Dark photon kinetic mixing effects for CDF W mass excess

Yu Cheng\textsuperscript{1,*}, Xiao-Gang He\textsuperscript{1,2†}, Fei Huang\textsuperscript{1‡}, Jin Sun\textsuperscript{1§}, and Zhi-Peng Xing\textsuperscript{1¶}

\textsuperscript{1}Tsung-Dao Lee Institute, and School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China and

\textsuperscript{2}National Center for Theoretical Sciences, and Department of Physics, National Taiwan University, Taipei 10617, Taiwan

(Dated: April 27, 2022)

Abstract

A new $U(1)_X$ gauge boson $X$ primarily interacting with a dark sector can have renormalizable kinetic mixing with the standard model (SM) $U(1)_Y$ gauge boson $Y$. This mixing besides introduces interactions of dark photon and dark sector with SM particles, it also modifies interactions among SM particles. The modified interactions can be casted into the oblique $S$, $T$ and $U$ parameters. We find that with the dark photon mass larger than the $Z$ boson mass, the kinetic mixing effects can reduce the tension of the W mass excess problem reported recently by CDF from 7$\sigma$ deviation to within 3$\sigma$ compared with theory prediction. If there is non-abelian kinetic mixing between $U(1)_X$ and $SU(2)_L$ gauge bosons, in simple renormalizable models of this type a triplet Higgs is required to generate the mixing. We find that this triplet with a vacuum expectation value of order 5 GeV can naturally explain the W mass excess.

\textsuperscript{*} chengyu@sjtu.edu.cn
\textsuperscript{†} hexg@sjtu.edu.cn
\textsuperscript{‡} fhuang@sjtu.edu.cn
\textsuperscript{§} 019072910096@sjtu.edu.cn
\textsuperscript{¶} zpxing@sjtu.edu.cn
I. INTRODUCTION

Recently CDF collaboration announced their new measurement of W boson mass with a value of $80,433.5 \pm 9.4$ MeV which is $7\sigma$ above the standard model (SM) prediction of $80,357 \pm 6$ MeV. This is a significant indication of new physics beyond the SM. A lot of efforts have been made to provide an explanation for this excess. Needless to say that better understanding of SM calculations, and also further experimental measurements are needed, nevertheless a lot of new ideas beyond the SM have merged to explain the W mass excess [3–49]. In this work we study effects of a class of well motivated dark photon models on the W mass.

A dark photon $X_\mu$ from a $U(1)_X$ gauge group primarily coupling to a dark sector can have kinetic mixing with the SM gauge boson. The kinetic mixing besides introduces interactions of dark photon and dark sector with SM particles, it also modifies interactions among SM particles which can be tested to high precision data obtained by various experiments. It has long been realized that a dark photon $X_\mu$ can mix with the $U(1)_Y$ gauge boson $Y_\mu$ in the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ through a renormalizable kinetic mixing term [50–53], $X^{\mu\nu}Y_{\mu\nu}$. Here $A^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. The phenomenological implications of this simple kinetic mixing have been studied extensively [54–58]. The kinetic mixing of the dark photon with the non-abelian gauge boson $W^a_\mu$, which transforms under the $SU(2)_L$ as a triplet represented by the superscript index “$a$”, has also been studied [59–64]. It turns out that this requires additional efforts because the simple naive kinetic mixing $X^{\mu\nu}W^a_{\mu\nu}$ term is not gauged invariant. One needs to introduce a scalar type of entity transforming also as a triplet to make the relevant term gauge invariant. The simplest one of such a entity is a scalar triplet $\Sigma^a$ transforming as $(1, 3, 0)$ under the $SU(3)_C \times SU(2)_L \times U(1)_Y$, with a non-zero vacuum expectation value (vev) $< \Sigma^0 > = v_\Sigma$. Renormalizable models have been constructed recently [63]. This type of model has some new interesting features, in particular CP violating kinetic mixing can also exist with testable consequences.

Both types of models mentioned above will modify the interactions of the SM particles and therefore produce deviations from the SM predictions which can be tested by experimental data. We find that the modified interactions can be casted into the oblique parameters $S$, $T$ and $U$ within the allowed parameter space, the kinetic mixing effects can help to explain the W mass excess of the recent CDF measurement. In the case of non-abelian kinetic mixing between $U(1)_X$ and $SU(2)_L$ gauge bosons, there are additional contributions to the W mass excess besides the kinetic mixing effects due to the vev of triplet Higgs required to generate the kinetic mixing. The triplet with a vev of order 5 GeV can naturally explain the W mass excess. We provide some details in the following.

II. S, T, U PARAMETERS IN ABELIAN KINETIC MIXING MODELS

With the kinetic mixing for the case of $U(1)_X \times U(1)_Y$, the kinetic terms of the bare fields $\tilde{X}$ and $\tilde{Y}$ and their interactions with other particles can be written as

$$L = -\frac{1}{4} \tilde{X}^{\mu\nu} \tilde{X}_{\mu\nu} - \frac{\sigma}{2} \tilde{X}^{\mu\nu} \tilde{Y}_{\mu\nu} - \frac{1}{4} \tilde{Y}^{\mu\nu} \tilde{Y}_{\mu\nu} + j^X_\mu \tilde{Y}_\mu + j^Y_\mu \tilde{X}_\mu .$$  \hspace{1cm} (1)
Here $j_X^\mu$ and $j_Y^\mu$ denote interaction currents of gauge fields $X$ and $Y$, respectively. The parameter $\sigma$ indicates the strength of the kinetic mixing.

