Quantifying the redshift space distortion of the bispectrum
III : Detection prospects of the multipole moments

Arindam Mazumdar\textsuperscript{1*}, Debanjan Sarkar\textsuperscript{2†}, Somnath Bharadwaj\textsuperscript{1,3‡}

\textsuperscript{1}Centre for Theoretical Studies, Indian Institute of Technology Kharagpur, Kharagpur - 721302, India
\textsuperscript{2}Department of Physics, Ben-Gurion University of the Negev, Be'er Sheva - 84105, Israel
\textsuperscript{3}Department of Physics, Indian Institute of Technology Kharagpur, Kharagpur - 721302, India

ABSTRACT

The redshift space anisotropy of the bispectrum is generally quantified using multipole moments. The possibility of measuring these multipoles in any survey depends on the level of statistical fluctuations. We present a formalism to compute the statistical fluctuations in the measurement of bispectrum multipoles for galaxy surveys. We consider specifications of a Euclid like galaxy survey and present two quantities: the signal-to-noise ratio (SNR) which quantifies the detectability of a multipole, and the rank correlation which quantifies the correlation in measurement errors between any two multipoles. Based on SNR values, we find that Euclid can potentially measure the bispectrum multipoles up to \( \ell = 4 \) across various triangle shapes, formed by the three \( \mathbf{k} \) vectors in Fourier space. In general, SNR is maximum for the linear triangles. SNR values also depend on the scales and redshifts of observation. While, \( \ell \leq 2 \) multipoles can be measured with SNR \( > 5 \) even at linear/quasi-linear \( (k \lesssim 0.1 \, \text{Mpc}^{-1}) \) scales, for \( \ell > 2 \) multipoles, we require to go to small scales or need to increase bin sizes. For most multipole pairs, the errors are only weakly correlated across much of the triangle shapes barring a few in the vicinity of squeezed and stretched triangles. This makes it possible to combine the measurements of different multipoles to increase the effective SNR.

**Key words:** methods: statistical – cosmology: theory – large-scale structures of Universe.

1 INTRODUCTION

Observations of the Cosmic Microwave Background (CMB) (Fergusson et al. 2012; Oppizzi et al. 2018; Akrami et al. 2020; Shiraishi 2019) and galaxy clustering (Feldman et al. 2001; Scoccimarro et al. 2004; Liguori et al. 2010; Ballardini et al. 2019) indicate that the primordial density fluctuations are consistent with the simplest models of inflation which predict these to be a Gaussian random field (Baumann 2009). The power spectrum is sufficient to quantify the statistics of a Gaussian random field for which all the higher order statistics are predicted to be zero. However, several inflationary scenarios also predict the primordial fluctuations to be non-Gaussian (primordial non-Gaussianity; Bartolo et al. 2004). Further, the non-linear evolution of initially Gaussian density fluctuations and also the non-linear biasing of the tracer fields (e.g. galaxies) both introduce ‘induced non-Gaussianity’ (Fry 1984; Bernardeau et al. 2002). The bispectrum, which is the Fourier transform of the three-point correlation function, is the lowest order statistics which is sensitive to the non-Gaussianity. Second order perturbation theory predicts that measurements of the bispectrum in the weakly non-linear regime can be used to constrain the bias parameters (Matarrese et al. 1997), and this framework has been applied in several galaxy surveys to extract the galaxy bias parameters (Feldman et al. 2001; Scoccimarro et al. 2001; Verde et al. 2002; Nishimichi et al. 2007; Gil-Marín et al. 2015). Further, measurements of the bispectrum allow us to break the degeneracy between the matter density parameter \( \Omega_m \) and the linear bias parameter \( b_1 \), something which is not possible by using the power spectrum alone (Scoccimarro et al. 1999). A precise measurement of the Baryon Acoustic Oscillations (BAO) in the bispectrum allows us to constrain the expansion rate of the universe Pearson & Samushia (2018). Upcoming galaxy surveys like Euclid (Blanchard et al. 2020), promises to measure the bispectrum with high precision, and thereby constrain the above parameters with unprecedented accuracy.

Redshift space distortion (RSD) is an important effect in galaxy redshift surveys (Kaiser 1987; Jackson 1972; Hamilton 1998). The line-of-sight (LoS) anisotropy in the redshift space power spectrum contains a wealth of cosmological information. For example, this can been used to: (i) measure the growth of structures \( f \) on linear scales (Loveday et al. 1996; Peacock et al. 2001; Hawkins et al. 2003; Guzzo et al. 2008), (ii) constrain the total density from massive neutrinos (Hu et al. 1998; Upadhye 2019), and (iii) test dark energy and modified gravity theories (Linder 2008; Song & Percival 2009; de la Torre et al. 2017; Johnson et al. 2016; Mueller et al. 2018).

The bispectrum also is affected by RSD, and the redshift space bispectrum also contains a wealth of cosmological in-
formation (Scoccimarro et al. 1999; Yankelevich & Porciani 2019; Hahn & Villaescusa-Navaarro 2021), and it is important to accurately model and quantify this. Hivon et al. (1995) and Verde et al. (1998) have formulated the initial theoretical framework for calculating the bispectrum in redshift space. However, they mainly focused on measuring the large scale bias parameter and the cosmological parameters, and they have not quantified the RSD anisotropy in general. Later, Scoccimarro et al. (1999) have quantified the anisotropy of the redshift space bispectrum utilizing spherical harmonics. Their work, however, was restricted only to the monopole and one quadrupole component ($\ell = 2, m = 0$).

The work of Hashimoto et al. (2017) also was limited to a single quadrupole component of the redshift space bispectrum. Nan et al. (2018) have calculated approximate analytical expressions for the higher angular multipole moments up to $\ell = 4$ based on the halo model, and their analysis is only limited to a few triangle configurations. Yankelevich & Porciani (2019) and Gualdi & Verde (2020) have analysed the combined ability of the redshift space power spectrum and bispectrum to constrain the cosmological parameters. Desjacques et al. (2018) have utilised effective field theory to model the redshift space bispectrum on quasi non-linear scales. The theoretical analysis of Clarkson et al. 2019 and de Weerd et al. (2020) suggests that relativistic effects will introduce a dipole anisotropy in the redshift space bispectrum on very large length-scales.

There has been some work towards developing fast estimators which quantify the anisotropy of the redshift space bispectrum. Slepian & Eisenstein (2017); Slepian & Eisenstein (2018) have introduced a technique to expand the redshift space three-point correlation function in terms of the products of two spherical harmonics. On the other hand, Sugiyama et al. (2019) have proposed a tri-polar spherical harmonic decomposition to quantify the anisotropy of the redshift space bispectrum, and as a demonstration they applied this to the Baryon Oscillation Spectroscopic Survey (BOSS) Data Release 12.

Our recent work (Bharadwaj et al. 2020, hereafter Paper I) presents a formalism to quantify the anisotropy of the redshift space bispectrum $B^\ell_\kappa(k_\alpha,k_\beta,k_\gamma)$ by decomposing it into multipole moments $\bar{B}^\ell_\kappa(k_1,\mu,t)$. Here $k_1$, the length of the largest side, and $\mu$ respectively quantify the size and shape of the triangle $(k_\alpha,k_\beta,k_\gamma)$. We have illustrated this formalism by quantifying the anisotropy due to linear RSD of the bispectrum arising from primordial non-Gaussianity. We have found that only the first four even $\ell$ multipoles are non-zero, for which we have presented explicit analytical expressions. These results are expected to be important to constrain $f_{\text{NL}}$ using the bispectrum measured from future redshift surveys. In a subsequent work Mazumdar et al. (2020) (hereafter Paper II), the same formalism was used to quantify the anisotropy of the induced redshift space bispectrum arising from Gaussian initial conditions. We present analytical expressions for all the multipole moments which are predicted to be non-zero ($\ell \leq 5, m \leq 6$) at second order perturbation theory. Considering triangles of all possible shapes, we have analysed the shape dependence of all the multipoles holding $k_1 = 0.2\text{Mpc}^{-1}$, $\beta_1 = 1$, $b_1 = 1$ and $\gamma_2 = 0$ fixed. Here $\beta_1, b_1$, and $\gamma_2$ quantify the linear redshift distortion parameter, linear bias and quadratic bias respectively. For most multipoles we find that the maxima or minima, or both, occur very close to the squeezed limit. Further, the absolute values $|\bar{B}^\ell_\kappa|$ are found to decrease rapidly if either $\ell$ or $m$ are increased. We have also provided rough estimates for measuring the various multipoles using upcoming galaxy redshift surveys. The present paper presents a detailed analysis for the prospects of measuring the various multipole moments using upcoming galaxy redshift surveys.

