Distributive Model of Maximum Permissible Emissions of Enterprises into the Atmosphere and Its Application

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Abstract. The paper discusses the problem of limiting enterprises’ emissions of harmful substances into the atmosphere taking into account the air pollution regulations. The cost model for reducing emissions of individual enterprises is formulated. The distributive model of maximum permissible emissions of enterprises is defined as the problem of minimizing the total cost of reducing emissions, the limitations of which are determined by the requirements for air pollution in residential areas. The disadvantages of the distributive model in the form of a linear programming problem are discussed. The presented model, in contrast to the linear programming model, is more resistant to data inaccuracies and provides a proportional distribution of the volume of permissible emissions for enterprises with identical characteristics. The solution of the direct minimization problem with inequality constraints is irrational due to its high dimension. A method for solving the direct problem based on the duality theory is obtained. As a rule, the dual problem has far less dimension. To solve it, the effective subgradient minimization method with stretching-compression of space is used. The distributive model is tested on real data. The example of solving an applied problem is given.

1. Introduction

In industrial centers, air pollution becomes one of the factors limiting the development of production. The maximum allowable concentration (MAC) of pollutants in the air of residential areas (Sanitary Rule 2.1.6.1032-01) is reflected in the sanitary-hygienic requirements to the purity of the air and supplemented by the possible values of the human health risk (Handbook 2.1.10.1920-04). The need to ensure restrictions on MAC leads to the concept of maximum permissible emissions (MPE) for enterprises approved in Russia (GOST 17.2.3.02-2014). Assessment of air pollution depending on the individual sources emissions of the enterprise is carried out using the Russian regulatory atmospheric dispersion model (Order 06.06.2017 № 273) based on [1]. For emissions within the MPE, the enterprise pays basic unit charges for each pollutant (Regulation 13.09.2016 N 913). For emissions in excess of MPE base payments are multiplied by a significant increase factor (5 or more). In this regard, it is important to establish the distributive models of MPE based on the set of sources that would meet the requirements on air pollution at the border of sanitary protection zone (SPZ) and over residential areas (RA) [1-12] and provide opportunities for meeting extra goals (maximizing profits, ensure the equivalence of MPE volumes for enterprises).

These problems include a large number of pollution sources (up to 10 000) and a significantly
smaller number (up to 1 000) of calculated points of pollutants concentration in excess of MAC of air pollutants [2]. Known approaches to emissions distribution are based on the mathematical formulation in the form of linear programming problem [3], where the objective function is maximized in the form of the total value of MPE, provided that each of the enterprises pays only the basic unit charges without the increasing coefficient. This is one of the possible formulations of the problem of constructing a distributive model, in which, due to the proportional nature of changes in profits related to changes in emissions, profit is maximized implicitly. The formulation in the form of a linear programming problem has significant drawbacks. As a rule, the input data of the problem are very noisy, which is complicated by the fact that the solution can be non-unique and unstable. If the solution is non-unique, the simplex method solution may be practically unacceptable.

The paper proposes a distribution model in which the MPE of enterprises are determined as a result of minimizing the total cost of emission abatement to ensure acceptable air pollution at the border of SPZ and over RA. This model, in contrast to the linear programming model, provides a proportional distribution of emissions for enterprises with identical characteristics. The problem can be solved by numerical optimization methods [5-10]. Due to the high dimension of the direct problem, we have developed a method based on the solution of the dual problem [10-14] by the method [15, 16] belonging to the class of relaxation subgradient minimization methods [17, 18]. The distributive model of MPE for enterprises is tested on real data. The paper presents an example of solving an applied problem.

