Radiation pressure mixing of large dust grains in protoplanetary disks
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Dusty disks around young stars are formed out of interstellar dust that consists of
amorphous, submicrometre grains. Yet the grains found in comets [1] and
meteorites [2], and traced in the spectra of young stars [3], include large crystalline
grains that must have undergone annealing or condensation at temperatures in
excess of 1,000 K, even though they are mixed with surrounding material that
never experienced temperatures as high as that [4]. This prompted theories of
large-scale mixing capable of transporting thermally altered grains from the inner,
hot part of accretion disks to outer, colder disk regions [5, 6, 7], but all have
assumptions that may be problematic [8, 9, 10, 11, 12]. Here I report that infrared
radiation arising from the dusty disk can loft grains bigger than one micrometre
out of the inner disk, whereupon they are pushed outwards by stellar radiation
pressure while gliding above the disk. Grains re-enter the disk at radii where it is
too cold to produce sufficient infrared radiation pressure support for a given grain
size and solid density. Properties of the observed disks suggest that this process
might be active in almost all young stellar objects and young brown dwarfs.

The history of thermal and compositional alternation of dust in dense dusty protoplanetary
disks around young pre-main-sequence (PMS) stars enables us to better understand
conditions that initiate formation of planets. One of the long standing problems arising from
this approach is the presence of crystalline dust in disk environments considered too cold for
crystallinity to occur. Thus, it has been suggested that silicates crystalize in the hot part
of the disk close to the central star and are transported outward into colder environment.
Currently favored theories of outward transport include: i) turbulent mixing [6], ii) ballistic
launching of particles in a dense wind created by interaction of the accretion disk with the
young star’s magnetic field (X-wind model) [5], and iii) mixing mediated by transient spiral
arms in marginally gravitationally unstable disks [7]. Although these theories sound promising
and may eventually result in the definitive solution to the problem of a large scale mixing,
they are so far hampered by theoretical assumptions needed for them to work. The turbulent
mixing requires a source of efficient turbulent viscosity and the magnetorotational instability
(MRI) is invoked as the most promising candidate, but large stretches of the disk are consid-
ered not sufficiently ionized to keep MRI active [8, 9, 10]. The X-wind model relies on the
theoretical notion of magnetic field configurations in the immediate vicinity of PMS stars and
high hopes are put on future observations to resolve this predicament [11]. The spiral arms
model is in the domain of discussions whether the underlying numerics, physical approxima-
tions and assumptions on the initial conditions are realistic enough to make results plausible
[8, 9, 12].

Unlike these theories, non-radial radiation pressure does not require additional assumptions
on the physical conditions in the disk because it stems from the basic radiative transfer
properties of optically thick dusty disks. It has been already shown that individual submicron grains do not move far away in the disk when pushed by radiation pressure because the force is primarily produced by radial stellar flux [13]. On the other hand, micron or larger grains are large enough to also have efficient interaction with the near infrared photons (NIR; equivalent to dust temperatures of $\sim 1000$-$2000$K) from the hot inner disk. Submicron grains are very inefficient emitters in NIR, hence, they overheat and sublimate further away from the inner disk surface. This leaves the surface populated only with large grains, while small grains can survive within the optically thick interior [14] or at larger disk radii. Direct imaging with NIR interferometers revealed that the observed location of inner disk rim is consistent with this description (see [15, 16, 17] and references therein).

In optically thick protoplanetary disks dust particles $\lesssim 1 mm$ are well coupled with the gas and their dynamics is dominated by the gas drag [13, 18]. Hence, dust motion is very similar to the gas orbital, almost Keplerian, motion. Radiation pressure force serves as a slow perturbation that leads to the rearrangement of dust orbits. In order to reconstruct the trajectory of particles pushed by radiation we need to derive the spatial orientation of radiation pressure vector. For that, we need estimates of the diffuse flux as the source of pressure asymmetry. I solve this using the two-layer formalism, which is a well established method utilized in problems involving protoplanetary disk emission [19].

