Fatigue Life Prediction of the Keel Structure of A Tsunami Buoy Using Spectral Fatigue Analysis Method

Angga Yustiawan¹*, Ketut Suastika¹, and Wibowo Harso Nugroho²

1. Department of Naval Architecture and Shipbuilding Engineering, Faculty of Marine Technology, Institut Teknologi Sepuluh Nopember (ITS), Surabaya 60111, Indonesia
2. UPT Balai Pengkajian dan Penelitian Hidrodinamika, BPP-Teknologi, Surabaya 60111, Indonesia

*E-mail: yustiawan@na.its.ac.id

Abstract

One of the components of the Indonesia Tsunami Early Warning System (InaTEWS) is a surface buoy. The surface buoy is exposed to dynamic and random loadings while operating at sea, particularly due to waves. Because of the cyclic nature of the wave load, this may result in a fatigue damage of the keel structure, which connects the mooring line with the buoy hull. The operating location of the buoy is off the Java South Coast at the coordinate (10.3998 S, 108.3417 E). To determine the stress transfer function, model tests were performed, measuring the buoy motions and the stress at the mooring line. A spectral fatigue analysis method is applied for the purpose of estimating the fatigue life of the keel structure. Utilizing the model-test results, the S-N curve obtained in a previous study and the wave data at the buoy location, it is found that the fatigue life of the keel structure is approximately 11 years.

Keywords: fatigue life, keel structure, spectral fatigue analysis, tsunami buoy

1. Introduction

Indonesia is located in a seismic active area due to the triple junction of three mega-tectonic plates, namely Eurasia plate, Indian Ocean-Australia plate, and Pacific Ocean plate. The relative movements of the three tectonic plates and two other plates, the Philippine Sea and the Carolinas, have resulted in frequent earthquakes in the border area and the meeting area of the plates, which can potentially trigger a tsunami [1].

One of the components of the Indonesia Tsunami Early Warning System (InaTEWS) is a surface buoy. A number of surface buoys are operated at several locations in the Indonesia tsunami-prone area, that is, in the Indian Ocean. The surface buoy is exposed to dynamic and random loadings, particularly due to waves while operating at sea.

To keep the surface buoy in its position, it is moored to the seabed by a mooring line. The mooring line is connected to the surface buoy by a keel structure (a plate structure with a hole as shown in Figs. 1 and 2). Because of the cyclic nature of the wave loading, this may result in fatigue damage or cracks of the keel structure, which ultimately can lead to a structural failure.

A fatigue life prediction of the keel structure of the tsunami buoy is necessary because the keel structure is made of used material (previously used ship-propeller shaft). It is important to have an estimate of the fatigue life of the keel structure in connection with maintenance
The structure that is considered in the analysis is Buoy No. 4 of BPPT, which is located off the Java South Coast at the coordinate (108.3417 E, 10.3998 S), as indicated in Fig. 3.

2. Experiment

The method of spectral fatigue analysis is applicable to structures which are excited by dynamic loading which has statistically stationary properties for a large number of stress cycles. An example of such loading is one due to waves.

The spectral method utilizes the stress spectrum to determine the number of stress cycles of various sizes. It is common to assume a Rayleigh distribution for the stress ranges in a given spectrum. The Rayleigh assumption is valid providing the bandwidth of the stress process is relatively small. Corrections to the Rayleigh approximation to account for bandwidth effects have been proposed by Wirsching and Light [4] and Ortiz and Chen [5]. Larsen and Lutes [6] proposed a single-moment spectral approach to account for bandwidth effects. In the present study, the Rayleigh approximation is applied because the stress process can be adequately described as a relatively small bandwidth process.

To determine the fatigue life of the keel structure, the wave data at the buoy location for a year are represented in the form of a wave scatter diagram. Buoy motion- and wave load tests were performed at the Indonesia Hydrodynamics Laboratory (IHL) BPPT, Surabaya, to obtain the stress transfer function of the keel structure. Fatigue tests, performed by Sahlan [2], resulted in a constant-amplitude S-N curve used as the basis in determining the fatigue life by using Palmgren-Miner’s rule.

