Estimation of the reliability function of the Rayleigh distribution using some robust and kernel methods

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Abstract. The research presents the reliability. It is defined as the probability of accomplishing any part of the system within a specified time and under the same circumstances. On the theoretical side, the reliability, the reliability function, and the cumulative function of failure are studied within the one-parameter Raleigh distribution. This research aims to discover many factors that are missed the reliability evaluation which causes constant interruptions of the machines in addition to the problems of data. The problem of the research is that there are many methods for estimating the reliability function but no one has suitable qualifications for most of these methods in the data such as the presence of anomalous values or extreme values or the appropriate distribution of these data is unknown. Therefore, the data need methods through which can be dealt with this problem. Two of the estimation methods have been used: the robust (estimator M) method and the nonparametric Kernel method. These estimation methods are derived to arrive at the formulas of their capabilities. A comparison of these estimations is made using the simulation method as it is implemented. Simulation experiments using different sample sizes and each experiment is repeated (1000) times to achieve the objective. The results are compared by using one of the most important statistical measures which is the mean of error squares (MSE). The best estimation method has been reached is the robust (M estimator) method. It has been shown that the estimation of the reliability function gradually decreases with time, and this is identical to the properties of this function.

Keywords: reliability function, Raleigh distribution, robust (M estimator), Kernel method.

1. Introduction

The reliability and its applications have great importance in the practical life of a human being. It is used to study the failure of devices and equipment that are related to his or her life because it reduces the maintenance costs of those devices and equipment. Precisely speaking, the development of science and technology around the world has led to the emergence of many electronic devices and the complex equipment and machinery that are used in many fields such as medicine, engineering, industry, telecommunications fields, astronautics, etc. But these devices are subject to failure and this would stop working with them, which leads to increase maintenance and perpetuating and reduced production. Consequently, this leads to losing a lot of money, time, and others. Therefore, the measurement of Reliability for any device would be the basis for the development and maintenance of most of these devices. So reliability is defined as “the probability of accomplishing any part of the system during a certain time and under special working conditions.” Sometimes, Maintenance means treating the damage before it occurs, which leads to reducing the total cost of work. This indicates the importance of reliability in our working life.
Knowing the reliability of each machine or device in any factory or institution makes it possible to predict the perfect total number of working and idle machines and devices at any time. As well as it studies the impact of holidays and sudden stops that the machines or devices are exposed to during their work. It also searches for methods and ways that ensure these equipment and machines achieve their aims for which they were designed or used. So during recent decades, many statisticians have been focused their attention to deal with anomalous data. More precisely, how to deal with the data that contains anomalous values, through the so-called robust estimation methods.

Immune capabilities are more efficient than the normal methods when anomalous values are found. They are also supposed to be very close to the capabilities of the normal methods when anomalous values are not found. On the other hand, it has been focused on nonparametric methods of reliability because of their importance in estimating the reliability function, especially when we cannot estimate a function. Reliability by parameterized methods due to the lack of the necessary conditions for them, so when knowing the distribution of failure times for any machine or device, it is easy to calculate reliability according to the usual and familiar parameterized methods. For all these reasons, some researchers have used more flexible methods than parametric methods for data analysis, namely the nonparametric methods which are inferential statistical methods that can be used to conclude the population under study in light of the sample regardless of the theoretical distribution of that population. These methods require assumptions and information about the distribution characteristics of the population to a lesser extent Parametric method. In addition to the time taken to analyze data in these methods is less than that of the parametric methods and thus leads to speeding up the results and benefit from them in practice.

