Radiation from a varying velocity charge in flight through a plate

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Abstract. The radiation from a particle traversing with varying velocity through a plate in straight motion (along the normal to its surface) has been investigated. It is assumed that the particle coordinate (inside the plate) is an arbitrary monotone function of time. The calculations are based on appropriate exact solutions of the Maxwell equations. It was shown that as a result of variation of particle velocity inside the plate (e.g., due to its slowing-down) the spectral-angular distribution of Cherenkov radiation generated by the particle inside the plate may be essentially affected. Possible practical application of this phenomenon is discussed.

1. Introduction

In the presence of substance the uniform and straight-line motion of a charged particle may go with radiation of electromagnetic waves (e.g., Cherenkov Radiation (CR), transition radiation) that are widely used in practice. In this regard, in investigations on the influence of matter on the charged particle radiation it is usually assumed that the particle motion is uniform and the calculations in this approximation well agree with experiment. In the present work our aim is to study the cases when this approximation is not correct and the nonuniformity of particle motion has to be taken into consideration.

General expressions concerning the electromagnetic field of a charged particle at its arbitrary motion in solid uniform medium have been derived earlier [1]. Analogous problem at the presence of two plane interfaces between media having different permittivities has been solved (see, e.g., [2, 3]). In these works, however, the influence of nonuniformity of particle motion on CR generated by the particle has not been investigated. In this work the effect of the nonuniformity of particle motion on CR generated by particle inside the plate has been studied and possible uses of this effect have been discussed.

In [5-7] the influence of particle slowing down on the angular distribution of CR generated by the particle has been studied. It was assumed that (i) the particle moves in an unlimited and transparent dielectric and (ii) CR (with the wavelength $\lambda_m$) is accumulated from a part of particle trajectory of $l \gg \lambda_m$ length. It was predicted that the structure and the width of angular distribution of CR depends (a) on the energy loss of particle in the dielectric, the initial energy of particle (heavy ion) [4-7] and (b)
on the ion mass (isotope effect [6,7]). A simple formula (see (17)) has been proposed in [7] allowing one to estimate the width of angular distribution of CR in case of \( l >> \lambda_m \). In this paper the influence of nonuniformity of particle motion (slowing down or acceleration stimulated by an external field) on the spectral-angular distribution of CR from the particle passing through a dielectric plate of finite thickness \( l \) was investigated. The absorption of radiation in the plate material is also taken into account.

2. Formulation of the problem

Now let us consider the motion of a charge \( q \) along OZ axis from \(-\infty < z < -l \) range (the range of \( \alpha = - \), see figure 1) that passes into a plate at some instant of time \( t_- \) ( \(- l < z < 0 \) range), travels in it with varying velocity \( v_q(t) \) and then, at some instant of time \( t_+ \), leaves the plate and continues its motion in \( 0 < z < +\infty \) range (\( \alpha = + \) range). In what follows it is assumed that inside the plate the particle coordinate \( z_q = z_0(t) \) is an arbitrary monotone function of time (such that there is \( t_q = t_0(z) \) function that is inverse for it for all \(-l < z < 0 \) and that the plate is made of radiation absorbing uniform and isotropic substance.

Without loss of generality it is also assumed that the space on both sides of the plate is filled with transparent uniform and isotropic substances (gases for instance) with dielectric permittivity and magnetic permeability \( \epsilon_\alpha \) and \( \mu_\alpha \) respectively. To simplify the calculations we shall assume that the motion of particle to the left and to the right of the plate is uniform (with velocities \( v_- \) and \( v_+ \) respectively).

As a result, we can represent the dielectric permittivity \( \epsilon \) and magnetic permeability \( \mu \) of the medium, the velocity \( v_q \) and coordinate \( z_q \) of the particle by means of following expressions:

\[
\psi = \begin{cases} 
\psi_{<} & z < -l \\
\psi_0(t) & -l < z < 0 \\
\psi_{>} & z > 0 
\end{cases}
\quad \text{and} \quad v_q = \begin{cases} 
v_{<} & t < t_- \\
v_0(t) & t_- < t < t_+ \\
v_{>} & t > t_+
\end{cases}
\quad \text{at} \quad t_- < t < t_+
\tag{1}
\]

where \( \psi = \epsilon, \mu \) and \( v_0 = \frac{dz_0}{dt} \). The parameters of laminated medium \( \epsilon \) and \( \mu \) will be assumed below to be independent of transverse coordinates \((x, y) \equiv \vec{r} \).

