Numerical study on coupling mode of mechanical and thermal drive

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Abstract. In modern gas centrifuge (GC), circulations driven by stationary scoops and temperature difference between end caps, also called as mechanical and thermal drive circulations, have very important effects on its separation performance. The flow patterns of mechanical and thermal drive circulations in a GC are different and whether the coupling mode of them is linear or nonlinear is not clear. In this research, an axial circulation in the model Iguassu GC is simulated for single mechanical, thermal, and both kind together drives. In addition, the product and waste baffles are also considered in the model GC, making the flow patterns much more complicated. The conditions of rotors with different aspect ratios are also investigated. We take the absolute value of the stream function along an axial cross section to define the intensity of various types of axial circulations. The simulation results demonstrate that the coupling mode of mechanical and thermal drive circulations in a GC is close to linear even though the circulations are reshaped by product and waste baffles. The result obtained may simplify the procedure of optimization for the separative power of a single GC for isotope separation.

1. Introduction

In modern gas centrifuge (GC), circulation is very important for the separation performance [1,2], and the circulation inside GC is usually driven by three kinds of phenomenon, two of them are much more significant and they are called mechanical and thermal drive. Mechanical drive is produced by stationary scoops and thermal drive is produced by temperature difference between end caps and along rotor wall. To make the circulation much more efficient to improve the separation performance of GC, baffles are usually introduced in GC to reshape the circulation, which makes the pattern of circulation much more complicated.

There have been a lot of researches studying on the flow field in gas centrifuge [3-7], all of the researches get the flow field with circulation by numerical simulation. Because the pressure and density gradients are very large due to the strong centrifugal force, which makes it impossible to measure the flow field by experimental method [8]. In these researches, the patterns of circulation driven by different drive have been clear and understandable, however, whether the coupling mode of them is linear or nonlinear is not clear enough, especially when we take the baffles to be considered. In other hands, the regulation of decay of circulation along axial direction inside GC for different length of rotor has not been studied yet. In this article, we use numerical method to simulate the 2D axial symmetric flow field in GC, based on the results of flow field, we studied the coupling mode of mechanical and thermal drive...
circulation and then summarized the regulations of decay of circulation along axial direction inside GC for different length of rotor.

2. Governing equations

The effect of mechanical and thermal drive only occurs in viscous flow zone, the flow can be described by N-S equations, which are the governing equations for all our cases. In this article, we introduce momentum sinks and energy sources to simulate the effect of scoops. The equations we solve are listed below.

$$\frac{1}{r} \cdot \frac{\partial}{\partial r} \left[ r (\rho + 1) \rho u \right] + \frac{\partial}{\partial z} \left[ (\rho + 1) \rho w \right] = M_s$$ (1)

$$\rho_0 (\rho + 1) \left[ u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} - \frac{(r + v)^2}{r} \right] + \frac{1}{2A} \frac{\partial}{\partial r} \left[ p_0 (p + 1) \right] + E_k G_r = F_{Sc}$$ (2)

$$\rho_0 (\rho + 1) \left[ u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} + u \left( 2 + \frac{v}{r} \right) \right] + E_k G_\theta = F_{Sc\theta}$$ (3)

$$\rho_0 (\rho + 1) \left( u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) + \frac{1}{2A} p_0 \frac{\partial}{\partial z} (p + 1) + E_k G_z = F_{Sc}$$ (4)

$$\rho_0 (\rho + 1) \left( u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} \right) - \frac{\gamma E_k}{Pr} D (\gamma - 1) p_0 (p + 1) \left( \frac{\partial u}{\partial r} + u + \frac{\partial w}{\partial z} \right) - 2 \lambda A E_k (\gamma - 1) \Phi = Q_S$$ (5)

In these equations, variables are in nondimensional form and represent the disturbances to the rigid body solution. Nondimensional number $E_k$, $Pr$ and $A$ represent Ekman number, Prandtl number and nondimensional velocity parameter, which are defined below:

$$E_k = \frac{\mu}{\rho \alpha \Omega r_a^2}, \quad Pr = \frac{\mu_c}{\rho \kappa}, \quad A = \frac{M \Omega^2 r_a^2}{2RT_0}$$ (6)