After electroweak symmetry breaking, $\hat{Y}$ and the neutral component of the $SU(2)_L$ gauge field $\hat{W}^3$ can be written in the combinations of the ordinary SM photon field $\hat{A}$ and the $Z$ boson field $\hat{Z}$ as follows

$$
\hat{Y}_\mu = \hat{c}_W \hat{A}_\mu - \hat{s}_W \hat{Z}_\mu , \quad \hat{W}^3_\mu = \hat{s}_W \hat{A}_\mu + \hat{c}_W \hat{Z}_\mu ,$$

(2)

where $\hat{c}_W \equiv \cos \theta_W$ and $\hat{s}_W \equiv \sin \theta_W$ with $\theta_W$ being the weak mixing angle. Meanwhile, the $\hat{Z}$ field receives a mass $m_Z$.

The general Lagrangian that describes $\hat{A}$, $\hat{Z}$ and $\hat{X}$ fields kinetic energy, and their interactions with the electromagnetic current $j_{em}^\mu$, neutral $Z$-boson current $j_Z^\mu$ and dark current $j_X^\mu$ is given by [57].

$$
\mathcal{L} = -\frac{1}{4} \hat{X}_{\mu\nu} \hat{X}^{\mu\nu} - \frac{1}{4} \hat{A}_{\mu\nu} \hat{A}^{\mu\nu} - \frac{1}{4} \hat{Z}_{\mu\nu} \hat{Z}^{\mu\nu} - \frac{1}{2} \sigma \hat{c}_W \hat{X}_{\mu\nu} \hat{A}^{\mu\nu} + \frac{1}{2} \sigma \hat{s}_W \hat{X}_{\mu\nu} \hat{Z}^{\mu\nu} + j_{em}^\mu \hat{A}_\mu + j_Z^\mu \hat{Z}_\mu + j_X^\mu \hat{X}_\mu + \frac{1}{2} m_Z^2 \hat{Z}_\mu \hat{Z}^\mu ,
$$

(3)

where the $Z$ boson mass term is included. Here the currents for fermions with charge $Q_f$ and weak isospin $I_3^f$ in the SM are given by

$$
j_{em}^\mu = - \sum_f e Q_f \tilde{f} \gamma^\mu f , \quad j_Z^\mu = \frac{\tilde{e}}{2 \hat{s}_W \hat{c}_W} \tilde{f} \sigma (g_\nu + g_A \gamma_5) f ,
$$

$$
g_\nu^f = I_3^f - 2 Q_f \hat{s}_W^2 , \quad g_A^f = I_3^f .
$$

(4)

Note that the $W$ boson field and its interactions are not affected directly.

The dark photon may be also massive. There are two popular ways of generating dark photon mass which give rise to different phenomenology. One of them is the “Higgs mechanism”, in which the $U(1)_X$ is broken by the vev of an SM singlet $S$, which is charged under $U(1)_X$. In this case, the mixing of Higgs doublet and the Higgs singlet offers the possibility of searching for dark photon at colliders in Higgs decays [54, 56, 57]. The other one is the “Stueckelberg mechanism” [58] in which an axionic scalar was introduced to allow a mass for $\hat{X}$ without breaking $U(1)_X$. In our later discussion our concern is that the dark photon is massive regardless where it comes from. We need to include a mass term $(1/2) m_X^2 \hat{X}_\mu \hat{X}^\mu$ in our discussions.

One can rewrite the Lagrangian to remove the kinetic mixing terms so that the gauge fields kinetic energy terms are in the canonical form. This way to do this is not unique as discussed in Ref. [57]. We choose to work with redefining the gauge fields such that photon has no interaction with $j_X^\mu$. In this case, one redefines the fields as the following

$$
\begin{pmatrix}
\hat{A} \\
\hat{Z} \\
\hat{X}
\end{pmatrix} =
\begin{pmatrix}
1 & \frac{-\sigma \hat{c}_W}{\sqrt{1-\sigma^2 \hat{c}_W^2}} & \frac{-\sigma \hat{c}_W}{\sqrt{1-\sigma^2 \hat{c}_W^2}} \\
0 & \frac{\hat{c}_W}{\sqrt{1-\sigma^2 \hat{c}_W^2}} & \frac{\hat{c}_W}{\sqrt{1-\sigma^2 \hat{c}_W^2}} \\
0 & \frac{\sqrt{1-\sigma^2 \hat{c}_W^2}}{\sqrt{1-\sigma^2 \hat{c}_W^2}} & \frac{1}{\sqrt{1-\sigma^2 \hat{c}_W^2}}
\end{pmatrix}
\begin{pmatrix}
\hat{A}' \\
\hat{Z}' \\
\hat{X}'
\end{pmatrix},
$$

