The minimum magnetic field of millisecond pulsars by accretion: application to X-ray neutron star SAX J1808.4-3658 in LMXB

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ABSTRACT

Based on the model of accretion induced the magnetic field decay of the neutron star (NS), the millisecond pulsars (MSPs) will own the minimum magnetic field when the NS magnetosphere radius shrinks to the stellar surface during the binary accretion phase. We find that this minimum magnetic field is related to the accretion rate \( \dot{M} \) as \( B_{\text{min}} \sim 2.0 \times 10^{7} \text{G}(M/0.1M_{\odot})^{1/2} \), where \( M_{\text{min}} = 4.6 \times 10^{15} \text{g/s} \) is the averaged minimum accretion rate required for the MSP formation and constrained by the long-term accretion time, which corresponds to the companion lifetime less than the Hubble time. The value of \( B_{\text{min}} \) is consistent with that of the observed radio MSPs and the accreting MSPs in low mass X-ray binaries, which can be found the case of the application on the minimum and present field strength of SAX J1808.4-3658. The prediction on the minimum magnetic field of MSPs would be the lowest field strength of NSs in universe, which could constrain the evolution mechanism of the magnetic field of accreting NSs.

Key words: accretion: accretion disks – binaries: close – X-rays: stars– stars: millisecond pulsars

1 INTRODUCTION

Millisecond pulsars (MSPs) are with the spin period (\( P \)) less than 10 ms and the magnetic field (\( B \)) around 10^{8.5} \text{G}. They are recycled pulsars (PSRs) through the accretion in the low mass X-ray binaries (LMXBs) (Stairs 2004; Lorimer 2008; van den Heuvel 2009, 2017; Manchester 2017). Since the first MSP (PSR B1937+21) discovered by Backer et al. in 1982, there have been over 300 MSPs recorded in ATNF pulsar catalogue until April of 2018 (Manchester et al. 2005). Their magnetic fields (\( B \)) are believed to decay from ~10^{10.5-15.5} \text{G} (Ho 2013; Luo et al. 2015; Kaspi 2017) to ~10^{5-9}\text{G} in LMXBs by accreting the mass of ~0.1–0.2M_{\odot}, which can be inferred from the observations (Bhattacharya & van den Heuvel 1991; Pinney & Kulkarni 1994; Bhattacharya & Srinivasan 1995; van den Heuvel 2004; Ruderman 2010; Zhang et al. 2011, 2016; Tauris 2015; Manchester 2017). As a comparison, the histogram of field strength of MSPs and normal PSRs are shown in Fig[1].

The model of a MSP formed during the accretion in LMXB was first proposed in 1980s (Alpar et al. 1982). Then, from the observational statistics, Taam and van den Heuvel (1986) found that the magnetic field of neutron star (NS) decayed inversely with the accretion mass, based on which, an empirical formula about the field strength evolution of the X-ray PSR with the accretion mass was presented by Shibazaki et al. (1989). Furthermore, by assuming the frozen magnetic field in the NS crust, the model of the accretion induced the magnetic field decay was proposed (Zhang & Kojima 2006), which was applied to simulate the B-P evolution of accreting NSs, whose results were suitable for those of the observed PSRs (Wang et al. 2011; Pan et al. 2013, 2015).

The magnetic field of MSPs can be estimated by different methods. For the radio and non-accreting MSPs, their magnetic fields can be calculated through the spin period and period derivative (\( P' \)): \( B \approx 3.2 \times 10^{19}(G)(PP')^{1/2} \) (Shapiro & Teukolsky 1983; Bhattacharya & van den Heuvel 1991). For the anomalous X-ray PSRs, the magnetic field can be obtained through modeling the non-thermal X-ray spectra with cyclotron and magnetic Compton scattering processes in the magnetosphere (Güver, Özel & Göğüş 2008). For the X-ray PSRs with high B (e.g. \( B \geq 10^{12} \text{G} \)), one can use the resonant electron cyclotron lines in the X-ray spectra (Caballero & Wilms 2012). For the accreting X-ray MSPs (AMXPs), the magnetic field can be derived through comparing the magnetico...
where \( \dot{p} \) is a rotational parameter of a pulsar catalogue until April of 2018). The samples almost follow a bimodal distribution (Camilo et al. 1994): centered at \( \sim 10^{12} \) G for normal pulsars labelled with the left-inclined bars and \( 10^{8.5} \) G for MSPs labelled with the right-inclined bars.

