Precision measurement of the $\eta$-mass at COSY-ANKE

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Abstract. Measurements on the mass of the $\eta$-meson performed at different experimental facilities over the last decade have resulted in very precise results but differ by up to 0.5 MeV/$c^2$, i.e., more than eight standard deviations. In order to clarify this situation a new high precision measurement of the $dp \rightarrow 3^He \eta$ reaction was conducted at the COoler SYnchrotron - COSY - of the Forschungszentrum Jülich using the ANKE magnetic spectrometer, with the aim to achieve a mass resolution of $\Delta m < 50$ keV/$c^2$. In order to measure the $\eta$ meson mass with high accuracy through the $dp \rightarrow 3^He \eta$ reaction, both the momentum of the circulating deuteron beam in COSY as well as of the emitted $^3$He nucleus has to be determined with high precision. The experimentally achieved resolution for the momenta of the $^3$He nuclei and the beam deuterons will allow to reach the envisaged $\eta$-meson mass resolution.

1. Introduction

Measurements of the mass of the $\eta$ meson performed at different experimental facilities over the last decade have resulted in very precise results which differ by up to 0.5 MeV/$c^2$, i.e., by more than eight standard deviations. The experiments that are no longer considered in the PDG tables [1] generally involve the identification of the $\eta$ as a missing-mass peak produced in a hadronic reaction. In order to see whether this is an intrinsic problem, and to clarify the situation more generally, a refined measurement of the $dp \rightarrow 3^He \eta$ reaction was proposed [2] at the Cooler Synchrotron COSY of the Forschungszentrum Jülich [3]. After producing the $\eta$ mesons through the $dp \rightarrow 3^He \eta$ reaction using a hydrogen cluster-jet target [4], the $^3$He were be detected with the ANKE magnetic spectrometer [5] that is located at an internal target position of the storage ring.

For this reaction of interest the total center of mass energy can be written as:

$$\sqrt{s} = \sqrt{(E_d + E_p)^2 - (\vec{p}_d + \vec{p}_p)^2} = \sqrt{2m_p\sqrt{m_{^3He}^2 + \vec{p}_{d}^2} + m_{^3He}^2 + m_{p}^2}$$  \hspace{1cm} (1)

With this energy $\sqrt{s}$ the center of mass momentum $p_f$ of emitted $^3$He nuclei of this two particle reaction is given by

$$p_f = \frac{[s - (m_{^3He} + m_{\eta})^2] \cdot [s - (m_{^3He} - m_{\eta})^2]}{2 \cdot \sqrt{s}}$$ \hspace{1cm} (2)

Since the proton, deuteron and helium masses are known with very high accuracy, the measured final state momentum $p_f$ only depends on the accelerator beam momentum $p_d$ and the mass of
the $\eta$ meson:

$$p_f = p_f(p_d, m_\eta)$$  \hspace{1cm} (3)

Therefore, the $\eta$ mass can be extracted from pure kinematics through the determination of the production threshold. This requires one both to identify the reaction threshold and to measure accurately the associated beam momentum. It should be noted that this method does not depend on the knowledge of total and differential cross sections which excludes corresponding uncertainties.

For the new $\eta$ mass proposal [2], the decision was taken to measure at thirteen fixed energies. To determine the mass using this kinematic method with a precision that is competitive with other recent measurements, i.e., $\Delta m_\eta < 50$ keV/$c^2$ [1], the associated beam momenta have to be fixed with an accuracy of $\Delta p/p < 10^{-4}$. This requires the thirteen beam momenta in the range of $3100 - 3200$ MeV/$c$ to be measured to better than 300 keV/$c$. In addition, based on results from earlier experiments at ANKE [6] it is known that the final state momentum $p_f$ can be extracted in the present case with an uncertainty of $\Delta p_f \sim 150$ keV.