1- Literature Review

2-1- Reliability Concept

There are several definitions of reliability and it can be considered that the first to define reliability is (Spearman, 1904), who defined it as “the amount of confidence in the instrument used in the measurement that gives us the same measurements when repeating the same event for consecutive times.” Also (Freeman, 1965) defined it as being the degree of confidence in the estimator for a certain time. As for Brown, 1983 he defined it as “the extent of consistency of measurement across time or the degree to which the measure is not affected by errors, "errors of measurement. It is defined by Best & James (2005, 2005) as “a design that ensures good quality of the estimator in the event of increased measurements of units observed.” On the mathematical side, the Reliability Function is defined as the probability that the machine will not fail until the time t, where t > 0). Let us assume that T is a non-negative random variable representing the time of failure and has a probability density function f (t), as well as an aggregate probability function F (t), symbolizing the reliability function R (t), and it is expressed mathematically as follows: [8]

\[ R(t) = \Pr(T > t) \quad \ldots \quad (1) \]

(t): The operating time, which is greater or equal to zero. (T): the accumulated time for the life of a particular device during the period (0, t), the T ears is a random variable representing the survival time of the experimental unit until the occurrence of failure, and the reliability function possesses several properties, including [9][10]

t is a value between zero and the integer one because it is a probability function, and in a mathematical sense it can be written as follows:[7]

\[ 0 \leq R(t) \leq 1 \quad \ldots \quad (2) \]

A monotonic function decreasing with time (inversely proportional to time), as the more time the machine works, the value of the reliability function decreases, in other words, that

\[ R(t_1) > R(t_2) > R(t_3) > \cdots > R(t_\infty) \quad \ldots \quad (3) \]

And that the value of the reliability function at zero time is (1) and its value begins to decrease gradually and monotonously, and its value becomes at the largest time (Max t) for the life of the machine is equal to zero, as follows:[11]

\[ R(t = 0) = 1 \]

\[ R(t = \text{Max } t) = 0 \]
If \( R(t) = 0 \), then the device will not work at all, and if the value of \( R(t) = 1 \), this is a clear indication that the device will continue to work until the time \( t \). This is a theoretical assumption only, and the figure below shows the relationship between time and the reliability function, the horizontal axis represents time, and the vertical axis represents the reliability function.

Figure (1) represents the reliability function [12]

Looking at the above figure, it is noticed that when \( t = 0 \), the value of the reliability function is higher than possible, and the value of the reliability function begins to decrease gradually and clearly as time progresses or the machine works until it approaches zero, and thus we can consider the device to have failed or stopped working.

2-2 Rayleigh Distribution

The Rayleigh distribution is one of the continuous distributions and one of the well-known and important models used in physical applications, as well as in single analyzes, error analyzes for various systems, and the study of failure time distribution. It was discovered by the English physicist (Lord Rayleigh). The formula for the Rayleigh distribution takes the following form:

\[
f(x, \lambda) = \frac{x}{\lambda} e^{-x^2/2\lambda} \quad x > 0, \quad \lambda > 0 \quad \ldots \quad (4)
\]

Since \( \lambda \) represents the scale parameter, and it is considered one of the single parameter distributions, and the cumulative function of the Raleigh distribution is:

\[
F(x) = 1 - e^{-x^2/2\lambda} \quad ; \quad 0 < X < \infty \quad \ldots \quad (5)
\]

Figure (2) the reliability function graphically for the Raleigh distribution

Thus, the reliability function for this distribution mathematically is as follows:

\[
R(t) = 1 - F(x) = 1 - \left(1 - e^{-x^2/2\lambda}\right)\ 
\therefore R(t) = e^{-x^2/2\lambda} \quad \ldots \quad (6)
\]

Thus, the reliability function according to the Raleigh distribution and when substituting time \( t \) instead of \( x \) becomes as follows:

\[
R(t) = e^{-t^2/2\lambda} \quad \ldots \quad (7)
\]

2- Estimation methods
2-1 Robust M. Estimation method

This method is called Robust White abbreviated to R. White is known that the idea of the OLS method relies on minimizing the sum of error squares as small as possible, i.e.:

$$\text{Min}_{a, b} \sum_{i=1}^{n} [Y_i - a - bX_i]^{2}$$

(8)

The method adopted by the M-estimate method is to reduce the following:

$$\text{Min}_{a, b} \sum_{i=1}^{n} P(Y_i - a - bX_i)$$

(9)

