Robust nonlinear observer design for permanent magnet synchronous motors

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Abstract
This paper is devoted to the application of a recently proposed globally convergent adaptive position observer to non-salient permanent magnet synchronous motors. Following the Dynamic Regressor Extension and Mixing Based Adaptive Observer (DREMBAO) approach, a new finite-time robust observer is presented that allows to track adaptively the rotor position by measuring only the currents and voltages and without knowledge of mechanical, electrical and magnetic parameters. A numerical example for the case with different rotor speeds and time-varying load torque is considered to reveal the advantages of the proposed approach in comparison with other existing methods. Experimental studies of the proposed robust nonlinear observer implementation are presented to illustrate the efficiency of the new design technique in different speed modes together with adaptive estimation of unknown parameters.

1 INTRODUCTION

One of the most challenging problems in drives control is the development of so-called sensorless controllers, that is designing control algorithms that ensure a good operation without the use of speed/position sensors. Numerous sensorless algorithms for permanent-magnet synchronous motors (PMSMs) have been proposed, and many of them are widely utilised in industrial, home, and traction applications due to their cost effectiveness and convenience in installation. In some applications (cranes, elevators, vacuum pumps), the installation of a shaft sensor is either inconvenient or not possible at all. In sensitive heating, ventilation and air conditioning systems, the use of sensorless controller reduces the cost and boosts their reliability and performance. Another benefit of sensorless control is the mechanical robustness due to the absence of cables, connectors and peripheral modules.

Sensorless techniques are classified largely into model-based and saliency based methods: the model-based method is a passive algorithm that does not utilise signal injection. Instead, it relies on the back-electromotive force (EMF). The model-based methods are advantageous in the medium and high-speed regions, and the implementation is relatively simple. However, the performance is poor at low speed, since the back-EMF is proportional to the speed. In a low-speed range, the magnitude of back-EMF is as low as the noise level. In that situation, the model-based algorithms are sensitive to temperature variation of motor coil resistance, IGBT on-drop, dead-time, EMF harmonics etc. For this reason, in low-speed operation, it is necessary to use signal injection methods [1]. See [2] for a general discussion on the topic and [3] for a recent experimental evaluation of different model-based algorithms.

There are two types of model-based observers depending on whether they estimate the back-EMF directly or the objective is...
to estimate the rotor flux. In contrast to the back-EMF, the rotor flux does not vanish at standstill, but remains constant. However, since initial values are unknown, the rotor flux observer must be constructed taking into account the stator current dynamics. Existing flux observers that are based on extended Kalman filters [4, 5] require the knowledge of the rotor speed; therefore, they must also incorporate a speed observer. In [6], a novel approach to design a nonlinear flux observer for surface-mount PMSM was proposed. The approach relies on two main features. First, exploiting the algebraic relation between flux and current, the authors propose to minimise a quadratic criterion implementing a gradient-descent based search. Second, using the fact that the flux derivative is computable from the measurement of currents and voltages, this term can be added to the derivative of the flux estimate. Strong stability properties of this observer were reported in [6] and later experimentally verified in [7]. Global stability was later proved—via a slight modification to the observer—in [8]. The gradient descent-based approach to state observation was, apparently, first proposed in [9], and has been recently used for flux observation of PMSMs in [10, 11].

Here, we use the recent results reported in [12] and [13–15] to extend the results of our previous works in several directions, with the main contributions of our work being as follows:

C1. We remove the need of open-loop integration of current and voltage—an operation that is well-known to be fragile as it is prone to the appearance of drifts.

C2. We design an adaptive flux observer that does not need the knowledge of the stator resistance and inductance. That is, we propose an algorithm that estimates the parameters and the systems state simultaneously.

C3. It is shown that the estimation of the unknown stator resistance and inductance converges in finite time, ensuring the effective asymptotic estimation of the rotor flux.

C4. The proposed design is robust in the sense that, in contrast to [6] and [16], it is not necessary to know the magnetic flux constant. Also, the algorithm is based solely on the electrical dynamics being, therefore, insensitive to load torque and rotor inertia variations.

To the best of our knowledge, the only results of adaptive observers for servo systems—without the deleterious injection of high-gain via sliding modes and/or the use of fractional powers—are the ones in [17] and [18] for electro-hydraulic servo systems with matched and mismatched disturbances. There are also results about disturbance observers for PMSM, for example [19], but in such papers, the rotor angle and/or angular velocity are assumed to be measurable.

