Two-photon exchange interaction from Dicke Hamiltonian under parametric modulation

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We consider the nonstationary circuit QED architecture in which a single-mode cavity interacts with \( N \) identical qubits, and some system parameters undergo a weak external perturbation. It is shown that in the dispersive regime one can engineer the two-photon exchange interaction by adjusting the frequency of harmonic modulation to (approximately) \( 2|\Delta_-| \), where \( \Delta_- \) is the average atom–field detuning. Closed analytic description is derived for the weak atom–field coupling regime, and numeric simulations indicate that the phenomenon can be observed in the present setups.

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I. INTRODUCTION

The area of circuit Quantum Electrodynamics (circuit QED) has grown to embrace a plethora of architectures with different kinds of multi-level atoms and sophisticated assemblies of interconnected 3D and 1D resonators and waveguides [11,12]. Diverse designs incorporating up to tens of Josephson Junctions give rise to superconducting artificial atoms with distinct properties regarding the dissipation mechanisms and the structure of energy levels, however, all they share the capability of coherently coupling to the Electromagnetic (EM) field [8–12]. Moreover, a single cavity mode can interact with several locally addressable artificial atoms [14,21] or an ensemble of trapped ultracold atoms [22,23].

Many types of superconducting artificial atoms allow for real-time manipulation of the energy levels or the atom–field coupling strength [24–30]. Combined with the ability of \textit{in situ} tuning the resonator’s frequency by external magnetic flux [31,32], such nonstationary circuit QED architectures give rise to a novel regime of light–matter interaction in which all the parameters in the Hamiltonian are controllable functions of time [33,34]. Using resonant perturbations one can induce creation and annihilation of photons or atomic excitations [35–40], generate entanglement [41–43], induce new forms of light–matter interaction [44,47], perform quantum simulations [48,49] and study other novel effects [50,53]. Some of the early proposals [33,54] have recently been verified experimentally, such as the one-photon exchange between the qubit and the field in the dispersive regime (reliant on the ‘rotating’ terms in the interaction Hamiltonian) [55,56] and generation of two quanta from vacuum due to the ‘counter-rotating terms’ (CRT) [57].

In this paper we describe another effect based on the rotating terms – the two-photon exchange interaction – that can be implemented in nonstationary circuit QED by modulating any system parameter with frequency \( \eta \approx 2|\Delta_-| \), where \( \Delta_- \) is the average atom–field detuning. We illustrate the phenomenon for the case of \( N > 1 \) off-resonant qubits described by the Dicke [58,59] or Tavis-Cummings [60,61] Hamiltonians, however, our approach can be straightforwardly generalized to an arbitrary multilevel atom in the ladder configuration [54]. Assuming the weak atom–field coupling regime, we derive a closed analytic description of the unitary dynamics (see Sec. II) and find a good agreement with numeric data even for moderate coupling strengths (Sec. II). We also show that our proposal can be implemented in the current circuit QED setups with weak dissipation and slightly different atoms (Sec. III A), and discuss manners to enhance the two-photon transition rate.

II. ANALYTIC RESULTS

Our system consists of a single mode of EM field interacting with \( N \) qubits, as described by the quantum Dicke model [58,59]

\[
\hat{H}/\hbar = \omega \hat{n} + \sum_{l=1}^{N} \left[ \frac{\Omega}{2} \hat{\sigma}_z^{(l)} + g(\hat{a} + \hat{a}^\dagger)(\hat{\sigma}_+^{(l)} + \hat{\sigma}_-^{(l)}) \right],
\]

where the index \( l \) labels the identical noninteracting atoms. We assume that the cavity frequency \( \omega \), the atomic transition frequency \( \Omega \) and the atom–cavity coupling strength \( g \) are externally prescribed functions of time (\( g \) is considered real). \( \hat{a} \) and \( \hat{a}^\dagger \) are the annihilation and creation operators and \( \hat{n} = \hat{a}^\dagger \hat{a} \) is the photon number operator. The qubit operators are \( \hat{\sigma}_z^{(l)} = |g^{(l)}\rangle \langle e^{(l)}| \), \( \hat{\sigma}_+^{(l)} = |e^{(l)}\rangle \langle g^{(l)}| \) and \( \hat{\sigma}_-^{(l)} = |e^{(l)}\rangle \langle e^{(l)}| - |g^{(l)}\rangle \langle g^{(l)}| \), where \( |g^{(l)}\rangle \) and \( |e^{(l)}\rangle \) denote the ground and excited states of the \( l \)-th qubit, respectively. In the absence of CRT \( \sum_{l=1}^{N} (\hat{a} \hat{\sigma}_-^{(l)} + \hat{a}^\dagger \hat{\sigma}_+^{(l)}) \) the Hamiltonian (1) is known as Tavis-Cummings Hamiltonian [60,61]. We stress that although our approach takes into account the CRT, the phenomenon described in this paper does not require their presence.