Symbols $G_r$, $G_\theta$, $G_z$, $D$ and $\Phi$ represent viscosity term in three directions, thermal diffusion term and viscous dissipation term, which are defined below:

$$G_r = -\frac{4}{3} \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} \right) - \frac{\partial^2 u}{\partial z^2} - \frac{1}{3} \frac{\partial^2 w}{\partial \theta^2}$$ (7)

$$G_\theta = -\frac{\partial^2 v}{\partial r^2} - \frac{\partial^2 v}{\partial z^2} - \frac{1}{r} \frac{\partial v}{\partial r} + \frac{v}{r^2}$$ (8)

$$G_z = -\frac{4}{3} \frac{\partial^2 w}{\partial z^2} - \frac{1}{3} \frac{\partial^2 u}{\partial \theta^2} - \frac{1}{3} \frac{\partial u}{\partial r} - \frac{\partial^2 w}{\partial r^2} - \frac{1}{r} \frac{\partial w}{\partial r}$$ (9)

$$D = -\frac{1}{r} \frac{\partial T}{\partial r} - \frac{\partial^2 T}{\partial r^2} - \frac{\partial^2 T}{\partial z^2}$$ (10)
In this article, only mechanic and thermal drive are studied, so the mass sources and sinks in equation (1) should be zero, which means $M_s = 0$. The effect of scoop is treated only along angular direction, so the momentum sources and sinks in equation (2)(4) should be zero too, which means $F_{sr} = 0$, $F_{sc} = 0$. According to the relationship of fluid friction and velocity, the angular momentum sink can be calculated by the equation below [9]:

$$\frac{1}{2}C_D(p+1)r_0(v + r)^2 dr dz = 2\pi F_{sr}^2 dr dz$$

(12)

where $C_D$ is the drag coefficient, in this article, we use $C_D = 0.3$.

3. Boundary conditions

Since we use sources and sinks to simulate the effect of scoop, there’s no need to specify special boundary conditions in all boundaries, we treat all boundaries as wall. The inner boundary is treated as slip wall and the other boundaries are treated as no slip walls. In the inner boundary, the temperature boundary condition is thermal isolation and in the other boundaries, we specify constant temperature values. All boundary conditions are shown below:

$$\frac{\partial \rho}{\partial r}|_{inner} = \frac{\partial r}{\partial r}|_{inner} = \frac{\partial T}{\partial r}|_{inner} = 0, \quad u_{inner} = 0$$

(13)

$$\frac{\partial \rho}{\partial n}|_{others} = 0, \quad \vec{U}_{others} = \vec{U}_{wall}, \quad T_{others} = T_{wall}$$

(14)

4. Simulation models and cases

The models we use in this article are based on the Iguassu centrifuge model, the parameters of this kind of centrifuge are listed in Table 1.

**Table 1. Parameters of Iguassu centrifuge.**

| Parameters                | Value         |
|---------------------------|---------------|
| Angular velocity $\Omega$ | 10000 rad/s   |
| Mean temperature $T_0$    | 300 K         |
| Pressure near side wall $p_w$ | 100 Torr   |
| Rotor radius $r_u$        | 0.06 m        |
| Typical rotor length $h$  | 0.48 m        |
| Inner boundary radius $r_{in}$ | 0.042 m  |

The working gas in this article is UF$_6$, some physical parameters of this kind of gas are listed in Table 2.
Table 2. Parameters of Iguassu centrifuge.

| Parameters                        | Value               |
|-----------------------------------|---------------------|
| Heat conductivity coefficient $\kappa$ | $6.1 \times 10^{-3}$ W/(m·K) |
| Viscosity coefficient $\mu$       | $1.83 \times 10^{-5}$ Pa·s |
| Relative molecular mass $M$       | 0.352 kg/mol        |
| Specific heat at constant pressure $c_p$ | 372.84 J/mol     |
| Specific heat ratio $\gamma$     | 1.067               |

The schematic graphic of one of the simulation models is shown in Figure 1.

Figure 1. Schematic graphic of simulation model.

In this article, to study the effect of aspect ratio of rotor on the decay of circulation and coupling mode of mechanical and thermal drive, we establish 6 models with different aspect ratios from 4 to 24, and we simulate the flow fields in conditions of pure mechanical drive, pure thermal drive and coupled mechanical and thermal drive to find whether the coupling mode of these two kinds of drive is linear or nonlinear.

5. Results and discussions

According to the flow field results, we use the distribution of stream function in an axial cross section to describe the driven circulation. The sum of stream function in condition of pure mechanical drive and pure thermal drive can represents the linear coupling mode of these two kinds of drive, and the result in condition of coupled mechanical and thermal drive represents the nonlinear mode, which is the real condition. The comparisons of these two kinds of result for different aspect ratios are shown from Figure 2 to Figure 7. All stream distributions are from axial cross section $Z=1/4h$ for each case.
Figure 2. Comparison of stream in cross section \( Z = \frac{1}{4}h \) for aspect ratio 4.

Figure 3. Comparison of stream in cross section \( Z = \frac{1}{4}h \) for aspect ratio 8.

Figure 4. Comparison of stream in cross section \( Z = \frac{1}{4}h \) for aspect ratio 12.
Figure 5. Comparison of stream in cross section $Z=1/4h$ for aspect ratio 16.

Figure 6. Comparison of stream in cross section $Z=1/4h$ for aspect ratio 20.

