Mode switching in the nearby Mira-like variable R Doradus

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ABSTRACT

We discuss visual observations spanning nearly 70 years of the nearby semiregular variable R Doradus. Using wavelet analysis, we show that the star switches back and forth between two pulsation modes having periods of 332 days and about 175 days, the latter with much smaller amplitude. Comparison with model calculations suggests that the two modes are the first and third radial overtone, with the physical diameter of the star making fundamental mode pulsation unlikely. The mode changes occur on a timescale of about 1000 d, which is too rapid be related to a change in the overall thermal structure of the star and may instead be related to weak chaos. The Hipparcos distance to R Dor is 62.4 ± 2.8 pc which, taken with its dominant 332-day period, places it exactly on the period-luminosity relation of Miras in the Large Magellanic Cloud. Our results imply first overtone pulsation for all Miras which fall on the P-L relation. We argue that semiregular variables with long periods may largely be a subset of Miras and should be included in studies of Mira behaviour. The semiregulars may contain the immediate evolutionary Mira progenitors, or stars may alternate between periods of semiregular and Mira behaviour.

Key words: stars: individual: R Dor – stars: individual: V Boo – stars: AGB and post-AGB – stars: oscillations – stars: variables: other

1 INTRODUCTION

Miras are large-amplitude, long-period variables located near the tip of the Asymptotic Giant Branch (AGB). Traditionally, stars are only considered Miras if their peak-to-peak amplitude at $V$ exceeds 2.5 mag. The periods are typically between 200 and 500 days, although OH/IR stars (a subset of the Miras which show large circumstellar extinction) have periods up to 2000 days. The periods are generally stable but the maximum and minimum magnitude can vary from cycle to cycle. Miras with periods longer than 300 days often show evidence for high mass-loss rates.

Mira variability is associated with the thermal-pulsing AGB, where the star alternates between periods of hydrogen and helium burning in a shell around the inert carbon/oxygen core. Mira pulsation occurs during the hydrogen shell-burning phase, when the star is more luminous, although it is possible that some stars also show Mira pulsations during the helium shell flash (the ‘pulse’; Wood & Zarro 1981; Zijlstra 1995).

The existence of a well-defined and narrow period–luminosity (P-L) relation for Miras in the LMC (Feast et al. 1989) is evidence that most Miras pulsate in the same mode. The identity of this mode, however, is still controversial. Temperatures and radii of Miras are consistent with first overtone: direct measurements of stellar angular diameters indicate that most Miras are larger and cooler than expected for fundamental mode pulsators (Tuthill et al. 1994; Feast 1996; van Leeuwen et al. 1997). On the other hand, the observed shock velocities in the CO lines are too large to easily be reconciled with any mode other than the fundamental (Wood 1990; Hinkle et al. 1997).

The semiregular variables differ from classical Miras in showing smaller amplitudes (< 2.5 mag peak-to-peak) and/or less regular pulsations, sometimes with multiple periods. Kerschbaum & Hron (1992, 1994) have argued that some stars classed as semiregulars are closely related to Miras, excluded only because of the restrictive classical definition. These stars could be important for our understanding of Miras and could include their immediate progenitors.

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R Doradus (HR 1492, M8 III) is classified as a semiregular (SRb) with a period of 338 d (Kholopov et al. 1988). This period is within the normal range for Miras, in contrast to most semiregulars, which have periods closer to 150 days. We have recently measured the angular diameter of R Dor to be $57 \pm 5$ mas, the largest of any star except the Sun (Bedding et al. 1997). We argued that this star is closely related to the Miras, in spite of its more complicated and smaller-amplitude variability, on the grounds that IRAS images show extended dust emission centred on R Dor (Young et al. 1993) and the IRAS LRS spectrum shows a weak silicate feature (Volk & Cohen 1989). Both indicate dust and mass loss, which are normally confined to Miras with periods of more than 300 days.

Here we discuss visual observations of R Dor spanning 70 years. We find that R Dor switches between two different modes, one with a Mira-like period and the other with a shorter period more typical of semiregulars. The Hipparcos distance is used to show that the longer period fits the Mira P-L relation.

