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Granular universe: statistical origin of special and doubly special relativity

Petr Jizba† and Fabio Scardigli‡
† FNSPE, Czech Technical University in Prague, Břehová 7, 115 19 Praha 1, Czech Republic
‡ Dipartimento di Matematica, Politecnico di Milano, Piazza Leonardo da Vinci 32, 20133 Milano, Italy
E-mail: p.jizba@fjfi.cvut.cz, fabio@phys.ntu.edu.tw

Abstract. We present a new theoretical evidence that a relativistically invariant quantum dynamics at large enough space-time scales can be derived from two inter-correlated genuinely non-relativistic stochastic processes that operate at different energy scales. This leads to Feynman amplitudes that are, in the Euclidean regime, identical to transition probability of a Brownian particle propagating through a granular space. Our observation implies a preferred frame and can have distinct experimental signatures. Ensuing implications for special and doubly-special relativity, quantum field theory, quantum gravity and cosmology are discussed.

1. Introduction

In recent years, the notion of emergence have been of vital importance to quantum field theory community for many reasons. Especially, in particle and condensed matter physics it embodies the essential feature of systems with several inter-correlated time or energy scales. In these systems, the observed macroscopic-scale dynamics and related degrees of freedom differ drastically from the actual underlying microscopic-scale physics [1]. Superstatistics provides an explicit realization of this paradigm: It predicts that the emergent behavior can be often regarded as a superposition of several statistical systems that operate at different spatio-temporal scales [2, 3]. In particular, many applications have recently been reported, in hydrodynamic turbulence [4], turbulence in quantum liquids [5], pattern forming systems [6] or scattering processes in high-energy physics [7].

The key conceptual premiss of the superstatistics is the existence of sufficient spatio-temporal scale separations between relevant dynamics, so that the system has enough time to relax to a local equilibrium state and to stay within it for some time. In practical applications we are typically concerned with two scales. Following Ref. [2], we consider an intensive parameter $\nu$ that fluctuates on a much larger time scale than the typical relaxation time of the local dynamics. The random variable $\nu$ can in practice identified, e.g., with the inverse temperature [2, 3], coupling constant [2], friction constant [8], volatility [9] or einbein [10]. On an intuitive ground, one may understand the superstatistics by using the adiabatic Ansatz. Namely, the system under consideration, during its evolution, travels within its state space $\mathcal{X}$ (described by state variable $a \in \mathcal{X}$) which is partitioned into small cells characterized by a sharp value of $\nu$. Within each cell, the system is described by the conditional distribution $p(a|\nu)$. As $\nu$ varies adiabatically from cell to cell, the joint distribution of finding the system with a sharp value of $\nu$ in the
state $A$ is $p(a, v) = p(a|v)p(v)$ (Bayes theorem). The resulting macro-scale (or emergent) statistical distribution $p(a)$ for finding a system in the state $a$ is obtained by marginalizing over the distribution of the variable being discarded, i.e.,

$$p(a) = \int \, dv \, p(a|v)p(v). \quad (1)$$

In mathematical language is $p(a)$ known as the marginal distribution. From technical point of view, the sufficient time scale separation between two relevant dynamics in a studied system allows to qualify superstatistics as a form of slow modulation [11].

In this paper, we recast the Feynman transition amplitude of a relativistic scalar particle into a form, which (after being analytically continued to imaginary times) coincides with a superstatistics marginal probability (1). The derivation is based on the Lévy–Khinchine theorem for infinitely divisible distributions [13, 12], and for illustration we consider the dynamics and the propagator of a Klein–Gordon (i.e., neutral spin–0) particle. Our reasonings can be also extended to charged spin–0, spin–$\frac{1}{2}$, Proca’s spin–1 particles and to higher-spin particles phrased via the Bargmann–Wigner wave equation [10]. Further generalization to external electromagnetic potential has been reported in Refs. [10, 14]. We further demonstrate that the above formulation can be looked at as if the particle would randomly propagate (in the sense of Brownian motion) through inhomogeneous or granular medium (“vacuum”) [14].

