The discovery \[1\] of the exotic baryon Θ+, with strangeness +1 and probable spin 1/2, recently supported by the observations of Θ++ in various experiments \[2, 3, 4, 5\], and the discovery \[2\] of the exotic isospin 3/2 baryon with strangeness -2, \[Σ_{3/2}\], have produced huge excitement in the high energy physics community.

The Θ+-baryon mass was successfully predicted in the “model-independent” way for the first time in Ref. \[7\]. However, it was the prediction of the narrow width of Θ+ in the chiral quark-soliton model of Ref. \[8\] that stimulated experimental searches. To estimate baryon multiplets \((8, 10, Σ0, Σ2, 27, \text{etc.})\) mass spectra, relevant mass differences and other baryon properties, various authors employed different types of methods and models \[9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33\].

The main aim of this Brief Report is the application of the minimal SU(3)\(_f\) extended Skyrme model \[13\] in an attempt to predict the 27\(_{3/2}\)-Σ\(_0\) mass splitting and the 27\(_{3/2}\)-Σ\(_0\) mass spectrum. The minimally extended Skyrme model uses only one free parameter, the Skyrmion charge \(e\), and only flavor symmetry breaking parameters the predictions are presented in tabular form. The predicted mass splitting 27\(_{3/2}\)-Σ\(_0\) is the smallest among all SU(3)\(_f\) baryonic multiplets, confirming earlier findings.

where the SB parameters \(\hat{x}, \beta', \delta'\) are given by \[13\]:

\[
\hat{x} = \frac{2m_0^2f_π^2}{m_0^2f_π^2} - 1, \quad \beta' = \frac{f_K^2 - f_π^2}{4(1 - \hat{x})},
\]

\[
\delta' = \frac{m_0^2f_π^2}{4(1 + \hat{x})}.
\]

The symmetry breaker \(\hat{x}\) was constructed systematically from the QCD mass term in the case of SU(3)\(_f\). The \(\delta'\) term is required to split pseudoscalar meson masses, while the \(\beta'\) term is required to split pseudoscalar decay constants (for details, see Ref. \[13\]).

To obtain the 27\(_{3/2}\)-Σ\(_0\) mass splittings and the 27\(_{3/2}\) mass spectrum, the following definition of the mass formulas are used:

\[
M_{Σ0}^{27}(x_0) = M^8 + \frac{3}{2λ_8(x_0)} - \frac{γ(x_0)}{2} δ_{B}^{Σ0},
\]

\[
M_B^{27}(x_0) = M^8 + \frac{3}{2λ_8(x_0)} + \frac{1}{λ_8(x_0)} - \frac{γ(x_0)}{2} δ_{B}^{Σ0},
\]

Here the experimental octet mean mass \(M^8 = \frac{1}{3} \sum_{B=1}^{8} M_B^8 = 1151\) MeV was used instead of \(M^8 = M_{\text{cal}}(x_0) + \frac{3}{2λ_8(x_0)}\). From experiment it also follows that the decuplet mean mass \(M^{10} = \frac{1}{10} \sum_{B=1}^{10} M_B^{10} = 1382\) MeV \[42\]. The splitting constants \(δ_{B}^{Σ0}\) and \(δ_{B}^{Σ0}\) are given in \[28\] and in Table 1. of Ref. \[26\], respectively. The moment of inertia \(λ_8\) for rotation in coordinate space, and the moment of inertia \(λ_8\) for flavor rotations in the direction of the strange degrees of freedom, except for the eighth direction, and the symmetry breaking quantity \(γ\), [the coefficient in the SB piece \(δ_{B} = -\frac{1}{2γ}(1 - D_{88})\) of a total collective Lagrangian \(L\),] are given in Ref. \[28\].

