Research Article

Vibration Response of Timoshenko Beam-Foundation Interaction Model under Accelerated Moving Load

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Received 15 June 2022; Revised 20 July 2022; Accepted 28 July 2022; Published 17 August 2022

Academic Editor: Madalina Dumitriu

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Simulation of an infinite Timoshenko beam subjected to accelerated moving load rested on the finite depth is assessed in this study. Then, the dynamic response of the infinite beam is illustrated on the various theoretical models, including the Winkler, Pasternak, and visco-elastic foundations. Furthermore, the effects of various damping, such as foundation damping, beam damping, and hysteretic damping (damping between soil grains), are also studied on the dynamic behavior of the beam. It has been worked out that the type of basement and its depth have a remarkable effect on the dynamic behavior. In addition, the load velocity will also cause a maximum displacement in the beam as the critical velocity approaches. The deflection of the beam on the basements increases when the velocity approaches the critical one, and the maximum displacement of the beam occurs under the load. Finally, it was seen that the presented diagrams for all three types of Winkler’s, Pasternak’s, and visco-elastic foundations follow the usual properties related to the critical velocity.

1. Introduction

Frequently, the dynamic behavior of structures under moving loads has played a significant role in engineering fields. Therefore, it is an extraordinary process to choose the most noticeable information in the dynamic behavior of structures to which one researcher should pay attention. The structures that are subjected to moving loads are susceptible to severe problems “oscillation, upward and downward displacement” in comparison with other structures. Also, the moving load’s velocity is an essential issue for the stability of the systems. For example, in high-speed train transport, the track structure can be described as a continuous beam on a uniform, homogeneous basement with vertical springs, which resist beam deflection. The critical velocity can be described as the velocity which causes the maximum deflection of the rail. Trains traveling at speeds close to the critical velocity cause significant deflections in the rail system “which in turn causes the interaction between rail and train” leads to more severe breakdowns such as train derailment and rail failure due to fatigue. In the analysis of the dynamic behavior of a structure, the excitation applied from the basement to the structure for the case where the structure relies on a rigid base and bedrock is the same excitation that existed before the construction of the structure at that point, as if the structure relies on soft soil, significant changes will occur in the seismic input of the structure. Therefore, the structure interacts with the surrounding soil and will cause changes in the movements of the structure. The point here is that the structure or basement may be modeled as elastic or visco-elastic or a semi-infinite system, which can even be nonlinear. The initial investigation of elastic base was presented by Timoshenko [1]. His study is about the behavior of the beam subjected to the moving load. The dynamic behavior of the Timoshenko beam subjected to the moving load on the Pasternak foundation was presented by Kargarnovin and Younesian [2]. The solutions for free vibration and the bending response of the beams on the Winkler and Pasternak foundations were provided by Ying et al. [3]. The spectral analysis of the beam is recommended by Gladysz and Sniady [4]. Moreover, by applying the differential transform method,
Balkaya et al. provided the dynamic response of the Timoshenko and Euler-Bernoulli beams on soil [5]. Motaghand et al. studied the complication of frequency analysis of beams on the Winkler foundation [6]. Also, the nonlinear responses of the clamped Euler-Bernoulli beams subjected to axial forces were concluded by Barari et al. [7]. The dynamic response of a composite beam subjected to a moving oscillatory mass is presented by Kargarnovin et al. [8]. The dynamic response of a non-uniform Timoshenko beam subjected to a moving mass [14]. Dimitrovová studied the dynamic behavior of the Euler-Bernoulli beam on a Winkler foundation for the critical velocity of the moving load [15]. Ghannadiasl and Rezaei Dolagh presented the dynamic response of the Euler-Bernoulli beam on a finite depth bases under a moving load [16, 17]. The influence of soil and structure interaction was also presented by them. Guerdouh and Khalfallah investigated the effects of soil properties on the seismic performance of structures [18]. Beskou et al. utilized the dynamic response of the elastic half-plane under a moving load [19]. For utility, (2) can be simplified into a differential equation of the Timoshenko beam with the rectangular section shown in Figure 1 is expressed by Li et al. [30] as below:

\[ EI \theta_{xx} + \kappa GA (w_x - \theta) - \gamma y_{tt} = 0, \]

