Superconducting films with antidot arrays - novel behavior of the critical current

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Novel behavior of the critical current density \( j_c \) of a regularly perforated superconducting film is
found, as a function of applied magnetic field \( H \). Previously pronounced peaks of \( j_c \) at matching
fields were always found to decrease with increasing \( H \). Here we found a reversal of this behavior for
particular geometrical parameters of the antidot lattice and/or temperature. This new phenomenon
is due to a strong “caging” of interstitial vortices between the pinned ones. We show that this
vortex-vortex interaction can be further tailored by an appropriate choice of the superconducting
material, described by the Ginzburg-Landau parameter \( \kappa \). In effective type-I samples we predict
that the peaks in \( j_c(H) \) at the matching fields are transformed into a step-like behavior.

I. INTRODUCTION

For practical applications of superconducting (SC) ma-
terials, the increase and, more generally, control of the
critical current in SC samples are of great importance.
In recent years much attention was given to the investi-
gation of superconducting films patterned with a regular
array of microholes (antidots), which have a profound in-
fluence on both the critical current and the critical mag-
netic fields. Due to the collective pinning to the
regular antidot array, vortices are forced to form rigid
lattices when their number “matches” integer and fra-
tional multiples of the number of pinning sites at fields
\( H_n = n\Phi_0/S \), and \( H_{p/q} = \frac{\Phi_0}{S} \) (where \( n, p, q \) are in-
tegers) respectively, where \( \Phi_0 \) is the flux quantum, and
\( S \) is the area of the unit cell of the antidot lattice. This
locking by the pinning array and the vortex lattice is responsible for the reduced mobility of the vor-
tices in applied drive and consequently the increased critical
current at integer and fractional matching fields, was
confirmed both by experiments (imaging \(^{7,8,38,39}\) magnetization and transport measurements \(^{13,14,38,39}\)) and molecular dynamics simulations \(^{38,39}\).

However, regardless on the imposed pinning profile, the vortices at interstitial sites always have high mobility \(^{5,8}\), show different dynamic regimes from the pinned ones, and their appearance is followed by a sharp drop in the critical current \(^{13,14,38,39}\). In this respect, the maximal occupation number of the antidots (saturation number \( n_s \)) becomes very important for any study of the critical current. In an early theoretical work, Mkrtchyan and Schmidt \(^{38,39}\) have shown that \( n_s \) depends only on the size of the holes. However, in the case of periodic pinning arrays this number depends also on the proximity of the holes, on temperature and on the applied field \(^{13,14,38,39}\).

Besides the pinning strength of the artificial lattice, vortex-vortex interactions are crucial for vortex dynamics. Most of the experiments on perforated superconducting films are carried out in the effective type-II regime \( (\kappa_\ast = 2\kappa^2/d \gg 1/\sqrt{2}, \kappa \) being the Ginzburg-Landau (GL) parameter, and \( d \) being the thickness of the SC film scaled to the coherence length \( \xi \), where vortices act like charged point-particles, and their interac-
tion with the periodic pinning potential can be described
using molecular dynamics simulations (MDS) \(^{14,13,14,38,39}\). However, the overlap of vortex cores (with sizes \( \sim \xi \), and the exact shape of the inter-vortex interaction (depending on the material properties reflected through \( \kappa \)), which are neglected in MDS, may significantly modify the equilibrium vortex structures and consequently the critical current.

In the present letter we investigate the critical current \( j_c \) of superconducting films with regular arrays of square antidots, taking into account all parameters relevant to the SC state, within the non-linear Ginzburg-Landau theory. This formalism allows us to analyze the \( j_c \) dependence on the geometrical parameters of the sample, material (even type-I) and temperature, and compare our results with existing experiments.

II. THEORETICAL APPROACH

We consider a thin superconducting film (of thickness \( d \)) with a regular lattice of square holes (side \( a \), period \( W \)) immersed in an insulating media with a perpendicular uniform applied field \( H \) (see the inset of Fig. 1). To describe this system, we solved the nonlinear Ginzburg-Landau (GL) equations for the order parameter \( \psi \) and the vector potential \( \vec{A} \) (in dimensionless units, and with temperature \( T \) taken explicitly in units of the zero field critical temperature \( T_{c0} \), see Ref. \(^{13,14,38,39}\) for more details):

\[
\left(-i\vec{\nabla} - \vec{A}\right)^2 \psi = \psi \left(1 - T - |\psi|^2\right), \tag{1}
\]

\[
-\kappa_\ast \Delta \vec{A} = \frac{1}{i} \left(\psi^* \vec{\nabla} \psi - \psi \vec{\nabla} \psi^*\right) - |\psi|^2 \vec{A}. \tag{2}
\]