Modeling the wave spectrum for each sea state is necessary to obtain the stress spectrum of the keel structure, which is required for the application of a spectral fatigue analysis. For that purpose, the wave scatter diagram is used as the basis for determining the wave spectra for different sea states using a JONSWAP model [7]:

\[
\tilde{S}_{\text{sw}}(\omega) = \frac{320.7 H_{\text{s1/3}}^2}{T_p^4} \omega^4 \exp \left[ -\frac{1930}{T_p^4} \omega^4 \right] \gamma^4
\]

(1)

where \(H_{\text{s1/3}}\) is the significant wave height, \(T_p\) is the peak period, \(\gamma\) is a peakedness parameter and \(A\) is an exponent.

The stress time-history recorded on the mooring line in the hydrodynamics model tests with \(H_s = 2\) m and \(T_p = 6\) sec is multiplied with the stress concentration factor...
(SCF) which is equal to 2.28 to obtain the stress at the keel structure. The SCF was calculated by Sahlan [2] and also reported by Sahlan et al. [3] using a finite element method. The stress time-history at the keel structure is then transformed into a stress spectrum (frequency domain) using Welch’s method [8].

The stress transfer function (or response amplitude operator, RAO) is obtained from the experimentally determined stress spectrum and the wave spectrum utilized in the model tests using the following relation:

\[ S_{\text{mm}}(\omega) = [\text{RAO}]^2 S_{\text{mm}}(\omega) \]  

(2)

where \( S_{\text{mm}}(\omega) \) is the stress spectrum and \( S_{\text{ww}}(\omega) \) is the wave spectrum.

The next step is to perform the calculation to obtain the stress spectrum according to the experimentally determined stress transfer function and the wave spectrum for each sea state in the wave scatter diagram. This is similar to changing the information of the stress characteristic in regular waves to the stress characteristic in random waves. The steps to calculate the fatigue life using the Rayleigh spectral method can be found, for example, in Baltrop and Adams [9].

For the fatigue life calculation it is necessary to calculate the area (zeroth moment, \( m_0 \)) and the second moment (\( m_2 \)) of the stress spectrum. In the present study, these moments are calculated by numerical integration by using a trapezium rule.

The area under the spectrum (\( m_0 \)) corresponds to the variance of the signal represented by the spectrum. (The variance is the mean square value of the signal calculated about its mean value.)

\[ m_0 = \int S_{\text{mm}}(\omega) d\omega \]  

(3)

\[ m_2 = \int S_{\text{mm}}(\omega) \omega^2 d\omega \]  

(4)

For a spectrum with rad/sec frequency-axis the mean zero-crossing period of the signal (\( T_z \)) is represented as

\[ T_z = \frac{2\pi \sqrt{m_0}}{m_2} \]  

(5)

The number of stress cycles (\( n \)) in time \( T \) seconds is therefore given by

\[ n = \frac{T}{T_z} \]  

(6)

The final equation for the damage in \( T \) seconds is

\[ D = \frac{n[m_0]}{A} \int \left[ \left( \frac{2 + m}{2} \right)^{1/n} \right] \sigma \]  

(7)

where \( A \) and \( m \) define the S-N curve:

\[ N = AS^{-m} \]  

(8)

The final equation for the damage rate for all sea states can also be written into

\[ D = \sum \frac{n}{N} = \sum \frac{n\sigma \varphi}{A} \]  

(9)

in which the constant amplitude stress range \( S \) is replaced by an effective fatigue stress range \( \sigma_{\text{efr}} \)

\[ \sigma_{\text{efr}} = \left( \frac{m_0}{m_2} \right)^{1/n} \left[ \frac{2}{2 + m} \right]^{1/n} \]  

