Scaling of Experimental Buoyancy Vortex Structures with Respect to Power Generation

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Abstract

A scaling or nondimensionalisation of atmospheric buoyancy vortices for power generation (based on the Oberbeck-Boussinesq assumption) is proposed that uses a published formulation from the study of Rotating Rayleigh-Benard Convection. This is combined with assumptions that the vortex flows are pseudo-cyclostrophic and that a radial Richardson number can serve as a predictor of the onset of Kelvin-Helmholtz instability leading to a transition to a turbulent plume, in order to locate the cold reservoir of the vortex when viewed as a heat engine. This permits the prediction of the behaviour of large vortices in atmosphere using data from experiments on small vortices.

1. Introduction

Atmospheric buoyancy vortices such as dust-devils, water-spouts and fire-whirls, are one instance of flows arising from rotating turbulent thermal convection [1, 2]. The use of an artificial vortex of this type as a source of carbon-neutral electricity generation is envisaged. Such vortices result from a combination of strong convection with rotation around the vertical axis (swirl). These properties are intermittent in the normal atmosphere, so natural vortices are transient. However steady-state convection and swirl can be engineered to produce continuous vortices. This has been demonstrated by others [3, 4].

At an industrial scale strong convection can be engineered by pumping water, taken from the secondary cooling circuits of existing thermal power stations, through nozzles to produce warm saturated airflows within the vortex core. The resulting convection of saturated air releases latent heat through condensation, which acts to increase the temperature difference from core to atmosphere as the core flows rise. This produces a vertical gradient of buoyancy which introduces vertical strain in the core-flows to act against radial diffusion and allow the core to rise to great height. Swirl is engineered using a ’swirl-henge’ of vertical aerofoils set around the heat-source in a ring. The angle of the aerofoils to the radial introduces swirl into the air drawn in by convection over the heat source. Swirl is then concentrated into the vortex core through the ‘end-wall effect’ arising from friction at the ground and conservation of angular momentum, acting on the vortex flows [5]. This produces an area of high wind-speeds at ground level, which can be used to drive a turbine for power generation, as shown in figure 1.
This method allows for carbon-neutral power generation from existing waste heat flows in light-wind and nil-wind conditions. It could complement existing wind generation in that it allows for the ‘wind-gap’ in generation capacity experienced in light-wind conditions to be reduced without grid-scale storage. It is expected to have low capital costs, as it does not require masts, deep foundations, large diameter turbines or high-torque gearboxes.

Preliminary analysis [5] suggests that such a Vortex Station with a ‘swirl-henge’ diameter of 20m would produce a vortex of 2m core diameter, allowing for 1MWe of generation at a conversion efficiency from heat input to electrical output of the order of 5%. This is based on a modification of published heat-engine models [6, 7] which assumes that the cold reservoir of the heat engine is at the top of the vortex. The vortex-top is the area where the coherent vortex-core flows break down to a turbulent plume.

More accurate prediction of the flow scales and available power requires a nondimensionalisation of the temperatures and forces of the vortex flows to allow laboratory results to predict performance at larger scale in atmosphere. This is addressed in the present paper. In section 2 our experimental methodology is described. In section 3 our preliminary results are discussed. In section 4 a potential nondimensionalisation of buoyancy vortices is suggested.

2. Experimental Methodology

2.1. Experimental Cabinet. The Buoyancy Vortex Extension Cabinet is shown in figure 2. Air is introduced into a cabinet at the base through vertical ducts of triangular section, to minimise disruption from drafts in the wind-tunnel hall. The crossflow extractor at the top generates sufficient flow up the cabinet to maintain stable temperature within 0.5 °C outside of the vortex. Our cabinet is taller (6m) than
others [3, 4] to minimise the influence of the extraction and allow the study of vortex extension by heating at height.

2.2. Heated Plate. Our vortices are formed over a 1m diameter, 16mm thick aluminium hotplate with a 2.5kW rated element, heating an area 700mm in diameter, insulated with 25mm of ceramic thermal insulation blanket underneath the plate and 2mm thick woven ceramic tape at the periphery. Power to the element is controlled using a Variac. Tests were also performed with smaller diameter electric heaters below the plate, which were not seen to change vortex structures significantly.

