Robust model predictive control employed to the container ship roll motion using fin-stabilizer

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Abstract: The aim of this paper is to find the non-linear behavior of a container ship roll motion by using fin-roll stabilizer and robust model predictive control (RMPC). To do so, numerical and analytical modeling has been introduced for the roll motion. Computational fluid dynamics method was employed to determine the hydrodynamic lift of the fin at various angles. RMPC was designed and used to control the non-linear roll motion in the presence of disturbances, uncertainties, which were caused by the irregular sea waves, and operational constraints of fin's actuator. To boost the validity of our results, the performance of this controller was compared with a conventional PID (Proportional-Integral-Derivative) controller. Simulation results indicated the significant amount of reduction in roll amplitude and roll rate.

Keywords: RMPC; non-linear modeling; container ship; CFD; fin-roll stabilizer

1. Introduction

The roll motion is the most important motions of a ship. The accelerations due to wave-induced roll motions negatively affect a container ship performances by making limitation in comfort, habitability, and safety. The roll stabilization systems have widely been studied for more than three decades, and various types of anti-rolling devices have been introduced to diminish the unfavorable roll motion. The active fin stabilizer has been taken into account as the most efficient anti-rolling technique for ships, which normally operates above definite speeds (Perez & Blanke, 2012).

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PUBLIC INTEREST STATEMENT

Among the motions of a ship at sea, roll motion is the most important one. The accelerations due to wave-induced roll motions negatively influence on a container ship performances by making limitation in comfort, workability, and safety. Also the main reason of ship's capsizing is the loss of roll stability caused by wave disturbance. Using RMPC controller with fin stabilizer may useful for the container ship that is encounter with disturbances and uncertainties due to the irregular sea waves and operational errors. This control method help to the significant mount of reduction in roll amplitude and roll rate.
The main reason of ship’s capsizing is the loss of roll stability caused by wave disturbance (Taylan, 1996). This disturbance results in uncertainty in roll model. Also there are two main limitations on fin stabilizers including saturation of the mechanical fin angle and the dynamic stall angle. The former causes an energy loss that is an important limitation for the container ship, while the latter is a non-linear phenomenon caused by unsteady hydrodynamic effects leading to the loss of roll moment and the control authority when the angle of attack exceeds a certain threshold. The dynamic stall depends on the lift coefficient and it changes with the effective angle of attack between the flow and fin (Ghaemi, Sun, & Kolmanovsky, 2009). The non-linear effects in ship roll dynamic are progressive. So, it should be considered in control design. Research on the non-linear roll motion model has also been carried out (Taylan, 2000).

The PID controller has mainly been used in the ship fin stabilizers (Surendran, Lee, & Kim, 2007). However, because of difficulty, non-linearity, and constriction of fin stabilizers in this method, attaining the optimal performance for control system is very intricate (Moradi & Malekizade, 2013). The results of using a combined neural network and PID for roll control of ship with small draught were presented (Ghassemi, Dadmarzi, Ghadimi, & Ommani, 2010) and later the robust control for horizontal plane motions of autonomous underwater vehicle was employed as well (Kamarlouei & Ghassemi, 2016). An especial control approach, which can be applied to concern with constrains, is optimal control. MPC and LQR (linear quadratic regulator) control algorithms are two main optimal control techniques applied to the ship roll motion. A linear model was used for the purpose of roll motion, and it strongly proposed the use of a MPC to prevent the development of non-linear effects by considering the possible constraints (Perez & Goodwin, 2008). Also in order to avoid dynamic stall, constrained control of fin stabilizers is done by applying MPC (Perez, 2005). Moreover, MPC is extensively used to control processes in industry due to its ability to cover state, input, and output constraints throughout the design of the controller. Design of MPC and its implementation to non-linear systems with or without constraints generally contain non-linear and non-convex programming methods leading to high on-line computing complexity (Razi & Haeri, 2015). Thus, in various applications, MPC is designed according to linearized or uncertain linear model presentations of the non-linear systems (Kothare, Balakrishnan, & Morari, 1996). The general MPC algorithms are not able to handle explicitly model uncertainties. Therefore, several methods are extended for Robust model predictive control (RMPC) including the model uncertainties in the problem formulation (Haeri, Taghirad, & Poursafar, 2010). The RMPC for constrained large-scale interconnected systems was employed (Lu, Zou, & Niu, 2016). A Lagrangian dual method was introduced to consider the optimization problem (Zhou, Li, Huang, & Xiao, 2015). In the same vein, for solving general constrained convex optimization problems, a generalized Hopfield network has been suggested (Li, Yu, Huang, Chen, & He, 2016). Both of the above-mentioned studies successfully applied the well-known hierarchical and distributed model predictive control four-tank benchmark. For an online utilization of the RMPC methods, linear matrix inequality (LMI) can be utilized because an LMI optimization problem can be employed online and covered constraints and uncertainties in order to robust stability (Jia, Krogh, & Stursberg, 2005).

