Quantization of coupled 1D vector modes in integrated photonic waveguides

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Abstract. A quantum mechanical analysis of the guided light in integrated photonics waveguides is presented. The analysis is made starting from one-dimensional (1D) guided vector modes by taking into account the modal orthonormalization property on a cross section of an optical waveguide, the vector structure of the guided optical modes and the reversal-time symmetry in order to quantize the 1D vector modes and to derive the quantum momentum operator and the Heisenberg equations. The results provide a quantum-consistent formulation of the linear and nonlinear quantum light propagations as a function of forward and backward creation and annihilation operators in integrated photonics. As an illustration, an application to an integrated nonlinear directional coupler is given, that is, both the nonlinear momentum and the Heisenberg equations of the nonlinear coupler are derived.
1. Introduction

In this work, we present a quantum mechanical analysis of the optical propagation corresponding to coupled one-dimensional (1D) vector modes in integrated waveguides [1], that is, modes spatially confined in 2D and therefore propagating in one spatial dimension; obviously these modes contain the particular case of 2D modes, that is, modes spatially confined in 1D and therefore with two free spatial dimensions for optical propagation. It is important to stress that the integrated photonic waveguides have as a common physical characteristic that the modes are confined in the order of the light wavelength. The optical 1D modes can be obtained in conventional integrated optical waveguides (which can be made in optical glasses, crystals, semiconductors, polymers, silicon, and so on) [2], in photonic crystal waveguides [3] and in nanophotonic waveguides (plasmonic modes) [4]; all these possible waveguide structures are considered in this work as different kinds of integrated photonic waveguides for which a common quantum mechanical formulation will be presented.

Recently, a preliminary study about quantization of coupled 2D modes in integrated optical waveguides was presented [5]. In this work, we generalize the above study to 1D vector modes in such a way that 1D modes are quantized in a macroscopic way and the quantum propagation is described by the spatial coupling of creation and annihilation operators associated with the guided modes by a previous calculation of a general quantum momentum operator with linear and nonlinear coupling terms. This analysis can be applied to different modal coupling problems between the optical guides, which form the basis of many linear and nonlinear integrated photonic devices such as integrated couplers, integrated gratings, integrated junctions, integrated interferometers and so on. These coupled modes optical devices provide very interesting classical dynamic properties (see [6] and references therein) and geometric properties [7], therefore, a proper quantization of the coupled modes propagation allows to increase the optical possibilities of these devices. It must be stressed that a proper quantization must be based on both the orthonormalization property of guided modes on the cross section of optical guides and the complex vector structure of the guided modes; in fact, as has been shown [5], the quantization results of propagation are only consistent when the longitudinal components of the guided modes are taken into account and when the orthonormalization...
property is used to construct the optical field operator. We will quantize the coupling of forward and backward guided modes propagating along an arbitrary $\eta$-direction (longitudinal direction), by calculating the quantum momentum operator as a function of the operators $\hat{a}_{\rho\sigma}$ and $\hat{a}_{\rho\sigma}^\dagger$, where $\rho\sigma$ is a double positive or negative discrete index, different from zero, characterizing the guided $\rho\sigma$-modes; for forward modes (and forward operators): $\rho > 0$, $\sigma > 0$ (this will be denoted as $\rho\sigma > 0$) and for backward modes (and backward operators): $\rho < 0$, $\sigma < 0$ (this will be denoted as $\rho\sigma < 0$). In this context, it is important to point out the pioneering works on macroscopic canonical quantization [8]–[11] based on mode functions; these works were directed toward the derivation of a quantum Hamiltonian operator and therefore on temporal interactions described by the dynamic of the operators $\hat{a}(t)$ and $\hat{a}^\dagger(t)$. Consequently, the orthonormalization condition used in the quantization process was obtained in the volume of a dielectric medium; likewise, these works described successful applications to optical devices formed with material media with longitudinal dependence of the electrical permittivity, such as optical cavities [10, 11]. In short, these works used, for the first time, a formalism based on classical nonplane modes. Nevertheless, in our case, we are interested in the derivation of the quantum momentum operator $\hat{M}$ and therefore on spatial interactions described by the $\eta$-spatial evolution of the operators $\hat{a}_{\rho\sigma}(\eta)$ and $\hat{a}_{\rho\sigma}^\dagger(\eta)$. Moreover, as was commented hereinbefore, a proper orthonormalization condition for guided modes must be defined and used on the cross section of the optical guides. Likewise, in our case, the arbitrary $\eta$-direction is the propagation direction and the $\xi\gamma$-plane is the plane where the guided modes are confined by a graded refractive index $n(\xi, \gamma)$, that is, 1D modes are considered, which contain as a particular case the 2D modes which are confined along one spatial dimension by means of a graded refractive index $n(\xi)$. The formalism developed in this work allows the description of both linear and nonlinear integrated optical devices in a fully consistent quantum way, in an analogous way to 3D optical devices such as beamsplitters, linear and nonlinear interferometers and so on, which, at present, have acquired importance for their many and different quantum applications [12]–[14].