Since $P$ is a symmetric convex function, we obtain from it immune capabilities. To reduce the above expression, the partial derivative concerning the parameters is taken and equal to zero, as follows:

$$\sum_{i=1}^{n} \psi(Y_i - a - bX_i) = 0$$

(10)

$$\sum_{i=1}^{n} \psi(Y_i - a - bX_i)X_i = 0$$

(10)

Since $\psi$ is the partial derivative of the function for the parameters, the equation ($\cdot \cdot \cdot$) can be solved in several ways, including the well-known numerical methods such as the Newton-Raphson Method. Or by relying on the method (Iteratively Weighted Least Squares) Frequency Least Weighted Squares (IWLS) according to the following:

$$\hat{\beta}_{\text{Rob.}} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (X'WX)^{-1}X'WY$$

(11)

$$Y = X\beta + e \quad ; \quad X = \begin{bmatrix} 1 \\ X \end{bmatrix}$$

$$W_{ii} = \frac{\psi(Y_i - a^* - b^*X_i)}{Y_i - a^* - b^*X_i}$$

(12)

Note that $W$ represents a matrix of diagonal weights, whose elements can be found according to the following formula:

Then I use these weights with the formula ($\cdot \cdot \cdot$) to get new values for

$$b^*_1 = \begin{bmatrix} a^*_1 \\ b^*_1 \end{bmatrix}$$

On the first repetition. Then previous values are prepared in a calculation and the repetition continues until we obtain very close by values with each other in the successive repetitions, then the last estimate from formula ($\cdot \cdot \cdot$) is the estimate of M.

In order for (M estimator) to have a scale-invariant, formula ($\cdot \cdot \cdot$) can be expressed as follows:

$$\text{Min} \sum_{i=1}^{n} \frac{P(Y_i - a - bX_i)}{\hat{\sigma}}$$

(13)

Accordingly, the equations shown in ($\cdot \cdot \cdot$) will be as follows:
\[
\sum_{i=1}^{n} \psi \left[ \frac{(Y_i - a - bX_i)}{\hat{\sigma}} \right] = 0
\]
\[
\sum_{i=1}^{n} \psi \left[ \frac{(Y_i - a - bX_i)}{\hat{\sigma}} \right] x_i = 0
\]

The diagonal weights of the W matrix shown in Formula (14) can be rewritten as follows:

\[
W_{ii} = \psi \left[ \frac{Y_i - a^o - b^o X_i}{\hat{\sigma}} \right] \left( \frac{Y_i - a^o - b^o X_i}{\hat{\sigma}} \right)
\]

As for the value of the standard estimator is estimated only once through a previous value before starting the repetition process by adopting one of the following formulas:

1. \( \hat{\sigma} = (2.1) \text{med}|e_i| \)
2. \( \hat{\sigma} = \frac{1}{n-4} \sum_{i=3}^{n-2} |e(i)| \)
3. \( \hat{\sigma} = 1.48[\text{med}|e_i - \text{med}e_i|] \)
4. \( \hat{\sigma} = \left[ \text{med}|e_i| / 0.6745 \right] \)

Since \( e_{(i)} \) represents the residuals, and \( e_i \) represents the ordered residuals, it is calculated as follows:

\[
e_i = y_i - a - bx_i
\]

As a result of the increased interest in this method (the M estimator) over the past decades makes many statisticians and researchers, such as ( Hample, Hinich, Talwar, and Rew, ) have developed and improved the immune estimations and suggested of many functions \( \Psi(\cdot) \). So that it gives the estimator of Non-Sensitivity and is not affected by the anomalous, and some of these functions can be illustrated in the following table [14]:

| Weight function | \( P(u) \) | \( \psi(u) \) | \( W(u) \) | Range |
|-----------------|----------|----------|----------|-------|
| A               | A2[1-cos(u/A)] 2A2 | A sin (u/A) | (u/A)-1 sin(u/A) | \(|u|\leq A\pi \) \( \left| u > A\pi \right| \) |
| B               | (B2/2)[1-(u/B)2] 0 | u [1-(u/B)2]2 | [1-(u/B)2]2 0 | \(|u|\leq B \) \( \left| u > B \right| \) |
| T               | u/2 0 | u 0 | 1 | \(|u|\leq T \) \( \left| u > T \right| \) |
| H               | u/2 0 | H Sin (u)H | H(u)-1 | \(|u|\leq H \) \( \left| u > H \right| \) |

In our research, since \( u \) represents the error whose value is to be reduced, this is the error of the model shown in equation (17).

