Quantized circular photoinduced orbital magnetization in multifold Weyl semimetal

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Multifold fermions are among the most promising candidates for directly probing the materials topology, such as the topological charges and Chern numbers. Recently, the quantized circular photogalvanic effect (CPGE) was predicted for the chiral Weyl semimetals (WSMs). Such a remarkable quantization has been generalized to the unconventional multifold fermions, and soon afterwards identified firstly for the multifold fermions in experiments. However, most of the rich topological properties of the multifold fermions remain largely unveiled so far. Here we propose the remarkable quantized circular photoinduced orbital magnetization (CPOM) for the multifold fermions. In particular, we demonstrate that the manifested quantizations are uniquely connected with the topological invariants for the spin-1 and spin-3/2 Weyl fermions. Furthermore, we show the quantization signature also holds for the general multifold fermions, which can be understood as a generic feature resulting from the intrinsic scale invariance. These interesting results are all numerically verified in this work, which can be experimentally checked in realistic multifold Weyl semimetals.

I. INTRODUCTION

Topological Weyl semimetals (WSMs) harboring Weyl fermions have been widely studied in recent years [1–5]. Generally, Weyl fermions are featured by linear band crossings at the so-called Weyl nodes, which individually act like monopole of Berry curvature and carry certain topological charge. Such intriguing Weyl topology give rise to a plethora of novel physical effects in WSMs [6–8], one of which is the bulk circular photogalvanic effect (CPGE) [9–20]. Most notably, the chiral WSMs in Pauli blocked regime (i.e., only a single Weyl node contributes) are predicted to manifest a frequency independent quantized CPGE, where the quantization uniquely relates to the integer topological charge of the Weyl node [21]. Obstructed by the presence of mirror symmetry, however, the experimental observation of this quantization signature remains to date as a challenge for the conventional WSMs.

The conventional Weyl fermions present twofold band degeneracy at the Weyl nodes, each of which carries single topological charge. In contrast, the unconventional Weyl fermions, termed as multifold fermions, reveal band degeneracies higher than two and possess high (≥ 2) topological charges [22–24]. A number of such multifold fermions have been predicted to exist and experimentally identified later in different chiral multifold semimetals [25–33]. Interestingly, the multifold semimetals do not possess any inversion symmetry, making them excellent platforms for probing a variety of nonlinear optical responses. For instance, the quantized CPGE previously proposed for conventional WSMs can also be exhibited by multifold fermions [34–38], and has been successfully demonstrated in multifold semimetals in the very recent experiments [39]. Yet, more extensive studies to reveal the rich topological properties of the multifold fermions are still highly desired.

The intrinsic orbital magnetization, similar to Berry curvature, performs fundamentally important role in many interesting phenomena such as the various linear anomalous transports and orbital magnetoelectric effect, etc [40–43]. Very recently, the electromagnetic field induced magnetization has generated mounting of interest. In linear regime, the electric field induced orbital magnetization have been experimentally observed in strained MoS₂ [44]. In addition, several works predict that the static magnetization can be generated through the so-called inverse Faraday effect in the nonlinear regime [45–48]. Here we propose another fascinating light induced orbital magnetization, namely the quantized circular photoinduced orbital magnetization (CPOM) for multifold fermions. In strike contrast to the earlier studies, our work reveals a quantization signature of the light induced orbital magnetization that remarkably relates to the topological charge of multifold fermions, exactly analogues to the quantized nonlinear photocurrent response, i.e., CPGE.