Returning to the quantum propagation problem in space, we must also point out that several fundamental studies have been made [15]–[19]. These studies have provided the background of the quantum theory of light propagation in space, that is, they have proven that the quantum operator which must be obtained, in order to describe quantum spatial propagation, is the momentum operator which in turn must be calculated by integrating the energy tensor element $T^{33}$ over the hyperplane $dx\,dy\,cdt$. Nevertheless, and to our knowledge, these kinds of studies were restricted to simple plane modes, and therefore they can be rigorously applied only to optical devices described by plane waves propagating along an arbitrary $\eta$-direction. The plane modes do not present longitudinal components like guided modes, and moreover, the corresponding field operators have a particular expression [17]. In fact, in this work, we will show that the above optical field operators are obtained as a limiting case of the optical field operators associated with guided modes.

On the other hand, several interesting and illustrative studies on non-classical light in guided mode coupling devices were presented [20]–[24]. Nevertheless, the corresponding momentum operators were proposed by transferring, in a straightforward way, the results obtained with plane waves, and therefore without taking into account either vector structure or orthonormalization property of guided modes. Likewise, analogies with optical quantum Hamiltonians, describing temporal interaction, were used to construct and propose momentum operators for coupled mode devices. This work extends the pioneering results obtained in [8]–[11] to the case of coupled mode quantum propagation in optical guides, justifies many
of the considerations introduced in [15]–[19] and generalizes the preliminary results presented in [5] to linear and nonlinear coupled 1D modes.

The primary aim of this work is to avoid methods based on heuristic arguments or mechanical analogies and thus to obtain ab initio the momentum operators for the quantum mechanical analysis of the optical propagation corresponding to coupled 1D modes in waveguides and consequently the Heisenberg equations governing the spatial coupling between the operators \( \hat{a}_{\rho\sigma}(\eta) \) and \( \hat{a}_{\rho\sigma}^+(\eta) \). The plan of the work is as follows: first of all, we briefly review the orthonormalization property of guided 1D vector modes present on a cross section of an optical guide (these modes are supported in channel guides, optical fibers and so on), which contain as a limiting case the guided 2D vector modes (which are supported in planar guides). Next, we calculate, in an explicit and a detailed way, the corresponding quantum momentum operators when there is no coupling between the guided modes, that is, the free field part of the momentum operator. Next, we will derive the momentum operator for modal coupling by the quantization of a classical (linear and nonlinear) polarization \( P(\xi, \gamma) \) perturbing the free momentum operator, and, by using the time-reversal symmetry, the momentum operator for a general linear contradirectional coupling will be obtained as a function of the operators \( \hat{a}_{\rho\sigma}(\eta) \) and \( \hat{a}_{\rho\sigma}^+(\eta) \). Finally, and as an illustration, an application to a well-known integrated nonlinear directional coupler is given, that is, both its momentum operator and its quantum Heisenberg equations are derived ab initio.

