Topics in chiral perturbation theory

ULF-G. MEIßNER
Centre de Recherches Nucléaires et Université Louis Pasteur de Strasbourg,
Physique Théorique, BP 20 Cr, 67037 Strasbourg Cedex 2, France

Abstract. I consider some selected topics in chiral perturbation theory (CHPT). For
the meson sector, emphasis is put on processes involving pions in the isospin zero S-wave
which require multi-loop calculations. The advantages and shortcomings of heavy baryon
CHPT are discussed. Some recent results on the structure of the baryons are also pre-
sented.

1. Introduction

This talk will be concerned with certain aspects of the standard model in the long-
distance regime. I will argue that there exists a rigorous calculational scheme and
that plenty of interesting and fundamental problems await a solution. I hope this
will trigger further detailed studies of these topics (and others which can only be
mentioned en passant).

Our starting point is the observation that in the three flavor sector, the QCD
Hamiltonian can be written as

\[ H_{\text{QCD}}^0 = H_{\text{QCD}}^0 + H_{\text{QCD}}^I \]

\[ H_{\text{QCD}}^I = \int d^3x \{ m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s \} \]

with \( H_{\text{QCD}}^0 \) symmetric under chiral \( SU(3)_L \times SU(3)_R \). On a typical hadronic scale,
say \( \rho = 770 \text{ MeV} \), the current quark masses \( m_q = m_u, m_d, m_s \) can be considered
as perturbations. The chiral symmetry of the Hamiltonian is spontaneously broken
down to its vectorial subgroup \( SU(3)_V \) with the occurrence of eight (almost) massless
pseudoscalar mesons, the Goldstone bosons (\( \varphi = \pi^+, \pi^0, \pi^-, K^+, K^-, K^0, \bar{K}^0, \eta \))

\[ M_q^2 = m_q B + \mathcal{O}(m_q^2) \]

with \( B = -<0|\bar{q}q|0> / F^2 \) and \( F \) the pion decay constant. In the confine-
ment (long-distance) regime, the properties of the standard model related to this
symmetry can be unambiguously worked out in terms of an effective Lagrangian,

\[ \mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{eff}}[U, \partial_\mu U, \ldots, \mathcal{M}] \]

with \( \mathcal{M} = \text{diag}(m_u, m_d, m_s) \) the quark mass matrix and the Goldstone bosons are
collected in the matrix-valued field \( U(x) = \exp\{i \sum_{a=1}^8 \varphi_a(x) \lambda^a / F \} \). Of course,
there is an infinity of possibilities of representing the non-linearly realized chiral symmetry. While the QCD Lagrangian is formulated in terms of quark and gluon fields and the rapid rise of the strong coupling constant $a_s(Q^2)$ with decreasing $Q^2$ forbids a systematic perturbative expansion, matters are different for the effective field theory (EFT) based on the effective Lagrangian (3). It can be written as a string of terms with increasing dimension,

$$L_{\text{eff}} = L_{\text{eff}}^{(2)} + L_{\text{eff}}^{(4)} + L_{\text{eff}}^{(6)} + \ldots$$

if one counts the quark masses as energy squared. To lowest order, the effective Lagrangian contains two parameters, $F_\pi$ and $B$. It is worth to stress that $B$ never appears alone but only in combination with the quark mass matrix, alas the pseudoscalar meson masses. Consequently, any matrix-element $< ME >$ for the interactions between the pseudoscalars can be written as

$$< ME >= c_0 \left( \frac{E}{\Lambda} \right)^2 + \left[ \sum_{i=1}^n (c_{1i}) + \text{(non-local)} \right] \left( \frac{E}{\Lambda} \right)^4 + \mathcal{O} \left( \frac{E}{\Lambda} \right)^6$$

This is obviously an energy expansion or, more precisely, a simultaneous expansion in small external momenta and quark masses. The first term on the r.h.s. of (5) leads to nothing but the well-known current algebra results, the pertinent coefficient $c_0$ can be entirely expressed in terms of $F_\pi$, the Goldstone masses and some numerical constants. As one of the most famous examples I quote Weinberg’s result for the $S$-wave, isospin zero scattering length $|1|$

$$a_0 = \frac{7M_\pi^2}{32\pi F_\pi^2}$$

which is such an interesting observable because it vanishes in the chiral limit $M_\pi \to 0$. At next-to-leading order, life is somewhat more complicated. As first shown by Weinberg [2] and discussed in detail by Gasser and Leutwyler [3], one has to account for meson loops which are naturally generated by the interactions. These lead to what I called "non-local" in (5). In fact, it can be shown straightforwardly that any $N$-loop contribution is suppressed with respect to the leading order result by $(E/\Lambda)^{2N}$. At $\mathcal{O}(E^4)$, the loop contributions do not introduce any new parameters. However, one also has to account for the contact terms of dimension four which are accompanied by a priori unknown coupling constants (the $c_{1i}$ in (5)). These so-called low-energy constants serve to renormalize the infinities related to the pion loops. Their finite pieces are then fixed from some experimental input. In the case of flavor SU(2), one has $n = 7$. Two of these constants are related to interactions between the pseudoscalars, three to quark mass insertions and the remaining two have to be determined from current matrix elements. The inclusion of gauge boson couplings to the Goldstone bosons is most simply and economically done in the framework of external background sources. Notice also that at order $E^4$ the chiral anomaly can be unambiguously included in the EFT. At order $E^6$, one has to consider loop diagrams with insertions from $L_{\text{eff}}^{(2)}$ and $L_{\text{eff}}^{(4)}$ as well as contact terms from $L_{\text{eff}}^{(6)}$ which introduces new couplings. Once the low-energy constants are fixed, the aspects of the dynamics of the standard model related to the chiral symmetry can be worked out systematically and unambiguously. Clearly, the EFT can only be applied below a typical scale $\Lambda \simeq M_\rho$ and higher loop calculations become more and
more cumbersome (but can't always be avoided as will be discussed below). This is the basic framework of CHPT in a nutshell. For more details, I refer to refs.[2,3], my review [4] and the extensive list of references given therein. It is worth pointing out that Leutwyler has recently given a more sound foundation of the effective Lagrangian approach by relating it directly to the pertinent Ward-Identities [5].

2. Meson-meson scattering and the mode of quark condensation

Pion-pion and pion-kaon scattering are the purest reactions between the pseudoscalar Goldstone bosons. The Goldstone theorem mandates that as the energy goes to zero, the interaction between the pseudoscalars vanishes. Consequently, \( \pi \pi \) and \( \pi K \) scattering are the optimal testing grounds for CHPT.

Let me first consider the chiral expansion of the isospin zero S-wave in \( \pi \pi \) scattering. In the standard formulation of CHPT, Gasser and Leutwyler have derived a low-energy theorem generalizing Weinberg's result (6) [6],

\[
\alpha_0 = 0.20 \pm 0.01
\]

which is compatible with the data, \( \alpha_0 = 0.23 \pm 0.08 \) [7]. The theoretical value (7) rests on the assumption that \( B \) is large, i.e. of the order of 1 GeV (from current values of the scalar quark condensates). However, if \( B \) happens to be small, say of the order of \( F \), one has to generalize the CHPT framework as proposed by Stern et al.[8]. In that case, the quark mass expansion of the Goldstone bosons takes the form

\[
M^2 = m_q B + m_s^2 A + O(m_q^2)
\]  

(8)

with the second term of comparable size to the first one. In ref.[9], this framework is discussed in more detail and a novel representation of the \( \pi \pi \) amplitude which is exact including order \( E^6 \) and allows to represent the whole \( \pi \pi \) scattering amplitude in terms of the S- and P-waves and six subtraction constants is given. The presently available data are not sufficiently accurate to disentangle these two possibilities. More light might be shed on this when the \( \phi \)-factory DAΦNE will be in operation (via precise measurements of \( K_{\ell 4} \)-decays) or if the proposed experiment to measure the lifetime of pionic molecules [10] will be done. It should also be pointed out that recent lattice QCD results seem to be at variance with the expansion (8), but this can only be considered as indicative [11]. Also, the experimentally well-fulfilled GMO relation for the pseudoscalar meson masses arises naturally in the conventional CHPT framework but requires parameter fine-tuning in case of a small value of \( B \sim 100 \) MeV. Novel high precision experiments at low energies are called for. This is an important question concerning our understanding of the standard model and it definitively should deserve more attention. For more details, I refer to sections 4.1 and 4.2 of ref.[4] as well as ref.[9].

