Chaotic Inflation with Right-handed Sneutrinos after Planck

Kazunori Nakayama\textsuperscript{a,c}, Fuminobu Takahashi\textsuperscript{b,c} and Tsutomu T. Yanagida\textsuperscript{c}

\textsuperscript{a}Department of Physics, University of Tokyo, Tokyo 113-0033, Japan
\textsuperscript{b}Department of Physics, Tohoku University, Sendai 980-8578, Japan
\textsuperscript{c}Kavli Institute for the Physics and Mathematics of the Universe (WPI), TODIAS, University of Tokyo, Kashiwa 277-8583, Japan

Abstract

We propose a chaotic inflation model in which the lightest right-handed sneutrino serves as the inflaton and the predicted values of the spectral index and tensor-to-scalar ratio are consistent with the Planck data. Interestingly, the observed magnitude of primordial density perturbations is naturally explained by the inflaton mass of order $10^{13}$ GeV, which is close to the right-handed neutrino mass scale suggested by the seesaw mechanism and the neutrino oscillation experiments. We find that the agreement of the two scales becomes even better in the neutrino mass anarchy. We show that the inflation model can be embedded into supergravity and discuss thermal history of the Universe after inflation such as non-thermal leptogenesis by the right-handed sneutrino decays and the modulus dynamics.
1 Introduction

The Planck results [1] confirmed the vanilla ΛCDM model with six cosmological parameters based on almost scale-invariant, adiabatic and Gaussian primordial density perturbations. This strongly suggests that our Universe experienced the inflationary epoch described by a simple (effectively) single-field inflation [2, 3].

The primordial density perturbations are parametrized by the spectral index $n_s$ and the tensor-to-scalar ratio $r$, and they are tightly constrained by the Planck data combined with other CMB and cosmological observations. Roughly speaking, $n_s$ and $r$ are sensitive to the shape and magnitude of the inflaton potential, respectively. It is known that $r$ is related to the field excursion of the inflaton, and the on-going and planned CMB observations will be able to probe $r \gtrsim 10^{-3}$, for which the inflaton field excursion exceeds the Planck scale. One of the large-field inflation is the chaotic inflation [4]. Intriguingly, the chaotic inflation based on the monomial potential is outside the $1\sigma$ allowed region, and in particular, the quadratic chaotic inflation is near the boundary of the $2\sigma$ allowed region. Interestingly, it was recently pointed out in Refs. [5, 6, 7, 8] (see Refs. [9, 10, 11] for early attempts) that the predicted values of $(n_s, r)$ lie inside the region allowed by the Planck data, if the quadratic inflaton potential is slightly modified at large field values.

While scalar fields are ubiquitous in theories beyond the standard model (SM) such as supersymmetry (SUSY) or string theory, the identity of the inflaton and its couplings to the SM sector are unknown. Here we consider a model in which one of the right-handed sneutrinos plays a role of the inflaton [12, 13, 14, 15], extending the original model by introducing a slight modification to the quadratic potential at large field values. The observed magnitude of the primordial density perturbations can be naturally explained by the sneutrino mass of $10^{13}$ GeV, which is close to the right-handed neutrino mass scale suggested by the seesaw mechanism [16] and the neutrino oscillation experiments. We will show that the agreement will be even better in the neutrino mass anarchy hypothesis in which the right-handed neutrino mass matrix is given by a random matrix [17, 18]. This model has an advantage over singlet inflation models, in that the inflaton has couplings with the leptons and Higgs fields, which enable the successful reheating. Moreover the baryon asymmetry generation through leptogenesis [19] naturally takes place.
2 Chaotic inflation with right-handed sneutrinos

2.1 A model in global SUSY

Let us first consider a chaotic inflation model with right-handed sneutrinos in a global SUSY framework. We will see shortly that it is possible to embed the model into supergravity without significant modifications.

We start with the following superpotential;

\[ W = \frac{1}{2} M_{ij} N_i N_j + \frac{1}{4} \lambda_{ijkl} N_i N_j N_k N_l + \cdots, \]

where \( N_i \) denotes a chiral superfields for the \( i \)-th right-handed neutrino, \( M_{ij} \) and \( \lambda_{ijkl} \) represent the mass and quartic coupling of the right-handed neutrinos, and the flavor indices are \( i, j = 1, 2, 3 \). For the moment we assume a minimal Kähler potential for \( N_i \). Here and in what follows we adopt the Planck units where the reduced Planck mass \( M_p \approx 2 \times 10^{18} \) GeV is set to be unity, unless explicitly shown otherwise for convenience.

