Excitonic condensate and quasiparticle transport in electron-hole bilayer systems

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(Dated: March 23, 2022)

PACS numbers: 73.20.Mf, 71.35.Lk

I. INTRODUCTION

Bilayer electron-hole systems undergo excitonic condensation when the distance $d$ between the layers is smaller than the typical distance between particles within a layer. All excitons in this condensate have a fixed dipole moment which points perpendicular to the layers, and therefore this condensate of dipoles couples to external electromagnetic fields. We study the transport properties of this dipolar condensate system based on a phenomenological model which takes into account contributions from the condensate and quasiparticles. We discuss, in particular, the drag and counterflow transport, in-plane Josephson effect, and noise in the in-plane currents in the condensate state.

Although these systems are not truly superfluid (the $U(1)$ symmetry associated with the phase of the exciton order parameter is not an exact symmetry) various signatures of excitonic condensation can be probed provided that the excitons are long-lived. At present, there are three main candidates for realization of excitonic condensation in double quantum wells: bilayers in which electron-hole plasma is created by optical pumping and then spatially separated by electric field to create indirect excitons, bilayer electron-electron quantum Hall systems near total filling factor one, and undoped bilayers in which carrier density in each layer can be independently controlled by an external gate. In the first system, the condensation is detected by photoluminescence and the voltage drop developed in each layer is measured. We calculate the resulting electric field in each layer by using a two-fluid model. In Sec. III we discuss the transport in bilayers in the presence of a weak link. This system

The plan of the paper is as follows. In the following section, we recall basics of excitonic condensation in bilayer system. We focus on transport experiments in which currents in the electron and the hole layers are externally fixed, and the voltage drop developed in each layer is measured. We calculate the resulting electric field in each layer by using a two-fluid model. In Sec. III we discuss the transport in bilayers in the presence of a weak link. This system

Just as $^3$He Cooper pairs have fixed orbital moment that is pointing in the same direction,\textsuperscript{12} Just as $^3$He is an orbital ferromagnet in the A-phase, the electron-hole condensate is a "ferromagnetic" dipolar fluid, in which all dipoles point in the same direction, are phase coherent, and therefore give rise to a nontrivial collective response to an applied magnetic field.

The formation and properties of excitonic condensates in double quantum wells is a subject of the ongoing debate. Although these systems are not truly superfluid (the $U(1)$ symmetry associated with the phase of the exciton order parameter is not an exact symmetry) various signatures of excitonic condensation can be probed provided that the excitons are long-lived. At present, there are three main candidates for realization of excitonic condensation in double quantum wells: bilayers in which electron-hole plasma is created by optical pumping and then spatially separated by electric field to create indirect excitons, bilayer electron-electron quantum Hall systems near total filling factor one, and undoped bilayers in which carrier density in each layer can be independently controlled by an external gate. In the first system, the condensation is detected \textit{a posteriori} by photoluminescence measurement\textsuperscript{15,18,17} of electron-hole recombination and transport measurements, which study the electromagnetic response of excitons, are not yet accessible.\textsuperscript{7} Therefore, an explicit demonstration of counterflow superfluidity in this system appears exceedingly difficult. The second system, quantum Hall bilayers, has been investigated by Eisenstein’s group\textsuperscript{13} and Shayegan’s group\textsuperscript{14}, and their remarkable results provide promising signatures of excitonic condensation, albeit over the vacuum of a fully-filled Landau level. Because of the non-trivial electromagnetic response of the “vacuum” underlying this condensate, the bilayer quantum Hall system is not a dipolar superfluid. In particular, it does not generate counterflow supercurrent in response to an in-plane magnetic field.

Therefore, in this paper, we focus on the third candidate. Independently contacted electron-hole bilayers have been experimentally investigated in the past decade,\textsuperscript{6} albeit in the (high density) region where exciton condensation does not occur. It is simply a matter of not too distant time when we will have electron-hole bilayers with low density and high mobility, which support the realization of excitonic condensate. In this paper we address electronic signatures of such a condensate that allow for a diagnostic of the condensate state, including its superfluid properties. We emphasize that our predictions do not necessarily have counterparts in the quantum Hall bilayers,\textsuperscript{13,14} because the mapping of quantum Hall bilayers on to excitonic superfluid ignores the response of the fully-filled Landau level vacuum.