There are generally five hydrodynamic moments acted on the ship in the roll motions, i.e. restoring moment, damping moment, inertia moment, wave moment, and anti-roll moment by fin stabilizer. Each of them should be calculated. In the present study, fin stabilizer was selected with span and chord and NACA0015 section. Computational fluid dynamics (CFD) method was also employed for the flow analysis so that the lift coefficient was determined at various angles. The excitation moment acted to the ship was considered by the irregular waves. The restoring moment was accounted for as a third-order polynomial function, and its coefficients were calculated by the GZ curve and the empirical formulae. A non-linear term was also taken into account for the roll damping moment. Furthermore, the linear damping coefficient, mass moment of inertia, and added mass moment of inertia were calculated by using the free roll decay test and empirical formulae.

The purpose of the present study was to employ the non-linear modeling and find their hydrodynamic moments as well as designing RMPC using LMI for fin-roll stabilizer in a container ship. Findings
of the study will be of great significance since it benefited from a controller that can consider all the properties of fin-roll model including non-linear dynamics, constraints, waves’ disturbance, and uncertainties in the design. Previous studies did not consider all the properties of model because of design limitations. In the following, Section 2 presents the non-linear modeling and coefficients extraction of a container ship. Section 3 explains the design of a RMPC controller for fin-roll stabilizer, while Section 4 presents the results and discussion. Finally, Section 5 provides the conclusions.

2. Non-linear modeling

It is supposed that the fin stabilizer is almost situated in amidship close to the center of gravity and there is a minimum coupling with the other motions. Therefore, the coupling with other motions can be considered and just one degree of freedom can be regarded. The model of roll motion is given (Perez & Blanke, 2012) as:

\[
I_{xx} \ddot{\phi} = \tau_d - \tau_f - \tau_h
\]  

(1)

\[
p = \phi
\]

where \( \phi, p, I_{xx}, \tau_f, \) and \( \tau_d \) are roll angle, roll rate, roll moment of inertia, moment created by fins, hydrodynamic moment and wave excitation moment, respectively. The moment due to the fins and the hydrodynamic moment are calculated for a container ship with characteristics which presented in the Tables 1 and 2 and the body plane illustrated in Figure 1.

2.1. The fins moment and CFD analysis

The fins moment can be determined by the following equation:

\[
\tau_f = \rho V^2 A_f R_f C_L(\alpha_e)
\]  

(2)

where \( \rho, V, A_f, R_f, \) and \( C_L(\alpha_e) \) are the density of the flow, ship speed, fin area, fin lever, lift coefficient, respectively. It is noted that the lift coefficient is a function of the effective angle of attack between the flow and the fin. The lift coefficient was computed by employing the CFD method as shown in Figure 2. The CFD analysis has been accomplished with Ansys Fluent package software with a hardware configuration, which contains four Parallel Processors and 32 GB RAM. The governing equations of fluid flow are flow separation, Reynolds averaged Navier-Stokes, and \( k-\varepsilon \). Finally, the lift coefficient was directly obtained from the model as well as the rate and pressure gradients.

| Parameter             | Amount | Unit |
|-----------------------|--------|------|
| Displacement          | 34,893 | ton  |
| Length                | 192.89 | m    |
| Beam                  | 32.207 | m    |
| Draft                 | 10.56  | m    |

Table 1. The specifications of the container ship

| Parameter          | Amount | Unit |
|--------------------|--------|------|
| Fin area           | 4.18   | m²   |
| Fin span           | 1.66   | m    |
| Mean chord         | 2.51   | m    |
| Aspect ratio       | 0.66   | -    |

