W Boson Inclusive Decays to Quarkonium and $B_{c}^{(*)}$ Meson at the LHC

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Abstract

In this paper, the production rates of quarkonia $\eta_c$, $J/\psi$, $\eta_b$, $\Upsilon$ and $B_{c}^{(*)}$ mesons through $W^+$ boson decay at the LHC are calculated, at the leading order in both the QCD coupling $\alpha_s$ and in $v$, the typical velocity of the heavy quark inside of mesons. It shows that a sizable number of quarkonia and $B_{c}^{(*)}$ mesons from $W^+$ boson decay will be produced at the LHC. Comparison with the predictions by using quark fragmentation mechanism is also discussed. Results show that, for the charmonium production through $W^+$ decay, the difference between predictions by the fragmentation mechanism and complete leading order calculation is around 3%, and it is insensitive to the uncertainties of theoretical parameters; however, for the bottomonium and $B_{c}^{(*)}$ productions, the difference cannot be ignored as the fragmentation mechanism is less applicable here due to the relatively large ratio $m_b/m_w$.

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I. INTRODUCTION

As the only charged gauge boson in the Standard Model (SM) whose mass is generated through the electroweak symmetry spontaneous breaking (EWSB), the precise measurements on $W$ boson’s mass and decay width offer a way to distinguish the different EWSB mechanisms, and effectively put a stringent constraints on the Higgs boson mass and New Physics beyond the SM\cite{1}. Furthermore, the study of the productions of $W$ bosons can be a probe for searching New Physics since the possible new particles may decay to $W$ bosons. By reason of that, recently, there have appeared several papers discussing “$W$ boson physics”, at Tevatron, ILC and LHC, respectively \cite{2}.

Before the LHC era, only two rare decays: $W^+ \rightarrow \pi^+\gamma$ and $W^+ \rightarrow D_s^+\gamma$, were experimentally studied, and the respective upper limits for their branching fractions are $8 \times 10^{-5}$ and $1.3 \times 10^{-3}$ \cite{3}. The LHC has been running for two years. Due to its high luminosity, the huge number of $W$ bosons are and will be produced. According to the estimations in \cite{1, 5, 6}, the production cross section of $W$ boson is around $\sim 10^2$ nb at the LHC with the center energy 7 TeV or 14 TeV. It is expected that hundreds of million $W$ bosons will be produced per year at the LHC with the conservative expectation of the luminosity $\sim 10^{32}$cm$^{-2}$s$^{-1}$ (which is two orders of magnitude smaller than the desired LHC luminosity $\sim 10^{34}$cm$^{-2}$s$^{-1}$). This makes the LHC also a $W$-boson factory which facilitates the precise experimental study on $W$ boson physics, especially $W$ boson rare decays.

In this article, we will discuss a class of $W$ boson inclusive decays to the S-wave quarkonium and $B_c$ meson, such as $\eta_c$, $J/\psi$, $\eta_b$, $\Upsilon$, $B_c$ and $B_{c}^{*}$. Because $W$ boson is so heavy and participates merely the electroweak interactions, the study on $W$ boson inclusive decays to quarkonia and $B_{c}^{(*)}$ mesons\footnote{Recently, the CDF collaboration just set a new 95% confidence level upper limit on the relative branching fraction $\Gamma(W^+ \rightarrow \pi^+\gamma)/\Gamma(W^+ \rightarrow e^+\nu)$ at $6.4 \times 10^{-5}$ which is a factor of 10 improvement over the previous limit \cite{4}.}, offers another great place for the precise test of the perturbative quantum chromodynamics (PQCD) and the quarkonium production mechanism. As a quarkonium is presumed to be a non-relativistic bound state of heavy quark and anti-quark, a wonderful theoretical tool to deal with the processes involving quarkonium is non-relativistic quantum chromodynamics (NRQCD), in which the low-energy interactions

\footnote{As in \cite{13, 14}, we treat $B_{c}^{(*)}$ as a non-relativistic bound state of $c$ and $\bar{b}$ in this paper.}
are organized by the expansion in \( v \), the typical relative velocity of the heavy quark and anti-quark inside of quarkonium. The inclusive production cross sections can be written as the product of the perturbatively calculable short-distance coefficients and the non-perturbative NRQCD matrix elements. At the leading order in \( v \), the only non-perturbative NRQCD matrix element of the S-wave quarkonium is proportional to the Schrödinger wave function at the origin squared.

On the other hand, since the W boson mass \( m_w \) is much greater than the heavy quark masses \( m_{b,c} \), the parton fragmentation may dominate the W boson inclusive decays to quarkonia and \( B_c^{(*)} \) mesons. The pioneer works in this field were done two decades ago, the universal fragmentation functions for different quarkonium and \( B_c \) meson were given in \([8, 10, 11]\). By these universal fragmentation functions, the direct production rate of quarkonium and \( B_c \) meson at large transverse momentum in any high-energy process can be approximated to be the product of the parton-level production rate and the universal fragmentation probability, and the corresponding computation is much easier than that done by within the NRQCD framework. In Ref.\([8–10]\), for the \( Z^0 \) inclusive decays to the S-wave charmonium, the authors did find great agreement between the results obtained by the parton fragmentation approximation and the complete leading order PQCD calculation.

However, in some processes, the fragmentation may not be the leading process, but main process when a certain condition of the fragmentation mechanism is not sufficiently fulfilled. In Ref.\([12]\) it was found that the fragmentation contribution is important but not dominant for top quark decays into quarkonium. Thus, it is worth examining if the fragmentation mechanism works in W boson inclusive decays to quarkonium and \( B_c \) meson.

In the following sections, we present the complete leading order calculation of \( W^+ \) boson decays to quarkonium \( \eta_c, J/\psi, \eta_b, \Upsilon \) and \( B_c^{(*)+} \) mesons; then we compare our results with that obtained by the fragmentation approximation; finally we discuss the theoretical uncertainties in our calculation.

II. FORMALISM

Some typical Feynman diagrams for quarkonia and \( B_c^{(*)} \) mesons hadronic production through \( W^+ \) decay are shown in Fig. \( \text{III} \). We will calculate them at the leading order in \( \alpha_s \).
FIG. 1: The typical Feynman diagrams for quarkonia and $B_s^{(*)}$ productions from $W$ decays. Some diagrams with the different gluon attachment are not shown.

and $v$. In NRQCD, a non-relativistic bound state of heavy quarks $Q$ and $\bar{Q}'$ is considered as an expansion of a series Fock states. The leading Fock state for the S-wave hadron is constructed as

$$|H_{[2s+1}^{J_s}J_j]}(p, \lambda)⟩ = \sqrt{2m_H} \sum_{i,j,\lambda_1,\lambda_2} \frac{\delta_{ij}}{\sqrt{2N_c}} C(J, \lambda; \frac{1}{2}, \lambda_1; \frac{1}{2}, \lambda_2)$$

$$\times \int \frac{d^3p_Q}{\sqrt{2E_Q2E_{Q'}}} \tilde{\Psi}_{2s+1,J_s}(p_Q)|Q_{i,\lambda_1}(p_Q)\bar{Q}'_{j,\lambda_2}(p_{Q'})⟩,$$

where $m_H$ denotes the mass of hadron and all the states are relativistically normalized. Here $p$ is momentum of hadron, $i, j$ stand for the color indices which run from $i, j = 1, ..., N_c$ with $N_c = 3$ for QCD, and $C(J, \lambda; 1/2, \lambda_1, 1/2, \lambda_2)$ is the C-G coefficient with $\lambda, \lambda_1$ and $\lambda_2$ being the third components of spin indices. The non-perturbative parameters $\tilde{\Psi}_{2s+1,J_s}(p_Q)$ is the Schrödinger wave functions in momentum space. Thus, to project out the amplitude for the S-wave hadronic states from the complete patron level amplitude at the leading order of $v$, practically we do the following replacements for the heavy quark $Q$ and -anti-quark $\bar{Q}'$ spinors

$$v_i(p_{Q'})\bar{u}_j(p_Q) \rightarrow \frac{\Psi_{1S_0}}{2\sqrt{m_Q+m_{Q'}}}\gamma^5(\hat{p}+m_Q+m_{Q'})\frac{\delta_{ij}}{\sqrt{N_c}},$$

$$v(p_{Q'})\bar{u}(p_Q) \rightarrow \frac{\Psi_{3S_1}}{2\sqrt{m_Q+m_{Q'}}}\ell^*(\hat{p}+m_Q+m_{Q'})\frac{\delta_{ij}}{\sqrt{N_c}}.$$
Here $\epsilon^\mu$ is the polarization vector of $^3S_1$ state, $\Psi_{2S_1}(0)$ is the Schrödinger wave function at the origin. At the heavy quark limit, we set $\Psi_{3S_1}(0) = \Psi_{1S_0}(0)$. Throughout the paper, we adopt the relation $m_H = m_Q + m_{Q'}$ since we are doing the LO calculation in $v$. Obviously, we have $m_Q = m_{Q'}$ for quarkonia, and $m_Q = m_b$, $m_{Q'} = m_c$ for $B_c^{(*)}$.

