Analysis of size effects on the Hoek-Brown failure criterion of intact granite samples

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Abstract. Scale or size effects of the uniaxial strength response in rock samples have been studied in detail in the past, so a good number of studies on different rocks are available. However, analyses on triaxial strength scale effects in rocks are scarce and they seldom address failure criteria (i.e. Hoek-Brown) evolution with specimen size. This obvious lack of data can be attributed to the difficulties of having available Hoek’s cell of different sizes. With the aim of filling this void, the authors have carried out sets of around 25 stress-strain triaxial tests on intact 30, 38, 54 and 84 mm diameter granite specimens, with various confinements (0.2 to 15 MPa), so reliable estimates of Hoek-Brown strength were obtained for every scale. We compare results with previous studies on UCS scale effects, showing a good correlation. Results suggest the studied granite undergoes a reverse size effect in terms of strength at low confinements. Indeed, the UCS increases as sample diameter increases up to around 50 mm, but decreases thereafter. However, results obtained put forward that this strength variation with scale tends to be mitigated for higher confinements where the scale effect may not be clearly recognised. So increased confinement can be associated with a decreased scale dependency component of strength.

1. Introduction
The mechanical behavior of rocks at micro and macroscopic scale is complex and inhomogeneous, something the authors attribute to geologic conditions, tectonic and weathering effects as well as to hydro-thermo-chemical processes. This is why it is not surprising that the mechanical response of rock specimens depends on the volume of material being loaded or strained. That is, rock strength shows significant scale effects. These effects are not well understood, yet they arise pervasively in rock mechanics and engineering practice in the determination of rock, joint and rock-mass properties.

Pinto da Cunha [1] emphasized the 3 different causes for mechanical parameter variation in rocks and rock masses linked to scale including: a) effects associated with jointing, typical of rock masses, b) shape effects caused by changes in system geometry and typically in specimen slenderness (a height to diameter ratio of at least 2:1 is recommended as per ISRM [2]) and c) intact rock size effects occurring unrelated to effects associated with geometry or jointing, which are the focus of this study.

Bieniawski [3] in cubic coal samples and other authors [4-6] in hard rock cylindrical specimens were among the first to study scale effects on UCS tests. More recent investigations [7,8] have further studied this issue showing that scale effects significantly vary in different rock types according to their strength, texture, occurrence of micro-flaws or alteration. All these studies suggest a decrease in intact rock strength with increasing specimen size in line with “weak-link” theories [9]. On the other hand, Masoumi et al [10] and Quiñones et al [11] observed that for unconfined strength tests the size-effect behavior of small specimens does not follow a commonly assumed size-effect model, in which the strength decreases as the specimen size increases. This observation effect was thought to be associated to large grain size in relation to sample size.
All of the cited studies focus on changes in peak strength under unconfined conditions. Brace [12] long ago noted the lack of knowledge on the effect of size on friction and, this seems to still be the case today. Hunt [13] and Medhurst and Brown [14] studied this behavior for gypsum and coal, respectively, sedimentary rocks behaving very differently to granite, and apart from this, only in a few cases the scale effect on triaxial tests has been reported [15, 16]. Masoumi [15] carried out triaxial compressive tests on Gosford sandstone specimens with 25, 50 and 96 mm diameters. The resulting size effect trends of peak strength for different confining pressures were similar to that observed for the UCS results, in which the peak strength increased and then decreased with an increase in size. The size dependency of the $m$ parameter [17] was assessed and showing a mild decrease with sample size. Walton [16] analyzed scale effects on triaxial tests in Stanstead granite specimens with diameter from 43 to 101 mm focusing on frictional and post-peak properties of intact rock. With respect to strength and disregarding smaller samples, the finding that UCS decreases with increased specimen size was consistent with classic studies. He also concluded, unlike the results reported by Masoumi [15] for sandstone, that the $m$ Hoek-Brown’s parameter increased with sample size. He identified an increased confinement dependency component of strength, but he was cautious in definitely stating it since other studies do not agree with this conclusion [18].

It seems that scale effects on triaxial tests are still not well understood, something mainly due to the scarcity of experimental data, as stated by Brace [12] long time ago, but that seems to still be the case [19]. Therefore, our main goal is trying to extend the available database by performing a significant amount of standard triaxial compressive strength tests on 30, 38, 54 and 85 mm diameter specimens of Blanco Mera granite. The aim of this testing program is having available relevant information concerning the scale effect on the triaxial strength of this already well-tested rock. This will be used to obtain realistic scale laws for analytical approaches and as a basis for matching numerical models [20].

