Monopole giant resonances and nuclear compressibility in relativistic mean field theory

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Abstract

Isoscalar and isovector monopole oscillations that correspond to giant resonances in spherical nuclei are described in the framework of time-dependent relativistic mean-field (RMF) theory. Excitation energies and the structure of eigenmodes are determined from a Fourier analysis of dynamical monopole moments and densities. The generator coordinate method, with generating functions that are solutions of constrained RMF calculations, is also used to calculate excitation energies and transition densities of giant monopole states. Calculations are performed with effective interactions which differ in their prediction of the nuclear matter compression modulus $K_{\text{nm}}$. Both time-dependent and constrained RMF results indicate that empirical GMR energies are best reproduced by an effective force with $K_{\text{nm}} \approx 270$ MeV.
1 Introduction

The nuclear matter compression modulus $K_{nm}$ is an important quantity in the description of properties of nuclei, supernovae explosions, neutron stars, and heavy ion collisions. In principle the value of $K_{nm}$ can be extracted from the experimental energies of isoscalar monopole vibrations (giant monopole resonances GMR) in nuclei. In a semi empirical macroscopic approach, the analysis is based upon a leptodermous expansion of the compression modulus of a nucleus, analogous to the liquid drop mass formula [1]. In principle such an expansion provides a “model independent” determination of $K_{nm}$. However, the macroscopic approach presents several ambiguities. The formula itself is based on the assumption that the breathing mode is a small amplitude vibration. More important, to correctly interpret the value of the volume term in the expansion, one has to assume a certain mode of vibration. It has been argued that a direct determination of the various contributions to the compression modulus through a fit of the breathing mode frequencies cannot provide an accurate value for $K_{nm}$. Recently it was shown by Shlomo and Youngblood [2] that the complete experimental data set on isoscalar monopole giant resonances does not limit the range of $K_{nm}$ to better than $200 - 350$ MeV. Microscopic calculations of GMR excitation energies might provide a more reliable approach to the determination of the nuclear matter compression modulus. One starts by constructing sets of effective interactions which differ mostly by their prediction for $K_{nm}$, but otherwise reproduce reasonably well experimental data on nuclear properties. Static properties of the ground state are calculated in the self-consistent mean field approximation, and RPA calculations are performed for the excitations [3, 4]. Non relativistic Hartree-Fock plus RPA calculations, using both Skyrme and Gogny effective interactions, indicate that the value of $K_{nm}$ should be in the range $210-220$ MeV. The breathing-mode giant monopole resonances...
have also been studied within the framework of the relativistic mean field (RMF) theory, using the generator coordinate method (GCM) [5]. It was shown that the GCM succeeds in describing the GMR energies in nuclei and that the empirical breathing mode energies of heavy nuclei can be reproduced by effective forces with $K_{\text{nn}} \approx 300$ MeV in the RMF theory.

In the present article we describe isoscalar and isovector monopole oscillations in spherical nuclei in the framework of time dependent relativistic mean field theory (RMFT). The model represents a relativistic generalization of the time-dependent Hartree-Fock approach. Nuclear dynamics is described by the simultaneous evolution of a single particle wave-functions in the time-dependent mean fields. Frequencies of eigenmodes are determined from a Fourier analysis of dynamical quantities. In this microscopic description, self-consistent mean-field calculations are performed for static ground-state properties, and time-dependent calculations for monopole excitations using the same parameter sets of the Lagrangian. A basic advantage of the time dependent model is that no assumption about the nature of the mode of vibrations has to be made. Another approach that goes beyond the HF+RPA approximation is provided by the Generator Coordinate Method. In the second part of the article we extend the model of Ref. [5], and calculate the excitation energies of giant monopole states with relativistic GCM. As generating functions we use products of Slater determinants, built from single-particle solutions of constrained RMF calculations, and coherent states that represent the meson fields.

The article is organized as follows. In Sec. 2 we present the essential features of the time-dependent relativistic mean-field model, and some details of its application to spherical nuclei. Results of time-dependent calculations for a series of doubly closed-shell nuclei and a set of effective interactions are discussed in Sec. 3. In Sec. 4 details of the relativistic GCM are explained. Sec. 5 contains a discussion of the systematics of energies
of giant monopole states that result from constrained RMF calculations with GCM. A summary of our results is presented in Sec. 6.

2 The time-dependent relativistic mean-field model

The dynamics of collective motion in nuclei is described in the framework of relativistic mean-field theory \[3, 4, 5, 8, 9, 10\]. Details of the time-dependent model are given in Refs. \[11, 12\]. Here we only outline its essential features. In relativistic quantum hadrodynamics the nucleons, described as Dirac particles, are coupled to exchange mesons and photons through an effective Lagrangian. The model is based on the one boson exchange description of the nucleon-nucleon interaction. The Lagrangian density of the model is given as

\[
\mathcal{L} = \bar{\psi} (i \gamma \cdot \partial - m) \psi + \frac{1}{2} (\partial \sigma)^2 - U(\sigma) \\
- \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^2 - \frac{1}{4} \tilde{R}_{\mu\nu} \tilde{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
- g_\sigma \bar{\psi} \sigma \psi - g_\omega \bar{\psi} \gamma \cdot \omega \psi - g_\rho \bar{\psi} \gamma \cdot \rho \tau \psi - e \bar{\psi} \gamma \cdot A \frac{1 - \tau_3}{2} \psi.
\]

(1)

The Dirac spinor \(\psi\) denotes the nucleon with mass \(m\). \(m_\sigma\), \(m_\omega\), and \(m_\rho\) are the masses of the \(\sigma\)-meson, the \(\omega\)-meson, and the \(\rho\)-meson, and \(g_\sigma\), \(g_\omega\), and \(g_\rho\) are the corresponding coupling constants for the mesons to the nucleon. \(U(\sigma)\) denotes the nonlinear \(\sigma\) self-interaction

\[
U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4,
\]

(2)

and \(\Omega^{\mu\nu}, \tilde{R}^{\mu\nu},\) and \(F^{\mu\nu}\) are field tensors \([8]\).

