Six-port type reflectometer based on reflection measurement system using a standing wave detector in the V-band

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Abstract A general vector network analyzer employs a super-heterodyne scheme to expand a measuring frequency band. The circuit structure is complicated because of implementing mixers, local oscillators, and amplifiers, therefore, the cost is not affordable. In order to simplify the circuit structure and be affordable, we employ a homodyne scheme, that is the six-port method. In this report, we demonstrated the performance of a six-port reflectometer in the V-band based on a waveguide standing wave detector.

Key words: V-band, six-port method, reflectometer, system parameter, standing wave detector

Classification: Microwave and millimeter wave devices, circuits, and hardware

1. Introduction

Current vector network analyzer (VNA) is based on a super-heterodyne scheme because it is easy to expand the measuring frequency bands. The expandability of the measuring frequency is realized by combining an intermediate frequency unit with an external frequency extending instrument. However, a measurement in the millimeter-wave band is complicated and expensive to prepare multiple frequency-multipliers and converters. Therefore, it is an entering barrier to small and medium sized enterprises (SMEs) to be necessity of a high capital investment for millimeter-wave measurements. An alternative is the homodyne scheme, which is the so-called six-port technique \([1–6]\), and many calibration methods \([7–23]\) for six-port circuits. It is combined with a linear six-port network and four power detectors, which make it possible to obtain vector parameters from the scalar power information. The six-port technique was developed in the early 1970s to realize a low-cost and accurate method to calculate the amplitude and phase for a device under test (DUT) in the microwave measurement.

In this report, we demonstrated a reflection measurement in the V-band based on a waveguide standing wave detector and spectrum analyzer with a harmonic mixer setup and then confirmed the feasibility of a low-cost millimeter-wave measurement configuration.

2. Measurement Theory

2.1 Six-port type reflectometer

A six-port type reflectometer (SPR) is shown in Fig.1. An oscillator is connected to the port \(P_1\), and a DUT is connected to the port \(P_2\), respectively. Four ports of \(P_3\), \(P_4\), \(P_5\), and \(P_6\) are power reading ports induced by the SPR.

![Fig. 1. Fundamental configuration of a six-port type reflectometer.](image)

Using an incident wave of \(a\) and a reflected wave of \(b\), the reflection coefficient \(\Gamma\) is described as

\[
\Gamma = \frac{a_2}{b_2}. \tag{1}
\]

The bold character describes a complex number. The reflection coefficient \(\Gamma\) of the DUT is obtained from the following theory. The SPR is configured by a linear time-invariant circuit and a single-mode incident wave traveling to the SPR, therefore, the incident waves of \(b_h\) at the four power detectors are given by the equation

\[
b_h = A_h a_2 + B_h b_2 \quad (h = 3, 4, 5, 6). \tag{2}
\]

Both \(A_h\) and \(B_h\) are complex constants in frequency dependence. The induced powers of \(P_h\) for the four detectors are proportional to the square of an absolute value of \(b_h\).

Using proportional constant of \(\alpha_h\), the induced powers are expressed as

\[
P_h = \alpha_h |b_h|^2 = \alpha_h |A_h a_2 + B_h b_2|^2. \tag{3}
\]
The power ratios of $P_3$ using $P_3$ on the port $P3$ is given by the equation

$$3P_3 = \frac{P_3}{P_3} = \frac{\alpha_k|A_k a_2 + B_k b_2|^2}{\alpha_3|A_3 a_2 + B_3 b_2|^2} = \frac{\alpha_k|B_k|^2 + \frac{A_k a_2 + B_k b_2}{\delta_3} + 1|^2}{\alpha_3|B_3|^2 + \frac{A_3 a_2 + B_3 b_2}{\delta_3} + 1|^2} = 3 K_i (\Gamma + 1)^2. \quad (4)$$

Three real numbers ($3 K_i$) and four complex numbers $k_i$ is system parameters for the SPR, as the following

$$3 K_i = \frac{\alpha_k|B_k|^2}{\alpha_3|B_3|^2}, \quad k_i = \frac{A_k}{B_k}. \quad (5)$$

The accuracy of these system parameters affects the DUT measurement result. In this study, we employed an easy phase shifter method based on a matched load and phase shifter [24, 25]. The system parameters have to be decided before the $\Gamma$ measurement. Equation (4) represents the intersection of three circles on the complex plane, which is equivalent to the reflection coefficient $\Gamma$.

