Screening Masses in SU(N) from
Wilson Renormalization Group

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Abstract

We apply a gauge invariant formulation of Wilson Renormalization Group (RG) to the computation of the Debye and transverse gluon masses in pure gauge SU(N) at high temperature. Following the Hard Thermal Loop effective field theory as a guideline, we develop an approximation scheme to the exact evolution equations.

The Debye mass receives sizable corrections compared to the leading order perturbative result, mainly due to the infrared singular behavior in the transverse gluon sector. A non-vanishing mass for the transverse gluons is found, which acts as an infrared regulator though not efficiently enough as to restore the validity of perturbation theory. Indeed, discussing the role of higher dimensional operators, we show that the gauge coupling for the transverse modes typically flows to non-perturbative values unless extremely high temperatures are reached.

After comparing our results with recent lattice simulations, we comment on the possibility of using this formulation of the RG as a tool to construct an effective field theory for the non-perturbative, long wavelength, transverse modes.
The study of the properties of a non-abelian plasma at high temperature is of great interest both in cosmology, concerning the quark-hadron phase transition and the properties of the symmetric phase of the Standard Model, and in particle physics, in connection with the possibility of creating the quark-gluon plasma in heavy ion collisions.

From a theoretical point of view, it has been known for a long time that perturbation theory is afflicted by severe infrared problems caused by the static transverse gluons, which remain massless at any order \( \mathcal{O}(g^0) \). As a consequence, the loop expansion ceases to be an expansion in powers of the coupling constant and, for instance, infinite loop orders contribute to the pressure at the same order \( \mathcal{O}(g^0) \).

The Debye (or longitudinal) mass describes the screening of chromo-electrostatic fields in the plasma, and its numerical value influences the probability of \( J/\Psi \) and \( \Upsilon \) formation, among the main signatures of quark-gluon plasma formation. At leading order in perturbation theory it is given by \( m_L^2 = Ng^2T^2/3 \) for \( SU(N) \). At next to leading order infrared divergencies show up, and the most one can compute is the coefficient of the term \( g^3 \log(m_L/m_T) \), \( m_T \) being the transverse (or magnetic) gluon mass, which at this level has to be introduced by hand \([3]\). The coefficients of the higher order terms in \( g \) are not computable in perturbation theory.

In this letter we will apply the Wilson Renormalization Group (RG) method introduced in refs. \([3,4]\) to the computation of the Debye mass \( m_L \). As we have seen, the value of \( m_L \) beyond leading order is influenced by the infrared cut-off in the transverse sector, \( m_T \). We will first derive a coupled system of RG flow equations for \( m_L \) and \( m_T \) describing the effect of the integration of thermal fluctuations at larger and larger length scales. Then, we will include in the system also the running of the coupling constants and of wave function renormalizations.

The RG formulation developed in refs. \([3,4]\) is based on the introduction of an infrared cut-off, \( \Lambda \), in the thermal sector of the theory, so that the modes of momentum \( |\vec{k}| \gg \Lambda \) are in thermal equilibrium at the temperature \( T \) while those with \( |\vec{k}| \ll \Lambda \) are frozen at \( T = 0 \). The RG flow equations then interpolate between the full (renormalized) quantum field theory at \( T = 0 \) in the \( \Lambda \to \infty \) limit, and the full quantum field theory in thermal equilibrium at the temperature \( T \) for \( \Lambda \to 0 \). Compared to other formulations of the Wilson RG in the literature \([5]\), we are then integrating out only the thermal fluctuations, all the quantum fluctuations being already included in the initial conditions of the RG flow. As discussed in detail in ref. \([4]\) the main advantage of introducing the cut-off only on the thermal sector is that BRST invariance is preserved. From a computational point of view, this allows us to use Slavnov-Taylor (ST) identities as a powerful constraint in approximating the exact evolution equations. From a physical point of view, we have a tool to derive an effective, gauge invariant, field theory, even for a non-zero value of the cut-off \( \Lambda \).

