Energy Harvesting Communications Using Dual Alternating Batteries

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Abstract

We consider an energy harvesting transmitter equipped with two batteries having finite storage capacities, communicating over an additive white Gaussian channel. The work is motivated by an observation that many practical batteries, when repeatedly charged after being partially discharged, suffer from degradation in the usable capacity. The capacity can be recovered by completely discharging the battery before charging it fully again. Hence, in this work, we impose the constraint that a battery must be charged (discharged) only after it is fully discharged (charged). Our goal is to maximize the long-term average throughput with non-causal and causal knowledge of the energy arrivals, which we assume to be Bernoulli. We propose two sub-optimal policies and obtain an upper bound on the performance gap ($G$) from the optimal long-term average throughput that is achieved with infinite capacity batteries. We find that $G$ remains constant as the amount of energy harvested per arrival increases. Numerically, we also find that $G$ decreases with the battery capacity faster than the inverse of the square root of the battery capacity.

I. INTRODUCTION

Recently, energy harvesting (EH) from natural and man-made sources for powering electronic devices has been gaining tremendous interest. In EH systems, energy arrives randomly and the harvested energy needs to be stored in batteries for reliable system operation. For efficient management of the harvested energy, several architectures have been proposed [1], [2]. When...
the system is equipped with a single battery, a dual-path architecture, wherein the harvested energy is simultaneously used to charge the battery and power the load directly, is shown to perform better than other architectures [2], [3]. However, in some cases, when directly powering the load from EH sources is infeasible, the performance with the single battery can significantly degrade because it is not possible to simultaneously save energy in the battery and power the load. Hence, energy harvesting is suspended while the battery energy is being used. In such cases, a two-battery solution can potentially far outperform the single battery solution [3].

In this work, we consider an EH transmitter equipped with two batteries having finite storage capacities. In our two-battery architecture, the batteries are charged and discharged alternately. That is, when a battery, referred to as the working battery, is being discharged, the harvested energy is stored in the other battery, referred to as the charging battery, as charging and discharging a battery simultaneously is not possible [4], [5]. In this case, one must switch the role of the batteries when the charging battery becomes full or when the working battery becomes empty, to avoid energy overflow and blank (no-transmission) periods, respectively. Depending on the energy arrival and consumption rates, these two events may not always occur simultaneously. This leads to charging (discharging) of batteries after discharging (charging) them partially. In practice, such a charging and discharging pattern affects the usable capacity of batteries. For instance, the usable capacity of many Nickel based batteries [6] and some Li-ion [7] batteries reduces at a significant rate with the number of charge/discharge cycles when a battery is repeatedly charged after a partial discharge. This phenomenon, referred to as the memory effect or voltage depression, can be avoided by discharging (charging) the battery completely before fully charging (discharging) it again [8]. In this work, we do not focus on modeling the voltage depression, but rather, we charge and discharge the batteries such that the voltage depression does not occur. With the above constraint, our goal in the work is to maximize the long-term average throughput.

In the EH communications literature, the impact of various battery limitations, such as inefficiency, finite capacity, internal resistance and leakage have been considered [4], [9], [10]. However, the impact of charging and discharging dynamics on the usable capacity of batteries has been scarcely studied. The authors in [10] characterize the interplay between the battery

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1Practical batteries cannot be simultaneously charged and discharged. The works which assume the batteries can be simultaneously charged and discharged, implicitly assume either the dual-path architecture or a two-battery framework where one of the batteries acts as an intermediate storage device.
discharge policy and the irreversible degradation (aging) of the storage capacity. We note that this phenomenon is different from voltage depression. Further, in [11], [12], the authors indirectly control the battery degradation by constraining the number of charge and discharge cycles per unit time. The work in this paper is related to [13] which considers an EH transmitter equipped with a single battery. In [13] a Bernoulli energy arrival model assumed, where either an energy packet with energy amount equal to the battery capacity arrives or no energy arrives in a slot. This implies, whenever a packet of energy arrives, the battery fills up completely. In this case, when a fresh energy packet arrives, the residual energy in the battery can be thought to be discarded instantaneously before replenishing it with the freshly arrived energy. Hence, we see that [13] implicitly accounts for the constraint considered in this paper. Further, we note that [13] implicitly assumes a two-battery solution. Our work generalizes [13] by accounting for the case when multiple energy arrivals are required to fill a battery completely. The main contributions of the paper are:

• We obtain optimal power allocations with non-causal (offline) and causal (online) knowledge of energy arrivals. For the online case, we note, discarding the residual energy when a fresh arrival occurs, as in [13], may be sub-optimal. We present a dynamic programming based policy with a better performance which optimally utilizes the remaining energy in the battery when a fresh energy packet arrives.

• We also propose two sub-optimal online policies for the finite capacity battery case. We show that the performance gap between the optimal policy for an ideal system equipped with infinite capacity batteries and the proposed policies with finite capacity batteries decays faster than the inverse of the square root of the battery capacity.

II. SYSTEM MODEL

We consider a transmitter equipped with two batteries, $B^I$ and $B^{II}$ (see Fig. 1). We assume that each battery has the storage capacity of $B_J$. The harvested energy is stored in one of the batteries before using it for the transmission [1]. Since it is not feasible to simultaneously charge
and discharge a battery \cite{4,5}. we operate the system as follows. When $B^i$ is being charged from the EH source, the transmitter draws power from $B^{II}$ and vice versa. Hence, at any point in time, the EH source and the transmitter will be connected to $a_1$ ($a_2$) and $b_2$ ($b_1$), respectively, as shown in Fig. 1. Based on the discussion in the introduction, we switch the role of batteries only when the working battery is completely discharged and the charging battery is completely charged.

We consider a time-slotted system with unit slot length. Energy arrivals in each slot are independent and identically distributed (i.i.d.) Bernoulli random variables with parameter $p$, where $B/N_B, N_B \in \mathbb{Z}^+$, units of energy is harvested per arrival. That is, the amount of energy harvested in slot $i$ is,

$$E_i = \begin{cases} \frac{B}{N_B} & \text{w.p. } p, \\ 0 & \text{w.p. } 1 - p, \end{cases}$$

(1)

where w.p. stands for with probability. Let $B^w_i$ and $B^c_i$ denote the amount of energy stored in the working and the charging battery at the start of slot $i$, respectively. We assume $B^w_1 = B$ and $B^c_1 = 0$. Let the transmit power in slot $i$ be $P_i$. Then,

$$B^c_{i+1} = \min(B^c_i + E_i, B), \quad i = 1, 2, \ldots$$

(2)

$$B^w_{i+1} = \max(B^w_i - P_i, 0), \quad i = 1, 2, \ldots$$

(3)

The min and max in the above equations capture the facts that the battery energy cannot exceed its capacity or become negative, respectively. The communication is over an additive white Gaussian noise channel having unit power spectral density. We assume the throughput with the constant transmit power of $P$ W in a slot is given by $\frac{1}{2} \log(1 + P)$ bits-per-second (bps). All the logarithms are to the base 2.

III. PROBLEM FORMULATION

We now formulate the long-term average throughput maximization problem subject to the constraint that the working (charging) battery must be discharged (charged) completely before switching the roles of the batteries. Since the harvested energy arrives randomly, the switching instants will be random. Due to the above constraint, the switching instants are the renewal instants of the battery state processes. Hence, we use the terms ‘switching’ and ‘renewal’ instants interchangeably. Let $L_k$ denote the length (in slots) of the $k$th renewal period. Let the number of slots required for the charging battery to accumulate $B$ units of energy in the $k$th renewal period.
be $C_k$ which is less than or equal to $L_k$ by our design. Since the system is reset after a renewal instant and because the energy arrivals and allocations are independent across renewal intervals, $L_1, L_2, \ldots$ form a sequence of i.i.d. random variables. The $k$th renewal occurs in $S_k$th slot, where,

$$S_k = S_{k-1} + L_k, \quad k = 2, 3, \ldots,$$

(4)

where we define $S_1 \overset{\Delta}{=} 1$. Further, due to the constraint described above, we must have,

$$B_{S_k}^w = B, \quad \text{and} \quad B_{S_k}^c = 0, \quad \forall \ k = 1, 2, \ldots$$

