Laser differential confocal radius measurement

Weiqian Zhao,* Ruoduan Sun, Lirong Qiu, and Dingguo Sha

Key Laboratory of Photoelectronic Imaging Technology and System (Ministry of Education of China), Opto-Electronic College, Beijing Institute of Technology, Beijing 100081, China

*zwq669@126.com

Abstract: A new laser differential confocal radius measurement (DCRM) is proposed for high precision measurement of radius. Based on the property of an axial intensity curve that the absolute zero precisely corresponds to the focus of the objective in a differential confocal system (DCS), DCRM uses the zero point of the DCS axial intensity curve to precisely identify the cat's-eye and confocal positions of the test lens, and measures the accurate distance between the two positions to achieve the high-precision measurement of radius of curvature (ROC). In comparison with the existing measurement methods, DCRM proposed has a high measurement precision, a strong environmental anti-interference capability and a low cost. The theoretical analyses and preliminary experimental results indicate that DCRM has a relative measurement error of better than 5ppm.

OCIS codes: (120.0120) Instrumentation, measurement, and metrology; (180.1790) Confocal microscopy; (220.4840) Testing.

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1. Introduction

Sphere component is widely used in precise optical systems as the most essential optical component, such as violet lithography, astronomical telescope and laser fusion program. The radius of curvature (ROC) is one of the most basic and important parameters in the sphere surface measurement, and the high precision ROC measurement is still a difficulty [1]. At present, the main methods used for ROC measurement include spherometer, Foucault test,
traveling microscope, focus retrocollimated interferometry, autocollimation, simple fiber-optic autocollimation method and figure measuring interferometer (FMI), and so on [2–6]. In the aforementioned methods, FMI is the most well-known method used for the ROC accurate measurement, which uses the interference fringe to identify the cat’s eye and confocal positions, and measures the accurate distance between the two positions to obtain the ROC. The common FMI uses PZT for phase-shifting, and has disadvantages as below. 1) It is difficult for the measurement beam to converge to a point due to the diffraction and the focal-depth restricts the accuracy of identification at the cat’s-eye and confocal positions. 2) The environmental vibration and airflow have an adverse effect on the stability of interference fringe at the cat’s-eye and confocal positions [7,9]. To improve the anti-interference capability, the instantaneous phase shifting interferometry was developed, but it is expensive and complex.

And therefore, a low-cost, compact laser differential confocal radius measurement (DCRM) is proposed, which has a high accuracy of the identifications at cat’s-eye and confocal points and a stronger anti-interference capability by using the differential confocal intensity detection [10,11]. In addition, DCRM is easy to be combined with a pupil filter and it can shorten the focal-depth to further improve the accuracy of the identifications at the cat’s-eye and confocal positions [12].

2. Measurement principle

2.1 DCRM principle

Based on the property that the zero point of the axial intensity curve corresponds to the focus of the objective in a differential confocal system (DCS) established by ourselves [10,11], DCRM as shown in Fig. 1 uses the null points $O_A$ and $O_B$ of DCS axial intensity curves $I_A$ and $I_B$ to make precise identifications at the cat’s-eye position A and confocal position B of the test sphere, and measures the distance between the two null positions to achieve the ROC accurate measurement.

As shown in Fig. 1, L_C is collimating lens, PBS is polarized beam splitter, P is $\lambda/4$, L_1 is standard focusing lens, L_2 is collecting lens, BS is beam splitter, S is point light source, D_1 and D_2 are detectors, P_1 and P_2 are pinholes, R is test lens, M is the offset of pinhole from the focus of L_2.

When the test lens R is moved near the cat’s-eye position A along the optical axis, the measurement beam is reflected along the backtracking by the lens R and then reflected on the collecting lens L_2 and BS by PBS, and the two measurement beam split by BS are received by detectors D_1 and D_2, respectively. Near the cat’s-eye position A, the overlap area of the measurement beam and the test surface is so small that the effect of the figure error on the DCS intensity curve is negligible. Then the signal $I_A(v,\varphi,u,u_M)$ obtained through the
differential subtraction of $I_{A1}(v,\varphi,u, + u_M)$ and $I_{A2}(v,\varphi,u, - u_M)$ received by the detectors $D_1$ and $D_2$ is [10,11]

$$I_v(\varphi, u, u_M) = I_v(\varphi, u, + u_M) - I_v(\varphi, u, - u_M)$$

$$= \frac{1}{4\pi} \int_{\varphi_0}^{\varphi_1} \int_{\theta_0}^{\theta_1} p_+(\rho, \theta) \cdot p_-(\rho, \theta) \cdot p_+(\rho, \pi + \theta) \cdot p_-(\rho, \pi + \theta) \cdot e^{i(\phi_1 - \phi_2)/2 - i\rho M} \cdot e^{-i\rho M} \rho d\rho d\theta$$

