Hydraulic resistance in open-channel flows over self-affine rough beds

Supplementary data

Hydraulic conditions covered during experiments

Table S1 Summary of hydraulic conditions in the AOCF flume (for R1 and $S = 0.1\%$)

| $S$ ($\times 10^{-3}$) | $H$ (mm) | $U$ (m/s) | $B/H$ | $H/\Delta$ | $F$ | $\Delta^+\times 10^3$ | $f_H$ | $f_R$ |
|------------------------|----------|-----------|-------|-------------|-----|-----------------------|-------|-------|
| 1.1E-03                | 29.7     | 0.138     | 39.7  | 5.0         | 0.26| 93                    | 0.132 | 0.126 |
| 1.0E-03                | 34.5     | 0.155     | 39.1  | 5.8         | 0.27| 98                    | 0.117 | 0.111 |
| 1.0E-03                | 39.9     | 0.172     | 39.6  | 6.7         | 0.27| 103                   | 0.106 | 0.099 |
| 1.0E-03                | 49.5     | 0.207     | 23.9  | 8.2         | 0.30| 117                   | 0.094 | 0.086 |
| 9.6E-04                | 59.7     | 0.232     | 19.8  | 9.9         | 0.30| 123                   | 0.083 | 0.076 |
| 9.8E-04                | 69.8     | 0.264     | 16.9  | 11.6        | 0.32| 135                   | 0.077 | 0.069 |
| 9.7E-04                | 79.5     | 0.290     | 14.8  | 13.2        | 0.33| 143                   | 0.072 | 0.063 |
| 1.0E-03                | 89.8     | 0.325     | 13.1  | 15.0        | 0.35| 155                   | 0.067 | 0.058 |
| 1.0E-03                | 99.4     | 0.351     | 11.9  | 16.6        | 0.36| 164                   | 0.064 | 0.055 |
| 1.0E-03                | 109.6    | 0.375     | 10.8  | 18.3        | 0.36| 170                   | 0.061 | 0.051 |
| 1.0E-03                | 119.7    | 0.401     | 9.9   | 19.9        | 0.37| 178                   | 0.059 | 0.049 |
| 9.9E-04                | 140.1    | 0.447     | 8.4   | 23.3        | 0.38| 192                   | 0.055 | 0.044 |
| 1.0E-03                | 159.8    | 0.494     | 7.4   | 26.6        | 0.39| 207                   | 0.052 | 0.041 |

$\uparrow\uparrow S$ is the mean water surface slope; $H$ is the mean flow depth; $U=Q/BH$ is the bulk velocity, $Q$ is the volumetric flow rate and $B$ is the flume width; $B/H$ is the aspect ratio; $H/\Delta$ is the relative submergence and $\Delta=4\sigma_z$ is the roughness height; $F=U/(gH)^{0.5}$ is the Froude number and $g=9.81\text{ m/s}^2$ is the acceleration due to gravity; $\Delta^+=\Delta u^*/\nu$ is the roughness Reynolds number, $\nu$ is the kinematic viscosity and $u^*=(gSH)^{0.5}$ is the shear velocity; $f_H=8gHS/U^2$; $f_R=8gRS/U^2$ where $R=BH/(B+2H)$ is the hydraulic radius.
Table S2 Summary of hydraulic conditions in the AOCF flume (for R1 and $S = 0.2\%$)

| $S$ (-) | $H$ (mm) | $U$ (ms$^{-1}$) | $B/H$ (-) | $H/\Delta$ (-) | $F$ (-) | $\Delta^*$ (-) | $f_u$ (-) | $f_k$ (-) |
|---------|-----------|----------------|------------|----------------|--------|--------------|--------|--------|
| 2.0E-03 | 25.2      | 0.161          | 46.9       | 4.2            | 0.32   | 117          | 0.154  | 0.147  |
| 2.1E-03 | 30.0      | 0.190          | 39.4       | 5.0            | 0.35   | 129          | 0.135  | 0.129  |
| 2.0E-03 | 35.1      | 0.217          | 33.6       | 5.8            | 0.37   | 139          | 0.119  | 0.113  |
| 2.0E-03 | 40.0      | 0.241          | 29.5       | 6.7            | 0.38   | 147          | 0.109  | 0.102  |
| 2.0E-03 | 49.8      | 0.289          | 23.7       | 8.3            | 0.41   | 163          | 0.094  | 0.086  |
| 2.0E-03 | 59.7      | 0.333          | 19.8       | 10.0           | 0.43   | 178          | 0.084  | 0.077  |
| 2.0E-03 | 70.0      | 0.376          | 16.9       | 11.7           | 0.45   | 193          | 0.077  | 0.069  |
| 2.0E-03 | 79.6      | 0.416          | 14.8       | 13.3           | 0.47   | 206          | 0.072  | 0.063  |
| 2.0E-03 | 89.5      | 0.455          | 13.2       | 14.9           | 0.49   | 218          | 0.068  | 0.059  |
| 2.0E-03 | 99.1      | 0.494          | 11.9       | 16.5           | 0.50   | 231          | 0.064  | 0.055  |
| 2.0E-03 | 110.0     | 0.530          | 10.7       | 18.3           | 0.51   | 241          | 0.061  | 0.051  |
| 2.0E-03 | 119.3     | 0.567          | 9.9        | 19.9           | 0.52   | 253          | 0.059  | 0.049  |
| 2.0E-03 | 139.4     | 0.633          | 8.5        | 23.2           | 0.54   | 273          | 0.055  | 0.045  |

