Hyperspectral Image Classification with Deep Metric Learning and Conditional Random Field

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Abstract

To improve the classification performance in the context of hyperspectral image processing, many works have been developed based on two common strategies, namely the spatial-spectral information integration and the utilization of neural networks. However, both strategies typically require more training data than the classical algorithms, aggregating the shortage of labeled samples. In this paper, we propose a novel framework that organically combines an existing spectrum-based deep metric learning model and the conditional random field algorithm. The deep metric learning model is supervised by center loss, and is used to produce spectrum-based features that gather more tightly within classes in Euclidean space. The conditional random field with Gaussian edge potentials, which is firstly proposed for image segmentation problem, is utilized to jointly account for both the geometry distance of two pixels and the Euclidean distance between their corresponding features extracted by the deep metric learning model. The final predictions are given by the conditional random field. Generally, the proposed framework is trained by spectra pixels at the deep metric learning stage, and utilizes the half handcrafted spatial features at the conditional random field stage. This settlement alleviates the shortage of training data to some extent. Experiments on two real hyperspectral images demonstrate the advantages of the proposed method in terms of both classification accuracy and computation cost.

1 Introduction

Hyperspectral images (HSI) are usually acquired by spaceborne or airborne sensors, recording the reflection or radiance spectra over hundreds of channels. They are usually formatted as data cubes. The height and width of an HSI are corresponding to the real world object under a specific resolution, while the depth is decided by the channels of the sensors. As a crucial task, the classification of HSI pixels attracts great attention for a long time [1,3]. Many early methods are based on classical machine learning algorithms and their variations, for instance, principal component analysis (PCA) [4,5], independent component analysis (ICA) [6], linear discriminant analysis (LDA) [7,8], support vector machine (SVM) [9], and sparse representation [10,11].

In recent years, neural networks (NN) have gained its popularity in many applications related to machine learning, due to its power in extracting deep and abstract representations from the original data. An increasing number of NN-based algorithms have been adapted to HSI classification task and achieved impressive results. Representatives of the earlier models are stacked autoencoder (SAE) [12], and deep belief network (DBN) [13]. With the advances in deep learning, various deep models have been applied to HSI classification, and have showed their power in producing self-learning features and in processing spatial data. This category of algorithms mainly include convolutional neural network (CNN) [14,16], recurrent neural network (RNN) [17,19], and deep metric learning (DML) [20,22], to name a few.

The conditional random field (CRF) is a probabilistic graphical based model that enables to characterize the contextual information among the labels and the data [23]. As an important application of CRF, image segmentation has also attracted attention in classifying HSI pixels [24,27]. In most of these works, the CRF has been integrated with CNNs as a post-processing step, processing the output features extracted by CNN encoders. To name a few, in [25], a restricted CRF algorithm is applied to refine the superpixel classification of CNN to the final pixel-wise classification results. In [27], the authors utilized CRF to improve the predictions on the CNN outputs and designed a specific deconvolutional network to produce the final classifications.

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In this paper, a framework that combines the DML and CRF algorithms is proposed. A DML model supervised with center loss is employed to extract spectrum-based features from individual pixels, and the CRF algorithm is applied to give final predictions by modeling both the spatial and spectral information among pixels based on the extracted features. To be more precise, our work has advantages in the following aspects:

- Intrinsic relations of DML and CRF algorithms help the classification accuracies to reach a high level. To the best of our knowledge, we are the first to introduce a framework that benefits from the underlying connections between DML and CRF.
- The setting of employing a spectral-based DML model and a handcraft spatial-based CRF algorithm keeps the framework simple. Compared to the CNN models, our framework is spectral-based in training phase and engages simpler model structure, thus alleviating the shortage of labeled HSI data raised in the CNN models [15, 20].
- In practice, by introducing the convolutional CRF (ConvCRF) [28], which implements the CRF inferences on GPU phase by convolutional operations, the proposed framework shows high efficiency in computation costs.

## 2 Proposed Framework

As two main parts of the proposed framework, DML and CRF algorithms are firstly introduced separately. Substantially, an overview of the whole DML-CRF framework is presented.

### 2.1 Deep Metric Learning

In [20], the center loss proposed in deep metric learning model [29] was firstly introduced to the HSI classification task. A 3-layered fully connected network was built to extract spectral features from the input data, as given in Fig. 1. The model is jointly supervised by cross-entropy loss and center loss, such that the features from the same class gather more tightly in Euclidean space. This model is adopted to encode the spectrum in our work.

