THE APPLICATION OF MODEL PREDICTIVE CONTROL ON STOCK PORTFOLIO OPTIMIZATION WITH PREDICTION BASED ON GEOMETRIC BROWNIAN MOTION-KALMAN FILTER

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Abstract. A stock portfolio is a collection of assets owned by investors, such as companies or individuals. The determination of the optimal stock portfolio is an important issue for the investors. Management of investors’ capital in a portfolio can be regarded as a dynamic optimal control problem. At the same time, the investors should also consider about the prediction of stock prices in the future time. Therefore, in this research, we propose Geometric Brownian Motion-Kalman Filter (GBM-KF) method to predict the future stock prices. Subsequently, the stock returns will be calculated based on the forecasting results of stock prices. Furthermore, Model Predictive Control (MPC) will be used to solve the portfolio optimization problem. It is noticeable that the management strategy of stock portfolio in this research considers the constraints on assets in the portfolio and the cost of transactions. Finally, a practical application of the solution is implemented on 3 company’s stocks. The simulation results show that the performance of the proposed controller satisfies the state’s and the control’s constraints. In addition, the amount of capital owned by the investor as the output of system shows a significant increase.

1. Introduction. An investment is an asset acquired with the objective of producing profit or income. Therefore, an investment can be defined as the current commitment of money in the expectation of obtaining future returns. In general, an investor expects to achieve a maximum return on a certain level of risk, or to earn a minimum risk on a certain level of return [2]. However, the greater returns normally can be achieved only at the price of higher risk.

Regarding to the investment in the financial markets, an investor has various options to invest his capital, and one of them is in the stock market. A stock is a...
form of security which can be used to describe the ownership of a company [2]. An investor typically decides to diversify his capital’s investment in various companies to create a stock portfolio as an effort to reduce the level of risk [9]. This portfolio is, usually, rebalanced by selling the existing stocks and using the fund to buy new securities which can increase the overall size of the portfolio. On the other hand, an investor may also sell his securities to reduce the size of his portfolio. Therefore, it is essential for the investor to manage his portfolio cautiously so that he can optimize his total capital in the portfolio.

Regarding to the stock portfolio management, there are two main issues that should be considered by an investor – the change in the future stock prices and stock portfolio optimization. Concerning on the former problem, an investor should have a strategy to forecast the stock prices in the future period. There are several tools that can be used to handle it, and one of them is Geometric Brownian Motion (GBM). Interestingly, this method can help the investor to predict the upcoming stock price movements based on the past data. However, one of the main issue on the application of GBM in predicting the movement of stock prices is the assumption of the constant parameters [11]. Therefore, the accuracy of the forecasted stock prices in the future is relatively poor. It is important to note that in assessing the accuracy of a forecasting method, we need a statistical measure and one of the widely used statistical measurement is Mean Absolute Percentage Error (MAPE). A lower value of MAPE suggests the more accuracy given by a such prediction method compared the other methods. According to Reddy (2016), the GBM model offers a relatively low Mean Absolute Percentage Error (MAPE) in the projection of daily stock prices during a one-week or, maximum, a two-week period. However, the errors tend to significantly increase when the period of prediction is longer than one month [11].

The GBM model should be updated over the time period to achieve more reliable forecasting results. In other words, we need to estimate the parameters of the GBM model based on the historical data, and one of the most common used estimation model is Kalman Filter (KF). The Kalman Filter is a state variable estimation method in linear discrete dynamical systems that can be used to predict one step ahead. Moreover, this method involves a model updating procedure. Therefore, by integrating the Kalman Filter method into the GBM model, the forecasting accuracy can have been improved. Guo et al. had used the Kalman Filter to improve the prediction results in estimating the traffic flow levels [8], while Galanis et.al applied the same method in forecasting the weather [7]. Furthermore, Cassola and Burlando utilized this method in the prediction of wind speed and wind energy [4].

