Cooperative Operation of the Fleet Operator and Incentive-Aware Customers in an On-Demand Delivery System: A Bilevel Approach

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Abstract—In this article, we study the cooperative operation problem between the fleet operator and incentive-aware customers in an on-demand delivery system. Specifically, the fleet operator offers discounts on transportation costs in exchange of customers’ delivery time flexibility. In order to capture the interaction between the fleet operator and customers, a novel bilevel optimization framework is proposed. By exploiting the strong duality, and the Karush–Kuhn–Tucker (KKT) optimality condition of customer optimization problems, we can reformulate the bilevel optimization problem as a mixed-integer nonlinear programming (MINLP) problem. Considering the inherent difficulties of MINLP, a computationally efficient algorithm, which combines the merits of Lagrangian dual decomposition and Benders decomposition, is devised to solve the resulting MINLP problem in a distributed manner. Finally, extensive numerical experiments demonstrate that the proposed cooperation scheme can decrease the delivery fees for the customers and reduce the operation cost of the fleet operator at the same time, thus leading to a win–win situation for both sides.

Index Terms—Benders dual decomposition (BDD), flexible time window, on-demand delivery system, valid cut.

I. INTRODUCTION

NOWADAYS, on-demand delivery has attracted increasing attentions [1]. Consumers want their products, not just delivered to the place they prefer but also delivered at the time of their choosing. The requirement of punctuality in this novel logistic paradigm introduces great challenges to the transportation service operator, incurring considerable difficulty in solving the vehicle routing problem of its transportation fleet [2]. In practice, different customers hold diverse attitude toward the punctuality, i.e., some customers have high demands of fast delivery while others are more flexible about the time of delivery. Therefore, the transportation fleet operator can exploit the difference of customer flexibility in delivery time to design a more cost-effective delivery strategy. It can be achieved by demand management techniques, where the fleet operator provides discounts to customers in exchange of delivery time flexibility. This cooperation between fleet operator and incentive-aware customers can not only decrease the operation cost of the operator but also bring financial benefits to the customers with flexibility.

The demand management strategy starts to be applied in the logistics industry during the last few years [3], [4], [5], [6], [7], [8], [9], [10], [11]. In [3], a compensation mechanism is introduced in the multiperiod vehicle routing problem with due dates, where customer with due dates exceeding the planning period may be postponed at a cost. Based on the previous work, Estrada-Moreno et al. [4] introduced the possibility to offer price discounts to gain service time flexibility. However, contrary to our research, the aforementioned papers assume that the price discounts are fixed by the operator, which cannot capture the individual preference of each customer. To study the single-item and uncapacitated lot-sizing problem, Li et al. [5] found out the value of offering price discounts in increasing delivery flexibility and reducing logistics cost. By extending the strategy of demand management to a variant of the multiperiod vehicle routing problem, where a service provider offers a discount to customers in exchange for delivery flexibility, Yildiz and Savelsbergh [6] used an exact dynamic programming algorithm to obtain the optimal results showcasing that the cost saving up to 30% can be achieved. Considering that the dynamic pricing scheme is able to substantially increase both revenue and the number of customers for the fleet [8], Ulmer [9] presented an anticipatory pricing and routing policy method that incentivizes customers to select delivery deadline options efficiently for the fleet to fulfill. To further exploit the benefits of the flexible delivery time, Strauss et al. [10] introduced flexible delivery time slots, defined as any combination of such regular time windows, and the reduced delivery charge for the customer providing flexible delivery time slots. Keskin et al. [11] introduced a new demand management technique in dynamic vehicle routing problems, i.e., outing, in which customers that have not yet been served can be actively encouraged to order a service sooner. Besides, [11] also proposes several strategies to determine the most relevant customers to tout.

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However, these aforementioned papers do not consider the interaction and negotiation between the fleet operator and customers. That is, the reaction of customers is not incorporated into the vehicle routing problem solved by the fleet operator. Considering that the fleet operator and incentive-aware customers need to work cooperatively to exploit the time flexibility, neglecting the customers’ reaction is not appropriate in practice. To this end, the research gap is bridged by this article.

To tackle the difficulties in the vehicle routing problem solved by the fleet operator, numerous papers have proposed a set of algorithms [4], [12], [13], [14], [15], [16], [17], [18], [19], [20]. The provided solution algorithms can be roughly classified as two categories: 1) the heuristics approaches [4], [13], [15] such as search-based algorithm [4], [13], [14], and learning-based heuristics method [21], [22] and 2) the exact methods, such as branch-and-cut method [16], [17], branch-and-price method [18], [19], column-and-row generation method [12], and Benders decomposition method [20]. In [4], a metaheuristic approach is proposed to find low-cost solution, which includes both the transportation costs and the cost of the price discounts offered. To determine a distribution plan to visit a set of customers, Larraín et al. [16] embedded a new family of valid inequalities within the framework of branch and bound to improve its performance. However, there are still a few limitations associated with these solution approaches. For instance, the heuristics-based algorithms obtain the near-optimal solution with a good performance on computation time, but without the theoretical guarantee on the solution quality. Regarding exact algorithms such as the Benders decomposition method, they can indeed yield optimal solutions but typically demand a substantial number of iterations. This can result in a significant computational burden. Hence, it is possible to substantially reduce the computation time of the Benders decomposition method by decreasing the number of iterations [23]. This motivation leads to the approach proposed in the subsequent sections.

In this article, we focus on the cooperative vehicle routing problem of a transportation fleet operator and incentive-aware customers. To achieve a better routing schedule, the fleet operator provides a transportation fee discount to the incentive-aware customers in exchange for delivery time flexibility. Note that with the adoption of a demand management strategy into the routing problem, the following twofold benefits emerge: 1) customers will receive the delivery cost savings with the discount offered and 2) when the customers provide flexible delivery time windows to the operator, more flexibility in deciding the optimal routing schedule can be obtained, leading to a reduced operation cost for the fleet operator.

The main contributions of this article are presented as follows.

