Inclusive jet production on the nucleus in the perturbative QCD with $N_c \rightarrow \infty$

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Abstract. Using the recently found hA amplitude, single and double inclusive jet production rates on the nucleus are studied in the perturbative QCD with $N_c \rightarrow \infty$. Jet multiplicities are found to grow with $A$ and energy as $A^{2/9} s^\Delta$ where $\Delta$ is the BFKL intercept. Long range correlations are found to behave as $A^{4/9} s^{2\Delta}$ and change from negative to positive at a certain high energy.

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1 Introduction

In the colour dipole approach of A.H. Mueller [1,2] with $N_c \to \infty$ the interaction with a heavy nucleus is exactly described by the sum of fan diagrams constructed of BFKL pomeron, each of them splitting into two [3,4]. The equation for this sum [3-6] has recently been studied perturbatively in [7], by asymptotic estimates in [8] and finally solved numerically in [6]. The results indicate that the cross-section of a particle on the nucleus saturates at high energies to its geometrical value $2\pi R_A^2$ corresponding to the scattering on a black disk. In itself this is not a very surprising result: since the days of the old Regge-Gribov theory it has been known that the supercritical pomeron fan diagrams with a triple pomeron vertex lead to a constant cross-section [9]. The new elements in the recent developments is that it has been understood that in the high-colour limit no splitting of the pomeron in more than two occurs and that the fan diagrams seem to be unitary by themselves so that they indeed represent the full scattering amplitude on the nucleus.

An interesting byproduct of the numerical solution in [6] is the found gluon density of the nucleus in the combined momentum-impact parameter space

$$\frac{\partial xG(x, q, b)}{\partial^2 q \partial^2 b} = \frac{N_c}{2\alpha_s \pi} h(y, q, b)$$

where the rapidity $y$ is related to $x$ by

$$y = \ln \frac{1}{x}$$

Function $h$ defined via the sum of BFKL fan diagrams below (Section 2) and found in [6] numerically is a soliton wave in $y - \ln q$ space moving towards higher rapidities with a constant velocity and preserving its nearly Gaussian shape. A fit to numerical data gives

$$h(y, q, b) = h_0 e^{-a(\xi - \xi_0(y,b))^2}$$

where $\xi = \ln q$, $h_0 \simeq 0.3$ and $a \simeq 0.3$ are universal and $\xi_0$ is nearly linear in $y$:

$$\xi_0 = c(b) + \Delta_0 y, \quad \Delta_0 = 2.3\bar{\alpha}$$

where we standardly define $\bar{\alpha} = \alpha_s N_c/\pi$. The term $c(b)$ actually depends on a dimensionless parameter $B$:

$$B = \pi \alpha_s^2 AT(b) R_N^2$$

where $R_N$ is the nucleon radius and $T(b)$ is the nucleus profile function. Numerical results for $c(b)$ are well described by

$$c(b) = -3.11 + (2/3) \ln B$$

Actually the gluon density does not really show itself in the total scattering cross-section at high rapidities. As mentioned, the cross-section becomes purely geometrical and insensitive to any dynamics. One expects however that the gluon density will be directly felt in production processes, where it is responsible for hard collisions giving rise to observable jets.
In this paper we study simplest of the production processes: the single and double inclusive jet production in \( hA \) collisions. It has to be noted that these observables were also studied for the fan diagrams in the old Regge-Gribov theory. The most important conclusion which was drawn from this study is that the multiplicities are strongly damped, as compared to the naive eikonal result. For \( hA \) scattering one obtains a multiplicity independent of \( A \) (instead of \( \sim A^{1/3} \)). As we shall see this result does not hold for the BFKL fan diagrams: the jet production rate grows with \( A \) although somewhat slower than in the eikonal approach, approximately as \( A^{2/9} \). The energy dependence remains essentially the same as in older studies: the production rate grows with rapidity \( Y \) essentially as \( \exp \Delta Y \), where \( \Delta \) is the BFKL intercept. With \( \alpha_s \sim 0.2 \) its value is close to 0.5, so that the multiplicities reach very high values at rapidities in the region \( 20 \div 30 \).

