Precise measurement of spin-averaged $\chi_{cJ}(1P)$ mass using photon conversions in $\psi(2S) \rightarrow \gamma \chi_{cJ}$

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(39) $(1\pm 3)\%$.
I. INTRODUCTION

Precise measurements of the spectrum and the decay properties of charmonia are essential to test Potential QCD models and QCD based approaches. There is renewed interest since the discovery of the X(3872) and the observations of the expected $\eta_c$(2S) and $h_C$ ($^1P_1$) states, and there has been recent progress, both theoretically and experimentally. There are more accurate determinations of the charmonium mass spectrum and radiative transition rates using both a relativistic quark model with relativistic corrections of order $v^2/c^2$ and a potential model with a semirelativistic approach. The $\psi(2S)$ mass and width have been redetermined with an updated radiative correction, and newly measured with better precision. In addition to previous measurements of $\chi_{cJ}$ states, two $\chi_{c0}$ measurements by E835 and new $\chi_{cJ}$ ($J=0,1,2$) measurements by CLEO have been recently published. Improved precision on $\chi_{cJ}$ masses is important for the determination of the singlet-triplet splitting, $M(^3P_J) - M(^3P_{cog})$, which is predicted by lattice QCD and nonrelativistic QCD. Here $M(^3P_{cog})$ is the spin-averaged $^3P_J$ mass for the $\chi_{cJ}$ states ($J=0,1,2$).

In this paper, results on the $\chi_{cJ}$ masses ($J=0,1,2$) and widths ($J=0,1$) from a measurement of the energy spectrum of inclusive photons in $\psi(2S)$ radiative decays, using photon conversions to improve the energy resolution, are presented. The measurement uses $14 \times 10^6 \psi(2S)$ events collected with the upgraded Beijing Spectrometer (BESII) at the BEPC Collider.

II. BES DETECTOR AND MONTE CARLO SIMULATION

The BESII detector is described elsewhere. A 12-layer vertex chamber (VC) surrounding the beam pipe provides hit information in trigger criteria for charged tracks. Charged particle momenta are determined with a resolution of $\sigma_p/p = 1.78\% + p^2$ (p in GeV/c) in a 40-layer cylindrical drift chamber (MDC). Particle identification is accomplished by measurements of ionization ($dE/dx$) in the MDC and time-of-flight (TOF) in a barrel-like array of 48 scintillation counters. The $dE/dx$ resolution is $\sigma_{dE/dx} = 8\%$; the TOF resolution is $\sigma_{TOF} = 200$ ps for hadrons. A 12-radiation-length barrel shower counter (BSC) measures energies of photons with a resolution of $\sigma_E/E = 21\%/\sqrt{E}$ ($E$ in GeV). A solenoidal coil supplies a 0.4 Tesla magnetic field over the tracking volume.

A Geant3 based Monte Carlo (MC) SIMBES, which simulates the detector response including interactions of secondary particles in the detector material, is used to determine the energy resolution and detection efficiency of photons reconstructed from their converted $e^+e^-$ pairs, as well as to optimize selection criteria and estimate backgrounds. Under the assumption of a pure $E_1$ transition for the $\psi(2S) \rightarrow \gamma \chi_{cJ}$ decays, the polar angle ($\theta$) distributions of the photons are given by $1 + k\cos^2\theta$ with $k = 1, -\frac{1}{2}, \frac{1}{2}$ for $J = 0, 1, 2$, respectively.

Good energy resolution for low energy photons is essential for precise measurements of $\chi_{cJ}$ masses and widths from fitting the photon spectrum of $\psi(2S)$ radiative decays. Momentum resolution of about 1.6 to 4.1 MeV/c can be obtained for low momentum electrons from 60 to 250 MeV/c. Photons from $\psi(2S) \rightarrow \gamma \chi_{cJ}$ decays have energies of about 261, 171, and 128 MeV for the $\chi_{cJ}$ final states ($J=0,1,2$), and the electrons produced in photon conversions occur in this low momentum region.

III. PHOTON RECONSTRUCTION AND SELECTION

We choose two oppositely charged tracks with each track having a good helix fit and a polar angle with $|\cos\theta| < 0.8$. The intersection of the electron and positron trajectories in the $xy$-plane (the beam line is the $z$ axis) is determined, and this point is taken as the photon conversion point (CP). The photon conversion length $R_{xy}$ is defined as the distance from the beam line to the CP in the $xy$-plane. Fig. shows the $R_{xy}$ distribution for photon conversions to $e^+e^-$ pairs in the BESII detector for hadronic events in the $58 \times 10^6 J/\psi$ event sample. The two broad peaks in Fig. correspond to the beampipe region, where the beampipe, the VC, and inner wall of the MDC are located. Combinatorial background from charged hadron tracks is also seen in the $R_{xy} < 2$ cm region. Equivalent materials in the beampipe wall, VC, VC outer wall, and the MDC inner wall are 0.536, 0.657, 0.375, and 1.107 in units of $0.01X_0$, respectively, where $X_0$ is a radiation length. The electron and positron directions are calculated at the photon conversion point, and their momenta are corrected to that point.