Throughout this paper, we use \((x_i, y_i)\) to denote the pixel \(x_i\) from the HSI \(X\), with the label \(y_i\). Let \(f(\cdot)\) be the function defined by the neural network, which outputs the extracted features. Use \(\tilde{p}(y|x_i)\) to denote the probability distribution that generated by applying the softmax function on the extracted features \(f(x_i)\). During the training stage, a joint loss \(L\) that sums the center loss \(L_c\) and the cross-entropy loss \(L_s\) is engaged. As the key part of DML, the center loss \(L_c\) is defined to measure the Euclidean distance between the produced features \(f(x_i)\) and its class centers \(c_{y_i}\), as

\[
L_c = \sum_i \| f(x_i) - c_{y_i} \|_2, \tag{1}
\]

where the class centers are formulated as

\[
c_{y_i} = \text{average}(\{ f(x_k) | y_k = y_i \}).
\]

At the testing stage, samples \(x_i\) are fed to the neural network \(f(\cdot)\). The outputs include both the extracted feature \(f(X)\) and the predicted probability distribution \(\tilde{p}(y|x_i)\), that are collected for the subsequential CRF step.
2.2 Conditional Random Field

The CRF algorithm plays an important role in image segmentation, with the merit of exploiting the global context information [23, 28, 30]. In this paper, we use the CRF with Gaussian edge potentials to fuse the spatial-spectral information, and give reasonable predictions of the HSI. The notations in this paper mainly follow the models of fully connected CRF in [30] and ConvCRF in [28].

Let \( \hat{y} \) be a choice of predictions over all the pixels in a HSI, the probability of \( \hat{y} \) is calculated from an energy function \( E(\hat{y}|f(X)) \) by a Gibbs distribution:

\[
P(\hat{y}|f(X)) = \frac{\exp(-E(\hat{y}|f(X)))}{Z(f(X))},
\]

where \( Z(f(X)) \) is the partition function [23]. In this algorithm, the energy function \( E(\hat{y}|f(X)) \) is set to have two parts, with

\[
E(\hat{y}|f(X)) = \sum_i \psi_u(\hat{y}_i|x_i) + \sum_{i<j} \psi_p(\hat{y}_i, \hat{y}_j|x_i, x_j),
\]

where \( \psi_u(\hat{y}_i|x_i) \) is the unary potential and \( \psi_p(\hat{y}_i, \hat{y}_j|x_i, x_j) \) is the pairwise potential. As in most applications of CRF, the unary potential is set as the cost of a pixel \( x_i \) taking label \( \hat{y}_i \), which is

\[
\psi_u(\hat{y}_i|x_i) = -\log(p(\hat{y}_i|x_i)).
\]

The pairwise potential is set to be

\[
\psi_p(\hat{y}_i, \hat{y}_j|x_i, x_j) = \mu(\hat{y}_i, \hat{y}_j)(w^{\text{app}}k_{\text{app}} + w^{\text{smo}}k_{\text{simo}}),
\]

where \( \mu(\hat{y}_i, \hat{y}_j) \) is called compatibility function and is given by the Potts model \( \mu(\hat{y}_i, \hat{y}_j) = |\hat{y}_i \neq \hat{y}_j| \), \( k_{\text{app}} \) and \( k_{\text{simo}} \) are respectively termed the appearance and smooth kernels, and \( w^{\text{app}} \) and \( w^{\text{simo}} \) are linear combination weights. If we denote the position of \( x_i \) as \( p_i \), the appearance kernel \( k_{\text{app}} \) in (3) is defined as

\[
k_{\text{app}}(\hat{y}_i, \hat{y}_j|x_i, x_j) = \exp\left(-\frac{|p_i - p_j|^2}{2\sigma^2} - \frac{|f(x_i) - f(x_j)|^2}{2\beta^2}\right)
\]

and the smoothness kernel \( k_{\text{simo}} \) is

\[
k_{\text{simo}}(\hat{y}_i, \hat{y}_j|x_i, x_j) = \exp\left(-\frac{|p_i - p_j|^2}{2\gamma^2}\right).
\]

As stated in [30], the appearance kernel is based on the observation that neighboring pixels with similar features tend to be from the same class, while the smoothness kernel helps to eliminate small isolated regions. It is noteworthy that the pairwise potential integrates both the spectral information and geometry information.

