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THE FIELD STRUCTURE OF VACUUM, MAXWELL EQUATIONS AND
RELATIVITY THEORY ASPECTS. Part 1

The authors dedicate this article to one of the mathematical and physical giants of the XX-th century -
academician Prof. Nikolai N. Bogolubov in memory of his 100th Birthday with great appreciation to his brilliant
talent and impressive impact to modern nonlinear mathematics and quantum physics

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Abstract

The vacuum structure and its modeling by means of field theoretic tools is analyzed. The Maxwell equations from the first principles are derived, the Lorentz force transformations with respect to non-inertial reference frames is discussed, some new interpretations of the relativity theory are presented.
1. **Introduction**

The nature of space-time and surrounding matter objects was and persists to be one of the most intriguing and challenging problems facing the mankind and natural scientists especially. As we know one of the most brilliant inventions in physics of XIX-th century was combining of electricity and magnetism within the Faraday-Maxwell electromagnetism theory. This theory explained the main physical laws of light propagation in space-time and posed new questions concerning the nature of vacuum. Nonetheless, almost all attempts aiming to unveil the real state of art of the vacuum problem appeared to be unsuccessful in spite of new ideas suggested by Mach, Lorentz, Poincaré, Einstein and some others physicists. Moreover, the non-usual way of treating the space-time devised by Einstein, in reality, favored to eclipsing both its nature and the related physical vacuum origin problems [1, 4, 5, 12, 17, 18], reducing them to some physically unmotivated formal mathematical principles and recipes, combined in the well known special relativity theory (SRT). The SRT appeared to be adapted to the only inertial reference systems and faced with hard problems of the electromagnetic Lorentz forces explanation and relationships between inertial and gravity forces. The latter was artificially "dissolved" by means of the well known equivalence principle owing to which the "inertial" mass of a material object was postulated to coincide with its "gravity" mass.

Simultaneously, the vacuum origin as a problem almost completely disappeared from the Einstein theory being replaced by the geometrization of the space-time nature and all related physical phenomena. Meanwhile, the impressive success of XX-th century quantum physics, especially of quantum electrodynamics, have demonstrated clearly enough [2, 3, 17] that the vacuum polarization and electron-positron annihilation phenomena make it possible to pose new questions about the space-time and vacuum structures, and further to revisit [3, 6, 8, 9] the existing points of view on them.

Below we try to unveil some nontrivial aspects of the real space-time and vacuum origin problems to derive from the natural field theory principles all of the well known Maxwell electromagnetism and relativity theories results, to show their relative or only visible coincidence with real physical phenomena and to feature new perspectives facing the modern fundamental physics.

2. **The Maxwell electromagnetism theory: new look and interpretation**

We start from the following field theoretical model of the vacuum considered as some physical reality imbedded into the standard three-dimensional Euclidean space reference system marked with three spatial coordinates \( r \in \mathbb{R}^3 \), endowed with the standard scalar product \( \langle \cdot, \cdot \rangle \), and parameterized by means of the scalar temporal parameter \( t \in \mathbb{R} \). The physical vacuum we will endow with a smooth enough four-vector potential function \((W, A) : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R} \), and material objects, imbedded into the vacuum, we will model (classically here) by means of the scalar density function \( \rho : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R} \) and the vector current density \( J : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}^3 \), being also smooth enough functions.

(i) The first field theory principle about the vacuum we accept sounds as follows: the four-vector function \((W, A) : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R} \) satisfies the standard continuity relationship

\[
\frac{1}{c} \frac{\partial W}{\partial t} + \langle \nabla, A \rangle > 0, \tag{2.1}
\]
where, by definition, $\nabla := \partial/\partial r$ is the usual gradient operator.

(ii) The second field theory principle we accept is a dynamical relationship on the scalar potential component $W : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$:

\[
\frac{1}{c^2} \frac{\partial^2 W}{\partial t^2} - \nabla^2 W = \rho,
\]

meaning the linear law of the small vacuum uniform and isotropic perturbations propagation in the space-time, understood here, evidently, as a first (linear) approximation in the case of weak enough fields.

