Crossbreed impact of double-diffusivity convection on peristaltic pumping of magneto Sisko nanofluids in non-uniform inclined channel: A bio-nanoengineering model

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Abstract
The consequences of double-diffusivity convection on the peristaltic transport of Sisko nanofluids in the non-uniform inclined channel and induced magnetic field are discussed in this article. The mathematical modeling of Sisko nanofluids with induced magnetic field and double-diffusivity convection is given. To simplify PDEs that are highly nonlinear in nature, the low but finite Reynolds number, and long wavelength estimation are used. The Numerical solution is calculated for the non-linear PDEs. The exact solution of concentration, temperature and nanoparticle are obtained. The effect of various physical parameters of flow quantities is shown in numerical and graphical data. The outcomes show that as the thermophoresis and Dufour parameters are raised, the profiles of temperature, concentration, and nanoparticle fraction all significantly increase.

Keywords
Nanofluids, peristaltic flow, thermal and concentration convection, induced magnetic field, non-uniform inclined channel, Sisko fluid
Introduction

Researchers have been particularly interested in studying peristaltic flow of Newtonian and non-Newtonian fluids to apply it in multiple domains and to peristaltic medical engineering problems too.1–7 Like Mekheimer and Abd elmaboud8 explained that the phenomenon is helpful to remove unwanted tissues such as cancer cells that can be destroyed through bioheat. In addition, peristaltic blood flow9 describe the phenomena of hyperthermia, in which the tissue is heated to 42°C–45°C, which will be destroyed. In addition to this, peristaltic flow is mostly found in ureter function, transportation of semi-solid food through intestine, passing of ova via fallopin tubes, transportation of seminal fluid into the cervix, bile motion along bile duct, motion of cilia, blood flow in arteries and veins, and urine excretion. In addition, mathematicians, biologists, and physicists became more interested in the problems of flow magnetohydrodynamics (MHD). Magnetohydrodynamics (MHD) works by the electromagnetic force of a magnetic field. These types of magneto-fluids include liquids, plasma, electrolytes, and salt water. Magnetohydrodynamics (MHD) also works on magnetic resonance imaging, hyperthermia, imaging resonance imaging, reducing bleeding during surgery, or the proliferation of cancer cells and the growth of magnetic tracers.10 In addition, the pump, hemodynamics, glandular ducts, bile duct, and intestinal tract are just a few applications of peristaltic flow with magnetic effects. Mekheimer11 described the effect created by the magnetic field on the (destructive) movement of the couple’s pressure on the limited channels and discussed arithmetic results. Bhatti et al.12 described blended impact of the magnetic force and slip conditions for peristaltic hemodynamics (“blood flow”) along a separate medium that is also consulted with pump signals and exact solutions for velocity distribution. Both Bhatti and Zeeshan13 analyzed the impact of slip and endoscope on peristaltic flow of fluid-shaped suspensions using the annulus (Some problems related to peristaltic flow and magnetic force impacts are listed in index).14–18

Currently, researchers are showing great interest in nanofluid flow as it has many applications in science, biomechanics, chemical, and nuclear industries. These liquids can be used in many engineering problems.19 Nanofluids mean the inclusion of a small number of nanometer particles that are said to be <100 nm in basic liquids such as oil, water, biofluids, ethylene, and lubricants. The name was first introduced by Choi20; supplemented small nanoparticles to the base liquid to enhance thermal conductivity. Eastmann et al.21 introduced heat transfer of copper nanoparticles of ethylene glycol nanofluid. Since then, the emergence of nanofluid development of the mathematical model by showing the features of Brownian diffusion. Sheikholeslami et al.22 examined the impact of viscosity dependent on time on free transfer of magnetohydrodynamic nanofluid transmission. Since then, several researchers have contributed to the latest developments in nanofluids.23–28 The content of nanofluid transport with peristaltic is rarely found in literature, although there are applications imperative for medical engineering programs. A study carried out by Akbar et al.,29 has elevated the peristaltic flow of nanofluid to a separating tube, which combines the temperature using a homotopy perturbation
method. Many important results of peristaltic transport of nanofluids flow can be found in Hayat et al.,30 Ramesh and Prakash,31 Riaz et al.,32 Abumandour et al.,33 Mekheimer et al.,34 and Ellahi et al.35

