Influence of Seismic Relative Wavelength on Strain Transfer Rate of Cross-section of Underground Structure

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Abstract: The seismic response of underground structures in nonlinear foundations was analyzed by using FLAC 3D software, and the effect of relative wavelength on the transverse strain transfer rate of structures is studied. The variation processes of the strain transfer rate can be divided into three distinct phases. In the first stage where the relative wavelength raises from 0.2 to 1.0, the strain transfer rate increases almost linearly with it; while in the second stage where the relative wavelength increases from 1.0 to 2.0, the growth trend is slowing down. And the strain transfer rate almost keeps a constant value during the last phase, in which stage the uniform strain field of soil can be used to analyze the strain transfer rate. According to the influence of relative wavelength and relative stiffness on the strain transfer rate, the calculation formula of the strain transfer rate is summarized.

1. Introduction

The underground structure must be able to withstand the static cover load and additional deformation caused by seismic motion. For various underground structures such as tunnels with circular or rectangular cross section, the most critical requirement is to withstand shear waves perpendicular to the tunnel axis [1], which can cause distortion of the tunnel section and generate axial forces and bending moments as well as displacements.

At present, the seismic responses of underground structures are mainly studied with two methods. One is the free field method, which takes the free field deformation of soil layer at the same position as the deformation of underground structure directly, thereby calculating the internal force. However, this assumption may lead to extremely conservative designs, especially among rigid structures in soft media. The second method is the soil-structure interaction method [2], which considers that underground structures have changed the free field deformation of the surrounding ground, and the structural response depends on the relative stiffness between the soil and structures. In this study, based on the deformation control principle, a calculation method of shear strain transfer rate of cross-section (namely, strain transfer rate) is proposed to provide a seismic response evaluation method for underground structures.

Strain transfer rate is the ratio of strain of underground structure to that of soil at the same position of structure, which has been studied by several Scholars. Zhang J Y [3] analyzed the influence of seismic wave parameters, soil and structure characteristics on strain transfer rate of underground structures by using two-dimensional dynamic finite element program SD4. Han W X [4] obtained the changing regularity of transverse shear strain transfer rate in different cross-section aspect ratio and structure-soil stiffness ratio by two-dimensional static methods. Research results above are consistent with the previous work of this study, showing that the soil-structure stiffness ratio is the key factor affecting the strain transfer rate, but the seismic wave amplitude and structural depth have a tiny
influence on it. However, the effect of ratio of seismic wavelength to structure size (namely, relative wavelength) on the strain transfer rate has not been fully studied.

In this study, the finite difference analysis software FLAC 3D is used to simulate the response of underground structures under earthquake, and the effect of relative seismic wavelength on the strain transfer rate is systematically analyzed. It is assumed that the rectangular structure is a linear elastic body with constrained interface, and the soil is an elastic-plastic body without considering excess pore pressure. Besides, the same interface is used in all analyses to eliminate the interference of additional variables.

2. Strain transfer rate

In this study, the cross section of rectangular structure is selected as the research object, and the rotation of the structure is considered. The maximum shear strain at the top and bottom of one side of the structure are calculated, and then divided by the maximum soil shear strain at the same position to obtain the strain transfer rate of the underground structure. The transverse section in the middle of the structure is selected as research object, and the shear strain transfer rate is defined as:

$$\alpha = \frac{\gamma_s}{\gamma_g}$$

where $\alpha$ is the strain transfer rate, $\gamma_s$ is the maximum shear strain of the structure, $\gamma_g$ is the maximum shear strain of the soil at the same position as the structure. $\gamma_s$ and $\gamma_g$ are defined as:

$$\gamma_s = \max \left\{ \left| \gamma_{s1} \right|, \left| \gamma_{s2} \right|, \ldots, \left| \gamma_{sn} \right| \right\}$$

$$\gamma_g = \max \left\{ \left| \gamma_{g1} \right|, \left| \gamma_{g2} \right|, \ldots, \left| \gamma_{gn} \right| \right\}$$

where, $\gamma_{g1}$ and $\gamma_{s1}$ are the shear strain of soil and structure in a certain dynamic time step, respectively. And the calculation formula of $\gamma_{gi}$ is:

$$\gamma_{gi} = \frac{\Delta L_i}{H}$$

where, $\Delta L_i$ is the horizontal displacement of the top of soil relative to the bottom of soil on the same side at a certain dynamic time step, $H$ is the height of structure, as shown in Figure 1.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Diagram of soil shear strain.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Diagram of structure shear strain.}
\end{figure}

Considering the rotation of the structure, the calculation formulas of $\gamma_{ai}$ are:

$$\gamma_{ai} = \gamma_{ai} \cdot \gamma_{2ai}$$

$$\gamma_{ai} = \frac{\Delta L_{ai}}{b}$$

$$\gamma_{2ai} = \frac{\Delta L_{2ai}}{a}$$

2
Where, $\Delta L_{1i}$ is the lateral displacement difference between left top point and left bottom point of structure in a certain dynamic time step, $\Delta L_{2i}$ is the vertical displacement difference between right bottom point and left bottom point of structure at a certain dynamic time step, $a$ and $b$ are the width and height of the structure, respectively, as shown in Figure 2.

