PROBING FOR THE ROOTS OF THE STANDARD MODEL

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ABSTRACT

The differences between the $N = 0$ and $N = 1$ standard models are emphasized in formulating their short distance extension. We sketch methods to reproduce many of the small numbers in the model in terms of scale ratios, applying see-saw like ideas to the breaking of chiral symmetries. We sketch how the $N = 1$ standard model, outfitted with an extra family Abelian symmetry to reproduce the mass hierarchies, naturally fits superstring models, by making use of generic non-renormalizable operators.

1. Introduction

In 1938, in one of his many remarkable insights, Oskar Klein started formulating what later came to be known as a Yang-Mills theory. Today, we know such theories provide the theoretical scheme for the interactions of the building blocks of matter, encapsulated in the Standard Model, in a remarkably compact description in terms of three gauge groups and eighteen parameters. Yet, it hardly looks like a fundamental theory: it has too many unconnected parts, but with enough similarity among them to lend credence to the belief that they probably are the chiral shards of a beautiful, more symmetric underlying structure. We need another Klein to point us in the right direction. If the Standard Model is indeed an effective low energy theory, it must come with an ultraviolet cut-off. The raison d'être and the value of this cut-off are the central question of fundamental theory. The absence of any experimental indication of its existence, indicates it must be at least of the order of hundreds of GeVs. At the higher end, Nature provides us with its own cut-off, the Planck scale, the largest cut-off we can presently imagine to the standard model. However it is far removed from present experimental scales.

Local field theories of gravity also break down at the Planck scale: their generalization to superstring theories may provide a cure by offering a well-managed deviation from space-time locality. One, the heterotic string remarkably contains the basic ingredients needed to reproduce the low energy world. Then, how do we match the standard model to string theory? One obvious obstacle is the disparity of scales: the standard model is known at or below hundreds of GeV’s, and string theory operates in the Planck region, seventeen orders of magnitude removed. It is quite possible that knowledge of the standard model alone, may not be sufficient to identify this match. In spite of a singular lack of uniqueness, low energy theories derived from superstrings show many generic features, which may survive in some form down to experimental energies. A spectacular feature is the presence of chiral
matter. Another is the possibility of supersymmetry at experimental scales. A more
detailed consequence is the presence of a number of vector-like particles, some with
electroweak quantum numbers, but with hitherto undetermined $\Delta I_W = 0$ masses.
Another is the existence of special non-renormalizable terms, used to compensate
for anomalies in the low energy theory.

In order to probe for the cut-off of the standard model, we need to conduct
experiments at higher energies. The present rate of progress is one order of mag-
nitude per human generation. Unless we find a way to prolong human life, this
method cannot be satisfactory for any one physicist. Theorists, on the other hand,
unimpaired by technical details, can perform gedanken experiments which are much
cheaper, and provide more immediate, albeit less believable, results. One tool for
this theoretical journey across the scales is the renormalization group. With it we
can continue the standard model to higher energies. If its extrapolated parameters show unreasonable behavior at some scale, it means that we have reached the
cut-off. The question of interest is simply the following: what is the value of this
cut-off? Is it just around the corner, is it at Planck scale, or...?

We start with a review of the standard model, and present arguments for ex-
tending its validity to much higher energies. We then discuss its supersymmetric
extension, the $N = 1$ standard model, which is perturbative all the way to the
Planck scale, where we can hope to match it with superstring models. We then
argue, based on our knowledge of the Yukawa sector, that certain types of non-
renormalizable terms, generic to superstrings, are needed to understand the pattern
of quark and lepton masses.

2. The $N = 0$ Standard Model

The $N = 0$ standard model is described by three Yang-Mills groups, each with
its own dimensionless gauge coupling, $\alpha_1$ for the hypercharge $U(1)$, $\alpha_3$ for QCD,
and $\alpha_2$ for the weak isospin $SU(2)$. QCD itself predicts strong CP violation, with
strength proportional to a fourth dimensionless parameter $\theta$.

The electroweak symmetry breaking Higgs sector contains two unknowns, a
dimensionless Higgs self-coupling, and the Higgs mass. The “measured” value of the
Fermi coupling accounts for one parameter, and the other is the value of the Higgs
mass, one of the two parameters of the model yet to be determined from experiment.
The Yukawa interactions between the fermions and the Higgs yields the nine masses
of the elementary fermions. This sector also contains three mixing angles which
monitor interfamily decays, and one phase which describes CP violation.