Our argument is based upon a recent observation [9, 10, 14] that the Euclidean path integral (PI) for relativistic particles may be interpreted as describing a doubly-stochastic process that operates at two separate spatio-temporal scales. The short spatial scale, which is much smaller than particle’s Compton length $\lambda_C = 1/mc$ ($h = 1$ is assumed throughout this paper), describes a Wiener (i.e., Galilean relativity) process with a sharp Newtonian mass. The large spatial scale, which is of order $\lambda_C$, corresponds to distances over which the fluctuating Newtonian mass changes appreciably. At scales much larger than $\lambda_C$ the particle evolves according to a genuine relativistic dynamics, with a sharp value of the mass coinciding with the Einstein rest mass. Particularly striking is the fact that when we average the particle’s velocity over the structural correlation distance (i.e., over particle’s $\lambda_C$) we obtain the velocity of light $c$. So the picture that emerges from this analysis is that the particle (with a non-zero mass!) propagates over the correlation distance $\lambda_C$ with an average velocity $c$, while at larger distance scales (i.e., when a more coarse grained view is taken) the particle propagates as a relativistic particle with a sharp mass and an average velocity that is subluminal. Quite remarkably, one can observe an identical behavior in the well-known Feynman’s checkerboard PI [16, 17] to which the transition amplitude (1) reduces in the case of a relativistic Dirac fermion in 1 + 1 dimensions [10, 14].

A considerably expanded presentation including the issue of reparametrization invariance, bibliography, and proofs of the main statements and formulas is presented in Ref. [14].

2. Superstatistics path integrals

When a conditional probability density function (PDF) is formulated through a PI, then it satisfies the Chapman–Kolmogorov equation (CKE) for continuous Markovian processes, namely [15]

$$P(x_b, t_b|x_a, t_a) = \int_{-\infty}^{\infty} \, dx \, P(x_b, t_b|x, t)P(x, t|x_a, t_a), \quad (2)$$

with $t$ being any time between $t_a$ and $t_b$. Conversely, any transition probability satisfying CKE possesses a PI representation [16]. In physics one often encounters probabilities formulated as a superposition of PI’s,

$$P(x_b, t_b|x_a, t_a) = \int_{0}^{\infty} \, dv \, \omega(v, t_b)P(x_b|x_a, t_b) \int_{x(t_a)=x_a}^{x(t_b)=x_b} Dx Dp \, e^{\int_{t_a}^{t_b} \, dv \, [i\hbar \frac{\partial}{\partial t} - V(p, x)]}. \quad (3)$$
Here $\omega(t, t_a)$ with $t_b = t_b - t_a$ is a normalized PDF defined on $\mathbb{R}^+ \times \mathbb{R}^+$. The form (3) typically appears in non-perturbative approximations to statistical partition functions, in polymer physics, in financial markets, in systems with reparametrization invariance, etc. The random variable $v$ is then related to the inverse temperature, coupling constant, volatility, vielbein, etc.

The existence of different time scales and the flow of the information from slow to fast degrees of freedom typically creates the irreversibility on the macroscopical level of the description. The corresponding information thus is not lost, but passes in a form incompatible with the Markovian description. The most general class of distributions $\omega(t, t_a)$ on $\mathbb{R}^+ \times \mathbb{R}^+$ for which the superposition of Markovian processes remain Markovian, i.e., when also $\hat{P}(x_b, t_b|x_a, t_a)$ satisfies the CKE (2), was found in Ref. [9]. The key is to note that in order to have (2) satisfied by $\hat{P}$, the rescaled PDF $w(v, t) \equiv \omega(v/t, t)/t$ should satisfy the CKE for homogeneous Markov process

$$w(v, t_1 + t_2) = \int_0^v dv' w(v', t_1)w(v - v', t_2).$$

Consequently the Laplace image fulfills the functional equation $\hat{w}(p_v, t_1 + t_2) = \hat{w}(p_v, t_2)\hat{w}(p_v, t_1)$, with $t_1, t_2 \in \mathbb{R}^+$. By assuming continuity in $t$, it follows that the multiplicative semigroup $\hat{w}(p_v, t)_{t \geq 0}$ satisfies $\hat{w}(p_v, t) = \{\hat{w}(p_v, 1)\}^t$. From this we see that the distribution of $v$ at $t$ is completely determined by the distribution of $v$ at $t = 1$. In addition, because $\hat{w}(p_v, 1) = \{\hat{w}(p_v, 1/n)\}^n$ for any $n \in \mathbb{N}^+$, $w(v, 1)$ is infinitely divisible. The Lévy–Khinchine theorem [13, 12] then ensures that $\log \hat{w}(p_v, t) \equiv -tF(p_v)$ must have the generic form

