Remarks on the interquark potential in the presence of a minimal length

Patricio Gaete

Departamento de Física and Centro Científico-Tecnológico de Valparaíso, Universidad Técnica Federico Santa María, Valparaíso, Chile

E-mail: patricio.gaete@usm.cl

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Abstract
We calculate the lowest-order corrections to the static potential for both Yang–Mills theory and gluodynamics in curved space–time, in the presence of a quantum gravity induced minimal length. Our analysis is carried out within stationary perturbation theory. As a consequence, the potential energy is the sum of a Yukawa-like potential and a linear potential for gluodynamics in curved space–time, leading to the confinement of static charges. Interestingly, we find that the coefficient of the linear term (’string tension’) is ultraviolet finite. We highlight the role played by the new quantum of length in our analysis.

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1. Introduction

The study of extensions of the standard model (SM), such as Lorentz invariance violation and fundamental length, have motivated a great deal of research effort in previous years [1–5]. Clearly, such studies are supported because the SM does not include a quantum theory of gravitation. In fact, there is the need for a new scenario which has been suggested to overcome theoretical difficulties in the quantum gravity research. In this respect we recall that string theories [6] provide a consistent framework to unify all fundamental interactions. Let us also mention here that in previous years the focus of quantum gravity has been towards effective models, which have helped to gain insight on a new and unconventional physics. In fact, these models incorporate one of the most important and general features, that is, a minimal length scale that acts as a regulator in the ultraviolet. Mention should be made, at this point, of different approaches which incorporate a minimal length [7–10]. Among these works, probably the most studied framework are quantum field theories allowing non-commuting position operators [11–16].

Another interesting observation is that most of the results in the noncommutative approach have been achieved using the so-called star product (Moyal product). However, as is known, there is another formulation of noncommutative quantum field theory. This alternative formulation is also known as quantum field theory in the presence of a minimal
length, which has been proposed in [17–19]. Basically, this approach is based on defining the fields as mean value over coherent states of the noncommutative plane, such that a star product need not be introduced. Subsequently, it has been shown that the coherent state approach can be summarized through the introduction of a new multiplication rule, which is known as the Voros star product [20–24]. In any case, the physics turns out be independent from the choice of product type [25]. An alternative view of these modifications is to consider them as a redefinition of the Fourier transform of the fields. Accordingly, the theory is ultraviolet finite and the cutoff is provided by the noncommutative parameter $\theta$. Note that the existence of a minimal length is determined by the noncommutative parameter $\theta$. In a general perspective, since one can incorporate a minimal length ($\sqrt{\theta}$) in space–time by assuming nontrivial coordinate commutation relations, we have then introduced a noncommutative geometry.

With this in mind, in previous studies [26, 27] we have considered the effect of the space–time noncommutativity on a physical observable. In fact, we have computed the static potential for axionic electrodynamics both in $(3+1)$ and $(2+1)$ space–time dimensions, in the presence of a minimal length. The point we wish to emphasize, however, is that our analysis leads to a well-defined noncommutative interaction energy. Indeed, in both cases we have obtained a fully ultraviolet finite static potential. Seen from such a perspective, the purpose of the present work is to extend the Abelian calculations to the non-Abelian case and find the corresponding static potential. Our calculations are done by using perturbation theory along the lines of [28–30]. An important advantage of this approach is that it provides a transparent description of the origin of asymptotic freedom in gauge theories. In short, we will calculate the lowest-order corrections to the static potential for both Yang–Mills theory and gluodynamics in curved space–time, in the presence of a minimal length.

2. Yang–Mills in the presence of a minimal length

As stated in the introduction, the main focus of this paper is to re-examine the interaction energy for gluodynamics in curved space–time in the presence of a minimal length. However, let us begin by considering a noncommutative version of Yang–Mills theory to illustrate the central features which will then be extended to noncommutative gluodynamics. Specifically, in this work we will focus our attention on the short-distance perturbative interaction potential.