A. $W^+ \to \eta_c$(or $J/\psi$) + $c + \bar{s}$

There are two Feynman diagrams for $W^+$ boson decay into a S-wave charmonium state associated with $c$ and $\bar{s}$. Implementing the Feynman rules of the Standard Model and projectors in Eq.(2), the amplitude for $W^+ \to \eta_c + c + \bar{s}$ is

$$M = \frac{16\pi g \alpha_s V_{cs}}{3} \frac{\Psi_{\eta_c}(0)}{2\sqrt{2}} (A_1 + A_2),$$

(4)

where

$$A_1 = \bar{u}(p_0) \gamma^\alpha \gamma^5 (2m_c + \not{p}) f_W (1 - \gamma^5) \frac{(-\not{p}_3 - \not{p}_5 - \not{p}_6)}{(p_3 + p_5 + p_6)^2} \gamma_\alpha v(p_5) \frac{1}{(p_3 + p_6)^2};$$

(5)

$$A_2 = \bar{u}(p_0) \gamma^\alpha \gamma^5 (2m_c + \not{p}) \gamma_\alpha \frac{(m_c + \not{p}_3 + \not{p}_4 + \not{p}_6)}{(p_3 + p_4 + p_6)^2 - m_c^2} f_W (1 - \gamma^5) v(p_5) \frac{1}{(p_3 + p_6)^2},$$

(6)

with $p$, $p_5$ and $p_6$ being the momenta of $\eta_c$ meson, $\bar{s}$ quark and $c$ quark, respectively; $p_3$ and $p_4$ the momenta of $\bar{c}$ quark and $c$ quark in $\eta_c$ meson; $V_{cs}$ the CKM matrix element; $g = e/\sin \theta_W$ with $\theta_W$ the Weinberg angle and $e$ the unit electro-charge; $\alpha \equiv e^2/(4\pi)$ and $\alpha_s = g_s^2/(4\pi)$ are the electromagnetic and the strong coupling constants, respectively; $\epsilon_W$ the polarization vector of $W^+$ boson. In this paper, we set $u$, $d$ and $s$ quarks massless, and ignore the dependence on the relative momentum between quark and anti-quark in meson, and set $p_3 = p_4 = p/2$. The spin-averaged partial decay width of $W^+ \to \eta_c + c + \bar{s}$ reads

$$d\Gamma = \frac{1}{2^8 m_w^3 \pi^5} \sum_{\text{spins}} |M|^2 d s_1 d s_2,$$

(7)

where $m_w$ is the mass of $W$ boson, $s_1 = (p + p_3)^2$ and $s_2 = (p_5 + p_6)^2$. The explicit analytic expressions for the square of the amplitude for $W^+ \to \eta_c + c + \bar{s}$ is given in Appendix A.

For the complete leading order calculation of $W^+ \to J/\psi + c + \bar{s}$, we just repeat the calculation that had been done in Ref.[9]. We find that there is a misprint in the term
−8xy^2r'M_W^2(1 + r - r' - r')^{-2} in \sum |A_2|^2 given in Appendix B of \cite{9}, which should be corrected to −8xy^2r'M_W^2(1 + r - r' - r')^{-2}.

B. \( W^+ \rightarrow \eta_b(\text{or } \Upsilon) + X \)

\( W^+ \rightarrow \eta_b(\text{or } \Upsilon) + c + \bar{b} \) is the leading order process of bottomonium production through \( W^+ \) decay in the expansion of \( \alpha_s \). However, such processes are the CKM-suppressed with the corresponding CKM factor \( V_{cb} = A\lambda^2 \) in the Wolfenstein parameterization where \( A = 0.813 \) and \( \lambda = 0.225 \). Numerically, the Wolfenstein parameter \( \lambda \sim \alpha_s(2m_c) = 0.26 \). Hence, we will consider some processes that are higher order in expansion of \( \alpha_s \) but lower order in expansion of the Wolfenstein parameter \( \lambda \). For instance, the process \( W^+ \rightarrow \eta_b + c + \bar{s} + g \) depicted by the last diagram in Fig. 1 is associated with the factor \( g_s^3V_{ud} \sim g^3_s\lambda^0 \) which is numerically comparable to the factor \( g_s^2V_{cb} \sim g^2_s\lambda^2 \) accompanied with the process \( W^+ \rightarrow \eta_b + c + \bar{b} \). Similarly, we also consider \( W^+ \rightarrow \Upsilon + c + \bar{s} + \gamma \) for \( W \) decays to \( \Upsilon \). Its amplitude is proportional to \( V_{ud}e^2 \), which is numerically comparable to the factor \( g_s^2V_{cb} \) accompanied with \( W^+ \rightarrow \Upsilon + c + \bar{b} \).

The calculation of \( \eta_b \) production is similar to the \( \eta_c \) production described in the previous subsection. Here we just show the results for the production of \( \Upsilon \). The complete amplitude for \( W^+ \) decay to \( \Upsilon \) with \( c \) and \( \bar{b} \) is

\[
M = \frac{16\pi}{3} \frac{g\alpha_sV_{cb}}{2\sqrt{2}} \Psi_{\Upsilon}(0)(A_1 + A_2),
\]

with

\[
A_1 = \bar{u}(p_5)\gamma^\alpha \frac{(m_c + p_3 + p_5 + p_6)}{(p_3 + p_5 + p_6)^2 - m^2_c} \frac{f_W(1 - \gamma^5)f^*_\Upsilon}{(2m_b + \bar{p})\gamma_\alpha v(p_6)} \frac{1}{(p_3 + p_6)^2},
\]

\[
A_2 = \bar{u}(p_5)\gamma^\alpha \frac{(m_b - p_3 - p_4 - p_6)}{(p_3 + p_4 + p_6)^2 - m^2_b} \frac{f_W(1 - \gamma^5)f^*_\Upsilon}{(2m_b + \bar{p})\gamma_\alpha v(p_6)} \frac{1}{(p_3 + p_6)^2},
\]

where \( p, p_5 \) and \( p_6 \) are the momenta of \( \Upsilon, c \) quark and \( \bar{b} \) quark, respectively; \( p_3 \) and \( p_4 \) are the momenta of \( b \) quark and \( \bar{b} \) quark in \( \Upsilon \) meson; \( \epsilon_\Upsilon \) is the polarization vector of \( \Upsilon \) boson. The explicit expression for the square of amplitude is given in Appendix B.
Because of the CKM suppression, the electroweak contributions depicted in Fig. 2 is required for the production of Υ as we argued above. Taking $W^+ \rightarrow \Upsilon + u + \bar{d}$ for example, the amplitude for this process is written as

$$M = \sqrt{3} g e^2 V_{ud} \Psi_\Upsilon(0) \frac{1}{2\sqrt{6}m_b} (A_1 + A_2 + A_3),$$

with

$$A_1 = \frac{\text{Tr}[\gamma_\alpha f_\Upsilon^\ast (2m_b + \not{p})]}{9\sqrt{2}(p_3 + p_4)^2} \times \bar{u}(p_0)\gamma^\alpha \frac{\not{p}_3 + \not{p}_4 + \not{p}_6}{(p_3 + p_4 + p_6)^2} f_W(1 - \gamma^5) v(p_5),$$

$$A_2 = -\frac{\text{Tr}[\gamma_\alpha f_\Upsilon^\ast (2m_b + \not{p})]}{18\sqrt{2}(p_3 + p_4)^2} \times \bar{u}(p_0) f_W(1 - \gamma^5) \frac{(-\not{p}_3 - \not{p}_4 - \not{p}_5)}{(p_3 + p_4 + p_5)^2} \gamma^\alpha v(p_5),$$

$$A_3 = -\frac{\text{Tr}[\gamma_\alpha f_\Upsilon^\ast (2m_b + \not{p})]}{6\sqrt{2}(p_3 + p_4)^2((p_3 + p_6)^2 - m_w^2)} \times \left( (p_5 + p_6 - p)_\mu g_{\alpha\nu} - (2p_5 + 2p_6 + p)_\alpha g_{\mu\nu} \ight. \left. + (p_5 + p_6 + 2p)_\nu g_{\alpha\mu} \right),$$

where $p$, $p_5$ and $p_6$ are the momenta of $\Upsilon$, $\bar{d}$ quark and $u$ quark, respectively; $p_3$ and $p_4$ are the momenta of $b$ quark and $\bar{b}$ quark in $\Upsilon$ meson. Here again we set $p_3 = p_4 = p/2$ in the calculation.

3 However, for that of $J/\psi$, the electroweak contribution can be ignored in LO, which is two orders of magnitude less than its QCD contribution.
$W^+ \to \eta_b + c + \bar{s} + g$ dominates the decay $W^+ \to \eta_b + X$. Its four-body decay amplitude is quite complicated, so we do not present the complete analytic expressions here but the numerical results in the next section. The potential infrared-divergences arising from the regions where the momentum of the gluon becomes soft vanish due to the “color-transparency”, and the potential collinear-divergences arising from the region where the momentum of the gluon becomes collinear to the $\eta_b$ and $c$-quark (or $\bar{s}$ quark) are regulated by the $b$ quark mass. Thus, the decay width of $W^+ \to \eta_b + c + \bar{s} + g$ is infrared-safe.

C. $W^+ \to B_c^{(*)} + b + \bar{s}$ and $W^+ \to B_c^{(*)} + c + \bar{c}$

There are two processes, namely $W^+ \to B_c^{(*)} + b + \bar{s}$ and $W^+ \to B_c^{(*)} + c + \bar{c}$, for $\bar{b}c$ bound states production in $W^+$ decay at the leading order. $W^+ \to B_c^{(*)} + g + g$ and $W^+ \to B_c^{(*)} + \gamma$ can also contribute, however their contributions vanish at the large $m_w$ limit, and thus are almost two and three orders of magnitude lower than $W^+ \to B_c^{(*)} + b + \bar{s}$ and $W^+ \to B_c^{(*)} + c + \bar{c}$ numerically.

