Brane New World and dS/CFT correspondence

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abstract

The occurrence of 5d de Sitter space with 4d de Sitter brane is discussed on classical and quantum level. It is shown that quantum effects maybe produced by dual CFT living on the brane. Moreover, gravity trapping on the brane is proved via the presentation of 5d dS gravity as 4d gravity coupled with gauge theory. This supports the dS/CFT correspondence. Some open questions in 5d dS/4d CFT correspondence are briefly discussed.

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1 Introduction

It became clear recently that holography should play the fundamental role in physics of 21st century. Moreover, there are significant changes in what we mean as holography by now. Indeed, roughly speaking we expect that sometimes (at some, not strictly specified conditions) the classical volume physics may describe the quantum boundary physics in less dimensions and vice-versa. The beatiful example of such holography is so-called AdS/CFT correspondence which occurs in string theory. Considering the four-dimensional QFT, it says that five-dimensional AdS gravity is equivalent to some four-dimensional super Yang-Mills theory living on the boundary of AdS space which is gravitational ground state.

The connection between braneworld and AdS/CFT correspondence maybe developed in Brane New World construction \cite{1, 2}. The quantum creation of braneworld thanks to conformal anomaly of four-dimensional quantum fields living on the brane occurs in such scenario.

Due to general character of holographic principle it is expected that it should be realized in different ways. In particular, one can change the bulk (AdS) space and try to realize holography for another manifolds. One of simplest candidates for bulk (apart from AdS) is constant curvature deSitter space. Then, holographic principle predicts that it should occur dS/CFT correspondence \cite{3, 4}.

The reason why AdS/CFT can be expected is the isometry of $d+1$-dimensional anti-de Sitter space, which is $SO(d, 2)$ symmetry. It is identical with the conformal symmetry of $d$-dimensional Minkowski space. We should note, however, $d+1$-dimensional de Sitter space has the isometry of $SO(d+1, 1)$ symmetry, which can be a conformal symmetry of $d$-dimensional Euclidean space. Then it might be natural to expect the correspondence between $d+1$-dimensional de Sitter space and $d$-dimensional euclidean conformal symmetry. In fact, the metric of $D = d+1$-dimensional anti de Sitter space (AdS) is given by

\[ ds^2_{\text{AdS}} = dr^2 + e^{2r} \left( -dt^2 + \sum_{i=1}^{d-1} (dx^i)^2 \right). \]  \hspace{1cm} (1)

In the above expression, the boundary of AdS lies at $r = \infty$. If one exchanges the radial coordinate $r$ and the time coordinate $t$, we obtain the metric of
the de Sitter space (dS):

\[ ds_{dS}^2 = -dt^2 + e^{2t} \sum_{i=1}^{d} (dx^i)^2 . \]  

Here \( x^d = r \). Then there is a boundary at \( t = \infty \), where the Euclidean conformal field theory (CFT) can live and one expects dS/CFT correspondence as one more manifestation of holographic principle. This may be very important as there are indications that our Universe has de Sitter phase in the past and in the future. Then, there appears very nice way to formulate some de Sitter gravitational physics in terms of the boundary QFT physics and vice-versa.

In the present contribution based mainly on ref.\[5\] we consider the possibility of classical and quantum creation of the inflationary brane in de Sitter bulk space in frames of mechanism of refs. \[1, 2\]. Moreover, the content of quantum fields on the brane may be chosen in such a way, that it corresponds to euclidean CFT dual to 5d dS bulk space. In this sense, one can understand that quantum creation of dS brane-world occurs in frames of dS/CFT correspondence. This is quite consistent, as is known (even in the absence of explicit CFT dual to 5d dS space) that holographic conformal anomaly from 5d dS space is proportional to the one of 4d super Yang-Mills theory\[6\]. It is interesting that brane de Sitter gravity (despite the fact that bulk represents not AdS but dS space) may be localized due to proposed dS/CFT correspondence.

\section{Classical and quantum de Sitter braneworlds}

The metric of 5 dimensional Euclidean de Sitter space that is 5d sphere is given by

\[ ds_{S_5}^2 = dy^2 + l^2 \sin^2 \frac{y}{l} d\Omega_4^2 . \]  

Here \( d\Omega_4^2 \) describes the metric of \( S_4 \) with unit radius. The coordinate \( y \) is defined in \( 0 \leq y \leq l\pi \). One also assumes the brane lies at \( y = y_0 \) and the bulk space is given by gluing two regions given by \( 0 \leq y < y_0 \).

