Heavy-to-light transition form factors and their relations in light-cone QCD sum rules

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Abstract

The improved light-cone QCD sum rules by using chiral current correlator is systematically reviewed and applied to the calculation of all the heavy-to-light form factors, including all the semileptonic and penguin ones. By choosing suitable chiral currents, the light-cone sum rules for all the form factors are greatly simplified and depend mainly on one leading twist distribution amplitude of the light meson. As a result, relations between these form factors arise naturally. At the considered accuracy these relations reproduce the results obtained in the literature. Moreover, since the explicit dependence on the leading twist distribution amplitudes is preserved, these relations may be more useful to simulate the experimental data and extract the information on the distribution amplitude.

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I. INTRODUCTION

Light-cone QCD sum rules have played an important role in the study of heavy-to-light transitions. One of the main uncertainties of this approach is due to the poorly known higher twist distribution amplitudes of the final meson. To eliminate this uncertainty, Refs. [1, 2] suggest to start from a correlator composed of chiral currents in studying the form factor $f_+(q^2)$ of the $B \to \pi l \nu$ process. When chiral currents are used, the opposite-parity state contributions add to the spectral density and make it more smooth and more like the perturbative one. As a result, the contributions from the twist-3 distribution amplitudes, $\phi_p(u)$ and $\phi_\sigma(u)$, disappear from the resulting light-cone sum rule. Moreover, it was found in Ref. [2] that all the twist-3 contributions, including that from the three-particle Fock state, do not appear in this improved sum rule. Thus up to twist-3 accuracy the sum rule for the form factor $f_+(q^2)$ depends only on the leading twist distribution amplitude, $\phi_\pi(u)$, so the result should be more stable [2]. Generally, one can prove that if chiral current is introduced, only distribution amplitudes of the same chirality are maintained. For the pseudoscalar meson, this immediately leads to the vanishing of all the twist-3 terms, since all of them are of the opposite chirality with the leading one. As a result, this improved sum rule should be more suitable to determine the moments of $\phi_\pi(u)$ from the experimental results for $f_+(q^2)$ [3], compared to ordinary sum rule [4]. This improved sum rule was further employed to study the semileptonic $B_s \to Kl\nu$ [5], $B \to \eta l\nu$ [6], and the $B(B_c) \to D l\nu$ [7, 8] decays. This method can be directly generalized to the calculation of the form factor $f_-$ and the $B \to P$ penguin form factor $f_T$, leading to similar sum rules as $f_+$. Since only the same distribution amplitude is involved, simple relations between them can be easily found.

Except for the weak transitions of $B(B_s)$ into a pseudoscalar meson, chiral currents were also utilized to calculate the radiative form factor $T_1$ of the $B \to V\gamma$ process [9, 10], where $V$ denotes a light vector meson. In this case, at the leading twist accuracy four distributions will contribute. The dominant contribution comes from the leading twist distribution amplitude of the transversely polarized vector meson, $\phi_\perp(u)$, which is chiral-odd, while the others are all chiral-even. As a result, when suitable chiral current is introduced, we are left with a simplified sum rule depending only on $\phi_\perp(u)$ at the leading twist accuracy. The numerical results for the form factor and the corresponding branching ratio depend crucially on the detail form of this distribution amplitude [10]. Combining with recent
experiment result of $B(B \to (\rho, \omega) + \gamma)$ [11], this sum rule can help to determine the properties corresponding distribution amplitude. Generalization to the other penguin form factors and the semileptonic form factors for the $B \to V$ transitions are also straightforward. Since all these sum rules depend mainly on one distribution amplitude, simple relations arise naturally between them.

Literally, the relations for the heavy-to-light form factors have been studied first by Stech [12] and Soares [13] in the spectator quark model, and more explicitly by Charles et. al [14], using light cone sum rules in the limit of heavy quark mass for the initial hadron and large energy for the final one. For the $B \to P$ transitions, our results coincide with the relations obtained in Ref. [14], while in the $B \to V$ case our relations are nearly the same as the leading power part of them. This is because in the later case only the leading twist contributions have been considered in our approach. Meanwhile, in our results the full dependence on the distribution amplitudes has been maintained. Thus the relations in our approach seems to be more general.

This paper is organized as follows. In the next section we review the approach of using chiral currents in the light-cone QCD sum rules, ad extend our previous results for $B \to Pl\nu$ and $B \to V\gamma$ processes to all the heavy-to-light form factors. A comparison of the relations between these form factors with other approaches is given in Sec.III. The last section is reserved for conclusion and discussion.

II. THE LIGHT-CONE QCD SUM RULES WITH CHIRAL CURRENTS

A. The $B \to P$ transition form factors

Chiral current was first introduced into light-cone sum rule in Ref. [1], in which the form factor $f_+(q^2)$ for $B \to \pi l\nu$ at zero momentum was calculated. The chiral current was also applied to the calculation of the B-meson decay constant $f_B$ in ordinary QCD sum rule, leading to a suppression of power corrections. A more explicit calculation of $f_+(q^2)$ for $B \to \pi l\nu$ up to twist-4 terms in this approach, was given in Ref. [2]. Let us first review their strategy.

