Numerical Simulation of Water Pollution Emergency Response Plan Comparison

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Abstract. In this paper, the numerical solution of concentration convection-diffusion equation is applied to the selection of emergency treatment schemes for water pollution, and the corresponding high-precision difference schemes are proposed. On this basis, taking the emergency treatment of oil spill in rivers as an example, the oil fence and the oil receiving device are corresponded to the first and second moving boundaries respectively. Four kinds of comparison schemes are given. Through numerical simulation of high-precision format, the time when oil spill is cleaned up in water area is obtained, and the efficient treatment scheme under the same parameters is selected. The results show that the numerical simulation method presented in this paper is reasonable and reliable, and provides a quick and accurate solution case for the selection of emergency treatment schemes for water pollution.

1. Introduction
At present, the study of concentration convection-diffusion equation is mainly focused on two aspects: high accuracy and fast calculation speed. It has been paid full attention in many research fields of environmental engineering. Liu Hongyi [1] and others applied the convection-diffusion model to the problem of pollutant emission from refuse incinerator, and simulated the modified model. Zheng ning [2] applied the governing equation to the temperature and oxygen concentration changes of coal during low temperature oxidation exothermic process, and simulated the spontaneous combustion process of coal. Huang Suiliang [3] simulated the migration and transformation of heavy metal pollutants in river water. However, little research has been done on oil spill pollution in water. The high-precision sub-scheme of concentration convection-diffusion equation under complex boundary conditions studied in this paper can be applied to the treatment of oil spill pollution. For the study of concentration diffusion equation, people prefer the Dirichelet boundary condition [4-6], while the real water pollution diffusion has the characteristics of Neumann boundary condition [7, 8]. The complex boundary conditions studied in this paper, that is, Neumann boundary conditions, is more practical.

2. Physical problems
2.1. The proposition of physical problems
In this paper, the containment fence and oil receiving device are used as moving boundary to deal with oil spill, and the application of the structure format in this paper is illustrated by taking the oil spill in river channel as an example. When the oil spill occurs, the oil fence and oil receiver are respectively
taken as the first and second mobile boundaries. Assuming that the channel is narrow, it can be considered that the concentration distribution of oil spill in the Y direction is consistent. Therefore, only the change of oil spill in the X direction can be considered. This physical problem can be simulated by one-dimensional concentration convection-diffusion equation, as shown in Figure 1.

![Figure 1. River oil spill schematic](image)

2.2. Construction of convection diffusion scheme with complex boundary concentration

In view of the above oil spill problem, we constructed a one-dimensional concentration convection diffusion equation with high accuracy.

\[
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = D \frac{\partial^2 C}{\partial x^2}
\]  

(1)

\[
\alpha_i C_{i+1}^{j+1} + \alpha_2 C_{i}^{j+1} + \alpha_3 C_{i-1}^{j+1} = \beta_1 C_{i+1}^{j} + C_i^{j} + \beta_2 C_{i-1}^{j}
\]  

(2)

Among them, \(D\) is diffusion coefficient, diffusion term is \(D \frac{\partial^2 C}{\partial x^2}\), \(u\) is convection velocity, the unit is \(m/s\), convection term is \(u \frac{\partial C}{\partial x}\). Among them, \(\gamma_1 = \frac{u \Delta t}{\Delta x}\) and \(\gamma_2 = \frac{D \Delta x}{\Delta t^2}\), the corresponding coefficients are as follows:

\[\alpha_i = -(6\gamma_1^4 - 72\gamma_1^3\gamma_2 - 6\gamma_1^2 + 216\gamma_1^2\gamma_2^2 - 4\gamma_1^2 - 72\gamma_1\gamma_2^2 + 12\gamma_1\gamma_2 - 144\gamma_2^3 + 24\gamma_2^4) / (72\gamma_1^4 - 144\gamma_1^3\gamma_2 + 60\gamma_1^2\gamma_2^2 + 16\gamma_1^2 + 288\gamma_2^3 - 240\gamma_2^4)\]

\[\alpha_2 = -(60\gamma_1^4 - 432\gamma_1^3\gamma_2^2 - 16\gamma_1^2 + 288\gamma_2^3 + 240\gamma_2^4) / (72\gamma_1^6 - 144\gamma_1^5\gamma_2 + 60\gamma_1^4\gamma_2^2 + 432\gamma_1^2\gamma_2^2 + 16\gamma_1^2 + 288\gamma_2^3 - 240\gamma_2^4)\]

\[\alpha_3 = -(6\gamma_1^4 + 72\gamma_1^3\gamma_2 - 6\gamma_1^2 + 216\gamma_1^2\gamma_2^2 - 4\gamma_1^2 + 72\gamma_1\gamma_2^2 - 12\gamma_1\gamma_2 - 144\gamma_2^3 + 24\gamma_2^4) / (72\gamma_1^6 - 144\gamma_1^5\gamma_2 - 60\gamma_1^4\gamma_2^2 + 432\gamma_1^2\gamma_2^2 + 16\gamma_1^2 + 288\gamma_2^3 - 240\gamma_2^4)\]

