Formation of millisecond pulsars with helium white dwarfs, ultra-compact X-ray binaries and gravitational wave sources

Hai-Liang Chen, 1,2,3* Thomas M. Tauris, 3,4 Zhanwen Han, 1,2,5,6 Xuefei Chen, 1,2,5,6†
Close-orbit low-mass X-ray binaries (LMXBs), radio binary millisecond pulsars (BMSPs) with extremely low-mass helium WDs (ELM He WDs) and ultra-compact X-ray binaries (UCXBs) are all part of the same evolutionary sequence. However, the formation and evolutionary link between these three different populations of neutron star (NS) binaries are not fully understood. In particular, a peculiar fine-tuning problem has previously been demonstrated for the formation of these systems.

In this investigation, we test a newly suggested magnetic braking prescription and model the formation and evolution of LMXBs. We compute a grid of binary evolution models and present the initial parameter space of the progenitor binaries which successfully evolve all the way to produce UCXBs. We find that the initial orbital period range of LMXBs, which evolve into detached NS+ELM He WD binaries and later UCXBs, becomes significantly wider compared to evolution with a standard magnetic braking prescription, and thus helps to relieve the fine-tuning problem.
Fine tuning problem: Only LMXBs in a very narrow range of initial orbital periods can reproduce the observed ELM He WDs, using standard prescriptions of orbital angular momentum losses via MB, mass loss and GWs. Istrate et al. (2014b)

**Code:** Modules for Experiments in Stellar Astrophysics (MESA) Paxton et al. 2019

**Assumptions:**
- The accretion rate of the NS is limited by Eddington mass-accretion rate
- The accretion efficiency of the NS is 0.30, i.e. 30% of the material transferred from the donor star is accreted by the NS
- Formation of circumbinary coplanar disk is not considered
Loss of angular momentum

Due to mass loss
\[
\frac{\dot{J}_{\text{ml}}}{J_{\text{orb}}} = \frac{\mu}{M_{\text{NS}}^2} \beta \frac{\dot{M}_2}{1 + q} \frac{\dot{M}_2}{M_2}
\]
where \(\mu\) is the reduced mass, \(\beta\) is the accretion efficiency and \(q = M_2/M_{\text{NS}}\).

Due to GW radiation
\[
\frac{\text{d}J_{\text{gw}}}{\text{d}t} = -\frac{32}{5} \frac{G^{7/2}}{c^5} \frac{M_{\text{NS}}^2 M_2^2 (M_{\text{NS}} + M_2)^{1/2}}{a^{7/2}}
\]
where \(\alpha\) is the binary separation.

Due to magnetic braking
\[
\frac{\text{d}J_{\text{mb}}}{\text{d}t} = \frac{\text{d}J_{\text{mb,Sk}}}{\text{d}t} \left( \frac{\omega_2}{\omega_{\odot}} \right)^\beta \left( \frac{\tau_{\text{conv}}}{\tau_{\odot,\text{conv}}} \right)^\xi \left( \frac{\dot{M}_{2,\text{wind}}}{M_{\odot,\text{wind}}} \right)^\alpha
\]
where \(\text{d}J_{\text{mb,Sk}}/\text{d}t\) is the standard Skumanich MB law, \(\omega_2\) is the angular velocity of the donor star, \(\tau_{\text{conv}}\) is the turnover time of convective eddies (measure of the depth of the convective zone) and \(\dot{M}_{2,\text{wind}}\) is the mass-loss rate of the donor star.

This stronger MB prescription will significantly reduce the binary separation on a shorter timescale compared to standard MB. Four cases are considered.

\[
(\beta, \xi, \alpha) = \begin{cases} 
(0, 0, 0) & \text{MB1 — standard MB} \\
(0, 2, 0) & \text{MB2 — convection-boosted MB} \\
(0, 2, 1) & \text{MB3 — intermediate MB} \\
(2, 4, 1) & \text{MB4 — wind-boosted MB}
\end{cases}
\]
Examples of binary evolution

In all three examples, the orbital periods decrease significantly, down to about 1 d (due to the relatively efficient MB prescription MB3) before RLO is initiated.

The system with the smallest initial orbital period never becomes detached from RLO in the LMXB stage and therefore does not produce a detached He WD to become a BMSP.

The second system after the LMXB stage becomes a detached binary (black circle) while within the grey area and is observed as a BMSP with an ELM He WD. It further evolves into an UCXB (onset of second RLO second red star) with a minimum $P_{\text{orb}}$ of 7 min.

The third system, after the LMXB stage becomes a detached binary (black circle) with a relatively large binary separation. Thereafter, the angular momentum loss due to GW radiation is not strong enough to make the system semi-detached and start another phase of RLO within Hubble time it will therefore not evolve to become a UCXB.

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Results

Binary systems with donor masses, $M_2 > 1.50 M_\odot$ can only evolve into BMSPs with He WD masses $> 0.2 M_\odot$. They will not evolve to a second RLO within a Hubble time.

For binary systems with initial $P_{\text{orb}} > 25$ d calculations stop because of numerical convergence problems shortly after the onset of RLO due to very large values of mass-transfer rates (development of CE).

The initial parameter space for $P_{\text{orb}}$ is significantly extended for binary systems that evolve into BMSPs and further into UCXBs (red stars).

Systems with larger initial orbital periods (prior to the LMXB phase) end up with systematically smaller orbital periods at the onset of the UCXB phase reaching a minimum between 5 - 9 min.
Results

Initial parameter space under the influence of MB

If the MB is turned completely off, there are no systems which can evolve into detached NS+HeWDs (and UCXBs).

For models with $\alpha=\xi=\beta=0$ (MB1), only systems with $M_2 = 1.25 M_\odot$ and initial orbital period, $P_{\text{orb}} = 2.985 - 2.990$ d can evolve into detached NS+He WDs and then UCXBs, in analogy with the findings of Istrate et al. (2014b).

For models with $\alpha=0, \xi=2, \beta=0$ (MB2), the parameter space for $P_{\text{orb}}$ is very narrow.

For models with $\alpha=0, \xi=2, \beta=1$ (MB3), the parameter space for $P_{\text{orb}}$ becomes significantly wider and thus helps to relieve the fine-tuning problem.

For models with $\alpha=2, \xi=4, \beta=1$ (MB4) the evolutionary tracks lead to numerical instabilities, and do not converge.
Results

Not so fast!

The $\alpha=0,\xi=2,\beta=1$ MB3 model cannot reproduce the wide-orbit BMSPs. The $\alpha=0,\xi=2,\beta=0$ MB2 model can reproduce the wide-orbit BMSPs but not the close-orbit BMSPs.

Conclusion: Although the MB3 prescription relieves the fine-tuning problem in producing UCXBs, in its present form it fails as a candidate for a universal MB prescription. Further investigations into alternative MB prescriptions are therefore needed.

THANK YOU