Collisionless distribution function of charged particles ensemble in a tokamak magnetic configuration with magnetic island

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Abstract. The collisionless distribution function of charged particle ensemble in the magnetic field of tokamak with a magnetic island is calculated. The calculation is based on the solution of the kinetic equation with source together with three-dimensional numerical calculations of charged particle trajectories. It is shown that in case of an inhomogeneous source trajectory, motion of trapped particles leads to anisotropization of the initially isotropic distribution of particle ensemble. The absence of contribution from the passing particles decreases the efficiency of spontaneous generation of a non-induction current in the magnetic island in comparison with the bootstrap effect in the system of nested magnetic surfaces.

1. Introduction
It is well known that the motion of charged particles, whose initial distribution is not a function of integrals of motion, leads to evolution of their distribution function and its spontaneous anisotropization. In particular, the consequence of this anisotropization in a tokamak toroidal-helical magnetic field is the effect of bootstrap current generation [1, 2], which has the key value for the non-inductive current drive in a stationary tokamak reactor [3]. The maximum possible bootstrap current density, which is generated due to the spontaneous anisotropization of charged particle ensemble as a result of their interaction with the axisymmetric tokamak magnetic field, was calculated in [4, 5] by solving the collisionless kinetic equation. Meanwhile, in modern tokamaks, three-dimensional effects play a crucial role. The appearance of magnetic islands under the influence of external resonant magnetic perturbations and/or the development of tearing instabilities drastically change plasma equilibrium and transport. The theory of related processes is poorly developed, primarily due to the absence of symmetry in the problem. In this paper, using the method proposed in [4], we calculate the collisionless distribution function in the center of magnetic island and make qualitative conclusions about the efficiency of bootstrap current generation in such system.

2. Calculation Method
We represent magnetic field in terms of the Hamiltonian function [6], which guarantees the fulfillment of the solenoidal condition for vector $\mathbf{B}$, by the construction: $2\pi \mathbf{B} = \left[\nabla \Phi \times \nabla \theta\right] + \left[\nabla \Psi \times \nabla \phi\right]$. Here, $\theta$ is the poloidal angle, $\phi$ is the toroidal angle, $\Phi$ is the toroidal magnetic flux function, and $\Psi$ is the poloidal magnetic flux function. The safety factor in this formulation is given by the expression:
To determine the concrete magnetic configuration, we define $\Phi$ and $\Psi$ as functions of coordinates $\{\rho, \phi, \theta\}$ of a quasi-cylindrical system that is related to the tokamak magnetic axis. The value $\rho = 0$ corresponds to the magnetic axis, $\rho = 1$ corresponds to the plasma boundary. The function $\Phi$ is chosen in such a way that the toroidal magnetic field decreases proportionally to $1/r$; $r$ is the distance from the geometrical axis of the torus, $r = R + \rho \cos \theta$ ($R$ is the major tokamak radius): $\Phi = \rho^2(1 - 2\rho \cos \theta/3R)/2$. The poloidal magnetic flux function $\Psi$ can be written in the following form:

$$\Psi = -\frac{\Phi}{1 + 2\Phi} + \Phi(1 - 2\Phi)A_{3/2} \cos(2\phi - 3\theta).$$

(1)

The first term in the right-hand side of expression (1), $\Psi_0 = -\Phi/(1 + 2\Phi)$, sets an unperturbed system of nested magnetic surfaces that are symmetric with respect to the equatorial plane of the tokamak, see figure 1, with the profile of safety factor $q = (1 + 2\Phi)^2$ ($q$ varies from 1 on the magnetic axis to 4 at the plasma boundary). The second term on the right-hand side of expression (1) describes the perturbed part of the Hamiltonian $\Psi$: the coefficient $A_{3/2}$ determines the amplitude of helical perturbation; $m = 3$ and $n = 2$ are the numbers of poloidal and toroidal harmonics, respectively; the factor $\Phi(1 - \Phi)$ ensures that the perturbation in the center and at the plasma boundary is zero. The appearance of a resonant helical perturbation ($A_{3/2} \neq 0$) leads to the splitting of the rational surface $q = 3/2$ with the formation of chain of islands in the middle of the plasma column, see figure 2.

