Reflection-less device allows electromagnetic warp drive

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Abstract

One of the striking properties of artificially structured materials is the negative refraction, an optical feature that known natural materials do not exhibit. Here, we propose a simple design, composed of two parallel layers of materials with different refraction indices $n_1 = -n_2$, that constructs perfect reflection-less devices. The electromagnetic waves can tunnel from one layer to the other, a feature that resembles a truncation of the physical space leading to an electromagnetic warp drive. Since the refractive indices do not require any large values, this method demonstrates for the first time the practical feasibility of guiding electromagnetic fields in complete absence of reflection phenomena and without degradation of transmission efficiency at all.
The recent advancement of meta-materials has extended electromagnetic properties and led to promising applications in disparate fields from telecommunications and semi-conductor engineering to medical imaging, defense and cloaking devices [1, 2, 3, 4, 5, 6, 7, 8, 9]. Meta-materials show new properties like the negative refraction, an optical feature that common materials do not exhibit [10]. Veselago showed that a material characterized by negative values of effective permittivity and permeability exhibits several reversed physics phenomena like the reversal of Snell’s law. However, and in spite of its physical interest, the application of negative refraction index has been mainly confined to the construction of perfect lens [11] and the modeling of perfect cloaking devices [12, 13]. Here, we propose a simple design that constructs perfect reflection-less devices where electromagnetic waves can tunnel from one side to the other side of the device. This perfect electromagnetic tunneling feature resembles a truncation of the physical space leading to a warp drive. The device consists of two parallel layers of materials with different refraction indices. While the first layer has a value \( n_1 \), the second layer consists of \( n_2 = -n_1 \). Furthermore, the absolute value of the refraction index can take any value, allowing practical applications using values close to 1. This simple device displays a striking reflection-less feature, with electromagnetic tunneling effect, that astonishingly has not been reported before. Since the values for the refractive indices do not require any large value, this method demonstrates for the first time the practical possibility to guide electromagnetic fields in complete absence of reflection phenomena. While perfect couplers have only been suggested using epsilon-near-zero (ENZ) medium [14], here we show that any pair of positive and negative refractive index can guide electromagnetic waves through a material without degradation of transmission efficiency at all.

Figure 1(a) shows a classical reflection phenomena. An electromagnetic wave (light rays) strikes a material and a reflected ray is observed. The law of reflection shows that the angle of incidence equals the angle of reflection \( \theta_i = \theta_r \). Physical laws show that reflection of light always occurs when light travels from a material of a given refractive index into a medium with a different refractive index. In general, only a fraction of the light is refracted since there is an unavoidable reflection from the interface. In sharp contrast, here we show that the proposed device is able to completely suppress the reflected light, allowing a perfect transmission of electromagnetic waves from one boundary (B1) to the other boundary (B3) (see Figure 1(b)). The only requirement is to have two media with different sign of refraction index but with the same absolute value as follows:

\[
n(x, y, z) = \begin{cases} 
  n_1 & (-a < x < 0) \\
  n_2 & (0 < x < a)
\end{cases}
\]  

where \( n_1 \) and \( n_2 \) are the refractive indices of the left and right-hand side materials with the condition:

\[
n_1 = -n_2. 
\]  

Here, we put the device in the uniform media (\( n = 1 \), for example, vacuum or air.) The device has three discontinuous boundaries of refractive index: the boundary between vacuum and the material \( n_1 \) (i.e., B1 \((x = -a)\)), the boundary between \( n_1 \) and \( n_2 \) (i.e., B2 \((x = 0)\)), and the boundary between \( n_2 \) and vacuum (i.e., B3 \((x = a)\)) (see Fig. 1). It is well known that a boundary between two media having different refractive index causes a reflection, which leads to degrade the transmission efficiency (Fig. 1(a)). However, here we show that in spite that our device has three boundaries (B1, B2 and B3), where refraction indices change, the resulting reflections are surprisingly and completely canceled each other out. This striking result is rooted in the combination of plus and minus refraction index [12, 13].

Let us consider the general case of the boundary between two different materials 1 and 2. Let \( \epsilon_1 \) and \( \mu_1 \) (resp. \( \epsilon_2 \) and \( \mu_2 \)) be permittivity and permeability of material 1 (resp. material 2). Then, the refractive index of material 1 (resp. material 2) is given by \( n_1 = \sqrt{\epsilon_1 \mu_1} \) (resp. \( n_2 = \sqrt{\epsilon_2 \mu_2} \)). By considering the standard electromagnetic theory [15], for the impedance matching case \((\sqrt{\frac{\epsilon_1}{\epsilon_2}} = \sqrt{\frac{\mu_1}{\mu_2}})\), the transmission ratio is given by

\[
T(\theta, \phi) = \frac{E_t}{E_i} = \frac{2 \cos \theta}{\cos \theta + \cos \phi}
\]
where $\theta$ is the incoming angle of light and $\phi$ is the outgoing (i.e., refraction) angle of light. Similarly, the reflection ratio is given by

$$R(\theta, \phi) = \frac{E_r}{E_i} = \frac{\cos \theta - \cos \phi}{\cos \theta + \cos \phi}$$

