The Range of the Kondo Cloud in Weakly Disordered Hosts

Gerd Bergmann and Richard S. Thompson
Department of Physics
University of Southern California
Los Angeles, California 90089-0484
e-mail: bergmann@usc.edu

October 29, 2014

Abstract

The calculation of the Kondo cloud is extended to disordered hosts. For a weakly disordered large three-dimensional host the structure of the ground state is very close to the pure host. However, the range of the disordered electron basis is much shorter. The extent of the Kondo cloud is essentially given by \( \sqrt{\xi_K l} \) where \( \xi_K \) is the Kondo length in the pure host and \( l \) the mean free path.

PACS: 75.20.Hr, 71.23.An, 71.27.+a
1 Introduction

The properties of magnetic impurities in a metallic host have fascinated physicists for a long time, particularly after the publication of Kondo’s paper \[1\] 50 years ago. In the meantime the field of the ”Kondo effect” has matured (see for example \[2\], \[3\], \[4\], \[5\]). One of the open questions is the so-called Kondo cloud. The idea is to divide the Kondo ground state, the singlet state \(\Psi_{SS}\), into two parts with opposite (net) d-spins. The proponents of the Kondo cloud argue that in each component there is a cloud of s-electron spins that compensates the d-spin. Such a cloud has been theoretically derived \[6\], \[7\], \[8\], \[9\], \[10\], \[11\], \[12\], \[13\], \[14\], \[15\], \[16\], \[17\], \[18\], \[19\], \[20\], \[21\], \[22\], \[23\], \[24\], \[25\], \[26\], \[27\]. This predicted Kondo cloud has not yet been experimentally detected.

If the host is pure (except for the magnetic impurity) then the range of this cloud is of the order of the Kondo length

\[
\xi_K = \frac{\hbar v_F}{k_B T_K}
\]

(\(k_B T_K = \text{Kondo energy, } v_F = \text{Fermi velocity of the s-electrons}\)).

One reason why an experimental detection of the Kondo cloud has been so difficult is the fact that the Kondo cloud is very dilute, about a single spin distributed over \(10^{14}\) host atoms in a pure three-dimensional host with a \(T_K\) of a few Kelvin. In this paper we discuss the possibility to reduce the Kondo length dramatically by using a disordered host with a finite mean free path \(l\). From superconductivity it is well known that a finite mean free path reduces the BCS coherence length from \(\xi_0 = \frac{\hbar v_F}{k_B T_c}\) to a much smaller length of \(\xi = \sqrt{\xi_0 l}\). Although the underlying physics of the Kondo effect and of superconductivity are quite different it is worthwhile to investigate the influence of disorder on the Kondo length and in particular the extension of the Kondo cloud.

The influence of disorder on the Kondo effect is investigated in a number of papers \[28\], \[29\], \[30\], \[31\]. Many focus on the extreme case of electron localization. In this case the magnetic impurity interacts only with a relatively small number of localized electrons, which in a way is similar to the case of small sample size.

In a weakly disordered host in three dimensions the electron density is in first approximation close to that of a pure host although the disorder causes some fluctuations. Any such fluctuation at the position of the magnetic impurity results in a change of the Kondo temperature. Therefore a finite degree of disorder in the host yields a spread of the Kondo temperatures for different realizations of the disorder. Such profiles of the Kondo temperature have been calculated \[30\].

