Decoherence of Rabi oscillations of electronic spin states in a double quantum dot

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We study the role of charge fluctuations in the decoherence of Rabi oscillations between spin states $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$ of two electrons in a double dot structure. We consider the effects of fluctuations in energy and in the quantum state of the system, both in the classical and quantum limit. The role of state fluctuations is shown to be of leading order at sufficiently high temperature, applicable to actual experiments. At low temperature the low frequency energy fluctuations are the only dominant contribution.

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I. INTRODUCTION

The keystone of quantum information processing is the coherent dynamics of the quantum logical bits (qubits). Although such coherent behavior is well established in atomic systems, it can be maintained only for very short time scales, of the order of few nanoseconds, in charge based solid state based systems. To overcome this problem one may employ the spin degree of freedom of the electrons residing in a quantum dot as a qubit. In fact, as a consequence of the confined geometry, the coherence time of the spin may be extended to be of the order of tens of microseconds, primarily restricted by the coupling of the nuclear spin environment via the hyperfine interaction. These results motivated the experimental progress in controlling electronic spin in GaAs gated quantum dots systems.

In a recent experiment the use of spin states of two electrons in a double dot as the holder of quantum information has been investigated. In that configuration the system is governed by (i) the hyperfine interaction which tends to mix singlet and triplet states and (ii) the exchange interaction which tends to preserve the total spin of the electron pair. The interplay between the two effects has been studied theoretically and analyzed experimentally. In particular Ref. reports Rabi oscillations between spin states driven (electrostatically) by tuning the exchange energy. Such oscillations (faster than the typical spin decoherence time) are mainly hindered by charge fluctuations.

In this paper we analyze the decoherence effects in the Rabi oscillations due to charge fluctuations. We consider both the effects of exchange energy fluctuations and fluctuations of the singlet hybridized state which is affected by charge fluctuations as well. In particular we calculate the time dependence of the Rabi oscillations in the presence of gate voltage and tunneling amplitude fluctuations, both in the classical (high temperature, Eq. (1)) and the quantum case (Eq. (13)). We describe the crossover of the decoherence rate between low and high temperature regimes, which can be relevant in the actual experiments. Classical energy fluctuations have been analyzed in Ref. 3.

II. THE MODEL

The system (Cf. Petta et al.) is schematically presented in Fig. 1. It consists of a gate confined semi-conducting double quantum dot. Tunnel barriers connect each dot to the adjacent reservoirs allowing dot-lead tunneling of electrons. The gate voltages, $V_T$, $V_L$, and $V_R$, control the tunnel between the dots, and the dots’
charge configuration \((n_L, n_R)\), respectively. It is possible to measure such a charge configuration using a quantum point contact (QPC) located near one of the dots. The dimensionless detuning parameter, \(\epsilon \equiv V_L - V_R\), controls the difference \(n_L - n_R\). In Ref. 3 the system was operated between \((1, 1)\) and \((0, 2)\). In the \((0, 2)\) charge configuration \((\epsilon = \epsilon_A)\), the antisymmetric nature of the electron wave function enforces the ground state of the system to be a singlet. The ground state of the system at \(\epsilon = \epsilon_B\), the dimensionless detuning parameter, is affected only by the fluctuations of \(\lambda \) and \(\rho \), which is reasonable for weak tunneling. Then \(\epsilon\) affects only the tunneling matrix element \(\lambda\), which is proportional to \(V_L - V_R\). The respective gates are controlled independently of each other, hence it is natural to assume that their fluctuations are independent. In principle it is possible to determine a correlation matrix for the fluctuations of the parameters in the Hamiltonian, \(\epsilon\) and \(\lambda\), by considering a specific potential form for the double dot. Instead we assume that \(V_T\) affects only the tunneling matrix elements, \(\lambda\), which is reasonable for weak tunneling. Then \(\epsilon\) is affected only by the fluctuations of \(V_L - V_R\). The Hamiltonian is

\[
\hat{H} = \hat{H}_0 \{\epsilon \rightarrow \epsilon + \xi_\epsilon(t), \lambda_s \rightarrow \lambda_s + \xi_\lambda(t)\} = \hat{H}_0 + E_0 \epsilon \langle T_0 | \mathcal{O} | T_0 \rangle
\]

where \(\xi_\epsilon(t)\) are Gaussian distributed with \(\langle \xi_\epsilon(t) \rangle = 0\). The typical dynamical scale of the nuclear environment is of the order of tens of microsecond, and therefore it acts as if it is a frozen external field over the duration of the experiment. With \(B_{NL}, B_{NR} \sim 5mT \ll B\), the hyperfine interaction is effective only at \(\epsilon \sim \epsilon_X\), where it can mix \(S_0\) and \(T_+\), and around \(\epsilon \sim \epsilon_B \ll 1\) where it mixes the low energy states \(S_g\) and \(T_0\):

\[
H_N = g_\mu_B (B_{NL} \cdot S_L + B_{NR} \cdot S_R).
\]

