Supersymmetry breaking and the radion in AdS$_4$ brane-worlds

Andrey Katz$^a$, Yael Shadmi$^a$, and Yuri Shirman$^b$

$^a$ Physics Department, Technion, Haifa 32000, Israel
$^b$ T-8, MS B285, LANL, Los Alamos, NM 87545

ABSTRACT

We compute the one-loop correction to the radion potential in the Randall-Sundrum model with detuned brane tensions, with supersymmetry broken by boundary conditions. We concentrate on the small warping limit, where the one-loop correction is significant. With pure supergravity, the correction is negative, but with bulk hypermultiplets, the correction can be positive, so that the 4d curvature can be lowered, with the radion stable. We use both the KK theory, and the 4d radion effective theory for this study.

andrey@physics.technion.ac.il, yshadmi@physics.technion.ac.il, shirman@lanl.gov
1 Introduction

It is well known by now that a 3-brane in AdS$_5$ can localize gravity [1], even when the brane tension is not tuned to the bulk cosmological constant as in the Randall-Sundrum (RS) model [2, 3]. The 4d theory can be either dS$_4$ or AdS$_4$ near the brane, with the 4d curvature proportional to the amount of detuning. In the latter case, if a second brane is added to the theory, the jump conditions for this brane constrain its position, so that the brane distance is determined by the bulk cosmological constant and two brane tensions [4], and, unlike in the RS model, the radion is stabilized.

Another qualitative difference between detuned and tuned brane systems has to do with supersymmetry breaking. Supersymmetric extensions of detuned brane models involve gravitino brane mass-terms, whose magnitudes are proportional to the amount of detuning [5]. When the phases of these two brane-terms differ, supersymmetry is broken [6]. This is clearly impossible in the tuned limit, where the brane terms vanish and their phases are ill-defined. Thus, the detuned theory, even with just pure gravity in the bulk, allows for spontaneous supersymmetry breaking, with the radion automatically stabilized. But these desirable features come at a price: the 4d theory is AdS$_4$, and so the 4d cosmological constant has the wrong sign for phenomenological applications. For model building purposes, it is therefore important to find modifications of the model that can lift the theory to Minkowski space without upsetting the radion stabilization. This has to be achieved while at the same time stabilizing the scalar partner of the radion, which, as we will explain, is a modulus of the classical theory.

In this paper, we will study the effects of supersymmetry-breaking quantum corrections on the radion potential. These effects are finite and calculable because the supersymmetry-breaking mechanism described above is non-local. While gravity gives a negative correction to the potential, hypermultiplets contribute with the opposite sign, so that the net contribution can offset, at least partially, the negative cosmological constant.

We focus here on models in which the 5d curvature, $k$, is much smaller than the inverse brane distance $1/R$. This complements the analysis of [7], which mostly considered the large warping case. For small warping, the loop correction to the potential can be important. This correction depends on two dimensionless parameters: the warp factor $kR$, and the 4d curvature in units of $k$, $1/(kL)$, which is proportional to the amount of detuning. Keeping $kL$ fixed and decreasing the warping, the loop correction should reproduce
the non-zero Casimir energy of flat orbifold models with broken supersymmetry. Thus, while the classical potential decreases for small warping, the loop correction does not. Furthermore, because it is a non-local effect, the supersymmetry breaking correction to the potential involves the warp factor $exp(-k\pi R)$, and is therefore suppressed for large warping. Note that since our framework is supersymmetric, there is no need for large warping in order to generate the weak-Planck scale hierarchy. Rather, our analysis is relevant for the problem of radius stabilization in models with an almost-flat extra dimension.

Our approach is complimentary to the approach of radion stabilization models based on the Casimir energy $[8, 9, 10, 11]$. While these typically start with zero radion potential at the classical level and use the Casimir energy in order to stabilize the radion, here we start with a small classical potential with the radion stabilized, and consider the Casimir energy as a correction to the cosmological constant. In the supersymmetric 4d theory, the radion is accompanied by the fifth component of the graviphoton, which is a modulus of the classical theory, even in the detuned case. As we will see, this field, which we will call $b$, is stabilized by the Casimir energy. In fact, because the graviphoton gauges a $U(1)_R$ symmetry of the 5d theory, under which the gravitino is charged, any phase difference of the gravitino brane terms can be compensated by a non-zero $b$, so the $b$ vacuum expectation value (VEV) breaks supersymmetry $[12]$. At one-loop, $b$ is stabilized either at the origin, corresponding to unbroken supersymmetry, or for maximal supersymmetry breaking.

There have been many studies of supersymmetry-breaking in brane worlds with AdS$_5$ bulk in the past few years (see for example $[13, 14, 15, 16]$). Some of these start with the tuned RS case but invoke additional sources of bulk and brane energies (from, say, a supersymmetry breaking sector), or brane gravitino mass terms, and rely on a supersymmetric 4d effective description. However, the unbroken 4d supersymmetry is determined by the bulk cosmological constant and brane tensions. Boundary terms can break this supersymmetry spontaneously if they are in certain ranges, but otherwise, as in the dS$_4$ case, break it explicitly$^2$. Our starting point in this paper is the supersymmetric 5d theory of $[5, 6]$, for which supersymmetry breaking is well understood. At low energies, this theory gives rise to a supersymmetric effective action for the radion supermultiplet, with a known

$^2$As is the case for ref. $[16]$. 
superpotential and Kähler potential \[17\]. We will use the two descriptions to calculate the potential at one-loop. In both cases, we assume small detunings, so that the 4d curvature, $1/L$, is small compared to the 4d Planck mass, $M_4$, and work to leading order in $1/L$.

