Dihadron Correlation in Jets Produced in Heavy-Ion Collisions

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Abstract

The difference between the structures of jets produced in heavy-ion and hadronic collisions can best be exhibited in the correlations between particles within those jets. We study the dihadron correlations in jets in the framework of parton recombination. Two types of triggers, $\pi^+$ and proton, are considered. It is shown that the recombination of thermal and shower partons makes the most important contribution to the spectra of the associated particles at intermediate $p_T$. In $pp$ collisions the only significant contribution arises from shower-shower recombination, which is negligible in heavy-ion collisions. Moments of the associated-particle distributions are calculated to provide simple summary of the jet structures for easy comparison with experiments.
1 Introduction

Recent experiments at RHIC have revealed extensive information on the effects of the dense medium on hadron production at large transverse momentum ($p_T$) [1, 2, 3]. The suppression of back-to-back correlation relative to the same-side correlation is a strong indication of substantial energy loss suffered by hard partons propagating through the medium [4]. It implies then that the jets detected in heavy-ion collisions are produced mainly near the surface of the medium so that the hard partons are less attenuated by the jet quenching effects [5]. If that is indeed the case, then a simple fragmentation model would predict that the structure of jets produced in heavy-ion collisions should be basically the same as that of jets in $pp$ collisions. That similarity has been shown to be absent in the data of more recent experiments [6]. The aim of this paper is to investigate the jet structure by examining the two-particle correlation within a jet produced by nuclear collisions at high energy. The difference from $pp$ jets is naturally caused by the presence of thermal partons in the jet environment, which we shall take into account.

The framework in which we shall study hadron production at large $p_T$ is parton recombination, which has been shown to explain some features of the data where fragmentation fails [8]-[11]. The anomaly associated with species dependence of the Cronin effect in d+Au collisions has also been resolved in the recombination picture [12, 13]. The main component that is new in this series of work is the inclusion of shower partons generated by hard partons. Whereas the recombination of two shower partons forms a hadron that can be identified with the fragmentation product in the conventional approach, the recombination of shower partons with thermal partons yield hadrons in the intermediate $p_T$ region that are
totally new. Here we go a step further. To study dihadrons in a jet we must consider four partons that recombine to form two hadrons; some of those partons will be thermal in order to have enhanced yield. Dihadrons formed without thermal partons correspond to those found in jets in vacuum, and are suppressed compared to those hadron pairs that involve the participation of thermal partons.

We shall consider two types of triggers, pion and proton, and calculate the distributions of the associated particles. Since we include the contributions from different species of hard partons, each having various flavors of shower partons, and since the four recombining partons have different momentum fractions that have to be permuted in the recombination formula, the combinatorial complication can result in a hundred terms or more. For that reason we limit our trigger to only $\pi^+$ and $p$, which are sufficient to reveal the properties of the jet structures. Detailed comparison of our results to current data is, however, difficult, since the experimental trigger at this stage consists of all charged hadrons, as are the associated particles [4, 6, 7]. Separating triggers to mesons and baryons has not resulted in any $p_T$ distributions for the associated particles [14]. Nevertheless, some coarse comparisons with the data can be made, and our results will be shown to be reasonable.

2 Single- and Two-particle Distributions

The hadronization process that we consider in order to calculate the dihadron correlation in a jet is parton recombination. The formalism for single-particle inclusive distribution at high $p_T$ is given in [11]. An essential ingredient in the hadronization process is the shower parton distributions (SPD), which give the probabilities of finding shower partons of various
flavors and momentum fractions in jets initiated by different hard partons [15]. Convolving the SPD’s, denoted by $S^j_i$, with the hard-scattered parton distributions $f_i(k)$ in heavy-ion collisions gives the shower component

$$S(q_1) = \xi \sum_i \int dk k f_i(k) S^j_i(q_1/k) ,$$

where the sum is over all hard partons, and $\xi$ is the average suppression factor due to energy loss in a dense medium, found to be 0.07 for central Au+Au collisions at $\sqrt{s} = 200$ GeV [11]. For two shower partons in the same jet we have

$$SS(q_1, q_2) = \xi \sum_i \int dk k f_i(k) \left\{ S^j_i \left( \frac{q_1}{k} \right), S^j_i \left( \frac{q_2}{k} \right) \right\} ,$$

where the curly brackets denote the symmetrization of the leading parton momentum fraction

$$\left\{ S^j_i(z_1), S^j_i(z_2) \right\} = \frac{1}{2} \left[ S^j_i(z_1) S^j_i \left( \frac{z_2}{1 - z_1} \right) + S^j_i \left( \frac{z_1}{1 - z_2} \right) S^j_i(z_2) \right].$$

This symmetrization is necessary, since either $j$ or $j'$ shower parton may be the leading parton in the jet, and Eq. (3) is a way to ensure that momentum conservation $z_1 + z_2 \leq 1$ is not violated. The SPD’s are determined by solving the recombination formula for the fragmentation functions (FF)

$$x D^M_i(x) = \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} \left\{ S^j_i(x_1), S^{j'}_i(x_2) \right\} R^{j'^j}_M(x_1, x_2, x) ,$$

where $R^{j'^j}_M$ is the recombination function (RF) for the hadronization process $j + j' \rightarrow M$.