2 Single jet inclusive production

In the BFKL framework, at fixed impact parameter \( b \), a single scattering contribution to the forward amplitude for the interaction of the projectile particle with a nucleus has a form

\[
A_1(Y, b) = isg^4AT(b) \int d^2r_1d^2r_2\rho(r)G(Y, r_1, r_2)\rho_N(r_2) \equiv 2is \int d^2r_1\rho(r_1)\Phi_1(y, r_1, b)
\]

Here \( \rho \) and \( \rho_N \) are the colour densities of the projectile and the target nucleon respectively. Function \( G \) is the forward BFKL Green function [10] taken at an overall rapidity \( Y \)

\[
G(Y, r, r') = \frac{rr'}{32\pi^2} \sum_{n=-\infty}^{+\infty} e^{in(\phi-\phi')} \int_{-\infty}^{\infty} \frac{d\nu e^{Y\omega(\nu)}}{[\nu^2 + (n-1)^2/4][\nu^2 + (n+1)^2/4]} (r/r')^{-2i\nu},
\]

where \( \phi \) and \( \phi' \) are the azimuthal angles and

\[
\omega(\nu) = \bar{\alpha}(\psi(1) - \text{Re}\psi(1/2 + i\nu))
\]

are the BFKL levels. Due to the azimuthal symmetry of the projectile colour density one may retain only the term with zero orbital momenta \( n = 0 \) in (5). The corresponding inclusive jet production rate

\[
I(y, k, b) = \frac{(2\pi)^2\partial\sigma}{\partial y\partial^2k\partial^2b}
\]

is obtained from the imaginary part of (4) divided by \( s \) after the substitution

\[
\int d^2rG(y_1, r_1, r)V_k(r)G(y, r, r_2)
\]

where \( r \)'s are relative distances between the gluons and \( V_k(r) \) is a vertex for the emission

\[
V_k(r) = \frac{4N_c\alpha_s}{k^2} \leftrightarrow e^{ikr} \rightarrow
\]

local in \( r \) (a differential operator, the arrows shows the direction of its action). The rapidity of the produced jet relative to the projectile is \( y_1 = Y - y \).
As mentioned, in the limit $N_c \to \infty$ the total scattering amplitude which includes multiple interactions in the target is given by the sum of fan diagrams shown in Fig. 1. To obtain the corresponding inclusive cross-section, one has to make the substitution (8) in one of the pomerons. However due to the well-known AGK rules all such substitutions in the pomerons below the upper splitting point (Fig. 2a) cancel, since the other branch, counting from the upper point, can be both "cut" and "uncut" (i.e. represent both real and virtual processes). The only left contribution is the one with the substitution (8) in the uppermost pomeron (Fig. 2b). It is clearly seen from Fig. 2b that the vertex $V_k$ is coupled from below to exactly the full amplitude $\Phi(y, r, b)$ which multiplies the projectile density in Fig. 1 and substitutes the single pomeron exchange contribution $\Phi_1(Y, r_1, b)$ in (4). Thus one obtains the inclusive cross-section for jet production on the nucleus at fixed $b$ as

$$I(y, k, b) = 2 \int d^2r'd^2r \rho(r')G(y_1, r', r)V_k(r)\Phi(y, r, b)$$

The total inclusive production rate as a function of $y$ and $k$ is obtained from (10) after the integration over $b$

Function $\phi(y, r, b) = \Phi(y, r, b)/(2\pi r^2)$, in the momentum space, satisfies a nonlinear equation [6]

$$\frac{\partial \phi(y, q, b)}{\partial \tilde{y}} = -H \phi(y, q, b) - \phi^2(y, q, b),$$

where $\tilde{y} = \tilde{\alpha}y$ and $H$ is the BFKL Hamiltonian for the so-called semi-amputated function [10]. As mentioned in the Introduction, this equation was solved numerically in [6].

The Laplace operator in (9) acting on $\Phi(y, r, b)$ gives

$$\nabla_r^2 \Phi(y, r, b) = 2\pi \nabla_r^2 r^2 \int \frac{d^2q}{(2\pi)^2} e^{iqr} \phi(y, q, b) = -2\pi \nabla_r^2 \int \frac{d^2q}{(2\pi)^2} e^{iqr} \nabla_r^2 \phi(y, q, b)$$

$$= 2\pi \int \frac{d^2q}{(2\pi)^2} e^{iqr} q^2 \nabla_q^2 \phi(y, q, b) = 2\pi h(y, r, b)$$

Note that in the integration by parts used to obtain the second expression one has to be careful, since function $\phi(y, q, b)$ has a logarithmic singularity at $q = 0$ of the form $-\ln q$. As a result, action of the Laplacian operator in $q$ generates a $\delta^2(q)$ term which in its turn leads to a constant in $\Phi$ (actually unity). However this term is annihilated by the Laplacian operator in $r$, so that no extra term arises from the integration by parts.

