Entanglement and decoherence of massive particles due to gravity

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Abstract

We analyze the dynamics of gravity-induced entanglement for $N$ massive particles. Considering a linear configuration of these particles, we investigate the entanglement between a specific pair of particles under the influence of the gravitational interaction between the massive particles. As the particle number increases, the specific particle pair decoheres more easily due to the gravitational interaction with other particles. The time scale of the gravity-induced decoherence is found analytically. We also discuss the entanglement dynamics of initially entangled particles, which exemplify the monogamy of gravity-induced entanglement.
I. INTRODUCTION

The unification of quantum mechanics with general relativity is one of the most fundamental problems in theoretical physics [1]. The superstring theory is promising as a candidate for theorizing quantum gravity (e.g., [2]). However, many unresolved issues remain. One of the reasons is that there are almost no experimental studies on quantum gravity. There are only a few experiments to test quantum mechanics in a classical gravitational field. The COW experiment was the first to show that a neutron follows quantum mechanics in a uniform gravitational field [3]. An experiment of a bouncing neutron in a uniform gravitational field has also been useful for investigating the quantum mechanics in a gravitational field (Ref. [4, 5], cf. [6]). However, the experiments so far have not answered the question of whether gravity follows quantum mechanics or not.

Recent advances in quantum sciences have opened the possibility of testing the quantum properties of gravity [7–9]. An interesting approach to test the superposition principle in a gravitational potential was proposed [10, 11], in which whether quantum entanglement is generated by gravity or not is studied. This proposal is based on a theorem in quantum information theory that quantum entanglement cannot be generated by local operations and classical communication (LOCC) [12]. For example, consider a quantum system composed of two subsystems A and B. Two local observers, Alice and Bob, perform arbitrary quantum operations on the subsystems A and B, respectively, and send classical information each other. This process called LOCC cannot increase the entanglement between the subsystems A and B. Hence, a non-local quantum operation is necessary to generate an entanglement between the two subsystems. The production of entanglement through gravity can be a test of the quantumness of gravity.

In Refs. [10, 11], an experimental test of gravity-induced entanglement in a matter-wave interferometer called the BMV experiment was proposed. In the experimental setup, two massive particles with spin are initially in superposed states, and the gravitational interaction between these particles induces quantum entanglement. The entanglement is then detected by measuring the spin correlations. For the feasible detection of entanglement, one requires the superposition of a mesoscopic particle. In Ref. [13], the authors provided an experimental setup for realizing such a superposition, and the authors of [14] discussed the origin of generating the quantum entanglement. These studies have stimulated many
works on testing the quantum properties of gravity \cite{15, 21}. Nguyen and Bernards proposed a similar setup to the BMV experiment \cite{21}. They assumed two separated masses, each of which is superposed in the direction perpendicular to their separation. This model is more easily analyzed because the gravitational interaction is simplified by the symmetry of the configuration.

In the present paper, we extend the model proposed by Nguyen and Bernards \cite{21} to include $N$ massive particles (see Fig. 1) arranged in a linear configuration. This arrangement enables us to compute the quantum state of the total system explicitly. Then, we investigate the many-body effects of gravity on the quantum entanglement. Because gravity is unscreened locally (gravity is a long-range force), it might be interesting to examine how a quantum system coupled to other massive particles is influenced by gravitational interaction. We show that the gravity-induced entanglement between a specific pair of particles is degraded by the decoherence due to the gravitational interaction with the other massive particles. We also find that entanglement monogamy appears in the model by assuming an initially entangled state. The features of decoherence and entanglement monogamy are demonstrated in the model of $N$ massive particles following the superposition principle.

This paper is organized as follows. In Sec. 2, we introduce the $N$-particle system on a straight line. Each particle is assumed to be in a superposed position state in the direction perpendicular to the straight line (see Fig. 1). We present the Hamiltonian of the system, which describes the gravitational interaction between the particles written in a simple form. We also present the reduced density matrix of a specific pair in the system, for which we evaluate the time-evolution of the entanglement negativity. In Sec. 3, we consider the case in which the initial state is an entangled state. This state demonstrates a monogamous behavior due to gravity. Sec. 4 presents the summary and conclusions. In Appendix A, we describe the construction of the Hamiltonian. In Appendix B, a review of the derivation of the Hamiltonian Eq. (3) is presented. In Appendix C, we present a proof of the negativity of the eigenvalues of the partial transposed density matrix Eq. (12). In Appendix D, we show the density matrix of the initially entangled state.
II. SYSTEM OF $N$ PARTICLES

In this section, we introduce the system of $N$ massive particles to investigate the quantum nature of gravity. These particles are placed at a separation of $d$ from their immediate neighbors. The $i$-th particle has mass $m_i$. Each particle is initially prepared as the superposition of two spatially localized states separated by distance $L$ along the same direction. We align the $N$ particles so that the superposition is along the vertical direction. This model, which is depicted in Fig. 1, is an extension of the model in Ref. [21] that considered the $N = 2$ case.

