Calculation method of multi-span tension stringing in transmission line

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Abstract. The process of multi-span tension stringing in transmission line is analysed based on elastic catenary equation. From the tower condition of transmission line, the conservation equations of span spacing and elevation difference are built for conductor, and the tension balance equations of conductor between different spans are got considering the rolling friction of the pulley. Added the external tension condition, the overall nonlinear equations of multi-span tension stringing are set up which can be solved by Newton iteration method. The iterative process is listed for the calculation of the conductor tension and sags in engineering problems. Compared the results with the numerical example, this method are proved the reliability. The calculation amount, efficiency and accuracy of this method can meet the needs of the practical engineering.

1. Preface
With the development of the Ultra-high voltage transmission line in China, the tension stringing technology has been widely accepted and adopted in the industry. During the whole stringing process of UHV, conductor tension must be kept large enough to raise the conductors off the ground, cross obstacles and facilities. This can avoid conductor damage, ensure the quality of the transmission lines, and minimize the influence for environment. Therefore, the construction technology of tension stringing must be applied in string operation. The selection of construction machinery and equipment is based on the conductor tension and traction, so the tension calculation method for conductor is important for engineering safe [1].

Nowadays, the major way of string construction calculation is the parabolic method [2], but this method has poor precision compared with actual measurement. Other method such as catenary method [3, 4] is more accurate, but the calculation is relatively complex. Therefore, the accurate calculation method of the tension and traction is proposed.

2. Condition of multi-span tension stringing
There are many important conditions for the process of tension stringing construction, such as the span spacing and elevation difference between towers in transmission line, and the material parameters of conductor.

2.1. Main parameter of transmission line
There are \( n \) continuous spans in the transmission line, and the span spacing is \( L_{si} \), elevation difference is \( h_{si}(i=1, \ldots, n) \), shown as figure 1. The unit gravity of conductor is \( q \), the product of cross-sectional area and elasticity modulus is \( EA_0 \).
Figure 1. Schematic diagram of the multi-span tension stringing.

The total length $s_0$ of the conductor can be obtained by measurement before the engineering construction.

Assuming that initial conductor length of each span is $s_i$, horizontal tension is $H_i$, vertical tension on point $A$ is $V_{Ai}$, so the vertical tension on point $B$ can be expressed as

$$V_{Bi} = V_{Ai} + q s_i \quad (i=1, \ldots, n).$$

The calculation formula of span spacing and elevation difference of conductor is [5]:

$$L_i = \frac{H_i}{q} \left[ \frac{q s_i}{EA_0} + \sinh \frac{V_{Ai}}{H_i} - \sinh \frac{V_{Bi}}{H_i} \right] \quad (i=1, \ldots, n).$$

2.2. Tension of conductor

One end of the conductor is pulled by traction engine and the other end by the tension machine, shown as figure 2. The tangential tension on the starting point $A$ and end point $B$ of each span are as follow:

$$T_{Ai} = \sqrt{H_i^2 + V_{Ai}^2} \quad (i=1, \ldots, n).$$

3. Analysis of multi-span tension stringing

The structural characteristics and the force balance of the conductor in the process of multi-span tension stringing are the key points of structural analysis.

3.1. Constant of spacing and elevation difference of each span

The spacing and elevation difference of each span is constant no matter how the length of conductor changes, so the equations about the spacing and elevation difference of conductor can be obtained.
Therefore, there are 2n equations of conductor for spacing and elevation difference of all spans.

3.2. Conductor tension balance besides stringing blocks

The conductor will be influenced by frictional force in tension stringing. The friction is equal to the difference of the tangential tensions of conductor on both sides of the pulley. The tangential tensions of conductor besides of the ith pulley is shown in figure 3.

![Figure 3. The conductor and pulley and the rolling friction.](image)

The friction is associated with the frictional parameter \( \mu \), the envelope angle \( \theta \) and the conductor tensions on both sides of pulley:

\[
F_i = F(\mu, \theta, T_{A_{i+1}}, T_{B_i})
\]  

(5)

3.2.1. The envelope angle. Envelope angle is the angle of contact arc of conductor, shown as figure 4.

\[
\theta = \tan^{-1}\left(\frac{V_{B_i}}{H_i}\right) - \tan^{-1}\left(\frac{V_{A_{i+1}}}{H_{i+1}}\right)
\]  

(6)

![Figure 4. Contact arc of conductor and pulley.](image)

3.2.2. The force of conductor acting on the pulley. Due to small contact part of the arc, the force \( N_p \) acting on pulley can be considered as the sum of both sides vertical force, as shown in figure 4.

\[
N_p = V_{A_{i+1}} - V_{B_i}
\]  

(7)

3.2.3. Direction of rolling friction. When \( T_{B_i} - T_{A_{i+1}} > 0 \), the direction of the rolling friction is along x axis, otherwise the opposite. Therefore, the rolling friction can be expressed as:

\[
F_i = \text{sign}(T_{B_i} - T_{A_{i+1}}) \mu F(\theta, N_p)
\]  

(8)

Establish the equilibrium equations of the conductor tangential tensions:

\[
T_{B_i} - T_{A_{i+1}} = F_i = \text{sign}(T_{B_i} - T_{A_{i+1}}) \mu F(\theta, N_p) \quad (i=1, \ldots, n-1)
\]  

(9)
3.2.4. The condition of traction tension. The end tension of conductor is the traction tension \( T_c \):

\[
T_c = T_0
\]  

(10)

4. Nonlinear equations

Taking the span spacing \( s_i \), horizontal tension \( H_i \), vertical tension \( V_{Ai} \) of conductor as the unknown quantities, the nonlinear equations of the conductor for the multi-span tension stringing can be expressed as

\[
F(X) = 0
\]  

(11)

Here \( X = [s_1, s_2, ..., s_n, H_1, H_2, ..., H_n, V_1, ..., V_n]^T \).