In fig.1, I show the phase-shift \( \delta_0 \) from threshold (280 MeV) to approximately 600 MeV [12]. One notices the rapid rise of the phase shift, and at 600 MeV it is already as large as 55 degrees and passes through 90 degrees at about 850 MeV. At energies below 600 MeV, the other partial waves do not exceed 15 degrees (in magnitude). This behaviour of \( \delta_0 \) is attributed to the so-called strong pionic final state interactions which I will discuss in section 3.
Figure 1. $\pi\pi$ scattering phase shift $\delta_0^0(s)$. The dashed line gives the tree result and the dashed-dotted the one-loop prediction. Also shown is the Roy equation band. The data can be traced back from ref.[12]. The double-dashed line corresponds to the one-loop result based on another definition of the phase-shift which differs at order $E^6$ from the one leading to the dashed-dotted line (and thus gives a measure of higher order corrections). On the right side of the hatched area, the one-loop corrections exceed 50 per cent of the tree result.
As indicated in fig.1, beyond 450 MeV the one loop corrections are half as big as the tree phase. Nevertheless, one can make a rather precise statement about the phase of the CP-violation parameter $\epsilon'[12], \Phi(\epsilon') = \frac{\pi}{2} - (\delta_0^0 - \delta_2^0)_{s=M_{K^0}^2} = (45 \pm 6)\degree$ (9)

This is due to the fact that the corrections to $\delta_2^0$ are of the same sign as the ones to $\delta_0^0$ and thus cancel. At tree level, $\Phi(\epsilon') = 37\degree$. The accuracy of the theoretical prediction is as good as the resent empirical one, $\Phi(\epsilon')_{\text{exp}} = (43 \pm 8)\degree$ [7]. Notice that it is much more difficult to get a precise number on $\Phi(\epsilon')$ from $K \rightarrow 2\pi$ decays because of the variety of isospin breaking effects one has to account for (this theme is touched upon in ref.[12]).

I briefly turn to the case of $\pi K$ scattering. Here, the empirical situation is even worse, which is very unfortunate. In the framework of conventional CHPT, the threshold behaviour of the low partial waves can be unambiguously predicted [13] since all low-energy constants in SU(3) are fixed. Furthermore, since the mass of the strange quark is of the order of the QCD scale–parameter, it is less obvious that the chiral expansion at next–to–leading order will be sufficiently accurate. Much improved empirical information of these threshold parameters might therefore lead to a better understanding of the three flavor CHPT. Another possibility is that the threshold of $\pi K$ scattering at 635 MeV is alreday so high that one has to connect CHPT constraints with dispersion theory. This concept has investigated in detail by Dobado and Pelaez [14] and certainly improves the prediction in the P–wave drastically. Another way of extending the EFT through the implicit inclusion of resonance degrees of freedom is discussed in ref.[62]. On the experimental side, a measurement of $\pi K$ molecule decays would certainly help to clarify the situation [11].

3. Two loops and beyond

The simplest object to study in detail the strong pionic final state interactions in the isospin zero S–wave is a three–point function, namely the so–called scalar form factor (ff) of the pion,

$$< \pi^a(p')\pi^b(p)|\bar{m}(\bar{u}u + \bar{d}d)|0> = \delta^{ab} \Gamma_\pi(s)M_\pi^2$$

with $s = (p + p')^2$. To one loop order, the scalar ff $\Gamma_{\pi,2}(s)$ has been given in ref.[3]. As shown in fig.2, closely about the two–pion cut, the real as well as the imaginary part of the one loop representation are at variance with the empirical information obtained from a dispersion–theoretical analysis [15]. However, unitarity allows one to write down a two–loop representation [16],

$$\Gamma_\pi(s) = d_0 + d_1 s + d_2 s^2 + \frac{s^3}{\pi} \int_4^{\infty} \frac{ds'}{s'^3} \sigma(s') \left\{ T_{\pi,2}^0(1 + \text{Re}\Gamma_{\pi,2}) + T_{\pi,4}^0 \right\}$$

where $T_{\pi,2}^0$ and $T_{\pi,4}^0$ are the tree and one loop representations of the $\pi\pi$ S–wave, isospin zero scattering matrix. Notice that the imaginary part of $\Gamma_\pi(s)$ to two
loops is entirely given in terms of known one loop amplitudes. The three subtraction constants appearing in (11) can be fixed from the empirical knowledge of the normalization, the slope and the curvature of the scalar $f_f$ at the origin. In the chiral expansion, these numbers are combinations of two low-energy constants from $L^{(4)}_{\text{eff}}$ and two from $L^{(6)}_{\text{eff}}$.

![Figure 2. Scalar form factor of the pion. The curves labelled '1', '2', 'O' and 'B' correspond to the chiral prediction to one-loop, to two-loops, the modified Omnès representation and the result of the dispersive analysis, respectively. The real part is shown in (a) and the imaginary part in (b).](image)

The turnover of the scalar $f_f$ at around 550 MeV can be understood if one rewrites (11) in an exponential form,

$$\text{Re} \Gamma_\mp(s) = P(s) \exp[\text{Re} \Delta_0(s)] \cos \delta_0 + O(E^6)$$

with $\text{Im} \Delta_0(s) = \delta_0 + O(E^6)$ fulfilling the final-state theorem at next-to-leading order. Although this representation is not unique, it allows to understand the vanishing of $\text{Re} \Gamma_\mp(s)$ at 680 MeV since the phase (in the loop approximation) passes through 90° at this energy thus forcing the turnover. Expanding $\cos \delta_0 = 1 - (\delta_0^2)^2 + \ldots = 1 + O(s^2/F^4)$ it becomes clear why this behaviour can only show up at two loop order (and higher). One can do even better and sum up all leading and next-to-leading logarithms by means of an Omnès representation [16]. This leads to
a further improvement in \( \text{Re}\Gamma_\pi(s) \) and allows to understand that the very accurate two loop result for \( \text{Im}\Gamma_\pi(s) \) is not spoiled by higher orders, these can be estimated from the improved chiral expansion of the scalar \( \sigma \) and are found to be small below 550 MeV. The physics behind all this is that the two-loop corrections lead to the two-pion cut with proper strength which dominates the scalar \( \sigma \) below 600 MeV. To go further one would have to include inelasticities (which start at order \( E^6 \)), in particular the strong coupling to the \( KK \) channel. It is also worth pointing out that the scalar \( \sigma \) can only be represented by a polynomial below \( s = 4M_\pi^2 \). Notice that in this energy range the normalized scalar \( \sigma \) varies from 1 to 1.4, signaling a large scalar radius of the pion. For comparison, the vector \( \sigma \) changes from 1 to 1.15 for \( 0 \leq s \leq 4M_\pi^2 \). In this way, unitarity allows to extend the range of CHPT, however, one has to be able to fix the pertinent subtraction constants (which is the equivalent to determining the corresponding low-energy constants).

