Extrinsic Curvature Induced 2-d Gravity

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Abstract

2-dimensional fermions are coupled to extrinsic geometry of a conformally immersed surface in $\mathbb{R}^3$ through gauge coupling. By integrating out the fermions, we obtain a WZNW action involving extrinsic curvature of the surface. Restricting the resulting effective action to surfaces of $h\sqrt{g} = 1$, an explicit form of the action invariant under Virasaro symmetry is obtained. This action is a sum of the geometric action for the Virasaro group and the light-cone action of 2-d gravity plus an interaction term. The central charges of the theory in both the left and right sectors are calculated.

$\LaTeX$ Typesetting

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1 Introduction

2-d gravity induced by a conformal field theory was first examined by Polyakov [1]. He found that there is an $Sl(2, R)$ current algebra. This opened the way to solve off-critical string theory. Knizhnik, Polyakov, and Zamolodchikov [2] exploited this hidden $Sl(2, R)$ symmetry to evaluate the scaling dimensions of planar random surfaces. Their results showed complete agreement with the numerical simulations [3].

The geometrical origin of $Sl(2, R)$ is intriguing. Polyakov [4] demonstrated how one might get diffeomorphisms out of restricted $Sl(2, R)$ gauge transformations. The mechanism resembles the phenomenon of transformation of iso-spin into ordinary spin in the presence of magnetic monopoles. It relies on background $Sl(2, R)$ gauge fields being partially gauge fixed. This leaves a residual gauge symmetry which solders iso-space and ordinary space yielding diffeomorphism. Extensions to other groups and gauge restrictions have been shown to yield W-algebras.

The present authors have explored [5] the possibility of realizing gauge fields in an intrinsic manner which admits partial gauge fixing in a geometrical context that leads to a theory of induced 2-d gravity. It was demonstrated that if one takes into account the extrinsic geometry of 2-d surfaces in d- dimensions then one can construct $SO(d)$ gauge fields from the components of the second fundamental form of the surface. This observation is of course well known in the form of Gauss-Codazzi equations of the immersed surface. What is interesting is that when we consider surfaces with the property $h\sqrt{g} = 1$ in the target space $\mathbb{R}^3$, where $h$ is the mean scalar curvature and $g$ is the determinant of the induced metric on the surface, there arises a hidden symmetry in the geometrical theory of such surfaces. This symmetry is precisely the Virasaro symmetry. A similar property exists in $\mathbb{R}^4$ also. Here the hidden symmetry is Virasaro⊕Virasoro, when one restricts to $h^i\sqrt{g} = 1/\sqrt{2}$ (i=1,2), where $h^i$ are the...
two principal mean scalar curvatures. Similar conclusions were arrived at in \([3]\) in the light cone gauge, whereas our results in \([4]\) are in the conformal gauge. Beyond \(\mathbb{R}^4\) it is not clear if there exists a hidden symmetry for any special choice of surfaces, but W-algebras have been obtained for surfaces immersed in affine spaces \([7]\).

It turns out to be extremely interesting to pursue further our observations in \([5]\). In this article we first obtain an action which is explicitly invariant under Virasaro symmetry. Our considerations are for surfaces in \(\mathbb{R}^3\) but we see no difficulty in principle to extending them to \(\mathbb{R}^4\). This action depends on the components \(H_{zz}\) and \(\frac{H_{zz}}{\sqrt{g}}\) of the second fundamental form as the basic fields, the first one playing the role of an induced metric while the second one that of the energy-momentum tensor. Actually, the procedure for obtaining this action was worked out by Polyakov in \([4]\). The resulting action is a WZNW action. One finds that the effective theory is described by a sum of the geometric action for the Virasaro group \([8]\), the action for 2-d gravity in the light cone gauge \([2]\), and an interaction term which is needed to make the action invariant under the Virasaro transformation. We then analyze the properties of the effective action under conformal transformations in the left and the right sectors and compute their central charges for the quantum theory of the extrinsic geometry. We find that the left sector is described by an energy momentum tensor \(T_{zz}\) which is a sum of \(T_{zz}\) of the light cone action and \(T_{zz}\) of the geometric action. They are both of the modified Sugawara form. The right sector, however, has only Virasaro symmetry and the energy-momentum tensor \(T_{zz}\) is that of the geometric action only. Luckily, the central charges for both these cases are known. Recently, Aoyama \([9]\) has calculated the central charges \(c_{L,R}\) of the geometric action, while \(c_L\) of the light cone action is known from \([4]\), and \([2]\). After a careful consideration of the contribution of the ghosts in both sectors, we find that the theory of induced 2-d
gravity induced from the extrinsic geometry in $\mathbb{R}^3$ works for conformal matter fields with $d > 1$ in both sectors. It has been remarked in [4] that the main stimulus for finding the gauge representation arises from the hope of describing $d > 1$ conformal fields in 2-d quantum gravity. Our results suggest that extrinsic curvature does indeed become fundamental variables in the case $d > 1$, rather than the intrinsic metric of the surface.