Table S3 Summary of hydraulic conditions in the AOCF flume (for R1 and $S = 0.3\%$)

| $S$ (-) | $H$ (mm) | $U$ (ms$^{-1}$) | $B/H$ (-) | $H/\Delta$ (-) | $F$ (-) | $\Delta^*$ (-) | $f_u$ (-) | $f_k$ (-) |
|---------|-----------|----------------|------------|----------------|--------|--------------|--------|--------|
| 3.0E-03 | 19.6      | 0.157          | 60.1       | 3.3            | 0.36   | 125          | 0.189  | 0.183  |
| 3.0E-03 | 24.1      | 0.191          | 48.9       | 4.0            | 0.39   | 137          | 0.156  | 0.149  |
| 3.0E-03 | 28.7      | 0.224          | 41.2       | 4.8            | 0.42   | 151          | 0.136  | 0.130  |
| 3.0E-03 | 33.6      | 0.259          | 35.1       | 5.6            | 0.45   | 163          | 0.120  | 0.114  |
| 3.0E-03 | 38.8      | 0.293          | 30.4       | 6.5            | 0.47   | 176          | 0.108  | 0.101  |
| 3.0E-03 | 49.5      | 0.351          | 23.9       | 8.2            | 0.50   | 198          | 0.095  | 0.088  |
| 3.0E-03 | 59.0      | 0.404          | 20.0       | 9.8            | 0.53   | 216          | 0.086  | 0.078  |
| 3.0E-03 | 69.2      | 0.456          | 17.1       | 11.5           | 0.55   | 232          | 0.078  | 0.070  |
| 3.0E-03 | 79.2      | 0.506          | 14.9       | 13.2           | 0.57   | 249          | 0.073  | 0.064  |
| 3.0E-03 | 89.2      | 0.557          | 13.2       | 14.9           | 0.60   | 266          | 0.069  | 0.060  |
| 3.0E-03 | 99.7      | 0.603          | 11.8       | 16.6           | 0.61   | 278          | 0.064  | 0.055  |
| 3.0E-03 | 109.7     | 0.647          | 10.8       | 18.3           | 0.62   | 293          | 0.062  | 0.052  |
| 3.0E-03 | 119.5     | 0.689          | 9.9        | 19.9           | 0.64   | 305          | 0.059  | 0.049  |
### Table S4 Summary of hydraulic conditions in the AOCF flume (for R2 and \( S = 0.1\% \))

| \( S \) (-) | \( H \) (mm) | \( U \) (ms\(^{-1}\)) | \( B/H \) (-) | \( H/\Delta \) (-) | \( F \) (-) | \( \Delta^* \) (-) | \( f_H \) (-) | \( f_R \) (-) |
|-------------|-------------|----------------|------------|----------------|--------|----------|--------|--------|
| 1.0E-03     | 30.0        | 0.146          | 39.3       | 5.0            | 0.27   | 91       | 0.115  | 0.110  |
| 1.0E-03     | 34.7        | 0.163          | 34.0       | 5.8            | 0.28   | 97       | 0.105  | 0.099  |
| 1.0E-03     | 39.8        | 0.181          | 29.6       | 6.6            | 0.29   | 104      | 0.099  | 0.093  |
| 1.0E-03     | 49.9        | 0.214          | 23.6       | 8.3            | 0.31   | 116      | 0.088  | 0.081  |
| 1.0E-03     | 59.5        | 0.246          | 19.8       | 9.9            | 0.32   | 127      | 0.079  | 0.072  |
| 9.9E-04     | 70.3        | 0.276          | 16.8       | 11.7           | 0.33   | 135      | 0.072  | 0.064  |
| 1.0E-03     | 79.7        | 0.310          | 14.8       | 13.3           | 0.35   | 147      | 0.068  | 0.060  |
| 1.0E-03     | 89.4        | 0.337          | 13.2       | 14.9           | 0.36   | 156      | 0.064  | 0.056  |
| 1.0E-03     | 99.8        | 0.362          | 11.8       | 16.6           | 0.37   | 163      | 0.061  | 0.052  |
| 1.0E-03     | 110.3       | 0.390          | 10.7       | 18.4           | 0.37   | 171      | 0.058  | 0.049  |
| 1.0E-03     | 119.8       | 0.413          | 9.8        | 20.0           | 0.38   | 178      | 0.056  | 0.047  |
| 1.0E-03     | 140.4       | 0.461          | 8.4        | 23.4           | 0.39   | 193      | 0.053  | 0.043  |
| 1.0E-03     | 160.4       | 0.506          | 7.4        | 26.7           | 0.40   | 207      | 0.051  | 0.040  |

### Table S5 Summary of hydraulic conditions in the AOCF flume (for R2 and \( S = 0.2\% \))

| \( S \) (-) | \( H \) (mm) | \( U \) (ms\(^{-1}\)) | \( B/H \) (-) | \( H/\Delta \) (-) | \( F \) (-) | \( \Delta^* \) (-) | \( f_H \) (-) | \( f_R \) (-) |
|-------------|-------------|----------------|------------|----------------|--------|----------|--------|--------|
| 2.0E-03     | 25.6        | 0.174          | 46.1       | 4.3            | 0.35   | 114      | 0.130  | 0.125  |
| 2.0E-03     | 30.5        | 0.204          | 38.7       | 5.1            | 0.37   | 126      | 0.116  | 0.110  |
| 2.0E-03     | 35.2        | 0.228          | 33.5       | 5.9            | 0.39   | 134      | 0.107  | 0.101  |
| 2.0E-03     | 39.9        | 0.251          | 29.6       | 6.6            | 0.40   | 143      | 0.099  | 0.093  |
| 2.0E-03     | 49.4        | 0.298          | 23.9       | 8.2            | 0.43   | 160      | 0.088  | 0.081  |
| 2.0E-03     | 59.4        | 0.343          | 19.9       | 9.9            | 0.45   | 175      | 0.079  | 0.072  |
| 2.0E-03     | 69.7        | 0.386          | 16.9       | 11.6           | 0.47   | 192      | 0.073  | 0.065  |
| 2.0E-03     | 79.7        | 0.426          | 14.8       | 13.3           | 0.48   | 204      | 0.068  | 0.060  |
| 2.0E-03     | 89.6        | 0.469          | 13.2       | 14.9           | 0.50   | 219      | 0.064  | 0.056  |
| 2.0E-03     | 99.5        | 0.508          | 11.9       | 16.6           | 0.51   | 231      | 0.061  | 0.052  |
| 2.0E-03     | 109.8       | 0.543          | 10.7       | 18.3           | 0.52   | 241      | 0.058  | 0.049  |
| 2.0E-03     | 119.9       | 0.581          | 9.8        | 20.0           | 0.54   | 252      | 0.056  | 0.047  |
| 2.0E-03     | 140.7       | 0.646          | 8.4        | 23.4           | 0.55   | 271      | 0.052  | 0.042  |
Table S6 Summary of hydraulic conditions in the AOCF flume (for R2 and \( S = 0.3\% \))