In Ref. [5] the inflation model with the superpotential

\[ W = \mu N^2 - \frac{\lambda}{3} N^3 \]

has been proposed under the name of Wess-Zumino inflation, in which an R-parity is explicitly broken. One of the advantages of our model (1) is that the R-parity is preserved.

In order to estimate the size of the interactions, let us express \( M_{ij} \) and \( \lambda_{ijkl} \) as

\[ M_{ij} = x_{ij} \Phi, \]
\[ \lambda_{ijkl} = y_{ijkl} \Phi^2, \]

where \( x_{ij} \) and \( y_{ijkl} \) are numerical coefficients of order unity, \( \Phi \) is a spurion field with B – L charge +2, and its expectation value represents the magnitude of the B – L breaking. To be concrete, we set \( \Phi \) to be \( \mathcal{O}(10^{-4}) \) as suggested by the seesaw mechanism [16] and the neutrino oscillation experiments.

The flavor structure is represented by \( x_{ij} \) and \( y_{ijkl} \), and we presume that they are complex-valued random matrices whose elements are of order unity, based on the neutrino mass anarchy hypothesis [17, 18, 20]. It is known that the observed large mixing angles for
neutrinos and the mild hierarchy for the mass squared differences can be nicely explained in the neutrino mass anarchy hypothesis.

Let us go to the mass eigenstate basis, \{\hat{N}_1, \hat{N}_2, \hat{N}_3\}, with mass eigenvalues \(M_1 \leq M_2 \leq M_3\). We identify the lightest right-handed sneutrino with the inflaton. Fig. 1 shows the probability distribution of the eigenvalues of the complex-valued symmetric random matrix \(x_{ij}\). As one can see the figure that the smallest eigenvalue typically ranges from 0.03 to 0.5. On the other hand, the flavor structure of the quartic couplings \(\lambda_{ijkl}\) are independent of the mass eigenstates, and so, we expect that \(|y_{1111}| \sim 1\) in this basis. Thus, the superpotential for the inflaton \(\phi \equiv \hat{N}_1\) is given by

\[
W = \frac{1}{2} M \phi^2 + \frac{1}{4} \lambda \phi^4,
\]

with

\[
M \equiv M_1 \sim 0.1 \Phi \sim 10^{-5},
\]

\[
\lambda \equiv \lambda_{1111} \sim \Phi^2 \sim 10^{-8},
\]

where we have dropped higher order terms, and we set \(M\) and \(\lambda\) real and positive for simplicity. We also assume that, during inflation, the heavier two mass eigenstates \(\hat{N}_2\) and \(\hat{N}_3\) are stabilized at SUSY minimum by their couplings with the inflaton \(\hat{N}_1\). The inflaton potential is given by

\[
V(\phi) = |M\phi + \lambda \phi^3|^2 = M^2 |\phi|^2 + \lambda M |\phi|^2 (\phi^2 + \phi^*^2) + \lambda^2 |\phi|^6
\]
Writing the inflaton field as $\phi = \varphi / \sqrt{2} e^{i\theta}$, the inflaton potential is minimized along $\cos 2\theta = 0$, namely, $\theta = \pi/4$. The inflaton potential along the radial component is

$$V(\varphi) = \frac{1}{2} M^2 \varphi^2 - \frac{1}{2} \lambda M \varphi^4 + \frac{1}{8} \lambda^2 \varphi^6,$$

It is a shifted version of the symmetry breaking potential. The inflation is possible if it initially sits in the vicinity of the local maximum at $\varphi = \sqrt{2M/\lambda}$.

We have numerically solved the inflaton dynamics and calculated the predicted $n_s$ and $r$ as shown by the red lines in Fig. 2 for the total e-folding number $N_e = 50$ and 60. The e-folding number depends on both the inflation scale and the thermal history after inflation. If there is a late-time entropy production by e.g. modulus decay, the e-folding number becomes smaller. As we shall see shortly, there is a modulus when we embed the present model into supergravity. Then the e-folding number is given $N_e \approx 54 - 55$, somewhere between the two lines. The black points correspond to the case of chaotic inflation with quadratic potential. One can see that, compared to the original quadratic chaotic inflation, the predicted $n_s$ and $r$ become smaller, thanks to the higher order terms in the inflaton potential. We have imposed the Planck normalization of the primordial density perturbations, and show how the parameters $M$ and $\lambda$ change in Fig. 3. Interestingly, the expected size of $M$ and $\lambda$ given by Eqs. (5) and (6) nicely match with the ranges favored by the Planck data. Note that the neutrino mass anarchy improves the agreement between the seesaw scale and the inflaton mass.