(10)

The fatigue life (F.L.) in years is given as

\[ F.L. = \frac{1}{D} \]  

(11)

For the purpose of determining the stress transfer function, motion tests were performed at the Indonesia Hydrodynamics Laboratory (IHL) BPPT, Surabaya [2-3]. The tests utilized a JONSWAP-type wave-spectrum with significant wave height \( H_s = 2.0 \) m and peak period \( T_p = 6.0 \) sec (see Fig. 4).
Furthermore, the prototype is scaled to the model with a ratio between model and prototype of 1:16. The duration of the measurement for a particular test is 15 minutes (prototype test duration of 60 minutes). The stress is measured at the mooring line by using stress gauges. The stress at the keel structure is obtained by multiplying the stress measured at the mooring line with the stress concentration factor (SCF). The load acting on the keel structure is only that due to the cyclic wave loading. The stress time-series at the keel structure is shown in Fig. 5.

3. Results and Discussion

The spectral fatigue analysis methods, including the Rayleigh method and those to account for the bandwidth effects, have been established in the marine engineering practice [4-6,9]. Applications of the methods in the ship design process are reported, among others, by Wang [10] and Nguyen et al. [11].

Wave data analysis. As common practice in marine engineering, the wave data at the buoy operating location were processed into a wave scatter diagram. The diagram shows the number of occurrences of a sea state represented by the range of significant wave height \(H_s\) and average wave period \(T_p\). The wave scatter diagram at the location of the buoy under consideration is shown in Fig. 6 (see Table 5 for more details).

Furthermore, each combination of \(H_s\) and \(T_p\) is used to determine the wave spectrum by using Eq. (1) so that the stress spectrum can be calculated by using Eq. (2) and utilizing the stress transfer function that was obtained from the model tests.

Determination of the stress transfer function. To determine the stress transfer function, the stress time history obtained from the model tests (see Fig. 5) was transformed into a stress spectrum (frequency domain) by using Welch’s method [8]. The resulting stress spectrum is shown in Fig. 7.

The stress transfer function is obtained from Eq. (2), given the stress spectrum (Fig. 7) and the water-surface elevation (wave) spectrum (Fig. 4). The resulting stress transfer function is shown in Fig. 8.

The frequency range that has been selected in the subsequent calculations of the stress spectra of the sea states (represented by the wave scatter diagram) lies between 0.86 rad/sec and 2.02 rad/sec because in this frequency range significant wave energy is present. Outside this frequency range the wave energy is negligible (the wave spectral densities tend to zero) so that the resulting values of the stress transfer function calculated from Eq. (2) is rather inaccurate.

Determination of the values of \(m\) and \(A\) from the S-N curve. Fatigue tests of the keel material (from used ship-propeller shaft) was conducted by Sahlan [2] to obtain the S-N curve. Generally, the S-N curve is displayed in a log-log scale. This is shown in Fig. 9. The value of the slope \(s\) of the S-N curve is \(s = -0.0944\) so that the value of \(m = -s^{-1} = 10.59\). The value of \(A\) is obtained from Eq. (8), which is equal to \(1.36 \times 10^{11}\).
Determination of the zeroth and second moments for a variety of sea states. The stress spectra are calculated from Eq. (2), given the wave spectra for different sea states as given in the wave scatter diagram and the stress transfer function obtained from the model tests.

The results of the calculations of the zeroth moment \( m_0 \) and the second moment \( m_2 \) for a variety of sea states using Eqs. (3) and (4) are shown in Tables 2 and 3, respectively.