This nonlinear equations can be solved by Newton iteration method [6].

5. Calculation process

The process of the method for the multi-span tension stringing is shown in figure 5.

6. Calculation cases

Taking the ±500kV Caopu-Changkou tension stringing construction project in references [7] as example. The traction tension is 14kN. Conductor parameters and the span information are shown in table 1 and table 2.

| Table 1. Conductor parameters. |
|--------------------------------|
| **Type of conductor** | LGJ-400/50 | **Unit weight (kg/m)** | 1.511 |
| **Cross-sectional area (m²)** | 20 | **Gravitational acceleration (kg/N)** | 10 |
| **Elasticity modulus (Pa)** | 15 | **Coefficient of tigidity** | 0.012 |

| Table 2. Information of span. |
|--------------------------------|
| **Tower number** | **Span spacing(m)** | **Elevation difference(m)** | **Tower number** | **Span spacing(m)** | **Elevation difference(m)** |
| N0 | N1 | 114 | -16.4 | N10 | N11 | 289 | 31.6 |
| N1 | N2 | 298 | 32.2 | N11 | N12 | 724 | 87 |
| N2 | N3 | 380 | 6.5 | N12 | N13 | 286 | 46.7 |
| N3 | N4 | 1035 | -162.1 | N13 | N14 | 476 | 73.2 |
| N4 | N5 | 810 | 149.7 | N14 | N15 | 610 | 127.2 |
| N5 | N6 | 171 | 21.4 | N15 | N16 | 694 | 56 |
According to the known condition, the calculated results of the conductor shape and tension are shown in figure 6.

![Figure 6. Calculated shape and horizontal tension of conductor.](image)

The comparison of conductor sag and length of each span are shown in table 3 between the results calculated by this method and data in the literature [7].

| Conductor sag of each span | Conductor length of each span |
|----------------------------|-------------------------------|
| Calculate result from literature(m) | Calculate result from this method(m) | Error(%) | Calculate result from literature(m) | Calculate result from this method(m) | Error(%) |
|-----------------------------|---------------------------------|----------|-----------------------------|---------------------------------|----------|
| 2.2744                      | 2.407                           | 5.8301   | 115.2                       | 114                             | -1.0530  |
| 15.5408                     | 17.308                          | 11.3714  | 300.3                       | 298                             | -0.7720  |
| 25.27                       | 27.9317                         | 10.5330  | 831.2                       | 810                             | -0.3160  |
| 187.4644                    | 208.8044                        | 11.3835  | 1070                        | 1035                            | -3.3820  |
| 114.8176                    | 124.587                         | 8.5086   | 834.4                       | 810                             | -3.0120  |
| 5.1172                      | 5.4627                          | 6.7517   | 172.4                       | 171                             | -0.8190  |
| 12.7576                     | 13.3717                         | 4.8136   | 270.5                       | 270                             | -0.1850  |
| 118.8208                    | 126.8477                        | 6.7555   | 835.5                       | 824                             | -1.3960  |
| 47.6848                     | 49.4378                         | 3.6762   | 524.9                       | 522                             | -0.5560  |
| 41.3344                     | 42.4979                         | 2.8148   | 489.9                       | 486                             | -0.8020  |
| 14.6162                     | 14.8857                         | 1.8438   | 291.2                       | 289                             | -0.7610  |
| 91.7308                     | 93.6205                         | 2.0600   | 736.9                       | 724                             | -1.7820  |
| 14.3144                     | 14.3106                         | -0.0270  | 290.3                       | 286                             | -1.5030  |
| 39.6508                     | 39.3831                         | -0.6750  | 483.8                       | 476                             | -1.6390  |
| 65.1176                     | 63.9066                         | -1.8600  | 627.7                       | 610                             | -2.9020  |
| 84.2864                     | 82.918                          | -1.6240  | 703.1                       | 694                             | -1.3110  |
| 47.32                       | 45.2745                         | -4.3230  | 524.2                       | 520                             | -0.8080  |
| 24.6094                     | 23.4323                         | -4.7830  | 376.1                       | 375                             | -0.2930  |
7.4264  6.9839  -5.9580  206.3  206  -0.1460  
64.6912  61.2765  -5.2780  612.9  608  -0.8060

The comparison shows that the results by this method agrees well with the data from literature [7], the maximum error of conductor sag of each span is 11.38%, the maximum error of conductor length of each span is 3.38%. That means the method is valid in calculation of multi-span tension stringing.

7. Summary
The nonlinear equations of multi-span tension stringing are present which include the conservation equations of span spacing and elevation difference and the tension balance equations of conductor. From the method, the traction, length and sag of conductor of each span in transmission line can be calculated to use in the selection of the traction machine.

The proposed method has good convergence and high calculation speed, which can fully meet the needs of engineering.

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