Theoretical Research on the Uniformity of Planetary Side Lapping Trajectory

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Abstract. The reduction to absurdity is adopted to analyze equation of planetary double-side lapping motion built by kinematics, and study the important parameter in the process of lapping-I (the ratio of grinding disc and the inner pin ring in rotary speed) satisfies the condition of uniformity of lapping trajectory of a point in workpiece relative to surface of grinding disc. The results show the ratio I that meet uniformity of lapping trajectory is a function of time t. On the other hand, for a given planetary double-side lapping machine, the I of constant ratio (rational or irrational) that satisfies uniformity of lapping trajectory does not exist unless it continues to change over the time. This research has laid a theoretical foundation for the further studying in uniformity of lapping trajectory, and also provided a new idea and method for the analysis of relative velocity between workpiece and grinding wheel, the wear uniformity of grinding wheel, and the study of workpiece surface quality.

Keywords: Planetary Double-side lapping; Trajectory; Wear; Uniformity.

1. Introduction
Using abrasive precision grinding and polishing technology is an important method of the ultra precision machining, to obtain the shape of the limit of accuracy, dimensional accuracy, surface roughness, surface integrity as the goal on the premise of not changing the workpiece material physical properties, have a wide application in machinery, automobile, electronics, communications, computer, laser, aerospace and other fields, is the key technology of precision machining parts of a mechanical seal ring, gauge block, hydraulic and pneumatic components, precision ceramics, monocrystalline silicon slice, sapphire, etc. The grinding motion trajectories reflects the relative motion of the grinding disc and the workpiece in the grinding process, which is the basis for an in-depth understanding of the grinding mechanism. It can be divided into two categories: one is the trajectories of movement of a point on the workpiece relative to the grinding disc, and the other is the trajectories of movement of a point on the grinding disc relative to the workpiece. The former is of great significance for revealing the grinding uniformity of the grinding disc, while the latter is directly related to the morphological features and texture distribution of the workpiece processed surface. The significance of studying the corresponding trajectory uniformity is reflected in the following aspects: ensuring that all points on the workpiece surface have the same material removal probability and ensuring uniform wear of the tool disc. Therefore, domestic and foreign scholars have conducted a lot of researches on the motion trajectories distribution and grinding mechanism of plane grinding and polishing. The results show that the driving mode and speed ratio are the main factors affecting the grinding uniformity. Yang changming et al. studied the relationship between the rotational speed of planet gear and solar wheel and the motion trajectory of abrasive particles and the quality of grinding plane. Zhao wenhong et al. studied the plane grinding uniformity by means of grinding with definite eccentric and indefinite eccentric. Wen donghui et al. carried out theoretical analysis on the influence of rotation speed ratio and eccentricity of grinding disc and workpiece on the
trajectories uniformity. The results showed that increasing the rotation speed ratio made the processing uniformity worse. Liu qing et al. [11] showed that x-y linkage of the workbench has an important impact on the trajectory; The theoretical research results of Yan zhen etal. [12] show that the introduction of yaw motion is conducive to improving the trajectories non-uniformity and improving the surface quality of the processed workpiece. Adam Barylski et al. [13] analyzed the influence of coupling relationship between velocity ratio $K_1$ of workpiece disc and grinding disc and cycle ratio $K_2$ of reciprocating cycle of workpiece disc and rotation cycle on trajectories distribution under linear swing drive, and optimized kinematic parameters to obtain better grinding trajectories uniformity.

No matter which kind of motion trajectory, when the grinding machine with a given driving mode is used for grinding processing, which motion parameters are closely related to the grinding motion trajectory, and what are the conditions for the uniformity of motion trajectory? Based on the kinematics analysis to establish the trajectories of movement of a point on the workpiece relative to the grinding disc in planetary double-side lapping, the reduction to absurdity is adopted to deduce mathematically the equation of motion trajectory, studied the important parameters in the process of grinding - I (the ratio of grinding disc and the inner pin ring in rotary speed) satisfying the conditions for the uniformity of motion trajectory, for the uniformity of planetary plane lapping movement laid a theoretical foundation for further study. It also provides a new idea and method for the analysis of the relative velocity between workpiece and grinding disc, the wear uniformity of grinding disc and the surface quality of workpiece. The method can also be used to evaluate the uniformity of the trajectory simulation of single-side grinding, single-side polishing and double-side polishing.