### 2.2 Embedding in supergravity

We consider the following Kähler and super-potentials [13];

$$K = \frac{3}{8} \ln \eta + \eta^2,$$

$$W = W(\phi_i)$$

with $\eta \equiv z + z^\dagger + |\phi_i|^2$, where $z$ is a modulus field and $\phi_i$ denotes chiral superfields in the model. Later we will identify $\phi_i$ with the right-handed neutrinos. The coefficients in $K$ are chosen so that $\eta$ is stabilized at $\eta = 3/4$ where the scalar potential vanishes [13].
\( 32 \eta (\partial_{\mu} \eta \partial^{\mu} \eta + I_{\mu} I^{\mu}) + \frac{16}{\eta} \eta^2 + \frac{3}{8 \eta} \partial_{\mu} \phi_i \partial^{\mu} \phi_i - V \) (11)

with

\[ V = \eta^2 e^{\eta^2} \left( \frac{8 \eta}{16 \eta^2 + 3} |W_i|^2 + \frac{(16 \eta^2 - 9)^2}{8(16 \eta^2 - 3)} |W|^2 \right), \] (12)

where \( W_i \equiv \partial W / \partial \phi_i \), and \( I_{\mu} \equiv i \partial_{\mu} (z - z^*) + i (\phi_i^* \partial_{\mu} \phi_i - \phi_i \partial_{\mu} \phi_i^*) \). We have assumed that the right-handed neutrinos are gauge-singlet and there is no D-term potential. It was shown in Ref. [13] that the modulus \( \eta \) is successfully stabilized during and after inflation, leading to the scalar potential with the same form in the global SUSY. The effective potential for
\( \phi_i \) is given by

\[
V = \frac{1}{2} \left( \frac{3}{4} \right)^{\frac{3}{2}} e^{\frac{9}{4} \pi} |\partial_{\phi_i} W|^2.
\]  

(13)

Then, assuming the superpotential of (4), we obtain the inflaton potential (8) after a trivial change of normalization of \( M \) and \( \lambda \) due to the numerical coefficient appearing in Eq. (13). The predicted values of the spectral index and the tensor-to-scalar ratio are the same as in Fig. 2.

3 Cosmology after inflation and phenomenological implications

3.1 Leptogenesis from inflaton decay

The inflaton, i.e. the lightest right-handed sneutrino, decays into leptons and Higgs through the yukawa coupling

\[
W = h_{1a} \tilde{N}_1 L_a H_u,
\]

(14)
where $L_\alpha$ and $H_u$ are chiral superfields for the lepton doublet and up-type Higgs. The reheating temperature is given by

$$T_R \simeq 6 \times 10^{11} \text{GeV} \left( \frac{M_{N_1}}{10^{13} \text{GeV}} \right)^{1/2} \left( \frac{\sqrt{\sum_\alpha |h_{1\alpha}|^2}}{10^{-3}} \right).$$

(15)

The CP violating decay of the sneutrino produces a non-zero lepton asymmetry \cite{12, 22, 23}, and the produced lepton asymmetry is evaluated as

$$\frac{n_L}{s} \simeq 7 \times 10^{-6} \left( \frac{T_R}{10^{11} \text{GeV}} \right) \left( \frac{m_{\nu_3}}{0.05 \text{eV}} \right) \delta_{\text{eff}} \Delta,$$

(16)

where $T_R$ is the reheating temperature, $m_{\nu_3}$ is the neutrino mass indicated by the measurements of the atmospheric neutrinos and $\delta_{\text{eff}}$ is the effective CP angle. The dilution factor by the modulus decay is represented by $\Delta$, which will be estimated later. The produced lepton asymmetry is converted to the baryon asymmetry through the sphaleron process. We shall see that the entropy production by the modulus decay can be suppressed for a sufficiently heavy modulus mass, and then the successful non-thermal leptogenesis by the decay of inflaton right-handed sneutrino is possible.

### 3.2 Modulus dynamics

Let us study the modulus dynamics to estimate the entropy production from the modulus decay. The minimum of the modulus $\eta$ is located at $\eta \simeq 3/4$ during inflation as $|W| \gg |W_\phi|$. The modulus mass at the minimum is larger than the Hubble parameter, i.e., $|W| \gg H$, and therefore it is stabilized at the minimum during inflation. After inflation, the inflaton F-term dominates over the superpotential, $|W_\phi| \gg |W|$, and the minimum of the modulus is shifted to $\eta \sim \sqrt{3}/4$ where the modulus mass is given by $|W_\phi| \sim H$.