In the present paper we restrict ourselves to small disorder, i.e. \(k_F l >> 1\), in three dimensions where the fluctuations are small. We consider a large host so that the spacing of the energy levels is much smaller than the Kondo energy. We use the fact that the Kondo cloud is already calculated for the pure host (see for example \[18\]) and show that indeed the Kondo cloud in a disordered host has a reduced extension of \(\sqrt{\xi_K l}\). We derive a relatively simple formula to calculate any polarization in the disordered host from the polarization in the pure host.
The basis of this paper is the solution of the Friedel-Anderson (FA) impurity problem in the FAIR approach by our group [32], [33], [34]. Following Wilson’s ingenious idea of reducing the number of states by assuming a conduction band with constant density of states we subdivide the energy band into cells \( \mathcal{C}_\nu \) which are represented by a single state \( \tilde{c}_\nu (r) \). Wilson also normalized the energy and momentum of the electron states. Since the presented arguments are not based on numerical calculations but on the discussion of physical properties we use in this paper (and only this paper) regular electron momenta \( k \) and energies \( E \). The wave vector \( k \) for the conduction band extends from \( 0 \leq k \leq 2k_F \). Like Wilson we use a linear dispersion relation between the energy \( E \) and the wave vector \( k \). In contrast to Wilson we don’t count the energy with respect to the Fermi level but set \( E = \hbar v_F k \) so that \( 0 \leq E \leq 2E_F \). The lower (occupied) half of the energy band is divided at the energies \( E_1, E_2, \ldots, E_n \) where \( E_\nu = E_F \left( 1 - \frac{1}{2^\nu} \right) \), forming energy cells \( \mathcal{C}_\nu = (E_{\nu-1}, E_\nu) \) or \( (k_{\nu-1}, k_\nu) \). The width of the energy cells is \( (E_\nu - E_{\nu-1}) = E_F 2^{-\nu} = \Delta \nu E_F \) where \( \Delta \nu = 2^{-\nu} \).

The energy band above the Fermi level is sub-divided in a mirrored fashion. Each cell \( \mathcal{C}_\nu \) contains \( Z_\nu \) eigenstates \( \tilde{\varphi}_j (r) \) of the disordered host. (Throughout this paper we denote the wave function \( \tilde{\varphi}_j (r) \), the creation operator \( \varphi_j^\dagger \) and the annihilation operator \( \tilde{\varphi}_j \) of the same state by the same symbol \( \varphi_j \) with different decorations).

Following Wilson we represent all the states in a cell \( \mathcal{C}_\nu \) by a linear superposition of the eigenstates \( \tilde{\varphi}_j (r) \)

\[
\tilde{c}_\nu (r) = \frac{1}{A} \sum_{j \in \mathcal{C}_\nu} \tilde{\varphi}_j^\dagger (0) \tilde{\varphi}_j (r), \quad A^2 = \sum_{j \in \mathcal{C}_\nu} |\tilde{\varphi}_j (0)|^2
\]

where the summation goes over all states \( j \) with energy \( \varepsilon_j \) in the cell \( \mathcal{C}_\nu \). Then the amplitude of \( \tilde{c}_\nu (r) \) at the origin is

\[
\tilde{c}_\nu (0) = \sqrt{\sum_{j \in \mathcal{C}_\nu} |\tilde{\varphi}_j (0)|^2}
\]

Since the cell \( \mathcal{C}_\nu \) originally contained \( Z_\nu \) electron states there are \( (Z_\nu - 1) \) states left. These states can be orthonormalized with respect to each other and the state \( \tilde{c}_\nu \). The resulting states we might call \( \tilde{c}_{\nu,\mu} \). If the composition of such a state \( \tilde{c}_{\nu,\mu} (r) \) is

\[
\tilde{c}_{\nu,\mu} (r) = \sum_{j \in \mathcal{C}_\nu} \alpha^j_{\nu,\mu} \tilde{\varphi}_j (r)
\]

then the orthogonality condition

\[
0 = \langle \tilde{c}_\nu | \tilde{c}_{\nu,\mu} \rangle = \int d^3r \frac{1}{A} \sum_{j \in \mathcal{C}_\nu} \tilde{\varphi}_j (0) \tilde{\varphi}_j^\dagger (r) \sum_{j' \in \mathcal{C}_\nu} \alpha^j_{\nu,\mu} \tilde{\varphi}_{j'} (r) = \frac{1}{A} \sum_{j \in \mathcal{C}_\nu} \sum_{j' \in \mathcal{C}_\nu} \alpha^j_{\nu,\mu} \tilde{\varphi}_j (0) = \frac{1}{A} \tilde{c}_{\nu,\mu} (0)
\]

yields that all the other states \( \tilde{c}_{\nu,\mu} (r) \) have vanishing amplitude at the origin and don’t interact with the magnetic impurity.