The effective field theory at the scale \( gT \) has been developed in refs. \([3,4]\) and is known as Hard Thermal Loop (HTL). As external momenta smaller than \( gT \) or next to leading order corrections are considered, the magnetic divergencies show up and the HTL resummation breaks down. Using the HTL effective theory as a starting point, the RG will guide us deeper in the infrared. As we will see, as \( \Lambda \) approaches \( m_T \) the gauge coupling in the transverse sector rises to non-perturbative values, then large corrections are to be expected to our results for \( m_T \). On the other hand, most of the renormalization of \( m_L \) takes place for larger values of \( \Lambda \), where the couplings are still small, and the approximation to the RG equation is still reliable.
2. In this letter we will be mainly interested in the longitudinal (or Debye) and transverse (or magnetic) masses, defined as the static poles of the full propagator,

\[ m_{L,T}^2 = \Pi_{L,T}(q_0 = 0, |\vec{q}|^2 = -m_{L,T}^2), \]  

where \( \Pi_{L,T} \) are obtained from the self-energy \( \Pi^{\mu\nu} \) as \( \Pi_L = \Pi^{00} \) and \( \Pi_T = -1/2 \Pi^{ii} \). As shown in ref. [7], the definitions (1) are gauge-independent. The same holds even in presence of the infrared cut-off, since it does not break BRST invariance.

The RG flow of the Debye and magnetic masses is described by the equations [4]:

\[ \Lambda \frac{\partial}{\partial \Lambda} m_{L,T}^2 = \frac{\Lambda \frac{\partial}{\partial q^2} \Pi_{L,T}(q)}{1 + \frac{\partial \Pi_{L,T}(q)}{\partial |\vec{q}|^2} |\vec{q}|^2 = -m_{L,T}^2}, \]

where the flow equations for \( \Pi_{L,T} \) are obtained by taking the appropriate components of that for the self-energy.

In the case of one particle irreducible vertices the RG flow equations can be obtained by the following simple recipe: 1) take the expression for the one loop correction to the desired vertex; 2) substitute the tree level propagators and vertices in it with the full, cut-off dependent ones; 3) take the derivative with respect to the explicit cut-off dependence (i.e. derive only the cut-off function in the propagators). For the self-energy we get [4]

\[ \Lambda \frac{\partial}{\partial \Lambda} \Pi_{\Lambda,\mu\nu}(q) = -\frac{i}{2} \int \frac{d^4k}{(2\pi)^4} K_{\Lambda,\rho\lambda}(k) \left[ \Gamma_{\Lambda,\mu\nu\rho\lambda}(q,-q,k,-k) + 2 \Gamma_{\Lambda,\mu\rho\eta}(q,-k,k-q) G_{\Lambda,\eta\delta}(k-q) \Gamma_{\Lambda,\nu\lambda\delta}(-q,k,q-k) \right] + \text{Ghost}. \]

The kernel,

\[ K_{\Lambda,\rho\lambda}(k) = \rho_{\Lambda,\rho\lambda}(k) \varepsilon(k_0) \Lambda \delta(|\vec{k}| - \Lambda) N_b(|k_0|), \]

contains the Bose-Einstein distribution function, \( N_b(|k_0|) \), and the full spectral function

\[ \rho_{\Lambda,\rho\lambda}(k) = -i \varepsilon(k_0) \text{Disc} G_{\Lambda,\rho\lambda}(k), \]

and \( G_{\Lambda,\rho\lambda}, \Gamma_{\Lambda,\mu\rho\eta}, \) and \( \Gamma_{\Lambda,\mu\nu\rho\lambda} \), are the full propagator and vertices computed at the scale \( \Lambda \).

Up to now no approximation has been performed, and the evolution equations (2) define the screening masses non-perturbatively. Of course, in order to solve them, one should know the exact propagator and vertices (including their momentum dependence) for any value of the cut-off \( \Lambda \). This requires managing an infinite system of differential equations for the self-energy and all higher order vertices, which is equivalent to solving finite temperature QCD.