Figure 1. Histogram of magnetic fields of 2636 pulsars (data from ATNF pulsar catalogue until April of 2018). The samples almost follow a bimodal distribution (Camilo et al. 1994): centered at \( \sim 10^{12} \) G for normal pulsars labelled with the left-inclined bars and \( 10^{8.5} \) G for MSPs labelled with the right-inclined bars.

Figure 2. Binary pulsars and millisecond pulsars (labelled with the dot and crossing symbols) distributed in the magnetic field versus spin period diagram. The ones with \( B < 10^{9} \) G are labelled with the quadrangles. Four solid lines represent the spin up lines with the accretion rates from \( 10^{18} \) g/s (upper) to \( 10^{5} \) g/s (bottom), and the dash line stands for the death line (Bhattacharya & van den Heuvel 1991). The upper-left small B-P diagram is the enlarged close-up of MSPs.

sphere radius to the co-rotation radius of accreting NS (Burderi et al. 1996, 2002), by which the magnetic fields of SAX J1808.4-3658 (Wijnands & van der Klis 1998) and XTE J1751-305 and XTE J0929-314 were estimated in the order of \( \sim 10^{11} \) G (Wijnands et al. 2005). The magnetic field of a NS in LMXB (NS/LMXB) can also be constrained with its detected frequency of kilo-Hertz quasi-periodic oscillation (kHz QPO, van der Klis 2000; Zhang 2004). In addition, if the frequency derivative of an accreting NS/LMXB is detected, the magnetic field can be estimated by the spin up formula of X-ray NS (Ghosh & Lamb 1979), by which the magnetic fields of a dozen NS/LMXBs are obtained as \( 10^{8-9} \) G with their frequency derivatives about \( \sim 10^{-14} \) Hz s\(^{-1}\) (Burderi & Di Salvo 2013).

In this paper, by the model of accretion induced the magnetic field decay of NS, we study the minimum field strength \( B_{\text{min}} \) of MSPs, which corresponds to the minimum accretion rate for the MSP formation in section 2. The consistence is investigated between our \( B_{\text{min}} \) of MSPs and that of the observed MSPs and AMXPs. We also figure out the magnetic field range of NS/LMXB whose companion is the possible upper limit mass. Furthermore, the minimum magnetic field of AMXPs is discussed, including an application for the minimum and present field strength of the first discovered AMXP, SAX J1808.4-3658. The conclusion and discussion are given in section 3.

2 MINIMUM MAGNETIC FIELD OF ACCRETING MSP

2.1 Accretion induced the magnetic field decay

During the binary accretion phase, a NS can evolve to be a MSP by accreting mass of \( \sim 0.1 - 0.2 M_{\odot} \) at least. With such accreted mass, the bottom magnetic field \( (B_{t}) \) of NS can be achieved when its magnetosphere radius shrinks from about a few thousand kilometers to the NS radius (van den Heuvel & Bitzaraki 1995). The estimated work of \( B_{t} \) was obtained by the model of the accretion induced field decay strength of NS (Zhang & Kojima 2006):

\[
B_{t} = 1.32 \times 10^{8} \left( \frac{M}{M_{\text{Edd}}} \right)^{1/2} m_{t}^{1/4} R_{e}^{5/4} \phi^{-7/4},
\]

where \( M \) is the accretion rate in units of the Eddington accretion rate \( \dot{M}_{\text{Edd}} \), \( m_{t} \) and \( R_{e} \) are the NS mass \( M \) and radius \( R \) in units of solar mass and \( 10^{6} \) cm, respectively. The parameter \( \phi \), always taken to be 0.5 (Li & Wang 1999; Ghosh & Lamb 1992), is a model dependent ratio between the magnetosphere radius \( R_{\text{ms}} \) and Alfvén radius \( R_{A} \) of NS (Shapiro & Teukolsky 1983; Frank et al. 2002):

\[
R_{A} = 1.7 \times 10^{8} \text{ (cm)} \left( \frac{M}{M_{\text{Edd}}} \right)^{-2/7} \mu_{30}^{4/7} m_{t}^{-1/7},
\]

where \( \mu_{30} = B_{t} R_{e}^{3} \) is the magnetic moment \( \mu \) in units of \( 10^{30} \) G cm\(^{3}\). \( B_{12} \) is the magnetic field \( B \) in units of \( 10^{12} \) G.