Another reaction which has attracted much attention recently is \( \gamma\gamma \to \pi^0\pi^0 \) in the threshold region. It belongs to the rare class of processes which are vanishing at tree level (since the photon can only couple to charged pions, one needs at least one loop) and do not involve any of the low-energy couplings from \( s \to 4M_\pi^2 \) at one loop order. Some years ago, Bijnens and Cornet [17] and Donoghue, Holstein and Lin [18] calculated the one-loop cross section and found that it is at variance with the Crystal ball data [19] even close to threshold (see fig.3).

This is another case where one has to account for the strong pionic final state interactions. At 400 MeV, one has

\[
\left( \frac{\sigma_{\exp}}{\sigma_{1\text{-loop}}} \right)^{1/2} = 1.3
\]

which is a typical correction in this channel (see discussion above on \( \alpha_0^0 \) and the scalar \( \sigma \)). In fact, dispersion theoretical calculations supplemented with current algebra constraints by Pennington [20] tend to give the trend of the data (see the shaded area in fig.3). An improved combination of chiral machinery and dispersion theory has been given by Donoghue and Holstein [21]. Even better, Bellucci, Gasser and Sainio [22] have performed a full two loop calculation. It involves some massive algebra and three new low-energy constants have been estimated from resonance exchange (the main contribution comes from the \( \omega \)). These couplings play, however, no role below 400 MeV. The solid line in fig.3 shows the two-loop result for the central values of the coupling constants. One finds a good agreement with the data up to \( E_{\pi\pi} = 700 \) MeV. This resolves the long-standing discrepancy between the chiral prediction and the data in the threshold region. For a more detailed discussion of these topics and the related neutral pion polarizabilities, I refer to ref.[22].

The last topic I briefly want to mention is the radiative kaon decay \( K_L \to \pi^0\gamma\gamma \) which has no tree-level contribution and is given by a finite one-loop calculation at order \( E^4 \) [23]. The predicted two-photon invariant mass spectrum turned out to be in amazing agreement with the later measurements [24]. However, the branching ratio which is also predicted was found about a factor three too small. Again, unitarity corrections work in the right direction. In recent work by Cohen, Ecker and Pich [25] and Kambor and Holstein [26] it is shown that unitarity corrections (eventually supplemented by a sizeable \( E^6 \) vector meson exchange contribution)
Figure 3. Cross section for $\gamma\gamma \rightarrow \pi^0\pi^0$. The chiral one and two loop predictions are given by the dotted and the solid line, in order. The hatched area is a dispersion-theoretical fit. The Crystal ball data are also shown. From [22].
can indeed close the gap between the empirical branching ratio and the CHPT prediction though not completely. These calculations are, however, not taking into account all effects beyond $E^4$ but they underline the importance of making use of dispersion theory in connection with CHPT. 5

4. Inclusion of baryons

While the chiral Lagrangian is particularly suited to investigate the properties of the pseudoscalar mesons, it can also be used to gain insight into the structure of the low-lying baryons. I will be brief on the formal aspects but rather refer to the reviews [4,27] and the extensive papers by Gasser, Sainio and Švarc [28] and Krause [29].

Let me first restrict myself to the two flavor sector, the pion–nucleon system. To lowest order $O(E)$, the effective Lagrangian takes the form

$$L_{\pi N} = \bar{\Psi}(i \gamma_\mu D^\mu - m + \frac{1}{2} g_A \gamma_\mu \gamma_5 u^\mu)\Psi$$

with $D_\mu$ the covariant derivative, $m$ the nucleon mass (in the chiral limit), $g_A$ the axial-vector coupling constant (in the chiral limit) and $u_\mu = iu^\dagger \nabla_\mu U u^\dagger, u = \sqrt{U}$ and $\Psi$ embodies the proton and neutron fields. The physics becomes most transparent if one expands the various terms in powers of the pion and external fields (like e.g. the photon). The vectorial coupling includes e.g. the photon–nucleon vertex and the two–pion seagull ("Weinberg term") whereas the axial-vector term in (14) leads to the pseudovector $\pi N$ coupling and the famous Kroll–Rudermann vertex (among others). The presence of the nucleon mass term, which is of comparable size to the chiral symmetry breaking scale, does not allow the nucleon four–momentum to be treated as small. This spoils the one–to–one correspondence between the loop and the energy expansion (for more details see refs.[27,28]). As pointed out in particular by Jenkins and Manohar [30], heavy quark EFT methods help to overcome this problem. Consider the nucleon as a very heavy, static source, i.e. non–relativistically. In that case, one can write its four–momentum as

$$p_\mu = mv_\mu + l_\mu$$

with $v_\mu$ the four–velocity ($v^2 = 1$) and $l_\mu$ a small off–shell momentum, $v\cdot l \ll m$. One can therefore write the nucleon wave function in terms of velocity eigenstates,

$$\Psi = \exp\{-imv \cdot x\}(H + h)$$

with $\sqrt{H} = H$ and $\sqrt{h} = -h$ (notice that I have interchanged the $H$ and the $h$ in comparison to the standard (funny) notation). If one now eliminates $h$ by use of its equation of motion (or by a Foldy–Wouthuysen transformation), one finds (for details, see e.g. ref.[31])

$$L_{\pi N} = \bar{\Psi}(i v \cdot D + g_A u \cdot S)H + O(1/m)$$

with $S_\mu$ the covariant spin–operator, $S_\mu = i\gamma_5 \sigma_\mu v^\nu/2$, known to our ancestors also as the Pauli–Lubanski vector. The cumbersome baryon mass term has disappeared and thus a consistent power counting emerges (this is discussed very nicely...
in Ecker’s lectures [32]). One also notices that all Dirac bilinears can be expressed in terms of $v_\mu$ and $S_\mu$ thus facilitating the algebra enormously. The one loop graphs contribute at order $E^3$. As will be discussed below, it is however mandatory to include the terms of order $E^4$ in the effective Lagrangian for accurate one-loop calculations, i.e.

$$L_{\pi N} = L_{\pi N}^{(1)} + L_{\pi N}^{(2)} + L_{\pi N}^{(3)} + L_{\pi N}^{(4)}$$

(18)

where I have not exhibited the meson Lagrangian discussed before. The coefficients accompanying $L_{\pi N}^{(2)}$ are all finite since the loops start to contribute at order $E^3$. A complete analysis of the divergence structure at order $E^3$ will soon be available [33]. A systematic analysis of nucleon properties to order $E^3$ can be found in [31] and some $E^4$ calculations have recently been performed. I will discuss one particular case in some more detail below. I will also elaborate on two yet unsolved problems in the heavy mass approach, one is related to the analytical structure of S-matrix elements (which does not appear in the relativistic formulation, see section 8) and the other is the extension to flavor SU(3) and the inclusion of decuplet fields (see sections 6 and 7).