2 Basic properties of $hg^{1/2} = 1$ surfaces

Although this section contains no new results, we feel that it is useful to recall the basic properties of $h\sqrt{g} = 1$ surfaces conformally immersed in $\mathbb{R}^3$ so that this article is self-contained. For details we refer the reader to [7].

The second fundamental form of a surface in $\mathbb{R}^d$ is defined by

$$\partial_\alpha \partial_\beta X^\mu = \Gamma^\gamma_{\alpha \beta} \partial_\gamma X^\mu + H^i_{\alpha \beta} N_i^\mu, \tag{1}$$

where $X^\mu(\xi^\alpha)$ are the string coordinates, $\Gamma^\gamma_{\alpha \beta}$ the affine connection defined by the induced metric $g_{\alpha \beta} = \partial_\alpha X^\mu \partial_\beta X^\mu$ and $N_i^\mu (i = 1...d - 2)$ are the normals to the string world sheet. We treat both the world sheet and the target space as Euclidean. The normals $N_i^\mu$ satisfy the following equations,

$$\partial_\alpha N_i^\mu = \tau^i_{\alpha j} N_j^\mu - H^i_{\alpha \gamma} \partial_\gamma X^\mu, \tag{2}$$

where $\tau^i_{\alpha}(\alpha = 1, 2)$ are the normal connections of the surface. (1) and (2) are also referred to as the Gauss-Codazzi equations. The scalar curvature $R$ of the world sheet is given as

$$R = (H^i_{\alpha})^2 - H^i_{\alpha \beta} H^i_{\beta}, \tag{3}$$
It is convenient to rewrite the structure equations (1) and (2) as follows;

$$\partial_z \hat{e}_i = (A_z)_{ij} \hat{e}_j \quad (i, j = 1, 2, 3). \quad (4)$$

A similar equation holds for the $\bar{z}$ derivative. Here $z = \xi_1 + i \xi_2$ and $\bar{z} = \xi_1 - i \xi_2$ are the local isothermal coordinates on the surface. $\hat{e}_i$ is a local orthonormal moving frame in which $\hat{e}_{1,2}$ are tangential and $\hat{e}_3$ is normal to the surface. $(A_z)_{ij}$ is antisymmetric in $(ij)$ whose elements are given in terms of $H^i_{\alpha \beta}$. It transforms under local $SO(3)$ gauge transformations

$$\hat{e}'_i = g_{ij} \hat{e}_j \quad (5)$$

as a gauge field. The gauge transformations generate $\hat{e}'_i$ which form a local moving frame to a different surface on which the induced metric is again in the conformal gauge. The two tangent vectors $\hat{e}_1$ and $\hat{e}_2$ to the surface are determined only up to an $SO(2)$ rotation. It is possible to choose these tangent vectors in such a way that $(A_z)_{12} = 0$. The explicit forms of $A_z$ and $A_{\bar{z}}$ after this rotation are given in [5]. If we now restrict to surfaces of $h \sqrt{g} = 1$ then the residual gauge transformation is a diffeomorphism. Of the three complex parameters determining $SO(3, C)$, two are determined in terms of the third. Then we find that $A_z^+ \equiv T_{zz} = \frac{H_{zz}}{\sqrt{g}}$ transforms as the energy-momentum tensor

$$\delta \epsilon^+ T_{zz} = -\partial_z^3 \epsilon^- - 2 \partial_z \epsilon^- T_{zz} - \epsilon^- \partial_z T_{zz} \quad (6)$$

