Stability analysis of Hoist Vertical Shiplift chamber under pitching motion

Shi Lang\textsuperscript{1,2}, Zhang Yang\textsuperscript{2}, Chengxiong Hao\textsuperscript{2}, Zhou Ji\textsuperscript{2}, Shiduan Wei\textsuperscript{1,2*} and Xia Re \textsuperscript{1,2*}

\textsuperscript{1} Key Laboratory of Hydraulic Machinery Transient (Wuhan University), Ministry of Education, Wuhan 430072, China
\textsuperscript{2} School of Power and Mechanical Engineering, Wuhan University, Wuhan 430072, China
\textsuperscript{*} Corresponding author’s e-mail: dwshi@whu.edu.cn; xiare@whu.edu.cn

Abstract. Aiming at the stability problem of large Hoist Vertical Shiplift chamber under pitching motion. A fluid dynamic model based on the Housner theory is built to study the sloshing in the ship chamber. The theoretical formulas of the capsizing moments are derived to research the effect of sloshing. To study the whole shiplift pitching stability, the system coupled by pitching motion, torsional vibration of main hoist and sloshing is built. Then, the critical distance of lifting points is obtained by using the Hurwitz theory. Taking seven built typical Hoist Vertical Shiplifts as examples, it is verified that the expression about the critical distance of lifting points is rational. The calculation results show that the sloshing in the ship chamber and the capsizing moment produced by the convective pressure acting on the sidewall of ship chamber could not be neglected in the design of the Hoist Vertical Shiplift. According to the successful experience of the Hoist Vertical Shiplift, it is suggested that the safety factor $S$ of the shiplift under pitching motion should be more than 1.5.

1. Introduction

Hoist Vertical Shiplift is an effective means to pass through high dams, and it is an important part of permanent navigation structures in water conservancy projects \cite{14}. Shiplift system is a complex system which is coupled by many subsystems. It is necessary to study the motion stability of such system \cite{11,12}. However, when the shiplift chamber is subjected to initial disturbance, the large mass of water in the chamber will produce huge capsizing moments due to sloshing, and there is a danger of the chamber being unstable and overturning. Once overturned, huge property loss and casualties will be generated. Therefore, it is urgent to analyze the stability of the whole shiplift system.

The dynamic characteristics of vertical shiplift are essentially different from those of general lifting machinery because of its large mass of water. The dynamic state description of the water in the shiplift chamber is the first problem to be solved in the dynamic analysis of the shiplift chamber structure and even the suspension system of the whole shiplift \cite{10}. Chen \cite{1} treated the water in the ship chamber as static water without considering the sloshing. From the principle of force balance, the calculation formula for the instability of the ship chamber was derived. Based on the Housner theory, Liao and Shi \cite{11,12} obtains the sloshing hydrodynamic pressure and capsizing moment caused by pitching motion,
but the impulse pressure and convective pressure acting on the side wall of the chamber are neglected in the calculation of hydrodynamic pressure. Shi et al. [16] establishes a coupled parametric nonlinear mathematical model of sloshing and vertical vibration of water-holding ship chamber suspension system under external motion based on the theory of hydrodynamic potential flow, and takes the hoisting scheme of Three Gorges shiplift as an example to simulate and calculate. However, the research on hydrodynamic pressure in ship chamber under pitching motion is not complete.

The coupling dynamic model of shiplift is also very important for the study of the pitching stability of the whole shiplift. Ruan and Cheng [17] established the nonlinear water wave equation and the ship-water-chamber coupling motion equation. The nonlinear water wave equation was decomposed by using perturbation theory. Then the coupled finite element discrete equation was obtained by Galerkin method, and the time domain analysis was carried out by combining the precise integration method. Li et al. [14] established a simplified mechanical model including the balance weight subsystem, the main lifting subsystem and the hull subsystem, derived the dynamic equations of the coupled system, and calculated the response of the water body, the hull and the motion of the ship under earthquake by using the mode superposition method. Cheng et al. [3] established the fluid-structure coupling system, the dynamic equation considering the fluid-structure coupling was established, and the natural vibration characteristics of the shiplift system was obtained, they discussed the influence of the balance weight mass, the lifting point position of the chamber, and the ship in the chamber on the natural vibration characteristics of the system, and calculated the critical lifting point position to maintain the stability of the system, but for a key index in the stability design of shiplift, critical distance of lifting points, there is no specific formula suitable for engineering application.

