On the Resummation of Large QCD Logarithms in $H \to \gamma\gamma$

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Abstract

We study the strong corrections to the Higgs coupling to two photons. This coupling is the dominant mechanism for Higgs production in photon-photon collisions. In addition, the two photon decay mode of the Higgs is an important and relatively background free channel of relevance at the LHC and the Tevatron. We develop a method for the resummation of large QCD corrections in the form of Sudakov-like logarithms of the type $\alpha_s \ln^2(p^2/m_H^2)$ and $\alpha_s \ln^{2p-1}(m^2/m_H^2)$ (where $m$ is the light quark mass) which can contribute to this process in certain models (for example, the MSSM for large $\tan\beta$) up to next-to-leading-logarithmic (NLL) accuracy. The NLL correction is moderate, the substantial part of which comes from terms not related to running coupling effects.

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1 Introduction

The origin of electroweak symmetry breaking is one of the central issues in particle physics. Within the Standard Model (SM), the solution to this problem is associated with the Higgs mechanism, which predicts a fundamental neutral scalar Higgs particle. The Higgs boson is the only SM elementary particle which has not been detected thus far. The precision electroweak measurements suggest the existence of a light Higgs boson in the mass region \[1\]

\[
113.5 < m_H < 200 \text{ GeV.} \tag{1}
\]

In this letter we discuss the Higgs-\(\gamma\gamma\) vertex, which is extremely important for Higgs physics, specially for Higgs boson masses in the range mentioned above. First, it is the main mechanism for Higgs production in photon-photon collisions \[2, 3, 4\]; second, it is a relatively background free decay mode for the Higgs at the LHC and the Tevatron and finally, the coupling is sensitive to new physics, and can be considered to be a counter of the number of new heavy particles. Because of these last two observations the Higgs-\(\gamma\gamma\) coupling should be very well calibrated. Thus, for example, the radiative corrections to this process should be under control.

The coupling of the Higgs boson to two photons is absent at tree level in the Standard Model. The first non-zero contribution arises from fermions and W boson loops. Because the Yukawa coupling of the quark is proportional to the quark mass, the contributions of the light quarks as well as charm and bottom quarks are well suppressed in comparison to the top quark loop contribution. The radiative corrections are well studied in this case: there is some literature devoted to the QCD and electroweak radiative corrections \[5, 6, 7, 8, 9\]. The only source of QCD corrections at the two loop order is the gluon corrections to the top quark loop. The QCD corrections are small for Higgs masses, \(m_H < 2m_t\), as shown by the explicit calculations in \[7, 5\]. For heavy Higgs masses, \(m_H > 2m_t\), the corrections are large (about 40%), although this mass range appears to be ruled out by the electroweak data.

It was observed in \[5\] that in the limit of large \(\frac{m^2_H}{m^2}\), where \(m\) is the quark mass, the Higgs-\(\gamma\gamma\) form factor gives the QCD double logarithmic asymptotic,

\[
F = F^{1\text{-loop}} (1 - \frac{\rho}{12}), \tag{2}
\]

with \(\rho = \frac{C_F}{2\pi} \ln^2 \left( \frac{m^2_H}{m^2} \right)\). For such a factor to be phenomenologically relevant one must consider the contribution to the form factor from light quark loops which as mentioned above are suppressed in the Standard Model. In fact, the \(b\) quark contribution is only about 4% of that of the top quark at lowest nontrivial order and decreases with radiative corrections. However, the contributions of the bottom quark are enhanced by \(\tan\beta\) in the super-symmetric extensions of the Standard Model, and therefore the limit \(m^2_H/m^2 \gg 1\), which we consider in this paper may be phenomenologically relevant quite apart from its inherent theoretical interest. Current estimates place allowed values of \(\tan\beta\) in a wide range \[10\]. For large values of \(\tan\beta \sim 30\), which are allowed, the bottom quark contributions to the Higgs-\(\gamma\gamma\) form factor can become comparable to that of the top quark in such models.