Now introduce the radiation energy

\[
W_\alpha = \int I_\alpha(\omega, \theta) d\omega d\theta,
\tag{2}
\]

penetrating the plane \( z = \text{const} \) distant from the plate during all the time of particle motion. It may be both to the left (\( \alpha = - \) range), and to the right (\( \alpha = + \) range) of the plate. In (2) \( I_\alpha(\omega, \theta) \) is the...
spectral-angular distribution of the radiation energy, propagating either against (\( \alpha = - \)) or along (\( \alpha = + \)) the direction of OZ axis. We shall focus on the investigation of some features of this function.

### 3. Stages of analytical calculations and finite expressions

Making allowance for the azimuthal symmetry of problem and performing the Fourier transform

\[
( f \equiv \vec{E}, \vec{H} ), \quad \text{the solution of the set of Maxwell equations is reduced to solution of a single equation [8]}
\]

\[
\left[ \varepsilon \frac{\partial}{\partial z} \left( \frac{1}{\varepsilon} \frac{\partial}{\partial z} \right) + \frac{\omega^2}{c^2} \varepsilon \mu + i^2 \right] \varepsilon \varepsilon_{\text{radiation}} = 4\pi \left( \varepsilon \frac{\partial}{\partial z} \left( \rho_{\text{radiation}} \right) - i \frac{\omega}{c} \varepsilon \mu j_{\text{radiation}} \right)
\]

(3)

(the basic equation of the problem under consideration). In this equation \( \varepsilon \equiv \varepsilon_{\text{rad}} \) and \( \mu \equiv \mu_{\text{rad}} \) are Fourier transforms of the dielectric permittivity and magnetic permeability (the allowance for the frequency dispersion is made, the contribution of spatial dispersion being considered negligibly small). Here we assume that \( \varepsilon_0 = \varepsilon_{\text{free}}, \mu_0 = \mu_{\text{free}}, \mu_0^* + i\mu_{\text{free}}^* \) are complex quantities (with due regard for the absorption of radiation in the plate material). Based on solution \( E_{\text{rad}} \) of equation (4), one can calculate the strengths of electric and magnetic field using the following formulae

\[
\vec{E}_{\text{rad}} = \frac{1}{(2\pi)^3} \int f(\vec{r}, t) \exp[i(\omega t - \vec{k} \cdot \vec{r})] d\vec{k} dt
\]

(3)

\[
\vec{H}_{\text{rad}} = \frac{\omega}{c} \left( \varepsilon \varepsilon_{\text{radiation}} + 4\pi \frac{i}{\omega} j_{\text{radiation}} \right) \vec{n} \times \vec{n}_z.
\]

(5)

Here \( \rho(\vec{r}, t) = q \delta(x) \delta(y) \delta[z - z_q(t)] \) and

\[
\vec{v}_q(z) \rho_{\text{rad}}(z) = \frac{q}{(2\pi)^3} \exp[i\omega t_q(z)] = j_{\text{radiation}}(z)
\]

(6)

is the current density associated with the charge (\( \vec{n}_z \) is the unit vector showing the direction of OZ axis).

