Constraints on the skewness coefficient of symmetric nuclear matter within the nonlinear relativistic mean field model

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Within the nonlinear relativistic mean field (NL-RMF) model, we show that both the pressure of symmetric nuclear matter at supra-saturation densities and the maximum mass of neutron stars are sensitive to the skewness coefficient \( J_0 \) of symmetric nuclear matter. Using experimental constraints on the pressure of symmetric nuclear matter at supra-saturation densities from flow data in heavy ion collisions and the astrophysical observation of a large mass neutron star PSR J0348+0432, with the former favoring a smaller \( J_0 \) while the latter a larger \( J_0 \), we extract a constraint of \(-494\text{MeV} \leq J_0 \leq -10\text{MeV}\) based on the NL-RMF model. This constraint is compared with the results obtained in other analyses.

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I. INTRODUCTION

Determination of the equation of state (EOS) of asymmetric nuclear matter (ANM) is one of fundamental questions in contemporary nuclear physics and astrophysics. The exact knowledge on the EOS of ANM provides important information on the in-medium nuclear effective interactions which play a central role in understanding the structure and decay properties of finite nuclei as well as the related dynamical problems in nuclear reactions \([1–16]\). The EOS of ANM also plays a decisive role in understanding a number of important issues in astrophysics including the structure and evolution of neutron stars as well as the mechanism of supernova explosion \([17–23]\). Conventionally, the EOS of ANM is given by the binding energy per nucleon as functions of nucleon density \( \rho \) and isospin asymmetry \( \delta \), i.e., \( E(\rho, \delta) \), and some bulk characteristic parameters defined at the saturation density \( \rho_0 \) of symmetric nuclear matter (SNM) are usually introduced to quantitatively characterize the EOS of ANM. For example, the energy \( E_0(\rho_0) \) and incompressibility \( K_0 \) of SNM as well as the symmetry energy \( E_{\text{sym}}(\rho_0) \) and its slope parameter \( L \) are the four famous lower-order bulk characteristic parameters of EOS of SNM. These bulk parameters defined at \( \rho_0 \) provide important information on both sub- and supra-saturation density behaviors of the EOS of ANM \([24, 25]\).

Based on the empirical liquid-drop-like model analyses of high precision data about nuclear masses, the \( E_0(\rho_0) \) is well known to be about \(-16\text{MeV}\). The incompressibility has been determined to be \( K_0 = 240 \pm 40 \text{MeV} \) from analyzing experimental data of nuclear giant monopole resonances (GMR) \([1–20, 29]\). For \( E_{\text{sym}}(\rho_0) \) and \( L \), the existing constraints extracted from terrestrial laboratory measurements and astrophysical observations are found to be essentially consistent with \( E_{\text{sym}}(\rho_0) = 32.5 \pm 2.5 \text{MeV} \) and \( L = 55 \pm 25 \text{MeV} \) (see, e.g., Refs. \([31, 32]\)). While these lower-order bulk characteristic parameters have been relatively well determined or in significant progress, our knowledge on the higher-order bulk characteristic parameters remains very limited. Following \( E_0(\rho_0) \), \( K_0 \), \( E_{\text{sym}}(\rho_0) \) and \( L \), the next bulk characteristic parameter should be the skewness coefficient \( J_0 \) (also denoted as \( K' \) or \( Q_0 \) in some literature) of SNM, which is related to the third-order density derivative of the binding energy per nucleon of SNM at \( \rho_0 \). The higher-order bulk characteristic parameter \( J_0 \) is expected to be important for the high density behaviors of nuclear matter EOS and thus may play an essential role in heavy ion collisions (HIC), the structure and evolution of neutron stars, supernova explosion, and gravitational wave radiation from merging of compact stars. To our best knowledge, so far there is very little experimental information on the \( J_0 \) parameter, and it is thus of great interest and critical importance to constrain the \( J_0 \) parameter, which is the main motivation of the present work.

Within the nonlinear relativistic mean field (RMF) model, we demonstrate in this work that the pressure of SNM at supra-saturation densities and the maximum mass of neutron stars provide good probes of the skewness coefficient \( J_0 \). In particular, combining the experimental constraints on the pressure of SNM at supra-saturation densities from flow data in HIC and the recent astrophysical observation of a large mass neutron star PSR J0348+0432, one can obtain a strong constraint on the \( J_0 \) parameter.

