FREE MOBILITY OF CAPITAL AND LABOR FORCE IN A TWO-COUNTRY MODEL: THE DYNAMIC GAME FOR GROWTH

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Abstract. In this paper, we consider a two-country and two-sector economy, where firms can choose to be innovative or not innovative, and workers to be skilled or unskilled. Using a dynamic game, we argue that exploiting the comparative advantages a country has in producing goods that use the most abundant factor of production, free mobility of capital and labor is beneficial for economic growth. However, if a country has a comparative advantage in a sector that uses intensely unskilled labor (which is the case of several underdeveloped economies), a poverty trap may arise. For this reason we argue that national Governments must ensure the technological development to improve competitiveness and therefore a social optimal use of the comparative advantages.

1. Introduction. Poverty traps are a well-known phenomenon in the economic literature. Several might be the causes: geographic location that prevents human and physical capital accumulation, like for example the lack of access to the sea that prevents the development of commercial relations with other countries, or the prevalence of certain types of diseases affecting health and therefore productivity of workers ([13])). High fertility rates are also a potential cause of poverty traps: fertility indeed tends to be inversely related to womens’ wages, or the most common

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proxy for wages, that is to say, education. Fertility rates are also inversely correlated with income per capita and the growth rate of income, and positively with illiteracy rates for adult females and poverty. A third potential cause might be credit market imperfections which limits entrepreneurs’ initiative, which may be due to the imperfect information about individual abilities or an imperfect enforcement of loan contracts. We will analyze the possibility of the emergence of poverty traps as a result of a short-term exploitation of comparative values in the framework of the free mobility of capital and labor.

When poverty is widespread, the lack of public capital may be another reason for a poor growth performance of a country, because population might be too poor to develop its own infrastructure. Similar considerations might be made for the government debt burden, which, if too high, may be a deterrent to private, public, domestic and foreign investments, because the perceived uncertainty and risk. Moreover, a high indebtedness may prevent economic development if servicing absorbs a large fraction of the resources that could be available for public investments and infrastructures. Quality of institutions is also a key important factor for economic growth: a poor enforcement of property rights might indeed prevent private investments. The lack of private investments may as well be a cause of a poor performance in terms of economic growth: although the potential costs associated with volatile short-term capital flows, foreign direct investments (in the following, FDIs) may carry some important benefits, especially if they are associated with other complementary policies ([4]). FDIs are usually concentrated to a small group of countries and sectors, and mainly linked to the exploitations of natural resources. Difficulties in attracting FDI reflect insufficient market size, poor infrastructures, political uncertainty and corruption. In this paper, we argue that domestic investments and FDIs must be accompanied by complementary policies which together lead to a path of sustained growth.

In this paper, we basically extend the imitative model introduced in [2], by allowing the presence of two countries with different comparative advantages, and free mobility of capital and workers. With respect to their original work, where poverty traps may emerge as a Pareto-dominated equilibria resulting from a coordination game between two populations of firms and workers in which the former prefer to use old technologies and be non-innovative, and the latter prefer not to invest in education, here we allow for the presence of international trade and free mobility of workers, letting the firms to choose the sectors in which getting specialized, in addition to being innovative or not, and workers are allowed to decide about the possibility to migrate, in addition to be skilled or unskilled. If indeed a country, in order to grow, must specialize in the sector that uses the most abundant factor of production and trade its goods internationally ([12]; [11] ; [10]), it can end up in a poverty trap depending on its decision about the sector in which it will specialize. So, as it is possible to argue, a poverty trap could arise not only as a coordination failure (implied by rational choices) between firms and workers as in [2], but also from the rational exploitation of comparative advantages in a multi-country economy, that basically translates in the choice of the sector in which getting specialized, if the cost of the primary factors of production are different between countries.

African and some Latin American countries are indeed very poor despite the huge availability of natural resources. One possible explanation can indeed be attributable to the specialization towards the extraction of natural resources, an activity that may possibly use intensively unskilled labor. So, the choice of the sector in
which one country can specialize can be a source of persistent poverty and economic stagnation if appropriate policies are carried out by the governments. So, if the choice of the sector in which getting specialized is somehow due to the existence of natural factors that make this sector more competitive in the global market, a government should finance the implementation of technologies that require more skilled labor and have higher productivity returns to avoid the possibility that a country end up in a poverty trap.

In our paper, we assume that our economy consists of two countries, and each one has access to two technologies, one innovative and the other not, and have the availability of two types of workers, one skilled and the other not. We assume free mobility of factors of production and countries interact within the framework of a globalized economy which, among other things, means accessing a broad international demand for products. However, while this condition implies that output produced by a single country always meets its demand, workers with different levels of education may not necessarily find the right outlet for their professional abilities and therefore migration between countries is allowed.

This work is organized as follows: section 2 describes the basic game assuming that there is no migration between countries. Section 3 removes this hypothesis and introduces the dynamic game with international mobility of factors of production. Contrary to our previous model, in order to allow for specialization and exploitation of the comparative advantages, we need to introduce two sectors of productions. In section 4 we introduce some considerations on technological poverty traps. Section 5 concludes.

2. Setup of the game. We consider an economy with two countries indexed $P$ and $Q$ with different factor endowments, where in each country there are two industries $A$ and $B$. There exist two types of firms, innovative ($I$-firms) and non-innovative ($\bar{I}$-firms), and workers can be of two types: skilled, $s$, and unskilled, $\bar{s}$. There is no borrowing and lending and no capital accumulation. There is free access to any available technology under different factor endowments. Markets in each country are perfectly competitive.

We assume strategic complementarities between innovative firms and skilled workers, and between unskilled workers and non-innovative firms. At time $t = t_0$ there is no labor and capital mobility between countries and the respective economies are in equilibrium. At time $t = t_1$, free circulation of capital and labor force is allowed.

