Observational Constraints of Modified Chaplygin Gas in Loop Quantum Cosmology

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We have considered the FRW universe in loop quantum cosmology (LQC) model filled with the dark matter (perfect fluid with negligible pressure) and the modified Chaplygin gas (MCG) type dark energy. We present the Hubble parameter in terms of the observable parameters $\Omega_{m0}$, $\Omega_{x0}$ and $H_0$ with the redshift $z$ and the other parameters like $A$, $B$, $C$ and $\alpha$. From Stern data set (12 points), we have obtained the bounds of the arbitrary parameters by minimizing the $\chi^2$ test. The best-fit values of the parameters are obtained by 66\%, 90\% and 99\% confidence levels. Next due to joint analysis with BAO and CMB observations, we have also obtained the bounds of the parameters ($B$, $C$) by fixing some other parameters $\alpha$ and $A$. From the best fit of distance modulus $\mu(z)$ for our theoretical MCG model in LQC, we concluded that our model is in agreement with the union2 sample data.

I. INTRODUCTION

The combinations of different observations astrophysical data continuously testing the theoretical models and the bounds of the parameters. Different observations of the SNeIa\textsuperscript{[1,4]}, large scale redshift surveys\textsuperscript{[5,6]}, the measurements of the cosmic microwave background (CMB)\textsuperscript{[7,8]} and WMAP\textsuperscript{[9,10]} indicate that our universe is presently expanding with acceleration. Standard big bang Cosmology with perfect fluid fails to accommodate the observational fact. In Einstein’s gravity, the cosmological constant $\Lambda$ (which has the equation of state $w_\Lambda = -1$) is a suitable candidate which derive the acceleration, but till now there is no proof of the origin of $\Lambda$. Now assume that there is some unknown matter which is responsible for this accelerating scenario which has the property that the positive energy density and sufficient negative pressure, know as dark energy\textsuperscript{[11,12]}. The scalar field or quintessence\textsuperscript{[13]} is one of the most favored candidate of dark energy which produce sufficient negative pressure to drive

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acceleration in which the potential dominates over the kinetic term. In the present cosmic concordance ΛCDM model the Universe is formed of \( \sim 26\% \) matter (baryonic + dark matter) and \( \sim 74\% \) of a smooth vacuum energy component. The thermal CMB component contributes only about 0.01\%, however, its angular power spectrum of temperature anisotropies encode important information about the structure formation process and other cosmic observables.

If we assume a flat universe and further assume that the only energy densities present are those corresponding to the non-relativistic dust-like matter and dark energy, then we need to know \( \Omega_m \) of the dust-like matter and \( H(z) \) to a very high accuracy in order to get a handle on \( \Omega_X \) or \( w_X \) of the dark energy [14, 15]. This can be a fairly strong degeneracy for determining \( w_X(z) \) from observations. TONRY data set with the 230 data points [16] along with the 23 points from Barris et al [17] are valid for \( z > 0.01 \). Another data set consists of all the 156 points in the “gold” sample of Riess et al [4], which includes the latest points observed by HST and this covers the redshift range \( 1 < z < 1.6 \). In Einstein’s gravity and in the flat model of the FRW universe, one finds \( \Omega_A + \Omega_m = 1 \), which are currently favoured strongly by CMBR data (for recent WMAP results, see [10]). In a simple analysis for the most recent RIESS data set gives a best-fit value of \( \Omega_m \) to be \( 0.31 \pm 0.04 \). This matches with the value \( \Omega_m = 0.29^{+0.05}_{-0.03} \) obtained by Riess et al [3]. In comparison, the best-fit \( \Omega_m \) for flat models was found to be \( 0.31 \pm 0.08 \) [14]. The flat concordance ΛCDM model remains an excellent fit to the Union2 data with the best-fit constant equation of state parameter \( w = -0.997^{+0.050}_{-0.054} (\text{stat})^{+0.077}_{-0.082} (\text{stat+sys together}) \) for a flat universe, or \( w = -1.038^{+0.056}_{-0.050} (\text{stat})^{+0.093}_{-0.097} (\text{stat+sys together}) \) with curvature [18]. Chaplygin gas is the more effective candidate of dark energy with equation of state \( p = -B/\rho \) [19] with \( B > 0 \). It has been generalized to the form \( p = -B/\rho^\alpha \) [20] and thereafter modified to the form \( p = A\rho - B/\rho^\alpha \) [21]. The MCG best fits with the 3 year WMAP and the SDSS data with the choice of parameters \( A = 0.085 \) and \( \alpha = 1.724 \) [22] which are improved constraints than the previous ones \(-0.35 < A < 0.025 \) [23].

