Application of a small deviation method in the study of the influence of external factors on gas turbine unit operation

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Abstract. A correction system for energy characteristics of gas turbine units (GTU) Siemens SGT-100 and Siemens SGT-300 has been developed. It includes a set of analytical and graphical dependencies of technical and economic indicators on each of the environmental parameters (ambient air temperature $t_a$, relative air humidity $\varphi_a$, atmospheric pressure $p_a$, etc.) and other external factors at fixed values of other conditions. The dependencies are obtained by mathematical processing of experimental data under various combinations of external factors. In practice, obtaining experimental data at fixed values of external factors on the operating equipment of the power plant is difficult, since there is always a change in the environmental parameters and technological parameters of the equipment being tested. In the present paper, this problem was solved by applying a small deviation method when processing test results. The dependence between the output characteristics of the object under study and the parameters of external factors was assumed to be linear in a relatively small range of changes in the parameters under study. The article presents the study results of the influence of environmental parameters on the gas turbine units performance by a small deviation method on the example of gas turbines Siemens SGT-100 (4.7 MW) and Siemens SGT-300 (7.9 MW). The calculations were performed using the experimental inspection results of a gas turbine power plant, the main equipment of which includes gas turbines Siemens SGT-100 (4.7 MW) and Siemens SGT-300 (7.9 MW). The inspection was conducted in order to develop regulatory and technical documentation on fuel use, including energy characteristics of equipment, a correction system for deviations of external factors, and models for calculating nominal and normative specific consumption of equivalent fuel for the supply of electric energy.

1. Introduction
Currently, gas turbine and combined-cycle technologies that include gas turbine and combined-cycle units of various brands and designs are widely used in large and small energy sectors to generate electric and thermal energy [1, 2, 3, 6].

One of the objectives of power plant operation is to normalize fuel consumption when planning the operation and to justify electricity and heat tariffs. Fuel consumption rationing is carried out by normative and technical documentation on fuel use, which includes the energy characteristics of the main and auxiliary equipment of the power plant.

Energy characteristics of gas turbine units contain a complex of technical and economic parameter dependencies of their operation on the electrical load at fixed values of external factors. In addition, they include a correction system for individual characteristics for changes in external factors or deviations of actual values of parameters and indicators from nominal values.
External factors in the operation of a gas turbine power plant are environmental parameters such as temperature, relative air humidity, and atmospheric pressure. To calculate the correction values of the gas turbine output for environmental parameter deviation from those accepted as normative, it is necessary, when developing energy characteristics, to conduct instrumental studies of operating equipment over the entire range of changes in a specific environmental parameter, for example, air temperature.

This requires a large number of experiments, each of which should include one parameter under study. However, other external factors should remain unchanged throughout the experiment. In order to evaluate the influence of individual parameters, the number of experiments should be large [1]. Often, in practice, these conditions cannot be achieved. Such dependencies are usually determined by the manufacturer of gas turbine units and are given as part of the typical energy characteristics.

The papers [3, 4] include the calculation results of thermal circuits at different ambient temperatures and graphs of the power dependence on environmental parameters. However, these studies do not allow one to clearly determine the dependence of the environmental parameter impact on the mode of gas turbine operation, since the calculations were performed at different values of heat supply to the combustion chamber of the gas turbine, i.e. at different values of the fuel consumption to keep the constant temperature of combustion products at the turbine inlet.

2. Method description

The solution to the above problem of determining the degree of environmental parameter impact on the operation of a gas turbine unit can be obtained by applying the small deviation method. The essence of the method is that the curve describing the behavior of the object under study is replaced by a line tangent at the point corresponding to the mode of the test. This method is used in the development of GTU equipment, and is also widely used in other areas of industry [8].