Good photons are selected. The photon conversion length must lie within the beampipe region, $2 < R_{xy} < 22$ cm, and the invariant mass of an $e^+e^-$ pair is required to satisfy $M_{e^+e^-} < 20$ MeV/c$^2$. Combinatorial background from
charged hadron tracks is further removed by requiring \( \cos \theta_{\text{defl}} > 0.9 \), where \( \theta_{\text{defl}} \) is the deflection angle between the photon momentum and photon track (a vector from the beam to the CP). To suppress background from beam-gas and beam-pipe interactions, the total energy in the event must satisfy \( E_{\text{tot}} > E_{\text{beam}}/2 \) and momentum asymmetry must satisfy \( d p_{\text{asym}} < 0.9 \). Here \( d p_{\text{asym}} \) is defined as a ratio of the vector sum to the scalar sum of the momenta of all charged and neutral tracks in the event. The observed photon energy spectrum from the \( \psi(2S) \) data after the selection of good photons is shown in Fig. 2. The spectrum shows the \( \chi_{cJ} \) states plus a large background. The sharp drop at low energy is mainly caused by low photon detection efficiency.

IV. \( dE/dx \) CORRECTION AND PHOTON ENERGY SCALE

Energy loss \( dE/dx \) by ionization for electrons traversing a small thickness of material with energy above a few tens of MeV can be described by the Bethe-Bloch equation \( 9 \). The \( dE/dx \) correction for charged particles, produced near the beamline and traversing the whole beampipe region, should take into account the full thickness of material in the region. However, \( e^+e^- \) pairs from photon conversions are mostly produced in the region where the VC outer wall and the MDC inner wall are located. Thus the effective thickness of material between the location, where a pair is produced, and first layer of the MDC wires must be estimated for each electron pair. The procedure to make \( dE/dx \) corrections for electrons has two steps: (1) A preliminary \( dE/dx \) correction using half the full thickness of all the materials in the beampipe region is made. Good photons are reconstructed, and their conversion lengths \( R_{xy} \) are calculated. (2) The final \( dE/dx \) corrections are estimated based on these \( R_{xy} \).

The energy scale of photons reconstructed from \( e^+e^- \) pairs is studied using simulated MC events and data. \( 63 \times 10^6 \pi^0 \) signal events are generated using MC technique, with same momentum and polar angular distributions as that
found from π⁰ data sample. A sample of π⁰ mesons decaying to two photons with both photons converting to e⁺e⁻ pairs is selected from 58 × 10⁶ J/ψ events. To suppress hadron contamination, electron identification is required and good photons are selected. Background is further suppressed with additional requirements on the photon energy, E_γ ≤ 1 GeV, and the opening angle between the two photons, 0.75 < |cosθ_{γγ}| < 0.97. The invariant mass distribution of two photons for both MC and data, after the specific dE/dx correction for electrons described above, is fitted with the improved Crystal Ball (ICB) function (defined in section V) plus a first order polynomial background. The results are shown in Fig. 3. The resulting π⁰ masses (134.47 ± 0.42) MeV/c² in data and (134.86 ± 0.20) MeV/c² in MC are consistent with the PDG value of 134.98 MeV/c². The corresponding mass resolutions (5.70 ± 0.58) MeV/c² in data and (5.55 ± 0.21) MeV/c² in MC agree within errors. The χ²/D.F. (degree of freedom) for the fits are 126/103 in data and 117/140 in MC.

\[ E_γ = \frac{(M_{ψ(2S)}^2 - M_{χ_{cJ}}^2)}{2M_{ψ(2S)}} \]

where \( M_{ψ(2S)} \) and \( M_{χ_{cJ}} \) are the masses of the \( ψ(2S) \) and \( χ_{cJ} \), respectively. The \( ψ(2S) \) and \( χ_{cJ} \) widths must be taken into account. \( M_{ψ(2S)} \) and \( M_{χ_{cJ}} \) are described by Breit-Wigner functions (2D problem). By taking \( x = M_{ψ(2S)} \), the probability density function (pdf) for the photon energy \( E_γ \) can be written as

\[ f_{pdf}(E_γ) = \int BW(x)BW(M_{χ_{cJ}}) \frac{x}{M_{χ_{cJ}}} dx, \]

where \( M_{χ_{cJ}} \) depends on \( E_γ \) by Eq. (1).

