Form factors and branching ratios of the FCNC

\[ B \to a_1 \ell^+ \ell^- \text{ decays} \]

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Abstract

We analyze the semileptonic \( B \to a_1 \ell^+ \ell^- , \ell = \tau, \mu, e \) transitions in the framework of the three-point QCD sum rules in the standard model. These rare decays governed by flavor-changing neutral current transition of \( b \to d \). Considering the quark condensate contributions, the relevant form factors as well as the branching fractions of these transitions are calculated.

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I. INTRODUCTION

The decays governed by flavor-changing neutral current (FCNC) transitions are very sensitive to the gauge structure of the standard model (SM) which provide an excellent way to test such a model. These decays, prohibited at the tree-level, take place at loop level by electroweak penguin and weak box diagrams. The FCNC transitions can be suppressed due to their proportionality to the small Cabibbo-Kobayashi-Maskawa matrix elements [1]. Among these, the FCNC semileptonic decays of the $B$ meson occupy a special place in both experimental measurements and theoretical studies for the precision test of the SM due to more simplicity.

So far, the form factors of the semileptonic decay $B \rightarrow a_1 \ell \nu$ have been studied via the different approaches such as the covariant light front quark model (LFQM) [2], the ISGW2 quark model [3], the constituent quark-meson model (CQM) [4], the QCD sum rules (SR) [5], and the light cone QCD sum rules (LCSR) [6]. However, the obtained results of these methods are different from each other.

In this work, we calculate the transition form factors of the FCNC semileptonic decays $B \rightarrow a_1(1260) \ell^+ \ell^- / \nu \bar{\nu}$ in the framework of the three-point QCD sum rules method (3PSR). Considering the transition form factors for such decays in the framework of different theoretical methods has two-fold importance:

1) A number of the physical observables such as branching ratio, the forward-backward asymmetry and lepton polarization asymmetry, which have important roles in testing the SM and searching for new physics beyond the SM, could be investigated.

2) These form factors can be also used to determine the factorization of amplitudes in the non-leptonic two-body decays.

On the other hand, any experimental measurements of the present quantities and a comparison with the theoretical predictions can give valuable information about the FCNC transitions and strong interactions in $B \rightarrow a_1 \ell^+ \ell^- / \nu \bar{\nu}$ decays.

The plan of the present paper is as follows: In Sec. II, we describe the sum rules method to calculate the form factors of the FCNC $B \rightarrow a_1$ transition. Section III is devoted to the numerical analysis of the form factors and branching ratio values of the semileptonic $B \rightarrow a_1$ decays, with and without the long-distance (LD) effects.
II. FORM FACTORS OF THE FCNC $B \to a_1$ TRANSITION IN 3PSR

To calculate the form factors of the FCNC $B \to a_1$ transition, within 3PSR method, we start with the following correlation functions constructed from the transition currents $J^V_\mu = \bar{d}\gamma_\mu(1 - \gamma_5)b$ and $J^T_\mu = \bar{d}i\sigma_{\mu\nu}q^\nu(1 + \gamma_5)b$ as follows:

$$\Pi^{V(T)}_{\mu\nu}(p^2, p'^2, q^2) = \int d^4x d^4y e^{-ipx}e^{ip'y}(0 \mid T[J^a_\nu(y)J^V(T)_\mu(0)J^{B\dagger}(x)] \mid 0), \tag{1}$$

where $J^B = \bar{u}\gamma_5b$, and $J^a_\nu = \bar{u}\gamma_\nu\gamma_5d$ are the interpolating currents of the initial and final meson states, respectively. In the QCD sum rules approach, we can obtain the correlation functions of Eq. (1) in two languages: the hadron language, which is the physical or phenomenological side, and the quark-gluon language called the QCD or theoretical side. Equating two sides and applying the double Borel transformations with respect to the momentum of the initial and final states to suppress the contribution of the higher states and continuum, we get sum rule expressions for our form factors. To drive the phenomenological part, two complete sets of intermediate states with the same quantum numbers as the currents $J^a_\nu$ and $J^B$ are inserted in Eq. (1). As a result of this procedure,

$$\Pi^{V(T)}_{\mu\nu}(p, p') = \frac{1}{p^2 - m_B^2} \frac{1}{p'^2 - m_{a_1}^2} \langle 0 \mid J^a_\nu | a_1 \rangle \langle a_1 | J^{V(T)}_\mu | B \rangle \langle B | J^{B\dagger} | 0 \rangle + \text{higher states}, \tag{2}$$