2 OBSERVATIONS BY A. JONES

One of us (AJ) has monitored R Dor over a 23-year period (1944–1967), producing about 1100 measurements (Fig. 1). R Dor is circumpolar from New Zealand, so there are no yearly gaps in the time series. Although fainter stars are observed using a home-made 317 mm $f/5$ Newtonian reflector, brighter stars such as R Dor are observed using a smaller finder telescope. The magnitudes are estimated by visual comparison with fields of bright standard stars. When making visual estimates, certain precautions were taken to eliminate possible errors which are particularly important for a star as red as R Dor. Observations of red stars are not made in conditions of bright moonlight. When observing a red star it is advisable to make quick glances, otherwise visual observations may overestimate its brightness. To make estimates, each star is brought to the centre of the field before noting its brightness. If the variable and a comparison star are not far apart, the observer’s head is oriented so that the line joining the stars is parallel to the eyes. Another issue to be wary of is that the star nearest the observer’s nose may seem a little brighter than is the case. When going to the telescope, every effort is made not to recall any previous measurements – the
observations reported in this paper have never been plotted by the observer.

Inspection of the data in Fig. 1 shows that the pulsation behaviour of R Dor changed significantly over time. The dashed curve shows a sinusoidal period of 332 d. The peak-to-peak amplitude of the pulsation reached up to 1.5 mag during the late 1940s, after which the variation was much more irregular and of lower amplitude. The presence of variations with a shorter period can be seen, particularly around JD 2434500.

The Fourier amplitude spectra are shown in Fig. 2, with the data taken both as a whole and also as two subsets. There are two strong periods of 332 days and about 175 days, with the longer period being almost absent in the second part of the time series.

2.1 Wavelet analysis

We have also analysed the observations using wavelets. This technique has been shown by several groups to be a useful tool for investigating period and amplitude changes in long period variables (Szatmáry & Vinkó 1992; Szatmáry et al. 1994; Gál & Szatmáry 1995b; Szatmáry et al. 1996; Foster 1996; Barthès & Mattei 1997). We have used the weighted wavelet Z-transform (WWZ) developed by Foster (1996) specifically for unevenly sampled data. We experimented with different values for the parameter $c$, which defines the tradeoff between time resolution and frequency resolution (Foster 1996), and settled on $c = 0.005$ as a good compromise.

The results are displayed in Fig. 3. The top panel shows the light curve, while the middle panel shows the WWZ (the bottom panel is discussed below). The two periods are clearly present, with periods of 332 d and 175 d, and the star appears to alternate between them. However, care must be taken in interpreting the WWZ if we wish to distinguish between mode switching, in which one period essentially replaces the other, and the case in which one period decreases in amplitude while the other remains roughly constant. As discussed by Foster (1996), the WWZ is an excellent locator of the signal frequency but a poor measure of amplitude. He recommends estimating the amplitude using the weighted wavelet amplitude (WWA), which is excellent for this purpose once the signal frequency is known, but is less good at finding the correct frequency.

To examine this, we have conducted a simulation which is shown in Fig. 4. The light curve (top panel) is the sum of two sinusoids with periods of 332 d and 183 d (the shorter period is slightly different to that in R Dor because the simulation was done before the real period was confirmed). The long-period signal has a semi-amplitude that decreases linearly from 0.6 mag to zero through the course of the time series. The shorter-period component has a constant semi-amplitude of 0.2 mag. The WWZ (middle panel) shows the two periods clearly. At the longer period the WWZ decreases with time, as would be expected for a decaying signal, but at the shorter period the WWZ shows an increase with time, despite the fact that the input signal has constant amplitude. The reason is that the WWZ essentially measures the significance of the period in question, which naturally increases as the other period becomes less dominant. The bottom panel shows the semi-amplitude at each of the two periods, calculated using the WWA. We see that, except for end effects, the WWA correctly recovers the amplitudes of the input signals. If we had relied on the WWZ alone, we might have inferred a gradual mode switch, whereas the WWA correctly shows that there has been no exchange of power.

Returning to R Dor, the WWA at each of the two periods is shown in the bottom panel of Fig. 3. Unlike the simulation, there is an exchange of power. The first switch occurs around JD 2434000, with simultaneous strengthening of the shorter period and weakening of the longer one. There is a less prominent reverse switch starting at about JD 2437000.