The form factors $f_+(q^2)$ and $f_-(q^2)$ for the semileptonic $B \to Pl\nu$ transition are usually
defined as:

\[
\langle P(p) | \bar{q}(p) \gamma_{\mu} b(B(p + q)) \rangle = 2 f_+(q^2) p_\mu + (f_+(q^2) + f_-(q^2)) q_\mu
\]  (1)

To obtain the relevant sum rules, one starts from the following correlation function:

\[
F_\mu(p, q) = i \int d^4 x e^{iqx} < P(p) | T \{ \bar{q}(x) \gamma_{\mu} b(x), \bar{b}(0)i\gamma_5 u(0) \} | 0 >
\]

\[
= F(p^2, (p + q)^2) p_\mu + \tilde{F}(p^2, (p + q)^2) q_\mu.
\]  (2)

The hadronic representation of this correlation function can be obtained by inserting a complete set of states including the \( B \)-meson ground state, higher resonances and non-resonant states with B-meson quantum numbers:

\[
F_\mu(p, q) = \frac{< P | \bar{q}_2 \gamma_{\mu} b | B > < B | \bar{b}i\gamma_5 q_1 | 0 >}{m_B^2 - (p + q)^2}
\]

\[+ \sum_h \frac{< P | \bar{s} \gamma_{\mu} b | h > < h | \bar{b}i\gamma_5 u | 0 >}{m_h^2 - (p + q)^2}
\]

\[= F(q^2, (p + q)^2) p_\mu + \tilde{F}(q^2, (p + q)^2) q_\mu.
\]  (3)

Replacing the infinite sum by a general dispersion relation in the momentum squared \((p+q)^2\) of the \( B \)-meson, one obtains:

\[
F(q^2, (p + q)^2) = \int_{m_B^2}^{\infty} \frac{\rho(q^2, s) ds}{s - (p + q)^2}
\]  (4)

where possible subtractions are neglected and the spectral density is given by

\[
\rho(q^2, s) = \delta(s - m_B^2)2f_+(q^2) \frac{m_B^2 f_B}{m_b} + \rho^h(q^2, s).
\]  (5)

\(\rho^h(p^2, s)\) denotes the spectral density of higher resonances and of the continuum of states and can be replaced by

\[
\rho^h(q^2, s) = \frac{1}{\pi} Im F_{QCD}(q^2, s) \Theta(s - s_0)
\]  (6)

invoking quark-hadron duality. Here \( s_0 \) is the threshold parameter, and \( Im F_{QCD}(p^2, s) \) is obtained from the imaginary part of the correlation function (2) calculated in QCD. This can be achieved by expanding the \( T \)- product of the current in (2) in the region of large space-like momenta \((p+q)^2 \ll 0\). The leading contribution arises from the contraction of the \( b \)-quark operators to the free \( b \)-quark propagator \(< 0 | \bar{b}b | 0 >\) and involves the following distribution amplitudes:

\[
< P(p) | \bar{q}_2(x) \gamma_{\mu} \gamma_5 q_1(0) | 0 > = -ip_\mu f_P \int_0^1 du e^{iupx} \phi(u)
\]  (7)
< P(p) | \bar{q}_2(x)i\gamma_5q_1(0) | 0 > = \frac{f_P m_P^2}{m_1 + m_2} \int_0^1 du e^{iu p x} \varphi_p(u) \tag{8}

< P(p) | \bar{u}(x)\sigma_{\mu\nu}\gamma_5 u(0) | 0 > = i(p_{\mu} x_{\nu} - p_{\nu} x_{\mu}) \frac{f_P m_P^2}{6(m_1 + m_2)} \int_0^1 du e^{iu p x} \varphi_\sigma(u) \tag{9}

where $m_1(m_2)$ is the current quark mass of $q_1(\bar{q}_2)$. For the invariant amplitude $F$ the QCD representation reads:

$$F_{QCD}(p^2, (p + q)^2) = -f_P m_b \int_0^1 du \frac{\varphi(u)}{(q + up)^2 - m_b^2} - \frac{f_P m_P^2}{m_1 + m_2} \left[ \int_0^1 du \left[ \frac{\varphi_p(u) u}{(q + up)^2 - m_b^2} + \frac{\varphi_\sigma(u)}{6((q + up)^2 - m_b^2)} \left( 2 - \frac{p^2 + m_b^2}{(q + up)^2 - m_b^2} \right) \right] \right].$$

Equating the Borel transformation of Eq. (4) and (10) we get the sum rule for the form factor $f_+(q^2)$:

$$f_+(q^2) = \frac{f_P m_b^2}{2f_B m_B^2} \int_0^1 \frac{du}{u} \frac{1}{\Delta_P} \exp \left[ \frac{m_B^2}{M_B^2} - \frac{m_b^2}{u M_B^2} - \frac{\bar{u}(q^2 - u m_P^2)}{u M_B^2} \right] \left[ \varphi(u) + \frac{\mu_P}{m_b} u \varphi_p(u) + \frac{\mu_P}{6m_b} \varphi_\sigma(u) \left( 2 + \frac{m_b^2 + q^2}{u M_B^2} \right) \right].$$

where $\mu_P = m_P^2 / (m_1 + m_2)$ and $\Delta_P$ is the solution to the equation $u s_0^B - m_b^2 - u \bar{u} m_P^2 = 0$ for $u \in [0, 1]$.

