Efficacy of Volume of Fluid Method in Computational Simulation of Sloshing Phenomenon
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Abstract. The present study deals with the analysis of stochastic transient pressure variations and wall impacts for scaled down models of LNG carriers by using the VOF technique for interface characterization. Numerical simulation of sloshing under sway using multiphase liquid-gas condition has been carried out. Use of VOF method for multiphase numerical simulation is compared with the results of previously performed SPH study and established sloshing experiments. The efficacies of the VOF interface capturing technique is established by comparison with high-resolution images taken during sloshing experiments performed under the same external excitation frequency, amplitude and fill level. The pressure data from experiments when compared with respective simulations involving SPH and VOF methods helps to capture the fluid flow behaviour and intricacies they can handle.

Keywords: Sloshing, VOF, SPH, LNG

1. Introduction
Liquefied Natural Gas (LNG) transportation through oil tankers in high erraticism of sea causes them to undergo sloshing [1]. Due to the scarcity of real time experimental data on sloshing in tankers [2], there have been many approaches for numerical simulation of sloshing phenomenon. The numerical techniques for capturing immiscible interfaces include front-tracking, VOF, Marker and Cell (MAC) method etc. For the liquid-gas interface capturing during sloshing, significant contribution has been from VOF, SPH and level set methods but the significance of MAC method in the development of interface capturing is nontrivial. VOF utilises Eulerian approach for flow analysis whereas level set method derives the interface by solving series of equations defining a volumetric fluid region. In MAC method, marker particles are holistically distributed inside a fluid region. By comparing the pressure at the centre of a cell with the free surface pressure, crude information about the interface can be determined inside specific cells [3]. The SPH method incorporates distribution of particles in the fluid domain to function like Lagrangian receivers to calculate flow variables.

With the mesh-free SPH model, Cao et al. [4] investigated the influence of structural parameters (amplitude, baffle height etc) on impact pressure whereas Rafiee [5] probed into the effects of using different incompressibility models in SPH based sloshing simulations. On the other hand, sloshing simulations capitalizing on VOF models have progressed through the application of unstructured grid-based discretization [6] to the formulation of two-phase improved VOF (iVOF) for achieving high accuracy [7]. Parametric analyses using VOF including the effects of tank fill level, excitation frequency etc have also been performed [8,9]. In addition to the models which are included in commercial software, there are examples of in-house solver developed for sloshing analysis [10].

This study aims to investigate the stochastic transient pressure variations and wall impacts for scaled down models of LNG carriers. The efficacy of VOF for interface characterization is established by comparison with high-resolution images taken during sloshing experiments. Instabilities like roof impact are also captured by fast camera and compared with the simulation contours.

2. Mathematical fluid flow analysis
2.1. Multiphase physics
VOF [11] is used wherever the interface deformations are highly irregular. In this method, a dependant variable characteristic to a given fluid in the domain is defined as the volume fraction(α) and linked to all cells. The change in α will be maximum in a direction normal to the interface boundary containing
the respective fluid. The normal direction can be determined by computing the derivatives of \( \alpha \) w.r.t. time and space.

\[
\frac{\partial \alpha}{\partial t} + u \frac{\partial \alpha}{\partial x} + v \frac{\partial \alpha}{\partial y} = 0 ; u, v \text{ represent velocities in } x \text{ and } y \text{ directions respectively}
\]

(1)

By computing \( \alpha \) and the normal for a particular cell, the interface can be constructed. \( \alpha \) can be evaluated by solving the continuity equation for the respective phase. For an incompressible flow without any interphase mass transfer and mass sources, this equation for fluid ‘q’ can be given as:

\[
\frac{\partial \alpha}{\partial t} + \nabla \alpha \vec{u} = 0 ; \text{ where } t=\text{time(s)} \text{ and } \vec{u} = \text{velocity vector(m/s)}
\]

(2)

For all other fluids, volume fraction can be calculated by:

\[
\sum_{q=1}^{n} \alpha_q = 1
\]