From the Lagrangian density \([11]\) the coupled equations of motion are derived, the Dirac equation for the nucleons:

\[
i \partial_t \psi_i = \left[ \alpha \left( -i \nabla - g_\omega \omega - g_\rho \tilde{\rho} \cdot \vec{A} - e \frac{1 - \tau_3}{2} A \right) + \beta (m + g_\sigma \sigma) \right] \psi_i.
\]

4
\[ + g_\omega \omega_0 + g_\rho \vec{\tau} \vec{\rho}_0 + e \frac{(1 - \tau_3)}{2} A_0 \right] \psi_i \]

and the Klein-Gordon equations for the mesons:

\[
\left( \partial_t^2 - \Delta + m^2_\sigma \right) \sigma = -g_\sigma \rho_s - g_2 \sigma^2 - g_3 \sigma^3 \]  

(4)

\[
\left( \partial_t^2 - \Delta + m^2_\omega \right) \omega_\mu = g_\omega j_\mu \]  

(5)

\[
\left( \partial_t^2 - \Delta + m^2_\rho \right) \vec{\rho}_\mu = g_\rho \vec{j}_\mu \]  

(6)

\[
\left( \partial_t^2 - \Delta \right) A_\mu = e j_{em}^\mu . \]  

(7)

In the relativistic mean field approximation the nucleons, described by a Slater determinant \( |\Phi\rangle \) of single-particle spinors \( \psi_i \), \( (i = 1, 2, ..., A) \), move independently in the classical meson fields. The sources of the fields are calculated in the no-sea approximation [12]:

- the scalar density

\[
\rho_s = \sum_{i=1}^{A} \bar{\psi}_i \psi_i, \]  

(8)

- the isoscalar baryon current

\[
j^\mu = \sum_{i=1}^{A} \bar{\psi}_i \gamma^\mu \psi_i, \]  

(9)

- the isovector baryon current

\[
\vec{j}^\mu = \sum_{i=1}^{A} \bar{\psi}_i \gamma^\mu \vec{\tau} \psi_i, \]  

(10)

- the electromagnetic current for the photon-field

\[
j_{em}^\mu = \sum_{i=1}^{A} \bar{\psi}_i \gamma^\mu \frac{1 - \tau_3}{2} \psi_i, \]  

(11)

where the summation is over all occupied states in the Slater determinant \( |\Phi\rangle \). Negative-energy states do not contribute to the densities in the no-sea approximation of the stationary solutions. However, negative energy contributions are included implicitly in the time-dependent calculation, since the Dirac equation is solved at each step in time for a different basis set. Negative energy components with respect to the original ground state
basis are taken into account automatically, even if at each time the no-sea approximation is applied. It is also assumed that nucleon single-particle states do not mix isospin. Because of charge conservation, only the 3-component of the isovector $\vec{\rho}$ contributes.

The ground state of a nucleus is described by the stationary solution of the coupled system of equations (3)–(7). It specifies part of the initial conditions for the time-dependent problem. In the present work we consider doubly closed-shell nuclei, i.e. nuclei that are spherical in the ground state. The nucleon spinor is in this case characterized by the angular momentum $j$, its $z$-projection $m$, the parity $\pi$, and the isospin $t_3 = \pm \frac{1}{2}$ for neutrons and protons [9]

$$\psi(r, t, s, t_3) = \frac{1}{r} \left( f(r)\Phi_{ljm}(\theta, \varphi, s) \right) e^{-iEt} \chi_{\tau}(t_3). \quad (12)$$

$\chi_{\tau}$ is the isospin function, the orbital angular momenta $l$ and $\tilde{l}$ are determined by $j$ and the parity $\pi$, $f(r)$ and $g(r)$ are radial functions, and $\Phi_{ljm}$ is the tensor product of the orbital and spin functions

$$\Phi_{ljm}(\theta, \varphi, s) = \sum_{m_s m_l} \frac{1}{2} m_l | j m \rangle Y_{lm}(\theta, \varphi) \chi_{m_s}(s). \quad (13)$$

For a given set of initial conditions, i.e. initial values for the densities and currents in Eqs. (8–11), the model describes the time evolution of a single particle wave-functions in the time-dependent mean fields. The description of nuclear dynamics as a time-dependent initial-value problem is intrinsically semi-classical, since there is no systematic procedure to derive the initial conditions that characterize the motion of a specific mode of the nuclear system. In principle the theory is quantized by the requirement that there exist time-periodic solutions of the equations of motion, which give integer multiples of Planck’s constant for the classical action along one period [13]. For giant resonances the time-dependence of collective dynamical quantities is actually not periodic, since generally giant resonances are not stationary states of the mean-field Hamiltonian. The coupling
of the mean-field to the particle continuum allows for the decay of giant resonances by direct escape of particles. In the limit of small amplitude oscillations, however, the energy obtained from the frequency of the oscillation coincides with the excitation energy of the collective state. In Refs. [12, 14, 13] we have studied oscillations of isovector dipole, isovector spin-dipole, isoscalar quadrupole, and isovector quadrupole character, which correspond to giant resonances in spherical nuclei. In Ref. [13] double giant isovector dipole and double giant isoscalar quadrupole resonances have been described in the framework of the time-dependent RMF theory and compared with recent experimental results. The model reproduces reasonably well the experimental data on energies and, for light nuclei, the widths of giant resonances. All the results are obtained using parameter sets that reproduce ground-state properties of nuclei, i.e. no new parameters are introduced in the model to specifically describe giant resonances. In the present study we apply the model to isoscalar and isovector monopole oscillations in spherical nuclei. In this microscopic description, self-consistent mean-field calculations are performed for static ground-state properties, and time-dependent calculations for monopole excitations. Using sets of effective interactions which differ mostly by their prediction of nuclear matter compressibility, and which otherwise provide reasonable results for ground state properties, we calculate the excitation energies of monopole resonances in a series of spherical nuclei.

In order to excite monopole oscillations in a doubly closed-shell nucleus, the spherical solution for the ground-state has to be initially compressed or radially expanded by scaling the radial coordinate. The amplitudes $f^{\text{mon}}$ and $g^{\text{mon}}$ of the Dirac spinor are defined

$$f^{\text{mon}}(r^{\text{mon}}) = (1 + a) \, f(r), \quad g^{\text{mon}}(r^{\text{mon}}) = (1 + a) \, g(r).$$  \hspace{1cm} (14)$$

The new coordinates are

$$r^{\text{mon}} = (1 + a) \, r.$$  \hspace{1cm} (15)
A reasonable choice for the parameter is $|a| \approx 0.05 - 0.1$. The energy transferred in this way to the ground state of the nucleus is somewhat above the giant resonance energy, and the resulting oscillations do not show anharmonicities associated with large amplitude motion. For the case of isoscalar oscillations the monopole deformations of the proton and neutron densities have the same sign. To excite isovector oscillations, the initial monopole deformation parameters of protons and neutrons must have opposite signs. After the initial monopole deformation, the proton and neutron densities have to be normalized.

It should also be emphasized that apart from the fact that we concentrate on monopole excitations no assumption about the radial nature of the mode of vibrations is made in the time-dependent calculation. We do not have to assume that the motion is adiabatic, nor that the mode corresponds to a scaling of the density. The frequency dependence of dynamical quantities and the transition densities are used to determine the structure of the eigenmodes. The frequency can be simply related to nuclear compressibility only if a single compression mode dominates.