Super-symmetric extensions of the Standard Model have been extensively discussed in the literature partly because of the possibility of resolving the hierarchy problem. The Minimal Super-symmetric Standard Model (MSSM) stabilizes the mass of the light Higgs bosons in the presence of high energy GUT scales. In the MSSM, spontaneous symmetry breaking is induced by two complex Higgs doublets leading to five Higgs particles, including light and heavy scalar [CP-even] particles \(h\) and \(H\); a pseudo-scalar [CP-odd] particle \(A\), and a pair of charged Higgs...
particles [11]. The lightest of these is predicted to be below \( m_Z \), although radiative corrections increase this limit up to 130 GeV due to large top quark contributions [12, 10]. We have such models in mind, wherein as discussed above the \( b \) quark and \( t \) quark contributions to the Higgs-\( \gamma\gamma \) form factor may become comparable. Then in order to understand the radiative corrections to the form factor it is important to better understand these large logarithmic corrections mentioned above and to find a method for resumming them.

The nature of the leading double logarithmic (DL) terms of the form \( \alpha_s^2 \ln^2(\frac{m_H}{m}) \) has been clarified in [13, 8]. It was shown that these logarithms are related to Sudakov form factor which enters into the one loop triangle diagram. In this letter we discuss how the resummation of these large logarithms to the next-to-leading-logarithmic (NLL) accuracy may be carried out.

As will be seen, the next to leading contribution is moderately small compared to the leading one, implying that the radiative corrections to the Higgs-\( \gamma\gamma \) form factor are under control for the case (like for example the MSSM for large \( \tan\beta \)) when the light quark contributions cannot be neglected.

This paper is organized as follows: In section 2 we discuss the resummation procedure of the leading double logarithms, and in section 3 we suggest a method for resumming the next to leading logarithmic contributions as well. Section 4 contains the numerical results and discussions. In section 5 we give our conclusions.

2 The Method

As mentioned earlier the Born coupling \( H-\gamma\gamma \) is zero in the SM as well as in the MSSM. At the one loop level, we find that the coupling is mediated by any charged heavy particle loop. In the case of the SM, the only contributions come from the quark loops and W-boson loops. The amplitude of the Higgs decay into two photons through quark loop can be presented in the form

\[
M(H \rightarrow \gamma\gamma) = (e^*_\mu)(e^*_\nu)d^{\mu\nu}(G_F\sqrt{2})^{1/2} \frac{\alpha}{4\pi} N_c \sum_q e_q^2 F_Q(t),
\]

(3)

here \( k_1 \) and \( k_2 \) are the momenta of the photons, \( e_1, e_2 \) are the corresponding polarization vectors, and \( s = (k_1+k_2)^2 = m_H^2, k_1^2 = k_2^2 = 0 \). The structure \( d^{\mu\nu} = (k_1 \cdot k_2)g^{\mu\nu} - k_2^\mu k_1^\nu \) can be constructed using QED gauge invariance and Lorenz invariance of the amplitude. The partial width is

\[
\Gamma(H \rightarrow \gamma\gamma) = \frac{G_F m_H^3}{128\sqrt{2}\pi^3} \left( \sum_Q N_C e_Q^2 g_Q F_Q + g_W F_W \right)^2,
\]

(4)

with [14, 13]

\[
F_Q = 2t^{-1}((1 - t^{-1})f(t) + 1), \quad F_W = -t^{-1}\left(3 + 2t + 3(2 - t^{-1})f(t)\right)
\]

(5)

and,

\[
f(t) = -\frac{1}{4} \left( \ln \sqrt{t} + \sqrt{t-1} \right)^2 \quad \text{at} \quad t = \left(\frac{m_H}{2m}\right)^2 > 1.
\]

In this letter we focus on the quark loop contributions only. These contributions will be common to all models considered. We wish to identify the leading and the next to leading logarithmic corrections and to present a procedure for resumming them.
The one loop DL correction arises when the quark line opposite the $Hq\bar{q}$ vertex is soft and this can be easily evaluated using the Sudakov parametrization \[16, 17\]; namely we decompose the soft quark momenta in terms of those along the hard photon momenta $k_1, k_2$ and transverse to it

$$l = \alpha k_1 + \beta k_2 + l_\perp, \quad s = (k_1 + k_2)^2 = m_H^2. \tag{7}$$

The DL contribution comes only from the region

$$m^2, |l_\perp|^2 \ll s|\alpha|, s|\beta| \ll s. \tag{8}$$

The loop integration in terms of the new variables reads

$$\int_\infty^{-\infty} d^4l = \frac{s}{2} \int_\infty^{-\infty} d\alpha \int_\infty^{-\infty} d\beta \int_0^{\pi} d\theta_{l_\perp}^2. \tag{9}$$