Let the volume of the host be \( V \) and the total number of electrons be \( 2Z \) (i.e. \( Z \) per spin). Then the average electron density per spin is \( n_0 = Z/V \). This electron density is
in good approximation homogeneously distributed over the energy range from the bottom of the band up to the Fermi energy, i.e. between 0 and \( E_F \) in the energy band. Since the width of the energy cell \( \mathcal{E}_\nu \) is \( \Delta_\nu E_F \) then this energy cell contributes the fraction \( \Delta_\nu n_0 \) to the electron density. On the other hand the density of the Wilson state \( \tilde{\varphi}_j (r) \) at the origin is \( \sum_j |\tilde{\varphi}_j (0)|^2 \). Since all the remaining band states in the cell \( \mathcal{E}_\nu \) have zero density at the origin it follows that
\[
\sum_{j \in \mathcal{E}_\nu} |\tilde{\varphi}_j (0)|^2 = \Delta_\nu n_0 = A^2
\]

Then the amplitude of the Wilson state at the impurity is \( A = \sqrt{\Delta_\nu n_0} \). This is the same amplitude as in the pure host. As long as we have a large volume (so that the energy level spacing of the \( \tilde{\varphi}_j (r) \) is much smaller than the Kondo energy) and neglect the small fluctuations in the local electron density due to the disorder we arrive at essentially the same Hamiltonian (equ. (2)) for the Wilson states as in the pure host. This is because the conduction electrons enter the Hamiltonian only through the energy of the Wilson states and their amplitude \( \tilde{\varphi}_\nu (r) \) at the impurity. The Hamiltonian of the FA-impurity is given by

\[
H_{FA} = \sum_\sigma \left\{ \sum_{\nu=1}^{N} \varepsilon_n c_{\nu \sigma}^{\dagger} c_{\nu \sigma} + E_d d_{\sigma}^{\dagger} d_{\sigma} + \sum_{\nu=1}^{N} V_{sd}(\nu) [d_{\sigma}^{\dagger} c_{\nu \sigma} + c_{\nu \sigma}^{\dagger} d_{\sigma}] \right\} + U n_{d,\uparrow} n_{d,\downarrow} \tag{2}
\]

where \( c_{\nu \sigma}^{\dagger} \) and \( d_{\sigma}^{\dagger} \) are the creation operators for the Wilson and d-states. \( V_{sd}(\nu) \) is the s-d-coupling which is proportional to \( \tilde{\varphi}_{\nu \sigma} (0) \) and \( U \) is the exchange interaction.

Therefore the mathematical form of the Kondo ground state is (in this approximation) the same for the disordered host as for the pure one. But one has to keep in mind that the almost identical properties of the Wilson states for the disordered and pure host are restricted to \( r = 0 \). At larger distances \( r >> l \) from the magnetic impurity the wave functions of the Wilson states for the pure and disordered host are quite different. This has important consequences for the extent of the Kondo cloud.

However, in the first step of the calculation we can essentially take the Wilson Hamiltonian of the FA-impurity in the Anderson model for the pure host. Since this Hamiltonian is already solved we can use this solution for the future discussion. We present the solution in the FAIR description \([32]\).

\[
\Psi_{SS} = \left[ A a_{0,\uparrow}^{\dagger} b_{0,\downarrow}^{\dagger} + B a_{0,\downarrow}^{\dagger} d_{\downarrow}^{\dagger} + C d_{\uparrow}^{\dagger} b_{0,\downarrow}^{\dagger} + D d_{\uparrow}^{\dagger} d_{\downarrow}^{\dagger} \right] |0_{\uparrow} \rangle |0_{\downarrow} \rangle \tag{3}
\]

where

\[
|0_{\uparrow} \rangle = \prod_{i=1}^{n-1} a_{i,\uparrow}^{\dagger} \prod_{i=1}^{n-1} b_{i,\downarrow} \langle \Phi_0 | , \quad |0_{\downarrow} \rangle = \prod_{i=1}^{n-1} b_{i,\uparrow} \prod_{i=1}^{n-1} a_{i,\downarrow} \langle \Phi_0 |
\]
are the half-filled polarized conduction bands, $\ket{\Phi_0}$ is the vacuum state\textsuperscript{1}. For the calculation of the Kondo cloud we divide the Kondo ground state in equ. (3) in two magnetic components, the top and the bottom part of the singlet solution. In zero magnetic field they have opposite s-electron polarization. This polarization is distributed over the occupied states which are composed of Wilson states.

