Research on control method of three-phase inverter based on LQR optimal tracking control

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Abstract. According to the linear quadratic optimal control principle, the idea of optimal tracking control is applied to the three-phase inverter control system, which can effectively improve the output characteristics of the system. Based on the establishment of mathematical model, the nonlinear load and load mutation are regarded as the uncertain factors affecting the performance of three-phase inverter, and the parameter optimization design of state feedback and input feedforward is carried out based on the linear quadratic optimal control method. The simulation results show that the LQR optimal tracking control can better adapt to the adverse effects of nonlinear load and load mutation than the traditional quasi-PR control, and the system has good harmonic performance, dynamic performance and the ability to resist load disturbance.

1. Introduction

With the continuous improvement of the permeability of distributed generation system, high-performance inverter plays an important role as the key link of power system access, which can convert direct current into high-quality alternating current for electrical equipment [1]. However, with the gradual improvement of the complexity and intelligence of electrical equipment, the abrupt and nonlinear characteristics of the electrical load put forward higher and higher requirements for the output of the inverter power supply. Therefore, the high performance control technology of voltage and current of three-phase inverter has become one of the important research directions in this field.

The traditional double closed-loop PI controller has been widely used in DC power supply and DC motor control [2], but there is a certain static error when PI control for three-phase AC power. PR controller is a controller based on resonance effect, and the control loop generated by it has an infinite gain mode at a particular frequency. In this way, any nonlinear influence or interference influence will be completely eliminated at this frequency, so the problem of AC control can be solved better [3]. The frequency adaptability of traditional PR control is not strong, and the control performance will be affected when the grid frequency shift occurs. In order to solve this problem, a certain frequency bandwidth can be added on the basis of PR control to form a quasi-proportional resonant control. In the case of nonlinear load and load mutation, the inverter using quasi-PR control has problems such as large output harmonic distortion and poor robustness [4].

In order to effectively improve the shortcomings of the quasi-PR control method, this paper studies an inverter control strategy based on the linear quadratic optimal control algorithm. The control
2. The mathematical model of three-phase inverter

The three-phase inverter studied in this paper adopts H-bridge inverter structure, and its main circuit topological structure is shown in Figure 1, in which the \( T_1 \)-\( T_6 \) switch tubes are power devices IGBT with inverse shunt diode, and the filter adopts LC filter. As for the capacitor connection structure of the filter, the short-circuit phase current is the geometric sum of the unshort-circuit two-phase current when the single-phase short-circuit occurs in the star-connected capacitor, and its value will not exceed three times of the rated current of the capacitor. When single phase short circuit occurs in triangular connection, the short circuit current will exceed many times of the rated current of the capacitor, which is easy to cause the expansion of the accident. Therefore, from the perspective of reliability design, it is more reasonable to adopt star connection of capacitor.

![Figure 1. Topological structure of main circuit of three-phase inverter.](image)

In Figure 1, the H-bridge inverter part can be approximately equivalent to the proportional part, with \( L_f \) as the output filter inductor, \( C_f \) as the output filter capacitor, and \( R_f \) as the equivalent impedance of the filter. Input voltage of DC side is \( U_{dc} \), output voltage of H-bridge is \( U_{Ta}, U_{Tb}, U_{Tc} \), capacitance voltage is \( U_{oa}, U_{ob}, U_{oc} \), inductance current is \( I_{La}, I_{Lb}, I_{Lc} \), output current is \( I_{oa}, I_{ob}, I_{oc} \).\(^8\)

The state space equation can be obtained from KCL and KVL as follows:

\[
\begin{bmatrix}
\dot{U}_o \\
\dot{I}_L
\end{bmatrix} =
\begin{bmatrix}
0 & \frac{1}{C_f} E_3 \\
-\frac{1}{L_f} E_3 & -\frac{R_f}{L_f} E_3 - \frac{1}{L_f} E_3
\end{bmatrix}
\begin{bmatrix}
U_o \\
I_L
\end{bmatrix} +
\begin{bmatrix}
0 & \frac{1}{C_f} E_3 \\
\frac{1}{L_f} E_3 & 0
\end{bmatrix}
\begin{bmatrix}
U_T \\
I_o
\end{bmatrix}
\]  \(\text{(1)}\)

