ON THE ANISOTROPY IN EXPANSION OF MAGNETIC FLUX TUBES IN THE SOLAR CORONA

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ABSTRACT

Most one-dimensional hydrodynamic models of plasma confined to magnetic flux tubes assume circular tube cross sections. We use potential field models to show that flux tubes in circumstances relevant to the solar corona do not, in general, maintain the same cross-sectional shape through their length and therefore the assumption of a circular cross section is rarely true. We support our hypothesis with mathematical reasoning and numerical experiments.

We demonstrate that lifting this assumption in favor of realistic, non-circular loops makes the apparent expansion of magnetic flux tubes consistent with that of observed coronal loops. We propose that in a bundle of ribbon-like loops, those that are viewed along the wide direction would stand out against those that are viewed across the wide direction due to the difference in their column depths. That result would impose a bias toward selecting loops that appear not to be expanding, seen projected in the plane of sky. An implication of this selection bias is that the preferentially selected non-circular loops would appear to have increased pressure scale heights even if they are resolved by current instruments.

Key words: magnetic fields – plasmas – Sun: corona – Sun: magnetic fields – Sun: UV radiation

Online-only material: animation, color figures

1. INTRODUCTION

Coronal loops appear as thin emitting strands in the solar atmosphere. They appear to follow the lines of the magnetic field in the corona and they are usually considered to be a manifestation of the hot and dense plasma confined within magnetic flux tubes. Because of the frozen-in condition, plasma cannot leave a flux tube through its sides so a local density enhancement could only get redistributed along the flux tube it is embedded in. The emission measure is proportional to the square of the density, so flux tubes with denser plasma produce more emission. As the thermal conductivity is much greater along the field than across it, a localized heating event heats the flux tube it is embedded in, but not its surroundings. Should the heating be sufficient such that the plasma emits an observable wavelength and should the plasma in a given flux tube be denser than its surroundings, imaging instruments would detect a coronal loop.

Coronal loops have been observed for several decades, yet some of their properties remain a mystery. For example, it is recognized that the width of coronal loops varies only weakly with height above the surface (e.g., Klimchuk 2000, and references therein). At the same time, if the emitting plasma is indeed confined to a magnetic flux tube, it should expand with height as the strength of the field drops and the flux is constant along the tube.

Various theories have been proposed to explain this phenomenon. For example, McClymont & Mikic (1994) showed that twisted flux tubes expand somewhat less with height than if they were untwisted, but these authors have also found that for currents typical for active region fields, the reported effect is not enough to agree with observations. In another study, Klimchuk et al. (2000) examined a twisted flux tube and concluded that the presence of electric currents is capable of enforcing circularity in the cross section of the tube. However, for any amount of twist they injected, only one of the shells of the tube was circular at the apex (and all other shells were oblate, as follows from comparing Figures 6 and 9 in their study). In particular, these authors found that the core of the tube was circular in the apex for a twist value slightly below \(2\pi\) (and for this value of twist, the surrounding shells were oblate at the apex). Klimchuk et al. speculated that for an increased amount of twist, the entire flux tube might become increasingly more circular. Their particular simulation was limited to twist values of \(\approx 2\pi\), but they mentioned the results of Mikic et al. (1990) to justify that twists of up to \(5\pi\) could be introduced to a flux tube without a loss of equilibrium. However, flux tubes simulated in a later study by Amari & Luciani (1999) became kink unstable at twists of \(\approx 2.7\pi\), which may be below the amount that needs to be injected to enforce circularity of the entire flux tube, according to Klimchuk et al.

It has also been shown that magnetic flux tubes expand less near separators (Plowman et al. 2009). This finding is consistent with observations that cool loop fans are associated with quasi-separatrix layers (Schrijver et al. 2010); however, solid evidence has yet to be collected that loops mainly form at the separators.

A hypothesis that has increasing gained popularity states that loops are composed of thin unresolved strands (DeForest 2007). The author argues that the lack of observed expansion of the loops with height can be explained in terms of the interactions between the resolution, the geometric cross section, and the diffuse background. This simple and elegant theory also explains several other puzzling aspects of coronal loops. Among these aspects is the fact that as the resolution of our instruments increases, progressively finer strands are reported in what were previously thought to be monolithic structures.

Another aspect explained by the thin strand theory is the apparent increase in the pressure scale height by several orders of magnitude in some EUV loops compared to theoretical predictions (e.g., Warren & Winebarger 2003; Winebarger et al. 2003, and references therein). In a simple model, DeForest (2007) shows that resolved and unresolved expanding strands would fade with height according to different power laws. We discuss this model in further detail later in the text. Also, an ensemble of thin strands was shown to be brighter than predicted by static
loops. This ensemble also appears to be isothermal in filter ratio measurements, which is consistent with observations (Warren et al. 2002). However, López Fuentes et al. (2008) argued that on the observations, it is possible to distinguish between unresolved but expanding strands and resolved but not expanding strands.

Every single one of the models we mentioned, as well as, to our knowledge, almost all other models of an isolated flux tube, make the assumption that the flux tube is circular in cross section (or that the cross section is constant and therefore its shape is irrelevant, e.g., Rosner et al. 1978). This situation mainly results from the fact that this assumption substantially simplifies calculations. In the absence of definite information about the cross-sectional shape of the loops, this simplification is reasonable to make.