Definition 2.1. The world economy, $W$, consists of two countries, two industries or branches, two types of firms and two types of workers, and symbolically is represented by the following set:

$$W = \{\{P, Q\}; \{A, B\}; \{I, \bar{I}\}; \{s, \bar{s}\}\}.$$  

Within each country, which we index by $h \in \{P, Q\}$, the choices of workers and the choices of firms can be represented in a $2-$strategies $\times 2-$populations normal(-strategic) form game. Accordingly, we denote by $\Gamma_h$ the game, with $S_{hw}$ and $S_{hf}$, respectively, the set of pure strategies of workers and firms in country $h$,

$$\Gamma_h = \{S_{hw} = \{hs_A, hs_B, h\bar{s}\}, \quad S_{hf} = \{hI_A, hI_B, h\bar{I}_A, h\bar{I}_B\}\}.$$  

Workers in country $h \in \{P, Q\}$ must choose between to work in industry $A$ or $B$ as skilled worker ($s_A$ or $s_B$) or as unskilled one $\bar{s}$ (indifferently from industry that
hired him). Analogously, firms of country \( h \in \{ P, Q \} \) must choose between to invest in industry \( A \) or \( B \) in innovation activities \( I \) or not to innovate \( \bar{I} \).

Country \( P \) is abundant in factors used productively in industry \( A \) and, country \( Q \) is abundant in factors used productively in industry \( B \). Hence, we can state the following.

**Definition 2.2.** The cost of production for each firm will be represented by a real function, which depends on branch, the type of firm and the country where the firm produce. These functions are denoted by: \( C_{hj} : [0, \infty) \to R \) for each \( j \in \{ A, B \}, f \in \{ I, \bar{I} \} \) and \( h \in \{ P, Q \} \). Countries have a comparative advantage in producing goods that use their most abundant factor of production. Hence, the cost of producing \( y > 0 \) units of output in industry \( A \) located in country \( P \) is lower than the cost of producing of \( y > 0 \) units of output in industry \( A \) located in country \( Q \). Symbolically:

\[
\max \{ c_{PAI}(y), c_{PAB}(y) \} < \min \{ c_{QAI}(y), c_{QAB}(y) \} \qquad (1)
\]

Let \( N_{hi} \) be the total number of workers in country \( h \) choosing the strategy \( i \in \{ s_A, s_B, \bar{s} \} \) and let \( N_h \) be the total number of workers in country \( h \in \{ P, Q \} \). Analogously, let \( F_{hj} \) be the total number of firms in country \( h \) choosing to be \( j \in \{ I_A, I_B, \bar{I}_A, \bar{I}_B \} \) (that is to say, innovative in sector \( A \), innovative in sector \( B \), non-innovative in sector \( A \) and non-innovative in sector \( B \)) and let \( F_h \) be the total number of firms in country \( h \in \{ P, Q \} \). Let \( \Delta_{hw} \) be the set of distribution over the set of pure strategies \( S_{hw} \) i.e:

\[
\Delta_{hw} = \{ (x_{hs_A}, x_{hs_B}, x_{\bar{s}}) : x_{hs_A} + x_{hs_B} + x_{\bar{s}} = 1; \text{and } x_{hi} \geq 0 \forall i \in \{ s_A, s_B, \bar{s} \} \}, \quad h \in \{ P, Q \}
\]

thus \( x_{hi} = N_{hi}/N_h \) is the fraction of individuals choosing to be \( i \) in the country \( h \). and let \( \Delta_{hf} \) be the set of distribution over the set of pure strategies \( S_{hf} \) i.e:

\[
\Delta_{hf} = \{ (y_{hIA}, y_{hIB}, y_{\bar{I}A}, y_{\bar{I}B}) : y_{hIA} + y_{hIB} + y_{\bar{I}A} + y_{\bar{I}B} = 1; \text{and } y_{hj} \geq 0 \forall j \in \{ I_A, I_B, \bar{I}_A, \bar{I}_B \} \}
\]

thus \( h_{hf} = F_{hj}/F_h \).

Sets \( \Delta_{hw} \) and \( \Delta_{hf} \) correspond to the mixed strategies over the set of pure strategies \( S_{hw} \) and \( S_{hf} \) respectively. A pair \( (x_h, y_h) \in \Delta_{hW} \times \Delta_{hf} \) represents a strategic profile or a state of the economy in country \( h \in \{ P, Q \} \).

Given a state \( (x_h, y_h), \forall \ h \in \{ P, Q \} \) and \( j \in \{ A, B \} \), we have that:

- For a fixed distribution \( y_h \), the expected conditional profit of a skilled worker in country \( h \) in industry \( j \) is given by

\[
\mathcal{E}(hs_j/y_h) = [w_{hs_j} + pr_{hj}]y_{hI_j} + w_{h\bar{s}}(1 - y_{hI_j}) - c_{hs}. \quad (2)
\]

where \( w_{hs_j} > 0 \) is wage earned by a skilled worker and \( pr_{hj} > 0 \) is a skill premium when hired by an innovative firm with probability \( y_{hI_j} \in [0, 1] \), \( w_{h\bar{s}} > 0 \) is wage earned by an unskilled worker when hired by a non-innovative firm with probability \( (1 - y_{hI_j}) \in [0, 1] \), and \( c_{hs} > 0 \) is education/training cost for being a skilled worker.
• Analogously, the expected profit of an unskilled worker in country $h$ and industry $j$ is given by

$$E(hs/y_h) = w_{hs}. \quad (3)$$

getting only the unskilled’s wage $w_{hs} > 0$. Notice that in this case there are not education costs. (For a discussion on the difference in salaries originating in the skills of workers see [6]), or [9].)

