Experimental investigation and numerical modeling of destruction dynamics of thin silicone oil layers under the local heating

E V Barakhovskaia¹,², S E Spesivtsev¹,² and D Y Kochkin¹,³

¹ Kutateladze Institute of Thermophysics, Russian Academy of Sciences, Prosp. Lavrentyeva 1, Novosibirsk, 630090, Russia
² Novosibirsk State University, Pirogova Str. 2, Novosibirsk, 630090, Russia
³ Novosibirsk State Technical University, Prosp. K. Marksa 20, Novosibirsk, 630073, Russia

E-mail: ella94@bk.ru

Abstract. The processes of heat and mass transfer in systems with liquid-gas interface are of interest to a wide range of problems. Destruction dynamics of thin horizontal layers of silicone oils were investigated using a confocal sensor on a 3D-positioning system. The numerical solution of the problem was obtained in the lubrication approximation theory for two-dimensional axisymmetric thermocapillary flow. The model takes into account the surface tension, viscosity, gravity and heat transfer in the substrate. The numerical algorithm for the joint solution of the energy equation and the evolution equation for the liquid layer thickness has been developed. The establishment method was used to obtain the stationary solutions. Experimental measurements and numerical calculations were made for silicone oils of different viscosities, heating power and initial thickness. The significant effect of the surface tension coefficient and its temperature coefficient on thermocapillary deformation was detected. It was experimentally established that the deformation value depends on a heat flux value. A liquid bump is formed at the boundary of the heating region that is also observed in numerically calculated profiles.

1. Introduction

The processes of heat and mass transfer in systems with liquid-gas interfacial surface are of interest to a wide range of fundamental and applied problems. At the present time, much attention is paid to studying the mechanisms that lead to dynamic deformations of the liquid-gas interfacial surface. Phenomena in horizontal liquid layers under the local heating have been studied in a number of papers, for example, in [1-3] the problem of stationary thermocapillary flow in a horizontal locally heated liquid layer is considered. The papers [4-5] address the problem of the liquid film flow with local heating from the inclined surface side. There are not many works that study a thin horizontal liquid layer under the local heating from the bottom side [6], but exactly in this formulation that the problem is directly related to one of the most demanded tasks in thermophysics - the problem of cooling microelectronic equipment [7-8]. The results of the experimental work are given in the study [9], in which the thermocapillary deformation and rupture of the horizontal water layer are first measured by a contactless measuring instrument, using the method of laser confocal microscopy.
Depending on the goal, the authors use different approaches to describe the processes occurring in thin liquid layers, and solve the problems arising in this process numerically. The presence of a large number of ongoing experiments confirms the relevance of the theoretical problem and the need for a deeper theoretical study and development of numerical simulation. Thus, theoretical and experimental studies are closely related and stimulate further research of problems in this field.

The aim of this work is theoretical and experimental study of thermocapillary deformations in the locally heated horizontal liquid layers with different properties using a confocal sensor on a 3D-positioning system and numerical modeling based on the thin layer approximation.

2. Description of the experiment

Thermocapillary deformation of dimethylpolysiloxane layer (further PMS or silicone oil) was investigated using experimental setup (figure 1). Working fluid from the syringe inputs on the substrate surface of the test cell, forming a horizontal liquid layer of 0.95 mm depth. The power of heating element is controlled by the power supply. Temperature in the test cell is measured by thermocouples (type K) connected to the measuring system with an accuracy of 0.1°C. Relative humidity and atmosphere temperature are measured using the thermohygrometer Testo 645 with an accuracy of 2% and 0.1°C, respectively. The heat flux density was determined by measuring the temperature difference between two different sections along the heater tip. Confocal system Micro-epsilon was used for measuring the layer thickness over the heating area. The system consists of the controller and the sensor. Sensors have the spatial resolution of 36 nm, the accuracy of 0.5 μm, the spot diameter of 9 μm and the measuring range of 3 mm. The maximum temporal resolution was 100 μs. The sensor is fixed on the three-dimensional positioning system with high-speed linear actuator on one of the horizontal axes. Linear actuator is connected to personal computer and controlled by special software. The sensor is moved with a speed of 104 mm/s in the range of ±5 mm from the center of substrate. The maximum moving distance is 28 mm. Also, the sensor can move along two other axes with the help of two hand-operated linear stages in the range of 50 mm in order to adjust the sensor position for thickness measurements. The test cell is installed in a horizontal position with the help of two-axis goniometer.

