Coupled quintessence and the coincidence problem

G. Mangano, G. Miele, and V. Pettorino

Dipartimento di Fisica, Universitá di Napoli “Federico II”, and INFN, Sezione di Napoli, Complesso Universitario di Monte S. Angelo, Via Cintia, I-80126 Naples, Italy

Abstract

We consider a model of interacting cosmological constant/quintessence, where dark matter and dark energy behave as, respectively, two coexisting phases of a fluid, a thermally excited Bose component and a condensate, respectively. In a simple phenomenological model for the dark components interaction we find that their energy density evolution is strongly coupled during the universe evolution. This feature provides a possible way out for the coincidence problem affecting many quintessence models.

PACS 98.80.-k, 98.80.Cq

The extensive astrophysical observations of Supernovae (SN) Ia provide a powerful tool to single out the main cosmological energy density contributions at redshift $z \sim 1$. The evidence for a presently accelerated cosmic expansion [1, 2] suggests the presence of a new dominant energy source whose equation of state $p = \omega \rho$ is compatible with a cosmological constant. The analysis of the recent experimental data favors in fact the range $-1 \leq \omega \leq -0.4$ [1, 3, 4]. In particular for a pure cosmological constant the favored energy density, in unit of the critical density, would be of the order of $\Omega_\Lambda \approx 0.7$. Unfortunately, this value would be several orders of magnitude smaller than what can
be predicted by our present knowledge of fundamental interactions. Moreover, it is also quite unnatural that ΩΛ is just of the same order of the cold dark matter term Ωm ≃ 0.3 today, while it has been negligible for, say, red-shifts larger than 1. This is a severe fine tuning problem, usually referred to as the coincidence problem.

A much more appealing scenario is to look at this additional term as due to a scalar field φ whose dynamics makes it negligible at high redshift and dominating the cosmic energy density at z ≤ 1 [5, 6, 7, 8]. However this wide class of model is not free from fine tuning problem, due to the arbitrariness in the choice of the scalar field potential, but it has nevertheless more appealing features. This is for example the case of tracker fields scenarios, where the scalar dynamics shows a pseudo-attractor solution, valid for a wide range of initial conditions and providing values of ωφ and Ωφ [5, 6] in good agreement with SN Ia observations.

Recently interacting quintessence scenarios have been extensively studied, considering a scalar field coupled with gravity, ordinary matter and dark matter [9, 10, 11, 12, 13, 14]. In particular, the recent data on CMBR anisotropy have provided a chance to constrain quintessence models [15, 16, 17, 18, 19, 20, 21, 22], though the high degree of arbitrariness on the scalar dynamics makes these constraints very model dependent. In this letter we assume a more phenomenological scenario, in which the quintessence and dark matter components are looked upon as two different components of the same fluid, a condensate contribution and a thermally Bose distributed “gas”, with energy densities ρΛ and ρm, respectively. We start considering the case where the condensate energy term corresponds to a pure cosmological constant, ω = −1. As it will be clear in the following, the main features of the model still hold for a more general equation of state with ω < 0. The gas component ρm is assumed to be cold matter, with vanishing pressure. Finally we consider for simplicity a spatially flat universe, Ωk = 0.

We parametrize the interaction in terms of powers of the two energy densities ρΛ and ρm. The time evolution of radiation energy density, ρr, condensate and dark matter are then dictated by the following equations

\[ \dot{\rho}_r = -4 \frac{\dot{a}}{a} \rho_r, \]  
\[ \dot{\rho}_m = -3 \frac{\dot{a}}{a} \rho_m - k M^{5-4\alpha-4\beta} \rho_m^\alpha \rho_\Lambda^\beta, \]  
\[ \dot{\rho}_\Lambda = k M^{5-4\alpha-4\beta} \rho_m^\alpha \rho_\Lambda^\beta, \]

where k is a dimensionless coupling constant and M a suitable mass parameter. In terms of the redshift z equations (1)-(3) can be conveniently rewritten as

