VMD description of $\tau \to (\omega, \phi)\pi^{-}\nu_{\tau}$
decays and the $\omega - \phi$ mixing angle

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Abstract

Using the vector meson dominance model we get predictions for the Cabibbo-favored $\tau^- \to \omega\pi^-\nu_{\tau}$ and $\tau^- \to \phi\pi^-\nu_{\tau}$ decays. We show how the measurements of these two decays can provide information on the nature of the violation of the OZI rule.

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τ⁻ → ωπ⁻νᵣ and τ⁻ → φπ⁻νᵣ are two Cabibbo-favored, marginal decay modes of the τ lepton whose precise measurements can give insight into the existence of axial second class currents [1] and on the nature of the violations to the OZI rule [2] in a clean environment, respectively.

The decay τ⁻ → ωπ⁻νᵣ has been studied previously in the context of the vector-meson dominance model [3], a low energy $U(3) \times U(3)$ chiral lagrangian model [4] and, most recently, using the heavy vector-meson chiral perturbation formalism [5]. The experimental information about this decay involves the measurement of the decay rate and the spectral function [6]. The angular distribution of the ωπ⁻ system was found to be consistent with a $J^P = 1^-$ state [6], which is typical of first class vector current and is in agreement [7] with data on $e^+e^- \rightarrow ωπ^0$ via the Conserved Vector Current (CVC) hypothesis.

Since the φ(1020) is almost a pure $s\bar{s}$ state, the decay τ⁻ → φπ⁻νᵣ is expected to be very suppressed with respect to τ⁻ → ωπ⁻νᵣ due to the OZI rule. Actually, the experimental upper limit on this decay indicates that $B(τ⁻ \rightarrow φπ⁻νᵣ) < 2.6 \times 10^{-4}$ at the 90 % C.L. [8]. Theoretical estimations based on the CVC hypothesis and using the $e^+e^- \rightarrow φπ^0$ data give $B(τ⁻ \rightarrow φπ⁻νᵣ) < 9 \times 10^{-4}$ [7]. Thus, the measurement of the φπ⁻ production relative to ωπ⁻ in τ decays can provide clean information on the nature of the violations to the OZI rule.

In this paper we examine these two decays of the τ lepton by using the model proposed in Ref. [3]. We first extract the relevant coupling constants from light meson decays and predict $B(τ⁻ \rightarrow ωπ⁻νᵣ) = (1.22 \pm 0.56)\%$ and $B(τ⁻ \rightarrow φπ⁻νᵣ) = (1.20 \pm 0.48) \times 10^{-5}$. Then we estimate the ratio of these two decays and find it to be sensitive to the $ω - φ$ mixing angle.

The lowest order amplitude for the τ⁻ → $Vπ⁻νᵣ$ decays can be written as

$$M = \frac{G_F V_{ud} l^μ}{\sqrt{2}} < Vπ^- |\bar{u}γ_μd|0 >,$$

where $l^μ$ is the V–A leptonic current and $V_{ud}$ is the $ud$ element of the Cabibbo-
Kobayashi-Maskawa matrix. The form of the hadronic matrix element of the vector current is fixed by Lorentz covariance to be:

$$< V \pi | \bar{u} \gamma_{\mu} d | 0 > = i F_V(s) \epsilon_{\mu\alpha\beta\gamma} \epsilon^{\ast \alpha} q_1^\beta q_2^\gamma $$ \hspace{1cm} (2)

where $q_{1,2}$ denote the four-momenta of $V$ and $\pi^-; \epsilon^*$ is the polarization four-vector of $V$, and $F_V(s)$ is the $s$-dependent ($s = (q_1 + q_2)^2$) form factor for the hadronic vertex.

The squared mass distribution of the hadronic system, can be written as follows,

$$\frac{d\Gamma}{ds} = \frac{G_F^2 |V_{ud}|^2}{1536 \pi^3 m^3} |F_V(s)|^2 \frac{(m^2 - s)^2}{s^2} (m^2 + 2s)(s^2 - 2s \Sigma^2 + \Delta^4)\frac{3}{2}$$ \hspace{1cm} (3)

where $m$ denotes the $\tau$ mass, $\Sigma^2 \equiv m_V^2 + m_\pi^2$ and $\Delta^2 \equiv m_V^2 - m_\pi^2$. The last factor in the r.h.s of Eq. (3) is characteristic of the $p$-wave of the $V\pi^-$ system. Eq. (3) above is in agreement with Eq. (5) of Ref. [3], after including a factor 1/2 missing in their Eq. (5).