(5)
to obtain the Lagrangian,

\[
\mathcal{L} = -\frac{1}{4} \tilde{X}^{\mu\nu} \tilde{X}_{\mu\nu} - \frac{1}{4} \tilde{A}^{\prime\mu} \tilde{A}^{\prime\mu} - \frac{1}{4} \tilde{Z}_{\mu} \tilde{Z}^\mu
\]

\[
+ j^\mu_{\mu m} \left( \tilde{A}^{\prime\mu} - \frac{\sigma^2 c_W c_W}{\sqrt{1 - \sigma^2}} \tilde{Z}^\mu - \frac{\sigma c_W}{\sqrt{1 - \sigma^2}} \tilde{X}^{\prime\mu} \right)
\]

\[
+ j^\mu_{\mu Z} \left( \frac{\sqrt{1 - \sigma^2 c_W^2} \tilde{Z}^{\prime\mu}}{\sqrt{1 - \sigma^2}} \right) + j^\mu_X \left( \frac{\sigma s_W}{\sqrt{1 - \sigma^2}} \tilde{Z}^{\prime\mu} + \frac{1}{\sqrt{1 - \sigma^2}} \tilde{X}^{\prime\mu} \right)
\]

\[
+ \frac{1}{2} m^2_Z \tilde{Z}_\mu \tilde{Z}^{\mu} + \frac{1}{2} m^2_X \left( \frac{\sigma s_W}{\sqrt{1 - \sigma^2}} \tilde{Z}^{\prime\mu} + \frac{1}{\sqrt{1 - \sigma^2}} \tilde{X}^{\prime\mu} \right)^2.
\]

We see that the field \( \tilde{A}^{\prime} \) is already the physical massless photon field \( A \) without the need of further mass diagonalization. However, the \( \tilde{Z}^{\prime} \) and \( \tilde{X}^{\prime} \) are mixed states. One needs to diagonalize the mass matrix in \( (\tilde{Z}^{\prime}, \tilde{X}^{\prime}) \) basis,

\[
\begin{pmatrix}
\frac{m^2_Z(1 - \sigma^2 c_W^2)^2 + m_X^4 \sigma^2 s_W^2}{(1 - \sigma^2)(1 - \sigma^2 c_W^2)} & \frac{m_X^2 \sigma s_W}{\sqrt{1 - \sigma^2}} \\
\frac{m_X^2 \sigma s_W}{\sqrt{1 - \sigma^2}} & \frac{m_X^2}{1 - \sigma^2 c_W^2}
\end{pmatrix}.
\]

To obtain the diagonalized fields \( Z \) and \( X \), we introduce the mixing angle as

\[
\begin{pmatrix}
Z \\
X
\end{pmatrix} = \begin{pmatrix}
c_\theta & s_\theta \\
-s_\theta & c_\theta
\end{pmatrix} \begin{pmatrix}
\tilde{Z}^{\prime} \\
\tilde{X}^{\prime}
\end{pmatrix},
\]

with \( c_\theta = \cos \theta, s_\theta = \sin \theta \), and

\[
\tan(2\theta) = \frac{2m^2_X \sigma s_W \sqrt{1 - \sigma^2}}{m^2_Z(1 - \sigma^2 c_W^2)^2 - m^2_X [1 - \sigma^2(1 + s_W^2)]}.
\]

The diagonal masses \( m^2_Z = m^2_Z(1 + \tilde{z}) \) and \( m^2_X \) corresponding to \( Z \) and \( X \) are given, respectively, by

\[
\tilde{m}^2_Z = \frac{m^2_Z(1 - \sigma^2 c_W^2)^2 + m_X^2 \sigma^2 s_W^2 c^2_\theta}{(1 - \sigma^2)(1 - \sigma^2 c_W^2)} + \frac{m^2_X}{1 - \sigma^2 c_W^2} s^2_\theta + 2s_\theta c_\theta \frac{m^2_X \sigma s_W}{\sqrt{1 - \sigma^2(1 - \sigma^2 c_W^2)}}
\]

\[
\tilde{m}^2_X = \frac{m^2_Z(1 - \sigma^2 c_W^2)^2 + m_X^2 \sigma^2 s_W^2}{(1 - \sigma^2)(1 - \sigma^2 c_W^2)} s^2_\theta + \frac{m^2_X}{1 - \sigma^2 c_W^2} c^2_\theta - 2s_\theta c_\theta \frac{m^2_X \sigma s_W}{\sqrt{1 - \sigma^2(1 - \sigma^2 c_W^2)}}.
\]
The resulting Lagrangian is given by

\[
\mathcal{L} = -\frac{1}{4} A_{\mu \nu} A^{\mu \nu} - \frac{1}{4} Z_{\mu \nu} Z^{\mu \nu} + \frac{1}{2} \bar{m}_Z^2 Z^\mu Z_\mu + j^\mu_{em} A_\mu - j^\mu_{em} \left( \frac{\sigma^2 \bar{s}_W \bar{c}_W}{\sqrt{1 - \sigma^2}} c_\theta + \frac{\sigma \bar{c}_W}{\sqrt{1 - \sigma^2}} s_\theta \right) Z_\mu + j^\mu_Z \sqrt{1 - \sigma^2} \frac{\bar{c}_W}{\sqrt{1 - \sigma^2}} c_\theta Z_\mu
\]