As discussed for instance in [3], RG equations can be solved iteratively, reproducing the usual loop expansion. Introducing in the RHS the \( \Lambda \)-independent expressions for the propagator and vertices at the loop \( l \) and integrating in \( \Lambda \) yields the results at the \( l+1 \) loop order. In this case, we recover the well known results \( m_L \equiv m_L^{10} = \sqrt{N/3}gT \) and \( m_T = 0 \) after the first iteration. However it is well known that in high temperature QCD the loop expansion ceases to be a sensible approximation already at the second loop, since contributions \( O(g^{3/2}T) \) to \( m_L \) are generated at each higher order [3, 2]. Concerning \( m_T \), it is easy to realize that starting from \( m_T = 0 \), no non-vanishing value can be generated at any order in the loop expansion.
Analogously, the HTL loop expansion at $l+1$ order can be reproduced by introducing the $\Lambda$-independent results obtained from the HTL effective theory at the $l$-loop in the RHS of eqs. (3). After the first iteration, the result of eq. (8) for $m_L$ discussed by Rebhan in ref. [2] is obtained.

In order to go beyond the HTL effective field theory and closer to the RG framework the next logical step is to promote the $\Lambda$-independent HTL propagator and vertices to $\Lambda$-dependent ones. Then, the contribution to $m_{L,T}$ at the scale $\Lambda$ will be given by the RHS of eqs. (3) with $m_{L,T}$ (and the coupling constant) evaluated at the same scale $\Lambda$. Moreover, we will require that the improved vertices and propagator satisfy the same tree level ST identities as the HTL ones [3].

The HTL propagator and vertices are complicated functions of the momenta [3], whose expressions simplify when the external energy vanishes. Since in this letter we are interested in computing the static quantities in (1) a great simplification to the flow equations can be obtained by rotating to the imaginary time [8]. The evolution equation (3) then takes the form (from now on we omit the $\Lambda$-dependence of the various quantities)

$$\Lambda \frac{\partial}{\partial \Lambda} \Pi_{\mu\nu}(q_0 = 0, |\vec{q}|) = T \sum_n F_{\mu\nu}(z = 2i\pi n T; |\vec{q}|; m_L, m_T)$$

$$- \int \frac{dk_0}{2\pi} F_{\mu\nu}(k_0; |\vec{q}|; m_L, m_T)$$

with

$$F_{\mu\nu}(k_0; |\vec{q}|; m_L, m_T) = -\frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \Lambda \delta(|\vec{k}| - \Lambda) G_{\rho\lambda}(k) [\Gamma_{\mu\nu\rho\lambda}(q, -q, k, -k)$$

$$+ 2\Gamma_{\mu\rho\lambda}(q, -k, k - q) G_{\eta\delta}(k - q) \Gamma_{\nu\lambda\delta}(-q, k, q - k)] + \text{Ghost.}$$

Notice that, due to the cancellation between the Matsubara term in the first line and the “$T = 0$” term in the second line, eq. (4) is free from ultraviolet divergencies, as it is manifest also in the form (3), which contains the Bose-Einstein function. This is of course to be expected, since these RG equations describe the effect of thermal fluctuations only.

The evolution equation for $m_L$ involves $F_{00}$. We approximate it by using tree level propagator and vertices for the non-zero Matsubara modes and in the “$T = 0$” part,

$$F_{00}(k_0; |\vec{q}|; m_L, m_T) \simeq F_{00}(k_0; |\vec{q}| = 0; m_L = 0, m_T = 0) \quad \text{for } n \neq 0.$$  

Analogously, in the evolution equation for $m_T$, we approximate $F_{ii}$ as

$$F_{ii}(k_0; |\vec{q}|; m_L, m_T) \simeq F_{ii}(k_0; |\vec{q}| = 0; m_L, m_T = 0) \quad \text{for } n \neq 0.$$  

These approximations will lead to $O(g^2 m_T^2 / T^2)$ and $O(g^2 m_L^2 / T^2)$ errors, respectively, since $O(g^2 m_{L,T} / T)$ terms may only come from the, infrared problematic, zero Matsubara mode.

The next-to-leading correction in the HTL resummed perturbation theory can be obtained by using resummed propagators and vertices for the zero mode only [3]. In the same spirit, we will employ HTL-inspired propagator and vertices only for $n = 0$.