Double diffusion of volatility of the separated liquid resting its size is determined by two different substances at different values. Stern36 confirmed that this arrangement is unstable or weed compaction rises to the ground. The double-diffusive convection used is called an essential element, and in many cases, it is a process of mixing at sea, where warm water and salt are often found above cold and fresh. In this case, the fast diffuser (temperature T) stabilizes, and the slow diffuser (salt S) relaxes, resulting in a double-sided salt of convection, which is the focus of our discussion. In addition to the use of oceanographic, dual-constructed convection is affected by heat and transport of various other geophysical and astrophysical fluid systems, from the magnetic melt37 to the inner planets and stars.38–41 Complete studies on double diffusion and its applications are listed in Alolaiyan et al.,42 Bég and Tripathi,43 Akram et al.,44 Akram and Afzal45 as well. The aim of the present study is to describe the influence of double diffusion on magnetic induction and peristaltic flow of Sisko nanofluids in an inclined non-uniform channel. In section 2 basic governing equations are defined. In section 3, mathematical formulations of Sisko fluid for two dimensional and directional flows of nanofluids under double-diffusivity convection is presented. The solution of proposed problem is discussed in section 4 whereas the graphical representation of obtained results is elaborated in section 5.

**Basic governing equations**

The basic equations of hydromagnetic Sisko nanofluid are given as follows:

(a) Maxwell’s equation

\[ \nabla \cdot \vec{E} = 0, \quad \nabla \cdot \vec{H} = 0, \quad (1) \]

\[ \nabla \times \vec{H} = \vec{J}, \quad \vec{J} = \sigma \{ \vec{E} + \mu_e (\vec{V} \times \vec{H}) \}, \quad (2) \]

\[ \nabla \times \vec{E} = - \mu_e \frac{\partial \vec{H}}{\partial t}, \quad (3) \]

From equations (1)–(3), the induction equation can then be attained as

\[ \frac{\partial \vec{H}^+}{\partial t} = \nabla \times \left( \vec{V} \times \vec{H}^+ \right) + \frac{1}{\chi} \nabla^2 \vec{H}^+. \quad (4) \]

(b) Continuity equation is

\[ \nabla \cdot \vec{V} = 0. \quad (5) \]
Naiver–Stoke equation is defined as

\[ \rho \left( \frac{dV}{dt} \right) = \nabla \cdot \tau - \mu_e \left( \vec{H}^+ \cdot \nabla \right) \vec{H}^+ - \nabla \left( \frac{1}{2} \mu_e \left( \vec{H}^+ \right)^2 \right) + g \left\{ (1 - \Theta_0) \rho_f \right\}. \]

\[ \{ \beta_T (T - T_0) + \beta_C (C - C_0) \} - \left( \rho_p - \rho_f \right) \left( \Theta - \Theta_0 \right) \].

(6)

For Sisko fluid, the stress tensor \( S \) is depicted by [5]

\[ \tau = -PI + S, \]

\[ S = \left( m + \eta(\sqrt{I})^{n-1} \right) A_1, A_1 = L + L^T, L = \text{grad} V, \Pi = \frac{1}{2} \text{trac}(A_i^2). \]

(8)

The thermal energy, solute concentration, and nanoparticle fraction [43] utilizing estimation of Oberbeck–Boussinesq, are deciphered as

\[ (pc)_f \left( \frac{dT}{dt} \right) = k \nabla^2 T + (pc)_p [D_B (\nabla \Theta \cdot \nabla T) + \left( \frac{D_T}{T_0} \right) \nabla^2 T] + D_{TC} \nabla^2 C, \]

(9)

\[ \frac{dC}{dt} = D_s \nabla^2 C + D_{TC} \nabla^2 T, \]

(10)

\[ \frac{d\Theta}{dt} = D_B \nabla^2 \Theta + \left( \frac{D_T}{T_0} \right) \nabla^2 T. \]

(11)

In the equations above, \( \left( \frac{d}{dt} \right)_p, p, V, k, \vec{J}, \vec{E}, \sigma, \mu_e, \rho_f, \rho_p, g, \rho_p, \beta_C, \beta_T, (pc)_f, \)

\( (pc)_p, C, T, \Theta, D_T, D_B, D_s, D_{CT}, D_{TC} \) denotes material derivative, pressure, velocity vector, thermal conductivity, current density, induced electric field, electric conductivity, magnetic permeability, density of fluid, density of fluid at \( T_0 \), acceleration, nanoparticle mass density, volumetric solutal expansion coefficient, volumetric thermal expansion coefficient, heat capacity of fluid, effective nanoparticle heat capacity, solutal concentration, temperature, volume fraction nanoparticle, coefficient of thermophoretic diffusion, coefficient of Brownian diffusion, solutal diffusively, Soret diffusively, and Dufour diffusively, respectively.