2. Grinding Machine Principle and Motion Trajectory Equation

2.1. Principles of the system

Figure 1 is a common structure schematic diagram of planetary double side lapping machine. The inner pin ring is driven by the motor through the reducer, and the outer pin ring is fixed on the machine base (there is also a grinding machine with the outer pin ring rotation), which can load multiple planet gear (workpiece holder) at the same time to drive multiple workpieces to move together, the upper and lower grinding disc is the consolidated abrasive grinding disc. The planet gear is driven by the inner pin ring to go around the center of the inner pin ring and rotate. The workpiece is placed in the hole of the planet gear, and the movement of the workpiece is driven by the planet gear. At the same time, under the action of the grinding force of the lower grinding disc and the upper grinding disc, rotation will also occur. Therefore, the movement of the workpiece is the synthesis of the planetary movement and rotation.

![Figure 1. Principle of double side grinding operation.](image)

According to the motion principle of the grinding machine, considering that the influence of the workpiece motion on the motion of the planet gear train is far less than that of the planet gear train on the motion of the workpiece, namely ignoring the rotation of the workpiece; At the same time, considering the similarity of the workpiece's relative motion with the upper grinding disc and the lower grinding disc, only the relative motion of the workpiece and the lower grinding disc is studied here. Therefore, it can be simplified into the relative motion schematic diagram of the workpiece and the lower grinding disc as shown in figure 2.
2.2. Mathematical formulation

In the plane perpendicular to the spindle of the machine, the rotation axis of the grinding disc and the inner pin rings is \( O_1 \), and the rotation axis of the planet gear is \( O_2 \). Take \( O_1 \) and \( O_2 \) as the origin of coordinates respectively, and set up the coordinate systems \( O_1X_1Y_1 \) and \( O_2X_2Y_2 \) in the plane, and fix them on the grinding disc and workpiece respectively. The \( O_1X_1 \) axis and \( O_2X_2 \) axis of the two coordinate systems are along the line direction of \( O_1O_2 \). The distance between \( O_1O_2 \) is the sum of the radius \( r \) of the inner pin pitch circle and the radius \( R \) of the planet wheel pitch circle, namely \((r+R)\).

The motion path equation of workpiece relative to the lower grinding disc is established by means of graph transformation. If any point \( P \) is selected on the workpiece with diameter \( d \), the initial distance between point \( P \) and point \( O_2 \) is \( RP \), and the Angle between the vector \( O_2P \) and the axis \( O_1X_1 \) is \( \alpha \), then the homogeneous coordinates of point dimensional vector to represent an n-dimensional vector method) can be expressed by the vector as:\[14\]:

\[
L = \begin{bmatrix} R_r \cos \alpha, R_r \sin \alpha \end{bmatrix}
\]

Let the angular velocity of the planet gear be \( \omega_H \), rotation angular velocity \( \omega_2 \), and the lower grinding disc angular velocity \( \omega_0 \). After time \( t \), the rotation Angle \( \theta_1 = \omega_1t \) of the planet gear around \( O_1 \) and the rotation Angle \( \theta_2 = \omega_2t \) of the vehicle around \( O_2 \) can be expressed by matrix as follows:

\[
T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ (r+R) & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta_1 & \sin \theta_1 \\ -\sin \theta_1 & \cos \theta_1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -(r+R) \cos \theta_1 & -(r+R) \sin \theta_1 & 1 \end{bmatrix}
\]

\[
T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -(r+R) & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta_2 & \sin \theta_2 \\ -\sin \theta_2 & \cos \theta_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ (r+R) \cos \theta_2 & (r+R) \sin \theta_2 & 1 \end{bmatrix}
\]