(3)

2.2 Momentum and continuity equations

The fundamental solution to the sloshing problem needs a solution to the conservation of mass and conservation of momentum (Navier-Stokes) equations mentioned below:

\[
\frac{\partial u_i}{\partial x_i} = 0 \quad \text{(Continuity)}
\]

(4)

\[
\rho \frac{\partial u_i}{\partial t} + \rho u_j \frac{\partial u_i}{\partial x_j} = F - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] \quad \text{(Momentum)}
\]

(5)

Where, \( p \)=Pressure in flow direction (Pa), \( F=\)Net body-force acting over the domain(N), \( u_i \) and \( u_j \) are the components of free stream velocity acting in \( x \)-direction and \( y \)-direction respectively (m/s). For sloshing, laminar flow model results in poor representation of flow features[8]. Turbulent flow models are therefore inevitable to capture sloshing flows. In turbulent flows, instantaneous velocity \( u_i \) is given by:

\[
u_i = U_i + u_i'
\]

(6)

\( U_i \) and \( u_i' \) are mean and fluctuating components of velocity respectively. Substituting \( u_i \) in Eq.(1)&(2) we obtain Reynolds Averaged Navier-Stokes equations (RANS) for an incompressible flow.

\[
\frac{\partial U_i}{\partial x_i} = 0
\]

(7)

\[
\rho \frac{\partial U_i}{\partial t} + \rho U_j \frac{\partial U_i}{\partial x_j} = F - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) + \tau \right]
\]

(8)

\[
\mu = \frac{\alpha_i k \rho}{\max(\alpha_i \omega_i \Omega F_i)}
\]

(9)
In the present study, SST $k-\omega$ turbulence model is used which acts as $k-\epsilon$ model for free stream computations and gives accurate values when solving in near wall region by using the normal $k-\omega$ model. This model further allows for effect of the turbulent shear stress transport in the following manner:

$$\tau = -\rho u_i u_j = \mu \Omega = \mu \frac{\partial u}{\partial y}$$

(10)

$\Omega$ is the strain rate magnitude;

$$\frac{d\tau}{dt} = \frac{\partial \tau}{\partial t} + u_k \frac{\partial \tau}{\partial x_k}$$

(11)

The transport equations for calculating $k$ and $\omega$ are given by Menter[12] as follows:

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = P_k - \beta k \omega + \frac{\partial}{\partial x_j} \left[ (v + \sigma_{\omega y}) \frac{\partial k}{\partial x_j} \right]$$

(12)

$$\frac{\partial \omega}{\partial t} + U_j \frac{\partial \omega}{\partial x_j} = \alpha \Omega^2 - \beta \omega^2 + \frac{\partial}{\partial x_j} \left[ (v + \sigma_{\omega y}) \frac{\partial \omega}{\partial x_j} \right] + 2(1 - F_1)\sigma_{\omega y} \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i}$$

(13)

Where the blending functions $F_1$ and $F_2$ are defined by:

$$F_1 = \tanh \left\{ \min \left[ \max \left( \frac{\sqrt{k}}{\beta \omega y}; \frac{500v}{y^2 \omega} \right) \frac{4\sigma_{\omega y}}{CD_{\omega y}} \right] \right\}$$

(14)

$$F_2 = \tanh \left[ \max \left( \frac{2\sqrt{k}}{\beta \omega y}; \frac{500v}{y^2 \omega} \right) \right]$$

(15)

$y$ is the distance to next surface and $v$ kinematic viscosity;

$$P_k = \min \left( \tau \frac{\partial U_i}{\partial x_j}; 10 \beta k \omega \right)$$

(16)

The cross-diffusion term is given by

$$CD_{\omega y} = \max \left( 2\rho \sigma_{\omega y} \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i}; 10^{-10} \right)$$

(17)

