Black-box modelling of gas turbine engine’s electrohydraulic fuel metering system.

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Abstract. The fuel and air flow rates are the main parameters affecting the gas turbine engine performance. The burned fuel in the combustor is the main source of heat energy. The fuel metering system (FMS) provide these engines with precise fuel mass flow rates required for regulation process. One of the most important and challenging concerns of engine design and control is, driving a valid model for the FMS. Physics-based models require well identification of the electrical and hydraulic system parameters. Measuring these parameters for the system under study, like the internal dimensions of these system components is very hard because of the system complicity and integrity. In this case it is preferred to treat the system as a black box. In this paper a transfer function model for an electrohydraulic FMS has been estimated and validated using MATLAB system identification toolbox (SIT). FMS closed loop measurements have been done for data set preparation process. Linear Variable Differential Transformer (LVDT) integrated in this system has been used for measuring the metering valve displacement which is proportionally represents the output mass flow rate. Linear variable differential transformer (LVDT) signal conditioning circuit has been designed and implemented to provide DC output voltage, proportional to the system output volume flow rate. Another conditioning circuit has been designed for output current amplification process required for electrical solenoid driving. The driven transfer function model has been validated using a different measured data set to insure a good representation of the physical system. Results show that the obtained model follows the real system with good accuracy and demonstrate the effectiveness of the transfer function modelling for the FMS.

KEYWORDS: Turboshaft gas turbine engine; fuel control unit modeling; system identification.

1. Introduction
Turboshaft gas turbine engines are responsible for supplying the power for various systems and play a vital role in marine and aerospace industries [1]. The fuel metering system (FMS) is the most imperative subsystem in the gas turbine engines. It controls the fuel mass flow rate provided to the combustion chamber which is the principal parameter that influences the engine output power.

In order to design the best control of a gas turbine engine, a valid transfer function / physics based mathematical model for the system with high accuracy should be implemented such that it generates realistic output. Physics based mathematical models are implemented amid the early stages of system design. Additionally, this methodology is utilized for simple system with possible assurance of its parameters. The other methodology is to drive transfer function mathematical model for predesigned
systems to such an extent that its parameters couldn’t be evaluated especially in reverse engineering areas.

Hydromechanical [2] [3] and electrohydraulic [4] FMSs are commonly used for gas turbine fuel control. However, electrohydraulic is such more entangled that it is progressively sensitive to external electrical noise from other subsystems as electrical motors and generators. Indeed, even its internal hydraulic parts are more convoluted, such that it expands the system non-linearity which requires more precise control.

Since sufficient information concerning a system is available so that a set of equations that accurately model the system can be derived. More often, a system is available only as a black box so that the relation between the system input and output is not so derivable using differential equations or physics based modeling. The main electrical and hydraulic parameters of the system under study couldn’t be identified accurately only as a black-box because of the system complicity, sensitivity and integrality.

Researches attempt to design accurate, robust and nonlinear models that could be utilized to control gas turbine engine and its subsystems. Linearization technics are widely used with nonlinear models for more costly efficient controller design and implementation. Furthermore, nonlinear model proposes more efficient representation of the real system; it requires more complex controller design process.

S. Chaturantabut et al. [5], A. N. Tudosie et al. [2] [3] introduced multiple empirical and numerical methods to interpolate the system behaviour utilizing linearized input signals to avoid nonlinearities evaluation.

A. Salehi et al. [1] proposed a control scheme for gas turbine engine fuel control unit. Wiener model was used as the structure of the engine. This structure composed of linear dynamic and nonlinear static blocks. Regarding to the authors, system identification tool was used for determining the linear part. However, the non-linear part was assumed to be known and studied only in the steady-state phase, such that the dynamic phase was not taken into consideration, which may affect the full system behaviour. Moreover, the authors use the same design input data for validation.

M. Basso et al. [6] built non-linear dynamic model for gas turbine engine including fuel control with system identification technique. In addition, NARX was adopted for parameters estimation. Authors concluded that all identified model's main regression terms are linear which reduce the control complexity. The model was validated on different set of data.

System identification based on MATLAB is highly efficient [7] and its system identification toolbox (SIT) is one of the commonly used tools for this purpose. There is a growing body of literature that recognizes the importance of using system identification tools not only to make this process easier, but to give researchers the advantage of comparing many system identification techniques on their systems and chose the more efficient one.

In this paper a test rig with conditioning circuit was designed and implemented for feedback measuring. In addition, another conditioning circuit was designed to amplify FMS solenoid driving current. Furthermore LabVIEW (version 14.0.1) GUI with acquisition system was utilized in final stage of measuring and data sets preparation. Finally transfer function model has been derived using SIT. Closed loop measurements have been implemented at three deferent step inputs across the system range. Two measured data sets have been used in the estimation process, and the third one has been used for model validation. The validation results showed that the proposed scheme behave accurately in different stages.