• For a fixed distribution $x_h$ of workers in country $h$, the expected profit of an innovative firm of industry $j$ in this country is given by

$$E(hI_j/x_h) = [gh_I_j(s_j) - w_{hs_j} - pr(hj)]x_{hs_j} + [gh_I_j(\bar{s}) - w_{s\bar{s}}](1 - x_{hI_j}). \quad (4)$$

where $gh_I_j(s_j) > 0$ is the gross profit when hiring skilled workers with probability $x_{hs_j} \in [0, 1]$, and $gh_I_j(\bar{s} > 0)$ is the same gross profit got when hiring unskilled workers with probability $(1 - x_{hI_j}) \in [0, 1]$.

• Analogously, the expected profit of a non-innovative firm in country $h$ in industry $j$ is given by

$$E(hI_j/x_h) = gh_I_j(\bar{s}) - w_s. \quad (5)$$

where $gh_I_j(\bar{s}) > 0$ is the gross profit from hiring unskilled workers.

Recall that by strategic complementarities, innovative firms prefer to hire skilled workers, and skilled workers prefer to be employed by innovative firms. Analogously this holds for non-innovative firms, which prefer to hire unskilled workers and the latter prefer to be hired by non-innovative firms. So, the interaction between firms and workers determines a coordination game where the former and the latter try to maximise their payoffs. Therefore, a pair of mixed strategies $(x^*_h, y^*_h) \in \Delta_{hw} \times \Delta_{hf}$ is a Nash equilibrium for the game, $\Gamma_h$, in country $h \in \{P, Q\}$, if and only if for all $s_i: x^*_{hs_i} > 0$ and for all $I_j : y^*_{hI_j} > 0$ we have that:

$$E(hs_i/y_h^*) \geq E(hl/y_h^*) \text{ for all } S_{hw} \text{ and}$$

$$E(hI_j/x_h^*) \geq E(hk/x_h^*) \text{ for all } S_{hf}.$$

Strategic complementarities imply that in each $h \in \{P, Q\}$ the strategic profiles

1. $x^1_h, y^1_h : x^1_h = (1, 0, 0, 0), y^1_h = (1, 0, 0, 0)$
2. $x^2_h, y^2_h : x^2_h = (0, 1, 0, 0), y^2_h = (0, 1, 0, 0)$
3. $x^3_h, y^3_h : x^3_h = (0, 0, 1, 0), y^3_h = (0, 0, 1, 0)$
4. $x^4_h, y^4_h : x^4_h = (0, 0, 0, 1), y^4_h = (0, 0, 0, 1)$

are Nash equilibria. The Nash equilibrium $(x^1_h, y^1_h)$ corresponds to the Nash equilibrium in pure strategies $(hs_A, I_{hA})$, i.e. given that workers are choosing to be skilled in branch $A$, firms prefer to be innovative in industry $A$ and reciprocally. The Nash equilibria $(x^2_h, y^2_h)$ corresponds to the Nash equilibrium in pure strategies $(hs_B, I_{hB})$, i.e. given that workers are choosing to be skilled in industry $B$, firms prefer to be innovative in industry $B$ and reciprocally. The Nash equilibria $(x^3_h, y^3_h)$ and $(x^4_h, y^4_h)$ correspond to the Nash equilibrium in pure strategies $(h\bar{s}, I_{hA})$ i.e. given that workers are choosing to be unskilled, firms prefer to be not to innovative.

**Proposition 1.** For the game $\Gamma_h$ defined above, there exists a threshold value such that firms and workers decide to be innovative and skilled, respectively.
Proof. Rational workers from country $h$ will prefer to be skilled in industry $j$ if and only if $\mathcal{E}(hs_j) > \mathcal{E}(hs_I) > w_{hs}$. i.e.: if and only if

$$y_{hI_j} > \frac{c_s}{w_{hs_j} - w_{hs} + pr_{hj}} = y_{hj}^T.$$  \hspace{1cm} (6)

Analogously, firms in country $h$ and industry $j$ prefer to be innovative $\mathcal{E}(hI_j) > \mathcal{E}(hI_I)$ if and only if

$$x_{hs_j} = \frac{g_{hI_j}(s) - g_{I_j}(s) - w_{sj} - w_{s} - pr(hj)}{g_{I_j}(s) - g_{I_j}(s)} = x_{hj}^T. \hspace{1cm} (7)$$

\begin{remark}
At time $t = t_1$ we assume free mobility of capital and workers, and the economies of both countries are in the equilibrium, that is $(x_P, y_P)$ for country $P$ and $(x_Q, y_Q)$ for country $Q$, such that $x_{hA}^* > 0, x_{hA}^* > 0, x_{hB}^* > 0$ and $y_{hA} > 0, y_{hB} > 0, h \in \{P, Q\}$. These inequalities mean that:

$$\mathcal{E}(hs_A/y_h^*) = \mathcal{E}(hs_B/y_h^*) = \mathcal{E}(hs_I/y_h^*)$$

$$\mathcal{E}(hI_A/x_h^*) = \mathcal{E}(hI_B/x_h^*) = \mathcal{E}(hI_I/x_h^*)$$

If we allow free mobility of capital and labor, these equalities do not hold anymore.
\end{remark}

3. Openness in the game. According with Definition 2 country $P$ is more competitive in industry $A$ than country $Q$, while the opposite happens in industry $B$.

We allow for perfect mobility of capital and labor between industries and countries. Once the free mobility of capital and labor is established, the set of strategies available to workers and firms is expanded.

