Multi-objective Planning for Distributed Generation with Consideration of the Randomness of DG output

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Abstract. DG as an alternative clean energy has a rapid development with the advantages of economy, environment protection and so on. But the randomness of power output has an influence on the security and stability of power system. At the same time, the trade-off among multiple optimization goals is often involved in the actual planning process. Therefore, using multi-objective optimization method to plan DG has wider application prospects. In this paper, a multi-objective optimization model of DG planning is established from three aspects of economy, safety and environmental protection. The flexible parameters of power system are introduced to represent the multi-objective optimization sub-functions and constraint conditions. Considering the influence of the stochastic feature of DG output, the multi-objective programming model is transformed into a flexible integrated optimization problem. One of the goals is lower cost, the others are safety and environmental protection. At the same time the objectives of economy, security and environmental protection are optimized comprehensively after DG integration, which are greatly adapted to the actual needs of the power grid planning department. In addition, the introduction of the static voltage stability margin optimization sub objective is of great significance for solving the problems caused by the uncertainty of load forecasting during the planning process.

1. Introduction
Under the background of a nationwide acceleration of clean energy replacement, Distributed Generation (DG) is raising more and more attention. Adding DG into power distributed network could perform an important function on peak-load shifting, improving voltage level, reducing the network loss and lightening environmental pollution [1,2]. However, with the integration of DG, the stochastic feature also brings negative effect on the safe and stable operation of power system [3,4]. Therefore, to fully take advantage of DG, rational planning of the integration of DG is vital and essential. It should be carefully considered to maximize the comprehensive benefit as well as meeting the technical requirements.

The planning issue of DG integration generally incorporates the site selection and size determination. Considering from the aspect of investment, an optimization model with the objective function of the total cost is built in reference [5,6], while the influence of DG integration to the safe and stable operation of power system is neglected. The minimum network loss is set as a goal in reference [7], but the investment cost and other elements are not taken into account. In reference [8],
optimizing site selection with the goal of maximizing DG penetration level is conducted from the perspective of clean energy utilization. Under this circumstances, environmental benefit is also maximized.

In reference [9,10], the optimization of site selection and size determination of DG planning in multi-stage are considered. Multi-objective programming model of DG is built in reference [11,12], but the stochastic feature of DG output is not discussed during the optimization process. In reference [13,14], multi-objective programming technique is applied, however, it is not enough to perform the optimization process only from two perspectives.

At present, the research on DG planning has made great progress and achievements. However, there are also some problems, mainly manifested in the following points: 1) The single-objective DG planning model only satisfies the requirements of the distribution system unilaterally and cannot meet the actual comprehensive requirements considering cost, safety and environmental protection. Therefore, using multi-objective optimization method has a broader application prospect. 2) The characteristic of strong randomness of DG output is not fully considered. Although the stochastic programming method can approximately simulate the characteristic of the strong randomness of DG output, it should not be ignored that there is an artificial approximation assumption for its distribution law, or the safety constraint conditions are allowed to be damaged to some extent, resulting in the danger of the system exceeding the limit.

In view of the problems existing in the research of DG planning, a multi-objective optimization model of DG planning is established from the perspectives of cost, safety and environmental protection. The flexible parameters of power system are introduced to represent the multi-objective optimization sub-functions and constraint conditions. Considering the stochastic effect of DG output, the mathematical model of DG multi-objective programming is transformed into a flexible two-objective comprehensive optimization problem. The constraint conditions are the same. One of the goals is to take the state variables and control variables as optimization variables for economy. The other is to take safety and environmental protection as the target, and state variables, control variables and flexibility parameters are set as the optimization variables. At the same time, the economy, security and environmental protection of the power system are optimized. Therefore, it is a comprehensive optimization to adapt to the actual needs of the power grid planning department.

In addition, considering that the distributed generation power should be mainly absorbed by nearby sources, the reverse power flow constraint is added in the planning model, that is, in principle, the reverse power supply to the superior voltage is not allowed.

2. Multi-objective optimization model of DG

With consideration of economy, safety and environmental protection, a multi-objective optimization model of DG planning is established in this study. Three distinct optimization targets, namely the investment cost of DG, the penetration level of DG and static voltage stability margin, were included in the optimization functions. To be precise, the investment cost of DG reflects the objective of economy. The penetration level of DG reflects the objective of environmental protection. The static voltage stability margin reflects the improvement of distributed system voltage security with the integration of DG.

