Is the apparent dichotomy between bursting activity of magnetars and radio pulsars real?

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Abstract.
Anomalous X-ray Pulsars (AXPs) and Soft Gamma-Ray Repeaters (SGRs) are a class of young neutron stars (NSs) characterized by high X-ray quiescent luminosities, short X-ray bursts, and giant flares (for SGRs). They are believed to be magnetars, i.e. NSs with magnetic fields \(\approx 10^{14} - 10^{15}\) G. The discovery of magnetar-like X-ray bursts from the young pulsar PSR J1846-0258 [1], with an inferred surface dipolar magnetic field of \(B_p = 4.9 \times 10^{13}\) G, lower than the traditionally considered magnetar range, and, more recently, by the discovery of SGR 0418+5729 with an even lower \(B_p = 7.5 \times 10^{12}\) G [2], well within the range of the rotation powered pulsars which do not display any bursting behaviour, has raised the obvious question: why some \textit{high-}B pulsars (PSR J1119-6127 and PSR J1814-1744, with \(B \approx 4 - 5 \times 10^{13}\) G) do not display any burst, while at least one case of \textit{low-}B NSs (SGR 0418+5729) does, if the magnetic field is their driving force?

1. Introduction
The majority of NSs are observed as radio-pulsars, with magnetic fields in the \(\approx 10^{11} - 10^{13}\) G range. They spin down due to dipole radiation losses, while converting a fraction of their rotational energy into electromagnetic radiation. The conventional wisdom is that, when the star is not accreting, its main energy source comes from rotational energy. This was shattered by the discovery of a peculiar class of sources characterized by long periods \((P \approx 2 - 11\) s) and high quiescent X-ray luminosities \((L_x \approx 10^{33} - 10^{35}\) erg s\(^{-1}\)), generally larger than their entire reservoir of rotational energy. These sources, historically classified as AXPs and SGRs, are also characterized by stochastic bursts of X-rays, releasing energies \(\approx 10^{39} - 10^{41}\) erg, although some cases do not show such activity (1E 1841-045). In the case of the SGRs, they are also characterized by more energetic \(\gamma\)-ray flares, with typical energetics \(\approx 10^{44} - 10^{45}\) erg. Several pieces of observations have allowed to establish that these peculiar objects are isolated, and hence the extra energy cannot be provided by accretion from a companion.

The most successful model to explain both the high quiescent X-ray luminosities, as well as the X-ray bursts and giant \(\gamma\)-ray flares is the \textit{magnetar} model [3, 4], in which SGRs and AXPs are believed to be endowed with large magnetic fields, \(B \approx 10^{14} - 10^{15}\) G. After a magnetar is born, the internal magnetic field is subject to a continuous process of evolution through the processes of Ohmic dissipation, Ambipolar diffusion, and Hall drift. Soon after birth (from hours
to days, at most) a solid crust is formed and protons in the liquid core undergo a transition to a superconducting phase. At this stage, the field evolution is likely to be governed by crustal processes, because the role of ambipolar diffusion in a superconducting core is questionable [5]. Dissipation of fields in the magnetar range can account for the enhanced persistent luminosities of AXPs and SGRs. These ideas have been confirmed by numerical simulations of coupled magneto-thermal evolution in the crust of NSs [6, 7].

The magnetic field also plays a fundamental role in the production of bursts and flares. The physical mechanisms responsible for these have been identified and amply discussed by [3, 4]. The remarkable analogy between tectonic activity of NSs (“starquakes”) and earthquakes on Earth was soon realized, because both phenomena share some statistical properties [8]. In the crust, magnetic stresses are generally balanced by elastic stresses. However, as the internal field evolves, local magnetic stresses can become too strong to be balanced by the elastic strength of the crust, which hence breaks. Then, after the starquake, a new equilibrium state is reached and the extra stored magnetic/elastic energy becomes available for powering the bursts and flares. According to the model by [3, 4], giant flares are likely to be the result of a global event, while small bursts are local phenomena.

Despite the success of the magnetar model in explaining some general features of the triggering mechanism of bursts and flares, some major questions have been left unanswered. In particular: what determines the frequency of the bursts? Why some objects display giant flares (SGRs), while others (AXPs) do not? How frequent are the different phenomena? Why does the burst location on the NS surface correlate with the pulsar phase (maximum of the quiescent X-ray lightcurve) for AXPs, while it is uncorrelated with phase for SGRs? (see e.g. [9] for a review). It is clear that the magnetic field alone, or at least the dipolar field component inferred through measurements of the period \( P \) and its derivative \( P' \), is not sufficient to account for this variety of behaviours that pulsars manifest. A unified physical framework is still lacking, and it constitutes a major puzzle in NS physics. Here we present a summary of recent works [10, 11] where we have provided the first estimates of the burst energetics and recurrence times based on long-term 2D simulations of the magneto-thermal evolution of the NS with realistic values of the breaking stress of the NS crust. Our approach allows us to identify some key elements of the magnetar phenomenology, and to shed light on what creates the variety of the – often puzzling – observed behaviour.