Table 2. Characteristics of the fin
2.2. The hydrodynamic moment

The hydrodynamic moment caused by the interaction between fluid and ship can be demonstrated as in the following:

\[
\tau_h = K_p \dot{\phi} + f_1(\phi, \dot{\phi}) + f_2(\phi)
\]  

(3)

where \(K_p\) regards a hydrodynamic moment in roll because of pressure variation that is proportionate to the roll accelerations, and the coefficient \(K_p\) is called roll added mass. \(f_1(\phi, \dot{\phi})\) is damping term and it can be represented as

\[
f_1(\phi, \dot{\phi}) = k_p \phi + k_{pip}[\dot{\phi}]^2
\]  

(4)

where \(k_p\) is a linear damping term, which includes forces due to wave making and linear skin friction, and the coefficient \(k_p\) is denoted a linear damping coefficient. \(k_{pip}[\dot{\phi}]\) is a non-linear damping term, which contains moments due to viscous effects, alike non-linear skin friction and eddy making due to flow separation, and the coefficient \(k_{pip}\) is denoted a non-linear damping coefficient. \(f_2(\phi)\) is the restoring moment term because of gravity and buoyancy, and it can be written as

\[
f_2(\phi) = \Delta GZ(\phi)
\]  

(5)
where $\Delta$ is the ship’s displacement and $GZ(\phi)$ is the restoring moment arm and it is the function of the roll angle. The GZ curve is an odd function and therefore represent with odd order polynomial. GZ curve of the container ship is shown in Figure 3. The right arm has been demonstrated by:

$$GZ(\phi) = c_1 \phi + c_3 \phi^3 + c_5 \phi^5$$  \tag{6}

where the coefficients $c_1$, $c_3$ and $c_5$ defined as follow (Taylan, 2000):

$$c_1 = \frac{d(GZ)}{d\phi} = GM, \quad c_3 = \frac{4}{\phi_v^4} \left(3A_{\phi v} - GM\phi_v^2\right), \quad c_5 = -\frac{3}{\phi_v^6} \left(4A_{\phi v} - GM\phi_v^2\right)$$  \tag{7}

where $GM$, $\phi_v$ and $A_{\phi v}$ are, the metacentric height, angle of vanishing stability and area under the GZ curve, respectively.

### 2.3. Calculation of hydrodynamic coefficients

Figure 4 shows the free roll decay curve. The first two peaks are $\phi_1$ and $\phi_2$. The non-dimensional damping coefficient (Crossland, 2003) is calculated as follows:

$$\zeta_\phi \equiv \frac{\ln(\phi_1/\phi_2)}{2\pi}$$  \tag{8}

Based on the results mentioned above and the following standard relationships, the hydrodynamic coefficients can be calculated as follows:

$$\omega_\phi = \frac{2\pi}{T_\phi}$$  \tag{9}

$$I_{xx} + K_\phi = \frac{\Delta GM}{\omega_\phi^2}$$  \tag{10}

![Figure 3. GZ curve of container ship.](image)

![Figure 4. Free roll decay test.](image)
where $K_{p\phi}$ is almost considered 5% of $k_p$ at a ship speed of 18 knots (Zhang & Andrews, 1999). The added mass moment of inertia is supposed to be 20% of the mass moment of inertia.

The wave exciting moment is specified as (Taylan, 2000):

$$\tau_W = I_{xx} \omega_e^2 \alpha_{max} \cos(\omega_e t)$$  \hspace{1cm} (12)

where $\alpha_{max}$ the maximum wave slope and $\omega_e$ is the wave encounter frequency, which described as follows:

$$\omega_e = \omega \left( 1 - \frac{\omega}{g} V \cos(\mu) \right)$$  \hspace{1cm} (13)

where $\omega$ and $\mu$ are the wave frequency and the wave encounter angle.