Focusing on the portfolio optimization problem, it is noticeable that several approaches have been discussed in literatures. For instance, in 1952, Harry Markowitz proposed the optimal portfolio method in managing a stock portfolio. In this research, he attempted to optimize the stock portfolio by optimizing the stocks’ returns while reducing the variance [9]. Another study was performed by Bielecki and Pliska who used a method of risk-sensitive control approach to administer the stock portfolio [1]. Primbs et. al then suggested a stochastic receding horizon control as a solution to a portfolio optimization problem. The proposed approach in this study showed a successful strategy in the optimization of a portfolio with the minimum transaction costs [10]. Once a portfolio management issue is described as an optimization problem, a control method can be applied. Among all the techniques of control methods, model predictive control (MPC) can be regarded as one of the
most effective method for dealing with the optimization problem. It is interesting to note that MPC is an optimal control method which objective is to design the system’s states and outputs towards desired values, by minimizing an objective function. Furthermore, MPC provides several benefits, including the ability to overcome all the constraints on the controls and the states of the system, as well as the capability to integrate all the variables into a single objective function [3].

This paper examined the utilization of Kalman Filter modified Geometric Brownian Motion (GBM-KF) on the stock prices prediction as well as the application of Model Predictive Control (MPC) on the stock portfolio optimization. There are two steps that will be discussed in this paper – predicting the stock prices and solving the optimal control problem in the portfolio optimization. In the former step, we used the GBM model to forecast the future stock prices. Subsequently, we combine GBM and KF method (GBM-KF) to increase the accuracy of the stock prices prediction. Once we obtain the forecast of stock prices based on the GBM-KF method, we can form a projected stock portfolio. At this point, we suggest the MPC approach to solve the optimal control problem in the stock portfolio optimization. Furthermore, the constraints on the system’s controls and states, as well as the transaction costs are also integrated in the mathematical modeling. In this study, the stock portfolio comprises of three different stocks, one risk-free asset, and one capital loan asset.

2. Geometric Brownian Motion-Kalman Filter implementation in forecasting the stock prices. In this paper, we used the daily closing price data from three different companies, namely Canon, Starbucks, and Microsoft. It should be noted that the data was collected from January 2019 to June 2019. To simplify the definition, we denote the value of the stock of Canon, Starbucks, and Microsoft as stock 1, stock 2, and stock 3, respectively.

![Figure 1. Daily Stock Price of Each Company](image1)

![Figure 2. Daily Stock Return of Each Company](image2)
Having obtained the daily stock prices for all the stocks, we then forecasted the stock prices for the next 30 trading days. Firstly, we used the Geometric Brownian Motion (GBM) model in predicting the stock prices, and subsequently improved the forecasting accuracy by implementing the Kalman Filter modified Geometric Brownian Motion (GBM-KF) method. For this reason, then, we divided our data into two groups, those were “in sample data” and “out sample data”. The “out sample data” was the last 30 days stock prices, whereas the “in sample data” was the remaining data. It should be noted that the “in sample data” was used to train the model, while the “out sample data” was used to assess the performance of the model in estimating the data.

As we have stated earlier, the model used to forecast the future stock prices is Geometric Brownian Motion (GBM). The stochastic differential equation of this model can be written as follows [5]:

\[
dS(t) = \mu S(t)dt + \sigma S(t)dW(t)
\]

where
- \( S(t) \) : stock price at time-\( t \)
- \( \mu \) : drift parameter
- \( \sigma \) : volatility parameter
- \( dW(t) \) : brownian motion

The solution of the stock price model in (1) is

\[
S(t + 1) = S(t)e^{(\mu - \frac{1}{2} \sigma^2)dt + \sigma \epsilon \sqrt{dt}}
\]

where \( \epsilon \sqrt{dt} = dW(t) \).

