The \textit{ab initio} no-core shell model

C. Forssén\textsuperscript{1*}, J. Christensson\textsuperscript{2}, P. Navrátil\textsuperscript{3}, S. Quaglioni\textsuperscript{3}, S. Reimann\textsuperscript{2}, J. Vary\textsuperscript{4}, S. Åberg\textsuperscript{2}

\textsuperscript{1} Fundamental Physics, Chalmers University of Technology, 412 96 Göteborg, Sweden
\textsuperscript{2} Mathematical Physics, LTH, Lund University, Box 118, 22 100 Lund, Sweden
\textsuperscript{3} Lawrence Livermore National Laboratory, P.O. Box 808, L-414, Livermore, CA 94551, USA
\textsuperscript{4} Department of Physics and Astronomy, Iowa State University, Ames, IA 50011, USA

Abstract. This contribution reviews a number of applications of the \textit{ab initio} no-core shell model (NCSM) within nuclear physics and beyond. We will highlight a nuclear-structure study of the $A = 12$ isobar using a chiral NN+3NF interaction. In the spirit of this workshop we will also mention the new development of the NCSM formalism to describe open channels and to approach the problem of nuclear reactions. Finally, we will illustrate the universality of the many-body problem by presenting the recent adaptation of the NCSM effective-interaction approach to study the many-boson problem in an external trapping potential with short-range interactions.

Introduction. A truly first-principles approach to the nuclear many-body problem requires a nuclear Hamiltonian that is based on the underlying theory of QCD. A candidate for providing the desired connection between QCD and the low-energy nuclear physics sector is chiral perturbation theory ($\chi$PT), see, e.g., the review by E. Epelbaum [1] and references therein. A very interesting observation from $\chi$PT is that three-nucleon forces (3NF) appear naturally already at the next-to-next-to leading order of the expansion. This chiral 3NF was recently implemented in nuclear many-body calculations as will be discussed in the next section.

Regardless of its origin, high-precision nuclear Hamiltonians are very difficult to implement when solving the nuclear many-body problem. At this workshop we have heard about a number of methods that are available to solve the few-body problem ($A = 3 - 4$) to basically numerical precision. For more than four particles there are only a handful of methods available when using modern, realistic interactions. Much effort has been spent in studying different unitary transformations of the interaction to make it tractable for actual many-body calculations. In particular, the \textit{ab initio} no-core shell model (NCSM) is usually

*E-mail address: christian.forssen@chalmers.se
combined with the cluster-approximated, Lee-Suzuki transformation to generate effective interactions, see e.g., Refs. [2]. In short, the NCSM is a general approach for studying strongly interacting, quantum many-body systems. It’s a matrix diagonalization technique to solve the translational invariant $A$-body problem in a finite harmonic oscillator basis. A particularly nice feature of the method is the flexibility of the harmonic-oscillator model space that implies basically no restrictions regarding the choice of Hamiltonian. Specifically, the NCSM method allows to test the modern $\chi$PT interactions in many-body calculations.

Recent NCSM Results. The $A = 12$ nuclear systems provide a challenge for modern $ab\ initio$ methods. The systems can potentially act as new benchmarks as relevant observables allow for sensitive tests of the nuclear Hamiltonians and the computed wave functions. The current level of our experimental understanding of $^{12}$C includes two bound states and the triple-alpha threshold at 7.3 MeV. Above this the picture becomes very complicated due to overlapping broad resonances. A central question concerns the possible existence of broad $0^+$ and $2^+$ resonances in this region. An important concept that attracts much theoretical interest is the interplay between triple-alpha and neutron-proton degrees of freedom. Studies of ground- and excited states in $A = 12$ systems are possible within the NCSM. These studies are particularly interesting since the chiral $3NF$ was recently implemented by P. Navrátíl et al. [3]. The inclusion of these terms in the NCSM gives the correct ordering of $T = 1$ states with the isobaric analogue of the $^{12}$B and $^{12}$N ground states being the lowest. It also provides the correct ordering of the $1^+$ and $4^+$ states although it over-corrects the spin-orbit strength [3]. Still, regardless of the interaction being used, these results demonstrate a limitation of the NCSM method. Whereas the spectrum and properties of shell-model like states are reproduced very nicely, states that are known to exhibit a high degree of clusterization are missing from the low-energy spectrum. They typically end up at much higher excitation energy and are far from converged.

