Comparative study of high-spin isomers in semi-magic $Z=50$ isotopic and $N=82$

isotonic chains

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A comparative study of high spin nuclear isomers observed in the semi-magic $Z=50$ isotopic and $N=82$ isotonic chains has been carried out. The 11/2$^-$, 10$^+$ and 27/2$^-$ isomers, which occur commonly in both the chains, display nearly identical systematics in excitation energy and half-life. An energy gap of ~ 4 MeV between the 0$^+$ ground states and 10$^+$ isomers and the 11/2$^-$ and 27/2$^-$ isomers exists before the mid-shell, which becomes a constant ~ 3 MeV after the mid-shell region. The large scale shell model calculations are able to reproduce the observed energy systematics for both the chains reasonably well. The shell model occupancies and the basic seniority rules have been used to fix the seniority quantum number. The seniority of all the isomeric states as well as those involved in the decay from/to the isomers have been assigned and the alignment properties are also discussed. The seniorities of the 10$^+$ and 27/2$^-$ isomeric states before the mid-shell are higher, which become lower after the mid-shell, due to the dominant role played by the $h_{11/2}$ orbital. The empirical systematics and the calculated results suggest that the change in the energy gap around the mid-shell, may be interpreted in terms of a change in the seniority of the isomeric states. However, the seniority of the 11/2$^-$ state remains conserved throughout both the chains. The systematics of the half-lives in both the chains have been understood on the basis of seniority and decay modes. Predictions for new isomers have also been made based on these systematics.

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I. INTRODUCTION

Nuclear isomers, which are metastable states of nuclei, may be regarded as a separate class of nuclei and their study has recently gained prominence because of many reasons, both fundamental as well as applied [1]. The experimental data on isomers have been growing significantly and the "Atlas of Nuclear Isomers" prepared by us lists more than 2450 isomers with a half-life cut-off ∼ 3 MeV after the mid-shell region.

The large scale shell model calculations are able to reproduce the observed energy systematics for both the chains reasonably well. The shell model occupancies and the basic seniority rules have been used to fix the seniority quantum number. The seniority of all the isomeric states as well as those involved in the decay from/to the isomers have been assigned and the alignment properties are also discussed. The seniorities of the 10$^+$ and 27/2$^-$ isomeric states before the mid-shell are higher, which become lower after the mid-shell, due to the dominant role played by the $h_{11/2}$ orbital. The empirical systematics and the calculated results suggest that the change in the energy gap around the mid-shell, may be interpreted in terms of a change in the seniority of the isomeric states. However, the seniority of the 11/2$^-$ state remains conserved throughout both the chains. The systematics of the half-lives in both the chains have been understood on the basis of seniority and decay modes. Predictions for new isomers have also been made based on these systematics.

We find that the nuclear isomers in semi-magic nuclei having proton number $Z=50$ and neutron number $N=82$ represent a very useful set of data. This also allows us to compare the behavior of the isomers in the Sn isotopes($N=54$ to 81) approaching the neutron-drip line with the isomers in the $N=82$ isotones($Z=51$ to 73) approaching the proton-drip line. In order to cover the full isotopic/isotonic chains, we have relaxed the cut-off limit of half-life at 10 ns and included all the known isomers in our study.

The level schemes of the $Z=50$, $^{119-130}$Sn isotopes have recently been measured by using the reactions induced by light ions, deep inelastic reactions, and fission fragment studies [12-24]. More recently Astier et al. [21, 22] has reported detailed level schemes of the $^{119-126}$Sn isotopes by using the binary fission fragmentation induced by heavy ions. The isomeric states 10$^+$ and 27/2$^-$ have been characterized as seniority $v=2$ and $v=3$ states respectively.

Similarly, the high-spin structure of the five $N=82$ isotones with $Z=54-58$ have also been studied by Astier et al. [23], where the isomeric states like 10$^+$ and 27/2$^-$, in even and odd mass isotopes respectively, have been described as broken proton pairs occupying the high-$j$ unique parity $h_{11/2}$ orbital. Iskra et al. [24] have also focused on high-seniority states in neutron-rich even-even Sn isotopes.

It may be noted that there exist some deformed collective states giving rise to an excited rotational band in even-even light mass Sn isotopes, interpreted as 2p-2h proton configuration. Such excited bands have been observed in the $^{116-118}$Sn isotopes [25, 31]. But the 10$^+$ isomeric state discussed in the present paper, exists as a yrast state, and does not support rotational struc-
In this paper, we present a comprehensive study of the systematics of the $11^2$, $10^+$ and $27^2$ isomeric states in the semi-magic $Z=50$ Sn-isotopes and $N=82$ isotones in section II. The energy and the half-life systematics of the nuclear isomers reveal many interesting features. We have carried out large scale shell model calculations to explain the systematic features and decipher their configurations, which we discuss in the section III.

The configuration and seniority assignments to these isomeric states have also been made with the validation of their measured excitation energy and half-life trend in the sections IV and V.

We then discuss the alignment of the $h_{11/2}$ neutrons/protons, particularly after the mid-shell in the section VII. This section also discusses the seniorities of the $2^+$, $12^+$, $15^2$, and $31^2$ states, after the mid-shell. On the basis of the present study, we claim that the isomeric states $11^2$, $10^+$ and $27^2$ may be identified as high-spin seniority isomers throughout both the chains. We find that the seniority appears to be a reasonably good quantum number in semi-magic isomers up to $j=11/2$.

The section VII gives an explanation of the experimental half-life systematics of these isomers in terms of the seniority and respective decay modes for both the chains. The $11^2$ isomeric states decay to the $7^2$, $5^2$, $3^2$, $1^2$ states; we have discussed their seniorities too. The seniority of the $8^+$ and $23^2$ states have also been assigned, to which the $10^+$ and $27^2$ isomers decay, in the same section. We also predict many new isomers and their properties from the observed and calculated systematic trends in the section VIII. The section IX summarizes and concludes the present work.

II. EXPERIMENTAL SYSTEMATICS OF $Z=50$ AND $N=82$ ISOMERS

On the basis of our compilation, we find that the $I=11^2$, $10^+$ and $27^2$ isomers are common to both the $Z=50$ isotopes as well as the $N=82$ isotones which covers a range of 32 nucleons. It is interesting that the same valence space, consisting of $0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$ and $0h_{11/2}$ orbitals, is involved in both the cases. While the neutrons occupy these orbitals in the $Z=50$ isomers, the protons take over that role in the $N=82$ isomers. This allows us to study the similarities and differences that may arise when protons are replaced by neutrons in the same set of orbitals.

We first discuss the experimental excitation energies and half-life systematics for both $Z=50$ and $N=82$ isomers. For the sake of simplicity and discussion, we divide the available valence space of 32 nucleons in three parts:

1. Nucleon numbers 51 to 58, before the mid-shell,
2. Nucleon numbers 59 to 66, the transition-region, and
3. Nucleon numbers 67 to 81, after the mid-shell.
A. Excitation energy systematics for the $Z=50$ isomers

We have plotted the measured excitation energies of the 11$^+$, 10$^+$ and 27$^+$ isomers for the $N=51$ to 81 chain of Sn-isotopes in the top panel of Fig. 1 where firm data are available.

1. Neutron numbers 51 to 58, before the mid-shell

A very limited experimental data are available for the isomers in this region. The 11$^+$ isomeric state is known only at $N=58$ and the 10$^+$ isomeric state is known at neutron numbers 54 and 56. The excitation energy of the $N=58$, 11$^+$ isomer is 1.67 MeV whereas the excitation energies of the 10$^+$ isomers are of the order of 4 MeV at both $N=54$ and 56. The 27$^+$ isomeric state is observed only at $N=57$ with energy $\sim 5$ MeV.

2. Neutron numbers 59 to 66, the transition-region

The excitation energies of all the three isomeric states show a very smooth variation as the neutron number varies from 59 to 66. Also, the energies of the 10$^+$ and 27$^+$ isomers begin to follow each other. The energy values change from approximately 1.3 and 4−5 MeV to the values 0.7 and 3−3.5 MeV for the 11$^+$, and 27$^+$ isomers respectively. As a result, the relative energy gap of $\sim 4$ MeV between the 11$^+$ and 27$^+$ isomers comes down to 2.5–3 MeV in going from the nucleon number 59 to 65. The 10$^+$ isomeric state lies at $\sim 4$ MeV from the 0$^+$ ground state, which reduces to 3.5 MeV from $N=64$ to 66. Therefore, the separation of 27$^+$ and 11$^+$ states is of the same order as the separation of 10$^+$ and 0$^+$ states. This suggests that the same structural change is involved in going to the 10$^+$ and 27$^+$ states from the 0$^+$ and 11$^+$ states, for the even-even and odd-A Sn-isotopes, respectively.

