Calculation and Analysis of Aerodynamic Parameters of Wing Based on NS Equation

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Abstract. The emergence of computational fluid dynamics has greatly simplified the research and design process of aircraft. At present, this technology has become the most popular research field in fluid mechanics and is of great significance to promote the development of China's aerospace and other undertakings. With the development of computational fluid mechanics, the numerical solution speed of the mean navier-stokes equations (RANS) has been greatly improved. In this paper, the M6 wing is taken as the calculation model, the RANS equation is coupled with the relevant calculation model, the pressure coefficient and aerodynamic characteristics are calculated and analyzed, and relevant conclusions are drawn.

1. The introduction
Computational fluid dynamics (CFD) solves the partial differential equation of fluid flow by computer, analyzes and explains the fluid flow quantitatively and qualitatively, and obtains the change rule of fluid flow [1]. Before the application of computational fluid dynamics in the field of aerospace, flight mechanics in aircraft research and design was mainly realized through theoretical analysis and wind tunnel experiments [2]. When computational fluid dynamics was adopted, the research and design process of aircraft was greatly simplified. At present, this technology has become the most popular research field in fluid mechanics and is of great significance to promote the development of China's aerospace and other undertakings.

NS (navier-stokes) equations are complex nonlinear partial differential equations used to describe the basic laws of fluid motion, among which the common one is the average navier-stokes equations (RANS)[3]. With the development of computational fluid mechanics, the numerical solution speed of the equations has been greatly improved. In this paper, the M6 wing is taken as the calculation model, the RANS equation is coupled with the relevant calculation model, the pressure coefficient and aerodynamic characteristics are calculated and analyzed, and relevant conclusions are drawn.

2. Governing equations and principles of numerical calculation

2.1. Governing equation principle
In computational fluid mechanics, in order to avoid the difference between the magnitudes of physical quantity of NS equation [4], the variables of NS equation are generally dimensionless in rectangular coordinate system. After processing, the general expression of 3d NS equation of compressed flow is shown in equation (1).
\[
\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial \xi} + \frac{\partial F}{\partial \eta} + \frac{\partial G}{\partial \zeta} = \frac{1}{\text{Re}} \left( \frac{\partial E_{v}}{\partial \xi} + \frac{\partial F_{v}}{\partial \eta} + \frac{\partial G_{v}}{\partial \zeta} \right)
\]

(1)

Where \( J \) is the row and column of the corresponding jacobian matrix, as shown in equation (2).

\[
J = \frac{\partial (\xi, \eta, \zeta, t)}{\partial (x, y, z, t)} = \begin{vmatrix}
\xi_x & \xi_y & \xi_z & \xi_t \\
\eta_x & \eta_y & \eta_z & \eta_t \\
\zeta_x & \zeta_y & \zeta_z & \zeta_t \\
0 & 0 & 0 & 1
\end{vmatrix}
\]

(2)

If the above transformation is applied to equation (1), the general form of the three-dimensional NS equation in the curvilinear coordinate system is:

\[
\frac{\partial Q}{\partial t} + \frac{\partial E'}{\partial \xi} + \frac{\partial F'}{\partial \eta} + \frac{\partial G'}{\partial \zeta} = \frac{1}{\text{Re}} \left( \frac{\partial E_{v}'}{\partial \xi} + \frac{\partial F_{v}'}{\partial \eta} + \frac{\partial G_{v}'}{\partial \zeta} \right)
\]

(3)

2.2. The turbulence model

Turbulence model is a set of closed equations to describe the average amount of turbulence, which is based on Reynolds average motion equation and pulsating motion equation, combining theory with experience and introducing a series of model assumptions. After adding turbulence model, the accuracy of RANS equation in calculating lift or drag force and moment is greatly improved. The SA model is mainly introduced here.

SA turbulence model is widely used to simulate solid wall turbulent flow with laminar flow. The core is to introduce related variables \( \bar{v} \) and obtain the viscosity coefficient of turbulence by solving \( \bar{v} \) the transport equation. The transport equation of \( \bar{v} \) can be summarized as followed:

\[
\frac{\partial \bar{v}}{\partial t} = C_{01}[1-h_{2}^{\gamma}]S[\bar{v}]+\frac{3}{2}[\nabla v][v+\bar{v}]\nabla \bar{v}+C_{62} \left( \nabla v \right)^{2}+C_{w}h_{w}-\frac{C_{nl}}{k} h_{2}\left( \frac{v}{d} \right)^{2}+h_{i} \Delta U^{2}
\]

(4)

2.3. Numerical method

At present, there are three main spatial discrete methods for numerical calculation of RANS equation, among which the implicit lu-sgs method has the advantages of high resolution, high precision and good stability, and is the most widely used.

