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Systematic search and ranking of physical contradictions using graph theory principles: Toward a systematic analysis of design strategies and their impacts

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Abstract

This paper presents three interconnected developments made during the course of a recent collective research work, the development of a systematic graph-based search tool for physical contradictions, a ranking approach for defining the order of criticality of the design contradictions and the associated analysis of the different design strategies that can be used to solve those contradictions or to enhance performance indicators. The systematic graph-based search for physical contradictions is using the set of elementary variables necessary to describe the system as basic input. The initial set is extracted based on taxonomy of variables combining classification work from NIST and classification of variables derived from the Bond Graph theory. The contradiction search method is in a second step classifying the set of variables into three categories: the constraint variables imposed to the designers by the context and the environment, the design variables on which the designer has the possibility to act and the performance variables that are used to evaluate the performance of the designed system. In a third step, interactions between variables are searched using two possibilities: a causal ordering algorithm developed during the course of the research or via a collective work of experts. The result of this step is a directed graph starting from the constraints variables and ending with the performance variables. In the fourth step objectives have to be assigned to the performance variables (minimal value, maximal value or target value). Those objectives are propagated back into the graph by analyzing the impact of the variables interacting with the performance variables. A physical contradiction is detected each time it is discovered that a design variable is associated with two contradictory objectives. Following this approach, a contradiction is represented as a node in the directed graph. It is possible to systematically map the different design strategies that can be used and to rank the possible impact of those design strategies. The article presents a concrete application of the approach on the case study of an air bearing and demonstrates the novelty of the approach to generate new viewpoints and insight in the analysis of the early stages of the development process. The potential impact of such type of design support is potentially very important. A future step will consist of developing a computer aided tool implementing the method.
1. Introduction

The objective of the present research work is to develop a systematic approach to analyze weaknesses of existing design solutions. The need this kind of examination is coming from the fact that effective and systematic innovation methods such as TRIZ in textbooks and in reality, use to focus on a limited amount of contradictions and those contradictions are often associated with different levels of granularity of the systems under analysis. How these contradictions have been found and selected does not use to be discussed.

This is a real issue when systems are becoming more and more complex and when it becomes more challenging to develop an overall understanding of the system behaviors and interactions. Developing a method to systematically search for contradictions in existing systems and to map those contradictions is potentially an important task. This would in particular allow a global analysis of the design strategies that can be used to develop new systems, incrementally improving old ones.

The present article is organized in the following manner: at first a brief state of the art related to contradictions in TRIZ is developed in order to present some more general principles associated with human cognition that are embedded in the concept of contradiction. Second, latest research associated to the field of graph representation in the field of logic is presented in this work. The concept of cause-effect relationship is especially addressed in this state of the art.

In a second part starting in section 4, the systematic approach proposed to search and map contradictions, is presented. At the same time the article illustrates the approach on the example of the case study of an air bearing. This case study has been used to show that even for components and systems developed during a long period of time such type of representation approach allows to systematically map the different design options available in practice. The usage of some TRIZ inventive principles is proposed at the end of the study to demonstrate the innovation potential of the approach.

A third part is shortly summarizing the article and is discussing the future developments of the approach presented in this short article.

2. Short state of the art

1.1. Contradictions

G. S. Altshuller [1] distinguished the following three types of contradictions: administrative contradictions, technical contradictions and physical contradictions. The two last types are of interest for this article.

Technical contradictions: An action is simultaneously useful and harmful or it causes Useful Function(s) and Harmful Function(s)

Physical contradictions: The physical contradiction implies inconsistent requirements to a physical condition of the same element of a Technical System (TS) or operation of a Technological Process (TP). For example, we want that an insulator in semiconductor chips has low dielectric constant \( k \) in order to reduce parasitic capacities and we want that insulator in semiconductor chips has high dielectric constant \( k \) in order to store information better.

The physical contradictions as well as the technical contradictions are usually crystallized during the problem analysis. Sometimes the technical contradictions can be obtained by analyzing techniques in the framework of Root Cause Analysis or Goldratt's Theory of Constraints [12].

3. Potential link with graph theory

Graphs are widely used to represent and model different aspects of the world. They are used in almost all the domains where representation and modeling are required. A graph is a visual representation formed of nodes and vertices. The nodes (or vertices) and edges represent many different elements such as networks of
communication, data organization, devices, flows, etc.

