Contextual objectivity and the quantum formalism.

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The “new orthodoxy” of quantum mechanics (QM) based on the decoherence approach requires many-worlds as an essential ingredient for logical consistency, and one may wonder what status to give to all these “other worlds”. Here we advocate that it is possible to build a consistent approach to QM where no other worlds are needed, and where the quantum formalism appears as a consequence of requiring the enumerability of physical properties. Such a quantization hypothesis is closely related to indistinguishability, and is deeply inconsistent with classical physics.

I. A SHORT STATUS REPORT.

The present orthodoxy of quantum mechanics - and even of physics in general - takes for granted that since the ultimate physical theory must a quantum theory, then it must be possible to reconstruct classical physics as a suitable approximation of quantum physics. For instance, classical physics should emerge from a suitable coarse-grained average done on localized objects, which is what we have at hand after a quantum system has undergone a decoherence mechanism.

Though the attempts done in that direction may be partially convincing, they hit what we consider to be an important conceptual difficulty, which is that any “decohered” quantum system sits in a “multiverse”, or “branching universe”, where the different possible outcomes of a quantum measurement (e.g. a Stern-Gerlach experiment) have exactly the same status. In order to interpret the trivial fact that only one result is found in a given measurement, and may decide about the whole future of our “unique” universe, one has to assume that we “live and perceive in one branch only”, and moreover that no further communication is possible between the various branches of the multiverse. Though one may consider this set of ideas as a definition of randomness, we cannot prevent ourselves to find it deeply unsatisfactory. Actually, it appears that such an attempt to give an ontological status to a quantum state leads to a complete mess-up of the very idea of ontology, since “all possibles” acquire exactly the same ontological status.

As a back-up, an alternative interpretation is the “old orthodoxy” provided by the Copenhagen interpretation of QM. Though this is really old-fashioned indeed, it has a modern version, that we can call the “minimalistic” interpretation of QM: it consists of looking at quantum states as algorithms for computing correlations between successive measurement outcomes. This view is quite nice and useful - and in some sense more solid than the previous ones - but it has the big disadvantage that it makes it very difficult to ask any question like: Why is QM the way it is? Such a question, having nothing to do with correlations between successive measurement outcomes, is simply heretic in this context, and thus cannot be answered. However, QM is now almost one century old, and the questions about how the baby was born should not be censored any longer.

II. AN ALTERNATIVE APPROACH.

A leading line of our alternative approach will be the following: since we know that the Copenhagen interpretation ripped to the bones - that is, the minimalistic interpretation - has always been quite successful, is there a way to see where it comes from - or in some sense, to make it necessary?

Actually, the Copenhagen interpretation has a built-in feature, which is the infamous “collapse of the wave function”. This collapse was reborn under a more civilized form in the minimalistic interpretation, which accepts only questions about correlations, and thus does not need to speak about any collapse. But one essential feature remains: a quantum measurement gives only one result, not many ones, in sharp opposition with the multiverse approach quoted above.

Therefore our line of attack will be the following: we will keep as close as possible to the minimalistic interpretation, and we will make more precise what actually occurs when a measurement is done: under idealized conditions, this simply defines the (quantum) state of the system, as explained in [1, 2, 3, 4]. As in the minimalistic interpretation, there is no need for adding any further collapse: all is already included in the fact that measurements define a state - and thus other measurements will simply define another state. Following this approach, a quantum state will be generally defined as the values of a set of physical properties, which can be predicted with certainty and measured reproducibly without changing the system[4]. The set of physical properties associated with all measurements needed to define a state constitutes a “context” (using the terminology of standard QM, this is a “complete set of commuting observables”). In a given context, all possible states constitute a set of “exclusive modalities”, that is, they are associated with measurement outcomes which can be unambiguously identified. This allows us to give an objective status to the quantum state, which is associated with predictions that are certain and observer-independent in a given context, hence the wording “contextual objectivity”.

Now the question is: why is this state a quantum state, and not a classical one? The intuitive answer we want to give is: “the state is quantum when the exclusive modalities are quantized, while the parameters defining the contexts are not”. To make it clearer through an ex-
ample, throwing a dice has a quantized outcome (a number between one and six) but also a quantized number of measurable physical quantities, because it is assumed that the dice always lies on one side. But if it is assumed that the dice can be oriented in space along any angles (using e.g. Euler angles), but that the measurement still gives only six outcomes whatever the dice orientation - then it becomes a quantum dice.

We may thus give the following reasonable postulate: a quantum system is characterized by the fact that any complete set of measurements will always provide a fixed number $N$ of possible exclusive modalities. The measurements themselves may depend on continuous parameters, so the number of possible measurements is in principle infinite. However, the number of mutually exclusive possible answers should be only $N$.

