Bridge Technology Condition Degradation Prediction Based on Bayes Dynamic Model

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Abstract. Establishing appropriate degradation model of bridge performance is the precondition of preventive maintenance. A Bayes dynamic model is proposed to predict the future performance of bridge. Bayes factor is used to evaluate whether the prediction mode is reasonable. The Bayes factors of Bayes dynamic models are tested through practical engineering. The results show that the Bayes dynamic model is fit for the bridge technology condition degradation prediction.

1. Introduction

As an important part of infrastructure system, bridges play a significant role in improving the economic, social, and environmental welfare of nations. During the service period of the structure, the bearing capacity gradually declines or even suddenly destroys due to aging (such as corrosion), natural disasters (such as earthquakes and hurricanes) and man-made extreme events (such as collisions and terrorist attacks). Therefore, it is of great significance to carry out scientific maintenance during the operation of the bridge[1, 2].

To implement preventive maintenance of bridges, managers can arrange management and maintenance funds more reasonably by preventive maintenance, it is necessary to effectively predict the degradation of bridge performance. In recent years, some achievements have been made in the prediction of bridge technical degradation. Zhang Yang et al.[3] proposed a multi-stage degradation model of bridge technical status based on Markov chain, which has achieved good prediction accuracy. However, this model only pays attention to objective assessment data of technical status and cannot conduct subjective intervention according to expert experience. Compared with the Markov model which cannot use subjective information, the Bayes theory-based[4] prediction model combines objective information with subjective information to realize prediction, a dynamic prediction model are established, and constantly modifies the prior model of relevant parameters by using the detection information under the current situation, so as to get a better match with the actual situation. The predictions are consistent and more accurate.

Bayes theory has been widely used in engineering field[5, 6]. Considering the advantages of the prediction model based on Bayes theory, more and more scholars have applied it to the field of bridge engineering[7-12]. Fan Xueping et al.[11] use the ultimate stress as an index to evaluate the performance of bridges and use Bayes dynamic model to predict it. The ultimate stress can well reflect whether the bearing capacity of the structure is beyond the limit, which is of great significance to the safety early warning of the structure. However, only one-step prediction of the stress cannot obtain the long-term evolution law of the ultimate stress.
Firstly, this paper introduces the idea of updating Bayes dynamic model. Then, according to the test data of practical engineering, Bayes linear growth and quadratic growth are established based on the technical condition score of bridges. Finally, the model is evaluated by Bayes factor.

2. Bayes dynamic model

2.1. Assumptions of Bayes Dynamic Model

The assumptions of Bayes dynamic model are as follows.

- State variables ($\theta_t$) is a Markov chain, $y_{t+1}$ is linear with $\theta_t$.
- Conditionally on $\theta_t$, $y_t$; $t = 1; 2; 3;...;T$, where T is the total monitored time, is independent of each other and $y_t$ depends on $\theta_t$ only.

The recurrence relationship between state variables and monitoring variables is shown in Figure 1.

![Figure 1 Dependence structure for a state-space model](image)

2.2. Establishment of Bayes Dynamic Linear Model

The Bayes dynamic model combines subjective and objective information to build a dynamic prediction model and uses the detection information under the current situation to constantly modify the prior model of the relevant parameters, to obtain more accurate and consistent prediction results with the actual situation.

The Bayes dynamic model consists of a system determined by two equations, which can be described as:

- How process observations are randomly dependent on current state parameters;
- How does the state parameter change with time, that is, the change and disturbance of the system's understanding of the law of things with time.

When the state parameters of Bayes dynamic model change linearly with time and the observation vectors and state parameters are normal random variables, the model is called normal dynamic linear model.

3. Bayes Dynamic Prediction of Bridge Technical Condition Score

3.1. Selection of parameters of Bayes dynamic model

For the Bayes dynamic model with linear growth, the parameters are selected as follows:

$$F_t = F = [1, 0]^T, G_t = G = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \forall t \in (0, 1, ..., T)$$

the observational equation, the equation of state and the initial priori information of the linear growth Bayes dynamic model can be obtained, as shown in equation (1) ~ (3):

1) Monitored equation

$$y_t = \mu_t + \nu_t \quad (1)$$

2) State equation

$$\theta_t = [\mu_t, \beta_t]; \quad \mu_t = \mu_{t-1} + \beta_{t-1} + \omega_{t1}; \quad \beta_t = \beta_{t-1} + \omega_{t2}, \omega_t = (\omega_{t1}, \omega_{t2})^T \quad (t = 1, 2, ..., T) \quad (2)$$

3) Initial prior distribution
Where $y_t$ is the monitored Bridge Technical Condition Score at time $t$, $\mu_t$ is the mean of the observed value at time $t$, $\beta_{t-1}$ is the rate of change of bridge technical condition score. All of them are normal random variables, $\nu_t$ is the error of monitor, $\omega_t$ is the fuzziness of recursive process.

### 3.2. Recursive Updating of Bayes Dynamic Model for Bridge Technical Status Scoring

Bayes dynamic model is suitable for probability recurrence of normal distribution. According to statistical data, see Table 2. Bridge technical status score conforms to normal distribution. In this paper, only one variable of bridge technical status score is studied. The recurrence process of normal Bayes dynamic model with one variable is as follows:

#### 3.2.1. Initial state

Initial priori information of time 0

$$(\theta_0 \mid D_0) \sim N[m_0, C_0]$$

#### 3.2.2. One-step prediction

1) Posterior distribution of state parameters at time $t-1$

$$(\theta_{t-1} \mid D_{t-1}) \sim N[m_{t-1}, C_{t-1}]$$

2) Prior distribution of state parameters at time $t$

$$(\theta_t \mid D_{t-1}) \sim N[a_t, R_t]$$

Where $a_t = G_t m_{t-1}, R_t = G_t C_{t-1} G_t^T + W_t$, $a_t$ is the one-step predicted value of the state parameter.