2. Classical analysis of guided 1D vector modes

Let us consider monochromatic guided 1D vector modes, that is, with only one temporal mode of frequency \( \omega \) in an optical waveguide characterized by a refractive index profile \( n^2(\xi, \gamma) \) and represented by a vector field solution with the following complex amplitude:

\[
E_{\rho\sigma}(\xi, \gamma, \eta, t) = E_{\rho\sigma}(\xi, \gamma, \eta) e^{-i\omega t} = E_{\rho\sigma}(\xi, \gamma) e^{i[\beta_{\rho\sigma}\eta - i\omega t]}, \tag{1a}
\]

\[
H_{\rho\sigma}(\xi, \gamma, \eta, t) = H_{\rho\sigma}(\xi, \gamma, \eta) e^{-i\omega t} = H_{\rho\sigma}(\xi, \gamma) e^{i[\beta_{\rho\sigma}\eta - i\omega t]}, \tag{1b}
\]

where \( \beta_{\rho\sigma} \) is the propagation constant of the \( \rho\sigma \)-mode with \( \rho\sigma = \pm m \pm n \) \((m, n > 0)\), and \( \{E_{\rho\sigma}(\xi, \gamma), H_{\rho\sigma}(\xi, \gamma)\} \) is the complex vector amplitude of the guided 1D modes. By taking into account the expressions (1a) and (1b), it is easy to derive from the Maxwell equations the following modal equations for the complex amplitudes of guided 1D vector modes

\[
\nabla_t \wedge E_{\rho\sigma} + i\beta_{\rho\sigma} u_\eta \wedge E_{\rho\sigma} = i\omega \mu H_{\rho\sigma}, \tag{2a}
\]

\[
\nabla_t \wedge H_{\rho\sigma} + i\beta_{\rho\sigma} u_\eta \wedge H_{\rho\sigma} = -i\omega \epsilon E_{\rho\sigma}, \tag{2b}
\]

where \( \nabla_t = (\partial_\xi, \partial_\gamma, 0) \) and \( u_\eta \) is a unit vector pointing in the \( \eta \)-direction, that is, perpendicular to a cross section of the optical waveguide. The amplitudes of the guided modes can be separated into the transverse field components \( \{E_{\eta\rho\sigma}(\xi, \gamma), H_{\eta\rho\sigma}(\xi, \gamma)\} \) and the longitudinal field components \( \{E_{l\rho\sigma}(\xi, \gamma), H_{l\rho\sigma}(\xi, \gamma)\} \) in such a way that from equations (2a) and (2b), and by using the standard procedures based on the Lorentz reciprocity theorem for optical waveguides [25], it is found that the norm of a guided \( \rho\sigma \)-mode, on a cross section \( \eta \) of the waveguide, is given by the expression [5, 25, 26]

\[
\| E_{\rho\sigma} \| = \| H_{\rho\sigma} \| = \left\{ 2 \operatorname{sgn}(\rho\sigma) \int \int \{E_{\rho\sigma} \wedge H_{\rho\sigma}^*\} u_\eta d\xi d\gamma \right\}^{1/2}, \tag{3}
\]
where the function $\text{sgn}(\rho \sigma)$ is defined to be $+1$ if $\rho \sigma > 0$, and $-1$ if $\rho \sigma < 0$. As will be shown, this function will justify one of the more important results used in quantum light propagation problems, i.e. the positivity of the momentum [17, 19]. Next, by denoting the transverse components of the normalized guided 1D vector mode as $e_{\rho \sigma}$ and $h_{\rho \sigma}$, the following quasi-complete orthonormalization condition, on a cross section of an optical guide, is obtained [25, 26]:

$$2 \text{sgn}(\rho \sigma) \int \int \{ e_{\rho \sigma} \land h_{\rho' \sigma'}^* \} u_{\eta} \, d\xi \, d\gamma = \delta_{|\rho \sigma|, |\rho' \sigma'|},$$

(4)

where $\delta_{|\rho \sigma|, |\rho' \sigma'|}$ is the Kronecker delta. Note that the orthonormalization condition is determined only by the transverse field components of the guided modes and it is a quasi-complete orthonormalization condition since for the cases $\rho \sigma = -\rho' - \sigma'$, equation (4) is not equal to zero; however, in spite of that, it will be a very useful relationship for obtaining the quantum momentum operator.