The magnitude of the applied field \( H \) is determined by the number of flux quanta piercing through the simulation region \( W_x \times W_y \). On the hole-edges we used boundary conditions corresponding to zero normal component of the superconducting current. Periodic boundary conditions \(^{13,14,38,39}\) are used around the square simulation region: \( \vec{A}(r + \hat{l}) = \vec{A}(r) + \vec{\nabla} \zeta(r) \) and \( \psi(r + \hat{l}) =
FIG. 1: (Color online) The critical current density $j_c$ (a) (in units of $j_0 = eH_2\xi/4\pi\lambda^2$) as a function of the applied magnetic field $H$ (in units of the first matching field $H_1$) for different antidot-sizes and fixed period $W = 8\xi$, and the contour plot of the Cooper-pair density at $H = H_2$ (b) and at $H = H_3$ (c) for $R = 2.6\xi$. The insets show the schematic view of the sample and the antidot occupation number $n_o$ as a function of the antidot-size $a$ at different matching fields (i.e. for $2-5$ vortices per unit cell).

\[ \psi \exp(2\pi i \zeta_i(\vec{r})/\Phi_0), \] where $\vec{l}_i$ ($i = x, y$) are lattice vectors and $\zeta_i$ is the gauge potential. We use the Landau gauge $\vec{A}_0 = H x \vec{e}_y$ for the external vector potential and $\zeta_x = HW_x y + C_x$, $\zeta_y = C_y$, with $C_x$, $C_y$, being constants. Generally speaking, depending on the geometry of the antidot lattice, one must minimize the energy with respect to the latter coefficients. The size of the supercell in our calculation is typically $4 \times 4$ unit cells (containing 16 holes, i.e. $W = 4W$). We solved the system of Eqs. (1)-(2) self-consistently using the numerical technique of Ref. [12]. In order to calculate the critical current, first we determine the ground vortex-state for a given applied magnetic field after multiple starts from a randomly generated initial Cooper-pair distribution. Then the applied current in the $x$-direction is simulated by adding a constant $A_{cx}$ to the vector potential of the applied external field. Note that the current $j_x$ in the sample resulting from the applied $A_{cx}$ is obtained after integration of the $x$-component of the induced supercurrents (calculated from Eq. (2)) over the $y = \text{const.}$ cross-section of the sample. With increasing $A_{cx}$, the critical current is reached when a stationary solution for Eqs. (1)-(2) ceases to exist (i.e. vortices are driven in motion by the Lorentz force).

III. INFLUENCE OF GEOMETRICAL PARAMETERS OF THE SAMPLE

The enforced stability of the vortex lattices in periodically perforated superconducting samples at integer and fractional matching fields leads to pronounced peaks in the critical current [1,2,3]. However, the exact shape of these lattices, and consequently their stability when locked to the pinning arrays, strongly depend on the parameters of the sample. For example, while small antidots can pin only one vortex, in larger holes multi-quantum vortices may become energetically preferable [10]. This reduces the number of interstitial vortices, whose higher mobility affects strongly the critical current of the sample. Therefore, in this section we investigate the critical current of our sample for different sizes of the antidots $a$ and antidot lattice period $W$.

FIG. 2: (Color online) The critical current density $j_c$ of the superconducting film at matching fields for different periods of the antidot lattice. The inset shows the antidot occupation number $n_o$ as a function of $W$ at different matching fields.
The lattice period is $W = 8\xi$, the film thickness $d = 0.1\mu m$ and GL parameter equals $\kappa = 1$ (corresponding roughly to Pb, Nb, or Al films). Note that, following the suggestion of Wahl, we took into account the influence of the perforation on the effective GL parameter $\kappa_s = 2\kappa^2/d(1-2a^2/W^2)$. As shown in Fig. 1 in the absence of applied field the samples with smaller antidots always have larger $j_c$, simply due to more superconducting material, i.e., larger screening. However, for $H \neq 0$, the critical current depends on the vortex structure in the sample. For small antidot-size $a = 1.5\xi$, where only one vortex can be captured by each hole (see the inset of Fig. 1), the $j_c(H)$ curve shows the expected maxima at integer matching fields $H_1, H_2, H_3$ and $H_4$ and at some of the fractional matching fields. As observed before, the peaks of the critical current at matching fields decrease with increasing magnetic field. However, a novel phenomenon is found for $2.5 \leq a/\xi \leq 2.8$: the critical current at the 3rd matching field is larger than the one at the 2nd matching field. Moreover, the critical currents for $H_2 < H < H_3$ are higher than those obtained for fields $H_1 < H < H_2$.