In the disordered case $\tilde{\psi}_\nu (r), \tilde{a}_i (r)$ and $\tilde{b}_i (r)$ are composed of the eigenstates of the disordered host which are in general unknown. Therefore the tricky part in the disordered host is to gain the essential information about the wave function of $\tilde{c}_\nu (r)$.

\section{The Pure Case}

In the following we want to compare the pure host with the dirty host. This consideration is more transparent when we don’t use Wilson’s dimensionless momenta and energies but the standard variables $k$ and $E$ for the momentum and energy of the electron.

In a sphere of radius $R$ the normalized eigenstates with finite amplitude at the origin have the form

$$\tilde{\varphi}_j (r) = \frac{1}{\sqrt{2\pi R}} \frac{1}{2i} r \sin k_j r$$

where $k_j = j \pi / R$ is a standard wave number and $k_\Delta = \pi / R$ is the step width of $k_j$. The averaged density $\rho_0 (r)$ (averaged over $\sin^2 (k_j r)$), integrated over a spherical shell of thickness $dr$ for $r >> 2\pi / k_j$ is

$$4\pi r^2 \rho_0 (r) dr = 4\pi r^2 dr \frac{1}{2\pi R} \frac{1}{2i} r \sin k_j r$$

The state $\tilde{\varphi}_j (r)$ can be split into an incoming and an outgoing wave. The latter has the form

$$\tilde{\varphi}_{j,o} (r) = \frac{1}{\sqrt{2\pi R}} \frac{1}{2i} r \exp (ik_j r)$$

and the incoming part is the conjugate complex state. The outflow of the outgoing wave is

$$J = 4\pi r^2 |\tilde{\varphi}_{j,o} (r)|^2 v_F = 4\pi r^2 \frac{1}{2\pi R} \frac{1}{2i} r \sin k_j r$$

The incoming part has the inflow which is equal and opposite. So the density is equal to

$$\rho_0 (r) = 2J / v_F = 1 / (2\pi r^2 R)$$

A Wilson state has the form

$$\tilde{c}_\nu (r) = \frac{1}{\sqrt{Z_\nu}} \sum_{j \in c_\nu} \frac{1}{\sqrt{2\pi R}} \frac{1}{2i} r \sin k_j r =$$

\textsuperscript{1}We arrive at the final $\Psi_{SS}$ by initially building the two \textit{fair} states $a_0^\dagger$ and $b_0^\dagger$ out of the Wilson basis $\{c_i^\dagger\}$. The \textit{fair} states define the full bases $\{a_i^\dagger\}$ and $\{b_i^\dagger\}$ uniquely. Then the energy expectation value of $\Psi_{SS}$ is calculated and the \textit{fair} states $a_0^\dagger$ and $b_0^\dagger$ are varied until $\Psi_{SS}$ with the lowest energy is obtained. The initial $a_0^\dagger$ and $b_0^\dagger$ can be arbitrary but different. An educated guess reduces the variation time. Details are in ref. [3].
with $Z_\nu = k_FR2^{-\nu}/\pi = k_FR\Delta_\nu/\pi$ and $
abla \Delta_\nu = \frac{R}{\pi} \int dk$

$$= \frac{1}{\pi} \sqrt{\frac{1}{2k_FR}\int \frac{kF(1-2^\nu)}{\Delta_\nu} r \sin (kr) dk}$$

Integration and some manipulations yield

$$\tilde{c}_\nu (r) = \frac{1}{\pi} \sqrt{\frac{\Delta_\nu}{2r}} \sin \left[ kF \left( 1 - \frac{2^\nu + 2^{\nu-1}}{2} \right) r \right] \left\{ \frac{1}{kF\Delta_\nu} \sin \left[ kF\Delta_\nu \frac{r}{2} \right] \right\}$$

The main contribution of this wave function is in the range $0 \leq r \leq 2/(\Delta_\nu k_F) \approx 2^\nu/k_F$ where the term in the right wavy bracket is of the order of one. In this range the term in the left square bracket is roughly normalized and the density $\rho_0 (r)$ for $r \gg 2/(\Delta_\nu k_F)$ approaches zero with increasing $r$ because of the $r^{-2}$ radial dependence of $\tilde{c}_\nu (r)$.

