Research on VRP of Waste Household Appliances for Recycling

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Abstract. In recent years, China has entered the peak of obsolescence of waste household appliances again with their faster upgrading and shorter life. The recycling of waste household appliances in reverse logistics will become a new area of China federation of logistics in the future. Transportation is an important part in the recycling activities. It is the key to reduce logistics costs that how to arrange efficiently for vehicles to recycle waste household appliances effectively and choose route reasonably. We establish a definite model of vehicle routing optimization of waste household appliances based on cost and apply the ant colony algorithm to solve the model in this paper. As the convergence speed of the ant colony algorithm is slow and it is easy to fall into local optimal solution, we improve the algorithm. Finally, this paper verifies the validity of the algorithm by an example.

1. Introduce
Now China’s household appliances have entered the peak of obsolescence as one of the largest household. According to statistics, the theory number of discarded home appliances in 2017 had reached 125,230,000. Among them, the number of television is 32,160,000; the refrigerator is 24,390,000; the washing machine is 16,200,000; the air conditioner is 27,230,000; the computer is 25,240,000[1]. The recycling of waste household appliances has become a new field of logistics development in China. Collecting the waste appliances properly can not only solve the situation of dwindling resources, but also it is of great significance to build the environment-friendly and development of circular economy society.

Reverse logistics which has the characteristics of dispersion, diversity and demand of uncertainty in China started relatively late comparing with foreign countries, which makes it difficult to do study reverse logistics[2]. In recent years, many scholars have been paying widespread attention to the vehicle routing problem(referred as VRP) for optimization, which is a key part in the rationalization of the design. Dantzig and Ramser[3] as mathematicians from America proposed VRP for the first time in 1959. They described VRP as a problem (of) how to arrange reasonably the distribution path for the shortest distance, the shortest delivery time, the least transportation costs and other objectives on the basis of customer's requirement such as delivery volume, delivery time, delivery place, vehicle load, maximum mileage of vehicle, etc[4].
2. The model of vehicle routing optimization of waste household appliances based on cost

2.1 Description the problem
VRP mainly means that we arrange the best route to complete the recycling of waste household appliances for customers from one or more centers after the site selection of the recycling center is finished. VRP is an optimal combination problem with constraints, which is closely related to the actual production and transportation activities. At present, more and more scholars have begun to model and solve it. However, most directly apply VRP in forward logistics to solve the problems from reverse logistics, short of researches in reverse logistics). We establish a model of vehicle routing optimization of waste household appliances based on cost in this paper. It is a practical significance study.

2.2 Constituent elements
VRP consists of customers, goods, vehicles, distribution centers, transportation network, constraint conditions and objective function.

Customers: It includes not only individual consumers, but also retailers, recycle bin and so on. The recycle bin studied in this paper mainly refers to that built at the recycling station of the street or neighborhood committees. Residents throw waste household appliances into the recycling station and exchange for some money. The workers in the recycling station contact the workers in recycling center through the recycling website of waste household appliances. Then the recycling center receives the recycling instructions and sends the car for recycling.

Goods: They are the objects of distribution and recovery in logistics network and mainly refer to the waste household appliances in the waste household appliances recycling system. The attributes of goods to be considered in VRP are: location, weight, volume, packaging, pick-up time, batch loading and so on. But we only take into account the location, volume and weight of the goods in this paper. The location of the goods determines the transportation route of the vehicles, and the volume and weight determine the stowage.

Vehicles: They are the main tools for transporting cargoes. In the VRP we mainly considers the type of vehicles, load, speed, maximum distance and parking location before and after distribution. Among them, the carrying capacity of the vehicle has two meanings: one is the maximum load; the other is the maximum capacity.

Distribution centers: The distribution center mentioned in this paper mainly refers to the recycling center, it can be one or more. It is the main place for collecting goods, distributing goods and loading. The next activity of reverse logistics will begin after the waste household appliances keep in reserve for a temporary time in the distribution centers.

Transportation network: It consists of three parts: vertices, undirected edges and directed arcs. The vertices mainly stand for distribution centers or customers. The attributes of edges and arcs contain distance, direction and restriction of traffic flow.