5. Baryon compton scattering

Compton scattering off the nucleon at low energies offers important information about the structure of these particles in the non-perturbative regime of QCD. The spin-averaged forward scattering amplitude for real photons in the nucleon rest frame can be expanded as a power series in the photon energy $\omega$,

$$T(\omega) = f_1(\omega^2) \vec{\epsilon}_f \cdot \vec{\epsilon}_i, \quad f_1(\omega^2) = a_0 + a_1 \omega^2 + a_2 \omega^4 + \ldots$$

(19)

where $\vec{\epsilon}_i,f$ are the polarization vectors of the initial and final photon, respectively, and due to crossing symmetry only even powers of $\omega$ occur. The Taylor coefficients $a_i$ encode the information about the nucleon structure. The first term in eq.(19), $a_0 = -e^2 Z^2/4\pi m$, dominates as the photon energy approaches zero, it is only sensitive to the charge $Z$ and the mass $m$ of the particle the photon scatters off (the Thomson limit). The term quadratic in the energy is equal to the sum of the so-called electric ($\tilde{\alpha}$) and magnetic ($\tilde{\beta}$) Compton polarizabilities, $a_1 = \tilde{\alpha} + \tilde{\beta}$. Corrections of higher order in $\omega$ start out with the term proportional to $a_2$. Over the last years, high precision measurements at Mainz, Illinois, Oak Ridge and Saskatoon [34] have lead to the following empirical values: $\tilde{\alpha}_p = (10.4 \pm 0.6) \cdot 10^{-4} \text{fm}^3$, $\tilde{\beta}_p = (3.8 \pm 0.6) \cdot 10^{-4} \text{fm}^3$, $\tilde{\alpha}_n = (12.3 \pm 1.3) \cdot 10^{-4} \text{fm}^3$, $\tilde{\beta}_n = (3.5 \pm 1.3) \cdot 10^{-4} \text{fm}^3$ making use of the dispersion sum rules [35] $(\tilde{\alpha} + \tilde{\beta})_p = (14.2 \pm 0.3) \cdot 10^{-4} \text{fm}^3$ and $(\tilde{\alpha} + \tilde{\beta})_n = (15.8 \pm 0.5) \cdot 10^{-4} \text{fm}^3$. The two outstanding features of these numbers are the fact that $(\tilde{\alpha} + \tilde{\beta})_p \simeq (\tilde{\alpha} + \tilde{\beta})_n$ and that the proton as well as the neutron behave essentially as (induced) electric dipoles.

In CHPT, it was found that to leading order in the chiral expansion the nucleon em polarizabilities are given by a few one loop diagrams (whose sum is finite) [36] with no counter term contributions (much like the reactions $K_L \to \pi^0\gamma\gamma$ and $\gamma\gamma \to \pi^0\pi^0$ discussed before). This calculation was later redone in the heavy mass approach [31]. The pertinent diagrams are shown in fig.4.
Figure 4. One-loop diagrams which lead to the nucleon em polarizabilities (20).

By isospin arguments, one finds that they will lead to the same polarizabilities for the proton and the neutron. The resulting expressions for $\tilde{\alpha}_p, n$ and $\tilde{\beta}_p, n$ contain therefore only parameters from the lowest order effective Lagrangian $L_{\pi N}^{(1)} + L_{\pi\pi}^{(2)}$ [31,36],

$$\tilde{\alpha}_p = \tilde{\alpha}_n = \frac{5e^2 g_A^2}{384\pi^2 F^2} \frac{1}{M^2} = 12.2 \cdot 10^{-4} \text{fm}^3, \quad \tilde{\beta}_p = \tilde{\beta}_n = \frac{\tilde{\alpha}_p}{10} = 1.2 \cdot 10^{-4} \text{fm}^3 \quad (20)$$

In the chiral limit, the em polarizabilities diverge. This is expected since the Yukawa suppression for massive pions turns into a long-range power-law fall-off as $M \to 0$. Clearly, the leading order CHPT results (20) explain the trends of the data. However, one might argue that the result for the magnetic polarizabilities is not very meaningful since one has not accounted for the strong $N\Delta$ M1 transition. In fact, this starts to contribute at order $E^4$ (and higher) in agreement with the decoupling theorem [37]. In ref.[38], a complete calculation of the em polarizabilities to $O(E^4)$ was given. At this order, contact terms from $L_{\pi N}^{(2,3,4)}$ enter. Some of them can be directly related to empirical information (viz $\pi N$ scattering [39]), others are estimated from resonance exchange, and this is where the $\Delta(1232)$ comes in. It should be stressed that the resonance saturation hypothesis to understand the values of the low-energy constants has only strictly been tested in the meson sector [40]. In the absence of sufficiently many accurate low-energy data, it serves as a working hypothesis in the baryon sector (this situation will be improved when more data will become available). The em polarizabilities to order $E^4$ take the form

$$(\tilde{\alpha}, \tilde{\beta})_{p,n} = \frac{C_1}{M^2} + C_2 \ln M^2 + C_3 \quad (21)$$

with $C_1 = 5e^2 g_A^2 / 384\pi^2 F^2$. The coefficient $C_2$ contains some low-energy constants from $L_{\pi N}^{(2)}$ and $C_3$ four novel ones from $L_{\pi N}^{(4)}$. The second and the third term in (21) are the new $E^4$ contributions. The $\Delta(1232)$ strongly dominates the constant $C_3$. The non-analytic loop contribution $\sim \ln M^2$ is potentially large. Indeed, in the case of $\tilde{\beta}_p$ the $\ln$ contribution is negative with a large coefficient and cancels most of the large positive one related to the $\Delta(1232)$ exchange. In ref.[41], a thorough study of the theoretical uncertainties entering the $E^4$ calculation was performed and the following results emerged

$$\tilde{\alpha}_p = 10.5 \pm 2.0, \quad \tilde{\beta}_p = 3.5 \pm 3.6, \quad \tilde{\alpha}_n = 13.4 \pm 1.5, \quad \tilde{\beta}_n = 7.8 \pm 3.6, \quad (22)$$
all in $10^{-4}\text{fm}^3$. For the electric polarizabilities the chiral expansion is well-behaved, i.e. the $E^4$ corrections amount to 14 (10) per cent for the proton (neutron). In the case of the magnetic polarizabilities large cancellations occur and a calculation at $\mathcal{O}(E^5)$ is called for. The large theoretical uncertainties stem mostly from the badly known off-shell parameters related to the $\pi N\Delta$ and $\gamma N\Delta$ dynamics and to some extent also from the contribution from strange ($K^+$) loops.

In fig.5, I show the real part of the proton forward Compton scattering amplitude $A_p(\omega) = -4\pi f_1(\omega^2)$ in comparison to the total photonucleon absorption cross section via

$$\text{Re } A_p(\omega) = \frac{e^2 Z^2}{m} - \frac{2\omega^2}{\pi} P \int_{\omega_0}^{\infty} d\omega' \frac{\sigma_{\text{tot}}(\omega')}{\omega'^2 - \omega^2}$$

(23)

The amplitude has a branch point related to the threshold energy for single pion photoproduction at

$$\omega_0 = M_\pi (1 + \frac{M_\pi}{2m})$$

(24)

Notice that to order $E^3$ the chiral representation [31] has a cut starting at $\omega_0 = M_\pi$ and only at $\mathcal{O}(E^4)$ one recovers the recoil correction $\sim 1/m$ in (24). This problem is inherent to the heavy mass formulation of baryon CHPT. In the relativistic formulation, the corresponding analytic structures (location of cut singularities) are always given correctly and do not depend on the order of the chiral expansion.

I will pick up this theme in section 8. Fig.5 also shows that the $E^4$ result for $A_p(\omega)$ reproduces the cusp at $\omega_0$. However, above that energy, the chiral prediction is at variance with the data. This can be traced back to the fact that the imaginary part changes sign at $\omega \simeq 180\text{ MeV}$. One would have to go to two loops to get an improved prediction for the imaginary part as already stressed in section 3.