This paper is organised as follows: The problem statement is described in Section 2. The principal result is presented in Section 3, where the finite-time observer design technique for the total flux is constructed for the case of the known resistance and inductance. The extended version for the case of unknown resistance is presented in Section 4 where DREMBAO-based finite-time estimates of the flux are obtained despite resistance uncertainty. In Section 5, the finite-time observer shown for the most robust setup is considered, where all parameters are unknown, including the inductance. In Section 6, numerical simulations are presented to show the advantages of the proposed approach compared with nearest similar algorithms. In Section 6, the experimental study results are also shown to confirm the efficiency and robustness of the proposed design technique.

Notation. Let us introduce basic notations used throughout the paper.

- \( \lambda = \cos(\lambda_\alpha, \lambda_\beta) \in \mathbb{R}^2 \) is the total flux,
- \( i = \cos(\bar{i}_\alpha, \bar{i}_\beta) \in \mathbb{R}^2 \) are the currents,
- \( v = \cos(\bar{v}_\alpha, \bar{v}_\beta) \in \mathbb{R}^2 \) are the voltages,
- \( \theta \in \mathbb{S} = [0, 2\pi) \) is the rotor position,
- \( \omega \in \mathbb{R} \) is the mechanical angular velocity,
- \( R > 0 \) is the stator windings resistance,
- \( L > 0 \) is the stator windings inductance,
- \( \lambda_m > 0 \) is the flux created by permanent magnets,
- \( n_p \in \mathbb{N} \) is the number of pole pairs,
- \( J > 0 \) is the rotor inertia,
- \( B > 0 \) is the viscous friction coefficient,
- \( \tau_L \in \mathbb{R} \) is the—possibly time-varying—load torque,
- \( \tau_e \) is the torque of electrical origin.

2 | PROBLEM FORMULATION

In the stationary \( \alpha\beta \) frame, the following equations describe the dynamics of unsaturated non-salient PMSM described by Krause [20] and Nam [2]:

\[
\dot{\lambda} = n - Ri,
\]

\[
J \dot{\omega} = -B\omega + \tau_e - \tau_L,
\]

\[
\dot{\theta} = \omega,
\]

where the total flux satisfies

\[
\lambda = LI + \lambda_m \begin{bmatrix} \cos(n_p \theta) \\ \sin(n_p \theta) \end{bmatrix},
\]

and the torque of electrical origin is given by

\[
\tau_e = n_p \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \lambda.
\]

The main problem in sensorless control is to reconstruct the rotor position \( \theta \) using measurements of the current \( i \) and the voltage \( n \). This problem can be reformulated as a problem of flux estimation noting that

\[
\theta = \frac{1}{n_p} \arctan \left\{ \frac{\lambda_\alpha - L_i \bar{i}_\alpha}{\lambda_\beta - L_i \bar{i}_\beta} \right\},
\]

which follows directly from (2).
The main objective of this work is to design a robust finite convergence time observer (FCTO) of the magnetic flux $\lambda$ using measurements of the current $i$ and voltage $v$. That is an algorithm to generate an estimate of the flux $\lambda(t)$, denoted $\lambda_{\text{FCTO}}(t)$, such that the following equality holds:

$$\dot{\lambda}_{\text{FCTO}}(t) = \lambda(t), \quad \forall t \geq t_0,$$

for some $t_0 \in (0, \infty)$.

As usual in parameter estimation and observation problems, the following forward completeness and boundedness assumptions are imposed.

**Assumption 1.** The stator voltage $v$ and the unknown external load torque $\tau_l$ are such that the trajectories of the PMSM model (1) exist for all $t > 0$ and are bounded.

It is clear that given a finite-time robust flux observer to calculate rotor position estimate $\hat{\theta}$ from

$$\hat{\theta} = \frac{1}{u_p} \arctan \left\{ \frac{\hat{\lambda}_x - Li_x}{\hat{\lambda}_y - Li_y} \right\},$$

and combining with a PLL-type speed estimator [2], one can implement a sensorless controller.

### 3 FINITE CONVERGENCE-TIME FLUX OBSERVER

In this section, we propose an FCTO for the flux assuming the stator resistance and inductance are known. The adaptive version, where this parameters are estimated, is given in the next sections.