3. Neutron numbers 67 to 81, after the mid-shell

As the neutron number increases and crosses the mid-shell near $N=66$, the 11$^+$ isomeric state briefly becomes the ground state for $N=73$, 75 and 77; it again becomes an isomeric state for $N>77$. The 10$^+$ and the 27$^+$ isomers belonging to even-even and even-odd nuclei respectively follow each other very closely throughout this region. This suggests that the configurations for the 10$^+$ and the 27$^+$ isomers are very similar in nature. Further, the separation in the excitation energies between 27$^+$ and 11$^+$ as well as 10$^+$ and 0$^+$ states remains constant, which again suggests that the same structural change is involved in going from 11$^+$ to 27$^+$ and 0$^+$ to 10$^+$ states throughout this isotopic region. As we shall show, this feature supports the interpretation of these isomers as seniority isomers.

We note two striking features for the Z=50 isomers from the previous discussion. Firstly, the energy gap between the 11$^+$ and 27$^+$ isomeric states is quite large and varies between $\sim 4$ MeV before the mid-shell. Similarly, the 10$^+$ state appears at $\sim 4$ MeV from the 0$^+$ ground state up to the mid-shell $N=64$. Secondly, near the mid-shell, a transition seems to be taking place where the excitation energies suddenly dip and the gap becomes $\sim 3$ MeV for 10$^+$ and 27$^+$ state, after the mid-shell.

B. Excitation energy systematics for the $N=82$ isomers

We have plotted a similar graph in the top panel of Fig. 2 for the measured excitation energies of the 11$^+$, 10$^+$ and 27$^+$ isomers in the N=82 isotopic chain having $Z=51$ to 81.

1. Proton numbers 51 to 58, before the mid-shell

Again, not many data are available for the 11$^+$ and 10$^+$ isomers. The 27$^+$ isomeric state has not yet been observed in this region. The 11$^+$ isomeric state is present at the proton number 51 with one extra particle to the closed shell configuration at $\sim 3$ MeV. The 10$^+$ isomeric state is observed at the proton numbers 56 and 58 at $\sim 4$ MeV.

2. Proton numbers 59 to 66, the transition-region

The 11$^+$ isomeric state exhibits a gradual decline in energy from $\sim 1.5$ MeV to $\sim 0$ MeV as the proton number increases from 59 to 65. It comes very close to the ground state at the proton number 65. The 10$^+$ isomer is observed at $Z=60$ and 64 at energy $\sim 4$ MeV, which declines to $\sim 3.5$ MeV at $Z=66$. The 27$^+$ isomers are not seen in this region.

3. Proton numbers 67 to 81, after the mid-shell

With the increase in proton number, the 11$^+$ isomeric state briefly becomes the ground state for $Z=67$, 69 and 71 just after the mid-shell i.e. $Z=66$; black dots have been used to denote them. It again becomes an isomeric state for $Z>71$. The 10$^+$ and the 27$^+$ isomers belonging to the even-even and even-odd nuclei of this isotonic chain follow each other very closely as in the Sn-isotopes. This again confirms the similar nature of the configurations for the 10$^+$ and the 27$^+$ isomeric states similar to what was observed for the isomers in the $Z=50$ isotopic chain. Again, the separation in the excitation energies remains nearly constant at $\sim 3$ MeV, which
suggests that the same mechanism is operative in going from $11/2^-$ to $27/2^-$ and $0^+$ to $10^+$ states throughout the isotonic chain.

To sum up, we note that the main features observed in the $Z=50$ isotopic chain are present in the $N=82$ isotonic chain also. We thus find a large spacing of $\sim 4$ MeV between the $11/2^-$ and $27/2^-$, and the $0^+$ and $10^+$ isomeric excitations. The large gap declines after the mid-shell suggesting the same kind of structural change in both $Z=50$ isotopic and $N=82$ isotonic chains. We shall show that this transition in both the regions at the mid-shell is due to the change in their quasiparticle (qp) configurations and the seniority quantum numbers.

FIG. 3: (Color online) Comparison of the half-lives of the isomeric states for $Z=50$ and $N=82$ chains. Black dots represent the ground states for $11/2^-$. Similar although valence protons are involved in one case while valence neutrons are involved in the other. The half-lives exhibit a rise near the mid-shell, attain a maximum value, and fall with increasing nucleon number.

The half-life of isomeric states $11/2^-$ shows a peak at $N=71$, and briefly becomes the ground state at $N=73-77$ for the $Sn$-isotopic chain. The $10^+$ and $27/2^-$ isomeric states also exhibit a maximum at neutron numbers 72 and 73 respectively. On the other hand, for the $N=82$ isomers, the peaks are observed at proton numbers 65, 70 and 71 for the isomeric states $11/2^-$, $10^+$ and $27/2^-$, respectively. It is noteworthy that the $11/2^-$ isomeric state for the $N=82$ isotonic chain, also shows the peak at $Z=65$ before becoming the ground state for the proton numbers $Z=67-71$. We note that the half-life of the isomeric state $10^+$, common to both the regions, attains a peak when the $h_{11/2}$ neutron/proton orbital is close to the half-filled configuration $[72]$. Same is true for the half-lives of the isomeric state $27/2^-$ in both the regions. Similar overall trend in the excitation energy and half-life for the $Sn$-isotopic chain and the $N=82$ isotonic chain also attests to the charge-symmetric nature of the interactions for the nuclei which have broadly different isospins.

It has been shown that the two body matrix elements of even tensor operators like quadrupole operator, between states with same seniority diminish at the mid-shell configuration $[71]$. As we shall show, the seniority remains reasonably good even for high-$j$ orbitals in the semi-magic isomeric states. Therefore, the electric quadrupole transitions between the same seniority states are expected to diminish when the valence shell is nearly half-filled. This argument can be applied to explain the isomerism of the $10^+$ and $27/2^-$ states in the present context. The isomeric states $10^+$ and $27/2^-$ mostly decay via $E2$ transitions to the lower lying $8^+$ and $23/2^-$ states, expected to be of the same seniority i.e., $\Delta\nu=0$ for both $Z=50$ isotopic and $N=82$ isotonic chain, giving rise to isomerism. The peaks of half-life for the $10^+$ and $27/2^-$ isomeric states lie near the nucleon numbers $70-73$, where the $h_{11/2}$ orbital is expected to get the half-filled occupancy in both the $Z=50$ and $N=82$ regions, and the $E2$ matrix elements almost vanish. We further elaborate these results in the section VII.

An understanding of the half-life systematics of the $11/2^-$ isomers is more complex. The $11/2^-$ isomeric state, for both the $Z=50$ and $N=82$ regions, are observed to decay via magnetic transitions $M2$ and higher, depending upon the available lower energy state. As we increase the nucleon numbers, the gamma decay energy keeps diminishing and the multipolarity rises to $M4$. As a result, the half-lives increase. The decay mode begins to change at the peaks in the half-lives, and $\beta$-decay/EC takes over. All the isomers become $\beta$-decaying isomers from $N=73-81$, for the $Z=50$ Sn-isotopic chain. This is also the reason that the half-lives of the $N=73-77$ isotopes, where the $11/2^-$ becomes the ground state, also follow the same half-life trend. However, the half-

C. The half-life systematics for the $Z=50$ and $N=82$ isomers

We have also plotted the respective measured half-lives (in $\mu$s) of these isomers with increasing nucleon number in Fig. 4. Due to the wide range of half-lives, we have introduced a break in the vertical scale, which is log after the break. The half-life systematics for the $Z=50$ isomers are plotted in the top panel, while the same for the $N=82$ isomers are plotted in the bottom panel. Here the black dots denote the situation where the $11/2^-$ state briefly becomes the ground state. It is very interesting to note that the ground state half-lives of $11/2^-$ state fit perfectly into the same trend as the isomeric half-lives. Further, the systematic trend in both the panels is very

FIG. 4: (Color online) Comparison of the half-lives of the isomeric states for $Z=50$ and $N=82$ chains. Black dots represent the ground states for $11/2^-$. Similar although valence protons are involved in one case while valence neutrons are involved in the other. The half-lives exhibit a rise near the mid-shell, attain a maximum value, and fall with increasing nucleon number.
lives still remain quite large because of the $\beta$-decaying mode. Similarly, the 11/2$^{-}$ isomeric state, for the $N=82$ isotonic chain, decays via EC-decay at proton numbers $Z=65$–69, then competes with $\alpha$-decay at $Z=71$, and finally becomes proton-decaying isomer for $Z=73$. Again, the change in the decay mode occurs at the $Z=65$ peak. The half-life of the $Z=73$ isotope, the last one to be observed, does not fit into the trend of the other isotones because of the proton-decay mode.