II. THE SKEWNESS COEFFICIENT \( J_0 \) IN NONLINEAR RMF MODEL

A. Nuclear matter characteristic parameters

The EOS of isospin asymmetric nuclear matter, namely \( E(\rho, \delta) \), can be expanded as a power series of
even-order terms in $\delta$ as
\[
E(\rho, \delta) \simeq E_0(\rho) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^4),
\]
where $E_0(\rho) = E(\rho, \delta = 0)$ is the EOS of symmetric nuclear matter, and the symmetry energy is expressed as
\[
E_{\text{sym}}(\rho) = \frac{1}{2} \frac{\partial^2 E(\rho, \delta)}{\partial \delta^2} \bigg|_{\delta=0}.
\]
Around the saturation density $\rho_0$, the $E_0(\rho)$ can be expanded, e.g., up to 3rd-order in density, as,
\[
E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2!} \chi^2 + \frac{J_0}{3!} \chi^3 + O(\chi^4),
\]
where $\chi = (\rho - \rho_0)/3\rho_0$ is a dimensionless variable characterizing the deviations of the density from the saturation density $\rho_0$. The first term $E_0(\rho_0)$ on the right-hand-side of Eq. (3) is the binding energy per nucleon in SNM at $\rho_0$ and the coefficients of other terms are
\[
K_0 = 9\rho_0^2 \frac{\partial^2 E(\rho_0)}{\partial \rho^2} \bigg|_{\rho=\rho_0},
\]
\[
J_0 = 27 \rho_0 \frac{\partial^3 E(\rho_0)}{\partial \rho^3} \bigg|_{\rho=\rho_0},
\]
where $K_0$ is the well-known incompressibility coefficient of SNM and $J_0$ is the skewness coefficient of SNM, i.e., the 3rd-order incompressibility coefficient of SNM (24 [25]).

Similarly, one can expand the $E_{\text{sym}}(\rho)$ around an arbitrary reference density $\rho_r$ as
\[
E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_r) + L(\rho_r)\chi_r + O(\chi_r^2),
\]
with $\chi_r = (\rho - \rho_r)/3\rho_r$, and the slope parameter of the symmetry energy at $\rho_r$ is expressed as
\[
L(\rho_r) = 3\rho_r \frac{\partial E_{\text{sym}}(\rho_r)}{\partial \rho} \bigg|_{\rho=\rho_r}.
\]
For $\rho_r = \rho_0$, the $L(\rho_r)$ is reduced to the conventional slope parameter $L = 3\rho_0 \partial E_{\text{sym}}(\rho)/\partial \rho|_{\rho=\rho_0}$.

If $\delta$ and $\chi$ are assumed to be small quantities on the same order, nuclear matter bulk characteristic parameters can then be classified accordingly in different orders. For example, $L$ and $J_0$ are on the same order-3, i.e., $\delta^3 \chi$ for $L$ and $\delta^2 \chi^3$ for $J_0$. In this sense, $E_0(\rho_0)$ is on the order-0, $K_0$ and $E_{\text{sym}}(\rho_0)$ are on the order-2. To see the role of $J_0$ in the EOS of SNM, one can re-write Eq. (3) in a slightly different form as
\[
E_0(\rho) \simeq E_0(\rho_0) + \frac{1}{2} K_0 \chi^2 \left(1 + \frac{\chi J_0}{3K_0}\right).
\]
Assuming $J_0$ has roughly the same magnitude as $K_0$, one can see that the contribution from the $J_0$ term to the EOS of SNM becomes comparable with that from the $K_0$ term if the baryon density is larger than about $3\rho_0$, corresponding to the typical densities inside a neutron star. On the other hand, the $J_0$ term plays a minor role for the EOS of SNM at sub-saturation densities relevant for nuclear structure properties. As we will see later, the pressure of SNM at supra-saturation densities and the maximum mass of neutron stars indeed display strong sensitivity on the $J_0$ parameter.

### B. Nuclear matter characteristic parameters in nonlinear RMF model

The nonlinear RMF model has made great success during the last decades in describing many nuclear phenomena (see, e.g., [34 16]). In the following, we briefly describe the nonlinear RMF model that we shall adopt in this work and present some useful expressions of nuclear matter characteristic parameters, especially the skewness coefficient $J_0$. The interacting Lagrangian of the nonlinear RMF model supplemented with couplings between the isoscalar and the isovector mesons reads (47 51)
\[
\mathcal{L} = \bar{\psi} \left[ m_0 i \not{\partial} - g_{\omega} \omega \not{\sigma} - g_{\rho} \not{\rho} \cdot \not{\tau} \right] \psi - \frac{1}{2} m_\omega^2 \sigma^2 + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma) + \frac{1}{2} m_\omega^2 \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{4} c_\omega (g_{\omega} \omega_{\mu\nu})^2 + \frac{1}{2} g_\rho^2 \rho_\mu \cdot \rho^\mu - \frac{1}{4} g_\rho^2 \rho_{\mu\nu} \rho^{\mu\nu} + \frac{1}{2} g_\omega^2 \omega_{\mu\nu} \Lambda \gamma g_{\omega} \omega_{\mu\nu},
\]
where $\omega_{\mu\nu} \equiv \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ and $\rho_{\mu\nu} \equiv \partial_\mu \rho_\nu - \partial_\nu \rho_\mu$ are strength tensors for $\omega$ field and $\rho$ field, respectively, $\psi, \sigma, \omega_{\mu\nu}, \rho_{\mu\nu}$ are nucleon field, isoscalar-scalar field, isoscalar-vector field and isovector-vector field, respectively, and the arrows denote the vector in isospin space, $U(\sigma) = b_\sigma M(3g_\sigma)3/3 + c_\sigma (g_\sigma)^2/4$ is the self interaction term for $\sigma$ field. $\Lambda_\gamma$ represents the coupling constant between the isovector $\rho$ meson and the isoscalar $\omega$ meson and it is important for the description of the density dependence of the symmetry energy. In addition, $M$ is the nucleon mass and $m_\sigma, m_\omega, m_\rho$ are masses of mesons.

