Research on global actual measurement of indoor surface flatness and verticality Based on sparse point cloud

Zhongyue Zhang\textsuperscript{1,4}, Huixing Zhou\textsuperscript{1,4}, Shun Wang\textsuperscript{2,4}, Yannan Lv\textsuperscript{1,4}, Xiaoyu Zheng\textsuperscript{1,4}, Langzhao Zeng\textsuperscript{3}

\textsuperscript{1}School of Mechanical-electronic and Vehicle Engineering, Beijing University of Civil Engineering and Architecture, Beijing 100044, China
\textsuperscript{2}School of Civil and Transportation Engineering, Beijing University of Civil Engineering and Architecture, Beijing 100044, China
\textsuperscript{3}School of Foreign Studies, Minzu University of China, Beijing 100081, China
\textsuperscript{4}Beijing Engineering Research Center of Monitoring for Construction Safety, Beijing 100044, China

Corresponding author’s e-mail address: 2108550020058@stu.bucea.edu.cn

Abstract. At present, it has become a trend to realize indoor actual measurement by reconstructing indoor 3D information with dense point cloud. Considering the large time consumption of traditional manual measurement and data overload of 3D reconstruction for a construction, the paper presents a global measurement method based on sparse point cloud for indoor actual measurement of flatness and verticality. First, given the point cloud is present with low density, our data size is much smaller than that in 3D reconstruction, which greatly reduces time consumption for data processing. Second, we calculate surface flatness degree by using a tolerance formula in machinery industry as reference after we conduct fitting of surfaces by least square method. As for the measurement of verticality, making use of the more orderly and effective point cloud clustering plus surface fitting based on sparse point cloud, we are able to calculate the verticality degree easily by a mathematical method. Compared with existing methods, this approach might feature a more tailored or specialized measurement method for actual measurement of flatness and verticality in acceptance of construction work in indoor scenes. To validate this approach, we present our experimental results and examine the systematic error by the mathematical assessment modelling. It is proved that the systematic error of the measurement method based on sparse point cloud is nearly ignorable.

1. Introduction
In a construction project, indoor actual measurement plays an important role in project tolerance management throughout a project planning, where the traditional manual measurement method is still mostly used [1]. However, it can be seen that more advanced measurement methods will give rise to higher efficiency and precision of measurement of quantities like flatness and verticality of surfaces [2]. At present, with the increase of labor cost and the progress of technology, automatic measurement methods are gradually replacing traditional manual measurement in acceptance of construction work [3].

The traditional manual measurement has inherent defects such as the limited measurement range by hand in indoor environment and the low time-efficiency and high cost of pure manual work. Under this background, we need to adopt a more global actual measurement method for flatness and verticality of
surfaces in indoor environment, which can get a more comprehensive and large-range measurement result of indoor structures. Therefore, merits of 3D LiDAR are highlighted. For example, 3D laser scanning equipment is able to analyze complicated scenes and realize 3D reconstruction of cultural relics, tunnels, bridges and other civil engineering constructions by extracting point cloud information of them. Thus, the acceptance of construction work is simplified [4].

However, comparatively speaking, for indoor measurement for flatness and verticality, 3D reconstruction method seems to unduly capture and process 3D scanning data from an indoor environment. As a matter of fact, a large number of relevant literatures focus on the indoor measurement method for varied scenes. Y Sun set up framework called patch graph convolution network (PGCNet), which can achieve effective performance equivalent to state-of-the-art for segmenting standard indoor scene dataset [5]. H Wu proposed to use few labelled samples to solve the problem of the requirement for abundant labelled samples for training with deep learning classification methods, which can enhance the efficiency of training [6]. Jung, J proposes a scheme for automated 3D geometric modeling of indoor structures, including regularized boundary by means of the constrained least-squares method to get better positional accuracies [7].

Despite their meaningful attempts to discuss approaches to point cloud clustering and information matching, they may be apt to ignorance of the relatively excessive time consumption of point cloud data collecting, considering that the scanned object is confined to indoor environment. Therefore, we may need to propose a new actual measurement method targeted at the measurement of flatness and verticality surfaces in indoor scenes. With reference to relevant literatures of a group of experts [8-10], we believe that the key goals of the specialized method include the following three points: 1). Searching measurement requirements or standards for flatness and verticality with a smaller size of point cloud data. 2). Achieving more simple and effective point cloud clustering. 3). Matching well the measurement results with standards in traditional and mature manual acceptance of construction work.

