Damage and fracture of fiber-reinforced ceramic-matrix composites under thermal fatigue loading in oxidizing atmosphere

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In this paper, the damage and fracture of fiber-reinforced ceramic-matrix composites (CMCs) subjected to thermal cyclic fatigue loading at elevated temperatures in oxidizing atmosphere are investigated. The temperature/cyclic dependent fiber/matrix interface shear stress is determined as a function of testing temperature, applied cycle number and material properties, which affects multiple thermal fatigue damage mechanisms. The microstress field of the thermal fatigue damaged composite is analyzed using the Budiansky-Hutchinson-Evans shear-lag model, considering matrix stochastic cracking, fiber/matrix interface debonding/sliding and fibers fracture. The matrix stochastic cracking, fiber/matrix interface debonding and sliding, fibers fracture in the interface damage zones are determined using the micromechanical approach. The relationships between the thermal fatigue cycling, multiple thermal fatigue damage mechanisms, fatigue hysteresis-based damage parameters and thermal fatigue lifetime are established. The effects of fiber/matrix interface properties, fiber radius, fiber volume fraction, matrix crack spacing and fatigue peak stress on thermal fatigue damage evolution in fiber-reinforced CMCs are analyzed. The thermal fatigue damage evolution and lifetime prediction of two different fiber-reinforced CMCs, i.e., cross-ply SiC/MAS and 2D SiC/SiC composites, subjected to thermal cycling fatigue in oxidizing atmosphere are predicted.

Key-words : Ceramic-matrix composites (CMCs), Thermal fatigue, Damage evolution, Hysteresis loops

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1. Introduction

The aeroengine is the largest weight expense in the aircraft. The exceptional high-temperature mechanical stability and excellent physical properties of ceramics make them a reasonable candidate for implementation in demanding environments such as a jet engine hot section.¹) Due to the brittle failure mode that occurs in these materials, measures must be taken to make the ceramics more durable, especially in applications where safety is of the highest concern based on the airworthiness requirements of the aircraft or aeroengine. The fiber-reinforced ceramic-matrix composites (CMCs) have been investigated and successfully implemented in many applications including exhaust nozzles and combustion chamber lining.²) General Electric has committed to using the long-fiber-reinforced CMCs for the next generation GE9x engine, and many other applications including land-based turbine and nuclear heat exchangers are being investigated.³)

As new materials, it is necessary to develop the models, methods and tools to predict the degradation, damage and failure mechanisms subjected to the cyclic loading at different temperatures and environments.⁴) For the real life applications, i.e., the turbine blades in a turbofan engine, dictate the need to determine the mechanical behavior of this material in an environment involving the thermal fatigue cycling under the constant load.⁵) Boccaccinia et al.⁷) investigated the damage evolution of NicalonSiC/Borosilicate (DURAN) glass-matrix composite subjected to thermal fatigue cycling between high-temperature (T = 700°C) and room-temperature in air condition. The flexural strength and Young’s modulus decreased and the internal friction increased with increasing of the applied cycle numbers, attributed to the viscous flow of the glass matrix, the oxidation of the fiber/matrix interface and fibers. Udayakumar et al.⁸) investigated the effect of thermal cycling on the mechanical properties of NicalonSiC/SiC composite at the elevated temperature between 413 and 1273 K. The matrix microcracks are generated under thermal fatigue cycling, leading to the oxidation of the BN interface and the strengthen of the fiber/matrix bonding. Mei and Cheng⁹) investigated the strain response of a C/SiC composite simultaneously subjected to the thermal fatigue cycling and mechanical cycling in an oxidizing atmosphere. The thermal and mechanical load cycling in an oxidizing atmosphere could cause the coating/matrix cracks, the degradation of the fiber/matrix interface and fibers oxidation. Under cyclic thermal loading at constant stress level, the fatigue stress-strain hysteresis
loops appear when the fiber repeatedly sliding relative to the matrix in different debonded zones. Reynaud et al.\textsuperscript{11} investigated the effects of temperature and oxidation on the mechanical hysteresis behavior in the long-fiber-reinforced CMCs. Li\textsuperscript{12} investigated the tension–tension cyclic fatigue behavior of C/SiC composite at room and elevated temperature of 800°C in air condition. The degradation rate of the fiber/matrix interface shear stress at 800°C is much higher than that at room temperature, leading to the greatly decreasing of the fatigue limit stress. Under thermal cyclic fatigue loading, oxidation will affect the fiber/matrix interface damage condition, and then the damage evolution and final fracture in fiber-reinforced CMCs. However, in the researches mentioned above, the temperature-dependent damage development of fiber-reinforced CMCs subjected to thermal fatigue cycling loading at elevated temperatures in oxidizing atmosphere has not been analyzed using the fatigue hysteresis-based damage parameters.

The objective of this paper is to investigate the damage and fracture of fiber-reinforced CMCs subjected to thermal cycling fatigue loading in oxidizing atmosphere. The Budiansky-Hutchinson-Evans shear-lag model is used to determine the microstress field of the thermal damaged composites, considering the damage mechanisms of matrix multicracking, fiber/matrix interface debonding and sliding, and fibers fracture. The matrix crack spacing, fiber/matrix interface debonding and sliding lengths, and broken fibers fraction subjected to thermal fatigue cycling are determined. The relationships between the thermal fatigue cycling, thermal fatigue damage mechanisms, fatigue hysteresis-based damage parameters and thermal fatigue lifetime are established. The effects of fiber/matrix interface properties, fiber radius, fiber volume fraction, matrix crack spacing and fatigue peak stress on thermal fatigue damage development of SiC/SiC composite at elevated temperature in oxidizing atmosphere are analyzed. The thermal fatigue damage evolution and thermal fatigue lifetime of two different fiber-reinforced CMCs, i.e., cross-ply SiC/MAS and 2D SiC/SiC composites subjected to thermal cycling are predicted.

