CLUSTERS AS TRACERS OF LARGE-SCALE STRUCTURE

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ABSTRACT. By virtue of their high galaxy space densities and their large spatial separations, clusters are efficient and accurate tracers of the large-scale density and velocity fields. Substantial progress has been made over the past decade in the construction of homogeneous, objectively derived cluster catalogs and in characterizing the spatial distribution of clusters. Consequently, the constraints on viable models for the growth of structure have been refined. A review of the status of cluster-based observations of large-scale structure is presented here, including discussions of the second and higher order moments, the dependence of clustering on richness (mass), recent and new measurements of bulk flows, and a new constraint on the cluster mass function in the range $0.7 < z < 1$.

1 Introduction

The study of the evolution of large-scale structure is fundamental to cosmology. When observations of the galaxy and cluster spatial distributions are combined with complementary spectroscopic and morphological information, one can probe the abundance and form of dark matter, the mean baryon and matter densities, the turnover scale in the perturbation power spectrum, and the formation epoch of galaxies and clusters. Clusters of galaxies are particularly well suited to these studies and, over the past decade, there have been substantial breakthroughs in and challenging questions raised about our understanding of large-scale structure from redshift and peculiar velocity surveys of clusters. In this review, I summarize the current constraints on the two-point cluster–cluster correlation function (and its dependence on cluster richness), the cluster power spectrum, the very large-scale distribution of clusters, bulk flow measurements, and the evolution of the cluster mass function. I also emphasize the importance of characterizing how one’s definition of a cluster ultimately affects the interpretation of observational and N-body simulation results. I conclude by highlighting several exciting cluster research programs now (or soon to be) underway which will provide new levels of accuracy in defining the relationship between the large-scale distribution and properties of clusters and the underlying astrophysics responsible for their formation and evolution.

2 The Case for Clusters As Tracers of LSS

There are several powerful arguments in support of using clusters to trace large-scale structure in the universe. Clusters lie $10 \sim 30$ times farther apart than galaxies, with intercluster separations in the range $30 \sim 100\, h^{-1}$ Mpc. This fact immediately makes them promising candidates for tracing large-scale structure because a relatively small sample can be used to probe great distances. Clusters are probably “close” to their initial formation positions $(\pm 10\, \text{(spec}/1000 \,\text{km}/\text{s})(t_0/10 \,\text{Gyr})\, h^{-1}\, \text{Mpc})$ because the typical peculiar velocity of an individual cluster is $500$ km/s or less (Watkins 1997). Furthermore, clustering of clusters is significantly amplified with respect to that for galaxies. This is, in part, due to the fact that clusters are located at highest peaks in matter distribution $(\rho_{CL}/\rho_c \gtrsim 100)$. This enables one to study features of the distribution that might otherwise be too weak to measure from galaxy redshift surveys (e.g., correlations on scales in excess of $30h^{-1}$ Mpc) and features which are dependent on the higher moments of the galaxy distribution. High order moments, for
example, are highly sensitive to the details of most biasing prescriptions. In addition, the dark matter on typical intercluster distance scales is likely still in the quasi-linear (or only mildly non-linear) regime and, thus, it is easier to associate present epoch constraints on the shape of the perturbation spectrum with its primordial counterpart. N-body simulations show that clusters are unbiased tracers of the underlying velocity field (Strauss et al. (1995); Gramann et al. (1995)) and provides further theoretical justification for the extensive observational efforts to measure large–scale flows (see Section 6). The observations have been motivated by the fact that relative redshift-independent distances to clusters can be measured to accuracies \(\sim 5\%\) owing to the large number of galaxies in each system. Clusters are also the exclusive sites for application of a number of secondary distance indicators (e.g, BCGs, SZ Effect) and large elliptical populations (allowing accurate fundamental plane measurements). Lastly, clusters can be easily detected out to \(z \sim 1\) (and beyond, if NIR data are available) owing to their high density contrast, red elliptical population, and bright member galaxies. This enables important constraints to be made on the evolution of large–scale structure and \(\Omega_c\).

3 Identifying Clusters

There are three basic ways to “define” what one means by a cluster of galaxies. They are:

**The Physical Definition** (at \(z \sim 0\)): Gravitationally bound, virialized system of dark matter, gas, and galaxies with a mass of at least \(\sim 10^{14} M_\odot\) inside a region \(\sim 1/h \text{Mpc}\) in radius.

