Analysis of students’ errors in responding to TIMSS domain algebra problem

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Abstract. The TIMSS results were referred by the Indonesian Ministry of Education and Culture to develop curriculum and textbooks of school mathematics. The purpose of this study was to understand the errors of students learning with the 2006 curriculum and students learning with the 2013 curriculum in answering the TIMSS algebra content domain. A test consisting of fifteen questions of content domain algebra and application was administered to the students. The wrong answer of the test was analysed to determine the weakness and strength of algebra learning conducted. The results showed that students studying with the 2013 curriculum were outperformed their counterpart in solving the problem of algebra content domain concerning expression of application. Misconceptions about algebra and arithmetic were also found in students’ answers. The misconceptions found in this study can be further developed into an instrument to evaluate pedagogical content knowledge.

1. Introduction
The TIMSS results from three consecutive years namely 1999, 2003, and 2007 show that Grade 8 Indonesian students are still problematic in solving the TIMSS problem of algebraic content domains [1]. To overcome this problem, the Ministry of Education developed a new curriculum which is called Kurikulum 2013 (curriculum 2013) to replace the 2006 curriculum. The new curriculum implies substantial changes in the mathematics textbooks. However, the impact cannot be seen directly as Grade 8 Indonesian students did not participate in TIMSS 2015. So, we have not obtained a picture of the students' learning ability with the 2013 curriculum in solving the TIMSS algebra domain problem. Therefore, this article is expected to fully support the recommendations aiming at improving the curriculum of junior mathematics subjects, learning process, assessment methods, and instruments learning textbooks, and teacher training (both pre-service and in-service training). The implication of this study is to use misconception found in students to be developed as an instrument to evaluate pedagogical content knowledge.

2. Method
The study was conducted in one junior high schools which use the 2013 curriculum for the 7th grade and 2006 curriculum for the 8th grade. The students of both grades already studied the topic algebraic forms & their operation in the previous semester. To obtain student performance in algebra domain, 15 problem items from the set of TIMSS 2011 problem ware translated to Indonesian and given to the students. The problem contained ten multiple choice questions & 5 essay questions. Problems 2, 4, 13,
and 15 have the main topic of equation/formulas and functions, while others are algebraic expressions. Problems 1, 2, 10, 14, and 15 have cognitive types of applying domains, while others have cognitive knowing. The answer were analysed based on the type of the problems. The proportion of answer for multiple choice question between the two group of students was compared based on the cognitive domain. The answer to essay problems was analysed qualitatively. Firstly, the answers one grouped base on the type of errors. Secondly, the grouped answers were analysed to investigate why the errors occurred. Some students were interviewed to confirm the analysis results.

3. Result and discussion

3.1. Results of student answers analysis on multiple choice questions

![Figure 1](https://example.com/figure1.png)

Figure 1. A comparison chart of 10 multiple choice question.

The graph of the comparison of students' answers to 10 multiple choice questions between students learning to use the 2006 curriculum and students studying with the 2013 curriculum is shown in Figure 1. The graph shows that students studying with the 2006 curriculum are superior in working on the TIMSS topic of algebraic expression that measures the domain knowledge and topic of equations/formulas and functions that measure the domain of application. While students who study with the curriculum 2013 are superior in working on the topic of algebraic expressions that measure the domain of application. This result is in line with the excellence of the 2013 textbook that has introduced a geometry representation of algebraic expressions and implementation problems involving the representation of algebraic expressions [2] so that students who have studied with the 2013 curriculum will easily answer questions 1 and 10. Besides, this may also occur because the 2013 student curriculum book contains more domain implementation questions than on knowledge domain issues [4, 5]. However, the percentage errors of students studying with the 2013 curriculum in solving domain knowledge problems outweigh the percentage of student errors in resolving domain implementation issues [6]. Therefore, the materials and questions presented in the 2013 student curriculum should also consider the domain of knowledge of algebraic expressions.