1. Capitalists/entrepreneurs/owners of firms need to choose, at the beginning of every period, the country in which to invest and whether to be innovative or not, so a capitalist may choose among the following strategies:

$$S_f = S_{fP} \times S_{fQ} = \{PI_A, PI_B, PI_A, PI_B, QI_A, QI_B, QI_A, QI_B\}$$

where $= S_{fP} = \{PI_A, PI_B, PI_A, PI_B\}$ and $S_{fQ} = \{QI_A, QI_B, QI_A, QI_B\}$ and

- $PI_A$ corresponds to the choice to invest in country $P$ as an innovative firm of branch $A$.
- $PI_B$ corresponds to the choice to invest in country $P$ as an innovative firm of branch $B$.
- $PI_A$ corresponds to the choice to invest in country $P$ as a non-innovative firm of branch $A$.
- $PI_B$ corresponds to the choice to invest in country $P$ as a non-innovative firm of branch $B$.
- Or alternatively to invest in country $Q$ as innovative or as a non-innovative firm in branch $A$, or $B$.

(Options that are denoted respectively by $QI_A, QI_B, QI_A$ and $QI_B$.)

We have that $S_f$ is the set of pure strategy for firms when trade is open. So $s_f \in S_f$ if and only if $s_f = hjf, f \in \{I, I\}, j \in \{A, B\}, h \in \{P, Q\}$.

2. Once that trade is open, workers will be able to choose the country ($P$ or $Q$), the branch ($A$ or $B$) and the degree of skill ($s$ - skill, or $s$ - unskill). So, after
free mobility is established, the set of pure strategies or possible behaviors is given by:

$$S_w = S_{Pw} \times S_{Qw} = \{P_{sA}, P_{sB}, P_{sA}, Q_{sB}, P_{sA}, Q_{sB}, Q_{sA}, Q_{sB}\}$$

Where $$S_{Pw} = \{P_{sA}, P_{sB}, P_{sA}, P_{sB}\}$$ and $$S_{Qw} = \{Q_{sA}, Q_{sB}, Q_{sA}, Q_{sB}\}$$.

So, $$s_w \in S_w$$ if and only if $$s_w = hw_j$$, $$w \in \{s, \tilde{s}\}$$, $$j \in \{A, B\}$$ and $$h \in \{P, Q\}$$.

The sets of mixed strategies for firms and workers are given by the sets of distributions over their respective types (pure strategies) in each country. Keep in mind that there is a probability that a firm in country $$k \neq h \in \{P, Q\}$$ will hire a worker from country $$h$$. This will be possible, only after the worker has emigrated.

So, when a worker of country $$h \in \{P, Q\}$$ chooses to remain in his own country or to emigrate, the expected value of each strategy $$i \in S_w$$ will be $$E(hi/x_h)$$ or $$E(ki/x_k)$$, $$k \neq h \in \{P, Q\}$$ depending on whether he chooses to stay in his country $$h$$ or to emigrate in country $$k$$. This is so, because we assume that regardless of the decision a worker makes (that is to say, emigrate or not) he will maintain, at least at the beginning, its specialty, that is to say, the branch of production and degree of specialization that he had at the time prior to make this decision. Analogously for the choices of investors. They must choose in which country to invest in the future, but they will maintain, at first, their characteristics at the time prior to this decision.

We consider that this distribution over the set of pure strategies $S_w$ and $S_f$ are coincident with the distribution of populations of firms and workers over their respective set of pure strategies in each country.

Assuming that the total number of workers is given, let us denote by $x_{hi}$, the fraction of $i$-workers ($i \in \{s, \tilde{s}\}$) in industry $j \in \{A, B\}$ from country $h \in \{P, Q\}$. So, the mixed strategy is given by the profile distribution of workers over their respective types and countries, i.e.

$$\Delta_w = \{xp_{sA}, xp_{sB}, xp_{sA}, x_{QsA}, x_{QsB}, x_{QsB}\}$$

So, $$\sum hi \in S_w x_{hi} = 1$$, $$0 \leq x_{hi} \leq 1$$

Analogously, for the distribution of the firms we have

$$\Delta_f = \{yp_{IA}, yp_{IA}, yp_{IA}, y_{QIA}, y_{QIA}, y_{QIA}\}$$

So, $$\sum f \in S_f y_{hf} = 1$$, $$0 \leq y_{hf} \leq 1$$. Strategic profiles $$(x, y) \in S_w \times S_f$$ correspond to the set of states of the 2-country economy, where free trade is allowed. Thus, $$x = (xp, xQ)$$ and $$y = (yp, yQ)$$, where the pair $$(x, y) = ((xp, xQ), (yp, yQ))$$ corresponds to an state of the open economy.

$$xp = (xp_{sA}, xp_{sA}, xp_{sB}, x_{QsB}), xQ = (x_{QsA}, x_{QsA}, x_{QsB}, x_{QsB})$$.

and

$$yp = (yp_{IA}, yp_{IA}, yp_{IA}, y_{QIA}), yQ = (y_{QIA}, y_{QIA}, y_{QIA}, y_{QIA})$$.

If workers or firms in country $$h$$ decide to migrate to country $$k$$, they must paid the cost of migration, $$c(h, k) = c(k, h) > 0$$.

### 3.1. A dynamic for the open economy.

Let us analyze now the game under free mobility of workers and capital between countries. Assume that the global economy is in state $$(x, y)$$.

We consider that a worker must first choose between to be skilled or unskilled, inside their own country, or in the foreign country. If he chooses to migrate, he must pay a cost $$c_w(h, k) = c_w(k, h) > 0$$. Analogously for investors, they must choose...
choose between investing in a innovative or in a non-innovative firm, the branch of industry and the country where to invest. If an investor in country \( h \) decides to invest in country \( k \), then, he must pay a cost \( c_{f}(h, k) = c_{f}(k, h) > 0 \).