In the above sum rule, the distribution amplitude $\varphi(u)$ is of twist-2, $\varphi_p(u)$ and $\varphi_\sigma(u)$ are of twist-3. There is also a twist-3 term from the following three-particle operator:

$$< P(p) | \bar{u}(x)G_{\mu\nu}(z)\sigma_{\rho\lambda}\gamma_5 u(0) | 0 > = if_{3p}[p_{\mu}(p_{\rho}g_{\lambda\nu} - p_{\lambda}g_{\rho\nu}) - q_{\nu}(q_{\rho}g_{\lambda\mu} - q_{\lambda}g_{\rho\mu})] \int \mathcal{D} \alpha \psi_3 K(\alpha) e^{i q (z \alpha_1 + z \alpha_2)}$$

where $G_{\mu\nu}(z) = (\lambda^c / 2) G_{\mu\nu}^c(z)$, $\mathcal{D} \alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(\alpha_1 + \alpha_2 + \alpha_3 - 1)$, $\lambda^c$ and $G_{\mu\nu}^c$ being the usual color matrices and the gluon field tensor. This term comes from the b-quark propagator including the interaction with gluons in first order:

$$< 0 | T\{b(x)\bar{b}(0)\} | 0 > = < 0 | T\{b(x)\bar{b}(0)\} | 0 > = -ig_s \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \int_0^1 du \left[ \frac{1}{2 (m_b^2 - k^2)} G_{\mu\nu}(ux)\sigma^{\mu\nu} + \frac{1}{m_b^2 - k^2} ux - \mu G_{\mu\nu}(ux) \gamma_\nu \right]$$
Substituting this propagator into the original correlation function and repeating the above process, we get the corresponding corrections to the form factor $f_+(q^2)$

$$f_{+G}(q^2) = \frac{f_{3\rho}m_b}{f_Bm_B^2} \int_0^1 \frac{du}{u} \int \mathcal{D}\alpha_i \Theta(\alpha_1 + u\alpha_3 - \Delta) \exp\left[\frac{m_B^2}{M_B^2}\right] \exp[-\frac{m_b^2 - q^2(1 - \alpha_1 - u\alpha_3)}{(\alpha_1 + u\alpha_3)M_B^2}] \frac{\varphi_{3\pi}(\alpha_i)}{(\alpha_1 + u\alpha_3)^2}.$$  \hfill (14)

Eq. (11) and (14) give the complete sum rule for $f_+(q^2)$ at the accuracy of twist-3. However, the twist-3 distribution amplitudes are poorly known at present, which introduce large uncertainties. To eliminate the twist-3 contributions, we can start from the following correlation function with the $B$-meson interpolating field $\bar{b}i\gamma_5 q$ replaced by the chiral current $\bar{b}i(1 + \gamma_5)q$

$$\Pi_\mu(p, q) = i \int d^4xe^{iqx} \langle P(p) | T\{ \gamma_\mu(1 + \gamma_5)b(x), \bar{b}(0)i(1 + \gamma_5)q_1(0) \} | 0 \rangle = \Pi(q^2, (p + q)^2)p_\mu + \tilde{\Pi}(q^2, (p + q)^2)q_\mu. \quad (15)$$

Now the scalar resonances corresponding to operator $\bar{b}q$, which is of opposite parity to the $B$-meson, also add to the spectral density. As a reflection of this fact, the QCD representation of this correlation function contains only one single distribution amplitude, at the accuracy of twist-3. In other words, all the twist-3 contributions for this correlator disappear automatically. More generally, one can prove that if chiral current is introduced in the correlator, only the distribution amplitudes of the same chirality remain in the final sum rule. In the pseudoscalar case, one can see that all the twist-3 distributions are of opposite chirality with the leading twist one, thus disappear automatically. So up to twist-3 accuracy, we obtain the sum rule depending on $\varphi(u)$ only:

$$f_+(q^2) = \frac{f_{3\rho}m_b^2}{f_Bm_B^2} \int_\Delta \frac{du}{u} \exp\left[\frac{m_B^2 - m_b^2 - \bar{u}(q^2 - um_{\pi^0})}{uM_B^2}\right]\varphi(u) \quad (16)$$

The sum rule for $f_-(q^2)$ can be obtained in the same way. Actually, the QCD calculation of the corresponding correlation function $\tilde{\Pi}(q^2, (p + q)^2)$ vanished at the twist-3 accuracy, leading to the following relation

$$f_-(q^2) = -f_+(q^2). \quad (17)$$

This method can be directly generalized to the calculation of the penguin form factor $f_T(q^2)$, which is defined as:

$$\langle P(p) | \bar{q}\gamma_\mu q^\nu(1 + \gamma_5)b | B(p + q) \rangle = i\frac{f_T(q^2)}{m_B + m_P} \left[(2p + q)_\mu q^2 - q_\mu(m_B^2 - m_P^2)\right] \quad (18)$$
Starting from the standard correlation function

\[ \Pi_\mu(p, q) = i \int d^4x \, e^{ipx} \langle P(p)|T\{\bar{q}_2(x)i\sigma_{\mu\nu}q^\nu(1 + \gamma_5)b(x)\bar{b}(0)i\gamma_5q_1(0)}|0 \rangle, \tag{19} \]

the corresponding sum rule has been derived in Ref. [16]:

\[ f_T(q^2) = \frac{m_b(m_B + m_P)f_P}{2f_Bm_B^2} \int_{\Delta_P}^1 \frac{du}{u} \exp\left[\frac{m_B^2}{M_B^2} - \frac{m_b^2 - \bar{u}(q^2 - um_B^2)}{uM_B^2}\right] \varphi(u) + \frac{\mu_p^2m_b}{3uM_B^2}\varphi(u) \tag{20} \]

where the twist-4 terms has been omitted. As in the semileptonic case, we simply replace the interpolating field \( \bar{b}\gamma_5q_1 \) by the left handed current \( \bar{b}(0)i(1 - \gamma_5)q(0) \). Thus we start from the following correlation function:

\[ \Pi_\mu(p, q) = i \int d^4x \, e^{ipx} \langle P(p)|T\{\bar{q}_2(x)i\sigma_{\mu\nu}q^\nu(1 + \gamma_5)b(x)\bar{b}(0)i(1 - \gamma_5)q_1(0)}|0 \rangle. \tag{21} \]

