Investigations on flexural wave propagation of a grillage structure using the multi-reflection method

Yanqiu Li¹, Runxin Zhou¹, Liangmei Liu¹ and Tao Chen¹*

¹College of Science, Harbin Engineering University, Harbin 150001, P.R. China
*Corresponding author, E-mail: taochen@hrbeu.edu.cn.

Abstract. The flexural wave propagation in a grillage structure with a propagating disturbance is studied firstly by the use of the multi-reflection method. The grillage structure consists of many beams intersecting each other at right angles. Each periodic binary beam consists of a finite repetition of alternating material A and material B. A propagating wave is incident upon a discontinuity of a beam along the x axis and gives rise to transmitted and reflected waves. Here all of the transmitted and reflected waves of given flexural wave incident upon the beam at some specified location are found and superposed, these waves travel to the other beams as a bending wave in the direction of their partially reflected and partially transmitted. Then the wave-fields of two beams along the y axis closest to external disturbance is set up, these two beams only need to consider the influence of bending waves on the x-beams on them. Next the wave-fields of two beams along the x axis closest to external disturbance is set up, the rest can be done in the same manner. Finally the wave-field of any beam at some specified location can be determined.

1. Introduction
Vibration is a common phenomenon in nature and engineering. In recent years, with the research and development of Engineering technology, vibration has been paid more and more attention in various engineering fields [1]. Grillage structure has strong stability, obvious weight reduction effect, high specific strength and stiffness [2]. It is well known that the study of structural vibration and noise control is of great significance to its engineering applications [3;4]. How to control harmful vibrations has become an urgent problem to solve. In the fields of transportation, aerospace and so on, the structural vibration and noise processing are very demanding. For example, when the satellite launch without vibration control will affect the life and even affect the stability of motion. Periodic grillage structure on the vibration and noise control has attracted the attention of researchers. In recent years, the periodic structure of materials and geometric parameters for elastic wave propagation has become a research hotspot.

In a continuous structure, vibration can be regarded as the superposition of waves traveling through the structure. The dynamic behavior of periodic grillage structures may be described in terms of waves and their propagation, reflection and transmission [5-9]. Therefore, the research method of periodic grillage structure may be used to investigate band gap characteristics of periodic grillage structures. and then, wave propagation characteristics may be studied based on the multi-reflection method [9-13]. This method and its applications are discussed in this paper.

This paper is organized as follows, A periodic grillage structure is adopted. And the results of injecting a propagating wave is concerned. All of the transmitted and reflected waves of flexural wave incident upon the grillage structures at some specified location are found and superposed using the multi-reflection method. The band structures of an infinite periodic grillage structure are presented by
the Bloch theorem. The relations of the wave-field of the grillage structure on adjacent identical elements are expressed by multi-reflection method and the Bloch theorem. The influences of the density and lattice constant on the band gaps of the grillage structures are considered. Finally, we list the conclusions from this study.

2. The multi-reflection method
As shown in figure 1, the grillage structure consists of many beams intersecting each other at right angles. Each periodic binary beam consists of a finite repetition of alternating material A with length \(a_1\) and material B with length \(a_2\). So the periodic beam’s lattice constant is \(a = a_1 + a_2\).

![Figure 1. The grillage structure.](image)

As the beginning to dealing with more complicated case, one may first study the wave-field setup between two discontinuities due to multiple reflections. According to the reference [11], we first give the relationship between the external excitation and the incident amplitude.

The system consists of a finite beam intersecting each other at right angles. The structure is divided into several parts to study. Firstly, as shown in Figure 2(a), the propagation characteristics of the elastic wave x-beam are first studied. Since the propagation characteristics of the elastic wave around the nearest beam are prioritized. Then the propagation characteristics of elastic wave vertical to the x beam are analyzed as shown in Figure 2 (b). Next, analyze the propagation characteristics of the elastic wave perpendicular to the x beam in Figure 2(c), at this time, both the Figure 2(b) and the Figure 2(c) need only consider the influence from the x beam. Then analyze the propagation characteristics of the elastic wave as shown in Figure 2(d). The x1 beam needs to consider the effect of the beam shown in Figure 2(c) on it. Next, the influence of the two beams shown in Figure 2(c) on the x(-1) beam is analyzed. The rest can be done in the same manner. Finally the wave-field of any beam at some specified location can be determined.
2.1. Wave reflection and transmission on structure 1