We use notations $|↑_i\rangle$ and $|↓_i\rangle$ to represent the states of the $i$-th particle at the left and right paths, respectively. We consider the case where the initial state of the total system is

$$|\Psi(0)\rangle = |\psi_1(0)\rangle \otimes \cdots \otimes |\psi_N(0)\rangle$$

where $|\psi_i(0)\rangle$ is the initial state of the $i$-th particle

$$|\psi_i(0)\rangle = \frac{1}{\sqrt{2}} (|↑_i\rangle + |↓_i\rangle).$$

FIG. 1: Sketch of our model of the $N$-particle system. Each particle is placed at a distance $d$ from its adjacent particles, and initially prepared in a superposition state with the position separated by distance $L$. The direction of the separation between each particle $d$ is orthogonal to the direction of the separation of the particle with its superposed position $L$. The $i$-th particle from the left ($1 \leq i \leq N$) has mass $m_i$. The particles interact with one another through gravity.
The initial state evolves under the gravitational interaction. The corresponding Hamiltonian is

\[ H = \sum_{i<j}^N H_{ij}, \quad H_{ij} = -\frac{\Delta_{ij}}{2} I_1 \otimes \cdots \otimes \sigma_z^{(i)} \otimes \cdots \otimes \sigma_z^{(j)} \otimes \cdots \otimes I_N \]  

(3)

where \( H_{ij} \) (up to a constant) describes the Newtonian potential between the \( i \)- and \( j \)-th particles and \( \Delta_{ij} \) for \( i<j \) is

\[ \Delta_{ij} = Gm_i m_j \left( \frac{1}{d(j-i)} - \frac{1}{\sqrt{(i-j)^2 + L^2}} \right). \]  

(4)

In Appendix A, we show that the familiar Hamiltonian from the Newtonian potential is described by the combination of \( H_{ij} \) and another term that only contributes to a total phase, which we omit.

The state of the total system at time \( t \) is \( |\Psi(t)\rangle = e^{-iHt/\hbar} |\Psi(0)\rangle \). The density matrix of the total system is obtained by explicit computation. Here, we focus on the entanglement between the 1st and 2nd particles. Tracing over the 3rd to the \( N \)-th particles in the density operator \( \rho(t) = |\Psi(t)\rangle \langle \Psi(t)| \), we obtain the reduced density matrix of the 1st and 2nd particles as

\[
\rho_{12}(t) = \text{Tr}_{3,\ldots,N}[\rho(t)] = \frac{1}{4} \begin{pmatrix}
1 & e^{i\Delta_{12}} & e^{i\Delta_{12}t} & e^{i\Delta_{12}t} \\
1 & 1 & e^{i\Delta_{12}t} & e^{i\Delta_{12}t} \\
e^{-i\Delta_{12}t} & e^{-i\Delta_{12}t} & 1 & e^{-i\Delta_{12}t} \\
e^{-i\Delta_{12}t} & e^{-i\Delta_{12}t} & e^{-i\Delta_{12}t} & 1 \\
\end{pmatrix}
\]

\[
= \frac{1}{4} \begin{pmatrix}
1 & e^{i\Delta_{12}} & e^{i\Delta_{12}t} & e^{i\Delta_{12}t} \\
1 & 1 & e^{i\Delta_{12}t} & e^{i\Delta_{12}t} \\
e^{-i\Delta_{12}t} & e^{-i\Delta_{12}t} & 1 & e^{-i\Delta_{12}t} \\
e^{-i\Delta_{12}t} & e^{-i\Delta_{12}t} & e^{-i\Delta_{12}t} & 1 \\
\end{pmatrix}
\]

(5)

Here, the order of the basis is \{\(|\uparrow_1\rangle |\uparrow_2\rangle, \ |\uparrow_1\rangle |\downarrow_2\rangle, \ |\downarrow_1\rangle |\uparrow_2\rangle, \ |\downarrow_1\rangle |\downarrow_2\rangle\}\}. In the following, we discuss the quantum entanglement due to Newtonian gravity for \( N = 3 \) and \( N > 3 \) after reviewing the case of \( N = 2 \).
A. Two-particle system ($N = 2$)

Here, we consider the system consisting of only two massive particles, which is the same as the model investigated in Ref. [21]. For $N = 2$, the initial state Eq. (1) is

$$|\Psi(0)\rangle = |\psi_1(0)\rangle \otimes |\psi_2(0)\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\rangle + |\downarrow_1\rangle) \otimes \frac{1}{\sqrt{2}}(|\uparrow_2\rangle + |\downarrow_2\rangle),$$

and the Hamiltonian of the two particles is

$$H = -\frac{\Delta_{12}}{2} \sigma_z^{(1)} \otimes \sigma_z^{(2)}, \quad \Delta_{12} = Gm_1m_2 \left(\frac{1}{d} - \frac{1}{\sqrt{d^2 + L^2}}\right).$$

The density matrix of a given pure state is

$$\rho(t) = e^{-iHt/\hbar} |\Psi(0)\rangle \langle \Psi(0)| e^{iHt/\hbar}$$

$$= \frac{1}{4} \begin{pmatrix}
1 & e^{i\Delta_{12}t} & e^{i\Delta_{12}t} & 1 \\
e^{-i\Delta_{12}t} & 1 & 1 & e^{-i\Delta_{12}t} \\
e^{-i\Delta_{12}t} & 1 & 1 & e^{-i\Delta_{12}t} \\
1 & e^{i\Delta_{12}t} & e^{i\Delta_{12}t} & 1
\end{pmatrix}. \quad (6)$$

We analyze the entanglement using the positive partial transpose (PPT) criterion [22]. According to this criterion, the state is entangled if at least one of the eigenvalues of the partial transposed matrix of the density matrix is negative. We now introduce the negativity defined as

$$\mathcal{N} = \sum_{\lambda_i < 0} |\lambda_i| \quad (7)$$

![FIG. 2: The negativity $\mathcal{N}$ computed from the partial transposed matrix Eq. (9) at $N = 2$ as a function of the dimensionless time $\Delta_{12}t/\hbar$. Because $\mathcal{N}$ is greater than or equal to zero, the state of two particles is entangled except only when $\mathcal{N} = 0.$](image-url)
where the $\{\lambda_i\}$s are the eigenvalues of the partial transposed matrix. The PPT criterion implies that the state is entangled if the negativity is positive, i.e., $\mathcal{N} > 0$. The eigenvalues of the partial transposed matrix are

$$
\lambda_{\pm} = \pm \frac{1}{2} \sin \left( \frac{\Delta_{12} t}{\hbar} \right), \quad \lambda'_{\pm} = \frac{1}{2} \left( 1 \pm \cos \left( \frac{\Delta_{12} t}{\hbar} \right) \right).
$$