| \( S \) (\( \times 10^{-3} \)) | \( H \) (mm) | \( U \) (m/s) | \( B/H \) (-) | \( H/\Delta \) (-) | \( F \) (-) | \( \Delta^* \) (-) | \( f_H \) (-) | \( f_R \) (-) |
|----------------|-----------|-----------|-------------|----------------|--------|--------|--------|--------|
| 3.0E-03        | 20.0      | 0.176     | 59.0        | 3.3            | 0.40   | 126    | 0.154  | 0.149  |
| 3.0E-03        | 24.3      | 0.206     | 48.6        | 4.0            | 0.42   | 138    | 0.134  | 0.128  |
| 3.0E-03        | 29.3      | 0.243     | 40.2        | 4.9            | 0.45   | 153    | 0.118  | 0.112  |
| 3.0E-03        | 34.2      | 0.276     | 34.5        | 5.7            | 0.48   | 165    | 0.106  | 0.101  |
| 3.0E-03        | 39.0      | 0.307     | 30.2        | 6.5            | 0.50   | 176    | 0.098  | 0.092  |
| 3.0E-03        | 49.6      | 0.363     | 23.8        | 8.3            | 0.52   | 197    | 0.088  | 0.081  |
| 3.0E-03        | 59.3      | 0.414     | 19.9        | 9.9            | 0.54   | 218    | 0.081  | 0.074  |
| 3.0E-03        | 69.0      | 0.466     | 17.1        | 11.5           | 0.57   | 234    | 0.074  | 0.066  |
| 3.0E-03        | 79.4      | 0.520     | 14.9        | 13.2           | 0.59   | 252    | 0.069  | 0.061  |
| 3.0E-03        | 89.5      | 0.572     | 13.2        | 14.9           | 0.61   | 269    | 0.065  | 0.056  |
| 3.0E-03        | 99.8      | 0.619     | 11.8        | 16.6           | 0.63   | 283    | 0.061  | 0.052  |
| 3.1E-03        | 109.9     | 0.667     | 10.7        | 18.3           | 0.64   | 299    | 0.059  | 0.050  |
| 3.0E-03        | 120.3     | 0.711     | 9.8         | 20.1           | 0.65   | 312    | 0.056  | 0.047  |

Table S7 Summary of hydraulic conditions in the AOCF flume (for R3 and \( S = 0.1\% \))

| \( S \) (\( \times 10^{-3} \)) | \( H \) (mm) | \( U \) (m/s) | \( B/H \) (-) | \( H/\Delta \) (-) | \( F \) (-) | \( \Delta^* \) (-) | \( f_H \) (-) | \( f_R \) (-) |
|----------------|-----------|-----------|-------------|----------------|--------|--------|--------|--------|
| 1.0E-03        | 29.9      | 0.161     | 39.4        | 5.0            | 0.30   | 90     | 0.091  | 0.087  |
| 1.0E-03        | 34.5      | 0.180     | 34.2        | 5.8            | 0.31   | 97     | 0.084  | 0.080  |
| 1.0E-03        | 39.4      | 0.201     | 30.0        | 6.6            | 0.32   | 104    | 0.078  | 0.073  |
| 1.0E-03        | 49.5      | 0.239     | 23.9        | 8.2            | 0.34   | 116    | 0.069  | 0.064  |
| 9.8E-04        | 59.7      | 0.272     | 19.8        | 10.0           | 0.36   | 125    | 0.062  | 0.056  |
| 1.0E-03        | 69.8      | 0.311     | 16.9        | 11.6           | 0.38   | 138    | 0.058  | 0.052  |
| 1.0E-03        | 79.6      | 0.342     | 14.8        | 13.3           | 0.39   | 147    | 0.055  | 0.048  |
| 1.0E-03        | 89.0      | 0.371     | 13.3        | 14.8           | 0.40   | 156    | 0.052  | 0.045  |
| 1.0E-03        | 99.1      | 0.398     | 11.9        | 16.5           | 0.40   | 163    | 0.050  | 0.043  |
| 1.0E-03        | 108.8     | 0.428     | 10.8        | 18.1           | 0.41   | 172    | 0.048  | 0.041  |
| 1.0E-03        | 119.4     | 0.456     | 9.9         | 19.9           | 0.42   | 180    | 0.046  | 0.038  |
| 1.0E-03        | 139.7     | 0.503     | 8.4         | 23.3           | 0.43   | 193    | 0.044  | 0.035  |
| 1.0E-03        | 159.8     | 0.552     | 7.4         | 26.6           | 0.44   | 208    | 0.042  | 0.033  |
Table S8 Summary of hydraulic conditions in the AOCF flume (for R3 and $S = 0.2\%$)

