A Constraint on the $x$ Dependence of the Light Antiquarks Ratio

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Abstract

We perform a careful study on the effect of the Pauli blocking to the light antiquark structure of the proton sea. We develop the formal expressions for the antiquark distributions, highlighting the role played by quark statistics and the vacuum structure. Ratios involving the antiquarks are calculated. In particular, it is found that $\Delta \bar{d}(x)/\Delta \bar{u}(x)$ should be negative and $x$ independent.

The first suggestion that the proton sea is not symmetric was put forward by Feynman and Field in 1977 [1]. They noticed that because there are more empty states for $d$ quarks in the proton, $\bar{d}d$ pairs would be created more easily than the $\bar{u}u$ pairs - the Pauli Blocking. In the early 1980’s, Thomas [2] noticed that a pion cloud in the proton having more $\pi^+$ than $\pi^-$ naturally produces an excess of $\bar{d}$ antiquarks. Then in the early 1990’s the first measurements of the Gottfried sum rule by the NMC [3] strongly suggested, in the context of the Parton model, that there was indeed an excess of $\bar{d}$ over $\bar{u}$ antiquarks. A flurry of papers followed, where most of the work was based on meson clouds (for a complete review on the Meson Cloud Model approach to the light antiquark asymmetry see [4]), with a
few exceptions where the Pauli Blocking was advocated \[5,6\]. Chiral quark models \[7\] and instantons \[8\] have also been used to explain the NMC data. By now, extensive experimental data is available and we have at our disposition not only data on the magnitude of $\bar{d} - \bar{u}$, as well as on its $x$ dependence \[9\].

In this letter we advocate the fundamental role played by the quark statistics in both the polarized and unpolarized antiquark distributions. In order to isolate such effects to the sea structure of the proton, we work out the formal definition of the antiquark distributions to a point where some actual predictions can be made without the introduction of a number of model parameters. From this point, we are able to study to what extent Pauli Blocking is a correction or is the main phenomenon behind the sea asymmetry of the light quarks.

We start with the formal definition of the antiquark distribution \[10\]:

$$7(x) = \frac{p^+}{2\pi} \int dz^- e^{-i x p^+ z^-} < P | \psi_+ (z^-) \psi_+^\dagger (0) | P > |_{z^+ = z^+_0 = 0}, \quad (0.1)$$

with $x > 0$. To calculate Eq. \(0.1\), we remember that the sum over all the possible spins of the $\psi_+$ operators provides:

$$\psi_+^\dagger \psi_+ = \psi_+^\dagger \uparrow \psi_+ \uparrow + \psi_+^\dagger \downarrow \psi_+ \downarrow = \frac{1}{2} \psi^\dagger (1 + \gamma_0 \gamma_3) \psi, \quad (0.2)$$

with

$$\psi(z) = \int \frac{d^3 k}{(2\pi)^3} \sum_\alpha [b_\alpha (k) u^{(\alpha)} (k) e^{-i k \cdot z} + d_\alpha^\dagger (k) v^{(\alpha)} (k) e^{i k \cdot z}]. \quad (0.3)$$

Using Eq. \(0.3\) in Eq. \(0.1\) we see that the term which involves the quark operators produces a restriction, in the form of $\delta (xp^+ + k^+)$, which forces it to not contribute to the antiquark distributions, once $x$ should be positive, ($p^+$ and $k^+$ are positive). Hence, taking a general for the Dirac spinor, with $f(k)$ and $g(k)$ its upper and lower components, respectively, the antiquark distribution is rewritten as\[9\]:

\[\text{As usual in the definition of parton distributions, we take the parton transverse momenta squared not to be large. Or, correspondingly, we take } k^2 > > k^2_\perp, \text{ implying that } \sigma \cdot \vec{k} \chi_\alpha = \pm \chi_\alpha \text{ (+ for } \alpha = \uparrow; \text{ - for } \alpha = \downarrow)\]
\[ q(x) = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} [f^2 + g^2 + 2f \cdot g] < p|d_+^\dagger(k)d_+(k) + d_+^\dagger(k)d_-(k)|p > e \delta(x - \frac{k^+}{p^+}). \] (0.4)

The calculation of the polarized distributions follows the same pattern. From the expression for the polarized antiquark distribution:

\[ \bar{q}(x) = \frac{p^+}{2\pi} \int dz e^{-ixp^+z^-} < P|\psi_+(z^-)\gamma_5\psi_+^\dagger(0)|P >_{z^+ = z^- = 0}, \] (0.5)

for \( x > 0 \), we get:

\[ \Delta \bar{q}(x) = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} [f^2 + g^2 + 2f \cdot g] < p|d_+^\dagger(k)d_+(k) - d_+^\dagger(k)d_-(k)|p > e \delta(x - \frac{k^+}{p^+}). \] (0.6)

