Vacuum-polarization near cosmic string in RS2 brane world

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Gravitational field of cosmic strings in theories with extra spatial dimensions must differ significantly from that in the Einstein’s theory. This means that all gravity induced properties of cosmic strings need to be revised too. Here we consider the effect of vacuum polarization outside a straight infinitely thin cosmic string embedded in a RS2 brane world. Perturbation technique combined with the method of dimensional regularization is used to calculate $\langle T_{\mu}^{\nu} \rangle_{\text{ren}}^{\text{vac}}$ for a massless scalar field.

I. INTRODUCTION

Now it is well understood that topological defects in quantum field theory may play an important role in physical phenomena at very different spacetime scales. In particular, vertex line defects describable on a macroscopic scale as cosmic strings are most likely to be actually generated at phase transitions in the early universe [1]. Recent cosmological data, in particular power spectrum of the microwave background, are not consistent with the density fluctuations predicted by the simplest cosmic string scenario. Nevertheless, some combination of density fluctuations produced by strings and inflation is possible. Moreover, gravitational properties of cosmic strings become the question of increasing interest because strings are predicted by many of the commonly considered models.

In a lot of applications radius of curvature of a string is much larger than its transverse diameter. So, string can be considered to be infinitely thin, and one can use the so-called world sheet approximation. In this approximation four-dimensional Einstein’s gravity leads to the result that the spacetime of an infinitely thin straight cosmic string is a direct product of the two-dimensional Minkowski plain and a cone. Corresponding Riemann tensor vanishes everywhere except on the world sheet of the string, where it has a $\delta$-like singularity. So, in this theory straight conical defects do not affect the local geometry of the spacetime but change its global properties, and the effects of conical structure on surrounding classical and quantized matter fields are purely nonlocal (topological). At the same time, it seems unlikely that at the sufficiently large energy scales Einstein’s theory describes gravity accurately. This means that alternative approaches, in particular those formulated in spacetime of more than four spacetime dimensions, must be considered too.

Until recently, Kaluza-Klein theories with a small size of extra dimensions have been of the most interest. At the present time, however, the brane world picture becomes more and more popular. In this approach our physical universe is considered to be a 3-brane embedded into a higher dimensional bulk (for the review of this problem see [2] and references therein). Thus, gravity becomes multidimensional while ordinary matter fields are still propagate in the world with three spatial dimensions.

Here we consider a so-called RS2 model with one infinite extra dimension and matter

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localized on a single positive tension brane \( \text{[3]} \). The main goal of our paper is to demonstrate once more that extra dimensions are not hidden, and there are nontrivial effects of the higher-dimensional gravity on the fields trapped to a brane.

This work is devoted to the effect of vacuum polarization near straight cosmic string imbedded in the brane of a RS2 world. Our investigation was stimulated by

i. the paper \( \text{[4]} \), in which a globally consistent expression for the linearized gravitational field in the Randall-Sundrum background with a matter on the brane is found;

ii. the paper \( \text{[5]} \), in which the gravitational properties of a straight cosmic string in the RS2 brane world are investigated;

iii. our paper \( \text{[6]} \), in which it is shown that perturbative methods can be used in investigation of vacuum polarization in the spacetime of a single string, and in the spacetime of multiple cosmic strings.

We restrict our consideration by the case of a massless scalar field, because it is obvious enough that for the fields of higher spin the result will be the same up to the numerical coefficient.

The paper is organized as follows. We intend to use the method of dimensional regularization. So, in Sec.II, the main results of the paper \( \text{[4]} \) are extended to the case of RS2 type world of dimension \( p+1+1 \). Then we consider the particular case when \( p \)-brane contains a multidimensional generalization of an “ordinary” straight string. Below we will use the term string for this co-dimension two object. In Sec. \( \text{[II]} \), we calculate the Euclidean Green’s function for a massless scalar field on the background under consideration. In Sec. \( \text{[IV]} \) the method of dimensional regularization is used to obtain the expression for the renormalized vacuum averaged energy-momentum tensor. In Sec. \( \text{[V]} \), we summarize our results.

In this paper, we use the metric with the signature \((-+\cdots+)\) and the system of units \( \hbar=c=1 \).