**The N-body Definition**: Peaks in a dark-matter dominated density field, perhaps satisfying additional constraints such as a minimum velocity dispersion, \(\sigma\), and/or minimum value of \(\sigma \rho\). The peaks can be located via Friends-of-Friends (FoF) or Local Overdensity (LO; nearest \(n\) particles) algorithms. The locations in real space are known precisely but the precise relationship between the dark and luminous matter distributions is poorly understood at present.

**The Observational Definition(s)**: These can vary substantially depending on number of available positional coordinates per galaxy (2D/3D) and on the wavelength of the survey (opt, IR, x-ray). Modern optical/IR searches identify density enhancements using FoF, LO, or matched filter algorithms in either 2 or 3 dimensions. X-ray searches look for sources with extended emission and/or cross-correlate all x-ray sources with optically-selected galaxy catalogs. Complications include projection effects (2D), redshift distortions (3D), sensitivity variations across the survey, and for the Abell (1958), Abell, Corwin, Olowin (ACO; 1989), and Zwicky et al. (1968) catalogs, some degree of human error.

Because the clustering properties of clusters will depend on how they were selected (e.g, Eke et al. 1996), any comparisons between observational datasets or between observations and simulations must quantify and correct for biases introduced by the selection process. This task is tractable for samples derived using well-understood and quantifiable selection criteria.

3.1 Cluster Catalogs

There have been at least 14 original cluster catalogs constructed to date and Table 1 summarizes their basic parameters. The impact of high speed, memory-rich computers and digital data is readily apparent: subsequent to 1992, cluster detection relies exclusively on the objective application of algorithms with accurately quantifiable selection functions (“Bulleted” catalogs). None the less, results from the visually derived Abell and ACO catalogs are still widely cited because they have been the only all sky cluster surveys available to date. The Zwicky cluster catalog, by contrast, is only infrequently used largely because their cluster finding procedure, visual identification of global isodensity contours twice as high as the mean contour, is fraught with many pitfalls including enhanced sensitivity to plate-to-plate photometric zeropoint variations. The cluster identification pre-
scription developed by Abell is a bit more robust and, indeed, the automated APM catalog is based upon a modified version of this approach.

Postman et al. (1986) demonstrated that the angular correlation functions of Abell and Zwicky clusters agree when appropriate subsamples of each catalog are chosen (corresponding to the spatial regime where the different algorithms identify similar types of clusters). Without such careful comparison, the \( \omega_{cc}(\theta) \) from Abell and Zwicky clusters differ significantly. A similar level of discrepancy has existed between the spatial correlation functions for the Abell/ACO clusters and that for the APM clusters (e.g. Dalton et al. 1992; Postman et al. 1992).

These differences are, in large part, due to differences in the respective selection criteria which result in different minimum richness limits in the catalogs (Bahcall & West 1992). To a lesser degree, differences in the large-scale structure statistics for the Abell/ACO and APM catalogs are due to projection effects which appear to be strongest in the original Abell catalog (Sutherland 1988; Sohn 1988; Efstathiou et al. 1992). A cross-correlation of the ACO and APM catalogs in a sector of sky covered by both reveals that for the APM clusters (e.g. Dalal et al. 1992), the angular correlation function (with the precise shape of the cluster–cluster spatial correlation function (with the precise shape of the two-point spatial correlation function, \( P(k) \), have been widely used to constrain the clustering properties of clusters. Although competing structure formation models can occasionally yield quite similar predictions for \( P(k) \) and \( \xi(r) \), they are none the less robust measures of clustering strength which provide basic information about the underlying matter distribution. They are also computationally straightforward to compute and substantial work has been dedicated to identifying the optimal estimators for these functions (Landy & Szalay 1993; Hamilton 1993; Peacock & Dodds 1994; Landy et al. 1996, Tegmark et al. 1998).

4 Second Order Cluster Correlations

Second order statistics such as the two-point spatial correlation function, \( \xi(r) \), and its Fourier transform, \( P(k) \), have been widely used to constrain the clustering properties of clusters. Although competing structure formation models can occasionally yield quite similar predictions for \( \xi(r) \) and \( P(k) \), they are none the less robust measures of clustering strength which provide basic information about the underlying matter distribution. They are also computationally straightforward to compute and substantial work has been dedicated to identifying the optimal estimators for these functions (Landy & Szalay 1993; Hamilton 1993; Peacock & Dodds 1994; Landy et al. 1996, Tegmark et al. 1998).

