When Tesla Meets Nash: Wireless Power Provision as a Public Good

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Abstract—The wireless power transfer (WPT) technology enables a cost-effective and sustainable energy supply in wireless networks. However, the broadcast nature of wireless signals makes them non-excludable public goods, which leads to potential free-riders among energy receivers. In this study, we formulate the wireless power provision problem as a public good provision problem, aiming to maximize the social welfare of a system of an energy transmitter (ET) and all the energy users (EUs), while considering their private information and self-interested behaviors. We propose a two-phase all-or-none scheme involving a low-complexity Power And Taxation (PAT) mechanism, which ensures voluntary participation, truthfulness, budget balance, and social optimality at every Nash equilibrium (NE). We propose a distributed PAT (D-PAT) algorithm to reach an NE, and prove its convergence by connecting the structure of NEs and that of the optimal solution to a related optimization problem. We further extend the analysis to a multi-channel system, which brings a further challenge due to the non-strict concavity of the agents’ payoffs. We propose a Multi-Channel PAT (M-PAT) mechanism and a distributed M-PAT (D-MPAT) algorithm to address the challenge. Simulation results show that, our design is most beneficial when there are more EUs with more homogeneous channel gains.

Index Terms—Wireless Power Transfer, Public Goods, Network Economics, Mechanism Design, Game Theory.

1 INTRODUCTION

1.1 Motivation

SINCE Nikola Tesla invented the wireless power transfer (WPT) technology, the far-field WPT has been developing very rapidly and has emerged as a promising solution to supply energy to low-power wireless devices. The far-field WPT allows energy users (EUs) to harvest energy remotely from the radio frequency signals radiated by energy transmitters (ETs) over the air. For example, Powercast has developed energy receivers that can harvest 10 microwatts (\(\mu\)W) RF power from a distance of 10 meters, which is sufficient to power the activities of many low-power devices, such as wireless sensors and RF identification (RFID) tags [2]. By flexibly adjusting the transmit power and time/frequency resource blocks used by an ET, the WPT can meet the dynamically changing real-time energy demands of multiple EUs simultaneously. The WPT is hence becoming an important building block of the next generation wireless systems [3], [4].

Fig. 1 shows an example of the WPT system, where an ET transmits power on three channels: GSM, 3G, and LTE. There are three EUs, each of which can harvest power on a subset of the channels. Here EUs can be heterogeneous in their channel conditions (due to different distances from the ET), energy consumption rates (due to different applications), and energy harvesting circuits (which result in different channel availabilities). Due to the heterogeneous characteristics, different EUs have different energy demands and value ET’s transmit power on different channels differently. In Fig. 1 for example, EU 3 is likely to have a higher energy demand than EU 2, since EU 3 is more energy-hungry with a lower battery status. Moreover, EU 1 is only equipped with energy harvesting circuits for the 3G channel, hence he cannot harvest energy on the other two channels.

To achieve the wide deployment of the WPT technology, one needs to address the potential economic challenges in the future WPT markets. One of such challenges is how an ET should allocate the power across different channels to balance his own operation cost and EUs’ heterogeneous power demands. There has been much excellent prior work tackling this issue from a centralized optimization point of view (e.g., [5]–[12]), assuming that EUs are unselfish and will always truthfully reveal their private information (such as the channel state information and harvested power requirements) to the ET. In practice, however, EUs may have their own interests as they may not be directly controlled by the ET, hence they may choose to misreport their private information if doing so can improve their own benefits. For example, if the ET’s goal is to ensure fairness among EUs in terms of their harvested power, an ET can report a smaller channel gain in order to receive more power than what he deserves. To our best knowledge, no existing work has addressed the network performance maximization problem in a WPT network under such a private information setting.
1.2 Challenges

To resolve the issue of private information, it is natural to consider a decentralized market solution (e.g., pricing mechanisms [13] and auctions [14]). For instance, in [14], the mobile users determine their data demands by responding to the market price and hence indirectly reveal their private information. Based on the users’ response to the market price, the mechanism would adjust the price to attain a market equilibrium. Such a mechanism works well in many network resource allocation problems, where each user only receives the benefit from the resource allocated to him. However, this may not work well in the WPT system.

More specifically, the resource in the WPT systems, the wireless power broadcast on each channel, is a non-excludable public good that is different from many previously considered wireless resources. Due to the broadcast nature of wireless signals, each EU’s received power only depends on his channel condition and ET’s broadcast power. Therefore, one EU harvesting power from the wireless signal does not affect the available energy to other EUs, hence wireless power is non-rivalrous and thus a public good. Furthermore, it is difficult to exclude some EUs from harvesting the energy once the wireless signal is transmitted, hence it is non-excludable. Hence, under a conventional market mechanism approach, the paying EUs would purchase the wireless power only to a self-satisfying level, while the remaining non-paying EUs may silently free-ride the wireless power without any payment. This eventually leads to an inefficient wireless power provision. Such a free-rider problem does not occur in wireless communication networks with unicast information (not power) transmissions (e.g. [13], [14]), because the unicast information data are private goods. That is, they are excludable due to message encryption and rivalrous because the data dedicated to one user typically does not benefit another user.

Although the WPT technology has been extensively studied in the literature, the solution to the economic challenges (including the nature of public goods and the private information) plays an indispensable role in paving the way for widely deployment and commercialization of the WPT networks.

1.3 Solution Approach and Contributions

Among solutions to the public good provision problems in the economics literature, the Lindahl taxation [15] is one of the approaches that can achieve an efficient public good provision in a complete information setting. In the private information setting, a promising solution is the Nash mechanism implementation of the Lindahl allocation. In particular, it achieves the social welfare optimum for the public good economy [16] at every Nash equilibrium (NE). The existing mechanisms ensure economic properties such as budget balance (e.g., [17], [18], [20]–[23]).

Nevertheless, there are several issues remaining unaddressed. First, the existing approaches cannot perfectly incentivize agents to voluntarily participate in the mechanism. Without such a property, there may exist free-riders opting out of the mechanism but still benefiting from the public goods [24]. Second, practical wireless power provision problems involve transmit power constraints (such as the maximum transmit power constraint of the ET). However, the existing distributed algorithms only guarantee to converge when there is no public good provision constraint (e.g., [20]–[23]). The challenge for such an algorithm design lies in discontinuity of agents’ payoffs, which makes the existing algorithmic approaches inapplicable in our case. This motivates us to propose a two-phase all-or-none scheme with proper economic mechanisms and the distributed algorithms to resolve the above two issues.

For a power transfer scenario with multiple channels (as in Fig. 1), we need to consider power allocation across different channels. Such a scenario further brings a technical challenge: each EU’s benefit may not be strictly concave in transmit power vector over all channels. Moreover, the power allocation problem is not separable across channels, since each agent’s benefit depends on the transmit power decisions across channels. This feature complicates the design of our mechanism and distributed algorithm. Based on the above discussions, we need to consider the following questions:

Question 1. How to design mechanisms that incentivize the participation of EUs and maximize the overall benefits of the WPT networks, for both single-channel and multi-channel settings?

Question 2. How to design algorithms for the ET and EUs to reach an NE, considering the public goods provision constraint?

We summarize our main contributions of this work as follows:

- **Problem Formulation:** To our best knowledge, this is the first work that addresses a wireless resource allocation problem from the perspective of non-excludable public goods. In particular, we solve the effective WPT provision problem by considering the EUs’ private information and selfish behaviors.

- **Mechanism Design:** We propose a two-phase all-or-none allocation scheme, and design a Power And Taxation (PAT) Mechanism that is significantly simpler than the existing Nash implementation schemes for public goods provision. Our scheme can incentivize the EUs to voluntarily participate in the mechanism, and can achieve several desirable economic properties such as efficiency and budget balance.

- **Distributed Algorithm Design:** For the case of a single-channel wireless power transfer, we propose a distributed PAT (D-PAT) Algorithm under which the decisions of ET and EUs are guaranteed to converge to an NE. We prove its convergence by mapping the NE to the saddle point of the Lagrangian of a related distributively solvable optimization problem. The proof methodology suggests a general distributed algorithm design methodology for computing the NE of our game.

- **Multi-Channel Extension:** For the more general case of multi-channel transfer, we present a Multi-Channel Power
and Taxation (MPAT) Mechanism which maintains the properties of the PAT Mechanism. We also design a distributed MPAT (D-MPAT) Algorithm and show its convergence to the NE even if agents have non-strictly concave payoffs.

- **Performance Evaluation:** Compared with a benchmark mechanism, the EU's average payoff under our proposed schemes significantly increases in the number of EUs increases. In addition, the proposed schemes are most beneficial when EUs have more homogeneous channels.

We organize the rest of the paper as follows. In Section 2, we review the related work. In Section 3, we introduce the system model and the problem formulation. In Section 4, we propose the constrained Lindahl allocation scheme. We propose the PAT Mechanism and the D-PAT Algorithm for a single-channel system in Section 5. We further propose the MPAT Mechanism and the D-MPAT Algorithm for a multi-channel system in Section 6. In Section 7, we provide numerical results to validate our analysis. Finally, we conclude our work in Section 8.

## 2 Related Work

Most of the early studies on WPT networks focused on system optimization with selfish users (e.g., [3]–[12]), where ETs and EUs are willing to truthfully report their private information and follow the optimization result. Specifically, references [5], [6] proposed efficient schemes to optimize the long-term ET placement, and references [7]–[12] considered the wireless resource allocation in WPT networks to optimize the communication performance.

To our best knowledge, there is only one recent work considering the game-theoretical analysis of the power provision problem in WPT networks with selfish EUs [25]. Specifically, Niyato et al. in [25] formulated a bidding game for a simple WPT system and analyzed the NE of the game. The equilibrium of the game does not maximize the system performance. Our work differs from [25] in that we aim to achieve a socially optimal system performance through mechanism design.

There are several related works on Nash implementation for public goods (e.g., [17], [18]). Specifically, Hurwicz in [17] presented a Nash mechanism that yields the social optimum for a public good economy, and the mechanism also ensures individually rationality and budget balance. Sharma et al. in [18] extended the results in [17] to a more general local public goods scenario with the CDMA networks as an example. However, references [17], [18] did not consider how the agents should iteratively update the messages to converge to the NE under private information. Another related study [19] considered the network security investment as a non-excludable public good and studied a mechanism. However, the studied scheme cannot always achieve the efficient public good provision as we do in our paper.

Only few papers proposed public goods mechanisms together with the corresponding updating processes that converge to the NE [20]–[23], where the best response dynamics [20]–[22] and the gradient-based dynamics [23] can provably converge to the NE under some technical conditions. However, these prior algorithms are not directly applicable in our model. This is because algorithms in [20]–[23] relied on the continuity of the best response dynamics or the gradient-based dynamics. Our constrained public good setting introduces the discontinuity, and hence requires new distributed algorithms for the constrained public good provision problem (due to the ET’s transmit power constraint).

Moreover, all the above studies did not consider the voluntary participation issue for non-excludable public goods. In our work, we will show that if the ET knows the total number of EUs, then a two-phase scheme can ensure voluntary participation.

## 3 System Model and Problem Formulation

In this section, we introduce the system model that captures several unique characteristics of the WPT problem. Accordingly, we formulate the public good provision problem, with the goal to maximize the social welfare.

### 3.1 System Model

**ET and EUs:** We consider a WPT system consisting of one ET transmitting power over \( N \) channels (bands) to a group of \( K \) EUs. Let \( N = \{1 \leq n \leq N\} \) be the set of channels and \( K = \{1 \leq k \leq K\} \) be the set of EUs. We further use agent 0 to denote the ET, hence the total set of decision makers in the system is \( K \cup \{0\} \). For the purpose of presentation, we will refer to the ET as “she” and an EU as “he”. The ET has an omnidirectional antenna and transmits wireless energy over the channels. Each EU has one energy receiver, but can potential receive energy from multiple channels. Different EUs can experience different time-varying channel conditions due to shadowing and fading. We consider a time period long enough such that the small-scale channel fading effects are averaged out and the channel conditions are stationary.

**Power and Cost:** The ET transmits a power of \( p_n \) (Watts) over channel \( n \). We denote \( \mathbf{p} = \{p_n\}_{n \in N} \) as the ET’s transmit power vector of all channels in \( N \). The ET incurs a cost of \( C(\mathbf{p}) \), which is a positive, increasing, continuously differentiable, and (not necessarily strictly) convex in \( \mathbf{p} \). The cost function can capture, for example, both the energy consumption cost and the maintenance cost for the ET’s operation. A strictly convex cost function may capture an increasing marginal cost of power transmission, which is due to several reasons such as:

- the electricity tariff structure in many countries [27];
- the increasing chance of transmitter failure as transmit power increases [28];
- the negative impact to other wireless networks due to a stronger interference [29].