It is fairly straightforward to extend the lowest order $E^3$ calculation to the three flavor sector. This allows to predict the hyperon polarizabilities which eventually will be measured at Fermilab and CERN via the Primakoff effect. In SU(3), one has two axial couplings and thus the lowest order effective meson–baryon Lagrangian reads

$$\mathcal{L}^{(1)}_{MB} = \text{Tr}(\bar{B}i\nu \cdot D B) + D \text{Tr}(\bar{B}S^\mu \{u_\mu, B\}) + F \text{Tr}(\bar{B}S^\mu [u_\mu, B])$$

(25)

where $U(x)$ now contains the eight pseudoscalar fields ($\pi, K, \eta$) and $B$ is a $3 \times 3$ matrix containing the low-lying baryon octet,

$$B = \begin{pmatrix}
\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda \\
\Sigma^+ \\
\Sigma^- \\
\Xi^- \\
\Xi^0 \\
= \frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda \\
= \frac{1}{\sqrt{6}} \Lambda
\end{pmatrix}$$

(26)

The numerical values for the two axial couplings $D$ and $F$ are $D \simeq 3/4$ and $F \simeq 1/2$ subject to the constraint $D + F = g_A = 1.26$. The hyperon polarizabilities can be calculated from the diagrams in fig.4 with $\pi$ and $K$ loops and one finds e.g. [42]

$$\bar{\alpha}_{\Sigma^+} = 6, \quad \bar{\alpha}_{\Sigma^+} = 9,$$

(27)

(in canonical units), i.e. the $\Sigma^+$ is expected to have a larger electric polarizability than the $\Sigma^-$ due to the kaon loop contributions. Quark model estimates give a
Figure 5. Forward spin-averaged Compton amplitude for the proton in comparison to the data. (a) $0 \leq \omega \leq 140$ MeV and (b) $140 \leq \omega \leq 210$ MeV.
similar pattern [43]. The $\Sigma^+$ is made of $u$ and $s$ quarks (which have opposite charges) and this allows for internal electric dipole excitations. In contrast, the charge-like $d$ and $s$ quarks in the $\Sigma^-$ tend to hinder such excitations leading to a small electric polarizability. The numbers given in (27) should be considered as first estimates since a complete $E^4$ calculation in SU(3) has yet to be performed. I will now take a critical look at the status of three flavor baryon CHPT.

6. Baryon masses and $\sigma$-terms

The simplest observables to investigate in flavor SU(3) are the baryon masses $m_N$, $m_\Lambda$, $m_\Sigma$, $m_\Xi$ and the three proton $\sigma$-terms defined via

$$\sigma_{\pi N}(t) = \langle \bar{m} < p'|\bar{u}u + \bar{d}d|p > \rangle$$

$$\sigma_{KN}^{(1)}(t) = \frac{1}{2}(\bar{m} + m_s) < p'|\bar{u}u + \bar{s}s|p >$$

$$\sigma_{KN}^{(2)}(t) = \frac{1}{2}(\bar{m} + m_s) < p'| - \bar{u}u + 2\bar{d}d + \bar{s}s|p >$$

with $t = (p' - p)^2$ the invariant momentum transfer squared and $\bar{m} = (m_u + m_d)/2$ the average light quark mass. At zero momentum transfer, the strange quark contribution to the proton mass is given by [44]

$$m_s < p|\bar{s}s|p > = \left( \frac{1}{2} - \frac{M_s^2}{4M_K^2} \right) \left[ 3\sigma_{KN}^{(1)}(0) + \sigma_{KN}^{(2)}(0) \right] + \left( \frac{1}{2} - \frac{M_K^2}{M_s^2} \right) \sigma_{\pi N}(0)$$

making use of the leading order meson mass formulæ $M_s^2 = 2mB$ and $M_K^2 = (\bar{m} + m_s)B$. This defines the scalar sector of baryon CHPT. To calculate the mass spectrum to order $E^3$, we need the symmetry breaking terms from

$$L_{MB}^{(2)} = b_D \text{Tr}(\tilde{B}[\chi_+, B]) + b_F \text{Tr}(\tilde{B}[\chi_+, B]) + b_0 \text{Tr}(\tilde{B}B)\text{Tr}(\chi_+)$$

with $\chi_+ = u^\dagger \chi u + u^\dagger \chi^\dagger u$ and $\chi = 2B(M + S)$ where $S$ denotes the nonet of external scalar sources. The constants $b_D$, $b_F$ and $b_0$ can be fixed from the knowledge of the baryon masses and the $\pi N \sigma$-term (or one of the $KN \sigma$-terms). The constant $b_0$ cannot be determined from the baryon mass spectrum alone since it contributes to all octet members in the same way. To this order in the chiral expansion, any baryon mass takes the form $[44,45]

$$m_B = m_0 - \frac{1}{24\pi F^2} \left[ \alpha_B^2 M_s^2 + \alpha_B^2 M_K^2 + \alpha_B^2 M_{N}^2 \right] + \gamma_D b_D + \gamma_F b_F - 2b_0(M_s^2 + 2M_K^2)(31)$$

The first term on the right hand side of (31) is the average octet mass in the chiral limit, the second one comprises the Goldstone boson loop contributions and the third term stems from the counter terms in (30) ("resonance physics"). Notice that the loop contribution is ultraviolet finite and non-analytic in the quark masses since $M_f^2 \sim m_f^{3/2}$. The constants $b_D$, $b_F$ and $b_0$ are therefore finite. A typical result at $O(E^3)$ from a least-square fit to $m_N$, $m_\Lambda$, $m_\Sigma$, $m_\Xi$ and $\sigma_{\pi N}(0) = 45$ Mev [46] is [44]

$$m_N = (0.965 - 0.018 - 0.264 + 0.248)\text{ GeV} = 0.936\text{ GeV}$$

$$m_\Lambda = (0.965 - 0.006 - 0.588 + 0.743)\text{ GeV} = 1.141\text{ GeV}$$

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where the various terms are the average octet mass, the pion loop, the \( K \) and \( \eta \) loop and the counterterm contributions, in order. A closer look at the results \((32)\) reveals that there are large cancellations between the strange loops and the counter terms. To have a more well-behaved chiral expansion, one might want to include the low-lying decuplet baryons as will be discussed below. At this order and within the accuracy of the \( E^3 \) calculation, the \( KN \sigma \)-terms turn out to be
\[
\sigma_{KN}^{(1)}(0) \simeq 200 \pm 50 \text{ MeV}, \quad \sigma_{KN}^{(2)}(0) \simeq 140 \pm 40 \text{ MeV}
\]
which is comparable to the first order perturbation theory analysis having no strange quarks, \(\sigma_{KN}^{(1)}(0) = 205 \text{ MeV} \) and \(\sigma_{KN}^{(2)}(0) = 63 \text{ MeV} \) \([47]\). At present, the \( KN \sigma \)-terms are not well determined. Since most of the phase shift data stem from kaon–nucleus scattering, it is of advantage to define them in terms of nuclear isospin, \(\sigma_{KN} = (3\sigma_{KN}^{(2)} + \sigma_{KN}^{(1)})/4 \) and \(\sigma_{KN}'' = (\sigma_{KN}^{(2)} - \sigma_{KN}^{(1)})/2 \). The best determinations available gives \(\sigma_{KN}^{(1)}(0) = 599 \pm 377 \text{ MeV} \) and \(\sigma_{KN}^{(2)}(0) = 87 \pm 66 \text{ MeV} \) \([48]\). This translates into \(\sigma_{KN}^{(1)}(0) = 469 \pm 390 \text{ MeV} \) and \(\sigma_{KN}^{(2)}(0) = 643 \pm 378 \text{ MeV} \). Let me finish this section with a remark on the calculation of \(m_N\) in a recent lattice simulation in quenched baryon CHPT \([49]\) (which is discussed in some detail by Golterman in these proceedings),
\[
m_N^{\text{CHPT}} = (0.97 - 0.5M_\pi + 3.4M_\pi^2 - 1.5M_\pi^3) \text{ GeV}
\]
\[
m_N^{\text{LFIT}} = (0.96 - 1.0M_\pi + 3.6M_\pi^2 - 2.0M_\pi^3) \text{ GeV}
\]
where LFIT denotes the fit to the lattice data. It is most significant that the negative curvature due to the \(M_\pi^2\) term from quenched CHPT and from the lattice fit are of the same magnitude. There is, however, a problem at small \(M_\pi\). The lattice data give a too large pion mass so that one has to plot \(m_N\) versus \(M_\pi\) and interpolate to the physical pion mass. On finds a hook at the lower end of this curve which sheds some doubts on the accuracy of the recently reported results by Butler et al. \([63]\). For more details on this, see Golterman \([64]\).

