Analysis of the “nagel effect” in reinforced concrete structures under torsion with bending

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Abstract. The physical essence of the “nagel effect” for main reinforcement in a spatial crack of reinforced concrete structures under the action of bending and torsion for the model of reinforced concrete structures deformation involves the formation of the first block and the second block of a spatial crack. The second block is located on the upper surface of the longitudinal reinforcement to the surface of normal stresses and the resultant force from crushing concrete and a special adhesion force to the surface of the reinforcement (and the additional effect of breaking the continuity) in the eccentricity to its center of the axis of the reinforcement for distributed variable bending moment (through the calculation of statically indefinable systems for the force method). It is important to allocate a special area of the left side for the longitudinal reinforcement in the first block of separation and the formation of a new longitudinal crack, which removes surface normal and tangential of stress adhesion. Consider the scheme—a beam with two fixed ends: to calculate it, take the main system obtained as a result of making a cut in the middle of the beam for given unit diagrams of bending moments: under the action of a vertical load; with linear displacements of the fixed ends (normal to the beam axis); with rotations of the fixed ends in construction mechanics.

1. Introduction
For more than 45 years, researchers have been interested in studying the “nagel” effect of reinforcement—a phenomenon observed in experiments related to the resistance of reinforced concrete elements to force and deformation effects. However, the physical nature of this phenomenon is still unclear and is determined from empirical considerations.

The physical nature of the "nagel" effect includes the parameter of the transverse force of the reinforcing bar $Q_s$, which characterizes the "nagel" effect and other main parameters [1] such as, taking into account the presence of several forces and displacements: the force $P$ (and $P_{1,t}$, for perpendicular plane) on the surface of the second block; the force of adhesion to the surface of the rebar $\tau_{ad}$ (and $\tau_{ad,t}$ for perpendicular plane) and a special force for the effect of breaking the continuity— is the opposite $\Delta T$; movement from the mutual shift of the crack banks $\Delta_1$ – for the vertical plane and $\Delta_2$ – for the perpendicular (horizontal) plane; movement from the width of the crack opening $\Delta_2 = a_{cr}$; the angle of rotation near the fixed end $\varphi_1$; the angle of twisting $\varphi_2$ is similar for the perpendicular (horizontal) plane; the variable longitudinal force in the slipping attachment end $N_s$ and other forces for the calculation models.

Under these conditions, revealing the essence of phenomena that allows us to build a computational
model for determining the "nagel" effect becomes an unusual and urgent task [2-13].

The "nagel" effect occurs in reinforced concrete structures from the action of bending, while in local areas near the spatial crack there are areas of crushing, punching, and separation.

To build the model of deformation of reinforced concrete structures, are proposed a scheme, which includes the first block and the second block of the spatial crack. Additional blocks are considered for cross-sections I-I and III-III. The area where the second block is crushed is located in the concrete on the upper surface of the longitudinal reinforcement (or transverse reinforcement). The area of separation of the first block is special and forms a new longitudinal crack – a new scheme for construction mechanics.

We will consider the forces arising from the "nagel effect" on the example of models of resistance of reinforced concrete elements subjected to the combined effect of torsion and bending – case 1 (figure 1 a, b) [14], as well as torsion, bending and transverse force – case 2 (figure 1 c, d) [15].

2. Materials and Methods

On the scheme (figure 2) for the “nagel effect”, we take into account several forces and displacements.

Consider the scheme – a beam with two fixed ends: to calculate it, take the main system (figure 2b), obtained as a result of making a cut in the middle of the beam and given unit diagrams of bending moments. Beam is under the action of a vertical load, with linear displacements of the fixed ends and with rotations of the pinches.

At the point where the beam is cut occur an internal forces: $X_1$ – vertical transverse force; $X_2$ – bending moment; $X_3$ – distributed variable bending moment from the coupling with the surface to the
center of the armature axis $X_4$ – horizontal transverse force; $X_5$ - torque $M_t$. Here $\Delta a_{crc,\text{max}}$ is the crack opening at a distance of two diameters from the axis of the reinforcing bar; $\Delta a_{crc,s}$ – the crack opening in the axis of the reinforcing bar. Increment $\Delta N_s$ for the surface (figure 3, c) is equal to: $\Delta N_s = 2\pi r_1 d_1 \tau_{ad} \omega$; here $\omega$ is a coefficient for the shear stresses from the adhesion $\tau_{ad}$ and $\tau_{ad,t}$ (figure 3, d).