So, a worker in country \( h \) will choose to follow the strategy \( i \in \{ S, \bar{S} \} \) if

\[
\mathcal{E}(h_i/y_h) \geq \max \left\{ \max_{j \in S_{h_w}} \{ \mathcal{E}(h_j/y_h) \}, \max_{j \in S_{k_w}} \{ \mathcal{E}(k_j/y_k) + c_{w}(hk) \} \right\}.
\]

Analogously for investors. So, an investor in country \( h \) must choose a strategy \( j \in S_{f} \). He must choose the country where to invest, i.e., a strategy \( j \in S_{f} \). He will choose the strategy \( j \) in \( h \) if and only if

\[
\mathcal{E}(h_j/x_h) > \max \{ \mathcal{E}(h_l/x_h), \mathcal{E}(k_j/x_k) + c(hk), \mathcal{E}(k_l/x_k) + c_{f}(hk) \}
\]

for all \( j \neq l \in \{ A, B \} \) and \( k \neq h \in \{ P, Q \} \).

A worker in country \( h \) prefers to be unskilled and not to emigrate if and only if

\[
w_{h\bar{s}} > \max \{ \mathcal{E}(h_s/y_h), \mathcal{E}(h_{s_l}/y_h), \mathcal{E}(k_{s_j}/y_k) + c_{w}(hk), w_{k\bar{s}} + c_{w}(hk) \}.
\]

A worker in country \( h \) will prefer to be skilled in industry \( j \) and remain in country \( h \), if

\[
\mathcal{E}(h_{s_j}/y_h) > \max \{ w_{h\bar{s}}, \mathcal{E}(h_{s_l}/y_h), w_{k\bar{s}} + c_{w}(hk), \mathcal{E}(k_{s_j}/y_k) + c_{w}(hk) \}.
\]

An investor in country \( h \) will prefer to invest in industry \( j \) in his own country if:

\[
\mathcal{E}(h_j/x_h) \geq \max \left\{ \max_{j \in S_{h_s}} \{ \mathcal{E}(h_j/x_h) \}, \max_{j \in S_{k_s}} \{ \mathcal{E}(k_j/x_k) + c_{f}(hk) \} \right\}
\]

where, according to our notation, country is represented by \( h \in \{ P, Q \} \) and industry by \( j \neq l \in \{ A, B \} \), where \( y_{hI_j} \) and \( x_{h_{s_j}} \) are the share of innovative firms and the share of skilled workers, respectively, in each country. Costs of education are given by, \( c_{h_{s_j}} \) and \( c_{h_{s_l}} \), and skill premia are \( pr_{hj} \) and \( pr_{hl} \), in the two industries \( j \neq l \). Let us consider, without loss of generality, that if salaries are uniform in every industry and each each country, i.e. \( w_{h_{s_j}} = w_{h_{s_l}} = w_{k\bar{s}} = w_{h_{\bar{s}}} = w_{h} \), the following proposition is a consequence of the assumption that workers and firms are rational, i.e. they choose their strategies looking for the expected value.

**Proposition 2.** A worker in country \( h \in \{ P, Q \} \) and industry \( j \in \{ A, B \} \) prefers to be skilled and remains (not to migrate) in his local country if:

\[
y_{hI_j} > \max \left\{ \frac{c_{h_{s_j}}}{pr_{hj}}, \frac{y_{hI_j} pr_{hl} + c_{h_{s_l}} - c_{h_{s_j}}}{pr_{hl}}, \frac{y_{kI_j} pr_{k_{s_j}} + c_{k_{s_j}} - c_{h_{s_j}} + c_{w}(hk)}{pr_{hl}} \right\} = y_{h_{\bar{s}_j}}.
\]

(11)

where \( y_{h_{\bar{s}_j}} \) is a threshold value, such that if the share of innovative firms \( y_{hI_j} \) is larger than this value, then workers from country \( h \) in industry \( j \) prefer to be skilled and remain in their own country. Notice that, as the costs of education \( c_{h_{s_j}} \) and \( c_{h_{s_l}} \) decrease and as the skill premia, \( pr_{hj} \) and \( pr_{hl} \), \( \forall j \neq l \in \{ A, B \} \), increases, this threshold decreases.

An investor in country \( h \) in industry \( j \) prefers to invest in innovative firms of industry \( j \) in his local country if:

\[
x_{h_{s_j}} > \max_{k \in \{ P, Q \}} \left\{ \frac{[\pi_{kI_j}(s_l) - \pi_{k_{hI_j}(s)}] x_{kI_j} + [\pi_{kI_j}(s_l) - \pi_{h_{hI_j}(s)}]}{\pi_{hI_j}(s) - \pi_{h_{hI_j}(s)}}, \pi_{kI_j}, \pi_{k_{hI_j}} \right\} = x_{h_{\bar{s}_j}},
\]

(12)

where \( l \neq j \in \{ A, B \} \).
where $x^T_{hj}$ is a threshold value, such that if the share of skilled workers, $x_{hsj}$, from country $h$ and industry $j$ is larger than this value, then firms’ investment in innovation activities is carried out in country and industry of origin.

**Proof.** Inequality (11) is verified if and only if the fraction of innovative firms in branch $j$ in his own country is bigger than $y^T_{hj}$, that is to say, if inequality (9) holds. Analogously for investors we have inequality (10).

Suppose that the global game is played repeatedly over an uncertain time span and that at the beginning of the period, workers and firms must choose their strategy for the current period. More precisely, we consider that at the end of each period workers and firms can change their strategies according to his own perception of the expected value of each feasible action. This choice is made according to the current perception that each agent has of the expected profits of each possible strategy to be follow. The dynamical system regulating the evolution of the economy, can be specified once that the imitation rules are established.

A reviewer (rational) worker chooses the strategy for the next period according to the expected value of the strategy. We assume that workers do not know the true distribution of firms, at the moment when they need to choose the future strategy. Urged by the peremptory term to make a decision appeal to an alternative method to the rational choice. So, this choice must be made within a framework of bounded rationality.

We assume that workers make this decision in accordance with a process of imitation, for example they do it as others do.