As a result of traversing material in the beam pipe region, the electron energy is smeared due to energy loss by ionization, and a long tail on the low side of the energy distribution is induced by bremsstrahlung radiation. Multiple scattering of electrons, especially at large angles, gives tails on both sides of the photon energy distribution of photon conversions. The photon energy resolution from photon conversions can be nicely modeled by our Geant3 MC simulation, and well fitted by the ICB function. The original Crystal Ball (CB) function has a Gaussian in its central and upper energy region but long tail at lower energy region. The improved CB function is defined as same as the CB function but has an additional tail at its upper side. The photon energy distributions from large MC samples of \( ψ(2S) \rightarrow γχ_{cJ} \) decays (\( J = 0, 1, 2 \)), with zero widths for both the \( ψ(2S) \) and \( χ_{cJ} \) states, are fitted to ICB functions and shown in Fig. 4. The χ²/D.F. from the fits are 103.8/93, 37.7/53 and 47.6/43 corresponding to \( χ_{c0}, χ_{c1}, χ_{c2} \) states. Five parameters in the ICB function, the photon energy resolution and four empirical parameters

V. PHYSICS PROBLEM AND DETECTOR RESOLUTION

FIG. 3: Invariant mass distribution of photon pair from J/ψ data (top) and Monte Carlo (bottom) events. The solid line is the fitted curve for signal plus background. The dashed line is the fitted curve for background.
to describe the tails on the lower and upper sides are determined from the fits and used as input parameters in the detector resolution function for each decay mode. Photon energy resolutions for the $\psi(2S) \rightarrow \gamma \chi_{cJ}$ decays ($J=0,1,2$) are found to be $3.78 \pm 0.04$, $2.58 \pm 0.05$, and $2.26 \pm 0.11$ MeV, respectively.

The energy dependencies of the photon detection efficiency and resolution are included in the fitting procedure. Normalization factors, masses $M_{\chi_{cJ}}$ and widths $\Gamma_{\chi_{cJ}}$ ($J=0,1,2$), are implicitly contained in the $E_1^3$ energy dependence included in the fitted signal shape. The detection efficiency $\epsilon_{\chi_{cJ}}(x)$ and energy resolution as a function of photon energy are included in the fitting. Normalization factors $E_{\gamma,\chi_{cJ}}$ and $\epsilon(E_{\gamma,\chi_{cJ}})$ are photon energy corresponding to fitted $\chi_{cJ}$ mass and efficiency at the photon energy, respectively. Notice that parameters, masses $M_{\chi_{cJ}}$ and widths $\Gamma_{\chi_{cJ}}$ ($J=0,1,2$), are implicitly contained in the $f_{pdf}(x)$ function, and detector resolution and tail parameters are in the $g_{res}(y)$ function. The likelihood function, $Lk(u; M_{\chi_{cJ}}, \Gamma_{\chi_{cJ}})$, is constructed with three $\chi_{cJ}$ signals plus threshold background:

$$Lk(u; M_{\chi_{cJ}}, \Gamma_{\chi_{cJ}}) = bg_{threshold}(u) + \sum_{J=0}^{2} A_J \cdot h_{pdf,\chi_{cJ}}(u; M_{\chi_{cJ}}, \Gamma_{\chi_{cJ}}).$$

Here $A_J$ is the observed area in each $\chi_{cJ}$ signal.

An input-output test is performed to verify the accuracy of the fitting algorithm for the 2D problem using MC events. The energy dependencies of the photon detection efficiency and resolution are included in the fitting procedure.

FIG. 4: Energy distributions of signal photons from $\psi(2S) \rightarrow \gamma \chi_{cJ}$ decays with zero widths of both $\psi(2S)$ and $\chi_{cJ}$ for $\chi_{c0}$ (left), $\chi_{c1}$ (mid) and $\chi_{c2}$ (right) final states fitted to the ICB function. The points are MC data. The solid line is the fitted curve.
A sample of MC events for the $\psi(2S) \rightarrow \gamma \chi_{cJ}$ decays with non-zero width for both the $\psi(2S)$ and $\chi_{cJ}$ are produced. The photon energy distribution is fitted with the 2D pdf function convoluted with the ICB resolution. The resulting masses and widths of the $\chi_{cJ}$ states in this test are consistent with the MC input parameters.