A short simplified solution is presented in figure 1 while a more rigorous derivation, which includes gravity, gas drag and radiation pressure, is described in Supplementary information §1. The result shows that the net radiation pressure force, which combines stellar and diffuse flux components, is directed exactly parallel to the disk surface irrespective of its curvature. This leads to a very interesting scenario. If the force is strong enough to move a large dust grain then such a large crystalline grain formed at the hot inner rim would glide over the disk surface toward colder disk regions until the diffuse disk flux becomes too “cold” (i.e. its peak wavelength is larger than the dust size), at which point the force keeping the dust afloat ceases.

Further insight into the nature of non-radial radiation pressure outflow requires a more detailed description of the disk structure and an advanced radiative transfer calculation. I started with preliminary modeling at such an advanced level. The first results are presented in figure 2. The model assumes dusty disk density structure of the form $\rho_d(R, z) \sim R^{-2} \exp(-z^2/2h^2)$, with the scale height $h = 1.67 \times 10^{-2} R^{1.25}$, where $R$ and $z$ are cylindrical coordinates scaled with the dust sublimation radius $R_{in}$ (the disk’s inner rim; see figure 1). The disk contains 0.1$\mu m$ and 2$\mu m$ olivine grains with the relative density ratio $10^4:1$ and the overall radial visual optical depth at $z = 0$ of 10,000. I performed a full 2D radiative transfer for the case of disk heating from a 10,000K star and 1,500K dust sublimation. Location of the dust sublimation disk surface is calculated self-consistently from the mutual exchange of infrared energy between 0.1$\mu m$ and 2$\mu m$ grains, resulting in $R_{in} = 44.7 R_*$ ($R_*$ is the stellar radius). Figure 2 shows the map of vertical radiation pressure on 2$\mu m$ grains and examples of grain trajectories. Results from this detailed approach confirm the plausibility of our theoretical arguments.

The ability of large grains to migrate along any disk curvature makes this theory independent of the ongoing debate on the geometrical structure of the inner disk region [15]. The popular view is that the inner sublimation edge is puffed up and curved [17]. The non-radial pressure would affect dust dynamics under such a disk curvature in the same way as in the
numerical example above, except that individual grains could decouple more easily from the inner disk and fly toward outer disk regions due to the disk’s self-shadowing \cite{20}.

Grains pushed by radiation create an outflow that operates at much shorter timescale than the local dust settling because radiation pressure is active in the region of lower gas density. In Supplementary information §2 I provide an estimate of the total amount of dust that flows outward in the disk surface layer. The outflow strength and range depend on the ratio, $\beta$, of radiation force tangentially to the disk surface over local gravity force (see Supplementary information §1 for a detailed description):

$$\beta \sim 0.4 \left[ \frac{L_\ast}{L_\odot} \right] \left[ \frac{M_\odot}{M_\ast} \right] \left[ \frac{3000\text{kg/m}^3}{\varrho_s} \right] \left[ \frac{\mu m}{a} \right],$$

where $a$ is the dust grain radius, $L_\ast$ is the stellar luminosity, $M_\ast$ is the stellar mass and $\varrho_s$ is the grain solid density. Grains with $\beta \gtrsim 0.5$ are gravitationally decoupled from the star and will be pushed away from the star as long as the diffuse flux keeps them afloat within the optically thin surface. Grains with $\beta \lesssim 0.5$ feel a “reduced” gravity and their settling is slowed down.

I made an attempt to estimate the spatial extent of significant vertical radiation pressure along the disk surface. I use a simplified, but illustrative model of the protoplanetary disk where the disk surface contains only single size grains. Results show (see figure 3 and Supplementary information §3) that significant dynamical effects from the non-radial radiation pressure are possible only for grains larger than about 1$\mu$m. Grains a few microns in size can be lifted out of the disk only at small disk radii where the disk is the hottest, but already 5$\mu$m grains can “glide” to large radii (over 1,000 stellar radii), provided that the radiation pressure is strong enough to push such a grain. The upper limit on grain size pushed that way is dictated by equation (1) that shows how the force decreases with grain size.