2. Experiment

In the experiment discussed here a vector polarized deuteron beam has been stored in COSY. The motion of the spin vector $\vec{S}$, defined in the rest frame of the particle, in a circular accelerator, synchrotron or storage ring, is given by the Thomas-BMT equation [7]. In a synchrotron without horizontal magnetic fields and where the electric field is always parallel to the particle motion, the spin motion is only a function of the transverse magnetic fields $\vec{B}_\perp$ of the accelerator. The deuteron spin precesses around the stable spin direction, which is given by the vertical fields of the guiding dipole magnets of the synchrotron. The number of spin precessions during a single circuit of the machine, the spin tune $\nu_s$, is proportional to the particle energy. In the coordinate basis of the moving particle, the spin tune is given by

$$\nu_s = G \gamma ,$$  \hspace{1cm} (4)

whereas, taking into account the extra spin precession during a single circuit of the machine, this becomes $\nu_s = 1 + G \gamma$ in the laboratory frame. Here $G = (g - 2)/2$ is the gyromagnetic anomaly of the particle, where $g$ is the gyromagnetic factor. For deuterons the gyromagnetic anomaly is $G_d = -0.1429872725(73)$. The beam polarization can be perturbed by a horizontal magnetic field in the synchrotron and, if the frequency of the perturbation coincides with the spin precession frequency, the beam depolarizes. A horizontal rf field from a solenoid or even a dipole can lead to rf-induced depolarizing resonances. The spin resonance frequency is given by [8]

$$f_r = (k + \gamma G)f_0 ,$$  \hspace{1cm} (5)

where $f_0$ is the revolution frequency of the beam, $\gamma G$ is the spin tune, and $k$ is an integer. If the rf frequency of the perturbation is close to $f_r$ then the polarization of the beam is maximally influenced. In our experiment we only considered the first spin resonance, i.e., $k = 1$, due to the frequency range of the rf solenoid. The kinematic $\gamma$-factor and thus the beam momentum can be determined on the basis of Eq. 5 purely by measuring both the revolution and spin-resonance frequencies.

The thirteen closely spaced energies studied near the $\eta$ threshold as well as at sub-threshold energies for background studies were divided into two so-called supercycles, each consisting of eight different beam energy settings. Each supercycle was used for five days of continuous Schottky data taking to study the long term stability of COSY and to take data in parallel for the $\eta$ meson mass determination. The reason for choosing supercycles instead of independent
measurements at fixed energy was to guarantee the same experimental conditions for each of the
beam energies. In this way the systematic uncertainties could be investigated in more detail.

Before starting each of the five day blocks, the individual beam energies were measured using
the spin resonance method. An example of a spin resonance spectrum at one energy is shown
in Fig. 1 [9]. Far away from the spin resonance, at 1.0116 MHz and 1.0120 MHz, a high beam
polarization was measured by the EDDA detector [10]. In contrast, if the frequency of the
solenoid coincided with the spin precession, the beam was maximally depolarized. The full
width at half maximum was in the region of 80-100 Hz for all energies. Unlike the earlier spin
resonance test measurement with a coasting beam, i.e., no cavities and no internal target [11],
the spin resonance spectra are not smooth. The structures, especially the double peak in the
center, are caused by the interaction of the deuteron beam with the barrier bucket cavity.

However, to study in more detail the shape of the spin resonance spectra, all 26 spectra were
fitted with gaussians and then shifted along the abscissa so that the mean value was zero [9].
They were then scaled to produce a uniform height. The resulting normalized spin resonance
spectrum is shown in the upper part of Fig 2. This is symmetric around zero and smooth, except
for the structure at the center. This region is shown in greater detail in the lower part of Fig. 2.
A structure with a symmetric double peak and an oscillation was observed in the center of the
spin resonance. However, it is important to note that the gaussian mean value, i.e., the spin
resonance frequency, is not influenced by this structure. This was checked by making a fit where
the data points at the center were neglected. The spin resonance frequencies \(f_r\) for all energies
were extracted from the spin resonance spectra by a gaussian fit which gave \(\chi^2/\text{ndf}\) in the region
of 2–3. The statistical uncertainties of the spin resonance frequencies are on the order of 1–2 Hz
at \(f_r \approx 1.01\) MHz.

