The structure of the screening layer near the cylindrical bodies emitting charged particles in a deep vacuum

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Abstract. The purpose of this paper is the exploration of the electric field structure near cylindrical bodies. The solution of the problem is reduced to solving the equation for the potential of a self-consistent electric field written in dimensionless form.

The behavior of the potential is found by the combined method (at the initial stage, analytically, and then numerically). In contrast to the works devoted to this subject, the dependence of the potential on the coordinates is determined at various values of the electric field strength on the surface of the bodies, which is considered as a parameter.

Such an approach made it possible to distinguish different regimes of electron flows motion: motion in an accelerating field, motion in a field with a minimum of the potential, and returned flow. Determining the position of the boundary between the second and third regimes made it possible to find the thickness of the shielding layer near isolated cylindrical bodies under conditions of deep vacuum.

Knowledge of the potential and magnitude of the screening layer have a practical value: for radio communications, for the smooth operation of electronic equipment on artificial celestial bodies, for studying the mechanisms of lunar soil erosion caused by electric fields, for studying the behavior of potential in complex dusty crystalline structures, as well as for constructing cylindrical diodes working in a given mode.

1. Introduction

It is known that material bodies can emit charged particles from its surface. Most often these particles are electrons. The causes and nature of the emission may be different, and the discharge may be accompanied by a number of specific phenomena: optical, acoustic, electromagnetic, etc.

The main types of electron emission under "normal" conditions include: thermoelectronic emission, photoemission, auto electronic, secondary electron emission, etc.

There is a quite wide range of issues and tasks in which mutual influence of electric fields created by external sources and moving streams of charged particles is essential. The first articles on this topic appeared at the beginning of the 20th century. Among them, first of all, it should be noted [1–5]. A sufficiently detailed analysis of these and other works was carried out in the monograph [6]. Most of these works relate to the field of radio electronics and electronic optics.
In the late 1950s, the range of self-consistent tasks expanded significantly due to tasks related to the exploration and conquest of outer space. Launched spacecraft began to be used to study natural resources of the Earth, Moon, Mars, small celestial planets, comets, etc.; as well as in meteorology, navigation, oceanology, communications and other areas of science and technology. Among the processes and phenomena that occur in the vicinity of spacecraft and natural celestial bodies are important phenomena associated with the shaping and the existence of screening near-surface layers consisting of charged particles. Theoretical studies in this direction are particularly active conducted by Soviet scientists [7–9]. American authors at the same time engaged in research near-surface screening layers of natural celestial bodies, in particular, the Moon [10, 11].

Recently, interest in internal self-consistent tasks has manifested itself in many areas of physics and technology: microwave radiation generators [12]; thermonuclear synthesis [13]; technological processes associated with the creation of semiconductor diodes and of semiconductors with more complex functions [14], etc.

2. Formulation of the self-consistent electric field calculating problem

If processes occurring near the body are stationary, then the potential of the electric field is determined by the Poisson equation:

\[ \Delta \varphi = -4\pi \rho_e, \]  

where \( \rho_e \) - volume electric charge density.

Langmuir received some exact and approximate solution using a set of additional assumptions [2, 3, 15].

The main assumptions made by Langmuir:
1. Gas-dynamic and electric fields are one-dimensional.
2. A moving medium is a one-component cold gas from charged particles (electrons).
3. The gas flow is stationary.
4. Collisions between particles are not taken into account.
5. Charged particles are absorbed on the surface of the electrodes.
6. The initial velocity of the electrons leaving the cathode is zero.

Further Langmuir wrote out one of the forms of the energy conservation law applied to one single particle, writing it in the form

\[ \frac{mv^2}{2} - e\varphi = C. \]  

where \( C \) is an arbitrary constant determined by the value of the electric field potential either on the surface of the cathode or at some other point of space. Finally, Langmuir determines the current density \( j \) by formula, which, generally speaking, is another assumption:

\[ j = -\rho_e v. \]  

The minus sign in (3) Langmuir introduces because by \( j \) he understood absolute current density value, under \( v \) - the absolute value of the speed of the departing electron, and \( \rho_e \) considered negative. Note that on the surface of the cathode, by assumption 6, relation (3) is not satisfied, although it is used in solving the problems.

For a cylindrical diode, the current density \( |\vec{j}| \) varies with the changing of the distance according to the following law:

\[ j_R = \frac{j_0 R_0}{R}, \]
where \(j_0\) is the electric current density on some surface with radius \(R_0\) (in particular, this surface can be taken as such a surface of the cylinder).