![Figure 1. Experimental rig: test cell and 3D-positioning system with high-speed linear actuator.](image-url)
3. Numerical modeling

A thin horizontal liquid layer of silicone oil under the local heating from the substrate side is considered. The numerical solution of the problem has been obtained in the lubrication approximation theory (long-wave approximation) for two-dimensional axisymmetric case. This approach eliminates the complexity of the problem, caused by the presence of the free surface. The model takes into account important parameters such as gravity, surface tension, viscosity, capillary pressure, thermocapillary effect and heat transfer in the substrate and liquid. Evaporation is neglected. Initially the liquid layer has flat surface and uniform temperature of the ambient \( T_a \). At the initial moment of time the heater is turned on, and the cuvette and liquid start to warm up. Thermocapillary flow and deformation of the liquid surface are formed, because of tangential stress on the surface of the liquid, caused by the inhomogeneity of liquid temperature. Deformations of the liquid surface are determined by the properties of the liquid, cuvette and heater. The scheme of the system is shown in figure 2.

The processes in the thin horizontal liquid layer under the local heating from the substrate side are described by the following system of equations (1)-(2) with boundary conditions (3)-(4).

Dynamics of thin films are well described by the evolution equation (1), which has been obtained using the lubrication approximation theory [2, 3, 10]. The temperature of the liquid layer and cuvette is determined by the energy equation (2) in cylindrical coordinates.

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \left[ \frac{h^3}{3\mu} \frac{\partial}{\partial r} \left( \rho gh + \sigma H \right) + \frac{h^2}{2\mu} \sigma_T \frac{\partial T}{\partial r} \right] \right) = 0, \quad (1)
\]

\[
\rho C_p \left( \frac{\partial T}{\partial t} + u \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( \lambda \left( r \frac{\partial T}{\partial r} \right) \right) \right) + \frac{\partial^2 T}{\partial z^2} + Q \quad (2)
\]

Equation (1) is a nonlinear differential equation in cylindrical coordinates of the first order in time and the fourth order in spatial variables relative to unknown function \( h(t,r) \). In equation (1) \( h \) is the film thickness, \( h_s \) is the velocity of the surface, \( q = \frac{h^3}{3\mu} f + \frac{h^2}{2\mu} \sigma_T \) is the local flow vector of liquid along the surface, \( f = grad \left( \rho gh + \sigma H \right) \) is the pressure gradient, \( \sigma_T = \sigma_T \) grad \( T \) is the thermocapillary tangent stress, \( \mu \) is the coefficient of dynamic viscosity, \( \rho \) is the density, \( g \) is the gravitational acceleration, \( \sigma_l \) is the surface tension, \( T \) is the temperature, \( H = \frac{h_s}{r} \left( 1 + h_s^2 \right)^{1/2} + \frac{h_s}{r} \left( 1 + h_s^2 \right)^{1/2} \) is the double mean curvature of the liquid surface, where \( h_s \) and \( h_l \) are the first and the second order derivatives with respect to \( r \).

In equation (2) \( \lambda \) is the thermal conductivity coefficient, \( C_p \) is the specific heat of the medium (solid or liquid), \( \rho \) is the density, \( u, v \) are the components of the velocity vector, \( Q \) is the bulk density of the heat sources, localized in the heater.

Boundary conditions for equation (1) have a clear physical meaning (\( R_c \) is the cuvette radius, \( t \) is time).

Figure 2. System scheme: liquid layer in the cuvette with local heater.
\( h_2 (t, 0) = 0 \)  the condition of the axial symmetry is given in the center of the cuvette
\( h_2 (t, R_c) = 0 \)  the contact angle is given on the border of the cuvette
\( q(t, 0) = 0 \)  flow rate is equal to 0 in the center
\( q(t, R_c) = 0 \)  the condition of impermeability of liquid through the walls.

Boundary conditions for equation (2) have form (index \( W \) determines the conditions at the bottom of the cuvette, index \( S \) determines the conditions on the free liquid surface, \( T_a \) is ambient temperature):

\[
\frac{\partial T}{\partial r} \bigg|_{r=0} = 0 \quad \text{- axial symmetry condition; \quad } \frac{\partial T}{\partial r} \bigg|_{r=R_c} = 0 \quad \text{- adiabatic right side wall}
\]
\[
\lambda \frac{\partial T}{\partial n} \bigg|_{rb} = \alpha_n (T_w - T_a) \quad \text{- the convective heat transfer coefficient is specified on the cuvette bottom (4)}
\]
\[
\lambda \frac{\partial T}{\partial n} \bigg|_{lc} = \alpha (T_S - T_a) \quad \text{- the convective heat transfer coefficient is specified on the free liquid surface}
\]

Initially liquid surface is flat and temperature of the liquid surface and cuvette is uniform:

\[
h_b |_{z=0} (r, z) = h_0, \quad T_b |_{z=0} (r, z) = T_a = \text{const}.
\]

For calculations there has been used splitting into physical processes, such as thermal conduction and liquid motion. The grid in the space variables in the liquid and solid phases is uniform. The time step is also uniform. The evolution equation of the liquid layer thickness (1) is approximated at grid nodes with finite volume method \([2, 3, 11, 13]\) with an implicit finite-difference scheme of first order in time and of second order in space. The implicit scheme is chosen to ensure stability. Then the obtained system of nonlinear algebraic equations is solved at each time step by the Newton method, and the Jacobians are calculated using numerical linearization \([2-3]\). The scheme has the second-order approximation for the spatial coordinates and the first-order in time. For calculating the temperature in the liquid, deformations of the surface are not taken into account. Since the heater is thin, we assume that the heat source is concentrated only in one layer of nodes. The energy equation (2) is approximated by a finite difference scheme using the fractional step method \([12-13]\). The obtained systems of linear algebraic equations at each step were solved by the tridiagonal matrix algorithm. Problem data satisfies sufficient conditions for determining the correctness and stability of tridiagonal matrix algorithm.