where $p$ and $p'$ are the momentum of the initial and final meson states, respectively. To get the transition matrix elements of the $B \to a_1$ with various quark models, we parameterize them in terms of the relevant form factors as

$$\langle a_1(p', \epsilon) \mid J^V_\mu \mid B(p) \rangle = \frac{1}{m_B + m_{a_1}} \left[ 2 F^V_A(q^2) i\varepsilon_{\mu\alpha\beta\epsilon}^* p^\alpha p'^\beta + F^V_0(q^2)(P.q)\epsilon^*_\mu \right. + \left. F^V_+(q^2)(\epsilon^*.p)P_\mu + F^V_-(q^2)(\epsilon^*.p)q_\mu \right],$$

$$\langle a_1(p', \epsilon) \mid J^T_\mu \mid B(p) \rangle = 2 F^T_A(q^2) i\varepsilon_{\mu\alpha\beta\epsilon}^* p^\alpha p'^\beta + F^T_0(q^2)(m_B^2 - m_{a_1}^2) \left[ \epsilon^*_\mu - \frac{1}{q^2}(\epsilon^*.q)q_\mu \right] + F^T_+(q^2)(\epsilon^*.p) \left[ P_\mu - \frac{1}{q^2}(P.p)q_\mu \right], \tag{3}$$

where $P = p + p'$ and $q = p - p'$. Also $m_{a_1}$ and $\epsilon$ are the mass and the four-polarization vector of the $a_1$ meson. The vacuum-to-meson transition matrix elements are defined in standard way, namely

$$\langle 0 \mid J^B \mid B \rangle = -if_B \frac{m_B^2}{m_b}, \quad \langle 0 \mid J^a_\nu \mid a_1 \rangle = i f_{a_1} m_{a_1} \epsilon_\nu. \tag{4}$$
Using Eq. (3), and Eq. (4) in Eq. (2), and performing summation over the polarization of the \( a_1 \) meson, we obtain

\[
\Pi_{\mu\nu}^V = -\frac{f_B m_B^2}{m_b} \frac{f_{a_1} m_{a_1}}{(p^2-m_B^2)(p'^2-m_{a_1}^2)} \times \left[ \frac{2F_A^V}{m_B+m_{a_1}} (q^2) i\varepsilon_{\mu\nu\alpha\beta} p^\alpha p'^\beta + F_0^V (q^2) (m_B-m_{a_1}) g_{\mu\nu} + \frac{F^V_+ (q^2)}{m_B+m_{a_1}} P_{\mu}\nu + \frac{F^V_0 (q^2)}{m_B+m_{a_1}} q_{\mu}\nu \right] + \text{excited states},
\]

\[
\Pi_{\mu\nu}^T = -\frac{f_B m_B^2}{m_b} \frac{f_{a_1} m_{a_1}}{(p^2-m_B^2)(p'^2-m_{a_1}^2)} \times \left[ 2F_A^{T+} (q^2) i\varepsilon_{\mu\nu\alpha\beta} p^\alpha p'^\beta + F_0^{T+} (q^2) (m_B-m_{a_1}) g_{\mu\nu} + \frac{F^{T+}_0 (q^2)}{m_B+m_{a_1}} P_{\mu}\nu \right] + \text{excited states}.
\]

(5)

To calculate the form factors \( F_i^{V(T)} (i = 0,+,-,A) \), we will choose the structures \( i\varepsilon_{\mu\nu\alpha\beta} p^\alpha p'^\beta, g_{\mu\nu}, P_{\mu}\nu, q_{\mu}\nu \), from \( \Pi_{\mu\nu}^V \) and \( i\varepsilon_{\mu\nu\alpha\beta} p^\alpha p'^\beta, g_{\mu\nu} \), and \( P_{\mu}\nu \) from \( \Pi_{\mu\nu}^T \), respectively. For simplicity, the correlations are written as

\[
\Pi_{\mu\nu}^V (p^2, p'^2, q^2) = \Pi_0^V g_{\mu\nu} + \Pi_+^V P_{\mu}\nu + \Pi_-^V q_{\mu}\nu + i\Pi_A^V \varepsilon_{\mu\nu\alpha\beta} p^\alpha p'^\beta + \cdots,
\]

\[
\Pi_{\mu\nu}^T (p^2, p'^2, q^2) = \Pi_0^T g_{\mu\nu} + \Pi_+^T P_{\mu}\nu + i\Pi_-^T \varepsilon_{\mu\nu\alpha\beta} p^\alpha p'^\beta + \cdots.
\]