The ability to measure any particular bispectrum multipole $\bar{B}^\ell_\kappa$ primarily depends on its amplitude, the galaxy number density and the extent of the survey volume. Until recently, measurements of the bispectrum were limited by the shot noise from the limited galaxy number density and the cosmic variance due to the limited volume of the galaxy redshift surveys (Scoccimarro et al. 2001; Verde et al. 2002; Croton et al. 2004; Jing & Boerner 2004; Kulkarni et al. 2007; Gaztanaga et al. 2009; Marin 2011). However, Gil-Marín et al. (2015, 2017); Philcox & Ivanov (2022) have recently measured the isotropic component (monopole) of the galaxy bispectrum using the Baryon Oscillation Spectroscopic Survey at a relatively high level of precision. They showed that it is possible to improve the constraints on the cosmological parameters by including the bispectrum along with the power spectrum. Upcoming galaxy surveys will cover unprecedented large volumes with high galaxy number density, and it is anticipated that this will enable us to precisely measure the higher multipoles of the bispectrum. For example, the Euclid space telescope (Blanchard et al. 2020) is expected to complete a wide survey that will measure $\sim 10^8$ galaxy redshifts over 15,000 square degrees on the sky in the range $z \lesssim 2.5$. Precise measurements of the galaxy bispectrum using Euclid are expected to place tight constraints on: (i) primordial non-Gaussianity (Fedeli et al. 2011; Dai & Xia 2020), (ii) neutrino masses (Chudaykin & Ivanov 2019; Hahn & Villaescusa-Navaarro 2021), (iii) modified gravity models (Bose et al. 2020), and (iv) all the other currently constrained standard cosmological parameters (Agarwal et al. 2021). The above estimates mostly rely on the real-space bispectrum or its redshift space monopole. It is important and interesting to also consider the higher multipoles of the bispectrum in order to utilize the full reach of this mission (Gualdi & Verde 2020).

In this paper we developed a formalism to calculate the statistical fluctuations expected in the bispectrum multipoles measured from any given galaxy redshift survey. We quantify these statistical fluctuations through the error covariance of the different multipoles. Here we have considered the specifications of the Euclid galaxy redshift survey for which we present the signal-to-noise ratio (SNR) with which the different multipoles are expected to be measured. We also present results for the correlations expected between the different multipole moments. The paper is organised as follows. In Section 2, we present the formalism for calculating the error covariance between different bispectrum multipoles. In Section 3 we present our main results, which include the dependence of bispectrum multipoles on triangle shapes, sizes and redshifts, possibility of measuring the different multipoles, and error covariance between the pair of multipoles in terms of rank correlation. Finally in Section 4, we summarize our findings and outline some future directions. In this paper, we use the Boltzmann code CLASS (Lesgourgues 2011; Blas et al. 2011) to calculate the input matter power spectrum for our analysis and assumed cosmological param-


\[ \delta^s(x) = \frac{1}{V} \sum_k e^{i \mathbf{k} \cdot \mathbf{x}} \Delta^s(k), \]

for which, in the absence of Poisson noise, the power spectrum is defined as

\[ \langle \Delta^s(k_1) \Delta^s(k_2) \rangle = V \delta_{k_1+k_2} P^s, \]

where \( \mu \) is the cosine of the angle between the Fourier mode \( k_1 \) and the line of sight (LoS) direction \( \hat{\mathbf{n}} \). We have considered \( \hat{\mathbf{n}} = \hat{z} \) here. The bispectrum is similarly defined as

\[ \langle \Delta^s(k_1) \Delta^s(k_2) \Delta^s(k_3) \rangle = V \delta_{k_1+k_2+k_3} B^s(k_1,k_2,k_3). \]

Throughout this paper we assume that the three vectors \((k_a,k_b,k_c)\), which form a closed triangle, are ordered such that \( k_1 \geq k_2 \geq k_3 \). In the absence of redshift space distortion the bispectrum depends only on the shape and size of the triangle formed by the three vectors \((k_1,k_2,k_3)\). Here, following Paper I, we parametrize this using \((k_1,\mu,\tau)\) where \( k_1 \) the length of the edge of the triangle \( k_1 \) is of length \( \mu = \cos \theta \) (fig. 1) and \( t = k_2/k_1 \) together quantify the shape of the triangle. Note that \( \mu \) and \( t \) are both bounded within the range \( 0.5 \leq \mu \leq t \leq 1 \) and \( 2 \mu t \geq 1 \). In the presence of redshift space distortion the bispectrum also depends on \( \mu_a = k_a \cdot \hat{n}/k_a \) where \( a = 1,2,3 \). Therefore, the bispectrum now depends on how the triangle is oriented with respect to \( \hat{n} \).

It is necessary to consider triangles of all possible orientations in order to quantify the anisotropy of the redshift space bispectrum. Here we start from a reference triangle in the \( x-z \) plane (fig. 1) for which we have

\[ k_1 = k_1 \hat{z}, \]

\[ k_2 = k_1 t [-\mu \hat{z} + \sqrt{1-\mu^2} \hat{x}], \]

\[ k_3 = -k_1 - k_2. \]

and \( k_3 = -k_1 - k_2 \). It is possible to obtain all possible orientations of the triangle by applying different rotations \( \hat{\mathbf{R}} \) to the reference triangle. We parameterize these rotations using \((\alpha,\beta,\gamma)\) the Euler angles which refer to successively rotating along the \( z, y \) and \( z \) axes respectively (Sakurai 1994). We then have

\[ \mu_1 = p_z \]

\[ \mu_2 = -\mu p_z + \sqrt{1-\mu^2} p_x \]

\[ \mu_3 = -[(1-\mu t)p_z + t \sqrt{1-\mu^2} p_x]/\sqrt{1-2\mu t + t^2} \]

where we have introduced an unit vector \( \hat{\mathbf{p}} = \hat{\mathbf{R}}^{-1} \hat{\mathbf{n}} \) which has components \( p_z = \cos(\beta) \) and \( p_x = -\sin(\beta) \cos(\gamma) \) respectively. Note that these expressions are independent of \( \alpha \) i.e. the axis of the first rotation \( (z) \) coincides with \( \hat{\mathbf{n}} \), and it does not affect the redshift space distortion. In summary, the redshift space bispectrum \( B^s(k_1,k_2,k_3) \) can be completely parameterized using \( B^s(k_1,\mu,\tau,\hat{\mathbf{p}}) \) where \( (k_1,\mu,\tau) \) quantify the size and shape dependence of the triangle while \( \hat{\mathbf{p}} \) quantifies its orientation with respect to \( \hat{\mathbf{n}} \).

The redshift space anisotropy of the bispectrum is quantified using multipole moments defined as

\[ B^m_m(k_1,\mu,\tau) = \frac{\sqrt{(2\ell+1)}}{4\pi} \int \bar{Y}^m_m(\hat{\mathbf{p}})^* B^s(k_1,\mu,\tau,\hat{\mathbf{p}}) d\Omega_p \]

where the integral over the solid angle \( d\Omega_p \) accounts for all possible orientations of the triangle. As discussed in Paper I, only the even \( \ell \) multipoles of the bispectrum are non-zero and these are all real valued.

In reality, we encounter a finite number of Fourier modes distributed on a regular grid. Here we define the bispectrum multipole estimator as a discrete sum over triangle configurations in Fourier space,

\[ B^m_m(k_1,\mu,\tau) = \sum_n w^m_m(\hat{\mathbf{p}}) \Delta^s(k_n) \Delta^s(k_1) \Delta^s(k_3) + c.c. \]

where \( k_a + k_b + k_c = 0 \) form a closed triangle, each value of \( n \) represents a different closed triangle, c.c. \( \equiv \) complex conjugate, and

\[ w^m_m(\hat{\mathbf{p}}) = \text{Re} \left[ \frac{2\ell+1}{4\pi} \sum_{n_1} |\bar{Y}^m_m(\hat{\mathbf{p}})|^2 \right]. \]

The sum over \( n \) covers triangles of all possible orientations, with shape and size in a bin of extent \( \Delta(k_1,\Delta\mu,\Delta t) \) around \((k_1,\mu,\tau)\). The exact binning scheme used for the present work is discussed later in this paper.