Among them:

\[
U_o = \begin{bmatrix} U_{oa} \\ U_{ob} \\ U_{oc} \end{bmatrix}, I_L = \begin{bmatrix} I_{La} \\ I_{Lb} \\ I_{Lc} \end{bmatrix}, U_T = \begin{bmatrix} U_{Ta} \\ U_{Tb} \\ U_{Tc} \end{bmatrix}, I_o = \begin{bmatrix} I_{oa} \\ I_{ob} \\ I_{oc} \end{bmatrix}, E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

Using the capacitor voltage and inductance current of LC filter as state variables, the continuous state space model can be obtained as follows:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t)
\end{align*}  \(\text{(2)}\)

Among them:
3. Linear quadratic (LQR) optimal control principle

Linear Quadratic (LQR) optimal control [5] plays a very important role in the research of modern control theory due to its characteristics of simple control scheme structure, fast response speed and small overshoot. The core idea of LQR optimal control method is that, for a linear time-invariant system such as three-phase inverter, within the whole time interval \([t_0, t_f]\), an optimal control law \(u(t)\) is found to minimize the following performance indexes by comprehensively considering the three aspects of deviation, control energy consumption and steady-state error in the process [6]:

\[
J = \frac{1}{2} \int_{t_0}^{t_f} \left[ x(U, t)^T Q x(U, t) + \frac{1}{2} \left[ x(U, t)^T Q x(U, t) + u(U, t)^T R u(U, t) \right] \right] dt
\]  

Among them: \(Q\) and \(Q\) is \(n \times n\) semi-positive definite symmetric constant weight matrix, \(R\) is \(r \times r\) positive definite symmetric constant weight matrix, \(t_0\) and \(t_f\) are the initial time and termination time of the control process respectively. \(L_x = \frac{1}{2} x(U, t)^T Q x(U, t)\) is used to limit the size of the output fluctuation in the process, which reflects the requirements for the dynamic process; \(L_x = \frac{1}{2} u(U, t)^T R u(U, t)\) reflects the constraint on the control energy; \(L_x = \frac{1}{2} x(U, t)^T Q x(U, t)\) corresponds to the terminal deviation, that is, the limitation of the steady-state control accuracy.

Theory analysis shows that the standard linear quadratic control is essentially a state feedback control method, simply using LQR controller to control the inverter although cannot get a better output stability, but the reference signal cannot be achieved effectively astatic tracking, so you need to increase output feedback on this basis, the output signal to track the reference signal tracking problem, is transformed into a regulation problem, control law is obtained by using the optimal control algorithm, in order to get better control effect.

4. LQR Optimal Tracking Control Strategy for Three-phase Inverter

The purpose of the no-static tracking control is to make the closed-loop system asymptotically stable, the output tracking error is zero and the external disturbance is suppressed simultaneously. According to the principle of internal mode, in order to realize the non-static tracking of three-phase voltage reference signal, the reference input and disturbance signal model must be preset in the controller.

The research object of this paper is the power frequency three-phase inverter power supply, the disturbance and reference signal are both sinusoidal signals with a frequency of 50 Hz, so the output feedback needs to include the sinusoidal signal model, namely resonant controller, to make the high gain near the fundamental frequency, so as to achieve no static error tracking. The resonance controller expression is:

\[
G(s) = \frac{K_3 s + K_4}{s^2 + \omega^2}
\]

Among them: \(K_3\) and \(K_4\) is the coefficient to be solved, is the feedback weighted value of the state variable of the resonant controller. \(\omega\) is the fundamental frequency of the perturbation and reference signal.

\[
x = \begin{bmatrix} U_0 \\ I_L \end{bmatrix}, \quad u = \begin{bmatrix} U^T \\ I_0 \end{bmatrix}, \quad y = U_0, \quad A = \begin{bmatrix} 0 & 1 \\ \frac{1}{C_f} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -\frac{1}{C_f} \\ \frac{1}{L_f} & \frac{1}{L_f} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}
\]
Figure 2 shows the block diagram of LQR optimal tracking control principle.

