Handling of nonlinear systems using filtered high-gain output feedback controller

Tong Ma | Yuqian Liu | Cyuansi Shih | Chengyu Cao

Summary
This paper develops a filtered high-gain output feedback controller for a class of nonlinear systems in the presence of unknown state-dependent and time-varying nonlinearities. It considers that the nonlinearities satisfy a semiglobal Lipschitz condition. The presence of high-gain observer in the adaptive law delivers a good property of disturbance rejection at the cost of peaking phenomenon as well as reduced robustness. The addition of filtering mechanism in the control law overcomes the cons of high-gain observer and makes it robust to uncertainties in modeling the nonlinear functions. In this way, the filtered high-gain output feedback controller realizes nonlinear time-varying uncertainty cancelation and good tracking delivering with guaranteed robustness. The simulation results demonstrate the high efficiency of our novel design for handling of a class of nonlinear systems in the presence of time-varying uncertainty when compared with saturated control signal.

KEYWORDS
filtering, high-gain, nonlinear, output feedback, robustness

1 | INTRODUCTION

Nonlinear systems subject to various time-varying uncertainties are widespread in engineering disciplines. The complexity of nonlinear feedback control challenges us to come up with systematic design procedures to meet control objectives and design specifications. This paper presents a novel filtered high-gain output feedback controller for handling of a class of time-invariant systems in the presence of nonlinear time-varying uncertainties.

Control of systems with nonlinear time-varying uncertainties in an output feedback setup remains a challenge in theory so far. Most of the existing output feedback results impose restrictive assumptions on nonlinearities. For example, in the work of Chen and Huang, nonlinearities can only depend on measurement y; in the work of Krishnamurthy et al, nonlinearities linearly depend on the unmeasured states; in the work of Gauthier et al, the nonlinear systems satisfy a global Lipschitz condition; and in the work of Qian and Lin, it proposed a feedback domination design method to construct a linear output compensator for output feedback control under the condition of a constant growth rate for nonlinear functions. Another popular approach is the internal model-based output feedback control scheme, which is used to handle the output regulation problem with desired trajectories generated by an exosystem. Adaptive control aims to handle large uncertainties, which is inevitable in most practical systems. However, its results for output feedback setup are quite limited and hard to extend to systems with relative order larger than one.
The Luenberger observer is a well-known approach for output feedback linear time-invariant systems. Researchers have done many works on handling with nonlinear systems using observers. For example, a fuzzy adaptive observer combined with backstepping design techniques is developed for a class of multiple-input–multiple-output nonlinear systems with unmeasured states in the work of Tong et al.\textsuperscript{7} In the work of Tami et al.,\textsuperscript{8} an estimation problem for a class of partially observable nonlinear systems is investigated, where the existence of a change of coordinates, which can transform the studied system into the proposed partial observer normal form, needs to be guaranteed. A sliding-mode observer is designed to estimate the coupled disturbances and system states of nonlinear systems in the work of Sun et al.\textsuperscript{9} In other works,\textsuperscript{10-12} an observer-based adaptive fuzzy controller is used to handle nonlinear systems with different kinds of unknowns. High-gain observer, using high gain in the Luenberger observer, works for a wide class of nonlinear systems and guarantees that the output feedback controller recovers the performance of the state feedback controller when the observer gain is sufficiently high by using a separation principle (see related works\textsuperscript{4,13-24}). High-gain observer is one of the few effective approaches in the control theory, which can handle nonlinear uncertain systems in a general output feedback setup. However, the high gain will result in high frequency oscillations because it generates aggressive signals both in magnitude and rate, this high frequency signals will excite the unmodeled dynamics and lead to unpredictable system behaviors once they enter the real system, which is known as the intrinsic feature of peaking phenomenon for high-gain observer. Another disadvantage of the high gain is that it reduces the time delay margin (see the works of Hovakimyan and Cao\textsuperscript{25} and Luo and Cao\textsuperscript{26}), one indicator of robustness. Right now, saturation signal or magnitude constraint is used to mitigate the negative effects of a high-gain observer and maintain stability.\textsuperscript{27} However, following the same design philosophy of L\textsubscript{1} adaptive control, a low-pass filtered mechanism will be a better way to overcome cons of high-gain observer (see other works\textsuperscript{25,27-40}).

L\textsubscript{1} adaptive control theory is a modification of model reference adaptive control that adopts fast and robust adaptation to reduce the undesired transient from the adaptation process and handle nonlinear time-varying uncertainties instead of constant or slowly varying parameters in conventional adaptive control. The L\textsubscript{1} algorithm consists of a state predictor, adaptive law, and control law. The state predictor is a dynamic system that consists of a linear system plus a vector of adaptive parameters that represent uncertainty estimates. The adaptive law is used to generate adaptive parameters such that the error between the predicted state and the real state is driven to zero at every integration time step, and the control law is designed such that the predicted output tracks a given reference. The prediction error can be made arbitrarily small by decreasing the integration time. However, this kind of fast adaptation often leads to aggressive control signals with large magnitudes and rates, as well as loss of time-delay margin, one of the system’s robustness indicators. To prevent the entire system from becoming a high-gain feedback loop, a low-pass filtering mechanism is added in the control law, which filters away aggressive signals and recovers robustness. The combination of high-gain and low-pass filter guarantees fast adaptation with satisfactory transient response for both input and output. We can take driving a car as an analogy. The adaptation loop is like our observation and the control loop is like our action on the steering wheels and gas paddle. We want our observation to be fast because we want to know the situation in advance for the preparation of next action, but we do not want aggressive movement on the steering wheels or hard brake on the gas paddle. The control actions should be slow down in our hand but the observation should be quick in our eyes.

The L\textsubscript{1} adaptive control can be extended to control output feedback systems more than relative degree one. However, some restrictions still apply. It is noted that the L\textsubscript{1} adaptive control shares the same structure as high-gain observers if we reorganize the equations (Figure 1). The observer is in fact a high-gain feedback in the internal loop, which plays the same role as fast adaptation. The cons of high-gain observer described previously are similar to the shortcoming of fast adaptation.