1) To the best of author’ knowledge, a novel bilevel vehicle routing model characterizing the cooperation between fleet operator and incentive-aware customers is proposed for the first time in the field of vehicle routing problem.
2) Due to the NP hardness of the bilevel vehicle routing problem (BVRP), exact reformulation techniques based on the Karush–Kuhn–Tucker (KKT) optimality condition are proposed, which is used to transform the BVRP as a single-level mixed-integer nonlinear programming (MINLP) problem.
3) In order to further reduce the computation complexity, these nonlinear terms in MINLP can be exactly linearized by exploiting the property of strong duality. Then, the MINLP is equivalently converted into a simpler mixed integer programming (MIP) problem.
4) To tackle with the inherent complexities of the MIP and protect the privacy of customers, we devise a novel decomposition method named as Benders dual decomposition (BDD) algorithm integrating the Lagrangian dual decomposition method with the generalized Benders decomposition (GBD) method. In comparison to the GBD method, the BDD method requires fewer iterations, leading to significant savings in computation time, which is theoretically proved and substantiated through simulation results. Furthermore, with the strengthened valid cuts, the proposed approach can provide the optimal solution and also achieve the distributed implementation.

The remainder of this article is summarized as follows. In Section II, We elaborate the system models, and the mathematical model of BVRP considering the flexible time windows. In order to tackle the NP hardness brought by the bilevel framework and computation difficulties from nonlinear terms in the objective function, we propose a set of accurate reformulation approaches to convert the primal bilevel mathematical model into the single-level MIP-based mathematical model in Section III. In addition, a computationally efficient decomposition-based algorithm, which combines the complementary advantages of Benders decomposition and Lagrangian dual decomposition method, is proposed in Section IV. We conduct simulation experiments in Section V to prove the validity of the proposed model and algorithm. Finally, in Section VI, we conclude this article. In this article, customers and requests are used interchangeably.

II. BILEVEL MODEL OF FLEET OPERATOR AND CUSTOMERS

In this article, a novel business model is considered where the transportation fleet operator and incentive-aware customers work cooperatively to achieve cost-effective goods delivery. In practice, among the scenario where a fleet operator serves massive customers, customers set the expected service start time (delivery or pickup time) individually before the fleet operator assigns a specific vehicle to serve them. As for customers who are insensitive to the service start time, to obtain the delivery (pickup) time flexibility for making a better routing schedule, the fleet operator is prone to provide the service fee discount. Meanwhile, customers who provide time flexibility for fleet operator will receive the delivery fee reduction. In this article, we propose a novel pricing mechanism to guide the behavior of fleet operator and customers. Unlike the similar papers considering the strategy of demand management [4], [6], [7], a bilevel optimization model is proposed to characterize the interaction between a company with massive customers and a fleet operator, which is shown in Fig. 1. In the following section, the details concerning lower level
customers model (follower problem) and upper level operator model (leader problem) are introduced first, then we present the bilevel model.

A. Mathematical Model of Customers

As the followers, to reduce the delivery fee, customers are willing to provide their time flexibility $\delta_i$ in exchange for a discount price $q_i$ set by the leader (fleet operator). In addition, $\delta_i$ could also cause inconvenience for customers due to the dispersed delivery time window and delayed delivery time. Then, we propose a convex function $I(\cdot)$ to quantify the inconvenience. Specifically, with the discount price $q_i$ given by the leader problem, the follower problem for each customer is shown as follows.

Problem 1 (Consumer Problem—Follower Problem):

$$\min_{\delta_i} I(\delta_i) - q_i \delta_i$$

s.t. $0 \leq \delta_i \leq \delta_i^*: \sigma_j, u_j$.  

Considering that the time sensitivity varies over different customers, a lower bound (0) and upper bound ($\delta_i^*$) constraint is also included in Problem (1). Besides, $\sigma_j$ and $u_j$ are the corresponding dual variables for these two constraints. Since the customer model Problem (1) satisfies Slater’s condition for the convexity of $I(\delta_i) - q_i \delta_i$ and the convex box constraints $0 \leq \delta_i \leq \delta_i^*$ in which $\delta_i^*$ is positive, the strong duality holds for Problem 1 [24].

B. Mathematical Model of Fleet Operator

We propose to use a directed graph to characterize the relation between depots and customers. The transportation network is modeled as a directed graph $G(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, v_n\} \cup \mathcal{R}$ denotes the start depot $v_1$, the end depot $v_n$, and the set of customer nodes $\mathcal{R}$, as well as $\mathcal{E}$ stands for the set of paths with $i, j \in \mathcal{V}$ denoting a path from node $i$ to node $j$. We denote the start depot and end depot by $v_1$ and $v_n$, respectively. Let $T_{ij}$ be the travel time of paths between nodes $i, j \in \mathcal{V}$. Besides, the binary variable $x^k_{ij}$ denotes whether vehicle $k$ is assigned to traverse path $ij$. Besides, $c_i$ stands for a unified cost vector consisting of both the negative delivery fee $-\mathcal{M}i$ and the vehicle usage fee $c_i$: $c_i = \begin{cases} -\mathcal{M}i, & \text{if } i \in \mathcal{R} \\ c_i, & \text{if } i = v_1 \end{cases}$.

The fleet operator, as a leader, aims to minimize its operation cost comprising of: 1) monetary value of travel time $\sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{E}} T_{ij} x^k_{ij}$ with $\Gamma$ denoting the monetary value of time; 2) the unified delivery cost defined above $\sum_{i \in \mathcal{R}} q_i \delta_i (\sum_{j \in \mathcal{E}} \sum_{k \in \mathcal{K}} x^k_{ij})$ provided to the customers.