2. Our approach to the problem.

We follow the evolution (in axial symmetry) of the magnetic field in a magnetar crust with the numerical code described in [12]. The initial configurations of the magnetic field include both a toroidal and a poloidal component. The temperature of the crust at different ages is given by the results of [7], where the interplay between the magnetic field evolution and the thermal evolution of magnetars was studied. For simplicity, and for numerical limitations, here we assume that the crust is isothermal. As the magnetic field evolves, the elastic crust moves through a series of equilibrium states in which its elastic stress \( \sigma_b(r, \theta, t) \) balances the (time-dependent) magnetic stress \( M_{ij}(r, \theta, t) \) in each direction. Assuming that equilibrium is reached at a certain time, \( t^{eq} \), we can define

\[
M_{ij}^{eq}(r, \theta) = \frac{B_i(r, \theta, t^{eq})B_j(r, \theta, t^{eq})}{4\pi} \sim \sigma_b(r, \theta, t^{eq}),
\]

where \( r \) and \( \theta \) indicate the radial and poloidal coordinates, respectively.

Recent molecular dynamical simulations [13] have provided a fit for the maximum stress that a NS crust can sustain

\[
\sigma_b^{max} = \left( 0.0195 - \frac{1.27}{\Gamma - 71} \right) n_i Z^2 e^2 a,
\]

where \( \Gamma = \frac{\mu}{\mu - 1} \) is the adiabatic index.
where $\Gamma = Z^2 e^2 / aT$ is the Coulomb coupling parameter, $a = [3/(4\pi n_i)]^{1/3}$ is the ion sphere radius, $n_i$ the ion number density, $Z$ the charge number, $e$ the electron charge, and $T$ is the temperature in energy units. This fit to results from extensive molecular dynamics was found to be in agreement with the Zhurkov model of strength, and provides a good approximation to the breaking stress for very long timescales $\approx 1 \text{ s} - 1 \text{ yr}$. We will extrapolate this result to even longer times, of sometimes $> 100 \text{ yr}$, but in any case, the breaking stress at very long timescales cannot exceed our adopted values [13].

As the magnetic field evolves under the influence of the dominant Hall drift term, and by Ohmic diffusion, local magnetic stresses in the crust can occasionally, at, say, a time $t_b$, depart from the equilibrium condition at a previous time $t_{eq}$ by an amount which is comparable to, or exceeds, the breaking stress of the crust,

$$M_{ij}(r, \theta, t_b) - M_{ij}^{eq}(r, \theta) \sim \sigma_b^{max}(r, t_b),$$

and hence the crust fractures. Here $\sigma_b^{max}$ is independent of the angle because of our assumption of isothermality.

Since our simulations follow the evolution of $B(r, \theta, t)$, and hence $M_{ij}(r, \theta, t)$, we can map the time-dependent regions in the NS crust which are subject to fractures. When this condition is fulfilled, a starquake will happen and a new equilibrium state will be reestablished. A fully consistent dynamical simulation of the NS starquake is out of our present capabilities. The typical timescales (nsec to sec) of individual burst/flare events are many orders of magnitude smaller than the long-term evolution timescales (the typical timesteps in our code are a few days or a week, to be able to follow the NS evolution up to $10^5 - 10^6$ years), hence we cannot follow dynamically the fracture propagation and model individual bursts. However, we can estimate the energy of an “outburst” (i.e. a collection of tens to hundreds of bursts occurring within $\approx 1$ week timescale) as follows. When a starquake happens, say at time $t = t_b$, a certain region of the crust in the vicinity of the point where critical conditions are reached will be affected. We consider that all surrounding regions where local magnetic stresses are close to the maximum (say by a fraction $\epsilon \sim 90\%$ or $95\%$) will be affected, i.e.

$$M_{ij}(r, \theta, t_b) - M_{ij}^{eq}(r, \theta) > \epsilon \sigma_b^{max}(r, t_b).$$