As a simple example, let us consider the case where a quantum measurement depending on one continuous angle $\alpha$ gives two results $+$ or $\cdot$. From our postulate, the same measurement done for the angle $\beta \neq \alpha$ will also give two results $+$ or $\cdot$. Classically the results for $\beta$ can be considered as “refinement” of the properties obtained for $\alpha$, and one should be able to define four exclusive modalities, $(\alpha : +, \beta : +)$, $(\alpha : +, \beta : -)$, $(\alpha : -, \beta : +)$, $(\alpha : -, \beta : -)$. But this contradicts our hypothesis (and also quantum mechanics): we have $\alpha$ and $\beta$ must be independent. Moreover, we assume that the number of exclusive modalities is only two, and therefore it cannot be split further by performing more measurements.

But why postulate that the number of exclusive measurement outcomes should be fixed? Actually, such a quantization postulate seems to be required to give a meaning to the fact that two systems are “identical”: two systems will be identical only if is possible to enumerate all their properties, and if they are all the same. It was very convincingly recognized by Leibnitz that identity cannot exist in classical physics, because one smaller detail is always able to distinguish two objects. He concluded that identity (or indistinguishability) is thus impossible, but here he was wrong: identity is possible, but quantum physics is required to give it a meaning.

**III. RECONSTRUCTING THE QUANTUM FORMALISM?**

Now, is the fact that $N$ is fixed enough to deduce the whole formalism of quantum mechanics? Though we cannot prove it fully presently, we are very tempted to answer positively. The main lines of the reasoning are the following:

(i) since we postulated that the $N$ exclusive modalities in one context cannot be “put together” with the $N$ exclusive modalities in another context, the modalities taken from two different contexts are essentially “non-exclusive”. In other words, it is impossible to “give more details” so that non-exclusive modalities would become exclusive, because we would end up in having more than $N$ exclusive modalities.

Therefore, the only meaningful question we can ask to the theory is: if the system is in modality $a_i$ of context $E$, what is the probability $p_{ij}$ to find it in modality $b_j$ of context $E'$? We emphasize that probabilities appear here to be logically needed for consistency: they are actually the only way to keep the hypothesis of having only $N$ exclusive modalities consistent with the infinite number of contexts. In some sense, this is the trade-off: we don’t want many worlds, but we absolutely need probabilities.

Then another question arises: is there any relation between the above $p_{ij}$, and the reciprocal probability $p_{ji}$ to find the system in modality $(a_i, E)$, knowing that it is in modality $(b_j, E')$? (it should be clear that in the framework of standard QM, one has $p_{ij} = p_{ji}$). Here we may consider that we are actually looking for a joint probability $p_{ij}$ connecting the modalities $(a_i, E)$ and $(b_j, E')$. However, the notion of a joint probability takes a special meaning, because the $N^2$ “events” $(a_i, E)$ and $(b_j, E')$ do not correspond to $N^2$ exclusive modalities for the system (this is forbidden by our main hypothesis). Nevertheless, in order to keep as close as possible to the notion of a symmetrical joint probability, we will simply assume that $p_{ij} = p_{ij}$. Then $p_{ij}$ depends symmetrically on the two contexts $E$ and $E'$, and correspond to the probability for the system to be in “both” modalities $(a_i, E)$ and $(b_j, E')$. This (non-classical) “both” is rigorously defined from the conditional probabilities, which keep a clear operational meaning in our case.

(ii) therefore, the theory must provide a $N \times N$ probability matrix $\Pi = \{p_{ij}\}$, giving the probabilities $p_{ij}$ connecting the modalities $(a_i, E)$ and $(b_j, E')$ as defined above. It is then simple to check that $\Pi$ is a doubly stochastic matrix (all lines and columns sum to one), which reduces to the identity if $E = E'$.

In order to manipulate these matrices, it is convenient to introduce orthogonal $N \times N$ projectors, by attributing a set of orthogonal projectors $\{\pi_i\}$ (resp. $\{\pi'_j\}$) to the exclusive modalities $\{a_i\}$ (resp. $\{b_j\}$) of the context $E$ (resp. $E'$). Then the question arises whether is possible to define an arbitrary doubly stochastic matrix as a function of these projectors. The answer is yes, by simply taking $: p_{ij} = Trace(\pi_i \pi'_j)$. This requires however that complex numbers (i.e. hermitian projectors) are used, because it is not possible to build an arbitrary doubly stochastic matrix from two sets of real projectors (there are not enough variables). Our projectors are therefore hermitian $N \times N$ matrices.

Now we are almost done, since a set of $N$ hermitian projectors $\{\pi_i\}$ can be associated with a set of rays in the
corresponding Hilbert space, that is with usual quantum states. Moreover, we already know that in context $E$ the measurement of the physical quantity $A$ will give with certainty the result $\alpha_i$ in state $\pi_i$. This is consistent with associating to $A$ the operator $\hat{A} = \sum_i \alpha_i \pi_i$, and thus recovering the standard definition of observables.