From \[9\] and \[10\] the 27\(_{3/2}\)-Σ\(_0\) mean mass splitting \(δ_{B}^{Σ0}\) is given by

\[
\Delta_{27}^{Σ0} = M_{Σ0}^{27} - M_{Σ0}^{27} = \frac{1}{2} \left[ \frac{3}{λ_8(x_0)} - \frac{1}{λ_8(x_0)} \right] = Δ_{B}^{Σ0} - \frac{1}{3} Δ_{B}^{Σ0},
\]
where $\Delta^{10}_{37}$ is also expressed in terms of the decuplet–octet $\Delta^{10}_{8}$ and the antidecuplet–octet $\Delta^{10}_{8}$ mean mass splittings, $\Sigma$. In the computations of the mean masses $\mathcal{M}_{\mathbf{37}}$ and $\mathcal{M}_{\mathbf{37/2}}$ the sum of $D_{88}$ diagonal elements over all components of irreducible representations cancels out because of the properties of the SU(3) Clebsh-Gordan coefficients.

The mass splittings between the same quark flavor content baryons of $\mathbf{27}_{3/2}$ and $\mathbf{10}$-plets are:

$$\delta_1 = M^{27}_{3/2}(\Theta_1) - M^{10}_{\mathbf{10}}(\Theta^+) = \Delta^{10}_{37} + \frac{3}{28} \gamma(x_0), \quad (6)$$

$$\delta_2 = M^{27}_{3/2}(N^*_2) - M^{10}_{\mathbf{10}}(N^+) = \Delta^{10}_{37} + \frac{5}{112} \gamma(x_0),$$

$$\delta_3 = M^{3/2}_{3/2}(N^*_4) - M^{10}_{\mathbf{10}}(N^+) = \Delta^{10}_{37} + \frac{5}{112} \gamma(x_0),$$

$$\delta_4 = M^{3/2}_{3/2}(\Sigma_2) - M^{10}_{\mathbf{10}}(\Sigma^+) = \Delta^{10}_{37} + \frac{1}{112} \gamma(x_0),$$

$$\delta_5 = M^{3/2}_{3/2}(\Sigma_1) - M^{10}_{\mathbf{10}}(\Sigma^+) = \Delta^{10}_{37} + \frac{1}{112} \gamma(x_0),$$

$$\delta_6 = M^{3/2}_{3/2}(\Sigma^*) - M^{10}_{\mathbf{10}}(\Sigma^+) = \Delta^{10}_{37} + \frac{1}{28} \gamma(x_0),$$

$$\delta_7 = M^{3/2}_{3/2}(\Xi^*_1) - M^{10}_{\mathbf{10}}(\Xi^*_1) = \Delta^{10}_{37} + \frac{3}{112} \gamma(x_0),$$

$$\delta_8 = M^{3/2}_{3/2}(\Xi^*_2) - M^{10}_{\mathbf{10}}(\Xi^*_2) = \Delta^{10}_{37} + \frac{3}{224} \gamma(x_0).$$

The $\Xi$ isoquartet and isodoublet from the $\mathbf{27}$, spin $3/2$, we mark as $\Xi^*_2$ and $\Xi^*_1$, to distinguish them from the the $\Xi$ isoquartet and isodoublet from the $\mathbf{10}$, spin $1/2$. We also mark the $\mathbf{27}$-plet isosinglet as $\Lambda^*$. Considering the SB parameters $\Sigma$, at this point we introduce three different dynamical assumptions based on the SB part of the Lagrangian producing three fits which will be used further in our numerical analysis:

(i) $m_\pi = m_K = 0$, $f_\pi = f_K = 93$ MeV

$$\Rightarrow \hat{x} = 1, \quad \hat{\beta} = 0;$$

(ii) $m_\pi = 138$, $m_K = 495$, $f_\pi = f_K = 93$ MeV

$$\Rightarrow \hat{x} = 24.73, \quad \hat{\beta} = 0, \quad \hat{\delta} = 4.12 \times 10^7 \text{MeV}^4;$$

(iii) $m_\pi = 138$, $m_K = 495$, $f_\pi = f_K = 93$, $f_K = 113$ MeV

$$\Rightarrow \hat{x} = 36.97, \quad \hat{\beta} = -28.6 \text{MeV}^2, \quad \hat{\delta} = 4.12 \times 10^7 \text{MeV}^4.$$