![Block diagram of LQR optimal tracking control principle](image)

In the case of considering external interference signals, the state space expression of the three-phase inverter system can be transformed into:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + Eh(t) \\
y(t) &=Cx(t) + Du(t)
\end{align*}
\]  

Among them: \(Eh(t)\) represents the external disturbance vector, \(D = 0\).

The state space expression of the resonant controller as output feedback is:

\[
\begin{align*}
\dot{\xi}(t) &= A_{\omega}\xi(t) + B_\omega e(t) \\
\phi(t) &= C_{\omega}\xi(t)
\end{align*}
\]  

Among them, \(\xi(t)\) is the state variable of the resonant controller, \(\phi(t)\) is the output of the resonant controller, \(e(t) = z(t) - y(t)\) is the error between the system output and the reference input.

\[
A_{\omega} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix}, B_\omega = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_{\omega} = \begin{bmatrix} K_3 & K_4 \end{bmatrix}, \omega = 2\pi f = 100\pi.
\]

According to Figure 2, the system control signal \(u(t)\) can be expressed as:

\[
u(t) = [K_1 \ K_2]x + [K_3 \ K_4]\xi = [K_1 \ K_2 \ K_3 \ K_4]\begin{bmatrix} x_{2 \times 1} \\ \xi_{2 \times 1} \end{bmatrix}
\]  

In order to solve the optimal tracking problem of the three-phase inverter output to the reference signal, it is necessary to combine the output feedback (Equation 6) and state feedback (Equation 5) in the three-phase inverter system, establish the system augmented state space equation [7], and finally solve it by using the standard linear quadratic optimal control algorithm. The augmented state space equation of three-phase inverter system is:

\[
\begin{align*}
\dot{X}(t) &= AX(t) + Bu(t) + \bar{E}H(t) \\
Y(t) &= C\dot{X}(t) + \bar{D}u(t)
\end{align*}
\]  

Among them: \(X(t) = \begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix}, Y(t) = \begin{bmatrix} y(t) \\ \phi(t) \end{bmatrix}, H(t) = \begin{bmatrix} h(t) \\ \phi(t) \end{bmatrix}, \bar{A} = \begin{bmatrix} A & 0 \\ -B_{\omega}C & A_{\omega} \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \bar{C} = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}, \bar{D} = \begin{bmatrix} D \\ 0 \end{bmatrix}.
\]

LQR optimal control method is used to find the optimal control law \(u(t)\), so that the following quadratic performance index takes a minimum value:

\[
J = \frac{1}{2} \int_0^T \left[ X^T(t)QX(t) + u^T(t)Ru(t) \right] dt
\]  

The optimal control can be obtained as:
Among them: $P$ is the semi-positive definite symmetric solution of the following Riccati equation:

$$P\dot{A} + \dot{A}^T P - PBR^{-1}B^T P + \dot{C}^T Q \dot{C} = 0$$

(11)

The feedback gain matrix can be obtained as follows:

$$K = [K_1, K_2, K_3, K_4] = -R^{-1}B^T P$$

5. Simulation and result analysis

In order to verify the effectiveness of the LQR optimal control method for three-phase inverter, a simulation model was established to analyse the system and compare it with the traditional quasi-proportional resonant control method. The values of relevant parameters for three-phase inverter simulation are shown in Table 1.

| Parameter                        | Value   |
|----------------------------------|---------|
| DC input voltage $U_{dc}$        | 720V    |
| Reference voltage $U_{ref}$      | 380V    |
| Filter capacitor $C$             | 100μF   |
| Filter inductance $L$            | 1000μH  |
| Filter resistance $R_m$          | 0.1Ω    |
| Pure resistive load $R_o$        | 10Ω     |
| Inductance in resistive load $L_o$| 2000μH  |
| Capacitance in nonlinear load $C_o$| 2000μF   |
| Sudden loading and unloading $R_z$| 100Ω    |
| Switching frequency $f_k$        | 10000Hz |
| Output voltage frequency $f$     | 50Hz    |
| Output angular frequency $\omega$| 100π    |

