Abstract—Once users have shared their data online, it is generally difficult for them to revoke access and ask for the data to be deleted. Machine learning (ML) exacerbates this problem because any model trained with said data may have memorized it, putting users at risk of a successful privacy attack exposing their information. Yet, having models unlearn is notoriously difficult. After a data point is removed from a training set, one often resorts to entirely retraining downstream models from scratch.

We introduce SISA training, a framework that decreases the number of model parameters affected by an unlearning request and caches intermediate outputs of the training algorithm to limit the number of model updates that need to be computed to have these parameters unlearn. This framework reduces the computational overhead associated with unlearning, even in the worst-case setting where unlearning requests are made uniformly across the training set. In some cases, we may have a prior on the distribution of unlearning requests that will be issued by users. We may take this prior into account to partition and order data accordingly and further decrease overhead from unlearning.

Our evaluation spans two datasets from different application domains, with corresponding motivations for unlearning. Under no distributional assumptions, we observe that SISA training improves unlearning for the Purchase dataset by 3.13×, and 1.658× for the SVHN dataset, over retraining from scratch. We also validate how knowledge of the unlearning distribution provides further improvements in retraining time by simulating a scenario where we model unlearning requests that come from users of a commercial product that is available in countries with varying sensitivity to privacy. Our work contributes to practical data governance in machine learning.

I. INTRODUCTION

Many applications of machine learning (ML) involve analyzing data that is collected from individuals. This data is often sensitive in nature and could include information like medical records [1] or personal emails [2]. Moreover, data pipelines are often not static [3]: new data is collected regularly and incrementally used to further refine existing models following the online learning paradigm [4].

Conversely, data may also need to be deleted. Recently introduced legislation, such as the General Data Protection Regulation (GDPR) in the European Union [5], the California Consumer Privacy Act [6] in the United States, and PIPEDA privacy legislation in Canada [7] include provisions that require the so-called right to be forgotten [8]. This requirement, which has been one of the most controversial in the GDPR, mandates that companies take “reasonable steps” to achieve “the erasure of personal data concerning [the individual]” [9]. The unprecedented scale at which ML is being applied on personal data motivates us to examine how this right to be forgotten can be efficiently implemented for ML systems.

Because ML models potentially memorize training data [10], [11], it is important to sanitize models trained on data that have been deleted. This problem is tangential to privacy-preserving machine learning—enforcing differential privacy [12] does not solve it. While algorithms with differential privacy guarantee a bound on how much individual training points contribute to the model and ensure that this contribution remains small [13], [14], there remains a non-zero contribution from each point. If this was not the case, the model would not be able to learn at all from training data (see § III). In contrast, forgetting requires that a particular training point have zero contribution to the model, which is orthogonal to the guarantee that differential privacy provides. Hence, privacy-preserving ML cannot be used to guarantee that models forget.

Having models forget necessitates knowledge of exactly how individual training points contributed to model parameter updates. Prior work showed this is possible when the learning algorithm queries data in an order that is decided prior to the start of learning [15]. When the dataset is instead queried adaptively, i.e., a given query depends on any queries made in the past, convergence of the approach is no longer guaranteed. In the adaptive setting, the divergence induced by this approach to have models forget is bounded only for models which require a small number of iterations for learning. Thus, the class of approaches introduced by Cao et al. [15] are probabilistic and do not scale to complex models, such as deep neural networks, which require a large number of adaptive iterations to learn.

A naive way to have such models provably forget is to retrain them from scratch. To avoid the large computational and time overhead associated with fully retraining models affected by training data erasure, our research seeks to hold ML to standards such as the right to be forgotten instead through the ability to unlearn. Given a trained model, unlearning assures the user that the model is no longer influenced in any way by data which the user elected to erase. Put another way, unlearning outputs a new model that could have been obtained without training on the user’s data in the first place.

Because of this strong definition, we do not consider the setting in which unlearning is used to mitigate poisoning attacks [16]–[18]; the guarantee we provide is far stricter than what would be needed for poisoning—i.e., that the loss of model accuracy due to the poisoning are mitigated. Instead, we focus on unlearning with strong guarantees in order to satisfy the right to be forgotten requirement in the GDPR.
Our **SISA** training approach, short for **S**harded, **I**solated, **S**liced, and **A**ggregated training, can be implemented with minimal modification to existing pipelines. First, we divide the training data in multiple shards such that a training point is included in a small number of shards only, ideally a single shard. Then, we train models in isolation on each of these shards, which limits the influence of a point to the models that were trained on shard(s) containing the point. Finally, when a request to unlearn a training point arrives, we need to retrain only the affected models. Because the shards are smaller than the entire training set, this decreases the retraining time to achieve unlearning. In addition, rather than training each model on the entire shard directly, we can divide each shard into slices and present slices incrementally during training. We save the state of model parameters before introducing each new slice, allowing us to start retraining the model from the last known parameter state that does not include the point to be unlearned—rather than a random initialization. Slicing further contributes to decreasing the time to unlearn, at the cost of storage overhead. At inference, we simply aggregate the predictions of models trained on each shard to label points.

To demonstrate that **SISA** training handles streams of unlearning requests effectively, we analytically show that it outperforms the naive approach of retraining from scratch whether the service provider chooses to process unlearning requests sequentially (i.e., immediately upon a user revoking access to their data) or in batches (i.e., the service provider buffers a few unlearning requests before processing them). Experimentally, we find that sharding the data into 20 shards, provides a speed-up of $3.13 \times$ and $1.658 \times$ for the Purchase and SVHN dataset respectively, if the number of unlearning requests is 0.003% of the total dataset sizes (respectively). This assumes that requests are made uniformly across the dataset. These gains can be achieved at no cost in terms of classification accuracy. Indeed, even when configured in its most accuracy-preserving settings, **SISA** training can handle orders of magnitude more unlearning requests than what Google expects would be required to implement the right-to-be-forgotten [19]. When we add in slicing, we find an additional speed-up of $1.428 \times$ and $1.176 \times$ for the Purchase and SVHN datasets respectively, if the number of unlearning requests is 0.001% of the total dataset sizes (respectively). While these numbers seem small, the savings in retraining times enable large organizations to benefit from economies of scale. We again validate experimentally that slicing has no impact on accuracy—it merely reorders data analyzed during training.

When faced with different distributions of unlearning requests, i.e., requests are not uniformly issued across the dataset, we present a refined variant of our approach, which assumes prior knowledge of the distribution of unlearning requests. We validate it in a scenario that models a company operating across multiple jurisdictions with varying legislation and sensitivities to privacy, and accordingly varying distributions of unlearning requests from users. We inform the design of our scenario with information publicly released by Google about its experience with the right to be forgotten [19]. Knowing this distribution enables us to further decrease expected unlearning time by placing the training points that will likely need to be unlearned in a way that reduces retraining time. The cost in terms of accuracy is either null or negligible, depending on the distribution of requests considered.

In summary, the contributions of this paper are:

- We formulate a new, intuitive definition of unlearning. Our definition also takes into account non-uniform distributions of unlearning requests.
- We introduce **SISA** training, a practical approach for unlearning that relies on data sharding and slicing to reduce the computational overhead of unlearning.
- We analytically derive the asymptotic reduction in time to unlearn with sharding and slicing when the service provider processes requests sequentially and in batches.
- We demonstrate that sharding and slicing combined do not impact accuracy, and that **SISA** training could be immediately applied to handle orders of magnitude more unlearning requests than what Google anticipates is required to implement the GDPR right to be forgotten [19].
- When the distribution of requests is known, **SISA** training enables us to further decrease expected unlearning time.