In the mean field approximation, after neglecting effects of fluctuation and correlation, meson fields are replaced by their expectation values, i.e., $\not{\sigma} \rightarrow \sigma$, $\not{\omega_0} \rightarrow \omega_{\mu\nu}$, $\not{\rho_0} \rightarrow \rho_{\mu\nu}$, where subscript "0" indicates zeroth component of the four-vector, superscript "(3)" indicates third component of the isospin. Furthermore, we also use in this work the non-sea approximation which neglects the effect due to negative energy states in the Dirac sea. The mean field equations are then expressed as
\[
m_\sigma^2 \overline{\sigma} = g_\sigma \left[ \rho_3 - b_\sigma M (g_\sigma \overline{\sigma})^2 - c_\sigma (g_\sigma \overline{\sigma})^3 \right],
\]
\[
m_\omega^2 \overline{\omega_0} = g_\omega \left[ \rho - c_\omega (g_\omega \overline{\omega_0})^3 - \Lambda_\gamma g_\omega \omega_0 \left(g_\rho \rho_0^{(3)}\right)^2 \right],
\]
\[
m_\rho^2 \rho_0^{(3)} = g_\rho \left[ \rho - \rho_3 - \Lambda_\gamma g_\rho \rho_0^{(3)} (g_\omega \omega_0)^2 \right],
\]
are the baryon density and scalar density, respectively, with the latter given by

$$\rho_s = \langle \psi \psi \rangle = \rho_{s,n} + \rho_{s,p},$$

(13)

In the above expression, we have

$$E_F^{J*} = \sqrt{k_F^{J*} + M^{J*2}}$$

and the nucleon Dirac mass is defined as

$$M^* \equiv M_{\text{dirac}}^* = M - g_\sigma \sigma.$$  

(15)

$k_F^* = k_F(1 + \tau_3^* / \delta)^{1/3}$ is the Fermi momentum with $\tau_3^* = +1$ for neutrons and $\tau_3^* = -1$ for protons, and $k_F = (3\pi^2 / 2)^{1/3}$ is the Fermi momentum for SNM at $\rho$.

The energy-momentum density tensor for the interacting Lagrangian density in Eq. (19) can be written as

$$T^{\mu\nu} = \psi \partial^\mu \psi \bar{\psi} \partial^\nu \bar{\psi} + \partial^\mu \rho \partial^\nu \bar{\rho} + \omega^\mu \omega^{\nu} \rho - \rho \omega^{\mu} \omega_{\nu} - \rho \omega^{\mu} \partial^\nu \bar{\rho} - \bar{E} g^{\mu\nu},$$

(16)

where $g^{\mu\nu} = (+, -, -, -)$ is the Minkowski metric. In the mean field approximation, the mean value of time (zero) component of the energy-momentum density tensor is the energy density of the nuclear matter system, i.e.,

$$\varepsilon = (T^{00})$$

$$\varepsilon = \varepsilon_{\text{kin}} + \varepsilon_{\text{kin}}^{\rho} + \frac{1}{2} \left[ m_2^2 \sigma^2 + m_2^2 \omega_0^2 + m_2^2 \left( \rho_0^3 \right)^2 \right]$$

$$+ \frac{1}{3} h_2 (g_\sigma \sigma)^4 + \frac{1}{4} c_\sigma (g_\sigma \sigma)^4 + \frac{3}{4} c_\omega (g_\omega \omega_0)^4$$

$$+ \frac{3}{2} \left( g_\rho \rho_0^3 \right)^2 \Lambda_v (g_\omega \omega_0)^2,$$

(17)

where

$$\varepsilon_{\text{kin}}^{J, *} = \frac{2}{(2\pi)^3} \int_0^{k_F^*} dk \sqrt{|k|^2 + M^{*2}}$$

$$= \frac{1}{\pi^2} \int_0^{k_F^*} k^2 dk \sqrt{k^2 + M^{*2}}$$

$$= \frac{1}{4} \left[ 3 E_F^{J*} \rho_J + M^* \rho_{s,J} \right], \quad J = \text{p,n},$$

