Mathematical model of vector potential and magnetic surface density of geophone SM-24

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Abstract. Geophone (SM-24) affecting its performance is simulated for studying its sensitivity degrading over the time. A linear mathematical model is to simulate velocity potential subjected to SM-24 magnetic ring and spring suspension. Analytic solution of the mathematical model proposes that velocity potential is associated to current distribution. An experiment study was conducted by totalling external resistor to improve the current distribution due to phase lag. Experiment results show that an increase of voltage output and amplitude is possible by adding 500 Ohm resistor.

1. Introduction
Geophone is used by geologist to distinguish gesture based on Faraday’s law of electromagnetic induction. Velocity sensing is a must for geophone in high precision and diverse application in many industries. In lithographic application, it is used in defining disruption of payload caused by movement of the wafer scanner, further geophone is also found in application for accurate position of a complex lens system. Commercial geophone today can detect inferior resonance frequency, however not as low as below 4.5 Hz, high noise to signal is found at lower frequency [1,2]. Geophone configured with high resonance frequency often have high dimensions and required extra power supply, it turn not cost effective to be used in area where there are no sources of power and in application which required long experimental time to obtain the required data.

Magnetic spring model of geophone is similar to Halback configuration for vibration isolation structure [4-6]. Difference between radial and axial magnetization gives a technique of force calculation on ring magnets [7, 8]. Based on Earnshaw’s theorem, the object which are under the influence of magnetic field, will experience force applied in an inverse-square [9, 10]. Regardless of the Earnshaw’s theorem, the permanency of passive magnet levitation is not conceivable. For example, the stability of magnetic levitation is possible base on diamagnetic material [11] and spin stabilization [12], or the use of elastic materials [13].

Due to mechanical system (spring) and high damping factor at an open circuit (the output signal tend to oscillate in undesirable manner), commercial geophones has limited bandwidth. Therefore, this article illustrated a study optimum damping ratio and sensitivity of geophone at low frequencies of 1 Hz.

2. Semi Analytic Modeling
Elementary principle of geophone mechanism is a mass suspended by mechanical springs. both the suspended mass/coil will oscillate proportional to the applied velocity when a velocity in vertical or horizontal direction is applied. Working principle is derived from Faraday’s law;
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\n
where \( E \) stand for electric field strength and \( B \) is magnetic flux density. For magnetic flux in a close surface, equation (1) can be rewritten in integral form

\[
\oint E \, dl = -\delta t \oint B \, ds
\]

Electromotive force (emf) is created by the magnetic field [14,15]. Referring to figure 1, geophones are constructed using cylindrical permanent magnet. By solving the magneto-static field equation, analytical explanation gives the vector potential model.

![Figure 1](image)

**Figure 1.** Model of ring magnet [19].

For frequencies below and above the resonance frequency, the oscillation of the mass is assumed constant. Given

\[
B = \nabla \times A, \quad \nabla \cdot B = 0 \quad \text{and} \quad \nabla \times H = J
\]

(2)

\( B \) is magnetic flux density, \( M \) is the magnetic field, and \( A \) is the Vector potential [16, 17, 18].

\[ \nabla \times M = J_M \]

is an expression of an equivalent magnetic surface current density. By assuming infinite domain, where no boundaries and no free current or \( J = 0 \). Poisson equation is then written as:

\[
A(r) = \frac{\mu_0}{4\pi} \int \frac{J(r')}{|r-r'|} \, dr'
\]

(3)

and current distribution

\[
J_M(r')\, dr' = J_M \cos \theta' Rd\theta' dz'
\]

with vector different

\[
|r - r'| = \sqrt{(r - R \cos \theta')^2 + (-R \sin \theta')^2 + (z - z')^2}
\]

\( \theta' \) is referred to phase lag due to current. Vector potential reads:

\[
A_\theta(r, z) = \frac{\mu_0 J_M}{4\pi} \int_{z' = h}^{h} \int_{\theta' = 0}^{2\pi} p d\theta' dz'
\]

with

\[
p = \frac{R \cos \theta'}{\sqrt{(z-z')^2 + R^2 + r^2 - 2R \cos \theta'}}
\]

(4)

From equations (3) and (4), we derive the following vector potential and current distribution relation:

\[
A_\theta \propto J_M
\]

(5)

Phase lag \( \theta' \) due to current distribution is solved by adding shunt resistor for some lower frequency commercial geophone.
3. Results and Discussions
Electromagnetic voltage generator or geophone with output proportional to the rate of change of magnetic flux; output is indirectly proportional to ground displacement and the velocity associated with the displacement, hence, geophone output is measured in volt/(m/sec) [19].

In this research, hypothesis is to show resistivity is related to vector potential (5) subject to magnetic surface current density \( J_m \) of a geophone (SM-24 model). Shunt resistor is used to study both input and output current density. Both the result of output of amplitude and voltages are compared to justify the hypothesis (5). Experimental results from prototype are recorded as following.

![Figure 2](image1.png)

Figure 2. (a) Output voltage of open circuit without shunt resistor. (b) Amplitude of open circuit without shunt resistor.

![Figure 3](image2.png)

Figure 3. (a) Output voltage of 500 Ohm (Ω) shunt resistor. (b) Amplitude of 500 Ohm (Ω) shunt resistor.

From the prototype results, the resistivity component that increase or decrease the amplitude is found related to the phase lag due to current distribution (4). Amplitude or displacement is referred to vector potential with relation (5). Figures 2 and 3 show the output with and without shunt resistor. 500 Ohm is chosen among the others, the response of SM-24 at lower frequency (Hz) is the best and consistent. This research is focused on phase lag due to current distribution at the ring magnet. However, the actual input current studies have not been completed.

4. Conclusion
Current distribution on ring magnet is distorted by phase lag. From the mathematical model, it is possible to relate vector potential with current distribution. Experimental result shown there’s a possibility of using SM-24 for low frequency logging by adding 500 Ohm resistor.

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