$$\log \hat{w}(p_v, t) = -t\left(ap_v + \int_0^\infty (1 - e^{-pu})\nu(du)\right),$$

where $a \geq 0$ is a drift constant and $\nu$ is some non-negative measure on $(0, \infty)$ satisfying $\int_{\mathbb{R}^+} \min(1, u)\nu(du) < \infty$. Finally the Laplace inverse of $\hat{w}(p_v, t)$ yields $\omega(v, t)$. Once $\omega(v, t)$ is found, then $\hat{P}(x_b, t_b|x_a, t_a)$ possesses a PI representation on its own. What is the form of the new Hamiltonian? To this end we rewrite (3) in Dirac operator form as [9]

$$\hat{P}(x_b, t_b|x_a, t_a) = \langle x_b | \int_0^\infty \! dv w(v, t_{ab})e^{-v\hat{H}} | x_a \rangle = \langle x_b | \{\hat{w}(\hat{H}, 1)\}^{t_{ab}} | x_a \rangle = \langle x_b | e^{-t_{ab}F(\hat{H})} | x_a \rangle.$$

From this we see that the identification $\hat{H}(p, x) = F(H(p, x))$ can be made. Here one might worry about the operator-ordering problem, not knowing in which temporal order $p$ and $x$ must be taken in $F$. For our purpose it suffices to note that when $H$ is $x$-independent, the former relation is exact. In more general instances the Weyl ordering is a natural choice since because in this case the required mid-point rule follows automatically and one does not need to invoke the gauge invariance [9, 29]. In situations when other non-trivial configuration space symmetries (such as non-holonomic symmetry) are required, other orderings might be more physical [9].

3. Statistical origin of relativistic quantum mechanics — granular universe

The Feynman transition amplitudes (or better its Euclidean version — transition probabilities) naturally fits into the structure of superstatistics PI’s discussed above.

Note first that the choice $a = 0$ and $\nu(du) = 1/(2\sqrt{\pi}u^{3/2})du$ leads to $F(p_v) = \sqrt{p_v}$. This identifies $w(v, t_{ab})$ with the (unshifted) Lévy distribution with the scale parameter $t_{ab}^2/2$. 

$$
Moreover, when \( H(p, x) = p^2 c^2 + m^2 c^4 \) then (3) can be cast into the form (cf. also Refs. [9, 10, 14])

\[
\int_{x(0)=x'}^{x(t)=x} Dx \frac{DP}{(2\pi)^D} \exp \left\{ \int_0^t d\tau \left[ ip \cdot \dot{x} - c \sqrt{p^2 + m^2 c^2} \right] \right\} = \int_{m}^{\infty} dm f_{\frac{1}{2}}(m, tc^2, tc^2 m^2) \int_{x(0)=x'}^{x(t)=x} Dx \frac{DP}{(2\pi)^D} \exp \left\{ \int_0^t d\tau \left[ ip \cdot \dot{x} - \frac{p^2}{2m} - mc^2 \right] \right\},
\]

(7)

where \( t_b - t_a \equiv t - 0 = t \), and

\[
f_p(z, a, b) = \frac{(a/b)^{p/2}}{2K_p(\sqrt{ab})} z^{p-1} e^{-(az+b/z)/2},
\]

(8)

is the generalized inverse Gaussian distribution [13] (\( K_p \) is the modified Bessel function of the second kind with index \( p \)). The left-hand side of (7) represents the PI for the free relativistic particle in the Newton–Wigner representation [18]. The genuine Klein–Gordon (KG) kernel which also contains negative-energy spectrum (reflecting the existence of the charge-conjugated solution, i.e., the antiparticle) can be received from Eq. (7) by considering the Feshbach–Villars representation of the KG equation and making the substitution [10]

\[
f_{\frac{1}{2}}(m, tc^2, tc^2 m^2) \rightarrow \frac{1 + \text{sgn}(t)\sigma_3}{2} f_{\frac{1}{2}}(m, |t|c^2, |t|c^2 m^2).
\]

(9)

The 2 \( \times \) 2 matrix nature of the smearing distribution (\( \sigma_3 \) is the usual 3rd Pauli matrix) naturally includes the Feynman–Stuckelberg causal boundary condition and accommodate thus both particles and antiparticles in a symmetric way [10, 19]. Note, that when partition function is to be calculated the trace will get rid of \( \text{sgn}(t) \) term and 1/2 is turned to 1.

