Double-couple earthquake source: symmetry and rotation

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SUMMARY
We consider statistical analysis of double-couple (DC) earthquake focal mechanism orientation. The symmetry of DC changes with its geometrical properties, and the number of 3-D rotations one DC source can be transformed into another depends on its symmetry. Four rotations exist in a general case of DC with the nodal-plane ambiguity, two transformations if the fault plane is known and one rotation if the sides of the fault plane are known. The symmetry of rotated objects is extensively analysed in statistical crystallographic texture studies, and we apply their results to analysing DC orientation. We consider theoretical probability distributions which can be used to approximate observational patterns of focal mechanisms. Uniform random rotation distributions for various DC sources are discussed, as well as two non-uniform distributions: the rotational Cauchy and von Mises–Fisher. We discuss how parameters of these rotations can be estimated by a statistical analysis of earthquake source properties in global seismicity. We also show how earthquake focal mechanism orientations can be displayed on the Rodrigues vector space.

Key words: Probability distributions; Earthquake source observations; Computational seismology; Statistical seismology; Fractures and faults.

1 INTRODUCTION
This paper addresses two problems: the random rotation of double-couple (DC) earthquake sources and how symmetry properties of these sources influence their rotation angle distribution and their display. Properties of earthquake focal mechanisms and methods for their determination are considered by Snoke (2003) and Gasperini & Vannucci (2003). Ekström et al. (2012, and references therein) discuss their extensive work on evaluating seismic moment tensors for global earthquakes.

In this paper, we consider only the DC earthquake focal mechanism. For tectonic events, non-DC mechanisms like the compensated linear vector dipole (CLVD) are likely due to various systematic and random errors in determining the mechanism (Frohlich & Davis 1999; Kagan 2003, 2009). These results suggest that routinely determined CLVD values would not reliably show the deviation of earthquake focal mechanisms from a standard DC model.

Snoke (2003) and Gasperini & Vannucci (2003) consider several equivalent representations for DC sources and their properties, and provide mathematical expressions for their mutual transformation. Krieger & Heimann (2012, and references therein) review routines for plotting moment tensors and focal mechanisms.

Two general techniques can be employed to study the 3-D rotation: orthonormal rotation matrices and normalized (unit) quaternions. The quaternion method has been used to evaluate these rotations in many investigations of earthquake focal mechanisms (see, e.g. Kagan 1991; Frohlich & Davis 1999; Kagan 2009; Kagan & Jackson 2011; Tape & Tape 2012). Kagan (2007) explains how ‘ordinary’ matrices and vectors can be used to obtain 3-D rotation parameters. Below we comment on the advantages and drawbacks of both methods.

Altmann’s (1986) book was a first monograph specifically dedicated to 3-D rotations [group SO(3)] and quaternions. At present, quaternions are widely used to describe rotations in space satellite and aeroplane dynamics (Kuipers 1999) and in simulations of virtual reality, robotics and automation (Hanson 2006; Dunn & Parberry 2011). These last three monographs explain quaternions in a more accessible manner. Many journal articles (see references in these monographs and in Kagan 2009) discuss practical application of quaternions for analysing the 3-D rotations.

However, the above publications do not consider the symmetry properties of rotated objects or how symmetry influences orientation analysis. As Kagan (1990, 1991, 2007, 2009) indicated, that because of the DC symmetry properties, the techniques considered in those publications cannot be used for the DC source orientation studies without major modifications. Depending on properties of DC earthquake focal mechanism, there are three types of earthquake DC source symmetry (Kagan 1990). The only scientific discipline where symmetry is extensively considered in 3-D rotation analysis is study of crystallographic (material) texture (Handscomb 1958; Mackenzie 1958; Grimmer 1979; Frank 1988; Heinz & Neumann 1991; Morawiec 2004; Meister & Schaeben 2005; Schaeben 2010).

Thus, this paper has four goals:

(1) We consider the symmetry of different representations of the DC earthquake source.
We present an introduction to the theory of the Rodrigues vector space, extensively used in the crystallographic texture analysis, and consider its application for displaying and describing the earthquake focal mechanism distribution.

We summarize theoretical statistical models of the DC source distributions.

As an illustration of the applicability of the above theoretical constructs to the statistical analysis of DC distributions, we update the results of the previous investigations of these statistical laws.

In this paper, the DC symmetry properties are described in Section 2. Section 3 considers 3-D rotation of the DC sources. Section 4 discusses applying the results of crystallographic texture analysis to DC source investigations. In Section 5, we consider theoretical probability distributions used to approximate observational patterns of focal mechanisms. Section 6 is dedicated to the statistical analysis of earthquake source properties in global seismicity. Using the results of Section 4, we also show how earthquake focal mechanism orientations can be displayed in the Rodrigues vector space. Sections 7 and 8 summarize our results.

2 FOCAL MECHANISM SYMMETRY

Depending on the known properties of DC earthquake focal mechanism, we consider three types of earthquake source symmetry (Kagan 1990):

(1) DC1—DC with no symmetry or the identity (I) symmetry, if the focal plane and its sides are known;
(2) DC2—DC with $C_2$, order 2 cyclic symmetry, that is, the focal plane is known, but its sides are not;
(3) DC4—DC with nodal planes that are not distinguishable; it has $D_2$, order 2 dihedral symmetry.

These earthquake source symmetries correspond to the following crystallographic symmetries considered in crystallographic texture analysis (see, for instance, Morawiec 2004). DC4 has an orthorhombic symmetry (as in a rectangular right parallelepiped or a rectangular box with unequal sides); DC2 has a monocline symmetry (as in a 3-D prism with two angles of 90° and one arbitrary angle); DC1 has a triclinic, or no symmetry.

