Correspondence between entropy-corrected holographic and Gauss-Bonnet
dark energy models

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Abstract

In the present work we investigate the cosmological implications of the entropy-corrected holographic dark energy (ECHDE) density in the Gauss-Bonnet framework. This is motivated from the loop quantum gravity corrections to the entropy-area law. Assuming the two cosmological scenarios are valid simultaneously, we show that there is a correspondence between the ECHDE scenario in flat universe and the phantom dark energy model in the framework of Gauss-Bonnet theory with a potential. This correspondence leads consistently to an accelerating universe.

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I. INTRODUCTION

Recent cosmological observations indicate that our universe is in accelerated expansion. These observations are those which is obtained by SNe Ia [1], WMAP [2], SDSS [3] and X-ray [4]. These observations also suggest that our universe is spatially flat, and consists of about 70% dark energy (DE) with negative pressure, 30% dust matter (cold dark matter plus baryons), and negligible radiation. In order to explain why the cosmic acceleration happens, many theories have been proposed. The simplest candidate of the dark energy is a tiny positive time-independent cosmological constant $\Lambda$, for which $\omega = -1$. However, it is difficult to understand why the cosmological constant is about 120 orders of magnitude smaller than its natural expectation (the Planck energy density). This is the so-called cosmological constant problem. Another puzzle of the dark energy is the cosmological coincidence problem: why are we living in an epoch in which the dark energy density and the dust matter energy are comparable?.

An alternative proposal for dark energy is the dynamical dark energy scenario. The dynamical nature of dark energy, at least in an effective level, can originate from various fields, such is a canonical scalar field (quintessence) [5], a phantom field, that is a scalar field with a negative sign of the kinetic term [6], or the combination of quintessence and phantom in a unified model named quintom [7]. Recently another paradigm has been constructed in the light of the holographic principle of quantum gravity theory, and thus it presents some interesting features of an underlying theory of dark energy [8]. This paradigm may simultaneously provide a solution to the coincidence problem [9]. The holographic dark energy model has been extended to include the spatial curvature contribution [10] and it has also been generalized in the braneworld framework [11]. Lastly, it has been tested and constrained by various astronomical observations [12–16]. Since holographic energy density corresponds to a dynamical cosmological constant, we need a dynamical framework, instead of general relativity, to consistently accommodate it. A proposal, closely related to the low-energy string effective action, is the scalar-Gauss-Bonnet gravity [17], which can be considered as a form of gravitational dark energy.

In the present paper we are interested in investigating the conditions under which we can obtain a correspondence between holographic and Gauss-Bonnet models of dark energy, i.e to examine holographic dark energy in a spatially flat Gauss-Bonnet universe.
II. GAUSS-BONNET DARK ENERGY

In this section we provide the basic Gauss-Bonnet model for dark energy [17–19]. In this framework, the candidate for dark energy is a scalar field $\phi$, which is moreover coupled to gravity through the higher-derivative (string-originated) Gauss-Bonnet term. The corresponding action is given by

$$S = \int d^4 x \sqrt{g} \left[ \frac{1}{2\kappa^2} R - \frac{\sigma}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + f(\phi) G' \right], \quad (1)$$

where $\kappa^2 = 8\pi G$ and $\sigma = \pm 1$. Also $f(\phi)$ is an arbitrary function of $\phi$ which denotes the coupling of the field with the geometry. For the sake of generality, we consider both behaviors of the scalar field i.e. canonical scalar field $\sigma = 1$, and $\sigma = -1$ which corresponds to phantom behavior. In the above expression (1), the quantity $G'$ represents the Gauss-Bonnet term which is explicitly written as:

$$G' \equiv R^2 - 4R_{\mu \nu} R^{\mu \nu} + R_{\mu \nu \rho \sigma} R^{\mu \nu \rho \sigma}, \quad (2)$$

where $R_{\mu \nu \rho \sigma}$, $R_{\mu \nu}$ and $R$ are respectively the Riemann and Ricci tensors and $R$ is the curvature scalar while $g_{\mu \nu}$ is the background metric. Motivated by several observational and empirical findings [1–4], we shall focus on the spatially flat Robertson-Walker universe with

$$ds^2 = -dt^2 + a(t)^2 (dr^2 + r^2 d\Omega^2), \quad (3)$$

where we took $k = 0$ in (1).