### 2.1 Calculation of the polarization

The Kondo ground state is an entanglement of two magnetic components with opposite net moment of the d-state. The Kondo cloud is given by the polarization of one of the two magnetic components. (We construct the ground state so that the first half in equ. (3) has a net negative d-spin.

$$\Psi_\downarrow = [ Aa^\dagger_{0,\uparrow}b_{0,\downarrow}^\dagger + Bb^\dagger_{0,\uparrow}d_{0,\downarrow}^\dagger + Cd^\dagger_{0,\uparrow}b_{0,\downarrow}^\dagger + Dd^\dagger_{0,\uparrow}d_{0,\downarrow}^\dagger ] |0_\uparrow,0_\downarrow\rangle$$

Here $|0_\uparrow,0_\downarrow\rangle = |0_\uparrow\rangle |0_\downarrow\rangle$ with $|0_\uparrow\rangle = \prod_{i=1}^n a^\dagger_{i,\uparrow} |\Phi_0\rangle$ is the (anti-symmetric) product of the occupied wave functions $\tilde{a}_{i,\uparrow} (r)$ which are composed of Wilson states

$$\tilde{a}_i (r) = \sum_\nu K_\nu K^\nu_i (r)$$

The matrix $K_\nu$ has been determined in the process of deriving the FAIR solution (3).

$$\tilde{a}_i (r) = \frac{1}{\pi} \frac{1}{r^2} \sum_\nu \sqrt{\frac{2}{\Delta_\nu k_F}} K_\nu \sin \left[ kF \left( 1 - \frac{2^\nu + 2^{\nu-1}}{2} \right) r \right] \sin \left[ kF\Delta_\nu \frac{r}{2} \right]$$

The orbital part of $\tilde{a}_{i,\uparrow} (r)$ and $\tilde{a}_{i,\downarrow} (r)$ are identical. Then the contribution of any wave function $\tilde{a}_{i,\uparrow} (r)$ in the occupied FAIR band $|0_\uparrow\rangle$ to the spin polarization of $\Psi_\downarrow$ is $\frac{1}{2} |\tilde{a}_i (r)|^2$.

The definitions of the wave functions $\tilde{b}_i (r)$ and their contribution to the polarization are equivalent.
The total polarization of the s-electrons in $\Psi_\downarrow$ is
\[
p^0 (r) \, dr = \frac{1}{2} \left[ |A|^2 \left( |\tilde{a}_0 (r)|^2 - |\tilde{b}_0 (r)|^2 \right) + |B|^2 |\tilde{a}_0 (r)|^2 - |C|^2 |\tilde{b}_0 (r)|^2 \right] \, dr \\
+ \frac{1}{2} \sum_{i=1}^n \left[ |\tilde{a}_i (r)|^2 - |\tilde{b}_i (r)|^2 \right] \, dr
\]

Within the FAIR theory this polarization $p^0 (r)$ in the pure host has been calculated [18].

3 Disordered Host

We consider a large weakly disordered host with $k_F l >> 1$. Furthermore the magnetic impurity (which we locate at the origin) should not be close to a normal impurity. In that case in the range $r < l/2$ the Wilson states $\tilde{c}_\nu (r)$ possess essentially the same wave functions as in the pure case. Within the cage of the nearest impurities the electrons are free and can be expressed as superpositions of plane waves. Let us assume that very close to the magnetic impurity the eigenstate of the disordered host $\tilde{c}_\nu (r)$ has the asymptotic form $\tilde{\varphi}_k (r) \cong \alpha_k e^{ik(r-r_0)}$. Then it contributes to the (appropriate) Wilson state $\tilde{c}_\nu (r)$ the component $\tilde{\varphi}_k (r) \tilde{\varphi}_k (0) \cong |\alpha_k|^2 e^{ikr}$.

The plane wave $\alpha_k e^{ikr}$ consists of Bessel functions
\[
|\alpha_k|^2 e^{ikr} = |\alpha_k|^2 4\pi \sum_{l=0}^\infty \sum_{m=-l}^l j_l (kr) Y_{l}^m (\theta_k, \phi_k) Y_{l}^{-m} (\theta_r, \phi_r)
\]

Only the $l = 0$ component is non-zero at the origin and equal to $|\alpha_k|^2 j_0 (kr) = |\alpha_k|^2 \sin (kr) / (kr)$. Summing over all eigenstates in the energy cell $C_\nu$ yields a constructive interference for the $l = 0$ components with a total amplitude of $\sum_{k \in C_\nu} |\alpha_k|^2 \sin (kr) / (kr)$ where $\sum_{k \in C_\nu} |\alpha_k|^2 \cong \sqrt{\Delta C}$.