The typical energy difference between them is \(J(\epsilon) = E_0 (\epsilon + \sqrt{\epsilon^2 + \lambda^2}) \ll \langle T_0 | H_N | S_g \rangle\). The ground state of the system at \(\epsilon = \epsilon_B\) is therefore

\[
\langle S_g (\epsilon = -1) \rangle / \sqrt{2} = |\uparrow \downarrow\rangle \text{ (for +)} \text{ or } |\uparrow \uparrow\rangle\text{ or } |\downarrow \downarrow\rangle \text{ (−): the spin in the two dots are oppositely oriented. Hereafter we consider } |\uparrow \downarrow\rangle\text{ state.}
\]

In the experiment described in Ref. 3 the detuning parameter is varied in time to induce Rabi oscillations between \(|\uparrow \downarrow\rangle\) and \(|\uparrow \uparrow\rangle\). The time dependence of the parameter \(\epsilon\) used to drive the oscillations is depicted in Fig. 1. The system is prepared in the state \(|\uparrow \downarrow\rangle\) (or equivalently \(|\downarrow \uparrow\rangle\)) at \(\epsilon = \epsilon_B\) (cf. Fig. 1). Subsequently, a gate voltage pulse at \(t = 0\) modifies \(\epsilon\) to a point where \(\langle T_0 | H_N | S_g \rangle \ll \langle \epsilon \rangle\), thus inducing oscillations between \(|\uparrow \downarrow\rangle\) and \(|\downarrow \uparrow\rangle\) over a time interval \(\tau\). The following manipulation of \(\epsilon\) (cf. Fig. 1) allows to relate the measured conductance of the QPC with the probability of finding the system in \(|\uparrow \downarrow\rangle\) right after the pulse, \(P(\tau) = \langle \uparrow \downarrow | \exp(-i \hat{H}_0 \tau) | \uparrow \downarrow \rangle^2\).

### III. CLASSICAL NOISE

The Rabi oscillations are obtained by tuning the energy difference \(J(\epsilon)\) between \(|S_g\rangle\) and \(|T_0\rangle\), which results in the different charge of the singlet and hybridized singlet. Rabi oscillations will therefore be extremely sensitive to an environment coupled to charge as opposed to the nuclear spin environment. Decoherence effects will originate both from fluctuations of the (exchange) energy \(J(\epsilon)\), analyzed in Ref. 3 and fluctuations of the hybridized singlet state \(|S_c(\epsilon)\rangle\). We analyze the Rabi oscillations taking into account fluctuations of \(V_L, V_R, V_T\). The respective gates are controlled independently of each other, hence it is natural to assume that their fluctuations are independent. The typical energy difference \(J(\epsilon)\) between \(|S_g\rangle\) and \(|T_0\rangle\) is small compared to the charging energy of the single \((1, 1)\) and \((0, 2)\) states, which can be written in the effective magnetic field in each of the \((s, t)\) and the charging energy \(e\) used to drive the oscillations is depicted in Fig. 1. The system is prepared in the state \(|\uparrow \downarrow\rangle\) (or equivalently \(|\downarrow \uparrow\rangle\)) at \(\epsilon = \epsilon_B\) (cf. Fig. 1). Subsequently, a gate voltage pulse at \(t = 0\) modifies \(\epsilon\) to a point where \(\langle T_0 | H_N | S_g \rangle \ll \langle \epsilon \rangle\), thus inducing oscillations between \(|\uparrow \downarrow\rangle\) and \(|\downarrow \uparrow\rangle\) over a time interval \(\tau\). The following manipulation of \(\epsilon\) (cf. Fig. 1) allows to relate the measured conductance of the QPC with the probability of finding the system in \(|\uparrow \downarrow\rangle\) right after the pulse, \(P(\tau) = \langle \uparrow \downarrow | \exp(-i \hat{H}_0 \tau) | \uparrow \downarrow \rangle^2\).

