Statistical physics of balance theory

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Introduction

(Social) Balance Theory

Methods and Application

Statistical Physics Approach
- Online World
- Cold War
- Hamiltonian
- Entropy

Conclusion
Proposed originally by Fritz Heider (1958).

- If two people have a positive (negative) relationship, their opinion about an object will be shared (differ)
- Friends tend to have similar preferences and common friends and enemies, and enemies tend to have the opposite.
Balance Theory applied to 3-cycles: 
*Your friend’s friend is your friend* and *your enemy’s enemy is your friend*. 

Discrepancies have been observed - Not all frustrated triads behave equally. So we study the 4 cases.

It has been measured that some unbalanced states are (sometimes) more common than balanced. What if we compare with a random network?
Comparative with a random network

Example: Timeseries of occupation probabilities in the political network of an Online World: Eve Online

- Balanced $[+ - -]$ (red) and $[+ + +]$ (magenta) are more common than random.
- Low Frustrated $[- - -]$ (green) is slightly more common than random.
- Strongly Frustrated $[+ + -]$ (blue) is much less common than random.
Each probability \( p_i \) is related to an energy of state \( i \) \( (E_i) \) and a degeneracy \( g(E_i) \).

\[
p_i = \frac{g(E_i)e^{-E_i/T}}{\sum_j g(E_j)e^{-E_j/T}}
\]

If all energies are the same, the probability matches with the situation in a random network.

\[
p_i = \frac{g(E_i)}{\sum_i g(E_i)}
\]

Lower (Higher) energies mean it would appear more often (less often) than in a random network:

The systems’ individual components "prefer" states with lower energies.
Political networks in Online World

\[ E_i / T = - \ln(p_i / g(E_i)) + cte \]

- Strongly Balanced [+ + +] (magenta) has the lowest energy.
- Low Frustrated [− − −] (green) and Low Balance [+ − −] (red) present similar energies.
- Strongly Frustrated [+ + −] (blue) has higher energy.
\[ \frac{E_i}{T} = -\ln\left(\frac{p_i}{g(E_i)}\right) + \text{cte} \]

- Strongly Balanced \([+++]\) (magenta) has the lower energy.
- Low Frustrated \([---]\) (green) and Low Balance \([+- -]\) (red) present similar energies, but low frustrate is slightly more common.
- Strongly Frustrated \([++-]\) (blue) has higher energy.
Can we describe the energies as a sum of energies related to the links?

| State | Energy |
|-------|--------|
| [++]  | $\alpha + 2\gamma + \omega$ |
| [---] | $\alpha - 3\gamma - 3\omega$ |
| [+-+] | $-\alpha + 2\gamma - \omega$ |
| [+++] | $-\alpha - 3\gamma + 3\omega$ |

- **three-body** $-\alpha s_{ij}s_{jk}s_{ki}$
- **two-body** $-\gamma(s_{ij}s_{jk} + s_{ij}s_{ki} + s_{jk}s_{ki})$
- **one-body** $\omega(s_{ij} + s_{jk} + s_{ki})$
Adjust of the Hamiltonian to the average of energies

| Parameter | EVE (SOV)   | EVE (+200)  | Cold War   | Middle East |
|-----------|-------------|-------------|------------|-------------|
| $\alpha$ (T) | 0.95 ± 0.03 | 1.02 ± 0.04 | 0.89 ± 0.07 | 1.09        |
| $\gamma$ (T)  | 0.38 ± 0.02 | 0.41 ± 0.02 | 0.61 ± 0.07 | 0.38        |
| $\omega$ (T)  | 0.18 ± 0.02 | 0.14 ± 0.02 | 0.14 ± 0.07 | 0.22        |
| Cte (T)      | 8.00 ± 0.03 | 8.04 ± 0.04 | 4.67 ± 0.07 | 2.70        |
Entropy associated with a system of discrete levels →
Information on the Homogeneity in the system:

\[ S = \sum_i p_i \ln(p_i) \]
Overview

- The proposed model allows one to quantitatively study social balance.
- We can separate the dynamical mechanisms (regulated by the energies) and stochastic aspects (random network).
- We find a persistent hierarchy, where the \([+ + +]\) \([+ + -]\) triad is the most (least) balanced.
- Shannon entropy \(\rightarrow\) detect the change points in the time series.
- Importance SBT (three-body forces); however, strong corrections via the two-body force.
Future Work: Extensions

- Transition Probabilities and their connection with the energy differences.
- Adding Neutral links: Introduce 6 new triads. \([0++], [0+-], [0--], [00+], [00-], [000] \).
Thanks for your attention.

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