New estimates of the CMB angular power spectra from the WMAP 5 yrs low resolution data

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ABSTRACT
A Quadratic Maximum Likelihood (QML) estimator is applied to the WMAP 5 year low resolution maps to compute the CMB angular power spectra at large scales for both temperature and polarization. Estimates and error bars for the six angular power spectra are provided up to $\ell = 32$ and compared, when possible, to those obtained by the WMAP team, without finding any inconsistency. The conditional likelihood slices are also computed for the $C_\ell$ of all the six power spectra from $\ell = 2$ to 10 through a pixel based likelihood code. Both the codes treat the covariance for $(T,Q,U)$ in a single matrix without employing any approximation. The inputs of both the codes (foreground reduced maps, related covariances and masks) are provided by the WMAP team. The peaks of the likelihood slices are always consistent with the QML estimates within the error bars, however an excellent agreement occurs when the QML estimates are used as fiducial power spectrum instead of the best-fit theoretical power spectrum. By the full computation of the conditional likelihood on the estimated spectra, the value of the temperature quadrupole $C_T^T$ is found to be less than $2 \sigma$ away from the WMAP 5 yrs $\Lambda$CDM best-fit value. The $BB$ spectrum is found well consistent with zero and upper limits on the B-modes are provided. The parity odd signals $TB$ and $EB$ are found consistent with zero.

Key words: cosmic microwave background - cosmology: theory - methods: numerical - methods: statistical - cosmology: observations

1 INTRODUCTION
The Cosmic Microwave Background (CMB hereafter) is a powerful tool for investigating the properties of the early and present Universe. Under hypothesis of Gaussianity and statistical isotropy, the main cosmological information coming from a CMB map is contained in its temperature and polarization angular power spectra (APS hereafter).

Many methods have been developed to give unbiased estimates of the CMB power spectra as Master (Hivon et al. 2002), Cross-Spectra (Saha et al. 2006, Polenta et al. 2005, Grain et al. 2009). At high multipoles ($\ell > 30$, Efstathiou 2004) the so called pseudo – $C_\ell$ algorithms are usually preferred to others techniques. These methods in fact implement the estimation of power spectral densities from periodograms (Hauser and Peebles 1974). Basically, they estimate the $C_\ell$ through the inverse Harmonical transform of a masked map that is then decom-
volved with geometrical kernels and corrected with a noise bias term. These techniques give unbiased estimates, and it has been shown they work successfully when applied to real data at high multipoles [Jones et al. 2004; Wu et al. 2004; Kuo et al. 2004; Dunkley et al. 2003]. However, it is well known that at low multipoles they are not optimal since they provide power spectra estimates with error bars larger than the minimum variance. Several strategies for measuring $C_\ell$ at low resolution have been developed and applied to CMB data with excellent results. These methods include the Quadratic Maximum Likelihood (QML) estimator [Tegmark 1997; Tegmark and de Oliveira-Costa 2001], and different sampling techniques such as Gibbs [Jewell et al. 2004; Wandelt et al. 2004; Eriksen et al. 2004], adaptive importance [Benabed et al. 2009] and Hamiltonian [Taylor et al. 2007].

In this paper, we apply a parallel implementation of the QML estimator [Tegmark 1997; Tegmark and de Oliveira-Costa 2001] on WMAP 5 year data. It is shown in [Efstathiou 2004] that at low multipoles, the intrinsic variance introduced by this technique is lower than the variance introduced by the the pseudo-$C_\ell$ methods. We also compute the conditional likelihood slices of the $C_\ell$ multipoles and compare the results with those obtained by the QML.

The largest angular scales of the CMB anisotropies probe the Physics of the early Universe. Therefore estimating its power spectra with optimal methods is crucial. The temperature and polarization low multipole pattern contains information on the reionization process, dark energy and the geometry of the universe. Moreover, determining at best the optical depth $\tau$, has an important impact on the amplitude and the spectrum of primordial perturbations, and therefore on inflationary models. Optimal methods at low resolution are also required by the observed value of the temperature quadrupole, anomalously low within the LCDM dominant model which remains however the simplest cosmological model preferred by data.

In this paper we perform an analysis of the power spectra from low resolution maps of the five year WMAP data providing $C_\ell$ and errors bars for all the 6 APS from $\ell = 2$ to 32 computed by our QML and we show the robustness of our results with respect to iterative estimates. We also give the conditional likelihood slices computed with a pixel based likelihood code from $\ell = 2$ to 10. The paper is organized as follows: in Section 2 we describe the main equation of the QML and in Section 3 we provide the basic expression for the likelihood. Section 4 is dedicated to the technical implementation with a brief description of the computational requirements both for the QML and for the likelihood codes. In Section 5 we report the results obtained from WMAP 5 year data and finally in Section 6 we draw our conclusions. As a convention, all the objects that are defined in pixel space are represented with bold face symbols.

