The use of Design of experiments to calculate the influence of planet oblateness in the temporary gravitational capture

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Abstract. This paper presents the study of the influence of the oblateness of one planet in the temporary gravitational capture of a particle (or a spacecraft) around the main celestial body of the system. The gravitational capture is a physical phenomenon where a celestial body (or a spacecraft), initially in a hyperbolic orbit with positive energy with respect to the main body of the system, is captured by another celestial body, changing the energy of the orbit to negative during a determined time. In this way, we have a temporary capture of this body or spacecraft. In the case of a spacecraft, it is possible to make this capture permanent by the application of a propulsion system to the spacecraft. This technique is done with savings over the traditional maneuvers performed without the support of the temporary capture, so it can help mission designers in the planning of the mission. As an example, the system used in the present paper is composed by the Sun-Jupiter-particle, and this system is modeled by the circular restricted three-body problem (CR3BP) plus the oblateness of Jupiter. To determine the effects of the oblateness in the dynamics of the particle, it is used the design of experiments to generate the data for a global analysis of the model. The oblateness of the planet is varied to show the effects of this parameter

1. Introduction

In this paper it is presented the study of the gravitational capture of a particle (or a spacecraft) by the planet Jupiter. The gravitational capture phenomenon occurs when a particle approaches a celestial body coming from an open hyperbolic orbit and has its motion modified to a closed elliptic orbit only due to gravity forces.

This phenomenon can happen in nature when particles or small celestial bodies, like comets or asteroids, approach the Solar System and the effects of large planets like Jupiter and/or Saturn remove slowly energy from this approaching body to generate its temporary capture with respect to the Sun.

In terms of aerospace applications, this effect has been explored in the literature and in real word missions. Several researches [1-24] showed the importance of this phenomenon in reducing the energy of a spacecraft going to the Moon and using the gravity of the Earth to reduce the fuel required to complete the transfer. The idea is to send the spacecraft to a first passage by the Moon to make a Swing-By [25-27], instead of a direct capture. After that, the spacecraft goes to a large distance from the Earth-moon system and then return for a gravitational capture maneuver, where the spacecraft approaches the Moon under the effects of the Earth, which generates a temporary gravitational capture of the spacecraft by the Moon. When close to the Moon, it is possible to apply an impulse that completes a permanent capture.
In the present paper, a study is made using a design of experiments [28] for offline simulations, focused in maximizing the total time that a body stays inside the sphere of influence [29] of Jupiter. The effect of the oblateness perturbs the orbit of the body in a significant form only if the body stays in orbit for a long time.

2. Mathematical Model
The system is modeled using the restricted three body problem [30]. In this model, the primaries describe circular motions around the barycenter of the system with constant angular velocity. The particle (or spacecraft) orbits the primaries, but do not interfere in their motion. Under these hypotheses, the equations of motion of the particle is given by,

\[
\vec{r}_3' = -\left( \frac{\mu_1}{r_{13}^3} \frac{r_{13}}{r_{23}^3} + \frac{\mu_2}{r_{23}^3} \right)
\]

where,

\[
r_{13}^2 = (x' + a'^*)^2 + y'^2 + z'^2
\]

\[
r_{23}^2 = (x' - b'^*)^2 + y'^2 + z'^2
\]

In those equations \(\vec{r}_{13}\) is the vector pointing from the first primary to the particle, \(\vec{r}_{23}\) is the vector pointing from the second primary to the particle, \(\mu_1\) and \(\mu_2\) are the gravitational parameters of the first and second primary, respectively, \(a'\) and \(b'\) are the x-axis coordinates of the first and second primary, respectively, and \(x'\), \(y'\) and \(z'\) are the coordinates of the particle. Applying a complex transformation, given by (3), in Eq. (1), and considering the oblateness effect of the first primary [31], we obtain Eqs. (4), (5) and (6) shown next.

\[
(\xi + i\eta) = (x' + iy')e^{i\eta t}
\]

\[
\dot{x'} - 2ny' = n^2x' - \mu_1 \frac{(x' + a')}{r_{13}^3} - \mu_2 \frac{(x' - b')}{r_{23}^3} - \mu_1 J_{2(1)} r_{p(1)}^2 \frac{(-3(x' + a'))}{2r_{13}^5} + \frac{15\pi^2 r_{23}^2 (x' - b')}{2r_{23}^5}
\]

\[
n^2y' - \mu_1 \frac{\dot{y}'}{r_{13}^3} - \mu_2 \frac{\dot{y}'}{r_{23}^3} - \mu_1 J_{2(1)} r_{p(1)}^2 \frac{(-3y')}{2r_{13}^5} + \frac{15\pi^2 y'}{2r_{23}^5} \]