| $S$ (-) | $H$ (mm) | $U$ (ms$^{-1}$) | $B/H$ (-) | $H/\Delta$ (-) | $F$ (-) | $\Delta^*$ (-) | $f_{h}$ (-) | $f_{r}$ (-) |
|---------|----------|-----------------|-----------|----------------|--------|----------------|---------|---------|
| 2.0E-03 | 25.6     | 0.204           | 46.0      | 4.3            | 0.41   | 119            | 0.097   | 0.093   |
| 2.0E-03 | 30.2     | 0.232           | 39.0      | 5.0            | 0.43   | 128            | 0.088   | 0.084   |
| 2.0E-03 | 35.2     | 0.263           | 33.5      | 5.9            | 0.45   | 139            | 0.081   | 0.076   |
| 2.0E-03 | 40.0     | 0.288           | 29.5      | 6.7            | 0.46   | 147            | 0.076   | 0.071   |
| 2.0E-03 | 50.0     | 0.341           | 23.6      | 8.3            | 0.49   | 166            | 0.068   | 0.063   |
| 2.0E-03 | 59.4     | 0.389           | 19.9      | 9.9            | 0.51   | 178            | 0.061   | 0.055   |
| 2.0E-03 | 69.5     | 0.438           | 17.0      | 11.6           | 0.53   | 193            | 0.057   | 0.051   |
| 2.0E-03 | 79.0     | 0.484           | 14.9      | 13.2           | 0.55   | 208            | 0.054   | 0.047   |
| 2.0E-03 | 89.4     | 0.528           | 13.2      | 14.9           | 0.56   | 220            | 0.051   | 0.044   |
| 2.0E-03 | 99.6     | 0.567           | 11.8      | 16.6           | 0.57   | 231            | 0.049   | 0.042   |
| 2.0E-03 | 109.3    | 0.606           | 10.8      | 18.2           | 0.59   | 242            | 0.047   | 0.040   |
| 2.0E-03 | 119.6    | 0.645           | 9.9       | 19.9           | 0.60   | 256            | 0.046   | 0.038   |

Table S9 Summary of hydraulic conditions in the AOCF flume (for R3 and $S = 0.3\%$)

| $S$ (-) | $H$ (mm) | $U$ (ms$^{-1}$) | $B/H$ (-) | $H/\Delta$ (-) | $F$ (-) | $\Delta^*$ (-) | $f_{h}$ (-) | $f_{r}$ (-) |
|---------|----------|-----------------|-----------|----------------|--------|----------------|---------|---------|
| 3.0E-03 | 18.9     | 0.193           | 62.5      | 3.1            | 0.45   | 126            | 0.120   | 0.116   |
| 3.0E-03 | 23.7     | 0.240           | 49.8      | 3.9            | 0.50   | 141            | 0.097   | 0.093   |
| 3.0E-03 | 29.2     | 0.280           | 40.5      | 4.9            | 0.52   | 157            | 0.088   | 0.084   |
| 3.0E-03 | 34.3     | 0.317           | 34.4      | 5.7            | 0.55   | 169            | 0.080   | 0.076   |
| 3.0E-03 | 39.3     | 0.350           | 30.0      | 6.5            | 0.56   | 181            | 0.075   | 0.070   |
| 3.0E-03 | 49.2     | 0.413           | 24.0      | 8.2            | 0.60   | 202            | 0.067   | 0.062   |
| 3.0E-03 | 59.2     | 0.480           | 19.9      | 9.9            | 0.63   | 222            | 0.060   | 0.055   |
| 3.0E-03 | 69.2     | 0.538           | 17.1      | 11.5           | 0.65   | 239            | 0.056   | 0.050   |
| 3.0E-03 | 79.2     | 0.594           | 14.9      | 13.2           | 0.67   | 255            | 0.052   | 0.046   |
| 3.0E-03 | 89.3     | 0.649           | 13.2      | 14.9           | 0.69   | 273            | 0.050   | 0.044   |
| 3.0E-03 | 99.5     | 0.700           | 11.9      | 16.6           | 0.71   | 287            | 0.048   | 0.041   |
| 3.0E-03 | 109.6    | 0.748           | 10.8      | 18.3           | 0.72   | 300            | 0.046   | 0.039   |
| 3.0E-03 | 119.2    | 0.791           | 9.9       | 19.9           | 0.73   | 311            | 0.045   | 0.037   |
Table S10 Summary of hydraulic conditions in the RS flume (for R1 and \( S = 0.1\% \))