**Transmit Power Constraints of the ET:** Let \( \sum_{n \in N} p_n \leq P_{\text{max}} \) represent the ET’s total power constraint over all channels, and let \( p_n \leq P_{\text{peak},n} \) represent the ET’s peak power constraint for channel \( n \). These two types of constraints capture the limitation of the physical circuits and regulations [6]. The transmit power \( \mathbf{p} \) thus lies in the set of

\[
\mathcal{P} = \left\{ \mathbf{p} : \sum_{n \in N} p_n \leq P_{\text{max}}, \ 0 \leq p_n \leq P_{\text{peak},n}, \ \forall n \in N \right\}.
\]
Both $C(p)$ and $P$ are ET’s private information and are not known by the EUs.

**Utility of EUs:** For EU $k \in K$, let $h_{k,n}$ denote his channel gain over channel $n$, let $h_k = \{h_{k,n}\}_{n \in N}$ denote his overall channel gain vector, and let $I_k$ denote the received ambient wireless power apart from the the ET’s transmitted wireless power (e.g. the wireless signal generated by the cellular networks). Here we assume that $h_k$ represents EU $k$’s long-term average channel gain. Given the ET’s transmit power $p$, EU $k$’s total received power is

$$q_k(p) = \sum_{n \in N} h_{k,n}p_n + I_k. \quad (2)$$

Each EU $k$ has a utility function $U_k(q_k(p))$, which is strictly concave, increasing, and continuous, which can reflect

- an EU’s overall valuation of the received power considering his battery status and energy consumption level;
- the nonlinear energy conversion efficiency of the physical circuits, such as the (concave) logistic functions [31];
- the fairness consideration as can be captured by the widely-used $\alpha$-fair utility function [32]. The fairness is an important consideration due to the “doubly-near-far” problem in the wireless powered communications [3].

Throughout this paper, we assume $I_k = 0$ for all EUs $k$ without loss of generality. Both $U_k(\cdot)$ and $h_k$ are EU $k$’s private information and are not known by the ET or other EUs.

### 3.2 Problem Formulation

We assume that a network regulator operates the ET and aims to optimize the system performance. Our analysis also applies to the case where the ET is a self-interested decision maker. This is because the ET also participates in the PAT Game defined in Section 5 (or the MPAT Game defined in Section 6) as a rational player to maximize her payoff. Specifically, the ET is interested in choosing the transmit power $p$ to solve the following Social Welfare Maximization (SWM) Problem

\[
\text{(SWM)} \quad \max_p \left( \sum_{k \in K} U_k(q_k(p)) - C(p) \right) \tag{3a} \\
\text{s.t.} \quad p \in P. \tag{3b}
\]

The objective function of the SWM Problem is concave and the constraint set is compact and convex.

To solve the SWM Problem in a centralized fashion, the ET needs to know the complete information of the EUs (i.e., utility functions $U_k(\cdot)$ and $q_k(p)$ for all $k$). This is difficult to achieve in practice, since EUs may not want to report their utility functions or their channel gains, as doing so may not maximize their own benefits. Hence, we need to design economic mechanisms to solve the SWM Problem, considering EUs’ private information and the public good nature of wireless power.

7. This is because for any arbitrary positive $I_k$ and $U_k(\cdot)$, we can always find another increasing, strictly concave, and continuous utility function $U_k(\cdot)$ such that $U_k(q_k(p) - I_k) = U_k(q_k(p))$ for all $p \in P$.

8. However, in the case where the ET is a self-interested decision maker, we need to introduce a third-party network regulator, who serves as a coordinator that collects the price and power proposals from the agents, for implementing the D-PAT Algorithm and the D-MPAT Algorithm. It is possible to further design profit-maximizing mechanisms by integrating the existing profit-maximizing mechanisms for private goods (e.g. [33]) and our proposed mechanisms. We will study this interesting direction in the future work.

### 3.3 Desirable Mechanism Properties

In this paper, we will design an economic mechanism that satisfies the following four desirable economic properties:

- **(E1) Efficiency:** Maximizes the social welfare, i.e., achieves the optimal solution of the SWM Problem.
- **(E2) Incentive compatibility:** An EU should (directly or indirectly) truthfully reveal his private marginal utility.
- **(E3) Voluntary participation:** An agent should get a non-negative payoff by participating in the market mechanism.
- **(E4) (Strong) Budget balance:** The total payment from the EUs equals the revenue obtained by the ET. In other words, if the mechanism is administrated by a third-party, then there is no need to inject money into the system.

We would like to clarify some aspects with respect to the existing mechanism design literature. First, for a direct mechanism, (E2) usually indicates that the mechanism can fully reveal the participants’ utility functions. However, [34] proved that there does not exist any public goods mechanism that satisfies (E1) and induces direct and truthful revelation at the same time. Hence, we focus on the indirect revelation, in the sense that the mechanism reveals EUs’ marginal utility at NEs.

Existing results on Nash mechanisms have focused on achieving (E1), (E2), and (E4), without considering (E3) [17], [18], [20]–[23]. The common assumption in these prior work is that there is a “government” that has enough power to force all agents to participate in the mechanism [17], [18], [20]–[23]. However, such mandatory participation may not be easily enforceable in reality [7].

Saijo and Yamato in [24] showed that there does not exist a mechanism that guarantees (E1), (E3), and (E4) simultaneously for non-excludable public goods. Nevertheless, the key assumption in [24] is that the consumers (which correspond to EUs in our model) can directly access the production technology and produce the non-excludable public goods themselves (even without participating in the mechanism). This assumption does not hold in our model, as we assume that EUs cannot access other energy sources other than the ET. Hence it is possible to design a mechanism that achieves (E1)-(E4), as shown in Section 5.

### 4 Constrained Lindahl Allocation Scheme

Before explaining our proposed mechanism, we first propose a constrained Lindahl allocation scheme in this section. We then show that, in a complete information setting, such an allocation scheme can lead to the self-enforcing and socially optimal transmit power in the WPT systems. Under incomplete information, we will further design mechanisms to implement the constrained Lindahl allocations (in Sections 5 and 6).

Consider a linear taxation scheme, under which there is a tax rate $R_{k,n}$ for each agent $k \in K \cup \{0\}$ on channel $n$. Let $R_k = \{R_{k,n}\}_{n \in N}$ denote the tax rate vector for agent $k \in K \cup \{0\}$. Based on the corresponding tax rate, each EU $k$ pays $p^T R_k = \sum_{n \in N} p_n R_{k,n}$ to the ET; the ET receives a reimbursement $-p^T R_0 = -\sum_{n \in N} p_n R_{0,n}$. For presentation simplicity, we let $p^T R_0$ denote the payment for the ET.

Suppose that the agent $k \in K \cup \{0\}$ can determine the transmit power considering his/her payment $p^T R_k$. Agent $k$ aims

9. In Japan, for example, it is mandatory to pay a TV public broadcasting fee if one owns a device that can receive a TV broadcast signal. The punishment for non-paying individuals, however, is practically non-existent due to the difficulty of enforcement.
to find the transmit power $p_k^{PM}$ to solve the payoff maximization problem:

$$p_k^{PM} = \begin{cases} \arg \max_{p \in \mathcal{P}} U_k(q_k(p)) - p^T \mathbf{R}_k, & \text{if } k \in \mathcal{K}, \\ \arg \max_{p \in \mathcal{P}} - p^T \mathbf{R}_0 - C'(p), & \text{if } k = 0. \end{cases} \quad (4)$$

For each EU $k$, we have that the optimal solution $p_k^{PM}$ satisfies

$$\nabla_p U_k(q_k(p_k^{PM})) = \mathbf{R}_k. \quad (5)$$

In the following, we introduce a specific taxation scheme, namely Lindahl tax [15]. Such a taxation scheme incentivizes agents to agree on provisioning the public goods to the socially optimal level, which constitutes a Lindahl allocation. However, the Lindahl tax scheme in the existing literature does not consider the public good provision constraint. Therefore, we propose the constrained Lindahl allocation scheme as follows.

**Scheme 1 (Constrained Lindahl Allocation Scheme).** The constrained Lindahl allocation is a tuple $\{(\mathbf{R}^*_k)_{k \in \mathcal{K} \cup \{0\}}, \mathbf{p}^o\)$:

1. The transmit power vector $\mathbf{p}^o$ is an optimal solution to the SWM Problem.
2. Lindahl tax rate $\mathbf{R}^*_k$ for each agent $k \in \mathcal{K} \cup \{0\}$ satisfies
   a. $\mathbf{R}^*_k = \nabla_p U_k(q_k(\mathbf{p}^o))$, for each EU $k \in \mathcal{K}$;
   b. $\mathbf{R}^*_0 = - \sum_{k \in \mathcal{K}} \mathbf{R}^*_k$, for the ET.

Intuitively, being charged the corresponding constrained Lindahl tax $\mathbf{R}^*_k$, each EU $k$’s optimal solution to (4) is the socially optimal transmit power $\mathbf{p}^o$, as shown from (5). Similarly, we can also show that the socially optimal transmit power $\mathbf{p}^o$ is also the optimal solution to the ET’s problem in (4). This indicates that under the constrained Lindahl tax, every agent prefers the socially optimal transmit power to all other power choices.

In Fig. 2 we present illustrative examples of the constrained Lindahl allocation scheme for a single-channel WPT system, with one ET and two EUs. The horizontal axis corresponds to the transmit power. The vertical axis represents each agent’s marginal utility (for the EUs) or cost (for the ET). Fig. 2(a) describes the case where the maximal transmit power $P_{max}$ is sufficiently large. In this case, from (3), the socially optimal transmit power $\mathbf{p}^o$ corresponds to the intersection of the ET’s marginal cost function $C'(p)$ and the two EUs’ aggregate marginal utility functions $\sum_{i \in \{1, 2\}} \partial U_i(h_i, p) / \partial p$. Fig. 2(b) corresponds to the more complicated case where $P_{max}$ is not large enough. More specifically, $P_{max}$ is smaller than the intersection point of the two curves $\sum_{i \in \{1, 2\}} \partial U_i(h_i, p) / \partial p$ and $C'(p)$, and the socially optimal transmit power $\mathbf{p}^o$ equals $P_{max}$ in this case.

As shown in both Fig. 2(a) and Fig. 2(b), the constrained Lindahl tax rate $\mathbf{R}^*_k$ for EU $k$ is exactly his marginal utility at the optimal transmit power. Note that if we substitute $\mathbf{R}^*_k$ into $\mathbf{R}_k$ in (4), we have $p_k^{PM} = \mathbf{p}^o$ from (5). That is, each EU’s best power purchase under the constrained Lindahl tax rate is to choose the optimal transmit power $\mathbf{p}^o$. In Fig. 2(a), when the constraint is slack, ET’s reimbursement per unit of power is $R^*_1 + R^*_2$, which is equal to her marginal cost $C'(p^o)$. From (4), we see that $p_k^{PM} = \mathbf{p}^o$. In Fig. 2(b), however, the constraint is tight. This means that ET’s reimbursement $R^*_1 + R^*_2$ is larger than the ET’s marginal cost $C'(p^o)$, which means that the ET aims to transmit as much as possible, i.e., $p_k^{PM} = \mathbf{p}^o = P_{max}$.

The interpretation of the constrained Lindahl allocation scheme is hence as follows. Under his constrained Lindahl tax rate, each agent $k \in \mathcal{K} \cup \{0\}$ experiences a “personalized market”. That is, each agent sees himself/herself capable of deciding the total amount of transmit power he/she will purchase/transmit (through solving (4)). In each agent’s personalized market, the socially optimal transmit power is the optimal decision according to (4). Therefore, the constrained Lindahl allocation scheme enables all agents to agree on the social optimum.

The above discussions assume that the ET knows the complete information regarding EUs’ utility functions. However, in reality, the ET cannot readily obtain such information. As a result, each EU has an incentive to under-report his utility to the ET to reduce his payment. This leads to the free-riding EUs. The under-reported utilities may distort the taxation scheme and lead to an inefficient transmit power (comparing with the socially optimal value). The consideration of such a realistic incomplete information scenario motivates us to design new mechanisms in Section 5 for the single-channel scenario and in Section 6 for the multi-channel case, to achieve the constrained Lindahl allocation at an equilibrium without relying on complete network information.

5 Single-Channel Nash Mechanism

In this section, we start with a single-channel scenario with incomplete information. For a better readability, we will drop the index $n$ in $h_k, n$ and $p_n$ in this section. We can express the feasible transmit power region as $\mathcal{P} = [0, P_{max}]$, where $P_{max}$ is the ET’s maximum transmit power. We further focus on the case where there are $K \geq 2$ EUs. Appendix B discusses the single-EU case.

We propose a two-phase all-or-none scheme, including a PAT Nash Mechanism in Phase II. We show that the agents’ equilibrium strategies coincide with constrained Lindahl allocation (given in Definition 1) and achieve (E1)-(E4). We further propose a D-PAT Algorithm that converges to an NE of an induced game, of which the challenge mainly lies in the transmit power constraint.

5.1 Two-Phase All-or-None Scheme

We propose a two-phase all-or-none scheme, as shown in Fig. 3. In Phase I, each EU sends a 1-bit message to the ET indicating whether or not he will participate in the Power and Tax (PAT) mechanism (to be described in Section 5.2). In Phase II, if all agents are willing to participate, the ET and the EUs will execute the PAT Mechanism in Phase II-Y; otherwise, the ET will transmit zero power and no trading occurs in Phase II-N. Here we adopt the following assumption throughout the paper:

**Assumption 1.** The ET knows the total number of EUs, $K$. 
Nash Mechanism

Next, we describe the PAT Mechanism to be executed in Phase II-Y.