Two of these parameters have dimensions, the Higgs mass, and the QCD con-
finement scale, obtained from $\alpha_3$ by dimensional transmutation. The QCD scale is a
tiny number in Planck units $\Lambda_{QCD} \sim 10^{-20} M_{Pl}$. This small number has a natural
explanation due to the logarithmic variation of the QCD coupling with scale.

The Higgs mass is unknown, although the electroweak order parameter is de-
terminated by the Fermi constant. In terms of the Planck mass it is also very small \( G_F^{-1/2} \sim 10^{-17} M_{Pl} \). The origin of this small number is a matter of much speculation. In perturbation theory the Higgs mass is of the same order of magnitude as the electroweak order parameter. The most natural idea is to relate this number to dimensional transmutation associated with new strong technicolor forces just beyond electroweak scales, so that the natural cut-off of the standard model is in the TeV range. This beautiful idea yields a satisfying natural explanation of this value, but fails to reproduce the values of the fermion masses.

Another class of extension of the standard model postulates supersymmetry\(^{(1)}\) at TeV scales. There, the electroweak order parameter is related to that of supersymmetry breaking. While not at first sight very economical, the breaking of supersymmetry automatically generates electroweak breaking\(^{(2)}\) in a wide class of theories. The beautiful ideas of technicolor can then be applied to supersymmetry breaking, without encountering the problem of fermion masses of technicolor applied to electroweak breaking.

Whatever the extension, there are many other numbers to explain, notably in the Yukawa sector of the theory. Quark and charged lepton masses break electroweak symmetry by \( \Delta I_W = 1/2 \), and \( |\Delta Y| = 1 \), the same quantum numbers as the electroweak order parameter, which also gives the W-boson its mass. In this sense charged fermion masses should be of the same order as the W mass. This is true only for the top quark, the others are unnaturally small

\[
\frac{m_{u,d}}{M_W} \sim \mathcal{O}(10^{-4}) ; \quad \frac{m_s}{M_W} \sim \mathcal{O}(10^{-3}) ; \quad \frac{m_c}{M_W} \sim \mathcal{O}(10^{-2}) ; \quad \frac{m_b}{M_W} \sim .05 .
\]

Similarly for the charged leptons,

\[
\frac{m_e}{M_W} \sim \mathcal{O}(10^{-5}) ; \quad \frac{m_{\mu}}{M_W} \sim \mathcal{O}(10^{-3}) ; \quad \frac{m_\tau}{M_W} \sim .02 ,
\]

which range from the tiny to the small. Neutrino masses are predicted to be exactly zero in the standard model only because of the global chiral lepton number symmetries. However there is mounting experimental evidence that neutrinos have masses. In the absence of new degrees of freedom they are of the Majorana kind, and break weak isospin by one unit, as \( \Delta I_W = 1 \). Direct experimental limits on neutrino masses indicate that they are at most extremely small: \( m_{\nu_e} < 10^{-17} M_W \).

The values of the three gauge parameters are known to great accuracy from measurements at low energy, although because of endemic problems associated with strong QCD, the color coupling is the least well known. Given these parameters, we can extrapolate the standard model to shorter distances, using the renormalization group perturbatively. The most interesting effect occurs in the extrapolation of the three gauge couplings. The hypercharge and weak isospin couplings meet at a scale of \( 10^{13} \) GeV, with a value \( \alpha^{-1} \approx 43 \), but at that scale, the QCD coupling is much larger, \( \alpha^{-1}_3 \approx 38 \). Thus, although the quantum numbers indicate possible
unification into a larger non-Abelian group, the gauge coupling do not follow suit in this naive extrapolation. Historically, before the couplings were measured to such accuracy, it was believed that all three did indeed unify in the ultraviolet. In the ultraviolet, the values of these couplings is less disparate than at experimental scales. Similarly, nothing spectacular occurs to the Yukawa couplings. For instance, the bottom quark and $\tau$ lepton Yukawa couplings meet around $10^9$ GeV, and part in the deeper ultraviolet. The situation is potentially more extreme in the Higgs sector because of the renormalization group behavior of the Higgs self coupling\(^{(3)}\).