The theoretical research proposed here is to use high-gain observer as the adaptive law in the L\textsubscript{1} adaptive architecture or, equivalently, add a filtering mechanism in high-gain observer to confine control signal’s bandwidth, magnitude, and rate. This will be the first special design of a controller that features the advantages of L\textsubscript{1} adaptive control (insertion of a low-pass filter at the input for robustness recovery) and high-gain observer (handling of nonlinear uncertainties). The presence of high-gain observer in the adaptive law delivers a good property of disturbance rejection at the cost of peaking phenomenon. The insertion of a low-pass filtering mechanism overcomes the peaking phenomenon and makes the high-gain observer robust to uncertainties in modeling nonlinear functions. The control law of the filtered high-gain output feedback controller consists of two components. As we can see from Figure 1, the second component is designed to compensate for the nonlinear disturbances, and the first component is designed to track a given command by applying inversion dynamics. In this way, the proposed technique guarantees that the output feedback controller stabilizes the system and achieves good tracking when the observer gain is sufficiently high.

This paper is organized as follows. Section 2 is the motivation statement and the engineering background for our main work. Section 3 gives the problem formulation. The structure of filtered high-gain output feedback controller is presented in Section 4, and Section 5 theoretically analyzes the nonlinear uncertainty handling capability and the tracking
FIGURE 1  Schemes of filtered high-gain output feedback controller [Colour figure can be viewed at wileyonlinelibrary.com]

performance of the novel design. Simulation results in Section 6 demonstrate the effectiveness of our controller design compared with saturated control signal, and Section 7 concludes the paper.

2 | MOTIVATION

Consider a single-input–single-output system, where the input $u$ and output $y$ are measured, but the system dynamics are unknown. Our objective is to draw a description with high fidelity of the system. To do this, a recursive integration mechanism is proposed here.

With $u$ and $y$ being measured, the derivative $\dot{y}$ is given by

$$\dot{y} = f_1(y, u).$$

If $\dot{y}$ is independent of $u$, remark $\dot{y} = y_1$, and then we continue to calculate the derivative of $y_1$, which is expressed as

$$\dot{y}_1 = f_2(y, y_1, u).$$

Once again, if $\dot{y}_1$ is independent of $u$, remark $\dot{y}_1 = y_2$, and repeat the derivation process. We can see that, if $u$ does not appear in any equations of $\dot{y}, \dot{y}_1, \ldots, \dot{y}_{n-2}$ until the equation of $\dot{y}_{n-1}$ is derived, then the system is identified as follows:

$$\dot{y} = y_1$$
$$\dot{y}_1 = y_2$$
$$\vdots$$
$$\dot{y}_{n-2} = y_{n-1}$$
$$\dot{y}_{n-1} = f_n(t, y, y_1, \ldots, y_{n-1}, u) + f(t, y, y_1, \ldots, y_{n-1}, u),$$

which is a chain of $n$ integrators and $f(t, y, y_1, \ldots, y_{n-1}, u)$ is the approximation error between the real system and the identified model, which is known as nonlinear uncertainty.

In many such problems, we can only measure some state variables in addition to those at the end of the chains of integrators. For example, the magnetic suspension system is modeled by

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = g - \frac{k}{m} x_2 - \frac{L_0 a x_3^2}{2m(a + x_1)^2}$$
$$\dot{x}_3 = \frac{1}{L(x_1)} \left[ -R x_3 + \frac{L_0 a x_2 x_3}{(a + x_1)^2} + u \right],$$

where $x_1$ is the ball position, $x_2$ is its velocity, and $x_3$ is the electromagnet current. In practice, the ball position $x_1$ and the electromagnet current $x_3$ are measured. For real-time monitoring and control purposes, we are motivated to develop an output feedback controller for this class of nonlinear systems.
3 PROBLEM FORMULATION

Consider the following single-input–single-output system:

\begin{align*}
\dot{x}(t) &= A_m x(t) + b_m u(t) + b_m f(t,x,u) \\
y(t) &= c_m x(t), \quad y(0) = y_0,
\end{align*}

or we can also expand the system as follows:

\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
& \quad \vdots \\
\dot{x}_{n-1} &= x_n \\
\dot{x}_n &= -a_0 x_1 - a_1 x_2 - \cdots - a_{n-1} x_n + u(t) + f(t, (x_1, x_2, \ldots, x_n), u(t)),
\end{align*}

where \(x(t) \in \mathbb{R}^n\) is the system state vector (unmeasured), \(u(t) \in \mathbb{R}\) is the control signal, \(y(t) \in \mathbb{R}\) is the system output, \(A_m\) is a known \(n \times n\) Hurwitz matrix, \(b_m, c_m \in \mathbb{R}^n\) are known constant vectors, system \(\Sigma_0 = (A_m, b_m, c_m)\) is controllable and observable, and \(f: \mathbb{R} \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}\) is an unknown nonlinear function.

**Assumption 1.** (Semiglobal Lipschitz condition on \(x\))

For any \(\delta > 0\), there exists \(L(\delta) > 0\) such that

\begin{align*}
|f(t,x,u) - f(t,\bar{x},u)| &\leq L(\delta) \|x - \bar{x}\|_\infty, \\
|f(t,0,u)| &\leq B,
\end{align*}

for all \(\|x\|_\infty \leq \delta\) and \(\|\bar{x}\|_\infty \leq \delta\) uniformly in \(u\) and \(t\).

Our final goal is to design an output feedback controller such that the system output \(y(t)\) tracks the desired system output \(y_{\text{des}}(t)\) described by

\begin{align*}
\dot{x}_{\text{des}}(t) &= A_m x_{\text{des}}(t) + b_m k_g r(t) \\
y_{\text{des}}(t) &= c_m x_{\text{des}}(t),
\end{align*}

where \(k_g = -(c_m A_m^{-1} b_m)^{-1}\) and \(r(t)\) is a given bounded reference input signal with \(\|r\|_{L_\infty} \leq \delta\).