The edges can be directed or not directed. If the edge is directed, a causality relation is defined. The structure of a plane can be represented by a directed graph. The vertices are the components and sub-systems of the plane and directed edges from sub-system A to sub-system B represent the causality of the flows linking A to B. For example if A is a computer and B the wings, the flows are represented by flow of fluid and flow of electricity. This representation is very general and can also represent mechanical connections between sub-systems. In the context of this article, a graph is used to represent a certain aspect of the topology of a real physical object (e.g. an air bearing). Different methods used in graph theory are used to process the graphs. Processing the graph may require the usage of computer systems. There are different ways to store graphs in a computer system. Theoretically one can distinguish between list and matrix structures.

In the present article, matrix representation of graphs is used extensively and different mathematic methods from linear algebra are applied (e.g. matrix manipulation in our case) such as matrix multiplication, sum of matrix columns and rows, computation of determinant, Cramer method, etc.. The manipulation of graphs in this article is mainly based on the usage of these elementary linear algebra methods. More advanced methods such as small world analysis are not used in this article. Nevertheless some features of complex graph analysis such as coupling and correlations structures are used in this article. The main theoretical reference used in this article is the book of Newman [9].

4. Proposed approach exemplified by the case study of an air bearing

The present article is voluntarily taking a systemic viewpoint. This systemic viewpoint is concretely exemplified using the case study of an air bearing.

4.1. Air bearing component: short presentation

With air bearings, virtually frictionless motion can be achieved. Due to that precision motion can be realized. Therefore air bearings are commonly used in 3D coordinate measurement machines and other machines requiring high precision positioning capabilities. Air bearings are usually aerostatic bearings that utilize an external air supply to create the air film between the two surfaces. Because contact is avoided, the bearing surfaces will not wear. The bearing surfaces are normally high quality machined surfaces, and the air gap between the surfaces are of the order of 5 μm to 10 μm.

A simple aerostatic bearing for flat surfaces consists of a body featuring a small-diameter nozzle connected to the air supply and providing the air flow to the gap between the bearing and its counter surface (see Figure 1). Higher load capacity can be achieved if the nozzle is connected to an air pocket. The load carrying capacity of the bearing is determined by the area of the bearing surface and the pressure distribution between the surfaces.

4.2. Listing the system variables and defining the system boundaries

In the present study, the boundaries of the study include the air bearing and its counter surface but not the air production system. This choice is the result of a pre-analysis not presented in this article. It should nevertheless be mentioned that one fundamental feature of the design process is its recursive nature and the possibility to go backward in this process. Consequently, if it appears later that the present definition of the boundaries limits the validity of the design strategies it is possible to reconsider this initial choice later.
Having defined the boundaries of the analysis, it will be necessary to define the initial set of variables used to describe the problem. The authors propose to use an adapted classification from Hirtz et al. [6] proposed by Coatanéa [4]. The interest of the approach is to propose a generic manner to list the variables of a problem by defining for each design problem different elements that are:

a. The domain, energies and fields related to the study,
b. Power variables (Efforts and flows),
c. State variables (Displacements and momentums),

From those initial elements, an initial list of variables can be established using the Tables 6 and 7, proposed by Coatanéa [4].

The last variables, that need to be described, are all the variables not yet present in this taxonomy but nevertheless fundamental to understand the behaviour of the system. Those variables are named by Coatanéa [4] connecting variables. They connect effort, flow, displacement and momentum into physical equations. The connecting variables are for example the dimensions of the air bearing influencing the behaviour of the system as well as the properties of the fluid to be used.

The following figure summarizes the key variables belonging to this category of connecting variables in the case of the air bearing.
This task has been accomplished in the case of the air bearing using a white box model [7] (white box is a term used in System Engineering and describing the fact that the content of the air bearing architecture and structure of the air bearing are known) for representing the system variables, the input variables and the output variables. Each of these three categories has to be populated. At the same time a detailed description of the units in form of basic quantities products is used to define the metric used to measure the variables.

Figure 3: White box model of the air bearing

The variables and base quantities are listed in the table below. The basic set of physical quantities, base units and symbols are presented using the SI system.

The basic symbols for the physical quantities in the first column on the left are used to represent the metrics of the variables of this problem.