## 2. Thermal fatigue damage models

The testing temperature affect the thermomechanical fatigue (TMF) behavior of long-fiber-reinforced CMCs, i.e., matrix microcracking, fiber/matrix interface debonding and thermal residual stress. If the radial thermal expansion coefficient of the matrix is higher than the coefficient of the fibers, at a testing temperature $T$ lower than the processing temperature $T_0$, i.e., $T < T_0$, the radial thermal residual stresses are compressive stresses. The temperature/cyclic-dependent fiber/matrix interface shear stress $[\tau_i(T, N)]$ can be determined using the following equation:\textsuperscript{11}

$$\tau_i(T, N) = \tau_0(N) + \mu \frac{[\alpha_{rt} - \alpha_{mt}](T_0 - T)}{A}$$

where $\tau_0$ denotes the cyclic-dependent fiber/matrix interface shear stress, and degrades with applied cycles due to the fiber/matrix interface wear or interface oxidation;\textsuperscript{6} $\mu$ denotes the fiber/matrix interface frictional coefficient; $\alpha_{rt}$ and $\alpha_{mt}$ denote the fiber and matrix radial thermal expansion coefficient, respectively; and $A$ is a constant depending on the elastic properties of the matrix and fibers.\textsuperscript{11} The fiber/matrix interface shear stress $\tau_i(N)$ decays after cycling from the initial value $\tau_{initial}$ to a steady-state value $\tau_{steady}$:

$$\tau_{steady} = (1 + b_0)(1 + b_0 N)^{-1}$$

where $b_0$ is a coefficient; and $j$ is an exponent which determines the rate at which interface shear stress drops with the number of cycle $N$.

### 2.1 Stress analysis

To analyze the micro stress distributions in the fiber and the matrix after thermal fatigue damage, a unit cell is extracted from the long-fiber-reinforced CMCs, as shown in Fig. 1. The unit cell contains a single fiber surrounded by a hollow cylinder of matrix. The fiber radius is $r_f$ and the matrix radius is $R = r_f V_f / V_m$. The length of the unit cell is $l_c/2$, which is just the half of the matrix crack space. The fiber/matrix interface debonded length is $l_d$. At the matrix cracking plane, the fiber carries the applied stress ($\sigma/V_f$), where $\sigma$ denotes the far-field applied stress and $V_f$ denotes the fiber volume fraction. The shear-lag model adopted by the Budiansky, Hutchinson and Evans\textsuperscript{13} is adopted to perform the stress and strain calculations in the fiber/matrix interface debonded region ($x \in [0, l_d]$) and fiber/matrix interface bonded region ($x \in [l_d, l_c/2]$).

$$\sigma_f(x) = \begin{cases} \sigma - \frac{2\tau_i(T)}{r_f} x, & x \in (0, l_d) \\ \sigma_{fo} + \left[ \frac{V_m}{V_f} \sigma_{mo} - 2\tau_i(T) \right] \exp \left( -\frac{x - l_d}{r_f} \right), & x \in (l_d, l_c/2) \end{cases}$$

$$\sigma_m(x) = \begin{cases} 2\tau_i(T) \frac{V_f}{V_m} \frac{x}{r_f}, & x \in (0, l_d) \\ \sigma_{mo} - \left[ \sigma_{mo} - 2\tau_i(T) \frac{V_f}{V_m} \frac{l_d}{r_f} \right] \exp \left( -\frac{x(l_d)}{r_f} \right), & x \in (l_d, l_c/2) \end{cases}$$

where $V_m$ denotes matrix volume fraction; and $\rho$ denotes the BHE shear-lag parameter.\textsuperscript{11}

![Fig. 1. The unit cell of the Budiansky-Hutchinson-Evans shear lag model.](image)
\[ \rho^2 = \frac{4E_s G_m}{V_m E_m E_f}\varphi \]  
(5)

where \( G_m \) denotes matrix shear modulus, and

\[ \varphi = -\frac{2 \ln V_f + V_m(3 - V_f)}{2V_m} \]  
(6)

\( \sigma_{f0} \) and \( \sigma_{m0} \) denote the fiber and matrix axial stress in the fiber/matrix interface bonded region, respectively.

\[ \sigma_{f0} = \frac{E_f}{E_c} \sigma_f + E_f(\alpha_{lc} - \alpha_{ft})\Delta T \]  
(7)

\[ \sigma_{m0} = \frac{E_m}{E_c} \sigma_m + E_m(\alpha_{lc} - \alpha_{lm})\Delta T \]  
(8)

where \( E_f, E_m \) and \( E_c \) denote the fiber, matrix and composite elastic modulus, respectively; \( \alpha_{ft}, \alpha_{lm} \) and \( \alpha_{lc} \) denote the fiber, matrix and composite axial thermal expansion coefficient, respectively; and \( \Delta T \) denotes the temperature difference between the fabricated temperature \( T_0 \) and testing temperature \( T_1 (\Delta T = T_1 - T_0) \).