| \( S (-) \) | \( H \) (mm) | \( U \) (ms\(^{-1}\)) | \( B/H \) (-) | \( H/\Delta \) (-) | \( F \) (-) | \( \Delta^+ \) (-) | \( f_H (-) \) | \( f_R (-) \) |
|-------------|-------------|----------------|-------------|-----------------|-----------|------------|----------|----------|
| 9.3E-04     | 35.9        | 0.146          | 11.1        | 6.0             | 0.25      | 102        | 0.122    | 0.103    |
| 9.6E-04     | 40.1        | 0.165          | 10.0        | 6.7             | 0.26      | 109        | 0.110    | 0.092    |
| 9.2E-04     | 45.5        | 0.181          | 8.8         | 7.6             | 0.27      | 114        | 0.100    | 0.082    |
| 9.3E-04     | 50.5        | 0.198          | 7.9         | 8.4             | 0.28      | 121        | 0.094    | 0.075    |
| 9.5E-04     | 55.5        | 0.215          | 7.2         | 9.3             | 0.29      | 128        | 0.090    | 0.071    |
| 9.3E-04     | 60.5        | 0.230          | 6.6         | 10.1            | 0.30      | 132        | 0.084    | 0.064    |
| 9.3E-04     | 66.0        | 0.241          | 6.1         | 11.0            | 0.30      | 137        | 0.083    | 0.062    |
| 8.9E-04     | 70.0        | 0.251          | 5.7         | 11.7            | 0.30      | 139        | 0.077    | 0.057    |
| 8.6E-04     | 75.4        | 0.263          | 5.3         | 12.6            | 0.31      | 141        | 0.074    | 0.053    |
| 8.5E-04     | 80.9        | 0.271          | 4.9         | 13.5            | 0.30      | 146        | 0.073    | 0.052    |
| 8.6E-04     | 86.3        | 0.289          | 4.6         | 14.4            | 0.31      | 151        | 0.070    | 0.049    |
| 8.6E-04     | 90.2        | 0.292          | 4.4         | 15.0            | 0.31      | 154        | 0.071    | 0.049    |
| 8.6E-04     | 95.0        | 0.310          | 4.2         | 15.8            | 0.32      | 158        | 0.067    | 0.045    |
| 9.6E-04     | 100.6       | 0.335          | 4.0         | 16.8            | 0.34      | 172        | 0.067    | 0.045    |
| 9.3E-04     | 105.0       | 0.340          | 3.8         | 17.5            | 0.34      | 173        | 0.066    | 0.043    |
| 8.6E-04     | 110.4       | 0.339          | 3.6         | 18.4            | 0.33      | 170        | 0.065    | 0.042    |
| 8.5E-04     | 115.6       | 0.353          | 3.5         | 19.3            | 0.33      | 173        | 0.062    | 0.039    |
| 8.6E-04     | 119.9       | 0.357          | 3.3         | 20.0            | 0.33      | 177        | 0.064    | 0.040    |
Table S11 Summary of hydraulic conditions in the RS flume (for R1 and $S = 0.2\%$)

| $S$ (\text{\text{-}}) | $H$ (mm) | $U$ (ms$^{-1}$) | $B/H$ (\text{-}) | $H/\Delta$ (\text{-}) | $F$ (\text{-}) | $\Delta^*$ (\text{-}) | $f_{nu}$ (\text{-}) | $f_{fr}$ (\text{-}) |
|-------------------|----------|----------------|-----------------|-----------------|-------------|----------------|----------------|----------------|
| 1.9E-03           | 30.7     | 0.181          | 13.0            | 5.1             | 0.33        | 128            | 0.136          | 0.118          |
| 1.9E-03           | 35.6     | 0.207          | 11.2            | 5.9             | 0.35        | 139            | 0.124          | 0.105          |
| 1.9E-03           | 40.4     | 0.233          | 9.9             | 6.7             | 0.37        | 148            | 0.111          | 0.092          |
| 1.9E-03           | 45.9     | 0.260          | 8.7             | 7.6             | 0.39        | 163            | 0.103          | 0.084          |
| 1.9E-03           | 50.8     | 0.283          | 7.9             | 8.5             | 0.40        | 167            | 0.096          | 0.077          |
| 1.9E-03           | 55.6     | 0.302          | 7.2             | 9.3             | 0.41        | 178            | 0.092          | 0.072          |
| 1.9E-03           | 60.4     | 0.323          | 6.6             | 10.1            | 0.42        | 180            | 0.086          | 0.066          |
| 1.9E-03           | 65.9     | 0.342          | 6.1             | 11.0            | 0.43        | 194            | 0.083          | 0.063          |
| 1.9E-03           | 70.8     | 0.369          | 5.7             | 11.8            | 0.44        | 196            | 0.078          | 0.058          |
| 1.9E-03           | 75.8     | 0.382          | 5.3             | 12.6            | 0.44        | 207            | 0.077          | 0.056          |
| 1.9E-03           | 80.8     | 0.400          | 4.9             | 13.5            | 0.45        | 207            | 0.074          | 0.053          |
| 1.9E-03           | 86.5     | 0.420          | 4.6             | 14.4            | 0.46        | 225            | 0.074          | 0.052          |
| 1.8E-03           | 90.9     | 0.431          | 4.4             | 15.2            | 0.46        | 217            | 0.071          | 0.049          |
| 2.0E-03           | 96.0     | 0.461          | 4.2             | 16.0            | 0.48        | 237            | 0.069          | 0.047          |
| 1.9E-03           | 100.7    | 0.471          | 4.0             | 16.8            | 0.47        | 238            | 0.067          | 0.045          |

Table S12 Summary of hydraulic conditions in the RS flume (for R1 and $S = 0.3\%$)