### 2.4. The fin dynamic model

Block diagram of the fin-roll closed loop control system is summarized in Figure 5. The roll angle and roll rate are measured by gyroscope. The active fin is actuated by Electro-Hydraulic Servomechanism which is called fin’s actuator. Fin as actuator has a first order model as (14). $\alpha_c$ is input variable of fin’s actuator as fin angle command and $\alpha_m$ is output variable as mechanical fin angle (Lee, Rhee, & Choi, 2011). According to Figure 5, $\alpha_m$ has two constraints, i.e. slew rate saturation and magnitude saturation.

$$T_e \dot{\alpha}_m + \alpha_m = K_{dc} \alpha_c$$  \hspace{1cm} (14)

where $K_{dc}$ is dc gain of the actuator and $T_e$ is time constant due to the delay between $\alpha_c$ and $\alpha_m$.

The relation between mechanical and effective angle of attack is specified as follows:

$$\alpha_e = -\alpha_f - \alpha_m, \quad \alpha_f = \arctan \left( \frac{R_f \rho}{V} \right) \approx \frac{R_f \rho}{V}$$  \hspace{1cm} (15)

where $\alpha_f$ is flow angle.

### 2.5. The state space model

According to (1)–(15), the non-linear model of roll motion can be summarized as (16).

$$\left( I_{xx} + K_p \right) \phi + k_p \phi + k_{p|\phi|} |\phi| + \Delta \left( c_1 \phi + c_3 \phi^3 + c_5 \phi^5 \right) = \tau_d - K_a \alpha_e$$

$$\alpha_e = -\frac{R_f \rho}{V} - \alpha_m$$  \hspace{1cm} (16)

Figure 5. Summarized block diagram of the fin-roll closed loop control system.
\[ T_e a_m + a_m = K_d a_c \]

State variables and output are defined as \( x(k) = [\phi(k), \, p(k), \, a_m(k)]^T \) and \( u(k) = \alpha(k) \).

Also the input and state constraints are considered as follows:

- Input constraint which contains the saturation of the mechanical fin angle:
  \[ |a_c| \leq a_{sat} \] (17)

- State constraints that are used in order to preventing the dynamic stall:
  \[ |a_e| = \left| \frac{R_f}{V} P + a_m \right| \leq a_{stall} \] (18)

Thus the state space model is defined as follows:

\[ x_1 = x_2 \]

\[ \dot{x}_2 = \frac{\Delta GM}{I_{xx} + K_p} \frac{k_p}{I_{xx} + K_p} x_1 + \frac{\left( \frac{k_p}{I_{xx} + K_p} \right) - k_p}{I_{xx} + K_p} x_2 + \frac{k_p}{I_{xx} + K_p} x_3 x_1 - \frac{\Delta}{I_{xx} + K_p} \left( c_3 x_1^3 + c_5 x_1^5 \right) + \frac{1}{I_{xx} + K_p} \tau_d \] (19)

\[ x_3 = -\frac{1}{T_e} x_3 + \frac{k_{dc}}{T_e} u \]

3. Robust model predictive control using LMI

At first, it is better to present the following lemmas for later use.

**Lemma 1 (Schur Complements)** The LMI

\[
\begin{bmatrix}
S(x) & D(x) \\
D(x)^T & O(x)
\end{bmatrix} > 0
\] (20)

where \( S(x) = S(x)^T \), \( O(x) = O(x)^T \) and \( D(x) \) are affine functions of \( x \), and is equivalent to

\[
O(x) > 0, \quad S(x) - D(x) O(x)^{-1} D(x)^T > 0
\] (21)

\[
S(x) > 0, \quad O(x) - D(x)^T S(x)^{-1} D(x) > 0
\] (22)

**Proof** Boyd, El Ghaoui, Feron, and Balakrishnan (1994).

**Lemma 2** Let \( L, \, \hat{L} \) be real constant matrices and \( P \) be a positive definite matrix of compatible dimensions.

\[
L^T P L + \hat{L}^T P \hat{L} \leq \epsilon L^T P L + \epsilon^{-1} \hat{L}^T P \hat{L}
\] (23)

Holds for any \( \epsilon > 0 \).

The non-linear discrete time system (19) with sample time \( T_s = 0.5 \) s is expressed as
\[ x(k + 1) = f(x(k), u(k), k) \]  
(24)

where \( k \) is the discrete time symbol, \( x(k) \in \mathbb{R}^n \) is the state vector, and \( u(k) \in \mathbb{R}^m \) is the control input.

