Comparative study of the finite-temperature thermodynamics of a unitary Fermi gas

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We study the finite-temperature thermodynamics of a unitary Fermi gas. The chemical potential, energy density and entropy are given analytically with the quasi-linear approximation. The ground state energy agrees with previous theoretical and experimental results. Recently, the generalized exclusion statistics is applied to the discussion of the finite-temperature unitary Fermi gas thermodynamics. A concrete comparison between the two different approaches is performed. Emphasis is made on the behavior of the entropy per particle. In physics, the slope of entropy gives the information for the effective fermion mass $m^*/m$ in the low temperature strong degenerate region. Compared with $m^*/m \approx 0.70 < 1$ given in terms of the generalized exclusion statistics, our quasi-linear approximation determines $m^*/m \approx 1.11 > 1$.

Keywords: Unitary Fermi gas thermodynamics; Quasi-linear approximation method; Generalized exclusion statistics

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I. INTRODUCTION

In recent years, the strongly interacting fermion physics becomes the focus of theoretical and experimental attention[1]. This is much attributed to the rapid progress of the atomic Fermi gas experiments.

By tuning the external magnetic field, one can control the $s$-wave scattering length $a$ or interaction strength between two atomic fermions. The crossover from Bardeen-Cooper-Schrieffer (BCS) to Bose-Einstein condensation (BEC) can be realized by the so-called Feshbach resonance[2]. At the resonance point, the scattering length can be singular with the existence of a zero-energy bound state. Although the scattering length is singular, the scattering cross-section is saturated as $\sigma \sim 4\pi/k^2$ (with $k$ being the relative momentum between two atomic fermions) due to the unitary property limit. The divergent scattering fermion thermodynamics is referred to as the unitary Fermi gas thermodynamics in the literature[3]. Dealing with the strongly interacting matter is related with a variety of realistic many-body topics.

Usually, the thermodynamics of dilute fermion system is determined by the two-body scattering length $a$, particle number density $n$ and temperature $T$. In the unitary limit with $a = \pm \infty$, the dynamical scattering limit should drop out in the thermodynamic quantities. At unitarity, the dynamical detail should not affect the thermodynamics; i.e., the unitary fermion system can manifest the universal properties[2].

Due to lack of any small expansion parameter, the unitary Fermi gas provides an intractable problem in statistical physics. The fundamental issue is on the zero-temperature ground state energy. Based on the dimensional analysis, the ground state energy should be proportional to that of the ideal Fermi gas with a universal constant $\xi = 1 + \beta$, which excites many theoretical and experimental efforts. The world average value of $\xi$ is $0.42 - 0.46[4, 5, 6, 7, 8]$. Recently, we have attempted a quasi-linear approximation method to explore the strongly interacting limit fermion thermodynamics[9]. The obtained ground state energy or the universal constant $\xi = \frac{7}{3}$ is reasonably consistent with some theoretical or experimental investigations.

Generally, the finite-temperature thermodynamics is as intriguing as the zero-temperature ground state energy. There have been several Monte Carlo finite-temperature calculations of a unitary Fermi gas[10, 11]. In the strongly correlation unitary fermions, the nonlinear quantum fluctuations/correlations compete with dynamical high order effects. In the weak degenerate Boltzmann regime, the nonlinear correlations make the second order virial coefficient $a_2$ vanish. To a great extent, the vanishing leading order quantum correction reflects the intermediate crossover characteristics of a unitary Fermi gas[9].

Can the intermediate characteristics be described in another way? In [12, 13], the generalized exclusion statistics was developed to describe the anyon behavior in the low-dimensional strongly correlation quantum system. Physically, the behavior of a unitary Fermi gas is between Bose gas and Fermi gas[10]. Similarly, the behavior of anyons is also between bosons and fermions. Can one use the anyons statistics to describe the intermediate unitary Fermi gas? Recently, the generalized exclusion statistics has been generalized to describe the unitary Fermi gas thermodynamics[14, 15]. As a hypothesis, the priority is that the thermodynamics at finite-temperature can be investigated quantitatively.