(5)

We now define the long-term average throughput as follows.

$$T \overset{\Delta}{=} \lim_{k \to \infty} \frac{1}{k} \mathbb{E} \left[ \sum_{i=1}^{k} \frac{1}{2} \log(1 + P_i) \right]$$

(6)

where the expectation is over all the possible sequences of energy arrivals. Our objective is to find the optimal power policy, $P_1, P_2, \ldots$, that maximizes the long-term throughput, i.e., the optimization problem of interest is given by,

$$\text{maximize} \quad T$$

(7a)

$$\text{subject to} \quad P_k \geq 0, \quad k = 1, 2, \ldots$$

(7b)

$$\sum_{k=S_{i-1}}^{S_i-1} P_k \leq B, \quad i = 2, 3, \ldots,$$

(7c)

where (7c) is because the maximum total amount of energy that can be consumed in a renewal period is $B$.

### IV. OPTIMAL SOLUTION

In this section, we obtain the long-term throughputs under various cases. Before we proceed, we note the following. Firstly, by renewal-reward theorem [14], the long-term average throughput in (6) is equal to,

$$T = \frac{\mathbb{E} \left[ \sum_{i=1}^{L} \frac{1}{2} \log(1 + P_i) \right]}{\mathbb{E}[L]},$$

(8)

where $L$ is the length of the renewal period, whose distribution is identical to $L_1, L_2, \ldots$. The expectation is with respect to the random variable $L$. Secondly, the random variable $C$, which is the number of slots required to accumulate $B$ units of energy, has the negative binomial distribution given by,

$$\text{Prob}(C = n) \overset{\Delta}{=} q_n = \binom{n-1}{n-N_B} p^{N_B} (1-p)^{n-N_B},$$

(9)

for $n \in \{N_B, N_B + 1, \ldots\}$ and $\mathbb{E}(C) = N_B/p$. 

A. An Upper-Bound on the Long-term Average Throughput

When no constraints are present, and the nodes are equipped with infinite capacity batteries, it is optimal to utilize $\epsilon$ smaller amount of power per slot on average, compared to the average harvesting rate, where $\epsilon > 0$ can be arbitrarily small. Further, by the concavity of the rate function, it is optimal to utilize an equal amount of energy $\mu = Bp/N_B$ in every slot, resulting in the long-term average throughput \[ T_{ub} = \frac{1}{2} \log(1 + \mu) \] (10)
The subscript ‘ub’ indicates that this performance is an upper bound on the performance with any other constraints.

B. Optimal Offline Policy

In the offline policy, we assume that the number of slots required to completely charge the battery is known at the start of the current renewal instant, i.e., the realization of $C_{k+1}$ is known at start of slot $S_k$ for $k \in \{1, 2, \ldots\}$. Due to the concavity of $\frac{1}{2} \log(1 + P)$ in $P$, it is optimal to transmit with a constant power over the $C_{k+1}$ slots. Hence, when $C_{k+1}$ is known, we can discharge the working battery such that it gets emptied in the same slot in which the charging battery fills up. Hence, we have $L_{k+1} = C_{k+1}$ and we transmit at the constant power of $B/L_{k+1}$ in the $k^{th}$ renewal. Hence, from (8) and (9), the optimal long-term throughput is given by
\[ T_{off} = \frac{p}{N_B} \sum_{n=N_B}^{\infty} \frac{nq_n}{2} \log \left(1 + \frac{B}{n}\right). \] (11)
It is easy to numerically compute the value of $T_{off}$.