\[ \frac{d\tilde{\rho}_r(z)}{dz} = \frac{4}{1 + z} \tilde{\rho}_r(z), \]  

where
\[
\frac{d\tilde{\rho}_m(z)}{dz} = \frac{1}{1+z} \left( 3\tilde{\rho}_m(z) + \lambda \frac{\tilde{\rho}_m^\alpha(z) \tilde{\rho}_\Lambda^\beta(z)}{\sqrt{\tilde{\rho}_r(z) + \tilde{\rho}_m(z) + \tilde{\rho}_\Lambda(z)}} \right),
\]
\[
\frac{d\tilde{\rho}_\Lambda(z)}{dz} = -\frac{\lambda}{1+z} \frac{\tilde{\rho}_m^\alpha(z) \tilde{\rho}_\Lambda^\beta(z)}{\sqrt{\tilde{\rho}_r(z) + \tilde{\rho}_m(z) + \tilde{\rho}_\Lambda(z)}},
\]

where \( \tilde{\rho}_i \equiv \rho_i/\rho_{\text{co}} \), with \( \rho_{\text{co}} = 3H_0^2M_{\text{Pl}}^2/(8\pi) = h^2 0.81 \times 10^{-46} \text{ GeV}^4 \) the present critical energy density, and

\[\lambda \equiv k \sqrt{\frac{3}{8\pi}} \left( \frac{M_{\text{Pl}}}{M} \right) \left( \frac{\rho_{\text{co}}}{M^4} \right)^{\alpha+\beta-3/2}.\]

By defining \( x \equiv \log_{10}(1+z) \) we then get

\[
\frac{d\tilde{\rho}_r(x)}{dx} = 4 \log(10) \tilde{\rho}_r(x),
\]
\[
\frac{d\tilde{\rho}_m(x)}{dx} = \log(10) \left( 3\tilde{\rho}_m(x) + \lambda \frac{\tilde{\rho}_m^\alpha(x) \tilde{\rho}_\Lambda^\beta(x)}{\sqrt{\tilde{\rho}_r(x) + \tilde{\rho}_m(x) + \tilde{\rho}_\Lambda(x)}} \right),
\]
\[
\frac{d\tilde{\rho}_\Lambda(x)}{dx} = -\lambda \log(10) \frac{\tilde{\rho}_m^\alpha(x) \tilde{\rho}_\Lambda^\beta(x)}{\sqrt{\tilde{\rho}_r(x) + \tilde{\rho}_m(x) + \tilde{\rho}_\Lambda(x)}}.
\]

Note that for the parameter \( \lambda < 0 \), the interaction term on the r.h.s. of (9) and (10) describes a continuous transfer of energy from the condensate to the ordinary non-relativistic particles. The corresponding scenario is a universe filled by both the condensate and a normal gas of dark particles, but with an effective mass of the condensate larger than the mass of thermal excitation in the gas phase. Direct decay of the condensate in dark matter particles is expected to be proportional to \( \rho_\Lambda \) only, while stimulated decay mechanism is of course proportional to number density of dark matter particles too. The natural expectation in this case is an interaction term with \( \alpha = \beta = 1 \). In the following we will keep varying these parameters, to study the general features of the model as function of \( \alpha \) and \( \beta \) and eventually study in more details the case where both are set equal to unity.

We will be mainly interested to the case \( \lambda < 0 \). The case \( \lambda > 0 \), which will be briefly treated in the following, also represents a possible framework. In this case, as the temperature lowers, a fraction of the energy stored in the gas of dark particles flows in the condensate, as during a temperature driven phase transition. Unfortunately this model suffers of the very same coincidence problem of many other quintessence scenario and thus it is much less appealing.