The explanation for this counterintuitive feature of $j_c$ lies in the hole occupation number $n_o$, i.e. the number of vortices inside the hole, and, consequently, in the saturation number $n_s$ of the holes ($n_s = n_o$ for larger fields). For holes with $2.5 \leq a \leq 2.8$, $n_s$ is equal to 1 ($n_s = a/4\xi$). Therefore, at $H = H_1$, all vortices are captured by the antidots. Analogously, at $H = H_2$, besides the pinned vortices, one vortex occupies each interstitial site. If the same analogy is followed further, one expects two vortices at each pinning site at the 3rd matching field. However, this depends also on the distance between neighboring holes. Namely, the period $W$ determines the distance between the interstitial vortices. In other words, for small $W$, this distance is small, vortex-vortex repulsion is large, and it may become energetically more favorable for the system to force one more flux-line into the hole rather than have two vortices at interstitial sides (see Figs. 1b,c). Therefore, our results show that the occupation number $n_o$ of the holes in a lattice depends not only on the size of the hole $a$, but also on the period $W$ and the number of vortices per unit-cell of the hole-lattice. For the parameters given above, $n_o = 2$ at $H = H_3$, i.e. only one vortex is located at each interstitial site, as for the case of $H = H_2$. Note that this interstitial vortex interacts repulsively with the pinned ones. This interaction is roughly twice as large at $H = H_3$ than at $H = H_2$, and the interstitial vortex becomes effectively “caged”. This “caging” effect has been found for $7\xi \leq W \leq 8.3\xi$ for the radius $R = 2.6\xi$. For $W > 8.3\xi$, $n_o$ and $n_s$ become independent of $W$ and for $W < 7\xi$, the larger suppression of superconductivity around the holes at $H = H_3$ becomes more significant than “caging”, and $j_c$ reverses again in favor of $H = H_2$.

With further increase of hole-size $a$, more vortices are captured by the holes. Consequently, in order to observe the “caging” effect, one needs to consider higher magnetic fields, i.e. for $n_o = 4$, at least $H > H_5$ is needed. In any case, this effect can always be realized at a given magnetic field by an adequate choice of $a$ and $W$. Fig. 2 shows the critical current density of our sample for different periods of the antidot lattice $W$ at $a = 2.6\xi$. For clarity, we plotted only the critical current at the matching fields. At $H = 0$ and $H = H_1$ (when there are no interstitial vortices) the critical current is an increasing function of $W$, because of the stronger screening by larger quantities of the superconducting material. For higher magnetic fields interstitial vortices nucleate more easily in a sample with larger $W$, which leads to a reduction of $j_c$. The dependence of $n_o$ (and thus also saturation number $n_s$) on $W$ is shown in the inset of Fig. 2 $n_o$ decreases from $n_o = 5$ to $n_o = 1$ at $H = H_5$, when we increase $W$ from $4\xi$ to $10\xi$.

IV. TEMPERATURE DEPENDENCE OF THE CRITICAL CURRENT

So far, we presented results of our calculations at fixed temperature, where all units were temperature dependent (e.g. distances were expressed in $\xi(T)$). In what follows, we consider a superconducting film with thickness $d = 20nm$, interhole distance $W = 1\mu m$, and antidot size $a = 0.33\mu m$. We choose the coherence length $\xi(0) = 40nm$ and the penetration depth $\lambda(0) = 42nm$, which are typical values for Pb films. We study the influence of temperature with the help of the right-side term of Eq. (1), which actually describes the temperature dependence of the coherence length $\xi(T) = \xi(0)/\sqrt{1-T/T_c(0)}$ and penetration depth $\lambda(T) = \lambda(0)/\sqrt{1-T/T_c(0)}$.

Fig. 3 shows the calculated critical current density of the sample as a function of the applied field normalized
to the first matching field at temperatures $T/T_{c0} = 0.86$, 0.88, 0.9, 0.92, 0.94, 0.96 and 0.98. As discussed in the previous section, the $j_c(H)$ curve shows pronounced maxima at matching fields with a substantial drop after the number of vortices per unit-cell exceeds the hole-saturation number (in our case, for $H > H_1$).

As expected, decreasing temperature results in an increase of the critical current for given magnetic field. However, the qualitative behavior of the $j_c(H)$ characteristics changes. Although the $W/a$ ratio remains the same, the size of vortices and the occupation number of the holes may change with temperature (as $\xi(T)$ changes). Actually, we found the same vortex structure at all temperatures $T > 0.8T_{c0}$, but $j_c(H_3)$ is found to be larger than $j_c(H_2)$ only in the temperatures range $0.8T_{c0} \leq T \leq 0.93T_{c0}$. At higher temperatures vortices become larger and suppression of superconductivity around the holes “masks” the critical current enhancement by the “caging” effect. This decrease of Cooper-pair density around the holes is also responsible for the decreased $j_c(H_1)/j_c(0)$ ratio with increasing temperature. At temperatures lower than $0.8T_{c0}$, the peaks of $j_c$ again decrease with magnetic field, since $W/\xi$ significantly increases and its influence on $n_o$ diminishes (see previous section).

