Abstract

One of the most important and surprising discoveries in cosmology in recent years is the realization that our Universe is dominated by a mysterious dark energy, which leads to an accelerating expansion of space-time. A simple generalization of the standard Friedmann-Robertson-Walker equations based on General Relativity and the Cosmological Principles with the inclusion of a number of closed extra dimensions reproduces the currently observed data on dark energy, without the introduction of any cosmological constant or new particles. In particular, with a few extra dimensions, we obtain the redshift dependence of the deceleration parameter, the dark energy equation of state, as well as the age of the Universe in agreement with data, with essentially no free parameter. This model predicts that the extra dimensions have been compactified throughout the cosmic history, and it therefore suggests that signals from early Universe may give promising signatures of extra dimensions.
Recent advances in cosmological observations produce a large set of high quality data from which an unprecedented detailed knowledge of the universe can be extracted. Furthermore, a 'cosmic concordance' \[1\] has emerged from several different and independent observations such as Type Ia supernovae \[2, 3\], cosmic microwave background (CMB) anisotropies \[4\] plus large scale galaxy surveys \[5\] and Sachs-Wolfe effects \[6\], pointing to a universe with flat space-time dominated by a mysterious dark energy, which accounts for about 70% of the total energy content. The remaining 30% contribution is largely due to dark matter. Furthermore, the dark energy is characterized by an equation of state (EOS), \( P_\Lambda = w \rho_\Lambda \), relating its pressure \( P_\Lambda \) and energy density \( \rho_\Lambda \), and \( w < -1 \) is concluded from data \[7, 8, 9\], which gives rise to a repulsive force and thus an accelerating expansion of space-time. The evolution of \( w \), \( w(z) \), has also been traced \[7, 8\], leading to the conclusion that the universe had undergone an earlier deceleration before changing to the current accelerating phase. In addition to the usual \( \Lambda \)CDM model, many alternatives have been constructed to account for this dark energy. These include various quintessence models \[10, 11, 12\], Chaplygin gas model \[13\], modified gravity and scalar-tensor theories \[14, 15\] and so on. The nature of dark energy has become one of the most important research topics in cosmology and contemporary physics.

On the other hand, theories of extra dimensions have received a lot of attention in recent years perhaps because string theory also requires more than four dimensions of space-time \[16\]. Various Braneworld models \[17\] have been proposed, in which the extra dimensions need not be small. The hope that these large extra dimensions may provide a simple solution to the hierarchy problem and that they may be observed by upcoming experiments has created much excitement in this subject \[18\]. Cosmological models involving extra dimensions \[19, 20, 21, 22, 23\] to explain the current cosmic acceleration are also beginning to appear.

In this article, we adopt the simplest generalization of general relativistic cosmology by incorporating a few extra spatial dimensions to show that such a minimal extension is already adequate to reproduce the most important cosmological features, \textit{i.e.} the observed age of the universe, \( t_0 \), and the evolution of both \( w \) and the deceleration parameter \( q \equiv -\frac{\ddot{a}}{a}\left(\frac{\dot{a}}{a}\right)^2 \). Based on these simple assumptions, dark energy is shown to be a possible manifestation of extra dimensions. It is further argued that the best chance to probe the extra dimensions lies in the early universe \[24\].

2
The base of the present model is a $1 + 3 + n$ dimensional space-time with $n$ being the number of extra dimensions. With the assumption of homogeneity and isotropy in both the ordinary and the extra dimensions, the generalized Robertson-Walker metric that describes this space-time takes the following form:

$$ds^2 = dt^2 - a^2(t) \left( \frac{dr_a^2}{1 - k_a r_a^2} + r_a^2 d\Omega_a^2 \right) - b^2(t) \left( \frac{dr_b^2}{1 - k_b r_b^2} + r_b^2 d\Omega_b^2 \right),$$

where $r_a(r_b), \Omega_a(\Omega_b)$ are the radial and angular coordinates of ordinary (extra) dimensions, $a(t), b(t)$ and $k_a, k_b$ are the scale factors and curvatures of the ordinary three-dimensional space and the extra dimensions, respectively. The matter content in the Universe is assumed to be perfect fluid and the corresponding stress-energy tensor is given by

$$T^M_N = \text{diag} \left( \bar{\rho}, -P_a, -P_a, -P_b, -P_b, \ldots \right),$$

where $\bar{\rho}$ denotes the high dimensional energy density and $\bar{P}_a, \bar{P}_b$ the pressure in the ordinary and extra dimensions. Then the Einstein equations lead to the following $1+3+n$ dimensional Friedmann-Robertson-Walker (FRW) equations ($\bar{G}$ is the higher dimensional gravitational constant):