2 THE QML METHOD

The QML method for Power Spectrum Estimation (PSE) of CMB anisotropies was introduced in [Tegmark 1997] and was extended to polarization in [Tegmark and de Oliveira-Costa 2001]. Given a map in temperature and polarization $x = (T, Q, U)$, the QML provides estimates $\hat{C}_\ell^X$ - with $X$ being one of $TT, EE, TE, BB, TB, EB$ - of the APS as:

$$\hat{C}_\ell^X = \sum_{\ell'} (F^{-1})_{\ell\ell'} X X' \left[ X' E_{X', X} - tr(N E_{X', X}) \right],$$

where the $F^X_{XY}$ is the Fisher matrix defined as

$$F^X_{XY} = \frac{1}{2} tr \left[ C^{-1} \frac{\partial C}{\partial C^X} C^{-1} \frac{\partial C}{\partial C^Y} \right].$$

The $E_X$ matrix is given by

$$E_X = \frac{1}{2} C^{-1} \frac{\partial C}{\partial C^X} C^{-1},$$

with $C = S(C_\ell) + N$ being the global covariance matrix (signal plus noise contribution, possibly extended to include residuals from foreground subtractions) and $C_\ell$ is called fiducial power spectrum. It can be shown that the QML method is unbiased in finding the APS $C_\ell^X, obs$ of the map $x$:

$$\langle \hat{C}_\ell^X \rangle = C_\ell^X, obs.$$

It is also optimal, since it can provide the smallest error bars allowed by the Fisher-Cramer-Rao inequality,

$$\langle \Delta \hat{C}_\ell^X \Delta \hat{C}_\ell^{X'} \rangle = (F^{-1})_{\ell\ell'} X X',$$

where the averages are meant to be over an ensemble of realizations.

We have verified by Montecarlo on simulated data that our implementation of the QML method (called BolPol, see Section 3) leads to unbiased minimum variance results as from Eqs. (3-4). We have also verified that Eq. (4) holds with $C_\ell \neq C_\ell^{obs}$, but the errors are no more minimum variance.

3 CONDITIONAL LIKELIHOOD (OR PROBABILITY) SLICES

Assuming that CMB anisotropies are Gaussian distributed, the likelihood for CMB temperature and polarization power spectra is given by [Bond, Jaffe and Knox 1998]:

$$L = exp \left[ -\frac{1}{2} x^T C^{-1} x \right] / \sqrt{(2\pi)^n det(C)},$$

where $n$ is the dimension of the vector $x$ (and depends on the number of pixels that are observed). It gives the probability to have the map $x$ given the model (i.e. $C_\ell$) needed to build the signal covariance matrix. The conditional likelihood slice of $C_\ell^X$ for given $\ell, X$ are obtained sampling Eq. (6) at different values of $C_\ell^X$ keeping $C_\ell^{Y, Z}$ fixed for $Y \neq \ell, X \neq X$. Our implementation in parallel fortran 90 of eq. (6) is called BoLike (see Section 4). It is important to stress that the covariance matrix in $x$ either in BolPol or BoLike is treated without any approximation and not separated in $T$ and $(Q, U)$ as in [Page et al. 2007]. BoLike can also be employed as a part of a likelihood code, to compute directly the posterior probability of a cosmological model at low $\ell$ directly in pixel space without relying on any approximation for the probability distributions of the $C_\ell^X$.

Other methods have been developed to compute the CMB likelihood for low resolution dataset, which can also be used to provide power spectrum estimates. One such approach is based on the Gibbs algorithm [Wandelt et al. 2004].
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Chu et al. [2005], which provides a computationally feasible framework to sample the posterior probability of the C_l given the data, P(C_l|y). Specifically, this is achieved by repeatedly drawing samples from the conditional distributions P(s|C_l,y) and P(s|y), s being a signal only CMB map. A Blackwell-Rao approximation can then be used to compute the CMB likelihood [Rudjord et al. 2007]. This method has been applied to the WMAP5 temperature data [Dunkley et al. 2003] and the WMAP3 temperature plus polarization data [Eriksen et al. 2004]. One feature of the Gibbs approach is that it can internally perform a parametric based component separation, and propagate model uncertainties to power spectra and likelihoods (Dickinson et al. 2007).