2. Experimental methodology

The testing program involves collecting data from tests on 4 size samples of Blanco Mera granite, a granular rock from NW Spain. This rock presents 20% quartz, 27% alkali feldspar, 35% plagioclase feldspar, 7% muscovite, 5% biotite, 4% chloride and 1% sericite. The grain size distribution is scattered with quartz grain sizes from 1 to 6 mm, feldspar crystals from 1 to 3 mm and smaller micas. This rock was tested in our lab for different purposes [21, 22], including studying UCS scale effects [11].

A large number of rock blocks where acquired, and around 30 cylindrical specimens were obtained for each group of sizes including 30, 38, 54 and 84 mm diameter specimens with length-to-diameter ratios of roughly 2. Standard triaxial tests were carried out for each group. All 4 test sets included 4 individual test groups for 0.2, 5, 10 and 15 MPa and 2-3 test groups for 2.5, 7.5 and 12.5, which makes about 25 tests for each set. Figure 1.a illustrates 4 typical samples for triaxial compressive testing.

Standard Hoek’s cells and sleeves were available for the 30 and 54 mm samples. The Hoek’s cell for the 38 mm and 84 mm samples were devised for this study together with sleeves (figure 1.b and 1.c). Some difficulties arose associated to the preparation of the larger sleeves to make them work properly without leakage. There exist now triaxial cells adaptable to different specimen sizes such as that provided by GCTS [15], but still for these cases, sleeves and platens should be adapted. Since they may not work so well in some cases (post-failure phase of the tests), we opted to work with traditional cells.

To perform compressive tests, a standard Servosis 2,000 kN press framework was used (figure 1.d). The system consists of a load cell, extensometers for displacement, load frame, hydraulic pumps for confinement control, test controller, test processor and a PC. The press works through a hydraulic pump pushing the bottom plate upward. The servo-control system was installed to manage the load or deformation rate. The rotation speed of the servomotor is adjusted automatically to provide the speed and pressure ordered by the control. The information is provided to the program by several available sensors, which were included in system. The control software used in this study was PCD2K, which was designed to work for standard conditions or to tailor tests according to user needs.
For the triaxial tests carried out, 2 radial and 2 longitudinal strain gauges were glued on every sample for strain recording. Additionally, the plate to plate distance was measured by means of 2 LVDTs and the volume of fluid entering and exiting the Hoek’s cell was also recorded with the idea of facilitating deformation analysis after failure.

Unloading-loading cycles were performed during testing, so evolving failure criteria and dilatancy on rock specimens could be derived from test results. In this analysis, only peak strength data will be analyzed in relation with specimen scale.

3. Results

Peak strength results for the 98 tests carried out are presented in table 1. In table 2, the mean of the peak strength results for each specimen’s size and each confinement tested are compiled, together with the coefficient of variation (standard deviation over mean value) expressed in percentage form.

4. Results interpretation

The peak strength results obtained will be interpreted in what follows regarding the intact rock scale effect. Figure 2.a graphs the mean peak strength results in relation to specimen scale for the different confinements tested. The confinement effect is clearly showing typical larger values for higher confinement, even if there is a crossing of the curves corresponding to 2.5 and 5 MPa undoubtedly due to natural variability.

In this graph of figure 2.a, it can be observed the double scale effect described by Masoumi [15] for the lower confinement cases. It consists on a strength increase from smaller samples (30 mm) to typical size samples (54 mm) and a forward strength decrease from this size to larger samples (84 mm). While this effect is clear for lower confinements, for higher confinement (10 to 15 MPa), it is not so significant, something well illustrated in figure 2.b, where the mean peak strength values obtained have been normalized in relation to the mean value of the standard size (54 mm diameter) specimens. It is clear in this graph a trend towards mitigating the scale effect for larger confinements, that is, strength tends to be more independent of scale sample for high confinement levels, which seems not to be in line with findings reported by other authors, such as Walton [16].
### Table 1. Values of peak strength for the triaxial compression tests.