The structure of (7) implies that \( \mathbf{m} \) can be interpreted as a Newtonian mass which takes on continuous values distributed according to \( f_{\frac{1}{2}}(m, tc^2, tc^2 m^2) \) with \( \langle m \rangle = m + 1/tc^2 \) and \( \text{var}(m) = m/ tc^2 + 2/t^2 c^4 \). Fluctuations of the Newtonian mass can be then depicted as originating from particle’s evolution in an inhomogeneous or granular (i.e., “polycrystalline”) medium. Granularity, as known, for example, from solid-state systems, typically leads to corrections in the local dispersion relation [20] and hence to alterations in the local effective mass. The following picture thus emerges: on the short-distance scale, a non-relativistic particle can be envisaged as propagating via classical Brownian motion through a single grain with a local mass \( \mathbf{m} \). This fast-time process has a time scale \( \sim 1/mc^2 \). An averaged value of the local time scale represents a transient temporal scale \( 1/mc^2 = 1/mc^2 \) which coincides with particle’s Compton time \( t_C \). At time scales much longer than \( t_C \) (large-distance scale), the probability that the particle encounters a grain which endows it with a mass \( \mathbf{m} \) is \( f_{\frac{1}{2}}(m, tc^2, tc^2 m^2) \). As a result one may view a single-particle relativistic theory as a single-particle non-relativistic theory where the particle’s Newtonian mass \( \mathbf{m} \) represents a fluctuating parameter which approaches on average the Einstein rest mass \( m \) in the large \( t \) limit. We stress that \( t \) should be understood as the observation time, a time after which the observation (position measurement) is made. In particular, during the period \( t \) the system remains unperturbed. One can thus justly expect that in the long run all mass fluctuations will be washed out and only a sharp time-independent effective mass will be perceived. The form of \( \langle m \rangle \) identifies the time scale at which this happens with \( t \sim 1/mc^2 \), i.e., with the Compton time \( t_C \). It should be stressed that above mass fluctuations have nothing to do with the Zitterbewegung which is caused by interference between positive- and negative-energy wave components. In our formulation both regimes are, however, decoupled.
We may also observe that by coarse-graining the velocity over the correlation time \( t_C \) we have
\[
\langle |\mathbf{v}| \rangle_{t_C} = \frac{\langle |\mathbf{p}| \rangle_{t_C}}{\langle m \rangle_{t_C}} = c. \tag{10}
\]
So on a short-time scale of order \( \lambda_C \) the Klein–Gordon particle propagates with an averaged velocity which is the speed of light \( c \). But if one checks the particle’s position at widely separated intervals (much larger than \( \lambda_C \)), then many directional reversals along a typical PI trajectory will take place, and the particle’s net velocity will be then less than \( c \)— as it should be for a massive particle. The particle then acquires a sharp mass equal to Einstein’s mass, and the process (not being hindered by fluctuating masses) is purely Brownian. This conclusion is in line with the well-known Feynman checkerboard picture \([14, 17]\) to which it reduces in the case of \((1 + 1)\) dimensional relativistic Dirac particle.

4. Robustness of the emergent special relativity

Understanding the robustness of the emergent Special Relativity under small variations in the mass-smearing distribution function \( f_{1/2} \) can guide the study of the relation between Einsteinian SR and other deformed variants of SR, such as Magueijo–Smolin and Amelino-Camelia’s doubly special relativity \([21, 22]\), or (quantum) \(\kappa\)-Poincaré deformation of relativistic kinematics \([23]\).