Mechanism 1. Power And Taxation (PAT) Mechanism

- **The message space:** Each agent $k \in \mathcal{K} \cup \{0\}$ sends a message $m_k \in \mathbb{R}^2$ to the ET:

$$m_k = (\gamma_k, b_k), \quad (6)$$

where $\gamma_k$ and $b_k$ are agent $k$’s power proposal and price proposal, respectively. Note that the ET (agent 0) also needs to send a message $m_0$ (to itself). We denote the message profile as $m = \{m_k\}_{k \in \mathcal{K} \cup \{0\}}$.

- **The outcome function:** The ET computes the transmit power $p$ based on the agents’ power proposals:

$$p(m) = \frac{1}{K+1} \sum_{k \in \mathcal{K} \cup \{0\}} \gamma_k. \quad (7)$$

The ET further computes the tax rate $R_k$ for agent $k \in \mathcal{K}$ based on the agents’ price proposals:

$$R_k(m) = b_{\omega(k+1)} - b_{\omega(k+2)}, \quad \forall k \in \mathcal{K} \cup \{0\}, \quad (8)$$

where $\omega(k) = \text{mod}(k, K+1)$, and mod is the modulo operator. The ET will announce $(p(m), R_k(m))$ to agent $k$, and the agent $k$ needs to pay the following tax to the ET,

$$t_k(m) = R_k(m)p(m), \quad \forall k \in \mathcal{K} \cup \{0\}. \quad (9)$$

Next we mention several key features of the PAT Mechanism. First, the determination of the transmit power $p$ in (7) depends on every agent’s power proposal. Second, agent $k$’s tax rate in (8) does not depend on his own price proposal $b_k$. Finally, the agents’ taxes in (9) cancel each other, i.e.,

$$\sum_{k \in \mathcal{K} \cup \{0\}} t_k(m) = 0, \quad \forall m \in \mathbb{R}^{2(K+1)}, \quad (10)$$

which means that the mechanism achieves the budget balance property (E4).

The PAT Mechanism is motivated by the Hurwicz mechanism [17]. In [17], each agent can only select the price proposal from $\mathbb{R}^+$, and the tax function is computed in a complicated function form: $t_k = R_kp + b_k(\gamma_k - \gamma_{\omega(k+1)})^2 - b_{\omega(k+1)}(\gamma_{\omega(k+1)} - \gamma_{\omega(k+2)})^2$. In our proposed scheme, the tax function is computed in the same way in (17) as in (7). A key contribution of this paper is that our proposed PAT Mechanism is considerably simpler than the Hurwicz mechanism in [17] but achieves the same desirable economic properties, as explained next.

5.3 Properties of the PAT Mechanism

In this subsection, we will prove that the PAT Mechanism achieves the economic properties of (E1) and (E3). Specifically, we will first analyze agents’ decisions in Phase II, assuming that every agent chooses to participate in Phase I. Then we return to Phase I to analyze agents’ participation decisions.

5.3.1 Analysis of Phase II

The PAT Mechanism induces a game among agents in Phase II, which we simply refer to as the PAT Game.

**Game 1. PAT Game (Induced by the PAT Mechanism in Phase II)**

- **Players:** all agents in $\mathcal{K} \cup \{0\}$.
- **Strategy:** $m_k \in \mathbb{R}^2$ described in (6) for each agent $k \in \mathcal{K} \cup \{0\}$.
- **Payoff function $J_k(p, t_k)$:** for each EU $k \in \mathcal{K}$,

$$J_k(p, t_k) = U_k(h_kp) - t_k; \quad (11)$$

for the ET (agent 0)

$$J_0(p, t_0) = \begin{cases} -C(p) - t_0, & \text{if } p \in \mathcal{P}, \\ -\infty, & \text{otherwise}. \end{cases} \quad (12)$$

The value $-\infty$ can be interpreted as an infinite penalty for the ET if she violates the maximum power constraint. Note that the ET’s payoff function in (12) is discontinuous due to the constraint $p \in \mathcal{P}$. This leads to a key challenge for the distributed algorithm design discussed later in Section 5.4.

**Definition 1 (Nash Equilibrium (NE)).** An NE of the PAT Game is a message profile $m^*$ that satisfies the following condition:

$$J_k(p(m^*), t_k(m^*)) \geq J_k(p(m_k, m_{-k}^*), t_k(m_k, m_{-k}^*)), \quad \forall m_k \in \mathbb{R}^2, \quad k \in \mathcal{K} \cup \{0\}, \quad (13)$$

where $m_{-k}^* = \{m_l\}_{l \neq k, l \in \mathcal{K} \cup \{0\}}$ is the NE message profile of all other agents except agent $k$.

Traditionally, an NE describes the agents’ stable strategic behaviors in a static game with complete information [17]. However, the agents in our WPT system do not know the private information (i.e., utilities, cost, or the transmit power constraint) of others. Here, we adopt the interpretation of [18], i.e., NE corresponds to the “stationary” messages profile of some message exchange process (to be described later in Section 5.4) that possesses the equilibrium property in (13).
To analyze an NE of the PAT Game, we summarize the sufficient and necessary conditions for an NE in Lemma 1 with its proof presented in Appendix A.2.

**Lemma 1.** A message profile \( m^* = \{ (\gamma^*_k, b^*_k) \}_{k \in K \cup \{0\}} \) is an NE if and only if the following conditions are satisfied,

\[
\gamma^*_k = (K + 1) \arg \max_{\gamma} J_k(p, R^*_k, p) - \sum_{l \neq k \in K \cup \{0\}} \gamma^*_l, \tag{14}
\]

for every agent \( k \in K \cup \{0\} \), where \( R^*_k \triangleq b^*_\omega(k+1) - b^*_\omega(k+2) \) is the NE tax rate for agent \( k \).

To understand Lemma 1, let \( p^* \) denote the NE transmit power. From (7), we have \( p^* = \sum_{l \in K \cup \{0\}} \gamma^*_l / (K + 1) \). This allows us to rewrite (14) as,

\[
p^* = \arg \max_{p} J_k(p, R^*_k, p), \quad \forall k \in K \cup \{0\}. \tag{15}
\]

Equation (15) implies that under the NE tax rates \( \{ R^*_k \}_{k \in K \cup \{0\}} \), the common NE transmit power \( p^* \) maximizes every agent's payoff. Otherwise, one agent \( k \) would have the incentive to adjust \( \gamma_k \) to change \( p(m^*) \) and improve his payoff. Therefore, an NE only occurs when all agents agree on the same transmit power.

We can show that there are multiple NEs for the PAT Game. To see this, given any \( (\gamma^*, b^*) \), we can add every \( b^*_k \) by the same constant, while the new message profile \( (\gamma^*, b^*) \) still satisfies the conditions described in (15) and thus is also an NE. However, we can show that the NE allocation \( (p^*, t^*) = (p(m^*), t(m^*)) \) is the same for all NEs, where \( p^* \) corresponds to the unique optimal solution of the SWM Problem. We can show that all NEs yield the unique constrained Lindahl Allocation in the following theorem (with its proof presented in Appendix A.3).

**Theorem 1** (Implementing Constrained Lindahl Allocation). There exist multiple NEs in the PAT Game, and all NEs correspond to the unique constrained Lindahl allocation.

Proving the existence of NEs involves constructing an NE \( m^* \) based on the optimal solution to the SWM Problem. Intuitively, the tax rates defined in (8) ensure that the sum of all tax rates is zero, i.e., \( \sum_{k \in K} R^*_k(m) = 0 \). Together with (15), we can derive the Karush-Kuhn-Tucker (KKT) conditions for the SWM Problem based on all agents' optimality conditions of (15). The remaining part of the proof for Theorem 1 involves showing that the NE condition in (15) can lead to a unique tax rate \( R^*_k \) for each agent.

The significance of Theorem 1 is three-fold. First, Theorem 1 shows that every NE of the PAT Game induced by the PAT Mechanism yields the socially optimal transmit power level as suggested in Definition 1. Second, Theorem 1 implies that the PAT Mechanism is incentive-compatible (E2). This is because the NE tax rates are the Lindahl taxes, which reveal every EU's marginal utility at the NEs by Definition 1. Third, each agent receives the same payoff at every NE due to the uniqueness of the Lindahl allocation. This means that each agent is indifferent to the choice among multiple NEs.

However, there should be an effective approach of selecting one NE from the multiple ones. Without such an agreement, the agents' distributed choices may not lead to an non-NE message profile. We will resolve the above issue through a distributed algorithm design in Section 5.4.

**5.3.2 Analysis of Phase I**

We now proceed to analyze agents' decisions in Phase I, where each agent compares the unique NE allocation \( (p^*, t^*) = (p(m^*), t(m^*)) \) (where everyone participates the PAT Mechanism) to the \((0, 0)\) allocation (where at least one agent chooses not to participate). We can show that each EU voluntarily participates, with its proof in Appendix A.4.

**Theorem 2** (Voluntary Participation). All agents will not be worse off by participating in the PAT Mechanism.

Theorem 2 implies that all EUs choose to participate in the PAT Mechanism in Phase I. The intuition is that, given arbitrary messages from other agents in the PAT Game, an EU \( k \) or the ET can always choose a power proposal \( \gamma_k \) so that the transmit power is zero (hence his tax is zero due to (9)). Such a choice is equivalent to the outcome where someone chooses not to participate in Phase I. Hence choosing to participate in Phase I is a weakly dominant strategy for each EU. Here we assume that each EU will voluntarily participate if he is not worse off by doing so. This is without loss of generality, since in practice we can let the ET offer an additional arbitrarily small amount of benefit \( \epsilon > 0 \) to every EU who chooses to participate to break the tie. In other words, this ensures that every EU can receive a strictly positive payoff improvement by choosing to participate. In addition, successfully inducing voluntary participation also relies on one of the fundamental assumptions of neoclassical economics: agents make decisions rationally; irrational EUs may make the all-or-none scheme fragile. We may tackle this issue using theories from behavioral economic (e.g., cognitive hierarchy [36]).

We note that the EUs' participation involves the energy consumption due to the communication overhead of the proposed algorithms. In this paper, we assume that such participation energy overhead is negligible for each EU. Specifically, the participation process is of a finite duration and the energy cost is one-time only. Once the ET decides the policy, the resultant wireless power transfer is over a much longer duration. Therefore, the energy harvested is much larger than the energy overhead for each EU (if he decides to participate) and thus we ignore the latter in our work for simplicity.

To summarize, we have shown that the two-phase all-or-none scheme and the PAT Mechanism together can achieve the desirable economic properties of (E1)-(E4). We will next propose a distributed algorithm, under which the agents can achieve the NE of the PAT Game.

**5.4 Distributed Algorithm to Achieve the NE**

As we mentioned previously, the private information setting and the NE selection issues make it difficult for the ET and EUs to directly compute their messages at an NE. Hence, we will propose an iterative distributed algorithm for the ET and EUs to exchange information and compute the NE. To prove the convergence of the algorithm, we will establish the connection between the NE of the PAT Game and the optimal primal-dual solution of a reformulation of the SWM Problem.