4 CONTROLLER DESIGN

The filtered high-gain output feedback controller is taken as

\begin{align*}
\dot{x}(t) &= A_m \hat{x}(t) + b_m u(t) + b_m f_0(t,\hat{x},u) + H(y(t) - c_m \hat{x}(t)) \\
\dot{y}(t) &= c_m \hat{x}(t), \quad \hat{y}(0) = \hat{y}_0 \\
H &= \begin{bmatrix} h_1 \ h_2 \ \cdots \ h_{n-1} \ h_n \end{bmatrix}^T = \begin{bmatrix} \alpha_1 / \epsilon \ \alpha_2 / \epsilon^2 \ \cdots \ \alpha_{n-1} / \epsilon^{n-1} \ \alpha_n / \epsilon^n \end{bmatrix}_{n \times 1}^T \\
u(s) &= k_g r(s) - C(s) L \{ f_0(t,\hat{x},u) \},
\end{align*}

where \(f_0: \mathbb{R} \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}\) is a nominal model of the unknown nonlinear function, \(H\) is the observer gain, \(\alpha_1, \alpha_2, \ldots, \alpha_n\) and \(\epsilon\) are positive constants with \(\epsilon \ll 1\), and \(C(s)\) is a low-pass filter with unit DC gain, \(L \{ f_0(t,\hat{x},u) \}\) is the Laplace transform of \(f_0(t,\hat{x},u)\). The design of high-gain observer is proposed in the work of Khalil,\(^4\) it is used to eliminate the dynamic errors caused by nonlinear disturbance estimation. Based on this idea, in our work, we use high-gain observer as the adaptive law in the \(L_\infty\) adaptive architecture or, equivalently, add a filtering mechanism in high-gain observer to confine control signal’s bandwidth, magnitude, and rate. The presence of high-gain observer in the adaptive law delivers a good property of disturbance rejection at the cost of peaking phenomenon. The insertion of a low-pass filtering mechanism overcomes the peaking phenomenon and makes the high-gain observer robust to uncertainties in modeling nonlinear functions, this is one novelty of our work. Besides that, our objective is to deliver a good tracking performance with the compensation of nonlinear disturbances, so we design a control law as in (6), this is another one novelty of our work.
Assumption 2. (Semiglobal Lipschitz condition on \( \dot{x} \))
For any \( \delta > 0 \), there exists \( L_0(\delta) > 0 \) such that
\[
|f_0(t, \dot{x}, u) - f_0(t, \ddot{x}, u)| \leq L_0(\delta) \|\dot{x} - \ddot{x}\|, \\
|f_0(t, 0, u)| \leq B_0,
\]
for all \( \|\dot{x}\| \leq \delta \) and \( \|\ddot{x}\| \leq \delta \) uniformly in \( u \) and \( t \).

For the proof of stability and uniform performance bounds, the choice of \( C(s) \) with system dynamics needs to ensure that there exists \( \gamma_x \) such that
\[
\|G_1(s) k \|_{L_1} \|r\|_{L_\infty} + \|(sI - A_0)^{-1} H c\|_{L_1} \gamma_x + \|G_1(s) (1 - C(s))\|_{L_1} (L_0(\gamma_x) \gamma_x + B_0) + \|(sI - A_0)^{-1}\|_{L_1} \|\dot{x}(0)\|_{L_\infty} + \gamma_y + \gamma_z < \gamma_x,
\]
where \( A_0 = A_m - H c_m, G_1(s) = (sI - A_0)^{-1} b_m \).

Define
\[
\gamma_y = \|G_1(s) k\|_{L_1} \|r\|_{L_\infty} + \|(sI - A_0)^{-1} H c\|_{L_1} \gamma_x + \|G_1(s) (1 - C(s))\|_{L_1} (L_0(\gamma_x) \gamma_x + B_0) + \|(sI - A_0)^{-1}\|_{L_1} \|\dot{x}(0)\|_{L_\infty}, \tag{9}
\]
\[
\gamma_z = \|s(sI - A_0)^{-1}\|_{L_1} \|\dot{y}(0)\|_{L_\infty} + \|s c_m G_1(s)\|_{L_1} (L(\gamma_x) \gamma_x + B + L_0(\gamma_x) \gamma_x + B_0), \tag{10}
\]
\[
\gamma_x = \|G_1(s)\|_{L_1} \|r\|_{L_\infty} + \|G_1(s) (1 - C(s))\|_{L_1} (L_0(\gamma_x) \gamma_x + B_0) + \|(sI - A_0)^{-1}\|_{L_1} \|\dot{x}(0)\|_{L_\infty} + \gamma_y + \gamma_z \tag{11}
\]
where \( S = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & \cdots & 1 \end{bmatrix}_{(n-1)\times n} \).

Due to the intrinsic feature of the high-gain observer, which is known as the peaking phenomenon, we introduce a low-pass filter \( C(s) \) here. Another way to avoid such unintended behavior is by saturating the control signal, as proposed in the work of Khalil and Grizzle,\(^{27}\) where the control signal is taken as
\[
u(t) = k_q r(t) - sat\left(f_0(t, \dot{x}, u)\right). \tag{12}
\]

A comparison between filtered controller and saturated controller will be given in Section 6.

5 | ANALYSIS

In this section, the performance of filtered high-gain output feedback controller will be analyzed in the perspectives of nonlinear uncertainty handling, robustness recovery, and tracking delivering.

5.1 | Disturbance rejection

The system in (1) is given by
\[
A_m = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 \\ -a_0 & -a_1 & \cdots & -a_{n-2} & -a_{n-1} \end{bmatrix}_{n \times n}, \tag{13}
\]
\[
b_m = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \end{bmatrix}_{n \times 1},
\]
\[
c_m = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}_{1 \times n},
\]
which is expressed in a controllable canonical form.
The error equation (15)-(17) in the singularly perturbed form. Toward that end, define the scaled estimation errors
\[ M_{\eta} \] which can also be written as
\[ \text{Theorem 1.} \quad \text{For the system described in (1) with defined matrix in (13), which is subject to the high-gain observer defined in (4)-(5), we have} \]
\[ \lim_{t \to 0} \| G_1(s) \|_{L_1} = 0, \] (14)
where \( G_1(s) \) is the transfer function from the disturbance term to the estimation error between two dynamic systems with smaller terms neglected.

