On hard electroproduction of mesons: helicity flip amplitudes with tensor gluon contributions

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Abstract

We consider hard electroproduction of mesons at twist-3 level and focus our attention on helicity flip amplitudes corresponding to the production of transversely polarized mesons by transverse photons. We demonstrate that helicity-flip amplitude for the transversely polarized particles are closely related to the contributions of the matrix elements of the tensor gluon operator which can serve as pure gluonic probe of the hadrons. Using standard factorisation approach and Wanzura-Wilczek approximation for the twist-3 matrix elements we find that such amplitudes do not have divergencies from the endpoint regions which appear in the other twist-3 amplitudes.

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Introduction

Hard exclusive reactions provide a unique opportunity to study the partonic structure of the hadrons, see e.g. recent reviews [1, 2] and references there in. In recent years exclusive electroproduction of mesons from nucleons has become a topic of broad interest. At higher energies a large amount of data has become available from experiments at DESY (HERA, HERMES) [3, 4, 5]. Further measurements are expected in future at DESY and CEBAF. From the theoretical side the progress in description of exclusive electroproduction has been possible due to factorisation theorem [6]. It states that the underlying photon-parton sub-processes are dominated by longitudinally polarized photons and large photon virtualities, $Q^2 \gg \Lambda_{QCD}^2$ and hence can be calculated perturbatively. Using such OPE approach productions of mesons by longitudinal photons have been studied in series of papers [6, 7, 8, 9, 10, 11, 12, 13, 14].

The situation with description of interactions of transversely polarized photons is less trivial. According to analysis carried out in [6] such amplitudes are higher twists effects, i.e. suppressed as $1/Q$ compared to the longitudinal photon. At the same time experimental measurements show clearly that helicity conserving amplitudes with transverse photon provide large contribution to observables at moderate values of $Q^2$. Hence one must deal with higher twists effects to understand the dynamics of the hard exclusive electroproduction. Unfortunately, straightforward application of pQCD is not possible because the factorisation is violated for the meson production by a transverse photon [6, 7, 15, 16].

The violation of the factorisation for the transverse case is closely related to the so-called end-point or soft contributions [6, 7, 15]. Using terminology of reduced diagrams [6, 15] the OPE and endpoint contributions for hard electroproduction process are shown in Fig. 1 and Fig. 2 respectively. In case of the transverse virtual photon or transverse outgoing meson the endpoint regime contributes to the same accuracy $1/Q^2$ as short distance configuration Fig. 1 [6, 7, 15]. Practically, the divergencies in the convolution integrals of the OPE-like configuration can be considered as a test for the possible contributions associated with endpoint diagram Fig. 2. A direct calculation carried out for the transverse $\rho$-meson [16] supports this observation. On the other hand, if the endpoint divergencies are absent in the convolution integrals one can hope that soft configurations give sub-leading contribution to an amplitude of this process.

In present paper we shall study helicity-flip amplitudes in the hard exclusive meson electroproduction. A typical feature of such processes is a contributions of the tensor gluon operators. Recently helicity-flip amplitudes have been the subject of theoretical research in DVCS [17, 18, 19], fragmentation [20] and meson production in $\gamma^*\gamma$ fusion [21, 22]. We shall demonstrate that helicity-flip amplitudes with tensor gluons in the exclusive meson electroproduction do not possess the endpoint divergencies and hence one can hope that these amplitudes can be described in the framework of QCD factorisation. For simplicity we shall consider scalar pion target, the generalization to the nucleon target is straightforward.

The remainder of this paper is organized as follows: in the next section we discuss the amplitude for the hard electroproduction of tensor mesons. The following section is devoted to discussion of the helicity-double-flip amplitude for the case of $\rho$-meson electroproduction. Finally, we conclude. Some technical details of present calculation are summarized in the Appendix.
Hard electroproduction of spin-2 meson with tensor polarization

The amplitude of the process

$$\gamma^*(q) + \pi(p) \rightarrow f_2(Q') + \pi(p'),$$  \hspace{1cm} (1)

is defined in terms of the matrix element of the electromagnetic current:

$$T^\mu = \langle \pi(p'), f_2(Q', \lambda) | J_{\text{em}}^\mu (0) | \pi(p) \rangle,$$  \hspace{1cm} (2)

where index $\lambda$ corresponds to the helicity of the tensor meson.

We shall consider the Bjorken limit, where collision energy $W$ and virtuality of the photon are large: $W \rightarrow \infty$, $-q^2 = Q^2 \rightarrow \infty$, but their ratio $Q^2/W$ is fixed. Let $p, p'$ and $q$ denote the momenta of the initial and final pions and photon, respectively. We introduce also light-cone vectors $n, n^*$ such that

$$n \cdot n = 0, \quad n^* \cdot n^* = 0, \quad n \cdot n^* = 1.$$

(3)

We shall work in the reference frame where the average nucleon momenta $P = \frac{1}{2}(p + p')$ and the virtual photon momentum $q$ are collinear along the $z$-axis and have opposite directions. To the twist-3 accuracy such a choice of the frame results in the following decomposition for the momenta:

$$P = \frac{1}{2}(p + p') = n^*, \quad \Delta = p' - p = -2\xi P + \Delta_\perp,$$

$$q = -2\xi P + \frac{Q^2}{4\xi} n, \quad Q' = q - \Delta = q' + \frac{m^2}{2(Pq')} n^* - \Delta_\perp,$$

$$Q'^2 = m^2, \quad q' = \frac{Q^2}{4\xi} n.$$

(4)

(5)

where $m = 1270$ MeV is the $f_2$-meson mass.