Problem 2 (Transportation Model—Leader Problem):

$$\min_{\mathcal{X}} \sum_{j \in \mathcal{V}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{E}} (\Gamma T_{ij} + c_i) x^k_{ij} + \sum_{j \in \mathcal{R}} q_i \delta_i \left( \sum_{j \in \mathcal{E}} \sum_{k \in \mathcal{K}} x^k_{ij} \right)$$

s.t. $\sum_{j \in \mathcal{V}} x^k_{ij} - \sum_{j \in \mathcal{V}} x^k_{ji} = b_i \quad \forall i \in \mathcal{V}, k \in \mathcal{K}$

$$\sum_{k \in \mathcal{K}} x^k_{ij} \leq 1 \quad \forall i \in \mathcal{R}$$

$$t_j \geq T_{ij} + t_i - M \left( 1 - x^k_{ij} \right) \quad \forall i \in \mathcal{V} \setminus v_n, j \in \mathcal{V} \setminus v_1, k \in \mathcal{K}$$

$$t_j \leq t_i + \delta_j \quad \forall j \in \mathcal{V} \setminus v_1$$

where in (2), $b_{ij} \neq v_1, v_n$ is 0 means that when $i \neq v_1, v_n$, $b_i = 0$.

In order to satisfy the physical requirements of the transportation system, vehicle flow constraints, pickup (delivery) time constraints, as well as the flexible time window constraints are formulated as (2)-(5) in the above mathematical formulation. Constraint (2) showcases the flow conservation constraint of vehicles. Specifically, a vehicle entering in the customer node has to exit out at the same customer node, and vehicles return to the end depot after starting at the start depot. In (4), each customer is served at most once by the vehicle. Time characteristics of customer are specified in (4), which states that the arrival time at customer nodes should not be later than the prescribed delivery time of customers. Besides, in (4), a constant $M$ with a large value has been introduced to linearize the original nonlinear constraint $t_j \geq (T_{ij} + t_i)x^k_{ij} \quad \forall i \in \mathcal{V} \setminus v_n, j \in \mathcal{V} \setminus v_1, k \in \mathcal{K}$. In addition, the flexible time window constraint is characterized by (5).

In the settings of Problem 2, we assume that there are only one start and end depots in the proposed model. In order to handle the real case of the multiple depots, the following multidepot transformation rule is devised as follows.

Multidepot Transformation Rule: If there are $m$ start depots and $m$ end depots in the considered fleet operation problem, we initially introduce a virtual start and end depots which are connected with the actual start and end depots of all vehicles. Subsequently, the travel time and route selection variables between the virtual depots and the actual depots are set as zero and one, respectively. Finally, all vehicles commence from the virtual start depot and conclude at the virtual end depot.

In Fig. 2, we consider such a fleet operation problem in which there are three start and end depots, and the implementation of multidepot transformation rule is also illustrated.
The customers model and fleet operator model in the above flexible time window. With the mathematical formulation to characterize the cooperative vehicle routing problem with a bilevel model of Operator and Customers.

With the proposed multidepot transformation rule, the fleet operation problem with multiple depots can be converted into the single-depot case and addressed by the proposed model.

C. Bilevel Model of Operator and Customers

Considering the interaction between the fleet operator and customers, a bilevel optimization framework is employed to characterize the cooperative vehicle routing problem with a flexible time window. With the mathematical formulation of the customers model and fleet operator model in the above sections, the BVRP is formulated as follows.

**Problem 3 (BVRP):**

$$\min \sum_{j \in \mathcal{K}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} (\Gamma T_{ij} + c_i) x_{ij} + \sum_{j \in \mathcal{R}} q_j \delta_j \sum_{i \in \mathcal{V}} \sum_{k \in \mathcal{K}} x_{ij}$$

s.t. $\delta_j^* \in \text{arg min} \{ I(\delta_j) - q_j \delta_j, 0 \leq \delta_j \leq \delta_j^* \} \forall j \in \mathcal{R}$

and (2)–(5)

where the customers problems are constraints to the upper level operator problem, such that the only members that are considered feasible must be both lower level optimal and satisfy upper level constraints.

D. Discussion

Here, the background of the proposed mathematical model and the difficulties of solving such a model are clarified as follows.

Note that the strict delivery time window may rule out the possibility of finding a good routing strategy. For example, even two customers that are geographically close may need to be served by two vehicles due to stringent delivery time constraints. Against this backdrop, the price discount is proposed as an incentive to encourage customers to provide time flexibility to the fleet operator, assisting the operator to lower its vehicle routing costs. To capture the cooperation that benefits both the fleet operator and customers, a mathematical model based on the bilevel optimization is developed.

However, even the simplest bilevel optimization problem is challenging to solve [25]. Moreover, in addition to the difficulties brought by the framework of the bilevel model, there is another complicating term $\sum_{j \in \mathcal{R}} q_j \delta_j$, which consists of the product of two continuous decision variables, in the objective function of BVRP. In the following section, a set of reformulation techniques is carefully devised in order to equivalently transform the original bilevel problem as a single-level MIP-based optimization problem.

III. MATHEMATICAL REFORMULATION OF BILEVEL OPTIMIZATION MODEL

In this section, the BVRP is reformulated as a single-level MIP with 1) the customer problem transformation which is achieved by equivalently replacing the customer problem with its KKT optimality condition and 2) the exact linearization approach of the nonlinear objective function.

A. Mathematical Reformulation of Customer Problem

The bilevel optimization problem, even for the convex case, has been shown to be NP-hard [25], [26]. Hence, to tackle the NP-hardness of the bilevel model, we propose to represent the follower problem, a convex optimization problem, with its KKT optimality condition. First, a set of nonnegative dual variables $(\mu_j, \sigma_j)$ is introduced for the constraints of customers model. As a result, the KKT condition of customers problem is derived and shown in

$$\nabla I(\delta_j) - q_j \sigma_j + \mu_j = 0 \forall j \in \mathcal{R} \quad (6a)$$

$$0 \leq \mu_j \perp (\delta_j - \delta_j^*) \geq 0 \forall j \in \mathcal{R} \quad (6b)$$

$$0 \leq \sigma_j \perp \delta_j \geq 0 \forall j \in \mathcal{R} \quad (6c)$$

where the expression “$a \perp b$” means at most one of $a$ and $b$ can take a strictly nonzero value with the other value being 0. The stationarity conditions are specified in (6a). These primal feasibility condition, dual feasibility condition, and complementary condition are showcased in (6b) and (6c). Then, original BVRP is reformulated as a single-level complicated vehicle routing problem (SCVRP) Problem 4.