### 2.1 The Error Covariance

We assume that the grid spacing in \( k \) is adequately fine so that various triangle orientations are adequately sampled which implies that \( \langle B^m_m(k_1,\mu,\tau) \rangle = B^m_m(k_1,\mu,\tau) \) where \( \langle \ldots \rangle \) denotes an ensemble average of different realisations of the random density field \( \Delta(k) \). In the present work we are interested in the statistical fluctuations (error)

\[ \Delta B^m_m = B^m_m(k_1,\mu,\tau) - \langle B^m_m(k_1,\mu,\tau) \rangle \]

which we quantify through the covariance

\[ C^{m'm'}_{m'm'} = \langle \Delta B^m_m \Delta B^{m'}_{m'} \rangle. \]
We calculate this using
\[ C_{\ell\ell^{'}}^{m\ell^{'}} + B_{\ell}^{m} B_{\ell^{'}}^{m^{'}} = (2V)^{-2} \sum_{n_1, n_2} w_n^{m}(\hat{\mathbf{p}}_{n_1}) w_{n^{'}}^{m^{'}}(\hat{\mathbf{p}}_{n_2}) \times \]
\[ \langle [\Delta^s(n_1) \Delta^s(n_1) \Delta^s(n_2) \Delta^s(n_2) \Delta^s(\hat{n}_1) \Delta^s(\hat{n}_2)] + \text{c.c.} \rangle \]
\[ + \langle \Delta^s(\hat{n}_1) \Delta^s(\hat{n}_1) \Delta^s(\hat{n}_2) \Delta^s(\hat{n}_2) \Delta^s(\hat{n}_2) \Delta^s(\hat{n}_2) \rangle \]
\[ + \text{c.c.} \rfloor . \]
\[ (11) \]

We have evaluated this using
\[ \langle \Delta^s(\hat{n}_1) \Delta^s(\hat{n}_1) \Delta^s(\hat{n}_2) \Delta^s(\hat{n}_2) \Delta^s(\hat{n}_2) \Delta^s(\hat{n}_2) \rangle = \]
\[ [1 + 3 \delta_{n_1, n_2}] V^2 B(\hat{n}_1, \hat{n}_1, \hat{n}_2, \hat{n}_2, \hat{n}_2, \hat{n}_2) \]
\[ + \delta_{n_1, -n_2} V^2 \langle P(\hat{n}_1) P(\hat{n}_1) P(\hat{n}_1) \rangle \]
\[ (12) \]

which assumes that the signal is weakly non-Gaussian whereby contributions from \( T^s \) the trispectrum and \( P^s \) the sixth order polyspectrum can be neglected. Here \( \delta_{n_1, n_2} \) is a Kronecker delta which has value one only when \( n_1 \) and \( n_2 \) refer to the same triangle and is zero otherwise. \( -n_2 \) refers to the same triangle as \( n_2 \) but the two are exactly oppositely oriented i.e. \( k_{1-n_2} = -k_{n_2} \), etc. Also note that \( \Delta^s(n_{-n_2}) = \Delta^s(n_2) \). Using these in eq. (10) we obtain
\[ C_{\ell\ell^{'}}^{m\ell^{'}} = \sum_n w_n^{m}(\hat{\mathbf{p}}_{n}) w_{n^{'}}^{m^{'}}(\hat{\mathbf{p}}_{n}) \times \]
\[ \{ (B(\hat{n}, \hat{n}, \hat{n})) B + \} \rfloor . \]
\[ (13) \]

which provides an estimate of the error covariance in the absence of Poisson noise.

### 2.2 Poisson noise

We now consider a discrete tracer (e.g. galaxies) whose density represents a Poisson sampling of the smooth filed which we have been discussing till now. We use \( \Delta_g(\hat{n}) \) to represent the corresponding density contrast in Fourier space. The discrete sampling of the smooth field gives rise to a shot noise (or Poisson noise) whose contribution to the power spectrum and the bispectrum are well studied (Peebles 1980; Scocciamaro et al. 2001; Smith 2009). Using \( \Delta_g(\hat{n}) \) instead of \( \Delta(\hat{n}) \) in eq. (2) to estimate the power spectrum, we have
\[ V^{-1} \langle \Delta_g^s(\hat{n}_1) \Delta_g^s(\hat{n}_1) \rangle = P_g^s(\hat{n}_1, \mu_1) + n_g^{-1} , \]
\[ (14) \]

where, \( P_g^s(\hat{n}, \mu) \) is the power spectrum of the smooth field \( \Delta(\hat{n}) \) and \( n_g \) is the number density of galaxies. Similarly considering the bispectrum we have
\[ V^{-1} \langle \Delta_g^s(\hat{n}_1) \Delta_g^s(\hat{n}_1) \Delta_g^s(\hat{n}_1) \rangle = B_g^s(\hat{n}_1, \hat{n}_2, \hat{n}_3) + n_g^{-1} \{ P_g^s(\hat{n}_1, \mu_1) + P_g^s(\hat{n}_2, \mu_2) + P_g^s(\hat{n}_3, \mu_3) \} + n_g^{-2} (15) \]

where it has been assumed that \( \hat{n}_1 + \hat{n}_2 + \hat{n}_3 = 0 \). It is now necessary to consider how eq. (13) for the bispectrum error covariance is modified if we account for the discrete sampling. We first consider the ensemble average of the product of six \( \Delta_g(\hat{n}) \) which is required to calculate the covariance (eq. 10). Here we use the shortened notation \( \Delta_{ag} \equiv \Delta_g(\hat{n}_a) \), \( \Delta_{ab} \equiv \Delta_g(\hat{n}_a + \hat{n}_b) \), \( \Delta_{abc} \equiv \Delta_g(\hat{n}_a + \hat{n}_b + \hat{n}_c) \), etc. We then have
\[ \langle \Delta_{ag} \Delta_{ag} \Delta_{ag} \Delta_{ag} \Delta_{ag} \Delta_{ag} \rangle = \]
\[ V n_g^{-1} + n_g^{-2} (\Delta_{ag} \Delta_{ag} \Delta_{ag} \Delta_{ag} \Delta_{ag} \Delta_{ag}) + \]
\[ n_g^{-3} (\Delta_{ag} \Delta_{ag} \Delta_{ag} \Delta_{ag} \Delta_{ag} \Delta_{ag}) + \]
\[ n_g^{-2} (\Delta_{ag} \Delta_{ag} \Delta_{ag} \Delta_{ag} \Delta_{ag} \Delta_{ag}) + \]
\[ n_g^{-4} (\Delta_{ag} \Delta_{ag} \Delta_{ag} \Delta_{ag} \Delta_{ag} \Delta_{ag}) + \text{combinations} \]
\[ (16) \]

where all the terms in the r.h.s. arise due to the discrete sampling, and for each term it is necessary to also consider all the distinct combinations of the indices \( a, b, c, \ldots \). Instead of embarking on a detailed calculation of the r.h.s., we present it in a schematic form which can be used for an order of magnitude estimate. Using \( C_{\ell\ell^{'}}^{m\ell^{'}} \) to represent the error covariance respectively in the presence and absence of shot noise, we have
\[ [C_{\ell\ell^{'}}^{m\ell^{'}}]_g - C_{\ell\ell^{'}}^{m\ell^{'}} \sim V^{-1} [n_g^{-1} + n_g^{-2} P_g^s + n_g^{-3} B_g^s] + \]
\[ n_g^{-2} [P_g^s]^2 + n_g^{-1} P_g^s B_g^s \]
\[ (17) \]