In recent years, loop quantum gravity (LQG) is a outstanding effort to describe the quantum effect of our universe. Nowadays several dark energy models are studied in the framework of loop quantum cosmology (LQC). Quintessence and phantom dark energy models [24, 25] have been studied in the cosmological evolution in LQC. When the Modified Chaplyning Gas coupled to dark matter in the universe is described in the frame work LQC by Jamil et al [26] who resolved the famous cosmic coincidence problem in modern cosmology. In another study [27]
the authors studied the model with an interacting phantom scalar field with an exponential potential and deduced that the future singularity appearing in the standard FRW cosmology can be avoided by loop quantum effects. Here we assume the FRW universe in LQC model filled with the dark matter and the MCG type dark energy. We present the Hubble parameter in terms of the observable parameters $\Omega_m$, $\Omega_x$ and $H_0$ with the redshift $z$. From Stern data set (12 points), we obtain the bounds of the arbitrary parameters by minimizing the $\chi^2$ test. The best-fit values of the parameters are obtained by 66%, 90% and 99% confidence levels. Next due to joint analysis with BAO and CMB observations, we also obtain the bounds and the best fit values of the parameters $(B, C)$ by fixing some other parameters $A$ and $\alpha$. From the best fit of distance modulus $\mu(z)$ for our theoretical MCG model in LQC, we concluded that our model is in agreement with the union2 sample data.

II. BASIC EQUATIONS AND SOLUTIONS FOR MCG IN LOOP QUANTUM COSMOLOGY

We consider the flat homogeneous and isotropic universe described by FRW metric, so the modified Einstein’s field equations in LQC are given by

$$H^2 = \frac{\rho}{3} \left(1 - \frac{\rho}{\rho_c}\right)$$  \hspace{1cm} (1)

and

$$\dot{H} = -\frac{1}{2}(\rho + p) \left(1 - \frac{2\rho}{\rho_c}\right)$$  \hspace{1cm} (2)

where $H$ is the Hubble parameter defined as $H = \frac{\dot{a}}{a}$ with $a$ is the scale factor. Where $\rho_c = \frac{\sqrt{3}}{16\pi G m_c}$ is called the critical loop quantum density, $\gamma$ is the dimensionless Barbero-Immirzi parameter. When the energy density of the universe becomes of the same order of the critical density $\rho_c$, this modification becomes dominant and the universe begins to bounce and then oscillate forever. Thus the big bang, big rip and other future singularities at semi classical regime can be avoided in LQC. Let us note here it has been suggested that $\gamma \sim 0.2375$ by the black hole thermodynamics in LQC. In LQG, this parameter is fixed by the requirement of the validity of Bekenstein-Hawking entropy for the Schwarzschild black hole. The physical solutions are allowed only for $\rho \leq \rho_c$. For $\rho = \rho_c$, it is called bounce. The maximum value of the Hubble factor $H$ is reached for $\rho_{max} = \frac{\rho_c}{2}$ and the maximum value of Hubble factor is $\frac{\rho_c}{12}$. 
Here $\rho = \rho_m + \rho_x$ and $p = p_x$, where $\rho_m$ is the density of matter (with vanishing pressure) and $\rho_x$, $p_x$ are respectively the energy density and pressure contribution of some dark energy. Now we consider the Universe is filled with Modified Chaplygin Gas (MCG) model whose equation of state (EOS) is given by

$$p_x = A\rho_x - \frac{B}{\rho_x^2}, B > 0, 0 \leq \alpha \leq 1 \tag{3}$$

We also consider the dark matter and and the dark energy are separately conserved and the conservation equations of dark matter and dark energy (MCG) are given by