From (2), we get

$$(\lambda - Li)^T (\lambda - Li) = \lambda^2_m,$$

which yields

$$\lambda^T \lambda - 2L \lambda^T i + L^2 i^T i - \lambda^2_m = 0.$$  

Applying the stable LTI filter to this identity

$$F(p) = \frac{v p}{p + v},$$

with $p := \frac{d}{dt}$ and $v > 0$, gives

$$\frac{v p}{p + v} \left[ \lambda^T \lambda + L^2 i^T i - 2L \lambda^T i \right] - \frac{v p}{p + v} [\lambda^2_m] = 0.$$  

Pulling the operator $p$ inside the brackets and doing some simple calculations allows us to rewrite the latter equation as

$$\frac{v}{p + v} \left[ 2L^2 \lambda^T \lambda + L^2 i^T i - 2L \lambda^T i \right] = 0.$$  

Replacing the flux derivative from the first equation of system (1) and grouping terms gives

$$2\frac{v}{p + v} \left[ \lambda^T \left( v - Ri - L \dot{\lambda} \right) + \frac{v}{p + v} \left[ L^2 \left( i^T i \right) - 2L i^T i \right] \right] + 2RL \frac{v}{p + v} \left[ i^T i \right] = 0.$$  

Notice that the unknown flux is inside a filtering operation in the first term of (5). In order to move it out of the filter, it we recall the Swapping Lemma [21]

$$\frac{v}{p + v} [x^T y] = \frac{x}{p + v} [x^T y] - \frac{1}{p + v} \left[ x \frac{y}{p + v} \right]^T.$$  

Applying this identity to (5) yields

$$\lambda^T \frac{v}{p + v} \left[ 2v - 2Ri - 2L \dot{\lambda} \right]$$

$$- \frac{1}{p + v} \left[ (v - Ri)^T \frac{v}{p + v} \left[ 2v - 2Ri - 2L \dot{\lambda} \right] \right]$$

$$+ \frac{v}{p + v} \left[ L^2 \left( i^T i \right) - 2L i^T i \right] + 2RL \frac{v}{p + v} \left[ i^T i \right] = 0,$$

where we have used again $\lambda^T = (v - Ri)^T$.

The last equation (6) is a linear regression

$$z_y = g_y^T \lambda,$$

where $z_y$ and $g_y$ are known functions—parametrised by $v$---and are given by

$$z_y := - \frac{1}{p + v} \left[ (v - Ri)^T \frac{v}{p + v} \left[ 2v - 2Ri - 2L \dot{\lambda} \right] \right]$$

$$+ \frac{v}{p + v} \left[ L^2 \left( i^T i \right) - 2L i^T i \right] + 2RL \frac{v}{p + v} \left[ i^T i \right]$$

$$g_y := \frac{v}{p + v} \left[ 2v - 2Ri - 2L \dot{\lambda} \right].$$

Next, following the DREM algorithm [13], we combine two regressor Equations (6) with different parameters $v_1 > 0, v_2 > 0$ and form the extended regression model

$$Y = Q \lambda,$$
where

\[ Y := \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}, \quad Q := \begin{pmatrix} g_{v_1}^T \\ g_{v_2}^T \end{pmatrix}. \]

Now, we recall that for any, possibly singular, square matrix \( A \), we have that \( \text{adj}(A)A = \det(A)I \), where \( \text{adj}[] \) is the adjoint matrix. Hence, following the mixing step of DREM, we multiply (8) from the left by \( \text{adj}[Q] \) to obtain two separate, scalar regressors, one for each of the unknowns, as

\[ \xi = \Delta \hat{\lambda}, \]  

(9)

where

\[ \Delta = \det[Q], \]

and

\[ \xi = \left( \begin{array}{c} \xi_\alpha \\ \xi_\beta \end{array} \right) := \text{adj}[Q]Y. \]

We are in position to present our first result, namely a FCTO for flux assuming that the resistance and the inductance are known.

**Proposition 1.** Consider the PMSM model described by (1) verifying Assumption 1. Assume also that the resistance \( R \) and the inductance \( L \) are known. Define the gradient descent flux estimator

\[ \dot{\lambda} = v - Ri + \gamma \Delta (\xi - \Delta \hat{\lambda}), \]  

(10)

where \( \gamma > 0 \) a design parameter. To complete the description of the FCTO introduce the dynamic extension

\[ \dot{w}_1 = -\gamma \Delta ^2 w_1, \quad w_1(0) = 1, \]  

\[ \dot{w}_2 = -\gamma \Delta ^2 w_2 + w_1(v - Ri), \quad w_2(0) = 0, \]  

(12)

and the clipping function to avoid singularity at \( t = 0 \)

\[ w_1(t) = \begin{cases} w_1(t) & \text{if } w_1(t) \in [0, \rho) \\ \rho & \text{if } w_1(t) \in [\rho, 1], \end{cases} \]  

(13)

where \( \rho \in (0, 1) \) is a designer chosen constant. The output of the FCTO defined as

\[ \hat{\lambda}^\text{FCTO} \]  

ensures (4) if the interval excitation condition

\[ \int_{0}^{t_1} \Delta ^2 ds \geq -\frac{1}{\gamma} \ln(\rho) \]

is satisfied for some \( t_1 \in (0, \infty) \).