where \( C_{\ell\ell^{'}}^{m\ell^{'}} \sim V [P_g^s]^3 + 3 [B_g^s]^2 \) (eq. 13). Here we consider estimates for the upcoming Euclid survey (Laureijs et al. 2011) which is expected to cover \( V \sim 10^6 \) Mpc$^3$. Following Yankelevich & Porciani (2019) we adopt \( n_g \sim 10^{-3} \) Mpc$^{-3}$ and a linear bias parameter \( b \sim 1 \) at \( z = 0.7 \). We have used Blas et al. (2011) to estimate \( P_g^s \sim 10^4 \) Mpc$^3$ and \( B_g^s \sim [P_g^s]^2 \) at \( k \sim 0.2 \) Mpc$^{-1}$ where we expect second order perturbation to be reasonably valid. Using these, we first consider eq. (14) where we have \( P_g^s \sim 10^4 \) Mpc$^3$ and \( n_g^{-1} \sim 10^3 \) Mpc$^3$. Similarly in eq. (15) we have \( B_g^s \sim 10^6 \) Mpc$^6$, whereas \( n_g^{-1} P_g^s \sim 10^5 \) Mpc$^5$ and \( n_g^{-2} P_g^s B_g^s \sim 10^6 \) Mpc$^6$. These estimates show that it is necessary to account for the shot noise in order to correctly estimate the power spectrum and the bispectrum. We now consider the error covariance for which we have \( V [P_g^s]^3 \sim 10^5 \) Mpc$^{12}$ and \( [B_g^s]^2 \sim 10^6 \) Mpc$^{12}$ whereby \( C_{\ell\ell^{'}}^{m\ell^{'}} \sim 10^7 \) Mpc$^{12}$ i.e. the error in the estimated bispectrum is dominated by the terms \( \sim V [P_g^s]^3 \). Considering the shot noise, we see that we have the largest contribution from \( n_g^{-1} P_g^s B_g^s \sim 10^6 \) Mpc$^{12}$ and the magnitude decreases with increasing power of \( n_g^{-1} \) with the smallest contribution coming from \( V^{-1} n_g^{-5} \sim 10^3 \) Mpc$^{12}$. Even if we consider a lower galaxy number density \( n_g \sim 10^{-4} \) Mpc$^{-3}$, we find that the terms \( V^{-1} [\ldots] \) are all \( \sim 10^{11} \) Mpc$^{12}$ and the two remaining terms \( \sim 10^{10} \) Mpc$^{12}$, all of which are several orders of magnitude smaller than the predicted value of \( C_{\ell\ell^{'}}^{m\ell^{'}} \). We conclude that it is quite reasonable to ignore the shot noise contribution, and we use \( C_{\ell\ell^{'}}^{m\ell^{'}} \) for the error estimates presented in the subsequent analysis.

### 2.3 Binning

We finally discuss the binning scheme which we have considered here. For the purpose of analytic predictions, it is convenient to assume that \( V \) is very large whereby \( dN_i \) the number of \( k \) modes in the interval \( d^3k \) is given by \( dN_i = (2\pi)^{-3} V d^3k \). The number of triangles \( dN_{\ell r} \) is then given by
\[ dN_{\ell r} = (2\pi)^{-6} V^2 d^3k_1 d^3k_2 , \]
\[ (18) \]
and we replace the sum in eq. (13) using \( \sum_n \rightarrow \int dN_{\ell r} \). Starting from \( k_1 = k_1 \hat{x} \) (fig. 1) we obtain all other possible vectors \( k_1 \) by changing the length \( k_1 \) or rotations through
the Euler angles \( \alpha \) and \( \beta \), and we have
\[
d^2k_1 = k_1^2 \, dk_1 \, d\beta \sin \beta \, da.
\] (19)

Considering \( \mathbf{k}_2 \), a change in length could occur through either a change in \( k_1 \) or in \( t \), whereas a change in orientation is associated with either a change in \( \mu = \cos \theta \) or \( \gamma \) the third Euler angle. We then have
\[
d^2k_2 = k_2^2 t^2 \, (dt \, dk_1 + k_1 \, dt) \, d\mu \, d\gamma.
\] (20)

Here we have considered bins of extent \( (\Delta k_1, \Delta \mu, \Delta t) \) centered around \((k_1, \mu, t)\). Using eq. (19) and eq. (20), we have
\[
dN_{tr} = (8\pi^2)^{-1} N_{tr} \, d\alpha \, \sin \beta \, d\beta \, d\gamma.
\] (21)

where \( N_{tr} \) the number of triangles in any particular bin is given by
\[
N_{tr} = (8\pi^4)^{-1} \langle V \hat{p}^3 \rangle t^2 \left[ \Delta \ln k_1 \left( t \Delta \ln k_1 + \Delta t \right) \Delta \mu \right]
\] (22)

Using these we have
\[
\sum_{n_1} | Y^n_{\ell m}(\hat{\mathbf{p}}_{n_1})|^2 = (4\pi)^{-1} N_{tr}
\] (23)

which we use in the expression for \( u^n_{\ell m} \) (eq. 8) to write eq. (13) as
\[
C_{\ell m n}^{cmr}(k_1, \mu, t) = \sqrt{\frac{(2\ell + 1)(2\ell' + 1)}{N_{tr}}} \int d\Omega \hat{p} \times \Re \left[ Y^n_{\ell m}(\hat{\mathbf{p}}) \right] \Re \left[ Y_{\ell m}^{\ell'}(\hat{\mathbf{p}}) \right] \left[ 3 |B^n(k_1, \mu, t, \hat{\mathbf{p}})|^2 + VP^n(k_1, \mu_1) P^s(k_2, \mu_2) P^s(k_3, \mu_3) \right].
\] (24)

Note that \( k_2, k_3, \mu_1, \mu_2, \mu_3 \) can all be calculated from \((k_1, \mu, t, \hat{\mathbf{p}})\) using eq. (4) and eq. (5). We have used eq. (24) to calculate the error estimates presented subsequently in this paper considering bins of extent \( (\Delta \ln k_1 = 0.1, \Delta \mu = 0.05, \Delta t = 0.05) \).

### 2.4 Second Order Induced Bispectrum

The induced bispectrum for any tracer in redshift space from second-order perturbation theory (2LPT) can be written as
\[
B_{2\ell PT}(k_1, k_2, k_3) = 2b_1^{-1} (1 + \beta_1 \mu_1^2)(1 + \beta_1 \mu_2^2) \left\{ F_2(k_1, k_2) + \frac{\gamma_2}{2} \mu_2 \beta_1 G_2(k_1, k_2) \right. \\
+ b_1 \beta_1 \mu_3 \frac{\mu_1}{k_1} \left[ 1 + \beta_1 \mu_2^2 \right] \\
+ \frac{\mu_2^2}{k_2} \left[ 1 + \beta_1 \mu_1^2 \right] \bigg\} P(k_1)P(k_2) + \text{cyc...},
\] (25)

where \( \beta_1 \) is the linear redshift distortion parameter, \( \gamma_2 = b_2/b_1 \) where \( b_1 \) is the linear bias and \( b_2 \) is the quadratic bias, and
\[
G_2(k_1, k_2) = \frac{3}{7} + \frac{k_1 \cdot k_2}{2} \left( \frac{1}{k_1^2} + \frac{1}{k_2^2} \right) + \frac{4}{7} \frac{(k_1 \cdot k_2)^2}{k_1^2 k_2^2},
\] (26)

refers to the second order kernel for the divergence of the peculiar velocity. Here we find it convenient to use the following notation
\[
F_2(k_1, k_2) = G_2(k_1, k_2) + \Delta G(k_1, k_2)
\] (27)

where
\[
\Delta G(k_1, k_2) = \frac{2}{7} \left[ 1 - \frac{(k_1 \cdot k_2)^2}{k_1^2 k_2^2} \right].
\] (28)

Note that, the notations used here are taken explicitly from Paper I and Paper II.

In Paper II, we have quantified the anisotropy of the induced redshift-space 2LPT bispectrum (eq. 25) in terms of the multipole moments \( B_{\ell m n}^{t_0}(k_1, \mu, t) \) defined in eq. (6). There we have also presented the formulas needed to calculate all the multipole moments that are predicted to be non-zero at second-order perturbation theory. Further, we have analysed the \( \mu - t \) dependence of \( B_{\ell m n}^{t_0}(k_1, \mu, t) \) at fixed \( k_1 = 0.2 \) Mpc\(^{-1}\) and for the parameter values \( \beta_1 = 1, b_1 = 1, \gamma_2 = 0 \). We find that the even multipole moments \( B_{\ell m n}^{00} \) and \( B_{\ell m n}^{20} \) show very similar behaviour where their values are positive in the whole \( \mu - t \) plane, the smallest values occur near the equatorial triangles and the largest values are found for the linear triangles. Two other even multipole moments \( B_{\ell m n}^{22} \) and \( B_{\ell m n}^{40} \) show positive values in the whole \( \mu - t \) plane, however, their shape dependence is different from the multipoles discussed above.

For the rest of the multipole moments, we find that they exhibit negative values at some parts of or through the entire \( \mu - t \) plane. Their contour patterns are also different from each other. We broadly see that, although the higher multipoles \( (\ell > 2) \) show rich variety of shape dependence, their amplitude \( | B_{\ell m n}^{t_0} | \) fall off sharply as \( \ell \) and \( m \) are increased.