In this study, based on the Housner theory, the capsizing moment expression considering impulse pressure and convection pressure is deduced, and the dynamic model considering pitching motion, sloshing and torsional vibration of main hoist is established. Taking seven typical Hoist Vertical Shiplifts as examples, the rationality of formula of Hoist Vertical Shiplift is verified.

2. Hydrodynamic pressure and capsizing moment

As shown in figure 1, shiplift chamber is a rectangular shallow water container with a width of $B$, $h$ is the depth of water, $\theta_0$ is the fluid free surface rotation angle, the Cartesian coordinate system is located at $O$ which is the fluid free surface center point, the total length is $2l$, the pitching motion angle is $\alpha$. 

![Figure 1: Location of the shiplift chamber and after pitching motions](image)

When the ship chamber is subjected to pitching motion, the shallow water sloshing. According to the Housner theory [5,6], the hydrodynamic pressure caused by sloshing can be divided into two parts, Impulsive pressure and Convective pressure.

2.1. Impulsive pressure

Pitching motion produces horizontal pulse motion with the acceleration of $u_0(t)$ on the side wall of the ship chamber. Under the action of Impulsive motion, Impulsive pressure is generated. As shown in figure 2, we assume that the water body is constrained by some vertical films (the shadow part in figure 2).
Figure 2. Impulsive pressure under pitching motion

According to the Housner theory, the expression of the Impulsive pressure inside the fluid is as follows:

\[
p = -\sqrt{3}\rho \ddot{u}_0 h \left[ \frac{y}{h} - \frac{1}{2} \left( \frac{y}{h} \right)^2 \right] \frac{\sinh \sqrt{3} \frac{x}{h}}{\cosh \sqrt{3} \frac{l}{h}}
\]

(1)

in which, the density of fluid is \( \rho \), and the \( u_0(t) \) is the horizontal acceleration acting on the side wall of the ship chamber.

The hydrodynamic pressure \( p_{tw} \) acting on the right wall and the hydrodynamic pressure \( p_{IB} \) acting on the bottom can be obtained by substituting \( x = L \) and \( y = h \) into Eq. (1) respectively.

\[
p_{tw} = -\sqrt{3}\rho \ddot{u}_0 h \left[ \frac{y}{h} - \frac{1}{2} \left( \frac{y}{h} \right)^2 \right] \frac{\sinh \sqrt{3} \frac{x}{h}}{\cosh \sqrt{3} \frac{l}{h}}
\]

(2)

\[
p_{IB} = -\frac{\sqrt{3}}{2}\rho \ddot{u}_0 h \frac{\sinh \sqrt{3} \frac{x}{h}}{\cosh \sqrt{3} \frac{l}{h}}
\]

(3)

Substitution of Eqs. (2)-(3) into \( M_{tw} = 2B \int_0^h p_{tw} (h - y) dy \), \( M_{IB} = B \int_1^l p_{IB} x dx \), the capsizing moments \( M_{tw} \) produced by the Impulsive pressure on the side wall and \( M_{IB} \) produced by the Impulsive pressure at the bottom can be obtained.

\[
M_{tw} = \frac{\sqrt{3}}{4}\rho Bu_0 h^3 \tanh \sqrt{3} \frac{l}{h}
\]

(4)

\[
M_{IB} = \rho Bu_0 h^2 l \left( 1 - \frac{\tanh \sqrt{3} \frac{l}{h}}{\sqrt{3} \frac{l}{h}} \right)
\]

(5)

2.2 Convective pressure

The fluid itself is excited into oscillations as \( \theta_0(t) \) while the ship chamber is subjected to pitching motion \( \alpha(t) \). According to the Housner theory, only the first-order modes of the fluid are considered. It is assumed that the water body consists of a thin slice of unit thickness (the shaded part in Fig. 3), the upper and lower surfaces of which are rigid films that can rotate freely.

