The Effects of Flavor Changing Neutral Current and Cabibbo-Kobayashi-Maskawa Matrix

Abstract

We study the effects of tree-level flavor changing neutral current and its present status with Cabibbo-Kobayashi-Maskawa matrix. Especially, we remark the effects of flavor changing neutral current on the unitary triangle of Cabibbo-Kobayashi-Maskawa matrix. In the case when the unitarity is nearly conserved, we find that the allowed FCNC in the $db$ sector is $|z_{d}^{db}/V_{CKM}^{td}V_{CKM}^{tb*}| \sim O(10^{-2})$, from the evaluation of the present bounds for the mixing size of neutral $B$ meson ($x_d$), and the $CP$ violation parameter in the neutral $K$ meson ($\epsilon$). Although the size is small, the new physics due to the tree-level FCNC in the $db$ sector is expected to be visible in the $B \to X_{d} l^{+} l^{-}$ and the $CP$ asymmetries in the neutral $B$ meson system. In the case when the unitarity is violated and the quadrangle is conserved, the FCNC is large enough, i.e. $|z_{d}^{db}/V_{CKM}^{td}V_{CKM}^{tb*}| \sim O(1)$. Then, a significant contribution is also expected in the inclusive $B \to X_{d} \gamma$ decay. As a typical model with this characteristic, we consider Vetor-like Quark Model, where the vector-like quarks are added into the three standard generations of quarks. An overview of the flavor physics in the SM is also given.
APPENDIX

A THE INTERACTIONS IN THE SM
A.1 The fermion and gauge sectors ........................................... 47
A.2 The Higgs sector .......................................................... 48
A.3 The Yukawa and mass sectors .......................................... 49
A.4 The gauge-fixing term ................................................... 50

B THE INTERACTIONS IN THE VQM
B.1 The Yukawa sector .......................................................... 50
B.2 The neutral gauge bosons sector ...................................... 52

C CKM MATRIX IN THE SM
C.1 Kobayashi-Maskawa parametrization .................................. 53
C.2 Wolfenstein parametrization .............................................. 55

D MASS MATRIX IN THE ODVQM
D.1 Natural suppression of tree-level FCNC’s .............................. 57
D.2 Natural existence of tree-level FCNC’s ............................... 57

E $b \to q \gamma$
E.1 In the SM ................................................................. 59
E.2 In the ODVQM .......................................................... 61

F NEUTRAL MESONS MIXING AND $CP$ VIOLATIONS
F.1 Neutral $K$ meson ....................................................... 65
F.2 Neutral $B$ meson ....................................................... 69

References ................................................................. 72
1 INTRODUCTION

In the present, within the Standard Model (henceforth SM)\footnote{Throughout this paper, the phrase Standard Model (SM) point to Minimal Electroweak Standard Model. Minimal means that the Higgs sector contains only one Higgs doublet.} the most of present high energy physics experiments can be explained relatively well. However, many phenomenas are still unexplained, e.g. fermion mass problem, \( CP \) violation, strong \( CP \) problem, Electric Dipole Moment and so on. In order to explain them, many models beyond the SM are proposed (refs. \[1\], \[2\] and \[3\]). Especially, in the sense of the existence of tree-level flavor changing neutral current (henceforth called FCNC), one can divide them into two types, i.e.

1. Models with tree-level FCNC (Vector-like Quark Model, etc.).

2. Models without tree-level FCNC (some Two Higgs Doublet Model, etc.).

In the SM, FCNC occurs only in the one loop and higher orders, but does not emerge in the tree-level.

The existence of FCNC’s in the SM have consequences in the rare decays of mesons, e.g. \( K \) and \( B \) mesons. As will be reviewed shortly in the first section, these rare decays have an important role to investigate the Cabibbo-Kobayashi-Maskawa matrix (henceforth called CKM matrix). The study of CKM matrix will lead us to have a better understanding about Yukawa sector in the SM, as well as new physics in the beyond SM. So in another word one can say that through study of the rare meson decays, we may learn some clues about Yukawa-sector of the electroweak theory which has not been understood well. In this meaning, it is important to test the various models beyond the SM.

In this paper, we study the effects of FCNC’s which occur in the tree-level by employing Vector-like Quark model (henceforth called VQM). This model is a possible minimum extension of the SM. In addition to the three standard generations of quarks, the vector-like quarks are introduced (see for example ref. \[3\]). In this model, there are two types of tree-level FCNC’s, i.e.,

1. Tree-level FCNC’s among the ordinary quarks and

2. Tree-level FCNC’s between the ordinary and the vector-like quarks.
They can be tested separately as discussed in refs. [4], [5] and references therein. By applying this model in the FCNC processes and rare decays, we investigate the effects of these FCNC’s especially on the triangle of CKM matrix that appears from the unitarity of CKM matrix. In this paper we concentrate on the $db$ sector of CKM matrix. In this paper, some neutral $K$ and $B$ mesons processes will be evaluated.

This paper is organized as follows. First, we make an overview of the SM and the induced flavor physics phenomenas, that is, unitarity of CKM matrix and the experimental bounds for the CKM matrix elements from $CP$ violation and rare decays in $K$ and $B$ mesons system. According to the processes, we discuss some experimental constraints on the unitarity of CKM matrix. Next, we give a brief introduction of the VQM. We show that in the model tree-level FCNC’s are appearing, and the sizes of FCNC’s are indicated as $z_{\alpha\beta}$. The upper-bounds for $z_{\alpha\beta}$ will be found from the $K$ and $B$ mesons processes, under an assumption that the tree-level FCNC’s are dominant. Further we evaluate the constraints for two cases,

1. Constraint when the unitarity is nearly conserved.

2. Constraint when the unitarity is violated and the quadrangle is conserved.

In the case (1), small FCNC is allowed from the evaluation of the experimental bounds of $B^0 - \bar{B}^0$ mixing and the $CP$ violation in the neutral $K$ meson. On the other hand, large FCNC is allowed in the case (2). Under the bounds for each case, the predictions for the neutral $B$ meson processes will be presented. The predictions for $B \to X_d l^+ l^-$ and $CP$ asymmetries in the neutral $B$ meson system show that significant contributions are expected in the model, although the FCNC is small. For large FCNC, a significant contribution is expected in the inclusive $B \to X_d \gamma$ decay. Additionally, we also confirm contribution of the new physics due to the VQM in the electroweak oblique correction parameters, $S, T, U$ (ref. [3]), and find that the contributions are negligible. Apart from the experimental constraints, we will also present theoretical studies on the FCNC’s, as a consequence of the behaviour of the mass matrices in the model.

All of the contents will be discussed in the main paper, and the detail calculations and explanations can be seen in the appendices.
2 FLAVOR PHYSICS IN THE SM : AN OVERVIEW

In this section, we will give an overview of the SM, and its implications on the flavor physics (see for example refs. [8], [9] and references therein).

2.1 The model

SM is a model which is based on the $SU(2) \otimes U(1)$ gauge group. In the SM, left-handed particles are introduced as $SU(2)$ doublet, but right-handed particles are as $U(1)$ singlet. The contents of the particles in this group are denoted as below,

- **Fermions**:
  
  \begin{align}
  \text{Quarks} & : \begin{pmatrix} u^i \\ d^i \end{pmatrix}_L, u^i_R, d^i_R \\
  \text{Leptons} & : \begin{pmatrix} \nu^i \\ \ell^i \end{pmatrix}_L, \ell^i_R.
  \end{align}

- **Gauge bosons**:
  
  \begin{align}
  \begin{pmatrix} W^{1}_{\mu} \\ W^{2}_{\mu} \\ W^{3}_{\mu} \end{pmatrix}, B_{\mu}.
  \end{align}

- **Scalars**:
  
  \[ \phi \equiv \left( \frac{1}{\sqrt{2}} \left( \chi^+ + \phi^0 \right) \right). \]

where $u^i$ and $d^i$ represent up-type ($u, c, t$) and down-type ($d, s, b$) quarks, with generation indices $i = 1, 2, 3$, and same for leptons $\ell^i$ ($e, \mu, \nu$) and neutrino $\nu^i$ ($\nu_e, \nu_\mu, \nu_\tau$). Throughout this paper, we use the following notations for chirality, $L \equiv \frac{1}{2}(1 - \gamma_5)$ and $R \equiv \frac{1}{2}(1 + \gamma_5)$. In the Minimal SM, we consider only one Higgs doublet to generate the fermion masses.

The various terms in the lagrangian can be written by demanding $SU(2) \otimes U(1)$ gauge invariance and lepton-quark universality of the electroweak interactions. It will reproduced in part in Appendix A. In relation with $K$ and $B$ physics, the main interest lies in the study of the flavor changing transitions, involving fermions, gauge bosons and Higgs fields.

As the results of Appendix A, we can write the related interactions for physics in the meson decays briefly as,

\[ \mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}} \left( V_{CKM}^{ij} \bar{u}^j \gamma^\mu L d^i + \bar{\nu}_i \gamma^\mu L \ell^i \right) W^+_\mu + h.c., \]

(5)
\[ \mathcal{L}_{\chi^\pm} = \frac{g}{\sqrt{2} M_W} \left[ V_{\text{CKM}}^{ij} \bar{u}^i (m_u L - m_d R) d^j + m_l \bar{\nu}^i L L^i \right] \chi^\pm + \text{h.c.}, \quad (6) \]
\[ \mathcal{L}_A = \frac{e}{3} \left[ 2 \bar{u}^i \gamma^\mu u^i - \bar{d}^i \gamma^\mu d^i - 3 \bar{\nu}^i \gamma^\mu \nu^i \right] A^\mu, \quad (7) \]
\[ \mathcal{L}_Z = \frac{g}{2 \cos \theta_W} \left\{ \bar{u}^i \gamma^\mu \left[ \left( 1 - \frac{4}{3} \sin^2 \theta_W \right) L - \frac{4}{3} \sin^2 \theta_W R \right] u^i \right. \\
\left. + \bar{d}^i \gamma^\mu \left[ \left( \frac{2}{3} \sin^2 \theta_W - 1 \right) L + \frac{2}{3} \sin^2 \theta_W R \right] d^i \right. \\
\left. + \bar{\nu}^i \gamma^\mu \left[ \left( 1 - 4 \sin^2 \theta_W \right) L - 4 \sin^2 \theta_W R \right] \nu^i \right. \\
\left. + \bar{l}^i \gamma^\mu \left[ \left( 2 \sin^2 \theta_W - 1 \right) L + 2 \sin^2 \theta_W R \right] l^i \right\} Z^\mu, \quad (8) \]

and the related triple gauge interactions are,
\[ \mathcal{L}_{W^\pm W^\mp A} = ie \left[ (\partial^\mu A^\nu - \partial^\nu A^\mu) W^\mu_+ W^\nu_- + A^\nu \left( \partial^\mu W^{\nu+} - \partial^\nu W^{\mu+} \right) W^\mu_- \\
- A^\nu \left( \partial^\mu W^{\nu-} - \partial^\nu W^{\mu-} \right) W^\mu_+ + A^\mu \left( W^\mu_+ \partial^\nu - \partial^\nu W^\mu_+ \right) W^\mu_- \right], \quad (9) \]
\[ \mathcal{L}_{\chi^\pm \chi^\mp A} = ie A^\mu \chi^\pm \partial^\mu \chi^\mp + \text{h.c.}. \quad (10) \]

Note here that the last two terms in the \( \mathcal{L}_{W^\pm W^\mp A} \) comes out from the gauge fixing terms (Eq. (159)).

The main goal of this section is to investigate the CKM matrix, \( V_{\text{CKM}} \), which appears in the above charge current interaction terms. It can be achieved by examining the \( CP \) violations and rare decays in the SM. This will be the aim of Sec. 2.2 and 2.4. In the Sec. 2.2 we will restudy the behaviour of CKM matrix in the SM, and extract all of considerable processes that should be used to test the CKM matrix.

### 2.2 CKM matrix

From the discussion in Appendix A.3, it is clear that the flavor changing transition in the SM emerge from the diagonalization of the quark mass matrices after spontaneous symmetry breaking (SSB). The Yukawa couplings in the SM are arbitrary complex numbers, and therefore there is no hope of understanding their origin within the SM. The origin of the differences of fermion masses is one of the outstanding problems in the particle physics. Perhaps, in a more fundamental framework these couplings may be derived from some deeper dynamics, for example this puzzle may be able to be worked out in the VQM framework as will be done in Sec. 3.5.1. Since the interpretation of the experimental results in the SM framework does
not require a prior knowledge of this connection, we shall assume the validity of CKM matrix as a consistent framework and study the consequences of this assumption in weak decays.

The matrix elements $V_{CKM}$ are determined by charged current coupling to the $W^{\pm}$ bosons and/or $\chi^{\pm}$. Symbolically it can be written as,

$$
V_{CKM} \equiv \begin{pmatrix}
V_{CKM}^{ud} & V_{CKM}^{us} & V_{CKM}^{ub} \\
V_{CKM}^{cd} & V_{CKM}^{cs} & V_{CKM}^{cb} \\
V_{CKM}^{td} & V_{CKM}^{ts} & V_{CKM}^{tb}
\end{pmatrix}.
$$

(11)

All of these elements have to be determined experimentally via $u^id^iW$ and $u^id^i\chi$ interactions. This is the principal task of experimental flavor physics.

Before going to the next section, let us give a parametrization that is usually used in relation with the experiment results. As done in Appendix C.1, in the $3 \times 3$ CKM matrix, after making any transformations, one phase is always remained. This phase leads to $CP$ violation in the flavor physics. However, under this fact and the condition that the CKM matrix is unitary, we can parametrize it in any arbitrary ways (Appendix C.1 and C.2). Here we employ an approximation but very popular form of the matrix $V_{CKM}$ due to Wolfenstein, that is

$$
V_{CKM} \approx \begin{pmatrix}
1 - \frac{1}{2} \lambda^2 & \lambda & A\lambda^3(\rho - i\eta + \frac{i}{2}\eta\lambda^2) \\
-\lambda & 1 - \frac{1}{2} \lambda^2 - i\eta A^2\lambda^4 & A\lambda^2(1 + i\eta\lambda^2) \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix}.
$$

(12)

This Wolfenstein parametrization still has three real parameters and one phase. This parametrization is an approximation form of the $V_{CKM}$, but has advantages especially in evaluating the CKM matrix elements phenomenologically.

Of the two generations part of $V_{CKM}$ in Eq. (11), four involving the $u,d,c,s$ quarks (Cabibbo angle) were already known before the discovery of bottom and top quarks. Two of the matrix elements involving the bottom quark, $V_{CKM}^{ub}$ and $V_{CKM}^{cb}$, have been measured in the decays of $B$ hadrons (ref. [10]). The remaining three elements, $V_{CKM}^{td}$, $V_{CKM}^{ts}$ and $V_{CKM}^{tb}$, are also accessible in $B$ decays. These will be discussed in Sec. 2.4.

2.3 Unitarity of CKM matrix

As stated before, the CKM matrix elements obey unitary constraints, $V_{CKM}V_{CKM}^\dagger = 1$. This has implications that any pair of rows or columns of CKM matrix are orthogonal, then leads to six orthogonal conditions, i.e.,
1. Orthogonality conditions on the columns:

\[ ds : \sum_{i=u,c,t} V_{CKM}^{id} V_{CKM}^{is} = 0, \]
\[ db : \sum_{i=u,c,t} V_{CKM}^{id} V_{CKM}^{ib} = 0, \]
\[ sb : \sum_{i=u,c,t} V_{CKM}^{is} V_{CKM}^{ib} = 0. \]  \(13\)

2. Orthogonality conditions on the rows:

\[ uc : \sum_{i=d,s,b} V_{CKM}^{ui} V_{CKM}^{ci} = 0, \]
\[ ut : \sum_{i=d,s,b} V_{CKM}^{ui} V_{CKM}^{ti} = 0, \]
\[ ct : \sum_{i=d,s,b} V_{CKM}^{ci} V_{CKM}^{ti} = 0. \] \(14\)

By using Wolfenstein parametrization, we can depict each of orthogonality condition in Eqs. (13) and (14) as a triangle. Thus, we obtain six triangles for them as depicted in Fig. (1). This will be derived quantitatively in the next section. We will discuss these triangles and the processes which determine them in Sec. 2.4. In general, the triangles are a consequence of the unitarity of CKM matrix, can be showed in a simple way (ref. \([11]\)). From Eqs. (13) and (14),

\[ V_{CKM}^{1i} V_{CKM}^{1j} + V_{CKM}^{2i} V_{CKM}^{2j} + V_{CKM}^{3i} V_{CKM}^{3j} = 0 \]  \(15\)

for \(i \neq j\). Now multiply it by \(V_{CKM}^{3i} V_{CKM}^{3j}\) and take the imaginary part, we obtain

\[ \text{Im} \left[ V_{CKM}^{3i} V_{CKM}^{3j} V_{CKM}^{1i} V_{CKM}^{1j} \right] + \text{Im} \left[ V_{CKM}^{3i} V_{CKM}^{3j} V_{CKM}^{2i} V_{CKM}^{2j} \right] = 0. \] \(16\)

Then,

\[ \text{Im} \left[ V_{CKM}^{3i} V_{CKM}^{3j} V_{CKM}^{1i} V_{CKM}^{1j} \right] = -\text{Im} \left[ V_{CKM}^{3i} V_{CKM}^{3j} V_{CKM}^{2i} V_{CKM}^{2j} \right]. \] \(17\)

In the same way, for \(i \neq j \neq k \neq l\),

\[ A = \pm \text{Im} \left[ V_{CKM}^{ki} V_{CKM}^{kj} V_{CKM}^{li} V_{CKM}^{lj} \right] \]
\[ = \pm \text{Im} \left[ |V_{CKM}^{ki} V_{CKM}^{kj}| e^{i\phi_1} \times |V_{CKM}^{li} V_{CKM}^{lj}| e^{-i\phi_2} \right] \]
\[ = \pm |V_{CKM}^{ki} V_{CKM}^{kj}| |V_{CKM}^{li} V_{CKM}^{lj}| \sin (\phi_1 - \phi_2) \] \(18\)
Figure 1: The CKM unitarity triangles following from the orthogonality of CKM matrix, where in the $ab$ sector, $\xi_i \equiv V_{CKM}^{ia}V_{CKM}^{ib^*}$ for the upper graphs, and $\xi_i \equiv V_{CKM}^{ai}V_{CKM}^{bi^*}$ for the lower ones.

is the area of triangle, where $\phi_1 - \phi_2$ is the angle between $V_{CKM}^{ki^*}V_{CKM}^{kj}$ and $V_{CKM}^{li}V_{CKM}^{lj^*}$.

Eq. (18) also indicates that $A$ is invariant under the phase redefinition (Appendices C.1 and C.2).

Finally, it is clear that a test of each triangle conservation should be a test of the unitarity of CKM matrix itself. This could be done by examining some processes which determine the CKM matrix elements. This will be the aim of the next subsection.

2.4 Experimental constraints on the CKM matrix

Unitarity constraints on the CKM matrix elements in Sec. 2.3 provide very powerful and interesting relations. We ought to test all of the triangles in Fig. 1 in order to test the Yukawa sector in the SM. In general, the sides of the triangles can be measured by FCCC and FCNC processes, and the angles are by $CP$ violations.
As stated in Sec. 2.2, apart from the already measured six CKM matrix elements, there are still three unknown elements. All of them are the elements in the third row of CKM matrix, and involving bottom and top quarks. Of course, it could be measured directly in the FCCC decays in the high energy experiment in the future, i.e. top decay processes. However, since the mass of top quark is very large, according to ref. [12], $m_t \approx 174 \pm 10^{+13}_{-12}$(GeV), we have difficulty realizing it in the present. This problem can be cleared in some $K$ and $B$ mesons processes. The reason is that, the $t \bar{d} W$ and/or $t \bar{d} \chi$ interactions appear in the one-loop order, and the top quark contributions in its internal lines are dominant.

In our opinion, the most interesting triangle is the $d b$ sector. The reason is, the $d b$ sector triangle will be measured confidently in the near future, especially when the $B$ factory and LHC are started. So, here we will concentrate on the triangle of $d b$ sector and do not go on discussing the other sectors.