Algorithm 1 illustrates the proposed iterative D-PAT Algorithm, with the following key steps. Each agent \( k \in K \cup \{0\} \) initializes his arbitrarily chosen message \( m_k(0) \in \mathbb{R}^2 \) (line 1). Then, the algorithm iteratively computes the messages until convergence. In each iteration, first each EU \( k \) sends his message to the ET (line 5). Then the ET computes each agent \( k \in K \cup \{0\} \)'s tax rate \( R_k(\tau) \), and sends \( R_k(\tau) \) together with agents \( \omega(k-1) \) and \( \omega(k-2) \)'s price proposals (lines 6, 7) to EU \( k \). Accordingly, each agent \( k \in K \cup \{0\} \) updates his power proposal and his price proposal (line 8), where \( \lfloor x \rfloor_b = \max(\min(b, x), a) \). Finally,
\begin{algorithm}
\caption{Distributively Compute the NE of the PAT Game (D-PAT Algorithm)}
\begin{algorithmic}[1]
\State Initialize the iteration index $\tau \leftarrow 0$ and step size $\{\rho(\tau)\}$.
\State Each agent $k \in K \cup \{0\}$ randomly initializes $m_k(0)$. The ET initializes the stopping criterion $\epsilon_1 > 0$ and $\epsilon_2 > 0$.
\State Set $\text{conv} \_\text{flag} \leftarrow 0$ \# initialize the convergence flag;
\State \While{$\text{conv} \_\text{flag} = 0$}
\State Set $\tau \leftarrow \tau + 1$;
\State Each EU $k \in K$ sends message $m_k(\tau)$ to the ET;
\State The ET computes the tax rate $R_k(\tau)$ from \eqref{eq:tax_rate}
\State the ET sends $R_k(\tau), \gamma_{\omega(k-1)}(\tau)$ and $\gamma_{\omega(k-2)}(\tau)$ to EU $k$, for each EU $k \in K$;
\State Each agent $k \in K \cup \{0\}$ computes $\gamma_k(\tau + 1)$ and $b_k(\tau + 1)$ as
\begin{equation}
\gamma_k(\tau + 1) = \begin{cases}
\arg \max_p J_k(p, R_k(p))^{\text{up}}, & \text{if } k \in K, \\
\arg \max_p J_k(p, R_k(p))^{\text{max}}, & \text{if } k = 0,
\end{cases}
\end{equation}
and
\begin{equation}
b_k(\tau + 1) = b_k(\tau) + \rho(\tau) \left( \gamma_{\omega(k-1)}(\tau) - \gamma_{\omega(k-2)}(\tau) \right),
\end{equation}
where $\rho(\tau)$ is the step size;
\If{$|b_k(\tau) - b_k(\tau - 1)| \leq \epsilon_1 |b_k(\tau - 1)|$ and $|\gamma_k(\tau) - \gamma_k(\tau - 1)| \leq \epsilon_2 |\gamma_k(\tau)|$, $\forall k \in K \cup \{0\}$}
\State $\text{conv} \_\text{flag} \leftarrow 1$;
\EndIf
\EndWhile
\State The ET computes $p(m(\tau))$ and $t(m(\tau))$ using \eqref{eq:power_proposal} and \eqref{eq:transmit_power}.
\end{algorithmic}
\end{algorithm}

Specifically, each agent $k \in K \cup \{0\}$ needs to send $\gamma_k$ and $b_k$ to the ET; the ET needs to send each EU $k$ her tax rate and two other agents’ proposals, for all $k \in K \cup \{0\}$. Hence the communication overhead is $O(K)$ per iteration. The computational complexity per iteration is $O(1)$ for each EU and $O(K)$ for the ET, since she computes the tax rates for all EUs with a complexity of $O(1)$.

### 5.5 The Convergence of the D-PAT Algorithm

There are two classes of existing dynamics that have been shown to converge to the NE of various public good provision mechanisms: the best-response dynamics (e.g. \cite{21, 23}) and the gradient-based dynamics (e.g. \cite{23}). These existing approaches, however, all assume that there are no constraints on public good provision. This assumption does not hold in our model, since we need to consider the maximum total transmit power constraint. Hence we need to find a new way to prove the convergence of our proposed D-PAT Algorithm.

The approach we take is to first reformulate the SWM Problem with a decomposition structure, then connect the saddle point of the Lagrangian of the reformulated problem and the NE of the PAT game. We will show that the D-PAT Algorithm converges to a saddle point and thus an NE of the PAT game.

#### 5.5.1 Problem Reformulation

Inspired by \cite{39}, we reformulate the SWM Problem by introducing auxiliary variables $\mathbf{\pi} = \{\pi_k\}_{k \in K \cup \{0\}}$, which decouple agents’ utility and cost functions:

\begin{equation}
\begin{aligned}
(R - \text{SWM}) \max & \sum_{k \in K} U_k(h_k \pi_k) - C(\pi_0) \\
\text{s.t.} & \pi_k = \pi_{\omega(k-1)}, \forall k \in K \cup \{0\}, \\
& \pi_0 \in \mathcal{P}.
\end{aligned}
\end{equation}

We can verify that the R-SWM Problem is equivalent to the SWM Problem and has a unique optimal solution.

Compared with the reformulation in \cite{39}, here we introduce the equality constraints in a different way to create the desired structure of the following Lagrangian.

#### 5.5.2 Lagrangian

We relax the equality constraints \eqref{eq:swm_eq} and define the Lagrangian of the R-SWM Problem as follows:

\begin{equation}
\mathcal{L}(\mathbf{\pi}, \beta) \triangleq \sum_{k \in K} U_k(h_k \pi_k) - C(\pi_0) - \sum_{k \in K \cup \{0\}} \beta_k \cdot \left( \pi_k - \pi_{\omega(k-1)} \right),
\end{equation}

where $\beta_k$ is the dual variable (or the consistency price \cite{39}) corresponding to the constraint $\pi_k = \pi_{\omega(k-1)}$.

#### 5.5.3 Dual Decomposition

The Lagrangian in \eqref{eq:lagrangian} has a nice dual decomposition structure, i.e., $\mathcal{L} = \sum_{k \in K \cup \{0\}} \mathcal{L}_k$, where $\mathcal{L}_k$ is the decomposed Lagrangian for each agent $k \in K \cup \{0\}$ as follows,

\begin{equation}
\mathcal{L}_k(\pi_k, \beta) = \begin{cases}
U_k(h_k \pi_k) - (\beta_k - \beta_{\omega(k+1)}) \pi_k, & \text{if } k \in K, \\
-C(\pi_k) - (\beta_k - \beta_{\omega(k+1)}) \pi_k, & \text{if } k = 0.
\end{cases}
\end{equation}

13. For example, an EU can set the upper bound to be the maximal transmit power of the local TV broadcast (e.g. 10 kW for the TV Tokyo).

14. Specifically, the reformulation in \cite{39} has the same form as in \cite{37}, as we will introduce soon.
Define $\pi^*(\beta) = \arg \max_{\pi \in \Gamma} \mathcal{L}(\pi, \beta)$, where $\Gamma \triangleq \{ \pi : \pi_0 \in \mathcal{P} \}$. Thus, the dual problem of the R-SWM Problem is

$$
\min_{\beta} \sum_{k \in K \cup \{0\}} \mathcal{L}_k(\pi_k^*(\beta), \beta).
$$

We define the saddle point of $\mathcal{L}$ as a tuple $(\pi^*, \beta^*)$ that satisfies:

$$
\mathcal{L}(\pi^*, \beta^*) \leq \mathcal{L}(\pi^*, \beta^*) \leq \mathcal{L}^* \forall \pi \in \Gamma, \beta \in \mathbb{R}^{K+1}. \tag{22}
$$

For such a saddle point, we can show that $\pi^*$ is the unique optimal solution to the R-SWM Problem and $\beta^*$ is the optimal solution to the dual problem in [21] [40, Chap. 5.4] [4].

5.5.4 Relation between the Saddle Point and the NE

If we set $p = \pi_k$ and $b_{\omega(k+1)} = \beta_k$ for all agents $k \in K \cup \{0\}$, then in the PAT Game the $J_k$ in (11) becomes exactly the decomposed Lagrangian $L_k$, i.e.,

$$
J_k(\pi_k, (\beta_k - \omega_{(k+1)})) = \mathcal{L}_k(\pi_k, \beta), \forall k \in K \cup \{0\}. \tag{23}
$$

Proposition 1 characterizes the relation between a saddle point for the Lagrangian in (19) and an NE of the PAT Game.

**Proposition 1.** For any saddle point $(\pi^*, \beta^*)$ defined in (22), the message profile $\tilde{\mathbf{m}} = \{(\gamma_k = \pi_k^*, b_k = \beta_k^* - \omega_{(k+1)}), \forall k \in K \cup \{0\}\}$ is an NE of the PAT Game.

We present the proof of Proposition 1 in Appendix A.5. Intuitively, Lemma 1 asserts that an NE only occurs if all agents have the same payoff-maximizing transmit power, given the equilibrium tax rate $R^*_k$. On the other hand, we attain the optimal dual solution $\beta^*$ only when the maximizer of the Lagrangian $\mathcal{L}(\pi^*, \beta^*)$ satisfies the equality constraint in the constraint in (18b). Together with the relation of $J_k$ and $\mathcal{L}_k$ in (23), we can see that Proposition 1 holds.

The significance of Proposition 1 is two-fold. First, Proposition 1 provides a new interpretation of the messages of the PAT Mechanism: the power proposal for each agent plays a role of the auxiliary variable, while the price proposal plays a role of the consistency price that pulls the auxiliary variables together.

Second, Proposition 1 also implies that for any distributed algorithm with a provable convergence guarantee to a saddle point of the Lagrangian in (19), we can design a corresponding distributed algorithm that converges to an NE of the PAT Game. This property allows us to exploit the convergence properties of well-designed optimization algorithms that the traditional approaches in [20]–[23] may not possess. Such a property also facilitates overcoming the additional technical challenge introduced in the multi-channel model in Section 6.

We are ready to show the convergence of the D-PAT Algorithm in the following theorem with the proof in Appendix A.6.

**Theorem 3.** When $C(p)$ is strictly convex and the step size $\rho(\tau)$ is diminishing [46] the D-PAT Algorithm converges to a saddle point of the Lagrangian in (19), hence an NE of the PAT Game.

The proof of Theorem 3 involves showing that the D-PAT Algorithm is the gradient method for solving the dual problem in (21). We can guarantee its convergence [41] if we employ the bounded gradients, which is satisfied due to the bounds in (16).

15. There are multiple optimal dual solutions $\beta^*$. To see this, given any saddle point of $(\pi^*, \beta^*)$, we can add every $\beta_k^*$ by the same constant, and the new tuple $(\pi^*, \beta^*)$ still satisfies the conditions described in (23) and thus is also a saddle point.

16. An example of the diminishing step size is $\kappa(\tau) = (1 + \tau)/(c + \tau)$ for a constant $c > 0$.

Note that the gradient method requires the strict concavity of each decomposed Lagrangian $L_k$. Thus, a linear cost function $C(p)$ cannot meet this requirement. However, we can adopt the algorithm to be introduced in Section 6.2 which guarantees its convergence even if $C(p)$ is linear.

6 Multi-Channel Nash Mechanism

We now turn to the problem for a general multi-channel WPT network, where the ET can transmit over $N \geq 1$ orthogonal channels. The multi-channel WPT network brings a new consideration of allocating power across available channels. We will consider the multi-channel extension to achieve (E1)–(E4).

Such a new algorithm design is non-trivial, because each agent’s payoff function couples the transmit power decision across all channels. In addition, agents’ payoff functions may not be strictly concave. Note that even if $U_k(\cdot)$ is a strictly concave in $q_k$, it may not be strictly concave in $p$ when $h_{k,n} = 0$ for some channel $n$. For example, consider a system with $N = 2$ channels and $U_1 = \log(1 + h_{1,1}p)$ (i.e., EU 1 only operates on channel 1). The Hessian matrix of $U_1$’s utility function with respect to $p$ is given by $H_1 = \begin{pmatrix} -h_{1,1}^2 & 0 \\ 0 & 0 \end{pmatrix}$. For every $p \in \mathcal{P}$, the Hessian is negative semi-definite but not negative definite. Hence, EU 1’s utility is concave but not strictly concave in $p$. Thus, we cannot directly adopt a gradient-based algorithm similar to the D-PAT Algorithm. Instead, we consider an algorithm based on the augmented Lagrangian method to distributively compute the NE.

6.1 Nash Mechanism

In this subsection, we design the Nash mechanism for the multi-channel network. We then show the proposed two-phase all-or-none scheme together with a new mechanism achieves the economic properties (E1)–(E4) for the multi-channel network.

We propose Mechanism 2, which is a generalization of the PAT Mechanism. Specifically, each agent submits a message for every channel even if he can only operate on a subset of all channels.

**Mechanism 2. Multi-Channel Power and Taxation (MPAT) Mechanism**

- **The message space**: Each agent $k \in K \cup \{0\}$ sends a message $\mathbf{m}_k \in \mathbb{R}^{2N}$ to the ET of the following form:

  $$
  \gamma_k \triangleq \{ \gamma_{k,n} \}_n \in N', \quad \mathbf{b}_k \triangleq \{ b_{k,n} \}_n \in N'. \tag{24b}
  $$

  where $\gamma_{k,n}$ and $b_{k,n}$ are agent $k$’s power proposal and price proposal for channel $n \in N'$, respectively. We denote the message profile as $\mathbf{m} = \{ \mathbf{m}_k \}_{k \in K \cup \{0\}}$.

- **The outcome function**: The ET announces the transmit power on each channel $n$ to every agent:

  $$
  p_n(\mathbf{m}) = \frac{1}{K+1} \sum_{k \in K \cup \{0\}} \gamma_{k,n}, \forall n \in N'. \tag{25}
  $$

  The ET further computes the tax rate $R_k(\mathbf{m}) = \{ R_{k,n}(\mathbf{m}) \}_{n \in N'}$ for agent $k \in K$ based on the agents’ price proposals: for every $k \in K \cup \{0\}$ and every $n \in N'$,

  $$
  R_{k,n}(\mathbf{m}) = b_{\omega(k+1),n} - b_{\omega(k+2),n}. \tag{26}
  $$

  The ET announces EU $k$’s tax $t_{k,n}(\mathbf{m}) = \{ t_{k,n}(\mathbf{m}) \}_{n \in N'}$: for every $k \in K \cup \{0\}$ and every $n \in N'$,

  $$
  t_{k,n}(\mathbf{m}) = R_{k,n}(\mathbf{m}) p_n(\mathbf{m}). \tag{27}
  $$


Equation (27) implies that $\sum_{k \in K \cup \{0\}} t_{k,n} = 0$, $\forall n \in N$. Thus, the MPAT Mechanism achieves the budget balance (E4). Similarly, the MPAT Mechanism induces the following MPAT Game among agents in Phase II.

