium nitrides. Only four specimens of the small specimens tested at $R = 0$ failed at oxide inclusions. Most of them failed at the surface or at titanium nitrides.

In the case of inclusion failure, the distance of the inclusions from the sample surface was determined. For the majority of inclusions, their distance to the sample surface is a multiple of their size $\sqrt{\text{area}}$. Here, no additional stress increase is to be expected due to a vicinity to the surface. In a total of five samples, inclusions were found near the sample surface. Near surface in this case means that the distance of the inclusion from the surface is less than its size $\sqrt{\text{area}}$, as described by Kolyshkin et al.\(^{[28]}\) In two of the small samples, which were tested at a stress ratio of $R = 0$, oxides were found near the surface. In the medium-sized samples, which were annealed at 180°C, one near-surface oxide was found at both stress ratios, and in the samples annealed at 240°C, one near-surface oxide was found at the $R$ ratio of $-1$. Due to the vicinity to the surface, an additional increase in stress or stress intensity is to be expected for these five specimens.

To take a closer look at the influence of the inclusions on the fatigue behavior, the stress intensity range is calculated for each failure-causing inclusion with Equation (1). For the five samples with inclusions close to the surface, $\alpha$ is set to 0.65. For the remaining samples with inclusions far from the surface, an $\alpha$ value of 0.5 is used. The residual stress measurements have shown that residual compressive stresses are only present in the region close to the surface. For the inclusions in the interior, the residual stresses can therefore be neglected. In the three middle-sized samples with inclusions near the surface, part of the inclusion is no longer within the residual compressive stress zone, so the residual stresses can also be neglected here. The same applies to the two small specimens with inclusions close to the surface. Hence, for the following calculations, the residual stresses are assumed to be zero.

The stress range $\Delta \sigma$ in Equation (1) corresponds to twice the stress amplitude $\sigma_a$. To be able to take the mean stresses into account when calculating the stress intensity factor range, an equivalent stress amplitude $\sigma_{ae}$ is used, which is calculated with Bomas’ fatigue criterion (Equation (5)). According to Bomas’ fatigue criterion, for a stress ratio of $R = -1$ and $R = 0$, the relation between the equivalent stress amplitude $\sigma_{ae}$ and the load stress amplitude $\sigma_a$ is shown in Equation (23) and (24), respectively. If residual stresses had to be taken into account, they would have been included in Equation (23) and (24) via the fatigue criterion.

$$\sigma_{ae} = \sigma_a$$

**Figure 3.** Exemplary SEM images of the four occurring causes of failure: a) Aluminum oxide, medium specimen, tempering $= 180^\circ$C, $R = 0$, $S_a = 570$ MPa/$N_f = 6.711.000$. b) Titanium nitride, medium specimen, tempering $= 180^\circ$C, $R = 0$, $S_a = 500$ MPa/$N_f = \text{run out} + S_a = 540$ MPa/$N_f = 7.351.800$. c) Nondefect, small specimen, tempering $= 180^\circ$C, $R = -1$, $S_a = 900$ MPa/$N_f = \text{run out} + S_a = 940$ MPa/$N_f = \text{run out} + S_a = 980$ MPa/$N_f = \text{run out} + S_a = 1020$ MPa/$N_f = 688.110$. d) Surface, medium specimen, tempering $= 180^\circ$C, $R = -1$, $S_a = 900$ MPa/$N_f = 75.100$. 
Taking into account that the stress range $\Delta \sigma$ corresponds to twice the equivalent stress amplitude $\sigma_{ae}$, the equivalent stress intensity factor range $K_{eq}$ is calculated for each oxide and each titanium nitride inclusion. Figure 4 shows the fatigue results of the specimens with inclusion failure from Figure 2, with the load stress amplitude replaced by the equivalent stress intensity factor range $K_{eq}$ on the ordinate. In this visualization of the fatigue test results and by not taking into account the mostly early failures at the surface, the trend that the fatigue life decreases with increasing load is more clearly visible than in the SN curve, where the stress amplitude is plotted as a load factor. Especially with the stress ratio of $R = 0$, it is noticeable that the stress intensity factor range $K_{eq}$ for the samples that failed at titanium nitrides is lower than that for samples that failed at oxides.

