Cooper pairs at above-critical current region

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Abstract It is generally believed that in a superconductor Cooper pairs are broken at above-critical current region, corresponding to the lost of superconductivity. We suggest that, under some circumstance, Cooper pairs could still exist above critical current, and that dissipation of the system is caused by the scattering of these pairs. The existence of Cooper pairs in this region can be revealed by investigating the temperature dependence of the electrical resistance.

Keywords critical current · cooper pairs · dissipation

1 Introduction

The formation of Cooper pairs in a superconductor is essential for superconductivity [1][2]. Cooper pairs are of bosonic nature, and it is believed that they generate a supercurrent in the same mechanism as helium atoms generate a superflow. It is clear that, once the cooper pairs in a superconductor are broken, superconductivity will be lost. An interesting question is that, can the system become dissipative without the breaking of cooper pairs?

From the view point of many-body physics, one can see there is a rather striking difference between the origin of Cooper pairs and the origin of superconductivity. Cooper pairs is due to some attractive interactions between fermions, while superconductivity, like superfluidity, has something to do with repulsive interactions between its composing bosons, i.e., the Cooper pairs [3][4]. This difference implies that the physical regime of Cooper pairs is not exactly the same as that of superconductivity. We suggest that, under some circumstances, Cooper pairs are not broken when one increases a supercurrent to above critical current. We shall also show that, at the above-critical current region, the electrical resistance, caused by the scattering of cooper pair, has a different temperature dependence from the case where the charge carriers of the current are fermions (electrons or holes). Thus the existence of Cooper pairs can be signified by the temperature behavior of the resistance.

Like the case of superfluidity [3][4][5], the dissipative behavior of a superconductor above a critical current $I_c$, can be explained naturally in terms of the many-body dispersion spectrum $E(I)$ of the charge carrier system ($E(I)$ is the lowest eigen energy at given current $I$). Beyond $I_c$, $E(I)$ is a monotonically increasing function of $I$ (see Fig. 1), thus a current can continuously lose its energy and momentum to the environment, corresponding to a dissipative decay process. This is contrast to the case in the $I < I_c$ regime where the supercurrents are metastable states, corresponding to the local minima of the $E(I)$ curve, whose decay is prevented by the energy barriers among the minima.

The dissipation mechanism illustrated above does not involves breaking of Cooper pairs. Moreover, the low-lying eigen energy states below $I_c$ can be Galileo boosted to generate the low-lying states above $I_c$ [3][5][6]. The Galileo boost, also could being viewed as center-of-mass-mention transformation, does not modify the inner structure of the states such as pair correlations. Thus, the Cooper pairs, present in states below $I_c$, survive the boost and exist in the states above $I_c$ [7].

In literature, Landau’s criterion of a superconductor is generally used for the analysis of transitional regime, which determines a critical velocity to be $\Delta/p_F$, where
Fig. 1 A schematic plot of the many-body dispersion spectrum of the charge carrier system in a superconductor. Region A is the superconducting region where there are metastable current-carrying states corresponding to the local minima of the curve; Region B is dissipative without metastable states. The many-body eigen energy states at region A can be mapped to the states at region B by Galileo transformation.

\( \Delta \) is the pairing gap and \( p_F \) is the Fermi momentum. Landau’s criterion requires an exchange of a quantum, with a large momentum (the order of \( p_F \)) and with a certain energy, between the superconducting charges and its environment. However, this exchange process could be inhibited for that the spectrum of the environment may generally be incompatible to absorb such an unusual quantum.

In a superconductor, the self-induced magnetic field of the supercurrent could lead to loss of superconductivity by breaking cooper pairs, (i.e., Silsbee effect [3]). In this case, there is an ‘external’ critical current determined by the critical magnetic field, and the Cooper pairs do not exist above critical current. However, Silsbee effect can be largely reduced by alignment of the currents and their geometries (see Fig. 2), thus one might be able to obtain an intrinsic critical supercurrent, like the case of superfluid \( ^4 \text{He} \), and reach a dissipative regime without breaking Cooper pairs. We shall assume the existence of Cooper pairs above critical current in follows.