2. Problem definition

In Russia, the system of managing the rates of pollutant emissions into the air depends on the MPE regulations. For each pollutant the MPE for the enterprise(s) is such rate of emissions $x_i$ from $i = 1, 2, ..., n$ air pollution sources (APS) at which in a given set of $m$ receptor points the following conditions are satisfied:

$$C_j = \sum_{i=1}^{n} A_{ji} x_i \leq b_j, \quad j = 1, 2, ..., m,$$  \hspace{1cm} (1)

where $C_j$ is the total pollutant concentration created by all APS, $b_j$ is the allowable concentration (MAC in the simplest case) at the $j$-th receptor point, $A_{ji}$ is the specific (per unit of emission rate) concentration from the $i$-th source at the $j$-th point. $A_{ji}$ has the sense of coefficient of influence ($i$-th source at the $j$-th point) and is calculated with the help of the atmospheric dispersion model depending on the type of source, its technical parameters and space coordinates, meteorological data, terrain relief coefficient and pollutant deposition parameters. The expressions (1) can be written in matrix form as a system of linear inequalities. The vector $x$ is one of the variants of MPE of an enterprise:

$$Ax \leq b, \quad x \in R^n, \quad b \in R^m, \quad A \in R^{m\times n}.$$  \hspace{1cm} (2)

The methodical literature [19, 20] which has recommendatory status (in contrast to the regulations) provides the methods of finding particular solutions of the system (2). The mentioned methods allow finding a unique solution, but these solutions are not based on the use of the objective function.

One of the approaches may consist in maximizing profits from total output. Due to the practical unavailability of the necessary information on production volumes and the actual unit cost of production in enterprises, we can assume that the profit is proportional to the volume of emissions. This assumption allows us to formulate the optimization linear programming problem (LPP) with respect to emissions, which is widely used in the formulation of optimization problems in the Russian and foreign literature [3, 21, 22]:

$$\max_{Ax \leq b} (p^T x), \quad p_i = 1, \quad i = 1, 2, ..., n.$$  \hspace{1cm} (3)

Here $a$ and $d$ are vectors of lower and upper limits of technical capabilities of enterprise emission
rates. The upper limits $d$ usually reflect the current emission state. Problem (3) seeks for the value of emissions $x$ from all APS at which their sum emission will reach its maximum value with saving normal level of air pollution. In particular, the goal (3) coincides with the main goal of enterprises when establishing the MPE standards, that is: determining the maximum volume of emissions for which only the basic unit charge payments should be made.

Let us formulate the maximization problem (3) as a minimization problem by replacing the sign of the objective function. This LPP has the following form: Let us denote $c_i = -p_i$, $i = 1, 2, \ldots, n$. The corresponding LPP has the following form:

$$\min (-c, x), \quad Ax \leq b, \quad x \in Q = \{x \mid a \leq x \leq d\}, \quad c, a, d, x \in \mathbb{R}^n, \quad b \in \mathbb{R}^m, \quad A \in \mathbb{R}^{m \times n}.$$  \(4\)

Here $x$ is the vector of the volume of emissions, and $Q$ is the set of restrictions for the enterprises.

The constraints of the problem (4) can have linear dependence, which is complicated by the errors of constraint coefficients and the approximate type of the objective function. Another problem is the uneven distribution of MPE volumes for enterprises with identical proportions. The solution of the problem (3) by the simplex method is determined by some basic acceptable solution [13] and may fail to meet the conditions of equivalence of restrictions for enterprises with identical influence of their emissions on air pollution. Let us consider the situation of unequal distribution of MPE volumes for identical enterprises at a hypothetical example with one restriction:

$$\min (-c_0 \sum_{i=1}^n x_i), \quad A_0 \sum_{i=1}^n x_i \leq b_0, \quad 0 \leq x_i \leq d_i \geq b_0 / A_0, \quad b_0, A_0 > 0, \quad i = 1, 2, \ldots, n.$$  \(5\)

Here we accept that $c_i = c_0$, $i = 1, 2, \ldots, n$. Component-wise restrictions in (5) are not active and do not affect the solution. The solution obtained by the simplex method will contain one non-zero component of the vector $x$, e.g. the first one: $x_i = b_0 / A_0$, $x_i = 0$, $i = 2, 3, \ldots, n$, which is determined by the initial basis solution. According to the statement of the problem (5), all enterprises are identical and only differ in the scale of production. Therefore, a more equitable distribution is determined by the same proportion of the top load for a particular enterprise:

$$x_i = b_0 d_i / (A_0 \sum_{j=1}^n d_j), \quad i = 1, 2, \ldots, n.$$  \(6\)

This example shows that the solution obtained by the simplex method may also be unacceptable because of the discrepancy between the method of solution and the statement of the applied problem.

