Monte Carlo Information-Oriented Planning (Revised version)

Vincent Thomas 1 and Géremy Hutin 2 and Olivier Buffet 3

Abstract. In this article, we discuss how to solve information-gathering problems expressed as ρ-POMDPs, an extension of Partially Observable Markov Decision Processes (POMDPs) whose reward ρ depends on the belief state. Point-based approaches used for solving POMDPs have been extended to solving ρ-POMDPs as belief MDPs when its reward ρ is convex in B or when it is Lipschitz-continuous. In the present paper, we build on the POMCP algorithm to propose a Monte Carlo Tree Search for ρ-POMDPs, aiming for an efficient on-line planner which can be used for any ρ function. Adaptations are required due to the belief-dependent rewards to (i) propagate more than one state at a time, and (ii) prevent biases in value estimates. An asymptotic convergence proof to ϵ-optimal values is given when ρ is continuous. Experiments are conducted to analyze the algorithms at hand and show that they outperform myopic approaches.

Preliminary note. This article is a revised version of the ECAI 2020 paper. This version better highlights the differences between our contribution, renamed ρ-POMCP(β), and the original ρ-POMCP algorithm [3]. We thus recommend reading and citing this revised version rather than the original ECAI 2020 paper.

1 INTRODUCTION

Many state-of-the-art algorithms for solving Partially Observable Markov Decision Processes (POMDPs) rely on turning the problem into a “fully observable” problem—namely a belief MDP—and exploiting the piece-wise linearity and convexity of the optimal value function in this new problem’s state space (here the belief space B) by maintaining generalizing function approximators. This approach has been extended to solving ρ-POMDPs as belief MDPs—i.e., problems whose performance criterion depends on the belief (e.g., active information gathering)—when the reward ρ itself is convex in B [2] or when ρ is Lipschitz-continuous [10].

In this paper, we propose two new algorithms for solving ρ-POMDPs which do not rely on properties of ρ such as its convexity or its Lipschitz-continuity, but are based on Monte Carlo sampling, are inspired by POMCP [21] and, while developed independently, extend the general direction proposed by Barto et al. [3]. These algorithms are : (1) ρ-beliefUCT, which applies UCT to the belief MDP; and (2) ρ-POMCP(β), which uses particle filters, i.e., samples particles during trajectories, to estimate the visited belief states and their associated rewards. We prove ρ-POMCP(β)’s asymptotic convergence and empirically assess these algorithms on various information-gathering problems.

The paper is organized as follows. Section 2 discusses related work on information-oriented control. Sec. 3 presents background (i) first on POMDPs, (ii) then on the Partially Observable Monte Carlo Planning (POMCP) algorithm, a Monte Carlo Tree Search approach for solving POMDPs, and (iii) finally on ρ-POMDPs. Sec. 4 describes our contribution: ρ-beliefUCT and ρ-POMCP(β), for solving ρ-POMDPs with MCTS techniques, and provides a proof of ρ-POMCP(β)’s convergence. Sec. 5 presents the conducted experiments and analyzes the results before concluding and giving some perspectives.

2 RELATED WORK

Early research on information-oriented control (IOC) involved problems formalized either (i) as POMDPs (as Elwertovich et al. [9] did recently, since an observation-dependent reward can be trivially recast as a state-dependent reward), or (ii) with belief-dependent rewards (and mostly ad-hoc solution techniques as [11,12]).

[Araya-López et al.] introduced ρ-POMDPs to easily formalize most IOC problems. They showed that a ρ-POMDP with convex belief-dependent reward ρ can be solved with modified point-based POMDP solvers exploiting the piece-wise linearity and convexity (PWLC) property, with error bounds that depend on the quality of the PWLC-approximation of ρ. More recently, Fehr et al. [10] applied the same approach as Araya-López et al., but for Lipschitz-continuous (LC)—rather than convex—belief-dependent rewards, demonstrating that, for finite horizons, the optimal value function is also LC. Then, deriving uniformly improvable lower- (and upper-)bounds led to two algorithms based on HSVI [22].

[Spaan et al.]’s POMDP-IR framework allows describing IOC problems with linear rewards which are provably equivalent to “PWLC” ρ-POMDPs (i.e., when ρ is PWLC) [19], and also leads to modified POMDP solvers. For its part, the general ρ-POMDP framework allows formalizing more problems—e.g., directly specifying an entropy-based criterion.