Similarly, grinding disc at an Angle \( \theta_0 = \omega_0t \) of rotation in time \( t \). If the grinding disc is regarded as immovable, it is equivalent to the rotation Angle of the planet gear around \( O_1 \) point in the direction opposite to the revolution of the planet gear, that is, the coordinate system \( O_2X_2Y_2 \) is finally rotated to the position of the coordinate system \( O_2'X_2'Y_2' \), and the corresponding transformation matrix is:

\[
T_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -(r+R) & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta_0 & \sin \theta_0 \\ -\sin \theta_0 & \cos \theta_0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ (r+R) \cos \theta_0 & (r+R) \sin \theta_0 & 1 \end{bmatrix}
\]
Therefore, the motion trajectory equation of any point P of the workpiece relative to the grinding disc is the motion of any point P of the workpiece relative to the grinding disc. It comes down to the coordinate $X_p, Y_p$ of point P in the coordinate system $O_1X_1Y_1$ after the transformation, which is expressed by the homogeneous coordinate $(X_p, Y_p, 1)$. Therefore, the coordinates $(X_p, Y_p)$ of any point P on the workpiece on the planet gear in $O_1X_1Y_1$ are:

$$
\begin{align*}
(X_p, Y_p, 1) &= L \times T_1 \times T_2 \times [R_p \cos \alpha, R_p \sin \alpha, 1] \\
&= \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 
\end{bmatrix} \\
&\times \begin{bmatrix}
\cos \theta_1 & \sin \theta_1 & 0 \\
-sin \theta_1 & \cos \theta_1 & 0 \\
0 & 0 & 1 
\end{bmatrix} \\
&\times \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
-\cos \theta_1 & \sin \theta_1 & 1 
\end{bmatrix} \\
&\times \begin{bmatrix}
\cos \theta_0 & \sin \theta_0 & 0 \\
0 & \cos \theta_0 & 0 \\
0 & 0 & 1 
\end{bmatrix} \\
&\times \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
-(r+R) & 0 & 1 
\end{bmatrix} \\
&= \begin{bmatrix}
R_p \cos(\theta_1 - \theta_2 - \theta_0 + \alpha) + (r+R) \cos(\theta_1 - \theta_0) \\
R_p \sin(\theta_1 - \theta_2 - \theta_0 + \alpha) - (r+R) \sin(\theta_1 - \theta_0) \\
0 
\end{bmatrix}
\end{align*}
$$

The above formula is simplified to get:

$$
\begin{align*}
X_p &= R_p \cos(\theta_1 - \theta_2 - \theta_0 + \alpha) + (r+R) \cos(\theta_1 - \theta_0) \\
Y_p &= R_p \sin(\theta_1 - \theta_2 - \theta_0 + \alpha) - (r+R) \sin(\theta_1 - \theta_0)
\end{align*}
$$

That is:

$$
\begin{align*}
X_p &= R_p \cos \left( (\omega_\theta - \omega_\phi) \ t + \alpha \right) + (r+R) \cos(\omega_\theta - \omega_\phi) \ t \\
Y_p &= R_p \sin \left( (\omega_\theta - \omega_\phi) \ t + \alpha \right) + (r+R) \sin(\omega_\theta - \omega_\phi) \ t
\end{align*}
$$

Formula (1) above is the trajectory equation of point P, that is, the motion equation of the workpiece at any point P relative to the grinding disc, and that's the basis for the following derivation.

Relative to the center radius $\rho$ of grinding disc:

$$
\rho = \sqrt{X_p^2 + Y_p^2}
$$

As can be seen from (1), the trajectory of point P relative to the grinding disc is a cluster of cycloid. For simplicity, set point P as the center of the workpiece, where $R_\rho$ is fixed. At this point, the distance that point P of the workpiece moves relative to the grinding disc in dt time:

$$
\begin{align*}
dX_p &= \int -R_p(\omega_\theta - \omega_\phi) \sin \left( (\omega_\theta - \omega_\phi) \ t + \alpha \right) - (r+R)(\omega_\theta - \omega_\phi) \sin(\omega_\theta - \omega_\phi) \ t \ dt \\
dY_p &= \int R_p(\omega_\theta - \omega_\phi) \cos \left( (\omega_\theta - \omega_\phi) \ t + \alpha \right) + (r+R)(\omega_\theta - \omega_\phi) \cos(\omega_\theta - \omega_\phi) \ t \ dt \\
dS &= \sqrt{dX_p^2 + dY_p^2} \ dt
\end{align*}
$$
The relationship between the revolution speed of the planet gear and the sun gear rotation speed, the planet gear and the rotation speed of the sun gear are as follows:

\[
\begin{align*}
\omega_z &= \frac{r(R - r)}{2R(r + R)} \omega_1 \\
\omega_H &= \frac{r}{2(r + R)} \omega_1
\end{align*}
\]