To this end, the initial point of our analysis is the following four-dimensional space–time Lagrangian:

$$L = -\frac{1}{4} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu}_a, \quad (1)$$

where $A_\mu^a(x) = A_\mu^a(x) T^a$ and $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$, with $f^{abc}$ the structure constants of the gauge group.

As already mentioned, our analysis is based on perturbation theory along the lines of [28–30]. In other words, we will work out the vacuum expectation value of the energy operator $H \langle \langle 0 | H | 0 \rangle \rangle$ at lowest order in $g$. In a case such as this, it is profitable to use the Coulomb gauge because the fields have a simple physical meaning. Thus the Hamiltonian corresponding to (1) reads

$$H = \frac{1}{2} \int d^3x \left[ (E^a) \right]^2 + (B^a)^2 \right] - \phi^a \nabla^2 \phi^a, \quad (2)$$

where the color-electric field $E^a$ has been separated into transverse and longitudinal parts: $E^a = E^a_T - \nabla \phi^a$.

It should be noted here, however, that by making use of Gauss’ law

$$\nabla^2 \phi^a = g (\rho^a - f^{abc} A^b \cdot E^c), \quad (3)$$

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we get
\[ \nabla^2 \phi^a = \left( g^2 \rho^a + g^2 f^{abp} A^b \cdot \nabla \frac{1}{\sqrt{2}} + g^3 f^{abc} A^b \cdot \nabla \frac{1}{\sqrt{2}} f^{chp} A^h \cdot \nabla \frac{1}{\sqrt{2}} \right) \left( \rho^p - f^{pde} A^d \cdot E^e_1 \right). \]

(4)

Next, the corresponding formulation of this theory in the presence of a minimal length [26, 27]. As a consequence, we will take the sources as \( \rho^a = \rho^a_0 + \rho^a_1 + \rho^a_2 \), where \( \rho^a_0(x) = r_0^a e^{i (x - y)^2} \delta^{(3)}(x - y) \) and \( \rho^a_1(x) = r_1^a e^{i (x - y)^2} \delta^{(3)}(x - y) \). As in [30], \( r_0^a \) and \( r_1^a \) are the color charges of a heavy antiquark \( \bar{q} \) and a quark \( q \) in a normalized color singlet state \( |\Psi\rangle = N^{-1/2} |q\rangle |\bar{q}\rangle \). Hence \( r_0^a r_1^a = \frac{1}{N} tr(T^a T^a) = -C_F \), where the anti-Hermitian generators \( T^a \) are in the fundamental representation of \( SU(N) \).

Having characterized the sources, we can now compute the expectation value of the energy operator \( H \). Thus, to order \( g^2 \) and \( g^3 \), we can then easily verify that
\[ V = V_1 + V_2, \]
where
\[ V_1 = -g^2 \int d^3 x (0) \rho^a_0 \frac{1}{\sqrt{2}} \rho^a_2 |0\rangle, \]
and
\[ V_2 = -3g^3 f^{abc} f^{chp} \int d^3 x (0) \rho^a_0 \frac{1}{\sqrt{2}} A^b \cdot \nabla \frac{1}{\sqrt{2}} A^h \cdot \nabla \frac{1}{\sqrt{2}} \rho^a_2 |0\rangle. \]

The \( V_1 \) term leads immediately to the result
\[ V_1 = -g^2 C_F \int \frac{d^3 k}{(2\pi)^3} \frac{e^{-i k \cdot r}}{k^2} e^{-i k \cdot r} = -g^2 C_F \frac{1}{4\pi^{3/2}} \frac{1}{r} \gamma \left( \frac{1}{2}, \frac{r^2}{\pi^2} \right), \]
where \( |r| \equiv |y - y'| = r \) and \( \gamma \left( \frac{1}{2}, \frac{r^2}{\pi^2} \right) \) is the lower incomplete Gamma function defined by the following integral representation
\[ \gamma \left( \frac{1}{2}, \frac{r^2}{\pi^2} \right) = \int_0^r \frac{d u}{u} u^{1/2} e^{-u}. \]

Hence we see that the term of order \( g^2 \) is just the regular Coulomb energy at the origin due to the color charges of the quarks.

