Research on phase diagram of traffic flow in a single loop roundabout under open boundary condition

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Abstract. The cellular automation traffic flow model of roundabout with inner-roundabout-lane, injection-roundabout-lane and extraction-roundabout-lane is used to study the traffic flow of single loop roundabout under open boundary conditions. Through computer simulation, the relationships between the entrance probability, exit probability, vehicle braking probability and system flow, the average speed of each lane and system in the open boundary conditions are studied. Besides, the space-time diagram of vehicles on the injection-roundabout-lane with different entrance exit probabilities and the system phase diagram are established. In the system phase diagram, the critical entry probability and exit probability values of each part of the phase diagram under different braking probability are given. The results show that the system traffic flow can be divided into three phases: the phase in which the system flow changes with the entrance probability, the phase in which the system flow changes with the exit probability, and the phase in which the system flow has nothing to do with the entrance probability and exit probability. This will provide some guidance for road management.

1. Introduction
In recent years, with more and more private cars, the increasing traffic flow can not meet the existing road traffic conditions. The resulting traffic congestion has become one of the most important and urgent problems in the process of smart city construction. These traffic jams not only attract people's attention, but also attract many research experts and scholars at home and abroad to analyse and study them. Among them, the most popular way to solve the problem of road traffic jam is to improve the scientific traffic planning and management mode. Because this can not only make full use of the existing transportation resources, but also improve the use efficiency of the existing facilities to the greatest extent. Therefore, these scholars pay more attention to the scientific theoretical research of traffic flow, and use the obtained traffic flow theoretical research results to guide the road planning and design, traffic flow control and management of actual traffic roads, and have achieved many satisfactory results of actual traffic flow control.

R. Marzoug et al. [1] used cellular automation model to simulate two-way intersections under open boundary conditions, and studied the traffic characteristics and probability of traffic accidents at two-way intersections. H. Echab et al. [2] studied a non signalized T-intersection under open boundary conditions using cellular automatic model. Among them, two different priority rules are introduced in the intersection to eliminate the congestion. The phase diagram and its variation with the left rear vehicle ratio are studied. In addition, the spatial, temporal and density distributions are also studied. R. Jorge L zapotecatl et al. [3] proposed a new method to realize self-organization and coordination of intersection traffic by using simple sensors based on cellular automation model. Xianyan Kuang and
Ziru Chen [4] analysed and studied the impact of the length of waiting area on traffic flow by establishing a cellular automation model of intersection with non-motor vehicle waiting area. Zhongbao Dai et al. [5] based on the optimization model of NaSch and BML coupling model, introduced the slow start rule, established the intersection cellular automation traffic flow model, and studied the intersection traffic flow model. Based on NaSch model, Dingqiang Yuan et al. [6] established a cellular automation model considering U-turn of vehicles at T-junction, and analysed the relationship between the position of U-turn lane and the flow of each lane. Kezhao Bai et al. [7] established a cellular automation traffic flow model of plane roundabout under open boundary conditions, and analysed the influence of boundary conditions on system flow and system phase diagram. Changsheng Zhu et al. [8] analysed and studied the effects of entrance probability, exit probability, maximum vehicle speed and braking probability on system flow based on cellular automation traffic flow Nash model under open boundary conditions, and studied the phase diagram of Nash model. Qilang Li et al. [9] established the traffic flow model of low-speed intersection composed of two single lanes, constructed the system phase diagram by using the local occupancy probability method, and gave the flow expression of each part of the phase diagram.

However, there are few studies on the traffic flow phase diagram at the single loop roundabout at home and abroad. Therefore, based on the work of the above scholars, this paper takes the single loop intersection as the prototype to establish the cellular automation model under the open boundary conditions, and studies the effects of the boundary conditions and braking probability on the traffic flow, the average speed of each lane and the system through computer simulation. Furthermore, the system phase diagram of the single loop roundabout is fully constructed and discussed.

2. Model and evolution rules

In the single loop roundabout section, considering the influence of drivers’ cautious driving characteristics (such as stable and cautious) and open boundary conditions, using cellular automation model, each lane is regarded as a one-dimensional discrete cell chain, and each cell is occupied or not occupied by a vehicle, so as to construct the traffic flow model of single loop roundabout as shown in Figure 1. Each direction of the model intersection is a two-way single lane, one of which is injection-roundabout-lane and the other is an extraction-roundabout-lane. Lane change is not allowed. The remaining four lanes at the intersection (Lane L9, Lane L10, Lane L11 and lane L12 respectively) are inner-roundabout-lane. Under the open boundary condition, in each time step, four vehicles traveling at the maximum speed enter injection-roundabout-lane in each direction with the entrance probability \( \alpha \), and participate in the vehicle evolution according to the operation rules of NaSch model. The
vehicles reaching the end of the extraction-roundabout-lane in each direction in each time step are removed from the system with the exit probability $\beta$.