2.1. Minimum optimization sub-function of total investment cost of DG

The total investment cost of DG is composed of comprehensive cost and installation cost. The objective function of investment cost is shown as equation (1).

$$\min f_i = \sum_{i=1}^{n_{DG}} (c_{i1} + c_{i2})P_i$$  \hspace{1cm} (1)

Where $n_{DG}$ is the total number of nodes that DG can be installed. $P_i$ is the rated capacity of the DG installed at node $i$. $c_{i1}$ and $c_{i2}$ represent the comprehensive cost and the installation cost per unit capacity of the DG installed at node $i$ respectively.
2.2. Maximum optimization sub-function of penetration level of DG

The concept of DG penetration level can be understood as the maximum DG power that can be integrated without affecting the safety, reliability and stability of the power grid. The objective function of penetration level is shown as equation (2).

$$\max_{i \in \Phi_g} P_{DG_i} = \sum_{i \in \Phi_g} P_{DG_i}$$ (2)

Where $P_{DG_i}$ is the DG capacity integrated through node $i$, and $\Phi_g$ represents the node set that DG can connect.

2.3. Maximum optimization sub-function of static voltage stability margin

Recent research shows that DG could play an important role in improving the static voltage stability margin of distribution system [15]. Hence, the voltage stability parameter in function (3) is developed to quantify the effect of DG.

$$L_b = 4 \left[ (XP_b - RQ_b)^2 + (XQ_b + RP_b)V_a^2 \right] / V_a^4$$ (3)

In equation (3), $L_b$ is the voltage stability parameter of branch $b$ (the start node is $a$, the end node is $b$); $X$ and $R$ respectively represents the reactance and resistance of branch $b$; $Q_b$ and $P_b$ respectively represents the reactive power and active power of branch $b$; $V_a$ is voltage amplitude of the start node.

The maximum voltage stability parameter of all branches is selected as the whole system voltage stability parameter $L$, as shown in equation (4).

$$L = \max(L_1, L_2, \cdots, L_{N-1})$$ (4)

Where $N$ is the total number of system nodes. The branch corresponding to $L$ is called the weakest branch of the system. When voltage collapse occurs in the system, it must be the weakest branch corresponding to $L$ that starts. Hence, the maximum optimization sub-function of static voltage stability margin could be equivalent to equation (5).

$$\min f_j = \min L$$ (5)

2.4. The constraint conditions

The constraint conditions include equality constraints and inequality constraints. Equality constraint is power flow equation, inequality constraint incorporates: node voltage constraint, branch power constraint, generator output constraint, DG output constraint, reverse power flow constraint and so on, as shown in equation (6).

$$\begin{align*}
V_{i^{\min}} &\leq V_i \leq V_{i^{\max}} \quad i \in \Phi \\
|P_{li}| &\leq P_{li}^{\max} \quad i \in \Phi \\
P_{DG_i} &\leq P_{DG_i}^{\max} \quad i \in \Phi_g \\
P_{li} &\leq P_{li}^{\max} \quad i \in \Phi \\
P_{DG_i} - P_{li} - P_{Di} &\leq \lambda S_i^{\max} \quad i \in \Phi_g 
\end{align*}$$ (6)

Where $V_{i^{\min}}$ and $V_{i^{\max}}$ represent the upper and lower limits of node $V_i$ respectively; $\Phi$ is a collection of distributed network nodes; $P_{li}$ is the active vector of a node, $P_{li}^{\max}$ is the upper limit of active vector; $P_{DG_i}$ is the active power of DG integrated into node $i$; $P_{DG_i}^{\max}$ is the maximum active power of DG integrated into node $i$, $\Phi_g$ is node set to be selected by DG; $P_{li}$ is the load of node $i$, $P_{Di}$ is the active power of the downstream network of node $i$; $S_i^{\max}$ is the upstream branch power directly connected with node $i$, $\lambda$ is the reverse power coefficient of DG and the recommendation number is 0.3.
In addition, the power of DG should be mainly absorbed by nearby sources. In principle, it is not allowed to send power back to the superior level, which means:

$$\sum_{i \in I} P_{DG_i} \leq \sum_{i \in I} P_{Li} \quad (7)$$

3. Flexibility of power system

According to reference [16], the flexibility of power system can be described as the adaptability and controllability of power system when the flexibility parameter δ changes. And the variation range of power system parameters could be described as equation (8).

$$T(\delta) = \left\{ \delta \mid \varepsilon^- (\delta) \leq y \leq \varepsilon^+ (\delta) \right\} \quad (8)$$

Where δ is the flexibility parameter that determines the variation range of parameters and the capability of flexibility. ε and ε', which are functions of δ, respectively represents the lower bound and the upper bound of the variation range of power system parameters.