Another crucial feature is that the change between context $E$ and context $E'$ appears to be associated with a unitary matrix $\Sigma$, so that $\pi'_j = \Sigma \pi_j \Sigma^\dagger$. Since these matrices must also act on the operators $\hat{A}$, one should require that they provide a representation (in the sense of groups theory) of the geometrical transformations acting of the true physical quantities. Depending on the geometrical group which is considered (rotation group, Galileo group, Poincaré group), one should thus be able to reconstruct various kinds of Hilbert spaces (angular momentum, non-relativistic QM, relativistic QM...).

It is also important to note that the matrices $\Sigma$ connecting different contexts should not commute, otherwise one would come back simply to a classical alternative between $N$ exclusive modalities. As said before, having a discretized number of exclusive modalities is not enough to get quantum mechanics, one needs also that the measured physical quantities (the contexts) are connected by non-commutative geometrical transformations.

Finally, an interesting question is where $\hbar$ comes in. This can be illustrated by taking the example of angular momentum, associated with 3-D rotations acting on the true physical quantities. Following our approach, we have to build a unitary representation of the rotation group, which is well known to be given by $U_u = \exp(-i \, u \cdot \hat{j})$. Here $u$ is a 3-D vector defining a rotation $R_u$, and $\hat{j}$ is a set of $3$ (dimensionless) $N \times N$ matrices $j_{x,y,z}$, obeying the angular momentum commutation relations obtained from the Lie algebra of the rotation group. By a standard calculation we can get the eigenvalues and eigenvectors of the complete set $\{\hat{J}_x, \hat{J}_y, \hat{J}_z\}$. Now, one can also physically measure the angular momentum of the system, by using a Stern and Gerlach apparatus, and thus construct the “physical” observable $\hat{J}$, where $\{\hat{J}_x, \hat{J}_y, \hat{J}_z\}$ can be associated to a given context. It appears then that $\hat{j}$ and $\hat{J}$ have exactly the same behaviour, up to a multiplicative constant which is precisely $\hbar$.

The rotation case is especially simple because $\hbar$ has the dimension of an angular momentum. But if one takes for instance translations, the rule is just the same : the unitary transformation is $T_u = \exp(-i \, a \cdot \hat{p})$ where $a$ has dimension $L$ (length) and $\hat{p}$ has thus dimension $1/L$. Then the observable $\hat{P}$ constructed from the measurements results will fit with $\hat{p}$ simply by writing $\hat{P} = \hbar \, \hat{p}$. This reasoning clearly shows that $\hbar$ does appear as the “unit” of quantization, which is required to connect the two (“probabilistic” and “geometrical”) definitions of an observable which appear in our construction.

Therefore, the dimension of $\hbar$ is an action because it has to match a physical observable $\hat{A} = \sum_i \alpha_i \pi_i$ onto an infinitesimal generator, which has the dimension of the inverse of the physical quantity conjugated to $\hat{A}$. We note that this provides a view of canonically conjugated observables directly in the framework of group theory.

### IV. CONCLUSION

Though many assertions are still to be proven, our program is to establish that the quantum formalism is a consequence of the quantization hypothesis, and that this formalism is actually the (only ?) way to make quantization consistent.

The quantum “strangeness” is thus rooted in the fact that “enumerability” of properties, which is closely related to indistinguishability, is deeply inconsistent with classical physics. We emphasize again that our “fixed $N$” postulate should not be confused with classical discreteness, which is an approximation always subject to further refinement. The fact that the “gain of knowledge” must stop because there is “nothing” between zero and one is actually the crucial quantum hypothesis.

As a conclusion, here are a few more remarks :
- in our approach a quantum measurement is the operation by which we define a quantum state, and there is essentially no difference between the beginning (“preparation of the state”) and the end (“collapse of the state”). Therefore the so-called collapse problem essentially vanishes. In case you feel unhappy with this, a convenient retreat is the minimalistic interpretation introduced above : QM provides a way to compute correlations between successive measurement results.

- similarly, there is no need for many worlds. A quantum state is always defined relative to a classical context, in which the state-defining measurement is carried out. If the new context is incompatible with the previous ones (that is, if the new modalities are non-exclusive with the previous ones), the state is reset as one among the new exclusive modalities. This does not result from any mysterious “branching of universes”, but it is the unavoidable consequence of the quantum postulate, and the only way to keep the enumerability of exclusive modalities consistent with the continous infinity of contexts.

- the usual question of the “boundary” between the classical and the quantum world is ill posed. Presently, quantum mechanics is always formulated in a classical context, where Leibnitz’s intuition is correct : properties are not enumerable, and objects are never identical. But at the quantum level Leibnitz is wrong, and just the opposite becomes true. This difference makes a quite decent boundary, which is enough for all practical purposes.

- our approach may also suggest new questions : is it possible to formulate QM without any reference to classical physics ? or to build a “time-ordered” theory where $p_{ijj} \neq p_{jki}$ (see above) ? can we give a simple geometrical meaning to all physical quantities in a unit system where $\hbar = 1$ ? Further considerations about entanglement, Bell’s inequalities, and related subjects can be found in previous eprints and publications [1, 2, 3, 4, 5].
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