Table 1. Determination of the Fatigue Life for \( H_s = 2 \) Meters and \( T_p = 6 \) Seconds

| Item [Unit] | Value       |
|-------------|-------------|
| \( m_0 \) [MPa²] | 1269.1461   |
| \( m_2 \) [MPa².sec^{-2}] | 1532.2478   |
| \( T_Z \) [sec] | 5.7184      |
| \( T \) [sec] | 3600        |
| \( n \) [cycles] | 629.5516    |
| \( m \) | 10.5928     |
| \( \sigma_f \Gamma \) [MPa] | 166.2023    |
| \( A \) | 1.36E+31    |
| \( D \) [year^{-1}] | 1.54E-05    |
| \( F.L. \) [year] | 7.52        |

Table 2. Values of \( m_0 \) [MPa²] for a Variety of Sea States

| \( H_s \) [m] | \( T_z \) [sec] |
|---------------|-----------------|
| 4.75          | 5.25            |
| 5.25          | 5.75            |
| 5.75          | 6.25            |

As shown in Table 1, the resulting fatigue life of the keel structure is 7.52 years. This value is close to that calculated by Sahlan [2] (see also Sahlan et al. [3]), using a time-domain analysis and Palmgren-Miner’s rule, which is equal to 7.51 years.

By reviewing the above results, it can be said that the fatigue life calculations using an analysis in the time domain and a spectral method (frequency domain) can provide results that are close to each other. This conclusion gives confidence in the application of the spectral method to estimate the fatigue life of the keel structure for varying sea states. In the following, the stress transfer function derived from the model tests is used to calculate the stress spectra for different sea states according to the wave scatter diagram.
Calculation of the mean zero-crossing period of the stress spectrum for a variety of sea states. Once the values of $m_0$ and $m_2$ have been determined, the mean zero-crossing period ($T_z$) of the stress process for a variety of sea states can be calculated using Eq. (5). The results are shown in Table 4.

Table 3. Values of $m_2$ [MPa²·sec⁻²] for a Variety of Sea States

| $H_s$ [m] | 4.75 | 5.25 | 5.75 | 6.25 |
|-----------|------|------|------|------|
| 0.75      | 227  | 298  | 244  | 145  |
| 1.25      | 631  | 827  | 677  | 402  |
| 1.75      | 1621 | 1328 | 789  |      |
| 2.25      | 2195 | 1304 |      |      |
| 2.75      |      |      | 1948 |      |
| 3.25      |      |      |      |      |
| 3.75      |      |      |      |      |
| 4.25      |      |      |      |      |
| 4.75      |      |      |      |      |

Calculation of the number of stress cycles for a variety of sea states. The number of stress cycles (n) in a duration of $T$ can be calculated using Eq. (6). The durations $T$ for all sea states considered are shown in Table 5 and the results of the calculations of n are shown in Table 6.

Table 4. Values of $T_z$ [sec] of the Stress Spectra for a Variety of Sea States

| $H_s$ [m] | 6.75 | 7.25 | 7.75 | 8.25 |
|-----------|------|------|------|------|
| 0.75      | 103  | 83   | 68   | 36   |
| 1.25      | 286  | 230  | 188  | 154  |
| 1.75      | 560  | 451  |      |      |
| 2.25      | 925  |      |      |      |
| 2.75      | 1382 | 1113 |      |      |
| 3.25      | 1930 | 1555 |      |      |
| 3.75      | 2070 | 1696 |      |      |
| 4.25      | 2179 | 1786 |      |      |
| 4.75      |      | 2231 |      |      |

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| 3.25      |      |      |      |      |
| 3.75      |      |      |      |      |
| 4.25      |      |      |      |      |
| 4.75      |      |      |      |      |

| $H_s$ [m] | 6.75 | 7.25 | 7.75 | 8.25 |
|-----------|------|------|------|------|
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| 1.25      | 286  | 230  | 188  | 154  |
| 1.75      | 560  | 451  |      |      |
| 2.25      | 925  |      |      |      |
| 2.75      | 1382 | 1113 |      |      |
| 3.25      | 1930 | 1555 |      |      |
| 3.75      | 2070 | 1696 |      |      |
| 4.25      | 2179 | 1786 |      |      |
| 4.75      |      | 2231 |      |      |