Figure 2. Amplitude spectra of the data in Fig. 1. The data were taken as a whole (top panel), and also in two parts (middle and lower panels), where the division was made at JD 2434500.

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Figure 3. Wavelet analysis of observations of R Dor by A. Jones. The top panel shows the light curve reproduced from Fig. 1. The middle panel shows the WWZ and the bottom panel shows the WWA evaluated at periods of 332 d (thick line) and 175 d (thin line). See text for more details.

which is also apparent in the raw light curve, possibly followed by another switch at the very end of the time series.

3 OTHER OBSERVATIONS

Further investigation of R Dor requires more data. A. Jones has recently resumed regular observations of R Dor and these are presented below, but there remains a gap of 30 yr which can only be filled by other observers. The archives of the Royal Astronomical Society of New Zealand (RASNZ) contain about 4000 measurements from other observers, which we have analysed. The largest individual contributions are 660, 400, 250, 340, 190, 170, 126, 124 and 123 measurements. There are 12 observers who contributed 50–100 measurements and 19 who contributed 20–50. Data from one of these observers were discarded because they were systematically fainter by 1 mag than the local trend, illustrating the variations between observers for extremely red stars and highlighting the desirability of relying on a single observer. Data from observers who contributed fewer than 20 measurements were not included.

The combined light curve, including the measurements by AJ (both old and new), is shown in Fig. 5. Also shown in the top panel are photographic magnitudes at maximum and minimum reported by Gaposchkin (1952), upon which the catalogued period is presumably based and which match the visual data quite well. We also show 96 photometric measurements by the Tycho instrument on the Hipparcos mission (ESA 1997; Høg et al. 1997). The accuracy of these space-based observations is much higher than the visual data, but they only span four years.

The wavelet analysis of the full data set (including the Tycho data but excluding the photographic extrema) is shown in Fig. 6. Both periods recur throughout the series and more mode switches are seen. The change between the different types of pulsation occurs on a time scale of about 1000 days.
4 RELATION TO THE MIRA P-L RELATION

On the basis R Dor is closely related to the Mira variables, we estimated the distance in Bedding et al. (1997) by fitting the catalogued period (338 d) to the LMC Mira P-L relation of Feast (1996). Our result of 61 ± 7 pc is now seen to be in excellent agreement with the Hipparcos distance of 62.4 ± 2.8 pc (parallax = 16.02 ± 0.69 mas; ESA 1997). No true Miras lie within about 100 pc and none has a distance measurement as accurate as that of R Dor.

Table 1 lists fundamental data for R Dor, derived using the Hipparcos distance and observations listed in Bedding et al. (1997). The uncertainty in the stellar radius is dominated by the error in the measured angular diameter. The uncertainty in the luminosity comes equally from the uncertainties in $M_{\text{bol}}$ and the distance.

Using these data, we can plot the position of R Dor relative to the Mira P-L relation. Figure 7 shows the result, where the lines indicates the P-L relations as given by Feast (1996):

$$M_K = -3.47 \log P + 0.91$$

$$M_{\text{bol}} = -3.00 \log P + 2.78,$$

where $P$ is the period in days. The small circles give the LMC data on which the relation is based (Feast et al. 1989), assuming a distance modulus to the LMC of 18.56. Note that the relation is better defined using the $K$-band magnitude than the bolometric magnitude, probably due to uncertainties in the bolometric corrections for such red stars. The LMC
stars with periods longer than about 400 days are known to be over-luminous with respect to the relation (see also Zijlstra et al. 1996).

R Dor is indicated by the filled circle, where the dominant 332-day period was used. The very small error bars leave no doubt that R Dor fits the relation extremely well. We confirm the suggestion by Bedding et al. (1997) that, in spite of its semiregular behaviour, R Dor can be considered a Mira-like variable and its evolutionary state must be very closely related to the regular large-amplitude Miras.