Table 1: List of variables and quantities used to measure them ([x] represents the dimension of the variable x)

| Input Variables                      | System Variables                                             | Output Variables                                             |
|--------------------------------------|--------------------------------------------------------------|--------------------------------------------------------------|
| Input pressure \([P] = M \cdot L^{-1} \cdot T^{-2}\) | Gap, \([h] = L\)                                            | Output pressure \([P_C] = M \cdot L^{-1} \cdot T^{-2}\)       |
| External force \([F] = M \cdot L \cdot T^{-2}\)      | Injection diameter, \([d] = L\)                             | Force to counter act \([F] = M \cdot L \cdot T^{-2}\)       |
| Input air flow \([q] = L^3 \cdot T^{-1}\)           | External diameter of the bearing, \([D_B] = L\)             | Output air flow \([q_0] = L^3 \cdot T^{-1}\)                |
| Atmospheric pressure \([P_{atm}] = M \cdot L^{-1} \cdot T^{-2}\) | Internal chamber diameter, \([D_I] = L\)                   |                                                             |
| Acceleration of gravity \([g] = L \cdot T^{-2}\)    | Mass, \([m] = M\)                                          |                                                             |
| Mass density \([\rho] = M \cdot L^{-3}\)            | Velocity of the counter surface \([V_{C}] = L \cdot T^{-1}\) |                                                             |
| Roughness of the counter surface, \([R_g] = L\)      |                                                             |                                                             |
| Air bearing stiffness \([K] = M \cdot T^{-2}\)       |                                                             |                                                             |

The next step consists of classifying the variables according to the level of control that can be achieved on them. In the description of the system boundaries and of the definition of the system boundaries and environment, a distinction is made between the variables that are controlled, the influenced variables and the non-controlled variables. To this initial list the authors propose to include the performance variables that are used by designers to access the quality of the design proposals. The performance variables are fundamental in a design context to take appropriate decisions regarding the potential design solutions. The categories used in this work are summarized in the figure below.
4.3. Graph representation of the cause-effect relations

For the analysis of the system, the cause-effects relations between variables must be described. In the exemplary case study of an air bearing the causal ordering of the variables was done by a group of air bearing specialists. They have generated the oriented graph of Figure 5. The non-controlled variables are located on the left of Figure 5. They form the only clustering group at this stage of the analysis. Non-controlled variables can only be influenced by variables of the same type since they are coming from outside the boundaries of the system. In our case, \( f_A \) and \( P \) are influencing each other. The design variables nodes are characterized by presence of more outgoing edges than incoming edges. The influenced design variables nodes are characterised by the presence of both incoming and outgoing edges. Finally, the performance variables nodes are usually located at the end of the chain of influencing edges. Considering these properties of variables, it is possible to cluster the variables by processing the adjacency matrix generated from Figure 5 and by considering both the direct and indirect levels of interactions. The processing approach is not presenting in this article due to space limitation. Nevertheless, the Figure 6 below presents the final clustering output of the variables.

Figure 5 shows that \( P_c, K, f_0 \) and \( F_c \) can be used as performance variables.

The other set of variables is composed of the non-controlled and design variables. It appears that \( d \) and \( h \) seems to be the design variables influencing most other variables. This means that if contradictions are found and solved on those variables, the main impact on the total performance of the air bearing system can be obtained. The injection diameter \( d \) is not influenced by other variables but \( h \) is influenced. This means that \( d \) can be more easily controlled than \( h \). Later in this analysis it appeared clearly that working on those variables imply a local optimization or local transformation of the design of the air bearing. It is very probable that stronger effects on the entire air bearing performance can be obtained by selecting other design strategies.

The edges in the Figure 5 are summarizing both the qualitative and physical laws governing the behaviour of the air bearing.
Figure 5: Cause-effect relations between variable for the air bearing without the classifications of variables into categories.

Figure 6: Representation of the levels of how variables are influenced and influencing other variables of the air bearing.

In Figure 6, the numbers on both axes represent the total number of direct and indirect incoming edges for the vertical axis and the total number of direct and indirect outgoing edges for the horizontal axis. The Figure 7 below is translating the clustering results from Figure 6, into a colour code applied to the nodes presented in Figure 4.
The validity of the graph in Figure 7 depends on the initial set of variables selected as well as on the completeness of the cause-effect relations analyses. A simple tool such as the dimensional analysis approach can be used in the case of the graph to validate the dimensional homogeneity of the graph. The method consists of analysing each of the nodes of the graph pointed out by arrows using the following approach described below and integrating research works presented by [8] [2] [11].