C. Optimal Online Policy

We now consider the online case with only causal knowledge of renewal instants. Since the precise time when the charging battery will get full is unknown, power is allocated based only on the distribution of energy arrivals. To obtain the optimal online policy, we adopt a dynamic programming framework. We now define the relevant quantities.

a) State space: The state of the system is defined by $3$-tuples $s \triangleq (b_1, b_2, \alpha)$, where $b_1$ and $b_2$ are the amounts of energy stored in batteries $B^I$ and $B^II$, respectively, and $\alpha$ is an indicator variable defined as follows.
\[ \alpha = \begin{cases} 
1 & \text{if } B^I \text{ is the working battery,} \\ 
0 & \text{if } B^I \text{ is the charging battery.}
\end{cases} \] (12)
By design, we note that when $\alpha = 1$ ($\alpha = 0$), $B^I$ is the charging (working) battery. The state space is given by

$$S = \{(b_1, b_2, \alpha) : 0 \leq b_1, b_2 \leq B, \alpha \in \{0, 1\}\}$$

(13)

b) Action space and reward: The action space the system can take in state $s = (b_1, b_2, \alpha) \in S$ is given by

$$\mathcal{A}(s) = \{P : 0 \leq P \leq b_1 \alpha + b_2 (1 - \alpha)\}$$

(14)

The constraint on $P$ in (14) is due to the fact that $P$ units are drawn from the working battery. The reward when the system takes action $P \in \mathcal{A}(s)$ in state $s \in S$ is given by $1/2 \log(1 + P)$.

c) State transition probability matrix: The next state $s' \triangleq (b'_1, b'_2, \alpha') \in S$ when the system takes action $P \in \mathcal{A}(s)$ in state $s = (b_1, b_2, \alpha) \in S$ is given by

$$\begin{cases} (b_1 - P \alpha, b_2 - P (1 - \alpha)) & \text{w.p. } 1 - p, \\ (b_1 - P \alpha + E (1 - \alpha), b_2 - P (1 - \alpha) + E \alpha) & \text{w.p. } p, \end{cases}$$

(15)

where we have accounted for the fact that, in a slot, the event that the transmitter harvests and stores $E$ units of energy in the charging battery occurs with probability $p$. As the role of the batteries must be switched when the working battery becomes empty and the charging battery becomes full, we have,

$$\alpha' = \begin{cases} 1 - \alpha & \text{if } ((b_1, b_2) = (0, B)) \text{ or } ((b_1, b_2) = (B, 0)), \\ \alpha & \text{otherwise.} \end{cases}$$

(16)

From (15) and (16), we can easily construct the probability transition matrix, $q(s'|P, s)$ for all $s \in S$ and $P \in \mathcal{A}(s)$.

d) Optimal value function: We consider $K$ slots for the optimization. We obtain the optimal value function $V_k(s)$ in slot $k \in \{1, \ldots, K\}$ and state $s \in S$ by solving the following Bellman equation.

$$V_k(s) = \max_{P \in \mathcal{A}(s)} \left\{ \frac{1}{2} \log(1 + P) + \sum_{s' \in S} q(s'|P, s) V_{k+1}(s') \right\}$$

(17)

for $k = K, K - 1, \ldots, 1$, where $V_{K+1}(s) \triangleq 0$. The optimal online throughput is then given by

$$T_{on} = \lim_{K \to \infty} \frac{V_K(s)}{K}$$

(18)
We note that the optimal online throughput, $T_{on}$, can also be obtained by solving the Bellman equation in the infinite horizon case with the discount factor arbitrarily close to one. Since the reward is bounded and the state space is finite, there exists an optimal stationary deterministic policy for \[17\], i.e., there exists a unique optimal action $P^*(s)$ in state $s$ independent of slot indices \[16\]. Hence, it suffices to search only in the set of all stationary deterministic policies.