### 2.2 Multiple matrix cracking

The cracking of the matrix depends upon the internal flaw inside of the matrix. The matrix cracking density increases with increasing of the applied stress above the initial matrix cracking stress of \( \sigma_{mc} \), and may eventually approach to the saturation at the applied stress of \( \sigma_{sat} \). There are four dominant damage models for predicting the matrix multiple cracking development inside of the long-fiber-reinforced CMCs, i.e., the maximum stress criterion, the energy balance approach, the critical matrix strain energy criterion and the statistical failure approach. The statistical failure approach assumes that the matrix cracking is governed by the statistical relation, which relates the size and spatial distribution of the matrix flaws to their relative propagation stress. The brittle nature of the matrix material and the possible formation of initial cracks distribution in the microstructure suggest that a statistical approach for matrix multicracking evolution is warranted in the long-fiber-reinforced CMCs. Using the statistical matrix cracking approach, the matrix crack spacing of \( l_c \) can be determined using the following equation.

\[ l_c = \frac{r_l V_m E_m}{V_f E_f} \times \frac{\sigma_R}{2\tau(T)} \Lambda \left(1 - \exp \left[-\frac{\sigma - (\sigma_{mc} - \sigma_{th})}{(\sigma_R - \sigma_{th}) - (\sigma_{mc} - \sigma_{th})}\right]\right)^{-1} \]  
(9)

where \( \sigma_R \) denotes the matrix cracking characteristic strength; \( \sigma_{mc} \) denotes the matrix cracking stress; \( \sigma_{th} \) denotes the matrix thermal residual stress; and \( \Lambda \) denotes final nominal crack space, which is a pure number and depends only on the micromechanical and statistical quantities characterizing the cracking. The final nominal crack spacing versus matrix Weibull modulus simulated by Monte Carlo method when \( \sigma_{mc}/\sigma_R = 0, 0.5, 0.75 \) and \( \sigma_{th}/\sigma_R = 0, 0.1, 0.2 \) are plotted in Fig. 2.

![Fig. 2. The final nominal matrix crack spacing versus matrix Weibull modulus of various \( \sigma_{mc}/\sigma_R \) and \( \sigma_{th}/\sigma_R \).](image)

### 2.3 Interface debonding and sliding

The fracture mechanics approach is adopted to determine the fiber/matrix interface debonded length \( l_d \), interface counter-slip length \( l_s \) and interface new-slip length \( l_z \), as shown in the following equation.

\[ \xi_d = \frac{1}{4\pi r_l} \frac{\partial w_f(0)}{\partial l_d} - \frac{1}{2} \int_0^l \tau_i(T) \frac{\partial w(x)}{\partial l_d} dx \]  
(10)

where \( \xi_d \) denotes the fiber/matrix interface debonded energy; \( F(=\pi r_l^2 \sigma / V_f) \) is the fiber load at the matrix cracking plane; \( w_f(0) \) denotes the fiber axial displacement at the matrix crack plane; and \( w(x) \) denotes the relative displacement between the fiber and the matrix.

The axial displacement of the fiber and the matrix, i.e., \( w_f(x) \) and \( w_m(x) \), are determined using the following equations.

\[ w_f(x) = \int_x^{l_f/2} \frac{\sigma_f}{V_f E_f} dx \]

\[ = \frac{\sigma}{V_f E_f} (l_d - x) - \frac{\tau_i(T)}{r_l E_f} (l_d^2 - x^2) \]

\[ = \frac{2 \tau_i(T)}{r_l} l_d^2 + \frac{r_l V_m E_m}{\rho V_f E_f} \sigma + \frac{\sigma}{E_c} (l_c/2 - l_d) \]  
(11)

\[ w_m(x) = \int_x^{l_f/2} \frac{\sigma_m}{E_m} dx \]

\[ = \frac{V_f}{V_m E_m r_l} (l_d^2 - x^2) + \frac{2 V_f \tau_i(T)}{\rho V_m E_m} l_d \]

\[ - \frac{r_l}{\rho E_c} \sigma + \frac{\sigma}{E_c} (l_c/2 - l_d) \]  
(12)

Using Eqs. (11) and (12), the relative displacement between the fiber and matrix is determined using the following equation.

\[ v(x) = |w_f(x) - w_m(x)| \]

\[ = \frac{\sigma}{V_f E_f} (l_d - x) - \frac{\tau_i(T) E_c}{V_m E_m E_f r_l} (l_d^2 - x^2) \]

\[ - \frac{2 \tau_i E_c l_d}{\rho V_m E_m E_f} r_l + \frac{r_l}{\rho V_f E_f} \sigma \]  
(13)
Substituting $w(x) = 0$ and $v(x)$ into Eq. (10), it leads to the following equation.

$$
\frac{E_v}{V_m E_m E_f} \frac{\partial^2}{\partial t^2} \tau_d + \left\{ \frac{E_v}{\rho V_m E_m E_f} \frac{\partial}{\partial t} \tau(T) \sigma - \frac{\tau(T) \sigma}{V_f E_f} \right\} l_d 
$$

$$
+ \frac{r_f V_m E_m}{4 V_f E_f E_c} \sigma^2
$$

$$
- \frac{r_f \tau(T)}{2 \rho V_f E_f} \sigma - \zeta_d = 0
$$

Solve Eq. (14), the fiber/matrix interface debonded length ($l_d$) is determined using the following equation.