Figure 3. Convective pressure under pitching motion

The components of any thin slice in \( x \) and \( y \) directions are \( u \) and \( v \). Under incompressible conditions, the fluid follows the continuity equation:

\[
v = x \dot{\theta}
\]

(6)

\[
\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 0
\]

(7)
where $\theta$ is the rotational angular velocity of $y$ film in the depth of water.

According to Hamilton's principle [7], $\dot{\theta}$ should satisfy the following equations:

$$\frac{\partial^2 \dot{\theta}}{\partial y^2} - \frac{I}{K} \dot{\theta} = 0$$

(8)

$$\frac{\partial^2 (\frac{\partial \theta}{\partial t})}{\partial y^2} \bigg|_{y=0} - \frac{g I}{K} \frac{\partial \theta}{\partial y} \bigg|_{y=0} = 0$$

(9)

in which the $I$ and $K$ are respectively related to the geometric dimensions of the chamber.

For the ship chamber under pitching motion, the boundary conditions for $\theta$ should be: When $y = 0$, $\theta = \theta_0$, $\dot{\theta} = \dot{\theta}_0$; when $y = h$, $\theta = \alpha$, $\dot{\theta} = \dot{\alpha}$. Under this boundary condition, Eqs. (10)-(12) can be obtained:

$$\theta = \theta_0 \cosh \frac{5y}{2l} + \frac{\alpha - \theta_0 \cos \frac{5h}{2l}}{\dot{\theta}_0 \sinh \frac{5h}{2l}} \sinh \frac{5y}{2l}$$

(10)

$$\dot{\theta}_0 + \omega^2 \theta_0 = \frac{\cosh \frac{5h}{2l}}{\omega^2}$$

(11)

$$\omega^2 = g \frac{5}{2l} \tanh \frac{5h}{2l}$$

(12)

where $\omega$ is the first natural frequency of the fluid in the chamber.

According to the Euler equation, the convective pressure $p_{CW}$ acting on the side wall and the convective pressure $p_{CB}$ acting on the bottom are:

$$p_{CW} = -\rho \frac{5l^2}{23} \left( \dot{\theta}_0 \sinh \frac{5y}{2l} + \frac{\alpha - \dot{\theta}_0 \cosh \frac{5h}{2l}}{\sinh \frac{5h}{2l}} \cosh \frac{5y}{2l} \right)$$

(15)

$$p_{CB} = -\frac{2}{5} \rho x \left[ \cosh \frac{5h}{2l} - \frac{1}{\sinh \frac{5h}{2l}} \frac{\alpha}{2} \left( \sinh \frac{5h}{2l} \dot{\theta}_0 \right) \right]$$

(16)

By substituting Eqs. (14)-(15) into $M_{CW} = B \int \theta^h p_{CW} (h - y) dy$ and $M_{CB} = B \int x p_{CB} dx$, the capsizing moment $M_{CW}$ generated by convective pressure on the side wall and $M_{CB}$ generated by convective pressure at the bottom can be obtained.
\[ M_{CW} = \frac{1}{3} B \rho l^3 \left[ -h + \frac{2}{5} l \sinh \left( \frac{5h}{2l} \right) \bar{\theta}_0 + \frac{\bar{\alpha} - \bar{\theta}_0 \cosh \left( \frac{5h}{2l} \right)}{\sinh \left( \frac{5h}{2l} \right)} \left( \cosh \left( \frac{5h}{2l} \right) - 1 \right) \right] \]  (17)

\[ M_{CB} = -\frac{8}{45} \rho Bl^4 \left[ \bar{\theta}_0 \sinh \left( \frac{5h}{2l} \right) + \frac{\bar{\alpha} - \bar{\theta}_0 \cosh \left( \frac{5h}{2l} \right)}{\sinh \left( \frac{5h}{2l} \right)} \left( \cosh \left( \frac{5h}{2l} \right) - 1 \right) \right] \]  (18)

### 2.3 Calculation of capsizing moment

The capsizing moment produced by the pitching motion consists of four parts: the capsizing moment \( M_{IW}, M_{IB}, M_{CW} \) and \( M_{CB} \), the four capsizing moments are calculated with specific examples.