The maximum shear strain of soil and structure is obtained by formula (2) and formula (3), respectively, and then the strain transfer rate of the structure cross section in the whole vibration process is calculated by formula (1).

3. Computational model and parameters

3.1. Soil and structure model

The rectangular frame underground C30 concrete structure in soft clay is taken as the research object, and the cross section in the middle of the structure is taken as the strain transfer rate analysis plane. According to the literature, the minimum calculation soil area on left and right sides of the cross-section is 2.5 times of the structure height. The upper surface of the structure is twelve meters deep in the soil and the mesh size of the numerical model $\Delta l$ is set at 0.5 meters. The model size of soil and structure is shown in Figure 3.

![Figure 3. Numerical model size of (a) soil and (b) structure (unit: m).](image)

| Density(Kg·m$^{-3}$) | Shear modulus (MPa) | Poisson's ratio | Cohesion(kPa) | Internal friction angle(°) |
|----------------------|---------------------|----------------|--------------|---------------------------|
| 1830                 | 7.69                | 0.3            | 16.8         | 11.2                      |

| Elastic modulus (GPa) | Poisson's ratio | Density (Kg·m$^{-3}$) | Length(m) | Width (m) | Height(m) |
|----------------------|----------------|-----------------------|-----------|-----------|-----------|
| 30                   | 0.2            | 2500                  | 50        | 100       | 25        |

The properties of the soil and structure are shown in table 1 and table 2. To simulate the non-linear characteristic of soil, hysteretic damping is introduced into FLAC3D. Secant modulus attenuation curve model adopts default parameters $L_1$ and $L_2$, according to empirical value. Besides, Rayleigh damping is used to simulate the hysteresis characteristics of soil and Rayleigh damping coefficients is set as follows: by calculating the undamped natural vibration of the model, the natural frequency is used as the center frequency of Rayleigh damping, among which minimum center frequency is 2.5Hz. The parameter of the rock-soil damping ratio is chosen as the minimum critical damping ratio [6]. The dynamic model parameters of soil are shown in Table 3.
Table 3. Dynamic model parameters of soil.

| Parameters | L1 (m/s) | L2 (m/s) | $\omega_{\text{max}}$ (Hz) | $\zeta_{\text{min}}$ |
|------------|---------|---------|-----------------|------------------|
| Values     | -3.156  | 1.904   | 2.5             | 0.05             |

The boundary conditions of the model have two setting types according to the situation of modeling analysis. When the static calculation is performed, the boundary conditions in the x, y, and z directions at the bottom of the model are fixed, and the sides of the model are fixed in the x and y directions. In the dynamic calculation, the absorption constraint seismic energy is realized by removing the z-direction fixed constraint at the bottom of the model, applying a Quiet boundary at the bottom and free field boundaries on both sides of the model to absorb seismic wave energy [5].

3.2. Seismic wavelength

The seismic wavelength varies in a large range with the change of seismic wave frequency and velocity. It is generally believed that the range of seismic wave frequency is 0.1-20 Hz [5] and that shear wave velocity is related to the properties of soil. The soil is assumed as an isotropic ideal viscoelastic body. In this study, in which situation the formula for calculating shear wave velocity is as follows:

$$C_s = \sqrt{\frac{G}{\rho}}$$

(8)

where, $C_s$ is the shear wave velocity, $G$ is the shear modulus of soil, $\rho$ is the soil density. According to Table 1, the calculated initial velocity of the shear wave in soil is 64.8m/s, which varies linearly with the depth of soil. The top and bottom wave velocities are automatically generated by the dynamic analysis module in FLAC 3D. The seismic wavelength can be calculated by the formula:

$$\lambda = \frac{C_s}{f}$$

(9)

where, $f$ is the shear wave frequency. In order to ensure the convergence and accuracy of the numerical calculation, the mesh size must be less than 1/10 of the wavelength corresponding to the input waveform with the highest frequency. Because the mesh size in the numerical model is set to 0.5 meters, the minimum wavelength of shear wave should not be less than 5 meters. According to the formula (4) and (6), it is apparent that the height of the structure is a key parameter for calculating the strain transfer rate. Therefore, the height of the structure (25m) is chosen as the representative size, and the simulated relative wavelength value is shown in Table 4.