Fig. 1 displays the geometry of the DC source (Aki & Richards 2002). It represents the quadrupolar ‘beachball’ radiation patterns of earthquakes. The focal plots involve painting on a sphere the sense of the first motion of the primary far-field P waves: solid for compressional motion and open for dilatational. The two orthogonal nodal planes separating these areas are the fault and the auxiliary planes. During routine determination of focal mechanisms, it is impossible to distinguish between these planes, a property called ‘nodal-plane ambiguity’. The planes’ intersection is the null-axis (called b-axis), the p-axis is in the middle of the open lune and the t-axis is in the middle of the closed lune. These three axes are called the ‘principal axes of an earthquake focal mechanism’, and their orientation defines the mechanism.

To make the focal mechanism picture unique, the eigenvectors are pointed down in seismological literature. However, the handedness of the coordinate system formed by the vectors can change as the result of such an assignment. The systems of the opposing handedness cannot be rotated one into another. In most of our considerations, we use the right-handed coordinate system placed at each earthquake centroid.

Fig. 2 displays four examples of the right-handed coordinate system for a DC4 source. The system can be arbitrarily rotated, and

Figure 1. Schematic (beachball) diagram of the DC earthquake focal mechanism and its quadrupole radiation patterns. The null (b) axis is orthogonal to the t- and p-axes, or it is located on the intersection of fault and auxiliary planes, that is, perpendicular to the paper sheet in this display. The n-axis is normal to the fault plane; u is a slip vector.

Figure 2. Four schematic diagrams of earthquake focal mechanism with the DC4 symmetry. The right-hand coordinate system is used. We show how three vectors can be arranged in a standard representation of a 3-D Cartesian coordinate system.
the handedness of the system is preserved. The left-handed system can be obtained in this picture if one inverts the direction of any individual axis (vector) or of all three axes. If the direction of two axes is reversed, the handedness of the system is preserved.

The earthquake focal plane can often be determined by inverting the higher rank point seismic moment tensors (McGuire et al. 2001; Chen et al. 2010) or by the aftershock pattern. The face/side (up/down or foot/hangingwall) of a focal plane generally is unknown. In such a case, the DC focal mechanism has a $C_2$ symmetry; we call it DC2. Finally, the face or the side of the focal plane can be known, as occurs in geological studies of earthquake faults; it is shown in Fig. 1 by symbols A and B. Such a source is called DC1. In our simulation of earthquake space-focal mechanism structure (Kagan 1982), the geometrical properties of each microdislocation is assumed to be known, thus the source is DC1.

Fig. 3 illustrates the difference between DC mechanisms of various symmetries. A vertical cylinder of one material, ‘A’, shown by the beige colour, is rotated counter-clockwise in a half-space of another material, ‘B’. Suppose that seismic events would be registered in two places of their contact, ‘1’ and ‘2’. If the focal plane is unknown, we would have two DC4 sources shown at the right and left ends of the diagram. If the focal planes, which are vertical lines in this plot, are known, the sources are DC2, and for the known sides of the fault planes two sources are DC1. One DC4 can be transformed into another by four rotations: the zero (0◦) rotation and three rotations (180◦, 0◦, 0◦) rotation angles for such focal mechanisms are likely to correspond to the ‘misorientation angles’ and the minimum rotation angle as the ‘disorientation angle’.

Kagan (1991) represented the orientation of a DC4 source by a normalized quaternion. When applied to the DC4 parametrization, the identity quaternion (zero rotation) is identified with the strike-slip DC4 source with plunge (Aki & Richards 2002) angles

$$\alpha_T = \alpha_F = 0^\circ, \quad \text{and} \quad \alpha_B = 90^\circ$$

and azimuths

$$\beta_T = 0^\circ, \quad \text{and} \quad \beta_F = 90^\circ$$

(Kagan 1991, 2005, 2009). Any other DC4 source corresponds to a quaternion describing the 3-D rotation from the reference DC4 source (eqs 1–3). Thus, the orientation of an arbitrary source may be considered as a rotation from the reference source.

Here, we consider how to compute the rotation of a DC1 source needed to align it with some reference DC1 source. It is unlikely that sufficient data would exist on DC1 sources for a statistical study of their distribution. Moreover, if the disorientation angle between two DC1 sources is large, it would be almost impossible to identify their fault planes and plane faces. However, in some cases we need to measure their angle of disorientation or the angular distance. Small rotation angles for such focal mechanisms are likely to correspond to the $\Phi_{\text{min}}$ for the DC4 source, as mentioned earlier.

To compute the disorientation of a DC1 source, we can modify our FORTRAN program listed in Kagan (1991). When a fault plane and its faces are known, a focal mechanism would be better specified through a fault plane geometry (Aki & Richards 2002, figs 4.13 and 4.20) with three angles: strike or azimuth ($\phi$), dip ($\delta$) and rake ($\lambda$). Usually, the range of these angles is taken as follows: $0^\circ \leq \phi < 360^\circ$, $0^\circ \leq \delta < 90^\circ$, $-180^\circ \leq \lambda < 180^\circ$. The problem arises when comparing two sources if the dip ($\delta$) of one focal plane exceeds $90^\circ$, so that a footwall of one mechanism becomes a hangingwall for another source (Aki & Richards 2002). To simplify the calculations in our program (see below), we extend the $\delta$ range to $180^\circ$.