| Shiplift names       | \( M_{IW} / (N \cdot m) \) | \( M_{IB} / (N \cdot m) \) | \( M_{CW} / (N \cdot m) \) | \( M_{CB} / (N \cdot m) \) |
|----------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| ShuiKou shiplift     | 72                          | 3710                        | 56000                       | 247000                      |
| PengShui shiplift    | 262                         | 6800                        | 105000                      | 1290000                     |
| GeHeyan shiplift     | 97                          | 2630                        | 40300                       | 545000                      |
| GouPitan shiplift    | 262                         | 6800                        | 105000                      | 1290000                     |
| GaoBazhou shiplift   | 97                          | 2630                        | 40300                       | 545000                      |
| TingZikou shiplift   | 70                          | 3600                        | 55000                       | 251000                      |
| YanTan shiplift      | 153                         | 3570                        | 55300                       | 214000                      |

It can be concluded from Table 1 that the difference between \( M_{IW} \) and \( M_{IB} \) is more than one order of magnitude compared with \( M_{CW} \) and \( M_{CB} \), and the influence of \( M_{IW} \) and \( M_{IB} \) on the calculation of hydrodynamic capsizing moment is very little. Therefore, when the hydrodynamic capsizing moment is calculated, only the influence of \( M_{CW} \) and \( M_{CB} \) is considered, while the influence of \( M_{IW} \) and \( M_{IB} \) is ignored.

The capsizing moment \( M \) produced by fluid consists of two parts: the hydrodynamic capsizing moment \( M_{CW} + M_{CB} \) and the capsizing moment \( M_s \) caused by uneven water depth.

\[ M = M_{CW} + M_{CB} + M_s = -J_s \bar{\theta}_0 - J_{s1} \bar{\alpha} + C_s \alpha \]  (19)

in which \( J_s \), \( J_{s1} \) and \( C_s \) can be expressed as:

\[ J_s = \frac{1}{3} B \rho l^3 h + \frac{18}{45} B \rho l^4 \left[ \sinh \left( \frac{5h}{2l} \right) \right] - \frac{1}{\tanh \left( \frac{5h}{2l} \right)} \left( \cosh \left( \frac{5h}{2l} \right) - 1 \right) \]  (19a)

\[ J_{s1} = \frac{18}{45} B \rho l^4 \left( \cosh \left( \frac{5h}{2l} \right) - 1 \right) \]  (19b)

\[ C_s = \frac{2}{3} \rho g B l^3 \]  (19c)

According to parameters of the large shiplift chamber, we can obtain \( \sqrt{\frac{5h}{2l}} \ll 1 \), \( \cosh \left( \sqrt{\frac{5h}{2l}} \right) \approx 1 \) and \( \sinh \left( \sqrt{\frac{5h}{2l}} \right) \approx \sqrt{\frac{5h}{2l}} \).
1, \cos h \left( \frac{\sqrt{2} h}{2} \right) - 1 \approx \frac{5}{4} \left( \frac{h}{2} \right)^2 \quad \text{and} \quad \sinh \left( \frac{\sqrt{2} h}{2} \right) \approx \frac{\sqrt{5} h}{2}. \quad \text{Since} \quad M \text{ can be simplified as:}

\[ M = -\frac{1}{6} B \rho l^3 \dot{h}_0 - \frac{1}{2} \rho B l^3 h \ddot{u} + C_2 \alpha \]

(20)

3. Dynamical model

As showed in figure 4, The dynamic model of the pitching motion, the torsional vibration of the main hoist and sloshing have been established from the work of Liao and Shi [11,12].

\[ J_1 \ddot{\phi}_1 + \gamma_1 \phi_1 + k \left( R \phi_1 - y - \frac{1}{2} a \alpha \right) R + C (\phi_1 - \phi_2) + \beta C \phi_1 = 0 \]  

(21a)

\[ J_1 \ddot{\phi}_2 + \gamma_1 \phi_2 + k \left( R \phi_2 - y + \frac{1}{2} a \alpha \right) R - C (\phi_1 - \phi_2) + \beta C \phi_2 = 0 \]  