\[ \text{Proof.} \quad \text{The estimation error between the real system and high-gain observer} \]
\[ \dot{x} = [\ddot{x}_1 \ddot{x}_2 \cdots \ddot{x}_n]^T = [x_1 - \hat{x}_1 \ x_2 - \hat{x}_2 \ \cdots \ \ x_n - \ddot{x}_n]^T \] (15)
satisfies the equation
\[ \dot{x}(t) = A_0 \ddot{x}(t) + b_m \delta(x, \ddot{x}, u), \] (16)
which can be expanded as
\[ \dot{x}_1(t) = -h_1 \ddot{x}_1(t) + \ddot{x}_2(t) \]
\[ \dot{x}_2(t) = -h_2 \ddot{x}_2(t) + \ddot{x}_3(t) \]
\[ \vdots \]
\[ \dot{x}_{n-1}(t) = -h_{n-1} \ddot{x}_1(t) + \ddot{x}_n(t) \]
\[ \dot{x}_n(t) = (-h_n - a_0) \ddot{x}_1(t) - a_1 \ddot{x}_2(t) - \cdots - a_{n-1} \ddot{x}_n(t) + \delta(x, \ddot{x}, u), \] (17)

where \( \delta(x, \ddot{x}, u) = f(t, x, u) - f_0(t, \ddot{x}, u), A_0 = A_m - HC_m \). We want to design the observer gain \( H \) such that \( \lim_{t \to \infty} \ddot{x}(t) = 0 \). In the absence of the disturbance term \( \delta(x, \ddot{x}, u) \), asymptotic error convergence is achieved by designing \( H \) such that
\[ A_0 = \begin{bmatrix}
-h_1 & 1 & 0 & \cdots & 0 \\
-h_2 & 0 & 1 & \cdots & 0 \\
\vdots & & & \ddots & \vdots \\
-h_{n-1} & 0 & \cdots & 0 & 1 \\
-h_n - a_0 & -a_1 & \cdots & -a_{n-1}
\end{bmatrix} \] (18)
is Hurwitz, which means the positive constants \( a_i \) in (5) are chosen such that the roots of \( \det(sI - A_0) = 0 \) are in the open left-half plane. In the presence of \( \delta(x, \ddot{x}, u) \), the observer gain \( H \) should be designed with additional goal of rejecting the effect of \( \delta(x, \ddot{x}, u) \) on \( \ddot{x} \). As we can see from (5), given a specified \( \varepsilon \), the observer gain \( H \) satisfies \( h_n \gg h_{n-1} \cdots \gg h_1 \gg 1 \).

With the smaller terms neglected, the transfer function from \( \delta(x, \ddot{x}, u) \) to \( \ddot{x}(t) \) can be expressed as
\[ G_1(s) = \frac{1}{s + h_1} \frac{s^{n-2} + h_1 s^{n-3} + \cdots + h_{n-1} s + h_n}{(s^n + h_1 s^{n-1} + \cdots + h_n s + h_n)}, \] (19)
which can also be written as
\[ G_1(s) = \frac{\varepsilon}{(\varepsilon s)^n + a_0(\varepsilon s)^{n-1} + \cdots + a_{n-1} (\varepsilon s) + a_n} \begin{bmatrix}
\varepsilon^{n-1} s + \varepsilon^{n-2} a_1 \\
\varepsilon^{n-1} s + \varepsilon^{n-2} a_1 s + \varepsilon a_{n-2} \\
\varepsilon^{n-1} s + \varepsilon^{n-2} a_1 s^2 + \varepsilon a_{n-2} s + \varepsilon a_{n-1} \\
\varepsilon^{n-1} s + \varepsilon^{n-2} a_1 s^3 + \varepsilon a_{n-2} s^2 + \varepsilon a_{n-1} s + a_{n-1}
\end{bmatrix}. \] (20)

Hence, \( \lim_{t \to 0} \| G_1(s) \|_{L_1} = 0 \). This completes the proof. \( \Box \)

Besides, the disturbance rejection property of the high-gain observer can be also seen in the time domain by representing the error equation (15)-(17) in the singularly perturbed form. Toward that end, define the scaled estimation errors
\[ \eta_1 = \frac{\ddot{x}_1}{\varepsilon^{n-1}}, \eta_2 = \frac{\ddot{x}_2}{\varepsilon^{n-2}}, \cdots, \eta_n = \frac{\ddot{x}_n}{\varepsilon}, \eta_n = \ddot{x}_n. \] (21)
The newly defined variables satisfy the singularly perturbed equation

\[\begin{align*}
\epsilon \dot{\eta}_1 &= -\alpha_1 \eta_1 + \eta_2 \\
\epsilon \dot{\eta}_2 &= -\alpha_2 \eta_1 + \eta_3 \\
&\vdots \\
\epsilon \dot{\eta}_{n-1} &= -\alpha_{n-1} \eta_1 + \eta_n \\
\epsilon \dot{\eta}_n &= -\alpha_n \eta_1 - a_0 \epsilon^n \eta_1 - a_1 \epsilon^{n-1} \eta_2 - \cdots - a_{n-1} \epsilon \eta_{n-1} + \epsilon \delta (x, \hat{x}, u). 
\end{align*}\]  

(22)

This equation shows clearly that reducing \(\epsilon\) diminishes the effect \(\delta\). It shows also that, for small \(\epsilon\), the scaled-estimation error \(\eta\) will be much faster than \(x\).

### 5.2 Filtering mechanism

For the purpose of good tracking delivering with guaranteed stability, the control signal should consist of two parts, the given reference tracking command and the disturbance compensation part.

