The Relativistic Green’s function model and charged-current inclusive neutrino-nucleus scattering at T2K kinematics

Andrea Meucci and Carlotta Giusti
Dipartimento di Fisica, Università degli Studi di Pavia and INFN, Sezione di Pavia, via A. Bassi 6, I-27100 Pavia, Italy
(Dated: January 15, 2015)

We compare the results of the relativistic Green’s function model with the experimental data of the charged-current inclusive differential neutrino-nucleus cross sections published by the T2K Collaboration. The model, which is able to describe both MINERνA and MiniBooNE charged-current quasielastic scattering data, underpredicts the inclusive T2K cross sections.

PACS numbers: 25.30.Pt; 13.15.+g; 24.10.Jv
Keywords: Neutrino scattering; Neutrino-induced reactions; Relativistic models

I. INTRODUCTION

In recent years many Collaborations have presented very interesting results of neutrino oscillations that aim at a precise determination of mass-squared splitting and mixing angles in $\nu_\mu$, disappearance and $\nu_e$ appearance measurements. Since modern experiments, such as MiniBooNE, SciBooNE, ArgoNeuT, MINERνA, and T2K, are performed with detectors made of medium-heavy nuclear targets, e.g. Carbon, Oxygen, Argon, or Iron, a clear understanding of neutrino-nucleus interactions, where all nuclear effects are well under control, is required for a proper analysis of experimental data. The recent progress, the questions and challenges in the physics of neutrino cross sections are reviewed in [1–3].

The first measurements of the charged-current quasielastic (CCQE) flux-averaged double-differential $\nu_\mu (\bar{\nu}_\mu)$ cross section on $^{12}$C in the few GeV region by the MiniBooNE Collaboration [4, 5] have raised extensive discussions that effects beyond the impulse approximation (IA) may play a significant role in this energy domain [6–8]. Multinucleon mechanisms, 2p-2h excitations and meson-exchange currents (MEC), and also RPA corrections appear essential to describe the data. Models based on the IA, which make use either of a realistic spectral function obtained within a nonrelativistic framework [18, 19] or of a relativistic IA (RIA) [20, 21], generally underestimate the MiniBooNE CCQE cross sections. Only the relativistic Green’s function (RGF) model is able to give a good description of the data [22]. Although under many aspects based on the RIA, the RGF model can recover contributions of final-state channels that are not included in other models based on the RIA. These contributions are recovered by the imaginary part of the relativistic optical potential that is used in the RGF model to describe final-state interactions (FSI).

The RGF model was originally developed within a non-relativistic [22, 23] and then a relativistic [24, 25] framework to describe FSI in the inclusive quasielastic (QE) electron scattering. The model was successfully tested against electron scattering data [25, 26, 27, 28] and it was later extended to neutrino-nucleus scattering, both in the charged-current (CC) [24, 30, 31] and in the neutral-current (NC) [37, 38] sector. Although different, the two situations present many similar aspects and the extension to neutrino scattering of the electron scattering formalism is straightforward. In the QE kinematic region, where the nuclear response to an electromagnetic probe is dominated by single-nucleon scattering with direct one-nucleon emission, a reliable description of the FSI effects between the ejected nucleon and the residual nucleus is very important for the comparison with data. In the RGF model FSI are described in the inclusive scattering consistently with the exclusive scattering by the same complex optical potential (OP) and the components of the nuclear response are obtained in terms of matrix elements of the same type as the distorted wave impulse approximation ones of the exclusive $(e, e'p)$ process, but involve eigenfunctions of the OP and of its Hermitian conjugate, where the opposite sign of the imaginary part gives in one case an absorption and in the other case a gain of strength. In the exclusive scattering, where only one channel is considered, the imaginary part gives an absorption that accounts for the flux lost to other channels. In the inclusive scattering, where all elastic and inelastic channels are included, the imaginary part redistributes the flux in all the channels and in the sum over all the channels the total flux is conserved. More details on the model can be found in our previous papers [24, 28, 32].