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$O(J(\epsilon)/\hbar \omega_s) = \sin^2 \theta$. Under this assumption the density matrix at any time $t > 0$ can be written as

$$\rho(t) = \frac{1}{2} \left[ |T_0 \rangle \langle T_0| + Y(t) |S_g \rangle \langle S_g| + (1 - Y(t)) |S_c \rangle \langle S_c| + (X(t) |S_g \rangle \langle T_0| + h.c.) \right],$$

in terms of the two functions $Y(t)$ and $X(t)$ describing the evolution of the state populations and coherence respectively. The explicit expressions, $X(t) = e^{i(J(\epsilon)/\hbar \gamma_t)t}$, $Y(t) = (1 + e^{-\gamma_t t})/2$, allow to determine the surviving probability $P(t) = \langle \uparrow \downarrow | \rho(t) | \uparrow \downarrow \rangle = 1/4[1 + Y(t) + 2 Re\{X(t)\}]$.

The measured probability consists of damped oscillations around a mean that approaches an asymptotic value. The decay of the oscillations is related only to the decoherence of $\rho(t)$, while the relaxation of the populations, encoded in $Y(t)$, determines the slow time variation of the mean. The dependence of the decay rates, $\gamma_1, \gamma_2$, on $\epsilon$ is quite different from what we would have obtained by simply accounting for fluctuation of $J(\epsilon)$, in which case

$$\gamma_1 = 0, \quad \gamma_2 = 4 \Gamma_c \sin^2 \theta + \Gamma_\lambda \cos^2 \theta,$$

- classical energy fluctuations only -

This means that the effects of fluctuations of the state $|S_g\rangle$ cannot be neglected with respect to (exchange) energy fluctuations. We also note that the asymptotic value of the probability is $P(\tau) = 3/8$, corresponding to a steady state density matrix with equally populated singlet states $|S_g\rangle, |S_c\rangle$. This is a signature of the high temperature ($k_B T \gg \hbar \omega_s$) limit, and is related to the assumption of classical gate voltage fluctuations. By contrast, at low temperature we expect that only the lower singlet level is populated. In the experiment oscillations with $J(\epsilon)$ ranging from tenths to few $\mu eV$ have been observed at $k_B T \sim 10 \mu eV$. We therefore expect that the condition $k_B T \sim \hbar \omega_s \gg J(\epsilon)$ is that obtained experimentally, in which case quantum state fluctuations are important and we explore their dependence on the temperature.

### IV. QUANTUM NOISE

In order to extend our analysis beyond the high temperature limit we need to consider the quantum nature of the gate voltage fluctuations. We therefore modify our classical model (Eq. (9)):

$$\hat{H} = \hat{H}_0 + \hat{V}_c A_c + \hat{V}_\lambda A_\lambda + H_{\text{bath}, c} + H_{\text{bath}, \lambda}.$$

Here the operators $\hat{V}_c(\lambda)$ acting on the system are the same as in Eq. (9) while the classical fluctuators are replaced by the operators $A_c(\lambda) = \sum a_{c(\lambda)}(b_{c(\lambda)}^\dagger + b_{c(\lambda)}^\dagger)$ of bosonic baths, $H_{\text{bath}, c(\lambda)} = \sum \hbar \omega_i b_{c(\lambda)}(b_{c(\lambda)}^\dagger)$. The effect of the reservoirs on the dynamics of the electrons in the double dot is entirely characterized by their symmetric and antisymmetric spectral functions

$$S_{\pm}(\omega) = 1/(2\pi) \int_{\mathbb{R}} d\omega' e^{i\omega\tau}[A_{c(\lambda)}(t), A_{c(\lambda)}(0)]_\pm.$$

We take both baths to be at equilibrium at the same temperature $k_B T = 1/\beta$. Their bosonic nature guarantees that $S^+(\omega) = \coth(\beta \hbar \omega/2) S^-(\omega)$, where $S^-(\omega)$ is temperature independent, $S^-(\omega) = \sum a_{c(\lambda)}^2/\hbar (\omega + \omega_i) - \delta(\omega - \omega_i)$.