The contributions of the $l > 0$ angular momenta cancel to zero in first approximation. This argument also works if the asymptotic form of $\tilde{\varphi}_k (r)$ is a standing wave.

When we sum over all disordered states in the energy cell $C_\nu$ we obtain within $r < l$ essentially the same state as in the pure host. It will not be perfectly spherical and is slightly disturbed by the back scattering from the nearest impurities. If one would perform impurity averaging at this point the state would become spherically symmetric, but it would also decay exponentially with increasing distance. However, we will perform the impurity averaging in a later stage.

First we recall that $\tilde{c}_\nu (r)$ (in the pure host) represents a superposition of a spherical incoming and outgoing wave. In the pure case the trajectories of the waves are radial beams. In the disordered host the scattering folds these trajectories. At each impurity the trajectory is split and folded (see Fig. 1) and a phase shift is attached to the wave function along the
trajectory.

Fig.1: a) The radial trajectory in a pure host. b) In the disordered host the trajectory splits at each impurity into different directions and picks up a phase shift. In both cases the wave function is plotted along the trajectories.

Before we discuss the density distribution we proceed to the basis states of the FAIR band, for example \( \tilde{a}_i (r) \). In the pure case we have calculated numerically the wave function \( \tilde{a}_i (r) \) using the Wilson states. Its value along the radial trajectory is known. In the disordered case we use the same same wave function \( \tilde{a}_i (r_t) \) along the trajectory where \( r_t \) is the path along the trajectory (not the distance from the impurity). Again at each impurity the trajectory (and the wave function) is split, its direction is changed and a phase shift is attached to the wave function along the next leg of the trajectory.

In Fig.2 a few folded trajectories are shown. While the (properly) integrated density of \( |\tilde{a}_i (r_t)|^2 \) over each leg of all trajectories is still normalized the crossing between different trajectories yields interferences.
This random propagation represents a quantum diffusion. A trajectory which in the pure host reaches the distance $r_0$ from the origin will have in an average $r_0/l$ collisions and reaches only a distance of $\sqrt{r_0/l} = \sqrt{r_0/l}$ from the origin. The detailed distribution of the charge $|\tilde{a}_i(r)|^2$ in the disordered host depends on the distribution of the impurities. Therefore we perform an impurity averaging. This averages the interferences to zero. Now we can calculate the averaged charge distribution of our FAIR state $\tilde{a}_i(r)$.

We consider in the pure host the in and outgoing waves of $\tilde{a}_i(r)$ in the spherical shell between $r_0$ and $r_0 + dr_0$. Let the density at the distance $r_0$ be $\rho_0^i(r_0)$, then the total charge in the spherical shell of thickness $dr_0$ is $4\pi r_0^2 \rho_0^i(r_0) dr_0$. In the disordered host this charge will obey a diffusion profile with the diffusion constant $D = \frac{1}{3} v_F l$ yielding

$$d\rho_{do}^i(r) = 4\pi r_0^2 \rho_0^i(r_0) \frac{1}{(\pi D t_0)^{3/2}} \exp \left( -\frac{r^2}{4Dt_0} \right) \, dr_0$$

where $t_0$ stands for $r_0/v_F$ which could be interpreted as a traveling time. We have to add the contribution from all spherical shells (which corresponds to an integration over $dr_0$). This yields the charge density $\rho_{do}^i(r)$ of $\tilde{a}_i(r)$ in the disordered host in terms of the charge density in the pure host $\rho_0^i(r_0)$.