Notice that I assume a solid spherical grain, which is a simplification of a more realistic fluffy dust aggregate \cite{21, 22}. Aggregates result in a much larger $\beta$ for the same grain size because they have a much lower grain density than the typical 3000$\text{kg/m}^3$ due to inclusion of vacuum into the grain structure. On the other hand, crystalline grains are largely transparent in the spectral range of stellar radiation \cite{1}, which would make radiation pressure ineffective. This remains an open problem for our theory, although crystalline grains incorporated into dust aggregates might have a non-transparent “glue” keeping the aggregate together, which would increase $\beta$ and mitigate these problems. Such “dustballs” are considered to be precursors of chondrules and CAIs in meteorites \cite{23}.

The main stellar parameter dictating the overall strength $\beta$ of radiation pressure effect on a grain is the luminosity-mass ratio $L_\ast/M_\ast$. Observations and evolutionary tracks indicate that $L_\ast/M_\ast \gtrsim 0.5$ (which gives $\beta \sim 0.4$ for a grain of 1$\mu$m diameter) in almost all young stellar objects, including brown dwarfs. Thus, non-radial radiation pressure is at least marginally relevant in all these objects, especially if a realistic dust aggregate model is taken into account. Moreover, at earlier evolutionary stages $\beta$ was larger because, according to stellar evolution models, the end of significant accretion (99% of the final mass) ends with $L_\ast/M_\ast > 10$ for stars $M_\ast \lesssim 1M_\odot$ \cite{24}.

Since crystallization is very efficient along the hot inner disk rim, radiation pressure mixing
of large grains would inevitably include the crystalline fraction and disperse such dust over the disk surface. Interestingly enough, such a correlation between large grains and crystalline fraction is detected in Herbig Ae stars (e.g. [25, 26]). This would be the most pronounced in the inner disk regions, closer to the inner rim, as it is indeed observed (e.g. [3, 27, 28]). With the help from disk turbulence, the surface of inner disk region is constantly replenished with new grains and the process continues as long as the radiation pressure is active.

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**Supplementary Information** is linked to the online version of the paper at www.nature.com/nature.

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Figure 1: Geometry of non-radial radiation pressure. The sketch shows a cross section of an optically thick protoplanetary dusty disk heated by a star. The disk has a central hole of radius $R_{in}$ where the dust overheats and sublimes away. According to the two-layer formalism [19], analysis of the disk emission in the near IR can be reduced to the disk’s optically thin surface, which is heated directly by the stellar radiation. In this approach the disk surface is replaced with a single temperature layer and we assume that the stellar radiation is completely absorbed within this layer. The disk interior is described as the second temperature layer, but it is heated only by infrared radiation from the surface layer and, therefore, it is much colder and does not contribute to the disk emission in the near and mid IR [19]. In optically thick passive disks we can use energy conservation at a surface point $(R, z)$ to set balance, $F_\star \sin \alpha = F^d$, between the bolometric stellar flux $F_\star$ intercepted by the disk at a grazing angle $\alpha$ and the outgoing disk radiation $F^d$ (IR emission and scattered stellar photons). In the approximation of geometrically thin disk surface we can assume that the entire local diffuse flux at the very surface is perpendicular to the surface. Grains that manage to decouple and move away from the surface would feel a reduced flux since the diffuse radiation streams out in all directions. We can decompose $F^d$ into radial, $F^d_r = -F_\star \sin^2 \alpha$, and azimuthal, $F^d_\perp = F_\star \sin \alpha \cos^2 \alpha$, components. If dust grains are big enough to have constant extinction in the wavelength range of $F^d_\lambda$ then the radiation pressure force becomes $\mathbf{F}_r \propto F_\star + F^d$. Using flux components from above gives the radial force $\mathbf{F}_r \propto F_\star \cos^2 \alpha$ and the azimuthal force $\mathbf{F}_\perp \propto F_\star \sin \alpha \cos \alpha$. Notice that this yields radiation pressure force directed exactly parallel to the disk surface, $\mathbf{F}_\perp / \mathbf{F}_r = \tan \alpha$, irrespective of the disk curvature. A more rigorous derivation is presented in Supplementary information §1, including dust dynamics due to gravity, gas drag and radiation pressure.
Figure 2: **Trajectory of dust grains under the influence of stellar gravity, gas drag and non-radial radiation pressure.** The colored background map shows the vertical $z$ component of the radiation pressure vector scaled with the value that the stellar pressure would have if the dusty disk were not there. Two stellar luminosity-mass ratios are used: $80L_{\odot}/M_{\odot}$ (white dashed line) and $25L_{\odot}/M_{\odot}$ (white solid line). Dust composition is olivine \[29\] of 2$\mu$m radius and solid density $3000 \text{ kg/m}^3$. Dust grains start their travel with a vertical upward motion until the gas density drops enough to loosen the influence of gas drag. After that the grain is ejected to a larger disk radius, where trajectory details depend on the strength and direction of radiation pressure. Trajectory is calculated numerically with the Runge-Kutta method. Radiation pressure is calculated numerically from 2D radiative transfer that includes dust absorption, scattering and emission. The disk consists of 0.1$\mu$m and 2$\mu$m grains that sublimate at 1500K, but the surface in this disk region is too hot for 0.1$\mu$m grains, which survive below the surface populated by 2$\mu$m grains. Spatial dimensions are scaled with the disk sublimation radius $R_{\text{in}}$ (see figure I). The disk gas and dust densities decrease exponentially with $z$. Red lines show the disk surfaces defined by the radial visual optical depth of 0.1 (dashed red line) and 1 (solid red line). Details of this numerical result will be shown in a separate publication.
Figure 3: Estimated strength of diffuse radiation pressure along the disk surface, indicating how far grains can travel. The estimated strength, $B$, is defined as the ratio of diffuse to stellar $\beta$ perpendicular to the disk surface (see equation 34 in Supplementary information §3 for details), at various distances from the star. Optical properties of the pushed grains and dust forming the disk surface are the same. The surface contains only one grain size and type. Lines show results for spherical grains of 0.5$\mu$m, 1$\mu$m and 5$\mu$m radius. Diffuse radiation pressure is important (i.e. $B \sim 1$) only for grains $\gtrsim 1$µm. Grains $\gtrsim 5$µm experience strong diffuse pressure over a large disk surface because of their efficient infrared absorption at longer wavelengths, while smaller micron grains can float only above the inner disk with the highest temperature. The dust is enstatite from [30]. Other compositions lead to qualitatively similar curves. Lines start at radii defined by 1,500K dust sublimation temperature.
SUPPLEMENTARY INFORMATION
1 Equations for non-radial radiation pressure dynamics