3) one-step prediction distribution at time t

$$(y_t \mid D_{t-1}) \sim N[f_t, Q]$$

Where $f_t = E(y_t \mid y_{t-1}) = F_t^T a_t$, $Q_t = \text{var}(y_t \mid y_{t-1}) = F_t^T R_t F_t + V_t$, $f_t$ is a one-step predictive value for evaluating the technical condition of bridges, $1/Q$ is one-step prediction accuracy.

4) Posterior Distribution of State Parameters at time t

$$(\theta_t \mid D_t) \sim N[m_t, C_t]$$

Where $m_t = E(\theta_t \mid D_t) = a_t + A_t e_t, C_t = \text{var}(\theta_t \mid D_t) = R_t - A_t A_t^T Q_t, A_t = R_t F_t Q_t^{-1}, e_t = y_t - f_t, e_t$ is One Step Prediction Error.

#### 3.3. Bayes factor

In this paper, Bayes factor is used as an index to evaluate the goodness of fit of prediction model. On the one hand, Bayes factor can reflect whether the detected data are abnormal data, on the other hand, it can also reflect the goodness and badness of fitting of prediction model.

For two models with the same statistical characteristics, one is a prediction model $M_0$. Another is alternative model $M_1$. The predicted distribution densities of the two methods are $p_0(\cdot)$ and $p_1(\cdot)$, Then, the Bayes factor is expressed as:

$$H_t = p_0(y_t \mid D_{t-1}) / p_1(y_t \mid D_{t-1})$$
References show that the probability density functions of one-step prediction model and alternative model used in this paper are as follows:

\[ p_0 = \left(2\pi\right)^{-0.5} \exp\left(-0.5e^2_{te}\right) \]  \hspace{1cm} (10)

\[ p_1 = \left(2\pi k^2\right)^{-0.5} \exp\left(-0.5e^2_{te} / k^2\right) \]  \hspace{1cm} (11)

Where \( e_{te} = (y_i - f_i) / Q_i^{0.5} \) is the standardized prediction error, \( k \) is standard deviation of \( e_{te} \), in this paper, \( k = 3 \).

4. Practical engineering verification

4.1. Engineering Background

Taking a 20-year-old reinforced concrete continuous girder bridge as an example, during its 20-year service period, the bridge is inspected regularly one year after opening to traffic, and then every three years. The bridge evaluation score is as follows: Table 1.

| Age | Score | Age | Score |
|-----|-------|-----|-------|
| 1   | 95    | 10  | 90    |
| 4   | 93    | 13  | 85    |
| 7   | 91    | 16  | 80    |
| 19  | 75    |

As Bayes updating method needs many sample data, which can be predicted according to the mean, variance and distribution of samples as parameters. In this case, it is expected to predict the technical status of a bridge, so many statistical data like bridges are needed to support it, in order to get more regular results.

| Age | Mean of score | Coefficient of variation | Distribution |
|-----|---------------|--------------------------|--------------|
| 1   | 96.67         | 0.55                     | Normal       |
| 4   | 94.37         | 0.7                      |              |
| 7   | 91.89         | 0.8                      |              |
| 10  | 89.23         | 0.94                     |              |
| 13  | 86.39         | 1.17                     |              |
| 16  | 83.37         | 1.27                     |              |
| 19  | 80.17         | 1.15                     |              |

Then, Combine the existing 302 non-overhauled bridges within 20 years of bridge technical status score statistics, as shown in Table 2. Among them, the mean data is used to build a priori model without information, to represent the cognition of Bayes dynamic model to the degradation of bridge technical condition without data; the coefficient of variation is used to fill the missing observation error term of observation data.

4.2. Prediction of Bridge Technical Status Score Based on Bayes Dynamic Model

4.2.1. Priori model

Without any information, a prior model based on experience or statistical data can be established. In this case, the prior model based on the statistical data of the bridge score of the periodic inspection (JTG H21-2004) is shown in Figure 2.

4.2.2. Posteriors model

The posteriori model is shown in Figure 3.
Fig. 4 shows the result of model updating. The Bayes dynamic model with quadratic growth can adapt to the data with non-linear change very well. In addition, the final update results can better describe the degradation law of technological status than the prior model. When the bridge score is between 40 and 60, the bridge will be divided into four types of bridges, and the four types of bridges need to be overhauled or renovated. Therefore, the performance threshold of this paper is the upper bound of the four types of bridges. As can be seen from Figure 4, bridges will degenerate to four types of bridges in the vicinity of the 27th year, and preventive maintenance can be carried out before 27 years to avoid overhaul or transformation.

4.3. Bayes Factor

It can be seen from Fig. 5 that the posterior Bayes factor of the observed data is higher than that of the prior Bayes factor, reflecting that the updated model is more in line with the actual situation.

5. Conclusions

- Bridge technical status score is used as bridge performance index, which fully combines the existing evaluation system and can be directly used in decision support of preventive maintenance of bridges.
- The changes of Bayes factors before and after the updating of the dynamic model are compared and analysed. The results show that the posterior Bayes factors are improved in varying degrees compared with the prior ones, reflecting that the updated model is more in line with the actual performance degradation of bridges.
The object of study in this paper is the scoring data of bridge technical condition without overhaul or transformation, which has strong regularity. For the weak regularity data, the applicability of Bayes dynamic model needs to be studied.

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