We here consider ρ-POMDPs, but not relying on generalizing (PWLC or LC) value function approximators as in previous work, so that any belief-dependent reward can be used. For instance, this allows addressing problems where the objective is to minimize the quantity of information of an adversary with known behaviour or where the objective is to gather information on specific state variables while maintaining a low quantity of information on confidential ones (like in medical or domotic fields). None of these problems can be solved by previous approaches on ρ-POMDP nor be modelled by POMDP-IR, which requires a PWLC reward function in B. To circumvent this difficulty, we build on Monte Carlo Tree Search (MCTS) approaches,
3 BACKGROUND

3.1 POMDPs

A POMDP \cite{Silver2010} is defined by a tuple $(S, A, Z, P, r, \gamma, b_0)$, where $S$, $A$ and $Z$ are finite sets of states, actions and observations; $P_{a,s}(s,s')$ gives the probability of transiting to state $s'$ and observing $z$ when applying action $a$ in state $s$ ($P_{a,s}$ is an $S \times S$ matrix); $r(s,a) \in \mathbb{R}$ is the reward associated to performing action $a$ in state $s$; $\gamma \in [0,1]$ is a discount factor; and $b_0$ is the initial belief state—i.e., the initial probability distribution over possible states. The objective is then to find a policy $\pi$ that prescribes actions depending on past actions and observations so as to maximize the expected discounted sum of rewards (here with an infinite temporal horizon).

To that end, a POMDP is often turned into a belief MDP $\langle B, A, P, r, \gamma, b_0 \rangle$, where $B$ is the belief space, $A$ is the same action set, and $P_{a,b}(b'|b,a) = P(b'|b,a) = \sum_{s \in S} b(s)P(a,s) r(s,a)$ are the induced transition and reward operators. This allows considering policies $\pi: B \rightarrow A$, and their value functions $V^\pi(b) = E[\sum_{t=0}^{\infty} \gamma^t r(b_t, \pi(b_t))] | b_0 = b$. Optimal policies maximize $V^\pi$ in all belief states reachable from $b_0$. Their value function $V^\ast$ is the fixed point of Bellman’s optimality operator \( (H) \) $HV : b \mapsto \max_a \left[ r(a,b) + \gamma \sum_{z} [P_{a,z}(b)||V^\ast(b')]], \right.$ and acting greedily with respect to $V^\ast$ provides such a policy.

3.2 MCTS for POMDPs

MCTS and UCT MCTS approaches \cite{Silver2010} are online, sampling-based algorithms, here described in the MDP framework (while they also serve in settings like sequential games). In MCTS, the tree representing possible futures from a starting state is progressively built by sampling trajectories in a non-uniform way. Each iteration consists in 4 steps: (i) selection: a trajectory is sampled in the tree according to an exploration strategy until a node not belonging to the tree is reached; (ii) expansion: this new node is added to the tree; (iii) simulation: this new node’s value is estimated by sampling a trajectory from this node according to a rollout policy; (iv) backpropagation: this estimate and the rewards received during the selection step are back-propagated to the visited nodes to update their statistics (value and number of visits). Upper Confidence Bound applied to trees (UCT) is an instance of MCTS where, when in a state node, the next action is selected using the Upper Confidence Bounds (UCB1) strategy, i.e., picking an action so as to maximize its estimated value increased by an exploration bonus $c_{t,a} = C_p \sqrt{\log(N(s))/N(s,a)}$, with $C_p > 0$ an exploration constant, $N(s)$ the number of past visits of node $s$, and $N(s,a)$ the number of selections of action $a$ when in node $s$.

POMCP Silver and Veness \cite{Silver2010} proposed the Partially Observable Monte Carlo Planning (POMCP) algorithm to apply MCTS to solving POMDPs. A POMDP is addressed through the corresponding belief MDP, a belief tree being made of alternating action and belief nodes as presented in Figure 1. The path to a belief node at depth $t$ follows the action-observation history $h_t = (a_0, z_0, a_1, z_1, \ldots, z_t)$ leading from the root belief to this belief $b(h_t)$.