For the hadronization of a hard parton in vacuum it is unnecessary to consider the shower partons, since they recombine to form $M$, as in Eq. (4), to recover the FF, from which the SPD’s are obtained. However, in the environment of thermal partons as in heavy-ion collisions the shower partons can recombine with the thermal partons, resulting in hadrons
that are dominant in the intermediate $p_T$ region because they benefit from the high density of the thermal partons as well as the higher momenta of the semi-hard shower partons. The invariant distribution of the thermal partons is parameterized by

$$T(q_1) = q_1 \frac{dN_{q}^{th}}{dq_1} = C q_1 \exp(-q_1/T),$$

where $T$ is the inverse slope enhanced by flow. Since we have no model to describe the hydrodynamical evolution of the bulk medium at low transverse momentum, the parameters $C$ and $T$ are determined by fitting the low-$p_T$ data. They are found to be [11]

$$C = 23.2 \text{ GeV}^{-1}, \quad T = 0.317 \text{ GeV},$$

for central Au+Au collisions at midrapidity and $\sqrt{s} = 200 \text{ GeV}$.

With both the thermal $T(q_1)$ and shower distributions $S(q_2)$ known, it is possible to calculate the hadron distribution in the 1D recombination formalism for the formation of a meson [16, 17]

$$p^0 \frac{dN_M}{dp} = \int \frac{dq_1}{q_1} \frac{dq_2}{q_2} F_{qq'}(q_1, q_2) R_M(q_1, q_2, p),$$

where $F_{qq'}(q_1, q_2)$ is the joint distribution of a quark $q$ at $q_1$ and an antiquark $\bar{q}'$ at $q_2$. For pion production at high $p_T$, we put $\vec{p}$ in the transverse plane, abbreviate $p_T$ as $p$, ignore pion mass and write [11]

$$\frac{dN_{\pi}}{pdp} = \frac{1}{p^3} \int_0^p dq_1 F_{\pi\pi'}(q_1, p - q_1),$$

where the RF [17, 18]

$$R_{\pi}(q_1, q_2, p) = \frac{q_1 q_2}{p^2} \delta \left( \frac{q_1}{p} + \frac{q_2}{p} - 1 \right),$$

5
has been used. For heavy-ion collisions $F_{q\bar{q}}$ can be written in the form

$$F_{q\bar{q}} = \mathcal{T}\mathcal{T} + \mathcal{T}\mathcal{S} + \mathcal{S}\mathcal{S},$$

(10)

where the possibility of two shower partons from two different jets is ignored. It is the $\mathcal{T}\mathcal{S}$ term in Eq. (10) that dominates at intermediate $p_T$, while the $\mathcal{T}\mathcal{T}$ and $\mathcal{S}\mathcal{S}$ terms are dominant in the low and very high $p_T$ regions, respectively.

Two-pion distribution can be obtained by a straightforward extension of the single-particle distribution given in Eq. (7), and one gets

$$\frac{dN_{\pi_1\pi_2}}{p_1p_2dp_1dp_2} = \frac{1}{p_1^2p_2^2} \int \left( \prod_{i=1}^{4} \frac{dq_i}{q_i} \right) F_4(q_1, q_2, q_3, q_4)R_{\pi_1}(q_1, q_2, p_1)R_{\pi_2}(q_3, q_4, p_2),$$

(11)

where sums over different combinations of partons contributing to the two RF’s are not exhibited explicitly. $F_4(q_1, q_2, q_3, q_4)$ is the joint distribution of two quarks and two antiquarks. If we are to study the two-pion distribution in a jet, then neither of the two pions should be the hadronization of thermal pions only. That is, each pion should contain at least one shower parton in order to qualify as a part of the jet. Using the terminology “thermal hadrons” to refer to the hadronization of thermal partons only, then in the experimental analysis of jet structure such thermal hadrons are regarded as background and are subtracted from the set of particles associated with a trigger. In our calculation of $dN_{\pi_1\pi_2}/dp_1dp_2$ associated with a jet, we simply leave out $\mathcal{T}\mathcal{T}$ contribution to any pion. Thus there are only two types of terms for $F_4$, which we represent schematically as

$$F_4 = (\mathcal{T}\mathcal{S})(\mathcal{T}\mathcal{S}) + (\mathcal{T}\mathcal{S})(\mathcal{S}\mathcal{S}),$$

(12)

where a term of the type $(\mathcal{S}\mathcal{S})(\mathcal{S}\mathcal{S})$ is omitted because without $\mathcal{T}$ it is negligible compared to $(\mathcal{T}\mathcal{S})(\mathcal{S}\mathcal{S})$ for hadron $p_T < 6$ GeV/c. In Eq. (12) the parentheses enclose the partons that
are to recombine. With that notation \((T S)(T S)\) is very different from \((T T)(SS)\), which contributes to a thermal hadron that we exclude from our consideration.