## II. BACKGROUND ON MACHINE LEARNING

We provide rudiments of machine learning as they apply to neural networks. We chose to study neural networks because they almost always generate the largest computational costs and require investments in dedicated accelerators [20], [21]; we also discuss other ML techniques in § IV. We surface aspects of training neural networks that make it difficult to unlearn without facing important computational costs.

Our efforts fall under the realm of supervised machine learning [22]. Tasks to be learned are defined in a space $\mathcal{Z}$ of the form $\mathcal{X} \times \mathcal{Y}$, where $\mathcal{X}$ is the sample space and $\mathcal{Y}$ is the output space. For example, $\mathcal{X}$ could be thought of as the space of images and $\mathcal{Y}$ as the labels of the images. When the task is a classification problem, labels are categorical.

Given a dataset of input-output pairs $(x, y) \in \mathcal{X} \times \mathcal{Y}$, the goal of a supervised learning algorithm is to find a model, i.e., a function $F: \mathcal{X} \mapsto \mathcal{Y}$ that maps these inputs to outputs. The learning algorithm that produces this model is an optimizer. It takes in a dataset, a hypothesis space, and an objective:

- **Dataset:** Consistent with the probably approximately correct (PAC) learning setting [23], we assume there is an underlying distribution on $\mathcal{Z}$ that describes the data; the learner has no direct knowledge of the distribution but has access to a dataset $\mathcal{D}$ that is drawn from it. This dataset $\mathcal{D}$ is further split into the training dataset $\mathcal{D}_{tr}$ and a holdout dataset called the test dataset $\mathcal{D}_{te}$ such that $\mathcal{D}_{te} \cup \mathcal{D}_{tr} = \mathcal{D}$ and $\mathcal{D}_{te} \cap \mathcal{D}_{tr} = \emptyset$.
- **Hypothesis space:** An hypothesis is a set of parameter values $w$, which together with the model architecture $F$ selected, represent one possible mapping $F_w: \mathcal{X} \mapsto \mathcal{Y}$ between inputs and outputs. In our case, the hypothesis...
is a neural network and its parameters, the weights that connect its different neurons (see below).

- **Objective:** Also known as the loss function, the objective characterizes how good any hypothesis is by measuring its empirical risk on the dataset, i.e., approximate the error of the model on the underlying task distribution of which we only have a few samples—the dataset. A common example is the cross-entropy loss, which measures how far a model’s outputs are from the label:

\[ l(x, y) = -\sum_{i=0}^{n-1} y_i \cdot \log(F_w(x)) \]

where \( n \) is the number of classes in the problem.

Given an architecture \( F \), a model \( F_w \) is found by searching for a set of parameters \( w \) that minimize the empirical risk of \( F_w \) on the training set \( D_{tr} \). Performance of the model is validated by measuring its accuracy on the test dataset \( D_{te} \).

We experiment with our approach using neural networks and deep learning [24]. Deep neural networks (DNNs) are non-parametric functions organized as layers. Each layer is made of neurons—elementary computing units that apply a non-linear activation function to the weighted average of their inputs. Neurons from a given layer are connected with weights to neurons of the previous layer. The layout of these layers and the weight vectors that connect them constitutes the architecture of the DNN, while the value of each individual weight (collectively denoted by \( w \)) is to be learned. A common choice of optimizer is the backpropagation algorithm [25]. The algorithm starts by assigning a random value to each weight.

Then, a data point is sampled from the dataset and the loss function is computed to compare the model’s prediction to the data point’s label. Each model parameter value is updated by multiplying the gradient of the loss function with respect to the parameter by a small constant called the learning rate. This algorithm enables learning and gradually improves the model’s predictions as more inputs are processed.

### III. Defining Unlearning

A requirement of privacy regulations such as the GDPR or the CCPA is that individuals whose data is housed by organizations have the right to request for this data to be erased. This requirement poses challenges to current machine learning technologies. We define the unlearning problem by examining these challenges, which then leads us to a formal model of the unlearning problem. We identify objectives for an effective approach to unlearning, which we use to show the ineffectiveness of existing “strawman” solutions.

#### A. Why is Unlearning Challenging?

One may wonder why unlearning is difficult to begin with. The reason unlearning is challenging stems from the complex and stochastic nature of the training methods used in modern ML pipelines. We further explain these reasons below.

1. **We have a limited understanding of how each data point impacts the model.** There exists no prior work that measures the influence of a particular training point on the parameters of a model. While research has attempted to trace a particular test-time prediction through the model’s architecture and back to its training data [26], [27], these techniques rely on influence functions, which involve expensive to compute second-order derivatives of the model’s training algorithm. Further, it is not obvious how to modify such influence functions so that they map the effect of a single training point on model parameters.

2. **Stochasticity in training.** A great deal of randomness exists in the training methods for complicated models such as deep neural networks (DNNs). For example, small batches of data (e.g., with 32 points) are randomly sampled from the dataset, and the ordering of batches varies between different epochs, i.e., passes of the training algorithm through the dataset. Further, training is often parallelized without explicit synchronization, meaning the inherent random ordering of parallel threads may make the training non-deterministic.

3. **Training is incremental.** Additionally, training is an incremental procedure where any given update reflects all updates that have occurred prior to it. For example, if a model is updated based on a particular training point (in a particular batch) at a particular epoch \( i \), all subsequent model updates will depend, in some implicit way, on that training point.

4. **Stochasticity in learning.** Intuitively, learning algorithms are designed to search for an optimal hypothesis in a vast hypothesis space. In the case of neural networks, this space contains all models that can be defined by setting the weights of a fixed neural network architecture. PAC learning theory suggests that the learned hypothesis is one of many hypotheses that minimize the empirical risk. For example, the common choice of optimizer for neural networks, stochastic gradient descent, is capable of converging to one of the many local minima for any convex loss function. Coupled with the stochasticity involved in training, it is very challenging to correlate a data point with the hypothesis learned from it.

#### B. Formalizing the Problem of Unlearning

We formalize the unlearning problem as a game between two entities: a service provider \( S \), and a user population \( U \). The service provider could be a large organization that collects information from various individuals (such as a company or hospital). This data is curated in the form of a dataset \( D \). The service provider uses this data for training and testing a machine learning model \( M \) in any way they desire. Any user \( u \in U \) can revoke access to their individual data \( d_u \subset D \). Observe that \( d_u \) can be a single element in the dataset, or a set of elements. Within a finite period of time, and in an inexpensive manner, the service provider has to erase the revoker’s data and modify any trained models \( M \) to produce \( M_{\neg d_u} \), where \( M_{\neg d_u} \) is some model that could plausibly have been trained if \( d_u \) were not in \( D \). Further, \( S \) must convince \( u \) that \( M_{\neg d_u} \) is such a model—a defense akin to that of plausible deniability. Access to data may be revoked by users sequentially, but the service provider may choose to perform data erasing in a batched fashion, as discussed in § [VII].

We illustrate this scenario in Figure [I]. From our earlier discussion, one can observe that given a dataset \( D \), it is possible to train one of several models that generalize well
Definition III.1. Let $M$ be a model, which the owner obtained by selecting an hypothesis that minimizes empirical risk over a training set $D = \{d_u : u \in U\}$ collected from population $U$. The owner unlearned the data $d_u$ of a user $u \in U$ if it releases a new model $M'$ that could have been obtained by selecting an hypothesis that minimizes empirical risk over $D \setminus d_u$.