### 2.4.1 Experimental constraints from FCCC processes

We briefly give the experimental results (refs. [10], [13] and [14]) of first and second rows of $V_{CKM}$ in Eq. (11). All of them are determined in the direct FCCC decays. The measurements of two generations part (Cabibbo matrix part) are accurate. On the other hand, $|V_{CKM}^{ud}|$ and $|V_{CKM}^{cd}|$ are less accurate, since there are theoretical uncertainties due to the puzzle of hadron matrix elements. However, because our interest is the FCNC processes, we will not discuss the FCCC processes in detail.

Here, we list the experiment results and the processes to determine them,

1. $|V_{CKM}^{ud}|$:  
   It is determined by nuclear beta decays, when compared to muon decay. One obtains,  
   $$ |V_{CKM}^{ud}| = 0.9744 \pm 0.0010 \text{.} $$  
   (19)

2. $|V_{CKM}^{us}|$:  
   There are mainly two ways to determine it, that is via $K_{e3}$ decays ($K^+ \rightarrow \pi^0 e^+\nu_e$ and $K_L^0 \rightarrow \pi^- e^+\nu_e$), and via semileptonic hyperon decays. One averages these two results to obtain,  
   $$ |V_{CKM}^{us}| = 0.2205 \pm 0.0018 \text{.} $$  
   (20)
3. $|V_{CKM}^{cd}|$:

The magnitude of $|V_{CKM}^{cd}|$ may be deduced from neutrino and antineutrino production of charm off valence $d$ quarks, and semileptonic strangeless $D$ decays ($D^0 \to \pi^- e^+ \nu_e$). But the second one is less accurate since the theoretical uncertainties in the form factor. From the first process,

$$|V_{CKM}^{cd}| = 0.204 \pm 0.017 .$$  \hspace{0.5cm} (21)

4. $|V_{CKM}^{cs}|$:

This is deduced from $D_{e3}$ decays ($D^0 \to K^- e^+ \nu_e$), analogous to $|V_{CKM}^{us}|$. It follows that,

$$|V_{CKM}^{cs}| = 1.02 \pm 0.18 .$$  \hspace{0.5cm} (22)

5. $|V_{CKM}^{cb}|$:

$|V_{CKM}^{cb}|$ can be gained from the inclusive semileptonic $B$ decays, that is $B \to X_c l \nu_l$ ($B^0 \to D^{*+} l^- \nu_l$, $B^- \to D^{*0} l^- \nu_l$). The present data have an accuracy of $\pm12\%$,

$$|V_{CKM}^{cb}| = 0.042 \pm 0.005 .$$  \hspace{0.5cm} (23)

6. $|V_{CKM}^{ub}|$:

Due to experimental progress, the fact that $|V_{CKM}^{ub}|$ can be shown to be non-zero as required to explain $CP$ violation, has been addressed by many groups. $|V_{CKM}^{ub}|$ is usually determined in the ratio $|V_{CKM}^{ub}/V_{CKM}^{cb}|$.

$$\frac{|V_{CKM}^{ub}|}{|V_{CKM}^{cb}|} = 0.08 \pm 0.02 .$$  \hspace{0.5cm} (24)

Note that, accurately measured $|V_{CKM}^{us}|$, have an important role in the Wolfenstein parametrization in Eq. [12], since the CKM matrix has been expanded in term of $|V_{CKM}^{us}| \equiv \lambda$.

On using Eq. (12), we can translate the above list to find the experimental constraints for the parameters in the Wolfenstein parametrized CKM matrix. The bounds for them can be summarized as,

1. $\lambda$:

$$|V_{CKM}^{us}| = \lambda \rightarrow \lambda = 0.2205 \pm 0.0018 .$$  \hspace{0.5cm} (25)
2. $A$:

$$|V_{CKM}^{cb}| = \lambda^2 \rightarrow A = 0.86 \pm 0.10. \quad (26)$$

3. $\rho$ and $\eta$:

$$\frac{|V_{CKM}^{ub}|}{|V_{CKM}^{cb}|} = \lambda \sqrt{\rho^2 + \eta^2} \rightarrow \sqrt{\rho^2 + \eta^2} = 0.36 \pm 0.09. \quad (27)$$

To get bounds for $\rho$ and $\eta$ separately, we need additional input from FCNC processes.

### 2.4.2 Experimental constraints from FCNC processes

In this section, we discuss the remaining CKM matrix elements which could be determined by FCNC processes. From Eq. (12), we should normalize each side of the triangle in Fig. (1) by $|V_{CKM}^{cd}V_{CKM}^{cb}|$ like below,

$$\frac{|V_{CKM}^{ud}V_{CKM}^{ub}|}{|V_{CKM}^{cd}V_{CKM}^{cb}|} \cong \sqrt{\rho^2 + \eta^2} \equiv |CA|, \quad (28)$$

$$\frac{|V_{CKM}^{td}V_{CKM}^{tb}|}{|V_{CKM}^{cd}V_{CKM}^{cb}|} \cong \sqrt{(1 - \rho)^2 + \eta^2} \equiv |AB|. \quad (29)$$

Therefore, in the complex plane ($\rho$, $\eta$), the triangle can be written as Fig. (2), by normalizing $|V_{CKM}^{cd}V_{CKM}^{cb}| \cong \lambda^3 \equiv 1$. Moreover, the side $|AB|$ can be rewritten in term of $|CA|$ by using Eqs. (28) and (29):

$$|AB| = \sqrt{1 - 2\rho + (\rho^2 + \eta^2)} = \sqrt{1 + |CA|^2 - 2|CA| \cos \gamma}. \quad (30)$$

Each angle in Fig. (2) can also be expressed in terms of $\rho$ and $\eta$ by using simple trigonometry (ref. [15]),

$$\sin 2\alpha = \frac{2\eta(\eta^2 + \rho^2 - \rho)}{(\rho^2 + \eta^2)((1 - \rho)^2 + \eta^2)} = \frac{2\sin \gamma(|CA| - \cos \gamma)}{|AB|^2}, \quad (31)$$

$$\sin 2\beta = \frac{2\eta(1 - \rho)}{(1 - \rho)^2 + \eta^2} = \frac{2|CA|\sin \gamma(1 - |CA| \cos \gamma)}{|AB|^2}, \quad (32)$$

$$\sin 2\gamma = \frac{2\rho\eta}{\rho^2 + \eta^2} = \frac{2\rho\eta}{|CA|^2}. \quad (33)$$
Figure 2: The CKM unitarity triangle in the $db$ sector.

In addition to the above relations, there is also a constraint from the feature of triangle, i.e.

$$\alpha + \beta + \gamma = 180^0. \tag{34}$$

The next question is, what the processes can be used to measured the sides and the angles of triangle in Fig. 2. Here, we list the processes which should determine the sides and the angles.

- $|CA|$: $B \to X_c l \nu_l$, · · ·
- $|AB|$: $B^0_d - \bar{B}^0_d$ mixing, $B \to X_d l^+ l^-$, $B \to X_d \gamma$, $B^0_d \to l^+ l^-$, · · ·
- $|AD|$: $e'/e$, $K_L \to \pi^0 e^+ e^-$, $K_L \to \pi^0 \nu \bar{\nu}$, · · ·
- $|AE|$: $(K^0_L \to \mu \bar{\mu})_{SD}$, · · ·
- $\alpha$: $B^0_d \to \pi^+ \pi^-$, $\rho^\pm \pi^\mp$, · · ·
- $\beta$: $B^0_d \to \psi K_s$, $D^\pm D^\mp$, · · ·
- $\gamma$: $B^0_s \to \rho^0 K_s$, · · ·
- The sign of $\rho$ and $\eta$: $\epsilon$. 

14
The effects of tree-level FCNC’s will be applied in the most reliable processes from the above list, that is \( B_d^0 - \bar{B}_d^0 \) mixing, \( CP \) violations in the \( B_d^0 \to \pi^+\pi^- \), \( \psi K_s \) and \( B_s^0 \to \rho^0 K_s \) decays and the parameter \( \epsilon \) in the neutral \( K \) meson system. The reason that the processes will be determined well in such experiments. So we are not discussing the others in either cases, SM and VQM, in this paper.

1) \( B \rightarrow X_q \gamma \) decay

Approximately, the short distance contribution of this decay can be expressed in the quark model by inclusive \( b \rightarrow q \gamma \) decay, since the bottom quark is heavy compared with the QCD scale. In relation to the side \( |AB| \) of triangle, we should observe the one-loop penguin diagram of \( b \rightarrow d \gamma \) which has been calculated in Appendix E.1. Unfortunately, in the inclusive \( b \rightarrow d \gamma \) decay, contribution from the penguin diagrams (Fig. (3)b) is smaller than from the tree-level diagram (Fig. (3)a), that is with order \( |V_{CKM}^{td}/V_{CKM}^{tb}| \propto O(\lambda) \). In the \( b \rightarrow s \gamma \) decay, the situation is different, since the tree-level is suppressed by order \( O(\lambda^3) \) due to the CKM matrix element \( V_{CKM}^{ub} \).

In order to make the magnetic moment type \( b \rightarrow d \gamma \) to be dominance, we exclude the tree-level contribution by improving the following condition (ref. [16])

\[
E_\gamma > \frac{m_B^2 - m_d^2}{2m_B} = 2.3(\text{GeV}). \tag{35}
\]

Thus, the penguin diagrams are to be the main contribution in the decay.

The SM prediction for the amplitude of \( b \rightarrow d \gamma \) decay is given in Appendix E.1. From the result, the branching ratio is,

\[
Br(b \rightarrow d \gamma)^{SM} \approx \frac{\alpha G_F^2 m_{B_d}^5 \tau_{B_d} Q_u^2}{128\pi^3} \left| V_{CKM}^{td} V_{CKM}^{tb*} \right|^2 \left| F_{bd\gamma}^{SM}(m_b) \right|^2 \]

\[
= 0.230 \times A^2 \lambda^6 \left( (1 - \rho)^2 + \eta^2 \right), \tag{36}
\]

where we neglect the down quark mass \( m_d \), because of \( m_d^2 \ll m_b^2 \). Here, we used \( m_t = 174(\text{GeV}) \) and \( m_{B_d} = 5373 \pm 4.2(\text{MeV}) \), and \( F_{bd\gamma}^{SM}(m_b) \) is given in Appendix E.1.

On the other hand, experimentally \( b \rightarrow d \gamma \) has been observed, but only the upper-bound have been obtained by CLEO collaboration (ref. [17]). The value is the sum of the branching
Figure 3: (a) Tree-level and (b) one-loop penguin diagram for the inclusive $b \to d\gamma$ decay, where $q$ denotes up-quarks.

ratio of the exclusive decays $B^-\to\rho^-\gamma$, $B\to\rho^0\gamma$ and $B^0\to\omega\gamma$, 

$$Br(b\to d\gamma)^{exp} < 6.3 \times 10^{-5}.$$ (37)

This result yields the upper-bound for the side $|AB|$ of triangle, 

$$\sqrt{(1-\rho)^2 + \eta^2} = |AB|^{SM} < 2.082.$$ (38)

Thus, it is clear that $b\to d\gamma$ itself is rather useless to determine the side $|AB|$, since it gives only the upper-bound. So it would better to determine it by $B_d^0 - \bar{B}_d^0$ mixing. However, this decay should be a powerful test for the sizes of FCNC’s in the future.

Apart from the above evaluation, it is also usual to consider a less model-dependent ratio of $b\to q\gamma$ decay, i.e.

$$\left(\frac{Br(b\to d\gamma)}{Br(b\to s\gamma)}\right)^{SM} = \left|\frac{V_{CKM}^{tb}V_{CKM}^{ts}}{V_{CKM}^{tb^*}V_{CKM}^{ts^*}}\right|^2 \cong \lambda^2 \left[(1-\rho)^2 + \eta^2\right].$$ (39)

Since the experimental result for $b\to s\gamma$ is, 

$$Br(b\to s\gamma)^{exp} = (2.32 \pm 0.51 \pm 0.29 \pm 0.32) \times 10^{-4}$$ (40)

from CLEO-II group (ref. [17]), one finds

$$|AB|^{SM} < 1.957.$$ (41)

This bound is better than the bound in Eq. (38).

Let us give a comment here about $b\to d\gamma$ decay. Present, experimentalists have difficulty measuring the decay (ref. [17]). But, here we hope that they can make a progress to measure
it in the future. At any cost, the decay is important to give a confident value for the side $|AB|$ together with $x_d$.

2) $K^0 - \bar{K}^0$ mixing

The small $CP$ violation in the neutral $K$ meson system can give some constraints to determine the CKM matrix. The violation has been observed, for example in the neutral $K$’s that decay into $(2\pi)_{I=0}$. One of them is described in the well-measured parameter $\epsilon$. Definition of the parameter can be seen in Appendix F.1, and is given in Eq. (269) as,

$$\epsilon^{SM} \approx \frac{G_F^2}{6\sqrt{2}\pi^2 \Delta M} m_K f_K^2 B_K M_W^2 \text{Im} \left( M_{12}^{SM} \right) e^{i\pi/4}$$

$$= e^{i\pi/4} \frac{G_F^2}{6\sqrt{2}\pi^2 \Delta M} m_K f_K^2 B_K M_W^2 A^2 \lambda^6 \eta$$

$$\left[ -\eta_c F_{\Delta S=2}^{SM}(x_c) + A^2 \lambda^4 (1 - \rho) \eta_t F_{\Delta S=2}^{SM}(x_t) + \eta_{ct} F_{\Delta S=2}^{SM}(x_c, x_t) \right]$$

by using Eq. (12) and dropping the second term in Eq. (269), since $2\text{Re} M_{12} \approx \Delta M^{exp} = 3.5 \times 10^{-15}$ is tiny. Thus, knowing $\epsilon$ experimentally will give the prediction for the CKM matrix elements. However, there is still theoretical uncertainties in the bag parameter $B_K$.

Experimentally,

$$\epsilon^{exp} = e^{i\pi/4} (2.258 \pm 0.018) \times 10^{-3}. \quad (43)$$

After putting the values for each parameter, the SM gives the prediction for $\epsilon$ as,

$$\epsilon^{SM} = e^{i\pi/4} \left( 3.82 \times 10^4 \right) B_K A^2 \lambda^6 \eta \left[ \left( 7.84 \times 10^{-4} \right) + 1.46 A^2 \lambda^4 (1 - \rho) \right]$$

for $f_K = 161$(MeV), $\eta_c = 0.85$, $\eta_{ct} = 0.36$, $\eta_t = 0.57$ and $m_t = 174$(GeV). In Fig. (5), we plot this equation with $B_K = 0.82 \pm 0.10$ from the recent lattice calculations. This constraint is important to determined the position of point A in the triangle, or the signs of $\rho$ and $\eta$ in the $(\rho, \eta)$ plane.

3) $B^0_q - \bar{B}^0_q$ mixing

Here, we discuss the $B^0_q - \bar{B}^0_q$ mixing, with $q = d, s$. Mixing in the $B^0_q - \bar{B}^0_q$ mixing involves the third generations and there is no reason to assume a small phase between $M_{12}$
and $\Gamma_{12}$ (Appendix F.2). However, the discussion is simplified because of Eq. (277), $\Gamma_{12}^{SM} \ll M_{12}^{SM}$. The SM predictions for the size of mixing is given in Eq. (282),

$$x_q^{SM} = \frac{G_F}{6\pi^2} m_B q M W^2 \tau_B q B_B \left| V_{CKM}^{tq} V_{CKM}^{tb*} \right|^2 \left| F_{\Delta B=2}^{SM}(x_t) \right|.$$  \hspace{1cm} (45)

On using Eq. (12), it yields,

$$q = d : \left| V_{CKM}^{td} V_{CKM}^{tb*} \right| \simeq A^2 \lambda^6 \left[ (1 - \rho)^2 + \eta^2 \right],$$  \hspace{1cm} (46)

$$q = s : \left| V_{CKM}^{ts} V_{CKM}^{tb*} \right| \simeq A^2 \lambda^4.$$  \hspace{1cm} (47)

Since the parameters $A$ and $\lambda$ have been determined in Eqs. (25) and (26), one is able to predict $x_s$ more confidently than $x_d$. But, experimentally $x_d$ is precisely measured yet.

The results for $x_d$ from the four experiments ARGUS, CLEO, ALEPH and DELPHI (refs. [10] and references therein) gives,

$$x_d^{exp} = 0.71 \pm 0.07$$  \hspace{1cm} (48)

by using $\tau_{B_d} = 1.44 \pm 0.15$(ps). Unfortunately, this precision is not matched by the theory, which is uncertain due to the imprecise knowledge of $f_{B_d}$ and $B_{B_d}$. The prediction of the SM is expressed as,

$$x_d^{SM} = \left( 2.51 \times 10^5 \right) \times f_{B_d}^2 B_{B_d} A^2 \lambda^6 \left[ (1 - \rho)^2 + \eta^2 \right].$$  \hspace{1cm} (49)

This equation determines the sides $|AB|$, and is depicted in Fig. (3) including the uncertainties, where $m_{B_d} = m_{B_s} = 5$(GeV), $\eta_{B_d} = \eta_{B_s} = 0.55$ and $m_t = 174$(GeV). In the Fig. (3) we adopt $160 \leq \sqrt{f_{B_d}^2 B_{B_d}} \leq 240$(MeV) from the recent lattice calculations and improved QCD sum rules. It yields,

$$0.631 \leq |AB|^{SM} \leq 1.386.$$  \hspace{1cm} (50)

Meanwhile, experimentally the mixing parameter $x_s$ is not determined yet, and only the lower-bound has been known (ref. [14]),

$$x_s^{exp} > 1.5.$$  \hspace{1cm} (51)

Thus, in the meaning of the triangle sides, $x_s$ itself does not work an important role. However, knowing $x_d$ and $x_s$ experimentally could reduce the theoretical parameters and obtain a less
model-dependent quantity,
\[ \left( \frac{x_d}{x_s} \right)^{SM} = \left| \frac{V_{CKM}^{td}}{V_{CKM}^{ts}} \right|^2 = \lambda^2 \left[ (1 - \rho)^2 + \eta^2 \right], \tag{52} \]
by assuming $SU(3)$ symmetry breaking is zero, i.e. $f_{B_q} B_{B_q}$ are same for $q = d, s$. Then, the bound for $|AB|$ is,
\[ |AB|^{SM} \leq 3.122. \tag{53} \]
So, the bound from single quantity $x_d$ is more reliable. But we expect a better constraint from this clean quantity when $x_s$ could be measured precisely in the future experiment.

4) $CP$ violations in $B_q^0 - \bar{B}_q^0$ mixing

We consider a neutral $B_q^0$ meson and its antiparticle $\bar{B}_q^0$. As final states $f$, we consider $f : \psi K_s, \pi^+ \pi^-, \rho K_s$ correspond to the angles $\alpha, \beta, \gamma$ of the triangles (Appendix F.2). All of $f$’s are considered as $CP$ even final states. We are interested in the neutral $B$’s decays into a $CP$ eigenstate $f$. Then, we shall use time-dependent asymmetry of each decay as given in Eq. (286),
\[ a_f(t) \equiv -\sin(\Delta M t) \sin \phi, \tag{54} \]
and the time-integrated asymmetry is,
\[ a_f \equiv \int_0^1 dt \, a_f(t) = -\frac{x_q}{1 + x_q^2} \sin \phi. \tag{55} \]
The detailed definition for $\sin \phi$ is given in Eq. (289).

We are now ready to give three explicit quantities for asymmetries that measure the angles $\alpha, \beta, \gamma$.

(a) $B_d^0 \to \psi K_s$:

The mixing phase in the $B_d^0 - \bar{B}_d^0$ mixing is given in Eq. (291) for $q = d$. The other phase is due to single final kaon $K_s$. Then we have to take into account the mixing phase in the $K^0 - \bar{K}^0$ mixing in Eq. (276). From the amplitude of $B_d^0 \to \psi K_s$ and its anti particle decays as depicted in Fig. (4a) and Eq. (290), the mixing phase is,
\[ \left( \frac{\hat{A}}{A} \right)^{SM}_{\psi K_s} = \left( \frac{V_{CKM}^{cs} V_{CKM}^{cb}}{V_{CKM}^{cs} V_{CKM}^{cb}} \right)^* . \tag{56} \]
Combining all of the phases into Eq. (289),

$$\sin \phi_{\psi K_s} = - \sin \left[ 2 \arg \left( \frac{-V_{CKM}^{cd}V_{CKM}^{cb}}{V_{CKM}^{td}V_{CKM}^{tb}} \right) \right]. \quad (57)$$

This gives the size of the angle between $\vec{A}\vec{B}$ and $\vec{B}\vec{C}$, that is $\phi_{\psi K_s} = -2\beta$. Experimentally, $\psi K_s$ mode is the only $CP$ eigenstates that has been observed so far.