For the case of spherical symmetry, the time-dependent Dirac equation (3) reduces to a set of coupled first-order partial differential equations for the complex amplitudes $f$ and $g$ of proton and neutron states

$$
\begin{align*}
\text{i} \partial_t f &= (V_0 + g_\sigma \sigma) f + (\partial_r - \frac{\kappa}{r} - iV_r) g \\
\text{i} \partial_t g &= (V_0 - g_\sigma \sigma - 2m) g - (\partial_r + \frac{\kappa}{r} - iV_r) f,
\end{align*}
$$

where $\kappa = \pm (j + \frac{1}{2})$ for $j = l \pm \frac{1}{2}$, and the indices 0 and $r$ denote the time and radial components of the vector field

$$
V^\mu = g_\omega \omega^\mu + g_\rho \rho^\mu + e \frac{(1 - \tau_3)}{2} A^\mu.
$$

For a given set of initial conditions, the equations of motion propagate the nuclear system.
in time. The potentials are solutions of the Klein-Gordon equations

\[
\left( -\frac{\partial^2}{\partial r^2} - \frac{2}{r} \frac{\partial}{\partial r} + m_\phi^2 \right) \phi(r) = s_\phi(r),
\]  

(19)

\( m_\phi \) are meson masses for \( \phi = \sigma, \omega \) and \( \rho \), and zero for the photon. The source terms are calculated from (8)–(11) using in each time step the latest values for the nucleon amplitudes. Retardation effects for the meson fields are not included, i.e. the time derivatives \( \partial_t^2 \) in the equations of motions for the meson fields are neglected. This is justified by the large masses in the meson propagators causing a short range of the corresponding meson exchange forces. At each step in time the meson fields and electromagnetic potentials are calculated from

\[
\phi(r) = \int_0^\infty G_{\phi}(r, r') s_\phi(r') r'^2 dr',
\]  

(20)

where for massive fields

\[
G_{\phi}(r, r') = \frac{1}{2m_\phi} \frac{1}{rr'} \left( e^{m_\phi |r-r'|} - e^{-m_\phi |r+r'|} \right),
\]  

(21)

and for the Coulomb field

\[
G_C(r, r') = \begin{cases} 
\frac{1}{r} & \text{for } r > r' \\
\frac{1}{r'} & \text{for } r < r'.
\end{cases}
\]  

(22)

The dynamical variables that characterize vibrations of a nucleus are defined as expectation values of single-particle operators in the time-dependent Slater determinant \( |\Phi(t)\rangle \) of occupied states. In the framework of the TDRMF model the wave-function of the nuclear system is a Slater determinant at all times. For isoscalar monopole vibrations, the time-dependent monopole moment is simply defined as:

\[
\langle r^2(t) \rangle = \frac{1}{A} \langle \Phi(t) | r^2 | \Phi(t) \rangle.
\]  

(23)
The corresponding Fourier transform determines the frequencies of eigenmodes. The transition density for a specific mode of oscillations is defined as the Fourier transform of the time-dependent baryon density at the corresponding frequency. The effective compression modulus $K_A$ for a nucleus of mass number $A$ is defined as

$$E_0 = \sqrt{\frac{\hbar^2 A K_A}{m < r^2 >_0}},$$

(24)

where $E_0$ is the energy of the isoscalar giant monopole resonance, $m$ is the nucleon mass, and $< r^2 >_0$ denotes the expectation value of $r^2$ in the unperturbed ground state. As in Ref. [4], Eq. (24) represents just a convenient parameterization of the dominant $A$-dependence for the energy of the giant monopole state.

3 Monopole oscillations in spherical nuclei

The study of isoscalar monopole resonances in nuclei provides an important source of information on the nuclear matter compressibility. We first consider three spherical nuclei from different regions of the periodic table: $^{40}$Ca, $^{90}$Zr and $^{208}$Pb. These nuclei differ not only in their masses, but also in the ratio of proton to neutron number. The complete experimental data set on isoscalar monopole giant resonances has been recently analyzed by Shlomo and Youngblood [2]. Using a semi empirical macroscopic approach to deduce from the systematics of GMR the systematics of nuclear compressibility, and to extrapolate the data to infinite nuclear matter, they have shown that the complete data set does not limit the range of $K_{nm}$ to better than 200 – 350 MeV. The GMR energy in $^{208}$Pb is rather well established at $13.7 \pm 0.3$ MeV. The average GMR energy for $^{90}$Zr as deduced from various experiments is around 16.5 MeV, and for $^{40}$Ca the value of the excitation energy adopted in the calculation was $18 \pm 1$ MeV [2]. In light nuclei, of course, not all monopole strength is seen.
We have performed time-dependent relativistic mean-field calculations for six sets of Lagrangian parameters: NL1 [15], NL3 [16], NL-SH [17], NL2 [18], HS [19], and L1 [20] (in order of increasing value of the nuclear matter compression modulus $K_{\text{nm}}$). The values of the parameters are given in Table 1. These effective forces have been frequently used to calculate properties of nuclear matter and of finite nuclei. In this respect our work parallels the generator coordinate calculations of giant monopole resonances and the constrained compressibility within the relativistic mean-field model of Ref. [5], and the non-relativistic self-consistent mean-field plus RPA calculation of nuclear compressibility with Gogny effective interactions of Ref. [4]. The idea is to restrict the possible values of the nuclear matter compression modulus, on the basis of the excitation energies of giant monopole states calculated with different effective interactions. In addition to the four non-linear sets NL1, NL3, NL-SH and NL2, we also include two older linear parameterizations HS and L1, which do not include the self-coupling of the sigma field. The sets NL1, NL-SH and NL2 have been extensively used in the description of properties of finite nuclei [9, 10].

In order to bridge the gap between NL1 ($K_{\text{nm}} = 211.7$ MeV), and NL-SH ($K_{\text{nm}} = 355.0$ MeV), we have also included a new effective interaction NL3 ($K_{\text{nm}} = 271.8$ MeV). This new parameter set has been derived recently [16] by fitting ground state properties of a large number of spherical nuclei, as well as nuclear matter properties. It appears that the NL3 effective interaction reproduces experimental data better than, for instance, NL1 or NL-SH.

Results of model calculations for $^{40}\text{Ca}$ are displayed in Figs. 1a and 1b: time-dependent monopole moments and the corresponding Fourier power spectra are shown. Time is measured in units of fm/$c$, and the energy $E = \hbar \omega$ in MeV. The numerical accuracy is $\Delta E = 2\pi \hbar c/T_{\text{final}} = 2\pi \hbar c/1000\text{fm} \approx 1.2$ MeV. The frequency of the isoscalar monopole oscillations generally increases with the nuclear matter compression modulus,
and the vibrations become more anharmonic. For the first three sets of parameters (NL1, NL3, and NL-SH) a single mode dominates. For NL2, HS and L1 the Fourier spectra are fragmented. The NL1 effective interaction reproduces especially well the ground-state properties of nuclei close to the stability line [9], and it appears that the frequency of monopole oscillations for the NL1 parameter set is very close to the expected experimental excitation energy of $18 \pm 1\text{ MeV}$.