According to Murakami, however, not only the equivalent stress intensity factor range $\Delta K_{eq}$ is dependent on the defect size, but also the threshold value $\Delta K_{th}$ of the steel itself. To take this effect into account when presenting the fatigue results, the individual threshold $\Delta K_{th}$ value is calculated for each specimen with inclusion failure using Equation (4). The equivalent stress intensity factor range $\Delta K_{eq}$ normalized to the respective threshold $\Delta K_{th}$ value is shown in Figure 5. They are the only samples that have a $\Delta K_{eq}/\Delta K_{th}$ ratio above 1.1.

The size distributions of the failure-causing inclusions from the fatigue fracture surfaces are shown in Figure 6 separately for oxides and titanium nitrides. In addition to the measured defect sizes and the corresponding exceedance probabilities, the distribution functions fit according to Equation (21) are shown as well. The corresponding distribution parameters are shown in Table 1.

As shown in Figure 6 and on the basis of the median size $\sqrt{\text{area}_0}$ of the fracture-causing oxide inclusions shown in Table 1, the crack-initiating oxide size increases with increasing specimen size. The median size of the crack-initiating inclusions in the small specimens tested at $R = -1$ and $R = 0$ was 25 or 20.7 $\mu$m, respectively. The medium-sized specimens showed a median critical inclusion size of about 30 $\mu$m and the large specimens of about 46 $\mu$m. The scatter parameter $c_0$ of the specimens ranged between 1.5 and 8.7. As a decreasing scatter parameter means a higher scatter, the large and the small specimens tested at $R = 0$ showed the highest scatter of fracture causing oxide inclusion sizes. The size distribution of the failure-causing titanium nitrides could only be determined for the small fatigue specimens for both $R$ ratios and for the medium-sized fatigue
specimen, which was annealed at 180 °C and loaded at $R = 0$. The titanium nitrides are smaller than the failure-causing oxide inclusions with a median of just below 10 μm for the small samples and 10.4 μm for the medium-sized samples. In the case of the medium specimens, which were tempered at 240 °C and tested at a stress ratio of $R = 0$, one sample failed at titanium nitride. This inclusion size fits with $\sqrt{\text{area}} = 10.3$ to the sizes determined in the previous mentioned batches. The scatter of the titanium nitride size tends to be smaller than the scatter of the oxide sizes.

Figure 5. Fatigue test result of the specimens with inclusion failure as a function of the equivalent stress intensity factor range $\Delta K_{eq}$ normalized to the individual threshold value $\Delta K_{th}$: (a) $R = -1$; (b) $R = 0$.

Figure 6. Size distributions of the crack-initiating oxide and titanium nitride inclusions.

Figure 7. Distribution function of the fatigue limits of the small, medium-sized, and large specimens at push–pull ($R = -1$) and alternating tensile loading ($R = 0$).
To obtain experimental fatigue limits, which are used for the evaluation of the calculated fatigue limits, the fatigue test results shown in Figure 2 are used as a basis. The fracture probabilities determined for each load horizon and the Weibull distributions adjusted to them are shown in Figure 7. The median $S_F$ and scatter parameter $m$ of the fatigue limit distribution (Equation (20)) adopted to the fatigue test results are shown in Table 1.

As was already evident from the SN diagram, the fatigue limit decreases with increasing specimen size. This is noticeable for both $R$ ratios. The higher tempering temperature of the medium specimen leads to a decrease in fatigue limit at both stress ratios. However, the reduction is lower at an $R$ ratio of 0. Note that the smaller the scatter parameter $m$ of the Weibull distribution, the greater the scatter of the fatigue limit. As shown in Figure 7 and Table 1, the small specimen tested at $R = 0$ shows a large scatter in comparison with the other test variants for which the scattering is at a similar level.