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I. INTRODUCTION

Bilayer quantum well systems, where (equilibrium) carriers in one layer are electrons and the other layer are holes, have been investigated extensively.\textsuperscript{1,2,3,4,5,6} These systems are one of the promising candidates for observing Bose-Einstein condensation of excitons, the bound states of electron-hole pairs.\textsuperscript{7,8,9,10} When the distance $d$ between layers is small compared to the typical distance $r_e$ between particles within each layer, excitons, resulting from the attractive Coulomb interaction between electrons and holes, form a dilute Bose gas and undergo condensation. Bilayer electron-hole systems have the added advantage that each exciton has a fixed dipole moment, which points in the same direction for all excitons. Therefore, we call this excitonic condensate a dipolar condensate.\textsuperscript{11} This property, which is unique to electron-hole bilayers, enables us to probe the nominally neutral condensate via electric and magnetic fields which are confined between the two layers.\textsuperscript{12} In this sense, the dipolar condensate resembles the A-phase of superfluid Helium-3, in which all $^3$He Cooper pairs have fixed orbital moment that is pointing in the same direction.\textsuperscript{13} Just as $^3$He is an orbital ferromagnet in the A-phase, the electron-hole condensate is a “ferromagnetic” dipolar fluid, in which all dipoles point in the same direction, are phase coherent, and therefore give rise to a nontrivial collective response to an applied magnetic field.

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is closely related to a Josephson junction. We present the results based on this analogy. In Sec. [V] we present the spectrum of current noise in these systems, in the absence of any external fields. We find that the current noise can provide a measure of the superfluid collective mode velocity. We conclude the paper with discussion in Sec. [V].

II. TRANSPORT

Let us consider a bilayer system with holes in the top layer and electrons in the bottom layer. We use a notation in which the hole (electron) coordinates are given by \( r_h = r + d/2 \), \( r_e = r - d/2 \), \( r \) is a 2-dimensional position vector, and \( d = d\hat{z} \) is a vector normal to the layers. The formation of excitonic condensate is signaled by a nonzero value of the order parameter \( \Delta(r) = \langle c_{h\uparrow}^\dagger c_{e\uparrow} \rangle = |\Delta|\exp[i\Phi(r)] \), where \( c_{h\uparrow} \) creates a hole in the top layer and \( c_{e\uparrow} \) creates an electron in the bottom layer. At low energies, the dipolar phase \( \Phi(r) \) is the only relevant degree of freedom. In the presence of gauge potentials, the dipolar phase transforms as \( \nabla \Phi(r) \sim \nabla \Phi(r) - e\mathbf{A}(r) \) where \( \mathbf{A}(r) = \mathbf{A}_e(r) - \mathbf{A}_h(r) \) is the difference between vector potentials in the electron-layer and the hole-layer (We use units such that \( h = 1 = c \)). This leads to a dipolar supercurrent in the condensate state, given by \( \mathbf{J}_d(r) = 2e\rho_d[\nabla \Phi(r) - e\mathbf{a}(r)] \), where \( \rho_d \) is the dipolar phase-stiffness. For a smoothly varying gauge potential, the antisymmetric combination \( \mathbf{a} \approx d\partial_z\mathbf{A} \) can be tuned by an in-plane magnetic field; in particular, a uniform in-plane field \( \mathbf{B}_{||} = -B_{||}\hat{y} \) leads to a uniform dipolar supercurrent \( \mathbf{J}_d = 2e^2\rho_d\mathbf{d} \times \mathbf{B}_{||} \).

![FIG. 1: (Color Online) Schematic electron-hole bilayer system, with fixed current \( J_0^h \) in the top (hole) layer and \( J_0^e \) in the bottom (electron) layer. We calculate the resulting electric fields (or voltage drops) which develop in each layer by using a two-fluid model.](image)

In this section, we consider transport experiments in which fixed currents \( \mathbf{J}_0^h \) and \( \mathbf{J}_0^e \) flow through the hole-layer and electron-layer, respectively. Our goal is to determine the resulting electric fields (or voltage drops) in the individual layers (Fig. [IV]). In general the currents are carried by the excitonic condensate and quasiparticles. We characterize the quasiparticle contributions in the linear-response regime as follows:

\[
\begin{align*}
\mathbf{J}_{\text{hp}}^0 &= \sigma_{hh} \mathbf{E}_h + \sigma_{he} \mathbf{E}_e, \\
\mathbf{J}_{\text{ep}}^0 &= \sigma_{ee} \mathbf{E}_e + \sigma_{eh} \mathbf{E}_h.
\end{align*}
\]