Then, dynamic system (24) can be divided into two parts as follows:

\[ x(k + 1) = Ax(k) + Bu(k) + \tilde{f}(x(k), u(k)) \]  
(25)

where 
\[
A = \frac{1}{I_s + k_s} \begin{bmatrix} 0 & 1 \\ -\Delta GM & k_p & k_s \\ 0 & 0 & -1/T_o \end{bmatrix}
\]

\[
B = \begin{bmatrix} 0 & 0 & k_{dc}/T_o \end{bmatrix}^T
\]

\[ \tilde{f}(x(k), u(k)) = f(x(k), u(k)) - Ax(k) - Bu(k) \]  
(26)

It is presumed that \( \tilde{f}(x(k), u(k), k) \) is a Lipschitz non-linear function and is limited as

\[ \tilde{f}(x(k), u(k), k) \leq \begin{bmatrix} x(k|k) \\ u(k|k) \end{bmatrix}^T M \begin{bmatrix} x(k|k) \\ u(k|k) \end{bmatrix} \]  
(27)

where \( M = W^TW \) is a symmetric and positive definite matrix.

The state and control constraints have the following conditions:

\[ x(k) \in \bar{X}, \quad u(k) \in \bar{U}, \quad k \geq 0 \]  
(28)

where \( \bar{X} \) and \( \bar{U} \) are compact sets in \( \mathbb{R}^n \) and \( \mathbb{R}^m \), respectively and both including the origin. For designing a state feedback control law \( u(k + i|k) = F(k)x(k+i|k), \quad i \geq 0 \) for the system (24), consider minimizing the following cost function at each sampling time \( k \):

\[ \min_{u(k+i|k)} J(k) = \sum_{i=0}^{\infty} x(k+i|k)^T Q_x x(k+i|k) + u(k+i|k)^T R_u u(k+i|k) \]  
(29)

Subject to (28).

where \( Q_x \) and \( R_u \) are weighting matrices. It is of value now to consider a quadratic Lyapunov function for the system at sampling time \( k \) in the form \( V(x(k|k) = x(k|k)^T P x(k|k) \) with \( P > 0 \) and \( V(0) = 0 \). Assume that \( V(x) \) satisfies the following robust stability limitation (Wan & Kothare, 2003).

\[ V(x(k+i|k)) - V(x(k+i|k)) \leq -x(k+i|k)^T Q_x x(k+i|k) - u(k+i|k)^T R_u u(k+i|k) \]  
(30)

via summing both sides of (26) from \( i = 0 \) to \( \infty \), we have

\[ x(\infty)^T P x(\infty) - x(k|k)^T P x(k|k) \leq -J(k) \]  
(31)

In order to have a limited cost function and asymptotic stability of the concluding closed loop system, \( x(\infty|k) \) must be zero. This signifies the closed-loop system asymptotically stability if and only if

\[ J(k) \leq x(k|k)^T P x(k|k) \leq \gamma \]  
(32)

where \( \gamma \) is considered as an upper bound for \( J(k) \) and is a positive scalar. Thus, the specified minimization problem in (29) can be rewritten as

\[ \min_{u(k+i|k)} \max_{x(k+i|k)} J(k) \]  
(33)

Subject to (28).
In this algorithm, at each time $k$, a state - feedback gain $F(k)$ is computed in the sense that the upper bound of $J(k)$ becomes minimum. Then, the first calculated input $u(k) = F(k)x(k)$ is utilized to model (24). The state $x(k + 1)$ is measured at the next sampling time, and the optimization problem is solved again to determine $F(k + 1)$.

Theorem 1 is presented to get a suitable $p > 0$ to fulfill (30) and the coinciding state feedback matrix $F$. Also the problem formulation for the suggested RMPC and optimization based on LMI to solve the determined problem are accomplished. Theorem 1 is solved again to determine $(k + 1)$ model (24). The state $x(k + 1)$ is measured at the next sampling time, and the optimization problem where are gained from the resolution of the optimization problem as follows:

$$\min_{\gamma, Q, Y}$$

Subject to

$$\begin{bmatrix} I & * \\ x(k) & Q \end{bmatrix} \geq 0$$

$$(35)$$

$$\begin{bmatrix} Q & * & * & * \\ \sqrt{1 + \varepsilon(AQ + BY)} & Q & * & * \\ \sqrt{1 + \varepsilon^{-2}WZ} & 0 & \varepsilon I & * \\ Q^2 & 0 & 0 & \gamma I \\ R_i^2 & 0 & 0 & \gamma I \end{bmatrix} \geq 0$$

$$(36)$$

$$Q - \varepsilon I \geq 0$$

$$(37)$$

where $Z = [Q; Y], \varepsilon$ is a positive design parameter, and the symbol $*$ stands instead of symmetric terms in the matrix.