We assume that the bath is weakly coupled to the system, and we determine the evolution of the density matrix to second order in $a_{c(\lambda)}$/$\omega_s$. Following the Bloch-Redfield approximation, we introduce a (short) bath correlation time, $\bar{\tau}$, characterizing the typical time scale at which any correlation of the system and the reservoir disappears. The time evolution of the reduced density matrix of the system, $\rho$, coarse grained at time scales $\Delta t \gg \bar{\tau}$, is Markovian. It is determined by the first order linear differential equation

$$\partial_t \rho_{a,b} = -i \omega_{a,b} \rho_{a,b} - \sum_{c,d} \mathcal{R}_{a,b,c,d} \rho_{a,c} e^{-i(\omega_{a,b} - \omega_{c,d})},$$

written in the basis of eigenstates of $\hat{H}_0$ where $\hbar \omega_{a,b} = \langle a| \hat{H}_0 |a\rangle - \langle b| \hat{H}_0 |b\rangle$ and $\mathcal{R}_{a,b,c,d}$ is the Bloch-Redfield tensor:

$$\mathcal{R}_{a,b,c,d} = \sum_{j=\epsilon,\lambda} \left[ \sum_n \delta_{b,d} \langle a| V_j n | n| V_j c | g_j (\omega_{c,n}) \right.$$}

$$\left. - \langle a| V_j c | d| V_j n | b | g_j (\omega_{b,d}) + \sum_n \delta_{a,c} \langle n| V_j n | n| V_j b | g_j (\omega_{n,d}) \right],$$

with $g(\omega) = 1/[S^+_j(\omega) + S^-_j(\omega)] - i \int_{\mathbb{R}} Pdx/(2\pi) (S^+_j(x) + S^-_j(x))/(x - \omega)$. At times $t \gtrsim \hbar / J(\epsilon) \gg \hbar / \omega_s$, neglecting terms of order $O(J/\omega_s)$, the sum in Eq. (11) involves only terms such that $\hbar (\omega_{a,b} - \omega_{c,d}) \ll 0$. Explicitly the only relevant entries of the Bloch-Redfield tensor are: $\mathcal{R}_{S_0, S_0, S_0, S_0} = -\mathcal{R}_{S_0, S_0, S_0, S_0}$, $\mathcal{R}_{S_0, S_0, S_0, S_0} = -\mathcal{R}_{S_0, S_0, S_0, S_0}$, $\mathcal{R}_{S_0, T_0, S_0, T_0} = \mathcal{R}_{T_0, S_0, T_0, S_0}$. It follows that state population and coherence evolve independently of each other and therefore the density matrix has the same form presented in Eq. (9), with different functions $X(t)$ and $Y(t)$.

Once the expression of $\rho(t)$ is known, it can be used to
calculate the survival probability,

\[ P_{qm}(\tau) = \frac{1}{8}[3 + e^{-\gamma_1 \tau} + \tanh \left( \frac{\hbar \omega_s}{2} \right) (1 - e^{-\gamma_1 \tau}) + 4 \cos((J(\epsilon)/\hbar + \Delta_J)\tau)e^{-\gamma_2 \tau}] , \quad (13) \]

\[ \gamma_1 = 2\pi[\sin^2(2\theta)S_{c}^+(\omega_s) + \cos^2(2\theta)S_{c}^+(\omega_s)]/\hbar^2 , \quad (14) \]

\[ \gamma_2 = \pi[\sin^2(2\theta)(S_{c}^+(\omega_s) - S_{c}^-(\omega_s)) + \cos^2(2\theta)\left( S_{a}^+(\omega_s) - S_{a}^-(\omega_s) \right) + (\cos(2\theta) - 1)^2S_{c}^+(0) + \sin^2(2\theta)S_{c}^+(0)]/(2\hbar^2) \], \quad (15) \]

and \( \hbar^2 \Delta_J = [\sin^2(2\theta) \int_{\mathbb{R}} P d\omega/\omega (S_{a}^-(\omega) - S_{a}^-(\omega)) + \int_{\mathbb{R}} P d\omega/(\omega + \omega_s) (\sin^2(2\theta)(S_{a}^+(\omega) - S_{a}^-(\omega)) + \cos^2(2\theta)(S_{a}^+(\omega) + S_{a}^-(\omega)))]/2 \). \( \Delta_J \) is a shift in the frequency of the Rabi oscillations that can be neglected compared with \( J(\epsilon)/\hbar \), consistent with our second order perturbation expansion. The Bloch-Redfield approximation employed implies that Eq. (13) is valid in the limit \( \gamma_1(2) \ll \omega_s/1.\tau \). We note that, unlike \( \gamma_2, \gamma_1 \) depends only on the symmetric (classical) correlators, \( S_{c}^+(\omega) \).

\( \gamma_2 \) consists of contributions from the bath correlation function at frequency \( \omega_s \) (which describes the relaxation process between the two singlet eigenstates and the corresponding contribution to the dephasing), and from the zero frequency correlation function (corresponding to the contribution of pure dephasing).