To achieve the three main goals of a specialized or tailored measurement of flatness and verticality of indoor surfaces, we propose three main solutions: 1) Referring to the concept of interspace of point clouds mentioned in ACI building tolerance standard [11], we abandon the high-density point cloud collection in 3D indoor reconstruction, but collect sparse point cloud to process and build a set of new standards for measurement of flatness and verticality of indoor surfaces. 2) This paper proves that sparse point cloud will help improve the orderliness and effectiveness of point clouds clustering and reveal the pattern of curve correlation. 3)least squares method is employed to make horizontal and vertical surface fitting so that calculation of flatness and verticality degree is achievable based on sparse point cloud. Based on fitted surfaces, we use for reference the assessment formula for the roughness of 3D surface in machinery industry to calculate the flatness degree of surfaces and calculate verticality degree by formula. By mathematical method rather than relatively time-consuming, less accurate and small-range manual measurement. Base on the consideration as above, we launch a research on the global indoor actual measurement of flatness and verticality based on sparse point cloud and carry an experiment on it.

2. Indoor global measurement method

2.1. Sparse points and the maximum interspace of point clouds

In order to obtain smaller size of point cloud data, we propose a method of extracting sparse point cloud. First, in the idealized construction model of 6 surfaces, we identify the largest fitted surface and calculate the number of points on it, marked as \( n_s \), and the \( s_{\text{max}} \) is the area of the largest surface. (see Figure 1). Calculate it in the formula \( s_0 \) as below. According to ACI building tolerance standard, the maximum interspace of point \( s = 303 \text{ mm} \) [13], so the value of \( s_0 \) cannot surpass 303mm. \( s_0 \) is estimated as the maximum interspace of point clouds captured in our experiment.

\[
s_0 = \sqrt{\frac{s_{\text{max}}}{n_s}} \quad (1)
\]
2.2 Mathematical method for calculating indoor surface flatness degree based on sparse-point cloud surface fitting

In the whole process of building construction acceptance, flatness information has different acceptance indicators in different stages of construction technology, but from the essence of the measurement information of these indicators, they are the collection of data used to solve flatness related problems in indoor information. Combined with the above analysis and comparison, the flatness problem can be regarded as a large-scale surface roughness. As shown in the figure (see Figure 2), taking the flatness of the wall as an example, the traditional measurement method with a guiding rule is to obtain the wall flatness by measuring the gap between the guiding rule and the wall. However, we measure the flatness of surfaces by sparse point cloud.

![Figure 1. sparse point cloud and interspace model.](image)

![Figure 2. Comparison between traditional flatness measurement and global flatness measurement.](image)

After the completion of the global point cloud data collection, they will be segmented according to the indoor point cloud clustering rules. In the ideal hexahedral indoor space, all the point clouds will be segmented into six clusters. According to 3D quadrant coordinate of point cloud clusters in the 3D coordinate system, they are marked as 6 surfaces. Each points cloud cluster represents a 3D fitted surface.

The wall surface equation is expressed as:

\[ Ax + By + Cz + D = 0 \]  
\[ \text{After transformation, the fitting plane of the wall surface can also be expressed as:} \]

\[ f(x, y, z) = ax + by + cz + 1 = 0 \]

For point set of point cloud cluster \( M \{(x_1, y_1, z_1), (x_2, y_2, z_2), ..., (x_n, y_n, z_n)\} \).

The best fitting plane of point cloud cluster \( M \) is:

\[ \sum_{i=1}^{n} [f(x, y, z) - f(x_i, y_i, z_i)]^2 = \text{min} \]  

So:

\[ g = \sum_{i=1}^{n} (ax_i + by_i + cz_i + 1)^2 = \text{min} \]

In order to make function \( g \) hold, partial derivatives of \( a, b \) and \( c \) should be obtained:

\[ \frac{\partial g}{\partial a} = 0, \frac{\partial g}{\partial b} = 0, \frac{\partial g}{\partial c} = 0 \]