The radiation pressure force pushing a grain of radius $a$ in direction $\hat{n}$ is

$$
\vec{F} = \frac{a^2 \pi}{c} \int Q^\text{ext}_\lambda \vec{F}_\lambda d\lambda,
$$

(2)

where $c$ is the speed of light, $Q^\text{ext}_\lambda$ is the extinction coefficient and $\vec{F}_\lambda$ is the total radiation flux in direction $\hat{n}$. The stellar contribution to the radiation pressure on a grain at distance $r$ from the star is

$$
\vec{F}_* = \frac{a^2 \pi}{c} \int Q^\text{ext}_\lambda \vec{F}_\lambda d\lambda = \frac{L_* a^2}{4 c r^2} \int Q^\text{ext}_\lambda f_\star d\lambda,
$$

(3)

where $L_*$ is the stellar luminosity and $f_\star$ is normalized shape of stellar spectrum $\int f_\star d\lambda = 1$. We use $\vec{F}_*$ to scale the force

$$
\frac{\vec{F}}{\vec{F}_*} = \frac{\frac{\int Q^\text{ext}_\lambda \vec{F}_\lambda d\lambda}{\int Q^\text{ext}_\lambda F_\star d\lambda}}{\frac{\int Q^\text{ext}_\lambda \vec{F}_\lambda d\lambda}{\int Q^\text{ext}_\lambda F_\star d\lambda}} = \hat{r} + \frac{\int Q^\text{ext}_\lambda \vec{F}_d d\lambda}{\int Q^\text{ext}_\lambda F_\star d\lambda}.\n$$

(4)

In the case of a negligible diffuse flux the pressure becomes identical to the stellar radiation force and $\vec{F} = \hat{r}$.