We expanded the \( \psi \) operators in terms of free quark and antiquark operators. Quarks in the proton, however, are not free and that can be translated into a complicated vacuum structure where the confined quarks live \[ q_{[7,11,12]} \]. In this environment, the free space vacuum is certainly not a good approximation for the ground state of the confined quark operators. Instead, one has to build a new vacuum structure based on bound quark operators. Following this direction, we extend the work of Tsushima, Thomas and Dunne \[ q_{[11]} \], to derive general forms for the antiquark distributions which incorporate the effects from the modified vacuum.

The bound state operators (denoted by a “*”) and the free state operators are related by a Bogolyubov transformation:

\[ b^* = A \cdot b + B \cdot d^\dagger, \] \[ d^*\dagger = C \cdot b + D \cdot d^\dagger, \] (0.7)

where the \( A, B, C, D \) factors are the overlaps between the bound and free states. As shown in \[ q_{[11]} \], these overlaps are non zero for a confining scalar potential, at least in 1+1 dimensions. The vacuum of the bound states are defined such that:

\[ b^*|0^* > = 0, \] \[ d^*\dagger|0^* > = 0. \] (0.8)

From Eqs. (0.8) and (0.9) it follows that:
\[ b|0^* > = - \frac{B}{DA - CB} |\overline{q}^* >, \]  
\[ d|0^* > = - \frac{C}{D^\dagger A^\dagger - C^\dagger B^\dagger} |q^* >, \]  
where \[ |\overline{q}^* >= d^\dagger|0^* >, \text{ and } |q^* >= b^\dagger|0^* >. \]  
Eqs. (0.10) are telling us that the modified vacuum is not empty but filled with quark-antiquark pairs.

In order to reach any conclusion regarding the sea quark structure of the proton, we should specify what is the state \[ |p> \] in Eqs. (0.4) and (0.6). The proton state, as a bound state of quarks, has to be built from bound state quark operators acting on the modified vacuum, as described by Eq. (0.9). The simplest form one can use for such state is that given by the \( SU(6) \) wave function:

\[ |p> \equiv F[b^\dagger]|0^*> \]
\[ = \frac{1}{18} \epsilon^{\alpha\beta\gamma} [b^\dagger(u,\uparrow,\alpha)b^\dagger(d,\downarrow,\beta) - b^\dagger(u,\downarrow,\alpha)b^\dagger(d,\uparrow,\beta)]b^\dagger(u,\uparrow,\gamma)|0^*>. \]

Because we know, from Eqs. (0.10) how the free quark operators act on the modified vacuum, we can readily calculate the distributions written in Eqs. (0.4) and (0.6). In particular, our interest is the antiquark distributions, which is calculated to be:

\[ \overline{q}^m(x) = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} [f^2 + g^2 + 2f \cdot g] \frac{|C|^2}{|BC - AD|^2} <q^*_m | F[b^\dagger] F[b^\dagger] | q^*_m > \delta(x - \frac{k^+}{p^+}), \]

where \( m \) stands for the possible spin projections. Notice that the antiquark distribution comes from the expectation values between the bound quark states. In the case of a proton state built from free quarks, there is no antiquark distributions, as \[ \frac{|C|^2}{|BC - AD|^2} \to 0. \]  
We can, of course, generate the sea quarks through perturbation theory once we know the quark-gluon vertex from QCD, a procedure which has actually been implemented before [13,14]. In this case, it happens that the quark-antiquark pairs generated from perturbative gluons produce an excess of \( \overline{u} \) antiquarks over \( \overline{d} \) antiquarks - in clear contradiction to the naive expectation from the Pauli principle. The solution to this dilemma is straightforward and will be presented in a forthcoming work.
To proceed, we need the calculation of the expectation value between the bound quark states. A direct calculation shows that:

\[
\begin{align*}
&\langle u^* | F^\dagger [b^{*\dagger}] F [b^{*\dagger}] | u^* \rangle = 4,
&\langle d^* | F^\dagger [b^{*\dagger}] F [b^{*\dagger}] | d^* \rangle = 5,
\end{align*}
\]