The system (8)-(10) can be numerically solved, as a function of \( x \), by fixing the values of \( \tilde{\rho}_r, \tilde{\rho}_m \) and \( \tilde{\rho}_\Lambda \) at present \( (x_i = 0) \), and then following the values of the energy densities back in time. In particular, to describe the features of this coupled system, we have chosen as reference values \( \tilde{\rho}_m(0) = \Omega_m = 0.3, \tilde{\rho}_\Lambda(0) = \Omega_\Lambda = 0.7 \) and the equivalence red-shift
$z_{eq} \simeq 3100$. This implies

$$\frac{\rho_r(z)}{\rho_m(z)} = \frac{\rho_r(0)}{\rho_m(0)} (1 + z) \implies \frac{\rho_r(0)}{\rho_m(0)} = \frac{1}{1 + z_{eq}}.$$  \hspace{1cm} (11)

It may seem that fixing the values of the energy densities parameters today is already a fine-tuning condition imposed to the evolution. This of course would be true if no interactions would be considered in the model. We will show in fact that, since $\rho_\Lambda$ and $\rho_m$ evolution are coupled, the fact that their ratio $\Omega_\Lambda/\Omega_m$ is of order one today represents a much less ad hoc condition for a wide range of the parameters $\alpha$, $\beta$ and $\lambda$. The fact that $\Omega_\Lambda \sim \Omega_m$ represent the main contribution to the energy density today is ultimately related to the fact that the interaction term keeps their evolution quite close throughout the universe expansion. We can say that the only remaining fine tuned condition is given by the value of the equivalence redshift, where radiation start becoming negligible.

In order to study the role of the free parameter $\alpha$, $\beta$, $\gamma$ on the evolution of the energy densities, we start fixing for example $\lambda = -10^{-3}$ and choose $\alpha = 1$ and $\beta = 0$. In Figure 1 are reported the energy densities for radiation $\tilde{\rho}_r$, matter $\tilde{\rho}_m$ and condensate $\tilde{\rho}_\Lambda$, as a function of $x$. For these values of free parameters, the interaction is due to $\tilde{\rho}_m$ only, but differently from the noninteraction case, the energy density of condensate here evolves in time. The evolution is of course enhanced for larger values of $\alpha$, but for $x \lesssim 2$ the condensate energy density $\tilde{\rho}_\Lambda$ reaches a constant value which, at present, provides an effective cosmological constant. Thus for $\beta = 0$ the model predicts scenarios very similar to other quintessence models and no new features overcoming the coincidence problem appear.

The behavior of the system completely changes if one takes non vanishing values for $\beta$. In this case, by increasing the value of $\beta$, the values of dark energy $\tilde{\rho}_\Lambda$ and dark matter $\tilde{\rho}_m$ are more and more connected. As it is clear from Figure 2, where the energy densities are plotted versus $x$ for different values of $\beta$ and for $\alpha = 1$ and $\lambda = -0.001$, due to the interaction term Eq.s (9) and (10), $\tilde{\rho}_m$ decreases for large redshift with the increasing of $\beta$, whereas $\tilde{\rho}_\Lambda$ still remains with larger values. As a consequence their evolution remain closer. The two quantities start to intersect each other for values of $\beta$ larger than 0.8. This is a relevant new feature of the model which solves the problem of coincidence, simply because the situation where the two densities $\tilde{\rho}_\Lambda$ and $\tilde{\rho}_m$ are of the same order of magnitude it is not a peculiarity of the present epoch but occurred many times in past.

In Figure 3 we show the plots corresponding to the non-interacting case $\lambda = 0$ and the evolution for $\lambda = -0.001$ and 0 for $\alpha$, $\beta = 1$. In this case for $0 < x < 2$ the two graphs exactly overlap, since in Eq. (9) the first term dominates whereas in Eq. (10) the l.h.s. is almost vanishing. This feature of the model is extremely important because provides a proper behavior of dark matter and energy for redshift smaller than $z_{eq}$ which does not affect large scale structures formation.