Observational evidence that coronal loops are circular in cross section is extremely difficult to obtain and there are not many studies that address this question. The difficulties stem from the fact that the corona is optically thin and what is observed is the superposition of a complicated background and multiple loops (or fragments of loops) along the line of sight (l.o.s.). Among the most frequently cited works is the study of Klimchuk (2000), who concluded that the loop profiles are typically simple and single-peaked and the width variations along the length of the loops are small. He argued that these results support the hypothesis that loops are circular in cross section.

There are also several studies that provide evidences to the contrary. For example, Wang & Sakurai (1998) examined flux tubes from several model fields (Low & Lou 1990) and concluded that in the presence of currents, the flux tubes tend to expand in an anisotropic manner. However, these authors found that the anisotropy is smaller for strongly sheared fields. López Fuentes et al. (2006) also mentioned that flux tubes could be highly non-circular in cross section. They, however, argued that in a large statistical sample of loops, the expansions seen in the “wide” and in the “narrow” directions would average out.

More evidence that loops are non-circular in cross section can be drawn from three-dimensional (3D) MHD models of coronal loops. For example, Gudiksen & Nordlund (2005) examined the shapes of the bases of flux tubes that were round at the apex and confirmed their theoretical argument that the expansion of flux tubes is not isotropic. Also, they found that in their simulations that the flux tubes that were initially round at the apex become "wrinkled" at the lower boundary. The studies of Mok et al. (2008) and Peter & Bingert (2012) do not address this issue directly; however, a visual examination of different renderings in their simulations, namely, the panels of Figure 3 of Mok et al. and Figures 1 and 5 in Peter & Bingert, suggests that the loops in these models also appear to have different widths when viewed from different directions.

A detailed comparison of the same set of loops viewed by the STEREO satellites might clarify this topic. However, besides the abovementioned issues of the optical thinness of the corona, such analysis would be greatly complicated by the choice of the separation angle between the satellites. It must be large enough for the difference in widths to be observable, yet small enough for the loops to still be identifiable on both images. Attempting such a study is not the intention of this paper; however we would still like to provide a qualitative example of such behavior of coronal structures. Figure 1 shows images from the two STEREO satellites taken at nearly the same time (on 2008-01-10T05:06, less than a minute apart when corrected for the light travel time and in the same wavelength of 171 Å). At the time these images were taken, the separation angle between the satellites was about 45°.

The following two figures show close-ups of two regions in these frames. Figure 2 shows an ephemeral region in the east portion of the images and a loop bundle that connects it to the ambient field. Note the different shape of this bundle when viewed from two different angles. We have marked the approximate locations of the two footpoints of this bundle, A and B, to guide the reader, and outlined the bundle in both images. The points A and B were picked on one of the images and then remapped onto the other one. If the bundle was circular in cross section, it would have the same width and the same l.o.s. depth when viewed from the side (STEREO A) and from the top (STEREO B). It is possible that the background obscures the true shape of the bundle. It is, however, just as possible that the bundle is simply non-circular in cross section. For example, such a difference in widths for different viewing angles is expected for a squished cylinder, whose cross section is elongated along one l.o.s. and across the other one. This option is favorably supported by the fact that on STEREO A (left image) the bundle is narrower and brighter, while on STEREO B (right image).
the bundle is wider and dimmer, even though the images are corrected for exposure and plotted on the same color scale.

Figure 3 shows two active regions and their interconnecting loops. As on the previous image, several points, C–H, were marked at the surface of the Sun to guide the reader. The arrows and the numbers 1–4 point to several coronal features. Loop 1, marked at the surface of the Sun to guide the reader. The arrows and the numbers 1–4 point to several coronal features. Loop 1 appears much thinner in the apex on STEREO A (left panel) than on STEREO B (right panel). Loop 2, which clearly connects points D and F on STEREO A, is nowhere to be found on STEREO B. A possibly corresponding feature is shown with a question mark; however, its footpoints are clearly D and E, not D and F. On the contrary, loop 4, connecting points D and E and extending more northward than point G on STEREO B, is hard to find (if at all) on STEREO A. Loop 3, connecting points G and H, appears much thinner on STEREO B than on STEREO A. Again, while it could be argued that all of these occurrences could be explained by the overlap of many features in an optically thin plasma, it is also possible that these loops are not circular in cross section.

In this study, we revisit the topic of the cross-sectional shape of coronal loops. We demonstrate that even flux tubes that are approximately round at some location do not maintain this shape further along their length in the corona. We also show how ignorance of this fact might affect our interpretation of coronal observations. In Section 2, we argue that there are no mathematical reasons for flux tubes to maintain the same cross-sectional shape—much in agreement with the arguments of Gudiksen & Nordlund (2005), but in more detail. In Section 3, we quantify the anisotropy in the expansion of flux tubes, using a model field based on a Helioseismic and Magnetic Imager (HMI) magnetogram. Section 4 demonstrates that the amount of oblateness can be sufficient to prevent us from observing the expansion of the loops and that selection criteria make the loops that lack apparent expansion appear brighter in the coronal images. In Section 5, we show that the anisotropy of the expansion might be responsible for higher apparent pressure scale heights, even in resolved coronal loops. Finally, Section 6 summarizes our findings and discusses their potential implication on our understanding of the structure of the corona and the mechanisms that heat it.

2. SQUASHING FACTOR—IN THE CORONA

The mapping, which magnetic field lines provide between two surfaces normal to the field, is not in general shape-conserving. This fact is usually described through a squashing factor $Q$, which is a dimensionless number normalized to be unity for shape-conserving mapping and increasing with increasing distortions (Titov & Hornig 2002).

