Comment on the energy dependence of the slope parameter

S. M. Troshin and N. E. Tyurin

NRC “Kurchatov Institute”-IHEP - Protvino, 142281, Russian Federation

received 27 September 2018; accepted in final form 22 October 2018
published online 22 November 2018

PACS 13.85.Dz - Elastic scattering
PACS 12.40.Jn – Regge theory, duality, absorptive/optical models

Abstract – We discuss the energy dependence of the slope parameter in elastic proton scattering. It is shown that unitarity generates the energy dependence of the slope parameter in geometrical models consistent with the experimental results including recent LHC data.

Introduction. – A speeding up of the energy increase of the slope parameter in elastic proton scattering is among the important discoveries performed at the LHC [1,2]. It has been found that the rate of the slope parameter increase gets larger with the energy growth compared to the rate at lower energies. This means a speedup of the interaction radius energy dependence in a transverse plane. The reason for this is directly connected to hadron interaction dynamics in the soft region and makes the studies in that direction important.

It is essential to take into account that hadrons are composite, extended objects and have form factors described by nontrivial functions. Their geometrical radii, contrary to the interaction ones, are energy independent, and determined by the minimal mass of the exchanged quanta responsible for the scattering [3]. The generation of the energy-dependent interaction radius is due to the unitarity condition in the direct reaction channel. Account of unitarity is performed by the unitarization of an input amplitude. This is a way to construct a final scattering amplitude obeying unitarity.

The need for unitarization has become evident at the time when the total cross-sections rise has been discovered. To reconcile the Regge model predictions to the experimental data one should introduce a pomeron pole contribution with the intercept $\alpha'(0)$ greater than unity. Such contribution would finally violate unitarity and, therefore, requires unitarization. The input amplitude of the Regge model with linear trajectory $\sim (s/s_0)^{\alpha(t)}$, however, includes diffraction cone shrinkage \textit{ab initio}, i.e., the slope parameter $B(s)$ logarithmically increases with the energy, $B(s) \sim \alpha'(0) \ln(s/s_0)$, $\alpha'(0) \neq 0$, while unitarity requires its double logarithmic asymptotic growth, $B(s) \sim \ln^2(s/s_0)$ if the total cross-section saturates the Froissart-Martin bound. To bring the slope parameter energy increase to requirements of unitarity bounds, one can assume that the slope of the pomeron trajectory $\alpha'(0)$ is an energy-dependent “effective” function (cf. [4]).

For the case of the input amplitude assumed by the geometrical models, there is a second reason for unitarization. The input amplitude itself does not imply growth of the slope parameter with energy in geometrical models. Only unitarization generates energy dependence of the slope parameter. This parameter grows with energy at its low and moderate values, where total cross-section does not increase. Unitarization makes the energy dependence of the slope parameter consistent with the experimental trend at such energies.

In this paper the origin of the $B(s)$ growth due to unitarity is discussed. We consider a class of the geometrical models operating with the amplitudes in the impact parameter representation (cf. for definition [5]).

Geometrical models and the slope parameter. – In these models an input amplitude which is a subject for subsequent unitarization is taken in a factorized form. The diffraction cone slope $B^0$ corresponding to such input amplitude does not depend on energy. It is determined by the geometrical radii of the colliding hadrons in the transverse plane. The geometrical radius of a hadron in its turn is determined by a minimal mass of the exchanged quanta responsible for the scattering [3]. The energy dependence of the actual slope $B(s)$ is generated then through the unitarization.

The studies of the geometrical properties of hadron interactions are important [6] for understanding the hadron dynamics ultimately related to the nonperturbative sector of QCD. Under this the unitarity leads to the energy...
dependence of the hadron interaction region in the transverse plane which initially has purely geometrical meaning. A physical interpretation of such mechanism can be found, e.g., in [7]. Thus, the final interaction radius appears to be energy dependent and so is the quantity

\[ B(s) = \frac{d}{dt} \ln \frac{d\sigma}{dt} |_{t=\omega}. \]

In geometrical models an input amplitude is commonly taken as an overlap of the matter distributions of the colliding hadrons \( D_1 \otimes D_2 \) following the pioneering paper by Chou and Yang [8]. It should be noted that the factorization results also from the tower diagrams calculations in the electrodynamics [9].