We consider the general case of simultaneous external modulation of all the system parameters as \( X = \ldots \)

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The index \( S \) stands within the subspace of a given value of Hamiltonian dressed states and eigenstates (\( \lambda_{k,n} \)). We expand the system state as
\[
|\psi\rangle = \sum_{k=0}^{N} \sum_{\Omega} \sum_{\mathcal{L}} \sum_{\mathcal{S}} \sum_{m} \sum_{s} \sum_{\lambda_m,S} \sum_{\tilde{\lambda}_{m,T,S}} e^{i\Phi_{m,s}(t)} e^{-i\tilde{\lambda}_{m,S} b_{m,s}(t)|\varphi_{m,s}\rangle} .
\]
where we defined the constant coefficients
\[
\Phi_{m,s}(t) = \sum_{k=0}^{N} \sum_{\Omega} \sum_{\mathcal{L}} \sum_{\mathcal{S}} \sum_{m} \sum_{s} \sum_{\lambda_m,S} \sum_{\tilde{\lambda}_{m,T,S}} e^{i\Phi_{m,s}(t)} e^{-i\tilde{\lambda}_{m,S} b_{m,s}(t)|\varphi_{m,s}\rangle} .
\]
and assumed that \( |\gamma_{L,k,n,T,S}|/\Delta_- \ll 1 \) for \( L = \omega, g, \Omega \). Under the resonant modulation frequency, \( \eta_{res} = \tilde{\lambda}_{m,T} - \lambda_{m,T} \). We get
\[
|\psi_{n,s}\rangle = N_{n,k} \left[ |\varphi_{n,k}\rangle + g_0 f_k \sqrt{\Delta_-} |\varphi_{n,k+1}\rangle \right] + g_0 f_k \sqrt{\Delta_-} \left[ |\varphi_{n,k+1}\rangle \right] + \frac{g_0^2 f_k \sqrt{\Delta_-}}{\Delta_-} \left[ |\varphi_{n,k+1}\rangle \right] + \frac{g_0^2 f_k \sqrt{\Delta_-}}{\Delta_-} \left[ |\varphi_{n,k+1}\rangle \right]
\]
where \( |\varphi_{n,k}\rangle \) is the frequency shift due to the counter-rotating terms in Eq. (3):
\[
u_{m,T} = g_0^2 \sum_{S} \left( \frac{\sum_{k=0}^{N} f_k \Lambda_{k,m,s,T}^2}{\lambda_{m,T} - \lambda_{m-2,s}} - \frac{\sum_{k=0}^{N} f_k \Lambda_{k,m+2,s,T}^2}{\lambda_{m+2,s} - \lambda_{m,T}} \right)
\]
where \( k = 0, 1, 2, \ldots, \min(n, N), K = n - k, \delta_- = g_0^2/\Delta_-, \)
\(|\varphi_{n,k}^{(0)}\rangle = |k, n-k\rangle \equiv |k\rangle_{\text{atom}} \otimes |n-k\rangle_{\text{field}}\) and \( \mathcal{N}_{n,k} \) is the normalization constant (\(|n\rangle_{\text{field}}\) is the cavity Fock state).

Actually, to evaluate the two-photon transition rate one needs the eigenstates to the fourth order in \( g_0/\Delta_- \), which are omitted here for brevity.

The two-photon exchange interaction \([62, 64]\) corresponds to the transition \(|\varphi_{n,k}\rangle \leftrightarrow |\varphi_{n,k+2}\rangle\), which represents (approximately) \(|k, n-k\rangle \leftrightarrow |k+2, n-k-2\rangle\).

To the lowest order in \( g_0/\Delta_- \) we obtain
\[
\Xi_{n,k,k+2} = D g_0 \left( \frac{g_0}{\Delta_-} \right)^3 \sqrt{(N-k)(N-k-1)} \times \sqrt{(k+1)(k+2)K(K-1)} \times \left( \frac{\varepsilon_\omega e^{-iD\phi_\omega} - \varepsilon_\Omega e^{-iD\phi_\Omega}}{\Delta_-} \right)^2,
\]
where \( D = \text{sign}(\Delta_-) \). The corresponding resonant modulation frequency reads
\[
\eta_r \approx 2 |\Delta_- + \delta_-(2N + 2n - 6k - 5)|.
\]

Notice that for a given value of \( k \) the other states \(|\{k, n-k\}, |k+2, n-k-2\rangle\) \((n' \neq n)\) are not affected by such modulation due to the condition \(|\delta_-| \gg |\Xi_{n,k,k+2}|\), as can be seen from Eq. [17].