Open quantum systems. A long-term vision for nuclear theory is to achieve a unified picture of the nuclear many-body system, including both bound and continuum states and the transitions between them. Preferably this picture should be grounded in the fundamental interactions between the constituent nucleons. In addition, the separation of scales known to occur in nuclear systems, should be properly described. This requires the simultaneous modeling of small-scale many-body degrees of freedom and large-scale few-body correlations. A possible route towards achieving such a microscopic picture of open channels and nuclear reactions is explored at Livermore by combining the NCSM formalism with resonating group methods (RGM) [4]. In the RGM approach the many-body wave function is decomposed into contributions from various channels that are distinguished by their different arrangement of the nucleons into clusters. By defining a set of antisymmetrized cluster basis functions, and diagonalizing the Hamiltonian in this space, one obtains a non-local, coupled-channels Schrödinger Equation for the relative motion of the clusters in the different channels. In Ref. [4] this approach was implemented and tested for certain $A = 4 – 5$ low-
energy, single-nucleon scattering problems. In particular, $n + ^4\text{He}$ scattering at low energies represents a convenient training ground for many-body scattering calculations. There is no $A = 5$ bound state, and single-channel scattering is valid up to rather high energies. There is a sharp, low-energy resonance in the $3/2^-$ channel, and a broader, high-energy resonance in the $1/2^-$ channel. Scattering in the $s$-wave channel is non-resonant but obviously depends critically on proper antisymmetrization. Phase shifts for both $n + ^4\text{He}$ and $p + ^4\text{He}$ scattering, calculated in the NCSM/RGM approach, are presented in Fig. 1. The method shows very good convergence behavior, but it’s clear that the position and widths of the $p$-wave resonances depend sensitively on the interaction model.

**Effective Interaction Approach to the Many-Boson Problem.** The emerging field of cold-atom physics has proven to be a very rich arena of research for few- and many-body physicists. Particle numbers can be varied, the interaction strength can in many cases be tuned through Feshbach resonances, and many different properties can be studied very cleanly in the laboratory. Nuclear physics techniques and tools have proven to be very useful to describe the physics of these systems. With trapping potentials that are very close to harmonic, the NCSM should be a perfect method. We recently adapted the NCSM formalism to describe a two-dimensional system of strongly interacting bosons [5]. A purely repulsive, short-ranged interaction was modeled with a Gaussian potential. Note that the different statistics of the bosonic many-body system required a complete rewrite of the NCSM suite of codes.

The success of the NCSM effective-interaction approach is demonstrated in Fig. 2. Ground- and excited-state energies are presented for a system of nine atoms. The NCSM results are compared to the much slower convergence of the standard configuration interaction (CI) method. The figure illustrates that stronger correlations within the system are obtained when increasing the interaction strength (right panel). In this case, the computed energies still show a slow decrease with increasing model space ($N_{\text{max}}$). Still, in comparison, the
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Figure 2. Energies for a system of nine bosons and total angular momentum \( L = 0 \), for different many-body space cutoffs \( (N_{\text{max}}) \). Repulsive Gaussian interactions with range \( \sigma = 0.1 \) and two different strengths \( (g) \) are used (oscillator units). The blue-dashed (red-solid) curves correspond to standard CI (effective interaction approach) calculations. From Ref. [5].

energies obtained from the standard CI calculations show a much slower convergence. These results represent an important first step of our new approach. Three-dimensional systems and higher particle numbers should also be within reach for future studies.

Conclusion. Recent applications of the ab initio NCSM within nuclear physics and beyond have been reviewed. In particular, we have demonstrated the study of chiral 3NF Hamiltonians in the p-shell, the treatment of open channels using the NCSM/RGM approach, and the effective-interaction approach to the many-boson problem.

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