Impact of uncertainties of unbound $^{10}\text{Li}$ on the ground state of two-neutron halo $^{11}\text{Li}$

Jagjit Singh$^1$ and W. Horiuchi$^2$

1 Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki 567-0047, Japan
2 Department of Physics, Hokkaido University, Sapporo, 060-0810 Japan

*jsingh@rcnp.osaka-u.ac.jp

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Abstract

Recently, the energy spectrum of $^{10}\text{Li}$ was measured upto 4.6 MeV, via $d(\text{^9Li}, \text{p})^{11}\text{Li}$, one-neutron transfer reaction. Considering the ambiguities on the $^{10}\text{Li}$ continuum spectrum with reference to new data, we report the configuration mixing in the ground state of the two-neutron halo nucleus $^{11}\text{Li}$ for two different choices of the $\text{^9Li} + \text{n}$ potential. For the present study, we employ a three-body (core + n + n) structure model developed for describing the two-neutron halo system by explicit coupling of unbound continuum states of the subsystem (core + n), and discuss the two-neutron correlations in the ground state of $^{11}\text{Li}$.

Contents

1 Introduction 2
2 Model Formulation 3
3 Two-body unbound subsystem (core + n) 3
4 Results and Discussions 5
5 Summary 8

References 9
1 Introduction

The light dripline nuclei lying away from the strip of stability, have gained prodigious attention of the nuclear physics community over the past few decades and a significant progress has been made both on experimental and theoretical sides to understand their exotic nature [1]. The one of the eye-catching phenomenon in some light dripline nuclei is the formation of halo, which is linked to the small binding energy of one or two valence nucleons [2,3]. Particularly two-neutron (2n) halo systems, consisting of a core and two weakly bound valence neutrons, demand a three-body description with proper treatment of continuum. The stability of such three-body (core+n+n) system is linked to the continuum spectrum of the two-body (core+n) subsystem. In this context, to explore the sensitivity of choice of a core+n potential with the configuration mixing in the ground state of three-body systems (core+n+n), we will discuss the results of the 2n-halo $^{11}$Li.

Although $^{11}$Li is the first observed two-neutron halo four decades ago [3]. Since then a lot of experimental and theoretical studies have been reported on structure of the $^{11}$Li. In order to understand the $^{11}$Li structure, the information over low-lying spectrum of $^{10}$Li is needed as a fundamental ingredient of three-body calculations. However, the $^{10}$Li structure was studied by various techniques such as fragmentation [4], $^{11}$Li$(p, d)^{10}$Li transfer reaction at TRIUMF [5], multi-neutron transfer [6] and pion absorption reactions [7]. Maximum of these studies report the low-lying $p_{1/2}$ neutron resonance with peak lying in the range of 500-700 keV. Also few of these studies reported the presence of s-wave virtual state close to the threshold with a scattering length in the range from $-20$ to $-30$ fm [4] and not much information is available on neutron $d$-wave.

Recently, the $^{10}$Li structure was investigated via $d(^{9}$Li, $p)^{11}$Li, one-neutron transfer reaction. This study reported $^{10}$Li energy spectrum up to 4.6 MeV, with the existence of $p_{1/2}$ resonance at $0.45\pm0.03$ MeV along with other two high lying structures at 1.5 and 2.9 MeV [8]. Also the role of $^{10}$Li resonances is investigated in the halo structure of $^{11}$Li via $^{11}$Li$(p, d)^{10}$Li transfer reaction at TRIUMF [5] and at the same facility the first conclusive evidence of a dipole resonance in $^{11}$Li having an isoscalar character has been reported [9,10]. In view of these new measurements and ambiguities over the experimental data, we aim to explore the sensitivity of the $^9$Li + n potential with the configuration mixing in the ground state of of three-body system ($^9$Li + n + n).