It is important for the interpretation of the spin resonance measurements to know to
what extent the positions of the observed spin resonance frequencies are stable over the finite
accelerator cycle in the presence of a thick internal target. Therefore, in a special measurement
the switch-on of the rf solenoid was delayed from 20 s to 178 s in order to investigate the
position of the spin resonance frequency close to the end of a long cycle. The observed data
(open symbols of Fig. 1) showed a resonance position which agreed with the data taken at the
beginning of the cycle to within 2 Hz.

In Fig. 3 the shifts between the first and second spin resonance measurements are shown
as red triangles for all thirteen energies. The frequencies in the first supercycle decrease by
Figure 2. Panel (a): The spin resonance spectra normalized by a gaussian. Panel (b): The same but with a binned abscissa in addition. The spin resonance shape is symmetric about zero and smooth except in the center, where a double peak structure is seen. This arises from the synchrotron oscillations of the beam particles caused by the interaction of the deuteron beam with the barrier bucket cavity. The inserts show the resonance valley in greater detail.
Figure 3. The spin resonance frequencies were measured twice, once before and once after the five days of data taking. The red triangles present the shift of the spin resonance frequency $f_r$ from the first to the second measurement. These shifts correspond to changes in the orbit length, which are shown as blue circles.

between 4 and 10 Hz for all energies, whereas for the second supercycle they increase in the range of $12 - 17$ Hz. These systematic shifts of the frequencies in the same direction indicate slight changes in the COSY settings. Because the revolution frequency was found to be stable the change is attributed to a shift in the orbit length $s$.

The velocity $v$ of the particle is the product of the revolution frequency and the orbit length $v = s f_0$. Using Eq. 5, the orbit length can be calculated from the revolution and the spin resonance frequencies:

$$s = c \left[ \frac{1}{f_0^2} - \left( \frac{G_d}{f_r - f_0} \right)^2 \right]^{\frac{1}{2}},$$

which allows the orbit lengths to be extracted with a precision better than 0.3 mm for every flat top. Since the nominal COSY circumference is 183.4 m, this gives a relative accuracy of $\Delta s/s \leq 2 \times 10^{-6}$. The precision is dominated by the uncertainty of the spin resonance frequency. The shift in the spin resonance frequency corresponds to a change in the orbit length of up to 3 mm, which is presented for all energies in Fig. 3 as blue circles. The shifts of the spin resonance frequencies of the first supercycle suggest an increase in the orbit length in the range of 0.7 – 1.6 mm and to a decrease in the range of 2.0 – 2.8 mm for the second supercycle [9].

To determine the precise beam momenta, the mean value of the two spin resonance measurements for every energy was calculated. These mean values differ by up to 10 Hz from the single spin resonance measurements. Nevertheless, in view of the observed shift of the spin resonance frequency, a very conservative systematic uncertainty of $\Delta f_r = 15$ Hz was assumed.

3. Results

The deuteron kinematic $\gamma$-factor and the beam momenta were calculated according to Eq. 7

$$\gamma = \frac{1}{G_d} \left( \frac{f_r}{f_0} - 1 \right),$$
$$p = m_d \beta \gamma = m_d \sqrt{\gamma^2 - 1}$$

from the knowledge of the revolution and the spin resonance frequencies. The accuracies to which both of the frequencies are determined are dominated by systematic effects. The revolution frequency measured by the Schottky spectrum analyzer has an uncertainty of $\Delta f_0 = 6$ Hz,
corresponding to one in the beam momentum of 50 keV/c. The error in the determination of the spin resonance arises from the small variations of the orbit length and \( \Delta f_r = 15 \, \text{Hz} \) corresponds to an uncertainty in the beam momentum of 164 keV/c. Because these systematic uncertainties are independent, they are added quadratically to give a total uncertainty \( \Delta p/p \leq 6 \times 10^{-5} \), i.e., a precision of 170 keV/c for beam momenta in the range of 3100 – 3200 MeV/c. This is over an order of magnitude better than ever reached before for a standard experiment in the COSY ring. The measurement of the beam momentum differed by up to 5 MeV/c from the nominal requested momentum.