The integration over the transverse momenta of the soft quark is performed by taking half of the residues in the corresponding propagator

$$\int \frac{d^4l}{l^2 - m^2 + i0} F = \int \frac{\pi s}{2} \frac{d\alpha d\beta d\theta_{l_\perp}^2}{s\alpha\beta - l_\perp^2 - m^2 + i0} F \to -i\pi^2 \frac{s}{2} \int d\alpha d\beta \Theta(s\alpha\beta - m^2) F. \tag{10}$$

In this manner the one loop amplitude can be calculated in DL approximation and result is,

$$F_{1-loop} = C \int \frac{1}{\alpha\beta} \Theta\left(\frac{m^2}{m_H^2}\right) = C\left(\frac{1}{2} \ln^2 \frac{m^2}{m_H^2}\right), \tag{11}$$

with $C = -\frac{4m^2}{m_H^2}$. As mentioned earlier, the infrared sensitive contributions come from a region where the fermion line opposite the Higgs-fermion-fermion vertex is soft. This is, of course, also the origin of the term proportional to quark mass in the expression for the Higgs-$\gamma\gamma$ form factor. In general, this one loop contribution gets radiative corrections and the additional DL contributions arise from the region of soft gluons. It is well known that to this accuracy they factorize and are independent of spin. Thus, aiming at DL accuracy we can use the eikonal approximation, and easily generalize the above to include all orders in the QCD coupling.

Let us briefly consider the resummation of these double logs for the Higgs-$\gamma\gamma$ form factor \[8\]. Some of the diagrams contributing to the next order in $\alpha_s$ are shown in Figs.(1). Only the double logarithms arising from Fig.(1a) actually exponentiate – other diagrams like those in Figs.(1b, 1c) do not contribute as discussed below. With Fig.(1a) we note that we may view the inner form factor as an off-shell Sudakov form factor (see Fig.2) with the ”external” quark legs labelled $p_1, p_2$ (see Fig.1).

$$p_1^2 = (k_1 + k)^2 = s\beta, \quad p_2^2 = (k - k_2)^2 = -s\alpha. \tag{12}$$

Here, we are using the Sudakov parametrization for the momenta $k = \alpha k_1 + \beta k_2 + k_\perp$.

In order to implement the resummation (see Fig.2) we use the expression for the off-shell quark-anti-quark Sudakov form factor \[18\]

$$S(p_1, p_2) = \exp\left(-\frac{C_F\alpha_s(\mu^2)}{2\pi} \ln\left(\frac{|p_1|^2}{s}\right) \ln\left(\frac{|p_2|^2}{s}\right)\right). \tag{13}$$
The kinematical region of interest is restricted by the kinematics of the one loop integral, and can be read off from eq. (8) and eq. (12):

\[ m^2 \ll |p_1|^2, |p_2|^2 \ll s. \] (14)

We have mentioned that diagrams like Figs.(1b, 1c) do not contribute to the accuracy we are interested in. Indeed all such diagrams are not included in eq.(13). It is important to note that, diagrams, like Figs.(1b, 1c) and their counterparts in higher orders that cannot be included in the off-shell Sudakov form factor are irrelevant not just to leading but also to the next to leading accuracy. To see this consider Fig.(1b): The soft fermion line is labelled with momenta \( p_1 + k - k_1 \). Using this fact together with \( k_1^2 = 0 \) and that for the infrared sensitive contribution, the lines should be nearly on-shell, we observe that there are no large scales \( \sim m_H \) associated with the vertex correction in the Feynman gauge. Similarly self-energy corrections cannot produce any large logarithms of the type \( \ln \frac{m^2}{m_H^2} \) in the Feynman gauge.

Now using Eq.(13) together with the above discussion, we have first to DL accuracy, the following for the resummation of the diagrams in Fig.(2),

\[ F = C \int_0^1 \int_0^1 d\alpha d\beta \Theta(\alpha\beta - \frac{m^2}{s}) \text{Exp} \left( -\frac{C_F\alpha_s}{2\pi} \ln |\alpha| \ln |\beta| \right). \] (15)

We transform the exponent into the power series and find that the integral of the \( n \)-th term will be of the form

\[ \int_0^1 d\xi_1 \int_0^{1-\xi_1} d\xi_2 \xi_1^{n+a} \xi_2^{n+b} = \frac{\Gamma(n+a+1)\Gamma(n+b+1)}{\Gamma(3+2n+a+b)}. \] (16)