(18)

is the kinetic part of the energy density. Similarly, the mean value of space components of the energy-momentum density tensor corresponds to the pressure of the system, i.e.,

$$P = \frac{1}{3} \sum_{J=1}^{3} (T^{11}) = P_{\text{kin}} + P_{\text{kin}}^{\rho}$$

$$- \frac{1}{2} \left[ m_2^2 \sigma^2 - m_2^2 \omega_0^2 - m_2^2 \left( \rho_0^3 \right)^2 \right]$$

$$- \frac{1}{3} h_2 (g_\sigma \sigma)^4 + \frac{1}{4} c_\sigma (g_\sigma \sigma)^4 + \frac{1}{4} c_\omega (g_\omega \omega_0)^4$$

$$+ \frac{1}{2} \left( g_\rho \rho_0^3 \right)^2 \Lambda_v (g_\omega \omega_0)^2,$$

(19)

where the kinetic part of pressure is given by

$$P_{\text{kin}}^{J} = \frac{1}{3\pi^2} \int_0^{k_F^*} \frac{k^4 \sqrt{k^2 + M^{*2}}}{Q_\omega E_F^{J*}} \quad J = \text{p,n},$$

(20)

The EOS of ANM can be calculated through the energy density $\varepsilon(\rho, \delta)$ by

$$E(\rho, \delta) = \frac{\varepsilon(\rho, \delta)}{\rho} - M.$$  

(21)

The EOS of SNM is just $E_0(\rho) \equiv E(\rho, \delta = 0)$, and the characteristic parameters $K_0$ and $J_0$ can be obtained from the following expressions

$$K_0(\rho) \equiv \int \frac{dE_0}{d\rho^2} = - \frac{9 \rho \sigma^2 M^{*2}}{Q_{\sigma} E_F^{\rho}} + \frac{9 \rho g_\omega^2}{Q_{\rho}} + \frac{3 k_F^2}{Q_{\omega}},$$

(22)

$$J_0(\rho) \equiv \int \frac{d^3 E_0}{d\rho^3} = \frac{3 k_F^4}{E_F^{\rho}} - \frac{3 k_F^4}{E_F^{\rho}} + \frac{27 \rho g_\omega^2 M^{*2} \rho^2}{Q_{\rho} E_F^{\rho}} \times$$

$$\left( \frac{3 \pi^2}{2 k_F E_F^{\rho}} + \frac{2 g_\omega^2}{Q_{\omega}} - \frac{g_\rho^2 M^{*2} \eta}{Q_{\omega} E_F^{\rho}} - \frac{2 g_\omega^2 M^{*2} \rho^2}{Q_{\sigma} E_F^{\rho}} + \frac{E_F^{\rho} \phi}{2 Q_{\sigma}} \right)$$

$$- \frac{162 \omega_0^2 g_\omega^4 \rho^2}{Q_{\omega}^3} - 9 K_0(\rho),$$

(23)

with

$$L_0(\rho) \equiv \int \frac{dE_0}{d\rho} = \left[ \frac{E_F^{\rho}}{4 \rho} - \frac{M^* \rho_0}{4} + g_\omega \omega_0 \right]$$

$$- \frac{1}{\rho} \left( \frac{3 \rho g_\omega^2 M^{*2}}{4} + U(\bar{\sigma}) + \frac{3 \rho g_\omega^4}{4} \right).$$

(24)

In the above expressions, we have

$$Q_{\sigma} = m_2^2 + g_\sigma^2 \left( \frac{3 \rho_0}{M^*} - \frac{3 \rho}{E_F^{\rho}} \right) + 2 b_\sigma M g_\omega^2 + 3 c_\sigma g_\omega^4 \sigma^2,$$

(25)

$$Q_{\omega} = m_2^2 + 3 c_\omega d_\omega \omega_0^2,$$

(26)

and

$$\eta = 3 g_\omega^3 \left( \frac{2 \rho \rho_0}{M^*} - \frac{3 \rho}{E_F^{\rho}} + \frac{M^* \rho}{E_F^{\rho}} \right) - 2 b_\omega M g_\omega^3 - 6 c_\omega g_\omega^4 \sigma,$$

(27)

$$\phi = \frac{2 g_\omega^2 k_F^2}{E_F^{\rho}}.$$  

(28)
with $E_F^* = (k_F^2 + M^*^2)^{1/2}$. To our best knowledge, Eq. (24) gives, for the first time [22], the analytical expression of the $J_0$ parameter in the nonlinear RMF model. In addition, we would like to point out that the general expression for $E_{sym}(\rho)$ and $L(\rho)$ in nonlinear RMF model has been derived in [53].