From the general viewpoint of statistical mechanics, calculating entropy is not a simple task. Either in classical or quantum theory, the entropy describes how the microscopic states are counted properly. From the quan-
tum degenerate viewpoint, the low-temperature behavior of the entropy is a characteristic quantity. For example, according to the Landau theory for the strong correlation Fermi-Liquid, the slope of entropy per particle versus temperature is related with the effective fermion mass \( m^*/m \). In physics, the dynamical parameter \( m^*/m \) is very important for the phase separation discussion for the asymmetric fermion system with unequal populations \([10, 11, 14, 15]\). Like the universal constant \( \xi = 1 + \beta \), the effective fermion mass \( m^*/m \) is another universal constant for the BCS-BEC crossover thermodynamics. Obviously, the physics beyond the mean-field theory should be reasonably well understood.

Unlike the ground state energy or the universal constant \( \xi \) with the world average value \( \xi \approx 0.44 \), the effective fermion mass is an unknown parameter up to now. For example, the effective fermion mass is estimated to be \( m^*/m \approx 1.04 \) with a quantum Monte Carlo calculation \([16]\). A quantitative study of the phase diagram at zero temperature along the BCS-BEC crossover using fixed-node diffusion Monte Carlo simulations shows \( m^*/m \approx 1.09 \) \([17]\). A many-body variational wave function with a T-matrix approximation leads to a larger value \( m^*/m \approx 1.17 \) \([18]\). What is the exact value of \( m^*/m \)?

In a quantitative way, we make a comparative study for the finite-temperature thermodynamic properties of the unitary fermion gas with the two formulations. The behavior of entropy per particle based on the quasi-linear approximation and the generalized exclusion statistics is discussed in detail. Indirectly, the effective fermion mass is determined from the entropy. The results are further compared with the Monte Carlo calculations.

The paper is organized in the following way. In Sec.II the relevant thermodynamic expressions are given by the quasi-linear approximation. Correspondingly, the thermodynamics given by the generalized exclusion statistics is presented in Sec.III. The numerical calculations and concrete comparisons between the two methods are given in Sec.IV. In this section, the entropy per particle and corresponding effective fermion mass \( m^*/m \) are discussed. In Sec.V we present the conclusion remarks.

II. THERMODYNAMICAL QUANTITIES GIVEN BY STATISTICAL DYNAMICS WITH QUASI-LINEAR APPROXIMATION

Strongly correlated matter under extreme conditions often requires the use of effective field theories in the description for the thermodynamic properties, independently of the energy scale under consideration. In the strongly interacting system, the central task is how to deal with the non-perturbative fluctuation and correlation effects. In Ref. [3], a quasi-linear approximation is taken to account for the non-local correlation effects on the unitary Fermi gas thermodynamics.

With the quasi-linear approximation method, the obtained grand thermodynamic potential \( \Omega(T, \mu) \) or pressure \( P = -\Omega/V \) can be described by the two coupled-parametric equations through the intermediate variable-effective chemical potential \( \mu^* \)

\[
P = \frac{2T}{\lambda^3} f_{5/2}(z') + \frac{\pi a_{eff}}{m} n^2 + n \mu_r, \tag{1}
\]

\[
\mu = \mu^* + \frac{2\pi a_{eff}}{m} n + \mu_r, \tag{2}
\]

In the above equations, \( \lambda = \sqrt{2\pi/(mT)} \) is the thermal de Broglie wavelength and \( m \) is the bare fermion mass (with natural units \( k_B = \hbar = 1 \) throughout the paper).