For this study, we use a three-body (core + n + n) structure model, developed for studying the weakly-bound ground and low-lying continuum states of Borromean systems sitting at the edge of neutron dripline [11]. In our approach, we start from the solution of the unbound subsystem (core + n) and the two-particle basis is constructed by explicit coupling of the two single-particle continuum wave functions. Initially, it was tested for the lightest 2n-halo $^6$He [12,13], heaviest known 2n-halo $^{22}$C [14] and 2n-unbound $^{20}$O [15] and has been successful in explaining the ground-state properties and the electric-dipole and quadrupole responses.

In this contribution, Sec. 2 briefly describes the formulation of our three-body structure model. In Sec. 3 we analyze the subsystem $^{10}$Li and fix the two different sets for $^9$Li + n potential, consistent with available experimental information. Section 4 presents our results for the three-body system, $^9$Li + n + n. Summary is made in Sec. 5.
2 Model Formulation

The three-body wave function for the $^9\text{Li} + n + n$ system is specified by the Hamiltonian

$$H = -\frac{\hbar^2}{2\mu} \sum_{i=1}^{2} \nabla_i^2 + \sum_{i=1}^{2} V_{\text{core}+n}(\vec{r}_i) + V_{12}(\vec{r}_1, \vec{r}_2),$$  \hspace{1cm} (1)

where $\mu = A_{c}m_N/(A_{c}+1)$ is the reduced mass, and $m_N$ and $A_{c} = 9$ are the nucleon mass and mass number of the core nucleus, respectively. $V_{\text{core}+n}$ is the core-neutron potential and $V_{12}$ is $n$-$n$ potential. The neutron single-particle unbound $s$-, $p$-, and $d$-wave continuum states of the subsystem ($^{10}\text{Li}$) are calculated in a simple shell model picture for different continuum energy $E_C$ by using the Dirac-delta normalization and are checked with a more refined phase-shift analysis. Each single-particle continuum wave function of $^{10}\text{Li}$ is given by

$$\phi_{\ell jm}(\vec{r}, E_C) = R_{\ell j}(r, E_C)[Y_{\ell}(\Omega) \times \chi_{1/2}^{(j)}].$$ \hspace{1cm} (2)

We use the mid-point method to discretize the continuum. The convergence of the results will be checked with the continuum energy cut $E_{\text{cut}}$ and $\Delta E$. These core $+ n$ continuum wave functions are used to construct the two-particle $^{11}\text{Li}$ states by proper angular momentum couplings and taking contribution from different configurations. The combined tensor product of these two continuum states is given by

$$\psi_{JM}(\vec{r}_1, \vec{r}_2) = \left[ \phi_{\ell_1 j_1}(\vec{r}_1, E_{C1}) \times \phi_{\ell_2 j_2}(\vec{r}_2, E_{C2}) \right]^{(J)}_{M}. \hspace{1cm} (3)$$

We use a density-dependent (DD) contact-delta pairing interaction [16], given by

$$V_{12} = \delta(\vec{r}_1 - \vec{r}_2) \left( v_0 + \frac{v_\rho}{1 + \exp [(r_1 - R_\rho)/a_\rho]} \right).$$ \hspace{1cm} (4)

The first term in Eq. (4) with $v_0$ simulates the free $n$-$n$ interaction, which is characterized by its strength and the second term in Eq. (4) represents density-dependent part of the interaction. The strengths $v_0$ and $v_\rho$ are scaled with the $\Delta E$ by following relation from Ref. [14]. The $v_\rho$ is the parameter which will be fixed to reproduce the ground-state energy. For a detailed formulation and calculation procedure one can refer to Refs. [11–13,17].