$\frac{H_{zz}}{\sqrt{g}}$ thus plays the role of energy momentum tensor for the extrinsic geometry induced 2-d gravity. An analysis of the transformation of $A_{\bar{z}}$ shows that $A_{\bar{z}}^- \equiv H_{\bar{z} \bar{z}}$ transforms as the metric tensor

$$\delta \epsilon^- H_{\bar{z} \bar{z}} = \partial_{\bar{z}} \epsilon^- + \epsilon^- \partial_{\bar{z}} H_{\bar{z} \bar{z}} - (\partial_{\bar{z}} \epsilon^-) H_{\bar{z} \bar{z}} \quad (7)$$
Finally the integrability condition which amounts to saying that $F_{zz} = 0 \ (A_z, A_{\bar{z}})$ is a pure gauge) leads to the anomaly equation of 2-d gravity:

$$\partial^3_z H_{zz} = \partial_z T_{zz} - 2(\partial_z H_{zz})T_{zz} - H_{zz}\partial_z T_{zz}$$

(8)

Similar results have been derived for surfaces in $\mathbb{R}^4$ [5] but we shall not need them here as we restrict to $\mathbb{R}^3$ in the sections below.

3 WZNW action for extrinsic geometry

This section contains results derived in [4] in the context of diffeomorphisms from gauge transformations. We need only to identify the gauge field with appropriate components of the extrinsic curvature tensor. For completeness and continuity we sketch this procedure. We wish to derive the WZNW action that exhibits explicitly the Virasaro symmetry in $h\sqrt{g} = 1$ surfaces. One couples the gauge fields discussed in the previous section to 2-d fermions in a gauge invariant manner and then integrate out the fermions. The resulting action is known to be a WZNW action and can be written as,

$$\Gamma_{\text{eff}} = \Gamma_-(A_z) + \Gamma_+(A_{\bar{z}}) - Tr \int A_z A_{\bar{z}} d^2 \xi,$$

(9)

where $A_z = h^{-1}\partial_z h$, and $A_{\bar{z}} = g^{-1}\partial_{\bar{z}} g$, $h, g \in SO(3, \mathbb{C})$. We next derive the form of (9) under the gauge restriction $A_{z}^{-} = 1(h\sqrt{g} = 1) \text{ and } A_{z}^{0} = 0$. Then,

$$\Gamma_-(A_{z}^{-} = 1, A_{z}^{0} = 0, A_{z}^{+} \equiv H_{zz}/\sqrt{g}) \equiv S_-(F_1),$$

can be explicitly constructed by parameterizing $A_{z}^{+}$ as,

$$A_{z}^{+} \equiv \frac{H_{zz}}{\sqrt{g}} = D_{z} F_{1} \\
\equiv \frac{\partial^3 F_{1}}{\partial_z F_{1}} - \frac{3}{2} \left( \frac{\partial^2 F_{1}}{\partial_z F_{1}} \right)^2,$$

(10)
to be,
\[ S_-(F_1) = \frac{1}{2} \int \left( \frac{\partial_z F_1}{\partial_z F_1} \right) \left[ \frac{\partial^3_z F_1}{\partial_z F_1} - 2 \left( \frac{\partial^2_z F_1}{\partial_z F_1} \right)^2 \right] dz \wedge d\bar{z}. \] (11)

What is appealing about this action is that the field \( F_1 \) has a geometrical interpretation. It is easily derived from our results [5] on Gauss map of 2-d surfaces into the Grassmannian manifold that \( F_1 \) in (11) is the \( CP^1(\simeq G_{2,3}) \) field that arises in the Gauss map. (11) is the geometric action for the Virasaro group studied by Alekseev and Shatashvili [8]. The quantum action \( S_+(H_{\pm \pm}) \) is defined by,
\[ \exp(-S_+(H_{\pm \pm})) = \int [dA] \delta(A^- - H_{\pm \pm}) \exp(-\Gamma_+(A^-) - \int A^+_\pm)). \] (12)

In the classical limit, evaluating the path integral at the stationary point,
\[ \frac{\delta \Gamma_+}{\delta A^0_\bar{z}} \equiv J^0_\bar{z} = 0 ; \quad \frac{\delta \Gamma_+}{\delta A^+_{\bar{z}}} \equiv J^+_{\bar{z}} = 1, \] (13)
we have,
\[ S^{cl}_+ = \min_{A^0_\bar{z},A^+_{\bar{z}}} \left[ \Gamma_+(A_{\bar{z}}) - \int A^+_\bar{z} \right]. \] (14)