The effective chemical potential \( \mu^* \) is introduced by the single-particle self-consistent equation. \( \mu^* \) makes the thermodynamic expressions appear as the standard Fermi integral formalism

\[
f_\nu(z') = \frac{1}{\Gamma(\nu)} \int_0^\infty \frac{x^{\nu-1}dx}{z' e^x + 1}, \tag{3}
\]

where \( \Gamma(\nu) \) is the gamma function, and \( z' = e^{\mu^*}/T \) is the effective fugacity. For example, the quasi-particle Fermi-Dirac distribution function gives the particle number density according to

\[
n = \frac{2}{\lambda^3} f_{3/2}(z'). \tag{4}
\]

In the coupled equations Eq.(1) and Eq.(2), the shorthand notations are defined as

\[
a_{eff} = -\frac{m}{2\pi n^2 D}, \quad m^2_D = \left( \frac{\partial n}{\partial \mu^*} \right)_T. \tag{5}
\]

The shift term \( \propto \mu_r \) characterizes the high order nonlinear contributions, which strictly ensures the energy-momentum conservation law. In the nonlinear approximation, this significant high order correction term can be fixed in a thermodynamic way. It is worthy noting that the term \( \propto \mu_r \) can be exactly canceled by each other in the Helmholtz free energy density

\[
\frac{F}{V} = f = -P + n \mu, \tag{6}
\]

where \( V \) is the system volume. However, the high order correlation term \( \propto \mu_r \) can be obtained in terms of the thermodynamic relations \( \Omega \)

\[
P = -\left( \frac{\partial F}{\partial V} \right)_{T,N} = -\left( \frac{\partial F}{\partial N} \right)_T = n^2 \left( \frac{\partial F}{\partial n} \right)_T, \tag{7}
\]

and

\[
\mu = \left( \frac{\partial F}{\partial N} \right)_{T,V} = \left( \frac{\partial F}{\partial V} \right)_T = \left( \frac{\partial f}{\partial n} \right)_T. \tag{8}
\]
Comparing those obtained from Eq. (7) and Eq. (8) with Eq. (1) and Eq. (2), the explicit expression of $\mu_r$ is

$$\mu_r = \frac{1}{2} \left( \frac{\partial m_3}{\partial n} \right)_T \left( \frac{2\pi a_s f_2}{m} \right)^2 n^2, \quad (9)$$

The integrated expressions of the pressure and chemical potential for the unitary Fermi gas are

$$P = \frac{2T}{\lambda^3} \left( f_{5/2}(z') - \frac{f_{3/2}^2(z')}{2f_{1/2}(z')} + \frac{f_{3/2}(z')f_{1/2}(z')}{2f_{1/2}^3(z')} \right), \quad (10)$$

$$\mu = \mu^* - T \frac{f_{3/2}(z')}{f_{1/2}(z')} + T \frac{f_{2}^2(z')f_{1/2}(z')}{2f_{1/2}^3(z')} . \quad (11)$$

In the quasi-linear approximation, the auxiliary implicit variable $\mu^*$ is introduced to characterize the non-linear fluctuation/correlation effects. As indicated by Eq. (10) and Eq. (11), the $\mu^*$ or $z'$ makes the realistic grand thermodynamic potential $\Omega(T, \mu)$ appear as the set of highly non-linear parametric equations, which can be represented by the standard Fermi integral. By eliminating the auxiliary variable $\mu^*$, the equation of state will uniquely be determined.

From the underlying grand thermodynamic potential-partition function, one can derive the analytical expressions for the entropy density $s = S/V$ and internal energy density $\epsilon = E/V$. The following partial derivative formulae will be used

$$\left( \frac{\partial \mu^*}{\partial T} \right)_n \left( \frac{\partial T}{\partial \mu^*} \right)_n = -1,$$

$$\left( \frac{\partial m_3}{\partial T} \right)_n \left( \frac{\partial T}{\partial m_3} \right)_n = \left( \frac{\partial m_3}{\partial \mu^*} \right)_T \left( \frac{\partial \mu^*}{\partial T} \right)_n . \quad (12)$$