Although the high-gain observer can diminish the effect of \(\delta (x, \hat{x}, u)\) with small \(\epsilon\), it will also bring unintended behavior to the system, which is known as the peaking phenomenon. We can take the following \(2 \times 2\) state-space model as an example and analyze its response in time domain:

\[\begin{align*}
\dot{x}_1 (t) &= x_2 (t) \\
\dot{x}_2 (t) &= -a_0 x_1 (t) - a_1 x_2 (t) + u (t) + f (t, x, u) \\
y (t) &= x_1 (t), y (0) = y_0.
\end{align*}\]  

(23)

The output feedback controller using high-gain observer is given by

\[\begin{align*}
\hat{x}_1 (t) &= \hat{x}_2 (t) + h_1 (y (t) - \hat{x}_1 (t)) \\
\hat{x}_2 (t) &= -a_0 \hat{x}_1 (t) - a_1 \hat{x}_2 (t) + u (t) + f_0 (t, \hat{x}, u) + h_2 (y (t) - \hat{x}_1 (t)) \\
\hat{y} (t) &= \hat{x}_1 (t), \hat{y} (0) = \hat{y}_0.
\end{align*}\]  

(24)

The estimation error satisfies the equation

\[\begin{align*}
\dot{\hat{x}}_1 (t) &= -h_1 \hat{x}_1 (t) + \hat{x}_2 (t) \\
\dot{\hat{x}}_2 (t) &= - (a_0 + h_2) \hat{x}_1 (t) - a_1 \hat{x}_2 (t) + \delta (x, \hat{x}, u).
\end{align*}\]  

(25)

From (25), we can get \(A_0 = \begin{bmatrix} -h_1 & 1 \\ - (a_0 + h_2) & -a_1 \end{bmatrix}\). Suppose \(\hat{y}_0 \neq y_0\), which means \(\hat{x}_1 (0) \neq x_1 (0)\). Then, calculate the transition matrix \(\exp(A_0 t)\) with the small terms neglected and note that its (2, 1) element is given by

\[\frac{-2h_2}{\sqrt{4h_2 - h_1^2}} e^{-h_1 t/2} \sin \left( \frac{t \sqrt{4h_2 - h_1^2}}{2} \right)\]  

(26)

when \(4h_2 > h_1^2\) and

\[\exp \left[ - \left( \frac{h_1 - \sqrt{h_1^2 - 4h_2}}{2} \right) t \right] - \exp \left[ - \left( \frac{h_1 + \sqrt{h_1^2 - 4h_2}}{2} \right) t \right]\]  

(27)

when \(4h_2 < h_1^2\). The amplitude of the exponential mode is greater than \(\sqrt{h_2}\) in the first case and \(h_2/h_1\) in the second case. Following (5), \(h_1 = \alpha_1/\epsilon, h_2 = \alpha_2/\epsilon^2\), we can notice that \(\hat{x} (0)\) will be \(O(1/\epsilon)\) due to \(\hat{x}_1 (0) \neq x_1 (0)\). Consequently, \(\hat{x} (t)\) will contain a term of the form \((1/\epsilon) e^{-at/\epsilon} \hat{x}_1 (0)\), which is the product of (26) or (27) and nonzero \(\hat{x} (0)\). Thus, if we increase \(h_1\) and \(h_2/h_1\), the exponential mode decays rapidly and it exhibits an impulse-like behavior where the transient peaks to \(O(1/\epsilon)\) values before it decays rapidly toward zero. In fact, the function \((1/\epsilon) e^{-at/\epsilon} \hat{x}_1 (0)\) approaches an impulse function as \(\epsilon\) tends to zero. Such peaking phenomenon is an intrinsic feature of any high-gain observer with \(h_2 \gg h_1 \gg 1\) and it could destabilize the system. To get a better feel for the peaking phenomenon, we simulate a \(2 \times 2\) system in Figure 2 by choosing \(\epsilon = 0.001\).
As we can see, the system state exhibits the intuitive behavior as we expected earlier. Another disadvantage of high-gain feedback is that it reduces the time delay margin, which is demonstrated in Figure 3.

Fortunately, these disadvantages of high-gain observer can be overcome by the addition of a filtering mechanism. That is why we introduce a low-pass filter in the control signal. Figure 4 demonstrates the effects of filtering mechanism comparing to a pure high-gain observer, which brings peaking phenomenon in control signal.

The low-pass filter filters away aggressive signals and recovers robustness. Hence, the generated control signal is confined to the control channel bandwidth and a reasonable time delay in the actuation and measurement channels can be tolerated.

We can take a scalar system as example

\[ \dot{x}(t) = -x(t) + u(t) + f(t, x, u), \quad x(0) = x_0. \]  

(28)

For the purpose of complete disturbance rejection, the unknown nonlinear function should be compensated by the control signal. Then, the controller is taken as

\[ \dot{x}(t) = -\dot{x}(t) + u(t) + \tilde{f}(t, \dot{x}(t), u(t)), \quad \dot{x}(0) = x_0 \]

\[ u(t) = -\tilde{f}(t, \dot{x}(t), u(t)) \]

\[ \tilde{f}(t, \dot{x}(t), u(t)) = -\Gamma (x(t) - \dot{x}(t)), \]

\[ \tilde{f}(0, \dot{x}(0), u(0)) = f_0, \quad \Gamma > 0. \]

(29)
The transfer function of this system is

\[ L_1 (s) = \frac{\Gamma}{s(s + 1)}. \]  

(30)

We can compute the gain crossover frequency

\[ \left| L_1 (j\omega_{gc}) \right| = \frac{\Gamma}{\omega_{gc} \sqrt{1 + \omega_{gc}^2}} = 1, \]  

(31)

which leads to the phase margin

\[ \phi = \pi + \angle L_1 (j\omega_{gc}) = \arctan \left( \frac{1}{\omega_{gc}} \right). \]  

(32)

It indicates that increasing \( \Gamma \) leads to higher gain crossover frequency and consequently reduces the phase margin. Although increasing \( \Gamma \) improves tracking performance and speeds up response, it obviously hurts the robustness of the closed-loop system. Here, we propose modifying the controller with a low-pass filter so that the trade-off between tracking and robustness is resolved and the high gain \( \Gamma \) can be easily increased for transient performance improvement without hurting the robustness of the closed-loop system.