Note that, the analysis in Paper II ignores the Finger-of-God (FoG) effect. At large scales linear perturbation theory and its second order extension are expected to provide a reasonably accurate description of the clustering of matter in real space. This however does not hold in redshift space where it is found that large peculiar velocities arising from highly non-linear small scale structures cause the large scale clustering pattern to appear elongated along the LoS. This is known as the Finger of God (FoG) effect, and it is important to include this in any realistic analysis of the redshift space power spectrum and bispectrum.

### 2.5 Finger of God Effect

In case of the power spectrum, the FoG effect is generally incorporated as an ad-hoc damping profile which multiplies the linear redshift space power spectrum. A number of profiles, namely Lorentzian, Gaussian, Lorentzian-squared etc., have been considered in the literature. Although the Lorentzian profile works well for the simulated data (Davis & Peebles 1982; Hamilton 1997; Hatton & Cole 1999; Seljak 2001; White 2001; Sarkar & Bharadwaj 2018, 2019), the Gaussian profile is expected to occur naturally (Bharadwaj 2001; Scoccimarro 2004; Hikage & Yamamoto 2013; Hikage et al. 2013; Okumura et al. 2015; Hikage & Yamamoto 2016). In this work, we consider a Gaussian profile for the FoG damping and model the FoG redshift-space power spectrum as (Peacock 1992)
\[
P^{\sigma}_{eOG}(k_1, \mu_1) = \exp \left[ -\left( k_1^2 \mu_1^2 \frac{\sigma^2}{2} \right) \right] \times P^{\sigma}_{ePT}(k_1, \mu_1)
\] (29)

where \( P^{\sigma}_{ePT}(k_1, \mu_1) = (1 + \beta_1 \mu_1^2)^2 P(k_1) \) is the linear redshift space power spectrum, and \( \sigma_p \) parametrizes the pairwise velocity dispersion is in units of comoving Mpc. We can equivalently use \( |\sigma_p h(A)| \) in units of \( \text{km s}^{-1} \). Note that on very large scales \( k_1 \sigma_p \ll 1 \), the damping factor.
exp \left[ -k_1^2 \mu_1^2 \sigma_v^2 / 2 \right] \approx 1, and the Kaiser effect \((1 + \beta_1 \mu_1^2)^2\) suffices to describe the RSD effect.

We similarly incorporate the FoG effect for the bispectrum as

\[
B_{\text{FoG}}(k_1, k_2, k_3) = \exp \left[ -(k_1^2 \mu_1^2 + k_2^2 \mu_2^2 + k_3^2 \mu_3^2 \sigma_v^2 / 2) \right] \times B_{2\text{PT}}(k_1, k_2, k_3)
\]

(30)

where we use eq. (25) to calculate \(B_{2\text{PT}}\). Due to the FoG damping term, we do not have closed form analytic expressions for the various multipole moments of \(B_{\text{FoG}}\). Here we have numerically integrated eq. (6) to compute the various multipole moments for which the results are presented in the following section.

3 RESULTS

In this section we present the shape and size dependence of the redshift space bispectrum, and also the corresponding statistical errors for an Euclid like survey. As discussed in Section 2, we have used the largest side \(k_1\) to quantify the size of the triangle, and we have used \(\mu = \cos \theta\) and \(t = k_2 / k_1\) (fig. 1) to quantify the shape. Following Fry (1984), we have defined the dimensionless bispectrum multipoles \((\text{Paper II})\)

\[
Q^m_\ell(k_1, \mu, t) = b_1 \frac{B^m_\ell(k_1, \mu, t)}{3 P(k_1)^2},
\]

(31)

which we have used to exhibit the results presented here.

Galaxy surveys like Euclid expect to cover a broad redshifts range from 0.001 to 2.5 (Ilić et al. 2021; Pozzetti et al. 2016), which can be divided into various redshift bins for the subsequent analysis. For the present work, we use the z bins as used in Yankelevich & Porciani (2019) and we also adopt their predicted values for the bias parameters \(b_1\) and \(\gamma_2\), the survey volume \(V\) and the pairwise velocity dispersion \(\sigma_v\). We have used the Boltzmann code CLASS (Lesgourgues 2011; Blas et al. 2011) to compute the real space matter power spectrum and the growth rate \(f\) for each \(z\) bin.

We have considered the \(z = 0.7\) redshift bin as the fiducial value for which most of our results are shown. Following Yankelevich & Porciani (2019), we have adopted the values \(b_1 = 1.18\), \(\gamma_2 = -0.9\), \(V = 8.97\) Gpc\(^3\) and \(\sigma_v = 0.97\) Mpc for this particular \(z\) bin. Figure 2 shows \(Q^m_\ell\) as functions of \(\mu\) and \(t\) at \(k_1 = 0.2\) Mpc\(^{-1}\) for the fiducial redshift \(z = 0.7\). Here \(k_1 = 0.2\) Mpc\(^{-1}\) is a sufficiently large length-scale where we may expect a combination of 2PT and FoG to provide a reasonably good description of the redshift space bispectrum. The cosmic variance is expected to increase if we consider small values of \(k_1\), whereas non-linear effects increase if we consider larger \(k_1\). Guided by this, we have mainly shown the results for \(k_1 = 0.2\) Mpc\(^{-1}\). We now briefly discuss the shapes of the triangle corresponding to different values of \(\mu\) and \(t\), the reader is referred to Paper I for further details. Considering any panel of fig. 2, the right boundary \(\mu = 1\) corresponds to linear triangles where the three sides are collinear. Here, the bottom right corner \((\mu, t) = (1, 0.5)\) corresponds to stretched triangles where \(k_2 = k_3 = k_1 / 2\), and the top right corner \((1, 1)\) corresponds to squeezed triangles where \(k_1 = k_2, k_3 \rightarrow 0\). The top boundary \(t = 1\) corresponds to L-isosceles triangles where the two larger sides have equal length \((k_1 = k_2)\), and the bottom boundary \(2 \mu t = 1\) corresponds to S-isosceles triangles where the two smaller sides have equal length \((k_2 = k_3)\). The top left corner \((0.5, 1)\) corresponds to equilateral triangles. The diagonal \(\mu = t\) corresponds to right-angled triangles, and the regions \(t > \mu\) and \(t < \mu\) correspond to acute and obtuse triangles respectively.

3.1 Predicted Multipoles Moments

In fig. 2, we have shown \(Q^m_\ell\) for \(\ell = 0, 2\) and 4. As mentioned earlier, only the even \(\ell\) multipoles are expected to be non-zero. Further, in 2PT we expect non-zero multipoles up to \(\ell = 8\) (Paper II), however the value of \(Q^m_\ell\) falls with increasing \(\ell\) and \(m\), and we have shown the main results only up to \(\ell = 4, m = 3\). As shown later in this section, we do not expect to have a statistically significant measurement of the higher multipoles for the survey parameters considered here. Note that \(Q^m_\ell\) and \(Q^m_\ell\) [FoG] respectively denote the results without and with the FoG. In all cases we find that the magnitude of \(Q^m_\ell\) [FoG]s is highly suppressed compared to \(Q^m_\ell\) due to the FoG effect. Considering \(Q^m_0\), we find that the the minima \((0.33)\) occurs for the equilateral triangles and the the maxima \((52.83)\) occurs close to the squeezed limit \((\mu \rightarrow 1, t = 0.865)\). The minimum and maximum values fall to 0.09 and 23.50 for \(Q^m_0\) [FoG]. We notice that the drop in value due to the FoG is more pronounced for equilateral triangles where the minima occurs. However, the overall patterns are visually very similar for \(Q^m_2\) and \(Q^m_2\) [FoG].

The contour pattern of \(Q^2_2\) shows features similar to those of \(Q^0_0\). Its peak value \((86.83)\) is larger than the peak value of \(Q^0_0\) and it occurs at \(\mu = 0.995, t = 0.795\). We observe that the FoG effect causes a larger suppression for the quadrupole than in monopole, and the maximum value of \(Q^2_2\) [FoG] drops to 11.95 which occurs at a slightly different location \((\mu = 0.995, t = 0.735)\). It is interesting to note that in the absence of the FoG damping \(Q^2_2\) is larger than than all the other multipoles including the monopole. However as soon as we apply the FoG effect, the peak value of \(Q^2_2\) [FoG] drops and \(Q^2_2\) [FoG] over takes it to become the multipole with the largest value. The entire contour pattern of \(Q^2_2\) [FoG] is quite different from that of \(Q^2_2\). Unlike \(Q^2_2\) which is positive everywhere, we see that \(Q^2_2\) [FoG] is negative over a region near the squeezed limit. The minimum values of \(Q^2_2\) \((0.23)\) and \(Q^2_2\) [FoG] \((-3.88)\) respectively occurs at the equilateral and squeezed limits, however the magnitude of \(Q^2_2\) [FoG] is still minimum for equilateral triangles.