As shown in Figure 2(a) an incident wave is given in material A of the segment (0). The incident waves are described by

\[ w_{im} = e^{-ik_{im}x} \]  

where \( k_{ix} = k_{y} = \left( \left( \rho S \right)/\left( El \right) \right)^{1/4} \), \( k_{x} = k_{y} = \left( \left( \rho S \right)/\left( El \right) \right)^{1/4} \), \( k_{x} \) and \( k_{y} \) are the bending wave numbers in the x and y beam, \( r \) and \( r' \) is the reflection coefficient \cite{16}. \( \rho \) denotes Young’s modulus, I the area moment of inertia, \( f \) frequency, \( \rho \) the density, S the cross-sectional area.

A set of positive-going waves produced at segment (0) and propagating in the positive x direction will give rise to transmitted and reflected waves when it impinges on discontinuity (1). Using the multiple reflection method to calculate, the process of calculating the wave fields of a single beam can be found in the reference, and is not repeated here. Multiple reflections produce a total wave \( W^{(0)}_{na+} \) and a total wave \( W^{(0)}_{na-} \) on segment (0) by superposition, where

\[ W^{(0)}_{na+} = \frac{re^{ik_{ix}x} \cdot e^{-2ik_{x}(l+n)}}{1 - rr'e^{-2ik_{nx}l}} \]  

\[ W^{(0)}_{na-} = \frac{e^{-ik_{ix}x}}{1 - rr'e^{-2ik_{nx}l}} \]  

where \( r \) and \( r' \) is the reflection coefficient. It can be seen from the literature that the reflection coefficients from part A to part B and from part B to part A are the same, so they are all expressed by \( r \), \( r' \) is the reflection coefficients at the intersection of two beams, \( l \) is the distance between the origin of the x system and the discontinuity (0).

Calculation, the initial approximate wave field of each section of the x beam can be expressed as follows:

\[ w_{na+}^{(i)}(x) = \sum_{i=0}^{\infty} w_{na+}^{(i)}(x) \quad w_{na+}^{(i)}(x) = \sum_{i=0}^{\infty} w_{na+}^{(i)}(x) \]  

\[ w_{na-}^{(i)}(x) = \sum_{i=0}^{\infty} w_{na-}^{(i)}(x) \quad w_{na-}^{(i)}(x) = \sum_{i=0}^{\infty} w_{na-}^{(i)}(x) \]  

\[ w_{(n-1)a+}^{(i)}(x) = \sum_{i=0}^{\infty} w_{(n-1)a+}^{(i)}(x) \quad w_{(n-1)a+}^{(i)}(x) = \sum_{i=0}^{\infty} w_{(n-1)a+}^{(i)}(x) \]  

\[ w_{(n-1)a-}^{(i)}(x) = \sum_{i=0}^{\infty} w_{(n-1)a-}^{(i)}(x) \quad w_{(n-1)a-}^{(i)}(x) = \sum_{i=0}^{\infty} w_{(n-1)a-}^{(i)}(x) \]  

(3)
A propagating wave is incident upon a discontinuity of a beam along the x axis and gives rise to transmitted and reflected waves. Here all of the transmitted and reflected waves of given flexural wave incident upon the beam at some specified location are found and superposed. These waves travel to the other beams as a bending wave in the direction of their partially reflected and partially transmitted.

2.2. Wave reflection and transmission on structure 2
We start out with a description of the vibrations of only two intersecting simple beams. For the sake of simplicity, we assume the connection to be rigid and the angle between the beams to be 90°.