2.3. Core Stabiliser. The circular baffle, seen in the centre of the hotplate in figure 4, is used to impose axial symmetry on the vortex flows and acts to stabilise the base of the vortex against lateral movement, keeping it within the turbine wheel. The vortex is also seen to be stabilised against side winds, which were introduced by opening one side of the cabinet. The vortex can be made to convect at more than 30° to the vertical, while staying within the turbine, using a core stabiliser of 280mm diameter, 100mm tall, stood 70mm above the hotplate.

2.4. Turbine wheel. A cylindrical turbine wheel of 100mm diameter with 12 off 20mm tall vertical vanes rotating around the vertical axis of the experiment is set in the inflow zone above the hotplate, to investigate the interaction between the turbine and vortex. The turbine vanes can be seen below the core stabiliser in figure 4. As might be expected from considerations of radial continuity, the turbine acts to reduce tangential velocity in the inflows, thus reducing swirl in the core. The 60° swirl vane setting, which would otherwise produce a two-cell vortex [8], produces a one-cell vortex in this case. The radial pressure drop across the turbine seems to increase lateral stability of the vortex core. The rate of rotation of the turbine was measured using a high-speed camera to allow estimation of Froude number.

2.5. Short swirl vanes. The hotplate is encircled by swirl vanes shown in figure 4, that can be set at 30°, 45°, 60° or 75° to the radial. The swirl vanes are shorter than those previously used [3, 4]. Little difference in vortex structure or height arising from using shorter vanes was found, except that the time taken for the vortex to self-organise is slightly longer. Figure 4 shows 12 off vanes in 4mm Perspex, 300mm tall by 150mm wide, set around a 1m diameter hotplate. Vanes of different heights up to 1800mm tall were tested before standardising on 300mm tall blades. The visualised vortex structures and preliminary measurements are unchanged. This suggests that the height of the swirl vanes is not a suitable vertical scale for nondimensionalisation and is also of practical importance for power generation since it shows that creating tall vortices does not require tall swirl vanes.

2.6. Computer controlled gantry. An instrument gantry allows placement of instruments radially and vertically within the experimental chamber. A thermocouple and hot-wire anemometer are shown mounted on a horizontal arm in figure 4. The use of the horizontal arm minimises disruption of the core but does not eliminate it.

2.7. Heating at height. The use of saturated flows in the core allows the use of infra-red heaters (6 off RS 196-6478 650W rated) as seen in figure 5. These heaters emit at wavelengths near the ~600nm peak in absorption of water vapour. Their radiation passes through the dry air around the core to preferentially heat the saturated core flows, thus allowing non-contact heating of the core at height to investigate extension of the vortex to greater aspect ratio.
2.8. **Flow visualisation.** Condensing water vapour is used for flow visualisation of the core. This allows the use of infra-red heaters to heat the core flows at height. The water vapour is produced by dripping a solution of e-vape liquid in water onto the hotplate outside the core. The solution is supplied by a peristaltic pump through a horizontal wand that can be seen at bottom left in figure 4. As the vapour is carried into the lower pressure area of the core, saturation and condensation result. This gives effective flow visualisation but complicates the instrumentation, since PIV using neutrally buoyant bubbles is not possible and temperature measurements cannot accurately assess density in a saturated mixture. It is planned to use ultrasonic anemometers mounted on the instrument gantry to allow non-contact measurement of radial profiles of core density and tangential velocity at different heights. Our existing estimates are based on preliminary measurements using thermocouple and thermal-anemometer probes, \( R \) as the radius of heating (700mm) and estimation of height of the vortex by inspection relative to features in the cabinet of known height.

3. **Results**

Figure 3 shows characteristic temperature fields from a DNS of RRBC [1] in terms of the nondimensionalisation described in section 4. Our experimental results fall within the QC regime under this nondimensionalisation. They show similar structures to those seen in the DNS in the QC regime, at similar values of Rossby number and Froude number.

Figures 4 and 5 show qualitative comparisons of structures obtained in our experiments and the DNS results [2]. Values of the gravitational Rossby number and Froude number in our experiment approximate those associated in the DNS with the transition from 3D to QC, but at a slightly higher
Rayleigh number. This suggests that cyclostrophic balance is the main determinant of vortex structure in the experiment.