The spectral function $\nu(s)$ for this decay is defined as follows [6],

$$\nu(s) = \frac{32 \pi^2 m^3}{G_F^2 |V_{ud}|^2 (m^2 - s)(m^2 + 2s)} \frac{d\Gamma}{ds}.$$ \hspace{1cm} (4)

In order to make predictions we need a specific model for $F_V(s)$. In the vector-meson dominance model, this form factor is given by (See Fig. 1)

$$F_V(s) = \Sigma_{V' = \rho, \ldots} \frac{g_{V'V\pi^-}}{m_{V'} - s - im_{V'} \Gamma_{V'}}$$ \hspace{1cm} (5)

where $g_{V'}$ denotes the coupling constant describing the $W^- - V'$ junction and $g_{V'V\pi^-}$ is the $V'V\pi^-$ coupling constant.

As is shown in Ref. [6], the experimental data on the spectral function requires the presence of at least one $\rho'$ in addition to the $\rho(770)$. Including these two resonances ($\rho + \rho'$), the form factor can be written as follows:

$$F_V(s) = \frac{g_{\rho\pi^-} g_{\rho\rho\pi^-}}{m_\rho^2 - s - im_\rho \Gamma_\rho} \left\{ 1 + \alpha_\nu \frac{m_\nu^2 - s - im_\nu \Gamma_\nu}{m_{\rho'}^2 - s - im_{\rho'} \Gamma_{\rho'}} \right\}$$ \hspace{1cm} (6)
where \( \alpha_V \equiv g_\rho g_\rho V_\pi / g_\rho g_\rho V_\pi \).

Thus, the data on the spectral function for \( \tau^- \rightarrow \omega \pi^- \nu_\tau \) can be used as a test of the form factor model. In order to fit the data on \( v(s) \), we have used in Eq. (6) the values \( g_\rho^- = \sqrt{2} m_\rho^2 / \gamma_\rho = (0.166 \pm 0.005) \) GeV\(^2 \) [3, 9], and \( g_{\rho^+ \omega \pi^-} = (13.5 \pm 2.5) \) GeV\(^{-1} \), which cover the range of values for this coupling as extracted from \( \rho^- \rightarrow \pi^- \gamma, \omega \rightarrow \pi^0 \gamma \) and \( \pi^0 \rightarrow \gamma \gamma \) using the isospin symmetry relation \( g_{\rho^- \omega \pi^-} = g_{\rho \omega \pi^0} \). We have fitted the data of Ref. [6] using the \( \rho(1450) \) or the \( \rho(1700) \) in addition to the \( \rho(770) \), leaving \( \alpha_\omega \) as a free parameter and allowing for an overall normalization factor \( N \) of the spectral function.

Although the data are rather poor, we find that a better fit can be obtained using the \( \rho + \rho(1450) \) combination. In this case the best fit gives:

\[
\alpha_\omega = -0.37 \pm 0.14 \quad (7)
\]

\[
N = 1.22 \pm 0.34 \quad (8)
\]

Although the normalization factor is consistent with unity, the experimental data for the spectral function could be systematically a 20 % higher than expected. Let us mention that the effects of taking an \( s \)-dependent width [3] for the \( \rho \) and \( \rho' \) Breit-Wigner forms in Eq. (6) does not change the results of the fit.

Using the above results into Eqs. (6) and (3) we obtain (for \( N = 1 \)):

\[
B(\tau^- \rightarrow \omega \pi^- \nu_\tau) = (1.22 \pm 0.56)\% \quad (9)
\]

where the uncertainty reflects the errors in \( g_{\rho \omega \pi}, g_\rho \) and \( \alpha_\omega \). The large uncertainty above is dominated by the error we have attributed to the \( \rho \omega \pi \) coupling constant. This result for the branching ratio is consistent (within errors) with the experimental value \( B(\tau^- \rightarrow \omega \pi^- \nu_\tau) = (1.6 \pm 0.5)\% \) [10] but it lies below the prediction based on the CVC hypothesis, \( B^{CVC}(\tau^- \rightarrow \omega \pi^- \nu_\tau) = (2.2 \pm 0.3)\% \) [7].