$$= \frac{1}{4\pi} \int_{\varphi_0}^{\varphi_1} \int_{\theta_0}^{\theta_1} p_+(\rho, \theta) \cdot p_-(\rho, \theta) \cdot p_+(\rho, \pi + \theta) \cdot p_-(\rho, \pi + \theta) \cdot e^{-i\rho M} \rho d\rho d\theta$$

where

$$u = \frac{\pi}{2\lambda} \left( \frac{D}{f^2} \right)^2 z$$

$$v = \frac{\pi}{2\lambda} \left( \frac{D}{f^2} \right)^2 r_0$$

where $\rho$ is the radial normalized radius of a pupil, $\theta$ is the angle of variable $\rho$ in the polar coordinate, $u$ is the axial normalized optical coordinate, $v$ is the lateral normalized optical coordinate, $\varphi$ is the angle of variable $v$ in the polar coordinate, $z$ is the axial displacement of the object, $r_0$ is the radial coordinate of the objective, $Df'$ is the relative aperture of DCS objective $L_1$, $p_+(\rho, \theta)$ is the pupil function of $L_1$, $p_-(\rho, \theta)$ is the pupil function of $L_2$ and $p_c(\rho, \theta)$ is the pupil function of $L_C$.

When $L_1$ and $L_2$ are the same in apertures, $p_c(\rho, \theta) = 1$, $p_1(\rho, \theta) = 1$, $p_2(\rho, \theta) = 1$, $I_d(0,0,u,u_M)$ obtained from Eq. (1) is

$$I_d(0,0,u,u_M) = \left[ \frac{\sin[(\varphi_1 + \rho_0)/4]}{(\varphi_1 + \rho_0)/4} \right]^2 - \left[ \frac{\sin[(\varphi_1 - \rho_0)/4]}{(\varphi_1 - \rho_0)/4} \right]^2.$$  (3)

And as shown in Fig. 1, the zero point $O_A$ of intensity curve $I_d(0,0,u,u_M)$ precisely corresponds to the cat's-eye position A of the test lens.

Near the confocal position B, the overlap area of the measurement beam and the test surface is so great that the effect of the figure error $\Phi(\rho, \theta)$ on the DCS intensity curve cannot be ignored, which introduced the wave-front error of $2\Phi(\rho, \theta)$ in the measurement. When the test lens $R$ is moved near the confocal position B along the optical axis, the signal $I_B(\varphi, \rho, u_M, \Phi)$ obtained through the differential subtraction of $I_B(\varphi, \rho, u_M, \Phi)$ and $I_{B2}(\varphi, \rho, u_M, \Phi)$ received by detectors $D_1$ and $D_2$ is [10,11]

$$I_v(\varphi, u, u_M, \Phi) = I_v(\varphi, u, + u_M, \Phi) - I_v(\varphi, u, - u_M, \Phi)$$

$$= \frac{1}{4\pi} \int_{\varphi_0}^{\varphi_1} \int_{\theta_0}^{\theta_1} p_+(\rho, \theta) \cdot p_+(\rho, \pi + \theta) \cdot p_-(\rho, \pi + \theta) \cdot e^{i(\phi_1 - \phi_2)/2 - i\rho M} \cdot e^{-i\rho M} \rho d\rho d\theta$$

$$= \frac{1}{4\pi} \int_{\varphi_0}^{\varphi_1} \int_{\theta_0}^{\theta_1} p_+(\rho, \theta) \cdot p_+(\rho, \pi + \theta) \cdot p_-(\rho, \pi + \theta) \cdot e^{-i\rho M} \rho d\rho d\theta$$

And as shown in Fig. 1, the zero point $O_B$ of intensity curve $I_B(0,0,u,u_M)$ precisely corresponds to the confocal position B of the test lens.
2.2 Effect of $\Phi(\rho, \theta)$ on the accuracy of identification at confocal position B

As shown in Fig. 2, figure error $\Phi(\rho, \theta)$ can make the average radius $r_{\text{avg}}$ (best-fit sphere) of the test lens $R$ deviate from $r$ and the DCRM intensity response curve change in shape under the high precision ROC measurement.

![Fig. 2. Effect of $\Phi(\rho, \theta)$ on identification precision at confocal position B.](image)

When several primary aberrations with the distinct effect on ROC measurement are considered, the figure error $\Phi(\rho, \theta)$ is [13]

$$
\Phi(\rho, \theta) \approx A_{040}\rho^4 + A_{022}\rho^2\cos^2\theta + A_{031}\rho^3\cos\theta + A_{120}\rho + A_{111}\rho\cos\theta
$$

where $A_{040}$ is the primary spherical aberration coefficient, $A_{022}$ is the astigmatism coefficient, $A_{031}$ is the coma coefficient, $A_{120}$ is the curvature of field coefficient and $A_{111}$ is the distortion coefficient.

$A_{040}\rho^4$ and $A_{022}\rho^2\cos^2\theta$ can incur the axial offset of best-fit sphere center of curvature according to the Displacement Theorem [13], and the normalized axial offset $\Delta$ is

$$
\Delta \approx \frac{2\pi}{\lambda}(2A_{040} + A_{022})
$$

$\Delta$ is the offset of $r_{\text{avg}}$ from $r$ caused by $\Phi(\rho, \theta)$.