4.1 The Cluster Auto-Correlation Function

The cluster–cluster spatial correlation function is often fit to a power-law of the form

\[
\xi_{cc}(r) = \left( \frac{r}{r_o} \right)^{-\gamma}
\]

This appears to be a reasonable model over \( 5 \leq r \leq 35 \, h^{-1} \) Mpc and results from several independent cluster catalogs yield \( 1.8 \leq \gamma \leq 2.2 \). Determinations of \( \xi_{cc}(r) \) from the Abell and APM catalogs are shown in Figure 4. Although the amplitude of \( \xi_{cc}(r) \) differs by factors between 6 to 30 from that for the galaxy auto-correlation function (with the precise ratio dependent on the sample compositions), the shape of the cluster and galaxy correlation functions are very similar. This similarity in shape is also seen in the respective power spectra, at least on scales less than 70 \( h^{-1} \) Mpc where the signal-to-noise ratio is high.

The most stringent constraints that \( \xi_{cc}(r) \) can place on structure formation models come from addressing the following questions:

Do \( r_o \) and \( \gamma \) change with the mass-scale of the systems being studied?

Are there deviations from a
Figure 1. The spatial distribution of a nearly volume limited sample of \( \sim 480 \) Abell and ACO clusters \((RC \geq 0)\). The axes are aligned with the Galactic coordinate system. The zone of avoidance at \(|b| = 13^\circ\) is shown.

Figure 2. The spatial correlation function for APM and Abell clusters, respectively. The best-fit power laws are shown.
power law (e.g., $\xi_{cc}(r) \leq 0$) and on what scales?

How do the amplitude and shape of $\xi_{cc}(r)$ vary with cosmic time?

Expectations are that, to some degree, $r_o$ must depend on the mass of the cluster; if structure grows by gravitational amplification of fluctuations in a Gaussian field, then collapsed objects form near peaks in this field and their clustering will depend on the height of the peak (Kaiser 1984; Barnes et al. 1985). Early data on $\xi_{gg}(r)$ and $\xi_{cc}(r)$ led Szalay & Schramm (1985) to propose a “scale-invariant” form given by

$$\xi_o(r) = \frac{1}{3}(r/d_i)^{-1.8}$$

and thus $r_o = 0.54d_i$ where $d_i$ is the mean interobject separation. Szapudi, Szalay, & Boschán (1992) demonstrated that amplification is a consequence of enhanced weighting of dense regions when deriving the higher moments of the density field (e.g., $\xi_{cc}(r)$). It is important to emphasize that the existence of a scale dependence to the correlation length does not imply that galaxies and clusters cannot both be tracers of the distribution of large-scale structure. Rather, it suggests that these two classes of systems trace the underlying matter differently.

Bahcall & West (1992; BW92) proposed that data for a wide range of catalogs of clusters and galaxies satisfy a scale-invariant relationship between $r_o$ and $d_i$ of the form $r_o = 0.4d_i$. Croft & Efstathiou (1994), however, could not reproduce a relationship between $r_o$ and $d_e$ (intercluster separation) as strong as the BW92 result using SCDM N-body simulations. They found a trend which showed little dependence of $r_o$ on $d_e$ when $d_e \gtrsim 30h^{-1}$ Mpc. A subsequent analysis by Croft et al. (1997), using an extended subset of the richest APM clusters, suggests that these systems show only a weak dependence on $r_o$ on $d_e$ that is consistent with low density CDM models.

In contrast, Walter & Klypin (1996) were able to reproduce a relationship between $r_o$ and $d_e$ which is as strong as the BW92 result from CHDM N-body simulations. However, those same simulations predict a dramatic decrease in the comoving space density of clusters as one looks back to $z = 0.5$, a prediction which is clearly not consistent with present observations (Postman et al. 1996; Carlberg et al. 1997). See also the results based on the Virgo Consortium simulations (Colberg et al., these proceedings).