For $W^+ \to B_c^+ + b + \bar{s}$, there are two Feynman diagrams and the corresponding amplitude is

$$M = \frac{16\pi g_\alpha s V_{cs}}{3 \cdot 2\sqrt{2} \cdot 2\sqrt{3(m_c + m_b)}} \Psi_{Bc}(0)(A_1 + A_2),$$

with

$$A_1 = \frac{1}{(p_3 + p_6)^2} \bar{u}(p_6)\gamma^\alpha\gamma^5(m_c + m_b + \hat{p})f_W(1 - \gamma^5)$$
$$\left(-\hat{p}_3 - \hat{p}_5 - \hat{p}_6\right) \gamma_\alpha v(p_5),$$

$$A_2 = \frac{1}{(p_3 + p_6)^2} \bar{u}(p_6)\gamma^\alpha\gamma^5(m_c + m_b + \hat{p})\gamma_\alpha$$
$$\frac{(m_c + \hat{p}_3 + \hat{p}_4 + \hat{p}_6)}{(p_3 + p_4 + p_6)^2 - m_c^2 f_W(1 - \gamma^5)v(p_5).}$$

Here $p$, $p_3$, $p_4$, $p_5$ and $p_6$ are, respectively, the momenta of $B_c^+$, $\bar{b}$ quark and $c$ quark in $B_c^+$ meson, $\bar{s}$ quark and $b$ quark. The corresponding amplitude at the leading order for $W^+ \to B_c^+ + c + \bar{c}$ is

$$M = \frac{16\pi g_\alpha s V_{cb}}{3 \cdot 2\sqrt{2} \cdot 2\sqrt{3(m_c + m_b)}} \Psi_{Bc}(0)(A_1 + A_2),$$

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with
\[
A_1 = \bar{u}(p_6)\gamma^\alpha \frac{(m_c + \hat{p}_4 + \hat{p}_5 + \hat{p}_6)}{(p_4 + p_5 + p_6)^2 - m_c^2} f_W(1 - \gamma^5)
= \frac{\gamma^5(m_c + m_b + \hat{p})\gamma_\alpha v(p_5)}{(p_4 + p_5)^2},
\]
(19)
\[
A_2 = \bar{u}(p_6)f_W(1 - \gamma^5) \frac{(m_b - \hat{p}_3 - \hat{p}_4 - \hat{p}_5)}{(p_3 + p_4 + p_5)^2 - m_b^2} \gamma^\alpha
= \frac{\gamma^5(m_c + m_b + \hat{p})\gamma_\alpha v(p_5)}{(p_4 + p_5)^2},
\]
(20)
where \( p, p_3, p_4, p_5 \) and \( p_6 \) are, respectively, the momenta of \( B_c, \bar{b} \) quark and \( c \) quark in \( B_c \) meson, \( \bar{c} \) quark and \( c \) quark.

Similarly, we can obtain the amplitudes for \( B_c^* \) production. In Appendix C, we detail the square of the amplitudes for both \( W^+ \to B_c^{(*)} + b + \bar{s} \) and \( W^+ \to B_c^{(*)} + c + \bar{c} \).

### III. NUMERICAL RESULTS AND ANALYSIS

#### A. Input parameters

Now we can employ the above formalism to evaluate the decay widths of \( W^+ \) boson to quarkonia and \( B_c^{(*)} \) mesons. We adopt the mass parameters for \( c \) quark, \( b \) quark and \( W \) boson as follows

\[
m_c = 1.50 \text{ GeV}, \quad m_b = 4.90 \text{ GeV} \quad \text{and} \quad m_w = 80.4 \text{ GeV}.
\]

The Schrödinger wave functions at the origin for \( J/\psi \) and \( \Upsilon \) are determined through their leptonic decay widths \( \Gamma_{ee} \), at the leading order in both \( \alpha_s \) and \( v \), which is [7].

\[
|\Psi_{J/\psi}(0)|^2 = \frac{m_{J/\psi}^2 \Gamma_{ee}}{16\pi \alpha^2 \epsilon_{c(b)}^2}.
\]

Taking \( \Gamma_{ee}^J = 5.55 \text{ keV} \), \( \Gamma_{ee}^\Upsilon = 1.34 \text{ keV} \), \( \alpha = 1/137 \) and \( \epsilon_c = 2/3 \), we obtain

\[
|\Psi_{J/\psi}(0)|^2 = 0.0447 \text{ GeV}^3 \quad \text{and} \quad |\Psi_{\Upsilon}(0)|^2 = 0.403 \text{ GeV}^3.
\]

In principle, we can extract \( \Psi_{\eta_c}(0) \) from the decay width of \( \eta_c \to \gamma\gamma \). However, since the experimental data uncertainty is still large, which is \( B(\eta_c \to \gamma\gamma) = (6.3 \pm 2.9) \times 10^{-5} \) [3],
TABLE I: Decay widths and branching fractions of quarkonium and $B_c$ meson through $W^+$ inclusive decays. Here the $q_i\bar{q}_j$ represents one kind of $c\bar{s}$, $b\bar{s}$, $c\bar{c}$, $b\bar{c}$, $u\bar{d}$, $\mu^+\nu_\mu$ and $c\bar{s}g$; the $Q_i\bar{Q}_j$ one kind of $c\bar{s}$ and $c\bar{b}$.

| $W^+ \to H q_i\bar{q}_j$ | $\Gamma(W^+ \to H q_i\bar{q}_j)$ (keV) | $\Gamma(W^+ \to H q_i\bar{q}_j) / \Gamma(W^+ \to Q_i\bar{Q}_j)$ |
|--------------------------|----------------------------------|----------------------------------|
| $W^+ \to J/\psi c\bar{s}$ | 90.4                            | $1.27 \times 10^{-4}$ |
| $W^+ \to \eta_c c\bar{s}$  | 87.5                            | $1.23 \times 10^{-4}$ |
| $W^+ \to B_c^+ b\bar{s}$  | 6.3                             | $8.84 \times 10^{-6}$ |
| $W^+ \to B_c^{++} b\bar{s}$ | 5.4                             | $7.61 \times 10^{-6}$ |
| $W^+ \to B_c^+ c\bar{c}$   | 0.4                             | $3.64 \times 10^{-6}$ |
| $W^+ \to B_c^{++} c\bar{c}$ | 0.5                             | $5.23 \times 10^{-4}$ |
| $W^+ \to \eta_b c\bar{b}$  | 0.015                           | $1.41 \times 10^{-5}$ |
| $W^+ \to \Upsilon c\bar{b}$ | 0.016                           | $1.49 \times 10^{-5}$ |
| $W^+ \to \Upsilon u\bar{d}$ | 0.014                           |                     |
| $W^+ \to \Upsilon \mu^+ \nu_\mu$ | 0.013                        |                     |
| $W^+ \to \eta_b c\bar{s}g$  | 0.517                           |                     |

The corresponding extraction of $\Psi_{\eta_b}(0)$ is not applicable in our numerical analysis. Instead, we set $\Psi_{\eta_b}(0) = \Psi_{J/\psi}(0)$, which is the consequence of the heavy quark spin symmetry in NRQCD at leading order $v^7$. Similarly, we adopt $\Psi_{\eta_b}(0) = \Psi_\Upsilon(0)$.

For $B_c^{(s)}$, we apply $|\Psi_{B_c}(0)|^2 = |\Psi_{B_c^s}(0)|^2 = 0.1307(\text{GeV})^3$ given in [14] by using the Buchmüller-Tye potential [15]. In addition, for consistency the leading order $\alpha_s$ running is adopted, i.e.

$$\alpha_s(\mu) = \frac{4\pi}{(11 - 2n_f/3) \ln(\mu^2/\Lambda_{QCD}^2)},$$

where we take $\Lambda_{QCD}^2 = 200\text{MeV}$ and $n_f = 4$ as in [14]. We choose the typical renormalization scale $\mu = 2m_c$ for charmonium and $B_c^{(s)}$ production, and $\mu = 2m_b$ for bottomonium production, correspondingly $\alpha_s(2m_c) = 0.26$ and $\alpha_s(2m_b) = 0.19$. 


B. Decay widths and feasibility at LHC

By the input parameters given above, we list the partial widths of $W$ decays to the S-wave quarkonia and $B_c^{(s)}$ mesons in Table I. Employing these partial widths, one can estimate the event numbers of quarkonium production through $W$ decays at the LHC. Considering that the LHC runs at the center-of-mass energy $\sqrt{s} = 14$ TeV with the luminosity $L_{p-p} = 10^{34}$ cm$^{-2}$s$^{-1}$, and the cross section $\sigma_{W^+} = 10^2$ nb at the LHC \cite{6}, the number of $W^+$ events per year is expected to be $3.07 \times 10^{10}$. Then we present the event rates of $W$ decays to quarkonia or $B_c^{(s)}$ in Table II.

The most readily identifiable quarkonia are $J/\psi$ and $\Upsilon$, because their leptonic decays have clear signals and relatively large branching fractions. With $B(\psi \rightarrow \mu\mu) = 5.93\%$, $B(\Upsilon \rightarrow \mu\mu) = 2.48\%$ and $B(\Upsilon \rightarrow \tau\tau) = 2.60\%$, we list the expected di-leptonic signals from $J/\psi$ and $\Upsilon$ decays in Table III. For experimental detection for such decays at the LHC, one has to look for the relevant events $pp \rightarrow W^+X$ with $W \rightarrow J/\psi(\mu^+\mu^-) + 2$ jets. According to \cite{16}, the efficiency of reconstruction of $J/\psi$ from its dimuon decay channel is around 40\% for $pp$ collision at LHC experiments. However, to tag whether such $J/\psi$ event is really from $W^+$ decay, one must reconstruct $W^+$ from $\mu^+\mu^- + 2$ jets event. Therefore, the additional information about relevant QCD background of such events could be crucial as well.

The most promising decay channels for reconstruction $\eta_c$ are $\eta_c \rightarrow K\bar{K}\pi$, $\eta\pi\pi$ and etc, which branching ratios are around a few percent. Thus, it should be possible to observe $W^+$ decays to $\eta_c$ at the LHC. Certainly, such measurements also suffers the difficulty of reconstruction of $W^+$ as mentioned above.

$B_c^+ \rightarrow J/\psi\pi^+$ is the most promising channels to identify $B_c^+$ at colliders. However, considering their small branching fractions and the efficiency of the event reconstruction, the measurements on $W$ decays to $B_c$ at the LHC would be quite difficult. With the similar reasoning, it would be barely possible to measure $W$ decays to $\eta_b$ at the LHC due to the lower branching fraction and the experimental difficulty for identifying $\eta_b$. 