On the other hand, we will assume dispersion-free and non-magnetic media and distinguish between modal electrical and magnetic energies stored per unit guide length and under temporal averaging by means of the following standard expressions:

$$W^e_{\rho \sigma} = W^e_{t \rho \sigma} + W^e_{l \rho \sigma} = \int \int d\xi \, d\gamma \, e E_{t \rho \sigma} E_{t \rho \sigma}^* + \int \int d\xi \, d\gamma \, e E_{l \rho \sigma} E_{l \rho \sigma}^*,$$

(5a)

$$W^m_{\rho \sigma} = W^m_{t \rho \sigma} + W^m_{l \rho \sigma} = \int \int d\xi \, d\gamma \, \mu_0 H_{t \rho \sigma} H_{t \rho \sigma}^* + \int \int d\xi \, d\gamma \, \mu_0 H_{l \rho \sigma} H_{l \rho \sigma}^*,$$

(5b)

where the energy fractions stored by the transverse ($t \equiv \xi \gamma$) and longitudinal ($l \equiv \eta$) modal field components have also been distinguished. Next, by forming scalar products of the modal equations (2a) and (2b) with the appropriate modal field components and also combining the results in an appropriate way we obtain the following relations:

$$\nabla_t (E_{t \rho \sigma} \land H_{t \rho \sigma}^*) + i \beta_{\rho \sigma} u_{\eta} (E_{t \rho \sigma} \land H_{t \rho \sigma}^*) = i \omega \mu_0 H_{t \rho \sigma} H_{t \rho \sigma}^* - i \omega e E_{l \rho \sigma} E_{l \rho \sigma}^*,$$

(6a)

$$\nabla_t (E_{t \rho \sigma} \land H_{t \rho \sigma}^*) + i \beta_{\rho \sigma} u_{\eta} (E_{l \rho \sigma} \land H_{l \rho \sigma}^*) = i \omega \mu_0 H_{l \rho \sigma} H_{l \rho \sigma}^* - i \omega e E_{l \rho \sigma} E_{l \rho \sigma}^*.$$

(6b)

Now, by integrating over the guide cross section and taking into account the 2D divergence theorem, we obtain the following relevant expression:

$$\text{sgn}(\rho \sigma) \| E_{\rho \sigma} \|^2 \beta_{\rho \sigma} \equiv \text{sgn}(\rho \sigma) \| H_{\rho \sigma} \|^2 \beta_{\rho \sigma} = \omega (W_{t \rho \sigma} - W_{l \rho \sigma})$$

(7)

with $P_{\rho \sigma} = \text{sgn}(\rho \sigma) \| E_{\rho \sigma} \|^2$ being the modal power or energy flow (positive for forward modes and negative for backward modes), and with $W_{t \rho \sigma} = W^e_{t \rho \sigma} + W^m_{t \rho \sigma}$ and $W_{l \rho \sigma} = W^e_{l \rho \sigma} + W^m_{l \rho \sigma}$, that is, the total transverse and total longitudinal energies, respectively.

### 3. Quantization of the free momentum of guided 1D vector modes

In this section, we will quantize the free momentum operator, that is, the momentum operator associated with the 1D vector modes without any interaction among them, in order to clarify the consistency of the method; the momentum operator for coupled modes will be derived later in the paper. First of all, we present the optical 1D vector mode field operator by using the normalized monochromatic modes, where the temporal mode fulfills a periodic condition in time such as was established in [17], that is, $E_{\rho \sigma} (\xi, \gamma, \eta, 0) = E_{\rho \sigma} (\xi, \gamma, \eta, T)$, where $T$ defines the mentioned periodic condition; next, the quantum free momentum operator is derived, and finally a simple canonical quantization of the momentum operator is presented.
3.1. Optical 1D vector mode field operator