II. GRAVITATIONAL FIELD OF A STRAIGHT COSMIC STRING IN RS2 SPACETIME OF DIMENSION \( p+1+1 \)

Let us start by the formulation of a linearized gravity on a \( p \)-brane imbedded in a bulk of dimension \( p+1+1 \). In this section we briefly reproduce the main ideas of the paper \( \text{[4]} \).

The action under consideration reads

\[
S = \frac{1}{16\pi G_{n+1}} \int d^n x \int dy \sqrt{-g} (R - 2\Lambda) + \frac{1}{16\pi G_{n+1}} \int_{\text{brane}} d^n x \sqrt{-\hat{g}} \left( \sigma + 16\pi G_{n+1} \mathcal{L}_{\text{matter}} \right),
\]

where \( n = p+1 \), \( g_{ab} \ (a, b = 0, 1, \ldots, n-1, y) \) is the metric of the bulk space-time, \( \hat{g}_{\mu\nu} \ (\mu, \nu = 0, 1, \ldots, n-1) \) is the induced metric on the brane, \( G_{n+1} \equiv 1/M^{n-1} \) and \( M \) are the bulk gravitational constant and the bulk Plank mass correspondingly. We shall denote coordinates on the \( p \)-brane as \( x^\mu \), \( y \) is coordinate transverse to the brane.

To obtain multidimensional solution of the Randall-Sundrum type we must choose

\[
\Lambda = -\frac{n(n-1)}{2} k^2 \quad \text{and} \quad \sigma = -4(n-1)k .
\]

In this case the metric

\[
ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2
\]
is the solutions for \( \mathcal{L}_{\text{matter}} = 0 \).

As it was shown in the paper [4], there is a gauge, in which the position of the 3-brane remains fixed at \( y = 0 \) even when matter is present. Proceeding along the same line, let us split up the bulk metric tensor as

\[
(g_{ab}) = \left( \hat{g}_{\mu \nu} \frac{N_\mu}{N_\nu} N \lambda N^\lambda + N^2 \right), \quad (g^{ab}) = \frac{1}{N^2} \left( N^2 \hat{g}^{\mu \nu} + N^\mu N^\nu - N^\mu \right).
\]

where \( N \) and \( N_\mu \) are the lapse function and shift vector, respectively.

For the linearization of the Einstein’s equations, we write the induced metric as

\[
\hat{g}_{\mu \nu} = e^{-2k|y|} (\eta_{\mu \nu} + \gamma_{\mu \nu}) ,
\]

and consider \( \gamma_{\mu \nu}, N_\mu \) and \( \phi \equiv N^2 - 1 \) to be small perturbations.

In this approximation the gauge conditions for the \( p \)-brane to remain at \( y = 0 \) have the form

\[
N_\mu = -\frac{\text{sgn} y}{2nk} \gamma, \quad \phi = -\frac{\text{sgn} y}{nk} \gamma_y ,
\]

\[
\tilde{\gamma}_{\mu \nu} = 0
\]

where \( \gamma = \eta^{\mu \nu} \gamma_{\mu \nu} \) and \( \tilde{\gamma}_{\mu \nu} = \gamma_{\mu \nu} - \frac{1}{n} \eta_{\mu \nu} \gamma \) is the traceless part of \( \gamma_{\mu \nu} \).

Following [4] we obtain that the resulting equation for the metric perturbation \( \tilde{\gamma}_{\mu \nu} \) is

\[
\partial_y (e^{-2k|y|} \partial_y \tilde{\gamma}_{\mu \nu}) - (n - 2)k e^{-2k|y|} \partial_y \tilde{\gamma}_{\mu \nu} + \Box \tilde{\gamma}_{\mu \nu} = -16\pi G_{n+1} \delta(y) \left[ t_{\mu \nu} - \frac{1}{n-1} \left( \eta_{\mu \nu} - \partial_\mu \partial_\nu \right) t \right]
\]

with a constrain

\[
\Box \gamma \bigg|_{y=0} = \frac{8\pi G_{n+1}nk}{n-1} t .
\]

In the last two equations \( t_{\mu \nu} \) is the energy-momentum tensor of a matter sitting on the brane and \( t = \eta^{\mu \nu} t_{\mu \nu} \).