Applying directly UCT on the belief-MDP would imply sampling trajectories $(b_0, a_0, b_1, a_1, \ldots, b_t)$ in belief space, thus requiring the complete POMDP model and a high computational cost to derive exact belief states. To prevent this cost, POMCP samples trajectories in state space, which only requires a simulator $G$ as a generative model of the POMDP. During the selection phase, the first state is sampled from the root belief estimate, then one alternates between (i) picking an action according to UCB1 (applied to estimated action values in the current belief node), and (ii) sampling a next state $s'$, observation $z$ and reward $r$ using $G(s,a)$. By accumulating values in the belief nodes, averaging over all simulated trajectories gives an estimate of the value $V(h)$ of the belief node $h$.

![Figure 1: Example of a belief tree with 2 actions: $a_1$ and $a_2$, and 2 observations: $z_1$ and $z_2$, with various quantities maintained by POMCP.](image-url)
Moreover, states are collected in each visited belief node in order to estimate the next root belief when an action is actually performed. By preventing the computation of exact beliefs $b(h)$ in each belief node, POMCP allows addressing large problems while preserving UCT’s asymptotic convergence.

Silver and Veness also proposed the PO-UCT algorithm as a first step towards POMCP. It differs from POMCP in that it does not collect states, but computes the belief state of the new root with an exact Bayes update.

3.3 $\rho$-POMDPs

$\rho$-POMDPs [3] differ from POMDPs in that their reward function $\rho$ is belief-dependent, thus allowing to define not only control-oriented criteria, but also information-oriented ones, thus generalizing POMDPs. The immediate reward $\rho$ is naturally defined as $\rho(b,a,b')$, the immediate reward associated to transitioning from belief $b$ to belief $b'$ after having performed action $a$.4

As presented in related work, [Araya-Lopez et al.][2] and Fehr et al. [10] have exploited PWLC and Lipschitz-continuous reward functions $\rho$ to solve general $\rho$-POMDPs. However, while many problems can be modeled with convex or Lipschitz-continuous $\rho$, this leaves us with a number of problems that cannot be solved with similar approximations or with the POMDP-IR approach (for instance, when we seek to minimize information or to find a compromise between gathering information and preserving privacy as presented in Sec.[2]).

Here, we propose to use the MCTS approach to address the general $\rho$-POMDP case with no constraints on the $\rho$ function. As an example, let us consider an agent monitoring a museum and whose aim is to let us consider an agent monitoring a museum and whose aim is to

4.2 $\rho$-POMCP($\beta$)

As a first approximation, the $\rho$-POMCP($\beta$) algorithm is similar to POMCP. During the selection step, trajectories are generated by sampling states and observations using generative model $G$. When visited, each belief node $h$ collects the state that has led to this node in order to build an estimate of the true belief state $b(h)$.

However, applying directly POMCP by only adding the current state of the trajectory to the belief node, as proposed in the original $\rho$-POMCP algorithm [3], may lead to poorly estimated immediate rewards during the first steps of the algorithm, thus causing the MCTS algorithm (i) to inefficiently spend time focusing on branches with over-estimated rewards and (ii) to slow down the exploration of branches with under-estimated rewards.
Algorithm 1: $\rho$-POMCP($\beta$)
Text in red highlights differences with POMCP.

$\rho$-POMCP($0$) = POMCP [3], i.e., w/o particle filter, setting $\beta' = \{(s')\}$.
Note that implementation details may differ.

\begin{align*}
\text{Fct Search}(h) & \quad \text{repeat} \\
& \quad \text{if } h = \text{empty then} \\
& \quad \quad \text{return } \emptyset \\
& \quad \text{else} \\
& \quad \quad \text{Simulate } (s, \beta, h, 0) \\
& \quad \quad \text{until TIMEOUT()} \\
& \quad \text{return } \arg \max_b V_{bb} \\
\end{align*}

\begin{align*}
\text{Fct Simulate} (s, \beta, h, \delta) & \quad \text{if } \gamma^\delta < \epsilon \text{ then return 0} \\
& \quad \text{if } h \notin T \text{ then return 0} \\
& \quad \text{forall } a \in A \text{ do} \\
& \quad \quad T'(ha) \leftarrow (N_{init}(ha), V_{init}(ha), \emptyset) \\
& \quad \text{return } \text{Rollout} (s, \beta, h, \delta) \\
& \quad \quad a \leftarrow \arg \max_b V_{hb} + \\
& \quad \quad \cV_{init}(N_{hb}) \\
& \quad \quad \beta' \leftarrow \text{PF}(\beta, a, z) \\
& \quad \quad \cup \{ (s', w_{s'}) \} \\
& \quad \quad B(h) \leftarrow B(h) \cup \beta' \\
& \quad \quad R \leftarrow \rho(B(h), a) \\
& \quad \quad + \gamma \cdot \text{Simulate} \\
& \quad \quad (s', \beta', haz, \delta + 1) \\
& \quad \quad N(h) \leftarrow N(h) + 1 \\
& \quad \quad N(ha) \leftarrow N(ha) + 1 \\
& \quad \quad V(ha) \leftarrow V(ha) + \\
& \quad \quad R - V(ha) \\
& \quad \quad \text{return } R \\
\end{align*}