\[ \kappa GA (w_x - \theta_x) - c_1 w_t - mw_{tt} - Nw_{xx} + \kappa GA P = \rho \delta (x - X_F (t)), \]  

where \( w \) and \( \theta \) indicates the transverse displacement and the rotation angle, respectively; \( EI, \kappa GA \) are the bending and the shear stiffness modules; \( \gamma \) is the rotational inertia \( \gamma = \rho I \); \( N \) is the axial force, \( m \) denotes the mass per unit length of the beam, \( P \) is the foundation pressure which will be replaced later, \( P \) and \( v \) are the moving load and its velocity. Moreover, \( c_1 \) is the beam damping coefficient and \( \delta \) shows the Dirac delta function. In this paper, the trajectory of the moving load equation \( X_F (t) \) is defined as below:

\[ X_F (t) = v_0 t + \frac{1}{2} a_0 t^2. \]

For utility, (2) can be simplified into a differential equation as follow:

\[ \theta_x = \frac{c_1}{kGA} - \frac{m}{kGA} w_x - \frac{N}{kGA} w_{xx} + P \]

[4]

Differentiating (1) with respect to \( x \) and using (4), we can obtain

\[ EI \theta_{xxx} + \kappa GA (w_{xx} - \theta_x) - \gamma y_{tt} = 0. \]  

According to (5), the \( \theta_{xx} \) and \( \theta_{tt} \) are calculated, and using (5), we can achieve
\[
\begin{align*}
&\left( EI - \frac{NEI}{\kappa GA} \right) w_{xxxx} + \frac{c_1 EI}{\kappa GA} w_{xxxt} + \left( \frac{\gamma N}{\kappa GA} - \frac{m EI}{\kappa GA} \right) w_{xxtx} + c_1 w_{xt} + mw_{xt} + N w_{xx} + \frac{c_1}{\kappa GA} w_{ttt} + \frac{ym}{\kappa GA} w_{tttt} \\
&= \frac{EI}{\kappa GA} \left( P_{xx} \delta (x - X_F(t)) + P \delta_{xx} (x - X_F(t)) \right) - \frac{y}{\kappa GA} \left( P_{xx} \delta (x - X_F(t)) + P \delta_{xx} (x - X_F(t)) \right) - P \delta (x - X_F(t)).
\end{align*}
\]

By converting the equations to the moving system \( s = x - X_F(t) \), (6) can be rewritten as below:

\[
\begin{align*}
&\left( EI - \frac{NEI}{\kappa GA} \right) w_{ssss} + (v + at) \frac{c_1 EI}{\kappa GA} w_{ssst} + (v + at)^2 \left( \frac{\gamma N}{\kappa GA} - \frac{m EI}{\kappa GA} \right) w_{sst} \\
&\quad - (v + at)c_1 w_{st} + (v + at)^2 mw_{st} + N w_{st} - (v + at)^3 \frac{c_1}{\kappa GA} w_{tst} + (v + at)^4 \frac{ym}{\kappa GA} w_{tttt} + p_s \\
&= \frac{EI}{\kappa GA} \left( P_{ss} \delta (s) + P \delta_{ss} (s) \right) - \frac{y}{\kappa GA} \left( (v + at)^2 P_{ss} \delta (s) + (v + at)^2 P \delta_{ss} (s) + a \delta \delta_{ss} (s) \right) - P \delta (s).
\end{align*}
\]

Also, in the vertical direction the dynamic equilibrium of the foundation is defined as below [15]:

\[
\frac{\partial^2 u}{\partial t^2} + c_f \frac{\partial u}{\partial t} = k_s H \frac{\partial^2 u}{\partial x^2} + G_s \frac{\partial^2 u}{\partial x^2},
\]

where \( c_f \) denotes the foundation viscous damping coefficient, \( u_s \) shows the vertical soil displacement, that is used to represent the effect of foundation damping correctly, \( \overline{p} \) presents the density of the soil, \( k_s \) is the stiffness, \( H \) depicts the soil depth, and \( G_s (\partial^2 u/\partial x^2) \) is used for the shear effect.