The constraint conditions of power system optimization model can be expressed by the following equation.

$$\begin{cases} \sum_{j \in J} g_j (x, u, y) = 0, & j \in J \\ \sum_{i \in I} h_i (x, u, y) \leq 0, & i \in I \end{cases} \quad (9)$$

Where x is the state variable, u is the control variable, and y is the uncertain parameter. The flexibility of these parameters can be expressed by equation (8). I represents the inequality constraint set that include node voltage constraint, branch power flow constraint, generator output constraint and some other constraints. J represents the equation constraint set that mainly incorporate the power balance equations.

Therefore, for $T(\delta)$, which has as many values as possible, the flexible constraint condition can be expressed as equation (10).

$$\max_{y \in T(\delta)} \min_{u \in I} \max_{i \in d} f_i (u, y) \leq 0 \quad (10)$$

In equation (10), max means to select the largest one from all inequality constraints. min means to improve the safety and reliability of the power system as much as possible by adjusting the control variable u: max $y \in T(\delta)$ indicates that the safety and reliability of power system in the worst conditions could be described by adjusting the corresponding parameters.

3.1. Flexible representation of penetration level of DG

Generally, DG output fluctuates randomly within a range, hence it could not be optimized as a deterministic variable. According to flexibility concepts, the flexibility of DG output belongs to the category of linear flexibility, and the variation range of its parameters can be described as equation (11).

$$T(\delta) = \left\{ \delta \mid P_{DG}^N - \Delta P_{DG}^- \leq P_{DG} \leq P_{DG}^N + \Delta P_{DG}^+ \right\} \quad (11)$$

Where $P_{DG}^N$ is the power fluctuation center of DG output, and $\Delta P_{DG}^-$ and $\Delta P_{DG}^+$ respectively represents the negative fluctuation deviation and the positive fluctuation deviation of DG output.

Research shows that the output power of DG has cluster effect which means it fluctuates in a stable range. Therefore, the upper and lower limits of the output power of DG is certain. Under flexible constraints of power system and with a given value of $\delta$, the maximum value of DG output power could meet the security and reliability constraints of power system. Then with the same value of $\delta$, any output power $P_{DG}$ could definitely meet security constraints. Hence, equation (11) can be simplified as follows:
Equation (12) eliminate the influence of negative fluctuation deviation of DG output power. The penetration level of DG is simplified to an optimization question of flexible parameter $\delta$. The fluctuation center $P_{DG}^N$ and positive fluctuation deviation $\Delta P_{DG}^+$ would only participate in the optimization as constant coefficients of $\delta$ and have no influence on the calculation results.

Finally, the maximum optimization sub-function of penetration level of DG is shown as equation (13).

$$\max P_{\text{total}} = \max f(\delta) = \sum_{i=1}^{m} \delta_i^2$$

### 3.2. Flexible representation of index of voltage stability margin

According to the definition of flexibility, the index of voltage stability margin belongs to linear flexibility. Hence the allowable range of uncertain variation could be described as equation (14).

$$L_{ui} - \delta_{ui} \Delta L_{ui} \leq L_{ui} \leq L_{ui} + \delta_{ui} \Delta L_{ui}$$

The derivation process of indicator $L$ illustrate that the smaller $L$, the better the voltage stability of system. Adversely, the larger $L$, the worse the voltage stability of system. When $L$ approaches 1.0, the system voltage would collapse. Therefore, the level of system voltage stability, namely the voltage stability margin, could be determined according to the distance between the value of $L$ and the critical value 1.0. Hence, equation (14) is equivalent to equation (15) when $\Delta L_{ui}$ is taken as a constant.

$$0 \leq L_{ui} \leq 1 - \delta_{ui} \Delta L_{ui}$$

Therefore, the maximum optimization sub-function of static voltage stability margin can be flexibly expressed as equation (16).

$$F(\delta) = \max(\delta_1, \delta_2, \ldots, \delta_{N-1})$$

The optimal solution of equation (16), which is $F(\delta^*)$, represents the flexible indicator of static voltage stability of power grid. And the value of $F(\delta^*)$ reflects the margin of static voltage stability of power system. The greater the flexible index, the stronger the ability of power system to control voltage when the changes of parameters are uncertain.