**Game 2. MPAT Game (Induced in Phase II)**
- **Players:** all agents in $K \cup \{0\}$.
- **Strategy:** $m_k \in \mathbb{R}^{2N}$ described in (24) for each agent $k \in K \cup \{0\}$.
- **Payoff function** $J_k(p, t_k) : \text{for each } E_k \in K,$
  \[ J_k(p, t_k) = U_k(q_k(p(m))) - \sum_{n \in N} t_{k,n}(m); \quad (28) \]
  for the ET (agent 0),
  \[ J_0(p, t_0) = \begin{cases} -C(p(m)) - \sum_{n \in N} t_{k,n}(m), & \text{if } p \in \mathcal{P}, \\ -\infty, & \text{otherwise}. \end{cases} \quad (29) \]

Different from the single-channel scenario, the multi-channel scenario may not admit a unique constrained Lindahl allocation. This is mainly due to the non-strict concavity of the objective function of the SWM Problem (3a). However, we present the following theorem with the proof in Appendix A.7.

**Theorem 4.** Each agent $k \in K \cup \{0\}$ receives the same payoff across different constrained Lindahl allocations.

We prove Theorem 4 by establishing the uniqueness of the received power for each EU, which leads to the uniqueness of each agent’s utility/cost and each agent’s total tax. This theorem indicates that each agent is insensitive to different constrained Lindahl allocations.

In the light of Theorem 4, we can show that the MPAT Mechanism achieves properties (E1)-(E3), with proofs in Appendices A.8 and A.9 respectively.

**Proposition 2 (Implementing Constrained Lindahl Allocations).** There exist multiple NEs in the MPAT Game, and each NE corresponds to a constrained Lindahl allocation.

**Proposition 3 (Voluntary Participation).** Each agent will participate in the MPAT Mechanism in Phase I.

To summarize, we have shown that every agent is indifferent to the choices of the NEs. Moreover, the two-phase all-or-none scheme and the MPAT Mechanism can achieve the desirable economic properties (E1)-(E4) for the multi-channel system. We next introduce the distributed algorithm.

### 6.2 Distributed Algorithm to Achieve the NE

In this subsection, we design the distributed algorithm for agents to compute an NE of the MPAT Game. We have shown that every NE leads to a constrained Lindahl allocation (Proposition 2) and all constrained Lindahl allocations are equivalent for every agent (Theorem 4). Therefore, agents just need to agree on reaching any of the NEs.

However, we cannot directly adopt a dual gradient-based algorithm similar to the D-PAT Algorithm, which requires the strict concavity of every agent’s payoff function. Instead, we propose an alternative approach in Algorithm 2 which ensures the convergence even if an agent’s payoff is not strictly concave. We will show Algorithm 2 is based on the Accelerated Distributed Augmented Lagrangians (ADAL) method [42] and prove its convergence in Section 6.3.

Algorithm 2: Distributed Algorithm to Reach the NE of the MPAT Game (D-MPAT Algorithm)

1. Initialize the iteration index $\tau \leftarrow 0$. Each agent $k \in K \cup \{0\}$ randomly initializes $m_k(0)$. The ET initializes the stopping criterion $\epsilon_1 > 0$ and $\epsilon_2 > 0$.
2. Set conv_flag $\leftarrow 0$ # initialize the convergence flag.
3. while conv_flag $= 0$ do
   4. Set $\tau \leftarrow \tau + 1$;
   5. Each EU $k \in K$ sends his message $m_k(\tau)$ to the ET;
   6. The ET computes the tax rate $R_k(\tau)$ in (26) for each agent $k \in K \cup \{0\}$;
   7. The ET sends $R_k(\tau), \gamma_{\omega(k-1)}(\tau)$ and $\gamma_{\omega(k-2)}(\tau)$ to EU $k$ for all $k \in K$;
   8. Each agent $k \in K \cup \{0\}$ computes $\gamma_k(\tau + 1)$ and $b_k(\tau + 1)$ as
     \[ \gamma_k(\tau + 1) = \gamma_k(\tau) + \sigma(\gamma_k(\tau) - \gamma_k(\tau)), \]
   9. \[ b_k(\tau + 1) = b_k(\tau) + \rho\sigma(\gamma_{\omega(k-1)}(\tau) - \gamma_{\omega(k-2)}(\tau)), \]
   where, for every agent $k \in K \cup \{0\}$,
   10. \[
       \gamma_k(\tau) = \begin{cases}
           \arg \max_{p} \left[ J_k(p, t_k(p, \tau)) \right] & \text{if } k \in K, \\
           \arg \max_{p \in \mathcal{P}} \left[ J_k(p, t_k(p, \tau)) \right] & \text{if } k = 0,
       \end{cases}
   \]
   11. \[ t_k(p, \tau) = \{ t_k(n, p(n, \tau)), t_k(n, p(n, \tau)) = R_k(\tau)p(n), \]
   12. $\rho = 1$, and $\sigma = 1/4$;
   13. if $|b_k,n(\tau) - b_k,n(\tau - 1)| \leq \epsilon_1|b_k,n(\tau - 1)|$ and $|\gamma_k,n(\tau) - \gamma_k,n(\tau - 1)| \leq \epsilon_2|\gamma_k,n(\tau - 1)|, \forall k \in K \cup \{0\}$ then
   14. conv_flag $\leftarrow 1$;
   15. The ET computes $p(m(\tau))$ and $t(m(\tau))$ using (25) and (27).

Algorithm 2 illustrates the proposed iterative D-MPAT Algorithm. The key difference compared with Algorithm 1 mainly lies in the updated of messages, as described in the following. First, for the power proposal update in (31), each agent maximizes his/her payoff minus a quadratic penalty (due to inconsistency with the agent $\omega(k-1)$’s power proposals); each agent updates the power proposals by (31). One main benefit of including the penalty term is that it ensures a unique solution of (32), therefore admits gradient-like updates of proposals as in (30) and (31). This resolves the drawback of the D-PAT algorithm of requiring the strict concavity to make the gradient-based algorithms feasible. Second, the price proposal update in (31) is similar to the D-PAT algorithm, which is designed to reduce the gaps of their power proposals according to (30).

Similar to the D-PAT Algorithm, the D-MPAT Algorithm should be executed in a synchronous fashion. In addition, the

17. Here the parameter $\sigma$ is the step size, which should be chosen in the interval $(0, 1/q)$, where $q$ is the number of agents coupled in the “most populated” constraint of the problem [42]. As we can observe, $q = 2$ in our case, so we set $\sigma = 1/4$. 

18. The ET computes $p(m(\tau))$ and $t(m(\tau))$ using (25) and (27).
6.3 Convergence of the D-MPAT Algorithm

Similar to the approach in Section 5.5, we prove the convergence of the D-MPAT Algorithm by reformulating the SWM Problem in (3a)-(3b). Then we demonstrate the connection between the saddle point of the augmented Lagrangian of the reformulation and the NE of the MPAT game. We next show that the D-MPAT Algorithm is an ADAL-based algorithm that converges to a saddle point and thus an NE of the MPAT game.

6.3.1 Problem Reformulation

We reformulate the SWM Problem by introducing auxiliary variables \( \pi = \{\pi_k\}_{k \in K} \), where \( \pi_k = \{\pi_{k,n}\}_{n \in N} \). They will help decouple agents’ utility/cost functions in the following reformulated problem:

\[
(R - \text{SWM} - M) \max_{\pi} \sum_{k \in K} U_k(q_k(\pi_k)) - C(\pi_0) \tag{33a}
\]

subject to \( \pi_k = \pi_{\omega(k-1)}, \forall k \in K \cup \{0\} \), \( \pi_0 \in P \). \tag{33b}

6.3.2 Lagrangian and Augmented Lagrangian

To show that Algorithm 2 is an ADAL-based algorithm, we consider the augmented Lagrangian function of the R-SWM-M Problem with the following decomposable structures [42]:

\[
L_\rho(\pi, \beta) = \sum_{k \in K} L_{\rho,k}(\pi_k, \beta) \tag{34}
\]

where \( L_{\rho,k}(\pi_k, \beta) \) is the agent \( k \)'s local augmented Lagrangian function, given by

\[
L_{\rho,k}(\pi_k, \beta) = \begin{cases} 
U_k(q_k(\pi_k)) - \sum_{n \in N} \pi_{k,n} \left[ \pi_{k,n} - \beta_{\omega(k-1),n} \right] & \text{if } k \in K, \\
C(\pi_0) - \sum_{n \in N} \pi_{k,n} \left[ \pi_{k,n} - \beta_{\omega(k-1),n} \right] & \text{if } k = 0. 
\end{cases} \tag{35}
\]

We observe that, if we let \( p = \pi_k, \gamma_{\omega(k-1)} = \pi_{\omega(k-1)}, \beta_k = b_{\omega(k-1)}, \text{ and } \beta_{\omega(k+1)} = b_{\omega(k+2)} \), then \( L_{\rho,k} \) is the objective that each agent \( k \in K \cup \{0\} \) maximizes in (32). Therefore, Algorithm 2 is the ADAL-based method, as described in [42].

We then define the saddle point of \( L_\rho \) as a tuple \( (\pi^*, \beta^*) \) that satisfies, for every \( \pi \in \Gamma \triangleq \{\pi : \pi_0 \in P\} \) and every \( \beta \in \mathbb{R}^{(K+1)N} \),

\[
L_\rho(\pi, \beta^*) \leq L_\rho(\pi^*, \beta^*) \leq L_\rho(\pi^*, \beta^*) \tag{36}
\]

6.3.3 Relation between the Saddle Point and the NE

We present the following result with its proof in Appendix A.10

**Theorem 5.** For any saddle point \( (\pi^*, \beta^*) \) satisfying (36), the message profile \( \pi^* = \{\pi_{k,n}^*\}_{k \in K, n \in N} \) is an NE of the MPAT Game. The D-MPAT Algorithm converges to a saddle point and thus the NE of the MPAT Game.

The proof is very similar to that of Proposition 1. Specifically, the set of the solution to R-SWM-M Problem that satisfies the KKT conditions is a subset of the NE message profile. In addition, we have shown that the D-MPAT Algorithm based on the ADAL method converges to a saddle point of the augmented Lagrangian in (34), hence an NE of the MPAT Game.

7 Numerical Results

Since we have proved the optimality and convergence of the proposed algorithms, here we numerically evaluate the convergence speed of proposed schemes. We further study the impacts of the number of EUs and the channel diversity on the performance of the proposed mechanisms.

7.1 Benchmarks

7.1.1 Distributed Pure Optimization (DPO) Algorithm

For the performance comparison purpose in terms of convergence speed in Section 7.3.1, we consider a distributed pure optimization (DPO) benchmark algorithm by adopting the following standard reformulation in [39]:

\[
\max_{\pi} \sum_{k \in K} U_k(q_k(\pi_k)) - C(\pi_0) \tag{37a}
\]

subject to \( \pi_k = \pi_0, \forall k \in K, \pi_0 \in P \). \tag{37b}

By doing so, we can use the algorithm in [42] to solve the problem. Note that such an algorithm is a pure optimization algorithm that relies on the strong assumption that EUs’ truthfully report their private information to achieve the social optimum.

7.1.2 Private Good Mechanism

For the performance comparison purpose in terms of the achievable social welfare and the EUs’ average payoff in Section 7.3.2 and 7.3.3, we consider a private good mechanism, which is a standard benchmark as considered in [16, Chap. 11. C]. That is, this benchmark treats the transmit power as a private good and ignores its public good nature. Specifically, EUs play a purchase game and each EU only pays for the transmit power that he requests. The market adjusts the price such that power supply equals the total power demand. Ignoring the wireless signals’ public good nature, the private good mechanism cannot prevent free-riders and may lead to inefficient power allocation.

We present the mechanism in details in Appendix B.

Reference [25] designed an interesting bidding mechanism for a WPT network with one channel, with which we also compare our PAT Mechanism in Appendix D.3.

7.2 Simulation Setup

We simulate the WPT operation in a time period of \( T = 1000 \) seconds. We assume that the ET’s cost function satisfies

\[
C(p) = eT \cdot \left( \sum_n p_n \right)^\gamma, \tag{38}
\]

where the exponential model captures that the failure rate (and thus the maintenance cost) of a transmitter grows exponentially [28]. We set \( \gamma = 1.1 \) and \( e = 0.5 \).

We adopt the following weighted \( \alpha \)-fair utility function [32] for each EU \( k \in K \),

\[
U_k \left( \sum_{n \in N} h_{k,n}p_n \right) = \frac{E_k}{B_k} \left( \sum_{n \in N} h_{k,n}p_n \right)^{1-\alpha} \cdot T, \tag{39}
\]

where \( E_k > 0 \) represents the energy consumption rate for EU \( k \) and \( B_k > 0 \) indicates the battery state of EU \( k \). Parameters \( B_k \) and \( E_k \) are uniformly and independently chosen from the intervals \([20, 50]\) and \([0.1, 0.3]\), respectively. The distance \( d_k \) between the ET and each EU \( k \) follows the independent and identically distributed (i.i.d.) uniform distribution from the interval \([1, r]\) (meter), where \( r \) is the cluster radius set to be 5 meter.
7.3 Results

7.3.1 Convergence

We first evaluate the convergence speed of the proposed algorithms and the DPO Algorithm mentioned in Section 7.1.1.