4.4. Dimensional analysis

Let \( y = \sum a_i x_i \) be a law. Then all \( a_i \) must have the same dimensions as \( y \). If \( a_i \) are dimensionless constants, then \( y \) must have the same dimension than \( x_i \). This is the principle of dimensional homogeneity. If the system of fundamental quantities needed in the law is in the form of 3 basic quantities namely the length \( L \), the mass \( M \) and the time \( T \) and if the dimension of the variables is a combination of the 3 basic dimensions then \( y \) has the form:

\[
Eq. 1 \quad y = C_1 L^{\alpha_1} M^{\alpha_2} T^{\alpha_3}
\]

This form is called the product theorem in which the constant \( C_1 \) and the exponents \( \alpha_1 \), \( \alpha_2 \) and \( \alpha_3 \) are dimensionless numbers.

When the dimensional validity of the graph has been verified, the next step consists of defining the objectives that are targeted on the performance variables. Traditionally three types of objectives are targeted. A performance variable should be maximized, minimized or a target value is expected. Using the exact same basic principle described above, it is possible to infer the impact of those performance variables on the other categories of variables implied in the same graph. The principle used to implement this principle is the...
It follows from the product theorem described in Equation 1, that every law which takes the form 
\[
y_0 = f(x_1, x_2, \ldots, x_n)
\]
can take the alternative form
\[
\Pi_0 = f(\Pi_1, \Pi_2, \ldots, \Pi_n)
\]
\(\Pi_i\) are dimensionless products. This alternative form is the final result of the dimensional analysis and is the consequence of the Vashy-Buckingham theorem. A dimensionless number is then a product which takes the following form:
\[
\Pi_i = y_i \left( x_1^{a_{i1}} x_2^{a_{i2}} x_3^{a_{i3}} \right)
\]
where \(\{x_1, x_2, x_3\}\) are called the influencing variables, \(\{y_1, y_2, y_3\}\) are called the influenced variables in our work and \(\{a_{ij}\} \leq i \leq n-r, x_2, x_3\) are the exponents. The denomination influenced variables has replaced the denomination performance variables used initially by several authors such as Bashkar and Nigam or Butterfield [3]. This choice made by the authors of this article reflects better the graph considerations included in this work. The graph representation associated with Equation 3 is the following.

![Figure 8: Representation of the Vashy-Buckingham theorem](image)

Figure 8: Representation of the Vashy-Buckingham theorem

After verifying the dimensional homogeneity, the next step of the computational approach consists of computing the coefficients of the dimensionless numbers presented in the last column of Table 3 (see below). To do so, basic results from linear algebra are used in this article. The Cramer’s method is used to compute those coefficients.

Cramer’s rule imposes two conditions to be able to compute the coefficients. First, the determinant of the system of equation to solve for each dimensionless number should by different from 0. Second, in order to compute a determinant, the manipulated matrixes should be a square matrix. In order to concretely exemplify the approach supported by an excel spread sheet, the first dimensionless number is considered.

\[
\Pi_1 = h, M^s, R_d^p, g^e, F^d
\]

From this dimensionless number, a matrix representation can be developed having the form presented in Table 2.
Table 2: Matrix representation of the dimensionless number $n_h$

| $n_h$ | Influenced variables [B] | Influencing variables (A) |
|-------|--------------------------|----------------------------|
|       | $h$                      | $M$ | $R_a$ | $g$ | $F$ |

- Base: L
  - $h$: 1
  - $M$: 1
  - $R_a$: 1
  - $g$: 1
  - $F$: 1

Table 3: Table of influencing variables and dimensionless numbers from Figure 7

| Case         | Influenced nodes                  | Initial set of influencing variables based on Figure 7 | Associated dimensionless numbers |
|--------------|-----------------------------------|-------------------------------------------------------|----------------------------------|
| Air bearing  | $h$ (gap)                         | $M, Ra, g, F, K$                                      | $Il_h = h, M, Ra^b, g^c, F^d$   |
|              | $P_c$ (Pressure between the counter surface and air bearing) | $P, d, h, f_o$                                         | $Il_{P_c} = P_c, P_c^{a1}, d^{b1}, h^{c1}, f_o^{d1}, F^{e1}$ |
|              | $K$ (Bearing Stiffness)           | $K, D_h, D_e, H, P_c, p$                              | $Il_K = K, D_h^{a2}, D_e^{b2}, h^{c2}, P_c^{d2}, p^{e2}$ |
|              | $f_o$ (Flow of air leaving the air bearing) | $d, P_{a,b}, P_c, h, V_c$                             | $Il_{f_o} = f_o, d^{a3}, P_{a,b}^{b3}, P_c^{c3}, h^{d3}, V_c^{e3}$ |
|              | $F_c$ (Lifting Force)             | $P_c, D_h, D_e$                                       | $Il_{F_c} = P_c, D_h^{a4}, D_e^{b4}, P_c^{c4}$ |