Constraints: The model constraints in this paper are mainly two: one is capacity constraints, the total demand (or supply) in per route does not exceed the capacity of the vehicle; the other is distance constraints, the maximum distance that per vehicle drives does not exceed a predetermined value.

Objective function: There are many objective functions in VRP. Common objective functions are minimizing the total transportation cost, including vehicle depreciation, fuel, salary, etc; minimizing the total distribution mileage, that means minimizing the total distribution distance of distribution vehicles; minimizing the number of distribution vehicles, which is minimizing the number of vehicles that need to be used for the same distribution. In addition, there exist maximizing service levels and minimizing tonne kilometers of vehicles. We can choose one or more objective functions in practice. However not all objective functions can be satisfied with conflicting each other. Therefore, we should select the appropriate objective function according to the specific distribution requirements.
2.3 Model assumptions

(1) For the sake of computation simplicity, this model only considers the single recovery center—the single distribution center. The model contains multiple recycling customer points. Each recovery truck starts from the recycling center, completes the recycling task by each customer point, and returns to the recycling center.

(2) The capacity of the recycling center is not limited.

(3) The amount of recovery from per customer point is known.

(4) The relative position coordinates between the recycling center and the recycling customer points are known, and the path length of each node is symmetrical.

(5) Each recycling customer point is served only once by a recycling vehicle.

(6) The load-carrying capacity of each recycling vehicle is known, and the recovery weight of each recycling customer point can not exceed the load-carrying capacity of a single vehicle.

(7) The total capacity of each recycling vehicle is known, and the recycling capacity of each recycling customer can not exceed the total capacity of a single vehicle.

(8) The maximum mileage of each recovery vehicle is known for a single task.

(9) The total number of goods recovered from each recycling vehicle can not exceed its load capacity.

(10) The total mileage of each vehicle can not exceed the maximum distance allowed by the vehicle.

(11) The recovered goods can be mixed together if they have no special requirements.

(12) There is a linear relationship between unit transportation cost and transportation distance in the model.

2.4 Symbolic description

(1) Parameters:

- $P$: It stands for the collection of all nodes—the recycling Center and the recycling customer points. $P = \{i\}$, $i=0$, means the recycling Center, $i=1, 2, ..., n$, means the recycling customer points.

- $S$: It stands for the collection of the recycling customer points. $S = \bigcup \{0\}$.

- $V$: It stands for the collection of the recycling vehicles. $V = \{k\}$, $K = 1, 2, ..., m$.

- $D$: It stands for the distance matrix between nodes. $D = [d_{ij}]_{n \times n}$.

- $d_{ij}$: It stands for the distance between nodes. $d_{ij} = 0, \forall i, j \in P$.

- $C_{ok}$: It stands for the fixed cost of the recovery vehicle $K$ and means the cost of increasing a recovery vehicle. In addition it mainly refers to the driver's salary and so on.

- $C_k$: It stands for the unit distance cost of the recovery vehicle $K$.

- $Z_{max}^k$: It stands for the maximum load capacity of the recovery vehicle $K$.

- $Z_{min}^k$: It stands for the minimum load capacity of the recovery vehicle $K$.

- $R_{max}^k$: It stands for the maximum volume of the recovery vehicle $K$.

- $R_{min}^k$: It stands for the minimum volume of the recovery vehicle $K$.

- $L_k$: It stands for maximum driving distance of the recovery vehicle $K$.

- $W$: It stands for the total weight of the goods from the recycling customer points.

- $V$: It stands for the total volume of the goods from the recycling customer points.

(2) Variables:
if the recovery vehicle K recycles the waste household appliances from the node i to the node j; Otherwise.

if the recovery vehicle K recycles the waste household appliances at the node i; Otherwise.

2.5 Mathematical model
The general objective of VRP is to complete the task of recycling waste household appliances from customers with the lowest cost of recycling operation. Its recovery and operation cost mainly includes vehicle transportation cost and vehicle fixed cost.