As can be seen from Eq. (1), the bottom magnetic field \( B_{t} \), which is also the minimum magnetic field of NS, is mainly depended on the accretion rate, and little affected by the NS mass and radius. During the NS evolution, if the minimum accretion rate can be deduced, its minimum magnetic field will be acquired. Thus, in the following subsection we will discuss the minimum accretion rate for the MSP formation and the corresponding magnetic field.

2.2 The minimum accretion rate and the corresponding magnetic field of NS

In the binary system, the accretion rate related to the accretion mass \( \Delta M \) and accretion time \( t_{\text{ac}} \) is

\[
M = \Delta M / t_{\text{ac}},
\]

where \( t_{\text{ac}} \) is associated with the age of the companion in the main sequence time \( T_{\text{MS}} \) (Shapiro & Teukolsky 1989):

\[
t_{\text{ac}} = \zeta T_{\text{MS}},
\]

\[
T_{\text{MS}} \simeq 1.3 \times 10^{10} \text{ (yr)} m_{t}^{-2.5},
\]

the parameter \( \zeta \) is usually taken as 10% (Shapiro & Teukolsky 1989), and \( m_{t} = M_{t}/M_{0} \) is the companion mass \( M_{t} \) in units of solar mass. If \( T_{\text{MS}} \) is closed to the Hubble age or the age of universe: \( t_{\text{H}} \simeq 1.38 \times 10^{10} \) yr (Planck et al. 2016), there will exist a
companion star in the main sequence with the minimum mass of about 1.0$M_\odot$ according to Eq. (6). Such result is consistent with the minimum companion mass of 0.8$M_\odot$ in LMXB estimated by van den Heuvel & Bitzaran (1995). When the least accretion mass of $\sim 0.1M_\odot$ of producing a MSP is acquired by the NS in LMXB, there will be the longest accretion time, which can result in a minimum accretion rate $M_{\min}$ with Eqs. (5) and (6). After the arrangement, the accretion rate $\dot{M}$ can be written with the minimum accretion rate $M_{\min}$:

$$M = M_{\min} m^{2.5} \left( \frac{\Delta M}{0.1M_\odot} \right) \left( \frac{\zeta}{0.1} \right)^{-1},$$

$$M_{\min} = 4.6 \times 10^{15} \, \text{g/s}. \tag{7}$$

If $M < M_{\min}$, the NS could not evolve to a MSP in LMXB. Thus, we call $M_{\min}$ to be the critical accretion rate for the MSP formation, which can be found the consistency with the B-P distribution of MSPs, as shown in Fig. 2. MSPs are almost above the spin-up line with $M > 10^{15} \, \text{g/s}$ and approximately gathered around the spin-up line with $M = 10^{15} \, \text{g/s}$. This scenario illustrates the accretion rate requested for the MSP formation is closed to the theory result of $M_{\min} = 4.6 \times 10^{15} \, \text{g/s}$, since the current B-P distribution of MSPs in Fig. 2 should be similar to that of their birth positions (Camilo et al. 1994).

Thus the minimum magnetic field of MSP can be expressed by Eq. (1) with $M_{\min}$ as:

$$B_{\min} = 2.0 \times 10^7 \, \text{(G)} \left( \frac{M}{M_{\min}} \right)^{1/2} \left( \frac{m}{1.4} \right)^{1/4} \left( \frac{R_c}{1.5} \right)^{-5/4} \left( \frac{\phi}{0.5} \right)^{-7/4}, \tag{9}$$

which shows a simply relation to the accretion rate $B_{\min} \sim 2.0 \times 10^7 \, \text{(G)} / (M / M_{\min})^{1/2}$. When taken the critical accretion rate $M_{\min}$ into consideration, the minimum magnetic field of all MSPs for conditions $m = 1.4$ and $R_c = 1.5$ is $B_{\min} \approx 2.0 \times 10^7 \, \text{G}$. It might also be the minimum magnetic field of all NSs in the universe. We list the minimum field strengths of MSPs in Table 1 that are influenced by the selected parameters: the accretion mass, NS radius and accretion rate.