Figure 8 summarizes the current situation. Part of the scatter in the figure is due to inconsistent comparisons between authors. The BW92 dependence of $r_o = 0.4d_i$ was based on power law fits to $\xi_{cc}(r)$ with $\gamma$ constrained to be 1.8. For the APM

### Table 1. Catalogs of Clusters of Galaxies

| Catalog              | Detection Passband | Approximate Redshift Range | Number of Clusters | Number with Spec. Redshifts |
|----------------------|---------------------|---------------------------|-------------------|-----------------------------|
| Abell (1958)         | Opt.                | $z \lesssim 0.3$          | ~2700             | ~1300                       |
| Zwicky et al. (1968) | Opt.                | $z \lesssim 0.3$          | ~9000             | ...                         |
| Shectman (1985)      | Opt.                | $z \lesssim 0.3$          | ~650              | ...                         |
| Gunn, Hoessel, Oke (1986) | Opt./NIR       | $z \lesssim 1$           | ~400              | ~50+                        |
| Abell, Corwin, Olowin (1989) | Opt.          | $z \lesssim 0.3$          | ~1350             | ~250+                       |
| Couch et al. (1991)  | Opt.                | $z \lesssim 0.6$          | ~100              | ~20                         |
| Henry et al. (1992)  | X-Ray               | $z \lesssim 0.6$          | ~95               | ~70+                        |
| Lumsden et al. (1992) | EDCC                | Opt.                      | ~700              | ~100+                       |
| Dalton et al. (1994a) | APM                 | Opt.                      | ~1000             | ~300+                       |
| Postman et al. (1996) | PDCS                | Opt./NIR                 | ~80               | ~20+                        |
| Scodeggio, Olsen, et al. (1998) | EIS            | $z \lesssim 1$           | ~250              | ...                         |
| Rosati et al. (1998) | X-Ray               | $z \lesssim 0.8$          | ~70               | ~60+                        |
| Vikhilin et al. (1998) | X-Ray              | $z \lesssim 0.6$          | ~200              | ...                         |
| Boehringer et al. (1998) | REFLEX            | X-Ray                     | ~450              | ~380+                       |

* = Automated Catalog
Figure 3. The spatial correlation length as a function of the intercluster separation for different cluster samples. At large $d_c$, we show the APM results when the slope, $\gamma$, is constrained to be 1.8. The relationships $r_o = \sqrt{d_c}$ and $r_o = 2.5\sqrt{d_c}$ are shown for comparison. Based on results from Bahcall & West (1992), Croft et al. (1997), Abadi, Lambas, & Muriel (1998), and this review.

results, Croft et al. (1997) allow $\gamma$ to be a free parameter and often obtain fits with $\gamma \approx 2.2$ for $d_c > 60h^{-1}$ Mpc. Since there exists a significant covariance between $r_o$ and $\gamma$, such comparisons must really be done at a common slope value. Indeed, when one fixes $\gamma$ at 1.8, the APM $r_o$ at $d_c > 60h^{-1}$ Mpc do increase as shown. A more substantial cause for scatter in Figure 3 is demonstrated by Eke et al. (1996) who find that $r_o$ can vary by up to 50% depending on the precise cluster identification procedure. They could reproduce either a strong or weak dependence of $r_o$ on $d_c$ depending on which cluster identification method was used and are able to reconcile the BW92 and the Croft et al. (1997) findings as consequences of the different selection procedures used by ACO and by the APM team.

In sum, a weak dependence of correlation amplitude on intercluster separation (and richness) is fairly well-established ($r_o \propto \sqrt{d_c}$ or $r_o \approx 0.2d_c$) both theoretically and observationally. The observational evidence for a stronger dependence (e.g., $r_o = 0.4d_c$) is from the angular clustering properties of Abell RC $\geq 2$ clusters, the clustering of x-ray bright Abell clusters (Abadi, Lambas, & Muriel 1998) and the supercluster correlation function (Bahcall & Burgett 1986) — all of which are derived from the Abell and ACO catalogs. The APM cluster results for $d_c > 60h^{-1}$ Mpc are based on < 60 clusters and, hence, are subject to possible systematic effects (as is any small catalog). The results for $d_c > 60$ thus require confirmation from larger redshift surveys (e.g., the extended APM, 2dF, and Sloan Digital Sky Survey [SDSS]). In any event, careful attention needs to be paid by both observers and theorists to the not so subtle effects of the cluster selection process on large-scale structure statistics before any physical in-
ferences are made based on the observed relationship between $r_o$ and $d_c$.