In a framework of limited rationality, instead of comparing alternatives before making a choice, decision makers often simply imitate the choices made by others. Imitation can be advantageous when the comparison of different alternatives is relatively expensive, or when decision makers do not have enough time or the possibility to collect all the information required for making a rational choice. See for instance [7]. This is a typical characteristic of the migration process as it is clearly highlighted in the socio-political literature about massive migration in Latin America.

According with these alternatives, they look for the behavior of the individual in their network, and follow the behavior of the most successful neighbor. We leave for later the discussion of a specific model of imitation, however, from now on we follow the approach introduced in [3]. Following this approach, we assume that the probability to choose one strategy, is directly related to the true expected value (unknown) of such a strategy. This is summarized in the following definition of the behavior rule:

**Definition 3.1. (The behavior rule)** The probability that a reviewer worker will switch strategy from $i \in S_w$ to $m \in S_w$ when the state of the economy is $(x, y) \in S_w \times S_f : x_i \neq 0$ is denoted by the expression $p(i \rightarrow m/(x, y))$ and verify the following rules:

$$p(i \rightarrow m/y) > p(i \rightarrow a/(y) \iff \mathcal{E}(m/y) > \mathcal{E}(a/y)$$

$$p(i \rightarrow a/y > 0 \iff \mathcal{E}(a/y) > \mathcal{E}(i/y) \text{ and}$$

$$p(i \rightarrow a/y) = 0 \text{ otherwise.}$$

\[ (13) \]
where by $E(i/y)$ we denote the expected value of the strategy $i \in S_w$ given the state $(x,y) \in \Delta_w \times \Delta_f$

Let $N_i(t)$ be the number of workers following strategy $i \in S_w$ in time $t$. Consider a total given number of workers (in every time $t$) $N = \sum_{i\in S_w} N_i(t)$.

Let us denote by $x_i(t) = \frac{N_i(t)}{N}$ the share of workers following strategy $i \in S_w$ in time $t$. Consider a total given number of workers (in every time $t$) $N = \sum_{i\in S_w} N_i(t)$.

Let us denote by $x_i(t) = \frac{N_i(t)}{N}$ the share of workers following the $i-$behavior, in time $t$. Then the expected flow for the sub-population of workers following $i$-strategy, in state $(x,y)$ can be represented by

$$\Delta_i = (\text{incoming flow to } i) - (\text{outgoing flow of } i) = \sum_{j\neq i} x_j p(j \rightarrow i/(x,y)) - x_i \sum_{j\neq i} p(i \rightarrow j/(x,y)).$$

Finally, assuming a continuous process of changes, we arrive to the following dynamical system modelling the evolution of each subpopulation of workers.

$$\dot{x}_{ihj}(t) = \sum_{j\neq i} x_j p(j \rightarrow i/(x,y)) - x_i \sum_{j\neq i} p(i \rightarrow j/(x,y)) x_{ihj}. \quad (14)$$

Workers prefer to follow the behavior with the greatest expected value (according with the rationality principle), so the probability that a reviewer worker chooses to migrate to change from industry or to become a skilled worker depends the expected benefits of each possible strategy. If worker’s information is complete, then they know these values and they will choose their future behavior according with such information. However if the information available for workers is incomplete or imperfect, they must create mechanisms or appeal to different sources of information before making future decisions with respect to their future behavior. In these cases, as we will argue later, imitation may be a natural and rational mechanism for choosing a behavior when there are no means or enough time to know or anticipate the future result of the current selection and the decision can not be delayed.

**Proposition 3.** Skilled workers in industry $j \in \{A,B\}$ in country $h \in \{P,Q\}$ prefer to migrate to country $k \in \{P,Q\}$ if

$$E(hs_j) + c_w(hk) < \max\{E(ks_j), E(k\bar{s})\}.$$

**Proof.** Recall that

$$E(is_j) = y_{it_j}(w_{ks_j} + pr_{ij}) + y_{i(-j)}w_{i\bar{s}}$$

$$E(i) = w_{i\bar{s}} : i \in \{P,Q\}$$

where $y_{h(-t_j)} = (1 - y_{hT_j})$ is the probability that a skilled worker of branch $j \in \{A, B\}$ will be hired by a different firm from an innovative firm of the $j$ branch in the country $h \in \{P, Q\}$.

Then, if $y_{kt_j}(t_1) > y_{kt_j}$ (\textsuperscript{\textdagger}) we have that

$$\max\{E(ks_j), E(k\bar{s})\} = E(ks_j)$$

So, under condition (\textsuperscript{\textdagger}) if a skilled worker in branch $j$ and country $h$ decides to migrate, he will work as skilled worker in branch $j$ and country $k.$ \qed

Notice that, the cost of producing $y > 0$ units of output in industry $j$ in a country where there is abundance of the specific input for production, $z_j > 0$, is lower than the cost of producing the same amount of output in country where this input is
relatively scarce. Then, in country $P$ the threshold value is lower for the industry $A$ than in the country $Q$ and inversely for the industry $B$. Then, the inequalities
\[ y^T_{PI_A} < y^T_{QI_A}, \quad y^T_{QI_B} < y^T_{PI_B}. \] (16)
hold. This means that innovations activities in a given branch are most likely in a country with comparative advantages in that branch. Consequently, we can infer that there is a natural tendency for skilled workers in each branch to migrate to those countries with comparative advantages in the industries in which they are specialized. This trend is determined once the threshold value in the corresponding country has been reached.

Assuming that the comparative advantages of each country are public knowledge, then there will be a migratory tendency of skilled workers from each branch to the countries with comparative advantages in that branch. We can summarize this conclusions in the following theorem.

**Theorem 3.2.** Under the hypothesis of public knowledge of comparative advantages, if country $P$ has a comparative advantage in industry $A$, while country $Q$ has a comparative advantage in industry $B$, then skilled workers of industry $A$ from country $Q$ tend to migrate towards the country $P$, while the skilled workers of industry $B$ from country $P$ tend to emigrate to the country $Q$. This holds even assuming that workers are bounded rational.