D. Proposed Suboptimal Online Policy

In this policy, we assume the energy remaining in the working battery is discarded once the charging battery accumulates $B$ units of energy. On the other hand, if the working battery gets completely discharged before the charging battery is full, the transmitter waits without transmission till the charging battery gets full. This assumption implies that the renewal instant is the same as the instant when the charging battery accumulates $B$ units of energy. Hence, the renewal length, $L$ is distributed identically as $C$ in \[9\]. Now, we obtain the optimal power allocation. Our analysis proceeds along the lines in \[13\], \[17\]. Let $\tilde{P}_i$ be the transmit power in slot $i \in \{1, \ldots, L\}$ after a renewal, i.e., $\tilde{P}_i \triangleq P_{S_{k+i-1}}$ for $k = 1, 2, \ldots$. Then, the throughput in \[8\] can be re-written as

$$T_{prop} = \frac{p}{N_B} \sum_{n=1}^{\infty} q_n \sum_{i=1}^{n} \frac{1}{2} \log(1 + \tilde{P}_i),$$

$$= \sum_{i=1}^{\infty} F_i(p, N_B) \frac{1}{2} \log(1 + \tilde{P}_i), \quad (19)$$

where $F_i(p, N_B) \triangleq (p/N_B) \sum_{n=1}^{\infty} q_n$. From \[7\], \[8\] and \[19\], to maximize the long-term average throughput in the online case, we need to solve the following optimization problem.

$$\text{maximize} \sum_{i=1}^{\infty} F_i(p, N_B) \frac{1}{2} \log(1 + \tilde{P}_i) \quad (20a)$$

$$\text{subject to} \sum_{i=1}^{\infty} \tilde{P}_i \leq B, \quad \tilde{P}_i \geq 0, \quad i = 1, 2, \ldots \quad (20b)$$

Clearly, \[20\] is a convex optimization problem. Hence, Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for optimality. Based on the KKT conditions, we obtain the optimal power allocations in the following theorem.

Lemma 1. Let

$$N = \max \left\{ n : \frac{\sum_{i=1}^{n} F_i(p, N_B)}{B + n} \leq F_n(p, N_B) \right\}. \quad (21)$$
For \( k = 1, 2, \ldots \), the optimal transmit power \( P^*_{S_{k+i-1}} \) is given by \( P^*_{S_{k+i-1}} = \tilde{P}_i^* \), where

\[
\tilde{P}_i^* = \begin{cases} 
\frac{(B+N)p}{N_B \sum_{i=1}^{i} F_i(p, N_B)} - 1 & \text{for } i = 1, \ldots, N_B, \\
\frac{(B+N)p}{\sum_{i=1}^{i} F_i(p, N_B)} - 1 & \text{for } i = N_B + 1, \ldots, N, \\
0 & \text{for } i > N.
\end{cases}
\]  

(22)

Proof. See Appendix A [18].

We note the probability that the charging battery fills up in less than \( N_B \) slots after a renewal is zero. Hence, it is intuitive that we must consume the available energy at the constant rate in the first \( N_B \) slots after a renewal, as suggested by (22). Further, when \( N_B = 1 \), we recover Theorem 1 of [13]. Since we are discarding the remaining energy in the working battery, the optimal solution in Lemma 1 is a sub-optimal online policy and clearly, we have, \( T^*_{\text{prop}} \leq T^\text{ub} \), where \( T^*_{\text{prop}} \) is obtained by substituting \( \tilde{P}_i^* \) from (22) into (19).

E. Simplified Suboptimal Online Policy

While preserving the structure of the policy in Lemma 1, we propose another policy where the power allocation in the \( i \)th slot after a renewal instant is given by

\[
\tilde{P}_i^* = BF_i(p, N_B) = \frac{Bp}{N_B} \sum_{n=i}^{\infty} q_n.
\]  

(23)

We discard the energy remaining in the working battery when the charging battery becomes full. Let \( T_\text{simp} \) denote the long-term average throughput obtained in this strategy. We now have the following result.

**Theorem 2.** The long-term average throughput with the power allocation policy in (23) satisfies

\[
T^\text{ub} \geq T^*_{\text{prop}} \geq T_\text{simp} \geq T^\text{ub} - G(N_B),
\]

(24)

where \( G(N_B) = \max_p - \frac{p}{2N_B} \sum_{i=1}^{\infty} (\sum_{n=i}^{\infty} q_n) \log (\sum_{n=i}^{\infty} q_n) \).

Proof. See Appendix B in [18].

We make the following remarks on the above theorem.