The squashing factor $Q$ is usually used to describe the mapping between regions of flux of positive and negative polarity on the photosphere. Its value is infrequently used in studies of magnetic topology, as its qualitative behavior sufficiently marks the location of topological features: $Q \to \infty$ defines separators and $Q \gg 1$ defines quasi-separator layers, or QSLs (Titov et al. 2002).

Defined this way, the squashing factor is not, in general, informative for the cross-sectional distortions of flux tubes in the corona. We illustrate this fact with several simple examples shown in Figure 4. These are three potential fields confined to half space with the lower boundary being a dipole (top row), an arcade (middle row), and a superposition of the two (bottom row). Due to the symmetry of these fields, the squashing factor for the photosphere–photosphere mapping is unity for all three of them (that is, a square on one polarity maps to a square on the other polarity). The left column shows four field lines initiated from the corners of a small square on the photosphere and the shape they map to at the apex. The right column shows the mapping set by the magnetic field from the lower boundary to the vertical plane in the middle of the computational domain (again, due to the symmetry of the system the field is normal to this plane at all points). We initiated a set of field lines from the corners of a square grid at one polarity at the lower boundary and found a set of points where these field lines crossed the vertical plane. Adjacent points are connected by straight lines, which in turn form a grid that the square grid at the photosphere maps to. The grid has gaps on the edges as we did not plot the grid cells that were partially outside of the domain. The top boundary on these plots is slightly below the boundary of the domain. Note that many grid cells on the vertical plane are substantially deformed. The corresponding flux tubes, while
Figure 3. Another portion of the images from Figure 1. As before, several points, C–H, are marked on the solar surface to guide the reader. The arrows and the numbers 1–4 point to several coronal features. Loop 1 appears much thinner in the apex on STEREO A (top panel) than on STEREO B (bottom panel). Loop 2, which clearly connects points D and F on STEREO A, is nowhere to be found on STEREO B (a possibly corresponding feature is shown with a question mark; however, its footpoints are clearly D and E, not D and F). On the contrary, loop 4, connecting points D and E and extending more northward than point G on STEREO B, is hard to find (if at all) on STEREO A. Loop 3, connecting points G and H, appears much thinner on STEREO B than on STEREO A. Again, while it could be argued that all of these occurrences could be explained by the overlap of many features in an optically thin plasma, it is also possible that these loops are not circular in cross section. (A color version of this figure is available in the online journal.)

having a square base, have an elongated cross section at the apex. Note also that the properties of the deformation depend on the configuration of the sources in the lower boundary. For example, in the arcade, flux tubes cannot expand in height along the translational direction of the arcade, so the expansion must take place entirely in the vertical direction (the arcade in our example is finite, so this effect vanishes toward its edges).

These three examples correspond to very simple fields. For more complex boundaries, the distortions of the cross section would in general be even stronger. Figure 5 shows a similar mapping for a potential field based on an HMI magnetogram onto a plane parallel to the l.o.s. We analyzed AR NOAA 11097 on 2010 August 15. (We have no reasons to consider it a special active region in any sense. The choice was based on the visibility of loops, the proximity to the central meridian, and the availability of high-resolution Solar Dynamics Observatory/HMI data, as we further argue on the importance of the fine-scale structure.) The top left panel shows where this plane crosses the plane of sky and top right panel shows where is it located in the computational domain (to construct the field, the magnetogram was remapped to the disk center and downsampled by a factor of two). Much like in the previous example, we initiate field lines at the corners of a regular grid (in this case, hexagonal) and calculate where these field lines cross the given plane. The bottom left and right panels show the mapping from the positive and negative polarities, respectively. The gaps in the grid are due to the fact that we do not initiate field lines from the points where field strength was $|B_z| < 100$ G.

Figure 5 thereby shows what the cross section of the flux tubes would look like along the l.o.s., if their bases were round. Most of the flux tubes undergo substantial distortions as they expand. If the coronal loops represent the emission of plasma in individual flux tubes, their cross sections must in general be anything but circular—even in this simple example. This effect is possibly even stronger in the actual coronal field. First, because the photosphere has a lot of fine structure that we missed by using a downsampled HMI magnetogram and, as Figures 4 and 5 show, the flux tubes tend to be more oblate near the boundaries of individual domains of connectivity. Second, the actual corona has currents flowing in it which might also affect the magnitude
Figure 4. Mapping of field lines from the lower boundary to the vertical plane for several simple potential fields: a dipole, an arcade, and a dipole between the poles of an arcade. In each case, field lines were initiated at the lower boundary from the positive polarity, in the corners of a square grid (with spacing less than one pixel on the lower boundary). On the left, we plot four such field lines from the corners of one square at the lower boundary. If we define a flux tube to have a square cross section at the lower boundary with these field lines in the corners of the square, the cross section of this flux tube in the apex would correspond to the shaded shape on the middle plane, which is simply a rectangle with corners set by the intersection of the “corner” field lines with this plane. Note how different the cross sections are for three different cases—and how different they all are from the starting square at the lower boundary! The right column shows the mapping of many field lines initiated from square grid onto this middle plane (the nodes on this plot are where the field lines intersect this plane and the adjacent nodes are connected showing the cross sections of flux tubes with square bases at the lower boundary).