Considering \(Q^2_2\), we find that this has negative values for most of the acute triangles and positive values for obtuse triangles, with a zero crossing somewhat above the line \(\mu = t\) which corresponds to right-angled triangles. The minima \((-0.43)\) and maxima \((19.24)\) respectively occur at \((\mu = 0.965, t \rightarrow 1)\) and \((\mu \rightarrow 1, t = 0.915)\) which are both very close to the squeezed limit. The FoG effect substantially changes the contour pattern, and we see that \(Q^2_2\) [FoG] is positive valued throughout. The maxima \((9.61)\) now occurs near the squeezed limit while the minima \((0)\) occurs for equilateral triangles.

In case of \(Q^2_2\), we find that the minimum value \((0.08)\) is near the stretched limit and the maximum value \((6.45)\) occurs close to the squeezed limit \((\mu \rightarrow 1, t = 0.975)\). The maximum \((4.84)\) and minimum \((0.04)\) values in case of \(Q^2_2\) [FoG] do not
change position, only the amplitudes drop. Overall, $Q_2^2$ and $Q_2^2[FoG]$ show similar contour patterns.

Considering $Q_0^4$, we see that it shows similar shape dependence as $Q_0^0$ and $Q_2^0$, only its maximum (21.5) and minimum (0.02) values are smaller. Comparing $Q_0^0[FoG]$ with $Q_0^0$, we see that the patterns are visually similar, however the values of $Q_0^0[FoG]$ are all negative and they are roughly an order of magnitude smaller. Comparing $Q_0^4[FoG]$ with $Q_0^4$, we see that the patterns are quite different, and the values of $Q_0^4[FoG]$ are roughly an order of magnitude smaller. We see that $Q_0^4[FoG]$ is negative near the squeezed limit where $|Q_0^4[FoG]|$ has a maxima (1.33), whereas the minima (0.30) occurs for equilateral triangles. In summary we note that the FoG effect has a significant impact on the redshift space bispectrum at the length-scales and redshifts of our interest, and we have included this in our subsequent analysis.

It is interesting to compare the $Q_m^\ell$ values (without FoG) shown in fig. 2 with those shown in Paper II. We note that the bias parameters and redshift $z = 0.7$ used here are different from those used in Paper II. We find that the contour patterns of $Q_0^0$, $Q_2^2$ and $Q_4^4$ look very similar in both fig. 2 and fig. 2

---

Figure 2. This shows $Q_m^\ell$ as functions of $\mu$ and $t$ at $k_1 = 0.2\text{Mpc}^{-1}$ for the fiducial redshift $z = 0.7$. $Q_m^\ell[FoG]$ means that the effect of FoG has been considered in bispectrum according to eq. (30). In other plots, where FoG is not mentioned in the parenthesis, the bispectrum is considered only up to the 2PT level (eq. 25). From the plots it is evident that FoG suppress the bispectrum significantly at $k_1 = 0.2\text{Mpc}^{-1}$.
and Paper II, however their values are $\geq 5$ times larger in Paper II which considers $z = 0$. On the other hand, the odd multipoles are found to be very different. For example, in Paper II the minima and maxima of $Q_2^3$ lie along the $\mu = 1$ and $t = 1$ lines respectively whereas it is exactly the opposite in fig. 2. This is also true for $Q_2^1$ where the contour patterns show completely opposite trends in Paper II and fig. 2. These differences are mainly due to the choice of the non-linear bias parameter $\gamma_2$ which is set to zero in Paper II whereas we have used $\gamma_2 = -0.9$ for fig. 2.

The entire discussion has been restricted to $k_1 = 0.2\text{Mpc}^{-1}$ till now. It is worth noting that the impact of the FoG effect falls exponentially as we move to lower $k$ (large scales), and it causes $\lesssim 5\%$ change in the bispectrum at $k_1 = 0.05\text{Mpc}^{-1}$ for $z = 0.7$. Second order perturbation theory (eq. 25) alone is adequate to model the redshift space bispectrum at $k_1 < 0.05\text{Mpc}^{-1}$, however we do not consider these small $k_1$ values in our work as most of the bispectrum multipoles become undetectable due to cosmic variance (as shown later).

3.2 SNR predictions

We now quantify the prospects of detecting the various bispectrum multipole moments $B_m^\mu$ using the signal-to-noise ratio (SNR) which is defined as

$$\text{[SNR]}_\ell^m = \frac{|B_\mu^m|}{\sqrt{\sigma^m_{\ell\ell m}}}.$$  \hspace{1cm} (32)

A value $\text{[SNR]}_\ell^m \geq 5$ (or possibly $\text{[SNR]}_\ell^m \geq 3$) would be considered as a statistically significant detection of the particular multipole moment. As mentioned earlier, we have considered bins of width ($\Delta \ln k_1 = 0.1, \Delta \mu = 0.05, \Delta t = 0.05$). We note that the signal to noise ratio will increase if we consider bins of larger widths. Considering a situation where our analysis predicts $\text{[SNR]}_\ell^m \approx 1$, it may still be possible to achieve a statistically significant detection by widening the bin width using ($\Delta \ln k_1 = 0.2, \Delta \mu = 0.1, \Delta t = 0.1$) which still retains a significant amount of the information regarding the length-scale and shape dependence of the bispectrum. Further the values of the various parameters used in our predictions are rather uncertain, and it is possible that the actual parameter values could result in a larger $\text{[SNR]}_\ell^m$. Motivated by these factors, we have shown the results for $\text{[SNR]}_\ell^m \geq 1$ where the signal and noise have equal amplitude, and we have also included this in the discussion.

Figure 3 shows $\text{[SNR]}_\ell^m$ for different multipoles with $k_1 = 0.2\text{Mpc}^{-1}$ and $z = 0.7$ fixed. The regions where $\text{[SNR]}_\ell^m < 1$ have been masked out from the plots, and the multipoles where $\text{[SNR]}_\ell^m$ never exceeds unity have not been shown. Broadly, the $\text{[SNR]}_\ell^m$ values decrease with increasing $\ell$, and for a fixed $\ell$ they decrease with increasing $m$. However, we find an exception that the maximum value of $\text{[SNR]}_\ell^2$ exceeds that of $\text{[SNR]}_\ell^0$. For all $\text{[SNR]}_\ell^m$ we find that the maximum value occurs for linear triangles ($\mu = 1$), however the value of $\ell$ corresponding to the maxima varies depending on $\ell$ and $m$. Further, in most cases $\text{[SNR]}_\ell^m$ decreases as the shape of the triangle is changed from linear ($\mu = 1$) to other obtuse triangles ($\mu > 1$) and then acute triangles ($\mu < 1$) and finally the equilateral triangle ($\mu, t = (0.5, 1)$). Considering $B_0^0$, we see that this can be detected at a high level of precision ($\text{[SNR]}_0^0 > 7$) for all triangle configurations, and $\text{[SNR]}_0^0$ is maximum (\approx 140) around $(1, 0.75)$. Considering $B_2^0$, we see

Figure 3. This shows the signal to noise $\text{[SNR]}_\ell^m$ (eq. 32) for different multipoles with $k_1 = 0.2\text{Mpc}^{-1}$ and $z = 0.7$ fixed. The regions where $\text{[SNR]}_\ell^m < 1$ have been masked out from the plots. We do not show the multipoles for which $\text{[SNR]}_\ell^m$ never exceeds unity. The vertical grey lines in the color bars mark the values $\text{[SNR]}_\ell^m = 5$.
that the SNR exceeds unity for most triangle configurations, however \( \text{SNR}^0 \) < 5 over a considerable region where \( t > \mu \). We also find a small region with \( \text{SNR}^0 \) < 1 near the squeezed limit where the values of \( B^0 \) have a zero crossing (fig. 2). The maximum value \( \text{SNR}^2 = 28.6 \) occurs around \((1, 0.6)\) which is near the stretched limit. Considering \( \text{SNR}^1 \), we see that the maximum value \((\approx 33.4)\) occurs around \((1, 0.85)\) and we have \( \text{SNR}^1 > 5 \) in a region surrounding this. As mentioned earlier, the maximum value of \( \text{SNR}^1 \) exceeds that of all the other multipole moments barring \( \text{SNR}^0 \), however the condition \( \text{SNR}^1 > 1 \) is satisfied for a relatively small range of shapes compared to \( \text{SNR}^0 \) and \( \text{SNR}^2 \). The maximum value of \( \text{SNR}^2 \) \((\approx 22.7)\) occurs for squeezed triangles, and we have \( \text{SNR}^2 > 5 \) in a region around this. The condition \( \text{SNR}^2 > 1 \) is satisfied over most of the \((\mu, t)\) space except for a small region near the stretched triangles. \( \text{SNR}^2 \) is very similar to \( \text{SNR}^1 \), except that the values are smaller and the maximum value now is \( \approx 7.1 \). The higher multipoles with \( \ell = 4 \) and \( m = 1, 2, 3 \) do not have \( \text{SNR}^m \) > 5 anywhere, however there is a rather large region where \( \text{SNR}^4 > 1 \). The maxima of \( \text{SNR}^4 \) \((\approx 3.6)\) occurs at the squeezed limit, whereas for \((m = 1, 3)\) the maxima \((2.9, 1.8)\) occur very close to the squeezed limit. The two latter multipoles satisfy \( \text{SNR}^4 > 1 \) in only a small region around the maxima.