We also define the transverse metric and antisymmetric transverse epsilon tensors: \[^6\]

$$g^\perp_{\mu \nu} = g^{\mu \nu} - n^\mu n^* \, n^\nu n^* \, n^\sigma n^* \, \epsilon^\perp_{\mu \nu}, \quad \epsilon^\perp_{\mu \nu} = \epsilon_{\mu \nu \alpha \beta} n^\alpha n^\beta n^* \, n^* \, n^\sigma n^*.$$  \hspace{1cm} (6)

We start from short discussion of the amplitude in the leading twist approximation. This amplitude describes the scattering of longitudinal photon and production of longitudinal meson. The factorisation theorem \[^6\] states that such amplitude is dominated by OPE-configuration shown in Fig. 1 and can be written in the form of integral convolution of a hard and soft collinear parts. The hard part is given by scattering amplitude of partonic subprocess and can be calculated in pQCD. The soft blocks are represented in terms of matrix elements of the light-cone operators. Typical diagrams contributing to our process are shown in Fig. 4 and Fig. 5.

Let us define twist-2 skewed parton distributions (SPD’s) and distribution amplitudes (DA’s) which parameterize soft matrix elements in the expression for the leading twist amplitude.

\[^6\] The Levi-Civita tensor $\epsilon_{\mu \nu \alpha \beta}$ is defined as the totally antisymmetric tensor with $\epsilon_{0123} = 1$.
Twist-2 SPD of the pion is given by \[ p' | \bar{\psi} \left( \frac{\lambda}{2} n \right) \gamma^\mu \psi \left( -\frac{\lambda}{2} n \right) | p \rangle = P^\mu \int_{-1}^{1} dx e^{ix\lambda} H(x, \xi), \] (7)

where, for the sake of simplicity, we do not write explicitly the \( t \)-dependence of the SPD, denoting \( H(x, \xi) \equiv H(x, \xi, t) \). Notice that this SPD is symmetric under the interchange of \( x \leftrightarrow -x \):

\[ H(-x, \xi) = H(x, \xi) \] (8)

Twist-2 DA’s for the tensor meson have been introduced in [21]. Taking into account the kinematics \([4]\) we obtain:

\[ \langle f_2(Q', \lambda) | \bar{\psi}(n^*) \gamma_\mu \psi(-n^*) | 0 \rangle = f_q m^2 g_\mu \epsilon^{(\lambda)}_{a\beta} n^* a^* b \int_{-1}^{1} du e^{iu(n^*)} \phi_q(u), \] (9)

where \( \bar{\psi} \psi = \bar{u}u + \bar{d}d \) and the symmetric and traceless polarization tensor \( \epsilon^{(\lambda)}_{a\beta} \) satisfies the condition \( \epsilon^{(\lambda)}_{a\beta} Q'_{\beta} = 0 \). Polarization sums can be calculated using

\[ \sum_{\lambda} \epsilon^{(\lambda)}_{\mu\nu} \left( \epsilon^{(\lambda)}_{\rho\sigma} \right)^* = \frac{1}{2} M_{\mu\rho} M_{\nu\sigma} + \frac{1}{2} M_{\mu\sigma} M_{\nu\rho} - \frac{1}{3} M_{\mu\nu} M_{\rho\sigma}, \] (10)

where \( M_{\mu\nu} = g_{\mu\nu} - Q'_\mu Q'_\nu/m^2 \) and the normalization is such that \( \epsilon^{(\lambda)}_{\mu\nu} (\epsilon^{(\lambda')}_{\mu\nu})^* = \delta_{\lambda\lambda'} \).

In addition, there exists leading twist gluon distribution amplitude:

\[ \langle f_2(Q', \lambda) | S_{\mu\nu} G^{a}_\mu(n^*) G^{a}_\nu(-n^*) | 0 \rangle = f^S g^{(\lambda)}_{\mu\nu} \int_{-1}^{1} du e^{iu(n^*)} \phi^S_g(u). \] (11)

Here \( S_{\mu\nu} \) stands for the symmetrisation in the two indices and removal of the trace: \( S_{\mu\nu} O_{\mu\nu} = \frac{1}{2} O_{\mu\nu} + \frac{1}{2} O_{\nu\mu} - \frac{1}{4} g_{\mu\nu} O_{\xi\xi} \).

The distribution amplitudes \( \phi_q(u) = -\phi_q(-u) \) and \( \phi^S_g(-u) = \phi^S_g(u) \) are the leading twist-2 distribution amplitudes for the tensor mesons with helicity \( \lambda = 0 \). The constants \( f_q \) and \( f^S_g \) are defined as the matrix elements of the local operators, for details see [21].