| $S$ (\text{\text{-}}) | $H$ (mm) | $U$ (ms$^{-1}$) | $B/H$ (\text{-}) | $H/\Delta$ (\text{-}) | $F$ (\text{-}) | $\Delta^*$ (\text{-}) | $f_{nu}$ (\text{-}) | $f_{fr}$ (\text{-}) |
|-------------------|----------|----------------|-----------------|-----------------|-------------|----------------|----------------|----------------|
| 2.9E-03           | 26.0     | 0.199          | 15.4            | 4.3             | 0.39        | 151            | 0.148          | 0.131          |
| 2.9E-03           | 30.5     | 0.223          | 13.1            | 5.1             | 0.41        | 164            | 0.139          | 0.120          |
| 2.9E-03           | 35.6     | 0.257          | 11.2            | 5.9             | 0.44        | 178            | 0.122          | 0.104          |
| 2.9E-03           | 40.7     | 0.287          | 9.8             | 6.8             | 0.45        | 192            | 0.115          | 0.095          |
| 3.0E-03           | 45.7     | 0.322          | 8.8             | 7.6             | 0.48        | 203            | 0.102          | 0.083          |
| 2.9E-03           | 50.8     | 0.346          | 7.9             | 8.5             | 0.49        | 211            | 0.096          | 0.077          |
| 2.9E-03           | 55.8     | 0.375          | 7.2             | 9.3             | 0.51        | 221            | 0.090          | 0.070          |
| 2.9E-03           | 61.3     | 0.401          | 6.5             | 10.2            | 0.52        | 232            | 0.087          | 0.066          |
| 2.9E-03           | 66.3     | 0.423          | 6.0             | 11.1            | 0.52        | 242            | 0.085          | 0.064          |
| 2.9E-03           | 71.0     | 0.450          | 5.6             | 11.8            | 0.54        | 250            | 0.081          | 0.060          |
| 3.0E-03           | 76.1     | 0.477          | 5.3             | 12.7            | 0.55        | 261            | 0.078          | 0.056          |
| 3.0E-03           | 80.6     | 0.501          | 5.0             | 13.4            | 0.56        | 268            | 0.075          | 0.053          |
| 3.0E-03           | 86.6     | 0.529          | 4.6             | 14.4            | 0.57        | 278            | 0.072          | 0.050          |
| 2.9E-03           | 90.5     | 0.534          | 4.4             | 15.1            | 0.57        | 280            | 0.072          | 0.049          |
Table S13 Summary of hydraulic conditions in the RS flume (for R2 and \( S = 0.1\% \))

| \( S (\) | \( H \) (mm) | \( U \) (m/s) | \( B/H \) (-) | \( H/\Delta (-) \) | \( \bar{F} \) (-) | \( \Delta^4 \) (-) | \( f_{\mu} \) (-) | \( f_k \) (-) |
|---|---|---|---|---|---|---|---|---|
| 9.2E-04 | 30.5 | 0.134 | 13.1 | 5.1 | 0.24 | 90 | 0.123 | 0.107 |
| 9.5E-04 | 36.0 | 0.158 | 11.1 | 6.0 | 0.27 | 99 | 0.107 | 0.091 |
| 9.2E-04 | 40.4 | 0.173 | 9.9 | 6.7 | 0.27 | 105 | 0.098 | 0.082 |
| 9.5E-04 | 45.8 | 0.189 | 8.7 | 7.6 | 0.28 | 112 | 0.096 | 0.078 |
| 8.6E-04 | 50.0 | 0.193 | 8.0 | 8.3 | 0.28 | 111 | 0.091 | 0.073 |
| 8.9E-04 | 55.2 | 0.214 | 7.2 | 9.2 | 0.29 | 118 | 0.084 | 0.066 |
| 9.5E-04 | 60.3 | 0.233 | 6.6 | 10.1 | 0.30 | 128 | 0.083 | 0.064 |
| 9.1E-04 | 65.4 | 0.241 | 6.1 | 10.9 | 0.30 | 130 | 0.081 | 0.061 |
| 9.3E-04 | 69.9 | 0.255 | 5.7 | 11.6 | 0.31 | 137 | 0.079 | 0.058 |
| 9.4E-04 | 75.5 | 0.272 | 5.3 | 12.6 | 0.32 | 142 | 0.075 | 0.055 |
| 9.2E-04 | 80.3 | 0.285 | 5.0 | 13.4 | 0.32 | 146 | 0.071 | 0.051 |
| 9.6E-04 | 85.8 | 0.297 | 4.7 | 14.3 | 0.32 | 153 | 0.073 | 0.051 |
| 9.5E-04 | 90.8 | 0.312 | 4.4 | 15.1 | 0.33 | 157 | 0.069 | 0.048 |
| 9.1E-04 | 96.2 | 0.319 | 4.2 | 16.0 | 0.33 | 158 | 0.067 | 0.045 |
| 9.6E-04 | 100.5 | 0.341 | 4.0 | 16.8 | 0.34 | 167 | 0.065 | 0.044 |
| 8.9E-04 | 105.6 | 0.340 | 3.8 | 17.6 | 0.33 | 163 | 0.064 | 0.042 |
| 9.4E-04 | 110.7 | 0.356 | 3.6 | 18.4 | 0.34 | 173 | 0.064 | 0.042 |
| 9.4E-04 | 115.2 | 0.366 | 3.5 | 19.2 | 0.34 | 176 | 0.063 | 0.040 |
| 8.9E-04 | 119.6 | 0.364 | 3.3 | 19.9 | 0.34 | 176 | 0.063 | 0.039 |
| $S$ (-) | $H$ (mm) | $U$ (ms$^{-1}$) | $B/H$ (-) | $H/\Delta$ (-) | $F$ (-) | $\Delta^*$ (-) | $f_m$ (-) | $f_R$ (-) |
|---------|---------|----------------|-----------|----------------|-------|---------------|--------|--------|
| 1.8E-03 | 25.6    | 0.162          | 15.6      | 4.3            | 0.32  | 119           | 0.140  | 0.124  |
| 1.9E-03 | 31.1    | 0.199          | 12.8      | 5.2            | 0.36  | 135           | 0.119  | 0.103  |
| 1.9E-03 | 35.7    | 0.220          | 11.2      | 5.9            | 0.37  | 142           | 0.109  | 0.093  |
| 1.9E-03 | 40.4    | 0.242          | 9.9       | 6.7            | 0.38  | 150           | 0.101  | 0.084  |
| 1.9E-03 | 45.6    | 0.266          | 8.8       | 7.6            | 0.40  | 160           | 0.097  | 0.079  |
| 1.9E-03 | 51.2    | 0.287          | 7.8       | 8.5            | 0.40  | 170           | 0.091  | 0.073  |
| 1.9E-03 | 55.8    | 0.314          | 7.2       | 9.3            | 0.42  | 181           | 0.086  | 0.067  |
| 1.9E-03 | 61.0    | 0.333          | 6.6       | 10.2           | 0.43  | 188           | 0.083  | 0.064  |
| 1.9E-03 | 65.5    | 0.352          | 6.1       | 10.9           | 0.44  | 195           | 0.079  | 0.060  |
| 1.9E-03 | 70.9    | 0.375          | 5.6       | 11.8           | 0.45  | 202           | 0.076  | 0.056  |
| 1.9E-03 | 75.8    | 0.391          | 5.3       | 12.6           | 0.45  | 211           | 0.075  | 0.054  |
| 1.9E-03 | 80.8    | 0.409          | 5.0       | 13.5           | 0.46  | 214           | 0.072  | 0.051  |
| 1.9E-03 | 86.2    | 0.427          | 4.6       | 14.4           | 0.46  | 224           | 0.071  | 0.050  |
| 1.9E-03 | 91.2    | 0.450          | 4.4       | 15.2           | 0.48  | 227           | 0.068  | 0.047  |
| 1.9E-03 | 96.0    | 0.458          | 4.2       | 16.0           | 0.47  | 235           | 0.068  | 0.046  |
| 2.0E-03 | 101.0   | 0.484          | 4.0       | 16.8           | 0.49  | 242           | 0.067  | 0.045  |