Let $p_2(\rho, \theta) = 1$, $p_1(\rho, \theta) = 1$ and $p_2(\rho, \theta) = 1$, $A_{040}\rho^4$ and $A_{022}\rho^2\cos^2\theta$ are substituted for Eq. (4), respectively, then

$$
I_{BA_{040}}(\nu_0, \theta, \nu_M, \Phi) = \left| \frac{1}{\pi} \int_0^{2\pi} \int_0^1 \int_0^{2\pi} \int_0^1 e^{i\nu_0(2\nu + \nu_M)/2} \cdot e^{-j\nu\cos(\theta - \phi)} \cdot e^{-j\nu\cos(\theta - \phi)} \rho d\rho d\theta \right|^2
$$

$$
I_{BA_{022}}(\nu_0, \theta, \nu_M, \Phi) = \left| \frac{1}{\pi} \int_0^{2\pi} \int_0^1 \int_0^{2\pi} \int_0^1 e^{i\nu_0(2\nu + \nu_M)/2} \cdot e^{-j\nu\cos(\theta - \phi)} \cdot e^{-j\nu\cos(\theta - \phi)} \rho d\rho d\theta \right|^2
$$

From Eq. (7) and (8), the intensity response curves $I_{BA_{040}}(0,0,\nu_M,\Phi)$ and $I_{BA_{022}}(0,0,\nu_M,\Phi)$ considering the primary spherical aberration $A_{040}\rho^4$ and primary astigmatism $A_{022}\rho^2\cos^2\theta$ at confocal positions B are shown in Fig. 3a) and Fig. 3b), respectively.
It can be seen from Fig. 3 that the relation between axial offset Δ of best-fit sphere centre of curvature $O'$ and the axial offsets of zero points of DCRM curves $I_{BA04}$ and $I_{BA02}$ is almost coincide with Eq. (6), i.e., the ROC obtained by DCRM is coincident with $r_{avg}$. And therefore, $Φ(ρ,θ)$ has a very small effect on the accuracy of the measurement by DCRM.

2.3 Effect of $Φ(ρ,θ)$ on identification sensitivity

Sensitivity $S_B(0,0,0,C,Φ)$ at zero point $O_B$ can be obtained by differentiating Eq. (4) on $u$ and is

$$S_B(0,0,0,C,Φ) = \frac{∂I_B(v,ρ,θ,u,υ,Φ)}{∂u} \bigg|_{v=0,ρ=0,θ=0,υ=C} \cdot \Phi.$$  \hspace{1cm} (9)

Sensitivity curve $S_B(0,0,0,C,Φ)$ obtained from Eq. (9) is shown in Fig. 4.

It can be seen from Fig. 4 that DCRM sensitivity $S_B(0,0,0,C,Φ)$ decreases as the aberration coefficient $A_n$ increases and $|S_B(0,0,0,C,Φ)| > 0.27$ which is half of that without aberrations when $A_{040} < 0.7λ$ and $A_{022} < 0.3λ$. And so, the accuracy of identification by DCRM entirely satisfies measurement requirements of the accuracy for the ROC measurement.

It is therefore that the accuracy of the measurement by DCRM is high and the effect of aberrations on DCRM is small as the surface figure precision of sphere measured is high when it is used for the high-precision radius measurement.

3. DCRM system using virtual pinhole for tracking

As shown in Fig. 1, DCRM requires that the pinhole center is coincident with the optical axis and the center of the Airy disk coming from the collecting lens $L_2$ exactly passes through the pinhole. In the measurement procedure, it is difficult to make the optical axes in coincidence between the test lens $R$ and DCRM, and the axes offset between them results in the lateral offset of the focus of collecting lens $L_2$. The lateral offset of the pinhole from the focus has an effect on DRCM intensity curve $I(z)$ result in an effect on the sensitivity and precision of the
identification. And the alignment error arising from the measurement line and datum line incur Abbe errors in DCRM.

In order to reduce the assembly difficulty of the test lens and measurement error caused by the assembly of the lens, a virtual pinhole (VPH) using CCD detection as shown in Fig. 5 is substituted for a conventional physical pinhole (PH).

![Fig. 5. Virtual pinhole detection principle.](image)

The Airy disk on the L2 focal plane is magnified and imaged on the CCD detector by microscope objective MO, and the centre coordinates \(P(x, y)\) of the Airy disk is obtained using image \(g(m, n)\) from CCD based on Eq. (10).

\[
\begin{align*}
x &= \left( \sum_{m,n} m \cdot g(m,n) \right) / \sum_{m,n} g(m,n) \\
y &= \left( \sum_{m,n} n \cdot g(m,n) \right) / \sum_{m,n} g(m,n)
\end{align*}
\]

where \(m\) and \(n\) the pixel coordinate of the image.