The update rules of cellular automation model vehicles at single loop roundabout are as follows:
- Evolution rules of vehicles on each lane.

NaSch model is a typical one-dimensional traffic flow cellular automation model, which is suitable for traffic flow simulation of single vehicle road section, and the simulation results of the model are consistent with the actual situation, so it has been widely used. Here, the evolution rules of vehicles on each lane adopt the evolution rules of NaSch model. Suppose, $v_n$ represents the running speed of the $n$-th vehicle, $x_n$ is the position of the $n$-th vehicle on a single lane, $V_{\text{max}}$ donates the maximum speed of the vehicle, $P_d$ is the braking probability and $x_{n-1} - x_n$ represents the distance between the $n$-th vehicle and the vehicle in front. The model includes the following four steps to update rules.

Step 1 Acceleration: $v_n \rightarrow \min(v_n + 1, V_{\text{max}})$
Step 2 Deceleration: $v_n \rightarrow \min(v_n, x_{n-1} - x_n - 1)$
Step 3 Randomization: $v_n \rightarrow \max(v_n - 1, 0)$ with the braking probability $P_d$
Step 4 Motion: $x_n \rightarrow x_n + v_n$

- Rules for vehicles to meet at the merging points at an intersection.

For multiple vehicles about to reach the merging points, the vehicles that arrive at the merging points first have priority; if the time is equal, vehicles close to the merging points have priority; if the time and distance required for multiple vehicles to reach the merging points are the same, vehicles in the inner-roundabout-lane can take priority.
- Rules for merging vehicles at diverging points at intersections.

For a non right turning vehicle approaching or at the diverging points, if its speed at the current time step is 0, in order to effectively avoid the deadlock state and simulate the situation that the vehicle changes out of the ring intersection when it is blocked at the intersection, it will turn right and drive into the out of the ring lane according to a certain turning probability $P_r$.

3. Numerical simulation results and analysis

We assume that $L_i$ represents the total number of discrete cells in the $i$-th lane, $N(t)$ is the total number of vehicles in the $i$-th lane at time $t$. So the system average speed $V(t)$ and system traffic flow $J(t)$ are as follows:

$$V(t) = \frac{1}{\sum_{i=1}^{12} N(t)} \sum_{i=1}^{12} \sum_{j=1}^{N_i(t)} V(t)$$

(1)

$$J(t) = \frac{\sum_{i=1}^{12} N_i(t) / L_i}{\sum_{i=1}^{12} N(t)} \sum_{i=1}^{12} \sum_{j=1}^{N_i(t)} V(t)$$

(2)

In the computer simulation, the number of cells in the injection-roundabout-lane and extraction-roundabout-lane which are in the four directions at the intersection is 100. The number of cells in the four inner-roundabout-lanes is 5, and the maximum speed of the vehicle is set to 3, that is, the vehicle can travel up to 3 cell distances each time. At the same time, the probability that two vehicles on the approach Lane choose the other three exits is $1 / 3$ and $P_r = 0.25$. Run 50000 time steps for each
sample, numerical simulation results of the next 10000 time steps are averaged in time and then take 10 samples for system average to eliminate the influence of randomness.

3.1. Influence of the entrance probability on system flow and average speed of each lane and system

As is depicted in Figure 2, when $P_d$ and $\alpha$ are constant, the system flow will first increase with the increase of $\alpha$, and then remain unchanged. This shows that the system flow is only affected by the entrance probability in a certain range. As shown in Figure 3, the average speed of each lane and system is higher when $0.3 \leq \alpha$. Moreover, the speed can ensure the smooth road of the roundabout at this time. When $\alpha > 0.3$, the average speed of each lane and system does not change significantly with the increase of the entrance probability, and is in a stable state. The speed of the injection-roundabout-lane and the inner-roundabout-lane is lower, in a relatively congested state. Nevertheless, the speed of the extraction-roundabout-lane is still high, in an unblocked state, indicating that the unblocked state of each lane is not the same when $\alpha$ is the same.

3.2. Influence of the exit probability on system flow and the average speed of each lane and system

As is described in Figure 4, when $P_d$ and $\beta$ remain unchanged, the system flow increases with the increase of $\beta$, and then remains unchanged. It shows that the exit probability only affects the flow change in a certain range. Meanwhile, when $P_d = 0.2$ and $\alpha \geq 0.4$ the relationship between system flow
and $\beta$ is consistent and is not affected by the change of $\alpha$. From Figure 5, when $0 < \beta < 0.73$, the average speed of each lane and system increases with the increase of $\beta$. The speed of the injection-roundabout-lane and the inner-roundabout-lane remains stable and does not change with the increase of $\beta$ when $\beta \geq 0.73$. At this time, the average speeds of these lanes are small, indicating that they are in a relatively congested state. However, the average speed of the extraction-roundabout-lane is still large, so the extraction-roundabout-lane is more conducive to vehicle traffic than other lanes, which indicates that when $\beta$ is the same, the unblocked state of each lane is also different.