III. RESULTS AND DISCUSSIONS

For the Lagrangian in Eq. (3), the properties of infinite nuclear matter is uniquely determined by $f_\sigma = g_\sigma/m_\sigma$, $b_\sigma$, $c_\sigma$, $f_\omega = g_\omega/m_\omega$, $c_\omega$, $f_\rho = g_\rho/m_\rho$, $\Lambda_V$, and $M$. If the nucleon mass in vacuum is set to be $M = 939$ MeV, one then has totally seven parameters to determine the properties of infinite nuclear matter in the nonlinear RMF model. Following the correlation analysis method proposed in [54] within the Skyrme-Hartree-Fock (SHF) approach, instead of using directly the seven microscopic parameters, i.e., $f_\sigma$, $b_\sigma$, $c_\sigma$, $f_\omega$, $c_\omega$, $f_\rho$, $\Lambda_V$, one can determine their values explicitly in terms of seven macroscopic quantities, i.e., $\rho_0, E_0(\rho_0), K_0, J_0, M^{\text{dirac}}_0 = M^{\text{dirac}}(\rho_0)$, $E_{sym}(\rho_0)$, and $L(\rho_0)$ where $\rho_0$ is the cross density whose value is fixed in this work at 0.11 fm$^{-3}$ [23]. Then, by varying individually these macroscopic quantities within their known ranges, one can examine transparently the correlation of nuclear matter properties with each individual macroscopic quantity. Recently, this simple correlation analysis method has been successfully applied to study the neutron skin [53, 54] and giant resonances of finite nuclei [53, 55], the higher-order bulk characteristics parameters of ANM (23), and the relationship between the nuclear matter symmetry energy and the symmetry energy coefficient in the mass formula [56]. We would like to point out although the seven macroscopic quantities defined above coherently act on the maximum mass of neutron stars and the pressure of the SNM, they are independent with each other in our analysis since we vary one quantity by keeping other six quantities fixed. This is one of the main advantages of our approach since the physics of these macroscopic quantities is different.

To examine the correlation of pressure of SNM at supra-saturation densities with each macroscopic quantity, we show in Fig. 1 the pressure of SNM $P(\rho)$ at $\rho = 3\rho_0$ from the nonlinear RMF model based on the FSUGold interaction (19) by varying individually $\rho_0, E_0(\rho_0), M^{\text{dirac}}_0, K_0, J_0$ within their empirical uncertain ranges, namely, varying one quantity at a time while keeping all others at their default values in FSUGold for which we have $\rho_0 = 0.148$ fm$^{-3}$, $E_0(\rho_0) = -16.3$ MeV, $M^{\text{dirac}}_0 = 0.61M$, $K_0 = 230$ MeV, $J_0 = -522.6$ MeV, $E_{sym}(\rho_0) = 27.11$ MeV, and $L(\rho_0) = 49.97$ MeV. It should be mentioned that the pressure of SNM is independent of the values of $E_{sym}(\rho_0)$ and $L(\rho_0)$. It is seen from Fig. 1 that the pressure of SNM $P(\rho)$ at $\rho = 3\rho_0$ increases with $\rho_0, M^{\text{dirac}}_0, K_0$ and $J_0$ while decreases with $E_0(\rho_0)$. In particular, the pressure of SNM $P(\rho)$ at $\rho = 3\rho_0$ displays a specially strong correlation with $J_0$. We note that the pressure of SNM $P(\rho)$ at other supra-saturation densities exhibits similar correlations with $\rho_0$, $E_0(\rho_0)$, $M^{\text{dirac}}_0$, $K_0$, and $J_0$. These features indicate that the pressure of SNM at supra-saturation densities is sensitive to the $J_0$ value, and thus the experimental constraints on the pressure of SNM at supra-saturation densities may provide important information on the $J_0$ value.

Since the pressure of SNM at supra-saturation densities is sensitive to the $J_0$ value, the maximum mass $M_{\text{max}}$ of static neutron stars is also expected to be sensitive to the $J_0$ value. The mass and radius of static neutron stars can be obtained from solving the Tolman-Oppenheimer-Volkoff (TOV) equations with a given neutron star matter EOS. A neutron star generally contains core, inner crust and outer crust from the center to surface. In this work, for the core where the baryon density is larger than the core-curst transition density $\rho_c$, we use the EOS of $\beta$-stable and charge neutral npeu matter obtained from the nonlinear RMF model. In the inner crust with densities between $\rho_{\text{out}}$ and $\rho_c$ where the nuclear pastas may exist, we construct its EOS (pressure $P$ as a function of energy density $E$) according to $P = a + bE^{1/3}$ because of our poor knowledge about its EOS from first principle [57, 58]. The $\rho_{\text{out}} = 2.46 \times 10^{-4}$ fm$^{-3}$ is the density separating the inner from the outer crust. The constants $a$ and $b$ are then easily determined by the pressure and energy density at $\rho_c$ and $\rho_{\text{out}}$ [53]. In this work, the $\rho_c$ is determined self-consistently within the nonlinear RMF model using the thermodynamical method (see, e.g., [51] for the details). In the outer crust with $6.93 \times 10^{-13}$ fm$^{-3} < \rho < \rho_{\text{out}}$, we use the EOS of BPS [54, 60], and in the region of $4.73 \times 10^{-15}$ fm$^{-3} < \rho < 6.93 \times 10^{-13}$ fm$^{-3}$ we use the EOS of FMT [59].