$$\dot{\rho}_m + 3H\rho_m = 0 \tag{4}$$

and

$$\dot{\rho}_x + 3H(\rho_x + p_x) = 0 \tag{5}$$

From first conservation equation (4) we have the solution of $\rho_m$ as

$$\rho_m = \rho_{m0}(1 + z)^3 \tag{6}$$

From the conservation equation (5) we have the solution of the energy density as

$$\rho_x = \left[ \frac{B}{A+1} + C(1 + z)^{3(\alpha+1)(A+1)} \right]^{\frac{1}{\alpha+1}} \tag{7}$$

where $C$ is the integrating constant, $z = \frac{1}{a} - 1$ is the cosmological redshift (choosing $a_0 = 1$) and the first constant term can be interpreted as the contribution of dark energy. So the above equation can be written as

$$\rho_x = \rho_{x0}\left[ \frac{B}{(1 + A)C + B} + \frac{(1 + A)C}{(1 + A)C + B} \right]^\frac{1}{\alpha+1} \times (1 + z)^{3(\alpha+1)(A+1)} \tag{8}$$

where $\rho_{x0}$ is the present value of the dark energy density.

In the next section, we shall investigate some bounds of the parameters in loop quantum cosmology by observational data fitting. The parameters are determined by $H(z)$-z (Stern), BAO and CMB data analysis [28–32]. We shall use the $\chi^2$ minimization technique (statistical data analysis) from Hubble-redshift data set to get the constraints of the parameters of MCG model in LQC.
III. OBSERVATIONAL DATA ANALYSIS MECHANISM

From the solution (8) of MCG and defining the dimensionless density parameters \( \Omega_{m0} = \frac{\rho_m}{3H_0^2} \) and \( \Omega_{x0} = \frac{\rho_x}{3H_0^2} \) we have the expression for Hubble parameter \( H \) in terms of redshift parameter \( z \) as follows \((8\pi G = c = 1)\)

\[
H(z) = H_0 \left[ \Omega_{x0} \left( \frac{B}{(1+A)C+B} + \frac{(1+A)C}{(1+A)C+B} \times \right. \right.
\]
\[
\times (1+z)^{3(\alpha+1)(A+1)} \left. \right]^{1/2} + \Omega_{m0}(1+z)^3 \right]^{1/2}
\]
\[
\times \left[ 1 - \frac{3H_0^2}{\rho_c} \left( \Omega_{x0} \left( \frac{B}{(1+A)C+B} + \frac{(1+A)C}{(1+A)C+B} \times \right. \right.
\]
\[
\times (1+z)^{3(\alpha+1)(A+1)} \left. \right]^{1/2} + \Omega_{m0}(1+z)^3 \right]^{1/2}
\]
\[
\quad \times \left[ 1 - \frac{3H_0^2}{\rho_c} \left( \Omega_{x0} \left( \frac{B}{(1+A)C+B} + \frac{(1+A)C}{(1+A)C+B} \times \right. \right.
\]
\[
\times (1+z)^{3(\alpha+1)(A+1)} \left. \right]^{1/2} + \Omega_{m0}(1+z)^3 \right]^{1/2}
\]

From equation (9), we see that the value of \( H \) depends on \( H_0, A, B, C, \alpha, z \) so the above equation can be written as

\[
H(z) = H_0 E(z)
\]

where

\[
E(z) = \left[ \Omega_{x0} \left( \frac{B}{(1+A)C+B} + \frac{(1+A)C}{(1+A)C+B} \times \right. \right.
\]
\[
\times (1+z)^{3(\alpha+1)(A+1)} \left. \right]^{1/2} + \Omega_{m0}(1+z)^3 \right]^{1/2}
\]
\[
\times \left[ 1 - \frac{3H_0^2}{\rho_c} \left( \Omega_{x0} \left( \frac{B}{(1+A)C+B} + \frac{(1+A)C}{(1+A)C+B} \times \right. \right.
\]
\[
\times (1+z)^{3(\alpha+1)(A+1)} \left. \right]^{1/2} + \Omega_{m0}(1+z)^3 \right]^{1/2}
\]

Now \( E(z) \) contains four unknown parameters \( A, B, C \) and \( \alpha \). Now we will fixing two parameters and by observational data set the relation between the other two parameters will obtain and find the bounds of the parameters.
Table 1: The Hubble parameter $H(z)$ and the standard error $\sigma(z)$ for different values of redshift $z$.