Starting from the 5d theory in section 2, we compute the shifts of the gravitino KK masses due to supersymmetry breaking. Because the contribution of a full KK supermultiplet to the Casimir energy vanishes for unbroken supersymmetry, these shifts allow us to derive the correction to the potential. Furthermore, these mass shifts are proportional to $1/L$, because, as explained above, supersymmetry can only be broken in the detuned case. To leading order in the 4d curvature we can therefore use flat-space propagators in the calculation, avoiding the complications of a full curved space calculation \[18, 19\].

In section 3 we turn to the 4d radion effective theory. The superpotential of this theory is proportional to $1/L$ \[17\] and is not corrected at one-loop. Therefore, to obtain the leading order contribution to the potential, only the tuned (RS) Kähler potential is needed. The tree-level Kähler potential was obtained in \[13, 17\] and one-loop corrections were calculated in \[20, 21, 22, 23\], so we can just use known results. The 4d radion theory also allows us to calculate the contribution of hypermultiplets to the potential. It would be interesting to incorporate hypermultiplets in the 5d supersymmetric theory, but we leave this for future work. Still, there is no reason to expect that the mere addition of hypermultiplets to the theory would break supersymmetry, and so it is safe to study them using the supersymmetric 4d effective theory. Again, only the RS Kähler potential is needed, and the 1-loop hypermultiplet contribution to this Kähler potential was discussed in \[23\].

## 2 Radion potential: KK calculation

We consider pure supergravity on a 5d orbifold, with AdS$_5$ bulk, and brane tensions detuned from the RS limit so that the 4d slices are AdS$_4$ \[5\]. (We summarize the elements of this theory in Appendix A for convenience.) For the 5d action to be supersymmetric, gravitino brane mass terms are required. Labeling the branes by 0 and $\pi$ according to their position in the 5th dimension, the brane mass terms, $\alpha_i$, are given by,

$$|\alpha_0|^2 = \frac{T}{T + T_0} = \frac{1}{4k^2L^2}, \quad \text{and} \quad |\alpha_\pi| = |\alpha_0|e^{k\pi R}, \quad (1)$$
where $T_0$ is the tension of the brane at zero, $L$ is the 4d radius of curvature, and $T \equiv 6M_5^3k$, with $M_5$ the 5d Planck scale and $k$ the 5d curvature. The distance between the branes, $R$, is determined by the brane tensions and $T$ [see eqn. (35)]. Note that supersymmetry gives an upper bound on the magnitudes of the two brane tensions.

When eqn. (1) is satisfied, the action preserves $N = 1$ supersymmetry locally at any point along the fifth dimension. Globally however, $N = 1$ supersymmetry is only preserved when solutions to the Killing spinor equations exist. This, in turn, requires that the phases of $\alpha_0$ and $\alpha_\pi$ are equal. When these phases differ, supersymmetry is spontaneously broken. Since the gravitino is charged under a gauged $U(1)$ subgroup of the $SU(2)_R$ symmetry of the bulk supergravity, we can rotate away the phase difference by turning on a non-zero VEV for the graviphoton field—the $U(1)$ gauge boson—in the fifth dimension. The condition for unbroken supersymmetry then becomes

$$|\alpha_\pi| e^{i\phi_\pi} = |\alpha_0| e^{i\phi_0} e^{i\kappa \pi (R - i\sqrt{6}B_5)} ,$$

where $B_5$ is the fifth component of the graviphoton. In this section we will set $B_5 = 0$ and work with the supersymmetry-breaking parameter

$$\phi = \phi_0 - \phi_\pi .$$

For unbroken supersymmetry, or $\phi = 0$, the contribution of each KK supermultiplet to the potential vanishes. A supersymmetry-breaking nonzero $\phi$ only affects the gravitini masses. Therefore, we can calculate the correction to the potential by considering only the gravitini KK tower,

$$\Delta V(\phi) = \Delta V_{\text{bosons}}(\phi) + \Delta V_{\text{fermions}}(\phi) = \Delta V_{\text{fermions}}(\phi) - V_{\text{fermions}}(\phi = 0) .$$

In order to calculate this correction, we need the KK gravitini masses, both in the supersymmetric case and with broken supersymmetry. In the tuned RS case, there is a pair of degenerate spin 3/2 states at each KK level. The degeneracy is lifted when the brane tensions are detuned. In Appendix B, we derive the gravitini mass shifts for small supersymmetry breaking, $\phi \ll 1$. To leading order in $1/L$, the two masses at each KK level are split by opposite amounts, and there is no correction to the potential. Expanding the masses to the next order in $1/L$ we find (see Appendix B)

$$m_{k}\pm = c_0^{(n)} \mp \frac{1}{kL} \left[c_1^{(n)} \pm c_{1,SB}^{(n)} \phi^2 \right] + \frac{1}{(kL)^2} \left[c_2^{(n)} \pm c_{2,SB}^{(n)} \phi^2 \right] + \mathcal{O}(\phi^4) .$$
The dimensionless coefficients $c$, which depend on $k$ and $R$, are given in Appendix B. In particular, $c_0^{(n)}$ is the mass of the $n$-th KK mode in the tuned RS model in units of $k$.

As expected, the mass shifts due to the supersymmetry breaking are proportional not just to $\phi^2$, but also to the 4d curvature $1/(kL)$. This reflects the fact that supersymmetry cannot be broken in the tuned case. We can therefore use flat-space propagators for the calculation. The curved-space propagators are the flat space ones plus $O(1/L^2)$ corrections. To leading order, if we keep these $1/L^2$ corrections in the propagators, we should keep just the zeroth order KK masses, and these give a zero result in (4).