We suppose that the real part of the elastic scattering amplitude is vanishingly small and can be neglected since the high-energy experimental data are consistent with the pure imaginary amplitude. We discuss the slope of the diffraction cone, \( B(s) \), which is determined by the mean value of the impact parameter squared \( b^2 \),

\[ \langle b^2 \rangle_{\text{tot}} = \frac{\int_0^\infty db \int_0^\infty dbf(s, b)}{\int_0^\infty dbf(s, b)}. \]  

(1)

Account for unitarity is performed by the schemes which provide an output amplitude \( f(s, b) \) limited by the unitarity limit \( f = 1 \) or by the black-disc limiting value of 1/2 [10]. Mechanisms generating the diffraction cone slope increase with energy are similar for the different approaches. Equation (1) and the unitarity

\[ \text{Im} F(s, t) = H_{el}(s, t) + H_{inel}(s, t), \]

where \( F(s, t) \) is the elastic scattering amplitude and \( H_{el, inel}(s, t) \) are the elastic and inelastic overlap functions, lead to the representation of \( B(s) \) as a sum of elastic and inelastic contributions, i.e.,

\[ B(s) = B_{el}(s) + B_{inel}(s). \]

As was said above, we assume that the real part of the scattering amplitude can be neglected in the qualitative consideration as well as possible spin flip amplitudes. Then the amplitude \( F(s, t) \) can be taken as a square root of the differential cross-section (with account for the relevant normalization factors). The functions \( H_{el, inel} \) can be obtained on the basis of the impact parameter analysis (cf., e.g., [11]) after Fourier-Bessel transformation.

The real part of the scattering amplitude cannot be directly extracted from the experimental data, but it has a small value at small values of the momentum transferred. Its possible restoration is discussed in [12].

Here (cf. [13]),

\[ B_{el, inel}(s) \sim \sigma_{el, inel} / \sigma_{tot}(s) \langle b^2 \rangle_{el, inel}(s). \]

Thus, the energy dependence of \( B(s) \) is determined by the cross-sections \( \sigma_{el, inel} \) and average values \( \langle b^2 \rangle_{el, inel}(s) \).

Averaging is going over corresponding overlap functions. The phase dependence is neglected, due to its unknownness. Different unitarization schemes provide different asymptotics for \( B_{el, inel}(s) \).

First, we consider the unitarization which incorporates the two scattering modes at high energy, absorptive and reflective ones, and assumes saturation of the unitarity limit by the partial amplitude at \( s \to \infty \) [14]. We discuss asymptotic energy dependences of \( B_{el, inel}(s) \).

The \( U \)-matrix [15] unitarization scheme incorporates both modes and the relation between the scattering amplitude \( f \) and the input amplitude \( u \) has rational form:

\[ f(s, b) = u(s, b) / [1 + u(s, b)], \]  

(2)

where \( u \) is a non-negative function.

In the geometrical models the \( u(s, b) \) has a factorized form:

\[ u(s, b) = g(s)\omega(b), \]  

(3)

where \( g(s) \sim s^\lambda \) at large values of \( s \) and \( \omega(b) \) exponentially decreases of at \( b \to \infty \). The power dependence on energy guarantees unitarity saturation \( f \to 1 \) at fixed \( b \) and the respective asymptotic growth of the total cross-section \( \sigma_{tot} \sim \ln^2 s \). Such forms of these functions can be justified by theoretical calculations based on massive quantum electrodynamics [16]. That factorized form and eq. (2) along with the function \( \omega(b) \) chosen to meet the analytical properties of the scattering amplitude lead to the following asymptotic dependences:

\[ B_{el}(s) \sim \ln^2 s \]  

(4)

and

\[ B_{inel}(s) \sim \ln s, \]  

(5)

since \( \langle b^2 \rangle_{el, inel}(s) \sim \ln^2 s \) and \( \sigma_{el, tot}(s) \sim \ln^2 s \), while \( \sigma_{inel}(s) \sim \ln s \) at \( s \to \infty \). If one applies those asymptotic dependences at available energies\(^1\), one should then interpret an observed speedup of the \( B(s) \) growth from \( \ln s \) to \( \ln^2 s \) as a transition between the two contributions into the slope.

In the framework of the geometrical considerations a typical way to construct the function \( \omega(b) \) as was already noted is to represent it as a convolution of the two matter distributions in a transverse plane as was proposed by Chou and Yang [8]:

\[ \omega(b) \sim D_1 \otimes D_2 \equiv \int D_1(b_1)D_2(b - b_1). \]  

(6)

This function can also be constructed by taking into account the hadron quark structure [17]. Thus, the following form was adopted in eq. (3):

\[ \omega(b) \sim \exp(-\mu b). \]  

(7)

\(^1\)It is a common practice, e.g., the Regge model is based on asymptotic dependence of Legendre polynomials, but its results are widely used at modern energies.
The value of the energy-independent parameter $\mu$ is determined by a particular chosen model, it can be assumed that $\mu = 2m_\pi$ based on the notion of hadrons’ peripheral pion cloud.