The final result at DL accuracy reads

\[ F_{DL} = F^{1-loop} \sum_{n=0}^{\infty} \frac{2\Gamma(n+1)}{\Gamma(2n+3)} (-\rho)^n, \] (17)

with \( \rho = \frac{C_F\alpha_s(s^2)}{2\pi} L^2, L = \ln \left( \frac{m^2}{s} \right) \). The index \( n \) shows the order of the amplitude in \( \alpha_s^n \). We can clearly identify the separate contributions of the fixed orders in \( \alpha_s \). On the other hand, if \( \rho \) is large all terms in the series are important, giving altogether some analytic function \( F_{DL}(\rho) \). This function is identified with a hyper-geometric function \( _2F_2(1,1;2,\frac{3}{2};z) \), namely

\[ F_{DL} = F^{1-loop} \sum_{n=0}^{\infty} \frac{2\Gamma(n+1)}{\Gamma(2n+3)} \left( -\rho \right)^n = _2F_2(1,1;2,\frac{3}{2};-\rho^4) F^{1-loop}, \] (18)

we recall here that, in general, the function \( _2F_2(a,b;c,d;z) \) is defined by a series

\[ _2F_2(a,b;c,d;z) = \sum_{k=0}^{\infty} \frac{(a)_k(b)_k z^k}{(c)_k(d)_k k!} \] (19)

Taking into account identities \( (1)_k = k!, (\frac{3}{2})_k = 2^{-2k} \frac{(2k+1)!}{k!} \), and \( (2)_k = \frac{\Gamma(2+k)}{\Gamma(2)} \), [13], we have

\[ _2F_2(1,1;2,\frac{3}{2};-\rho^4) = \sum_{k=0}^{\infty} \frac{(k!)^2 (-\rho)^k}{(2k+1)!(k+1)!} = \sum_{k=0}^{\infty} \frac{2\Gamma(k+1)}{\Gamma(2k+3)} \left( -\rho \right)^k. \] (20)
This gives us the final DL result, eq. (18). For large values of the parameter $\rho$ the function $F_{DL}$ has the following asymptotic,

$$F_{DL}(\rho) = \frac{2 \ln(2\rho)}{\rho} \Gamma^{1-loop}.$$  \hspace{1cm} (21)

We see that despite the fact that perturbation theory blows up at large $\rho$, the resummed result gives a smooth well defined function.

### 3 Next-to-leading-logarithmic accuracy

It is possible to develop this approach to achieve next-to-leading-logarithmic accuracy. For this we need an expression for the Sudakov form factor with NLL accuracy. In fact, such an analysis already exists in the literature [20] for the case when the two external fermion lines are off-shell by the same amount, i.e., $p_1^2 = p_2^2 = \rho^2$:

$$S_{NLL}(p,p) = \text{Exp} \left( -\frac{C_F \alpha_s(p^2)}{2\pi} \ln^2 \left( \frac{s}{|p|^2} \right) + \frac{3C_F \alpha_s(p^2)}{4\pi} \ln \left( \frac{s}{|p|^2} \right) \right).$$  \hspace{1cm} (22)

For our purposes, we need to extend the analysis to take into account that $p_1^2 \neq p_2^2$. It is easy to see that for the region, eq. (14), the proof of factorization and exponentiation given in [20] goes through with straightforward changes. The one major change involves the normalization of the coupling. We have studied one and two loop diagrams for the $Hq\bar{q}$ vertex with slightly off-shell quarks with the following result:

$$S_{NNL}(p_1, p_2) = \text{Exp} \left( -\frac{\alpha_s(\nu^2)C_F}{2\pi} \left[ \ln \left( \frac{|p_1|^2}{s} \right) \ln \left( \frac{|p_2|^2}{s} \right) + \frac{3}{4} \ln \left( \frac{|p_1|^2}{s} \right) + \frac{3}{4} \ln \left( \frac{|p_2|^2}{s} \right) \right] \right),$$  \hspace{1cm} (23)

with the normalization of the coupling constant determined to be $\nu^2 = \sqrt{|p_1^2||p_2^2|}$. We show only the double and single IR logarithms in eq. (23). In order to understand this normalization, we have to consider the diagram shown in Fig. (23), where we are keeping track of the $n_f$ dependent pieces only since they are separately gauge invariant. Such diagrams can be accounted for by considering the following gluon propagator