In addition to two-pion correlation we shall also study the correlation between a pion and a proton in a jet. The single-baryon distribution has the general form

\[
p^0 \frac{dN_B}{dp} = \int \left( \prod_{i=1}^{3} \frac{dq_i}{q_i} \right) F_3(q_1, q_2, q_3) R_B(q_1, q_2, q_3, p) \tag{13}
\]

where the RF for proton is \([8]\)

\[
R_p(q_1, q_2, q_3, p) = \frac{g_{\alpha\beta}}{6} \left( y_1 y_2 \right)^{\alpha+1} y_3^{\beta+1} \delta \left( \sum_{i=1}^{3} y_i - 1 \right), \quad y_i = \frac{q_i}{p} \tag{14}
\]

\[
g_{\alpha\beta} = \left[ B(\alpha + 1, \alpha + \beta + 2) B(\alpha + 1, \beta + 1) \right]^{-1} \tag{15}
\]

with \(\alpha = 1.75\) and \(\beta = 1.05\) \([19]\). Thus the \(\pi p\) joint distribution in a high-\(p_T\) jet is

\[
\frac{dN_{\pi p}}{p_1 p_2 dp_1 dp_2} = \frac{1}{p_1^2 p_2^2} \int \left( \prod_{i=1}^{5} \frac{dq_i}{q_i} \right) F_5(q_1, q_2, q_3, q_4, q_5) R_\pi(q_1, q_2, p_1) R_p(q_3, q_4, q_5, p_2). \tag{16}
\]

The 5-parton distribution \(F_5\) has the schematic form

\[
F_5 = (T S)(T T S) + (T S)(T S S), \tag{17}
\]

where we have omitted terms of the type \((T S)(SSS)\), \((SS)(T SS)\) and \((SS)(SSS)\) because they are all negligible compared to the ones in Eq. (17).

In the next section we examine in detail the multi-parton distributions \(F_4\) and \(F_5\) and how they contribute to the associated particle distributions, when either the pion or the proton is used as a trigger.
3 Distributions of Associated Particles

To select the appropriate two-particle distributions to calculate, let us examine the type of quantities that have been measured experimentally. There exist data from RHIC experiments that give some properties of charged particles associated with triggers detected in certain $p_T$ ranges [6, 7, 14] We shall therefore calculate distributions with similar kinematic ranges. Because of detection limitations there is at present no particle identification in either the trigger or the associated particles. We shall, nevertheless, do the calculations for specific particle species in anticipation for the corresponding data that will become available in the future. In particular, we shall consider $\pi^+$ and $p$ as trigger particles, and $\pi^\pm$ and $p$ as associated particles. Considering all charged particles would involve overwhelming complications and uncertainties without gaining clarity.

The two-particle distributions given in Eqs. (11) and (16) do not explicitly specify what the two particles at $p_1$ and $p_2$ are within the same jet. The jet momentum $k$, which is the momentum of the initiating hard parton, is imbedded in the expression for the shower partons, Eq. (2), and is integrated over all values in a collision. When the two-parton distribution, such as that given in Eq. (10), is generalized to $F_4$ in Eq. (12), it is important to make sure that the hard partons in each of $S$ in (12) are one and the same, and similarly in Eq. (17). To make this point explicit, it is helpful to write the two-particle distribution within one jet as $dN^{(i)}_{h_1h_2}(k)/dp_1dp_2$, so that all shower partons are associated with the hard parton $i$ at momentum $k$. Then the particle ($h_2$) distribution associated to a trigger particle
\( (h_1) \) at \( p_1 \) is

\[
\left. \frac{dN_{h_2}}{dp_2} \right|_{h_1(p_1)} = \frac{\sum_i \int dk k f_i(k) \frac{dN^{(h_2)}_{h_1}}{dp_1 dp_2}(k)}{\sum_i \int dk k f_i(k) \frac{dN^{(h_1)}_{h_2}}{dp_3}(k)}. \tag{18}
\]

Further integration of Eq. (18) over \( p_1 \) in the specified range of the trigger momentum yields the distribution of the associated particles that is measured. Note that the energy loss suppression factor \( \xi \) is cancelled in Eq. (18). In the following our expressions for \( S \) will not contain \( \xi \sum_i \int dk k f_i(k) \) that appears in Eqs. (1) and (2), since it is now shown explicitly in Eq. (18).