If dust grains are big enough to have a constant extinction in the wavelength range of $F^d_\lambda$ then the radiation pressure force becomes $\vec{F} \propto \vec{F}_* + \vec{F}_d$. The disk flux is perpendicular to the disk surface and we can decompose it into radial and azimuthal components (see figure [1])

$$
F^d_r = -F_* \sin^2 \alpha, \quad \text{(5)}
$$

$$
F^d_\perp = F_* \sin \alpha \cos \alpha. \quad \text{(6)}
$$

Using these components gives the radial force

$$
\vec{F}_r \propto F_* - F_* \sin^2 \alpha = F_* \cos^2 \alpha \quad \text{(7)}
$$

and the azimuthal force

$$
\vec{F}_\perp \propto F_* \sin \alpha \cos \alpha. \quad \text{(8)}
$$

We see from this analysis that the radiation pressure force is directed exactly parallel to the disk surface $\vec{F}_\perp / \vec{F}_r = \tan \alpha$ irrespective of the disk curvature. This leads to a very interesting scenario. If the force is strong enough to move a big dust grain then such a big crystalline grain formed at the hot inner rim would glide over the disk surface toward colder disk regions until the diffuse disk flux becomes too “cold” (i.e. its peak wavelength is larger than the dust size), at which point the force keeping the dust afloat ceases.

The “strength” of the radiation pressure force is measured by the ratio of the radiation pressure in direction $\hat{n}$ and the local gravity force

$$
\frac{\vec{F}}{\vec{g}} = \frac{\cancel{\frac{a^2 \pi}{c} \int Q^\text{ext}_\lambda \vec{F}_\lambda d\lambda}}{GM_* m_d / r^2} = \frac{L_* a^2}{4 c G M_* m_d} \frac{\int Q^\text{ext}_\lambda f_\star d\lambda}{\int Q^\text{ext}_\lambda F_\star d\lambda}, \quad \text{(9)}
$$

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which yields

\[ \overline{\beta} = 0.2 \left[ \frac{L_*}{L_\odot} \right] \left[ \frac{M_\odot}{M_*} \right] \left[ \frac{3000 \text{kg/m}^3}{\rho_s} \right] \left[ \frac{\mu m}{a} \right] \overline{f} \int Q_{\lambda}^{ext} f_\lambda d\lambda, \tag{10} \]

where \( \rho_s \) is the solid density of a grain and \( L_\odot \) and \( M_\odot \) are solar luminosity and mass. Grains with \( \beta \cos \alpha \geq 0.5 \) are gravitationally decoupled from the star and will be pushed away from the star as long as the diffuse flux keeps them afloat within the optically thin surface. Grains with \( \beta \cos \alpha < 0.5 \) “feel” a reduced gravitational force and shift to a larger stable orbit.

The pressure vector \( \overline{f} \) is close to unity and, as described above, points tangentially to the disk surface. Stellar radiation peaks at wavelengths smaller than big grains, hence we can approximate \( Q_{\lambda}^{ext} \sim 2 \) and \( \int Q_{\lambda}^{ext} f_\lambda d\lambda \sim 2 \). This value of \( \int Q_{\lambda}^{ext} f_\lambda d\lambda \) is correct independently of all other grain properties (chemical composition or shape) when the grain size is much larger than the wavelength (so called “extinction paradox”). In cases when the grain size is larger by a factor of a few, \( Q_{\lambda}^{ext} \) can vary between \( \sim 1 \) and \( \sim 4 \) due to the contribution of anisotropic scattering. We see from this that the stellar luminosity to mass ratio and grain density dictate the size of a grain capable of migrating out of the hot inner disk rim.

The equation of motion of a particle in a gaseous medium includes forces of gravity, gas drag in the Epstein regime and radiation pressure [13, 31]

\[ \ddot{R} = -G \frac{M_*}{r^3} \overline{f} - \frac{\dot{\rho}_g}{\rho_s} \frac{c_s}{a} \left( \overline{v}_f - \overline{v}_g \right) + \overline{\beta} G \frac{M_*}{r^2}, \tag{11} \]

where \( \rho_g \) and \( \overline{v}_g \) are the local gas density and velocity, respectively, and \( c_s \) is the local speed of sound. We can assume that on the time scales of interest the gas has no radial or vertical velocities and rotates around \( \hat{z} \) axis with the Keplerian angular velocity. We expand this equation in a cylindrical coordinate system \((R, \phi, z)\)