2.2.1. Multiple reflections between two discontinuities. As shown in the Figure 2(b) the calculation the bending waves of transmission to part B of y beam can be described by

\[ W_{y} = \frac{d_{y}e^{-ik_{y}a} + d_{y}d_{e}e^{-2ik_{y}a}r_{e}^{-2ik_{y}(l+2a)}}{(1 - rr)e^{-2ik_{y}a} + (1 - rr)e^{-2ik_{y}a}r_{e}^{-2ik_{y}(l+2a)}} \]

where \( d \) and \( d_{y} \) transmission coefficients for bending waves [15], and \( d_{y} \) is the transmission coefficients at the intersection of two beams, \( d_{y} \) and \( d_{y} \) were expressed by x beam transmission to the transmission coefficient of y beams.

Then at the discontinuity (1), a reflected component \( W_{y} \) is generated

\[ W_{y_{1}} = \frac{r_{e}d_{y}e^{-ik_{y}a}e^{-2ik_{y}(l+2a)}}{(1 - rr)e^{-2ik_{y}a} + (1 - rr)e^{-2ik_{y}a}r_{e}^{-2ik_{y}(l+2a)}} + \frac{d_{y}d_{e}e^{-2ik_{y}a}r_{e}^{-2ik_{y}(l+2a)}}{(1 - rr)e^{-2ik_{y}a} + (1 - rr)e^{-2ik_{y}a}r_{e}^{-2ik_{y}(l+2a)}} \]

The reflected portion \( W_{y_{1}} \) will travel in the negative y direction until it is incident upon discontinuity (0), and there will be transmitted and reflected. The reflected portion

\[ W_{y_{2}} = \frac{rr_{e}d_{y}e^{-ik_{y}a}e^{-2ik_{y}(l+2a)}}{(1 - rr)e^{-2ik_{y}a} + (1 - rr)e^{-2ik_{y}a}r_{e}^{-2ik_{y}(l+2a)}} + \frac{d_{y}d_{e}e^{-2ik_{y}a}r_{e}^{-2ik_{y}(l+2a)}}{(1 - rr)e^{-2ik_{y}a} + (1 - rr)e^{-2ik_{y}a}r_{e}^{-2ik_{y}(l+2a)}} \]

The reflected waves \( W_{y_{2}} \) will again travel in the positive x direction and will suffer the same situation as the initial incident waves. There will be transmitted and reflected on discontinuity (1). The reflected portion

\[ W_{y_{3}} = \frac{r_{e}d_{y}e^{-ik_{y}a}e^{-2ik_{y}(l+2a)}}{(1 - rr)e^{-2ik_{y}a} + (1 - rr)e^{-2ik_{y}a}r_{e}^{-2ik_{y}(l+2a)}} + \frac{d_{y}d_{e}e^{-2ik_{y}a}r_{e}^{-2ik_{y}(l+2a)}}{(1 - rr)e^{-2ik_{y}a} + (1 - rr)e^{-2ik_{y}a}r_{e}^{-2ik_{y}(l+2a)}} \]

So two set of waves will be result in the region between the two discontinuities. One consists of an infinite number of portions traveling in the positive x direction, and the other consists of a same number traveling in the negative x direction. Multiple reflections produce a total wave \( W_{y_{1}} \), and a total wave \( W_{y_{1}} \) on segment (0) by superposition, where

\[ W_{y_{1}} = \frac{d_{y}e^{-ik_{y}a} + d_{y}d_{e}e^{-2ik_{y}a}r_{e}^{-2ik_{y}(l+2a)}}{(1 - rr)e^{-2ik_{y}a} + (1 - rr)e^{-2ik_{y}a}r_{e}^{-2ik_{y}(l+2a)}} + \frac{rr_{e}d_{y}e^{-ik_{y}a}e^{-2ik_{y}(l+2a)}}{(1 - rr)e^{-2ik_{y}a} + (1 - rr)e^{-2ik_{y}a}r_{e}^{-2ik_{y}(l+2a)}} + \frac{d_{y}d_{e}e^{-2ik_{y}a}r_{e}^{-2ik_{y}(l+2a)}}{(1 - rr)e^{-2ik_{y}a} + (1 - rr)e^{-2ik_{y}a}r_{e}^{-2ik_{y}(l+2a)}} + \frac{rr_{e}d_{y}e^{-ik_{y}a}e^{-2ik_{y}(l+2a)}}{(1 - rr)e^{-2ik_{y}a} + (1 - rr)e^{-2ik_{y}a}r_{e}^{-2ik_{y}(l+2a)}} + \frac{d_{y}d_{e}e^{-2ik_{y}a}r_{e}^{-2ik_{y}(l+2a)}}{(1 - rr)e^{-2ik_{y}a} + (1 - rr)e^{-2ik_{y}a}r_{e}^{-2ik_{y}(l+2a)}} \]
According to current documents, If $r$ is a convergent matrix \([11]\) (i.e., $k \to \infty, r^k \to 0$), then