Finally, as the inflaton oscillation amplitude decreases, the superpotential becomes larger than the inflaton F-term, $|W_\phi| \ll |W| \sim m_{3/2}$, where $m_{3/2}$ is the gravitino mass in the low energy. Then the minimum again moves to $\eta \simeq 3/4$ where the modulus mass is $\simeq 6m_{3/2}$. In this process the modulus starts to oscillate with an amplitude of order

---

1 Note that if the reheating temperature exceeds the inflaton mass, one needs to take account of the dissipation effect as well as non-perturbative particle production to estimate the precise reheating temperature \cite{21}. In this case thermal leptogenesis, instead of the non-thermal one, takes place.
η_i \sim 0.1$. Thus the Universe will be dominated by the modulus coherent oscillations soon after the reheating, and there is a cosmological moduli problem \cite{24,25}.

Let us study the modulus decay processes. To be concrete we express the modulus $z$ as

$$
 z \equiv \frac{\tau + ia}{\sqrt{2K_{zz}}},
$$

where $\tau$ and $a$ are canonically normalized real and imaginary components, and $K_{zz} = 4/3$ at the potential minimum, where the modulus $\tau$ has a mass $m_\tau \simeq 6m_{3/2}$ while the axion $a$ remains massless. The modulus $\tau$ decays into a pair of gravitinos with the rate \cite{26},

$$
\Gamma(\tau \to 2\psi_{3/2}) \simeq \frac{1}{96\pi} \frac{m_\tau^5}{m_{3/2}^2} \left(1 - \frac{4m_{3/2}^2}{m_\tau^2}\right)^{3/2} \left(1 - 6 \frac{m_{3/2}^2}{m_\tau^2} + \mathcal{O}\left(\frac{m_\tau^4}{m_\tau^4}\right)\right),
$$

Note that the gravitino production rate is enhanced by a factor of $m_\tau^2/m_{3/2}^2$ due to the longitudinal component. The modulus $\tau$ can also decay into a pair of axions $a$ with the rate \cite{27}

$$
\Gamma(\tau \to 2a) = \frac{1}{64\pi} \frac{K_{zz}^2}{K_{zz}^3} m_\tau^3,
$$

where we have used $K_{zz} = 16/9$ at the potential minimum in the second equality. Therefore, if the modulus does not have any other interactions, it mainly decays into gravitinos, which dominate the Universe for a while and then decay into lighter degrees of freedom including the standard model particles \cite{28}. The entropy dilution factor $\Delta(<1)$ is given by\footnote{The total e-folding number becomes close to 50 when there is a large entropy production by the modulus decay.}

$$
\Delta = \min \left[1, \frac{3m_\tau T_{3/2}}{B_{3/2} m_{3/2} T_R \eta_i} \right] \simeq \min \left[1, 7 \times 10^{-5} \left(\frac{0.1}{\eta_i}\right)^2 \left(\frac{m_{3/2}}{10^9 \text{GeV}}\right)^{3/2} \left(\frac{10^{11} \text{GeV}}{T_R}\right)\right],
$$

where $B_{3/2}$ is the branching fraction of the modulus decay into gravitinos, and $T_{3/2}$ is the decay temperature of the gravitinos. In the second equality we set $B_{3/2} \simeq 1$ and used the
gravitino decay rate given by

\[ \Gamma_{3/2} \simeq \frac{193}{384\pi} m_{3/2}^3, \]  

(22)

assuming that it decays into the standard model particles and their superpartners. The gravitino decay temperature is estimated as

\[ T_{3/2} \simeq 4 \text{ TeV} \left( \frac{m_{3/2}}{10^9 \text{ GeV}} \right)^\frac{3}{2}. \]  

(23)

Therefore, one needs a heavy gravitino mass, \( m_{3/2} \gtrsim 10^{9-10} \text{ GeV} \), for successful leptogenesis. Note that the final baryon asymmetry becomes independent of the reheating temperature. The abundance of axions produced by the modulus decay is diluted by the gravitino decay and its contribution to the effective neutrino species is given by \( \Delta N_{\text{eff}} \sim 0.03 \).

In this minimal set-up, the SUSY breaking is not mediated to the SM sector. In particular, there are no anomaly mediation contributions \cite{29}. We can generate soft SUSY breaking masses for the superpartners of the SM particles by introducing an extra SUSY breaking sector whose effect is transmitted to the SM sector by gauge interactions. The soft SUSY breaking mass scale can be of order TeV, and some of the superparticles may be within the reach of LHC.