| 30 mm | 38 mm | 54 mm | 84 mm |
|-------|-------|-------|-------|
| \( \sigma_3 \) [MPa] | \( \sigma_1 \) [MPa] | \( \sigma_3 \) [MPa] | \( \sigma_1 \) [MPa] | \( \sigma_3 \) [MPa] | \( \sigma_1 \) [MPa] | \( \sigma_3 \) [MPa] | \( \sigma_1 \) [MPa] |
| 0.2 | 102 | 0.2 | 123.25 | 0.2 | 141.78 | 0.2 | 61.40 |
| 0.2 | 75.24 | 0.2 | 119.9 | 0.2 | 108.74 | 0.2 | 89.31 |
| 0.2 | 100.2 | 0.2 | 125.46 | 0.2 | 147.68 | 0.2 | 74.46 |
| 0.2 | 104.2 | 0.2 | 111 | 0.2 | 126.87 | 0.2 | 118.05 |
| 2.5 | 147.8 | 2.5 | 163.68 | 2.5 | 191.67 | 2.5 | 125.94 |
| 2.5 | 87.57 | 2.5 | 141.62 | 2.5 | 188.14 | 2.5 | 143.39 |
| 2.5 | 165.17 | 2.5 | 141.65 | 2.5 | 158.05 |
| 5 | 150.26 | 5 | 201.11 | 5 | 140.7 | 5 | 177.78 |
| 5 | 169.98 | 5 | 158.28 | 5 | 177.6 | 5 | 177.86 |
| 5 | 163.42 | 5 | 196.89 | 5 | 169.6 | 5 | 154.90 |
| 5 | 200.3 | 5 | 154.14 | 5 | 169.63 | 5 | 143.74 |
| 7.5 | 121.76 | 7.5 | 228.27 | 7.5 | 209.18 | 7.5 | 192.64 |
| 7.5 | 196.52 | 7.5 | 162.06 | 7.5 | 246.51 | 7.5 | 206.92 |
| 7.5 | 211.46 | 7.5 | 236.28 | 7.5 | 191.75 | 7.5 | 241.75 |
| 10 | 228.44 | 10 | 234.45 | 10 | 231.38 | 10 | 222.77 |
| 10 | 206.27 | 10 | 245.39 | 10 | 250.71 | 10 | 200.01 |
| 10 | 249.75 | 10 | 250.61 | 10 | 257.82 | 10 | 202.42 |
| 10 | 242.03 | 10 | 244.43 | 10 | 200.66 | 10 | 240.37 |
| 12.5 | 275.14 | 12.5 | 276.39 | 12.5 | 254.42 | 12.5 | 235.88 |
| 12.5 | 236.22 | 12.5 | 275.96 | 12.5 | 267.72 | 12.5 | 259.18 |
| 15 | 305.35 | 15 | 296.49 | 15 | 223.98 | 15 | 302.62 |
| 15 | 294.73 | 15 | 288.05 | 15 | 295.28 | 15 | 295.40 |
| 15 | 285.33 | 15 | 287.53 | 15 | 274.29 | 15 | 242.11 |
| 15 | 293.66 | 15 | 302.15 | 15 | 318.75 | 15 | 274.08 |

### Table 2. Mean values and coefficients of variation of strength for every size and confinement.

| 30 mm | 38 mm | 54 mm | 84 mm |
|-------|-------|-------|-------|
| \( \sigma_1 \) [MPa] | Mean [MPa] | CV | Mean [MPa] | CV | Mean [MPa] | CV | Mean [MPa] | CV |
| 0.2 | 95.4 | 14.2% | 119.9 | 5.3% | 131.3 | 13.2% | 85.8 | 28.4% |
| 2.5 | 117.7 | 36.2% | 156.8 | 8.4% | 173.8 | 16.1% | 142.5 | 11.3% |
| 5 | 171.0 | 12.4% | 177.6 | 14.0% | 164.4 | 9.9% | 163.6 | 10.4% |
| 7.5 | 176.6 | 27.2% | 208.9 | 19.5% | 215.8 | 13.0% | 213.8 | 11.8% |
| 10 | 231.6 | 8.2% | 243.7 | 2.8% | 235.1 | 10.9% | 216.4 | 8.8% |
| 12.5 | 255.7 | 10.8% | 280.8 | 2.9% | 261.1 | 3.6% | 239.8 | 7.4% |
| 15 | 297.7 | 29.0% | 293.6 | 2.4% | 278.1 | 14.5% | 278.6 | 9.7% |
| Average | 19.7% | 7.9% | 11.7% | 12.5% |
Figure 2. a) Triaxial strength for different confinements and scales and b) Similar graphs where triaxial strength values are normalized.