$$(rr) e^{-2ik_{2m+1}} e^{l+} \to 0 \quad n \to \infty,$$

so there is $$(rr) e^{-2ik_{2m+1}} e^{l+} \to 1 \quad n \to \infty.$$

Initial approximations of wave-field on segment (0) can be obtained. Equations (8) and (9) can be rewritten as

$$W_{y_{1nb-}}^{(0)} = \frac{d_{y}e^{-ik_{2m+1}}}{(1-rr e^{-2ik_{2m+1}})} \left(1-rr e^{-2ik_{2m+1}}\right)^2 + \frac{rd_{y}d_{x}e^{-ik_{2m+1}} \cdot e^{-2ik_{2m+1}}}{(1-rr e^{-2ik_{2m+1}})} \left(1-rr e^{-2ik_{2m+1}}\right)^2 + \cdots \quad (8)$$

$$W_{y_{1nb-}}^{(0)} = \frac{d_{y}e^{-ik_{2m+1}}}{(1-rr e^{-2ik_{2m+1}})} \left(1-rr e^{-2ik_{2m+1}}\right)^2 + \frac{rd_{y}d_{x}e^{-ik_{2m+1}} \cdot e^{-2ik_{2m+1}}}{(1-rr e^{-2ik_{2m+1}})} \left(1-rr e^{-2ik_{2m+1}}\right)^2 + \cdots \quad (9)$$

Above Section neglected transmitted waves across the boundaries of segment (0) in Figure 2(a). If waves are transmitted in only rightward direction, then wave-field established on segment (0) $W_{y_{1nb+}}$ will be transmitted past discontinuity (1), and the total transmission will suffer multiple reflections between discontinuity (1) and (2). Multiple reflections produce a total wave $W_{y_{1nr+}}^{(0)}$ and a total wave $W_{y_{1nr-}}^{(0)}$ on segment (1) by superposition, where

$$W_{y_{1nr+}}^{(0)} = \frac{dd_{y}d_{x}e^{-ik_{2m+1}}}{(1-rr e^{-2ik_{2m+1}})} \left(1-rr e^{-2ik_{2m+1}}\right)^2 + \frac{rd_{x}d_{x}d_{x}e^{-ik_{2m+1}} \cdot e^{-2ik_{2m+1}}}{(1-rr e^{-2ik_{2m+1}})} \left(1-rr e^{-2ik_{2m+1}}\right)^2 + \cdots \quad (10)$$

$$W_{y_{1nr-}}^{(0)} = \frac{dd_{y}d_{x}e^{-ik_{2m+1}}}{(1-rr e^{-2ik_{2m+1}})} \left(1-rr e^{-2ik_{2m+1}}\right)^2 + \frac{rd_{x}d_{x}d_{x}e^{-ik_{2m+1}} \cdot e^{-2ik_{2m+1}}}{(1-rr e^{-2ik_{2m+1}})} \left(1-rr e^{-2ik_{2m+1}}\right)^2 + \cdots \quad (11)$$

Let the region between discontinuity (n) and discontinuity (n+1) be called segment (n) then wave-field on segment (n) can also be established.

2.2.2. Bidirectional propagation. Above Section give initial approximations that neglect the effects of propagation in the upward direction on each segment when waves are transmitted in only downward direction. The problem is taken into account subsequently.