In DRS models a further invariant scale \( \ell \) is introduced, besides the usual speed of light \( c \), and \( \ell \) is typically considered to be of order of the Planck length. A small variation \( \delta f_{1/2} \) of the smearing function originates the new Hamiltonian
\[
\bar{H} = \frac{\epsilon_1}{4} + \left(1 + \frac{\epsilon_0}{2}\right) \sqrt{p^2 c^2 + m^2 c^4 + \frac{\epsilon_2}{4}}, \tag{11}
\]
with \( \epsilon_1 = -2 \left(1 + \epsilon_0/2\right) \sqrt{2} \) (see Ref. \([14]\) for details). By setting
\[
\epsilon_1 = 2 \left(\sqrt{1 - \frac{1}{1 - c^2 m^2 \ell^2}} - 1\right), \quad \epsilon_2 = \frac{4c^6 m^4 \ell^2}{1 - c^2 m^2 \ell^2},
\]
the new Hamiltonian \( \bar{H} \) can be easily identified with
\[
\bar{H} = c \frac{-m^2 c^2 \ell + \sqrt{p^2(1 - m^2 c^2 \ell^2) + m^2 c^4}}{1 - m^2 c^2 \ell^2}, \tag{12}
\]
which coincides with the Magueijo–Smolin’s doubly special relativistic Hamiltonian, in, say, its version \([24]\). For the last Hamiltonian a relation analogous to (7) holds, where now the smearing function has the form \( f_{1/2}(m, tc^2 \lambda, tc^2 m^2 \lambda) \) with \( \lambda = 1/(1 - m^2 c^2 \ell^2) \). The correlation distance \( 1/mc \lambda \) can be naturally assumed as the minimal size \( L_{\text{GR\text{AI\text{N}}}} \) of the “grain of space” of the polycrystalline medium, which is linked to the new invariant scale \( \ell \) by
\[
L_{\text{GR\text{AI\text{N}}}} := \frac{1}{mc \lambda} = \lambda_C (1 - m^2 c^2 \ell^2). \tag{13}
\]
By tuning the size \( L_{\text{GR\text{AI\text{N}}}} \) of these "grains of space" it is possible to pass continuously from Lorenz symmetry to other different symmetries, as those enjoyed by DSR models. We can in principle speculate that each large scale symmetry could originate from a specific kind of space(time) granularity.
5. Generalization to quantum fields

The transition probability presented in (6) was constructed under assumption that the Hamiltonian $H$ represents a single particle. Of course, the single-particle relativistic quantum theory is logically untenable, since a multi-particle production is allowed whenever the particle reaches the threshold energy for pair production. In addition, Leutwyler’s no-interaction theorem [25] prohibits interaction for any finite number of particles in the context of relativistic mechanics. To evade the no-interaction theorem it is necessary to have an infinite number of degrees of freedom to describe interaction. The latter is typically achieved via local quantum field theories (QFT).

It should be underlined in this context that the PI for a single relativistic particle is still a perfectly legitimate building block even in QFT. Recall that in the standard perturbative treatment of, say, generating functional for a scalar field each Feynman diagram is composed of integrals over product of free correlation functions (Feynman’s correlators):

$$\Delta_F(y, ct_y; z, ct_z) = \frac{1}{4} \int_{-\infty}^{\infty} d\tau \text{sgn}(\tau - t_y) \bar{P}(y, \tau | z, t_z),$$

and may thus be considered as a functional of the PI $\bar{P}(x', t' | x, t)$. In fact, QFT in general, can be viewed as a grand-canonical ensemble of fluctuating particle histories (worldlines) where Feynman diagrammatic representation of quantum fields depicts directly the pictures of the world-lines in a grand-canonical ensemble. This the so-called “worldline quantization” of particle physics is epitomized, e.g., in Feynman’s worldline representation of the one-loop affective action in quantum electrodynamics [26], in Kleinert’s disorder field theory [27] or in the Bern–Kosower and Strassler “string-inspired” approaches to QFT [28].

Because of (14), the relationship between bosonic Bern–Kosower Green’s function $G_B(\tau_1, \tau_2)$ and the PI $\bar{P}(x', t' | x, t)$ can be found easily through the known functional relation between $G_B$ and $\Delta_F$, cf. Refs. [28].