(iii) The third principle is similar to the first one and means simply the natural continuity condition for the density and current density functions:

\[
\frac{\partial \rho}{\partial t} + < \nabla, J > = 0.
\]

We need to note here that the vacuum field perturbations velocity parameter $c > 0$, used above, coincides with the vacuum light velocity as we are trying to derive successfully from these first principles the well known Maxwell electromagnetism field equations, to analyze the related Lorentz forces and special relativity relationships. To do this, we first combine equations (2.1) and (2.2):

\[
\frac{1}{c^2} \frac{\partial^2 W}{\partial t^2} = -< \nabla, \frac{1}{c} \frac{\partial A}{\partial t} > = < \nabla, \nabla W > + \rho,
\]

whence

\[
< \nabla, -\frac{1}{c} \frac{\partial A}{\partial t} - \nabla W > = \rho.
\]

Having put, by definition,

\[
E := -\frac{1}{c} \frac{\partial A}{\partial t} - \nabla W,
\]

we obtain the first material Maxwell equation

\[
< \nabla, E > = \rho
\]

for the electric field $E : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3$. Having now applied the rotor-operation $\nabla \times$ to expression (2.5) we obtain the first Maxwell field equation

\[
\frac{1}{c} \frac{\partial B}{\partial t} - \nabla \times E = 0
\]

on the magnetic field vector function $B : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3$, defined as and

\[
B := \nabla \times A.
\]

To derive the second Maxwell field equation we will make use of (2.8), (2.1) and (2.5):

\[
\nabla \times B = \nabla \times (\nabla \times A) = \nabla < \nabla, A > - \nabla^2 A =
\]

\[
= \nabla (-\frac{1}{c} \frac{\partial W}{\partial t}) - \nabla^2 A = \frac{1}{c} \frac{\partial}{\partial t} (-\nabla W - \frac{1}{c} \frac{\partial A}{\partial t} + \frac{1}{c} \frac{\partial A}{\partial t}) - \nabla^2 A =
\]

\[
= \frac{1}{c} \frac{\partial E}{\partial t} + \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} - \nabla^2 A).
\]

We have from (2.4), (2.6) and (2.9) that

\[
< \nabla, \frac{1}{c} \frac{\partial E}{\partial t} > = \frac{1}{c} \frac{\partial \rho}{\partial t} = -\frac{1}{c} < \nabla, J >,
\]
or
\[
(2.10) \quad < \nabla, -\frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} - \nabla \left( \frac{1}{c} \frac{\partial W}{\partial t} \right) + \frac{1}{c} J > = 0.
\]
Making now use of (2.1), from (2.10) we obtain that
\[
(2.11) \quad < \nabla, -\frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} - \nabla \left( \frac{1}{c} \frac{\partial W}{\partial t} \right) + \nabla^2 A + \nabla \times (\nabla \times A) + \frac{1}{c} J > = 0.
\]
Thereby, equation (2.11) yields
\[
(2.12) \quad \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} - \nabla^2 A = \frac{1}{c} (J + \nabla \times S)
\]
or for some smooth vector function \( S : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3 \). Here we need to note that continuity equation (2.3) is defined, concerning the current density vector \( J : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3 \), up to a vorticity expression, that is \( J \simeq J + \nabla \times S \) and equation (2.12) finally can be written down as
\[
(2.13) \quad \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} - \nabla^2 A = \frac{1}{c} J.
\]
Having substituted now (2.13) into (2.9) we obtain the second Maxwell field equation
\[
(2.14) \quad \nabla \times B - \frac{1}{c} \frac{\partial E}{\partial t} = \frac{1}{c} J.
\]
In addition, from (2.8) one also finds the no-magnetic charge relationship
\[
(2.15) \quad < \nabla, B > = 0.
\]
Thus, we have derived all the Maxwell electromagnetic field equations from our three main principles (2.1), (2.2) and (2.3). The success of our undertaking will be more impressive if we adapt our results to those following from the well known relativity theory in the case of point charges or masses. Below we will try to demonstrate the corresponding derivations based on some completely new physical conceptions of the vacuum medium first discussed in [7].