$$
l_d(T) = \frac{r_f}{2} \left( \frac{V_m E_m \sigma}{V_f E_f \tau(T)} - \frac{1}{\rho} \right) - \sqrt{\left( \frac{r_f}{2 \rho} \right)^2 + \frac{r_f V_m E_m \sigma}{E_c [\tau(T)]^2} \zeta_d}
$$

The temperature-dependent fiber/matrix interface counter-slip length ($y$) and interface new-slip length ($z$) can also be determined using the fracture mechanics approach, as shown by the following equations.

$$
y(T) = \frac{1}{2} \left( l_d(\sigma_{\max}) - \frac{r_f}{2} \left( \frac{V_m E_m \sigma}{V_f E_c \tau(T)} - \frac{1}{\rho} \right) \right)
$$

$$
- \sqrt{\left( \frac{r_f}{2 \rho} \right)^2 + \frac{r_f V_m E_m \sigma}{E_c [\tau(T)]^2} \zeta_d}
$$

$$
z(T) = y(\sigma_{\min}) = \frac{1}{2} \left( l_d - \frac{r_f}{2} \left( \frac{V_m E_m \sigma}{V_f E_c \tau(T)} - \frac{1}{\rho} \right) \right)
$$

$$
- \sqrt{\left( \frac{r_f}{2 \rho} \right)^2 + \frac{r_f V_m E_m \sigma}{E_c [\tau(T)]^2} \zeta_d}
$$

2.4 Hysteresis-based damage parameters

If matrix multicracking and fiber/matrix interface debonding are present upon first loading, the fatigue hysteresis loops develop as a result of energy dissipation through the frictional slip between fibers and the matrix upon unloading and subsequent reloading. Upon unloading, the counter slip occurs in the fiber/matrix interface debonded region. The fiber/matrix interface debonded region can be divided into two regions, i.e., the interface counter-slip region and the interface slip region, as shown in Fig. 3(a). The fiber/matrix interface counter-slip length is defined as $y$. Upon reloading, new slip occurs in the fiber/matrix interface debonded region. The fiber/matrix interface debonded region can be divided into three regions, i.e., the interface new-slip region, the interface counter-slip region and the interface slip region, as shown in Fig. 3(b). The interface new-slip region is defined as $z$. Based on the debonding and sliding condition between the fiber and the matrix in the debonded regions, the fatigue hysteresis loops of the long-fiber-reinforced CMCs can be divided into four different cases, including:

1. Case 1, partially fiber/matrix interface debonding and the fiber sliding completely relative to the matrix in the interface debonded region;
2. Case 2, partially fiber/matrix interface debonding and the fiber sliding partially relative to the matrix in the interface debonded region;
3. Case 3, completely fiber/matrix interface debonding and the fiber sliding partially relative to the matrix in the interface debonded region;
4. Case 4, completely fiber/matrix interface debonding and the fiber sliding completely relative to the matrix in the interface debonded region.

When the damage forms within the composite, the composite strain is determined using the following equation, which assumes that the composite strain $\varepsilon_c$ is equivalent to the average strain in an undamaged fiber.

$$
\varepsilon_c = \frac{2}{E_i l_c} \int_0^{l_c/2} \sigma(x) dx - (\alpha_l - \alpha_i) \Delta T
$$
When the fiber/matrix interface partially debonds, the unloading strain $\varepsilon_{\text{unloading}}$ and reloading strain $\varepsilon_{\text{reloading}}$ are determined using the following equations.

\[
\varepsilon_{\text{unloading}} = \frac{\sigma}{V_t E_t} + \frac{4}{V_t E_t} \frac{\tau_i(T) y^2}{r_l E_t} - 2 \frac{\tau_i(T)(2y-l_c)(2y+l_d-l_c)}{r_l E_t} - (\alpha_{\text{bc}} - \alpha_{\text{fl}}) \Delta T
\]

\[
\varepsilon_{\text{reloading}} = \frac{\sigma}{V_t E_t} - 4 \frac{\tau_i(T) y^2}{r_l E_t} + 4 \frac{\tau_i(T)(y-2z)^2}{r_l E_t} + 2 \frac{\tau_i(T)(l_d/2-2y+2z)}{r_l E_t} - (\alpha_{\text{bc}} - \alpha_{\text{fl}}) \Delta T
\]

When the fiber/matrix interface completely debonds, the unloading strain $\varepsilon_{\text{unloading}}$ and reloading strain $\varepsilon_{\text{reloading}}$ are determined using the following equations.

\[
\varepsilon_{\text{unloading}} = \frac{\sigma}{V_t E_t} + 4 \frac{\tau_i(T) y^2}{r_l E_t} - 2 \frac{\tau_i(T)(2y-l_c/2)^2}{r_l E_t} - (\alpha_{\text{bc}} - \alpha_{\text{fl}}) \Delta T
\]

\[
\varepsilon_{\text{reloading}} = \frac{\sigma}{V_t E_t} - 4 \frac{\tau_i(T) y^2}{r_l E_t} + 4 \frac{\tau_i(T)(y-2z)^2}{r_l E_t} + 2 \frac{\tau_i(T)(l_d/2-2y+2z)}{r_l E_t} - (\alpha_{\text{bc}} - \alpha_{\text{fl}}) \Delta T
\]

The area associated with the fatigue hysteresis loops is the dissipated energy ($U_e$) during corresponding cycle, which is defined by the following equation.

\[
U_e = \int_{\varepsilon_{\text{min}}}^{\varepsilon_{\text{max}}} [\varepsilon_{\text{unloading}}(\sigma) - \varepsilon_{\text{reloading}}(\sigma)] d\sigma
\]

where $\varepsilon_{\text{unloading}}$ and $\varepsilon_{\text{reloading}}$ denote the unloading and reloading strain, respectively. Substituting the unloading and reloading strains of Eqs. (19)-(22) corresponding to the fiber/matrix interface partially and completely debonding into Eq. (23), the fatigue hysteresis dissipated energy $U_e$ can be obtained.