In regions \( 0 < z < +\infty \) (\( \alpha = + \)) and \( -\infty < z < -l \) (\( \alpha = - \)) equation (4) must have the following solutions:

\[
E_{\text{rad}} = E_{\text{free}} + \frac{iq}{2\pi^2 \omega} a_s \exp[\pm i\omega \sigma_s(z - z_s) / c], \quad \sigma_s = \sqrt{\varepsilon_s \mu_s - u^2 c^2 / \omega^2},
\]

(7)

where the 1-st summand is the known field of a charge uniformly moving in a homogeneous and solid medium with parameters \( \varepsilon_s, \mu_s \), and the 2-nd one describes the free field (radiation field); \( z_s = 0, z_s = -l \) [8-10]. Far from the plate (\( z \to \pm\infty \)) the spectral-angular distribution of radiation energy is determined by an expression [8,10]

\[
I_\omega(\omega, \theta) = \frac{2q^2 \cos^2 \theta}{\pi c \sin \theta} \left| a_\omega(\omega, u) \right|^2, \quad \text{where} \quad u^2 = \frac{\sigma^2}{c^2} \varepsilon_\omega \mu_\omega \sin^2 \theta,
\]

(8)

and \( \theta \) is the angle giving the direction of radiation with respect to \( z = +\infty \) and \( z = -\infty \) for the forward (\( \alpha = + \)) and backward radiations (\( \alpha = - \)) respectively. For calculation of \( a_\omega \) it would be adequate to equate (7) to the complete solution of equation (4) that is valid for all \( -\infty < z < +\infty \). One can determine this solution, e.g., by using the Green function method.

Dropping the intermediate calculations, we give the final expression

\[
a_\omega = \frac{\phi_\omega \varepsilon_0}{2\varepsilon, \sigma_\omega} \left[ \left( \eta + \eta_- \right) \frac{c}{\varepsilon, \varepsilon_-} - \frac{\eta}{1 - \sigma_\omega \varepsilon_- / c} + \frac{\eta}{1 + \sigma_\omega \varepsilon_- / c} \right] \frac{1}{k_v +} + \\
+ \left( \frac{\sigma_\omega}{1 - \varepsilon, \varepsilon_- / c} \right) \exp[i\omega(t - t_s)] - \frac{i\omega}{2c-k_-} \left\{ 0 \right. \\
\left. \int \xi_\omega(z) \exp[i\omega(t_s(z) - t)] dz \right\}.
\]

(9)
The expression for $a_i$ is obtained from (9) first by means of replacing $v_a$ by $-v_a$, and second, + – in all the indices. The following notations are introduced in (9):

$$4\eta = (1+k_c)(1+k_c)\exp[-i\omega\sigma_0/c] + (1-k_c)(1-k_c)\exp[i\omega\sigma_0/c], \quad \sigma_i = \sqrt{\varepsilon_i - u^2c^2/\omega^2}, \quad i = 0, \pm$$

$$4\eta_\pm = (1-k_c)(1+k_c)\exp[i\omega\sigma_0/c] + (1+k_c)(1-k_c)\exp[-i\omega\sigma_0/c], \quad k_\pm = \frac{\varepsilon_0 \sigma_\pm}{\varepsilon_\pm \sigma_0}, \quad \chi_\pm = \frac{c}{\varepsilon_\pm \varepsilon_0} - \frac{\mu_\pm}{\varepsilon_0}$$

$$\xi_\pm = (1+k_c)\left(\mu_0 - \frac{\sigma_0 c}{\varepsilon_0 v_0}\right)\exp[-i\omega\sigma_0(z+l)/c] + (1-k_c)\left(\mu_0 + \frac{\sigma_0 c}{\varepsilon_0 v_0}\right)\exp[i\omega\sigma_0(z+l)/c] \quad (10)$$

$$\xi_\mp = (1+k_c)\left(\mu_0 + \frac{\sigma_0 c}{\varepsilon_0 v_0}\right)\exp[i\omega\sigma_0z/c] + (1-k_c)\left(\mu_0 - \frac{\sigma_0 c}{\varepsilon_0 v_0}\right)\exp[-i\omega\sigma_0z/c]$$

and, at last, $\phi_\pm = -\exp(i\omega\sigma_0)$ is an inessential phase factor. In the particular case of $v_\pm = v_\pm(t) = \text{const}$ and $\varepsilon_\pm = \mu_\pm = 1$, the well known [8-10] expression for CR and transition radiation from a particle leaving the plate for vacuum is obtained from (9).