As a cousin effect of CPGE, the CPOM in this work is derived similarly to the injection current in CPGE [9, 10, 21]. We find that, the CPOM [see Eq. (6)] directly depends on the transfer of the orbital moments (i.e., the non-zero difference between the orbital moments) rather than the velocity difference found previously for the injection current. Because of the inexistence of the non-zero difference among the orbital moments, we point out that, the CPOM always vanishes in two-band systems, e.g., the conventional spin-1/2 WSMs. Instead, here we study the CPOM for the multifold fermions with multi energy bands. This is significantly different from the previous studies on nonlinear optical responses, which mostly are conducted for two-band models. Next we ap-
mately the CPOM for isotropic spin-$j$ ($j = 1, 3/2$) fermions, i.e., the SU(2) symmetric multifold fermions. Interestingly, we find the trace of the CPOM tensor [see Eq. (11)] is merely determined by the relevant Chern numbers besides the Fermi velocity and some fundamental constants. This is the most remarkable finding of our work. It explicitly reveals the quantization signature of CPOM for spin-$j$ fermions, and more importantly, uncovers another fascinating optical effect in addition to CPGE that can fingerprint the topological invariants. Moreover, we also study the CPOM for the more general multifold fermions that do not satisfy the SU(2) symmetry (i.e., anisotropic multifold fermions), where the CPOMs are surprisingly found to manifest quantized plateaus as well. Comparing to spin-$j$ fermions, the quantized values in these scenarios are accordingly reduced for multifold fermions with different anisotropies. By involving the scale-invariant property of the general multifold fermions and CPOM [Eqs. (8, 11-13)], we argue that, such a quantization signature is preserved universally for realistic WSMs. We also show that, the optical transitions in our scheme in generating the light induced orbital magnetization for (either isotropic or anisotropic) multifold fermions, are confined by $|n - m| = 1$ with $m, n$ being the band indices, and only the lowest two or the topmost two bands are responsible for the non-zero CPOM (as schematically shown in Fig. 1). The features mentioned above are also numerically verified in this work (see Figs. 2, 3).

The rest of the paper is organized as follows. In Sec. II, we derive the analytical equations for the CPOM based on density matrix approach. We then generalize the derived results of CPOM for the SU(2) symmetric spin-$j$ fermions in Sec. IIIA, where we present the remarkable equations for the quantized CPOM. Then it follows by the detailed analysis based on the explicit models of the quantized CPOM for threefold and fourfold fermions in Sec. IIIB. We also briefly discuss the quantized CPOM for the sixfold fermions in Sec. IIIB. Finally we end with a brief discussion and conclusion in Sec. IV.

II. FORMALISM OF CPOM

Within the context of semiclassical interpretation, the orbital magnetization can be understood as a combined effect of the orbital moment and the boundary current, which originate from the local self-rotation and global center-of-mass motion of the electron wave packet, respectively [40]. Normally, the illuminated circular light is believed to focus inside the bulk of material, such that the boundary current effect is inconsequential and only the contribution from the orbital moment is involved in the light-magnetization interaction. Therefore, we will center us upon the orbital magnetization resulting from the orbital moment in the following discussions.

For a wave packet with center of mass $\langle \hat{r}_b \rangle$, the orbital momentum operator can be defined as

$$\hat{\mu}^a = -\frac{e}{c} \epsilon^{abc} \{\hat{r}_b - \langle \hat{r}_b \rangle, \hat{v}_c\}. \quad (1)$$

Here, $\hat{r}_b$ and $\hat{v}_c$ are the position and velocity operator, respectively. The repeated Cartesian indexes $a, b, \cdots$ imply the Einstein summation hereafter. Thus the orbital moment is found as [40],

$$\mu^a_n (k) = -\epsilon^{abc} \frac{ie}{2\hbar} \langle \partial_b u_{nk} \rangle \left( \hat{H}_k - \epsilon_{nk} \right) \langle \partial_c u_{nk} \rangle, \quad (2)$$

where $\partial_b = \partial/\partial k_b$, and $\epsilon_{nk}$ is the electron energy corresponding to the Bloch state $|u_{nk}\rangle$ with band index $n$ at crystal momentum $k$. Note that, the orbital moment $\mu^a_{nk}$ transforms exactly like the Berry curvature under symmetry operations, which vanishes only when the time reversal symmetry and inversion symmetry are both preserved. A net thermodynamic average of $\mu^a_{nk}$ straightforwardly gives the orbital magnetization in our scheme. Of more interest in this work is the effect of the orbital magnetization on the nonlinear optical responses.

A permanent magnetization can result from the interactions between the medium and circularly polarized light [45–48]. Consider a monochromatic incident light defined by $E_\theta (t) = |E_\theta (\omega) e^{-i\omega t} + E_\theta (-\omega) e^{i\omega t}|/2$, where $E_\theta (\omega)$ is the complex amplitude of the oscillating electric field at angular frequency $\omega$. Under the excitation of $E_\theta (t)$, the light induced static magnetization can be effectively written as

$$M_a = \sum_{\pm} \chi^{a}_{\pm \omega} (-\omega, \omega) E_\theta (-\omega) E_c (\omega), \quad (3)$$