The entropy is derived according to

$$s = \frac{1}{n} \left( \frac{\partial P}{\partial T} \right)_n = \frac{5}{2} f_{5/2}(z') - \ln z' + \frac{3}{4} \frac{f_{-1/2}(z')f_{3/2}(z')}{f_{1/2}^3(z')} - \frac{f_{3/2}(z')}{f_{1/2}^3(z')} . \quad (13)$$

Correspondingly, the explicit energy density expression is calibrated to be

$$\epsilon = \frac{3T}{\lambda^3} \left( f_{5/2}(z') - \frac{f_{3/2}^2(z')}{2f_{1/2}(z')} + \frac{f_{3/2}(z')f_{1/2}(z')}{2f_{1/2}^3(z')} \right) . \quad (14)$$

Essentially, the entropy density includes the high order nonlinear contribution. What we want to emphasize is that the third law of thermodynamics is exactly ensured as expected. The analytical analysis indicates that the energy density at zero-temperature gives the dimensionless universal coefficient according to $\xi = \mu/E_F = \frac{\pi}{\lambda}$ or $E/(\frac{4}{9}N E_F) = \xi$, where the Fermi energy is $E_F = (3\pi^2 n)^{2/3}/(2m)$ and $T_F$ is the Fermi characteristic temperature in the unit Boltzmann constant. The universal coefficient $\xi = \frac{\pi}{\lambda}$ has attracted much attention in the literature and is reasonably consistent with some Monte Carlo calculations [4, 10].

### III. Thermodynamics Given by the Generalized Exclusion Statistics

#### A. Generalized exclusion statistics

The generalized exclusion statistics is proposed in [12] and [13]. If the dimensional of the Hilbert space is $d$ and the particle number is $N$, then $d$ and $N$ are connected by $\Delta d = -g \Delta N$, where the shift of the single-particle states number is $\Delta d$. The shift of the particle number for identical particle system is $\Delta N$ and $g$ is a statistical parameter, which denotes the ability of one particle to exclude other particles in occupying single-particle state. When $g = 0$ the intermediate statistics returns to the Bose-Einstein statistics and $g = 1$ to the Fermi-Dirac statistics.

For anyons, the number of quantum states $W$ of $N$ identical particles occupying a group of $G$ states are determined by the interpolated statistical weights of the Bose-Einstein and Fermi-Dirac statistics. A simple formula with the generalized exclusion statistics is used to describe the microscopic quantum states [12]

$$W = \frac{[G + (N - 1)(1 - g)]!}{G! [G - gN - (1 - g)]!} . \quad (15)$$

One can divide the one-particle states into a large number of cells with $G \gg 1$ states in each cell, and calculate the number with $N_i$ particles in the $i$-th cell. The total energy and the total number of particles are fixed and given as

$$E = \sum_i N_i \epsilon_i , N = \sum_i N_i , \quad (16)$$

with $\epsilon_i$ defined as the energy of particle of species $i$. By generalizing Eq. (15), we have

$$W = \prod_i \frac{[G_i + (N_i - 1)(1 - g)]!}{N_i! [G_i - gN_i - (1 - g)]!} . \quad (17)$$

We consider a grand canonical ensemble at temperature $T$. For very large $G_i \gg 1$ and $N_i \gg 1$, using the Stirling formula $\ln N! = N(\ln N - 1)$, and introducing the average occupation number defined by $\bar{N_i} = N_i/G_i$,
one has
\[
\ln W = \sum_i \ln \left[ \frac{[G_i + (N_i - 1)(1-g)]!}{N_i! [G_i - gN_i - (1-g)]!} \right]
\]
\[
\simeq \sum_i \left[ G_i (1 + (1-g)\bar{N}_i) \ln G_i (1 + (1-g)\bar{N}_i)
- G_i (1-g\bar{N}_i) \ln G_i (1-g\bar{N}_i) - G_i \bar{N}_i \ln G_i \bar{N}_i \right].
\]
(18)