We draw the attention of the reader to two important aspects of the definition. First, the definition captures inherent stochasticity in learning: it is possible for multiple hypotheses to minimize empirical risk over a training set. As illustrated by models $M_A$ and $M_C$ in Figure 1, two models having different parameters does not imply that they were trained with a different dataset. Conversely, two models trained with a different dataset do not necessarily have different parameters. Second, the definition does not necessarily require that the owner retrain the model $M'$ from scratch on $D \setminus d_u$, as long as they are able to provide evidence that model $M'$ could have been trained from scratch on $D \setminus d_u$. This opens up directions for approaches that produce certificates proving that $M'$ could have been obtained from $D \setminus d_u$. We leave determining such certification procedures to future work.

C. Goals of Unlearning

A service provider would have to implement a strategy to perform unlearning. The simple strategy we have discussed thus far i.e. training a model from scratch on the dataset without the point being unlearned is very powerful. We refer to this strategy as the baseline strategy. However, for large dataset sizes, such an approach will quickly become intractable (in terms of time and computational resources expended). For example, to be compliant with GDPR/CCPA, organizations will have to retrain models very frequently. Thus, any new strategy should meet the following requirements.

G1. Intelligibility: Conceptually, the baseline strategy is very easy to understand and implement. Similarly, any unlearning strategy should be intelligible; this requirement ensures that the strategy is easy to debug by non-experts.

G2. Comparable Accuracy: It is conceivable that the accuracy of the model degrades, even in the baseline of retraining from scratch, if the fraction of training points that need to be unlearned becomes too large. Even if there is no component of the approach that explicitly promotes high accuracy, any unlearning strategy should introduce a (small) bounded accuracy gap in comparison to the baseline strategy for any number of points unlearned.

G3. Reduced Unlearning Time: The strategy should have provably lower time than the baseline for unlearning any number of points.

G4. Provable Guarantees: Like the baseline, any new strategy should provide provable guarantees that any number of points have been unlearned (and do not influence model parameters). Additionally, such a guarantee should be intuitive and easy to understand for non-experts.

G5. Model Agnostic: The new strategy for unlearning should be general i.e. should provide the aforementioned guarantees independent of the nature/complexity of the model.

G6. Limited Overhead: Any new unlearning strategy should not introduce additional overhead to what are already computationally-intense training procedures.

D. Strawman Solutions

Based on the requirements discussed earlier, we propose several strawman candidates for an unlearning strategy. The goals specified in the parentheses are the goals the strawman solutions that we do not meet.

1. Differential Privacy: Proposed by Dwork et al. [29], differential privacy offers probabilistic guarantees about the privacy
of individual records in a database. While several efforts [14], [30] have shown that it is possible to learn in a differentially private manner, i.e. the presence or absence of a point in a training set does not impact the learning algorithm substantially, this approach does not provide concrete guarantees (G4) for unlearning a point (because of its probabilistic nature). Put another way, our definition of unlearning would require that the model were trained with an $\varepsilon$-differential privacy guarantee of $\varepsilon = 0$. This would prevent the model from learning anything from any of its training points.

Other mechanisms relax the definition of differential privacy to provide certificates of data removal. This includes the recent and concurrent proposal by Guo et al. [51]. They do so by performing unlearning using a one-step Newton update; updating the parameters using the influence function (27) of the point to be removed. By doing so, they are able remove the influence of the data point on the parameters. While such a mechanism introduces a small residue, this is masked by adding noise (similar to approaches in differential privacy). Their certificate gives theoretical guarantees that a model obtained by unlearning a data point is indistinguishable from a model with the data point. However, their definition of unlearning coupled with their mechanisms to achieve it mean that they fall short of achieving (G4) and (G5).

2. Homomorphic Encryption: Homomorphic encryption is a set of schemes that enable computation on encrypted data without having to decrypt it [32]. Such mechanisms provide confidentiality. However, the connection between homomorphic encryption and improved retraining times for unlearning is unclear (G3); to the best of our knowledge, there is no learning algorithm that can operate on encrypted (or seemingly random) inputs. Additionally, such schemes [33–35] have a large computational overhead (G6) and are often used only for inference and not training.

3. Statistical Query Learning: Cao et al. [15] model unlearning in the statistical query learning framework [36]. By doing so, they are able to unlearn a point when the learning algorithm queries data in an order decided prior to the start of learning. In this setting, it is possible to know exactly how individual training points contributed to model parameter updates. However, their approach is not general (G5) and does not easily scale to more complex models (such as those considered in this work). Indeed, these models are trained using adaptive statistical query algorithms which make queries that depend on all queries previously made. In this setting, the approach of Cao et al. [15] diverges in an unbounded way unless the number of queries made is small, which is not the case for the deep neural networks we experiment with.

4. Decremental Learning: Ginart et al. [37] consider the problem from a data-protection regulation standpoint. They present a rigorous formal definition of data erasure completeness which can be relaxed into a distance-bounded definition. Deletion time complexity bounds are also provided. They note that the deletion problem is orthogonal to the privacy issue, which means deletion capability does not imply privacy nor vice versa. However, it is unclear if the approach presented (Quantized k-Means) is applicable (G5) and scalable (G6) for all model classes.

IV. THE SISA APPROACH

Our discussion so far motivates why retraining from scratch on the dataset omitting data points that need to be unlearned is the most straightforward way to provide provable guarantees. However, this naive strategy is inefficient in the presence of large datasets or models with high capacity that take time to train. We present our approach, SISA (or Sharded, Isolated, Sliced, Aggregated) training, which overcomes these issues.

A. The SISA Approach to Training

As illustrated in Figure 2, SISA training replaces the model being learned several times where each replica receives a disjoint shard of the dataset—similar to current distributed training strategies [38], [39]. We refer to each replica as a constituent model. However, SISA training deviates from current strategies in the way incremental model updates are propagated or shared. The key difference is that there is no flow of information between constituent models. For example, if each constituent model is a deep neural network (DNN) trained with stochastic gradient descent (SGD), then gradients computed on each constituent are not shared between different constituents; each constituent is trained in isolation. This ensures that the influence of a shard (and the data points that form it) is restricted to the model that is being trained using it. Each shard is further partitioned into slices, where each
constituent model is trained incrementally (and iteratively) with an increasing number of slices. At inference, the test point is fed to each constituent and all the constituents’ responses are aggregated by a strategy similar to the ones used in ML ensembles.

When a data point is to be unlearned, only the constituent model whose dataset contains this point is affected. A data point is unlearned from a particular shard and a particular slice in the shard. Retraining can start from the last parameter state saved prior to including the slice containing the data point to be unlearned: only the models that are trained using the slice containing the unlearned point need to be retrained. We will describe each component in more detail in §IV-B.

Observe that we make no assumptions about the nature of the constituent models or if the constituents are homogeneous (i.e. the same model or hypothesis class) or heterogeneous (i.e. different models or hypothesis class). Sharding is possible for any model or hypothesis class: it has no impact on how training is performed beyond the smaller set of data each model has access to. Slicing is possible for any iterative learning algorithm that is stateful: the algorithm should be such that it can continue to learn from its current state when presented with new data. Gradient descent naturally falls under that category. However, decision tree learning is a counter-example of a technique that does not benefit from slicing, because it greedily chooses a feature to add to the decision tree based on how well it splits the data according to a metric like Gini impurity. For this reason, when a new slice of data is added, the tree must be constructed again from scratch. In summary, slicing can be used for any model that is trained through gradient descent: e.g., logistic regression and neural networks, but also support vector machines in some cases [40].

The key requirement of our training strategy is that the updates obtained during the iterative training process are not exchanged between different constituents. Intuitively, such an approach may seem detrimental to improving the generalization capabilities of the model; each constituent is trained on a (significantly) smaller portion of the dataset, and it is unclear how much information is lost by doing so, especially when the learned updates are not shared. We evaluate this aspect in §VII and show that one can partition the training set such that there is minimal impact on prediction accuracy.