It is interesting to note that from our discussion in section 2, \( A^-_{\bar{z}} \) after gauge rotation is indeed given by \( H_{\pm \pm} \), where \( H_{\pm \pm} \) is the component of the second fundamental form. It is shown in [1] that (14) may be given an explicit form by,
\[ S^{cl}_+(F_2) = -\frac{1}{2} \int \left[ \left( \partial^2_z F_2 \right) \frac{\partial_z F_2}{\partial_z F_2} - \left( \partial^2_{\bar{z}} F_2 \right)^2 \frac{\partial_{\bar{z}} F_2}{\partial_{\bar{z}} F_2} \right] dz \wedge d\bar{z}. \] (15)

This is precisely the light-cone gauge action of 2-d gravity [1, 2]. Indeed \( J^0_{\bar{z}} = 1 \) and \( J^+_{\bar{z}} = 0 \) play the role of light cone gauge fixing conditions on the metric, namely, \( h_{--} = 0, h_{+-} = 1 \). In (15), \( F_2 \) is related to \( H_{\pm \pm} \) by,
\[ H_{\pm \pm} = \frac{\partial_z F_2}{\partial_{\bar{z}} F_2}. \] (16)
The complete action (9) is thus given by,

$$\Gamma_{\text{eff}}(F_1, F_2) = \frac{1}{2} \int \partial z F_1 \left[ \frac{\partial^2 F_1}{\partial z F_1} - 2 \left( \frac{\partial^2 F_1}{\partial z F_1} \right)^2 \right]$$

$$- \frac{1}{2} \int \partial z^2 F_2 \left[ \frac{\partial^2 F_2}{\partial z F_2} - \left( \frac{\partial^2 F_2}{\partial z F_2} \right)^2 \frac{\partial z F_2}{\partial z F_2} \right]$$

$$- \int \frac{\partial z F_2}{\partial z F_2} D_z F_1. \quad (17)$$

$$\Gamma_{\text{eff}}$$ can be shown to be explicitly invariant under transformations (6) and (7) of $H_{\bar{z}z} \sqrt{g}$ and $H_{zz}$ respectively. Thus (17) is the extrinsic curvature induced 2-d gravity action that exhibits Virasaro symmetry claimed in [5]. It is worth remarking that this action contains both the geometric and the light-cone actions with a gauge invariant coupling. Incidentally, the last term in (17) may be written as $\int |H|^2 \sqrt{g} d^2 \xi$ which is precisely the extrinsic curvature action. The equation of motion following from (17) is,

$$\nabla_{\bar{z}} \left( \frac{H_{zz}}{\sqrt{g}} \right) = \partial^3_{\bar{z}} H_{zz}, \quad (18)$$

where,

$$\nabla_{\bar{z}} = \partial_{\bar{z}} - 2(\partial_{\bar{z}} H_{zz}) - H_{zz} \partial_{\bar{z}}. \quad (19)$$

The main result then is that we have an action described by the fields $H_{zz} = \frac{\partial_{\bar{z}} F_2}{\partial z F_2}$ and $\frac{H_{zz}}{\sqrt{g}} = D_z F_1$, which has Virasaro symmetry. This action is basically a superposition of the geometric and the light-cone 2-d gravity actions and hence is the statement that extrinsic geometry induces an effective 2-d gravity theory.
4 Properties of $\Gamma_{eff}$ under conformal transformations

Following Aoyama [9] we assign conformal weights $(\Delta_{\bar{z}}, \Delta_z) = (0, 0)$ to the fields $F_1$ and $F_2$. In the $\bar{z}$ sector, the conformal transformations are given by,

$$\delta_{\text{conf}}^\bar{z} F_i = \bar{\epsilon}(\bar{z}) \partial_{\bar{z}} F_i, \quad (i = 1, 2). \tag{20}$$