In Fig. 4 we plot the agents’ power proposals achieved by the D-PAT Algorithm for a system with $K = 3$ EUs and $N = 1$ channel with a carrier frequency of 915 MHz. We set the upper bounds for the power proposal to be $P_{k,\text{up}} = 5$ for all agents. The power proposals converge to the socially optimal transmit power. This is because we design the algorithms to satisfy the equality constraints in (18b). In addition, for the D-PAT Algorithm, the EU 1’s submitted power proposal is 5 at the beginning. This happens since EU 2 receives a negative tax rate $\bar{R}_1$ (not shown in the figure) and thus submits the power proposal as large as possible.

In Fig. 5 we plot the agents’ power proposals achieved by the D-MPAT Algorithm for a system with $K = 2$ EUs and $N = 2$ channels with carrier frequencies of 915 MHz and 950 MHz, respectively. We observe that the power proposals converge to the optimal transmit power on each corresponding channel.

We further assess the convergence speed of the D-MPAT Algorithm for different numbers of the EUs and channels. We set the convergence parameters $\epsilon_1 = \epsilon_2 = 0.05$. We show that the number of iterations for the D-MPAT Algorithm to converge unless stated otherwise. The channel gain follows the long-term path-loss model, $h_{k,n} = a_{k,n} \phi_n d_{k,n}^{-\gamma}$, where $a_{k,n}$ is a binary parameter indicating whether EU $k$ operates on channel $n$ or not; $\phi_n$ denotes a positive parameter related to carrier frequency. Parameter $a_{k,n}$ follows the i.i.d. Bernoulli distribution, which equals 1 with probability $\text{Prob}$ and equals 0 with probability $1 - \text{Prob}$. Parameter $\phi_n$ satisfies $\phi_n = (2.39 \times 10^7/C F_n)^2$ and $C F_n$ is the carrier frequency of channel $n$ to be specified later. We set $\text{Prob} = 0.8$ here, and we study the impact of $\text{Prob}$ on the performances in Appendix B. These parameters do not change during the time period of interest.

7.3.2 Impact of the Number of EUs

We then study the impact of the number of EUs $K$ on the performance of the proposed MPAT Mechanism and the private good mechanism introduced in Section 7.1.2. Specifically, for the single-channel scenario, both the DPO Algorithm and the D-MPAT Algorithm converge within 40 iterations when $K \leq 7$. Moreover, the number of iterations increases slightly in $K$, for both algorithms. We observe a similar trend for the two-channel scenario.

**Observation 1.** Despite of the lack of complete information, our MPAT Algorithm can elicit users’ truthful information without much degradation of convergence speed.

In Fig. 7 we can see that the average EUs’ payoff achieved by both schemes increase in $K$. Moreover, when $K$ becomes larger, the performance gap between $P_{\text{max}} = 1$ W and $P_{\text{max}} = 4$ W also becomes larger for the MPAT Mechanism. This is because as $K$ becomes larger, a larger $P_{\text{max}}$ can allow a larger transmit power that provides more benefits to more EUs in the MPAT Mechanism at the social optimal solution. However, for the private mechanism, a larger $K$ does not significantly increase the demand or the social welfare, since the free-rider issue of the private good mechanism leads to an inefficient power provision. Moreover, the social welfare improvement of the proposed MPAT Mechanism (compared with the private good mechanism) increases in $K$, reaching 170% when $K = 15$ and $P_{\text{max}} = 4$ W.

In Fig. 8 we can see that the average EUs’ payoff achieved by both schemes increase in $K$. For the private good mechanism, the EUs’ payoff slightly increases in $K$. This is due to the free-rider issue in the private good mechanism, i.e., each additional EU...
tends to free-ride but not purchase wireless power, leading to an insufficient transmit power provision level. On the other hand, the proposed MPAT Mechanism can lead to a significant improvement in the EUs’ average payoff. Hence, it shows the significant social welfare benefit of preventing the free-riders.

**Observation 2.** Compared with the private good mechanism, the MPAT Mechanism can lead to significantly more EUs’ average payoff improvement when the number of EUs increases.

As today’s wireless networks are becoming increasingly denser, we believe that the proposed schemes will bring significant benefit to the overall system performance.

### 7.3.3 Impact of the Channel Diversity

We next study the impact of the channel diversity on the achievable social welfare, where the carrier frequencies are \{865, 890, 915, 950\} MHz, respectively. The trend in terms of average EUs’ payoff is similar to the social welfare and will be presented in Appendix C.

Fig. 9 compares the social welfare of the two schemes under different cluster radius \(r\), with \(N = 4\) channels and \(K = 10\) EUs. A larger cluster radius \(r\) implies that EUs’ channel conditions are more diverse.

In Fig. 9 for both schemes, the achievable social welfare decreases as \(r\) increases, since EUs experience a small channel gain due to the larger distance. Moreover, the performance gaps between the MPAT Mechanism and the private good mechanism decrease in \(r\). As the cluster radius becomes larger, the diversity (difference) of EUs’ utility also increases. For instance, when one EU has much better channel gains than the other EUs, purchasing power by this EU alone (as in the private good mechanism) can achieve a social welfare close to the optimum.

**Observation 3.** The performance benefit of the MPAT Mechanism is most significant when EUs have comparable channel gains.

### 8 Conclusion

Due to their broadcast nature, wireless signals are non-excludable public goods in the WPT networks. We formulated the first public good problem for the WPT networks. We proposed a simple Nash implementation PAT Mechanism (for a single-channel scenario) and an MPAT Mechanism (for a multi-channel scenario), considering agents’ selfish behaviors and private information. We then established the connection between the optimal solution of a reformulated optimization problem and the equilibrium induced by the mechanism. This leads to a general framework that allows us to adopt a wide range of distributed optimization algorithms to compute the equilibrium induced by some carefully designed mechanisms, and ensure the convergence under some fairly general conditions (such as the non-strict concavity and payoff discontinuity in this paper).

For the future work, it is interesting to consider the mechanism design for simultaneous information and power transfer networks. This requires us to design a new mechanism that involves the effective provision of both public good (wireless power) and private good (information).

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APPENDIX A
PROOFS

To facilitate the following proofs, we start by presenting the Karush-Kuhn-Tucker (KKT) conditions for the SWM Problem.

A.1 KKT Conditions for SWM Problem

We present the KKT conditions for the single-channel case and the multi-channel case separately as follows.

A.1.1 The Single-Channel Case

\[
\sum_{k \in \mathcal{K}} \frac{\partial U_k(h_k p)}{\partial p} - \frac{\partial C(p)}{\partial p} - \lambda + \mu = 0, \quad (40a)
\]

\[
\lambda(p - P_{\text{max}}) = 0, \quad (40b)
\]

\[
\mu p = 0, \quad (40c)
\]

\[
p \in \mathcal{P}, \quad (40d)
\]

\[
\lambda, \mu \geq 0, \quad (40e)
\]

where \(\lambda\) is the dual variable corresponding to the constraint \(p \leq P_{\text{max}}\), and \(\mu\) is the dual variable corresponding to the constraint \(0 \leq p\).

A.1.2 The Multi-Channel Case

\[
\sum_{k \in \mathcal{K}} \frac{\partial U_k(g_k(p))}{\partial p_n} - \frac{\partial C(p)}{\partial p_n} - \lambda_n - \nu + \mu_n = 0, \quad \forall n \in \mathcal{N}, \quad (41a)
\]

\[
\lambda_n(p_n - P_{\text{peak},n}) = 0, \quad \forall n \in \mathcal{N}, \quad (41b)
\]

\[
\nu \left(\sum_{m \in \mathcal{N}} p_m - P_{\text{max}}\right) = 0, \quad (41c)
\]

\[
\mu_n p_n = 0, \quad \forall n \in \mathcal{N}, \quad (41d)
\]

\[
p \in \mathcal{P}, \quad (41e)
\]

\[
\lambda_n, \mu_n \geq 0, \quad \forall n \in \mathcal{N}, \quad (41f)
\]

\[
\nu \geq 0 \quad (41g)
\]

where \(\lambda_n\) is the dual variable corresponding to the constraint \(p \leq P_{\text{peak},n}\), \(\mu_n\) is the dual variable corresponding to the constraint \(0 \leq p_n\), and \(\nu\) is the dual variable corresponding to the constraint \(\sum_{m \in \mathcal{N}} p_m \leq P_{\text{max}}\).

It is easily to verify that the Slater’s condition holds for the SWM Problem in (40a)-(40e). Hence, the above KKT conditions are both sufficient and necessary for the global optimality of the corresponding SWM Problems.

A.2 Proof of Lemma 1

To prove the necessity of (14), we consider the following agent \(k\)’s payoff maximization problem:

\[
\max_{\gamma_k} J_k \left( \gamma_k + \sum_{l \neq k, l \in \mathcal{K} \cup \{0\}} \gamma_l^* \right), \quad (42)
\]

and a simplified problem:

\[
\max_{\gamma_k} J_k(p, R_k^*(p)). \quad (43)
\]

We can see that the optimal values of (42) and (43) are the same, since for any \(p\) and \(\gamma_k\) \(\forall l \neq k, l \in \mathcal{K} \cup \{0\}\) \(\gamma_l^*\), there always exists a \(\gamma_k\) such that \((K + 1)p = \gamma_k + \sum_{l \neq k, l \in \mathcal{K} \cup \{0\}} \gamma_l^*\). Therefore, let \(\gamma_k^*\) be agent \(k\)’s NE power proposal (and hence the optimal solution to (42)) and \(p^*\) be the optimal solution to (43). The fact that problems in (42) and (43) having the same optimal solutions implies

\[
\gamma_k^* + \sum_{l \neq k, l \in \mathcal{K} \cup \{0\}} \gamma_l^* = (K + 1)p^*. \quad (44)
\]

This further leads to

\[
\gamma_k^* = (K + 1) \arg \max_p J_k(p, R_k^*(p)) - \sum_{l \neq k, l \in \mathcal{K} \cup \{0\}} \gamma_l^*. \quad (45)
\]

This completes the proof.

A.3 Proof of Theorem 1

To prove Theorem 1, we first rewrite the necessary and sufficient conditions for (14) in Lemma 1 and then prove the existence and efficiency.

We rewrite the necessary and sufficient conditions for (14) in Lemma 1 in the following:

\[
\frac{1}{K + 1} \sum_{l \in \mathcal{K} \cup \{0\}} \gamma_l^* = p^*, \quad (46a)
\]

\[
\frac{\partial U_k(h_k p^*)}{\partial p} - b_{\text{peak},(k+1)}^* - b_{\text{peak},(k+2)}^* = 0, \quad k \in \mathcal{K}, \quad (46b)
\]

\[
-C^*(p^*) - b_{\text{peak}}^* - b_{\text{peak},n}^* - \lambda + \mu = 0, \quad (46c)
\]

\[
\lambda(p^* - P_{\text{max}}) = 0, \quad (46d)
\]

\[
\mu p^* \geq 0, \quad (46e)
\]

\[
\lambda, \nu, \lambda_n \geq 0, \quad (46f)
\]

where (46b) is the first-order condition for each EU’s payoff maximization problem, and (46a)-(46c) are the KKT conditions of the ET’s payoff maximization problem.

A.3.1 Existence

Let \((p^*, \lambda^*, \mu^*)\) be the solution to the KKT conditions in (40a)-(40e). There always exists a message profile \((m^* = \{\gamma_k^*, b_k^*\})_{k \in \mathcal{K} \cup \{0\}}, \hat{\mu}^*, \hat{\lambda}^*\) such that \(\hat{\mu}^* = \mu^*, \hat{\lambda}^* = \lambda^*, \gamma_k^* = p^*\), for all \(k \in \mathcal{K} \cup \{0\}\), and

\[
b_k^* = \begin{cases} 
-\frac{\partial C(p^*)}{\partial p} - \hat{\lambda}^* - \hat{\mu}^*, & \text{if } k = 1, \\
0, & \text{if } k = 2, \\
-\sum_{l=1}^{k-2} \frac{\partial U_l(h_k p^*)}{\partial p}, & \text{if } 3 \leq k \leq 0.
\end{cases} \quad (47)
\]

We observe that the message profile \((m^* = \{\gamma_k^*, b_k^*\})_{k \in \mathcal{K} \cup \{0\}}, \hat{\mu}^*, \hat{\lambda}^*\) satisfies (46a)-(46f). This indicates that it is an NE. This completes the proof of existence.

