Algodynamics: Teaching Algorithms using Interactive Transition Systems

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The importance of algorithms and data structures in computer science curricula has been amply recognized. For many students, however, gaining a good understanding of algorithms remains a challenge. Because of the automated nature of sequential algorithms there is an inherent tension in directly applying the 'learning by doing' approach. This partly explains the limitations of efforts like algorithm animation and code tracing.

Algodynamics, the approach we propose and advocate, situates algorithms within the framework of transition systems and their dynamics and offers an attractive approach for teaching algorithms. Algodynamics starts with the premise that the key ideas underlying an algorithm can be identified and packaged into interactive transition systems. The algorithm when 'opened up', reveals a transition system, shorn of most control aspects, enriched instead with interaction. The design of an algorithm can be carried out by constructing a series of interactive systems, progressively trading interactivity with automation. These transition systems constitute a family of notional machines.

We illustrate the algodynamics approach by considering Bubblesort. A sequence of five interactive transition systems culminate in the classic Bubblesort algorithm. The exercise of constructing the individual systems also pays off when coding Bubblesort: a highly modular implementation whose primitives are borrowed from the transition systems. The transition systems used for Bubblesort have been implemented as interactive experiments. These web based implementations are easy to build. The simplicity and flexibility afforded by the algodynamics framework makes it an attractive option to teach algorithms in an interactive way.

CCS Concepts: • Social and professional topics → Computer science education; Computational thinking; Model curricula; • Applied computing → Interactive learning environments.

Additional Key Words and Phrases: Transition Systems, Algorithms, Pedagogy, Learning-by-Doing, Step-wise Refinement

1 INTRODUCTION

Algorithms form an essential component of recommended curricula in computer science[23], computer engineering[22] and software engineering[31]. On the other hand, students continue to face difficulty with algorithmic problem solving: in problem formulation, notation for expressing solutions, logical reasoning, tracing the execution, and understanding the behaviour of their solutions[30].

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Investigation into the sources of this difficulty has mostly focused around programming [30, 33, 48] rather than algorithms directly. This is not surprising; algorithms are ultimately implemented as programs. Mental models are useful in solving problems [19], but in the absence of a clear, structured and simple notation and a standard abstract model, students try to construct ad-hoc mental models built from programming constructs or pseudo-code [7].

The central role of an abstract machine in defining the concept of an algorithm has been emphasized by Denning [14]. Other recent efforts have also sought to reconsider the role of notional machines, which abstract the workings of a particular programming language, paradigm or hardware architecture [15, 18, 39].

We take the position that algorithms need their own, tailor-made abstract machines. An algorithm-specific abstract machine, as opposed to a ‘general computer’, directly works with the abstractions inherent in the problem and the high-level operations used by the algorithm.

It is widely understood that interaction is an essential component of ‘active learning’ and ‘learning by doing’ [11, 34]. The student’s own experience with computation is mostly defined by interaction with digital devices like mobile phones. It is natural, therefore to expect abstract machines, as well, to incorporate interactivity.

Computer scientists already have a powerful formalism to study interactive abstract machines: transition systems. Such systems exhibit deterministic and non-deterministic behaviour, are terminating or non-terminating, and can model both sequential and concurrent systems. In computer science, they are used in the study of verification and control of reactive and embedded systems [24, 41]. Their potential as abstract machines for building mental models of algorithms and data structures execution remains mostly unexplored.

In this paper, we propose *Algodynamics*, a pedagogical framework that situates algorithms in the design space of interactive transition systems. In trying to bring interaction to algorithms, however, we face the following apparent challenge: an algorithm is non-interactive, (i.e., automated), sequential and terminating. Its notional machine inherently does not lend itself to interaction. However, seen as an transition system, an algorithm consists of a trivial action: next and is situated at the non-interactive end of the spectrum of transition systems. In algodynamics, the understanding of an algorithm is approached via a sequence of interactive transition systems culminating in the algorithm. Figure 1 tries to capture this idea.

By bringing in transition systems, we bring in an alternative notational and reasoning framework that is abstract, precise and compact. The framework allows the expression of mental models that align closer to the problem and the high-level operations rather than the implicit architectural model of a programming language (functions, while loops, etc.)