The model constants are defined as:
\[
\alpha_1 = \frac{5}{9}, \alpha_2 = 0.44, \beta_1 = \frac{3}{40}, \beta_2 = 0.0828, \beta^n = 0.09, \sigma_{k1} = 0.85, \sigma_{k2} = 1, \sigma_{vol} = 0.5, \sigma_{w2} = 0.856
\]

In order to calculate \( \alpha \) in respective cells for a time-dependent problem, the explicit time-step formulation for evaluating \( \alpha \) for the current time-step is as follows:

\[
\frac{\alpha^{n+1}_q - \rho_q^{n+1}}{\Delta t} + \sum_f (\rho_q^n U_f^n \alpha_q^{n, f}) = 0
\]

(18)

Where \( n \) denotes time-step, \( \alpha_q^{n, f} \) = apparent volume fraction for q phase, \( U_t \) = volume flux through faces of cell, based on normal velocity. For free surface representation, the geometric reconstruction scheme is employed.

3. Computational modelling

3.1 Visualization and analysis of sloshing instabilities

The maximum density ratio of LNG (470 kg/m3) to vapour natural gas (0.7 kg/m3) is near to 675. In this study, air (1.225 kg/m3) is taken as primary gaseous phase and water (998.2 kg/m3) is the secondary liquid phase with liquid to gas density ratio of 814. As a consequence, a factor of safety arises between the maximum pressure obtained by this study and the actual LNG sloshing impacts. The meshing is performed in ICEM tool of ANSYS. The two-dimensional analysis is performed for a simple rectangular geometry without any baffles or curvature which acts as a representative cross section plane of a cuboidal tank. 1:150 (0.26m×0.18m) scaled down model [13] of an actual 125000 m3 tank with 50% fill level by height is used for comparing the efficacy of VOF in interface representation (Model A in figure 1). While 1:30 (1.3m×0.9m) model [14] with 70% fill by height is used for peer-to-peer comparison of pressure data computed by using VOF with the experimental and SPH results by [15] (Model B in figure 1).

![Figure 1](image1.png)

For case A, an unstructured grid with 46800 nodes and 93160 quadrilateral faces is found sufficient for capturing sloshing accurately. Figure 2 compares simulated and experimental wave formation realizing the mesh accuracy. For B, unstructured grid with 292500 nodes and 483900 quadrilateral faces was used. The pressure values from the simulation are in comparable accordance with the experimental data obtained by Rafiee [15], therefore this grid is selected. In this study the excitation of the tank B from rest is performed with a constant amplitude of 0.1 m whereas Rafiee [15] applied a gradual and sinusoidal increase in the amplitude of motion from 0 to 0.1 m across ten initial cycles of oscillation which consumed a total time of 13.5 s with 0.74 Hz oscillation frequency. Thus, the pressure variations considered are after these ten cycles.
3.2 Discretization schemes
Pressure Implicit with Splitting of Operator (PISO) method is employed to solve pressure-velocity coupling in the pressure-based solver using Non-Iterative Time Advancement (NITA) to improve the overall time accuracy at the expense of increased computational time. VOF simulations are performed by limiting the CFL number to a maximum value of 2 for confining numerical diffusion. The Green Gauss Cell Based scheme for gradients evaluation of scalar $\phi$ at cell center $\phi_c$ is written as following:

$$
\nabla \phi = \frac{1}{\nu} \sum_{i=1}^{f} \phi_f A_f
$$

(19)

Where $\nu$ is the number of cells on the face ($f$) and $\phi_f$ is calculated as below:

$$
\phi_f = \frac{\phi_{i0} + \phi_{i1}}{2}
$$

(20)

Further, PRESTO scheme is used for pressure interpolation and First Order Upwind scheme is used for spatial discretization of momentum, turbulent kinetic energy and dissipation rate.