The density for all FAIR states $\tilde{a}_i(r)$, and $\tilde{b}_i(r)$ has been calculated in the pure case in [18]. In complete analogy these densities in the pure host can be translated into the disordered case using relation (8). The total averaged polarization $\overline{p_{do}}(r)$ of the state $\Psi_\downarrow$ is
then

\[ \overline{p_{do}}(r) = 4\pi \int_0^\infty r_0^2 p_0(r_0) \frac{1}{\left(\pi l r_0/3\right)^{3/2}} \exp \left(-\frac{3r^2}{4lr_0}\right) dr_0 \] (9)

Within this model the Kondo cloud polarization in a disordered host can be obtained from the polarization in a pure host by means of equ. (9). Any polarization \( p_0(r) \) in the pure host can be transferred into the disordered host.

The range of the polarization in a disordered host is of the order of \( \sqrt{\xi_K l} \) where \( \xi_K \) is the Kondo length in the pure host.

For a Kondo system with a Kondo temperature of \( T_K \approx 2K \) and a host with a Fermi velocity of about \( v_F \approx 3 \times 10^6 m/s \) the Kondo length (in a pure host) is about \( \xi_K = 10\mu \). This incloses a sphere of volume \( 4 \times 10^3 \mu^3 \). If the host has a mean free path of \( l = 10nm \) then one obtains for the dirty host a Kondo length of about \( \xi_{K\text{do}} \approx 0.3\mu \). Now the Kondo cloud is distributed over volume which is smaller by a factor of \( 4 \times 10^4 \). This improves the chance to detect the Kondo cloud experimentally although some of the previous experimental methods might not be applicable in the disordered host.

4 Conclusions

The Kondo cloud in a weakly disordered host is investigated. The two magnetic components which are entangled in the ground state are artificially separated and the spin polarization of one of the components is calculated. If the magnetic impurity is well separated from the non-magnetic impurities then within a sphere of radius \( l/2 \) (i.e. half the mean free path) the ground-state wave functions in the disordered host are very close to those of the pure host. These wave functions are superpositions of incoming and outgoing spherical waves. In the pure host their propagation is radial and the difference between their densities for spin up and down give the polarization of the Kondo cloud. In the disordered host the trajectories are split at the impurities and represent quantum diffusion. Averaging over the impurity position permits us to calculate the polarization in the disordered host from the known polarization in the pure host. The spatial extension is reduced from the Kondo length \( \xi_K \) in the pure host to \( \sqrt{\xi_K l} \) in the disordered host.
References

[1] J. Kondo, Prog. Theor. Phys. 32, 37 (1964), Resistance Minimum in dilute magnetic alloys.

[2] A. C. Hewson, The Kondo problem to heavy Fermions, Cambridge University Press, 1993.

[3] R. Bulla, T. A. Costi, and T. Pruschke, Rev. Mod. Phys. 80, 395 (2008), Numerical renormalization group method for quantum impurity systems.

[4] C. Latta, F. Haupt, M. Hanl, A. Weichselbaum, M. Claassen, W. Wuester, P. Fallahi, S. Faelt, L. Glazman, J. von Delft, H. E. Türeci, and A. Imamoglu, Nature 474, 627 (2011), Quantum quench of Kondo correlations in optical absorption.

[5] G. Bergmann, J. Supercond. Nov. Magn. 25, 609 (2012), A Compact Treatment of Singular Impurities Using the Artificial Friedel Resonance (FAIR) Method.

[6] G. Horwitz, S. Alexander, and M. Fibich., Phys. Rev. 168, 495 (1968), Singlet-Triplet Formulation for Dilute Magnetic Alloys. I. s-d Exchange Model.

[7] P. W. Anderson and G. Yuval, Phys. Rev. Lett. 23, 89 (1969), Exact Results in the Kondo Problem: Equivalence to a Classical One-Dimensional Coulomb Gas.

[8] J. B. Boyce, and C. P. Slichter, Phys. Rev. Lett. 32, 61 (1974), Conduction-Electron Spin Density around Fe Impurities in Cu above and below TK.

[9] E. S. Sorensen, and I. Affleck, Phys. Rev. B 53, 9153 (1996), Scaling theory of the Kondo screening cloud.

[10] V. Barzykin and I. Affleck, Phys. Rev. Lett. 76, 4959 (1996), The Kondo Screening Cloud: What Can We Learn from Perturbation Theory?

[11] I. Affleck, and P. Simon, Phys. Rev. Lett. 86, 2854 (2001), Detecting the Kondo Screening Cloud Around a Quantum Dot.