**Mathematical formulation**

In this problem, the incompressible hydro-magnetic flow of a non-uniform channel of an electrically conducting Sisko nanofluid is considered. The \( X—\text{axis} \) is drawn along the propagation of wave and its \( Y—\text{axis} \) is orthogonal. It is also assumed that the channel is slanted at an angle of \( \alpha \). The channel upper wall has temperature \( T_0 \), the concentration of solute \( C_0 \) and nanoparticle concentration \( \Theta_0 \) while
lower wall has a temperature $T_1$, solute concentration $C_1$ and nanoparticle concentration $Y_1$. A sustained magnetic field of $H_0$ intensity tends to act perpendicularly, resulting in the induced magnetic field $H(h_X(X, Y, t), H_0 + h_y(X, Y, t), 0)$ and the total magnetic field $H^+(h_X(X, Y, t), H_0 + h_y(X, Y, t), 0)$. The geometry of the current problem is drawn in Figure 1.

The geometrical shape of the deformations of wall is depicted as

$$\tilde{R}(X, t) = \tilde{a}(X) + \tilde{b} \sin \left( \frac{2\pi}{\lambda} (X - ct) \right),$$

where $\tilde{a}(X) = \tilde{b}_0 + \tilde{b}_1 X$, and $\lambda, a, \tilde{b}_0, \tilde{b}, c, t$ represents wavelength, channel half width at axial distance $X$, half width at inlet, wave amplitude, wave speed and time respectively, and ($\tilde{b}_1 \ll 1$) is constant.

The velocity field for current problem is $V = (U(X, Y, t), V(X, Y, t), 0)$, so equations (5)–(11) are defined in laboratory frame $(X, Y)$ as

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0,$$

$$\rho \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial X} + V \frac{\partial}{\partial Y} \right) U = - \frac{\partial P}{\partial X} + \frac{\partial S_{XX}}{\partial X} + \frac{\partial S_{XY}}{\partial Y} - \frac{\mu_e}{2} \left( \frac{\partial H^+}{\partial Y} \right)^2.$$ 

Figure 1. Geometry of the problem.
\[ + \mu_e \left( \frac{\partial h_x}{\partial x} + \frac{\partial h_y}{\partial y} + H_0 \frac{\partial h_x}{\partial y} \right) + \rho g \sin \alpha \]

\[ + g \left\{ \left( 1 - \Theta_0 \right) \rho_f (T - T_0) + \beta_c (C - C_0) \right\} - \left( \rho_p - \rho_f \right) (\Theta - \Theta_0), \]

\[ \rho \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} + V \frac{\partial}{\partial y} \right) V = - \frac{\partial P}{\partial Y} + \frac{\partial S_{xy}}{\partial x} + \frac{\partial S_{yy}}{\partial Y} - \mu_e \left( \frac{\partial H^+}{\partial Y} \right) \]

\[ + \mu_e \left( \frac{\partial h_y}{\partial x} + h_y \frac{\partial h_y}{\partial y} + H_0 \frac{\partial h_y}{\partial y} \right) - \rho g \cos \alpha, \]

\[ (\rho c)_f \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} + V \frac{\partial}{\partial y} \right) T = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \rho \left\{ D_B \left( \frac{\partial \Theta}{\partial x} + \frac{\partial \Theta}{\partial y} \right) \right\}, \]

\[ \left( \frac{D_T}{T_0} \right) \left[ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right] + D_{TC} \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right), \]

\[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} + V \frac{\partial}{\partial y} \right) C = D_s \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + D_{TC} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \]

\[ \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} + V \frac{\partial}{\partial y} \right) \Theta = D_B \left( \frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} \right) + \left( \frac{D_T}{T_0} \right) \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \]