$$3 \left[ \left( \frac{\dot{a}}{a} \right)^2 + \frac{k_a}{a^2} \right] = 8\pi \bar{G} \bar{\rho} + \bar{\rho}_{\text{eff}},$$

$$2 \frac{\ddot{a}}{a} + \left[ \left( \frac{\dot{a}}{a} \right)^2 + \frac{k_a}{a^2} \right] = -8\pi \bar{G} \bar{P}_a - \bar{P}_{a,\text{eff}},$$

$$3 \frac{\ddot{a}}{a} + 3 \left[ \left( \frac{\dot{a}}{a} \right)^2 + \frac{k_a}{a^2} \right] = -8\pi \bar{G} \bar{P}_b - \bar{P}_{b,\text{eff}},$$

in which

$$\bar{\rho}_{\text{eff}} \equiv -\frac{n(n - 1)}{2} \left[ \left( \frac{\dot{b}}{b} \right)^2 + \frac{k_b}{b^2} \right] - 3n \frac{\dot{a} \dot{b}}{a b},$$

$$\bar{P}_{a,\text{eff}} \equiv n \frac{\dot{b}}{b} + \frac{n(n - 1)}{2} \left[ \left( \frac{\dot{b}}{b} \right)^2 + \frac{k_b}{b^2} \right] + 2n \frac{\dot{a} \dot{b}}{a b},$$

$$\bar{P}_{b,\text{eff}} \equiv (n - 1) \frac{\dot{b}}{b} + \frac{(n - 1)(n - 2)}{2} \left[ \left( \frac{\dot{b}}{b} \right)^2 + \frac{k_b}{b^2} \right] + 3(n - 1) \frac{\dot{a} \dot{b}}{a b}.$$

Recall that in theories involving extra dimensions the $1 + 3$ D energy density and Newtonian gravitational constant $\rho, G_N$ are related with $\bar{\rho}, \bar{G}$ by $\rho = \bar{\rho} V^n$ and $G_N = \bar{G}/V^n$ with $V^n$ being the volume of extra dimensions \cite{25}, we could replace the $\bar{G} \bar{\rho}$ in Eq. (3) by $G_N \rho$.

Obviously, the left-hand sides of Eqs. (3) and (4) are just the same as those in the $1 + 3$ D FRW equations. The effects of the extra dimensions are summarized in $\bar{\rho}_{\text{eff}}$ and $\bar{P}_{a,\text{eff}}$. 

3
which could be interpreted as the energy density and pressure of the geometry-induced matter first proposed by Einstein et al. [26, 27] and later developed by Wesson et al. [28]. In this sense the 1 + 3 D FRW equations are reproduced as parts of the higher dimensional ones and naturally we ask ourselves whether the observed dark energy can be explained by the effects of the extra dimensions.

The generalized FRW equations determine uniquely the evolutions of $a$ and $b$ once a complete set of initial conditions is given. A complete study of the various possibilities in the generalized FRW model will be presented in Ref. 24. The present work focuses on the case of $k_a = 0$, $k_b = 1$, $\bar{P}_a = \bar{P}_b = 0$ and $n = 7$. It is different from the model discussed in Ref. 23 where flat extra dimensions and non-flat ordinary dimensions are considered; the main results are also considerably different. The choices of $k_a$ and $\bar{P}_a$ follow from observational data, which imply that currently the ordinary dimensions are flat and matter is mostly non-relativistic. Our solutions do not depend on $n$ sensitively and the case of $n = 7$ is simply chosen for the sake of illustration.

The initial conditions of the generalized FRW equations in Eqs. (3)-(5) are taken from the current observed values of the Hubble parameter $H_0 = \dot{a}_0/a_0$, the deceleration parameter $q_0 = -\ddot{a}_0/a_0 H_0^2$, and the cosmological matter density parameter $\Omega_{m0} = 8\pi G \rho_0/3 H_0^2$, where a subscript '$0'$ indicates present day ($z = 0$) value. The Hubble parameter has been measured with many different techniques. For instance, the HST Key project [29] gives an estimation of $h = 0.72 \pm 0.03 \pm 0.07$ ($h \equiv H_0/100\text{Kms}^{-1}\text{Mpc}^{-1}$) based on the traditional distance ladder approach, Treu and Koopmans [30] obtain $h = 0.59^{+0.12}_{-0.07} \pm 0.03$ using gravitational lens time delay method, Reese et al. [31] combines the Sunyaev-Zel’dovich and X-ray flux measurements of galaxy clusters to find $h = 0.6 \pm 0.04^{+0.13}_{-0.18}$, and Jimenez et al. [32] use the constraints from the absolute ages of Galactic stars and the observed position of the first peak in the CMB angular power spectrum to obtain $h = 0.69 \pm 0.12$. We adopt the range $0.58 < h < 0.70$ suggested by these measurements. The current deceleration factor is extracted from the allowed values in Ref. 8: $q_0 \simeq -1.3 \sim -0.4$ (with $-0.6 \gtrsim q_0 \gtrsim -1.1$ at the 1$\sigma$ C. L.). Finally, we adopt the typical value of $\Omega_{m0} = 0.3$. With the values of $h, q_0, \Omega_{m0}$ and thus $\dot{a}_0/a_0, \ddot{a}_0/a_0$ fixed, Eqs. (3)-(8) are used as simple algebraic equations to solve for the values of $\dot{b}_0/b_0$, $\ddot{b}_0/b_0$ and $k_b/b_0^2 = 1/b_0^2$. The initial conditions are then completely specified such that the same group of equations can be integrated backward in
FIG. 1: The time evolution of $a$ and $b$ with $a_0$ set to be 1 by convention. Note that the largeness of $b$ is only the result of normalizing the arbitrary (positive) $k_b$ to be 1 and such that $a$ and $b$ are not comparable physically. The current time is $t = 0$.