Students trace their programs to understand how their algorithm runs. Algodynamics connects it to a sequence of transitions. Because the elements of the trace operate on higher abstract actions, the cognitive load on the student is reduced.

At each stage in the process leading to the construction of the algorithm, we have a complete, interactive system that the student can play with, reason about and develop insight about the problem and the solution and explore other solutions and problems in the vicinity. This approach encourages creative experimentation (the highest level of Bloom’s taxonomy). In the programming part of the lab, the student is simply coding, in a systematic manner, the transition systems she has already understood.

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In the rest of the paper, we review the literature on the difficulties of teaching and learning algorithms through programming (Section 2) and then include a short self-contained, introduction to transition systems and algodynamics (Section 3). We illustrate the algodynamics approach by exploring the Bubblesort algorithm (Section 4). Conclusions and future work are presented in Section 5.

2 RELATED WORK

In this section, we briefly review the challenges students face that are related to problem solving, programming and algorithms.

Learning programming continues to be difficult, as evidenced by various studies on pass rates of introductory programming courses across various countries [10, 29, 47]. The difficulties of novice programmers has been widely researched [15, 40], including in a recent literature survey by Medeiros [30], who groups the difficulties of novice programmers into 4 categories: 1) problem formulation, 2) solution expression, 3) solution execution and evaluation, and 4) behavior.

Difficulties in problem formulation include problem solving, the abstract nature of programming, and algorithmic and logical reasoning. These resonate with Winslow’s discussion on novice vs. expert [48]: “Experts tend to approach a program through its objects rather than control structures...experts use algorithms, rather than a specific syntax, abstracting from a particular language to the general concept.” Robins [33] lays out the novice vs. expert differences in great detail, citing several studies. He indicates “the most important deficits relate to the underlying issues of problem solving, design, and expressing a solution/design as an actual program.” Fix [16] studies mental representation of programs by novices and experts and finds that the novice’s representation lacked the characteristics evidenced by the expert’s representation.

Program comprehension is another area cited as difficult for novices, as evidenced by their ability to trace the program and also explain it in English [26]. Vainio [43] studies the causes for poor tracing skills and describes 4 specific difficulties students face, including Inability to use external representations and inability to raise abstraction level. Lopez [28] and Lister [27] also
conclude that those who can’t trace well also can’t explain the code, and those who perform reasonably well at code writing have also acquired the ability to trace and explain.

Summarizing these observations, it seems clear that students need to acquire the following skills to become better at problem solving: 1) **Abstraction and Modeling** - ability to model the problem and think at an abstract level, using external representations as needed, 2) **Program comprehension and tracing** - ability to understand the program by tracing and explaining in English, and 3) **Algorithmic reasoning** - ability to reason logically rather than via program syntax and control structure.

An algorithm course in an undergraduate program is expected to address the above concerns. However, given the challenges above, it is important to first review the ways in which the teaching of algorithms is approached and to then propose pedagogical improvements that address the challenges above.

The concerns with the way algorithms are taught are not new. Levitin [25] argued to reconsider the way we teach design and analysis of algorithms and incorporate more problem-solving, almost 20 years ago. While the constructivist approach has been espoused for teaching programming [9, 50] and constructionism [20] for experiential learning, we do not see evidence in the literature of knowledge construction approaches when teaching algorithms. Leading textbooks on algorithms [12, 35, 36, 38] teach algorithms in a fait accompli manner - algorithms are presented in real code or pseudo-code, and the applicable strategy is pointed out (‘divide and conquer’, ’backtracking’, ’greedy’, ’dynamic’, etc.). Computer Science Curricula 2013 [23] approaches Algorithms and Complexity Body of Knowledge along similar lines. Use of code makes algorithm understanding hard for the same reason novice programmers find programming hard. Use of pseudo-code makes it only somewhat easier due to the lack of notational and semantic standards [13]. Such an approach to teaching doesn’t help the students in constructing their own knowledge about algorithm design.

A common and frequently discussed, approach in the literature for algorithm teaching is algorithm visualization (AV) [1–3, 37, 44]. While AV does offer better engagement and explainability, the outcomes and usage haven’t been significant enough to label it as an effective approach to teaching algorithms [21, 42, 45]. One of the challenges could be the limited role of interactivity in these visualizations. While there is evidence that adding interactivity to visualizations can positively impact understanding, [32, 46], most of the algorithm visualizations available today lack interactivity.