| $z$ | $H(z)$ | $\sigma(z)$ |
|-----|--------|-------------|
| 0   | 73     | ± 8         |
| 0.1 | 69     | ± 12        |
| 0.17| 83     | ± 8         |
| 0.27| 77     | ± 14        |
| 0.4 | 95     | ± 17.4      |
| 0.48| 90     | ± 60        |
| 0.88| 97     | ± 40.4      |
| 0.9 | 117    | ± 23        |
| 1.3 | 168    | ± 17.4      |
| 1.43| 177    | ± 18.2      |
| 1.53| 140    | ± 14        |
| 1.75| 202    | ± 40.4      |

A. Analysis with Stern ($H(z)$−$z$) Data Set

Using observed value of Hubble parameter at different redshifts (twelve data points) listed in observed Hubble data by [33] we analyze the model. The Hubble parameter $H(z)$ and the standard error $\sigma(z)$ for different values of redshift $z$ are given in Table 1. For this purpose we first form the $\chi^2$ statistics as a sum of standard normal distribution as follows:

$$\chi^2_{\text{Stern}} = \sum \frac{(H(z) - H_{\text{obs}}(z))^2}{\sigma^2(z)}$$  \hspace{1cm} (12)

where $H(z)$ and $H_{\text{obs}}(z)$ are theoretical and observational values of Hubble parameter at different redshifts respectively and $\sigma(z)$ is the corresponding error for the particular observation given in table 1. Here, $H_{\text{obs}}$ is a nuisance parameter and can be safely marginalized. We consider the present value of Hubble parameter $H_0 = 72 \pm 8$ Kms$^{-1}$ Mpc$^{-1}$ and a fixed prior distribution. Here we shall determine the parameters $A, B, C$ and $\alpha$ from minimizing the above distribution $\chi^2_{\text{Stern}}$. Fixing the two parameters $C, \alpha$, the relation between the other parameters $A, B$ can
be determined by the observational data. The probability distribution function in terms of the parameters $A, B, C$ and $\alpha$ can be written as

$$L = \int e^{-\frac{1}{2}\chi^2_{\text{Stern}}P(H_0)}dH_0$$

(13)

where $P(H_0)$ is the prior distribution function for $H_0$. We now plot the graph for different confidence levels. In early stage the Chaplygin Gas follow the equation of state $P = A\rho$ where $A \leq 1$. So, as per our theoretical model the two parameters should satisfy the two inequalities $A \leq 1$ and $B > 0$. Now our best fit analysis with Stern observational data support the theoretical range of the parameters. The 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) contours are plotted in figures 1, 2 and 4 for $\alpha = 0.5$ and $A = 1, 1/3, -1/3$. The best fit values of $B$ and $C$ are tabulated in Table 2.

| $A$   | $B$   | $C$   | $\chi^2_{\text{min}}$ |
|-------|-------|-------|------------------------|
| 1     | 0.904 | 0.565 | 10.828                 |
| $\frac{1}{3}$ | 0.561 | 0.778 | 8.230                  |
| $-\frac{1}{3}$ | 0.849 | 0.599 | 7.057                  |

Table 2: $H(z)$-z (Stern): The best fit values of $B, C$ and the minimum values of $\chi^2$ for different values of $A$.

**B. Joint Analysis with Stern + BAO Data Sets**

The method of joint analysis, the Baryon Acoustic Oscillation (BAO) peak parameter value has been proposed by [34] and we shall use their approach. Sloan Digital Sky Survey (SDSS) survey is one of the first redshift survey by which the BAO signal has been directly detected at a scale $\sim 100$ MPc. The said analysis is actually the combination of angular diameter distance and Hubble parameter at that redshift. This analysis is independent of the measurement of $H_0$ and not containing any particular dark energy. Here we examine the parameters $B$ and $C$ for Chaplygin gas model from the measurements of the BAO peak for low redshift (with range $0 < z < 0.35$) using standard $\chi^2$ analysis. The error is corresponding to the standard deviation, where we consider Gaussian distribution. Low-redshift distance measurements is a lightly dependent on different cosmological parameters, the equation of state of dark energy and
Figs. 1 - 3 show that the variation of $C$ with $B$ for $\alpha = 0.5$, $\Omega_m = 0.29$, $\Omega_{x0} = 0.72$ with $A = 1, 1/3, -1/3$ respectively for different confidence levels. The 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) contours are plotted in these figures for the $H(z)$-$z$ (Stern) analysis.