Remark 2.1. It is interesting to analyze a partial case of the first field theory vacuum principle (2.1) when the following conservation law for the scalar potential field function \( W : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R} \) holds:
\[
(2.16) \quad \frac{d}{dt} \int_{\Omega_t} W d^3 r = 0,
\]
where \( \Omega_t \subset \mathbb{R}^3 \) is ny open domain in space \( \mathbb{R}^3 \) with the smooth boundary \( \partial \Omega_t \) for all \( t \in \mathbb{R} \) and \( d^3 r \) is the standard volume measure in \( \mathbb{R}^3 \) in a vicinity of the point \( r \in \Omega_t \).

Having calculated expression (2.16) we obtain the following equivalent continuity equation
\[
(2.17) \quad \frac{1}{c} \frac{\partial W}{\partial t} + < \nabla, \frac{v}{c} W > = 0,
\]
where \( \nabla := \nabla_r \) and \( v := dr/dt \) is the velocity vector of a vacuum medium perturbation at point \( r \in \mathbb{R}^3 \) carrying the field potential quantity \( W \). Comparing now equations (2.1), (2.17) and using equation (2.3) we can make the suitable identifications:
\[
(2.18) \quad A = \frac{v}{c} W, \quad J = \rho v,
\]
well known from the classical superconductivity theory \[4\]. Thus, we face with a new physical interpretation of the conservative electromagnetic field theory when the vector potential \(A : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}^3\) completely determines via expression \((2.18)\) by the scalar field potential function \(W : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}\). It is also evident that all of the Maxwell electromagnetism filed equations derived above hold too in the case \((2.18)\), as it was first demonstrated in \[7\] (but with some mathematical inaccuracies).

Consider now the conservation equation \((2.16)\) jointly with the related integral momentum conservation relationship

\[
\frac{d}{dt} \int_{\Omega} (\frac{vW}{c^2}) d^3 r = 0, \quad \Omega|_{t=0} = \Omega_0,
\]

where as above \(\Omega_t \subset \mathbb{R}^3\) is for any time \(t \in \mathbb{R}\) an open domain with the smooth boundary \(\partial \Omega_t\), whose evolution is governed by the equation

\[
\frac{dr}{dt} = v(r, t)
\]

for all \(x \in \Omega_t\) and \(t \in \mathbb{R}\), as well as by the initial state of the boundary \(\partial \Omega_0\). As a result of relationship \((2.19)\) one obtains the new continuity equation

\[
\frac{d(vW)}{dt} + vW < \nabla, v >= 0.
\]

Making now use of \((2.17)\) in the equivalent form

\[
\frac{dW}{dt} + W < \nabla, v >= 0,
\]

we obtain finally a very interesting local conservation relationship

\[
\frac{dv}{dt} = 0
\]

on the vacuum matter velocity \(v = \frac{dr}{dt}\), holding for all values of the time parameter \(t \in \mathbb{R}\). As it is easy to observe, the obtained relationship completely coincides with the well known hydrodynamic equation \[10\] of ideal compressible liquid without any external exertion, that is any external forces and field "pressure" are equal identically to zero. We received a natural enough result that the propagation velocity of the vacuum field matter is constant and equals exactly \(v = c\), that is the light velocity in the vacuum, if to recall the starting wave equation \((2.2)\) owing to which the small vacuum field matter perturbations propagate in the space with the light velocity.

3. Special relativity and dynamical field equations

From classical electrodynamics we know that the main dynamical relationship relates the particle mass acceleration with the Lorentz force which strongly depends on the absolute charge velocity. For the electrodynamics to be independent on the reference system physicists were forced to reject the Galilean transformations and replace them with the artificial Lorentz transformations. This resulted later in the Einstein relativity theory which has partly reconciled the problems concerned with deriving true dynamical equations for a charged point particle.

We will now start from the scalar field vacuum medium function \(W : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}\) in the conservation condition case \((2.16)\) discussed above. This means, obviously, that the vacuum medium field vector
potential \( A : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}^3 \), charge and current densities \((\rho, J) : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}^3 \times \mathbb{R}\) are related owing to expressions \(2.18\).