Let us consider first \( \pi^+ \) trigger, and \( \pi^+, \pi^- \), and \( p \) associated with it. The initiating hard parton \( i \) can be \( u, d, s, \bar{u}, \bar{d}, \bar{s} \) and \( g \). For every given \( i \), the shower parton \( j \) can be \( u, d, \bar{u}, \bar{d} \). There are therefore 28 possible SPD’s. For \( \pi^+ \pi^+ \) in a jet, \( F_4 \) for four partons has three terms

\[
F_{\pi^+\pi^+}^4 = (TS)(TS) + (TS)(SS) + (SS)(TS), \tag{19}
\]

where the first pair of parentheses in each term correspond to the trigger, the second the associated particle. We omit the term \( (SS)(SS) \) because it is negligible. Note that it is the only term contributing to two particles in a jet produced in \( pp \) collisions, since there are no thermal partons of any significance in the environment. Herein lies already the difference between jets produced in heavy-ion and hadronic collisions, without even any quantitative details to be investigated. For \( \pi^+ \pi^+ \), \( j \) should only be \( u \) or \( \bar{d} \) within each of the six sets of parentheses; the thermal partons should just be given the flavor of the complement to make a \( \pi^+ \), i.e., \( u \bar{d} \). Thus \( (TS)(TS) \) is a symbolic short-hand for

\[
(TS)(TS) = [(T_u S_{\bar{d}}) + (S_u T_{\bar{d}})] \cdot [(T_u S_{\bar{d}}) + (S_u T_{\bar{d}})], \tag{20}
\]
where the subscript denotes the flavor. For the other terms in Eq. (19) we have

\[(\mathcal{T}S)(SS) = ((\mathcal{T}_u S_d + (S_u \mathcal{T}_d)) (S_u S_d)),\]  

(21)

\[(SS)(\mathcal{T}S) = (S_u S_d) [(\mathcal{T}_u S_d + (S_u \mathcal{T}_d))].\]  

(22)

For \(\pi^+\pi^-\) production we have in a similar way

\[F_{\pi^+\pi^-} = [(\mathcal{T}_u S_d + (S_u \mathcal{T}_d)) \cdot [(\mathcal{T}_u \mathcal{T}_u S_d) + 2(\mathcal{T}_u S_u \mathcal{T}_d) + 2(\mathcal{T}_u S_u S_d) + (S_u S_u \mathcal{T}_d)].\]  

(23)

It should be noted that in the above expression \(u (\bar{d})\) have momenta \(q_1 (q_2)\), and \(\bar{u} (d)\) have momenta \(q_3 (q_4)\).

In the case of \(\pi^+p\) correlation there is an extra \(\mathcal{T}\) or \(\mathcal{S}\), as shown in Eq. (17). Hence, the 5-parton distribution has the form

\[F_{\pi^+p} = [(\mathcal{T}_u S_d + (S_u \mathcal{T}_d)) \cdot [(\mathcal{T}_u \mathcal{T}_u S_d) + 2(\mathcal{T}_u S_u \mathcal{T}_d) + 2(\mathcal{T}_u S_u S_d) + (S_u S_u \mathcal{T}_d)].\]  

(24)

The momenta of the quarks in the proton are in the order \(u(q_3)u(q_4)d(q_5)\). The two factors of 2 in the above equation arise from the fact that \(\mathcal{T}_u(q_3)S_u(q_4)\mathcal{T}_d(q_5)\) makes the same contribution as \(S_u(q_3)\mathcal{T}_u(q_4)d(q_5)\). That is, the two \(u\) quarks can each receive contributions from the thermal and shower sources. The same holds for \(\mathcal{T}_u S_u S_d\) and \(S_u \mathcal{T}_u S_d\) also.

For proton trigger we shall consider only \(\pi^+\) and \(\pi^-\) as the associated particles. For \(\pi^+\) associated with \(p\), \(F_{\pi^+p}^p\) is trivially related to \(F_{\pi^+p}^p\) simply by interchanging the positions of the parentheses in Eq. (24), i.e., interchanging trigger and associated particle, and \(p_1 \leftrightarrow p_2\). For \(\pi^-\) associated with \(p\) we have

\[F_{\pi^-p}^p = [(\mathcal{T}_u \mathcal{T}_u S_d) + 2(\mathcal{T}_u S_u \mathcal{T}_d) + 2(\mathcal{T}_u S_u S_d) + (S_u S_u \mathcal{T}_d)] \cdot [(\mathcal{T}_u S_d) + (S_u \mathcal{T}_d)].\]  

(25)
The thermal partons are flavor independent, so Eq. (5) can be used for $u$, $d$ and their antiquarks. The situation with the shower partons are far more complicated. Their distributions depend on the species of the initial hard parton $i$ and the shower parton $j$. However, by not considering the production of kaons and hyperons, there are only three basic SPD’s: $K$, $L$, and $G$. We can write $S^j_i$ in the matrix form

\[
S^j_i = \begin{pmatrix} K & L \\ L & K \\ L & L \\ G & G \end{pmatrix}
\]