- Firstly, \( G(1) \approx 0.72 \) and we recover Proposition 3 of [13].
- Further, numerically, we can show that \( \max_{N_B} G(N_B) \approx 0.72 \). Hence, the long-term average throughputs in the simplified and the proposed suboptimal policies are at most 0.72 bits away.
from the upper bound $T_{\text{ub}}$ for any value of $p$, $B$ and $N_B$, under the Bernoulli energy harvesting model.

- Finally, when the parameters $E$ and $p$ are fixed, $N_B$ increases proportionately with $B$, as $N_B = B/E$. Based on numerical analysis, we find the bound $G(N_B)$ decreases monotonically at a rate faster than the inverse of the square root of $N_B$ and $B$.

\section*{F. Constant Power Policy}

In this policy, we consume the energy available in the battery at a constant rate of $B/(\lfloor N_B p^{-1} \rfloor)$ as long as the battery is not empty, i.e., the power allocation in the $i$th slot after a renewal instant is given by

\[ \bar{P}_i = \frac{B}{\lfloor N_B p^{-1} \rfloor} \text{, for } i = 1, \ldots, \lfloor N_B p^{-1} \rfloor. \]  

(25)

We discard the remaining energy in the working battery when the charging battery becomes full. We denote the long-term average throughput of the policy by $T_{\text{const}}$. Several variants of this policy have been considered in the literature \cite{13} and references therein, and they have been shown to perform competitively with optimal policies. Specifically, the version proposed in \cite{19} has been shown to approach $T_{\text{ub}}$ asymptotically with $B$. Hence, we compare suboptimal online policies with the constant power policy.
In Fig. 2, we plot the variation of the long-term average throughput with the battery capacity in the optimal offline, optimal online, proposed suboptimal and simplified suboptimal online policies. We note as the battery capacity increases, the performance gap between offline and online policies decreases. Further, the performance gap of the proposed suboptimal online policy from the upper bound, $T_{ub}$, tends to zero as $B$ increases much faster than the simplified suboptimal policy.

In Fig. 3, we plot the variation of the long-term average throughput with the energy harvested per arrival in optimal offline, proposed suboptimal online, simplified suboptimal online and constant power policies, along with upper and lower bounds from (24). Although the performance of the constant power policy is slightly better than the simplified suboptimal policy for smaller values of $E$, the proposed suboptimal policy always performs better than the constant power policy. Further, as $E$ increases, the performance of the constant power policy diverges while the offline, proposed and the simplified suboptimal policies are always within 0.35 bits from the upper bound. We note $G(N_B)|_{N_B=4} \approx 0.35$.

In Fig. 4, we plot the variation of the long-term average throughputs with the energy arrival probability, $p$, for a fixed mean harvesting rate of $\mu$. Note that the lower the value of $p$, the higher is the amount of energy harvested per arrival. In this case, the long-term average throughputs are low as the optimal power allocation may not be constant across the slots. For the higher
values of \( p \), the harvested energy arrives more uniformly and the power allocation can be nearly constant, and, hence, the performance is better.

VI. CONCLUSIONS

We considered an energy harvesting transmitter equipped with two batteries. In order to avoid recoverable capacity degradation of the batteries, we charge (discharge) a battery only when it is completely discharged (charged). With this constraint, we obtained long-term average throughputs in offline and online policies. We proposed two online sub-optimal policies, and found that their performance gap from the unconstrained optimal performance decreases with the battery capacity faster than the inverse of the square root of the battery capacity.

APPENDIX

A. Proof of Lemma 1

The Lagrangian of (20) is given by

\[
L = - \sum_{i=1}^{\infty} F_i(p, N_B) \frac{1}{2} \log(1 + \tilde{P}_i) + \lambda \left( \sum_{i=1}^{\infty} \tilde{P}_i - B \right) - \sum_{i=1}^{\infty} \mu_i \tilde{P}_i,
\]

(26)

where \( \lambda, \mu_i \geq 0 \) are Lagrange multipliers. Differentiating \( L \) with respect to \( \tilde{P}_i \), we have the following stationarity condition.