Function $h(y, q, b)$, which is the gluon density, up to a trivial factor (Eq. (1)) is just a Fourier transform of $h(y, r, b)$:

$$h(y, q, b) = q^2 \nabla_q^2 \phi(y, q, b)$$

So we find the inclusive cross-section in the form

$$I(y, k, b) = \frac{16\pi \alpha_s N_c}{k^2} \int d^2r'd^2r \rho(r') \nabla_r^2 G(y_1, r', r)e^{ihr}h(y, r, b)$$

We observe that the definition of $h(y, q, b)$ as the gluon density introduced in [6] is supported by its role in the jet production on the nucleus.
Applying the Laplace operator to the Green function (5) (with only \( n = 0 \) terms retained) we obtain

\[
\nabla_r^2 G(y_1, r', r) = -\frac{r'}{8\pi^2 r} \int_{\infty}^{\infty} \frac{d\nu e^{y_1 \omega(\nu)}}{(\nu + i/2)^2} (r'/r)^{-2\nu},
\]

(15)

We are going to study the production rate at \( y_1 \gg 1 \), that is far from the projectile fragmentation region. Then we can use the asymptotics of (15) at high \( y_1 \), which comes from \( \nu \sim 0 \). Presenting \( \omega(\nu) \) in the standard manner

\[
\omega(\nu) = \Delta y - \beta \nu^2
\]

(16)

where

\[
\Delta = \bar{\alpha} 4 \ln 2, \quad \text{and} \quad \beta = \bar{\alpha} 14 \zeta(3)
\]

(17)

we get for (15)

\[
\nabla_r^2 G(y_1, r', r) \simeq \frac{r'}{2\pi^2 r} e^{\Delta y_1} \sqrt{\frac{\pi}{\beta y_1}}
\]

(18)

With (18), integration over \( r' \) converts \( r' \) into the average transverse dimension of the projectile \( \langle r \rangle_P = R_P \). Doing the integration over \( r \) we obtain

\[
I(y, k, b) = \frac{8\bar{\alpha}}{k^2} R_P e^{\Delta y_1} \sqrt{\frac{\pi}{\beta y_1}} F(y, k, b)
\]

(19)

where

\[
F(y, k, b) = \int \frac{d^2 q}{2\pi} h(y, q, b) = \int \frac{d^2 q}{2\pi} h(y, q, b) e^{ikr}
\]

(20)

As mentioned in the Introduction, the gluon density \( h(y, q, b) \) has a sharp peak at \( q = q_0(y, b) = \exp \xi_0(y, b) \) and is practically zero everywhere else. With growing \( y \) \( q_0(y, b) \) grows exponentially, so that at high \( y \) we can safely neglect \( k \) in the denominator of (20) as compared to \( q \). Then \( F \) becomes independent of \( k \) and given by

\[
F(y, b) = \int \frac{d^2 q}{2\pi q} h(y, q, b) = \int \frac{d^2 q}{2\pi} h(y, q, b)
\]

(21)

where \( \xi = \ln q \). One easily finds that

\[
\int d\xi h(y, q, b) = 1
\]

(22)

Indeed Eq. (13) can be rewritten as

\[
h(y, q, b) = \frac{d^2}{d\xi^2} \phi(y, q, b)
\]

(23)

Putting (23) into (21) and taking into account that at \( \xi \to +\infty \phi = \text{const} - \xi \) and at \( \xi \to +\infty \phi \to 0 \) we obtain (22). So (21) in fact has a meaning of the average gluon momentum in the nucleus \( \langle q \rangle \). Since \( h \) has a sharp maximum at \( \xi = \xi_0(y, b) \) given by (3) one expects that \( F(y, b) \) will approximately be given just by \( q_0(y, b) \). Numerical
integration gives more accurate values which are well fitted by the formula similar to (3) but with slightly different parameters:

$$F(y, b) = q_1(y, b) = 1.2 B^{2/3} \frac{e^{\Delta_1 y}}{R_N \sqrt{y}}, \quad \Delta_1 = 2.38\bar{\alpha}$$

(24)

With this approximate form of $F$ we find the final expression for the inclusive rate production (in (GeV/c)$^{-2}$)

$$I(y, k, b) = 1.2 \frac{8\bar{\alpha}}{\pi} \frac{R_P}{R_N} \sqrt{\pi \alpha_s^2 A T(b)}_N^{2/3} \frac{e^{\Delta Y - \epsilon} y}{\sqrt{\beta y (Y - y)}}$$

(25)

where

$$\epsilon = \Delta - \Delta_1 = 0.39\bar{\alpha}$$

and is relatively small. As we observe, the inclusive jet production is maximal in nucleus fragmentation region and slowly falls as one moves to higher $y$.