The fatigue hysteresis width $\Delta \varepsilon$ is defined by the following equation.

\[
\Delta \varepsilon = \varepsilon_{\text{c,unload}} \left(\frac{\varepsilon_{\text{min}} + \varepsilon_{\text{max}}}{2}\right) - \varepsilon_{\text{c,reload}} \left(\frac{\varepsilon_{\text{min}} + \varepsilon_{\text{max}}}{2}\right)
\]

The fatigue hysteresis modulus $E$ is defined by the following equation.

\[
E = \frac{\varepsilon_{\text{max}} - \varepsilon_{\text{min}}}{\varepsilon_{\text{c,\sigma_{\max}}} - \varepsilon_{\text{c,\sigma_{\min}}}}
\]

3. Thermal fatigue lifetime prediction model

Under thermal cyclic fatigue loading at elevated temperatures, fibers fracture occurs due to the gradual fiber/matrix interface wear and interface oxidation. The Global Load Sharing assumption is used to determine the stress carried by the intact and fracture fibers.
oxidation model and fiber strength degradation model with Eqs. (33) and (34). When the fibers broken fraction approaches to the critical value, the composite fatigue fractures.

4. Results and discussions

The ceramic composite systems of SiC/SiC composite is used for the case study. For the SiC/SiC composite, the material properties are given by:27) $V_f = 30\%$, $E_f = 230$ GPa, $E_m = 300$ GPa, $r_f = 7.5\mu m$, $\zeta_m = 0.1 J/m^2$, $\alpha_f = 2.9 \times 10^{-6}/K$, $\alpha_t = 3.9 \times 10^{-6}/K$, $\alpha_{tm} = 4.6 \times 10^{-6}/K$, $\alpha_{im} = 2.0 \times 10^{-6}/K$ and $\sigma_0 = 2.6$ GPa. In the present analysis, the thermal fatigue damage evolution of fatigue hysteresis dissipated energy, fatigue hysteresis modulus, fatigue hysteresis width and fiber/matrix interface debonding ratio are considered. These damage parameters relate with the damage and fracture of the fiber-reinforced CMCs subjected to the thermal cyclic fatigue loading at elevated temperature. When the fibers broken fraction approaches to the critical value, the composite fatigue fracture, and the critical values for the damage parameters can also be determined. Li28) developed an approach to predict the final fracture of carbon fiber-reinforced CMCs under cyclic fatigue loading using the fatigue hysteresis dissipated energy-based parameters. The critical fatigue hysteresis dissipated energy-based damage parameters for composite fracture have been determined. The effects of the fiber/matrix interface properties, fiber radius, fiber volume fraction, matrix crack spacing and fatigue peak stress levels on the temperature-dependent thermal fatigue damage evolution of SiC/SiC composite subjected to thermal cyclic and constant loading are discussed.

4.1 Effect of fiber/matrix interface properties

The effect of fiber/matrix interface shear stress ($\tau_0 = 5$ and 10 MPa) on the temperature-dependent thermal fatigue damage evolution of the fatigue hysteresis dissipated energy ($U_c$), fatigue hysteresis modulus ($E$), fatigue hysteresis width ($\Delta\varepsilon$) and interface debonding ratio ($2l_d/l_o$) versus the cyclic temperature curves under $\sigma_{\max} = 200$ MPa at the temperature range from $Tem = 100^\circ$C to $Tem = 800^\circ$C are shown in Fig. 4. When the cyclic dependent fiber/matrix interface shear stress decreases from $\tau_0 = 10$ to 5 MPa, the fatigue hysteresis dissipated energy and fatigue hysteresis width increase, and the fatigue hysteresis modulus decreases, due to the increasing fiber/matrix interface debonding and sliding length.

The effect of fiber/matrix interface frictional coefficient ($\mu = 0.1$ and 0.5) on the temperature-dependent thermal fatigue damage evolution of the fatigue hysteresis dissipated energy ($U_c$), fatigue hysteresis modulus ($E$), fatigue hysteresis width ($\Delta\varepsilon$) and fiber/matrix interface debonding ratio ($2l_d/l_o$) versus cyclic temperature curves under $\sigma_{\max} = 200$ MPa at the temperature range from $Tem = 100^\circ$C to $Tem = 800^\circ$C are shown in Fig. 5. When the fiber/matrix interface frictional coefficient increases from $\mu = 0.1$ to 0.5, the fatigue hysteresis dissipated energy and fatigue hysteresis width decrease, and the fatigue hysteresis modulus increases, due to the decreasing fiber/matrix interface debonding and sliding length.

4.2 Effect of fiber radius

The effect of the fiber radius ($r_f = 6$ and 9$\mu m$) on the temperature-dependent thermal fatigue damage evolution of the fatigue hysteresis dissipated energy ($U_c$), fatigue hysteresis modulus ($E$), fatigue hysteresis width ($\Delta\varepsilon$) and fiber/matrix interface debonding ratio ($2l_d/l_o$) versus cyclic temperature curves under $\sigma_{\max} = 300$ MPa and the temperature range from $Tem = 100^\circ$C to $Tem = 800^\circ$C are shown in Fig. 6. When the fiber radius increases from $r_f = 6$ to 9$\mu m$, the fatigue hysteresis dissipated energy and fatigue hysteresis width increase, and the fatigue hysteresis modulus decreases, due to the increasing fiber/matrix interface debonding and sliding length.