$\rho_{max}$. This convergence proof holds in particular for $\rho$-POMCP($0$), thus answering an open question by demonstrating that the original $\rho$-POMCP [3] asymptotically converges to an optimal solution.

**Theorem 1** Let $p$ be continuous in $h$ and bounded by $\rho_{max}$, and $\epsilon > 0$, then the root action values computed by $\rho$-POMCP($\beta$) converge asymptotically to $\epsilon$-optimal action values.

**Proof** Since $p$ is bounded and the criterion is $\gamma$-discounted, the $\epsilon$-convergence allows reasoning with a finite horizon only, even though the problem horizon is infinite (with an infinite belief space). To do so, let us consider the tree of depth $\delta = \frac{\ln(1 - \epsilon)}{\ln(1 - \gamma)}$, and assume that the root belief estimate is exact. Due to UCB1, all nodes in that tree are visited infinitely often. Each belief-node $h$ then collects infinitely many particles ($|\beta| + 1$ at each visit) and, due to the use of particle filters applied from the root node, the probability distribution induced by cumulative bags $B(h)$ converges to the true belief state $b(h)$. Let us prove by induction that, at any depth $d \leq \delta$, the action value estimates are bounded by $\rho_{max}$. $\epsilon$. Trivially, any node’s value estimate is absolutely bounded by $\rho_{max}$, so that, in particular, the bound is $\gamma^{-d} \epsilon$ at depth $\delta$ (cf. def. of $\delta$). Let us now assume that the induction property holds at depth $d \in \{1, \ldots, \delta\}$. Then for any node $h$ at depth $d - 1$, (i) due to $p$’s continuity, $\rho(h(a))$ is correctly estimated, (ii) since each action is infinitely selected, by induction, the action values converge to $\gamma^{d-1}$-optimal values, and (iii) UCB1 selects the optimal action infinitely more often than other actions, thus $V(h)$ converges to a $\gamma^{d-1}$-optimal value.
4.3 ℓ−POMCP (β) variants

**Studied variants** The current difference between POMCP and ℓ−POMCP (β) lies in the way particles are collected in order to estimate the reward obtained at each transition. But value updates are updated in the same way. However, these updates can be improved, leading to several ℓ−POMCP (β) variants, since, during its execution, ℓ−POMCP builds increasingly better estimates of the true belief states in visited belief nodes. To do so, we first discuss what is computed by updates performed by POMCP (and vanilla ℓ−POMCP (β)), and then present two variants we developed, which Bargiacchi had already proposed [3].

**Computations performed by POMCP** If we ignore the node initialization in POMCP, then, when considering a node-action pair ha, the value stored in V(ha) averages, over N(ha) samples/visits:

- \[ \sum_{s \in B_{ha}} r(s, a) \text{: the total reward over the states } s \text{ that were sampled while action } a \text{ was picked in } h (\text{denoted this set of states}); \]
- \[ \sum_{s \in B_{ha}} N(h, a, z) \text{: the total return of the rollouts generated from observation } z \text{ after picking } a \text{ in } h \text{ (set } Z_{ha}); \]
- \[ \sum_{s \in Z_{ha}} N(haz) \text{: the number of times action } a \text{ was followed by observation } z \text{ in node } h \text{ (while } N(haz) \text{ is the number of updates of node } haz); \]
- \[ \sum_{s \in Z_{ha}} \sum_{s' \in A} N(haza')V(haza') \text{: the sum of the values of the children nodes, weighted by their visit counts (which also includes the rollouts performed from these nodes).} \]

By introducing belief node value estimates V(h), initialized with rollout values (for N(h) = 1), this leads to the following formulas:

\[
V(h) \leftarrow \frac{1}{N(h)} \left[ \text{Rollout}(h) + \sum_{a' \in A} N(ha')V(ha') \right];
\]

\[
V(ha) \leftarrow \frac{1}{N(ha)} \left[ \sum_{s \in B_{ha}} r(s, a) + \gamma \sum_{z \in Z_{ha}} N(haz)V(haz) \right].
\]

**Last-value-update ℓ−POMCP (β)** Thus, in POMCP, V(ha) is a moving average that “contains” an estimate of r(b(h), a), that estimate being computed as \( \sum_{s \in B_{ha}} r(s, a) \div N(ha) \).