(8)

$\lambda_{\pm}'$s are always positive, i.e., $\lambda_{\pm}' > 0$. In contrast, either $\lambda_+$ or $\lambda_-$ is always negative or zero. Therefore, the negativity is

$$
\mathcal{N} = \frac{1}{2} \left| \sin \left( \frac{\Delta_{12} t}{\hbar} \right) \right|.
$$

(9)

Fig. 2 shows $\mathcal{N}$ as a function of the dimensionless time $\Delta_{12} t/\pi \hbar$. The two particles periodically oscillate between the maximally entangled and non-entangled states. This is because the effects of the environment have not been considered [21].

B. Three-particle system ($N = 3$)

We next consider the system with three massive particles $N = 3$ and focus on the entanglement between two particles in this system. The initial state of the three particles is

$$
|\Psi(0)\rangle = |\psi_1(0)\rangle \otimes |\psi_2(0)\rangle \otimes |\psi_3(0)\rangle
$$

$$
= \frac{1}{\sqrt{2}}(|\uparrow_1\rangle + |\downarrow_1\rangle) \otimes \frac{1}{\sqrt{2}}(|\uparrow_2\rangle + |\downarrow_2\rangle) \otimes \frac{1}{\sqrt{2}}(|\uparrow_3\rangle + |\downarrow_3\rangle).
$$

(10)

![Fig. 3: Behavior of $\mathcal{N}$ as a function of the dimensionless time $\Delta_{12} t/\pi \hbar$ for $L/d = 1$ when the three particles have the same mass. In the case of $N = 3$, $\mathcal{N}$ either takes positive values or is zero; hence, the 1st and 2nd particles are always entangled except when $\mathcal{N} = 0.$]
The unitary evolution of this system is governed by $U(t) = \exp[-iHt/\hbar]$ with the Hamiltonian $H = H_{12} + H_{13} + H_{23}$. By tracing over the 3rd particle, the reduced density matrix of the two particles (the density matrix Eq. (5) for $N = 3$) is obtained as

$$\rho_{12}(t) = \frac{1}{4} \begin{pmatrix}
1 & e^{i\Delta_{12}t} \cos(\frac{\Delta_{23}t}{\hbar}) & e^{i\Delta_{12}t} \cos(\frac{\Delta_{13}t}{\hbar}) & \cos(\frac{\Delta_{11} + \Delta_{23}t}{\hbar}) \\
e^{-i\Delta_{12}t} \cos(\frac{\Delta_{23}t}{\hbar}) & 1 & \cos(\frac{\Delta_{13} - \Delta_{23}t}{\hbar}) & e^{-i\Delta_{12}t} \cos(\frac{\Delta_{13}t}{\hbar}) \\
e^{-i\Delta_{12}t} \cos(\frac{\Delta_{23}t}{\hbar}) & \cos(\frac{\Delta_{13} - \Delta_{23}t}{\hbar}) & 1 & e^{-i\Delta_{12}t} \cos(\frac{\Delta_{23}t}{\hbar}) \\
\cos(\frac{\Delta_{13} + \Delta_{23}t}{\hbar}) & e^{i\Delta_{12}t} \cos(\frac{\Delta_{23}t}{\hbar}) & e^{i\Delta_{12}t} \cos(\frac{\Delta_{13}t}{\hbar}) & 1
\end{pmatrix}.$$  

(11)

We investigate the entanglement between the 1st and 2nd particles based on the PPT criterion. We can compute the eigenvalues of the partial transposed matrix in Eq. (11). Two of the four eigenvalues, Eq. (C1) and Eq. (C2), are presented in Appendix B. As shown in Appendix B, the negativity can be written as

$$\mathcal{N} = -\frac{1}{4} \left(1 - \left|\cos(\frac{\Delta_{13}t}{\hbar}) \cos(\frac{\Delta_{23}t}{\hbar})\right| \right.$$ 

$$- \sqrt{1 + \cos^2(\frac{\Delta_{13}t}{\hbar}) \cos^2(\frac{\Delta_{23}t}{\hbar}) - 2 \cos(2\frac{\Delta_{12}t}{\hbar}) \cos(\frac{\Delta_{13}t}{\hbar}) \cos(\frac{\Delta_{23}t}{\hbar})} \right).$$  

(12)

Fig. 3 shows $\mathcal{N}$ as a function of the dimensionless time $\Delta_{12}t/\pi\hbar$. Here, we assume that the three particles have the same mass and distance between each particle is equal to the superposition distance, i.e., $L = d$. We find that the negativity $\mathcal{N}$ is positive (the 1st and 2nd particles are entangled) except at the zeros that appear periodically. The maximum value of $\mathcal{N}$ varies and is smaller than $1/2$, unlike the case for $N = 2$. These differences are caused by the gravitational interaction with the 3rd particle. The reduction of the entanglement can be understood as being due to the gravity-induced entanglement with the additional 3rd particle that plays the role of the environment.