Repeating the procedure as in the previous case, the sum rule can be obtained immediately:

\[ f_T(q^2) = \frac{m_b(m_B + m_P)f_P}{f_Bm_B^2} \int_{\Delta_P}^1 \frac{du}{u} \exp\left[\frac{m_B^2}{M_B^2} - \frac{m_b^2 - \bar{u}(q^2 - um_B^2)}{uM_B^2}\right] \varphi(u) \tag{22} \]

Comparing with the sum rule for \( f_+ \), the following relation can be easily found:

\[ f_T(q^2) = \frac{m_B + m_P}{m_b} f_+(q^2) \tag{23} \]

These relations between \( f_+ \), \( f_- \), and \( f_T \) have been confirmed by the numerical results in the light-cone QCD sum rules [17]. However, to make these relations manifest in the ordinary light-cone sum rules, one needs to take certain limits, as we will see in the next section.

**B. The \( B \to V \) transition form factors**

In this subsection we will attempt to generalize the idea of chiral current to the \( B \to V \) transitions, where \( V \) denotes a light vector meson. At leading-twist accuracy, one will encounter the following distributions: [18]:

\[ \langle 0|\bar{\psi}_2(z)\gamma_\mu\psi_1(-z)|V(P, \lambda)\rangle = f_V m_V \left[ p_\mu e^{(\lambda) \cdot z} \int_0^1 du e^{i\xi p \cdot z} \phi_{||}(u) \right.
\]

\[ + \left. e^{(\lambda) \cdot z} \int_0^1 du e^{i\xi p \cdot z} \phi_{\perp}(u) \right] \tag{24} \]
\[
\langle 0 | \psi_2 (z) \gamma_\mu \gamma_5 \psi_1 (-z) | V (P, \lambda) \rangle = \frac{1}{2} \left( f_V - f_V^T \frac{m_1 + m_2}{m_V} \right) m_V \epsilon_\nu \gamma_\lambda \epsilon_\mu \gamma_5 \psi_1 (-z) V (P, \lambda) \int_0^1 du \, e^{i \xi p^2} g_\perp (u). \tag{25}
\]

\[
\langle 0 | \bar{\psi}_2 (z) \sigma_{\mu \nu} \psi_1 (-z) | V (P, \lambda) \rangle = i \int \frac{d^4 x}{(2\pi)^4} e^{i q x} < V (p, \lambda) | T \bar{\psi}_2 (x) \sigma_{\mu \nu} \gamma_5 q^\nu b (x), \bar{b} (0) i \gamma_5 \psi_1 (0) | 0 > = \frac{i \epsilon_{\mu \nu \rho \sigma} (\lambda)}{p_{B B}^2} T ((p + q)^2) \tag{26}
\]

where \( \xi = 2u - 1, p_\mu = P_\mu - \frac{1}{2} z_\mu \frac{m_B^2}{m_V}. \) The function \( \phi_\parallel (u) \) and \( \phi_\perp (u) \) give the leading twist distributions in the fraction of total momentum carried by the quark in transversely and longitudinally polarized mesons, respectively. The functions \( g_\perp (\epsilon) \) and \( g_\parallel (\epsilon) \) are always identified to be twist-3 from power counting, but in fact they contain contributions of both operators of twist-2 and twist-3 \[19\]. Notice that \( \phi_\perp (u) \) is chiral-odd, while the other three are all chiral-even. Therefore, by suitably choosing the chiral current one may also obtain simplified sum rules at the leading-twist accuracy. Let’s demonstrate this procedure by reviewing the calculation of the penguin form factor first. The relevant form factors are defined as following:

\[
\langle V (p, \lambda) | \bar{\psi}_2 (x) \sigma_{\mu \nu} q^\nu (1 + \gamma_5) b (x) | B (p_B) \rangle = i \epsilon_{\mu \nu \rho \sigma} \epsilon_\lambda \rho \sigma \frac{p_{B B}^2}{p_{B B}^2} 2 T_1 (q^2)
\]

\[
+ T_2 (q^2) \left\{ \epsilon_\lambda (m_B^2 - m_V^2) - (\epsilon_\lambda) (p_B (p_B + p)_\mu) \right\}
\]

\[
+ T_3 (q^2) (\epsilon_\lambda) (p_B) \left\{ p_\mu - \frac{q^2}{m_B^2 - m_V^2} (p_B + p)_\mu, \right\}
\]

where

\[
T_1 (0) = T_2 (0). \tag{28}
\]

The decay width for the \( B \rightarrow V \gamma \) process is mainly determined by \( T_1 (0) \), so we can just focus on \( T_1 (q^2) \) only:

\[
\langle V (p, \lambda) | \bar{\psi}_2 (x) \sigma_{\mu \nu} q^\nu \gamma_5 b (x) | B (p_B) \rangle = i \epsilon_{\mu \nu \rho \sigma} \epsilon_\lambda \rho \sigma \frac{p_{B B}^2}{p_{B B}^2} 2 T_1 (q^2). \tag{29}
\]