**Proof.** Given the assumptions on the comparative advantages, inequality (16) follows, then
\[ PR[y^T_{PI_A} < y^T_{QI_A}(t_1)] > PR[y^T_{QI_A} < y^T_{QI_A}(t_1)] \]
i.e; the probability that the number of innovative firms in branch $A$ in country $P$ overcomes the threshold value is greater than the probability that the corresponding threshold value will be exceeded in country $Q$.

Assuming that the cost of migration is low enough, it follows that
\[ E(Qs_A) + c_{QP} < E(Ps_A) \] and
\[ E(Ps_B) + c_{PQ} < E(Qs_B). \]

**Corollary 1.** This means that country $P$ at time $t = t_0$ specializes in industry $A$ and country $Q$ becomes specialized in industry $B$ due to the fact that country $P$ has a comparative advantage in industry $A$ and $Q$ in industry $B$, and agents follow an imitative behavior.

4. **Falling in a technological poverty trap.** Specialization in economics refers to countries that focus on production of limited types of goods, according to what they can produce best or at a lower cost. Adam Smith, in his book “An Inquiry into the Nature and Causes of the Wealth of Nations”, explains and clarifies this concept. Smith in [12] illustrates the benefits of specialization and the division of labor when describing a pin factory, in which each worker performs a single specialized task. Adam Smith argues that specialization increases output because workers do not lose time shifting among different tasks. Smith also believes that specialized workers were more likely to innovate, to create tools or machinery to make their tasks even more efficient. We agree, partially, with this ideas. It is important to notice that specialization (especially if it comes from the abundance and the relative cheapness of a given resource) does not mean necessarily a maximization of social
welfare. As we will see, this specialization, together with international trade, can lead, under particular initial conditions, to a poverty trap. The easily achieved success in the international markets from a natural advantage can lead to stagnation, rather than to encourage technological development.

Consider now the case:

$$x_{sAP}(t) < x_{sAP}^T$$ and $$x_{sBQ}(t) > x_{sBQ}^T$$

where country $P$ has comparative advantage in industry $A$, (according with our model, the degree of specialization increases on time) and the supply of skilled workers is lower than the threshold value. Therefore, firms prefer to be non-innovative and complementarily workers prefer to be unskilled because $\mathcal{E}(PnsA) > \mathcal{E}(PsA)$. Then according to the rationality principle (see (13)) we have that:

$$p(PnsA/PsA) > 0 \text{ and } p(PsA/PnsA) = 0$$

i.e. a reviewer skilled worker from country $P$ within industry $A$ imitates to become unskilled.

Since the probability that a worker is a reviewer increases inversely with the performance of her current strategy, then $\gamma_{PA}^P > 0$, and the probability that a skilled worker within industry $A$ becomes unskilled worker in $A$ is positive, i.e.

$$p(PsA \to PnsA) = \gamma_{sAP}p(PnsA/PsA) > 0$$

while the reverse probability is zero: $p(PnsA \to PsA) = \gamma_{sAP}p(PsA/PnsA) = 0$.

Hence, the flow from the subpopulation of skilled workers to the other subpopulation is positive, and given that the $\mathcal{E}(sAP) < \mathcal{E}(nsAP)$ there is not a positive flow to the subpopulation of skilled workers in industry $A$ an country $P$ because the expected value to be unskilled in industry $A$ and country $P$ is higher than the expected value of to be skilled in such branch and country. Moreover there exists an important flow from this subpopulation to the others. Using the equation (14) and considering $P((PsA/ihj) = 0 \forall ihj$ we obtain:

$$\dot{x}_{sAP} = -\gamma_{sAP} \left[ \sum_{jih \neq sAP} P(jih/PsA) \right]$$
Reciprocally, given that the fraction of innovative firms in branch $B$ and country $Q$ is higher than the threshold value, workers prefer to be skilled, and so $\dot{x}_{sB}^Q > 0$. This implies that the productivity of innovative firms in branch $B$ and country $Q$ increases and firms prefer to become innovative.

Moreover for $p(QsB/PnsA)$ positive and small enough or zero, then $\dot{x}_{sA}^P > 0$ and as we argued before $\dot{x}_{sA}^P < 0$. The result is that the country is evolving to a low level equilibrium in branch A, however, according with the assumption that

$$\max \{c_{AI}(y), c_{ANI}(y)\} < \min \{c_{AQ}(y), c_{ANIQ}(y)\}$$

country $P$ remains specialized in branch A, but the evolution is to a low level equilibrium.

With similar arguments it is possible to argue that for branch $B$ in country $Q$, the economy is evolving to a low-level equilibrium, in this case both countries are in a poverty trap, given by the imitative behavior of the economic agents, and stimulated by international trade.

International trade is the means by which countries can get specialized when they have a comparative advantage. In our case, country $P$ has comparative advantage in industry $A$. However, in order to develop the branch with comparative advantage means to lose the most innovative firms and skilled workers, at least in this industry. This means that country $P$ can be caught in a poverty trap.

So, let us now consider the following definition of a poverty trap. (See [1].)

**Definition 4.1.** A poverty trap is a Pareto dominated Nash equilibrium resulting from a coordination game that is at the same time an asymptotically stable equilibrium of a dynamical system whose solutions correspond to the future evolution of the economy.

In our case the dynamical system is given by equations (18). Consequently, from the point of view of the economy of a country, we can introduce the following definition.

**Definition 4.2.** We say that a country is in a poverty trap if the initial conditions are in the basin of attraction of a Pareto-dominated equilibrium. Which means that rational behavior of the economic agents is not enough to reach a superior welfare level. Even more, along the time the individual rational behavior, makes it more and more harder to get out of the poverty trap in which the country is immersed.

In our model, this happens if at $t_0$, the fraction of innovative firms in country $h$ is below the threshold value $y_{hj}^T$ given by equation (11).