The most apparent difference between RRBC and the experimental buoyancy vortices seen in figure 4 is that the aspect ratio in RRBC is set by the geometry of the rotating domain, while the aspect ratio of the buoyancy vortex is set by the height at which the coherent core breaks down to a turbulent plume. The structure of experimental vortices formed above hotplates in the absence of heating at height is well determined in most respects [3, 4], but the height of the vortex is not explained. It is found that vortex height can be extended by additional heating at height, which gives increased tangential velocities at the ground. This is of importance to the possibility of using a buoyancy vortex in the open atmosphere to drive a turbine.

![Regime diagram in the gravitational Rossby number to Froude number space for Ra=10^8.](image)

**Fig. 3** Regime diagram in the gravitational Rossby number to Froude number space for Ra=10^8, showing temperature fields in the three-dimensional (3D), (QG), (QC) and (CC) regimes: adapted with permission from [1].

In the absence of heating at height, buoyancy vortex cores show a characteristic conical vortex structure with a constant cone angle, \( \frac{\partial R_t}{\partial z} = \text{constant} \) for \( R_t \), the radius of maximum tangential velocity [4]. The same structure is evident in the present experiments, as shown in figure 4. A discontinuity arising at the top of the vortex is observed involving a breakdown to a turbulent plume with an associated increase in entrainment and cone angle.
With infra-red heating the height of core breakdown to the plume was extended from ~1.5m to ~2.5m, with a smaller apparent cone angle and turbine speed increased from 135rpm to 163rpm. In both cases, power to hotplate = 2kW, 60° swirl vane angle, Ra~10⁹, Ro~1, Fr~0.5, as shown in figure 6. The 6 off infra-red heaters are 650W rated devices. The power absorbed in the core is unknown at this time. This preliminary result suggests that the height of core breakdown may serve as a suitable vertical length scale for nondimensionalisation. The height of the swirl-vanes does not seem to give a suitable vertical length scale.

Fig. 4 Comparison of flow structures in our experiments and in the DNS of RRBC Experiment Ra~10⁹, Ro~1, Fr~0.5: DNS Ra=10⁸, Ro=1, Fr=0.5 [1].

Fig. 5 Comparison of instantaneous instabilities in our experiments and in the DNS of RRBC Experiment Ra~10⁹, Ro~1, Fr~0.5: DNS Ra=10⁸, Ro=1, Fr=0.5 [2] 6 off infra-red heaters are shown at left.
Initial estimates from our experiments suggest $Ri_r > 0.25$ at the base of the vortex and $Ri_r \approx 0.25$ at the breakdown to the plume, as discussed in section 4.1.1. Future work will investigate whether this condition can predict vortex height in the experiment. If so, it will allow a nondimensionalisation for buoyancy vortices, using the height of core breakdown as the vertical length scale.

Fig. 6 Comparison of core breakdown height with (right) and without (left) heating at height. The swirl vane support ring is at 1.8m.

In our experiments transient instabilities occur, which appear to be inertial waves that travel up the core and cause premature vortex breakdown to the plume, making the vortex core shorter for a period of some seconds. The vortices then recover to a characteristic quasi-steady-state height and then breakdown again after a period of minutes. The behaviour is not periodic but does repeat over time.

4. Nondimensionalisation

Rotating Rayleigh-Benard convection (RRBC) has been widely used as a paradigm model of rotating turbulent thermal convection [1]. RRBC laboratory experiments involve a fluid rotated around a vertical axis while being heated from below and cooled from above and has been widely studied [2]. Although the boundary conditions are different our experiments suggest the inner flows in RBBC and atmospheric buoyancy vortices are sufficiently similar that the non-dimensionalising ratios and quantities developed in studying RRBC may be applied to atmospheric buoyancy vortices, with some revision.

Previous authors [2] use a non-dimensionalising scheme progressing from continuity (equation 2a), the Navier-Stokes equations using the Oberbeck-Boussinesq approximation (equation 2b) and the temperature equation (2c) in Direct Numerical Simulations (DNS) of RRBC. They present results of DNS models at $Ra = 10^7$ and $10^8$ for the parameter space $0.025 \leq Ro_g \leq \infty$ and $0 \leq Fr \leq 10$, which includes Rayleigh numbers, Rossby numbers and Froude numbers similar to those seen in the present experiments. The Oberbeck-Boussinesq approximation, considering density effects under strong
gradients as well as buoyancy, is used to write a version of the Navier-Stokes equation that is suited to analysis of rotating turbulent thermal convection.