4. Numerical results and their analysis

Below we shall confine ourselves to the consideration of the simplest case when the force

$$F = \frac{d}{dt} \frac{mv_0}{\sqrt{1-v_0^2/c^2}} \quad (11)$$

with which the plate material slows down the particle motion is constant: $F(t) = \text{const}$. In this case [11] the velocity $v_0$ and coordinate $z = z_0(t)$ of the particle are determined from equalities

$$v_0 = \sqrt{1-1/\gamma^2}, \quad \gamma(z) = \gamma_\pm + (z+l)(\gamma_+ - \gamma_-)/l \quad (12)$$

and the corresponding instant of time $t = t_0(z)$ is described by the function

$$t_0(z) = t_0 + \left(\sqrt{\gamma^2 - 1} - \sqrt{\gamma^2 - 1}/l\right)/c(\gamma_+ - \gamma_-). \quad (13)$$

We shall also assume that (a) the plate is made of molten quartz with $\varepsilon_0 = 3.78 \cdot (1+0.0001\cdot i)$ and $\mu_0 = 1$ \quad (14)

in the frequency interval $\sim 10^{10}$ GHz, (b) the Lorentz factor $\gamma$ of particle (electron) at incidence on the plate is $\gamma_\pm = 2$ and that (c) the thickness of plate is such that at an escape from plate the Lorentz factor of particle is reduced (owing to the slowing down) to the value of $\gamma_+ = 1.5$. As for such a choice of the values of parameters inside the plate the Cherenkov condition is observed

$$c/v_0(t)/\sqrt{\varepsilon_0} < 1 \quad (15)$$

and the Cherenkov radiation is generated during all the time period of particle flight inside the plate.

The spectral-angular distribution of the energy radiated in the forward direction by electron ($\alpha = +$) as shown in figure 2 is added up of CR and transition radiation generated at an incidence on and escape of particle from the plate.

For simplification of analysis we additionally show in figure 3 the spectral (at the left) and the angular (right) distributions of $I_\omega(\alpha, \theta)$ function. The oscillations in the radiation spectrum (in figure 3, at the left) are due to the superposition and interference of electromagnetic oscillations generated by the particle at the flight through the plate of finite thickness. The first maximum is at $\omega_{\text{max}}l/c = 5$.

So, with known value of $\omega_{\text{max}}$ one can determine the plate thickness $l$ from this equality.

If the particle were not slowed down in the plate, its energy would stay constant: $\gamma = \gamma(z) = \text{const}$.

In figure 3 the upper curve $b$ on the right corresponds to this case. As is seen from the comparison of $a$ and $b$ curves in figure 3, the slowing down of particle notably reduces the radiation energy.
Spectral-angular distribution of the energy of total radiation in the forward direction $I_{\omega}(\omega, \theta)$ at the flight of relativistic electron through the plate (molten quartz). In calculations, the allowance for slowing down of particle inside the plate is made under the assumption that the force, with which the plate material slows down the particle motion is constant. The electron energy at the incidence is $E = 2mc^2$ and is $E = 1.5mc^2$ at the escape from the plate.

**Figure 2.** Spectral-angular distribution of the energy of total radiation in the forward direction $I_{\omega}(\omega, \theta)$ at the flight of relativistic electron through the plate (molten quartz). In calculations, the allowance for slowing down of particle inside the plate is made under the assumption that the force, with which the plate material slows down the particle motion is constant. The electron energy at the incidence is $E = 2mc^2$ and is $E = 1.5mc^2$ at the escape from the plate.

**Figure 3.** Leftwards is the spectral distribution of the same radiation as that in figure 2 in $\theta = 50^\circ$ direction. On the right is the angular distribution of the same radiation as that in figure 2 at the intended frequency $\omega l/c = 5$ (curve a). Curve b (on the right) is the angular distribution of $I_{\omega}(\omega, \theta)$ function for $\omega l/c = 5$ and uniform motion of the electron inside the plate with $\gamma = \gamma(z) = \text{const}$.