(4)

First, we analyze the discontinuity of the boundary $B_2$ (i.e., the boundary between positive and negative refraction indices). By simply applying Snell’s law to Eq. 2, we obtain $\theta = -\phi$. In this case, by considering Eq. (4), we can easily see that there is no reflection at all for all incident angles and polarizations at $B_2$.

Similarly, the reflection ratio is given by

$$R(\theta, \phi) = \frac{E_r}{E_i} = \frac{\cos \theta - \cos \phi}{\cos \theta + \cos \phi}$$

(4)

Secondly, let us consider that the light enters from vacuum to material $n_1$ through boundary $B_1$ (see continuous black line in Fig. 1(b)). At a first look at Eq. (4), it seems that there should be some reflections at boundary $B_1$. This contribution is given by Eq. (4). However, we will show that this reflection is completely canceled out by summing up all the reflections that take place at the boundary $B_3$.

Next, let us consider the case when the light enters from left vacuum to material $n_1$ through the boundary $B_1$. This light ray is bent at the interface $B_2$ and is reflected from the interface $B_3$. Then, it returns to the interface $B_1$ and goes out to left vacuum through the interface $B_1$ (see red dashed line in Fig. 1(b)). In this case, the ratio is given by

$$A_1 = T(\theta, \phi)R(\phi, \theta)T(\phi, \theta)$$

(5)

However, the light rays can be reflected again from the interface $B_1$ and go back two times between the interfaces $B_1$ and $B_3$, and finally come back to vacuum through the interface $B_1$ (see green and blue dashed-dotted lines in Fig. 1(b)). For the case that the light rays go back two times, the ratio is given by $A_2 = T(i, j)R^2(j, i)T(j, i)$. In fact, this process leads to an infinite series of contribution to the reflection ratio. Thus, more generally, for going back $n$ times, the ratio can read as $A_n = T(i, j)R^{2n-1}(j, i)T(j, i)$. Therefore, the total sum of all contributions is

$$A_{tot} = T(\theta, \phi)\left\{\sum_{i=1}^{\infty} R^{2n-1}(\phi, \theta)\right\}T(\phi, \theta)$$

$$= \frac{\cos \phi - \cos \theta}{\cos \theta + \cos \phi}$$

(6)

Unexpectedly, the sum of all contributions cancels the original reflection $R(\theta, \phi)$ of Eq. (4). Therefore, there is no reflection at boundary $B_1$ at all.

We can also discuss about the transmission ratio. The leading contribution comes from the case that the light enters from vacuum to the material $n_1$ and goes through $n_1$ and $n_2$ and finally goes out through the interface $B_3$ (see continuous black line in Fig. 1(b)). This contribution of transmission ratio is given by

$$B_1 = T(\theta, \phi)T(\phi, \theta)$$

(7)

By following a similar argument, we can illustrate another cases. The light enters into $n_1$ through $B_1$ and is reflected from $B_3$. Then, it goes back to $B_1$ and is reflected again. Finally, it returns to right vacuum through $B_3$. This contribution is given by $B_2 = T(\theta, \phi)R^2(\phi, \theta)T(\phi, \theta)$. Indeed, there are also infinite series of reflections and refractions that contribute to the transmission ratio. The total transmission ratio reads as

$$B_{tot} = T(\theta, \phi)\left\{\sum_{i=0}^{\infty} R^{2n}(\phi, \theta)\right\}T(\phi, \theta)$$

$$= 1$$

(8)

which shows that the light is completely transmitted from $B_1$ to $B_2$.

On the other hand, it is worth mentioning how the phase velocity behaves in the device. The negative refractive index has opposite sign of phase velocity to the positive refractive index. The phase shift in the positive refractive index region $n_1$ is exactly canceled out by the phase shift in the negative refractive index region $n_2$. Therefore, in the proposed device, the phase seems to jump instantaneously from one side to another.
In conclusion, there is no reflection for all the interfaces B1, B2 and B3 and phase seems to jump from one side (B1) to the other side (B3) instantaneously. The device behaves as a perfect electromagnetic tunneling device. More concretely, the physical space between B1 and B2 seems to be truncated leading to a warp drive.