TABLE II: The expected annual event numbers of the quarkonium and $B_c$ meson from $W^+$ decay at the LHC with $\sqrt{s} = 14 \text{TeV}$ and $L_{pp} = 10^{34}\text{cm}^{-2}\text{s}^{-1}$.

|          | $J/\psi$ | $\eta_c$ | $B_c^+$ | $B_{c^*}^+$ | $\eta_b$ | $\Upsilon$ |
|----------|----------|----------|---------|------------|----------|------------|
| Events number ($\times 10^4$) | 133      | 129      | 9.8     | 8.8        | 1.5      | 0.12       |

TABLE III: The corresponding annual leptonic events of $J/\psi$ and $\Upsilon$.

|          | $J/\psi$ | $\Upsilon$ |
|----------|----------|------------|
| Decay channel | $e^+e^-$ | $\mu^+\mu^-$ | $e^+e^-$ | $\mu^+\mu^-$ | $\tau^+\tau^-$ |
| Events     | $7.9\times10^4$ | $7.9\times10^4$ | 30       | 30         | 31        |

C. Comparisons with the fragmentation mechanism

Since $m_w \gg m_b, m_c$, one can argue that the fragmentation mechanism may be the dominant contribution to the inclusive decay rate of the $W^+$ into the quarkonium and $B_c$ meson which survives in the limit $m_w/m_{b,c} \to \infty$. In the fragmentation mechanism, the hadron $H$ with energy $E$ is produced by the fragment of a type $i$ parton with energy $E/z$ ($z$ is the longitudinal momentum fraction of $H$ relative to type $i$ parton) directly from the decay of $W^+$ boson. The possibility of $i \to H$ is presumed to be described by the universal fragmentation function $D_{i \to H}(z)$. Putting all the possible parton fragmentation together, the differential decay width of $H$ inclusive production can be written as

$$
d\Gamma(W^+ \to H(E) + X) = \sum_i \int_0^1 dz d\hat{\Gamma}(W^+ \to i(E/z) + X, \mu) D_{i \to H}(z, \mu)
+ O\left(\frac{m_H}{m_w}\right), \tag{21}
$$

where $d\hat{\Gamma}$ is the parton level differential decay width, and the sum is over the parton type $i$ and the hadron $H$’s longitudinal momentum fraction $z$. At the leading order of $\alpha_s$, $d\hat{\Gamma}$ does not depend on the longitudinal momentum fraction $z$. Thus, the total decay width turns to be

$$
\Gamma(W^+ \to H(E) + X) = \sum_i \hat{\Gamma}(W^+ \to i(E/z) + X) P_{i \to H},
$$
\[
P_{i \rightarrow H} \equiv \int_{0}^{1} dz D_{i \rightarrow H}(z, \mu),
\]

where \(P_{i \rightarrow H}\) is the so-called fragmentation possibility.

After paying the price of \(\mathcal{O}(m_{H}/m_{w})\) power corrections, Eq. (21) has a number of advantages in calculation. The parton level differential decay width \(d\hat{\Gamma}\) is easy to be calculated and dependent only on the typical short-distance scale \(m_{w}\). The fragmentation functions are universal and dependent on the hadronization scale, and therefore, they can be either extracted from the experimental measurements or calculated from certain phenomenological models. Fortunately, the fragmentation functions for the S-wave quarkonium and \(B_{c}\) meson can be calculated perturbatively [10, 11], together with all the non-perturbative hadronization effects being parameterized into the Schrödinger wave functions of the mesons at origin at the non-relativistic limit.

In calculation we take the following fragmentation probabilities from [10, 11]:

\[
P(c \rightarrow \psi) = \frac{32\alpha_{s}^{2}(2m_{c})|\Psi_{\psi}(0)|^{2}}{27m_{c}^{3}} \left( \frac{1189}{30} - 57 \ln 2 \right),
\]

\[
P(c \rightarrow \eta_{c}) = \frac{32\alpha_{s}^{2}(2m_{c})|\Psi_{\eta_{c}}(0)|^{2}}{27m_{c}^{3}} \left( \frac{773}{30} - 37 \ln 2 \right),
\]

\[
P(\bar{b} \rightarrow B_{c}^{+}) = \frac{8\alpha_{s}^{2}(2m_{c})|\Psi_{B_{c}^{+}}(0)|^{2}}{27m_{c}^{3}} f \left( \frac{m_{c}}{m_{b} + m_{c}} \right),
\]

\[
P(\bar{b} \rightarrow B_{c}^{++}) = \frac{8\alpha_{s}^{2}(2m_{c})|\Psi_{B_{c}^{++}}(0)|^{2}}{27m_{c}^{3}} g \left( \frac{m_{c}}{m_{b} + m_{c}} \right),
\]

where the function \(f(r)\) and \(g(r)\) are

\[
f(r) = \frac{8 + 13r + 228r^{2} - 212r^{3} + 53r^{4}}{15(1 - r)^{5}}
\]
\[+ \frac{r(1 + 8r + r^{2} - 6r^{3} + 2r^{4})}{(1 - r)^{6}} \ln(r),\]

\[
g(r) = \frac{24 + 109r - 126r^{2} + 174r^{3} + 89r^{4}}{15(1 - r)^{5}}
\]
\[+ \frac{r(7 - 4r + 3r^{2} + 10r^{3} + 2r^{4})}{(1 - r)^{6}} \ln(r).\]

\(P_{b \rightarrow \Upsilon}\) and \(P_{b \rightarrow \eta_{b}}\) can be obtained from (23) and (24) by substituting the mass \(m_{b}\) for \(m_{c}\). And \(P_{c \rightarrow B_{c}^{(*)}+}\) can be also got from (25) and (26) by interchanging \(m_{b}\) and \(m_{c}\).
TABLE IV: The branching fractions for the quarkonium and $B_c$ meson production predicted by the complete leading order calculation and fragmentation approximation and their comparisons. 

| Branching Fractions | LO calculation ($B$) | Fragmentation ($B^*$) | $\frac{B^*-B}{B}$ |
|---------------------|---------------------|----------------------|-----------------|
| $\Gamma(W^+ \rightarrow J/\psi c\bar{s})/\Gamma(W^+ \rightarrow c\bar{s})$ | $1.27 \times 10^{-4}$ | $1.31 \times 10^{-4}$ | 3.3% |
| $\Gamma(W^+ \rightarrow \eta_c c\bar{s})/\Gamma(W^+ \rightarrow c\bar{s})$ | $1.23 \times 10^{-4}$ | $1.27 \times 10^{-4}$ | 3.6% |
| $\Gamma(W^+ \rightarrow B_c^{+} b\bar{s})/\Gamma(W^+ \rightarrow c\bar{s})$ | $8.84 \times 10^{-6}$ | $1.03 \times 10^{-5}$ | 17% |
| $\Gamma(W^+ \rightarrow B_c^{*+} b\bar{s})/\Gamma(W^+ \rightarrow c\bar{s})$ | $7.61 \times 10^{-6}$ | $8.92 \times 10^{-6}$ | 17% |
| $\Gamma(W^+ \rightarrow B_c^{+} c\bar{c})/\Gamma(W^+ \rightarrow c\bar{b})$ | $3.64 \times 10^{-4}$ | $3.85 \times 10^{-4}$ | 6% |
| $\Gamma(W^+ \rightarrow B_c^{*+} c\bar{c})/\Gamma(W^+ \rightarrow c\bar{b})$ | $5.23 \times 10^{-4}$ | $5.41 \times 10^{-4}$ | 3% |
| $\Gamma(W^+ \rightarrow \eta_c b\bar{b})/\Gamma(W^+ \rightarrow c\bar{b})$ | $1.41 \times 10^{-5}$ | $1.76 \times 10^{-5}$ | 24% |
| $\Gamma(W^+ \rightarrow \Upsilon c\bar{b})/\Gamma(W^+ \rightarrow c\bar{b})$ | $1.49 \times 10^{-5}$ | $1.81 \times 10^{-5}$ | 21% |

The numerical results on the decay widths of $W$ inclusive decays to quarkonium and $B_c$ meson by using the fragmentation approximation are listed in Table IV. We also list the comparisons between the predictions given by the fragmentation approximation and the complete leading order calculations. We find that the difference between the predictions by the two methods is negligible for the charmonium productions. However, for the bottomonium and $B_c^{(*)}$ productions, the differences is sizable but understandable, since the power corrections to the fragmentation mechanism is order of $2m_b/m_w$, which is numerically around 10%.

D. Theoretical uncertainties

At last, we investigate the theoretical uncertainties in our calculations for the $J/\psi$ and $\eta_c$ production from $W$ boson decay. The main sources of uncertainties include the wave function at the origin, the strong coupling constant and the quark mass. After some numerical check, we find that the biggest uncertainty in our results is from the choice of the charm quark mass. Here we present the dependence of our results on the charm quark mass in Fig. 3. It shows that we can get a production rate of charmonium which is 1.6 times as much as we had calculated if we adopt $m_c = 1.27$ GeV. Besides, the production rate from fragmentation is a slightly bigger than that from complete leading order computation. The difference between
the LO calculation and the fragmentation approximation is insensitive to the charm mass. The second largest uncertainty is due to the choice of renormalization scale as mentioned in the front of this section. Also we lay out their relations in Fig. 3. The dependence on the renormalization scale can be reduced by considering the NLO QCD corrections and employing the Altarelli-Parisi equations to resum the large logarithms.

![Graph](image1)

**FIG. 3:** The partial width of the process $\Gamma(W^+ \rightarrow J/\psi c\bar{s})$ versus the mass of charm quark (upper, $\mu = 3$ GeV) and the renormalization scale (lower, $m_c = 1.5$ GeV). The dashed line corresponds to the prediction by the complete LO calculation, while the solid line corresponds to that by the fragmentation approximation.