B. Techniques

1. Sharding: By dividing the data into disjoint fragments and training a constituent model on each smaller data fragment, we are able to distribute the training cost. While this means our approach naturally benefits from parallelism across shards, we do not take this into account in our analysis and experiments, out of fairness to the baseline of retraining a model from scratch (which could also be accelerated by distributing the computation across multiple machines).

For the remainder of this section, we assume that we have no prior information associated with the probabilities with which each individual point might be unlearned. In such a scenario, a dataset $D$ can be uniformly partitioned into $S$ shards such that $\cap_i D_i = \emptyset$ and $\cup_i D_i = D$. For each shard $D_i$, a model (denoted $M_i$) is trained using the entirety of the data available in $D_i$. In §VIII we explore the scenario where the distribution of unlearning requests is known to the service provider.

Observe that user $u$'s data-point $d_u$ can lie in each of the $S$ shards with equal probability. Moreover, one of the parameters of the training can be whether each $d_u$ be part of only one shard or several. For simplicity, we will assume that each $d_u$ is part of only one shard, as this maximizes the savings in unlearning time. We discuss this further in §IX. If the user desires for $d_u$ to be unlearned, then the service provider has to (a) first locate the shard in which $d_u$ is located, referred to as $D_u$, and (b) retrain from scratch the corresponding model on $D_u \setminus d_u$; this will result in a new model $M'_u$. In comparison, the naive approach would entail retraining the model from scratch on $D \setminus d_u$. Since $|D| >> |D_u|$, the time required for retraining (henceforth referred to as retraining time) in the naive scenario is far greater than in our proposal. Specifically, our proposal provides an expected speed-up of $S$.\footnote{For a single unlearning request.}

2. Isolation: Observe that based on the proposal detailed earlier, the training of each shard occurs in isolation. Instead, in typical distributed training algorithms [38], [39], training updates from each constituent are shared between each other and training iterates based on a joint update. By not doing so, we potentially degrade the generalization ability of the overall model (comprising of all constituents). However, we demonstrate that for appropriate choices of the number of shards, this does not occur in practice. Isolation is a subtle, yet powerful construction that enables us to give concrete, provable, and intuitive guarantees with respect to unlearning.

Indeed, through isolation, we can guarantee that a data point only impacts parameters of the models that were trained on shards that include this data point. This guarantee holds in the face of all stochasticity found in training and learning, which is what enables us to obtain strong privacy guarantees, even for models like deep neural networks—this is an essential distinction from from prior efforts [15].

3. Slicing: By further dividing data dedicated for each model (i.e. each shard) and incrementally tuning (and storing) the parameter state of a model, we obtain additional time savings. Specifically, each shard $D_k$ is further uniformly partitioned into $R$ disjoint slices such that $\cap_i D_{k,i} = \emptyset$ and $\cup_i D_{k,i} = D_k$. We perform training for $E$ epochs to obtain $M_k$ as follows:

\begin{enumerate}
\item At step 1, train the model using random initialization using only $D_{k,1}$, for $E_1$ epochs. Let us refer to the resulting model as $M_{k,1}$. Save the state of parameters associated with this model.
\item At step 2, tune the model $M_{k,1}$ using $D_{k,1} \cup D_{k,2}$, for $E_2$ epochs. Let us refer to the resulting model as $M_{k,2}$. Save the parameter state.
\item At step $R$, tune the model $M_{k,R-1}$ using $\cup_i D_{k,i}$, for $E_R$ epochs. Let us refer to the resulting final model as $M_{k,R} = M_k$. Save the parameter state.
\end{enumerate}
As before, observe that if user $u$’s data-point $d_u$ lies in shard $D_{k,u}$, then it can lie in any of the $R$ slices with equal probability. Thus, if the user desires for $d_u$ to be unlearned, then the service provider has to (a) first locate the slice in which $d_u$ is located, referred to as $D_{k,u}$, and (b) perform the training procedure as specified above from step $u$ onwards using $D_{k,u} \setminus d_u$; this will result in a new model $M'_{k,u}$. This provides an expected speed-up of $(\frac{R+1}{2}) \times$ compared to using just the strategy without slicing.

It is also worth noting that the duration of training for the constituent models with and without data slicing is different when they have the same number of epochs. Each epoch takes less time when only a subset of the slices is being trained on; on the other hand, training incremental combinations of slices takes longer because the training process is repeated after each slice is added. In order to train models with and without slicing for the same amount of time, we introduce the following relationship between the number of epochs with and without slicing. Let $e'$ be the number of epochs without slicing; we seek to find the number of epochs $e = \frac{R}{\sum_{i=1}^{R} e_i}$ to train a model with $R$ slices, where $e_i$ is the number of epochs required to train data of size $\frac{N}{R}$. We make a simplifying assumption: we assume that each slice is trained equally long i.e., $\forall i, e_i = \frac{e}{R}$. We also assume that the training time is estimated solely based on the amount of training data (as detailed in §V). Thus, we obtain:

$$e' = \sum_{i=1}^{R} \frac{e_i}{R}$$

By simplifying this equation, we get:

$$e \leftarrow 2 \cdot \frac{R}{R+1} \cdot e'$$

The speed-up provided by slicing comes at no expense beyond the overhead induced by storing the state of parameters before each slice is introduced in training.

4. Aggregation: At inference time, predictions from various constituent models can be used to provide an overall prediction. How these predictions are aggregated is determined by an aggregation strategy. For example, the overall prediction could be the majority vote of all the constituents’ votes. Observe that by varying the aggregation strategy, one can increase or decrease the accuracy of the overall model. In our implementation, the aggregation strategy can easily be modified to one of several candidates depending on the accuracy requirements.

Take-away. In summary, the techniques discussed here provide an expected speed-up of $\frac{S(R+1)}{2}$ in terms of retraining time (for one unlearning request); for large values of the number of shards $S$ and the number of slices $R$, this technique is considerably more efficient than the naive approach. However, our approach introduces several challenges.

C. Challenges

We make no assumptions about (a) the nature of unlearning requests, (b) the nature of training algorithms, and (c) the nature of data distribution within both the shards and slices. This results in several challenges which we discuss below.

1) Weak Learners: Because each model is trained on a shard that is smaller than the original dataset, these models could be weak learners [41]; in other words, their accuracy will be lower than a single model trained on the entire dataset. The primary reason for why this accuracy gap could exist is that when each constituent model is trained on very limited data; if the model has high-capacity (as is the case with DNNs), this might result in overfitting to the small training dataset. Some of this accuracy will be recovered by aggregation operation. However, we instantiate our approach assuming that the constituent models trained on shards are all trained with the same architecture. Because this departs from prior work on ML ensembles, which typically involves an ensemble of heterogeneous models [42], we may not obtain as large benefits from aggregation as is typically the case.

In addition to the limited capabilities of each weak learner, the uniform split of data to each $D_k$ provides no information for the data service provider to choose a subset of the constituent models for prediction. Therefore, the service provider resorts to using all models in the ensemble for prediction and aggregating their results. However, there are several candidates for aggregation, each with their own pros and cons.

Note that in our experiments, we observe that we do not encounter weak learners in the regime of number of shards that we operate in. This observation is explained by the large dataset size (of 310k and 600k points, respectively). Because the cost of unlearning incurs significant overhead on the service provider in the large dataset setting, the weak learner challenge may not present itself in practice for the datasets which one would want to use SISA training for.

2) Hidden Overheads: On the other hand, both sharding and slicing should have a negligible impact on the total time spent training, prior to unlearning, compared to the baseline of training a single model on the entire dataset. The same goes for computational resources needed for training. Wall-clock time overhead may arise from initializing multiple models as the number of shards increases or interrupting training to save the state of each model, before a new slice is introduced.