(21b)

\[ J_2 \ddot{\alpha} + \gamma_2 \ddot{\alpha} - \frac{1}{2} a k \left( R \phi_1 - y - \frac{1}{2} a \alpha \right) + \frac{1}{2} a k \left( R \phi_2 - y + \frac{1}{2} a \alpha \right) = M \]  

(21c)

where \( \gamma_1 \) is the damping coefficient of the torsional vibration of the main hoist, \( \gamma_1 = 2 \xi_1 \omega_1 J_1 \), \( \xi_1 \) is the damping ratio of the torsional vibration of the main hoist, \( \omega_1 \) is the natural frequency of the torsional vibration of the main hoist, \( J_1 \) is the equivalent moment of inertia of the main hoist system on the upstream (or downstream) side of the shiplift converted to the low speed shaft; \( \gamma_2 \) is the damping coefficient of the pitching motion of the shiplift chamber, \( \gamma_2 = 2 \xi_2 \omega_2 J_2 \), \( \xi_2 \) is the damping ratio of the torsional vibration of the shiplift chamber, \( J_2 \) is the moment of inertia for a chamber carrying a pitching motion around the center of mass. \( \omega_2 \) is the natural frequency of torsional vibration; \( a \) is the center distance of the vertical lifting point of the main hoist; \( \phi_1, \phi_2 \) are the angles of the main hoist drum (the upper or downstream sides) on the low speed shaft; \( R \) is the drum radius; \( \beta C \) is the equivalent stiffness which is transformed from drive shaft to low speed shaft. The drive shaft is from the main hoist motor to the drum; \( K \) is the sum of the rope stiffness for the upstream (or downstream) side.

By using the method of variable substitution, and order \( \phi = \phi_1 - \phi_2 \), The coupling dynamic equations of torsional vibration, pitching motion and sloshing are obtained by substituting the variables of the Eqs. (11)-(12) for Eq. (21).

\[ \ddot{\phi} + C_{10} \phi + C_{11} \phi - C_{12} \alpha = 0 \]  

(22a)

\[ \ddot{\alpha} + C_{20} \ddot{\alpha} - C_{21} \phi + C_{22} \alpha + C_{23} \dot{\theta}_0 + C_{24} \theta_0 = 0 \]  

(22b)

\[ \ddot{\theta}_0 + C_{31} \theta_0 - C_{32} \ddot{\alpha} = 0 \]  

(22c)

where \( C_{10} = \frac{\gamma_1}{J_1} = 2 \xi_1 \sqrt{C_{11}} = 0.2 \sqrt{C_{11}} \), \( C_{11} = \frac{kr^2 + (\beta + 2) c}{J_1} \), \( C_{12} = \frac{ak}{J_1} \), \( C_{20} = \frac{\gamma_2}{(J_2 + J_{s1})} \)

\[ 2 \xi_2 \sqrt{C_{22}} = 0.1 \sqrt{C_{22}} \]  

\[ C_{21} = \frac{3}{2} \frac{akR}{(J_2 + J_{s1})} \]  

\[ C_{22} = \frac{3}{2} \frac{ka^2 - c_4}{J_2 + J_{s1}} \]  

\[ C_{23} = \frac{J_s}{(J_2 + J_{s1})} \]  

\[ C_{24} = - \frac{c_4}{(J_2 + J_{s1})} \]  

\[ C_{31} = \frac{g}{K} \tanh \left( \frac{\sqrt{2}}{K} h \right) \]  

\[ C_{32} = \frac{1}{c_5 \sqrt{K} h} \]  