In terms of these functions expression for leading twist amplitude reads:

\[ T_{\gamma \ell \rightarrow f_2(0)}^\mu = \frac{(2\xi P + q')^\mu}{(P q')} \frac{m^2}{(P q')} \frac{(4\pi e_\alpha)}{4N_c} \frac{1}{\xi} \int_{-1}^{1} dx H(x, \xi) C^-(x, \xi) \int_{-1}^{1} du \frac{1}{1 - u^2} \left\{ \frac{1}{2} C_F f_q u \phi_q(u) + f^S_g \phi^S_g(u) \right\}, \] (12)

where

\[ C^\pm(x, \xi) = \frac{1}{x - \xi + i0} \pm \frac{1}{x + \xi - i0}. \] (13)

\(^\dagger\)The gauge link between points on the light-cone is not shown but always assumed.
The flavor dependence in Eq.(12) can be restored easily by substitution:

$$H(x, \xi) \rightarrow e_u H_u(x, \xi) + e_d H_d(x, \xi),$$

(14)

where $e_f$ is charge of a quark of flavor $f = u, d$ in units of the proton charge ($e_u = 2/3, e_d = -1/3$).

All convolution integrals in (12) are well defined as it should be according to factorisation theorem. It is obvious, that amplitude (12) can be considered as a particular limit of more general situation of production of two pions from longitudinal photon. Such process have been considered in [13] and our result (12) is in agreement with that presented in [13].

At the twist three level § there exist three amplitudes with helicity flip: $\gamma_\perp(\pm 1)\pi \rightarrow f_2(0)\pi$, $\gamma_L(0)\pi \rightarrow f_2(\pm 1)\pi$ and $\gamma_\perp(\pm 1)\pi \rightarrow f_2(\pm 2)\pi$. In what follows we shall consider helicity flip amplitude describing transition $\gamma_\perp(\pm 1)\pi \rightarrow f_2(\pm 2)\pi$ because tensor gluon contribution can appear only in this case. Let us note that diagrams like in Fig. 5 can not contribute to the twist-3 accuracy to such amplitude because meson state with $\lambda = \pm 2$ constructed from two quarks in P-wave is suppressed by extra power of large $Q^2$ in comparison with gluons carrying equal helicities.

The space time development of partonic process with such gluons is shown in Fig.3. For simplicity we refer to the region where all momentum fractions are positive and hence the partonic subprocess is

$$\gamma^* + q \rightarrow gg + q$$

(15)

As one can see from that picture, in the pure collinear kinematics the projections of the angular momentum $J_z$ in initial and final states are not balanced. To match them we must consider diagrams with emission of the transverse gluons (genuine twist-3 contributions) or include kinematical effects due to orbital motion of the quarks. In what follows we neglect for simplicity the genuine twist-3 contributions and consider only kinematical or Wandzura-Wilczek (WW) contributions to the twist-3 matrix elements.

In our calculations we shall use following matrix elements. The tensor meson with helicities $\lambda = \pm 2$ is created by two collinear gluons with equal helicities, see Fig. 3. Corresponding twist-2 matrix element have already been considered in [21]:

$$\langle f_2(Q', \lambda)| S_{\mu\nu}G_{\ast \lambda}(n^\ast)G_{\ast \lambda}(-n^\ast)|0 \rangle = f_g^{T} e^{\lambda}(q'n^\ast)^{2} \int_{-1}^{1} du e^{iu(q'n^\ast)}\phi^{T}_g(u),$$

(16)

where we used notation $G_{\ast \lambda} = n^\ast G_{\xi \mu}$.

The distribution amplitude $\phi^{T}_g(u)$ is symmetric with respect to the interchange of $u \leftrightarrow -u$ and describes the momentum fraction distribution of the two gluons in the $f_2$-meson. The asymptotic distribution at large scales is equal to

$$\phi^{T,as}_g(u) = \frac{15}{16(1-u^2)^2}.$$

(17)

The constant $f_g^{T}$ is renormalized multiplicatively [11, 23, 24]:

$$f_g^{T}(Q^2) = f_g^{T}(\mu^2)L^{-1+6N_c/\beta_0},$$

(18)

§We adopt here kinematical definition of twist, i.e. terms suppressed by $1/Q^2$ are of twist-3.
where $L = \alpha_s(Q^2)/\alpha_s(\mu^2)$, $\beta_0 = 11/3N_c - 2/3n_f$.

Expressions for the pion matrix elements must be written up to twist-3 accuracy. Corresponding results have been obtained in [23, 26, 27, 28]. We shall follow notation adopted in [26, 29]:

\[
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p' | \bar{\psi} \left( -\frac{\lambda}{2} \right) \gamma^\mu \psi \left( \frac{\lambda}{2} \right) | p \rangle = P^\mu H(x, \xi) + \Delta^\mu H_3(x, \xi),
\]

(19)

\[
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p' | \bar{\psi} \left( -\frac{\lambda}{2} \right) \gamma_\mu \gamma_5 \psi \left( \frac{\lambda}{2} \right) | p \rangle = i\varepsilon_{\mu\alpha\beta\delta} \Delta^\alpha P^\beta n^\delta H_A(x, \xi).
\]

(20)