$$D_{\mu\nu}^{ab} = -i\delta^{ab} \left( g_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2} \right) \frac{1}{k^2} \frac{1}{1 + \Pi(k^2)},$$  \hspace{1cm} (24)

where $\Pi(k^2)$ is the vacuum polarization by the gluon; at the one loop level it is simply $\Pi = \frac{\alpha_s \beta_0}{4\pi} \ln \left( \frac{k^2}{\mu^2} e^C \right)$, $\beta_0 = 11 - \frac{2}{3} n_f$, $C$ being a scheme-dependent constant ($\overline{MS}$ scheme $C = -\frac{5}{3}$).

The diagram Fig.3 corresponds to the first term in the expansion of the gluon propagator in $\alpha_s$. The $n_f$ part of this result is, as mentioned earlier, a gauge invariant part of the complete set of two loop diagrams.

Because the effects of the running coupling gives only single logarithmic terms it is enough to consider the remaining integrals to DL accuracy. Namely, we may trace only the terms proportional to $\ln \left( \frac{p_1^2}{s} \right) \ln \left( \frac{p_2^2}{s} \right)$ from Fig.3. At DL accuracy the spin structure of the amplitude is simple, so that one needs to consider the scalar integral only,

$$I = 1 + \frac{\alpha_s(\mu^2)C_F}{2\pi} (2p_1 p_2) \int \frac{d^4k}{(2\pi)^4} \frac{1}{(p_1 + k)^2 (p_2 - k)^2} \left( 1 - \frac{\alpha_s \beta_0}{4\pi} \ln \left( \frac{k^2}{\mu^2} \right) \right) i.$$  \hspace{1cm} (25)
To evaluate this, we consider a slightly more general integral

\[ J = i \int \frac{d^4k}{(2\pi)^4} \frac{1}{(p_1 + k)^2} \frac{1}{(p_2 - k)(k^2)^{1+\Delta}}, \]  

which after expanding in $\Delta$ will give us the desired integral in $I$. Using Feynman parameters, this integral is reduced to

\[ J = -\frac{1}{(4\pi)^2} \int_0^1 dy \frac{1}{A\Delta} \mu^{2\Delta} \left[ e^{-\Delta \ln(A+B)} - e^{-\Delta \ln(B)} \right], \]

with

\[ A(y) = p_2^2 y^2 + 2p_1 p_2 y + p_1^2, \]
\[ B(y) = -2p_1 p_2 y - p_2^2 - p_1^2, \]
\[ A(y) + B(y) = p_2^2 y(-1 + y). \]

The function $A(y)$ has two zeros, $y_\pm$:

\[ A = p_2^2(y - y_+)(y - y_-), \quad y_\pm = \frac{-2p_1 p_2 \pm \sqrt{(2p_1 p_2)^2 - 4p_1^2 p_2^2}}{2p_2^2}. \]

For very small virtualities, $p_1^2, p_2^2 \to 0$, the roots are simplified to $y_+ = \frac{p_1}{p_2}, y_- = -\frac{p_1}{p_2}$. Expanding the integrand of $J$ in $\Delta$ up to second order we have

\[ J = \frac{1}{(4\pi)^2} \int_0^1 dy \frac{1}{p_2^2(y - y_+)^2(y - y_-)^2} \mu^{2\Delta} \left( \ln \frac{p_2^2 y(1 - y)}{(2p_1 p_2 + p_1^2) y + p_1^2} \right) + \frac{\Delta}{2} \left[ - \ln^2(p_2^2 y(1 - y)) + \ln^2((2p_1 p_2 + p_2^2) y + p_1^2) \right]. \]

The final integration over $y$ is simple, the result is

\[ J = \frac{1}{(4\pi)^2 2p_1 p_2} \left( - \ln \frac{|p_1|^2}{s} \ln \frac{|p_2|^2}{s} + \frac{\Delta}{2} \ln \frac{|p_1|^2}{s} \ln \frac{|p_2|^2}{s} \ln \left( \frac{|p_2^2||p_2^2|}{\mu^4} \right) \right). \]