And then, the VPH with a given radius \(r_{VPH}\) is centered at point \(P(x, y)\), the grey summation of pixels within the circle of VPH is calculated, and the circle is equivalent to the physical pinhole of a confocal microscopy system and its size and shape can be adjusted by the software.

DCRM system (DCRMS) using CCD tracking detection by VPH is established as shown in Fig. 6 based on the measurement principle shown in Fig. 1 and VPH technique.

![Fig. 6. DCRMS using VPH tracking.](image)
In the measurement procedure, DCRM processes image \( g(m,n) \) using Eq. (10) to obtain the intensity centre of the Airy disk in real time and automatically track the center position \( P(x,y) \) of the Airy disk. It makes the PH centers coincident with the centre of Airy disk to eliminate the effect of PH lateral offset on position precision at confocal point and simplify the adjustment of the test lens \( R \).

4. Determination of DCRMs key parameters and their effects

4.1 VPH size

As pinhole size decreases, the system sensitive increases while the energy received by the detector decreases. Hence, in a high system sensitive area, a larger pinhole should be selected to ensure enough energy passes through the pinhole. Analysis and calculation indicate that the pinhole size must satisfy Eq. (11), and an optimum pinhole should be in the range of 3–10\( \mu m \) in diameter [10,11].

\[
d \leq \frac{2.5\lambda}{\pi \sin \alpha}
\]  

(11)

where \( d \) is the pinhole diameter, \( \lambda \), the optical wavelength, \( \sin \alpha \), the numerical aperture of \( L_2 \).

If the MO magnification is \( N \), the pixels corresponding to the VPH diameter satisfies

\[
d_{\text{VPH}} = \frac{N \cdot d}{p} \leq \frac{2.5N\lambda}{\pi p \sin \alpha}
\]

(12)

where \( p \) is the pixel size of CCD.

4.2 VPH offset

As shown in Fig. 7, it is difficult for the offset of the two VPH to be equal in the amount under the adjustment of DCRM light-path. Assuming the offset of the two detectors are \( M + \delta \) and \( -M \) while their corresponding normalized offset are \( u_M + u_\delta \) and \( u_M \), and the offset of zeros \( O_A \) and \( O_B \) from focuses A and B are \( \Delta z_A \) and \( \Delta z_B \), respectively, under the measurement while their normalized value are \( u_A \) and \( u_B \).

![Fig. 7. Light-path principle with different offset of two detectors.](image)

When \( L_1 \) and \( L_2 \) are the same in apertures, \( p_c(\rho,\theta) = 1 \), \( p_l(\rho,\theta) = 1 \), \( p_s(\rho,\theta) = 1 \) and the effect of figure error \( \Phi \) is neglected, the axial responses at points \( O_A \) and \( O_B \) obtained using Eq. (3) and Eq. (4) satisfy
\[
I_A(0,0,u_A,u_M) = \left[ \frac{\sin\left(\frac{2u_A + u_x + u_M}{4}\right)}{2} \right]^2 - \left[ \frac{\sin\left(\frac{2u_A - u_M}{4}\right)}{2} \right]^2 = 0. \quad (13)
\]

\[
I_B(0,0,u_B,u_M,\Phi) = \left[ \frac{\sin\left(\frac{2u_B + u_x + u_M}{4}\right)}{2} \right]^2 - \left[ \frac{\sin\left(\frac{2u_B - u_M}{4}\right)}{2} \right]^2 = 0. \quad (14)
\]

From Eq. (13) and Eq. (14),

\[
u_A = u_B = -\frac{u_M}{4}. \quad (15)
\]

It can be seen from Eqs. (2) and (15) that the offset \(\Delta z_A\) and \(\Delta z_B\) of points \(O_A\) and \(O_B\) are equal, the radius measurement error caused by the VPH axial offset \(\delta\) is

\[
\Delta r_\delta = \Delta z_A - \Delta z_B = 0. \quad (16)
\]

It is obvious that in the DCRMS light-path adjustment, \(\Delta z_A\) and \(\Delta z_B\) caused by the offset of the detectors are the same in the size and direction so that they can counteract each other. And so, the adjustment difficulty of the detectors reduces greatly in DCRMS because the measurement error caused by \(\delta\) is small.

4.3 Illumination collimation system

As shown in Fig. 8, it is difficult in the DCRMS light-path adjustment for the illumination collimation system to ensure the pinhole and the focus of collimation lens \(L_C\) in coincidence. This changes the defocused pupil function \(p_C(\rho,\theta)\) of collimating lens \(L_C\) so that the curves \(I_A(0,0,u_A,u_M)\) and \(I_B(0,0,u_B,u_M,\Phi)\) are translated along the axial direction.

Assuming the PH offset is \(\Delta z_C\) while its corresponding normalized offset is \(u_C\), and the offset of zeros \(O_A\) and \(O_B\) from focuses A and B are respectively \(\Delta z_A\) and \(\Delta z_B\) in the measurement while their normalized value are \(u_A\) and \(u_B\). Then defocused pupil function \(p_C(\rho,\theta)\) of the collimating lens \(L_C\) satisfies

\[
p_C(\rho,\theta) = e^{i\rho^2(u_C/2)}. \quad (17)
\]

Fig. 8. Effect of beam parallel of illumination collimating system.