Table 5. Values of $T$ [Hour] for a Variety of Sea States

| $H_s$ [m] | 4.75 | 5.25 | 5.75 | 6.25 |
|-----------|------|------|------|------|
| 0.75      | 36   | 409  | 209  | 27   |
| 1.25      | 4    | 1673 | 1704 | 729  |
| 1.75      | 62   | 1473 | 699  |      |
| 2.25      | 6    | 815  |      |      |
| 2.75      |      | 7    |      |      |
| 3.25      |      |      |      |      |
| 3.75      |      |      |      |      |
| 4.25      |      |      |      |      |
| 4.75      |      |      |      |      |

Table 6. Values of n [cycles] for a Variety of Sea States

| $H_s$ [m] | 4.75 | 5.25 | 5.75 | 6.25 |
|-----------|------|------|------|------|
| 0.75      | 22184| 231488| 115133| 15170|
| 1.25      | 2465 | 946893| 938694| 409602|
| 1.75      | 1.75 | 35091 | 811442 | 392746 |
| 2.25      | 2.25 | 3305  | 457922 |      |
| 2.75      | 2.75 | 5     | 3933  |      |
| 3.25      | 3.25 |      |      |      |
| 3.75      | 3.75 |      |      |      |
| 4.25      | 4.25 |      |      |      |
| 4.75      | 4.75 |      |      |      |

| $H_s$ [m] | 6.75 | 7.25 | 7.75 | 8.25 |
|-----------|------|------|------|------|
| 0.75      | 6    | 5    | 6    | 7    |
| 1.25      | 134  | 10   | 10   | 4    |
| 1.75      | 252  | 59   |      |      |
| 2.25      | 94   |      |      |      |
| 2.75      | 114  | 34   |      |      |
| 3.25      | 5    | 86   |      |      |
| 3.75      | 6    | 25   |      |      |
| 4.25      | 9    | 16   |      |      |
| 4.75      | 9    | 16   |      |      |
Calculation of the effective fatigue stress-range for a variety of sea states. The effective fatigue stress-range ($\sigma_{efr}$) for a variety of sea states can be calculated using Eq. (10). The results are shown in Table 7.

| $H_s$ [m] | $T_z$ [sec] | $\sigma_{efr}$ [MPa] |
|-----------|-------------|----------------------|
| 0.75      | 4.75        | 76                   |
| 1.25      | 5.25        | 95                   |
| 1.75      | 5.75        | 134                  |
| 2.25      | 6.25        | 210                  |
| 3.25      | 6.75        | 59                   |
| 3.75      | 7.25        | 65                   |
| 4.25      | 7.75        | 195                  |
| 4.75      | 8.25        | 225                  |

Determination of the fatigue life of the keel structure. According to Table 8, the accumulation of damage is 0.0898 for a year, so that the fatigue life of the keel structure calculated from Eq. (11) is $F.L. = \frac{1}{0.0898} = 11.14$ years, which is approximately 11 years.

4. Conclusions

A spectral fatigue analysis method has been applied to estimate the fatigue life of the keel structure of a tsunami buoy. A comparison between results obtained from a time-domain analysis and a spectral fatigue analysis (frequency domain) has shown that the two estimates are close to each other. Generally, the spectral method is preferred to the time-domain analysis because it can account for variations in sea states in which the structure is operated in an elegant and straightforward manner.

Utilizing the model-test results, the S-N curve obtained by Sahlan [2] and the wave data at the buoy operating location, at the coordinate (108.3417 E, 10.3998 S), it is found that the fatigue life of the keel structure of the tsunami buoy is approximately 11 years.

In the present study, corrosion effects have not been included in the estimation of the fatigue life of the keel structure. Inclusion of these would result in a smaller fatigue life than the result presented above. A further study is required to include such effects in the assessment of the fatigue life of the keel structure of the tsunami buoy.

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