(b) $B_d^0 \rightarrow \pi^+\pi^-$:

The mixing phase for this mode is,

$$\frac{A}{\tilde{A}}_{\pi^+\pi^-}^{SM} = \frac{\left( V_{CKM}^{ud}V_{CKM}^{ub} \right)^*}{V_{CKM}^{td}V_{CKM}^{tb}^*}. \quad (58)$$

Similar with the $\psi K_s$ mode,

$$\sin \phi_{\pi^+\pi^-} = - \sin \left[ 2 \arg \left( \frac{-V_{CKM}^{td}V_{CKM}^{tb}}{V_{CKM}^{ud}V_{CKM}^{ub}^*} \right) \right]. \quad (59)$$

It yields, $\phi_{\pi^+\pi^-} = -2\alpha$. 

Figure 4: Diagrams which responsible for (a) $B_d^0 \rightarrow \psi K_s$, (b) $B_s^0 \rightarrow \rho K_s$ and (c) $B_d^0 \rightarrow \pi^+\pi^-$ decays.
Figure 5: The experimental bounds for the CKM unitarity triangle of the $db$ sector, with $ub$ and $lb$ denote upper-bound and lower-bound respectively.

(c) $B_s^0 \rightarrow \rho K_s$:

The mixing phase for this mode is same with $\pi^+ \pi^-$, since the quark subprocess is same, but the mixing from the neutral $B$ mixing is given in Eq. (291) for $q = s$. After taking into account the phase mixing from $K_s$ in Eq. (276), we obtain

$$\sin \phi_{\rho K_s} = -\sin \left[ 2 \arg \left( \frac{V_{CKM}^{ud} V_{CKM}^{ub^*}}{V_{CKM}^{cd} V_{CKM}^{cb^*}} \right) \right]. \quad (60)$$

Then, this gives $\phi_{\rho K_s} = -2 \gamma$.

However, all of these asymmetries, which give the sizes of angles, could be determined in the near future experiments, $B$ factory etc.

2.5 The triangle in the SM

In the preceding sections, we have reevaluated the constraints from some direct CC processes for two generations elements, and the NC processes for the others. The last task is to describe the experimental constraints on the sides and angles of the triangle. We depicted them in Fig. [1].
From Eqs. (27) and (50), the bound for the sides $|AB|$ and $|CA|$ in the SM are
\begin{align}
0.631 & \leq |AB|^{SM} < 1.386, \\
0.25 & \leq |CA|^{SM} < 0.45
\end{align}
respectively. The bound of $|CA|$ is depicted as circles with centre point $(0,0)$ in Fig. (5). Constraints from the mixing size $x_d$ is plotted in Fig. (5) as circles with center point $(1,0)$ by using the bound in Eq. (61). On the other hand, $\epsilon$ does not have a direct influence to the sides or angles, but it has determined where the point A lies. The bound has been depicted as hyperbolas according to Eq. (61). From Fig. (6), point A must lie in the above $\rho$ axis.

We note here that the wide allowed region in the bounds are mostly due to the theoretical uncertainties appear in the hadronic matrix elements.

On the other hand, the determination of the angles are still difficult. However, the knowledge of $\rho$ and $\eta$ will give predictions for the sizes of the angles, by assuming that the triangle is perfectly satisfied. Here we display them in the ($\rho, \eta$) plane by employing the relations in Eqs. (30) $\sim$ (34). The relations yield,
\begin{align}
\sin 2\alpha & \approx \frac{2\eta \left(1 - \frac{\rho^2}{\sqrt{\lambda}}\right)}{|AB|^2 |CA|},
\end{align}
by eliminating \( \sin 2\beta \). In Fig. (6), the equations are shown with its dependence on the angles for the reliable values of \( \rho \) and \( \eta \). The maximum allowed regions for \( \rho \) and \( \eta \) have been found from the bounds in the Fig. (5). The uncertainties in the figure are due to the bounds of the sides \( |AB| \) and \( |CA| \). We can comment that the figures are not sensitive for the uncertainty of \( |AB| \), but rather sensitive for \( |CA| \). This has implication that the difference between upper-bound and lower-bound in \( \sin 2\beta \) is not visible. This leads that a better prediction for \( |CA| \), i.e. \( V_{CKM}^{ub} \), must be expected in the future.
3 FLAVOR PHYSICS WITH TREE-LEVEL FCNC

As stated in Sec. 2, in the SM FCNC processes occur through flavor changing in the CC interactions as the consequences of CKM matrix. It means that in the SM, FCNC processes have only appeared at one-loop and/or higher orders. This is usually called as GIM mechanism, which naturally brings to the realization of the suppression of FCNC processes. However, in some models beyond the SM, the tree-level FCNC is possibly realized, in contrast with the models which forbid the tree-level FCNC.

In this paper, we employ a model, which realizes the tree-level FCNC in a simple way, that is called as VQM. In fact, the calculations and results are not available for the world of VQM only, but also for the other models with similar features. We will do similar procedures in Sec. 2.

3.1 The model

The model is a simplest extension of the SM, and has been proposed for example in ref. [3]. In addition to the three standard generations of quarks, we introduce one down-type and one up-type quarks, and let the particle content in the other sectors are unaltered. Then the particle content in the quark sector becomes,

\[
\left( \begin{array}{c}
u_i \\
d_i \\
 \end{array} \right)_L, t_{L'}, b_{L'}, u_R^\alpha, d_R^\alpha
\]

where \( \alpha \) and \( i \) are generation indices with \( \alpha = 1, 2, 3, 4 \) and \( i = 1, 2, 3 \). Here, \( t' \) and \( b' \) are fourth generation up-type (\( Q_t' = 2/3 \)) and down-type (\( Q_b' = -1/3 \)) vector-like quarks.

The existence of vector-like quarks will change the NC interactions. In addition to the interactions in Sec. 2.1, there are new interactions in the Z boson, photon and neutral Higgs sectors as below (Appendix B),

\[
\mathcal{L}_A = \frac{g}{2\cos\theta_W} \left\{ u^\alpha \gamma^\mu \left[ \left( \frac{4}{3} \sin^2 \theta_W \delta^{\alpha\beta} L - \frac{4}{3} \sin^2 \theta_W \delta^{\alpha\beta} R \right) u^\beta \\ + \bar{d}^\alpha \gamma^\mu \left[ \left( \frac{2}{3} \sin^2 \theta_W \delta^{\alpha\beta} - z_d^{\alpha\beta} \right) L + \frac{2}{3} \sin^2 \theta_W \delta^{\alpha\beta} R \right] d^\beta \right\} A_\mu,
\]

\[
\mathcal{L}_Z = \frac{g}{2M_W} \left[ z_u^{\alpha\beta} \bar{u}^\alpha \left( m_u L + m_u R \right) u^\beta + z_d^{\alpha\beta} \bar{d}^\alpha \left( m_d L + m_d R \right) d^\beta \right] Z_\mu,
\]

\[
\mathcal{L}_H = \frac{g}{2M_W} \left[ z_u^{\alpha\beta} \bar{u}^\alpha \left( m_u L + m_u R \right) u^\beta + z_d^{\alpha\beta} \bar{d}^\alpha \left( m_d L + m_d R \right) d^\beta \right] H,
\]
\[ \mathcal{L}_{\chi^0} = \frac{-i g}{2M_W} \left[ z_u^{\alpha \beta} \bar{u}^\alpha (m_{u^\alpha} L - m_{u^\beta} R) u^\beta - z_d^{\alpha \beta} \bar{d}^\alpha (m_{d^\alpha} L - m_{d^\beta} R) d^\beta \right] \chi^0. \]  

(69)

where,

\[ z_d^{\alpha \beta} \equiv \sum_{i = d, s, b} V^\alpha_i V_{i\beta}^{*} = \delta^{\alpha \beta} - V^{\alpha b} V_{\beta b}^{*}, \]  

(70)

\[ z_u^{\alpha \beta} \equiv \sum_{i = d, s, b} U^\alpha_i U_{i\beta}^{*} = \delta^{\alpha \beta} - U^{\alpha b} U_{\beta b}^{*}. \]  

(71)

From the above lagrangians, it is clear that the tree-level FCNC’s have been appearing. The tree-level FCNC’s occur through the couplings \( z_d \) and \( z_u \), which have similar role with \( V_{CKM} \) in the SM to realize flavor changing among the quarks. The difference is, \( z_u(d) \) are among the quarks with same charge, and \( V_{CKM} \) is among the quarks with different charges.

Since the main interest of the paper is the unitarity of CKM matrix, especially in the \( db \) sector, we simplify the discussion by ignoring the up-type vector-like quark, and considering only one down-type vector-like quark. However, the up-type vector-like quark has an important role in connection with the effects of tree-level FCNC’s on the orthogonal conditions in Eq. 14. The problem is the up-quarks contained meson, like \( D \) meson, processes must be considered. Then, it is expected that, for example in the \( D^0 - \bar{D}^0 \) mixing, the processes are small in the SM, since the quarks that appear in the internal line are light. The heaviest one is the bottom quark, but it is still not heavy enough compared with the top quark that appears in the internal line in the \( K \) and \( B \) mesons processes. So, in the next subsection, we will discuss the CKM matrix in the VQM with only one down-type vector-like quark (henceforth called ODVQM).

### 3.2 CKM matrix

Since the vector-like quark is belong to the \( SU(2) \) singlet group, there is no interaction like \( u^i b' W \) and/or \( u^i b' \chi \) in the bare lagrangian. But, redefining the quark fields will generate the interaction which are mediated by the modified CKM matrix as in Eq. 167,

\[ V_{CKM}^{ij} \equiv \sum_{i = 1}^{3} U^{ij} V^{i3*}. \]  

(72)
The interesting feature of Eq. (72) is the unitarity violation of CKM matrix. This modified CKM matrix gives the following relation,

\[
\left(V_{CKM} V_{CKM}^\dagger\right)^{\alpha\beta} = \sum_{i=1}^{3} V^{\alpha i} V^{\beta i*} = z_d^{\alpha\beta},
\]

by using Eq. (70), and the unitarity \( UU^\dagger = 1 \). Our interest is the \( db \) sector, where the relation becomes,

\[
z_d^{db} = \sum_{i=u,c,t} V_{CKM}^{id} V_{CKM}^{ib*}.
\]

Compared with Eq. (13), it can be said that \( z_d^{db} \) indicates the unitarity violation of CKM matrix.

### 3.3 Experimental constraints on the FCNC’s

We are discussing the main content of the paper. The employed experiments are same with ones in the SM case in the preceding section. In addition to the standard \( K \) and \( B \) mesons processes, the constraints from \( S,T,U \) parameters are also confirmed briefly.

#### 3.3.1 Tree-level FCNC dominance

Before making observation of the effects of tree-level FCNC’s on the CKM matrix, let us impose upper-bounds for the mixings in the ODVQM. The constraints can be made by assuming that the processes are dominated by \( Z \) exchange tree-level FCNC diagrams.

The effective four Fermi interactions for the \( Z \) and \( W^\pm \) exchanges tree-level processes can be generated from the interactions in Eqs. (5) and (67) and their mass terms, that is

\[
\mathcal{L}_{eff}^{Z_{ODVQM}} = \sqrt{2} G_F z_d^{\alpha\beta} \left( \bar{d}^a \gamma_\mu L d^\beta \right) \left[ \bar{\nu}^j \gamma_\mu L \nu^j + \bar{\nu} \gamma_\mu L \bar{\nu} - \frac{1}{2} z_d \bar{\eta} \gamma_\mu L \eta \right],
\]

\[
\mathcal{L}_{eff}^{W^\pm} = 2 \sqrt{2} G_F V_{CKM}^{\alpha\beta} \left( \bar{u}^\alpha \gamma_\mu L d^\beta \right) \left( \bar{\nu}_l^\dagger \gamma_\mu L \nu_l^i \right) + \text{h.c.}.
\]

We also give four fermi interaction for \( W^\pm \) here, since most of the FCNC processes that are considered below are normalized by FCCC processes.

1) *Upper-bound for \( |z_d^{ds}| \)*
The upper-bound can be found from $K^+ \to \pi^+ \nu \bar{\nu}$ decay. This decay can be expressed in the quark subprocess by $s \to d \nu \bar{\nu}$. This decay is usually determined in the ratio

\[
\frac{Br(K^+ \to \pi^+ \nu \bar{\nu})_{ODVQM}}{Br(K^+ \to \pi^0 e^+ \nu_e)} = \frac{\langle \pi^+ | L_{eff} | K^+ \rangle}{\langle \pi^0 | L_{eff}^{ODVQM} | K^+ \rangle} = \frac{3}{2} \left( \frac{z_d^{ds}}{V_{CKM}^{us}} \right)^2
\]

for all three neutrinos in the final state. Here, we have used Eqs. (75) and (76), and the $SU(2)$ Clebsh-Gordan coefficients to evaluate the hadron matrix elements.

The recent experiment results give,

\[
Br(K^+ \to \pi^+ \nu \bar{\nu})_{exp} \leq 5.2 \times 10^{-9},
\]
\[
Br(K^+ \to \pi^0 e^+ \nu_e)_{exp} = 4.82 \times 10^{-2}.\]

These yield

\[
\left| \frac{z_d^{ds}}{V_{CKM}^{us}} \right| \leq 2.7 \times 10^{-4}.
\]

Then, for $|V_{CKM}^{us}|$ in Eq. (29), the upper-bound for $z_d^{ds}$ is,

\[
\left| z_d^{ds} \right| \leq 6.0 \times 10^{-5}.
\]

2) Upper-bound for $|z_d^{qb}|$

We work with $B \to X_q l^+ l^-$ decay to get the upper-bound of $|z_q^{db}|$ with $q = d, s$. Because the matrix elements of $B$ meson is not evaluated well, let us consider the decay in the quark model of $b \to q l^+ l^-$. In order to reduce the theoretical uncertainties from the $m_{B_q}$, the following ratio has been usually used,

\[
\frac{Br(B \to X_q l^+ l^-)^{ODVQM}}{Br(B \to X_c l^+ \ell^-)} \propto \frac{Br(b \to q l^+ l^-)^{ODVQM}}{Br(b \to c l^+ \ell^-)} \approx \frac{1}{4P(m_c^2)} \left| \frac{z_d^{qb}}{V_{CKM}^{sb}} \right|^2 \left[ (1 - 2 \sin^2 \theta_W)^2 + 4 \sin^2 \theta_W \right]
\]

where $P(m_c^2) \approx 0.55$ is the phase factor. The detailed calculation can be seen in ref. [18].

Experimentally the final state has not been tagged, so we can think that the upper-bounds for $q = d, s$ are same, i.e.

\[
Br(B \to X_q l^+ l^-)_{exp} \leq 5 \times 10^{-5}.
\]
Because of
\[ \text{Br} (B \rightarrow X_c l \nu)^{\exp} = 0.12, \]  
we obtain
\[ \left| \frac{z_{dq}^{qb}}{V_{CKM}^{cb}} \right| \leq 0.043, \]  
and
\[ |z_{dq}^{gb}| \leq 2.0 \times 10^{-3}, \]  
for \( |V_{CKM}^{cb}| \) in Eq. (23).

Anyway, there is also a constraint from \( B_q^0 - \bar{B}_q^0 \) mixing. Especially from \( B_d^0 - \bar{B}_d^0 \) mixing we can expect more precise bound, since experimentally \( x_d \) has been measured precisely.

The mixing in the \( Z \) tree diagram dominance is,
\[ x_q^{ODVQM} = \frac{2\sqrt{3}G_F}{3} m_{B_q} \tau_{B_q} \eta_{B_q} f_{B_q}^2 B_{B_q} \left| z_{dq}^{gb} \right|^2 \]  
Using the experiment results in Eqs. (48) and (51), one finds
\[ |z_{ds}^{sb}| > 1.1 \times 10^{-3}, \]  
\[ |z_{dd}^{db}| = 7 \times 10^{-4}. \]  
However, one should not trust in the bounds, since we need \( CP \) violations occur in the neutral \( K \) and \( B \) mesons system. So we must not ignore the CC diagrams or take \( |V_{CKM}^{tq}| \approx 0 \) into account.

The above bounds show only the absolute upper-bound for each mixing, but are less useful itself. It should better to present them in the ratios that correspond to the less model-dependent ratios of some processes. This can be achieved with the help of quadrangle relation in Eq. (23), and of course, by using the bounds of the first and second rows matrix elements in Sec. 2.4.1 and the preceding section. Our aim is to derive the ratios between the mixings of FCNC and FCCC. Especially our interest is the ratio \( \left| z_{d_1d_2}/V_{CKM}^{td_1} V_{CKM}^{td_2} \right| \), with \( d_i = d, s, b \). The reasons that most of the FCCC processes are dominated by the contribution of top exchange in the internal line. As the first step, rewrite Eq. (73),
\[ \frac{z_{d_1d_2}}{V_{CKM}^{w_2d_2}} = \frac{V_{CKM}^{u_1d_1} V_{CKM}^{u_1d_2}}{V_{CKM}^{w_2d_2} + V_{CKM}^{u_2d_1} + V_{CKM}^{r_1d_1} V_{CKM}^{r_2d_2}}. \]
where \( u_i = u, c \). Further equation,

\[
\left| \frac{z^{d_1 d_2}}{V_{CKM}^{td_1} V_{CKM}^{td_2^*}} \right| = \left| \frac{z^{d_1 d_2}}{V_{CKM}^{u_2 d_2}} \right| \times \left| \frac{V_{CKM}^{u_2 d_2}}{V_{CKM}^{td_1} V_{CKM}^{td_2^*}} \right|
\]

leads to the wanted ratios, since the upper-bounds of each ratio in r.h.s. have been determined.

Therefore, on using Eqs. (19) \( \sim \) (24), (80) and (85), we obtain

\[
d_1 = d, \quad d_2 = b \quad \text{and} \quad u_1 = u, u_2 = c \quad \rightarrow \quad \left| \frac{z^{db}}{V_{CKM}^{td} V_{CKM}^{tb^*}} \right| \leq 2.15,
\]

\[
d_1 = s, \quad d_2 = b \quad \text{and} \quad u_1 = u, u_2 = c \quad \rightarrow \quad \left| \frac{z^{sb}}{V_{CKM}^{ts} V_{CKM}^{tb^*}} \right| \leq 0.05,
\]

\[
d_1 = d, \quad d_2 = s \quad \text{and} \quad u_1 = c, \ u_2 = u \quad \rightarrow \quad \left| \frac{z^{ds}}{V_{CKM}^{td} V_{CKM}^{ts^*}} \right| \leq 1.13 \times 10^{-3}.
\]

Nevertheless, in the \( K \) meson system, the contribution of charm quark in the internal line is not negligible. So we need also the mixing ratio of \( z^{ds} \) and \( V_{CKM}^{cd} V_{CKM}^{cs^*} \). This can be verified easily by using the fact that it is larger than the top’s one. Then from Eq. (94),

\[
\left| \frac{z^{ds}}{V_{CKM}^{cd} V_{CKM}^{cs^*}} \right| \ll 1.13 \times 10^{-3}.
\]

According to the above upper-bounds, the sizes of tree-level FCNC’s are large enough in the \( db \) sector, and conversely negligible in the \( ds \) sector. We will use these ratios frequently in the next. Therefore, we can conclude here that the tree-level FCNC’s do not give significant contributions to the \( K \) meson system. Then the constraints for the CKM matrix from the neutral \( K \) meson system are same with the SM ones that already done in the preceding sections. On the other hand, the FCNC’s in the \( qb \) sectors are large enough to have significant contributions for some processes in the neutral \( B \) meson system. This will be discussed in the next section.

### 3.3.2 Quadrangle relation

According to the results in the preceding section, the tree-level FCNC should not contribute to the neutral \( K \) meson system in a good approximation. It means that \( V_{CKM}^{ts}, \ V_{CKM}^{cs} \) and \( V_{CKM}^{cd} \) are non-zero, and the unitarity in the \( ds \) sector is conserved. On the other hand, the matter is different in the \( db \) sector, since \( z^{db} \) can be comparable with \( V_{CKM}^{td} V_{CKM}^{tb^*} \).
So the unitarity will be violated, and the relation,

\[ V_{CKM}^{ud} V_{CKM}^{ub}^* + V_{CKM}^{cd} V_{CKM}^{cb}^* + V_{CKM}^{td} V_{CKM}^{tb}^* = z_d^{db} \tag{96} \]

from Eq. (73), is materialized. This relation changes the unitary triangle of CKM matrix in the SM to be quadrangle as depicted in Fig. (7).