Substituting Eq. (17) for Eqs. (1) and (4), when \(L_1\) and \(L_2\) are the same in apertures and the figure precision of the test lens is very high, the axial responses at points \(O_A\) and \(O_B\) obtained satisfy
From Eqs. (18) and (19),

\[ u_A = u_B = -\frac{u_C}{2}. \]  \hspace{1cm} (20)

It can be seen from Eq. (2) and Eq. (20) that when the offset \( \Delta z_A \) and \( \Delta z_B \) of points \( O_A \) and \( O_B \) are equal, they can counteract each other. The radius measurement error caused by \( \Delta z_C \) is

\[ \Delta r_c = \Delta z_A - \Delta z_B = 0. \]  \hspace{1cm} (21)

Hence, it greatly reduces the adjustment difficulty of DCRMS detectors because the measurement error caused by \( \Delta z_C \) is very small.

4.4 Nonlinear calibration of CCD detectors

The zero point is obtained through the differential subtraction of the normalized VPH data, so the difference of two CCDs in intensity response has little effect on the position of the zero point.

However, the nonlinearity of the intensity response has an effect on the determination of the zero point. Due to the discreteness of data in the measurement, the zero point can hardly be captured exactly, so the CCD nonlinearity must be calibrated and a linear curve can be fitted by the data near the zero point. And the calibration setup on CCD nonlinearity is shown in Fig. 9.

The test chart in accordance with ISO14524:99 are placed at the outlet of the integrating sphere, and a set of grey values are shown in Table 1.
Table 1. Nonlinear calibration on CCD response

| Normalized luminance | Normalized grey value of CCD | Normalized luminance | Normalized grey value of CCD |
|----------------------|-----------------------------|----------------------|-----------------------------|
| 0.001                | 0.000                       | 0.200                | 0.535                       |
| 0.005                | 0.000                       | 0.302                | 0.646                       |
| 0.017                | 0.000                       | 0.457                | 0.774                       |
| 0.041                | 0.103                       | 0.661                | 0.850                       |
| 0.072                | 0.249                       | 0.813                | 0.935                       |
| 0.132                | 0.393                       | 1.000                | 1.000                       |

A set of grey data shown in Table 1 is obtained from the test chart using MTV-2821 CCD being calibrated. And the nonlinear calibration function $R^{-1}(y)$ shown in Fig. 10 are obtained through third-order polynomial fitting to the grey data, where $x$ is the normalized luminance of the test chart and $y$ is the normalized grey value of CCD output.

![Nonlinear calibration function $R^{-1}(y)$](image1)

Fig. 10. Nonlinear calibration of CCD response curve.

The nonlinear calibration on the image $g(m,n)$ coming from CCD is made using $R^{-1}(y)$ in DCRMS and the response curves before and after calibration are shown in Fig. 11.

![Intensity response curves before and after calibration on CCD](image2)

Fig. 11. Intensity response curves before and after calibration on CCD. a) Intensity response curves of two CCD b) DCS intensity curve.

It can be seen from Fig. 11 that the VPH response curve $I(z)$ is more coincident with the theory in shape after the calibration, its side lobe is suppressed obviously, its linearity is improved and its half width at half of a maximum is diminished.

4.5 Adjustment and correction of axial alignment errors

As shown in Fig. 12, the measurement axis $d$ of a distance measurement interferometer (DMI), the motion axis of translation stages $m$ and radius axis of lens measured $i$ should be...
coincident, whereas the angles between them always exist in practice. The angle between axes \( t \) and \( m \) is \( \beta \) and the angle between axes \( m \) and \( d \) is \( \gamma \), the ROC measurement error is [3]

\[
\sigma_{\text{axial}} = r(1 - \cos \beta \cdot \cos \gamma).
\]  

(22)

It can be seen from Eq. (22) that \( \beta \) and \( \gamma \) should be diminished to reduce the ROC measurement error \( \sigma_{\text{axial}} \).

The angle \( \gamma \) is diminished through the careful alignment of DMI measurement axis \( d \) and motion axis of translation stage \( m \) and it is adjusted only at the time of establishing the system. And the correction of angle \( \beta \) is essential because it is produced by the placement of the test lens and should be adjusted for each test lens.

The adjustment of angle \( \beta \) in DCRMS is shown in Fig. 12, the center coordinate of the spot \( p_A(x,y) \) is obtained using VPH when the test lens is translated to the cat's-eye position A, then the center coordinate of the spot \( p_B(x,y) \) is obtained using VPH when the test lens is translated to the confocal position B, and thereby obtaining the lateral displacement \( \varepsilon \) of the spot center.