### 3.3. Flexible representation of constraints

After the penetration level of DG is expressed flexibly, uncertain parameters would be contained in the constraints of power system optimization. Therefore, the inequality constraint conditions of equation (6) can be flexibly performed as the following:

$$\max \min \max_{P_{DG}, P_i, Q_i, \delta} \begin{cases} P_i - P_i^{\max} \leq 0 \\ V_i^{\min} - V_i \leq 0 \\ V_i - V_i^{\max} \leq 0 \\ P_{\text{min}}^{Gi} - P_{\text{Gi}} \leq 0 \\ Q_{\text{min}}^{Gi} - Q_{\text{Gi}} \leq 0 \\ Q_{\text{Gi}} - Q_{\text{max}}^{Gi} \leq 0 \\ P_{DGi} - P_{DGi}^{\max} \leq 0 \\ P_{DGi} - P_{DGi} - \lambda S_i^{\max} \leq 0 \end{cases}$$
4. Multi-objective programming model with consideration of stochastic feature of DG
Based on the flexibility representation of penetration level and static voltage stability margin of DG, the output power of DG and the voltage stability margin are set to be variable $y$, investment cost objective function $f_{\text{value}} = -\min f_i$ is introduced, the multi-objective planning flexible mathematic model could be expressed as following:

$$
\begin{align*}
\max f_{\text{value}} \\
\max \eta^T(\delta - \delta_0) \\
\max \min \max_{\epsilon_n, \epsilon_n, \delta_0} \left\{ \begin{array}{l}
P_i - P_i^{\max} \leq 0 \\
V_i^{\min} - V_i \leq 0 \\
V_i - V_i^{\max} \leq 0 \\
P_{\text{DG}} - P_{\text{DG}}^{\max} \leq 0 \\
Q_{\text{DG}} - Q_{\text{DG}}^{\max} \leq 0 \\
Q_{\text{DG}} - Q_{\text{DG}}^{\max} \leq 0 \\
P_{\text{DG}} - P_{\text{DG}}^N = 0 \\
Q_{\text{DG}} - Q_{\text{DG}}^N = 0 \\
I_a \leq 1 - \delta_a \Delta I_a \\
\end{array} \right.
\end{align*}
$$

(18)

Where, $\eta = [\eta_L, \eta_{DG}]^T$ stands for weighting coefficient of flexible parameters and $\eta_L + \eta_{DG} = 1$. $\delta_0 = [\delta_{\text{L},0}, \delta_{\text{DG},0}]^T$ is multiscale reference value of flexible parameters and it could be measured by statistic data.

5. Mathematical model solution
As showed in equation (18), programming model is a multi-objective optimization problem and could be equivalent to:

$$
\begin{align*}
&\max f(x, u, y) \\
&\max \eta^T(\delta - \delta_0) \\
&\max \min \max_{y, u, k, i} h_k(x, u, y) \leq 0 \\
&s.t.: g(x, u, y) = 0 \\
&y \in T(\delta)
\end{align*}
$$

(19)

In equation (19), the objective functions are conflicted with each other. A trade-off decision has to be made. The equation (19) could be departed into two sub-problem:

Sub-problem 1:

$$
\begin{align*}
&\max f(x, u, y) \\
&\max \phi(\delta) = \min v \\
&s.t.: h_i(x, u, y) \leq v \quad i \in I \\
&g(x, u, y) = 0
\end{align*}
$$

(20)

Sub-problem 2:
In the sub-problem 1, flexible parameter $\delta$ of DG power is a constant and let $v$ be its max value, which is the critical value 0, then equation (20) could be simplified to equation (22).

$$\max f(x,u,y)$$
$$\max \eta^T(\delta - \delta_0)$$
$$\max v$$
$$s.t. \begin{align*}
  h_i(x,u,y) &\leq v & i \in I \\
v &\leq 0 \\
g(x,u,y) &= 0 \\
y &\in T(\delta)
\end{align*}$$

(21)

In the sub-problem 2, when the maximum value of $v$ equals to critical value 0, equation (21) could be simplified to equation (23).

$$\max f(x,u,y)$$
$$s.t. \begin{align*}
  h_i(x,u,y) &\leq 0 & i \in I \\
g(x,u,y) &= 0
\end{align*}$$