where $\chi^{a}_{\pm \omega} (-\omega, \omega)$ is the three-rank susceptibility tensor. Note that, $M_a$ given above belongs to a second-order response, which corresponds to the leading contribution of the photoinduced static magnetization. Using the density matrix approach, this susceptibility tensor can be explicitly written as

$$\chi^{a}_{\pm \omega} (-\omega, \omega) = \frac{e^2}{\omega^2} \sum_{lmn} \frac{d\theta}{(2\pi)^3} \left[ \frac{f_{nl} f_{ln} a_{nm} b_{ml}}{(\epsilon_{lm} + i\eta) (\epsilon_{ln} + \hbar\omega + i\eta)} \right. \right.$$

$$+ \left. \left. \frac{f_{nl} v_{ln} a_{nb} b_{ml}}{(\epsilon_{ln} - i\eta) (\epsilon_{lm} - \hbar\omega - i\eta)} \right]\right), \quad (4)$$

where $\eta = \hbar/\tau$ is introduced to cover the relaxation process with relaxation time $\tau$, $f_{nl} = f_n - f_l$ with $f_{nl}$ is the Fermi-Dirac distribution function for Bloch electrons in band $n$ ($l$), $\epsilon_{ln} = \epsilon_l - \epsilon_n$ indicates the energy differences between band $l$ and $m$, $v_{mn}^b$ and $\mu_{ln}^a$ are the matrix elements expressed in the band basis for the velocity operator $v^a_k = \hbar^{-1} \partial_k H_k$ and orbital moment operator [see Eq. (1)], respectively.

The susceptibility tensor satisfies $\chi^{a}_{\omega} (-\omega, \omega) = \chi^{a}_{\omega} (\omega, -\omega)^*$ since ultimately the physical observable quantities are real-valued. Also, Eq. (3) remains unchanged as simultaneously exchange $b \leftrightarrow c$ and $\omega \leftrightarrow -\omega$. 
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$$H = v_F \hbar \mathbf{k} \cdot \mathbf{S},$$

where $v_F$ is the Fermi velocity and $\mathbf{k}$ is the momentum
measured from the band crossing points. Matrix $\mathbf{S}$ is

![FIG. 1. Schematic band structures for spin-$j$ Weyl fermions with $j = 1/2, 1, 3/2$. Panel (a) depicts the conventional spin-1/2 Weyl fermion, where the Chern numbers of the bands are $C = -1, 1$ from low to high. The wave pockets of the conduction and the valence band carry exactly the same orbital moment, i.e., $\mu_c = \mu_v$. Consequently, the photoinduced transition between the two bands cannot give finite CPOM based on Eq. (7). For the spin-1 fermion in panel (b), however, the nearest two bands correspond to two different orbital moments. Hence it exhibit a nonzero CPOM. The Chern numbers assigned to each band are $C = -2, 0, 2$ from low to high. Similarly, a finite CPOM can exist as well for the spin-3/2 in panel (c). The non-zero difference between the orbital moments can either come from the lower two bands with Chern number $C = -3, -1$, or from the higher two bands with Chern numbers $C = 1, 3$. Note that, the middle two bands for spin-3/2 fermion, showing the same orbital moment, does not allow any optical transition. All these three panels show isotropic dispersion relations.]

a generalization of the Pauli matrix $\sigma$ applied for the
conventional spin-1/2 Weyl fermions. The case with matrices satisfying $[S_a, S_b] = i \epsilon^{abc} S_c$ is of particular interest to us, where the depicted multifold fermions are called spin-$j$ Weyl fermions hereafter. The spin-$j$ Weyl fermions with $N$-dimensional ($N = 2j + 1$) spin representation $S_a$ is SU(2) symmetric. On the other hand, the SU(2) symmetry is broken for multifold fermions with matrices that do not form a closed Lie algebra. Such anisotropic multifold
fermions, however, can be adiabatically connected to the spin-$j$ Weyl fermions, as will be discussed later [see Fig. 2(a) and 4(a)].