Two further quantities, the beam momentum smearing \( \delta p/p \) and the smearing of the orbit length \( \delta s/s \), can be extracted from the spin resonance spectra. The width of the spin resonance spectra depends only on the strength of the resonance and the momentum smearing. In our case a resonance strength of \( \epsilon = 3.2 \times 10^6 \) leads to a spin resonance with a FWHM width of 9.1 Hz. This is much smaller than the observed width of 80-100 Hz, which is therefore dominated by the momentum spread of the beam. Assuming a gaussian distribution in the revolution frequency with a FWHM = 40 – 50 Hz, and neglecting all other contributions, the width of the spin resonance distribution requires a momentum spread of \( (\delta p/p)_{\text{rms}} \approx 2 \times 10^{-4} \). This upper limit on the beam momentum width corresponds to a smearing of the orbit length of \( (\delta s/s)_{\text{rms}} \approx 4 \times 10^{-5} \).

The momentum spread could be checked from the frequency slip factor \( \eta \), which was measured at each energy. Using \( \delta p/p = 1/\eta \times (\delta f_0/f_0) \) this leads for example at \( p_{\text{nominal}} = 3.1625 \, \text{GeV/c} \) to \( (\delta p/p)_{\text{rms}} = 1.4 \times 10^{-4} \), which is consistent with the value obtained from the resonance distribution [9].

In a next step the final state momenta of emitted \(^3\text{He} \) nuclei have to extracted for all beam momenta. In Fig. 4 one momentum distribution of \(^3\text{He} \) nuclei obtained at a beam momentum of \( p = 3.159 \, \text{GeV/c} \) is shown. A signal of the \( dp \rightarrow ^3\text{He} \eta \) reaction is clearly visible on top of a broad distribution, originating mainly from multi-pion production. By using data obtained at sub-threshold energies a background subtraction can be performed with highest accuracy.

In Fig. 5 the extracted momenta of the \(^3\text{He} \) nuclei \( p_f \) are plotted against the determined deuteron beam momenta \( p_d \). Due to the high precision of the data, the size of the error bars is in the order of the width of the solid fit line. Thus, for visibility reasons much larger symbol sizes are used in this figure. The shown solid line is a fit to the data using Eqs. 1-3 with the

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**Figure 4.** Center of mass momentum distribution of detected \(^3\text{He} \) nuclei at a COSY beam momentum of \( p_d = 3.159 \, \text{GeV/c} \). The background distribution, originating mainly from multi-pion production, can be described well by data recorded at sub-threshold energies. A background subtraction leads to a clean signal from the \( dp \rightarrow ^3\text{He} \eta \) reaction.
mass of the $\eta$ meson as fit parameter. Although preliminary, the present data clearly indicate that with this discussed measurement an extraction of the $\eta$ meson mass can be performed with high accuracy.

4. Conclusions and outlook
It was possible to determine the beam momenta with an accuracy of $\Delta p/p \leq 6 \times 10^{-5}$, i.e., the thirteen momenta in the range $3100\text{–}3200$ MeV/c were measured with precisions of $\approx 170$ keV/c, a feat never before achieved at COSY. The actual precision was limited by the systematic variations of the orbit length and the characteristics of the Schottky spectrum analyzer.

The orbit length could be extracted from the revolution and spin resonance frequencies with an accuracy of $\Delta s/s \leq 2 \times 10^{-6}$. Thus for COSY, with a circumference of 183.4 m, the orbit length could be measured with a precision below 0.3 mm. This may allow one to obtain a better knowledge of the orbit behavior in COSY. In addition, the distributions in the beam momentum and orbit length could be extracted from the data. The results are also sensitive to the synchrotron oscillations which lead to synchrotron side-band resonances and so these can also be studied with the spin resonance method. These results were achieved using a deuteron beam, but there are no in-principle reasons why the depolarization technique should not be applicable to proton beams at COSY with same success.

The final state momenta of emitted $^3$He nuclei can be determined with high accuracy using
the ANKE spectrometer. In our particular case the obtained resolutions should allow the mass of the $\eta$ meson to be measured with a precision of $\Delta m_\eta \leq 50 \text{keV}/c^2$.

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