5 MODELS FOR R DORADUS

The internal structure of AGB stars, where most of the mass is centrally concentrated and the radius is determined by a highly extended envelope, leads to a large ratio between the
periods of the fundamental pulsation mode and the first overtone (e.g., Wood 1975). It also leads one to expect that a fundamental mode pulsator will have a much smaller radius than a star with the same mass and luminosity that is pulsating in an overtone. This difference has been used to distinguish between fundamental and overtone pulsators for those (relatively few) Galactic Miras that have a measured angular diameter. By estimating distances using the LMC P-L relation, Feast (1996) concluded that the fundamental mode could be excluded. Using Hipparcos distances, van Leeuwen et al. (1997) confirmed this conclusion for most stars, and they found two whose radii appeared consistent with fundamental mode (but see also Barthès 1998).

In Bedding et al. (1997) we performed this test for R Dor using our measured angular diameter and the distance from the P-L relation to estimate a mass. We used the standard pulsation equation,

\[ Q = P \left( \frac{M}{M_\odot} \right)^{\frac{1}{2}} \left( \frac{R}{R_\odot} \right)^{-\frac{3}{2}}, \]

where \( P \) is the period in days, \( R \) and \( M \) are the stellar radius and mass in solar units and \( Q \) is the pulsation constant (in days), which is about 0.04 d for the first overtone and 0.09 d for the fundamental mode (e.g., Fox & Wood 1982).

Using the revised radius and period in Table 1, we can repeat this calculation. In first overtone, the stellar mass is found to be \( M = 0.8 \pm 0.25 M_\odot \), while in fundamental mode we find \( M = 4.1 \pm 1.4 M_\odot \). As in Bedding et al. (1997), we find that fundamental mode pulsation implies a mass for R Dor that is higher than expected from its period (Feast 1989; Vassiliadis & Wood 1993). For this reason, overtone pulsation is strongly favoured.

The fact that R Dor shows two well-defined periods may help in the mode identification. Table 2 lists models calculated by P. Wood (private communication) for the stellar parameters of R Dor. The calculations assumed a core mass of 0.62 \( M_\odot \) and abundances of helium and heavier element of \( Y = 0.30 \) and \( Z = 0.02 \), respectively. The first three rows show models that were calculated assuming that the primary period (332 d) corresponds to the fundamental mode. For each model, a mass was chosen and the ratio \( l/H_p \) of the mixing length to the pressure scale height was adjusted to give the correct period. The resulting effective tempera-
ture, together with the periods of the other modes ($P_i$) and the mode growth rates per period ($GR_i$) and $Q$ values, are shown in the table.

The next three models were calculated in the same way, but assuming that the primary period corresponds to the first overtone. The growth rate for the fundamental mode ($GR_0$) is very high in these models, implying that the star would be unstable. This can be taken as evidence against first overtone pulsation, within the constraints of the model.

These models confirm that fundamental mode pulsation cannot occur at the measured stellar radius unless the mass is greater than about $3M_\odot$. If we do identify the dominant period in R Dor with the fundamental mode then we must identify the observed secondary period with the first. If we instead identify the dominant period with overtone pulsation, the models imply that the secondary period is third overtone or higher.

The final row in Table 2 is a model calculated by Barthès & Tuchman (1994) for the Mira variable S CMi. Coincidentally, the period of this star (334 d) is very close to that of R Dor. The model was calculated assuming that the dominant period corresponds to the first overtone. The model luminosity, while somewhat lower than that of R Dor, lies only just outside our estimated ($1\sigma$) uncertainty range. Interestingly, the period of the third overtone in the model is in excellent agreement with our observed secondary period in R Dor. Better agreement is possible because the period ratios (i.e., the $Q$ values) are significantly different from those of Wood. The model growth rates also imply that the first and third overtones should be stronger than the second. Note, however, that the effective temperature of this model is significantly lower than that of R Dor (and hence the radius is larger), so it is certainly not an ideal model. Finally, we note that models by Barthès & Mattei (1997) for o Cet give similar period ratios. The difference between these models and those of Wood resides in the choice of mixing length, which is a matter of some uncertainty.

In summary, while we cannot definitely identify the pulsation modes in R Dor, the model of Barthès & Tuchman (1994) reproduces the two measured periods very well as the first and third overtones. Since the dominant period of R Dor places it exactly on the Mira P-L relation, our results imply that this relation most likely corresponds to first overtone pulsation.