Although we will justify the expression of the modal field operator associated with a spatial multimode regime later, one can write, by taking into account the usual field operator in quantum optics [12]–[14], [27], the modal field operator \{\hat{e}, \hat{h}\} as a superposition of normalized guided vector mode operators [5], that is

\[
\hat{e} = \sum_{\rho} \hat{e}_{\rho} = \sum_{\rho} (\hat{e}_{\rho}^{(+)} + \hat{e}_{\rho}^{(-)}) = \sum_{\rho} \frac{\sqrt{\hbar \omega}}{\sqrt{c T}} \left\{ \frac{1}{\| E_{\rho} \|} \left[ \hat{a}_{\rho} E_{\rho} e^{-i\lambda t} + \hat{a}_{\rho}^\dagger E_{\rho}^* e^{i\lambda t} \right] \right\}, \tag{8a}
\]

\[
\hat{h} = \sum_{\rho} \hat{h}_{\rho} = \sum_{\rho} (\hat{h}_{\rho}^{(+)}) + \hat{h}_{\rho}^{(-)}) = \sum_{\rho} \frac{\sqrt{\hbar \omega}}{\sqrt{c T}} \left\{ \frac{1}{\| H_{\rho} \|} \left[ \hat{a}_{\rho} H_{\rho} e^{-i\lambda t} + \hat{a}_{\rho}^\dagger H_{\rho}^* e^{i\lambda t} \right] \right\}, \tag{8b}
\]

where the superscripts (+) and (−) denote the well-known field operators with positive and negative frequencies, respectively [12, 13]. The factor \((c T)^{1/2}\), introduced in [17], can be regarded as a normalization condition of the temporal mode. Moreover, note that the creation \(\hat{a}\) and annihilation \(\hat{a}^\dagger\) operators are dimensionless as usual. On the other hand, we must note that the norm, given by equation (3), represents a generalization of the transmission conditions claimed in [15]–[17]. In these works it was assumed that the electrical field must be divided by \(n^{1/2}\) and the magnetic field multiplied by \(n^{1/2}\); however, we can prove that these factors are obtained as a particular case of the modal norms for plane modes in a homogeneous medium with a constant index \(n\) and therefore with a propagation constant equal to \(\beta = \omega n/c\). If we assume, for the sake of simplicity, that the plane waves have amplitude equal to one on a cross section \(A\) then the modal norms, obtained from equation (3), for both transverse electric (TE) and transverse magnetic (TM) plane modes are as follows [5]: \(\| E_{\rho} \|_{TE} = (2\epsilon_0/\mu_0)^{1/2} A n^{1/2}\) and \(\| E_{\rho} \|_{TM} = (2\mu_0/\epsilon_0)^{1/2} A n^{1/2}\). These expressions are, excepting constants related to the system of units, identical to those assumed in [15]–[17] by using arguments of continuity of the field; however, as has just been shown, they can be obtained in a natural way from the modal norms. Likewise, with the above results, \(\| E_{\rho} \|_{TE}\) and \(\| E_{\rho} \|_{TM}\), it is easy to prove that the operators \{\hat{e}_{\rho}, \hat{h}_{\rho}\} contain, as a particular case, the usual expression of the operator associated with a mode plane \(\sigma\) with amplitude unity, propagating in free space along the \(\eta\)-direction and considered in a volume \(V = A c T\), that is, if we consider the case of a TE mode, then by taking into account that in the vacuum the density of energy is related to the density of the energy flow [28] by the constant \(c\), then the norm \(\| E_{\rho} \|_{V}\) in the volume \(V\) must be related to the propagating mode norm as follows: \((c T)^{1/2}\| E_{\rho} \|_{TE} = c^{1/2}\| E_{\rho} \|_{V}\), therefore, \(\| E_{\rho} \|_{V} = (2\epsilon_0 A c T)^{1/2} = (2\epsilon_0 V)^{1/2}\), which allows us to obtain the usual operator field for a plane mode in a volume \(V\) [12]–[14].