One can define the eigenstates for spin-$j$ Weyl fermions as $|j, j_n\rangle$, where $\mathbf{k} = k / k$ is the direction of the quantized axis of interest, $j$ is the total orbital quantum number, and $j_n$ is the projected orbital quantum number along $k$ for the $n$th band. Before moving to the CPOM for spin-$j$ Weyl fermions, we notice that the optical transition is allowed only between the nearest two bands. To quickly prove this point, one can directly compute the velocity matrix elements involved in the optical transition, i.e., $v^a_{mn}/v_F = \langle j, j_m| S_a |j, j_n\rangle$. By rotating the axes, it follows $v^a_{mn}/v_F = \langle j, j_m| \lambda_1 S_z + \lambda_2 S_- + \lambda_3 S_+ |j, j_n\rangle$, where $S_\pm = S_z \pm i S_y$ are the ladder operators, and $\lambda_1, 2, 3$ are constant coefficients. Based on this equation, one can conclude that the velocity element is nonzero only if $|m - n| \leq 1$. Furthermore, since only the off-diagonal components (i.e., $m \neq n$) of the velocity matrix are relevant to the CPOM [see Eq. (7)], the effective optical

III. GENERAL THEORY OF CPOM FOR MULTIFOLD FERMIONS

A. Quantized CPOM for spin-$j$ Weyl fermions

In condensed matter systems, unlike the much higher
symmetry requirement of Poincaré group in fundamental
particle physics, the emergent quasiparticles only need
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transition can occur if and only if |m - n| = 1.

To reveal the CPOM for spin-\( j \) Weyl fermions, we first derive the analytical expressions for the orbital moment for the eigenstates \( |\hat k, j, j_n\rangle \),

\[
\mu_n(k) = ev_F \frac{\hbar}{2k} \left[ j(j + 1) - \frac{1}{4} C_n^2 \right].
\]

(9)

Here, relations \( \partial_j |k\hat z, j, j_n\rangle = -ik^{-1} \Sigma_y |k\hat z, j, j_n\rangle \) and \( \partial_j |k\hat z, j, j_n\rangle = ik^{-1} \Sigma_y |k\hat z, j, j_n\rangle \) have been used, and \( C_n = 2n\tau F / |v_F| \) denotes the Chern number of the \( nth \) band. Note that, for a given spin-\( j \) system, \( \mu_n \) uniquely depends on the Chern number of each band. Without loss of generality, we set the Fermi level \( E_F < 0 \) in the following discussions. A non-zero orbital moment difference of the given two bands is prerequisite for generating nonzero CPOM. Combining with the optical transition of the given two bands is prerequisite for generating nonzero CPOM. Consequently, we arrive at

\[
\text{Tr} (\lambda_\omega) = \lambda^{-1} \text{Tr} (\lambda_\omega).
\]

(12)

The above equation also indicates the CPOM takes the following form

\[
\omega \text{Tr} [\beta] = D,
\]

(13)

where \( D \) is a material dependent constant. This implies that, even for the anisotropic multifold fermions, a CPOM plateau is expected to be observed.

In the next section, we will apply the derived CPOM formula Eq. (7) to the concrete models for the threefold, fourfold Weyl fermions. The sixfold Weyl fermion is also discussed. Most importantly, we will show that the CPOM plateau can exist for all multifold fermions.

**B. CPOM plateau for general threefold, fourfold and sixfold fermions**

The most general Hamiltonian for the threefold fermion protected by crystal symmetries is given by [23]

\[
H_{3f}(\phi, k) = hv_F \begin{bmatrix}
0 & e^{i\phi} k_x & e^{-i\phi} k_y \\
e^{-i\phi} k_x & 0 & e^{i\phi} k_z \\
e^{i\phi} k_y & e^{-i\phi} k_z & 0
\end{bmatrix},
\]

(14)

where \( \phi \) is real-valued material-dependent parameter. Since \( v_F \) only changes the overall energy scale, the phase diagram of \( H_{3f} \) is parameterized by \( \phi \) as shown in Fig. 2(a). The SU(2) symmetric spin-1 phases are marked in triangle, where \( \phi = (4n \mp 1) \pi / 6 \) and the sign \( \pm \) correspond to the opposite chirality of the fermions (i.e., \( H_{3f} = \pm hv_F k \cdot S \)). The spin-1 phase is protected by time-reversal symmetry. As gradually adjusting \( \phi \) from the spin-1 phase with positive chirality to the one with negative chirality, there exist the phase boundary \( \phi_0 = n \pi / 3 \) as shown in dashed lines in Fig. 2(a), where the bands become degenerate along directions \( |k_x| = |k_y| = |k_z| \). Apart from the phase boundary, the general threefold [SU(2) asymmetric] Weyl fermion can be adiabatically connected to the spin-1 fermion within each sector of the phase diagram. The threefold fermions are allowed to exist in SGs. (195-199) and SGs. (207-214) in the absence of the spin-orbital coupling (SOC) [24], and in SG. 199 and SG. 214 in the presence of SOC [23, 24].