For large shiplift, \( C_{23} \ll 1 \), \( C_{31} \approx \frac{5 \rho h}{24 l^2} \), \( C_{32} \approx 1 \) and \( C_{11} \gg C_{22}, C_{31} \gg C_{32}, C_{11} \gg C_{24}, C_{11} \gg C_{31} \).
4. Stability analysis for shiplift system
In order to analyze the stability of the system, the Eqs. (27)-(28) are changed as follows
\[
\begin{align*}
(C_3 C_{32} + 1) \ddot{\alpha} + C_{20} \dot{\alpha} + C_{22} \alpha - C_{24} \varphi - (C_{23} C_{31} - C_{24}) \theta_0 &= 0 \quad (23a) \\
(1 + C_{32} C_{23}) \ddot{\theta}_0 + (C_{31} + C_{32} C_{24}) \theta_0 + C_{32} C_{20} \dot{\alpha} + C_{32} C_{22} \alpha - C_{32} C_{21} \varphi &= 0 \quad (23b)
\end{align*}
\]
It is difficult to judge the stability of the system directly by the coupling dynamic equations of torsional vibration, pitching motion and sloshing. So we transform the dynamic equation into matrix form. Let \[x_1 = \dot{\varphi}, \ x_2 = \varphi, \ x_3 = \dot{\alpha}, \ x_4 = \alpha, \ x_5 = \dot{\theta}_0 \text{ and } x_6 = \theta_0.\] The Eq. (22a), Eq. (23a) and Eq. (23b) can be changed as
\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4 \\
\dot{x}_5 \\
\dot{x}_6
\end{bmatrix} =
\begin{bmatrix}
-C_{10} & -C_{11} & 0 & 0 & C_{12} & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & -C_{20} & -C_{22} & 0 & 0 & C_{23} - C_{24} \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -C_{20} & 1 & 0 & 0 \\
0 & 0 & 0 & -C_{20} & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix}
\]
By analyzing the stability of the matrix characteristic equation, the stability of the system can be judged. The matrix characteristic equation is as follows:
\[
r^6 + a_1 r^5 + a_2 r^4 + a_3 r^3 + a_4 r^2 + a_5 r + a_6 = 0 \quad (24)
\]
in which
\[
a_1 = \frac{C_{10} (C_{33} C_{32} + 1) + C_{20}}{1 + C_{33} C_{32}} \approx 0.2 \sqrt{C_{11}} + 0.1 \sqrt{C_{22}} \quad (24a)
\]
\[
a_2 = \frac{C_{22} + C_{11} + C_{31} + C_{10} C_{20} + C_{24} C_{32} + C_{11} C_{23} C_{32}}{1 + C_{33} C_{32}} \approx C_{11} + 0.02 \sqrt{C_{22} C_{11}} \quad (24b)
\]
\[
a_3 = \frac{C_{10} (C_{24} C_{32} + C_{31} + C_{22}) + (C_{31} + C_{11}) C_{20}}{1 + C_{33} C_{32}} \approx 0.1 C_{11} \sqrt{C_{22}} \quad (24c)
\]
\[
a_4 = \frac{-C_{12} C_{21} + (C_{10} C_{20} + C_{22}) C_{31} + C_{11} (C_{22} + C_{31} + C_{24} C_{32})}{1 + C_{33} C_{32}} \approx C_{11} C_{20} C_{31} \quad (24d)
\]
\[
a_5 = \frac{(C_{11} C_{22} - C_{12} C_{21}) C_{31}}{1 + C_{33} C_{32}} \approx C_{11} C_{22} C_{31} \quad (24e)
\]
\[
a_6 = \frac{(C_{11} C_{22} - C_{12} C_{21}) C_{31}}{1 + C_{33} C_{32}} \approx C_{11} C_{22} C_{31} \quad (24f)
\]
According to Hurwitz theory, the stability of the zero solution of the homogeneous linear differential Eq. (24) is judged. The necessary and sufficient conditions for the stability of the whole system are as follows:
\[
\begin{align*}
\Delta_1 &= a_1 > 0 \quad (25a) \\
\Delta_2 &= \begin{vmatrix}
a_1 & 1 \\
a_3 & a_2
\end{vmatrix} > 0 \quad (25b) \\
\Delta_3 &= \begin{vmatrix}
a_3 & a_2 & a_1 \\
a_5 & a_4 & a_3
\end{vmatrix} > 0 \quad (25c) \\
\Delta_4 &= \begin{vmatrix}
a_1 & 1 & 0 & 0 \\
a_3 & a_2 & a_1 & 1 \\
a_5 & a_4 & a_3 & a_2 \\
0 & a_6 & a_5 & a_4
\end{vmatrix} > 0 \quad (25d)
\end{align*}
\]
For the shiplift, Eq. (25a)-(25c) is automatically met, and Eq. (25d) can be expressed as
\[
\Delta_4 = a_1 a_2 (a_3 a_4 - a_2 a_5) - a_1^2 (a_4^2 - a_2 a_6) + a_1 (a_4 a_5 - a_3 a_6) - a_3 (a_3 a_4 - a_2 a_5) \\
+ a_5 (a_1 a_4 - a_5) > 0
\]
It is not difficult to find that \( a_5 a_2 (a_2 a_4 - a_2 a_5) \) is much larger than the other four terms, so the stability condition of the system can be expressed as \( a_3 a_4 - a_2 a_5 > 0 \).