We see, that the first term in this equation reproduces the DL result from eq.\((13)\) and eq.\((23)\). It can be checked, that the second term suggests the normalization of the coupling constant to be, $\nu^2 = \sqrt{|p_1^2||p_2^2|}$. Indeed, returning to the integral $I$, we find

\[ I = 1 - \frac{\alpha_s(\mu^2) C_F}{2\pi} \ln \frac{|p_1|^2}{s} \ln \frac{|p_2|^2}{s} \left[ 1 - \frac{\alpha_s(\mu^2) \beta_0}{4\pi} \ln \left( \frac{\sqrt{|p_1^2||p_2^2|}}{\mu^2} \right) \right]. \]

It is clear that the last logarithm, containing the $\beta_0$ term, can be absorbed into the running coupling, giving $\alpha_s(\nu^2)$ with the normalization point $\nu^2 = \sqrt{|p_1^2||p_2^2|}$. The exponentiation of the integral $I$ will give us the final off-shell Sudakov exponent, eq.\((23)\).

In order to get single logarithms in eq.\((23)\) we have to include the numerator and the spin structure. We do not present the details of these calculations here. Instead we note, that all logarithms we have accounted for are of infrared origin, $s \gg p_1^2, p_2^2 \to 0$. We do not show
the UV logarithms which come as a result of the normalization of the quark mass, $m$, or the quark Yukawa coupling, $g_{Hqq}$. These logarithms are of the form $\gamma \ln \frac{\mu}{s}$, and are related to the anomalous dimensions of the quark mass and the Yukawa coupling, $\gamma$, and can be traced separately from the IR logs. Such terms can be omitted if the Yukawa coupling and related to it the quark mass in the leading order result are normalized at a large scale $\mu^2 = s$. The formula eq. (23) reproduces the expression for the Sudakov form factor at non-equal virtualities at DL accuracy derived by Carrazone et. al. in [18], eq. (13), as well as at NLL with equal virtualities $p^2 = p_1^2 = p_2^2$ derived by Smilga in [20], eq. (22).

In addition, the normalization point $\nu^2 = |p_1^2||p_2^2|$ that we find reproduces that of the NLL results with equal virtualities $p^2 = p_1^2 = p_2^2$ derived by Smilga in [20], eq. (22).

In our opinion, this scale, $\nu^2 = |p_1^2||p_2^2|$, has a very transparent origin. The vertex of the interaction of a soft gluon with an off-shell quark ($p_1^2$) is described by the coupling $g(p_1^2)$. In the situation of gluon-exchange between two quarks with different virtualities, we have an effective coupling $g(p_1^2)g(p_2^2)$. Using the running of the coupling $\alpha(\mu^2) = 4\pi\alpha_s(\mu^2)$, at one loop level, $\alpha(\mu^2) = \alpha_s(\nu^2)/(1 + \alpha_s(\nu^2)/(4\pi)\ln(\nu^2/\nu^2))$, we will find that the effective coupling $g(p_1^2)g(p_2^2)$ is reduced to $\alpha_s(\sqrt{|p_1^2||p_2^2|})$. This coincides with our previous results.

As a next step, we include this form factor inside the one loop triangle diagram and calculate the last one loop integration with the form factor which now accounts for all large logarithms with NLL accuracy. The final result for the next-to-leading-logarithmic form factor reads

$$F_Q = F_{DL} + F_{NLL},$$

(36)

with $F_{DL}$ from eq. (17) and

$$F_{NLL} = \frac{1}{L} F_{1-loop} \sum_{\nu=1}^{\infty} \frac{\Gamma(n+1)}{\Gamma(2n+2)} (-\rho)^n \left( 3 - \frac{n+1}{C_F 2n+2} \frac{\ln(s/\mu^2)}{L} \right),$$

(37)

with $\beta_0 = 11 - \frac{2n_f}{3}$, $n_f$ is a number of light flavors. Because the typical virtuality is harder than $(2m_b)^2$, we take number of light flavors to be $n_f = 5$. Because the Yukawa coupling and the related quark mass in the leading order result are normalized at a large scale $\mu^2 = s$, we have to use

$$F_{1-loop} = -\frac{2m(m_H^2)}{m_H^2} \cdot \frac{m^2}{m_H^2} \ln^2 \frac{m^2}{m_H^2},$$

(38)

in $F_{DL}$, where $m(m_H^2)$ is the $\overline{MS}$ mass and $m$ is the pole mass. Note, that the second mass in $F_{1-loop}$ is the pole one - it comes from the t-channel soft quark propagator of the sub-diagram $q\bar{q} \rightarrow \gamma\gamma$. This amplitude does not have any anomalous dimensions and, therefore, $\mu$ independent. The difference between the running mass and the pole mass should be accounted only at $O(\alpha_s^L 2n_f - 1)$.

