Single Spin Asymmetry in Electroproduction of Scalar or Pseudoscalar Meson Production off the Scalar Target

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Abstract
We discuss the electroproduction of scalar (0\textsuperscript{++}) or pseudoscalar (0\textsuperscript{−+) meson production off the scalar target. The most general formulation of the differential cross section for the 0\textsuperscript{−+} or 0\textsuperscript{++} meson production process involves only one or two hadronic form factors, respectively, on a scalar target. The Rosenbluth type separation of the differential cross section provides the explicit relation between the hadronic form factors and the different parts of the differential cross section in a completely model-independent manner. The absence of the single spin asymmetry for the pseudoscalar meson production provides the benchmark for the experimental data analysis. The measurement of the single spin asymmetry for the scalar meson production may provide a clear criterion whether the leading-twist formulation of the generalized parton distribution is in agreement with the most general formulation of the hadronic tensor.

Keywords: electromagnetic meson production, single spin asymmetry

While the virtual Compton scattering process is coherent with the Bethe-Heitler process, the meson electroproduction process offers a unique experimental determination of the hadronic structures for the study of QCD and strong interactions. In particular, coherent electroproduction of the scalar (0\textsuperscript{++}) or pseudoscalar (0\textsuperscript{−+}) meson production off a scalar target (e.g. the 4\textsuperscript{He nucleus) provides an excellent experimental terrain to discuss the fundamental nature of the hadron physics without involving many complications from the spin degrees of freedom. We discuss in this work two benchmark examples (0\textsuperscript{++} vs. 0\textsuperscript{−+}) that provide a unique interface between the theoretical framework and the experimental measurements of physical observables.

To establish the notation for the electroproduction of meson \textbf{m} off the scalar target \textbf{h}, we write

\[ e(k) + h(P) \rightarrow e'(k') + h'(P') + m(q'), \]

and the virtual photon momentum is defined to be \( q = k - k' \), see Fig. \textsuperscript{[1]}. In the target rest frame (TRF) presented in Ref. \textsuperscript{[1]}, the differential electroproduction cross section is given by

\[ d\sigma \equiv \frac{d^5\sigma}{dydxdt\phi_k\phi_{q'}} = \kappa\langle |\mathcal{M}|^2 \rangle, \]

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where

\[ \kappa \equiv \frac{1}{(2\pi)^3} \frac{y x}{32 Q^2 \sqrt{1 + \left(\frac{2Mx}{Q}\right)^2}}. \]  

Here, \( y = P \cdot q/P \cdot k \), \( t = (P - P')^2 \) and \( x = Q^2/(2P \cdot q) = Q^2/(2M \nu) \) with \( Q^2 = -q^2 \), the target mass \( M \) and the virtual photon energy \( \nu \) in TRF. For the one-photon-exchange process, the transition amplitude \( \mathcal{M} \) can be expressed as the invariant product of the leptonic current \( eL^\mu = e \bar{u}(k', s') \gamma^\mu u(k, s) \) and the hadronic current \( eJ^\mu \) mediated by the photon propagator, i.e., \( \mathcal{M} = e^2 L \cdot J/q^2 \). As discussed in Ref. \[1\], by using the reduced three momenta product obtained from the \( q \cdot J = 0 \) relation, we get the following invariant amplitude squared

\[
\langle |\mathcal{M}|^2 \rangle = \left( \frac{e^2}{q^2} \right)^2 \mathcal{L}^{\mu\nu} \mathcal{H}_{\mu\nu}
\]

\[ = \left( \frac{e^2}{q^2} \right)^2 \left[ \frac{2q^2}{\epsilon - 1} \langle |\tau_{fi}|^2 \rangle + 2i \lambda \epsilon^{\mu\nu\alpha\beta} k_\alpha k'_\beta J^\dagger_\mu J_\nu \right], \tag{4} \]

where the hadronic tensor is given by

\[ \mathcal{H}_{\mu\nu} = J^\dagger_\mu J_\nu \tag{5} \]

and the leptonic tensor including the electron beam polarization \( \lambda \) is given by

\[ \mathcal{L}^{\mu\nu} = q^2 \Lambda^{\mu\nu} + 2i \lambda \epsilon^{\mu\nu\alpha\beta} k_\alpha k'_\beta \tag{6} \]

with \( \Lambda^{\mu\nu} = g^{\mu\nu} + \frac{2}{q^2} (k^\mu k'^\nu + k'^\mu k^\nu) \). Here, \( \mathcal{L}^{\mu\nu} \) and \( \mathcal{H}_{\mu\nu} \) are contracted to yield Eq. (4) with