In other approaches based on the RIA, FSI are included in the emitted nucleon state with real potentials, either retaining only the real part of the relativistic energy-dependent complex OP, or using distorted waves obtained with the same relativistic energy-independent mean-field potential considered in describing the initial nucleon state (RMF) [23, 40, 41]. In the relativistic plane-wave impulse approximation (RPWIA) FSI are neglected.

The results of these different descriptions of FSI have been compared in [30] for the inclusive QE electron scattering, in [32] for the CCQE neutrino scattering, and in [24, 33] with the CCQE and NC elastic MiniBooNE data. Both RGF and RMF models can describe successfully electron scattering data and their related scaling functions. Both models are able to provide a satisfactory...
description of the CCQE MINERνA data [36, 41]. In the case of the MiniBooNE CCQE data, both models reproduce the shape of the experimental cross sections, but only the RGF gives cross sections of the same magnitude as the experimental ones without the need to increase the world average value of the axial mass $M_A$ [24, 36]. The larger RGF cross sections are due to the translation to the inclusive strength of the overall effect of inelastic channels, including rescattering and some multinucleon contributions, that are recovered in the model by the imaginary part of the OP can include pion-absorption and pion-emission processes, that should have already been included in the RMF and in other models based on the IA.

The optical potential is a powerful tool to recover and include important contributions. The availability of phenomenological relativistic OP’s, obtained through a fit of elastic proton-nucleus scattering data, is essential to make RGF calculations feasible, but the use of a phenomenological OP does not allow us to disentangle the role of a specific inelastic channel and can therefore introduce uncertainties and ambiguities in the interpretation of the RGF results. The imaginary part can recover, to some extent, contributions beyond direct one-nucleon emission, such as, for instance, rescattering of the outgoing nucleon and some multinucleon processes, which can be included in CCQE measurements, but the RGF model is based on the use of a one-body nuclear current and does not contain MEC mechanisms that in other models have been found to be significant. On the other hand, the imaginary part of the OP can include pion-absorption and pion-emission processes, that should have already been subtracted in the MiniBooNE analysis. It has been written in [3] that the good agreement of the RGF results with the MiniBooNE data “should be interpreted with care” and that “it would be very interesting to confront the RGF results with the fully CC-inclusive data” [3]. The comparison with the fully CC-inclusive data, which include also pion production, is the motivation of the present paper.

The T2K collaboration has recently published [42] new results on the CC-inclusive double differential cross section on $^{12}$C, which includes also pion production, and of the CCQE cross section [43]. The T2K $\nu_\mu$ energy range is the same as for MiniBooNE, the beam peaks at ~ 600 MeV, similar to that of MiniBooNE, but it is significantly narrower and receives almost negligible contributions for energies larger than 1 GeV. In view of these differences, the analysis of T2K data is another useful and independent test for a theoretical description.

In this paper we compare the results of the RGF model with the CC-inclusive $\nu_\mu$ and $\nu_e$ T2K cross sections on $^{12}$C [42, 43]. As a first step, we compare our results with the CCQE $\nu_\mu$-$^{12}$C cross sections measured by the MiniBooNE and T2K collaborations, for which there is consistency between the two experiments within the current statistical and systematic uncertainties [42]. Then, we consider the flux-averaged CC-inclusive $\nu_\mu$ and $\nu_e$ differential cross sections from T2K with the aim to investigate whether the effects of the inelastic channels recovered in the RGF model by the relativistic OP give or do not give enough strength to reproduce these data.

II. RESULTS

In all the calculations presented in this work we have adopted the standard value for the nucleon axial mass $M_A = 1.03$ GeV/$c^2$. The bound nucleon states are taken as self-consistent Dirac-Hartree solutions derived within a relativistic mean field approach using a Lagrangian containing $\sigma$, $\omega$, and $\rho$ mesons [46–50]. We have used two different parametrizations for the relativistic OP of $^{12}$C that is adopted in our RGF calculations: the Energy-Dependent and A-Independent EDAI (where the $E$ represents the energy and the $A$ the atomic number) OP of [51], and the more recent Democratic (DEM) phenomenological OP of [52]. The EDAI OP is a single-nucleus parametrization, which is constructed to better reproduce the elastic proton-$^{12}$C phenomenology, whereas the DEM parametrization is a global parametrization, which depends on the atomic number $A$ and is obtained through a fit to more than 200 data sets of elastic proton-nucleus scattering data on a wide range of nuclei and that is not limited to doubly closed shell nuclei. In comparison with electron scattering data, the DEM parametrization produces in general good results for doubly magic nuclei and less good but still acceptable results for nuclei with a number of nucleons far from the magic numbers [31, 53].