C. $N$-particle system ($N > 3$)

In this subsection, we consider an $N$-particle system with more than three particles, i.e., $N > 3$. We can compute the entanglement from the reduced density matrix Eq. (5) with respect to the 1st and 2nd particles. It is easy to obtain the eigenvalues of the partial transposed matrix in Eq. (5). We find that two of the four eigenvalues can be negative. Here, we assume that all of the particles have the same mass $m$. Figure 4 demonstrates the
evolution of the negativity and the four eigenvalues of the partial transposed matrix in Eq. (5) for \( N = 10 \) and \( L/d = 1 \). Compared to the \( N = 3 \) case, we see that the eigenvalues take negative values only for a short period after the initial time. One of the two eigenvalues takes negative values at \( t \lesssim 2\pi\hbar/\Delta_{12} \), but both eigenvalues then become positive. This means that the entanglement between the 1st and 2nd particles disappears at \( t \gtrsim 2\pi\hbar/\Delta_{12} \). The gravitational interaction generates the entanglement between the 1st and 2nd particles, as well as the entanglement between these two and the other particles. The result exemplifies the decoherence phenomenon due to the gravity, although this decoherence is investigated in the framework of an open quantum system [23].

![Graph](image)

**FIG. 4:** (left panel): Behavior of negativity \( \mathcal{N} \) as a function of the dimensionless time \( \Delta_{12}t/\pi\hbar \). Here, we assume ten particles (\( N = 10 \)) with the same mass, and \( L/d = 1 \). The negativity takes positive values for only a short period of \( t \lesssim 2\pi\hbar/\Delta_{12} \). (right panel): Behavior of the corresponding four eigenvalues (red solid line, blue dashed line, green dotted line, and purple dot-dash line). The negativity is positive if at least one of the eigenvalues is negative, and zero if all eigenvalues are positive.

The entanglement dynamics depend on ratio \( L/d \) between the length scale of the superposition \( L \) and distance between the adjacent particles \( d \). Figure shows the time evolution of the negativity and eigenvalues of the partial transposed matrix Eq. (5) at \( L/d = 10^{-3} \) (upper panels) and \( L/d = 10^3 \) (lower panels). The early entangled phase at \( L/d = 10^3 \) lasts for a shorter time than at \( L/d = 1 \) and \( L/d = 10^{-3} \). This is because the particles are close to one another when \( L \gg d \), and the 1st and 2nd particles rapidly decohere because of the gravitational interaction with the other particles. In contrast, when \( L \ll d \), the entanglement between the two particles is less likely to be affected by the other particles because the
\[ N = 10 \]
\[ L / d = 10^{-3} \]

\[ N = 10 \]
\[ L / d = 10^{3} \]

\[ \lambda \]

\[ \Delta_{12} \pi \hbar \]

\[ \Delta_{12} \pi \hbar \]

\[ \lambda \]

\[ \lambda \]

\[ \Delta_{12} \pi \hbar \]

\[ \Delta_{12} \pi \hbar \]

FIG. 5: Same as Fig. 4 but with \( L / d = 10^{-3} \) (upper left and right panels), and with \( L / d = 10^{3} \) (lower left and right panels).

particles are far from one another. Hence, the entangled phase lasts longer than the other cases of \( L / d = 1 \) and \( L / d = 10^{3} \).

Let us examine the decoherence behavior analytically by taking the limit of ratio \( L / d \). As the off-diagonal components of Eq. 5 characterize the coherence of the two particles, the decay time of these components determines that of the entanglement. From the inequality \( \cos \theta \leq e^{-\theta^{2}/2} \) for \( 0 \leq \theta \leq \pi / 2 \), the absolute values of the off-diagonal components of Eq. 5 satisfy

\[ |\rho_{12}| = |\rho_{34}| = \prod_{i=3}^{N} \cos \left[ \frac{\Delta_{12} i t}{\hbar} \right] \leq \exp \left[ -\sum_{i=3}^{N} \frac{\Delta_{12} i t^{2}}{2\hbar^{2}} \right], \]

\[ (13) \]

\[ |\rho_{13}| = |\rho_{24}| = \prod_{i=3}^{\infty} \cos \left[ \frac{\Delta_{11} i t}{\hbar} \right] \leq \exp \left[ -\sum_{i=3}^{N} \frac{\Delta_{11} i t^{2}}{2\hbar^{2}} \right], \]

\[ (14) \]

\[ |\rho_{14}| = \prod_{i=3}^{N} \cos \left[ \frac{(\Delta_{11} + \Delta_{21}) i t}{\hbar} \right] \leq \exp \left[ -\sum_{i=3}^{N} \frac{(\Delta_{11} + \Delta_{21})^{2} i t^{2}}{2\hbar^{2}} \right], \]

\[ (15) \]

\[ |\rho_{23}| = \prod_{i=3}^{N} \cos \left[ \frac{(\Delta_{11} - \Delta_{21}) i t}{\hbar} \right] \leq \exp \left[ -\sum_{i=3}^{N} \frac{(\Delta_{11} - \Delta_{21})^{2} i t^{2}}{2\hbar^{2}} \right], \]

\[ (16) \]
for $0 \leq (\Delta_{1i} \pm \Delta_{2j})t/h \leq \pi/2$. These inequalities enable us to examine the behavior of the off-diagonal components. Under condition $L \gg d$, approximation $\Delta_{ij} \sim Gm^2/[d(j-i)]$ can be taken, and we can estimate the upper bounds of the absolute values of the off-diagonal components at $N \to \infty$ as

$$\rho_{12} = |\rho_{34}| \leq e^{-\sum_{i,j=1}^{\infty} \frac{1}{(i-2)^2} \left( \frac{Gm^2 i^2}{\Delta_{12}} \right)^2} = e^{-\zeta(2) \left( \frac{Gm^2 i^2}{\Delta_{12}} \right)^2},$$

(17)

$$\rho_{13} = |\rho_{24}| \leq e^{-\sum_{i,j=1}^{\infty} \frac{1}{(i-1)^2} \left( \frac{Gm^2 i^2}{\Delta_{12}} \right)^2} = e^{-\zeta(2) - 1 \left( \frac{Gm^2 i^2}{\Delta_{12}} \right)^2},$$