In the WW-approximation twist-3 SPD's $H_3(x, \xi)$ and $H_A(x, \xi)$ can be rewritten in terms of twist-2 function $H(x, \xi)$. Corresponding expressions read:

\[
H_3^{WW}(x, \xi) = -\frac{1}{2} \int_{-1}^1 du W_+(x, u, \xi) \frac{\partial}{\partial \xi} H(u, \xi) - \frac{1}{2} \int_{-1}^1 du W_-(x, u, \xi) \frac{\partial}{\partial u} H(u, \xi),
\]

(21)

\[
H_A^{WW}(x, \xi) = \frac{1}{2\xi} \int_{-1}^1 du W_-(x, u, \xi) \left( \xi \frac{\partial}{\partial \xi} + u \frac{\partial}{\partial u} \right) H(u, \xi),
\]

(22)

where WW-kernels read

\[
W_\pm(x, u, \xi) = \frac{1}{2} \left\{ \theta(x > \xi) \frac{\theta(u > \xi)}{u - \xi} - \theta(x < \xi) \frac{\theta(u < \xi)}{u - \xi} \right\} \pm \frac{1}{2} \left\{ \theta(x > -\xi) \frac{\theta(u > \xi)}{u + \xi} - \theta(x < -\xi) \frac{\theta(u < \xi)}{u + \xi} \right\}.
\]

(23)

Typical diagrams contributing to the amplitude $\gamma_{\perp}(\pm 1)\pi \rightarrow f_2(\pm 2)\pi$ at twist-3 level are depicted in the Fig. 4. Using matrix elements (16), (19) and (20) we obtained:

\[
T_{\gamma_{\perp}(\pm 1)\rightarrow f_2(\pm 2)}^\mu = e^{(\mu_{\perp} \cdot \nu)} \Delta_{\perp \nu} T_2(Q^2, \xi),
\]

(24)

\[
T_2(Q^2, \xi) = \frac{f_g^T}{Q^2} (4\pi \alpha_s) \frac{8}{N_c} \int_{-1}^1 du \frac{\phi_u^T(u)}{(1 - u^2)^2} \times \int_{-1}^1 dx C^-(x, \xi) \left\{ \left( x \frac{\partial}{\partial x} + \xi \frac{\partial}{\partial \xi} \right) H(x, \xi) - \xi H_3(x, \xi) - x H_A(x, \xi) \right\},
\]

(25)

where $e^{(\mu_{\perp} \cdot \nu)}$ is symmetric and traceless transverse projection of the polarization vector $e_{\mu\nu}^{(\perp)}$.

The flavor dependence in Eq.(24) can be restored by same substitution as in (14).

Consider the convolution integrals which appear in the RHS of (24). The integral with meson distribution amplitude has now stronger endpoint singularities in the coefficient function in comparison with twist-2 case (12). But we expect that gluon distribution amplitude vanishes faster than $u$ as $u \rightarrow \pm 1$. At least asymptotical shape (17) supports such expectations. Then singularities at $u = \pm 1$ of the coefficient function is compensated by endpoint behavior of the gluonic wave function and the integral is convergent.

The second integral, defining convolution with pion SPD’s is also well defined. To see this, one has to inspect properties of the convolution integrals which appear in (24) at the points
$x = \pm \xi$. Consider first combination like $(x \frac{\partial}{\partial x} + \xi \frac{\partial}{\partial \xi}) H(x, \xi)$. It has no discontinuities at the points $x = \pm \xi$. One can see this easily integrating by parts and using that $H(x = \pm 1, \xi) = 0$:

$$\int_{-1}^{1} du C^{-}(x, \xi) \left( x \frac{\partial}{\partial x} + \xi \frac{\partial}{\partial \xi} \right) H(x, \xi) = \xi \frac{\partial}{\partial \xi} \int_{-1}^{1} dx C^{-}(x, \xi) H(x, \xi).$$  \hspace{1cm} (26)

where RHS is well defined. The remaining part of the convolution integral with SPD in (24) depends on the combination $\xi H_3(x, \xi) + x H_A(x, \xi)$ which is free from discontinuities at the points $x = \pm \xi$. One can prove it in WW-approximation using (21) and (22).

In general case, the factorisation for twist-3 amplitude for the meson production is broken as it have been discussed in [9]. This is closely related to the fact, that in addition to the OPE-like diagram Fig. 1, there is contribution of the same order from other reduced diagram depicted in Fig. 2. This is exactly the, so-called, endpoint contribution discussed in [7, 15]. For instance, the divergencies of the convolution integrals which were found in twist-3 calculations for the exclusive production of transversely polarized vector mesons [16] can be considered as a signal that contributions from the regions depicted in Fig. 2 must be taken into account. We expect that such situation takes place also for other twist-3 helicity amplitudes which were mentioned above, namely, $\gamma_1^* \pi \rightarrow f_2(\pm 1)\pi$, $\gamma_1^*(\pm 1)\pi \rightarrow f_2(0)\pi$.