Since the total $hA$ cross-section saturates at large $Y$ an immediate consequence of (25) is that the multiplicity grows like exp $\Delta Y$, that is, as the cross-section generated by a single pomeron exchange. This conclusion is also true for the inclusive production from the sum of fan diagrams in the old Regge-Gribov theory with a local pomeron. However this theory predicts a much stronger $y$-dependence, corresponding to (25) with $\epsilon = \Delta$.

As to the $A$-dependence, a novel element in the BFKL fan diagrams is factor $[\pi \alpha_s^2 AT(b) R_N^2]^{2/3}$. For a nucleus with constant density inside a sphere of radius $R_A = A^{1/3} R_0$ its integration over all $b$ leads to a factor

$$2\pi R_A^2 A^{2/9} \frac{3}{8} \gamma, \quad \gamma = \left( \frac{3R_N^2}{2R_0^2} \right)^{2/3}$$

(26)

Since the total $hA$ cross-section at large $Y$ saturates to its black-disc limit $2\pi R_A^2$ the jet multiplicity results proportional to $A^{2/9}$.

$$\mu(y, k) = \frac{1}{\sigma_{hA}^{\text{tot}}} \int d^2 b I(y, k, b) = 1.2 A^{2/9} \frac{3\bar{\alpha}}{\pi} \frac{R_P}{R_N} \frac{\gamma}{\sqrt{\beta y (Y - y)}}$$

(27)

So unlike the fan diagrams in the old Regge-Gribov theory, the BFKL model predicts rising of the multiplicities with $A$, although somewhat weaker than according to the eikonal model ($\propto A^{1/3}$).

Although the gluon distribution in the nucleus results radically changed as compared to the single nucleon, in particular, strongly damped at small momenta, this seems to have no effect on the $k$-dependence of the jet production rate at moderate $k << q_1(y, b)$: it remains proportional to $1/k^2$ and growing as $k \to 0$. The inclusive cross-section integrated over all $k$ remains infinite, as for a single BFKL pomeron exchange. Thus discussing the integrated cross-section we have to cutoff the spectrum from below by some $k_{\text{min}}$. The formal expression for this cross-section can be trivially found integrating (19) over all $k > k_{\text{min}}$. A simple estimate of its magnitude can be made in the logarithmic approximation in the large parameter $q_1(y, b)/k_{\text{min}}$. Evidently

$$\int \frac{d^2 k}{(2\pi)^2 k^2} F(y, k, b) \theta(k - k_{\text{min}}) = \int \frac{d^2 q}{2\pi} \mu(y, q, b) \int \frac{d^2 k}{(2\pi)^2 k^2} \frac{1}{|k + q|} \theta(k - k_{\text{min}})$$
\[ I(y, b) \equiv \int \frac{d^2 k}{(2\pi)^2} \theta(k - k_{\min}) = \frac{4\bar{\alpha}}{\pi} R \rho e^{\Delta(Y - y)} \sqrt{\frac{\pi}{\beta(Y - y)}} q_1(y, b) \ln \frac{q_1(y, b)}{k_{\min}} \]

with \( q_1(y, b) \) given by (24).

At high \( y \) the logarithmic factor reduces to \( \Delta_1 y \). Its effect, opposite to the exponential, is to flatten the distribution \( I(y) \) as compared to \( I(y, k) \). The \( A \) dependence evidently becomes somewhat stronger. Numerical integration of \( I(k, y, b) \) with function \( h \) found in [6] gives results which more or less agree with this approximate estimates. At \( \bar{\alpha}Y = 6 \) for lead \( (A = 207) \) we obtain multiplicities which in the region \( 2 \leq \bar{\alpha}y \leq 5 \) very slowly grow from \( 1.13 \times 10^5 \) to \( 1.26 \times 10^5 \). The found \( A \) dependence can be fitted by \( \sim A^{0.6} \). The large magnitude of the multiplicity is due to the exponential factor \( \exp \Delta Y = \exp(\bar{\alpha}Y4\ln 2) \).

### 3 Double inclusive cross-sections

Passing to double inclusive cross-sections and using the AGK rules we find two contributions schematically shown in Figs. 3 a.b.

A simpler contribution Fig. 3a corresponds to emission of both gluons from the upper pomeron. To obtain it we have to make two insertions (8) in the upper BFKL Green function. We get

\[ I_a(y_1, k_1, y_2, k_2, b) \equiv \left[ \frac{(2\pi)^4}{\partial y_1 \partial^2 k_1 \partial y_2 \partial^2 k_2 \partial^2 b} \right]_{\text{Fig.3a}} \]

\[ = 2 \int d^2r'd^2r_1 d^2r_2 \rho(r') G(Y - y_1, r', r_1) G(y_1 - y_2, r_1, r_2) V_{k_1}(r_1) V_{k_2}(r_2) \Phi(y_2, r_2, b) \]

where we assumed that \( Y >> y_1 >> y_2 >> 1 \) in the lab. system.