(6)

Now, we consider the theoretical part of the sum rules. For this aim, each \( \Pi_i^{V(T)} \) function is defined in terms of the perturbative and nonperturbative parts as

\[
\Pi_i^{V(T)} (p^2, p'^2, q^2) = \Pi_{\text{per}}^{V(T)} (p^2, p'^2, q^2) + \Pi_{\text{nonper}}^{V(T)} (p^2, p'^2, q^2).
\]

(7)

For the perturbative part, the bare-loop diagrams are considered. With the help of the double dispersion representation, the bare-loop contribution is written as

\[
\Pi_{\text{per}}^{V(T)} = -\frac{1}{(2\pi)^2} \int ds' \int ds \frac{\rho_i^{V(T)} (s, s', q^2)}{(s-p^2)(s'-p'^2)} + \text{subtraction terms},
\]

where \( \rho \) is spectral density. The spectral density is obtained from the usual Feynman integral for the bare-loop by replacing \( \frac{1}{p^2-m_b^2} \to -2\pi i\delta(p^2-m_b^2) \). After standard calculations for the spectral densities \( \rho_i^{V(T)} \), where \( i \) is related to each structure in Eq. (3), we have

\[
\rho_0^V = -\frac{3}{2} s' \Lambda^{-3} \left( 2 ss' - 2 \Delta^2 + 2 \Delta u - u^2 \right) m_b,
\]

\[
\rho_\pm^V = -\frac{3}{2} s' \Lambda^{-5} \left( 12 u\Delta s' - 4 ss'^2 - 2 u^2 s' - 12 s' \Delta^2 \pm 2 sus' \pm 6 u\Delta^2 \pm u^3 \mp 6 u^2 \Delta \right) m_b,
\]

\[
\rho_A^V = 3 s' \Lambda^{-3} (u - 2 \Delta) m_b,
\]

\[
\rho_0^T = \frac{3}{2} s' \Lambda^{-3} \left( 2 s^2 s' - 2 s \Delta^2 + 2 s \Delta u - su^2 - 4 ss' \Delta + sus' + u \Delta^2 \right),
\]

\[
\rho_\pm^T = \frac{3}{2} s' \Lambda^{-5} \left( 12 u\Delta s' - 4 ss'^2 - 2 u^2 s' - 12 s' \Delta^2 \pm 2 sus' \pm 6 u\Delta^2 \pm u^3 \mp 6 u^2 \Delta \right) m_b,
\]

\[
\rho_A^T = 3 s' \Lambda^{-3} (u - 2 \Delta) m_b.
\]
\[ \rho_+^T = \frac{3}{2} s' \Lambda^{-5} \left( 4 s^2 s' + 2 u s^2 s' + 6 u s^2 s' - 8 s s' \Delta + 8 \Delta u s s' - 4 s s' \Delta^2 - 7 s u^2 s' + s u^3 \right. \\
\left. - 6 s u^2 \Delta + 6 s u \Delta^2 + 6 u \Delta^2 s' - 4 u^2 \Delta s' + 4 \Delta u^3 - 5 u^2 \Delta^2 \right), \]
\[ \rho_A^T = -3 s' \Lambda^{-3} (u - 2 \Delta) m_b^2, \] (8)

where \( u = s + s' - q^2 \), \( \Lambda = \sqrt{u^2 - 4 s s'} \), and \( \Delta = s - m_b^2 \).

Now, the nonperturbative part contributions to the correlation functions are discussed (Eq. (11)). In QCD, the three point correlation function can be evaluated by the operator product expansion (OPE) in the deep Euclidean region. Up to dimension 6, the operators are determined by the contribution of the bare-loop, and power corrections coming from dimension-3 \( \langle \bar{\psi} \psi \rangle \), dimension-4 \( \langle G^2 \rangle \), dimension-5 \( m_b^2 \langle \bar{\psi} \psi \rangle \), and dimension-6 \( \langle \bar{\psi} \psi \rangle^2 \) operators \[5\]. The bare-loop diagrams, perturbative part of the correlation functions, are discussed before. For the nonperturbative part contributions, our calculations show that the contributions coming from \( \langle G^2 \rangle \) and \( \langle \bar{\psi} \psi \rangle^2 \) are very small in comparison with the contributions of dimension-3 and 5 that, their contributions can be easily ignored. We introduce the nonperturbative part contributions as