**Problem 4 (SCVRP):**

$$\min \sum_{j \in \mathcal{K}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} (\Gamma T_{ij} + c_i) x_{ij} + \sum_{j \in \mathcal{R}} q_j \delta_j \sum_{i \in \mathcal{V}} \sum_{k \in \mathcal{K}} x_{ij}$$

s.t. (2)–(6).

Note that there are still some hard-to-solve terms both in the objective function and constraints of Problem 4. Thus, we propose a set of exact linearization approaches to handle the difficulties from these nonlinear terms of Problem 4 in the later section.

B. Exact Linearization Approach for Nonlinear Terms

In this section, in order to decrease the computation time of Problem 4, which is an MINLP problem, we propose a set of equivalent linearization methods: 1) linearizing the nonlinear complementary constraint by introducing additional binary variables and 2) exploiting the power of strong duality to accurately linearize the nonlinear term $q_j \delta_j$.

1) Linearization of Complementary Optimization Constraint: Due to the nonlinear nature of complementary constraints (6b) and (6c) which render Problem 4 hard to solve, we linearize these nonlinear terms by introducing...
There is still a nonlinear term for convenience, we denote $\phi(\delta_j) = I_j(\delta_j) + (u_j - q_j - \sigma_j)\delta_j - u_j\delta_j$.

The dual function can be obtained as follows $^1$:

$$ g(u_j, \sigma_j) = \inf_{\delta_j} I_j(\delta_j) + (u_j - q_j - \sigma_j)\delta_j - u_j\delta_j $$

$$ q_{\delta_j} = I_j(\delta_j) + u_j\delta_j - \phi^*(\delta_j^*) $$

Then, the nonlinear terms $q_{\delta_j}$ in the objective function of Problem 4 can be exactly linearized by replacing $q_{\delta_j}$ with $I_j(\delta_j) + u_j\delta_j - \phi^*(\delta_j^*)$. As a result, to handle the resulting bilinear terms $(I_j(\delta_j) + u_j\delta_j - \phi^*(\delta_j^*))x_{ij}^*$ in the objective function, the Big-M method is employed. Specifically, a continuous auxiliary $\eta_j$ and addition constraint (8) are introduced

$$ \eta_j \geq I_j(\delta_j) + u_j\delta_j - \phi^*(\delta_j^*) - M\left(1 - \sum_{k \in K} \sum_{i \in \mathcal{I}} x_{ij}^* \right) \forall j \in \mathcal{R}. $$

Following the above linearization techniques, Problem 4 can be equivalently reformulated as the single-level tractable vehicle routing problem (STVRP) Problem 5 which essentially is an MIP problem.

**Problem 5 (STVRP):**

$$ \min_{X} \sum_{k \in K} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} (\Gamma T_{ij} + c_i)x_{ij}^* + \sum_{j \in \mathcal{R}} \eta_j $$

s.t. (2)—(5) and (6a), (7), (8).

where we denote $X = \{x_{ij}^*, \eta, q_j, \delta_j, u_j, \sigma_j, \omega_j^1, \omega_j^2, \eta_j\}$ for simplicity. However, since the transportation service operator and customers are independent entities, customers may not be willing to provide private information to the operator, thus hindering the centralized implementation of the solution method. Hence, to protect the privacy of both sides, we propose a decomposition-based solution algorithm to solve Problem 5 in the following section. Finally, to help the reader understand the underlying idea behind the proposed equivalent reformulation approach, a flowchart is shown in Fig. 3.

**IV. DECOMPOSITION-BASED APPROACH**

In the aforementioned section, the bilevel optimization problem of transportation service operator and customers has been exactly converted into a single-level tractable MIP problem which can be directly solved by commercial solvers. However, it is difficult to gather the parameters of the single-level tractable formulation from the transportation service operator and customers simultaneously, since they are independent entities with their own interests. Thus, to achieve a global solution, we devise a decomposition-based algorithm named as BDD method, which solves the resultant Problem 5 in a distributed manner. Note that the BDD method combines the complementary merits of Lagrangian dual decomposition and Benders decomposition method [23], [28], [29], [30]. Specifically, by exploiting the power of the Lagrangian dual decomposition method, we devise a novel form of subproblem which is an MIP problem. With the newly formulated subproblem, valid Benders cuts are generated which are stronger than the classic Benders cuts derived from the GBD method.

**A. Generalized Benders Decomposition Approach**

Considering that the BDD method is based on the GBD method, thus, to demonstrate the performance of the proposed BDD approach, the details of the GBD method are illustrated first as follows [28], [31], [32].

1) **Master Problem:** By relaxing (4), (5), (6a), (7), and (8), the resulting master problem (MP) which is a relaxed version of Problem 5, provides the lower bound for Problem 5 $^2$

$$ \min_{x_{ij}^*, \omega_j^m} \sum_{k \in K} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \Theta + (c_i + \Gamma T_{ij})x_{ij}^* $$

s.t. (2)—(4)

where $\Theta$ is an auxiliary variable representing the lower bound of primal subproblem (PSP).

$^2$In a later section, we denote $m \in M = \{1, 2\}$. 

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1 For convenience, we denote $\phi(\delta_j) = I_j(\delta_j) + (u_j - q_j - \sigma_j)\delta_j$. 