Adhesion to the surface of the rebar $\tau_{ad}$ (diagram—the cubic parabola) and a special force for the effect of breaking the continuity, $-\Delta T$ (f2 a, b); movement from the mutual shift of the crack banks $\Delta_1$, $\Delta_3$ (figure 2 a); movement from the width of the crack opening $\Delta_2=a_{crc}$ (figure 2a); the angle of rotation near the fixed end $\phi_1=a\tan\left(\frac{\Delta a_{crc,\text{max}}}{2d}\right)$ (figure 2a); the angle of twisting $\phi_2$; the variable longitudinal force in the slipping attachment end $N_i$ (figure 2 a). The scheme is a beam with two fixed ends at $l=3d$, with distances of 0.25 d – for $\Delta T; 0.5$ d to $P_1$ and $P_1,1; 1.25$ d – for the adhesion $\tau_{ad}$ and $\tau_{ad,t}$ (figure 2a).

**Figure 2.** Calculation scheme of the “nagel effect”: loading scheme (a); equivalent system (forces $X_1$, $X_2$, $X_3$, $X_4$, $X_5$, and external loads ($P_1$, $\Delta T$, $\tau_{ad}$, $\Delta N_s$), displacements ($\Delta_1$, $\Delta_2$, $\Delta_3$) and rotation near the fixed ends ($\phi_1$, $\phi_2$) (b); unit diagrams of forces $X_1$, $X_2$, $X_3$, $X_4$, $X_5$ (c-h).

**Figure 3.** The surface of the main upper reinforcement for the second block is under the normal concrete stresses $q_1$, (parabola) and the resultant force $P_1$ (for concrete crush) and the adhesion stresses $\tau_{ad}$ (the cubic parabola) on the surface of rebar: a – cubic parabola diagram and a special force for the effect of breaking the continuity $\Delta T$; b, c – a cell nodes of normal stress $q_1$ and adhesion $\tau_{ad}$; d – the surface area of the longitudinal reinforcement and concrete local normal stress $q_1$ and tangential stress $\tau_{ad}$ to the eccentricity (e) to the center O of the rebar; d-bending moment of the main reinforcement $M_\text{e}=X_2$ and the Bernoulli hypothesis; e - dependency graph $\sigma_s$-$\varepsilon_s$.

Normal stresses on surface $q_1=2\pi r_1 d_1 \sigma_{scr} \omega$. Here $\omega$ is a coefficient for the normal stresses $q_1$ for the surface (figure 1 c).

The angle of rotation near the fixed end can be found:
\[
\tan(\varphi_1) = \frac{\Delta \text{arc}c \cdot s_{\text{arc},\text{max}}}{2d} \\
\varphi_1 = \tan(\frac{\Delta \text{arc}c \cdot s_{\text{arc},\text{max}}}{2d})
\]

Single diagrams of bending moments for all internal forces are given at figure 2 c-h. In this case, the force diagram from \(X_3\) and \(X_3, f\) instead of a cubic parabola can be approximated by a rectangle and a trapezoid for simplification (figure 2 e, h).

**Figure 4.** Calculation scheme for the beam equivalent system (a), the main system scheme for applying external forces, displacements and rotation of the fixed end at some angle (a, c, e, g), diagrams of moments caused by applied loads, displacements and rotation of the fixed end at some angle – b - \(M_{P_1}\); d - \(M_{N_s}\), f - \(M_{\Delta_1}\), h - \(M_{\varphi_1}\).

\[
N_{s,1} = \sigma_{s,k} \cdot A_s + \frac{\Delta_2}{l_{\text{arc}}} \cdot E_s \cdot A_s \rightarrow N_{s,1} = \frac{\Delta_2}{l_{\text{arc}}} \cdot E_s \cdot A_s\]

\[
N_s(x) = f(\tau_{ad}) + f(\Delta_2) = N_{s,\text{max}} \cdot 2\pi \cdot 1.25d \cdot \tau_{ad} + \frac{\Delta_2}{l_{\text{arc}}} \cdot E_s \cdot A_s
\]

Schemes, which allow taking into account forces \(N_s(x)\) in the main system and the corresponding diagrams of moments caused by displacements \(\frac{\Delta_2}{l_{\text{arc}}}\) and \(\frac{\Delta_2}{l_{\text{arc}}} \cdot E_s \cdot A_s\) in a form of the sine wave. In this case, the variable force \(N_s(x)\) is a function of the adhesion \((\tau_{ad})\) and \(\frac{\Delta_2}{l_{\text{arc}}} \cdot E_s \cdot A_s\) and the corresponding moment diagram \((M_{s,X3} = X_3 = N_s(x) \cdot e)\).
3. Results