![Fig. 12. Adjustment of angle \( \beta \) between optical axis \( t \) of the lens and motion axis \( m \).](image)

Angle \( \beta \) is obtained using the following equations, and

\[
\tan \beta \approx \frac{\varepsilon}{2r_{\text{DMI}}} \quad (23)
\]

\[
\varepsilon = \frac{n \cdot p \cdot f_{1,1}}{N \cdot f_{1,2}} \quad (24)
\]

where \( n \) is the pixel number corresponding to the lateral displacement of spot center from position A to position B, \( f_{1,1} \) is the focal length of standard focusing lens \( L_1 \) and \( f_{1,2} \) is the focal length of collecting lens \( L_2 \).

In the practical measurement, ROC measurement error caused by angle \( \beta \) can be corrected using Eq. (25).

\[
r = \frac{r_{\text{DMI}}}{\cos \beta} = \frac{r_{\text{DMI}}}{\cos(\text{arc} \tan \beta))} \approx r_{\text{DMI}} / \cos \left( \frac{\varepsilon}{2r_{\text{DMI}}} \right) = r_{\text{DMI}} / \cos \left( \frac{n \times p \times f_{1,1}}{2N \times f_{1,2} \times r_{\text{DMI}}} \right) \quad (25)
\]

where \( r_{\text{DMI}} \) is the radius measured by DMI.

The ROC measurement error \( \sigma_{\text{axial}} \) and the adjustment difficulty of the test lens reduce greatly because it caused by angle \( \beta \) can be corrected.
5. Measurement errors

The main errors having effects on ROC measurement are the distance measurement error of laser interferometer, position errors at cat's-eye position A and confocal position B, wavefront aberrations of sphere measured and axial alignment errors, and so on [3,14,15].

5.1 Distance measurement error $\sigma_d$

The distance $AB$ between cat's-eye position A and confocal position B can be obtained by DMI, and its measurement precision is highly dependent on the deadpath error [8,9]. For the distance measurement, the variation of atmosphere in the laboratory is compensated by monitoring the temperature, pressure, and humidity real-timely. The measurement errors consist of: 1) $\sigma_{pa}$ for the barometer (pa), 2) $\sigma_K$ for the thermometer (°C), and 3) $\sigma_H$ for the hygrometer (% relative humidity). The distance measurement error satisfies:

$$\sigma_d = \sigma_{\text{air}} \sqrt{\left(2.68 \times 10^{-9} \sigma_{pa}\right)^2 + \left(9.27 \times 10^{-7} \sigma_K\right)^2 + \left(1 \times 10^{-9} \sigma_H\right)^2}$$  \hspace{1cm} (26)

where $\sigma_{\text{air}}$ is the refraction index error of air, $L_{\text{deadpath}}$ is the difference between the DMI measurement and reference path lengths.

5.2 Null error $\sigma_z$

When the figure precision of the test sphere is high, the figure error is negligible, and the identification sensitivity $S(0,0,u_{M})$ is calculated by differentiating Eq. (3) on $u$.

$$S(0,0,u_{M}) = \frac{\partial I_A(0,0,u_{M})}{\partial u} \bigg|_{u=0} = 2 \cdot \frac{\sin(\frac{4u_{M}}{\pi})}{4\pi} \cdot \left[\left(\frac{u_{M}}{\lambda}\right) \cos\left(\frac{u_{M}}{\lambda}\right) - \sin\left(\frac{u_{M}}{\lambda}\right)\right].$$ \hspace{1cm} (27)

The sensitivity of the differential intensity curve is the largest and $S_{\text{max}} = -0.54$ at the zero point when $u_{M} = 5.21$ [10,11]. And the DCS null error $\delta z$ and the relative aperture $D/f'$ of confocal objective satisfy Eq. (28).

$$\sigma_z = \frac{\delta I_A(0,0,u_{M})}{\pi(D/f')^2} = \frac{2\lambda}{\pi(D/f')^2} = \frac{1.18\lambda}{\text{SNR} \cdot (D/f')^2}$$ \hspace{1cm} (28)

where $\delta I_A(0,0,u_{M})$ is the random noise of normalized differential signal caused by CCD of VPH, SNR is the signal noise ratio of VPH and their relation is $\text{SNR} = 1/\delta I_A(0,u_{M},u_{M})$. It can be seen from Eq. (28) that the measurement precision increases as $D/f'$ increases.

5.3 Axial alignment errors $\sigma_{\text{axial}}$

The residual error $\delta \beta$ obtained using Eq. (25) after the adjustment and correction is

$$\delta \beta = \arctan\left(\frac{\delta n}{2r}\right) \approx \frac{\delta n \cdot p \cdot f_{l1}}{2N \cdot f_{l2} \cdot r}$$ \hspace{1cm} (29)

where $p$ is the pixel size of CCD.

The position precision of the spot center is $\delta n = 0.1$ through the sub-pixel subdivision. When $N = 10$, $p = 8\mu m$, $f_{l1} = 9\ mm$ and $f_{l2} = 160\ mm$, the residual error obtained using Eq. (29) is $\delta \beta = 0.000023 \text{mrad}$ with radius $r = 100\ mm$. Hence, the measurement error of $r$ caused by $\delta \beta$ is negligible.