### 4.2. Inclusion Size Prediction

As the fatigue specimens failed at oxide inclusions and at titanium nitride inclusions, both inclusion types have been regarded for the inclusion size prediction. It can be assumed that the globular oxides as well as the cube-shaped edgey titanium nitrides have the same projected area regardless of the microsection orientation. The largest globular oxides and largest titanium nitrides found in longitudinal sections measuring $S_0 = 210 \, \text{mm}^2$ were used as input data in this study. In Figure 8, the determined equivalent inclusion diameters $d$ are shown. They range between 9.5 and 51 $\mu\text{m}$. The titanium nitrides found are usually smaller than the oxides, with a size of 6.8–15 $\mu\text{m}$. The Gumbel and the Fréchet distribution according to Equation (9) and (10) have been fit to the metallographical examination results. In the case of the oxides, the Fréchet distribution describes the oxide sizes more accurately than the Gumbel distribution, especially in the region of the largest inclusion. In the case of titanium nitrides, both distribution functions show a similar course. Overall, the Gumbel distribution is somewhat steeper than the Fréchet distribution in the region of the largest inclusions. The size distribution of the titanium nitrides in the area of the largest inclusions is better described by the Gumbel distribution than by the Fréchet distribution. The corresponding distribution parameters are shown in Table 2.

The corresponding reference volumes are calculated for the Fréchet distribution with Equation (14) and for the Gumbel distribution with Equation (15). For the titanium nitrides, the reference volume for both distribution functions is about 2.1 $\text{mm}^3$. For the oxides, a reference volume of 3.58 $\text{mm}^3$ is determined based on the Fréchet distribution and a reference volume of 3.83 $\text{mm}^3$ is calculated based on the Gumbel distribution.

Based on the parameters of both distribution functions, the expected largest oxide and titanium nitride inclusion size was calculated for the different sample types. Equation (13) was used to determine the largest inclusion with the parameters of the Fréchet distribution. The largest inclusion based on the Gumbel distribution was determined using Equation (12). For this purpose, the highly stressed volumes of the specimens shown in Table 1 were used. It is important to note that the calculated equivalent diameters are converted to area using Equation (7).

Sections in specimens with compressive residual stresses have a reduced probability of failure compared with equally loaded specimens without residual stresses. These consequently reduce the highly stressed volume of the fatigue specimens. However, as the specimens in this study have only a very thin compressive residual stress zone in the near-surface region, the total highly stressed volume from Table 1 was used for the inclusion size prediction. The calculated sizes of the largest inclusion are shown in Table 3.

### Table 2. Fréchet and Gumbel distribution parameters of the largest oxide and largest titanium nitride inclusions found in the cross sections.

| Inclusion type | Fréchet $V_0$ [mm$^3$] | Gumbel $V_0$ [mm$^3$] |
|----------------|-------------------------|------------------------|
| Oxide         | 3.58                    | 3.83                   |
| TiN           | 3.83                    | 4.14                   |

### Table 3. Experimentally determined median inclusion sizes and calculated median inclusion sizes.

| Batch            | Median inclusion size $\sqrt{\text{area}}$ [\mu m] | $\sqrt{\text{area}}$ [\mu m] |
|------------------|--------------------------------------------------|-------------------------------|
|                  | Oxide                                            | Titanium nitride              |
| Experiment Gumbel| Fréchet                                          | Experiment Gumbel              |
| Small 180 °C $R = -1$ | 25.0 21.6 22.7 9.0 11.6 12.4 | Small 180 °C $R = -1$ | 20.7 9.5 |
| Small 180 °C $R = 0$  | 20.7 9.5                                          | Medium 180 °C $R = -1$ | 30.1 38.6 74.7 16.8 23.4 |
| Medium 180 °C $R = -1$ | 30.1 38.6 74.7 16.8 23.4 | Medium 180 °C $R = 0$  | 27.8 10.4 |
| Medium 240 °C $R = -1$ | 31.3                                              | Medium 240 °C $R = 0$  | 28.9 10.3 |
| Medium 240 °C $R = 0$  | 28.9 10.3                                         | Large 180 °C $R = -1$ | 45.8 53.5 212.8 21.3 41.1 |
| Large 180 °C $R = -1$ | 45.8 53.5 212.8 21.3 41.1 | Large 180 °C $R = 0$  | 25.8 10.3 |

a) only one specimen failed at a titanium nitride.
oxides and largest titanium nitrides are shown in Table 3. For comparison, the medians of the oxide sizes found in the fracture surfaces and, in the case that this cause of failure occurred, the experimentally determined titanium nitride sizes are also shown here. A direct comparison of the calculated and experimentally determined inclusion sizes is shown in Figure 9.