We now consider the \( k_1 \) dependence of \( \text{SNR}^0 \) with \( z = 0.7 \) fixed, as shown in fig. 4. The smallest value of \( k_1 \) roughly corresponds to the linear extent of the survey. The results are shown for only three different triangle shapes namely squeezed, stretched and equilateral. However, we can combine these with fig. 3 to qualitatively infer the \( k_1 \) dependence expected for other triangle configurations. The number of triangles in any bin scales as \( N_{1r} \propto k_1^3 \) (eq. 22), and we expect the cosmic variance to scale as \( k_1^{-3} \) (eq. 24) as \( k_1 \) is increased. Based on this we may expect \( \text{SNR}^0 \) to increase monotonically with increasing \( k_1 \). This is broadly true for all the triangles in fig. 4, except for the sharp dips which are seen to occur for a few of the multipoles at certain values of \( k_1 \). As seen in fig. 2, some of the multipoles have both positive and negative values, and the dips in fig. 4 correspond to the zero crossings which are also reflected in \( \text{SNR}^0 \). We now discuss the prospects of detecting the various multipoles at different values of \( k_1 \). Here we use \( k_1 [5] \) to denote the smallest value of \( k_1 \) where the condition \( \text{SNR}^m > 5 \) is satisfied, and generally we expect the \( B^m \) to be detectable for all \( k_1 \geq k_1 [5] \). Considering \( B^m \) we see that \( k_1 [5] = 0.05 \text{Mpc}^{-1} \) for the squeezed and stretched triangles, and \( k_1 [5] = 0.1 \text{Mpc}^{-1} \) for equilateral triangles. Overall, we expect to detect \( B^0 \) for all linear triangles and some obtuse triangles at \( k_1 \geq 0.05 \text{Mpc}^{-1} \), and for all triangle at \( k_1 \geq 0.1 \text{Mpc}^{-1} \). Note that \( B^0 \) is predicted to be negative at large \( k_1 \) for equilateral triangles. We find a small \( k_1 \) range around \( k_1 \approx 0.4 \text{Mpc}^{-1} \) where we do not expect to detect \( B^0 \) for equilateral and possibly some of the acute triangles. Considering \( B^2 \), we have \( k_1 [5] = 0.05 \text{Mpc}^{-1} \) and \( 0.1 \text{Mpc}^{-1} \) for stretched and squeezed triangles respectively. The SNR falls off towards equilateral triangles where we have \( \text{SNR} > 1 \) for \( k_1 \geq 0.1 \text{Mpc}^{-1} \) and \( k_1 [5] = 0.6 \text{Mpc}^{-1} \). Overall we expect \( B^2 \) to be detectable in the vicinity of the stretched limit for \( k_1 \geq 0.05 \text{Mpc}^{-1} \), and for several other linear and obtuse triangles for \( k_1 \geq 0.1 \text{Mpc}^{-1} \). Considering acute and equilateral triangles, it will be possible to detect \( B^2 \) at \( k_1 \geq 0.1 \text{Mpc}^{-1} \) if we increase the bin widths. For all shapes, \( B^2 \) is predicted to be negative at large \( k_1 \). The transition from positive to negative values occurs at \( k_1 \approx 0.1 \text{Mpc}^{-1} \) for squeezed triangles, and then spreads to other shapes (stretched and then equilateral) as \( k_1 \) is increased. As seen in fig. 3, it will not be possible to detect \( B^2 \) in a few \( \mu - t \) bins where the zero crossing occurs. The exact \( \mu - t \) locations of these bins will shift with \( k_1 \). Considering \( B^2 \) and \( B^2 \) together, we see that \( k_1 [5] \approx 0.1 \text{Mpc}^{-1} \) for squeezed triangles, and we expect to detect these multipoles for several linear and obtuse triangles for \( k_1 \geq 0.1 \text{Mpc}^{-1} \). Further, in all three cases \( (\mu, t) \) \( B^m \) shown here \((\ell = 4, m = 0, 1, 2, 3) \) all have \( k_1 [5] \) in the range \( 0.2 - 0.3 \text{Mpc}^{-1} \) for squeezed triangles, and we expect to detect these multipoles for several linear and obtuse triangles near the squeezed limit. We note that \( B^2 \) also has \( k_1 [5] \) in the range \( 0.2 - 0.3 \text{Mpc}^{-1} \) for stretched triangles, and the region of \( \mu - t \) space where this will be detected is relative large compared to the other \( \ell = 4 \) multipoles. We also note that \( B^2 \) and \( B^2 \) both satisfy \( \text{SNR} > 1 \) for the three triangle shapes shown in fig. 4 for \( k_1 \geq 0.3 \text{Mpc}^{-1} \), and we expect to detect these over the entire \( \mu - t \) space if we increase the bin widths.

Figure 5 shows how \( \text{SNR}^m \) varies with redshift with \( k_1 = 0.2 \text{Mpc}^{-1} \) fixed. Note that the bias parameters \( b_1, \gamma_2 \), as well as the survey volume \( V \) change with changing redshift, and we have used the redshift dependent values from Table 1 of (Yankelevich & Porciani 2019). We see that the survey volume \( V \) increases with \( z \), and we expect the cosmic variance at a fixed \( k_1 \) to decrease with increasing \( z \). As a result of this, \( \text{SNR}^m \) is expected to increase monotonically with redshift. However, there are two more effects that determine \( \text{SNR}^m \) at each \( z \): (i) the amplitude of the bispectrum decreases with increasing \( z \), and (ii) the FoG suppression also decreases with increasing \( z \). The bias parameters also affect the \( z \) dependence of \( \text{SNR}^m \). We see that for many of the multipoles \( \text{SNR}^m \) increase slightly with increasing \( z \) whereas there are some where the opposite occurs, and for a few cases we also have dips in \( \text{SNR}^m \) at some values of \( z \). As discussed earlier, these dips correspond to the zero crossings of \( B^m \). We now discuss the prospects of detecting the different multipoles at the various redshifts shown here. Considering \( B^0 \), we see that this is detectable with \( \text{SNR}^0 > 5 \) at all the redshifts for all the three triangles considered. \( B^0 \) is detectable with \( \text{SNR}^0 > 5 \) at all redshifts for stretched triangles. For squeezed triangles, the above is true for \( z < 1.5 \), whereas for equilateral triangles \( 1 < \text{SNR}^0 < 5 \) at all redshifts. Considering \( B^2 \) and \( B^2 \), we see that they will be detected with \( \text{SNR}^m > 5 \) at all the redshifts only for squeezed triangles. For the stretched and equilateral triangles, \( \text{SNR}^2 \) and \( \text{SNR}^2 \) values are below 5 at all redshifts. All the \( \ell > 2 \) multipoles cannot be observed with \( \text{SNR}^m > 5 \) at any redshift and for the triangle shapes considered. Note that, the above discussion is true only for for three specific triangle shapes at \( k_1 = 0.2 \text{Mpc}^{-1} \). However, as we have discussed earlier, it is possible to detect the higher order multipoles with \( \text{SNR}^m > 5 \) by considering triangles of other shapes or somewhat larger \( k_1 \) values or larger bin widths.