In Figs. 2a and 2b the results for $^{90}\text{Zr}$ are shown. The frequencies increase with $K_{nm}$, the motion is more anharmonic and more damped. The damping of the monopole moments comes from the coupling to the continuum, and there is also a contribution from the damping via the mean-field (Landau damping). The experimental GMR excitation energy for $^{90}\text{Zr}$ of 16.5 MeV is found between the values calculated for the NL1 and NL3 parameter sets. For NL1 (15.6 MeV), NL3 (19.4 MeV), NL-SH (21.1 MeV) and the lower frequency of NL2 (21.6 MeV) we display in Fig. 3 the corresponding transition densities. For NL1, NL3 and NL-SH the transition densities show a radial dependence characteristic for the “breathing” mode: a change of the density in the volume at the expense of that on the surface. The details depend on the underlying shell structure. An unusual radial dependence is found for the NL2 parameterization. The transition density, both for protons and neutrons, has a minimum at $r = 0$, of the same sign as on the surface. The monopole oscillations at the center of the nucleus are in phase with surface oscillations. It seems that a similar behavior is found for HS and L1, although due to fragmentation it is difficult to decide at which frequency to plot the transition density.

In Ref. [4] it was stressed that, rather than on the systematics over the whole periodic table, the determination of the nuclear compressibility relies more on a good measurement and microscopic calculations of GMR in a single heavy nucleus such as $^{208}\text{Pb}$. The results of TDRMF calculations for $^{208}\text{Pb}$ are presented in Figs. 4-7. In Figs. 4a and 4b we
display the monopole moments and their Fourier power spectra. As one would expect for a heavy nucleus, there is very little fragmentation and a single mode dominates for all parameter sets. The experimental excitation energy $13.7 \pm 0.3$ MeV is very close to the frequency of oscillations obtained with the NL3 parameter set: 14.1 MeV. As in the case of $^{90}$Zr, the calculated excitation energy for the NL1 parameter set ($K_{nm} = 211.7$ MeV), is approximately 1 MeV lower than the average experimental value.

The effective compression modulus $K_A$ of $^{208}$Pb is displayed in Fig. 5 as a function of $K_{nm}$. The observed behavior of $K_A$ is almost identical to the constrained compressibility calculated in the relativistic mean-field model using the generator coordinate method [5]. The deviation from the almost linear behavior, which is observed for the HS linear parameter set, is slightly more pronounced in the present time-dependent calculation.

The transition densities that correspond to the main peaks calculated for the six effective interactions are displayed in Figs. 6a and 6b. In the left columns we plot the dynamical transition densities, calculated as Fourier transforms of the time-dependent baryon densities. On the right hand sides the scaling densities are shown [3]

\[
\rho_S^{(p)}(r) = 3\rho_0^{(p)} + r \frac{d}{dr}\rho_0^{(p)}(r),
\]

where $\rho_0^{(p)}$ denotes the ground-state vector density for protons (neutrons). In the scaling model the transition densities follow from a simple radial scaling of the ground-state density, both the central density and the surface thickness vary. We notice that the scaling transition densities are almost identical, with a possible exception for L1, for all effective interactions. They do not provide any information about the dynamics of isoscalar monopole vibrations. On the other hand, the radial shape of the transition densities that result from time-dependent calculations depends very much on the value
of the nuclear matter compression modulus. As we have already seen for $^{90}$Zr, starting with NL2 ($K_{\text{nm}} = 399.2$ MeV), a minimum develops in the center of the nucleus. It also appears that the oscillations of proton and neutron densities are out of phase at $r = 0$. Thus not only does the frequency of oscillations increase with $K_{\text{nm}}$, but also the radial dependence of the density oscillations changes dramatically.

In order to understand better this behavior, we separate in Fig. 7 the volume and surface contributions to the transition densities. Following the procedure of Ref. [3] we define a velocity field associated with the collective motion

$$u(r) = -\frac{r}{r^3 \rho_0(r)} \int_0^r r'^2 \rho_T(r', E_0) dr'$$

where $\rho_0$ is the ground-state density, and $\rho_T(r', E_0)$ denotes the transition densities shown in Fig. 6. The transition density is separated into two components

$$\rho_T^\text{vol}(r, E_0) = \rho_0 \nabla u = \rho_0 \frac{1}{r^2} \frac{d}{dr}(r^2 u)$$

$$\rho_T^\text{surf}(r, E_0) = u \nabla \rho_0 = u \frac{\rho_0}{dr}.$$ 

The resulting volume and surface transition densities are shown in Fig. 7 for all six effective interactions. We notice that the surface contribution does not depend much on the parameter set used, that is, on the nuclear matter incompressibility. The volume transition density, as one would expect, is very sensitive to the value of $K_{\text{nm}}$. A very interesting phenomenon is the formation of standing waves in the bulk. It starts already for NL2, but is clearly observed for HS and L1.

The effective interactions NL1 and NL3 seem to produce GMR excitation energies which are quite close to the experimental values. We have therefore calculated, for these two parameter sets, the isoscalar giant monopole resonances in a number of doubly closed-shell nuclei: $^{56}$Ni, $^{100,132}$Sn, $^{122}$Zr, $^{146}$Gd. The results, together with those already
discussed, are shown in Fig. 8. The energies of giant monopole states are determined from the Fourier spectra of the time-dependent monopole moments, and are displayed as function of the mass number. We notice that the NL1 excitation energies are systematically lower, but that otherwise the two effective interactions produce very similar dependence on the mass number. The empirical curve $E_x \approx 80 A^{-1/3} \text{MeV}$ is also included in the figure, and it follows very closely the excitation energies calculated with the NL3 parameter set.

Experimental data on isovector giant monopole resonances are much less known. The systematics of excitation energies will not, in general, depend on the nuclear matter compression modulus. More likely, energies will depend on the coefficient of asymmetry energy. Isovector excitations are therefore outside the main topic of the present study. Nevertheless we have calculated the eigenfrequencies of isovector monopole modes, and compared them with available data on energies of giant monopole resonances. The time-dependent isovector monopole moments

$$<r_p^2> - <r_n^2>$$

and the corresponding Fourier spectra for $^{208}\text{Pb}$ are displayed in Fig. 9. Compared to the isoscalar vibrations (Fig. 4), we notice that the isovector oscillations are more damped and the anharmonicities are more pronounced. Consequently, the Fourier spectra are more fragmented. In each figure only the energy of the main peak is displayed. The four non-linear parameter sets produce similar Fourier spectra, with most of the strength between 25 MeV and 30 MeV. The fragmentation of the Fourier spectra is more pronounced for the linear effective interactions HS and L1, with a considerable amount of strength shifted in the energy region 30 – 40 MeV. The results of time-dependent calculations should be compared with the experimental value for the IV GMR in $^{208}\text{Pb}$: $26 \pm 3 \text{ MeV}$ [21]. We
have also calculated the time-dependent isovector monopole moments for $^{40}\text{Ca}$ and $^{90}\text{Zr}$. Although the oscillations are more anharmonic as compared to $^{208}\text{Pb}$, the corresponding Fourier spectra are generally in agreement with the experimental data on the IV GMR: 31.1±2.2 MeV for $^{40}\text{Ca}$ and 28.5±2.6 MeV for $^{90}\text{Zr}$ [21].