However, we observed that such divergencies are absent in the expression (23). We suppose that it could be related to a possibility that OPE-like contribution dominates at twist-3 accuracy in such helicity-flip amplitude. To see it rigorously one has, of course, to prove that the possible endpoint contributions corresponding to Fig. 2 is sub-leading. We shall not address this question in details in the paper. Some naive arguments in support of our speculation can be given on the basis of conservation of angular momentum. To get the spin projection $S_z = 2$ the quark and antiquark have to be in a $P$-wave state while the gluons can be in $S$-wave. On the light-cone, however, contribution of the orbital angular momentum is higher-twist and amplitude corresponding to helicity state $\lambda = \pm 2$ is dominated at large photon virtualities by gluon contribution. Of course, such arguments can not guarantee the absence of the soft configurations even if contributions from the OPE-like configurations Fig. 1 are free from the divergencies. In this paper we adopt the point of view which states that if factorisation is not violated, i.e. there are no divergencies in the convolution integrals, then possible non-OPE configurations are sub-leading. From this assumption it follows that helicity flip amplitude $\gamma_1^*(\pm 1)\pi \rightarrow f_2(\pm 2)\pi$ can be described in the framework of factorisation approach.

Since $f_2$ decays in two pions with the branching ratio about 85%, one natural possibility to measure the amplitude of interest is via the hard exclusive two-pion electroproduction $\gamma + p \rightarrow p\pi\pi\pi$. This process has been considered recently at the leading twist approximation in [13]. Let $k_1$ and $k_2$ be the momenta of the two pions in the final state and $k = (k_1 - k_2)$ is their relative momentum. The form factor of interest $T_2(Q^2, \xi)$ (24) is related to the corresponding two-pion amplitude $A^{\pm}_{\gamma(\pm 1)\rightarrow \pi\pi(\pm 2)}$ in the region $(k_1 + k_2)^2 \sim m^2$ as

$$A^{\pm}_{\gamma(\pm 1)\rightarrow \pi\pi(\pm 2)} = \varepsilon_\mu(\pm)(k_\mu^\perp k_\nu^\perp - \frac{1}{2} g^{\mu\nu} k_\perp^2) \Delta_{\mu\nu} \frac{g_{f_2\pi\pi}}{m^2} \frac{T_2(Q^2, \xi)}{m^2 - (k_1 + k_2)^2},$$  \hspace{1cm} (27)

*We use notation $\varepsilon_\mu(\pm)A^\mu \equiv A^\pm$, where $\varepsilon_\mu(\pm)$ denotes the transverse polarization vector of the virtual photon.
where $g_{f_2\pi\pi}$ is the corresponding decay constant:

$$\langle \pi(k_1)\pi(k_2)|f_2(P,\lambda)\rangle = \frac{g_{f_2\pi\pi}(\lambda)}{m}e^{\alpha\beta}k^\alpha k^\beta. \quad (28)$$

Hence $A^\pm_{\pi(\pm 1)\rightarrow \pi(\pm 2)}$ can be separated using its nontrivial dependence on the azimuthal angle $\phi_{\pi\pi}$ between lepton and decay pion planes. Moreover, $A_\pm$ is symmetric under the interchange of the pion momenta (isoscalar state) and can be measured by the interference with the contribution of the two-pion production in isovector state that is antisymmetric to the interchange of the pion momenta: In the difference of cross sections $\sigma(k_1, k_2) - \sigma(k_2, k_1)$ only the interference term survives. Some estimates of the interference for the leading twist cross sections have been presented in [13, 32].

**Hard electroproduction of $\rho$-meson with double-helicity-flip**

In this section we consider another possibility to get twist-3 amplitude with tensor gluon operator. It is possible in the process of $\rho$-meson production with double-helicity-flip: $\gamma^*(\pm 1)\pi \rightarrow \rho(\mp 1)\pi$. Such amplitude is responsible for the violation of the s-channel helicity conservation (SCHC) [33]. The effects of the violation of the SCHC in electroproduction have been considered in various models in [34, 33, 35]. Our approach is different because it is based on the application of OPE. In such approach double-helicity-flip production of the $\rho$-meson is analogous, in some sense, to helicity flip amplitude in DVCS [17, 18]. But in DVCS such amplitude is leading twist-2 contribution. In case of electroproduction this amplitude is non-leading twist-3 effect, i.e. behaves as $1/Q^2$ because $\rho$ meson is a composite particle.

The amplitude of the process

$$\gamma^*(q) + \pi^\pm(p) \rightarrow \rho_0(Q') + \pi^\pm(p'), \quad (29)$$

is defined in terms of the matrix element of the electromagnetic current:

$$M^\mu = \langle \pi(p'), \rho(Q', \lambda)|J^\mu_{e.m.}(0)|\pi(p)\rangle, \quad (30)$$

Decomposition of momenta can be obtained from (4) using substitution $m \rightarrow m_\rho$ where $m_\rho$ is the $\rho$-meson mass.

Typical diagrams contributing to $\rho$-meson helicity flip amplitude are depicted in Fig. 6. The space time development of the corresponding hard partonic subprocess is shown in the Fig. 7. For simplicity, we consider only situation when all momentum fractions are positive. Again, we observe that for pure collinear scattering process there is a difference of angular momentum projections between initial and final partonic states. Therefore, one may conclude that amplitude of this process is suppressed at least by $1/Q$ relative to twist-2 amplitude of pure longitudinal $\rho$-meson production $\gamma_L^*\pi \rightarrow \rho_L\pi$. To obtain the leading contribution to the helicity flip amplitude one must take into account the twist-3 contributions to the $\rho$-meson matrix elements. Corresponding distribution amplitudes have been introduced in [37, 38]. As in the previous case, in our calculations we neglect genuine twist-3 effects.