(1) objective function

\[ F = \min \left( \sum_{k \in V} C_{ik} + \sum_{i \in P} \sum_{j \in P} \sum_{k \in V} C_k d_{ijk} x_{ijk} \right) \]  

Subject to :

\[ \sum_{k \in V} y_{ik} = 1 \quad i \in S \]  

\[ \sum_{j \in P} x_{ijk} = y_{ik} \quad i \in P, k \in V \]  

\[ \sum_{i \in P} x_{ijk} = y_{jk} \quad j \in P, k \in V \]  

\[ \sum_{i \in S} w_i y_{ik} \leq Z_{\text{max}}^k \quad k \in V \]  

\[ \sum_{i \in S} w_i y_{ik} \geq Z_{\text{min}}^k \quad k \in V \]  

\[ \sum_{i \in S} v_i y_{ik} \leq R_{\text{max}}^k \quad k \in V \]  

\[ \sum_{i \in S} v_i y_{ik} \geq R_{\text{min}}^k \quad k \in V \]  

\[ \sum_{i \in P} d_{ijk} x_{ijk} \leq L_k \quad k \in V \]  

(12)

if the recovery vehicle K recycles the waste household appliances from the node i to the node j; Otherwise.

if the recovery vehicle K recycles the waste household appliances at the node i; Otherwise.

The formula (3) is the objective function of the model, which represents the minimum operating cost of recovery; Constrain(4) means that each recycling customer point can be served once. Constrain(5), (6) ensure that the vehicles entering and leaving a certain customer point are the same vehicle, and ensure that each node is served only once. Constrain(7), (8) are maximum and minimum...
carrying capacity for vehicles. Constrain (9), (10) are maximum and minimum volume for vehicles. Constrain (11) stands for maximum vehicle distance. Constrain (12), (13) stands for the variable $x_{ijk}$ and $y_{ik}$.

3. Algorithm selection

VRP is a typical NP-hard problem. The scale of the problem increases exponentially with the growth of recycling nodes. As a result, computer processing becomes more difficult. It is very difficult to obtain the optimal solution. At present, there are many methods to solve the VRP, which can be roughly divided into accurate algorithms and heuristic algorithms. Among them, heuristic algorithm is divided into traditional heuristic algorithm and modern heuristic algorithm. The algorithm used in this paper is the Ant colony Algorithm of modern heuristic algorithm. The ant colony algorithm is slow in convergence speed and easy to fall into local optimum. In order to overcome these shortcomings, an improved ant colony algorithm is proposed in this paper.

3.1 The summary of ant colony algorithm

The ant colony algorithm was first put forward by Italy scholar called M.Dorigo and others in 1991. It is an optimization algorithm for simulating ant colony in finding the best path during foraging process. In general, it is also a stochastic heuristic method, which can be used to solve different optimization problems of combination with universality and robustness. Furthermore, it is a method based on global optimization.

3.2 Improved ant colony algorithm

Ant colony algorithm converges to the optimal path by pheromone accumulation and updating. It has the characteristics of distribution, parallelism and global convergence. But the lack of pheromones in the early stage leads to a slow algorithm speed. In addition, the ant colony algorithm tends to converge too early so the search is trapped in the local optimal solution. In order to overcome these two shortcomings of ant colony algorithm, we have improved the ant colony algorithm in the following aspects.

(1) Dynamic change of evaporation coefficient $\rho$. The value of the evaporation coefficient determines the convergence speed and the global shrinkage ability of the algorithm. It is inversely proportional to the convergence rate and is directly proportional to the global search ability. In accordance with the basic ant colony algorithm, the probability of unseen paths being selected will be relatively reduced if $\rho$ is a large constant. Moreover, it also affects the global search capability. If $\rho$ is too small, it will affect the convergence speed of the algorithm. So we make some appropriate adjustments for improving the algorithm. At the beginning of the algorithm, $\rho$ is larger. We increase the influence of information concentration and accelerate the convergence speed of the algorithm. Because we expect to find a better solution as soon as possible. When the algorithm stagnates, $\rho$ should be reduced. We reduce the influence of pheromone on ant colony and increase the searching ability of ant colony on solution space to get rid of the constraint of local optimal solution. We adopt the strategy of dynamically changing the evaporation coefficient $\rho$ and calculate a threshold $N_{c1}$ according to simulation experiment in the paper.