The results are as follows:
\[ a \sum_{i=0}^{n} x_i^2 + b \sum_{i=0}^{n} x_i y_i + c \sum_{i=0}^{n} x_i z_i = -\sum_{i=0}^{n} x_i \]
\[ a \sum_{i=0}^{n} x_i^2 + b \sum_{i=0}^{n} y_i^2 + c \sum_{i=0}^{n} y_i z_i = -\sum_{i=0}^{n} y_i \]  \hspace{1cm} (6)
\[ a \sum_{i=0}^{n} x_i z_i + b \sum_{i=0}^{n} z_i y_i + c \sum_{i=0}^{n} z_i^2 = -\sum_{i=0}^{n} z_i \]

\[ Q = \begin{bmatrix}
\sum_{i=0}^{n} x_i^2 & \sum_{i=0}^{n} x_i y_i & \sum_{i=0}^{n} x_i z_i \\
\sum_{i=0}^{n} x_i y_i & \sum_{i=0}^{n} y_i^2 & \sum_{i=0}^{n} y_i z_i \\
\sum_{i=0}^{n} x_i z_i & \sum_{i=0}^{n} z_i y_i & \sum_{i=0}^{n} z_i^2 
\end{bmatrix} \]  \hspace{1cm} (7)

\[ X = [a, b, c]^T \]  \hspace{1cm} (8)

\[ K = \begin{bmatrix}
-\sum_{i=0}^{n} x_i \\
-\sum_{i=0}^{n} y_i \\
-\sum_{i=0}^{n} z_i 
\end{bmatrix} \]  \hspace{1cm} (9)

Coefficient of plane equation:
\[ X = Q^{-1}K \]  \hspace{1cm} (10)

For \( a, b, c \) in \( X \) set:
\[ a = \frac{A}{D}, b = \frac{B}{D}, c = \frac{C}{D} \]  \hspace{1cm} (11)

Deviation set of all point clouds in point cloud cluster \( \{d_i\} \):
\[ d_i = \left| Ax_i + By_i + Cz_i + D \right| \sqrt{A^2 + B^2 + C^2} \]  \hspace{1cm} (12)
\[ m_0 = \sqrt{\frac{\sum_{i=1}^{n} d_i^2}{n-3}} \]  \hspace{1cm} (13)

2.3 Mathematical method for calculating surface verticality degree based on sparse-point cloud surface fitting
In the traditional manual measurement of indoor surface verticality, guiding ruler is the key tool. During construction acceptance, the guiding ruler is placed basically vertical to the ground (see Figure 3), specifically, with the bubble in the guiding ruler is located at the middle of the horizontal ruler scale. As shown in the figure, the length of the guiding ruler is known to be \( r \), the maximum allowable deviation distance is \( e \), and the guiding ruler is vertical to ground at 90 degrees. Therefore, \( \gamma \) (the deviation angle between the wall and the guiding rule) can be figured out.
\[ l = \sqrt{r^2 + e^2} \]  \hspace{1cm} (14)
\[ \gamma = \arccos\left(\frac{r^2 + l^2 - e^2}{2 \times r \times l}\right) \]  \hspace{1cm} (15)
The actual angle between the wall and the ground is $\theta$:

$$\theta = 90^\circ + \arccos\left(\frac{r^2 + l^2 - e^2}{2 \times r \times l}\right)$$

(16)

In the process of global actual measurement of indoor surface verticality, with the indoor surface fitting equation (the plane fitting equation is from least squares), we measure the angle between every couple of surfaces. The fitting equation is as follows.

For two surfaces, their surface fitting equations are as:

$$A_s x + B_s y + C_s z + D_s = 0, A_j x + B_j y + C_j z + D_j = 0$$

(17)

Solving the angle between two surfaces:

$$\theta = \arccos\left(\frac{A_s A_j + B_s B_j + C_s C_j}{\sqrt{A_s^2 + B_s^2 + C_s^2} \sqrt{A_j^2 + B_j^2 + C_j^2}}\right)$$

(18)

In practical measurement, the surface selection depends on indoor structure. In many cases, ceiling is the alternative to ground.

3. Application of the global actual measurement method based on sparse point cloud in our experiment

3.1 Experiment conditions

In order to examine the effectiveness of the global actual measurement of indoor surface flatness and verticality based on sparse point cloud, we applied the global measurement method in indoor scenes in this experiment. In this experiment, the point cloud data is collected by our innovated 3D LiDAR in an existing home covering a floor area of 17 square meters (see Figure 4) with its structure in conformity to the 3D model mentioned in the paper and then is processed on MATLAB 2016B in Windows10.

3.2 The analysis of time consumption in visualization system

In the global actual measurement of surface flatness and verticality, with the growth of the point cloud data size, the indoor conditions will be well revealed by data visualization in MATLAB. However, the growth of data size gives rise to the load of algorithm. (see Figure 5)
To explicitly examine the time-efficiency of sparse point clouds in the processing of data, this paper takes the visualization program of extracting point cloud data of varied sizes as examples by calculating the time-consumption of it with MATLAB. (see Figure 6) The results are shown in the graph above—the two times rise of the data size of point cloud will lead to exponential rise of the time consumption of program operation. Thus, based on the premise that measurement is undertaken in circumstances in line with standard acquirements, the smaller size of point cloud, the more time-friendly.