So far we have assumed a specific form of the Kähler potential, which however may be subject to various corrections such as graviton-gravitino loops. It is however difficult to quantify such effects on the moduli stabilization and the contributions to the soft SUSY breaking masses from an effective field theory point of view. In general, we expect that the soft SUSY breaking masses for sfermions will be a few orders of magnitude smaller than the gravitino mass, if such corrections are induced radiatively. Such heavy sfermion mass, especially the stop mass, of order \( 10^6-7 \text{ GeV} \) is consistent with the SM-like Higgs boson of mass near 126 GeV \cite{30, 31}. On the other hand, unless \( z \) has a direct coupling to the SM gauge fields (which will be considered below), the gaugino mass remains significantly suppressed and it arises only at the two-loop level and given by \( \sim m_{3/2}^3 \) \cite{32}, where we have neglected loop factors. Therefore we need to invoke an additional SUSY breaking and its mediation to the visible sector, in order to generate a sizable gluino mass \( \gtrsim \text{TeV} \). The resultant soft mass spectrum resembles that in split SUSY \cite{32} or pure gravity mediation \cite{33, 34} scenarios.
Another possible extension is to introduce couplings of $z$. Let us here briefly discuss what happens if $z$ is coupled to the SM gauge sector. We introduce the following coupling to the gauge bosons:

$$\mathcal{L} = \int d^2\theta \frac{z}{M} W^\alpha W_\alpha + \text{h.c.}$$

(24)

where $M$ is an effective cutoff scale and $W^\alpha$ denotes the SM gauge superfield. We assume that the gauge superfields are canonically normalized, which is not modified as $\langle z \rangle \ll M$. The gaugino mass is generated by the above interaction,

$$m_\lambda = 2 K z \bar{z} \frac{m_{3/2}}{M} = \frac{3m_{3/2}}{M}.$$  

(25)

For instance, the gaugino mass of $\mathcal{O}(1)$ TeV is generated for $M \simeq 10^6 M_p$ and $m_{3/2} \simeq 10^9$ GeV. In the minimal set-up, the sfermion masses dominantly come from the renormalization group evolution effect as in the gaugino mediation model \[35, 36\]. Thus the squark/slepton masses are suppressed by a loop factor compared with the gauginos. For a sufficiently large $M$, the soft SUSY breaking masses can be of $\mathcal{O}(1 - 10)$ TeV. The 126 GeV Higgs boson mass can be explained in such a setup \[37\].

The above coupling induces the decay of modulus into the gauge boson as

$$\Gamma(z \to A_\mu A_\mu) \simeq \frac{3N_g}{32\pi} \frac{m_z^3}{M^2},$$

(26)

where $N_g$ is the number of gauge bosons, and we have $N_g = 12$ in the SM. The modulus decays also into gauginos with a similar rate. The partial decay rate into the SM gauge sector is smaller than that into gravitinos unless $M$ is much smaller than the Planck scale, and the above estimate on the entropy dilution factor remains almost unchanged.

The axion $a$ becomes the QCD axion as it acquires a mass from the QCD instanton effect through Eq. \(24\). The axion decay constant $f_a$ is related to the effective cut-off $M$ as

$$f_a = \frac{\sqrt{2K z \bar{z}}}{32\pi^2} M,$$

(27)

and the axion mass is given by

$$m_a \simeq 5 \times 10^{-16} \text{eV} \left(\frac{10^6 M_p}{M}\right).$$

(28)
However the axion isocurvature perturbation becomes too large in this case no matter how the initial misalignment angle is tuned, because of the high inflation scale \[^{38, 39}\]. One solution to this problem is to introduce a coupling of \( z \) to another hidden strong gauge group so that the axion gets a heavy mass of \( \sim \Lambda^2/M \) during inflation, where \( \Lambda \) is the dynamical scale of the hidden gauge group. If the hidden-gauge group remains strongly coupled in the low-energy, the axion does not solve the strong CP problem, and even if it is produced by the coherent oscillations, it decays into the SM gauge bosons, thus avoiding the isocurvature constraint. Note that one can choose the value of \( \Lambda \) so that it does not modify the moduli stabilization significantly. Alternatively, if the hidden-gauge group becomes weakly-coupled in the low-energy somehow by e.g. non-trivial dynamics of a dilation field or hidden Higgs fields \[^{40}\], the axion may be able to solve the strong CP problem, avoiding the isocurvature constraint \[^{41}\].

Lastly let us comment on the lightest SUSY particle (LSP). The SUSY particles are produced by the gravitino decays. Since the gravitino decay temperature is higher than TeV for \( m_{3/2} \gtrsim 10^9 \text{GeV} \) (see Eq. \((23)\)), the LSPs are thermalized if their mass is of order 100 to 1000 GeV. Then a right amount of dark matter can be explained by the thermal relic of the LSPs. On the other hand, if the LSP mass is much larger than TeV, the thermal relic abundance likely exceeds the observed dark matter abundance, and one would need to introduce a small amount of R-parity violation.