(18)

$$\rho_{14} \leq e^{-\sum_{i,j=1}^{\infty} \frac{1}{(i-1)^2} \left( \frac{Gm^2 i^2}{\Delta_{12}} \right)^2} = e^{-\zeta(2) + 1 \left( \frac{Gm^2 i^2}{\Delta_{12}} \right)^2},$$

(19)

$$\rho_{23} \leq e^{-\sum_{i,j=1}^{\infty} \frac{1}{(i-1)^2} \left( \frac{Gm^2 i^2}{\Delta_{12}} \right)^2} = e^{-\zeta(2) - 3 \left( \frac{Gm^2 i^2}{\Delta_{12}} \right)^2},$$

(20)

where $\zeta(n)$ is the zeta function. Therefore, we may write the decoherence time of our model when $L \gg d$ as

$$t_D \sim \frac{dh}{Gm^2} \sim \frac{\hbar}{\Delta_{12}}.$$  

(21)

In the lower left panel of Fig. 5 which corresponds to $L/d = 10^3$, the negativity becomes zero when $\Delta_{12} t/\hbar \pi \sim \mathcal{O}(1)$. This can be roughly explained by the decoherence time Eq. (21). Conversely, when $L \ll d$, the upper bounds are evaluated as

$$\rho_{12} = |\rho_{34}| \leq e^{-\sum_{i,j=1}^{\infty} \frac{1}{(i-2)^2} \left( \frac{Gm^2 i^2}{dh} \right)^2} = e^{-\frac{\zeta(6)}{8} \left( \frac{Gm^2 i^2}{dh} \right)^2},$$

(22)

$$\rho_{13} = |\rho_{24}| \leq e^{-\sum_{i,j=1}^{\infty} \frac{1}{(i-1)^2} \left( \frac{Gm^2 i^2}{dh} \right)^2} = e^{-\frac{\zeta(6) - 1}{8} \left( \frac{Gm^2 i^2}{dh} \right)^2},$$

(23)

$$\rho_{14} \leq e^{-\sum_{i,j=1}^{\infty} \frac{1}{(i-1)^2} \left( \frac{Gm^2 i^2}{dh} \right)^2} = e^{-\frac{1}{8} (2\zeta(6) - 12\zeta(2) + 19) \left( \frac{Gm^2 i^2}{dh} \right)^2},$$

(24)

$$\rho_{23} \leq e^{-\sum_{i,j=1}^{\infty} \frac{1}{(i-1)^2} \left( \frac{Gm^2 i^2}{dh} \right)^2} = e^{-\frac{1}{8} (2\zeta(6) + 12\zeta(2) - 21) \left( \frac{Gm^2 i^2}{dh} \right)^2},$$

(25)

where we used approximation $\Delta_{ij} \sim Gm^2 L^2/[2d(j-i)]^3$ and considered the limit where $N$ goes to infinity. Therefore, the decoherence time is approximately estimated as

$$t_D \sim \frac{d^3 h}{Gm^2 L^2} \sim \frac{\hbar}{2\Delta_{12}}.$$  

(26)

In the left panel of Fig. 5 which corresponds to $L/d = 10^{-3}$, the negativity takes a value of zero when $\Delta_{12} t/\hbar \pi \sim \mathcal{O}(1)$. This time scale also roughly corresponds to the decay time (26) of the off-diagonal components.
Thus, the decoherence time can be roughly evaluated by the decay rate of the off-diagonal components of the reduced density matrix. The above results show that the decoherence time of our model does not strongly depend on the ratio of $L/d$, as long as the number of particles is sufficiently large. This might be due to the special characteristics of the one-dimensional configuration of our model. It is difficult to extend the above analysis to models with two- or three-dimensional configurations, and we do not consider such higher-dimensional models in this paper.

III. MONOGAMY OF INITIALLY ENTANGLED STATE

The results in the previous section demonstrate that the entanglement among a specific subsystem and the other systems causes quantum decoherence due to gravitational interaction. This feature can also be understood as an effect of the monogamy for quantum systems. Here, we focus on the monogamy due to gravity by considering the system prepared in an initial state in which the specific subsystem is entangled.

Specifically in this section, we consider the system consisting of three particles prepared in initial states in which the 2nd and 3rd particles are initially entangled, but are not entangled with the 1st particle. Namely, we consider the initial states

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\rangle + |\downarrow_1\rangle) \otimes (a|\uparrow_2\rangle |\downarrow_3\rangle + b|\downarrow_3\rangle |\uparrow_3\rangle),$$

where $|a|^2 + |b|^2 = 1$. Then, we analyze whether entanglement monogamy arises due to gravity. Entanglement monogamy is the condition in which if the 2nd and 3rd particles are maximally entangled, then the 1st particle cannot be entangled with the 2nd or 3rd particle [24]. The Hamiltonian is given by Eq. (3) with $N = 3$, and the evolved state at the time $t$ is $|\Psi(t)\rangle = e^{-i\mathcal{H}t/\hbar} |\Psi(0)\rangle$. The density matrix of the total system is given by Eq. (D2) in Appendix D. The reduced density matrix is given by

$$\rho_{12}(t) = \text{Tr}_3[|\Psi(t)\rangle \langle \Psi(t)|].$$

$$= \frac{1}{2} \begin{pmatrix}
|a|^2 & 0 & |a|^2 e^{-i\Delta_{12-13}/\hbar} & 0 \\
0 & |b|^2 & 0 & |b|^2 e^{i\Delta_{12-13}/\hbar} \\
|a|^2 e^{i\Delta_{12-13}/\hbar} & 0 & |a|^2 & 0 \\
0 & |b|^2 e^{-i\Delta_{12-13}/\hbar} & 0 & |b|^2
\end{pmatrix}. \quad (28)$$
We find that all the eigenvalues of the partially transposed matrix are positive; hence, there is no entanglement between the 1st and 2nd particles. Similarly, there is no entanglement between the 1st and 3rd particles. This clearly shows the appearance of monogamy in the entanglement through the gravitational interaction. Interestingly, this property appears even when the 2nd and 3rd particles are not maximally entangled.