We can consider two cases, depending on the value of the Higgs mass. If it is below 135 GeV\(^{(4)}\), the self-coupling turns negative somewhere below Planck scale. This results in a loss of perturbation theory, with a potential unbounded from below. Using the recently announced value of the top quark mass, a Higgs mass of 120 GeV means that “instability” sets in at 1 TeV, indicating some new physics at that scale. When operative, this bound provides a low (with respect to Planck mass) energy cut-off for the standard model.

If the Higgs mass is above 200 GeV, its self-coupling rises dramatically towards its Landau pole at a relatively low energy scale. It means loss of perturbative control of the theory, and sets an upper bound on the Higgs mass since there is no evidence of any strong electroweak coupling at experimental scales. Strong coupling implies the Higgs is a composite; in the technicolor scenario it is a condensate of techniquarks. There is a tiny range of intermediate values for the Higgs mass for which both the instability and triviality bounds are pushed to scales beyond the Planck length, and there is no standard model prediction of new physics; the cut-off may well be indeed the Planck scale.

However this is not an entirely satisfactory situation because of the dependence of the various standard model parameters on the cut-off. Quantum fluctuations \textit{additively} renormalize the Higgs mass with a term linearly proportional to the cut-off. Thus even if the Higgs mass is in a region that does not \textit{technically} require new physics below Planck mass, its value is unnaturally small, if Planck mass is the cut-off. Only for a low cut-off is its value natural. Thus we have two possibilities, either expect a low cut-off, or find a way to alter the cut-off dependence of the electroweak order parameter. We already know such an example: the cut-off dependence of any chiral fermion mass is only logarithmic. The reason is chiral symmetry, which is recovered by setting the fermion mass to zero. It affords a protection mechanism which weakens its cut-off dependence.

\section*{3. The $N = 1$ Standard Model}

Supersymmetry avoids the \textit{technical} naturalness problem by linking any fermion to a boson of the same mass. With exact supersymmetry, the boson mass finds itself protected by the chiral symmetry of the fermion. As long as supersymmetry is broken at energies in the range of TeV, this is enough protection to produce a low Higgs mass. This might seem to be small progress, since a new symmetry has
been introduced to relax the strong cut-off dependence, a symmetry which has to be broken itself at a small scale, $V_{SUSSY} \sim 10^{-15} M_{Pl}$.

In the $N=1$ standard model, there are only gauge and Yukawa coupling constants, and their values at experimental scales are such that none blow up below Planck mass. In particular, the perky Higgs self-coupling is replaced by the square of gauge and Yukawa couplings. The great theoretical advantage of the $N=1$ standard model is to allow the perturbative extrapolation all the way to Planck scale, opening the way for comparison with fundamental theory!

There are tantalizing hints of simplicity in the extrapolation of the couplings. Firstly the gauge couplings seem to be much closer to unification, and at a scale large enough not to be invalidated by proton decay bounds. The hypercharge and weak isospin couplings meet at a scale of the order of $10^{16}$ GeV, with a value $\alpha^{-1} \approx 25$, and the QCD coupling is much closer to, if not right on the same value$^{(5)}$. It may still be a shade higher than the others, with $(\alpha^{-1} - \alpha_3^{-1}) \leq 1.5$.

The second remarkable thing is that with simple boundary conditions at or near Planck mass, inspired by universal soft supersymmetry breaking, the renormalization group drives one of the Higgs masses to imaginary values in the infrared. This in turns triggers electroweak breaking$^{(2)}$, made possible by the large top quark mass!

The Higgs mass is not arbitrarily high in the minimal supersymmetric extension. At tree-level, it is predicted to be below the Z-mass, but it suffers large radiative corrections due to the top Yukawa coupling, raising it above the Z, but not by an arbitrarily large amount$^{(6)}$.

This general scheme allows us to study the pattern of fermion masses at these shorter distances; there are more regularities with supersymmetry. For instance, the bottom quark and $\tau$ masses seem to unify at or around $10^{16-17}$ GeV$^{(7)}$, the same scale where the gauge couplings converge.

The most striking aspect of the fermion masses is that only one chiral family has large masses, leading us to consider theories where the tree-level Yukawa matrices are simply of the form

$$Y_{u,d,e} = y_{t,b,\tau} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$  

These matrices imply a global chiral symmetry, $U(2)_L \times U(2)_R$, in each charged sector. The hierarchy between the bottom and top quark masses requires explanation. In the $N=1$ model, it is linked to another parameter which comes from the Higgs sector, the ratio of the vev of the two Higgs. Hence it may not pertain to properties of the Yukawa matrices. Why are the other two families so light? Starting from the rank two Yukawa matrices, we must find a scheme by which the zeros get filled, presumably in higher orders of perturbation theory. In order to
see how this might come about, let us examine one well-known case in which small numbers are naturally generated, the see-saw mechanism\(^8\).