We can compute the elastic energy stored in that portion of the crust volume, and assume that it will be released at once (i.e., in one timestep, $\approx 1$ week) and that the affected region will return immediately to equilibrium. The new equilibrium stresses are reset, and the process is repeated as the magnetic field evolves (we neglect the feedback produced by the local deposition of energy, which will require a much more complex modeling). This allows us to obtain the energy available in a given event, the time interval between events, and the location of the fracture. It should be stressed that this represents the total energy of an “outburst”, that may be released in one very energetic flare, or many small bursts, or a combination of both. Also, this estimate is to be taken as an upper limit to the event energetics. Depending on the location and local physical conditions (density, temperature), part of the energy is lost by neutrino emission, part of it can be transferred to the magnetosphere, and part of it results in local heating and radiated by photons from the surface on a much longer timescale. Typically, energy released in the inner crust (above neutron drip) is more easily lost in the form of neutrinos while energy released near the surface can have a very high efficiency and a direct observational impact in thermal (surface) and non-thermal (magnetospheric) radiation.

The energy available after a starquake can therefore be estimated as the elastic energy corresponding to a static shear strain $\Psi$, which can be well approximated by [4]

$$E_b(t_b) = \frac{1}{2} \int dV \mu \Psi^2 = \Sigma_i j \int dV \left[ \frac{M_{ij}(r, \theta, t_b) - M_{ij}^{eq}(r, \theta)}{\mu(r, t_b)} \right]^2$$
where we have used the fact that the yield strain of the crust in the region where breaking conditions are fulfilled is approximated by

$$\Psi_{ij} \approx \frac{M_{ij}(r, \theta, t_b) - M_{ij}^{eq}(r, \theta)}{\mu(r, t_b)},$$

and the shear modulus $\mu$ is given by [14]

$$\mu(r, t) = \left(0.01106 - \frac{28.7}{r^{1.3}}\right) n_i \frac{Z^2 e^2}{a}. \quad (7)$$

The integral in Eq.(5) is computed over the volume $dV$ for which condition (4) on the stresses in any direction is satisfied.

3. Results.

We now report the most relevant results of a number of cases that we studied. As a baseline model we have chosen a NS born with an initial dipolar field of $B_p = 8 \times 10^{14}$ G (at the pole) and an internal toroidal field of $B_t = 2 \times 10^{15}$ G at maximum, similar to models used in previous studies and close to what one expects to be the final geometry after MHD equilibrium is reached in a hot, liquid proto-neutron star. In this particular model, at the age of $10^4$ yrs the dipolar field has decreased to about 20% of its initial value, while its internal toroidal field has been dissipated by more than a factor of 2. Of course other initial conditions modify quantitatively the result we show in the following, but the general trends remain nearly the same. The evolution is followed for $10^5$ years. During this time, we monitor the frequency and angular distribution of magnetic stresses in the NS crust, and the energy released any time local magnetic stresses become strong enough to fracture the crust.

![Figure 1](image.png)

**Figure 1.** Outburst properties of an object with initial magnetic field components $B_p = 8 \times 10^{14}$ G and $B_t = 2 \times 10^{15}$ G during three different periods of its lifetime: 400-1600 yr (labeled “SGR”), 7-10 kyr (labeled as “AXP”), and 60-100 kry (labeled as “old AXP”). We have fixed $\epsilon = 0.9$.

The main results about frequency and distribution of energies available to be released following crust fractures are summarized in Fig. 1. We remind again that the time step of our simulation is $\approx 1$ week, thus groups of bursts that would occur on timescales shorter than
this are classified as a single outburst in our simulations. We selected three representative periods in a magnetar life, involving each one the same total number of events (1000). The first one is labeled "SGR" and spans the interval between 400 and 1600 yrs. The second case is labeled "AXP" and covers the period $\approx 7 - 10$ kyrs, and the third period is named "old AXP" and corresponds to the events recorded from 60 to 100 kyrs. The first important result is that there is a strong difference in the energetics and recurrence time as the star evolves. This is because the crustal magnetic field evolves more rapidly initially due to the Hall drift, while at late times its geometry has rearranged into a more quasi-steady state and the evolution is slower.

