A Cell Library Methodology for Co-Design of MEMS Package-Device

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Abstract. Results from both experimental and numerical studies have already shown the significant package effects on the performance and the reliability of MEMS devices. To enable the theoretical predictions of these packaging effects, a compact joint model of an entire packaged MEMS device is established based on the Cell Library Concept and the Nodal Analysis Method. This modeling methodology greatly simplifies the package-device co-design work compared with the conventional numerical ways, and its precision is guaranteed due to its clarity in the physical nature.

1. Introduction

The knowledge in device-level modeling of MEMS systems has become increasingly mature with the intensive development and research of conventional MEMS devices. Since MEMS are intrinsically sensitive to the structural stresses, thermal mismatch between the coupled package-device structures will have a significant impact on the performance and reliability of the devices, i.e. the thermally induced package effects. Many works have validated this kind of effects through FEM simulations and experimental observations [1-4]. Some of them managed to optimize quantitatively the packaged devices through careful calibrations [3,4]. By far, however, there still lacks a compact theoretical method to characterize the package-level behaviors of the packaged systems. To obtain a theoretical solution for the modeling of MEMS package-device systems, we have to notice some other related works. Rabinovich[1,5] first applied the co-design idea to FEM simulations by partitioning the packaged structure into the device cell and the substrate cell. Kobrinksy [6] further extracted the anchor cell from the device in his study on the buckling behavior of a doubly clamped microbeam. These works have already involved the modeling idea of the novel Cell Library Concept although they are still not enough for an integrated study of the entire package-device system.

This paper presents a complete joint model of the packaged MEMS system and theoretically solves the coupling problem between the package and device structure. The whole packaged system are first partitioned into three major cells, the device, the anchor and the substrate (including the chip, adhesive and the package substrate). Each cell is then modeled individually according to their specific natures and properly combined together to enable the package-level simulations of the device performance. Examples of the package effects on the major parameters of a doubly clamped microbeam are presented to demonstrate the application of this modeling methodology. This methodology is expected to simplify the numerous calculations in conventional FEM simulations and improve the predictability and the efficiency of the design tools.
2. Modeling of the Package-Device System

A conventional packaged MEMS system could be simplified as the structure depicted in Fig.1. The MEMS device is constrained to the chip layer by the anchor. The chip layer and the package substrate layer are joint together using a certain kind of adhesive material, like the epoxy, the Au-Si eutecticum and so on. Such heterogeneous structure will experience thermoelastic strains under thermal loads and introduce additional geometric deformations along the top surface of the chip layer, which in turn affects the responses of the device significantly. This phenomenon is well known as the thermally induced package effect and has been validated by numerous experiments and simulations. Despite of all of these validations, there still lacks a compact theoretical framework to investigate the coupling behaviors between the package and the device system, which hinders the efficiency and the accuracy of the design work. Starting from the mature modeling method of conventional MEMS devices, we try to extend the valuable Cell Library Concept and the compact Nodal Analysis Method to a more general domain in which new cells of the anchor and the substrate are further included as well as the existing device cells. The modeling of these new cells will be specified as followed and an effective way of their combinations into a joint model of the packaged system will also be introduced.

2.1. Modeling of the Substrate Cell

Fig.2 shows the 2D model of the conventional substrate cell, namely a die bonding structure with an adhesive interlayer between the chip layer and the package substrate layer. Assume the structure undergoes a small, elastic deformation as the environmental temperature changes, our objective is to obtain the precise distribution of the elastic strain along the top surface of the chip layer.

Timoshenko [7] first considered this kind of multilayered thermoelastic problem and gave a compact and precise closed-form solution for stacked membranes. But his solution cannot deal with the strain distribution of either the edge area, or the high aspect ratio layers. Chen [8] and Suhir [9] modified his method and proposed the adhesive assumption and the interface compliance assumption respectively to handle the edge area issue. Other works followed their assumptions. However with their goals to predict the magnitude of the interfacial stress, these works are not totally suitable for the precise estimation of the strain distribution along the chip surface. Chen’s model is adopted as the basis of our study and proper modifications are made to obtain an analytical solution for the surface strain.

In the Cartesian coordinate system in Fig.2, the fundamental equations are established by treating the chip and the substrate layers as classical beams, and the adhesive layer as a simplified elastic entity. The contribution of the axial stress to the bending stiffness is neglected due to the relatively large rotary inertia of the bonding structure.