### 2.2 Calculation of system parameters using matched load and phase shifter

In order to obtain $\Gamma$ for a DUT, system parameters in equation (4) have to be calculated in advance. First, three real system parameters, $3 K_i$, should be given. If a matched load is connected to the port $P2$, the incident wave will be zero ($a_2 = 0$). Then, equation (3) is expressed as

$$P_h = \alpha_h|B_h b_2|^2 = P_{hr}. \quad (6)$$

The $P_{hr}$ denotes the four standard port power readings. Using the standard port power, real system parameters are expressed as

$$3 K_i = \frac{\alpha_k|B_k|^2}{\alpha_3|B_3|^2} = \frac{\alpha_k|B_k|^2}{\alpha_3|B_3|^2} = \frac{P_{hr}}{P_{hr}}. \quad (7)$$

Next, the normalized port power is obtained by dividing $P_h$ of each port with $P_{hr}$ of the standard port power

$$\overline{P_h} = \frac{P_h}{P_{hr}} = \frac{\alpha_h|A_h a_2 + B_h b_2|^2}{\alpha_k|B_h b_2|^2} = \frac{|A_h a_2 + B_h b_2|^2}{|B_h b_2|^2} = \frac{1}{\delta_3} (\Gamma + 1)^2. \quad (8)$$

According to the easy phase shifter method, the port power readings of $P_0$, $P_1$, and $P_2$ are measured by varying the phase value of the variable short on a reference plane. First, a standard phase value is set for $\overline{P_{hr}}$, which is equivalent to $\Gamma = -1$. Next, concerning both $P_{hr}$ and $P_{hr}$ are measured by each phase gap of $120^\circ$, approximately. Consequently, the reflection coefficient from the reference plane is expressed as

$$\Gamma_n = |\Gamma_n| e^{j\theta_n} (n = 0, 1, 2),$$

and each reflection coefficient

$$\Gamma_0 = -1. \quad \text{Therefore, the normalized port power readings for each phase value } \theta_n \text{ are expressed as}$$

$$\overline{P_{hn}} = |k_h + \frac{1}{\Gamma_n} e^{-j\theta_n}|^2. \quad (9)$$

The complex system parameters $k_h$ in equation (9) represents three circles on the complex plane. Each circle is $(-1/|\Gamma_n|) e^{-j\theta_n}$ and each radius is $\sqrt{\overline{P_{hn}}}$. The expanded equation (9) is given by

$$\overline{P_{hn}} = |1 + |\Gamma_n| k_h e^{j\theta_n}|^2$$

$$= 1 + |\Gamma_n| k_h e^{j\theta_n} + |\Gamma_n| k_h e^{-j\theta_n} + (|\Gamma_n| |k_h|)^2. \quad (10)$$

Because of the three phase values $\theta_n$, a simultaneous equation can be established using a matrix form.

$$\begin{pmatrix} \overline{P_{h0}} - 1 \\ \overline{P_{h1}} - 1 \\ \overline{P_{h2}} - 1 \end{pmatrix} = \begin{pmatrix} |\Gamma_0|^2 \\ |\Gamma_1|^2 \\ |\Gamma_2|^2 \end{pmatrix} e^{j\theta_0} + |\Gamma_0| k_h e^{-j\theta_0} + (|\Gamma_0| |k_h|)^2 \begin{pmatrix} k_0 \\ k_1 \\ k_2 \end{pmatrix}$$

From equation (11), four complex system parameters $k_i$ are obtained. Each system parameter forms a circle on the complex plane and then they have theoretically an intersection, which is the calculated system parameter.