In Fig. 1 we show our calculated CCQE $\nu_\mu$-$^{12}$C cross sections per target neutron as a function of the neutrino energy compared with the MiniBooNE [4] and T2K [43].
Figure 2. Flux-averaged CC-inclusive double differential $\nu_\mu$-$^{12}$C cross sections per target nucleon as a function of the muon momentum. The data are from T2K [42].

Although the results of these two independent measurements can be consistently compared in the entire range of energies, with the only exception of the T2K datum in the energy bin $1 - 1.5$ GeV, we observe that the average magnitude of the MiniBooNE dataset is larger than that of the T2K one. The differences between the RGF-EDAI and RGF-DEM results are sizable. These differences are due to the different imaginary parts of the two OPs, particularly for the energies considered in kinematics with the lowest scattering angles and the largest kinetic energies of the muon [24]. The RGF-EDAI cross section is larger than the RGF-DEM one, in better agreement with the MiniBooNE data and in agreement with both MiniBooNE and T2K cross sections within the error bars in the entire energy range of the data. The RGF-DEM cross section underpredicts the MiniBooNE data at low $E_\nu$ and it is in better agreement with the T2K data. The RPWIA cross section, which is also shown in the figure for a comparison, is similar to the RGF-DEM one. We note that other models based on the IA give in general results somewhat lower than the RPWIA one and therefore lower than the data.

In Fig. 2 we present the CC-inclusive double differential $\nu_\mu$-$^{12}$C cross section $d^2\sigma/(dP_\mu d\cos \vartheta_\mu)$ as a function of the outgoing muon momentum transfer $P_\mu$ for four different bins in the scattering angle. The calculated cross sections are flux-averaged over the T2K $\nu_\mu$ flux [54] and compared with the experimental data of [42].

The RPWIA results in Fig. 2 are approximately 50% lower than the data. Also the RGF results underestimate the data. Both RGF-EDAI and RGF-DEM cross sections are generally lower than the data, although within the error bars for low values of $P_\mu$ and large angular bins. A satisfactory agreement with the data is obtained with the model of [55], which includes np-nh excitations and single-pion production. In the RGF model the imaginary part of the OP can include the excitation of multinucleon channels. We cannot exclude that it can contain some contribution due to pion emission, we cannot disentangle and evaluate the relevance of this contribution, but in
any case the results in Fig. 2 indicate that this is not enough to reproduce CC-inclusive data.

We note that for the most forward angular bin (0.94 \( \leq \cos \theta_s \leq 1.00 \)) the RGF results are significantly smaller than the RPWIA ones. In this small bin the transverse and charge-isovector contributions are suppressed and the longitudinal response gives the main contribution to the cross section. In addition, it has been shown in [56] that models based on quasi-free scattering cannot describe properly this kinematic situation where \( \sim 1/2 \) of the total cross section arises from excitation energies below \( \sim 50 \text{ MeV} \). If we consider that in the RGF calculations collective effects are neglected, it is not surprising that our results are significantly lower than the data in the forward angular bin.

In Fig. 3 and Fig. 4 the calculated CC-inclusive \( \nu_e \)-\( ^{12}\text{C} \) differential cross sections are displayed as a function of the electron momentum and scattering angle, respectively, and compared with the T2K data of [45]. For sake of simplicity the calculations have been performed only for the reduced phase-space (momentum \( > 550 \text{ MeV/c} \) and \( \cos \vartheta > 0.72 \)) and not for the full phase-space of T2K. The T2K \( \bar{\nu}_e \) beam peaks at \( \sim 500 \text{ MeV} \) and, in contrast to the \( \nu_\mu \) one, extends to energies larger than 1 GeV. Thus the CC \( \nu_e \) cross section at T2K may receive also higher-order contributions like two-pion production. Also in this case our calculations are significantly lower than the data. The RPWIA and the two RGF results are very similar in Fig. 3 where the three curves practically overlap, while in Fig. 4 the differences are small but visible. The fact that collective effects are not included in our model is one of the reasons of the large underestimation of the experimental data at smaller angle in Fig. 3.