Whence it follows for a cylindrical diode:

\[
\rho_e = -\frac{j_0 R_0}{R} \sqrt{\frac{m_e}{2e \varphi(R)}} ; \tag{5}
\]

In the formula (5) - \(m_e\) and \(e\) - the mass and charge of the electron.

Substituting the formula (5) in the equation (1) we obtain:

\[
\Delta \varphi = 4\pi j_0 R_0 \sqrt{\frac{m_e}{2e \varphi(R)}} . \tag{6}
\]

In (6), we make the transition to dimensionless variables. Let’s assign the index 1 to dimensionless variables. The specified transition is carried out using the formulas:

\[
\varphi = \frac{m_e v_0^2}{2e} \varphi_1 , \quad R = l R_1 . \tag{7}
\]

After substituting formulas (7) into equation (6), we obtain the following equation:

\[
\frac{d}{dR_1} \left( R_1 \frac{d\varphi_1}{dR_1} \right) = \frac{8\pi l e^2 n_0 R_0}{m_e v_0^2} \frac{1}{\sqrt{\varphi_1}} . \tag{8}
\]

or

\[
\frac{d}{dR_1} \left( R_1 \frac{d\varphi_1}{dR_1} \right) = l \frac{R_0}{r_D^2} \frac{1}{\sqrt{\varphi_1}} . \tag{9}
\]

where \(r_D = \sqrt{m_e v_0^2/(2\pi n_0 e^2)}\) - normalized in a certain way the Debye radius in the electron flow near the surface of the body.

We assume that \(l = r_D^2/R_0\). Then the coefficient on the right side equation (9) will be equal to unity and the equation will not contain any dimensional or dimensionless parameters. It will look like:

\[
\frac{d}{dR_1} \left( R_1 \frac{d\varphi_1}{dR_1} \right) = \frac{1}{\sqrt{\varphi_1}} . \tag{10}
\]

3. The exact solution and methods for obtaining approximate solutions

There is a solution that has a simple form among the solutions of equation (10):

\[
\varphi_1 = \left( \frac{3}{2} \right)^{4/3} R_1^{2/3} . \tag{11}
\]

You can make sure of this by substituting (11) into (10) and doing the necessary calculations.

The equation (10) for the potential \(\varphi_1\) can be transposed to the integral form:

\[
\varphi_1 = 1 + R_{10} E_{10} \ln(R_1/R_{10}) + \int_{R_{10}}^{R_1} \frac{dR_2}{R_2} \int_{R_{10}}^{R_2} \frac{dR_3}{\sqrt{\varphi_1(R_3)}} . \tag{12}
\]

Further progress in solving this problem depends on the method of the setting of the second boundary condition for the potential \(\varphi(x)\). Most often for the equations of type (10) consider the following two ways of specifying the boundary conditions.

The first is to set the electric field on the left border flow, in other words, at the point where the value of the electric fields potential is set.
\[
\frac{d\varphi_1}{dR_1} \bigg|_{R_1=R_{10}} = E_{10},
\]

(13)

where \(E_{10}\) is a given value that determines (up to a sign) electric field strength at \(R_1 = R_{10}\). \(E_{10}\) can accept any real values that are compatible with the scope of the considered model. The case \(E_{10} = 0\) conform to conditions of full electric field screening.

This formulation corresponds to the Cauchy problem.

In the dimensionless form (13) looks as follows:

\[
\frac{d\varphi_1}{dR_1} \bigg|_{R_1=R_{10}} = 4 \cdot R_E \frac{R_D}{R_0},
\]

(14)

where

\[
R_E = \sqrt{\frac{E_0^2}{8\pi n_0 m_e v_0^2}}; \quad E_0 = \frac{d\varphi}{dR} \bigg|_{R=R_0}.
\]

(15)

\(R_E\) is called the electric pressure number.

The second way to select the boundary condition is to set the value of the potential on the surface of the anode (the boundary problem for equation (10)).

An approach based on solving the Cauchy problem for equation (10) is considered in [16]. The consecutive solution of the boundary problem was carried out in [17].

4. Results and conclusions

The general solution of equation (10) in analytical form could not be found. Therefore, for further research we used numerical methods for solving equations (10). Integral curves that determine the dependence of the potential from coordinates, are shown in Figures 1 and 2. Figure 1 refers to the case monotonously increasing potential and solutions having a minimum.

**Figure 1.** Dependence of \(\varphi_1\) on \(R_1\) for initial \(E_{10}: -2 \leq E_{10} \leq 5\).

**Figure 2.** Dependence of \(\varphi_1\) on \(R_1\) for initial \(E_{10}: -3 \leq E_{10} \leq -1\).

Values of potentials and coordinates were taken in dimensionless form. Different the values of the electric field on the surface of a cylindrical body correspond to different integral curves.