Similarly as in Fig. 1 we plot in Fig. 2 the maximum mass $M_{\text{max}}$ of static neutron stars from the nonlinear RMF model based on the FSUGold interaction by varying individually $\rho_0, E_0(\rho_0), M^{\text{dirac}}_0, K_0, J_0, E_{sym}(\rho_0)$, and $L(\rho_0)$ within their empirical uncertain ranges. Indeed, one can see that the $M_{\text{max}}$ displays a very strong positive correlation with the $J_0$ parameter. In addition, the $M_{\text{max}}$ exhibits weak positive correlation with the $M^{\text{dirac}}_0$ and $K_0$, and weak negative correlation with the $\rho_0$ and $E_0(\rho_0)$. It is interesting to see that the $M_{\text{max}}$ is essentially independent of the values of $E_{sym}(\rho_0)$ and $L(\rho_0)$, implying that, in the nonlinear RMF model, the $M_{\text{max}}$ is basically determined by the isoscalar part of the nuclear matter EOS. Since the seven microscopic parameters change with the macroscopic quantities, it is thus not surprising to see that the maximum mass of a neutron star based on the FSUGold interaction by varying macroscopic quantities may be totally different from the default one from FSUGold, which is about $1.74M_\odot$. These features indicate that the observed largest mass of neutron stars may put important constraint on the $J_0$ value. We would like to point out that the interaction FSUGold is only used in Figs. 1 and 2 as a reference for the correlation analyses and using other RMF interactions will not change our conclusion.
work the following empirical uncertain ranges for these ranges. For the nonlinear RMF model, we use in this $\rho$ by varying individually $\rho_0$ (a), $E_0(\rho_0)$ (b), $M_{\text{dirac}}^0$ (c), $K_0$ (d), and $J_0$ (e).

![FIG. 1: Pressure of SNM at $\rho = 3\rho_0$ from the nonlinear RMF model based on the FSUGold interaction by varying individually $\rho_0$ (a), $E_0(\rho_0)$ (b), $M_{\text{dirac}}^0$ (c), $K_0$ (d), and $J_0$ (e).](image)

Experimentally, the pressure of SNM at supra-saturation densities (from $2\rho_0$ to about $5\rho_0$) has been constrained by measurements of collective flows in HIC (3), which is shown as a band in the left window of Fig. 1. In the nonlinear RMF model, if one only changes the $J_0$ value while the other 6 macroscopic quantities are kept at their values in the FSUGold interaction, one can find that the $J_0$ value should be in the range of $-985 \text{ MeV} \leq J_0 \leq -327 \text{ MeV}$ to be consistent with the flow data in HIC (3). However, keeping the other 6 macroscopic quantities at their values in the FSUGold interaction is obviously a strong assumption because the extraction of the $J_0$ value from the flow data in HIC will also depends on the values of $\rho_0,$ $E_0(\rho_0),$ $M_{\text{dirac}}^0,$ and $K_0,$ which can be varied within their empirical uncertain ranges. For the nonlinear RMF model, we use in this work the following empirical uncertain ranges for these macroscopic quantities, i.e., $\rho_0 = 0.153 \pm 0.008 \text{ fm}^{-3}$, $E_0(\rho_0) = -16.2 \pm 0.3 \text{ MeV}$, $M_{\text{dirac}}^0/M = 0.61 \pm 0.04,$ and $K_0 = 230 \pm 20 \text{ MeV},$ which represent the typical uncertain ranges known or predicted from different interactions in the nonlinear RMF model (50).

