Large Amplitude Harmonic Vibration under Pulse Doppler Measurement

N Hermawan¹,²*, T Ishii³, Y Yamakoshi⁴ and Y Saijo¹

¹Graduate School of Biomedical Engineering, Tohoku University, 6-6-12 Aramaki Aza Aoba, Aoba-ku, Sendai, Miyagi 980-8579, Japan
²Biomedical Engineering Department, Institut Teknologi Sepuluh Nopember, Gedung A, B, C, dan AJ, Kampus ITS Keputih, Sukolilo, Surabaya, Jawa Timur 60111, Indonesia
³Frontier Research Institute for Interdisciplinary Sciences, Tohoku University, 6-3 Aramaki Aza Aoba, Aoba-ku, Sendai 980-8578, Japan
⁴Faculty of Science and Technology, Gunma University, 1-5-1 Tenjin-cho, Kiryu-shi, Gunma 376-8515, Japan

*norma.hermawan@bme.its.ac.id

Abstract. A flow velocity or tissue motion can be measured by pulse Doppler method. However, when the method is applied to a sinusoid motion or harmonic vibration, the estimation result is deviated. In this paper, the generalized theory of pulse Doppler measurement to the sinusoid motion is explained and the relation between the estimation deviation and the real value is given. Furthermore, a special case of binary velocity estimation is expanded and simulated with different parameters. This simulation result suggested that different selection of parameters may fundamentally affect the output of pulse Doppler estimation.

1. Introduction
The elasticity of biological tissue with a negligible viscosity is proportional to the square of a shear wave velocity that propagates in it. This fundamental principle underlies a method of elastography by using Doppler technique [1]. In later research, a method for estimating the harmonic vibration from pulse Doppler in time domain was proposed [2]. As one of underlying foundation of the color Doppler shear wave elastography [3], it is interesting to reformulate the harmonic vibration observation by pulse Doppler method, by which, the more comprehensive explanation of binary pattern phenomenon is given. Understanding the theory provides a fundamental consideration when determining parameters in the variants of color Doppler elastography.

The original derivation of the color Doppler elastography indices that the pulse Doppler autocorrelator ensemble size, the shear wave displacement amplitude and shear wave frequency must be controlled in a certain condition. An investigation found that the resulted binary pattern at suggested condition is a special case of a more general situation. In this paper, the general case of color Doppler shear wave elastography is described and simulation result of some possible parameters are presented. The clinical application of the method in elastography includes a potential development of diagnostic tool for identifying disease that is represented by distinct elasticity such as cancer or Adhesive Capsulitis (AC) of the shoulder [4]. It also can be used in tandem with other method such as B-mode speckle tracking analysis [5] to improve the robustness of the disease detection.
2. Observation of sinusoid displacement by pulse Doppler

In a pulse Doppler method, an object velocity is estimated from phase difference of a successive ultrasonic wave at a given duration. Suppose an ultrasound sequence is transmitted with an interval of every pulse $\Delta t = \frac{1}{PRF}$. The phase of ultrasound echo is shifted by a moving reflector, and it is used to estimate its velocity by

$$\dot{v} = \frac{c}{4\pi f_0 \Delta t} \Delta \phi_i ,$$  \hspace{1cm} (1)

where $c$ is sound speed and $f_0$ is the ultrasound frequency. A quadrature detector receives a RF signal in the form of

$$D_i = I_i + jQ_i ,$$  \hspace{1cm} (2)

where

$$I_i = a \cos \left( \phi_0 + \frac{2\pi f_0}{c} 2v_i \Delta t \right) \quad \text{and}$$

$$Q_i = a \sin \left( \phi_0 + \frac{2\pi f_0}{c} 2v_i \Delta t \right) .$$  \hspace{1cm} (3)

The phase difference between the $i^{th}$ and the $(i + 1)^{th}$ received ultrasonic waves to solve equation (1) is obtained by,

$$\Delta \phi_i = \arg (D_{i+1}D_i^*) .$$  \hspace{1cm} (4)