[12] I. P. Simon and I. Affleck, Phys. Rev. Lett. 89, 206602 (2002), Finite-Size Effects in Conductance Measurements on Quantum Dots.

[13] J. Kroha, S. Kirchner, G. Sellier, P. Woelfle, D. Ehm, F. Reinert, S. Hübner and C. Geibel, Physica E: 18, 69 (2003), Structure and transport in multi-orbital Kondo systems.

[14] R. G. Pereira, N. Laflorencie, I. Affleck, and B. I. Halperin, arXiv:cond-mat/0612635 (2007), Kondo Screening Cloud and Charge Quantization in Mesoscopic Devices.

[15] L. Borda, Phys. Rev. B 75, 041307(R) (2007), Kondo screening cloud in a one-dimensional wire: Numerical renormalization group study.
[16] J. Simonin, arXiv:0708. 3604 (2007) , Looking for the Kondo cloud

[17] I. Affleck, L. Borda, H. Saleur, Phys. Rev. B 77, 180404(R) (2008) , Friedel oscillations and the Kondo screening cloud

[18] G. Bergmann, Phys. Rev. B 77, 104401 (2008) , Quantitative calculation of the spatial extension of the Kondo cloud

[19] G. Bergmann, Phys. Rev. B 78, 195124 (2008) , Friedel oscillations of Kondo impurities: A comparison

[20] G. Bergmann, Eur. Phys. J. B 75, 497 (2010) , Density of states in the magnetic ground state of the Friedel-Anderson impurity

[21] G. Bergmann and R. S. Thompson, Eur. Phys. J. B 84, 273 (2011) , Numerical Calculation of the Fidelity for the Kondo and the Friedel-Anderson Impurities

[22] A. Holzner, I. P. McCulloch, U. Schollwöck1, J. v. Delft, and F. Heidrich-Meisner, Phys. Rev. B 80, 205114 (2009) , Kondo screening cloud in the single-impurity Anderson model: A density matrix renormalization group study

[23] G. Bergmann, and Y. Tao, Eur. Phys. J. B 73, 95 (2010) , Oscillations of the magnetic polarization in a Kondo impurity at finite magnetic fields

[24] C. A. Buesser, G. B. Martins, L. Costa Ribeiro, E. Vernek, E. V. Anda, and E. Dagotto, Phys. Rev. B 81, 045111 (2010) , Numerical analysis of the spatial range of the Kondo effect

[25] C. A. Buesser, G. B. Martins, L. C. Ribeiro, E. Vernek, E. V. Anda, and E. Dagotto, Phys. Rev. B 81, 045111 (2010) , Numerical analysis of the spatial range of the Kondo effect

[26] M. Vojta, L. Fritz and R. Bulla, Europhys. Lett. 90, 27006 (2010) , Gate-controlled Kondo screening in graphene: Quantum criticality and electron-hole asymmetry

[27] J. Park, S.-S. B. Lee, Y. Oreg, and H.-S. Sim, Phys. Rev. Lett. 110, 246603 (2013) , How to Directly Measure a Kondo Cloud’s Length

[28] J. R. Schrieffer, J. Appl. Phys. 38, 1143 (1967) , The Kondo Effect—The Link Between Magnetic and Nonmagnetic Impurities in Metals?

[29] V. Dobrosavljevi?, T. R. Kirkpatrick, and B. G. Kotliar, Phys. Rev. Lett. 69, 1113 (1992) , Kondo effect in disordered systems

[30] S. Kettemann and E. R. Mucciolo, Phys. Rev. B 75, 184407 (2007), Disorder-quenched Kondo effect in mesoscopic electronic systems
[31] S. Kettemann, E. R. Mucciolo, I. Varga, and K. Slevin, Phys. Rev. B 85, 115112 (2012), Kondo-Anderson transitions

[32] G. Bergmann, Phys. Rev. B 74, 144420 (2006), Compact Approximate Solution to the Friedel-Anderson Impurity Problem

[33] G. Bergmann, Phys. Rev. B 73, 092418 (2006), A Critical Analysis of the Mean-Field Approximation for the Calculation of the Magnetic Moment in the Friedel-Anderson Impurity Model

[34] G. Bergmann and L. Zhang, Phys. Rev. B 76, 064401 (2007), A Compact Approximate Solution to the Kondo Problem