**Proof.** To simplify the notation in the proof, we define the vector signal

\[ H(i, v) := v - Ri, \]

that replaced in (10) yields

\[ \dot{\lambda} = H(i, v) + \gamma \Delta (\xi - \Delta \hat{\lambda}). \]  

(15)

Using (9), we get the well-known error Equation for (15)

\[ \dot{\lambda} = -\gamma \Delta ^2 \hat{\lambda}, \]  

(16)

where we defined \( \hat{\lambda} := \lambda - \hat{\lambda} \). Solving the error Equation (16) yields

\[ \hat{\lambda}(t) = e^{-\gamma \int_{0}^{t} \Delta ^2 ds} (\hat{\lambda}(0) - \hat{\lambda}(0)). \]  

(17)

Integrating (11), we get

\[ w_1(t) = e^{-\gamma \int_{0}^{t} \Delta ^2 ds}, \]

that replaced in (17) yields

\[ \lambda(t) - \hat{\lambda}(t) = w_1(t) \left( \lambda(t) - \int_{0}^{t} H(i(s), v(s))ds - \hat{\lambda}(0) \right), \]  

(18)

where we used the fact that

\[ \hat{\lambda}(0) = \lambda(t) - \int_{0}^{t} H(i(s), v(s))ds. \]

Now, we make the key observation that the solution of the differential Equation (12) can be obtained as

\[ w_2(t) = \int_{0}^{t} e^{-\gamma \int_{0}^{s} \Delta ^2 ds} w_1(\sigma) H(i(\sigma), v(\sigma))d\sigma \]

\[ = \int_{0}^{t} e^{-\gamma \int_{0}^{s} \Delta ^2 ds} e^{-\gamma \int_{0}^{s} \Delta ^2 ds} H(i(\sigma), v(\sigma))d\sigma \]

\[ = \int_{0}^{t} e^{-\gamma \int_{0}^{s} \Delta ^2 ds} H(i(\sigma), v(\sigma))d\sigma \]

\[ = w_1(t) \int_{0}^{t} H(i(\sigma), v(\sigma))d\sigma. \]

Replacing the latter in (18) yields

\[ \lambda(t) - \hat{\lambda}(t) = w_1(t) \left( \lambda(t) - \hat{\lambda}(0) \right) - w_2(t). \]

According to the definition of (13), we have that \( w_1(t) > \rho > 0 \) for all \( t \geq t_1 \). Consequently, for \( t \geq t_1 \) we can write

\[ \lambda(t) = \frac{1}{1 - w_1(t)} [\hat{\lambda}(t) - \hat{\lambda}(0)w_1(t) - w_2(t)]. \]
The proof is completed, from (14), noting that \( w_1(t) = w_i(t) \) for all \( t \geq t_1 \).

4 ROBUST FLUX OBSERVER WITH UNKNOWN RESISTANCE

In this section, we show that the flux and the stator resistance \( R \) can be estimated simultaneously using a more sophisticated parameterisation of the PMSM model and applying the DREM procedure [13]; hence \( R \) may be assumed to be unknown.

Combining terms with respect to unknown flux and resistance, the Equation (6) can be rewritten in the form

\[
2 \frac{1}{\rho + \nu} \left[ v^T (\dot{\phi}_2 - L \dot{\phi}_1) \right] - L^2 \dot{\phi}_3 + 2 I \dot{\phi}_4
\]

\[= \lambda^T (2 \phi_2 - 2I \phi_1) + R \lambda^T (-2 \phi_1)\]

\[+ R \left( \frac{1}{\rho + \nu} \left[ v^T 2 \phi_1 + i^T (2 \phi_2 - 2I \phi_1) \right] + 2 I \phi_3 \right)\]

\[+ R^2 \frac{1}{\rho + \nu} \left[ i^T (-2 \phi_1) \right],\]

where we introduce auxiliary and computable signals

\[
\phi_1 = \frac{v}{\rho + \nu} [i], \quad \phi_2 = \frac{v}{\rho + \nu} [v],
\]

\[
\phi_3 = \frac{v}{\rho + \nu} [i^T i], \quad \phi_4 = \frac{v}{\rho + \nu} [v^T i].
\]