Here and further in the text, we make the assumption that the base is round, but this assumption is done merely for the simplicity of the argument. Indeed, suppose that the flux tubes are round at the apex instead. The mapping from the apex to the base is described by the inverse of the mapping from the base to the apex and so the footpoints of flux tubes would have to be oblate. Likewise, a flux tube with a round cross section at a...
Figure 5. Mapping of field lines from the lower boundary to a vertical plane in a manner similar to Figure 4. In this case, the field lines were initiated from corners of regular hexagons at the lower boundary. The plane was set by the Atmospheric Imaging Assembly (AIA) l.o.s. and an arbitrary line in the plane of the sky that visually passes through the tops of many loops (the red line in the top left panel). The top left panel shows the AIA 171 Å image and the corresponding HMI magnetogram. The top right panel shows the position of this plane in the 3D computational domain. The points at which the field lines cross this plane are calculated and plotted for the field lines initiated from negative (bottom left) and positive (bottom right) polarities at the lower boundary. Adjacent points are connected to form polygons—these would correspond to cross sections of flux tubes in the active region, should these flux tubes be regular hexagons at the lower boundary. The gaps in the plots reflect that we did not trace field lines from where $|B_z|$ was below a 100 G threshold. We encourage readers of the electronic edition to zoom in the two bottom panels to examine the cross sections of the low-lying flux tubes. Very few flux tubes could still be described as regular hexagons at the apices (here, we mean more of a qualitative description of a visual impression; we make quantitative estimates in Section 3). As we argue in Section 4, this fact may have substantial impact on the visibility of the corresponding coronal loops.

(A color version of this figure is available in the online journal.)

given point along its length would in general be oblate at other points.

3. STATISTICS

We further investigate how much flux tubes deform at the apex if their base cross section is circular. For this investigation, we perform the following exercise. We consider a set of flux tubes that fill the entire cube of the field from Figure 5 (minus the volume with a field strength at the base below the 100 G threshold). We initiate field lines on the positive (or the negative) polarity at the nodes of the hexagonal grid. The hexagons the field lines define are considered the bases of the flux tubes. We further follow these flux tubes along their axes and examine what their shape at the point of maximal expansion. The shapes in Figures 4 and 5 illustrate the distortions along the line of sight. However, they could not be used for quantifying the distortions rigorously because (a) they are not, in general, at the apices and (b) they are not, in general, normal to the axes of the flux tubes. Therefore, we develop a different calculation for a more rigorous approach.

We perform a further analysis as follows. For each flux tube, we initiate an axis field line at the center of the hexagonal base
and define the “apex” point as the point of maximal expansion (calculated at the point where the field strength along the axis is at a minimum). We then construct a plane perpendicular to the axis at this point and calculate where the six corner field lines cross this plane. The result is a planar hexagonal polygon (not regular in general). These shapes are shown in Figure 6 for flux tubes starting from positive (left panel) and negative (right panel) polarities. Online materials include an animation of these plots to show them being rotated. As before, the gaps in the plots correspond to the areas where the field strength at the base was below the 100 G threshold.

The estimated magnetic flux is well conserved along these model flux tubes, as demonstrated in Figure 7. This figure shows a scatter plot of the magnetic flux at the apex of each flux tube versus the magnetic flux at the base. In both cases, we use the thin-tube approximation and we assume that the field strength does not vary along the cross section. We furthermore assume that the cross-sectional area is approximately that of the polygon with corners at the six corner field lines. Possible reasons for the flux variation along the flux tube include, for example, a substantial variation of the magnetic field strength on a cross section, inaccuracies in the numerical integration of field lines (especially around topological divides), and the shape of the cross section becoming too distorted to be adequately treated as a polygon. We find that these effects are small for the majority of flux tubes, as shown in Figure 7. We further concentrate on the 8636 flux tubes out of 9050 (about 95%) for which the discrepancy between the flux at the apex and the flux at the base is less than 50 G pixel$^2$.

The cross sections of flux tubes at the apices are then fit with an ellipse using the least-squares method (Fitzgibbon et al. 2002), yielding two semiaxes $a_{\text{apx}}$ and $b_{\text{apx}}$ (where $a_{\text{apx}} > b_{\text{apx}}$). We also calculate what the width of each flux tube would have been if the expansion were isotropic as $r_{\text{apx}} = \sqrt{A/\pi}$, where $A$ is the cross-sectional area.

The results are summarized in Figures 8 and 9 and Table 1. Figure 8 shows histograms of $a_{\text{apx}}$ (cyan), $b_{\text{apx}}$ (orange), and $r_{\text{apx}}$ (black). The top panel shows a histogram in the conventional sense, that is, it is a number of flux tubes (all of which have the same base areas) in each bin and the bottom panel shows the total flux in all flux tubes in the same bin. The semi-axes and radii are given in pixels, and the base of each flux tube is a hexagon of $r_0 = a_0 = b_0 = 0.5$ pixels. In both plots, it is clear that the flux tube expansion is substantially different in the two directions normal to the field: a typical (in full width at half maximum sense) expansion factor is about 1–4 times in $a_{\text{apx}}$ and about

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**Figure 6.** Same set of magnetic field lines from Figure 5, but this time every individual set of six field lines was cut at the apex by a plane perpendicular to the direction of the field at the central field line. We define the apex as the point along a flux tube where the field strength is minimal and therefore the cross-sectional area of the flux tube is maximal. (These plots may be viewed from different angles in the animations available in the online materials.)

(Animations and color version of this figure are available in the online journal.)

**Figure 7.** Comparison of flux at the base and the flux at the apex for the six-sided flux tubes. This plot demonstrates that flux is well conserved for most of the “tubes” and therefore the approximation we make for their cross-sectional shape holds for most of them. For further analysis, we discard those tubes for which the fluxes differ by more than 50 G pixel$^2$ (414 flux tubes out of a total of 9050, or about 5%). The 414 discarded points are plotted in gray and the remaining points are plotted in black.