3.3 The experiment results of the global actual measurement of indoor surface flatness and verticality.

3.3.1 The accuracy of the global actual measurement of surface flatness based on sparse point cloud
Weighing the time of processing point cloud and the concept of sparse point cloud model, we choose the indoor point cloud with the number of point clouds (n = 3000) for experimental measurement. In this magnitude, the maximum distance of any two points cloud is s = 114.184mm, which is below the maximum number of coefficient point cloud model. In this paper, the plane fitting and surface roughness solution calculation are by the equations (2)-(13). In the experiment, the sftool toolbox in MATLAB is used to complete the three-dimensional surface fitting. As shown in Table 1, by the means of the global
measurement proposed, the indoor flatness is \( m_0 \), and the flatness result obtained by traditional manual measurement in the same scene is \( m_c \). Taking the traditional manual measurement results as the true value, the measurement error of this method is less than 1.6 mm.

**Table 1. Flatness measurement results of different planes.**

| Measurement objects | The fitting plane | \( m_0 \) | \( m_c \) |
|---------------------|------------------|---------|---------|
| Plane A             | 192.3x-0.009961y-z-330100=0 | 3.781   | 2.45    |
| Plane B             | 0.0001934x-1194y-z+2423000=0 | 1.3352  | 2.1     |
| Plane C             | 5.237e^x-7.944e^y-z+1246=0 | 1.435   | 1.8     |
| Plane D             | -216.6x-0.04866y-z+412000=0 | 3.444   | 2.28    |
| Plane E             | 0.09517x-813.8y-z-1697000=0 | 1.397   | 1.95    |
| Plane F             | -2.752e^x-1.002e^y-z-1280=0 | 1.4347  | 1.64    |

3.3.2 The measurement of verticality of fitted walls.

After the measurement for indoor horizontal surfaces, we gain a formula for the measurement results of all the fitted surfaces. Based on these equations (14)-(18), we are able to figure out the verticality of fitted surfaces according to the formula, the entire measurement results of verticality in the experiment are revealed in the following Table 2. It proves that the measurement error of the global actual measurement of fitted surfaces verticality do not surpass 0.12 degrees.

**Table 2. Errors of global verticality measurement with manual results as standard.**

| Measurement objects | Measurement results of global measurement method (°) | Measurement results of manual method (°) | Error (°) |
|---------------------|------------------------------------------------------|-----------------------------------------|-----------|
| A                   | 89.7018                                              | 89.8056                                 | 0.1038    |
| B                   | 89.9526                                              | 89.9654                                 | 0.0128    |
| D                   | 89.7018                                              | 89.8056                                 | 0.1038    |
| E                   | 89.9526                                              | 89.9654                                 | 0.0128    |

4. Conclusion

In conclusion, global actual measurement of indoor surface flatness and verticality based on sparse point cloud is a comprehensive and specialized measurement method for indoor measurement in construction works. It has following merits, validated by the experiment results present in this paper: 1). It is able to cover larger measurement range than traditional manual measurement method. 2). Smaller size of point cloud data reduces the load of data processing, making measurement more efficient. For instance, in our experiment to measure a room in size of 4120×3620×2526, we merely need to process 3000 points clouds, superior to 3D LiDAR measurement in the aspect of time consumption. 3). It achieves higher orderliness of point cloud clustering because it is performed in visualization system in MATLAB based on sparse point cloud. 4). Surface fitting based on sparse point cloud allows us to figure out flatness and verticality of indoor surfaces in mathematical method. With surfaces fitting by least square method, the distance deviation of points from the fitted surfaces is able to be used in the calculation of flatness by referring to a tolerance formula. Plus, the verticality of indoor surfaces, typically walls to horizontal surface, is calculated by the angle between two fitted surfaces. Furthermore, the flatness measurement error is less than 1.6mm and verticality error less than 0.12 degrees. Additionally, the systematic error of the method is examined through a mathematical assessment modelling.

Because of the lower cost and efficient method, researching for the global actual measurement based on sparse point cloud is meaningful to realize the automatic measurement in construction. Although this paper proposed the indoor actual measurement method with sparse point cloud and made a simple attempt, there are still a lot of related work needed to do, including: indoor point cloud cluster segmentation of sparse point cloud, influence of cloud density at different points on measurement...
accuracy and how to reduce the systematic error of measurement equipment etc. Our team will continue to research relevant issues in the future.

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