In the DNS [2] the domain is rotated about its vertical axis with angular velocity \( \Omega = \Omega \hat{e}_z \). The molecular kinematic viscosity \( \nu \) and thermal diffusivity \( \kappa \) are assumed to be constant for the fluid. Density is expressed as a Taylor expansion around the mean temperature:

\[
\rho = \rho_m (1 - \alpha (T - T_m)), \quad \alpha \equiv -\frac{1}{\rho_m} \frac{\partial \rho}{\partial T} = \text{the isobaric expansion coefficient.} \quad (1)
\]

The higher order term \( \rho_m \alpha (T - T_m) \) is neglected except in the gradient terms for gravitational and centrifugal acceleration, resulting from buoyancy forces. The mean term \( \rho_m \) is absorbed into the pressure term. The other terms are turbulent diffusion and Coriolis acceleration.

The governing equations are then written in cylindrical coordinates as:

\[
\nabla \cdot \mathbf{u} = 0 \quad (2a)
\]

\[
D_t \mathbf{u} = \nu \nabla^2 \mathbf{u} - \nabla p + 2 \Omega \mathbf{u} \times \hat{e}_z - \Omega^2 r \alpha (T - T_m) \hat{e}_r + g \alpha (T - T_m) \hat{e}_z \quad (2b)
\]

\[
D_t T = \kappa \nabla^2 T \quad (2c)
\]

where \( \mathbf{u} \) is the velocity vector, \( \nu \) is the kinematic viscosity, \( p \) is pressure, \( r \) is radius within the vortex, \( \alpha \) is the isobaric expansion coefficient, \( T \) is absolute temperature, \( \hat{e}_r \) and \( \hat{e}_z \) are the radial and vertical unit vectors, and \( \kappa \) is the thermal diffusivity.

The effect of the gravitational and centrifugal terms is to drive warmer, less dense air upwards and radially inwards. These equations are expressed in non-dimensional form as:

\[
\nabla \cdot \mathbf{\bar{u}} = 0 \quad (3a)
\]

\[
D_t \mathbf{\bar{u}} = \frac{\Pr^{1/2}}{\Ra^{1/2} \gamma^{3/2}} \nabla^2 \mathbf{\bar{u}} - \nabla \bar{p} + \frac{\gamma^2}{Rog} \mathbf{\bar{u}} \times \hat{e}_z - \Fr \bar{T} \hat{e}_r + \bar{T} \hat{e}_z \quad (3b)
\]

\[
D_t \bar{T} = \frac{1}{\Ra^{1/2} \Pr^{1/2} \gamma^{3/2}} \nabla^2 \bar{T} \quad (3c)
\]

Reference scales are used: the domain radius \( R \), the imposed vertical temperature difference \( \Delta \), and the velocity \( V = \sqrt{\alpha g \Delta R} \), so \( \bar{r} = r/R \), \( \bar{\bar{T}} = \frac{T - T_m}{\Delta} \), \( \bar{\mathbf{u}} = \mathbf{u}/V \), \( \bar{\bar{p}} = p/\rho V^2 \) and \( \bar{t} = t \cdot V/R \).

The boundary conditions are assumed to be no-slip on all walls, the sidewall adiabatic and the top and bottom boundaries isothermal at \( \bar{T}_{\text{top}} = -\frac{1}{2} \), \( \bar{T}_{\text{bottom}} = \frac{1}{2} \). The fluid layer height is \( \Delta \).