**IV. CONCLUSIONS**

In this paper, we show that there will be around $10^6 J/\Psi$ and $\eta_c$, $10^4 B_c^{(s)}$ mesons and $10^3 \Upsilon$ and $\eta_b$ annually produced from $W$ boson inclusive decay at the LHC, if the machine works at the center energy $\sqrt{s} = 14$ TeV and with the luminosity $\mathcal{L}_{p-p} = 10^{34}$ cm$^{-2}$s$^{-1}$. And the fragmentation mechanism works well for $W$ boson inclusive decays to the S-wave charmonium, bottomonium and $B_c^{(s)}$ mesons. The numerical calculation shows that the difference between fragmentation approximation and complete LO calculation for $W$ boson decays to charmonium is around 3%, and it is not magnified by considering the uncertainties of theoretical parameters. But for $W$ decays to $B_c^{(s)}$ mesons and bottomonium, the differences rise, but it is understandable.
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Appendix A: The square of the amplitudes for $W^+ \rightarrow \eta_c + c + \bar{s}$

Here we introduce parameters

\[ s_{10} = p_1 \cdot p, \quad s_{15} = p_1 \cdot p_5 \]

with $p_1$, $p$ and $p_5$ being the momenta of $W^+$, $\eta_c$ and $\bar{s}$ quark, respectively.

\[ \Sigma |A_1|^2 = \frac{1}{(m_c^2(m_c^2 - m_w^2) - s_{10})^2 (m_c^2 - m_w^2 + 2s_{15})^2)} \times (64(27m_c^6m_w^2 + (m_c^2 - 2s_{15})^2)(m_c^4 - 2m_w^2(s_{10} - 2s_{15})) \]

\[ -4s_{15}(s_{10} + s_{15}) - m_c^2(m_w^2 - 2s_{15})(7m_w^4) \]

\[ -8s_{15}(5s_{10} + s_{15}) - 2m_w^2(10s_{10} + s_{15})) \]

\[ -3m_c^4(7m_w^4 + 4(3s_{10} - 5s_{15})s_{15} + 2m_w^2(3s_{10} + 8s_{15}))) , \]

\[ \Sigma |A_2|^2 = \frac{1}{(m_c^2(m_c^2 - m_w^2) + 2s_{15})^4)} \times (64(27m_c^6m_w^2 + (m_c^2 - 2s_{15})^2)(m_c^4 - 2m_w^2(s_{10} - 2s_{15})) \]

\[ -4s_{15}(s_{10} + s_{15}) - m_c^2(m_w^2 - 2s_{15})(7m_w^4) \]

\[ -8s_{15}(5s_{10} + s_{15}) - 2m_w^2(10s_{10} + s_{15})) \]

\[ -3m_c^4(7m_w^4 + 4(3s_{10} - 5s_{15})s_{15} + 2m_w^2(3s_{10} + 8s_{15}))) , \]
Here we define parameters:

\[ s_{10} = p_1 \cdot p, \quad s_{15} = p_1 \cdot p_5 \]

with \( p_1, p \) and \( p_5 \) being the momenta of \( W^+ \), \( Y \) and \( c \) quark, respectively.

\[
\Sigma |A_1|^2 = \frac{1}{(m_w^2(m_b^2 - m_c^2 + m_w^2 - s_{10})^2(-m_b^2 + m_c^2 + m_w^2 - 2s_{15})^2) \times (64(66m_0^6m_w^2 - 2m_c^4m_w^2 + m_w^8 - 8m_w^6s_{10} + 35m_w^4s_{10}^2)}
\]

\[
= \frac{1}{(-46m_w^2s_{10}^3 - 4m_w^6s_{15} + 20m_w^4s_{10}s_{15} - 20m_w^2s_{10}s_{15})}
\]

\[
-4s_{10}s_{15} + 4m_w^4s_{15} - 8m_w^2s_{10}s_{15} - 4s_{10}s_{15}^2
\]

\[
-m_w^4(3m_w^4 + m_w^2(22s_{10} - 8s_{15}) - s_{10}(7s_{10} + 8s_{15}))
\]

\[
+m_b^4(42m_w^2m_w^2 + m_w^4 + 3s_{10} - 32s_{10}s_{15} + 16s_{15}^2
\]

\[
-2m_w^2(97s_{10} + 32s_{15})) - 2m_w^2(m_w^4(15s_{10})
\]

\[
-2s_{10} + m_w^2(-9s_{10}^2 - 28s_{10}s_{15} + 4s_{15})
\]

\[
+m_s(11s_{10} + 18s_{10}s_{15} + 8s_{15}) - 2m_b^2(5m_w^4m_c^2 + 2m_w^6
\]

\[
+m_s^2(s_{10} - 10s_{15}) + m_w^4(27s_{10} + 10s_{15})
\]

\[
-m_w^2(85s_{10} + 36s_{10}s_{15} + 4s_{15}) - m_w^2(25m_w^4 + 23s_{10}
\]

\[
+28s_{10}s_{15} + 8s_{15}^2 - 4m_w^2(7s_{10} + 5s_{15}))
\]

\[
\text{Appendix B: The square of the amplitudes for } W^+ \to Y + c + \bar{b}
\]
First we present the squared matrix elements for $W^+ \rightarrow B^+_c + b + s$. We introduce

\[ s_{10} = p_1 \cdot p, \quad s_{15} = p_1 \cdot p_5 \]

with $p_1$, $p$ and $p_5$ being the momenta of $W^+$, $B^+_c$ and $s$ quark, respectively.
\[
= \frac{1}{m_b^2 m_w^2 (m_b^2 + m_w^2 + m_c^2 (m_c^2 + m_w^2 - 2 s_{10}))^2} \\
\times \frac{1}{(m_c^2 - m_w^2 + 2 s_{15})^2} (16 (m_b + m_c)^2 (2 m_b^5 m_c m_w^2 (m_c^2 - m_c^4 - 2 m_b^2 (s_{10} - 2 s_{15}) + 8 s_{15} (s_{10} + s_{15})) - m_c^2 (m_c^2 + m_w^2 - 2 s_{10})) (3 m_c^4 m_w^2 - m_b^6 + 4 m_b^4 (s_{10} + s_{15}) - 8 s_{10} s_{15} (s_{10} + s_{15}) - 4 m_b^2 (s_{10} + s_{10} s_{15} + s_{15}^2) + m_c^2 (-2 m_c^4 + 4 m_w^2 (s_{10} + s_{15})) + 4 (-s_{10}^2 + s_{10} s_{15} + s_{15}^2)) - 2 m_b m_c (7 m_c^6 m_w^2 + m_c^4 (-m_c^2 - 4 s_{10}^2 + 12 s_{10} s_{15} + 8 s_{15}^2 + 2 m_w^2 (s_{10} + 6 s_{15})) - m_c^2 (m_c^2 + m_w^2 - 2 s_{10}) (m_c^4 - 2 m_w^2 (3 s_{10} + 2 s_{15}) + 4 (2 s_{10}^2 + 3 s_{10} s_{15} + s_{15}^2)) - m_c^2 (5 m_w^6 + 8 s_{10} s_{15} (3 s_{10} + 2 s_{15}) - 2 m_w^4 (11 s_{10} + 8 s_{15}) + 4 m_w^2 (6 s_{10}^2 + 6 s_{10} s_{15} + s_{15}^2))) + m_c^2 (-22 m_b^6 m_w^2 + m_c^4 (11 m_w^4 - 8 s_{15} (5 s_{10} + 3 s_{15}) - m_c^2 (7 s_{10} + 13 s_{15})) + m_c^2 (5 m_w^6 + 8 s_{10} s_{15} (s_{10} + s_{15}) - m_c^4 (29 s_{10} + 22 s_{15}) + 12 m_w^2 (3 s_{10}^2 + 4 s_{10} s_{15} + s_{15}^2)) + m_w^2 (m_c^6 - 16 s_{10} (s_{10} + s_{15})^2 - 2 m_w^4 (5 s_{10} + 2 s_{15}) + 4 m_w^2 (6 s_{10}^2 + 7 s_{10} s_{15} + s_{15}^2))) - 2 m_b^3 m_c (6 m_c^4 m_w^2 + m_c^2 (-5 m_c^4 + 4 s_{15} (3 s_{10} + 2 s_{15}) + m_w^2 (20 s_{10} + 26 s_{15})) - m_c^2 (m_c^4 - 12 m_w^2 (s_{10} + s_{15}) + 4 (4 s_{10}^2 + 8 s_{10} s_{15} + 3 s_{15}^2))))
\]

\[\Sigma |A_2|^2\]

\[
= \frac{1}{(m_b^2 m_w^2 (m_b^2 - m_w^2 + 2 s_{15})^4)} \times (16 (m_b + m_c)^2 (2 m_b^5 m_c m_w^2 (m_c^2 - m_b^4 - 2 s_{10} (m_b^2 - 2 s_{15}) (m_b^4 - 8 s_{10} s_{15} (s_{10} + s_{15}) + 2 m_w^2 (2 s_{10} + s_{15}))) + (m_b^2 - m_w^2 + 2 s_{15}) (4 m_c^4 m_w^2)
\]

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\[-(m_w^2 - 2s_{15})(m_w^4 - 2m_w^2(s_{10} - 2s_{15}) - 4s_{15}(s_{10} + s_{15}))
\+ m_c^2(3m_w^4 - 4s_{15}(4s_{10} + 3s_{15}) + m_w^2(-8s_{10} + 12s_{15}))
\+ m_b^2(9m_c^4m_w^2 - (m_w^2 - 2s_{15})(m_w^4 - 4(s_{10} - 2s_{15})s_{15}
\- 2m_w^2(s_{10} + 7s_{15})) - 2m_c^2(4m_w^4 + 2(s_{10} - 10s_{15})s_{15}
\+ m_w^2(s_{10} + 22s_{15}))))),
\]

\[
\Sigma \text{Re}(A_1A_2^*)
\]
\[
= \frac{1}{m_b^2m_w^2(m_b^2 + m_c^2 + m_c(m_w^2 + m_w^2 - 2s_{10}))}
\times \frac{1}{(m_w^2 - m_b^2 + 2s_{15})^3(-16(m_b + m_c)^2(2m_b^4m_c(m_c^2m_w^2
\- 3m_b^4 - 2m_b^2s_{15} + 2s_{15}^2) - m_b^3(3m_c^4m_w^2
\+ 2m_c^2(3m_w^4 + 2(s_{10} - 2s_{15})s_{15} + 5m_w^2(s_{10} + 3s_{15}))
\+ m_w^2(3m_w^4 + 4s_{15}(s_{10} + 2s_{15}) - 2m_w^2(3s_{10} + 5s_{15})))
\- 4m_b^2m_c(3m_c^4m_w^2 + 2m_w^6 + 2s_{15}(2s_{10} + s_{15})
\- 2m_w^2(s_{10} + 3s_{15}) + m_w^2(-2s_{10} + 3s_{15})
\+ m_c^2(-2m_w^4 + s_{15}(7s_{10} + s_{15}) + m_w^2(4s_{10} + 11s_{15})))
\+ m_c(-((m_w^2 - 2s_{10})(m_w^4 - 4s_{15}^2)(m_w^2 - 2(s_{10} + s_{15})))
\+ m_c^4(m_w^4 - 2m_w^2(s_{10} - s_{15}) - 4s_{15}(3s_{10} + 2s_{15}))
\- 2m_c^2(m_w^4(s_{10} - 2s_{15}) + 4s_{15}(-3s_{10} - s_{10}s_{15} + s_{15}^2)
\+ m_w^2(-2s_{10}^2 + 8s_{10}s_{15} + 2s_{15}^2))) + m_b(-5m_c^2m_w^2
\+ m_c^4(3m_w^4 - 2m_w^2(s_{10} + 2s_{15}) - 8s_{15}(5s_{10} + 2s_{15}))
\- m_w^2(m_w^2 - 2s_{15})(3m_w^4 - 2m_w^2(5s_{10} + 4s_{15})
\+ 4(2s_{10}^2 + 3s_{10}s_{15} + s_{15}^2)) + m_b^2(m_w^6 - 6m_w^4(2s_{10} + s_{15})
\- 8s_{15}(-2s_{10}^2 + 3s_{10}s_{15} + 2s_{15}^2) + 4m_w^2(4s_{10}
\+ 5s_{10}s_{15} + 4s_{15}^2))))\).}