\[ \ddot{R} = R \ddot{\phi}^2 - \frac{GM_*}{(R^2 + z^2)^{3/2}} \mu \dot{R} + \beta R \frac{GM_*}{R^2 + z^2}, \tag{12} \]

\[ \ddot{\phi} = -\frac{\dot{R}}{R} \dot{\phi} - \mu \left( \dot{\phi} - \frac{v_g}{R} \right), \tag{13} \]

\[ \ddot{z} = -\frac{GM_* z}{(R^2 + z^2)^{3/2}} - \mu \dot{z} + \beta z \frac{GM_*}{R^2 + z^2}, \tag{14} \]

where we use \( \overline{\beta} = \beta R \dot{R} + \beta_z \dot{z} \) and

\[ \mu = \frac{\dot{\rho}_g}{\rho_s} \frac{c_s}{a}. \tag{15} \]

We work in the regime \( \mu \gg \dot{R}/R \) where particles are strongly coupled with the gas and have a short gas drag stopping time. Hence, in equation [13] we can assume that dust and gas have the same angular motion similar to the Keplerian speed \( \Omega_K^2 = GM_*/R^3 \) [13]. Replacing \( \dot{\phi} \) in equation [12] with \( \Omega_K \) and using

\[ \frac{GM_*}{R^2} - \frac{GM_*}{(R^2 + z^2)^{3/2}} \sim \frac{GM_*}{R^2} \frac{3z^2}{2R^2} \ll \frac{\beta R}{R^2 + z^2} \tag{16} \]

yields the solution

\[ \dot{R} = \frac{GM_*}{\mu} \frac{\beta R}{R^2 + z^2}. \tag{17} \]
Similarly, from equation 14 we derive

\[ \dot{z} = \frac{GM_\ast}{\mu} \frac{1}{R^2 + z^2} \left( \beta_z - \frac{z}{\sqrt{R^2 + z^2}} \right). \]  

(18)

These are velocities of big dust particles in the optically thin disk surface under the influence of stellar and diffuse radiation pressure.

Direction of trajectories in the \( R - z \) plane is equal to the ratio

\[ \frac{\dot{z}}{\dot{R}} = \frac{\beta_z - \sin \theta}{\beta_R}, \]  

(19)

where \( \sin \theta = z/\sqrt{R^2 + z^2} \). From equations 7 and 8 it follows

\[ \frac{\beta_z}{\beta_R} = \frac{\beta_\ast \sin \theta + \beta_\perp \cos \theta}{\beta_\ast \cos \theta - \beta_\perp \sin \theta} = \frac{\sin \theta \cos \alpha + \cos \theta \sin \alpha}{\cos \theta \cos \alpha - \sin \theta \sin \alpha}. \]  

(20)

In the inner disk region \( \cos \theta \cos \alpha \gg \sin \theta \sin \alpha \), which yields

\[ \frac{\dot{z}}{\dot{R}} \sim \tan \theta + \tan \alpha - \frac{\sin \theta}{\beta_R}. \]  

(21)

Notice that under a strong radiation pressure force (i.e. \( \sin \theta/\beta_R \ll 1 \)) \( \tan \theta + \tan \alpha \) is exactly the curvature of the disk surface for small angles \( \alpha \) and \( \theta \) (figure 1). The same is true for the inner disk rim where \( \alpha \) is not small, but \( \theta \ll 1 \), and the trajectory becomes \( \dot{z}/\dot{R} \sim \tan \alpha \).