We can now consider the contribution of the $n$-th KK mode to the Casimir energy. As argued above, this contribution is of the form (4), namely, the difference between the fermion contributions with and without supersymmetry breaking. Substituting the masses (5) we have

$$\Delta V = 4 \frac{1}{L^2} \phi^2 \times$$

$$\sum_n \int \frac{d^4p}{(2\pi)^4} \left[ 2 \frac{k^2 (c_0^{(n)})^2 c_1^{(n)} c_{1,SB}^{(n)}}{[p^2 + (m_0^{(n)})^2]^2} - \frac{c_0^{(n)} c_{2,SB}^{(n)}}{p^2 + (m_0^{(n)})^2} - \frac{c_1^{(n)} c_{1,SB}^{(n)}}{p^2 + (m_0^{(n)})^2} \right],$$

up to $1/L^4$ terms. Here $m_0^{(n)} = kc_0^{(n)}$. Note that this correction is linear in the supersymmetry-breaking mass shifts, and therefore in the supersymmetry-breaking scale. This can be seen clearly in equation (4). One power of $1/L$ in this equation comes from the fact that the correction is proportional to the 4d curvature, while the remaining $\phi^2/L$ comes from the supersymmetry breaking scale. However, the supersymmetry breaking scale must also involve the warp factor $exp(-k\pi R)$ because it is a non-local effect. Therefore, the correction to the potential can only be significant for small warping. Furthermore, for small $k$ we approach flat space, where the classical potential for the radion vanishes, so radiative corrections may be comparable to the tree level terms. We will therefore choose

$$\frac{1}{L} \ll k \ll \frac{1}{R}.$$  

In terms of the 5d scales, this corresponds to taking $k \ll M_5$, with the ratios $T_0/T, T_\pi/T$ held fixed, so that the radius $R$ is fixed [see eqn. (35)] and $1/(kL)$ is a small number.

With this choice we can calculate the correction (4) analytically. We then
As expected, the correction is cut-off by the first KK mass, which is roughly \(1/R\), and goes to zero for zero 4d curvature, for which the supersymmetry-breaking shifts vanish. Apart from the overall \(1/(kL)^2\), the correction does not involve \(k\): it appears at zeroth order in the warping \(kR\). This is not surprising, because, as mentioned above, the Casimir energy is non-zero in flat space in the presence of supersymmetry-breaking.

Indeed, the result \((8)\) reproduces the Casimir energy of flat orbifold models, with supersymmetry broken by brane superpotentials \([25, 26]\), in the limit that the brane superpotentials are small. In the flat orbifold models of \([25]\), the brane superpotentials can be arbitrary. Here they are dictated by supersymmetry, and proportional to the detuning, \(1/(kL)\), which we take to be small [see eqn. \((11)\)]. Note that for small warping, the brane terms can only differ by a phase, \(\alpha_\pi \sim e^{i\phi}\alpha_0\). The comparison to flat orbifold models is in fact non-trivial, because the physics is quite different. In particular the KK spectrum \((5)\) is different from the spectrum of flat orbifold models. Effectively however, the KK contributions to the potential take the same form in both cases.

Thus, for small warping, the one-loop correction to the potential is important. We will return to this point in the next section, where we discuss the behavior of the one-loop improved potential starting from the 4d radion theory.

We note that, while we could only calculate \(\Delta V\) analytically to leading order in \(kR\), we can compute it numerically for any warping, using eqns. \((60)-(64)\) for the gravitino mass shifts.

At this point, we can deduce the mass of the graviphoton field \(b\) (defined in terms of \(B_5\) according to \((38)\)). As mentioned above, a non-zero \(\phi\) can be rotated away by a VEV of \(b\), which is a modulus of the classical theory. Given also that \(b\) is periodic, with period \(2/(3k)\) \([17]\), we can plausibly replace \(\phi \to \phi - 3\pi k b\) in eqn. \((8)\). This guess will be borne out by the analysis of the next section.
3 Radion potential: 4d radion theory

We can also evaluate the correction to the energy using the 4d radion theory following the approach of [7]. This will allow us to obtain the potential for finite values of the supersymmetry breaking phase $\phi$ and to derive its dependence on the graviphoton zero-mode. It will also allow us to include the effect of bulk matter fields.

The effective 4d action for the radion was found in [17] by matching to the 5d theory at tree-level. The theory has the superpotential\footnote{We rescale the superpotential and Kähler potential of [17] as $K \to K + 3M^2_4 \ln(1 - e^{-2k\pi R})$, $W \to \frac{W}{(1 - e^{-2k\pi R})^{3/2}}$.},

$$W = \frac{1}{1 - e^{-2k\pi R}} \frac{M^2_4}{L} \left(1 - e^{i\phi} e^{k\pi R} e^{-3k\pi T}\right), \quad (9)$$

where $T$ is the radion superfield, whose scalar component is $r + ib$, where $r$ is the radion and $b$ is the graviphoton Wilson line (see Appendix A for their definitions in terms of the 5d theory).

The superpotential (9) is proportional to $1/L$, and vanishes when the brane tensions are tuned. Indeed, this superpotential is essentially the sum of the two brane superpotentials $\alpha_0$ and $\alpha_\pi$, “weighted” by the appropriate warp factor. In the tuned case, $\alpha_0 = \alpha_\pi = 0$ and the superpotential vanishes.

If we are only interested in the tree-level potential to leading order in the 4d curvature, we can therefore use the RS Kähler potential

$$K = -3M^2_4 \ln \left(\frac{1 - e^{-k\pi(T + \bar{T})}}{1 - e^{-2k\pi R}} \right), \quad (10)$$

neglecting $\mathcal{O}(1/(M_4 L)^2)$ terms.