It would indeed be interesting to look at the detailed configurations and other properties which finally lead to the almost identical experimental systematics of the excitation energies and half-lives in both the $Z=50$ and $N=82$ regions. We have, therefore, carried out large scale shell-model calculations for these single-closed shell nuclei, which give detailed information about their configurations and allow us to fix the seniority of the isomeric states, confirming the assignments made in the earlier works. The theoretical results presented in the next section strongly support the interpretation given above and highlight the importance of seniority in the semi-magic isomers.

III. SHELL MODEL CALCULATIONS

The calculations involving many identical nucleons in a single-$j$ configuration are easier to deal with in the nuclear shell model \cite{6, 7}. However, the calculations for actual nuclei generally involve many particles in different orbitals. This results in the configuration mixing of different orbitals depending on their relative positions and the interaction matrix elements.

A. Calculation details

Neutrons in the $Z=50$ isotopes and protons in the $N=82$ isotopes occupy the same valence space consisting of 0$g_{7/2}$, 1$d_{5/2}$, 1$d_{3/2}$, 2$s_{1/2}$ and 0$h_{11/2}$ orbitals between the magic proton/neutron numbers 50 and 82. We have carried out large scale shell model calculations in the region of single closed shell nuclei for $Z=50$ isotopes and $N=82$ isotones. The calculations have been performed by using the Nushell code \cite{33} along with the SN100PN \cite{34} interaction. The proton and neutron single particle energies have been taken as 0.8072, 1.5623, 3.3160, 3.2238, 3.6051 MeV and $-10.6089, -10.2893, -8.7167, -8.6944, -8.8152$ MeV for the available 0$g_{7/2}$, 1$d_{5/2}$, 1$d_{3/2}$, 2$s_{1/2}$ and 0$h_{11/2}$ valence orbitals respectively.

The calculations have been carried out for the three regions as used in the previous discussion of systematics i.e. from nucleon number 51 to 58, the mid-shell transition region from 59 to 66 and after the mid-shell having nucleon number $>66$.

1. Nucleon numbers 51 to 58, before the mid-shell

We have done the calculations with full open valence space up to nucleon number 58 for this region, where valence nucleons can occupy any of the available 0$g_{7/2}$, 1$d_{5/2}$, 1$d_{3/2}$, 2$s_{1/2}$ and 0$h_{11/2}$ valence orbitals.

2. Nucleon numbers 59 to 66, the transition-region

We introduce some constraints to reduce the dimensions by adding some particles to $g_{7/2}$ and $d_{5/2}$ orbitals so that the calculations remain tractable and could be done in a reasonable time. Although we already have 8 particles which could fill up the $g_{7/2}$ orbital completely, there are strong chances of mixing of $g_{7/2}$ and $d_{5/2}$ orbitals. Further, the $h_{11/2}$ orbital is required to get the desired isomeric negative spin state in this transition region. We, therefore, allow the $g_{7/2}$ to be filled by 4–8 particles.

3. Nucleon numbers 67 to 81, after the mid-shell

We have truncated the valence space by freezing the $g_{7/2}$ orbital after the mid-shell region, i.e. nucleon number $>66$. It reduces the dimensions of the calculations significantly without affecting the desired results. The remaining particles are then allowed to occupy the next four available orbitals i.e. 1$d_{5/2}$, 1$d_{3/2}$, 2$s_{1/2}$ and 0$h_{11/2}$.

B. Results

The bottom panels of Fig. 1 and 2 present the calculated excitation energies for the $Z=50$ and $N=82$ isomers respectively. We find that all the three isomeric negative spin states 11$g_{7/2}$, 11$d_{5/2}$, 11$d_{3/2}$, 11$s_{1/2}$ and 11$h_{11/2}$ are systematically smaller than the experimental values.

A few general comments on the generation of the required spins of these isomers are in order. We note that the spin 11/2$^{-}$ can be generated only for the nucleon numbers 51 to 81 and necessarily requires the involvement of the $h_{11/2}$ orbital. Similarly, the 10$^{+}$ and the 27/2$^{-}$ spins can be generated only for nucleon numbers from 52 to 80 and 53 to 79, respectively. This is an outcome of the fact that it is not possible to generate the
spin $10^+$ at the nucleon number 51 or, 81 with one particle / hole in the closed shell configurations at $50/82$. Further, the $10^+$ spin at nucleon number 52 and 80 can only be generated by two particles or two holes in the $h_{11/2}$ orbital. Similarly, the $27/2^-$ spin at nucleon numbers 53 and 79 can only be generated by three particles or three holes in the $h_{11/2}$ orbital.

The detailed features of the calculated excitation energies for both the $Z=50$ and $N=82$ isomers are discussed below:

1. **Nucleon numbers 51 to 58, before the mid-shell**

   The relative energy gap between the $11/2^-$ and $27/2^-$ states is calculated to be $\sim 4$ MeV. There is a sudden jump in the energy of $27/2^-$ at the nucleon number 53 for both $Z=50$ and $N=82$ isomers. This may be understood as the spin $27/2^-$ can be generated only by having all the three nucleons in the $h_{11/2}$ orbital which lies much higher in energy than the Fermi level. Similarly, the calculated energy of the $10^+$ isomeric states relative to the $0^+$ ground state is $\sim 4$ MeV, same as the experimentally observed difference. The calculated energy of the $10^+$ state also exhibits a jump at the nucleon number 52. The same reason is valid here also since the $10^+$ state with two particles in the valence orbital can only be produced by putting them into the $h_{11/2}$ orbital, which makes it higher in energy.

2. **Nucleon numbers 59 to 66, the transition-region**

   The observed trend of the experimental data is well reproduced by the calculated results. The calculated energy difference between the $11/2^-$ and $27/2^-$ states declines from $3-4$ MeV to $\sim 2$ MeV in going from nucleon numbers 59 to 65, which is smaller than the experimentally observed value. The $10^+$ isomeric state, which follows the $27/2^-$ state, also shows a fall in energy in this region, which is again smaller than the experimental values.

3. **Nucleon numbers 67 to 81, after the mid-shell**

   As the nucleon number crosses the mid-shell configuration at 66, the $11/2^-$ state becomes the ground state in the calculations. The energy difference between the $0^+$ and $10^+$ states and, the $11/2^-$ and $27/2^-$ isomers becomes $\sim 2$ MeV and remains constant throughout the range from 67 to 81, smaller than the measured values. Further, the $10^+$ states closely follow the $27/2^-$ state in a very smooth manner. It clearly suggests that the similar type of structure is responsible for the $10^+$ and $27/2^-$ states. The $h_{11/2}$ orbital is expected to play the main role in deciding the configuration of the isomeric states in this region since the $g_{7/2}$ remains completely filled.

![FIG. 4: (Color online) Variation of occupancy corresponding to the maximum partition for the $Z=50$ isotopes.](image1)

![FIG. 5: (Color online) Variation of occupancy corresponding to the maximum partition for the $N=82$ isotopes.](image2)

**IV. THE WAVE FUNCTIONS AND THE CONFIGURATION ASSIGNMENTS**

We now discuss the wave functions obtained from the shell model calculations and assign the configuration to...
the states by using the occupancies to the maximum partition.

A. \( Z=50 \) isomers

The wave functions obtained from the shell model calculations are analyzed for the maximum partition and presented in the form of a bar chart for the Sn-isotopes in Fig. 4. We have plotted the calculated configurations of the \( 0^+, 10^+ \) and \( 11/2^-, 27/2^- \) states corresponding to the even-even and odd-A Sn-isotopes respectively. The bottom panel of the plot shows the occupancies for the \( 10^+ \) and \( 27/2^- \) states for each even and odd neutron number, which appear alternately on the x-axis. We can easily read the calculated occupancies of the different orbitals by following the color coding. For example, the red color corresponds to the \( d_{5/2} \) orbital. Its absence in a bar implies no particle in the \( d_{5/2} \) orbital in the configuration of that isotope. Thus the final configuration of the \( 10^+ \) state of \( {^{122}\text{Sn}} \) can be read from Fig. 4 as \{\( g_{7/2}^0 \ d_{5/2}^0 \ d_{3/2}^0 \ h_{11/2}^6 \)\}. The top panel of the Fig. 4 similarly exhibits the occupancies for the maximum partition in the wave function of the \( 0^+ \) and \( 11/2^- \) states in the Sn-isotopes. We note that the nature of the wave function of the \( 0^+ \) ground state of an even-even Sn-isotope is very similar to the \( 11/2^- \) state in the neighboring odd-A Sn-isotope having one extra neutron. Similarly, the nature of the wave function of the \( 10^+ \) state in an even-even Sn-isotope is very similar to the \( 27/2^- \) state in the neighboring odd-A Sn-isotope having one extra neutron. It is also noteworthy that wave functions in the chain of isotopes change very systematically, by maintaining an order.