3.2. Results of Analysis on Student Answers on Essay Questions

The problem 11 and 12 presented in Table 1 is to discuss the topic of equations/formulas and functions of the knowledge domain. The students that occurred in table 2 indicate that the student made a mistake in the process of solving the problem [2]. Students who answered 10, made an error in arithmetic calculations. Students who 

\[
\frac{0}{10}
\]

have a misunderstanding of integer execution operations by fractional numbers. The problem 12 discusses the topic of algebra on the domain expression of knowledge. The problem measures the skill of simplifying the expression of fractional algebraic expressions. Students who answered 

\[
\frac{3x^3}{14}
\]

have misconceptions on fractions. So, the students
do it also on algebraic fractions. The students also have the misconception in summing algebraic expressions. Students who answered $\frac{9x^3}{8}$, have only misconceptions in summing algebraic expressions.

| Problem | Error found | Student work |
|---------|-------------|--------------|
| No 11. Use the formula $y = 100 - \frac{100}{1+t}$ to find the value of $y$ when $t = 9$. Answer: __________ |
| Simplify the terms of algebraic expressions that only contain variables. Do not use fractional reduction rules with integers correctly. |
| $\frac{11}{10} \cdot \frac{100}{10} - \frac{9}{10} - \frac{9}{10}$
| $= \frac{9}{10}$ |
| No 12. Simplify the expression $\frac{3x}{8} + \frac{x}{4} + \frac{x}{2}$. Show your work Answer: __________ |
| Equates the denominator by summing the denominator of the algebraic fractional term and directly sums the numerator of the algebraic fractional term. Directly sums the numerator of the algebraic fractional term. |
| $\frac{3x}{8} + \frac{x}{4} + \frac{x}{2} = \frac{3x}{2} + \frac{2x}{2} + \frac{6}{2} = \frac{9x}{2}$ |

The problem 13 and 14 presented in Table 2 discusses the topic of equations/ formulas and functions application domain. The students who answered 74 made a mistake in the planning phase of problem-solving [7]. Students who received $25 + 8$ answers had misconceptions about the distributive nature of factoring. Students who $8ab$ answered have a misconception in simplifying the algebraic expression. The problem 14 discuss the topic of equations/ formulas and functions of the knowledge domain. Students who answered $\frac{51}{5}$ and $12b$ have misconceptions in the distributive nature. Students who answered 4 had misconception in understanding algebraic expressions, whereas students who answered 742 handled the numbers as mere entities rather than quantities [8].
Table 2. Errors students make on the number 13 and number 14.

| Problem | Error found | Student work |
|---------|-------------|--------------|
| 13. a + b = 25 | Specify the specific values on variables a and variables b. | ![Image](image1.png) |
| What is the value of 2a + 2b + 4? | Error in factoring | ![Image](image2.png) |
| Answer: | Simplify two unlike terms into one term. | ![Image](image3.png) |
| 14. k = 7 and l = 10 | Meaning 3kl as 3k + 3l | ![Image](image4.png) |
| What is the value of P when P = \frac{3kl}{5}? | Meaning 3kl as 3(k×l) = (3×k)×(3×l) | ![Image](image5.png) |
| Answer: | Meaning 3kl as 3 + k + l | ![Image](image6.png) |
| | Meaning 3kl combined number 3, k and l as a new number. | ![Image](image7.png) |

Problem 15 presented in Table 3 discusses the topic of equation/ formulas and function domain applying. All student responses show that students have started from the planning phase of problem-solving. Therefore, column error done is a classification strategy used by students. All students cannot model the situation into suitable algebraic equations [1], which is the most significant percentage of error when students solve the problem [7]. Student responses on the first line indicate that students have misconceptions about simplifying algebraic expressions. The student's answer on the third line shows that the students made a mistake in the process skill phase [7].
Table 3. Errors students make on the number 15.