3. **The distributive model based on the quadratic dependence of the cost of emission abatement**

One of the possible ways of formation of the target function consists in finding a variant of MPE (2) at which the costs of emission abatement would be minimal. The values of current emissions $x_i^0$, $i = 1, 2, \ldots, n$ serve as the upper limits for the enterprises emission rates $d_i$. Typically, the upper limit displays the current emissions $x_i^0 = d_i$. Let us define the change in the MPE of the enterprise per unit volume of current emissions:

$$\Delta_i = (d_i - x_i) / d_i, \quad i = 1, 2, \ldots, n.$$  \(7\)

Let us present the cost of reconfiguring production per unit volume of emissions in the form of a quadratic model:

$$f_i = c_i \Delta_i + \varepsilon_i \Delta_i^2 / 2, \quad i = 1, 2, \ldots, n.$$  \(8\)

Here, the components $c_i$ of the vector $c$ characterize the unit cost of emissions abatement of all APS from the current volume of emissions $x_i^0 = d_i$ to the desired values of $x_i$, and the components $\varepsilon_i$ of the
vector $\epsilon$ characterize the quadratic part of the unit cost of emission abatement. Since there is no detailed information about the parameters in (8), let us assume that they are the same: $c_i = c_0 \epsilon_i = \epsilon_0$, $i = 1, 2, ..., n$.

The total costs of reconfiguring production with the account of (8) will have the following form:

$$F_0(x) = c_0 \sum_{i=1}^n d_i A_i + \epsilon_0 \sum_{i=1}^n d_i A_i^2 / 2 = c_0 \sum_{i=1}^n (d_i - x_i) + \frac{\epsilon_0}{2} \sum_{i=1}^n (d_i - x_i)^2 / d_i.$$  

(9)

Since fixed costs are not reduced, let us exclude them from the formula (9). As the result, we get:

$$F(x) = -(c_0 + \epsilon_0) \sum_{i=1}^n x_i + \frac{\epsilon_0}{2} \sum_{i=1}^n x_i^2 / d_i.$$  

(10)

Let us formulate the following optimization problem based on (10):

$$\min F(x), \quad Ax \leq b, \quad x \in Q = \{x | a \leq x \leq d\}.$$  

(11)

Analogous to (5), let us consider the problem of minimizing the function (10) with constraints from (5):

$$\min F(x), \quad A \sum_{i=1}^n x_i \leq b_0, \quad 0 \leq x_i \leq d_i \geq b_i / A_i, \quad b_i, A_i > 0, \quad i = 1, 2, ..., n.$$  

(12)

Let us write the Lagrange function of the problem (12)

$$L(x, y) = F(x) + y(A \sum_{i=1}^n x_i - b_0) =$$

$$= -(c_0 + \epsilon_0) \sum_{i=1}^n x_i + \frac{\epsilon_0}{2} \sum_{i=1}^n x_i^2 / d_i + y(A \sum_{i=1}^n x_i - b_0).$$

Using extremum conditions

$$\frac{\partial L(x, y)}{\partial x_i} = -(c_0 + \epsilon_0) + \epsilon_0 x_i / d_i + yA_0 = 0, \quad i = 1, 2, ..., n,$$

we get

$$x_i = d_i (c_0 + \epsilon_0 - yA_0) / \epsilon_0, \quad i = 1, 2, ..., n.$$  

(13)