The equation of motion for the scale factor is [18]:

$$\frac{\sigma}{2} \ddot{a}^2 - V(\phi) + 16f'(\phi)\dot{\phi} \ddot{\phi} \frac{\ddot{a}}{a} + 8 \left[ f'(\phi) \dot{\phi} + f''(\phi) \ddot{\phi}^2 \right] H^2 = p_\Lambda, \quad (4)$$

while for the scalar field, we have

$$\sigma \left[ \dddot{\phi} + 3H \ddot{\phi} + \frac{V'(\phi)}{\sigma} \right] = 24f'(\phi) H^2 \frac{\ddot{a}}{a}, \quad (5)$$

Moreover we have a constraint equation, namely:

$$\frac{\sigma}{2} \dot{\phi}^2 + V(\phi) - 24f'(\phi) \dot{\phi} H^3 = \rho_\Lambda. \quad (6)$$

In the expressions (4) and (6) above, $p_\Lambda$ and $\rho_\Lambda$ are the pressure and energy density due to the scalar field and the Gauss Bonnet interaction [19], which are identified as the corresponding quantities of dark energy.
III. ENTROPY CORRECTED HOLOGRAPHIC DARK ENERGY

The black hole entropy plays a central role in the derivation of holographic dark energy (HDE). Indeed, the definition and derivation of holographic energy density depends on the entropy-area relationship \( S \sim A \sim L^2 \) of black holes in Einstein’s gravity, where \( A \sim L^2 \) represents the area of the horizon. However, this definition can be modified from the inclusion of quantum effects, motivated from the loop quantum gravity (LQG). The quantum corrections provided to the entropy-area relationship leads to the curvature correction in the Einstein-Hilbert action and vice versa [20].

The corrected entropy takes the form [21]

\[
S = \frac{A}{4} + \tilde{\gamma} \ln \left( \frac{A}{4} \right) + \tilde{\beta},
\]

where \( \tilde{\gamma} \) and \( \tilde{\beta} \) are dimensionless constants of order unity. The exact values of these constants are not yet determined and still debatable in loop quantum cosmology. These corrections arise in the black hole entropy in LQG due to thermal equilibrium fluctuations and quantum fluctuations [22]. It is very interesting if one can determine the coefficient in front of log correction term by observational constraints. This term also appears in a model of entropic cosmology which unifies the inflation and late time acceleration, see [23], and it was found the coefficient might be extremely large due to current cosmological constraint, which inevitably brought a fine tuning problem to entropy corrected models. Taking the corrected entropy-area relation (7) into account, the energy density of the HDE will be modified as well. On this basis, Wei [24] proposed the energy density of the so-called “entropy-corrected holographic dark energy” (ECHDE) in the form

\[
\rho_\Lambda = 3c^2R_h^{-2} + \gamma R_h^{-4}\ln(R_h^2) + \beta R_h^{-4}, \tag{8}
\]

in units where \( M_p^2 = 8\pi G = 1 \), and \( c \) is a constant which value is determined by observational fit. The future event horizon \( R_h \) is defined as

\[
R_h = a \int_t^\infty \frac{dt}{a} = a \int_a^\infty \frac{da}{H a^2}, \tag{9}
\]

which leads to results compatible with observations. Furthermore, we can define the dimensionless dark energy as:

\[
\Omega_\Lambda \equiv \frac{\rho_\Lambda}{3H^2} = \frac{3c^2 + \gamma R_h^{-2}\ln(R_h^2) + \beta R_h^{-2}}{3H^2R_h^2}. \tag{10}
\]

In the case of a dark-energy dominated universe, dark energy evolves according to the conservation law

\[
\dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = 0, \tag{11}
\]
or equivalently
\[ \dot{\Omega}_\Lambda = -\frac{2\dot{H}}{3H^2R_h^2}(3c^2 + \gamma R_h^{-2} \ln(R_h^2 + \beta R_h^{-2}) + \frac{HR_h - 1}{3H^2R_h^3} [-6c^2 + 2\gamma R_h^{-2} - 4\gamma R_h^{-2} \ln R_h^2 - 4\beta R_h^{-3}], \tag{12} \]

where the equation of state is
\[ p_\Lambda = \left[ -1 - \frac{2\gamma R_h^{-2} - 4\gamma R_h^{-2} \ln(R_h^2) - 4\beta R_h^{-2} - 6c^2}{3(3c^2 + \gamma R_h^{-2} \ln(R_h^2) + \beta R_h^{-2})} \right] \left\{ 1 - \sqrt{\frac{3\Omega_\Lambda}{3c^2 + \gamma R_h^{-2} \ln(R_h^2) + \beta R_h^{-2}}} \right\} \rho_\Lambda, \tag{13} \]

which leads straightforwardly to an index of the equation of state of the form:
\[ w_\Lambda = -1 - \frac{2\gamma R_h^{-2} - 4\gamma R_h^{-2} \ln(R_h^2) - 4\beta R_h^{-2} - 6c^2}{3(3c^2 + \gamma R_h^{-2} \ln(R_h^2) + \beta R_h^{-2})} \left[ 1 - \sqrt{\frac{3\Omega_\Lambda}{3c^2 + \gamma R_h^{-2} \ln(R_h^2) + \beta R_h^{-2}}} \right]. \tag{14} \]

**IV. CORRESPONDENCE BETWEEN ECHDE AND GAUSS-BONNET DARK ENERGY MODELS**

We want to obtain the conditions under which there is a correspondence between the Gauss-Bonnet dark energy model and the entropy corrected holographic dark energy scenario, in the flat background space. In particular, to determine an appropriate Gauss-Bonnet potential which makes the two pictures to coincide with each other.