For \( W^+ \to B_c^{*+} + b + \bar{s} \), we introduce

\[ s_{10} = p_1 \cdot p, \quad s_{15} = p_1 \cdot p_5 \]

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with $p_1, p$ and $p_5$ being the momenta of $W^+$, $B_c^{*+}$ and $s$ quark, respectively.

\[
\Sigma |A_1|^2 = \frac{1}{m_b^2 m_w^2 (m_b (m_c^2 + m_w^2) + m_c (m_c^2 + m_w^2 - 2 s_{10}))^2} \left( \frac{1}{(m_c^2 - m_w^2 + 2 s_{15})^2} \right) (16 (m_b + m_c)^2 (6 m_b^3 m_c m_w^2 (m_c^2 - m_w^2 - 2 s_{15}) + 2 m_b^3 m_c (40 m_c^4 m_w^2 - m_w^2 (3 m_w^4 - 48 s_{10}^2 - 48 s_{10} s_{15}) + 4 s_{15}^2 + 4 m_w^2 (3 s_{10} + s_{15})) - m_c^2 (5 m_w^4 + 4 (5 s_{10} - 2 s_{15}) s_{15} + 2 m_w (50 s_{10} + 23 s_{15})) + m_b^4 (35 m_c^4 m_w^2 + m_w^2 (-3 m_w^4 + 8 (3 s_{10} - s_{15}) s_{15} + m_w^2 (6 s_{10} + 4 s_{15}) - 2 m_c^2 (8 m_w^4 + 2 (3 s_{10} - s_{15}) s_{15} + m_w^2 (27 s_{10} + 28 s_{15}))) + m_c^2 (m_c^2 + m_w^2 - 2 s_{10} (9 m_c^4 m_w^2 + m_w^6 - 4 m_w^4 (s_{10} + s_{15}) + 8 s_{10} s_{15} (s_{10} + s_{15}) + 4 m_w^2 (5 s_{10}^2 + s_{10} s_{15} + s_{15}^2) - 2 m_c^2 (m_w^4 + 2 m_c^2 (7 s_{10} + s_{15} - 2 s_{10} s_{15} + s_{15})) + m_b m_c (23 m_c^6 m_w^2 + m_c^2 (s_{15} - 2 s_{10} s_{15} + s_{15})) + 2 m_c^2 (21 s_{10} + 8 s_{15}) + 12 m_w^2 (10 s_{10}^2 + 2 s_{10} s_{15} + s_{15}^2)) + m_c^2 (m_w^2 - 2 s_{10} (m_c^4 - m_w^2 (47 s_{10} + 6 s_{15}) + 4 (11 m_w^4 + 88 m_c^6 m_w^2 + m_c^4 (1 + 4 m_w^2 (72 s_{10} + 17 s_{15})) + m_w^2 (s_{15}^2 - 2 s_{10} + s_{15} + 16 s_{10} (3 s_{10} + 2 s_{10} s_{15} + s_{15}^2) + 4 m_w^2 (10 s_{10}^2 + 7 s_{10} s_{15} + s_{15}^2)) - 2 m_c^2 (2 m_w^6 - 8 s_{10}^2 s_{15} + m_w^4 (43 s_{10} + 18 s_{15}) - 4 m_w^2 (33 s_{10}^2 + 14 s_{10} s_{15} + s_{15}^2)))))),
\]

\[
\Sigma |A_2|^2 = \frac{1}{(m_c^2 m_w^2 (m_c^2 - m_w^2 + 2 s_{15})^4) (16 (m_b + m_c)^2 (6 m_b^3 m_c m_w^2 (m_c^2 - m_w^2 - 2 s_{15}) - m_b^3 m_c (40 m_c^4 m_w^2 - m_w^2 (3 m_w^4 - 48 s_{10}^2 - 48 s_{10} s_{15}) + 4 s_{15}^2 + 4 m_w^2 (3 s_{10} + s_{15})) - m_c^2 (5 m_w^4 + 4 (5 s_{10} - 2 s_{15}) s_{15} + 2 m_w (50 s_{10} + 23 s_{15})) + m_b^4 (35 m_c^4 m_w^2 + m_w^2 (-3 m_w^4 + 8 (3 s_{10} - s_{15}) s_{15} + m_w^2 (6 s_{10} + 4 s_{15}) - 2 m_c^2 (8 m_w^4 + 2 (3 s_{10} - s_{15}) s_{15} + m_w^2 (27 s_{10} + 28 s_{15}))) + m_c^2 (m_c^2 + m_w^2 - 2 s_{10} (9 m_c^4 m_w^2 + m_w^6 - 4 m_w^4 (s_{10} + s_{15}) + 8 s_{10} s_{15} (s_{10} + s_{15}) + 4 m_w^2 (5 s_{10}^2 + s_{10} s_{15} + s_{15}^2) - 2 m_c^2 (m_w^4 + 2 m_c^2 (7 s_{10} + s_{15} - 2 s_{10} s_{15} + s_{15})) + m_b m_c (23 m_c^6 m_w^2 + m_c^2 (s_{15} - 2 s_{10} s_{15} + s_{15})) + 2 m_c^2 (21 s_{10} + 8 s_{15}) + 12 m_w^2 (10 s_{10}^2 + 2 s_{10} s_{15} + s_{15}^2)) + m_c^2 (m_w^2 - 2 s_{10} (m_c^4 - m_w^2 (47 s_{10} + 6 s_{15}) + 4 (11 m_w^4 + 88 m_c^6 m_w^2 + m_c^4 (1 + 4 m_w^2 (72 s_{10} + 17 s_{15})) + m_w^2 (s_{15}^2 - 2 s_{10} + s_{15} + 16 s_{10} (3 s_{10} + 2 s_{10} s_{15} + s_{15}^2) + 4 m_w^2 (10 s_{10}^2 + 7 s_{10} s_{15} + s_{15}^2)) - 2 m_c^2 (2 m_w^6 - 8 s_{10}^2 s_{15} + m_w^4 (43 s_{10} + 18 s_{15}) - 4 m_w^2 (33 s_{10}^2 + 14 s_{10} s_{15} + s_{15}^2)))))).
\]
\[ \Sigma \text{Re}(A_1A_2^*) \]

\[
\frac{1}{m_b^2 m_w^2 (m_b (m_c^2 + m_w^2) + m_c (m_c^2 + m_w^2 - 2s_{10}))} \\
\times \frac{1}{(m_c^2 - m_w^2 + 2s_{15})^3} (-16 (m_b + m_c)^2 (6 m_b^4 m_c (m_c^2 m_w^2 - 3 m_w^4 \nonumber \\
- 2 m_b^2 s_{15} + 2 s_{15}^2) + m_b^3 (31 m_b^4 m_w^2 - 2 m_b^2 (25 m_w^4 + 6 (s_{10}) \nonumber \\
- 4 s_{15}) s_{15} + 3 m_b^2 (5 s_{10} + 7 s_{15}) + m_b^2 (-9 m_w^4 - 4 s_{15} (3 s_{10}) \nonumber \\
+ 2 s_{15}) + 2 m_b^2 (9 s_{10} + 7 s_{15})) + 4 m_b^2 m_c (10 m_c^4 m_w^2 \nonumber \\
- 7 m_w^6 - 2 s_{15}^2 (2 s_{10} + s_{15}) + 2 m_w^4 (5 s_{10} + 6 s_{15}) \nonumber \\
+ m_b^2 (6 s_{10}^2 - 4 s_{10} s_{15} - 7 s_{15}^2) - m_c^2 (6 m_w^2 + (s_{10} - 17 s_{15}) s_{15} \nonumber \\
+ m_c^2 (20 s_{10} + 23 s_{15})) + m_c (4 m_c^4 m_w^2 - (m_w^2 - 2 s_{10})(m_w^4 \nonumber \\
- 4 s_{15}^2) (m_w^2 - 2 (s_{10} + s_{15})) + m_c^4 (5 m_w^4 - 4 (s_{10} - 2 s_{15}) s_{15} \nonumber \\
- 14 m_w^2 (s_{10} + s_{15})) - 2 m_c^2 (-6 m_w^2 (s_{10} + s_{15})^2 \nonumber \\
+ 3 m_w^4 (s_{10} + 2 s_{15}) + 4 s_{15} (-s_{10}^2 + 3 s_{10} s_{15} + s_{15}^2))) \nonumber \\
+m_b (21 m_c^6 m_w^2 + m_c^4 (7 m_w^4 + 8 s_{15} (-2 s_{10} + 5 s_{15}) \nonumber \\
- 2 m_w^2 (29 s_{10} + 32 s_{15})) - m_w^2 (m_w^2 - 2 s_{15}) (3 m_w^4 \nonumber \\
- 2 m_w^2 (5 s_{10} + 4 s_{15}) + 4 (2 s_{10}^2 + 3 s_{10} s_{15} + s_{15}^2)) \nonumber \\
+ m_w^2 (-13 m_w^6 + m_w^4 (4 s_{10} - 2 s_{15}) - 8 s_{15}^2 (7 s_{10} + 2 s_{15}) \nonumber \\
+ 4 m_w^2 (10 s_{10}^2 + 13 s_{10} s_{15} + 4 s_{15}^2))) ). \nonumber 
\]