We can multiply the single diagrams of bending moments from the displacement $\frac{d_1}{2}$ (as well as all single diagrams from displacement $\frac{d_2}{2}$ for a perpendicular plane) and the displacement $\Delta_2$:

$$N_{s,2} = N_{s,\text{max}} - 2\pi r \cdot 1.25d \cdot \frac{X}{e} \cdot \omega + \frac{\Delta_2}{l_{\text{crc}}} \cdot E_s \cdot A_s = \sigma_{s,k} \cdot A_s + \Delta T \cdot 0.25d \cdot 1.25d \cdot \frac{X}{e} \cdot \omega + \frac{\Delta_2}{l_{\text{crc}}} \cdot E_s \cdot A_s$$ (4)

Here adhesion from the surface to the center axis of the upper reinforcement $r_{ad} = \frac{X}{e_{\text{ver}}}$ (figure 1,c); $e_{\text{ver}}$ – the vertical eccentricity, from the surface of the reinforcement toits center axis.

Then we obtain the internal forces $X_1$, $X_3$, $X_{3,1}$, $X_4$, $X_5$ from the calculation of related statically indeterminate systems (6), (7) by the force method:

$$\begin{align*}
X_1 \delta_{11} + X_3 \delta_{13} + X_{1, p} = & \ 0 \\
X_1 \delta_{31} + X_3 \delta_{33} + X_{3, p} = & \ 0 \\
X_3 \delta_{43} + X_4 \delta_{44} + X_{4, p} = & \ 0 \\
X_3 \delta_{53} + X_5 \delta_{55} + X_{5, p} = & \ 0
\end{align*}$$

(5)

(6)

Unknowns $X_2$, $X_3$, $X_{3,1}$, $X_4$, $X_5$ we can find from related systems of equations

$$\begin{align*}
X_2 \delta_{22} + X_3 \delta_{23} + X_{2, p} = & \ 0 \\
X_2 \delta_{32} + X_3 \delta_{33} + X_{3, p} = & \ 0 \\
X_3 \delta_{43} + X_4 \delta_{44} + X_{4, p} = & \ 0 \\
X_3 \delta_{53} + X_5 \delta_{55} + X_{5, p} = & \ 0
\end{align*}$$

(7)

(8)

Coefficients $\delta_{11}$, $\delta_{13}$, $\delta_{31}$, $\delta_{33}$, $\delta_{22}$, $\delta_{23}$, $\delta_{43}$, $\delta_{44}$, $\delta_{53}$, $\delta_{55}$ and the load members $\Delta_{1, p}$, $\Delta_{2, p}$, $\Delta_{3, p}$, $\Delta_{4, p}$, $\Delta_{5, p}$ of these equations are equal to:

$$\delta_{11} = 2 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{12}$$

$$\delta_{13} = \frac{1}{E_l} \cdot \left[ 0 + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot N_{s,1} \cdot e \right] = \frac{N_{s,1} \cdot e^2}{8 \cdot E_l}$$

(10)

$$\delta_{22} = \frac{1}{E_l} \cdot 1 \cdot 1 \cdot 1 = \frac{1}{E_l}$$

(11)

$$\delta_{23} = \frac{1}{E_l} \cdot \left[ 1 \cdot 1 \cdot N_{s,1} \cdot e \right] = \frac{N_{s,1} \cdot e}{E_l}$$

(12)

$$\delta_{31} = \frac{1}{E_l} \cdot \left[ 0 + N_{s,1} \cdot e \cdot 0.25d \cdot 0.125d + N_{s,2} \cdot s_{\max} \right]$$

$$+ \frac{1}{2} \cdot \frac{N_{s,\max}}{2 \cdot N_{s,1} \cdot e} \cdot \frac{e^2 \cdot 0.31 \cdot N_{s,1} + 0.75 \cdot N_{s,2} \cdot s_{\max}}{2 \cdot s_{\max}}$$

(13)

$$\delta_{32} = \frac{1}{E_l} \cdot \left[ 0 + N_{s,1} \cdot e \cdot 0.25d \cdot 1 + N \cdot N_{s,\max} \right]$$

$$+ \frac{N_{s,2} \cdot e}{2} \cdot 0.625d \cdot 1 = \frac{e}{E_l} \cdot \left[ 0.25 \cdot N_{s,1} + 0.94 \cdot N_{s,2} \cdot s_{\max} \right]$$

(14)

$$\delta_{33} = \frac{1}{E_l} \cdot \left[ 0 + N_{s,1} \cdot e \cdot 0.25d \cdot N_{s,1} \cdot e \cdot + N_{s,2} \cdot s_{\max} \right]$$
Here $EI=E_sI_s$, for the main reinforcement bar (beam).

One of possible cases of the beam load is the movement of the fixed end by an amount $\Delta_2$ in the direction perpendicular to the axis of the rod $AB$.