To derive the light-cone sum rule for \( T_1 (q^2) \), usually one choose the following correlation function based on Eq. \([61]\):

\[
T_\mu (p, q) = i \int d^4 \xi e^{i \xi x} < V (p, \lambda) | T \bar{\psi}_2 (x) \sigma_{\mu \nu} q^\nu b (x), \bar{b} (0) i \gamma_5 \psi_1 (0) | 0 > = \frac{i \epsilon_{\mu \nu \rho \sigma} \epsilon_\lambda \rho \sigma}{p_{B B}^2} T ((p + q)^2) \tag{30}
\]
A standard procedure leads to the following sum rule \[20\]:

\[
\frac{f_B m_B^2}{m_b + m_q} 2T_1(0)e^{-(m_B^2 - m_q^2)/M_B^2} = \\
= \int_0^1 du \frac{1}{u} \exp \left[ -\frac{\bar{u}}{M_B^2} \left( \frac{m_B^2}{u} + m_V^2 \right) \right] \theta \left[ s_0 - \frac{m_B^2}{u} - \bar{u} m_V^2 \right] \left\{ \frac{m_B^2}{u} \phi_{\perp}(u, \mu) \right\} + \frac{u m_V f_V g_\perp^{(v)}(u, \mu) + \frac{m_B^2 - u^2 m_V^2 + u M_B^2}{4 u M_B^2} m_V f_V g_\perp^{(a)}(u, \mu) }{4 u M_B^2} .
\]

Now we try to simplify the sum rule by introducing suitable chiral current \[9\]. First, notice that Eq. \((27)\) can be simplified when \(q^2 = 0\):

\[
< V(p, \lambda) | \bar{\psi}\sigma_{\mu\nu}(1 + \gamma_5)q^\nu b | B(p + q) > \\
= 2 \{ -ie_{\mu\nu\alpha\beta} e^{(\lambda)} q_a p^\beta + p \cdot q e_\mu^{(\lambda)} - q \cdot e^{(\lambda)} p_\mu \} T_1(0) + q \cdot e^{(\lambda)} q_\mu [T_3(0) - T_1(0)] .
\]

Starting from this definition, one can construct the following correlator by choosing the right-handed current \(\bar{b}i(1 + \gamma_5)q_1\) for the \(B\) meson:

\[
F_\mu(p, q) = i \int d^4 x e^{iqx} < V(p, \lambda) | \bar{T} \psi_2(x) \sigma_{\mu\nu}(1 + \gamma_5)q^\nu \bar{b}(x), \bar{b}(0)i(1 + \gamma_5)\psi_1(0)|0 > \\
= 2 \{ -ie_{\mu\nu\alpha\beta} e^{(\lambda)} q_a p^\beta + 2 p \cdot q e_\mu^{(\lambda)} - 2 q \cdot e^{(\lambda)} p_\mu \} F \left[ (p + q)^2 \right] + ...
\]

Then the following simplified sum rule can be obtained:

\[
T_1(0) = \frac{m_B^2}{m_B^2} \frac{f_B^2}{f_V^2} \phi_{\perp}(u) \int_{\Delta^2}^1 du \frac{\phi_{\perp}(u)}{u} \exp \left[ -\frac{m_B^2 + \bar{u} m_V^2}{u M_B^2} \right] ,
\]

where \(\Delta^2\) is the solution to the equation \(us_0 - m_B^2 - \bar{u}m_V^2 = 0\) for \(u \in [0, 1]\). So at the leading-twist accuracy we obtained a sum rule depending on the distribution \(\phi_{\perp}\) only, similar to the \(B \rightarrow P\) case. As a result, the final numerical results for \(T_1(0)\) and the branching ratio depend crucially on \(\phi_{\perp}\). This fact can be utilized to determine the properties of \(\phi_{\perp}\) from the experimental results of the corresponding decay process.

The sum rule for the form factor \(T_1\) for finite value of \(q^2\) can be obtained from Eq. \((34)\) by trivial modifications. The result is given as follows:

\[
T_1(q^2) = \frac{f_B m_B^2}{f_V^2} e^{-m_B^2/M_B^2} \\
\times \int_{\Delta^2}^1 du \frac{1}{u} \exp \left[ -\frac{m_B^2 + \bar{u} q^2 - u m_V^2}{u M_B^2} \right] \phi_{\perp}(u) ,
\]

\[35\]
where

$$\Delta \nu = \left[ \sqrt{(s_0^B - q^2 - m_{\nu}^2)^2 + 4m_{\nu}^2(m_0^B - q^2) - (s_0^B - q^2 - m_{\nu}^2)} \right] / (2m_{\nu}^2),$$

is the solution to $u_0^B = m_0^B - u\tilde{m}^B + \tilde{u}q^2 = 0$ for $u \in [0, 1]$. Generalization to the sum rules for other two form factors $T_2$ and $T_3$ is also straightforward. First one can show that the omitted terms of Eq. (33) are identically zero at the considered accuracy. Further decomposing Eq. (33) in the following form:

$$F_{\mu}(p, q) = i \int d^4xe^{iqx} < V(p, \lambda)T\psi_2(x)\sigma_{\mu\nu}(1 + \gamma_5)q^\nu b(x), \bar{b}(0)i(1 + \gamma_5)\psi_1(0)|0 >$$

$$= \left[ 2i\epsilon_{\mu\alpha\beta\gamma}e^{(\lambda)\nu}q^\alpha p^\beta \right] F \left[ (p + q)^2 \right]$$

$$+ \left[ e^{(\lambda)}(m_B^2 - m_{\nu}^2) - (e^{(\lambda)}p_B)(p_B + p)_\mu \right] \left( 1 - \frac{q^2}{m_B^2 - m_{\nu}^2} \right) F \left[ (p + q)^2 \right]$$

$$+ \left[ e^{(\lambda)}p_B q_\mu - \frac{q^2}{m_B^2 - m_{\nu}^2} (p_B + p)_\mu \right] F \left[ (p + q)^2 \right]$$

one immediately reads out the relations:

$$T_2(q^2) = \left( 1 - \frac{q^2}{m_B^2 - m_{\nu}^2} \right) T_1(q^2)$$

$$T_3(q^2) = T_1(q^2)$$

(38)