2 In a later section, we denote $m \in M = \{1, 2\}$.
2) **Primal Benders Subproblem:** With the integer solution obtained from solving (MP), the primal subproblem can be formulated as follows with the copies of MP decision variables $\lambda_{ij}^k$, $\Omega_j^m$, which has been used in previous research [29], [31], [33], [34]

$$\min \sum_{j \in R} \eta_j$$ \hspace{1cm} (PSP)

s.t. \hspace{1cm} $\lambda_{ij}^k = \lambda_{ij}^k \hspace{1cm} \forall i, j \in \mathcal{V}, k \in \mathcal{K} : \chi_{ij}^k$ \hspace{1cm} (9a)

$$\omega_{mij}^k = \Omega_{ij}^m \hspace{1cm} \forall i \in \mathcal{R}, m \in \mathcal{M} : \lambda_{ij}^m$$ \hspace{1cm} (9b)

$$\sum_{k \in \mathcal{K} j \in \mathcal{V}} \lambda_{ij}^k - \sum_{k \in \mathcal{K} j \in \mathcal{V}} \lambda_{ij}^k = b_{ij} \hspace{1cm} \forall i \in \mathcal{V}; k \in \mathcal{K}$$ \hspace{1cm} (9c)

$$\textstyle b_{ij} = 1, b_{ij} = -1, b_{ij} \neq 1, v_{ij} = 0$$ \hspace{1cm} (9d)

$$\sum_{k \in \mathcal{K} j \in \mathcal{V}} \lambda_{ij}^k \leq 1 \hspace{1cm} \forall i \in \mathcal{R}$$ \hspace{1cm} (9e)

$$t_j \geq T_j + t_i - M\left(1 - \lambda_{ij}^k\right) \forall i \in \mathcal{V} \setminus \mathcal{V}_n$$ \hspace{1cm} (9f)

$$t_j \leq T_j + t_i + \delta_j \forall i \in \mathcal{V} \setminus \mathcal{V}_n$$ \hspace{1cm} (9g)

$$\sum_{j \in \mathcal{V}} (\delta_j) - a_j - c_j + u_j = 0 \forall \mathcal{R}$$ \hspace{1cm} (9h)

$$\sum_{j \in \mathcal{V}} (\delta_j) = M \left(1 - \delta_j\right) \forall j \in \mathcal{V}$$ \hspace{1cm} (9i)

$$\sum_{j \in \mathcal{V}} (\delta_j) = M \left(1 - \Omega_j^m\right) \forall j \in \mathcal{V}$$ \hspace{1cm} (9j)

In Lemma 1, it is shown that a valid optimality cut can be generated by optimizing an MIP-based subproblem.

**Lemma 1:** Given the linear relaxation solution of MP $x_{ij}^k, \omega_{mij}^k \in \mathbb{R}$, and dual multipliers $\lambda_{ij}^k, \lambda_{mij}^k \in \mathbb{R}$, solving the Lagrangian relaxation-based subproblem (LSP), an MIP problem, provides the optimal solution $\left(\Omega_j^m, \lambda_{ij}^k, \eta_j\right)$

$$\min \sum_{k \in \mathcal{K} i \in \mathcal{V} j \in \mathcal{V}} \omega_{mij}^k - \sum_{k \in \mathcal{K} i \in \mathcal{V} j \in \mathcal{V}} \lambda_{ij}^k \chi_{ij}^k$$ \hspace{1cm} (LSP)

s.t. \hspace{1cm} (9c), . . . , (9j), $\lambda_{ij}^k, \Omega_j^m \in \mathbb{B}$

Then

$$\sum_{k \in \mathcal{K} i \in \mathcal{V} j \in \mathcal{V}} \omega_{mij}^k - \Omega_j^m \geq 0$$ \hspace{1cm} (10)

is a valid optimality cut for the MP.

The proof is in Appendix A.

**B. Benders Dual Decomposition Approach**

To reduce the computation time of the GBD method, by exploiting the property of Lagrangian dual decomposition, we propose a novel decomposition method, BDD algorithm, which iteratively strengthens the MP with stronger optimality and feasibility cut.

Specifically, in order to strengthen the classic generalized Benders cut, two constraints (9c) and (9b) are relaxed into the objective function by introducing two dual multipliers $\lambda_{ij}^k$ and $\lambda_{mj}^m$. The Lagrangian dual problem is formulated as follows:

$$\max \min_{\lambda_{ij}^k} \sum_{k \in \mathcal{K} i \in \mathcal{V} j \in \mathcal{V}} \sum_{i \in \mathcal{V}} \eta_j - \sum_{i \in \mathcal{V}} \lambda_{ij}^k \chi_{ij}^k$$ \hspace{1cm} (10)

s.t. \hspace{1cm} (9c), . . . , (9j).

By applying this relaxation step, integrality requirements can be imposed on any subset of the variables $\lambda_{ij}^k$ and $\Omega_j^m$ given they are no longer equal to $x_{ij}^k$ and $\omega_{mij}^k$, respectively.

The proof is in Appendix B.

As for the feasibility valid cut, the same rule applies.

**Lemma 2:** Given the infeasible linear relaxation solution of (MP) $x_{ij}^k, \omega_{mij}^k \notin \mathbb{X}_{pp}$, and dual multipliers $\lambda_{mij}^m, \lambda_{ij}^k \in \mathbb{R}$, solving the (FSP), which is an MIP problem and shown in Appendix C, provides the optimal solution $\left(\Omega_j^m, \chi_{ij}^k\right)$.

Then

$$0 \geq \sum_{k \in \mathcal{K} i \in \mathcal{V} j \in \mathcal{V}} S_{ij} + \sum_{k \in \mathcal{K} i \in \mathcal{V} \in \mathcal{V}} S_{rk}$$ \hspace{1cm} (11)

$$+ \sum_{i \in \mathcal{V}} S_{ij} + \sum_{i \in \mathcal{V}} S_{ij}$$ \hspace{1cm} (12)

is a valid feasibility cut for the MP.
With these stronger Benders cuts (11) and (12), compared with the GBD method, the performance of the proposed BDD method is theoretically enhanced, which will be validated in later simulation results. The BDD method is able to achieve the global optimal solution, which is shown in Theorem 2.

**Theorem 2:** With the strengthened Benders cuts (11) and (12), the proposed BDD method can still obtain the global optimal solution.

**Proof:** As demonstrated in Lemmas 1 and 2, the proposed optimality and feasibility cuts are valid cuts for the MP which has been proven in Appendix A. This implies that the proposed cuts do not eliminate any feasible integer solutions [35]. Consequently, our approach can attain the optimal solution, similar to the GBD method [32].