4 Discussion and Conclusions

In this paper we have revisited the chaotic inflation model in which the lightest right-handed sneutrino plays the role of the inflaton. The model predicts a rather large tensor-to-scalar ratio, which is within the reach of the future and on-going B-mode search experiments. Furthermore, the inflaton naturally reheats the SM particles and non-thermal leptogenesis takes place naturally.

We have also embedded the right-handed sneutrino inflation model in a supergravity framework, and shown that the inflaton dynamics is same as in the global SUSY case for a certain class of the Kähler potential. The price we have to pay for obtaining the inflaton potential as in the global SUSY is the existence of a modulus field, which causes
a cosmological moduli problem. We have shown that that the gravitino mass should be sufficiently heavy, i.e. \( m_{3/2} \gtrsim 10^9 \text{GeV} \), for successful leptogenesis, since otherwise the modulus (and gravitino) decay would dilute the baryon asymmetry too much.

The soft mass spectrum in the visible sector depends on the precise form of the Kähler potential. As long as it is given by Eq. (9) (a more general form will be discussed in Appendix), the structure of the visible sector is essentially same as in the global SUSY, and the SUSY breaking effect is not mediated to the visible sector. In particular, there is no anomaly mediation contribution. In this case we need to invoke an additional SUSY breaking and its mediation mechanism to the visible sector. We however note that the Kähler potential could receive various corrections such as graviton-gravitino and moduli loops. In this case, we expect that sfermions obtain a SUSY breaking mass a few orders of magnitude smaller than the gravitino mass, while the gaugino mass remains significantly suppressed, which requires an additional SUSY breaking and its mediation mechanism. The resultant soft SUSY mass spectrum will be similar to those in the split SUSY and pure gravity mediation scenarios.

Some comments are in order. We have dropped higher order terms in (4). This is justified as the inflaton has a mass about one order of magnitude smaller than the naively expected value. Otherwise, higher order terms are generically non-negligible where the first term and the second term in (4) become comparable to each other. That said, it is in principle possible that the higher order terms modify the inflaton potential. It may be possible to lift the inflaton potential at large field values so that there is no local minimum. In this case, there will be no problem of choosing the initial position of the inflaton near the local maximum. In particular, the inflaton potential can be flatter at large field values, which will lead to a smaller tensor-to-scalar ratio in better agreement with the observation.

Acknowledgments

We are grateful to John Ellis for fruitful discussion in the early stage of the project, and we thank Hitoshi Murayama for discussion. This work was supported by the Grant-in-Aid for Scientific Research on Innovative Areas (No. 21111006 [KN and FT], No.23104008
A Condition for successful chaotic inflation

In this Appendix we derive conditions for successful chaotic inflation with $|\phi| \gg M_P$. Let us consider the following Kähler potential and superpotential:

\[ K = f(\eta), \]  
\[ W = W(\phi), \]

where

\[ \eta = z + z^\dagger + c|\phi|^2, \]

with a numerical constant $c$. It exhibits the Heisenberg symmetry for $c = 1$. The kinetic term is given by

\[ \mathcal{L}_{\text{kin}} = \frac{f''}{4} [(\partial \eta)^2 + I_\mu I^\mu] + cf'|\partial \phi|^2, \]

where $I_\mu = i\partial_\mu(z - z^\dagger) + ic(\phi^\dagger \partial_\mu \phi - \phi \partial_\mu \phi^\dagger)$. The scalar potential is given by

\[ V = e^f \left[ \frac{1}{cf f' f''} |W_\phi|^2 + \left( \frac{f'^2}{f''} - 3 \right) |W|^2 \right], \]

where the prime denotes the derivative with respect to $\eta$. For chaotic inflation to happen, we demand the following relations at $\eta = \eta_{\text{min}}$:

\[ F(\eta_{\text{min}}) \equiv \left[ \frac{f'^2}{f''} - 3 \right]_{\eta = \eta_{\text{min}}} = 0, \]

\[ F'(\eta_{\text{min}}) = \left[ \frac{f'(2f'' f' - f' f'')}{f''} \right]_{\eta = \eta_{\text{min}}} = 0, \]

\[ F''(\eta_{\text{min}}) = \left[ \frac{f''}{f''} (3f'' f'' - f' f''') \right]_{\eta = \eta_{\text{min}}} > 0. \]

If these are satisfied, $\eta$ is stabilized at $\eta = \eta_{\text{min}}$, where the dangerous second term in (33) vanishes. Then the potential for $\phi$ may resemble that in the global SUSY case even for $|\phi| \gg 1$. 
To be more concrete, let us assume the following form:

\[ f(\eta) = a \ln \eta + b \eta + d\eta^2, \]  