4 Constrained relativistic mean-field calculations

A very useful method for description of excited states in nuclei is provided by constrained mean-field calculations. In the framework of the relativistic mean-field theory, constrained calculations were performed for the ground state of $^{24}\text{Mg}$, using two quadratic constraints [22]. By analyzing the curvature of the energy surface near the ground state, in Ref. [23] constrained RMF calculations were used to determine the dependence of the excitation energy of giant monopole states on the nuclear compressibility. In Ref. [5] this approach has been generalized by applying the generator coordinate method (GCM) to the RMF. The GCM takes into account correlations produced by collective motion of the nucleons.

In the present study we further extend the method of Ref. [5], by using a more general ansatz for the generating functions of the GCM. Specifically, in Ref. [5] Slater determinants of nucleon single-particle spinors, resulting from constrained RMF calculations, were used as generating functions. Here the bosonic sector is explicitly taken into account. The generating functions will be defined as direct products of Slater determinants built from single-particle spinors and the coherent states that represent the meson fields. Furthermore, the investigation is extended to include the isovector ($T = 1$) giant monopole states.
The GCM $A$-particle trial wave-function $\Psi_{\text{GCM}}(\mathbf{r}_1, \ldots, \mathbf{r}_A)$ is written in the form

$$
\Psi_{\text{GCM}}(\mathbf{r}_1, \ldots, \mathbf{r}_A) = \int f(q) \Psi(\mathbf{r}_1, \ldots, \mathbf{r}_A; q) \, dq ,
$$

(29)

where $\Psi(\mathbf{r}_1, \ldots, \mathbf{r}_A; q)$ are the “generating functions”, and $f(q)$ is the “generator” or “weight function” that depends on the “generator coordinate” $q$. From the variation of the energy of the system with respect to $f(q)$, the Hill-Wheeler integral equation [24] for the weight function $f(q)$ is derived

$$
\int \left[ \mathcal{H}(q, q') - EN(q, q') \right] f(q') \, dq' = 0 ,
$$

(30)

where

$$
\mathcal{H}(q, q') = \langle \Psi(q) | \hat{H} | \Psi(q') \rangle ,
$$

(31)

and

$$
\mathcal{N}(q, q') = \langle \Psi(q) | \Psi(q') \rangle ,
$$

(32)

are the Hamiltonian and normalization kernels, respectively. The solutions of the Hill-Wheeler equation are the discrete eigenvalues $E_n$ and eigenfunctions $f_n(q)$, which determine the nuclear wave-functions for the ground and excited states (for details see Ref. [26]).

The relativistic extension of the GCM uses the self-consistent solutions of the equations of motion of the Lagrangian density [1]. The generating functions are defined as products of Slater determinants $\Phi(q)$, built from constrained RMF solutions for the Dirac single-nucleon spinors $\psi_i(\mathbf{r}; q)$, and the coherent states of the $\sigma$, $\omega$, and $\rho$ meson field and the electromagnetic field

$$
|\Psi(q)\rangle = |\Phi(q)\rangle \otimes |\sigma(q)\rangle \otimes |\omega_\mu(q)\rangle \otimes |\rho_\mu(q)\rangle \otimes |A_\mu(q)\rangle .
$$

(33)

The general expression for the boson coherent state reads [25]:

$$
|\phi_\mu\rangle = C^{-\frac{1}{2}} \exp \left[ \int d^3k \, \omega(k) \phi_\mu(k) a^+(k) \right] |0\rangle ,
$$

(34)
where $\phi_{\mu}$ denotes the $\sigma$, $\omega_{\mu}$, $\rho_{\mu}$ and $A_{\mu}$ fields, $\phi_{\mu}(k)$ is the Fourier-transform of the field $\phi_{\mu}(r)$,

$$\phi_{\mu}(k) = \omega(k) \int d^3r \phi_{\mu}(r)e^{ikr}, \quad (35)$$

$$\omega(k) = k_0 = \sqrt{k^2 + m_{\phi_{\mu}}^2}, \quad (36)$$

and

$$d^3k = \frac{d^3k}{(2\pi)^32\omega(k)} = \frac{d^4k}{(2\pi)^4}2(2\pi)\delta(k \cdot k - m_{\phi_{\mu}}^2)\theta(k^0). \quad (37)$$

$a^+(k)$ is the creation operator of a boson with momentum $k$ and $C$ is the normalization constant. Onishi’s formulas [26] are used in the calculation of the integral kernels (31) and (32) in the solution of the Hill-Wheeler equation. The normalization kernel can be written as product of fermionic and bosonic factors

$$N(q, q') \equiv N_D(q, q')N_M(q, q'), \quad (38)$$

where

$$N_D(q, q') \equiv \langle \Phi(q)|\Phi(q') \rangle = \det\{N_{ij}(q, q')\}, \quad (39)$$

with the matrix

$$N_{ij}(q, q') = \langle \psi_i(q)|\psi_j(q') \rangle, \quad (40)$$

where $|\psi_i(q)\rangle$ denotes the single-particle spinors. The boson factor

$$N_M(q, q') = \langle \sigma(q)|\sigma(q') \rangle \langle \omega_{\mu}(q)|\omega_{\mu}(q') \rangle \langle \rho_{\mu}(q)|\rho_{\mu}(q') \rangle \langle A_{\mu}(q)|A_{\mu}(q') \rangle \quad (41)$$

can be calculated from the overlap kernel

$$\langle \phi_{\mu}(q)|\phi_{\mu}(q') \rangle = \exp \left( -\frac{1}{2} \int d^3k \left| \phi_{\mu}(k; q) - \phi_{\mu}(k; q') \right|^2 \right). \quad (42)$$