If the face/side of a fault plane is unknown, as shown in Fig. 3, we need to calculate the second angle of the rotation for the DC2 source. As with the DC1 source, the data on DC2 sources are sparse and insufficient for a statistical study, but we need a technique to measure their angles of disorientation. An easy way to accomplish this measurement would be to change the strike of the fault plane by a small angle and change the rake sign. The modified DC1ROT FOR program is available at http://jumpy.igpp.ucla.edu/~kagan/dc1rot.for.

In this program, we use the quaternion technique to determine the rotation angle and the rotation axis parameters to transform one DC source into another. Quaternions are used because for rotation angle $\Phi$ close or equal to $180^\circ$, the matrix method cannot determine the rotation axis parameters (Kagan 2007).

Kagan (2009, appendices A and B) discusses normalized quaternions and their relation to the DC4 source. Representations for the DC4 by seismic moment tensors as well as by orthonormal matrices are considered. A quaternion representation allows a relatively easy determination of the rotation angle and the axis parameters for the DC4 earthquake focal mechanism (Kagan 1991).

If only the rotation angle is needed, then one can use a scalar (dot) product of two quaternions (Hanson 2006, p. 65; Dunn & Parberry 2011, p. 255) to determine the angle

$$\cos(\Phi/2) = q^* \cdot q = q_0^* q_0 + q_1^* q_1 + q_2^* q_2 + q_3^* q_3,$$
where \( q' \) are normalized quaternions for each DC source and \( q_i \) are the quaternion's components.

In our program (Kagan 1991), we first compute the orthonormal matrix for each DC source and then determine the corresponding normalized quaternion. There is a possibility of losing precision when converting a matrix to a quaternion (Shepperd 1978; Horn 1987). A certain computation technique should be applied to avoid this. We used a similar technique in our programs DCROT.FOR (Kagan 1991, the end of the SUBROUTINE QUATFPS) and in DC1ROT.FOR (see above).

4 EARTHQUAKE FOCAL MECHANISM AND CRYSTALLOGRAPHIC TEXTURE STATISTICS

Frank (1988) proposes using the Rodrigues vector space to represent 3-D rotation of symmetrical objects. The 3-D Rodrigues vector \( r = (r_1, i, 2, 3) \) is calculated as

\[
\mathbf{r} = \mathbf{n} \tan(\Phi/2) \quad \text{for} \quad 0 \leq \Phi \leq 180^\circ, \tag{5}
\]

where \( \Phi \) is the rotation angle and \( \mathbf{n} \) is the rotation axis. This representation has an advantage: under any transformation of the Rodrigues map corresponding to a change of the reference orientation, straight lines transform into straight lines, and planes into planes. Each straight line segment in the space corresponds to a rotation around the fixed axis, the length of the segment connecting the origin to the point is equal to tan (\( \Phi / 2 \)).

For an object with the non-identity symmetry, accepted points lie in a region around the origin, which is called ‘the fundamental zone of the map’. It is a polyhedron, bounded by the planes which are orientationally equidistant between the origin and the neighbouring equivalent point by a symmetry rotation to the origin. Any points lying outside one of these planes have an equivalent point lying inside the fundamental zone (Frank 1988). For an orthorhombic crystal with three orthogonal axes, the fundamental zone is a cube, with its six faces orthogonal to the axes at a distance from the origin of \( \tan 45^\circ = 1 \) (Frank 1988). The cube is surrounded by three neighbouring zones, each divided into two at infinity.

Frank (1988), Neumann (1992) and Morawiec & Field (1996) propose using the Rodrigues vector space to display the disorientation of symmetric objects in a fundamental zone as a point in the space. The point coordinates are calculated as follows: the length of a vector \( \zeta \) is

\[
\zeta = \tan(\Phi_{\text{min}}/2), \tag{6}
\]

where \( \Phi_{\text{min}} \) is the minimum rotation angle. Three coordinates of a point in the zone are

\[
x_1 = \zeta \times \sin(\theta) \sin(\phi);
\]

\[
x_2 = \zeta \times \sin(\theta) \cos(\phi);
\]

\[
x_3 = \zeta \times \cos(\theta), \tag{7}
\]

where \( \theta \) is the colatitude, and \( \phi \) is the azimuth of the rotation axis. As shown in Fig. 4, we identify two points on the p-axis; similarly, \( x_2 \) is a point on t and \( x_3 \) on b.

Fig. 4 shows the fundamental zone for a DC4 sound (Heinz & Neumann 1991, fig. 7). It is a cube with corner coordinates \( x_1 = \pm 1.0; x_2 = \pm 1.0; x_3 = \pm 1.0 \). Owing to the DC4 symmetry and statistical source exchange symmetry (i.e. rotation of \( s_1 \) source into \( s_2 \) is equivalent to rotation of \( s_3 \) source into \( s_1 \)), an octant of the cube contains full information about the orientation distribution for uniformly random rotation. This octant is called the ‘MacKenzie cell’ (Morawiec & Field 1996, see also their fig. 1 displaying the cells for the \( D_3 \) and \( D_4 \) symmetries).

Each point ‘inside’ the cube uniquely corresponds to a certain orientation/rotation with a minimum rotation angle \( \Phi_{\text{min}} \leq 120^\circ \). The points inside the inscribed sphere of the cube correspond to the rotations with angles \( \Phi_{\text{min}} \leq 90^\circ \). The other three symmetrically equivalent rotations are situated outside the fundamental zone. For example, the point of the zero rotation is located at the cube centre, whereas three other rotation points are at infinity: \( x_1 = \pm \infty; x_2 = \pm \infty; x_3 = \pm \infty \), corresponding to 180° rotations. These points at \( \pm \infty \) are equivalent (Altmann 1986). Similarly, for any point inside the cube, three points outside correspond to the rotations with angles \( \Phi > \Phi_{\text{min}} \).