Here \( \sigma_{ee}, \sigma_{hh} \) are longitudinal conductivities and \( \sigma_{he}, \sigma_{eh} \) are the drag conductivities. Adding the condensate contribution gives the following equations for currents in the two layers.

\[
\begin{align*}
\mathbf{J}_h^0 &= +e\rho_d (\nabla \phi - e\mathbf{a}) - (\sigma_{hh} \partial_t \mathbf{A}_h + \sigma_{he} \partial_t \mathbf{A}_e), \\
\mathbf{J}_e^0 &= -e\rho_d (\nabla \phi - e\mathbf{a}) - (\sigma_{ee} \partial_t \mathbf{A}_e + \sigma_{eh} \partial_t \mathbf{A}_h),
\end{align*}
\]

where we have used Maxwell’s equation \( \mathbf{E} = -\partial_t \mathbf{A} \). It is convenient to rewrite the two equations, [III] and [IV], in terms of their sum and difference, leading to the equations for antisymmetric gauge potential \( \mathbf{A} = (\mathbf{A}_h + \mathbf{A}_e) \).

\[
\begin{align*}
\mathbf{J}_s^0 &= -(\sigma_s + \eta_s) \partial_t \mathbf{A} - (\sigma_d - \eta_d) \partial_t \mathbf{a}, \\
\mathbf{J}_d^0 &= +2e\rho_d (\nabla \Phi_d - e\mathbf{a}) - (\sigma_s - \eta_s) \partial_t \mathbf{a} - (\sigma_d + \eta_d) \partial_t \mathbf{A}.
\end{align*}
\]

Here, \( \sigma_s(d) = (\sigma_{ee} \pm \sigma_{hh}), \eta_s(d) = (\sigma_{he} \pm \sigma_{eh}) \), and \( \mathbf{J}_s^0 = (\mathbf{J}_h^0 + \mathbf{J}_e^0) \) and \( \mathbf{J}_d^0 = (\mathbf{J}_h^0 - \mathbf{J}_e^0) \) are the external currents in the symmetric and antisymmetric channel, respectively.
It is straightforward to solve these equations and obtain the time dependence of vector potentials $A_h(t)$ and $A_e(t)$ within each layer, provided that the supercurrent contribution is stationary. The resulting electric fields in the two layers are given by

$$E_h(t) = \frac{J^0_s}{2(\sigma_s + \eta_s)} + \frac{1}{2}(1 - \sigma_{ds})\alpha e^{-\alpha t}\tilde{a}(0),$$

$$E_e(t) = \frac{J^0_s}{2(\sigma_s + \eta_s)} - \frac{1}{2}(1 + \sigma_{ds})\alpha e^{-\alpha t}\tilde{a}(0).$$

Here, $\sigma_{ds} = (\sigma_d - \eta_d)/(\sigma_s + \eta_s)$, the decay rate $\alpha = \frac{2e^2\rho_d}{[(\sigma_s - \eta_s) - \sigma_{ds}(\sigma_d + \eta_d)]}$, and we have defined

$$\tilde{a} = \frac{1}{2e^2\rho_d} [J^0_s - 2e\rho_d\nabla\Phi + \left(\frac{\sigma_d + \eta_d}{\sigma_s + \eta_s}\right)J^0_s].$$

which decays exponentially, $\frac{\partial}{\partial t}\tilde{a}(t) = -\alpha\tilde{a}(t)$. Decay rate $\alpha$ is positive since typically $\sigma_s > \eta_s$ and $\sigma_{ds}$ is small. Eqs. 7 and 8 are the main results in this section. It follows that, at long times $t \gg \alpha^{-1}$, electric fields in the two layers are identical irrespective of individual currents flowing in each layer. Their strength is determined by current in the symmetric channel $J^0_s$. This is consistent with the fact that the dissipationless condensate contributes only to current in the antisymmetric channel.