With the Gumbel distribution, the oxide sizes are much better predicted than with the Fréchet distribution. The latter clearly overestimates the oxide size. The deviation increases with increasing sample size. Based on the Gumbel distribution, a satisfactory correlation is obtained for the oxide inclusions for all sample sizes. In the fracture surfaces, the failure-causing titanium nitride inclusions are smaller than the failure-causing oxide inclusions. Based on both distribution functions, titanium nitride inclusion sizes are calculated that are smaller than the expected oxide inclusions. For those batches where a comparison with the experimentally determined titanium nitrides is possible, a more accurate prediction is made on the basis of the Gumbel distribution than with the Fréchet distribution. With the Fréchet distribution, the titanium nitride size found in the fracture surfaces is overestimated, as it is the case with the oxides.

### 4.3. Fatigue Limit Calculation

The calculation of the fatigue limit of the samples is based on the calculated largest oxide and titanium nitride inclusions present inside the samples. The possible failure of the samples at the surface or at inclusions near the surface is not taken into account in this calculation approach. As the samples from this investigation only have a thin surface-near layer with significant residual compressive stresses and the failure at near-surface inclusions is not considered in this calculation, the residual stresses are not taken into account in the further calculation. Hence, for the calculation of the fatigue limit, only the mean stress must be taken into account at the alternate tensile loading.

By equating the equivalent stress amplitude \( \sigma_{ae} \) from Equation (23) or (24) with the uniaxial mean stress free fatigue strength \( \sigma_f \) of Equation (17), the permanently bearable load stress amplitude \( \sigma_a \) can be calculated for the respective stress ratio \( R \).

\[
\sigma_a(P_f; R = 0) = \sigma_f(R = 0) = \frac{1.64 \cdot (H_v + 95)}{\sqrt{\text{area}^{0.116}}} \cdot (1 + (2 \cdot \alpha/3)) \tag{26}
\]

Based on the oxide sizes and titanium nitride sizes shown in Table 3 and the hardness shown in Table 1, the fatigue limit of the different fatigue specimens is calculated for both inclusion types. This means that two fatigue limits are calculated for each sample. The parameter \( \alpha_m \) was determined for the respective hardness of the samples with Equation (6). Figure 10 and Table 4 compare the calculated and experimentally determined fatigue limits. The fatigue limits calculated on the basis of the Gumbel distribution are above the respective fatigue limits calculated with the Fréchet distribution. On the basis of the oxide size distribution, smaller fatigue limits are calculated on the basis of the titanium nitride size distribution. Therefore, based on the model, failure on oxides is expected.

For a comparison with the experimentally determined fatigue limits, the oxide-based calculated fatigue limits are the most suitable. Here it can be seen that the fatigue limits are better described with the oxide sizes determined on the basis of the Fréchet distribution than with the Gumbel distribution. Based on the oxide sizes determined with the Gumbel distribution, the fatigue strengths are overestimated. Altogether, the calculation model is able to illustrate the respective influencing factors on the fatigue strength well.

In the case that two or more inclusion types have to be considered at the same time as failure cause, the fatigue behavior of the specimens can be described additionally in a different way with a combined calculation approach. The survival probability of the sample \( P_{\text{survival}} \) in this investigation can be described as a function of the failure-critical inclusion size \( d_{\text{critical}} \) based on the distribution function of the largest oxides in the microsection \( F_{\text{Oxide}} \) and the distribution function of the largest titanium nitrides in the microsection \( F_{\text{TiN}} \).
The survival probability of the sample therefore depends on the probability of the sample not exceeding the critical defect size \(d_{\text{critical}}\). When an inclusion becomes critical dependent not only on the material but also on the load level, the critical inclusion size can be calculated for the push–pull and for the alternate tensile loading by rearranging Equation (25) or (26) according to the defect size. Taking into account the relationship between equivalent diameter \(d\) and \(\sqrt{\text{area}}\) in Equation (7), the critical defect size \(d_{\text{critical}}\) is shown in Equation (28) for push–pull loading and in Equation (29) for alternate tensile loading.