The GBM forecasting results were collected using historical data. It is noticeable that, in forecasting the stock prices of “out sample data”, this model assumed the constant drift and volatility parameters. However, the estimation of stock prices had a relatively poor accuracy. In order to improve the accuracy of the stock price forecasts, the GBM model should be enhanced by integrating it with Kalman Filter (KF) since KF can help to minimize the covariance of estimation error. The KF method was firstly introduced by Rudolph E. Kalman in the 1960s through his classic study about a recursive solution to linear discrete data filtering problems [12]. This approach could be implemented to both a linear system or a nonlinear system. The Kalman Filter’s algorithm is shown in the Table 1, whereas the thorough process of implementing the Kalman Filter’s algorithm can be seen in [7]. Since the model discussed on (1) is a continuous model, then the following algorithm provided in Table 1 refers to [11].

| System Model and Measurement Model | System model : \( x_{k+1} = f(x_k, u_k, k) + Gw_k \) |
|-----------------------------------|-----------------------------------------------|
| Measurement model                | Measurement model : \( z_k = h(x_k, k) + v_k \) |
| Assumption                       | Assumption : \( x(0) \sim X(x_0, P_0) ; w(k) \sim N(0, Q_k) \); \( v_k \sim N(0, R) \) |
| Initialization                   | \( \hat{x}(0) = \tilde{x}_0 ; P(0) = P_0 \) |

**Table 1. Kalman Filter Algorithm**
**Table 1. Kalman Filter Algorithm (continued)**

| Time Predict | Estimation : $\hat{x}_{k+1} = f(\hat{x}_k, u_k)$ |
|--------------|--------------------------------------------------|
| Covariance   | $P_{k+1} = AP_kA^T + G_kQ_kG_k^T$               |
| Measurement Update | Kalman gain : $K_{k+1} = P_{k+1}H^T(H_k+1P_{k+1}H^T + R_{k+1})^{-1}$ |
|              | Estimation : $\tilde{x}_{k+1} = \hat{x}_{k+1} + K_{k+1}(z_{k+1} - H\hat{x}_{k+1})$ |
|              | Error covariance : $P_{k+1} = (I - K_{k+1}H)P_{k+1}$ |

**Figure 3. Stock’s Forecasting Results**

The purpose of integrating GBM and KF models was to update the drift and volatility parameters over the time, which indicated that those parameters would change as the new data was inputted. Eventually, the GBM-KF model could improve the forecasting results by incorporating the error corrections from GBM’s direct output. Figure 3 depicts the forecasting results, while Table 2 indicates the comparison of the MAPE between GBM and GBM-KF method.

| Stock       | GBM | GBM-KF |
|-------------|-----|--------|
| Stock 1 (Canon) | 1.01 | 0.16   |
| Stock 2 (Starbucks) | 0.59 | 0.097  |
| Stock 3 (Microsoft) | 1.23 | 0.1    |

**Table 2. MAPE (%) GBM vs GBM-KF**

It can clearly be seen from Table 2 that GBM-KF approach outperforms the GBM method in predicting the stock prices for three companies. By estimating the drift and volatility parameters in the GBM model, the GBM-KF approach successfully reduced the error between the actual price and the forecasting price. Finally, the stock price forecasts based on GBM-KF approach would be used in the development of our stock portfolio.
3. **Mathematical model of stock portfolio management.** At the initial time, an investor had a capital that could be invested in \( n \) different risky assets during the investment period. In general, the capital was fund/money (in US$) and can be in a risk-free state, which usually is a bank account. As a consequence, this account can generate a constant yield. The investor’s capital deposited in the bank account is represented by the \((n+1)^{th}\) asset. During the investment period, the investor has to rebalance his portfolio to obtain the optimal return. We describe this problem in the discrete form and define the time steps as \( t = 1, 2, \cdots, T \), with \( T \) being the end of planning horizon.

The model of stock portfolio management for the \( n \) risky assets can be defined by [6]:

\[
x_i(k+1) = [1 + \eta_i(k)] [x_i(k) + p_i(k) - q_i(k)], \quad i = 1, 2, \cdots, n
\tag{3}
\]

where

- \( \eta_i(k) \): return in the \( i \)-th risky asset at time-\( k \)
- \( p_i(k) \): the total transferred money from the bank account into the \( i \)-th risky asset at time-\( k \)
- \( q_i(k) \): the total transferred money from the \( i \)-th risky asset into the bank account at time-\( k \)
- \( x_i(k) \): total capital invested in the \( i \)-th risky asset at time-\( k \)