\[ \Pi_{\text{nonper}}^{V(T)} = \langle u \bar{u} \rangle C_i^{V(T)}, \] (9)

where \( \langle u \bar{u} \rangle = -(0.240 \pm 0.010)^3 \text{ GeV}^3 \). After some straightforward calculations, the explicit expressions for \( C_i^{V(T)} \), are given as

\[ C_0^V = \frac{(m_b^2 - q^2)}{2 r r'} - m_b^2 \left[ \frac{1}{6 r r'} + \frac{m_b^2 - q^2}{6 r r'^2} + 3 m_b^2 - 4 q^2 \frac{2}{12 r^2 r'} + \frac{m_b^4 - m_b^3 q^2}{4 r^3 r'} \right], \]
\[ C_+^V = - \frac{1}{r r'} - m_b^2 \left[ \frac{1}{3 r^2 r'} - \frac{m_b^2 - q^2}{3 r^2 r'^2} + \frac{m_b^2}{2 r^3 r'} \right], \]
\[ C_-^V = \frac{1}{r r'} - m_b^2 \left[ \frac{1}{r^2 r'} + \frac{m_b^2 - q^2}{3 r^2 r'^2} + \frac{m_b^2}{2 r^3 r'} \right], \]
\[ C_A^V = \frac{1}{r r'} - m_b^2 \left[ \frac{1}{3 r^2 r'} + \frac{m_b^2 - q^2}{3 r^2 r'^2} + \frac{m_b^2}{2 r^3 r'} \right], \]
\[ C_0^T = \frac{(-m_b^3 + m_b q^2)}{2 r r'} - m_b^2 \left[ \frac{1}{4 r r'} + \frac{m_b(m_b^2 - q^2)}{6 r r'^2} - \frac{m_b(4 m_b^2 - 5 q^2)}{12 r^2 r'} - \frac{m_b(m_b^2 - q^2)^2}{6 r^2 r'^2} \right. \]
\left. - \frac{m_b^5 - m_b^4 q^2}{4 r^3 r'} \right], \]
\[ C_+^T = \frac{m_b}{2 r r'} - \frac{m_b^2}{3 r^2 r'} \frac{2 m_b (m_b^2 - q^2)}{8 r^2 r'^2} + \frac{m_b^3}{4 r^3 r'}, \]
\[ C_A^T = - \frac{m_b}{r r'} - m_b^2 \left[ - \frac{m_b}{2 r^2 r'} - \frac{m_b(m_b^2 - q^2)}{3 r^2 r'^2} - \frac{m_b^3}{2 r^3 r'} \right], \] (10)
where \( r = p^2 - m_0^2 \), \( r' = p'^2 \), and \( m_0 = (0.8 \pm 0.2) \text{GeV} \).

The next step is to apply the Borel transformations as

\[
B_{k^2}(M^2)(\frac{1}{k^2 - m^2})^n = \frac{(-1)^n e^{-m^2/M^2}}{\Gamma(n)} (M^2)^n,
\]

with respect to the \( p^2(p^2 \to M_1^2) \) and \( p'^2(p'^2 \to M_2^2) \) on the phenomenological as well as the perturbative and nonperturbative parts of the correlation functions and equate these two representations of the correlations. The following sum rules for the form factors are derived

\[
F^V_{i}(q^2) = -\frac{m_b}{f_B m_B f_a} e^{m_B^2/M^2} e^{m_a^2/M^2} \times \left\{ -\frac{1}{4\pi^2} \int_0^{s_0} ds' \int_{s_L}^{s_0} ds \rho_i^V(q^2) e^{-s/M^2} e^{-s'/M^2} + \langle \bar{u}u \rangle \times B_{p^2}(M_1^2) B_{p'^2}(M_2^2) C_i^V(q^2) \right\},
\]

where

\[
F_A^V(q^2) = \frac{2F_A^V(q^2)}{m_B + m_a}, \quad F_0^V(q^2) = F_0^V(q^2)(m_B - m_a),
\]

\[
F_A^V(q^2) = \frac{F_A^V(q^2)}{m_B + m_a}, \quad F_0^V(q^2) = F_0^V(q^2),
\]

\[
F_0^V(q^2) = \frac{2F_A^V(q^2)}{m_B + m_a}, \quad F_0^V(q^2) = F_0^V(q^2)(m_B - m_a),
\]

\[
F_0^V(q^2) = \frac{F_A^V(q^2)}{m_B + m_a}, \quad F_0^V(q^2) = F_0^V(q^2).
\]