As a comparison, the 16 MSPs with the magnetic fields lower than $10^8 \, \text{G}$ are listed in Table 2, where the parameters of $P$, $\dot{P}$ and $B$ are taken from ATNF pulsar catalogue (Manchester et al. 2005). From Table 2, we find that even the lowest observed B of MSPs, e.g., $B = 4.5 \times 10^7 \, \text{G}$ of PSR J1938+2012, is still higher than our calculated value of $B_{\min}$. It might be attributed to the fact that the magnetosphere of PSR J1938+2021’s has not been compressed onto the the stellar surface in the accretion phase, otherwise its current spin period would be about one millisecond (van der Klis 2000; Zhang 2004). What is more, the field strength of the observed radio MSPs in ATNF data catalogue are deduced by the magnetic dipole model, which would also lead to the difference from our calculated $B_{\min}$. For the AMXPs, the minimum field strengths of the 14 AMXPs calculated by Mukherjee et al. (2015) are also lower than our $B_{\min}$. The explanation might be that $B_{\min}$ of MSPs in this paper is based on the mean value of the minimum accretion rate during the accretion, and the work by Mukherjee et al. (2015) was made with the condition of the lowest X-ray luminosity of those 14 AMXPs.

### 2.3 The field strength range of MSP corresponding to the upper limit mass of $M_c$ in main sequence in LMXB

During the accretion, the maximum accretion mass is expected to be captured by the NS/LMXB. However, a sizeable part of the companion mass loses from the system. Thus the accreted mass of NS will be a fraction to the companion mass with a ratio coefficient $f$:

$$\Delta m = f m_c, \tag{10}$$

and the accretion rate will be:

$$\dot{M} = \frac{M_c}{t_{ac}} \simeq 4.8 \times 10^{10} \, \text{(g/s)} \left( \frac{\zeta}{0.1} \right)^{-1} f m_c^{3.5}. \tag{11}$$

where $f$ may be in the order of 0.5 as argued by van den Heuvel and Bitzaran (1995). Since the mass statistics of NSs and MSPs illustrates that the accreted mass is about $0.1 - 0.2 M_\odot$ for the MSP formation at least (Zhang et al. 2011; Kiziltan et al. 2013; Ozel & Freire 2016; Antoniadis et al. 2016), we prefer $f$ to be about $0.1 - 0.2$.

A NS/LMXB with $m = 1.4$ and $R_c = 1$ for $M_c > 1.0 M_\odot$ possesses the accretion rate larger than $M_{\min}$. Meanwhile it cannot accrete material over Eddington-limiting, e.g., $M < M_{\text{Edd}} = 2 \times 10^{16} \, \text{g/s}$. With the limit of the accretion rate $\dot{M} = M_{\text{Edd}}$, one can derive an upper limit companion mass in the main sequence according to Eq. (11):

$$M_c^{\text{max}} \simeq 5.6 M_\odot \left( \frac{f}{0.1} \right)^{-2/7}. \tag{12}$$

With $0.1 < f < 0.2$, $M_c^{\text{max}}$ will be about $4.6 M_\odot$ to $5.6 M_\odot$. Hence a possible mass distribution of the companion in main sequence in LMXB is $1.0 M_\odot < M_c < M_c^{\text{max}}$, corresponding to the accretion rate of $M_{\min} < \dot{M} < M_{\text{Edd}}$. In such a LMXB, the NS will evolve to be a MSP with the magnetic field of $2.0 \times 10^7 \, \text{G} \leq B \leq 2.8 \times 10^9 \, \text{G}$. While for $M_c > M_c^{\text{max}}$, the accretion time might be too short for the binary system to produce a MSP.

### 2.4 On the minimum magnetic fields of AMXPs

AMXPs exhibit the X-ray outburst during the accreting phase with the luminosity floating a few magnitudes higher than that in the quiescence phase (Wijnands & van der Klis 1998; van der Klis 2000, 2006; Mukherjee et al. 2015). The variation of the luminosity leads

### Table 1. The minimum magnetic fields of MSPs with the different radii and accretion rates

| $B$ (10$^7$ G) | $R$ (km) |
|---------------|---------|
| 4.6 ($\Delta M = 0.1M_\odot$) | 2.0 | 3.3 |
| 9.2 ($\Delta M = 0.2M_\odot$) | 2.8 | 4.6 |