The only new element in this equation is the second BFKL Green function with Laplacians applied in both coordinates. Using (5) we find

\[ \Delta_1 \Delta_2 G(y, r_1, r_2) = \frac{1}{2\pi^2 r_1 r_2} \int d\nu e^{i\omega(\nu)} (\frac{r_1}{r_2})^{-2i\nu} \]

Combining this with (15) for the first Green function (with the integration variable \( \nu_1 \)) we find an integral over \( r_1 \):

\[ \int d^2 r_1 e^{i k_1 r_1} r_1^{-2i\nu + 2i\nu_1} = \pi(k_1/2)^{2i(\nu - \nu_1)} \frac{1}{(\nu_1 - \nu - i\omega) \Gamma(1 - i\nu_1 + i\nu) \Gamma(1 + i\nu - i\omega)} \]

As before we limit ourselves with the leading contribution as both \( Y - y_1 \) and \( y_1 - y_2 \) become large, which correspond to small values of both \( \nu \) and \( \nu_1 \). Correspondingly we put \( \nu = \nu_1 = 0 \) everywhere except in the exponents \( \omega(\nu_1)(Y - y_1) \) and \( \omega(\nu)(y_1 - y_2) \).
and in the denominator of (32). We find that only the real part of (32) contributes, which is equivalent to taking

\[ \frac{1}{i(\nu_1 - \nu - i0)} \rightarrow \pi \delta(\nu_1 - \nu) \]

Using (16) we finally get for the double inclusive cross-section (30)

\[ I_a(y_1, k_1, y_2, k_2, b) = 16\pi R_P \frac{\tilde{A}^2}{k_1^2 k_2^2} e^{\Delta(y_1 - y_2)} \sqrt{\frac{\pi}{\beta(Y - y_2)}} F(y_2, k_2, b) \]

(33)

where function \( F(y, k, b) \) was defined in Section 2 (Eq.(20)). A remarkable feature of this cross-section is that it depends only on the rapidity of the slower jet \( y_2 \) and does not depend on \( y_1 \) altogether. In the approximation (24) the dependence on \( y_2 \) results the same as for the single inclusive cross-section, namely \( \exp(-\epsilon y_2) \), so that the cross-section slowly diminishes as \( y_2 \) increases.

Using approximation (24), integrating over \( b \) and dividing by \( \sigma_{tot}^{AA} \) we get the corresponding multiplicity distribution

\[ \mu_a(y_1, k_1, y_2, k_2) = 1.2 A^{2/9} \frac{R_P}{R_N} \frac{\tilde{A}^2}{k_1^2 k_2^2} e^{\Delta Y - \epsilon y_2} \sqrt{\frac{\pi}{\beta y_2(Y - y_2)}} \]

(34)

where \( \gamma \) was defined in (26).

The second contribution (Fig. 3b) corresponds to emission of both jets just after the first splitting of the upper pomeron. To write it we first take the contribution from all non-trivial fan diagrams to \( \Phi \) [6]

\[ \Phi(Y, r', b) = -\frac{g^2 N_c}{8\pi^3} \int_0^Y dy' \int \prod_{i=1}^3 d^2 x_i \delta^2(x_1 + x_2 + x_3) \]

\[ \frac{x_3^4}{x_1^2 x_2^2} G(Y - y', r', x_3) \Phi(y', x_1, b) \Phi(y', x_2, b) \]

(35)

where we denoted \( x_i, i = 1, 2, 3 \) the transverse coordinates of the triple pomeron vertex. According to the AGK rules the corresponding contribution to the absorptive part with both lower branches cut is twice (35) with the opposite sign. To finally obtain the double inclusive cross-section we have to insert into both \( \Phi \)'s on the right-hand side vertexes \( V_k(r) \) as was done in (10). In this way we obtain the second part of the double inclusive cross-section as

\[ I_b(y_1, k_1, y_2, k_2, b) = \frac{g^2 N_c}{2\pi^3} \int d^2 r' \rho(r') d^2 r_1 d^2 r_2 \int_0^Y dy' \int \prod_{i=1}^3 d^2 x_i \delta^2(x_1 + x_2 + x_3) \]

\[ \frac{x_3^4}{x_1^2 x_2^2} G(Y - y', r', x_3) G(y' - y_1, x_1, r_1) V_{k_1}(r_1) \Phi(y_1, r_1, b) G(y - y_2, x_2, r_2) V_{k_2} \Phi(y_2, r_2, b) \]

(36)

We again assume \( Y >> y_1 >> y_2 >> 1 \) so that the rapidity of the splitting point \( y' \) varies from \( y_1 \) to \( Y \).