However, for certain point configurations up to four equal \( \Phi_{\text{min}} \) appear on the cube boundary. To demonstrate the symmetry representation in the Rodrigues space and the appearance of these equal minimum rotation angles in Figs 5–7, we show complicated trajectories of DC4 source rotation in the space.

For example, when a point moving orthogonally from the origin reaches a cube face, it simultaneously appears on the opposite face: two 90° rotations produce the same effect (Fig. 5). This means that when we determine the minimum angle \( \Phi_{\text{min}} \) for cyan point rotations shown in Fig. 5 using the program developed by Kagan (1991), we find two equal solutions. The remaining two angles are greater than \( \Phi_{\text{min}} \) (see also the previous section).

If a point on one face moves to an edge, the ‘identical’ point on the opposite face simultaneously moves to another edge until both points reach the middle of the edges. This orientation corresponds to the rotation \( \Phi \approx 109.5^\circ \), and Fig. 6 shows that there are three equivalent points at the edges. The third point appears as it moves from the outside of the cube to the third edge. As in Fig. 5, this means that three equal angles \( \Phi_{\text{min}} \) would be obtained. Finally, when a point is at a vertex, as shown in Fig. 7, three other vertices correspond to the same rotation \( \Phi = 120^\circ \) (Frank 1988), that is, all four rotation angles are \( \Phi_{\text{min}} \).

This arrangement of the orientations for the rotation angles \( \Phi \geq 90^\circ \) describes a complex topology for DC4 source rotation. This topology involves projective or Möbius transformation (Altmann 1986; Frank 1988). Full analysis of the DC4 source orientation, when and if performed, would involve very intricate investigations of rotation angle transformations due to the source symmetry.
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Figure 5. Fundamental zone display for DC4 source. Two opposite faces of the fundamental cube (Fig. 4) are shown. Colours show two face points corresponding to one source orientation with the angle $\Phi_1 \geq 90^\circ$. The cyan central points correspond to two equivalent rotations $\Phi_1 = 90^\circ$. When rotation point moves to the edge of one face an equivalent point on the opposite face also moves to another edge, resulting in rotation with an angle 109.5$^\circ$ (see Fig. 6).

Figure 6. Fundamental zone display for DC4 source. Colours show four sets of three edge points corresponding to one source disorientation with the angle $\Phi_1 \approx 109.5^\circ$. Compare to Fig. 5 where two edge points are result of points moving on opposing faces. The third point appears as it moves from outside the fundamental cube to the third edge. Three other sets of points are similarly produced.

The Rodrigues space has no special advantages in displaying the orientation distribution for two other sources: DC2 and DC1. For DC2 source which has $C_2$ symmetry, the fundamental zone is a layer bounded by two planes perpendicular to the $b$-axis, each at the distance $\tan(45^\circ) = 1$ from 0. For a DC1 source, the whole Rodrigues space up to infinity is included. In these cases, other spaces are more convenient in displaying the 3-D rotation distribution (Frank 1988; Morawiec & Field 1996). Altmann (1986, pp. 164–176) explains the projective or Möbius topology of rotations in the quaternion parametric ball for non-symmetrical objects.

5 ROTATION ANGLE DISTRIBUTIONS

5.1 Uniform random rotation of DC sources

The distribution of the uniform random rotation for DC sources constitutes a reference for distributions occurring in earthquake focal mechanisms where we expect the distributions to be only partially random. These stochastic distributions can be analytically calculated by taking into account the sources’ symmetry.

A distribution of the minimum angle $\Phi_{\text{min}}$ for a uniform random rotation of the DC4 source was obtained by Kagan (1990, eqs. 3.1–3.3), using the results by Handscomb (1958) and Mackenzie (1958) for the random disorientation of two cubes. The probability density function (PDF) is

$$f(\Phi) \begin{cases} (4/\pi)(1 - \cos \Phi) & \text{for } 0 \leq \Phi \leq 90^\circ; \\ (4/\pi)(3 \sin \Phi + 2 \cos \Phi - 2) & \text{for } 90^\circ \leq \Phi \leq \Phi_S \end{cases}$$

and

$$f(\Phi) \begin{cases} (4/\pi) \left\{ 3 \sin \Phi + 2 \cos \Phi - 2 \\ - (6/\pi) \left\{ 2 \sin \Phi \arccos \left( \frac{1 + \cos \Phi}{-2 \cos \Phi} \right)^{1/2} \\ - (1 - \cos \Phi) \arccos \left( \frac{1 + \cos \Phi}{-2 \cos \Phi} \right) \right\} \right\} & \text{for } \Phi_S \leq \Phi \leq 120.0^\circ, \end{cases}$$

where

$$\Phi_S = 2 \arccos \left( 3^{-1/2} \right) = \arccos \left( \frac{1}{3} \right) \approx 109.47^\circ.$$ (11)

For the DC2 source, a similar PDF is

$$f(\Phi) = \frac{2}{\pi} \left[ 1 - \cos(\Phi) \right] \text{ for } 0 \leq \Phi \leq 90^\circ;$$ (12)

and

$$f(\Phi) = \frac{2}{\pi} \sin(\Phi) \text{ for } 90^\circ \leq \Phi \leq 180^\circ.$$ (13)
For the DC1 source, the function is
\[ f(\Phi) = (1/\pi)[1 - \cos(\Phi)] \quad \text{for} \quad 0 \leq \Phi \leq 180^\circ. \] (14)

Grimmer (1979), also following Handscomb (1958) and Mackenzie (1958) results, obtained similar analytic expressions for a completely random rotation of orthorhombic, monoclinic and triclinic crystals (equivalent in symmetry to the DC earthquake source with various restrictions described above). He listed median angles as well as their mean and standard deviations for all these distributions. Morawiec (1995, 2004, pp. 117–119) derived these distributions by integration in the Rodrigues space.