Mathematically, the final prediction is obtained by

\[
y^* = \arg \max_{\hat{y}} P(\hat{y}|f(X)),
\]

which is, however, hard to compute. Usually, a method of mean field approximation [30] is used to approximately calculate the results. In [28], the authors assumed that the pairwise potentials only take effect when the Manhattan distance between \( x_i \) and \( x_j \) is less than the so-called filter-size \( k \). Under this assumption, the mean field inference algorithm is implemented on GPU phase and could be calculated more efficiently. This inference algorithm is referred as ConvCRF. Readers may refer [28, 30] for more detailed definitions and calculations of CRF.

2.3 A Summary

In general, we first use a DML model to generate spectrum-based features \( f(x_i) \), as well as the preliminary predictions \( \tilde{p}(y|x_i) \). Then, \( \tilde{p}(y|x_i) \) is reformulated as the unary function of CRF by [2]. The pairwise potentials, which include the appearance and smooth kernels, are expressed by [23] using features \( f(x_i) \) and the corresponding pixels’ positions \( p_i \). Finally, the ConvCRF algorithm is adopted to make CRF inference, producing the final predictions of the whole image.
In essence, it is the intrinsic connection between the center loss of DML and the appearance kernel in CRF that contributes to the performance of the proposed framework. Compared to the features extracted by the conventional NN models, the features extracted by DML with center loss gather more tightly within the same class in Euclidean space, i.e., pixels from the same class tend to be encoded as more similar features. Meanwhile, the appearance kernel \( f(x_i) \) is well designed to rely on the Euclidean distances between features. When compared to CRFs that rely on raw pixel spectra or features from plain NN models, the existence of center loss in DML rationalize the CRF algorithm in our framework and enhance the final classification results.

3 Experiments

3.1 Datasets Description

The experiments are carried out on two well-known HSI datasets, namely the Pavia University scene and the Salinas scene collected by ROSIS sensor and AVRIS sensor, respectively. The Pavia University scene used in experiments has a size \((610, 340, 103)\) in (Height, Width, Bands). The spatial resolution is 1.3 m, while the band depth covers the wavelength from 0.43 μm to 0.86 μm with 12 noisy and water bands removed. Regarding the Salinas scene, the size of image is \((512, 217, 204)\) after removing 20 water absorb bands. The spatial resolution is 3.7 m, and the spectra covers a bandwidth range from 0.4 μm to 2.5 μm.

3.2 Experimental Settings and Results

To show the advantage of combining DML with center loss and CRF, comparative experiments are carried out by using DML-NN-CRF, and the proposed framework DML-CRF. Here, the only difference between the NN model and the DML model is the absence of center loss in the former. Moreover, two state-of-art methods, namely 3D-CNN [15] and CSFF (DML-CSFF) [21] are also compared as baselines of HSI classification algorithms. Experiments are perforated with deep learning platforms Caffe [31] and PyTorch [32], on a machine equipped with CPU of Intel Xeon E5-2660@2.6GHz and GPU of NVIDIA TitanX.

As for the DML model, the length of the extracted feature is set to be \(\text{len}(f(x_i)) = 32\). The hyperparameters, such as learning rate, balance weight \(\lambda\), and etc., are all chosen as default values, as in the original paper [20]. Regarding the CRF algorithm in NN-CRF and DML-CRF, there are five parameters \(w^{\text{app}}, w^{\text{smo}}, \theta_\alpha, \theta_\beta,\) and \(\beta\). According to [30], the performances of CRF in terms of classification accuracies are relatively robust to these five parameters. Therefore, the default setting,

\[
w^{\text{app}} = 10, \ w^{\text{smo}} = 3, \ \theta_\alpha = 0.1, \ \theta_\beta = 80, \ \theta_\gamma = 3,
\]

in [28,30] are used directly. The only hyperparameter to be set is the filter size \(k\) in ConvCRF, which is chosen as \(k = 7\) for Pavia University scene and \(k = 15\) for Salinas scene. An analysis of these variables is given in Section 3.3. The comparing methods 3D-CNN and CSFF (DML-CSFF) are implemented by following [15,21].

If no otherwise specified, the training samples used in all the experiments follow the same preprocessing procedure. Each dataset is firstly normalized to have zero mean and unit variance. The training set is formed by randomly chosen 200 pixels per class. For 3D-CNN, 200 HSI patches from each class are randomly chosen instead. To avoid overfitting effects, virtual samples are generated and used by the linear combination of the pixels with same class labels, with \(x = qx_1 + (1-q)x_2\), in all the neural networks. The classification performance is evaluated by three metrics, namely overall accuracy (OA), average accuracy (AA), and the kappa coefficient (κ).