In the high temperature limit, \( \beta \hbar \omega_s \to 0 \), \( S_{c}^+(\omega) \) is negligibly small as compared with \( S_{c}^+(\omega) \), we thus expect a classical result. If we furthermore assume an Ohmic bath, i.e. \( S_{c}^+(\omega) = \alpha(\omega)\hbar^2\omega \), such that at high temperature \( S_{c}^+(\omega) \sim S_{c}^+(0) \) for \( \omega < \omega_s \), Eq. (13) reduces to Eq. (4) with

\[ \gamma_1 = 2\pi/\hbar[S_{c}^+(0) \sin^2(2\theta) + S_{c}^+(0) \cos^2(2\theta)] , \quad (16) \]

- quantal high \( T \).

We in fact recover the classical result for white noise fluctuations (Eq. (7)) by identifying \( \Gamma_{c} = \pi/(2\hbar^2)S_{c}^+(\omega) \). At low temperature, \( \beta \hbar \omega_s \gg 1 \), \( S_{c}^+(\omega_s) \approx S_{c}^+(\omega) \) and the quantum nature of the bath becomes important. The rate \( \gamma_1 \) disappears from the expression for \( \tilde{P}(t) \) and we also note the finite frequency contribution to \( \gamma_2 \) vanishes. Only the zero frequency component of the spectral density of the bath (which is responsible for pure dephasing) survives,

\[ \gamma_2 = \pi/(2\hbar^2)(4S_{c}^+(0) \sin^2(2\theta) + S_{c}^+(0) \cos^2(2\theta)) \] \quad (17)

- quantal low \( T \).

In this limit the dependence of \( \gamma_2 \) on \( \theta \) can be explained entirely in terms of classical fluctuations of the oscillation frequency, \( J(\epsilon) \) (cf. Eq. (5)). Indeed, the effects of fluctuations of the state \( |S_g\rangle \) do involve transitions between the latter state and the singlet excited state, yet these transitions are exponentially suppressed by \( e^{-\beta \omega_s} \). Note however that experimentally the regime \( \beta \hbar \omega_s \gtrsim 1 \) can be reached, hence the effect of fluctuations of the state \( |S_g\rangle \) can be of interest. In particular this fluctuations affect the steady state value of the survival probability which is a function of \( \beta \omega_s \), \( P_{qm}(\tau \to \infty) = 1/8[3 + \tanh(\beta \hbar \omega_s/2)] \) (cf. Fig. 2) and can be directly observed in the experiments.

A comparison with the experimental results of Ref. is obtained assuming Ohmic baths with spectral densities \( S_{c}^+(\omega) = \alpha(\omega)\hbar^2 \omega \). These properly describe charge fluctuations due to the external circuit controlling the gate voltages. An analysis of the possible scenarios \( \alpha_c \lesssim \alpha \), shows that the experimental fact that the number of visible oscillations as function of \( J(\epsilon) \) is constant is correctly reproduced for \( \alpha_c \lesssim \alpha \) (cf. insets in Fig. 2). The time dependence of \( P_{qm}(\tau) \) is depicted in Fig. 2 for different values of \( J(\epsilon) \). We obtain a fit with the experimental data for \( \alpha_c = \alpha \approx 7 \times 10^{-3} \), consistent with the strength of electromagnetic environment in other solid state systems.

\[ \text{V. EXTENDED MODEL} \]