The behavior shown by the energy densities plots for $\lambda = -0.001$ and $\beta = 1$, reported
in Figures 2 and 3 can be easily understood. By increasing the redshift \(x\) the adimensionalized Hubble parameter \(\dot{H} = \sqrt{\dot{\rho}_r + \dot{\rho}_m + \dot{\rho}_\Lambda}\) at the denominator of Eq.s (9) and (10) also increases, but less than the product of energy densities at the numerator does. Thus for large values of \(\tilde{\rho}_m\) the interaction term in Eq. (6) starts to be effective, consequently \(\tilde{\rho}_\Lambda\) increases hence enhancing the interaction term. This occurs till \(\tilde{\rho}_\Lambda\) reaches the dark matter contribution (at \(x \sim 3\) for this choice of parameters). From this value on \(\tilde{\rho}_m\) feels the presence of \(\tilde{\rho}_\Lambda\) and starts to decrease whereas the dark energy remains constant. This behavior ends when the first term in the r.h.s of (9) becomes the dominant one and thus \(\tilde{\rho}_m\) starts to increase again and so the cycle repeats.

In Figures 4(a) and 4(b) we report the quantity \(\log_{10}(\tilde{\rho}_m/\tilde{\rho}_\Lambda)\) versus \(x\) for \(\beta = 0.6\) and \(\beta = 0.9\), respectively. The other parameters are fixed to be \(\alpha = 1\) and \(\lambda - 0.001\) as for the plots of Figure 2. As it is clear from Figures 4 for \(\beta = 0.6\) the interaction term is too small to produce sizeable effects. This situation drastically changes for \(\beta = 1\) where the ratio oscillates with peaks and dips corresponding to the cycles of the Figure 2. Actually the value \(\beta \sim 1\) is the value which most naturally fit the stimulated decay scenario previously outlined.

The peculiar features of the model do not critically depend on the value of \(\lambda\). In Figure 5 the energy density plots are shown for several values of \(\lambda\) in the range \(-1 < \lambda < -10^{-9}\). As it is clear from the figure, the redshift at the equivalence increases for smaller value of \(|\lambda|\). If one imposes the equivalence redshift in the interval \(z_{eq} = 3100^{+600}_{-400}\) [23] then \(\lambda\) must satisfy the bound \(|\lambda| < 10^{-4}\). Reminding the definition of \(\lambda\) (see Eq.(7)), it is possible to recast the bound on this parameter, for a given choice of \(\alpha\) and \(\beta\), in a bound for the mass parameter \(M\) and the dimensionless coupling \(k\) defined in Eq.(7). In particular for \(\alpha = \beta = 1\) one would get
\[
k \sqrt{\frac{3\rho_{eq} M_{Pl}}{8\pi}} \lesssim 10^{-4} \quad \Rightarrow \quad k \lesssim \frac{2.6}{h} \left(\frac{M}{GeV}\right)^3.
\] (12)
For values of the mass scale \(M\) already larger than, say, few GeV, Eq.(12) is not constraining at all the coupling \(k\).

The main result of SN Ia data is that expansion is accelerated for \(z \simeq 1\). In order to understand if our model is indeed compatible with this result we consider the effective equation of state of state of the primordial fluid \(w(x)\), defined as
\[
w(x) \equiv \frac{\tilde{p}_r(x) + \tilde{p}_m(x) + \tilde{p}_\Lambda(x)}{\tilde{\rho}_r(x) + \tilde{\rho}_m(x) + \tilde{\rho}_\Lambda(x)} = \frac{\tilde{\rho}_r(x)}{\tilde{\rho}_M(x) + \tilde{\rho}_R(x) + \tilde{\rho}_\Lambda(x)}.
\] (13)
In particular a phase of accelerated expansion is guaranteed if \(w < -1/3\). It is also worth noticing that recent data on the supernova SN1997ff at \(z = 1.7\) indicates, moreover, that the universe expansion for this redshift is still decelerated. This is actually in beautiful agreement with our results, as can be seen in Figure 6, at least for this choice of parameters. Figures 6, and 8 and 9 also shows that the accelerated phase show that acceleration
occurs for \( z \leq 1 \) without assuming any particular fine-tuning of free parameters. The behavior of \( w(x) \) for larger redshifts and for two different values of \( \lambda \) is reported in Figure 7. The deep occurring for \( x \sim 3.5 \) is a genuine effect of the interaction terms, which is more evident for large values of \( \beta \) and/or of \( \lambda \) as shown in Figures 8 and 9, respectively. Note that the deep appears for \(|\lambda| \geq 10^{-5}\), and with the increasing of \(|\lambda|\) moves to smaller values of \( x \).