Now, it is easy to show that for \( k > 0 \) (Randall-Sundrum case) and \( q^2 > 0 \) the final solution for \( \tilde{\gamma}_{\mu \nu} \) in the momentum space reads

\[
\tilde{\gamma}_{\mu \nu}(q, y) = \frac{8\pi G_{n+1}}{|q|} \left[ t_{\mu \nu} - \frac{1}{n-1} \left( \eta_{\mu \nu} - \frac{q_\mu q_\nu}{q^2} \right) t \right] e^{nk|y|/2} K_{n/2} \left( \frac{e^{k|y|/2}}{K_{n/2-1}(|q|/k)} \right).
\]

In \( n = p + 1 \) spacetime dimensions the energy-momentum tensor of a straight infinitely thin string (object of co-dimension two) has the form

\[
t_{\mu \nu} = -\mu \delta^2(\vec{x}) \left( \eta_{\mu \nu} - \sum_{i=1}^2 \delta_{\mu i} \delta_{\nu i} \right),
\]

where \( \vec{x} = (x^1, x^2) \) and \( \mu \) is the energy per unit volume of the string.
For this particular case the traceless part of the metric on the brane takes the form
\[
\tilde{\gamma}_{\mu\nu}(x, 0) = 8\pi G_{n+1}\mu \int \frac{d^2\vec{q}}{(2\pi)^2} e^{i\vec{q}\vec{x}} \frac{K_{n/2}(q/k)}{qK_{n/2-1}(q/k)} W_{\mu\nu}(\vec{q}) ,
\]
(12)
where \(\vec{q} = (q_1, q_2)\), \(q = |\vec{q}|\) and
\[
W_{\mu\nu}(\vec{q}) = \frac{1}{n-1} \left[ (n-1) \sum_{i=1}^{2} \delta_{i\mu} \delta_{i\nu} - \eta_{\mu\nu} - (n-2) \frac{q_\mu q_\nu}{q^2} \right]
\]
(13)
with \(q^\mu = (0, q^1, q^2, 0, \ldots, 0)\). As for the trace of the metric, it can be calculated using Eq. (9)
\[
\gamma(x) = 8\pi kG_{n+1}\mu n\frac{n(n-2)}{n-1} \int \frac{d^2\vec{q}}{(2\pi)^2} e^{i\vec{q}\vec{x}}
\]
(14)
The solution obtained (12) can be written in the form \(\tilde{\gamma}_{\mu\nu}(x, 0) = \tilde{\gamma}_{\mu\nu}^0(x) + \tilde{\gamma}_{\mu\nu}^1(x)\),
\[
\tilde{\gamma}_{\mu\nu}^0(x) = 8\pi G_{n+1}(n-2)k\mu \int \frac{d^2\vec{q}}{(2\pi)^2} e^{i\vec{q}\vec{x}} \frac{1}{q^2} W_{\mu\nu}(\vec{q}) ,
\]
(15)
\[
\tilde{\gamma}_{\mu\nu}^1(x) = 8\pi G_{n+1} \int \frac{d^2\vec{q}}{(2\pi)^2} e^{i\vec{q}\vec{x}} K_{n/2-2}(q/k) \frac{qK_{n/2-1}(q/k)}{qK_{n/2-1}(q/k)} W_{\mu\nu}(\vec{q}) ,
\]
(16)
where \(\tilde{\gamma}_{\mu\nu}^0\) is the solution corresponding to the case of a string in the \(n\)-dimensional Einstein’s gravity, while the second term is the contribution from the bulk.

Now we can obtain the expressions for the intrinsic Ricci tensor
\[
\hat{R}_{\mu\nu} = 8\pi G_{n+1} k\mu \frac{n-2}{2} \frac{\delta^2(\vec{x})}{2} \sum_{i=1}^{2} \delta_{i\mu} \delta_{i\nu} - \frac{1}{2} \Box \gamma_{\mu\nu}^1(x) .
\]
(17)
and for the scalar curvature
\[
\hat{R} = 16\pi G_{n+1} k\mu \frac{n-2}{2} \frac{\delta^2(\vec{x})}{2} .
\]
(18)
We see that, in contrast to its four-dimensional equivalent, the spacetime of a string on a brane is not Ricci flat, but the (linearized) intrinsic scalar curvature is still equal to zero everywhere outside the core.