Another proposed approach to estimate the APS and likelihood at large angular scales is based on adaptive importance sampling. The idea is to model each single likelihood at large angular scales is based on adaptive im-

5 DESCRIPTION OF THE WMAP 5 YEARS DATASET AND RESULTS

In this section we describe the data set that we have considered. We use the ILC map in T, the foreground cleaned maps in (Q, U), the noise covariance matrix in (Q, U) and the masks at N_{side} = 16 publicly available at the LAMBDA web site [3]. The temperature map is the ILC map at N_{side} = 16 smoothed at 9.8 degrees. We have added a random noise realization with variance of 1\mu K^2 as suggested in Dunkley et al. [2009]. Consistently the noise covariance matrix for TT is taken to be diagonal with variance equal to 1\mu K^2. We have explicitly checked that the estimates do not depend on the noise realization we added on the temperature map. The temperature map has been masked with a mask covering \sim 16\% of the sky and the monopole and the dipole have been subtracted from the observed sky by means of the Healpix routine “remove-dipole” (Gorski et al. 2003) that works in pixel space. The polarization maps for Q and U are provided by the WMAP team at the same resolution N_{side} = 16. The inverse of the masked noise covariance matrix for the polarization part is available at the same resolution. We have followed the procedure explained at the LAMBDA web site (see footnote 4) to obtain the direct noise covariance for the observed pixels. The used polarization mask is larger than the temperature mask and covers \sim 26\% of the sky. The noise covariance matrices for TQ and TU are not provided and we set them to zero. In this work we use the WMAP 5 yrs publicly available data products to derive our spectral estimates. The uncertainties due to foreground cleaning cannot be taken into account since are not provided explicitly by the WMAP team. The QML estimates have been obtained by constructing the signal covariance matrix by using fiducial C_l is up to 3N_{side} and we self-consistently compute C_{X X} up to 3N_{side}. We show the QML estimates up to 2N_{side}, which is a conservative choice for \ell-range, not to incur into discretization errors.

We present the results obtained by our implementation of the QML estimator on the low resolution WMAP5 maps described in the previous section and those obtained by the WMAP team in Figs. 1-4, respectively for TT, EE, TE and BB spectra. In the top panels we show the estimates of BolPol with error bars (dark blue or red, see below) and the pseudo-C_l estimates obtained by the WMAP team with error bars (light blue). The BolPol estimates in dark blue are obtained by using as fiducial the theoretical WMAP5 best-fit [Dunkley et al. 2009], a $\Lambda$CDM cosmological model with $\Omega_b h^2 = 0.0227$, $\Omega_c h^2 = 0.108$, $H_0 = 72.4\text{ km s}^{-1}\text{ Mpc}^{-1}$, $\tau = 0.089$, $n_s = 0.961$, $A_s = 2.41 \times 10^{-9}$ (at $k = 0.002\text{ Mpc}^{-1}$). Error bars loose dependence on the fiducial model by iterating the QML: we use the first run of BolPol to obtain another set of estimates with relative

1 http://healpix.jpl.nasa.gov/. For the reader not familiar with the Healpix notation, N_{side} is related to the number of pixels N_{pix} by N_{pix} = 12N_{side}^2.
2 Note that N_{side} = 16 is not the highest resolution that BolPol is able to consider. Currently BolPol is able to process maps of N_{side} = 32.
Figure 1. Estimates of TT angular power spectrum from WMAP 5 year data at low resolution. Upper panel: BolPol estimates (dark blue diamonds) with error bars (dark blue), iterated BolPol estimates (red diamonds) with error bars (red), WMAP pseudo-$C_\ell$ estimates (light blue cross) with error bars (light blue). Lower panel: differences between the sets of estimates in unit of sigma (same conventions as upper panel for the colors).

error bar, which is plotted in red. The iterated estimates are always very close to those obtained with the WMAP5 best-fit as fiducial model: this means that the QML estimated are sufficiently stable with respect to iteration. The same does not happen to the error bars: in particular the error on the TT quadrupole is substantially smaller, and a decrease of the error also occurs for $\ell = 3, 4$ in temperature. Our estimate for the octupole in BB is consistent with zero and very different from the one obtained by WMAP (note however that the WMAP likelihood slice for $C_{BB}$ is not anomalous as the WMAP pseudo-$C_\ell$ estimate (Nolta et al. 2009)).

In the lower panels of Figs. 1-4 we show the differences between the sets of estimates in unit of sigma (same conventions as upper panel for the colors).

We list now the reduced $\chi^2$ values for the iterated BolPol estimates from $\ell = 2$ to $\ell = 32$ with the fiducial input model: $\chi^2_{TT} = 4.423$ but excluding the quadrupole this value decreases to 1.079 recovering so the anomaly of the low quadrupole value of TT; for the other reduced $\chi^2$-values we find $\chi^2_{TE} = 0.785$, $\chi^2_{EE} = 1.422$, $\chi^2_{BB} = 1.607$.