Thus, the slope parameter $B(s)$ has leading energy dependence at $s \to \infty$,

$$B(s) \sim \ln^2 s,$$

(8)

which is related to the elastic contribution and can be considered in favor of the reflective scattering mode. Absorption provides here a subleading contribution in the form of eq. (5).

On the other hand, there is no way to discriminate elastic and inelastic contributions into $B(s)$ at $s \to \infty$ when only the absorptive scattering mode is assumed. In this mode $f \to 1/2$ at $s \to \infty$ and $b$ fixed. In this case both contributions $B_{el}(s)$ and $B_{inel}(s)$ are proportional to $\ln^2 s$ at $s \to \infty$ and have similar energy dependences at finite energies. It is a typical situation with $B(s)$ behavior in the unitarization schemes based on eikonal or continued unitarity [10]. At the present time only the impact parameter analysis can help to discriminate different unitarization schemes. Indeed, much higher energies are needed for such a discrimination under that study of the integral observables.

Rigorously speaking, what said above is relevant for the asymptotic energy region. Despite the strong temptation to use these results at the available energies, it should be realized that the experimental data do not belong to this region. A conclusion on the indication of the reflective scattering mode appearance would be legitimate at higher energy values since we are dealing with integrated quantities without information on the distribution of the interaction probabilities over the impact parameter. At the moment only at the highest LHC energy the function $u(s,b=0)$ approaches unity or has a little bit higher value. Below such energies one can expand the scattering amplitude $f$ over the function $u$ according to eq. (2). Keeping only the first and second terms in this expansion, the slope parameter $B(s)$ can be approximated in geometrical models at small and moderate energies (i.e., in the preasymptotic energy region) in the form

$$B(s) \approx a + bs^\lambda.$$

(9)

The comparison of eq. (9) with the experimental data (see fig. 1) allows one to get a reasonable value for the $\chi^2/ndf$ around 0.8 and the value of a parameter $\lambda$ to be at $\lambda \simeq 0.1$.

Adhering to the geometrical models, we should conclude also that the observed total cross-section increase at the accelerator energies is of a preasymptotic nature. The recently measured [18] small value of the real to imaginary parts ratio of the elastic scattering amplitude can be interpreted as a slowing-down of total cross-section increase due to the beginning of the transition to the energies where the effects of unitarization start to be important.

***

We are grateful to the referees for the valuable remarks and comments.

REFERENCES

[1] The TOTEM Collaboration (G. Antchev et al.), hep-ex/1712.06153 preprint (2017).
[2] ATLAS Collaboration (Aaboud M. et al.), Phys. Lett. B, 761 (2016) 158.
[3] Yukawa H., Proc. Phys. Math. Soc. Jpn., 17 (1935) 48.
[4] Schegelsky V. A. and Ryskin M. G., Phys. Rev. D, 85 (2012) 094024.
[5] Henyey F. S., Phys. Lett. B, 45 (1973) 363.
[6] Blankenbecler R., Goldberger M. L., Phys. Rev., 126 (1962) 766.
[7] Stodolsky L., SLAC-PUB-0864 preprint (1971).
[8] Chou T. T., Yang C. N., Phys. Rev. Lett., 20 (1968) 1213.
[9] Bourrely C., Soffer J., Wu T. T., Phys. Rev. D, 19 (1979) 3249.
[10] Glushko N. I., Kobylnsky N. A., Martynov E. S., Shelest V. P., Sov. J. Nucl. Phys., 38 (1983) 106.
[11] Alkin A., Martynov E., Kovalenko O.,Troshin S.M., Phys. Rev. D, 89 (2014) 091501(R).
[12] Troshin S.M., Tyurin N.E., Mod. Phys. Lett. A, 32 (2017) 1750028.
[13] Ajduk Z., Nuovo Cimento A, 15 (1973) 390.
[14] Troshin S. M., Tyurin N. E., Int. J. Mod. Phys. A, 22 (2007) 4437.
[15] Savrin V. I., Tyurin N. E., Khristalev O. A., Part. Nucl., 7 (1976) 21.
[16] Cheng H., Wu T. T., Phys. Rev. Lett., 24 (1970) 1456.
[17] Troshin S. M., Tyurin N. E., Phys. Rev. D, 49 (1994) 4427.
[18] The TOTEM Collaboration (Antchev G. et al.), CERN-EP-2017-335.