In order to prove robust stability of the closed loop, we need to establish the following lemma.

**Lemma 3 (Feasibility)** Any feasible solution of the optimization in Theorem 1 at time $k$ is also feasible for all times $t > k$. Thus if the optimization problem in Theorem 1 is feasible at time $k$ then it is feasible for all times $t > k$.

**Proof** The proof is the same as proved in Lemma 2 by Kothare et al. (1996).

It should be noted that (35) is positive definite condition of $P$ matrix in order to satisfy robust stability and (36) is asymptotically stability condition of closed-loop system. Proofs (35) and (36) follow from the theorem 1 proved in the following. Proof (Theorem 1) The proof is based on (Haeri et al., 2010).

To obtain (36), the modified quadratic function $V$ required to fulfill (30). Substituting the state space (25), in inequality (30) results in

$$x(k + 1|k)Q x(k + 1|k) + u(k + 1|k)R_u u(k + 1|k)$$

$$- x(k + 1|k)^T P x(k + 1|k) + \{ A x(k + 1|k) + B u(k + 1|k) + f(x(k + 1|k), u(k + 1|k)) \}^T$$

$$\times P \{ A x(k + 1|k) + B u(k + 1|k) + f(x(k + 1|k), u(k + 1|k)) \} \leq 0$$

$$(38)$$

Defining the function $h(x, u)$ as
\( h(x, u) = \{Ax(k + i|k) + Bu(k + i|k) + f(x(k + i|k), u(k + i|k))\}^T \)

\( \times P\{Ax(k + i|k) + Bu(k + i|k) + f(x(k + i|k), u(k + i|k))\} \)

\( = \{Ax(k + i|k) + Bu(k + i|k)\}^T P\{Ax(k + i|k) + Bu(k + i|k)\} \)

\( + \{Ax(k + i|k) + Bu(k + i|k)\}^T P\{f(x(k + i|k), u(k + i|k))\} \)

\( + \{f(x(k + i|k), u(k + i|k))\}^T P\{Ax(k + i|k) + Bu(k + i|k)\} \)

\( + \{f(x(k + i|k), u(k + i|k))\}^T P\{f(x(k + i|k), u(k + i|k))\} \)

And applying Lemma 2, the upper bound of \( h(x, u) \) becomes

\[
h(x, u) \leq (1 + \epsilon)\{Ax(k + i|k) + Bu(k + i|k)\}^T P\{Ax(k + i|k) + Bu(k + i|k)\} \]

\( + (1 + \epsilon^{-1})\{f(x(k + i|k), u(k + i|k))\}^T P\{f(x(k + i|k), u(k + i|k))\} \)

Consider \( P \leq \lambda_{\text{max}} I \leq \mu I \)

where \( \lambda_{\text{max}} \) is the maximum eigenvalue of \( P \) and \( \mu I \) is the corresponding upper bound, then

\[
h(x, u) \leq (1 + \epsilon)\{Ax(k + i|k) + Bu(k + i|k)\}^T P\{Ax(k + i|k) + Bu(k + i|k)\} \]

\( + (1 + \epsilon^{-1})\mu\{f(x(k + i|k), u(k + i|k))\}^T f(x(k + i|k), u(k + i|k)) \)

The term involving \( f(., .) \) in the above equation is bounded as

\[
\hat{f}(x(k + i|k), u(k + i|k))^T f(x(k + i|k), u(k + i|k)) \leq [x(k + i|k)^T W^T W(x(k + i|k); u(k + i|k)] \]

Then

\[
h(x, u) \leq (1 + \epsilon)\{Ax(k + i|k) + Bu(k + i|k)\}^T P\{Ax(k + i|k) + Bu(k + i|k)\} \]