3.3 Dynamic mesh and excitation parameters
The erratic motion of tank fluid will peak when the external excitation frequency equalises with the resonance frequency irrespective of the applied varying motion. The linear excitation of tank near infinitesimal variation from the natural frequency is sufficient to capture universal sloshing. Therefore, in the present study the effect of only sway motion of the tank is studied. The sway motion was simulated by using the slider-crank analogue with the help of in cylinder function of FLUENT 15.0. Position $x$ of the grid swaying with an excitation frequency $\omega'$ at time $t$ is given as:

$$
x = r_c \cos \omega' t
$$

(21)

where $r_c$ is the sway amplitude. (0.025 m for A and 0.1 m for B)

Here $\omega'$ is selected within 1% margin of natural frequency $\omega$ which is calculated with Eq.(22).

$$
\omega^2 = \frac{\pi g}{l} \tanh \left( \frac{\pi h}{l} \right)
$$

(22)

as validated near resonance in[9] for the first natural frequency.

For the dynamic motion of mesh, a cell moving to centroid of its neighbouring cell is used. Laplacian smoothing is used to calculate the position of a mesh node $\hat{x}_i$ after its motion as follows [16]:

Figure 2. Comparison of wave development phenomenon by experimental and computational (VOF) methods for 10% and 50% fill level (Sway Frequency- 1.1 Hz, Amplitude- 0.025 m)
\[ x_i = \frac{\sum_{j=1}^{n} \bar{x}_j}{n} \]  

where \( n \) = total number of neighbouring nodes \( \bar{x}_j \).

After an iteration, respective new node position \( \bar{x}_{j+1} \) is calculated by:

\[ \bar{x}_{j+1} = \bar{x}_j (1 - \beta) + \bar{x}_i (\beta) \]  

\( \beta = 1 \) is the node relaxation factor for rigid mesh.

### 3.4 Monitor specifications and experimental apparatus

For model B, dynamic monitors were placed at specific positions of the tank wall to extract accurate values of maximum and average impact pressures from the transient simulation. The comparison with SPH and experimental results is done for the monitors P1, P2, P3 and P4. Rest of the monitors are evenly distributed through the entire ullage area and tank roof to get overall estimation of maximum pressure. Table 1 shows the co-ordinates and respective denotations for all the monitors.

| MONITOR | EXPERIMENTAL & SPH NAME | X CO-ORDINATE (m) | Y CO-ORDINATE (m) |
|---------|--------------------------|------------------|------------------|
| P1      | P093                     | 0                | 0.45             |
| P2      | P180                     | 0                | 0.63             |
| P3      | P178                     | 0                | 0.72             |
| P4      | P098                     | 0.15             | 0.9              |
| P5      | -                        | 0.3              | 0.9              |
| P6      | -                        | 0.65             | 0.9              |
| P7      | -                        | 1                | 0.9              |
| P8      | -                        | 1.15             | 0.9              |
| P9      | -                        | 1.3              | 0.72             |
| P10     | -                        | 1.3              | 0.63             |
| P11     | -                        | 1.3              | 0.45             |

In order to generate sway motion of the tank, a six degree of freedom system manufactured by MOOG is used. For capturing sloshing instabilities, a Phantom v5.2 camera (12 GB memory, capable of acquiring upto 1000 fps at the maximum resolution of 1152×896 pixels in monochrome at a depth of 12 bits/pixel is used. The objective lens is a 50 mm fixed focal Nikkor lens.

### 4. Results

#### 4.1 Visualization and analysis of sloshing instabilities

A transient flow visualisation is created to correlate sloshing with maximum pressure and time of impact. The roof of the LNG vessel has less mechanical strength than other sections, therefore it becomes quintessential to analyse the roof impacts and their timings. One of the most important phenomena which occurs during the partially filled transportation of LNG vessel is roof impact. The magnitude of the pressure impacts decreases as water moves along the length of the roof. In order to understand how liquid accumulates at the roof during impact, the flow development can be visualised from figure 3. It shows that the fluid impacts on the upper left corner of the wall which exerts a reaction force on the liquid causing a change in its momentum due to which it travels along the roof. This liquid travelling along the roof is also responsible for sloshing loads on the roof of the vessel. High impact pressures during roof impacts are recorded mostly at tank corners. The impact between the sloshing fluid and the tank roof is less likely to occur at lower filling ratios.
4.2 Impact pressure discussion