5.2 Non-uniform distributions of random rotations

Two non-uniform rotation angle distributions are useful in analysing earthquake focal mechanism rotation: the rotational Cauchy law (Kagan 1982, 1992) and von Mises–Fisher/Bingham rotational distribution (Kagan 1992, 2000; Schauben 1996; Mardia & Jupp 2000, pp. 289–292; Morawiec 2004, pp. 88–89).

The Cauchy distribution is especially important for representing earthquake rupture geometry, since it can be shown by theoretical arguments (Zolotarev 1986, pp. 45–46; Kagan, 1990) that the Cauchy distribution represents the degree of ‘incoherence’ or ‘complexity’ in a set of earthquake focal mechanisms. The cumulative rotational Cauchy distribution can be written as (Kagan 1992, 1990)

\[ f(\Phi) = \frac{2}{\pi} \left[ \frac{\kappa A^2(1 + A^2)}{(\kappa^2 + A^2)^2} \right] \]
\[ = \frac{4\kappa}{\pi} \left[ 1 - \cos(\Phi) \right] \left[ 1 + \kappa^2 + (\kappa^2 - 1) \cos(\Phi) \right]^{-1}, \quad \text{for} \quad 0 \leq \Phi \leq 180^\circ, \] (15)

where \( A = \tan(\Phi/2) \). The scale parameter \( \kappa \) of the Cauchy distribution represents the degree of ‘incoherence’ or ‘complexity’ in a set of earthquake focal mechanisms. The cumulative rotational Cauchy distribution can be written as

\[ F(\Phi) = \frac{2}{\pi} \left[ \arctan(A/\kappa) - \frac{A \times \kappa}{A^2 + \kappa^2} \right]. \] (16)

The Cauchy distribution is assumed to be axisymmetric on the quaternion hypersphere \( S^3 \). This means that the rotation axis poles are distributed uniformly over a regular \( S^2 \) sphere. For a general case, the axes distribution for earthquake focal mechanisms may need to be specified as non-uniform. In that case, certain rotations would be preferred depending on the focal mechanism of a reference event. Moreover, in our previous investigations the rotation angle and the pole distributions were considered as independent; in reality these PDFs cannot be factored and should be studied as a joint distribution. However, we have not yet advanced to this stage (see Section 6).

The von Mises–Fisher distribution for the 3-D orientation is widely discussed in literature (Schaeben 1996; Mardia & Jupp 2000; Morawiec 2004). Schaeben (1996) and Morawiec (2004) show that this distribution is essentially equivalent to the Bingham distribution. The von Mises–Fisher distribution is a Gaussian-shaped function concentrated near the zero angle. This distribution can be implemented to model random errors in determining focal mechanisms. The distribution has many forms. However, even the simplest axially symmetric expressions, due to the complexity of normalization, represent difficult computation.

For small values of the angle standard error \( \sigma_\phi \), the von Mises–Fisher-type distribution is equivalent to the rotational Maxwell law used by Kagan & Knopoff (1985, eq. A3) and Kagan (1992, eq. 12). The latter distribution is obtained by generating a 3-D normally distributed random variable \( u \) with the standard deviation \( \sigma_\phi \) and then calculating the unit quaternion

\[ q_0 = \frac{1}{\sqrt{1 + u_1^2 + u_2^2 + u_3^2}}, \]
\[ q_i = u_i \frac{1}{\sqrt{1 + u_1^2 + u_2^2 + u_3^2}} \quad \text{for} \quad i = 1, 2, 3. \] (17)

The 3-D rotation angle is calculated

\[ \Phi = 2 \arccos(q_0) \approx 2 \arccos\left(1 - \frac{1}{2}(u_1^2 + u_2^2 + u_3^2)\right) \]
\[ \approx 2 \arcsin\left(u_1^2 + u_2^2 + u_3^2\right) \approx 2\sqrt{u_1^2 + u_2^2 + u_3^2}. \] (18)

This equation is twice the length of a vector in the 3-D space.

Since components of vector \( u \) are normally distributed, the angle \( \Phi \) (in degrees) follows the Maxwell distribution with

\[ \sigma_\phi = 360 \sigma_\phi / \pi, \] (19)

where we assume that all components of \( \sigma_\phi \) are equal (\( \sigma_{\phi_1} = \sigma_{\phi_2} = \sigma_{\phi_3} = \sigma_\phi \)). For \( 0^\circ \leq \Phi \leq 180^\circ \), the Maxwell PDF is

\[ \psi(\Phi) = \sqrt{\frac{2}{\pi}} \frac{\Phi^2}{\sigma_\phi} \exp\left[-\Phi^2 / (2\sigma_\phi^2)\right]. \] (20)

This equation describes the distribution of a vector length in three dimensions, if the vector components have a Gaussian distribution with a zero mean and a standard error \( \sigma_\phi \). The Maxwell cumulative distribution function (CDF) is

\[ \Psi(\Phi) = \text{erf} \left( \frac{\Phi}{\sigma_\phi \sqrt{2}} \right) - \sqrt{\frac{2}{\pi}} \frac{\Phi}{\sigma_\phi} \exp\left[-\Phi^2 / (2\sigma_\phi^2)\right]. \] (21)

where \( \text{erf}(\cdot) \) is an error function.