B. \( N=82 \) isomers

Similarly, we have presented the wave functions having maximum partition for the \( N=82 \) isotonic chain in Fig. 5. The bar chart is showing the calculated configurations of the \( 0^+ \), \( 11/2^- \) and \( 10^+, 27/2^- \) states for the \( N=82 \) isotones. The bottom panel exhibits the variation of occupancies with maximum partition for the \( 10^+ \) and \( 27/2^- \) states at every alternate even and odd proton numbers. The same color coding helps to follow the calculated occupancies for the \( N=82 \) isotones also. In a similar fashion, the top panel of Fig. 5 shows the occupancies for the maximum partition in the wave function of the \( 0^+ \) and \( 11/2^- \) states in the \( N=82 \) isotones. Again the nature of the wave function of the \( 0^+ \) ground state of an even-even \( N=82 \) isotope is quite similar to the \( 11/2^- \) state in the neighboring odd-A \( N=82 \) isotope having an extra proton. Similarly, the nature of the wave function of the \( 10^+ \) state in a given even-even \( N=82 \) isotope is very similar to the \( 27/2^- \) state in the neighboring odd-A \( N=82 \) isotope having an extra proton. The wave functions in the chain of isotopes also change very systematically in an ordered way similar to the chain of Sn-isotopes. This comparison highlights the origin of the similar systematic features in both the regions.

V. THE AVERAGE PARTICLE NUMBERS AND THE SENIORITY ASSIGNMENTS

A. \( Z=50 \) isomers

Further insight into the configuration and structure of the various isomeric states can be obtained by plotting the average particle numbers in each of the valence orbitals for the Sn-isotopes, see Fig. 6. The four sections of the graph show the variation of the average particle number with increasing neutron number for the \( 0^+, 10^+ \) and \( 11/2^-, 27/2^- \) states corresponding to the even-even and the odd-A Sn isotopes respectively. The average particle number increases in a regular way with the increase in the neutron number as expected. However, the average particle number for the \( d_{3/2} \) and \( s_{1/2} \) orbitals is almost zero up to \( N=66 \), i.e. the mid-shell; it rises, thereafter, and changes in a very smooth and regular way, making negligible contribution to the desired spin states. There is a competition between the remaining three \( g_{7/2} \), \( d_{5/2} \) and \( h_{11/2} \) orbitals, which play an important role in deciding the final configuration of the desired spin state. The spin states \( 11/2^- \) and \( 27/2^- \) always need the involvement of the unique parity \( h_{11/2} \) orbital to get the negative parity.

The average particle number variation of the orbitals involved in generating the \( 0^+ \) and \( 11/2^- \) state corresponding to an even-even and the odd-A Sn-isotope respectively, has almost similar behavior. The observed variation along with the structure of the wave function confirms that the \( 11/2^- \) spin arises from the odd-neutron in the \( h_{11/2} \) orbital in each odd-A Sn-isotope. It also suggests that the odd neutron in the odd-A Sn-isotopes is totally aligned.

1. Basic seniority rules and arguments

To explain the seniority of the isomeric states, we make use of the basic thumb rules for assigning the seniority. In simple terms, seniority is a quantum number, defined as the number of unpaired nucleons in a given nuclear state. This concept was first introduced by Racah in the atomic context. It has been shown that seniority remains conserved up to \( j=7/2 \) in single-\( j \) orbital, in the generalized seniority scheme. The spin-states generated by nucleons in single-\( j \) shell having \( j=7/2 \) or less, do not have the chance of seniority mixing due to single possibility of seniority, which generates them. For higher-\( j \) values i.e. \( j>7/2 \), the same spin-states may belong to different seniority. It has, however, been shown
recently that the \( j = 9/2 \) and higher-\( j \) shells may also have partially conserved seniority \[8–11\].

For the semi-magic nuclei, the yrast states will have the lowest seniority if a single-\( j \) shell is involved \[7, 8\]. It is also well known that each even-even nucleus has its ground state with separation of two states having same \( \Delta v \). The ground state \( 0^+ \) i.e. pair correlated state and each odd-A nucleus has its ground state with \( v=1 \) for a single-\( j \) shell configuration. Also the energy separation of two states having same \( \Delta v \) does not depend on the number of the particles in the \( j \) shell for good seniority \[8–11\]. Here \( \Delta v \) can be zero also.

2. Seniority

From the previous arguments and the rules, the seniority appears to be a reasonably good quantum number for the \( 11/2^- \) state, which is an isomeric as well as a ground state in the semi-magic nuclei. It is known that the \( 0^+ \) ground state has the seniority \( v=0 \) for each even-even Sn-isotope. We may, therefore, assign the seniority \( v=1 \) for the yrast \( 11/2^- \) isomeric state, coming from the last odd-particle in the \( h_{11/2} \), throughout the odd-A Sn-isotopic chain, which is also in line with the similar structure of the wave functions of the \( 0^+ \) and \( 11/2^- \) states in even-even and odd-A Sn-isotopes respectively.

In a similar fashion, the variation of the average particle number for the isomeric states \( 10^+ \) and \( 27/2^- \) is observed to be similar in nature. From the plots and the structure of the wave function, we conclude that the \( 27/2^- \) isomeric state up to \( N=61 \) (except at \( N=53 \) originates from one neutron in the \( h_{11/2} \) orbital, and one broken pair each in the \( g_{7/2} \) and \( d_{5/2} \) orbitals. We also conclude that the seniority \( v \) of the isomeric state \( 27/2^- \) becomes large i.e. \( v=5 \) in the range \( N=55 \) to 61. In the isotopic range \( N=63–65 \), the average occupancy of the \( h_{11/2} \) orbital becomes \( h_{11/2}^3 \) for producing the isomeric state \( 27/2^- \), which results in a seniority \( v=3 \). For \( N > 65 \), the particles keep on adding to the \( h_{11/2} \) orbital, as the \( g_{7/2} \) and \( d_{5/2} \) orbitals are almost full. The energy difference between the \( 11/2^- \) and \( 27/2^- \) states stays nearly constant, suggesting that same mechanism is responsible in the generation of the \( 27/2^- \) state from the \( 11/2^- \) state. This confirms that the change in seniority is \( \Delta v=2 \) in going from \( 11/2^- \) to \( 27/2^- \). Seniority, therefore, remains conserved at \( v=3 \) for the isomeric state \( 27/2^- \) in the range \( N=63–79 \). It, therefore, emerges that the se-
FIG. 7: (Color online) The average particle number variation for different orbitals with proton number for the 11/2−, 27/2−, 0+ and 10+ states in the N=82 isotones.

We may extend the same argument for the 10+ isomeric states in the Sn-isotopic chain. It is seen to have the h_{11/2} configuration from N=54−56, the total spin coming from the breaking of one neutron pair each in the g_{7/2} orbital and the d_{5/2} orbital. The seniority v=4 may be assigned for the 10+ state in N=54−56 isotopes. The h_{11/2} orbital again plays the main role in the generation of the desired 10+ spin state, having average of 2 particles from N=58 to 66 with seniority as v=2, i.e. by breaking up of one neutron pair of the h_{11/2} orbital. For N>66, the particles keep on adding to the h_{11/2} orbital having filled up the g_{7/2} and d_{5/2} orbitals. The relative placement of the 0+ and 10+ states remains nearly constant, due to the fact that the seniority stays conserved at v=2 for the 10+ state, from N=58 and onwards. The change in seniority before and after the mid-shell can explain the change in the energy difference between the 0+ and 10+. The orbitals g_{7/2}, d_{5/2} and h_{11/2}, thus, play the main role in deciding the features of the 10+ and 27/2− isomers. Astier et al. [21, 22] have recently assigned the same configurations and seniorities for the Sn-isotopes from N=69 and onwards and our studies confirm the same.

B. N=82 isomers

We present in Fig. 7 the average particle number variation for the 0+, 10+, 11/2−, 27/2− states for the even-even and odd-A N=82 isotopes respectively. The d_{3/2} and s_{1/2} orbitals again make negligible contribution to the final configuration of these isomers. The contribution of the remaining g_{7/2}, d_{5/2} and h_{11/2} orbitals exhibit strong similarities with the results for the Sn-isotopes. However, a difference arises in the occupation of the h_{11/2} orbital which now remains vacant up to Z=62 for the 10+ state. The 10+ isomer arises by the breaking of one pair of proton each in g_{7/2} and d_{5/2}. The seniority of the 10+ state up to Z=62 (except at Z=52) becomes v=4, which declines to v=2 afterwards. This also confirms the previous assignments made by Astier et al. [22] for the N=82 isotonic chain with Z=54−58. This change in seniority occurs due to the different scheme of single par-
ticle energies for protons as compared to the neutrons. The relative position of $h_{11/2}$ orbital in the proton valence is higher than in the neutron valence space. This further leads to a proton configuration with seniority $v=1$ having an odd proton in $h_{11/2}$ for obtaining the 11/2$^-$ state throughout the N=82 isotonic chain. We also find that the 27/2$^-$ state has a seniority $v=5$ before the mid-shell, which finally becomes $v=3$ having three unpaired protons in the $h_{11/2}$ orbital after the mid-shell, similar to the Sn-isotopic chain.