By changing coordinate to moving coordinate, we get the following equation:

\[
\overline{p}(v + at)^2 \frac{\partial^2 u}{\partial s^2} - c_f (v + at) \frac{\partial u}{\partial s} = k_s H \frac{\partial^2 u}{\partial x^2} + G_s \frac{\partial^2 u}{\partial x^2}.
\]

The boundary conditions are satisfied by the relative displacement, which makes the resolvability easier, and can be defined as follows:

\[
z = \zeta H, u = u_s (1 - \zeta), u_s (s, H, t) = 0.
\]

Moreover, by considering \( \chi = \sqrt{k_s / 4EI} \) “the moving coordinate can be transferred to the dimensionless coordinate \( \xi = \chi s \)” and dividing the equation by the static displacement \( w_{st} = \overline{p} \Delta / 2k_s \), we arrive at

\[
\frac{\partial^2 \hat{u}}{\partial \xi^2} - \frac{\partial \hat{u}}{\partial \xi} + \frac{\eta_f}{\alpha^2} \frac{\partial^2 \hat{u}}{\partial \xi^2} - \frac{1}{\alpha^2} \frac{\partial^2 \hat{u}}{\partial \xi^2} = 0,
\]

where \( \eta_f = c_f H / \sqrt{k_s m}, \eta_s = v_s / v_{cr} \) shows the shear coefficient. The term \( v_s \) is the velocity of shear wave, \( \mu \) is the mass ratio, \( \mu = \sqrt{\overline{p} H / m} \), and \( \alpha = (v_{ox} + a v_t) / v_{cr} \) is the velocity ratio. The critical velocity of the Timoshenko beam \( v_{cr} \) is calculated by Dimitrová [15] as follows [31]:

\[
v_{cr} = \sqrt{k (EI \Delta - 2 (r G A)^2) + 2 G A (\sqrt{kr^2 G A} - k EI G A) \Delta^2 / m \Delta^2},
\]

where \( \Delta = k r^2 - G A, r, A, \) and \( G \) refer to the radius of gyration of the beam cross-section, the reduced cross-sectional area, and the shear modulus of the beam, respectively.
The following relation can be considered under the homogeneous conditions:

\[
\tilde{u}_r = \sum_{j=1}^{\infty} U_j \sin(j \pi \zeta). \tag{13}
\]

Therefore, multiplication with one mode shape, substitution and integration from 0 to 1 depth, and using Fourier transform, we arrive at

\[
U_j^* = \frac{\omega^2 2j \pi (1 - (\zeta_j / \alpha)^2)}{-\omega^2 (1 - (\zeta_j / \alpha)^2) - i \omega \xi_j / \alpha} + (j \pi / \alpha^2) W^* \tag{14}
\]

\[
\left( EI - \frac{NEI}{kGA} + (v + at)^2 \left( \frac{\gamma N}{kGA} - y - \frac{mEI}{kGA} \right) \right) \omega_{,ssss} + \left( N + (v + at)^2 \right) m \omega_{,ss} - (v + at) \omega_{,s} - (1 - i \eta_h \kappa) \left( \sum_{j=1}^{\infty} j \pi u_j - \omega \right)
\]

\[
\left. \frac{EI}{kGA} \left( P_{,s} \delta(s) + P_{,s} \delta(s) \right) - \frac{\gamma}{kGA} \left. \left. \left( v + at \right)^2 s \omega_{,ss} + (v + at)^2 s \omega_{,s} + a \omega \right) \right) \right) - P \delta(s). \tag{16}
\]

Changing to dimensionless condition, \( N = 2\eta_N \sqrt{k_{st}EI} \), \( (v + at) = av_{cr} \), and \( c_1 = 2\eta_h \sqrt{mk_{st}} \), (17) is obtained. Furthermore, we have

\[
\bar{w}_{\xi\xi} = \frac{8}{k_{st} A_1} a v_{cr} A_2 \left( \eta_h \sqrt{mk_{st}} \right) \left( \frac{k_{st}}{4EI} \right)^3 \bar{w}_{\xi\xi} + \left( 2 \sqrt{k_{st}EI} \eta_N + \left( av_{cr} \right)^2 m \right) \frac{4}{k_{st} A_1} \sqrt{k_{st} / 4EI} \bar{w}_{\xi\xi}
\]

\[
-v_{cr} \left( \eta_h \sqrt{mk_{st}} \right) \frac{8}{k_{st} A_1} \left( \frac{k_{st}}{4EI} \right) \bar{w}_{\xi} + \frac{4}{A_1} (1 - i \eta_h) \left( \bar{w} - \sum_{j=1}^{\infty} j \pi U_j \right)
\]