In the subsequent section, we provide clarification on some implementation details.

### C. Implementation Details of Benders Dual Decomposition Approach

Given the theoretical results clarified in Theorem 1, the solution of linear programming (LP) relaxation (MP) can derive stronger Benders cut than the solution of (MP).

1) **Fractional Solution:** At the early stage of the proposed iterative algorithm, to quickly derive valid cuts, we first solve the LP relaxation of the (MP) with classical cuts, which is a strategy that was originally devised by McDaniel and Devine [28], [36].

2) **Stronger Cut Generation:** The strengthened Benders cuts (11), (12) are generated in the following steps: 1) the value of dual multipliers \( \lambda \) is obtained by solving the (PSP) and 2) with the dual multipliers \( \lambda \), stronger Benders cuts are generated by solving (LSP).

Thanks to the power of stronger optimality and feasibility cut, compared with the GBD method, the proposed BDD method holds a faster convergence rate for the entire solution process. Finally, the solution procedures of the proposed BDD are shown in Fig. 4. Within the first \( T \) iteration, the valid cuts from the proposed BDD method are generated. Besides, the implementation details of GBD are shown on the right-hand side of Fig. 4. The pseudocode of the proposed algorithm is showcased at Algorithm 1 for a clear illustration.

### V. Numerical Results

In this section, extensive simulations are carried out to validate 1) the superiority of the proposed bilevel model in saving the delivery cost for customers and reducing the operation cost
TABLE I
NUMERICAL DATA FOR SIMULATIONS

| Parameter                        | Variable | Value  |
|----------------------------------|----------|--------|
| The slope of the inconvenience function | $\gamma_{1,2}$ | 0.5-0.3 |
| The intercept of the inconvenience function | $\chi_{1,2}$ | -0.01, 0.01 |
| Time flexibility                  | $\delta$ | 1 hour |
| Vehicle usage cost               | $c_v$    | $99$ |
| Delivery fee                      | $M_i$    | $9.05$ $\$$ |

for the operator as well as 2) the performance of the proposed BDD method in computation time savings and the reduction of the number of iterations compared with the GBD method.

A. Parameter Settings

To evaluate the performance of the proposed algorithm, we test our model on the map of Belgium\(^3\) consisting of the geographic information of 1000 nodes, which is modified from the data set of Yao et al.\(^37\). In terms of parameter settings, revenue from serving a customer is set as $9.05\(^4\) and the usage cost for vehicle is set as $99.\(^5\) The following numerical experiments are all implemented 50 times for each instance,\(^6\) and statistical results are shown and analyzed. For instance, when the number of customers is 101, we choose these 101 customers randomly from the 1000-node map at each run of simulation experiments. Thanks to the different road topologies of randomly selected nodes, the performance of the proposed BDD method over various road topologies is demonstrated. Besides, for brevity, we model the inconvenience function as a two-segment convex piecewise linear function in simulations, \(I_\delta(\delta_j) = \max_{n\in\{1,2\}} \gamma_n \delta_j + \chi_n\). Note that the inconvenience function can be easily extended to any form of convex optimization problem as explained in the above theoretical analysis. In addition, Table I showcases the details of numerical data. We implement all optimization methods with Python 3.7 on Intel Core i9-10980XE CPU 3.00 GHz × 36 with 64 GB of memory.

B. Performance of Bilevel Model

Compared with the (VRPTW) model in Appendix D, the performance of BVRP is evaluated in terms of the operation cost reduction for the fleet operator, and the delivery fee saving of customers.

1) Superiority of Bilevel Model in the Operation Cost Reduction of Fleet Operator: As illustrated in Fig. 5, comparison between the proposed BVRP and (VRPTW)\(^7\) showcases that the proposed bilevel optimization model has a good performance on the operation cost saving of fleet operator. Note that with the increase of the number of customers, the reduction of operation cost becomes larger, which can explained as that more customers potentially provides more time flexibility for the scheduling of vehicle routing problem.

2) Superiority of Bilevel Model in the Delivery Fee Savings of Customers: As shown in Fig. 6, the highest value of average delivery fee reductions of customers has exceeded 25%, which demonstrates the power of BVRP in the delivery fee reductions. In addition, the value of average delivery fee reductions becomes larger with the increase of \(\gamma\). The reason has been clarified in the above section.

C. Performance of Proposed BDD Method

In this section, the error bar including the information of mean and standard deviation in which the length of the error bar is 2 times of the standard deviation, and the specific markers denote the mean values, is employed to show the effectiveness of the proposed BDD. Compared with the GBD, the performance of the proposed BDD method is evaluated.

\(^3\)http://www.vrp-rep.org/datasets/item/2017-0001.html
\(^4\)https://www.fedex.com/en-us/shipping/one-rate.html
\(^5\)https://www.kayak.com/United-States-Car-Rentals.253.crc.html
\(^6\)Only considering five vehicles. The instances with more vehicles are evaluated in a later section.
\(^7\)The details concerning vehicle routing problem without incorporating time flexibility are represented in Appendix D.
Fig. 7. Mean and standard deviation value of iteration number of GBD and BDD over 50 times of experiments.

Fig. 8. Mean and standard deviation value of computation time of GBD, BCP, and BDD over 50 times of experiments.

in the following two aspects: 1) the number of iterations and 2) the computation time. Due to the power of stronger Benders cuts, the number of iterations of the BDD method will be less than the GBD method inherently, which has been illustrated in Fig. 7.

Consequently, compared with the GBD method, the BDD method, which involves in the computation of mixed integer subproblem, still achieves better performances over all instances in terms of the computation time as illustrated in Fig. 8. In order to further evaluate the performance of the proposed BDD in terms of the reduction of computation time, the branch cut and price method which is the state-of-the-art algorithm in the vehicle routing problem [38], [39], [40], [41], [42], [43] is used to verify the effectiveness of the BDD in Fig. 8. As illustrated in Fig. 8, compared with the state-of-the-art solution method, the proposed BDD method achieves a good performance in reducing the computation time with a decrease of nearly two orders of magnitude.