Now we turn our attention to two transport experiments.

i) First we focus on the drag setup, in which a fixed current $J^0_s$ flows through the drive-layer and the voltage drop across the second layer, namely the drag-layer, is measured, $J^0_s = J^0$ and $J^0_e = 0$. We find that the voltage in the drag layer will be given by

$$V_{\text{drag}} = \frac{LJ^0_s}{2(\sigma_s + \eta_s)}$$

where $L$ is the linear sample size. When the system is deep in the superfluid phase (when the temperature $T \to 0$ or when $d/r_s \to 0$), the density of quasiparticles is vanishingly small and therefore quasiparticle conductivity vanishes. In that regime, therefore, $V_{\text{drag}} \to \infty$; in other words, it is increasingly difficult to maintain a nonzero current in the drive layer while keeping the drag layer open. We point out that the temperature dependence of the drag-voltage is determined by the temperature dependence of quasiparticle conductivities, and typically it will be different from the Coulomb drag which dominates when the system is not a dipolar superfluid.

ii) The second experiment is the counterflow setup in which equal and opposite currents flow through the two layers, $J^0_s = J^0$ and $J^0_e = -J^0$. In this case, we find that the voltage drop across either layers is zero. This is expected since, in the present case, the current is carried entirely by the condensate and not by the quasiparticles. We also find that the transient electric fields in the two layers are given by

$$E_h(t) = \frac{1}{2}(1 - \sigma_{ds})\alpha e^{-\alpha t}\tilde{a}(0),$$

$$E_e(t) = -\frac{1}{2}(1 + \sigma_{ds})\alpha e^{-\alpha t}\tilde{a}(0).$$

In particular, for symmetric electron-hole bilayers, $\sigma_{ds} = 0$, these fields are equal and opposite. These predictions, along with the check that the long-time electric fields in the two layers are indeed independent of currents in individual layers (as long as the current in the symmetric channel is constant), will provide robust check of the on-set of excitonic condensation in these systems.

III. IN-PLANE JOSEPHSON EFFECT

Now we discuss transport across a weak link in the presence of voltage applied across the link. This effect was proposed in the case of bilayer quantum Hall systems a long time ago, but it has not yet been experimentally observed. Here we focus on bilayer electron-hole systems with conventional tunneling across the weak links within each layer. In a bilayer system with vanishingly small bare recombination rate, the dipolar phase is fixed spontaneously.
across the sample and it varies across the weak link with little energy cost. The action for the dipolar phase \( \Phi(r,t) \) is given by

\[
S = \int_{r,t} \left[ \frac{C}{2} \left( \partial_t \Phi - e a_0 \right)^2 - \frac{\rho_d}{2} (\nabla \Phi - e a)^2 \right]
\]

(14)

where \( C \) is the capacitance, and \( a_0(r,t) = a_\alpha(r,t) - a_\beta(r,t) \) is the difference between potentials \( V_\alpha \) \((\alpha = L, R)\) in the two layers, which naturally couples to the time-derivative of the dipolar phase. We consider a system where the dipolar phase is uniform on the two sides and has spatial gradients only near the weak link (Fig. 2). In the regime \( C \gg \rho_d \), the time evolution of dipolar phase is given by \( \Phi_\alpha(t) = \Phi_{0\alpha} + e \int dt' V_\alpha(t') \). Therefore, the dipolar phase difference evolves as

\[
\Phi_d(t) = \Phi_L(t) - \Phi_R(t) = \Phi_{0d} + e \int dt' [V_L(t') - V_R(t')].
\]

(15)

In analogy with a Josephson junction, the dipolar current across the weak link is given by \( J(t) = J_c \sin \Phi_d(t) \). Since it is possible to have a nonzero phase difference \( \Phi_{0d} \neq 0 \) in the absence of interlayer voltage, the system can exhibit in-plane (condensate) current across the weak link.

![Diagram of in-plane Josephson effect in electron-hole bilayers.](image)

**FIG. 2:** (Color Online) In-plane Josephson effect in electron-hole bilayers. Dipolar phases on two sides of the weak link are uniform, and the time-evolution of the dipolar phase difference \( \Phi_d \) is determined by the voltage difference \( V_L - V_R \).