To obtain a more consistent interpretation of strength, the authors have fit the Hoek-Brown failure criterion parameters to the triaxial strength results obtained for every size group of samples. The values of Hoek-Brown peak strength $\sigma_{ci}$ and $m$ are computed according to the classic peak Hoek-Brown failure criterion for intact rock where $s=1$ [5]:

$$\sigma_1 = \sigma_3 + \sigma_{ci} \left( \frac{m}{\sigma_{ci}} \sigma_3 + 1 \right)^{0.5}$$

This equation can be rewritten as:

$$(\sigma_1 - \sigma_3)^2 = m \sigma_{ci}^2 \sigma_3 + \sigma_{ci}^2$$

And changing the variables to $y = (\sigma_1 - \sigma_3)^2$ and $y = \sigma_{ci}$, the equation becomes:

$$y = Ax + B,$$

where $\sigma_{ci} = \sqrt{B}$ and $m = \frac{A}{\sigma_{ci}}$.

Based on these equations, $\sigma_{ci}$ and $m$ can be obtained by fitting a line to the transformed data set. The results of the fits including the parameters and regression coefficients together with the graphed original data in $\sigma_1$-$\sigma_3$ axes are presented in figure 3 for every specimen size. The scattering of the data can be observed visually and correlated with the provided coefficients of regression.

Figure 3 shows how uniaxial compressive strength grows from 30 mm, to 38 mm and to 54 mm, and then decreases. For 15 MPa confinement all results are similar unlike for lower confinement cases. Since the peak strength for high confinements is similar in all cases, but the unconfined strength is lower for smaller samples, then it reaches a maximum and then it decreases again, it is not surprising that the parameter $m$ values follows a trend opposite to that of $\sigma_{ci}$ in relation to specimen diameter. That is, $m$ is very high for 30 mm samples ($m=76.1$), it diminishes for 38 mm samples and even more so for 54 mm samples ($m=29.0$), but it grows for larger 84 mm diameter samples ($m=42.4$).

This can be tracked in figure 4.a, where the failure criteria corresponding to every size have been graphed together. Whereas a significant scattering is observed for low confinement levels, all curves tend to converge for higher strength levels. Figure 4.b shows the described trends of Hoek-Brown parameters ($\sigma_{ci}$ and $m$) with specimen size.
In what concerns uniaxial compressive strength, obtained results are compared against a previous study on the scale effects on Blanco Mera granite samples [11]. In that study, UCS results were fit to a unified size effect law (USEL) as considered by Masoumi et al [10] for unconfined strength of intact rock. The set of unconfined compressive strength of Blanco Mera granite samples are graphed in figure 5, together with the fit of the so-called USEL approach. The compressive strength results obtained for low confined samples (0.2 MPa) in this study are also graphed in this same figure. The authors have also plot the values corresponding to the fits of $\sigma_{ci}$ for every sample size tested. It is clear that the
observed trends are the same as those reported in previous studies, even if due to the natural variability of the rock (particularly relevant for unconfined and small cases, as demonstrated by the values of the coefficients of variation in table 2), results from the two, differently focused, studies are slightly different.

![Graph](image)

**Figure 5.** UCS test results on Blanco Mera granite samples and USEL fit to those results (Quiñones et al (2017)) and low confinement results and fit $\sigma_{ci}$ values according to this study.

5. **Conclusions**

Even if a good number of studies have analysed scale effects on unconfined intact rock strength, only a few have focused on triaxial strength. In this study, about 25 triaxial strength tests at different confinements ranging from 0.2 to 15 MPa have been performed for four different sample sizes from 30 to 84 mm diameter. Only peak strength is analysed within the framework of this study.

Based on the strength values of these tests, Hoek-Brown failure criterion envelopes have been fit to strength results for every size and the corresponding parameters, intact rock strength ($\sigma_{ci}$) and $m$, have been computed. Intact rock strength values show a trend similar to that detected for unconfined tests where a general decreasing trend of strength with sample size is observed over a threshold sample scale corresponding to 40-50 mm diameter sample. For smaller samples, the opposite trend is observed, so strength increases with sample size. This double trend has been observed in other rocks. Whereas the decreasing trend is attributed to the so-called weak-link theory, where flaws in the sample can be easily found for larger specimens, the general although scattered inverse scale effect in smaller samples is thought to be associated to effects of large grain sizes in relation to sample sizes.

Not many results were available regarding the evolution of the parameter $m$ with sample size. The authors have found a trend opposite to that of intact rock strength. That is in the range 30 to 54 mm samples the $m$ parameter tends to decrease with scale, whereas for larger samples the parameter tends to grow with scale. Results show that strength at high confinements tends to be more or less equal regardless sample size as shown by Habib and Vouille [18], so the trend of the parameter $m$ related to friction will produce larger values when the unconfined strength is low (for small, but also for large samples) and smaller for higher unconfined strengths (standard size). Such a result has not been clearly reported so far, so further experimental evidence will contribute to check its validity.
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