\[
+ j^\mu_X \left( \frac{\sigma \bar{s}_W}{\sqrt{1 - \sigma^2}} c_\theta + \frac{1}{\sqrt{1 - \sigma^2}} s_\theta \right) Z_\mu - \frac{1}{4} X_{\mu \nu} X^{\mu \nu} + \frac{1}{2} \bar{m}_X^2 X_\mu X^\mu + j^\mu X \left( -\frac{\sigma \bar{s}_W}{\sqrt{1 - \sigma^2}} s_\theta + \frac{1}{\sqrt{1 - \sigma^2}} c_\theta \right) X_\mu.
\]

\[
+ j^\mu_{em} \left( \frac{\sigma^2 \bar{s}_W \bar{c}_W}{\sqrt{1 - \sigma^2}} s_\theta - \frac{\sigma \bar{c}_W}{\sqrt{1 - \sigma^2}} c_\theta \right) X_\mu - j^\mu_Z \sqrt{1 - \sigma^2} \frac{1 - \sigma^2 \bar{c}_W}{\sqrt{1 - \sigma^2}} s_\theta X_\mu.
\]

(11)

If one just considers dark photon kinetic mixing effects on SM particles, the relevant terms are the first two lines in the above Lagrangian. The rest terms involving dark sectors will not be directly related. To compare with precision experimental data and address the W mass excess, we now recast the dark photon effects in terms of oblique parameters. We find that the oblique \( S, T \) and \( U \) parameters will be generated. The derivation for the oblique parameters can be arrived at by first writing the modifications to the SM Lagrangian in the following way \[65\],

\[
L = \frac{1}{2} (1 + z - C) m_Z^2 Z^\mu Z_\mu + (1 + w - z) m_W^2 W^{\mu \dagger} W_\mu
\]

\[
+ \left( 1 - \frac{A}{2} \right) j^\mu_{em} A_\mu + \left( 1 - \frac{C}{2} \right) (j^\mu_Z + G j^\mu_{em}) Z_\mu + \left( 1 - \frac{B}{2} \right) j^\mu_W W^{\mu \dagger} + h.c. \) .\]

(12)

where \( j^\mu_W = -\bar{e} / \sqrt{2} \bar{s}_W \bar{e}^\gamma \gamma LV_{KM} f^d \). Normalizing the fields and charges to the physical ones, one obtains the relations

\[
\alpha S = 4 \pi c_w^2 (A - C) - 4 s_w c_w (c_w^2 - s_w^2) G, \quad \alpha T = w - z, \quad \alpha U = 4 \pi c_w^2 (s_w^2 A - B + c_w^2 C - 2 s_w c_w G).
\]

(13)

To the leading order, \( \bar{s}_w \) and \( \bar{c}_w \) can be replaced by \( s_w \) and \( c_w \) in the above \( C \) and \( G \).

In our case, since there are no modifications to the \( W^\pm_\mu \), whose coupling and bare mass, \( B \) and \( w \) are both zero. Also we see from Eq. (11) that there exists no modification for photon interaction, therefore \( A = 0 \). We obtain

\[
C = 2 \left( 1 - \frac{\sqrt{1 - \sigma^2} \bar{c}_W}{\sqrt{1 - \sigma^2} \bar{c}_W} \right), \quad G = \frac{\sigma^2 \bar{s}_W \bar{c}_W}{1 - \sigma^2 \bar{c}_W} - \frac{\sigma \bar{c}_W \sqrt{1 - \sigma^2} s_\theta}{1 - \sigma^2 \bar{c}_W} c_\theta, \quad z = C + C. \]

(14)
We obtain oblique parameters to the first order in $\sigma^2$ as

$$aS = \frac{4s_W^2c_W^2\sigma^2}{1 - m_X^2/m_Z^2} \left( 1 - \frac{s_W^2}{1 - m_X^2/m_Z^2} \right),$$

$$aT = -\sigma^2s_W^2 \frac{m_X^2/m_Z^2}{(1 - m_X^2/m_Z^2)^2},$$

$$aU = 4s_W^4c_W^2\sigma^2 \left( -\frac{1 - 2m_X^2/m_Z^2}{(1 - m_X^2/m_Z^2)^2} + \frac{2}{1 - m_X^2/m_Z^2} \right). \quad (15)$$

The above leads to correction to the W mass as

$$\Delta m_W^2 = m_Z^2c_W^2 \left( -\frac{aS}{2(c_W^2 - s_W^2)} + \frac{c_W^2aT}{(c_W^2 - s_W^2)} + \frac{aU}{4s_W^2} \right)$$

$$= -m_Z^2c_W^2 \frac{m_Z^2(1 - s_W^2)\sigma^2s_W^2}{(m_X^2 - m_Z^2)(-1 + 2s_W^2)}. \quad (16)$$