where

\[ S_{XX} = 2 \frac{\partial U}{\partial x} \left( m + \eta \left\{ 2 \left( \frac{\partial U}{\partial x} \right)^2 + \left( \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial x} \right)^2 + 2 \left( \frac{\partial V}{\partial Y} \right)^2 \right\} \right), \]

\[ S_{XY} = \left( \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial x} \right) \left( m + \eta \left\{ 2 \left( \frac{\partial U}{\partial x} \right)^2 + \left( \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial x} \right)^2 + 2 \left( \frac{\partial V}{\partial Y} \right)^2 \right\} \right), \]

\[ S_{YY} = 2 \frac{\partial V}{\partial y} \left( m + \eta \left\{ 2 \left( \frac{\partial U}{\partial x} \right)^2 + \left( \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial x} \right)^2 + 2 \left( \frac{\partial V}{\partial Y} \right)^2 \right\} \right), \]

In fixed frames \((X, Y)\), flow is unsteady, but in wave frames \((x, y)\) motion is steady. The relationship between these two frames is defined as

\[ u = U - c, x = X - ct, v = V, y = Y, p(x, y) = P(X, Y, t), \]

Defining dimensionless quantities

\[ \bar{y} = \frac{y}{b_0}, \bar{x} = \frac{x}{\lambda}, \bar{v} = \frac{v}{c}, \delta = \frac{h_0}{\lambda}, \bar{p} = \frac{\rho g \delta}{\mu c^2}, \bar{t} = \frac{ct}{\lambda}, \text{Re} = \frac{\rho c b_0}{\mu}, \bar{r} = \frac{R}{b_0}, \]
\[
\theta = \frac{T - T_0}{T_1 - T_0}, \quad \gamma = \frac{C - C_0}{C_1 - C_0}, \quad \Omega = \frac{\Theta - \Theta_0}{\Theta_1 - \Theta_0}, \quad u = \frac{\partial \Psi}{\partial y},
\]
\[
\nu = -\delta \frac{\partial \Psi}{\partial x}, \quad h_x = \frac{\partial \Phi}{\partial y}, \quad h_y = -\delta \frac{\partial \Phi}{\partial x},
\]
\[
Pr = \frac{(\rho c)_p u}{k}, \quad Ln = \frac{v}{D_B}, \quad N_{CT} = \frac{D_{CT}(T_1 - T_0)}{(C_1 - C_0)D_s}, \quad N_{TC} = \frac{D_{CT}(C_1 - C_0)}{k(T_1 - T_0)}, \quad \eta = \frac{1}{m(\frac{b_0}{c})^{n-1}},
\]
\[
G_{rt} = \frac{\tilde{b}_0^2(1 - \Theta_0)(T_1 - T_0)p_f b_T}{\mu_0 c}, \quad G_{rc} = \frac{g(1 - \Theta_0)p_f \beta_s(C_1 - C_0)\tilde{b}_0^2}{\mu_0 c},
\]
\[
\tilde{S} = \frac{\tilde{b}_0}{\mu c} S, \quad Le = \frac{v}{D_s},
\]
\[
N_b = \frac{(\rho c)_p D_B(\Theta_1 - \Theta_0)}{k}, \quad N_t = \frac{(\rho c)_p D_T(T_1 - T_0)}{T_0 k}, \quad G_{rF} = \frac{g(p_f - p_r)(\Theta_1 - \Theta_0)\tilde{b}_0^2}{\mu_0 c},
\]
\[
(21)
\]

Where \( Re, Pr, \delta, G_{rt}, N_b, G_{rc}, Ln, Le, N_t, N_{CT}, N_{TC}, \gamma, \theta, \Omega, G_{rF} \) stands for Reynolds number, Prandtl number, wave number, thermal Grashof number, Brownian motion, solutal Grashof number, nanofluid Lewis number, Lewis number, thermophoresis parameters, Soret parameter, Dufour parameter, solutal (species) concentration, temperature, nanoparticle fraction, and nanoparticle Grashof number, respectively.

The dimensionless form of equation (12) is
\[
r(x) = 1 + \omega x + \beta \sin(2\pi x),
\]
(22)

where \( \beta = \frac{k}{b_0} \) stands for amplitude ratio or occlusion and \( \omega = \frac{\tilde{b}_1}{b_0} \).