To clearly demonstrate the performance of BDD, we compare the iteration processes of GBD and BDD in Fig. 9. It is evident from the figure that BDD converges to the optimal solution in approximately 35 iterations, whereas GBD requires over 160 iterations to reach the optimal solution.

D. Parameter Sensitivity Test on the Time Flexibility

To explore the impact of time flexibility on the operation cost reduction and the delivery fee saving, we conduct three sets of simulation experiments with the increasing value of $\delta$ from 0.5 to 1.5 h. In Fig. 10, the operation cost decreases with the increase of $\delta$ over all instances, which reveals that more time flexibility is beneficial for the reduction of operation cost. As a result, the larger time flexibility helps the customers save more delivery fees, which is validated in Fig. 11.

E. Scalability of BDD Method

In this section, extensive experiments are carried out to evaluate the scalability of the proposed BDD algorithm by
characterize the cooperation between the fleet operator and incentive-aware customers for the first time, in which the fleet operator provides price discounts in exchange of customers’ time flexibility. Due to the inherent difficulties of the bilevel optimization model, we aim to exactly reformulate the BVRP as a single-level MIP by exploiting the power of the KKT optimality condition. In addition, a solution method with low computation complexity is proposed to solve the resulting single-level vehicle routing problem. Finally, to evaluate the performance of the proposed bilevel optimization model and the solution method, we carry out extensive numerical experiments, which validate the performance of the proposed bilevel model on the operation cost saving of fleet operator and the delivery fee reduction of customers, and the efficacy of proposed solution algorithm in computation speed.

APPENDIX A

PROOF OF LEMMA 1

Proof: The Benders cut (11) is valid if any \((\Theta, x_{ij}^k, \Omega_j^m)\) satisfying

\[
\Theta \geq \min_{x_{ij}^k, \Omega_j^m} \left\{ \sum_{k \in K} \sum_{i \in V} \sum_{j \in V} \eta_j : (9), x_{ij}^k, \Omega_j^m \in \mathbb{B} \right\} \tag{13}
\]

also satisfies the cut. Given any \((\Theta, x_{ij}^k, \Omega_j^m)\) satisfying the above inequality, we have

\[
\Theta \geq \min_{x_{ij}^k, \Omega_j^m} \left\{ \sum_{k \in K} \sum_{i \in V} \sum_{j \in V} \eta_j : (9), x_{ij}^k, \Omega_j^m \in \mathbb{B} \right\}
\]

\[
\geq \max_{x_{ij}^k, \Omega_j^m} \left\{ \sum_{j \in V} \sum_{m \in M} \lambda_{om}^m \omega_j^m + \sum_{i \in V} \sum_{j \in V} \lambda_{xm}^m \eta_j - \sum_{i \in V} \sum_{j \in V} \lambda_{x}^j x_{ij}^k + \sum_{j \in V} \sum_{m \in M} \eta_j \right\}
\]

\[
= \max_{x_{ij}^k, \Omega_j^m} \left\{ \sum_{j \in V} \sum_{m \in M} \lambda_{om}^m (\omega_j^m - \omega_j^{m^*}) + \sum_{i \in V} \sum_{j \in V} \lambda_{xm}^m \eta_j - \sum_{i \in V} \sum_{j \in V} \lambda_{x}^j x_{ij}^k + \sum_{j \in V} \sum_{m \in M} \eta_j \right\}
\]

\[
= \max_{x_{ij}^k, \Omega_j^m} \left\{ \sum_{j \in V} \sum_{m \in M} \lambda_{om}^m (\omega_j^m - \omega_j^{m^*}) + \sum_{i \in V} \sum_{j \in V} \lambda_{xm}^m \eta_j - \sum_{i \in V} \sum_{j \in V} \lambda_{x}^j x_{ij}^k + \sum_{j \in V} \sum_{m \in M} \eta_j \right\}
\]
where the second line follows from weak duality and the fourth line follows from the optimality of \((\tilde{\eta}_j, \tilde{\lambda}_ij^k, \tilde{\Omega}_ij^m)\). Thus, the proposed Benders cut is valid.

**APPENDIX B**

**Proof of Theorem 1**

Proof: Given feasible linearization relaxation solution of MP \(\omega_i^{m*}, x_{ij}^{k*}\), and dual variables \(\lambda_{\omega_i}^{m*}, \lambda_{ij}^{k*}\) from (PSP), we have

\[
\Theta \geq \max_{\lambda_{\omega_i}^{m*}, \lambda_{ij}^{k*} \in \mathbb{R}} \left\{ \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} \lambda_{\omega_i}^{m*} \omega_i^{m*} + \sum_{i,j \in \mathcal{V}} \lambda_{ij}^{k*} x_{ij}^{k*} \right\}
\]

\[
+ \min_{\tilde{\lambda}_{\omega_i}^{m*}, \tilde{\lambda}_{ij}^{k*} \in \mathbb{R}} \left\{ \sum_{j \in \mathcal{V}} \sum_{i \in \mathcal{V}} \eta_j + \sum_{i,j \in \mathcal{V}} \tilde{\lambda}_{ij}^{k*} (x_{ij}^{k*} - \tilde{x}_{ij}^{k}) \right\}
\]

\[
- \sum_{j \in \mathcal{V}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} \omega_j^{m*} \Omega_j^{m*} \} \right\}
\]

\[
= \min_{\tilde{\lambda}_{\omega_i}^{m*}, \tilde{\lambda}_{ij}^{k*} \in \mathbb{R}} \left\{ \sum_{j \in \mathcal{V}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} \eta_j - \sum_{i,j \in \mathcal{V}} \sum_{j \in \mathcal{V}} \tilde{\lambda}_{ij}^{k*} (x_{ij}^{k*} - \tilde{x}_{ij}^{k}) \right\}
\]

\[
- \sum_{j \in \mathcal{V}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} \omega_j^{m*} \Omega_j^{m*} \} \right\}
\]