A combined fit of the three photon spectra corresponding to the $\psi(2S) \rightarrow \gamma \chi_{c0}, \gamma \chi_{c1}, \gamma \chi_{c2}$ decays is performed to three 2D pdf functions (see Eq. 2), each convoluted with its ICB resolution function, plus threshold background. The $\chi_{c2}$ width is fixed in the fit due to the limited statistics. The $\chi^2$ and D.F. (degrees of freedom) from the fit are 521.0 and (520-13), where total number of free parameters is 13. The effect of the beam energy spread [20] in the measurement is also included, but is found to be negligible due to the narrow width of the $\psi(2S)$ state. A study shows that the bin size (0.5 or 0.2 MeV) in the binned fits slightly affects the fitted masses and widths of the $\chi_{cJ}$ states. The difference in the results due to different bin sizes are added to the systematic error. The results of the binned fit (0.5 MeV/bin) are shown in Fig. 2.

VII. SYSTEMATIC ERRORS

Samples of QED radiative two photon events with one photon converting to an $e^+e^-$ pair are selected for both data and MC simulation. The two photons are required to be emitted back-to-back. The fitted photon energy in data is different from the expected MC value by $-1.2\sigma$ ($\sigma = 0.86$ MeV), which has a relative error at the same level as a correction factor $s = 0.9975 \pm 0.0007$ for the magnetic field [21]. Thus a relative precision of 0.0007 is added as the systematic error in the photon energy determination.

The selection of $\pi^0 \rightarrow \gamma \gamma$ events with both photons converting to $e^+e^-$ pairs from $58 \times 10^6$ $J/\psi$ events yields a data sample of 503 $\pi^0$ mesons. A MC sample of $\pi^0$ mesons is generated with the same momentum and polar angular distributions as found from the $\pi^0$ data sample. The $\pi^0$ mass resolutions determined from the data and MC are in good agreement; their difference is $0.15 \pm 0.06$ MeV/$c^2$. We assume that the photon energy resolution and uncertainty in the direction of the photon momentum each contribute half in the $\pi^0$ mass resolution. Hence, the difference $\Delta \sigma_{M,0}$ of the $\pi^0$ mass resolutions between MC and data from the uncertainty of the photon energy resolution lies within $(-0.29, +0.59)$ MeV/$c^2$ with a probability of 68.3%. We assume $\Delta \sigma_{E,0}(\chi_{cJ})/\sigma_{E,0}(\chi_{cJ}, MC) = \Delta \sigma_{M,0}/\sigma_{M,0}(MC)$, where $\sigma_{E,0}(\chi_{cJ}, MC)$ and $\Delta \sigma_{E,0}(\chi_{cJ})$ are the MC photon energy resolution and the difference between MC and data for $\chi_{cJ}$ final states, and $\sigma_{M,0}(MC)$ and $\Delta \sigma_{M,0}$ are $\pi^0$ mass resolution in MC and the difference between MC and data. Thus $1\sigma$ confidence intervals of $\Delta \sigma_{E,0}(\chi_{cJ})$ for the $\psi(2S) \rightarrow \gamma \chi_{c0}, \gamma \chi_{c1}$ decays are estimated to be $(-0.20, +0.40)$ and $(-0.14, +0.27)$ MeV, which are used to estimate systematic errors in the determination of the $\chi_{c0}$ and $\chi_{c1}$ widths.

The effect of the background shape uncertainty is studied using $\psi(2S)$ data and $\psi(2S) \rightarrow anything$ MC [22]. The relative differences in background shape parameters between floated and fixed widths of the $\chi_{c0, c1}$ states are determined in fits for MC data, and fed back to correct background parameters in the fits for data. The difference between results for $\psi(2S)$ data with the background shape floated and fixed is taken as a systematic error. In addition, our MC study with non-zero width of both $\psi(2S)$ and $\chi_{cJ}$ shows that differences in the fitted masses from input values for the $\chi_{c0}$ and $\chi_{c1}$ are $0.12 \pm 0.06$ and $0.06 \pm 0.03$ MeV/$c^2$, while that for the $\chi_{c2}$ is as large as $0.31 \pm 0.06$ MeV/$c^2$. The differences are attributed to uncertainties in the energy loss correction for low momentum electrons. The systematic errors, including the contributions from the uncertainties of the photon detection efficiency, are summarized in the Table I.