Using equations (20) and (21), equation (13) is automatically satisfied and equations (14)–(19) in wave frame (after bars dropping) becomes
\[
Re\delta(\Psi_{xy}\Psi_y - \Psi_{yy}\Psi_x) = -\frac{\partial p_m}{\partial x} + \delta \frac{\partial S_{xx}}{\partial x} + \delta \frac{\partial S_{xy}}{\partial y} + \frac{Re}{Fr} \sin \alpha + ReS_1^2\Phi_{yy}
\]
\[
+ ReS_1^2 \delta(\Phi_{y}\Phi_{xy} - \Phi_{x}\Phi_{yy}) + G_{rt}\theta + G_{rc}\gamma - G_{rF}\Omega,
\]
(23)
\[
Re\delta^3(\Psi_{xy}\Psi_x - \Psi_{xx}\Psi_y) = -\frac{\partial p_m}{\partial y} + \delta^2 \frac{\partial S_{xx}}{\partial x} + \delta \frac{\partial S_{yy}}{\partial y} - \frac{Re}{Fr} \cos \alpha + Re\delta^2 S_1^2\Phi_{xy}
\]
\[
- ReS_1^2 \delta^3(\Phi_{y}\Phi_{xx} - \Phi_{x}\Phi_{xy}),
\]
(24)
\[
RePr\delta(\theta_x\Psi_y - \theta_y\Psi_x) = (\theta_{xy} + \delta^2 \theta_{xx}) + N_{TC}(\delta^2 \gamma_{xx} + \gamma_{yy})
\]
\[
+ N_b(\delta^2 \Omega_x\theta_x + \Omega_y\theta_y) + N_t(\delta^2(\theta_x)^2 + (\theta_y)^2),
\]
(25)
\[
\text{Re} \delta \text{Le} (\gamma_x \Psi_y - \gamma_y \Psi_x) = (\delta^2 \gamma_{xx} + \gamma_{yy}) + N_C T (\delta^2 \theta_{xx} + \theta_{yy}), \quad (26)
\]

\[
\text{Re} \delta \text{Ln}(\Psi_x \Omega_y - \Psi_y \Omega_x) = (\delta^2 \Omega_{xx} + \Omega_{yy}) + \frac{N_f}{N_b} (\delta^2 \theta_{xx} + \theta_{yy}), \quad (27)
\]

\[
\Psi_y - \delta (\Phi_x \Psi_y - \Phi_y \Psi_x) + \frac{1}{R_m} (\delta^2 \Phi_{xx} + \Phi_{yy}) = E. \quad (28)
\]

where

\[
S_{xx} = 2 \delta \left( 1 + \eta \left\{ 2 \delta^2 \left( \frac{\partial^2 \Psi}{\partial x \partial y} \right)^2 + \left( \frac{\partial^2 \Psi}{\partial y^2} - \delta^2 \frac{\partial^2 \Psi}{\partial x^2} \right)^2 + 2 \delta^2 \left( \frac{\partial^2 \Psi}{\partial x \partial y} \right)^2 \right\} \right)^{\frac{1}{2}} \frac{\partial^2 \Psi}{\partial x \partial y},
\]

\[
S_{xy} = \left( 1 + \eta \left\{ 2 \delta^2 \left( \frac{\partial^2 \Psi}{\partial x \partial y} \right)^2 + \left( \frac{\partial^2 \Psi}{\partial y^2} - \delta^2 \frac{\partial^2 \Psi}{\partial x^2} \right)^2 + 2 \delta^2 \left( \frac{\partial^2 \Psi}{\partial x \partial y} \right)^2 \right\} \right)^{\frac{1}{2}} \left( \frac{\partial^2 \Psi}{\partial y^2} - \delta^2 \frac{\partial^2 \Psi}{\partial x^2} \right),
\]

\[
S_{yy} = -2 \delta \left( 1 + \eta \left\{ 2 \delta^2 \left( \frac{\partial^2 \Psi}{\partial x \partial y} \right)^2 + \left( \frac{\partial^2 \Psi}{\partial y^2} - \delta^2 \frac{\partial^2 \Psi}{\partial x^2} \right)^2 + 2 \delta^2 \left( \frac{\partial^2 \Psi}{\partial x \partial y} \right)^2 \right\} \right)^{\frac{1}{2}} \frac{\partial^2 \Psi}{\partial x \partial y},
\]