Concerning the prediction for the age of the universe, as function of \( \tilde{\rho}_\Lambda \) and \( \tilde{\rho}_m \), this does not provide any severe bound on the free parameter ranges. For all choices of the parameters which satisfy \( z_{eq} = 3100^{+600}_{-400} \) we have checked that the result is the same, in the present uncertainty range, of a customary non-interacting cosmological constant. This is simply due to the fact that, during the last few red-shifts, which mainly contribute to the total age of the universe, \( \rho_\Lambda \) and \( \rho_m \) effectively behave as a cosmological constant and non interacting cold matter.

The choice \( \omega_\Lambda = -1 \), even if affects of course the time evolution of dark matter and dark energy, is not crucial to get the main features of \( \tilde{\rho}_\Lambda \) and \( \tilde{\rho}_m \) evolution. In particular a different choice for \( \omega_\Lambda \), provided that \( \omega \leq -0.4 \) is still producing the behaviour already discussed, with \( \tilde{\rho}_\Lambda \) and \( \tilde{\rho}_m \) following and intersecting each other during the universe evolution. This is shown in Figure 10, where the energy densities versus \( x \) are plotted for \( \alpha = \beta = 1, \lambda = -0.001 \) and \( \omega_\Lambda = -0.5 \) and \( -0.8 \) respectively.

As stated above the class of models with \( \lambda > 0 \) also describe a possibly interesting scenario. In this case, in fact, the condensate is not produced till the redshift (temperature) does not go under a critical value, \( x_c \). This is shown in Figure 11 where for the corresponding choice of parameters \( x_c \sim 2.8 \). For \( x \) larger than \( x_c \), \( \tilde{\rho}_\Lambda \) is vanishing, and \( \tilde{\rho}_m \) and \( \tilde{\rho}_r \) have the same behavior they would have in absence of interaction. At \( x = x_c \) the fluid undergoes a phase transition and part of the energy density stored in dark matter flows in the condensate till it start dominate at low red-shifts. Unfortunately this scenario is still affected by the coincidence problem. The value of \( x_c \) where the phase transition takes place is determined by the values of \( \rho_m \) and \( \rho_\Lambda \) today or, viceversa, it should be accurately tuned to provide the observed values for the matter/cosmological constant energy ratio today.

To summarize, the coupled quintessence framework provides an interesting scenario where the mutual interaction between dark matter and dark energy components shows a quite close evolution of their energy densities during the universe expansion. This remarkable feature represents a simple way out to the coincidence problem, since it makes the present situation of \( \rho_\Lambda \sim \rho_m \) a cyclic condition which have occurred many times in the past. The detail of the energy density evolution is of course a function of the assumed interaction term. We have considered a simple model where the dark energy is provided by a condensate component of a fluid. The interaction with the gas of non relativistic particle (dark matter) is then due to stimulated decay processes of the condensate into
gas particles. Though the model is still lacking a detailed microscopic description, it is nevertheless quite phenomenologically well motivated, and it would be worth analyzing their properties in more details, in view of the natural way it deals with the cosmic coincidence problem.

Acknowledgements
We would like to thank S. Matarrese for useful discussions.