We easily find

\[ x_3^2 \nabla_3^4 G(y, r', x_3) = \frac{r'}{2\pi^2 x_3} \int d\nu_3 e^{\nu_3 (\nu_3)} \left( \frac{r'}{x_3} \right)^{2i\nu_3} \]

(37)
Using this and representation (15) for the other two BFKL Green functions we find an integral over \( x_i, i = 1, 2, 3 \)

\[
J(\nu_i) = \int \prod_{i=1}^{3} d^2 x_i \delta^2(x_1 + x_2 + x_3) \frac{1}{x_1 x_2 x_3} x_1^{-2i\nu_1} x_2^{-2i\nu_2} x_3^{-2i\nu_3} \quad (38)
\]

This integral diverges at large \( x_i \)’s for real \( \nu \)’s. In fact, presenting the \( \delta \) function as an integral over momentum \( q \) we find

\[
J = 2\pi \prod_{i=1}^{3} 2^{-2i\nu_i} \frac{\Gamma(1/2 - i\nu_i)}{\Gamma(1/2 + i\nu_i)} \int \frac{d^2 q}{q^2} q^{2(\nu_1 + \nu_2 + \nu_3)} \quad (39)
\]

which does not exist for real \( \nu \) due to the divergence at \( q = 0 \). To overcome this difficulty we have to shift the integration contours in \( \nu \)’s into the lower half-plane, using the fact that integrands are analytic in the strip \(-i/2 < \text{Im}\, \nu < +i/2\). We choose to do it in a symmetric manner putting

\[
\nu_i = -i/6 + \bar{\nu}_i, \quad i = 1, 2, 3 \quad (40)
\]

With such a choice we get an extra factor \( q \) in the integral over \( q \), which then gives a \( \delta \) function:

\[
J = \pi^2 \prod_{i=1}^{3} \frac{\Gamma(1/2 - i\bar{\nu}_i)}{\Gamma(1/2 + i\bar{\nu}_i)} \delta(\bar{\nu}_1 + \bar{\nu}_2 + \bar{\nu}_3) \quad (41)
\]

Now we pass to the integration over the rapidity of the splitting point. The integral is done trivially:

\[
\int_{y_1}^{Y} dy e^{y(\omega_1 + \omega_2 - \omega_3)} = \frac{e^{Y(\omega_1 + \omega_2 - \omega_3)} - e^{y_1(\omega_1 + \omega_2 - \omega_3)}}{\omega_1 + \omega_2 - \omega_3} \quad (42)
\]

where we denote \( \omega_1 = \omega(\nu_1) \) etc. Multiplying by the rest exponential factors we find the overall rapidity factor as

\[
e^{(Y-y_1)\omega_1 + (Y-y_2)\omega_2} - e^{(Y-y_1)\omega_3 + (y_1-y_2)\omega_2} \quad (43)
\]

Note that each of the two terms depends only on two of the three \( \nu \) variables. Therefore integrating over \( \nu \)’s we can always take as independent variables just those which are present in the exponents in (43).

Having this in mind we again use the fact that all rapidity differences in the exponents are large and use the saddle point method. Although the initial contours of integration in the independent \( \nu \)’s lie along the lines \( \text{Im}\, \nu = -1/6 \), the saddle points remain at \( \nu = 0 \), that is at \( \bar{\nu} = (1/6)i \). This means that independent \( \nu \)’s will be close to zero, as usual. The only price we have to pay for the shift in the contours of integration will be the value of the dependent \( \nu \) at the saddle point.