A.3.2 Efficiency

Combining the first-order conditions for all EUs’ payoff maximization problems in (46b) and that for the ET’s problem in (46c), we have

\[
\sum_{k \in \mathcal{K}} \frac{\partial U_k(h_k p^*)}{\partial p} - \frac{\partial C(p^*)}{\partial p} - \hat{\lambda}^* - \hat{\mu}^* = 0. \quad (48)
\]

We can find that (48) and (46c)-(46f) are equivalent to the KKT conditions for the SWM Problem in (40a)-(40e). In other words, for every NE \(m^*\), the resulted \((p^*, \hat{\mu}^*, \hat{\lambda}^*)\) satisfies the KKT conditions in (40a)-(40e). Thus, at any NE, the socially optimal transmit power is achieved.
Moreover, since there exists a unique socially optimal transmit power due to the strict concavity of the objective of the SWM Problem, all NEs lead to the same socially optimal transmit power $p^o$. 

### 4.3.3 Lindahl Tax Rate

The tax rate for each EU is given by $R^*_{k} = b^{*}_{o(k+1)} - b^{*}_{o(k)} = U'_{k}(p^*)$. Since the socially optimal transmit power is unique, the tax rate for each EU is also unique due to \ref{46b}. Therefore, the tax rate for the ET $R^*_{0} = -\sum_{k \in K} R^*_{k}$ is also unique.

#### A.4 Proof of Theorem \[2\]

By the definition of NE, we obtain $J_k(p(m_k, m^*_{-k}), t_k(m_k, m^*_{-k})) \leq J_k(p^*, t^*_k), \forall m_k \in M_k$. \hspace{1cm} (49)

Since \ref{49} holds for all $m_k = (\gamma_k, b_k)$, substituting $\frac{1}{1+\gamma_k} = \bar{\rho}$ and the tax in \ref{4}, we have

$$J_k(\bar{\rho}, R^*_{k} \bar{\rho}) \leq J_k(p^*, t^*_k), \forall \bar{\rho} \in \mathbb{R}, \forall k \in K \cup \{0\}. \hspace{1cm} (50)$$

For $\bar{\rho} = 0$, \ref{50} further implies that

$$J_k(0, 0) \leq J_k(p^*, t^*_k), \forall k \in K \cup \{0\}, \hspace{1cm} (51)$$

which means that each agent weakly prefers the allocation ($p^*, t^*_k$) when everyone participates, compared to the allocation when someone chooses not to participate (0, 0). Hence, no matter other agents choose to participate or not, it is a weakly dominant strategy for each agent $k$ to choose participating in the PAT Mechanism.

#### A.5 Proof of Proposition \[1\]

The KKT conditions of the R-SWM Problem are given by

$$\frac{\partial U_k(q_k(\pi^*_k))}{\partial \pi^*_k} - \beta^*_k + \beta^{*}_{\omega(k+1)} = 0, \forall k \in K, \hspace{1cm} (52a)$$

$$-\frac{\partial C((\pi^*_0))}{\partial \pi^*_0} - \beta^*_0 + \beta^*_1 - \hat{\lambda} + \hat{\mu} = 0, \hspace{1cm} (52b)$$

$$\hat{\lambda}(\pi^*_0 - P_{\max}) = 0, \hspace{1cm} (52c)$$

$$\hat{\mu} = 0, \hspace{1cm} (52d)$$

$$\hat{\lambda} \geq 0, \forall k \in K \cup \{0\}. \hspace{1cm} (52e)$$

Let $\gamma_k = \pi^*_k = p^o$, for all $k \in K \cup \{0\}$. We show that $\{\gamma_k\}_{k \in K \cup \{0\}}$ satisfies \ref{46a}. In addition, substituting $b^{*}_{o(k+1)}$ into $\beta^*_k$, we have that \ref{52a} and \ref{52c} have exactly the same structure as \ref{46b} and \ref{46c}, which implies that $m = \{(\gamma_k = \pi^*_k, b_k = \beta^{*}_{\omega(k-1)})\}$ satisfies the NE conditions in \ref{46b}-\ref{46d} and thus is an NE.

#### A.6 Proof of Theorem \[3\]

Let $\gamma_k = \pi^*_k$ and $b_k = \beta^{*}_{\omega(k-1)}$. We observe that the D-PAT Algorithm is a dual-based gradient method for solving the following problem:

$$\max \ U_k(q_k(\pi^*_k)) - C(\pi^*_0) \hspace{1cm} (53a)$$

s.t. $\pi_k = \pi^{*}_{\omega(k-1)}, \forall k \in K \cup \{0\}, \hspace{1cm} (53b)$

$\pi^*_0 \in \mathcal{P}, \hspace{1cm} (53c)$

$0 \leq \pi_k \leq P^{up}_{K}, \forall k \in K. \hspace{1cm} (53d)$

Note that if EUs select large enough power upper bounds $\{P^{up}_{K}\}$, the problem in \ref{53a}-\ref{53d} yields exactly the same solution to that of the R-SWM Problem. Specifically, \ref{16} is the primal update which computes the dual function given the dual variable in each iteration, and \ref{17} is the dual update to gradually solve the dual problem in \ref{21}.

By \ref{41}, the gradient method converges to the optimal solution if we employ (i) a diminishing step size and (ii) the bounded gradients. The updated $\gamma_k$ is bounded due to \ref{16} and hence the gradient is bounded. Thus, we have shown that the D-PAT Algorithm converges to the optimal solution of \ref{53a}-\ref{53d} and thus to an NE of the PAT Game according to Proposition \ref{1}

#### A.7 Proof of Theorem \[4\]

Consider a reformulation of the SWM Problem in \ref{3} by introducing the auxiliary received power vector $z \in \mathbb{R}^K$:

$$\max \ \sum_{k \in K} U_k(z_k) - C(p), \hspace{1cm} \text{s.t.} \quad z_k = h^T_k p, \ \forall k \in K, \hspace{1cm} p \in \mathcal{P}.$$ 

To prove \ref{4} we will prove that optimal received power vector $z^*$ is unique by contradiction (though the solution $(z^*, p^*)$ to the above optimization problem is not unique). We then prove the uniqueness of the total tax for each agent at the NEs.

#### A.7.1 Uniqueness of $z^*$

To prove the uniqueness of $z^*$, we suppose that there exist two optimal solutions $(z^*_1, p^*_1)$ and $(z^*_2, p^*_2)$ such that $z^*_1 \neq z^*_2$ (but $p^*_1$ may or may not be equal to $p^*_2$). Due to the strict concavity of $U_k(z_k)$ and the convexity of $C(p)$, we have that

$$C \left( \frac{p^*_1 + p^*_2}{2} \right) \leq \frac{1}{2} \left( C(p^*_1) + C(p^*_2) \right),$$

$$\sum_{k \in K} U_k(z^*_1, k) + \sum_{k \in K} U_k(z^*_2, k) \geq \frac{1}{2} \left( \sum_{k \in K} U_k(z^*_1, k) + \sum_{k \in K} U_k(z^*_2, k) \right).$$

In addition, it is easy to verify that $(z^*_1/2 + z^*_2/2, p^*_1/2 + p^*_2/2)$ is also a feasible solution, which leads to an objective value of

$$\sum_{k \in K} U_k(z^*_1, k) + \sum_{k \in K} U_k(z^*_2, k) = \frac{1}{2} \left( C(p^*_1) + C(p^*_2) \right).$$

where $v^*$ is the objective value achieved by optimal solution $(z^*_1, p^*_1)$ or $(z^*_2, p^*_2)$. The above result contradicts to the fact that $(z^*_1, p^*_1)$ and $(z^*_2, p^*_2)$ are optimal. Hence, we have proved the uniqueness of the optimal received power vector $z^*$. This means that all optimal solution leads to the same utility for each EU and hence the same cost for the ET.

#### A.7.2 Uniqueness of each agent's total tax

Next, we will prove the uniqueness of the total tax for each agent at the NEs. The total tax for each EU $k$ is

$$\sum_{n \in N} t^*_{k,n} = \sum_{n} \frac{\partial U_k(q_k(p^*))}{\partial p_n} p^*_n = \frac{\partial U_k(z^*_k)}{\partial z_k} \sum_{n \in N} h_{k,n} p^*_n = \frac{\partial U_k(z^*_k)}{\partial z_k} z^*_k, \hspace{1cm} (55)$$

which is unique due to the uniqueness of $z^*_k$. The ET’s total tax $\sum_{n} t^*_{0,n}$ is thus also unique. Therefore, we have proved that each agent $k \in K \cup \{0\}$ receives the same payoff at every constrained Lindahl allocation.
A.8 Proof of Proposition 2

First, similar to Lemma 1 and (15) for the single-channel system, it follows that the equilibrium transmit power vector \( \mathbf{p}^* \) must maximize every agent’s payoff. Otherwise, some agent can change his power proposal to improve his payoff. Hence, the sufficient and necessary conditions for the NEs of the MPAT Game are given by: for every channel \( n \in \mathcal{N} \),

\[
\frac{1}{K + 1} \sum_{k \in \mathcal{K} \cup \{0\}} \gamma_{k,n} = p_n^*, \quad (56a)
\]

\[
\frac{\partial U_k(q_k(p^*))}{\partial p_n} - b_{k,\omega(k+1),n}^* + b_{k,\omega(k+2),n}^* + \lambda_n - \bar{\nu} + \bar{\mu}_n = 0, \quad (56b)
\]

\[
-\frac{\partial C(p^*)}{\partial p_n} - b_{1,n}^* + b_{2,n}^* + \bar{\lambda}_n - \bar{\nu} + \bar{\mu}_n = 0, \quad (56c)
\]

\[
\bar{\lambda}_n(p_n^* - P_{\text{peak}}) = 0, \quad (56d)
\]

\[
\bar{\nu} \left( \sum_{m \in \mathcal{N}} p_m^* - P_{\max} \right) = 0, \quad (56e)
\]

\[
\bar{\mu}_n p_n^* \geq 0, \quad (56f)
\]

\[
\bar{\mu}_n, p_n^*, \bar{\lambda}_n, \bar{\nu} \geq 0, \quad (56g)
\]

where \( \bar{\nu} \) and \( \bar{\lambda}_n \) are the dual variables corresponding to the total power constraint and peak power constraints for the ET’s payoff maximization problem, respectively. Equation (56a) is EUs’ first order conditions and (56c)-(56g) are the ET’s KKT conditions.

Combining (56b) and (56c), it follows that

\[
\sum_{k \in \mathcal{K}} \frac{\partial U_k(q_k(p^*))}{\partial p_n} - \frac{\partial C(p^*)}{\partial p_n} - \bar{\lambda}_n + \bar{\nu} + \bar{\mu}_n = 0, \quad \forall n \in \mathcal{N} \quad (57)
\]

Additionally, (57) and (56d)-(56g) constitute the KKT conditions for the SWM Problem in (41a)-(41g), indicating that the every NE transmit power \( \mathbf{p}^* \) is exactly the socially optimal transmit power \( \mathbf{p}^0 \) to the SWM Problem.

A.9 Proof of Proposition 3

Substituting \( \frac{1}{K+1} (\gamma_{k,n} + \sum_{l \in \mathcal{K} \cup \{0\}} \gamma_{l,n}^*) = \bar{p}_n \) and the tax in (27) into the definition of NE, we obtain that

\[
J_k(\bar{p}, \bar{t}_k) \leq J_k(p^*, t_k^*), \quad \forall \bar{p} \in \mathbb{R}^N, \forall k \in \mathcal{K} \cup \{0\}, \quad (58)
\]

where \( \bar{t}_k = \{t_{k,n}\}_{n \in \mathcal{N}} \) and \( \bar{t}_{k,n} = R_{k,n}^* \bar{p}_n \), for all \( n \in \mathcal{N} \) and all \( k \in \mathcal{K} \cup \{0\} \). For an all-zero vector \( \bar{p} = 0 \), (58) further implies that

\[
J_k(0, 0) \leq J_k(p^*, t_k^*), \quad \forall k \in \mathcal{K} \cup \{0\}, \quad (59)
\]

which means that each agent \( k \) always weakly prefers to participating in the MPAT Mechanism, independent of other agents’ choices.

A.10 Proof of Theorem 5

The KKT conditions of the R-SWM-M Problem are given by: for every channel \( n \in \mathcal{N} \),

\[
\frac{\partial U_k(q_k(p^*))}{\partial p_n} - \beta_{k,n}^* + \beta_{\omega(k+1),n}^* = 0, \quad \forall k \in \mathcal{K}, \quad (60a)
\]

\[
-\frac{\partial C(p^*)}{\partial p_n} - \beta_{0,n}^* + \beta_{1,n}^* - \bar{\lambda}_n - \bar{\nu} + \bar{\mu}_n = 0, \quad (60b)
\]

\[
\bar{\lambda}_n(p_0^* - P_{\text{peak}}) = 0, \quad (60c)
\]

\[
\bar{\nu} \left( \sum_{m \in \mathcal{N}} p_{m,n}^* - P_{\max} \right) = 0, \quad (60d)
\]

\[
\bar{\mu}_n p_0^* = 0, \quad (60e)
\]

\[
\pi_{k,n}, \nu_n, \bar{\lambda}_n \geq 0, \quad \forall k \in \mathcal{K} \cup \{0\}, \quad (60f)
\]

Letting \( \gamma_k = \pi_k^* = p_k^* \), for all \( k \in \mathcal{K} \cup \{0\} \). We can show that \( \{\gamma_k\}_{k \in \mathcal{K} \cup \{0\}} \) satisfies (60a). In addition, substituting \( b_{\omega(k+1)} \) into \( \beta_{l}^* \), we have that (60a)-(60f) have exactly the same structure as (46b)-(46f), which implies that \( \bar{m} = \{\gamma_k = \pi_k^*, b_k = b_{\omega(k+1)}\}_{k \in \mathcal{K} \cup \{0\}} \) satisfies the NE conditions in (55b)-(55g) and thus is an NE.