The numerical algorithm for the joint solution of the energy equation and the evolution equation for the liquid layer thickness has been developed. The calculations are performed sequentially. The time step for the evolution equation is done after the time step for the energy equation. The mathematical model accounted such defining parameters as the geometry of the problem, parameters of the liquid, properties of the substrate and the heater materials, heating methods. The calculation results have shown that all these parameters have a significant impact on the distribution of the heat and deformations of the liquid surface. Stationary solutions have been obtained by the establishment method.

4. Analysis of calculations and experimental results

Numerical calculations were made for two marks of silicone oil: PMS-5 and PMS-100. The depth of the thermocapillary deformation depends strongly on the initial thickness of the layer and the oil viscosity \([14]\).

Stationary states for different types of silicone oil under the local heating are shown in figure 3a. It was obtained that viscosity affects slightly the deformation changes. Numerical results in figure 4 show the influence of viscosity, surface tension coefficient and its temperature coefficient on thermocapillary deformation. Differences in the calculation results for PMS-5 and PMS-100 are
explained by differences in the coefficients of surface tension $\sigma$ and its temperature coefficient $\sigma_T$ (figure 4). PMS-5 has a lower surface tension and, at the same time, depends more on temperature than PMS-100.

The thickness distributions for PMS-5 and PMS-100 along the cuvette of radius $R_{cuvette}$ at different heating power, after 2 seconds from the start of heating, are shown in figure 3b. The unsteady state of the process is visible in the case of PMS-100, where a liquid bump is located at the boundary of the heater, formed due to displacement of the liquid from the center of the cuvette. Since the liquid PMS-100 is more viscous, then within 2 seconds the layer does not have time to spread under the influence of gravity and surface tension.

Dependences of the silicone oil thickness on the thermocapillary deformation depth have been measured experimentally. Evolution of the layer deformation profile is shown in figure 5. Critical power $q_{cr}$ was found for different initial thickness of liquid layer. It is worth noting that a liquid bump is also observed in experimentally measured deformation profiles, being in agreement with theoretical calculations.

![Figure 3](image1.png)

**Figure 3.** The distributions of the liquid film dimensionless thickness along the $R_{cuvette} = 18$ mm for different types of silicone oil. ($R_{heater} = 0.8$ mm, $h_0 = 300$ µm) (a) stationary case: $t_{heating} = 100$ s, $Q = 16.5$ mW; (b) for different heating values $Q = 16.5$ mW (dash line); $Q = 0.03$ W (solid line), $t_{heating} = 2$ s.

![Figure 4](image2.png)

**Figure 4.** The influence of viscosity, surface tension coefficient and its temperature coefficient on thermocapillary deformation (stationary case: $R_{cuvette} = 18$ mm, $R_{heater} = 0.8$ mm, $h_0 = 300$ µm, $t_{heating} = 100$ s, $Q = 16.5$ mW).
Figure 5. Thickness distribution of the PMS-100 along the cuvette for different initial thickness and heating power (a) $h_0 = 300 \, \mu m$, $q_{cr} = 55 \, W/cm^2$, (b) $h_0 = 400 \, \mu m$, $q_{cr} = 68 \, W/cm^2$ (c) $h_0 = 500 \, \mu m$, $q_{cr} = 82 \, W/cm^2$.

5. Conclusion
Deformations in locally heated horizontal layers of silicone oils of different viscosities have been measured and calculated numerically. Numerical solutions have been obtained in lubrication approximation theory for silicone oils: PMS-5 and PMS-100. The dimensionless distributions of the liquid film thickness along the cuvette for different types of silicone oil and different heating power were obtained. The significant effect of the surface tension coefficient and its temperature coefficient on thermocapillary deformation of silicone oils was detected. In unsteady state of the process there is the liquid bump formed due to displacement of the liquid from the center of the cuvette. Dependencies of the depth of thermocapillary deformation on the initial layer thickness for silicone oils with different viscosities were obtained using confocal sensor on a 3D-positioning system with high-speed linear actuator. It was established that the deformation value depends on a heat flux value. A liquid bump is formed at the boundary of the heating region that is also observed in numerically calculated profiles.

Acknowledgments
The study was financially supported by the Russian Science Foundation (Project 14-19-01755).

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