When we turn the fixed end $B$ by an angle $\varphi_1$ in the main system, we obtain the movements in the directions of unknown forces.

The load members of equations (from forces $P_i$ and load movements $\Delta_j, j=1,2,3$) $\Delta_{1,p}$, $\Delta_{2,p}$, $\Delta_{3,p}$, $\Delta_{4,p}$, $\Delta_{5,p}$ in the main system in the directions of unknown forces are equal to:

$$\Delta_{1,p} = \begin{bmatrix} 1 \ 1 \ 1 \ 2 \ 2 \ 2 \ 0 \ + \ 1 \ 1 \ 1 \ 2 \ 2 \ 2 \ d \ p_1 \ 1 \ d \ + \ 1 \ 1 \ 1 \ 2 \ 2 \ 3 \ N_{s,1} \ 2 \ 2 \ 2 \ 2 \ \frac{\Delta_1}{2} \ + \\
+ \ 1 \ 1 \ 1 \ 2 \ 2 \ 3 \ N_{s,2} \ 2 \ \frac{\Delta_1}{2} \ + \ 2 \ 1 \ 1 \ 1 \ 2 \ 2 \ 2 \ \frac{6E_I}{9d^2} \ \frac{\Delta_1}{3} \ + \ 1 \ 1 \ 1 \ 2 \ 2 \ 3 \ \frac{4E_I}{3d} \ \varphi_1 \ + \\
+ \ 1 \ 1 \ 1 \ 2 \ 2 \ 2 \ \frac{2E_I}{3d} \ \varphi_1 \ - \ \frac{9d^4}{16} \ \frac{1}{2} P_1 + \ \frac{9d^4}{24} \ \Delta_1 \ \left( N_{s,1} + N_{s,2} \right) - \ \frac{1}{3} \ \frac{4E_I}{3d} \ \varphi_1 = \ \frac{d}{2} P_1 + E_I \ \varphi_1 ;
\end{bmatrix}$$

$$\Delta_{2,p} = l \cdot 1 \ d \ - \ \frac{1}{2} \ P_1 \ 1 \ d + 0 + 0 + l \cdot 1 \ l \ \frac{1}{4} E_I \ \frac{4E_I}{3d} \ \varphi_1 = \ \frac{d}{2} P_1 + E_I \ \varphi_1 ;$$

$$\Delta_{3,p} = \begin{bmatrix} 0 - 0.625d \ N \left( \frac{N_{s,2}}{4} \right) + \ N \left( \frac{N_{s,2}}{2} \right) \ e \cdot 0.625d \ P_1 \ 1 \ d \ 0.68 \ + \\
+ \ \left[ -N_{s,1} \ e \cdot 0.25d \ 0.08 \cdot N_{s,2} \ \frac{1}{2} \ N_{s,1} \ e \cdot 0.25d \ 0.08 \ \frac{6E_I}{9d^2} \ \frac{\Delta_1}{2} \ + \\
- \left( \frac{N_{s,2}}{4} \right) \ e \cdot 0.25d \ 0.08 \ \frac{6E_I}{9d^2} \ \frac{\Delta_1}{2} \ + \\
+ \ \left[ N \left( \frac{N_{s,2}}{4} \right) + N \left( \frac{N_{s,2}}{2} \right) \ 0.625d \ 0.78 \ \frac{6E_I}{9d^2} \ \frac{\Delta_1}{2} \ + \\
+ \ \left[ -N_{s,1} \ e \cdot 0.25d \ 0.18 \ \frac{4E_I}{3d} \ \varphi_1 + N \left( \frac{2E_I}{3d} \right) \ \varphi_1 \ - \ \left( \frac{N_{s,2}}{4} \right) \ e \cdot 0.25d \ 0.68 \ \frac{2E_I}{3d} \ \varphi_1 \right] \right] \right] \right] ;$$
\begin{equation}
\Delta_4, p = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 0 \\
1 & 2 & 2 & 2 \\
2 & 2 & 2 & 3 \\
N_{s_2} \cdot \Delta_3 \end{bmatrix} + 2 \begin{bmatrix} 1 & 1 & 1 & 2 \\
1 & 2 & 2 & 2 \\
2 & 2 & 2 & 3 \\
6E1 \cdot \Delta_3 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\
2 & 2 & 2 \\
2 & 2 & 2 \\
3 \cdot 4EI \cdot \phi_2^+ \\
\end{bmatrix}
\end{equation}

\begin{equation}
\Delta_5, p = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 0 \\
1 & 2 & 2 & 2 \\
2 & 2 & 2 & 3 \\
9d^2 \cdot \Delta_2 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\
2 & 2 & 2 \\
2 & 2 & 2 \\
4d - 3d \cdot \phi_2^+ \\
\end{bmatrix}
\end{equation}