Computing the change in $\Gamma_{eff}$ under (20), we find,

$$\delta \Gamma_{eff} = -\int \bar{\epsilon}(\bar{z}) \left\{ \frac{\partial z F_1}{\partial F_1} \partial_{\bar{z}} (D_{\bar{z}} F_1) + \frac{\partial z F_2}{\partial F_2} \partial_{\bar{z}}^3 \left( \frac{\partial_{\bar{z}} F_2}{\partial_{\bar{z}} F_2} \right) \right\} \equiv -\int \bar{\epsilon}(\bar{z}) \partial_{\bar{z}} T_{\bar{z}z}, \tag{21}$$

where,

$$T_{\bar{z}z} \equiv T_{\bar{z}z}(F_1) + T_{\bar{z}z}(F_2)$$

$$= -\frac{1}{2} \left\{ \left( \frac{\partial z F_1}{\partial F_1} \right)^2 - 2(\partial z F_1) \left( \frac{\partial^2 z F_1}{(\partial z F_1)^2} - \frac{(\partial^2 z F_1) \partial z \partial_{\bar{z}} z F_1}{(\partial z F_1)^3} \right) \right\}$$

$$+ \left\{ \frac{\partial z F_2}{\partial F_2} \partial_{\bar{z}}^2 \left( \frac{\partial z F_2}{\partial F_2} \right) - \frac{1}{2} \left( \partial z \frac{\partial z F_2}{\partial F_2} \right)^2 \right\}. \tag{22}$$

It is interesting to note that the interaction term in (17) is invariant under conformal transformations in the $\bar{z}$ sector. $T_{\bar{z}z}(F_1)$ agrees with the expression in [9] where it is shown that it is of the Sugawara form for $SL(2, R)$ currents. $T_{\bar{z}z}(F_2)$ agrees with the expression in KPZ[2]. Thus the total energy momentum tensor in the $\bar{z}$ sector is a sum of the Sugawara forms of $T_{\bar{z}z}$ for the geometric ($F_1$) and the light cone ($F_2$) actions.

In the $z$-sector the conformal transformations are given by,

$$\delta_{\text{conf}}^z F_i = \epsilon(z) \partial_z F_i, \quad (i = 1, 2). \tag{23}$$
Computing the change in the action, we find,

$$\delta \Gamma_{\text{eff}} = - \int \epsilon(z) \partial \bar{z} (D_z F_1).$$

(24)

It is important to note that the energy momentum tensor in the $z$-sector is,

$$T_{zz} = D_z F_1.$$ 

(25)

The variation of $S(F_2)$ cancels that of the interaction term in $L_7$. This result agrees with the interpretation earlier in section 2, of $D_z F_1 = \frac{H_{zz}}{\sqrt{g}}$ as the energy momentum tensor for the Virasaro symmetry. Thus to summarize, we have found that the effective 2-d gravity induced by the extrinsic geometry has Virasaro symmetry in the $z$-sector and Virasaro-Kac-Moody symmetry in the $\bar{z}$-sector.

5 Quantum properties of $\Gamma_{\text{eff}}$

In the $\bar{z}$ sector, we found that the total energy momentum tensor of our theory is a sum of that of the geometric action and that of the light-cone action. The geometric action in this sector as well as in the $z$ sector has been quantized by Aoyoma [9], while the light-cone action has been quantized by KPZ [2]. We can therefore make use of their results to describe the quantum theory of extrinsic curvature induced 2-d gravity in the $\bar{z}$ sector.

The quantization of the light-cone action in the $z$ sector has not been done so far. Fortunately, as we found above, the energy momentum tensor of our theory in the $z$ sector is identical to that of the geometric action given by $D_z F_1$, and as remarked above this sector of the geometric action has been quantized in [3].

Let us discuss the $\bar{z}$ sector first. In the quantum theory of the geometric action the conformal weights of the field $F_1$ is changed in accordance with the observation that
\[ \partial_z F_1 = \exp(\frac{\sqrt{2}}{k} \phi), \] tells us that it should be \((1, 0)\), since in the classical Liouville theory, \(\exp(\frac{\sqrt{2}}{k} \phi)\) has conformal weight \((1, 1)\). Accordingly, we take,

\[ \delta_{\text{conf}}^\bar{\bar{\epsilon}} F_1 = \epsilon(\bar{\bar{z}}) \partial_{\bar{\bar{z}}} F_1 + (\partial_{\bar{\bar{z}}} \epsilon(\bar{\bar{z}})) F_1. \quad (26) \]