### Table I: Summary of systematic errors in the determination of the $\chi_{cJ}$ masses and widths (in MeV/$c^2$).

| source                      | $M_{\chi_{c0}}$ | $\Gamma_{\chi_{c0}}$ | $M_{\chi_{c1}}$ | $\Gamma_{\chi_{c1}}$ | $M_{\chi_{c2}}$ |
|-----------------------------|-----------------|-----------------------|-----------------|-----------------------|-----------------|
| background shape            | 0.03            | 0.8                   | 0.02            | 0.07                  | 0.04            |
| correction in magnetic field| 0.19            | 0.13                  | 0.09            |                       |                 |
| MC simulation in $\sigma_{E, \gamma}$ | +0.29 | +0.25                  | -0.76           | -0.77                  |                 |
| different bin size          | 0.02            | 0.02                  | 0.01            | 0.01                  | 0.02            |
| photon energy correction    | 0.18            | 0.09                  | 0.37            |                       |                 |
| efficiency uncertainty      | 0.04            | 0.03                  | 0.01            | 0.00                  | 0.04            |
| error of $M_{\psi(2S)}$     | 0.034           | 0.034                 | 0.034           |                       |                 |
| total                       | 0.27            | 0.16                  | 0.16            | 0.14                  | 0.39            |
VIII. RESULTS AND DISCUSSION

With good energy resolution for low energy photons obtained using photon conversions, the precise measurement of the masses and widths of $\chi_{cJ} (J = 0, 1, 2)$ states from inclusive $\psi(2S)$ radiative decays can be obtained. The masses and widths are determined to be $M_{\chi_{c0}} = 3414.21 \pm 0.39 \pm 0.27$, $M_{\chi_{c1}} = 3510.30 \pm 0.14 \pm 0.16$, $M_{\chi_{c2}} = 3555.70 \pm 0.59 \pm 0.39$ MeV/$c^2$, $\Gamma_{\chi_{c0}} = 12.6^{+1.5+0.9}_{-1.6-1.1}$ and $\Gamma_{\chi_{c1}} = 1.39^{+0.40+0.26}_{-0.38-0.77}$ MeV/$c^2$. The mass splittings in the $\chi_{cJ}(1P)$ triplet and their ratio are found to be $\Delta M_{2J} = M_{\chi_{c2}} - M_{\chi_{c1}} = 45.40 \pm 0.61 \pm 0.42$ MeV/$c^2$, $\Delta M_{1J} = M_{\chi_{c1}} - M_{\chi_{c0}} = 96.09 \pm 0.41 \pm 0.31$ MeV/$c^2$ and $\rho(\chi_{cJ}) = \Delta M_{2J}/\Delta M_{1J} = 0.472 \pm 0.006 \pm 0.004$. For the first time, the spin-averaged $^3P_J$ mass (weighted with the factors $2J + 1$) for the $\chi_{cJ}$ states is precisely measured in one experiment and determined to be $M(3P_{c0}) = 3524.85 \pm 0.32 \pm 0.30$ MeV/$c^2$. The first errors in the results are statistical and the second are systematic. Correlations are taken into account in estimations of both statistical and systematic errors for the $\Delta M_{2J}$, $\Delta M_{1J}$, $\rho(\chi_{cJ})$ and $M(3P_{c0})$. Correlation coefficients between mass parameters for statistical error are obtained from the error matrix of the combined fit, and that for systematic error are all assumed to be 1.

The $\chi_{cJ}$ masses ($J=0,1,2$) determined here are consistent with the recent measurements by CLEO [11], but have smaller systematic errors. The precisions for the $\chi_{c0}$ and $\chi_{c1}$ masses are compatible with those of previous measurements by E835 [10] and E760 [9], while that for the $\chi_{c2}$ mass is not as good as theirs due to low statistics. Note that our $\chi_{c0}$ mass is lower than that measured by the E835 via $\chi_{c0} \to \gamma J/\psi$ decay by 1.2 MeV (corresponding to 1.8σ), but agrees with their later measurement via $\chi_{c0} \to \pi^\pm \pi^\mp$ decay. The width of the $\chi_{cJ}$ states ($J=0,1$) determined here are also consistent with their values; larger errors in our widths are caused by limited statistics for both signal photons and inclusive $\pi^0$ mesons.

IX. ACKNOWLEDGMENT

The BES collaboration thanks the staff of BEPC for their hard efforts and the members of IHEP computing center for their helpful assistance, and also T.P. Li for helpful discussion on 2D pdf function. This work is supported in part by the National Natural Science Foundation of China under contracts Nos. 19991480, 10225524, 10225525, the Chinese Academy of Sciences under contract No. KJ 95T-03, the 100 Talents Program of CAS under Contract Nos. U-11, U-24, U-25, and the Knowledge Innovation Project of CAS under Contract Nos. U-602, U-34 (IHEP); by the National Natural Science Foundation of China under Contract No.10175060 (USTC); and No.10225522 (Tsinghua University); and by the U.S. Department of Energy under Contract No.DE-FG02-04ER41291 (U Hawaii).

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