The major problem with these non-uniform random distributions is that they do not consider the symmetry of the rotated object. When rotation angles are small, the distribution is concentrated around the zero angle neighbourhood. For the DC4 source as shown in Fig. 4, almost all distribution density would be inside the fundamental zone. However, for more spread-out angle distributions, we should account for cases where the rotation angle exceeds the maximum angles (see, e.g. eqs 8–11). Then, the distribution would be folded back into the fundamental zone.

Mason & Schuh (2009) propose to convolve angle distribution with appropriate 3-D spherical or 4-D hyperspherical harmonics to obtain a new angle distribution which fits into the fundamental zone. It is not clear whether such calculations can be made analytically. Simulation seems the only practical way to transform both Cauchy and von Mises/Fisher distributions for the \( \Phi_0 \) symmetric case (i.e. for maximum \( \Phi_{\text{min}} \) rotation angle 120°). Kagan (1992, fig. 3c) produced such distributions. Fig. 8 plots the appropriate Cauchy distribution which is reduced to \( \Phi_{\text{min}} \leq 120^\circ \).
6 FOCAL MECHANISMS STATISTICS

6.1 Disorientation angle statistics

Since there is no general model of earthquake focal mechanism distribution, we need to study the distribution of mechanisms in earthquake catalogues empirically to infer their properties. How various tectonic and geometrical factors shape the distribution of earthquake sources should be studied as well. Such investigations are difficult because we are dealing with a multidimensional stochastic point process: earthquake size, occurrence time, location and source parameters serve as potential inputs to the distributions.

In this paper, we are mostly interested in the distributions of rotation between two earthquake focal mechanisms. Even if we fix earthquake time, space and magnitude interval, the DC rotation distribution depends on at least three variables: the rotation angle and two spherical coordinates of a rotation axis pole. Displaying all three degrees of freedom in a distribution presents a difficult problem. Therefore, in our previous investigations we studied partial distributions. For example, Kagan (1992, figs 6–9, 2009, figs 9–10) obtained various distributions of the rotation angle $\Phi_{\text{min}}$ between two focal mechanisms. Below we first update our most important results on the distribution of the rotation angle $\Phi_{\text{min}}$, and then analyse a 3-D distribution of rotation angle and the axes in the Rodrigues space.

We used the Global Centroid Moment Tensor catalogue (Ekström et al. 2012), referred to subsequently as GCMT. This catalogue employs relatively consistent methods and reports tensor focal mechanisms. The GCMT catalogue started in 1977, and is complete only for earthquakes with magnitudes of about 5.8 and larger. The present catalogue contains more than 36 000 earthquake entries from 1977 January 1 to 2011 December 31.

Fig. 8 displays cumulative distributions of the rotation angle $\Phi_{\text{min}}$ for shallow earthquake pairs with the magnitude threshold $m_t = 5.0$ that are separated by a distance of less than 50 km. We study whether the rotation of focal mechanisms depends on where the second earthquake ($s_2$) of a pair is situated with regard to the first event ($s_1$). Thus, we measure the rotation angle for centroids located in 30° cones around each principal axis of the first event (see curves, marked the $t$, $p$- and $b$-axes).

The curves in Fig. 8 are narrowly clustered, with about 95 per cent of angles less than 90°, within an inscribed sphere of the fundamental zone (Fig. 4). This pattern can be compared to the uniform DC rotational Cauchy distribution (eqs 15–16). This distribution is characterized by a parameter $\kappa$; a smaller $\kappa$-value corresponds to the rotation angle $\Phi_{\text{min}}$ concentrated closer to zero. Thus, regardless of spatial orientation, all earthquakes have focal mechanisms similar to a nearby event. Earthquakes in the cone around the $b$-axis correspond to a smaller $\kappa$-value than events near the other axes. These results are similar to those shown in fig. 6 by Kagan (1992) or fig. 9 by Kagan (2009).

Figs 9 and 10 show the disorientation angle distribution for the magnitude cut-off $m_t = 5.8$. As may be expected, the higher magnitude, the angles are concentrated closer to zero, and the difference between the curves corresponding to various cones increases. Maxwell distribution curves are shown to illustrate possible behaviour of the angle distributions near zero. The $\sigma_\Phi = 7.5°$ parameter of the distribution is small, compared to the distribution range ($0° \leq \Phi \leq 120°$). Therefore, the curves are concentrated close to zero; we do not need simulation to consider the curve behaviour for large values of $\Phi_{\text{min}}$, as done, for example, in fig. 3(c) by Kagan (1992).

The difference in the distribution curves corresponding to various focal mechanism axes suggests that the Cauchy distribution parameter $\kappa$ depends upon the geometry of a fault system. Contrary

![Figure 8](https://example.com/figure8.png)

**Figure 8.** Cumulative distributions of rotation angles for pairs of focal mechanisms of shallow earthquakes (depth 0–70 km) in the GCMT catalogue 1977 January 1–2011 December 31; centroids are separated by distances between 0 and 50 km, magnitude threshold $m_t = 5.0$. The total number of events is 26 986. Lines from left to right: filled circles are centroids in 30° cones around the $b$-axis; dashed red line is for the Cauchy rotation with $\kappa = 0.1$; circles—all centroids; crosses—centroids in 30° cones around the $p$-axis; $x$-signs—centroids in 30° cones around the $t$-axis; right yellow solid line is for the random rotation.
to our assumptions (eq. 15), poles of rotation axes are not uniformly distributed over the $S^2$ sphere.