Consider now jointly vacuum field medium conservation equations \(2.17\) and \(2.2\) at the density \(\rho = 0\):
\[
- \frac{1}{c^2} \frac{\partial^2 W}{\partial t^2} = \frac{1}{c^2} \frac{\partial}{\partial t} \left( \frac{\partial W}{\partial t} \right) = \frac{1}{c^2} \frac{\partial}{\partial t} \left( < \nabla, v W > \right) = < \nabla, \frac{\partial}{c^2} W > = - < \nabla, \nabla W >.
\]
(3.1)

From relationship \(3.1\) one follows that
\[
\frac{\partial}{\partial t} \left( \frac{W_v}{c^2} \right) + \nabla W = \nabla \times F,
\]
where \(F : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}^3\) is some smooth function, which we put, by definition, to be zero owing to the \textit{a priori} assumed vortexless vacuum medium dynamics. So, our dynamical equation on the vacuum medium scalar field function \(W : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}\) looks like
\[
\frac{\partial}{\partial t} \left( \frac{W_v}{c^2} \right) + \nabla W = 0.
\]
(3.3)

Consider now a charged point particle \(q\) in a space point \(r = R(t) := R_0 + \int_0^t u dt \in \mathbb{R}^3\), depending on time parameter \(t \in \mathbb{R}\) and position \(R_0 \in \mathbb{R}^3\) at initial point \(t = 0\). Since the vacuum medium field is described by means of the potential field function \(W : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}\), which is naturally disturbed by the charged particle \(q\), we will model this fact approximately as the following resulting functional relationship:
\[
W(r, t) = \tilde{W}(r, R(t))
\]
for some scalar function \(\tilde{W} : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}\). This function must satisfy equation \(2.17\), that is
\[
< \frac{\partial \tilde{W}}{\partial R}, u > + < \nabla, \tilde{W} v > = 0.
\]
(3.5)

As we are interested in the function \(\tilde{W} : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}\) as \(r \rightarrow R(t) \in \mathbb{R}^3\), where the charged point particle is located, we obtain from \(3.5\) that
\[
< \frac{\partial \tilde{W}}{\partial R}, u > |_{r \rightarrow R(t)} + \tilde{W} < \nabla, v > |_{r \rightarrow R(t)} = 0,
\]
giving rise to the relationship
\[
\frac{\partial \tilde{W}}{\partial R} = - \frac{\partial \tilde{W}}{\partial r}
\]
(3.6)
as \(r \rightarrow R(t)\), since \(v(r \rightarrow R(t)) \rightarrow dR(t)/dt := u(t)\) and \(< \nabla, v > |_{r \rightarrow R(t)} \rightarrow < \nabla, u(t) > |_{r \rightarrow R(t)} = 0\) for all \(t \in \mathbb{R}\).

Returning now to equation \(3.3\) we can write down, owing to \(3.6\), that
\[
\frac{1}{c^2} \left( \frac{\partial W_v}{\partial R} + \tilde{W} \frac{\partial v}{\partial t} \right) |_{r \rightarrow R(t)} = \frac{1}{c^2} \left( - < \frac{\partial \tilde{W}}{\partial R}, v > + \tilde{W} \frac{\partial v}{\partial t} \right) |_{r \rightarrow R(t)} = \frac{1}{c^2} \left( < \frac{\partial \tilde{W}}{\partial R}, u > + \tilde{W} \frac{\partial u}{\partial t} \right) |_{r \rightarrow R(t)} = \frac{1}{c^2} \frac{d}{dt} (\tilde{W} u) = - \frac{\partial \tilde{W}}{\partial R} |_{r \rightarrow R(t)} = \frac{\partial \tilde{W}}{\partial t},
\]
(3.7)

where we put, by definition, \(\tilde{W} := \tilde{W}(r, R(t)) |_{r \rightarrow R_0}\). Thus, we obtained from \(3.7\) that the function \(\tilde{W} : \mathbb{R}^3 \rightarrow \mathbb{R}\) satisfies the determining dynamical equation
\[
\frac{d}{dt} \left( - \frac{\tilde{W}}{c^2} u \right) = - \frac{\partial \tilde{W}}{\partial R}
\]
(3.8)
at point \(R(t) \in \mathbb{R}^3\) of the point charge \(q\) location.
Now we need to proceed with our calculations ahead and would like to interpret the quantity \(-\frac{\bar{W}}{c^2}\) as the real "dynamical" mass of our point charge \(q\) at point \(R(t) \in \mathbb{R}^3\), that is

\[
m := -\frac{\bar{W}}{c^2}.
\]

Then, using (3.9) we can rewrite equation (3.8) as

\[
\frac{dp}{dt} = -\frac{\partial \bar{W}}{\partial R},
\]

where the quantity \(p := mu\) has the natural momentum interpretation.