\[
d_{\text{critical}}(R = -1) = \frac{2}{\sqrt{\pi}} \left( \frac{1.64 \cdot (H_V + 95)}{\sigma_a} \right)^{1/0.116}
\]  

\[
d_{\text{critical}}(R = -1) = \frac{2}{\sqrt{\pi}} \left( \frac{1.64 \cdot (H_V + 95)}{\sigma_a \cdot (1 + (2 \cdot \alpha/3))} \right)^{1/0.116}
\]  

From summary, this approach determines which defect size is failure critical at a given load. Based on the inclusion size distributions determined on the microsections, the probability of having a defect size smaller than this is determined, separately for oxides and titanium nitrides. The load stress amplitude \(\sigma_a\) that leads to a survival probability of \(P_{\text{survival}} = 0.5\) corresponds to the fatigue limit of the specimen. The fatigue limits calculated with this combined approach are shown in Figure 11 and in Table 4.

With the combined calculation approach, almost the same fatigue limits are calculated as with the calculation approach based only on the size distribution of the oxides. Only in the case of small specimens a slightly lower fatigue limit is calculated here than with the oxide approach.

5. Discussion

In this article, a prediction method for the fatigue limit of high-strength steels is described. The method combines the calculation of the failure-critical inclusions size in specimens based on metallographic investigations and a fatigue limit calculation approach which uses the failure-critical inclusions size as input parameter. The applied model parameters were adapted to a database of fatigue data from several already completed research projects.\(^{[16]}\) The calculation approach includes the integration of Bomas’ fatigue criterion to be able to consider any stress condition.\(^{[4]}\) This approach has the advantage that the required model parameters are adapted in advance to a broad data basis including results from different steels. On the one hand, this means that no reference tests have to be conducted to determine a suitable model parameter set. On the other hand, as the previously determined model parameter set was suitable to satisfactorily describe the fatigue limit of case hardening steels, quenched and tempered steels, and rolling-bearing steels, it is expected that this data set is suitable for a fatigue limit prediction of different steels.

The fatigue tests conducted experimentally on the specimens made of 100Cr6 steel show that different causes of failure can occur. Besides mainly oxides, crack initiation also occurred on titanium nitrides or at the surface. When looking at the lifetimes in the SN diagram in Figure 2, it is noticeable that there is a large scatter in different batches on several load horizons. In some cases, the lifetimes on one load horizon differ by more than two decades. This wide scatter is caused, among other things, by the competing causes of failure. The failures at the surface usually occur earlier than failures at inclusions in the interior. However, even if only the specimens that failed at inclusions are considered, in some cases, there is still a large difference.
between the fracture load cycle numbers on a single load horizon. By plotting the number of cycles to failure as a function of the equivalent stress intensity factor range $\Delta K_{eq}$ (Figure 4), the fatigue life scatter for specimens stressed with a similar equivalent stress intensity factor range decreases significantly for most batches. This shows that the fracture-mechanical evaluation of the inclusions, as it is also used in the calculation method presented, is useful. It is shown in Figure 4 that the titanium nitrides lead to failure at somewhat lower stress intensities than the oxides. At first glance, this suggests that the titanium nitride inclusions have a more detrimental effect on the fatigue limit than the oxides. According to Murakami, however, not only the stress intensity depends on the defect size but also the threshold value of the material itself. The titanium nitrides that cause failure are usually smaller than the oxides that cause failure (Figure 6). Consequently, the threshold value $\Delta K_{th}$ of the steel according to Equation (4) is also lower for the small titanium nitrides than for the oxides. If the calculated stress intensity factor ranges $\Delta K_{eq}$ are related to the respective threshold values, the difference between oxides and titanium nitrides disappears, as shown in Figure 5. In addition, the normalization to the individual threshold value also leads to a further decrease in the fatigue life scatter of the oxides compared with the non-normalized data shown in Figure 4. This proves that the application of a defect size-dependent threshold value in fatigue limit calculation is a suitable approach. Furthermore, Figure 5 shows that if the inclusion sizes are included in the assessment of harmfulness, no distinction needs to be made between the inclusion types.