The matrix $A$, introduced in Table 2, formed by the influencing variables is not squared. In order to be able to apply Cramer’s method, variables should be combined to form a 3x3 matrix. The combination principles should simply avoid having columns which are multiple of others and columns or lines containing only 0. The new matrix $A$ generated from this combination of variables is presented in Table 4. It should be noted by the readers that it is not the only possible combination. A heuristic to develop the combinations is presented later in this article.

Table 4: Square matrix $[A]$

| $n_h$ | Influenced variables [B] | Influencing variables (A) |
|-------|--------------------------|----------------------------|
|       | $h$                      | $M, Ra$ | $g$ | $F$ |

- Base: L
  - $h$: 1
  - $M$: 1
  - $Ra$: 1
  - $g$: 1
  - $F$: 1

The approach used after that stage to compute the coefficients, is the traditional Cramer’s method yielding the following approximated coefficients in the case of Table 4.

$$Il_h = h, M^{a1}, Ra^{b1}, g^{c1}, F^{d1}$$

The coefficients demonstrate the dimensional homogeneity of the network of variables influencing $h$ in Figure 7. It is not a complete proof of the validity of the model but it helps according to the authors detecting potential missing variables or completely irrelevant variables added to models. This aspect is studied more deeply below.

The usage of the clustering approach requires some guidelines to group the variables (i.e. like in Table 4). An important first heuristic rule is proposed at this stage:
1- It is advised to create clusters of grouped variables which are dimensionally rich (i.e. minimizing the number of 0 in the matrix $[A]$ and combining variables of different types instead of variables measured using similar units).

The other important heuristic rule to apply when grouping variables is the following:

2- Group variables that are activated at the same time in the behavior of the studied system.

The description of the air bearing behaviour can exemplify the rule.

A pressure $P$ is introduced in the air bearing through the canal of diameter $d$. This is leading to a pressure $P_c$. This pressure $P_c$ is acting simultaneously on the air bearing surface of diameters $D_I$ and $D_E$ creating a lifting force $F_c$, this lifting force $F_c$, if superior to $F$ is generating a gap $h$. An output flow of compressed air $f_o$ escapes through this gap diminishing the pressure $P_c$ and the lifting force $F_c$ which then diminishes $h$ and the output flow of air $f_o$.

The timeline (Figure 9) summarizes this description. Using the information provided by Figure 6, one will notice that two clusters of variables are appearing ($P$, $d$) and ($P_c$, $D_E$, $D_I$). These two clusters influence $P_c$ and $K$, respectively, in Figs. 6 and 7. This means concretely that the dimensionless groups used to describe the variables influencing $P_c$ and $K$, respectively should be formed of this cluster when using Cramer’s method.

![Timeline](image.png)

**Figure 9: Timeline summarizing the activation of the major variables describing the behaviour of the air bearing**

In the graph generated by the experts group in Figure 7 and 7, the output pressure $P_c$ is a hub influenced by several variables but in turn also influencing several variables. The Table 5 is summarizing the different variables associated with $P_c$. 

Table 5: Initial matrix for $n_{Pc} = Pc \cdot P^{\alpha_1} \cdot d^{b_1} \cdot h^{c_1} \cdot fo^{d_1} \cdot p^{e_1}$

| $n_{Pc}$ | Influenced variables [B] | Influencing variables (A) |
|----------|--------------------------|--------------------------|
|          | $Pc$ | $P$ | $d$ | $h$ | $fo$ | $p$ |
| Base dimensions |
| L | 1 | 1 | 1 | 1 | 3 | -3 |
| T | -2 | -2 | 0 | 0 | -1 | 0 |
| M | 1 | 1 | 0 | 0 | 0 | 1 |

The Table 6 below is presenting a clustering approach which is not following the two clustering principles proposed above to compute $Pc$.