With this Kähler potential and superpotential, the order parameter for supersymmetry breaking is [17]

$$D_T W \propto (1 - e^{i(\phi - 3k\pi b)}), \quad (11)$$

so that supersymmetry is broken for $3k\pi b \neq \phi$. However, it is easy to verify that the tree-level potential is independent of the phase in (11). This is a very peculiar theory: Its Kähler potential is that of no-scale supergravity, and similarly its potential is $b$-independent. But whereas the no-scale model has a constant superpotential, and supersymmetry broken everywhere, here
the superpotential is $b$-dependent, and supersymmetry remains unbroken for $3k\pi b = \phi$.

At the loop level, the superpotential is not corrected. This is not quite the familiar "non-renormalization" of the superpotential—in fact, the superpotential (9), which is obtained by matching to the 5d theory at the KK scale, involves $M_4^2$, $R$, $L$, and $k$, all of which are physical, renormalized quantities. To obtain the supersymmetric low-energy radion theory at one-loop, one should again match to the 5d theory, this time at one-loop. As mentioned above, the superpotential (9) is essentially the sum of the two brane-superpotentials, weighted by the warp superfield $\exp(-3k\pi T)$. Since the brane superpotentials are not renormalized, and the warping is dictated by the symmetry of the 5-dimensional space, the superpotential (9) remains unchanged.

To compute the potential at one-loop we therefore just need the one-loop correction to the Kähler potential. Furthermore, to leading order in $1/L$, the Kähler potential is just the RS Kähler potential, which depends only on the combination $T + \bar{T}$.

$$V = e^{K/M_4^2} |W_0|^2 M_4^2 \left\{ M_4^2 K^{-1}_{TT} \left[ \frac{K_T}{M_4^2} + e^{\pi k(R-3r)} (3\pi k - \frac{K_T}{M_4^2}) \right] \right\}^2 - 3 \left[ 1 - e^{\pi k(R-3r)} \right]^2 - 4e^{\pi k(R-3r)} \sin^2 \frac{3\pi k b - \phi}{2} \left[ \frac{K_T}{K_{TT}} (3\pi k - \frac{K_T}{M_4^2}) + 3 \right] ,$$

(12)

where $W_0 = M_4^2 / ((1 - e^{-2\pi R})L)$. Writing $K = -3M_4^2 \log(\Omega/3M_4^2)$, the supersymmetry-breaking, or $b$-dependent part of the potential simplifies further,\footnote{At tree-level, this holds to all orders in $1/L$. At the loop-level, the Kähler potential will have terms proportional to $1/L^2$. Such terms are suppressed compared to the terms we discuss here by $1/(M_4 L)^2$.}

$$\Delta V = -4 \cdot 3^4 |W_0|^2 M_4^4 e^{-2\pi k R} \frac{\Omega'' + \Omega'''}{\Omega^2 (\Omega' \Omega - (\Omega')^2)} \sin^2 \frac{3\pi k b - \phi}{2} ,$$

(13)

where the primes denote derivatives with respect to $k\pi T$\footnote{With a slight abuse of notations we use double prime to refer to the derivative with respect to $T$ and $\bar{T}$. Since $\Omega = \Omega(T + \bar{T})$, this is the same as the second derivative with respect to $T$.}.
\( \Omega' + \Omega'' = 0 \), so we can set \( \Omega = \Omega_{\text{tree}} \) everywhere else to obtain, up to terms of the order \( \mathcal{O}(1/(M_4L)^4) \),

\[
\Delta V = 4e^{2\pi kR} \frac{1}{L^2} (\Delta \Omega' + \Delta \Omega'') \sin^2 \frac{3\pi kb - \phi}{2},
\]

(14)

where \( \Delta \Omega \) is the one-loop correction to \( \Omega \).

The potential is therefore extremized for either zero or maximal supersymmetry breaking. Which one of these is the minimum/maximum depends on the one-loop correction to the Kähler potential. At the minimum/maximum of the potential, the \( b \) mass-squared is given by

\[
m_b^2 = \pm 3^5 e^{-2k\pi R} |W_0|^2 M_4^2 \frac{\Omega' + \Omega''}{(\Omega\Omega'' - \Omega')^2} \]

\[
= \pm 3e^{2k\pi R} \frac{|W_0|^2}{M_4^6} (1 - e^{-2\pi kR})^4 (\Delta \Omega' + \Delta \Omega''),
\]

(15)

where we again used the fact that \( \Omega' + \Omega'' = 0 \) at tree level and substituted \( \Omega = \Omega_{\text{tree}} \) everywhere else.

With pure gravity in the bulk, the one-loop correction to the Kähler function is given by the following integral \[23\],

\[
\Delta \Omega_{\text{gravity}} = \frac{k^2 a^2}{4\pi^2} \int_0^\infty dy \, y \log \left( 1 - \frac{K_1(ya_\pi)I_1(y)}{K_1(ya_\pi)I_1(y)} \right),
\]

\[
a_\pi = e^{-k\pi(T + \bar{T})}.
\]

(16)

The \( b \)-dependent potential can then be written as an integral over a combination of modified Bessel functions, and evaluated numerically for any value of \( kR \). Alternatively, since we are interested in small warping, with

\[
\frac{1}{L} \ll k \ll \frac{1}{R},
\]

(17)

we can expand the integrand for \( \exp(-2\pi kR) \sim 1 \). The leading order term in this limit is \[21, 23\]

\[
\Delta \Omega = -\frac{\zeta(3)}{4\pi^4} \frac{1}{(T + \bar{T})^2}.
\]

(18)

6The next term in the expansion is \# \frac{k}{T + \bar{T}} \), where the coefficient can be calculated using (18).
Using the superpotential (9) we then find,

$$\Delta V = -\frac{3\zeta(3)}{8\pi^6} \frac{1}{(kL)^2} \frac{R^2}{r^6} \sin^2 \left(\frac{3\pi kb}{2} - \frac{\phi}{2}\right),$$

(19)

which matches (8) for $b = 0$ and small $\phi$. As we mentioned at the end of the previous section, the one-loop, supersymmetry breaking contribution to the potential appears at zeroth order in $kR$. In contrast, the tree-level potential near $r = R$ is order $(kR)^2$, because it should vanish in the flat space limit. Near the extrema with $r = R$ we then have

$$\frac{\Delta V}{V_{\text{tree}}} = \frac{\zeta(3)}{8\pi^6} \frac{1}{(kR)^2} \frac{1}{(M_4 R)^2} \frac{1}{x^6} \sin^2 \frac{3\pi kb - \phi}{2},$$

(20)

so that the loop suppression is partly compensated by inverse powers of $kR$. Thus, for small values of $kR$, the quantum correction significantly modifies the radion potential and cosmological constant.