The Hamiltonian kernel $H(q, q')$ reads

$$H(q, q') = N(q, q') \int H(r; q, q')dr, \quad (43)$$
where, for the case of spherical symmetry, the energy density kernel $\mathcal{H}(r; q, q')$ is given as

$$
\mathcal{H}_{\text{RMF}}(r; q, q') = \sum_{i,j=1}^{A} N_{ji}^{-1} \bar{\psi}_{i}^{+}(r; q)\{ -i\alpha \nabla \} \psi_{j}(r; q') + M\rho_{s}(r; q, q')
$$

\[+ \frac{1}{2} g_{s}\rho_{s}(r; q, q')\sigma(r; q, q') + \frac{1}{2} g_{\alpha}\rho_{\alpha}(r; q, q')\omega^{0}(r; q, q') \]

\[+ \frac{1}{2} g_{p}\rho_{p}(r; q, q')\rho^{0}(r; q, q') + \frac{1}{2} e\rho_{p}(r; q, q')A^{0}(r; q, q') \]

\[+ \frac{1}{2}\left( (\nabla\sigma(r; q, q'))^{2} + m_{\sigma}^{2}\sigma^{2}(r; q, q') \right) + \frac{1}{3} g_{2}\sigma^{3} + \frac{1}{4} g_{3}\sigma^{4} \]

\[- \frac{1}{2}\left( (\nabla\omega^{0}(r; q, q'))^{2} + m_{\omega}^{2}(\omega^{0}(r; q, q'))^{2} \right) \]

\[- \frac{1}{2}\left( (\nabla\rho^{0}(r; q, q'))^{2} + m_{\rho}^{2}(\rho^{0}(r; q, q'))^{2} \right) \]

\[- \frac{1}{2}(\nabla A^{0}(r; q, q'))^{2}, \quad (44) \]

with the density matrices

$$
\rho_{s}(r; q, q') = \sum_{i,j=1}^{A} N_{ji}^{-1} \bar{\psi}_{i}(r; q)\psi_{j}(r; q') \quad (45)
$$

$$
\rho_{v}(r; q, q') = \sum_{i,j=1}^{A} N_{ji}^{-1} \bar{\psi}_{i}^{+}(r; q)\psi_{j}(r; q') \quad (46)
$$

$$
\rho_{3}(r; q, q') = \sum_{i,j=1}^{A} N_{ji}^{-1} \bar{\psi}_{i}^{+}(r; q)\tau_{3}\psi_{j}(r; q') \quad (47)
$$

$$
\rho_{p}(r; q, q') = \sum_{i,j=1}^{A} N_{ji}^{-1} \bar{\psi}_{i}^{+}(r; q)\left(1 - \frac{\tau_{3}}{2}\right)\psi_{j}(r; q') \quad (48)
$$

The densities are sources for the Klein-Gordon equations. Their solutions are the meson fields that appear in Eq. (44)

$$
\phi_{\mu}(r; q, q') = \frac{1}{2}(\bar{\phi}_{\mu}(r; q) + \phi_{\mu}(r; q')) \quad (49)
$$

5 Isoscalar and isovector Giant Monopole Resonances

Solutions of constrained RMF calculations have fixed expectation value $Q$ of some observable $\hat{Q}$. The constraining operator $\hat{Q}$ is coupled to the Dirac Hamiltonian through a
Lagrange multiplier \( q \)

\[
\left( \hat{h}_D - q \hat{Q} \right) \psi_i(\vec{r}; q) = \varepsilon_i \psi_i(\vec{r}; q),
\]

(50)

with the Dirac Hamiltonian

\[
\hat{h}_D = -i\alpha \nabla + \beta((M + g_\sigma \sigma) + g_\omega \gamma^\mu \omega_\mu + g_\rho \gamma^\mu \tau_\rho \tilde{\rho}_\mu + e \gamma^\mu \frac{(1 - \tau_3)}{2} A_\mu).
\]

(51)

The constrained Dirac equation and the corresponding Klein-Gordon equations for the fields are solved self-consistently. In relativistic GCM calculations the Lagrange multiplier \( q \) is chosen as the generator coordinate. The constraint operator reads

\[
\hat{Q} \equiv \hat{Q}^T = \sum_{i=1}^{A} (2t_i)^T r_i^2, \quad T = 0, 1
\]

(52)

and it relates the generator coordinate to nuclear \textit{rms} radii

\[
\langle \hat{Q} \rangle_{T=0} = \langle r^2 \rangle = \frac{1}{A} \int r^2 \rho_v(\vec{r}; q) d^3 r,
\]

\[
\langle \hat{Q} \rangle_{T=1} = \langle r_n^2 \rangle - \langle r_p^2 \rangle = \frac{1}{N} \int r^2 \rho_v^n(\vec{r}; q) d^3 r - \frac{1}{Z} \int r^2 \rho_v^p(\vec{r}; q) d^3 r.
\]

(53)

Using a Hartree code in coordinate space, constrained RMF calculations for the spherical symmetric case are performed for a number of values of the Lagrange multipliers \( q \). The allowed values of \( q \) are limited by the requirement that the coupled system of Dirac and Klein-Gordon equations converges to a self-consistent solution.

From a set of constrained RMF solutions (single-particle spinors of occupied states, meson and electromagnetic fields), the normalization (32) and energy (31) kernels are calculated. Solutions of the associated integral Hill-Wheeler equation determine the nuclear ground and excited states. The method of “symmetrical orthogonalization” \[26, 27\] is applied to the generalized eigenvalue problem. The Hamiltonian kernel is projected onto the regular (collective) subspace of the norm kernel. A series of collective eigenenergies of the constrained system and the corresponding nuclear wave-functions are obtained, which
can be interpreted as the correlated ground-state solution and excited monopole states. Furthermore, nuclear densities in the ground-state and first excited state are calculated, as well as the transition densities

$$\rho_{fi}(r) = \int dq dq' f_f(q) f_i(q') N(q, q') \rho_v(r; q, q') ,$$

(54)

where $f$ and $i$ denote the resonance and the ground-state, respectively. Using the four non-linear effective interactions NL1, NL3, NL-SH and NL2, calculations have been performed for a number of spherical nuclei.

In Table 2 the isoscalar monopole excitation energies are displayed. For the NL1 parameter set we also include in the second row the excitation energies obtained with constrained GCM calculations from Ref. [5]. It appears that the inclusion of meson coherent states in the generating functions lowers the calculated excitation energies by approximately 0.5 MeV. Compared to the results of time-dependent calculations of Sec. 3, the energies in Table 2 are lower by more than 1 MeV, and in some cases the difference is larger than 2 MeV. The reason is that in constrained GCM calculations one implicitly assumes that the motion is adiabatic. In the time-dependent calculations on the other hand, no assumption about the nature of the mode of vibration is made. The comparison between constrained and time-dependent mean-field calculations strongly emphasizes the fact that we have to understand how the nucleus vibrates in order to relate the excitation energies of monopole states to the nuclear matter compression modulus.