\( + (1 + \epsilon^{-1})\mu\{f(x(k + i|k), u(k + i|k))\}^T f(x(k + i|k), u(k + i|k)) \)

In order to satisfy (30) for all \( i \geq 0 \), we should guarantee that the following equation is negative

\[
x(k + i|k)^T Q x(k + i|k) + u(k + i|k)^T R u(k + i|k) - x(k + i|k)^T P x(k + i|k) \]

\( + (1 + \epsilon)\{Ax(k + i|k) + Bu(k + i|k)\}^T P\{Ax(k + i|k) + Bu(k + i|k)\} \]

\( + (1 + \epsilon^{-1})\mu\{x(k + i|k)^T u(k + i|k)\}^T W^T W(x(k + i|k); u(k + i|k)] < 0 \)

Replacing \( u(k + i|k) \) by \( F x(k + i|k) \), (44) is rewritten as

\[
(1 + \epsilon)x(k + i|k)^T (A + BF)^T P(A + BF) x(k + i|k) - x(k + i|k)^T P x(k + i|k) \]

\( + x(k + i|k)^T Q x(k + i|k) + x(k + i|k)^T R F x(k + i|k) \]

\( + (1 + \epsilon^{-1})\mu x(k + i|k)^T [I F^T] W^T W[I; F] x(k + i|k) < 0 \)

That is satisfied for all \( i \geq 0 \) if

\[
(1 + \epsilon)(A + BF)^T P(A + BF) - P + Q + F^T R F + (1 + \epsilon^{-1})\mu[I F^T] W^T W[I; F] < 0 \]

Substituting \( Q = \gamma P^{-1}, Q > 0, Y = FA \) and \( \xi = \gamma I^{-1} \), Premultiplying (46) by \( Q \), and then applying Schur complements, (46) becomes

\[
\begin{bmatrix}
  Q & * & * & * & * \\
  \sqrt{1 + \epsilon (AQ + BY)} & Q & * & * & * \\
  \sqrt{1 + \epsilon^{-1}} W Z & 0 & \xi I & * & * \\
  Q_{\xi}^T Q & 0 & 0 & \gamma I & * \\
  R_{\gamma}^2 Y & 0 & 0 & 0 & \gamma I \\
\end{bmatrix} \geq 0
\]
Applying Schur complements to (32), we derive

\[ Q - \phi I \succeq 0 \]

Applying Schur complements to (32), we derive

\[
\begin{bmatrix}
I & x(k) \\
Q & -Q^{-1}
\end{bmatrix} \succeq 0
\]

By solving the inequalities of (35) and (36), the solution of the convex programming problem (29) provides a feedback gain \( F \). The control law achieved by this means guarantees the closed loop stability for the system described by (24). In this algorithm, the stability domain that is defined by an ellipsoidal invariant set

\[ S = \{ x | x^T Q^{-1} x \leq 1 \} \]

is reevaluated at a new iteration until it becomes constant. Thus, the algorithm converges to a local minimum for each sampling time.

In industrial processes, there are constraints on input states and output that should be considered throughout the control law design.

The constraint on the control input in the form of

\[ \| u(k+i|k) \|_2 \leq u_{\text{max},2}, \ i \geq 0 \]  

From (32), we know that the states \( x(k+i|k) \), \( i \geq 0 \) determine an ellipsoidal invariant set

\[ S = \{ x | x^T Q^{-1} x \leq 1 \} \]

Therefore,

\[ \| u(k+i|k) \|_2^2 = \| Fx(k+i|k) \|_2^2 = \| YQ^{-1/2} (Q^{-1/2} x(k+i|k)) \|_2^2 \leq \| YQ^{-1/2} \|_2^2 \]  

From (47) and (49), the input two-norm constraint in (47) is rewritten as

\[ Y^T Q^{-1} Y - u_{\text{max}}^2 I \preceq 0 \]

Applying Schur complements, (50) is equivalent to

\[
\begin{bmatrix}
u_{\text{max}}^2 I & * \\
Y^T & Q
\end{bmatrix} \succeq 0
\]

This is an LMI and can easily be combined with problem (29).