In vanilla ℓ−POMCP (β), the sampled r at a current time step is replaced with an estimate of \( \rho(B(h)) \) as \( \rho(B(\phi(i))(h)) \), where \( B(N(h)) \) is the cumulative bag after the first N(h) visits. In this case, V(ha) includes thus (among other elements) an average of successive estimates (i.e., it computes \( \sum_{i=1}^{N(ha)} \rho(B(\phi(i))(h)) \), where \( \phi(i) \) is the ith visited of h where a was selected). However, it would seem more appropriate to instead use the reward associated to the last estimated belief \( \rho(B(\phi(N(ha)))(h)) \), which is a less biased estimate.

This proposed variant, called last-value-update ℓ−POMCP (β) (or “true ℓ−POMCP (β)”), fixes this rather easily by replacing the update of V(ha) in Algorithm 2B by

\[
V(ha) \leftarrow \frac{N(ha) - 1}{N(ha)} [V(ha) - \rho^{prev}(h, a)] + \rho(B(h), a) + \frac{1}{N(ha)} \gamma. \text{SIMULATE}(s', haz, δ + 1),
\]

where \( \rho^{prev}(h, a) \) is the previous value of the reward when ha was last experienced (thus needs to be stored).

**Last-value-update ℓ−POMCP (β)** But then, V(ha) also “contains” estimates of average rewards for future time steps, which suffer from the same issue. To fix this, **SIMULATE** should not return a sample return, but an estimate of the average return.

The updates in the backpropagation step consist then in re-estimating all the values of the visited belief nodes by using the reward obtained at each transition and the initial rollout, which needs to have been previously stored. This is done in last-value-update ℓ−POMCP (β) variant (or “ℓ−POMCP (β)”) through the following formulas:

\[
V(h) \leftarrow \frac{1}{N(h)} \left[ \text{Rollout}(h) + \sum_{a} N(ha)V(ha) \right],
\]

\[
V(ha) \leftarrow \rho(B(h), a) + \frac{\gamma}{N(ha)} \sum_{s} [N(haz), V(haz)].
\]

5 EXPERIMENTS

### 5.1 Benchmark Problems

POMDPs being a subclass of ℓ−POMDPs, first benchmark problems we consider are the classical Tiger, Tiger-Grid and Halway2 problems [19] (as per Cassandra’s POMDP page) and instances of Rock Sampling [23] with several grids of n × n cells, and n rocks to sample (where n equals 4, 6 and 8). A reward of +100 (resp. −100) is given for sampling a good (resp. bad) rock.

Then, a known issue with information-gathering problems is that a simple myopic strategy may often give very good results [5][20]. In order to assess the proposed algorithms, we had to provide problems where myopic strategies encounter difficulties.

We first proposed the **Museum problem**, inspired by [19], where an agent has to continuously localize a visitor in a toric grid environment (4 × 4 in our experiments). The state corresponds to the visitor’s unknown location. At each time step, the visitor stays still with probability 0.6, and moves to 1 of his 4 neighboring cells with probability 0.4 (chosen uniformly). The agent acts by activating a camera in any location. It then receives a deterministic observation: “present” if the visitor is at this location, “close” if s/he is in a neighboring cell, and “absent” if s/he is further away. Additionally: the immediate reward corresponds to the negentropy of the belief: \( \rho(b, a, b') = -H(b') = \sum_{s} b(s). \log (b(s)) \); the initial belief b0 is uniform over all cells; and \( \gamma = 0.95 \). The interest of this problem lies in the large number of actions (one per cell). We also used a variant, **Museum Threshold**, as proposed in Sec. 4.3 where the (non-continuous) reward is based on a threshold function on the belief state (with \( \alpha = 0.8 \)) which is null in most of the belief space.