Vertical radiation pressure \( \beta_z \) reduces the influence of gravity on big grains, which results in expansion of optically thin disk surface. This is equivalent to a disk where gravity on a given grain size is reduced by a factor of \( 1 - \beta_z \), which increases the scale height by \( 1/\sqrt{1 - \beta_z} \). Grains with \( \beta_z \geq 1 \) can decouple from the dense gaseous disk if they reach heights where gas drag does not dominate the dust dynamics. Such a vertical expansion works only with big grains and optically thin dust. *If too much dust enters this zone and makes it optically thick, the radiation pressure decreases and the expansion subdues.*
2 Dust mass flowing in the disk surface

The outflow velocity is (see equations 17 and 18):

\[ v = \sqrt{R^2 + \dot{z}^2} \sim \frac{GM_*}{\mu(z_s)} \frac{\beta}{R^2 + \dot{z}^2}. \] (22)

We assume that \( v \) is constant within the optically thin surface that starts from the height \( z_s \). For the surface populated by grains of average radius \( a \) the net dust mass flux is [13]

\[ \dot{M}_{\text{sur}} = 2 \int_{z_s}^{\infty} v(R, z) \varrho_d(R, z) 2\pi R dz, \] (23)

where \( \varrho_d(R, z) \) is the dust number density at \((R, z)\). The factor of two comes from the disk having two sides. Using \( \mu \) from equation 15 and gas to dust ratio \( \xi = \varrho_g/\varrho_d \), the mass flux becomes

\[ \dot{M}_{\text{sur}} = 4\pi GM_* \varrho_s a \frac{\beta}{c_s \xi} R \int_{z_s}^{\infty} \frac{dz}{R^2 + \dot{z}^2}. \] (24)

The assumption of a constat sound speed \( c_s \sim 2000 \text{m/s} \) is correct within about \( \pm 700 \text{m/s} \) for temperatures considered here. Solving the integral and neglecting \( z_s/R \ll \pi/2 \) yields

\[ \dot{M}_{\text{sur}} = \frac{0.021}{\xi} \frac{M_\odot}{\text{year}} \left[ \frac{M_*}{M_\odot} \right] \left[ \frac{\varrho_s}{3000 \text{kg/m}^3} \right] \left[ \frac{a}{\mu \text{m}} \right] \beta. \] (25)

We replace \( \beta \) with equation 10 and get

\[ \dot{M}_{\text{sur}} = \frac{8 \times 10^{-3}}{\xi} \frac{M_\odot}{\text{year}} \left[ \frac{L_*}{L_\odot} \right]. \] (26)

where we assume \( \int Q^\text{ext}_s f_\lambda d\lambda \sim 2 \) and \( |\vec{f}| \sim 1 \).

Estimated amount of dust from equation 26 that flows outward in the disk surface layer depends critically on the gas to dust ratio \( \xi \), which is very uncertain in the inner disk region because of the interplay between dust sublimation, growth and settling. For the standard \( \xi = 100 \) the outflow transfers one Earth mass within \( \sim 13,000 \) years for \( L_* = 1L_\odot \) and \( \sim 13 \) years for \( L_* = 100L_\odot \). If the dust outflow becomes larger than the disk accretion inflow then the dusty disk starts to erode from its inner rim outward. According to our estimation, such an erosion happens when the total disk accretion is \( \lesssim 2.4 \times 10^{-8}[L_*/L_\odot]M_\odot/\text{yr} \). This value is about the same as the observed accretion rate averages in T Tauri [32] and Herbig Ae [33] stars, while in more luminous Herbig Be stars the limit becomes very high. Interestingly, observations indicate structural differences in the inner disk geometry between low and high luminosity young stellar objects [15, 16].

The timescale for an outflow of dust from cylindrical radius \( R \) to \( R' \gg R \) can be derived from equation 22. The radial component of the velocity is

\[ v_R \sim \frac{GM_* \beta}{\mu(z_s)} \frac{R}{(R^2 + \dot{z}^2)^{3/2}} \sim \frac{GM_* \beta}{\mu(z_s)} \frac{1}{R^2}, \] (27)

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which gives the time for a grain transport from $R$ to $R' > R$

$$t_{rad}(R, R') = \int_R^{R'} dR/v_R = \frac{\mu(z_s) R^3}{3 G M_\ast \beta} \left[ \left( \frac{R'}{R} \right)^3 - 1 \right] = \frac{\mu(z_s)}{3 \beta \Omega_K^2(R)} \left[ \left( \frac{R'}{R} \right)^3 - 1 \right],$$

(28)

where $\mu(z)$ is defined in equation 15, $z_s$ is the height where optically thin surface starts and $\Omega_K(R)$ is the Keplerian speed at $R$. In comparison, the local dust settling time is $t_{set}(R) = \mu/\Omega_K^2(R)$ [18], where $\mu$ goes over all $z$, yielding $t_{set} \sim 10^5 \text{yr}$ for $\mu m$ sized particles in typical disks. Radiation pressure is active in the region $z \sim z_s$ where $\mu$ is much smaller than in the disk interior of small $z$. Therefore, it operates at much smaller timescales than $t_{set}$ when $R' \sim R$, but becomes comparable $t_{rad}(R, R') \sim t_{set}$ when $R'/R \gtrsim 10$. 