On the other hand, the 1st particle is entangled with the composite system of the 2nd and 3rd particles. We partially transpose the density matrix Eq. (D2) in Appendix D concerning the 1st particle to analyze the entanglement between the 1st particle and the system comprising the 2nd and 3rd particles. We then obtain the eight eigenvalues of the partial transposed matrix, from which the negativity is given by

$$N_{1-(2,3)} = |a||b| \left| \sin \left[ \frac{\Delta_{12} - \Delta_{13}}{\hbar} t \right] \right|. \quad (29)$$

This negativity has the same form as that of the system of the two particles Eq. (9) when $a = b = 1/\sqrt{2}$. Thus, the entanglement between the 1st particle and the composite system of the 2nd and 3rd particles can be regarded as the entanglement for the $N = 2$ case. This results from the initial reduction of the underlying states. When we choose the initial state as Eq. (27), the underlying basis states of the 2nd and 3rd particles are $|\uparrow_2\rangle |\downarrow_3\rangle$ and $|\downarrow_2\rangle |\uparrow_3\rangle$. In contrast, when we choose the initial state as Eq. (10), the underlying basis states are $\{ |\uparrow_2\rangle |\uparrow_3\rangle, |\uparrow_2\rangle |\downarrow_3\rangle, |\downarrow_2\rangle |\uparrow_3\rangle, |\downarrow_2\rangle |\downarrow_3\rangle \}$. Additionally, we take the partial trace of the density matrix of the total system $|\Psi(t)\rangle \langle \Psi(t)|$ with the initial condition of Eq. (27) with respect to the 1st particle to focus on the entanglement between the 2nd and 3rd particles. We then find the following negativity of the partially transposed matrix:

$$N_{2-3} = |a||b| \cos \left[ \frac{\Delta_{12} - \Delta_{13}}{\hbar} t \right]. \quad (30)$$

From the above results, Eqs. (29) and (30), it can be seen that as $N_{1-2,3}$ increases, $N_{2-3}$ decreases and vice-versa. These properties can be considered as the effects of the monogamy of entanglement.

As a special case, we consider the case of $\Delta_{12} = \Delta_{13}$ in which the mass of the 2nd particle differs from that of the 3rd particle. Then, from Eqs. (29) and (30), we have

$$N_{1-2,3} = 0, N_{2-3} = |a||b|. \quad (31)$$
That is, the 1st particle is never entangled with the system of the 2nd and 3rd particles, and the 2nd particle is always entangled with the 3rd particle. In this situation, the gravitational interaction is

\[
H_{12} = -\frac{\Delta_{12}}{2} \sigma_z^{(1)} \otimes \sigma_z^{(2)} \otimes I_3
\]

\[
H_{13} = -\frac{\Delta_{12}}{2} \sigma_z^{(1)} \otimes I_2 \otimes \sigma_z^{(3)}.
\]

The system of the 1st and 2nd particles then evolves in the same way as the system of the 1st and 3rd particles. Therefore, the state of the 1st particle evolves in the same way as the 2nd and 3rd particles.

IV. SUMMARY AND CONCLUSION

We have investigated the quantum many-body effect in the entanglement between a multi-particle system due to gravity. Our model is the simplest one-dimensional extension of the work of Ref. [21]. This simplicity allows us to analyze the system exactly. We found that a specific particle pair in the three-particle system produces a periodic entanglement. For a \(N(> 3)\)-particle system, quantum entanglement in a specific particle pair may be present at earlier times, however the entanglement tends to disappear through entanglement with other particles due to gravity, which plays the role of the environment. This can be regarded as a decoherence phenomenon due to the gravity. We estimated the characteristic time of this type of decoherence for the first time. Furthermore, we discussed the monogamy of the entanglement by considering a system in which two out of the three particles in the system are prepared in an initially entangled state. These phenomena of quantum many-body systems will be useful for testing the quantum nature of gravity.

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Appendix A: GRAVITATIONAL INTERACTION BETWEEN TWO PARTICLES

The Hamiltonian representing the gravitational potential between the $i$-th and $j$-th particles can be written as

\[
H_{ij} = -Gm_im_j \begin{pmatrix}
\frac{1}{d|j-i|} & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{d^2|j-i|^2 + L^2}} & 0 & 0 \\
0 & 0 & \frac{1}{\sqrt{d^2|j-i|^2 + L^2}} & 0 \\
0 & 0 & 0 & \frac{1}{d|j-i|}
\end{pmatrix}
\]

\[
= -\frac{Gm_im_j}{2} \left( \frac{1}{d|j-i|} - \frac{1}{\sqrt{d^2|j-i|^2 + L^2}} \right) \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
= -\frac{Gm_im_j}{2} \left( \frac{1}{d|j-i|} + \frac{1}{\sqrt{d^2|j-i|^2 + L^2}} \right) \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