time to obtain $a(t), b(t), q(t), w(t) \equiv \bar{P}_{a,eff}(t)/\bar{\rho}_{eff}(t)$ and the cosmic age $t_0$.

Figure 1 shows the time evolutions of $a$ and $b$, with $h = 0.62$, $q_0 = -1.1$ and $a$ set to be 1 at present ($t = 0$). Qualitatively speaking, the present model reproduces the observed deceleration-acceleration transition for the ordinary dimensions in which the extra dimensions shrink continuously throughout the cosmic history. The cosmic age $t_0$, i.e. the time between $a = 0$ and $a = 1$ (for our realistic purpose the duration of the inflationary and radiation-dominated eras is so tiny compared with the matter-dominated period that we could treat the latter as $t_0$ approximately), is found from the figure to be $\approx 12.6$ Gyr in good agreement with values quoted in literature: a lower limit of cosmic age is given by the age of the oldest globular clusters plus $0.2 \sim 0.3$ Gyr [32], leading to a typical value of 12 Gyr [33]. To observe the reality of our model, we show in Fig. 2 the dependence of model-predicted $t_0$ on $q_0$ and $h$ for the $n = 7$ case with the observational bounds from $q_0$ and $t_0$ marked by grey area. This result shows that a large allowed region exists that gives
FIG. 2: The dependence of the model-predicted cosmic age $t_0$ on different choices of $q_0$, $h$, $\Omega_{m0}$ and $n$. The grey patch represents the allowed area of the parameters according to observations: $-0.6 \gtrsim q_0 \gtrsim -1.1$ as the 1σ C. L. region given in Ref. 8 and $t_0$ is taken to be $\geq 12.0$ Gyr. The values of parameters are labelled aside.

solutions consistent with observed data. It is emphasized that, although $n$ is a free chosen parameter, its variation does not change the results significantly. For example, the cosmic age in the case of $n = 3$ differs from that for $n = 7$ by only $\sim 5\%$.

A tighter constraint on a model of the dark energy is the redshift dependence of its EOS, $w(z)$. For the chosen set of parameters above, we show the $w(z)$ calculated from our model in Fig. 3. We obtain $w_0 = w(z = 0) \simeq -1.5$ and $w'_0 = dw(z = 0)/dz \simeq 4$. These results agree with several recent analyses of supernova data, including those given by De Pietro & Claeskens [7]: $w_0 \simeq -1.4$, $-12 < w'_0 < 12$, Riess et al. [8]: $w_0 = -1.31^{+0.22}_{-0.28}$ and other parametric reconstructions of $w(z)$ using the current observational data [34, 35, 36, 37], but not very well with the reported value of Ref. 8: $w'_0 = 1.48^{+0.81}_{-0.90}$. However, the present results lie at the boundary of the 1σ contour of the estimation using the gold sample without HST data in Ref. 8. This discrepancy could be due to their simple assumption in the fitting
FIG. 3: The redshift dependence of dark energy EOS, $w(z)$. Two sets of observational results ($z \geq 0$) extracted from Supernovae data are represented by error bars and grey region $^{34,35}$ and the model prediction by solid curve. Case of a cosmological constant ($w(z) = 1$) is denoted by dashed line. Current state of the Universe in the $w - z$ plane is also marked.

relation $w(z) = w_0 + w'_0 z$, which is valid only when $z$ is small. If $w(z)$ bends down as $z$ increases, as the present model (Fig. 3) suggests, the said fitting will underestimate $w'_0$. This underestimation may explain why the contours in the $w_0 - w'_0$ plane shift downward when higher-redshift HST discovered data are included in Ref. 8. It is predicted that a lower $w'_0$ will be found when even higher-redshift data are used.