4.3 Effect of fiber volume fraction

The effect of fiber volume fraction ($V_f = 30$ and 35$\%$) on the temperature-dependent thermal fatigue damage evolution of the fatigue hysteresis dissipated energy ($U_c$), fatigue hysteresis modulus ($E$), fatigue hysteresis width ($\Delta\varepsilon$) and fiber/matrix interface debonding ratio ($2l_d/l_o$) versus the cyclic temperature curves under $\sigma_{\max} = 300$ MPa and the temperature range from $Tem = 100^\circ$C to $Tem = 800^\circ$C are shown in Fig. 7. When the fiber volume fraction increases from $V_f = 30$ to 35$\%$, the fatigue hysteresis dissipated energy and fatigue hysteresis width decrease, and the fatigue hysteresis modulus increases, due to the decreasing fiber/matrix interface debonding and sliding length.

4.4 Effect of matrix crack spacing

The effect of the matrix crack spacing ($l_c = 200$ and 300$\mu m$) on the temperature-dependent thermal fatigue damage evolution of the fatigue hysteresis dissipated energy ($U_c$), fatigue hysteresis modulus ($E$), fatigue hysteresis width ($\Delta\varepsilon$) and fiber/matrix interface debonding ratio ($2l_d/l_o$) versus the cyclic temperature curves under $\sigma_{\max} = 300$ MPa and the temperature range from $Tem = 100^\circ$C to $Tem = 800^\circ$C are shown in Fig. 8. When the matrix crack spacing increases from $l_c = 200$ to 300$\mu m$, the fatigue hysteresis dissipated energy and fatigue hysteresis width decrease, and the fatigue hysteresis modulus increases, due to the decreasing fiber/matrix interface debonding and sliding range.

4.5 Effect of fatigue peak stress

The effect of fatigue peak stress ($\sigma_{\max} = 200$ and 250 MPa) on the temperature-dependent thermal fatigue damage evolution of the fatigue hysteresis dissipated energy ($U_c$), fatigue hysteresis modulus ($E$), fatigue hysteresis width ($\Delta\varepsilon$) and interface debonding ratio ($2l_d/l_o$) versus cyclic temperature curves at the temperature range from $Tem = 100^\circ$C to $Tem = 800^\circ$C are shown in Fig. 9. When the fatigue peak stress increases from $\sigma_{\max} = 200$ to 250 MPa, the fatigue hysteresis dissipated energy and fatigue hysteresis width increase, and the fatigue hysteresis modu-
lus decreases, due to the increasing fiber/matrix interface debonding and sliding length.

5. Experimental comparisons

The temperature-dependent thermal fatigue damage, fracture and lifetime prediction of two different fiber-reinforced CMCs, i.e., cross-ply SiC/MAS and 2D SiC/SiC composites, subjected to thermal cycling fatigue at elevated temperature in oxidizing atmosphere are predicted.

5.1 Cross-ply SiC/MAS composite

Reynaud et al.\textsuperscript{11}) investigated the cyclic fatigue behavior of cross-ply SiC/MAS at elevated temperatures in inert atmosphere. The temperature-dependent damage evolution of cross-ply SiC/MAS is predicted. The experimental
fatigue hysteresis dissipated energy versus applied cycle number curves of cross-ply SiC/MAS under $\sigma_{\text{max}} = 110 \text{ MPa}$ at 600, 800 and 1000°C in inert atmosphere are shown in Fig. 10. The fatigue hysteresis dissipated energy decreases with test temperature. The material properties are given by: $V_f = 32\%$, $E_f = 230 \text{ GPa}$, $E_m = 75 \text{ GPa}$, $r_f = 7.5 \mu\text{m}$, $\zeta_4 = 0.1 \text{ J/m}^2$, $\alpha_f = 2.9 \times 10^{-6}/\text{K}$, $\alpha_m = \alpha_\text{im} = 1.2 \times 10^{-6}/\text{K}$, and $\sigma_0 = 2.6 \text{ GPa}$. The fatigue hysteresis dissipated energy corresponding to different applied cycle numbers and test temperatures are predicted, as shown in Fig. 11. When $N = 1$, the experimental hysteresis dissipated energy decreases from 34.9 kJ/m$^3$ at 1000°C, 18.8 kJ/m$^3$ at 800°C, to 3.9 kJ/m$^3$ at 600°C, corresponding to the interface shear stress of

![Fig. 5](image-url)
Fig. 6. (a) The hysteresis energy versus temperature curves; (b) the hysteresis modulus versus temperature curve; (c) the hysteresis width versus temperature curves; and (d) the interface debonded length versus temperature curves of SiC/SiC composite subjected to thermal cycle under constant loading for different fiber volume fraction of $r_f = 6$ and $9 \mu m$.

Fig. 7. (a) The hysteresis energy versus temperature curves; (b) the hysteresis modulus versus temperature curve; (c) the hysteresis width versus temperature curves; and (d) the interface debonded length versus temperature curves of SiC/SiC composite subjected to thermal cycle under constant loading for different fiber volume fraction of $V_f = 30$ and $35\%$. 
Fig. 8. (a) The hysteresis energy versus temperature curves; (b) the hysteresis modulus versus temperature curve; (c) the hysteresis width versus temperature curves; and (d) the interface debonded length versus temperature curves of SiC/SiC composite subjected to thermal cycle under constant loading for different fiber volume fraction of \( \frac{l_c}{d} = 200 \) and 300 \( \mu \text{m} \).