4.2 The Zero Crossing of $\xi(r)$

Deviations in $\xi(r)$ from a single power law behavior are, in principle, sensitive tests for the shape of the primordial fluctuation power spectrum. One such deviation is the scale at which the correlation amplitude goes to zero (Klypin & Rhee 1994; KR94). The amplitude of $\xi_{cc}(r)$ appears to be positive at least out to $40h^{-1}$ Mpc and possibly out to scales of $60h^{-1}$ Mpc (Postman et al. 1992; Olivier et al. 1993; KR94; Dalton et al. 1994b; Boehringer et al. 1998). However, on scales from $60 - 100 h^{-1}$ Mpc, it is very unlikely ($2 - 3\sigma$ level) that $\xi_{cc}(r) > 0$. This result is seen in the Abell/ACO, APM, and REFLEX (x-ray selected) cluster catalogs. Systematic effects, such as the integral constraint for a finite sample (which forces $\xi(r)$ to eventually become negative) or small errors in the mean cluster number density, appear not to be large enough to fully explain this zero crossing. One important implication of this result may be that $\xi_{gg}(r)$ is also positive out to at least $40h^{-1}$ Mpc (Simpson et al. 1992). Indeed, preliminary results from the 2dF redshift survey (Maddox, these proceedings) and the ESO Slice Project (Guzzo, these proceedings) both find positive $\xi_{gg}(r)$ out to at least $35h^{-1}$ Mpc. Such observations put severe constraints on CDM models. As noted by KR94, ACDM models (e.g., Kofman, Gnedin, & Bahcall 1993) predict a zero crossing at $r_z = 16.5(\Omega h^2)^{-1}$ Mpc. If the above observations hold up, then this suggests that $\Omega h$ lies in the range $0.28 - 0.41$.

4.3 The Cluster Power Spectrum

The power spectrum of clusters, $P(k)$, provides a complementary constraint on their clustering properties: broad features in the correlation function are narrow in Fourier space and vice versa. Furthermore, errors in $P(k)$ are easier to estimate correctly and the results are somewhat less sensitive to uncertainties in the mean space density of clusters than those for $\xi(r)$. Current constraints on the cluster power spectrum are shown in Figure 4. The shape of cluster power spectrum, like its inverse Fourier transform $\xi(r)$, is consistent with the shape of the power spectrum of optical, IRAS, and radio galaxies at $k > 0.04h$ Mpc$^{-1}$ (Peacock & Dodds 1994; Einasto et al. 1997; Retzlaff et al. 1997; Tadros et al. 1998). This suggests, again, that clusters and galaxies are tracing similar perturbations in the matter distribution. The turnover in $P(k)$ is detected (but not with high significance) for $k < 0.03h$ Mpc$^{-1}$. The SDSS, 2dF, and other large redshift surveys should eventually yield dramatically improved constraints on the turnover, a feature which depends upon the horizon scale at the epoch of matter-radiation equilibrium.

The amplitude of $P(k)$ for Abell/ACO clusters is, on average, a factor of $2 - 3$ higher than that for APM clusters, consistent with differences seen in their respective $\xi(r)$. In turn, the APM cluster $P(k)$ amplitude is about $6 - 8 \times$ higher than that derived for galaxies in the Las Campanas Redshift Survey (Lin et al. 1996; LCRS). The observed shape of $P(k)$ is reasonably well represented by MDM models ($0.2 < \Omega \nu < 0.3$, low-density CDM models ($\Omega h \sim 0.3 \pm 0.1$), and/or $\Lambda$CDM models ($\Lambda \sim 0.3$) (Borgani et al. 1996). The apparently strong feature seen at $120 \pm 15h^{-1}$ Mpc in the Abell/ACO $P(k)$ (Einasto et al. 1997) is not seen in the $P(k)$ derived from either APM clusters (Tadros et al. 1998) or REFLEX x-ray selected clusters (Boehringer et al. 1998). A subsequent analysis of the Abell/ACO $P(k)$ by Retzlaff et al. (1998) find that the feature is not statistically significant when sample variance is properly accounted for. None the less, a statistically significant feature is detected in the 2D LCRS $P(k)$ at around $100h^{-1}$ Mpc (Landy et al. 1996) and spikes continue to be found in the galaxy redshift distribution on similar scales in new pencil beam surveys (Broadhurst et al. 1995). These features, which are not reproduced by most non-baryonic matter dominated models, are presumably due to characteristic scales of voids and
Figure 4. Power spectra for APM galaxies (Maddox et al. (1996)), APM clusters (Tadros et al. (1998)), and Abell/ACO clusters (large squares are Retzlaff et al. (1997); small squares are Einasto et al. (1997)). The galaxy power spectrum has been normalized to match the amplitude of the APM cluster power spectrum. \( P(k) \) for two models also shown.