Here, the order of the basis is \{\ket{\uparrow_i} \ket{\uparrow_j}, \ket{\uparrow_i} \ket{\downarrow_j}, \ket{\downarrow_i} \ket{\uparrow_j}, \ket{\downarrow_i} \ket{\downarrow_j}\}. Using the Pauli matrices of the individual two-level systems $\sigma_z^{(i)}$ and $\sigma_z^{(j)}$, and the unit matrix, the Hamiltonian can be written as

\[
H_{ij} = -\frac{\Delta_{ij}}{2} \sigma_z^{(i)} \otimes \sigma_z^{(j)} - \frac{\Delta_C}{2} I_i \otimes I_j, \quad (A1)
\]

where $\Delta_{ij}$ and $\Delta_C$ are

\[
\Delta_{ij} = Gm_im_j \left( \frac{1}{d|j-i|} - \frac{1}{\sqrt{d^2|j-i|^2 + L^2}} \right), \quad (A2)
\]

\[
\Delta_C = Gm_im_j \left( \frac{1}{d|j-i|} + \frac{1}{\sqrt{d^2|j-i|^2 + L^2}} \right). \quad (A3)
\]

Here, by considering $\rho(t) = \ket{\Psi(t)} \bra{\Psi(t)}$, the second term of Eq. (A1) is canceled; thus, we consider only the first term in the Hamiltonian.
Appendix B: DERIVATION OF DENSITY MATRIX FOR THE SYSTEM OF \( N \) PARTICLES

The density matrix for the \( N \)-particle system is given by

\[
\rho(t) = |\Psi(t)\rangle \langle \Psi(t)| = e^{-iHt/\hbar} |\Psi(0)\rangle \langle \Psi(0)| e^{iHt/\hbar}
\]  

(B1)

where the initial state \( |\Psi(0)\rangle \) and Hamiltonian \( H \) are given by Eqs. (1) and (3), respectively. We focus on the system comprising the 1st and 2nd particles by tracing over the Hilbert space of the other particles. The reduced density matrix is then

\[
\rho_{12}(t) = \text{Tr}_{3,\ldots,N}[\rho(t)].
\]  

(B2)

By using the Bloch representation, the density matrix is given by

\[
\rho_{12}(t) = \frac{1}{4} \sum_{i,j} \lambda_{ij} \sigma_i^{(1)} \otimes \sigma_j^{(2)}, \quad (i, j = 0, 1, 2, 3)
\]  

(B3)

where \( \sigma_0 = I \) and the others are Pauli matrices. Determining coefficients \( \lambda_{ij} \) gives the expression for the reduced density matrix \( \rho_{12}(t) \). By using \( \text{Tr}[\sigma_i \sigma_j] = 2\delta_{ij} \), the coefficients \( \lambda_{ij} \) are

\[
\lambda_{ij} = \text{Tr}_{1,2}[\sigma_i^{(1)} \otimes \sigma_j^{(2)} \rho_{12}(t)]
\]

\[
= \text{Tr}_{1,2}[\sigma_i^{(1)} \otimes \sigma_j^{(2)} \text{Tr}_{3,\ldots,N}[\rho(t)]]
\]

\[
= \text{Tr}_{1,\ldots,N}[\sigma_i^{(1)} \otimes \sigma_j^{(2)}[\rho(t)]]
\]

\[
= \langle \Psi(t)| \sigma_i^{(1)} \otimes \sigma_j^{(2)} |\Psi(t)\rangle.
\]  

(B4)
After performing complex calculations, we obtain \( \lambda_{ij} = 0 \) except for

\[
\lambda_{00} = 1,
\]

\[
\lambda_{01} = \cos\left(\frac{\Delta_{12} t}{\hbar}\right) \prod_{i=3}^{N} \cos\left(\frac{\Delta_{2i} t}{\hbar}\right),
\]

\[
\lambda_{10} = \cos\left(\frac{\Delta_{12} t}{\hbar}\right) \prod_{i=3}^{N} \cos\left(\frac{\Delta_{1i} t}{\hbar}\right),
\]

\[
\lambda_{11} = \frac{1}{2} \left[ \prod_{i=3}^{N} \cos\left(\frac{\Delta_{1i} + \Delta_{2i} t}{\hbar}\right) + \prod_{i=3}^{N} \cos\left(\frac{\Delta_{1i} - \Delta_{2i} t}{\hbar}\right) \right],
\]

\[
\lambda_{22} = -\frac{1}{2} \left[ \prod_{i=3}^{N} \cos\left(\frac{\Delta_{1i} + \Delta_{2i} t}{\hbar}\right) - \prod_{i=3}^{N} \cos\left(\frac{\Delta_{1i} - \Delta_{2i} t}{\hbar}\right) \right],
\]

\[
\lambda_{23} = -\sin\left(\frac{\Delta_{12} t}{\hbar}\right) \prod_{i=3}^{N} \cos\left(\frac{\Delta_{1i} t}{\hbar}\right),
\]

\[
\lambda_{32} = -\sin\left(\frac{\Delta_{12} t}{\hbar}\right) \prod_{i=3}^{N} \cos\left(\frac{\Delta_{2i} t}{\hbar}\right).
\]

We finally derive the density matrix, Eq. (5), by substituting these coefficients into Eq. (B3).