The $\arg$ function is analogous to the arctangent function in four quadrants. When it is applied to a successive ultrasonic wave with ensemble size of $N$ by autocorrelation technique [6], the average phase shift is

$$\Delta \phi_i = \begin{cases} 
\tan^{-1} \frac{E_U}{E_L} + \pi & \text{if } E_U > 0, E_L < 0, \\
\tan^{-1} \frac{E_U}{E_L} & \text{if } E_L > 0, \\
\tan^{-1} \frac{E_U}{E_L} - \pi & \text{if } E_U < 0, E_L < 0,
\end{cases}$$  \hspace{1cm} (5)

where

$$E_U = \sum_{i=-\frac{N}{2}}^{\frac{N}{2}} I_i Q_{i+1} - I_{i+1}Q_i , \quad \text{and}$$

$$E_L = \sum_{i=-\frac{N}{2}}^{\frac{N}{2}} I_{i+1}I_i + Q_{i+1}Q_i .$$  \hspace{1cm} (6)

Given an autocorrelator integration duration of $T_a$, the pulse repetition interval is denoted by a relation,

$$\Delta t = \frac{T_a}{N} .$$  \hspace{1cm} (7)

When an autocorrelation algorithm is used to measure a sinusoid displacement
\[ \xi_i = \xi_m \sin(\omega_b t + \phi_b) \]  
(8)

with a maximum displacement \( \xi_m \) and the shear wave angular frequency \( \omega_b = 2\pi f_b = \frac{2\pi}{T_b} \), the \( i \)th phase of the echo is

\[ \phi_i = \phi_0 + \frac{2\pi f_0}{c} \ast 2\xi_i \]  
(9)

Therefore, the quadrature detector receives RF signals with

\[ I_i = a \cos\left(\phi_0 + \frac{4\pi f_0}{c} \ast \xi_i\right) \quad \text{and} \]
\[ Q_i = a \sin\left(\phi_0 + \frac{4\pi f_0}{c} \ast \xi_i\right) \]  
(10)

\( \Delta \phi_i \) is obtained by substituting (10) for \( I \) and \( Q \) in the equation (6). After completing the derivation, the flow velocity of sinusoid vibration in pulse Doppler system is obtained as a function of autocorrelator integration duration \( T_a \) and shear wave angular frequency \( \omega_b \). In this derivation, the ensemble size \( N \) of the autocorrelation does not matter when it is normalized by the pulse repetition interval. Hence,

\[ \bar{\nu} = \frac{2\xi_m \cos(\omega_b t + \phi_b) \sin\left(\omega_b \frac{T_a}{2}\right)}{T_a} \]  
(11)

Since a flow velocity is a derivative of displacement, it can be calculated from the pulse Doppler measurement by

\[ \nu = \bar{\nu} \frac{T_a \omega_b}{2 \sin\left(\omega_b \frac{T_a}{2}\right)} \]  
(12)

![Figure 1. Velocity index \( \beta_v \) as a function of normalized integration time \( T_a/T_b \).](image-url)
A ratio of the real vibration velocity and the measured velocity may be denoted by an index

$$\beta_v = \frac{\bar{v}}{v} = 2 \sin \left( \frac{\omega_b T_a}{2} \right) ,$$

(13)

which is a function of normalized integration time $T_a/T_b$. Since the ultrasound phase shift is calculated by arctangent function, it is mandatory to keep $-\pi/2 < \Delta \phi_i < \pi/2$ to avoid aliasing. Therefore, the constraint for maximum shear wave displacement is

$$\omega_b \xi_m < \frac{\lambda}{8\Delta t} .$$

(14)

The velocity of small amplitude sinusoid vibration appears lower by pulse Doppler estimation. The Figure 1 reveals graphical plot of velocity index $\beta_v$ in respect to the normalized autocorrelator integration time $T_a/T_b$ as a representation of equation (13). The graph suggested that the higher PRF will result in better approximation to the velocity. Figure 2(a) shows sinusoid velocity plot as a derivative of sinusoid displacement with an amplitude $\xi_m = 1.2664 \times 10^{-5}$ and frequency $\omega_b = 200$ Hz. A pulse Doppler estimation of a sinusoid velocity is displayed in the Figure 2(b) for different values of PRF, when $N = 4$. It can be viewed from the image that the estimated velocity approaches zero if normalized integration $T_a/T_b$ is an integer number.