APPENDIX B

BENCHMARK: PRIVATE GOOD MECHANISM

In this section, we consider a standard benchmark mechanism in (16) Chap. 11. C, to illustrate the inefficiency of private provision of public goods. In this case, each EU only pays for the transmit power that he requests. In other words, we treat the transmit power as a private good and ignore its non-rivalrous and non-excludable public good nature.

The private good mechanism leads to a state of the economy where

- EUs play a purchase game, where each EU \( k \) chooses his power demand \( x_k \) to maximize his payoff function (i.e., utility minus payment), considering a given market price \( \theta \) and transmit power requested by other EUs.
- The ET acts as a profit maximizer and chooses a supplied transmit power \( \mathbf{p} \) to maximize his profit (i.e., revenue minus cost), given the market price \( \theta \).
- The market adjusts the market price \( \theta \) such that the power supply equals the power demand.

We characterize the equilibrium of the private good mechanism as follows:

**Definition 2** (Benchmark Equilibrium (BE)). A BE is market allocation specified by the equilibrium market price and the equilibrium transmit power \( (\mathbf{p}^*, \theta^*) \) such that

\[
x_k^*(\theta^*, x_{-k}^*) \in \arg \max_{x_k \geq 0} \left[ U_k \left( q_k \left( x_k + \sum_{j \neq k, j \in \mathcal{K}} x_j^* \right) \right) - \theta^* x_k \right], \quad \forall k \in \mathcal{K} \quad (61)
\]

\[
\mathbf{p}^*(\theta^*) \in \arg \max_{\mathbf{p} \in P} \left[ \theta^* \mathbf{p} - C(\mathbf{p}) \right], \quad (62)
\]

where \( \theta^* \) is selected such that

\[
\sum_{k \in \mathcal{K}} x_k^*(\theta^*) = \mathbf{p}^*(\theta^*). \quad (63)
\]
AlGORITHM 3: Algorithm to Reach the BE

1. Initialize the iteration index $\tau \leftarrow 0$. Each agent $k \in K \cup \{0\}$ randomly initializes $p(0) \succ 0$. The ET initializes the stopping criterion $\epsilon$.
2. Set $\text{conv\_flag} \leftarrow 0$ # initialize the convergence flag.
3. while $\text{conv\_flag} = 0$ do
   4. Each EU $k$ computes the gradient of his utility function with respect to the power $\nabla_p U_k(q_k(p(t)))$ and sends the value to the ET;
   5. The ET computes the gradient of her utility function with respect to the power $\nabla_p C(p(t))$;
   6. The ET computes $p(t + 1)$ by
      \[
      p(t + 1) = \{p(t) + \alpha(t) \left[ \max_k \nabla_p U_k(q_k(p(t))) - \nabla_p C(p(t)) \right]\}_p,
      \tag{64}
      \]
      and broadcasts it to all EUs, where $[\cdot]_p$ denotes the projection onto the feasible convex set $\mathcal{P}$;
   7. Set $\tau \leftarrow \tau + 1$;
   8. if $||p(t + 1) - p(t)||_2 \leq \epsilon||p(t)||_2$ then
      9. $\text{conv\_flag} \leftarrow 1$;
   10. end
11. end
12. Compute the BE price vector $\theta^*$ by
\[
\theta^* = \max_k \nabla_p U_k(q_k(p^*)).
\tag{65}
\]

The reason that we select the above BE is that if there are no public good effects (i.e., an EU $k$ cannot benefit from another EU $j$’s contribution $x_j$), the BE becomes the well-known competitive equilibrium. This can lead to a social optimum for a private good economy [16]. However, for the public goods economy, the BE can be highly inefficient in general, i.e., far away from the maximum social welfare that can be achieved.

To demonstrate its inefficiency and design an algorithm to compute the BE, we conduct the following analysis:

- When there exists a $p^*(\theta^*)$ that is an interior point of $\mathcal{P}$, we have
\[
\theta^* = \nabla_p C(p^*) = \max_k \nabla_p U_k(q_k(p^*)),
\tag{66}
\]

where
\[
\max_k \nabla_p U_k(q_k(p)) \leq \left( \max_k \frac{\partial U_k(q_k(p))}{\partial p_1}, ..., \max_k \frac{\partial U_k(q_k(p))}{\partial p_N} \right)^T.
\tag{67}
\]

- From (66), we can see that, if $K \geq 2$, the sum marginal utility of all EUs is larger than the ET’s marginal cost at the BE on each channel $n$, i.e. [18]
\[
\sum_{k \in K} \nabla_p U_k(q_k(p^*)) > \nabla_p C(p^*).
\tag{68}
\]

In other words, the optimal solution to the SWM Problem (which would equalize the two terms in (68) should be a larger transmit power than the one at the BE.

18. For any two vectors $a$ and $b$, $a \succ b$ means that $a$ is componentwisely larger than $b$.

Motivated by (66), we consider a projected subgradient-based algorithm to compute the BE, as summarized in Algorithm 3. We explain the key steps in the following. Each EU sends the gradient of his utility to the ET (line 5). Then, the ET computes the subgradient of her cost and updates the price vector according to (64). This direction of price update aims to equalize $\nabla_p C(p)$ and $\max_k \nabla_p U_k(p)$. The ET checks the termination criterion (line 8). The termination happens if the absolute changes of the power updates are small, determined by a positive constant $\epsilon > 0$. Finally, the ET sets the BE price to EUs’ maximal marginal utility on each channel, as in (65).

Although it is difficult to analytically prove Algorithm 3’s convergence to a BE, we can numerically show its convergence, as demonstrated in Fig. 10. Specifically, in Fig. 10 we show the cumulative distribution function (CDF) of the number of iterations for Algorithm 3 to converge, with different accuracy constants $\epsilon$. We show that Algorithm 3 always converges within 200 and 2,000 steps when $\epsilon = 10^{-2}$ and $10^{-3}$, respectively.

APPENDIX C

EXTENSION TO THE SINGLE-EU MODEL

C.1 Extension to the Single-EU Model

In this section, we discuss how to extend our analysis to the single-EU model.

Note that we cannot directly apply our previous PAT/MPAT mechanisms where there is only one EU, since the tax rates defined in (8) and (27) require at least three agents (i.e., at least two EUs). Hence we need to perform some novel transformation of the problem.

The key idea of such a transformation is to introduce a virtual agent into the system. The virtual agent has an index $v$, as the constrained Lindahl allocation with the virtual agent, and define \((\{R_k^v\}_{k \in \{0,1,2\}}, p^v)\) as the constrained Lindahl allocation with the virtual agent. We can show the following equivalent results:

**Proposition 4.** For agents 0 and 1, the constrained Lindahl allocations with and without the virtual agent are the same, i.e., \((\{R_k^v\}_{k \in \{0,1\}}, p^v) = (\{R_k^0\}_{k \in \{0,1\}}, p^0)\).

**Proof:** Since the additional virtual agent’s utility is $U_2 = 0$, the equilibrium transmit power $p^v$ is also the optimal solution to the SWM Problem in the system with the virtual agent. That is, $p^v = p^*$. Thus, $R_1^v = \nabla_p U_k(q_k(p^*)) = \nabla_p U_k(q_k(p^*)) = R_1^*$. Moreover, it is readily verified that $R_2^v = 0$, which indicates that
\[
R_0^v = -R_1^v = -R_1^* - R_2^v = R_0^*.
\]
This completes the proof.

The intuition is that the virtual agent has a constant zero utility, which does not change the optimal transmit power vector or the Lindahl tax rates. Therefore, with the introduced virtual agent, we can still use the PAT/MPAT mechanisms to achieve the constrained Lindahl allocation and thus the socially optimal transmit power.

C.2 Extension to the Multi-ET Model

In the case of a multi-ET scenario where multiple ETs are transmitting over orthogonal channels (as studied in [37]), our proposed mechanisms and the corresponding algorithms can be easily extended to that case. Specifically, all ETs and all EUs submit their messages described in (24) to a central ET. Then, the central ET determines the transmit power and the taxes according to (25)-(27). We can show that the proposed PAT/MPAT Mechanisms and the corresponding PAT/MPAT Algorithms can also achieve the properties (E1)-(E4).

In the case where ETs can potentially transmit over the same channels, the problem becomes extremely challenging, since the SWM Problem becomes an energy beamforming problem or a coordinated multi-energy transmission problem with distributed antennas. This makes the SWM problem non-convex, which is challenging to solve even if the ETs have complete information. We hence leave the multi-ET system for future work.

C.3 Extension to the Unknown $K$ Model

To further extend our current model to the case where the ET cannot know the total number of EUs, we can utilize the mechanism design based on repeated games [37, Chap 2.3] to exploit each EU’s local knowledge of the existence of his neighbor EUs. Consider an undirected knowledge graph, where each node corresponds to an agent; each edge implies that the nodes incident to the edge know the existence of each other. Notice that such a knowledge graph might correspond to, for example, the topology of a wireless sensor network, since the transmitter and the receiver in a sensor network need to know the existence of each other.

Starting from the current PAT Mechanism, we can further propose a new mechanism that requires each EU to report his participation decision to his neighbors. The new mechanism corresponds to a new game, which is a repeated game consisting of infinitely many repetitions of the current game studied in our paper. In this case, the EUs can play “trigger strategy” to punish free-riding neighbors [37, Chap 2.3]. With a properly designed punishment strategy, there exists a subgame perfect equilibrium, where every EU would always choose to participate in the mechanism until he detects a free-riding neighbor and would send a non-participation signal to the ET. Based on our current all-or-none scheme, upon receiving any non-participation signal, the ET would suspend transmitting wireless power to punish the free-riders. This means that when taking into his neighbors’ trigger strategies into account, no EU has the incentive to change from the trigger strategy to a free-riding strategy of keeping silent. Hence, assuming that each EU has at least one neighbor on the knowledge graph, it is possible to eliminate free-riders by exploiting the repeated interactions among the EUs. Such an assumption is much weaker than the assumption of the ET’s complete knowledge of the total number of EUs.

APPENDIX D

SUPPLEMENTARY SIMULATION RESULTS

In this section, we present supplementary simulation results. We further study the impacts of the probability of channel availability and the channel diversity.

D.1 Impact of Channel Availability

Figs. 11 and 12 illustrate the impact of $\text{Prob}_\text{A}$, the probability that one EU is available to harvest energy on a certain channel. As shown in Fig. 11 for both schemes, the achievable social welfare increases as $\text{Prob}_\text{A}$ becomes larger, since EUs can benefit more from the wireless power across different channels. Moreover, the performance gain of the MPAT Mechanism compared with the private good mechanism also increases in $\text{Prob}_\text{A}$. Specifically, when $\text{Prob}_\text{A} = 1$ and $P_{\text{max}} = 4$ W, the performance improvement is 160%. In Fig. 12, we observe a similar trend as in Fig. 11. That is, the achievable EUs’ average payoff improvement of the MPAT Mechanism compared with the private good mechanism also increases as $\text{Prob}_\text{A}$ increases.

D.2 Impact of Channel Diversity

We study the impact of channel diversity on the EUs’ average payoff. As shown in Fig. 13, EUs’ average payoff decreases as $r$
Fig. 14. Social welfare comparison with the PAT Mechanism and the bidding mechanism in [25].

increases. Moreover, the EUs’ average payoff gain of the MPAT Mechanism compared with the private good mechanism decreases in $r$. This also shows that the performance in terms of the EUs’ average payoff improvement of the MPAT Mechanism is also most significant when EUs have comparable channel gains.

D.3 Comparison with Bidding Mechanism in [25]

We compare the proposed PAT Mechanism with the bidding mechanism proposed in [25], we set $r = 5m$ and $CF_1 = 915$ MHz. In Fig. 14 we can see that the social welfare of both schemes increases in the number of EUs $K$. Moreover, when $K$ becomes larger, the performance gap between $P_{\text{max}} = 1W$ and $P_{\text{max}} = 4W$ also becomes larger for the PAT Mechanism. On the other hand, the maximum power constraint has almost no impact on the bidding mechanism. This is because as $K$ becomes larger, a larger $P_{\text{max}}$ can allow a larger transmit power that provides more benefits to more EUs in the PAT Mechanism. However, the bidding mechanism would incentivize the free-riders due to the similar reason under the benchmark mechanism. Hence, only one EU with the largest marginal utility will purchase power, and a larger $K$ does not significantly increase the demand.