For example, $S^u_u = K$, $S^u_d = L$, $S^u_s = L$, $S^u_g = G$. Antiquarks are like quarks of different flavors, i.e, $S^\bar{u}_\bar{u} = K$, $S^\bar{u}_\bar{u} = L$, $S^\bar{d}_\bar{d} = L$, etc. Note that hard $s$ quark is included, though not shower $s$ quark. The parametrizations of the SPD’s are given in Ref. [15], in which one can also find a thorough discussion of why $j = g$ is excluded due to gluon conversion. In Eqs. (20)-(25) the label $i$ for hard partons is suppressed, but is shown explicitly in Eq. (18). The subscripts of $S$ in those equations correspond to the label $j$ above. Thus for every fixed $i$ all the $S_j$ distributions in Eqs. (20)-(25) can be rewritten as $K$, $L$ and $G$. When $i$ is changed, the translation also changes. Thus, for example, when $i = u$, $\bar{u}$, $d$, $\bar{d}$, $s$, $\bar{s}$, and $g$, the $(TS)$ for $\pi^+$ in Eq. (20) becomes

\[
(TS)_{i=u} = (TL) + (KT) ,
\]

\[
(TS)_{i=\bar{u},d,s,\bar{s}} = (TL) + (LT) ,
\]

\[
(TS)_{i=d} = (TK) + (LT) ,
\]
\[(T S)_{i=g} = (T G) + (G T) . \] (30)

The index of \( T \) has been omitted because of flavor independence. In the case of proton, if \( i = u \), then only one shower \( u \) quark can be valence, the other must be in the sea; consequently, we have for the \( T SS \) part in Eq. (24), for example,

\[(T S S)_{i=u} = 2(T K L) + (K L T) , \] (31)

\[(T S S)_{i=d} = 2(T L K) + (L L T) , \] (32)

while, for \( i = \bar{u}, \bar{d}, s \) and \( \bar{s} \), \( K \)'s above are replaced by \( L \)'s, and for \( i = g \), both \( K \) and \( L \) are replaced by \( G \).

The application of Eqs. (27)-(30) to (20)-(22) must take into account the consideration that there can only be one valence quark in a jet, and it can be in either the trigger or the associated particle. Thus, for example, \((T S)(T S)\) in Eq. (20) for \( \pi^+\pi^+ \) should have the explicit form, for \( i = u \),

\[
[(T S)(T S)]_{i=u} = \frac{1}{2} \{[(T L) + (K T)] \cdot [(T L) + (L T)]
\]

\[
+ [(T L) + (L T)] \cdot [(T L) + (K T)] \} , \] (33)

whereas, for \( \pi^+p \), the term \((T S)(T S S)\) in Eq. (24) becomes

\[
[(T S)(T S S)]_{i=u} = \frac{1}{2} \{[(T L) + (K T)] \cdot [2(T L L) + (L L T)]
\]

\[
+ [(T L) + (L T)] \cdot [2(T K L) + (K L T)] \} . \] (34)

So far the complications above are due to the differences in SPD’s for different \( i \) and \( j \). Further complications arise when the constraint due to momentum conservation is to
be applied. Since the sum of the momenta of shower partons in a jet cannot exceed the hard parton momentum $k$, the momentum fractions in the arguments of the SPD’s cannot be independent. In Eqs. (12) and (17) there are terms where $S$ appears twice or thrice. Let the momenta of the two-quark case be denoted by $q_a$ and $q_b$, and let us consider the term $(K_T + L_T)$ in Eq. (33) for illustration. The constraint applies only to $K(z_a)$ and $L(z_b)$, where $z_{a,b} = q_{a,b}/k$, while the momenta of $T$ are independent. If $K$ has the leading momentum $q_a$, then the maximum momentum of $L$ is $k - q_a$, and vice-versa. Thus we use the symmetrized combination given in Eq. (3)

$$\{K(z_a), L(z_b)\} \equiv \frac{1}{2} \left[ K(z_a)L\left(\frac{z_b}{1-z_a}\right) + K\left(\frac{z_a}{1-z_b}\right)L(z_b) \right].$$  \hfill (35)$$

In the case when there are three SPD’s, as in Eq. (34), we symmetrize as follows

$$\{K(z_a), L(z_b), L(z_c)\} \equiv \frac{1}{3} \left[ \right.$$

$$K(z_a) \left\{ L\left(\frac{z_b}{1-z_a}\right), L\left(\frac{z_c}{1-z_a}\right) \right\}$$

$$+ L(z_b) \left\{ K\left(\frac{z_a}{1-z_b}\right), L\left(\frac{z_c}{1-z_b}\right) \right\}$$

$$+ \left\{ K\left(\frac{z_a}{1-z_c}\right), L\left(\frac{z_b}{1-z_c}\right) \right\} L(z_c) \] \).$$  \hfill (36)$$

The procedure for our calculation is now completely specified.

### 4 Results

To provide an understanding of the order of magnitude of the various terms, let us first give the result of $\pi^+\pi^+$ correlation in central Au+Au collisions at $\sqrt{s} = 200$ GeV. Since the STAR data [4] are for the trigger momentum in the range $4 < p_T < 6$ GeV/c, we calculate

$$\frac{dN_{\pi^+}^{(\pi^+)}}{dp_2} = \int_4^6 dp_1 \frac{dN_{\pi^+}}{dp_2} \bigg|_{\pi^+(p_1)}.$$