2. System description

The system under study is part of the fuel management and control system of AGT1500 gas turbine engine of ground equipment. This engine has 1500 horsepower (HP) output maximum power at 22500 RPM power turbine speed and delivers an output torque up to 5017 newton meters. In this study, our interest is the electrohydraulic FMS which accurately controls the fuel mass flow rate supplied to the engine.
Figure 1 shows the functional scheme of the FMS under study. The system derived by vane pump (1) runs on diesel fuel. The pressurized fuel is pumped into a differential pressure regulator (2). The differential regulated pressure is applied on the variable area of the metering unit (3). The constant pressure difference applying on the both sides of the metering area (7), makes this area is the only affecting parameter on the fuel output mass flow rate. The fuel nozzle (5) is responsible for spraying this output flow rate inside the combustion chamber. The metering valve (6) displacement determines linearly the metering cross section area.

In order to measure the metering valve displacement, an LVDT sensor (8) is integrated with the system. This feedback electrical signal represents linearly the output mass fuel flow rate of the FMS. The hydraulic amplifier (4) serves to convert the small electrical proportional solenoid (9) core displacement into a considerable pressure force. This force pilot controls the metering valve movement.

Figure 1. FMS of the engine under study

3. Experimental work
Accurate measurements for the real system steady-state and dynamic response are required for modeling and validation processes. A test rig and an acquisition system have been designed and implemented to build a complete measuring system as shown in figure 2.
3.1. Test rig setup
In order to facilitate modeling of the electrohydraulic FMS, a test rig has been built as shown in figure 3. It consists of hydraulic parts (fuel tank, fuel filters, the electromechanical fuel pump and the fuel injector). All these parts are genuine for good representation of the real system. The system is driven by an AC motor with inverter to control the motor speed.
3.2. Acquisition system

Experimental measurements are necessary to understand the system behavior in several operation conditions which essential for the modeling process. Acquisition system has been used for this purpose. It consists of three main parts; conditioning circuits, NI USB DAQ and LabVIEW (version 14.0.1) graphical user interface (GUI). LVDT feedback signal conditioning achieved using (AD698) the integrated circuit and its required external components. This IC generates the required AC signal for LVDT primary coil excitation process and measure the output voltage of the secondary coil as shown in figure 4 it calculates the ratio between the output RMS voltage of the secondary coil and the primary coil excitation RMS voltage then generates output DC voltage proportional to the LVDT core displacement.

![Figure 4. FMS AD698 conditioning circuit](image)

The solenoid maximum driving current is 1A. This current can’t be supplied by the NI DAQ digital to analog (DAC) channels. So a current amplification circuit is necessary for solenoid driving. OP548 operational amplifier circuit used for this purpose. The circuit scheme shown in figure 5 has been design and implemented to amplify the current with a voltage gain equal one by using zero ohm resistor (R₂) and 10 kΩ resistor (R₁). The MCU analogue voltage considered as the input voltage (V_{IN}) of this amplification circuit. The amplifier output voltage (V_o) is driving the fuel flow proportional solenoid which has internal resistance (Z_L) of 10Ω.

All conditioning and required power circuits integrated in one printed circuit board (PCB) has been designed and implemented as shown in figure 6.

The measurements were made using a DAQ connected to the conditioning PCB and an interface program designed for this purpose on LabVIEW.

3.2.1. Data sets preparation. Accurate experimental measurements are fundamental for estimation and validation processes. A key aspect of driving a valid model using system identification technic is the proper selection of the estimation data set. Because of the system nonlinearity and instability at certain regions, the system closed loop response has been measured. This response is measured with three different step inputs across the system range.

Fuel flow request voltage signal (WFR), actual fuel flow feedback voltage signal (WFA) and the solenoid input voltage (VIN). The pound per hour (PPH) unit has been used in the engine manufacturer technical manual. So, in this study this unit has been chosen for the output mass flow rate representation to make the comparison process between the manufacturer FMS specifications and the study results more easily. The multiplication factor between the voltage representation and the PPH representation of the mass flow rate has been determined by measuring the output mass flow rate
at different points. These measurements show that converting the voltage representation of the mass flow rate to pound per hour (PPH) is achieved by multiplying the voltage value by 100.