\( s_0 \) and \( s_0' \) are the continuum thresholds in the \( B \) and \( a_1 \) meson channels, respectively. \( s_L \), the lower limit of the integration over \( s \), is: \( m_b^2 + \frac{m_a^2}{m_B - q^2} \).

III. NUMERICAL ANALYSIS

In this section, we present our numerical analysis of the form factors \( F_i^V(q^2) \) \( (i = 0, +, -, A) \). We choose the values of the quark, lepton, and meson masses and also the leptonic decay constants as: \( m_b = 4.8 \text{ GeV} \), \( m_\mu = 0.105 \text{ GeV} \), \( m_\tau = 1.776 \text{ GeV} \), \( m_{a_1} = 1.260 \text{ GeV} \), \( m_B = 5.280 \text{ GeV} \), \( f_{a_1} = (238 \pm 10) \text{ MeV} \). For the value of the \( f_B \), we shall use \( f_B = 140 \text{ MeV} \). This value of \( f_B \) corresponds to the case where \( \mathcal{O}(\alpha_s) \) corrections are not taken into account (see \[11\], \[12\]). The values of continuum thresholds \( s_0 \) and \( s_0' \) are taken to be \( s_0 = (35 \pm 2) \text{ GeV} \) \[13\] and \( s_0' = (2.55 \pm 0.15) \text{ GeV} \) \[10\].

The expressions for the form factors in Eq. \[12\] contain also the Borel mass parameters \( M_1^2 \) and \( M_2^2 \). These are mathematical objects, so the physical quantities, i.e., the form factors, should be independent of them. The working regions for \( M_1^2 \) and \( M_2^2 \) are determined
by requiring that the contributions of the higher states and continuum be effectively sup-
pressed, and therefore it guarantees that the contributions of higher dimensional operators
are small. We found good stability of the sum rules in the interval $8 \text{ GeV}^2 \leq M_1^2 \leq 15 \text{ GeV}^2$
and $2.5 \text{ GeV}^2 \leq M_2^2 \leq 4 \text{ GeV}^2$.

Equation (12) shows the $q^2$ dependence of the form factors in the region where the sum
rule is valid. To extend these results to the full region, we look for parametrization of the
form factors in such a way that in the validity region of the 3PSR, this parametrization
coincides with the sum rules prediction. We use two following sufficient parametrizations
of the form factors with respect to $q^2$ as:

$$F_i^{(1)}(q^2) = \frac{1}{1 - (\frac{q^2}{m_B^2})} \sum_{k=0}^{2} b_k \left[ z^k + (-1)^k \frac{k}{3} z^4 \right].$$

where $z = \frac{\sqrt{t_+ - q^2 - \sqrt{t_+ - t_0}}}{\sqrt{t_+ - q^2 + \sqrt{t_+ - t_0}}}$, $t_+ = (m_B + m_{a_1})^2$ and $t_0 = (m_B + m_{a_1})(\sqrt{m_B} - \sqrt{m_{a_1}})^2$ [14],

and also

$$F_i^{(2)}(q^2) = \frac{f_i(0)}{1 - \alpha \left( \frac{q^2}{m_B^2} \right) + \beta \left( \frac{q^2}{m_B^2} \right)^2}.$$  

We evaluated the values of the parameters $b_k (k = 1, ..., 3)$ of the first and $f_i(0)$, $\alpha$, $\beta$ of the second fit function for each transition form factor of the $B \rightarrow a_1$ decay, taking
$M_1^2 = 10 \text{ GeV}^2$ and $M_2^2 = 3 \text{ GeV}^2$. Tables I and III show the values of the $b_k$ and $f_i(0)$, $\alpha$, $\beta$ for the form factors.