Now utilizing long wavelength and low but finite estimation of the Reynolds number, the equations (23)–(29) becomes

\[
0 = -\frac{\partial p}{\partial x} + \frac{\partial S_{xy}}{\partial y} + \frac{\text{Re} \delta S_{1} \Phi_{yy}}{\text{Fr}^2 \sin \alpha} + \text{Re} S_{1} \Phi_{xy} + G_{t} \theta + G_{r} \gamma - G_{r} \Omega, \quad (30)
\]

\[
0 = -\frac{\partial p}{\partial y}, \quad (31)
\]

\[
\frac{\partial^2 \theta}{\partial y^2} + N_{rC} \frac{\partial^2 \gamma}{\partial y^2} + N_b \left( \frac{\partial \Omega \partial \theta}{\partial y \partial y} \right) + N_t \left( \frac{\partial \theta}{\partial y} \right)^2 = 0, \quad (32)
\]

\[
\frac{\partial^2 \gamma}{\partial y^2} + N_C T \frac{\partial^2 \theta}{\partial y^2} = 0, \quad (33)
\]

\[
\frac{\partial^2 \Omega}{\partial y^2} + N_t \frac{\partial^2 \theta}{\partial y^2} = 0. \quad (34)
\]

\[
\frac{\partial^2 \Phi}{\partial y^2} = R_m \left( E - \frac{\partial \Psi}{\partial y} \right). \quad (35)
\]

where
\[ S_{xy} = \left( 1 + \eta \left\{ \left( \frac{\partial^2 \Psi}{\partial y^2} \right)^2 \right\}^{\frac{1}{2}} \right) \frac{\partial^2 \Psi}{\partial y^2}. \]  

(36)

Removing pressure from equations (30) and (31) yields

\[
\frac{\partial^2}{\partial y^2} \left[ 1 + \eta \left\{ \left( \frac{\partial^2 \Psi}{\partial y^2} \right)^2 \right\}^{\frac{1}{2}} \right] \frac{\partial^2 \Psi}{\partial y^2} + Re S_1^2 \Phi_{yy} + \frac{Gr_t}{Re} \frac{\partial \theta}{\partial y} + \frac{Gr_c}{Re} \frac{\partial \gamma}{\partial y} - \frac{Gr_r}{Re} \frac{\partial \Omega}{\partial y} = 0,
\]

(37)

\[
\frac{\partial^4 \Psi}{\partial y^4} + \eta \frac{\partial^2}{\partial y^2} \left\{ \left( \frac{\partial^2 \Psi}{\partial y^2} \right)^2 \right\} - Re S_1^2 R_m \frac{\partial^2 \Psi}{\partial y^2} + \frac{Gr_t}{Re} \frac{\partial \theta}{\partial y} + \frac{Gr_c}{Re} \frac{\partial \gamma}{\partial y} - \frac{Gr_r}{Re} \frac{\partial \Omega}{\partial y} = 0.
\]

(38)

The boundary conditions for the proposed problem in wave frame are defined as follows:

\[ \Psi = 0, \quad \frac{\partial^2 \Psi}{\partial y^2} = 0 \text{ on } y = 0, \]

\[ \Theta = F, \quad \frac{\partial \Theta}{\partial y} = -1 \text{ on } y = r(x) = 1 + \omega x + \beta \sin(2\pi x), \]

(39)

\[ \Theta = 0, \quad \frac{\partial \Theta}{\partial y} = 0 \text{ and } \Theta = 0 \text{ on } y = r(x), \]

(40)

\[ \theta = 0, \quad \frac{\partial \theta}{\partial y} = 0 \text{ and } \theta = 1 \text{ on } y = r(x), \]

(41)

\[ \Omega = 0, \quad \frac{\partial \Omega}{\partial y} = 0 \text{ and } \Omega = 1 \text{ on } y = r(x), \]

(42)

\[ \gamma = 0, \quad \frac{\partial \gamma}{\partial y} = 1 \text{ on } y = r(x). \]

(43)

Here, \( F \) is the time mean wave frame flow rate that can be related with the time mean flow \( Q \) by \( Q = F + 1 \) and \( F = \int_0^r \frac{\partial \psi}{\partial y} dy \).