Table S15 Summary of hydraulic conditions in the RS flume (for R2 and $S = 0.3\%$)

| $S$ (-) | $H$ (mm) | $U$ (ms$^{-1}$) | $B/H$ (-) | $H/\Delta$ (-) | $F$ (-) | $\Delta^*$ (-) | $f_m$ (-) | $f_R$ (-) |
|---------|---------|----------------|-----------|----------------|-------|---------------|--------|--------|
| 2.9E-03 | 21.2    | 0.169          | 18.8      | 3.5            | 0.37  | 135           | 0.171  | 0.154  |
| 3.0E-03 | 26.5    | 0.205          | 15.1      | 4.4            | 0.40  | 151           | 0.145  | 0.128  |
| 2.9E-03 | 30.8    | 0.239          | 13.0      | 5.1            | 0.44  | 162           | 0.123  | 0.107  |
| 2.9E-03 | 36.2    | 0.270          | 11.1      | 6.0            | 0.45  | 176           | 0.113  | 0.096  |
| 3.0E-03 | 41.2    | 0.301          | 9.7       | 6.9            | 0.47  | 189           | 0.106  | 0.088  |
| 2.9E-03 | 46.2    | 0.327          | 8.7       | 7.7            | 0.49  | 199           | 0.100  | 0.081  |
| 3.0E-03 | 51.0    | 0.355          | 7.8       | 8.5            | 0.50  | 210           | 0.094  | 0.075  |
| 2.9E-03 | 56.7    | 0.384          | 7.1       | 9.4            | 0.51  | 219           | 0.089  | 0.069  |
| 3.0E-03 | 61.1    | 0.413          | 6.5       | 10.2           | 0.53  | 231           | 0.084  | 0.064  |
| 2.9E-03 | 66.3    | 0.435          | 6.0       | 11.0           | 0.54  | 237           | 0.081  | 0.061  |
| 3.0E-03 | 70.9    | 0.456          | 5.6       | 11.8           | 0.55  | 247           | 0.079  | 0.059  |
| 2.9E-03 | 76.5    | 0.478          | 5.2       | 12.7           | 0.55  | 251           | 0.075  | 0.054  |
| 3.0E-03 | 81.2    | 0.505          | 4.9       | 13.5           | 0.57  | 265           | 0.074  | 0.053  |
| 2.9E-03 | 86.2    | 0.521          | 4.6       | 14.4           | 0.57  | 266           | 0.071  | 0.050  |
| 2.9E-03 | 90.3    | 0.540          | 4.4       | 15.0           | 0.57  | 274           | 0.070  | 0.048  |
Table S16 Summary of hydraulic conditions in the RS flume (for R3 and $S = 0.1\%$)

| $S$ (-) | $H$ (mm) | $U$ (m s$^{-1}$) | $B/H$ (-) | $H/\Delta$ (-) | $\Phi$ (-) | $\Delta^+$ (-) | $f_h$ (-) | $f_r$ (-) |
|---------|---------|-------------|-----------|---------------|-----------|----------------|-----------|---------|
| 9.7E-04 | 29.4    | 0.158       | 13.6      | 4.9           | 0.29      | 85             | 0.090     | 0.078   |
| 9.7E-04 | 35.0    | 0.182       | 11.4      | 5.8           | 0.31      | 92             | 0.081     | 0.069   |
| 9.7E-04 | 39.8    | 0.198       | 10.1      | 6.6           | 0.32      | 98             | 0.078     | 0.065   |
| 9.7E-04 | 45.5    | 0.223       | 8.8       | 7.6           | 0.33      | 105            | 0.070     | 0.057   |
| 9.7E-04 | 49.5    | 0.234       | 8.1       | 8.2           | 0.34      | 114            | 0.069     | 0.055   |
| 9.7E-04 | 55.6    | 0.257       | 7.2       | 9.3           | 0.35      | 121            | 0.064     | 0.050   |
| 9.7E-04 | 59.9    | 0.272       | 6.7       | 10.0          | 0.35      | 126            | 0.062     | 0.048   |
| 9.7E-04 | 65.5    | 0.289       | 6.1       | 10.9          | 0.36      | 132            | 0.060     | 0.045   |
| 9.7E-04 | 69.9    | 0.301       | 5.7       | 11.7          | 0.36      | 136            | 0.059     | 0.044   |
| 9.7E-04 | 75.1    | 0.317       | 5.3       | 12.5          | 0.37      | 141            | 0.057     | 0.041   |
| 9.7E-04 | 80.4    | 0.328       | 5.0       | 13.4          | 0.37      | 146            | 0.057     | 0.041   |
| 9.7E-04 | 85.4    | 0.341       | 4.7       | 14.2          | 0.37      | 150            | 0.056     | 0.039   |
| 9.7E-04 | 89.6    | 0.353       | 4.5       | 14.9          | 0.38      | 154            | 0.055     | 0.038   |
| 9.7E-04 | 95.7    | 0.371       | 4.2       | 15.9          | 0.38      | 159            | 0.053     | 0.036   |
| 9.7E-04 | 100.4   | 0.383       | 4.0       | 16.7          | 0.39      | 163            | 0.052     | 0.035   |
| 9.7E-04 | 104.8   | 0.392       | 3.8       | 17.5          | 0.39      | 168            | 0.052     | 0.034   |
| 9.7E-04 | 109.4   | 0.403       | 3.7       | 18.2          | 0.39      | 169            | 0.052     | 0.033   |
| 9.7E-04 | 113.9   | 0.411       | 3.5       | 19.0          | 0.39      | 172            | 0.051     | 0.033   |
Table S17 Summary of hydraulic conditions in the RS flume (for R3 and S = 0.2%)