(35)

with numerical coefficients \(a, b\) and \(d\). From Eqs. (34), we find

\[ 2d\eta_{\text{min}}^2 = (3a^2)^{1/3} + a, \]  

(36)

and

\[ 4d^2\eta_{\text{min}}^3 - 6ad\eta_{\text{min}} - ab = 0. \]  

(37)

We also have

\[ F''(\eta_{\text{min}}) = 6 \left( 4d + \frac{b}{4\eta_{\text{min}}} \right). \]  

(38)

For example, if we take \(b = 0\) and \(d = 1\), we find \(a = 3/8\) and \(\eta_{\text{min}} = 3/4\) as found in Ref. [13]. If we take \(b = 0\) and \(d = 0\), we find \(a = -3\) as in the no-scale form, although \(\eta\) is massless and not stabilized since \(F''(\eta_{\text{min}}) = 0\) in this limit. Fig. 4 shows contours of \(b\) satisfying conditions (36)-(37) on \((a, d)\)-plane. It is checked that \(F''(\eta_{\text{min}}) > 0\) for all the parameter ranges.

**References**

[1] P. A. R. Ade et al. [Planck Collaboration], arXiv:1303.5082 [astro-ph.CO].
[2] A. H. Guth, Phys. Rev. D 23, 347-356 (1981); A. A. Starobinsky, Phys. Lett. B 91 (1980) 99; K. Sato, Mon. Not. Roy. Astron. Soc. 195, 467-479 (1981).

[3] A. D. Linde, Phys. Lett. B 108 (1982) 389; A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982).

[4] A. D. Linde, Phys. Lett. B 129, 177 (1983).

[5] D. Croon, J. Ellis and N. E. Mavromatos, Physics Letters B 724, 165 (2013) arXiv:1303.6253 [astro-ph.CO].

[6] K. Nakayama, F. Takahashi and T. T. Yanagida, Phys. Lett. B725, 111 (2013) arXiv:1303.7315 [hep-ph]; JCAP 1308, 038 (2013) arXiv:1305.5099 [hep-ph].

[7] J. Ellis, D. V. Nanopoulos and K. A. Olive, Phys. Rev. Lett. 111, 111301 (2013) arXiv:1305.1247 [hep-th]. JCAP 1310, 009 (2013) arXiv:1307.3537 [hep-th]; arXiv:1310.4770 [hep-ph].

[8] R. Kallosh and A. Linde, JCAP 1306, 027 (2013) arXiv:1306.3211 [hep-th]; JCAP 1306, 028 (2013) arXiv:1306.3214 [hep-th].

[9] C. Destri, H. J. de Vega and N. G. Sanchez, Phys. Rev. D 77, 043509 (2008) astro-ph/0703417.

[10] A. D. Linde, Pisma Zh. Eksp. Teor. Fiz. 37, 606 (1983) [JETP Lett. 37, 724 (1983)]; Phys. Lett. B 132, 317 (1983).

[11] R. Kallosh and A. D. Linde, JCAP 0704, 017 (2007) arXiv:0704.0647 [hep-th].

[12] H. Murayama, H. Suzuki, T. Yanagida and J. 'i. Yokoyama, Phys. Rev. Lett. 70, 1912 (1993).

[13] H. Murayama, H. Suzuki, T. Yanagida and J. 'i. Yokoyama, Phys. Rev. D 50, 2356 (1994) [hep-ph/9311326].

[14] J. R. Ellis, M. Raidal and T. Yanagida, Phys. Lett. B 581, 9 (2004) hep-ph/0303242.

[15] S. Antusch, M. Bastero-Gil, K. Dutta, S. F. King and P. M. Kostka, Phys. Lett. B 679, 428 (2009) arXiv:0905.0905 [hep-th].

[16] T. Yanagida, in Proceedings of the “Workshop on the Unified Theory and the Baryon Number in the Universe”, Tsukuba, Japan, Feb. 13-14, 1979, edited by O. Sawada and
A. Sugamoto, KEK report KEK-79-18, p. 95, and “Horizontal Symmetry And Masses Of Neutrinos”, Prog. Theor. Phys. 64 (1980) 1103; M. Gell-Mann, P. Ramond and R. Slansky, in “Supergravity” (North-Holland, Amsterdam, 1979) eds. D. Z. Freedom and P. van Nieuwenhuizen, Print-80-0576 (CERN); S. L. Glashow, NATO Adv. Study Inst. Ser. B Phys. 59, 687 (1980); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980); see also P. Minkowski, Phys. Lett. B 67, 421 (1977).

[17] L. J. Hall, H. Murayama and N. Weiner, Phys. Rev. Lett. 84, 2572 (2000) [hep-ph/9911341].