3.3 Correlation between different multipoles

It is important to note that the estimated values of the different multipole moments are expected to be correlated. We
have quantified the correlations in the measurement errors between any two multipoles using the rank correlation

$$R_{m'm'}^{\mu\ell\ell'} = \frac{C_{m'm'}^{\mu\ell\ell'}}{\sqrt{C_{m'm'}^{\mu\ell\ell'} C_{m'm'}^{\ell\ell'}}}. \quad (33)$$

with $-1 \leq R_{m'm'}^{\mu\ell\ell'} \leq 1$. Here positive and negative values indicate that the errors are correlated and anti-correlated respectively. We show the rank correlations between different multipoles in fig. 6. We find that for most multipole pairs, the errors are only weakly correlated ($|R_{m'm'}^{\mu\ell\ell'}| < 0.1$) across much of the $\mu - t$ plane. However, there are a few exceptions which we highlight below. We first consider $R_{00}^{00}$ which refers to $B_0^0$ and $B_2^0$ which have the highest SNR. We see that the measurement errors in these two multipoles are correlated for obtuse triangles and anti-correlated for acute triangles. $R_{00}^{00}$ shows the highest correlation (0.42) in the stretched limit, and the highest anti-correlation ($-0.1$) in the squeezed limit. Considering $R_{02}^{02}$, we see that the measurement errors of the pair $(B_0^0, B_2^2)$ are correlated with values $\sim 0.25$ in the vicinity of the squeezed limit. Considering $R_{22}^{02}$, we see that this show maximum anti-correlation ($-0.18$) in the squeezed limit. $R_{24}^{02}$ shows the highest correlation (0.33) and anti-correlation ($-0.1$) in the stretched and squeezed limits respectively. We have $|R_{m'm'}^{\mu\ell\ell'}| < 0.1$ across the entire $\mu - t$ plane for all the other multipoles not shown here. Overall, the errors in the various multipoles are weakly correlated for most triangle shapes barring a few in the vicinity of squeezed and stretched triangles.

Figure 7 shows the redshift dependence (from $z = 0.7$ to 2) of the four rank correlations shown in fig. 6 for three specific triangle shapes with $k_1 = 0.2\text{ Mpc}^{-1}$ fixed. Considering
triangles. For squeezed (stretched) triangles $R_{02}^{02} \sim 0.25$ (0.0) for $k_1 \leq 0.4 \text{Mpc}^{-1}$ beyond which both sharply declines to $\sim -0.25$ at $k_1 = 1 \text{Mpc}^{-1}$. In contrast, $R_{02}^{02}$ shows exactly the opposite $k_1$ dependence in that we have anti-correlations for small $k_1$ and correlations at large $k_1$, with a zero crossing whose $k_1$ value depends on the shape of the triangle. Considering squeezed, stretched and equilateral triangles we respectively have $R_{02}^{02} \sim -0.1, 0.0$ and $0.2$ at small $k_1$ ($\leq 0.1 \text{Mpc}^{-1}$), whereas these values are $\sim 0.25, 0.25$ and 1.0 for large $k_1$ ($\leq 0.1 \text{Mpc}^{-1}$).

4 DISCUSSION AND CONCLUSION

The redshift space anisotropy of the bispectrum is generally quantized using multipole moments as defined in eq. (6). The possibility of measuring these multipoles in any survey depends on the level of statistical fluctuations. In this paper, we developed a formalism to compute the statistical fluctuations in the measurement of bispectrum multipoles for galaxy surveys. We quantify the fluctuations through the covariance as defined in eq. (24) which assumes the flat sky approximation. We consider the specifications of a Euclid like galaxy survey to present our results. We mainly consider two quantities: (i) the signal-to-noise ratio(SNR) $[\text{SNR}]^m$ which quantifies the detectability of a multipole $B^m_i$ (eq. 32), and (ii) the rank correlation which quantifies the correlation in measurement errors between any two multipoles (eq. 33). We show, how these quantities depend on the triangle configurations in $k$ space, as well as their evolution in redshift. We also show how our results change as we introduce the FoG effect.

We find that the FoG effect plays a crucial role at length scales $k > 0.05 \text{Mpc}^{-1}$ for $z = 0.7$. This suppresses the values of bispectrum multipoles and the suppression is in general stronger for the higher order multipoles. The amplitude of the monopole $B^m_0$ predicted by 2PT is reduced by half when FoG effect is introduced. For $B^m_2$, the amplitude is suppressed by almost an order of magnitude in presence of FoG effect. This FoG suppression, however, is not very important at small $k$ or at high $z$. We see that in general $[\text{SNR}]^m$ values decrease with increasing $\ell$, and for a fixed $\ell$ they decrease with increasing $m$. Considering all $[\text{SNR}]^m$, we find that the maximum value occurs for linear triangles ($\mu = 1$), however the value of $t$ corresponding to the maxima varies depending on $\ell$ and $m$. Also, in most of the cases $[\text{SNR}]^m_\ell$ decreases as the shape of the triangle is changed from linear ($\mu = 1$) to other obtuse triangles ($\mu > t$) and then acute triangles ($\mu < t$) and finally the equilateral triangle ($\mu, t = (0.5, 1)$). We note that, at large, $[\text{SNR}]^m_\ell$ increases with $k_1$ and $z$.

Considering individual multipoles, we expect to detect $B^m_0$ for all the triangles at $k_1 > 0.1 \text{Mpc}^{-1}$ across the redshifts considered here. On the other hand, we expect $B^m_2$ to be detectable in the vicinity of the stretched limit for $k_1 > 0.5 \text{Mpc}^{-1}$ at all the redshifts, and for several other linear and obtuse triangles for $k_1 \geq 0.1$ up to $z = 1.5$. Detection of $B^m_3$ for acute and equilateral triangles, as well as for obtuse triangles at $z > 1.5$, is possible if we increase the bin size. Detection of $B^m_3$ at all redshifts is possible only for squeezed triangles at $k_1 \geq 0.1 \text{Mpc}^{-1}$. For other triangle shapes, detection is possible at higher $k_1$. Considering $B^m_4$ multipoles, we see that the detection is possible across the
redshift range only for a few linear and obtuse triangles near the squeezed limit at $k_1 > 0.2 \text{Mpc}^{-1}$. Note that, for all the multipoles, the possibility of detection, or the $\text{[SNR]}^m$ values, can be increased by increasing the bin size in $k_1, \mu$, or $t$.

Considering $\mathcal{R}_{\ell\ell'}^{m_1}$, we find that for most multipole pairs the errors are only weakly correlated (with $|\mathcal{R}_{\ell\ell'}^{m_1}| < 0.1$) across much of the $\mu - t$ plane barring a few in the vicinity of squeezed and stretched triangles. For a fixed triangle shape, the $\mathcal{R}_{\ell\ell'}^{m_1}$ values are practically independent of $k_1$ for $k_1 \lesssim 0.05 \text{Mpc}^{-1}$ and show a strong $k_1$ dependence for $k_1 > 0.1 \text{Mpc}^{-1}$ where FoG effect is really important. For a fixed triangle shape and $k_1$, $\mathcal{R}_{\ell\ell'}^{m_1}$ evolve moderately with redshift.

We, therefore, conclude that the future surveys like Euclid can potentially measure the higher order redshift space bispectrum multipoles (up to $\ell = 4$), beyond the isotropic component (monopole), across various triangle shapes and sizes. The signal-to-noise or significance of these measurements, however, depend on the scales and redshifts of observation. The signal-to-noise also determines the information content of the individual multipoles. Significant measurements of $\ell \leq 2$ multipoles are possible even at $k_1 \lesssim 0.1 \text{Mpc}^{-1}$ across the $z$ range $0.7 - 2$. These scales are particularly important as the FoG suppression is minimum here. Here, we expect to measure the $\ell \leq 2$ multipoles with highest signal-to-noise for linear and obtuse triangles. For $\ell > 2$ multipoles, we require to go to large $k_1$ for significant detection or we need large bin size to increase the signal-to-noise. Due to the weak correlation (at $k_1 < 0.2 \text{Mpc}^{-1}$) between the errors of multipole pairs for most of the triangle shapes, it is possible to combine different multipole to increase the information content. This becomes particularly important when we try to
extract cosmological parameters from bispectrum measurements. Following our analysis, we expect to reduce the errors on the cosmological parameters when we combine the higher multipoles along with the monopole results (Gil-Marín et al. 2015, 2017; Philcox & Ivanov 2022). This we plan to study in future. Finally, we reiterate that in Euclid like surveys, we expect to measure bispectrum multipoles up to $\ell = 4$ by suitably choosing the scale and redshifts of observation.

DATA AVAILABILITY

The data generated during this work will be made available upon reasonable request to the authors.

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