have the ability to measure the Hubble constant $H_0$ directly. The BAO peak parameter may be defined by

$$A = \frac{\sqrt{\Omega_m}}{E(z_1)^{1/3}} \left( \frac{1}{z_1} \int_0^{z_1} \frac{dz}{E(z)} \right)^{2/3}$$

Here $E(z) = H(z)/H_0$ is the normalized Hubble parameter, the redshift $z_1 = 0.35$ is the typical redshift of the SDSS sample and the integration term is the dimensionless comoving distance to the redshift $z_1$. The value of the parameter $A$ for the flat model of the universe is given by $A = 0.469 \pm 0.017$ using SDSS data from luminous red galaxies survey. Now the $\chi^2$ function for the BAO measurement can be written as

$$\chi^2_{BAO} = \frac{(A - 0.469)^2}{(0.017)^2}$$

Now the total joint data analysis (Stern+BAO) for the $\chi^2$ function may be defined by

$$\chi^2_{total} = \chi^2_{Stern} + \chi^2_{BAO}$$

According to our analysis the joint scheme gives the best fit values of $B$ and $C$ in Table 3. Finally we draw the contours $A$ vs $B$ for the 66% (solid, blue), 90% (dashed, red) and 99%
The contours are drawn for 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) confidence levels in figs. 4 - 6 which show the variations of $C$ against $B$ for $\alpha = 0.5$, $\Omega_{0m} = 0.29$, $\Omega_{x0} = 0.72$ with $A = 1, 1/3, -1/3$ respectively for the $H(z)$-z+BAO joint analysis.

(dashed, black) confidence limits depicted in figures 4 – 6 for $\alpha = 0.5$ and $A = 1, 1/3, -1/3$.

| $A$  | $B$   | $C$   | $\chi^2_{min}$ |
|------|-------|-------|-----------------|
| 1    | 0.735 | 0.610 | 827.909         |
| $\frac{1}{3}$ | 0.921 | 0.735 | 768.499         |
| $-\frac{1}{3}$ | 0.585 | 0.998 | 767.440         |

Table 3: $H(z)$-z (Stern) + BAO : The best fit values of $B$, $C$ and the minimum values of $\chi^2$ for different values of $A$.

C. Joint Analysis with Stern + BAO + CMB Data Sets

One interesting geometrical probe of dark energy can be determined by the angular scale of the first acoustic peak through angular scale of the sound horizon at the surface of last scattering which is encoded in the CMB power spectrum Cosmic Microwave Background (CMB) shift parameter is defined by $\Delta$. It is not sensitive with respect to perturbations but are suitable to constrain model parameter. The CMB power spectrum first peak is the shift parameter which is given by
The contours are drawn for 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) confidence levels in figs. 7 - 9 which show the variations of $C$ against $B$ for 

$\alpha = 0.5, C = 0.01, \Omega_{0m} = 0.29, \Omega_{x0} = 0.72$ with $A = 1, 1/3, -1/3$ respectively for the 

$H(z)$-BAO+CMB analysis.

$$R = \sqrt{\Omega_m} \int_{0}^{z_2} \frac{dz}{E(z)}$$

(17)

where $z_2$ is the value of redshift at the last scattering surface. From WMAP7 data of the work of Komatsu et al [38] the value of the parameter has obtained as $R = 1.726 \pm 0.018$ at the redshift $z = 1091.3$. Now the $\chi^2$ function for the CMB measurement can be written as

$$\chi^2_{CMB} = \frac{(R - 1.726)^2}{(0.018)^2}$$

(18)

Now when we consider three cosmological tests together, the total joint data analysis (Stern+BAO+CMB) for the $\chi^2$ function may be defined by

$$\chi^2_{TOTAL} = \chi^2_{Stern} + \chi^2_{BAO} + \chi^2_{CMB}$$

(19)

Now the best fit values of $B$ and $C$ for joint analysis of BAO and CMB with Stern observational data support the theoretical range of the parameters given in Table 4. The 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) contours are plotted in figures 7-9 for $\alpha = 0.5$ and $A = 1, 1/3, -1/3.$
Table 4: $H(z)$-z (Stern) + BAO + CMB: The best fit values of $B$, $C$ and the minimum values of $\chi^2$ for different values of $A$.