Through the Lagrange multiplier method, the most probable distribution of \( \bar{N}_i \) is determined by
\[
\frac{\partial}{\partial \bar{N}_i} [\ln W - \sum_i G_i \bar{N}_i (\epsilon_i - \mu)/T] = 0,
\]
(19)
with chemical potential \( \mu \). It follows that
\[
\bar{N}_i e^{(\epsilon_i - \mu)/T} = [1 + (1-g)\bar{N}_i]^{(1-g)} (1-g\bar{N}_i)^g.
\]
(20)

Setting \( \omega_i = 1/\bar{N}_i - g \), we have the anyon statistical distribution
\[
\bar{N}_i = \frac{1}{\omega_i + g},
\]
(21)
where \( \omega \) obeys the relation
\[
\omega^g (1+\omega)^{1-g} = e^{(\epsilon - \mu)/T}.
\]
(22)

One can define \( \omega_0 \) of \( \omega \) at \( \epsilon = 0 \) with Eq. (22)
\[
\mu = -T \ln [\omega_0^g (1 + \omega_0)^{1-g}].
\]
(23)

The relation between \( \mu \) and \( T \) has been established indirectly through \( \omega_0 \) and \( g \). From Eq. (22), the \( \omega \) and \( \omega_0 \) are related with each other through single-particle energy \( \epsilon \)
\[
\epsilon = T \ln \left[ \left( \frac{\omega}{\omega_0} \right)^g \left( \frac{1 + \omega}{1 + \omega_0} \right)^{1-g} \right],
\]
(24)
which gives
\[
d\epsilon = \frac{T (g + \omega)}{\omega (1 + \omega)} d\omega.
\]
(25)

For \( T = 0 \), the average occupation number can be explicitly indicated as
\[
\bar{N} = \begin{cases} 
0, & \text{if } \epsilon > \mu; \\
\frac{1}{g}, & \text{if } \epsilon < \mu,
\end{cases}
\]
(26)
which is quite similar to the Fermi-Dirac statistics.

### B. Particle number and energy densities

In the anyon statistics, the density of states is also given by
\[
D(\epsilon) = \alpha (2m)^{3/2} V \epsilon^{1/2}/(4\pi^2),
\]
(27)
where \( \alpha \) is the degree of the spin degeneracy and \( m \) is the bare fermion mass.

At \( T = 0 \), the particle number is explicitly given by
\[
N = \frac{1}{g} \int_0^{\tilde{E}_F} D(\epsilon) d\epsilon = \frac{\alpha (2m)^{3/2}}{6\pi^2} V E_F^{3/2},
\]
(28)
where \( \tilde{E}_F \) is related with the Fermi energy \( E_F \) through \( \tilde{E}_F = g^{2/3} E_F \). With the \( \tilde{E}_F \) symbol, the system energy can be represented as
\[
E = \frac{1}{g} \int_0^{\tilde{E}_F} \epsilon D(\epsilon) d\epsilon = \frac{3}{5} g^{2/3} N E_F.
\]
(29)

As we will see, once \( g \) is fixed, one can discuss the general finite-temperature thermodynamic properties. Therefore, the essential task in the generalized exclusion statistics is fixing the statistical factor \( g \). This can be determined by the zero-temperature ground state energy or the universal constant \( \xi \) according to \( \xi = g^{2/3} \). Various theoretical or experimental attempts have been made in the literature for determining the ground state energy. With the universal coefficient \( \xi = \frac{3}{4} \), the expected statistical factor can be identified to be \( g = \frac{4}{3} \).