7. Inclusion of the decouplet in the EFT

The low-lying decuplet is only separated by \(\Delta = 231 \text{ MeV}\) from the octet baryons, which is just \((5/2)F_\pi\) (notice that I have not used the conventional argument \(\Delta = (5/3)M_\pi\) since the splitting stays finite in the chiral limit much like \(F_\pi\) and not at all like \(M_\pi\) and considerably smaller than the kaon or \(\eta\) mass. One therefore expects the excitations of these resonances to play an important role even at low energies. This is also backed by phenomenological models of the nucleon in which the \(\Delta(1232)\) excitations play an important role. In addition, there are large \(N_C\) arguments \([50]\) which, however, have to be taken \textit{cum grano salis} since the chiral and infinite number of color limits do not commute. In the meson sector, the first resonances are the vector mesons \(\rho\) and \(\omega\) at about 800 MeV, \textit{i.e.} they are considerably heavier than the Goldstone bosons. However, it is not only the small octet–decuplet splitting which plays a role. One should also notice that the \(\Delta(1232)\) coupling to the \(\pi N\) system is very large, \(g_{\pi\Delta N} \simeq 2g_{\pi NN}\) with \(g_{\pi NN} = 13.5\). Similarly, the \(\gamma\Delta N\) coupling is very strong. Would the \(\Delta(1232)\) (or the decuplet) be weakly coupled to the nucleon (octet), its role would be very different. It was therefore argued by Jenkins and Manohar \([51]\) to include the spin-3/2 decuplet in the
effective theory from the start. Denote by $T^\mu$ a Rarita-Schwinger fields in the heavy mass formulation satisfying $v \cdot T = 0$. The effective Lagrangian of the spin-3/2 fields at lowest order reads

$$L_{MBT} = -i\bar{T}^\mu v \cdot \partial T_\mu + \Delta \bar{T}^\mu T_\mu + \frac{C}{2} (\bar{T}^\mu u_\mu B + \bar{B} u_\mu T^\mu).$$

where we have suppressed the flavor SU(3) indices. For an explicit expression see ref.[65]. Notice that there is a remaining mass dependence which comes from the average decuplet-octet splitting $\Delta$ which does not vanish in the chiral limit. The constant $C$ is fixed from the decay $\Delta \rightarrow N \pi$ or the average of some strong decuplet decays, $|C| = 1.5 \ldots 1.9$. The decuplet propagator carries the information about the mass splitting $\Delta$ and reads

$$\frac{iP_{\mu\nu}}{v \cdot l - \Delta + i\varepsilon}$$

with $P_{\mu\nu}$ a projector (for a review, see ref.[52]). The appearance of the mass splitting $\Delta$ spoils the exact one-to-one correspondence between the loop and low-energy expansion. The two scales $F_\pi$ and $\Delta$ which are both non-vanishing in the chiral limit enter the loop calculations and they can combine in the form $(\Delta/F_\pi)^2$. The breakdown of the consistent chiral counting in the presence of the decuplet is seen in the loop contribution to the baryon mass. The loop diagrams with intermediate decuplet states which naively count as order $E^4$ renormalize the average octet baryon mass even in the chiral limit by an infinite amount. Therefore one has to add a counter term of chiral power $E^0$ to keep the value $m_0$ fixed

$$\delta L^{(0)}_{MB} = -\delta m_0 \text{ Tr}(\bar{B} B)$$

$$\delta m_0 = \frac{10C^2\Delta^3}{3F_\pi^2} \left[ L + \frac{1}{16\pi^2} \left( \ln \frac{2\Delta}{\lambda} - \frac{5}{6} \right) \right]$$

$$L = \frac{\lambda^{d-4}}{16\pi^2} \left[ \frac{1}{d-4} + \frac{1}{2} (\gamma_E - \ln 4\pi - 1) \right]$$

with $\lambda$ the scale introduced in dimensional regularization and $\gamma_E$ the Euler-Mascheroni constant. This mass shift is similar to the one in the relativistic version of pion-nucleon CHPT, where the non-vanishing nucleon mass in the chiral limit leads to the same kind of complications [28]. The inclusion of the decuplet fields has three effects on the mass formulae (31). First there is an infinite loop contribution with decuplet intermediate states and, second, an infinite renormalization of the order $E^2$ of the low-energy constants $b_D, b_F$ and $b_0$ plus a finite contribution which starts out at $O(E^4)$. The constants $b_D, b_F$ and $b_0$ have to be renormalized as follows:

$$b_D = b_D^*(\lambda) - \frac{\Delta C^2}{2F_\pi^2} L,$$

$$b_F = b_F^*(\lambda) + \frac{5\Delta C^2}{12F_\pi^2} L,$$

$$b_0 = b_0^*(\lambda) + \frac{7\Delta C^2}{6F_\pi^2} L,$$

where the finite pieces $b_D, F, b_0^*(\lambda)$ are then determined by the fitting procedure. The explicit form of the decuplet contributions to the octet masses can be found in
In table 1, I show some results of these fits. It is obvious that simply taking the decuplet to account for the $E^4$ (and higher) contributions does not lead to a consistent picture of the scalar sector of CHPT (notice that in ref.[45] some tadpole diagrams with insertions from $\mathcal{L}_{MB}^{(2)}$ have also been included but that does not alter these conclusions). As already stressed a couple of times, a complete $E^4$ calculation should be performed. For doing that, it might be easier to use the decuplet to estimate some low-energy constants rather than taking it as dynamical dof's in the EFT. The role of the decuplet contributions has also been critically examined by Luty and White [53].

### Table 1. Results of the calculation including the full decuplet intermediate states. The values of $D, F, \Delta$ and $C$ are input. All dimensionful numbers are in MeV. Empirically, $\Delta\sigma_{\pi N} = 15$ MeV and $\Sigma_p = m_s < p|\bar{s}s|p > = 130$ MeV [46].

| $D$ | $F$ | $\Delta$ | $C$ | $\sigma_{KN}^{(1)}(0)$ | $\sigma_{KN}^{(2)}(0)$ | $\Delta\sigma_{\pi N}$ | $\Sigma_p$ |
|-----|-----|-------|----|----------------|----------------|----------------|-------|
| 0.75 | 0.50 | 293 | 1.8 | 38 | -23 | 14.3 | 513 |
| 0.75 | 0.50 | 293 | 1.5 | 86 | 28 | 12.3 | 419 |
| 0.75 | 0.50 | 231 | 1.8 | 67 | 10 | 15.1 | 455 |
| 0.75 | 0.50 | 231 | 1.5 | 106 | 51 | 12.8 | 379 |
| 0.56 | $2D/3$ | 293 | $2D$ | 244 | 150 | 6.8 | 132 |
| 0.56 | $2D/3$ | 231 | $2D$ | 255 | 163 | 7.1 | 110 |

8. Spectral distribution of the nucleon form factors

When discussing the forward Compton amplitude, I mentioned that the corresponding branch point related to the one–pion threshold has itself a $1/m$ expansion in the heavy mass formulation which disturbs the analytical structure of the amplitude. To take a closer look at this problem, let us consider the chiral expansion of the so–called (isovector) Pauli form factor $F^Y_V(t)$. It is defined by the matrix–element of the isovector–vector quark current,

$$< p'|\bar{q}\gamma_\mu \tau^a \frac{2}{2} q|p > = \bar{u}(p') \left[ \gamma_\mu F^Y_1(t) + \frac{i\sigma_{\mu\nu}k^\nu}{2m} F^Y_2(t) \right] \tau^a \bar{u}(p)$$

with $k = p' - p$ and $t = k^2$. As first observed by Frazer and Fulco [54] and discussed in detail by Höhler and Pietarinen [55] the imaginary part of $F^Y_2(t)$ exhibits a strong enhancement very close to threshold ($t = 4M^2$) as shown in fig.6a. The imaginary part of the isovector nucleon form factors inherit the singularity on the second sheet.
due to the projection of the Born term (at $t_0 = 4M_\pi^2(1 - M_\pi^2/4m^2) = 3.98M_\pi^2$) in $\pi N$ scattering (from diagrams of the type $\gamma \rightarrow \pi\pi \rightarrow N\bar{N}$).