Now we consider the semileptonic decay $B \to VL\nu$. Generalization of the chiral current method to this kind of process has been attempted in Ref. [21], where the interpolating field for the heavy meson was chosen to be the left-handed chiral current. The resulting sum rules contain the chiral-even terms of $\phi_\parallel$, $g_\nu^\perp$ and $g_\perp^a$, but the dominant chiral-odd one, $\phi_\perp$, is eliminated. From the above calculation for the penguin form factors, it can be found that in order to maintain the dominant contribution one should choose the right-handed $\bar{b}(0)i(1 + \gamma_5)q(0)$ instead. Let us specify this procedure more explicitly.

The form factors for the $B \to VL\nu$ process can be defined as:

$$\langle V(p, \lambda)|(V - A)_\mu|B \rangle = -i(m_B + m_{\nu})A_1(q^2)e^{(\lambda)}_\mu + \frac{iA_+(q^2)}{m_B + m_{\rho}}(e^{(\lambda)}p_B)(p_B + p)_\mu$$

$$+ \frac{iA_-(q^2)}{m_B + m_{\nu}}(e^{(\lambda)}p_B)(p_B - p)_\mu + \frac{2V(q^2)}{m_B + m_{\nu}}\epsilon^\alpha_\mu \epsilon^{a\beta\gamma}(e^{(\lambda)}p_Bp_\beta p_\gamma),$$

(39)

where $(V - A)_\mu = \bar{\psi}(z)\gamma_\mu(1 - \gamma_5)b$ is the corresponding weak current, $\lambda$ is the polarization vector of the vector meson, and $q = p_B - p$ is the momentum transfer to the leptons. Replacing the $B$ by the ordinary interpolating field $j_B^a = \bar{b}i\gamma_5\psi_1$, one can consider the
following correlator:

\[
\Pi_{\mu}(p, q) = i \int d^4x e^{iqx} \langle V(p, \lambda)|T(V - A)_{\mu}(z)j_B^\dagger(0)|0\rangle \\
= -i\Pi_{1}((p + q)^2)e^{\lambda} + i\Pi_{+}((p + q)^2)(e^{\lambda}p_B)(p_B + p)_{\mu} \\
+\Pi_{V}((p + q)^2)\epsilon^{\nu\alpha\beta}e^{(\lambda)}_{\nu}p_B\epsilon_{\alpha\beta} + \ldots,
\]

where the term corresponding to the form factor \(A_{-}\) is omitted for simplicity. Repeating the procedure described in the previous section, one obtain the following sum rules \[22\]:

\[
A_{1}(q^2) = \frac{m_b}{f_B(m_B + m_V)m_B^2} \exp\left\{ \frac{m_B^2 - m_b^2}{M_B^2} \right\} \int_0^1 du \exp\left\{ \frac{\bar{u}}{uM_B^2} (q^2 - m_b^2 - um_V^2) \right\} \\
\Theta[c(u, s_0^B)] \left\{ f_{V}^{\dagger}(\mu)\phi_{\perp}(u) \frac{1}{2u} (m_b^2 - q^2 + u^2m_V^2) + f_V m_b m_V g_{\perp}^{(v)}(u) \right\},
\]

\[
A_{+}(q^2) = \frac{m_b(m_B + m_V)}{f_B m_B^2} \exp\left\{ \frac{m_B^2 - m_b^2}{M_B^2} \right\} \int_0^1 du \exp\left\{ \frac{\bar{u}}{uM_B^2} (q^2 - m_b^2 - um_V^2) \right\} \\
\left\{ \frac{1}{2} f_{V}^{\dagger}(\mu)\phi_{\perp}(u)\Theta[c(u, s_0^B)] + f_V m_b m_V \Phi_{\parallel}(u) \frac{1}{uM_B^2} \Theta[c(u, s_0^B)] + \delta[c(u, s_0^B)] \right\},
\]

\[
V(q^2) = \frac{m_b(m_B + m_V)}{2f_B m_B^2} \exp\left\{ \frac{m_B^2 - m_b^2}{M_B^2} \right\} \int_0^1 du \exp\left\{ \frac{\bar{u}}{uM_B^2} (q^2 - m_b^2 - um_V^2) \right\} \\
\left\{ f_{V}^{T}\phi_{\perp}(u)\Theta[c(u, s_0^B)] + \frac{1}{2} f_V m_b m_V g_{\perp}^{(a)}(u) \frac{1}{uM_B^2} \Theta[c(u, s_0^B)] + \delta[c(u, s_0^B)] \right\},
\]

where the definition

\[
\Phi_{\parallel}(u, \mu) = \frac{1}{2} \left[ \bar{u} \int_0^u dv \frac{\phi_{\parallel}(v, \mu)}{v} - u \int_u^1 dv \frac{\phi_{\parallel}(v, \mu)}{v} \right]
\]

has been used, and \(c(u, s_0^B) = us_0^B - m_b^2 + q^2\bar{u} - u\bar{u}m_V^2\).