Now we can discuss various Josephson effects based on the time-dependence of the applied voltage. We start with \( V_L - V_R = V_0 + V_1 \cos \omega t \), which leads to the phase-difference evolution \( \Phi_d(t) = \Phi_{0d} + e V_0 t + (e V_1 / \omega) \sin \omega t \). Thus, for \( V_1 = 0 \), we get the ac Josephson effect, where the in-plane current oscillates in the presence of a dc voltage, \( J(t) = J_c \sin(\Phi_{0d} + e V_0 t) \). On the other hand, for an ac voltage, we find that the Josephson current shows Shapiro steps at frequencies \( \omega_n = e V_0 + n \omega \),

\[
J(t) = J_c \sum_{n=-\infty}^{\infty} J_n(e V_1 / \omega) \sin(\Phi_{0d} + \omega_n t)
\]

(16)

where weight of the current with frequency \( \omega_n \) is given by \( J_n(e V_1 / \omega) \), the Bessel function of order \( n \). Let us assume that the frequency \( \omega \) is fixed, vary the amplitude \( V_1 \) of the applied voltage, and focus on the first two harmonics, at frequencies \( \omega_0 = \Phi_{0d} + e V_0 \) and \( \omega_1 = \omega_0 + \omega \). The strengths of these two components are given by \( J_0(e V_1 / \omega) \) and \( J_1(e V_1 / \omega) \) respectively. Thus, as we sweep the voltage amplitude \( V_1 \), the frequencies of the two harmonics are unchanged, but their strengths vary. In particular, when \( e V_1 / \omega = 0 \), only the first harmonic contributes, whereas for \( e V_1 / \omega \approx 2 \) (the first zero of \( J_0 \)), only the second harmonic contributes and the ratio of these harmonics is 1.0:0.6. One can produce the same Shapiro steps by applying an in-plane ac magnetic field. The ac magnetic field produces an ac in-plane electric field and would have the same effect as an ac voltage, as discussed above.

We conclude this section with a brief derivation of the critical current \( J_c \) in terms of the microscopic Hamiltonian for tunneling across the weak link. The tunneling Hamiltonian is given by

\[
H_t = \int_{r \in L, r' \in R} \left[ T^e_{rr'} \hat{c}^\dagger_{er} \hat{c}_{er'} + T^h_{rr'} \hat{c}^\dagger_{hr} \hat{c}_{hr'} + \text{h.c.} \right].
\]

(17)

Here, \( \hat{c}^\dagger_{er} \) \((\hat{c}_{hr})\) creates an electron (hole) in the bottom (top) layer at position \( r \), and \( T^e \) \((T^h)\) are the electron (hole) tunneling matrix elements. The tunneling rate across the weak-link is given by Fermi’s golden rule, \( \gamma \propto H_t^2 \). However, in the limit of vanishing potential difference across the link, the only term which remains nonzero is

\[
\gamma_0 \propto T^e T^h \langle c_{eL} c_{eR} c_{hL} c_{hR} \rangle \sim T^e T^h \langle c_{eL} c_{hL} \rangle \langle c_{eR} c_{hR} \rangle \sim T^e T^h \Delta_L \Delta_R^*.
\]

(18)
Thus the critical current $J_s \propto \gamma_0$ is proportional to both electron and hole tunneling matrix elements and it is necessary to have weak links in both layers for the in-plane Josephson effect to occur.\cite{footnote} We emphasize that in conventional Josephson junctions in superconductors, the tunneling matrix elements are typically spin independent and a systematic study of tunnel junctions in which the tunneling for the up and down spins varies significantly has not been performed. On the other hand, bilayer electron-hole systems offer junctions where the tunneling for the two constituents of the exciton (electrons and holes) can be independently controlled.