On the other hand, observe that the slicing strategy introduces a storage overhead for storing the model’s parameter state before each slice is added to the training set.

3) Hyperparameter Search: Additionally, sharding and slicing may require that the service provider revisit some hyperparameter choices made on the entire dataset. For instance, sharding and slicing may require training with a different number of epochs. Slicing could also negatively interact with batching when the service provider is using a large number of slices—because each slice will be increasingly smaller.

If each constituent model requires a different set of hyperparameters for optimal performance, then as the number of models (of the order $O(S.R)$) increases, performing hyperparameter tuning is a truly challenging problem. Training $O(S.R)$ models, depending on the hyperparameter search needed to optimize for these challenges may introduce a computational overhead, as it may be greater than that in the naive retraining scenario. We note that hyperparameters are
shared across constituent models when data is split uniformly across shards. In that case, one only needs to train $O(R)$ models to tune the hyperparameters for slicing.

**Take-away.** We revisit these challenges in § VII discuss the various solutions we explored for each of the problems listed above, and highlight insights we gained from them.

V. MEASURING TIME

One important aspect of our evaluation is the measurement of time. Reporting time measured empirically in our experiments could potentially be misleading; there are several sources of error whilst measuring wall-clock time that are implementation-dependent. In particular, we need to handle variance of training time across hardware with different ML accelerators. To this end, we provide tight asymptotic bounds for time in the presence of sharding and slicing.

Under the assumption that all other hyperparameters stay the same, then the retraining time is proportional to the number of points that need to be used in retraining. Therefore, the following analysis calculates the expectation of the number of data points needed for retraining, given an unlearning request, as the number of shards and slices varies.

A. MEASURING TIME FOR SHARDING

Observe that for each erasure request, a single model is retrained when it arrives sequentially whereas multiple models are retrained when the requests arrive as a batch.

1. Sequential Setting: In the sequential setting, we make two assumptions: (a) the training data is shuffled and evenly split into $S$ shards and (b) each unlearning request can require any of the $S$ shards to be retrained, with equal probability, at any step. We do not assume that the shard size remains constant as unlearning requests arrive; we explicitly calculate the expectation of the number of points needed to be used for retraining.

To achieve our desired result, we make another assumption: the number of data points in each shard changes only slightly, enough so as to ensure that each shard is equally likely to be impacted by an incoming unlearning request.

If the sharding is uniform, then each model has (roughly) the same number of initial training data $\frac{N}{S}$; it is obvious that the first unlearning request will result in retraining of $\frac{N}{S} - 1$ points for the one shard that is affected. For the second unlearning request, there will be two cases: the shard affected in the first unlearning request is affected again, which will result in retraining $\frac{N}{S} - 2$ data points with a probability $\frac{1}{S}$, or any other shard is impacted resulting in retraining $\frac{N}{S} - 1$ data points with probability $\frac{S-1}{S}$. Thus, inductively, we can see that for the $i^{th}$ unlearning request, the probability that $\frac{N}{S} - 1 - j$ points (for $0 \leq j \leq i - 1$) are retrained is:

$$\sum_{i=1}^{K} \sum_{j=0}^{i-1} \binom{i-1}{j} \left( \frac{1}{S} \right)^j \left( 1 - \frac{1}{S} \right)^{i-j-1} \left( \frac{N}{S} - 1 - j \right)$$

This expression can be simplified using the binomial theorem, as described in the appendix to obtain:

$$\left( \frac{N}{S} - 1 \right) K - \frac{K(K-1)}{2S}$$

An upper bound for the above equation can be obtained if we assume that after each unlearning request, the size of each shard remains constant; thus, the cost of any step is $\frac{N}{S}$. We then have a linear bound for the total cost: $\frac{N}{S} K$. This bound also shows that by doubling the number of shards, the number of data points that need retraining will be divided by two.

2. Batch Setting: Alternatively, the service provider could aggregate unlearning requests into a batch, and service the batch. In such a batched setting, we introduce the random variables $(s_i)_{i=1}^{K}$, which denotes the index of the shard impacted by an unlearning request $i$ (where $K$ is the number of unlearning requests in the batch). We assume that these random variables are i.i.d. and follow a uniform distribution over $[1, S]$, where $S$ denotes the number of shards. The cost of unlearning the batch is $C = \sum_{j=1}^{S} (\frac{N}{S} - u_i)h_j$, where $(u_j)_{j=1}^{S}$ are the random variables which indicate the number of times a shard of index $j$ is impacted, and $(h_j)_{j=1}^{S}$ are the Bernouilli random variables indicating if a shard of index $j$ is impacted by an unlearning request. We can show that $(u_j)_{j=1}^{S}$ follows a binomial distribution $B(K, \frac{1}{S})$. Thus, the expected cost is:

$$\mathbb{E}[C] = N(1 - (1 - \frac{1}{S})^K) - K$$

Asymptotically, $\mathbb{E}(C) \sim N(1 - \exp(\frac{K}{S}))$ where $\tau = (-\ln(1 - \frac{1}{S}))^{-1}$ when $K \to 0$, and $\mathbb{E}(C) \sim N - K$ when $K \to +\infty$. Thus, the benefits of sharding are most noticeable when $K << N$. We visualize the asymptotic behavior of this time analysis in Figure 11 (see Appendix).

B. MEASURING TIME FOR SLICING

Our analysis of slicing differs from the analysis we presented for sharding because unlike shards, which are independent, a slice depends on all slices observed before them. Again, we distinguish two cases: the first, the service provider processes unlearning requests sequentially, and in the second, requests are processed in batches.

a) Sequential Setting: The case where unlearning requests are processed as a stream is easier to analyze. Since we assume that the time for retraining a model is proportional to the number of points needed to be retrained, we need to find the expectation of the number of slices that will need to be retrained for a single unlearning request.

Let $\gamma$ be the number of points contained per slice. If we sample an unlearning request from the uniform distribution
and where \( N \) is the number of points in the dataset, the expected slice index which we should retrain from is:

\[
\frac{U(0, N-1)}{\gamma}
\]

The expected number of slices we need to retrain is then equal to the number of slices, \( R \), to which we subtract the expected index of the slice which contains the point to be unlearned. We multiply this number of slices to be retrained by the number of points, \( \gamma \), in each slice to obtain:

\[
\left( R - \mathbb{E} \left[ \frac{U(0, N-1)}{\gamma} \right] \right) + 1 \cdot \gamma
\]  

(1)

which is an upper bound on the expected number of points to be retrained within a shard—upon receiving a single unlearning request. The upper bound is due to the approximation we make about the number of points per slice \( \frac{N}{R\gamma} \) remaining constant throughout unlearning. In practice, we show in § VI that this approximation is acceptable given that the number of requested handled is negligible compared to the total number of points included in the training set.

b) Batch Setting: Here, the service provider waits to receive multiple unlearning requests before processing them. As before, we denote the number of unlearning requests processed in a batch as \( K \). In this case, we need to find the expected minimum value over multiple draws of a random variable to compute the index of the slice from which we will have to restart training. Each unlearning request can still be modelled as a random draw from a uniform distribution \( U(0, N-1) \). However, the model will now have to be retrained from the slice which contains an unlearning request and has the smallest index – all iterations of training on slices that follow it were impacted by the point included in this slice.

To compute the minimum slice index among all slices affected by the \( K \) unlearning requests, we make the simplifying assumption that multiple unlearning requests are sampled from a uniform distribution \( U(0, N-1) \) with replacement. Although this assumption does not hold (the same point would not be asked to be unlearned multiple times), we verify numerically that it does not significantly affect our estimate. It is intuitive to see why given that the number of requests is orders of magnitude smaller than the number of points in the training set.