The equilibriums of the force and the moment of each layer are given by

\[
E_i\frac{d^2\bar{\epsilon}(x)}{dx^2} = \frac{dF_i(x)}{dx} = (-1)^{i-1}\frac{\tau_i(x)}{h_i}
\]  

(1)
The horizontal strains at the interface of each layer are given by

\[-E_i' \frac{d^2 v_i(x)}{dx^2} = \frac{dM_i(x)}{dx} = Q_i(x) - \tau_i(x) \frac{h_i + h_{i+1}}{2} = (-1)^{i-1} \int \sigma(x) dx - \tau_i(x) \frac{h_i + h_{i+1}}{2}\]

(2)

The horizontal strains at the interface of each layer are given by

\[\frac{d u_i(x)}{dx} = \frac{d \tau_i(x)}{dx} + \frac{d}{dx} \int (-1)^{i-1} \frac{d^2 v_i(x)}{dx^2} h_i \frac{h_i + h_{i+1}}{2} + \alpha \Delta T\]

(3)

Neglecting the thermal expansion of the adhesive layer, the displacement continuity at the interface of the layers is described as

\[u_i(x) - u_i(x) = k_i \tau_i(x) - \frac{h_i}{G_i} \tau_i(x)\]

(4)

\[w_i(x) - w_i(x) = k_i \sigma_i(x) - \frac{h_i}{E_i} \sigma_i(x)\]

(5)

And the boundary conditions at the free edge are

\[F_i \left( \frac{L_{sub}}{2} \right) = 0 \quad M_i \left( \frac{L_{sub}}{2} \right) = 0 \quad Q_i \left( \frac{L_{sub}}{2} \right) = 0\]

(6)

where \( E' = E_i / (1 - v_i^2) \), \( G_i = E_i / [2(1 + v_i)] \). \( E_i \) is the Young’s modulus, \( h_i \) is the thickness of the beam layer, \( \alpha_i \) is the thermal expansion coefficient, \( u_i \) is the horizontal displacement at the interface, \( \tau_i \) is the axial displacement, \( v_i \) is the vertical displacement, \( F_i \) is the horizontal force, \( Q_i \) is the shear force and \( M_i \) is the bending moment. Subscript \( i = 0,1,2 \) corresponds to the adhesive layer, the chip layer and the package substrate layer, respectively. \( k_i \) and \( k_i \) represent the equivalent horizontal and vertical interfacial compliance. \( \tau_i \) and \( \sigma_i \) are the interfacial shear stress and peeling stress.

Rearrange Equations (1) to (6) into a seventh-order ordinary differential equation of shear stress \( \tau_0 \), from which an analytical solution can be obtained upon the boundary and symmetric conditions. From this solution, the other concerned results can be derived including the peeling stress, the displacements and the strain distribution along the chip surface, which is denoted by

\[e_{\tau_0}(x) = \frac{du_0(x)}{dx} = \frac{d \tau_0(x)}{dx} - \frac{d^2 w_0(x)}{dx^2} \frac{h_0}{2}\]

(7)

The thermal expansion of the adhesive layer can also be taken into account by modifying the boundary conditions as

\[F_0 \left( \frac{L_{sub}}{2} \right) = -\frac{F_0}{2} \quad M_0 \left( \frac{L_{sub}}{2} \right) = (-1)^{-i-1} \frac{F_i}{2} \frac{h_i + h_{i+1}}{2} \quad Q_0 \left( \frac{L_{sub}}{2} \right) = 0\]

(8)

where \( F_0 = \frac{\alpha_0 + \alpha_i - 2 \alpha_{i+1}}{2} \Delta T \left( \frac{1}{E_i h_i} + \frac{1}{E_j h_j} + \frac{1}{E_k h_k} \right) \)

Compared with Chen’s model, this modified model adjusts the elastic behavior of each layer and adds the consideration of the thermal expansion of the adhesive layer. It is easily shown that these modifications increase the validity and veracity of the solution by correlating the model with the Timoshenko model in the middle area (i.e. \(|x| < L - 1.5h\)) of the structure. Similar methods have been expanded to the axisymmetric coordinate system to estimate the 3D results, which can be treated as the exact solution in the middle area of a large die, and the first-order approximation of a small die.