### III. $S$, $T$, $U$ PARAMETERS IN NON-ABELIAN KINETIC MIXING MODELS

We now discuss how the W mass is modified in a class of non-abelian kinetic mixing models. This is the class of models in which kinetic mixing between the $U(1)_X$ gauge boson $\tilde{X}_\mu$ and $SU(2)_L$ gauge boson $W_\mu$ can be induced. Here $W_\mu$ transforms as a $SU(2)_L$ triplet. To realize such kinetic mixing, the group index “$a$” needs to be balanced which can be achieved easily by introducing a scalar triplet $\Sigma^a$. With the help of $\Sigma^a$ the kinetic mixing terms of the following forms can be gauge invariant

$$\tilde{X}^{\mu\nu} \tilde{W}^a_{\mu\nu} \Sigma^a, \quad \epsilon^{\mu\nu\alpha\beta} \tilde{X}_{\mu\nu} \tilde{W}^a_{\alpha\beta} \Sigma^a. \quad (17)$$

The component fields of $W^a$ and $\Sigma^a$ are given by

$$\sigma^a \tilde{W}^a_\mu = \left( \tilde{W}^a_\mu, \sqrt{2}\tilde{W}^a_\mu W^3, -\tilde{W}^a_\mu W^3 \right), \quad \sigma^a \Sigma^a = \left( \Sigma^0, \sqrt{2}\Sigma^+, -\Sigma^0 \right). \quad (18)$$

Here $\sigma^a$ are the Pauli matrices. When the $\Sigma^a$ neutral component ($\Sigma^0$) develops a non-zero vev $<\Sigma^0> = v_\Sigma$, the kinetic mixing in the usual form from the first term, $\sqrt{2}\tilde{X}^{\mu\nu}\tilde{W}^3_{\mu\nu}v_\Sigma$, and a new form from the second term, $\sqrt{2}\epsilon^{\mu\nu\alpha\beta} \tilde{X}_{\mu\nu} \tilde{W}^3_{\alpha\beta}v_\Sigma$ will be induced.

Note that had one replaces $W^3_{\alpha\beta}$ by $\tilde{Y}_{\alpha\beta}$, then $\epsilon^{\mu\nu\alpha\beta} \tilde{X}_{\mu\nu} \tilde{Y}_{\alpha\beta} = 2\partial^\mu(\tilde{X}^{\nu}\tilde{Y}_{\mu})$ which would have no perturbative effects due to $\tilde{W}^3_{\mu\nu} = s_W\tilde{A}_{\mu\nu} + c_W\tilde{Z}^{\mu\nu} + ig(W^-W^+ - W^+W^-)$. The term $\epsilon^{\mu\nu\alpha\beta} \tilde{X}_{\mu\nu} \tilde{W}^3_{\alpha\beta}$ cannot be written just as a total derivative one, but also a new term $2ige^{\mu\nu\alpha\beta} \tilde{X}_{\mu\nu} (W^-W^+) + \text{terms}$ then has physical effects. Some interesting implications have been studied in Ref. [63, 64].

The operators in Eq. (17) are dimension 5 ones which is nonrenormalizable. If one insists on renormalizability of the model, additional ingredients need to be introduced to generate them at the loop level. A specific renormalizable model has been constructed recently [64]. However, the kinetic mixing parameters generated are too small [64] to make a significant impact on W mass, and can be neglected. But in this class of models, there are still two
FIG. 1: (a) The CDF allowed regions in $m_X - |\sigma|$ plane. The allowed parameter space is shown in black line for central value, the 1$\sigma$, 2$\sigma$ and 3$\sigma$ ranges are also shown. (b) The $S$, $T$ and $U$ parameters as functions of $|\sigma|$ for $m_X$ in the range of 200 − 300 GeV. The size of observables decrease when $m_X$ increases.

contributions which can affect the W mass significantly. One is the possible mixing term $-(1/2)\sigma X^{\mu\nu} \tilde{Y}_{\mu\nu}$ discussed earlier which generates the $S$, $T$, and $U$ parameters given in Eq. (15). Another one is the non-zero $v_\Sigma$ of the triplet $\Sigma_a$ generated modification to the electroweak precision parameter $\rho$. We have \cite{2, 18}

\[ \rho = 1 + \frac{4v_\Sigma^2}{v^2} = 1 + \alpha T_\Sigma, \] (19)

where $v = 246$ GeV is the SM Higgs vev.

The term $T_\Sigma = 4v_\Sigma^2/\alpha v^2$ is an addition to the $T$ parameter which needs to be considered in this class of model. Therefore for this class of models Eq.(16) is modified to

\[
\Delta m_W^2 = m_Z c_W^2 \left( \frac{-\alpha S}{2(c_W^2 - s_W^2)} + \frac{c_W^2 \alpha T}{(c_W^2 - s_W^2)} + \frac{\alpha U}{4s_W^2} \right) \\
= -m_Z c_W^2 \frac{m_Z^2 (1 - s_W^2) \sigma^2 s_W^2}{(m_X^2 - m_Z^2)^2 (-1 + 2s_W^2)} + m_Z^2 c_W^2 \frac{c_W^2}{c_W^2 - s_W^2} \frac{4v_\Sigma^2}{v^2}. \] (20)

IV. NUMERICAL ANALYSIS AND CONCLUSIONS

We are now ready to put things together to analyze whether the dark photon models can accommodate the W mass excess indicated by the recent CDF result. The CDF result is 7$\sigma$ above the SM prediction, which implies that the new contributions must have $\Delta m_W^2 > 0$, so that $\Delta m_W^{CDF} = \sqrt{(m_W^{SM})^2 \Delta m_W^2} - m_W^{SM} \approx (\Delta m_W^2/(m_W^{SM})^2/2)$ to produce the required 70 MeV becomes possible.