We start with the action \( S \) which is the sum of the Einstein-Hilbert action \( S_{EH} \) with positive cosmological constant, the Gibbons-Hawking surface term
$S_{GH}$, the surface counter term $S_{1}$ and the trace anomaly induced action $\mathcal{W}$:

$$S = S_{EH} + S_{GH} + 2S_{1} + \mathcal{W}, \quad S_{EH} = \frac{1}{16\pi G} \int d^{5}x \sqrt{g(5)} \left( R_{(5)} - \frac{12}{l^{2}} \right),$$

$$S_{GH} = \frac{1}{8\pi G} \int d^{4}x \sqrt{g(4)} \nabla_{\mu}n^{\mu}, \quad S_{1} = -\frac{3}{8\pi Gl} \int d^{4}x \sqrt{g(4)},$$

$$\mathcal{W} = b \int d^{4}x \sqrt{\tilde{g}} F A + b' \int d^{4}x \sqrt{\tilde{g}} \left\{ A \left[ 2\tilde{\Box} + \tilde{R}_{\mu\nu}\nabla_{\mu}\nabla_{\nu} - \frac{4}{3}\tilde{R}\tilde{\Box}^{2} \right] \right\}$$

$$+ \frac{2}{3}(\tilde{\nabla}_{\mu}\tilde{R})\tilde{\nabla}_{\mu} A + \left( \tilde{G} - \frac{2}{3}\tilde{R}\right) A \right\}$$

$$- \frac{1}{12} \left\{ b'' + \frac{2}{3}(b + b') \right\} \int d^{4}x \sqrt{\tilde{g}} \left[ \tilde{R} - 6\tilde{\Box} A - 6(\tilde{\nabla}_{\mu} A)(\tilde{\nabla}^{\mu} A) \right]^{2}. \quad (4)$$

Here the quantities in the 5 dimensional bulk spacetime are specified by the suffices (5) and those in the boundary 4 dimensional spacetime by (4). The factor 2 in front of $S_{1}$ in (4) is coming from that we have two bulk regions which are connected with each other by the brane. In (4), $n^{\mu}$ is the unit vector normal to the boundary. In (4), one chooses the 4 dimensional boundary metric as

$$g(4)_{\mu\nu} = e^{2A} \tilde{g}_{\mu\nu} \quad (5)$$

and we specify the quantities with $\tilde{g}_{\mu\nu}$ by using $\tilde{}$. $\tilde{G}$ ($\tilde{F}$) are the Gauss-Bonnet invariant and the square of the Weyl tensor.

In the effective action (10) induced by brane quantum matter, in general, with $N$ real scalar, $N_{1/2}$ Dirac spinor, $N_{1}$ vector fields, $N_{2}$ (= 0 or 1) gravitons and $N_{HD}$ higher derivative conformal scalars, $b$, $b'$ and $b''$ are

$$b = \frac{N + 6N_{1/2} + 12N_{1} + 611N_{2} - 8N_{HD}}{120(4\pi)^{2}},$$

$$b' = -\frac{N + 11N_{1/2} + 62N_{1} + 1411N_{2} - 28N_{HD}}{360(4\pi)^{2}}, \quad b'' = 0. \quad (6)$$

For typical examples motivated by AdS/CFT (and presumably by dS/CFT because central charges are the same in AdS/CFT or dS/CFT) correspondence one has: a) $\mathcal{N} = 4$ SU($N$) SYM theory $b = -b' = \frac{N^{2} - 1}{4(4\pi)^{2}}$, b) $\mathcal{N} = 2$ Sp($N$) theory $b = \frac{12N^{2} + 18N - 2}{24(4\pi)^{2}}, \quad b' = -\frac{12N^{2} + 12N - 1}{24(4\pi)^{2}}$. Note that $b'$ is negative in the above cases.