An additional impediment that makes it hard harness interactivity and create game-like environments for algorithm teaching is that algorithms are inherently automatic and non-interactive.

We believe that an effective approach to teaching algorithms needs to focus on answering these questions: 1) How do we enable students to create abstract mental representations of algorithms? 2) How do we help students trace better and reason about the program? and 3) How do we let the students interact with the algorithm infrastructure and learn the principles of algorithm design through such interactions?

In his 1971 seminal paper [49], Niklaus Wirth says: "Clearly, programming courses should teach methods of design and construction, and the selected examples should be such that a gradual development can be nicely demonstrated."

In this paper, we take inspiration from the approach Wirth suggests for programming and apply it towards the design and construction of algorithms. The method we employ is interactive...
transition systems. The gradual development is demonstrated by the sequence of transition systems, each addressing a design concern and deciding on a data representation or control strategy. This way, students understand the nature of the algorithm and also engage with it sufficiently to develop their own understanding.

3 TRANSITION SYSTEMS AND THE ALGODYNAMICS APPROACH

A transition system is a model to understand how quantities of interest change when subjected to outside forces called inputs or actions. The set of all possible values of the tuple of quantities constitute the state space of the system. Dynamics studies how the state changes when subjected to a series of actions over time. Algodynamics models algorithms as transition systems and studies their dynamics. In the algodynamics approach, finding a solution to a computation problem reduces to finding a sequence of the actions that steer a system from an initial state to its final state, the desired state.

Algodynamics explores the space between interactive systems and algorithms. In this space exist several transition systems, each presenting itself as an interactive laboratory that reveals some important aspect of the algorithm. The laboratory keeps the student engaged and encouraged to discover interactively new ways to solve a problem. Because a transition system is interactive, the student can play with the system by choosing an action (akin to making a move in a game) and seeing its effect on the system’s state. What sequence of actions to apply is left to the student, and the student may devise her own strategy to choose these moves. From merely ‘observing’, or ‘tracing’ how an algorithm runs, the student now takes control and steers the system towards the solution. The student is now ‘learning by doing.’

The opposite of ‘interactive’ is ‘automated’. A transition system is said to be automated if there is only one action that can be performed on the state. Algorithms are automated systems. In the algodynamics approach, the student is introduced to a gradually developed series of increasingly less interactive transition systems which culminate in the algorithm. At each stage the student is trading interaction for automation. This allows the student to see the design of an algorithm emerge in an incremental manner.

3.1 Brief introduction to Transition Systems

Our definition of a transition system is one that has now become standard[6, 8, 41]. A transition system (TS) is a six-tuple \((X, X_0, U, \delta, Y, h)\) where \(X\) is a set called the state space; \(X_0\) is a subset of \(X\) called the set of initial states; \(U\) is a set called the set of actions; \(\delta : X \times U \rightarrow 2^X\) is the transition function or dynamics; if \(\delta(x, u) = x'\) we write \(x \xrightarrow{u} x'\), which is called a transition. \(Y\) is a set called the view (or observation) space and \(h : X \rightarrow Y\) is a map called the view map. A TS is automated if the action set is a singleton. The set of states, observation and actions could each be finite, infinite or continuous. A system is said to be deterministic if \(|\delta(x, u)| \leq 1\) for all \(x\) and \(u\). For a deterministic system, whenever \(x \xrightarrow{u} x'\), \(x'\) is unique. A state \(x \in X\) is said to be terminating if \(\delta(x, u) = \emptyset\) for all \(u \in U\). A run is a sequence of linked transitions \(x_0 \xrightarrow{u_1} x_1 \xrightarrow{u_2} x_2 \cdots x_{n-1} \xrightarrow{u_n} x_n\). For a run as above the sequence of states \((x_0, x_1, \cdots, x_n)\) is called a trajectory and the sequence of observations in the set \(Y\), namely, \((h(x_0), h(x_1), \cdots, h(x_n))\) is called a trace. A TS is terminating if it has no infinitely long runs.

