Construction of the mathematical concept of pseudo thinking students

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Abstract. Thinking process is a process that begins with the acceptance of information, information processing and information calling in memory with structural changes that include concepts or knowledges. The concept or knowledge is individually constructed by each individual. While, students construct a mathematical concept, students may experience pseudo thinking. Pseudo thinking is a thinking process that results in an answer to a problem or construction to a concept “that is not true”. Pseudo thinking can be classified into two forms there are true pseudo and false pseudo. The construction of mathematical concepts in students of pseudo thinking should be immediately known because the error will have an impact on the next construction of mathematical concepts and to correct the errors it requires knowledge of the source of the error. Therefore, in this article will be discussed thinking process in constructing of mathematical concepts in students who experience pseudo thinking.

1. Introduction
The very interesting thing to discuss when learning math is how students construct mathematical concepts and solve math problems. Constructing mathematical concepts mean building mathematical knowledge through the attribution of one concept to another. The process of building knowledge in mathematics is done continuously so that it becomes a complete knowledge for students.

There are two possible outcomes when students construct mathematical concepts namely success or failure. The failed construction indicates the presence of student's difficulty in constructing mathematical concepts. These difficulties are often reflected in the form of mistakes made by the students. Such errors might be a visible results of the construction process rather than an output of actual mental activity [1]. The study of students' mistakes in constructing mathematical concepts has been largely undertaken by researchers [2,3,4].

Among students' mistake in constructing mathematical concepts is their tendency to take the form of pseudo thinking, the true pseudo thinking and false pseudo thinking [4]. The true pseudo thinking occurs when students get correct answers without being able to justify the answer. In other words, after being explored deeply, it turns out that what students think does not match the substance of the concept (wrong reasoning). The false pseudo thinking occurs when students give incorrect answers, but after undergoing the process of reflection, students are able to correct the answer and to reason correctly [4]. Research related to pseudo thinking have been largely undertaken by researchers [5,6,7]. Based on the results known that students who experience pseudo thinking needs to know the process
of thinking when solving math problems. This is due to pseudo thinking is a unreal thinking, so the correct answer may not necessarily result from the right thinking process and incorrect answers are not necessarily generated from the wrong thinking process [4]. Pseudo thinking is a thinking process that generates answers to problems or constructs of "incorrect" concepts [8]. The construction of concept does not represent the actual thought.

Pseudo thinking has been studied by many researchers using different terms and contexts. Pseudo analytic versus Pseudo conceptual in the context of routine mathematical problem solving [9]. Established Experience (EE) versus Plausible Reasoning (PR) in the context of problem solving non routine [10]. Direct Translation Approach (DTA) versus Meaning Based Approach (MBA) in the context of solving word problems [11]. Dual Process Theory of Kahneman (Process System 1 versus Process System 2) in the context of solving algebra problems [12].

The results of preliminary study conducted by researchers by looking at the results of students' task when completing the daily test of straight line equation material in grade VIII E at SMP N 16 Surakarta. When the researcher did the interview to ask how student got the answer, he or she still did not understand what he wrote correctly. It shows that the student has not understood what he or she wrote correctly. This is called the pseudo thinking process, a state in which students do not really use their minds to solve math problems. In other words, the construction process undertaken by students in solving the mathematical problems is not in accordance with the mathematical concepts requested on the matter.

The pseudo-thinking process of constructing mathematical concepts needs to be known, because the result of concept construction do not match the students 'mental activity (students' wrong reasoning). Students' mistakes in constructing mathematical concepts in students experiencing pseudo thinking need to get some attentions, because if it isn’t resolved then the error will have an impact on the construction of subsequent mathematical concepts. This is due to the mathematical concept that one has relevance with other mathematical concepts. Therefore, in this research will be described pseudo thinking process on the equation of straight-line equation.

2. Research Method
This research is a qualitative research. In this research will be described pseudo thinking process in constructing mathematical concepts on straight-line equation. This study involved ten junior high school students of 8th grade. The instruments used in this research are the task sheets and interviews. Data collection techniques in this study were conducted with think aloud method, where students were given a task sheet to be completed, students were asked to solve problems in front of the researcher, and in the process of solving the problem, the students revealed what they were thinking. To obtain the data validation techniques in this research, one of the steps used is triangulation method. Triangulation method is done by comparing the result of student task and interview result.

In this study, to obtain the subject of research, the researcher gives the task sheet to the students. Further, the researchers classified the students' answers into groups of correct answers and incorrect answers based on the final answers the students gave on the completion sheet. Students who involved in this study were students who experience true pseudo thinking and false pseudo thinking. To determine which students are experiencing true pseudo thinking, the students with correct answer are asked to justify the answer, if the student is unable to justify the answer, the student is chosen to be the research subject (true pseudo thinking). Meanwhile, to determine students who experience false pseudo thinking. The students with incorrect answer are given the opportunity to reflect or re-examine the work. If after the reflection, students with incorrect answer are able to correct the answer to be the correct answer, the students with incorrect answer are chosen to be the subject of research (false pseudo thinking).