Let us first consider the chiral expansion of $\text{Im} F_2^V(t)$ in the relativistic formulation of baryon CHPT. Following Gasser et al. [28], one has

$$\text{Im} F_2^V(t) = \frac{8g_A^2}{F_\pi^2} m^4 \left[ 4 \text{Im}\gamma_4(t) + \text{Im}\Gamma_4(t) \right]$$

where the loop functions and their imaginary parts $\gamma_4$ and $\Gamma_4$ are given in ref.[28]. For our purpose, we only need $\text{Im} \gamma_4(t)$ since its threshold is the two-pion cut whereas $\text{Im} \Gamma_4(t)$ only starts to contribute at $t = 4m^2$. The resulting imaginary part for $\text{Im} F_2^V(t)/t^2$ is shown in fig.6b (solid line). One sees that the strong increase at threshold is reproduced (see also the remarks in ref.[28]) since the chiral representation of $\text{Im} \gamma_4(t)$ indeed has the proper analytical structure, i.e. the singularity on the second sheet at $t_0$. The chiral representation of $\text{Im} F_2^V(t)/t^2$ does not stay constant on the left shoulder of the $p$-resonance but rather drops. This is due to the fact that in the one loop approximation, one is only sensitive to the first term in the chiral expansion of the pion charge form factor $F_2^V(t)$. In fig.6a I also show calculations with $F_2^V(t) = 1$ (dashed line) and $F_2^V(t) = 1 + 1$, $<r^2>$, $t/6$ (dash-dotted line). These curves resemble very much the chiral expansion. To reiterate, this particular example shows that in the relativistic version of baryon CHPT the pertinent analytical structures of current and S-matrix elements are given correctly.

Let us now consider the heavy mass approach. The corresponding imaginary part follows from ref.[31],

$$\text{Im} F_2^V(t) = \frac{g_A^2 m}{8F_\pi^2} \left( \frac{1}{4} - \frac{M_\pi^2}{t} \right) \sqrt{t} \Theta(t - 4M_\pi^2)$$

Here, the imaginary part comes form a $\ln(2M_\pi - \sqrt{t})$ which has a branch point at $t = 4M_\pi^2$ (chiefly because to lowest order in the $1/m$ expansion the threshold energy of $\pi N$ scattering is $\omega_0 = M_\pi$ [39] and the corresponding left-handed cut starts there). This also leads to an enhancement of the imaginary part of $F_2^V(t)$ as shown by the dashed line in fig.6b. The enhancement is stronger than in the relativistic case. Stated differently, to this order in the chiral expansion the analytic structure is not given correctly much like in the case of the forward Compton scattering amplitude discussed before. One should therefore perform an order $E^0$ calculation in the heavy mass approach to have a sufficiently accurate and correct representation of the isovector nucleon form factors. A two loop calculation will also answer the yet unresolved question whether or not in the isoscalar channel there is an enhancement around $t = 9M_\pi^2$. State of the art dispersion theoretical analysis of the nucleon form factors assume only a set of poles in the corresponding spectral distributions [56]. Finally, I wish to stress that in this context the matching formalism discussed in ref.[31] starts to play a role (which allows to relate matrix-elements in the heavy mass and relativistic formulation of CHPT).

9. Aspects of electroweak pion production

In this section, I will discuss some physics aspects related to threshold pion produc-
Topics in chiral perturbation theory

Figure 6. (a) Dispersion-theoretical result for $\text{Im} \frac{F^V_2(t)}{t^2}$ ($t$ in units of the pion mass, denoted $\mu$ here). As D-function, the inverse pion form factor is used, $D(t) = 1/F^V_\pi(t)$. The dash-dotted and dashed lines are explained in the text. (b) Chiral representation in the relativistic formulation of baryon CHPT [28] (solid line) and in the heavy mass approach (dashed line) [31].
tion by electroweak interactions. This is of particular interest for the now existing generation of CW electron machines like at Mainz, Bates ....

Let me first consider the production of one single pion by the isovector axial current. As particularly stressed by Adler [57], a unified treatment of em and weak pion production allows to relate information from neutrino–nucleon and electron–nucleon scattering experiments. Obviously, by using PCAC, the coupling of the weak axial current to the nucleon in the initial state and a nucleon plus a pion in the final state is closely related to πN scattering. Consider now a reaction of the type ν(k₁) + N(p₁) → l(k₂) + N(p₂) + π⁺(q) and define k = k₁ − k₂. The threshold energy squared is s = (p₁ + k)² = (m + Mₚ)². At threshold and in the πN cms frame, one can express the pertinent matrix–element in terms of six S–wave multipoles,

\[ T^{(±)} \cdot ε = 4π(1 + μ)χ² \left[ ε_0 L^{(±)}_{0+} + ε \cdot k H^{(±)}_{0+} + iε \cdot (k × ε) M^{(±)}_{0+} \right] \chi_1 \]  (42)

with \( χ_{1,2} \) two–component Pauli spinors, \( μ = Mₚ/m \) the ratio of the pion and nucleon mass and \( ε_μ ≈ υ_iγ_μυ_u \) the axial polarization vector. The superscript '±' refers to the isospin even/odd part of the amplitude. In ref.[58], we derived the chiral expansions of these threshold multipoles to order \( E^3 \). Of particular interest is the multipole \( L^{(±)}_{0+} \) since it is directly proportional to the so–called scalar form factor of the nucleon, \( σ_{πN}(t) \sim < p'|i\bar{u}(\bar{u} + d)l|p > [58], \)

\[ L^{(+)}_{0+} = \frac{1}{3πMₚFₚ} \left\{ \frac{σ_{πN}(k² - Mₚ²) - \frac{1}{4}σ_{πN}(0)}{a^+Fₚ} - \frac{A}{16πmFₚ} + C_L^{(+)}M² + O(q²) \right\} \]  (43)

The constant \( C_L^{(+)} \) subsumes numerous \( k² \)-independent kinematical, loop and counter term contributions. If one assumes \( C_L^{(+)} \) to be of the order of 1 GeV⁻³, the term proportional to the scalar form factor dominates the amplitude (43) in the threshold region. This might offer another determination of this much discussed quantity. However, an analysis including also higher order effects has to be performed to find out how cleanly this multipole can be separated in neutrino–induced pion production and how large the corresponding cross section is. It is furthermore interesting to note that although \( L^{(+)}_{0+} \) vanishes at the photon point \( k² = 0 \) in the chiral limit, its slope nevertheless stays finite - this is a particular effect due to chiral loops.