**Solution methodology**

*Exact solution*

The exact solution of temperature, volume fraction of the nanoparticle and concentration of solutal (species) that fulfills the boundary conditions (41 – 43) is

\[ \theta = \frac{e^{-\lambda y} - 1}{e^{-\lambda r} - 1}. \]

(44)

\[ \Omega = \frac{N_t (e^{-\lambda y} - 1)}{N_t (1 - e^{-\lambda r})} + \frac{y \left( \frac{N_t}{N_b} + 1 \right)}{r}. \]

(45)
\[ \gamma = \frac{N_{CT}(e^{-\lambda y} - 1)}{1 - e^{-\lambda r}} + \frac{y(N_{CT} + 1)}{r}. \]  

(46)

where

\[ \lambda = \frac{N_t + N_b}{r(1 - N_{CT}N_{TC})}. \]  

(47)

**Numerical solution**

The exact solutions of equations (30), (35), and (38) is difficult due to non-linearity nature of partial differential equations. Numerical solution of the non-linear equations are calculated using Mathematica software. Thus, by numerical approximation to solutions, graphical illustration is achieved.

**Graphical evaluation**

In this part, the graphical results of the existing problems are thoroughly discussed. To address the highly non-linear equations, numerical solution is computed, and graphical illustration is shown to see impact of concentration, temperature, nanoparticle fraction, pressure gradient, pressure rise, magnetic force function and velocity of numerous interesting physical parameters. To observe the graphical behavior of thermophoresis \( N_t \) and Dufour \( N_{TC} \) parameters on temperature, concentration, and nanoparticle fraction Figures 2 to 7 are plotted. It is illustrated in Figures 2 and 3 that there is an increase in temperature profile by increasing the
values of the parameters of thermophoresis and Dufour. This is because parameters of thermophoresis and Dufour reveal a direct relationship with temperature. Figures 4 and 5 are drawn to see the impact of Dufour and thermophoresis parameters on concentration. It’s shown in Figures 4 and 5 that magnitude of the concentration profile increases by increasing the values of the parameters of
Dufour and thermophoresis. Similar behavior is observed for the case of nanoparticle fraction. It’s depicted from Figures 6 and 7 that magnitude of the nanoparticle fraction increases due to the increasing values of thermophoresis and Dufour parameters. This is because parameters of thermophoresis and Dufour reveal a direct relationship with nanoparticle fraction and concentration. Figures 8 to 10 are represented to see consequence of pressure rise on non-Newtonian parameter $h$, $v$ and $Fr$. To study the pressure rise effects we divided the peristaltic regions into three parts, namely, retrograde ($\Delta p>0, Q<0$), peristaltic ($\Delta p>0, Q>0$), and copumping ($\Delta p<0, Q>0$) regions. It is illustrated in Figure 8 that in retrograde pumping region, pressure rise increases due to increasing values of non-Newtonian parameter $h$, whereas behavior in peristaltic and copumping regions is quite reverse. Here pressure rise decreases by increasing $\Box$. It’s noted in Figure 9 that by increasing $\omega$ values there is a decrease in pressure rise in retrograde and peristaltic pumping regions, while in copumping region pressure rise increases. Figure 10 is drawn to viewing the impact of pressure rise for increasing Froude number $Fr$. It’s shown in Figure 10 that by increasing $Fr$ values the pressure rise decreases in all peristaltic pumping regions. To witness the impact of pressure gradient on $\eta, \omega$, and $R_m$, Figures 11 to 13 are sketched. It is noted in Figure 11 that in the region when $x \in [0, 0.17]$ and $x \in [0.4, 1]$ the pressure gradient increases by increasing non-Newtonian parameter $\eta$ values but when $x \in [0.18, 0.4]$ the pressure gradient decreases. It is clarified from Figure 12 that behavior of pressure gradient for case of $\omega$ is opposite as compared with non-Newtonian parameter $\eta$. In this case pressure gradient decreases when $x \in [0, 0.1]$ and $x \in [0.35, 1]$ by increasing $\omega$ values but when $x \in [0.1, 0.35]$ the pressure gradient decreases. Figure 13 examines the
pressure gradient impact on $R_m$. It is noticed that magnitude values of pressure gradient decrease by increasing $R_m$ values (see Figure 13). Velocity profile behavior is observed in Figures 14 and 15 for different values of $N_b$ and $Q$. It is illustrated in Figure 14 that increasing $N_b$ values increase the magnitude of the velocity profile when $y \in [0, 0.6]$, whereas when $y \in [0.6, 1.2]$ the magnitude values of velocity
profile decreases. It is noted in Figure 15 that by increasing volume flow rate $Q$ the magnitude value of velocity profile increases. The effects of $E$ and $R_m$ on magnetic force are shown in Figures 15 and 17. This can be observed from Figure 16 that by increasing $E$ values, magnetic force feature decreases. It is illustrated in Figure 17 that there is increase in magnetic force function by increasing $R_m$ values.