In the standard model, the neutrino Majorana mass matrix is zero at tree-level. A detailed examination shows that these zeros are protected from quantum corrections by conservation of chiral global lepton number for each species. In the see-saw mechanism, the usual neutrinos are mixed with new electroweak singlet fields (neutral leptons), by \(\Delta I_{\omega} = 1/2\) terms, of electroweak breaking strength, which gives them the same lepton numbers. These new particles are free to acquire \(\Delta I_{\omega} = 0\) Majorana masses, \(M\), of any magnitude, in particular well above the electroweak scale. Upon diagonalization, this generates a mass for the familiar neutrinos, depressed from typical electroweak values by the ratio of scale \(m/M\), where \(m\) is the electroweak order parameter. A scale ratio between electroweak and chiral lepton number breaking is used to generate a small number.

We can imagine a similar analysis for the charged Yukawa matrices, where the zeros are also protected by chiral symmetries. We first couple the massless fermions of the first two families to new fermions with similar quantum numbers, thereby sharing with them the chiral symmetries. Unlike the neutral case, these new fermions have electroweak charges, and can only have Dirac masses, which break the chiral symmetry. However Dirac masses require vector-like partners (this differs from the neutral sector). Thus it may well be that the small numbers in the mass ratios are to be explained in terms of scale ratios, underlying the need for extrapolating the standard model many orders of magnitude. We would also require vector-like particles with \(\Delta I_{\omega} = 0\) masses, at the higher scales.

The order parameter of supersymmetry breaking is the mass difference between a particle and its superpartner. For supersymmetry to make sense in the context of the standard model, this split has to be of the order of a TeV. How does this come about? It may be useful to draw on an analogy with chiral symmetry which, while broken by masses, is still an extraordinarily useful concept.

Chiral symmetry is spontaneously broken both by the strong interactions, and by electroweak breaking. The latter induces masses for the quarks. In the limited context of the effective low energy theory of the Strong Interactions, this breaking may be viewed as an explicit breaking, in the sense it gives a mass to the associated Nambu-Goldstone boson.

We can only speculate on the nature of the exact mechanism that causes supersymmetry breaking. A popular mechanism is akin to technicolor where a hidden strong color forms a gaugino condensate, which in supergravity, breaks supersymmetry spontaneously. However, in the context of the effective low energy theory, this breaking can be parametrized by explicit breaking terms.

Chiral symmetry starts with massless pseudoscalar mesons (from the its spontaneous breaking by QCD). They decouple from matter at zero momentum (low energy theorems). Supersymmetry starts with massless superparticles, such as gluinos, etc... They couple to matter in a manner predetermined by the supersymmetry.
Explicit chiral symmetry breaking is introduced in the chiral Lagrangean by soft mass terms. Their effect is to give masses to the pions, and the general covariance properties of these soft breaking terms is reflected in measureable sum rules among the pseudoscalars, such as the Gell-Mann-Okubo formula. Similarly, supersymmetry breaking is introduced in the $N = 1$ standard model Lagrangean by soft terms. These give masses to all the superpartners. However the precise form of these terms is a question subject to experimental test, for it will imply sum rules among the superpartners. After a number of measurements equal to the number of soft parameters, the theory becomes predictive. Not only the idea of supersymmetry, but also the precise form of its breaking will be tested at the LHC and the NLC.

In the case of chiral symmetry, we understand the explicit breaking terms as coming from quark condensates caused by QCD. In supersymmetry, we may hope to arrive at an equally simple understanding. Perhaps gaugino condensates may form in a hidden sector linked to us by the universal gravitational interactions only. In that case we may expect great simplicity in the breaking terms, yielding recognizable patterns among the superpartner masses.

The real advantage of low energy supersymmetry is to allow for an extrapolation of the standard model to Planck scale. This raises the hope of matching it to a more fundamental theory. Of these, none is as beautiful as superstring theory. Thus we limit our discussion to some of its generic features which might be of use in the phenomenological discussion that will take place in the next decade.