When using a small deviation method, the increment of function is replaced by its differential [1]:

$$\Delta y \approx \frac{dy}{dx} \Delta x$$

or

$$dy = \frac{\Delta y}{y} \approx \frac{dy}{dx} \frac{\Delta x}{x} = K\delta x,$$

where $\delta x$, $\delta y$ – the increment of the argument and the increment of the function; $K$ – influence coefficient of the increment of the argument on the increment of the function.

Influence coefficient $K$ is calculated at initial values of $x$ and $y$. The linearization of the equations describing the operation of gas turbine units is carried out by their consequent logarithmation and differentiation.

The paper presents the results of testing and processing of experimental data by a small deviation method for two types of GTU - Siemens SGT-100 and SGT-300. The tests included measuring the parameters of the GTU operation and the values of external factors during the testing period under various electrical loads at the power plant. Table 1 shows the measured values for three typical modes of operation of gas turbine units.

### Table 1. Test results of GTU Siemens SGT-100 and SGT-300

| №  | Parameter                          | Unit of measure | Siemens MW | SGT-100 4.7 | SGT-300 7.9 |
|----|-----------------------------------|-----------------|------------|-------------|-------------|
|    |                                   |                 | Mode 1     | Mode 2      | Mode 3      | Mode 1     | Mode 2 | Mode 3 |
| 1  | GTU fuel flow                     | nm³/h           | 910.6      | 974.8       | 1162.2      | 1395.0     | 1668.4 | 2035.8 |
| 2  | Fuel temperature                  | °C              | 41.0       | 41.0        | 40.3        | 41.0       | 42.0   | 41.3   |
| 3  | Fuel pressure on the combustion chamber | bar         | 11.7       | 12.6        | 13.7        | 11.8       | 12.8   | 13.7   |
| 4  | Ambient air temperature           | °C              | 8.2        | 7.2         | 6.5         | 6.9        | 6.7    | 6.8    |
A SGT type gas turbine unit is a single-shaft unit consisting of a compressor, a combustion system and a gas turbine. The GTU compressor is axial, single-stage, and has 10 compression stages. The first four rows of compressor stator blades are entrance guide adjustable blades designed to facilitate the compressor starting system and prevent compressor surging. The blade angle is set by outside transfer mechanism (drive) of blades. The guide blades were in the same position during the tests in all operating modes.

The combustion system uses 6 reverse-flow combustion chambers. The combustion chambers are arranged symmetrically at an angle of 30° to the axis of the gas turbine. The gas turbine has 2 stages: high pressure stage and low pressure stage. The high-pressure stage generates the power needed to drive the compressor, and the low-pressure stage provides the output power to drive the electric generator.

The system of equations describing the operation of a gas turbine unit on the example of a single-shaft SGT-100 GTU is as follows:

- turbine power equation:
  \[ N_t = G_s c_p T_g \left(1 - \frac{1}{\pi_t} \right) \eta_t \quad (1) \]

- compressor power equation:
  \[ N_c = G_c c_p T_u \left(\frac{p_m^e - 1}{\eta_c} \right) \quad (2) \]

- equation of the relationship between the air temperature at the inlet and outlet of the compressor:
  \[ T_u'' = T_u \left(1 + \frac{\pi_m^e - 1}{\eta_c} \right) \quad (3) \]

- equation of the relationship between the exhaust gases temperature in the turbine and the temperature of gases at the turbine inlet:

| No. | Parameter                                      | Value       |
|-----|-----------------------------------------------|-------------|
| 5   | Atmospheric pressure                          | mm Hg       |
| 6   | Air humidity                                  | %           |
| 7   | GTU generator load                            | kW          |
| 8   | Generator voltage                             | W           |
| 9   | Power factor                                  |             |
| 10  | Pressure drop in the air pollution control device | mm H2O    |
| 11  | Compressor inlet temperature                  | °C          |
| 12  | Compressor outlet pressure                    | bar         |
| 13  | Excess pressure at the turbine outlet         | bar         |
| 14  | Gas temperature at the turbine outlet         | °C          |
| 15  | Rotor speed                                   | r/min       |
| 16  | Air capacity                                  | m³/hr       |
| 17  | Combustion value                              | MJ / m³     |
| 18  | Fuel density                                  | kg/m³       |