In Equation 3 of the Appendix, we show that the expected value of the minimum across \( K \) draws \( \chi_0, \ldots, \chi_{K-1} \) from a uniform distribution \( U(0, \alpha) \) is: \( \mathbb{E}[\min(\chi_0, \ldots, \chi_{K-1})] = \frac{\beta + K \alpha}{\alpha + K} \). From this, we derive that the expected value of the minimum index of the slice from which we need to retrain from, after having received \( K \) unlearning requests, is \( \frac{R + K - 1}{K + 1} \). If we repeat the reasoning that led to Equation 1 in the sequential case, we obtain the number of points to be retrained for \( K \) requests:

\[
\left( R - \frac{R + K \cdot 1}{K + 1} \right) + 1 \cdot \gamma = \frac{R (K + 1)}{K + 1} \cdot \gamma
\]

(2)

When \( K \ll R \), we have asymptotically \( \frac{R K + 1}{K + 1} \rightarrow R \geq 1, R \), which implies that the expected speed-up is equal to the number of slices when the number of requests processed in a batch is sufficiently small. Instead, as the number of requests in a batch becomes predominant over \( R \), we gradually lose all speed-ups and have to retrain the entire shard. We detail the proof in Appendix E.

VI. IMPLEMENTATION DETAILS

A.Datasets

We utilize the SVHN and the Purchase dataset for our experiments. These two datasets were chosen to showcase that the SISA training approach is agnostic to the domain of the learning task (vision and not vision).

SVHN: The SVHN dataset is used for vision tasks and contains 630420 \( 32 \times 32 \times 3 \) sized images for 10 classes. It includes images of house numbers collected by the Google Maps Street View service. Hence, it is understandable that some individuals whose house was photographed may request for the photograph to be erased. For instance, Google Street View has limited availability in Germany because it is perceived as a privacy violation there.

Purchase: Similar to the methodology of Shokri et al. [45] §6, we curated the Purchase dataset by choosing the top 600 most purchased items based on the category attribute. We then created 600 binary features from these items. This process resulted in a dataset of 310000 records. We then used k-means clustering to segment users into 2 clusters. Again, it is clear that users may not permit their purchase history to be used for training models and would accordingly revoke access to their purchase history, triggering an unlearning request.

It is important to note that training (and naively unlearning points from) datasets of this size induces a significant overhead to the service provider.

B. Models & Experimental Setup

For simplicity, we use the same models for (a) the baseline and (b) the SISA training scheme. For the SVHN dataset, we use the ResNet-16 architecture. For the Purchase dataset, we use a DNN with a single hidden layer comprising of 128 nodes and the tanh activation. Hyperparameters are also kept constant across the baseline and SISA training scheme, with the exception of the number of epochs, which we set accordingly to the calculation presented in § IV when slicing. A majority vote is used for aggregation.

We run our experiments on the SVHN dataset in parallel on a SLURM cluster of 216 GPUs. The cluster is composed of 44 nodes, each equipped with 4 P100 Nvidia GPUs with 12G dedicated memory, and 10 nodes each equipped with 4 T4 Nvidia GPUs with 16G of dedicated memory. Each server is equipped with 32 Intel Xeon Silver 4110 CPUs with 8 cores each and 192G of RAM. For the Purchase dataset, we run our experiments on two smaller servers each equipped with one Nvidia GeForce RTX 2080 GPU with 12G of dedicated memory, 12 Intel Xeon W-2133 CPUs with 6 cores each and 16G of RAM. Across both servers, the underlying OS is Ubuntu 18.04.2 LTS 64 bit. We use PyTorch v1.3.1 with CUDA 10.1 and Python 3.6.
VII. Evaluation

Our evaluation is designed to understand the limitations of SISA training in the scenario where the service provider has no information about the nature of the distribution of the unlearning requests i.e. in the uniform setting. In § VIII we utilize explicit knowledge of this distribution (modeled based on recent public insight from Google [19]) to verify that it improves retraining time. All code (and model checkpoints) will be made public. In this section, our experiments tease apart each component of the SISA training approach. We perform an ablation study to answer the following questions:

1) What is the impact of sharding on accuracy for varying numbers of unlearning requests?
2) What is the impact of slicing on accuracy for varying numbers of unlearning requests?
3) Does SISA training improve the retraining time (compared to the baseline)?

From our analysis, we draw the following insights on the applicability of SISA training in practical settings:

1) We observe that the sharding component of SISA training induces accuracy degradation as (a) the number of unlearning requests increases and (b) the number of shards increases. This accuracy degradation stems from the decrease in the number of data points per shard caused by both (a) and (b). Thus, we are able to determine a shard size that preserves accuracy and improves retraining time (refer § VII-A).

2) We observe that the slicing does not induce accuracy degradation so long as the number of epochs required for training are recalibrated (refer § VII-A).

3) Even in the worst-case scenario (with no knowledge of the distribution of unlearning requests), for a certain number of unlearning requests, just sharding (with no slicing) significantly outperforms the baseline. This effect is enhanced with slicing (at a nominal increase in storage overhead — linear in the number of slices). If the number of requests exceeds this threshold, SISA training gracefully degrades to the performance of the baseline. We can analytically obtain this threshold (refer § VII-B) based on our theoretical analysis in § V.

A. The Big Picture

To understand the gains provided by our approach, we first stress test the approach to understand how it works for very large number of shards and a very large number of unlearning requests. We then analyze how slicing helps.

1) Impact of Sharding: As discussed earlier, increasing the number of shards (S) in the ensemble provides an expected unlearning speed-up (refer § V). However, we wish to understand the impact of sharding on accuracy. To this end, we utilize SISA training for a large number of unlearning requests.

From Figures 3 and 4 we make the following observations: (a) by increasing the number of shards to a value greater than 20, we observe a more noticeable decrease in accuracy (greater than 5 percentage points) and (b) increasing the percentage of unlearning requests (as a function of the dataset) beyond 20% causes a steeper drop in accuracy. Both these cases can be attributed to the decreasing volumes of data. Increasing the number of shards translates to lesser volumes of data in each shard. If the number of shards is greater than 20, we observe that the weak learner challenge identified in § IV manifests itself. Similarly, increasing the number of unlearning requests results in a decrease in the number of data points per shard (after unlearning), resulting in an accuracy degradation.

The key takeaway is that it is essential to ensure each shard has sufficiently many data points (before and after unlearning) so that the accuracy of the constituent models is high.

2) Impact of Slicing: From Figure 5 we observe that slicing does not have detrimental impact on model accuracy in comparison to the approach without slicing if the training time is the same for both approaches. We ensure that training time is the same by setting the number of epochs for slicing based on the calculations in § IV. Combined with the analysis in § V it is clear that slicing reduces the retraining time so long as the storage overhead for storing the model state after each slice is acceptable (which is linear in the number of slices).