3.2. Quantum free momentum operator

Next, we will evaluate the quantum momentum operator by integrating, over the hyperplane \(d\xi d\gamma d\tau\), the energy tensor element \(T^{33}\) [17, 25, 29, 30], whose classical expression, by using the normalized 1D vector modes, can be written as follows:

\[
T^{33} = \frac{1}{2} \left[ (\epsilon_0 n^2 e_1 e_1 - \epsilon_0 n^2 e_2 e_2) + (\mu_0 h_1 h_1 - \mu_0 h_2 h_2) \right]. \tag{9}
\]

Therefore, the free quantum momentum operator for the 1D vector modes takes the form [5, 17]:

\[
\hat{M}_O = \frac{1}{2} \int \int \int_T \left[ (\epsilon_0 n^2 \hat{e}_1 \hat{e}_1 - \epsilon_0 n^2 \hat{e}_2 \hat{e}_2) + (\mu_0 \hat{h}_1 \hat{h}_1 - \mu_0 \hat{h}_2 \hat{h}_2) \right] d\xi d\gamma d\tau = \hat{M}_O^e + \hat{M}_O^\mu. \tag{10}
\]
where the operator $\hat{M}_O$ can be separated into the electric and magnetic contributions $\hat{M}_e^O$ and $\hat{M}_m^O$ respectively. Likewise, it is important to note the presence of the longitudinal components, which are essential to derive the quantum momentum operator, as was previously made clear for 2D vector modes TE and TM [5], which are a limiting case of the 1D modes analyzed in this work. Now, by inserting equations (8a) and (8b) into equation (10), we can distinguish between the terms (crossing terms) which rapidly oscillate over time as follows: $\exp\{\pm 2i\omega t\}$. Therefore, the temporal integration (which is equivalent to a temporal averaging) of all these terms is equal to zero, and on the other hand, there are terms (non-crossing terms) whose temporal dependence has been cancelled, that is, temporal integration over the period $T$ is equal to one, therefore the electric and magnetic contributions to the quantum momentum operator can be rewritten as follows:

$$\hat{M}_e^O = \frac{\hbar \omega}{2\|E_{\rho\sigma}\|^2} \sum_{\rho\sigma} \int \int \left[ \epsilon_{\rho\sigma} n^2 (E_{\rho\sigma} E_{\rho\sigma}^* - E_{\rho\sigma}^* E_{\rho\sigma}) (\hat{a}_{\rho\sigma} \hat{a}_{\rho\sigma}^\dagger + \hat{a}_{\rho\sigma}^\dagger \hat{a}_{\rho\sigma}) \right] \, d\xi \, d\gamma, \quad (11a)$$

$$\hat{M}_m^O = \frac{\hbar \omega}{2\|H_{\rho\sigma}\|^2} \sum_{\rho\sigma} \int \int \left[ \mu_{\rho\sigma} (H_{\rho\sigma} H_{\rho\sigma}^* - H_{\rho\sigma}^* H_{\rho\sigma}) (\hat{a}_{\rho\sigma} \hat{a}_{\rho\sigma} + \hat{a}_{\rho\sigma}^\dagger \hat{a}_{\rho\sigma}^\dagger) \right] \, d\xi \, d\gamma. \quad (11b)$$

Taking into account equation (7), the above integrals can be identified with the $\rho\sigma$-modal electric and magnetic energy flows and therefore the sum of the equations (11a) and (11b) gives the following free momentum operator:

$$\hat{M}_O = \hat{M}_e^O + \hat{M}_m^O = \sum_{\rho\sigma} \frac{\hbar}{2} \text{sgn}(\rho\sigma) \beta_{\rho\sigma} (\hat{a}_{\rho\sigma} \hat{a}_{\rho\sigma}^\dagger + \hat{a}_{\rho\sigma}^\dagger \hat{a}_{\rho\sigma}). \quad (12)$$

Quantum relations and commutation rules for forward and backward modes were derived in [5] by using the reversal-time symmetry of the Maxwell’s equations and the $\eta$-reversal symmetry of modes, that is,

$$\hat{a}_{\rho\sigma < 0} = -\hat{a}_{\rho\sigma < 0}^\dagger, \quad \hat{a}_{\rho\sigma > 0}^\dagger = -\hat{a}_{\rho\sigma > 0}. \quad (13a)$$

$$[\hat{a}_{\rho\sigma}, \hat{a}_{\rho'\sigma'}^\dagger] = \text{sgn}(\rho\sigma) \delta_{|\rho| |\rho'|} \delta_{|\rho| |\rho'|}. \quad (13b)$$