Fig. 4 shows the redshift dependence of the deceleration parameter $q(z)$. The deceleration parameter is positive and nearly constant ($\sim 0.8$) for large $z$ but changes rapidly to a negative value ($\sim -3.8$) between $z \sim 1$ and $z \sim -0.5$ with the transition from deceleration ($q > 0$) to acceleration ($q < 0$) occurring at $z \sim 0.3$, consistent with the data-fitting result given in Ref. 33. The current state of the universe in the $q_0 - q'_0$ plane is around $(-1.1, 5)$, slightly outside the $3\sigma$ C. L. according to Ref. 8, which again is expected to be improved if the simple linear fitting is modified to include higher order terms.
FIG. 4: Same as Fig. 3, but for the redshift dependence of $q$. Grey region represents observational data \cite{34,35} and solid (dashed) line the prediction of our ($\Lambda$CDM) model. The states of the Universe in the $q - z$ plane at current time and at the time of deceleration-acceleration transition are marked.

The robustness of the present model could be shown in the flow plot \cite{23} of the solutions of Eqs. (3)-(8), as in Fig. 5 for a pressureless $n = 7$ universe. For $k_a = 0$, the evolution of any solution can be represented on a 2D plane formed by the two parameters $(X_b \equiv k_b/b^2 u^2, Y \equiv v/u)$ where $u \equiv \dot{a}/a$ and $v \equiv \dot{b}/b$. The arrows indicate the direction of time evolution. Four representative paths are shown as white lines labelled 1-4, and the regions corresponding to $\rho > (\rho <) 0$ and $\ddot{a} > (\ddot{a} <) 0$ are also marked. Only solutions flowing in the $\rho > 0$ region are considered. There are four finite fixed points, labelled A-D, with C being the only stable one. Both of the flow patterns 1 and 2 possess a transition from deceleration to acceleration in agreement with observational results while they differ in that pattern 2 also exhibits an initial acceleration phase. However, the current Hubble constant and deceleration parameter indicate that pattern 1 offers a better description of the observable universe, because the cosmic age associated with the pattern 2 solution is typically

8
FIG. 5: Flow plot showing the evolution of solutions on the \((X_b \equiv k_b/b^2 u^2, Y \equiv v/u)\) plane. Regions corresponding to \(\rho < (>) 0\) and \(\ddot{a} < (>) 0\) are shown. Four representative evolution paths are given by white lines (1-4) and four fixed pointed are indicated (A-D).

much shorter than the lower bound imposed by globular cluster age \([4, 33]\). It is noted that the early deceleration to later acceleration evolution of ordinary dimensions in our model is a robust feature with respect to small perturbations in the cosmological parameters and, again, varying \(n\) does not alter the flow patterns significantly. But on the other hand, the curvature of the extra dimensions is critical: solutions corresponding to \(k_b \leq 0\) exhibit (1) either a strictly accelerating or decelerating phase or (2) a transition from acceleration to deceleration in contradiction to observation.

The concept of a running Newtonian gravitational constant has received some attention in recent literature. If gravity propagates freely in all of the ordinary and extra dimensions,
the effective 4D Newtonian constant \( G_N \) should vary with time according to

\[
\frac{\dot{G}_N}{G_N} = -n \frac{\dot{b}}{b}.
\]  

(9)

It seems that Eq. (9) poses a severe constraint on \( \dot{b}/b \). However, it is doubtful whether constraints on \( \dot{G}_N/G_N \) derived from laboratory, solar system, pulsar tests and so on are applicable to our cosmological model: these tests are dominated by local gravitational fields and are more appropriately described by static metrics, rather than the cosmological metric given in Eq. (1). As a result, they do not ‘feel’ the effect of cosmological changes such as \( \dot{a} \). And similarly, \( \dot{b} \), being cosmological in nature, will be not easily probed by these tests. So we will not consider this specious constraint on the time-evolution of extra dimensions in our model.

The results of the present work may have important implications in cosmology. First of all, the present model shows that the extra dimensions compactify throughout the cosmic history, with an exponential deflationary phase extending up to \( z \sim 10 \). It suggests that the most prominent signature of extra dimensions is likely to be found in cosmological signals, such as CMB. Secondly, the main point of this article is that dark energy is a possible manifestation of extra dimensions. Lastly, the fate of the universe in our model is characterized by a continuously accelerating expansion of the ordinary dimensions accompanied by accelerating compactification of the extra dimensions. Eventually, \( b \) becomes so small that quantum gravity dominates.

In summary, we have shown that a generalized FRW cosmology which includes a few extra closed spatial dimensions predicts the observed expansion history of the universe without any cosmological constant. Dark energy arises as the effect of the evolution of the extra dimensions, and its observed equation of state is reproduced with essentially no free parameter. The solution is robust. It suggests that much more prominent effects of the extra dimensions may be observable in cosmic signals from the early universe.

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