For the general finite-temperature scenario, the particle number and energy can be rewritten as
\[
N = \int_0^{\tilde{E}_F} \frac{D(\epsilon) d\epsilon}{\omega + g},
\]
(30)
\[
E = \int_0^{\tilde{E}_F} \frac{\epsilon D(\epsilon) d\epsilon}{\omega + g}.
\]
(31)

By replacing Eq. (24)-Eq. (26) and Eq. (28) into Eq. (30) and Eq. (31), one can have
\[
\frac{3}{2} \left( \frac{T}{T_F} \right)^{3/2} a(\omega_0) = 1,
\]
(32)
\[
\frac{E}{N E_F} = \frac{3}{2} \left( \frac{T}{T_F} \right)^{5/2} b(\omega_0),
\]
(33)
\[
a(\omega_0) = \int_{\omega_0}^{\infty} \frac{d\omega}{\omega (1 + \omega)} \ln \left( \frac{\omega}{\omega_0} \right)^g \left( \frac{1 + \omega}{1 + \omega_0} \right)^{1-g} \left( \frac{1 + \omega}{1 + \omega_0} \right)^{1-g} \]
(34)
\[
b(\omega_0) = \int_{\omega_0}^{\infty} \frac{d\omega}{\omega (1 + \omega)} \ln \left( \frac{\omega}{\omega_0} \right)^g \left( \frac{1 + \omega}{1 + \omega_0} \right)^{1-g} \left( \frac{1 + \omega}{1 + \omega_0} \right)^{1-g}.
\]
(35)

Eq. (32) determines \( \omega_0 \) for a given temperature \( T \). \( E/(N E_F) \) can be obtained by a given \( \omega_0 \) through Eq. (33).

For giving the explicit entropy density expression with the generalized exclusion statistics in the next subsection, let us make further discussion for the energy density. By eliminating \( N \) with Eq. (28) and Eq. (33), the energy can be alternatively expressed as
\[
E = \frac{\alpha (2m)^{3/2}}{4\pi^2} V T^{5/2} b(\omega_0).
\]
(34)
The partial derivative of the internal energy $E$ to $T$ for fixed $\mu$ is given by

$$\left( \frac{\partial E}{\partial T} \right)_\mu = \frac{\alpha V (2m)^{3/2}}{4\pi^2} T^{3/2} \left[ \frac{5}{2} b(\omega_0) + T \left( \frac{\partial b(\omega_0)}{\partial T} \right)_\mu \right].$$

(35)

Furthermore, the variable $\omega_0$ of the integral function $b(\omega_0)$ can be converted into $\mu$ and $T$ through Eq.(23)

$$b(\omega_0, \mu, T) = \int_\omega^\infty \frac{d\omega}{\omega(1+\omega)} \left[ \ln(\omega^g(1+\omega)^{1-g}) + \frac{\mu}{T} \right]^{3/2}.$$

(36)

Therefore, one can have

$$\left( \frac{\partial b}{\partial T} \right)_\mu = \frac{3}{2T} \ln[\omega_0^g(1+\omega_0)^{1-g}] a(\omega_0).$$

(37)

C. Entropy per particle

Due to the scaling properties, the thermodynamics of a unitary Fermi gas also satisfies the ideal gas virial theorem $P = \frac{2}{3} E V$.

(38)

According to the thermodynamic relation for the entropy $S$ and pressure $P$, one can have

$$S = \frac{2}{3} \left( \frac{\partial E}{\partial T} \right)_\mu.$$

(39)

By substituting Eq.(35) and Eq.(37) into Eq.(39), the explicit expression for the entropy per particle is derived to be

$$\frac{S}{N} = \frac{5}{2} \left( \frac{T}{T_F} \right)^{3/2} b(\omega_0) + \ln[\omega_0^g(1+\omega_0)^{1-g}],$$

(40)

where $\omega_0$ is given by Eq.(32) for a given $T$.

IV. NUMERICAL RESULTS AND COMPARISONS

Based on the above analytical expressions, we will give the numerical results.

