Dependence of tsunami wave maximal heights along a coastline on the source orientation

M Lavrentiev\textsuperscript{1,2}, K Lysakov\textsuperscript{1,2}, An Marchuk\textsuperscript{1,3}, K Oblaukhov\textsuperscript{3} and M Shadrin\textsuperscript{1,2}

\textsuperscript{1} Institute of Automation and Electrometry SB RAS, Ac. Koptyug pr. 1, 630090, Novosibirsk, Russia
\textsuperscript{2} Novosibirsk State University, Pirogov st., 1, 630090, Novosibirsk, Russia
\textsuperscript{3} Institute of Computational Mathematics and Mathematical Geophysics SB RAS, Ac. Lavrentiev pr., 6, 630090, Novosibirsk, Russia

E-mail: mmlavrentiev@gmail.com

Abstract. The authors address an important problem of timely evaluation of tsunami danger after offshore underwater earthquake. In case of Japan it takes nearly 20 minutes for the wave to approach the nearest coast after the seismic event. In this paper we continue to study the influence of tsunami source parameters on the distribution of the wave heights along the coast. As a rule, tsunami wave heights have sharp maximums at selected points of the coast. So, after seismic event the wave may be dangerous only at a few locations. Numerical tests show dependence of the distribution of such maximal wave heights from the orientation of the initial sea bed (sea surface) displacement, which has a form close to reconstruction of historic event at South-East of Japan.

1. Introduction

Numerical experiments, as well as field observations after historical seismic events, show that tsunami wave heights have rather sharp spikes along the coastal line. Therefore, in case of near field tsunami, which is not dramatically strong (say, for the earthquake magnitude 7.5-8.0), wave is really dangerous only at certain locations. The question that arises is whether it is possible to specify the regions of potential danger shortly after a seismic event.

In the presented paper we study numerically the wave heights distribution at South-West of Japan, the region, there strong tsunami is expected according to [1]. Wave propagation is simulated by a version of shallow water system, given in [2,3]. Implementation of Mac-Cormack finite difference scheme is used for numerical experiments. Earlier, this implementation has been tested by the authors. A rather good agreement with the known exact solutions of shallow water system (special cases of bottom slope and parabolic depth profiles) was observed [4,5].

The rest of the paper is arranged as follows. We first describe the mathematical model, numerical approximation, and digital bathymetry in use. Then, in section 3, numerical experiments are presented. The same elliptic tsunami source has two different locations and the distribution of tsunami wave maximal heights is given for two different orientation of the source at each location. For comparison, the spherical tsunami source was also studied. Numerically obtained results are then discussed. As it is observed, both source location and orientation are crucial for particular localization of the wave height spikes along the coast.
2. Mathematical model and numerical scheme

2.1. Mathematical model

Following [3], we use the following equivalent form of a shallow water system (which does not take into account such external forces as sea bed friction, Coriolis forces and others):

\[ H_y + (uH)_x + (vH)_y = 0 \]
\[ u_x + uu_x + vu_y + gH_x = gD_x \]
\[ v_x + uv_x + vv_y + gH_y = gD_y \]

where \( H(x,y,t) = \eta(x,y,t) + D(x,y) \) is the entire height of water column, \( \eta \) being the sea surface disturbance (wave height), \( D(x,y) \) is depth (which is supposed to be known at all grid points), \( u \) and \( v \) components of velocity vector, \( g \) is acceleration of gravity.

2.2. Mac-Cormack numerical scheme

The shallow water equations (1) at the mesh nodes \( \Omega \) on the \( n \)-th time step will be approximated with the help of explicit two-step Mac-Cormack finite difference scheme of the second order approximation:

1st step:

\[
\begin{align*}
\hat{H}_j^{n+1} - H_j^n & \frac{\tau}{\Delta x} + \frac{H_y^n u_j^n - H_x^{n-1} u_{j-1}^n}{\Delta x} + \frac{H_{y_j}^n v_j^n - H_{y_j}^{n-1} v_{j-1}^n}{\Delta y} = 0, \\
\frac{\hat{u}_j^{n+1} - u_j^n}{\tau} + \frac{u_j^n u_j^n - u_{j-1}^n}{\Delta x} + \frac{v_j^n u_j^n - u_{j+1}^n}{\Delta x} & + g \frac{\eta_j^n - \eta_{j-1}^n}{\Delta y} = 0, \\
\frac{\hat{v}_j^{n+1} - v_j^n}{\tau} + \frac{u_j^n v_j^n - v_{j-1}^n}{\Delta x} + \frac{v_j^n v_j^n - v_{j+1}^n}{\Delta y} & + g \frac{\eta_j^n - \eta_{j-1}^n}{\Delta y} = 0.
\end{align*}
\]