Then, we present the squared matrix elements for $W^+ \rightarrow B_c^+ + c + \bar{c}$. Here we introduce

\[ s_{10} = p_1 \cdot p, \quad s_{15} = p_1 \cdot p_5 \]

with $p_1$, $p$ and $p_5$ being the momenta of $W^+$, $B_c^+$ and $\bar{c}$ quark, respectively.

\[ \Sigma |A_1|^2 \]
\[
\frac{1}{(m_b^3 + m_c^3 m_c - m_c^3 m_w - m_c m_w^2 + m_b (-m_c^2 + m_w^2 - 2 s_{10}))^2} \\
\times \frac{1}{m_b^2 m_c^2 (m_b^2 - 2 m_c^2 + 2 m_w^2 - 2 s_{10} - 2 s_{15})^2} \\
\times (-16(m_b + m_c)^2 (3m_b^4 m_w^2 + 14m_b^7 m_c m_w^2) \\
-2m_b^5 m_c (9m_c^2 m_w^2 - 19m_c^4 + 8s_{10} - 4s_{10}s_{15} - 8s_{15}^2) \\
+14m_w^2 (s_{10} + 2s_{15})) + m_b^6 (14m_c^2 m_w^2 + 9m_w^4) \\
-2m_w^2 (5s_{10} + 6s_{15}) + 4(-s_{10}^2 + s_{10}s_{15} + s_{15}^2)) \\
-2m_b^3 m_c (19m_c^4 m_w^2 - 7m_b^6 + m_w^4 (26s_{10} + 8s_{15}) \\
-4m_w^2 (s_{10}^2 + 12s_{10}s_{15} - 2s_{15}) + 8s_{10} (-s_{10}^2 + s_{10}s_{15} + 2s_{15}^2) \\
-2m_c^2 (9m_w^4 - 2s_{10}^2 + 2m_w^2 (s_{10} - 10s_{15}) + 4s_{10}s_{15} \\
+8s_{15}^2)) - m_b^2 (30m_c^6 m_w^2 - m_c^4 (23m_w^4 - 12s_{10}) \\
+2m_w^2 (s_{10} - 34s_{15}) + 44s_{10}s_{15} + 28s_{15}^2) \\
+(m_w^2 - 2s_{10}) (m_w^2 - 2s_{15}) (m_w^2 + m_w^4 (4s_{10} - 2s_{15}) \\
-4s_{10} (s_{10} + s_{15})) - 4m_c^2 (3m_w^6 - 12m_w^4 s_{10} \\
-4s_{10}s_{15} (s_{10} + s_{15}) + m_w^2 (3s_{10}^2 + 20s_{10}s_{15} - 6s_{15}^2))) \\
+m_c^2 (-7m_c^6 m_w^2 - m_w (m^2_w - 2s_{15}) (m^4_w + 8s_{10}s_{15}) \\
-2m_w^2 (s_{10} + s_{15}) + m_c^2 (-m_w^6 + 8s_{10}^3 - 4m_w^4 (s_{10} - 2s_{15}) \\
+4m_w^2 (s_{10}^2 + 2s_{10}s_{15} - 2s_{15}^2)) + m_c^4 (9m_w^4 \\
-2m_w^2 (3s_{10} + 10s_{15}) + 4 (3s_{10}^2 + 3s_{10}s_{15} + s_{15}^2))) \\
-2m_b^6 m_c (11m_c^6 m_w + m_w^2 (m_w^2 - 2s_{10}) (m_w^2 - 2s_{15}) (m_w^2 \\
+2s_{10} - 2s_{15}) - m_c^2 (m_w^6 - 8s_{10}s_{15} + m_w^4 (-10s_{10} + 8s_{15}) \\
+4m_w^2 (2s_{10} + 4s_{10}s_{15} - 3s_{15}^2)) - m_c^4 (11m_w^4 \\
-2m_w^2 (3s_{10} + 14s_{15}) + 4 (s_{10}^2 + 5s_{10}s_{15} + 2s_{15}^2))) \\
+m_b^6 (-44m_c^4 m_w^2 + 5m_b^6 - 4m_b^4 (5s_{10} + 2s_{15}) \\
+8m_w^2 s_{10} (s_{10} + 5s_{15}) + 8s_{10} (s_{10}^2 - 2s_{10}s_{15} - 2s_{15}^2) \\
+m_c^2 (55m_c^4 - 2m_w^2 (9s_{10} + 46s_{15}) \\
+4 (-3s_{10}^2 + s_{10}s_{15} + 7s_{15}^2)))),
\]
\[
\Sigma |A_2|^2 \\
= \frac{1}{m_b^2 m_w^2 (m_b^2 - m_c^2 + m_w^2 - 2s_{10} - 2s_{15})^4} \\
\times (16(m_b + m_c)^2(4m_b^6 m_w^2 + 12m_b^5 m_c m_w^2 - 6m_c^2 m_w^2 \\
+ (m_w^2 - 2(s_{10} + s_{15})^2(m_w^4 - 2m_w^2(s_{10} - 2s_{15}) \\
- 4s_{15}(s_{10} + s_{15}) - m_c(-13m_w^4 + 8m_w^2(s_{10} + s_{15}) \\
+ 12(s_{10} + s_{15})^2) + m_b^4(2m_c^2 m_w^2 + 7m_w^4 \\
+ 4m_w^2(-5s_{10} + s_{15}) + 4(s_{10}^2 - 2s_{10} s_{15} - 3s_{15}^2)) \\
+ 2m_c^3 m_c(m_w^4 - 2m_w^2(13s_{10} + 7s_{15}) + 8(2s_{10} + 3s_{10} s_{15} \\
+ s_{15})) - 2m_c^2(4m_b^6 + 4(s_{10} + s_{15})^2(3s_{10} + 4s_{15}) \\
- m_w^4(7s_{10} + 4s_{15}) - 8m_w^2(s_{10}^2 + 3s_{10} s_{15} + 2s_{15}^2)) \\
+ 2m_b m_c(2m_c^4 m_w^2 - m_w^6 - 16s_{10}(s_{10} + s_{15})^2 \\
+ m_w^4(-8s_{10} + 4s_{15}) + 4m_w^2(7s_{10}^2 + 6s_{10} s_{15} - s_{15}^2) \\
+ m_c^2(-m_w^4 + 2m_w^2(s_{10} - 5s_{15}) + 8s_{15}(s_{10} + s_{15}))) \\
+ 2m_b^2(8m_c^4 m_w^2 + 2m_w^6 - 4(s_{10} - 4s_{15})(s_{10} + s_{15})^2 \\
+ m_w^4(-11s_{10} + 4s_{15}) + 8m_w^2(2s_{10}^2 - s_{10} s_{15} - 3s_{15}^2) \\
- 2m_b^2(5m_c^4 + m_w^4(5s_{10} + 11s_{15}) \\
- 2(5s_{10}^2 + 12s_{10} s_{15} + 7s_{15}^2))))),
\]

\[
\Sigma \text{Re}(A_1 A_2^*) \\
= \frac{1}{m_b^3 + m_b^2 m_c - m_c^2 + m_c m_w^2 + m_b(-m_c^2 + m_w^2 - 2s_{10}) \\
\times \frac{1}{m_c^2 m_w^2 (m_b^2 - m_c^2 + m_w^2 - 2s_{10} - 2s_{15})^3} \\
\times (16(m_b + m_c)^2(5m_b^6 m_c m_w^2 + m_b^5(10m_c^2 m_w^2 + 5m_w^4 \\
- 4s_{10}^2 - 14m_w^2 s_{15} + 4s_{10} s_{15} + 8s_{15}^2) + m_b^4 m_c(-17m_w^2 m_w^2 \\
+ 17m_w^4 + 6m_w^2(s_{10} - 6s_{15}) - 8(3s_{10}^2 + s_{10} s_{15} - 2s_{15}^2)) \\
+ m_b^2 m_c(-29m_c^4 m_w^2 + 5m_w^6 - 2m_w^4(3s_{10} + 11s_{15}) \\
- 4m_w^2(7s_{10}^2 - 9s_{10} s_{15} - 8s_{15}^2) + 8(3s_{10} + 2s_{10} s_{15})}}
\]

\[-3s_{10}s_{15}^2 - 2s_{15}^3\] \[+ m_c^2(9m_w^4 - 2(s_{10} + 2s_{15})^2\]
\[+ m_w^2(3s_{10} + 4s_{15})) - 2m_b^2(26m_c^4m_w^2 - 4m_w^6\]
\[-14m_w^2s_{15}(2s_{10} + s_{15}) + m_w^4(9s_{10} + 14s_{15})\]
\[+ 4(-s_{10}^3 + 2s_{10}s_{15} + 4s_{10}s_{15}^2 + s_{15}^3) + m_c^2(-15m_w^4\]
\[+ 2s_{10}(5s_{10} + 6s_{15}) + m_w^2(s_{10} + 18s_{15}))\]
\[+ m_c(9m_b^6m_w^2 + m_w^2(m_w^2 - 2s_{15})(m_w^4 - 4m_w^2s_{10} + 4s_{10}^2 - 4s_{15}^2)\]
\[+ m_c^4(-11m_w^4 + 28m_w^2(s_{10} + s_{15}) - 4s_{10}(s_{10} + 2s_{15}))\]
\[+ m_w^2(m_w^6 - 8s_{10}s_{15}(s_{10} + s_{15}) - 2m_w^4(6s_{10} + 7s_{15})\]
\[+ 4m_w(3s_{10}^2 + 8s_{10}s_{15} + 6s_{15})) + m_b(10m_b^6m_w^2\]
\[+ (m_w^2 - 2s_{10})(m_w^2 - 2s_{15})(3m_w^4 - 8m_w^2(s_{10} + s_{15})\]
\[+ 4(s_{10} + s_{15})^2) - m_c^4(11m_w^4 + 8s_{10}^2 + 12s_{10}s_{15} + 8s_{15}^2\]
\[+ 2m_w^2(21s_{10} + 25s_{15})) - 2m_c^2(m_w^6 + 4m_w^4s_{15}\]
\[+ 2m_w^2(3s_{10}^2 - 3s_{10}s_{15} - 7s_{15}^2) - 4(s_{10} + 2s_{10}s_{15} - s_{15}^3)))).\]