Thus, the expression of the center critical distance \( a_{cp} \) of the lifting hoist can be obtained.

\[
a_{cp} = \frac{\sqrt{2(P + C_s)}}{k} \tag{26}
\]

in which:

\[
P = \frac{\sqrt{T_1 + T_2 - T_1}}{4(\beta + 2)^2 C^2} \tag{26a}
\]

\[
T_1 = 2.5 \times 10^{-3}[k R^2 + (\beta + 2)C(J_2 + J_{s1})] \frac{g^2 h^2}{l^4} \tag{26b}
\]

\[
T_2 = 4[(\beta + 2)C][(\beta + 2)C + 2k R^2] C_s \tag{26c}
\]

5. Calculation example

In order to illustrate the rationality of the formula \( a_{cp} \) for calculating the critical distance of lifting points proposed in this paper, seven typical Hoist Vertical Shiplifts are compared and analyzed, as shown in Table 2.

| Shiplift names        | \( a_0/\text{m} \) | \( a_{cs}/\text{m} \) | \( a_c/\text{m} \) | \( a_{cp}/\text{m} \) | \( S \) | \( \frac{a_{cp} - a_{cs}}{a_{cp}} \times 100 \) | \( \frac{a_{cp} - a_c}{a_{cp}} \times 100 \) |
|-----------------------|-------------------|-------------------|-------------------|-------------------|------|---------------------------------|---------------------------------|
| ShuiKou shiplift      | 75.0              | 21.2              | 21.4              | 30.2              | 2.5  | 29.8                            | 29.1                            |
| PenShui shiplift      | 36.0              | 8.3               | 8.4               | 11.8              | 3.1  | 29.7                            | 28.8                            |
| The first step of GeHeyan shiplift | 23.6          | 5.3               | 5.3               | 7.5               | 3.2  | 29.3                            | 29.3                            |
| The second step of GeHeyan shiplift | 24.0          | 5.1               | 5.2               | 7.3               | 3.3  | 30.1                            | 30.1                            |
| The second step of GouPitan shiplift | 36.2          | 11.6              | 11.7              | 16.6              | 2.2  | 30.1                            | 29.5                            |
| GaoBazhou shiplift    | 26.0              | 5.1               | 5.1               | 7.3               | 3.6  | 30.1                            | 30.1                            |
| TingZikou shiplift    | 60.1              | 26.1              | 26.5              | 37.5              | 1.6  | 30.4                            | 29.3                            |

In Table 2, \( a_0 \) is the design the critical distance of lifting points \([2,9]\); \( a_{cs} \) is the critical distance of lifting points neglecting water sloshing \([9]\); \( a_c \) is the critical distance of lifting points neglecting the capsizing moment \( M_{CW} \) \([4]\); \( S \) is the pitching motion stability safety factor, defined as \( S = a_{cp}/a_0 \).

Comparing \( a_{cp} \) with \( a_{cs} \), the effect of sloshing in the chamber on the critical distance of lifting points is 29.3 to 30.4%, which indicates that sloshing in the chamber could not be neglected in the design of Hoist Vertical Shiplift.

Comparing \( a_{cp} \) with \( a_c \), the influence of the capsizing moment \( M_{CW} \) produced by the convective pressure in the side wall of the chamber on the critical distance of lifting points \( a_{cp} \) is 28.8 to 30.1%, which indicates that \( M_{CW} \) could not be neglected in the design of the Hoist Vertical Shiplift.