**IV. NOISE IN THE IN-PLANE CURRENT**

In the last two sections, we considered transport in a bilayer system when the system is driven by external fields. In this section, we focus on noise in the system when it is not driven. We consider the noise in the in-plane current and show that it, combined with the measurement of counterflow superfluidity, provides a direct measure of the superfluid velocity of the excitonic condensate. Noise in any observable is a probe of the excitation spectrum of the system. Therefore, in condensate state, we expect the noise to probe the low-lying excitations, namely the collective sound mode of the dipolar superfluid. To this end, let us consider noise correlations between currents in the two layers. We start with the symmetrized current-current correlator,

$$C_{ij}^{eh}(r, t) = \langle J_{ie}(r, t)J_{jh}(0) \rangle + \langle J_{jh}(0)J_{ie}(r, t) \rangle = 2\text{Re}\langle T J_{ie}(r, t)J_{jh}(0) \rangle$$

(19)

where $i, j$ denote 2-dimensional Cartesian components of the current density and $T$ stands for time ordering. This correlator corresponds to autocorrelation between current fluctuations at two probes separated by $r$ at times $t$ apart, when the system is in equilibrium without any external voltage applied. At low temperature, the quasiparticle contribution to the fluctuations is vanishingly small, assuming a fully gapped spectrum. Therefore, we can approximate the current fluctuations as $\delta J_{k(e)} = \pm \epsilon \rho_d \delta \nabla \Phi$, where the dynamics of the dipolar phase $\Phi$ is governed by action $[14]$. It is straightforward to evaluate the time-ordered current-current correlator in momentum space

$$\langle T J_{i+}^*(k, \omega)J_{j-}(k\omega) \rangle = (\epsilon \rho_d)^2 k_i k_j \frac{i}{C(\omega^2 - \nu_c^2 k^2)}$$

(20)

where $\nu_c = \sqrt{\rho_d/C}$ is the velocity of the superfluid sound mode. Therefore, the real-space current-current correlator is given by

$$C_{ij}^{eh}(r, t) = -(\epsilon \rho_d)^2 \int_\Lambda \frac{d^2 k}{(2\pi)^2} \frac{k_i k_j}{k} \cos(k \cdot r - \nu_c k |t|)$$

(21)

where $\Lambda \sim r_c^{-1}$ is the momentum cutoff beyond which the hydrodynamic description of the excitonic condensate fails. It follows from Eq. (21) that the current-current correlator vanishes for $i \neq j$.

![Diagram](image)

**FIG. 3:** (Color Online) Noise measurement in non-driven bilayer system. Current fluctuations in the hole and electron layers are (anti)correlated in the condensate state, where fluctuations are primarily due to the superfluid sound mode. Therefore, the noise power spectrum probes the properties of this sound mode.

In experiments, it is natural to consider the correlator for (integrated) current fluctuations at the edge of the sample. Consider, for example,

$$C_{xx}^{eh}(x, t) = \int dy C_{xx}^{eh}(r, t) = -(\epsilon \rho_d)^2 \int dy \int \frac{d^2 k}{(2\pi)^2} k_x^2 \cos(k \cdot r - \nu_c k |t|).$$

(22)

The integration along the $y$-axis only retains the $k_y = 0$ component and the noise correlator becomes

$$C_{xx}^{eh}(x, t) = -(\epsilon \rho_d)^2 \int_\Lambda \frac{dk_x}{(2\pi)} |k_x| \cos(k_x x - \nu_c k_x |t|).$$

(23)
It is straightforward to evaluate the integral and we get

$$C_{xx}^{eh}(x, t) = -\frac{(\epsilon \rho_d \Lambda)^2}{C v_c} \left[ \frac{\sin(x_+ \Lambda)}{x_+ \Lambda} + \frac{\sin(x_- \Lambda)}{x_- \Lambda} - \frac{(1 - \cos(x_+ \Lambda))}{(x_+ \Lambda)^2} - \frac{(1 - \cos(x_- \Lambda))}{(x_- \Lambda)^2} \right],$$

(24)

where we have defined auxiliary variables $x_{\pm} = (x \pm v_c t)$.

Eq. (24) has several interesting features. The correlator is only a function of $x_{\pm} = (x \pm v_c t)$ and is symmetric in them. This is expected since the dipolar action, Eq. (14), is Lorentz invariant in the absence of external fields. When the two probes measure total current fluctuations at the same co-ordinate, $x = 0$ and $x_{\pm} = \pm v_c t$, the correlator simplifies to

$$C_{xx}^{eh}(0, t) = -2 \epsilon v_c^2 \Lambda^2 \rho_d \rho_c \left[ \frac{\sin(v_c \Lambda t)}{v_c \Lambda t} - \frac{(1 - \cos(v_c \Lambda t))}{(v_c \Lambda t)^2} \right].$$

(25)