Two measured data have been combined to build the estimation data set as shown in figure 7. One is the measured closed loop response of the system for 4.2V (420 PPH) and the other is for 7.2V (720 PPH) step input. Many researchers use the same data set used for estimation, to validate their driven models which is considered as a weak point in their studies because the model was trained to fit this data. So, in order to test the model accurately, a new data set should be introduced to the model to check its efficiency. As presented in [8], it is preferred to use different data set for validation to insure that the system response was predicted accurately.

Figure 5. Amplification circuit

Figure 6. Conditioning and power circuits PCB
The third measured data set shown in figure 8 was used for model validation process. It has been measured for the system closed loop response for 5.7V (570 PPH) step input request.

**Figure 7.** Estimation data set

**Figure 8.** Validation data set
3.2.2. Model estimation and validation. The system identification is performed using Matlab’s SIT (R2017a) by many researchers in different fields ([9], [10] and [11]). The toolbox has a useful graphical user interface (GUI) which is used to import the time domain data set showed in figure 7. The toolbox is used to estimate time continuous transfer function from this imported data.

The single-input single-output continuous time linear time-invariant system can be described by Equation (1) [12].

\[ y_u(t) = G(p)u(t) \]  

with

\[ G(p) = \frac{B(p)}{A(p)} \]

\[ B(p) = b_0 + b_1p + \cdots + b_mp^m \]

\[ A(p) = a_0 + a_1p + \cdots + a_np^n, \quad a_n = 1, \quad n \geq m \]

where \( u(t) \) is the input signal and \( y_u(t) \) the system response to \( u(t) \), \( p \) is the differential operator \( (p_x(t) = \frac{dx(t)}{dt}) \).

Equation (1) describes the output at all values of the continuous-time variable \( t \) and can also be written as in the free noise case:

\[ a_0y_u(t) + a_1y_u^{(1)}(t) + \cdots + y_u^{(n)}(t) = b_0u(t) + b_1u^{(1)}(t) + \cdots + b_mu^{(m)}(t) \]  

(2)

Applying the one-sided Laplace transform to both sides of (2) yields

\[ \sum_{i=0}^{n-1} a_is^iY_u(s) + s^nY_u(s) = \sum_{i=0}^{m} b_is^iU(s) + \sum_{i=0}^{n-1} c_is^i \quad (3) \]

Or

\[ A(s)Y_u(s) = B(s)U(s) + C(s) \]  

(4)

where \( S \) represents the Laplace variable while \( Y_u(s) \) and \( U(s) \) are respectively the Laplace transform of \( y_u(t) \) and \( u(t) \). The coefficient \( c_i \) depend on the initial conditions. Assume now that a causal analogue pre-filter has a Laplace transforms \( F(s) \). Applying the filter to both sides of (3) yields

\[ \sum_{i=0}^{n-1} a_iY_{uf}^i(s) + s^nY_{uf}(s) = \sum_{i=0}^{m} b_iU_{uf}^i(s) + \sum_{i=0}^{n-1} c_i\zeta_{uf}^i(s) \]  

(5)

where

\[ Y_{uf}^i(s) = s^iF(s)Y_u(s), \quad U_{uf}^i(s) = F(s)U(s) \quad \text{and} \quad \zeta_{uf}^i(s) = s^iF(s) \]

Denoting the inverse Laplace transforms as
\[
\sum_{l=0}^{n-1} a_l y_{uf}^{(l)}(t) + y_{uf}^{(n)}(t) = \sum_{l=0}^{m} b_l u_f^{(l)}(t) + \sum_{l=0}^{n-1} c_l \xi_f^{(l)}(t)
\]  
(6)

At the time-instant \( t = t_k \), considering the additive noise on the output measurement, Equation (4) can be rewritten as

\[
\sum_{l=0}^{n-1} a_l y_{uf}^{(l)}(t_k) + y_{uf}^{(n)}(t_k) = \sum_{l=0}^{m} b_l u_f^{(l)}(t_k) + \sum_{l=0}^{n-1} c_l \xi_f^{(l)}(t_k) + \varepsilon_{EE}(t_k, \theta)
\]
(7)

where \( \varepsilon_{EE}(t_k, \theta) \) denotes the equation error also termed as ‘generalized equation error’ [13]. To estimate the parameters \( a_1, b_1 \) Equation (5) can be reformulated using the transformed variables into standard linear regression form as

\[
y_f^{(n)}(t_k) = \varphi_f^T(t_k) \theta + \varepsilon_{EE}(t_k, \theta)
\]
(8)

with

\[
\varphi_f^T(t_k) = \begin{bmatrix} -y_f^{(n-1)}(t_k) & \ldots & -y_f(t_k) & u_f^{(m)}(t_k) & \ldots & u_f(t_k) \end{bmatrix}
\]
(9)

\[
\theta = [a_{n-1} \ldots a_0 \ b_m \ldots b_0]^T
\]
(10)