$$\rho = \begin{cases} \frac{N_{\text{max}} - N_c}{N_{\text{max}}} + \rho_0, & N_c > N_{c1} \\ \rho_0, & N_c < N_{c1} \end{cases}$$

(14)
represents cycle times, means the maximum cycle times. On account of the idea of genetic algorithm we propose a global variable in the whole operation process, which is the ant trajectory with the best value of the objective function found so far. Thus we prevent the algorithm from discarding the optimal solution when the initial evaporation coefficient is large. After each iteration, will be compared with the current optimal scheme. When it is found that the target value after the iteration is better, will be replaced by the current target value. When we calculate pheromone intensity, we will replace the worst ant walking trajectory. In this way, it is possible to converge quickly to a relatively better solution.

(2) max-min ant colony algorithm
The information quantity of each edge may appear maximum or minimum after a certain search with pheromone updating, the maximum will make the search premature, and the minimum is not conducive to the global search. We apply max-min ant colony algorithm (MMAS) to solve VRP in this paper in order to improve these shortcomings. It keeps the number of possible residual pheromones between and on each optimization path and retains the optimal path after each cycle. The main steps are as follows:
(1) After the iteration, only the information from the path of the optimal solution is updated to make better use of the historical information.
(2) For fear of avoiding premature convergence to solutions that are not globally optimal, the possible concentrations of exogenous hormones in each path are limited between and . If it goes beyond the range, it will be compulsively set to or . It can effectively avoid the amount of information on one path is much more than that on the other so that all the ants are concentrated on the same path and the algorithm will not spread.
(3) The initial concentration of exogenous hormone on each path is set to at the initial time. The algorithm will have a ability to find better solutions when is smaller. All ants will update the information on the path after they complete one iteration.

4. Illustrative example
According to the establishment and theoretical analysis of the model of VRP, as well as improvement of ant colony algorithm, we verify ant colony algorithm to solve such problems is effective by the following example.
A recycling center (numbered 0) recovers waste household appliances from the 20 fixed customers (numbered 1, 2, ..., 20) within a radius of 50km. The coordinates of each customer point (we establish a coordinate system from the origin of the recycling center, distance unit: km), the volume and weight of the items to be recycled are fixed and known, as shown in the table. The distance between the recycling center, the customer points and the customer points can be calculated by the Euclidean distance formula between two points. Currently we have designed and optimized the vehicle distribution and route in the recycling center. It is known that the vehicle type is the same, the maximum load is , the largest volume is , the minimum load is , the minimum volume is , the maximum permitted driving distance is , the fuel consumption per vehicle is 30L/100km, the average unit price of gasoline is 5.18 yuan/L, the salary of the driver that they drive out once a day on average is 1000 yuan every month, and the average daily departure time is 1.6 yuan per kilometer,
the fixed cost of each vehicle can be obtained after average and rounding, and the unit distance cost is \( C_k = 1.6 \text{ yuan/km} \), \( k = 1, 2, \ldots, m \).

(1) We apply the improved ant colony algorithm to optimize the recovery path of waste household appliances. The parameters of the algorithm are shown in Table 1: The number of ant is \( m = 10 \), \( \rho = 0.1 \), the number of elite ant is \( \theta = 2 \). We solve the above examples by C language programming in the running environment of Intel(R) Core(TM) 2 Duo 3.3 GHz, 4GB and get a satisfactory feasible solution, namely scheme 1 and scheme 2, as shown in Figure 1 and Figure 2.