The classification results with mean and standard deviation over five runs are reported in TABLE 1. As shown in the first three columns, the absence of either DML or CRF deteriorates the classification accuracies. Compared to the state-of-the-art methods, the proposed DML-CRF still leads to comparable results. The proposed DML-CRF outperforms 3D-CNN with a large margin on both datasets. When compared to CSFF, DML-CRF performs better in all the metrics on the Pavia University scene. On Salinas scene, DML-CRF surpasses CSFF in terms of AA, but is slightly inferior to CSFF in terms of OA and κ. The testing time of the comparing methods is given in TABLE 2. We observe the DML-CRF is overwhelmingly faster than CSFF and several times faster than 3D-CNN, thanks to the implementation of ConvCRF on GPU.

3.3 Parameter Optimization

This subsection mainly discusses the parameters and hyperparameters of CRF. For DML model, more details on the behaviors of hyperparameters can be found in [20]. We verify the robustness of CRF to parameters \(w^{\text{app}},\)

\[\text{http://www.ehu.eus/ccwintco/index.php?title=Hyperspectral_Remote_Sensing_Scenes}\]
Table 1: Classification accuracies (averaged over 5 runs) of DML, NN-CRF, DML-CRF, 3D-CNN, and CSFF on Pavia University scene and Salinas scene

|          | DML   | NN-CRF | DML-CRF | 3D-CNN | CSFF  |
|----------|-------|--------|---------|--------|-------|
|          | OA(%) | AA(%)  | κ       | OA(%)  | AA(%)  | κ     |
| Pavia Univ.  | 93.67 ± 0.31 | 98.78 ± 0.15 | 99.10 ± 0.10 | 98.14 ± 0.10 | 98.90 ± 0.14 |
|           | 93.64 ± 0.22 | 97.53 ± 0.18 | 98.72 ± 0.20 | 97.32 ± 0.68 | 98.49 ± 0.13 |
| Salinas  | 92.72 ± 0.44 | 97.86 ± 0.28 | 98.12 ± 0.21 | 95.91 ± 0.87 | 98.53 ± 0.34 |
|          | 97.09 ± 0.16 | 99.25 ± 0.11 | 99.26 ± 0.08 | 98.79 ± 0.29 | 99.02 ± 0.20 |
|          | 0.9153 ± 0.0014 | 98.38 ± 0.20 | 0.9880 ± 0.0014 | 0.9687 ± 0.0015 | 0.9852 ± 0.0018 |

Table 2: Comparison of testing time (in seconds)

|          | 3D-CNN | CSFF | DML-CRF |
|----------|--------|------|---------|
| Pavia Univ.  | 50.94  | 1779.45 | 8.68 |
| Salinas  | 45.69  | 5453.36 | 17.51 |

Figure 2: Classification accuracies in terms of AA and OA, along with varying parameters $w_{app}$, $w_{smo}$, $\theta_\alpha$, $\theta_\beta$, and $\theta_\gamma$, on Pavia University and Salinas scenes.

Figure 3: Classification accuracies in terms of AA and OA, along with varying hyperparameter $k$, on Pavia University and Salinas scenes.
To this end, we first anchor the default values by $w^{app} = 10, w^{smo} = 3, \theta_\alpha = 0.1, \theta_\beta = 80, \theta_\gamma = 3$. And then, several experiments are tested by varying each parameter. The relationships between every parameter and the classification performance are presented in Fig. 2. It is obvious that the classification performance is relatively robust to the parameters.

Regarding the only hyperparameter $k$ in ConvCRF, it controls the size of the spatial information that CRF takes into account. The effect of $k$ on the classification accuracies are shown in Fig. 3. As expected, larger filter sizes lead to higher accuracies, but also require more computation cost.

4 Conclusion

In this paper, we proposed a framework that combines DML and CRF. The DML model is used to extract features from pixels of HSIs. The advantage of center loss reduces the Euclidean distances between the extracted features which share the same class label. Later, the CRF algorithm is applied to give predictions over the whole HSI by using both the extracted features and their position information. Contrast experiments demonstrated that the absence of either DML or CRF declines the classification performance. Moreover, the proposed framework provides comparable results to the state-of-art methods in both classification accuracy and computation cost. Additional experiments are performed to show the effects of varying parameters and hyperparameters on the classification accuracies.

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