(A color version of this figure is available in the online journal.)
Table 1

|                      | Median Peak FWHM Range | Median Peak FWHM Range |
|----------------------|-------------------------|-------------------------|
| Number of Flux Tubes | Circular $r_{apx}$ (pixel) | 2.4 1.6 0.7–2.5          |
|                      | Ellipse $a_{apx}$ (pixel)  | 1.3 0.7 0.4–1.6          |
|                      | Ellipse $b_{apx}$ (pixel)  | 5.6 3.0 1.6–6.8          |
| Aspect ratio $a_{apx}/b_{apx}$ | 4.5 2.2 1.3–5.4          |

Notes. The first conclusion is that flux tubes are substantially oblate at their apices with a typical aspect ratio of 2–5. The second conclusion is that most of the expansion takes place along one direction only. Given the base radius of $r_0 = a_0 = b_0 = 0.5$ pixels, the flux tubes expand at most by a factor of 4 in one of the directions and over a factor of 15 in the other one.

3–20 in $b_{apx}$, while if oblateness is unaccounted for, the typical expansion factors would have been around 2–5 times. Figure 9 shows the histogram (top panel) and the flux distribution (bottom panel) of $a_{apx}/b_{apx}$, the typical range of this quantity is 1.5–5 and the median is 4.5.

These numbers likely underestimate the actual oblateness in the cross section of coronal flux tubes. There are several reasons for this result, some of which we list below. One reason is that the actual magnetic field at the photosphere probably has a finer structure than that which appears in our lower boundary data (which consists of an HMI magnetogram downsampled by a factor of two). By doing this downsampling, we possibly missed many small-scale topological divides. As Figure 5 shows, the flux tubes become more distorted near topological divides, in agreement with the definition of these divides via squashing factor. The other factor contributing to a possible underestimation is that we chose a potential (or current-free) field model. Current sheets are associated with additional structure in the connectivity (Büchner 2006) so the topological structure of the actual corona is in general more complex than that of the potential field.

Figure 8. Statistics of the apex expansion in the elongated (cyan) and shortened (orange) directions. The top plot shows the number of flux tubes that fall in the $a_{apx}$ and $b_{apx}$ bins and the bottom plot shows the total flux in each bin. The black curve for each plot corresponds to what the expansion would have been if the cross section was circular, that is, it corresponds to $\Gamma = \sqrt{A_{apx}/A_0}$. The cross-sectional radius at the base is $r_0 = a_0 = b_0 = 0.5$ pixels and is shown with a dashed vertical line.

(A color version of this figure is available in the online journal.)

Of course, the base hexagons lie in the horizontal ($z = 0$) plane and so are not perpendicular to the axis of the corresponding flux tubes. It could be argued that flux tubes that are inclined at the base might in fact be oblate in the initial cross section, thus introducing a bias in our statistics. If this bias was strong, there would be a strong correlation between the field inclination and the oblateness at the apex. Figure 10 demonstrates that such a correlation is not substantial, with a Spearman correlation coefficient $\rho = -0.14$. This result suggests that the oblateness of a flux tube at its base does not systematically contribute to its oblateness at its apex.

4. EXAMPLE

To better understand the effects caused by the oblateness of the loops, we simulate two coronal loops taken from the potential field shown in Figure 5. The bases of the flux tubes are regular hexagons of slightly different diameters so that they have similar fluxes; they also have similar lengths and locations within the domain. Figure 11 shows a 3D sketch of these two flux tubes (top left) and renderings of the emission measures of
Figure 10. Value of \( a_{\text{apx}}/b_{\text{apx}} \) does not appear to be strongly correlated with the inclination of the field at the base of the flux tube. Spearman’s correlation coefficient is \( \rho = -0.14 \).

The flux tubes are rendered as follows. As before, we initiate six field lines from the corners of a regular hexagon and one field line in its center. For many points along this central field line, we construct slices as described above and so split the flux tube into a set of thin volumes contained between consecutive slices (Figure 12, left panel). Each volume is assigned an “intensity” multiplier (e.g., an emission measure). The contribution of this volume to the rendered image intensity at a given pixel is calculated as the volume of the portion of the slice that projects into the given pixel times the intensity multiplier of the volume (Figure 12, right panel). This quantity is an equivalent to the l.o.s. depth of the feature times its filling factor (to get the actual emission measure, this quantity needs to be divided over the pixel area, which is the same for all pixels and so is irrelevant in our analysis). The rendered image is then a sum of the renderings of the individual volumes of all loops on the image.

We assume isothermal hydrostatic atmospheres, so the intensity multiplier at each slice was simply \( n^2 \propto e^{-z/z_0} \). The scale height \( z_0 = 40 \) pixels \( \approx 35 \) Mm is the same for both loops.

The displays show a square root of the intensity for all three renderings, and the color scale is also the same for all three plots.

Note that in the \( x-y \) projection, loop 1 (marked as a red triangle in all three renderings) appears brighter and thinner than loop 2 (marked as a blue square). The opposite is true for loop 2 (marked as a blue square).