2 Analysis and Results

The electrical resistance in a superconductor above \( I_c \) and below \( T_c \) is mainly caused by the scattering properties of Cooper pairs with phonons. We shall discuss the temperature behavior of the resistance. For simplicity, we assume physical properties of the superconductor are isotropic. First, we consider the dispersion relation of a Cooper pair. The dispersion is linear at small momentum \( q \), i.e., \( \varepsilon(q) \approx v |q| \) (we approximate \( v \) by the critical velocity of the supercurrent \( v_c \)). At large \( q \), the energy of a Cooper pair is approximately \( q^2 / 2m_c \) where \( m_c \) is the mass of a Cooper pair assumed to be \( 2m_e \) (\( m_e \) is the mass of an electron). A general form of dispersion

\[
\varepsilon(q) = \sqrt{v^2 c^2 q^2 + \left( q^2 / 4m_e \right)^2}
\]

can be used for approximation for all \( q \) values [9]. We consider the leading scattering process in which one phonon is adsorbed or emitted by a Cooper pair (see Fig. 3). The total energy and momentum should be conserved,

\[
\varepsilon(q_1) = \varepsilon(q_2) \pm \hbar c s p
\]

\( q_1 = q_2 \pm p \)

Where \( q_1 \) (\( q_2 \)) is the momentum of a Cooper pair before (after) the scattering, \( p \) is the momentum of the phonon, \( c_s \) is the sound velocity, and \( + (-) \) corresponds to the emission (absorption) of the phonon.

Combining Eq. 1, Eq. 2 and Eq. 3, one gets,

\[
\sqrt{v^2 c^2 q_1^2 + \left( q_1^2 / 4m_e \right)^2} = \sqrt{v^2 c^2 q_1^2 + \left( q_1^2 / 4m_e \right)^2} \pm \hbar c_s |q_1 - q_2|
\]
In most superconductors, $v_c$, can be estimated using critical current density [10], is roughly the orders of ten meters per second or below. Thus $v_c$ is orders of magnitude smaller than $c_s$. One then can realize that unless the $q_1$ ($q_2$) in the phonon emission (absorption) process is roughly equal or larger than $2m_e c_s$, Eq. 4 can not be satisfied. for a Cooper pair with a momentum of $2m_e c_s$, the energy is roughly $m_e c_s^2$ and the corresponding temperature is $T^* = m_e c_s^2 / k_B$ where $k_B$ is the Boltzmann constant. Thus at low temperature $T < T^*$, one-phonon scattering process is absent, and the scattering process of Cooper pairs must involve at least two phonons.

One can invoke Boltzmann transport equation to determine the temperature dependence of resistance of the Cooper pair system. However, unlike the case of a fermionic system where the existence of Fermi surface and Pauli blocking are essential, the analysis of the (bosonic) Cooper pair system can be simplified. At $T > T^*$, the one-phonon scattering process can bring the momentum of a Cooper pair (with an energy of $k_B T$ or less) to zero or to the opposite direction, thus effectively causing the dissipation of the current. The probability of one phonon process in low temperature is proportional to $T$ (see, e.g., [11]), thus the resistance $R$ is linear in $T$. At $T < T^*$, one can find that probability of two-phonons process is proportional to $T^2$, thus $R \propto T^2$. In metals where temperature dependence of resistance is determined by electron phonon scattering, the power law of $R(T)$ at low temperature is different, with an exponent being 5 (see, e.g., [11][12][13]). Thus one can distinguish between Cooper pairs and electrons by checking the powering law of $R(T)$ in the above critical current regime.

In the so called strange metal phase of some high-$T_c$ cuprates, the resistance has also a linear temperature dependence. One might wildly speculate that this linear behavior could be caused by the scattering of Cooper pairs. In order for this speculation to be valid, Cooper pairs shall exist above $T_c$ in some systems. An example is that cold Fermi atom gases which can be tuned to go through BCS-BEC crossover. In the BEC side, the binding energy of Cooper pairs (or molecules) can be orders of magnitude larger than superfluid transition temperature $T_c$, since $T_c$ can be made small by decreasing the density of atom gases.

3 Conclusions

we suggest that, under some circumstance, Cooper pairs could exist in a superconductor above critical current, and the system has a different power law of $R(T)$ from the case where electrons are current carriers.

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