Fig. 9. (a) The hysteresis energy versus temperature curves; (b) the hysteresis modulus versus temperature curve; (c) the hysteresis width versus temperature curves; and (d) the interface debonded length versus temperature curves of SiC/SiC composite subjected to thermal cycle under constant loading for different fiber volume fraction of \( \sigma_{\text{max}} = 200 \) and 250 MPa.
17.8 MPa at 1000°C, 13.6 MPa at 800°C, and 9.4 MPa at 600°C. When $N = 10$, the experimental fatigue hysteresis dissipated energy decreases from 11.1 kJ/m$^3$ at 1000°C, 3.5 kJ/m$^3$ at 800°C, to 1.1 kJ/m$^3$ at 600°C, corresponding to the fiber/matrix interface shear stress of 15.4 MPa at 1000°C, 10.4 MPa at 800°C, and 5.3 MPa at 600°C. When $N = 100$, the experimental fatigue hysteresis dissipated energy decreases from 8.8 kJ/m$^3$ at 1000°C, 2.0 kJ/m$^3$ at 800°C, to 0.62 kJ/m$^3$ at 600°C, corresponding to the fiber/matrix interface shear stress of 13.2 MPa at 1000°C, 7.8 MPa at 800°C, and 2.3 MPa at 600°C. When $N = 1000$, the experimental fatigue hysteresis dissipated energy decreases from 7.6 kJ/m$^3$ at 1000°C, 0.6 kJ/m$^3$ at 800°C, to 0.37 kJ/m$^3$ at 600°C, corresponding to the fiber/matrix interface shear stress of 12.1 MPa at 1000°C, 6.1 MPa at 800°C, and 0.22 MPa at 600°C.

Worthem\textsuperscript{29} investigated the TMF behavior of cross-ply SiC/MAS composite under in-phase (IP) and out-of-phase (OP) cyclic loadings with respect to thermal cycling between 600 and 1100°C. 3.5 kJ/m$^3$ at 800°C, to 1.1 kJ/m$^3$ at 600°C, corresponding to the fiber/matrix interface shear stress of 15.4 MPa at 1000°C, 10.4 MPa at 800°C, and 5.3 MPa at 600°C. When $N = 100$, the experimental fatigue hysteresis dissipated energy decreases from 8.8 kJ/m$^3$ at 1000°C, 2.0 kJ/m$^3$ at 800°C, to 0.62 kJ/m$^3$ at 600°C, corresponding to the fiber/matrix interface shear stress of 13.2 MPa at 1000°C, 7.8 MPa at 800°C, and 2.3 MPa at 600°C. When $N = 1000$, the experimental fatigue hysteresis dissipated energy decreases from 7.6 kJ/m$^3$ at 1000°C, 0.6 kJ/m$^3$ at 800°C, to 0.37 kJ/m$^3$ at 600°C, corresponding to the fiber/matrix interface shear stress of 12.1 MPa at 1000°C, 6.1 MPa at 800°C, and 0.22 MPa at 600°C.

Fig. 11. (a) The experimental and theoretical fatigue hysteresis dissipated energy versus temperature curves; and (b) the fiber/matrix interface shear stress versus temperature curve of SiC/MAS composite at the applied cycle number of $N = 1, 10, 100$ and 1000 under $\sigma_{\text{max}} = 110$ MPa at 600, 800, and 1000°C in inert atmosphere.

Fig. 12. (a) The experimental and theoretical fatigue life curves; and (b) the broken fibers fraction versus cycle number curves at $\sigma_{\text{max}} = 90$ and 100 MPa for SiC/MAS composite subjected to IP TMF with respect to thermal cycling between 600 and 1100°C.
\[ \sigma_{\text{max}} = 100 \text{MPa}, \text{the broken fibers fraction increases from} \]
0.3\% at the first applied cycle to 25\% at the 301th applied cycle. When the broken fibers fraction approaches to the critical value, the composite fatigue fractures.

The experimental and theoretical predicted fatigue life S–N curves of cross-ply SiC/MAS composite subjected to the OP TMF loading are shown in Fig. 13(a). The broken fibers fraction versus applied cycle number curves at \( \sigma_{\text{max}} = 90 \) and 100 MPa are shown in Fig. 13(b). When \( \sigma_{\text{max}} = 90 \text{MPa}, \) the broken fibers fraction increases from 0.16\% at the first applied cycle to 25\% at the 63th applied cycle; and when \( \sigma_{\text{max}} = 100 \text{MPa}, \) the broken fibers fraction increases from 0.33\% at the first applied cycle to 25\% at the 35th applied cycle. When the broken fibers fraction approaches to the critical value, the composite fatigue fractures.