4. Beyond to Superstrings?

Superstring theories provide us, in the words of S. Fubini, with a glimpse of 21st century mathematics. They are not understood with any depth, but some of their generic features, applied to low energy theories can be identified. They yield effective low energy gauge theories, valid below a scale $M_U$, related to the gauge coupling through the formula (9)

$$M_U \approx 2.5 \sqrt{\alpha_U} \times 10^{18} \text{ GeV}.$$  

With $M_X = 10^{16} \text{ GeV}$, and $\alpha^{-1}_U < \alpha^{-1}_X \approx 25$, this implies that contact with the superstring can be made provided that $M_U/M_X > 50$, so that there is a slight discrepancy with the apparent gauge unification scale.

Their second feature is to produce at lower energies remnants of $27$ and $\overline{27}$ representations of $E_6$, reducing in the effective low energy theory to three chiral families and many vector-like particles, with similar quantum numbers, which may be used in see-saw like mechanisms to generate small numbers in the Yukawa matrices. It also means many intermediate thresholds between the supersymmetry and unification scales, which is expected since the effective gauge group is usually larger than the standard model’s.
A third feature is the existence of a local $U(1)$ symmetry, with anomaly cancelled through the Green-Schwarz mechanism. This symmetry, though broken close to the Planck scale, may be discernable in the extrapolated low energy standard model. Ibáñez\(^{(10)}\) has argued that this symmetry can be used to fix the weak mixing angle in superstring theories. Following Ibáñez and Ross\(^{(11)}\), we argue\(^{(12)}\) that this Abelian symmetry sets the dimensions of the Froggatt and Nielsen\(^{(13)}\) Yukawa operators.

Are any of these features present in the extrapolated low energy theory? Consider first the unification of the gauge couplings. It is predicated on two assumptions: that the weak hypercharge coupling is normalized to its unification into a higher rank Lie group, such as $SU(5)$, $SO(10)$ or $E_6^{(14)}$, and on the absence of intermediate thresholds with matter carrying strong or electroweak quantum numbers between 1 TeV and $10^{16}$ GeV. The gauge couplings may not exactly unify at $M_X$, and we may want to alter this simple picture by requiring at least one intermediate threshold between the SUSY scale and the illusory unification scale at $M_X$ to obtain unification at the string scale $M_U^{(15)}$. At one-loop, the couplings $\alpha_{-i}(t)$ for the three gauge groups, ($i = 1, 2, 3$ for $U(1)_Y$, $SU(2)_L$, and $SU(3)^C$, respectively) run with scale according to

$$
\alpha_{-i}(t) = \alpha_{-i}(t_X) + \frac{b_i}{2\pi}(t - t_X),
$$

where $t = \ln(\mu/\mu_0)$, $t_X = \ln(M_X/\mu_0)$, and $\mu_0$ is an arbitrary reference energy. For the three families and two Higgs doublets of the minimal supersymmetric standard model, we have $b_1 = -33/5$; $b_2 = -1$; $b_3 = 3$. Since the low energies values of $\alpha_1$ and $\alpha_2$ are known with the greatest accuracy, we use their trajectories to define $t_X$ as the scale at which they meet:

$$
\alpha_{-X}^{-1} \equiv \alpha_{-1}^{-1}(t_X) = \alpha_{-2}^{-1}(t_X).
$$

The extrapolated data show that $\alpha_{-X}^{-1} \approx 25$, with $M_X \approx 10^{16}$ GeV. We do not assume precisely the same value for $\alpha_3(t_X)$ at that scale; rather we set

$$
\alpha_{-X}^{-1} = \alpha_{-3}^{-1}(t_X) + \Delta.
$$

Present uncertainties on the QCD coupling suggest that

$$
|\Delta| \leq 1.5.
$$

Suppose there is an intermediate threshold above supersymmetry at

$$
t_I = \ln(M_I/\mu_0); \quad t_I < t_X,
$$

caused by new vector-like particles with electroweak singlet masses at $M_I$. Their effect is to alter the $b_i$ coefficients:

$$
b_i \rightarrow b_i - \delta_i, \quad i = 1, 2, 3.
$$
By requiring unification at $M_U$, we find the constraints

$$\frac{r}{14} = \frac{t_U - t_X}{t_U - t_I}, \quad \frac{q}{4} = \frac{t_U - t_X - \pi \Delta/2}{t_U - t_I},$$

written in terms of

$$q \equiv \delta_3 - \delta_2 \quad \text{and} \quad \frac{2}{5} r \equiv \delta_2 - \delta_1 .$$