\[
\frac{1}{P} A_1 \frac{8}{A_1} \left( \frac{k_{st}}{4EI} \right)^2 \left( \frac{P_{,s} \delta(\xi) + P_{,s} \delta(\xi)}{P_{,s} \delta(\xi)} - \frac{8}{A_1} \frac{\gamma}{kGA} \frac{\kappa_{st}}{4EI} \delta(\xi) \right) - \frac{8}{A_1} \delta(\xi), \tag{17}
\]

where \( A_1 \) and \( A_2 \) are clarified as below:

\[
A_1 = \left( 1 - \frac{N}{kGA} + \frac{(v + at)^2}{EI} \left( \frac{\gamma N}{kGA} - y - \frac{mEI}{kGA} \right) + \frac{(v + at)^4}{EI} \right) \frac{\gamma}{kGA},
\]

\[
A_2 = \left( \frac{EI}{kGA} - (v + at)^2 \frac{\gamma}{kGA} \right). \tag{18}
\]

By the Fourier transform, we have

\[
W^* = \frac{-\omega^2 A_2 (8/A_1) \chi^2 \left( -\sqrt{2 \pi} \omega \text{Sign}[\omega] \right) + i (8/A_1) \left( \gamma / kGA \right) \alpha \chi (\sqrt{\pi / 2} \text{Sign}[\omega]) - (8/A_1)}{\lambda + (4/A_1) (1 - i \eta_h) S} \tag{19}
\]

where \( \text{Sign}[\omega] \) gives \(-1, 0, 1\) depending on whether \( [\omega] \) is negative, zero, or positive. The expressions \( \lambda \) and \( S \) are defined as below:
1. **Numerical Examples**

In order to verify the present study with previous research, the Timoshenko beam subjected to a moving load is considered with the particular values provided in Table 1. The deflection shape of the beam is shown in Figure 1 for the various load velocities.

From Figure 2, it is seen that the greater the velocity, the greater the displacement of the beam. For instance, in the diagrams shown, when the load velocity value is 1, the relative displacement occurs in a smaller area of the beam. However, as its value increases to 1.5, the displacement affects the longer length of the beam.

### Table 1: Numerical data.

| Parameter | Value       |
|-----------|-------------|
| Young's modulus (E) | 206910 (MPa) |
| Force (P) | 144.6 (kN) |
| Shear modulus (G) | 79580.77 (MPa) |
| Poisson's ratio (ν) | 0.3 |
| Cross-sectional area (A) | 86.13 (cm²) |
| Density (ρ) | 7.82 (g/cm³) |
| Shear coefficient (k) | 0.41 |
| Moment inertia (I) | 3950 (cm⁴) |
| Radius of gyration (r) | 0.0677 (m) |
| Mass per unit length (m) | 59.9352 (kg/m) |
| Soil stiffness (k) | 20 (MPa) |

![Figure 2: Velocity effect on the deflection shape of the beam.](image)

\[
\lambda = \omega^4 - i\omega^3 \left( \frac{8}{k_{st} A_1} A_2 \left( \eta_b \sqrt{m k_{st}} \right) \eta_c \right) - i\omega^2 \left( 2 \sqrt{k_{st} E I} \eta_c + \left( \alpha v_T \right)^2 \right) \left( \frac{4}{k_{st} A_1} \right)^2 (\xi_j \sum_{j=1}^{N} \eta_j - j\pi)^2, \tag{20}
\]

### 3.1. The Influence of Foundation Type on the Dynamic Response of the Beam

A beam with the features of Table 2 is considered for this purpose. By solving the governing equation of the beam, the displacement of the Timoshenko beam for the three types of foundations, including the classical Winkler’s (η₆ = 0 and μ = 0), Pasternak’s (μ = 0), and visco-elastic (η₆ = 0.529) is investigated and shown in Figure 2. From Figure 3, it is seen that the type of foundation affects the displacement of the beam remarkably, which also changes the dynamic response of the beam and provides a different behavior to the type of foundation. Like the Euler-Bernoulli beam, the displacement of the Timoshenko beam on the Winkler foundation is more remarkable than both
Pasternak's and visco-elastic foundations due to the absence of shear wave propagation and the shear ratio.