2.2. Modeling of the Anchor Cell

An anchor provides the mechanical connection between the device and the substrate. It transfers the substrate deformation to the device boundary with a certain magnitude of compliance. In MEMS, there are two conventional types of anchor, the surface-micromachined step-up anchor and the bulk-
micromachined wedge-like anchor, as shown in Fig. 3(a) and Fig. 3(b), respectively. The modeling of the step-up anchor will be illustrated as followed.

As for the step-up anchor, we follow the similar modeling method used by Kobrinsky [6], but make some improvements. As in Fig. 3(a), let the substrate undergo a small tension with the strain $\epsilon_{\text{sub}}$, and the governing equations for the joint structure under this tension can be formulated as

$$
\epsilon_{\text{sub}} = \frac{P}{E_b t^3/12} \left[ S_1, S_2, S_3 \right] \left[ P \right] = \left[ \frac{P L_b}{2 E_b t} \right] + \left[ M L_b \right] + \left[ \epsilon_{\text{sub}} \times \frac{L_b}{2} \right]
$$

where $E_b = E_b/(1 - \nu_b)^2$, $E_b$ and $\nu_b$ are the Young’s modulus and Poisson ratio of the beam material. $I_b = t^3/12$ is the rotary inertia of the beam, $[A_a]$ is the compliance matrix of the anchor. $S_1 \sim S_3$ are the undetermined matrix element. According to the Reciprocity Theorem the matrix must be symmetric. The force $P$ and moment $M$ at node $J$ constitute the generalized force $[Q]$. And the translational DOF $u_J$ and rotational DOF $\phi_J$ at node $J$ constitute the generalized displacement $\{q_J\}$. The maximum deflection of the doubly-clamped beam can be derived form Eqs. (9) as:

$$
\varepsilon_{\text{max}} = \frac{3 \epsilon_{\text{sub}}}{4 t^3} \frac{L_b^3}{3 S_1 + 6 S_2/2 S_3} + \frac{S_4 S_5}{S_3} + S_2
$$

When varying $L_b$ under a certain magnitude of $\epsilon_{\text{sub}}$, a set of FEM simulation data about the dependence of $\varepsilon_{\text{max}}$ on $L_b$ can be obtained. The Least Square Method is adopted to extract the fitting parameters $S_1 \sim S_3$ from the calculated data. This modeling approach differs from the Kobrinsky model in that it tries to figure out the compliance matrix of the anchor from the behavior of the beam, and involves the connection compliance between the anchor and the substrate. These have altogether improved the credibility and veracity of the modeling method.

As for the wedge-like anchor, a similar method can be applied to extract another compliance matrix numerically. However, we can also implement the basic analytical solutions about the wedge-like entities in Elastic Mechanics to obtain the analytical form of the compliance matrix. This modeling method has already been demonstrated and the results agree well with the numerical one.

2.3. Combination of the joint model
Since the thickness of the device are usually far less than the thickness of the substrate, the influence of the device on the deformation of the substrate can be reasonably neglected. Hence the substrate deformation $\{q_b\}$ can be expressed as the sum of the anchor deformation $\{q_a\}$ and the device deformation $\{q_d\}$.
\{q_i\} = \{q_i\} + \{q_i\} = [A_i]{Q} + [A_i]{Q}

(11)

The substrate deformation can be calculated from the substrate model, namely the Eqs.(2) and the Eqs.(7). It includes the average normal strain and the average curvature along the chip surface

$$
\varepsilon_n = \frac{1}{L} \int_0^L \varepsilon_n \kappa(x) \, dx \\
\kappa_n = \frac{1}{L} \int_0^L \kappa(x) \frac{d^2 w_i(x)}{dx^2} \, dx
$$

(12)

where \(x_0\) is the central location of the device. \([A_i]\) represents the compliance matrix of the device, which can be derived numerically or analytically from the mature mechanics theory. This joint model can be used to calculate the package-level response of the device system with much less time and reasonable accuracy.

3. Case Studies

The structure of a doubly clamped beam is widely used in various fields of MEMS applications, including the RF switch, the inertia device and the test structure. Pull-in voltage is one of the most important design parameters of this structure. Here we use the joint model to study the package effects on the pull-in voltage as an illustration for the modeling concept. The structure of the joint model is also shown in Fig.3(a).