[18] N. Haba and H. Murayama, Phys. Rev. D 63, 053010 (2001) [hep-ph/0009174].

[19] M. Fukugita and T. Yanagida, Phys. Lett. B 174, 45 (1986).

[20] K. S. Jeong and F. Takahashi, JHEP 1207, 170 (2012) [arXiv:1204.5453 [hep-ph]].

[21] K. Mukaida and K. Nakayama, JCAP 1301, 017 (2013) [arXiv:1208.3399 [hep-ph]]; JCAP 1303, 002 (2013) [arXiv:1212.4985 [hep-ph]].

[22] H. Murayama and T. Yanagida, Phys. Lett. B 322, 349 (1994) [hep-ph/9310297].

[23] K. Hamaguchi, H. Murayama and T. Yanagida, Phys. Rev. D 65, 043512 (2002) [hep-ph/0109030].

[24] G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby, G. G. Ross, Phys. Lett. B 131, 59 (1983); J. R. Ellis, D. V. Nanopoulos, M. Quiros, Phys. Lett. B 174, 176 (1986); A. S. Goncharov, A. D. Linde, M. I. Vysotsky, Phys. Lett. B 147, 279 (1984).

[25] B. de Carlos, J. A. Casas, F. Quevedo and E. Roulet, Phys. Lett. B 318, 447 (1993) [hep-ph/9308325]; T. Banks, D. B. Kaplan and A. E. Nelson, Phys. Rev. D 49, 779 (1994) [hep-ph/9308292].

[26] M. Endo, K. Hamaguchi, F. Takahashi, Phys. Rev. Lett. 96, 211301 (2006) [hep-ph/0602061]; Phys. Rev. D74, 023531 (2006) [hep-ph/0605091]; S. Nakamura, M. Yamaguchi, Phys. Lett. B638, 389-395 (2006) [hep-ph/0602081]; M. Dine, R. Kitano, A. Morisse, Y. Shirman, Phys. Rev. D73, 123518 (2006) [hep-ph/0604140].

[27] T. Higaki, K. Nakayama and F. Takahashi, JHEP 1307, 005 (2013) [arXiv:1304.7987 [hep-ph]].
[28] K. S. Jeong and F. Takahashi, JHEP 1301, 173 (2013) [arXiv:1210.4077 [hep-ph]].

[29] K. -I. Izawa, T. Kugo and T. T. Yanagida, Prog. Theor. Phys. 125, 261 (2011) [arXiv:1008.4641 [hep-ph]].

[30] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 716, 1 (2012) [arXiv:1207.7214 [hep-ex]].

[31] S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 716, 30 (2012) [arXiv:1207.7235 [hep-ex]].

[32] N. Arkani-Hamed, S. Dimopoulos, G. F. Giudice and A. Romanino, Nucl. Phys. B 709, 3 (2005) [hep-ph/0409232].

[33] M. Ibe and T. T. Yanagida, Phys. Lett. B 709, 374 (2012) [arXiv:1112.2462 [hep-ph]].

[34] M. Ibe, S. Matsumoto and T. T. Yanagida, Phys. Rev. D 85, 095011 (2012) [arXiv:1202.2253 [hep-ph]].

[35] K. Inoue, M. Kawasaki, M. Yamaguchi and T. Yanagida, Phys. Rev. D 45, 328 (1992).

[36] D. E. Kaplan, G. D. Kribs and M. Schmaltz, Phys. Rev. D 62, 035010 (2000) [hep-ph/9911293]; Z. Chacko, M. A. Luty, A. E. Nelson and E. Ponton, JHEP 0001, 003 (2000) [hep-ph/9911323].

[37] T. Moroi, T. T. Yanagida and N. Yokozaki, Phys. Lett. B 719, 148 (2013) [arXiv:1211.4676 [hep-ph]]; T. T. Yanagida and N. Yokozaki, Phys. Lett. B 722, 355 (2013) [arXiv:1301.1137 [hep-ph]].

[38] M. Kawasaki, K. Nakayama, T. Sekiguchi, T. Suyama and F. Takahashi, JCAP 0811, 019 (2008) [arXiv:0808.0009 [astro-ph]].

[39] C. Hikage, M. Kawasaki, T. Sekiguchi and T. Takahashi, JCAP 1307, 007 (2013) [arXiv:1211.1095, arXiv:1211.1095 [astro-ph.CO]].

[40] G. R. Dvali, [hep-ph/9505253].

[41] K. S. Jeong and F. Takahashi, Phys. Lett. B 727, 448 (2013) [arXiv:1304.8131 [hep-ph]].