We now come to the \( V_2 \) term, which is given by
\[ V_2 = 3g^3 C_F tr \int \frac{d^3 k}{(2\pi)^3} \frac{e^{-i k \cdot r} T(k)}{k^2}, \]
where
\[ T(k) = \int \frac{d^3 \rho}{(2\pi)^3} \frac{1}{2|p|(|p - k|^2)} \left( 1 - \frac{(p \cdot k)^2}{p^2 k^2} \right). \]

(11)

To get equation (10) we have expressed the \( A^a \) fields in terms of a normal mode expansion: \( A^a(x, t) = \int \frac{d^3 \rho}{(2\pi)^3} \sum \lambda \phi^a (p, \lambda) e^{-i p \cdot x} + |a^0 (p, \lambda) e^{i p \cdot x} \rangle \). Also we have used \[ [a^a (p, \lambda), a^b (l, \sigma)] = \delta^{ab} \delta^{\lambda \sigma} \delta^{(3)}(p - l) \] and \( \sum \phi^a (p, \lambda) e^{i p \cdot x} (k, \lambda) = \delta^{ij} - \frac{k^i k^j}{k^2} \). It is worth recalling here that the correction term of order \( g^3 \) represents an anti-screening effect and makes the interquark potential weaker at short distances, which is in the origin of asymptotic freedom in QCD. Incidentally, it is of interest to note that the anti-screening effect is due to the instantaneous Coulomb interaction of the quarks.

Our next task is to compute the integral (11). In fact, when this integral is performed it is found that by introducing a cutoff \( (A) \) is just sufficient to extract the logarithmic divergence.
Thus, the corresponding integration gives \( \mathcal{I}(k) = \frac{1}{(2\pi)^3} \ln(k^2) \). This then implies that the integral of equation (10) can be cast under the form:

\[
V_2 = g^4 C_F C_A \int \frac{d^3 k}{(2\pi)^3} \frac{e^{-i k \cdot r}}{k^2} - \frac{12}{48\pi^2} \ln \left(\frac{k^2}{\Lambda^2}\right). \tag{12}
\]

Now, we move on to compute the integral (12). To do this, it is advantageous to introduce a new auxiliary function \( F \) [31]:

\[
F(r, \Lambda, s) \equiv \Lambda^{2s} \int \frac{d^3 k}{(2\pi)^3} \frac{e^{-i k \cdot r}}{(k^2)^{1+s}} e^{-i k \cdot r}, \tag{13}
\]

such that the \( V_2 \) term takes the form

\[
V_2 = -\frac{12}{48\pi^2} g^4 C_F C_A \frac{\partial}{\partial s} [F(r, \Lambda, s)]_{s=0}. \tag{14}
\]

We also note that

\[
F(r, \Lambda, s) = \Lambda^{2s} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{(k^2)^{1+s}} \int_0^{r/\Lambda} dy \frac{y^{1/2}}{4\pi^2} \left( \frac{r^2}{4y} - \theta \right)^s e^{-\gamma}. \tag{15}
\]

Hence we see that, at leading order in \( \theta \), the function \( F \) is then

\[
F(r, \Lambda, s) = \frac{(\Lambda r)^{2s} \Gamma(1/2 + s)}{4\pi^2 r^{(1/2 - s)}} - \frac{s (\Lambda r)^{2s} \Gamma(3/2 + s)}{r^{(3/2 - s)}} \tag{16}
\]

With this, we can write (14) also as

\[
V_2 = -\frac{1}{\Lambda^{3/2}} g^4 \frac{12}{48\pi^2} C_F C_A \frac{1}{r} \ln(\Lambda r) r^{(1/2 - s)} + s \frac{g^4}{\pi^2} \frac{12}{48\pi^2} C_F C_A \frac{1}{r} \gamma(1/2, r/\Lambda). \tag{17}
\]

We draw attention to the fact that the previous result is finite at order \( \theta \). In fact, one comment is pertinent in this context. Since we have considered the quantization of the full theory, without expanding in powers of \( \theta \) and no truncation was made, we get a finite result. The \( \theta \) expansion of the function \( F \) is only for reasons of calculation. This has nothing to do with the \( \theta \) expansion through the Moyal * product, as we have explained in [26].