13
where the RHS is defined by Eq. (18). The result is shown in Fig. 1. The dashed line indicates the contribution from the first term in Eq. (19) that has the structure $(T S)(T S)$, while the dash-dot line represents the next two terms in Eq. (19) that are of the form $(T S)(S S) + (S S)(T S)$. The solid line is their sum. Clearly, the overall distribution is dominated by the component that involves two thermal partons due to the high density of those soft partons. Each time a thermal parton is replaced by a shower parton in their recombination, the yield is lower. For that reason we have not bothered to calculate the contribution from $(S S)(S S)$, which corresponds to the double application of the fragmentation function. Presumably such contributions can become important at very large $p_T$, where the effects of thermal parton are insignificant. To generate four shower partons in a jet resulting in two pions each with $p_T > 4$ GeV/c would require a much harder collision than is necessary if some thermal partons can participate. The issue here is not which channels a hard parton hadronizes into, given a value of $k$ (as one considers in fragmentation), but rather, given two pions at $p_1$ and $p_2$, what the most favorable value of $k$ is in the environment of dense thermal partons (as one considers in recombination).

We next consider the dependence on the density of thermal partons. Since we have already investigated d+Au collisions in connection with the Cronin effect [12], where the soft parton distributions (called thermal also) have been determined for various centralities, we calculate $\pi^+\pi^+$ correlation for three cases: central Au+Au (0-5%), central d+Au (0-20%) and peripheral d+Au (60-90%). The result is shown in Fig. 2, where the three cases are represented, respectively, by solid, dashed and dash-dot lines. Evidently, the density of thermal partons has a crucial effect on the yield of the associated particles. Since peripheral (60-90%) d+Au collisions are almost equivalent to pp collisions, we can see directly from Fig.
that the structures of jets in central Au+Au and pp collisions are drastically different. If we plot the ratio of the spectra for central Au+Au to peripheral d+Au in linear scale, we get the solid curve in Fig. 3. The ratio of central d+Au to peripheral d+Au is shown by the dashed line. The former ratio exceeds 5 around $p_T = 2$ GeV/c and remains large throughout the intermediate $p_T$ range. We now have strong evidence that the structure of jets in nuclear collisions is very different from that in hadronic collisions.

We now consider $\pi^+$, $\pi^-$ and $p$ production associated with a $\pi^+$ trigger in the 4 to 6 GeV/c range. Fig. 4 shows the three contributions by the thin solid ($\pi^+$), dashed ($\pi^-$) and dash-dot lines ($p$), with the sum indicated by the thick solid line. The data points are from STAR, which includes all charged hadrons in both the trigger and the associated particles [6]. Since what is calculated is not exactly what is measured, one should not expect perfect agreement. However, $\pi^+$ is a dominant component of the trigger, and the data average over different trigger particles, whereas the different associated particles are summed. Thus what is calculated should not differ greatly from what is measured. Indeed, the agreement is very good both in normalization and in shape. It is therefore reasonable to infer that our approach has captured the essence of the physics of hadronization.

It is interesting to note that the yield of $\pi^-$ is higher than that of $\pi^+$ when the trigger is $\pi^+$, as is evident in Fig. 4. The reason is that when $i$ is summed over all hard parton species, the cases when $i = d$ and $\bar{u}$ can give rise to valence shower partons that enhance the $\pi^-$ production through the $K$ distribution, but not the $\pi^+$ production. When $i = u$ and $\bar{d}$, the trigger uses up the valence shower parton to form $\pi^+$ so the associated particle, whether $\pi^+$ or $\pi^-$, has to be formed by the sea shower parton through the $L$ distribution, resulting in no big difference between $\pi^+$ or $\pi^-$. Thus adding up the contributions from all hard partons
results in more $\pi^-$ than $\pi^+$ in a $\pi^+$ triggered jet. This is a prediction that can be checked by experiments with good particle identification.

We also note that the proton yield in Fig. 4 is greater than the $\pi^+$ yield in the 2-3 GeV/c range because a $u$ or $d$ shower parton can recombine with two thermal partons, thereby increasing the $p/\pi$ ratio for the same reason that the ratio exceeds 1 without trigger [8]-[11]. However, the proton yield is less than the $\pi^-$ yield in the $\pi^+$ triggered jet, since $\pi^+p$ does not have the advantage of the $\bar{u}$-initiated jet that enhances the $\pi^+\pi^-$ production.

Finally, we come to the proton trigger and show in Fig. 5 the result of our calculation for the associated particles being $\pi^+$ and $\pi^-$, in dashed and dash-dot lines, respectively. The solid line is their sum. The normalization and shape of the total distribution for the associated particles are roughly the same as those in Fig. 4 for $\pi^+$ trigger. The $\pi^+$ and $\pi^-$ components have no noticeable difference (bearing in mind that the $\pi^-$ curve is lowered by a factor of 2 to avoid overlap). That is reasonable, since $i = \bar{d}$ and $\bar{u}$ favor $\pi^+$ and $\pi^-$ equally, while $i = u$ and $d$ are both used in the trigger, leaving $\pi^+$ and $\pi^-$ again on comparable footing. These features can also be checked directly by experiments when particle identification is improved.