For vector-like matter generated from superstrings, $q$ and $r$ are integers. The value of the gauge coupling at unification is now

$$\alpha_{-1}^{-1} = \alpha_{X}^{-1} - \frac{1}{2\pi} [\delta_2(t_U - t_I) + t_U - t_X] .$$

These equations have solutions for non-exotic matter. For instance when $\Delta = 0.82$ with $r = 5$, $q = 1$, we get

$$M_U = 7.5 \times 10^{17}\text{GeV} ; \quad M_I = 4.4 \times 10^{12}\text{GeV} ; \quad \alpha_{-1}^{-1} = 11 .$$

However most solutions do not allow large $M_X/M_I$.

In realistic superstring models, the assumption of one intermediate scale is probably not justified. For several intermediate thresholds, by applying these equations repeatedly, we obtain similar equations, with $q$ and $r$ replaced by average quantities which are no longer integers. It might seem rather surprising that in the MSSM the gauge couplings should appear to be nicely headed for unification at $M_X$, only to be redirected to a new meeting place at $M_U$, but the apparent perverseness of this situation allows us put some non-trivial constraints on the scenario.

Let us now turn to the last topic, the possibility of an Abelian gauge symmetry, with anomaly cancelled by the Green-Schwarz mechanism. This gauged Abelian symmetry can play a role in determining the dimensions of the entries of the Yukawa matrices\(^{(11,12)}\). In an effective low energy theory, anomalies not cancelled by particles with masses lower than the cut-off, will require non-renormalizable terms for cancellation. Thus if we can identify an anomalous symmetry in the low energy theory, we can hope to learn something about the the theory beyond the cut-off.

The most general Abelian charge that can be assigned to the particles of the Minimal Supersymmetric Standard Model, with $\mu$ term, can be written as

$$X = X_0 + X_3 + \sqrt{3}X_8 ,$$

where $X_0$ is the family independent part, $X_3$ is along $\lambda_3$, and $X_8$ is along $\lambda_8$. We set

$$X_i = (a_i, b_i, c_i, d_i, e_i) ,$$

where $i = 0, 3, 8$, and the entries correspond to the components in the family space of the fields $Q, \bar{u}, \bar{d}, L$, and $\bar{e}$, respectively. Both Higgs doublets have the same zero
X-charge, without loss of generality, since an imbalance can be created by mixing in the hypercharge $Y$.

With the tree-level Yukawa coupling only to the third family, we obtain the constraints

$$
\frac{a_0 + b_0}{3} = 2(a_8 + b_8), \quad \frac{a_0 + c_0}{3} = 2(a_8 + c_8), \quad \frac{d_0 + e_0}{3} = 2(d_8 + e_8).
$$

The other entries in the Yukawa matrices are much smaller, forbidden by $X$ symmetry to appear at tree level. We assume they appear in the low energy effective theory as non-renormalizable operators, and that the excess X-charge at each of their entries is made up by powers of a single electroweak singlet field. A typical term would be of the form

$$Q_i \pi_j H_u \left( \frac{\theta}{M} \right)^{n_{ij}},$$

where $\theta$ is some field with unit X-charge, $M$ is some large scale, and $n_{ij}$ is needed for $X$ conservation. In order to produce a small coefficient, the $i$th and $j$th fermions need to go through a number of intermediate steps to interact. The larger the number steps, the larger $n_{ij}$, and the smaller the entry in the effective Yukawa matrix. This approach was advocated long ago by Froggatt and Nielsen (13). This yields approximate zeros in the matrices, creating textures (16). For example, in the charge 2/3 sector,

$$n_{12} = 3(a_8 + b_8) + a_3 - b_3.$$

Since $\theta$ may have a large expectation value, it may be accompanied by its vector-like partner $\bar{\theta}$, with opposite charge, showing that the exponents $n_{ij}$ need not be positive, but if all the $n_{ij}$ are positive, several interesting phenomenological consequences follow (12). First the $n_{ij}$ exponents are not all independent, resulting in order of magnitude estimates among the Yukawa matrix elements

$$
(Y)_{11} \sim \frac{(Y)_{13}(Y)_{31}}{(Y)_{33}},
$$

$$
(Y)_{22} \sim \frac{(Y)_{23}(Y)_{32}}{(Y)_{33}},
$$

valid for each of the three charge sectors. These relations are consistent with many of the allowed textures. Another important consequence is that the X-charge of the determinant in each charge sector is independent of the texture coefficients that distinguish between the two lightest families