### 2.3 Calculation of reflection coefficient $\Gamma$ for a DUT

In this subsection, the reflection coefficient $\Gamma$ for a DUT is calculated using the four calculated complex system parameters. First, equation (8) is expanded as

$$\overline{P_h} = |k_h \Gamma|^2 + k_h \Gamma + k_h \Gamma + 1$$

$$\overline{P_h} - 1 = |k_h \Gamma|^2 + k_h \Gamma + k_h \Gamma. \quad (12)$$

Using the matrix form, the $\Gamma$ is based on four circles on the complex plane, as the following equation. The intersection of these circles is the reflection coefficient $\Gamma$.

$$\begin{pmatrix} \overline{P_3} - 1 \\ \overline{P_4} - 1 \\ \overline{P_5} - 1 \\ \overline{P_6} - 1 \end{pmatrix} = \begin{pmatrix} |k_3|^2 \\ |k_4|^2 \\ |k_5|^2 \\ |k_6|^2 \end{pmatrix} e^{j\theta_3} + \begin{pmatrix} k_3 \k_4 \k_5 \k_6 \end{pmatrix} e^{-j\theta_3} \begin{pmatrix} |\Gamma|^2 \\ \Gamma \end{pmatrix} \quad (13)$$

### 3. Experimental setup for V-band reflectometer

To confirm the measurement theory in the previous section in the V-band, we configured the SPR function as shown in Fig. 1, using a standing wave detector in the U-band (40–60 GHz). A reflectometer in the V-band based on the NRD guide [29, 30] was already proposed, we employed a waveguide standing wave detector acquiring a measurement bandwidth. The detector detects the standing waves inside a waveguide putting a probe, as shown in Fig. 2, and then, it can act as a six-port type reflectometer. In fact, by adjusting
the position of the probe, the detector can play a role of a six-port type reflectometer.

Fig. 3 shows a reflection coefficient measurement system in the V-band based on the U-band standing wave detector. The detector is a SPC Electronics 3S601A U-band standing wave detector, which complies with the EIAJ standard waveguide flange of WR-1500. Therefore, a conversion waveguide from the EIAJ standard to the EIA standard and a WR-19 to WR-15 tapered waveguide are prepared. The picked up power from the detector is measured using the Rohde & Schwarz signal spectrum analyzer FSW67 via a WR-15 waveguide to a 1.85 mm coaxial transducer with a low-loss coaxial cable.

Actually, the SPR has four measuring ports \( P_3, P_4, P_5, P_6 \), as shown in Fig. 1, in this study, these ports are set at the positions of 0.0 mm, 2.0 mm, 4.0 mm, and 6.0 mm on the standing wave detector. These positions are adjusted using a dial gauge on the standing wave detector. The induced power at the four ports is measured using a VNA in advance. To obtain the four system parameters in equation 11, connecting the variable short to the port \( P_2 \), four powers are measured for the position of 0.0 mm, 2.0 mm, 4.0 mm, and 6.0 mm, respectively, on the standing wave detector while changing the other three positions of the variable short. Since the loss in the waveguide short is negligible small, adapting \( \Gamma_n = e^{j\theta_n} \) (\( n = 0, 1, 2 \)).

4. Experimental setup

4.1 Measurement methods

In the measurement, a combination of a tapered waveguide transition from WR-15 to WR-10, and a WR-10 matched load is prepared. Using the DUT, the transition behavior can be observed at the cut-off frequency of 59.015 GHz in the TE\(_{10}\) fundamental mode of the WR-10. Therefore, the measurement frequency is set by 56.0–64.0 GHz and a step of 0.5 GHz. The reflection coefficient on the SPR is compared with the measured reflection coefficient using a Keysight N5247A [31] VNA.