Switching off SU(3)$_f$ symmetry breaking, which corresponds to case (i), the absolute masses of each member of the multiplet become equal for each fixed $\hat{x}$. In the chiral limit,

$$x_0 = \frac{\sqrt{15}}{4} \Rightarrow \Delta^{10}_{37} = \delta_1, \ldots, \delta_8 = \frac{52e^3f_\pi}{285\sqrt{30}\pi^2} \cdot \delta_2 \geq 10.$$  \quad (8)

For example, from Table I and Table II one would have $M^{27}_{3/2} = 1808$ MeV and $\Delta^{10}_{37} = 32.6$ MeV, for $\epsilon = 4.7$.

The mass splittings $\delta$ and $\hat{\delta}$ as functions of two different dynamical assumptions, (ii,iii), and the Skyrmie charge $\epsilon$ are given in Table II. We have chosen four values of the Skyrmie charge $\epsilon = 3.4, 4.2, 4.4, 4.7$ because in the minimal approach they give the best fit for the nucleon axial coupling constant $g_A = 1.25$. The mass splitting $\Delta^{10}_{37}$ is 231 MeV, and the penta-quark masses $\mathcal{M}_{\mathbf{37}} = 1540$ MeV and $\mathcal{M}_{\mathbf{37}}^{\exp} = 1861$ MeV, respectively.

Assuming equal spacing for antidecuplets, from the recent experimental data ($\mathcal{M}_{\mathbf{37}}^{\exp} = 1540$ MeV and $\mathcal{M}_{\mathbf{37}}^{\exp} = 1861$ MeV), in Ref. 28 we have found the following masses of antidecuplets $\mathcal{M}_{N^*} = 1647$ MeV, $\mathcal{M}_{\Xi^*_1} = 1745$ MeV, the mean mass $\mathcal{M}_{\mathbf{37}} = \frac{1}{8} \sum_{B=1}^{10} M_B^{\mathbf{37}} = 1754$ MeV and the mass difference $\Delta^{10}_{37} = 603$ MeV. Taking 603 MeV, bonafide, as an “experimental” estimates for $\Delta^{10}_{37}$, together with $\Delta^{10}_{37} = 30$ MeV. It turns out from Table III that only $\epsilon \approx 3.2$, in the most realistic case (iii), could account for the small value of $\Delta^{10}_{37}$. However, $\epsilon = 3.2$ gives too small values for $\Delta^{10}_{37}$ and $\Delta^{10}_{37}$. Using 1754 MeV for the $\mathbf{37}$-plet mean mass and the predicted range for the mean mass splitting $30 < \Delta^{10}_{37} < 95$ MeV, we find the range for the $\mathbf{37}_{3/2}$-plet mean mass as $1784 \leq \mathcal{M}_{\mathbf{37}}^{\exp} \leq 1849$ MeV, which is approximately placed into the center of the $\mathbf{37}_{3/2}$-plet mass spectrum displayed in Fig. 4 of Ref. 12 (for A and B fits), and in Fig. 4 of Ref. 28. Careful inspection of the results for the $\mathbf{37}_{3/2}$-plet mass spectrum from Fig. 4 of Ref. 12 shows approximate agreement with our results, $\delta_1, \ldots, \delta_8$, for $4.2 \leq \epsilon \leq 4.7$ fit (iii), presented in Table III.