### III. CONCLUSIONS

In this paper we have compared the predictions of the RGF model with the CCQE and CC-inclusive \( \nu_\mu \) and \( \nu_e \) scattering T2K data. The RGF model is able to give a satisfactory description of inclusive QE electron scattering cross sections and of the CCQE MiniBooNE and MINER\( \nu\)A data, both for \( \nu_\mu \) and \( \bar{\nu}_\mu \) scattering, without the need to increase the standard value of the axial mass.

The RGF results are usually larger than the results of other models based on the IA. In the RGF model FSI are described using a complex energy-dependent relativistic OP whose imaginary part includes the overall effect of the inelastic channels, which give different contributions at different energies, and makes the RGF results sensitive to the kinematic conditions of the calculations. With a complex OP the model can include all the available final-state channels and not only direct one-nucleon emission processes. The important role of contributions other than direct one-nucleon emission has been confirmed by different and independent models in the case of CCQE MiniBooNE cross sections, but the same conclusion is doubtful in the case of MINER\( \nu\)A data.

The RGF model can include contributions of final-state channels like, e.g., rescattering processes of the nucleon in its way out of the nucleus, non-nucleonic \( \Delta \) excitations, which may arise during nucleon propagation, with or without real-pion production, and also some multi-nucleon processes. These contributions are not incorporated explicitly in the model with a microscopic calculation: they can be recovered, to some extent, at a phenomenological level by the imaginary part of the phe-
nomenologial OP which is adopted in the RGF calculation. The use of a phenomenological OP does not allow us to disentangle and evaluate the role of a specific reaction process. Different available parametrizations of the phenomenological relativistic OP can introduce uncertainties in the predictions of the model. The determination of a theoretical OP, which fulfills the dispersion relations in the whole energy region of interest, would be very useful to reduce the theoretical uncertainties.

In the RGF model the nuclear response is written in terms of the single-particle optical-model Green’s function. This result is obtained retaining only the one-body part of the nuclear current. The inclusion of two-body MEC would require an extended model based on the two-body Green’s function, whose evaluation represents a very hard task.

The imaginary part of the OP can include pion-production processes that should have already been subtracted in the analysis of CCQE data. In this paper we have shown that the fully CC-inclusive T2K cross sections, which include pion production, are clearly underestimated by the RGF calculations. Even if we cannot disentangle the pion-production contribution that can be included in the phenomenological OP, this is not enough to reproduce the CC-inclusive T2K data. If we consider that the RGF model was developed to describe FSI in the inclusive QE scattering and that it is able to give a reasonably good agreement with QE electron and CCQE neutrino-scattering data, the present comparison with T2K data can be interpreted as an indication that the pion-production channel gives only a minor contribution to the RGF results.

The full and explicit inclusion of multinucleon channels is required to successfully reproduce CC-inclusive data. Other models, which explicitly include multinucleon emission channels, obtain a satisfactory agreement with the T2K data [55].

ACKNOWLEDGMENTS

We thank Marco Martini for interesting and useful discussions.