The obtained equation (3.10) is very interesting from the dynamical point of view. Really, from equation (3.10) we obtain that

\[
\langle u, \frac{d}{dt}(mu) \rangle = c^2 \langle \frac{\partial m}{\partial R}, u \rangle = c^2 \frac{dm}{dt}.
\]

As a result of (3.11) we easily derive, following [7], the conservative relationship

\[
\frac{d}{dt} \left( m \sqrt{1 - \frac{u^2}{c^2}} \right) = 0
\]

for all \(t \in \mathbb{R}\). Thereby, the quantity

\[
m \sqrt{1 - \frac{u^2}{c^2}} = m_0
\]

is constant for all \(t \in \mathbb{R}\), giving rise to the well known relativistic expression for the mass of a point particle:

\[
m = \frac{m_0}{\sqrt{1 - \frac{u^2}{c^2}}},
\]

As we can see, the point particle mass \(m\) depends, in reality, not on the coordinate \(R(t) \in \mathbb{R}^3\) of the point particle \(q\), but on its velocity \(u := dR(t)/dt\). Since the field potential \(\bar{W} : \mathbb{R}^3 \to \mathbb{R}\) consists of two parts

\[
\bar{W} = \bar{W}_0 + \Delta \bar{w},
\]

where \(\bar{W}_0 : \mathbb{R}^3 \to \mathbb{R}\) is constant and responsible for the external influence of all long distant objects in the Universe upon the point particle \(q\) and \(\Delta \bar{w} : \mathbb{R}^3 \to \mathbb{R}\) is responsible for the local field potential perturbation by the point charge \(q\) and its closest ambient neighborhood. Then, obviously,

\[
\Delta m := m - m_0 = -\Delta \bar{w}/c^2
\]

is the strictly dynamical mass component belonging to the point particle \(q\). Moreover, since the full momentum \(p = mu\) satisfies equation (3.10), one can easily obtain that the quantity

\[
\bar{W}^2 - p^2 c^2 = E_0^2
\]

is not depending on time \(t \in \mathbb{R}\), that is \(dE_0/dt = 0\), where \(E_0 := m_0c^2\). The result (3.17) demonstrates us the important property of the energy essence: the point particle \(q\) is, in reality, endowed with the only dynamical energy \(\Delta E := \Delta mc^2\). Concerning the so called "internal" particle energy \(E_0 = m_0c^2\) we see that it has nothing to do with the real particle energy, since its origin is determined completely owing to the long distant objects of the Universe and could not be used for any physical processes, contrary to the known Einstein theory statements about a "huge" internal energy stored inside the particle mass.

Equivalently, the Einstein theory statement about the equivalence of the mass and the "internal energy of particle appears to be senseless, since the main part of the field potential function \(\bar{W} : \mathbb{R}^3 \to \mathbb{R}\) at the
location point of the particle $q$ is constant and owes to the long distant objects in the Universe, which obviously can not be used for so called "practical applications".

Nonetheless, we have observed above, as a by-product, the well known relativistic effect of the particle mass growth depending on the particle velocity in the form (3.14). As it was already mentioned in [7] this "mass growth" is, in reality, completely of dynamical nature and is not a consequence of the Lorentz transformations, as it was stated within the Einstein SRT. Moreover, we can state that all of so called relativistic effects have also nothing to do with both the mentioned above Lorentz transformations and with such artificial effects as length shortening and time slowing”. There is also no reasonable cause to identify the particle mass with its real energy and vice versa. Concerning the interesting physical effect called particles annihilation”we need here to stress that it has also nothing to do with the transformation of particles masses into energy. The field theoretical explanation of this phenomenon consists in creating their very special bonding state, whose interaction with ambient objects is vanishing. As a result the visible inertial or dynamical mass of this bound state is also zero, what the experiment shows, and nothing else. Inversely, if an intensive enough gamma-quant meets such a bound state of two particles, it can break them back into two separate particles, what the experiment also shows to happen. Here we can recall a similar analogy borrowed from the modern quantum physics of infinite particle systems described by means of the second quantization scenario [10, 18] suggested in 1932 by V. Fock. Within this scenario there also realize creating-annihilation effects which are present owing to the inter-particle interaction forces. Moreover, as we know from the modern superfluidity and superconductivity theories within this picture one can describe special bound states of particles, so called Cooper pairs”, whose interaction to each other completely vanishes and whose combined mass strongly differs from the sum of the suitable components and equals the so called effective compound mass, depending strongly on the potential field intensity inside the superfluid or superconductor matter.