5.2 2D SiC/SiC composite

Reynaud et al.\(^{(11)}\) investigated the cyclic fatigue behavior of 2D SiC/SiC composites at elevated temperatures in inert atmosphere. The temperature-dependent damage evolution of 2D SiC/SiC composites at elevated temperatures are predicted. The experimental fatigue hysteresis dissipated energy versus the applied cycle number curves of 2D SiC/SiC composite under \( \sigma_{\text{max}} = 130 \text{MPa} \) at 600, 800, and 1000°C in inert atmosphere are shown in Fig. 14. The fatigue hysteresis dissipated energy decreases with testing temperature. The material properties are given by:\(^{(27)}\) \( V_f = 40\%, E_f = 230 \text{ GPa}, E_m = 350 \text{ GPa}, r_1 = 7.5 \text{ mm}, \eta = 1.0 \text{ J/m}^2, \alpha_{\text{rm}} = 2.9 \times 10^{-6}/\text{K}, \alpha_{\text{rl}} = 3.9 \times 10^{-6}/\text{K}, \alpha_{\text{lm}} = 4.6 \times 10^{-6}/\text{K}, \alpha_{\text{ll}} = 2 \times 10^{-6}/\text{K}, \) and \( \sigma_0 = 2.6 \text{ GPa} \). The fatigue hysteresis dissipated energy corresponding to different applied cycle numbers and test temperatures are predicted, as shown in Fig. 15. When \( N = 10 \), the experimental fatigue hysteresis dissipated energy decreases from 10.2 kJ/m\(^3\) at 1000°C, 9.2 kJ/m\(^3\) at 800°C, to 5.4 kJ/m\(^3\) at 600°C, corresponding to the fiber/matrix interface shear stress of 24.9 MPa at 1000°C, 34.7 MPa at 800°C, and 44.6 MPa at 600°C. When \( N = 100 \), the experimental fatigue hysteresis dissipated energy decreases from 10.2 kJ/m\(^3\) at 1000°C, 9.4 kJ/m\(^3\) at 800°C, to 5.5 kJ/m\(^3\) at 600°C, corresponding to the fiber/matrix interface shear stress of 22.4 MPa at 1000°C, 31.2 MPa at 800°C, and 40.1 MPa at 600°C. When \( N = 1000 \), the experimental fatigue hysteresis dissipated energy decreases from 11.5 kJ/m\(^3\) at 1000°C, 10.6 kJ/m\(^3\) at 800°C, to 5.8 kJ/m\(^3\) at 600°C, corresponding to the fiber/matrix interface shear stress of 18.9 MPa at 1000°C, 26.8 MPa at 800°C, and 34.7 MPa at 600°C. When \( N = 100000 \), the experimental fatigue hysteresis dissipated energy decreases from 21.5 kJ/m\(^3\) at 1000°C, 15.4 kJ/m\(^3\) at 800°C, to 7.4 kJ/m\(^3\) at 600°C, corresponding to the fiber/matrix interface shear stress of 11.9 MPa at 1000°C, 19.8 MPa at 800°C, and 27.6 MPa at 600°C.

Worthem\(^{(29)}\) investigated the TMF behavior of 2D SiC/SiC composite under IP and OP cyclic loadings with respect to thermal cycling between 600 and 1100°C. The experimental and theoretical predicted fatigue life S–N curves of 2D SiC/SiC composite subjected to the IP TMF loading are shown in Fig. 16(a). The broken fibers fraction versus cycle number curves at \( \sigma_{\text{max}} = 50 \) and 60 MPa are shown in Fig. 16(b). When \( \sigma_{\text{max}} = 50 \text{ MPa}, \) the broken fibers fraction increases from 0.002\% at the first applied
cycle to 25% at the 505th applied cycle; and when $\sigma_{\text{max}} = 60$ MPa, the broken fibers fraction increases from 0.01% at the first applied cycle to 25% at the 225th applied cycle. When the broken fibers fraction approaches to the critical value, the composite fatigue fractures.

The experimental and theoretical predicted fatigue life S–N curves of 2D SiC/SiC composite subjected to the OP TMF loading are shown in Fig. 17(a). The broken fibers fraction versus cycle number curves at $\sigma_{\text{max}} = 50$ and 60 MPa are shown in Fig. 17(b). When $\sigma_{\text{max}} = 50$ MPa, the broken fibers fraction increases from 0.002% at the first applied cycle to 25% at the 192th applied cycle; and when $\sigma_{\text{max}} = 60$ MPa, the broken fibers fraction increases from 0.01% at the first applied cycle to 25% at the 103th applied cycle. When the broken fibers fraction approaches to the critical value, the composite fatigue fractures.

6. Conclusions

In this paper, the damage and fracture of fiber-reinforced CMCs subjected to thermal cycling at elevated temperatures in oxidizing atmosphere has been investigated. The relationships between thermal fatigue cycling, damage mechanisms, fatigue hysteresis-based damage parameters and thermal fatigue lifetime have been established. The effects of fiber/matrix interface properties, fiber radius, fiber volume fraction, matrix crack spacing and fatigue peak stress on the temperature-dependent thermal fatigue damage evolution of SiC/SiC composite subjected to thermal cycling fatigue loading have been analyzed. The temperature-dependent damage, fracture and lifetime prediction of cross-ply SiC/MAS and 2D SiC/SiC composites subjected to thermal fatigue cycling loading at elevated temperatures have been predicted.

1) With increasing fiber/matrix interface frictional coefficient, fiber volume fraction and matrix crack spacing, the fatigue hysteresis dissipated energy and fatigue hysteresis width decrease, and the fatigue hysteresis modulus increases, due to the decreasing fiber/matrix interface debonding and sliding range.

2) With increasing fiber radius and fatigue peak stress, the fatigue hysteresis dissipated energy and fatigue hysteresis width increase, and the fatigue hysteresis modulus decreases, due to the increasing fiber/matrix interface debonding and sliding length.
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