Since the CVC hypothesis is expected to be exact in the limit of isospin symmetry, one should expect small deviations from the result of Ref. [7].
There are two probably reasons for this discrepancy. On the one hand, the cross section data on $e^+e^- \rightarrow \omega\pi^0$ [11] used to get the CVC prediction for $\tau^- \rightarrow \omega\pi^-\nu_\tau$ was measured only for $1 \text{ GeV} \leq E_{cm} \leq 1.4 \text{ GeV}$, and an extrapolation is required for the kinematical range in $\tau$ decay. Actually, the model used in [7] to extrapolate the $e^+e^-$ data seems to be in disagreement with the measurement of the spectral function in $\tau^- \rightarrow \omega\pi^-\nu_\tau$ at higher $s$ values (see Ref. [6]). This suggests that the errors in the CVC prediction for $\tau^- \rightarrow \omega\pi^-\nu_\tau$ could have been underestimated. On the other hand, let us comment that if the normalization in the data for the spectral function reported in [6] are indeed larger by 20% (see Eq. (8)), this would increase our prediction by the same amount.

In order to predict the branching ratio for $\tau^- \rightarrow \phi\pi^-\nu_\tau$, we can rely on the above model and use the $\rho^+\phi\pi^-$ coupling extracted from $\phi \rightarrow \rho\pi$:

$$g_{\rho^+\phi\pi^-} = (1.10 \pm 0.03) \text{ GeV}^{-1}. \quad (10)$$

Observe that we have to divide the $\phi \rightarrow \rho\pi$ decay rate reported in [10] by a factor of 3 in order to account for the three isospin channels allowed in $\phi \rightarrow \rho\pi$ decay.

Since the spectral function of $\tau^- \rightarrow \phi\pi^-\nu_\tau$ has not been measured yet, we will assume that $\alpha_\phi = \alpha_\omega$ in Eq. (6), which is equivalent to require that:

$$\frac{g_{\rho\phi\pi}}{g_{\rho\omega\pi}} = \frac{g_{\rho\phi\pi}}{g_{\rho'\omega\pi}}. \quad (11)$$

This relation can be obtained if we assume a $U(3)$ invariant coupling for the $V'VP$ vertex and we replace the $1^3S_1$ nonet of vector mesons (the $\rho$) by the $2^3S_1$ nonet of vector mesons (the $\rho'$) (in the $n^{2S+1}L_J$ spectroscopic notation. See for example p.1320 in [10]).

Using Eq. (10) into (6) and (3) and relying on the above assumptions, we obtain:

$$B(\tau^- \rightarrow \phi\pi^-\nu_\tau) = (1.20 \pm 0.48) \times 10^{-5} \quad (12)$$

which lies one order of magnitude below the experimental upper limit reported in Ref. [8], and almost two order of magnitude below the upper limit
obtained using the CVC hypothesis [7]. The error in Eq. (12) is dominated by the error in $\alpha_\phi$.

We now focus our discussion on the $\omega - \phi$ mixing angle. As is well known (see p.1320 in Ref. [10]), the physical states $\omega$ and $\phi$, can be written in terms of the octet ($V_8$) and singlet ($V_0$) states of SU(3) as follows:

$$\omega = V_8 \sin \theta_V + V_0 \cos \theta_V$$
$$\phi = V_8 \cos \theta_V - V_0 \sin \theta_V$$  \hspace{1cm} (13)

or, if we define $\delta \equiv \theta_V - \theta_I$, which measures the deviation from the ideal mixing angle $\theta_I = \arctan(1/\sqrt{2}) \approx 35.3^\circ$, we can write:

$$\omega = \cos \delta \frac{1}{\sqrt{2}} (\bar{u}u + \bar{d}d) - \sin \delta \bar{s}s$$
$$\phi = -\sin \delta \frac{1}{\sqrt{2}} (\bar{u}u + \bar{d}d) - \cos \delta \bar{s}s.$$  \hspace{1cm} (14)

The value obtained from the quadratic mass formula for vector mesons is $\delta \approx 4^\circ$ (p. 1320 in [10]).

The SU(3) invariant lagrangian for the $V'VP$ interactions including singlet vector states can be written as follows:

$$L_{V'VP} = G_{V'VP}^8 d_{abc} \epsilon^{\alpha\beta\gamma\delta} P^a \partial_\alpha V^b_\beta \partial_\gamma V^c_\delta + \sqrt{2} G_{V'VP}^0 \delta_{ab} \epsilon^{\alpha\beta\gamma\delta} P^a (\partial_\alpha V^b_\beta \partial_\gamma V^0_\delta + V \leftrightarrow V')$$  \hspace{1cm} (15)

where $P^a(V^a)$ $a = 1, \cdots, 8$ denote the octet of pseudoscalar (vector) mesons, and $V^0$ is the SU(3) singlet vector meson. $G_{V'VP}^i$ ($i = 8, 0$) are the corresponding octet and singlet coupling constants.