Sheets in the galaxy distribution. The lack of a significant detection of this feature in cluster power spectra may be a consequence their sparser sampling of the density field. For instance, a direct comparison between the galaxy distribution from the extended (\( R \leq 15.4, N_{\text{gal}} \sim 6000 \)) CFA redshift survey (Geller 1998) and the Abell cluster distribution in the same volume, reveals that several prominent features in the galaxy distribution which contribute to the peaks originally found by Broadhurst et al. (1990) are not traced by the clusters.

Narrow, large amplitude features in \( P(k) \) are surprising yet intriguing. There is presently no theoretical consensus on the origin of preferred scale lengths. Eisenstein et al. (1998) hypothesize that such excess power could be a consequence of baryonic acoustic oscillations in adiabatic models. However, they note that this would require a substantial error in currently favored values of cosmological parameters. Einasto et al. (1997) simply conclude that our present understanding of the formation of large-scale structure requires substantial revision.

5 High-Order (\( N \geq 3 \)) Statistics

Higher order statistics potentially provide some of the best constraints on the degree of biasing (e.g., Jing 1997) and, thus, on the reliability of clusters as tracers of the mass. The high-order moments of cluster distributions have already been shown to be non-zero. For example, ACO clusters exhibit hierarchical clustering behavior given by

\[
\xi_N(r_1, \ldots, r_N) = \sum_{\alpha} Q_N^{(\alpha)} \sum_{ij} \Pi^{N-1}_i \xi_2(r_{ij})
\]
up to 6th order (Cappi & Maurogordato 1995) with \( Q_3 \approx 1.0 \). APM 
clusters display a similar hierarchical behavior, at least up to 4th order 
(Gaztanaga, Croft, & Dalton 1995).

The deprojected \( s_N \)'s (\( \xi_N = \sum N \xi^{-1}_N \)) for APM galaxies (Gaz- 
tanaga 1994) and ACO clusters are quite similar in amplitude which sug- 
gests that ACO clusters and APM galaxies are sampling the same un- 
derlying matter distribution but that the biasing between galaxies and 
clusters is non-linear. The \( S_{3,4} \) values for ACO and APM clusters are (3.1, 
22) and (\( \sim 2, \sim 8 \)), respectively.

6 Very Large Scale (> 200 h\(^{-1}\) Mpc) Structure

The large intercluster spacing and the enhanced amplitude of their clus- 
tering makes the study of structure on very large scales possible, in prin- 
ciple. In practice, the signals on these scales are small and errors in mod- 
eling systematic effects such as photometric zeropoint variations, sam- 
ple variance, or Galactic reddening can yield artificial signals of compa- 
rable amplitudes. Tully (1987) first proposed the detection of very large- 
scale alignment of the local Abell cluster distribution with the Super- 
galactic plane. Postman et al. (1989) countered that this effect (based on 
available data at the time) was not statistically significant (\( < 2 \sigma \)) and 
could be expected from sample with the observed \( \xi(r) \). Tully et al. (1992) 
extended their analysis to a full-sky sample of Abell and ACO clusters 
and still found an alignment with the Supergalactic plane, claimed to 
be significant at the 6-sigma level, extending to scales of 450h\(^{-1}\) Mpc. 
The structure is only a small amplitude fluctuation (\( \delta \rho/\rho \leq 0.015 \)), if 
indeed real. Scaramella (1992) also used Abell/ACO clusters to study 
power on 600h\(^{-1}\) Mpc scales and found relatively low values for for 
density fluctuations, consistent with limits on the fluctuations in the cos- 
mic microwave background (CMB). The study of very large scale struc- 
ture performed directly from cluster redshift surveys will not likely ad- 
vance much further until automated wide-area, homogeneous cluster cat- 
als, like those expected from the SDSS or the extended APM sur- 
vey (\~900 clusters; Tadros 1998), be- 
come available.