Substitute (4) and let \( I = \frac{\omega_H}{\omega_1} \) into (3), get:

\[
dS = \sqrt{dY_p^2 + dY_x^2} dt
\]

\[
= \alpha_1 \left[ \frac{r(r + 2R) - I}{2R(r + R)} R_p^2 + \left( \frac{r}{2(r + R)} - I \right)^2 (r + R)^2 \right] dt
\]

(5)

\[
= \alpha_1 \left[ +2R_p(r + R) \left( \frac{r(r + 2R) - I}{2R(r + R)} \right) - I \left( \frac{r}{2(r + R)} - I \right) \cos \left( \frac{r}{2R} \omega_1 t + \alpha \right) \right] dt
\]

3. Analysis of Conditional of Uniform Distribution of Grinding Trajectory

At any point P of the workpiece, the trajectory density relative to the grinding disc is uniformly distributed, that is, the arc appearance of the trajectory is completely uniform for the radius of the grinding disc. In other words, the derivative of the trajectory arc length S with respect to the radius of the grinding disc (that is, the change rate of the track arc length with the radius) should be linearly related to \([14]\):

\[
\frac{dS}{d\rho} = K \rho
\]

According to (2):

\[
d \rho = - \frac{R_p(r + R) r \omega_1}{2R} \times \frac{\sin \left( \frac{r}{2R} \omega_1 t + \alpha \right)}{\sqrt{R_p^2 + (r + R)^2 + 2R_p(r + R) \cos \left( \frac{r}{2R} \omega_1 t + \alpha \right)}} dt
\]

(7)

Substitute (5) and (6) and replace \( a = \frac{r}{2R} \), \( b = r + R \), \( i_1 = \frac{[r(r + 2R)/2R(r + R)] - I}{2} \), \( i_2 = \frac{r}{2(r + R)} - I \) into (7), get:

\[
K = \frac{dS}{\rho d \rho} = \frac{\alpha_1 \left[ t_1^2 R_p^2 + t_2 b^2 + 2R_p b t \sin (\omega_1 t + \alpha) \right] \cos (\omega_1 t + \alpha) \times \frac{\sin (\omega_1 t + \alpha)}{\sqrt{R_p^2 + b^2 + 2R_p b \cos (\omega_1 t + \alpha)}}}{abR_p \sin (\omega_1 t + \alpha)}
\]

(8)

According to equation (7), if the linear relation is true, K should be a constant, i.e:

\[
\frac{dK}{dt} = 0
\]

\[
\frac{dK}{dt} = \alpha_1 \left[ \frac{b}{i_1^2 R_p} \times \frac{\cos (\omega_1 t + \alpha) + \cos^2 (\omega_1 t + \alpha)}{i_2 \sin^2 (\omega_1 t + \alpha) \sqrt{i_1^2 R_p^2 + i_2 b^2 + 2R_p b t \sin (\omega_1 t + \alpha)}} \right] = 0
\]

(9)

That is:
\[1 + \frac{i_1^2 R_p^2 + i_2^2 b_i^2}{R_p b_i i_2} \cos(a \omega t + \alpha) + \cos^2(a \omega t + \alpha) = 0 \]  \hspace{1cm} (10)

It can be seen that (10) is a quadratic equation with one unknown of \(\cos(a \omega t + \alpha)\), and the solution is:

Solution 1:
\[\cos(a \omega t + \alpha) = \frac{-2i_1^2 R_p^2}{2i_1 i_2 R_p b} = \frac{-i_1 R_p}{b_i} \]  \hspace{1cm} (11)

Solution 2:
\[\cos(a \omega t + \alpha) = \frac{-2b_i^2 i_2^2}{2i_1 i_2 R_p b} = \frac{-b_i}{i_1} \]  \hspace{1cm} (12)

As can be seen from (11) and (12), the two solutions are reciprocal to each other, so only one of them can be solved.