\[
z' = -\mu_1 \frac{z'}{r_{13}^3} - \mu_2 \frac{z'}{r_{23}^3} - \mu_1 J_{2(1)} r_{p(1)}^2 \frac{(-3z')}{2r_{13}^5} + \frac{15\pi^2 z'}{2r_{23}^5}
\]

in those equations, \(J_{2(1)}, J_{2(2)}, r_{p(1)}, r_{p(2)}\) are the values of \(J_2\) and the primary ratios 1 and 2, respectively. It is adopted only the oblateness effect in the first primary, because we are considering only systems with low values of \(\mu\), so the effects of the oblateness of the smaller primary is negligible.

Being \(x', y'\) and \(z'\) the components of the position vector in the rotating system, \(t'\) the time, \(m_1\) and \(m_2\) the masses of the first and second primary, respectively, and \(C_1\) and \(C_2\) defining the dimensionless variables described in (7), equations (4), (5) and (6) turns into (8), (9) and (10), respectively.
where $r_{13}$ and $r_{23}$ are given, respectively, by (11) and (12).

$$r_{13} = \sqrt{(x + \mu)^2 + y^2 + z^2}$$

(11)

$$r_{23} = \sqrt{(x - 1 + \mu)^2 + y^2 + z^2}$$

(12)

3. **Design of experiments**

The design of experiments is related to the flow used to succeed an investigation, which can be theoretical/numerical or experimental, in order to obtain the best information of a model through its results. The information of interest can be, for example, the sensitivity of a model to the variation of its input parameters, as is the case of the present work.

A good design of experiments usually has a factorial form. In other words, it demonstrates that the best way to obtain the sensitivity of the parameters in a mathematical model is not to vary one by one and to analyze the variation of the response as a function of the variation of the input, but varying the inputs in order to compose a factorial experiment.

There are complete factorial experiments, which are those in which all possible binary combinations of a variable are exercised, and there are those where not all combinations are performed. In this work we will use a complete factorial study.

There are also cases in which two variables present confusion between themselves. Confusion is a term used in statistics when it is not possible, through the response of a system, to identify which variable proves such an effect.

4. **Results**

The first step is to find the variables that affect the temporary gravitational capture. Based in the mathematical model used, they are:

- Non-dimensional gravitational constant ($\mu$)
- Primary ratio ($r_p$)
• Initial condition of position (end capture condition).
• Velocity in the final capture condition ($v(t = 0)$)
• Argument of perigee ($\omega$)
• Orbit inclination ($i$)
• Right Ascension of the ascending node ($\Omega$)
• Oblateness the primary ($J_2$)

A simulation was performed considering the following factorial experiment: for the $\mu$ variable, values of 1.1, 2.0 and 10 were simulated; for the velocity ($v$) the values were 0.1, 0.5 and 1.0; for orbit types it was considered direct and retrograde cases; for the values of $\omega$, we used $0^\circ$, $90^\circ$, $180^\circ$ and $270^\circ$; for the inclination we used $0^\circ$, $30^\circ$, $60^\circ$ and $90^\circ$ and, finally, for the right ascension of the ascending node the values used were $0^\circ$, $90^\circ$ and $180^\circ$. The times for escape or collision were obtained and are shown in Fig. (1). The main effects and interaction maps between the variables are shown in Figs. (2) and (3), respectively.

It is observed that the time to escape from the sphere of influence is directly proportional to the nondimensional gravitational constant ($\mu$), to the oblateness of the planet ($J_2$), to the distance of the planet ($r_p$), to the distance between the primaries ($\Delta r_p$) and the true anomaly ($f$) and inversely proportional to the velocity ($v$), inclination ($i$) and right ascension of the ascending node ($\Omega$). It can also be observed that there are strong interactions between the distance from the planet ($\Delta r_p$) and the velocity ($v$) and between the inclination ($i$) and the oblateness ($J_2$).

The strength of the interaction between two variables is higher when the angle formed between the lines in Fig. (3) is large. When they do not intersect themselves, the interaction between the variables is weak or nonexistent.

Finally, to expose only the effect of the oblateness of a planet in the temporary gravitational capture, Table (1) is constructed to show the time required by the particle to escape from Jupiter and cases of other hypothetic planets with 10 and 50 times the oblateness of Jupiter. It is shown the respective times and the percentage increment with respect to the baseline (only $J_2$).

**Figure 1.** Time to collide and to escape for a simulation of temporary gravitational capture in Jupiter, considering tridimensional orbits.
**Figure 2.** Main effects map of the parameters that influence the temporary gravitational capture in Jupiter, generated through design of experiments.