However, there is also a considerable case when the above quadrangle relation would be violated in the \( db \) sector. It means that the size of \( z_d^{db} \) is not large enough such that comparable with the other terms in the relation. We are showing it now. The first task is evaluating the parametrization of the modified CKM matrix in Eq. (72). To compare it with the SM one, it should better to adopt the same way of Wolfenstein parametrization.

Employ the definitions of \( z_d^{\alpha \beta} \) in Eq. (70), the bounds in Eqs. (92), (93) and (94),

\[
\begin{align*}
|z_d^{ds}| &= |V^{db'} V^{sb'}^*|, \\
|z_d^{sb}| &= |V^{sb'} V^{bb'}^*|, \\
|z_d^{db}| &= |V^{db'} V^{bb'}^*|, \\
\end{align*}
\tag{97}
\]

that yields, from the bounds in Eqs. (81), (88) and (89),

\[
\begin{align*}
\frac{|z_d^{ds}|}{|z_d^{sb}|} &= \frac{|V^{db'}|}{|V^{bb'}|} < 5.5 \times 10^{-2}, \\
\frac{|z_d^{db}|}{|z_d^{sb}|} &= \frac{|V^{db'}|}{|V^{bb'}|} < 6.4 \times 10^{-1}, \\
\frac{|z_d^{ds}|}{|z_d^{db}|} &= \frac{|V^{sb'}|}{|V^{bb'}|} < 8.6 \times 10^{-2}. \\
\end{align*}
\tag{98, 99, 100}
\]

Now, we are going on parametrizing the CKM matrix in the model. For simplicity, without loss of generality let us assume that

1. \( V^{db'} \), \( V^{sb'} \) and \( V^{bb'} \) are real.

2. The \( 3 \times 3 \) part of CKM matrix is generated from the unitary matrix that diagonalizes the up-quark fields and normalized by \( V^{bb'} \)
Figure 7: The ideal quadrangle of the $db$ sector with the shortest $|DB|$ in the small FCNC case.

Under these assumptions, the CKM matrix can be written by the unitary matrices $U$ and $V$ as following,

$$V_{CKM} = U V^\dagger$$

\[ \equiv \begin{pmatrix} V_{CKM}^{ad} & V_{CKM}^{us} & V_{CKM}^{ub} \\ V_{CKM}^{cd} & V_{CKM}^{cs} & V_{CKM}^{cb} \\ V_{CKM}^{td} & V_{CKM}^{ts} & V_{CKM}^{tb} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & \bar{V}_{db}'/V_{bb}' \\ 0 & 1 & 0 & \bar{V}_{sb}'/V_{bb}' \\ 0 & 0 & 1 & 1 \end{pmatrix} \]

\[ \equiv \begin{pmatrix} V_{CKM}^{ad} & V_{CKM}^{us} & V_{CKM}^{ub} \\ V_{CKM}^{cd} & V_{CKM}^{cs} & V_{CKM}^{cb} \\ V_{CKM}^{td} & V_{CKM}^{ts} & V_{CKM}^{tb} \end{pmatrix} \left( V_{db}'/V_{bb}' + V_{ckm}^{ub} \right) \]

(101)

with $U$ and $V$ are defined in Eq. (72). We have kept only $3 \times 4$ part of $V^\dagger$. On using the upper-bounds in Eq. (98), the fourth column can be written in term of $\lambda$ up to order $O(\lambda^4)$ roughly,

$$V_{CKM}^{(4)} < \begin{pmatrix} \lambda^3 \\ \lambda^2 \\ 1 \end{pmatrix} ,$$

(102)

from the bounds in Eqs. (98) and (100). But, from Eqs. (93), (94) and (95) we hope that, at least, $ds$ sector is unitary. So it must have value with order smaller than $O(\lambda^4)$ to conserve the unitarity of CKM matrix in the $ds$ sector. For the other sectors, especially $db$ sector, the unitarity are violated. When we concern a case where the unitarities in all sectors are
conserved, i.e. $V_{CKM}^{(4)} \propto \lambda^4$, we will find a new bound for CKM matrix elements as done below.

In the case, we can adopt the parametrization in Eq. (12) for the $3 \times 3$ part of the modified CKM matrix in the model without any change. Then, the side $|\text{AD}|$ in Fig. (5) must be proportional to $|z_{d}^{db}|$ when the quadrangle is conserved. As notations, put the angles as $\alpha'$, $\beta'$, $\gamma'$ in place of $\alpha$, $\beta$ and $\gamma$ in the SM, and call the angle between $\vec{AD}$ and $\vec{DB}$ as $\theta_d$. Derivation of this angle will be given in Sec. 3.3.3. Further, more precise bound for $|V_{CKM}^{td}V_{CKM}^{tb*}|$ can be found from the figure, that is the shortest line from point B to the hyperbola $\epsilon^{tb}$. It gives,

$$0.513 \leq |\text{DB}|^{QDVQM} \leq 1.386 \, ,$$

and

$$0.0041 \leq |V_{CKM}^{td}V_{CKM}^{tb*}| \leq 0.0109 \, ,$$

(103)

where the upper-bounds is found from $x_d^{ab}$ when $|z_d^{db}| \cong 0$ as derived in Eq. (61) by using Eqs. (25) and (26).

Hence, under the assumption that the quadrangle relation is conserved, the size of $|z_d^{db}|$ must have same order with $|V_{CKM}^{td}V_{CKM}^{tb*}|$. However, employing the bound of $|V_{CKM}^{td}V_{CKM}^{tb*}|$ in Eq. (104) into the well-measured mixing size $x_d$ will show that it can not be realized. This will be done in the next.

### 3.3.3 $B_q^0 - \bar{B}_{q}^0$ mixing

The new contribution for $x_q$ in the ODVQM is coming from the tree-level $\Delta B = 2$ diagrams in Fig. (17b) (ref. [19]). In fact, the $\chi^0$ and $H$ exchange diagrams must be considered here. But, the diagram will be suppressed by a factor $m^2/M^2$ with $m$ is external line quark mass, and $M$ is $H$ or $\chi^0$ mass.

After doing a calculation for the $Z$ tree diagram as done in Sec. 3.3.1, and combining Eqs. (45) and (87),

$$x_q^{QDVQM} = \frac{G_F^2}{6\pi^2} m_{B_q} M^2 \tau_{B_q} \eta_{B_q} f_{B_q}^2 B_{B_q} \left| V_{CKM}^{tq}V_{CKM}^{tb*} \right|^2 \left| F_{\Delta B = 2}^{SM} (x_t) \right| \Delta_q, \quad (105)$$

where $x_q^{QDVQM}$ is normalized by the SM contribution. Here we adopt same notations as used
Figure 8: The upper-bound for the mixing ratio $|z_{d}^{db}/V_{CKM}^{td}V_{CKM}^{tb}|$ from $x_{d}$ in the conserved unitarity case.

in ref. [19],

$$\Delta_{q} \equiv 1 + r_{q} e^{2i\theta_{q}},$$

(106)

where, $\theta_{q}$ has been introduced in Sec. 3.4.1, and

$$r_{q} \equiv \frac{4\sqrt{2}\pi}{GF M_{W}^{2}} \left| F_{\Delta B=2,SM}^{q}(x_{t}) \right| \left| \frac{z_{d}^{qb}}{V_{CKM}^{q}V_{CKM}^{tb}} \right|^{2}.$$  

(107)

The absolute value of $\Delta_{q}$ can be written as,

$$|\Delta_{q}| = \sqrt{1 + r_{q}^{2} + 2 r_{q} \cos 2\theta_{q}}.$$  

(108)

Note here that the QCD correction factor $\eta_{Bq}$ is approximated to be same for the $Z$ contribution, since QCD correction above the scale of $M_{Z}$ are negligible.

The appearance of phase $\theta_{d} (\neq 180^0)$ reconstructs the unitary triangle as depicted in Fig. (6). Then, Eq. (105) becomes

$$x_{d}^{ODVQM} = (2.51 \times 10^{5}) \times f_{B_{d}}^{2} B_{B_{d}} A^{2} \lambda^{6} \left[ (1 - \rho)^{2} + \eta^{2} \right] \left( 1 + r_{d} e^{2i\theta_{d}} \right),$$

(109)

with

$$r_{d} \approx 514.48 \left| \frac{z_{d}^{db}}{V_{CKM}^{td}V_{CKM}^{tb}^{*}} \right|^{2},$$

(110)
where the same values as in Eq. (49) have been used. We can see here that Eq. (103) is sensitive for the term $r_d$. The reason is clear, since contribution of the SM is one order higher than that of the ODVQM. So the contribution of the ODVQM may be comparable with the SM one even for small $\left|z_d^d/V_{CKM}^{tq}V_{CKM}^{tb^*}\right| < O(10^{-2})$.

Finally, in the case when the unitarity is conserved, substituting the value of Eq. (103) or Eq. (104) into Eq. (109) gives the upper-bound for the mixing ratio. Fig. (8) shows the allowed values for mixing ratio as a function of $\theta_d$. Then, the upper-bound is,

$$\left|\frac{z_d^d}{V_{CKM}^{td}V_{CKM}^{tb^*}}\right| \leq 0.032.$$  (111)

This bound justifies the result in the preceding section when the unitarity is nearly conserved. This will be used to analyze some processes with small FCNC in Sec. 3.4.

### 3.3.4 $S, T, U$ parameters

Before going to predict some processes in the neutral $B$ meson system, we will confirm the well-known $S, T, U$ parameters with the new effects from the ODVQM (ref. [6]). $S, T, U$ parameters indicate the effects of oblique corrections from new physics as depicted in Fig. (9). According to the original definitions in ref. [7], the $S, T, U$ parameters have been defined as linear combinations of gauge bosons, $W^i_\mu$ and $B_\mu$ ($i = 1, 2, 3$). The combinations are,

$$S \equiv -16\pi \frac{d}{dq^2}\Pi^{new}_{3Y}(q^2)\bigg|_{q^2=0},$$  (112)

$$\alpha T \equiv \frac{g^2 + g'^2}{M^2_Z} \left[\Pi^{new}_{11}(0) - \Pi^{new}_{33}(0)\right],$$  (113)

$$U \equiv 16\pi \frac{d}{dq^2}\left[\Pi^{new}_{11}(q^2) - \Pi^{new}_{33}(q^2)\right]\bigg|_{q^2=0},$$  (114)

where $\Pi^{new}_{ij}$ are two point functions of two gauge bosons, $G^i_\mu$ and $G^j_\mu$, due to new physics. Here $g'$ and $g$ are the couplings for $U(1)$ and $SU(2)$ gauge fields respectively.

Fortunately, in the model there is no interaction between vector-like quark and the $SU(2)_L$ gauge bosons, $W^i_\mu$. So it is clear that without including any mixings in the internal fermions like Fig. (9b), the $S, T, U$ parameters in the model are exactly zero. It means that in the model, the $\rho$ parameter is nearly conserved, since $\alpha T \equiv \rho - 1 \simeq 0$.

However, it may be worthwhile to concern the effects of the mixings as verified in ref. [6]. We note here that, the contributions of the lowest order of mixings in the internal fermions
in Fig. (9b) would be suppressed, roughly by a factor \( (m_{q_i}/m_{b'})^2 \). The reason is simple, since according to Eq. (198), the mixings between ordinary quarks \( d^i \) and vector-like quark \( b' \) are proportional to \( m_{q_i} \). On the other hand, a factor \( 1/m_{b'}^2 \) appears from the heavy fermion propagators. So, we can say that the model would not give significant alteration for \( S, T, U \) parameters. Then we could not expect more strict bounds for the sizes of FCNC’s from the parameters.

### 3.4 The effects of tree-level FCNC’s on the neutral \( B \) meson system

The next task after deriving the bounds for each mixing in the preceding section, is reevaluating some processes in the neutral \( B \) meson system in the model. We will give predictions for \( B \to X_d \gamma \) (refs. [4] and [27]), \( B \to X_d l^+ l^- \) and the \( CP \) violations in the neutral \( B \) meson system (ref. [19]). All of processes has not been measured yet.

Especially, the \( B \to X_d l^+ l^- \) and the \( CP \) violations in the neutral \( B \) meson system are appropriate processes to verify the model. The reason is, in the processes significant tree-level FCNC’s contributions are expected, since the new contributions appear as \( Z \) exchange tree diagrams. So, the contribution will be one order larger than the CC mediated diagrams’ ones. On the other hand, the situation is different in the \( B \to X_d \gamma \) process, since all of contributions from CC mediated and NC mediated diagrams, appear in the one-loop level. Therefore, we need a mixing ratio with order \( O(1) \) to make the NC contributions to be comparable. But, the \( B \to X_d \gamma \) is still important to search the mass of down-type vector-like quark, since the heavy vector-like quark contributes in the internal line of NC mediated diagrams. Unfortunately, when the upper-bound for the mixing ratio in the \( db \) sector is suppressed, i.e. \( \sim O(10^{-2}) \) (Eq. (11)), the tree-level FCNC contribution should not be
Figure 10: The branching-ratio for the inclusive $B \to X_d \gamma$ decay as a function of $\theta_d$.

visible in the $B \to X_d \gamma$.

Experimentally, the rate of $B \to X_d \gamma$ is expected to be larger than $B \to X_d l^+ l^-$, roughly by a factor with order $1/\alpha \cong O(10^2)$. It has consequence that $B \to X_d \gamma$ is easier to observe. So, for the present we will also give the predictions for the decay.

### 3.4.1 $B \to X_d \gamma$

Similar with the SM case, here we assumpt the quark subprocess $b \to d \gamma$ decay. The new contributions that appear in the ODVQM, are mainly from the $Z$ and $\chi^0$ mediated penguin diagrams with down-quarks in the internal line.

Adopt the result in Appendix E.2, the branching ratio is predicted as,

$$ Br(b \to d\gamma)^{\text{ODVQM}} \cong \frac{\alpha G_F^2}{128 \pi^4} m_b^5 \tau_B Q_u^2 |V_{CKM}^{td} V_{CKM}^{tb*}|^2 |F_{bd\gamma}^{\text{ODVQM}}(m_b)|^2 $$

$$ = 0.252 \times A^2 \chi^6 \left[(1 - \rho)^2 + \eta^2\right] |F_{bd\gamma}^{\text{ODVQM}}(m_b)|^2, \quad (115) $$

where the QCD corrected function $F_{bd\gamma}^{\text{ODVQM}}(m_b)$ is given in Appendix E.2

$$ F_{bd\gamma}^{\text{ODVQM}}(m_b) \equiv F_L^T(m_b) \cong F_R^T(m_b) $nless\phantom{)}$$

$$ \cong 0.67 (F - 0.42 F^9 - 0.88), \quad (116) $$
because approximately the values of $F$ and $F^g$ are the following,

$$F \equiv F_L \cong F_R$$

$$\sim -0.5861 + \frac{Q_d}{Q_u} \left| \frac{z_{d}^{db}}{V_{CKM}^{td}V_{CKM}^{tb^*}} \right| e^{i\theta_d} \left[ 1 + \frac{2}{3} \times 0.234 \times (-1.667) \right. \\
+ \left( z_{d}^{dd} + z_{d}^{bb} \right) 0.333 - \left| V^{tb'} \right|^2 \left( F_{NC}^2 (r_{b'}, w_{b'}) + F_{NC}^3 (r_{b'}) \right) \right],$$  \hspace{1cm} (117)

$$F^g \equiv F^g_L \cong F^g_R$$

$$\sim -0.097 + \frac{Q_d}{Q_u} \left| \frac{z_{d}^{db}}{V_{CKM}^{td}V_{CKM}^{tb^*}} \right| e^{i\theta_d} \left[ \frac{5}{3} + \frac{2}{3} \times 0.234 \times (-1.667) \right. \\
+ \left( z_{d}^{dd} + z_{d}^{bb} \right) 0.333 - \left| V^{tb'} \right|^2 \left( F_{NC}^2 (r_{b'}, w_{b'}) + F_{NC}^3 (r_{b'}) \right) \right],$$  \hspace{1cm} (118)

where $\theta_d \equiv \arg \left( z_{d}^{db}/V_{CKM}^{td}V_{CKM}^{tb^*} \right)$ and $m_t = 174$(GeV). The existence of the mixing ratio in the NC parts makes the contribution to be suppressed for small mixing ratio in Eq. (111). We note here that the function $F_{bd \gamma}^{ODVQM}$ has dependence on the down-type vector-like quark mass $m_{b'}$ due to the functions $F_{NC}^2$ and $F_{NC}^3$ in Appendix (E.2).

We give the figures for the branching-ratio in Fig. (10) for $m_{b'} = 2$(TeV), and Fig. (11) for $\theta_d = 0^0, 180^0$. Here, $\left| z_{d}^{dvdv} \right| = 1$, $m_H = 1$(TeV), mixing ratio = 2, and $\left| V_{CKM}^{td}V_{CKM}^{tb^*} \right|$ as in Eq. (104). Anyway, the prediction of the SM for the branching ratio is,

$$8.971 \times 10^{-6} \leq Br^{SM} \leq 2.641 \times 10^{-5}.$$  \hspace{1cm} (119)
Figure 12: The differential branching ratio for $B \to X_d l^+ l^-$ as a function of the scaled invariant dilepton mass ($\hat{s} \equiv m^2/m_b^2$).

3.4.2 $B \to X_d l^+ l^-$

As stated before, the $B \to X_d l^+ l^-$ decay is an appropriate test for the model, even for the tiny mixing ratio. Because, in the model the decay also occurs at $Z$ mediated tree diagram, as well as one-loop level CC diagrams of off-shell $b \to q \gamma$ as explained in Appendix E. Note that, the one-loop NC diagrams should not give significant contribution for small mixing ratio as shown in the inclusive $B \to X_d \gamma$ decay.

The detailed calculation for the CC diagrams contribution can be seen in ref. [18]. In the reference, the calculation have been done including the effects of resonances of $J/\psi$ and $\psi'$. In the small FCNC case, we just need to concern the contribution from $Z$ mediated tree diagram that the result has been given in Eq. (82). The mixing ratio appears again if we combine both contributions, and normalize them by the CC contribution. Since the equation is too complicated, we do not write it again in the paper. The readers are expected to cite the reference for the calculation of CC diagrams. Here, we just give a figure that shows the effects of tree-level FCNC in the differential branching ratio of the $B \to X_d l^+ l^-$ decay including long distance effects due to $J/\psi$ and $\psi'$ resonances. We have put $m_t = 174$(GeV), mixing
\[ \sin 2\alpha' \]

Figure 13: The dependence of \( \sin 2\alpha' \) on the \( \theta_d \).

ratio \( \sim 0.025 \), and the CKM matrix elements as in Sec. 2.4.1 in the figure. The values of the other parameters are same with ref. [18].

### 3.4.3 \textit{CP} violations in \( B_q^0 - \bar{B}_q^0 \) mixing

The effects of vector-like quark in the \textit{CP} violated decays in Sec. 2.4.2 appear in the mixing phase in \( B_q^0 - \bar{B}_q^0 \) mixing. As done in ref. [19], the tree Z exchange diagram generates new mixing phase from Eq. (106), that is

\[ \arg \Delta_q = \tan^{-1} \left( \frac{r_q \sin 2\theta_q}{1 + r_q \cos 2\theta_q} \right). \] (120)

Therefore, the mixing phase \( \phi_f \) in Sec. 2.4.2 should be changed as,

\[ \phi_f' \equiv \phi_f + \arg \Delta_q \] (121)

in the ODVQM, with \( r_q \) is given in Eq. (107).

As discussed in the preceding section, the contributions of FCNC’s are significant for the angles, even for the sizes are small. The reason is clear, since the mixing ratio in Eq. (110) is multiplicated by a factor with order \( O(10^2) \). The same situation can be said for \( q = s \) and its mixing ratio in Eq. (113). The dependence of the angles on the \( \theta_d \) has been discussed in ref. [19], and the figures are given again in Figs. (13) and (14). The figures are drawn by
using Eq. (120). In the figures, we have put \( r_d \equiv 0.32 \) for mixing ratio \( \sim 0.025 \). The values of the angles in the SM are \( \sin 2\alpha = -0.59 \) and \( \sin 2\beta = 0.69 \) for the most reliable pair of \( (\rho, \eta) \), that is \( \rho = 0.25 \) and \( \eta = 0.3 \). From this equation and the figures, it is clear that the contribution of tree-level FCNC may be constructive or destructive depend on the \( \theta_d \). The SM results will be restored again at \( \theta = n\pi \) with \( n \) is any arbitrary integer number.