Subject to the conditions of feasibility $A \sum_{i=1}^n x_i = b_0$, we shall find

$$y = \left( c_0 + \epsilon_0 - b_0 \epsilon_0 / (A_0 \sum_{i=1}^n d_i) \right) / A_0.$$  

Finally, using the latter in (13), we shall find the formula (6)

$$x_i = d_i ((c_0 + \epsilon_0) - (c_0 + \epsilon_0 - b_0 \epsilon_0 / (A_0 \sum_{i=1}^n d_i))) / \epsilon_0 = d_i b_0 / (A_0 \sum_{i=1}^n d_i), \quad i = 1, 2, ..., n.$$  

Thus, using the quadratic function of costs leads to equivalent distribution of MPE for enterprises with identical characteristics.

4. **Problem-solving procedure**

Let us formulate the optimization problem (11) with the function (10):

$$\min \left[ -(c_0 + \epsilon_0) \sum_{i=1}^n x_i + \frac{\epsilon_0}{2} \sum_{i=1}^n x_i^2 / d_i \right], \quad Ax \leq b, \quad x \in Q = \{x | a \leq x \leq d\}.$$  

(14)

Let us write the Lagrange function to the problem (12):

$$L(x, y) = -(c_0 + \epsilon_0) \sum_{i=1}^n x_i + \frac{\epsilon_0}{2} \sum_{i=1}^n x_i^2 / d_i + (y, Ax - b), \quad y \geq 0, \quad y \in R^m, \quad x \in Q.$$  

(15)
Since the number $m$ of constraints is much less than the number $n$ of variables we obtain a solution to the direct problem (14) based on the solution of its adjoint problem.

According to [13] we introduce an adjoint function

$$
\theta(y) = -\min_{x \in \mathbb{Q}} L(x, y), \quad y \geq 0, \quad y \in \mathbb{R}^n.
$$

(15)

The minus in (15) is set in order to further deal with the problem of minimizing the adjoint function. Denote by $x(y)$ the solution of the minimization problem (15) by $x$ at a fixed $y$.

If there are no restrictions on the variables $x$, then by solving the system $\nabla_y L(x, y) = 0$, we get

$$
\frac{\partial L(x, y)}{\partial x_i} = -(c_i + \epsilon_0) + \epsilon_0 x_i / d_i + [A^y x_i] = 0, \quad i = 1, 2, ..., n,
$$

subject to the limitations the solution will look like

$$
x_i^*(y) = \begin{cases} 
    a_i, & \text{if } x_i^* \leq a_i, \\
    d_i, & \text{if } x_i^* \geq d_i, \\
    x_i^*, & \text{if } a_i \leq x_i^* \leq d_i,
\end{cases} \quad i = 1, 2, ..., n.
$$

The function $\theta(y)$ is convex, piecewise linear, everywhere defined and by virtue of the duality theorem [5] the original problem is equivalent to the unconditional minimization of adjoint function. If the influences of emissions at the receptor points of some of the enterprises are identical, that is, if the corresponding columns of the restriction matrix are equal, the respective values of $a_i$ in (16) will be the same, and the expressions for $x_i^*$ will be proportional to the maximum emissions $d_i$ of enterprises. This means a more equitable distribution of workloads for the enterprises.

Non-smooth minimization problem

$$
\min_{y \in \mathbb{Q}} \theta(y)
$$

(17)

could be solved by subgradient methods [15, 16]. This approach, under the condition $m \ll n$, has an advantage over the solution of the direct problem. When solving tests to the problem (14) $c_0 = 1$ was set, at the same time, the value $\epsilon_0$ was chosen so that the solution by the value of the objective function was close to the theoretical one. In practical problems the value of $\epsilon_0$ was chosen on the basis of expert analysis of problem solutions with different parameters.

