Comment on “Stable Quantum Computation of Unstable Classical Chaos”

Christof Zalka

Department of Combinatorics and Optimization, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1

In a recent letter, Georgeot and Shepelyansky claim to have shown that there is a big advantage in using a quantum computer to simulate and study classical chaotic systems, over using a conventional (classical) computer. (Actually, the authors made this claim already in the second part of an earlier letter.)

Most of the paper is devoted to showing that the quantum simulation is much less affected by errors than the classical one. This is based on a clearly inappropriate comparison of the two cases. The authors consider the computationally simple, but chaotic, “Arnold cat map”, which is an area-preserving bijective map of a two-dimensional torus onto itself. They also consider a discretized version of this map which can be simulated exactly on a digital computer. The authors then propose to compute this discretized map on a quantum computer on many possible initial values simultaneously, utilizing “quantum parallelism”.

Strangely they then compare this quantum computation of the discretized map to a classical simulation of the full continuous map. Because they imagine the classical simulation to be done with fixed-precision arithmetic, the chaotic dynamics will quickly amplify rounding errors, so that after a few iterations of the map, the results will be totally wrong. It is then clear that the quantum simulation fares better, even when considering noise acting on the qubits, which the authors do. For this comparison, see e.g. figure 1 (left classical, right quantum) or the last paragraph of the paper.

Thus, in effect, the authors reach their conclusion in favor of the quantum simulation by demanding more from the classical than from the quantum simulation, specifically, that it should do an accurate simulation for any real initial values. Actually, the issue of rounding errors is not specific to classical or quantum computation, but is typical for digital computation. Note that the authors consider the usual kind of quantum computer consisting of qubits which is digital, namely, binary.

Apart from the discussion of errors, the authors make an unsubstantiated claim just before the last paragraph. They say that their quantum simulation lets one obtain “global quantities inaccessible by classical computation”. They propose to model the initial superposition of input values according to some (classical) probability density distribution (how, exactly, they do not say). They then want to extract information of interest about the final density, for example, by applying a quantum Fourier Transformation to the final superposition before observing the quantum computer.

Although this may allow the extraction of some information about the power spectrum of the final distribution, it is by no means clear that this could not be done just as efficiently on a classical computer. In particular, note that for the kind of quantum simulation described here, we need a reversible discrete map. This would, for example, allow one to evolve a few closely spaced final values backwards (on a classical computer), thereby permitting one to obtain information about the “fine structure” of a final density.

At any rate, it is an extraordinary claim, worthy of careful justification, to have found a new application for which a quantum computer provides exponential advantage over the known classical algorithms. This is especially true for “openly given” problems, thus without “black boxes” or limitations on communication etc. So far not much more than Shor’s algorithm(s) and, not surprisingly, the simulation of quantum systems achieved that.

* Electronic address: zalka@uwaterloo.ca
[1] B. Georgeot and D.L. Shepelyansky, Phys. Rev. Lett. 86 (23), 5393 (2001) (also quant-ph/0101004)
[2] B. Georgeot and D.L. Shepelyansky, Phys. Rev. Lett. 86 (13), 2890 (2001) (also quant-ph/0010005)