A TS is generally interactive. A user can choose an initial state \(x_0\), and a sequence of actions \(u_1, u_2, \cdots, u_n\) to construct a run and see what the trajectory looks like. She may like to choose
actions that result in a specified trajectory. As remarked earlier a TS is said to be automated if there is only one action in $U$. In such a case the unique action is generally denoted by $next$ and there is no more any choice for the user in the construction of trajectories. The choice of an initial state determines the entire trajectory.

A computation problem is specified by choosing a view space $Y$, an initial subset $Y_0$ of $Y$, a final subset $Y_\omega$ of $Y$, and a map $f : X_0 \rightarrow Y_\omega$ called the specification map. An interactive solution of a computation problem is a TS whose traces begin with some $y_0 \in Y_0$ and end in $f(y_0) \in Y_\omega$. If an interactive solution can be designed to be automated, terminating and deterministic, then the solution is called an algorithm.

In Algodynamics, we take the specification of a computation problem as stated above and try to construct tentatively an interactive solution with some wide and suitable choice of data types and actions to construct $X$ and $U$, followed by the full transition system. Then we progressively refine it to generate a sequence of interactive solutions that ends with an algorithm.

4 ILLUSTRATING THE ALGDYNAMICS APPROACH WITH BUBBLESORT

We apply the algodynamics approach to explore the specification of the sorting problem and the Bubblesort algorithm. Our concern here is not Bubblesort’s efficiency (which is poor), nor its persistent use in algorithm pedagogy despite its many shortcomings[4], but its mechanics, which is simple, but not trivial.

The sorting problem may be specified as building an algorithmic system with initial observation $a^0$, an $n$-sized array of numbers to be sorted, and in whose terminal state, we observe $sort(a^0)$, its sorted permutation. The goal is to build an algorithmic transition system for Bubblesort and implement it as a program.

4.1 A sequence of transition systems for Bubblesort

We suggest a sequence of five transition systems $B_1$ to $B_5$ to explore the design of Bubblesort. The sequence is but one of many possible trajectories in the design space that contains Bubblesort. Each system in the sequence $B_1$ to $B_5$ highlights a key decision in the design of Bubblesort. We present sample runs, and briefly note the key properties of the system. We include screen-shots of online experiments for some of the systems. A full exposition should include mathematical arguments of correctness. We elide them here in the interest of brevity.

State variables and Notation: We use $a$ to denote the array state variable. The size of $a$ is assumed to be $n$ and element indexing is zero-based. In addition, we assume array index variables $0 \leq i < j < n$ and $0 \leq b \leq n$ which appear as part of the state vector in specific transition systems.

4.1.1 Transition System $B_1$: “Swap Machine”. Bubblesort rests on a key primitive: transposing or swapping elements in an array. The first system $B_1$ is designed around this idea. We assume $swap(a, i, j)$ denotes the result of swapping $a_i$ with $a_j$ in $a$. The state vector of $B_1$ consists of an array $a$. An action in $B_1$ is of the form $swap(i, j)$, where $0 \leq i < j < n$. The dynamics of $B_1$ is captured by the transition relation $a \xrightarrow{swap(i,j)}_{B_1} a'$ iff $a' = swap(a, i, j)$.

Here are example runs in $B_1$ starting with the array $[8, 6, 7, 4]$. The underlines identify elements being swapped.
Fig. 2. Online experiment based on transition system $B_1$ showing the swapping of elements

\[
\begin{align*}
[8, 6, 7, 4] & \xrightarrow{\text{swap}(0,3)} B_1 \rightarrow [4, 6, 7, 8] \\
[8, 6, 7, 4] & \xrightarrow{\text{swap}(1,2)} B_1 \rightarrow [8, 7, 6, 4] \xrightarrow{\text{swap}(0,3)} B_1 \rightarrow [4, 7, 6, 8]
\end{align*}
\]

The runs may be easily created with paper and pencil by the student and on the blackboard by the teacher. In addition, the student may create and explore these interactively through a virtual experiment implementing this system. (See Figure 2.)

Note that $B_1$ is interactive, deterministic and non-terminating. We are free to swap any two elements. This can sometimes, result in a very short run to sort an array (Run 1). The freedom also makes $B_1$ versatile. We can now use $B_1$ to not just sort, but also reverse a sequence (Run 2). With some practice, the student could conjecture that $B_1$ may be used to obtain any permutation of the array. The more inquisitive student may want to know why $B_1$ ‘works’ for sorting. The more mathematically oriented student may be encouraged to explore how this relates to the theory of permutation groups[17].