3.2. The Dynamic Behavior of the Timoshenko and Euler-Bernoulli Beams on Various Foundations

3.2.1. Classical Winkler’s Foundation. A beam with the specifications of Table 1 for the Euler-Bernoulli beam and Table 2 for the Timoshenko one is considered. By assuming \( \nu = 0 \) and \( \mu = 0 \), the foundation is converted to the Winkler’s one. The displacement of the two beams on the classical Winkler’s foundation is obtained and provided in Figure 4. From Figure 4, it is seen that the displacement of the Timoshenko beam is close to the Euler-Bernoulli one.

3.2.2. Pasternak’s Foundation. For this purpose, a beam with the data of Table 1 for the Euler-Bernoulli beam and Table 2 for the Timoshenko one is considered. Assuming \( \mu = 0 \), the foundation is converted to the Pasternak’s one. The displacement of the two beams on this foundation is compared and provided in Figure 5. By comparing the diagrams obtained in Figure 5, it can be seen that, like the Winkler’s foundation, the displacement of the two beams on the
Pasternak’s foundation is not noticeable. The behavior of both beams is almost close to each other.

3.2.3. Visco-Elastic Foundation. To model and compare the two theories on the visco-elastic foundation, the numerical input data of Table 1 for the Euler-Bernoulli beam and Table 2 for the Timoshenko beam are summarized. Defining \( \eta_h = 0.529 \) (visco-elastic foundation), the behavior of the Euler-Bernoulli and the Timoshenko beams is compared and shown in Figure 6. From Figure 6, it can be seen that, like the previous two foundations, i.e., Winkler’s and Pasternak’s, the behavior of the two beams on the visco-elastic foundation is not significant. Also, the displacement of both beams is the same as in the previous cases and they are almost close to each other. As a result, comparing the behavior of the Euler-Bernoulli and Timoshenko beams, it is observed that the displacement of both beams on the Winkler’s foundation is greater than the other two ones.

Then, the maximum downward and upward displacement of the diagrams obtained in Figures 4–6 in the two foundations of Winkler’s and Pasternak’s for the critical velocity of \( v_{E-B}^{cr} = 497 \text{ (m/s)} \) and \( v_{T}^{cr} = 497.336 \text{ (m/s)} \), and the
A load velocity of 323 m/s is obtained and compared in Figure 7. To show the influence of critical velocity on the maximum displacement of the beam, the load velocity is reduced to 165 m/s. Again, the maximum downward and upward displacement for both theories on the Winkler’s foundation is provided in Figure 8.

Comparing the diagrams in Figures 7 and 8 for the two types of foundations, Winkler’s and Pasternak’s, it can be seen that in both Euler-Bernoulli and Timoshenko theories, as getting closer to the critical velocity —497 m/s for the Euler-Bernoulli beam and 497.336 m/s for the Timoshenko beam “the displacement of both beams increases in both directions.” Also, by reducing the load velocity from 323 meters per second to 165 meters per second, the maximum downward and upward displacement in both beams decreased.
4. Conclusion

The dynamic behavior of Timoshenko and Euler-Bernoulli beams with infinite length under accelerated moving load on a foundation with finite depth was investigated and compared in this paper. In the present study, merely the dynamic equilibrium in the vertical direction is considered. The governing equations of the beam on the foundation, including Winkler’s, Pasternak’s, and visco-elastic, were calculated and presented by considering the effects of beam damping, foundation damping, and hysteretic damping. Considering the interaction of the soil and the governing equations, the physics of the problem is modeled with mathematical equations. Finally, the governing differential equations are converted to algebraic equations using the Fourier transform method. Then, the proposed model is validated, and using the considered method, the dynamic response of the beam on the soil bases is identified. In this paper, considering the shear influence in a simplified form, the beam deflection shape is obtained for various velocities of the applied moving load. Also, the effect of different soil depths on the response of the Timoshenko beam is investigated. It was found that the dynamic behavior of the beam changes dramatically based on the type of foundation and its depth. Furthermore, the displacement obtained for each foundation increases with increasing load speed and approaching the critical state. The most significant displacement occurs just below the applied load. On the other hand, in the low mass ratio, it was also observed that the critical velocity approaches the classical equation.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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