The input parameters of the DNS (Eq. 4) are:

- the Rayleigh Number: \( Ra = \frac{\Gamma_d}{\Gamma_c} \), where \( \Gamma_d \) is the timescale for thermal transport by diffusion and \( \Gamma_c \) is the timescale for thermal transport by convection at velocity \( V \),
- the Prandtl number of the fluid,
- the rotational Froude number, describing the importance of centrifugation.

\[
Ra \equiv \frac{\alpha g \Delta H^3}{\kappa \nu} = \frac{\Gamma_d}{\Gamma_c}, \quad Pr \equiv \frac{\nu}{\kappa}, \quad Fr \equiv \frac{\Omega^2 R}{g}. \quad (4)
\]
Rotation is characterised using a gravitational Rossby number ($R_{og}$) derived from the relationship of rotation and inertia:

$$R_{og} \equiv \frac{\Gamma_{g}}{\Gamma_{gb}} = \frac{\sqrt{\frac{\Delta}{g \Delta H}}}{2 \Omega H},$$

(5)

A centrifugal Rossby number is derived:

$$R_{oc} \equiv \frac{\Gamma_{c}}{\Gamma_{cb}} = \sqrt{\frac{\Delta}{2}} = \sqrt{\frac{\gamma}{Fr}}$$

(6)

where $\gamma$ is the aspect ratio of the domain.

The relevant timescales include:

$$\Gamma_{\Omega} = \frac{1}{2\Omega},$$

the Coriolis timescale,

$$\Gamma_{gb} = \Gamma_{ff} = \frac{H}{\sqrt{ag\Delta H}}$$

the gravitational buoyancy (or free-fall) timescale,

$$\Gamma_{cb} = \frac{R}{\sqrt{a\Delta R^2}}$$

the centrifugal buoyancy timescale.

(7)

An analysis of the resulting non-dimensionalised flow structures with respect to these timescales is shown in Figure 7, separating the event space into differing regimes [2]. The quasi-centrifugal regime (QC) holds where $\Gamma_{cb} \ll \Gamma_{\Omega}$ and $\Gamma_{cb} \ll \Gamma_{ff}$. This implies that centrifugal buoyancy dominates the flows within the QC regime.

Fig. 7 Regime diagram in the inverse gravitational Rossby number versus Froude number space, showing three-dimensional (3D), quasi-geostrophic (QG), quasi-centrifugal (QC) and Coriolis-centrifugal (CC) regimes related to timescales: reproduced with permission [2].
4.1. Proposed Nondimensionalisation for Buoyancy Vortices

Given that the height of the buoyancy vortex is not obviously connected to the radial scale or the height of the swirl vanes A modification of the nondimensionalisation presented in the DNS study [2] is suggested with the substitution of:

\[ V = \sqrt{g\alpha \Delta_p H_p}, \]

for the reference velocity, where \( H_p \) is the height of the breakdown to a plume and \( \Delta_p \) is the difference in virtual temperature between the air entering the base of the vortex core and that entering the base of the plume. The base of the plume is taken to be the cold reservoir of the heat-engine driving the vortex flows. There is some support for this in the literature. The authors analysed tangential wind-speeds in dust-devils [9] and found a correlation where:

\[ V_t \propto \sqrt{\text{CAPE}(H_s)\cdot H_s}, \]

(8)

where \( H_s \) is the height of the superadiabatic layer in the atmosphere, this seems to also be the height of the core before breakdown [9]. CAPE(Hs) is the Convectively Available Potential Energy available up to that height.

An analytic solution of the Navier-Stokes equation [10] for a vortex rising in a saturated atmosphere of virtual potential temperature \( \theta \) and depth \( \bar{h} \), under constant lapse-rate divergence produces a similar velocity scaling:

\[ \frac{v}{\bar{h}} \propto \beta = \left[ \frac{\partial \bar{\theta}}{\partial z} \right]^{1/2} \approx \sqrt{\frac{g}{T} \frac{\delta T}{\bar{h}}}, \]

(9)

which is equivalent to:

\[ V \propto \sqrt{g\alpha \Delta \bar{h}}. \]

The axial momentum within a buoyancy vortex is modelled as [4]:

\[ \frac{d}{dz} \int_0^\infty \rho \left( w^2 - \frac{1}{2} v^2 \right) r dr = \int_0^\infty \rho \cdot g' r dr, \]

(10)

where \( g' = ((\rho_\infty - \rho(r,z))/\rho_\infty)g \) is the gravitational acceleration or specific buoyancy.