In the case of abelian kinetic mixing, we see from Eq. (16) that in order to have $\Delta m_W^2 > 0$, the dark photon mass $m_X$ must be larger than $m_Z$. We therefore confine our analysis in this range for $m_X$. When $m_X$ becomes larger, a larger $|\sigma|$ is needed. We plot the allowed ranges
in Fig. 1a for $m_X$ and $\sigma$ within the central value, 1$\sigma$, 2$\sigma$ and 3$\sigma$ boundaries, respectively. We see that there are ranges in the $m_X - |\sigma|$ plane which can solve the W mass excess problem. CMS has searched [66] for dark photon in the range of $m_X$ below about 200 GeV decaying to $\mu^+\mu^-$ final states and gives a stringent constraint on $\sigma < 10^{-3} - 10^{-2}$. With this $\sigma$ limit, we see that to address the W mass excess problem, $m_X$ needs to be larger than 200 GeV and also $\sigma$ needs to be larger than 0.2. Since our expansion parameter is $\sigma^2$, we consider $\sigma$ in the range of 0.2 to 0.3 is a reasonable range. With improved high energy search similar to what carried out by CMS, the model can be more stringently constrained.

Global fit of electroweak precision data, has given constraint on $S$, $T$, and $U$ separately. Recently, the results of EW global fit with CDF W mass obtain the oblique parameters: $S$, $T$, and $U$. We take the fit values from Ref. [3] for comparison: $S = 0.06 \pm 0.1$, $T = 0.11 \pm 0.12$, $U = 0.14 \pm 0.09$. As shown in Fig. 1b, it turns out that although we can obtain the CDF measured W mass, but it is not possible to satisfy the bounds on the $S$, $T$ and $U$ parameters within 2$\sigma$ allowed ranges. But within 3$\sigma$ allowed ranges, the abelian kinetic mixing effect can accommodate the CDF W mass measurement.

In the non-abelian kinetic mixing case, with the help of $v_\Sigma$ in the range of a few GeV, the model can easily accommodate the CDF W mass excess with very small $\sigma$. We now discuss how a non-zero $v_\Sigma$ affects the model parameters. In this case, from Fig. 2a we see that a $v_\Sigma$ in the range of a few GeV can help solve the CDF W mass excess problem even with a very small kinetic mixing $\sigma$. The expressions for $S$ and $U$ are not changed compared with abelian kinetic mixing case, but the total $T$ needs to add an additional $T_\Sigma$. This result in changing the relative size of the parameters for a given $m_W$. Without $T_\Sigma$, $m_X$ cannot be too much larger than the CMS lower bound of 200 GeV, and $\sigma$ cannot be much smaller than 0.2 or so. When one includes $T_\Sigma$ in the analysis, a much larger $m_X$ and also a smaller $\sigma$ can be allowed if the model is required to solve the W mass excess. In this case, the absolute values of $S$ and $U$ can be made small to satisfy the global fit allowed ranges. This is shown in Fig. 2b.
Before summary, we would like to comment about a possible consequence of the Z boson couples to dark sector. If the dark sector particles are enough light, Z can decay into them to enhance the invisible width. As an example, we assume that there is a vector-like fermion $j^\mu_X = \tilde{g}\tilde{f}\gamma^\mu f$ coupling to the original $X^\mu$. After normalizing the couplings and fields, we have

$$L_{int} = \tilde{g}\left(\frac{\sigma s_W}{\sqrt{1 - \sigma^2}}\frac{s_W}{\sqrt{1 - \sigma^2 c_W^2}} + \frac{1}{\sqrt{1 - \sigma^2 c_W^2}} s_W\right)\tilde{f}\gamma^\mu f Z^\mu \approx \tilde{g}\frac{\sigma s_W m_Z^2}{m_Z^2 - m_X^2} \tilde{f}\gamma^\mu f Z^\mu. \quad (21)$$

This interaction gives a invisible decay width for $Z \to f\tilde{f}$

$$\Gamma = \frac{\tilde{g}^2}{12\pi} \frac{\sigma^2 s_W^2}{(1 - m_X^2/m_Z^2)^2} m_Z \sqrt{1 - \frac{4m_f^2}{m_Z^2}} \left(1 + \frac{2m_f^2}{m_Z^2}\right). \quad (22)$$

For the fermion with a very small mass, if fixing $m_X = 250$ GeV, and $\tilde{g} = g_Y = 0.356$, we obtain the branching ratio as $2.8 \times 10^{-5}(\sigma/0.2)^2(\tilde{g}/g_Y)^2$. Using the Z decay width in Ref. [2], one obtains $Br^{new}(Z \to \text{invisible}) = 2.3 \times 10^{-5}$. The Z invisible decay width agrees with SM prediction well. As long as $\sigma\tilde{g}$ is smaller than 0.65, one can safely satisfy the data.