### 3.5 Theoretical studies on the FCNC

In this subsection, we will show that there are considerable constraints due to the features of mass matrix in the VQM. The idea started from the possibility of deriving the mass differences in the quark sector by introducing the vector-like quarks. The reason that, the sizes of FCNC’s, which are expressed as \( z_{u(d)} \), have been related with the unitary matrices which diagonalize the mass terms in the Yukawa sector. The relations have been given in Eqs. (70) and (71). Unfortunately, the form of these unitary matrices are not unique. Then, the sizes of FCNC’s will depend on the assumptions which have been made in generating the matrices. Here we will give the theoretical studies for the FCNC’s in the down-quark sector \( (z_d) \), since our interest in this paper is the triangle in the \( db \) sector. Let the uniqueless of the matrices as an outstanding problem that must be solved in the future. However, the studies
will support the phenomenological result in Sec. 3.3.2, i.e. the case when the FCNC’s are small.

From the results in Appendix D, we can divide the constraints into two classes, that is

1. Theoretical constraints which make the FCNC’s to be exactly zero (refs. [4] and [21]).
2. Theoretical constraints which give non-zero FCNC’s (ref. [5]).

For the detailed derivations, the class (1) is given in the Appendix D.1, and Appendix D.2 for the class (2).

In order to show that, let us start with the following down-type mass matrix without loss of generality,

\[
M_d^0 = \begin{pmatrix}
  m_d^0 & 0 & 0 & 0 \\
  0 & m_s^0 & 0 & 0 \\
  0 & 0 & m_b^0 & 0 \\
  J_d & J_s & J_b & m_b^0 \\
\end{pmatrix},
\]

where \( m^0 \) is the mass parameters in the weak basis. It can be shown that with an appropriate choice of weak basis, we can always transform the down-type mass matrix in this particular form as has been proved in Eq. (201). To derive the unitary matrix \( V \) which diagonalizes the down-quark mass, we need to analyze the following matrix,

\[
M_d^0 M_d^{0\dagger} = \begin{pmatrix}
  m_d^0 m_d^0 & 0 & 0 & m_d^0 J_d^* \\
  0 & m_s^0 m_s^0 & 0 & m_s^0 J_s^* \\
  0 & 0 & m_b^0 m_b^0 & m_b^0 J_b^* \\
  m_d^0 J_d & m_s^0 J_s & m_b^0 J_b & M^2 \\
\end{pmatrix},
\]

where, \( M \equiv \sqrt{m_b^0 + |J_d|^2 + |J_s|^2 + |J_b|^2} \). The necessary condition to have non-zero \( z_d^{ij} \) is the presence of \( J_i \) and \( J_j \). But, this is not sufficient condition here as occured in the class (1).

### 3.5.1 Natural suppression of FCNC’s

First, let see the class (1). This case is equivalent with the simplest form of Eq. (123), that is

\[
M_d^0 M_d^{0\dagger} = \begin{pmatrix}
  m_d^0 m_d^0 & 0 & 0 & m_d^0 J_d^* \\
  0 & m_s^0 m_s^0 & 0 & m_s^0 J_s^* \\
  0 & 0 & m_b^0 m_b^0 & m_b^0 J_b^* \\
  m_d^0 J_d & m_s^0 J_s & m_b^0 J_b & 2 |J|^2 + m^2 \\
\end{pmatrix},
\]
by supposing that $J_d = J_b \equiv J$, $J_s = 0$ and $m_d^0 = m_b^0 = m_{b'}^0 \equiv m^0$. In this case, we concentrate on the FCNC between $d$ and $b$ quarks because of the presence of $J_d$ and $J_b$. Moreover, at $m^0 \ll J$, we find a unitary matrix $V$ which diagonalizes Eq. (123) from Eq. (208),

$$V \approx \frac{1}{\sqrt{2}} \begin{pmatrix}
-1 & 0 & -1 & \frac{2m_d^0 J^*}{2|J|^2} \\
0 & \sqrt{2} & 0 & 0 \\
1 & 0 & -1 & 0 \\
\sqrt{2m_d^0 J} & 0 & \sqrt{2m_b^0 J} & \sqrt{2}
\end{pmatrix}. \quad (125)$$

Under the above approximation, the mass eigenvalues are found by the diagonalized $M_d^0 M_d^{0\dagger}$,

$$M_d M_d^{\dagger} \equiv V M_d^0 M_d^{0\dagger} V^\dagger \equiv \text{diag} \left( m_d^2, m_s^2, m_b^2, m_{b'}^2 \right) \simeq \text{diag} \left( \frac{m_0^4}{2|J|^2}, m_s^0, m_0^2, 2|J|^2 \right). \quad (126)$$

Here we denote $m_\alpha \ (\alpha = d, s, b, b')$ as the physical mass.

Finally, under the assumption which are considered, for example the mixing $z_{db}$ vanishes since from Eqs. (125) and (70), $|z_{db}| = |V_{db}' V_{bb}'^*| = 0$. However, there are some interesting points in Eq. (126),

1. Despite of assuming the same masses for $d$, $b$ and $b'$ quarks first, we can derive the mass differences of the physical masses by introducing only two parameters $m^0$ and $J$ in the lagrangian.

2. There is a constraint on the ratio of the mass mixing $J$ and the theoretical mass $m^0$, that is

$$\frac{2|J|^2}{m^0} \approx \frac{m_b^2}{m_d^2} \sim 2.5 \times 10^5. \quad (127)$$

Thus, $|J| \approx 1.77$(TeV).

3. From the above value of $|J|$, the mass of the down-type vector-like quark can be predicted to be $m_{b'} \approx 2.5$(TeV).

### 3.5.2 Natural existence of FCNC’s

In this section, we will show the case where theoretically $z_{u(d)}$ are non-zero and may be enhanced in many cases. The same approach with Appendix D.1 will be done. The difference
is just the assumptions which have been made to generate the eigenvalues and eigenvectors.

We start with an assumption that $J_s = 0$. In fact, this assumption is needed to make the mass matrix in Eq. (123) to be simply solved, since we do not need solve a four by four matrix.

$$
\begin{bmatrix}
  m_d^0 & 0 & 0 & m_d^0 J_d^* \\
  0 & m_s^0 & 0 & 0 \\
  0 & 0 & m_b^0 & m_b^0 J_b^* \\
  m_d^0 J_d & 0 & m_b^0 J_b & M^2
\end{bmatrix}
$$

Here, $M^2 \equiv \sqrt{m_b^2 + |J_d|^2 + |J_b|^2}$. However, even for under this assumption we still have difficulty solving the mass matrix exactly. Here, further simplification can be made by making the following assumptions. Suppose that $m_i \ll M$, so the mass matrix can be expanded in term of $m_i/M$. Then, let us consider two radical cases,

1. $m_d^0 \approx m_b^0$, and
2. $m_d^0 \ll m_b^0$.

In the case (1), we find the mass eigenvalues from Appendix D.2 as,

$$
M_d M_d^\dagger \cong \text{diag}\left(\frac{m_d^0 m_b^0}{M^2} \left[1 - 2r \frac{|J_b|^2 - |J_d|^2}{|J_b|^2 + |J_d|^2}\right], m_s^0, m_b^0 \left[1 - 2r \frac{|J_b|^2 - |J_d|^2}{|J_b|^2 + |J_d|^2}\right]^{-1}, M^2\right),
$$

where $m \equiv \frac{1}{2} (m_b^0 + m_d^0)$ and $r \equiv (m_b^0 - m_d^0)/(2m)$. Moreover, from the eigenvectors the mixing in $db$ sector is obtained,

$$
|z_d^{db}| \cong r \frac{m_d m_b}{m_b^0 M} J_d J_b \frac{2 |J_d|^2 \left(m_b^2 + |J_d|^2\right) + |J_b|^2 \left(2 |J_d|^2 - |J_b|^2\right)}{|J_d|^2 |J_b|^2}.
$$

From Eq. (130), it is easily understood that the discussion in the Sec. 3.5.1 is contained, since it corresponds to $r = 0$. Thus, in this case the existence of FCNC has a sensitivity for the size of $r$.

In the second case, the mass eigenvalues are found as the following,

$$
M_d M_d^\dagger \cong \text{diag}\left(\frac{m_d^0 m_b^0}{m_b^2 + |J_d|^2}, m_s^0, \frac{m_b^0 \left(m_b^0 + |J_d|^2\right)}{M^2}, M^2\right).
$$

The size of FCNC should be,

$$
|z_d^{db}| \cong r \frac{m_d m_b}{m_b^2 m_b^0 M} J_d J_b \left(1 + \frac{|J_b|^2}{m_b^0 + |J_d|^2}\right).
$$
Therefore, the size of FCNC could be enhanced by adjusting the parameters $J_d$, $J_b$ and $m_{\nu}^0$ under a condition that down-type vector-like quark mass should be $m_{\nu} \geq 46$(GeV) from $e^+ e^-$ collider and $m_{\nu} \geq 86$(GeV) from hadron collider (ref. [10]).
4 SUMMARY AND CONCLUSIONS

We have restudied the unitarity of CKM matrix in the SM by employing profitably the triangle that is consequences of the orthogonal conditions in Sec. 2. The study has been concentrated on the triangle of $db$ sector. To evaluate it the main processes and the present results of experiments have been used.

Further evaluation have been done to examine the effects of tree-level FCNC. The situation is realized in VQM model in Sec. 3.1. In relation to the $db$ sector, we have concerned a special case of the model, where only one down-type vector-like quark has been added. In the model, the unitary triangle has been changed to be quadrangle. From the upper-bound of each mixing under the assumption that the tree-level FCNC’s are dominant, we have found that the neutral $K$ meson processes would not be influenced. But the matter is different in the neutral $B$ meson system. However, in the case when the FCNC is small, more precise bound has been found for the mixing ratio in the $db$ sector from the $B_d^0 - \bar{B}_d^0$ mixing and the lower-bound of $\epsilon$, i.e. mixing ratio $\sim O(10^{-2})$. This has consequence that the unitary triangle in the $db$ sector is also nearly conserved, that is

$$V_{CKM}^{ud}V_{CKM}^{ub} + V_{CKM}^{cd}V_{CKM}^{cb} + V_{CKM}^{td}V_{CKM}^{tb} = z_{d}^{db} \approx 0,$$

as well as the unitary triangles in the $ds$ and $sb$ sectors. Additionally, by a rough dimension analysis we found that $S,T,U$ parameters give no significant constraints for the tree-level FCNC’s in the model.

In the last, we made some predictions for not yet measured processes in the neutral $B$ meson system. On using the allowed values of the the mixing ratio in the case when the FCNC is small, the predictions for $B \rightarrow X_d l^+ l^-$ and $CP$ asymmetries in the neutral $B$ meson system have been made. The significant contributions on these processes are expected. On the other hand, a prediction for inclusive $B \rightarrow X_d \gamma$ decay has been made for large FCNC case. The important point is that this process has dependence on the down-type vector-like quark mass. Lastly, it can be said that, the present status of experiments still approve the possibilities of new physics beyond the SM, in this case is ODVQM.

In addition to the experimental constraints, we argued the relations between the sizes of
FCNC’s and the down-quark mass matrices. From these relations, there are some considerable theoretical constraints depend on the form of down-quark mass matrix. As typical examples, we have concerned two radical cases, when the FCNC’s are exactly vanish, and the small FCNC’s are exist. In either cases the results suggest that the sizes of FCNC’s must be small. However, the relations are not unique, so it is less powerful to use them seriously when we examine the triangle in the ODVQM. On the other hand, the approaches may give an interesting mechanism, for example to generate the mass differences of fermions.

The present paper, however discuss a part of Yukawa sector. Next studies for the other sectors of unitary triangles, especially for the up-quarks sector are expected. We are looking forward to the progress of experiments on the up-quark contained mesons ($D$ meson, etc), since the heavy up-type vector-like quarks will have significant contributions on the processes.

**ACKNOWLEDGMENTS**

We would like to thank T. Morozumi for his guidance and collaboration in this research, and also to T. Muta for his assistance and useful discussion during the period of our attendance at Elementary Particle Physics Laboratory - Hiroshima University. Moreover, we feel grateful to all of our laboratory members, especialy for J. Kodaira, T. Onogi, T. Goto, S. Hashimoto, T. Inagaki and T. Yoshikawa for their private teachings, and E. Nakamoto for her warmth assistance. The study of the author is supported by a grant from Overseas Fellowship Program, BPPT-Indonesia.
APPENDIX

A THE INTERACTIONS IN THE SM

We will not reproduce all of the terms of lagrangian here. The aim of this appendix is to appreciate how flavor changing transitions emerge in the VQM. More detailed procedure could be seen in ref. [8] and references therein.

Through the calculations of the lagrangians in the Appendices (A) and (B), we use the following notations,

\[ \psi_L : \text{doublet fermions} \]
\[ q : \text{singlet fermions} \]
\[ D_\mu \equiv \partial_\mu - ig T^a W^a_\mu - ig_s Y B_\mu, \quad (134) \]
\[ W^a_{\mu\nu} \equiv \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g e^{abc} W^b_\mu W^c_\nu, \quad (135) \]
\[ B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu, \quad (136) \]
\[ \tilde{\phi} \equiv \left( \begin{array}{c} v + \phi^{0*} \\ -\chi^- \end{array} \right), \quad (137) \]
\[ \phi^0 \equiv H + i \chi^0. \quad (138) \]

Here, \( T^a \equiv \frac{1}{2} \sigma^a \) are three Pauli matrices \( (a = 1, 2, 3) \), \( Y \) denotes isospin, \( g \) is \( SU(2) \) coupling constant, \( g' \) is \( U(1) \) coupling constant, and \( e \equiv g \sin \theta_W = g' \cos \theta_W \) with \( \theta_W \) is Weinberg angle. Meanwhile, \( \tilde{\phi} \equiv i T_2 \phi^* \) and \( \chi^- \equiv (\chi^+)^* \). Weak eigenstates are denoted by \( \tilde{\psi}, \tilde{q} \) and so on.

A.1 The fermion and gauge sectors

The kinetic term for the fermions is,

\[ \mathcal{L}_F = i \tilde{\bar{q}} \gamma^\mu D_\mu \tilde{q}, \quad (139) \]

where \( D_\mu \) is defined in Eq. (134).

After making redefinition of the gauge fields in order to get the physical masses, the explanation has been omitted here, it can be rewritten as,

\[ \mathcal{L}_{NC} = e J^\mu_Q A_\mu + \frac{g}{\cos \theta_W} J^\mu Z_\mu, \quad (140) \]
Here, $J_Q^\mu$, $J^\mu$ and $J^{\mu\pm}$ are currents for neutral and charge change interactions, $Q \equiv T_3 + Y$, and

$$W^\pm_\mu \equiv \frac{1}{\sqrt{2}} \left( W^1_\mu + i W^2_\mu \right),$$  \hspace{1cm} (142)  \\
$A_\mu \equiv \sin \theta_W W^3_\mu + \cos \theta_W B_\mu,$ \hspace{1cm} (143)  \\
$Z_\mu \equiv \cos \theta_W W^3_\mu - \cos \theta_W B_\mu,$ \hspace{1cm} (144)  \\
$J_Q^\mu \equiv Q_q \bar{\tilde{q}} \gamma_\mu \tilde{q},$ \hspace{1cm} (145)  \\
$J^\mu \equiv \bar{\tilde{q}} \gamma_\mu \left( T_3 - Q_q \sin^2 \theta_W \right) L - Q_q \sin^2 \theta_W R \tilde{q} = \frac{1}{2} \gamma_\mu \left( T_3 - Q_q \sin^2 \theta_W \right) L - Q_q \sin^2 \theta_W R \tilde{q},$ \hspace{1cm} (146)  \\
$J^{\mu\pm} \equiv \bar{\tilde{q}} \gamma_\mu (T_1 \pm iT_2) L \tilde{q}.$ \hspace{1cm} (147)

In the SM, there are two kinds of gauge boson. One is belong to $SU(2)$, and the other one is $U(1)$ gauge group. The kinetic terms for them are,

$$\mathcal{L}_G = \frac{1}{4} \left( W^{\mu\nu A} W^{\mu\nu A} + B^{\mu\nu A} B^{\mu\nu A} \right).$$  \hspace{1cm} (148)

Substituting Eqs. (134), (135) and (136),

$$\mathcal{L}_{W^\pm W^\mp A} = ie \left[ \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu} \right] W_\mu^+ W_\nu^- + A^{\nu} \left( \partial^{\mu} W^{\nu+} - \partial^{\nu} W^{\mu+} \right) W_\mu^- - A^{\nu} \left( \partial_\mu W_\nu^- - \partial_\nu W_\mu^- \right) W^{\mu+}.$$  \hspace{1cm} (149)

Note here that the others $W^\pm W^\mp A$ terms also appear in the gauge fixing lagrangian in Eq. (159).

### A.2 The Higgs sector

For the Higgs sector, we can write it as below,

$$\mathcal{L}_Y = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi),$$  \hspace{1cm} (150)

where, $V(\phi)$ is the potential term, $V(\phi) \equiv -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$. This potential term will give the mass of the physical Higgs. By substituting Eqs. (4), (134) and (137),

$$\mathcal{L}_Y = \left\{ \partial_\mu + ig \sin \theta_W A_\mu + \frac{ig}{2 \cos \theta_W \left( \cos^2 \theta_W - \sin^2 \theta_W \right)} Z_\mu \right\} \chi^-$$
\[ + i M_W W_\mu^- + \frac{ig}{2} \left( H - i \chi_0 \right) W_\mu^- \}
\times \left\{ \left[ \partial^\mu - ig \sin \theta_W A^\mu - \frac{ig}{2 \cos \theta_W} \left( \cos^2 \theta_W - \sin^2 \theta_W \right) Z^\mu \right] \chi^+ 
- i M_W W_\mu^+ - \frac{ig}{2} \left( H + i \chi_0 \right) W_\mu^+ \right\}
+ \frac{1}{2} \left[ W_\mu^+ \chi^- - i M_Z Z_\mu + \left( \partial^\mu - \frac{ig}{2 \cos \theta_W} Z^\mu \right) \left( H - i \chi_0 \right) \right]
\times \left[ W_\mu^- \chi^+ + i M_Z Z_\mu + \left( \partial^\mu + \frac{ig}{2 \cos \theta_W} Z^\mu \right) \left( H + i \chi_0 \right) \right] - V(\phi). \]  

(151)

Here, \( W^\pm \) and \( Z \) bosons masses are defined as,

\[ M_W^2 \equiv \frac{g^2 v^2}{4}, \quad M_Z^2 \equiv \frac{g^2 v^2}{8 \cos^2 \theta_W}, \]  

(152)

where \( v \) is the vacuum-expectation-value (VEV) of Higgs, \( \langle \phi \rangle \equiv v \).

### A.3 The Yukawa and mass sectors

The most important term here is the lagrangian of Yukawa sector. This is the standard mechanism in the SM to generate the fermion masses. The lagrangian for Yukawa sector with up and down fermions is,

\[ \mathcal{L}_Y = -f_{d}^{ij} \bar{\psi}_L^i \phi \tilde{d}^j_R - f_{u}^{ij} \bar{\psi}_L^i \phi \tilde{u}^j_R + h.c. \]  

(153)

Here, \( f_{d}^{i\alpha} \) and \( f_{u}^{i\alpha} \) are the Yukawa couplings which give the masses of fermions through the vacuum expectation value of Higgs doublet.