Let us therefore examine the full potential. For small $k$, the tree plus loop potential is given by,

$$V = \frac{3M_4}{L^2} \left[1 - 2x - \frac{\zeta(3)}{8\pi^6} \frac{1}{(M_4 R)^2} \frac{1}{x^6} \sin^2 \frac{3\pi kb}{2}\right],$$

(21)

where

$$x = \frac{r}{R}.$$  

(22)

As we saw above, the one-loop correction with pure gravity is always negative. Supersymmetry is unbroken for $b = 0$. This vacuum is a stable saddle point with a positive mass squared for the radion, and a negative mass squared for $b$, which is still above the Breitenlohner-Freedman bound \[27\],

$$m_b^2 = -\frac{9\zeta(3)}{8\pi^4} \frac{1}{(M_4 R)^2} \frac{1}{L^2} > -\frac{9}{4L^2}.$$  

(23)

The 4d cosmological constant and radion VEV are not modified in this vacuum\(^7\).

For $b = 1/(3k)$, supersymmetry is maximally broken. Surprisingly, when the loop correction is sufficiently small, the potential has two extrema in the

\(^7\)Note that we implicitly imposed this matching condition in eqn. 18 in order to determine the non-calculable contributions to $\Delta \Omega$. 

10
region \( r < R \). One is a minimum, while the other is a saddle point, which is stable for a small range of \( kR \). In either supersymmetry-breaking vacuum, the effect of the quantum corrections is to lower the 4d cosmological constant.

We can get a positive contribution to the potential for a sufficient number of bulk hypermultiplets\(^8\). These correct the Kähler function by

\[
\Delta \Omega_{\text{hyper}} = N_H \frac{k^2 a^2}{8\pi^2} \int_0^\infty dy \, y \log \left( 1 - \frac{I_{[c+1/2]}(ya_{\pi}) K_{[c+1/2]}(y)}{K_{[c+1/2]}(ya_{\pi}) I_{[c+1/2]}(y)} \right),
\]

where \( c \) is related to the bulk mass of the hypers, and \( N_H \) denotes the number of hypermultiplets. For the special case \( c = 1/2 \), this contribution coincides with eqn (16), up to a factor of \(-1/2\). Thus, with \( N_H > 2 \) such hypermultiplets, the supersymmetry breaking contribution is positive, and for small warping,

\[
\Delta V = \left( \frac{N_H}{2} - 1 \right) \frac{3\zeta(3)}{8\pi^6} \frac{1}{(kL)^2} \frac{R^2}{r^6} \sin^2 \left( \frac{3\pi kb}{2} - \frac{\phi}{2} \right).
\]

For a few hypermultiplets\(^9\), maximal supersymmetry breaking is always a stable saddle point of the potential, since, again, the stability bound \(27\) is satisfied

\[
m_b^2 > -\frac{9}{4L_{\text{eff}}^2}.
\]

where \( L_{\text{eff}} \) is the net 4d curvature at the new minimum of the potential. We then have a very interesting situation. At tree level, the radius is stabilized due to the small detuning of the two brane tensions. For maximal supersymmetry breaking, the negative tree-level potential can be reduced with the radion and \( b \) both stable. In principle, with a suitable choice of \( kR \), the net 4d cosmological constant can be made arbitrarily small. As long as this net cosmological constant is negative, the new vacuum is stable, because eqn. (26) is satisfied. For phenomenological purposes however, there is no need to set the net cosmological constant strictly to zero. Once the MSSM is embedded into the model, it would contribute a cosmological constant of order \( \tilde{m}^4 \), where \( \tilde{m} \) is the typical MSSM soft mass. This mass would be proportional to the supersymmetry-breaking scale \( L^{-1} \). So the relevant question

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\(^8\)The contribution of vector supermultiplets is negative \(7\).

\(^9\)Note that for two hypermultiplets with the special mass \( c = 1/2 \), the potential remains flat to leading order in our expansion.
is whether the sum of the classical and quantum contributions to the potential that we considered can be made smaller or comparable to the MSSM contribution. While detailed model building is beyond the scope of this paper, it seems that in the minimal framework we are considering, with just \( c = 1/2 \) hypermultiplets, this cannot be achieved. This is related to the fact that both the supersymmetry-breaking scale and the cosmological constant are proportional to a single scale, \( L^{-1} \), so that the form of the potential is determined by a single dimensionless parameter, namely, \( kR \).

We close this section with a comment regarding the periodicity of the potential. So far we have written the potential in terms of \( b \), which has mass dimension \(-1\). Switching to the canonically normalized field,

\[
b_c = \frac{\pi k e^{-k\pi R}}{1 - e^{-2k\pi R} M_4 b},
\]

the period is roughly

\[
\frac{\pi e^{-k\pi R}}{1 - e^{-2k\pi R} M_4},
\]

which is always above the KK scale. In particular, in the limit \( kR \ll 1 \), the period of \( b_c \) is \( M_4/(3kR) \), which is much larger than the Planck scale. Thus, we can only use the 4d effective theory to study small fluctuations of \( b \) around either the supersymmetric or supersymmetry-breaking extremum.