Notice that (19) is a regression-like model of six unknowns

\[
\xi_v = q_v^T \mu,
\]

where

\[
\mu = \begin{bmatrix}
\lambda \\
R \lambda \\
R \\
R^2
\end{bmatrix}, \quad q_v = \begin{bmatrix}
q_{1,v} \\
q_{2,v} \\
q_{3,v} \\
q_{4,v}
\end{bmatrix}
\]

\[
q_{1,v} = 2 \phi_2 - 2I \dot{\phi}_1, \quad q_{2,v} = -2 \phi_1,
\]

\[
q_3(v) = \frac{1}{\rho + \nu} \left[ v^T 2 \phi_1 + i^T q_1 \right] + 2 I \phi_3,
\]

\[
q_4(v) = \frac{1}{\rho + \nu} \left[ -2 i^T \phi_1 \right],
\]

\[
\xi(v) = \frac{1}{\rho + \nu} \left[ v^T q_1 \right] - L^2 \dot{\phi}_3 + 2 I \dot{\phi}_4.
\]

We underscore the fact that, unlike classical regression equations where the unknown quantities are constant parameters, in the unknown vector \( \mu \) of (20), there are states. We will show below that this difficulty can be overcome using DREM.

Next, following the DREM algorithm [13] combining six regressor Equations (20) with different parameters \( v_k > 0, k \in \{1, 2, ..., 6\} \), we form the extended regression model

\[
Y_R = Q_R \mu,
\]

where

\[
Y_R := \begin{bmatrix}
\xi_1 \\
\xi_2 \\
\xi_3 \\
\xi_4 \\
\xi_5 \\
\xi_6
\end{bmatrix}, \quad Q_R := \begin{bmatrix}
q_{1,v_1} \\
q_{2,v_2} \\
q_{3,v_3} \\
q_{4,v_4} \\
q_{5,v_5} \\
q_{6,v_6}
\end{bmatrix}.
\]

Multiplying (21) from the left by the adjoint matrix of \( Q_R \) results in six separate regressor models for each of the unknowns

\[
\sigma_k = \Delta_{jk} \mu_k,
\]

where

\[
\Delta_R = \det(Q_R),
\]

\[
\sigma := \text{col}[\sigma_1, ..., \sigma_6] = \text{adj}(Q_R)Y_R.
\]

Now, we can introduce an adaptive FCTO—that estimates the flux and the resistance—assuming that only the inductance is known.

**Proposition 2.** Consider the PMSM model described by (1) verifying Assumption 1. Assume also that the inductance \( L \) is known. Define the gradient descent resistance estimator with the dynamic extension

\[
\dot{R} = \gamma_R (\sigma_5 - \Delta_5 \dot{R}),
\]

\[
\dot{R}^\text{FT} = \frac{R - R(0)w_i(t)}{1 - w_i(t)},
\]

and flux observer

\[
\dot{\lambda} = v - \lambda^\text{FT} i + \gamma_\lambda (\sigma_1 - \Delta_\lambda \dot{\lambda}),
\]

\[
\dot{\lambda}^\text{FCTO} = \frac{\dot{\lambda} - \lambda(0)w_i(t) - w_2}{1 - w_i(t)},
\]

where \( \gamma_R > 0 \) and \( \gamma_\lambda > 0 \) are design parameters, and the signals \( w_2(t) \) and \( w_i(t) \) are defined by (12) and (13), respectively.

The estimates (24) and (26) satisfy

\[
\dot{R}^\text{FT} = R, \quad \forall t \geq t_2,
\]

\[
\dot{\lambda}^\text{FCTO} = \lambda(t), \quad \forall t \geq t_2,
\]
if the interval excitation condition

\[ \int_0^{t_2} \Delta^2_R(s) ds \geq -\frac{1}{\gamma} \ln(\rho) \]

is fulfilled for some \( t_2 \in (0, \infty) \).

**Proof.** The proof is similar to the proof of Proposition 1.

From (23) and (22), where \( \sigma_1 = \Delta_R^2 \), the error for the resistance estimate is

\[ \hat{R} = -\gamma \Delta^2_R R; \]

hence, one can obtain (see [12])

\[ \hat{R} = (1 - w_i) R + w_i \hat{R}(0). \]

Therefore, for the estimate \( \hat{R}^{FT} \) defined in (24), we have

\[ \hat{R}^{FT}(t) = R, \quad \forall t \geq t_2. \]

Combining (22) and (25) we get the error equation for the flux

\[ \dot{\lambda} = -\gamma \Delta^2_R \lambda - (R - \hat{R}^{FT}) i. \quad (27) \]

Since \( R - \hat{R}^{FT} \) becomes zero in finite time, (27) coincides with (16) and the remaining necessary arguments follow the ones used in Proposition 1.

\[ \square \]

## 5 | GENERALIZATION OF ROBUST FLUX OBSERVER DESIGN

In this section, we consider the general setup for the case with unknown resistance and inductance. Also it is important to emphasize that the rotor inertia \( J \), the viscous friction \( B \), the load torque \( \tau_L \), and the flux created by permanent magnet \( \lambda_p \) are not needed in our flux and position observer design.