5  Atmospheric pressure mm Hg 751.5 751.3 750.9 752.0 751.8 751.3
6  Air humidity % 63.2 73.5 77.3 75.3 78.3 81.0
7  GTU generator load kW 2150.3 3200.0 4208.3 3621.0 5369.0 6938.3
8  Generator voltage W 6289.3 6289.7 6290.3 6314.7 6320.0 6319.0
9  Power factor 0.88 0.94 0.97 0.98 0.99 0.99
10 Pressure drop in the air pollution control device mm H2O 28.00 28.33 29.33 42.67 42.00 42.33
11 Compressor inlet temperature °C 10.67 9.67 9.00 9.00 9.33 10.00
12 Compressor outlet pressure bar 12.30 12.95 13.56 12.61 13.23 13.69
13 Excess pressure at the turbine outlet bar 0.90 0.89 0.89 0.93 0.92 0.90
14 Gas temperature at the turbine outlet °C 363.00 436.00 516.33 351.33 425.67 496.00
15 Rotor speed r/min 17527 17531 17552 14129 14154 14132
16 Air capacity m³/hr 55868 56689 56265 85863 81936 83449
17 Combustion value MJ / m³ 41.07 41.07 41.07 41.07 41.07 41.07
18 Fuel density kg/m³ 0.934 0.934 0.934 0.934 0.934 0.934

(1) \[ N_t = G_s c_p T_g \left(1 - \frac{1}{\pi_t} \right) \eta_t \]

(2) \[ N_c = G_c c_p T_u \left(\frac{p_m^e - 1}{\eta_c} \right) \]

(3) \[ T_u'' = T_u \left(1 + \frac{\pi_m^e - 1}{\eta_c} \right) \]

Journal of Physics: Conference Series 1683 (2020) 042006 doi:10.1088/1742-6596/1683/4/042006
\[ T_g'' = T_g' - T_g' \left(1 - \frac{1}{\pi_m} \right) \eta_t; \]  

(4)

- the power balance of the GTU units:

\[ N_{el} = N_i - N_c - \Delta N_{mec} - \Delta N_{e.g.}; \]  

(5)

- heat balance in the GTU combustion chamber:

\[ T_g'' = \frac{QB_g + G_a c_{pa} T_g'}{G_g c_{pg}}, \]  

(6)

Formulas (1) – (6) have the following notations:

- \( N_i, N_c \) — turbine power and compressor power, kW;
- \( N_{el} \) — GTU electric power, kW;
- \( \Delta N_{mec}, \Delta N_{e.g.} \) — mechanical losses and losses in the electric generator, kW;
- \( G_g, G_a \) — mass consumption of combustion products and air, kg / s;
- \( c_p \) — average Isobaric heat capacity of the working fluid (air, combustion products), kJ/(kg·K);
- \( T_{a'}, T_{a''} \) — air temperature at the compressor inlet and the compressor output, K;
- \( T_k', T_g'' \) — gas turbine inlet temperature of combustion products and gas turbine outlet temperature of combustion products, K;
- \( \pi_c, \pi_t \) — air compression degree in the compressor, combustion products expansion degree in the turbine;
- \( m = \frac{k - 1}{k} \), where \( k \) — adiabatic index of air and combustion products, respectively;
- \( \eta_c, \eta_t \) — efficiency of compressor and gas turbine;
- \( Q \) — lowest combustion value, kJ / kg;
- \( B_g \) — fuel consumption, kg/s.