This correlator has a power spectrum

$$C_{xx}^{eh}(\omega) = -\frac{2\pi \epsilon v_c^2 \rho_d}{v_c} |\omega| \theta(v_c \Lambda - |\omega|).$$

(26)

Thus, the power spectrum of the current noise is linear with a slope which is proportional to the superfluid density and inversely proportional to the superfluid collective-mode velocity $v_c$. Eq. (26), combined with the counterflow superfluidity result, $J_d = 2 \epsilon \rho_d d \rho B \parallel$, provides a direct measurement of the superfluid velocity $v_c$. The power spectrum vanishes beyond $\omega_0 = v_c \Lambda \propto v_c \rho_c^{-1}$ because the hydrodynamic description is not valid beyond momentum $\Lambda$ and therefore it cannot probe frequencies higher than $v_c \Lambda$. A similar calculation of current fluctuations within a single layer gives $C_{xx}^{ee}(r, t) = C_{xx}^{eh}(r, t) = -C_{xx}^{eh}(r, t)$. This result is crucially based on the system being in the condensate state. We emphasize that this result will not hold in bilayer quantum Hall systems in the phase-coherent state. When the two layers are weakly coupled, current fluctuations in the two layers will have qualitatively different nature. Thus, measurement of the in-plane current noise can provide yet another signature of the excitonic condensate state and it’s power spectrum can provide a measure of the superfluid collective mode velocity and it’s evolution near the phase boundary.

V. DISCUSSION

We have considered the transport and noise in dipolar excitonic condensate in electron-hole bilayers. These bilayers are a promising candidate for the realization of excitonic condensate. In principle, the condensation of excitons is not well-defined because excitons are metastable bound states of electron-hole pairs which eventually decay. However, with present semiconductor heterostructures, it possible to create electron-hole bilayers with very low recombination rates and therefore probe properties of the excitonic condensate without destroying it. We find that the dipolar phase has a number of nontrivial transport properties that can lead to the condensate detection.

We have discussed the drag and counterflow transport features of these bilayers taking into account the effect of condensate and quasiparticles. We find that for the counterflow setup, the steady-state current is carried solely the condensate and hence there is no voltage drop in either layer, in the ideal case. In contrast, for the drag experiment, we find that the electric fields in both layers are the same. These two results are specific cases of our primary result, Eqs. (11) and (14), which shows that the electric fields in the two layers are the same irrespective of the current distributions in the electron and the hole layers, and is determined only by their sum total. We also discussed the analog of in-plane Josephson effect and showed that the existence of weak links in both layers is instrumental to it.

We have demonstrated how noise spectroscopy of in-plane current fluctuations in the electron-hole bilayers can provide a signature of the condensate state as well as a direct measurement of the collective mode velocity $v_c$. The current fluctuations at two probes, one in the electron layer and other in the hole layer, will be naturally correlated if ground state consists of excitons and the low-lying excitations are superfluid collective modes. On the other hand, if the two layers are uncoupled, these fluctuations will be uncorrelated. We find that the power spectrum of the current noise is linear with slope $-2 \epsilon v_c^2 \rho_d/v_c$. The noise spectrum, combined with the measurement of dipolar supercurrent $J_d = 2 \epsilon \rho_d d \rho B \parallel$, will allow us to extract information about both the dipolar phase stiffness $\rho_d$ and the collective mode velocity $v_c$.

This power spectrum will change significantly when the bilayer system undergoes a quantum phase transition with increasing $d/r_s$, going from excitonic condensate to weakly coupled layers. We emphasize that the noise spectroscopy is complementary to the transport experiments. In transport experiments, the bilayer system is perturbed using external fields and the response of excitonic condensate is measured. The noise spectroscopy, on the other hand, is performed on a bilayer system in equilibrium, where statistical and quantum fluctuations provide the
probe of low-lying excitations of the system. These complimentary measurements will provide various signatures of
the superfluid condensate and deepen our understanding of excitonic condensates in semiconductors.

It is a pleasure to acknowledge discussions with Peter Littlewood. The work at LANL was supported by the DOE.
Sandia National Laboratories is a multi-program laboratory operated by Sandia Corporation, a Lockheed-Martin
Company, for the U. S. Department of Energy under Contract No. DE-AC04-94AL85000.

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