(0.12)

where the sum over the spins have been performed. Thus it follows that there is an excess of down antiquarks in the proton compared to the up antiquarks. Now we can write down the explicit expression for the difference:

\[
\overline{d}(x) - \overline{u}(x) = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \left[ f^2 + g^2 + 2f \cdot g \right] \frac{|C|^2}{|BC - AD|^2} \delta(x - \frac{k^+}{p^+}),
\]

(0.13)

which is positive definite. Although we do not know how to calculate Eq. (0.13) for the realistic (3 + 1) dimensions case, we can make a direct comparison to the experimental result by calculating the ratio:

\[
\frac{\overline{d}(x)}{\overline{u}(x)} = \frac{5}{4}.
\]

(0.14)

The experimental result is not an \(x\) independent curve. However, we know that in the small \(x\) region, where the number of perturbative antiquarks is quite large, the ratio should approach 1. In the intermediate \(x\) region the number of perturbative antiquarks diminishes drastically and the constant ratio, around the 5/4 of Eq. (0.14), should appear. However, inside this region we have the meson cloud, which is known to give its main contribution around the \(x = 0.2\) region. The deviation from 5/4 is maximum exactly in this region. Nevertheless, the interesting point is that the result encapsulated by Eq. (0.14) is fundamental for any calculation of the sea asymmetry effect.

Having established the fundamental role played by the quark statistics, along with the vacuum structure, to the observed sea asymmetry, we discuss the role of the Pauli Blocking in the polarized sea. As before, we calculate the expectation value for the number of polarized antiquarks of a given flavor in the proton using the quark distribution as given in (0.11). However, instead of summing over the spins, we now have to take the difference. The matrix elements in Eq. (0.11) when calculated for \(m = \uparrow\) and \(m = \downarrow\) gives a factor of 4/3 for the \(\overline{u}\) antiquarks and 1/3 for the \(\overline{d}\) antiquarks. Hence, we have for the polarized distributions:
\[ \Delta \pi (x) = \frac{41}{32} \int \frac{d^3k}{(2\pi)^3} \left[ f^2 + g^2 + 2f \cdot g \right] \frac{|C|^2}{|BC - AD|^2} \delta \left( x - \frac{k^+}{p^+} \right), \]

\[ \Delta \bar{d}(x) = -\frac{11}{32} \int \frac{d^3k}{(2\pi)^3} \left[ f^2 + g^2 + 2f \cdot g \right] \frac{|C|^2}{|BC - AD|^2} \delta \left( x + \frac{k^+}{p^+} \right). \]

Note that the ratios, like

\[ \frac{\Delta \bar{d}(x) - \Delta \pi (x)}{\bar{d}(x) - \pi (x)} = -\frac{5}{3}, \]

are \( x \) independent up to pionic corrections to the unpolarized distributions and quark mass corrections to all distributions. A different choice for the wave function, Eq. (0.11), would render a different numerical factor, but it would not introduce an extra \( x \) dependence in the ratio. On the other hand, the ratio

\[ \frac{\Delta \bar{d}(x)}{\Delta \pi (x)} = -\frac{1}{4} \]

should not be affected by pions (as they do not contribute to the polarized distributions). In this case, the experimental value for \( \Delta \bar{d}(x)/\Delta \pi (x) \) should be very close to a straight line.

We have seen that developing the formal expression for the antiquark distributions, Eq. (0.1), as far as possible in terms of the quark operators, allows us to derive some general conclusions about the physics responsible for the antiquark asymmetries in the proton sea. We see that the vacuum structure inside the proton, as expressed by Eqs. (0.10), is decisive in reaching this conclusion. However, it is not the final story, as Eqs. (0.10) would exist even if quarks were bosons. The fact that quarks are fermions is fundamental for the observed asymmetry in the number of the unpolarized light quarks in the proton sea, as can be seen from Eqs. (0.12). For the polarized case, the quark statistics is more than fundamental, it is probably the only sizeable effect to be measured. Of course, to extract numbers we need to model the proton wave function. Using the SU(6) quark wave function for the proton, we see that the \( x \) dependence cancels when ratios are taken. The result embodied by Eq. (0.18) is made more important when we remember the discussion after the \( \bar{d}(x)/\pi (x) \) was calculated: pion dressing of the proton wave function is the effect that gives the \( x \) dependence of the unpolarized ratio. It means that the ratio would be different from the unity even in a world.
without pions. In the polarized case, apart from quark mass effects in the large $x$ region, Eq. (0.18) should hold. In this way, the polarized ratio is expected to be $x$ independent.

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