4.1.2 Transition system $B_2$: “Order Machine”. $B_1$ allows too much freedom and lacks direction. Playing with $B_1$, the student might realise that sorting can be done by swapping only those elements that are out of order (an operation we call ‘ordering’). The system $B_2$ captures this insight.

The transition system $B_2$ has the array $a$ as its sole state variable. There is only one type of action in $B_2$: order($i, j$), where, again $0 \leq i < j < n$. The transition relation is $a \xrightarrow{\text{order}(i, j)} B_2 a' \iff a_i > a_j$ and $a' = \text{swap}(a, i, j)$.

Interacting with the virtual experiment implementing $B_2$ should help the student conjecture that every run in $B_2$ terminates and the terminal state always is a sorted array. Of course, the mathematically inclined student may wish to prove this property.

4.1.3 Transition System $B_3$: “Order Adjacent machine”. How do we choose indices for ordering? The simplest strategy picks adjacent elements. The transition system $B_3$ incorporates this choice.
Therefore, $B_3$ has as its state variable the array $a$ and single type of action $\text{adj}(i)$ where $0 \leq i < n - 1$. Its dynamics is described by the transitions in $B_2$: $a \xrightarrow{\text{adj}(i)} B_3 \xrightarrow{\text{order}(i+1)} B_2 \xrightarrow{a'}. $

### 4.1.4 Transition System $B_4$: “Bubble” Machine

$B_3$ still leaves undecided the strategy for selecting the next action. This problem is addressed by the system $B_4$, which adopts a simple, linear traversal strategy to automatically locate the next index. Note that now the choice of which index to consider for swapping adjacent elements is no longer available. It is automated via an index variable $i$ in $B_4$, $i$, initialised to 0 is maintained as part of $B_4$'s state, along with the sequence $a$.

To achieve the linear incremental strategy, $B_4$ comes equipped with an action $\text{inc}$. $\text{inc}$ orders the adjacent elements at $i$ and $i + 1$ and then automatically increments $i$. A sequence of $\text{inc}$ actions sweep the array $a$ starting from $i = 0$.

A single sweep of the array is insufficient for sorting. Therefore, the student is given the option of resetting the index any time. A reset action heralds the beginning of a new sweep cycle. This is accomplished by the reset action:

For someone interested in observing only the array variable in $B_4$, the view function $h_{B_4}$ is just $h_{B_4}(a, i) = a$. The transitions for $B_4$ are: $(a, i) \xrightarrow{\text{reset}} B_4 \xrightarrow{(a, 0)} B_4 \xrightarrow{(a, i+1)} B_4$ iff $i < n - 1$ and $a_i \leq a_{i+1}$, and $(a, i) \xrightarrow{\text{inc}} B_4 \xrightarrow{(a', i+1)} B_4$ iff $i < n - 1$ and $a \xrightarrow{\text{adj}(i)} B_4 \xrightarrow{a'} B_4$.

Playing with $B_4$ by tracing out a few runs, an observant student may notice the following phenomenon: the index $i$ carries along with it the maximum element in the array swept so far. The largest element ‘bubbles its’ way towards the right end of the array.

However, the bubbling may be interrupted by a reset at any step. Furthermore, the reset could be invoked infinitely often. $B_4$ is thus no longer a terminating system.

### 4.1.5 Transition System $B_5$: “Bubblesort” Machine

A student could discover that repeated sweeps of the $B_4$ machine can be used to arrange the maximum element of an array at position $n - 1$, the next maximum at position $n - 2$, and so on. This divides the array at any step into an unsorted part followed by a sorted part. Whenever the sweep index $i$ reaches the boundary between the two parts, a reset could be triggered. The system $B_5$ implements this strategy for resetting. The state vector of $B_5$ now includes an additional boundary variable $b$, that ranges from 0 to $n$. A boundary value of $k$ means that (a) all elements $a_k$ to $a_{n-1}$ are sorted and (b) $a_k$ is greater than or equal to all elements $a_0$ to $a_{k-1}$.