3.3. Discussion on the space-time diagram of vehicles on the injection-roundabout-lane under different entrance probabilities

As can be seen from Figure 6, when $\alpha$ approaches the critical value 0.3, congestion occurs only in the 30 cells connected to the inner-roundabout-lane. However, the blocking area increased significantly when $\alpha$ is greater than the critical value, and it leads to congestion on the whole injection-roundabout-lane. This is due to the fact that the speed of the vehicle in the system is low and has been saturated with the increase of $\alpha$.

3.4. Discussion on the system phase diagram

As can be seen from Figure 7, the system phase diagram shows the different states of the system under different $\alpha$ and $\beta$. The phase diagram can be divided into three regions: I, II, and III. In region I, the system is in a free state. In region II, the system is in a partially congested state, and in region III, the system is in a heavily congested state.
The system phase diagram shown in Figure 7 can be made through the influence on the entrance probability, exit probability, system flow and the average speed. Among them, zone I represents the phase whose flow is independent of the change of $\alpha$ and determined with the determination of $\beta$, and the system flow can be improved by increasing $\alpha$ in this region. Zone II represents the phase whose flow is independent of the change of $\beta$ and determined with the determination of $\alpha$, and the system flow can be improved by increasing $\beta$ in this region. Zone III represents the phase whose flow is independent of the changes of $\alpha$ and $\beta$. At the same time, as can be seen from Figure 7, when $P_d$ becomes larger, the area of region I of the phase diagram becomes larger and the area of region II decreases. This shows that when $P_d$ becomes larger, each lane is more likely to be blocked, which will make $\beta$ have a more significant impact on the system flow.

4. Conclusion
Single loop roundabout is a common non signal control road intersection in daily life. It is loved by people because it can reduce the waiting time caused by signal lights. However, when the traffic flow is large, the congestion problem has been perplexing us. Through computer simulation, it is concluded that both $\alpha$ and $\beta$ affect the system flow within a certain range, and the critical value of $\alpha$ at the intersection is small, which shows that the single loop roundabout is more suitable for the intersection with small traffic flow to a certain extent, so as to ensure the smoothness of the road. Simultaneously, the phase diagram of the system is obtained through analysis and simulation, and change the specific values of $\alpha$ and $\beta$ as needed to adjust the system flow and provide reference for actual traffic control, such as $\alpha = 0.3$ and $\beta \geq 0.73$ can ensure the maximum flow of the intersection system and keep each lane unblocked.

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References
[1] Rachid Marzoug,Hicham Echab,Noureddine Lakouari,Hamid Ez-Zahraouy.Car accidents at the intersection with speed limit Zone and Open Boundary Conditions[J].Cellular Automata,2016,9863:303-311.
[2] H.Echab,N.Lakouari,H. Ez-Zahraouy,A.Benyoussef.Phase diagram of a non-signalized T-shaped intersection[J].Physica A: Statistical Mechanics and its Applications,2016,461:674-682.
[3] Jorge L Zapotecatl,David A Rosenblueth,Carlos Gershenson.Deliberative self-organizing traffic lights with elementary cellular automata[J].Complexity,2017,2017:1-15.
[4] Xianyan kuang, Ziru Chen.Traffic flow model of signalized intersection with non-motor vehicle waiting area[J].Journal of Transportation Systems Engineering and Information Technology,2019,19(4):179-186.
[5] Zhongbao Dai,Weifang Li,Haixin Lin,Weibin Hu.Traffic flow model at crossroad and application[J].Journal of Shantou University(Natural Science),2018,33(1):49-56.
[6] Dingqiang Yuan,Mingmin Guo,Peng Zhang,Cellular automaton for U-turn vehicles at T-crossing[J].Journal of Shanghai University(Natural Science edition), 2020. 26(4):595-605.
[7] Bai Kezhao, Kuang Hua, Liu Muren, Kong Lingjiang. Study on phase diagram of traffic flow at circular intersection under open boundary conditions [J]. Acta physica Sinica,2010, 59 (9): 5990-5995.
[8] Changsheng Zhu,Bing Wang,Junqiang Huang,Jie Wang.Study on phase diagram based on NS model[J].Computer Engineering and Applications,2014,50(21):48-51.
[9] Qilang Li,Xiaoyan Sun,Binghong Wang,Muren Liu,Phase diagrams of the crossroad traffic model with low velocity vehicles[J]. Acta physica Sinica,2010, 59(9):5996-6002.