It is observed that, with a 10 times increase in the oblateness of the planet, the time for capture increases from 3% to 10% and, with an increase of 50 times, the increase is from 50% to 60%.

5. **Conclusions**

The effects of the oblateness of a planet in the temporary gravitational capture of a particle around a main celestial body are studied. To find conditions where the trajectory and the total time inside the sphere of influence it is necessary a planet or cloud of particles with higher oblateness values.

In this paper we used a design of experiments to propose a sequence of simulations to acquire easily the effects of each parameter in the model. It was showed that it is possible to evaluate a model in this way, identifying not only the sensibility of the model to each variable but also the interaction between the variables.

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Table 1. Times to escape from capture assuming different values for the oblateness of the planet.

|   | Time to escape \((J_2)\) | Time to escape \((10 \times J_2)\) | Δ Time to escape \((10 \times J_2)\) | Time to escape \((50 \times J_2)\) | Δ Time to escape \((50 \times J_2)\) |
|---|---------------------------|---------------------------------|---------------------------------|---------------------------|--------------------------|
| 1 | 0.381                     | 0.393                           | 3.1%                            | 0.577                     | 51.4%                    |
| 2 | 0.381                     | 0.393                           | 3.1%                            | 0.577                     | 51.4%                    |
| 3 | 0.383                     | 0.404                           | 5.5%                            | 0.591                     | 54.3%                    |
| 4 | 0.383                     | 0.403                           | 5.2%                            | 0.593                     | 54.8%                    |
| 5 | 0.382                     | 0.402                           | 5.2%                            | 0.599                     | 56.8%                    |
| 6 | 0.381                     | 0.393                           | 3.1%                            | 0.577                     | 54.3%                    |
| 7 | 0.383                     | 0.404                           | 5.5%                            | 0.591                     | 54.3%                    |
| 8 | 0.383                     | 0.403                           | 5.2%                            | 0.593                     | 54.8%                    |
| 9 | 0.382                     | 0.402                           | 5.2%                            | 0.599                     | 56.8%                    |
| 10| 0.381                    | 0.393                           | 3.1%                            | 0.577                     | 56.8%                    |
| 11| 0.383                    | 0.408                           | 6.5%                            | 0.620                     | 61.9%                    |
| 12| 0.383                    | 0.408                           | 5.5%                            | 0.620                     | 59.1%                    |
| 13| 0.383                    | 0.408                           | 6.5%                            | 0.600                     | 56.7%                    |
| 14| 0.383                    | 0.408                           | 6.5%                            | 0.602                     | 57.2%                    |
| 15| 0.383                    | 0.409                           | 6.8%                            | 0.609                     | 59.0%                    |
| 16| 0.383                    | 0.408                           | 6.5%                            | 0.620                     | 61.9%                    |
| 17| 0.383                    | 0.408                           | 6.5%                            | 0.600                     | 56.7%                    |
| 18| 0.383                    | 0.408                           | 6.5%                            | 0.600                     | 56.7%                    |
| 19| 0.383                    | 0.408                           | 6.5%                            | 0.602                     | 57.2%                    |
| 20| 0.383                    | 0.409                           | 6.8%                            | 0.609                     | 59.0%                    |
| 21| 0.383                    | 0.408                           | 6.5%                            | 0.620                     | 61.9%                    |
| 22| 0.384                    | 0.417                           | 8.6%                            | 0.609                     | 58.6%                    |
| 23| 0.384                    | 0.419                           | 8.9%                            | 0.611                     | 59.1%                    |
| 24| 0.384                    | 0.423                           | 10.2%                           | 0.617                     | 60.7%                    |
| 25| 0.384                    | 0.417                           | 8.6%                            | 0.609                     | 58.6%                    |
| 26| 0.384                    | 0.417                           | 8.6%                            | 0.609                     | 58.6%                    |
| 27| 0.384                    | 0.417                           | 8.9%                            | 0.611                     | 59.1%                    |
| 28| 0.384                    | 0.423                           | 10.2%                           | 0.617                     | 60.7%                    |
Figure 3. Interaction map of the parameters that influence the temporary gravitational capture in Jupiter, generated through design of experiments. Legend for the colors: $\mu$: black represents 0.1 and red represents 0.5, $\Delta C_3$: black represents 0.1 and red represents 1.0, $J_2$: black represents 0.01 and red represents 1.00, $r_p$: black represents 0.0001 and red represents 0.0100, sense of orbit: black represents retrograde orbits and red represents direct orbits.

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