3 Two-body unbound subsystem (core $+ n$)

The investigation of the two-body (core $+ n$) subsystem is crucial in understanding the three-body system (core $+ n + n$). The interaction of the core with the valence neutron ($n$) plays a fundamental role in the binding mechanism of the three-body system. The elementary concern over the choice of a core $+ n$ potential is the ambiguities in the experimental information about the core $+ n$ system. We employ the following core $+ n$ potential

$$V_{\text{core}+n} = \left( -V_0^\ell + V_{\ell s} \ell \cdot \frac{1}{r} \frac{d}{dr} \right) \frac{1}{1 + \exp \left( \frac{r - R_c}{a} \right)}, \hspace{1cm} (5)$$

where $R_c = r_0A_c^{1/3}$ with $r_0$ and $a$ are the radius and diffuseness parameter of the Woods-Saxon potential. The values of $r_0 = 1.27 \text{ fm}$ and $a = 0.67 \text{ fm}$ are adopted from Refs. [16,18].
Table 1: Parameter sets of the core-$n$ potential for $\ell = 0, 1, 2$ states of a $^9\text{Li} + n$ system. The possible resonances with resonance energy $E_R$ and decay width $\Gamma$ in MeV are also tabulated.

| Set | $\ell_j$ | $V_0^\ell$ (MeV) | $V_{ts}$ (MeV) | $E_R$ (MeV) | $\Gamma$ (MeV) |
|-----|---------|-----------------|---------------|-------------|-------------|
| A   | $s_{1/2}$ | 50.50           | –             | –           | –           |
|     | $p_{1/2}$ | 40.00           | 21.02         | 0.46        | 0.36        |
|     | $d_{5/2}$ | 47.50           | 21.02         | 2.98        | 1.39        |
| B   | $s_{1/2}$ | 47.50           | –             | –           | –           |
|     | $p_{1/2}$ | 40.00           | 21.02         | 0.46        | 0.36        |
|     | $d_{5/2}$ | 47.50           | 21.02         | 2.98        | 1.39        |

Figure 1: $^9\text{Li}+n$ phase shifts for $1/2^-$ and $5/2^+$ states corresponding to core+$n$ potential tabulated in Table 1.

For the present calculations we ignore the spin of the core $^9\text{Li}$. The neutron number 6 is assumed for the neutron core configuration given by $(0s_{1/2})^2(0p_{3/2})^4$. The four valence neutron continuum orbits, i.e., $p_{1/2}$, $d_{5/2}$, $s_{1/2}$ and $d_{3/2}$ are considered in the present calculations for $^{10}\text{Li}$. $^{10}\text{Li}$ is interesting in the sense that it shows inversion of $s_{1/2}$ and $p_{1/2}$ levels.

The scattering length of the virtual $s$-state, position and width of low-lying $p$-resonance along with higher lying $\ell = 2$ resonance vary from experiment to experiment. In the view of the new experimental measurements [5, 8], we use two different potential sets for core + $n$ potential, which are tabulated in Table 1. The only difference between our two sets A and B is we use different $s$-wave depth ($V_0^0$), leading to different scattering length of the $s_{1/2}$ virtual state, which further effect the $s$-wave component in ground state of $^{11}\text{Li}$. In our set A the $s$-wave potential is deep enough to increase the $s$-component dominance in the ground state of $^{11}\text{Li}$ in comparison to set B. Our both sets reproduces the observed $p_{1/2}$ resonance at
0.45 MeV consistent with Ref. [8] and the $d_{5/2}$ resonance, that lies at higher energy around 2.98 MeV, this position is consistent with the high-lying structure of $^{10}\text{Li}$ reported in Ref. [8]. The phase-shifts corresponding to these resonances are shown in Fig. 1. Similar potentials are used also in Refs. [16,18].

### 4 Results and Discussions

The three-body model with two non-interacting particles in the above single-particle levels of $^{10}\text{Li}$, produces different parity states, when two neutrons are placed in different unbound orbits mentioned in Sec. 3 (for details see Table. 2). The corresponding oscillatory single-particle continuum wave functions for $s_{1/2}$, $p_{1/2}$, $d_{5/2}$, and $d_{3/2}$ states are plotted in Fig. 2. The four configurations ($s_{1/2})^2$, ($p_{1/2})^2$, ($d_{5/2})^2$, ($d_{3/2})^2 couple to $J^\pi = 0^+$ for $^{11}\text{Li}$.