The first correction terms on segment (0) are due to the upward propagating components on all segments to the right of the segment (0) when only leftward propagation is permitted. $W_{y_{1nb-}}^{(0)}$ on segment (1) will be transmitted past discontinuity (1), and the total transmission will suffer multiple reflections between discontinuity (0) and (1). Multiple reflections produce a total wave on segment (0); $W_{y_{1nr+}}^{(0)}$ on segment (2) will be transmitted past discontinuity (2), and the total transmission will suffer multiple reflections between discontinuity (1) and (2), then will be transmitted past discontinuity (1), and the total transmission will suffer multiple reflections between discontinuity (0)
and (1). Multiple reflections produce a total wave on segment (0). By the same token, the first correction terms \( w_{y_{lab}}^{(1)} \) and \( w_{y_{lab}}^{(2)} \) can be obtained by superposing the waves resulting on segment (0)

\[
\begin{align*}
W_{y_{lab}}^{(1)} &= \left( \frac{dd_{a}e^{-ik_{a}y}}{(1 - r_{e}^{2}e^{-k_{a}y_{0}})} \left( \frac{1}{1 - r_{e}^{2}e^{-k_{a}y_{0}} - i(1 + a)} \right) \right) + \frac{1}{dd_{a}} \left( \frac{(1 - r_{e}^{2}e^{-k_{a}y_{0}})}{(1 - r_{e}^{2}e^{-k_{a}y_{0}})} \right) \\
W_{y_{lab}}^{(2)} &= \left( \frac{dd_{a}e^{-ik_{a}y}}{(1 - r_{e}^{2}e^{-k_{a}y_{0}})} \left( \frac{1}{1 - r_{e}^{2}e^{-k_{a}y_{0}} - i(1 + a)} \right) \right) + \frac{1}{dd_{a}} \left( \frac{(1 - r_{e}^{2}e^{-k_{a}y_{0}})}{(1 - r_{e}^{2}e^{-k_{a}y_{0}})} \right)
\end{align*}
\]

After calculation, the first approximate wave field of each segment can be expressed as follows:

\[
W_{y_{lab}}^{(1)} = \left( \frac{d_{a}e^{-ik_{a}y}}{(1 - r_{e}^{2}e^{-k_{a}y_{0}})} \left( \frac{1}{1 - r_{e}^{2}e^{-k_{a}y_{0}} - i(1 + a)} \right) \right) + \frac{1}{dd_{a}} \left( \frac{(1 - r_{e}^{2}e^{-k_{a}y_{0}})}{(1 - r_{e}^{2}e^{-k_{a}y_{0}})} \right)
\]

\[
W_{y_{lab}}^{(2)} = \left( \frac{d_{a}e^{-ik_{a}y}}{(1 - r_{e}^{2}e^{-k_{a}y_{0}})} \left( \frac{1}{1 - r_{e}^{2}e^{-k_{a}y_{0}} - i(1 + a)} \right) \right) + \frac{1}{dd_{a}} \left( \frac{(1 - r_{e}^{2}e^{-k_{a}y_{0}})}{(1 - r_{e}^{2}e^{-k_{a}y_{0}})} \right)
\]

Similar to the above considered case, the second correction terms on a given segment can be obtained. The second correction terms on segment (0) are due to the downward propagating components on all segments to the up of the segment (0), \( w_{y_{lab}}^{(1)} \) and \( w_{y_{lab}}^{(2)} \) can be obtained by superposing the waves resulting on segment (0).

The third and the fourth correction terms can also be obtained by means of this iteration procedure. The third correction terms on a given segment are due to the upward propagating components of the second correction terms on all segments to the down of the segment when only upward propagation is permitted. After many calculations, the third correction terms can be expressed as follows:

\[
W_{y_{lab}}^{(3)} = \left( \frac{d_{a}e^{-ik_{a}y}}{(1 - r_{e}^{2}e^{-k_{a}y_{0}})} \left( \frac{1}{1 - r_{e}^{2}e^{-k_{a}y_{0}} - i(1 + a)} \right) \right) + \frac{1}{dd_{a}} \left( \frac{(1 - r_{e}^{2}e^{-k_{a}y_{0}})}{(1 - r_{e}^{2}e^{-k_{a}y_{0}})} \right)
\]