7 The Large-scale Velocity Field

A complementary approach (and perhaps a more promising one given 
current cluster catalogs) to studying very large-scale structure using clus- 
ters is through the mapping of the large-scale velocity field. Currently, 
at least 7 independent cluster-based peculiar velocity surveys, all reaching 
scales of 100h\(^{-1}\) Mpc or larger, are either complete or in progress (see 
Table 1). Inferences about the underlying mass distribution from pecu- 
liar velocity surveys are less susceptible to incompleteness effects and 
radial density gradients than those from redshift surveys. However, pe- 
culiar velocity surveys require highly accurate photometric and spectro- 
scopic calibrations and extremely homogeneous data (see Strauss, these 
proceedings). Careful characterization of the systematic errors and the 
effects of sparse sampling are also re- 
quired (e.g, Lauer & Postman 1994; 
Feldman & Watkins 1994). Figure 5 
summarizes the current constraints on bulk flow amplitudes from both 
galaxy and cluster based surveys. The constraints on the largest scales 
are nearly all from cluster-based sur- 
veys. Included in the plot are two new results. The exciting results of 
a 700 km/s flow at 8000 km/s depth 
from Hudson et al. are discussed else- 
where in these proceedings. Willick 
(1998) reports the measurement of 
\( v_{Bulk} = 900 \pm 375 \) km/s (1\( \sigma \) er- 
ror) in the redshift range 9000 \( \leq 
\) \( cz \leq 12,000 \) km/s based on a Tully- 
Fisher survey of 15 rich Abell clus- 
ters. Neither of these two results 
are consistent with the direction of 
Lauer & Postman (1994; LP) re- 
sult. They may, however, be con- 
sistent with each other. Indeed, no 
other work to date has corroborated 
the LP bulk flow (see also Wegner 
et al. 1998) and Saglia, these pro- 
cedings). This may suggest that ei- 
ther the original LP BCG sample is 
a statistical fluke or an additional 
parameter is required for accurate 
BCG distance estimation (e.g, Hud- 
son & Ebeling 1997). The extended
The amplitude of derived bulk flows from recent galaxy and cluster-based peculiar velocity surveys. The results from cluster-based surveys are indicated by the filled data points.

BCG survey by Lauer, Postman, & Strauss (1999) will provide a good test. In contrast, Dale et al. (1997) find no evidence for a large bulk flow at 8000 km/s. The inconsistent results for the amplitude and direction of large-scale bulk flows argue that the convergence scale is not yet well-established. However, the quality and quantity of data from the ongoing surveys, including promising results from space-based SBF studies, should be sufficiently good that much better constraints will be available within the next two years or so.

8 The Cluster Mass Function at z > 0.7

The advent of the Keck telescope and the Low-Resolution Imaging Spectrograph have revolutionized spectroscopic surveys of distant (z > 0.7) clusters. This capability has now enabled us (Oke, Postman, & Lubin 1998) to provide a preliminary constraint on the normalization of the cluster mass function (CMF) in range 0.76 < z < 0.92 based on data for 3 clusters and between 22 and 36 cluster members for each system. The specifics of the mass estimation techniques are described in Postman, Lubin, Oke (1998). The new constraints on the CMF are based on the clusters CL1324+3011 (z = 0.76), CL1604+4304 (z = 0.90), and CL1604+4321 (z = 0.92). The latter two are part of a supercluster. All 3 clusters are from the Gunn, Hoessel, Oke (1986; GHO) catalog and their kinematic masses are all in excess of $4.5 \times 10^{14} h^{-1} M_\odot$ within their central $1h^{-1}$ Mpc regions. If we make the quite conservative assumption that these are the only 3 clusters this massive within the entire GHO catalog ($\sim 72$ deg$^2$), then we find that, for $\Omega_0 = 0.2$, a lower limit on the CMF in the range $0.7 < z < 1$ is

$$N(\geq M = 4.5 \times 10^{14} h^{-1} M_\odot) > 1.1 \times 10^{-7} h^3 \text{Mpc}^{-3}$$