Table 1: The dates of collecting center and collecting stations

| node | coordinate | weight | volume | node | coordinate | weight | volume |
|------|------------|--------|--------|------|------------|--------|--------|
| 0    | (0, 0)     | 0      | 4      | 11   | (-45, -35) | 1.8    | 2.6    |
| 1    | (10, 20)   | 1.5    | 3      | 12   | (-15, -30) | 2      | 3.2    |
| 2    | (30, 35)   | 2      | 6      | 13   | (-15, -35) | 1.2    | 5      |
| 3    | (30, 25)   | 3      | 2.5    | 14   | (-20, -25) | 1.8    | 2.1    |
| 4    | (40, 15)   | 1      | 4      | 15   | (-30, -40) | 2      | 4      |
| 5    | (45, 0)    | 2.5    | 1.5    | 16   | (30, -30)  | 3.2    | 2      |
| 6    | (40, 1.5)  | 2.5    | 1.5    | 17   | (20, -10)  | 2      | 1.6    |
| 7    | (35, 25)   | 2      | 3      | 18   | (58, -5)   | 1      | 2      |
| 8    | (15, 20)   | 2      | 3.5    | 19   | (10, -15)  | 1.9    | 2.5    |
| 9    | (-25, 10)  | 2      | 4.5    | 20   | (-40, -25) | 3      | 5      |
| 10   | (-15, 10)  | 2      | 4      |      |            |        |        |

Fig. 1: The solving scheme 1 of ant colony algorithm
The scheme 1:
The recovery path of vehicle1 is 0—1—2—3—4—5—18—0.
The recovery path of vehicle2 is 0—8—9—6—7—0.
The recovery path of vehicle3 is 0—10—14—12—13—15—11—0.
The recovery path of vehicle4 is 0—20—16—17—19—0.
The total mileage is 489 km and the objective function is 1467 yuan.

The scheme 2:
The recovery path of vehicle1 is 0—1—2—3—4—5—0.
The recovery path of vehicle2 is 0—8—7—6—9—10—0.
The recovery path of vehicle3 is 0—11—15—13—12—0.
The recovery path of vehicle4 is 0—17—18—20—16—19—0.
The total mileage is 479 km and the objective function is 1437 yuan.

(2) In order to test the validity of the ant colony algorithm, we select Lingo14.0 as the traditional algorithm of optimization and compare the results with the ant colony algorithm. We solve the above examples by Lingo14.0 in the running environment of Intel(R) Core(TM) 2Duo 3.3GHz, 4GB. The solution is shown below (the specific scheme is shown in Figure 3).

The scheme 3:
The recovery path of vehicle1 is 0—1—2—3—4—5—0.
The recovery path of vehicle2 is 0—8—7—6—9—10—0.
The recovery path of vehicle3 is 0—11—15—13—12—0.
The recovery path of vehicle4 is 0—18—20—16—17—19—0.
The total mileage is 471 km and the objective function is 1413 yuan.
5. Conclusion
The improved ant colony algorithm can solve many groups of satisfactory solutions, while the traditional algorithm can only solve a group of optimal solutions. Strictly speaking the optimal solution obtained by the traditional optimization algorithm is the optimal path and the minimum cost. However, the optimal solution may not be the solution most in line with the actual situation in fact. There is no room for decision-makers to choose. For the decision-maker who wants to make decision on the basis of the solutions, there is a risk of finding a set of impractical solutions to the real problem by using the traditional optimization method, which makes it impossible for him to make decision. But if the decision-maker adopts the intelligent optimization method, he can select the "optimal solution" from many groups of "satisfactory solution" which accords with the actual conditions and has practical significance. The results of the two algorithms show that the solution obtained by ant colony algorithm is also feasible and approximate to the optimal solution obtained by the traditional method. The number of vehicles recovered by the two methods is the same, but the customer service points of each vehicle are different, which makes the total operation process and the value of objective function different. There exit two forms, one form is new-built, the other one is expanded-built for the location problems. Expanded-built can utilize the distribution points in distribution network of forward logistics, then expand its scale on the basis of the origin distribution points. It can make full use of the existing facilities, so the costs of expansion are generally lower than the new cost. Generally, enterprises regard the lowest cost of operation as the optimal conditions in the recovery system. Operation costs considered in this paper are mainly composed of two parts: (1) the cost to build new-built or expanded-built collection stations and collection centers; (2) transportation cost that waste household appliances shipped from the customer to the recycling stations and shipped from the Recycling stations to recycling centers.

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