\[\beta_1 = -(36\gamma_1^6 + 36\gamma_1^5\gamma_2 - 72\gamma_1^4\gamma_2 + 6\gamma_1^4 - 72\gamma_1^3\gamma_2^2 - 6\gamma_1^3 + 216\gamma_1^3\gamma_2^2 - 4\gamma_1^3 + 72\gamma_1\gamma_2^2 + 12\gamma_1\gamma_2 + 288\gamma_2^3 - 240\gamma_2^4) / (72\gamma_1^6 - 144\gamma_1^5\gamma_2 - 60\gamma_1^4\gamma_2^2 + 432\gamma_1^2\gamma_2^2 + 16\gamma_1^2 + 288\gamma_2^3 - 240\gamma_2^4)\]

\[\beta_2 = -(36\gamma_1^6 - 36\gamma_1^5\gamma_2 + 6\gamma_1^4 + 72\gamma_1^3\gamma_2^2 - 4\gamma_1^3 + 72\gamma_1\gamma_2^2 - 12\gamma_1\gamma_2 + 144\gamma_2^3 + 24\gamma_2^4) / (72\gamma_1^6 - 144\gamma_1^5\gamma_2 - 60\gamma_1^4\gamma_2^2 + 432\gamma_1^2\gamma_2^2 + 16\gamma_1^2 + 288\gamma_2^3 - 240\gamma_2^4)\]

The above is a general expression of the high-precision scheme for one-dimensional concentration convection-diffusion equation. Next, the high-precision scheme for complex boundary convection-diffusion equation is derived. By Taylor expansion of point \(C_{i+2}\), \(C_{i-2}\), at boundary point \(C_i\), the expression of boundary point relationship is obtained by combining the two equations with the high-precision general expression, i.e. formula (2):
Among them, the expression $a_1 = \alpha_1, a_2 = \alpha_2, a_3 = \alpha_3, F_i^1 = \beta_1 C_i^{e+1} + C_i^o + \beta_2 C_i^{e+1}, \quad \frac{\partial C}{\partial x} = s$

### 2.3. Accuracy verification of complex boundary schemes

\[
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = D \frac{\partial^2 C}{\partial x^2} + u \frac{\partial}{\partial x} (\sin(\pi x) + \pi \cos(\pi x))
\]

\[
C(1, t) = 0; \quad C(x, 0) = \sin(\pi x)e^{\frac{\Delta t}{2D}}
\]

The analytical solution for the upper form problem is:

\[
C(x, t) = \sin(\pi x)e^{-\frac{(2\pi x)^2 + (\frac{u}{D})^2}{4D^2} + \frac{u}{2D}}
\]

The numerical solution can be shown in Figure 2 as a result of the numerical solution and the absolute error of the exact solution. From Figure 2 it can be seen that the maximum error order of the numerical solution and the exact solution reaches order $10^{-9}$, that is, the error meets the accuracy requirement of $O(\tau^2 + h^4)$. The numerical results are valid. The finite difference scheme of concentration convection-diffusion equation constructed in this paper meets the accuracy requirement under complex boundary conditions.

![Figure 2. Absolute error of numerical solution and exact solution](image)

### 2.4. Selection of alternatives

At this point, we propose four alternatives.

- **Scheme 1**: Both the oil fence and the oil receiving device are fixed at the upper and lower boundaries of the first type respectively;
- **Scheme 2**: The oil fence moves downstream as the second type boundary...
and the oil receiving device is fixed downstream as the first type boundary; Scheme 3: Both the oil fence and the oil receiving device are fixed as the first type boundary. Boundary, the fence is fixed in the upstream position, and the oil receiving device moves upstream in the downstream position. Scheme 4: As the second boundary, the fence moves downstream from the upstream position and the oil receiving device moves upstream as the first boundary.

Assuming that the oil concentration at the initial time is \( C(x,0) = \sin(\pi x) \), the oil spill treatment time is 0.2s. In order to achieve higher simulation accuracy, small grids, i.e. \( \Delta x \) and \( \Delta t \) (\( \Delta x \) is the space step, \( \Delta t \) is the time step), are used to determine \( \Delta x = 0.0005 / 0.001m \) and \( \Delta t = 0.0001s \). The movement speed of the device is \( u_{device} = 5m/s \) or \( u_{device} = 10m/s \) respectively. The velocity of water flow is \( u = 0.5, 10m/s \), and the diffusion coefficient \( D = 1, 2, 3, 4, 5m^2/s \) only considers the oil spill diffusion under ideal conditions. It is stipulated that when the surplus of oil spills is less than \( 10^{-2} \), we believe that the oil spill is completely disposed. Taking the moving velocity of the device as \( u_{device} = 10m/s \), diffusion coefficient \( D = 5m^2/s \) and flow velocity \( u = 10m/s \) as examples, the simulation results of scheme 1 and scheme 2 are shown as follows:

![Figure 3. Oil distribution in scheme 1](image1)

![Figure 4. Oil distribution in scheme 2](image2)