The propagator we need is

$$\Delta_{\mu\nu}|_{k_0 = 0} = \frac{Z_L}{|\vec{k}|^2 + m_L^2} g_{\mu\nu} g_{\rho\sigma}$$

$$+ \frac{Z_T}{|\vec{k}|^2 + m_T^2} \left( g_{\mu\nu} - g_{\mu\rho} g_{\nu\sigma} + \frac{k_{\mu} k_{\nu}}{|\vec{k}|^2} \right) - \frac{k_{\mu} k_{\nu}}{|\vec{k}|^4} \bigg|_{k_0 = 0}, \quad (5)$$
where, compared to the true HTL propagator [2] we have \( \Lambda \)-dependent \( m_{L,T} \) and \( Z_{L,T} \). The wave function renormalizations are defined, as usual, as \( Z_{L,T} = 1 + \frac{\partial}{\partial q^2} \Pi_{L,T}(q_0 = 0, |q|^2) \big|_{|q|^2 = m_{L,T}^2} \).

In the HTL approximation, the three and four gluon vertices, evaluated at zero external energy, reduce to their tree level expressions. Here, the presence of a non-zero \( m_T \) forces us to modify the trilinear vertex in order to preserve the gauge invariance of the evolution equations [2]. Requiring that the new vertex is related to the propagator [2] by the tree-level ST identity does not fix it completely. The transverse component of the vertex could be in principle determined by studying its RG equation. In order to limit the number of flow equations, in this letter we will instead fix the transverse part by imposing that the improved vertex has the same form as that obtained by Alexanian and Nair [2],

\[
\Gamma_{\mu\nu\rho}(q,k,-q-k)|_{q_0=k_0=0} = g[v_{\mu\nu\rho}(q,k,-q-k) + V_{\mu\nu\rho}(q,k,-q-k)]
\]

where

\[
v_{\mu\nu\rho}(q,k,-q-k) = (q-k)_\rho g_{\mu\nu} + (2q+k)_\rho g_{\mu\nu} - (q+k)_\rho g_{\mu\nu}
\]

and

\[
V_{\mu\nu\rho}(q,k,-q-k) = V_{\mu\rho}(q,k,-q-k) = V_{\nu\rho}(q,k,-q-k) = 0,
\]

\[
V_{ij\mu}(q,k,-q-k) = -m_T^2 A \left[B k_i k_j k_\mu + C (q_i q_j k_\mu + q_i q_\mu k_j + q_\mu q_j k_i)\right] - (q \leftrightarrow k)
\]

with

\[
A = \frac{1}{k^2 q^2 - (q-k)^2}, \quad B = \frac{q k q}{k^2 (q+k)^2}, \quad C = \frac{k(q+k)}{(q+k)^2}.
\]

Note that the vertex correction \( V_{\mu\nu\rho} \) does not appear in the evolution equation for \( m_L \) but only in that for \( m_T \). Inserting eqs. (2), (3), and the tree level expressions for the four-gluon vertex and the ghost propagator and vertices in the on-shell evolution equations [2] the gauge parameter dependence drops, as can be checked by explicit calculation.

In the flow equations [2] the vertices are evaluated at external momenta of order \( \Lambda \). As a first step, the thermal contribution to the renormalization of the coupling constant will be neglected, and the dependence on the external momentum will be taken into account by using the \( T = 0 \) beta function at two loops,

\[
g^{-2}(\Lambda) = \frac{11 N}{24\pi^2} \log \left( \frac{\Lambda}{\Lambda_{MS}} \right) + \frac{17 N}{88\pi^2} \log \left[ 2 \log \left( \frac{\Lambda}{\Lambda_{MS}} \right) \right]
\]

for \( \Lambda > m_L \), and \( g(\Lambda) = g(m_L) \) for \( \Lambda \leq m_L \). The wave function renormalizations \( Z_{L,T} \) will be fixed at \( Z_{L,T} = 1 \).