(22)

For the sub-problem 2, Lagrange function could be made as shown in equation (24).

$$L_u(x,u,y) = \max_{x,u} f(x,u,y) + \alpha^T g(x,u,y) + \beta^T h(x,u,y)$$

(24)

The dual problem is shown in equation (25).

$$L_d(\alpha,\beta) = \min_{\alpha,\beta} \max_{x,u} f(x,u,y) + \alpha^T g(x,u,y) + \beta^T h(x,u,y)$$

(25)

The sub-problem 2 provides the lower limit of the optimal value of optimization problem. Dual problem reflects the complementary relaxation constraint in K-T condition, which provides the upper limit of the optimal value of optimization problem. Thus, the following equation (26) is established:

$$L_u(x,u,y) \leq L(x,u,\alpha,\beta) \leq L_d(x,u,\alpha,\beta)$$

(26)

Where: $x_0, u_0, \alpha_0, \beta_0$ respectively represent the original value of each variables and parameters; $x, u, \alpha, \beta$ respectively represent the optimal value of each variables and parameters. When $f, h, g$ are convex functions, the equal sigh in equation (26) could be established. In power system analysis, $f, h, g$ could be approximately thought to be convex functions within limits. Therefore, the following equation appears:

$$f(x,u,y) + \alpha^T g(x,u,y) + \beta^T h(x,u,y) \leq f(x_0,u_0,y_0) + \alpha^T g(x_0,u_0,y_0) + \beta^T h(x_0,u_0,y_0)$$

(27)

Which is:

$$f(x,u,y) \leq f(x_0,u_0,y_0) + \alpha^T [g(x_0,u_0,y_0) - g(x,u,y)] + \beta^T [h(x_0,u_0,y_0) - h(x,u,y)]$$

(28)

The equation (28) could be re-written in the following way:

$$f(x,u,y) \leq f_0 + \Delta f$$

(29)
Where

$$
\Delta f = \alpha^T [g(x_0, u_0, y_0) - g(x, u, y)] + \beta^T [h(x_0, u_0, y_0) - h(x, u, y)]
$$

(30)

The equation (29) stands for the optimal condition for optimize the sub-problem of equation (22) under certain parameter $\delta$. Applying equation (29) to equation (22), the sub-problem 2 could be presented as the following:

$$
\begin{align*}
\max_{x, u} & \quad f(x, u) \\
\text{s.t.} & \quad h_i(x, u, y) \leq 0, \quad i \in I \\
 & \quad g(x, u, y) = 0 \\
 & \quad y \in T(\delta)
\end{align*}
$$

(31)

Thus it can be seen that multi-objective optimization problem could be transformed into two optimization sub problem with each single objective. One optimization objective is to lower the investment cost; the other objective is to improve the level of security and environmental protection. It should be noted that sub problem of equation (22) should be solved firstly before solving the comprehensive optimization problem. The parameters of $f_0$ and $\Delta f$ could be ascertainment. Then the equation (31) could be solved. The solution of the original problem can be obtained by solving the two sub-problems (22) and (31) through cross iteration.

In addition, equation (18) is a typical multi-objective non-linear optimization problem. There is no absolute optimal solution. The solution most satisfying to the engineering reality could only be selected from the Pareto optimal frontier set of the optimization problem. DEMPSO algorithm in reference [17] is applied in this paper to optimize the siting and sizing problem of DG integration.

6. Case study

In this paper, the modified 43-node distribution network in reference [18] is adopted as the test system (the voltage level is 100 kV). The configuration is shown in figure 1. Corresponding to the year of planning level, the active load is 349 MW and the reactive load is 154 Mvar. The planning total capacity of DG don’t exceed fifty percent of the total load of the system. The deviation of the active power of DG is set to be 5 MW and the power factor is 0.9. The node set of DG installation is [2, 3, 4…43]. The comprehensive cost and installation cost of DG per unit capacity are 0.7×106 dollars MW-1 and 0.2×106 dollars MW-1 respectively in reference [19].

![Figure 1. The modified 43-node distribution network](image)

In order to verify the influence of randomness of DG output on penetration level of DG, deterministic analysis method in reference [20] and the approach purposed in this paper are applied
respective to program the optimal sub target of penetration level of DG. The results are illustrated in table 1.