The energy spectrum for spin-1 Weyl fermion is isotropic as shown in Fig. 2(b) resulting from the SU(2) symmetry of the Hamiltonian. The orbital moment \( \mu_n^z \) is also presented, and exhibits divergent features as indicated by Eq. (9). The Chern number of each band is two times of its corresponding \( j_n \) giving \( C_n = (-2, 0, 2) \), respectively. Due to the optical transition rule, as discussed in Sec. III A, only the lower two bands are responsible for the CPOM [also see Fig. 2(d)]. The numerical result for the spin-1 case is given in Fig. 2(f), which shows that the CPOM is quantized at \( \omega \text{Tr} [\beta] = 8 \beta_0 \), consistent with
Eq. (11). Note that, the quantization plateau always exists for $\hbar \omega > |E_F|$ since the optical transition surface $S_{21}$ is always closed for spin-1 Weyl fermions.

Next, we discuss the CPOM for the general threefold fermions. Without loss of generality, here we consider $\phi = 1.3 \times \pi/3$. The band structure and the orbital moment are shown in Fig. 2(e), which presents evident anisotropic features. In general, the anisotropic threefold fermions have no analogy to the spin-1 angular momentum, and the optical transitions can occur between any two bands. However, the optical transition rule $|m - n| = 1$ is approximately valid in a large region of the phase diagram. The transition rate for $H_{3f}$ with $\phi = 1.3 \times \pi/3$ is shown in Fig. 2(e), where $\Omega_{13}$ that dissatisfies angular moment conservation is found to be negligible compared to $\Omega_{12}$. Thus, it is reasonable to only consider the optical transition of the lower two bands even for the general multifold fermions.

The numerical results of CPOM for general threefold fermions are shown in Fig. 2(f). Surprisingly, though not exactly quantized to $8\beta_0$, the CPOM for other anisotropic threefold fermions can still manifest as CPOM plateaus at some other reduced but finite values. Note that, such a CPOM plateau can exist as long as $S_{21}$ forms a closed transition surface for the incident light with energy $\hbar \omega$. In addition, there exists a critical $\phi$, above which $S_{21}$ cannot be closed under any range of energy of the incident light. The CPOM plateau continuously decrease as $\phi$ gradually approaches the phase boundary at $\phi = \pi/3$, several such cases are shown in Fig. 2(f). It can be understood as follows, that in contrast to the spin-1 phase which is an isotropic surface (not shown here), the optical transition surface $S_{21}$ is in the shape of tetrahedron. A concrete example with $\phi = 1.3 \times \pi/3$ is shown in Fig. 4. Obviously, the orbital moment difference $\Delta_{12}$ is mainly distributed along direction (111), whereas the Berry curvature along a different direction (111). As a result, the product of these two quantities, which is responsible for CPOM, turns out to be smaller than that of the isotropic phase.

By far, we have systematically analyzed the CPOM plateau for the threefold fermions, either for the SU(2) symmetric spin-1 phases protected by time reversal symmetry or the other anisotropic threefold fermions phases that are not SU(2) symmetric. In what follows, we will show the CPOM plateau for the fourfold fermions in a similar manner.

The most general Hamiltonian for the fourfold Weyl

![FIG. 2. (a) Phase diagram for the threefold fermions in Eq. (14) parameterized by $\phi$ along azimuthal direction. The positive $v_F$ is assumed in (a). The phase diagram is divided into different phase sectors by the phase boundaries shown by the dashed lines. The general phases for anisotropic threefold fermions in each sector are adiabatically connected with the SU(2) symmetric spin-1 phases indicated by the triangles. (b-c) show the band structures for the threefold fermions in the isotropic [i.e., SU(2) symmetric] phase with $\phi = \pi/2$, and in a general anisotropic phase with $\phi = 1.3 \times \pi/3$, respectively. The colors represent the distribution of the orbital moment $\mu_z^\alpha$ associated to each band. (d) The optical translation rate $\Omega_{1m}$ in Eq. (7) for spin-1 fermions, where $\Omega_{13}$ is zero. (e) Same to (d) but for the anisotropic threefold fermions with $\phi = 1.3 \times \pi/3$. Here $\Omega_{13}$ is nonzero, but is considerably smaller than $\Omega_{12}$. (f) The CPOM $\omega\text{Tr}([\beta])$ for threefold fermions plotted as a function of the incident light energy $\hbar \omega$ with different $\phi$ within the phase sector indicated by blue line in panel (a). For the spin-1 phase, the CPOM is quantized perfectly to $8\beta_0$ (the dark blue line), while for the general phases [\phi = (1.4, 1.3, 1.2) \times \pi/3], the CPOMs are still quantized but to different reduced values. The energy windows manifesting the quantized plateaus gradually shrinks with parameter $\phi$ approaching the phase boundary ($\phi = \pi/3$ in our case). Here, the Fermi level is adjusted to $E_F = -0.1\text{eV}$ to form a non-vanishing optical transition surface, and we set $\hbar v_F = 2.0\text{eV} \cdot \text{Å}$.](image)
fermions with negative chirality within the phase sector SOC, and are not allowed in the absence of SOC [23, 24].