Consider an operating procedures for linearization of the equations describing the gas turbine unit operation on the example of the turbine power equation (1)

At the first stage, equation (1) is logarithmized:

\[ \ln N_i = \ln G_g + \ln c_p + \ln T_g' + \ln \left(1 - \frac{1}{\pi_m} \right) + \ln \eta_t. \]  

(7)

At the second stage, the equation obtained (7) is differentiated by the corresponding variable, the influence of which must be studied. As a result, we get influence coefficients:

\[ K_1 = \frac{d(\ln N_i)}{dG_g} G_g = 1; \]  

(8)

\[ K_2 = \frac{d(\ln N_i)}{dc_p} c_p = 1; \]  

(9)

\[ K_3 = \frac{d(\ln N_i)}{dT_g} T_g' = 1; \]  

(10)
\[ K_k = \frac{d(\ln N_t)}{d\pi_t} \pi_t = \frac{m}{\pi_t^{m-1}}; \]  
\[ K_5 = \frac{d(\ln N_t)}{dm} m = \frac{m \ln \pi_t}{\pi_t^{m-1}}; \]  
\[ K_6 = \frac{d(\ln N_t)}{d\eta_t} \eta_t = 1. \]

Therefore, equation (1) in a form of dependence of the relative increment of the function on the relative increments of the arguments is as follows:

\[ \delta N_t = K_1 \delta G_g + K_2 \delta c_p + K_3 \delta T_g + K_4 \delta \pi_t + K_5 \delta m + K_6 \delta \eta_t; \]

Equation (14) establishes a link of a relative change value of gas turbine power (as a separate unit) on any of the parameters (gases flow, expansion degree of the working fluid in the turbine, gas temperature at the turbine inlet, etc.) included in equation (1).

In this example, the influence coefficients \( K \) are set for only one of the equations (1) - (6), but for completeness and correctness of calculations it is necessary to linearize all the equations describing the gas turbine unit operation.

In addition, in engineering calculations the system of equations (8) – (13) must be supplemented with dependencies that take into account not only the ambient temperature, but also atmospheric pressure and relative air humidity, since these parameters to a greater or lesser extent depend on the density of air, and hence the air-mass flow and combustion products.

It is obvious that such operations require a large amount of calculations to obtain equations in relative increments of type (14) in analytical form, therefore, such calculations are recommended in computer algebra systems.

Coefficients of environmental parameter influence on the characteristics of the GTU were calculated based on the tests results of gas turbine units Siemens SGT-100 (4.7 MW) and Siemens SGT-300 (7.9 MW) conducted in a wide range of electrical loads. Table 2 shows the numerical values of some of the influence coefficients.

| Influence coefficient                        | Value of influence coefficient at GTU electric power , kW |
|---------------------------------------------|---------------------------------------------------------|
| Ambient air temperature on compressor power |                                                          |
| Atmospheric pressure on the compressor power|                                                          |
| Relative air humidity on the compressor power|                                                          |
| Ambient air temperature on the compressor outlet temperature | |
| Compression ratio on the compressor outlet temperature | |
| Compression degree on compressor power      |                                                          |
| Ambient air temperature on mass air flow    |                                                          |
| Atmospheric pressure on air mass flow       |                                                          |

|                  | Siemens SGT-100 4.7 | Siemens SGT-300 7.9 |
|------------------|---------------------|---------------------|
| 2150             | 1.108               | 0.305               |
| 3200             | 1.116               | 0.305               |
| 4208             | 1.117               | 0.304               |
| 3621             | 1.114               | 0.305               |
| 5369             | 1.120               | 0.305               |
| 6938             | 1.128               | 0.304               |
The influence coefficients shown in Table 2 can be used both to bring the results of gas turbine tests to fixed environmental conditions in accordance with ISO 2314:2009, and to study the influence of external factors on the operation of a gas turbine unit. A chart of modes and a correction system for the GTU deviation indicators from changes in external factors from conditions in accordance with ISO 2314:2009 were developed on the basis of test results, as well as using a small deviation method. Figure 1 shows a chart of the operating modes of the GTU *Siemens SGT-100 4.7 MW* with an operating time of ~100,000 hours.