The seniority assignment for both the Z=50 isotopic chain and N=82 isotopic chain along with the unpaired nucleon configurations have been summarized in the Tables [1] and [2] respectively. The number of particles in any configuration represents the number of unpaired nucleons in the respective orbital for the desired spin state, i.e., all the other nucleons are paired, contributing to zero angular momentum.

VI. ALIGNED NATURE OF THE $h_{11/2}$ PROTONS/NEUTRONS

Further evidence for the seniority of various isomers and the states which decay to the isomers by $\gamma-$ transitions, is obtained from the alignments. For this purpose, we compare the $\gamma-$ transition energies and their ratios in the even-even and odd-A isotopic/isotonic nuclei respectively.

A. The Experimental evidence

We have listed the measured gamma energies associated with the transitions $\Delta E_{0^+}^{2^+}$ and $\Delta E_{11/2}^{15/2^-}$ for the even-even and odd-A Sn-isotopes in Table III [2, 21, 33, 56]. We have used the notation $\Delta E_{0^+}^{2^+}$ for the $(2^+ \rightarrow 0^+)$ $\gamma-$ transition energy; and similarly for others. The ratio of these transitions denoted as $R(15 : 2)$ = $\Delta E_{11/2}^{15/2^-}/\Delta E_{0^+}^{2^+}$ remains $\sim 1$ for the $^{114-125}$Sn isotopes. The 11/2$^-$ state in odd-A Sn-isotopes and the 0$^+$ state in the neighboring even-even Sn-isotope have great similarity in their wave functions, as already pointed out. This suggests the alignment of the odd-neutron in the $h_{11/2}$ orbital, producing the 11/2$^-$ spin state.

Similarly, the observed gammas $\Delta E_{10^+}^{12^+}$ and $\Delta E_{27/2}^{31/2^-}$ in even-even and odd-A Sn isotopes have also been compared in Table III for $^{118-128}$Sn isotopes, recently reported by Astier [21, 53]. Fotiades et al. [52] have compared the almost identical energies and similar structure involved in the $(2^+ \rightarrow 0^+)$ and $(12^+ \rightarrow 10^+)$ $\gamma-$ transitions within the same isotopes of $^{116-128}$Sn, and suggested that the 10$^+$ isomeric state comes from the two aligned neutrons in the $h_{11/2}$ orbital. We have calculated the ratio of the transitions in odd-A Sn-isotope and its even-even core Sn-isotope, denoted as $R(31 : 12)$ = $\Delta E_{27/2}^{31/2^-}/\Delta E_{10^+}^{12^+}$, which is also $\sim 1$. This further confirms the similar nature of the 27/2$^-$ state in the odd-A Sn-isotope to the 10$^+$ state in even-even Sn-isotope after the mid-shell as already pointed out. We, therefore, conclude that the 10$^+$ and 27/2$^-$ isomeric states come from the aligned two and three–neutrons in the $h_{11/2}$ orbital respectively.

By using the rules discussed in the previous section, we may assign the seniority after the mid-shell as $v=0$ for the 0$^+$ states, $v=1$ for the 11/2$^-$ states, $v=2$ for the 2$^+$ states and $v=3$ for the 15/2$^-$ states. Similarly, we assign the seniority $v=2$ for the 10$^+$ states, $v=3$ for the 27/2$^-$ states, $v=4$ for the 12$^+$ states, and $v=5$ for the 31/2$^-$ states. The same seniority difference $\Delta v=2$ between the 15/2$^-$ and 11/2$^-$ states, and for the 10$^+$ and 0$^+$ states gives their corresponding ratio $R(15 : 2)$ as $\sim 1$. The difference $\Delta v=2$ also holds for the 31/2$^-$ and 27/2$^-$ states, and for the 12$^+$ and 10$^+$ states, which makes the ratio $R(31 : 12)$ $\sim 1$, as expected.

B. The Calculated evidence

1. Z=50 isomers

We have also tabulated in Table IV the calculated gammas associated with the transitions $\Delta E_{0^+}^{2^+}$, $\Delta E_{11/2}^{15/2^-}$ and $\Delta E_{10^+}^{12^+}$, $\Delta E_{27/2}^{31/2^-}$ in the even-even and odd-A Sn-isotopes. We note that the calculated ratios $R(15 : 2)$ and $R(31 : 12)$ also remain $\sim 1$. The calculations also suggest that the one, two, and three $h_{11/2}$ neutrons align to produce the 11/2$^-$, 10$^+$ and 27/2$^-$ isomeric states respectively, after the mid-shell. Due to this, the 10$^+$ and 27/2$^-$ states closely follow each other after the mid-shell. However, it does not happen before the mid-shell due to the configuration mixings of the $g_{7/2}$ and $d_{5/2}$ orbitals with the $h_{11/2}$ orbital.

2. N=82 isomers

We summarize the calculated gammas $\Delta E_{0^+}^{2^+}$, $\Delta E_{11/2}^{15/2^-}$ and $\Delta E_{10^+}^{12^+}$, $\Delta E_{27/2}^{31/2^-}$ in the even-even and odd-A N=82 isotonic chain in Table IV. The ratios $R(15 : 2)$ and $R(31 : 12)$ again become $\sim 1$. The alignment of $h_{11/2}$ protons takes place after the mid-shell, similar to the $h_{11/2}$ neutrons in the Sn-isotopic chain. We can, therefore, draw the same conclusions for the N=82 isotonic chain as for the Z=50 isotopic chain. The similar configurations and seniorities lead to the similar kind of alignment of $h_{11/2}$ neutrons/protons for the 11/2$^-$, 10$^+$ and 27/2$^-$ isomeric states in the both Z=50 and N=82 regions.
TABLE I: Seniority assignments to the isomeric states $11/2^-$, $10^+$ and $27/2^-$ for the Sn-isotopic chain, where the unpaired neutrons in the respective orbitals are listed as configuration.

| Isotope | Configuration | Seniority | Isotope | Configuration | Seniority | Isotope | Configuration | Seniority |
|---------|---------------|-----------|---------|---------------|-----------|---------|---------------|-----------|
| $^{102}$Sn | $h_{11/2}^2$ | 2 | $^{103}$Sn | $h_{11/2}^1$ | 1 | $^{104}$Sn | $h_{11/2}^3$ | 3 |
| $^{104}$Sn | $d_{5/2}^2$ | 4 | $^{105}$Sn | $h_{11/2}^1$ | 1 | $^{106}$Sn | $g_{7/2}^2$ | $d_{5/2}^2$ |
| $^{108}$Sn | $g_{7/2}^2$ | 4 | $^{107}$Sn | $h_{11/2}^1$ | 1 | $^{109}$Sn | $g_{7/2}^2$ | $d_{5/2}^2$ |
| $^{110}$Sn | $h_{11/2}$ | 2 | $^{111}$Sn | $h_{11/2}^1$ | 1 | $^{112}$Sn | $g_{7/2}^2$ | $d_{5/2}^2$ |
| $^{112}$Sn | $h_{11/2}$ | 2 | $^{113}$Sn | $h_{11/2}^1$ | 1 | $^{114}$Sn | $h_{11/2}^1$ | 1 |
| $^{114}$Sn | $h_{11/2}$ | 2 | $^{115}$Sn | $h_{11/2}^1$ | 1 | $^{116}$Sn | $h_{11/2}^1$ | 1 |
| $^{118}$Sn | $h_{11/2}$ | 2 | $^{119}$Sn | $h_{11/2}^1$ | 1 | $^{120}$Sn | $h_{11/2}^1$ | 1 |
| $^{122}$Sn | $h_{11/2}$ | 2 | $^{121}$Sn | $h_{11/2}^1$ | 1 | $^{124}$Sn | $h_{11/2}^1$ | 1 |
| $^{126}$Sn | $h_{11/2}$ | 2 | $^{127}$Sn | $h_{11/2}^1$ | 1 | $^{128}$Sn | $h_{11/2}^1$ | 1 |