It is easily shown \cite{9} that \(T'_{\bar{\bar{z}}}(F_1)\) is given by,

\[ T'_{\bar{\bar{z}}}(F_1) = T_{\bar{\bar{z}}}(F_1) - \partial_{\bar{\bar{z}}} J_0^0, \quad (27) \]

where \(T_{\bar{\bar{z}}}(F_1)\) is defined in (22) and,

\[ J_0^0 = -\frac{1}{2} \left\{ F_1 \frac{\partial_{\bar{\bar{z}}}^2 \partial_{z} F_1}{(\partial_{\bar{\bar{z}}} F_1)^2} - (\partial_{\bar{\bar{z}}}^2 F_1) \frac{\partial_{z} \partial_{\bar{\bar{z}}} F_1}{(\partial_{\bar{\bar{z}}} F_1)^3} - \frac{\partial_{\bar{\bar{z}}} \partial_{z} F_1}{\partial_{\bar{\bar{z}}} F_1} \right\}. \quad (28) \]

It is known from \cite{9} that the central charge of this energy momentum tensor is,

\[ c_{\bar{\bar{z}}}(F_1) = 15 - 6(k + 2) - \frac{6}{k + 2}. \quad (29) \]

The results of KPZ \cite{2} can be used to find the central charge of \(T_{\bar{\bar{z}}}(F_2)\). Recall that \(S_{+}^{cl}(H_{\bar{\bar{z}}}^{+})\) has been obtained by imposing the conditions (13), which can be identified with the light-cone gauge conditions \(h_{--} = 0\) and \(h_{+-} = 1\) in equation 6 of \cite{2}. Thus the improved energy momentum tensor is as given by KPZ \cite{4}. The central charge of this theory is found to be same as (29),

\[ c_{\bar{\bar{z}}}(F_2) = c_{\bar{\bar{z}}}(F_1). \]

Let us examine the ghost sector of our theory. Considering first the geometric action, we recall that it was obtained by the choice \(A_{\bar{\bar{z}}}^{-} \equiv h \sqrt{g} = 1\), and \(A_{\bar{\bar{z}}}^{0} = 0\). In the quantum theory they manifest as gauge restrictions and by identifying \(A_{\bar{\bar{z}}}^{-} = 1\) with \(h_{+-} = 1\) and \(A_{\bar{\bar{z}}}^{0} = 0\) with \(h_{-} = 0\) gauge choice in 2-d gravity, we find that the ghosts contribute \(-28\) to the central charge. The argument above suggest that
the ghost contribution to the central charge for the light-cone action is also \(-28\).

Denoting the central charge of the matter by \(d\), we thus find,
\[
\mathcal{c}_{\text{tot}}^z = d - 56 + 2(15 - 6(k + 2) - \frac{6}{k + 2}).
\]  

(30)

By equating this to zero, we find,
\[
k + 2 = \frac{d - 26 \pm ((2 - d)(50 - d))^\frac{1}{2}}{24}.
\]

(31)

Because of the doubling of the gravity and ghost contributions, we find that the theory of induced 2-d gravity makes sense for \(d \leq 2\).

Turning now to the \(z\)-sector, our theory is equivalent to the \(z\)-sector of the geometric action. From the result of Aoyama [9] we have,
\[
\mathcal{c}_z = 13 - 6(k + 2) - \frac{6}{k + 2}.
\]

(32)

The ghost contribution is once again \(-28\). Thus, we have,
\[
\mathcal{c}_{\text{tot}}^z = d - 28 + 13 - 6(k + 2) - \frac{6}{k + 2}.
\]

(33)

From (33) we find,
\[
k + 2 = \frac{d - 15 \pm ((3 - d)(27 - d))^\frac{1}{2}}{12}.
\]

(34)

From this we note that the theory in \(z\)-sector works for \(d \leq 3\).

6 Summary

We have shown that the extrinsic curvature of immersed surfaces with the property \(h\sqrt{g} = 1\) induce an effective 2-d gravity theory described by a sum of the geometric and light cone actions in a gauge invariant way. The central charges in both the left and right sectors are calculable and the theory makes sense for \(d \geq 1\) in both the sectors.
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