After \(a, b, i_1, i_2\) substituting back into (12) and arranging, we can get:
\[
I = \frac{(r + R) + \cos\left(\frac{r}{2R} \frac{\omega t + \alpha}{\omega} R_p\right)}{\cos\left(\frac{r}{2R} \omega t + \alpha R_p\right)} = \frac{e + \cos\left(\frac{r}{2R} \omega t + \alpha R_p\right)}{R_p}
\]  \hspace{1cm} (13)

\(r\) - the radius of the sun gear pitch circle
\(R\) - the radius of the planet gear pitch circle
\(R_P\) - the radius of any point of the workpiece relative to the center of the planet gear
\(\omega\) - the rotation speed of the sun gear
\(e\) - eccentricity, that is, the distance between the center of the planet gear and the grinding disc.

The formula is based on the assumption that point \(P\) on the workpiece (here is the center point of the workpiece) is uniform with respect to the trajectory of the grinding disc.

4. Results and Discussion

Equation (13) is derived by using the reductive method -- first, it is assumed that the trajectory density of any point of the workpiece relative to the grinding disc is uniformly distributed - that is, the trajectory arc length \(S\) of \(P\) point of the workpiece relative to any radius \(R\) of the grinding disc is linearly related to its center radius of the grinding disc through mathematical derivation. This means that in order to achieve uniform distribution of the workpiece's trajectory density relative to the grinding disc at any point, (13) must be true. The (13) shows that meet of uniformity of lapping trajectory the rotating speed ratio \(I\) (grinding disc angular velocity \(\omega_0\) and the ratio of the sun gear angular velocity \(\omega_1\)) is the function of \(r, R, R_P\) and \(t\). For a given grinding machine and workpiece processing, \(r, R, R_P\) has been determined, that is constant, only the sun gear angular velocity \(\omega_1\) and grinding disc angular velocity \(\omega_0\) is the operating parameters can be changed. More importantly, the rotating speed ratio \(I\) is a function of the grinding time \(t\). This indicates that in order to satisfy the uniformity of lapping trajectory of the workpiece relative to the grinding disc, the rotating speed ratio must change with time. On the other hand, corresponding to a constant speed ratio (whether rational or irrational), the uniformity of lapping trajectory the workpiece relative to the grinding disc is impossible to achieve a linear uniform distribution. The effect of time \(t\) can be ignored only if and only if the infinitesimal in \(\frac{r}{2R} \omega_1\) is close to zero and the \(\cos\left(\frac{r}{2R} \omega t + \alpha\right)\) amplitudes close to a straight line, but this is impossible in reality.

In order to discuss the influence of rotation speed ratio on the motion trajectory uniformity, formula (1) can be converted into the following formula:
Taking the AC700F planar CNC grinding machine of planetary pin gear of Peter Wolters company in Germany as an example, this paper discusses and analyzes the influence of the rotational speed ratio $I$ of grinding disc and sun gear on the workpiece's motion trajectory relative to the grinding disc. Its parameters are as follows: sun gear pitch radius $R_1$ = 137.85 mm; the pitch radius of the planet gear is $R_P$ = 90.5 mm; For the sake of representativeness and convenience of calculation, the workpiece center point is taken as the starting point of motion, That is, $R_p$ is the pitch circle radius of the workpiece hole distribution on the planet gear $R_p$ = 90.5 mm. Considering that the influence of initial Angle on the trajectory distribution uniformity can be ignored, we can set $\alpha = 0$. Take the sun gear speed $n_1$ = 100 rpm, $t$ = 0.42 s and substitute the above parameters into (13) and (14) , Matlab software was used for trajectory simulation to obtain the workpiece center point relative to the grinding disc motion trajectory distribution in figure 3.

\[
\begin{align*}
X_p &= R_p \cos \left[ \frac{r(t+2R)}{2R} - I \alpha t + \alpha \right] + (t+R) \cos \left( \frac{r}{2(t+R)} - I \right) \alpha t \\
Y_p &= R_p \sin \left[ \frac{r(t+2R)}{2R} - I \alpha t + \alpha \right] + (t+R) \sin \left( \frac{r}{2(t+R)} - I \right) \alpha t
\end{align*}
\]  

(14)

Figure 3. Workpiece center point relative to grinding disc tract distribution.