| Problem | Error found | Student work |
|---------|-------------|--------------|
| No. 15  | Simplify every available algebraic expression on the problem. | ![Image](image1.png) |
| A piece of wood was 40 cm long. It was cut into three pieces. The lengths in cm are 2x – 5, x + 7, x + 6 | Substituting a particular value in variable x, to determine the answer. | ![Image](image2.png) |
| A. What is the length of the longest piece? Answer: _____ cm | Create three equations separately to determine the value of each algebraic expression. | ![Image](image3.png) |
| B. Show your work. If you use a calculator, you still must describe all the steps you used to obtain your answer. | | |

The way students in answer question number twelve were in line with Dhamlini and Kibirige’s research [9]. The student answers the question by multiplying the numerators and adding the denominator respectively. This case indicates that students failed to use algorithms for fractions using least common division (LCD) which shows misconceptions on numerators (multiplying instead of adding) and errors adding different denominators leading to incorrect LCD (error leading to misunderstanding) [9]. The case also indicates that the students have generalized the multiplication rule against the addition rule [10]. Student's answer number thirteen shows that students seem to understand only that two different letters always have different numerical values, but have not realized that two different letters can have the same numerical value [11]. So that students answer about number fourteen by specifying two numbers specific for two different variables, 12 for a and 13 for b. This case can be regarded as the effect of a limited conceptual understanding of the meaning of the letter as a variable [12]. When students simplify the algebraic expressions given in Problem number fifteen, it shows that students assume that x + 7 is a process which interpreted as “adding x and seven.” Students have not assumed x + 7 as an object because it contains operation symbol. Students consider that the symbol “+” or “−” as an invitation to do something [13]. They also consider not to be part of the answer. Students also consider x as the unit of measurement to replace cm.

3.3. Implication

The instrument development was conducted in three stages. Firstly, presenting the student's questions and answers; secondly, submitting questions asked to teachers predicting students' thinking, what students have comprehended and what students have not comprehended; and thirdly, asking how the teacher helps the student [14]. Examples of such questions are available in Table 4. The teacher's response to student misconceptions that have been identified is to re-explain, provide situations that cause cognitive conflict, and guide students to see their mistakes [15], while the types of questions that
teachers have identified are instructional questions, investigative questions, and inadequate/incompetent questions [16]

| Table 4. The instrument for Evaluation Pedagogical Content Knowledge. |
|---|
| 1. Simplify the expression $\frac{3x}{8} + \frac{x}{4} + \frac{x}{2}$. Show your work. |
| Ani’s solution: $\frac{3x}{8} + \frac{x}{4} + \frac{x}{2} = \frac{3x^3}{14}$ |
| Budi’s solution: $\frac{3x}{8} + \frac{x}{4} + \frac{x}{2} = \frac{9x^3}{8}$ |
| a) What does/ does not the student comprehend? |
| b) How could you show/ explain to the student that this answer is incorrect? |
| c) How could you help them? |

4. Conclusion

Results of the analysis show that students who studied under the 2013 curriculum are superior in solving problems of algebra application expression domain. However, they are lacking in solving algebraic expression problems of knowledge domains and problems of equations/formulas and functions that measure application domains. The 2013 student curriculum textbook should be designed by considering the strength of the 2006 curriculum textbook, which is proven to facilitate students well to solve problems of algebraic expression domain and problems of equations/formulas and functions measuring application domains. Moreover, there are misconceptions found when students solved TIMSS problems of algebraic content domain namely misconceptions about fractional meanings and fractional operations, misconceptions of algebraic expressions and in simplifying algebraic expressions, as well as misconceptions about distributive properties. Besides, we also found a variety of incorrect student strategies in solving the problem. It is possible that various parties can utilize this knowledge, such as being used to develop a mathematics curriculum for students, mathematics curriculum of student-teacher, mathematics textbook design, as well as to create an instrument of the national exam and improve teacher training material.

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