Some comments on the derivation of the eq. (37) are in order. First, we have used eq. (10) in the derivation. Second, the first term, the factor 3, comes from single logarithmic terms (two last terms in eq. (23)), which are not related to the running of $\alpha_s$, whereas the last two terms in eq. (37) are related to the running of the coupling constant:

$$\beta_0 \ln(\sqrt{|p_1^2||p_2^2|}/\mu^2) = \beta_0 \left( \frac{1}{2} \ln(|\alpha|) + \frac{1}{2} \ln(|\beta|) + \ln \frac{s}{\mu^2} \right).$$

(39)
It is interesting, that if we chose the normalization scale in eq. (23) to be $k_1^2$, we would get a slightly different result. Such a normalization has been used in [1]. Using the fact that $k_1^2 = \alpha_s \beta_s \mu$, the normalization scale in the Sudakov form factor for this choice of the normalization becomes $\mu^2 = \frac{|p_1^2| |p_2^2|}{s \mu^2}$. Going to $F(\rho)$, the only difference in the calculations will enter through the term $\beta_0 \ln(\frac{|p_1^2| |p_2^2|}{s \mu^2}) = \beta_0 (\ln(\alpha_s) + \ln(|\beta|) + \ln \frac{\mu^2}{s \mu^2})$ (compare with eq.(39)). It would give a factor two larger non-logarithmic contribution proportional to $\beta_0$ in the final result for $F_Q$, eq.(41). It is worth mentioning, that this scale, $\mu^2 = \frac{|p_1^2| |p_2^2|}{s \mu^2}$, does not agree with results of Smilga at $p_1^2 = p_2^2 = p^2$, $\mu^2 = p^2$ nor with our scale $\mu^2 = \sqrt{|p_1^2||p_2^2|}$. We stress, that all our results, especially eq.(37) are valid only for very large L, $|L| \gg 1$.

We may expand the expression for $F_Q$ at the two loop level. Redefining all masses through $m(\mu^2)$ by using

$$m = m(\mu^2) \left( 1 + \frac{\alpha_s C_F}{2\pi} \left[ \frac{3}{2} \ln \frac{\mu^2}{m^2} + 2 \right] \right),$$

we have

$$F_Q = -2\frac{m^2(\mu)}{m_H^2} \ln^2 \frac{m^2}{m_H^2} \left( 1 + \frac{C_F \alpha_s}{2\pi} \left[ -\frac{1}{12} \ln^2 \frac{m^2}{m_H^2} + \ln \frac{m^2}{m_H^2} + 3 \ln \frac{\mu^2}{m^2} \right] \right).$$

This expanded result at two loops is in agreement with that in [22], and can be viewed as a powerful check of our new resummed result.

4 Numerical results and discussions

Having at hand all analytical NLL results for the form factor $F_Q$ we turn now to the numerical analysis. In order to get some estimates, we have used $m_b = 4.5$ GeV and the coupling constant normalized at $\alpha_s(m_Z) = 0.118$ [21]. First, in Fig.4, we show the ratio of two amplitudes, $R(m, m_H, \mu) = \frac{F_Q(H \rightarrow \gamma\gamma)}{F_{1-loop}}$, as a function of the Higgs mass. The result of the purely double logarithmic resummation is presented at different normalization scales: at the soft scales $\mu^2 = m_b^2$, $9m_b^2$, by the dashed-dotted and the short-dashed curves, at the hard scale $\mu^2 = s$ by the dashed line, and at the intermediate scale $\mu^2 = s x^{0.4}$, $x = m_b^2/m_H^2$ by the dotted curve. We see that the DL resummed result depends substantially on $\mu$. In order to improve the stability in $\mu$ we may include the $\beta_0$ term, which has to make the $\mu$ dependence of the results smoother. We have checked that this is indeed the case. As an example, we show the DL results plus the $\beta_0$-part of the NLL correction, by the solid line in Fig.4, choosing $\mu^2 = s$. We note that, omitting the non-$\beta_0$ part is not so meaningful and the $\beta_0$ term has to be treated in the same way as the other NLL radiative corrections, specially, since as we will see later, its effects are smaller than other NLL corrections.