6 DISCUSSION

6.1 Other cases of mode switching

Mode switching in a true Mira variable has not been observed. There have been reports of one or more secondary periods in the following Miras: S CMi and χ Cyg (Barthès & Tuchman 1994), BS Lyr (Mantegazza 1996) and o Cet (Barthès & Mattei 1997). However, the secondary periods have much smaller amplitudes than the dominant period, making it difficult to be certain of their reality. Mattei et al. (1997) recently reported secondary periods in two Miras (T Col and T Eri) and in both cases the period ratio is 1.9.

Among semiregulars, several definitely show two periods. Note that the star Z Aur, whose period changes were reported by Lacy (1973) and discussed by Wood (1975), is actually a yellow supergiant of the RV Tauri class of variables (Joy 1952) and is not relevant to this discussion. Cad-

Figure 7. The position of R Dor (filled circle) relative to the P-L relation for LMC Miras.
Table 2. Theoretical models of Mira variables, showing periods (in days) and growth rates per period

| $M$ ($M_\odot$) | $T_{\text{eff}}$ (K) | $R$ ($R_\odot$) | $L/H_P$ | Fundamental | 1st overtone | 2nd overtone | 3rd overtone |
|---------------|-----------------|----------------|--------|-------------|-------------|-------------|-------------|
|               | $P_0$           | $Q_0$ | $GR_0$ | $P_1$ | $Q_1$ | $GR_1$ | $P_2$ | $Q_2$ | $GR_2$ | $P_3$ | $Q_3$ | $GR_3$ |
| 1.5           | 3138            | 279   | 2.68   | 332.7 | 0.087 | 0.11   | 154.7 | 0.041 | 0.20   | 120.2 | 0.032 | -0.009 | 96.9  | 0.025 | 0.054 |
| 2.0           | 2970            | 312   | 2.00   | 334.6 | 0.086 | -0.030 | 159.1 | 0.041 | 0.12   | 107.8 | 0.028 | 0.028  | 96.5  | 0.025 | 0.030 |
| 3.0           | 2787            | 354   | 1.39   | 334.0 | 0.087 | -0.036 | 155.8 | 0.040 | 0.052  | 97.5  | 0.025 | 0.086  | 83.5  | 0.022 | -0.021 |
| 0.8           | 3038            | 298   | 3.41   | 601.4 | 0.105 | 3.5    | 332.1 | 0.058 | 0.0051 | 227.3 | 0.039 | 0.20   | 190.8 | 0.033 | 0.44  |
| 1.0           | 2776            | 357   | 2.54   | 779.2 | 0.115 | 2.7    | 332.5 | 0.049 | 0.0012 | 259.3 | 0.038 | 0.30   | 219.6 | 0.033 | 0.10  |
| 1.5           | 2387            | 483   | 1.48   | 1276.3| 0.147 | 1.24   | 335.5 | 0.039 | 0.30   | 276.0 | 0.032 | -0.0076| 234.0 | 0.027 | -0.013 |

Models with $L = 6800 L_\odot$ (P. Wood, private communication)

Models with $L = 5775 L_\odot$ (Barthès & Tuchman 1994)

| $M$ ($M_\odot$) | $T_{\text{eff}}$ (K) | $R$ ($R_\odot$) | $L/H_P$ | Fundamental | 1st overtone | 2nd overtone | 3rd overtone |
|---------------|-----------------|----------------|--------|-------------|-------------|-------------|-------------|
|               | $P_0$           | $Q_0$ | $GR_0$ | $P_1$ | $Q_1$ | $GR_1$ | $P_2$ | $Q_2$ | $GR_2$ | $P_3$ | $Q_3$ | $GR_3$ |
| 1.15          | 2295            | 482   | 1.2    | 1658.3 | 0.169 | 1.68a  | 334.6 | 0.034 | 0.50a  | 236.7 | 0.024 | 0.036a | 176.1 | 0.018 | 0.19a |

*growth rates in Barthès & Tuchman (1994) are per day and have been converted to per cycle.