Figure 7. Comparison of stream in cross section $Z=1/4h$ for aspect ratio 24.
We can see that there are indeed some nonlinear effects during the coupling of thermal and mechanical drive circulations, however these effects are not significant. The linear coupling results are almost the same to the nonlinear coupling results for every case, from very short rotor to very long rotor.

The absolute value of max differences between linear and nonlinear coupling results for each case are listed in Table 3.

**Table 3.** The absolute value of max difference between two kinds of coupling mode.

| Aspect ratio | Max difference |
|--------------|----------------|
| 4            | 4.78×10^{-7}   |
| 8            | 2.52×10^{-7}   |
| 12           | 2.55×10^{-7}   |
| 16           | 2.93×10^{-7}   |
| 20           | 2.96×10^{-7}   |
| 24           | 2.88×10^{-7}   |

We can see that the difference between two kinds of coupling mode is relatively small comparing to the max value of stream.

Not the same as the definition of intensity of circulation in the reference paper [10], in this article, we use the quantity of circulation in an axial cross section to define the intensity of circulation in order to study the effect of aspect ratio on decay of thermal and mechanical drive. The quantity of circulation can be defined by equation below, and it is the most direct description of the intensity of circulation for not only total reflux GC but also for GC with feed and extractions.

\[
L_m = \frac{1}{2} \int_{r_i}^{r_e} \omega r^2 \rho \omega dr
\]

(15)

The decay regulations of mechanical drive circulation for different aspect ratios of rotor are shown in Figure 8.

**Figure 8.** Decay regulations of mechanical drive circulation for different aspect ratios of rotor.

We can see that with the increasing of aspect ratio of rotor, the intensity of mechanical drive circulation at lower axial cross section becomes more and more strong, but has less and less significant changes at higher axial cross section near the product baffle, which means that the decay of mechanical drive along axial direction is faster in long rotor than that in short rotor.

The decay regulations of thermal drive circulation for different aspect ratios of rotor are shown in Figure 9.
Figure 9. Decay regulations of thermal drive circulation for different aspect ratios of rotor.

We can see that the effect of aspect ratio of rotor on decay of thermal drive is pretty different with that on mechanical drive. Similar to the regulations of mechanical drive, with the increasing of aspect ratio of rotor, the density of thermal drive circulation becomes stronger and stronger but the decay regulations of different aspect ratio of rotor are very similar. This phenomenon can be caused by the axial global distribution of thermal drive effect, which is different with the axial local distribution of mechanical drive effect.

6. Conclusions
In this article, we simulate the flow fields of total reflux gas centrifuge for different aspect ratios of rotors, using these results of flow fields, we studied the coupling mode of mechanical drive and thermal drive circulations and the effects of aspect ratio of rotor on decay of mechanical and thermal drive. Here come the conclusions of our studies.

1) There are indeed some nonlinear effects on coupling of mechanical and thermal drive circulations, but the nonlinear factors are not significant so the coupling mode of mechanical and thermal drive circulation is almost linear.
2) As the coupling mode of mechanical and thermal drive is approximately linear, we can study these two kinds of drive separately and then combine the results linearly.
3) The aspect ratio of rotor affects the decay of mechanical drive evidently, but does not affect decay of thermal drive too much.
4) With the increasing of aspect ratio of rotor, the intensity of mechanical drive circulation at lower axial cross section becomes more and more strong, but the decay becomes more and more fast.
5) With the increasing of aspect ratio of rotor, the intensity of thermal drive circulation also becomes more and more strong, but there is not significant difference among them for the decay regulations.

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References
[1] Jiang D, Zeng S and Fu R 2005 J. of Tsinghua University (Science and Technology) 12
[2] Borisevich V D and Potanin E P 2018 J. Phys.: Conf. Ser. 1099 012011
[3] Kai T 1977 J. Nucl. Sci. Technol 14
[4] Omnes P 2007 *Computers & fluids* **36** 1028-1039
[5] Borisevich V D, Levin E V and Naumochkin V V 1989 *Fluid Dynamics* **24** 520–4
[6] Lu Xu, Shi Zeng, Niannian Chen et al 2016 *Atomic Energy Sci and Technol.* **50** 385–9
[7] Doneddu F, Roblin P and Wood H G 2000 *Separation Sci. and Technol.* **35** 1207–21
[8] Bogovalov S V, Borisevich V D, Borman V D et al. 2013 *Computers & Fluids* **86** 177–84
[9] Yunan Zhang 2017 *Numerical Study of the Flow Field with Strong Disturbances, the Hydraulics and Separation Characteristics of a Gas Centrifuge* (Tsinghua University)
[10] Gu Z, Jiang D, Zeng S and Borisevich V D 2017 *Numerical Study on Decay of Endcap Drive in a Rotating Cylinder* (14th International Workshop on Separation Phenomena in Liquids and Gases. Stresa, Italy)