| $A$   | $B$  | $C$  | $\chi^2_{\text{min}}$ |
|-------|------|------|------------------------|
| 1     | 0.735| 0.457| 10022.6                |
| $\frac{1}{3}$ | 0.692 | 0.694 | 9963.5                |
| $-\frac{1}{3}$ | 0.731 | 0.599 | 9962.11               |

D. Redshift-Magnitude Observations from Supernovae Type Ia

The Supernova Type Ia experiments provided the main evidence for the existence of dark energy. Since 1995, two teams of High-z Supernova Search and the Supernova Cosmology Project have discovered several type Ia supernovas at the high redshifts [1–4]. The observations directly measure the distance modulus of a Supernovae and its redshift $z$ [39, 40]. Now, take recent observational data, including SNe Ia which consists of 557 data points and belongs to the Union2 sample [18].

From the observations, the luminosity distance $d_L(z)$ determines the dark energy density and is defined by

$$d_L(z) = (1 + z)H_0 \int_0^z \frac{dz'}{H(z')}$$

(20)

and the distance modulus (distance between absolute and apparent luminosity of a distance object) for Supernovas is given by

$$\mu(z) = 5 \log_{10} \left[ \frac{d_L(z)/H_0}{1 \text{ MPc}} \right] + 25$$

(21)

The best fit of distance modulus as a function $\mu(z)$ of redshift $z$ for our theoretical model and the Supernova Type Ia Union2 sample are drawn in figure 10 for our best fit values of $\alpha$, $A$, $B$ and $C$. From the curves, we see that the theoretical MCG model in LQC is in agreement with the union2 sample data.
In fig. 10, $u(z)$ vs $z$ is plotted for our model (solid line) and the Union2 sample (dotted points).

IV. DISCUSSIONS

Modified Chaplygin gas (MCG) is one of the candidate of unified dark matter-dark energy model. We have considered the FRW universe in loop quantum cosmology (LQC) model filled with the dark matter (perfect fluid with negligible pressure) and the modified Chaplygin gas (MCG) type dark energy. We present the Hubble parameter in terms of the observable parameters $\Omega_{m0}$, $\Omega_x$, and $H_0$ with the redshift $z$ and the other parameters like $A$, $B$, $C$, and $\alpha$. We have chosen the observed values of $\Omega_{m0} = 0.28$, $\Omega_x = 0.72$, and $H_0 = 72$ Kms$^{-1}$ Mpc$^{-1}$. From Stern data set (12 points), we have obtained the bounds of the arbitrary parameters by minimizing the $\chi^2$ test. Next due to joint analysis of BAO and CMB observations, we have also obtained the best fit values and the bounds of the parameters $(B, C)$ by fixing some other parameters $A = 1, 1/3, -1/3$ and $\alpha = 0.5$. The best-fit values and bounds of the parameters are obtained by 66%, 90%, and 99% confidence levels are shown in figures 1-9 for Stern, Stern+BAO and Stern+BAO+CMB analysis. The distance modulus $\mu(z)$ against redshift $z$ has been drawn in figure 10 for our theoretical model of the MCG in LQC for the best fit values of the parameters and the observed SNe Ia Union2 data sample. So our predicted theoretical MCG model in LQC permitted the observational data sets. The observations do in fact severely constrain the nature of allowed composition of matter-energy by constraining the range of the values of the parameters for a physically viable MCG in LQC model. We have checked that when $\rho_c$ is large, the best fit values of the parameters and other results of LQC model in MCG
coincide with the results of the ref. [29] in Einstein’s gravity. When $\rho_c$ is small, the best fit values of the parameters and the bounds of parameters spaces in different confidence levels in LQC distinguished from Einstein’s gravity for MCG dark energy model. Also, in particular, if we consider the generalized Chaplygin gas ($A = 0$), the best fit value of critical Barbero-Immirzi parameter $\gamma$ is 0.2486, where we have assumed the values of other parameters $\alpha = 0.5$ and $B = 0.561$ for our convenience. In summary, the conclusion of this discussion suggests that even though the effect that quantum aspect of gravity have on the CMB are small, cosmological observation can put upper bounds on the magnitude of the correction coming from quantum gravity that may be closer to the theoretical expectation than what one would expect.

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