The case B of figure 1 is simulated for the comparison study of sloshing loads for the highly stochastic sloshing problem. The SPH method uses discrete particles that are carried with the flow. The hydrodynamic and other quantities are evaluated at the particle positions and are calculated from a weighted-average of the values on other local particles. Therefore, each particle is essentially “smoothed” over a finite volume of fixed mass, and in this way, these SPH codes are naturally adaptive with density. In this study, the SPH data which is used for comparison is a multiphase, two-dimensional case of water and air weakly compressible SPH. Table 2 lists the maximum impact pressure values as obtained by VOF, experimental and SPH method. For the P1 monitor, the maximum impact pressure value as calculated from VOF simulations are in sublime agreement with the experimental results. The VOF method underpredicts this value by an infinitesimally small value (0.81%), whereas the SPH values bear a considerable underprediction of 23.87% from the experimental values. For P2 monitor, VOF overpredicts by 12% but high level of disparity between SPH and experimental results is observed which undercalculates the maximum impact values 42%. This underprediction for SPH is maximum for the P3 monitor, but here VOF also underpredicts by similar margin. Both methods overpredict the pressure values for P4 monitor but error in VOF is greater than that of the SPH counterpart. Therefore, out of the four cases, maximum sloshing load values encountered by the tank wall as calculated by VOF simulations are in better agreement with the experimental results for two cases. For one case both methods calculate with a similar error margin but for the last case (P4), SPH yields better results. Also, the issue of underprediction of maximum impact pressure by SPH method is predominant for three out of four cases. By calculating the mean of the percentage error in calculating the maximum loads, it is concluded that VOF underpredicts the values by an average of 3.96% whereas, SPH by 26.08%. Therefore, the accuracy of VOF can be deemed better for calculation of this parameter.

Table 2: Comparison of Maximum Impact Pressure Values and Time of maximum impact for VOF and SPH with Experimental Values

| Monitor | Expt. max. impact (kPa) | VOF impact pressure | SPH impact pressure | Expt. impact time (s) | VOF impact time | SPH impact time |
|---------|--------------------------|---------------------|---------------------|-----------------------|----------------|-----------------|
|         | Value (kPa) | Error (%) | Value (kPa) | Error (%) | Value (s) | Error % | Value (s) | Error % |
| P1(P093)| 8.21        | 8.143    | -0.81     | 6.25     | -23.87 | 7.29    | 8.82     | 20.98   | 0.97 | -86.69   |
| P2(P180)| 22.3        | 25.15    | 12.78     | 12.85    | -42.37 | 3.39    | 3.36     | -0.88   | 2.11 | -37.76   |
| P3(P178)| 28.08       | 15.61    | -44.40    | 15.74    | -43.95 | 3.22    | 1.61     | -50     | 8.14 | 152.79   |
| P4(098)| 38.06       | 44.38    | 16.60     | 40.29    | 5.86   | 3.64    | 3.73     | 2.47    | 6.79 | 86.53    |
| Average|             | -3.96    | -26.08    |           |         |         | -6.88    |         | 28.72 |

Representative pressure histograms are plotted in figure 4 and time at which the tank encounters this impact is mentioned in table 2. Regarding the time history of sloshing loads, for the P1
monitor, the time of primary maximum impact during experimental study is in better coherence with that of the VOF analysis.

Besides these, a higher value of maximum impact pressure is found at a location P8 other than that reported by Rafiee [15] during SPH simulations. Figure 5 shows the maximum pressure encountered at all the dynamic monitors. Another notable observation is that co-ordinates of the monitors in which two peaks (Figure 5) are observed on the roof at 0.15 m offset from both the upper corners.