Then we solve the system of equations (5):

\begin{align}
X_1 \delta_{11} + X_3 \delta_{13} + \Delta_1, p &= 0 \\
X_1 \delta_{31} + X_3 \delta_{33} + \Delta_3, p &= 0
\end{align}

\begin{equation}
X_1 = \frac{-X_3 \delta_{13} - \Delta_1, p}{\delta_{11}}
\end{equation}

\begin{equation}
X_3 = \frac{-X_3 \delta_{13} - \Delta_3, p}{\delta_{11}}
\end{equation}

After substitution, we obtain:

\begin{align}
X_3 \left[ \frac{N_{s_1}, e^2 \cdot l^2}{8EI} - \frac{12EI}{l^3} \cdot \left( - \frac{e \cdot d^2}{EI} \right) \right] + [0.031 \cdot N_{s_1} + 0.75 \cdot N_{s_2} \cdot e^2 \cdot d^2 \cdot s_{s_{\text{max}}}, s_{\text{max}}] \\
+ 0.26 \cdot N_{s_2}^2 + 0.31 \cdot N_{s_2} \cdot N_{s_1} \left( -0.01 \cdot N_{s_2} \cdot \Delta_1 + 0.01 \cdot \frac{EI \cdot \Delta_1}{d} - 0.06 \cdot EI \cdot \phi_1 \right)_{s_{\text{max}}}
\end{align}

\begin{align}
- N \left( -0.625d - 0.11d \cdot N_{s_2} \cdot \Delta_1 + 0.1 \cdot \frac{EI \cdot \Delta_1}{d} + 0.026 \cdot EI \cdot \phi_1 \right)_{s_{\text{max}}}
\end{align}

\begin{align}
N_{s_2} \left[ \frac{N_{s_1}, e^2 \cdot l^2}{8EI} - \frac{12EI}{l^3} \cdot \left( - \frac{e \cdot d^2}{EI} \right) \right] + [0.031 \cdot N_{s_1} + 0.75 \cdot N_{s_2} \cdot e^2 \cdot d^2 \cdot s_{s_{\text{max}}}, s_{\text{max}}] \\
+ 0.26 \cdot N_{s_2}^2 + 0.31 \cdot N_{s_2} \cdot N_{s_1} \left( -0.01 \cdot N_{s_2} \cdot \Delta_1 + 0.01 \cdot \frac{EI \cdot \Delta_1}{d} - 0.06 \cdot EI \cdot \phi_1 \right)_{s_{\text{max}}}
\end{align}

\begin{align}
\Delta_4, p = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 0 \\
1 & 2 & 2 & 2 \\
2 & 2 & 2 & 3 \\
N_{s_2} \cdot \Delta_3 \end{bmatrix} + 2 \begin{bmatrix} 1 & 1 & 1 & 2 \\
1 & 2 & 2 & 2 \\
2 & 2 & 2 & 3 \\
6E1 \cdot \Delta_3 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\
2 & 2 & 2 \\
2 & 2 & 2 \\
3 \cdot 4EI \cdot \phi_2^+ \\
\end{bmatrix}
\end{align}

\begin{align}
\Delta_5, p = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 0 \\
1 & 2 & 2 & 2 \\
2 & 2 & 2 & 3 \\
9d^2 \cdot \Delta_2 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\
2 & 2 & 2 \\
2 & 2 & 2 \\
4d - 3d \cdot \phi_2^+ \\
\end{bmatrix}
\end{align}
Then we solve the related system (6), given that \( X_{3,t} = X_3 \cdot \eta \) :

\[
X_{3,t} \delta_{33} + X_4 \delta_{44} + A_4, p = 0
\]
\[
X_{3, t} \delta_{53} + X_5 \delta_{55} + A_5, p = 0
\]

Then we solve the related system (6), given that \( X_{3,t} = X_3 \cdot \eta \) :

\[
X_4 = \frac{-X_3 \delta_{33} - A_4, p}{\delta_{44}};
\]
\[
X_5 = \frac{-X_3 \delta_{33} - A_5, p}{\delta_{55}};
\]