The thermal renormalization of \( m_L/T \) as a function of the cut-off \( \Lambda \) can be read from Fig.1. The effect of the fluctuations integrated out at scales \( \Lambda/T \gtrsim O(1) \) is negligible due to Boltzmann suppression. Around \( \Lambda/T = O(1) \) the HTL contribution is rapidly built up. Stopping the running at this scale basically reproduces the HTL effective field theory results. Going to lower energy scales the infrared divergencies in the magnetic sector start showing up and, depending on the value of \( m_T \), their effect may eventually overcome the leading perturbative result. This is clearly seen from the upper line, where we have set \( m_T = 0 \).
In next-to-leading order perturbative computations the generation of a $O(g^2T)$ magnetic mass by non-perturbative effects is usually invoked as a mechanism to regulate the infrared divergencies coming from the transverse gluon sector. The computation of $m_L$ at next-to-leading order gives \[ m^2_L = \frac{N}{3} g^2 T^2 \left( 1 + \frac{N}{2\pi} g \log \frac{m_L}{m_T} + Cg + O(g^2) \right) \]
where $C$ is a coefficient receiving contributions at any loop order and therefore not computable in perturbation theory. If we introduce by hand an ad hoc $m_T = O(g^2T)$, then perturbation theory can tell us the coefficient of the $g \log(1/g)$ term, but in any case it becomes meaningless beyond this order.

Fixing $m_T = g^2(T)T$, where $g(T)$ is obtained by computing eq. (7) at the thermal scale of perturbation theory, $\Lambda_{th} = 4\pi T \exp(-\gamma_E) \simeq 7.055 T$ \[\text{[10]},\] the lowest line in Fig.1 is obtained. In Fig. 2 $m_L/T$ is plotted as a function of the temperature. The lowest line corresponds again to $m_T = g^2(T)T$. Notice that it agrees with the next-to-leading order perturbative result (eq. (8) with $C = 0$) only for very high temperatures $T \gtrsim 10^{10}\Lambda_{MS}$. The next step is to let $m_T$ run, studying the coupled system of RG equations for $m_L$ and $m_T$. We note immediately that, at this level of the approximation, $m_T = 0$ is a fixed point of the RG flow. In fact, in this case, the RHS of the flow equation reduces to the one-loop expression which, upon integration, gives $m_T = 0$.

The possibility of a thermal renormalization of the magnetic mass is then related to its initial value at $T = 0$. The structure of the gluon propagator is still subject of intense study, and no definite answers are available. However different groups have found evidence for the existence of a pole at non-zero momentum of $O(\Lambda_{MS})$ \[\text{[11]}\]. In the following we will assume that such a pole indeed exists, and will use $m_T = m_L = \eta \Lambda_{MS}$, with $\eta$ a coefficient.
Figure 2: Plot of $m_L/T$ vs. $\log_{10}(T/\Lambda_{\overline{MS}})$ for $SU(3)$. The lowest line is obtained by fixing $m_T = g^2(T)T$, the "HTL" line represents eq.(8) computed for $g = g(T)$ and $C = 0$, and the "HTL + NP" (dotted line) is the same equation but for $C = 1.3$. The solid line is the result of the RG system for $m_L$ and $m_T$.

of $O(1)$, as initial value for the running. Increasing $T$, the sensitivity to $\eta$ becomes weaker and weaker, since changing this parameter is equivalent to rescaling $\Lambda_{\overline{MS}}$, which enters the problem only in the argument of the logarithm in eq. (7).

The result is plotted with the continuos line in Fig. 2. A comparison with the next-to-leading expression of eq. (8) (dotted line), computed for $g = g(T)$, allows us to determine the non-perturbative coefficient $C$ as

$$C = \begin{cases} 1.1 & \text{for } SU(2) \\ 1.3 & \text{for } SU(3) \end{cases}$$