In a majority of the samples with inclusion failure, the inclusions were in the interior of the samples. Only in five samples the inclusion-causing failure was located near the surface of the sample. When calculating the stress intensity, a factor $\sigma$ of 0.65 was used for these near-surface inclusions instead of the value of 0.5. This gives the inclusions higher harmfulness compared with the internal inclusions. In Figure 5, the five samples failing at near-surface inclusions are the only samples that have a stress intensity ratio of more than 1.1. At the same time, it is shown in Figure 5 that these specimens achieve some of the lowest numbers of cycles to failure, which prove the higher harmfulness of inclusions that are close to the surface compared with the internal inclusions. For an optimal calculation of the fatigue limit, the sample should therefore be divided into a near-surface region and an inner region. The volume of the near-surface region is small compared with the volume of the inner region. Furthermore, the residual compressive stresses in the surface layer of the specimens studied here compensate for the harmfulness of the inclusions near the surface. As the low failure rate of near-surface inclusions shows, the restriction to the internal inclusions conducted in this work is a simplification, but it is acceptable due to the existing high residual compressive stresses in the near-surface region of the samples.

The fracture surface investigations of the fatigue specimens have shown that oxides and titanium nitrides are the failure-causing inclusion types. As shown in Figure 6, the failure-causing titanium nitrides are smaller than the failure-causing oxides. From the small to the medium-sized sample, there is a very small increase in the size of the failure-causing titanium nitrides. For the oxides, a clear increase in the failure-causing inclusion size with increasing sample size can be seen. At the same time, the fatigue limit decreases with increasing specimen volume, as shown in Figure 7, especially in the comparison of the medium-sized and the large specimens. The small sample shows only a slightly higher fatigue strength than the medium-sized sample. However, this fact can be explained by the comparatively large proportion of small samples failing at titanium nitrides and the surface as a competing cause of failure to the oxides. It can be assumed that some of these samples would probably have broken at higher cycle numbers or higher loads on oxides, if the weak points surfaces and titanium nitrides were not present. In summary, the investigation shows that a statistical size effect is present in the inclusions and that this also has a measurable influence on the fatigue limit. This must be taken into account when predicting the fatigue limit.

To be able to conduct an optimal fatigue limit prediction, all inclusion types would have to be taken into account, especially if no information on the cause of the failure is available in advance. In the case of this study, it is known from experimental fatigue tests that the oxides are the dominant causes of failure, and titanium nitrides occur as a cause of failure as well. Therefore, this study was limited to these two inclusion types. With the Fréchet distribution, the size of the largest oxides in the microsections can be described more precisely than with the Gumbel distribution (Figure 8). In the area of the largest and therefore most critical oxide inclusions, too little scatter is expected with Gumbel distribution. In contrast, the largest distribution of titanium nitrides is better described by the Gumbel distribution than by the Fréchet distribution. Nevertheless, based on the Gumbel distribution, the failure critical oxide and titanium nitride sizes in the fatigue samples are predicted better than with the Fréchet distribution (Figure 9). With the Fréchet distribution, the oxide size is clearly underestimated for medium and large samples. One possible explanation for this may be that the globular oxide inclusions have different formation histories. This could mean that they obey different distributions. However, this difference is not visible in the cross section by light microscopy. If these intermixed oxides are described undifferentiated with a single distribution function, too much scatter is determined. Consequently, too large oxides are predicted for the fatigue specimens. The Gumbel distribution generally proceeds above the Fréchet distribution in the area of the largest inclusions, as shown in Figure 8. This means that on the basis of Gumbel distribution, smaller inclusions are predicted for a volume that is larger than the reference volume than with the Fréchet distribution. It is possible that the Gumbel distribution, whether mixed or not, is better suited for inclusion size prediction than the Fréchet distribution. To confirm this assumption, further steel batches will have to be examined in the future. Furthermore, the exact chemical composition of the largest oxides in the cross section should be examined more closely with scanning electron microscopy to obtain an indication of possible differences.