Table 4. Selected relative wavelength.

| Wavelength $\lambda$ (m) | 5   | 10  | 15  | 20  | 25  | 37.5 | 50  | 75  | 125 | 250 | 500 | 1250 | 2500 |
|-------------------------|-----|-----|-----|-----|-----|------|-----|-----|-----|-----|-----|------|------|
| Relative wavelength $\hat{K}_\lambda$ | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.5  | 2.0 | 3.0 | 5.0 | 10  | 20  | 50   | 100  |

3.3. Seismic load

The simulated transverse shear seismic wave is a horizontal vibration sinusoidal wave exerted on the bottom of the model, of which the velocity-time formula is:

$$v_s = \lambda \sin(2\pi ft)$$

(10)

where, $v_s$ is the tangential velocity of sinusoidal wave, $\lambda$ is the amplitude of tangential velocity, $f$ is the frequency of vibration wave. The Velocity input cannot be directly exerted on the viscous boundary in dynamic calculation, which needs to be transformed into stress load by the formula:

$$\sigma_s = 2\rho C_s v_s$$

(11)
where, \( \sigma_s \) is the stress loading, \( \rho \) is soil density, \( C_s \) is the propagation velocity of vibration wave. According to formula (8), (10) and (11), the expression of stress load is:

\[
\sigma_s = 2(\rho C_s) v_s = \sqrt{2\rho \lambda} \sin(2\pi f t)
\]

(12)

The vibration direction is horizontal (Y) and the propagating direction is vertical (Z).

4. Result and discussion

The numerical simulation is carried out with the stiffness ratio of the structure at 1.0, 1.5, 2.0 and 2.5, respectively, to obtain the change rules of the strain transfer rate with the relative wavelength, which is shown in figure 4. The variation processes of the strain transfer rate are similar under different stiffness ratios, which can be divided into three distinct phases. In the first stage where \( K_{\lambda} \) raise from 0.2 to 1.0, the strain transfer rate increases almost linearly with it. While in the second stage where \( K_{\lambda} \) increases from 1.0 to 2.0, the growth trend is slowing down. During the first two phases, the strain transfer rates increase from 0.5808, 0.4543, 0.3863 and 0.3400 to 0.9327, 0.7275, 0.6000 and 0.5050 respectively, with the maximum growth rate at 60.48%. In the third stage in which \( K_{\lambda} \) have exceed 2.0, however, strain transfer rate is almost independent of relative wavelength and the biggest growth rate reach merely at 3.48%.

In plane strain problems, it is generally assumed that the seismic wavelength is much larger than the cross section size of the structure, and the soil shear strain shear the same value within the scope of the structure. Fig. 4 shows that when the seismic wavelength is larger than 2.0 times the structure height, the strain transfer rate of the structure can be calculated according to the uniform strain field.

\[
\alpha = (0.58 \cdot K + 0.48)^{-1} - (0.5538 + 0.1299 \cdot K) \left[ 1 + \left( 2.5 \cdot K_{\lambda} \right)^2 \right]^{-1}
\]

(13)

where, \( \alpha \) is the strain transfer rate, \( K_{\lambda} \) is the relative wavelength, \( K \) is the stiffness ratio. The strain transfer rate calculated by formula (13) fit well with the numerical simulation results, as show in Figure 5. When the relative wavelength becomes very large, the second term of the formula is negligible. In this condition, the equation (13) can be simplified as follows:

\[
\alpha = (0.58 \cdot K + 0.48)^{-1}
\]

(14)

It is consistent with the formula proposed by Han W X [4], in whose research the effect of relative wavelength was neglected and the strain field around the structure was assumed to be uniform.
5. Conclusion
The three-dimensional elastic-plastic constitutive model of underground structures is studied by dynamic response analysis method using FLAC 3D software, and the effect of seismic wavelength on the strain transfer rate of cross-section of the structure in non-linear foundation is studied. The numerical simulation results show that the strain transfer rate under different stiffness ratios presents similar changing trends, and can be divided into three stages. The strain transfer rate increases almost linearly with the seismic wavelength in the first stage where $K_{\lambda}$ raises from 0.2 to 1.0, and the growths rate slows down in the second stage where $K_{\lambda}$ increases to 2.0. During the last phase, the strain transfer rate remains almost constant and can be calculated according to the uniform strain field. The empirical formula of strain transfer rate with respect to relative wavelength and stiffness ratio established coincides with the simulated results, which can be simplified when $K_{\lambda}$ is very large, providing a express method to calculate it. It is noteworthy that the strain transfer rate would not achieve at 1 when the stiffness ratio comes to 1, which may be caused by the viscous boundary effect and the non-linear characteristics of soil and needs to be further studied.

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