Take the first term in (43), for which \( \nu_1 \) and \( \nu_2 \) are independent variables and \( \nu_3 \) is the dependent one. At the saddle point we shall have \( \nu_1 = \nu_2 = 0 \) and \( \nu_3 = -(1/6)i - \bar{\nu}_1 - \bar{\nu}_2 = -(1/2)i \). Accordingly we put \( \nu_1 = \nu_2 = 0 \) and \( \nu_3 = -(1/2)i \) in all terms except in the exponent and the singular \( \Gamma \)-function:

\[
\Gamma(1/2 - i\nu_3) = \Gamma(i\nu_1 + i\nu_2) = \frac{\Gamma(1 + i\nu_1 + i\nu_2)}{i(\nu_1 + \nu_2 - i0)} \sim \frac{1}{i(\nu_1 + \nu_2 - i0)} \quad (44)
\]
Again only the real part contributes, so that (43) actually gives

$$\pi \delta(\nu_1 + \nu_2)$$

Using (16) and integrating over \(\nu_1\) and \(\nu_2\) we obtain a factor

$$\frac{1}{\Delta} e^{\Delta (2Y - y_1 - y_2)} \sqrt{\frac{\pi}{\beta(2Y - y_1 - y_2)}}$$

and an extra factor \(r'\) due to the fact that \(\nu_3 = -(1/2)i\) at the saddle point.

For the second term in (43) the independent variables are \(\nu_2\) and \(\nu_3\) which are zero at the saddle point. The dependent \(\nu_1\) is equal to \(-(1/2)i\) at the saddle point. The \(\Gamma\) functions in (41) will give \(\pi \delta(\nu_2 + \nu_3)\), so that the integration over \(\nu_2\) and \(\nu_3\) will give a factor

$$\frac{1}{\Delta} e^{\Delta (Y - y_2)} \sqrt{\frac{\pi}{\beta(Y - y_2)}}$$

and we shall have an extra factor \(r_1\) due to \(\nu_1 = -(1/2)i\) at the saddle point.

Other factors in (36) are straightforward. Combining all of them we find the expression for the double inclusive cross-section corresponding to Fig. 2b in the form

$$I_b(y_1, k_1, y_2, k_2, b) = \frac{4}{\ln 2} \frac{\bar{\alpha}^2}{k_1^2 k_2^2} e^{\Delta (2Y - y_1 - y_2)} \sqrt{\frac{\pi}{\beta(2Y - y_1 - y_2)}} F(y_2, k_2, r)$$

$$\left[ \langle r^2 \rangle_p F(y_1, k_1, b) - R \rho e^{-\Delta (Y - y_1)} \sqrt{\frac{2Y - y_1 - y_2}{Y - y_2}} h(y_1, k_1, b) \right]$$

where

$$\langle r^2 \rangle_p = \int d^2 r r^2 \rho(r)$$

The second term is evidently exponentially small as compared to the first and we can safely neglect it, having in mind all approximation already made. So our final expression is

$$I_b(y_1, k_1, y_2, k_2, b) =$$

$$\frac{4}{\ln 2} \frac{\langle r^2 \rangle_p \bar{\alpha}^2}{k_1^2 k_2^2} e^{\Delta (2Y - y_1 - y_2)} \sqrt{\frac{\pi}{\beta(2Y - y_1 - y_2)}} F(y_1, k_1, b) F(y_2, k_2, r)$$

Recalling our expression (19) for the single inclusive cross-section we can rewrite it as

$$I_b(y_1, k_1, y_2, k_2, b) = \frac{1}{16 \ln 2} \frac{\langle r^2 \rangle_p}{\langle r \rangle^2_p} \sqrt{\beta Y - y_1 - y_2} \frac{\beta (Y - y_1) (Y - y_2)}{2Y - y_1 - y_2} I(y_1, k_1, b) I(y_2, k_2, b)$$

After the integration over \(b\) with the approximation (24) we get the corresponding multiplicity distribution:

$$\mu_b(y_1, k_1, y_2, k_2) = 1.44A^{4/9} \frac{6}{5 \ln 2} \frac{\langle r^2 \rangle_p}{R_N^{2/3}} \frac{\bar{\alpha}^2}{k_1^2 k_2^2} e^{2\Delta Y - \epsilon(y_1 - y_2)} \sqrt{\frac{\pi}{\beta y_1 y_2 (2Y - y_1 - y_2)}}$$

Evidently at high \(Y\) the part \(\mu_b\) dominates due to the factor \(\exp 2\Delta Y\) as compared with only \(\exp \Delta Y\) in \(\mu_a\)
Correlations are determined by the ratio

\[ R(Y, y_1, y_2) = \frac{\mu(y_1, k_1, y_2, k_2)}{\mu(y_1, k_1)\mu(y_2, k_2)} \]  

(52)

(evidently it does not depend on \( k_1 \) and \( k_2 \)). Neglecting the part \( \mu_a \) we find from (27) and (51)

\[ R(Y, y_1, y_2) = \frac{2}{15 \ln 2} \frac{\langle r^2 \rangle_P}{R_P^2} \sqrt{\frac{\beta (Y - y_1)(Y - y_2)}{\pi 2Y - y_1 - y_2}} \]  