fers. (195-199) and SGs. (207-214) in the presence of geral phases can be adiabatically connected to the nearby Weyl fermions. Similar to threefold fermions, other gen-

nal phases can be parameterized by $\phi = \arctan(-3)$ along special directions. Also, the band’s Chern number with $\phi = \arctan(-3)$ has already been ruled out by earlier

works on the fermion classification [23]. Conclusively, we have analyzed the quantized CPOM for all the possible multifold fermions in this work.

IV. DISCUSSIONS AND CONCLUSIONS

In this work, we systematically demonstrate the quan-
tization signatures of the CPOM for multifold fermions, including the threefold, fourfold and sixfold fermions, ei-
ther within or beyond the regime of SU(2) symmetric phases. Closely analogous to the quantized CPGE that has been successfully observed in experiments [39], the quantized CPOM proposed in this work can also be experimentally probed in the realistic multifold semimetals through the state-of-the-art magneto-optical Kerr microscopy [51, 52]. To better estimate the magnitude of CPOM, a response coefficient $G$ can be defined from $M = GI$. Here $I = \varepsilon_0 c E_0^2/2$ is the intensity of the incident light, $\varepsilon_0$ is the permittivity of the vacuum, and $c$ is the speed of light. Combining with Eq. (5), we have $G = \frac{2\pi a}{\varepsilon_0}$. By taking a constant relaxation time $\tau = 1$ ps, we estimate the response coefficient $G$ is of the magnitude $\sim 10^5 \mu_B \AA^{-1}/W$. Such a quantity is experimentally measurable, which describes the amount of orbital mo-
ment that can be generated per Watt of the light and per depth of penetration.
Moreover, we want to stress that the quantized CPOM proposed in this work does not rely on the chirality or helicity [see Eq. (11)], thus is different from the other earlier proposed effects in multifold fermions [21, 46, 53]. We also want to mention that the results obtained in this work remain valid for all types of multifold fermions described by the linearized \( k \cdot p \) Hamiltonians. To more accurately characterize the quantized CPOM in the realistic materials, possible improvements to our discussions might necessitate considering the tight-binding models or even first principle calculations, and also the more realistic estimations of the scattering processes, etc. These interesting topics, however, are beyond the scope of this current work, and are left for future studies.

In summary, we show the multifold fermions can reveal a quantized orbital magnetization induced by circularly polarized light. In particular, for those SU(2) symmetric spin-\( j \) phases like the spin-1, 3/2 fermions, the CPOM are exactly quantized to some integers determined by topological Chern numbers apart from some constant parameters. Moreover, we show that the CPOM for general multifold fermions can also manifest as plateaus quantized to some other noninteger values. To the best of our knowledge, our work for the first time reveals the quantization behaviors in the regard of orbital magnetization. The exciting result in our work also provides an alternative way to probe the topological invariant and other relevant topological properties of the multifold fermions. Additionally, the result obtained in this work for multifold fermions can serve as a brand new mechanism to effectively induce and even precisely manipulate the magnetization in bulk materials as well. Of both the theoretical and experimental importance, it is believed our work will stimulate more relevant future studies.

**ACKNOWLEDGMENTS**

This work is supported by the National Key R&D Program of China (Grants No. 2020YFA0308800), the NSF of China (Grants No. 11734003, No. 12061131002 and No. 12004035), the Strategic Priority Research Program of the Chinese Academy of Sciences (Grant No. XDB30000000), and the Beijing Institute of Technology Research Fund Program for Young Scholars.

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See Supplemental Materials for the details about the derivation of Eq. (7).

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