As a general fact, we observe that the corrections to $m_L$ are larger than those obtained by fixing $m_T = g^2 T$. This is due to the fact that the values of $m_T$ given by the RG are generally much smaller than $g^2 T$, so the decoupling of the RG flows takes place deeper in the infrared. It can be seen explicitly from the dashed line in Fig. 1. A $m_T = O(g^2 T)$ naturally emerges from gap equations, [12, 9] where the $\Lambda$-dependence of $m_T$ inside the loop integral is neglected. Once this dependence is kept, the simple scaling law $m_T = O(g^2 T)$ is lost. We can understand it by looking at the approximate form of the flow equation for $m_T$ obtained in the limit $\Lambda \gg m_T$,

$$\Lambda \frac{\partial}{\partial \Lambda} m_T^2 = -K g^2 T \frac{m_T^2}{\Lambda},$$

with $K = 55/8\pi^2 \approx 0.697$ in the case of $SU(3)$. We may also neglect the $\Lambda$-dependence of $g$ and the contribution of the running for $\Lambda$ below $m_T$, as we can verify by comparison with the full numerical results.
The gap equation approximation corresponds to integrating eq. (9) with a \( \Lambda \)-independent \( m_T = m_{T,\text{gap}} \) such that

\[
\frac{m_{T,\text{gap}}^2}{T^2} = \frac{m_T^2(\Lambda = \infty)}{T^2} + K g^2 \frac{m_{T,\text{gap}}^2}{T} \int_{m_{T,\text{gap}}}^{\infty} \frac{d\Lambda}{\Lambda^2} \\
\simeq K g^2 \frac{m_{T,\text{gap}}}{T} \quad (T \gg m_T(\Lambda = \infty) = \eta \Lambda_{\overline{MS}}),
\]

leading to the non-vanishing solution \( m_{T,\text{gap}} = K g^2 T \).

On the other hand, integrating eq. (9) down to \( \Lambda = m_T \) without further approximations, gives the equation

\[
m_T^2 = m_T^2(\Lambda = \infty) \exp \left( K g^2 T/m_T \right), \tag{10}
\]

which always gives \( m_T \ll m_{T,\text{gap}} \) for \( T \gg \eta \Lambda_{\overline{MS}} \).

From the full RG equations we obtain values for \( m_T \) which are in reasonably good agreement with the solutions of eq. (10) and, in the range of temperatures considered, are very well approximated by the law (in \( SU(3) \))

\[
m_T = 0.128 g^2 T(1 + 2.26 g \log g).
\]

3. In order to assess the reliability of our approximation to the RG flow equations for \( m_L \) and \( m_T \) we must consider the thermal running of higher dimensional operators. In particular, we will study the running of the gauge coupling constant in order to determine the efficiency of the screening masses as infrared regulators.

In the flow equations for \( m_L \) and \( m_T \) only two types of trilinear vertices appear, \( \Gamma_{00i} \) and \( \Gamma_{ijk} \), where \( i,j,k = 1,2,3 \). The RG equations for these two vertices differ remarkably in the infrared, since that for \( \Gamma_{ijk} \) receives contribution from the loop in which the three circulating gluons are all transverse, of mass \( m_T \), whereas in the loops contributing to the running of \( \Gamma_{00i} \) at least one longitudinal gluon appears. As a consequence, it is convenient to modify the parameterization of the trilinear vertex in eq. (3) by introducing two different coupling constants, \( g_L \) for \( \Gamma_{00i} \), and \( g_T \) for \( \Gamma_{ijk} \). Of course, due to the presence of the thermal bath, this is not in contradiction with Lorentz invariance.

From the RG flow of \( \Gamma_{\mu
\nu\rho} \) we extract those of \( g_{L,T} \), defined as

\[
g_L = \lim_{q \to k} \frac{(q+k)^\rho}{q^2 - k^2} \Gamma_{00\rho}(q,k,-q-k) \bigg|_{q^0 = 0, \sqrt{q^2} = -m_L^2},
\]

\[
g_T = \lim_{q \to k} \frac{(q+k)^\rho}{2(q^2 - k^2)} \Gamma_{iij}(q,k,-q-k) \bigg|_{q^0 = 0, \sqrt{q^2} = -m_T^2}, \tag{11}
\]

The running of the couplings is shown in Fig. 3. Notice the impressive difference, induced by thermal effects, between \( g_L \) and \( g_T \). This is due to the fact that graphs with only transverse gluons in the loop contribute to the running of \( g_T \) but not the ones of \( g_L \). When \( m_T \ll \Lambda \ll T \) the transverse sector of the theory is in the three-dimensional regime, and the coupling constant \( g_T \) grows as \( 1/\Lambda \) in the infrared. The non-vanishing \( m_T \) leads to decoupling for \( \Lambda \lesssim m_T \), but it takes place when the coupling \( g_T \) has already grown to non-perturbative values.
Figure 3: RG flows for $g_L$ and $g_T$ defined as in eq. (11) for $SU(3)$ and $T = 10^4 \Lambda_{MS}$.