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$^2$ The coefficient of $S_{1}$ cannot be determined from the condition of finiteness of the action as in AdS/CFT. However, using the renormalization group method as in [8] this coefficient can be determined uniquely [5].
The metric of $S_4$ with the unit radius is given by
\[ d\Omega_4^2 = d\chi^2 + \sin^2 \chi d\Omega_3^2. \] (7)

Here $d\Omega_3^2$ is described by the metric of 3 dimensional unit sphere. If one changes the coordinate $\chi$ to $\sigma$ by $\sin \chi = \pm \frac{1}{\cosh \sigma}$, one obtains
\[ d\Omega_4^2 = \frac{1}{\cosh^2 \sigma} \left( d\sigma^2 + d\Omega_3^2 \right). \] (8)

Then one assumes the metric of 5 dimensional space time as follows:
\[ ds^2 = dy^2 + e^{2A(y, \sigma)} g_{\mu\nu} dx^\mu dx^\nu, \quad \tilde{g}_{\mu\nu} dx^\mu dx^\nu \equiv l^2 \left( d\sigma^2 + d\Omega_3^2 \right) \] (9)

and one identifies $A$ and $\tilde{g}$ in (3) with those in (5). Then $\tilde{F} = \tilde{G} = 0$, $R = \frac{6}{l^2}$ etc. Due to the assumption (9), the actions in (4) have the following forms:
\[ S_{EH} = \frac{l^4 V_3}{16\pi G} \int dy d\sigma \left\{ \left( -8\partial_y^2 A - 20(\partial_y A)^2 \right) e^{4A} + \left( -6\partial_\sigma^2 A - 6(\partial_\sigma A)^2 + 6 \right) e^{2A} - \frac{12}{l^2} e^{4A} \right\}, \]
\[ S_{GH} = \frac{l^4 V_3}{2\pi G} \int d\sigma e^{4A} \partial_y A, \quad S_1 = -\frac{3l^3 V_3}{8\pi G} \int d\sigma e^{4A}, \]
\[ W = V_3 \int d\sigma \left[ b' A \left( 2\partial_\sigma^4 A - 8\partial_\sigma^2 A \right) - 2(b + b') \left( 1 - \partial_\sigma^2 A - (\partial_\sigma A)^2 \right)^2 \right]. \] (10)

Here $V_3$ is the volume or area of the unit 3 sphere.

In the bulk, one obtains the following equation of motion from $S_{EH}$ by the variation over $A$:
\[ 0 = \left( -24\partial_y^2 A - 48(\partial_y A)^2 - \frac{48}{l^2} \right) e^{4A} + \frac{1}{l^2} \left( -12\partial_\sigma^2 A - 12(\partial_\sigma A)^2 + 12 \right) e^{2A}, \] (11)

which corresponds to one of the Einstein equations. Then one finds solutions, $A_S$, which correspond to the metric $dS_5$ in (3) with (8).

\[ A = A_S = \ln \sin \frac{y}{l} - \ln \cosh \sigma. \] (12)

\footnote{If we Wick-rotate the metric by $\sigma \to it$, we obtain the metric of de Sitter space:
\[ d\Omega_4^2 \to ds_{dS}^2 = \frac{1}{\cos^2 t} \left( -dt^2 + d\Omega_3^2 \right). \]}
On the brane at the boundary, one gets the following equation:

\[ 0 = \frac{48l^4}{16\pi G} \left( \partial_y A - \frac{1}{l} \right) e^{4A} + b' \left( 4\partial_y^4 A - 16\partial_y^2 A \right) - 4(b + b') \left( \partial_y^4 A + 2\partial_y^2 A - 6(\partial_y A)^2 \partial_y^2 A \right). \] (13)

We should note that the contributions from \( S_{\text{EH}} \) and \( S_{\text{GH}} \) are twice from the naive values since we have two bulk regions which are connected with each other by the brane. Substituting the bulk solution \( A = A_S \) in (12) into (13) and defining the radius \( R \) of the brane by \( R \equiv l \sin \frac{y_0}{l} \), one obtains

\[ 0 = \frac{1}{\pi G} \left( \frac{1}{R} \sqrt{1 - \frac{R^2}{l^2} - \frac{1}{l}} \right) R^4 + 8b'. \] (14)