Now we replace the \(j_B^1 = \bar{b}\gamma_5\psi\) in Eq.(40) by the right-handed current \(j_B^{R1} = \bar{b}(1 + \gamma_5)\psi_1\), and the corresponding correlator becomes:

\[
\Pi_{\mu}(p, q) = -i \int d^4x e^{iqx} \langle V(p, \lambda)|T\{\bar{\psi}_2(x)\gamma_{\mu}(1 - \gamma_5)b(x), \bar{b}_1(0)(1 + \gamma_5)\psi_1(0)\}|0\rangle \\
= \Gamma^1\epsilon^{(\lambda)}_{\mu} - \Gamma^+(e^{(\lambda)}q)(2p + q)q_{\mu} - \Gamma^- (e^{(\lambda)}q)q_{\mu} + i\Gamma V\epsilon_{\alpha\beta\gamma}e^{(\lambda)}\epsilon_{\alpha\beta\gamma}q_{\mu},
\]

(45)
A direct calculation leads to the following simplified sum rules:

\[
A_1(q^2) = \frac{f_V^T m_b}{f_B m_B} e^{m_b^2/M_B^2} \times \int_{\Delta_V}^1 \frac{du}{u} \exp \left[ -\frac{m_b^2 - \bar{u}(q^2 - u m_V^2)}{u M_B^2} \right] \frac{m_B^2 - q^2 + u^2 m_V^2}{u m_B (m_B + m_V)} \phi_\perp(u),
\]

(46)

\[
A_\pm(q^2) = \frac{f_V^T m_b}{f_B m_B} e^{m_b^2/M_B^2} \times \int_{\Delta_V}^1 \frac{du}{u} \exp \left[ -\frac{m_b^2 - \bar{u}(q^2 - u m_V^2)}{u M_B^2} \right] \frac{(m_B + m_V)}{m_B} \phi_\perp(u),
\]

(47)

\[
A_\pm(q^2) = -A_\mp(q^2),
\]

(48)

\[
V(q^2) = A_+(q^2),
\]

(49)

where

\[
\Delta_V = \left[ \sqrt{(s_0^B - q^2 - m_V^2)^2 + 4 m_V^2 (m_b^2 - q^2) - (s_0^B - q^2 - m_b^2)^2} \right]/(2 m_V^2),
\]

(51)

is the solution to \(us_0^B - m_b^2 - \bar{u} m_V^2 + \bar{u} q^2 = 0\) for \(u \in [0, 1]\). Just as the improved sum rules for the penguin form factors, these sum rules contain only the transverse distribution amplitude \(\phi_\perp(u)\), the chiral-even terms involving \(\phi_\parallel(u)\), \(g_1^v\) and \(g_1^a\) are completely eliminated.

In Ref. [7] and Ref. [23] we have attempted to apply these sum rules in the \(B \to D l \nu\) process and the semileptonic decays of the \(B_c\)-meson. The results for some channels, such as the \(B \to D l \nu\) and \(B_c \to D(D^*) l \nu, B_c \to J/\psi(\eta_c) l \nu\), are roughly consistent with other approaches. However, the best test background for these sum rules should be the heavy-to-light transitions, so in the following section we will compare our results with those derived in other approaches, such as those in Ref. [12] and Ref. [14].

III. COMPARISON WITH OTHER APPROACHES

By using a constituent quark model approach and assuming simple properties of the spectator quark, the semileptonic heavy-to-light form factors are shown to be related by a
single universal function \[12\]. In our definitions, these relations read:

\begin{align*}
  f_+(q^2) &= R(q^2, m_F) \\
  f_-(q^2) &= -R(q^2, m_F) \\
  A_1(q^2) &= \frac{2E_F}{m_B + m_F} R(q^2, m_F) \\
  A_+(q^2, m_F) &= \frac{m_B + m_F}{m_B} \frac{E_F - m_F^2}{m_F} R(q^2, m_F) \\
  A_-(q^2, m_F) &= -\frac{m_B + m_F}{m_B} \frac{E_F + m_F^2}{m_F} R(q^2, m_F) \\
  V(q^2, m_F) &= \frac{m_B + m_F}{m_B} R(q^2, m_F).
\end{align*}

where \( E_F = \frac{1}{2m_B} (m_B^2 + m_F^2 - q^2) \) is the energy of the final state and \( m_F \) denotes the mass. Furthermore, by employing the Isgur-Wise relations [24] between the semileptonic and the radiative form factors one can obtain \[12\]

\[ T_1(q^2) = R(q^2, m_F) \] (53)

These relations were further studied in Ref. [13] and [25].

Later, a more rigorous study of the form factor relations was done in Ref. [14]. Based on a light-cone sum rule calculation in the limit of heavy mass for the initial hadron and large energy for the final one, all the form factors are shown to depend on three independent
functions. Again we write the relations in our present definition, which are as follows:

\[ f_\perp(q^2) = \zeta(m_B, E_T) , \]
\[ f_\perp(q^2) = -\zeta(m_B, E_T) , \]
\[ f_T(q^2) = \left( 1 + \frac{m_T}{m_B} \right) \zeta(m_B, E_T) , \]
\[ A_1(q^2) = \frac{2E_T}{M_B + m_Y} \zeta_\perp(m_B, E_Y) , \]
\[ A_\perp(q^2) = \left( 1 + \frac{m_Y}{m_B} \right) \left[ \zeta_\perp(m_B, E_Y) - \frac{m_Y}{E_Y} \zeta_\parallel(m_B, E_Y) \right] , \]
\[ A_\perp(q^2) = -\left( 1 + \frac{m_Y}{m_B} \right) \left[ \zeta_\perp(m_B, E_Y) - \frac{m_Y}{E_Y} \zeta_\parallel(m_B, E_Y) \right] , \]
\[ V(q^2) = \left( 1 + \frac{m_Y}{m_B} \right) \zeta_\perp(m_B, E_Y) , \]
\[ T_1(q^2) = \zeta_\parallel(m_B, E_Y) , \]
\[ T_2(q^2) = \left( 1 - \frac{q^2}{m_B^2 - m_Y^2} \right) \zeta_\perp(m_B, E_Y) , \]
\[ T_3(q^2) = \zeta_\perp(m_B, E_Y) - \frac{m_Y}{E_Y} \left( 1 - \frac{m_B^2}{m_Y^2} \right) \zeta_\parallel(m_B, E_Y) . \]