4. RELATIVITY PRINCIPLES REVISITING

It is a well known fact that the Einstein special relativity theory is applicable only for physical processes related to each other by means of the inertial reference systems, moving with constant velocities. In this case one can make use of the Lorentz transformations and calculate the components of suitable four-vectors and the resulting mass growth of particles owing to formula (3.11). A nontrivial problem arises when we wish to analyze these quantities with respect to non-inertial reference systems moving with some nonzero acceleration. Below we will revisit this problem from the devised above vacuum field theory scenario and show that the whole special relativity theory emerges as its partial case or by-product and is free of the artificial ”inertial reference systems” problems mentioned above.

Really, our vacuum field theory structure is described by dynamical equation (3.8), which we would like to investigate in a neighborhood of two interacting to each other point particles $q_1$ at point $R_1(t) \in \mathbb{R}^3$ and $q_2$ at point $R_2(t) \in \mathbb{R}^3$, respectively. As it was already done in Section 2 we assume that the vacuum potential field function $W : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$ can be representable as $W = \hat{W}(r; R_1(t), R_2(t))$ for some function
\( \dot{W} : \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R} \) and all \( t \in \mathbb{R} \). Then, based on continuity equation (2.17) we obtain
\[
< \frac{\partial \dot{W}}{\partial R_1}, u_1 > + < \frac{\partial \dot{W}}{\partial R_2}, u_2 > + < \frac{\partial \dot{W}}{\partial r}, v > + \dot{W} < \nabla, v > = 0.
\]

We will now be interested in the potential field function \( \tilde{W} : \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R} \) in a vicinity of the relative vector \( R(t) := R_2(t) - R_1(t) \in \mathbb{R}^3 \), keeping in mind that the interaction between particles \( q_1 \) and \( q_2 \) depends on this relative interparticle distance \( R(t) \in \mathbb{R}^3 \). From (4.1) as the relative distance \( r \to R(t) \in \mathbb{R}^3 \) we derive easily that
\[
\frac{\partial \tilde{W}}{\partial R_1} + \frac{\partial \tilde{W}}{\partial R_2} \bigg|_{r \to R(t)} = 0, \quad \frac{\partial \tilde{W}}{\partial R} + \frac{\partial \tilde{W}}{\partial r} \bigg|_{r \to R(t)} = 0.
\]

Combining relationships (4.2) with dynamical field equations (3.3) we obtain that
\[
\left( -\frac{\dot{W}}{c^2} \right) = \left( -\frac{\dot{W}}{c^2} \right) = -\frac{\partial \tilde{W}}{\partial R} \bigg|_{r \to R(t)} = \frac{\partial \tilde{W}}{\partial R} \bigg|_{r \to R(t)},
\]
whence one derives the new dynamical equation
\[
\frac{d}{dt} \left( -\frac{\dot{W}}{c^2} (u_2 - u_1) \right) = -\frac{\partial \tilde{W}}{\partial R}
\]
on the resulting function \( \hat{W} := \tilde{W} |_{r \to R(t)} \).

Equation (4.3) possesses a very important feature of depending on the only relative quantities not depending on the reference system. Moreover, we have not on the whole met the necessity to use other transformations of coordinates different from the Galilean transformations. We mention here that dynamical equation (4.3) was also derived in [7] making use of some not completely true relationships and mathematical manipulations. But the main corollary of [7] and our derivation, saying that equation (4.3) fits for all reference systems, both inertial and accelerated, appears to be fundamental and give rise to new unexpected results in the modern electrodynamics. Below we will proceed to one of very important relativity physics aspect, concerned with the well known Lorentz force expression measuring the action exerted by external electromagnetic field on a charged point particle \( q \) at space point \( R_2(t) \in \mathbb{R}^3 \) for any time moment \( t \in \mathbb{R} \).