\[
\langle |\tau_{fi}|^2 \rangle = \frac{1}{2} (|H_x|^2 + |H_y|^2) + \frac{\epsilon}{2} (|H_x|^2 - |H_y|^2) + \epsilon_L |H_z|^2 - \sqrt{\frac{1}{2}} \epsilon_L (1 + \epsilon) (H_x^* H_z + H_z^* H_z), \tag{7} \]
where $H_i = J_i(i = x, y, z)$, $\epsilon = \frac{A_{xx} - A_{yy}}{A_{xx} + A_{yy}} = -\frac{2M^2 x^2 + 2Q^2 (y - 1)}{2M^2 x^2 + 2Q^2 (y^2 - 2y + 2)}$ and $\epsilon_L = \frac{Q^2}{M^2} \epsilon$ as given by Eq. (16) of Ref. [1]. The last terms in Eqs. (4) and (6) for the case of a polarized electron beam with $\lambda = \pm 1$ depending on the electron spin are related with the single spin asymmetry (SSA). Due to the absence of the interference with the Bethe-Heitler process, the SSA of the meson electroproduction is a direct measure of any asymmetry within the hadronic tensor, i.e., $H_{\mu\nu} \neq H_{\nu\mu}$.

In parallel to the Levi-Civita symbol $\epsilon^{\mu\nu\alpha\beta}$, we have recently introduced in Ref. [2] the backbone of the Compton tensor defined by

$$d_{\mu\nu\alpha\beta} = g_{\mu\nu} g_{\alpha\beta} - g_{\mu\alpha} g_{\nu\beta},$$

(8)

which may be used to construct pieces of “DNA” for the virtual Compton scattering as well as the meson electroproduction by contracting with the three basis four vectors such as $q, \bar{P} = P + P'$ and $\Delta = P - P' = q' - q$. The most general hadronic tensor structures obtained by our “DNA” method in virtual Compton scattering off the scalar target are in complete agreement with the previous results by Metz [3] and further comparisons with other methods [4] and results of general hadronic tensors for the nucleon target [5] are underway. In the present work of the meson electroproduction off the scalar target, we note that the hadronic current for the pseudoscalar $(0^+)$ meson production is governed by a single hadronic form factor defined by

$$J_{PS}^\mu = F_{PS} \epsilon_{\mu\nu\alpha\beta} q_\nu \bar{P}_\alpha \Delta_\beta,$$

(9)

while the hadronic current for the scalar $(0^{++})$ meson production involves two hadronic form factors given by

$$J_S^\mu = (S_q q_\alpha + S_{\bar{P}} \bar{P}_\alpha) d_{\mu\nu\alpha\beta} q_\beta \Delta_\nu,$$

(10)

where the hadronic form factors $F_{PS}, S_q$ and $S_{\bar{P}}$ are dependent on the Lorentz invariant variables $Q^2, x$ and $t = \Delta^2$. Redefining the scalar hadronic form factors $F_1$ and $F_2$ for the later convenience as

$$F_1 = S_q - S_{\bar{P}},$$
$$F_2 = S_{\bar{P}},$$

(11)

we get the hadronic current for the scalar $(0^{++})$ meson production as

$$J_S^\mu = F_1 (q^2 \Delta^\mu - q^\mu q \cdot \Delta) + F_2 [(\bar{P} \cdot q + q^2) \Delta^\mu - (\bar{P}^\mu + q^\mu) q \cdot \Delta],$$

(12)

which reduces to the usual electromagnetic current $J^\mu \propto (P + P')^\mu$ for the case of no meson production, i.e. $q' = 0$. The electromagnetic current conservation is assured of course both for the electroproduction of pseudoscalar $(0^-)$ and scalar $(0^{++})$ mesons owing to $q_\mu J_{PS}^\mu = 0$ and $q_\mu J_S^\mu = 0$, respectively.

For the pseudoscalar meson production case, we should note that the SSA term is zero because, owing to the fact that only a single hadronic form factor occurs, the hadronic tensor
is symmetric:

\[ \mathcal{H}_{\mu\nu} = |F_{PS}|^2 \epsilon_{\mu\alpha\beta\gamma} \epsilon_{\nu\alpha'\beta'\gamma'} q^\alpha \tilde{P}^\beta \Delta_{\gamma} q^{\alpha'} \tilde{P}^{\beta'} \Delta'_{\gamma'} = \mathcal{H}_{\nu\mu}, \]  

and contracts with the antisymmetric leptonic tensor \(2i\lambda \epsilon^{\mu\nu\alpha\beta} k_\alpha k'_\beta\) for the SSA given by Eq. (11), i.e.