After substitution, we obtain:

\[
X_4 = X_3 \cdot \eta \cdot \frac{-\delta_{43} \cdot \Delta_{4, p}}{\delta_{44}};
\]

\[
X_4 = X_3 \cdot \eta \cdot 1.5 \cdot N_{s, 1} \cdot (0.25 \cdot d \cdot EI \cdot P_{1, t} + 0.16 \cdot \frac{\Delta_1}{d} \cdot EI \cdot (N_{s, 1} + N_{s, 2}) \cdot 0.44 \cdot \frac{\Delta_1}{d} + 0.59 \cdot \frac{\varphi_1}{d^2});
\]
Solving a system of equations (7):

\[
\begin{align*}
X_2 & = -\frac{X_3\delta_{22} + \Delta_{2,p}}{\delta_{22}};
X_3 & = \frac{X_3\delta_{32} + \Delta_{3,p}}{\delta_{32}};
\end{align*}
\]

After substitution, we obtain:

\[
X_3 = \frac{N_{s,1}e^{-d}EI}{EI}\cdot \left( \frac{dP_1EI}{6} + \frac{EI^2\varphi_1}{3d} \right) \cdot \left( \frac{\Delta_{s,1}}{\varphi_{s,1}} \right) \cdot \left[ \frac{0.25\cdot N_{s,1} + 0.94\cdot N_{s,2}}{EI \cdot s,1, s,2, s, max} \right]
\]

\[
-0.26\cdot N_{s,2} + 0.31\cdot N_{s,2}\cdot N_{s, max} - N_{s,1}\cdot e \left( -0.01\cdot N_{s,2}\cdot \Delta_1 + 0.01\cdot \frac{EI\Delta_1}{d} - 0.06\cdot EI\varphi_1 \right) - \left[ \frac{-N_{s,1}\cdot e\cdot d}{EI} \cdot \frac{0.25\cdot N_{s,1} + 0.94\cdot N_{s,2}}{EI \cdot s,1, s,2, s, max} \right] - \frac{\Delta_{s,2}}{\varphi_{s,2}} - \frac{\Delta_{s,1}}{\varphi_{s,1}}
\]

\[
X_3 = \frac{N_{s,1}e^{-d}EI}{EI}\cdot \left( \frac{dP_1EI}{6} + \frac{EI^2\varphi_1}{3d} \right) \cdot \left( \frac{\Delta_{s,2}}{\varphi_{s,2}} \right) \cdot \left[ \frac{0.25\cdot N_{s,1} + 0.94\cdot N_{s,2}}{EI \cdot s,1, s,2, s, max} \right]
\]

\[
X_2 = \frac{X_3\delta_{23} + \Delta_{2,p}}{\delta_{22}} = \frac{X_3\delta_{32} + \Delta_{3,p}}{\delta_{32}} = \frac{X_3\delta_{33} + \Delta_{3,p}}{\delta_{33}} = \frac{X_3\delta_{23} + \Delta_{2,p}}{\delta_{23}}
\]

\[
= \frac{d}{EI}\cdot \frac{0.5\cdot N_{s,1}^2 + 0.94\cdot N_{s,2}s_{s, max}}{s,1, s,2, s, max}
\]
Then we solve the related system (8), given that $X_{3,t} = X_3 \cdot \eta$:

\[
\begin{align*}
X_3,4_3 x_4 + x_4 \delta_4 + \Delta_4 p &= 0; \\
X_3,5_5 x_5 + x_5 \delta_5 + \Delta_5 p &= 0.
\end{align*}
\]

After substitution, we obtain:

\[
\begin{align*}
X_4 &= X_3 \cdot \eta \cdot \frac{-\delta_4}{\delta_4}; \\
X_5 &= X_3 \cdot \eta \cdot \frac{-\delta_5}{\delta_5};
\end{align*}
\]

From here, the adhesion with the surface of thereinforcements $(\tau_2^2 + \tau_1^2, \tau_{ad,1}^2) = (X_{3,3, sum})^{1/2}$; $e_{sum} = (e_{ver}^2 + e_{hor}^2)^{1/2}$ - eccentricity from the surface of the armature to its center axis.

Then $M = X_{3,3, sum} (\tau_2^2 + \tau_1^2, \tau_{ad,1}^2) = (X_{3,3, sum})^{1/2}$; moment from adhesion with the surface of the reinforcement.

The support reactions and reference moments will be equal:

\[
\begin{align*}
R_{A, ver} &= -X_1; R_{B, ver} = X_1; \\
R_{A, hor} &= -X_4; R_{B, hor} = X_4; \\
M_A &= N_3 \cdot e_{ver}; M_B = N_3 \cdot e_{hor}; \\
M_{A, hor} &= N_3 \cdot e_{hor}; M_{B, hor} = N_3 \cdot e_{hor}.
\end{align*}
\]

Here, $e_{ver}$ – the eccentricity in the vertical plane; $e_{hor}$ – the eccentricity in the perpendicular (horizontal) plane.