The state constraints inclusive of

\[ |x_j| \leq x_{j,\text{max}}, \ j = 1, 2, \ldots, n \]

are fulfilled if there exists a symmetric matrix \( X \) so that

\[ X - Q \succeq 0 \] with \( X_{jj} \leq x_{j,\text{max}}^2, \ j = 1, 2, \ldots, n \)

The LMIs (51) and (52) can simply be combined with the minimization problem in Theorem 1.

4. Results and discussions

The sailing condition is supposed for a random sea and beam sea conditions at the forward speed 18 knots. The exciting moment is calculated by the Matlab code according to (12) and (13) and the results are depicted in Figure 6. According to the empirical formulas and the specifications of the container ship with fin, the obtained values are \( C_L = 0.61 \), \( dC_L/d\alpha_e = 1.762 \ (\text{rad}^{-1}) \), and \( \alpha_{\text{stall}} = 0.62 \ (\text{rad}) \).

By using the roll decay test, the first two peaks are \( \phi_1 = 10 \) and \( \phi_2 = 4.5 \). The non-dimensional damping coefficient and roll natural period are obtained as \( \zeta_o = 0.127 \) and \( T_o = 15 \) s based on (8).
The calculated restoring and damping moment coefficients for the container ship are presented in Table 3. In addition, the sea water density $\rho = 1,025 \text{ (kg/m}^3)$; gravity constant $g = 9.81 \text{ (m/s}^2)$ are considered. Controller design parameters are

$$R = 0.01 \quad Q = \text{diag}(1, 0.1, 0.1), \quad \varepsilon = 0.01, \quad W = \text{diag}(0.001, 0.001, 0.001)$$

The closed loop fin-roll system is simulated by employing the RMPC controller with fin for initial roll angle $12^\circ$. Simulation results were compared with the results of the PID (Surendran et al., 2007) angle and roll rate in the RMPC controller is smaller and smoother than PID.

The characteristics of simulation results for two controllers are summarized in Table 4, which demonstrates that roll angle and roll rate are improved about 80% in comparison with the results of PID controller. In other words, results indicate that the use of RMPC control responses rate was increased and the amount of overshoot and responses amplitude were reduced as well. It is noteworthy that improvement has been made in spite of considering the irregular wave disturbance and constraints in the design of RMPC. More importantly, RMPC will be robust against sea state changes leading to waves' disturbance and uncertainty in the model, while PID cannot guarantee this capability.
5. Conclusions
In this paper, the non-linear modelling was derived for the fin-roll motion. The coefficients of non-linear model, the non-linear damping, and restoring terms were computed by means of the empirical formulas and numerical calculations by using the CFD method and MATLAB software. By considering the NACA0015 section, flow analysis and lift coefficient computation were presented by CFD method. A RMPC was designed to satisfy the operational constraints of the roll motion including the mechanical fin angle and the dynamic stall saturations in order to achieve the desired performance. The simulation results for two RMPC and PID controllers were presented and compared in the presence of irregular wave. The simulation results showed that due to considering the wave disturbance and constraints in controller design, the RMPC reduced the RMS value of the roll motion. Although the results of this study revealed that disturbance rejection was done, further studies might be needed to see if adding $H_{\infty}$ constraint contains, i.e. transfer function from wave disturbance to roll angle to RMPC algorithm and using one state observer for the combination of RMPC algorithm with adaptive disturbance model can result in better achievements.

Figure 8. The roll rate response for the RMPC in comparison with PID controller and uncontrolled with initial roll angle 12°.

Table 3. The calculated coefficients for the container ship

| Amount          | Factor |
|-----------------|--------|
| 5.5             | $c_1$  |
| -1.5            | $c_3$  |
| 0.074           | $c_5$  |
| $1.2 \times 10^8$ (N.m.s) | $k_s$  |
| $11 \times 10^6$ (kg m²)   | $I_{xx} + k_p$ |
| $0.06 \times 10^8$ (N.m.s) | $k_{pss}$ |

Table 4. Comparison of the performance of controllers

| Max. roll rate (°/s) | Max. roll angle (°) | Controller |
|----------------------|---------------------|------------|
| 48                   | 50                  | Uncontrolled |
| 8                    | 12                  | PID controller |
| 2                    | 3                   | RMPC        |

Table 3. The calculated coefficients for the container ship

Table 4. Comparison of the performance of controllers
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