\[ \epsilon^{\mu\nu\alpha\beta} k_\alpha k'_\beta \mathcal{H}_{\mu\nu} = 0. \]  

The situation here is very different from the \(\pi^0\) electroproduction off a proton target in which several hadronic form factors are involved. The status of the data and phenomenology in the generalized parton distribution (GPD) approach of deeply virtual meson production (DVMP) on the nucleon has been reviewed in Ref. [6]. The GPD formulation has been applied to the deeply virtual Compton scattering (DVCS) process off the pion [7], as well as off nuclei up to spin-1 [9], and further refined off a spinless target [10]. The coherent vs. incoherent DVCS processes off the spin 0 nuclei have also been discussed with respect to the nuclear medium modification of hadrons in terms of the GPD formulation [11]. In clear distinction from the recent SSA measurement of DVCS off 4\(^4\)He [12], however, the meson electroproduction process discussed here doesn’t have any interference with the Bethe-Heitler process. As far as a single hadronic form factor governs the hadronic current, the SSA of the meson electroproduction should vanish in general regardless of the complexity in the hadronic form factor. We thus note that the SSA of the coherent pseudoscalar (e.g. \(\pi^0\)) meson electroproduction off the scalar target (e.g. the \(4^4\)He nucleus) vanishes due to the symmetry given by Eq. (14): i.e.,

\[ \frac{d\sigma_{PS}^{\lambda=+1} - d\sigma_{PS}^{\lambda=-1}}{d\sigma_{PS}^{\lambda=+1} + d\sigma_{PS}^{\lambda=-1}} = 0. \]  

Moreover, in the TRF kinematics [1] defining the azimuthal angle \(\phi\) between the leptonic plane and the hadronic plane taking the virtual photon direction as \(\hat{z}\)-direction, the hadronic current for the pseudoscalar \((0^-+)\) meson production given by Eq. (9) yields \(H_z = 0\) in Eq. (7). Regardless of the electron beam polarization \(\lambda\), the differential cross section for the pseudoscalar meson (e.g. \(\pi^0\)) production is thus given by

\[ d\sigma^{PS} = d\sigma^{PS}_U + d\sigma^{PS}_P \epsilon \cos 2\phi = d\sigma^{PS}_U (1 - \epsilon \cos 2\phi), \]  

where

\[ d\sigma^{PS}_U = -d\sigma^{PS}_P \]

\[ = \kappa e^4 |F_{PS}(Q^2, t, x)|^2 \sin^2 \theta \]

\[ = \frac{4M^2 x^2 + Q^2}{4M^2 x^2 (1 - \epsilon)} \left[ (4M^2 x^2 + Q^2) 
\right. \]

\[ \left. \left[ x^2 (t^2 - 4m^2 M^2) + Q^4 + 2Q^2 t x \right] \right], \]  

with the meson mass \(m\) and the lab angle \(\theta\) for the meson production in the hadronic plane. This provides the Rosenbluth type separation of the differential cross section for the electroproduction of the pseudoscalar meson, from which the pseudoscalar meson form factor \(F_{PS}(Q^2, t, x)\) may be extracted directly from the experimental data of the differential cross section if available.
For the scalar meson production case, however, the SSA term doesn’t vanish as there are two independent hadronic form factors \( F_1(Q^2, t, x) \) and \( F_2(Q^2, t, x) \) given by Eq. (12), which are complex in general. The differential cross section for the scalar meson production is given by

\[
d\sigma^S_X = d\sigma^S_U + d\sigma^S_P \epsilon \cos(2\phi) + d\sigma^S_L \epsilon_L + d\sigma^S_I \cos \phi \sqrt{\epsilon_L(1 + \epsilon)} + \lambda d\sigma^S_{SSA},
\]

where \( d\sigma^S_U = d\sigma^S_P \) and

\[
\begin{bmatrix}
d\sigma^S_U \\
d\sigma^S_L \\
d\sigma^S_I \\
d\sigma^S_{SSA}
\end{bmatrix} = 
\begin{bmatrix}
U_1 & U_2 & U_3 & 0 \\
L_1 & L_2 & L_3 & 0 \\
I_1 & I_2 & I_3 & 0 \\
0 & 0 & 0 & S_A
\end{bmatrix} 
\begin{bmatrix}
|F_1|^2 \\
|F_2|^2 \\
F_{12}^+ \\
F_{12}^-
\end{bmatrix}
\]

with \( F_{12}^\pm = F_1 F_2^* \pm F_2 F_1^* \). The matrix elements in Eq. (19) are obtained as follows:

\[
U_1 = \frac{\kappa e^4 \sin^2 \theta Q^2}{4M^2 x^2(1 - \epsilon)} (x^2 (t^2 - 4m^2 M^2) + Q^4 + 2Q^2 t x),
\]

\[
U_2 = \frac{\kappa e^4 \sin^2 \theta Q^2 (x - 1)^2}{4M^2 x^4(1 - \epsilon)} (x^2 (t^2 - 4m^2 M^2) + Q^4 + 2Q^2 t x),
\]

\[
U_3 = \sqrt{U_1 U_2},
\]

\[
L_1 = \frac{\kappa e^4 Q^4}{8M^2 x^2(1 - \epsilon)(4M^2 x^2 + Q^2)} (m^2 + Q^2 + t(2x - 1))^2,
\]

\[
L_2 = \frac{\kappa e^4 (m^2 (4M^2 x + Q^2) + Q^2 (4M^2 x + 2tx - 3t) - 4M^2 tx + Q^4)^2}{8M^2 x^2(1 - \epsilon)(4M^2 x^2 + Q^2)},
\]

\[
L_3 = \sqrt{L_1 L_2},
\]

\[
I_1 = \frac{\kappa e^4 I_c \tan \theta Q^2 (m^2 + Q^2 + t(2x - 1))}{2M^2 x^2(1 - \epsilon)(4M^2 x^2 + Q^2)},
\]

\[
I_2 = \frac{\kappa e^4 I_c \tan \theta (x - 1)}{2M^2 x^3(1 - \epsilon)(4M^2 x^2 + Q^2)} \times \left[ m^2 (4M^2 x + Q^2) + Q^2 (4M^2 x + 2tx - 3t) - 4M^2 tx + Q^4 \right],
\]

\[
I_3 = \frac{\kappa e^4 I_c \tan \theta}{4M^2 x^3(1 - \epsilon)(4M^2 x^2 + Q^2)} \left[ m^2 (4M^2 x^2 + Q^2 (2x - 1)) + Q^2 (4M^2 x^2 + 4tx^2 - 6tx + t) - 4M^2 tx^2 + Q^4 (2x - 1) \right],
\]

\[
S_A = \frac{\kappa e^4 \sin \theta \sin \phi}{2M x^2 y} (m^2 + Q^2 - t) \times \sqrt{Q^2(y - 1) + M^2 x^2 y^2 \sqrt{x^2 (t^2 - 4m^2 M^2) + Q^4 + 2Q^2 tx}},
\]
where \( I_c = 2M^2x^2(t - m^2) + Q^2x(2M^2x + t) + Q^4 \) and \( \cos \theta = \frac{I_c}{q \sqrt{(4M^2x^2 + Q^2)(x^2(t^2 - 4m^2M^2) + Q^4 + 2Q^2tx)}} \). Thus, the SSA of the coherent scalar meson electroproduction off the scalar target is given by

\[
\frac{d\sigma_S^{\Delta} + d\sigma_S^{\Delta}}{d\sigma_S^{\Delta} + d\sigma_S^{\Delta}} = \frac{d\sigma_S^{\Delta}S}{\sigma_S^{\Delta}S(1 + \epsilon \cos(2\phi)) + \sigma_S^{\Delta}L + \sigma_S^{\Delta} \cos \phi \sqrt{\epsilon_L}(1 + \epsilon)},
\]

which is proportional to \( F_1F_2^* - F_2F_1^* \). As \( F_1F_2^* - F_2F_1^* \neq 0 \) in general, the SSA of the scalar meson (e.g. \( f_0(980) \)) electroproduction is not expected to vanish. For the kinematic region where at least one of \( F_1 \) and \( F_2 \) develops the imaginary part, the SSA shouldn’t vanish. The nonvanishing SSA measured in DVCS off \( ^4\text{He} \) [12] indicates that the imaginary part of the hadronic amplitude is accessible in the current experimental regime. Therefore, it will be very interesting to compare the experimental data on the SSAs between the \( \pi^0 \) electroproduction and the \( f_0(980) \) electroproduction off the \( ^4\text{He} \) nucleus. We note that Eqs. (18) - (20) provide the Rosenbluth type separation of the differential cross section for the electroproduction of the scalar meson, from which the scalar meson form factors \( F_1(Q^2, t, x) \) and \( F_2(Q^2, t, x) \) can be directly extracted from the experimental data. In principle, the experimental data can reveal both the real part and the imaginary part of \( F_1(Q^2, t, x) \) and \( F_2(Q^2, t, x) \) through Eqs. (18) - (20) and the consistency with the SSA given by Eq. (21) can be checked for the kinematic region where any of these form factors is found to develop the imaginary part.