Interestingly enough, this $j_c(H_{n+1})$ vs. $j_c(H_n)$ reversing behavior was recently found experimentally, but not noticed. In Fig. 6 of Ref. a clear enhancement of the critical current at $H/H_1 = 3$ was found which is larger than the one at $H/H_2$ as is similar to the behavior found in Fig. 8 for $T/T_{c0} < 0.94$. A quantitative comparison between theory and experiment is difficult because of the different determination of $j_c$. In our calculations we assume normal state as soon as vortices are set in motion, whereas in transport measurements a certain value of threshold voltage determines the critical current.

V. EFFECTIVE TYPE-I VS. TYPE-II BEHAVIOR OF THE CRITICAL CURRENT

In the previous sections we have shown that higher mobility of interstitial vortices leads to a dramatic decrease of the critical current. Therefore, keeping the interstitial sites of the superconductor vortex-free is essential for an improvement of $j_c$. In this respect, we consider effectively type-I superconductors, where (i) the screening of the magnetic field is enhanced (i.e. vortices will be compressed in the holes), and (ii) the interaction of vortices becomes attractive (depends on $\ln \kappa_s$). As an example we considered a sample, with antidot size $a = 0.5\mu m$, lattice period $W = 1.5\mu m$, $\xi_0 = 40\text{nm}$ and $\lambda_0 = 10\text{nm}$. We fine-tuned the effective vortex-vortex interaction by varying the thickness of the sample, i.e. changing from type-II to type-I behavior with increasing $d$. Fig. 4 shows the critical current of the sample for four values of the film thickness: $d = 10\text{nm}$ (solid dots), $d = 50\text{nm}$ (open dots), $d = 100\text{nm}$ (solid squares) and $d = 150\text{nm}$ (open squares) at $T = 0.97T_{c0}$. For $d = 10\text{nm}$, the sample is still in the type-II regime ($\kappa_s = 2.887$), and the critical current shows a peak-like behavior at the matching fields. The drop in the critical current for $H > H_1$ is caused by the interstitial vortices which is larger with increasing $\kappa_s$. Note that the “caging” effect is also present for these values of the parameters. When the film thickness is increased ($d = 50\text{nm}$, $\kappa_s = 0.577$) the critical current density is higher for $H < H_2$. For $H < H_1$, this increase is achieved due to a stronger Meissner effect. At fields $H_1 < H < H_2$ the increase of $j_c$ is more apparent, as all vortices are captured by the holes (except for $d = 10\text{nm}$, where $n_o = 1$). As soon as interstitial vortices appear in the sample ($H > H_2$), $j_c$ becomes even smaller than the one for $d = 10\text{nm}$. This inversion of $j_c$ clearly demonstrates the higher mobility of interstitial vortices in type-I superconductors.

Looking at the $j_c(H)$ curve as a whole for $d = 50\text{nm}$, we observed a pronounced step-like behavior. Note that in type-I samples the matching between the number of flux-lines and the number of antidots does not lead to a peak-like increase of the critical current. Namely, regardless of their number, additional flux lines are doubly pinned by the attractive hole-potentia and the attractive interaction with previously pinned vortices. The “step” in $j_c(H)$ occurs only when the number of vortices per hole $n_o$ changes (more vortices and consequently a larger suppression of $\psi$ around the hole), or when interstitial vortices appear. The effect of an increase of $n_o$ diminishes as $d$ increases (i.e. $\kappa_s$ decreases and screening increases), or when temperature is lowered (as discussed before). This tendency is illustrated by solid and open dots in Fig. 4 and ultimately leads to a two-step $j_c(H)$ curve, with larger critical current for $H < H_n$, and smaller $j_c$ for $H > H_n$, where $n = n_o$.

![FIG. 4: (Color online) The critical current density $j_c$ as a function of the applied magnetic field for different thickness of the sample. Remaining parameters are listed in the figure.](image-url)
VI. CONCLUSIONS

We studied the critical current of a superconducting film containing an array of antidots in a uniform magnetic field, as a function of all relevant parameters. We found that the well-known $j_c$ enhancement by artificial vortex pinning strongly depends on the antidot occupation number $n_o$. The latter is determined not only by the size of the antidots as commonly believed, but also by their spacing and the applied field. As a consequence, when the parameter conditions are met, the critical current becomes larger at higher matching fields, contrary to the conventional behavior. Such a feature is a result of the “caging” of interstitial vortices between the larger number of pinned ones. This effect is strongly influenced by temperature, which agrees with a recent experiment.

Additionally, the interactions in this system can be tailored by the effective Ginzburg-Landau parameter $\kappa_\ast$. In effectively type-I samples, vortices attract, $j_c$ increases and peaks at matching fields diminish due to the enforced pinning at all fields. As a result, a novel step-like behavior of the critical current is found, with a sharp drop at higher fields, when interstitial vortices appear.

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