Table 6: Clustered matrix not following the heuristic principle

| $n_{Pc}$ | Influenced variables [B] | Influencing variables (A) |
|----------|--------------------------|--------------------------|
|          | $Pc$ | $P$. $d$. $h$. $fo$. $p$ |
| Base dimensions |
| L | 1 | 1 | 2 | 0 |
| T | -2 | -2 | 0 | -1 |
| M | 1 | 1 | 0 | 1 |

The “wrong” clustering of Table 6 is generating the dimensionless number:

$II_{Pc} = Pc \cdot P^{\alpha_1} \cdot d^{b_1} \cdot h^{c_1} \cdot fo^{d_1} \cdot p^{e_1}$

According to this dimensionless number, the output pressure $Pc$ in the air bearing is solely influenced by the input pressure $P$. The pressure drops due to the diameters; mass density of the fluid and flow that are known to have an impact on the output pressure are not grasped by this initial computation. The computation is valid dimensionally but in practice some physical phenomena that nevertheless really impact the output pressure $Pc$ are not integrated in the dimensionless number. If the proposed heuristic is applied, another clustering emerges in Table 7. This leads to the following clustering minimizing the number of 0 in matrix $A$.

Table 7: Clustering matrix following the heuristic principle

| $n_{Pc}$ | Influenced variables [B] | Influencing variables (A) |
|----------|--------------------------|--------------------------|
|          | $Pc$ | $P$. $d$. $fo$. $h$. $p$ |
| Base dimensions |
| L | 1 | 2 | 4 | -3 |
| T | -2 | -2 | -1 | 0 |
| M | 1 | 1 | 0 | 1 |

After applying the Cramer method the coefficient of the equation are computed and it leads to the following dimensionless number:

$II_{Pc} = Pc \cdot P^{1.33} \cdot d^{1.33} \cdot h^{0.67} \cdot fo^{0.67} \cdot p^{0.33}$

In this dimensionless number all the variables are actively participating to the explanation of the phenomenon. It is also possible to see that the contribution to $Pc$ is proportional to the value of the exponent, according to the
rearranged equation below. $P$ and $d$ are participating positively to $P_c$ when $f_o$ and $p$ are acting negatively. $f_o$ as a bigger impact than $p$.

$$P_c = ll_{pc}P^{1.33}, d^{1.33}, h^{-0.67}, f_o^{-0.67}, p^{0.33}$$

The exponents give a direct evaluation of the contributions of individual variables to the target variable $P_c$. The exponent does not take into account the order of magnitude of the different variables of the equation. Nevertheless the exponent consideration can be used further for qualitative considerations later in a complementary study. The present article is not taking benefit of this observation. Another element that can be detected using this dimensional homogeneity approach is the evaluation of the completeness of a set of variables. For example, the variables influencing $F_c$ in Figure 7 are considered without taking into account the mass density of the fluid $p$. The Table 8 is directly resulting from Figure 7. The set of influencing variables is clearly inappropriate according the Cramer’s rule. Indeed, $D_I$ and $D_E$ are generating two similar columns in $A$ and consequently a null determinant. If those variables are combined, the matrix $A$ becomes a 2x3 matrix. This implies that another variable should be added to the initial set of influencing variables from Figure 7. This variable should contain base dimensions different from 0 for $T$ (Time) or $M$ (Mass). Several potential variables from our initial set of variables can influence $F_c$ directly (Table 8). The most probable variable is the fluid mass density $p$. This is resulting to Table 9 in which the proposed heuristic has been considered.

Table 8: Initial matrix for $H_{pc} = F_c, D_I, D_E, P_c$

| $n$ | $F_c$ | Influenced variables [B] | Influencing variables (A) |
|-----|------|------------------------|------------------------|
|     |      | $F_c$ | $D_I$ | $D_E$ | $P_c$ |
| Base dimensions | L | 1 | 1 | 1 | -1 |
|     | T | -2 | 0 | 0 | -2 |
|     | M | 1 | 0 | 0 | 1 |

Table 9: Expended initial set of influencing variables by adjunction of $p$

| $n$ | $F_c$ | Influenced variables [B] | Influencing variables (A) |
|-----|------|------------------------|------------------------|
|     |      | $F_c$ | $D_I, P_c$ | $D_E$ | $p$ |
| Base dimensions | L | 1 | 0 | 1 | -3 |
|     | T | -2 | -2 | 0 | 0 |
|     | M | 1 | 1 | 0 | 1 |

The computation using the Cramer’s method is providing the following coefficients.