In the fatigue limit calculation approach used, the fatigue limit decreases as the inclusion size increases. Consequently, a lower fatigue limit is calculated on the basis of the oxides than on the basis of the titanium nitrides. As there are both titanium nitrides and oxides in the fatigue specimens, the larger inclusions, in this case the oxides, are decisive for the achievable fatigue limit.
As shown in Figure 5, the transition from nonfractured to fractured samples is at a $\Delta K_{eq}/\Delta K_{th}$ ratio of 0.9 or at a ratio of 0.8 for the large samples, respectively. That is, the approach chosen slightly overestimates the fatigue limit of the samples investigated in this project. In an ideal prediction, the fracture–no fracture transition would be at a $\Delta K_{eq}/\Delta K_{th}$ ratio of 1. Consequently, as shown in Figure 10, an overestimated fatigue limit is calculated for most specimens. Therefore, more accurate fatigue limit predictions are made on the basis of the oxide Fréchet distribution, as the overestimation of the fatigue limit is compensated by the oxides that are determined as too large. In general, it can be assumed that the actual fatigue limit is sometimes overestimated and sometimes underestimated with the chosen calculation approach, as not all influencing factors on the fatigue limit are taken into account with this approach. In this particular case, the incorrect oxide size determined on the basis of the Fréchet distribution compensates for the deviation in the fatigue limit calculation. However, if the calculation approach had underestimated the fatigue limit, the inclusion sizes based on the Fréchet distribution would have further increased the fatigue limit deviation. Therefore, the inclusion sizes determined on the basis of the Gumbel distribution should be used for the fatigue limit calculation, as this allows a more accurate inclusion size prediction according to the current status.

In the combined fatigue limit calculation approach, the size distributions of the oxides and titanium nitrides are considered simultaneously. This approach is particularly important when similar inclusion sizes are predicted for a specific sample type. In this case, both causes of failure occur in direct competition with each other and the resulting fatigue limit is lower than the fatigue limits calculated for each inclusion type individually. Only in the case of small specimens a slightly lower fatigue limit is calculated than with the oxide approach. That means, a few crack initiations at titanium nitrides are expected in the small samples. With increasing sample size, however, the oxide inclusions are dominant and determine the fatigue limit. In the experiment, titanium nitride failures occurred in the small samples and in two batches of the medium-sized samples. The calculation model slightly underestimates the failure probability on titanium nitrides, but the tendency is correctly captured.

6. Conclusion

Based on the investigation conducted, the following conclusions can be drawn. 1) Different causes of failure can occur when specimens or components are subjected to fatigue loading. For an optimal fatigue limit prediction, all possible causes of failure must therefore be taken into account, especially if it is not clear in advance which failure causes are to be expected. 2) For specimens of different sizes, an increase in the size of the failure-causing inclusion can be seen within the highly stressed specimen volume. When predicting the defect size from metallographic sections, especially with regard to larger components, it is essential to take this effect into account. 3) The inclusion size and the hardness of the steel are the dominant influencing factors on the fatigue limit. 4) The size of the failure-critical inclusions can be calculated on the basis of inclusion sizes determined in metallographically investigated microsections. Here, the statistical size effect must be taken into account. 5) Possibly the oxide inclusions found in the steel have different origins. In this case, it would be more accurate to describe the different oxide inclusions with separate distribution functions. A more precise prediction of the size of the inclusions would be the consequence. 6) By linking the inclusion size prediction with the calculation approach for the fatigue limit as a function of the defect size, it is possible to calculate the fatigue limit of specimens. Only metallographically determined inclusion sizes and the steel hardness are required as input data. With this approach, different $R$ ratios, different specimen sizes, different steels, and different heat-treatment conditions can be considered. 7) To be able to estimate the fatigue limit of a component that is subject to cyclic loading, only metallographically determined inclusion sizes and the hardness of the steel matrix are required. 8) Using the combined calculation approach, it is possible to consider several types of inclusions in the fatigue limit prediction, which represent competing causes of failure.