5. Example of a practical task

To show the benefit of distributive model we make an example of solving the practical problem of determining the MPE for several nearby plants with a common territory of SPZ in one of the industrial cities of Kuzbass, Russia. A set of 64 sources (48 point, 3 linear and 13 area) emit several pollutants including nitrogen dioxide (MAC = 0.2 mg/m$^3$). To calculate the air pollution and the values of MPE (vector $x$) for nitrogen dioxide, the ERA software package was used (more https://lpp.ru/).

Atmospheric dispersion model, constituent in the package, is admitted in Russia and the Republic of Kazakhstan to use in industrial designs.

Dispersion modeling on regular receptor grid with the step of 100 m with existing emission rates shows that there is a zone of pollution with excess of MAC which covers a part of SPZ and the nearest areas or RA. Points with air pollution exceeding the MAC along SPZ and over RA (altogether 270 point) are automatically selected by the special software procedure. To prepare the solution of the problem (3) in the selected points were calculated the coefficients of influence $A_{ji}$, forming a matrix $A$.

All elements $b_j$ of the $b$ vector in this example are equal to the MAC, but it is a possibility to specify their individual values (for example, 0.8 MAC in recreational areas). Vectors and limitations on the scope of the search of the solution $x$ is given as follows: the vector $d$ represents for all the 64 sources
the set of existing emission rates (in which were carried out the initial calculation) and the vector \( a = 0.2d \). This means that the MPE for each source cannot exceed its existing emission, and the emission abatement at any source cannot be greater than 80%. Thus, the formulation of the problem (3), which were solved by various methods, has been carried out.

The methods recommended in [19, 20] were built-in the software package ERA rather long ago. Let us point once more that mentioned methods allow to find a unique solution of the system (2), but are not optimization. Moreover the absence of an objective function in methods [19, 20] leads to difficulties in explanation the sense of finding solutions.

Recently, the software support of distributive model described in this paper was added to the software package ERA and rather grate number of practical tasks have been solving with it using. It is quite clear that optimization model practically always (except for the trivial case with one source) gives a more effective solution to the problem of finding the MPE for an enterprise or a group of enterprises. Table 1 shows total MPE characteristics were obtained by all three methods for our example.

### Table 1. A comparison of total value of MPE and required emission abatement were obtained by several methods.

| Method               | Number of APS with abatement of emission | Sum value of existing emissions, g/s | Sum value of MPE, g/s | Required emission abatement, g/s |
|----------------------|-----------------------------------------|-------------------------------------|-----------------------|---------------------------------|
| MRN-87, [19]         | 17                                      | 70.94                               | 43.96                 | 26.98                           |
| The order N 66, [20] | 8                                       | 70.94                               | 48.04                 | 22.9                           |
| Distributive model   | 8                                       | 70.94                               | 57.43                 | 13.51                           |

Thus, it can be seen that the solution of the optimization problem by using the distributive model gives a much larger than [19, 20] the total value of MPE for the sources of the industrial zone. This makes it possible to reduce both the costs of achieving MPE in proportion to the required emission abatement and the annual payments by reducing the share of total emission for which the fee is charged with increasing coefficients.

6. Conclusion

The paper presents a distribution model of maximum permissible emissions of harmful substances into the atmosphere for enterprises, taking into account the regulations for air pollution in residential areas adopted in Russia. The model is defined as a nonlinear programming problem of minimizing the total cost of emission abatement, whose limitations are determined by sanitary requirements on air pollution. The proposed model, in contrast to the linear programming model, is more resistant to data inaccuracies and provides for proportional distribution of emissions for enterprises with identical air pollution influence.

Due to the high dimension of the direct nonlinear programming problem, we developed an algorithm for finding a solution to its dual problem, which has far less dimension. To solve the nonsmooth dual problem, we used the effective subgradient methods with a change in the space metric, designed to minimize undifferentiated functions. The computational experiment confirms the efficiency of the solution based on the distributive model and the computational efficiency of the proposed solution method. We also provided an example of the solution to the applied problem of regulating the emissions for the industrial zone enterprises, showing the advantages of the optimization approach.

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