We see that the second order in \( g_0/\Delta_- \) the same frequency \( \eta_r \) couples the pair of states \(|0\rangle \otimes |0\rangle \leftrightarrow |2, 2\rangle\) and \(|1, 6\rangle \leftrightarrow |3, 4\rangle\); we verified that for certain values of parameters this fact persists to the fourth order in \( g_0/\Delta_- \) as well.

III. NUMERIC RESULTS

To check our analytic predictions we solved numerically the Schrödinger equation for the original Hamiltonian [1]. In Fig. 1 we compare the exact numeric results, with and without CRT, to the approximate formulas [13] - [14]. We plot the average number of photons \( n_{\text{ph}} = \langle \psi | n | \psi \rangle \) and the average number of atomic excitations \( n_{\text{at}} = \langle \psi | \sum_{k=1}^N k a_{k,k} | \psi \rangle \) for the initial state \(|0\rangle \otimes |\alpha\rangle\) and parameters \( N = 2, g_0\sqrt{N}/\omega_\Omega = 8 \times 10^{-2}, \Delta_- = -9 g_0 \sqrt{N}, \varepsilon_\omega/g_0 = 10^{-1}, \phi_\omega = 0, \varepsilon_\Omega = \varepsilon_\omega = 0 \).

As expected, the resonant modulation frequencies vary depending on whether the CRT are taken into account or not: \( \eta_r = 2|\Delta_-| \times 1.0678 \) with CRT and \( \eta_r = 2|\Delta_-| \times 1.0540 \) without CRT. The analytic and numeric results agree qualitatively, though there is a roughly 20% difference in the analytic and actual transition rates \( |\Xi_{5,0,2}| \).

Such discrepancy is not surprising, since for the above parameters the required inequalities \( \sqrt{2n} g_0/\Delta_- \ll 1 \) and \( |\Delta_-| \ll |\Omega| \) are only barely satisfied [64]. The apparent broad width of the curves is explained by fast low-amplitude oscillations due to the off-resonant photon exchange inherent to the dispersive regime, as inferred from the wavefunction [4] and the expression for the dressed states [15].

In Fig. 2 we consider a more realistic initial state \(|0\rangle \otimes |\alpha\rangle\), where \(|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^\infty \alpha^n/\sqrt{n!} \) stands for the cavity coherent state with \( \alpha = \sqrt{5.5} \) so that the initial probability of 5 photons is \( P_{\text{ph}}(5) \approx 0.17 \).

For the sake of compactness we only present the exact numeric results in the presence of CRT. We set \( N = 6 \) and consider the g-modulation (with parameters \( g_0\sqrt{N}/\omega_\Omega = 8 \times 10^{-2}, \Delta_- = -9 g_0 \sqrt{N}, \varepsilon_\omega/g_0 = 10^{-1}, \phi_\omega = 0 \) and \( \eta_r = 2|\Delta_-| \times 1.0389 \)), as well as the simultaneous modulation of \( g \) and \( \Omega \) with the additional parameters \( \varepsilon_\Omega/\Delta_- = 10^{-1} \) and \( \phi_\Omega = \pi \) (in this case \( \eta_r = 2|\Delta_-| \times 1.0388 \)). These modulation frequencies were
The modulation drives the transition $|0, 4\rangle \leftrightarrow |2, 2\rangle$.

$$\hat{H}/\hbar = \omega \hat{n} + \sum_{l=1}^{2} \left[ \frac{\Omega(l)}{2} \hat{\sigma}^z(l) + g^{(l)}(\hat{a} + \hat{a}^\dagger)(\hat{\sigma}^x(l) + \hat{\sigma}^z(l)) \right],$$

where $\hat{\rho}$ is the total density operator and $\hat{L}$ is the Liouvillean. To get a rough estimative we solved numerically the ‘standard’ phenomenological master equation [54] for zero-temperature reservoirs [66]

$$\dot{\hat{\rho}} = \kappa D[\hat{a}^\dagger \hat{a}] \hat{\rho} + \sum_{l=1}^{2} \left( \gamma^{(l)} \hat{D}[\hat{a}^\dagger \hat{a}] + \gamma^{(l)}_\rho \hat{D}[\hat{\sigma}^z(l)] \right),$$

where $\kappa$, $\gamma^{(l)}$, and $\gamma^{(l)}_\rho$ denote the cavity damping and the $l$-th qubit’s relaxation and pure dephasing rates, respectively.