\( P(u) \): a function in terms of \( u \); \( \psi(u) \): the weight function aimed at minimizing the error.
W(u): The matrix of weights used to obtain estimates having the least error.

As for the constants of specified pieces in the (Range) column, they are used to modify the efficiency of the resulting estimators and take the values shown in the following table:

| Weight function | A      | B      | T      | H      |
|-----------------|--------|--------|--------|--------|
| Segment constants| 1.339  | 4.680  | 2.790  | 1.340  |

And that the distribution of the estimator M is an asymptotic normal distribution with mean $\hat{\beta}_R$. And a covariance and covariance matrix $\text{Var} - \text{Cov}(\hat{\beta}_R)$, and according to the following formula:

$$\text{Var} - \text{Cov}(\hat{\beta}_R) = \sigma^2_R (X'X)^{-1}$$

(18)

$$\sigma^2_R = \frac{E(\psi^2)}{E(\psi')}^2$$ and $\hat{\sigma}^2_R = \frac{1}{n} \sum_{i=1}^{n} \psi^2(r_i) \frac{1}{\left[ \frac{1}{n} \sum_{i=1}^{n} \psi'(r_i) \right]^2}$

(19)

Note that $\hat{\sigma}^2_R$ is a biased estimate, and to make it an unbiased estimate, it is made based on the following formula:

$$\hat{\sigma}^2_R = R^2 \frac{1}{n^2} \sum_{i=1}^{n} \psi^2(r_i) \left[ \frac{1}{n} \sum_{i=1}^{n} \psi'(r_i) \right]$$ and $R = 1 + \frac{\rho \text{Var}(\psi')}{nE(\psi')^2}$

(20)

and after obtaining the estimator M of the model parameters, that is:

$$\hat{Y} = \hat{a} + \hat{b}X$$

(21)

Accordingly, the estimated distribution of Raleigh for this method is:

$$F_X = 1 - e^{-\left(\frac{x^2}{2\lambda}\right)} \Rightarrow \text{log}(1 - F_X) = \frac{x^2}{2\lambda} \Rightarrow \text{log(1 - F_X)}^{-1} = 2 \log x - \log(2\lambda)$$

$$\hat{a} = -\log(2\lambda) \Rightarrow 2\lambda = e^{-\hat{a}x} \therefore \lambda = \frac{e^{-\hat{a}x}}{2}$$

(22)

We obtain the estimator of M for the reliability function $\hat{R}_m(t)$ by using the following formula:

$$\hat{R}_m(t) = e^{-\left(\frac{t}{2\lambda}\right)}$$

(23)

2-2 Kernel Estimator Method (K.E) 

This method (Kernel estimator) is one of the nonparametric methods for estimating the aggregate functions, reliability function, spectrum functions, regression, etc. It was proposed by the researchers (Rosenblatt and Parzen). It aims to revise the data in a way that makes obtaining estimators with characteristics close to the properties of the parameters. There are different types of nonparametric (Kernel) functions, and Gaussian Kernel functions will be used, which are defined by the following formula:
\[ K(x) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-x^2}{2}\right), \quad 0 \leq x \leq 1 \quad (24) \]

Although the Kernel functions are important for obtaining capabilities that converge with the statistical inferential properties, the researcher (Hardle) emphasized that selecting the functions is not the most important step in the estimation method, but rather selecting the introductory parameter \( h \), (bandwidth) is Most importantly, that is, selecting the appropriate bandwidth equals selecting the best kernel function.