Figure 3 shows the dependence of the particle flow rate \(V_1\) on the coordinate \(R_1\). Figure 4 shows the dependence of the absolute value of the bulk density of electric charge \(\rho_1\) on the coordinate \(R_1\).

The following conclusions were made on the basis of the obtained solutions.

1) In the acceptable region of \((R_1, \varphi_1)\) there are three subregions \(\Omega_1, \Omega_2\) and \(\Omega_3\), such that In \(\Omega_1\), the boundary problem has a unique solution.
Figure 3. Dependence of velocity $V_1$ on distance $R_1$ for $E_{10}$.

Figure 4. Dependence of density $\rho_1$ on distance $R_1$ for $E_{10}$.

In $\Omega_2$, the boundary problem has two and only two solutions.

In $\Omega_3$ there are no solutions of the boundary problem (excluding return particles).

2) For a cylindrical diode (as for a flat one) there are values $E_{10}$, at which the so-called "virtual cathode" is formed.

3) Unlike flat symmetry in the case of cylindrical symmetry the electric field strength tends to zero at infinity for all possible solutions to the Cauchy problem.

4) In the region $\Omega_2$, the curves $\varphi_1(R_1)$ correspond to the different initial values of the electric field strength on the surface of the body can, generally speaking, overlap. This suggests that solutions of the original equation (10), which are stable in the small, with large perturbations can become unstable.

5) For an isolated cylindrical body emitting particles of the same sign in vacuum there is a near-surface screening layer of finite thickness, its thickness is in order of $2.9 \cdot r_D^2/R_0$.

The developed methodology and the obtained results give answers to the questions formulated in the abstract. The given formulas and graphs can be compared with the results obtained experimentally. So, for example, in [18] the sizes of the screening layers were determined by the Laser Photodetachment Signals (LPD) method. The experimental data and estimates of the screening layer thickness are in good agreement with the results obtained in this article.

References
[1] Child C D 1911 Phys. Ref. Ser. 1. 32, N 5 492–511
[2] Langmuir I 1913 Phys. Ref. Ser. 1. 2 450–486
[3] Langmuir I and Blodgett K B 1923 Phys. Ref. 22 347–356
[4] Boguslavskiy S A 1961 Izbrannyi Trudi po Fizike [Selected Works of Physics], in Russian (Moscow: PhysMathGis)
[5] Bursian R T and Pavlov V I 1923 Zhurnal russkogo fisiko-himicheskogo obsvestva [Journal of Rus. Phys. Chem. Society], in Russian 55, v. 1-3 71–80
[6] Kirstein P T, Kino G S and EWaters W 1967 Space-charge flow (NY: McGraw-Hill)
[7] Kurt V G and Moroz V V 1961 Iskustvennie Sputniki Zemli [Artificial Earth Satellites], in Russian 7 78–83
[8] Moskalenko A M 1964 Geomagnetism i Aeronomiya 14 260–268
[9] Allpert Y L, Gurevich A V and Pitaevskiy A P 1964 Iskustvennie Sputniki Zemli v razrezhennoi plazme [Artificial satellites of the Earth in rarefied plasma], in Russian (Moscow: Nauka)
[10] Ūpik E J and Singer S F 1960 J. Geophys. Res. 65 3065–3070
[11] Walbridge E J 1973 J. Geophys. Res. 78 3668–3687
[12] Dubinov A E and Selemir V D 2002 J. Radiotechnica i Electronica [J. Radio engineering and electronics], in Russian 47 645–672
[13] Ottinger P F, Goodrich P J, Hinshelwood D, Mosher D, Neri J, Rose D, Stephanakis S and Young F 1992 Proc. of the IEEE 80 1010–1018
[14] Hernandes E 1998 Cryst. Res. Technol. 33 285–289
[15] Langmuir I 1914 Phys. Zeitschrift 15 348, 516
[16] Gunko Y F and Kolesnikova E N 2004 Sbornik Modeli neodnorodnykh sred: Fizicheskaya mekhanika [Models of inhomogeneous media: Physical mechanics], in Russian (St.Petersburg: Ed. Spb. State University)
[17] Kuznetsov V I and Ender A Y 2013 Tech.Phys. 58 1705–1714
[18] Kajita S, Kado S, Okamoto A, Shikama T, Iida Y, Yamasaki D and Tanaka S 2004 Measurement of electron sheath thickness and collection region of electric probe using laser photodetachment signals 12th International Congress on Plasma Physics, 25-29 October 2004, Nice (France) URL https://hal.archives-ouvertes.fr/hal-0000196