References

[1] S. Perlmutter et al., Astrophys. J. 517 (1999) 556.
[2] A.G. Riess et al., Astrophys. J. 116 (1998) 1009.
[3] M.S. Turner and M. White, Phys. Rev. D 56 (1997) R4439.
[4] L. Wang, R. Caldwell, J. Ostriker, and P. Steinhardt, Astrophys. J. 530 (2000) 17.
[5] C. Wetterich, Nucl. Phys. B302 (1988) 668.
[6] B. Ratra and P.J.E. Peebles, Phys. Rev. D 37 (1988) 3406.
[7] J. Frieman, C.T. Hill, A. Stebbins, and I. Waga, Phys. Rev. Lett. 75 (1995) 2077.
[8] R.R. Caldwell, R. Dave, and P.J. Steinhardt, Phys. Rev. Lett. 80 (1998) 1582.
[9] S.M. Carroll, Phys. Rev. Lett. 81 (1998) 3067.
[10] T. Damour, G.W. Gibbons, and C. Gundlach, Phys. Rev. Lett. 64 (1990) 123.
[11] J.-P. Uzan, Phys. Rev. D 59 (1999) 123510.
[12] T. Chiba, Phys. Rev. D 60 (1999) 083508.
[13] X. Chen and M. Kamionkowski, Phys. Rev. D 60 (1999) 104036.
[14] C. Baccigalupi, F. Perrotta, and S. Matarrese, Phys. Rev. D 61 (2000) 023507.
[15] L. Amendola, Phys. Rev. D 62 (2000) 043511.
[16] C.B. Netterfield et al., Astrophys. J. 571 (2002) 604.
[17] A.T. Lee et al., Astrophys. J. 561 (2001) L1.
[18] N.W. Halverson et al., Astrophys. J. 568 (2002) 38.
[19] P.S. Corasaniti and E. Copeland, Phys. Rev. D 65 (2002) 043004.
[20] R. Bean and A. Melchiorri, Phys. Rev. D 65 (2002) 041302.
[21] D. Tocchini-Valentini and L. Amendola, Phys. Rev. D 65 (2002) 063508.
[22] L. Amendola, C. Quercellini, D. Tocchini-Valentini, and A. Pasqui, astro-ph/0205097.
[23] R. Bowen, S. H. Hansen, A. Melchiorri, J. Silk, and R. Trotta, MNRAS 334 (2002) 760.
Figure 1: Energy densities in unit of $\rho_c$ for $\alpha = 1$, $\beta = 0$ and $\lambda = -0.001$. 
Figure 2: Energy densities in unit of $\rho_{\odot}$ for $\alpha = 1$ and $\lambda = -0.001$ and varying $\beta$; the corresponding value of $\beta$ is reported on the top of each graph.
Figure 3: Energy densities in unit of $\rho_0$ for $\lambda = 0, -0.001$ with $\alpha = 1$ and $\beta = 1$.

Figure 4: The $\log_{10}[\tilde{\rho}_m/\tilde{\rho}_\Lambda]$ versus $x$ for the two values $\beta = 0.6$ (Fig. (a)) and $\beta = 0.9$ (Fig. (b)). The values of $\alpha$ and $\lambda$ are the same of Figure 2.
Figure 5: Energy densities in unit of $\rho_{c0}$ for varying $\lambda$; the corresponding value of $\lambda$ is reported on the top of each graph. The quantities $\alpha$ and $\beta$ are both fixed to be equal to 1.
Figure 6: Total equation of state for $\lambda = -0.001$ and $\alpha = \beta = 1$. The behaviour of $w$ is plotted for a range of $z$ near the present epoch.

Figure 7: Total equation of state for $\lambda = 0$ ($w_0$) and $\lambda = -0.001$($w$) with $\alpha = \beta = 1$. 
Figure 8: Total equation of state for different values of $\beta$, with $\alpha = 1$ and $\lambda = -0.001$. 
Figure 9: Total equation of state for different values of $\lambda$, for $\alpha = \beta = 1$. 
Figure 10: Energy densities in unit of $\rho_{c0}$ for $\alpha = \beta = 1$, $\lambda = -0.001$ and $\omega_\Lambda = -0.5$ and $-0.8$ respectively.

Figure 11: Energy densities in critical units for $\lambda = 0.0014$ and $\alpha = \beta = 1$. 