In the foregoing we have presented the distributions of the associated particles, which offer more details than the overall yields. The latter provide a short and useful summary of the jet structure that is easier to measure. We therefore calculate the three lowest moments of the distributions that have already been obtained

$$M_n^{(h_1)} = \int_{0.5}^{4.5} dp_2 p_2^n \frac{dN^{(h_1)}}{dp_2},$$

where $dN^{(h_1)}/dp_2$ is the $p_T$ distribution of all the particles associated with trigger $h_1$ that
we have calculated: $\pi^+$, $\pi^-$ and $p$ for $h_1 = \pi^+$, and $\pi^+$ and $\pi^-$ for $h_1 = p$. The lower limit of the integral in Eq. (38) is set at 0.5 GeV/c because our calculated result is not reliable for $p_T < 0.5$ GeV/c; the upper limit is set at 4.5 GeV/c, since we do not want it to exceed the average of the trigger momentum that is between 4 and 6 GeV/c. Thus, by definition, $M_{0}^{(h_1)}$ is a measure of the average number of particles associated with trigger $h_1$, $M_{1}^{(h_1)}$ being the total scalar $p_T$ of those particles, and $M_{2}^{(h_1)}$ the total $p_T^2$ of them. The last quantity is insensitive to the low $p_T$ behavior of $dN^{(h_1)}/dp_T$, and is a good measure for comparison between theory and experiment. Our results on $M_{n}^{(h_1)}$ for central Au+Au collisions are shown in Table I. Note that the values of the three moments change by mildly decreasing factors

| $n$ | 0 | 1 | 2 |
|-----|---|---|---|
| $h_1 = \pi^+$ | 1.394 | 1.707 | 2.703 |
| $h_1 = p$ | 0.882 | 0.999 | 1.450 |

(0.63, 0.59, 0.54) when the trigger is changed from $\pi^+$ to $p$. This is a feature that can more easily be checked by experiments than the distributions $dN^{(h_1)}/dp_T$ themselves.

To compare the above results with what one can expect from $pp$ collisions, we can return to what we have already calculated, i.e., $\pi^+\pi^+$ correlation in peripheral d+Au collisions, since that is very close to $pp$ collisions. Indeed, for an appreciation of the centrality dependence we calculate the moments of the three distributions shown in Fig. 2. The results are given in Table II. The ratios $R = [\text{Au+Au (central)}]/[\text{d+Au (peripheral)}]$ are 4.8, 4.9 and 4.9 for $n = 0, 1, 2$, respectively. They are very similar and roughly 5. We expect that if all particles
Table 2: Values of the moments $M_n^{(\pi^+)}$ for central $\pi^+$ associated particle only in three colliding systems

|       | 0    | 1    | 2    |
|-------|------|------|------|
| Au+Au (central) | 0.428 | 0.498 | 0.754 |
| d+Au (central)   | 0.200 | 0.223 | 0.331 |
| d+Au (peripheral)| 0.089 | 0.101 | 0.153 |

associated with the trigger are included, the ratio will remain about the same.

There are some data on total charged multiplicity and total scalar $p_T$, but they include the trigger [6]. The ratio for [Au+Au (central)]/$pp$ on multiplicity is $\sim 1.3$, and on scalar $p_T$ is $\sim 1.5$. By subtracting out the trigger contribution, it is possible to see a rough agreement with our result; however, we leave the quantification of the comparison to the experiments.

5 Conclusion

We have studied the structure of jets produced in heavy-in collisions by calculating dihadron correlation in the framework of parton recombination. Since the jets are produced in the environment of dense partonic medium, they are different from the ones produced in $pp$ collisions and $e^+e^-$ annihilation. The interaction between the hard and soft partons is very important. It is taken into account in our study by allowing shower partons to recombine with the thermal partons. Since our formalism has been applied successfully to single-particle spectra in previous studies, we have no freedom to adjust any part of our treatment of the
two-particle distributions, nor is there any free parameter to vary. All the results shown in
the previous section are predictions.

In our approach to hadron production at high and intermediate $p_T$ the effect of energy loss
by the hard partons traversing the dense medium is represented by a multiplicative factor $\xi$. That factor is cancelled in our definition of the associated particle distribution, which is the
ratio of the two-particle distribution to the trigger-particle distribution. Thus the dihadron
correlation in a jet that we calculate is independent of the degree of jet quenching. It can be
compared to the corresponding data on the near-side jet, since such jets are produced by hard
collisions near the surface facing the detector and suffer minimal energy loss. Unfortunately,
inadequate particle identification renders unfeasible direct comparison between the currently
available data with our predictions.

In principle, it is possible to calculate what has currently been measured, i.e., all charged
hadrons in the trigger and other particles in the jet. In practice, the task would be dauntingly
complicated and involve many more terms than what we have already considered, including
thermal partons whose parametrizations have been less reliably determined. Besides, the
current data are in a passing phase; better particle identification is forthcoming. What
we have calculated are for clean triggers and associated particles, and can be effectively
compared with future data.