Moreover, in the measurement, it is expected that there is no intersection of the three circles because of uncertainty of calibration standards and manual operation. Thus, the \( \Gamma \) of the DUT is obtained from equation 11 using a radical center.

\[
\begin{pmatrix}
    k_3^2 & k_3 & 1 & 1 \\
    k_3 & 1 & 1 & 1 \\
    1 & 1 & 1 & 1 \\
    1 & 1 & 1 & 1
\end{pmatrix}
\begin{pmatrix}
    |\Gamma_0|^2 & |\Gamma_1|^2 & |\Gamma_2|^2 & |\Gamma_3|^2 \\
    e^{-j\theta_0} & e^{-j\theta_1} & e^{-j\theta_2} & e^{-j\theta_3} \\
    e^{j\theta_0} & e^{j\theta_1} & e^{j\theta_2} & e^{j\theta_3} \\
    k_3 & k_3 & k_3 & k_3
\end{pmatrix}
\frac{k_3}{k_3}
\]

From this, solving the above simultaneous equation, it is possible to obtain the system parameter of \( k_3 \). For \( h = 4, 5 \), it is also possible to obtain the remaining two system parameters.
Fig. 5. Calculated complex system parameter obtaining intersection of three circles. Centers are phases of variable short, and radii are normalized port powers.

Fig. 6. Calculated complex system parameters of SPR in V-band at 60 GHz.

Fig. 7. $\Gamma$ at 60 GHz.

Fig. 8. SPR including directional coupler.
4.2 Calculation method using pre-acquired magnitude of reflection coefficient

The $\Gamma$ of a DUT is obtained using the four measured powers on the four ports, as described in the previous section. In this subsection, we attempted to use the pre-acquired magnitude of a reflection coefficient using a directional coupler for obtaining the phase information of the DUT. The experimental setup is shown in Fig.8. The directional coupler is added to the output port of the standing wave detector and then the magnitude $|\Gamma|$ of the DUT is measured using a FSW67 signal spectrum analyzer. Using the magnitude, $\Gamma$ is also obtained by solving the following simultaneous equation.

\[
\begin{align*}
\left( \begin{array}{c}
P_3 \\
P_4
\end{array} \right) &= \left( \begin{array}{c}
|t_3\Gamma |^2 + 1 \\
|t_4\Gamma |^2 + 1
\end{array} \right) \\
\left( \begin{array}{c}
\frac{t_3}{P_3} \\
\frac{t_4}{P_4}
\end{array} \right) &= \left( \begin{array}{c}
\Gamma \\
\Gamma
\end{array} \right)
\end{align*}
\]

Equation 15 is changed to a matrix form and then the phase information can be obtained using the equation.

\[
\begin{pmatrix}
\frac{t_3}{P_3} & \frac{t_3}{P_3} \\
\frac{t_4}{P_4} & \frac{t_4}{P_4}
\end{pmatrix}
\begin{pmatrix}
\Gamma \\
\Gamma
\end{pmatrix}
= \begin{pmatrix}
\frac{P_3}{|t_3\Gamma|^2 - 1} \\
\frac{P_4}{|t_4\Gamma|^2 - 1}
\end{pmatrix}
\]

4.3 Results and discussion

The $\Gamma$ was compared with the results on the Keysight N5247A [31], six-port SPR, and six-port SPR with a directional coupler, as shown in Fig.9. The DUT was the combination of a tapered waveguide transition and matched waveguide load. The transition behavior close to 59 GHz was obviously confirmed for each measurement result. The above results are good agreement with the N5247A measurement result.

5. Conclusion

In this study, we verified that $\Gamma$ measurement of a DUT can be available using a SPR in the V-band. The measurement results are corresponding to the results measured by a standard super-heterodyne based VNA. In this study, we used a standing wave detector to pick up four power readings in manual, mechanically. The configuration can be exchanged with a passive waveguide component. Although the six-port method is operated at a limited frequency band because of the homodyne configuration and a cut-off behavior of waveguide, it is possible to build an affordable millimeter-wave vector measurement instrument. It mentions the six-port vector measurement method can remove an entering barrier for the SMEs.

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