(53)

If the two jets, forward and backward in the c.m. system, are taken symmetric, that is,

\[ y_1 = \frac{1}{2} Y + y, \quad y_2 = \frac{1}{2} Y - y \]  

(54)

Eq. (53) becomes

\[ R(Y, y) = \frac{1}{15 \ln 2} \frac{\langle r^2 \rangle_P}{R_P^2} \sqrt{\frac{\beta}{\pi} \left( Y - \frac{4y^2}{Y} \right)} \]  

(55)

With the rapidity gap between the jet fixed and growing energy the term \( 4y^2/Y \) can be neglected, so that the right-hand side of (55) becomes independent of the gap \( 2y \) and proportional to \( \sqrt{Y} \). This means that eventually it inevitably becomes greater than unity, so that the correlations become positive. The concrete threshold for this depends on the values of \( \bar{\alpha} \) and ratio \( \langle r^2 \rangle / R_P^2 \). With the Yukawa distribution of colour in the projectile the last ratio is equal 2. Taking \( \alpha_s = 0.2 \) we get for (55)

\[ R \simeq 0.195 \sqrt{Y} \]  

(56)

So with these values correlations remain negative up to quite high energies corresponding to \( Y < 25 \).

Note that in the framework of the old Regge-Gribov theory, due to the assumed relations between the parameters and smallness of the intercept \( \Delta \), it was argued that the ratio \( R \) for the part \( \mu_b \) is identically equal to unity, so that correlations are entirely generated by the part \( \mu_a \) [9]. In the present approach this does not hold. Although the factors exponential in rapidities are indeed the same in \( \mu_b \) and the product \( \mu(y_1, k_1)\mu(y_2, k_2) \), the power factors are not, so that their final \( Y \)-dependence is different. Also the relation between the coefficients is not fixed but depends on \( \alpha_s \). As a result the ratio \( R \) for \( \mu_b \) is different from unity and depends on energy and \( \alpha_s \), so that the correlations become mainly determined by \( \mu_b \) and thus much stronger than assumed in earlier studies.

4 Conclusions

We have studied the inclusive jet production off the nucleus in the perturbative QCD with \( N_c \to \infty \), in which the total hA amplitude is exactly given by a sum of fan diagrams constructed of BFKL pomerons. We have used the numerical results obtained for this sum in our earlier paper [6]. Our main results are the following.

Our formulas confirm interpretation of function \( h(y, q, b) \) defined by Eq. (13) as the gluon density in the nucleus. In [6] this was done by comparison with the perturbative
expression for the structure function, the validity of which for large values of the gluon density can be questioned.

The inclusive jet production rate is found to rapidly grow with energy in the same manner as the cross-section generated by a single pomeron exchange. This result is an immediate consequence of the saturation of the total hA cross-section, and was also found in earlier studies of the fan diagrams with local pomerons.

However in contrast to these older studies, our jet production rate is $A$ dependent: jet multiplicities grow roughly as $A^{2/9}$. So the $A$ dependence is somewhat reduced as compared to the eikonal result $A^{1/3}$, but not so strongly as thought before. This result may have direct consequences for particle production in nucleus-nucleus collisions, where we may expect the multiplicities grow as $A^{10/9}$ for identical nuclei, less than in the eikonal approach ($A^{4/3}$) but greater than in the older fan diagram estimates ($A^{2/3}$).

The double inclusive jet production is dominated by the contribution from Fig. 2b. Unlike older studies, this contribution is not cancelled in the correlation function and leads to correlations which start negative at lower energies and then change to positive at a certain (rather high) energy. Their magnitude corresponds to a double pomeron exchange and so grows very large at high energies.

In conclusion we want to remark that since the gluon distribution in the nucleus is strongly shifted to higher momenta, the notorious difficulty of the diffusion towards very small momenta seems to be overcome. This problem always raised doubts of the validity of the perturbative treatment of hadronic amplitudes: even if the initial wave function is centered at high-momenta, its evolution in rapidity inevitably introduces low momentum contributions, for which the perturbative treatment is not valid. The large nucleus seems to damp these small momenta contributions, so that the use of the perturbation theory becomes justified.

5 References

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6 Figure captions

Fig. 1. The amplitude for hA scattering. Lines denote BFKL pomerons.
Fig. 2. Inclusive jet production from the lower (a) and upper (b) pomerons.
Fig. 3. Double inclusive jet production from the upper pomeron (a) and immediately
after the first splitting (b).
Fig. 1
Fig. 2a

Fig. 2b
Fig. 3a

Fig. 3b