From \( N \) available samples of the input and output signals, the least-squares (LS) estimate that minimizes the sum of the squared errors is given by

\[
\hat{\theta}_N^{LS} = \left[ \sum_{i=1}^{N} \varphi_f(t_k) \varphi_f^T(t_k) \right]^{-1} \sum_{i=1}^{N} \varphi_f(t_k)y_f^{(n)}(t_k)
\]
(11)

It is however well known that the conventional least squares method delivers biased estimates in the presence of general cases of measurement noise. One of the simplest solutions to the asymptotic bias problem associated with the basic LS algorithms to use instrumental variable (IV) methods since they do not require a priori knowledge of the noise statistics. A bootstrap estimation of IV type where the instrumental variable is built from an auxiliary model is considered here [14]. The instrument is given by:

\[
\hat{\varphi}_f^T(t_k) = \begin{bmatrix} -\hat{y}_{uf}^{(n-1)}(t_k) & \ldots & -\hat{y}_{uf}(t_k) & u_f^{(m)}(t_k) & \ldots & u_f(t_k) \end{bmatrix}
\]
(12)

where

\[
\hat{y}_{uf}(t_k) = F(p) \hat{y}_u(t_k)
\]
(13)

Subject to zero initial conditions and \( \hat{y}_u(t_k) \) is the noise-free output calculated from

\[
\hat{y}_u(t_k) = G(p, \hat{\theta}_N^{LS})u(t_k)
\]
(14)

The IV-based estimated parameters are then given by
\[ \hat{\theta}_N^{IV} = \left[ \sum_{i=1}^{N} \hat{\Phi}(t_k)\hat{\Phi}^T(t_k) \right]^{-1} \sum_{i=1}^{N} \hat{\Phi}(t_k)y^{(n)}_f(t_k) \] (15)

After estimation of the transfer function model unknown coefficients, the transfer function formula can be driven based on Equation (3) with zero initial condition as following

\[ Y_u(s)\left(\sum_{i=0}^{n-1} a_i s^i + s^n\right) = U(s)\left(\sum_{i=0}^{m} b_i s^i\right) \]

\[ G(s) = \frac{b_m s^m + b_{(m-1)} s^{(m-1)} + \cdots + b_0}{s^n + a_{(n-1)} s^{(n-1)} + \cdots + a_0} \] (16)

where

- \( G(s) \) System transfer function.
- \( (a_0, a_1, \ldots, a_n) \) numerator coefficients
- \( (b_0, b_1, \ldots, b_m) \) denominator coefficients
- \( (m) \) number of the system zeros
- \( (n) \) number of the system poles

These coefficients have been estimated using SIT for several system orders from second to fifth order with all probabilities of poles and zeros numbers. Table (1) illustrates the best fit results of comparing the validation measured data with the predicted output of these estimated transfer functions.

| Order | 2\(^{nd}\) | 3\(^{rd}\) | 4\(^{th}\) | 5\(^{th}\) | 6\(^{th}\) |
|-------|------------|------------|------------|------------|------------|
| i     | 2          | 1          | 3          | 4          | 6          |
| k     | 2          | 3          | 4          | 5          | 6          |
| Fit   | 43.38%     | 57.62%     | 82.36%     | 83.46%     | 79.15%     |

What stands out in this table is the high nonlinearity of this system which makes the linear representation can’t achieve fit higher than 84%. The difference between the fourth and fifth order transfer functions fit so, the fourth order one has been selected to simplify the controller design process. The fourth order estimated transfer function of the system is

\[ G(s) = \frac{5.2455 s^3 + 106.15 s^2 - 4.2325 + 10.4}{s^4 + 10.65 s^3 + 54.15 s^2 - 1.1475 + 4.644} \] (17)

Although the fit percentage is not very high but figure 9 shows the good tracing of the model output.
4. Conclusion

Gas turbine engine FMS introduced to this study for modelling. Complete acquisition system includes the required conditioning circuit was the link between the test rig and the PC LabVIEW for measurements recording. The measured data signals were utilized to drive transfer function model of gas turbine engine electromechanical FMS. The black box modelling technique was applied via MATLAB’s system identification toolbox (SIT). Estimation and validation data sets were prepared based on closed loop system response measurements. These measurements have been applied on a test rig was implemented for this purpose. The validation results showed accurate fitting with high tractability of the actual system output.

The driven model could be considered as a validated model for the described system. Furthermore, it could be used to design and compare the effect of different control techniques on the system behaviour for further development.

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