**Table 2: Possible configurations of $^{11}\text{Li}$ arising from two neutrons in $s$-, $p$- and $d$-orbitals.**

|       | $s_{1/2}$ | $p_{1/2}$ | $d_{3/2}$ | $d_{5/2}$ |
|-------|-----------|-----------|-----------|-----------|
| $s_{1/2}$ | 0$^+$    | 0$^-$, 1$^-$ | 1$^+$, 2$^+$ | 2$^+$, 3$^+$ |
| $p_{1/2}$ | 0$^+$    |            | 1$^-$, 2$^-$ | 2$^-$, 3$^-$ |
| $d_{3/2}$ | 0$^+$, 2$^+$ |           | 1$^+$, 2$^+$, 3$^+$, 4$^+$ |
| $d_{5/2}$ |          |           | 0$^+$, 2$^+$, 4$^+$ |

Figure 2: $^{9}\text{Li}+n$ continuum waves as a function of radial variable for continuum energies 1, 2.5 and 5 MeV, respectively.

The continuum single-particle wavefunctions are calculated with energies from 0.0 to 5.0 MeV and normalized to a delta for the spd-states of $^{10}\text{Li}$ on a radial grid which varies from 0.1 to 100.0 fm with the $^{9}\text{Li}+n$ potential discussed in Sec. 3. In the three-body calculations, along
with the core + n potential the other important ingredient is the n-n interaction. We use the DD contact-delta pairing interaction, with the only adjustable parameter being \( v_\rho \). The two particle states are formed using mid-point method with an energy spacing of 2.0, 0.5, 0.25 and 0.1 MeV corresponding to block basis dimensions of \( N = 5, 10, 20 \) and 50, respectively, and the matrix elements of the pairing interaction are calculated. In Fig. 3, the eigenspectrum for \( J = 0^+ \) case is presented and from figure it is clear that with increase in basis dimensions the superfluous bound states moves into the continuum. The biggest adopted basis size gives a fairly dense continuum in the region of interest.

In the DD contact-delta pairing interaction (defined by Eq. (4)), the strength of the DI part is given as \( v_0 = 2\pi^2 \frac{b^2}{m_N} \frac{2a_{nn}}{\pi - 2k_c a_{nn}} \), where \( a_{nn} \) is the scattering length for the free neutron-neutron scattering and \( k_c \) is related to the cutoff energy, \( e_c \), as \( k_c = \sqrt{\frac{m_N e_c}{\hbar^2}} \). We use \( a_{nn} = -15 \text{ fm} \) and \( e_c = 30 \text{ MeV} \) [16], which leads to \( v_0 = 857.2 \text{ MeV fm}^3 \). For the parameters of the DD part, we determine them so as to reproduce the two-neutron separation energy of \(^{11}\text{Li}, S_{2n} = -0.369 \text{ MeV} \) [19]. The values of the parameters that we employ are \( R_\rho = 1.25 \times A_c^{1/3} \) (\( A_c = 9 \)) and \( v_\rho = 862.5 \) and 861.75 MeV fm\(^3\) for set A and B, respectively.

We report the percentage configuration mixing in the ground state of \(^{11}\text{Li} \) in Table 3. We found that for Set A for which \( V_0^0 \) is deeper shows dominance of \((s_{1/2})^2\) configuration in the ground state leading to formation of s-neutron halo. Whereas for Set B for which \( V_0^0 \) is shallower shows dominance of \((p_{1/2})^2\) configuration in the ground state leading to formation of p-neutron halo. The preliminary numbers for calculated matter radii with these potential sets are 3.53 and 3.24 fm for Set A and B, respectively. These results of configuration mixing and matter radii are consistent with the results of Refs. [16,20] for \(^{11}\text{Li} \). The detailed investigation of the configuration mixing with inclusion of core spin is in progress.
Table 3: Components of the ground state of $^{11}$Li in %, with the model parameters energy cut, $E_{cut} = 5$ MeV and bin size, $\Delta E = 0.1$ MeV. The core+n potential used are tabulated in Table 1.