The angle $\gamma$ between DMI and motion stage can be less than 0.5 mrad through the careful adjustment [3].

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Radius measurement error $\sigma_{\text{axial}}$ caused by the misalignment of the DMI measurement axis $d$, the motion axis of translation stages $m$ and radius axis of lens measured $r$ is

$$\sigma_{\text{axial}} = r(1 - \cos \beta \cdot \cos \gamma) \approx r(1 - \cos \gamma) = r(1 - \cos 0.0005) \approx 1.24 \times 10^{-7} r \quad (30)$$

### 5.4 Figure error $\sigma_{\text{figure}}$

The analyses indicate that the effect of figure error on identification precision at DCRMS confocal position B is negligible for the test sphere with high precision.

The cat’s-eye point does not correspond to the best-fit sphere and it lies in a small area of the test lens surface, so that a figure error at wavelength level is introduced. And the error $\sigma_{\text{figure}}$ can be corrected by the figure of the test sphere obtained by phase shift interferometer, and the figure error after the correction satisfies [3]:

$$\sigma_{\text{figure}} \approx 0.1 \cdot \text{PV} \quad (31)$$

### 5.5 Synthesis error

Considering the effect of the aforementioned several errors on the measurement results, the total measurement error $\sigma_r$ is

$$\sigma_r \approx \sqrt{\sigma_L^2 + 2 \cdot \sigma_z^2 + \sigma_{\text{axial}}^2 + \sigma_{\text{figure}}^2} \quad (32)$$

When the parameters of the test lens are $r = 100\text{mm}$, $D = 20\text{mm}$ and PV = $0.1\lambda$, the magnification of objective used for VPH is $N = 10$, the CCD pixel size is $p = 8\mu\text{m}$ and signal noise ratio is $\text{SNR} = 150:1$, the errors obtained using Eqs. (26)–(31) are $\sigma_L \approx 0.1\mu\text{m}$, $\sigma_z \approx 0.1\mu\text{m}$, $\sigma_{\text{axial}} \approx 0.01\mu\text{m}$ and $\sigma_{\text{figure}} \approx 0.01\mu\text{m}$, respectively.

The system measurement error obtained using Eq. (32) is

$$\sigma_r \approx \sqrt{(0.1)^2 + 2 \cdot (0.1)^2 + (0.01)^2 + (0.01)^2} \mu\text{m} \approx 0.17 \mu\text{m} \quad (33)$$

Its relative error is

$$\delta_r \approx \frac{\sigma_r}{r} \times 100\% \approx \frac{0.17}{100 \times 1000} \times 100\% = 0.00017\% \approx 2\text{ppm} \quad (34)$$

The system relative measurement error can be better than 5ppm with the consideration of environment and some negligible errors.

### 6. Experiments

#### 6.1 Experiment setup

The experimental setup shown in Fig. 13 is established based on Fig. 6 to verify the validity of DCRM proposed.
Fig. 13. Experimental setup. (1) X80 laser interferometer produced by RENISHAW, (2) Interferometry measurement prisms, (3) Air bearing slider, (4) Test lens, (5) Standard focusing lens, (6) Collimator, (7) He-Ne laser, (8) Single-mode fiber, (9) CCD confocal detection system 1, (10) CCD confocal detection system 2, (11) Drive motor, (12) Material sensor, (13) Air sensor, (14) Monitor, (15) Image capture and soft process system.

The environmental conditions in the measurement laboratory are: pressure = (102540 ± 60) Pa, temperature = (20 ± 0.2) °C and relative humidity = (40 ± 4) %. In the experiment, the objective lens of M-20X, N.A. = 0.40 and $f_{L1} = 9.0\text{mm}$ produced by NEWPORT is used as standard focusing $L_1$, the focal length of collecting lens $L_2$ is $f_{L2} = 160\text{mm}$, the CCD used is MTV-2821, the magnification of VPH microscope objective is $N = 10 \times$, DMI used is X80 laser interferometer produced by RENISHAW, and motion rail used is high-precision air bearing slider of 300mm in range and 0.1µm in straightness. The test sphere used is concave, its diameter is 42mm and the ROC of its front surface is $r = -36.697\text{mm}$ obtained by the traveling microscope with the uncertainty of 0.002mm [2].