TABLE II: Seniority assignments to the isomeric states $11/2^-$, $10^+$ and $27/2^-$ for the $N=82$ isotonic chain, where the unpaired protons in the respective orbitals are listed as configuration.

| Isotope | Configuration | Seniority | Isotope | Configuration | Seniority | Isotope | Configuration | Seniority |
|---------|---------------|-----------|---------|---------------|-----------|---------|---------------|-----------|
| $^{134}$Te | $h_{11/2}^2$ | 2 | $^{135}$I | $h_{11/2}^1$ | 1 | $^{136}$Xe | $d_{5/2}^2$ | $h_{11/2}^1$ |
| $^{136}$Xe | $g_{7/2}^2$ | 4 | $^{137}$Cs | $h_{11/2}^1$ | 1 | $^{138}$Ba | $g_{7/2}^2$ | $d_{5/2}^2$ |
| $^{138}$Ba | $g_{7/2}^2$ | 4 | $^{139}$La | $h_{11/2}^1$ | 1 | $^{140}$Ce | $g_{7/2}^2$ | $d_{5/2}^2$ |
| $^{140}$Ce | $g_{7/2}^2$ | 4 | $^{141}$Pr | $h_{11/2}^1$ | 1 | $^{142}$Nd | $g_{7/2}^2$ | $d_{5/2}^2$ |
| $^{142}$Nd | $g_{7/2}^2$ | 4 | $^{143}$Pm | $h_{11/2}^1$ | 1 | $^{144}$Sm | $g_{7/2}^2$ | $d_{5/2}^2$ |
| $^{144}$Sm | $g_{7/2}^2$ | 4 | $^{145}$Eu | $h_{11/2}^1$ | 1 | $^{146}$Gd | $h_{11/2}^1$ | 1 |
| $^{146}$Gd | $h_{11/2}^2$ | 2 | $^{147}$Tb | $h_{11/2}^1$ | 1 | $^{148}$Dy | $h_{11/2}^1$ | 1 |
| $^{148}$Dy | $h_{11/2}$ | 2 | $^{149}$Ho | $h_{11/2}^1$ | 1 | $^{150}$Er | $h_{11/2}$ | 2 |
| $^{150}$Er | $h_{11/2}$ | 2 | $^{151}$Tm | $h_{11/2}^1$ | 1 | $^{152}$Yb | $h_{11/2}^1$ | 2 |
| $^{152}$Yb | $h_{11/2}$ | 2 | $^{153}$Lu | $h_{11/2}^1$ | 1 | $^{154}$Hf | $h_{11/2}$ | 2 |
| $^{154}$Hf | $h_{11/2}$ | 2 | $^{155}$Ta | $h_{11/2}^1$ | 1 | $^{156}$W | $h_{11/2}$ | 2 |
| $^{156}$W | $h_{11/2}$ | 2 | $^{157}$Re | $h_{11/2}^1$ | 1 | $^{158}$Os | $h_{11/2}$ | 2 |
| $^{158}$Os | $h_{11/2}$ | 2 | $^{159}$Ir | $h_{11/2}^1$ | 1 | $^{160}$Pt | $h_{11/2}$ | 2 |


VII. INTERPRETATION OF THE HALF-LIFE SYSTEMATICS

The experimental half-life systematics for the isomeric states $11/2^-$, $10^+$ and $27/2^-$, in both the $Z=50$ and $N=82$ chains, show a similar trend. The seniority assignment to the states, in which the isomers decay, will now be discussed on the basis of the seniority rules. We have already noted that these isomeric states show different behavior before and after the mid-shell due to the availability of different orbitals for generating the desired spin states. Therefore, we discuss both the regions separately for the sake of simplicity.

A. After the mid-shell

The isomeric nuclei behave as single-$j$ shell nuclei after the mid-shell, where the unique-parity $h_{11/2}$ orbital dominates. We first choose this region to discuss the role of seniority and half-lives of these isomeric states in both the isotopic and isotonic chains. We find that the $10^+$ and $27/2^-$ isomers mostly decay via $E2$ transitions to the lower $8^+$ and $23/2^-$ states, respectively. Also, the calculated wave functions and occupancies suggest that both the $10^+$ and $8^+$ states have the same seniority, $v=2$, after the mid-shell, a condition required for seniority isomers because the transition between same seniority states is
TABLE III: Comparison of the experimentally measured $\Delta E_{0^+}^2$, $\Delta E_{10^+}^{12}$ and $\Delta E_{11/2^-}^{15}$, $\Delta E_{27/2^-}^{31}$ $\gamma$-transitions in even-even and odd-A Sn-isotopes for $N \geq 64$. All the energies are in MeV. $R(15 : 2)$ is defined as $\Delta E_{11/2^-}^{15} / \Delta E_{0^+}^2$ and $R(31 : 12)$ is defined as $\Delta E_{27/2^-}^{31} / \Delta E_{10^+}^{12}$.

| Isotope  | $\Delta E_{0^+}^2$ | $\Delta E_{10^+}^{12}$ | Isotope  | $\Delta E_{11/2^-}^{15}$ | $\Delta E_{27/2^-}^{31}$ | $R(15 : 2)$ | $R(31 : 12)$ |
|----------|--------------------|------------------------|----------|--------------------------|------------------------|------------|-------------|
| $^{114}\text{Sn}$ | 1.300              |                        | $^{115}\text{Sn}$ | 1.312                     |                        | 1.0089     |              |
| $^{116}\text{Sn}$ | 1.294              |                        | $^{117}\text{Sn}$ | 1.279                     |                        | 0.9887     |              |
| $^{118}\text{Sn}$ | 1.230              | 1.237                  | $^{119}\text{Sn}$ | 1.220                     | 1.179                  | 0.9921     | 0.953       |
| $^{120}\text{Sn}$ | 1.171              | 1.190                  | $^{121}\text{Sn}$ | 1.151                     | 1.083                  | 0.9827     | 0.910       |
| $^{122}\text{Sn}$ | 1.141              | 1.103                  | $^{123}\text{Sn}$ | 1.107                     | 1.043                  | 0.9706     | 0.946       |
| $^{124}\text{Sn}$ | 1.132              | 1.047                  | $^{125}\text{Sn}$ | 1.088                     | 0.924                  | 0.9614     | 0.883       |

TABLE IV: Comparison of the calculated $\Delta E_{0^+}^2$, $\Delta E_{10^+}^{12}$ and $\Delta E_{11/2^-}^{15}$, $\Delta E_{27/2^-}^{31}$ $\gamma$-transitions in even-even and odd-A Sn-isotopes for $N \geq 64$. All the energies are in MeV.

| Isotope  | $\Delta E_{0^+}^2$ | $\Delta E_{10^+}^{12}$ | Isotope  | $\Delta E_{11/2^-}^{15}$ | $\Delta E_{27/2^-}^{31}$ | $R(15 : 2)$ | $R(31 : 12)$ |
|----------|--------------------|------------------------|----------|--------------------------|------------------------|------------|-------------|
| $^{114}\text{Sn}$ | 1.508              | 0.853                  | $^{115}\text{Sn}$ | 1.463                     | 0.782                  | 0.970       | 0.916       |
| $^{116}\text{Sn}$ | 0.878              | 1.120                  | $^{117}\text{Sn}$ | 0.889                     | 0.921                  | 1.012       | 0.822       |
| $^{118}\text{Sn}$ | 0.988              | 1.007                  | $^{119}\text{Sn}$ | 0.942                     | 1.011                  | 0.953       | 1.004       |
| $^{120}\text{Sn}$ | 0.939              | 0.905                  | $^{121}\text{Sn}$ | 0.872                     | 0.871                  | 0.928       | 0.962       |
| $^{122}\text{Sn}$ | 0.888              | 0.822                  | $^{123}\text{Sn}$ | 0.827                     | 0.841                  | 0.931       | 1.023       |
| $^{124}\text{Sn}$ | 0.863              | 0.830                  | $^{125}\text{Sn}$ | 0.840                     | 0.827                  | 0.973       | 0.996       |
| $^{126}\text{Sn}$ | 0.892              | 0.813                  | $^{127}\text{Sn}$ | 0.857                     | 0.812                  | 0.960       | 0.999       |
| $^{128}\text{Sn}$ | 0.964              | 0.892                  | $^{129}\text{Sn}$ | 1.014                     |                        | 1.051       |              |

The 10$^+$ isomeric states mostly decay to the available 8$^+$ states, before the mid-shell too. The 8$^+$ states are also found to change their seniority in the same way as the 10$^+$ isomeric states, on the basis of the calculated occupancies and the seniority rules, for both the chains. There is an exception at $^{108}\text{Sn}$, where the calculations suggest the seniority $v=4$ for the 8$^+$ state, which differs from the seniority $v=2$ for the 10$^+$ isomeric state. However, there are always good chances of seniority mixing due to the contributions from the multi-$j$ orbitals. Still, the yrast state in the semi-magic nuclei will be dominated by the lowest possible seniority. As a result, the 10$^+$ state becomes a seniority isomer before the mid-shell as well as the $E2$ transitions diminish for the same seniority states. It is also found that the excitation energy for the 10$^+$ isomeric states is independent of the particle number, which supports the interpretation.