In **active-localization** problems, an agent is in a toric grid with white and black cells. At each step, it can move to a neighboring cell or observe the color of its current cell. Active localization is difficult for myopic strategies as they see no (immediate) benefit in moving. Additionally: observations and transitions are deterministic; b0 is uniform over all cells; the reward corresponds to the entropy difference between b and \( b' \): \( \rho(b, a, b') = -H(b') + H(b) \); and \( \gamma = 0.95 \). Several configurations have been studied (cf. Fig. 3): (i) MazeCross, where black cells make it easy to localize oneself; (ii) MazeLines, where the agent cannot localize itself within the striped region and has to plan several steps ahead to look for the spot in the empty region; and (iii) MazeHole, which requires the agent to reason one step in advance to search for the missing black cell in this regular configuration; and (iv) MazeDots, identical to MazeHole except that black cells are separated
by several white cells. MazeDots_nxn corresponds to the cell configuration used for GridX and GridNotX, and (f) the obstacle configuration for SeekAndSeek.

Figure 3: The various cell configurations used for active localization problems; from left to right: (a) MazeCross, (b) MazeLines, (c) MazeHole and (d) MazeDots; (e) corresponds to the cell configuration used for GridX and GridNotX, and (f) the obstacle configuration for SeekAndSeek.

5.2 Results

Influence of rollouts

Preliminary experiments have been conducted with random rollouts. For the same number of descents, ρ-POMCP(β) with random rollouts (stopped when ρ > 0.1) lasts between 5 and 10 times longer than without any rollout (setting the value of new nodes to 0). Moreover, the gain from using a pure random rollout policy was also rarely significant. In some problems (Tiger), it even reduced the observed performance. We thus focus on ρ-POMCP(β) and ρ-beliefUCT without rollouts. Note that, due to implementation details, Bargiacchi’s ρ-POMCP [3] may exhibit a different behavior than ρ-POMCP(0).

Comparison with myopic strategies

Table 1 presents results comparing the purely Random strategy, Look-ahead strategies, ρ-beliefUCT, and ρ-POMCP(β) on the benchmark problems. The look-ahead-H algorithms perform dynamic programming over all possible futures for a fixed finite horizon H by using the complete ρ-POMDP model. The pure myopic strategy, where the agent maximizes its immediate reward, corresponds to Look-ahead-1, whereas Look-ahead-3 corresponds to anticipating all consequences 3 time-steps in advance. Both ρ-MCTS algorithms (ρ-POMCP(β) and ρ-beliefUCT) use a fixed number of descents nbDescents = 10,000, without any rollout (setting the value of new nodes to 0) and use a specific constant UCB for each problem as specified in Tab.1 (usually (Rmax − Rmin)/(1 − γ)). For ρ-POMCP(β), [β] = 50 and importance sampling was used.

(a) Tab.1 shows that Look-ahead-1 is close to the best value only on Tiger, GridX and Museum entropy. Regarding museum problems, the myopic strategy is less efficient than look-ahead-3 in Museum Threshold due to the sparsity of non-zero rewards.

(b) It is known that Look-ahead-1 often gives good results and usually constitute a very good baseline [12]. But, we have also compared our approaches to the more challenging Look-ahead-3 strategy. In this case, ρ-POMCP(β) and ρ-beliefUCT give better results than Look-ahead-1 and similar results to Look-ahead-3. In most cases, the difference between Look-ahead-1 and ρ-POMCP(β) is significant.

(c) ρ-POMCP(β) and ρ-beliefUCT provide the same results for this number of descents and take usually a lot more time than the Look-ahead algorithms. ρ-beliefUCT is usually faster than ρ-POMCP(β), but this depends on the problem since computational costs of these two algorithms come from different operations. In ρ-beliefUCT, this cost is due to the computation of exact beliefs when a new belief node is added. In ρ-POMCP(β), it is due to the generation of small bags β at each transition. That is why, whereas the time needed by ρ-POMCP(β) is more regular (except for the yet-unexplained case of GridX), the time needed by ρ-beliefUCT largely depends on |S|.

For instance, belief computation is quick in Tiger or CameraClean, but requires more time in Active Localization problems, where belief states include many states with a non-zero probability. Sec.4 proposes tentative solutions to ρ-POMCP(β)’s high computational cost.