**Appendix C: EIGENVALUES OF PARTIAL TRANSPOSED MATRIX FOR THE SYSTEM OF THREE PARTICLES**

We find that the four eigenvalues of the partial transposed matrix of Eq. (11) can be written as

\[
\lambda_{\pm} = \frac{1}{4} \left[ 1 \pm \cos\left(\frac{\Delta_{13} t}{\hbar}\right) \cos\left(\frac{\Delta_{23} t}{\hbar}\right) \right. \\
- \sqrt{1 + \cos^2\left(\frac{\Delta_{13} t}{\hbar}\right) \cos^2\left(\frac{\Delta_{23} t}{\hbar}\right) \pm 2 \cos\left(\frac{\Delta_{13} t}{\hbar}\right) \cos\left(\frac{\Delta_{13} t}{\hbar}\right) \cos\left(\frac{\Delta_{23} t}{\hbar}\right) \cos\left(\frac{\Delta_{23} t}{\hbar}\right)}
\]

\[
\lambda'_{\pm} = \frac{1}{4} \left[ 1 \pm \cos\left(\frac{\Delta_{13} t}{\hbar}\right) \cos\left(\frac{\Delta_{23} t}{\hbar}\right) \right. \\
+ \sqrt{1 + \cos^2\left(\frac{\Delta_{13} t}{\hbar}\right) \cos^2\left(\frac{\Delta_{23} t}{\hbar}\right) \pm 2 \cos\left(\frac{\Delta_{13} t}{\hbar}\right) \cos\left(\frac{\Delta_{13} t}{\hbar}\right) \cos\left(\frac{\Delta_{23} t}{\hbar}\right) \cos\left(\frac{\Delta_{23} t}{\hbar}\right)}
\]

where the \( \lambda'_{\pm} \)'s are always positive or zero. We hence consider \( \lambda_{\pm} \). When we assume \( \lambda_{\pm} \geq 0 \), the following inequality holds:

\[
\left(1 - \cos\left(\frac{2\Delta_{12} t}{\hbar}\right)\right) \cos\left(\frac{\Delta_{13} t}{\hbar}\right) \cos\left(\frac{\Delta_{23} t}{\hbar}\right) \geq 0.
\]
Because $1 - \cos\left(\frac{2\Delta_{12}}{\hbar} t\right) \geq 0$, we have $\cos\left(\frac{\Delta_{13}}{\hbar} t\right) \cos\left(\frac{\Delta_{23}}{\hbar} t\right) \geq 0$. We then obtain the inequality

\[\begin{align*}
(1 - \cos\left(\frac{\Delta_{13}}{\hbar} t\right) \cos\left(\frac{\Delta_{23}}{\hbar} t\right)) & \geq 0, \\
(1 + \cos^2\left(\frac{\Delta_{13}}{\hbar} t\right) \cos^2\left(\frac{\Delta_{23}}{\hbar} t\right) - 2 \cos\left(\frac{2\Delta_{12}}{\hbar} t\right) \cos\left(\frac{\Delta_{13}}{\hbar} t\right) \cos\left(\frac{\Delta_{23}}{\hbar} t\right)) & = 2(1 - \cos\left(\frac{2\Delta_{12}}{\hbar} t\right) \cos\left(\frac{\Delta_{13}}{\hbar} t\right) \cos\left(\frac{\Delta_{23}}{\hbar} t\right)) \\
& \leq 0,
\end{align*}\]

where the last inequality holds by $\cos\left(\frac{\Delta_{13}}{\hbar} t\right) \cos\left(\frac{\Delta_{23}}{\hbar} t\right) \geq 0$. Thus, we have

\[1 - \cos\left(\frac{\Delta_{13}}{\hbar} t\right) \cos\left(\frac{\Delta_{23}}{\hbar} t\right) \leq \sqrt{1 + \cos^2\left(\frac{\Delta_{13}}{\hbar} t\right) \cos^2\left(\frac{\Delta_{23}}{\hbar} t\right) - 2 \cos\left(\frac{2\Delta_{12}}{\hbar} t\right) \cos\left(\frac{\Delta_{13}}{\hbar} t\right) \cos\left(\frac{\Delta_{23}}{\hbar} t\right)},
\]

which implies that $\lambda_- \leq 0$. Similarly, when we assume $\lambda_+ \geq 0$, we have $\cos\left(\frac{\Delta_{13}}{\hbar} t\right) \cos\left(\frac{\Delta_{23}}{\hbar} t\right) \leq 0$, which leads to $\lambda_+ \leq 0$. Therefore, one of the eigenvalues $\lambda_{\pm}$ necessarily takes negative values; thus, the negativity is given by Eq. (12).

**Appendix D: DENSITY MATRIX OF THE INITIALLY ENTANGLLED SYSTEM**

We consider the three-particle system in which two particles are initially entangled. The density matrix is

\[\rho(t) = e^{-i\mathcal{H}t/\hbar} |\Psi(0)\rangle \langle \Psi(0)| e^{i\mathcal{H}t/\hbar}, \tag{D1}\]

where the initial state and Hamiltonian for $N = 3$ are given by Eqs. (27) and (3), respectively. By evaluating this expression, we eventually obtain

\[
\rho(t) = \frac{1}{2} \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & |a|^2 & a^*b & 0 & 0 & 0 & 0 & 0 \\
0 & a^*b e^{-i(\Delta_{12} - \Delta_{13})t/\hbar} & |b|^2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & a^*b & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & |a|^2 & a^*b e^{-i(\Delta_{12} - \Delta_{13})t/\hbar} & 0 & 0 & 0 \\
0 & 0 & a^*b & 0 & 0 & |b|^2 e^{-i(\Delta_{12} - \Delta_{13})t/\hbar} & 0 & 0 \\
0 & 0 & 0 & 0 & a^*b & 0 & |b|^2 & 0
\end{pmatrix} \tag{D2}\]
where the order of the basis is \{\ket{\uparrow_1\uparrow_2\uparrow_3}, \ket{\uparrow_1\uparrow_2\downarrow_3}, \ket{\uparrow_1\downarrow_2\uparrow_3}, \ket{\downarrow_1\uparrow_2\uparrow_3}, \ket{\downarrow_1\uparrow_2\downarrow_3}, \ket{\downarrow_1\downarrow_2\uparrow_3}, \ket{\downarrow_1\downarrow_2\downarrow_3}\}.

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