B. Understanding the Regime

The results presented in § VII-A are exhaustive, and cover a diverse number of shards, slices, and unlearning requests. However, not all these configurations are interesting, as some have a detrimental impact on accuracy (as discussed above). By fixing the number of shards based on our analysis earlier,
we can bound the accuracy degradation. However, we wish to understand if there are improvements in retraining time for any number of unlearning requests given this fixed number of shards. Our time analysis in §V suggests otherwise. Based on this analysis, we plot the time required for retraining as a function of the number of retraining requests (refer to Figure 6a). We observe that for both datasets, the regime where the SISA training approach provides the most retraining benefits is when the number of unlearning requests (as a function of the size of the total dataset) is less than 0.075 i.e. the percentage of data to unlearn $\leq 0.075\%$ of the dataset. If the number of unlearning requests exceeds this value, then the SISA training approach gracefully degrades to the performance of the naive baseline. Utilizing only slicing, we observe that the regime where slicing provides gains is when the number of unlearning requests is less than 0.003% of the dataset (refer Figure 6b). Thus, to extract benefit from both approaches, the ideal number of unlearning requests would be the minimum of the two. While the regime we provide gains in ($\leq 0.003\%$) may seem very small, recent work by Bertram et al. [19] shows that in practice, the number of unlearning requests (as a function of the size of the total dataset) is much smaller, and is in the order of $10^{-6}$. Additionally, large organizations operate on datasets which are much larger than those in our experiments; the (narrow) regime in which SISA training provides a benefit still provides significant cost reductions.

Consistent with our expectation, the combination of slicing and sharding does not decrease accuracy in the regime under consideration (see Figures 9 and 10 in Appendix).

VIII. DISTRIBUTIONAL KNOWLEDGE

In this section, we relax our assumptions and discuss how additional knowledge of the distribution of unlearning requests can be beneficial to the service provider. Specifically, we wish to understand (a) if we can estimate those data points that are more likely to be unlearned than others based on auxiliary information and (b) how this knowledge can be used a priori to minimize the retraining time and accuracy degradation.

We believe that an owner’s request for unlearning may vary depending on (a) how their data is used and by whom the data is used, (b) the general perception of the surrounding (geographic) population, and (c) incidents related to data misuse etc. For example, machine learning models are not adept at dealing with bias; data owners from those populations...
who are biased against may wish to request for their data to be erased. As before, we assume the existence of a data owner $u \in U$, and the data point generated by $u$ to be $d_u$. We denote the probability of user $u$ requesting to have their data erased as $p(u)$. By aggregating users who are likely to request data erasure into shards of small sizes, intuitively, we would be able to reduce the retraining time.

To illustrate, consider a population split between two groups: the first group $H$ having a high probability $p_H$ of being unlearned and the second group $L$ having a low probability $p_L$ of being unlearned, with $p_H \gg p_L$. If we follow the uniform sharding of § IV each shard will contain points from both groups $H$ and $L$. Because points from $H$ are very likely to be unlearned, and each shard contains at least a few points from group $H$, it is very likely that all shards will have to be unlearned—even if the number of unlearning requests is low. This scenario is illustrated in Figure 7. Alternatively, if we know the population will follow such a distribution of unlearning requests, we can adapt our sharding strategy to concentrate all points from members of group $H$ in relatively few partitions. This strategy ultimately reduces the total number of shards whose models need to be retrained. We now apply this intuition to a more realistic scenario.

**A. Realistic Scenario**

Modeling realistic distributions of unlearning requests is a challenging proposition; prior work in this space is limited. Without data to determine the parameters for a known distribution, such as a Gaussian, or to learn an underlying distribution, we design the following scenario based on insight from the recent work published by Google [19]. Specifically, we propose a scenario where we assume that an organization with access to data records from a large number of data owners operates across various countries, with owners in each region having varied privacy expectations. We assume the existence of $N$ countries with varied populations; the dataset $D$ comprises of per-country datasets $D_c$ for each country $c$. Thus, we have $\cap_c D_c = \emptyset$ and $\cup_c D_c = D$. Each data owner in the country $c$ has a fixed probability (denoted $p_c$) for issuing a data erasure request i.e., $\forall d_u \in D_c, p(u) = p_c$. Thus, the data owner issuing an unlearning request can be modeled as a Bernoulli trial.

It is important to note that this technique can be generalized to any distribution so long as it is known by the service provider. Specifically, after selecting a distribution $\nu$ that models the unlearning requests from a population $U$, we randomly sample from this distribution to assign the probability $p(u)$ with which each $u \in U$ wishes to perform data erasure. Each data point is still a Bernoulli trial; however, the sum of these independent Bernoulli trials can be modelled by a Poisson binomial distribution. Armed with this knowledge, we can evaluate the expected number of unlearning requests for this shard $D_i$, over $n$ trials, as $E[X_i] = np_i$, where $p = \frac{\sum_{u \in U} p(u)}{|D_i|}$, and $E[X_i]$ denotes the expectation with which shard $D_i$ is unlearned. By selecting those users $u \in U$ and their corresponding data elements $d_u$, to create shard $D_i$, such that $E[X_i] < C$ for any constant $C \leq 1$, we expect to not have to retrain a model trained using shard $D_i$. DNNs typically require large data volumes for training; we attempt to create few data shards, with more data in each shard.

**B. Distribution-Aware Sharding**

a) Approach: This motivates distribution-aware sharding, where the service provider can create shards in a way so as to minimize the time required for retraining. We discuss one such approach in Algorithm [1] under the following assumptions: (a) the distribution of unlearning requests is known precisely, and (b) this distribution is relatively constant over

![Graph showing impact of sharding and slicing on retraining time](image-url)

Fig. 6: Impact of sharding and slicing on retraining time, as measured by the changes induced in the number of points needed for retraining (which is a proxy for retraining time). Observe that below a particular number of unlearning requests, both sharding and slicing provide noticeable improvements. Afterwards, both gracefully degrade to the performance of the baseline.

![Graph showing distribution of unlearning requests](image-url)

Fig. 7: Example of how a service provider aware of the distribution of unlearning requests may adapt to outperform uniform sharding.
a time interval. Recall that each data point \(d_u \in D\) has an associated probability \(p(u)\) with which it may be erased. We first sort the data points in the order of their erasure probability, and points to a shard \(D_t\) till the desired value of \(\mathbb{E}[D_t]\) is reached. Once this value is exceeded, we create a new shard \(D_{t+1}\) and restart the procedure with the residual data \(D \setminus D_t\).

By enforcing a uniform cumulative probability of unlearning across shards, Algorithm 1 naturally aggregates the training points that are likely to require unlearning into a fewer shards that are also smaller in size.

### Algorithm 1 Distribution-Aware Sharding

**Input:** Dataset \(D\), constant \(C\)

1. **procedure** ShardData\((D, C)\)
2. \(\text{sort } \{d_u\}_{i=1}^{|D|} \text{ by } p(u)\)
3. \(i \leftarrow 0\)
4. create empty shard \(D_i\)
5. for \(j \leftarrow 0\) to \(|D|\) do
6. remove \(d_u\) with lowest \(p(u)\) from \(D\)
7. \(D_i = D_i \cup d_u\)
8. if \(\mathbb{E}[x_i] \geq C\) then
9. \(D_i = D_i \setminus d_u\)
10. \(i \leftarrow i + 1\)
11. create empty shard \(D_i\)
12. \(D_i = D_i \cup d_u\)
13. end if
14. end for
15. end procedure

b) Results: As done for our motivating example, Figure 8 plots the number of points to be retrained with respect to the number of unlearning requests for both uniform and distribution-aware sharding. In expectation, the distribution-aware strategy decreases the number of points to be retrained. Yet, because this strategy creates shards of unequal size, we also need to evaluate the accuracy of our predictions aggregated across constituent models. We find that the aggregate achieves about 94.4% prediction accuracy in the regime of unlearning requests we consider, which is one percent point lower than uniform sharding, at 95.7%. This result means that distribution-aware sharding incurs a trade-off of accuracy for decreased unlearning overhead. We leave to future work the exploration of alternatives to majority voting aggregation that would cope with imbalanced shard sizes induced by distribution-aware sharding, to bridge this gap in accuracy.