\[
W_{y_{lab}}^{(4)} = \left( \frac{d_{a}e^{-ik_{a}y}}{(1 - r_{e}^{2}e^{-k_{a}y_{0}})} \left( \frac{1}{1 - r_{e}^{2}e^{-k_{a}y_{0}} - i(1 + a)} \right) \right) + \frac{1}{dd_{a}} \left( \frac{(1 - r_{e}^{2}e^{-k_{a}y_{0}})}{(1 - r_{e}^{2}e^{-k_{a}y_{0}})} \right)
\]
By calculation, the initial approximate wave field of each section of the y1 beam can be expressed as follows:

\[
\begin{align*}
    w_{ny1a+}^{(j)}(y) &= \sum_{i=0}^{\infty} w_{ny1a+}^{(i)}(y), \\
    w_{ny1a-}^{(j)}(y) &= \sum_{i=0}^{\infty} w_{ny1a-}^{(i)}(y), \\
    w_{ny1b+}^{(j)}(y) &= \sum_{i=0}^{\infty} w_{ny1b+}^{(i)}(y), \\
    w_{ny1b-}^{(j)}(y) &= \sum_{i=0}^{\infty} w_{ny1b-}^{(i)}(y), \\
    w_{[n+1]y1a+}^{(j)}(y) &= \sum_{i=0}^{\infty} w_{[n+1]y1a+}^{(i)}(y), \\
    w_{[n+1]y1a-}^{(j)}(y) &= \sum_{i=0}^{\infty} w_{[n+1]y1a-}^{(i)}(y), \\
    w_{[n-1]y1b+}^{(j)}(y) &= \sum_{i=0}^{\infty} w_{[n-1]y1b+}^{(i)}(y), \\
    w_{[n-1]y1b-}^{(j)}(y) &= \sum_{i=0}^{\infty} w_{[n-1]y1b-}^{(i)}(y).
\end{align*}
\] (14)

The symmetric systems considered above have the special feature that the new wave type is excited only in the intersecting beam. One can easily verify that this is not always so. In fact, whenever the system is asymmetric, both wave types are excited in each beam section.

### 2.3. Wave reflection and transmission on structure 3 and structure 4

Then, we analyze the wave fields of y(-1) beam. We consider a system of beams as shown in Figure 2c. The calculation process is the same as above, and the y(-1) beam only considers the influence of the x beam on it. The incident waves on the y(-1) beam are described by

\[
w_{y(-1)a}^{(j)} = \frac{d_{y0}e^{ik_{y}z_{a}}\cdot e^{-2ik_{y}z_{0}}}{(1-rr_{e}^{2k_{y}z_{0}})} + \frac{d_{y0}e^{ik_{y}z_{a}}\cdot e^{-2ik_{y}z_{0}}}{1-rr_{e}^{2k_{y}z_{0}}}
\] (15)

Similar to the above considered case, the incident wave propagates to each section of y(-1) beam after multiple reflections.

So the wave field of each segment on the y(-1) beam can be described by

\[
\begin{align*}
    w_{ny(-1)a+}^{(j)}(y) &= \sum_{i=0}^{\infty} w_{ny(-1)a+}^{(i)}(y), \\
    w_{ny(-1)a-}^{(j)}(y) &= \sum_{i=0}^{\infty} w_{ny(-1)a-}^{(i)}(y), \\
    w_{ny(-1)b+}^{(j)}(y) &= \sum_{i=0}^{\infty} w_{ny(-1)b+}^{(i)}(y), \\
    w_{ny(-1)b-}^{(j)}(y) &= \sum_{i=0}^{\infty} w_{ny(-1)b-}^{(i)}(y), \\
    w_{(n+1)y(-1)a+}^{(j)}(y) &= \sum_{i=0}^{\infty} w_{(n+1)y(-1)a+}^{(i)}(y), \\
    w_{(n+1)y(-1)a-}^{(j)}(y) &= \sum_{i=0}^{\infty} w_{(n+1)y(-1)a-}^{(i)}(y), \\
    w_{(n+1)y(-1)b+}^{(j)}(y) &= \sum_{i=0}^{\infty} w_{(n+1)y(-1)b+}^{(i)}(y), \\
    w_{(n+1)y(-1)b-}^{(j)}(y) &= \sum_{i=0}^{\infty} w_{(n+1)y(-1)b-}^{(i)}(y), \\
    w_{(n-1)y(-1)a+}^{(j)}(y) &= \sum_{i=0}^{\infty} w_{(n-1)y(-1)a+}^{(i)}(y), \\
    w_{(n-1)y(-1)a-}^{(j)}(y) &= \sum_{i=0}^{\infty} w_{(n-1)y(-1)a-}^{(i)}(y), \\
    w_{(n-1)y(-1)b+}^{(j)}(y) &= \sum_{i=0}^{\infty} w_{(n-1)y(-1)b+}^{(i)}(y), \\
    w_{(n-1)y(-1)b-}^{(j)}(y) &= \sum_{i=0}^{\infty} w_{(n-1)y(-1)b-}^{(i)}(y).
\end{align*}
\] (16)