With this insight, it is now possible to define the dynamics of $B_5$. The state vector $(a, i, b)$ is initialised to $(a^0, 0, n)$. The sweep index $i$ varies from 0 to $b - 1$. At each step, the sweep index $i$ is incremented, until it reaches $b - 1$. After that $i$ is reset to 0. Simultaneously, the boundary index $b$ is decremented. The system terminates when $b$ is less than or equal to 1. $B_5$ has only one action next. making it an automated system. The observable of $B_5$ is the array $a$. The dynamics of $B_5$ is defined below:

$$
(a, i, b) \xrightarrow{\text{next}} B_5 \quad (a', i', b) \quad \text{iff } i < b - 1 \text{ and } (a, i) \xrightarrow{\text{inc}} B_4 \quad (a', i')
$$

$$
(a, i, b) \xrightarrow{\text{next}} B_5 \quad (a', i', b - 1) \quad \text{iff } i = b - 1 \text{ and } (a, i) \xrightarrow{\text{reset}} B_4 \quad (a', i')
$$

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Fig. 3. Screenshot of the Bubblesort algorithm (system $B_5$) displaying the values of array, index and boundary state variables during a sorting run.

![Bubblesort Machine](image)

Table 1. The sorting of $[8, 6, 7, 4]$ witnessed by runs in the various transition systems $B_1$ to $B_5$. The boundary variable $b$ is part of only $B_5$’s state. The index variable $i$ (underlined) is part of only $B_5$’s and $B_4$’s state.

| State $a, i, b$ | $B_5$ | $B_4$ | $B_3$ | $B_2$ | $B_1$ |
|----------------|-------|-------|-------|-------|-------|
| $[8, 6, 7, 4], 0, 4$ | next | inc | adj(0) | order(0, 1) | swap(0, 1) |
| $[6, 7, 4, 3], 0, 3$ | next | inc | adj(1) | order(1, 2) | swap(1, 2) |
| $[6, 7, 8, 4], 2, 4$ | next | inc | adj(2) | order(2, 3) | swap(2, 3) |
| $[6, 7, 4, 8], 3, 4$ | next | reset | | | |
| $[6, 7, 4, 8], 1, 3$ | next | inc | adj(1) | order(1, 2) | swap(1, 2) |
| $[6, 7, 4, 8], 2, 3$ | next | reset | | | |
| $[6, 4, 7, 8], 0, 2$ | next | inc | adj(0) | order(0, 1) | swap(0, 1) |
| $[4, 6, 7, 8], 1, 2$ | next | reset | | | |
| $[4, 6, 7, 8], 0, 1$ | | | | | |

An interactive experiment for $B_5$ allows the student to step through the bubblesort algorithm. A screenshot of the algorithm in progress is shown in Figure 3.

Whenever $i$ reaches $b - 1$, it is automatically reset, $b$ is decremented and as a result, the sorted segmented of the array grows by one. A formal correctness proof uses induction on the transition systems is not difficult, but omitted.

Table 1 compares the runs amongst the five transition systems $B_1$ to $B_5$ on the input array $[8, 7, 6, 4]$. It is now a simple exercise for the student to improve the Bubblesort algorithm by incorporating a done? boolean state variable that terminates the algorithm as soon as there is a sweep with no swaps.

### 4.2 Coding the transition systems into a program

The process of translation consists of systematically coding the actions of each transition system. Each action maps to a function whose arguments consist of the state variables. The final transition system $B_5$ contains a single while loop along with a termination condition. A translation to Python code is show below:
Prototype implementations of the five transitions systems suggested in this paper are available online[5].

5 CONCLUSIONS AND FUTURE WORK

We have introduced the algodynamics approach to teaching and designing of algorithms. We have illustrated the approach using Bubblesort. We have applied this approach on other sorting and searching algorithms, algorithms on trees, recursion and also concurrent algorithms. These will be reported elsewhere.

For the student, the algodynamics approach shows a clear way to think about the algorithm, in an interactive way. It encourages the teachers to construct their own pathways of transition systems to explain the design of an algorithm.

Field level trials (with suitably designed teaching kits), both for teachers and students are essential to test the efficacy of this approach and remain to be done. Incorporating the algodynamics approach in the syllabi of the algorithms course will require the introduction of transition systems earlier in the curriculum and integrating it with the course on automata theory. How this is to be done remains to be explored.

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