Comparing the pure Skyrmie model prediction of Ref. 12 (fits A and B in Figure 4) with our results from Table III we have found that our case (iii) with $4.3 < \epsilon < 4.7$
TABLE II: The $27_{3/2}$ mass spectrum (MeV) as functions of the Skyrme charge $e$ and for fits (ii), (iii).

| Fit  | (ii) | (iii) |
|------|------|------|
| $e$  | 3.2  | 3.2  |
| $\Theta$ | 1343 | 1343 |
| $\Sigma$ | 1388 | 1388 |
| $\Lambda$ | 1478 | 1478 |
| $\Xi$ | 1522 | 1522 |
| $\Omega$ | 1657 | 1657 |

supports fit B, and for $4.4 \leq e \leq 4.6$ agrees nicely with fit A. Both fits A and B from $\Theta$, $\Sigma$, $\Lambda$, $\Xi$, $\Omega$ lie between $4.0 \leq e \leq 4.6$ for case (ii). Case (iii) with $4.2 \leq e \leq 4.7$ also supports the results presented in Table 1. of Ref. 26. From Table II we conclude that the best fit for the $27_{3/2}$ baryon mass spectrum, as a function of $e$ and for $f_K \neq f_\pi$, would lie between $e \simeq 4.2$ and $e \simeq 4.7$, just like that for the octet, decuplet and anti-decuplet mass spectra in Table I. The masses of $\Lambda^*$ and $\Xi^*_2$ are equal owing to the absence of anomalous moments of inertia in the model used. Note, however, that the anomalous moments of inertia contributions are estimated to be at best $\sim 1\%$ for the $\Xi^*_2$ mass, for example.

Next we comment on possible effects coming from the mixing between exotic rotational excitations and vibrational (or radial) excitations in the minimal SU(3)$_f$ extended Skyrme model. Let us note that, in the case of the $27_{3/2}$-plet, states with $Y = \pm 2$ and $Y = +1$, $I = 3/2$ do not mix with neither 8, 10 or 27, nor with their vibrational excitations. They will have vibrational excitations themselves, but, as results of Ref. 32 indicate, such vibrations are expected to have minor influence on “base” states. Therefore for these states our predictions are correct within the approximations made, i.e. by neglecting $1/N_c$ corrections to $\mathcal{L}_{SB}$. All other states will be subject to mixing. However, their masses, given in Table II, represents the predictions under no mixing assumption. Considering the question of the decay width calculations, the Skyrme model is too crude to give reliable predictions for the widths 26. Here the $1/N_c$ corrections, missing in the present approach, are of primary importance.

For the simplest version of the total Lagrangian, the results given in Tables I and II do agree well with the other Skyrme model based estimates 27. In particular, our approach is similar to 14, 16.

As has been discussed in 43, although the symmetry breaking effects are generally very important, the main effect comes from the $D_{SB}$ term confirming the results of 16, 14, 28, 32. In our approach, in the language of 32, the reduction of the influence of the so called $\Xi_{32}$ cloud was taken into account by inclusion of the SB term $(1 - D_{SB})$ in mass formulae 8, 16, 17.

It is clear from Table II that for fixed $e$ the difference between fits (ii) and (iii) is crucial for the correct description of the mass splittings 13, 16. For small mass splittings the contribution of the term proportional to $(f_K^2 - f_\pi^2)$ in the Lagrangian $\mathcal{L}$ plays a major role.

The $27_{3/2}$ mass splittings are the quantities whose measured values, together with measurements of the decay modes branching ratios, would determine the spins, 3/2 or 1/2, of observed objects, like $\Xi_{3/2}^*$, thus placing it into the right SU(3)$_f$ representation. We do expect that experimental analysis, considering other members of the 10 and $27_{3/2}$-plets, should also be performed.

We hope that the present calculation, taken together with the analogous calculation in 13, 14, 16, 26, 27, 28, 29 will contribute to the understanding of the overall picture of the baryonic mass spectrum and mass splittings in the Skyrme model, as well as to further computations of other nonperturbative, dimension-6 operator matrix elements between different baryon states 14, 15.

Since the splittings 13, 16 represent the smallest splittings among splittings between the SU(3)$_f$ multiplets 8, 10, 27, 35 and 38 we would urge our colleagues to continue experimental analysis of penta-quark spectral and decay modes and find the penta-quark members of the $27_{3/2}$-plet which would mix with or lie just above the penta-quark family of the 10-plet.

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