The transaction costs can be defined as \( \alpha \) and \( \beta \), where they correspond to the transaction cost on the stock purchasing and on the stock selling, respectively. The evolution of the risk-free asset, which is the bank account, can be illustrated as follows:

\[
x_{n+1}(k+1) = [1 + r_1(k)] [x_{n+1}(k) - (1 - \alpha) \sum_{i=1}^{n} p_i(k) + (1 - \beta) \sum_{i=1}^{n} q_i(k)]
\tag{4}
\]

where

- \( r_1(k) \): yield of the bank account

Finally, the evolution of the investor’s capital can be expressed as:

\[
y(k) = \sum_{i=1}^{n+1} x_i(k)
\tag{5}
\]

As we can see on (5), the total investor’s capital is the sum of the invested capital in each of risky assets and the risk-free asset. This model also includes \( \alpha \) and \( \beta \), which penalize the transaction costs in terms of stock’s yield.

Regarding to the mathematical model of the stock portfolio management, we define the state variable as \( \mathbf{x} \), which includes \( x_1, \cdots, x_n, x_{n+1} \). The \( x_1, x_2, \cdots, x_n \) variables indicate the total of investor’s capital in the first, second, \( \cdots \), and \( n \)-th risky asset, respectively. It is also important to note that \( x_{n+1} \) describes the total capital of investor in the risk-free asset. Subsequently, the control variables are represented by \( \mathbf{u} \) including \( p_i \) and \( q_i \), where \( i = 1, 2, \cdots, n \). Eventually, the output variable is denoted by \( \mathbf{y} \) that shows the total of investor’s capital in all assets. Based on the description above, we can rewrite (3), (4), and (5) into the matrices as follows:

\[
x(k+1) = A\mathbf{x}(k) + B\mathbf{u}(k)
\tag{6}
\]

\[
y(k) = C\mathbf{x}(k)
\tag{7}
\]
where
\[
x(k + 1) = \begin{bmatrix} x_1(k + 1) \\ \vdots \\ x_n(k + 1) \\ x_{n+1}(k + 1) \end{bmatrix}, \quad u(k) = \begin{bmatrix} p_1(k) \\ \vdots \\ p_n(k) \\ q_1(k) \\ \vdots \\ q_n(k) \end{bmatrix},
\]
\[
A = \begin{bmatrix} 1 + \eta_1(k) & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1 + \eta_n(k) & 0 \\ 0 & \cdots & 0 & 1 + r_1(k) \end{bmatrix},
\]
\[
B = \begin{bmatrix} 1 + \eta_1(k) & \cdots & 0 & - (1 + \eta_1(k)) & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1 + \eta_n(k) & 0 & - (1 + \eta_n(k)) & 0 \\ (1 + r_1(k))(-1 - \alpha) & \cdots & (1 + r_1(k))(-1 - \alpha) & (1 + r_1(k))(1 + \beta) & \cdots & (1 + r_1(k))(1 + \beta) & 1 + r_1(k) \end{bmatrix},
\]
\[
C = [1 \cdots 1 - 1].
\]

In the portfolio optimization, we define the constraints of the control variables as:
\[
p_i(k) \geq 0, \text{ where } i = 1, 2, \cdots, n \quad (8)
\]
\[
q_i(k) \geq 0, \text{ where } i = 1, 2, \cdots, n \quad (9)
\]

In addition, the \(x_{n+1}\) variable cannot be a negative value, that will mean loans in the account. Therefore, we define a constraint for variable \(x_{n+1}\) as
\[
x_{n+1}(k) - (1 + \alpha) \sum_{i=1}^{n} p_i(k) + (1 - \beta) \sum_{i=1}^{n} q_i(k) \geq 0 \quad (10)
\]

Finally, we assume that the short sales are not permitted when rebalancing the portfolio. For this reason, we define the constraint as follows:
\[
x_i(k) + p_i(k) - q_i(k) \geq 0 \quad (11)
\]

4. **The implementation of model predictive control in the stock portfolio optimization.** As a main base of the prediction for the MPC, we can write the model in a discrete state space equations which is generally defined as:
\[
x(k + 1) = Ax(k) + Bu(k) \quad (12)
\]
\[
y(k) = Cx(k) \quad (13)
\]

where \(x(k), u(k),\) and \(y(k)\) denote the state, the control, and the output of the system, respectively. The \(A\) matrix describes the states evolution, whereas \(B\) indicates the matrix that relates the system’s inputs and states. Finally, \(C\) denotes the matrix which relates the states and the output of the system.