In Fig. 10 the calculated energies of giant monopole states are plotted as functions of $A^{-1/3}$. An average linear dependence is observed for all four parameter sets. The deviations from a pure linear dependence are relatively small for NL1 and NL3. They are more pronounced for the effective interactions NL-SH and NL2, for which the nuclear matter compression modulus $K_{nm} > 300$ MeV. The effective nuclear compression moduli $K_A$ [24]
of $^{16}$O, $^{40}$Ca, $^{90}$Zr and $^{208}$Pb are shown in Fig. 11 as functions of $K_{nm}$. The GCM results for $E_0$ and $<r^2>_0$ are used in (24). We did not calculate $K_A$ from the curvature of the energy surface, and then $E_0$ from (24), as was done in Refs. [23, 5]. The relation (24) is used to obtain an estimate for the values $K_A$ that result from GCM calculation, in which the dynamics of the system is described by the non diagonal matrix elements of the energy kernel. Due to the correlations that these matrix elements produce, the calculated values of $K_A$ are somewhat lower than the corresponding results of Ref. [5]. For $^{208}$Pb the values of $K_A$ can be compared with results of time-dependent calculations (see Fig. 5). The constrained GCM transition densities (54) for $^{208}$Pb are shown in Fig. 12. Similar in shape to those calculated in the scaling model (Fig. 6), the constrained transition densities display a radial behavior which does not significantly depend on the effective interaction. In contrast to the results of time-dependent calculation, the dynamics of isoscalar monopole vibrations which is described by the constrained transition densities does not depend on the effective compression modulus. Again, this is due to the assumption of adiabatic motion, which is inherent in the constrained GCM approach.

In Table 3 we display the excitation energies of isovector monopole states in $^{40}$Ca, $^{90}$Zr and $^{208}$Pb, and compare the theoretical values with experimental data on isovector giant monopole resonances [21]. The calculations have been performed for the four nonlinear parameter sets, and in the second column the corresponding asymmetry energy coefficients $a_{sym}$ are included. We notice that the calculated excitation energies, like the results of time-dependent calculation, do not depend very much on the effective interaction. In particular, no simple relation can be established between the coefficient of asymmetry energy and the excitation energies of isovector states. The calculated values for $^{40}$Ca and $^{90}$Zr are close to the experimental excitation energies. For $^{208}$Pb, however, constrained GCM calculations produce results which are almost 10 MeV lower that the
experimental IV GMR. This is in contrast with the results of time-dependent calculation which center around 28 MeV, in agreement with experimental data. This result might indicate that in a heavy nucleus such as $^{208}\text{Pb}$ the assumption of adiabatic motion is not justified for the isovector mode. On the other hand, the constrained GCM results might also indicate that some strength should be expected around 17 MeV for the isovector monopole mode in $^{208}\text{Pb}$. The corresponding time-dependent results (Fig. 9) display some strength below the main peak in the Fourier power spectra, but only very little below 20 MeV. Finally, the isovector transition densities for $^{208}\text{Pb}$ are shown Fig. 13. As we have already seen for the isoscalar mode, the transition densities depend very little on the effective interactions.

6 Summary

Relativistic mean field theory has been used to investigate the monopole eigenmodes of a number of spherical closed shell nuclei, from $^{16}\text{O}$ to the heavy nucleus $^{208}\text{Pb}$. Two microscopic methods have been applied:

a) Time-dependent relativistic mean-field calculations. Starting from the self-consistent mean-field solution for the ground state and a set of appropriate initial conditions, the full set of time-dependent RMF equations is solved and dynamical variables are analyzed. Collective isoscalar and isovector monopole oscillations are studied, which correspond to giant resonances in spherical nuclei. Excitation energies and the structure of eigenmodes are determined from a Fourier analysis of dynamical monopole moments and densities.

b) The generator coordinate method, with generating functions that are solutions of constrained RMF calculations, is used to calculate excitation energies and transition
densities of giant monopole states. Constrained RMF calculations, with $r^2$ (T=0 and T=1) as the constraint operator, determine the stationary wave-functions with different values of nuclear rms radii. Using the generator coordinate method, the RMF-Hamiltonian is diagonalized in the basis of these solutions. The lowest eigenmode corresponds to the correlated ground state. The solution for the first excited state describes the giant monopole resonance.

Calculations have been performed for six different parameter sets of the Lagrangian. Two of them correspond to older effective forces, which do not include nonlinear self-interaction of the $\sigma$-field. The others are more modern parametrizations, which have been used with great success in recent years for a unified description of properties of nuclei over the periodic table (binding energies, radii, isotopic shifts, deformation parameters, moments of inertia in superdeformed nuclei, nuclear halos at the neutron drip line). The six parameter sets differ essentially in the deduced value for the incompressibility of nuclear matter. Therefore, from the energy spectra and transition densities calculated with these effective forces, it has been possible to study the connection between the incompressibility of nuclear matter and the breathing mode energy of spherical nuclei.

For the isoscalar mode we have found an almost linear relation between the excitation energy of the monopole resonance and the nuclear matter compression modulus. GCM calculations based on constrained solutions of the RMF equations yield, in general, slightly lower excitations energies as compared with results of time-dependent calculations. This can be understood from the assumption of adiabatic motion which is implicitly contained in constrained RMF calculations. No assumption about the nature of the mode is made in the time-dependent model. With both methods, the effective interactions NL1 and NL3 produce GMR excitation energies which are close to the experimental data. Based on the very precise experimental value for the isoscalar GMR energy in the heavy nucleus $^{208}$Pb,
which is rather well reproduced by the set NL3, we have derived a value of approximately 270 MeV for the incompressibility of nuclear matter. For a series of doubly closed-shell nuclei, the effective interaction NL3 reproduces the empirical result $E_x \approx 80 A^{-1/3}$ MeV.

Transition densities provide valuable information on the structure of the different modes. It turns out that there are essential differences between the two methods. The constrained GCM transition densities are similar to those of the simple scaling model, and display a radial behavior which does not significantly depend on the effective force. This is not the case for the time-dependent RMF calculations. Here the radial shape of the transition densities depends very much on the value of the nuclear matter compression modulus. Only for the parameter sets NL1, NL3, and NLSH, the transition densities display a radial shape characteristic for the breathing mode. For the other sets we find essential deviations in the nuclear interior, sometimes even standing waves. Only the surface component of the transition density is rather independent on the effective force.

As expected, for the isovector mode we have found no relation between the excitation energy of the giant resonance and the calculated incompressibility of nuclear matter. More damping and more fragmentation in the spectra is observed, compared to the isoscalar case. Excitation energies that result from time-dependent calculations are in good agreement with available experimental data. The constrained RMF results agree with experiment only for light and medium heavy nuclei. For $^{208}$Pb we find results which are almost 10 MeV lower than the experimental T=1 GMR. This might indicate that in heavy nuclei the assumption of adiabatic motion is not justified for the isovector mode.

Summarizing our investigations in the framework of relativistic mean field theory, we conclude that the nuclear matter compression modulus $K_{nm} \approx 270$ MeV is in reasonable agreement with the available data on spherical nuclei.
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Figure Captions

Fig. 1a Time-dependent isoscalar monopole moments $< r^2 >(t)$ and the corresponding Fourier power spectra for $^{40}$Ca. The parameter sets are NL1, NL3 and NL-SH.

Fig. 1b Same as in Fig. 1a, but for the effective interactions NL2, HS and L1.