On the contrary, country $Q$, using its comparative advantage, becomes an high tech and developed country. So, the exploitation of comparative advantages can give place to divergent processes of growth, even if both countries are taking advantages from international trade.

Following this line of thought, we introduce a definition of technological poverty trap.

**Definition 4.3.** A country is in a technological poverty trap if: i) firms prefer not to be innovative, that is, there are no incentives for technological development (including the branch where it has a comparative advantage), ii) the working population prefers not to be skilled. Decision based on the non-existence of incentives for a rational worker to choose to be qualified. and iii) this is a self-reinforcing process.
An opposite scenario is offered by the industrial specialization in country $Q$ and sector $B$, where the supply of skilled workers is higher than the threshold value, and then $\mathcal{E}(sBQ) > \mathcal{E}(nsBQ)$, this means that $\dot{x}_{sBQ} > 0$. Moreover, since $\mathcal{E}(QsB) > \mathcal{E}(PsB)$, a skilled worker within industry $B$ from country $P$ will prefer to migrate to country $Q$, and this eventuality depends on the cost $c_{AB}$, and obviously on the expected income differentials between countries. Much of the analysis made in [8] can be considered here for understanding the phenomena of migration between countries or branch in the same country. A sketch of this case is represented in Figure 2. The following inequality summarizes the above situation:

$$p(PnsA/PsA) > p(QsA/PsA) > p(PsB/PsA) > 0$$

The first inequality holds because of the comparative advantage of country $P$ in industry $A$. The second one holds because we assume that the cost $C_{AB}$ is high enough, and from the previous assumption (17) it follows that

$$p(PnsA/QsA) > p(PsA/QsA) > p(QsB/QsA) > 0$$

The last two inequalities depend on the cost $c_{AB}$.

**Remark 2.** Note that the social result of the industrial specialization within each country is divergent. Industry $A$ in country $P$ is evolving to a poverty trap. On the contrary, industry $B$ in country $Q$ is going to a high-level equilibrium. International trade exacerbates the poor social performance of country $P$’s economy. Despite the fact that this country is successful in international trade because of its comparative advantage, it evolves into a poorly industrialized equilibrium with low human capital development.

**Remark 3.** The particular case of the $P$–country evolving to a poverty trap calls for participation in the economy of a central authority or policy maker, provided that the objectives of such an intervention are clear and bounded in time.

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1. It should be noted that the cost of moving from one country to another depends on many factors. This cost can be high even in the case where workers stay within the same branch, because some specializations acquired in the country of origin can not be recognized in the country of destination.
4.1. **Overcoming the poverty trap.** The main question is, if it is possible to overcome a technological poverty trap. Next we argue on this possibility.

According to our analysis, a technological trap of poverty is the result of adverse internal conditions for the future development of a country, but which, from a short-term point of view, appear as favorable. Moreover, when this appreciation is reinforced by the development of international trade that allows a fast growth in countries with comparative advantages. Even when its expression is the development of an industry that generates little added value or technologically backward.

There are several countries in which the growing export of crude raw materials generates growth (increases in GDP) but without repercussions on the technological development of the country, rather the opposite happens. Initial conditions of a country are the responsible for condemning a country to low levels of future growth, once the comparative advantages have been exhausted. Therefore, a benevolent social planner must take care of threshold values, overcoming them is necessary so that the country does not remain in the trap of poverty and can begin a process of technological development, being less expensive in the branch where comparative advantage exists.

Since the threshold value in country $P$ for this industry, given by $x_{sAP}^T$, is lower than the corresponding value for the same industry in country $Q$, $x_{sAQ}^T$, i.e.: $x_{sAP}^T < x_{sAQ}^T$ then, in order to take the country off from the path of low growth (the poverty trap), an intervention by the policy maker in industry $A$ is more efficient than an intervention in industry $B$.

The objective of public intervention may be aimed at complementing markets’ action, developing a program of incentives for investments in R&D, education, or directly for skilled workers, in industry in which the country has an advantage. The fulfillment of this objective can be facilitated by the existence international trade that allows the possibility of complete specialization.

5. **Concluding remarks.** In this model we show that technological poverty traps and, in general, inequality between countries may arise as a consequence of specialization of a country towards sectors that use intensely unskilled labor, in an open market economy with free mobility of workers. This fact suggests that appropriate policies promoting technological development may be needed in order to avoid such a trap. A government must indeed ensure that the national economy becomes able to use technological development to improve competitiveness and, at the same time, to engage in paths towards sustained growth by exploiting the comparative advantages.

Many African, Asian and Latin American countries despite their abundance in natural resources, are trapped in poverty trap. Several of them are specialized in sectors that largely make use of unskilled labor abound see for instance [14]. In the opposite side, other countries, instead, have exploited their natural resources and

\[\text{http://projects-beta.worldbank.org/en/results/2014/08/06/innovation-as-key-to-diversify-the-economy-in-peru}\]
invested this returns in research aimed at improving technology which allows to extract at lower costs and/or guarantees sustainability of the whole process. Finland, as well as Norway, are valid examples of how governments have managed to keep these countries on paths of sustained growth and welfare despite their specialization at least in origin were focused in wood processing and production, and oil extraction, respectively, that is to say, in principle, sectors in which the use of unskilled labor abounds. Their technological developments were externalities of improvements in that kind of productions, with the help of the government who subsidized research when private incentives were too low. Finland, for example, developed strong industries such as pulp and paper, chemical and electronics (especially the production of electric motors and industrial automation), and all of them are someway the result of the exploitation of its more abundant resource, that is to say, forests.

So, a government that wants to promote growth should subsidize R&D (if this activity is found not sufficiently profitable by the existing private firms) in order make firms willing to pay more for hiring skilled labor, which has a higher productivity, because this makes less likely a path of this country towards a poverty trap.

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