Substitute the scalar (Eqs. (1) and (138)), and pick up the VEV terms, the would-be mass terms are presented as,

\[ \mathcal{L}_{mass} = -f_{d}^{ij} \bar{\psi}_L^i \sqrt{v} \tilde{d}^j_R - f_{u}^{ij} \bar{\psi}_L^i \sqrt{v} \tilde{u}^j_R + h.c.. \]  

(154)

The fermion fields should be diagonalized as,

\[ f_{d}^{ij} \sqrt{v} \equiv (V_L^\dagger m_d V_R)^{ij}, \quad f_{u}^{ij} \sqrt{v} \equiv (U_L^\dagger m_u U_R)^{ij}, \]  

(155)

where \( V \) and \( U \) are 3 \( \times \) 3 unitary matrices which relate the weak-eigenstates to mass-eigenstates as,

\[ d^j_L \equiv V^{ij} \tilde{d}^j_L, \]  

(156)

\[ u^j_L \equiv U^{ij} \tilde{u}^j_L. \]  

(157)
Redefinitions of the fermion fields in Eqs. (156) and (157) make a principal change in the CC lagrangian in Eq. (141), without changing the NC one. The effect of the diagonalizations is usually expressed as (Eq. (180)),

\[ V_{CKM}^{ij} \equiv \sum_{k=1}^{3} U_{ki} V_{kj}^*, \]  

(158)

and called as the Cabibbo-Kobayashi-Maskawa matrix (CKM matrix). Through this CKM matrix, flavor changing in the CC interactions in the SM has occurred.

A.4 The gauge-fixing term

About the gauge-fixing term, we employ the procedure of ref. [22],

\[ \mathcal{L}_{GF} \equiv -\frac{1}{\xi} \left( f_W f_W^\dagger + f_Z^2 \right), \]  

(159)

where,

\[ f_W \equiv \partial_\mu W^{\mu -} + ieA_\mu W^{\mu -} + i\xi M_W \chi^- , \]  

(160)

\[ f_Z \equiv \frac{1}{\sqrt{2}} \left( \partial_\mu Z^{\mu} + \xi M_Z \chi^0 \right). \]  

(161)

According to ref. [22], the advantage of this gauge-fixing is the absence of $W^\pm \chi^\mp A$ interaction in the full lagrangian. Therefore the number of diagrams is reduced without changing the propagator of each field, for example in the $b \rightarrow s(d)\gamma$ process.

B THE INTERACTIONS IN THE VQM

In this appendix, mainly we will give the lagrangian for the $Z$ (ref. [3]) and the neutral Higgs (ref. [4]) sector, and show how the tree-level FCNC’s comes out. We do similar procedures as in Appendix A, and gauge fixing lagrangian is as Eq. (159).

B.1 The Yukawa sector

In principle, the Yukawa sector is same with Eq. (153). The problem is how to generate the mass of vector-like quarks. In general, there are two solutions for the problem, that is

1. Introducing one singlet Higgs, and

2. Putting a bare mass for each vector-like quark in the lagrangian.
Here, we adopt the second solution. However the discussion of our interest is not altered by the choice.

Adding the bare mass terms into the Yukawa sector lagrangian in Eq. (153),
\[
\mathcal{L}_Y = - f_d^{i\alpha} \bar{\psi}_i^L \phi \bar{d}_R^{\alpha} - f_d^{4\alpha} \bar{d}_L d_R^{\alpha} v' - f_u^{i\alpha} \bar{\psi}_i^L \phi \bar{u}_R^{\alpha} - f_u^{4\alpha} \bar{u}_L u_R^{\alpha} v' + h.c. .
\] (162)

The terms which are proportionated to \( f_d^{4\alpha} \) and \( f_u^{4\alpha} \) are bare mass terms. These terms contain both diagonal couplings for vector-like quarks \((t', b')\), and off-diagonal couplings between left-handed \( SU(2) \) vector-like quarks \((t_L', b_L')\) and right-handed vector-like quarks \((u_L^i, d_L^i)\). We diagonalize the Yukawa couplings as,
\[
f_d^{i\alpha} \sqrt{v} \equiv (V_L^\dagger m_d V_R)^{i\alpha}, \quad f_d^{4\alpha} v' \equiv (V_L^\dagger m_d V_R)^{4\alpha}, \quad (163)
\]
\[
f_u^{i\alpha} \sqrt{v} \equiv (U_L^\dagger m_u U_R)^{i\alpha}, \quad f_u^{4\alpha} v' \equiv (U_L^\dagger m_u U_R)^{4\alpha}, \quad (164)
\]
where \( V \) and \( U \) are \( 4 \times 4 \) unitary matrices which relate the weak-eigenstates to mass-eigenstates and generate CKM matrix as,
\[
d_L^{\beta} \equiv V^{\beta\alpha} \bar{d}_L^{\alpha}, \quad (165)
\]
\[
u_L^{\beta} \equiv U^{\beta\alpha} \bar{u}_L^{\alpha}, \quad (166)
\]
\[
V_{CKM}^{\alpha\beta} \equiv \sum_{i=1}^{3} U^{i\alpha} V^{i\beta*} . \quad (167)
\]

Substituting the above relations into the lagrangian in Eq. (162), the neutral Higgs sector becomes,
\[
\mathcal{L}_{\phi^0} = - \frac{1}{v} m_d^{\beta} V^{\alpha i} V^{\beta i*} \bar{d}_L^\alpha d_R^\beta \phi^0 - \frac{1}{v} m_u^{\beta} U^{\alpha i} U^{\beta i*} \bar{u}_L^\alpha u_R^\beta \phi^0 + h.c. .
\] (168)

With the following definitions,
\[
z_d^{\alpha\beta} \equiv \sum_{i=1}^{3} V^{\alpha i} V^{\beta i*} = \delta^{\alpha\beta} - V^{\alpha 4} V^{\beta 4*}, \quad (169)
\]
\[
z_u^{\alpha\beta} \equiv \sum_{i=1}^{3} U^{\alpha i} U^{\beta i*} = \delta^{\alpha\beta} - U^{\alpha 4} U^{\beta 4*}, \quad (170)
\]
the neutral Higgs sector (Eq. 168) can be written as,
\[
\mathcal{L}_{\phi^0} = - \frac{1}{v} m_d^{\beta} z_d^{\alpha\beta} \bar{d}_L^\alpha d_R^\beta \varphi^0 - \frac{1}{v} m_u^{\beta} z_u^{\alpha\beta} \bar{u}_L^\alpha u_R^\beta \varphi^0 + h.c. .
\] (171)
It is apparent that if there are non-zero off-diagonal elements in $z_d^{\alpha\beta}$ and $z_u^{\alpha\beta}$, FCNC's arise in the neutral Higgs sector,

$$\mathcal{L}_H = -\frac{g}{2M_W} \left[ z_u^{\alpha\beta} \bar{u}^\alpha (m_u L + m_d R) u^\beta + z_d^{\alpha\beta} \bar{d}^\alpha (m_d L + m_d R) d^\beta \right] H, \quad (172)$$

$$\mathcal{L}_{\chi^0} = -\frac{ig}{2M_W} \left[ z_u^{\alpha\beta} \bar{u}^\alpha (m_u L - m_d R) u^\beta - z_d^{\alpha\beta} \bar{d}^\alpha (m_d L - m_d R) d^\beta \right] \chi^0. \quad (173)$$

**B.2 The neutral gauge bosons sector**

The lagrangian in these sectors can be found easily. By using Eqs. (140), (145) and (146), and inserting the new vector-like quarks, the current for $A$ sector is,

$$J_Q^\mu \equiv Q_q \bar{\tilde{q}} \gamma^\mu \tilde{q} + Q_{b'} \bar{\tilde{b}}' \gamma^\mu \tilde{b}' + Q_t' \bar{\tilde{t}}' \gamma^\mu \tilde{t}'$$

$$= \frac{1}{3} \left( 2\bar{u}^\alpha \gamma^\mu u^\alpha - \bar{d}^\alpha \gamma^\mu d^\alpha \right) A_\mu, \quad (174)$$

after making diagonalization of the quark fields through Eqs. (165) and (166).

For the $Z$ sector, we obtain,

$$J^\mu \equiv \bar{\tilde{q}} \gamma^\mu T_3 \tilde{q} - \sin^2 \theta_W \left( Q_q \bar{\tilde{q}} \gamma^\mu \tilde{q} + Q_{b'} \bar{\tilde{b}}' \gamma^\mu \tilde{b}' + Q_t' \bar{\tilde{t}}' \gamma^\mu \tilde{t}' \right). \quad (175)$$

Diagonalizing the quarks field by Eqs. (165) and (166),

$$J^\mu = \frac{1}{2} \left\{ \bar{u}^\alpha \gamma^\mu \left[ \left( z_u^{\alpha\beta} - \frac{4}{3} \sin^2 \theta_W \delta^{\alpha\beta} \right) L - \frac{4}{3} \sin^2 \theta_W \delta^{\alpha\beta} R \right] u^\beta \right.\right.$$  

$$+ \left. \bar{d}^\alpha \gamma^\mu \left[ \left( \frac{2}{3} \sin^2 \theta_W \delta^{\alpha\beta} - z_d^{\alpha\beta} \right) L + \frac{2}{3} \sin^2 \theta_W \delta^{\alpha\beta} R \right] d^\beta \right\}, \quad (176)$$

by using definitions in Eqs. (169) and (170).

**C CKM MATRIX IN THE SM**

In this appendix, we give two usually employed parametrizations. By these parametrizations, it will also be shown that there is only one remaining phase in the CKM matrix.

In the Yukawa sector lagrangian in Eq. (153), suppose that the up-quark sector is diagonalized first,

$$U_L^{*ij} f_u^{jk} \frac{v}{\sqrt{2}} U_R^{kl} \equiv m_{ij}^{ul} \quad (177)$$
Generally, when the matrices $U_L$ diagonalizes the mass matrix in the up-quark sector, the mass matrix in the down-quark sector is not automatically diagonalized, that is

$$ L_{d-mass} = U_L^{ij} f_d^{jk} \frac{v}{\sqrt{2}} V_R^{kl} \bar{d}_L^i d_R^j, \quad (178) $$

is not diagonalized. So, in order to make it diagonalized, the down-quark fields should be redefined again,

$$ m_d^{il} \equiv U_L'^{ijm} f_d^{jk} \frac{v}{\sqrt{2}} V_R^{kl} \bar{d}_L^i d_R^j, \quad (179) $$

where $V_L$ is newly introduced unitary matrix. Thus, the relation between weak-eigenstate and mass-eigenstate of $SU(2)$ doublet $d$ field becomes,

$$ d_L^i \equiv (U_L U'_L)^{il} d'_L^l, \quad (180) $$

where $d'_L$ is the mass eigenstate of down-quark. Write $U_L U'_L \equiv V_L$, we obtain the CKM matrix as Eq. (158) in the CC interactions.

### C.1 Kobayashi-Maskawa parametrization

This Appendix is based on refs. [23] and [26]. The task is to parametrize the CKM matrix which has been defined in Eq. (158). The CKM matrix is expressed as a multiplication of three unitary matrices. Since a unitary matrix could be considered as a rotation without loss of generality, we can suppose the matrices as rotations with three different angles in three dimensions space,

$$ R(\theta_1, \theta_2, \theta_3) = R_y(\theta_3) R_z(\theta_2) R_z(\theta_1), \quad (181) $$

where $R_i(\theta_j)$ is a rotation about $i$ with angle $\theta_j$. For convenience, it can be expressed as,

$$ R(\theta_1, \theta_2, \theta_3) = R_z(\theta_3) R_y(\theta_2) R_z(\theta_1). \quad (182) $$

Since as stated above, the matrices can be rewritten as rotation, let us express the matrices like below,

$$ R_z(\theta_{1(3)}) \equiv \begin{pmatrix} c_{1(3)} & -s_{1(3)} & 0 \\ s_{1(3)} e^{i \varphi_{1(3)}} & c_{1(3)} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (183) $$

53
Thus, we can redefine $c_i = \cos \theta_i$ and $s_i = \sin \theta_i$. Then, we can make $R_z(\theta_{1(3)})$ to be real, since for example,

\[
\begin{pmatrix}
1 & 0 \\
0 & e^{-i\varphi}
\end{pmatrix} \times \begin{pmatrix}
c & -s e^{-i\varphi} \\
s e^{i\varphi} & c
\end{pmatrix} \times \begin{pmatrix}
e^{i\varphi'} & 0 \\
0 & e^{i(\varphi + \varphi')}
\end{pmatrix} \propto \begin{pmatrix}
c & -s \\
s & c
\end{pmatrix} : \text{real}.
\]

Thus, we can redefine

\[
R' (\theta_1 \theta_2 \theta_3) \equiv U R (\theta_1 \theta_2 \theta_3) U'
\]

\[
= U R_z(\theta_3) R_y(\theta_2) R_z(\theta_1) U'
\]

\[
= \left( U R_z(\theta_3) U^{*\prime} \right) \left( U^\prime R_y(\theta_2) U^{*\prime} \right) \left( U^{\prime\prime} R_z(\theta_1) U' \right)
\]

\[
= R'_z(\theta_3) R'_y(\theta_2) R'_z(\theta_1),
\]

with $U$, $U'$, $U''$, $U'''$ are diagonal unitary matrices like the matrices in Eq. (187), and

\[
R'_z(\theta_{1(3)}) \equiv \begin{pmatrix}
c_{1(3)} & -s_{1(3)} & 0 \\
s_{1(3)} & c_{1(3)} & 0 \\
0 & 0 & 1
\end{pmatrix}.
\]

Further redefinition for $R'_y(\theta_2)$ is done by multiplying on either of $R' (\theta_1 \theta_2 \theta_3)$ by a matrix of the form,

\[
U''' \equiv \begin{pmatrix}
\beta & 0 & 0 \\
0 & \alpha & 0 \\
0 & 0 & \beta
\end{pmatrix},
\]

where $U'''$ commutes with $R'_z(\theta_{1(3)})$ and $|\alpha| = |\beta| = 1$. Then it yields,

\[
R'' (\theta_1 \theta_2 \theta_3) \equiv U''' R' (\theta_1 \theta_2 \theta_3) U'^{\prime\prime}\!
\]

\[
= R'_z(\theta_3) \left( U''' R'_y(\theta_2) U'^{\prime\prime}\!ight) R'_z(\theta_1)
\]

\[
= R'_z(\theta_3) R''_y(\theta_2) R'_z(\theta_1),
\]

where

\[
R''_y(\theta_2) \equiv \begin{pmatrix}
c_2 & 0 & -s_2 \\
0 & e^{i\delta} & 0 \\
s_2 & 0 & c_2
\end{pmatrix}.
\]

It is clear that just only one phase $\delta$ has been remained in the CKM matrix. Finally, the CKM matrix in the Kobayashi-Maskawa parametrization is given by multiplying the matrices $R'_z(\theta_{1(3)})$ and $R''_y(\theta_2)$ of Eqs. (187) and (190),

\[
V_{CKM} = \begin{pmatrix}
c_{12}c_{3} - s_{12}s_{3}e^{i\delta} & -s_{12}c_{3} - c_{12}s_{3}e^{i\delta} & -s_{2}c_{3} \\
c_{12}s_{3} + s_{12}c_{3}e^{i\delta} & -c_{12}s_{3} + c_{12}c_{3}e^{i\delta} & -s_{2}s_{3} \\
c_{1}s_{2} & -s_{1}s_{2} & c_{2}
\end{pmatrix}.
\]

54
C.2 Wolfenstein parametrization

This Appendix is based on ref. [24]. This parametrization is based on the phenomenological approximation. The main principle is the fact that $|V_{CKM}^{us}| \approx \sin \theta_C \approx 0.22$ is quite well determined, where $\theta_C$ is Cabibbo angle. Then, by defining $|V_{CKM}^{us}| \equiv \lambda$, it could be considered an expansion of $V_{CKM}$ in powers of $\lambda$. The experiment bound for $|V_{CKM}^{cb}|$ in Eq. (23) suggests that $|V_{CKM}^{cb}|$ is order of $\lambda^2$ rather than $\lambda$, so it should be parametrized as $|V_{CKM}^{cb}| \equiv A \lambda^2$ with $A$ is a constant which is determined by experiment bound on $|V_{CKM}^{cb}|$. Note here that the elements of $V_{CKM}^{ub}$ and $V_{CKM}^{td}$ are zero in order $O(\lambda^2)$.

It leads that we have to make an approximation up to order $O(\lambda^3)$. Unitarity of CKM matrix then describes the following form,

$$V_{CKM} \approx \begin{pmatrix}
1 - \frac{1}{2} \lambda^2 & \lambda & A \lambda^3 (\rho - i \eta) \\
-\frac{1}{2} \lambda & 1 - \frac{1}{2} \lambda^2 & A \lambda^2 \\
A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 & 1
\end{pmatrix} + O(\lambda^4). \quad (192)
$$

Here, two new parameters $\rho$ and $\eta$ must be introduced to satisfy the unitary condition. The detailed prove must be seen in ref. [24].

However, the parametrization up to $O(\lambda^2)$ is too rough to describe the $CP$ violation. The reason is that, generally all $CP$ violating quantities are proportional to,

$$J_{CP} = \text{Im} \left( V_{CKM}^{cb} V_{CKM}^{us} V_{CKM}^{ub} V_{CKM}^{cs}^* \right) \quad (193)$$

as derived in ref. [30]. Then, to get non-zero result, one additionally needs the leading $CP$ violating pieces of $V_{CKM}^{cs}$ and $V_{CKM}^{cb}$. For this, the unitarity up to $O(\lambda^5)$ in the imaginary parts has to be satisfied. The Wolfenstein parametrization up to this order is then given as Eq. (12).

D MASS MATRIX IN THE ODVQM

Let us start from the Yukawa sector lagrangian in Eq. (162) and the diagonalizations of Yukawa couplings in Eqs. (163) and (164). The first task is to derive the quark mass matrices. Pick up the mass terms of down-quark sector in Eq. (162), since the same procedure can be done in the up-quark sector.

$$\mathcal{L}_{d\text{-mass}} = -\frac{v}{2} f_d \bar{d}_i \gamma_i \tilde{d}_R - v' f_d \bar{d}_i A \gamma_i d_R$$
Redefine each field by the following transformations,

\[ \tilde{\psi}_L^i \equiv A_L^{ij} \psi_L^j \longrightarrow \tilde{d}_L^i \equiv A_L^{ij} d_L^j, \]
\[ \tilde{d}_L^4 \equiv e^{-i \theta_d} d_L^4, \]
\[ \tilde{d}_R^\alpha \equiv B_R^{\alpha \beta} d_R^\beta, \]

where \( A_L \) and \( B_R \) are arbitrary unitary matrices. Substituting them into Eq. (194),

\[ L_{d-mass} = - \left( \tilde{d}_L^j \tilde{d}_L^4 \right) \left( \frac{v}{2} f_d^{j \alpha} d_L^4 \right) B_R^{\alpha \beta} \]
\[ \equiv - \left( \tilde{d}_L^j \tilde{d}_L^4 \right) \left( m_0^{i \beta} m_0^{4 \beta} \right) d_R^\alpha \]

where,

\[ m_0^{i \beta} \equiv A_L^{ji} \frac{v}{2} f_d^{j \alpha} B_R^{\alpha \beta} = \begin{pmatrix} \text{Re} [m_d^0] & 0 & 0 & 0 \\ 0 & \text{Re} [m_s^0] & 0 & 0 \\ 0 & 0 & \text{Re} [m_b^0] & 0 \end{pmatrix}, \]

\[ m_0^{4 \beta} \equiv e^{-i \theta_d} v' f_d^{4 \alpha} B_R^{\alpha \beta} = e^{-i \theta_d} \begin{pmatrix} J_d & J_s & J_b & m_{b'} \end{pmatrix} \]
\[ = \begin{pmatrix} J_d & J_s & J_b & \text{Re} [m_{b'}] \end{pmatrix}. \]

Here, \( J_i \) are generally complex numbers. Phases in the imaginary number \( f_d \) are cancelled by the phases contained in the unitary matrices which have been multiplied on either. Therefore, we obtain the mass matrix of down-quarks as,

\[ M_d^0 = \begin{pmatrix} m_0^{i \beta} \\ m_0^{4 \beta} \end{pmatrix} = \begin{pmatrix} m_d^0 & 0 & 0 & 0 \\ 0 & m_s^0 & 0 & 0 \\ 0 & 0 & m_b^0 & 0 \\ J_d & J_s & J_b & m_{b'} \end{pmatrix}, \]

where we leave out the symbols Re and Im as knowledge, i.e. \( m_\alpha^0 \) are real and \( J_i \) are complex quantities.