Figure 4. Variation of rotational speed retio satisfying the uniformity of motion trajectory.
It can be judged from the figure that the track point distribution is uniform, but if it exceeds $t=0.45\text{s}$, the track will be very chaotic, because the rotational speed ratio is close to infinity. In the same way, Matlab software was used for trajectory simulation to obtain rotational speed ratio of the corresponding changing with the time as shown in figure 4, there are the actual value of the mutation (in theory to plus or minus up) in the change cycle, is clearly impossible, but you can take the middle period ($t=0.62\text{s}$ to $1.52\text{s}$) by getting rid of the extremum and access to basic meet the requirements of trajectory distribution uniformity, shown in figure 5 to ignore the extreme value point and the simulation time $t=18\text{s}$ track points distribution, subject to further research on this aspect.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure5}
\caption{Movement point distribution of workpiece center.}
\end{figure}

By inverting (13), equation (15) can be obtained:

$$
I_2 = \frac{\frac{r}{2} + \left(\frac{r}{2R} + \frac{r}{2(r+R)}\right) \cos \left(\frac{r}{2R} \omega_t + \alpha\right) R_p}{(r+R) + \cos \left(\frac{r}{2R} \omega_t + \alpha\right) R_p}
$$

(15)

Similarly, by substituting the above parameters into (15), the change curve of rotational speed ratio with time as shown in figure 6 can be obtained. It can be seen from the curve that it is a time-dependent trig function with zero and negative values.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure6}
\caption{Movement point distribution of workpiece center.}
\end{figure}
Similarly, the rotational speed ratio (15) was substituted into the trajectory (14), and the above related parameters were substituted. The grinding parameters were taken as \( n_1 = 100 \text{rpm} \) of the sun gear, and the grinding time \( t = 20 \text{s} \) to obtain the workpiece center's motion trajectory curve relative to the grinding disc as shown in figure 7. It can be judged from the figure that the distribution of its trajectory points is uniform, but there is a ring blank belt between them. What causes it remains to be studied in the next step.

![Figure 7. Workpiece center point relative to grinding disc relative to grinding disc.](image)

The uniformity of the grinding trajectory is the premise to ensure the uniform wear of the grinding disc or the improvement of the flatness of the workpiece, but it is not enough to ensure the uniform wear of the grinding disc or the removal of the workpiece by the uniform material of the machining surface. The decisive factor should be the relative speed between the workpiece and the grinding disc. According to Preston equation, the grinding disc wear or workpiece material removal rate is a function of the relative velocity. In the next step, according to the above ideas, the condition that the relative velocity density of the workpiece at point \( P \) relative to the grinding disc is uniformly distributed can be studied. In other words, the derivative of the synthetic relative velocity with respect to the radius of the grinding disc (that is, the rate of change of the synthetic relative velocity with the radius) should be linearly related to the radius of the grinding disc.

5. Conclusion

Based on the reductionist method, the motion equation of planetary plane grinding established by kinematics is analyzed, and the important parameter in the grinding process -- rotational speed ratio \( I \) (grinding disc rotational speed and sun gear rotational speed ratio) is studied to meet the condition that the motion trajectory uniformity of any point on the workpiece relative to the grinding disc is satisfied, and the following conclusions are drawn:

(1) the rotational speed ratio \( I \), which satisfies the motion trajectory uniformity of any point on the workpiece relative to the grinding disc, is not only related to the structural parameters of the grinding mechanism (eccentricity, radius of the sun gear, radius of the planet gear, coordinates of the point on the workpiece), but also a function of time \( t \).

(2) for a given planetary plane grinding grinder, the fixed speed ratio \( I \) (whether rational or irrational) that satisfies the motion trajectory uniformity of the planetary grinding disc does not exist. If the rotational speed ratio changes periodically with time, the possibility of motion trajectory uniformity distribution exists.

(3) this study lays a theoretical foundation for the further study on the uniformity of the motion trajectory of planar grinding, and also provides a new idea and method for the analysis of the relative velocity between workpiece and grinding disc, the wear uniformity of grinding disc, and the study on the surface quality of workpiece.
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