3. Result analysis

By numerical simulation, the time data of four emergency treatment schemes for oil spill in rivers at different diffusion coefficients and flow velocities are obtained and listed. Taking schemes 2 and 4 as examples, as shown in tables 1-2:

| \( D \) | \( u=0m/s \) | \( u=5m/s \) | \( u=10m/s \) |
|---|---|---|---|
| \( u_{device}=5m/s \) | \( u_{device}=10m/s \) | \( u_{device}=5m/s \) | \( u_{device}=10m/s \) | \( u_{device}=5m/s \) | \( u_{device}=10m/s \) |
| 1 | 0.1520 | 0.0827 | 0.1464 | 0.0801 | 0.1410 | 0.0782 |
| 2 | 0.1394 | 0.0761 | 0.1340 | 0.0784 | 0.1290 | 0.0760 |
| 3 | 0.1268 | 0.0696 | 0.1239 | 0.0691 | 0.1210 | 0.0679 |
| 4 | 0.1143 | 0.0623 | 0.1126 | 0.0622 | 0.1068 | 0.0614 |
| 5 | 0.1015 | 0.0567 | 0.0985 | 0.0442 | 0.0954 | 0.0438 |
Table 2. Scheme 4 Time required to treat oil spills at different oil spill diffusion coefficients D and different water flow u

| D     | $u_{ave}=5\,m/s$ | $u_{ave}=10\,m/s$ |
|-------|------------------|-------------------|
| $u=0\,m/s$ | 0.0876 | 0.0438 |
| $u=5\,m/s$ | 0.0869 | 0.0434 |
| $u=10\,m/s$ | 0.0860 | 0.0430 |
| $u=5\,m/s$ | 0.0760 | 0.0392 |
| $u=10\,m/s$ | 0.0754 | 0.0389 |
| $u=10\,m/s$ | 0.0747 | 0.0385 |
| $u=5\,m/s$ | 0.0677 | 0.0356 |
| $u=10\,m/s$ | 0.0673 | 0.0353 |
| $u=10\,m/s$ | 0.0668 | 0.0349 |
| $u=5\,m/s$ | 0.0612 | 0.0327 |
| $u=10\,m/s$ | 0.0604 | 0.0325 |
| $u=10\,m/s$ | 0.0598 | 0.0322 |
| $u=5\,m/s$ | 0.0559 | 0.0303 |
| $u=10\,m/s$ | 0.0556 | 0.0301 |
| $u=10\,m/s$ | 0.0553 | 0.0329 |

It can be seen from Table 1 that all the oil spill treatment time in Table 1 is longer than that in Table 2. This shows that the oil spill treatment effect of Scheme 4 is better. It can be clearly seen that when the moving speed of the treatment unit doubles, although the economic cost increases, the oil spill treatment time is significantly shortened. Comparing the four schemes, the moving speed of the oil spill treatment device is $u=10\,m/s$, and the diffusion coefficient $D=5\,m^2/s$ is taken as an example, and table 3 is listed.

Table 3. The four scheme has a mobile speed of 10 diffusivity $D=5$ and oil spill processing time under different u conditions.

| time (s) | $u=0\,m/s$ | $u=5\,m/s$ | $u=10\,m/s$ |
|----------|------------|-------------|-------------|
| scheme 1 | 0.1775     | 0.1734      | 0.1619      |
| scheme 2 | 0.0567     | 0.0442      | 0.0438      |
| scheme 3 | 0.0568     | 0.0562      | 0.0553      |
| scheme 4 | 0.0303     | 0.0301      | 0.0329      |

According to Table 3, the treatment time of oil spill in Scheme 1 is longer and the treatment efficiency is not high. Because there is no need to move the treatment equipment, the economic cost is the lowest, thus reducing the cost. The treatment time of oil spill in Scheme 2 and Scheme 3 is almost the same, which is much shorter than that in Scheme 1, and the treatment efficiency is higher, and the two schemes are economical. The cost is a little high and requires continuous movement of the oil fence or oil collection device. According to the data, the treatment time of scheme 2 is slightly shorter than that of scheme 3, and the treatment effect of scheme 2 is better when the economic cost is similar; scheme 4 has the least treatment time and the best treatment efficiency, but the corresponding economic cost is the highest.

In summary, in scheme 1, the oil fence and the oil collection device are not moving and the economic cost is greatly reduced, but the efficiency of oil spill emergency treatment is too low to be suitable for application; while in scheme 4, the oil spill contamination is handled simultaneously by the oil spill fence and the oil collection device, although the economic cost is high, but the efficiency of oil spill emergency treatment is the most efficient.

4. Summary

Firstly, the numerical model of one-dimensional high-precision finite difference scheme for concentration convection-diffusion equation is established for emergency treatment of water pollution, and the second boundary scheme for concentration convection-diffusion equation is constructed. The verification scheme meets the high-precision requirements, that is, the error meets the accuracy and accuracy requirements. Taking the emergency treatment of oil spill in the river as an example, the numerical calculation is carried out. The results show that scheme 4, i.e. oil fence and oil receiving device, is the first and second boundary selection scheme, with high economic cost, but the oil spill
treatment time is the shortest and the efficiency is the highest, which provides a theoretical basis for solving the problem of water pollution emergency treatment.

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