Figure 11. Sketch of two flux tubes (top left) and the renderings of the corresponding synthetic coronal loops in the \( x-y \), \( y-z \), and \( x-z \) projections (as marked). The field lines forming these tubes were initiated as two regular hexagons at the opposite polarities. The renderings show the emission measure corresponding to a hydrostatic atmosphere with scale height of 40 pixels. The apex of loop 1 is marked as a red triangle and the apex of loop 2 is marked as a blue square in all renderings. Note that loop 1 appears brighter and non-expanding in the \( x-y \) projection, while the opposite occurs in the \( x-z \) projection; see Figures 13–15 for the numerical justification. In fact, given these two projections alone, it is easy to mistakenly believe that there are two loops, where one is wide and expanding and the other one is thin and non-expanding. In fact, the two loops have similar radii and they both expand—but in different directions. Note also how the projection effect impacts the perceived density scale height.

(A color version of this figure is available in the online journal.)
the $x-z$ projection: loop 2 appears brighter and thinner. At the same time, loop 1 does not appear to expand with height in the $x-y$ projection and loop 2 does not appear to expand in the $x-z$ projection. An observer who is unaware of this effect and who wishes to study the expansion of coronal loops is more likely to select loop 1 over loop 2 if he/she is looking at the $x-y$ projection, simply because loop 1 stands out and appears visible from end to end. When looking at the $x-z$ projection, such an observer might be more likely to select loop 2 for analysis for the same reasons, especially if there are more loops in the background.

Further insight into the selection factors imposed by the viewing angle and the implications for the inferred expansion of the loops can be obtained from Figures 13–15. Figure 13 shows the computed column emission measures for each pixel of these two loops evaluated along the axis in various projections (for this exercise, the loops were rendered separately to eliminate possible background issues). This figure shows that the relative intensities would indeed make, for example, loop 1 stand out over loop 2 at the apex in the $x-y$ projection, and vice versa (though somewhat less) in the $x-z$ projection.

Figures 14 and 15 demonstrate that, first, flux is indeed conserved in these flux tubes, and second, their expansion is highly anisotropic. Third, these figures demonstrate that the actual expansion cannot be measured from any single perspective. We calculate the flux along the tube as $\Phi_i = |B_i|A_i$, where $A_i$ is the area of the polygon forming the cross section and $|B_i|$ is evaluated at the axis; the top panels of Figures 14 and 15 show that the flux is indeed constant along the tubes. These panels also show the cross sections along the tubes in several locations, demonstrating that they are substantially different from the regular starting hexagons. The middle panels show the inferred cross section from the three renderings (blue squares for $x-z$, green triangles for $y-z$, and red diamonds for $x-y$). We simulated the procedure of inferring the width of the loops from the observations (López Fuentes et al. 2006; López Fuentes et al. 2008) in ideal circumstances: we rendered each loop separately (so there was no background) and fit Gaussian profiles to slits across the projected loops’ axes. The resulting widths are consistent with those in Figure 11; for instance, loop 1 expands in the $x-z$ rendering and does not expand in the $x-y$ rendering—that is, it does not expand in the projection in which it appears brighter than loop 2 (Figure 13, top panel). The units of width are chosen to be pixels on the rendered image intensity at a given pixel is calculated as the volume of the portion of the slice that projects into the given pixel times the intensity multiplier of the slice (e.g., an emission measure). This quantity is equivalent to the line-of-sight depth of the feature times its filling factor.

**Figure 12.** Left panel: a schematic drawing of a “slice” volume along a flux tube, enclosed between two planes normal to the axis of the tube. A cross section of a flux tube is assumed to be a hexagon (not regular in general). Field lines are traced from the corners of the base cross section (a regular hexagon) and from the center of the base. For a given point along the axis field line (dashed line), a cross section is set by the intersection of the plane normal to the axis with the “corner” field lines. Right panel: the contribution of a given slice to the rendered image intensity at a given pixel is calculated as the volume of the portion of the slice that projects into the given pixel times the intensity multiplier of the slice (e.g., an emission measure). This quantity is equivalent to the line-of-sight depth of the feature times its filling factor.
on the coronal image that might be attributed to the oblate cross section of the coronal loops.

5. EFFECT ON THE OBSERVED INTENSITY SCALE HEIGHT

As we mentioned before, the oblateness of a loop cross section may introduce a bias in our interpretation of the observations when deriving a pressure scale height. The reason for this bias is that to derive a density, an estimate of a column depth is usually needed. As we showed in Section 4, this column depth does not in general follow from the width of the loop.

To understand this fact, consider a vertical flux tube filled with a hydrostatic isothermal atmosphere. When the tube is viewed from the side, its emission measure at a given image pixel along its projected axis will be \( EM(z) \propto n^2 d \), where \( d \) is the column depth. If the diameter of the flux tube is constant with height Rosner et al. e.g., as in 1978, and in the first column of Figure 19), then \( EM(z) \propto n^2(z) \). The flux tube expansion with height increases the geometrical column depth and therefore \( EM(z) \) drops more slowly for a given pressure scale height (Figure 19, Column 2). Flux \( \Phi \) is constant along the tube, so if the cross section is circular, then the column depth is \( d(z) = 2 \sqrt{\Phi} / \pi B(z) \) and in this case \( EM(z) \propto n^2 B^{-1/2} \).