mus et al. (1991) found evidence for mode switching in three semiregular variables (RV And, S Aql and U Boo). They suggested that these stars appear to switch between large-amplitude, long-period Mira-like oscillations and short-period, lower-amplitude ones, as we have found for R Dor. Percy & Desjardins (1996) observed mode switching in the SRb star W Boo from 25 d to 50 d. Szatmáry et al. (1996) list ten semiregulars that have two periods and Gál & Szatmáry (1995a) suggest that some of these have visual light curve variations that may be connected with mode switching. Most recently, Mattei et al. (1997) listed 28 semiregulars with two periods (4 in common with the list of Szatmáry et al. 1996) and suggested that multiple periods are common in semiregulars. Period ratios all lie within the range 1.7 to 2.0 and R Doradus, with a ratio of 1.8, is no exception. We note that this narrow range of ratios is consistent with the conclusion that all these stars are pulsating in the same pair of modes.

The star most similar in behaviour to R Dor is probably V Boo. Szatmáry et al. (1996) discuss the visual light curve of this star in some detail and show that the amplitude of the primary mode (period 258 d) has decreased with time. They discuss the secondary period (137 d), which they say has constant amplitude, leading them to conclude that mode switching has not occurred. However, inspection of the light curve (their Fig. 1) reveals similarities to R Dor: a strong long period which then gives way to double peaks of smaller amplitude.

We have analysed visual measurements of V Boo made by the AFOEV (Association Francaise des Observateurs d’Etoiles Variables) and the VSOLJ (Variable Stars Observers League in Japan). This is a subset of the data analysed by Szatmáry et al. (1996), except that the database now extends to JD 2451000, giving an extra 1000 days of measurements. Data from observers who contributed fewer than 50 measurements were not included. The observations (9456 in total) were binned into 5-day averages, yielding 2551 data points. The light curve and wavelet analysis are shown in Fig. 8. The gradual decrease in amplitude of the primary period is confirmed. The amplitude of the secondary period, while variable, does not correlate (or anti-correlate) very strongly with that of the primary period through most of the time series, although there is an indication of mode switching near the end. It appears that V Boo has changed from a true Mira to a semiregular, as suggested by Szatmáry et al. (1996), and may be entering a mode-switching phase similar to that currently being shown by R Dor.

Although there are still only a few reported cases of mode switching behaviour, it may not be unusual. Identification of mode switching in variables with such long periods requires a long baseline of observations. The fact that mode switching is seen in R Dor, which is the nearest Mira-like star, is consistent with it being fairly common. Evolutionary, periodic mode switching may occur immediately prior to the phase of fully regular Mira pulsations. Although the evolution from semiregular to Mira has not been studied previously, the case of R Dor suggests that this change takes place over an extended time, and that mass loss begins well before the change is completed. Alternatively, R Dor may belong to a separate class of Mira-like variables which shows intermittent pulsation behaviour.

6.2 Models for mode switching

Three theoretical models are available which include Mira mode switching. Keeley (1970) has presented a model in which a Mira switches between the fundamental and the first overtone and back again. The change is gradual, extending over tens of (overtone) periods, and is linked to a change in luminosity.

Ya’ari & Tuchman (1996) discuss semi-stable models, where a seemingly stable solution after several tens of cycles adjusts its internal energy structure and therefore its pulsation behaviour. In their non-linear models, the star changes over a period of several hundred years (approximately the thermal time scale of the envelope) from first overtone to a new fundamental mode, where the latter has a shorter period than predicted from the earlier overtone pulsation. This re-
Figure 8. Wavelet transform for V Boo based on the AFOEV and VSOLJ data. Same layout as Fig. 3, except that the periods in the lower panel (WAA) are 258 d (thick line) and 137 d (thin line).

sult questions the usefulness of mode analysis. The model star pulsates for the first 300 years in an overtone, after which the fundamental mode grows and becomes dominant. During this time the fundamental mode changes in period from 508 to 330 days.

Both these models predict the change from overtone to fundamental to be gradual, over tens of cycles, which is unlike the behaviour we see in R Dor. It does not appear that the mode switching in R Dor occurs on the thermal time scale of the envelope, and the thermal structure of the star can be assumed constant over the mode change. We note that in both the above models, the switch is between the first overtone and fundamental mode, whereas in R Dor it appears to be between first and third or higher overtones. The fact that R Dor fits the Mira P-L relation so well strongly argues against effects of non-linear growth. Both the low pulsation amplitude and the short mode life time in R Dor may keep the star within the linear regime. This may argue against the Ya’ari & Tuchman model for all Miras which agree with the P-L relation.