$\text{charge } \frac{2}{3} : 6(a_8 + b_8) \equiv U, \text{ charge } -\frac{1}{3} : 6(a_8 + c_8) \equiv D, \text{ charge } -1 : 6(d_8 + e_8) \equiv L.$
Let the value of $\frac{\theta}{M}$ be a small parameter $\lambda$. In the simplest case, this parameter would be the same for all three charge sectors. Then we have

$$\frac{m_d m_s m_b}{m_e m^\mu m^\tau} \sim \mathcal{O}(\lambda^{(D-L)}) .$$

It is more difficult to compare the up and down sectors in this way since we do not know the value of $\tan \beta$, which sets the normalization between the two sectors

$$\frac{m_u m_e m_t}{m_d m_s m_b} \sim (\frac{y_t}{y_b})^3 \tan^3 \beta \times \mathcal{O}(\lambda^{(U-D)}) .$$

Since this ratio is much larger than one, it means either that $\tan \beta$ is itself large, with $U$ close to $D$, or that $\tan \beta$ is not large, but $D > U$.

It can be shown\(^{(12)}\) that with one electroweak singlet field, the X symmetry has to be anomalous to reproduce the features of the data. The three chiral families contribute to the mixed gauge anomalies as follows

$$C_3 = 2a_0 + b_0 + c_0 ,$$

$$C_2 = 3a_0 + d_0 ,$$

$$C_1 = \frac{1}{3}a_0 + \frac{8}{3}b_0 + \frac{2}{3}c_0 + d_0 + 2e_0 .$$

The subscript denotes the gauge group of the Standard Model, i.e. $1 \sim U(1)$, $2 \sim SU(2)$, and $3 \sim SU(3)$. The X-charge also has a mixed gravitational anomaly, which is simply the trace of the X-charge,

$$C_g = (6a_0 + 3b_0 + 3c_0 + 2d_0 + e_0) + C'_g ,$$

where $C'_g$ is the contribution from the particles that do not appear in the minimal $N = 1$ model. The last anomaly coefficient is that of the X-charge itself, $C_X$, which is the sum of the cubes of the X-charge.

It was suggested by Ibàñez\(^{(10)}\), that an anomalous $U(1)$ symmetry, with its anomalies cancelled through the Green-Schwarz mechanism, is capable of relating the ratio of gauge couplings to the ratios of anomaly coefficients

$$\frac{C_i}{k_i} = \frac{C_X}{k_X} = \frac{C_g}{k_g} ,$$

which relates the Weinberg angle to the anomaly coefficients, without the use of Grand Unification. The $k_i$ are the Kac-Moody levels; are integers for the non-Abelian factors only. The mixed $YX X$ anomaly, however, must vanish by itself.

We demand that the non-Abelian gauge groups have the same Kac-Moody levels, which means that

$$C_2 = C_3 \quad \text{or} \quad d_0 = b_0 + c_0 - a_0 .$$
Secondly we require that at or near the unification or string scale, the Weinberg angle have the value
\[ \sin^2 \theta_W = \frac{3}{8}, \]
which translates into the further constraint
\[ 5C_2 = 3C_1 \quad \text{or} \quad e_0 = 2a_0 - b_0. \]

These equations are sufficient to infer that \( L = D \), which implies, remarkably enough, that the products of the charged lepton masses is of the same order of magnitude as that of the down-type quarks\(^{(12)}\). This provides a remarkable link between the value of the weak mixing angle and the ratio of down quark to charged lepton masses.

This formalism has been used\(^{(11)}\) to generate symmetric textures, of the kind found to be allowed by experiment \(^{(16)}\). Work is in progress to determine how these equations constrain possible textures. One result is that it appear to be difficult to generate acceptable constraints, without invoking Green-Schwarz cancellation. In that case, this particular way of generating textures would require the type of mechanism that is generic to superstrings!

The following examples have shown how several problems with the standard model might require a superstring explanation. While it is clearly too soon to claim to have made the connection, we think we are on the way to asymptotic beauty.

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