2nd step:

\[
\begin{align*}
\frac{H_y^{n+1} - (\hat{H}_j^{n+1} + H_j^n)}{2} & + \frac{H_x^{n+1} - H_{x+1}^{n+1}}{\Delta x} + \frac{H_{y+1}^{n+1} - H_y^{n+1} - H_{y+1}^{n+1} + H_y^n}{\Delta y} = 0, \\
\frac{u_j^{n+1} - (\hat{u}_j^{n+1} + u_j^n)}{2} & + u_j^n \frac{\hat{u}_j^{n+1} - \hat{u}_j^{n+1}}{\Delta x} + v_j^n \frac{\hat{u}_j^{n+1} - \hat{u}_j^{n+1}}{\Delta x} + g \frac{\hat{\eta}_j^{n+1} - \hat{\eta}_j^n}{\Delta y} = 0, \\
\frac{v_j^{n+1} - (\hat{v}_j^{n+1} + v_j^n)}{2} & + u_j^n \frac{\hat{v}_j^{n+1} - \hat{v}_j^{n+1}}{\Delta x} + v_j^n \frac{\hat{v}_j^{n+1} - \hat{v}_j^{n+1}}{\Delta y} + g \frac{\hat{\eta}_j^{n+1} - \hat{\eta}_j^n}{\Delta y} = 0.
\end{align*}
\]

Usually, the real tsunami wave simulation is performed in a spherical or geodetic coordinate system \((\lambda, \phi)\), where \( \lambda \) is the longitude and \( \phi \) is the latitude. Accordingly, the following relations are used to calculate the differences \( \Delta x \) and \( \Delta y \):

\[
\begin{align*}
\Delta x_j & = \frac{\pi (\lambda_{j+1} - \lambda_j)}{180^\circ} R_E \cos \phi_j, \\
\Delta y_j & = \frac{\pi (\phi_{j+1} - \phi_j)}{180^\circ} R_E.
\end{align*}
\]

where \( R_E \) stands for the Earth radius.
Indeed, in order to calculate the values of the sought functions at point \((i,j,n+1)\) the values at 3 points of the previous time step \((i,j,n), (i-1,j,n), \) and \((i,j-1,n)\) are used during the first step in (2), and at the points \((i,j,n), (i+1,j,n), \) and \((i,j+1,n)\) during the second step in (3). However, the proposed version to realize the three-point calculation stencil seems to be preferable compared to the one from the MOST software package [3].

2.3. Digital bathymetry
Numerical experiments were arranged at the gridded bathymetry around Kii Peninsula and Shikoku Island (southern part of Japan) developed using JDOSS 500m Gridded Bathymetry Data around Japan [6]. The above bathymetry and the computational grid have the following characteristics: (1) Domain size is 3000x2496 points; (2) Grid steps are 0.003 and 0.002 arc degrees (which means 280,6 and 223 meters, respectively); (3) Array covers the area between 131° and 140° E, 30,01° and 35° N; (4) Time step used in computations is equal to 0.5 sec.

3. Numerical experiments
To understand how wave height distribution along the coast depends on a shape and orientation of the tsunami source, a series of numerical calculations was carried out. The used model sources (initial seabed displacement areas) have form of ellipse with a ratio of lengths of axes 3:1. They were located over a continental slope close to Shikoku Island (figure 2) and Kii Peninsula (figure 6).

The initial sea surface elevation in grid-points inside the model tsunami source was as follows:

\[
H(i,j) = (1 + \cos(\pi \cdot \arg(i,j))) \cdot H_0 / 2
\]

where \(H_0\) is the water surface displacement at the central point \((i_0, j_0)\) of the ellipse. The parameter \(\arg(i, j)\) represents the ratio between the distance from the grid-point to the centre of ellipse and the distance from this centre to the ellipse border in this direction

\[
\arg(i,j) = \left( \frac{(i-i_0)\Delta x \cdot \cos(\beta) + (j-j_0)\Delta y \cdot \sin(\beta)}{r_1} \right)^2 + \left( \frac{(j-j_0)\Delta y \cdot \cos(\beta) + (i-i_0)\Delta x \cdot \sin(\beta)}{r_2} \right)^2.
\]

Here \(r_1, r_2\) are the ellipse axis lengths, \(\Delta x, \Delta y\) are the grid-steps and \(\beta\) is the long axis azimuth. Figure 1 shows the shape of 3 meters height ellipsoidal model source with the axis length ratio equal to 3, and the water height distribution along the ellipse axis.

![Figure 1. The shape (below) and cross-section (above) of a model ellipsoidal tsunami source.](image-url)
wave heights at coastline areas, which meet the continuation of ellipse short axis. It has been generally confirmed by our numerical experiments with a number of elliptic tsunami sources, long axis was 300 km, short axis – 100 km.