The same procedure can be done in the up-quark sector respectively. Strictly, the mass matrix for up-quark sector becomes,

\[ M_u^0 = \begin{pmatrix} m_u^0 & 0 & 0 & 0 \\ 0 & m_c^0 & 0 & 0 \\ 0 & 0 & m_t^0 & 0 \\ J_u & J_c & J_t & m_{u'} \end{pmatrix}, \]
D.1 Natural suppression of tree-level FCNC’s

With the procedure in refs. [4] and [21], we try to derive the unitary matrix which diagonalize Eq. (123) in the case (1). When we concentrate on the mixing $z_d z_d^*$, we can make an assumption that $J_d = J_b \equiv J$, $J_s = 0$ and $m_d^0 = m_b^0 = m_b^0 \equiv m^0$. Under these assumptions, the discussion becomes simpler since it does not need to calculate the eigenvalues and eigenvectors of four by four matrix. Further, it can be reduced to the $2 \times 2$ matrix problem, that is

$$M_d^0 M_d^{0\dagger} = \begin{pmatrix} m_0^2 & m_0^0 J^* \\ m_0^0 J & 2 |J|^2 \end{pmatrix}.$$  

The problems are to find the mass eigenvalues $M_d M_d^{\dagger}$, with $M_d M_d^{\dagger} \equiv V M_d^0 M_d^{0\dagger} V^{\dagger}$. So we have to calculate the eigenvalues and eigenvectors of this matrix. From the definition of $M_d M_d^{\dagger}$, the eigenfunction is written as,

$$\left( M_d^0 M_d^{0\dagger} \right) V_i^{\dagger} = \lambda_i V_i^{\dagger},$$

with $\lambda_i \equiv \left(M_d M_d^{\dagger}\right)_i$. After any calculations, the eigenvalues are,

$$\lambda_1 = \frac{m_o^4}{2 |J|^2} \equiv m_d^2,$$

$$\lambda_2 = m_b^2 \equiv m_b^2,$$

$$\lambda_3 = 2 |J|^2 - \frac{m_o^4}{2 |J|^2} \equiv m_b^2,$$

and $m_s^{02} \equiv m_s^2$ respectively. Meanwhile, the eigenvectors correspond to the eigenvalues can strictly be found,

$$V^{\dagger} \cong \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 0 & 1 & \frac{\sqrt{2}m_0^0}{2 |J|^2} \\ 0 & \sqrt{2} & 0 & 0 \\ -1 & 0 & -1 & \frac{\sqrt{2}m_0^0}{2 |J|^2} \\ \frac{2m_0^0 J^*}{2 |J|^2} & 0 & 0 & \sqrt{2} \end{pmatrix}.$$  

D.2 Natural existence of tree-level FCNC’s

In this appendix, we will show the case where theoretically $z_u(d)$ are non-zero as done in ref. [5]. The same approach with Appendix D.1 will be done, by changing the assumption, that
is $J_s = 0$. This assumption is needed to avoid solving $4 \times 4$ mass matrix of Eq. (123).

$$M^0_d M^0_d = \begin{pmatrix}
    m^0_d & 0 & 0 & m^0_d J^*_d \\
    0 & m^0_s & 0 & 0 \\
    0 & 0 & m^0_b & m^0_b J^*_b \\
    m^0_d J^*_d & 0 & m^0_b J^*_b & M^2 \\
\end{pmatrix} \rightarrow \begin{pmatrix}
    m^0_d & 0 & m^0_d J^*_d \\
    0 & m^0_b & m^0_b J^*_b \\
    m^0_d J^*_d & m^0_b J^*_b & M^2 \\
\end{pmatrix}. \tag{209}
$$

Further assumption is $m_i \ll M$, then the matrix can be expanded in term of $m_i/M$,

$$M^0_d M^0_d = M^2 \begin{pmatrix}
    m^0_d / M^2 & 0 & m^0_d J^*_d / M^2 \\
    0 & m^0_b / M^2 & m^0_b J^*_b / M^2 \\
    m^0_d J^*_d / M^2 & m^0_b J^*_b / M^2 & 1 \\
\end{pmatrix}, \tag{210}
$$

where, $M = \sqrt{m^0_b + |J_d|^2 + |J_s|^2 + |J_b|^2}$. By solving the eigenvalues in order to order,

$$\left( M^0_d M^0_d \right)^{(j)} V^{\dagger}_{i}^{(j)} = \lambda_{i}^{(j)} V^{\dagger}_{i}^{(j)}, \tag{211}
$$

with $i = 1, 2, 3, 4$ are the column numbers and $j = 0, 1, 2$ are the order indices. Since the calculation is too long and complicated, we do not give it here. Note that, the calculation to first order is sufficient to derive the mixing, but up to second order calculation is required to derive the mass eigenvalues. The fourth eigenvector, which should be used to get $z_d^{db}$, is found as,

$$V^{\dagger}_{4} \approx \frac{1}{M^2} \begin{pmatrix}
    m^0_d J^*_d \\
    m^0_b J^*_b \\
    x \\
\end{pmatrix}, \tag{212}
$$

where $x$ is an undetermined number in the calculation to first order. So, it yields

$$|z_d^{db}| \approx \frac{m^0_d m^0_b J^*_d J^*_b}{M^4} \neq 0. \tag{213}
$$

Of course, this result has not finished yet, since the mass eigenvalues must be substituted into the equation. However, by this equation, at least, it is clear that non-zero mixing is possible for finite values of ordinary quarks masses.

\[ b \rightarrow q \gamma \]

We will give a detail calculation for both on-shell and off-shell $b \rightarrow q \gamma$ processes. The on-shell process is responsible for the inclusive $B \rightarrow X_q \gamma$ decay, while for the $b \rightarrow q l^+ l^-$ decay both of them are responsible. As stated in Appendix A, the diagrams that responsible for the $b \rightarrow q \gamma$ ($q = d, s$) decays are one-loop penguin diagrams as depicted in Fig. (15). The
Figure 15: Diagrams related to $b \rightarrow q \gamma$ in the SM and ODVQM.

$W^\pm X^\mp A$ vertices contained penguin diagrams disappear due to the gauge fixing in Eq. \(159\).

The $b \rightarrow q g$ diagrams, which are needed in the calculation of QCD correction, are realized by the third and fourth diagrams with replacing the external photon lines by the gluon lines.

Technically, the calculation have been done in on-shell renormalization up to second order in the external momenta. The counter terms are realized by $W^\pm$, $\chi^\pm$, $Z$, $H$ and $\chi^0$ exchange quark self-energy diagrams, and symbolically depicted in the first diagram in Fig. \(15\). More detailed calculations can be seen in refs. \[16\], \[22\] and \[25\].

E.1 In the SM

In the SM, the must-be calculated diagrams are the CC diagrams, i.e. the second and third diagrams in Fig. \(15\). The content of the would-be counter term diagrams are $W^\pm$ and $\chi^\pm$ exchange quark self-energy diagrams.

After making a straightforward calculation, the effective hamiltonian for $b \rightarrow q \gamma$ is given
as the following,
\[ H_{eff}^{SM} = \frac{G_F e}{8\sqrt{2}\pi^2} Q_u V_{CKM}^{tq} V_{CKM}^{tb*} \left\{ F_1^{CC}(x_t) [\gamma_\mu, \gamma_\nu] (m_q L + m_b R) \\
+ F_3^{CC}(x_t) \left( q_\mu \gamma_\nu q_\bar{\nu} \right) L \right\} . \]

The functions are,
\[ F_1^{CC}(x_t) = x_t \frac{-7 + 5x_t + 8x_t^2}{8(1 - x_t)^3} - 3x_t^2 \frac{(2 - 3x_t) \ln x_t}{4(1 - x_t)^4}, \]
\[ F_3^{CC}(x_t) = \frac{2}{3} \ln x_t + \frac{25 - 19x_t}{24(1 - x_t)^3} - \frac{x_t^2}{18(1 - x_t)^4} \ln x_t, \]

with \( x_t \equiv (m_t/M_W)^2 \) and \( Q_u = 2/3 \). We note here that, the cancellation of divergences are independent on the GIM mechanism. The GIM mechanism just works for making simplification of the light internal quarks (up and charm) terms, since \( m_u \) and \( m_c \) are lighter than \( M_W \), then approximately \( x_u \approx x_c \approx 0 \). On using orthogonal condition in Eq. (13),
\[ V_{CKM}^{uq} V_{CKM}^{qb*} + V_{CKM}^{cq} V_{CKM}^{cb*} = - V_{CKM}^{tq} V_{CKM}^{tb*}, \]

the up and charm quarks terms can be rewritten in term of \( V_{CKM}^{tq} V_{CKM}^{tb*} \).

In order to complete the calculation, let us consider the QCD correction up to leading logarithm (refs. [16] and [22]). The QCD corrected amplitude for on-shell \( b \rightarrow q \gamma \) is,
\[ T^{SM} = \frac{G_F e}{8\sqrt{2}\pi^2} Q_u V_{CKM}^{tq} V_{CKM}^{tb*} F_{bq\gamma}^{SM}(m_b) \bar{q}(p') [\gamma_\mu, \gamma_\nu] (m_q L + m_b R) b(p) e^\mu, \]

with,
\[ F_{bq\gamma}^{SM}(m_b) \approx 0.67 \left[ F_1^{CC} - 0.42 F_2^{CC} - 0.88 \right]. \]

Here we suppose \( \Lambda_{QCD} = 150(\text{MeV}) \) for flavor number \( n_f = 5 \), \( \mu_0 = M_W \) and \( \mu = m_b \). The functions \( F_2^{CC} \) are induced by \( b \rightarrow q g \) diagrams, that is
\[ F_2^{CC}(x_t) = -x_t \frac{2 + 5x_t - x_t^2}{8(1 - x_t)^3} - \frac{3x_t^2 \ln x_t}{4(1 - x_t)^4}. \]

Note here that QCD 'correction' in the \( B \rightarrow X_q \gamma \) decay is large. There is an enhancement for about one order of the branching ratio. But the enhancement will be smaller as top-quark mass is larger. The detailed behaviour of the QCD correction in these processes can be seen in refs. [10] and [24].
E.2 In the ODVQM

The calculation of the decay in the ODVQM is similar with in the SM as done in refs. [4] and [27]. However, the problem is coming from the unitarity violation of CKM matrix as written in Eq. (74). Then in place of Eq. (217), we have

$$V_{CKM}^{uq} V_{CKM}^{ub} \ast + V_{CKM}^{eq} V_{CKM}^{eb} \ast = -V_{CKM}^{tq} V_{CKM}^{tb} \ast + z_d^{qb}.$$ (221)

The effective hamiltonian for $b \to q \gamma$ in the model is given as the following,

$$H_{eff}^{ODVQM} = \frac{G_F e}{8\sqrt{2}\pi^2} Q_u V_{CKM}^{tq} V_{CKM}^{tb} \frac{2}{3} \sin^2 \theta_W F_1^{NC}(r_q) + F \left( q_\mu, \dot{q}_\mu \right) L \right\}.$$ (222)

where we normalized it by the CC contribution. The contained functions are,

$$F_L \equiv \left( F_1^{CC}(x_t) \right)^{ODVQM} + Q_d \frac{Q_d}{Q_u V_{CKM}^{tq} V_{CKM}^{tb}} \left[ \frac{2}{3} \sin^2 \theta_W F_1^{NC}(r_q) + z_d^{qb} \left( \sum F_2^{NC}(r_q, w_q) + F_3^{NC}(r_q) \right) \right]$$

$$+ z_d^{qq} \left( F_2^{NC}(r_q, w_q) + F_3^{NC}(r_q) \right) + z_d^{bb} \left( F_2^{NC}(r_b, w_b) + F_3^{NC}(r_b) \right) \right\},$$ (223)

$$F_R \equiv \left( F_1^{CC}(x_t) \right)^{ODVQM} + Q_d \frac{Q_d}{Q_u V_{CKM}^{tq} V_{CKM}^{tb}} \left[ \frac{2}{3} \sin^2 \theta_W F_1^{NC}(r_b) + z_d^{qb} \left( \sum F_2^{NC}(r_b, w_b) + F_3^{NC}(r_b) \right) \right]$$
\(-|V_{ib}V_{ib}^\ast|^2 \left( F_2^{NC}(r_\alpha, w_\alpha) + F_3^{NC}(r_\alpha) \right) \),

\[ F = (F_3^{CC}(x_t))^{ODVQM} + \frac{Q_d}{Q_u} \frac{z_d^{qb}}{V_{CKM}^{tb} V_{CKM}^{tb\ast}} \left[ \frac{2}{3} \sin^2 \theta_W \left( F_4^{NC}(r_q) + F_5^{NC}(r_b) \right) \right] + |V_{ib}V_{ib}^\ast|^2 \left( F_4^{NC}(r_\alpha) - r_\alpha F_5^{NC}(r_\alpha) - w_\alpha F_5^{NC}(w_\alpha) \right) \] .

These equations are obtained by using the upper-bounds in Sec. 3.3.1. The contributions of \(q = d\) for \(q = s\) and \(q = s\) for \(q = d\) can be neglected, since \(|z_d^{ds}z_d^{db}|\) and \(|z_d^{ds}z_d^{sb}|\) are tiny. Hence, the remaining mixings are \(z_d^{qq}z_d^{qb}, z_d^{qb}z_d^{bb}\) and \(z_d^{q\ast b}z_d^{b\ast b}\). Furthermore, rewriting the mixing between vector-like quark and ordinary quarks,

\[ z_d^{q\ast b}z_d^{b\ast b} = V_{q\ast b}V_{q\ast b}^\ast V_{b\ast b} V_{b\ast b}^\ast = -z_d^{qb} |V_{ib}V_{ib}^\ast|^2 , \]

gives simplification of functions as in the above equations.

On using Eq. (221), the modified functions \(F_i^{CC}\) from CC contributions are,

\[ (F_1^{CC}(x_t))^{ODVQM} = x_t \left[ \frac{-7 + 5x_t + 8x_t^2}{8(1 - x_t)^3} - 3x_t \frac{2 - 3x_t \ln x_t}{4(1 - x_t)^4} \right] + \frac{1}{2} \frac{z_d^{qb}}{V_{CKM}^{tb} V_{CKM}^{tb\ast}} , \]

\[ (F_2^{CC}(x_t))^{ODVQM} = -x_t \left[ \frac{2 + 5x_t - x_t^2}{8(1 - x_t)^3} - \frac{3x_t^2 \ln x_t}{4(1 - x_t)^4} - \frac{5}{6} \frac{z_d^{qb}}{V_{CKM}^{tb} V_{CKM}^{tb\ast}} \right] , \]

\[ (F_3^{CC}(x_t))^{ODVQM} = \frac{2}{3} \ln x_t + x_t^2 \left[ \frac{25 - 19x_t}{24(1 - x_t)^3} - x_t \frac{5x_t^2 - 2x_t - 6}{18(1 - x_t)^4} \ln x_t \right] + \frac{33}{18} \frac{z_d^{qb}}{V_{CKM}^{tb} V_{CKM}^{tb\ast}} . \]

On the other hand, the NC contribution functions are,

\[ F_1^{NC}(r_\alpha) = -10 + 15r_\alpha + 6r_\alpha^2 - 11r_\alpha^3 - 6r_\alpha (3 - 4r_\alpha) \ln r_\alpha, \]

\[ F_2^{NC}(r_\alpha, w_\alpha) = r_\alpha \left[ -20 + 39r_\alpha - 24r_\alpha^2 + 5r_\alpha^3 - 6(2 - r_\alpha) \ln r_\alpha \right] + w_\alpha \left[ -16 + 45w_\alpha - 36w_\alpha^2 + 7w_\alpha^3 - 6(2 - 3w_\alpha) \ln w_\alpha \right] , \]

\[ F_3^{NC}(r_\alpha) = \frac{-4 + 9r_\alpha - 5r_\alpha^3 - 6(1 - 2r_\alpha) \ln r_\alpha}{12(1 - r_\alpha)^4} , \]

\[ F_4^{NC}(r_\alpha) = \frac{2 + 27r_\alpha - 54r_\alpha^2 + 25r_\alpha^3 - (12 - 54r_\alpha + 36r_\alpha^2) \ln r_\alpha}{36(1 - r_\alpha)^4} , \]

\[ F_5^{NC}(r_\alpha) = \frac{-28 + 27r_\alpha + r_\alpha^3 - 6(2 - 3r_\alpha) \ln r_\alpha}{36(1 - r_\alpha)^4} . \]
We denote $Q_u = 2/3$, $Q_d = -1/3$, $x_\alpha \equiv m_\alpha^2/M_W^2$, $r_\alpha \equiv m_\alpha^2/M_Z^2$ and $w_\alpha \equiv m_\alpha^2/M_H^2$.

Note that, $F_1^{CC}$ and $F_3^{CC}$ are $W^\pm$ and $\chi^\pm$ exchange; $F_1^{NC}$, $F_3^{NC}$ and $F_4^{NC}$ are $Z$ exchange; $F_2^{NC}$ and $F_5^{NC}$ are $\chi^0$ and $H$ exchange diagrams contribution. In $F_2^{NC}$, the first term comes from $\chi^0$ exchange, while the second term comes from $H$ exchange diagram. The dependence on the vector-like quark mass in the inclusive $B \to X_q \gamma$ decay occur in the functions $F_2^{NC}$ and $F_3^{NC}$. The dependence is depicted in Fig. [12] with $m_H = 750\,(\text{GeV})$.

Finally, the QCD corrected amplitude for on-shell $b \to q \gamma$ in the ODVQM becomes,

$$T_{ODVQM}^{b\gamma} = \frac{G_F^e}{8\sqrt{2}\pi^2} Q_u V_{CKM}^{tq} V_{CKM}^{tb^*} \bar{q}(p') \left[ \gamma_\mu, \gamma_\nu \right] \left( F_{b\gamma}^L(m_b)m_q L + F_{b\gamma}^R(m_b)m_b R \right) b(p) e^\mu,$$

where $F_L^T$ and $F_R^T$ are defined as,

$$F_{b\gamma}^L(m_b) \equiv 0.67 \left[ F_L - 0.42 F_L^g - 0.88 \right],$$

$$F_{b\gamma}^R(m_b) \equiv 0.67 \left[ F_R - 0.42 F_R^g - 0.88 \right].$$

The functions $F_{L(R)}^g$ are given as the following,

$$F_{L}^g \equiv \left( F_{CC}^{LC}(x_L) \right)^{ODVQM} + \frac{Q_d}{Q_u} V_{CKM}^{tq} V_{CKM}^{tb^*} \left[ \frac{2}{3} \sin^2 \theta_W F_1^{NC}(r_q) \right.$$

$$+ \left. z_{d}^{qq} \left( F_2^{NC}(r_q, w_q) + F_3^{NC}(r_q) \right) + z_{d}^{bb} \left( F_2^{NC}(r_b, w_b) + F_3^{NC}(r_b) \right) \right]$$

$$- \left. \left| V_{t'b'} \right|^2 \left( F_2^{NC}(r_{t'}, w_{t'}) + F_3^{NC}(r_{t'}) \right) \right),$$

$$F_{R}^g \equiv \left( F_{CC}^{RC}(x_R) \right)^{ODVQM} + \frac{Q_d}{Q_u} V_{CKM}^{tq} V_{CKM}^{tb^*} \left[ \frac{2}{3} \sin^2 \theta_W F_1^{NC}(r_b) \right.$$

$$+ \left. z_{d}^{qq} \left( F_2^{NC}(r_q, w_q) + F_3^{NC}(r_q) \right) + z_{d}^{bb} \left( F_2^{NC}(r_b, w_b) + F_3^{NC}(r_b) \right) \right]$$

$$- \left. \left| V_{t'b'} \right|^2 \left( F_2^{NC}(r_{t'}, w_{t'}) + F_3^{NC}(r_{t'}) \right) \right).$$

### F NEUTRAL MESONS MIXING AND CP VIOLATIONS

In the first part of this appendix, we shall give a common explanation that hold for either mixings in the neutral $K$ and $B$ mesons system. The discussion is based on the refs. [7], [20], [28] and [24].

Consider an arbitrary neutral meson $P^0$ and its antiparticle $\bar{P}^0$. The neutral $P$ meson state could be expressed as a linear combination of them as,

$$\left| P \right> \equiv a \left| P^0 \right> + b \left| \bar{P}^0 \right>,$$

(240)
and is governed by the time-dependent Schrodinger equation

\[ H |P\rangle = E |P\rangle \]

\[ \equiv \left(M - \frac{i}{2}\Gamma\right) |P\rangle . \quad (241) \]

Here, \( M \) and \( \Gamma \) are \( 2 \times 2 \) matrices. \( M \) is the Hermitian part of \( H \) and would-be a mass matrix. \( \Gamma \) is anti-Hermitian part of \( H \), and describes the exponential decay of the \( P \) meson.