$$H_{pc} = F_c, D_I^{-1}, D_E^{-1}, P_c^{-1}, p^0$$

The mass density $p$ has a 0 coefficient. All the other valid combinations used for the clustering are providing the same results. This tend to prove that even if the addition of a new variable is necessary for applying the Cramer’s rule, the added variable is not playing a role in the physical phenomena. This has been verified by checking that all the other combinations provide the same results. For the two other dimensionless combinations required from Table 3, $n_K$ and $n_{fo}$, the same heuristic has been applied in order to minimize the number of 0 in matrix $A$. It seems that this principle is not sufficient; indeed different results can be found for $n_K$ and $n_{fo}$. Some of these results will generate dimensionless groups where several parameters have a zero coefficient. The authors propose to add to the
two heuristic rules a third rule in order to select the most appropriate result for the dimensionless groups when several potential results are available:

3- The selected dimensionless groups should be chosen so that the number of exponents equal to 0 is minimized.

In the case of the air bearing, the rule seems to match well with the proposals made by the experts group. The two remaining computed dimensionless groups computed according to this heuristic are the following:

\[ II_{R} = K \cdot D_{E}^{-1} \cdot D_{T}^{-1} \cdot h^{4} \cdot P_{c}^{-1} \cdot p^{0} \]

\[ II_{fo} = f_{o} \cdot d^{-1} \cdot P_{adm}^{-1} \cdot P_{c}^{-1} \cdot h^{-1} \cdot V c^{-1} \]

The graph established in Figure 7 is obviously validated using the method presented above. The next section will present the definition of the objectives attached to the performance variable group and their propagations backward in the influencing network. The consequence of the orientation of the edges of Figure 7 is not treated in this article. It should be mentioned that a causal ordering algorithm has been already proposed by Coatanéa et al. [5] as an extension of an existing algorithm proposed by Peng and Sheng [10]. The algorithm is not fully capable yet of generating the entire causal ordering of the variables but can be used interactively with a group of users.

### 4.5. Search for physical contradictions using the mathematic propagation machinery

Mathematic tools have been developed by Bashkar and Nigam [3] in order to analyse the interactions between variables forming a dimensionless product. The development of dimensionless groups in the previous section is used in this section to develop the propagation mechanism. In this tools, the following is useful for us in order to infer the nature of the qualitative relation between variables. A specific dimensionless group can be expressed in the following manner according to Equation 3:

A specific dimensionless group can be expressed in the following manner according to Equation 3:

\[ y_{i} = \prod_{k} x_{j}^{-\alpha_{ij}} \cdot x_{i}^{-\alpha_{ai}} \cdot x_{m}^{-\alpha_{mi}} \]

This equation can be written:

Eq. 4: \[ \frac{y_{i}}{x_{j}} = \prod_{k} \frac{x_{j}^{-\alpha_{ij}}}{x_{j}} \cdot \frac{x_{i}^{-\alpha_{ai}}}{x_{i}} \cdot \frac{x_{m}^{-\alpha_{mi}}}{x_{m}}. \]

The derivative of this dimensionless group has the form:

Eq. 5: \[ \frac{\partial y_{i}}{\partial x_{j}} = -\prod_{k} \alpha_{ij} \cdot \frac{x_{j}^{-\alpha_{ij}}}{x_{j}} \cdot \frac{x_{i}^{-\alpha_{ai}}}{x_{i}} \cdot \frac{x_{m}^{-\alpha_{mi}}}{x_{m}} \]

Consequently the derivative takes the general form:

Eq. 6: \[ \frac{\partial y_{i}}{\partial x_{j}} = -\alpha_{ij} \cdot \frac{y_{i}}{x_{j}} \]
Knowing the sign of the exponents, $\alpha_{ij}$, the sign of the intra dimensionless group derivative can be determined by using the Equation 6.

Using the dimensionless numbers defined in the previous section, it is possible to define the signs of the derivatives linking the different variables of the air bearing model presented above.

For example, the objective to maximize $F_c$ is propagated backward considering the dimensionless groups $\Pi_{F_c} = F_c, D_E^{-1}, D_I^{-1}, P_C^{-1}$.

Using Equation 6, the following derivatives can be defined.

$$\frac{\partial F_c}{\partial D_E} = +1 \frac{F_c}{D_E}$$

$$\frac{\partial F_c}{\partial D_I} = +1 \frac{F_c}{D_I}$$

$$\frac{\partial F_c}{\partial P_C} = +1 \frac{F_c}{P_C}$$

It means that $F_c$ and $D_E$ are varying in the same direction. If we want to maximize $F_c$, there is also needs to maximize $D_E, D_I$ and $P_C$.