Let us first consider the simple Gauss-Bonnet solutions acquired in [18, 19]. In this case \( f(\phi) \) is given as [17]
\[ f(\phi) = f_0 e^{\frac{2\phi}{\phi_0}}. \tag{15} \]

In addition, we assume that the scale factor behaves as \( a = a_0 h_0 \), and similarly to [18] we will examine both \( h_0 \)-sign cases. However, when \( h_0 \) is negative the scale factor does not correspond to expanding universe but to shrinking one. If one changes the direction of time as \( t \to -t \), the expanding universe whose scale factor is given by \( a = a_0 (-t)^{h_0} \) emerges \(^1\). Since \( h_0 \) is not an integer in general, there is one remaining difficulty concerning the sign of \( t \). To avoid the apparent inconsistency, we may further shift the origin of the time as \( t \to -t \to t_s - t \). Then the time \( t \) can be positive as long as \( t < t_s \), and we can consistently write \( a = a_0 (t_s - t)^{h_0} \). Thus, we can finally write [18]
\[ H = \frac{h_0}{t}, \quad \phi = \phi_0 \ln \left( \frac{t}{t_1} \right), \tag{16} \]

\(^1\) For this form of scale factor one could obtain an interesting phenomenon when \( t \) arrives \( t_s \), i.e., a big rip singularity [22]. So this is an important scenario and also its relation with other cosmological singularities [23].
when $h_0 > 0$ or

$$H = \frac{-h_0}{t_s - t}, \quad \phi = \phi_0 \ln \frac{t_s - t}{t_1}$$

(17)

when $h_0 < 0$, with an undetermined constant $t_1$.

Let us first investigate the positive-$h_0$ case. If we establish a correspondence between the holographic dark energy and Gauss-Bonnet approach, then using dark energy density equation (9) and relation (10), together with expressions (16), we can easily derive the scalar potential term as

$$V = \frac{e^{\frac{-2\phi}{\phi_0}}}{t_1^4} \left( 3\Omega h_0^2 + \frac{48f_0 h_0^3}{t_1^2} - \frac{\sigma \phi_0^2}{2} \right).$$

(18)

Note that expressions (16) allow for an elimination of time $t$ in terms of the scalar field $\phi$. Furthermore, by substituting $\phi$, and $H$ from (16), $f(\phi)$ from (15) and $V(\phi)$ from (18) into (5) we obtain:

$$-3\sigma h_0 \phi_0 + \frac{6\Lambda h_0^3}{\phi_0} + \frac{96f_0 h_0^3}{\phi_0 t_1^2} - 3h_0^2 \frac{d\Omega}{d\phi} + \frac{48f_0 h_0^3 (h_0 - 1)}{\phi_0 t_1^2} = 0$$

(19)

where

$$\frac{d\Omega}{d\phi} = \frac{d\Omega}{dt} \frac{t}{\phi_0} = \frac{d\Omega}{dt} \frac{t_1}{\phi_0} e^{\frac{\phi}{\phi_0}}.$$

(20)

Now, under the ansatz $a = a_0 t^{h_0}$ it is easy to see from (9) that in order for $R_h$ to be finite, $h_0$ has to be greater than 1. In such a case we straightforwardly find:

$$R_h = \frac{t}{h_0 - 1}.$$

(21)

$$\Omega_A = \frac{(h_0 - 1)^2}{3h_0^2} \left[ 3c^2 + \gamma \left( \frac{t_1 e^{\frac{\phi}{\phi_0}}}{h_0 - 1} \right)^{-2} \ln \left\{ \left( \frac{t_1 e^{\frac{\phi}{\phi_0}}}{h_0 - 1} \right)^2 \right\} + \beta \left( \frac{t_1 e^{\frac{\phi}{\phi_0}}}{h_0 - 1} \right)^{-2} \right],$$