Similarly, the 27$^+$ isomeric states also have larger seniority before the mid-shell due to the $g_{7/2}$ and $d_{5/2}$ orbitals. The 27$^+$ isomeric state have the approximate seniority $v=5$ for the nucleon numbers 55 to 61, otherwise the seniority remains $v=3$, in both the chains. We also assign an approximate seniority $v=3$ to the yrast 23$^+$ states, similar to the above discussion.

We also find that the half-life of 10$^+$ isomers in even-even nuclei are $\sim 100$ times larger than the half-life of the known 27$^+$ isomers in odd-A nuclei, before the

B. Before the mid-shell

The situation becomes rather complex before the mid-shell due to the multi-$j$ shell configurations for the 10$^+$ and 27$^+$ isomeric states in both the $Z=50$ isotopic and $N=82$ isotonic chains. The multi-$j$ shell occupancies lead to a seniority mixing in generating the desired spin states. The 10$^+$ isomeric states have been shown to have a seniority $v=2$ except at the $N=54$ and 56, which have a seniority $v=4$ for the Sn-isotopic chain. Similarly, the seniority of the $N=82$ isotonic chain is also $v=2$ except for $Z=54-62$, where it becomes $v=4$.

hindered. Same is true for the 27$^+$ and 23$^+$ states too, having the seniority $v=3$, after the mid-shell. Further, the electric quadrupole transitions between the two same seniority states have been shown to nearly vanish when the valence shell is close to the half-filled $\frac{5}{2}$-$\frac{7}{2}$. Therefore, both the isomeric states 10$^+$ and 27$^+$ show a peak near the nucleon numbers 70–73, where the $h_{11/2}$ orbital becomes half-filled. This is true for both the $Z=50$ isotopic and the $N=82$ isotonic chains. The isomeric states 10$^+$ and 27$^+$, therefore, behave as seniority isomers, having almost conserved seniority as is also evident from the constant relative energy of these isomeric states which is particle number independent.
TABLE V: Comparison of the calculated $\Delta E_{0^+}^{2+}$, $\Delta E_{10^+}^{12^+}$ and $\Delta E_{11^+}^{15/2^-}$, $\Delta E_{27^2/2_2^-}^{31/2^-}$ $\gamma$-transitions in even-even and odd-A $N=82$ isotones for $Z \geq 64$. All the energies are in MeV.

| Isotope | $\Delta E_{0^+}^{2+}$ (MeV) | $\Delta E_{10^+}^{12^+}$ (MeV) | Isotope | $\Delta E_{11^+}^{15/2^-}$ (MeV) | $\Delta E_{27^2/2_2^-}^{31/2^-}$ (MeV) | $R(15:2)$ | $R(31:12)$ |
|---------|-----------------------------|-----------------------------|---------|-----------------------------|-----------------------------|-------------|-------------|
| $^{146}$Gd | 2.212 | 0.994 | $^{147}$Tb | 2.152 | 0.920 | 0.972 | 0.925 |
| $^{148}$Dy | 1.088 | 1.293 | $^{149}$Ho | 1.182 | 1.501 | 1.086 | 1.160 |
| $^{150}$Er | 1.162 | 1.090 | $^{151}$Tm | 1.067 | 1.000 | 0.918 | 0.917 |
| $^{152}$Yb | 1.102 | 0.981 | $^{153}$Lu | 1.006 | 0.902 | 0.913 | 0.919 |
| $^{154}$Hf | 1.068 | 0.923 | $^{155}$Ta | 0.974 | 0.845 | 0.912 | 0.915 |
| $^{156}$W | 1.057 | 0.899 | $^{157}$Re | 0.971 | 0.780 | 0.919 | 0.868 |
| $^{158}$Os | 1.069 | 0.887 | $^{159}$Ir | 0.989 | 0.681 | 0.925 | 0.768 |
| $^{160}$Pt | 1.114 | 0.867 | $^{161}$Au | 1.022 | 0.917 | | |

TABLE VI: Predictions of the isomeric states $11/2^-$, $10^+$ and $27/2^-$ for the Sn-isotopic chain. All the energies are in MeV.

| Isotope | Energy | Half-life | Isotope | Energy | Half-life | Isotope | Energy | Half-life |
|---------|--------|-----------|---------|--------|----------|---------|--------|----------|
| $^{102}$Sn | $\sim 6 - 7$ | $\sim 10 - 200ps$ | $^{103}$Sn | $\sim 3$ | $\sim 10 - 30ps$ | $^{105}$Sn | $\sim 7 - 8$ | $\sim 1 - 20ps$ |
| $^{108}$Sn | $\sim 4 - 4.5$ | $\sim 10 - 200ps$ | $^{109}$Sn | $\sim 2 - 3$ | $\sim 10 - 30ps$ | $^{105}$Sn | $\sim 5 - 6$ | $\sim 1 - 20ps$ |
| $^{110}$Sn | $\sim 4 - 4.5$ | $\sim 10 - 200ps$ | $^{110}$Sn | $\sim 2 - 3$ | $\sim 10 - 30ps$ | $^{110}$Sn | $\sim 5 - 6$ | $\sim 1 - 20ps$ |
| $^{112}$Sn | $\sim 4 - 4.5$ | $\sim 10 - 200ps$ | $^{111}$Sn | $\sim 5$ | $\sim 1 - 20ps$ | $^{113}$Sn | $\sim 5$ | $\sim 1 - 10ns$ |
| $^{117}$Sn | $\sim 4$ | $\sim 10 - 200ps$ | $^{117}$Sn | $\sim 4$ | $\sim 10 - 200ps$ | | | |

mid-shell. This may be a direct consequence of a larger seniority mixing in the $27/2^-$ states of the odd-A nuclei as compared to the $10^+$ states in even-even nuclei.

We have already discussed that the seniority $v=1$ for the $11/2^-$ isomeric state, throughout the whole $Z=50$ and $N=82$ chains. We find that the $11/2^-$ state also follows the same trend in half-life as others. The $11/2^-$ state decays to the different available states like, $7/2^+$, $5/2^-$, $3/2^+$ or, $1/2^+$, all of which have the seniority $v=1$ from $g_{7/2}$, $d_{5/2}$, $d_{3/2}$ and $s_{1/2}$ orbitals, respectively. The $11/2^-$ mostly decays via gamma decay before the mid-shell, and changes its decay mode to the beta/EC near the mid-shell as one crosses the stability region. As a result, the half-lives continue to remain large after the mid-shell.

VIII. PREDICTIONS OF POSSIBLE NEW ISOMERS

Similar configurations, occupancies and wave functions lead to similar trends in the excitation energy and the half-life for both the $Z=50$ and $N=82$ chains. Therefore, the knowledge of basic key features of one region enables us to make predictions for the unknown isomers/isomeric properties in the other region. It is well known that the population of nuclei at the drip lines is more difficult than near the stability line. The systematic features become a great tool to make a guess for the isomers in the near-drip line nuclei. We can see that the $11/2^-$, $10^+$ and $27/2^-$ isomeric states are observed with many gaps for the $Z=50$ isotopic/ $N=82$ isotonic chains within the same valence space $50–82$. The above results and discussion strongly suggest the existence of new isomers in the gaps, with almost similar excitation energies and half-lives.

We have summarized our predictions for the new isomers in Tables VI and VII. These tables open the way for new possible experiments to explore the new isomers. We have also listed the available experimental data of the yrast $11/2^-$, $10^+$ and $27/2^-$ states with their excitation energies in the table VII which are yet to be characterised as isomeric states. They mostly follow the systematics presented in this work. These states are most likely to be isomers which needs to be confirmed by lifetime measurements.

IX. SUMMARY AND CONCLUSIONS

To summarize, we have presented a comprehensive overview of the observed excitation-energy and half-life systematics of the $11/2^-$, $10^+$ and $27/2^-$ isomeric states for both the $Z=50$ isotopic and $N=82$ isotonic chains, which display strikingly identical behavior. A constant energy gap of $\sim 4$ MeV between the $0^+$ and $10^+$ states and, the $11/2^-$ and $27/2^-$ isomeric states exists before the mid-shell, which becomes $\sim 3$ MeV after the mid-shell. We have used the large scale shell model calcula-
We also find that the calculated wave functions and the states have a good seniority throughout both the chains. On the other hand, we find that some seniority mixing starts before the mid-shell, due to the multi-shell, because of the dominant nature of the unique-parity $h_{11/2}$ nucleons. On the other hand, we find that some seniority mixing starts before the mid-shell, due to the multi-$j$ shell configurations.