The use of the (Kernel) estimator method requires determining the introductory parameter \( h \), as this parameter greatly affects the bias and variance, and increasing the introductory parameter. This leads to increase bias and reduce variance, and vice versa, and as a result, the introductory parameter affects the degree of smoothing of the estimated curve and its proximity to The true curve, and for this, you must fulfill the following conditions:

1. \( \lim_{n \to \infty} h = 0 \)
2. \( \lim_{n \to \infty} nh = \infty \quad (25) \)

The introductory parameter of the Kernel method can be either constant or variable. The constant means the use of the single introductory parameter along the real line used to estimate the reliability function. It is estimated according to one of the estimation methods shown later. While the variable means the use of different values of the bandwidth and the length of the real line when estimating the reliability function. Sometimes, the use of the variable bandwidth becomes useless. The use of the fixed preliminary parameter is estimated by one of the estimation methods, shown in the following sub-sections.

Assuming that \((t_1, t_2, ..., t_n)\) is a sample of independent observations of a similar distribution with a probability function defined as follows:

\[ \hat{f}(t) = \frac{1}{nh} \sum_{i=1}^{n} K \left( \frac{t - T_i}{h} \right), \quad i = 1, 2, ..., n \quad (26) \]

\( K(x) \): The Kernel function represents and is calculated from equation No. (24). It is a real, identical, specific, and constant function that has many names, including a function (weight, shape, basic). it fulfils the following conditions:

1. \( \int_{-\infty}^{\infty} K(x)dx = 1 \)
2. \( \int_{-\infty}^{\infty} xK(x)dx = 0 \)
3. \( \int_{-\infty}^{\infty} x^2K(x)dx > 0 \quad (27) \)

t: represents the estimation time of the reliability function. ; \( n \): the size of the studied sample.

\( h \): represents the bandwidth parameter, which is fixed and estimated according to the following steps:

\[ h = 1.06 \hat{\sigma} n^{-1/5} \quad (28) \]

\( \hat{\sigma} \): Estimate the static introductory parameter:

\[ \hat{\sigma} = \min \left[ S, \frac{D}{1.349} \right] \quad (29) \]

\[ D = X_{(0.75n)} - X_{(0.25n)} \quad (30) \]
We obtain an estimate of a function for (Kernel), from which we can obtain an estimate of the cumulative function of (Kernel) as follows:

\[ \hat{F}_{(K,F)}(t) = \frac{1}{nh} \sum_{i=1}^{n} \int_{-\infty}^{t} K\left(\frac{y - T_i}{h}\right) dy \]  

(31)

Thus, an estimate of the reliability function for (Kernel) can be obtained according to the following formula:

\[ \hat{R}_{(K,F)}(t) = 1 - \left[ \frac{1}{nh} \sum_{i=1}^{n} \int_{-\infty}^{t} K\left(\frac{y - T_i}{h}\right) dy \right] \]  

(32)

After estimating the reliability function by the aforementioned methods, we have to compare them through the results and find out what is the best method in terms of practical application. Here, a comparison will be made between the studied estimation methods, and for the comparison between estimation methods, the statistical criterion mean squares error (MSE) \(^{[18]}\)

\[ \text{MSE} \left( \hat{R}(t) \right) = \frac{1}{L} \sum_{i=1}^{L} \left( \hat{R}_i(t) - R(t) \right)^2 \]  

(33)

Where \(L\) represents the number of iterations of each experiment.

3- Simulation

The use of the simulation method to compare between the studied or proposed methods by researchers to know the best method. So it has been focused on it in this chapter. It represents a comparison between some immune and non-scientific methods for estimating the reliability function. It presents a theoretical study and determines the methods that are appropriate for estimating.

There are several different methods of simulation, including the analog method, the mixed procedure, the inverse transformation method. The inverse transformation method is used to generate random views (data) from a single theoretical community that simulates the real community, as the random numbers are formulated, The researcher used one of these methods, which is the inverse conversion method.

The simulation program was written using (MATLAB-18) application, and the results of the analysis were shown in Tables 2 and 3.