The interpretation of the derivatives is the following; if we want to maximize $F_c$, there is also needs to maximize $D_E, D_I$ and $P_C$.

The same principle is applied on all the other 5 dimensionless groups computed in the previous section.

The Figure 10 has been derived from this calculation. The propagation method has been applied in this example to the design variables in green and blue but not for the non-controlled variables in black.
In Figure 10, the propagation backward from the objectives set to the objective variables (i.e. red variables) are also generating objectives on the other variables such as the design variables (i.e. blue and green variables). If contradictory objectives are resulting from these propagations, it is resulting in a physical contradiction.

The Figure 10 can be simplified to show more explicitly the main contradictions (see Figure 11).

The next step of the analysis consists of listing and selecting between the different design strategies that can be used to overcome those three major contradictions in the design of air bearings. This work has already been done inside our research group. It is not presented in this paper but is the topic of publications under preparation at the moment.
5. Discussion-Conclusion

The approach developed in this article offers a method that can be supported by a computer analysis tool for systematically highlighting technical and physical contradictions. Indeed they are both appearing in the graph in Figure 11. The authors claim that the future possibilities embedded in the present research in terms of systematic mapping of design strategies are important. Nevertheless, the level of innovation addressed by the present article implies the existence of an existing system or at least of sufficiently precisely described concepts of solutions. The types of contradictions addressed directly in the case study of the article are the physical contradictions which are the ones visible at the most detailed level of representation of system architecture. The next step of the approach introduced in this article is consisting of listing all the potential design strategies that can be employed and most importantly to evaluate the potential impact associated with the usage of a certain design strategy. The main interest of the article is lying in the fact that this research work is trying to develop connections between different methods and approaches to form a coherent whole that can be supported by means of interactive computer tools. In this work, the authors have tried to demonstrate that TRIZ, graph theory and dimensional analysis can form parts of a
coherent holistic approach supporting the design process. In future publications, the rest of the approach will be presented and the innovative results associated with the usage of this method in the air bearing context will be presented, too.

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References

[1] Altshuller, G. S., To Find an Idea, Nauka, Novosibirsk 1986 (In Russian).
[2] Barenblatt G. I., Similarity, Self-similarity and Intermediate Asymptotics, Consultant Bureau, Plenum, New York, 1979.
[3] Bhattachar R., Nigam A., Qualitative physics using dimensional analysis, Artificial Intelligence, vol. 45, pp. 73-111, 1990.
[4] Coatanéa E., Conceptual Modelling of Life Cycle Design: A Modelling and Evaluation Method Based on Analogies and Dimensionless Numbers, PhD dissertation, ISBN 951-22-7853-7, ISBN 951-22-7852-9, 2005.
[5] Nonsiri S., Coatanéa E., Medyna G., Noack A., Kiviluoma P., Calonius O., An Early Modelling and Simulation approach for Fast evaluation of early design concepts, Journal of Engineering Design, 2013, (submitted).
[6] Hirtz, J. Stone R. B., McAdams D. A., Szykman S. and Wood K. L., A functional basis for engineering design: Reconciling and evolving previous efforts, Springer-Verlag 2002, Research in Engineering Design 13, pp. 65–82, 2002.
[7] INCOSE Systems Engineering Handbook V3.2, INCOSE Editor 2010.
[8] Matz W., Le principe de similitude en Génie Chimique, Dunod, Paris, 1959.
[9] Newman M. E. J., Networks: An Introduction. Oxford: Oxford University Press, 2010. ISBN 0-19-920665-1.
[10] Shen, Q., Peng, Combining Dimensional Analysis and Heuristics for Causal Ordering – In Memory of Dr RobMilne – Rob Milne: A Tribute to a Pioneering AI Scientist, Entrepreneur and Mountaineer, A. Bundy and S. Wilson (Eds.), IOS Press, 2006.
[11] Sonin A.A., The physical basis of dimensional analysis, 2nd edition, Department of Mechanical Engineering MIT Cambridge, MA 02139, 2001.
[12] Wilson, P. F., Dell, L. D., and Anderson, G. F., Root cause analysis: a tool for total quality management; ASQC Quality Press, Milwaukee, 1993.