Now, by taking into account equation (13b) and choosing a positive contribution of the vacuum fluctuations, the free momentum operator can be rewritten in a normal order as follows:

$$\hat{M}_O = \sum_{\rho\sigma} \hbar \text{sgn}(\rho\sigma) \beta_{\rho\sigma} \hat{a}_{\rho\sigma}^\dagger \hat{a}_{\rho\sigma}, \quad (14)$$

where the fluctuations of the vacuum have been cancelled; moreover, this result contains important physical content: the function $\text{sgn}(\rho\sigma)$ ensures the positivity of momentum, which was assumed in [18, 19], and now it is a natural consequence of the modal norms.

3.3. A simple canonical quantization

As a complementary view of the momentum quantization, we present a simple macroscopic canonical quantization analogous to the one used for the Hamiltonian quantization. We start by
obtaining the classical momentum of a 1D vector mode $\rho \sigma$ by using the complex representation of the modal field, compatible with the modal norm given by equation (3), that is,

$$E_{\rho \sigma}(\xi, \gamma, t) = E_{\rho \sigma}(\xi, \gamma, t) + E^*_{\rho \sigma}(\xi, \gamma, t),$$

$$H_{\rho \sigma}(\xi, \gamma, t) = H_{\rho \sigma}(\xi, \gamma, t) + H^*_{\rho \sigma}(\xi, \gamma, t),$$

which must be inserted into the energy tensor element $T^{33}$ (now without using normalized modes) and integrated over the hypercross section of the guide $d\xi \, dy \, c \, dt$. Therefore, by making the temporal integration in the interval $[0, T]$ the following free classical momentum is obtained:

$$M_{O_{\rho \sigma}} = cT \int \int d\xi \, dy \, \left[ \epsilon_o n^2 (E_{\rho \sigma} E^*_{\rho \sigma} - E_{\rho \sigma} E^*_{\rho \sigma}) + \mu_o (H_{\rho \sigma} H^*_{\rho \sigma} - H_{\rho \sigma} H^*_{\rho \sigma}) \right].$$

(16)

Now, by taking into account equation (7) the following expression is obtained for the classical momentum:

$$M_{O_{\rho \sigma}} = \text{sgn}(\rho \sigma) \frac{\beta_{\rho \sigma}}{\omega} \parallel E_{\rho \sigma} \parallel^2 cT,$$

(17)

where $\parallel E_{\rho \sigma} \parallel^2$ is given by equation (3); the above expression is the starting point for obtaining a simple canonical quantization of the momentum and the 1D vector modal fields simultaneously; that is, we rewrite the 1D vector mode fields as follows:

$$E_{\rho \sigma}(\xi, \gamma, \eta, t) = E_{\rho \sigma}(\eta) e^{\rho \sigma}(\xi, \gamma) e^{-i\omega t},$$

$$H_{\rho \sigma}(\xi, \gamma, \eta, t) = E_{\rho \sigma}(\eta) h^{\rho \sigma}(\xi, \gamma) e^{-i\omega t},$$

(18a)

(18b)

where $E_{\rho \gamma}$ is a constant complex amplitude and $e_{\rho \sigma}^{\rho \sigma}$ and $h_{\rho \sigma}^{\rho \sigma}$ are the normalized modes. Now, by inserting these expressions into equation (17), and taking into account the modal normalization, we obtain

$$M_{O_{\rho \sigma}} = \text{sgn}(\rho \sigma) \frac{\beta_{\rho \sigma}}{\omega} cT E_{\rho \sigma}(\eta) E^*_{\rho \sigma}(\eta).$$

(19)