Following the approach proposed in Section 3, we combine terms in the Equation (6) with respect to the unknown flux, resistance and inductance, which yields

\[ \frac{1}{\rho + \nu} \left[ \nu^T \phi_2 \right] = \]

\[ = \lambda^T \phi_2 - R \lambda^T \phi_1 + L \lambda^T \phi_1 + R \left( \frac{1}{\rho + \nu} \left[ \nu^T \phi_1 + \nu^T \phi_2 \right] \right) \]

\[ + L \left( \phi_3 - \frac{1}{\rho + \nu} \left[ \nu^T \phi_1 \right] \right) + RL \left( \phi_3 - \frac{1}{\rho + \nu} \left[ \nu^T \phi_1 \right] \right) \]

\[ - R^2 \frac{1}{\rho + \nu} \left[ i^T \phi_1 \right] + \frac{1}{2} L^2 \phi_3. \]

One can see that (19) is a regression model of 11 unknowns

\[ \tilde{\xi}_v = \tilde{q}_v^T \tilde{\mu}, \quad (28) \]

where

\[ \tilde{\mu} = \left( \lambda \ R \lambda \ I \lambda \ R \ L \ RL \ R^2 \ L^2 \right)^T, \]

\[ \tilde{q}_v = \text{col}(q_{1,v}, q_{2,v}, ..., q_{11,v}), \]

\[ \tilde{q}_{1,v} = \phi_2, \quad \tilde{q}_{2,v} = -\phi_3, \quad \tilde{q}_{3,v} = -\phi_4, \]

\[ \tilde{q}_{4,v} = \frac{1}{\rho + \nu} \left[ \nu^T \phi_1 + \nu^T \phi_2 \right], \]

\[ \tilde{q}_{5,v} = -\phi_4 + \frac{1}{\rho + \nu} \left[ \nu^T \phi_1 \right], \]

\[ \tilde{q}_{6,v} = \phi_3 - \frac{1}{\rho + \nu} \left[ i^T \phi_1 \right], \quad \tilde{q}_{7,v} = -\frac{1}{\rho + \nu} \left[ i^T \phi_1 \right], \]

\[ \tilde{q}_{8,v} = \frac{1}{\rho + \nu} \left[ i^T \phi_1 \right]. \]

Next, repeating the DREM algorithm [13] and combining 11 regressor equations (28) with different parameters \( v_k > 0, k = \{1, 2, ..., 11\} \), we form the extended regression model

\[ \tilde{Y} = \tilde{Q} \tilde{\mu}, \]

where

\[ \tilde{Y} := \left( \tilde{\xi}_v^1 \quad \tilde{\xi}_v^2 \right), \quad \tilde{Q} := \left( \tilde{q}_v^1 \quad \tilde{q}_v^2 \right). \]

Multiplying (21) from the left by the adjoint matrix of \( \tilde{Q} \) results in 11 separate regressor models for each of unknowns

\[ \tilde{\sigma}_k = \tilde{A} \tilde{\mu}_k, \]

where \( \tilde{A} = \text{def}(\tilde{Q}) \) and \( \tilde{\sigma} := \text{col}(\tilde{\sigma}_1, ..., \tilde{\sigma}_11) = \text{adj}(\tilde{Q}) \) is computed using adjoint matrix of \( \tilde{Q} \).

The general robust flux observer is given by

\[ \dot{\hat{R}} = \gamma_b \Delta (\sigma_7 - \Delta \hat{R}), \quad \dot{\hat{L}} = \gamma_b \Delta (\sigma_8 - \Delta \hat{L}), \quad (29) \]

\[ \dot{\hat{\lambda}} = \nu - \hat{R} \dot{i} + \gamma_3 \Delta (\sigma_{1,2} - \Delta \hat{\lambda}), \quad (30) \]

where \( \gamma_{b,L} > 0 \) are design parameters.