On using Eqs. (240) and (241), and multiplying the l.h.s. by \( \langle \bar{P} | H | P \rangle = E \langle \bar{P} | P \rangle \),

\[ \langle \bar{P} | H | P \rangle = E \langle \bar{P} | P \rangle . \quad (242) \]

Generally, the states \( |P^0\rangle \) and \( |\bar{P}^0\rangle \) are orthogonal. This orthogonality causes,

\[ E \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \langle P^0 | H | P^0 \rangle & \langle P^0 | H | \bar{P}^0 \rangle \\ \langle \bar{P}^0 | H | P^0 \rangle & \langle \bar{P}^0 | H | \bar{P}^0 \rangle \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} M_0 - \frac{i}{2}\Gamma_0 & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M_0 - \frac{i}{2}\Gamma_0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}, \quad (243) \]

since CPT invariance, \( \langle P^0 | H | P^0 \rangle = \langle \bar{P}^0 | H | \bar{P}^0 \rangle \), guarantees \( M_{11} = M_{22} \equiv M_0 \) and \( \Gamma_{11} = \Gamma_{22} \equiv \Gamma_0 \). From the above equation, the eigenvalues and eigenvectors are found as,

\[ E_\pm = M_0 - \frac{i}{2}\Gamma_0 \pm \sqrt{ \left(M_{12} - \frac{i}{2}\Gamma_{12}\right) \left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)} , \]

\[ \begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\sqrt{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}} \begin{pmatrix} \sqrt{M_{12} - \frac{i}{2}\Gamma_{12}} \\ \pm \sqrt{M_{12} - \frac{i}{2}\Gamma_{12}} \end{pmatrix}. \quad (244) \]

On using Eq. (244), we can define the eigenstates for the neutral \( P \) meson mass matrix as,

\[ |P_\pm \rangle \equiv p |P^0\rangle \pm q |\bar{P}^0\rangle , \quad (245) \]

with eigenvalues \( E_\pm \equiv M_\pm - \frac{i}{2}\Gamma_\pm \). Further, the following definition is usually used in place of \( E_\pm \),

\[ \Delta E \equiv E_- - E_+ \]

\[ \equiv \Delta M - \frac{i}{2}\Delta \Gamma \]

\[ = 2\sqrt{ \left(M_{12} - \frac{i}{2}\Gamma_{12}\right) \left(M_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)} . \quad (246) \]
It causes,

\[ \Delta M = 2 \text{Re} \sqrt{(M_{12} - \frac{i}{2} \Gamma_{12}) \left( M_{12}^* - \frac{i}{2} \Gamma_{12}^* \right)}, \quad (247) \]

\[ \Delta \Gamma = -4 \text{Im} \sqrt{(M_{12} - \frac{i}{2} \Gamma_{12}) \left( M_{12}^* - \frac{i}{2} \Gamma_{12}^* \right)} \]

\[ = \frac{4}{\Delta M} \text{Re} \left( M_{12} \Gamma_{12}^* \right), \quad (248) \]

\[ \frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}}} = \frac{1 - \epsilon_t}{1 + \epsilon_t}. \quad (249) \]

The parameter \( \epsilon_t \) in Eq. (249) indicates the mixing that occurs due to the off-diagonal elements in the mass matrix (Eq. (243)), even for all \( \Gamma \)'s were zero. By definition in Eq. (249), Eq. (245) can be rewritten again,

\[ |P_\pm\rangle = \frac{1}{\sqrt{2 \left( 1 + |\epsilon_t|^2 \right)}} \left[ (1 + \epsilon_t) |P^0\rangle \pm (1 - \epsilon_t) |\bar{P}^0\rangle \right]. \quad (250) \]

Let us make a convention for charge conjugation as below,

\[ C |P^0\rangle = - |\bar{P}^0\rangle, \quad C |\bar{P}^0\rangle = - |P^0\rangle. \quad (251) \]

It yields under \( CP \) transformation,

\[ CP |P^0\rangle = |\bar{P}^0\rangle, \quad CP |\bar{P}^0\rangle = |P^0\rangle. \quad (252) \]

because the \( P \)'s are pseudoscalars. Then, the parameter \( \epsilon_t \) in Eq. (249) indicates the \( CP \) violation in the neutral \( P \) meson system. In particular, the case when \( \epsilon_t \approx 0 \), or no relative phase between \( M_{12} \) and \( \Gamma_{12} \), means the absence of the \( CP \) violation, and Eq. (250) would be \( CP \) odd and even states respectively,

\[ |P_+\rangle = \frac{1}{\sqrt{2}} \left( |P^0\rangle + |\bar{P}^0\rangle \right) \rightarrow CP: +, \quad (253) \]

\[ |P_-\rangle = \frac{1}{\sqrt{2}} \left( |P^0\rangle - |\bar{P}^0\rangle \right) \rightarrow CP: -. \quad (254) \]

F.1 Neutral \( K \) meson

We make further discussion in the case of the neutral \( K \) meson by replacing the above \( P \) with \( K \). The description of mixing in the \( K^0 - \bar{K}^0 \) mixing is simplified by the fact that it is accounted for, to a good approximation, by physics of two generations of quark. So
the direct $CP$ violation can be ignored, and the mass eigenstates coincide with the $CP$ eigenstates. Because of $2\pi$ is $CP$ even while $3\pi$ is $CP$ odd, only

$$K_+ \rightarrow \pi^0 \pi^0, \pi^+ \pi^- , \tag{255}$$

$$K_- \rightarrow \pi^0 \pi^0 \pi^0, \pi^+ \pi^- \pi^0 \tag{256}$$
decays can be realized when the $CP$ is conserved. Conversely, a small $K_- \rightarrow 2\pi$ decay will provide $CP$ violation in the $K$ meson system. It is customary to call $K_+$ as $K_S$ and $K_-$ as $K_L$, since two-pion final state has much larger phase than the three-pion final state. Then, $K_+$ that decays into $2\pi$, decays much faster than $K_-$. Experimentally,

$$\tau_S = 0.9 \times 10^{-10}(s) , \tag{257}$$

$$\tau_L = 5.2 \times 10^{-8}(s) , \tag{258}$$

$$\Delta M_{K}^{exp} \equiv M_{K_L} - M_{K_S} \cong 3.5 \times 10^{-12}(\text{MeV}) . \tag{259}$$

Thus, the relation

$$\Delta \Gamma \cong -2\Delta M \tag{260}$$
is found, since

$$\Gamma_S^{exp} = 7.4 \times 10^{-12}(\text{MeV}) , \, \Gamma_L^{exp} = 1.3 \times 10^{-14}(\text{MeV}) , \tag{261}$$
then, $\Delta \Gamma \equiv \Gamma_L - \Gamma_S \cong -\Gamma_S$.

Now, we are ready to evaluate the indirect $CP$ violation in the neutral $K$ meson system. This can be described by rewriting Eq. (250) as,

$$|K_S\rangle = \frac{1}{\sqrt{1 + |\epsilon_t|^2}} \left( |K_1\rangle + \epsilon_t |K_2\rangle \right) , \tag{262}$$

$$|K_L\rangle = \frac{1}{\sqrt{1 + |\epsilon_t|^2}} \left( |K_2\rangle + \epsilon_t |K_1\rangle \right) , \tag{263}$$

where

$$|K_{1(2)\rangle} \equiv \frac{1}{\sqrt{2}} \left( |K^0\rangle \pm |\bar{K}^0\rangle \right) \tag{264}$$
are $CP$ eigenstates. Thus, since $K_S$ and $K_L$ are not $CP$ eigenstates, they may decay into $3\pi$ and $2\pi$. 66
Figure 17: $\Delta P = 2$ diagrams for $P^0 - \bar{P}^0$ mixing in the (a) SM and (b) ODVQM, with $P = B, K$.

Experimentally measured parameter of $CP$ violation in the neutral $K$ meson system is defined as,

$$
\epsilon \equiv \frac{\langle (2\pi)_{I=0} | H_{\Delta S=2} | K_L \rangle}{\langle (2\pi)_{I=0} | H_{\Delta S=2} | K_S \rangle}. 
$$

(265)

This quantity is different with $\epsilon_t$ in Eq. (247), that is independent of the phase convention. In the SM, $M_{12}$ and $\Gamma_{12}$ are nearly real, then

$$
\Delta M^{SM} \approx 2 \text{Re} M_{12}^{SM},
$$

(266)

$$
\Delta \Gamma^{SM} \approx 2 \text{Re} \Gamma_{12}^{SM},
$$

(267)

$$
\epsilon_t \approx \frac{i \text{Im} M_{12}^{SM} + \frac{1}{2} \text{Im} \Gamma_{12}^{SM}}{\Delta M^{SM} - \frac{1}{2} \Delta \Gamma^{SM}} \approx e^{i\pi/4} \frac{\text{Im} M_{12}^{SM}}{\sqrt{2} \Delta M^{SM}}.
$$

(268)

Therefore, using Eqs. (264) and (265),

$$
\epsilon = \frac{i \text{Im} A_0 + \epsilon_t \text{Re} A_0}{\text{Re} A_0 + \epsilon_t \text{Im} A_0} 
\approx \epsilon_t + \frac{i \text{Im} A_0}{\text{Re} A_0} 
\approx \frac{e^{i\pi/4}}{\sqrt{2} \Delta M^{SM}} \left( \text{Im} M_{12}^{SM} + 2 \frac{\text{Im} A_0}{\text{Re} A_0} \text{Re} M_{12}^{SM} \right),
$$

(269)

where $A_i \equiv \langle (2\pi)_{I=i} | H_{\Delta S=2} | K^0 \rangle$, and use $|\epsilon_t \text{Im} A_0| \ll |\text{Re} A_0|$, $\text{Im} \Gamma_{12}^{SM} \propto -2(\text{Re} A_0)(\text{Im} A_0)$, and $(1/2)\Delta \Gamma^{SM} \approx \text{Re} \Gamma_{12}^{SM} \propto (\text{Re} A_0)^2$.

We shall give $M_{12}$ in the SM which can be found by calculating the matrix elements of
the $\Delta S = 2$ effective Hamiltonian as,

$$M_{12}^{SM} = \langle K^0 \left| H_{\Delta S = 2}^{SM} \right| \bar{K}^0 \rangle + \text{(long distance effects)}, \quad (270)$$

where $H_{\Delta S = 2}^{SM}$ is found from the calculation of the diagrams in Fig. (17 a) with $d_1 = d$ and $d_2 = s$,

$$H_{\Delta S = 2}^{SM} = \frac{G_F^2 M_W^2}{4\pi^2} \sum_{i,j=c,t} \lambda_i \lambda_j \eta_{ij} F_{\Delta S = 2}^{SM}(x_i, x_j) \left( \bar{s} \gamma_\mu L d \right)^2 + \text{h.c.}, \quad (271)$$

where the $u$ quark and the external momenta have been ignored, since $m_u \ll M_W$. Here, $\eta_{ij}$ is the QCD correction in the diagrams, $\lambda_i \equiv V_{CKM}^{id} V_{CKM}^{is*}$, the function $F_{\Delta S = 2}^{SM}$'s are (ref. [24]),

$$F_{\Delta S = 2}^{SM}(x, y) = \left[ \frac{1}{4} + \frac{3}{2(1-x)} - \frac{3}{4(1-x)^2} \right] \frac{xy}{x-y} + (x \leftrightarrow y) - \frac{3xy}{4(1-y)(1-y)}, \quad (272)$$

$$F_{\Delta S = 2}^{SM}(x_i) = x_i \left[ \frac{1}{4} + \frac{9}{4(1-x_t)} - \frac{3}{2(1-x_t)^2} \right] + \frac{3}{2} \left[ \frac{x_t}{x_t - 1} \right]^3 \ln x_t, \quad (273)$$

The puzzle of hadron matrix element here is evaluated by PCAC, and for simplicity ignore the long distance effects,

$$\langle K^0 \left| (\bar{s} \gamma_\mu L d)^2 \right| \bar{K}^0 \rangle \approx \frac{1}{3} m_K f_K^2 B_K, \quad (274)$$

where $f_K$ and $B_K$ are the decay constant of $K$ meson and the bag parameter. Lastly, combining all of the above results,

$$M_{12}^{SM} = \frac{G_F^2}{12\pi^2} m_K f_K^2 B_K M_W^2 \left[ \lambda_c^2 \eta_c F_{\Delta S = 2}^{SM}(x_c) + \lambda_t^2 \eta_t F_{\Delta S = 2}^{SM}(x_t) + 2\lambda_c \lambda_t \eta_{ct} F_{\Delta S = 2}^{SM}(x_c, x_t) \right] \quad (275)$$

Therefore, substituting this equation into Eq. (269) will give a constraint for the contained CKM matrix elements.

On the other hand, we also need the quantity in Eq. (249) which is frequently used in the calculation of $CP$ violations in the neutral $B$ meson system. Moreover, $t$ quark and mixing $c - t$ quarks term in Eq. (275) can be ignored too, because of suppression due to $|V_{CKM}^{id} V_{CKM}^{is*}| \approx O(\lambda^5)$. Therefore, from Eq. (249) approximately,

$$\left( \frac{q}{p} \right)^{SM}_{K^0} = \frac{\left( V_{CKM}^{id} V_{CKM}^{cs*} \right)^*}{V_{CKM}^{id} V_{CKM}^{cs*}}, \quad (276)$$
F.2 Neutral $B$ meson

In principle, the description of mixing in the neutral $B_q$ meson system is similar with the $K$’s one. We just change $s$ by $b$ and $d$ by $q$, or $d_1 = q$ and $d_2 = b$ in Fig. (17). The main purpose here is to derive the $CP$ asymmetries when the neutral $B_q$’s decay into arbitrary final state $f$.

In the SM framework, the important differences with the $K$ meson are that the contribution in the internal line is just coming from top quark only, because of the small masses of up and charm quarks, and the CKM matrix elements of up and charm quarks have same order with the top one. The second one is,

$$|\Gamma_{12}^{SM}| \ll |M_{12}^{SM}|,$$  \hspace{1cm} (277)

which can be strictly showed by calculating the real and imaginary parts of the diagrams in Fig. (17 a), i.e.

$$\left| \frac{\Gamma_{12}}{M_{12}} \right|^{SM} \approx \left| \frac{\pi}{3F_{\Delta B = 2}^{SM}(x_t)} \right| \frac{m_b^2}{m_t^2} \ll 1,$$  \hspace{1cm} (278)

where $F_{\Delta B = 2}^{SM}(x_t) \equiv F_{\Delta S = 2}^{SM}(x_t)/x_t$, and $F_{\Delta S = 2}^{SM}(x_t)$ has been given in Eq. (273). However, when $m_u \neq m_c$, a small $CP$ violation in the $B_0^q - \bar{B}_0^q$ will occur. It can be seen in the calculation of imaginary part without ignoring the light quarks, that is

$$\Gamma_{12}^{SM} \approx \frac{G_F^2m_b^3f_{B_q}B_{B_q}}{36\pi} \left| V_{CKM}^{tq}V_{CKM}^{tb*} \right|^2 \left[ 1 + \frac{8}{3} \frac{m_c^2}{m_b^2} \frac{V_{CKM}^{tq}V_{CKM}^{cB}}{V_{CKM}^{tq}V_{CKM}^{cB*}} \right],$$  \hspace{1cm} (279)

by using GIM mechanism and the fact that $m_u \ll m_c \ll m_t$. Since the real part is

$$M_{12}^{SM} \approx \frac{G_F^2m_t^2m_bf_{B_q}B_{B_q}}{12\pi^2} \left| V_{CKM}^{tq}V_{CKM}^{tb*} \right|^2 \left| F_{\Delta B = 2}^{SM}(x_t) \right|,$$  \hspace{1cm} (280)

one finds their ratio as,

$$\left( \frac{\Gamma_{12}}{M_{12}} \right)^{SM} \approx \frac{\pi}{3} \frac{1}{F_{\Delta B = 2}^{SM}(x_t)} \frac{m_b^2}{m_t^2} \left[ 1 + \frac{8}{3} \frac{m_c^2}{m_b^2} \frac{V_{CKM}^{tq}V_{CKM}^{cB}}{V_{CKM}^{tq}V_{CKM}^{cB*}} \right].$$  \hspace{1cm} (281)

The second term indicates the small $CP$ violation if $m_c \neq m_b$. However, we are not interested in this small $CP$ violation, and the detailed discussion can be seen in ref. 28. From Eqs. (247) and (277), the mass splitting approximately becomes $\Delta M_q^{SM} \approx 2 \left| M_{12}^{SM} \right|$. This has
consequences for the size of mixing in the neutral $B_q$ meson to be written as,

$$
x_{q}^{SM} = \frac{\Delta M_{q}^{SM}}{\Gamma_B} = \frac{G_F^2 m_{B_q} M_W^2 \tau_{B_q} \eta_{B_q} f_{B_q}^2 B_{B_q}}{2} \left| V_{CKM}^* t_q V_{CKM}^{0\ell}\right|^2 \left| F_{B=2}^{SM}(x_{t}) \right|. \quad (282)
$$

Now, we are going on evaluating the $CP$ violations in the neutral $B_q$ meson system. First, the time-evolution neutral $B$ states can be written by using Eqs. (245), (248) and (247) as,

$$
|B_q^0(t)\rangle = g_+(t) |B_q^0\rangle + \frac{q}{p} g_-(t) |\bar{B}_q^0\rangle, \quad (283)
$$

$$
|\bar{B}_q^0(t)\rangle = \frac{p}{q} g_-(t) |B_q^0\rangle + g_+(t) |\bar{B}_q^0\rangle, \quad (284)
$$

where, the initial condition is, at $t = 0$, $B_q^0(t)$ ($\bar{B}_q^0(t)$) would be pure $B_q^0$ ($\bar{B}_q^0$), and

$$
g_\pm(t) \equiv \frac{1}{2} e^{-i(M_L-i/2\Gamma_L)t} \left( 1 \pm e^{-i(\Delta M-i/2\Delta\Gamma)t} \right). \quad (285)
$$

The next step is defining the quantity that measures the $CP$ violations. This can be done by concerning the asymmetries when the $B_q^0$‘s decay into any final state $f$. It is defined as,

$$
a_f(t) \equiv \frac{\Gamma \left( B_q^0(t) \to f \right) - \Gamma \left( \bar{B}_q^0(t) \to f \right)}{\Gamma \left( B_q^0(t) \to f \right) + \Gamma \left( \bar{B}_q^0(t) \to f \right)}. \quad (286)
$$

The next task is then calculating the amplitude of the decays. On using Eqs. (277), (283), (284) and (285), one finds

$$
|A_f(t)|^2 \quad + \quad |\bar{A}_f(t)|^2 \quad \propto \quad \left( 1 \pm \frac{p}{q} \right)^2 \left[ \frac{1}{2} \left( 1 + \left| \frac{q}{p} \bar{A}_f \right|^2 \right) + 2 \text{Im} \left( \frac{q}{p} \bar{A}_f \right) \sin(\Delta M t) e^{-1/2\Delta\Gamma t} \right]

+ \left( 1 \mp \frac{p}{q} \right) \cos(\Delta M t) \left( 1 - \left| \frac{q}{p} \bar{A}_f \right|^2 \right) e^{-1/2\Delta\Gamma t}, \quad (287)
$$

with $A_f \equiv \langle f | H | B_q^0 \rangle$ and $\bar{A}_f \equiv \langle f | H | \bar{B}_q^0 \rangle$. Substituting them into Eq. (286) yields

$$
a_f(t) \equiv - \sin(\Delta M t) \sin \phi_f. \quad (288)
$$

Here,

$$
\sin \phi_f \equiv \text{Im} \left[ \frac{q}{p} \left( \frac{A}{A} \right) \right], \quad (289)
$$

$$
\left( \frac{A}{A} \right)_f \equiv \frac{\langle f | H | B_q^0 \rangle}{\langle f | H | B_q^0 \rangle}. \quad (290)
$$
Nevertheless, in the neutral $B$ meson system, the quantity in Eq. (249) can be found by calculating the real part of Fig. (17) same with the neutral $K$ meson. But, here the top quark dominance approximation can be done by the GIM without any ambiguities. Thus,

\[
\left( \frac{q}{p} \right)^{SM}_{B_q^0} = \left( \frac{V_{CKM}^{tq}V_{CKM}^{tb}}{V_{CKM}^{tq}V_{CKM}^{tb}} \right)^*.
\]  

(291)
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