Table 1. DG power planning results with and without randomness.

| Algorithm                          | DG penetration level/MW | Planning result          |
|------------------------------------|-------------------------|--------------------------|
| deterministic analysis method      | 170                     | 5(4),6(5),17(3),26(2),29(2) |
|                                    |                         | 32(3),36(6),37(3),38(6)  |
| Method proposed in this paper      | 130                     | 5(3),6(3),17(3),26(1),29(1),32(2),36(4),37(1),38(6) |

Note: the number outside the brackets is the number of busbar where DG installed, the number inside the brackets means the optimal installed capacity, the unit is 5MW.

The results in table 1 illustrate that at a certain load level the acceptable total power of most nodes and systems decreased to some extent after considering the stochastic feature of DG output. The reason is that in a certain system of which the conventional unit parameters margins are adjustable, the results calculated by the deterministic analysis method merely represent that the system satisfies the security and stability constraints under the current level of power. Because of the randomness of DG output, there could be one or multiple values of the power of DG at this maximum power level. Then the matching solution could not be found, results in the security and stability constraints of the system being damaged. The system is in danger of exceeding its limits. Especially, when the adjustable margin of system control parameters could satisfy the random variation of DG output, which means any power value at the current power level has a matching control parameter, then the optimal solution of two methods will be equal, as shown in nodes 17, 32, 38 and so on.

Moreover, in practical planning, balance among multiple objectives are generally involved according to the will of decision maker. Table 2 compares the different results under different planning targets by using the method proposed in this paper with consideration of the stochastic feature of DG output. Case 1 considers the optimal investment cost. Case 2 tends to have an optimal penetration level. Optimal indicator of static voltage stability margin is emphasized in case 3. In case 4, an integrated decision-making optimization of three sub-target is conducted under the condition of equal weight.

Table 2. DG power planning results with and without randomness.

| Planning case | Investment cost/10^5dollars | Penetration level/MW | Voltage index | Planning result                      |
|---------------|------------------------------|----------------------|---------------|--------------------------------------|
| Case 1        | 112.5                        | 110                  | 0.06          | 3(2),6(1),14(2),18(2),26(1),32(4),33(3),36(2),37(3),38(2) |
| Case 2        | 121.2                        | 130                  | 0.05          | 5(3),6(3),17(3),26(1),29(1),32(2),36(4),37(1),38(6) |
| Case 3        | 124.3                        | 125                  | 0.04          | 2(1),3(3),9(2),18(2),27(4),31(1),32(2),33(1),34(2),35(3),36(3),42(1) |
| Case 4        | 118.2                        | 120                  | 0.05          | 16(2),17(3),18(1),27(5),33(4),36(2),40(1),41(3),42(3) |

Note: the number outside the brackets is the number of busbar where DG installed, the number inside the brackets means the optimal installed capacity, the unit is 5MW.

According to table 2, case 1 emphasizes the optimization of investment cost but the performance of DG penetration level and the improvement of system static voltage indicator is not so satisfying. Case 2 focuses on better penetration level of DG but the corresponding investment cost is high. In case 3, the importance of improving the static steady-state voltage is emphasized, while to some extent the penetration level is limited and a higher price of DG has been paid. In case 4, multi-objective integrated flexible programming is adopted. Though the corresponding three sub-targets are not optimal, the overall satisfaction of the planning strategy is better than that of a single goal. The relationship among sub-objectives could be well coordinated, so as to provide a better planning strategy of DG for the decision makers and the actual planning needs could be greatly met.
7. Conclusions
In this paper, multi-objective optimization model of DG planning is built from three perspectives of economy, security and environmental protection with consideration of stochastic feature of DG output. By introducing the flexibility parameters of power system, the multi-objective optimization sub-functions and constraint conditions are flexibly expressed to form the two-objective comprehensive optimization problem. Through the comparative analysis of examples, the following conclusions can be drawn.

(1) The flexible expression of DG power could effectively solve the uncertainty problems of DG output in the optimization planning, deal with the constraint conditions with uncertain parameters more flexibly, and can reasonably deal with the influence of the randomness of DG output on the system security and stability.

(2) In this paper, the optimal site selection of DG is comprehensively modeled from the three perspectives of DG investment cost, DG penetration level and static voltage stability margin. Based on the power system security and stability constraints, the security, environmental protection and economy are all taken into account and it is more adapt to the actual planning needs.

(3) The introduction of the subitems of static voltage stability margin optimization is of great significance to solve the problems caused by the uncertainty of load prediction in the planning process.

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