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||To avoid confusion we stress that “pion plane” is the plane which is defined by pions from decay $f_2 \rightarrow \pi\pi$.||
and consider only corresponding WW-contributions. We define chiral even two-particle distribution amplitudes of the \( \rho \)-meson as

\[
\langle \rho(Q', \lambda)|\bar{\psi}(n^*)\gamma_\mu\psi(-n^*)|0\rangle = f_\rho m_\rho q_\mu \frac{(e^*n^*)}{(q' n^*)} \int_{-1}^{1} du \phi(u) e^{i(q' n^*)u} \\
+ f_\rho m_\rho e_{\perp\mu} \int_{-1}^{1} du G(u) e^{i(q' n^*)u},
\]

(31)

\[
\langle \rho(Q', \lambda)|\bar{\psi}(n^*)\gamma_5\gamma_\mu\psi(-n^*)|0\rangle = \frac{i}{2} f_\rho m_\rho e_{\mu\alpha\beta} e^{*\gamma(n^*)\nu} \int_{-1}^{1} du \bar{G}(u) e^{i(q' n^*)u},
\]

(32)

where \( f_\rho \) and \( e^*_\mu \) denote the decay constant and polarization vector of a \( \rho \)-meson and \( \bar{\psi}\psi = \bar{u}u - \bar{d}d \). Twist-3 distributions amplitudes \( G(u) \) and \( \bar{G}(u) \) in the WW-approximation are given by following relations:

\[
G(u) = \frac{1}{2} \int_{-1}^{u} dw \frac{\phi(w)}{1 - w} + \frac{1}{2} \int_{u}^{1} dw \frac{\phi(w)}{1 + w},
\]

(33)

\[
\bar{G}(u) = \int_{-1}^{u} dw \frac{\phi(w)}{1 - w} - \int_{u}^{1} dw \frac{\phi(w)}{1 + w},
\]

(34)

Gluon SPD is defined by tensor gluon operator in the following way:

\[
\langle p'|S_{\mu\nu}G_{\lambda}(\frac{\lambda}{2}n)G_{\lambda}(\frac{-\lambda}{2}n)|p\rangle = S_{\mu\nu}\Delta_{\perp\mu}\Delta_{\perp\nu}/f_\pi^2 \int_{-1}^{1} dx e^{i\pi\lambda} G^T(x, \xi),
\]

(35)

where we introduced a factor \( 1/f_\pi^2 \) in order to have the dimensionless combination \( \Delta_{\perp\mu}\Delta_{\perp\nu}/f_\pi^2 \). The SPD \( G^T(w, \xi) \) is symmetric to the interchange of \( x \leftrightarrow -x \) and at large scale \( \mu \to \infty \) the leading contribution equals to

\[
G^T_{as}(x, \xi) = G^T(\mu) \frac{15}{16} \frac{1}{\xi^5} (x^2 - \xi^2)^2 \theta(|x| \leq \xi).
\]

(36)

The dimensionless constant \( G^T(\mu) \) is renormalized multiplicatively with the same anomalous dimension as \( f_g^T \) in (18).

A straightforward calculation of Feynman diagrams, Fig. 7, gives:

\[
M^\mu = \frac{f_\rho m_\rho}{Q^2} (4\pi e\alpha_s) \frac{S_{\mu\nu}\Delta_{\perp\mu}\Delta_{\perp\nu}}{f_\pi^2} \frac{2\xi^2}{N_c} \times \int_{-1}^{1} du \frac{\phi(u)}{1 - u^2} \int_{-1}^{1} dx \frac{G^T(x, \xi)}{(x - \xi + i0)^2(x + \xi - i0)^2}
\]

(37)

Let us first examine the properties of the convolution integrals. The twist-2 \( \rho \)-meson distribution amplitude which enters into the answer can be taken close to its asymptotic shape:

\[
\phi(u) = \frac{3}{4}(1 - u^2).
\]

(38)
Then one can see that corresponding convolution integral in (37) is well defined. It is interesting that possible contributions of twist-3 distributions amplitude $G(u)$ and $\tilde{G}(u)$ cancel in the sum of all diagrams. We do not see special reasons for this, except that it can be a consequence of the WW-relations (33) and (34). Let us note, that this cancellation is not related to the question about behavior at the points $u = \pm 1$. From the calculation one can see that contribution of each diagram is nonsingular because twist-3 distributions $G(u)$ and $\tilde{G}(u)$ enter in a combination which nullifies at the end-points. More detailed discussion of this question is given in the Appendix.

The second integral in (37) is well defined if tensor gluon SPD $G^T(w, \xi)$ and its first derivative have no discontinuities at the points $x = \pm \xi$. At present the detailed properties of this gluon SPD are poorly known. Therefore, the one possible procedure to get information about behavior at these points is to study the asymptotic $\mu \to \infty$ limit of SPD evolution. The asymptotic expression $G^T_{as}(x, \xi)$ is given by (36). Using this expression one obtains that corresponding convolution integral is free from singularities. Assuming that the analytical properties at the points $x = \pm \xi$ of the non-asymptotic amplitude are the same, i.e. at least first derivative is smooth function, we conclude that integral with tensor gluon SPD converges. Therefore we observe again that the convolution integrals of the twist-3 amplitude with tensor gluons do not have divergencies at the end-points.