Comparing Figs. 1 and 3 we see that the evolution of the coupling constants affects our results for $m_L$ and $m_T$ in a different way. First, notice that in the flow equation for $m_L$ only the vertex $\Gamma_{00i}$, and then the safer coupling $g_L$, appears explicitly. The effect of $g_T$ is felt by $m_L$ only indirectly, through the running of $m_T$. Moreover, the running of $m_L$ is almost completely decoupled before $g_T$ starts its crazy rise. As a consequence, the non-perturbative regime for $g_T$ does not affect very much our results for $m_L$, since there is no time enough to communicate it to the longitudinal sector.

On the other hand, since most of the running of $m_T$ takes place when $g_T$ is also running the inclusion of higher order operators gives larger corrections to $m_T/T$. We estimate that they are typically of $O(1)$, corresponding to a few percent corrections to $m_L/T$.

4. Summarizing, we have employed the Wilson Renormalization Group at finite temperature introduced in refs. [3, 4] to study the gluon self-energy in high temperature QCD. In order to keep contact with the perturbative and HTL-resummed results, we have developed an approximation scheme based on the use of HTL-inspired Ansätze for the propagator and vertices appearing in the RG equations.

We have concentrated on $m_L$ and $m_T$, the Debye and magnetic masses, deriving a system of differential equations for them, the coupling constants and the wave function renormalizations. We obtain large corrections to $m_L$ with respect to the leading order result, $m_{LO}^L = \sqrt{N/3} g T$, mainly due to infrared effects in the transverse sector.

Our results for $m_L$ in pure gauge $SU(2)$ are in good agreement with the lattice results of ref. [3] on the pole mass of the longitudinal gluon propagator in the Landau gauge. For instance, for $T/\Lambda_{MS} = 4000$ we find $m_L/T = 1.12$ whereas they obtain $m_L/T = 1.1 - 1.3$, the leading order perturbative value being $m_L/T \simeq 0.84$. Lattice computations of the
screening masses have been reported also by Kajantie et al. in ref. [15], where the long distance behaviour of certain gauge-invariant operators is studied. In this case corrections a factor two to three larger than ours and those of ref. [13] are generally found. As discussed in [14, 13] a possible explanation of this discrepancy could be that the effective thermal mass extracted from gauge-invariant composite operators arises from the superposition of several decoupled gluons.

Concerning $m_T$, results much smaller than the value $g^2 T$ obtained from gap equations are typically found, and this has been ascribed to the scale dependence of $m_T$.

The inclusion of the running of the coupling constants typically leads to sizable corrections in the transverse sector, since the relevant coupling rises to non-perturbative values in the infrared. This result clearly indicates that the dynamics of the transverse modes of wavelength $\sim 1/m_T$ is highly non-perturbative.

In perturbation theory, the non-perturbative modes circulate in the loop at any order, leading to the infrared problem that we have already discussed. The same statement that the HTL give the effective theory at the scale $gT$ is meaningful only at leading order, since at higher orders the longer scales come into play. On the other hand, in a RG framework, the scale $\Lambda$ can be used to keep the long wavelength modes under control, allowing the definition of effective theories at larger and larger scales in a clean way. This procedure usually runs into problems for gauge theories, since the introduction of the cut-off $\Lambda$ breaks gauge invariance. In our framework this is not the case, since the cut-off is imposed on the thermal, on-shell, sector of the theory only [4]. The construction of effective field theories along the lines illustrated in ref. [16] for the scalar theory can then be properly carried out in this context.

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