In Fig. 4 we compare the dynamics for the ideal and realistic scenarios under the $g$-modulations and the initial cavity coherent state $|g^{(1)}, g^{(2)}\rangle \otimes |\alpha\rangle$, where $\alpha = \sqrt{3}$. For the realistic case we set: $g^{(l)} = g_0^{(l)} + \varepsilon^{(l)} \sin(\eta l)$, $g_0^{(1)}/\omega_0 = 5.66 \times 10^{-2}$, $g_0^{(2)} = 1.01 g_0^{(1)}$, $\varepsilon^{(1)}/g_0^{(1)} = 0.1$, $\Delta^{(1)} = \omega_0 - \Omega^{(1)} = -0.72 \omega_0$, $\Delta^{(2)} = 1.02 \Delta^{(1)}$, $\kappa/\omega_0^{(1)} = \gamma^{(1)}/g_0^{(1)} = 5 \times 10^{-5}$, $\gamma^{(1)}_\rho = \gamma^{(1)}$, and $\eta_l = 2|\Delta^{(1)}| \times 1.0632$. For the ideal case $g_0^{(1)}/\omega_0 = 5.66 \times 10^{-2}$, $\varepsilon^{(1)}/g_0^{(1)} = 0.1$, $\Delta^{(1)} = -0.72 \omega_0$, and $\eta_l = 2|\Delta^{(1)}| \times 1.0531$ (the modulation frequencies were chosen to induce the transition $|0, 4\rangle \leftrightarrow |2, 2\rangle$). Such parameters are compatible with the current circuit QED architectures, where typically $\omega_0/2\pi = 10$ GHz [11, 10, 12, 14, 16, 19, 26, 57]. We see that for initial times, $t \lesssim 1 \mu s$, the two-photon exchange can be proved via measurements of the average number of atomic excitations $n_{at}$ or the probabilities $P_{at}(2)$, $P_{ph}(2)$.

A. Simulation under realistic conditions

The above numeric results apply to an ideal situation, namely, strictly identical atoms and dissipation-free environment. To assess the experimental feasibility of our proposal in circuit QED, we consider a realistic scenario of two slightly different artificial atoms coupled to a single-mode waveguide resonator under weak Markovian dissipation. The dynamics is now governed by the master equation

$$\frac{d\hat{\rho}}{dt} = \frac{1}{i\hbar}[\hat{H}, \hat{\rho}] + \hat{L}\hat{\rho}$$

Figure 3: (Color online) Exact numeric dynamics of probabilities $P_{ph}(k)$ and $P_{at}(m)$ for $N = 6$ and the simultaneous modulation of $g$ and $\Omega$. The initial state is $|0\rangle \otimes |\alpha\rangle$, $\alpha = \sqrt{3.5}$, and the modulation induces the transition $|0, 5\rangle \rightarrow |2, 3\rangle$.

To attest that the periodic behavior of $n_{ph}$ and $n_{at}$ indeed corresponds to a two-photon exchange, in Fig. 3 we plot the probability $P_{ph}(k)$ of $k$ photons and the probability $P_{at}(m)$ of $m$ atomic excitations under the simultaneous $g$- and $\Omega$-modulations (discussed in Fig. 2). We observe that the main transition occurs between the states $|0, 5\rangle$ and $|2, 3\rangle$, although there are unwanted couplings between other states owing to the off-resonant one-photon exchange. As an example, we illustrate small oscillations between the states $|2, 3\rangle \leftrightarrow |3, 2\rangle$, inferred from the periodic oscillation of probabilities $P_{ph}(2)$ and $P_{at}(3)$ at the same rate as the probabilities $P_{ph}(3)$ and $P_{at}(2)$.
and $P_{ph}(\Delta)$, whereas the measurement of the average photon number is of little help due to overwhelming effects of dissipation.

IV. CONCLUSIONS

We showed analytically and numerically that effective two-photon exchange interaction between a single cavity mode and $N > 1$ off-resonant qubits can be achieved by externally modulating any system parameter at frequency $\eta \approx 2|\Delta_-|$, where $\Delta_-$ is the average atom–field detuning. This effect originates from the ‘rotating’ terms in the Dicke (or Tavis-Cummings) Hamiltonian, but the associated transition rate is quite small due to the multiplicative factor $(g_0/\Delta_-)^2$. Closed analytical description was derived under the assumption of weak atom–field coupling, and a good agreement with exact numeric results was observed even for moderate coupling strengths. For a simultaneous modulation of different parameters the transition rate can be increased by properly adjusting the initial phases. Regarding the experimental feasibility, we demonstrated that for $N = 2$ our proposal can be implemented in the current circuit QED architectures on the timescales $\sim 1 \mu$s, which could be further reduced through an increase in the modulation amplitudes, atom–field coupling strength or the number of qubits.

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