We now want to consider the \( (g^2) \) screening contribution to the potential, which is due to the exchange of transverse gluons. This effect makes the interquark potential stronger at short distances. With this in mind, from perturbation theory, we start by writing

\[
V_2^* = 2g^4 f_{abc} f_{def} \sum_{n=2}^{\text{gluon}} \frac{1}{k^n} \int d^3 x \int d^3 w \langle 0 | \rho_2 A^b \cdot E_f^e | n \rangle_x \times \langle n | \rho_1 A^c \cdot E_f^e | 0 \rangle_w, \tag{18}
\]

where, by construction, \( \langle 0 | \rho_2 A^b \cdot E_f^e | n \rangle \) is the matrix element in the basis of states in which the non-perturbated Hamiltonian is diagonal. In order to evaluate equation (18), we note that the intermediate state \( | n \rangle \) must contain a pair of transverse gluons, since the terms \( A^b \cdot E_f^e \) must create and destroy dynamical gluon pairs. Taking this remark into account, we may write two gluon states as

\[
\sum_{n=2}^{\text{gluon}} | n \rangle \langle n | = \frac{1}{2} \sum_{k, l} \sum_{\lambda, \sigma} \int d^3 k' \int d^3 l' a^e_{\lambda \sigma}(k, \lambda) a^{f \dagger} (l, \sigma) | 0 \rangle | 0 \rangle | a' (l, \sigma) a'' (k, \lambda) \rangle. \tag{19}
\]

Substituting equation (19) into equation (18) and following our earlier procedure, the \( V_2^* \) term reduces to

\[
V_2^* = -C_A C_F g^4 \int \frac{d^3 k}{(2\pi)^3} \frac{e^{-i k \cdot r}}{k^2} e^{i k \cdot r} \mathcal{I}(k). \tag{20}
\]
where
\[
I(k) = \int \frac{d^3l}{(2\pi)^3} \frac{(|l| - |l - k|)^2}{4|l - k|(|l| + |l - k|)} \left\{ 1 + \frac{|(l - k) - l|^2}{F(l - k)^2} \right\}. \tag{21}
\]

Integrating now over \( k \), one then obtains \( I(k) = \frac{1}{4\pi^2} \ln \left( \frac{k^2}{\lambda^2} \right) \). As a consequence, the \( V_2^* \) term becomes
\[
V_2^* = -g^2 C_F C_A \int \frac{d^3k}{(2\pi)^3} \frac{e^{-g k^2}}{k^2} e^{-ik \cdot r} \frac{1}{4\pi^2} \ln \left( \frac{k^2}{\Lambda^2} \right). \tag{22}
\]
It is straightforward to see that this integral is exactly the one obtained in expression (12).

By putting together equations (8), (12) and (22), we obtain for the total interquark potential
\[
V = -g^2 C_F \int \frac{d^3k}{(2\pi)^3} \frac{e^{-g k^2}}{k^2} \left\{ 1 + g^2 C_A \frac{11}{48\pi^2} \ln \left( \frac{\Lambda^2}{k^2} \right) \right\} e^{-ik \cdot r}. \tag{23}
\]
By means of the function \( F^* \) we evaluate the interquark potential in position space. Accordingly, one finds
\[
V = -g^2 C_F \frac{1}{4\pi^2 r} \left( 1 + g^2 C_A \frac{11}{24\pi^2} \ln (\Lambda r) \right) \gamma \left( \frac{\gamma}{2}, \frac{r^2}{\Lambda^2} \right) + \theta g^2 C_F C_A \frac{11}{48\pi^2} \frac{1}{2} r \gamma \left( \frac{\gamma}{2}, \frac{r^2}{\omega} \right). \tag{24}
\]
An immediate consequence of this is that for \( \theta = 0 \) one obtains the known interquark potential at lowest order in \( g [28] \).