In order to demonstrate our theoretical findings, we have conducted a computer simulation using commercial software COMSOL. We simulated incident plane waves at a given angle of incidence and measured the $E_z$ component. We set two computational experiments to show the reflection-less effect. The first simulation contains two materials with the same refraction index ($n_1 = n_2 = 2$). Incoming waves strike the medium, change trajectory according Snell’s law and are finally transmitted (see Fig. 2(a)). The subtraction of the incident wave reveals the existence of a reflected wave as predicted by the physical laws (see Fig. 2(c)). It is worth noticing that the $E_z$ component of the left hand side vacuum in Fig. 2(a) is deformed by a combination of incident wave and reflected wave. Now, let us consider a second experiment. Here, although we also have two media, the refractive index has the same absolute value but different sign ($n_1 = 2$ and $n_2 = -2$). This difference leads to a striking result. As shown in Fig. 2(b), the incident and transmitted waves are identical. This is highlighted in Fig. 2(d), because when an incident wave is removed, no reflected wave is observed in the left vacuum, showing no degradation of transmission efficiency at all. We show the incoming wave in vacuum without device in Fig. 3 for comparison purposes.

It is worth mentioning that our device has also a so-called electromagnetic tunneling effect. Recent works have shown that a material with electric permittivity close to zero can behave like a perfect coupler [3], where electromagnetic waves can tunnel through a material. This effect also leads to an absence of degradation efficiency. However, these proposed theoretical analyses and experiments are based on close to zero values for permittivity. In contrast, our device behaves exactly like an electromagnetic tunneling device for any range of permittivity values. Figs. 2(b) and 2(d) show that the electromagnetic wave performs a warp drive from one side to another.

Beyond purely electromagnetic devices, reflection also occurs at the surface of transparent media, such as glasses. Many real devices common in our daily life have screens and surfaces that cannot escape from reflection phenomena. Several examples are iphone, television and PC screens, car glasses and optical devices in general. In particular, for higher incident angles, the reflection is so intense that can make the screen unreadable even though we use anti-glare screen protectors. It seems impossible to eliminate the effect of all reflection for any angle and any polarization in isotropic media. Although a design for a reflection-less screen seems impossible, our findings shows that a reflection-less isotropic device is possible simply by combining a positive and negative refraction index. Amazingly, at each boundary, there is no reflection at all for any arbitrary and polarization angle. The structure and design of our proposed design is very simple, and its practical realization is anticipated since it only requires one condition: any pair of positive and negative refraction index with the same absolute value. In addition to the mentioned screen and glasses applications for visual range of wavelength, the striking cancellation of reflection phenomena could be very fundamental to the design of many novel applications in electromagnetic devices from microwave passive devices and defense systems to solar panels and wireless communications wherever distortions arising from reflections degrade the transmission efficiency.

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Figure 1: (a) The trajectory of light rays through media with refractive index $n_1 = n_2 = 2$. It represents a typical example of refraction and reflection in conventional materials. (b) The trajectory of light rays in our proposed device through media with refractive index $n_1 = 2$ and $n_2 = -2$. The line colors correspond with those of (a), but having different light path. The black solid line represents an incident light ray entering materials $n_1$, $n_2$. The light ray is reflected from B1, bent at B2 and refracted at B3. Then the light ray (red dashed line) is reflected again from B3 and goes out to left vacuum through B1. Next, the light ray is reflected again from B1 (green dash-dotted line) and a certain fraction of the light goes out to right vacuum through B3. The remaining light (blue dash-dotted line) is reflected at B3 again and goes back to the left vacuum through B1. All the reflected and outgoing light rays start at the same point respectively.
Figure 2: In all cases, the incoming plane waves, defined as TE mode and with plane of incidence parallel to $xy$-plane, enter a material from left to right with incident angles ($\pi/3$). (a) $n_1 = 2$, $n_2 = 2$ case. $E_z$-component is represented as an incident light wave entering the device from left to right. A wave distortion in left vacuum is observed due to the combination of incident wave and reflected wave. (b) The proposed device with $n_1 = 2$, $n_2 = -2$. $E_z$-component is represented as in (a), but with opposite sign of $n_2$. The wave pattern of the left vacuum is exactly the same as that of the right vacuum. It shows a perfect tunneling effect or warp drive (i.e., no reflection and perfect transmission of light). (c) $n_1 = 2$, $n_2 = 2$ case. The scattered $E_z$-component of case (a) is represented (i.e., this is the same as (a), but we remove incident wave and show only scattered component). It highlights the existence of reflection in the left vacuum. (d) $n_1 = 2$, $n_2 = -2$. The scattered $E_z$-component of case (b) is represented (i.e., this is the same as (b), but we remove incident wave and show only scattered component). There is no reflected wave at all in the left vacuum.
Figure 3: For comparison with Fig. 2, $E_z$-component is represented for case $n_1 = n_2 = 1$ (i.e., there is no device. The simulation settings are the same as Fig. 2.