It is evident from the results of our study that the dihadron correlation is dominated
by the components that involve the highest number of thermal partons. The recombination
mechanism boosts the yield when high-density thermal partons are included, but also boosts
the $p_T$ of the product when the semi-hard shower partons are involved. The effect has
already been shown to operate in the single-particle distributions, but now exhibits itself
more conspicuously in dihadron correlations in jets, since at least four partons are involved, two of which can be thermal. The difference between jets produced in heavy-ion collisions compared to those produced in pp collisions is huge, as evidenced by the large ratio shown by the solid line in Fig. 3 and by the ratios of the moments in Table II – about 5 between the first and third rows of values.

It is hard to see how the properties of the dihadron correlations in jets that we have found can be reproduced in any fragmentation model even with medium modification of the fragmentation function, whose focus has been on the effect of energy loss [20]. There are terms in our recombination formulas, when the shower parton momenta are symmetrized, that cannot be written in factorizable forms involving products of two FF’s. Leaving aside such technical details, let us accept the conceivable possibility that perturbative branching of a hard parton can generate hard shower partons, and that they can further fragment by means of suitable modified FF’s that mimic $T_S$ recombination. If it is successful in the end, it seems that the scheme would have lost the original advantage of the fragmentation approach that relies on the universality of the FF’s. If the modification of a FF depends sensitively on the detailed properties of the partonic medium, then, by comparison, $T_S$ recombination would seem to be a more direct and physically cogent approach to hadronization. Besides, as pointed out earlier, energy loss is not the issue in dihadron correlation.

There are limitations to the formalism that we have used for our calculations. We have not considered the $Q^2$ dependence of dihadron correlation, since the SPD’s used are for $Q$ fixed at 10 GeV/c [15]. That limitation is not a matter of principle, but of practice. To account for the $Q^2$ evolution of the SPD’s is a worthwhile problem in its own right, inasmuch as the same problem for the FF’s has been pursued for decades [21, 22]. To apply that evolutionary
property to the dihadron correlation would be prohibitively complicated. Attempts have been initiated to investigate the evolutionary aspect of dihadron distribution of the fragmentation process in the operator formalism [23]. An input on the initial distribution for such an evolution would still have to involve the type of consideration presented here. Furthermore, how the thermal partons are to be incorporated in that approach is not clear.

Another limitation is rooted in our formalism. Since we have relied entirely on our 1D formulation of recombination, it is not possible in the same formalism to address the question of angular correlation in a jet. Since jets are 3D objects, it is obvious that longitudinal-transverse correlation within a jet can contain information about the properties of the recombining partons that we cannot probe in the simplified 1D formalism. Clearly, despite the substantial progress that has been made in our line of investigation, there remains much ground to improve and generalize in the study of the physics of hadronization in heavy-ion collisions.

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Figure Captions

Fig. 1. Transverse momentum distribution of $\pi^+$ associated with a $\pi^+$ trigger. The contribution from terms of the form $(TS)(TS)$ is shown in dashed line, while the contribution from terms of the form $(TS)(SS)$ is shown in dashed-dot line. The solid line is the sum of all components.

Fig. 2. Associated particle ($\pi^+$) distribution with $\pi^+$ trigger for central Au+Au (solid), central d+Au (dashed) and peripheral d+Au collisions (dash-dot line).

Fig. 3. The ratio of central Au+Au to peripheral d+Au collisions (solid line) and that of central d+Au to peripheral d+Au collisions (dashed line).

Fig. 4. Transverse momentum distributions of $\pi^+$, $\pi^-$ and $p$, associated with a $\pi^+$ trigger. The data are from STAR [6] for all charged hadrons in the trigger and associated particles.

Fig. 5. Transverse momentum distributions of $\pi^+$ and $\pi^-$ (lowered by a factor of 2) associated with proton trigger.
\[ \frac{dN}{dp_T} \left[ \text{GeV/c}^{-1} \right] \]

dependent variable \( \frac{dN}{dp_T} \) against \( p_T^2 \) for different categories:

- \( \pi^+ \) trigger
- \( \pi^+ \) associated

Graph shows the following lines:
- **sum**
- **TTSS terms**
- **TSSS terms**
The graph shows the dependence of the invariant cross-section $dN/dp_T$ on $p_T^2$ for different centralities and trigger types in Au+Au and d+Au collisions. The x-axis represents $p_T^2 (\text{GeV/c})^2$, while the y-axis represents the cross-section in units of $[(\text{GeV/c})^{-1}]$. The lines indicate different centrality classes: central Au+Au (solid), central d+Au (dashed), peripheral d+Au (dotted), and trigger types as indicated.
AuAu central
dAu peripheral
dAu central
dAu peripheral

Ratio

\( p_T^2 \text{ (GeV/c)}^3 \)
\[ \frac{dN}{dp_T} \text{[(GeV/c)]}^{-1} \]

- STAR (all charged)
- \( \pi^+ \) trigger
- sum (associated)
- \( \pi^- \)
- \( \pi^+ \)
- \( p \)
\[ \frac{dN}{dp_T} \left[ \text{GeV/c}^{-1} \right] \]

- p trigger
- sum (associated)
- \( \pi^+ \)
- \( \pi^- \) (/2)