In young SGRs, the typical energies released after each starquake are of the order of $10^{44}$ erg and the typical event rate $\sim 1$/yr. Note that this does not necessarily imply that a flare will be observed. We remind that this is the maximum energy available released in the interior, but the mechanism to transfer this energy to the surface and magnetosphere may not work in many cases, especially when the fracture occurs in the inner crust. From our results, we observed that more than 90% of the fractures occur close to the surface, simply because the crust is less strong there, so that we expect a relatively high efficiency of the process. A more detailed study of the efficiency of the energy transfer is out of the scope of this work, which is aimed to describe the expected behavior in statistical terms. For "AXP-like" objects the energetics is shifted to lower values and a second peak appears at about $10^{41}$ ergs, and the waiting time between outbursts increases to a few years. This trend is more pronounced as the object gets older, and for old AXPs the recurrence time becomes of the order of tens of years and nearly half the events are low-energy events. We find that, independently of age, the longest waiting times correspond to the most energetic, flare-type, events ($E > 10^{44}$ erg). Hence giant flares are expected to be less frequent than the less energetic bursts. Indeed, while giant flares have been observed in only 3 objects, bursts have been observed in more than a dozen. The physical reason behind the bimodal distribution is the directionality of stresses. We found that fractures associated to the magnetic stress component $M_{\theta,\phi}$ are more frequent, but they are mostly associated to low energy events $\approx 10^{41}$ erg. On the other hand, fractures caused by the $M_{r,\phi}$ component are responsible for most of the $E > 10^{44}$ erg events. The events associated to large values of $M_{r,\phi}$ span the whole range of energies, but they become very rare after the "SGR" stage, so that the long term energy distribution becomes more clearly bimodal.

It is interesting to compare the general trends predicted by our simulations with the observations of magnetars so far. We find that, for a given $B$-field strength and configuration at birth, younger objects are generally more active, both in event frequency and strength, and hence a natural sequence for a magnetar is to evolve from an SGR to an AXP-like object. Interestingly, this age sequence is also what observations have been hinting at [15]. Overall, we find that the energetics of the "SGR-like" objects tend to be higher than the ones of the "AXP-like" objects, again in agreement with observations [16]. Our predicted clustering of the energy distribution (around $10^{40} - 10^{41}$ and $10^{44}$ erg) is suggestive of the observed dichotomy outbursts/giant flares, with much rarer events in between [17]. Our waiting time distributions are roughly log-normal. A direct comparison with observations cannot however be made at this stage, due to the very limited statistics. However, if the secular waiting time distribution of starquakes (which is what we predict) has a similar behaviour to the waiting time distribution of bursts within an outburst, then our results would again match the observations [8].

Further connections to the observations can be made by inspection of the right panels of Fig. 1, where we show the angular distribution of the bursts. While for "SGRs" and "old AXPs" there is no clear trend, it is interesting to note that for the sample named "AXPs" most of the fractures happen at small polar angles. If the polar region is hotter than the equator, our results suggest a correlation between burst location and pulsar phase. The properties of this object are typical of what is routinely classified as an AXP [9]. The reason for this particular feature at middle ages is probably related to the evolution of the internal toroidal field, as
already discussed in [12]. The Hall drift displaces the internal toroidal field towards the poles, in a typical timescale of $10^3 - 10^4$ yrs (the Hall timescale), but after a few Hall timescales the magnetic field is reconfigured in a more stable, steady-state. Maybe this characteristic location of starquakes closer to the pole is reflecting this fact, but we cannot make a definite claim about this point. Consistent simulations that include the local heating and the temperature variations produced by the energy release are in progress and will be presented elsewhere.

4. Conclusions.
The results from the study cases presented in this letter bear a direct relevance for the interpretation of the overall magnetar phenomenology. Overall, an “SGR-like” object tends to have a higher dipolar field than an “AXP-like” object or a “high-$B$ radio pulsar”, but there is no fundamental separation among what constitutes the apparent different classes. Among the key elements that create the variety of observed phenomena, the age is probably more important that a small variation in the magnetic field strength. SGR-like flares can be produced even with low $B_p$ if the internal toroidal field is sufficiently large, but those events are in any case much less frequent in old, low-field NSs than in young, high-field magnetars.

In summary, we have found that there is no fundamental separation among what constitutes the apparently different classes of “normal radio pulsars”, of “high-$B$ radio pulsars”, of “high-$B$ AXPs”, “high-$B$ SGRs”, or “low-$B$ SGRs/AXPs”. We have been able to identify some key elements that create the variety of observed phenomenology. In particular, while what we infer from measurements of $P$ and $\dot{P}$ is only the dipolar $B$-field component, $B_p$, both the dipolar component and the strength of the hidden toroidal component are similarly important. The importance of a toroidal component had already been pointed out in the earlier work of [4]; they noticed how an internally wound $B$-field can more easily stress the NS crust, as our results have shown, and it is also a natural outcome of a strong $\alpha$-dynamo. More importantly, our results show that the age (or the evolutionary stage) is the most relevant feature to understand different levels of activity in different families of NSs. The same object may behave differently at different times, without in principle having a different nature. Our simulations have been able to quantify this effect, explain key observational properties, and give a step forward to connect and unify the entire pulsar/magnetar phenomenology.

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