[1] J. A. Formaggio and G. P. Zeller, Rev. Mod. Phys. 84, 1307 (2012)
[2] J. G. Morfin, J. Nieves, and J. T. Sobczyk, Adv. High Energy Phys. 2012, 934597 (2012)
[3] L. Alvarez-Ruso, Y. Hayato, and J. Nieves, New Journal of Physics 16, 075015 (2014)
[4] A. A. Aguilar-Arevalo et al. (MiniBooNE Collaboration), Phys. Rev. D 81, 092005 (2010).
[5] A. A. Aguilar-Arevalo et al. (MiniBooNE Collaboration), Phys. Rev. D 88, 032001 (2013).
[6] T. Leitner, O. Buss, L. Alvarez-Ruso, and U. Mosel, Phys. Rev. C 79, 034601 (2009).
[7] M. Martini, M. Ericson, G. Chanfray, and J. Marteau, Phys. Rev. C 80, 065501 (2009).
[8] M. Martini, M. Ericson, G. Chanfray, and J. Marteau, Phys. Rev. C 81, 045502 (2010).
[9] M. Martini and M. Ericson, Phys. Rev. C 87, 065501 (2013).
[10] T. Leitner and U. Mosel, Phys. Rev. C 81, 064614 (2010).
[11] A. M. Ankowski and O. Benhar, Phys. Rev. C 83, 054616 (2011).
[12] E. Fernandez Martinez and D. Meloni, Physics Letters B 697, 477 (2011).
[13] J. E. Amaro, M. B. Barbaro, J. A. Caballero, and T. W. Donnelly, Phys. Rev. Lett. 108, 152501 (2012).
[14] J. E. Amaro, M. B. Barbaro, J. A. Caballero, T. W. Donnelly, and J. M. Udiás, Phys. Rev. D 84, 033004 (2011).
[35] A. Meucci, C. Giusti, and M. Vorabbi, Phys. Rev. D 85, 093002 (2012).
[36] A. Meucci and C. Giusti, Phys. Rev. D 89, 117301 (2014).
[37] A. Meucci, C. Giusti, and F. D. Pacati, Phys. Rev. D 84, 113003 (2011).
[38] R. González-Jiménez, J. A. Caballero, A. Meucci, C. Giusti, M. B. Barbaro, M. V. Ivanov, and J. M. Udías, Phys. Rev. C 88, 025502 (2013).
[39] A. Meucci and C. Giusti, Phys. Rev. D 89, 057302 (2014).
[40] J. A. Caballero, J. E. Amaro, M. B. Barbaro, T. W. Donnelly, C. Maieron, and J. M. Udías, Phys. Rev. Lett. 95, 252502 (2005).
[41] G. D. Megias, M. V. Ivanov, R. González-Jiménez, M. B. Barbaro, J. A. Caballero, T. W. Donnelly, and J. M. Udías, Phys. Rev. D89, 093002 (2014).
[42] K. Abe et al. (T2K Collaboration), Phys. Rev. D 87, 092003 (2013).
[43] K. Abe et al. (T2K Collaboration), (2014), arXiv:1411.6264 [hep-ex].
[44] K. Abe et al. (T2K Collaboration), Data release for T2K 2014 $\nu_e$ CC inclusive cross-section measurement, http://t2k-experiment.org/results/nd280-nue-xs-2014
[45] K. Abe et al. (T2K Collaboration), Phys. Rev. Lett. 113, 241803 (2014).
[46] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. 16, 1 (1986).
[47] P. G. Reinhard, Rep. Prog. Phys. 52, 439 (1989).
[48] P. Ring, Prog. Part. Nucl. Phys. 37, 193 (1996).
[49] G. A. Lalazissis, J. König, and P. Ring, Phys. Rev. C 55, 540 (1997).
[50] B. D. Serot and J. D. Walecka, Int. J. Mod. Phys. E6, 515 (1997).
[51] E. D. Cooper, S. Hama, B. C. Clark, and R. L. Mercer, Phys. Rev. C 47, 297 (1993).
[52] E. D. Cooper, S. Hama, and B. C. Clark, Phys. Rev. C 80, 034605 (2009).
[53] A. Meucci, M. Vorabbi, C. Giusti, F. D. Pacati, and P. Finelli, Phys. Rev. C 89, 034604 (2014).
[54] K. Abe et al. (T2K Collaboration), Phys. Rev. D 87, 012001 (2013).
[55] M. Martini and M. Ericson, Phys. Rev. C 90, 025501 (2014).
[56] J. E. Amaro, M. B. Barbaro, J. A. Caballero, T. W. Donnelly, and C. F. Williamson, Phys. Lett. B 696, 151 (2011).