The three universal form factors \( \zeta(M, E) \), \( \zeta_\parallel(M, E) \) and \( \zeta_\perp(M, E) \) are given by:

\[ \zeta(M, E) = \frac{1}{f_B} \frac{1}{2E^2} \left[ -f_P\phi'(1)I_2(\omega_0, \mu_0) + \frac{f_P m_T^2}{m_Q + m_Q} \phi_p(1)I_1(\omega_0, \mu_0) \right] , \]
\[ \zeta_\parallel(M, E) = \frac{1}{f_B} \frac{1}{2E^2} \left[ -f_V\phi'_\parallel(1)I_2(\omega_0, \mu_0) + f_V m_V h'_\parallel(1)I_1(\omega_0, \mu_0) \right] , \]
\[ \zeta_\perp(M, E) = \frac{1}{f_B} \frac{1}{2E^2} \left[ -f_V\phi'_\perp(1)I_2(\omega_0, \mu_0) + f_V m_V g'_\perp(1)I_1(\omega_0, \mu_0) \right] . \]

with the functions \( I_j(\omega_0, \mu_0) \) defined by:

\[ I_j(\omega_0, \mu_0) = \int_0^{\omega_0} d\omega \omega^j \exp \left[ \frac{2}{\mu_0} (\bar{\Lambda} - \omega) \right] \quad j = 1, 2 \]

Here the parameters \( \bar{\Lambda}, \mu_0 \) and \( \omega_0 \) are related to the ordinary parameters:

\[ \bar{\Lambda} = m_B - m_b \]
\[ M_B^2 = m_b \mu_0 \]
\[ s_0^B = (m_b + \omega_0)^2 \]

These relations are confirmed in the Soft-Collinear Effective Theory [26]. The above relations (54)-(63) are quite similar to those (Eq. (52) and Eq. (53)) obtained by Stech. Actually, if
one impose $\zeta = \zeta_\perp = \zeta_\parallel$, these two set of relation almost coincide except some ambiguity in the sub-leading terms $\sim m'_F/m_B$ or $m'_F/E_F$ \[14\]. Although from the expressions \[64\]-\[66\] one can not find general reasons for this relation to hold, the numerical results for these three form factors may support it, because the decay constant and the leading twist distribution amplitudes of the pseudoscalar, the longitudinally and the transversely polarized vector meson are not quite different. This explains in some sense the consistence of the relations obtained by Stech with the lattice data \[27\].

Now compare our results with those given in Eqs. \[54\]-\[63\]. For the $B \to P$ transitions form factors the relations from the two approaches are exactly the same. As have been mentioned in previous section, these relations were also confirmed by the numerical results in the light-cone sum rule calculation \[17\]. For the $B \to V$ transitions, our leading-twist results are also very similar as the leading power part of Eqs. \[57\]-\[63\]. The only difference is in the extra factor $2E_V/(m_B + m_V)$ for $A_1$. In our result this factor is $u$-dependent and inside the integral over $u$ \[46\]. However, when $E_V$ is taken to be very large, $\Delta_V \to 1$ this factor $\frac{m_B^2 - q^2 + u^2 m_V^2}{u m_B (m_B + m_V)} \sim \frac{2E_V}{m_B + m_V}$ and factors out. Thus at the considered accuracy, our approach by using the chiral currents reproduces naturally the corresponding relations obtained from other approaches, and at the same time preserves the full dependence on the leading twist distribution amplitudes. So our relations can be directly utilized to simulate the experimental data and extract the corresponding information on the distribution amplitudes.

IV. CONCLUSION

The improving approach of using chiral currents in the light-cone QCD sum rules is systematic reviewed and successfully generalized to all heavy-to-light weak transition. The resulting light-cone sum rules for all the semileptonic and penguin form factors depend only on one leading twist distribution amplitude, up to twist-3 accuracy for the $B \to P$ transitions and to leading-twist accuracy in the $B \to V$ case. The other contributions disappear automatically since they have the opposite chirality with the dominant one. Since the poorly-known twist-3 distribution amplitudes are eliminated, these sum rules should be more stable than the ordinary one. A systematic numerical calculation of the heavy-to-light form factors using these sum rules is in process. Moreover, if the form factors is known very well experimentally, one can also utilize these sum rules to study the properties of the
leading twist distribution amplitudes [3].

Since only one leading distribution amplitude is involved, simple relations for all the form factors arise naturally in our approach. At the considered accuracy these relations reproduce the results obtained by using light-cone sum rules in the limit of heavy quark mass for the initial hadron and large energy for the final one, and at the same time preserve the full dependence on the leading twist distribution amplitude. Therefore these relations may be more useful to simulate the experimental data and extract the information of the leading twist distribution amplitudes.

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