Considering the stress-stiffening effect, the Eqs.(11) can be expanded into a detailed expression as:

$$
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\phi_0
\end{bmatrix} = \frac{1}{E'} \begin{bmatrix} S_1 & S_2 & P \\ S_2 & S_3 & M \\ P & M & \text{tr}(D)
\end{bmatrix} \begin{bmatrix} P \\ qL \frac{E}{E'} \frac{1}{2} \left( \frac{d v(x)}{dx} \right)^2 \end{bmatrix} + \begin{bmatrix} \frac{P}{2} \frac{E}{E'} \frac{1}{2} \left( \frac{d v(x)}{dx} \right)^2 \\ \frac{qL}{2P} \left( \frac{q}{k_0} \left( \frac{12M}{E' P^2} \right) \tanh(k_0 \frac{L}{2}) \right) \end{bmatrix}
$$

(13)

where \(k_0 = \frac{12P}{E' P^2} \), \(\phi_0 = -\kappa_n \frac{L}{2}\) represents the initial slope of the anchor under the substrate bending, \(\varepsilon_x\) represents the process induced residual strain of the beam, and \(v_q(x)\) is the vertical deflection of the beam, formulated by:

$$
v_q(x) = \frac{qL^2}{8P} \frac{1}{k_0} \left( \frac{q}{P} - \frac{12M}{E' P} \right) - \frac{q^2}{2P} + \frac{1}{k_0} \left( \frac{q}{P} \left( \frac{12M}{E' P^2} \right) \cosh(k_0 \frac{L}{2}) \right) \cosh(k_0 \frac{L}{2})
$$

(14)

Iterations are carried out between the Eqs.(13) and (14) till the converged solutions of \(P\), \(M\) and \(v_q(x)\) are obtained.

Estimation approach for the pull-in voltage \(V_{th}\) using the joint model is as follows. First an equivalent initial stress \(\sigma_{eq}\) is obtained by solving the stress value in the packaged beam near the pull-in location. Next an equivalent spring constant \(K_{eq}\) is extracted numerically by substituting the equivalent initial stress into the basic Eqs.(13) and (14) and solving the load-deflection relationship within the linear, small deformation domain. The obtained equivalent spring constant is then substituted into a closed-form expression to calculate the pull-in voltage after the package process

$$
V_{th} = \sqrt{2.95K_{eq} g_0^3 \over \varepsilon_0 (1 + 0.42g_0/w)}
$$

(15)

where \(\varepsilon_0\) is the dielectric constant and \(w\) is the width of the beam. Eqs.(15) modifies the Osterberg’s pull-in voltage formula[10] for ideally clamped beam by introducing the equivalent spring constant and revising the fitting coefficient. The modified formula expands the previous assumptions of rigid substrate, rigid anchor and small deformation, and is more applicable for the doubly-clamped beam fabricated with the conventional surface-micromachined process.

Fig.4 shows the simulated package effects on the pull-in voltage. It is clearly seen that the effects of the package process on the device response is the combination of multiple factors. Omission of
these factors will reduce the accuracy, repeatability and predictability of the designed performance, making it difficult for the testing and calibration of the packaged devices. It is relevant to mention that the epoxy-adhering technique on the FR4 substrate, which is often adopted in the laboratory research, has actually a larger and more ambiguous effect on the device response due to the greater thermal mismatch and the smaller bending stiffness of the coupled package-device structure.

The Cell Library Method presented in this paper provides a concise and flexible way to simulate the package-device co-design problem. New cells of joint model can be added and modified for various applications and the modeling of each cell can be optimized analytically or numerically considering the practical requirements.

![Fig.4 Package effects on the pull-in voltage of the doubly clamped beam with respect to (a) device location; (b) ambient temperature. Soft bonding indicates epoxy-adhesive bonding on FR4 substrate; hard bonding indicates Au-Si eutectic bonding on ceramic substrate.](image)

4. Conclusions
In this paper, a joint model based on the Cell Library Method is presented to theoretically solve the MEMS package-device co-design problem. Package effects on the pull-in voltage of a doubly clamped micro-beam are studied using this method. Results show that the packaging effects are both significant and complicated. Neglecting or misestimating these effects in the device design will decrease the repeatability and predictability of the responses of the packaged devices and increase the difficulty of the calibration.

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