For $W^+ \rightarrow B_c^{*+} + c + \bar{c}$, we introduce parameters:

$$s_{10} = p_1 \cdot p, \quad s_{15} = p_1 \cdot p_5$$

with $p_1$, $p$ and $p_5$ being the momenta of $W^+$, $B_c^{*+}$ and $\bar{c}$ quark, respectively.

$$\Sigma|A_1|^2 = \frac{1}{(m_b^3 + m_b^2m_c - m_c^3 + m_c^2m_w + m_b(-m_c^2 + m_w^2 - 2s_{10}))^2} \times \frac{1}{m_b^2m_w^2(m_b^2 - m_c^2 + m_w^2 - 2s_{10} - 2s_{15})^2} \times (16(m_b + m_c)^2(9m_b^8m_w^2 + 46m_b^7m_c^2m_w^2 + m_b(74m_c^2m_w^2 \]
\[+ 7m_b^4 - 2m_b^2(27s_{10} + 2s_{15}) + 4(3s_{10}^2 + 3s_{10}s_{15} + s_{15}^2))\[+ 2m_b^5m_c(27m_c^2m_w^2 + 5m_w^4 - 2m_w^2(55s_{10} + 2s_{15})\[+ 4(6s_{10}^2 + 7s_{10}s_{15} + 2s_{15}^2)) + m_b^4(68m_c^4m_w^2 - m_w^6\[+ 8m_b^2(12s_{10}^2 + s_{10}s_{15} + s_{15}^2) + m_c^2(-39m_w^4 + 6s_{10}^2\]
\[+ 92s_{10}s_{15} + 28s_{15}^2 + m_w^2(-254s_{10} + 44s_{15}))\]

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\[-2m_bm_c(7m_c^6m_w^2 - m_c^2(m_w^2 - 2s_{10})(m_w^4 + 2m_w^2(s_{10} - 2s_{15}))) + 4(4s_{10}^2 - s_{10}s_{15} + s_{15}^2)) + m_c^4(-21m_w^4w - 10m_c^2(7s_{10} + 2s_{15}) - 4(s_{10}^2 - 5s_{10}s_{15} + 2s_{15}^2)) + m_c^2(15m_w^6 - 8s_{10}(s_{10} - 2s_{15})s_{15} - 2m_w^4(35s_{10} + 12s_{15}))) + 4m_w^2(14s_{10}^2 + 24s_{10}s_{15} + 3s_{15}^2)) + 2m_c^3m_c(53m_c^4m_w^2 - 5m_c^2 + 24s_{10}(s_{10} + s_{15}) - 2m_w^4(13s_{10} + 4s_{15})) + 12m_w^2(11s_{10}^2 + s_{15}^2) + m_c^2(-42m_w^4 - 28m_w^2(s_{10} - 2s_{15}))) + 4(7s_{10}^2 + 6s_{10}s_{15} + 4s_{15}^2)) + m_c^4(54m_w^6m_w^2 + (m_w^2 - 2s_{10})(m_c^2 + 4m_w^4(s_{10} - s_{15}) + 8s_{10}s_{15}(s_{10} + s_{15})) + 4m_w^2(3s_{10}^2 - 3s_{10}s_{15} + s_{15}^2)) - 4m_c^2(7m_w^6 - 4m_w^2(s_{10} + 2s_{15}))) - 4m_w^2(3s_{10} + s_{15}) + m_c^2(-35s_{10}^2 + 28s_{10}s_{15} - 2s_{15}^2) + 4s_{10}(s_{10}^2 + s_{10}s_{15} + s_{15}^2)) + m_c^4(-23m_w^2 + m_w^2(-34s_{10} + 52s_{15}) + 4(13s_{10}^2 - 7s_{10}s_{15} + 7s_{15}^2)) + m_c^4(-23m_w^4 + m_w^2(74s_{10} + 28s_{15})) + 4(3s_{10}^2 + 3s_{10}s_{15} - s_{15}^2)) - m_w^2(m^6w^2 - 2m_w^4(s_{10} + 2s_{15})) - 16s_{10}(2s_{10}^2 + s_{15}^2) + 4m_w^2(4s_{10}^2 + 3s_{10}s_{15} + s_{15}^2)) + m_c^2(-11m_w^6 - 4m_w^4(17s_{10} + 6s_{15})) + 4m_w^2(21s_{10}^2 + 22s_{10}s_{15} + 4s_{15}^2) + 8(s_{10}^2 + 2s_{10}s_{15}^2))))).

\[\Sigma |A_2|^2 = \frac{1}{m_c^2m_w^2(m_c^2 - m_c^2 + m_w^2 - 2s_{10} - 2s_{15})^4 \times (16(m_c^4 + m_c^2)^2 + 8m_b^5m_e^2m_t^2 + (m_c^4(m_w^4 - 4s_{10}^2) + 4m_w^2(s_{10} - 2s_{15}) + 4s_{15}^2) + (m_w^2 - 2(s_{10} + s_{15}))^2(m_w^4 - 2m_w^2(s_{10} - 2s_{15} - 4s_{15}(s_{10} + s_{15})) - 2m_c^2(m_w^6 + 4s_{10}(s_{10} + s_{15})^2 - m_w^4(s_{10} + 4s_{15}) - 4m_w^2(s_{10} - s_{15}^2)) + m_c^2(12m_w^2m_c^2 - m_w^4 - 4m_w^2(s_{10} + s_{15}) + 4(3s_{10} + 4s_{10}s_{15} + s_{15}^2)) + 2m_c^3m_c(12m_w^2m_c^2 - 9m_w^4 - 6m_w^2(5s_{10} + 3s_{15})) + 4m_w^2(21s_{10}^2 + 22s_{10}s_{15} + 4s_{15}^2) + 8(s_{10}^2 + 2s_{10}s_{15}^2))))).\]
\[
+8(4s_{10}^2 + 7s_{10}s_{15} + 3s_{15}^2)) + 2m_b^2(17m_c^4m_w^2 - m_w^6
-12s_{10}(s_{10} + s_{15})^2 + m_w^4(-5s_{10} + 4s_{15}) + 4m_w^2(5s_{10}^2
+4s_{10}s_{15} - s_{15}^2) + m_c^2(-16m_w^4 - 34m_w^2(s_{10} + s_{15})
+44(s_{10} + s_{15})^2)) + 2m_b m_c(8m_c^4m_w^2 - m_w^6
-16s_{10}(s_{10} + s_{15})^2 + m_w^4(-8s_{10} + 4s_{15})
+4m_c^2(7s_{10}^2 + 6s_{10}s_{15} - s_{15}^2) + m_c^2(-7m_w^4
-2m_w^2(5s_{10} + 11s_{15}) + 8(2s_{10}^2 + 5s_{10}s_{15} + 3s_{15}^2))))),

\[\Sigma \text{Re}(A_1 A_2^*) \]

\[
= \frac{1}{m_b^3 + m_b^2m_c - m_c^3 + m_c m_w^2 + m_b(-m_c^2 + m_w^2 - 2s_{10})}
\times \frac{1}{m_c^2 m_w^2(m_b^2 - m_c^2 + m_w^2 - 2s_{10} - 2s_{15})^3}
\times (-16(m_b + m_c)^2(4m_b^2m_w^2 + 21m_b m_c m_w^2 + m_b^5(38m_c^2 m_w^2
-m_w^4 - 2m_w^2(10s_{10} + s_{15}) + 4(3s_{10}^2 + 5s_{10}s_{15} + 2s_{15}^2))}
+m_c^5(39m_c^2 m_w^2 - 17m_w^4 - 2m_w^2(45s_{10} + 8s_{15})
+8(7s_{10}^2 + 12s_{10}s_{15} + 5s_{15}^2)) + m_b m_c(35m_c^4m_w^2 - 15m_w^6
+18m_w^2(s_{10} + s_{15}) - 8(5s_{10} - 2s_{15})(s_{10} + s_{15})^2
+4m_w^2(17s_{10}^2 - s_{10}s_{15} - 8s_{15}^2) + m_w^2(-26m_w^4 + 52s_{10}^2
+64s_{10}s_{15} + 32s_{15}^2 - 6m_w^2(15s_{10} + 8s_{15})))) + 2m_b^3(20m_c^4m_w^2
-4m_w^6 + 3m_w^4(s_{10} + 2s_{15}) + 2m_w^2(8s_{10}^2 - 3s_{15}^2)
+4(-3s_{10}^3 - 4s_{10}s_{15}^2 + s_{15}^3) + m_c^2(-19m_w^4 + 38s_{10}^2
+68s_{10}s_{15} + 32s_{15}^2 - m_w^2(69s_{10} + 22s_{15})))) + m_c(m_c^6m_w^2
+m_c^4(3m_w^4 - 4m_w^2s_{10} + 4s_{10}^2 - 8s_{15}) - m_w^2(m_w^2 - 2s_{15})(m_w^4
-4m_w^2s_{10} + 4s_{10}^2 - 4s_{15}^2) + m_c^2(-3m_w^6 + 8s_{10}s_{15}(s_{10} + s_{15})
+2m_w^4(6s_{10} + 5s_{15}) - 4m_w^2(s_{10}^2 + 6s_{10}s_{15} + 6s_{15}^2))}
+m_b(14m_c^6m_w^2 - (m_w^2 - 2s_{10})(m_w^2 - 2s_{15})(3m_w^4
-8m_w^2(s_{10} + s_{15}) + 4(s_{10} + s_{15})^2) - m_c^4(m_w^4
+2m_w^2(13s_{10} + 9s_{15}) - 4(6s_{10}^2 + s_{10}s_{15} - 2s_{15}^2)) - 2m_c^2(5m_w^6

27
\[-4m_w^4(3s_{10} + 2s_{15}) + m_w^2(-14s_{10}^2 + 14s_{10}s_{15} + 22s_{15}^2) \\
+ 4(s_{10}^3 + 2s_{10}s_{15} - s_{15}^3))\]

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