III. EUCLIDEAN GREEN’S FUNCTION

Euclidean Green’s function of a scalar field in a spacetime of the dimension \(n\) is the solution of the Poisson equation
\[
\left( \hat{g}_{E}^{\mu\nu} \hat{\nabla}_{\mu} \hat{\nabla}_{\nu} - \xi \hat{R} \right) G_E(x, x') = -\frac{\delta^n(x-x')}{\sqrt{\hat{g}_E(x)}}
\]
(19)
where \(\hat{g}_E^{\mu\nu}\) is obtained from the metric under consideration by the Wick rotation.
Proceeding along the same line as in the paper [5], let us write this equation in the form
\[ \Delta_0^n G^E(x, x') = -\delta^n(x - x') - V G^E(x - x') , \] (20)
where \( \Delta_0^n \equiv \delta^{\mu\nu} \partial_\mu \partial_\nu \) is the Laplace operator in the \( n \)-dimensional Euclidean space, and

\[ V = \partial_\mu \left( \sqrt{g_E} \hat{g}^{\mu\nu}_E \partial_\nu \right) - \delta^{\mu\nu} \partial_\mu \partial_\nu - \xi \hat{R} . \] (21)

When the use of perturbation theory is justified, the solution of the equation (20) can be written as
\[ G^E = G^E_0 + G^E_0 V G^E_0 + G^E_0 V G^E_0 V G^E_0 + \ldots , \] (22)
where \( G^E_0 \) is the Green’s function of the Poisson equation in the \( n \)-dimensional Euclidean space.

If we restrict ourselves by the first-order correction to the Green’s function, we can represent the perturbation operator \( V \) in the form
\[ V = -\gamma^{\mu\nu} \partial_\mu \partial_\nu + \frac{n - 2}{2n} \delta^{\mu\nu} (\partial_\mu \gamma \partial_\nu + \gamma \partial_\mu \partial_\nu) - \xi \hat{R} . \] (23)

In the last equation \( \hat{R} \) is the first-order correction to the intrinsic scalar curvature, and the explicit form of the linearized metric, obtained in the previous section, was taken into account.

As it was shown in the previous section, (see (18)), \( \hat{R} \) is equal to zero outside the world sheet of a string, where it has a \( \delta \)-like singularity. Eq.(19) is ill-defined for such a potential. The possible way to avoid this problem is to smooth out the singularity [7, 8, 9]. As the result, one finds that when (at the end of calculations) the regularization of the singularity is removed, the dependence of the Green’s functions on \( \xi \) vanishes. So, this term can be ruled out of \( V \).

Now using equations (22), (23) and the explicit expression for the function \( G^E_0 \), we find that the first-order correction to the Euclidean Green’s function \( G^E_1 = G^E_0 V G^E_0 \) reads
\[ G^E_1 = G_{\text{Eins}} + G_{\text{bulk}} , \] (24)
where

\[ G_{\text{Eins}} = -8\pi G_{n+1} k \mu (n - 2) \int \frac{d^2 \vec{q}}{(2\pi)^2} \frac{e^{i\vec{q}\vec{x}}}{q^2} \int \frac{d^n p}{(2\pi)^n} \frac{e^{-ip(x-x')}}{p^2(q-p)^2} \sum_{i=3}^n p_i^2 \] (25)

coincides with the the expression obtained for the first-order corrections to the Green’s function in the \( n \)-dimensional locally flat conical space [4], while

\[ G_{\text{bulk}} = 8\pi G_{n+1} \mu \int \frac{d^2 \vec{q}}{(2\pi)^2} e^{i\vec{q}\vec{x}} K_{n/2-2} (q/k) \int \frac{d^n p}{(2\pi)^n} e^{-ip(x-x')} W_{\mu\nu}(\vec{q}) p^\mu p^\nu \] (26)

is the contribution from the bulk.
IV. CALCULATION OF THE VACUUM EXPECTATION VALUE $\langle T^\mu_\nu \rangle_{\text{ren vac}}$

To calculate the vacuum expectation value $\langle T^\mu_\nu \rangle_{\text{vac}}$, let us start from the formal expression

$$\langle T^\mu_\nu \rangle_{\text{vac}} = \lim_{x' \to x} D^\nu_\mu(x, x') G^E(x, x'),$$