![Chart of modes of GTU Siemens SGT-100 4.7 MW](image-url)

**Figure 1.** Chart of modes of GTU Siemens SGT-100 4.7 MW
Using the chart of modes for the ambient air temperature and electric power of the gas turbine unit one can determine the following:

- heat flow per turbine (to the combustion chamber), MW;
- combustion products temperature at the turbine inlet, °C;
- exhaust gas temperature, °C;
- flow of combustion products (gases); kg/s.

In practice, the mode charts and graphs that show the influence coefficients should be used as follows. At a known ambient temperature (°C) and the actual power at the terminals of a generator (MW) we can determine the following: the combustion products temperature at the turbine inlet (°C), the combustion products temperature at the turbine outlet (°C) and the calculated heat flow to the combustion chamber (MW). The chart of modes presents numerous acceptable modes of gas turbine unit operation.

The influence coefficients obtained allow one to calculate the operating mode of a gas turbine unit at various environmental parameters, but at a constant value of heat supply to the combustion chamber of the gas turbine unit. As a result, the dependences of the correction coefficients (Fig. 2, 3, 4) to the electric power and combustion products flow of gas turbine units on the deviation value of the environment parameters determined by ISO 2314:2009 were obtained.

Therefore, the correction curves are used to determine the values of GTU power changes caused by the deviation of one or more specific external factors.

For example, if the ambient temperature deviates from +15 °C (conditions in accordance with ISO 2314: 2009) to +10 °C, the correction coefficient for the GTU power will be \( K_{1N} \approx 1.007 \) (Fig. 2), i.e. the GTU power will increase by approximately 0.7%, provided that the amount of heat supplied to the combustion chamber and, consequently, fuel flow remain unchanged.

\[
N_{el}^5 = N_{el}^{+15} \times K_{1N}.
\] (15)

**Figure 2.** Graph of the dependence of correction coefficients on the ambient temperature for GTU Siemens SGT-100 4.7 MW: \( K_{1N} \) – correction coefficient to GTU power; \( K_{1NG} \) – correction coefficient to mass flow of combustion products
**Figure 3.** Graph of the dependence of correction coefficients on atmospheric pressure for GTU Siemens SGT-100 4.7 MW: $K_{2N}$ – correction coefficient to GTU power; $K_{2N}G_g$ – correction coefficient to mass flow of combustion products.

**Figure 4.** Graph of the dependence of correction coefficients on air humidity for GTU Siemens SGT-100 4.7 MW: $K_{3N}$ – correction coefficient to GTU power; $K_{3N}G_g$ – correction coefficient to mass flow of combustion products.
Similarly, we can describe the effect of atmospheric pressure, relative humidity on the GTU operation according to the graphs presented in Fig. 3, 4. At lower atmospheric pressure, the air density and mass flow rate decreases which causes a decrease in the power demand of the compressor and increase GTU capacity at a constant value of supplied heat volume.

The influence of relative humidity on the GTU electrical power and the flow of combustion products is significantly lower, since the air density is less dependent on relative humidity than on temperature and atmospheric pressure.

The dependences obtained qualitatively correspond to the research results presented in [3, 6] for other models of gas turbine units.

3. Conclusions
The use of the small deviation method allows one to significantly reduce the number of testings performed on the existing equipment when compiling the energy characteristics of the equipment, as well as when studying the impact of any of the parameters on the GTU operation as a whole.

The obtained dependences of correction coefficients for the electric power and the GTU combustion product flow on the value of environmental parameter deviation qualitatively and quantitatively characterize the influence of these parameters on the GTU electric power. If the supplied heat value is constant, these dependencies make it possible to analyze the external factors impact on the GTU operation mode.

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Acknowledgments
The research was carried out with the financial support of the Council for grants of the President of the Russian Federation in the framework of the scientific project MK-2614.2019.8.