According to the successful experience of hoisting vertical shiplift, \( S > 1.5 \) is recommended.
6. Conclusion

Based on the Housner theory, this paper establishes the dynamic model of sloshing in the shiplift chamber, and derives the theoretical formula about capsizing moment. Then the coupled dynamic composed by torsional vibration, pitching motion and sloshing is established in order to analyze the system stability. Using the Hurwitz theory, critical distance of lifting points \( a_{cp} \) is obtained. And the rationality of the formula is verified by taking seven built typical Hoist Vertical Shiplifts as examples, from which the conclusions are drawn:

1) The influence of sloshing in the chamber on \( a_{cp} \) is 29.3-30.4\%. Therefore, in the design of Hoist Vertical Shiplift, the capsizing moment caused by the water sloshing could not be ignored.

2) The influence of capsizing moment \( M_{GW} \) is 28.8-30.1\%. Since in the design of Hoist Vertical Shiplift, the capsizing moment caused by convective pressure in the side wall of the chamber could not be ignored.

3) The formula of critical distance of lifting points has significant engineering guiding value.

4) According to the successful experience of Hoist Vertical Shiplift, it is suggested that the safety factor of pitching stability should be \( S > 1.5 \).

Conflict State

No potential conflict of interest is reported by the authors.

Acknowledgments

This study was supported by the National Key R&D Program of China (No.2016YFC0402002), and the authors gratefully acknowledge this support.

References

[1] Chen, J.Z., Bao G.J., Ma G.Y. (1996) Stability analysis of Vertical Fully Balanced Shiplift chamber. Hydro-Science and Engineering, 301-308.

[2] Cheng, X.H., Shi, D.W., Li, H.X., Xia, R, Zhang, Y., Zhou, J. (2018) Stability and parameters influence study of Fully Balanced Hoist Vertical ShipLift. Structural Engineering and Mechanics, 66: 583-594.

[3] Cheng, G.D., Li, H.T., Ruan, S.L. (2005) Free vibration characteristics and stability analysis of Shiplift system. Journal of Mechanical Strength, 27(3):276-281.

[4] GB 51177-2016. (2016) The Ministry of Water Resources of the People’s Republic of China. Beijing, China.

[5] Granam, E.W., Rodriguez, A.M. (1952) Characteristics of fuel motion which affect airplane dynamics. Journal of Applied Mechanics, (19):381-388.

[6] Housner, G.W. (1957) Dynamic pressures on accelerated fluid containers. Bulletin of the Seismological Society of America, 47(1):15-35.

[7] Ju, R.C., Zeng, X.C. (1983) Theory of coupled vibration of elastic structures and liquids. In: Huang, R.H. (Eds.), Seismological Press, Beijing. pp.40-123.

[8] Liao, L.K. (2014) Safety analysis and design of Full Balanced Hoist Vertical ShipLift. Structural Engineering and Mechanics, 49(3):311-327.

[9] Liao, L.K., Zhang, S.K. (2006) Analysis on 2-D nonlinear slosh of ship-chamber. Journal of Ship Mechanic, 47-55.

[10] Liao, L.K., Shi, D.W. (1996) Coupling analysis of shiplift under pitching motion and main engine torsional vibration. Yangtze River, 19-22.

[11] Liao, L.K., Shi, D.W. (1996) Stability analysis of Shiplift under suspension condition. Water Conservancy & Electric Power Machinery, 9-12.

[12] Li, J.Z., Bao, G.J., Ma, G.Y. (1996) Stability analysis of Vertical Fully Balanced Shiplift chamber. Hydro-Science and Engineering, 301-308.

[13] Li, H.T., Cheng, G.D., Ruan, S.L. (2005) Seismic response of Shiplift system. Journal of Dalian University of Technology, 45(4):473-479.
[14] Shi, D.W., Peng, H., Zhao, T.Z., Cheng S.X. (2015) Finite element and experimental analysis of pinion bracket-assembly of Three Gorges project shiplift. Journal of Central South University, 22:1307-1309.

[15] Shi, D.W., Song, Z., Liao, L.K. (2003) Coupling analysis of ship-chamber of shiplift and parametric vibration of water. Engineering Journal of Wuhan University, 36(1):77-80.

[16] Ruan, S.L., Cheng, G.D. (2003) Calculation of ship-water-chamber coupled system in the shiplift with finite element method in time domain. Chinese Journal of Computational Mechanics, 20(3):290-294.