### Table 2. The 16 radio MSPs with the low magnetic fields of $< 10^8$ G (Data from ATNF pulsar catalogue)

| No. | Name | $P$ (ms) | $\dot{P}$ ($10^{-11}$ s$^{-1}$) | $B$ (10$^7$ G) |
|-----|------|---------|----------------|-------------|
| 1   | J1938+2012 | 2.63 | 0.75 | 4.50 |
| 2   | J2229+2643 | 2.98 | 1.52 | 6.81 |
| 3   | J1327-0755 | 2.68 | 1.77 | 6.97 |
| 4   | J1017-7156 | 2.34 | 2.22 | 7.29 |
| 5   | J1216-6410 | 3.54 | 1.62 | 7.65 |
| 6   | J0514-4002A | 4.99 | 1.17 | 7.73 |
| 7   | J1544+4937 | 2.16 | 2.93 | 8.05 |
| 8   | J1745+1017 | 2.65 | 2.73 | 8.61 |
| 9   | J1836-2354A | 3.35 | 2.32 | 8.92 |
| 10  | J2317+1459 | 3.45 | 2.43 | 9.26 |
| 11  | J1640+2224 | 3.16 | 2.82 | 9.55 |
| 12  | J1101-6424 | 5.11 | 1.80 | 9.70 |
| 13  | J1906+0055 | 2.79 | 3.32 | 9.74 |
| 14  | J0034-0534 | 1.88 | 4.97 | 9.77 |
| 15  | J1910-5959A | 3.27 | 2.95 | 9.93 |
| 16  | J0636+5129 | 2.87 | 3.38 | 9.96 |
to the changing of the accretion rate, thus the magnetic field of AMXPs can be derived by equating the magnetosphere radius with the co-rotation radius $R_{\text{co}}$ of NS (Burderi et al. 1996; Zhang & Kojima 2006):
\[ B = B_1 \left( \frac{R_{\text{co}}}{R} \right)^{7/4}, \]
\[ R_{\text{co}} \approx 1.5 \times 10^6 \text{ cm} \times \left( \frac{P}{\text{ms}} \right)^{1/2}, \]
where $P$ is the spin period of AMXPs in units of milliseconds. When $R_{\text{co}}$ equals the NS radius, the AMXP will own the minimum magnetic field. For 19 AMXPs with the spin periods between $1.7 - 6.1$ ms (Patruno et al. 2017), their $R_{\text{co}}$ are in the range of $24 - 56$ km, which are larger than the radii of NSs of $\sim 10 - 15$ km. Thus, it can be concluded that the magnetic fields of 19 AMXPs have not achieved their bottom values.

Another method for judging the minimum magnetic field of AMXPs is comparing their spin frequencies to the kilohertz quasi-periodic oscillation (kHz QPO) frequencies. The upper limit kHz QPO frequency of a AMSP is believed to be the Keplerian frequency with the orbital accreting matter (van der Klis 2000; Zhang 2004):
\[ \nu_k = \left( \frac{G M}{4 \pi^2 R^3} \right)^{1/2} \approx 1839 \text{ Hz} \left( \frac{\dot{M}}{R_0} \right)^{1/2}, \]
where $G$ is the gravitational constant. For the NS with $m = 1.4$ and $R_0 = 1.5$, the orbital frequency $\nu_k$ is 1184 Hz. Some AMXPs are with their upper kHz QPO frequency over 1000 Hz (van der Klis 2000, 2006; Wang et al. 2017), which can be fairly compared with $\nu_k$. However, the spin frequency of 19 AMXPs is from 164 Hz to 599 Hz (Patruno et al. 2017), which shows a deep gap to their upper kHz QPO frequency or $\nu_k$, hence we conclude that the minimum magnetic fields of 19 AMXPs have not yet achieved. If the co-rotation radius approaches the stellar radius, AMXPs will own the minimum magnetic field through the accretion.

### 2.5 The minimum magnetic field of SAX J1808.4-3658

SAX J1808.4-3658 is the first AMXP discovered by Wijnands & van der Klis (1998). Since its discovery, seven outbursts have been recorded until 2015 (Sanna et al. 2017). With the valuable observed information, we try to find the mean values of mass, radius and accretion rate to estimate the minimum magnetic field and present field value of SAX J1808.4-3658.

There are many mass researches on SAX J1808.4-3658. Leahy et al. (2008) constrained the source mass to be $1.3 M_\odot$, that was almost consistent with the result $1.35 M_\odot$ by Chakrabarty & Morgan (1998). Elebert et al. (2009) derived the mass range of this source to be $0.6 - 1.8 M_\odot$. While the latest research on source mass was about $0.97 M_\odot$ (Wang et al. 2013), that is similar to the lower mass limit $- 1.0 M_\odot$ with the data of 1998, 2002 and 2005 outbursts proposed by Morsink & Leahy 2011. Based on these mass researches, a proximately mean value of mass is taken to be $1.3 M_\odot$ for the magnetic field calculation of SAX J1808.4-3658.