In the binary pattern color Doppler elastography, the estimated velocity is either zero or maximum velocity [3]. A close inspection to the original derivation gives an obvious insight that the method operates at integer $T_a/T_b$ value. The output is produced since the vibration amplitude is larger than $\lambda/8$. The given ensemble size $N$ of multiple of 4 and PRF is $4f_b$ implies that the method is not a unique situation. By varying the value of ensemble size and the ratio of shear wave frequency and the PRF, the other forms of distinct pattern are possible.
3. Simulation
In this simulation, different values of ensemble size are simulated to obtain different patterns. The ratio of $T_a/T_b$ is kept at integer value and the displacement amplitude is varied from $\lambda/64$ to $\lambda$.

3.1. Odd multiple of Ensemble Size
The selection of PRF and ensemble size in this simulation is formulated by $PRF = kf_b$ and $N = (2n-1)k$ respectively, where $n$ is an integer number and $k$ is 3, 4, 5, 6, 7 and 8. The simulation result in the Figure 3 shows the different patterns of velocity for different amplitude of displacement. It can be seen that the binary patterns are produced at even value of $k$.

3.2. Even multiple of Ensemble Size
On the other simulation when the ensemble size $N$ is chosen at even value $(2n)k$, a similar pattern is shown. The result in the Figure 4 is clearly an inverted version of the previous simulation in 3.1. In this simulation the same parameter of $PRF = kf_b$ is used.

4. Conclusion
In most situations, the velocity estimation error can be corrected by the velocity index $\beta_v$ factor based on a priory knowledge of $T_a$ and $T_b$. In a condition when the ratio $T_a/T_b$ is an integer the displacement amplitude is small, the pulse Doppler produce zero flow velocity estimation. At higher amplitude, the estimation produces different result due to aliasing. This fact brings up further possibilities and has been exploited on the binary pattern elastography [3]. It is revealed for the first time in this paper that the previously presented binary pattern is only one among many others. Different pattern of velocity estimations may appear, depending on the selection of PRF scale in respect to the vibration frequency $f_b$, as well as ensemble size $N$. This result implies that careful consideration about ensembles size must be taken such as in the application of color Doppler shear wave elastography.

Figure 3. Velocity estimation at different vibration amplitude and odd multiple of ensemble size.
Figure 4. Velocity estimation at different vibration amplitude and even multiple of ensemble size.

References

[1] Yamakoshi Y, Sato J and Sato T 1990 Ultrasonic imaging of internal vibration of soft tissue under forced vibration. *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control* **32**(2) 45-53.

[2] Huang SR, Lerner RM and Parker KJ 1992 Time domain Doppler estimators of the amplitude of vibrating targets. *The Journal of the Acoustical Society of America* **91**(2) 965-974.

[3] Yamakoshi Y, Kasahara T, Iijima T and Yuminaka Y 2015 Shear Wave Wavefront Mapping Using Ultrasound Color Flow Imaging. *Ultrasonic Imaging* **37**(4) 323-340.

[4] Wu CH, Chen WS and Wang TG 2016 Elasticity of the Coracohumeral Ligament in Patients with Adesive Capsulitis of the Shoulder. *Radiology* **278**(2) 458-464.

[5] Hermawan N, Fujiwara M, Hagiwara Y and Saijo Y 2020 Visualization of Shoulder Ligaments Motion by Ultrasound Speckle Tracking Method 42nd Annual International Conference of the IEEE Engineering in Medicine & Biology Society (EMBC) 2084-87.

[6] Kasai C, Namekawa K, Koyano A and Omoto R 1985 Real-time Two-dimensional Blood Flow Imaging Using an Autocorrelation Technique. *IEEE Transactions on Sonic and Ultrasonic SU-32*(3) 458-464.