| S (-) | $H$ (mm) | $U$ (ms$^{-1}$) | $B/H$ (-) | $H/\Delta$ (-) | $F$ (-) | $\Delta$' (-) | $f_{ln}$ (-) | $f_r$ (-) |
|-------|----------|----------------|------------|----------------|--------|--------------|------------|----------|
| 2.0E-03 | 25.3     | 0.199          | 15.8       | 4.2            | 0.40   | 117          | 0.101      | 0.090    |
| 2.0E-03 | 29.9     | 0.227          | 13.4       | 5.0            | 0.42   | 127          | 0.091      | 0.079    |
| 2.0E-03 | 35.1     | 0.259          | 11.4       | 5.8            | 0.44   | 137          | 0.083      | 0.070    |
| 2.0E-03 | 40.1     | 0.287          | 10.0       | 6.7            | 0.46   | 147          | 0.077      | 0.064    |
| 2.0E-03 | 45.0     | 0.311          | 8.9        | 7.5            | 0.47   | 156          | 0.074      | 0.060    |
| 2.0E-03 | 50.1     | 0.338          | 8.0        | 8.3            | 0.48   | 164          | 0.069      | 0.055    |
| 2.0E-03 | 55.1     | 0.363          | 7.3        | 9.2            | 0.49   | 173          | 0.066      | 0.052    |
| 2.0E-03 | 59.9     | 0.387          | 6.7        | 10.0           | 0.50   | 180          | 0.063      | 0.049    |
| 2.0E-03 | 65.3     | 0.411          | 6.1        | 10.9           | 0.51   | 188          | 0.061      | 0.046    |
| 2.0E-03 | 69.4     | 0.434          | 5.8        | 11.6           | 0.53   | 195          | 0.058      | 0.043    |
| 2.0E-03 | 75.7     | 0.453          | 5.3        | 12.6           | 0.53   | 208          | 0.058      | 0.042    |
| 2.0E-03 | 80.5     | 0.473          | 5.0        | 13.4           | 0.53   | 215          | 0.057      | 0.041    |
| 2.0E-03 | 85.8     | 0.495          | 4.7        | 14.3           | 0.54   | 221          | 0.055      | 0.039    |
| 2.0E-03 | 90.8     | 0.506          | 4.4        | 15.1           | 0.54   | 227          | 0.056      | 0.039    |
| 2.0E-03 | 94.3     | 0.523          | 4.2        | 15.7           | 0.54   | 230          | 0.055      | 0.037    |

Table S18 Summary of hydraulic conditions in the RS flume (for R3 and S = 0.3%)

| S (-) | $H$ (mm) | $U$ (ms$^{-1}$) | $B/H$ (-) | $H/\Delta$ (-) | $F$ (-) | $\Delta$' (-) | $f_{ln}$ (-) | $f_r$ (-) |
|-------|----------|----------------|------------|----------------|--------|--------------|------------|----------|
| 2.9E-03 | 19.2     | 0.191          | 20.9       | 3.2            | 0.44   | 123          | 0.121      | 0.111    |
| 2.9E-03 | 25.1     | 0.245          | 15.9       | 4.2            | 0.49   | 141          | 0.096      | 0.085    |
| 2.9E-03 | 30.1     | 0.282          | 13.3       | 5.0            | 0.52   | 154          | 0.087      | 0.076    |
| 2.9E-03 | 35.1     | 0.316          | 11.4       | 5.9            | 0.54   | 166          | 0.081      | 0.069    |
| 2.9E-03 | 40.3     | 0.355          | 9.9        | 6.7            | 0.56   | 177          | 0.074      | 0.061    |
| 2.9E-03 | 45.5     | 0.388          | 8.8        | 7.6            | 0.58   | 188          | 0.070      | 0.057    |
| 2.9E-03 | 50.3     | 0.412          | 8.0        | 8.4            | 0.59   | 198          | 0.068      | 0.054    |
| 2.9E-03 | 54.3     | 0.438          | 7.4        | 9.0            | 0.60   | 205          | 0.065      | 0.051    |
| 2.9E-03 | 60.6     | 0.474          | 6.6        | 10.1           | 0.61   | 215          | 0.062      | 0.048    |
| 2.9E-03 | 65.3     | 0.492          | 6.1        | 10.9           | 0.61   | 223          | 0.062      | 0.047    |
| 2.9E-03 | 70.4     | 0.517          | 5.7        | 11.7           | 0.62   | 239          | 0.061      | 0.045    |