\[
\geq \sum_{j \in \mathcal{V}} \sum_{i \in \mathcal{V}} \eta_j + \sum_{i,j \in \mathcal{V}} \sum_{j \in \mathcal{V}} \lambda_{ij}^{k*} (x_{ij}^{k*} - \tilde{x}_{ij}^{k})
\]

\[
+ \sum_{j \in \mathcal{V}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} \omega_j^{m*} \Omega_j^{m*} \}
\]

\[
\geq \sum_{j \in \mathcal{V}} \sum_{i \in \mathcal{V}} \eta_j + \sum_{i,j \in \mathcal{V}} \sum_{j \in \mathcal{V}} \lambda_{ij}^{k*} (x_{ij}^{k*} - \tilde{x}_{ij}^{k})
\]

\[
+ \sum_{j \in \mathcal{V}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} \omega_j^{m*} \Omega_j^{m*} \}
\]

\[
to \text{quantify the tightness between these two optimality cuts. Due to the positive value of } \Xi, \text{ we conclude that the derived optimality cut is } \Xi \text{ tighter than the classic generalized optimality cut.}
\]

**APPENDIX C**

**Feasibility Subproblem of Benders Dual Decomposition Method**

We denote the decision variables set of feasibility subproblem by \(\mathcal{X}_{fp} = \{ S \in \mathcal{R}_+, X \setminus (x_{ij}^k, \omega_i^{m*}), \lambda_{ij}^{k*}, \Omega_j^{m*} \}\)

\[
\min_{\lambda_{\omega_i}^{m*}, \lambda_{ij}^{k*}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} S_{ij} + \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} \sum_{r \in \{15,16\}} S_{ijr} + \sum_{j \in \mathcal{V}} \sum_{r \in \{12, 14, 17\}} S_{ijr} + \sum_{i \in \mathcal{V}} \lambda_{ij}^{k*} (x_{ij}^{k*} - x_{ij}^{k*})
\]

\[
+ \sum_{i \in \mathcal{V}} \lambda_{ij}^{m*} (\Omega_j^{m*} - \omega_j^{m*}) \}
\]

\[
\text{(FSP)}
\]

s.t. \(t_j + S_{ij} \geq t_i^* + t_j - M (1 - \lambda_{ij}^{k*}) \)

\(\forall i \in \mathcal{V} \setminus v_0, j \in \mathcal{V} \setminus v_1, k \in \mathcal{K}\)

\(t_j \leq t_j + S_{ij} \quad \forall j \in \mathcal{V} \setminus v_1 \)

\(t_j \leq t_j + \delta_j^* + S_{ij} \quad \forall j \in \mathcal{V} \setminus v_1 \)

\(q_j := \nabla I(\delta_j) - q_j - \sigma_j + u_j + S_{ij} \quad \forall j \in \mathcal{R} \)

\(0 \leq \delta_j - \delta_j^* + S_{ij} \quad \forall j \in \mathcal{R} \)

\(0 \leq u_j + S_{ij} \quad \forall j \in \mathcal{R} \)

\(0 \leq \delta_j - \delta_j^* + S_{ij} \quad \forall j \in \mathcal{R} \)

\(\sigma_j \leq M (1 - \lambda_j^* + S_{ij}) \quad \forall j \in \mathcal{R} \)

\(0 \leq \delta_j + S_{ij} \quad \forall j \in \mathcal{R} \)

\(\delta_j \leq M \lambda_j^* + S_{ij} \quad \forall j \in \mathcal{R} \)

\(0 \leq \sigma_j + S_{ij} \quad \forall j \in \mathcal{R} \)

\(\sigma_j \leq M (1 - \lambda_j^* + S_{ij}) \quad \forall j \in \mathcal{R} \)

\(\eta_j + S_{ij} \geq I_j(\delta_j) + u_j \delta_j - \phi^*(\delta_j) \quad \forall j \in \mathcal{R} \)

\(- M \left(1 - \sum_{k \in \mathcal{K}} \lambda_{ij}^{k*}\right) \quad \forall j \in \mathcal{R} \)

\(\sum_{j \in \mathcal{V}} \lambda_{ij}^{k*} - \sum_{j \in \mathcal{V}} \lambda_{ij}^{k*} \leq b_i + S_{15i} \quad \forall i \in \mathcal{V}; \quad b_i \leq \sum_{j \in \mathcal{V}} \lambda_{ij}^{k*} - \sum_{j \in \mathcal{V}} \lambda_{ij}^{k*} + S_{16i} \quad \forall i \in \mathcal{V}; \quad k \in \mathcal{K} \)
\[
\sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{V}} \alpha_{ij}^k \leq 1 + \sum_{i \in \mathcal{R}} \delta_{1i} \quad \forall i \in \mathcal{R}
\]

\[
\alpha_{ij}^k, \Omega_{ij}^k \in \{0, 1\}.
\]

### APPENDIX D

**Mathematical Model Without Incorporating Time Flexibility**

The detailed mathematical model of the fleet operator without considering the time flexibility is shown as follows:

\[
\begin{align*}
\min & \quad \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} (t_{ij} + c_{ij}) x_{ij}^k \quad \text{(VRPTW)} \\
\text{s.t.} & \quad \sum_{j \in \mathcal{V}} x_{ij}^k - \sum_{i \in \mathcal{V}} x_{ji}^k = b_i \quad \forall i \in \mathcal{V}; k \in \mathcal{K} \\
& \quad b_{v_i} = 1, b_{v_n} = -1, b_{ij \neq p = v_1, v_n} = 0, \\
& \quad \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{V}} x_{ij}^k \leq 1 \quad \forall i \in \mathcal{R} \\
& \quad t_j \geq t_{ij} + t_i - M(1 - x_{ij}^k) \quad \forall i \in \mathcal{V} \setminus \{v_n\}; j \in \mathcal{V} \setminus \{v_1\}; k \in \mathcal{K} \\
& \quad t_j' \leq t_{ij} \quad \forall j \in \mathcal{V} \setminus \{v_1\}.
\end{align*}
\]

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