The tensor structure $S_{\mu\nu}\Delta^\mu_+\Delta^\nu_+$ signals a $\cos^2\phi$ term in the cross section where $\phi$ is angle between lepton and hadron planes. The azimuthal dependence can be used to make a direct extraction of the double-spin-flip amplitude. The detailed description of the polarization density matrix for the $\rho^0$-meson can be found in [39].

Assuming that factorisation takes place we conclude that electroproduction of $\rho$-meson with helicity flip provides a good opportunity to study effects due to transverse gluons in a target. Concerning to possible higher twist corrections we expect that their effect can not be large. Parametrically, a ratio of twist-3 to higher twist amplitudes behaves like $f_{\rho^0}\rho^0 \frac{M^2}{Q^2} : f_{\rho^0}\rho^0 \frac{M^2}{Q^2}$, where $M^2$ is some soft scale, for instance, $\rho$-meson mass or a nucleon mass for nucleon target. This estimate is based on considerations of possible partonic contributions with nonzero orbital momentum. At this point one finds a difference with DVCS helicity flip amplitude. As it was shown in [40] helicity flip amplitude in DVCS can be very sensitive to the power corrections related to orbital motion of quarks.

Conclusions

We have calculated twist-3 helicity-flip amplitudes for the hard meson electroproduction in the Wanzura-Wilczek approximation on a scalar pion target. We considered two cases corresponding to production of C-even tensor meson $f_2$ and C-odd vector meson $\rho$. We have found that both amplitudes receive contributions from tensor gluon operator. In both cases convolution integrals in the amplitudes are free of the endpoint singularities at least for the asymptotical approximations of the non-perturbative matrix elements of the tensor gluon operator. We conclude therefore that basing on this observation we may suppose that factorisation is not violated for these twist-3 amplitudes.
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APPENDIX: Calculation of the $\rho$-meson helicity-double-flip amplitude

Here we briefly describe the calculations of the $\rho$-meson amplitude with helicity flip and discuss contributions of the twist-3 functions $G(u)$ and $\tilde{G}(u)$. The diagrams contributing to our amplitude are depicted in Fig. 6. Consider as example calculation of diagram $D_3$. Assuming for simplicity that $N_f = 1$ and taking into account definition of the amplitude (37) we obtain:

$$D_3^\mu = e(i)^2 \left( \frac{i}{2\pi^2} \right)^2 \int d^4x \int d^4y \langle \pi(p') , \rho(Q', \lambda) | \tilde{\psi}(x) \hat{A}(x) t^a \gamma^\mu \frac{\gamma^\nu - \gamma^\nu}{y^4} \hat{A}^b(y) t^b \psi(y) | \pi(p) \rangle,$$ (A.1)

where we use notation $\hat{A} = \gamma^\mu A_\mu$ and $t^a$ denotes the generators of the SU($N_c$) group which satisfy $\text{Tr}(t^a t^b) = \frac{1}{2} \delta^{ab}$. Rewriting this expression in terms of the two matrix elements: $\langle p' | A(x) A(y) | p \rangle$ and $\langle Q' | \tilde{\psi}(x) \gamma^\nu \psi(y) | 0 \rangle$ we obtain matrix elements with gluon fields. Remind that we need only twist-2 tensor projection. Using axial gauge we have (15):

$$\langle p' | S_{\alpha \beta} A^\alpha(x) A^\beta(y) | p \rangle = \frac{\Delta_{\perp}^{(\alpha \beta)}}{f^2} \int dx \frac{G^\perp(x, \xi)}{(x - \xi + i0)(x + \xi - i0)} e^{i(P_x)(x-\xi)-(P_y)(x+\xi)} ,$$ (A.2)

where $S_{\alpha \beta} \Delta_\perp^{(\alpha \beta)} \equiv \Delta_\perp^{(\alpha \beta)}$. For the quark matrix element it is convenient to use Fierz identities **:

$$\langle Q' | \tilde{\psi}(x) \gamma^\nu \psi(y) | 0 \rangle = \frac{1}{4} \text{Tr} \left\{ \gamma_0 \Gamma \right\} \langle Q' | \tilde{\psi}(x) \gamma^\nu \psi(y) | 0 \rangle - \frac{1}{4} \text{Tr} \left\{ \gamma_0 \gamma_5 \Gamma \right\} \langle Q' | \tilde{\psi}(x) \gamma^\nu \gamma_5 \psi(y) | 0 \rangle ,$$ (A.3)

where $\Gamma$ is generic Dirac matrix structure. Perfoming these transformations and substituting gluon matrix element in (A.1) we arrive at:

$$D_3^\mu = -\frac{e}{4\pi^4} \frac{1}{2N_c f^2} \int dx \frac{G^\perp(x, \xi)}{(x - \xi + i0)(x + \xi - i0)} \int d^4x \int d^4y e^{i(P_x)(x-\xi)-(P_y)(x+\xi)} \frac{\Delta_{\perp}^{(\alpha \beta)}}{x^4y^4}$$

$$\times \left\{ \frac{1}{4} \text{Tr} \left\{ \gamma_0 \gamma_\alpha \hat{x} \gamma^\mu \hat{y} \gamma_\beta \right\} \langle Q' | \tilde{\psi}(x) \gamma^\nu \psi(y) | 0 \rangle - \frac{1}{4} \text{Tr} \left\{ \gamma_0 \gamma_5 \gamma_\alpha \hat{x} \gamma^\mu \hat{y} \gamma_5 \gamma_\beta \right\} \langle Q' | \tilde{\psi}(x) \gamma^\nu \gamma_5 \psi(y) | 0 \rangle \right\} .$$