To summarize, we have studied the recent CDF measurement of W mass on two classes of dark photon models, one is the abelian kinetic mixing case due to a dark photon abelian $U(1)_X$ and SM $U(1)_Y$ gauge boson mixing, and another one is the non-abelian kinetic mixing from a dark photon $U(1)_X$ and another non-abelian SM $SU(2)_L$ gauge boson mixing. This mixing besides introduces interactions of dark photon and dark sector with SM particles, it also modifies interactions among SM particles. We recast these modifications into the well know oblique $S$, $T$, and $U$ parameters. We find that with the dark photon mass larger than the Z boson mass, the kinetic mixing effects can reduce the tension of the W mass excess problem from 7σ to within 3σ compared with theory prediction. If there is non-abelian kinetic mixing between $U(1)_X$ and $SU(2)_L$ gauge bosons, in simple renormalizable models of this type a triplet Higgs is required to generate the mixing. We find that this triplet with a vacuum expectation value of order 5 GeV can naturally explain the W mass excess.

**Acknowledgments**

This work was supported in part by Key Laboratory for Particle Physics, Astrophysics and Cosmology, Ministry of Education, and Shanghai Key Laboratory for Particle Physics and Cosmology (Grant No. 15DZ2272100), and in part by the NSFC (Grant Nos. 11735010, 11975149, and 12090064). XGH was supported in part by the MOST (Grant No. MOST 106-2112-M-002-003-MY3). ZPX was supported by the NSFC (Nos. 12147147).

**Note added**

After our paper appeared, Holdom brought us the attention of Ref. [67] where the same $S$, $T$, $U$ parameters had been calculated. Our expressions of $S$, $T$, $U$ parameters agree with
each other.