The bending moment in the main reinforcement in the vertical plane $M_4 = X_2$. At this time the dependence "$\sigma_4 - \varepsilon_4$" (figure 3, f) and the Bernoulli hypothesis (figure 3, e) are true. Then:

\[
M_4 = X_2 = \sigma_4 \varepsilon_4 A_4
\]

The bending moment in the main reinforcement in the perpendicular (horizontal) plane $M_{s,t} = X_5$. At this time the dependence "$\sigma_5 - \varepsilon_5$" (figure 3, f) and the Bernoulli hypothesis (figure 3, e) are true. Then:

\[
M_5 = X_5 = \sigma_5 \varepsilon_5 A_5
\]
\[ M_{s,t} = X_3 = \sigma_s \omega_s z_s A_s \eta \]  \hspace{1cm} (44)

4. Discussion

We consider the scheme of a beam with two fixed ends. For calculations we take the main system obtained as a result of making a cut in the middle of the beam. For this system we can make the single diagrams of bending moments from the action of vertical and horizontal loads (and displacements), from the linear displacements of the fixed ends (normal to the beam axis), from rotations at the fixed end.

In this case the scheme for the "nagel effect" allow us to take into account several forces and displacements: adhesion to the surface of the rebar \( \tau_{ad} \) (cubic parabola) and a special force for the effect of breaking the continuity, – \( \Delta T \) (figure 2 a, b); movement from the mutual shift of the crack banks \( \Delta_1, \Delta_3 \); movement from the width of the crack opening \( \Delta_2 = a_{cr} \); the angle of rotation near the fixed end \( \varphi_1 = a \tan \left( \frac{\Delta_{acc}}{a_{max}} \right) \); the angle of twisting \( \varphi_2 \); the variable longitudinal force in the slipping attachment end \( N_b \). The scheme is a beam with two fixed ends at \( l = 3d \), with distances of 0.25 \( d \) – for \( \Delta T \); 0.5 \( d \) to \( P_1 \); and \( P_1, i \); 1.25 \( d \) – for the adhesion \( \tau_{ad} \) and \( \tau_{ad,t} \).

We can multiply the single diagrams of bending moments from the displacement \( \Delta_2 \) (as well as single diagrams from displacement \( \Delta_2 \) for a perpendicular plane) and the displacement \( \Delta_2 \):

At the point where the beam is cut occur an internal forces: \( X_1 \) – vertical transverse force \( X_2 \) – bending moment; \( X_3 \) – distributed variable bending moment from the coupling with the surface to the center of the armature axis \( X_4 \) – horizontal transverse force; \( X_5 \) – torque \( M_t \); \( X_3 = M_{s,X3} \).

From here, the adhesion with the reinforcement \( \tau_{ad} = \frac{X_3}{e_{cr}} \) and \( \tau_{ad,t} = \frac{X_3}{e_{hor}} \) causes distributed variable bending and torque moments. The unknowns \( \tau_{ad} \) (and \( \tau_{ad,t} \)) can be found from the canonical equations for structural mechanics.

5. Conclusion

The physical essence of the “nagel effect” for main reinforcement in a spatial crack of reinforced concrete structures under the action of bending and torsion for the model of deformation of reinforced concrete structures involves the formation of the first block and the second block of a spatial crack.

The surface of the main upper reinforcement for the second block is under the normal concrete stresses \( q_1 \) (parabola) and the resultant force \( P_1 \) (for concrete crush) and the adhesion stresses \( \tau_{ad} \) (the cubic parabola) on the surface of rebar and a special force for the effect of breaking the continuity, – \( \Delta T \).

Here the distributed variable bending moment \( M_{s,X3} = X_3 \) can be found through the calculation of statically indeterminate systems with the force method.

It is important to allocate a special area of the left side for the longitudinal reinforcement in the first block of separation and the formation of a new longitudinal crack, which removes surface normal and tangential stress from adhesion.

Adhesion with the reinforcement \( \tau_{ad} = \frac{X_3}{e_{cr}} \) causes distributed variable bending moment \( M_{s,X3} = X_{3,t} = X_3 \eta \). The unknowns \( \tau_{ad} \) (and \( \tau_{ad,t} \)) can be found from the canonical equations for structural mechanics.

At the point where the beam is cut occur an internal forces: \( X_1 \) – vertical transverse force \( X_2 \) – bending moment; \( X_3 \) – distributed variable bending moment from the coupling with the surface to the center of the armature axis \( X_4 \) – horizontal transverse force; \( X_5 \) – torque \( M_t \); \( X_3 = M_{s,X3} \).