6. Implications in cosmology and gravity

When space-time is curved, a metric tensor enters in both PI’s in (7) in a different way, yielding different “counterterms” [15, 29]. For instance, in Bastianelli–van Nieuwenhuizen’s time slicing regularization scheme [29] one has (when $\hbar$ is reintroduced)

$$\frac{p^2}{2m} \rightarrow \frac{g^{ij} p_i p_j}{2m} + \frac{\hbar^2}{8m} (R + g^{ij} \Gamma^m_{il} \Gamma^l_{jm}),$$

$$\sqrt{p^2 + m^2 c^2} \rightarrow \sqrt{g^{ij} p_i p_j + \frac{\hbar^2}{4} (R + g^{ij} \Gamma^m_{il} \Gamma^l_{jm}) + m^2 c^2} + \frac{\hbar^4 \Phi(R, \partial R, \partial^2 R)}{4} + O(\hbar^6).$$

Here $g^{ij}$, $R$ and $\Gamma^j_{kl}$ are the (space-like) pull-back metric tensor, the scalar curvature and Christoffel symbol respectively. The non-vanishing counterterm $\Phi(\ldots)$ has the explicit form

$$\Phi(R, \partial R, \partial^2 R) = \frac{1}{16} (\partial_i \partial_j g^{ij})^2 - \frac{1}{16} \partial_i \partial_j \partial_k \partial_l \left( g^{ij} g^{kl} \right) - \frac{1}{8} \left( \partial_i \partial_j g^{ij} \right) \left( R + g^{mn} \Gamma^l_{mk} \Gamma^k_{nl} \right) + \frac{1}{8} \partial_i \partial_j \left[ g^{ij} \left( R + g^{mn} \Gamma^l_{mk} \Gamma^k_{nl} \right) \right].$$

When (15) is inserted into (7), the superstatistics identity breaks down as can be explicitly checked to the lowest order in $\hbar$. The respective sides of the equation will thus have different physical implications. Because the Einstein equivalence principle requires that the
local spacetime structure can be identified with the Minkowski spacetime possessing Lorentz symmetry, one might assume the validity of (7) at least locally. However, in different space-time points one has, in general, a different typical length scale of the local inertial frames, depending on the gravitational field. The characteristic size of the local inertial (i.e. Minkowski) frame is of order $1/|K|^{1/4}$ where $K = R_{αβσδ}R^{αβσδ}$ is the Kretschmann invariant and $R_{αβσδ}$ is the Riemann curvature. Relation (7) tells us that the special relativistic description breaks down in regions of size smaller than $λ_C$. For curvatures large enough, namely for strong gravitational fields, the size of the local inertial frame can become smaller than $λ_C$, that is $1/|K|^{1/4} \lesssim λ_C$. In such regions the special relativistic description is no more valid, and according to (7) must be replaced by a Newtonian description of the events. For instance, in Schwarzschild geometry we have $K = 12r_s^2/r^6$, and the breakdown should be expected at radial distances $r \lesssim (λ^2_C r_s)^{1/3}$ ($r_s$ is the Schwarzschild radius) which are — apart from the hypothetical case of micro-black holes (where $λ_C \simeq r_s$) — always deeply buried below the Schwarzschild event horizon. In the cosmologically relevant Friedmann-Lemaître-Robertson-Walker (FLRW) geometry, we have $K = 12(\dot{a}^2 + a^2 \ddot{a}^2)/(ac)^4$, and the breakdown happens when $(\dot{a}^2 + a^2 \ddot{a}^2) \gtrsim (ac/λ_C)^4$, where $a(t)$ is the FLRW scale factor of the Universe and $\dot{a} = da/dt$. Applying the well-known Vilenkin–Ford model [30] for inflationary cosmology, where $a(t)$ is given by: $a(t) = A\sqrt{\sinh(Bt)}$ with $B = 2c\sqrt{\Lambda/3}$ ($\Lambda$ is the cosmological constant), we obtain a temporal bound on the validity of local Lorentz invariance, which, expressed in FLRW time, is

$$t \lesssim \frac{1}{B} \arcsinh \left[ \frac{Bλ_C}{(8c^4 - (Bλ_C)^4)^{1/4}} \right] \equiv \bar{t}.$$  