This constraint is consistent with estimates made by Bahcall, Fan, & Cen
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Table 2. Current Cluster-based Peculiar Velocity Surveys

| Investigators              | D.I. | $N_{clus}$ | Depth (km/s) | $\sigma_D$/cluster |
|---------------------------|------|------------|--------------|---------------------|
| Dale et al. (1997)        | TF   | ~ 50       | 18,000       | ~ 7%                |
| Gibbons, Fruchter, Bothun (1998) | FP   | 20         | 11,000       | ~ 6%                |
| Hudson et al. (1998; SMAC) | FP   | ~ 60       | 12,000       | 7%                  |
| Lauer & Postman (1994)    | BCG  | 119        | 15,000       | 16%                 |
| Lauer, Postman, Strauss (1999) | FP   | ~ 500      | 24,000       | 16%                 |
| Tonry et al. (1998)       | SBF  | 11         | 10,000       | < 5%                |
| Wegner et al. (1998; EFAR)| FP   | 84         | 15,000       | 8%                  |
| Willick (1998)            | TF   | 15         | 12,000       | ~ 5%                |

(1997) and provides additional observational support for a low-density universe. While this constraint is relatively crude (± factors of 2 − 20), the discovery of similarly massive systems (e.g, MS1054-03 and MS1137+66) at similarly high redshifts will likely continue to grow as observations of distant clusters progress.

9 Cosmological Implications and Future Developments

The general conclusions one can draw from the ensemble of cluster data and simulations discussed above are that

1. Clusters are reliable tracers of the underlying mass but trace it differently from galaxies. In particular, clusters trace the large–scale structure sparsely since they are relatively rare objects. The biasing between galaxies and clusters is non-linear and is dependent on their intrinsic properties (e.g. the central mass of the cluster).

2. The statistically significant power seen in the cluster distribution on scales between $30 - 60 h^{-1}$ Mpc implies that galaxies are also likely to exhibit correlations on the same scales. Indeed, the larger galaxy redshift surveys (LCRS, 2dF, ESO Slice survey) now confirm that $\xi_{gg}(r) > 0$ at least to $35 h^{-1}$ Mpc.

3. The cluster observations seem to favor MDM models ($0.2 \lesssim \Omega_c \lesssim 0.3$), low–density CDM models ($\Omega h \sim 0.3 \pm 0.1$), and/or ΛCDM models ($\Lambda \sim 0.3$). The exception would be if the large-amplitude, large–scale bulk flows persist, in which case somewhat higher values for $\Omega$ are required.

4. Massive (few $\times 10^{14} h^{-1} M_\odot$) clusters exist at $z > 0.7$ in an abundance that is hard to reconcile with $\Omega = 1$ models.

There is still much we need to learn about large–scale structure formation and evolution and a number of exciting developments over the next 3 to 5 years will help. New, larger objective cluster catalogs will soon be available from surveys such as the SDSS, 2dF, and extended APM. Using them, we should be able to constrain the cluster power spectrum with unprecedented accuracy to scales approaching 1 Gpc. These catalogs will also enable more extensive, direct comparisons between the galaxy and cluster distributions in identical volumes and will allow us to establish, with significantly better accuracy, the dependence of the clustering properties of clusters on their intrinsic parameters. Joint x-ray/optical cluster searches (e.g, Donahue et al. 1999) should elucidate the nature of cluster evolution at intermediate redshifts ($z \lesssim 1$).

Deep, wide-area galaxy surveys (e.g, Postman et al. 1998b; Jannuzi, Dey, et al. 1998) will provide important and new measurements of the evolution of large–scale structure out to $z = 1$ and beyond. These same surveys, coupled with deeper x-ray surveys, should prove profitable for the continued identification of massive clusters with $z > 0.8$, with the corresponding implications for structure formation models. The completion of several independent cluster-based peculiar velocity surveys which
all probe $\sim 100 - 200 h^{-1}$ Mpc scales but with different techniques should, hopefully, provide a better consensus on the convergence scale and the origin of the CMB dipole motion. Lastly, but as important as any of the above observational efforts, the new billion particle simulations, like those being pioneered by the Virgo Consortium, with high spatial resolution and spanning a large dynamic range in cosmic time will provide much more accurate and refined model predictions.

Acknowledgments

I thank Tod Lauer, Michael Strauss, István Szapudi, Michael Vogelezang, Neta Bahcall, and Harald Ebeling for the lively discussions on various aspects of this review. A special thanks to Jeff Willick for allowing me to be the first to publicly present his preliminary bulk flow result and to Helen Tadros for providing an electronic version of the APM cluster $P(k)$.

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