3. Result and Discussion
In this research, there were ten students of 8th grade in junior high school required to complete the task sheet of straight-line equation material. The results are categorized as presented in table 1.
Table 1. Categories of Students’ Job Results

| Category           | Students |
|--------------------|----------|
| Pseudo thinking    | 7        |
| Totally right      | 1        |
| Totally wrong      | 2        |

Based on table 1, it appears that there were still many students who experience pseudo thinking, as many as 7 students out of 10 (70%). However, in this article the description of students’ pseudo thinking process in constructing mathematical concepts was focused on student answers when determining the gradient and determining the equation of a straight line through two points.

Figure 1 is a picture of the students’ work in determining the gradient that indicates that the student was in true pseudo thinking.

![Figure 1. The Student's Answer Determines The Gradient](image)

When determining the gradient, subjects illustrate the gradient formula which is $\frac{a}{b}$. Furthermore, researcher conducted conversations with the students to explore and clarify the thinking process of students in determining the gradient. The dialogue between researcher and students are shown below:

| Researcher : Are you sure that the formula for determining the straight line gradient is $\frac{a}{b}$? |
| Student : Yes, miss ... |
| Researcher : What is the reason? |
| Student : yaaa, because usually illustrated like that by my teacher, miss .. |
| Researcher : Hmm ..., $a$ and $b$ what does that mean? |
| Student : $a$ and $b$ are variables, miss |

Researcher continued the dialogue with students by asking questions that led to cognitive conflict.

| Researcher : Do you know the form of the straight line equation $y = mx + c$? |
| Student : yes miss, it's a common form of straight line equation. |
| Researcher : Then, what is the gradient of the equation $y = mx + c$? |
| Student : Yes, it is clear that the gradient is '$m$' |
| Researcher : What is '$m$'? |
| Student : $m$ is gradient, miss |
| Researcher : I mean, what is '$m$'? is it variable or the coefficient of $x$? |
| Student : (pause) '$m$' is a variable, miss.. |
Researcher continued the question

| Researcher : Is \( ax + by + c = 0 \) a straight-line equation? |
|---------------------------------------------------------------|
| Student : Yes miss, it is a straight-line equation other than a general form. |
| Researcher : Then what's the gradient? |
| Student : (The student replied quickly) \( \frac{-a}{b} \) miss |
| Researcher : Why is it \( \frac{-a}{b} \)? |
| Student : Yaa.., because my teacher used to teach that the formula determines the gradient is \( \frac{-a}{b} \) (student was unable to give a reason).

Based on the dialogue, it can be inferred that student experienced true pseudo thinking. Student was able to give a correct answer when asked to determine the gradient of the straight-line equation, but after undergoing further thinking, it turns out what the student thought was not in accordance with the substance of the concept in determining the gradient. The reason of the students in determining the gradient is to use the formula \( \frac{-a}{b} \) by looking at \( a \) and \( b \) as variable. The constructs \( a \) and \( b \) as variables represent one form of error because \( a \) and \( b \) (in the context of a straight-line equation) not the type of variable but the coefficients of \( x \) and \( y \). Furthermore, based on the results of the interview, the student's mistake occurred also happened when the student declared \( m \) as variable and the student was unable to give the reason why the gradient of the straight-line equation was \( ax + by + c = 0 \) is \( \frac{-a}{b} \). Therefore, the concept of construction in determining the gradient was not intact or what is commonly called as the construction hole. The construction hole is a hole in the student's thinking structure caused by an incomplete concept construction process [1]. The concept construction process that determines the gradient of students with true pseudo thinking is presented in figure 2.

Fault of true pseudo thinking process in constructing the concept occurs because students just "imitate" procedures done by the teacher often without knowing the reason why such procedures are used. As the result, students' reasoning in constructing mathematical concepts does not develop optimally as they assume that to solve the problem, they can simply choose the procedure of completion in accordance with the problems given previously.

Furthermore, false pseudo thinking occurs when students were asked to determine the equation of a straight-line through two known points. The following results of the student's work in solving the problem of determining the equation of the straight-line through the points A (3, -2) and point B (6.0) are presented in figure 2.

![Figure 2. The Student's Answer Determines The Equation of A Straight-Line Through Two Points](image-url)
Based on the results of the answers given by students, students are experiencing the process of System 1 (S1) which is fast, automatic, without effort, unconscious, and inflexible [12]. This can be seen from the student's answer when determining the gradient through two points until it finally determines the equation of a straight-line through two points. Student also experienced the process of thinking "fuzzy memory" or remembering vaguely [9]. Student remembered that determining the equation of a straight-line through two points can be done by determining the gradient of the line first, but the student’s memory is only vague. Therefore, the answers produced were incorrect. After the reflection student was able to improve the answer to be the correct answer. In other words, student experienced a false pseudo thinking. A dialogue showing that student experienced false pseudo thinking is presented below.