Interestingly, it is observed that the term proportional to \( \theta \) is ultraviolet finite. Another important point to be mentioned in our discussion comes from the explicit appearance of the cutoff \( \Lambda \) in (24). Evidently, at distances higher than \( 1/\Lambda \) the interaction potential (24) is convergent. Nevertheless, at distances lower than \( 1/\Lambda \) the static potential is divergent. This result immediately shows that the interaction energy concept is not suitable in this region. In other words, \( 1/\Lambda \) naturally defines a critical distance below which the field regime prevails, that is, the distance scale for which we can no longer describe the interaction by using the potential or force concept, but only by the field one. Also, (below \( 1/\Lambda \)) our description must be supplemented by degrees of freedom that are truncated below the cutoff \( \Lambda \).

3. Gluodynamics in curved space–time in the presence of a minimal length

We now pass to the calculation of the interquark potential for gluodynamics in curved space–time [32]. In other words, we wish to explore the effects of including a minimal length on the nature of the potential. The corresponding theory is governed by the Lagrangian density:
\[
\mathcal{L} = \frac{|\epsilon_V|}{m^2} \frac{1}{2} e^{x/2} (\partial_\mu \chi)^2 + |\epsilon_V| e^x (1 - \chi) - e^x (1 - \chi) \frac{1}{4} F_{\mu \nu}^a F^{a \mu \nu}, \tag{25}
\]
where the real scalar field \( \chi \) of mass \( m \) represents the dilaton, and \( -|\epsilon_V| \) is the vacuum energy density. Thus, after retaining only the leading quadratic term and integrating over the \( \chi \) field, we arrive at the following effective Lagrangian:
\[
\mathcal{L}_{\text{eff}} = -\frac{1}{4} F_{\mu \nu}^a \left( 1 + \frac{m^2}{\Delta} \right) F_{\mu \nu}^{a} + |\epsilon_V|. \tag{26}
\]
We have omitted all the technical details and refer the reader to [32] for them.

Now, proceeding as before, the Hamiltonian of the theory is given by
\[
H = -\frac{1}{2} \int d^3x \phi^a (\nabla^2 - m^2) \phi^a. \tag{27}
\]
In analogy with (4) we have
\[(\nabla^2 - m^2)\phi^0 = \left(g\delta^{ab} + g^2 f^{abc} A^b \cdot \nabla \frac{1}{\sqrt{2}} + g^3 f^{abc} A^b \cdot \nabla \frac{1}{\sqrt{2}} f^{e} A^e \cdot \nabla \frac{1}{\sqrt{2}}\right) \times \left(\rho^a - f^{abc} A^c \cdot \left[\nabla^2 - m^2\right] E^a_0\right). \tag{28}\]

We are now in a position to evaluate the interquark potential for gluodynamics in curved space–time. In this case, we see that the expectation value \(\langle 0|H|0\rangle\) reads:
\[V = V_1 + V_2. \tag{29}\]

The term \(V_1 (V_1 \equiv V_1^{(1)} + V_1^{(2)})\) is now given by
\[V_1^{(1)} = g^2 \int d^3x |0\rangle \rho^a_1 \frac{1}{\sqrt{2} - m} \rho^a_2 |0\rangle, \tag{30}\]

and
\[V_1^{(2)} = 3g^4 f^{abc} f^{e} \int d^3x |0\rangle \rho^a_1 \frac{1}{\sqrt{2} - m} A^b \cdot \nabla \frac{1}{\sqrt{2}} A^e \cdot \nabla \frac{1}{\sqrt{2}} \rho^a_2 |0\rangle. \tag{31}\]