To do this we put, owing to the vacuum field theory, that the resulting potential field function \( \hat{W} : \mathbb{R}^3 \to \mathbb{R} \) can be representable in a vicinity of the charged point particle \( q \) as
\[
\hat{W} = \hat{W}_0 + q \phi,
\]
where \( \phi : \mathbb{R}^3 \to \mathbb{R} \) is a suitable local electromagnetic field potential and \( \hat{W}_0 : \mathbb{R}^3 \to \mathbb{R} \) is a constant vacuum field potential owing to the particle interaction with the external distant Universe objects. Then, having
substituted (4.4) into main dynamical field equation (4.3) we obtain that

\[
\frac{d}{dt}(\overline{W}/c^2 u) = \frac{d}{dt}(\overline{W}/c^2 v) - \nabla \overline{W} = -\nabla \overline{W} + \frac{\partial}{\partial t}(-\overline{W}/c^2 v) + \langle u, \nabla \rangle (-\overline{W}/c^2 v) =
\]

\[
-\nabla \overline{W} + \frac{1}{c} \frac{\partial}{\partial t}(\overline{W}/c) - u \times (v \times \nabla \overline{W}) - \langle u, v \rangle \nabla \overline{W} =
\]

\[
-\nabla \overline{W}(1 + \frac{\langle u, v \rangle}{c^2}) + \frac{1}{c} \frac{\partial}{\partial t}(\overline{W}/c) - \frac{1}{c^2} u \times (v \times \nabla \overline{W}) =
\]

\[
-\frac{q}{c} \overline{\varphi}(1 + \frac{\langle u, v \rangle}{c^2}) - \frac{q}{c} \frac{\partial A}{\partial t} + \frac{q}{c} u \times (\nabla \times A),
\]

(4.5)

where we denoted \( u := u_2, \ v := u_1, \nabla := \partial/\partial R_2 = \partial/\partial R \) and \( A := \varphi v/c \), being the related magnetic potential. Since we have already shown that the Lorentz force

\[
F := \frac{d}{dt}(\overline{W}/c^2 u) = \frac{d}{dt}\left(\frac{m_0}{\sqrt{1 - u^2/c^2}}\right)
\]

is given by expression (4.5), it can be rewritten down in the form

\[
F = \frac{d}{dt}\left(\frac{m_0}{\sqrt{1 - u^2/c^2}}\right) = qE + \frac{q}{c} u \times B - \frac{q}{c^2} \nabla \varphi \langle u, v \rangle =
\]

(4.6)

which was derived also in [7] and where we put, by definition, \( E := -\nabla \varphi - \frac{1}{c} \frac{\partial A}{\partial t}, \ B := \nabla \times A, \) being respectively the suitable electric and magnetic vector fields.

5. Conclusion

The resulting expression (4.6) is almost completely equivalent to the well known classical Lorentz force \( F \) up to the additional “inertial” term

\[
F_c := -\frac{q}{c} \nabla \langle u, A \rangle,
\]

(5.1)

which is absent in the relativistic theory. Namely, owing to the absence of term (5.1) the classical relativistic Lorentz force expression was not invariant with respect to any reference frame transformations, except inertial ones. And, as it was noticed in [7], owing only to this fact the relativistic physics faced with many difficulties during the past century and the physicists were forced to use the artificial Lorentz transformations and related with them visible length shortening and time slowing effects. Moreover, they gave rise to such strange enough and non-adequate notions as non-Euclidean time-spaces [11, 13, 14], black holes [19, 20, 11, 17] and some other nonphysical objects. Concerning the results described above we could state that the vacuum field theory approach of [7] to fundamental physical phenomena is really a powerful tool in hands of researchers, who wish to penetrate into the hidden properties of the surrounding us Universe. As the microscopical quantum level of describing the vacuum field matter structure is, with no doubt, very important, we see the next challenging steps in understanding the backgrounds of quantum processes from the approach devised in [7] and in this work and in deriving new physical relationships, which will help us to explain the Nature more deeply and adequately.
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