## 4 Conclusions and Outlook

In this paper we studied a supersymmetric orbifold model with detuned brane tensions. We examined the KK gravitino spectrum, and calculated the shifts of the KK masses due to detuning and supersymmetry breaking. We then calculated the one-loop correction to the potential, using both the KK spectrum and the low-energy radion theory.

Throughout we take the detuning to be small, so that the 4d curvature is the lowest scale in the problem. In addition, we concentrate on the small warping case, since in this case the loop suppression of the Casimir energy is partly compensated by powers of \( kR \). We note that higher loops will be suppressed by the usual loop factors compared to the one-loop contribution. The parameter \( kR \) only appears in the ratio of the one-loop correction to the tree-level potential. As we explained, the reason is that the loop correction
remains finite as we decrease the warping, while the tree-level potential is proportional to the 5d curvature.

We found that both the radion and graviphoton are stabilized at one loop, with supersymmetry either unbroken or maximally broken. In the presence of a few bulk hypermultiplets, supersymmetry breaking gives a significant positive contribution to the potential. In the minimal framework we considered, the form of the potential is determined by the parameter $kR$. Embedding the MSSM into this framework, the typical soft mass would be of order $L^{-1}$ at most. Then, even though the classical and loop contribution can almost cancel, the net 4d cosmological constant is never smaller than the typical cosmological constant generated by the MSSM. It would be interesting to study models with hypermultiplets of different bulk masses, where the potential is more complicated and there is more freedom in the choice of parameters.

While we have only studied here the two-brane detuned system, it would be interesting to explore the supersymmetric single-brane model, with detuned brane tension. This model is a supersymmetric generalization of the Randall-Karch model, in which 4d gravity emerges through an ultra-light graviton KK mode. The supersymmetric version of this theory therefore looks like 4d massive supergravity in AdS$_4$. While the mass of the ultra-light “graviton” is below the AdS$_4$ curvature, it would be interesting to understand the structure of this theory from a purely theoretical point of view.

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A  The framework

In this appendix we present the elements of the theory, summarizing the results of [5, 6, 12, 17] for completeness. The bulk action is the supersymmetric AdS$_5$ action,

\[ S_{\text{bulk}} = M_5^3 \int d^5x \sqrt{G} \left[ -\frac{1}{2} R + 6k^2 + i \bar{\Psi}_M^j \Gamma^{MNK} D_N \Psi_K^i \right. \]

\[ \left. -\frac{3}{2} k \vec{q} \cdot \vec{\sigma}_i^j \bar{\Psi}_M^j \Sigma^{MN} \Psi_N^j - \frac{1}{4} F_{MN} F^{MN} - i \frac{\sqrt{6}}{16} F_{MN} (2 \bar{\Psi}_M^i \bar{\Psi}_N^i + \bar{\Psi}_P^i \Gamma^{MNPQ} \Psi_Q^i) \right. \]

\[ \left. - \frac{1}{6\sqrt{6}} \epsilon^{MNPQK} F_{MN} F_{PQ} B_K + \frac{\sqrt{6}}{4} k \vec{q} \cdot \vec{\sigma}_i^j B_N \bar{\Psi}_M^i \Gamma^{MKN} \Psi_K^j \right] , \]

where $M_5$ is the 5d Planck scale, and $k$ is the 5d bulk cosmological constant. The unit vector $\vec{q}$ defines the gauged $U(1)$ subgroup of $SU(2)_R$ and we will set it to $(0, 0, 1)$ for simplicity. The graviphoton, $B_M$, is the gauge boson of this $U(1)_R$. Because of the 5d curvature, the gravitino is charged under the $U(1)_R$. Note that a non-zero $B_5$ gives a non-zero mass to the 4d gravitino.

We also define for convenience:

\[ T = 6M_5^3 k. \]  

(28)

Apart from brane tensions, the brane action contains gravitino mass terms,

\[ S_{\text{brane}} = - \int d^4x dx_5 e_4 \left[ T_0 + 2\alpha_0 (\psi_{m1} \sigma^{mn} \psi_{n1} + h.c.) \right] \delta(x_5) \]  

\[ - \int d^4x dx_5 e_4 \left[ T_\pi - 2\alpha_\pi (\psi_{m1} \sigma^{mn} \psi_{n1} + h.c.) \right] \delta(x_5 - R) . \]  

(29)

In order for the bulk plus branes action to be locally supersymmetric, the gravitino brane mass terms must satisfy

\[ T_i = \frac{1 - |\alpha_i|^2}{1 + |\alpha_i|^2} T . \]  

(30)

Therefore, the absolute values of the brane tensions cannot be bigger than the bulk cosmological constant:

\[ |T_{0,\pi}| \leq T . \]  

(31)

The inequality is saturated for the tuned RS case.
The resulting 4d theory can be either AdS\textsubscript{4} or Mink\textsubscript{4}, with the metric,
\[
 ds^2 = a^2(x_5)\hat{g}_{\mu\nu}dx^\mu dx^\nu - dx_5^2 ,
\] (32)
where \( \hat{g}_{\mu\nu} \) denotes the standard AdS\textsubscript{4} or Mink\textsubscript{4} metric in Poincare coordinates, and where the warp factor is given by
\[
a(x_5) = e^{-kx_5} + \frac{1}{4k^2L^2}e^{kx_5} .
\] (33)
Here \( L \) is the 4d curvature radius, given by
\[
 \frac{1}{4k^2L^2} = \frac{T - T_0}{T + T_0} .
\] (34)
The brane distance is
\[
 R = \frac{1}{2k\pi} \ln \frac{(T + T_0)(T + T_\pi)}{(T - T_0)(T - T_\pi)} ,
\] (35)
and the 4d Planck scale is
\[
 M_4^2 = \frac{M_5^3}{k}(1 - e^{-2k\pi R}) .
\] (36)
Note that the requirement of local supersymmetry excludes the dS\textsubscript{4} case, since the latter implies \( T_0 > T \), in conflict with (30).