From the above lagrangian and using the definition given in Eqs. (13)–(14) for the physical states we can derive

$$g_{\rho^+\phi\pi^-} = \frac{G_{V'VP}^8}{3} \left\{ \sqrt{2} (1 - r) \cos \delta - (1 + 2r) \sin \delta \right\}$$  \hspace{1cm} (16)
$$g_{\rho^+\omega\pi^-} = \frac{G_{V'VP}^8}{3} \left\{ \sqrt{2} (1 - r) \sin \delta + (1 + 2r) \cos \delta \right\}$$  \hspace{1cm} (17)
where \( r \equiv G^0_{V'V}/G^0_{V'VP} \neq 1 \) accounts for deviations from nonet symmetry of vector mesons.

If we introduce now these couplings into Eq. (6) and use the results given in Eqs. (9) and (12) we can build the following ratio:

\[
R_{\omega\phi} \equiv \frac{B(\tau^- \to \phi\pi^- \nu_{\tau})}{B(\tau^- \to \omega\pi^- \nu_{\tau})} \\
\approx 0.1482 \left| \frac{\sqrt{2} \left( \frac{1-r}{1+2r} \right) - \tan \delta}{\sqrt{2} \left( \frac{1-r}{1+2r} \right) \tan \delta + 1} \right|^2
\]

(18)

Note that the above result reduces to the simple model proposed in Ref. [12] (namely, that violations to the OZI rule arises purely from \( \omega - \phi \) mixing) in the limit that \( r = 1 \), because in this case

\[
R_{\omega\phi} = \tan^2 \delta \cdot f
\]

(19)

where \( f \) is a kinematical factor.

In Table 1 we show the results for \( R_{\omega\phi} \), Eq. (18), as a function of the angle \( \delta \) when we allow for a \( \pm 20\% \) deviation from nonet symmetry \((r = 1)\). We can observe that, when \( r = 0.8 \) or 1, the relative production of \( \phi\pi^-/\omega\pi^- \) in tau decays is very sensitive to deviations from the ideal mixing angle. In the special case that \( \delta = 4^0 \) and \( r = 1 \), and using the experimental branching ratio for \( \tau^- \to \omega\pi^- \nu_{\tau} \), we obtain from Table 1 \( B(\tau^- \to \phi\pi^- \nu_{\tau}) \approx 1.16 \times 10^{-5} \), which is consistent with the estimated value in Eq. (12).

In summary, we have given a description of the \( \tau^- \to (\omega, \phi)\pi^- \nu_{\tau} \) in the framework of the vector dominance model. The branching fraction of \( \tau^- \to \omega\pi^- \nu_{\tau} \) is found to be consistent with the present experimental value [10] and it lies below the prediction based on the CVC hypothesis. Our prediction for the \( \phi\pi^- \) mode \( B(\tau^- \to \phi\pi^- \nu_{\tau}) = (1.20 \pm 0.48) \times 10^{-5} \) lies one order of magnitude below the present experimental upper limit given in Ref. [8] and could be measured at a \( \tau \)-charm factory. It is also shown that the simultaneous measurements of the \( \omega\pi^- \) and \( \phi\pi^- \) decay rates, can provide useful information on the \( \omega - \phi \) mixing angle.
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TABLE CAPTIONS

1. Predictions for $R_{\omega\phi}$, Eq. (18), as a function of the mixing angle $\delta$ for three different choices of $r$.

| $\delta$ (°) | $r = 0.8$     | $r = 1.0$     | $r = 1.2$     |
|-------------|--------------|--------------|--------------|
| 1           | $1.23 \times 10^{-3}$ | $4.52 \times 10^{-5}$ | $1.51 \times 10^{-3}$ |
| 2           | $8.02 \times 10^{-4}$  | $1.81 \times 10^{-4}$ | $2.08 \times 10^{-3}$ |
| 3           | $4.66 \times 10^{-4}$  | $4.07 \times 10^{-4}$ | $2.75 \times 10^{-3}$ |
| 4           | $2.20 \times 10^{-4}$  | $7.25 \times 10^{-4}$ | $3.52 \times 10^{-3}$ |
| 5           | $6.60 \times 10^{-5}$  | $1.13 \times 10^{-3}$ | $4.38 \times 10^{-3}$ |

Table 1
\[ V = \sum_{V'=\rho, \rho', \ldots} V' = \rho, \rho', \ldots \]

Fig. 1: VMD contributions to $\tau^- \rightarrow V \pi^- \nu_\tau$. 