In Fig. 5, we plot the first and iterated BolPol estimates and relative error bars (in blue and red, respectively) for TB and EB. We do not plot the estimates by the WMAP team since these are not provided in the LAMBDA site. Also for these parity-odd correlators the QML estimates are very stable with respect to the iteration. Note how the error bars in TB change (due to substantially different fiducial model in TT in the iterated run), whereas those in EB do not (since the fiducial EE and BB spectra are mainly unchanged during the iteration). The TB null reduced $\chi^2$ for $\ell = 2 - 23$ is 1.34 to be compared with 0.97 quoted for $\ell = 24 - 450$ by the WMAP team. The EB null reduced $\chi^2$ for $\ell = 2 - 23$ is 1.14.

At low multipoles, symmetric error bars, such as those provided by the Fisher matrix, are just an approximation, since we know that the likelihood for $C_\ell$ is far from a symmetric Gaussian. We therefore evaluate the conditional likelihood slices for the six spectra from $\ell = 2$ to 10; we present these results in Figs. 6-7. As for the QML, we compute the slices on the WMAP5 best-fit (blue points) and on the BolPol estimates (orange points obtained with the same fiducial used for the iterated QML run). It is important to note that the peaks of the likelihood slices are very different for the two sets of conditionings: the quadrupole in temperature is the most striking example. We do not observe such dependence on the fiducial model in the estimates in
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Figure 2. Estimates of EE angular power spectrum from WMAP 5 year data at low resolution. Upper panel: BolPol estimates (dark blue diamonds) with error bars (dark blue), iterated BolPol estimates (red diamonds) with error bars (red), WMAP pseudo-$C_\ell$ estimates (light blue cross) with error bars (light blue). Lower panel: differences between the sets of estimates in unit of sigma (same conventions as upper panel for the colors).

6 DISCUSSIONS AND CONCLUSIONS

We have performed a new estimate of the CMB angular power spectra at low multipoles from low resolution maps of the five years WMAP data.

The QML estimates are found in agreement with the pseudo-$C_\ell$ WMAP ones: the best agreement is for the TT spectrum and differences at the level of $2-3\sigma$ are found in EE and BB.
Figure 3. Estimates of TE angular power spectrum from WMAP 5 year data at low resolution. Upper panel: BolPol estimates (dark blue diamonds) with error bars (dark blue), iterated BolPol estimates (red diamonds) with error bars (red), WMAP pseudo-$C_\ell$ estimates (light blue cross) with error bars (light blue). Lower panel: differences between the sets of estimates in unit of sigma (same conventions as upper panel for the colors).

Figure 8. Upper bounds on BB. y axis: Likelihood slice for BB. x axis $\ell(\ell+1)C_{\ell\ell}^{BB}/(2\pi)$ binning from $\ell = 2$ to 6. Dark blue diamonds represent the likelihood slice obtained sampling on band powers. Light blue crosses represent the likelihood slice obtained sampling on power spectrum. See also the text.

Whereas the QML is not used by the WMAP team, a pixel likelihood code is employed and results are given in Dunkley et al. (2009), Nolta et al. (2009). The difference between the conditional likelihood slices given here and those obtained by the WMAP team are due to our exact treatment of the covariance matrix: the approximation used by the WMAP team requires less computational resources but has the drawback of allowing negative values for conditional slices of EE and BB whereas ours are always positive. The origin of the difference in the constraint in $C_\ell^{BB}$ for $\ell = 2 - 6$ is partially due to this difference.

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Figure 4. Estimates of TT angular power spectrum from WMAP 5 year data at low resolution. Upper panel: BolPol estimates (dark blue diamonds) with error bars (dark blue), iterated BolPol estimates (red diamonds) with error bars (red), WMAP pseudo-$C_\ell$ estimates (light blue cross) with error bars (light blue). Lower panel: differences between the sets of estimates in unit of sigma (same conventions as upper panel for the colors).

Figure 5. BolPol estimates of TB (upper panel) and EB (lower panel) angular power spectra from WMAP 5 year data at low resolution. Dark blue symbols are for the not iterated case and red for the iterated case.
Figure 6. Likelihood Slices for TT, TE and EE from $\ell = 2$ to 10 for the WMAP 5 year data at low resolution (i.e. nside = 16). Blue slices are for the not iterated case and the red ones for the iterated case. The blue plus represent the not iterated BolPol estimate with error bars (blue horizontal line) and the red plus the iterated ones.
Figure 7. Likelihood Slices for BB, TB and EB from $\ell = 2$ to 10 for the WMAP 5 year data at low resolution (i.e. nside = 16). Blue slices are for the not iterated case and the red ones for the iterated case. The blue plus represent the not iterated BolPol estimate with error bars (blue horizontal line) and the red plus the iterated ones.

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