Another reaction of interest is the photoproduction of two pions in the threshold region. It gives complimentary information to the extensive studies of single pion photo– and electroproduction performed over the last few years. Dahm and Drechsel [59] were the first to systematically study the process \( γN → ππN \) in a chiral field theory. To be specific, they considered Weinberg’s pion–nucleon Lagrangian [60] coupling in the photon via minimal substitution. At threshold, the transition current takes the form

\[ T \cdot ε \bigg|_{thr} = iε \cdot (\bar{ε} \times ε) [M₁δ_{ab} + M₂δ_{ab}τ₃ + M₃(τₐδₗ₃ + τₖδₐ₃)] \]  (44)

where 'a,b' are the pion isospin indices and the \( τ \)'s act on the nucleon. The explicit form of the corresponding five–fold differential cross section is given in [59]. Here,
we are interested in the chiral expansion of the multipoles $M_1$, $M_2$ and $M_3$, i.e. their expansion in powers of $M_\pi$. In general, the reaction $\gamma N \rightarrow \pi\pi N$ involves five independent Mandelstam variables. At threshold, the kinematics is simplified since the pion four-momenta are equal, $q_1 = q_2 = (M_\pi, 0, 0, 0)$. In heavy fermion CHPT the calculation furthermore simplifies if one works in the Coulomb gauge $\epsilon \cdot v = 0$ and realizes that $S \cdot q_1 = S \cdot q_2 = 0$ [61]. The lowest order result stems from tree diagrams with one insertion from $\mathcal{L}^{(2)}_{\pi N}$ as shown in fig.7.

![Diagram](image)

Figure 7. Tree diagrams which lead to eq.(45). The box denotes an insertion from $\mathcal{L}^{(2)}_{\pi N}$. Solid, dashed and wiggly lines denote nucleons, pions and photons, respectively.

One finds

$$M_1 = \mathcal{O}(M_\pi), \quad M_2 = \frac{e}{4m^2F_\pi^2}(2g_\omega^2 - 1 - \kappa_V) + \mathcal{O}(M_\pi), \quad M_3 = \frac{-M_2}{2}. \quad (45)$$

These differ from the results in [59] by the terms proportional to $\kappa_V$ and are a factor two smaller in magnitude. The reason is that gauging the Weinberg Lagrangian can not generate the anomalous couplings of the photon. Of course, simply calculating these tree diagrams is not sufficient, one has to at least work out the $\mathcal{O}(M_\pi)$ corrections. These are (i) kinematical corrections of the type $M_\pi/m$, (ii) contributions from one-loop diagrams, (iii) further insertions from $\mathcal{L}^{(2,3)}_{\pi N}$ and (iv) contributions from tree diagrams with intermediate $\Delta(1232)$ states. As a preliminary result, let me consider the corrections of the first two types (a more thorough discussion can be found in ref.[61]). Although the $\Delta(1232)$ is very close to the two pion production threshold (the energy difference being 17 MeV), the potentially large diagrams with small energy denominators of the type $m_\Delta - (m_N + 2M_\pi)$ are suppressed by corresponding numerators. The corrections of type (i) and (ii) lead to [61]

$$M_1 = \frac{eg_\omega^2 M_\pi}{4m^2F_\pi^2} = 0.019 \text{ fm}^3$$

$$M_2 = \frac{eM_\pi}{4m^2F_\pi^2}(g_\omega^2 - \kappa_V) + \frac{eg_\omega^2 M_\pi}{64\pi F_\pi^4} \left[ \frac{3\pi}{2} + i(2\sqrt{3} - \ln(2 + \sqrt{3})) \right]$$
For $M_1$, the loops do not contribute to order $M_\pi$ whereas for $M_{2,3}$ they lead to a complex correction. This due to some loop diagrams involving two or three pion propagators which acquire an imaginary part for \( \omega > M_\pi \) (here, \( \omega = 2M_\pi \)). The first term on the r.h.s. of (46) stems from the kinematical corrections. Comparing the numbers in (46) to the lowest order results $M_1 = 0$, $M_2 = -2M_3 = 0.084$ fm$^3$, one sees that the $M_\pi$ corrections are large. One might therefore question the whole approach, but it is conceivable that once the type $$(iv)$$ corrections from the $\Delta$ are included, the dominant physics will be under control and subsequent higher order corrections play a minor role. This topic is under investigation [61]. To get an idea about the role of the loop and kinematical corrections, let us consider the specific final states like in $\gamma p \rightarrow \pi^+\pi^- p$. In that case, the cross section is proportional to the quantity $|M_{00}| = |(M_1 + M_2 + 2M_3)(M_1 + M_2 + 2M_3)^*|^{1/2}$ and similarly for the other channels. In table 2, I show $|M_{ij}|$ in fm$^3$ for the lowest order (45) and with the $M_\pi$ corrections (46). The most prominent result is that the double $\pi^0$ channels, which are vanishing to lowest order, are in fact dominant after the inclusion of the $O(M_\pi)$ corrections.

Table 2. Contribution of the threshold multipoles to two pion production channels in fm$^3$ (no phase space factors are accounted for).

| process          | \(O(1)\) | \(O(1) + O(M_\pi)\) |
|------------------|---------|---------------------|
| \(\gamma p \rightarrow \pi^+\pi^- p\) | 0.084   | 0.125               |
| \(\gamma p \rightarrow \pi^+\pi^0 n\) | 0.059   | 0.108               |
| \(\gamma p \rightarrow \pi^0\pi^0 p\) | 0.000   | 0.271               |
| \(\gamma n \rightarrow \pi^+\pi^- n\) | 0.084   | 0.099               |
| \(\gamma n \rightarrow \pi^-\pi^0 p\) | 0.059   | 0.108               |
| \(\gamma n \rightarrow \pi^0\pi^0 n\) | 0.000   | 0.238               |

This is completely different from the single photoproduction case in which the final states including a charged pion have the largest cross sections. An experimental verification of this pattern would be of utmost interest. It also persists when one includes the $\Delta$-corrections at this order in the chiral expansion [61]. The corresponding total cross sections for the various channels can be compactly written as
with $E_\gamma$ the photon energy in the lab frame, $\eta_1, 2, 3$ isospin factors (like e.g. $\eta_1 = \eta_2 = 1, \eta_3 = 2$ for the $p\pi^0\pi^0$ final state) and $\xi$ is a Bose factor ($= 1/2$ in case of equal particles in the final state, one otherwise). The threshold energy is $E_\gamma^{\text{thr}} = 2M_\gamma(1 + \mu) = 320.7$ MeV with $\mu = M_\pi/m$. The formula (47) is only valid close to threshold assuming that the amplitude in the threshold region can be approximated by the threshold amplitude. Furthermore, the three-body phase space has been approximated by an analytical expression which is good within a few percent. For $E_\gamma = 330$ MeV, i.e. 10 MeV above threshold, the total cross section is 0.35, 0.15 and 0.11 nbarn for $\gamma p \rightarrow p\pi^0\pi^0, p\pi^+\pi^-$ and $n\pi^+\pi^0$, in order. Of course, these numbers should only be taken as a first approximation due to the assumptions going into their calculation. It is therefore a tough experimental task to measure these reactions close to threshold and verify the effects of the chiral loops.

10. Brief outlook

In this lecture I could only give a glimpse of the many facets of chiral perturbation theory. The role of effective Lagrangian methods in the parametrization of physics beyond the standard model and in the context of longitudinal vector boson scattering to test the Higgs sector of the electroweak symmetry breaking has been discussed here by Burgess [66] and Phillips [67]. Another widely discussed topic is the combined application of chiral symmetry and heavy quark effective field theory methods (some references can be traced back from [4]). Furthermore, these methods allow also to make precise statements for finite temperature and volume effects and much more. There remain many open theoretical problems and challenging experimental tasks to further tighten our understanding of the strong interactions at momentum scales were they are really strong.

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