**Figure 8.** Effect of $\eta$ on pressure rise $\Delta p$.

**Figure 9.** Effect of $\omega$ on pressure rise $\Delta p$. 
An interesting phenomenon is trapping which occurs in peristaltic supported flows. It is defined as the formation of internally moving fluid mass besieged by the streamlines of peristaltic waves. At the exalted flow rates and significant occlusions, streamlines catch the fluid mass bolus and push it ahead along with peristaltic waves. Figures 18 to 20 describes that the trapped bolus size decreases for high values of $G_{rt}$, $\eta$ and $Q$.

![Figure 10. Effect of $Fr$ on pressure rise $\Delta p$.](image1)

![Figure 11. Effect of $\eta$ on pressure gradient $dp/dx$.](image2)
This section addresses the final remarks on the present issue. The impact of double-diffusivity convection on peristaltic flow of Sisko nanofluids in inclined non-uniform channel in the presence of induced magnetic field is discussed.

**Figure 12.** Effect of $\omega$ on pressure gradient $dp/dx$.

**Figure 13.** Effect of $R_m$ on pressure gradient $dp/dx$.

## Concluding remarks

This section addresses the final remarks on the present issue. The impact of double-diffusivity convection on peristaltic flow of Sisko nanofluids in inclined non-uniform channel in the presence of induced magnetic field is discussed.
Mathematical modeling of Sisko nanofluids with induced magnetic field and double-diffusivity convection are described in detail. For the problem under consideration, exact and numerical solutions are presented. The main findings are as follows.

Figure 14. Effect of $N_b$ on velocity profile $u$.

Figure 15. Effect of $Q$ on velocity profile $u$.

Mathematical modeling of Sisko nanofluids with induced magnetic field and double-diffusivity convection are described in detail. For the problem under consideration, exact and numerical solutions are presented. The main findings are as follows.
The temperature profile, magnitude value of concentration profile, and magnitude value of nanoparticle fraction significantly rise as the thermophoresis and Dufour parameters are increased.

Increasing non-Newtonian parameter \( h \) the pressure rise increases in retrograde pumping region, whereas in peristaltic and copumping regions pressure rise decreases.

**Figure 16.** Effect of \( E \) on magnetic force function \( \Phi \).

**Figure 17.** Effect of \( R_m \) on magnetic force function \( \Phi \).

- The temperature profile, magnitude value of concentration profile, and magnitude value of nanoparticle fraction significantly rise as the thermophoresis and Dufour parameters are increased.
- Increasing non-Newtonian parameter \( \eta \) the pressure rise increases in retrograde pumping region, whereas in peristaltic and copumping regions pressure rise decreases.
Figure 18. Streamlines for various $G_r$ values.

Figure 19. Streamlines for various $\eta$ values.

Figure 20. Streamlines for various $Q$ values.
Increasing non-uniform parameter \( \omega \) the pressure rise decreases in retrograde and peristaltic pumping regions, while in copumping region pressure rise increases.

By increasing \( Fr \) values the pressure rise decreases in all the peristaltic pumping regions.

The pressure gradient increases by increasing non-Newtonian parameter \( \eta \).

By increasing \( E \), the magnetic force function decreases and by increasing \( R_m \) magnetic force function increases.

The trapped bolus size reduces when higher values of \( G_{rt}, \eta, \) and \( Q \) are considered.

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