Increment \( \Delta N_b \) for the surface of rebar \( \Delta N_b = 2 \pi r d \tau_{ad} \omega \); and \( \Delta N_{s,T} = 2 \pi r d \tau_{ad} \Delta T \).

The bending moment in the main reinforcement in the vertical plane \( M_s = X_2 \). At this time the dependence “\( \sigma_s \cdot \omega_s \)” and the Bernoulli hypothesis are true, \( M_s = X_2 = \sigma_s \omega_s z_s A_s \).
The torque moment in the main reinforcement in the perpendicular (horizontal) plane $M_{\text{hor}} = \sigma_4 \cdot e_4$. At this time the dependence $\sigma_2 \cdot e_2$ and the Bernoulli hypothesis are true, $M_{\text{hor}} = \sigma_4 \cdot e_4$.

The support reactions and support moments are equal to: $R_{A,\text{ver}} = -X_1 \cdot e_1$; $R_{B,\text{ver}} = X_1 \cdot e_1$; $R_{A,\text{hor}} = -X_5 \cdot e_5$; $R_{B,\text{hor}} = X_4 \cdot e_4$; $M_A = N_{s,1} \cdot e_{\text{ver}}$; $M_B = N_{s,2} \cdot e_{\text{ver}}$; $M_{A,t} = N_{s,1} \cdot e_{\text{hor}}$; $M_{B,t} = N_{s,2} \cdot e_{\text{hor}}$.

References
[1] Kolchunov V I, E.I.Zazdravnyh, 1996 Calculation model of the “nagel effect” in a reinforced concrete element. News of higher educational institutions, 10, pp 18-25
[2] Kolchunov V I, Dem’yansov A I, N.V. Naumov N V, 2019 Calculation of the stiffness of reinforced concrete structures under the action of torsion and bending. IOP Conf. Series: Journal of Physics, (2019), 1425, article 012077, pp 1-10
[3] Kolchunov V I, Dem’yansov A I, 2020. The second stage of the stress-strain state of reinforced concrete constructions under the action of torsion with bending (Theory). IOP Conf. Series: Materials Science and Engineering, 753, article 032056, pp 1-9
[4] Dem’yansov A I, Kolchunov V I, 2017, M.Mihailov, The calculation models of static and dynamic deformation reinforced concrete constructions in torsion with bending at the time of the spatial crack formation. Building and Reconstruction, 3, pp13-22
[5] Kolchunov V I, Dem’yansov A I, 2018. Bringing the experimental data of reinforced concrete structures crack resistance in correspondence with their theoretical values. Science and construction, 15(1), pp 42-49
[6] Suvorov A, 2018. An overview of the possibilities of nonlinear deformation model in analyzing the formation and development of the nagel effect. Urban Studies, 2, pp 70-75. DOI: 10.7256/2310-8673.2018.2.23385
[7] Kodysh E N, Nikitin I K, Trekin H H, 2010 Calculation of reinforced concrete structures made of heavy concrete for strength, crack resistance and deformations. Moscow: ASV, p 348
[8] Golyshnev A B 2009. The resistance of reinforced concrete. Kiev: Basis, p 432(2009)[In Russian]
[9] Vishnu H. Jariwalaa, Paresh V. 2013 Patel, Sharadkumar P. Purohit. Strengthening of RC Beams subjected to Combined Torsion and Bending with GFRP Composites, Procedia Engineering, 51, pp 282-289
[10] Adheena Thomas, Afia S. Hameed 2017 An Experimental Study on Combined Flexural and Torsional Behaviour of RC Beams, International Research Journal of Engineering and Technology, 04, iss. 05, pp 1367-70
[11] KhalidounRahal, 2007 Combined Torsion and Bending in Reinforced and Prestressed Concrete beams Using Simplified Method for Combined Stress-Resultants, ACI Structural Journal, 104, 4, pp 402-411
[12] Pettersen J S 2014 Non-Linear Finite Element Analyses of Reinforced Concrete with Large Scale Elements : Including a Case Study of a Structural Wall, Norwegian University of Science and Technology, p 85
[13] H.Nahvi H, M.Jabbari M 2005Crack detection in beams using experimental modal data and finite element model, International Journal of Mechanical Sciences, 47, pp 1477-97
[14] Dem’yansov A I, Kolchunov V I, The second stage of stressed-deformed condition of reinforced concrete structures when turning with bending (case 2), Construction Mechanics and Calculation of Structures, 4(285), pp 20-30
[15] Kolchunov V I, Dem’yansov A I The second stage of the stressed-deformed condition of reinforced concrete structures when of turning with bending (Case 2), International Journal for Computational Civil and Structural Engineering, 15(4), 66-82. DOI: 10.22337/2587-9618-2019-15-4-66-82