The finite time observer in general robust scenario may be straightforwardly completed with algorithms (24) and (26), where \( \hat{R}, \hat{\lambda} \) are defined in (29), (30) correspondingly.
The described flux observers (14) and (26) are compared with the adaptive observer proposed in [10], which uses a special representation of the PMSM dynamics and filtering to recast the problem as classical linear regression estimation problem:

\[
\dot{\lambda} = L \dot{i} + q_a + \xi_{8,9},
\]

with

\[
q_a = \xi_{1,2} - R \xi_{3,4} - L \dot{i},
\]

\[
\dot{\xi}_{1,2} = \nu,
\]

\[
\dot{\xi}_{3,4} = i,
\]

\[
\dot{\xi}_{8,9} = \gamma_a \Omega (\nu_a - \Omega^T \xi_{8,9}),
\]

\[
\nu_a = -\alpha_a (|q_a|^2 + \xi_5),
\]

\[
\dot{\xi}_5 = -\alpha_a (|q_a|^2 + \xi_5),
\]

\[
\Omega = \alpha_a (2q_a - \xi_6,7),
\]

\[
\dot{\xi}_{6,7} = \alpha_a (2q_a - \xi_{6,7}),
\]

\[
\alpha_a > 0, \gamma_a > 0 \text{ are design parameters.}
\]

The simulation has been performed in MATLAB Simulink. The model of the drive uses the parameters of the ESTUN EMG-10ASA22. The motor is controlled by the standard field-orient control, and flux observer performance is investigated. The motor model and controller parameters are presented in Table 1. The load torque is modelled as a biased sinusoidal signal, which is shown in Figure 2a. The speed reference signal is depicted in Figure 2b.
We investigate the observer behaviour under motor parameter uncertainty and increase the stator resistance in the model to 2 ohm. The nominal value from the Table 1 is used in the observers [10] and (14).

Additionally, the electrical angle is estimated based on (3) substituting a corresponding flux estimate (14), (26), (31):

$$\hat{\theta}_z = \arctan \left( \frac{\lambda_{\alpha}}{\lambda_{\beta} - L_{d\beta}} \right).$$

The velocity is estimated with the help of a standard PLL-type speed estimator [2]:

$$\dot{\chi}_1 = K_p (\hat{\theta} - \chi_1) + K_r \chi_2,$$
$$\dot{\chi}_2 = \hat{\omega} - \chi_1,$$
$$\hat{\omega} = K_p (\hat{\theta} - \chi_1) + K_r \chi_2,$$

where $\hat{\theta} = \frac{1}{n_s} \hat{\theta}_z$, $K_p = 20$ and $K_r = 10$ are proportional and integral gains, respectively.

In the first case the simulation parameters are the following:

- Observer (31): $\alpha_a = 100, \gamma_a = 1$. 

---

**TABLE 2** Coefficient for filter $F(p)$

| $i$ | $a_i$  | $k_i$  |
|-----|--------|--------|
| 1   | 1494.99| 1502.5 |
| 2   | 638.04 | 655.4  |
| 3   | 396.34 | 423.8  |
| 4   | 279.40 | 317.1  |
| 5   | 208.51 | 256.9  |
| 6   | 159.56 | 219    |
| 7   | 122.64 | 193.8  |
| 8   | 92.89  | 176.4  |
| 9   | 67.63  | 164.5  |
| 10  | 45.20  | 156.7  |
| 11  | 24.48  | 152    |

**FIGURE 3** The estimates from the first simulation
Observer (14): \( \gamma_\alpha = \gamma_\beta = 100, \nu_1 = 10, \nu_2 = 20. \)
- Observer (26): \( \gamma_\alpha = \gamma_\beta = \gamma_R = 100, \nu_1 = 10, \nu_2 = 20, \nu_3 = 40, \nu_4 = 60, \nu_5 = 80, \nu_6 = 100, \hat{R} = 1. \)

All initial conditions are equal to zero, except the ones mentioned above.

The external load torque and reference speed signals are depicted in Figure 2. Figure 3 shows the transients of the estimates. The observers (31) and (14) are robust to parameter uncertainties and provide bounded estimation errors. The flux estimation error for the adaptive observer (26), where the estimate of the stator resistance (23) is used, converges to zero as expected. In Figure 3b, the resistance value and estimate are depicted. Without measurement noises and for the known value of inductance, the estimation error converges to zero in finite time.

The electrical angle estimate error and velocity estimates are depicted in Figures 3c and 3d. The electrical angle estimate error also converges to zero, when it is calculated using (26). The difference between the speed estimates is neglectable.

In the second case, the simulation parameters are the following:
- \( F(p) = \frac{k_0}{p_0^2 + \omega^2}; \)
- The coefficients of the filter \( F(p) \) are given in Table 2;
- \( k_\epsilon = 1450; \)
- \( \gamma = \gamma_\alpha = \gamma_\beta = 1; \gamma_R = 1.2; \gamma_L = 2.7; \)
- \( R(0) = 1; \)
- \( L(0) = 0.001; \)
- \( \hat{\lambda}_0(0) = \hat{\lambda}_0(0) = 1; \)

All initial conditions are equal to zero except \( \hat{\lambda}_0, \hat{\lambda}_1, \hat{R} \) and \( \hat{L} \).