**TABLE I: The values of the $b_k$ related to $F_i^{(1)}(q^2)$.**

| Parameter | $F_0^{(1)V}$ | $F_+^{(1)V}$ | $F_-^{(1)V}$ | $F_A^{(1)V}$ | $F_0^{(1)T}$ | $F_+^{(1)T}$ | $F_A^{(1)T}$ |
|-----------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| $b_0$     | 0.28         | -0.30        | 0.35         | 0.44         | -0.21        | 0.33         | -0.33        |
| $b_1$     | 2.80         | -1.79        | 1.77         | 0.80         | -2.14        | 1.42         | -0.60        |
| $b_2$     | 15.52        | 0.94         | 0.09         | 3.89         | -11.34       | -0.04        | -2.90        |

So far, several authors have calculated the form factors of the $B \rightarrow a_1 \ell \nu$ decay via the
different approaches. For a comparison, the form factor predictions of the other approaches
at $q^2 = 0$ are shown in Table. III.

The dependence of the form factors, $F_i^{(1)V(T)}(q^2)$ and $F_i^{(2)V(T)}(q^2)$ on $q^2$ extracted from
the fit functions, Eqs. (13) and (14), are given in Figs. (1) and (2), respectively.
TABLE II: The values of the $f_i(0)$, $\alpha$ and $\beta$ connected to $F_i^{(2)}(q^2)$.

| Parameter | $F_0^{(2)V}$ | $F_+^{(2)V}$ | $F_-^{(2)V}$ | $F_A^{(2)V}$ | $F_0^{(2)T}$ | $F_+^{(2)T}$ | $F_-^{(2)T}$ |
|-----------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $f_i(0)$  | 0.49          | -0.41         | 0.46          | 0.50          | -0.37         | 0.41          | -0.37         |
| $\alpha$  | -0.52         | 0.34          | 0.37          | 0.58          | -0.50         | 0.44          | 0.58          |
| $\beta$   | 0.38          | 0.14          | -0.04         | -0.39         | 0.48          | -0.10         | -0.40         |

TABLE III: Transition form factors of the $B \to a_1 \ell \nu$ at $q^2 = 0$ in various models. The results of the LFQM and ISGW2 have been rescaled according to the form factor definition in Eq. (3).

| Model      | $F_0^V(0)$ | $F_+^V(0)$ | $F_-^V(0)$ | $F_A^V(0)$ |
|------------|------------|------------|------------|------------|
| LFQM[2]    | 0.37       | 0.29       | 0.13       | 0.41       |
| ISGW2 [3]  | 0.54       | -0.81      | 1.01       | 0.34       |
| CQM [4]    | 1.32       | 0.34       | 1.20       | 0.09       |
| SR [5]     | -0.68      | -0.33      | -0.23      | -0.41      |

In the standard model, the rare semileptonic $B \to a_1 \ell^+ \ell^-$ and $B \to \rho \ell^+ \ell^-$ decays are described via loop transitions, $b \to d \ell^+ \ell^-$ at quark-level. Both mesons $a_1$ and $\rho$ have the same quark content, but different masses and parities, i.e., $\rho$ is a vector $(1^-)$ and $a_1$ is a axial vector $(1^+)$. Therefore transition form factors of the $B \to a_1$ decays are not the same as those for $B \to \rho$, exactly. A comparison between the form factors of the $B \to a_1 \ell^+ \ell^-$ and $B \to \rho \ell^+ \ell^-$ decays at $q^2 = 0$ are shown in Table IV.

FIG. 1: The form factors $F_i^{(1)V}$ and $F_i^{(1)T}$ on $q^2$. 

8
FIG. 2: The form factors $F_i^{(2)V}$ and $F_i^{(2)T}$ on $q^2$.

### TABLE IV: The form factors of the $B \to a_1 \ell^+ \ell^-$ via the SR (This work) and $B \to \rho \ell^+ \ell^-$ in the LCSR [15] at $q^2 = 0$.

| Mode          | $F_0^V(0)$ | $F_+^V(0)$ | $F_1^V(0)$ | $F_0^T(0)$ | $F_+^T(0)$ | $F_T^T(0)$ |
|---------------|------------|------------|------------|------------|------------|------------|
| $B \to a_1 \ell^+ \ell^-$ | 0.49       | -0.41      | 0.46       | 0.50       | -0.37      | 0.41       |
| $B \to \rho \ell^+ \ell^-$ | 0.24       | 0.22       | 0.30       | 0.32       | 0.27       | 0.18       |