Acknowledgements

The IGF Project (18973 N) of the Arbeitsgemeinschaft Wärmebehandlung und Werkstofftechnik e. V. (AWT) was funded by the Arbeitsgemeinschaft industrieller Forschungsvereinigungen Otto von Guericke e. V. (AiF) through the IGF program of the Bundesministerium für Wirtschaft und Energie (BMWi) due to a resolution of the Deutscher Bundestag. The authors are grateful for financial support. The authors would like to dedicate this publication to Professor Dr.-Ing. habil. Wolfgang Bleck on the occasion of his 70th birthday. Together, they have been investigating the case of influences of different sizes of inclusions since 1998, though the aim back then was more to pin grain boundaries with small inclusions to increase the grain size stability than to investigate the initiation of fractures. Still, cleanliness was also always a topic of discussions and sometimes destroyed the hopes in showing a significantly better fatigue limit based on grain size stabilization. Because of this we are confident that Professor Bleck will enjoy this article in remembrance of a long research friendship. Congratulations and all the best.

Open access funding enabled and organized by Projekt DEAL.

Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

Research data are not shared.

Keywords

fatigue behavior, fatigue failure at inclusions, fatigue limit calculations, inclusion size predictions, multiaxial fatigue

Received: April 29, 2021
Revised: July 21, 2021
Published online: August 31, 2021

[1] J. Rösler, H. Harders, M. Bäker, *Mechanisches Verhalten der Werkstoffe*, Vieweg + Teubner Verlag, Wiesbaden 2008.
[2] A. Melander, M. Rolffson, A. Nordgren, B. Jansson, H. Hedberg, T. Lund, Scand. J. Metall. 1991, 20, 229.
[3] H. Bomas, M. Schleicher, Fatigue Fract. Eng. Mater. Struct. 2005, 28, 983.
[4] H. Bomas, M. Bacher-Hoechst, R. Kienzler, S. Kunow, G. Loewisch, F. Muehleder, R. Schroeder, Fatigue Fract. Eng. Mater. Struct. 2010, 33, 126.
[5] J. Ma, B. Zhang, D. Xu, E. H. Han, W. Ke, Int. J. Fatigue 2010, 32, 1116.
[6] U. Krupp, K. Koschella, A. Giertler, Procedia Struct. Integrity 2019, 23, 517.
[7] Y. Murakami, Metal Fatigue: Effects of Small Defects and Nonmetallic Inclusions, Elsevier, Oxford, 2002.
[8] [15] Y. Murakami, Y. Yamashita, Procedia Eng. 2014, 74, 6.
[16] J. Schumacher, B. Clausen, H.-W. Zoch, MATEC Web Conf. 2018, 165, 14003.
[17] B. Crossland, in Proc. of the Inter. Conf. on the Fatigue of Metals, Institute of Mechanical Engineers, London 1956, p. 138.
[18] K. Dang Van, B. Griveau, O. Message, in Biaxial And Multiaxial Fatigue EGF 3 (Eds: M. W. Brown, K. J. Miller), Mechanical Engineering Publications, London, England 1989, p. 479.
[19] G. Sines, in Metal Fatigue (Eds: G. Sines, J. L. Waisman), MCGraw-Hill Book Co. Inc., New York 1959, p. 145.
[20] R. Thumser, S. Kleemann, J. W. Bergmann, A. Kleemann, Int. J. Fatigue 2012, 41, 52.
[21] G. Shi, H. V. Atkinson, S. M. Sellars, C. W. Anderson, Acta Metall. 1999, 47, 1455.
[22] C. W. Anderson, J. de Maré, H. Rootzén, Acta Mater. 2005, 53, 2295.
[23] J. Schumacher, H. Bomas, H.-W. Zoch, Int. J. Fatigue 2012, 41, 119.
[24] F. Meurling, A. Melander, M. Tidesten, L. Westin, Int. J. Fatigue 2001, 23, 215.
[25] W. Ziebart, K. Heckel, Z. Werkstofftech. 1977, 8, 105.
[26] J. Huster, Ph. D. Thesis, Universität der Bundeswehr München 1988.
[27] H. Bomas, T. Linkewitz, P. Mayr, Fatigue Fract. Eng. Mater. Struct. 1999, 22, 733.
[28] A. Kolyshkin, A. Grigorescue, E. Kaufmann, M. Zimmermann, H.-J. Christ, Procedia Struct. Integrity 2016, 2, 1083.