The implicit lu-sgs method decomparts the block diagonal matrix into the product of upper triangle matrix and lower triangle matrix, and improves the computational efficiency of the solving process. In the NS equation, the implicit lu-sgs method is adopted for discretization. After the first-order discretization of time, the NS equation shown in equation (3) becomes:

\[
\frac{Q^{n+1} - Q^{n}}{\Delta t} + \left( \frac{\partial F}{\partial \xi} + \frac{\partial G}{\partial \eta} + \frac{\partial H}{\partial \zeta} \right)^{n+1} = \left( \frac{\partial E_{v}}{\partial \xi} + \frac{\partial F_{v}}{\partial \eta} + \frac{\partial G_{v}}{\partial \zeta} \right)^{n}
\]

(5)
3. Numerical analysis

The M6 wing is a swept-back half wing with no torsion along the spanwise position. In this paper, the M6 wing is taken as the calculation model, and the specific position of the pressure point of the wing is shown in table 1.

| N  | 1   | 2   | 3   | 4   | 5   | 6   |
|----|-----|-----|-----|-----|-----|-----|
| y/b| 0.20| 0.43| 0.65| 0.81| 0.90| 0.95|

After selecting the calculation model, NS equation was adopted as the main control equation in this paper, and the implicit LU-SGS method in the time discrete method was adopted in the numerical calculation method. The turbulence model was SA model, and the pressure coefficient and aerodynamic characteristics were calculated by adding min-mod limiter and modified 3rd-upwind bias limiter and multi-grid technology, respectively.

3.1. Calculation and analysis of pressure coefficient

Table 2 for the pressure coefficient under different limiter yan wing six typical battle like the location of the pressure coefficient distribution table, compare the results with the experimental data analysis can be seen from the table in the different limiter will have little impact on the pressure coefficient distribution and shock wave position, min - mod limiter in capture the shock wave position slightly better than modified 3rd - Upwind offset limiter capture the shock wave position.

| Parameter                     | Min-mod  | Modified 3rd-upwind |
|-------------------------------|----------|----------------------|
| Pressure coefficient effect   | Less affected | Less affected        |
| Shock wave location fitting effect | Good    | Normal               |

3.2. Convergence analysis

Figure1~ Figure3 are the comparison curves of residual value convergence curve, downward lift coefficient convergence curve and resistance coefficient convergence curve under different limiter respectively. It can be seen from the comparative analysis results in FIG. 1 that under modified 3rd-upwind bias limiter, the calculation tends to be stable around 2200 steps, and the convergence error is about 10-9. Under the min-mod limiter, the calculation tends to be stable at about 2000 steps, and the convergence error is about 10-7.5. According to the analysis results in FIG. 2 and FIG. 3, the convergence curves of lift coefficient and drag coefficient under different limiter almost coincide. In summary, it can be concluded from the above analysis that: the modified 33-upwind bias limiter has high stability, and the convergence speed of min-mod limiter is fast; the effect of the limiter on the convergence of residual value is obvious, and the effect on the convergence of lift force and resistance is not significant.
3.3. Aerodynamic analysis
By comparing the size of variation coefficient [5], the influence of different limiter on aerodynamic characteristics of M6 wing surface can be analyzed, and the variation coefficient is proportional to the...
influence degree of aerodynamic force. Table 3 shows the aerodynamic calculation results of models with different limiter.

**Table.3 Aerodynamic calculation results under different limiter**

| Aerodynamic coefficient         | Min-mod | Modified 3rd | The average | The standard deviation | Coefficient of variation |
|--------------------------------|---------|--------------|-------------|------------------------|-------------------------|
| Lift                           | 0.26758 | 0.26689      | 0.26725     | 0.00035                | 0.00131                  |
| Resistance                     | 0.01731 | 0.01745      | 0.01739     | 0.00003                | 0.00256                  |
| Differential pressure resistance| 0.01175 | 0.01185      | 0.01181     | 0.00005                | 0.00463                  |
| Viscous resistance             | 0.00556 | 0.00553      | 0.00555     | 0.00001                | 0.00175                  |

By analyzing the calculation results in table 3, it can be concluded that under the above calculation conditions and with different limiter, the coefficient of resistance variation is large, among which the coefficient of pressure difference resistance is the largest among the four coefficients. Through analysis, it can be concluded that the above limiter has little effect on lift coefficient, big effect on drag coefficient and obvious effect on pressure difference drag coefficient.

### 4. Conclusion

In this paper, by studying the accuracy of numerical calculation based on RANS equation, we focus on the influence of the limiter on the calculation results. The full turbulence numerical calculation of the RANS equation coupling SA model of M6 wing shape was carried out with different restrictors, and the effects of different restrictors on its pressure coefficient distribution, convergence, aerodynamic characteristics and other aspects were compared and analyzed. The following conclusions were drawn through comparison with experimental data:

1. The pressure coefficient distribution and shock wave position predicted by the two limiter are almost the same, which are in good agreement with the experimental data.
2. The shock wave quality predicted by min mod limiter was relatively good;
3. The convergence and accuracy of modified 3rd-upwind bias limiter are better than the calculated results of min mod limiter.

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