Table 2 shows the values of the reliability function for ten iterations and the estimation methods for the first and second case with the assumed parameter \((\lambda = 1)\) and \((\lambda = 0.8)\) respectively, and the sample size \(n = 15\).

| n  | \(x\)  | Real \((R(t))\) | M-E | K-E | \(\lambda = 1\) | \(\lambda = 0.8\) |
|----|-------|----------------|-----|-----|----------------|-----------------|
| 10 | 1.234 | 1.9923         | 1.41 | 1.12 | 1.9999         | 1.9931          |
|    | 1.234 | 0.9841         | 1.48 | 1.18 | 1.9999         | 1.9931          |
|    | 1.234 | 1.4309         | 1.49 | 1.19 | 1.9999         | 1.9931          |
|    | 1.234 | 1.3769         | 1.49 | 1.19 | 1.9999         | 1.9931          |
|    | 1.234 | 1.3769         | 1.49 | 1.19 | 1.9999         | 1.9931          |
|    | 1.234 | 1.3769         | 1.49 | 1.19 | 1.9999         | 1.9931          |
|    | 1.234 | 1.3769         | 1.49 | 1.19 | 1.9999         | 1.9931          |
|    | 1.234 | 1.3769         | 1.49 | 1.19 | 1.9999         | 1.9931          |
|    | 1.234 | 1.3769         | 1.49 | 1.19 | 1.9999         | 1.9931          |

MSE | 1.1111 | 1.5777          | 1.1111 | 1.5777 | 1.1111 | 1.5777 |
Table 3 shows the values of the reliability function for ten iterations and the estimation methods for the first and second case with the assumed parameter ($\lambda = 1$) and ($\lambda = 0.8$) respectively, and the sample size $n = 25$.

| n  | X       | Real (R(t)) | M-E   | K-E   | n  | x       | Real (R(t)) | M-E | K-E |
|----|---------|-------------|-------|-------|----|---------|-------------|------|------|
| 25 | 0.1128  | 0.9911      | 0.9911| 1.000 | 20 | 0.330   | 0.9877      | 0.9824| 0.9999|
|    | 0.1887  | 0.9820      | 0.9999|       |    | 0.337   | 0.9854      | 0.9824| 0.9998|
|    | 0.2044  | 0.9831      | 0.9831| 0.9999|    | 0.309   | 0.9870      | 0.9824| 0.9999|
|    | 0.2847  | 0.9741      | 0.9998|       |    | 0.500   | 0.9780      | 0.9778| 0.9999|
|    | 0.7707  | 0.9004      | 0.9997|       |    | 0.14     | 0.9110      | 0.8919| 0.9979|
|    | 0.9468  | 0.9468      | 0.9990|       |    | 0.523   | 0.9837      | 0.9773| 0.9997|
|    | 0.9887  | 0.9779      | 0.9992|       |    | 0.510   | 0.9851      | 0.9778| 0.9997|
|    | 0.7434  | 0.9488      | 0.9988|       |    | 0.269   | 0.8741      | 0.8787| 0.9984|
|    | 0.9768  | 0.9421      | 0.9980|       |    | 0.524   | 0.8741      | 0.8787| 0.9984|
|    | 0.9912  | 0.9912      | 0.9990|       |    | 0.522   | 0.8741      | 0.8787| 0.9984|
| MSE|         | 1.0000      | 0.9932|       | MSE|         | 1.0000      | 0.9932|       |

The results of the tables 2 and 3 showed that the (M-Method) is the best method for estimating because it has the lowest MSE and percentages according to the sequence of preference for it.

4- Conclusion

Through a simulation study to compare the parameterized Kernel method and the Robust M.Estimation methods, it was observed that the Robust M.Estimation methods are better using the mean error squares MSE criterion comparison for the use of models and different sample sizes, other methods can be used to estimate the reliability function, such as the Nadaraya-Watson or Bayes methods, and other methods. And comparison with Robust M.Estimation methods.

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