Distributions of maximal wave heights at the entire computational domain for the model 3 m high source in front of Shikoku Island are shown in figures 2 and 3. Case of the long axis of ellipse along the coast is presented in figure 2, while figure 3 demonstrates the case of the long axis orthogonal to the coast.

![Figure 2](image2.png)

**Figure 2.** Distribution of tsunami maximal heights all around computational domain. Tsunami model source near Shikoku Island. Long axis of ellipse is “parallel” to the coastline.

![Figure 3](image3.png)

**Figure 3.** Distribution of tsunami maximal heights all around computational domain. Tsunami model source at Shikoku Island. Long axis of ellipse is “orthogonal” to the coastline.

Detailed distributions of wave height maximums (in centimeters) along the coast are presented in figures 4 and 5.
Figure 4. Detailed maximal wave heights distribution (in centimeters) along the shoreline. Elliptic tsunami model source near Shikoku Island. Long axis of ellipse is “parallel” to the coastline.

Figure 5. Detailed maximal wave heights distribution (in centimeters) along the shoreline. Long axis of ellipse near Shikoku Island is “orthogonal” to the coastline.

As was noted, in case of “parallel” source orientation the maximal wave heights (up to 450 cm) are recorded just opposite the source center position (figure 3). In case of the “orthogonal” source orientation, maximal wave heights (up to 400 cm) are recorded at the edges of the Shikoku island and along the Kyushu coast (figure 4). On the Kii peninsula and near Hamamatsu wave heights not exceed 100 cm. Similar numerical experiments were arranged for tsunami model source near Kii Peninsula. Results are presented in figure 6 (the “parallel” source) and in figure 7 (the “orthogonal” source).

Figure 6. Distribution of the wave maximal heights (in centimeters) in grid points. Elliptic tsunami model source near Kii peninsula. Long axis of ellipse is “parallel” to the coastline.
Figure 7. Distribution of tsunami maximal heights all around computational domain. Tsunami model source at Kii peninsula. Long axis of ellipse is “orthogonal” to the coastline.

Detailed distributions of the wave maximal heights along the coast of Kii Peninsula and Hamamatsu area for the above numerical experiments are demonstrated in figure 8. As expected, the highest waves (up to 5 m) in case of “parallel” source were detected (figure 8). For the “orthogonal” source the wave height there are twice lower, but it has been approximately increased twice at Shikoku Island and near Hamamatsu (figure 8).

Figure 8. Detailed distribution of the wave maximal heights in case of source location near Kii Peninsula. Long axis of ellipse is “parallel” to the coastline (above) and “orthogonal” to the coastline (below).
In order to understand the role of the wave energy radiation directivity for wave heights near a shore the numerical experiment on tsunami generated by the round-shaped source has been carried out. The total potential energy of this source is equal to the ellipsoidal one used for previous calculations. Calculated wave height maxima distribution all around the computational domain is presented in figure 9, and tsunami heights along the coastline are drown in figure 10.

**Figure 9.** Distribution of tsunami maximal heights all around computational domain. Tsunami round-shaped source is situated near Shikoku Island.

**Figure 10.** Detailed distribution of the wave maximal heights along the shoreline. Tsunami round-shaped source near Shikoku Island.

Due to the uniform wave energy radiation by the rounded source to all directions one can see much more even distribution of wave maxima along the entire coastline. Wave amplitude is slightly decreasing when moving off the source center projection along a shoreline.

### 4. Discussion

According to the results of numerical experiments the distribution of tsunami height near the shore is strongly depends on the source shape and orientation of its long axis. For the extended sources oriented by a long axis parallel to the coastline, the zone of maximum heights of waves is located in the neighborhood of a projection of the source center to the coastline. Such orientation of the center is typical for subduction zones therefore in such cases the most dangerous areas of the coast are located just opposite to the center of the extended source. And, on the contrary, outside of a projection of such source tsunami waves have considerably lower height that should be taken into account by tsunami...
warning services. If a source has round shape or will extend in the orthogonal direction to the coastline, then the high amplitude of waves can be observed on segment of the coast which length considerably exceeds the source size. At the same time local features of a bottom relief are responsible for distribution of wave height maxima along the shoreline. For example, extremely high waves can be registered on capes which represent continuations of the underwater ridges working as tsunami wave-guides [7].

5. Conclusion

The influence of orientation of the model ellipsoidal source on distribution of tsunami height maxima along the coast of the Shikoku Island of and the Kii Peninsula in the southern part of Japan is considered. Results of numerical calculations show significant influence of a shape of initial water surface elevation on wave heights on various sites of the coastline. At the same time, in some points of the shoreline the height of impacting tsunami wave is mostly determined by local features of a bottom relief.

In addition, series of calculations carried out by means of new computer architecture (FPGA) showed a possibility of fast assessment (several tens of seconds) of the expected tsunami heights on long segments of the coast that gives perspective for such a hardware usage in tsunami warning centers.

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