(27)

where $D^\nu_\mu$ is a differential operator whose explicit form is determined by the classical expression for the stress-energy tensor. For a massless scalar field with arbitrary coupling and in the lowest order with respect to $\mu$, we have

$$D^\nu_\mu = (1 - 2\xi) \nabla^\nu_\mu - 2\xi \nabla^\nu_\mu + \left(2\xi - \frac{1}{2}\right) \delta^\nu_\mu \nabla^\lambda.$$

(28)

Substituting the expression for $G^E$ into (27), we obtain, that in our approximation the vacuum expectation value of the energy-momentum tensor consists of two parts

$$\langle T^\mu_\nu \rangle_{\text{vac}} = \langle T^\mu_\nu \rangle_{\text{Einst}} + \langle T^\mu_\nu \rangle_{\text{bulk}},$$

where

$$\langle T^\mu_\nu \rangle_{\text{Einst}} = -8\pi G_{n+1}(n - 2)k\mu \int \frac{d^2q}{(2\pi)^2} \frac{e^{i\vec{q}\vec{x}}}{q^2} \int \frac{d^n p}{(2\pi)^n} \frac{D^\mu_\nu(p, q)}{p^2(p - q)^2} \sum_{i=3}^n p_i^2,$$

(29)

$$D^\mu_\nu(p, q) = p_\mu p_\nu - (1 + 2\xi)q_\mu p_\nu + 2\xi q_\mu q_\nu - \left(\xi - \frac{1}{4}\right) q^2 \delta^\nu_\mu,$$

(30)

and

$$\langle T^\mu_\nu \rangle_{\text{bulk}} = 8\pi G_{n+1}k\mu \int \frac{d^2q}{(2\pi)^2} e^{i\vec{q}\vec{x}} \frac{K_{n/2-2}(q/k)}{q K_{n/2-1}(q/k)} \int \frac{d^n p}{(2\pi)^n} \frac{D^\mu_\nu(p, q)}{p^2(p - q)^2} W_\alpha^\beta p^\alpha p^\beta.$$

(31)

Subsequent calculations can be performed using the method of dimensional regularization along the same line as in the case of the four-dimensional Einstein’s theory. One must perform the replacement $n \to D = n - 2\xi$ and multiply the regularized vacuum averaged stress-energy tensor by $(\lambda)^{n-D}$ ($\lambda$ is an arbitrary parameter with the dimension of mass) to restore the dimension of $\langle T^\mu_\nu \rangle_{\text{vac}}$. After the regularization all the integrations in the Eq. (31) can be performed explicitly. Corresponding calculations can be found in the paper \cite{6}. Using the results of this paper we can write that for the case of 3-brane ($n = 4$) the zero-zero component reads

$$\langle T^0_0 \rangle_{\text{Einst}} = \frac{G_{4\mu}}{90\pi^2 r^4} + \left(\xi - \frac{1}{6}\right) \frac{4G_{4\mu}}{\pi^2 r^4}.$$  

(32)

Regularized contribution from the bulk has the form

$$\langle T^0_0 \rangle_{\text{bulk}} = \frac{8\pi G_{n+1}k\mu}{4(D+1)(D/2-1)} \Gamma(2 - D/2) \frac{\Gamma^2(D/2)}{\Gamma(D)} \times$$

$$\times \frac{1}{(4\pi)^{D/2}} \int \frac{d^2q}{(2\pi)^2} e^{i\vec{q}\vec{x}} \frac{K_{D/2-2}(q/k)}{q K_{D/2-1}(q/k)} \frac{q^n}{\lambda} \frac{W_\alpha^\beta}{r^D}.$$  

(33)
It is well known [10], that in the smooth region of a spacetime

\[ \langle T^\mu_\nu \rangle_{\text{div}} = -\frac{1}{(4\pi)^{D/2}} \left[ \frac{1}{D-4} + \frac{1}{2} \left\{ C + \ln \left( \frac{\lambda_0^2}{\lambda^2} \right) \right\} \right] \times \]

\[ \times \left( \frac{1}{90} H^\mu_\nu - \frac{1}{90} (2)^H^\mu_\nu + \left( \frac{1}{6} - \xi \right)^2 (1)^H^\mu_\nu \right) . \] (34)