One sees that eq. (14) does not depend on \( b \). First we should note \( 0 \leq R \leq l \) by definition. Even in the case that there is no quantum contribution from the matter on the brane, that is, \( b' = 0 \), Eq. (14) has a solution:

\[ R^2 = R_0^2 \equiv \frac{l^2}{2} \text{ or } \frac{y_0}{l} = \frac{\pi}{4}, \frac{3\pi}{4}. \] (15)

In Eq. (14), the first term \( \frac{R^3}{\pi G} \sqrt{1 - \frac{R^2}{l^2}} \) corresponds to the gravity, which makes the radius \( R \) larger. On the other hand, the second term \( -\frac{R^4}{\pi G l} \) corresponds to the tension, which makes \( R \) smaller. When \( R < R_0 \), gravity becomes larger than the tension and when \( R > R_0 \), vice versa. Then both of the solutions in (15) are stable. Although it is not clear from (14), \( R = l \left( \frac{\pi}{l} = \frac{y_0}{l} \right) \) corresponds to the local maximum. Hence, the possibility of creation of inflationary brane in de Sitter bulk is possible already on classical level, even in situation when brane tension is fixed by holographic RG. That is qualitatively different from the case of AdS bulk where only quantum effects led to creation of inflationary brane \([2, 1]\) (when brane tension was not free parameter). Of course, if brane tension is taken to be arbitrary, i.e. the coefficient of \( S_1 \) is not fixed then there appears more possibilities.

Let us make several remarks about properties of dS brane-world. There is an excellent explanation \([1]\) why gravity is trapped on the brane in the AdS spacetime. This uses AdS_5/CFT_4 correspondence. This can be generalized to the brane in dS spacetime by using proposed dS/CFT correspondence.

In \([8]\) it has been shown that the bulk action diverges in de Sitter space when we substitute the classical solution, which is the fluctuation around
the de Sitter space in (2). In other words, counterterms are necessary again. The divergence occurs since the volume of the space diverges when \( t \to \infty \) (or \( t \to -\infty \) after replacing \( t \) by \(-t\) in another patch). Then we should put the counterterms on the space-like branes which lie at \( t \to \pm \infty \). Therefore dS/CFT correspondence should be given by

\[
e^{-W_{\text{CFT}}} = \int [dg][d\varphi]e^{-S_{\text{dS grav}}}, \quad S_{\text{dS grav}} = S_{\text{EH}} + S_{\text{GH}} + S_1 + S_2 + \cdots,
\]

\[
S_{\text{EH}} = \frac{1}{16\pi G} \int d^5x \sqrt{-g(5)} \left( R(5) - \frac{12}{l^2} + \cdots \right),
\]

\[
S_{\text{GH}} = \frac{1}{8\pi G} \int_{M^+_{4} + M^-_{4}} d^4x \sqrt{g(4)} \nabla^\mu n^\mu, \tag{16}
\]

\[
S_1 = \frac{3}{8\pi Gl} \int_{M^+_{4} + M^-_{4}} d^4x \sqrt{g(4)}, \quad S_2 = \frac{l}{32\pi G} \int_{M^+_{4} + M^-_{4}} d^4x \sqrt{g(4)} \left( R(4) + \cdots \right), \cdots.
\]

Here \( S_1, S_2, \cdots \) correspond to the surface counterterms, which cancel the divergences in the bulk action and \( M^\pm_4 \) expresses the boundary at \( t \to \pm \infty \).

Let us consider two copies of the de Sitter spaces \( \text{dS}_1(1) \) and \( \text{dS}_2(2) \). We also put one or two of the space-like branes, which can be regarded as boundaries connecting the two bulk de Sitter spaces, at finite \( t \). Then if one takes the following action \( S \) instead of \( S_{\text{dS grav}} \),

\[
S = S_{\text{EH}} + S_{\text{GH}} + 2S_1 = S_{\text{dS grav}} + S_1 - S_2 - \cdots, \tag{17}
\]

we obtain the following boundary theory in terms of the partition function:

\[
\int_{\text{dS}_1(1) + \text{dS}_2(1) + M^+_{4} + M^-_{4}} [dg][d\varphi]e^{-S} = \left( \int_{\text{dS}_1} [dg][d\varphi]e^{-S_{\text{EH}} - S_{\text{GH}} - S_1} \right)^2 = \left( \int_{\text{dS}_2} [dg][d\varphi]e^{-S_{\text{grav}}} \right)^2 = e^{-2W_{\text{CFT}} + 2S_2 + \cdots} \tag{18}
\]