Based on the pressure of SNM constrained by flow data in HIC (3), to extract the upper limit of the $J_0$ value, one should use the values of $\rho_0,$ $E_0(\rho_0),$ $M_{\text{dirac}}^0,$ and $K_0$ that make the resulting pressure of SNM as small as possible when $J_0$ is fixed. This can be obtained by using $\rho_0 = 0.145 \text{ fm}^{-3}$, $E_0(\rho_0) = -15.9 \text{ MeV}$, $M_{\text{dirac}}^0/M = 0.57,$ and $K_0 = 210 \text{ MeV},$ denoted as set “S”, since the pressure of SNM $P(\rho/\rho_0)$ at supra-saturation densities increases with $\rho_0,$ $M_{\text{dirac}}^0,$ and $K_0$ while decreases with $E_0(\rho_0)$ as shown in Fig. 1. With the set “S” for $\rho_0,$ $E_0(\rho_0),$ $M_{\text{dirac}}^0,$ and $K_0,$ one can find the upper limit of $J_0 = -10 \text{ MeV}$ for the $J_0$ value, which is indicated by solid line in the left window of Fig. 1. For $J_0 > -10 \text{ MeV},$ the model would over-predict the pressure of SNM constrained by flow data in HIC (3). Similarly, one can obtain the lower limit of the $J_0$ value by using the values of $\rho_0,$ $E_0(\rho_0),$ $M_{\text{dirac}}^0,$ and $K_0$ that make the resulting pressure of SNM as large as possible when $J_0$ is fixed, and this can be obtained with $\rho_0 = 0.161 \text{ fm}^{-3},$ $E_0(\rho_0) = -16.5 \text{ MeV},$ $M_{\text{dirac}}^0/M = 0.65,$ and $K_0 = 250 \text{ MeV},$ denoted as set “H”. Using the set “H” for $\rho_0,$ $E_0(\rho_0),$ $M_{\text{dirac}}^0,$ and $K_0,$ one can extract the lower limit of $J_0 = -1280 \text{ MeV},$ which is indicated by dashed line in the left window of Fig. 1. The model would under-predict the pressure of SNM constrained by flow data in HIC (3) if $J_0 < -1280 \text{ MeV}$. Therefore, from the pressure of SNM constrained by flow data in HIC (3), one can extract the constraint of $-1280 \text{ MeV} \leq J_0 \leq -10 \text{ MeV}$.

Recently, a new neutron star PSR J0348+0432 with a mass of $2.01 \pm 0.04 M_\odot$ was discovered (61), and this
FIG. 3: (Color online) Left window: Pressure of SNM as a function of baryon density. The solid (dashed) line is the prediction from the nonlinear RMF model with $J_0 = -10$ (−1280) MeV and the set “S (H)" for $\rho_0$, $E_0(\rho_0)$, $M_{\text{dirac}}^0$, and $K_0$. The band represents the constraints from flow data in HIC [3]. Right window: The maximum mass of static neutron stars as a function of $J_0$ in the nonlinear RMF model with the set “NS-H" for $\rho_0$, $E_0(\rho_0)$, $M_{\text{dirac}}^0$, $K_0$, $E_{\text{sym}}(\rho_c)$ and $L(\rho_c)$. The band represents mass $2.01 \pm 0.04 M_\odot$ for PSR J0348+0432 [61].

neutron star is only the second pulsar with a precisely determined mass around 2$M_\odot$ after PSR J1614-2230 [62] and sets a new record of the maximum mass of neutron stars. The lower mass limit of 1.97$M_\odot$ for PSR J0348+0432 thus may set a lower limit of the $J_0$ value below which the model cannot predict a neutron star with mass equal or above 1.97$M_\odot$. To extract the lower limit of the $J_0$ value from the observed heaviest neutron star PSR J0348+0432, one can use the values of $\rho_0$, $E_0(\rho_0)$, $M_{\text{dirac}}^0$, $K_0$, $E_{\text{sym}}(\rho_c)$ and $L(\rho_c)$ that make the resulting maximum mass of neutron stars as large as possible when $J_0$ is fixed, and from Fig. 2 this can be obtained with $\rho_0 = 0.145$ fm$^{-3}$, $E_0(\rho_0) = -16.5$ MeV, $K_0 = 250$ MeV, $M_{\text{dirac}}^0/M = 0.65$, $E_{\text{sym}}(\rho_c) = 26$ MeV and $L(\rho_c) = 60$ MeV, denoted as set “NS-H". This leads to a lower limit of $J_0 = -494$ MeV for the $J_0$ value as shown in the right window of Fig. 3 where the maximum mass of neutron stars is plotted as a function of $J_0$ when the other 6 macroscopic quantities are fixed at their values as in set “NS-H". For $J_0 < -494$ MeV, the maximum mass of static neutron stars predicted in the nonlinear RMF model would be always smaller than 1.97$M_\odot$. It should be pointed out that here the interior of neutron stars has been assumed to be npeμ matter. New degrees of freedom such as hyperons or quark matter that could be present in the interior of neutron stars usually soften the EOS of neutron star matter and thus a larger $J_0$ value would be necessary to obtain a neutron star with mass of 1.97$M_\odot$. Therefore, including the new degrees of freedom in neutron stars will be consistent with the constraint of $J_0 \geq -494$ MeV.