As was shown in [26], the \(V_1^{(1)}\) term reduces to
\[V_1^{(1)} = -g^2 C_F e^{m^2} \frac{1}{4\pi} \left[\int_{r}^{\infty} e^{-mr} \frac{1}{\sqrt{2} - m^2} \right], \tag{32}\]

while, following the same steps as those of the preceding section, the \(V_1^{(2)}\) term turns out to be
\[V_1^{(2)} = -g^4 C_F C_A \frac{12}{48\pi^2} \rho^a \frac{1}{\sqrt{2} - m} \left[\Lambda^2 \int_{\rho^a}^{\infty} d^3k \frac{1}{(2\pi)^3} \right] \tag{33}\]

For our purposes it is sufficient to retain the leading quadratic term in \(k\). Thus, the \(V_1^{(2)}\) term simplifies to
\[V_1^{(2)} = -g^4 C_F C_A \frac{12}{48\pi^2} \frac{1}{(2\pi)^3} \frac{1}{(k^2 + m^2)} \rho^a \tag{34}\]

where \(F\) was defined in (13).

Now we turn our attention to the \(V_2\) term, which this time is expressed as
\[V_2 = 2g^4 \sum_{n=2}^{\text{gluon}} \frac{1}{E_n} f^{abc} f^{def} \int d^3x \int d^3w \int d^3l \sum_{\lambda, \sigma} \frac{1}{E_n} \rho^a \frac{1}{\sqrt{2} - m} A^b \cdot \left(\frac{\nabla^2 - m^2}{\sqrt{2}}\right) E^a_0 |0\rangle \tag{35}\]

Unlike in the previous subsection, this latter term consists of two parts, namely:
\[V_2^{(1)} = g^4 f^{abc} f^{def} \int d^3x \int d^3w \int d^3k \int d^3l \sum_{\lambda, \sigma} \frac{1}{E_n} \rho^a \frac{1}{\sqrt{2} - m} A^b \cdot E^a_0 |0\rangle \tag{36}\]

and
\[V_2^{(2)} = m^4 g^4 f^{abc} f^{def} \int d^3x \int d^3w \int d^3k \int d^3l \sum_{\lambda, \sigma} \frac{1}{E_n} \rho^a \frac{1}{\sqrt{2} - m} A^b \cdot E^a_0 |0\rangle. \tag{37}\]
Following our earlier procedure, these expressions can be conveniently rewritten as

\begin{equation}
V^{(1)}_2 = -g^4 C_F C_A \int \frac{d^3k}{(2\pi)^3} \frac{e^{-\theta k^2}}{k^2 + m^2/2} \frac{1}{48\pi^2} \ln \left( \frac{k^2}{\Lambda^2} \right) e^{ikr},
\end{equation}

and

\begin{equation}
V^{(2)}_2 = -m^2 g^4 C_F C_A \int \frac{d^3k}{(2\pi)^3} \frac{e^{-\theta k^2}}{k^2 + m^2/2} \frac{1}{48\pi^2} \ln \left( \frac{k^2}{\Lambda^2} \right) e^{ikr}.
\end{equation}

Again, since we are interested in estimating the lowest-order correction to the interquark potential, we will retain only the leading quadratic terms in the expressions (38) and (39), namely,

\begin{equation}
V^{(1)}_2 = -g^4 C_F C_A \int \frac{d^3k}{(2\pi)^3} \frac{e^{-\theta k^2}}{k^2} \frac{1}{48\pi^2} \ln \left( \frac{k^2}{\Lambda^2} \right) e^{ikr},
\end{equation}

and

\begin{equation}
V^{(2)}_2 = -m^2 g^4 C_F C_A \int \frac{d^3k}{(2\pi)^3} \frac{e^{-\theta k^2}}{k^2} \frac{1}{48\pi^2} e^{ikr}.
\end{equation}