The modified controller is given by

\[ \dot{x}(t) = -\dot{x}(t) + u(t) + \ddot{f}(t, \dot{x}(t), u(t)), \quad x(0) = x_0 \]

\[ u(t) = -C(s) \dot{f}(t, \dot{x}(t), u(t)) \]

\[ \dot{f}(t, \dot{x}(t), u(t)) = -\Gamma (x(t) - \dot{x}(t)) \]

\[ \dot{f}(0, \dot{x}(0), u(0)) = f_0, \quad \Gamma > 0. \]  

(33)

where \( C(s) = \omega_c / (s + \omega_c) \), we can get the transfer function of this closed-loop system

\[ L_2 (s) = \frac{\Gamma C(s)}{(s(s + 1) + \Gamma(1 - C(s)))}. \]  

(34)

It can be noticed that, as \( \Gamma \to \infty \), (34) can be rewritten as follows:

\[ L_3 (s) = \frac{C(s)}{(1 - C(s))} = \frac{\omega_c}{s}. \]  

(35)

It can be clearly seen that this transfer function has an infinite gain margin and a phase margin of \( \pi/2 \). From the aforementioned analysis, we can see that the low-pass filter can eliminate the high frequency components in the control signal, which are brought by the high-gain observer, and recover the robustness of the system. The combination of low-pass filter and high-gain observer realizes nonlinear disturbance rejection with guaranteed stability.
5.3 | Stability and tracking analysis

**Theorem 2.** Given the system in (1) and the controller defined in (4)-(6) subject to (8), if \( \|x(0)\|_\infty < \gamma_x \) and \( \hat{x}(0) \) in the output predictor is chosen such that \( \|\hat{x}(0)\|_\infty < \gamma_y \), \( \|\tilde{y}(0)\|_\infty \leq \gamma_f \), and \( \|\tilde{z}(0)\|_\infty \leq \gamma_z \), then we have

\[
\begin{align*}
\|\tilde{y}\|_{L_w} & \leq \gamma_f \quad (36) \\
\|\tilde{z}\|_{L_w} & \leq \gamma_z \quad (37) \\
\|x\|_{L_w} & < \gamma_x \quad (38) \\
\|u\|_{L_w} & < \gamma_u \quad (39) \\
\|x_{\text{des}} - x\|_{L_w} & < \gamma_x \quad (40) \\
\|y_{\text{des}} - y\|_{L_w} & < \gamma_y \quad (41)
\end{align*}
\]

where \( x(t) = \begin{bmatrix} y(t) \\ z(t) \end{bmatrix} \), \( \hat{x}(t) = \begin{bmatrix} \hat{y}(t) \\ \hat{z}(t) \end{bmatrix} \), \( \tilde{x}(t) = x(t) - \hat{x}(t) = \begin{bmatrix} \tilde{y}(t) \\ \tilde{z}(t) \end{bmatrix} \), \( \gamma_y \), \( \gamma_f \), and \( \gamma_z \) are respectively introduced in (9)-(11), \( \gamma_x \) is a positive constant, and

\[
\begin{align*}
\gamma_u & = \left\| k_g r \right\|_{L_w} + \| C(s) \|_{L_1} \left( L_0(\gamma_y) \gamma_x + B_0 \right) \\
\gamma_x & = \| H_1(s) k_x \|_{L_1} \| r \|_{L_w} + \left\| (sI - A_m)^{-1} x_{\text{des}}(0) \right\|_{L_w} + \gamma_x \\
\gamma_y & = \| c_m \|_{L_w} \gamma_y.
\end{align*}
\]

**Proof.** Since \( \|x(0)\|_\infty < \gamma_x \) and \( x(t) \) is continuous, then assuming the opposite implies that there exists \( t' \) such that

\[
\|x(t')\|_\infty = \gamma_x,
\]

which implies that \( \|x(t')\|_{L_w} = \gamma_x \) holds.

Since \( \|\hat{x}(0)\|_\infty < \gamma_y \) and \( \hat{x}(t) \) is continuous, then assuming the opposite implies that there exists \( t'' \) such that

\[
\|\hat{x}(t'')\|_\infty = \gamma_y,
\]

which implies that \( \|\hat{x}(t'')\|_{L_w} = \gamma_y \) holds.

Assuming \( t' < t'' \). At first, we prove that, for all \( t \leq t' \), one has

\[
\|\tilde{y}\|_{L_w} \leq \gamma_f, \quad \|\tilde{z}\|_{L_w} \leq \gamma_z.
\]

It follows from (16) that

\[
\tilde{y}(s) = c_m(sI - A_0)^{-1} \begin{bmatrix} \hat{y}(0) \\ \hat{z}(0) \end{bmatrix} + c_m G_1(s) L \{ \delta(x, \hat{x}, u) \}
\]

\[
\tilde{z}(s) = S(sI - A_0)^{-1} \begin{bmatrix} \hat{y}(0) \\ \hat{z}(0) \end{bmatrix} + SG_1(s) L \{ \delta(x, \hat{x}, u) \}.
\]

Due to the Lipschitz conditions on \( x \) and \( \hat{x} \), we have

\[
|\delta(x, \hat{x}, u)| = |f(t, x, u) - \hat{f}(t, \hat{x}, \hat{u})| \leq |f(t, x, u)| + |\hat{f}(t, \hat{x}, \hat{u})| \leq (L(\gamma_x) \gamma_x + B) + (L_0(\gamma_y) \gamma_x + B_0).
\]

Because

\[
\|\tilde{y}(0)\|_\infty \leq \gamma_f, \quad \|\tilde{z}(0)\|_\infty \leq \gamma_z.
\]
then, for all $t \in [0, t']$, it follows from (50)-(53) and definitions of $\gamma_2$ in (10) and $\gamma_2$ in (11) that
\begin{equation}
\|\hat{y}_i\|_{L_\infty} \leq \gamma_{\hat{y}_i}, \quad \|\hat{x}_i\|_{L_\infty} \leq \gamma_{\hat{x}}.
\end{equation}

Since $\hat{x}(t) = x(t) - \hat{x}(t)$, we have
\begin{equation}
\|x(t)\| \leq \|\hat{x}(t)\| + \|\hat{x}(t)\|.
\end{equation}

It is known that
\begin{equation}
\hat{x}(t) = \left[\begin{array}{c} \hat{y}(t) \\ \hat{z}(t) \end{array}\right].
\end{equation}

Then, the upper bound of $\hat{x}(t)$ is written as
\begin{equation}
\|\hat{x}(t)\|_{\infty} \leq \|\hat{y}(t)\|_{\infty} + \|\hat{z}(t)\|_{\infty}.
\end{equation}

Furthermore, it follows from (4) that
\begin{equation}
\hat{x}(s) = (sI - A_0)^{-1}b_m \hat{r}(s) + \left((sI - A_0)^{-1} H_{cm}x(s) + (sI - A_0)^{-1} b_m (1 - C(s))L(f_0(t, \hat{x}, u)) + (sI - A_0)^{-1} \hat{x}(0) \right).
\end{equation}