Now, suppose that \( B(z) \) drops with height but that the flux tube expands only in one direction and that its cross section is an ellipse with semiaxes \( a > b \) and area \( \pi ab \). In this case, when viewed perpendicular to the expanding direction, the column depth would stay constant and \( EM(z) \propto n^2 \), just like for a non-expanding tube. When viewed along the direction of expansion, \( \Phi \) would fade as \( EM(z) \propto n^2 B^{-1} \) (Figure 19, Columns 3 and 4). In the unresolved case in which the tube...
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Figure 15. Flux, cross sections, and inferred and actual expansions of loop 2 (the notation is the same as in Figure 14). Note that the $x−y$ projection allows one to observe the expansion of loop 2, but not loop 1, while in the $x−z$ projection the expansion of loop 1 is clear, but loop 2 exhibits little to no width variations. Note also that neither of the inferred widths allows one to access the actual expansions of the loops.

(A color version of this figure is available in the online journal.)

Figure 16. Rendering of a set of several hundred flux tubes filled with hydrostatic isothermal atmospheres (with the same scale height) viewed in a plane-of-sky projection. The grayscale corresponds to the square of the column emission measure in arbitrary units. The flux tubes are initiated in the $z=3$ pixel plane and the radius at the base is 0.5 pixels on the rendered image, so all of the flux tubes are resolved at the base. Note how tubes that do not expand a lot in the plane of the sky appear brighter and stand out from their neighbors, which do expand in this projection—this result is particularly evident in the close-up panel labeled #1. Also, note the characteristic elongated bright spots where the loops turn toward the observer (close-up #2).

is thinner than the pixel width $\Delta x_{\text{pix}}$ in the plane of sky, as in Column 5 in Figure 19, its emission times its filling factor would be equivalent to the emission of a tube 1 pixel wide with a depth $d_{\text{eff}} = A/\Delta x_{\text{pix}}$. In this case, $EM(z) \propto n^2 B^{-1}$ regardless of the cross section shape, and the emission measure is the same as in the case of a resolved tube expanding along the l.o.s. only.

Figure 20 shows these three power laws, $n^2$, $n^2 B^{-1/2}$, and $n^2 B^{-3}$, plotted against $z$ (essentially revisiting Figure 5 in DeForest 2007). All three models assume $n = \exp(-z/z_0)$ with $z_0 = 50$ Mm and $B \propto 1/z^3$, with the coefficient chosen such that all three curves intersect at one pressure scale height. Both the unresolved case and the l.o.s. only expansion case in Figure 19 ($EM \propto n^2 B^{-1}$) can be an order of magnitude brighter than the constant cross-sectional curve ($EM \propto n^2$), which is in agreement with the observations (Warren & Winebarger 2003), for example, on our plot at two pressure scale heights. The anisotropic expansion model however, is supported by the study of López Fuentes et al. (2008), who showed that it is possible to distinguish between expanding, expanding but unresolved, and resolved but not expanding loops. We also demonstrate, that, depending on orientation, the scale height might even be lower than that of an isotropically expanding loop. In fact, the same fan-like structure, which is not entirely planar but has “ripples” on it, might appear in the plane of sky as a combination of dim underdense diffuse emission and thin overdense bright strands—despite having a uniform pressure scale height!

6. SUMMARY AND DISCUSSION

We have demonstrated that magnetic flux tubes may exhibit great variation in their cross-sectional shapes along their lengths. In particular, flux tubes with round bases at the lower boundary become oblate at their apices in the potential field models that we examined. The direction and the amount of oblateness varies significantly between different field models and even within a single model. This result, in principle, might render a study of the cross-sectional shape based on one selected flux tube inconclusive. Therefore, an analysis of large statistical samples of flux tubes is required to draw definite conclusions about the shapes of these tubes.
We examined a large sample of thin flux tubes in a potential field model of an active region. Our conclusion is that at least in the studied model, the aspect ratio at the apices of the initially round (at the base) tubes varies from 1.5–5. The distribution peaks at 2 but it is highly skewed and has a median around 4.5. We verified that the oblateness is only slightly influenced by the inclination of the field at the base and is therefore not strongly modulated by the initial cross-sectional shape of the flux tube.

We also examined the shape distortions for the end-to-end mapping of the flux tubes and concluded that, in general in our model field, these distortions are even stronger than the end-to-apex distortions. This result in principle implies that for a given coronal loop, two footpoints might be substantially different shapes. This finding, in turn, could affect heating models that assume that heat is deposited on both footpoints. These analyses are however beyond the scope of this paper; we chose to focus here on the apex properties of the coronal loops.

These oblateness values can strongly influence the perceived expansion of the flux tubes, such that the expansion depends on the viewing angle. In particular, we demonstrate that for two flux tubes, drawn from the model field and viewed from the top, that one of the loops expands and the other does not, whereas the opposite is seen when the system is viewed from the side.

We address the arguments of Klimchuk (2000), who concluded that coronal loops must be generally circular in cross section. The flux tubes in our rendering exercise, while strongly oblate in cross section, produced simple, single peak profiles in the rendered images. We measured widths of these flux tubes when viewed from different angles using the technique currently used in the literature (e.g., López Fuentes et al. 2008), namely, by fitting the profiles to Gaussian curves.

We demonstrated that the widths, inferred from a simulated image, do not correlate with the actual width of these flux tubes. Nor do they correlate with the diameters of circles of the same area as the flux tubes. In the two projections in which the flux tubes are clearly visible, their widths vary rather modestly, again, despite their strong oblateness.

We also make a point that for oblate, optically thin flux tubes, variations in width are associated with variations in brightness: when viewed along the “wide” side, a ribbon appears thin and bright, and when viewed perpendicular, the same ribbon appears wide but dim. This result also addresses the argument of López Fuentes et al. (2006), who pointed out that even if loops were oblate, a spread in orientations with respect to the l.o.s. would still allow one to observe their expansion—on average. This point is an important conclusion of this paper, as it implies that for any given viewing angle, there will be selection criteria that will make some ribbons stand out when they are viewed along the “wide” side.