6.2 Rodrigues space statistics and display

A major problem in the orientation visualizing is the high dimensionality of the 3-D rotation space: the orthogonal matrices are characterized by nine values, the seismic moment tensor requires five or six variables and the normalized quaternion needs four values. The real number of degrees of freedom for a 3-D rotation is three. Thus, in principle, an orientation distribution can be shown in a 3-D diagram.

We obtain a distribution diagram for a set of disorientations. One way to display such a diagram of the fundamental zone is through stereopairs (Neumann 1992). Morawiec & Field (1996) display the distribution of disorientation parameters by points in some parallel sections of the fundamental zone.
Fig. 11 shows a distribution for randomly rotated DC4 sources in a central section ($-0.05 \leq x_3 \leq 0.05$) of the fundamental zone of the Rodrigues space. The points in the inscribed blue circle of the square correspond to the rotations with angles $\Phi_{\text{min}} \leq 90^\circ$, whereas for the square corners the angle $\Phi_{\text{min}} \approx 109.5^\circ$ (11). Fig. 12 displays a similar distribution of the earthquake focal mechanism orientation in the GCMT catalogue. We show how the second earthquake ($s_2$) of a pair is rotated in regard to the earlier event ($s_1$). Since the disorientation of these two events may not be symmetric in time we display it in a full Rodrigues space. As Figs 8–10 demonstrate, the distribution of the rotation angles for earthquake sources is strongly concentrated close to 0°.

If we compare Figs 11 and 12, this concentration is marked in the fundamental zone display.

Table 1 summarizes earthquake focal mechanism disorientation patterns in the fundamental zone of DC4. The total number of events $N$ with the magnitude above the threshold is shown, as well as the total number of pairs $N_p$ with centroids separated by less than 50 km. $N_c$ is the pair numbers in the central zone shown in Fig. 12. Although the central section ($-0.05 \leq x_3 \leq 0.05$) occupies only a small part of the fundamental cube, close to 50 per cent of the pairs are there due to a tight concentration of rotation angles near the zero value. For simulated random events in the same central zone, the number $N_c$ is about 7 per cent of the total (see Fig. 11).

We also display the correlation coefficients of the point scatter field. Whereas for earthquake focal mechanisms, the coefficients $\rho_{bp}$ and $\rho_{bt}$ are close to zero, the $\rho_{pt}$ and $\rho_{pt}^\prime$ coefficients are non-zero, testifying to a certain pattern of focal mechanism rotation.

Kagan (2009, fig. 11) also shows that rotation axes are concentrated closer to the t-axis. All correlation coefficients for the simulated random mechanisms are around zero. A more detailed statistical analysis of this pattern will be carried out in our future work.

The values of the average rotation angle and its standard deviation ($\bar{\Phi} \pm \sigma_\Phi$) show that for larger earthquakes both variables are smaller. This may be caused by a higher accuracy in determining focal mechanism for stronger shocks (Kagan 2003). For simulated focal mechanisms, the $\bar{\Phi} \pm \sigma_\Phi$ values are close to the theoretical estimates for orthorhombic symmetry – $75.16^\circ \pm 20.85^\circ$ (Grimmer 1979, table 5).

6.3 Distributions of rotation axes

Mackenzie (1964) derived the distribution of the rotation axes for cubic symmetry. Morawiec (1996) obtained distributions of rotation axes for any symmetric object encountered in crystallographic texture analysis. Using his results, we can write down the distribution of rotation axes for the $D_2$ symmetry: the DC4 source. The Mackenzie cell is shown in Fig. 4. We designate the coordinate axes as $x_i$, $i = 1, 2, 3$, and the distribution depends on distance from the origin. As seen in Fig. 4, the distribution should have a threefold cyclic symmetry $C_3$ around the origin or around the cube vertex. Then, the PDF for the axes length is

$$p(\rho) = \frac{16}{\pi^2} \left[ \arctan(\rho) - \rho/(1 + \rho^2) \right],$$

(22)
Figure 12. Disorientation distribution in the fundamental zone of the Rodrigues space for shallow earthquakes in the GCMT catalogue. Centroids are separated by distances between 0 and 50 km; magnitude threshold $m_t = 5.8$; the total number of events is 6160. The points are shown in the central section of the fundamental zone $-0.05 \leq x_3 \leq 0.05$ (see Fig. 4). The total number of earthquake pairs is 19 397; 8725 pair points are in the central section. X-axis corresponds to $x_1$, Y to $x_2$.

Table 1. Properties of disorientation point scatter in the fundamental zone of a DC4 source.

| No. | $m_t$ | $N$   | $N_p$ | $N_c$ | $\rho_{pt}$ | $\rho_{bp}$ | $\rho_{bt}$ | $\rho_{pt}'$ | $\overline{\Phi} \pm \sigma_{\Phi}$ |
|-----|-------|-------|-------|-------|-------------|-------------|-------------|-------------|----------------------------------|
| 1   | 5.6   | 9615  | 43    | 18    | 0.126       | 0.026       | 0.013       | 0.170       | 31.6 ± 27.5                     |
| 2   | 5.8   | 6160  | 19    | 36    | 0.117       | 0.022       | -0.001      | 0.127       | 29.9 ± 26.9                     |
| 3   | 6.25  | 2154  | 274   | 124   | 0.274       | -0.016      | -0.026      | 0.257       | 28.5 ± 26.1                     |
| 4   | Simul.| 25 000| 25    | 000   | -0.012      | -0.002      | 0.002       | -0.015      | 75.0 ± 21.0                     |

Notes: The GCMT catalogue time interval is 1977 January 1–2011 December 31. $N$ is the total number of events with magnitude $m \geq m_t$; $N_p$ is the total number of event pairs; $N_c$ is the number of event pairs in the central section; $\rho_{pt}$, $\rho_{bp}$ and $\rho_{bt}$ are the correlation coefficients for all points; $\rho_{pt}'$ is the correlation coefficient for all points within the central section; $\overline{\Phi} \pm \sigma_{\Phi}$ is the average disorientation angle and its standard deviation.

where $\rho = \sqrt{x_1^2 + x_2^2 + x_3^2}$ is the distance from the origin [0, 0, 0] to the walls of the fundamental zone. For small value of $\rho$, $p(\rho) \propto \rho^3$.