In the Fig.5, we present $R = \frac{F_Q(H \rightarrow \gamma\gamma)}{F_{1-loop}}$ as a function of the Higgs mass (in GeV) to DL and NLL accuracy. In this figure the result of the DL resummation is shown by the dashed line and the result of the resummation to NLL accuracy is shown by the solid line. We see that the correction is moderate and positive. The substantial contribution comes from the non $\beta_0$ part as mentioned previously. It is easy to understand the size and the sign of the DL and SL effects. In fact, the typical value for $\alpha_s \ln \frac{m^2}{m_H^2} \approx 1$, but the numerical factor $\frac{1}{2\pi}$ makes the parameter $\rho$ to be $\rho \approx 0.1$. That is the size of the DL corrections. The relative size of the SL corrections in comparison to the DL contributions is estimated as $\frac{1}{L}$, so that the absolute correction is of
order 5%. We assume the range $m_\ell = 100 - 500$ GeV for the Higgs mass. The sign of SL and DL corrections in the Sudakov form factor are different, being negative for DL and positive for the SL in the exponent. That in turn implies the positive sign for NLL effects in eq. (37).

The normalization point in the final result for the Sudakov form factor is 

$$\nu^2 = \sqrt{|p_1^2||p_2^2|},$$

which corresponds to some $\mu^2$, such that $m_0^2 \ll \mu^2 \ll s$. We see from eq. (37) that the $\beta_0$ terms are zero at

$$\mu^2 = s \left( \frac{m_b^2}{s} \right)^{a} \quad \text{with} \quad a = \frac{n + 1}{3n + 2}. \quad (42)$$

The function $a(n)$ changes with $n$ in the interval from $\frac{1}{3}$ up to $\frac{2}{5}$, so that the typical value of $a$ is about $a = 0.4$. That is why the choice of $\mu^2 = s \nu^{0.4}$ in the DL results, reproduces the DL result plus the $\beta_0$ NLL result very well. Finally, we stress that the NLL corrections are only moderate and the substantial part comes from effects which are not related to the running of the coupling constant.

5 Conclusion

In this letter we have studied the logarithmic QCD corrections to the Higgs coupling to two photons. We have developed a method for the resummation of large QCD corrections in the form of Sudakov-like logarithms of the type $\alpha_s^n \ln^{2n-1} \left( \frac{m}{m_\ell} \right)$ which can contribute to this process in certain models, such as the MSSM for large $\tan \beta$, up to next-to-leading-logarithmic (NLL) accuracy. Our main result is eq. (37). The NLL correction to the form factor is moderate, of order 5%, the substantial part of which comes from terms not related to running coupling effects.

The new type of QCD corrections in turn imply that there are additional uncertainties in the QCD correction to $H \to \gamma \gamma$ in the MSSM with large $\tan \beta$. This QCD correction is important only for the MSSM - it does not show up in the Standard Model where the light quark contributions are suppressed.

Some of the ideas presented here can also be applied to the process $\gamma \gamma \to b \bar{b}$. The details will be published elsewhere.

Acknowledgements

We thank G. Kane, G. Korchemsky, M. Melles, K. Melnikov, M. Spira, S. Rigolin, E. Yao for helpful discussions. This work has been supported in part by the US Department of Energy.
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Figure 1: Some diagrams, which represented QCD corrections to the $H \to \gamma\gamma$ decay.

Figure 2: Diagrams contributing to DL and NLL in higher orders.

Figure 3: The diagram responsible for the normalization scale setting in the coupling constant.
Figure 4: The ratio $\frac{F_Q(H\rightarrow\gamma\gamma)}{F_{1\text{-loop}}}$ as a function of the Higgs mass. The result of DL resummation plotted at the different normalization scales $\mu$. The dotted-dashed curve corresponds to $\mu^2 = m_b^2$, the short-dashed curve to the $\mu^2 = 9m_b^2$, the dashed curve to $\mu^2 = s$, the dotted curve to $\mu^2 = sx^{0.4}$. In addition, we show the DL result contribution with the $\beta_0$ part of the NLL correction at $\mu^2 = s$ by the solid curve.

Figure 5: The ratio $\frac{F_Q(H\rightarrow\gamma\gamma)}{F_{1\text{-loop}}}$ as a function of the Higgs mass. The result of the DL resummation is shown by the dashed line and the the result of the resummation to NLL accuracy is shown by the solid line, $\mu^2 = s$. 