In position space we then have

\begin{equation}
V^{(1)}_2 = g^4 \frac{1}{48\pi^2} C_F C_A \frac{\partial}{\partial s} \left[ \mathcal{F} \left( r, \Lambda, s \right) \right]_{s=0},
\end{equation}

and

\begin{equation}
V^{(2)}_2 = \frac{m^2 g^4}{32\sqrt{2\pi}^2} \left\{ r \gamma(1/2, r^2/\theta) + 2\sqrt{\theta} e^{-r^2/\theta} + \frac{2\theta}{r} r \gamma(1/2, r^2/\theta) \right\}.
\end{equation}

Finally, by putting together equations (32), (34), (42) and (43), we obtain for the total interquark potential

\begin{equation}
V = -g^4 C_F \frac{e^{m^2/\theta}}{4\pi^2} \frac{1}{r} \left\{ e^{-mr} - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} d^3 \xi e^{-\xi^2 - mr^2/\theta} \right\}
\end{equation}

\begin{equation}
\quad - \frac{g^4 C_A C_F}{16\pi^2} \frac{1}{r} \left[ \frac{11}{6} \ln(\Lambda r) - \frac{m^2 \theta}{\sqrt{2}} \right] \gamma(1/2, r^2/\theta) + \frac{11g^4 C_A C_F}{48\pi^2} \frac{\theta}{r^3} r \gamma(1/2, r^2/\theta)
\end{equation}

\begin{equation}
\quad + \frac{g^4 m^2 C_A C_F}{16\sqrt{2\pi}^2} \sqrt{\theta} e^{-r^2/\theta} + \frac{g^4 m^2 C_A C_F}{32\sqrt{2\pi}^2} r \gamma(1/2, r^2/\theta).
\end{equation}

Here, in contrast to our previous analysis [32], the coefficient of the linear term (‘string tension’) is ultraviolet finite. This result displays a marked qualitative departure from its commutative counterpart [32]. However, one can easily verify that in the limit marrow0 the confinement term vanishes, as it should. Furthermore, it is straightforward to see that the terms proportional to $g^2$ and to $\theta$ are all ultraviolet finite. In addition, the cutoff $\Lambda$ appears again in the above expression. In this regard we refer to our comments in the previous subsection.

### 4. Final remarks

To conclude, within stationary perturbation theory, we have studied the confinement versus the screening issue for gluodynamics in curved space–time in the presence of a minimal length. We have found that the static potential is the sum of a Yukawa-type and a linear potential, leading to the confinement of static charges. More interestingly, it was shown that the coefficient of the linear term is ultraviolet finite, where the new quantum of length was crucial to obtain this result. At this point, we would like to remark that our model for gluodynamics is an effective description that comes out upon integration over the dilaton field, whose excitation

\[ V = -\frac{1}{6 \pi^2} \int d^3 \xi \frac{1}{2} \left( \frac{11}{6} \ln(\Lambda r) - \frac{m^2 \theta}{\sqrt{2}} \right) \gamma(1/2, r^2/\theta) + \frac{11g^4 C_A C_F}{48\pi^2} \frac{\theta}{r^3} r \gamma(1/2, r^2/\theta) \]

\[ + \frac{g^4 m^2 C_A C_F}{16\sqrt{2\pi}^2} \sqrt{\theta} e^{-r^2/\theta} + \frac{g^4 m^2 C_A C_F}{32\sqrt{2\pi}^2} r \gamma(1/2, r^2/\theta). \]
is massive. We also draw attention to the role played by dilaton in yielding confinement: its mass contributes linearly to the string tension.

Also, from (24) it is easy see that the introduction of a minimal length preserves the asymptotically free behavior at short distance, characterizing the non-Abelian character of the strong interactions.

As a final remark it should be mentioned that a correct identification of physical degrees of freedom is a fundamental ingredient for understanding the physics hidden in gauge theories. This then implies that, once the identification has been made, the computation of the potential is achieved by means of Gauss’ law.

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