Then, we arrive at the following upper bound of $\hat{x}(t)$
\begin{equation}
\|\hat{x}(t)\|_{L_\infty} < \|G_1(s) k_g\|_{L_1} \|r\|_{L_\infty} + \left((sI - A_0)^{-1} H_{cm}\right)_{L_1} \gamma_x
\end{equation}
\begin{equation}
+ \|G_1(s) (1 - C(s))L_0(\gamma_\hat{x}) \gamma_x + B_0\| + \left((sI - A_0)^{-1}\right)_{L_1} \|\hat{x}(0)\|_{L_\infty}.
\end{equation}

Finally, following from (55), (57), (59), we obtain the upper bound of $x(t)$
\begin{equation}
\|x(t)\|_{L_\infty} < \|G_1(s) k_g\|_{L_1} \|r\|_{L_\infty} + \left((sI - A_0)^{-1} H_{cm}\right)_{L_1} \gamma_x
\end{equation}
\begin{equation}
+ \|G_1(s) (1 - C(s))L_0(\gamma_\hat{x}) \gamma_x + B_0\| + \left((sI - A_0)^{-1}\right)_{L_1} \|\hat{x}(0)\|_{L_\infty} + \gamma_\hat{x} + \gamma_\hat{z}.
\end{equation}

By considering stability condition, (60) becomes
\begin{equation}
\|x(t)\|_{L_\infty} < \gamma_x,
\end{equation}

which contradicts (45) and proves (38).

Following from (38), (59), at $t = t'$, we further obtain
\begin{equation}
\|\hat{x}(t')\|_{L_\infty} < \gamma_{\hat{x}},
\end{equation}

which contradicts (47).

Following from (38), (54), (62), we further obtain results (36), (37).

With the definition of control signal in (6) and Assumption 2, we have
\begin{equation}
\|u\|_{L_\infty} < \|k_g\|_{L_\infty} + \|C(s)\|_{L_1} (L_0(\gamma_\hat{x}) \gamma_x + B_0),
\end{equation}

which leads to the result in (39).

Deduct (1) from (3), we derive the Laplace transform as follows:
\begin{equation}
x_{des}(s) - x(s) = (sI - A_m)^{-1} \left(b_m k_g \hat{r}(s) + x_{des}(0)\right) - x(s).
\end{equation}

Based on the result derived in (38), we have
\begin{equation}
\|x_{des} - x\|_{L_\infty} < \|H_1(s) k_g\|_{L_1} \|r\|_{L_\infty} + \left((sI - A_m)^{-1} x_{des}(0)\right)_{L_\infty} + \gamma_x,
\end{equation}

which proves (40) and leads to (41)
\begin{equation}
\|y_{des} - y\|_{L_\infty} < \|c_m\|_{L_\infty} \gamma_x.
\end{equation}

The proof is completed.
Remark 1. Because $\lim_{t \to 0} \|G_1(s)\|_{L_1} = 0$ and $A_0$ is Hurwitz, when $t \to \infty$, $\tilde{y}(t) = 0$, $\tilde{z}(t) = 0$. By making the bandwidth of low pass filter $C(s)$ large enough, the control law (6) can ensure that $\lim_{t \to 0} \hat{y}(s) = y_{des}(s)$. The result (36) and (37) in Theorem 2 together with the control law guarantees that the difference between $y(t)$ and $y_{des}(t)$ is bounded, as we can see from (41).

6 | SIMULATIONS

6.1 | Guidance on picking up design parameters

Many people care about the impact of design parameters of the high-gain observer on the stability and tracking performance of the system. There is no doubt that the key point for the design of the filtered high-gain output feedback controller is how to find the high-gain vector $H$. Generally speaking, you can pick up $\varepsilon$ between [0.001, 0.1], however, it also depends on specified cases. If you choose a bigger $\varepsilon$, the error dynamics due to nonlinear disturbance estimation will not be eliminated; another case is when you are handling with a nonlinear system of high-order using the filtered high-gain output feedback controller, if you choose a smaller $\varepsilon$, it will really hurt the robustness of the system.

Here, I still take the system in (23) as an example to further on the picking up of design parameters. I think many people will care about the case where $4h_2 = h_1^2$, because, when $4h_2 = h_1^2$, it means the error dynamics have two identical eigenvalues. Hence, I want to explore further on the stability of the system in such case and gain some insight into the picking up of design parameters.

If we neglect the smaller terms, we can derive the response of the error dynamics in (25) in the time domain as follows:

$$\begin{bmatrix} 0 \hat{x}_1(t) \\ 0 \hat{x}_2(t) \end{bmatrix} = \begin{bmatrix} e^{-\frac{h_1 t}{2}} - \frac{h_2 t}{2} e^{-\frac{h_1 t}{2}} & te^{-\frac{h_1 t}{2}} \\ -h_2 t e^{-\frac{h_1 t}{2}} & e^{-\frac{h_1 t}{2}} + \frac{h_1}{2} e^{-\frac{h_2 t}{2}} \end{bmatrix} \begin{bmatrix} 0 \hat{x}_1(0) \\ 0 \hat{x}_2(0) \end{bmatrix} + \begin{bmatrix} f_0^t e^{-\frac{h_1 t}{2} (t-\tau)} + \frac{h_1}{2} e^{-\frac{h_2 t}{2} (t-\tau)} d\tau \\ f_0^t e^{-\frac{h_1 t}{2} (t-\tau)} + \frac{h_1}{2} e^{-\frac{h_2 t}{2} (t-\tau)} d\tau \end{bmatrix} \delta (x, \bar{x}, u). \tag{67}$$

We can see clearly from Equation (67), although $t$ is really small, the high gain $h_1, h_2$ are really large, hence, the elements in the transfer matrix are still quite large, which may lead to the divergence of the error dynamics due to the nonlinear disturbance. In practice, in the case where $4h_2 = h_1^2$, if we take the smaller terms into consideration, we can get the Laplace transformation as follows:

$$(sI - A_0)^{-1} = \begin{bmatrix} -a_1 & -1 \\ a_0 + h_2 & -h_1 \end{bmatrix} / (s + (a_1 + h_1)/2)^2 + a_1 h_1/2 + a_0 - a_1^2/4. \tag{68}$$

If we want to avoid the same case as in (67), where the error dynamics have two identical eigenvalues, we can pick up an $h_1$ to satisfy $a_1h_1/2 + a_0 - a_1^2/4 \neq 0$, and $h_2 \gg h_1 \gg 1$. Indeed, given the nonlinear system, we can test in a prior step whether the chosen high-gain vector can lead to good tracking delivery, meanwhile, maintain the robustness of the system.