The simulation results are depicted in Figures 4–6. Using estimate stator resistance in flux estimation, the last one converges as expected. As seen from the figures, the estimated resistance and inductance converge to their true value.

In Figure 4a, we show the norm of the flux estimation error, and as you can see in Figure 5a, the electricity angle error is converging to zero.
The transients of the resistance and induction estimates are presented in Figures 6a and 6b.

### 6.1 Experimental investigation

An experimental investigation has been performed on the setup, which is depicted in Figure 1. It consists of a D1400 motor with eight pairs of poles produced by JSC “DIAKONT,” magnetic powder brake, two optical encoders and a torque sensor, which is based on a strain gauge. The nominal values of the motor are given in Table 3.

The motor is controlled by standard field-orient control using rotor position measurements from the encoder and the calculated value of the velocity. Values of the currents and voltages in $\alpha\beta$-frame, the calculated value of the velocity and measured angle have been recorded. This data is used to investigate the performance of the proposed observers in MATLAB Simulink. The flux is not measured, and we cannot compare estimates with measurements. Using flux estimates, we calculate the electrical angle and velocity estimates and compare it with the measured values.

The observer parameters in this case are the following:

- $F(p) = \frac{kk_p}{p^2 + a}$;
- The coefficients of the filter $F(p)$ are given in Table 4.
- $k_p = 1450$;
- $\gamma = \gamma_\alpha = \gamma_\beta = 2$, $\gamma_R = 0.4$, $\gamma_L = 0.06$;
- FTO parameters: $\hat{\eta}_k = 0.5$, $\hat{\eta}_R = 0.925$, $\hat{\eta}_L = 0.925$;
- $\hat{R}(0) = 1$;
- $\hat{L}(0) = 0.003$;
- $\hat{\lambda}_\alpha(0) = \hat{\lambda}_\beta(0) = 1$.

All initial conditions are equal to zero except $\hat{R}$, $\hat{L}$, $\hat{\lambda}$.

The results of the experimental investigation based on measured current and voltage signals are depicted in Figures 7–10 for different velocities of the rotor. In Figures 7 and 8, results are shown for the case where desired velocity is a sinusoidal function of time $\omega^* = 30 + 10 \cos(25.12 t)$; in Figures 9 and 10, results are shown for the desired velocity which is a periodical step function from 20 to 40 rad/s.

In Figures 7d and 9d, we show the flux estimate. Figures 7c and 9c show the estimate of the rotor electrical angle. The transitions for the resistance and induction estimates are shown in
FIGURE 7  Transients of the state estimates for the experimental investigation based on measured current and voltage signals for the sinusoidal desired velocity

Figures 8a and 10a and Figures 8b and 10b. The estimates do not converge to the constant value. One of the explanations is that those parameters are non-stationary, and that the sensitivity to noise is high.

The considered result demonstrates that the proposed observers provide reliable estimates of the rotor velocity, rotor position, and also usable estimates of the rotor resistance and inductance.
This paper proposes a new finite-time robust flux observer of the PMSM. Using a novel parametrisation, we obtain an equation in linear regression form. Applying DREM approach [13] gives a set of scalar equations for $\alpha, \beta$ flux components, which are used to construct DREMBAO flux observer. Outputs of this observer are used to obtain finite-time flux estimate.
most general robust scenario with unknown resistance, inductance, permanent magnet flux, inertia, viscous friction and load torque is considered, and finite-time DREMBAO based flux and position observer is proposed.

Based on the flux estimate, the position is reconstructed and further used in PLL-type speed observer [2, 10] to estimate the rotor angular velocity. Simulation results are given to reveal the high performance of the proposed approach. Experimental studies demonstrate that the observers provide the usable position and rotor speed estimates for the sensorless version of the field-orient controller.

It can be noticed that there is a moment of time when finite-time convergence is applied. It is mean that after this moment we have a gradient observer and after the abrupt change of parameters, the convergence will be exponential or asymptotic, which depends on the level of system excitation.

Another moment, which should be highlighted, is that the excitation requirements of the generalised observer is not well defined. The excitation level can be easily checked by value of $\dot{e}_x$. However, it is hard to provide description of the operation conditions, where the excitation conditions are fulfilled. This problem will be investigated in future works.

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