(22)

and

$$w_A = -1 - \frac{2\gamma \left( \frac{t_1 e^{\frac{\phi}{\phi_0}}}{h_0 - 1} \right)^{-2} - 4\gamma \left( \frac{t_1 e^{\frac{\phi}{\phi_0}}}{h_0 - 1} \right)^{-2} \ln \left\{ \left( \frac{t_1 e^{\frac{\phi}{\phi_0}}}{h_0 - 1} \right)^2 \right\} - 4\beta \left( \frac{t_1 e^{\frac{\phi}{\phi_0}}}{h_0 - 1} \right)^{-2} - 6c^2}{3(3c^2 + \gamma \left( \frac{t_1 e^{\frac{\phi}{\phi_0}}}{h_0 - 1} \right)^{-2} \ln \left\{ \left( \frac{t_1 e^{\frac{\phi}{\phi_0}}}{h_0 - 1} \right)^2 \right\} + \beta \left( \frac{t_1 e^{\frac{\phi}{\phi_0}}}{h_0 - 1} \right)^{-2}} \times \left[ 1 - \sqrt{\frac{3\Omega_A}{3c^2 + \gamma \left( \frac{t_1 e^{\frac{\phi}{\phi_0}}}{h_0 - 1} \right)^{-2} \ln \left\{ \left( \frac{t_1 e^{\frac{\phi}{\phi_0}}}{h_0 - 1} \right)^2 \right\} + \beta \left( \frac{t_1 e^{\frac{\phi}{\phi_0}}}{h_0 - 1} \right)^{-2}}} \right].$$

(23)

Let us proceed to the investigation of the negative-$h_0$ case. Repeating the same steps, but imposing relations (17) we find that

$$V = \frac{e^{\frac{-2\phi}{\phi_0}}}{t_1^4} \left( 3\Omega h_0^2 + \frac{48f_0 h_0^3}{t_1^2} - \frac{\sigma \phi_0^2}{2} \right),$$

(24)
and
\[ -2\sigma \phi_0 - 3\sigma h_0 \phi_0 + \frac{6\Omega h_0^3}{\phi_0} - \frac{96f_0 h_0^3}{\phi_0 t_1^2} - 3h_0^2 \frac{d\Omega}{d\phi} + \frac{48f_0 h_0^3 (h_0 - 1)}{\phi_0 t_1^2} = 0, \] (25)
where
\[ \frac{d\Omega}{d\phi} = \frac{d\Omega}{dt} \frac{t_s - t}{\phi_0} = - \frac{d\Omega}{dt} \frac{t_1}{\phi_0} e^{\frac{\phi}{\phi_0}}. \] (26)

Now, under the ansatz \( a = a_0 (t_s - t)^{h_0} \) we can see from (21) that \( R_h \) is always finite if \( h_0 \) is negative, which is just the case under investigation. Then we have:
\[ R_h = \frac{t_s - t}{1 - h_0}. \] (27)

\[ \Omega_\Lambda = \frac{(h_0 - 1)^2}{3h_0^2} [3c^2 + \gamma \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 \ln \left\{ \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 \right\} + \frac{1}{\beta} \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2], \] (28)
and therefore
\[ w_\Lambda = -1 - \frac{2\gamma \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 - 4\gamma \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 \ln \left\{ \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 \right\} - 4\beta \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 - 6c^2}{3(3c^2 + \gamma \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 \ln \left\{ \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 \right\} + \beta \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2}\right]} \times \frac{3\Omega_\Lambda}{\sqrt{3c^2 + \gamma \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 \ln \left\{ \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 \right\} + \beta \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2}}. \] (29)

We also mention the conditions under which phantom crossing [27] can be realized in the present scenario:
\[ 2\gamma \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 - 4\gamma \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 \ln \left\{ \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 \right\} - 4\beta \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 - 6c^2 > 0, \] (30)
\[ 3\Omega_\Lambda < 3c^2 + \gamma \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 \ln \left\{ \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 \right\} + \beta \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2. \] (31)
and
\[ 2\gamma \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 - 4\gamma \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 \ln \left\{ \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 \right\} - 4\beta \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 - 6c^2 < 0, \] (32)
\[ 3\Omega_\Lambda > 3c^2 + \gamma \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 \ln \left\{ \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2 \right\} + \beta \left( \frac{t_1 e^{\frac{\phi_0}{\phi_0} - 1}}{h_0 - 1} \right)^2. \] (33)
Thus phantom crossing is possible if either conditions [30] and [31] or [32] and [33] are satisfied. This implies that the model of entropy corrected holographic dark energy gives its equation of state across \(-1\), consistent with the Gauss-Bonnet model for the correspondence to be generically applicable.
V. CONCLUSIONS

Within the different candidates to play the role of the dark energy, the entropy-corrected holographic dark energy model, has emerged as a possible model with EoS across $-1$. In the present paper we have studied cosmological application of holographic dark energy density in the Gauss-Bonnet gravity framework. By considering the entropy-corrected holographic energy density as a dynamical cosmological constant, we have obtained the equation of state for the holographic energy density in the Gauss-Bonnet framework. After that we have studied the conditions under which we can obtain a correspondence between entropy-corrected holographic and Gauss-Bonnet models of dark energy.

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