A. Internal energy and chemical potential

From Eq.(32) and Eq.(33), the energy per particle versus the rescaled temperature can be solved. As indicated by Fig[1] the internal energies for the unitary Fermi gas based on the quasi-linear approximation and the generalized exclusion statistics have similar analytical properties; i.e., the internal energy increases with the increase of temperature. The two approaches both show that the energy density of a unitary Fermi gas is lower than that of the ideal Fermi gas. However, the shift of the internal energy given by the quasi-linear approximation is more quicker than that determined by the generalized exclusion statistics model.

![Fig. 1: The internal energy per particle versus the rescaled temperature. The solid curve denotes that for the ideal Fermi gas, and the short-dashed one is that given by the quasi-linear approximation. The long-dashed curve represents the result in terms of the generalized exclusion statistics model. The dots and solid squares are the Monte Carlo calculations [10] and [11], respectively.](image)

With Eq.(32) and Eq.(23), we have also shown the chemical potential versus the rescaled temperature in Fig[2]. The chemical potential given by the two formalisms decreases with the increase of temperature. The departure of them is getting bigger with the increasing temperature.

The results for the energy per particle shown in Fig[1] and Fig[2] in terms of the two different analytical approaches are reasonably consistent with the Monte Carlo calculations [10, 11], while the chemical potential differs explicitly from the Monte Carlo result [11] for $T/T_F > 0.8$.

![Fig. 2: Physical chemical potential versus the rescaled temperature. The line-styles are similar to Figure.1.](image)
B. Entropy

![Graph showing entropy per particle versus the rescaled temperature.](image)

FIG. 3: Entropy per particle versus the rescaled temperature. The line-styles are similar to Figure 1. The Monte Carlo simulation result is extracted from Ref. [16].

With Eq. (32) and Eq. (40), the entropy per particle curve versus the rescaled temperature is presented in Fig. 1. The quasi-linear approximation predicts that the curve is higher than that of the ideal Fermi gas, while the generalized exclusion statistics model gives lower values compared with that of the ideal Fermi gas. With the increase of temperature, the entropy per particle given by the generalized exclusion statistics is getting closer to and almost overlaps with that of the ideal Fermi gas. In terms of the quasi-linear approximation, the ratio of entropy to that of the ideal Fermi gas approaches a constant in the Boltzmann regime.

Especially, in the low-temperature strong degenerate regime, the slope of the entropy per particle versus the scaled temperature given by these two approaches is different. The low-temperature behavior is determined by the effective fermion mass according to the Landau theory of strongly correlated Fermi-liquid. In turn, from the entropy curve, one can derive the effective fermion mass indirectly. The careful study shows that the quasi-linear approximation indicates \( m^*/m \approx 1.11 > 1 \), while the latter predicts \( m^*/m \approx 0.70 < 1 \). Compared with the latter, the quasi-linear approximation result is more consistent with the Monte Carlo calculations \( m^*/m \sim 1.04 - 1.09 \) [16, 17].

V. CONCLUSION

In terms of the quasi-linear approximation method and generalized exclusion statistics model, the internal energies, chemical potentials and entropies of a unitary Fermi gas have been analyzed in detail. The two different approximations give similar behavior for the internal energies and chemical potentials of a unitary Fermi gas.

The entropy is an important characteristic quantity in statistical mechanics. The entropy by the quasi-linear approximation is higher than that of the ideal non-interacting fermion gas. In the Boltzmann regime, the entropy curve given by the generalized exclusion statistics gets closer towards and almost overlaps with that of the ideal Fermi gas. The entropy given by the quasi-linear approximation is getting far away from that of the ideal Fermi gas and the ratio of entropy to that of the ideal Fermi gas approaches a constant.

According to the quasi-particle viewpoint of the Landau Fermi-Liquid theory, the slope of entropy per particle determines the effective fermion mass in the low-temperature strong degenerate region. The numerical analysis demonstrates that the generalized exclusion statistics model gives \( m^*/m \approx 0.70 < 1 \). The developed quasi-linear approximation predicts \( m^*/m \approx 1.11 > 1 \), which is closer to the updating Monte Carlo investigations.

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