Now, we would like to evaluate the branching ratio values for the $B \to a_1 \ell^+ \ell^-$ decays. The expressions of the differential decay width $d\Gamma/dq^2$ for the $B \to a_1 \nu \bar{\nu}$ and $B \to a_1 \ell^+ \ell^-$ decays can be found in [16, 17]. These expressions contain the Wilson coefficients $C_7^{\text{eff}}$, $C_9^{\text{eff}}$, $C_{10}$, and also the CKM matrix elements $V_{tb}$ and $V_{td}$. The effective Wilson coefficients $C_9^{\text{eff}}(q^2)$, are given as

$$C_9^{\text{eff}}(q^2) = C_9 + Y(q^2).$$

The function $Y(q^2)$ contains the short-distance (SD) contributions, $Y_{\text{per}}(q^2)$, as well as the LD contributions coming from the real $c\bar{c}$ intermediate states called charmonium resonances. Two resonances, $J/\psi$ and $\psi'$, are narrow and the last four resonances, $\psi(3370)$, $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$, are above the $D\bar{D}$-threshold and as a consequence the width is much larger. The explicit expressions of the $Y(q^2)$ can be found in [18] (see also [19, 20]). Considering $C_7^{\text{eff}} = -0.313$, $C_{10} = -4.669$, $|V_{tb}V_{td}^*| = 0.008$ [19], and the form factors related to the fit functions, Eqs. (13) and (14), and after numerical analysis, the branching ratios for the $B \to a_1 \ell^+ \ell^-/\nu \bar{\nu}$ are obtained as presented in Table V. In this table, we show only the values obtained considering the SD effects contributing to the Wilson coefficient.
$C^\text{eff}$ for charged lepton case.

TABLE V: The branching ratios of the semileptonic $B \to a_1\ell^+\ell^-$ decays, considering two groups of the form factors. 1 and 2 stand for the form factors, $F^{(1)V}(T)$ and $F^{(2)V}(T)$, respectively.

| Mode | form factors | Value |
|------|--------------|-------|
| $\text{Br}(B \to a_1\nu\bar{\nu}) \times 10^8$ | 1/2 | $7.41 \pm 2.44$
| & | $7.78 \pm 2.32$
| $\text{Br}(B \to a_1e^+e^-) \times 10^8$ | 1/2 | $2.75 \pm 0.58$
| & | $2.90 \pm 0.95$
| $\text{Br}(B \to a_1\mu^+\mu^-) \times 10^8$ | 1/2 | $2.54 \pm 0.47$
| & | $2.70 \pm 0.89$
| $\text{Br}(B \to a_1\tau^+\tau^-) \times 10^9$ | 1/2 | $0.37 \pm 0.09$
| & | $0.33 \pm 0.10$

In this part, we would like to present the branching ratio values including LD effects. Due to our calculations $q^2 < m^2_{\psi(4040)}$, we introduce some cuts around the narrow resonances of the $J/\psi$ and $\psi'$, and study the following three regions for muon:

\begin{align}
\text{I} : & \quad 2m_\mu \leq \sqrt{q^2} \leq M_{J/\psi} - 0.20, \\
\text{II} : & \quad M_{J/\psi} + 0.04 \leq \sqrt{q^2} \leq M_{\psi'} - 0.10, \\
\text{III} : & \quad M_{\psi'} + 0.02 \leq \sqrt{q^2} \leq m_B - m_{a_1},
\end{align}

and the following two for tau:

\begin{align}
\text{I} : & \quad 2m_\tau \leq \sqrt{q^2} \leq M_{\psi'} - 0.02, \\
\text{II} : & \quad M_{\psi'} + 0.02 \leq \sqrt{q^2} \leq m_B - m_{a_1}.
\end{align}

In Table VI we present the branching ratios for muon and tau obtained using the regions shown in Eqs. (16-17), respectively. In our calculations, two groups of the form factors are considered. Here, we should also stress that the results obtained for the electron are very close to the results of the muon and for this reason, we only present the branching ratios for muon in our table. Considering the form factors, $F^{(1)V}(T)$ and $F^{(2)V}(T)$, the dependency of the differential branching ratios on $q^2$ with and without LD effects for charged lepton case is shown in Fig. 3. In this figure, the solid and dash-dotted lines show the results without and with the LD effects, respectively, using the form factors, $F^{(1)V}(T)$. Also the circles and stars are the same as those lines but considering $F^{(2)V}(T)$.