### IX. Discussion

**Unlearning in the Absence of Isolation.** Conceptually, SISA training borrows elements from distributed training and ensemble learning. As discussed earlier, the divide from ensemble learning stems from the fact that each constituent model in SISA training is obtained in isolation. Ensemble learning approaches utilize boosting algorithms \([46]\), even for ensembles of neural networks \([47]\), to enhance accuracy.

**Data Replication.** Thus far, we have assumed that all shards are disjoint. Empirical evidence suggests that beyond a certain data volume (i.e. shard size), there is performance degradation in each constituent model when datasets are too small. One way to alleviate this problem without collecting more data is through data replication. There are several considerations for data replication. One must decide which data point is replicated such that the accuracy of the constituent models is increased. This selection is a challenging problem, akin to the issues that underlie active learning \([48]\). One must also factor in if access to the replicated data point is likely to be revoked; if that is the case, one would intuitively wish to reduce the replication of such a point to limit overhead on unlearning. Understanding these trade-offs is of interest and is future work.

**Is All Data Useful?** Neural networks require large datasets. However, not all of this data is useful \([49]\). As discussed earlier, understanding the importance of each data point towards the final model parameters learned is a challenging problem. A relatively simpler problem is that of core-set selection, where the objective is to choose a subset of the dataset that will produce a hypothesis that is as performant as one obtained while using the entire dataset \([50], [51]\). Core-sets can help reduce the cost of learning; consequently, they can also improve the cost of unlearning as well.

**Verified Unlearning.** We assume that the service provider performs unlearning in an honest manner. Our approach provides an intuitive and provable guarantee under the assumption that the data owner believes the service provider, due to the inherent stochasticity in learning (refer Figure 1). However, under certain adversarial settings, this trust need not be the case. As stated earlier, there is no way to measure the influence of a data point on the model parameters. Even worse, these models are often proprietary. Thus, understanding if the unlearning procedure can be verified, similar to approaches in other domains \([52], [54]\), is of merit.

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Appendix

A. Impact of both Sharding and Slicing

Observe that, as expected, a combination of both slicing and sharding do not degrade the accuracy (refer Figures 9 and 10).

B. Simulation of Sharding Time Analysis

We simulate the time analysis described in §V-A to create a visualization of the sharding time analysis. This provides the reader with intuition as to the regime where SISA training provides greatest benefits (the region where the two curves do not meet).

C. Minimum of Draws from a Uniform

Let us write $\chi_0, \ldots, \chi_{K-1}$ the $K$ draws we make from a uniform $U(\alpha, \beta)$. We would like to compute the expectation of the minimum of these draws, which we write $M = \min(\chi_0, \ldots, \chi_{K-1})$.

Proof. Our proof follows material found online [55]. We begin by writing down the cumulative distribution function of $M$:

$$F_M(x) = P(M \leq x) = 1 - P(M > x) = 1 - P(\chi_0 > x)P(\chi_1 > x) \ldots P(\chi_{U-1} > x) = \begin{cases} 0 & \text{if } x < \alpha \\ 1 - \left(\frac{\beta - x}{\beta - \alpha}\right)^U & \text{if } \alpha \leq x \leq \beta \\ 1 & \text{if } x > \beta \end{cases}$$

Where the penultimate line holds because the draws are independent. We now take the derivative and obtain the density function:

$$f_M(x) = \begin{cases} \frac{\beta - x}{\beta - \alpha}^U - \frac{\beta - \alpha}{(\beta - \alpha)^U} & \text{if } \alpha \leq x \leq \beta \\ 0 & \text{if } x < \alpha \text{ or } x > \beta \end{cases}$$
To obtain the expectation of \( M \), we integrate:

\[
E[M] = \int_{-\infty}^{\infty} xf(x)dx = \frac{\beta + Kl}{K+1}
\]

(3)

**D. Sequential Time Analysis of Sharding**

**Proof.** Assume we have to retrain \( N - 1 - j \) points. There are \((i-1)\) ways to choose those \( j \) points and place them in the retrained shard. Once they are chosen, all the other points being unlearned have to be scattered among the \((S - 1)\) remaining shards. The probability of unlearning those \( N - 1 - j \) points on a specific shard is \((i-1)\)\((S - 1)\)\(^{i-1}\). Thus, the expected cost of an unlearning step is \( \sum_{j=0}^{K} (i-1) \sum_{j=0}^{K} (S - 1)\)\(^{i-1}\)\((S - 1)\)\(^{-j}\)\((S - 1)\)\(^{-j}\)

We can sum the expected cost of all steps to compute the expected total cost of the sequence (linearity of the expected value).

\[
\sum_{i=1}^{K} \sum_{j=0}^{K} (i-1) \sum_{j=0}^{K} (S - 1)\)\(^{i-1}\)\((S - 1)\)\(^{-j}\)\((S - 1)\)\(^{-j}\)
\]

We can use the fact that \( j^{(i-1)} = (i-2)\)\(^{(i-1)}\) and apply the binomial theorem to both inner sums after reindexing the second inner sum.

\[
= (S - 1)\)\(^{-j}\)\((K-1)\)\(^{-j}\)
\]

(4)

**E. Batched Time Analysis of Sharding**

**Proof.** We denote \( S \) the number of shards and \( K \) the number of points in the batch. Let \((s_i)_{i \in [1,K]}\) be random variables that give the index of the shard hit by each point in the batch. We assume that those variables are iid. and that:

\[
\forall i \in [1,K], s_i \sim U(0,S)
\]

We can define \((h_j)_{j \in [1,S]}\) which are Bernoulli random variables whose value is one when the shard \( j \) is hit. Obviously, we have:

\[
h_j = 0 \iff \forall i \in [1,K], s_i \neq j
\]

Thus \( P(h_j = 0) = \left(1 - \frac{1}{S}\right)^K \). We define the total cost of retraining as the number of points that need to be retrained while processing the batch. We can easily derive an approximation of the cost by assuming the shards remain balanced this is true when \( K << N \):

\[
C = \sum_{j=1}^{S} h_j |\text{shard}_{j}| = \sum_{j=1}^{S} h_j \frac{N - K}{S}
\]

And using the expected value of a Bernoulli random variable and the fact that the expected value is linear:

\[
E(C) = (N - K) \left(1 - \left(1 - \frac{1}{S}\right)^K\right)
\]

Though this approximation is close enough to the real value, deriving a more accurate formula is fairly easy. We first define \((u_j)_{j \in [0,K]}\), the random variables that count the number of hits for each shard. They count the number of successes in a repetition of independent Bernoulli experiments, namely counting the number of times \( s_i = j \), when \( i \) varies from 1 to \( U \). Thus:

\[
\forall j \in [1,S], u_j \sim B(K, \frac{1}{S})
\]

Furthermore, the random vector \( u \) is clearly multinomial: each component counts the number of successes for a specific class (hit a certain shard). This is comparable to rolling a \( S \)-faces die \( K \) times. The expected cost becomes:

\[
C = \sum_{j=1}^{S} \frac{N}{S} h_j \left(\frac{N}{S} - u_j\right) = \sum_{j=1}^{S} \left(\frac{Nh_j}{S} - u_j h_j\right)
\]

By construction,

\[
h_j = 0 \iff u_j = 0
\]

Thus \( u_j h_j = u_j \) and:

\[
C = \sum_{j=1}^{S} \left(\frac{Nh_j}{S} - u_j\right)
\]

Using the linearity of the expected value and the expected values of Bernoulli and binomial random variables,

\[
E(C) = \sum_{j=1}^{S} \left(\frac{N}{S} \left(1 - \left(1 - \frac{1}{S}\right)^K\right) - \frac{K}{S}\right)
\]

\[
= N \left(1 - \left(1 - \frac{1}{S}\right)^K\right) - K
\]

□