Fig. 2a Time-dependent isoscalar monopole moments $< r^2 >(t)$ and the corresponding Fourier power spectra for $^{90}$Zr. The parameter sets are NL1, NL3 and NL-SH.

Fig. 2b Same as in Fig. 2a, but for the effective interactions NL2, HS and L1.

Fig. 3 Transition densities for the isoscalar monopole states in $^{90}$Zr.

Fig. 4a Time-dependent isoscalar monopole moments $< r^2 >(t)$ and the corresponding Fourier power spectra for $^{208}$Pb. The parameter sets are NL1, NL3 and NL-SH.

Fig. 4b Same as in Fig. 4a, but for the effective interactions NL2, HS and L1.

Fig. 5 The compression modulus $K_A (24)$ of $^{208}$Pb as function of the nuclear matter compression modulus $K_{nm}$. The excitation energies result from time-dependent relativistic mean-field calculations.

Fig. 6a Transition densities calculated as Fourier transforms of time-dependent baryon densities (left column), and scaling transition densities (right column), for the isoscalar monopole states in $^{208}$Pb. The parameter sets are NL1, NL3 and NL-SH. Solid lines denote proton densities, and dashed lines correspond to neutron transition densities.

Fig. 6b Same as in Fig. 6a, but for the effective interactions NL2, HS and L1.
Fig. 7 Volume and surface transition densities for the isoscalar monopole states in $^{208}$Pb. Solid lines correspond to proton densities, and dashed lines denote neutron transition densities.

Fig. 8 Excitation energies of isoscalar giant monopole resonances in doubly closed-shell nuclei as function of the mass number. The effective interactions are: NL1 (squares) and NL3 (circles). The solid curve corresponds to the empirical relation $\approx 80 A^{-1/3}$ MeV.

Fig. 9a Time-dependent isovector monopole moments $<r_p^2>-<r_n^2>$ and the corresponding Fourier power spectra for $^{208}$Pb. The parameter sets are NL1, NL3 and NL-SH.

Fig. 9b Same as in Fig. 9a, but for the effective interactions NL2, HS and L1.

Fig. 10 Constrained relativistic GCM excitation energies of isoscalar monopole states in doubly closed-shell nuclei as function of mass number. The effective interactions are NL1, NL3, NL-SH and NL2.

Fig. 11 The effective nuclear compression modulus $K_A$ ($^{234}$) of $^{16}$O, $^{40}$Ca, $^{90}$Zr and $^{208}$Pb as function of $K_{nm}$. The excitation energies of monopole states are calculated with the constrained relativistic GCM.

Fig. 12 Constrained GCM transition densities for the isoscalar monopole states in $^{208}$Pb.

Fig. 13 Constrained GCM transition densities for the isovector monopole states in $^{208}$Pb.
Table 1: Parameter sets for the effective Lagrangian.

|     | NL1  | NL3  | NL-SH | NL2  | HS   | L1   |
|-----|------|------|-------|------|------|------|
| \(m\) [MeV] | 938.0 | 939.0 | 939.0 | 938.0 | 939.0 | 938.0 |
| \(m_\sigma\) [MeV] | 492.25 | 508.194 | 526.0592 | 504.89 | 520.0 | 550.0 |
| \(m_\omega\) [MeV] | 795.335 | 782.501 | 783.0 | 780.0 | 783.0 | 783.0 |
| \(m_\rho\) [MeV] | 763.0 | 763.0 | 763.0 | 763.0 | 770.0 | 0.0  |
| \(g_\sigma\) | 10.138 | 10.217 | 10.44355 | 9.111 | 10.47 | 10.30 |
| \(g_\omega\) | 13.285 | 12.868 | 12.9451 | 11.493 | 13.80 | 12.60 |
| \(g_\rho\) | 4.975 | 4.474 | 4.3828 | 5.507 | 4.04  |      |
| \(g_2\) [fm\(^{-1}\)] | −12.172 | −10.431 | −6.9099 | −2.304 |      |      |
| \(g_3\) | −36.265 | −28.885 | −15.8337 | 13.783 |      |      |
| \(K_{nm}\) [MeV] | 211.7 | 271.8 | 355.0 | 399.2 | 545.0 | 626.3 |

Table 2: Constrained GCM energies of isoscalar monopole states. The values of \(K_{nm}\) and the excitation energies are in MeV.

| \(K_{nm}\) | \(16\,^O\) | \(40\,^Ca\) | \(48\,^Ca\) | \(90\,^Zr\) | \(208\,^Pb\) |
|-----|------|-------|-------|-------|--------|
| 1   | NL-1 | 211.7 | 20.2  | 16.6  | 15.9   | 14.1   | 11.0   |
| 2   | NL-1 | 211.7 | 20.6  | 17.1  |        | 14.7   | 11.7   |
| 3   | NL-3 | 271.8 | 22.6  | 19.6  | 18.9   | 16.9   | 13.0   |
| 4   | NL-SH| 355.0 | 25.0  | 22.0  | 21.5   | 19.5   | 15.0   |
| 5   | NL-2 | 399.2 | 27.1  | 24.4  | 23.0   | 21.9   | 16.6   |

Table 3: Constrained GCM energies of isovector monopole states. The values of \(a_{sym}\) and the excitation energies are in MeV.

| \(a_{sym}\) | \(40\,^Ca\) | \(90\,^Zr\) | \(208\,^Pb\) |
|-----|-------|-------|--------|
| NL-1 | 43.5  | 29.0  | 26.3   | 16.5   |
| NL-3 | 37.4  | 28.6  | 27.4   | 18.0   |
| NL-SH| 36.1  | 28.5  | 27.9   | 18.4   |
| NL-2 | 43.9  | 30.3  | 28.8   | 16.9   |
| exp. | 31.1±2.2 | 28.5±2.6 | 26.0±3.0 |
\begin{align*}
\langle r^2 \rangle_{\text{Pb NL1}} &\approx 34 \text{ fm}^2 \\
\langle r^2 \rangle_{\text{Pb NL3}} &\approx 34 \text{ fm}^2 \\
\langle r^2 \rangle_{\text{Pb NLSH}} &\approx 34 \text{ fm}^2
\end{align*}

\begin{align*}
E_{\text{Pb NL1}} &\approx 12.4 \text{ MeV} \\
E_{\text{Pb NL3}} &\approx 14.1 \text{ MeV} \\
E_{\text{Pb NLSH}} &\approx 16.1 \text{ MeV}
\end{align*}
$^{208}\text{Pb}$ – compression modulus

$K_A$ [MeV] vs. $K_{nm}$ [MeV]

- NL1
- NL2
- NL3
- NL–SH
- HS
- L1
\[ \langle r_p^2 \rangle - \langle r_n^2 \rangle \text{ [fm}^2\text{]} \]

- \( ^{208}\text{Pb NL1} \)
- \( ^{208}\text{Pb NL3} \)
- \( ^{208}\text{Pb NLSH} \)