Since \( S_2 \) can be regarded as the Einstein-Hilbert action on the boundary, the gravity on the boundary becomes dynamical. In other words, there is strong indication that braneworld model under consideration at some conditions may be effectively described by 4d gravity interacting with some gauge theory. In fact, the explicit proof that indeed gravity trapping occurs on dS brane in 5d dS bulk is given in [9] and standard Newton potential is induced there [10]. This is extremely powerful argument in favor of dS/CFT correspondence.
Now we consider the quantum effects ($b' \neq 0$ case) on the brane in (14). Let us define a function $F(R^2)$ as follows:

$$F(R^2) = \frac{1}{\pi G} \left( \frac{1}{R} \sqrt{1 - \frac{R^2}{l^2}} - \frac{1}{l} \right) R^4 .$$

(19)

Then one can easily find

$$F(0) = F \left( \frac{l^2}{2} \right) = 0 , \quad F(l^2) = -\frac{l^3}{\pi G} ,$$

$$F(R^2) > 0 \quad \text{when } 0 < R^2 < \frac{l^2}{2} \quad \text{and} \quad F(R^2) < 0 \quad \text{when } \frac{l^2}{2} < R^2 \leq l^2 .$$

(20)

The function $F(R^2)$ has a maximum

$$F = F_m \equiv \frac{l^3}{16\pi G} \left( -26 + 35 \sqrt{1 - \frac{9}{50}} \right)$$

when

$$R^2 = R_m^2 \equiv \frac{5l^2}{4} \left( 1 - \sqrt{1 - \frac{9}{50}} \right) < \frac{l^2}{2} .$$

(21)

(22)

The above results tell

1. When $-8b' > F_m$ or $-8b' < -\frac{l^3}{\pi G}$, there is no solution in Eq.(14). That is, the quantum effect exhibits the creation of the inflationary brane world.

2. When $0 < -8b' < F_m$, there appear two solutions in (14). The solution with larger radius $R$ corresponds to the classical one in (15) but the radius $\bar{R}$ in the solution is smaller then that in the classical one. In other words, quantum effects try to prevent inflation. The solution with smaller radius can be regarded as the solution generated by only quantum effects on the brane. Anyway the radii $R$ in both of the solutions are smaller than that in the classical one (15). Since $\frac{1}{R}$ corresponds to the rate of the expansion of the universe when $S_4$ is Wick-rotated into 4d de Sitter space, the quantum effect makes the rate larger.
3. When $0 > -8b' > -\frac{a^2}{5G}$, which is rather exotic case since usually $b'$ is negative as in (3), Eq. (14) has unique solution corresponding to the solution in the classical case (15) and the quantum effect on the brane makes the radius $R$ larger.

The de Sitter brane may be thought as inflationary brane. Moreover, we got the alternative description of the dS braneworld as some 4d gravity with matter. Then principal possibility of the end of brane inflation maybe established [5].

3 Discussion

The consideration of 5d deSitter braneworld above strongly indicates to 5d dS/4d CFT correspondence. In many respects as is shown in refs. [11] it is similar to 5d AdS/4d CFT correspondence. Moreover, similar questions as in AdS/CFT correspondence maybe addressed also in dS/CFT correspondence.

Unfortunately, unlike to string-motivated AdS/CFT duality one cannot present yet the consistent and satisfactory model of CFT (for which even central charge is known [3]) dual to 5d dS space. In many respects this slow down the development of dS/CFT correspondence. It is quite possible also that such situation indicates to necessity of essential modification of what we mean as CFT now (see one explicit example in [4]). In particular, it is expected that such dual CFT may have some problems with unitarity, and (or) with tachyons. From another side, many properties of such CFT are known from gravitational description. In particular, even Wilson loop [12] in such CFT maybe found explicitly. Unlike to AdS/CFT the quark-antiquark potential indicates to the possibility of confinement. In this respect, dS/CFT correspondence may favor QCD better that AdS holographic description.

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