The condition (30) restricts just the magnitudes of the gravitino brane mass terms, but not their phases. When \( \alpha_0, \alpha_\pi \) have different phases, there is no solution to the Killing spinor equations (valid in the bulk and on the branes) and supersymmetry is spontaneously broken. Actually, this is only true for \( B_5 = 0 \). Allowing some constant \( B_5 \) background, the condition for unbroken supersymmetry becomes,
\[
 \alpha_\pi = \alpha_0 e^{k\pi(R - i\sqrt{6}B_5)} .
\] (37)
Writing \( \alpha_i = |\alpha_i|e^{i\phi_i} \), we see that the phase difference
\[
 \phi = \phi_0 - \phi_\pi ,
\]
can be compensated by a shift of \( B_5 \), as expected from the \( U(1)_R \) invariance of the 5d theory.
For the tuned case, \( T_0 = -T_\pi = T \), we recover RS1, with zero 4d curvature, and with the radius \( R \) undetermined. Furthermore, the gravitino mass terms \( \alpha_0 \) and \( \alpha_\pi \) vanish in this case, so that equation (37) is always satisfied, and supersymmetry is preserved.

To conclude this appendix we relate the brane distance \( R \) and \( B_5 \) to the radion superfield \( T = r + ib \) used in section 3: the graviphoton Wilson line is defined as
\[
b = \frac{1}{\sqrt{6\pi}} \int_{-\pi R}^{\pi R} B_5 dx_5 ,
\]
and the radion
\[
r = \frac{1}{2\pi R} \int_{-\pi R}^{\pi R} \sqrt{G_{55}} dx_5 .
\]

\section{The KK mass shift}

In this appendix we derive the mass shifts of the gravitini KK modes by expanding around the tuned RS case. We will do so by solving the appropriate Schrödinger problem for the gravitini. To write down the gravitini equations of motion, we rescale the 4d coordinates as
\[
\tilde{x}^\mu = \frac{2T}{T + T_0} x^\mu .
\]

At the end of the day, we will therefore need to rescale the masses we find by
\[
m \rightarrow \frac{2T}{T + T_0} m .
\]

The gravitini equations of motion are then
\[
\frac{d\hat{a} b_1}{dr} + \frac{3k}{2} \hat{a} b_1 + \frac{d\hat{a}}{dr} b_1 = \hat{m} \hat{b}_2 \tag{42}
\]
\[
\frac{d\hat{a} b_2}{dr} - \frac{3k}{2} \hat{a} b_2 + \frac{d\hat{a}}{dr} b_2 = -\hat{m} \hat{b}_1 , \tag{43}
\]
where \( b_1, b_2 \) are gravitini wave-functions and
\[
\hat{a}(r) = \frac{T + T_0}{2T} a(r) , \tag{44}
\]
\footnote{With this rescaling, we reproduce the conventions of 6.}
with the boundary conditions:

\[ b_2(r = 0) = \alpha_0 b_1(r = 0) \]  \hspace{1cm} (45)

\[ b_2(r = \pi R) = \alpha_0 b_2(r = \pi R) . \] \hspace{1cm} (46)

Note that \( R \) in these equations is the radius given by eqn. (35).

To convert these equations to a Schrödinger-like problem we go to coordinates:

\[ z(r) = \frac{2}{kA} \arctan \left( \sqrt{\frac{T - T_0}{T + T_0}} e^{kr} \right) , \] \hspace{1cm} (47)

where

\[ A^2 = 1 - \frac{T_0^2}{T^2} , \] \hspace{1cm} (48)

and rescale

\[ b_i(z) = \frac{1}{kA} \sin(kAz) \psi_i(z) . \] \hspace{1cm} (49)

The \( \psi_2 \) equation of motion then takes the form

\[ \left( -\partial_z^2 + V_2(z) \right) \psi_2(z) = m^2 \psi_2(z) , \] \hspace{1cm} (50)

with

\[ V_2(z) = -\frac{3k^2A^2 \cos(kAz)}{2 \sin^2(kAz)} + \frac{9k^2A^2}{4 \sin^2(kAz)} \frac{1}{\sin^2(kAz)} . \] \hspace{1cm} (51)

The second wavefunction is completely determined by equation (42) as

\[ \ddot{\psi}_1 = -\frac{\dot{\psi}_2}{m} + \frac{5}{2mz} \psi_2 + \frac{\psi_2 d \ln a}{m} dz . \] \hspace{1cm} (52)

For completeness we also write down the profile function in these new coordinates:

\[ a(z) = \frac{A}{\sin(kAz)} ; \] \hspace{1cm} (53)

Since we want to perturb around the tuned case, it is useful at this stage to define the following quantities

\[ \epsilon \equiv \sqrt{\frac{T - T_0}{T}} \] \hspace{1cm} (54)

\[ \delta \equiv \sqrt{\frac{T - T_\pi}{T}} , \] \hspace{1cm} (55)
and we require both $\epsilon \ll 1$ and $\delta \ll 1$. Note also that
\[ \delta = e^{\pi kR} \epsilon, \] (56)
and we keep $R$ finite.