By using the presently known [31] value $Λ \simeq 10^{-52}$ m$^{-2}$ and the $τ$-lepton Compton’s wavelength $λ_C \simeq 6.7 \times 10^{-16}$ m (yielding the tightest upper bound on $t$), we obtain $\bar{t} \simeq 4 \times 10^{-24}$ s. Note that, since $Bλ_C \ll c$, then $\bar{t} \simeq λ_C/c = t_C$. Such a violation of the local Lorentz invariance naturally breaks the particle-antiparticle symmetry since there is no unified theory of particles and antiparticles in the non-relativistic physics — formally one has two separate theories. If the resulting matter-antimatter asymmetry provides a large enough CP asymmetry then this might have essential consequences in the early Universe, e.g., for leptogenesis. In this respect, $\bar{t}$ is compatible with the nonthermal leptogenesis period that typically dates between $10^{-26}$–$10^{-19}$ s after the Big Bang.

7. Conclusions and perspectives

The presented picture of emergent special relativity, where the observed space-time symmetries have a superstatistics basis, suggests an alternative reading of relativistic signals phrased in terms of quantum-mechanical probabilities. As such, it also makes explicit how the SR invariance is encoded in the grain smearing distribution. Notably, the exact LS of a space-time has no fundamental significance in our analysis, as it is only an accidental symmetry of the coarse-grained configuration space in which a particle executes a standard Wiener process. In passage from grain to particle’s Newtonian mass fluctuates according to an inverse Gaussian distribution. The observed inertial mass of the particle is thus not a fundamental constant, but it reflects the particle’s interaction with the granular vacuum (cosmic field). This, in a sense, supports Mach’s view of the phenomenon of inertia.

Interactions can be included in our framework in two different ways. The interaction with a background field (such as electromagnetic field) can be directly treated with the superstatistics prescription (7), see [10]. On the other hand, the multi-particle interactions can be consistently formulated by “embedding” the relativistic PI in QFT via the worldline quantization. Such an embedding may help to study several cosmological implications of systems with granular
space. If any of such systems quickly flows to the infrared fixed point, any direct effect due to
the space discreteness, and related SR violation, might be insignificant on cosmological scales
(where Lorentz and diffeomorphism invariance are restored), while it might be crucial in the
early Universe, e.g., for leptogenesis and the ensuing baryogenesis. Consequences on the detailed
structure of the Cosmic Microwave Background spectrum will be explored in future work.

The presented approach implies a preferred frame. In this connection it is worth of noting
that, despite the fact that (7) is not manifestly LS invariant, one may use the St¨ uckelberg trick
and introduce a new fictitious variable into the PI (7), in such a way that the new action will
have the reparametrization symmetry, but will still be dynamically equivalent to the original
action. For relevant details see Ref. [14]. By not knowing the source, one may then view this
artificial gauge invariance as being a fundamental or even a defining property of the relativistic
theory. One might, however, equally well, proclaim the “polycrystalline” picture as being a basic
(or primitive) edifice of SR and view the reparametrization symmetry as a mere artefact of an
artificial redundancy that is allowed in our description. It is this second view that we favored
here.

The observant reader could notice that the presented scenario cannot directly accommodate
the massless particles such as photons (identity (7) holds true only for $m \neq 0$). One possibility
would be to use the PI representation of Polyakov–Wheeler for massless particles and try to
construct a similar superstatistics duality as in the case of massive particles. This procedure is,
however, not without technical difficulties and currently is under investigation. Conceptually is
far more simpler to assume that the photon has a small mass. At present, there are a number
of experimental limits to the mass of the photons [32]. For instance, tests based on Coulomb’s
law and the galactic vector potential set the upper limit of $m_\gamma \lesssim 10^{-18}$ eV/c$^2 \lesssim 10^{-57}$ g.
This gives the domain correlation distance for the photon $\sim 1/m_\gamma c^2 \sim 10^{43}$ m which is bigger than
the radius of observable Universe ($\sim 10^{26}$ m) and so in this picture the photon mass does not
fluctuate — it is a quasi-invariant.

Finally, it is hoped that our approach should reinforce the links between the superstatistics
paradigm and the approach to quantum gravity based on stochastic quantization [33]. In
particular, the outlined granular space could be a natural model for the noise terms in a Parisi–
Wu stochastic-like quantization approach to gravity.

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