Combining the constraint of $-1280$ MeV $\leq J_0 \leq -10$ MeV from the pressure of SNM constrained by flow data in HIC [3] which favors a smaller $J_0$ value and the constraint of $J_0 \geq -494$ MeV from the recently discovered heaviest neutron star PSR J0348+0432 [61] which favors a larger $J_0$ value, one can extract the following constraint for the $J_0$ parameter

$-494$ MeV $\leq J_0 \leq -10$ MeV.

(29)

It should be emphasized that the constraint $-494$ MeV $\leq J_0 \leq -10$ MeV represents a conservative extraction based on flow data in HIC [3] and the recently discovered heaviest neutron star PSR J0348+0432 [61] in the nonlinear RMF model. This is because we have not considered the possible correlations existed among the $\rho_0$, $E_0(\rho_0)$, $M_{\text{dirac}}^0$, $K_0$, $E_{\text{sym}}(\rho_c)$ and $L(\rho_c)$, and have simply set simultaneously their values in the boundary of their empirical uncertain ranges. Considering the correlations possibly existed among the $\rho_0$, $E_0(\rho_0)$, $M_{\text{dirac}}^0$, $K_0$, $E_{\text{sym}}(\rho_c)$ and $L(\rho_c)$ should further narrow the constraint $-494$ MeV $\leq J_0 \leq -10$ MeV, and it will be interesting to see the quantitative constraint on the $J_0$ parameter based on the data of finite nuclei, neutron stars, and heavy-ion collisions using the exhaustive statistical analysis method although this is beyond the scope of the present work. The conservative constraint $-494$ MeV $\leq J_0 \leq -10$ MeV obtained in the present work indicates that, if the $J_0$ value is out of the region $-494$ MeV $\leq J_0 \leq -10$ MeV, the nonlinear RMF model either cannot predict the pressure of SNM constrained by flow data in HIC [3] or cannot describe the recently discovered heaviest neutron star PSR J0348+0432 [61]. It is worth mentioning that the constraint on the $J_0$ depends on our knowledge of the other six quantities. Any improvement on these six macroscopic quantities will make the range for the $J_0$ constraint narrower. In addition, the extracted constraint on the $J_0$ could depend on the form of the energy density functional, and it will be interesting to see how the constraint changes if other energy density functionals are used.

In Fig. 4 we show the comparison of the $J_0$ constraint obtained in our analysis with those obtained with other analyses and/or other methods ([22, 24, 63, 64]), including the constraint of $J_0 = -700$ ± 500 MeV obtained by [22] from the analysis of nuclear GMR, the constraint of $J_0 = -280^{+72}_{-110} (\pm 500^{+170}_{-296})$ MeV obtained by [64] from analyzing a heterogeneous data set of six neutron stars using a Markov chain Monte Carlo algorithm within a Bayesian framework by assuming $r_\text{ph} \gg R$ ($r_\text{ph} = R$) where $r_\text{ph}$ is the photospheric radius at the time the flux is evaluated and $R$ is the stellar radius, the constraint of $J_0 = -355 \pm 95$ MeV deduced by [24] based on a correlation analysis method within SHF energy density functional, and the constraint of $J_0 = -390 \pm 90$ MeV deduced by [22] who used the similar method as [24]. It is seen that the constrained region of $J_0$ obtained in the present work has a remarkable overlap with those existing in the literature. In particular, our present constraint from the relativistic model is nicely consistent with the constraints
deduced from the non-relativistic Skyrme-Hartree-Fock approach and they all indicate that the $J_0$ parameter should be larger than about $-500$ MeV.

**IV. SUMMARY**

Within the nonlinear relativistic mean field model, using macroscopic nuclear matter characteristic parameters instead of the microscopic coupling constants as direct input quantities, we have demonstrated that the pressure of symmetric nuclear matter at supra-saturation densities and the maximum mass of neutron stars provide useful probes for the skewness coefficient $J_0$ of symmetric nuclear matter. In particular, using the existing experimental constraints on the pressure of symmetric nuclear matter at supra-saturation densities from flow data in heavy ion collisions and the astrophysical observation of recently discovered heaviest neutron star PSR J0348+0432, with the former requiring a smaller $J_0$ while the latter a larger $J_0$, we have extracted a constraint of $-494\text{MeV} \leq J_0 \leq -10\text{MeV}$.

We have compared the present constraint with the results obtained in other analyses, and found they are nicely in agreement. In particular, our present constraint from the relativistic model is nicely consistent with the constraints deduced from the non-relativistic Skyrme-Hartree-Fock approach and they all indicate that the $J_0$ parameter cannot be too small, namely, it should be larger than about $-500$ MeV. The present constraint on the $J_0$ parameter provides important information on the high density behaviors of the EOS of symmetric nuclear matter, and also may be potentially useful for the determination of the high density behaviors of the EOS of asymmetric nuclear matter, especially the high density symmetry energy.

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