6.2 Experiment

As shown in Fig. 6, when the test lens $R$ is moved along the optical axis under the measurement, DCRMS uses the null $O_A$ of DCS intensity curve $I_A$ to precisely determine the cat's-eye position A. The null $O_A$ of DCS intensity curve $I_A$ as shown in Fig. 14 corresponds to the cat's-eye position A, and the position coordinate corresponding to $O_A$ is 36.69542mm. Then, the pixel coordinate of the spot centre obtained by VPH $p_A(x,y)$ is (376.6, 309.1).
Similarly, when the test lens $R$ is moved near the confocal position $B$ under the measurement, DCRMS uses the null $O_{B}$ of DCS curve $I_{B}$ to precisely determine the confocal position $B$, and the position coordinate corresponding to $O_{B}$ is $-0.00093\,\text{mm}$. Then, the pixel coordinate of the spot centre obtained by VPH $p_{B}(x,y)$ is $(383.7, 280.5)$. So, the distance between $O_{A}$ and $O_{B}$ is $r_{DMI} \approx -36.6964\,\text{mm}$. As shown in Fig. 12, the pixel distance between $p_{A}(x,y)$ and $p_{B}(x,y)$ is

$$n = \sqrt{(376.6 - 383.7)^2 + (309.1 - 280.5)^2} \approx 29.5.$$  \hspace{1cm} (35)

Then, ROC of the test lens obtained using Eq. (25) is

$$r = r_{DMI} \cos \left( \frac{n \cdot p}{2N \cdot r_{DMI}} \right) = \frac{-0.00093 - 36.69542}{2 \times 10 \times (-0.00093 - 36.69542)} \approx -36.6964\,\text{mm}$$  \hspace{1cm} (36)

The figure error of the test lens obtained by Zygo interferometer is shown in Fig. 15, PV = 0.208\,$\lambda$ and the radius deviation caused by the figure error is 0.037\,$\lambda$. So, the radius obtained after error correction is

$$r = -36.6964 + 0.037 \times 0.0006328 \approx -36.6964\,\text{mm}.$$  \hspace{1cm} (37)

The air sensor of X80 is used to monitor the temperature, pressure and humidity real-timely for compensating the laboratory environment. The errors are: 1) $\sigma_{pa} = 50\,$pa for the
barometer, 2) $\sigma_K = 0.1^\circ C$ for the thermometer, and 3) $\sigma_H = 3\%$ for the hygrometer. The DCRM deadpath is $L_{\text{deadpath}} = 0.6m$. The distance measurement error obtained by Eq. (26) is

$$
\sigma_x = \sqrt{(L_{\text{deadpath}} + r)^2 \cdot \left[ (2.68 \times 10^{-9} \sigma_p)^2 + (-9.27 \times 10^{-7} \sigma_e)^2 + (-1 \times 10^{-4} \sigma_r)^2 \right]}. 
$$

(38)

$$
= \sqrt{(600^2 + 36.6964^2) \cdot \left[ (2.68 \times 10^{-9} \times 50)^2 + (-9.27 \times 10^{-7} \times 0.1)^2 + (-1 \times 10^{-4} \times 3)^2 \right]} \approx 0.001\text{mm}
$$

The axial error caused by the misalignment obtained by Eq. (30) is

$$
\sigma_{\text{axial}} = 1.24 \times 10^{-7} r = 1.24 \times 10^{-7} \times (-36.6964) = -5\text{nm}.
$$

(39)

The relative aperture of the objective lens M-20X is 0.87, the CCD pixel size is $p = 8\mu m$ and signal noise ratio is $SNR = 150:1$, the null error obtained using Eq. (28) is

$$
\sigma_t = -\frac{1.18\lambda}{SNR \cdot (D / f')^2} = -\frac{1.18 \times 0.6328}{150 \times 0.87^2} = -0.007\mu m.
$$

(40)

The figure error of the test lens is $PV = 0.208\lambda$, and the radius error caused by the figure error obtained using Eq. (28) is

$$
\sigma_{\text{figure}} \approx 0.1 \cdot PV = 0.1 \times 0.208 \times 0.6328 = 0.013\mu m.
$$

(41)

The whole measurement error obtained using Eq. (32) is

$$
\sigma_r \approx \sigma_L^2 + 2 \cdot \sigma_{\varepsilon}^2 + \sigma_{\text{axial}}^2 + \sigma_{\text{figure}}^2
$$

$$
= \sqrt{(0.1)^2 + 2 \times (-0.007)^2 + (-0.005)^2 + (0.013)^2} \mu m \approx 0.10\mu m.
$$

(42)

Its relative error is

$$
\delta_r \approx \frac{\sigma_r}{r} \times 100\% \approx \frac{0.10}{36.6964 \times 1000} \times 100\% = 0.00027\% \approx 3\text{ppm}.
$$

(43)

7. Conclusions

The differential confocal radius measurement proposed uses the zero point of DCS axial intensity response to precisely identify the cat's-eye position and confocal position as to achieve the high-precision ROC measurement. The preliminary experiments indicate that the relative error of the differential confocal radius measurement proposed is better than 5ppm, and the approach proposed

1) improves obviously the identification precision at cat's-eye position and confocal position because its linearity and sensitivity are the best at the zero of DCS intensity response curve.

2) has a strong environmental anti-interference capability because it uses the DCS intensity response curve obtained through differential subtraction as focusing criterion.

3) is easily combined with pupil filtering to suppress the focal-depth of standard lens, and consequently improves the identification sensitivity.

The approach proposed has the potential to be a high-accuracy, compact, simple, and low-cost detector for implementations of the radius bench method.

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