Next, as shown in Figure 2(d), we consider the wave fields of x1 beam, the incident wave of this beam needs to consider the influence from y1 and y(-1) beams. It can be described by
different for different frequency values.

number of iterations can be obtained within the desired accuracy. That is, error between propagation

We know

3.

there is the following expression on the x beam.

The above expression also has a similar

Similarly, the following formula can be found

The above expression also has a similar expression in the y beam as follows

According to the Bloch theorem \( \overline{W}_{y(n-1)x} = e^{\mu y_n} \overline{W}_{y(n-1)x+n} \), the following formula can be obtained.

3. Numerical simulation and discussion

We know \( W_{n\alpha} = \sum_{n=0}^{\infty} W_{n\alpha}^{(i)} \), \( W_{(n-1)\alpha} = \sum_{n=0}^{\infty} W_{(n-1)\alpha}^{(i)} \), Substituting it in equation (23) and equation (26) number of iterations can be obtained within the desired accuracy. That is, error between propagation constants obtained by in equation (23) and equation (26). It is noted that number of iterations may be different for different frequency values.

\[
W_{d1/1a} = \frac{r_1^3 d_1^2 d_{\alpha\beta} e^{ik_{\alpha\beta}x} e^{-2ik_{\alpha\beta}y}}{(1 - rr_1 e^{-2ik_{\alpha\beta}x})^2} + \frac{r_1^3 d_1^2 d_{\alpha\beta} e^{ik_{\alpha\beta}y} e^{-2ik_{\alpha\beta}x}}{(1 - rr_1 e^{-2ik_{\alpha\beta}y})^2}
\]

\[
W_{d1x(-1)i} = \frac{d_\alpha d_\beta e^{ik_{\alpha\beta}x} e^{-2ik_{\alpha\beta}y}}{(1 - rr_1 e^{-2ik_{\alpha\beta}x})^2} + \frac{d_\alpha d_\beta e^{-ik_{\alpha\beta}x} e^{-2ik_{\alpha\beta}y}}{(1 - rr_1 e^{-2ik_{\alpha\beta}y})^2}
\]

(17)

\[
W_{(n-1)a+} = \exp(-ik_{\alpha}a)W_{na+}
\]

(18)

So we have the following relation

\[
W_{x(n-1)a+} = e^{i\mu a} e^{ik_{\alpha}x} W_{x(n-1)a+}
\]

(20)

\[
W_{y(n-1)a+} = \exp(-ik_{\alpha}a)W_{y(n-1)a+}
\]

(22)

\[
W_{y(n-1)a-} = e^{i\mu a} e^{-ik_{\alpha}y} W_{y(n-1)a-}
\]

(23)

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{image.png}
\caption{The band structure of a periodic}
\end{figure}
From this section, numerical calculations of finite periodic beams with epoxy as material A and iron as material B are performed. The young’s modulus and density of epoxy (A) are represented as $E_1 = 4.43 \times 10^4 \text{N/m}^2$, $\rho_1 = 1142 \text{kg/m}^3$. The young’s modulus and density of iron (B) are represented as $E_2 = 12.15 \times 10^5 \text{N/m}^2$, $\rho_2 = 7780 \text{kg/m}^3$, and the moment of inertia is $I = 1.067 \times 10^{-10} \text{m}^4$. The beam cross-sectional area is $S = 8 \times 10^{-4} \text{m}^2$. The lattice constant is chosen to be $a = 0.2 \text{m}$, $a_i = a_j$. The band structure of an infinite periodic grillage structure is depicted in Figure 3.