[1] CDF Collaboration, T. Aaltonen et al., Science 376 no. 6589, (2022) 170–176.
[2] Particle Data Group Collaboration, P. A. Zyla et al., “Review of Particle Physics,” PTEP 2020 no. 8, (2020) 083C01.
[3] C. T. Lu, L. Wu, Y. Wu and B. Zhu, [arXiv:2204.03796 [hep-ph]].
[4] P. Athron, A. Fowlie, C. T. Lu, L. Wu, Y. Wu and B. Zhu, [arXiv:2204.03996 [hep-ph]].
[5] G. W. Yuan, L. Zu, L. Feng and Y. F. Cai, [arXiv:2204.04183 [hep-ph]].
[6] A. Strumia, [arXiv:2204.04191 [hep-ph]].
[7] J. M. Yang and Y. Zhang, [arXiv:2204.04202 [hep-ph]].
[8] J. de Blas, M. Pierini, L. Reina and L. Silvestrini, [arXiv:2204.04204 [hep-ph]].
[9] X. K. Du, Z. Li, F. Wang and Y. K. Zhang, [arXiv:2204.04286 [hep-ph]].
[10] T. P. Tang, M. Abdughani, L. Feng, Y. L. S. Tsai and Y. Z. Fan, [arXiv:2204.04356 [hep-ph]].
[11] G. Cacciapaglia and F. Sannino, [arXiv:2204.04514 [hep-ph]].
[12] M. Blennow, P. Coloma, E. Fernández-Martínez and M. González-López, [arXiv:2204.04559 [hep-ph]].
[13] B. Y. Zhu, S. Li, J. G. Cheng, R. L. Li and Y. F. Liang, [arXiv:2204.04688 [astro-ph.HE]].
[14] K. Sakurai, F. Takahashi and W. Yin, [arXiv:2204.04770 [hep-ph]].
[15] J. Fan, L. Li, T. Liu and K. F. Lyu, [arXiv:2204.04805 [hep-ph]].
[16] X. Liu, S. Y. Guo, B. Zhu and Y. Li, [arXiv:2204.04834 [hep-ph]].
[17] H. M. Lee and K. Yamashita, [arXiv:2204.05024 [hep-ph]].
[18] Y. Cheng, X. G. He, Z. L. Huang and M. W. Li, [arXiv:2204.05031 [hep-ph]].
[19] H. Song, W. Su and M. Zhang, [arXiv:2204.05085 [hep-ph]].
[20] E. Bagnaschi, J. Ellis, M. Madigan, K. Mimasu, V. Sanz and T. You, [arXiv:2204.05260 [hep-ph]].
[21] A. Paul and M. Valli, [arXiv:2204.05267 [hep-ph]].
[22] H. Bahl, J. Braathen and G. Weiglein, [arXiv:2204.05269 [hep-ph]].
[23] P. Asadi, C. Cesarotti, K. Fraser, S. Homiller and A. Parikh, [arXiv:2204.05283 [hep-ph]].
[24] L. Di Luzio, R. Gröber and P. Paradisi, [arXiv:2204.05284 [hep-ph]].
[25] P. Athron, M. Bach, D. H. J. Jacob, W. Kotlarski, D. Stöckinger and A. Voigt, [arXiv:2204.05285 [hep-ph]].
[26] J. Gu, Z. Liu, T. Ma and J. Shu, [arXiv:2204.05296 [hep-ph]].
[27] J. J. Heckman, [arXiv:2204.05302 [hep-ph]].
[28] K. S. Babu, S. Jana and V. P. K., [arXiv:2204.05303 [hep-ph]].
[29] M. Endo and S. Mishima, [arXiv:2204.05965 [hep-ph]].
[30] T. Biekötter, S. Heinemeyer and G. Weiglein, [arXiv:2204.05975 [hep-ph]].
[31] R. Balkin, E. Madge, T. Menzo, G. Perez, Y. Soreq and J. Zupan, [arXiv:2204.05992 [hep-ph]].
[32] N. V. Krasnikov, [arXiv:2204.06327 [hep-ph]].
[33] X. F. Han, F. Wang, L. Wang, J. M. Yang and Y. Zhang, [arXiv:2204.06505 [hep-ph]].
[34] J. Kawamura, S. Okawa and Y. Omura, [arXiv:2204.07022 [hep-ph]].
[35] A. Ghoshal, N. Okada, S. Okada, D. Raut, Q. Shafi and A. Thapa, [arXiv:2204.07138 [hep-ph]].
[36] P. F. Perez, H. H. Patel and A. D. Plascencia, [arXiv:2204.07144 [hep-ph]].
[37] K. I. Nagao, T. Nomura and H. Okada, [arXiv:2204.07411 [hep-ph]].
[38] K. Y. Zhang and W. Z. Feng, [arXiv:2204.08067 [hep-ph]].
[39] N. D. Barrie, C. Han and H. Murayama, [arXiv:2204.08202 [hep-ph]].
[40] D. Borah, S. Mahapatra, D. Nanda and N. Sahu, [arXiv:2204.08266 [hep-ph]].
[41] T. A. Chowdhury, J. Heeck, S. Saad and A. Thapa, [arXiv:2204.08390 [hep-ph]].
[42] G. Arcadi and A. Djouadi, [arXiv:2204.08406 [hep-ph]].
[43] L. M. Carpenter, T. Murphy and M. J. Smylie, [arXiv:2204.08546 [hep-ph]].
[44] O. Popov and R. Srivastava, [arXiv:2204.08568 [hep-ph]].
[45] K. Ghorbani and P. Ghorbani, [arXiv:2204.09001 [hep-ph]].
[46] M. Du, Z. Liu and P. Nath, [arXiv:2204.09024 [hep-ph]].
[47] Y. P. Zeng, C. Cai, Y. H. Su and H. H. Zhang, [arXiv:2204.09487 [hep-ph]].
[48] S. Baek, [arXiv:2204.09585 [hep-ph]].
[49] D. Borah, S. Mahapatra and N. Sahu, [arXiv:2204.09671 [hep-ph]].
[50] L. B. Okun, Sov. Phys. JETP 56 (1982), 502 ITEP-48-1982.
[51] P. Galison and A. Manohar, Phys. Lett. B 136 (1984), 279-283 doi:10.1016/0370-2693(84)91161-4
[52] B. Holdom, Phys. Lett. B 166 (1986), 196-198 doi:10.1016/0370-2693(86)91377-8
[53] R. Foot and X. G. He, Phys. Lett. B 267 (1991), 509-512 doi:10.1016/0370-2693(91)90901-2
[54] D. Curtin, R. Essig, S. Gori and J. Shelton, JHEP 02 (2015), 157.
[55] M. He, X. G. He and C. K. Huang, Int. J. Mod. Phys. A 32, no.23n24, 1750138 (2017) [arXiv:1701.08614 [hep-ph]].
[56] M. He, X. G. He, C. K. Huang and G. Li, JHEP 03, 139 (2018) [arXiv:1712.09095 [hep-ph]].
[57] J. X. Pan, M. He, X. G. He and G. Li, Nucl. Phys. B 953, 114968 (2020).
[58] B. Kors and P. Nath, Phys. Lett. B 586 (2004), 366-372.
[59] F. Chen, J. M. Cline and A. R. Frey, Phys. Rev. D 79 (2009), 063530.
[60] F. Chen, J. M. Cline and A. R. Frey, Phys. Rev. D 80 (2009), 083516.
[61] G. Barello, S. Chang and C. A. Newby, Phys. Rev. D 94 (2016) no.5, 055018.
[62] C. A. Arguelles, X. G. He, G. Ovanesyan, T. Peng and M. J. Ramsey-Musolf, Phys. Lett. B 770 (2017), 101-107.
[63] K. Fuyuto, X. G. He, G. Li and M. Ramsey-Musolf, Phys. Rev. D 101 (2020) no.7, 075016.
[64] Y. Cheng, X. G. He, M. J. Ramsey-Musolf and J. Sun, [arXiv:2104.11563 [hep-ph]].
[65] C. P. Burgess, S. Godfrey, H. Konig, D. London and I. Maksymyk, Phys. Rev. D 50 (1994), 7011-7024.
[66] CMS collaboration, https://inspirehep.net/literature/1748026.
[67] B. Holdom, Phys. Lett. B 259, 329-334 (1991).