In general, the objective function can be defined as [13]:
\[
J(u(k), e(k)) = \sum_{j=1}^{N_p} e(k + j)^T Q e(k + j) + u(k + j)^T R u(k + j) \quad (14)
\]

where \(e(k + j) = y(k) - r(k),\) with \(r(k)\) and \(N_p\) indicate the reference trajectory and the prediction horizon, respectively. It should be noted that \(u(k + j)\) represents the value of the control of system at time step \(k + j,\) while \(e(k + j)\) is the error at
time step \( k + j \). Furthermore, \( Q \) and \( R \) are positive semidefinite matrices for error and control, respectively.

The goal of the MPC controller is to find the optimal controller \( u^* \) which can minimize the objective function. Since the optimal control used in MPC is in a quadratic-programming form, then we can reexpress the objective function in (14) as follows:

\[
\min J(\hat{u}(k)) = \hat{u}^T(k)H\hat{u}(k) + 2f^T\hat{u}(k)
\]

where

\[
H = \hat{B}^T\hat{Q}\hat{B} + \hat{R}
\]

\[
f = \hat{B}^T\hat{Q}(\hat{A}\hat{x}(k) - r(k))
\]

\[
\hat{u}(k) = [u(k), u(k + 1), \ldots, u(k + N_p - 1)]^T
\]

\[
\hat{B} = \begin{bmatrix}
C B & 0 & 0 & 0 & \cdots & 0 \\
C A B & C B & 0 & 0 & \cdots & 0 \\
C A^2 B & C A B & C B & 0 & \cdots & 0 \\
C A^3 B & C A^2 B & C A B & C B & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
C A^{N_p-1} B & C A^{N_p-2} B & C A^{N_p-3} B & C A^{N_p-4} B & \cdots & C B
\end{bmatrix}_{N_p \times N_p}
\]

\[
\hat{A} = \begin{bmatrix}
C A & \vdots & \vdots & \vdots & \cdots & \vdots \\
\vdots & C A & \vdots & \vdots & \cdots & \vdots \\
\vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
\vdots & \vdots & \cdots & C A & \vdots & \vdots \\
\vdots & \vdots & \cdots & \vdots & C A & \vdots \\
\vdots & \vdots & \cdots & \vdots & \cdots & C A^2
\end{bmatrix}_{N_p \times 1}
\]

\[
\hat{Q} = \begin{bmatrix}
Q & 0 & 0 & \cdots & 0 \\
0 & Q & 0 & \cdots & 0 \\
0 & 0 & Q & \cdots & 0 \\
0 & 0 & 0 & \ddots & \vdots \\
0 & 0 & 0 & \cdots & Q
\end{bmatrix}_{N_p \times N_p}
\]

\[
\hat{R} = \begin{bmatrix}
R & 0 & 0 & \cdots & 0 \\
0 & R & 0 & \cdots & 0 \\
0 & 0 & R & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & R
\end{bmatrix}_{N_p \times N_p}
\]

The optimal control of the system at the time step \( k \) can be described as:

\[
\hat{u}^*(k) = [u^*(k), u^*(k + 1), \ldots, u^*(k + N_p - 1)]^T
\]

After obtaining the optimal control value at time step \( k \), we can use the MPC method to predict the system’s states and output at time step \( k + 1 \). Furthermore, the value of the output is also included into the optimization process in order to achieve the optimal control value at the next-time step. This process is repeated until the system’s output follows the reference trajectory that has been defined at the beginning of the period.

5. Numerical simulation in portfolio optimization problem. At the initial condition, we define all the values of the control to be zero. It means that \( p_i = q_i = 0 \), where \( i = 1, 2, 3 \). Furthermore, the value for the initial state variables are given as \( x(0) = [x_1(0), x_2(0), x_3(0), x_4(0)]^T = [0, 0, 0, 1 \times 10^5]^T \).

It is important to note that the data from one of international banks in South East Asia, whereas the other parameters had been chosen by the author.