| Set | $l_j$ | Present work | Reference [20] |
|-----|-------|--------------|----------------|
| A   | $(s_{1/2})^2$ | 55.5          | 64.0           |
|     | $(p_{1/2})^2$ | 33.1          | 30.0           |
|     | $(d_{5/2})^2$ | 7.1           | 3.0            |
| B   | $(s_{1/2})^2$ | 24.5          | 27.0           |
|     | $(p_{1/2})^2$ | 59.6          | 67.0           |
|     | $(d_{5/2})^2$ | 9.1           | 3.0            |

Figure 4: Two-particle density for the ground state of $^{11}$Li for Set A (upper-panel) and Set B (lower-panel) as a function $r_1 = r_2 = r$ and the opening angle between the valence neutrons $\theta_{12}$ for settings mentioned in caption of Table 3.

The two particle density of $^{11}$Li as a function of two radial coordinates, $r_1$ and $r_2$, for valence neutrons, and the angle between them, $\theta_{12}$ in the LS-coupling scheme is given by

$$\rho(r_1, r_2, \theta_{12}) = \rho^{S=0}(r_1, r_2, \theta_{12}) + \rho^{S=1}(r_1, r_2, \theta_{12})$$  (6)
The explicit expression for \( S = 0 \) component is given by \([16, 21]\)

\[
\rho^{S=0}(r_1, r_2, \theta_{12}) = \frac{1}{8\pi} \sum_{L} \sum_{\ell,j} \sum_{\ell',j'} \hat{\ell} \hat{\ell}' \hat{L} \hat{L} \left( \begin{array}{c} \ell & \ell' & L \\ 0 & 0 & 0 \end{array} \right)^2 (-1)^{\ell' + \ell'} \sqrt{\frac{2j + 1}{2\ell + 1}} \sqrt{\frac{2j' + 1}{2\ell' + 1}} \psi_{\ell j}(r_1, r_2) \psi_{\ell' j'}(r_1, r_2) Y_{L0}(\theta_{12})
\]

(7)

where \( \hat{\ell} = \sqrt{2\ell + 1} \) and \( \psi_{\ell j}(r_1, r_2) \) is the radial part of the two-particle wave function which is determined from Eq. (3) by making use of Eqs. (5) and (6) of \([13]\).

Figure 4 shows the two-particle density plotted as a function of the radius \( r_1 = r_2 = r \) and their opening angle \( \theta_{12} \), with a weight factor of \( 4\pi r^2 \cdot 2\pi r^2 \sin\theta_{12} \) for both Sets A (upper panel) and B (lower panel). The distribution at smaller and larger \( \theta_{12} \) are referred to as “di-neutron” and “cigar-like” configurations, respectively. One can see in Fig. 4 that the two-particle density is well concentrated around \( \theta_{12} \leq 90^\circ \) for both Sets A (upper panel) and B (lower panel), which is the clear indication of the di-neutron correlation. The di-neutron component has a relatively higher density in comparison to the small cigar-like component for both sets in the ground state of \(^{11}\text{Li}\). The two peak structure in the two-particle density is attributed to the mixing of the s- and p-wave components \( (\ell \leq 1) \) in the ground state of \(^{11}\text{Li}\).

5 Summary

In the present study we report the emergence of bound 2\( n \)-halo ground state of \(^{11}\text{Li}\) from the coupling of four unbound spd-waves in the continuum of \(^{10}\text{Li}\) due to the presence of pairing interaction. The configuration mixing in the ground state of \(^{11}\text{Li}\) has been reported for the two particular choices of core+\( n \) potential, fixed in the view of the available recent experimental data. Also, the 2\( n \)-neutron correlation for this system showing prominence of the di-neutron component is discussed. Investigations with different choices of pairing interactions and inclusion of spin of core (\(^{9}\text{Li}\)) are in progress and will be reported elsewhere.

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