Figure 4. Pressure histograms for Experimental, VOF and SPH methods for P1 monitor

Figure 5. Values of Maximum Impact Pressure observed at all 11 monitors

5. Conclusions
In this study, the simulation of sloshing phenomenon is performed for two scaled-down models. The key conclusions are summarized below.

a) VOF method exhibits high efficacy in capturing physical representation of interface, transient change in its nature and the calculation of concerned flow parameters.

b) For each value of maximum impact load calculated by VOF, the error, in comparison with experimental results, is average of 4% underprediction, whereas, for SPH, this error is around 26% underprediction. For pinpointing the time of maximum impact, VOF yields more accurate values all the four cases. VOF calculated the impacts at a time earlier to the experimental counterpart by 7% while SPH calculated the maximum impact late by a margin of 29%. Therefore, VOF provides more agreeable results for both the calculations.

c) The area proximal to ullage corners is highly susceptible to severe impacts. Maximum impact pressure is observed at an offset of 0.15 m from the upper corners on the tank roof. Therefore, this area should be critically designed, analysed and monitored for structural failure.

References
[1] Shin Y, Kim JW, Lee H, et al. Sloshing Impact of LNG Cargoes in Membrane Containment Systems in the Partially Filled Condition.
[2] Perić M, Zorn T. Simulation of sloshing loads on moving tanks. Proc Int Conf Offshore Mech Arct Eng - OMAE. American Society of Mechanical Engineers Digital Collection; 2005. p. 1017–1026.
[3] Welch JE, Harlow FH, Shannon JP, et al. THE MAC METHOD- A COMPUTING TECHNIQUE FOR SOLVING VISCOUS, INCOMPRESSIBLE, TRANSIENT FLUID-FLOW PROBLEMS INVOLVING FREE SURFACES. 1965.
[4] Cao XY, Ming FR, Zhang AM. Sloshing in a rectangular tank based on SPH simulation. Appl Ocean Res. 2014;47:241–254.
[5] Rafiee A, Dutykh D, Dias F. Numerical Simulation of Wave Impact on a Rigid Wall Using a Two-phase Compressible SPH Method. Procedia IUTAM. Elsevier B.V.; 2015. p. 123–137.
[6] YANG C, LÖHNER R, LU H. An unstructured-grid based volume-of-fluid method for extreme
wave and freely-floating structure interactions. J Hydrodyn. 2006;18:415–422.

[7] Wemmenhove R, Luppes R, Loots E, et al. Modeling two-phase flow with offshore applications. Proc Int Conf Offshore Mech Arct Eng - OMAE. 2005. p. 993–1001.

[8] Von Bergheim P, Thiagarajan KP. The air-water sloshing problem: Parametric studies on excitation magnitude and frequency. Proc Int Conf Offshore Mech Arct Eng - OMAE. 2008. p. 603–609.

[9] Thiagarajan KP, Rakshit D, Repalle N. The airwater sloshing problem: Fundamental analysis and parametric studies on excitation and fill levels. Ocean Eng. 2011;38:498–508.

[10] Ming PJ, Duan WY. Numerical simulation of sloshing in rectangular tank with VOF based on unstructured grids. J Hydrodyn. 2010;22:856–864.

[11] Hirt CW, Nichols BD. Volume of fluid (VOF) method for the dynamics of free boundaries. J Comput Phys. 1981;39:201–225.

[12] Menter F. Zonal Two Equation k-w Turbulence Models For Aerodynamic Flows. 23rd Fluid Dyn Plasmadynamics, Lasers Conf [Internet]. Reston, Virigina: American Institute of Aeronautics and Astronautics; 1993 [cited 2020 Jun 10]. Available from: http://arc.aiaa.org/doi/10.2514/6.1993-2906.

[13] Rakshit D. Thermodynamic analysis of two-phase fluids stored in stationary and slowly moving tanks. 2012.

[14] Bass III RL, Bowles EB, Cox PA. LIQUID DYNAMIC LOADS IN LNG CARGO TANKS. 1980;

[15] Rafiee A. SPH modeling of multi-phase and energetic flows. 2011.

[16] Herrmann L.R. Laplacian-isoparametric grid generation scheme. J Eng Mech Div. 1976;102:749–907.