The control actions based on MPC optimization are presented in the Figure 4. It can clearly be seen that the system’s controls during the period are within the constraints value. It demonstrates that the controller tries to achieve the best performance of the system. With regard to Figure 5, one can see the dynamic of investor’s total capital in each risky asset. Interestingly, the daily stock return of each stock plays a significant role in the change of the invested capital in each asset. It indicates that the system in the MPC method tries to minimize the loss when the
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| Variable | $\alpha$ | $\beta$ | $r_1$ | $r_2$ | $x(0)$ | $N_p$ |
|----------|----------|----------|-------|-------|--------|-------|
| Value    | 0.0002   | 0.0002   | 0.0003 | 0.00031 | $[0, 0, 0, 1 \times 10^5]^T$ | 10    |

| Variable | $Q$ | $R$ | $r(k)$ | $p_{\text{max}}$ | $q_{\text{max}}$ |
|----------|-----|-----|--------|------------------|------------------|
| Value    | 1   | 0.1 | $10^6$ | $10^5$           | $10^5$           |

Table 3. Parameters of Stock Portfolio

Figure 4. Control Variables in Portfolio Optimization

Figure 5. The Dynamic of Total Invested Capital in Each Stock

daily price of the stocks decline. On the other hand, the controller takes an effort to maximize the return of the stock when the daily stock prices increase. As a result, the total of investor’s capital grows substantially during the time period. Figure 6 indicates the amount of invested capital in the risk-free asset during the investment period. It can be seen that during the first 17 days, there was an insignificant change on the invested capital in the risk-free asset. This condition may be affected by the limitation of the initial capital and the constraints in the control variables. However, during the last 5 days, the total capital in the risk-free asset indicates a significant change since the investor has received the returns from the stocks’ trading during the previous 10 days. Finally, the investor’s capital in all assets can
The MPC controller may achieve the best performance when the reference value is relatively large. This value should be theoretically infinite, requiring the MPC controller to perform the control behaviour within the constraints. As a result, the amount of investor’s capital increases substantially due to the highest return on each investment. At the initial condition, the total amount of invested capital is US$ $1 \times 10^5$. This amount, then, grows to reach a value of around US$ 1.1272 \times 10^5$ at the end of investment period. As a comparison, when an investor decided to invest his capital in the bank, then, at the end of period he will receive a mere US$ 1.0009 \times 10^5$. A relatively big difference between the investor’s capital in the portfolio and in the bank during the period indicates that investing in stock’s portfolio is more profitable than investing in the bank.

With regard to the implementation of model predictive control in the portfolio optimization, it indicates a positive outcome. The performance of MPC reflects an effective approach, enabling MPC to follow a strategy which closes to that of stock market traders. The MPC controller will take an action to sell the stocks when the prices go down, and buy them when the stocks indicate an increasing trend. Furthermore, the constrained solutions given by MPC are effective since the

6. Conclusion. In this paper, we demonstrate how the Kalman-Filter (KF) model can be used to improve the forecast of stock prices based on Geometric Brownian Motion (GBM) approach. We may conclude that, from the MAPE of forecasting results, the GBM-KF outperforms the GBM model in forecasting daily stock prices. Following that, we used the stock price prediction based on the GBM-KF model to calculate the each stock’s return, which then will be used in the portfolio optimization.
all controllers consider the transaction cost and the limited amount of assets that can be traded. These constraints provide more opportunities for MPC to tailor its optimization approach to the needs of investors. In addition, the MPC strategy also provides an optimal solution based on the cost function and the problem’s constraints. Interestingly, the initial conditions of the states should be chosen to prevent an unfeasible solution given by the MPC constraints.

Ultimately, the simulation results show that the MPC is well-applied to solving the portfolio optimization problem. In this case, the MPC is capable of designing the best controller with the aim of controlling the investor’s capital in the stock portfolio investment. Furthermore, the MPC acts as a decision maker in determining the best stock investment strategy. As a result, the total capital of investor in all assets increased at the end of the period. It is also important to note that different optimization strategies may result in different solutions to the same problem. It has a relatively high dependency on the cost function, the constrains values, and the reference trajectory.

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