In the last expression \( \lambda_0 \) is the infrared cutoff which must be introduced in the case of massless fields, and for our metric been taken in the linear approximation we find

\[ H^\mu_\nu = -4n^0 \hat{R}^\mu_\nu + 2 \hat{R}^\mu_\nu \] (35)

\[ (1)^H^\mu_\nu = 2 \hat{R}^\mu_\nu - 2 \delta^\mu_\nu \Delta^0_0 \hat{R} \] (36)

\[ (2)^H^\mu_\nu = \hat{R}^\mu_\nu - \frac{1}{2} \delta^\mu_\nu \Delta^0_0 \hat{R} - \Delta^0_0 \hat{R}^\mu_\nu . \] (37)

As we know, outside the core of the string \( \hat{R} = 0 \) and \( \hat{R}^\mu_\nu = -\Delta^0_0 \gamma^1_{\mu\mu} / 2 \). Combining all the results above, for the case of 3-brane we get

\[ \langle T^\mu_\nu \rangle^\text{ren}_{\text{bulk}} = -\frac{1}{(4\pi)^2} \frac{G_4 \mu}{90} \int \frac{d^2 \vec{q}}{(2\pi)^2} \frac{e^{i\vec{q} \vec{x}} K_0(q/k)}{q k K_1(q/k)} q^4 \ln \left( \frac{q}{\lambda} \right) \times \]

\[ \times \left[ 3 \sum_{i=1}^2 \delta^\mu_i \delta^\nu_i - \delta^\mu_\nu - 2 \frac{q^\mu q^\nu}{q^2} \right] . \] (38)

From Eq (38) we can obtain that at large distances, when \( r \gg k^{-1} \), the contribution from the bulk is negligible compared to (32).

\[ \langle T^0_0 \rangle^\text{ren}_{\text{bulk}} = \frac{1}{(4\pi)^2} \frac{64 \mu}{45} \frac{1}{k^2 r^6} \left[ \ln 2 + 3 - C - 2 \ln(r \lambda) \right] , \] (39)

while near the core of the string, at \( r \ll k^{-1} \),

\[ \langle T^0_0 \rangle^\text{ren}_{\text{bulk}} = \frac{1}{(4\pi)^2} \frac{G_5 \mu}{5} \frac{1}{r^5} \left[ - \ln 2 + \frac{8}{3} - C - \ln(r \lambda) \right] . \] (40)

We see, that contrary to the case of the Einstein’s gravity the expression for the total vacuum averaged \( \langle T^\mu_\nu \rangle^\text{ren}_{\text{vac}} \) contains the \( \lambda \)-dependent term which appears due to the renormalization procedure.

V. CONCLUSIONS

In this paper we consider the vacuum polarization effects outside a straight cosmic string in a RS2 brane world. It is shown, that in the lowest order of the perturbation theory \( \langle T^\mu_\nu \rangle^\text{ren}_{\text{vac}} \) consists of two parts. The first term depends on the angular deficit only and reproduces the result obtained previously in the Einstein’s gravity. This contribution is uniquely defined because of the local flatness of the cosmic string spacetime in four dimensions. The second contribution depends on the scale \( k \) and arises due to the possibility of gravity to propagate
in the five-dimensional bulk. Relative contribution of this term tends to zero as $kr \to \infty$, but it becomes significant at short distances. Another interesting feature is the appearance of an arbitrary mass scale $\lambda$ in this part of $\langle T_{\mu\nu} \rangle_{\text{ren vac}}$. From the first glance the loss of uniqueness must lead to some problems because the ordinary approach to the back reaction problem consists of solving the semiclassical Einstein’s equations with $\langle T_{\mu\nu} \rangle_{\text{ren vac}}$ in its right-hand side. Analogous question was considered in the case of a global monopole in the four-dimensional Einstein’s theory [11], and as in the case of the global monopole, it can be shown that any variation of $\lambda$ may be absorbed by the coefficients before the fourth-order terms appearing in the left-hand side of the one-loop Einstein’s equations.

VI. ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research, grant 99-02-16132.

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