Expanding the potential $V_2(z)$ in powers of $\epsilon$, we solve the equations of motion for $\psi_2$. Writing $\psi_2$ as an expansion in $\epsilon$, we get
\[ \psi_2 = G(z^{1/2}J_1(mz) + \epsilon^2 \frac{k^2}{m^2} z^{3/2} J_2(mz) - \epsilon^2 \frac{k^2}{m^2} z^{1/2} J_1(mz) + H(J \leftrightarrow Y), \] (57)
where $G$ and $H$ are constants, which must be determined from the boundary conditions. Using the matching condition (52) we also have
\[ \psi_1 = \bar{G}(\sqrt{z}J_2(mz) + \epsilon^2 \frac{k^2}{m^2} \sqrt{z}J_2(mz) - \frac{\epsilon^2}{3} \frac{k^2}{m^2} z^{3/2} J_1(mz)) + \bar{H}(J \leftrightarrow Y). \] (58)
Expanding also $R$, $\alpha_0$ and $\alpha_\pi$ we get a system of four linear homogeneous equations for $G$ and $H$. The masses can be computed from the requirement that the characteristic determinant of the system should vanish.

Finally we should rescale the results by (51). Writing
\[ m^{(n)}_{\pm} = m^{(n)}_0 \pm \frac{1}{kL} c^{(n)}_1 \pm \frac{1}{kL} c^{(n)}_{1,SB} \phi^2 + \frac{1}{(kL)^2} c^{(n)}_2 + \frac{1}{(kL)^2} c^{(n)}_{2,SB} \phi^2, \] (59)
we find that the shift due to detuning, with no supersymmetry breaking, is $n$-independent,
\[ c^{(n)}_1 = \frac{1}{2}, \] (60)
while the leading order shift due to supersymmetry breaking is
\[ c^{(n)}_{1,SB} = -\frac{1}{4} \frac{e^{k\pi R} \Delta^{(n)}_{12} \Delta^{(n)}_{21}}{(e^{k\pi R} \Delta^{(n)}_{12} + \Delta^{(n)}_{21})^2}. \] (61)
where
\[ \Delta^{(n)}_{ij} \equiv \begin{vmatrix} J_i(c^{(n)}_0) & Y_i(c^{(n)}_0) \\ J_j(e^{k\pi R} c^{(n)}_0) & Y_j(e^{k\pi R} c^{(n)}_0) \end{vmatrix}, \] (62)
where $c^{(n)}_0 = m^{(n)}_0 / k$. Note that $\Delta^{(n)}_{11} = 0$. At the next order
\[ c^{(n)}_2 = -\frac{1}{12 \sqrt{2}} \left[ \frac{e^{3k\pi R} \Delta^{(n)}_{12} + 4 \Delta^{(n)}_{21} + 3 e^{k\pi R} \Delta^{(n)}_{12}}{e^{k\pi R} \Delta^{(n)}_{12} + \Delta^{(n)}_{21}} c^{(n)}_0 - 3 \sqrt{2} c^{(n)}_0 - \frac{3}{c^{(n)}_0} \right], \] (63)
and
\[ c_{2,SB}^{(n)} = -\frac{e^{k\pi R} \Delta_{12}^{(n)} \Delta_{21}^{(n)} (-3e^{k\pi R} \Delta_{12}^{(n)} - 3\Delta_{21}^{(n)} + 2e^{k\pi R} c_0^{(n)} \Delta_{22}^{(n)})}{4\sqrt{2}(e^{k\pi R} \Delta_{12}^{(n)} + \Delta_{21}^{(n)})^3} \frac{1}{c_0^{(n)}}. \]  

(64)

At the lowest level, one gravitino state is projected out by the orbifolding, and we are left with a gravitino with two degrees of freedom, and a radion fermion with two degrees of freedom, both of mass
\[ m = \frac{1}{L}. \]  

(65)

When supersymmetry is broken, the radion fermion is eaten and the 4d gravitino has four degrees of freedom with mass
\[ m_{zm} = \frac{1}{L} + \frac{1}{2L} \frac{e^{2\pi kR}}{(e^{2\pi kR} - 1)^2} \phi^2. \]  

(66)

Using (66) we can define
\[ c_1^{(0)} = 1, \quad c_{1,SB}^{(0)} = \frac{e^{2\pi kR}}{2(e^{2\pi kR} - 1)^2}. \]  

(67)

Since the mass of this mode is zero in the tuned case, its contribution to the vacuum energy starts at order $1/L^2$, so we don’t need higher corrections.

**C Casimir energy in flat limit**

Here we evaluate the correction (6) for small warping. In this limit,
\[ c_0^{(n)} = \frac{n\pi}{e^{\pi kR} - 1} + \frac{3(e^{\pi kR} - 1)}{8n\pi e^{\pi kR}} + \mathcal{O}((e^{\pi kR} - 1)^3), \]  

(68)

where the leading term is just the flat space result, and
\[ c_1^{(n)} c_{1,SB}^{(n)} \rightarrow -\frac{e^{\pi kR} \phi^2}{4(e^{\pi kR} - 1)^2} + \mathcal{O}(1) \]  

(69)

\[ c_0^{(n)} c_{2,SB}^{(n)} \rightarrow \mathcal{O}(1). \]  

(70)
Thus the leading order of (6) gives the dominant contribution. Substituting these results in (6) we have:

$$\Delta V = -\frac{1}{(kL)^2} \frac{2}{(\pi R)^2} \phi^2.$$

(71)

$$\int \frac{d^4p}{(2\pi)^4} \left[ \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{(p^2 + (n/R)^2)^2} - \sum_{n=1}^{\infty} \frac{(n/R)^2}{(p^2 + (n/R)^2)^2} + \frac{1}{4p^2} \right].$$

The last term is the contribution of the zero mode, and cancels the quadratic divergence from the first two sums. Performing the sums by standard techniques we obtain:

$$\Delta V = -\frac{3\zeta(3)}{2^5\pi^2} \frac{1}{(\pi R)^4} \frac{1}{(kL)^2} \phi^2.$$

(72)

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