We also conclude that the isomeric states $10^+$ and $27/2^-$ states have a higher seniority before the mid-shell, which becomes lower after the mid-shell. The change in the seniority is responsible for the change in the energy gap around the mid-shell. On the other hand, the $11/2^-$ states have a good seniority throughout both the chains. We also find that the calculated wave functions and the average occupancies of the $11/2^-$ and $27/2^-$ states in the odd-$A$ nuclei, have a structure which is very similar to the $0^+$ and $10^+$ states of the neighboring even-even nuclei for both the chains.

By using the alignment pattern of $h_{11/2}$ nucleons after the mid-shell, we have been able to assign the seniority for the $2^+, 12^+, 15/2^-$ and $31/2^-\alpha$ states, which decay to the $0^+, 10^+$, $11/2^-$ and $27/2^-$ states, respectively. We have also assigned the approximate seniorities to the $7/2^+, 5/2^+, 3/2^+, 1/2^+, 8^+$ and $23/2^-\alpha$ states, in which the isomeric states mostly decay.

As a consequence, we have also been able to explain the half-life systematics of these isomeric states in terms of their configurations, seniorities and the changing decay modes. The largest value of half-life for the $11/2^-\alpha$ isomers occurs when the decay mode begins to change. We find that the half-lives of the $10^+$ and $27/2^-\alpha$ isomers have the largest values when the valence orbital $h_{11/2}$ becomes half-filled. To conclude, the present work highlights the importance of seniority in the semi-magic isomers.

To sum up, we find that the Sn-isotopes approaching the neutron-drip line behave in a way very similar to the $N=82$ isotones approaching the proton-drip line. The similarity also holds to a significant extent for the neutron-deficient Sn-isotopes and the proton-deficient $N=82$ isotones. Therefore, this study also tests the charge-symmetric nature of the nuclear forces in the isomeric states for the nuclei which have widely different isospins. On the basis of these studies, we have made predictions for several unknown isomers, their excitation energies and half-lives for future experiments.

**TABLE VII**: Predictions of the isomeric states $11/2^-, 10^+$ and $27/2^-$ for the $N=82$ isotonic chain. All the energies are in MeV.

| Isotope | $10^+$ Energy | $10^+$ Half-life | $11/2^-$ Energy | $11/2^-$ Half-life | $27/2^-$ Energy | $27/2^-$ Half-life |
|---------|----------------|------------------|-----------------|-------------------|-----------------|-------------------|
| $^{134}\text{Tc}$ | $\sim 6 - 7$ | $\sim 100 - 500\text{ps}$ | $^{135}\text{I}$ | $\sim 2 - 2.5$ | $\sim 10 - 50\text{ps}$ | $^{135}\text{I}$ | $\sim 8 - 9$ | $\sim 10 - 50\text{ps}$ |
| $^{136}\text{Xe}$ | $\sim 3.5 - 4$ | $\sim 100 - 500\text{ps}$ | $^{137}\text{Cs}$ | $\sim 2 - 2.5$ | $\sim 100\text{ps}$ | $^{137}\text{Cs}$ | $\sim 4 - 5$ | $\sim 10 - 50\text{ps}$ |
| $^{141}\text{Sm}$ | $\sim 4 - 4.5$ | $\sim 100 - 500\text{ps}$ | $^{139}\text{La}$ | $\sim 1.5 - 2$ | $\sim 1\text{ns}$ | $^{139}\text{La}$ | $\sim 4 - 5$ | $\sim 50 - 100\text{ps}$ |

**TABLE VIII**: A list of the $11/2^-, 10^+$ and $27/2^-$ states from the available experimental data [23, 36] which may be possible isomeric states on the basis of our predictions. All the energies are in MeV.

| $Z=50$ | $N=82$ |
|--------|--------|
| $^{108}\text{Sn}$ | $10^+$ | 4.256 | $^{134}\text{Tc}$ | $(10^+)$ | 5.621 |
| $^{110}\text{Sn}$ | (10) | 4.317 | $^{136}\text{Xe}$ | $10^+$ | 3.485 |
| $^{112}\text{Sn}$ | $10^+$ | 4.680 | $^{139}\text{La}$ | $11/2^-$ | 1.420 |
| $^{105}\text{Sn}$ | $(27/2^-)$ | 5.874 | $^{137}\text{Cs}$ | $(27/2^-)$ | 4.408 |
| $^{109}\text{Sn}$ | $27/2^-$ | 5.285 | $^{139}\text{La}$ | $(27/2^-)$ | 4.627 |
| $^{111}\text{Sn}$ | $27/2^-$ | 4.839 | $^{143}\text{Pm}$ | $27/2^-$ | 4.580 |
| $^{115}\text{Sn}$ | $27/2^-$ | 4.866 | $^{145}\text{Eu}$ | $27/2^-$ | 4.967 |
| $^{117}\text{Sn}$ | $(27/2^-)$ | 3.824 | $^{147}\text{Tb}$ | $(27/2^-)$ | 3.889 |
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[1] P. Walker and G. Dracoulis, Nature 399, 35 (1999).
[2] A. K. Jain et al., Atlas of Nuclear Isomers, to be published.
[3] A. K. Jain, Invited talk at the 6th Asian Nuclear Physics Association Symposium, VECC, Kolkata, 19-21 February, 2014.
[4] A. K. Jain, Invited talk at the International Workshop on Nuclear Theory, Rila Mountains, Bulgaria, 22-28 June, 2014.
[5] G. Racah et al., Phys. Rev. 61, 186 (1942); Phys. Rev. 63, 367 (1943).
[6] R. D. Lawson, Theory of the Nuclear Shell Model, (Oxford University Press, New York, 1980).
[7] I. Talmi, Simple Models of Complex Nuclei. The Shell Model and Interacting Boson Model (Harwood, Academic, Chur, Switzerland, 1993).
[8] P. Van Isacker, Journal of Physics: Conf. Series 322, 012003 (2011).
[9] P. Van Isacker, AIP Conf. Proc. 1323, 141 (2010).
[10] L. Zamick and P. Van Isacker, Phys. Rev. C 78, 044327 (2008).
[11] C. Qi, Z.X. Xu, R.J. Liotta, Nucl. Phys. A 884, 21 (2012).
[12] B. Fogelberg, K. Heyde, J. Sau, Nucl. Phys. A 352, 157 (1981).
[13] P. J. Daly et al., Z. Phys. A 323, 245 (1986).
[14] R. Broda et al., Phys. Rev. Lett. 68, 1671 (1992).
[15] R. Mayer et al., Phys. Lett. B 336, 308 (1994).
[16] P. J. Daly et al., Phys. Scr. T 56, 94 (1995).
[17] J. A. Pinston et al., Phys. Rev. C 61, 024312 (2000).
[18] C. T. Zhang et al., Phys. Rev. C 62, 057305 (2000).
[19] R. L. Loeve et al., Phys. Rev. C 77, 064313 (2008).
[20] S. Pietri et al., Phys. Rev. C 83, 044328 (2011).
[21] A. Astier, Journal of Physics: Conf. Series 420, 012055 (2013).
[22] A. Astier et al., Phys. Rev. C 85, 054316 (2012).
[23] A. Astier et al., Phys. Rev. C 85, 064316 (2012).
[24] L. W. Iskra et al., Phys. Rev. C 89, 044324 (2014).
[25] J. Bron et al., Nucl. Phys. A 318, 335 (1979).
[26] A. Van Poelgeest et al., Nucl. Phys. A 346, 70 (1980).
[27] H. Harada et al., Phys. Lett. B 207, 1 (1988).
[28] A. Savelius et al., Nucl. Phys. A 637, 491 (1998).
[29] J. Gableske et al., Nucl. Phys. A 691, 551 (2001).
[30] M. Wolinska-Cichocka et al., Eur. Phys. J. A 24, 259 (2005).
[31] S. Y. Wang et al., Phys. Rev. C 81, 017301 (2010).
[32] N. Fotiades et al., Phys. Rev. C 84, 054310 (2011).
[33] B. A. Brown and W. D. M. Rae, Nushell@MSU, MSU-NSCL report (2007).
[34] B. A. Brown, N. J. Stone, J. R. Stone, I. S. Towner, and M. Hjorth-Jensen, Phys. Rev. C 71, 044317 (2005).
[35] A. Astier, EPJ Web of Conf. 62, 01008 (2013).
[36] ENSDF data base, www.nndc.bnl.gov/ensdf/