| Researcher : Why do you define a gradient through two points using the formula \(-\frac{y_2 - y_1}{x_2 - x_1}\) ? |
|---------------------------------------------------------------|
| Student : The formula determines the gradient through two points is \(-\frac{y_2 - y_1}{x_2 - x_1}\). |
| Researcher : Are you sure of your answer ? |
| Student : (pause) may I try again, miss? |
| Researcher : Please ... |
| Student : Determining the gradient through two points means \(m = \frac{\text{selisih ordinat A dan E}}{\text{selisih absis A dan B}}\) or |

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**Figure 3. Construction Process with The Concept of True Pseudo Thinking**

The equation is known in the general form \(y = mx + c\). Its gradient \(m\)

Given the equation other than the general form that is \(ax+by+c = 0\)

Its gradient \(-\frac{a}{b}\)

**Determines gradient from** \(ax+by+c = 0\), **Change the equation** \(ax+by+c = 0\) **into** \(y = mx + c\).

\[
\begin{align*}
\leftrightarrow & \quad by = -ax - c \\
\leftrightarrow & \quad y = \frac{-ax - c}{b} \\
\leftrightarrow & \quad y = -\frac{a}{b} x - \frac{c}{b}
\end{align*}
\]

Therefore, the equation \(ax + by + c = 0\) can be changed to \(y = -\frac{a}{b} x - \frac{c}{b}\). So the gradient of \(ax+by+c = 0\) is \(-\frac{a}{b}\).
The researcher continued the interview with the student

**Researcher:** Why, in determining the equation of the straight-line, do you should use the equation of line $y = mx + c$? Can you explain?

**Student:** Ehmmm .... because it is the equation of the line that I know, ma'am...

because straight-line equation material mmmm... it is already... I was looking and on the matter which shows that the line through two points namely point A (3, -2) and B (6,0). So, I can choose one of the points to use in determining the equation of the straight-line. Next, I substitute the value of $m$ and substitute one of the points of point A (3, -2) into the equation $y = mx + c$, to obtain the value of $c$. When the value of $c$ has been obtained it will obtain a straight-line equation.

**Researcher:** Hmm..., Did you forget about the concept of determining the equation of a straight line through two known points?

**Student:** (With a smile) Yes miss, I forgot that. Therefore, I first determined the gradient and then used the equation of line $y = mx + c$.

**Researcher:** Do you remember the concept of determining the equation of a straight-line through a known point?

**Student:** (While recalling the formula) If I'm not mistaken, the formula is $y - y_1 = m(x - x_1)$. Is that right miss?

**Researcher:** That is right. Do you think the formula $y - y_1 = m(x - x_1)$ can be used to determine the equation of a straight-line through two points?

**Student:** (Pause for a moment) Seems to be like that, ma’am ... May I try it again?

**Researcher:** Please...

**Student:** $y - y_1 = m(x - x_1)$. The formula to determine the gradient is $m = \frac{y_2 - y_1}{x_2 - x_1}$. Substitute the value of $m$ to the equation $y - y_1 = m(x - x_1)$. Therefore, we obtain $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$. As on the matter, note that the line through two points, which is point A (3, -2) and B (6, 0). So, we assume that point A $(x_1, y_1)$ and point B $(x_2, y_2)$. Substitute point A and point B into the equation $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ (students fix the answer to $y - (-2) = \frac{0-(-2)}{6-3}(x - 3)$).

So, The equation of straight-line that obtained is $y + 2 = \frac{2}{3}(x - 3)$ ataup $y = \frac{2}{3}x - 4$. Is it correct, miss?

**Researcher:** Yes, it is.

Based on the dialogue it is known that the student was actually able to correctly answer the problem, but because the student's thinking process was vague in constructing the concept of determining the equation of the straight-line, the answer was wrong. This is the student's answer after do reflection presented in figure 4.

False pseudo thinking process was spontaneous, without any reflection, and it was used to infer the answer also known as Direct Translated Approach thinking process [11] and thinking processes of students was also fast and unconscious [12]. In fact, when reflection was done, the student was able to correct the answer, in other words the students experienced a false pseudo thinking.
The concept of construction process of determining the equation of a straight-line through two points done by the student with false pseudo thinking is presented in figure 5.

The use of the pseudo-thinking process in constructing a mathematical concept was wrong because student's thinking processes were spontaneous, losing control on what he thought or did, and such process gave the vague procedure. Therefore, the student's answer was incorrect. However, after the reflection, student was able to correct the answer.

4. Conclusion
Based on the results of the research, students 'mistakes in constructing mathematical concepts of students experiencing pseudo thinking occur because the students' thought processes are memorizing, spontaneous, not controlling what they think or doing, and remembering the procedures that occur
vaguely. Consequently, mathematical concepts constructed by students are not intact or are called construction holes. Students’ mistakes in constructing mathematical concepts in students experiencing pseudo thinking need attention, because if not addressed then the error will have an impact on the construction of later mathematical concepts. This is because the mathematical concept that one has a relationship with other mathematical concepts.

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