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3 Extent of the diffuse radiation pressure

We compare values of $\beta$ perpendicular to the disk surface originating from diffuse ($\beta_{IR}$) and stellar radiation ($\beta^* \sin \alpha$)

$$B = \frac{\beta_{IR}}{\beta^* \sin \alpha} = \frac{\int Q'_\lambda F^d_\lambda d\lambda}{\int Q'\lambda F^*_\lambda d\lambda \sin \alpha},$$

(29)

where $Q'_\lambda$ is the extinction coefficient of a dust grain pushed by the radiation pressure. For a given distance $r$ from the star, the stellar component can be rewritten as

$$\int Q'_\lambda F^* d\lambda = \left(\frac{R^*}{r}\right)^2 \sigma_{SB} T^{*4} \langle Q' \rangle_{T^*},$$

(30)

where $\sigma_{SB}$ is the Stefan-Boltzmann constant, $R^*$ and $T^*$ are the stellar radius and temperature, respectively, and $\langle Q' \rangle_{T^*}$ is averaged $Q'_\lambda$ over the stellar spectrum.

The diffuse radiation is difficult to specify because it depends on structural properties of a particular protoplanetary disk, hence it suffers from various modeling uncertainties. But we know that the most inner dusty disk region contains only big grains in its hot surface layer because small grains overheat at these distances and sublimate under direct stellar radiation. Hence, here we consider a disk surface populated with big grains. The radial thickness of disk surface is defined by complete absorption of stellar radiation, where we use the surface radial optical depth at $\lambda=0.55\mu m$ as $\tau_V \sim 1$. The diffuse radiation is approximated according to the approach described in [19]

$$\int Q'_\lambda F^d_\lambda d\lambda = \sigma_{SB} T^4 \int Q'_\lambda b_\lambda(T) \epsilon_{IR} \sin \alpha \ d\lambda,$$

(31)

where $T$ is the surface dust temperature, $b_\lambda(T)$ is normalized Planck function and $\epsilon_{IR} \sin \alpha$ is the surface thickness in infrared. Unlike small grains where $\epsilon_{IR} = Q^{abs}_\lambda / Q^{abs}_V < 1$ always holds, big grains have

$$\epsilon_{IR} = \begin{cases} 
Q^{abs}_\lambda / Q^{abs}_V & \text{when } Q^{abs}_\lambda / Q^{abs}_V < 1 \\
1 & \text{when } Q^{abs}_\lambda / Q^{abs}_V \geq 1
\end{cases}$$

(32)

where $Q^{abs}_\lambda$ is the absorption coefficient of surface dust. The surface temperature is dominated by stellar heating, hence we can estimate the temperature from the equilibrium between stellar absorption and optically thin infrared emission.

$$\left(\frac{T}{T^*}\right)^4 = \left(\frac{R^*}{r}\right)^2 \frac{\langle Q^{abs} \rangle_{T^*}}{\langle Q^{abs}\rangle_T},$$

(33)

where $\langle Q^{abs} \rangle_T$ is the Planck average at temperature $T$. Combining 30, 31 and 33 with 29 gives

$$B = \frac{\beta_{IR}}{\beta^* \sin \alpha} = \frac{\langle Q^{abs} \rangle_{T^*}}{\langle Q^{abs}\rangle_T} \frac{\langle Q' \epsilon_{IR} \rangle_T}{\langle Q' \rangle_{T^*}}.$$ 

(34)

1Equation 33 holds for the optically thin surface of an optically thick disk, while optically thin disks have a factor of 1/4 in the right hand side of the equation. For the source of this difference between optically thin and thick disks see [14, 34].

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Supplementary information references

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