We would like to compare the branching ratio values of these decays with those of the related processes, such as $B_s \to \phi\ell^+\ell^-$ decays governed by the flavor-changing neutral current
TABLE VI: The branching ratios of the semileptonic $B \to a_1 \ell^+ \ell^-$ decays including LD effects in three regions. 1 and 2 stand for the form factors, $F_i^{(1) V(T)}$ and $F_i^{(2) V(T)}$, respectively.

| Mode                        | form factors | I          | II         | III         | I+II+III    |
|-----------------------------|--------------|------------|------------|-------------|-------------|
| $Br(B \to a_1 \mu^+ \mu^-) \times 10^8$ | 1/2          | 2.07±0.68  | 0.27±0.09  | 0.08±0.03   | 2.42±0.80   |
| $Br(B \to a_1 \tau^+ \tau^-) \times 10^9$ | 1/2          | undefined  | 0.11±0.04  | 0.15±0.05   | 0.26±0.09   |

FIG. 3: The differential branching ratios of the semileptonic $B \to a_1$ decays on $q^2$ with and without LD effects.

transition of $b \to s$. The branching fraction predictions for the $B_s \to \phi$ decays in different approaches are shown in Table VII. The comparison between the two processes explicitly indicates that the branching ratios suppression of the $B_s \to a_1$ decays. As mentioned previously, the FCNC transitions can be suppressed due to their proportionality to the small

TABLE VII: The branching ratio values of the semileptonic $B_s \to \phi \ell^+ \ell^-$ decays in various methods.

| Mode                        | Method      | I          | II         | III         | I+II+III    | EXP         |
|-----------------------------|-------------|------------|------------|-------------|-------------|-------------|
| $Br(B_s \to \phi \mu^+ \mu^-) \times 10^7$ | 3PSR[17]    | 8.93       | 1.39       | 2.29        | 12.6        | $12.3^{+4.0}_{-3.4}$ |
|                             | LFQM[16]    | 7.91       | 1.88       | 2.56        | 12.4        |             |
|                             | CQM[16]     | 8.30       | 1.83       | 2.23        | 12.4        |             |
| $Br(B_s \to \phi \tau^+ \tau^-) \times 10^8$ | 3PSR[17]    | undefined  | 0.13       | 8.46        | 8.59        |             |
|                             | LFQM[16]    | undefined  | 0.48       | 8.87        | 9.35        |             |
|                             | CQM[16]     | undefined  | 0.48       | 8.31        | 8.79        |             |
The Cabibbo-Kobayashi-Maskawa matrix elements. The difference between the branching ratio values of the two different processes, $B \rightarrow a_1$ and $B_s \rightarrow \phi$, can be due to the difference between the matrix elements, $|V_{tb}V_{td}^*| = 0.008$ and $|V_{tb}V_{ts}^*| = 0.038$, respectively.

Finally, we want to calculate the longitudinal lepton polarization asymmetry and the forward-backward asymmetry for the considered decays. The expressions of the longitudinal lepton polarization asymmetry and the forward-backward asymmetry, $P_L$ and $A_{FB}$, are given in [16, 17]:

The dependence of the longitudinal lepton polarization and the forward-backward asymmetries for the $B \rightarrow a_1l^+l^-$ decays on the transferred momentum square $q^2$ with and without LD effects are plotted in Figs. (4) and (5), respectively.

The measurement of these quantities in the FCNC transitions are difficult. In Ref. [21],
measurements of the BABAR are presented for the FCNC decays, $B \to K^* \ell^+ \ell^-$ including branching fractions, isospin asymmetries, direct CP violation, and lepton flavor universality for dilepton masses below and above the $J/\psi$ resonance. Furthermore, BABAR results from an angular analysis in $B \to K^* \ell^+ \ell^-$ are reported in which both the $K^*$ longitudinal polarization and the lepton forward-backward asymmetry are measured for dilepton masses below and above the $J/\psi$ resonance.

In summary, the transition form factors of the semileptonic $B \to a_1 \ell^+ \ell^- / \nu \bar{\nu}$ decays were investigated in the 3PSR approach. Considering both the SD and LD effects contributing to the Wilson coefficient $C_{9}^{\text{eff}}$ for charged lepton case, we estimated the branching ratio values for these decays. Also, for a better analysis, the dependence of the longitudinal lepton polarization and forward-backward asymmetries of these decays on $q^2$ were plotted.

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