Our findings also addressed the increased pressure scale height observed in some bright EUV loops (e.g., Warren & Winebarger 2003; Winebarger et al. 2003, and references therein), at the same time being consistent with evidence that
| Expansion          | 1. None                      | 2. Isotropic                     | 3. Anisotropic ⊥ to the l.o.s. | 4. Anisotropic along the l.o.s. | 5. Unresolved ⊥ to the l.o.s. |
|--------------------|------------------------------|----------------------------------|-------------------------------|---------------------------------|-------------------------------|
| Flux               | irrelevant                   | $\Phi \propto B d^2$            | $\Phi \propto Bab$, $b = const$ | $\Phi = BA$                     |
| Column depth       | $d(z) = \text{const}$       | $d(z) \propto B^{-1/2}$         | $d(z) = \text{const}$         | $d(z) \propto B^{-1}$          |
| $EM(z) \propto$    | $n^2$                        | $n^2 B^{-1/2}$                   | $n^2$                         | $n^2 B^{-1}$                   |

Figure 19. Impact of the expansion of coronal loops on the observed density scale height.

Figure 20. Model of how the emission measure scales with height for the flux tubes in Figure 19. The functions plotted are $n^2$ (dotted line), $n^2 B^{-1/2}$ (dashed line), and $n^2 B^{-1}$ (solid line). We assume a hydrostatic atmosphere $n \propto \exp(-z/z_0)$ with $z_0 = 50$ Mm for all curves and the field strength dropping like that of a point dipole, $B(z) = z_0/z^3$ (the constant for $B$ is chosen such that all three curves intersect at one density scale height).

Figure 21. Without prior knowledge of how magnetic flux tubes are distorted at a particular location, it would be hard to tell apart the emission produced by the two scenarios sketched in this figure. The first one is a result of several small-scale events that supply the corona with hot plasma; they are small enough to stay unresolved (or barely resolved) at the apex, and the plasma they supply is dense enough so that the structure is visible even with a low filling factor. The second scenario is a result of a single coherent event, but the flux tube in which this event is embedded gets elongated at the apex; such an event might not be much smaller than the perceived width of the structure at the apex. As the edges of the flux tube might not, in general, be smooth, the fluctuations in the line-of-sight depth might produce small “ripples” in the otherwise resolved structure.
individual strands are in fact resolved (López Fuentes et al. 2008; Aschwanden & Nightingale 2005). As we showed in the previous section, a resolved but highly oblate loop might demonstrate the same enhancement of apparent scale height as would an unresolved loop.

The immediate implications of our study are the following: without an analysis of the shape distortions between the bases and the apices of the coronal loops, we cannot put constraints on the size of the loop base, nor can we evaluate by how much the plasma in the loops is denser compared to the diffuse background. For example (see Figure 21), the same non-expanding coronal loop could be composed of a set of thin unresolved strands with small bases (so that they are unresolved at the apex) and high density (to compensate for the low filling factor), or it could be a single coherent structure of a larger size at the base, which does not expand along the l.o.s. Moreover, if the cross section is not a smooth shape but rather has a "wrinkled" edge (e.g., Gudiksen & Nordlund 2005 reported that in their model field, the circular cross sections of thin flux tubes deformed into rippled shapes further along the axes), the individual "wrinkles" may, due to the increased column depth, appear brighter. Therefore, when viewed at a higher resolution, both systems might resolve into thin strands. Figure 22 shows at such features on the Sun. Even if we consider that some (or all) of these are bundles of strands, these bundles appear to be elongated in cross section, which by itself is consistent with our findings.

Our study is focused on the properties of magnetic flux tubes; however, care must be taken to distinguish between flux tubes and observed coronal loops. Loops fill only a fraction of the coronal volume and the selection mechanisms that determine which flux tubes get filled with hot and dense plasma are a topic of ongoing research. It might in principle be possible that, for example, plasma is more likely to get heated in flux tubes that exhibit a relatively isotropic expansion (in our experiment, as Figure 9 shows, out of roughly 9000 flux tubes, there are about a thousand for which $a_{\text{apx}}/b_{\text{apx}} \lesssim 2$). In that case, coronal loops might predominantly have a circular cross section even if for flux tubes this fact is not generally true. Evidence contradicting the above statement is that loop fans are associated with topological divides and therefore regions of high, rather than low, distortions (Schrijver et al. 2010). It might also be the case that the events that result in the deposition of hot plasma into the atmosphere happen predominantly at coronal heights, and are approximately round across the field—in the corona. Our preliminary experiments suggest that most of the shape distortion happens relatively close to the lower boundary (e.g., see the cross sections of our simulated loops in Figures 14 and 15), so flux tubes round at their apices might undergo less shape distortion at their higher portions, in which case they would have highly distorted footpoints. If observational evidence in favor of either scenario becomes available, that might be very important for our understanding of the mechanism of coronal heating. In the absence of such, our analysis gives us reasons to believe that loops should not, in general, be assumed to have circular cross sections throughout their length.

Overall, our study allows one to explain the constant width of coronal loops as well as their excessive pressure scale height using fewer assumptions. Indeed, rather than involving additional entities in the explanation (such as size of the strands, coronal currents, and so on), we lift the assumption of the circular cross section. We show that the aforementioned properties of loops might be a natural consequence of a simple property of magnetic fields that has not been analyzed to date.

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