The multi-reflection method is used to calculate the relationship between any point of the periodic grillage structure of iron/epoxy resin and the incident amplitude. A good agreement between the multi-reflection method and finite element method results on frequency response of the grillage structure validates the reliability of the multi-reflection method. The frequency response diagram of the periodic grillage structure is also drawn, as shown in Figure 4. Wen et al. [8-11] use the finite element method got the first two gap range as $2242 - 3166 \text{ Hz}$ and $4298 - 6030 \text{Hz}$. It is repeated here for the sake of completeness and comparison.

A lateral disturbance of unit amplitude is applied on segment (0), frequency response function $w_5$ and $w_{10}$ at the end of segment (5) and (10) is considered. $w_1$ is frequency response function at the end of segment (1), and the results are shown in Figure 4. By comparison, it can be found that the band gap feature is more obvious as the position of the frequency response and the distance from the external excitation are increased.

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Figure 4. Frequency response of a finite grillage structure. $l = 3a$, (a) segment (5), (b) segment (10).

Figure 5. Band structure of two materials. (a) part B material is lead. (b) part B material is titanium.
As we known, the material parameters have an important influence on the band gap characteristics of the periodic structure [16]. In general, the relative band gap of periodic materials will change with the change of materials. Next, we will analyze whether the periodic grid structure has this characteristic. The control variable method is used to keep the remaining material parameters and structural parameters unchanged. Material A is always epoxy resin, and change material B. In Figure 5(a), the B material is used for lead, and in Figure 5 (b), the B material is used as titanium. From Figure 5, it can be seen that the frequency region has changed obviously, the waveform has not changed obviously, but the width of the relative band gap has changed.

Lattice constant is an important parameter for studying the structure of periodic grillage structure [17]. In phononic crystals, with the change of lattice constants in the periodic structure, the near-band and pass-band will change obviously. However, no one has studied the influence of the lattice constants on the relative band gap size of the periodic grillage structure. Next, the relationship between the relative band gap width and the lattice constants of the periodic grillage structure is studied. With the remaining parameters unchanged, the lattice constants are changed, and the changes of band gap characteristics under different lattice constants are analyzed.

Therefore, next, keep the remaining parameters unchanged, change the lattice constant, and analyze the changes of the band gap characteristics under different lattice constants. Figure 6 (a) is the image when the lattice constant is \( a = 0.2\, mm \), and Figure 6(b) is the image when the lattice constant is \( a = 0.4\, mm \) and Figure 6 (c) is the image when the lattice constant is \( a = 0.6\, mm \). Four white regions in the figure represent four band gaps. It can be seen from these three graphs that when the lattice constant changes, the frequency regions of the band gaps will change, but the waveform will not change.

![Figure 6](image)

**Figure 6.** Band structure for an infinite periodic grillage structure. (a) \( a = 0.2\, mm \), (b) \( a = 0.4\, mm \), (c) \( a = 0.6\, mm \).
4. Conclusions
The flexural wave propagation in grillage structure with a disturbance is studied theoretically as well as numerically. The relation between the wave-field of the incident wave and the wave-field of resulting waves on any beam is expressed. The application of the results to analysis a grillage structure with finite intersecting beams with a propagating disturbance is then demonstrated. This type of propagation may consists of bending waves in special frequency bands, in a combination of longitudinal and bending waves. From these results, the following conclusions can be drawn:

(1) Determine the wave field at any particular location of any beam. Find the function between the amplitude of any segment is established using multi-reflection method.
(2) As the position of the frequency response is increased and the distance from the external excitation is increased, the band gap feature is more obvious.
(3) When the lattice constant changes, the frequency region of the band gap will change, but the waveform will not change, and the relative band gap width remains stable.

These systematic studies are valuable for enriching the calculation methods and analyzing the propagation characteristics of waves. The research results are meaningful in the theory as well as its application to vibration control.

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