Based on an analysis of radiation spectrum some effective value $E_{\text{eff}}$ of particle energy $E = \gamma mc^2$ is determined, such that the spectrum of CR for uniform motion with this energy would best approximate the “actual” radiation spectrum (in which the allowance for slowing down of particle is made). The results of our numerical calculations ascertain that this effective value of particle energy is determined by means of the following equality:

$$E_{\text{eff}} \equiv (E_{\gamma} + E_{\gamma})/2.$$  \hfill (16)

The energy losses of particle $E_{\gamma} - E_{\gamma}$ due to its slowing in plate may be determined from (16) using the value of $E_{\text{eff}}$ and, e.g., $E_{\gamma}$. As a result, the force $F$ slowing down the particle motion inside the plate, is also determined.

Formula

$$\Delta \theta_{\parallel} = \arccos(c / v_{\parallel}\sqrt{\gamma}) - \arccos(c / v_{\parallel}\sqrt{\gamma})$$  \hfill (17)

allows to estimate the portion of the width of angular distribution of CR, which is due to the slowing down of particle in the plate. According to [7] formula (17) describes good enough the width of the angular distribution of CR (with wavelength $\lambda_{\omega}$) in the case when the length of the particle trajectory...
\( l \gg \lambda_m \) \((\lambda_m = 2\pi c / (\omega_0 \sqrt{\varepsilon_0})\) is the wavelength of CR in an unlimited transparent medium with permittivity \( \varepsilon_0 \). As seen from the data shown on the right-hand side in figure 3 the half-width \( \Delta \theta \approx 50^\circ \) of the angular distribution of CR is greater than the value \( \Delta \theta \approx 7.2^\circ \) calculated by formula (17). This is understandable, since plotted in the right-hand side in figure 3 is the case \( l \approx \lambda_m \). In this case, CR is only formed when the particle is flying out of the plate so one cannot neglect the contribution of transition radiation.

It is worthwhile to add that CR from a relativistic particle travelling a limited trajectory of length \( l \) in a gas has been investigated theoretically and experimentally in [12,13]. In these papers the alternative criterion \( v_0 (\varepsilon_0^{1/2} - \lambda_m / l) > c \) for CR generation has been proposed (cf. with (15)). We rewrite this criterion as \( l > \lambda_m / (\varepsilon_0^{1/2} - c / v_0) \). For the right-hand side case in figure 3 it leads to \( l > 1.4\lambda_m \) where \( \lambda_m = 2\pi c / (\omega_{\text{max}} \sqrt{\varepsilon_0}) \). This inequality indicates directly that CR just begins to form when the particle left the plate of \( l \approx \lambda_m \) length.

5. Conclusions

The electromagnetic field of a charged particle in flight through a plate along the normal to its surface in straight-line motion at varying velocity with no restrictions on the variation law of this velocity (save the fact that its projection on the direction of motion should be a constant-sign function of time) has been investigated based on appropriate exact solutions of the Maxwell equations. The obtained analytical expressions are essentially simpler than those to be derived from earlier reported general formulae (e.g., [3]). The influence of irregularities in particle motion on CR generated by this particle inside the plate was not dealt with in [3]. This deficiency is filled up in the present work.

It was shown that the variation of particle velocity inside the plate (e.g., due to the slowing down of particle) may essentially influence the spectral-angular distribution of Cherenkov radiation of particle. This fact may be used for practical purposes, particularly for simultaneous measurement of three quantities, - the force that slows down the particle motion in plate, the thickness of the plate and the refractive index of the plate material.

In this paper we do not take into account the effects due to the multiple scattering of charged particle and ignore fluctuations of energy losses in plate substance. Numerical calculations were carried out on the assumption that the particle decelerating force \( F \) is not changed when the particle moves in the plate. In reality \( F \) depends on the velocity of particle and therefore it should vary during the slowing down. This fact is taken into consideration, for example, in [7]. A detailed study of this problem for a particle traversing through a plate requires a separate consideration.

Acknowledgments

The authors are thankful to the anonymous reviewer, the valuable comments of which helped to improve the statement of problem.

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