A modification of this term is useful for taller vortices, where the atmospheric density varies with height:

\[ g^*(z) = ((\rho(\infty,z) - \rho(r,z))/\rho(\infty,z))g. \]

(11)

It follows that:

\[ \int_0^H g^*(z) \ dz = \text{CAPE}(H). \]

(12)

Equations 9 and 11 involve the idea of lapse-rate divergence, wherein the core flows cool more slowly in rising than the surrounding atmosphere, leading to lower density in the core than in the surroundings. It is consistent with the idea of CAPE, which is widely used in meteorology.

Lapse rate divergence arises in a dry vortex, such as a dust-devil, with a core rising at the adiabatic lapse-rate in a super-adiabatic layer. It arises in a condensing vortex, such as a water-spout or tornado, with a core rising at the pseudo-adiabatic lapse-rate in a temperate atmosphere. In a natural atmospheric
vortex, it is suggested that lapse-rate divergence plays the same role as heating at height in our experiments. Lapse rate divergence or core heating at height introduce axial strain in the vortex core that acts to concentrate it in rising, against the effects of entrainment and radial diffusion. Lapse rate divergence cannot easily be scaled down, so artificial heating at height becomes necessary in our experiment.

Since the diameter of heating under the hotplate does not seem to dictate the structure of the vortex (see section 2.2) it is proposed that the value of $R$ used for nondimensionalising should be taken as the radius of the swirl vanes, as used in previous work [3, 4], although this makes comparisons with the DNS of RRBC more difficult [1, 2].

4.1.1. Richardson Number and Kelvin Helmholtz Instability

The Richardson Number ($Ri$) is widely used in studying geophysical flows to predict the onset of entrainment and turbulent mixing in shear flows under density gradients. It is the dimensionless number that expresses the ratio of buoyancy to the flow shear. In the study of these flows, instances with $Ri > 0.25$ are expected to be stable, breaking down at $Ri = 0.25$ via a Kelvin Helmholtz instability with subsequent production of turbulent energy [7].

If the vortex core flows are quasi-centrifugal, the point at which breakdown to the plume occurs should be predictable using a radial Richardson number. A transition at the top of the vortex is expected, similar to a Kelvin Helmholtz instability, leading to the production of turbulent energy and the creation of a turbulent plume immediately above the coherent vortex core. The Richardson number is given by:

$$\text{Radial Richardson Number} \equiv \frac{\text{buoyancy}}{\text{flow shear}} = Ri_r = \frac{a}{\rho} \frac{\partial \rho}{\partial r} r^2,$$

where $a = \frac{v^2}{r}$ is the centripetal acceleration, with the expectation that the breakdown to a plume may be associated with a condition $Ri_r \leq 0.25$. If the breakdown to the plume can be predicted in this way, this then allows estimation of $H_p$ and $\Delta p$ to complete the nondimensionalisation.

5. Discussion

The characteristic and instantaneous flow fields in our experiments appear similar to those seen within the QC regime for similar values of $(Fr, Rog)$ [1, 2]. The trends are also similar, with experimental vortex cores becoming slimmer, and less subject to transient instabilities at the higher Rossby and Froude numbers arising from increased hot-plate temperature. The transient waves noted in section 3 have a low wave number, which would support the use of LES models to explore them. We plan to develop such models, while validating them against experimental data.

The usefulness of the suggested nondimensionalisation in modelling buoyancy vortices could be tested by comparing CFD models using compressible models, and models using the Oberbeck-Boussinesq models, with and without the centrifugal term. To the extent that the Oberbeck-Boussinesq models with the centrifugal term capture the same behaviours as the compressible models, this would support the usefulness of the nondimensionalisation.

6. Conclusions

It is concluded that the height of the vortex does not exhibit a simple relationship with the radial dimensions in the experiment or the height of the swirl vanes. It seems to be the height of onset of vortex core breakdown to a turbulent plume, occurring at a height which may be predicted through the use of a radial Richardson number. A modification of the reference scales for velocity, radius, height, and...
temperature difference used in a published nondimensionalisation of Rotating Rayleigh-Benard convection is suggested to allow analysis of buoyancy vortices. This formulation permits the prediction of the required heat-input power and temperature and vortex station size to generate an artificial atmospheric vortex that could drive a turbine to provide carbon-neutral power generation of a desired electrical power, based on experimental results at much smaller scale.

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