To calculate the collisionless distribution function, $f$, we use the method that has been proposed and implemented in a series of works [4, 5, 7]. We describe the evolution of distribution function of charged particle ensemble in a magnetic by the collisionless kinetic equation with source $S$:

$$\frac{df}{dt} = \frac{df}{dt} + v \nabla f + \frac{Ze}{m} \frac{[v \times B]}{c} \frac{df}{dv} = S = \delta(t)n_0(r)\gamma_0(v)$$

(2)

Here, $\delta(t)$ is the delta function of time, $v$ is the velocity, $Z$ is the charge number, $m$ is the mass of the ensemble particles, $e$ is the elementary charge, and $c$ is the speed of light. The function $\gamma_0(v)$ defines the initial particle velocity distribution, and the function $n_0(r)$ is the initial distribution in the coordinate space. By integrating equation (2), we obtain:

$$f = \Theta(t)n_0(r_0)\gamma_0(v_0), \quad \Theta(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

(3)
where, $\Theta(t)$ is the Heaviside function, $\mathbf{r}_0 = \mathbf{r}(t = 0), \mathbf{v}_0 = \mathbf{v}(t = 0)$ are the initial position and velocity of particle, respectively. Having numerically calculated the trajectories of charged particles $\mathbf{r} = \mathbf{r}(\mathbf{r}_0, \mathbf{v}_0, t), \mathbf{v} = \mathbf{v}(\mathbf{r}_0, \mathbf{v}_0, t)$, we can reconstruct the dependences $\mathbf{r}_0 = \mathbf{r}_0(\mathbf{r}, \mathbf{v}, t)$ and $\mathbf{v}_0 = \mathbf{v}_0(\mathbf{r}, \mathbf{v}, t)$, and, thus, find the Eulerian representation of the function $f: f(\mathbf{r}, \mathbf{v}, t) = \Theta(t)h_0(\mathbf{r}_0(\mathbf{r}, \mathbf{v}, t)|\mathbf{v}_0(\mathbf{r}, \mathbf{v}, t))$.

The distribution function $f(\mathbf{r}, \mathbf{v}, t)$ is clearly non-stationary. To obtain its stationary value (the analogue of the distribution function calculated by the perturbation theory in the case of rare collisions [8]), we average expression (3) over time: $\langle f(\mathbf{r}, \mathbf{v}, t) \rangle = \int_0^T n_0(\mathbf{r}_0(\mathbf{r}, \mathbf{v}, t))\gamma_0(v_0(\mathbf{r}, \mathbf{v}, t))dt / T$. At times that exceed the characteristic times of charged particles motion along the trajectory, the distribution function $f(\mathbf{r}, \mathbf{v}) = \lim_{T \to \infty} \langle f(\mathbf{r}, \mathbf{v}, t) \rangle$ no longer depends on time (convergence is provided by the conditional periodicity of trajectories) and is determined by the integrals of motion – see [7].

Further, in calculations, we use the isotropic Maxwellian particle velocity distribution $\gamma_0(v) = \gamma_0(v^2) = (4\pi T)^{3/2} \exp(-v/v_T)^2$ as the source, ($v_T$ is the thermal velocity of particles). The initial concentration of particles is considered as a function of the magnetic surface label in the unperturbed magnetic configuration $n_0(r) = n_0(\Psi_0)$, namely: $n_0 = n_A \left(1 - \left(4\cdot\Psi_0^2\right)^2\right)$ ($n_A$ is the concentration of particles on the magnetic axis, the factor before $\Psi_0$ ensures that the plasma concentration at the boundary is zero). This choice of the source allows us to investigate changes in the distribution function of charged particles that are associated with the formation of a magnetic island. Below, calculations are given for protons with an energy of 1 keV, magnetic field on the magnetic axis $B_0 = 2$ T, and aspect ratio equal three. The direction of toroidal magnetic field coincides with the direction of toroidal current.