Burderi and King (1998) calculated the NS radius of SAX J1808.4-3658 to be 15 km according to the radius restriction $R \sim 13.8 m^{1/3}$ km with $m = 1.3 M_\odot$. While Papitto et al. (2009) proposed the radius to be 18 km with $m = 1.4 M_\odot$ by fitting with the disk-line profile, whose emission was identified from the neutral (or mildly ionized) iron. According to such radius results, the mean value of NS radius for SAX J1808.4-3658 is selected to be 16.5 km.

Patruno et al. (2017a) deduced the luminosity of SAX J1808.4-3658 was about $10^{36}$ erg/s, which was confirmed through the bolometric flux of the first five times outburst (Galloway & Cumming 2006). Other researches on the source luminosity were $(4.7 - 6.4) \times 10^{35}$ erg/s during the short-live peak (Hartman et al. 2008) or $6.6 \times 10^{36}$ erg/s with the source distance 3.5 kpc during the outburst stage (Hartman et al. 2008; Papitto et al. 2009). These results give the source a mean value of the luminosity to be $10^{36}$ erg/s, which corresponds to the accretion rate to be $10^{16}$ g/s.

Thus, with the mean values of $m = 1.3$, $R_0 = 1.65$ and $M = 10^{35}$ erg/s from the above discussion for SAX J1808.4-3658, we derive its minimum field strength $B_{\text{min}} \approx 2.5 \times 10^8$ G and present field strength $B \approx 7.1 \times 10^8$ G, according to Eqs. (9) and (13). Thus, present magnetic field approaches to our result $2 \times 10^8$ G, and the present magnetic field approaches to $\sim 10^8$ G that was evaluated by Wijnands & van der Klis (1998) and Chen (2017). SAX J1808.4-3658 is still in the spinning-up phase (Di salvo & Burderi 2003), and it will continue the field decay process until the magnetosphere collides its surface.

### 3 CONCLUSION AND DISCUSSION

The minimum magnetic field of MSP appears when the NS magnetosphere is compressed onto the stellar surface, according to the model of accretion induced the field strength decay. We find that the minimum field strength is proportionally related to the accretion rate, shown as $B_{\text{min}} \approx 2.0 \times 10^7 G (\dot{M}/M_{\text{min}})$ for a MSP with $m = 1.4$, $R_0 = 1.5$ and $M_{\text{min}} \approx 4.6 \times 10^7$ g/s. $M_{\text{min}}$ is the critical minimum accretion rate for the MSP formation, which is constrained by two conditions: the accretion mass 0.1$M_\odot$ required for the MSP formation at least, and the longest accretion time about 10% of the main-sequence time of the companion star that closes to the Hubble age or the age of universe.

With the accretion rate $M_{\text{min}}$, the minimum magnetic field $2.0 \times 10^7$ g/s of a MSP is obtained, that is closed to the minimum field strength of the observed MSPs, as shown in Fig. 3 and Table 2. Thus, we propose that the calculated minimum magnetic field of MSP is also the minimum field strength of NS, which would be a limitation of the magnetic field decay of NS in the universe. When the accretion rate is from $M_{\text{min}}$ to $M_{\text{field}}$ in LMXBs, the magnetic field of MSPs will be $2.0 \times 10^7 G \leq B \leq 2.8 \times 10^8 G$, which gives a requirement for the companions mass range in main sequence to be about $1.0 M_\odot$ to 4.6 or 5.6$M_\odot$.

Some approximations exist during the estimation of $B_{\text{min}}$, such as the accretion time and ratio between the accretion disk radius and Alfvén radius of the NS, which may cause $B_{\text{min}}$ a little change: (a) the accretion time is a fraction of the companion lifetime (10%). The relaxation of this condition will modify $B_{\text{min}}$; (b) the ratio between the magnetic sphere and Alfvén radius is 0.5 as a usual choice (Ghosh & Lamb 1979; Wang 1996; Ho et al. 2017). A higher one, e.g. 0.8 ~ 1 (Li & Wang 1999), could also arise $B_{\text{min}}$.

When comparing $B_{\text{min}}$ to the estimated field strengths of AMXPs, all AMXPs are with the higher magnetic field ($\sim 10^9$ G) than the obtained minimum field value, e.g., the field strength calculation of the first AMXP, SAX J1808.4-3658. Its mean values of mass, radius and accretion rate of the source are selected based on the researches on its seven bursts in the past 20 years. With such conditions, the present magnetic field of the source is calculated to be $7.1 \times 10^8 G$, and the minimum value is $2.5 \times 10^8 G$. The results illustrate that SAX J1808.4-3658 will continue the evolution till the magnetic field decays to the minimum value.
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