In contrast to our general formulation with the two independent hadronic form factors for the electroproduction of the scalar \((0^{++})\) meson, the leading twist GPD formulation yields a single GPD and thus provides the zero SSA, \( d\sigma_S^{\Delta}S = 0 \). The situation here is very different from the DVMP on the nucleon which involves more than one leading twist GPDs [6]. As discussed in our review [13], the original leading twist GPD formulations [14, 15, 16] are limited to the kinematic region \(|t| < < Q^2\). The leading twist formulation in Ref. [15] adopts a specific relation among the particle momenta given by \( q = q' - \zeta P \) or \( P' = (1 - \zeta)P \), where \( \zeta \) is the skewness in the GPD formulation [14] given by \( \zeta = \Delta^+/P^+ \). If we apply this leading twist relation \( q = q' - \zeta P \) to our general formulation given by Eq. (12), the scalar meson current gets reduced to

\[
J_S^L = \zeta(F_1 + F_2)(q^2P^\mu - q^\mu q \cdot P),
\]

where the two independent form factors merge together to yield effectively only one hadronic form factor that corresponds to a single leading twist GPD. Thus, the reduced formulation with a single hadronic form factor corresponding to a single leading twist GPD results in the symmetric hadronic tensor \( H_{\mu
u} = H_{\nu\mu} \) as in the case of the pseudoscalar meson electroproduction and yields the vanishing SSA as it contracts with the antisymmetric leptonic tensor \( 2i\lambda\epsilon^{\mu\nu\alpha\beta}k_\alpha k_\beta \). The coherent experimental measurement to judge whether the SSA of the scalar meson (e.g. \( f_0(980) \)) electroproduction off the scalar target (e.g. the \( ^4\text{He} \) nucleus) vanishes or not would provide a unique opportunity to distinguish between the leading twist GPD formulation and our general formulation presented in this work. In this respect, not only pseudoscalar but also scalar meson electroproduction measurements off a scalar target are highly desired to pin down the viable roadmap on the analyses of precision experimental data, e.g. from the
JLab 12 GeV upgrade. An exactly solvable hadronic model calculation is currently underway to explore the kinematic regions where all the hadronic form factors discussed here develop imaginary parts, and explicitly demonstrate the extraction of the hadronic form factors from our general formulation of the hadronic currents.

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References

[1] R. A. Williams, C.-R. Ji and S. R. Cotanch, Phys. Rev. C 46, 1617 (1990).
[2] C.-R. Ji and B.L.G. Bakker, PoS QCDEV2017, 038 (2017); B.L.G. Bakker and C.-R. Ji, Few Body Syst. 58, no.1, 8 (2017).
[3] A. Metz, Virtuelle Comptonstreuung und die Polarisierbarkeiten des Nukleons (in German), PhD thesis, Universität Mainz, 1997.
[4] G. Eichmann, C. S. Fischer and W. Heupel, Phys. Rev. D 92 (2015) 056006; G. Eichmann, “Baryon spectroscopy and structure with functional methods”, presentation in Light Cone 2018 at JLab, May 14-18, 2018.
[5] D. Drechsel, G. Knöchlein, A. Yu. Korchin, A. Metz and S. Scherer, Phys. Rev. C57 (1998) 941; R.Tarrach, Nuovo Cim A28, (1975) 409.
[6] L. Favart, M. Guidal, T. Horn and P. Kroll, arXiv:1511.04535v2 [hep-ph] April 11, 2018.
[7] A.V. Belitsky, D. Müller, A. Kirchner and A. Schäfer, Phys. Rev. D64 (2001) 116002.
[8] V. Guzey and M. Strikman, Phys. Rev. C 68 (2003) 015204.
[9] A. Kirchner and D. Müller, Eur. Phys. J. C32 (2003) 347.
[10] A.V. Belitsky and D. Müller, Phys. Rev. D79 (2009) 0141017.
[11] S. Liuti and S. K. Taneja, Phys. Rev. C 72, 034902 (2005).
[12] M. Hattawy et al. (CLAS Collaboration), Phys. Rev. Lett. 119, 202004 (2017).
[13] C.-R. Ji and B.L.G. Bakker, Int. J. Mod. Phys. E 22, No. 2 (2013) 1330002.
[14] X. D. Ji, Phys. Rev. Lett. 78 (1997) 610; Phys. Rev. D 55 (1997) 7114.

[15] A. V. Radyushkin, Phys. Lett. B 380 (1996) 417; A.V. Radyushkin, Phys. Rev. D 56 (1997) 5524.

[16] D. Müller, D. Robaschik, B. Geyer, F. M. Dittes and J. Horejsi, Fortsch. Phys. 42 (1994) 101.