(d) In problems with larger state or observation spaces, like Hallway2 and TigerGrid, ρ-MCTS algorithms are faster than Look-ahead-3 (by a factor between 8 and 20 depending on the problem) while achieving a higher performance. It shows that ρ-MCTS algorithms manage to deal with problems even with a high branching factor

8 To prevent side-effects, when several actions share the same highest value, the performed action is randomly selected among them.
(where Look-ahead-3 cannot compete) by focusing their descents on interesting branches.

(e) Finally, results from Rocksampling are difficult to analyze since they highly depend on the rock locations (different for v1, v2 and v3). The random action selection in Look-ahead gives no feedback policy for reaching rocks, whereas ρ-MCTS requires more time to reach the interesting rewards (cf. Tab. 2).

Results with fixed time-budget Table 2 presents results regarding the influence of β with a fixed time-budget of 1 s by action (except last column). When β increases, the number of descents performed by ρ-POMCP(β) naturally decreases (since trajectory generation is slower), but this has no clear impact on the ρ-discounted cumulated value. Several problems exhibit different behaviors. In CameraClean and Tiger, there is a significant performance gap between β = 0 and β = 5. This might come from the strongly stochastic observation process which requires better belief state estimates to act correctly. In Rocksampling, small values of β give better results. This may be due to overestimation which favors exploitation, and more descents allowing to reach a greater depth. Finally, results obtained with LostOrFound need to be commented in detail. With a UCB constant of 35, ρ-POMCP(β) needs a large bag to give good results. In this problem, convex rewards (found status) are compared with concave rewards (lost status). However, when we try to maximize information (convex reward), a poorly estimated belief lead to an optimistic return and ρ-POMCP(β) is attracted by the corresponding branch. On the contrary, with concave rewards, the estimated return is low and will be an incentive not to explore this branch anymore. Both these effects lead ρ-POMCP(β) to fail to find a good policy when β is low. When the UCB constant increases, this effect disappears due to favored exploration. In (almost) all cases, when the time-budget increases (like 10 s in the last column), ρ-POMCP(β) manages to generate the highest cumulated return (higher than Look-ahead-3 from Tab. 1).

Comparison between rejection and importance sampling (IS) We have conducted experiments with ρ-POMCP(β) algorithms to test the interest of using importance sampling instead of rejection sampling. All experiments were conducted with 200 runs of 40 actions, 10 000 descents per action, |β| = 20, and UCB constants from previous tables. The cumulated values are the same and rejection sampling requires approximately 20% more time on small problems (the table is not presented in this article). However, for problems with larger observation spaces or with highly stochastic observation process, like TigerGrid, Hallway2 or Museum problems, rejection sampling requires much more time (from 2 to 8 times more than IS, i.e., from 48 s to 381 s for TigerGrid problem) because of a high reject rate.

Results with proposed variants To speed up convergence, lru-ρ-POMCP(β) and lrvu-ρ-POMCP(β) variants (cf. Sec 4.3) have been investigated replacing moving averages by up-to-date estimates, which are less biased in our setting. However, up to now, experiments with fixed time budgets (100 ms, 200 ms, 1 s and 10 s) have not shown any significant improvement.

6 DISCUSSION

In this article, we proposed two algorithms, ρ-POMCP(β) and ρ-beliefUCT, to address ρ-POMDPs without constraints on the reward function. ρ-POMCP(β) (and trivially ρ-beliefUCT) is proved to asymptotically converge when ρ is continuous and bounded. ρ-MCTS algorithms are thus particularly useful when considering non-convex non-Lipschitz-continuous rewards which cannot be addressed by previous approaches (like [2], [10] or [13]).

Conducted experiments show that both algorithms give better results than the proposed baseline. The difference between ρ-POMCP(β) and original ρ-POMCP (β = 0 in Table 2) is less significant. However, we proposed problems (like LostOrFound) where the
use of a particle filter generates better results with a fixed-time budget (due to possibly better estimated beliefs when using small particle bags).

This advantage of \( \rho \)-POMCP(\( \beta \)) over original \( \rho \)-POMCP would probably increase with a better estimated beliefs accessible from the available time budget. We have observed that \( \rho \)-POMCP(\( \beta \)) is time consuming due to the many sampled particles. One promising direction would be to save time by letting a cumulative bag \( B(h) \) grow sub-linearly (rather than linearly) with the number of visits \( N(h) \), i.e., by considering dynamic \( \beta \). This will require modifying the way particles are generated, which is anonymous reviewers, for their feedback.

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