The third applicable model involves the effects of chaos on the stellar structure and was developed by Icke et al. (1992), based on work by, e.g., Buchler & Goupil (1988). The model is described as a driven oscillator, with the driving mechanism located well below the photosphere. They show that the pulsations becomes less regular as the envelope mass decreases, moving from stable to multi-periodic to chaotic solutions.

The chaotic solution of Icke et al. (1992) corresponds to weak chaos, where orbits associated with different modes intersect because of small perturbations to the star. The strongest effects are found near maximum radius when the velocity in the atmosphere is near zero. The star can extend its stay here causing characteristic humps on the light curve near maxima (Icke et al., their Figure 9). This model appears to fit R Dor quite well. In particular, the model depicted in the bottom panel of their Fig. 13 shares features with R Dor: the growing bump on the declining part of the light curve, and the shift between modes on a short time scale. Chaotic models can show very sudden changes in oscillation charac-
teristics, sometimes after a long sequence of regular oscillations. The presence of chaotic variations may therefore be common to Miras.

It has been pointed out by V. Icke (private communication) that in such a chaotic solution, the switch between modes is made easier if the radial order \( n \) changes by an even number. He argues that a change between adjacent modes, such as the fundamental and first overtone, requires the two modes to be in anti-phase in order to match the velocities at the stellar surface. Internally, these two modes will have opposite velocities over much of the star and the switch would have a large energy requirement. This transition can therefore be considered as ‘forbidden.’ Two modes with \( \Delta n = 2 \) are much more similar inside the star and a switch between them will be energetically favoured. This simple consideration agrees well with our suggestion that switching in R Dor is between the first and third overtones.

Interestingly, in the Icke et al. (1992) model, R Dor-like behaviour comes after stable Mira behaviour, as the envelope mass decreases. This agrees with the extended IRAS shell around R Dor indicating a long mass-loss history, and also with the behaviour of V Boo. It is the only model in which semiregulars are Mira descendants rather than Mira progenitors.

7 CONCLUSIONS

We have shown that R Dor switches between two pulsation modes, on a time scale of a few cycles. The longer mode is shown to fit the Mira P-L relation extremely well, which shows that semiregulars and Miras are closely related. The Hipparcos parallax for R Dor places it only 62 pc away, making it the nearest Mira-like star. It clearly makes sense to include long-period semiregulars such as R Dor in studies of Mira behaviour.

From stellar models, we make a possible identification for the two modes as the first and third overtones, implying that Mira variables on the P-L relation pulsate in the first overtone. The physical diameter of R Dor excludes fundamental mode pulsation for reasonable masses. We suggest that the change between semiregulars and Miras is due to a mode switch between third and first overtone, or between higher overtones. However, by ruling out fundamental mode pulsation we are left with the problem of large observed shock velocities. Until this is explained, the controversy over the mode of Mira pulsation cannot be laid to rest.

Models giving mode switching through non-linear growth of the fundamental mode and an adjustment of the thermal structure of the star predict a much longer time scale for switching than observed. In other words, the change in R Dor occurs too rapidly to be related to a change in the overall thermal structure of the star. Chaotic effects discussed by Icke et al. (1992) appear to fit R Dor very well. They predict that irregular behaviour becomes stronger when the envelope mass of the star decreases. This agrees with the evidence for extensive past mass loss and implies that R Dor, and perhaps other semiregulars, may have evolved from true Miras.

ACKNOWLEDGEMENTS

We appreciate the efforts of Ranald McIntosh and Frank Bateson in maintaining the RASNZ database. We thank Peter Wood for the pulsation calculations reported in Table 2. We also thank him and Gordon Robertson for comments on the manuscript. Data for V Boo were obtained from the AFOEV database, operated at CDS, France and the VSOLJ database in Japan. TRB is grateful to the Australian Research Council for financial support. AAZ thanks ESO for financial support.

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