At the next step one has to substitute into (A.4) twist-3 expressions (31) and (32) for the quark matrix elements and perfom coordinate-space integrations. The calculation of the other diagrams can be done using the similar approach. The final results can be written as follows:

$$D_3^\mu = \frac{\Delta_{\perp}^{(\mu \nu)}}{Q^2} e^\nu C \int dx \frac{(-1)G^\perp(x, \xi)}{(x - \xi + i0)(x + \xi - i0)} \int du \frac{G(u) - u}{u - \phi(u)} \left\{ G(u) - u \tilde{G}(u) - \phi(u) \right\} ,$$ (A.4)

**We define Dirac matrix $\gamma_5$ as $\gamma_5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3$ and $\text{Tr} \{ \gamma_5 \gamma_\mu \gamma_\rho \gamma_\nu \gamma_\delta \} = 4i \varepsilon_{\mu \rho \nu \delta \gamma}$.**
\[
 D_2^\mu = \frac{\Delta^{(\mu\nu)}}{Q^2} \epsilon_\nu^* C \int_{-1}^{1} dx \frac{G_T(x, \xi)}{(x - \xi + i0)^2(x + \xi - i0)} \int_{-1}^{1} \frac{du}{1 - u^2} \left[ G(u) - \frac{u}{2} \tilde{G}(u) - \phi(u) \right] \tag{A.5}
\]

\[
 D_3^\mu = \frac{\Delta^{(\mu\nu)}}{Q^2} \epsilon_\nu^* C \int_{-1}^{1} dx \frac{G_T(x, \xi)}{(x - \xi + i0)^2(x + \xi - i0)^2} (-2\xi) \int_{-1}^{1} \frac{du}{1 - u^2} \left[ G(u) - \frac{u}{2} \tilde{G}(u) \right] \tag{A.6}
\]

where the common factor \( C = \frac{f_{am} f_{am}}{f_{\pi}} (4\pi e \alpha_S) \frac{2 \xi}{N_c} \). We have also used the properties of the WW-contributions (33) and (34) to rewrite the contributions \( D_1^\mu \) and \( D_2^\mu \) in the form presented in (A.4) and (A.5). Using (33) and (34) one finds that functions \( G(u) \) and \( \tilde{G}(u) \) do not equal zero at the end points:

\[
 G(\pm 1) = \frac{1}{2} \int_{-1}^{1} du \frac{\phi(u)}{1 - u}, \quad \tilde{G}(\pm 1) = \pm \int_{-1}^{1} du \frac{\phi(u)}{1 - u}, \tag{A.7}
\]

In our case they enter into the expressions (A.4)–(A.6) in the combination \( G(u) - \frac{u}{2} \tilde{G}(u) \) which nullifies at the points \( u = \pm 1 \). The latter is very important for convergence of the corresponding integrals in (A.4)–(A.6). However, in the sum of the all contributions these WW-terms cancel:

\[
 D_1^\mu + D_2^\mu + D_3^\mu = \frac{\Delta^{(\mu\nu)}}{Q^2} \epsilon_\nu^* C (-2\xi) \int_{-1}^{1} dx \frac{G_T(x, \xi)}{(x - \xi + i0)^2(x + \xi - i0)^2} \int_{-1}^{1} \frac{du}{1 - u^2} \frac{\phi(u)}{1 - u}. \tag{A.8}
\]

As we note in the main body of the paper it may be related to a property of the WW-approximation. To get final answer (37) from (A.8) one has to take into account the flavor structure which reduces to multiplication of (A.8) on the factor \( \frac{1}{2}(e_u - e_d) = \frac{1}{2} \).

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Figure 1: OPE-like regime for the hard electroproduction of meson.

Figure 2: Endpoint regime.

Figure 3: The partonic subprocess for the production of the $f_2$-meson by tensor gluons (left) and its space time development (right). The arrows above each partonic line denote the projection of the spin of the particle on $z$-axis.
Figure 4: Typical diagrams for electroproduction of $f_2$ with gluon distribution amplitudes.

Figure 5: Typical diagrams for electroproduction of $f_2$ with quark distribution amplitude.
Figure 6: Diagrams for the electroproduction of $\rho$-meson with double-helicity-flip. Their contributions are denoted as D1, D2 and D3 respectively.

Figure 7: The partonic subprocess for the double-helicity-flip amplitude of the $\rho$-meson (left) and its space time development (right). The arrows above each partonic line denote the projection of the spin of the particle on $z$-axis.