\[
\frac{\partial L}{\partial \tilde{P}_i} = - \frac{F_i(p, N_B)}{2 \ln 2(1 + \tilde{P}_i)} + \lambda - \mu_i = 0, \quad i = 1, 2, \ldots
\]

(27)
Further, the complementary slackness conditions, $\lambda \left( \sum_{i=1}^{\infty} \tilde{P}_i - B \right) = 0$ and $\mu_i \tilde{P}_i = 0$, must be satisfied in the optimal solution. Hence, whenever $\tilde{P}_i > 0$, we must have, $\mu_i = 0$. From (27), we get,

$$\tilde{P}_i = \frac{F_i(p, N_B)}{2\lambda \ln 2} - 1, \quad i = 1, 2, \ldots,$$

where the optimal $\lambda$ can be found from (20b), as shown below.

**Solving for the optimal $\lambda$:** Due to the total energy constraint, we have,

$$B = \sum_{i=1}^{N} \tilde{P}_i = \sum_{i=1}^{N} \left( \frac{F_i(p, N_B)}{2\lambda \ln 2} - 1 \right),$$

$$= \frac{1}{2\lambda \ln 2} \sum_{i=1}^{N} F_i(p, N_B) - N,$$

$$\Rightarrow \lambda^* = \frac{\sum_{i=1}^{N} F_i(p, N_B)}{2 \ln 2 (B + N)},$$

where $N$ is the last slot where $\tilde{P}_i > 0$. From (28) and the expression for $\lambda^*$, we find that $N$ is the largest $n$ that satisfies

$$\sum_{i=1}^{n} F_i(p, N_B) \leq F_n(p, N_B)$$

Hence, from (28),

$$\tilde{P}_i = \frac{(B + N)F_i(p, N_B)}{\sum_{i=1}^{N} F_i(p, N_B)} - 1, \quad i = 1, 2, \ldots, N$$

The transmit power in the rest of the slots are zero. Further, noting that $F_i(p, N_B) = p/N_B$ for $i = 1, \ldots, N_B$, we obtain (22).
B. Proof of Theorem 2

\[ T_{\text{simp}} = \frac{\mathbb{E}\left[ \sum_{i=1}^{L} \frac{1}{2} \log(1 + \tilde{P}_i) \right]}{\mathbb{E}(L)}, \]

\[ = \frac{\mathbb{E}\left[ \sum_{i=1}^{L} \frac{1}{2} \log(1 + \frac{Bp}{N_B} \sum_{n=1}^{\infty} q_n) \right]}{\mathbb{E}(L)}, \]

\[ \geq a \frac{\mathbb{E}\left[ L \frac{1}{2} \log(1 + \frac{Bp}{N_B}) \right]}{\mathbb{E}(L)} + \frac{\mathbb{E}\left[ \sum_{i=1}^{L} \frac{1}{2} \log \left( \sum_{n=1}^{\infty} q_n \right) \right]}{\mathbb{E}(L)}, \]

\[ = \frac{1}{2} \log(1 + \frac{Bp}{N_B}) + \frac{p}{N_B} \sum_{i=1}^{\infty} q_i \frac{1}{2} \log \left( \sum_{n=1}^{\infty} q_n \right), \]

\[ = \frac{1}{2} \log(1 + \frac{Bp}{N_B}) + \frac{p}{N_B} \sum_{i=1}^{\infty} \left( \sum_{n=1}^{\infty} q_n \right) \frac{1}{2} \log \left( \sum_{n=1}^{\infty} q_n \right), \]

\[ \geq \frac{1}{2} \log(1 + \mu) - G(N_B) \]  

(34)

where (a) is because \( \log(1 + \alpha x) \geq \log(1 + x) + \log(\alpha) \) for \( 0 \leq \alpha \leq 1 \) and we note \( 0 \leq \sum_{n=1}^{\infty} q_n \leq 1 \), and we define \( G(N_B) \triangleq \max_p(-p/N_B \sum_{i=1}^{\infty} (\sum_{n=1}^{\infty} q_n) \frac{1}{2} \log (\sum_{n=1}^{\infty} q_n)). \)

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