The previous analysis is now extended to include the lowest energy triplet state in the charge configuration \( (0,2), |T_0\rangle \). This might be necessary if the energy of
\[ T_0 \] is comparable with the Coulomb energy \( E_0 \). The new Hamiltonian reads, \( H'_0 = H_0 + E_0(\lambda_i | T_0 \rangle \langle T_0 | + \text{h.c.}) + E_0(\delta - \lambda_i \epsilon) | T_0 \rangle \langle T_0 | \), where \( \delta \) is the excitation energy to the triplet state in the \((0, 2)\) configuration. Owing to electron tunneling, \( \lambda_i \), the two triplet states, \( | T_0 \rangle \) and \( | T_0' \rangle \), do hybridize (cf. Fig. 1(c)). The energy spectrum of singlet and triplet states in the subspace \( S_2 \) is 0 is depicted in Fig. 1(c). The energies of the hybridized triplet states, \( T_0(\epsilon) = -\sin \varphi | T_0 \rangle + \cos \varphi | T_0' \rangle \) and \( T_0(\epsilon) = \cos \varphi | T_0 \rangle + \sin \varphi | T_0' \rangle \), are \( E_0(\delta - \lambda_i \epsilon)/2 = E_0(\delta + \lambda_i \epsilon)/2 \) respectively, with tan \( \varphi = \sqrt{(\epsilon - \delta)^2 + \lambda_i^2}/\lambda_i \). The Hamiltonian for this model includes four parameters, \( \epsilon, \delta, \lambda_i, \lambda_f \) which depend on three fluctuating gate voltages only, \( V_L, V_R, V_T \). In principle it is possible to determine a correlation matrix for the fluctuations of the parameters in the Hamiltonian by considering a specific potential form for the double dot. Instead we assume that \( V_T \) affects only the tunneling matrix elements, which is reasonable for weak tunneling. The fluctuations of \( \lambda_i \rightarrow \lambda_i + \xi_i(t) \) and \( \lambda_f \rightarrow \lambda_f + \xi_f(t) \) will then depend on the same bath and will therefore be correlated, \( \xi_i(t) = f_c \xi_f(t) \) with \( f_c = (\partial \lambda_i/\partial V_T)/(\partial \lambda_f/\partial V_L) \). At the same time the gate voltage difference \( V_L - V_R \) will affects only \( \epsilon \). The density matrix now evolves as (cf. Eq. 13) \( \rho'(t) = \rho(t) + 1/2 W(t) | T_0 \rangle \langle T_0 | + 1/2 (1 - W(t)) | T_0 \rangle \langle T_0 | \), and the probability of finding the system in the \( | T_0 \rangle \) at time \( \tau \) is \( P_{\text{qm}}(\tau) = P_{\text{qm}}(\tau) - 1/8 (1 - \tan(\beta \hbar \omega_1/2)) (1 - e^{-\gamma_2 \tau}) \), with \( \gamma_2 = 2 \pi^2/\hbar^2 [\sin^2(2\varphi) S_c^z(\omega_1) + J^2 \cos^2(2\varphi) S_c^z(\omega_2) \] and \( \gamma_2 \) replaced by \( \gamma_2 = \gamma_2/4 (1 - \tan(\beta \hbar \omega_1)) + \gamma_2/4 (1 - \tan(\beta \hbar \omega_2)) + \pi/(2\hbar^2) [\cos^2(2\varphi) - \cos^2(2\varphi) S_c^z(0) + (\sin^2(2\varphi) - f \sin^2(2\varphi)) S_c^z(0)] \). The physical mechanism that induces decoherence in \( P(\tau) \) is the same described in the previous paragraphs. Similarly to the decoherence in the singlet subspace, fluctuations in \( | T_0 \rangle \) do involve now the exited triplet state \( | T_0 \rangle \), an effect that is small in \( e^{-\beta \hbar \omega_1}/2 \). Note that, even at low temperature, \( k_B T \ll \hbar \omega_1 \), fluctuations in the energy \( J(\epsilon) \) (cf. Fig. 1(c)) modify \( P_{\text{qm}}(\tau) \). Remarkably in this case, even in the presence of a “sweet point” \( \theta = \varphi \), while \( \partial J = 0 \), fluctuations of the tunneling rates are important due to the difference between electron tunneling in the triplet and singlet states.

VI. CONCLUSIONS

We have presented here a simple model describing the effect of charge fluctuations on Rabi oscillations between spin states \( | T_0 \rangle \) and \( | T_0 \rangle \) of electrons in a double dot. We have accounted for decoherence effects due to both energy and quantum state fluctuations, by including the quantum effects of a fluctuating environment within the Born-Markov approximation —Eqs. (13–15). We have shown that not only in the high temperature limit does the result reproduce that of classical fluctuations (compare Eq. (16) to Eq. (1)), but also the low temperature result has a classical interpretation in terms of energy fluctuations only (not state fluctuations, compare Eq. (13) to Eq. (3)). In fact the role of the state fluctuations is significant at a temperature that exceeds the singlet excitation energy, a regime which is accessible experimentally. Note that at high temperature the “classical limit” refers to classical environmental induced fluctuations. The latter can still cause fluctuations in the quantum state of the system. At low temperature state fluctuations are frozen out.

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Such a model can better fit the experimentally observed behavior of $J(\epsilon)$ around $\epsilon \sim 0$. Furthermore one can identify a “sweet point” where $\partial_\epsilon J = 0$ at which fluctuations in $\epsilon$ are less effective.