3. Distribution function in the center of magnetic island

Let us investigate the time-averaged distribution function that is formed in the center of magnetic island in the plane $\phi = 0$. Figure 3 shows the dependence of $f$ on the toroidal velocity (cosine of the pitch angle) for a fixed velocity module and phase of Larmor rotation. For comparison, the figure also shows the form of distribution function in the system of nested magnetic surfaces (curve 1), which are obtained in [7]. The dashed line shows the distribution of particles from a homogeneous source with $n_0 = n_A$. In the absence of the initial concentration gradient, the distribution function (3) is a function of integral of motion (energy), and consequently, is stationary: as in the initial moment of time, it remains isotropic.

The distribution function in the center of magnetic island consists of two regions that correspond to the passing and trapped particles: the practically horizontal sections of the curve that are close to the initial distribution correspond to passing particles, and the central strongly anisotropic sloping section corresponds to trapped particles. The established form of distribution function is determined by the initial source inhomogeneity and by the trajectories of ensemble particles, see [7].

Let us compare the distribution function that is formed in the center of magnetic island (curve 2) with the distribution function on rational magnetic surface with $q = 3/2$ in the unperturbed magnetic configuration (curve 1). Their general view is very similar. The appearance of magnetic island does not change the boundaries between the trapped and passing particles, as evidenced by the same position of "spikes" on curves 1 and 2. The region of trapped particle destruction of magnetic configuration leads to the appearance of asymmetry in particle contribution with positive and negative pitch angles: in the center of magnetic island, particles with a positive toroidal velocity predominate, while in the system of nested magnetic surfaces, the fractions of particles with $v_\phi > 0$ and with $v_\phi < 0$ are the same (curve 1 is practically symmetric with respect to the initial value of distribution function). The main effect is related to passing particles. If in the unperturbed system the passing particles, due to the finite deviation of their trajectories from the magnetic surfaces, made an appreciable contribution to the anisotropization of the distribution function, in the center of magnetic island their
contribution is negligibly small: in the region of passing particles, the distribution function is practically indistinguishable from the initial isotropic distribution of the source function.

To explain this effect, we calculate the instantaneous radial deviation of the passing particle trajectory, which originates in the center of magnetic island, from the unperturbed magnetic surface $q = 3/2$, and compare the results with particle motion in unperturbed magnetic configuration. An example of the obtained dependences for the passing particle with $v_\phi/v = -0.9$ is shown in figure 4. One oscillation period of curve 1 corresponds to one bounce period of the passing particle. One can easily see that in the presence of magnetic island the passing particle deviates both outside and inside from the starting magnetic surface. As a result, the average radial deviation of passing particle is completely neutralized, that explains the obtained form of the distribution function.

**Figure 3.** Normalized distribution function: (1) in the system of nested magnetic surfaces on the surface $q = 3/2$, (2) in the center of magnetic island, $\varphi = 0$. The dashed line shows the distribution from a homogeneous source with $n_0 = n_A$. The initial phase of the Larmor rotation is zero, $v = v_T$.

**Figure 4.** Time dependence of the radial deviation of the passing particle with $v_\phi/v = -0.9$ from the unperturbed magnetic surface in the unperturbed (1) and island (2) magnetic configurations. The dashed lines show the mean values. Time is measured in cyclotron periods; the deviation is normalized on the Larmor radius.

4. Conclusions
The evolution of isotropic velocity distribution of charged particles during the collisionless motion in a toroidal-helical tokamak field with a magnetic island is considered. It is shown that the inhomogeneity of the initial particle distribution leads to anisotropization of its distribution function. In the center of magnetic island, the anisotropization is due to the motion of trapped particles. The absence of contribution from passing particles means a decrease in the efficiency of spontaneous generation of the non-induction current in magnetic island in comparison with the bootstrap effect in the system of nested magnetic surfaces [4]. The asymmetry of established distribution with a predominant contribution from trapped particles with positive pitch angles indicates an increase in the particle concentration in the center of magnetic island as a result of the considered evolution.

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