When solving the three-phase inverter control system, $h(t) = z(t) = \theta$ can be set. The feedback gain matrix $K$ can be obtained by using the command $K = \text{lqr}([\dot{A}, \dot{B}, \dot{C}, R])$ in the MATLAB control toolbox. In order to obtain a good control effect and make the system output track the reference signal quickly, more attention should be paid to the control of state variables. Therefore, the weight $Q$ of state variables is much greater than the weight $R$ of input variables. Based on this, weight matrices $Q$ and $R$ are selected as follows:

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 10^9 & 0 \\ 0 & 0 & 0 & 10^9 \end{bmatrix}, R = \begin{bmatrix} 1 \end{bmatrix}_{4 \times 4}$$

The feedback gain matrix is obtained as follows:

$$K = \begin{bmatrix} 0.0015 & 0.1009 & -99.4958 & -4792.4 \end{bmatrix}$$

According to the design idea of quasi-PR control parameters [8], the controller optimization parameter is determined as $K_p = 20, K_i = 40, \omega_c = 5$, bring them into the simulation model of the three-phase inverter quasi-PR control system. All the coefficients in $K$ are put into the simulation model of three-phase inverter LQR control system. In the case of resistive load, resistive load and nonlinear load, the load was suddenly added at 0.1s, and the load was suddenly unloaded at 0.15s. The two
control strategies were simulated and compared to compare the output voltage waveform of the three-phase inverter under the two control strategies. The results are shown in Figure 3-8. Figure 3 and Figure 4 respectively show the three-phase voltage output under the two control methods with resistive load. (a) is output voltage waveform of three-phase inverter, (b) is output voltage harmonics of three-phase inverter. It can be clearly seen that, compared with the quasi-PR control, the optimal tracking control using LQR can reduce the output voltage harmonics more effectively, and the harmonic distortion rate is only 0.34%. The dynamic performance of the two control methods has little difference, and the stability is restored after 5ms of sudden load loading and 35ms of sudden load unloading.

![Figure 3](image1.png)

**Figure 3.** Output voltage under quasi-PR control with resistive load

![Figure 4](image2.png)

**Figure 4.** Output voltage of LQR optimal tracking control with resistive load

Figure 5 and Figure 6 respectively show the three-phase voltage output under the two control methods with resistive load. (a) is output voltage waveform of three-phase inverter, (b) is output voltage harmonics of three-phase inverter. Similar to the case of resistive load, the optimal tracking control of LQR can reduce the output voltage harmonics more effectively, and the harmonic distortion rate is only 0.40%.

![Figure 5](image3.png)

**Figure 5.** Output voltage under quasi-PR control with resistive load
Figure 6. Output voltage of LQR optimal tracking control with resistive load

Figure 7 and Figure 8 respectively show the three-phase voltage output under the two control methods with nonlinear load. (a) is output voltage waveform of three-phase inverter, (b) is output voltage harmonics of three-phase inverter. It can be seen that the nonlinear load poses a great challenge to the output of the three-phase inverter, and large harmonics are generated under the two control methods. However, the optimal tracking control of LQR can reduce the output voltage harmonics more effectively and has better output stability, which has obvious advantages compared with the quasi-PR control.

Figure 7. Output voltage under quasi-PR control with nonlinear load

Figure 8. Output voltage of LQR optimal tracking control with nonlinear load

6. Conclusion
In this paper, the three-phase inverter as the research object, through the simulation comparison with the traditional quasi-PR control method, verified the effectiveness of the LQR optimal tracking control strategy. Simulation results show that the proposed control strategy significantly improves the harmonic and steady state performance of the output voltage with nonlinear load and load mutation, and the system has good robustness and anti-load disturbance ability.
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