6.2 | Numerical simulation results and analysis

Consider the system in (1) with

$$A = \begin{bmatrix} 0 & 10 \\ -1 & -1.4 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad c^T = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad y(0) = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}. \tag{69}$$

Here, $x = [x_1, x_2]^T \in \mathbb{R}^2$ is an unmeasured vector, the control objective is to design a high-gain observer $H$ and control signal $u(t)$ such that the system can achieve nonlinear disturbance rejection and good tracking delivering with guaranteed stability. In the implementation of controller, set integration step $T = 1e - 3$, the observer gain is designed as $H = \begin{bmatrix} 100 & 0 \end{bmatrix},$ a low-pass filter is chosen as $C(s) = 20/(s + 20)$, the saturation points are $-5$ and $5$. The simulation results for the filtered high-gain output feedback controller and the saturated controller are demonstrated as follows.

The simulation results in Figure 5, Figure 6, Figure 7, and Figure 8 demonstrate the performance of the filtered high-gain output feedback controller given different tracking commands and nonlinear disturbances.
The simulation results in Figure 9, Figure 10, Figure 11 and Figure 12 demonstrate the performance of saturated controller given different tracking commands and nonlinear disturbances, which are used to compare with the filtered high-gain output feedback controller.

Remark 2. It demonstrates clearly that the filtered high-gain output feedback controller delivers good tracking performance with satisfactory robustness. In the case 1 where \( r(t) = 1 \) and \( f(t, x, u) = x_1 x_2 + u \), the control input \( u \) of the filtered high-gain output feedback controller is within [0.75, 1] (Figure 5); however, for the saturated controller, the control input \( u \) ranges between [0.2, 1.2] (Figure 9). In the case 2 where \( r(t) = \sin(0.3t) \) and \( f(t, x, u) = \sin(x_1^2) + u x_2 + x_1 \cos(x_2) \), the control input \( u \) of the filtered high-gain output feedback controller is within \([-1.2, 0.4]\) (Figure 6); however, for the saturated controller, the control input \( u \) ranges between \([-6, 6]\) (Figure 10). In the case 3 where \( r(t) = \sin(0.3t) \) and \( f(t, x, u) = x_1 x_2 + u x_1 \), the control input \( u \) of the filtered high-gain output feedback controller is within \([-50, 10]\) (Figure 11). In the case 4 where \( r(t) = \sin(0.3t) \) and \( f(t, x, u) = x_1 x_2 + u^2 x_1 \), the control input \( u \) of the filtered high-gain output feedback controller is within \([-0.8, 0.6]\) (Figure 8); however, for the saturated controller, the control input \( u \) demonstrates unexpected behavior (Figure 12). Given all the cases, compare the performance of the filtered high-gain output feedback controller and the saturated controller, although both controllers can deliver good tracking performance, it can be obviously seen that the saturated controller has higher requirement for the input due to the peaking phenomenon. Especially in case 3
FIGURE 7  Performance for \( r(t) = \sin(0.3t) \) and \( f(t, x, u) = x_1 x_2 + u x_1 \) [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 8  Performance for \( r(t) = \sin(0.3t) \) and \( f(t, x, u) = x_1 x_2 + u^2 x_1 \) [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 9  Performance for \( r(t) = 1 \) and \( f(t, x, u) = x_1 x_2 + u \) [Colour figure can be viewed at wileyonlinelibrary.com]
FIGURE 10  Performance for \( r(t) = \sin(0.3t) \) and \( f(t, x, u) = \sin(x_1^2) + ux_2 + x_1\cos(x_2) \) [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 11  Performance for \( r(t) = \sin(0.3t) \) and \( f(t, x, u) = x_1x_2 + ux_1 \) [Colour figure can be viewed at wileyonlinelibrary.com]

FIGURE 12  Performance for \( r(t) = \sin(0.3t) \) and \( f(t, x, u) = x_1x_2 + u^2x_1 \) [Colour figure can be viewed at wileyonlinelibrary.com]
and case 4, the time history of control input in these two cases is not desired. The higher requirement on control input means higher cost in practical industrial applications. Just as we mentioned earlier, the filtering mechanism added in high-gain observer confines control signal's bandwidth, magnitude, and rate, and a reasonable time delay is yielded. However, for the saturated controller, it can only set a limitation on the magnitude, which does not do any help to the aggressive signals with high rate or high frequency. Therefore, we can conclude that the filtered high-gain output feedback controller is a better choice than the saturated controller.

7 | CONCLUSIONS

Nonlinear systems subject to uncertainties are widespread in engineering disciplines. Output feedback control design for these systems to meet the objectives of tracking and robustness is a challenging task. Faced with such challenge, our proposed controller design not only delivers a good property of disturbance rejection with the presence of high-gain observer but also guarantees robustness of the system with addition of a filtering mechanism at the input. It is expected that this novel filtered high-gain architecture can handle a broad class of output feedback systems with nonlinear model inaccuracies and time-varying uncertainties. The knowledge obtained is essential for not only engineers understanding but also control of practical systems. It can help the monitoring, control, and optimization of industry systems for improved efficiency and economy.

Extensive algorithm modification and theoretical work are still needed to push the boundaries of the proposed architecture. It will also lead to a better understanding of the role of high-gain plays in the feedback theory. The hypotheses are (i) high-gain is the key to realize separation principles for nonlinear time-invariant systems; and (ii) additional filtering mechanism can realize the trade-off between robustness and performance while maintaining the separation principles.

Although the proposed control strategy can handle uncertainties, it does not mean a black-box controller design is feasible. A reasonable dynamic model with some fidelity is still required and, in fact, is needed for almost any control theory.

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ORCID

Tong Ma http://orcid.org/0000-0003-3419-8222

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