Its values at $\rho = 1$, $\rho = \sqrt{2}$ and $\rho = \sqrt{3}$ correspond to appropriate values for the $D_2$ symmetry (Morawiec 1996, table 2). These $\rho$-values correspond to the rotation angles $\Phi = 90^\circ$, $109.5^\circ$ and $120.0^\circ$, respectively. For $\Phi \leq 90^\circ$, the rotation axes are distributed uniformly over the $S^2$ sphere, but they intersect the sphere near the cube vertex close to $\Phi = 120^\circ$.

In Fig. 13, we show the distribution of the rotation poles for the second earthquake focal mechanism on a reference sphere of the first event. Because of the symmetry of the DC4 source, we reflect the point pattern on our reference sphere at the planes perpendicular to all axes. Thus, the distribution can be shown on a spherical octant. We use the Lambert azimuthal equal-area projection. The points concentrate near the projections of the far edges of the MacKenzie cell (see Fig. 4) and around the cube vertex which corresponds to the disorientation angle $\Phi = 120^\circ$.

These are only 18 pairs of earthquakes with the rotation angle $109.5^\circ \leq \Phi \leq 120^\circ$. Fig. 12 demonstrates that the number of event pairs decreases strongly as the rotation angle $\Phi_{\text{min}}$ approaches $109.5^\circ$. No pairs exceeding this value are shown in the plot, they are found close to the cube vertices (see Fig. 4).

7 DISCUSSION

Quantitative study of earthquake focal mechanisms is an important pre-requisite for understanding earthquake rupture. Though these investigations began in the mid-1950s, publications have been...
mostly descriptive until now; relatively little modelling and rigorous statistical analysis have been performed. A major difficulty in analysing focal mechanisms is both the high dimensionality and non-commutativity of the 3-D rotations. This presents a major challenge in analysing a set of focal mechanisms.

Several papers (Kagan 1992, 2000, 2009) have investigated statistical distributions of earthquake focal mechanisms. We found that the disorientation angle is close to zero for spatially close earthquakes, and the angle decreases if the interearthquake time interval approaches zero. We also showed (Kagan 2009) that the CLVD component of focal mechanism tensor is either zero or close to zero for most geometric barriers proposed as common features in an earthquake fault system.

However, the major challenges in describing and understanding the distributions of focal mechanisms still remain. As we see from Figs 8–10, the angle distribution is not axially symmetric: in certain directions \( \Phi_{\min} \) is larger than in others. Thus, the distributions used to approximate the angle pattern, like the rotational Cauchy distribution (eqs 15–16), need to be made more complex.

The distribution of rotation axes was not investigated as thoroughly as that for disorientation angles. There is still no theoretical model for approximating empirical data, but applying the Rodrigues space may render such analysis more manageable.

However, even these limited results contribute significantly to understanding of earthquake focal mechanism properties and allows certain quantitative applications for seismic risk evaluation. Kagan & Jackson (2011) explain that the forecasted tensor focal mechanism enables calculating an ensemble of seismograms for each point of interest on the Earth’s surface. Moreover, the focal mechanism distribution allows us to estimate fault plane orientation for past earthquakes, through which we can identify a preferred rupture direction for future events.

The angle \( \Phi_{\min} \) has also been used to directly compare moment tensors from two different earthquakes (Okal et al. 2011). It has as well been applied in comparing moment tensors computed for the same events through different techniques (Frohlich & Davis 1999; Pondrelli et al. 2007; Yang et al. 2012). Such comparisons can help to refine the moment tensor algorithms and lower their computational cost, since they can reveal the relative importance of various assumptions implicit in the algorithms.

Study of focal mechanism rotations carried out in previous sections neglected to account for general geometry of focal zone and, in particular, for the relation of focal mechanisms to the Earth’s surface. Only pairwise DC rotations in infinite space have been considered. More complete investigations of source orientations in geographical as well as tectonic reference frames, though much more difficult to accomplish, should also be done. Perhaps research techniques in crystallographic texture analysis (see, for instance, Morawiec 2004) can be applied in these studies.

8 CONCLUSIONS

(1) The symmetry properties of an earthquake DC focal mechanism are considered. Given available seismological and geological information, three symmetries are possible: \( D_2 \) (dihedral symmetry), \( C_2 \) (cyclic symmetry) and \( I \) (identity). Determining the orientation or disorientation of the source depends on its symmetry.
(2) Quaternion representation is the most convenient tool for analysing a DC 3-D rotation. A 3-D rotation requires at least three degrees of freedom for its characterization.

(3) Several theoretical distributions to describe a 3-D rotation of DCs are presented: random rotation, rotational Cauchy and von Mises–Fisher.

(4) The Rodrigues space, so extensively used in crystallographic texture analysis is applied to display and analyse earthquake focal mechanism distribution. This space allows us to represent 3-D patterns of how symmetric objects are oriented.

(5) We illustrate the proposed methods by statistically analysing the GCMT catalogue of earthquake focal mechanisms.

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