Let us choose new canonical variables in order to obtain a classical momentum which can be identified with a well-known mechanical problem, that is, a harmonic oscillator, but now the role of time will be played by the longitudinal variable $\eta$ and the role of frequency will be played by the propagation constant $\beta_{\rho \sigma}$. Therefore, let us consider

$$E_{\rho \sigma}(\eta) = \frac{1}{\sqrt{2cT \text{sgn}(\rho \sigma) \beta_{\rho \sigma} / \omega}} (\beta_{\rho \sigma} q_{\rho \sigma} + ip_{\rho \sigma}),$$

$$E^*_{\rho \sigma}(\eta) = \frac{1}{\sqrt{2cT \text{sgn}(\rho \sigma) \beta_{\rho \sigma} / \omega}} (\beta_{\rho \sigma} q_{\rho \sigma} - ip_{\rho \sigma}),$$

(20a)

(20b)

accordingly the classical momentum fulfills the equation of a (spatial) harmonic oscillator, that is,

$$M_{O_{\rho \sigma}} = \frac{1}{2} \left( p_{\rho \sigma}^2 + \beta_{\rho \sigma}^2 q_{\rho \sigma}^2 \right),$$

(21)

which can be also easily obtained by starting from the modal wave equation for 1D vector modes propagating along the $\eta$-direction. That is:

$$\frac{\partial^2 E_{\rho \sigma}}{\partial \eta^2} + \beta_{\rho \sigma}^2 E_{\rho \sigma} = 0,$$

(22)
which clearly suggests a spatial harmonic oscillator, that is, with \( \eta \) substituting the time \( t \) and \( \beta_{\rho\sigma} \) substituting the frequency \( \omega \) of a temporal harmonic oscillator. It must be stressed that spatial harmonic and anharmonic oscillators appear in a natural way in linear and nonlinear classical optical propagation problems \([31,32]\), which can be described by means of a functional variational formulation based on spatial density functionals (spatial Lagrangians) \([32]\) that is fully compatible with the canonical results used in this subsection.

Next, by using the principle of quantization in the classical momentum operator given by equation \((21)\) and the following change of the operators:

\[
\hat{a}_{\rho\sigma} = \frac{1}{\sqrt{2\hbar \text{sgn}(\rho\sigma) \beta_{\rho\sigma}}} (\beta_{\rho\sigma} \hat{q}_{\rho\sigma} + i \hat{p}_{\rho\sigma}),
\]

\[
\hat{a}_{\rho\sigma}^\dagger = \frac{1}{\sqrt{2\hbar \text{sgn}(\rho\sigma) \beta_{\rho\sigma}}} (\beta_{\rho\sigma} \hat{q}_{\rho\sigma} - i \hat{p}_{\rho\sigma}),
\]

the complex amplitude \( E_{o\rho\sigma} \) can be quantized and accordingly the 1D vector modal field can be too, that is, from equations \((20a)\) and \((23a)\), we obtain

\[
\hat{e}^{(+)}(\xi, \gamma, \eta, t) = \hat{E}_{o\rho\gamma}(\eta) e_{\rho\sigma}(\xi, \gamma) e^{-i\omega t} = \left(\frac{\hbar}{cT}\right)^{1/2} \frac{E_{\rho\sigma}}{\|E_{\rho\sigma}\|} \hat{a}_{\rho\sigma},
\]

\[
\hat{e}^{(-)}(\xi, \gamma, \eta, t) = \hat{E}_{o\rho\gamma}^\dagger(\eta) e_{\rho\sigma}(\xi, \gamma) e^{+i\omega t} = \left(\frac{\hbar}{cT}\right)^{1/2} \frac{E_{\rho\sigma}}{\|E_{\rho\sigma}\|} \hat{a}_{\rho\sigma}^\dagger.
\]

It is obvious that the sum of the above equations gives the optical 1D vector mode field \( \hat{e}_{\rho\sigma} \) defined in equation \((8a)\) as we have proved. This ends the simple canonical quantization, where spatial harmonic oscillators \( \rho\sigma \) have been quantized, that is, when we consider the spatial propagation of a 1D vector mode, then the optical field fulfills the equation of a classical \( \eta \)-spatial harmonic oscillator with spatial ‘frequency’ \( \beta_{\rho\sigma} \), which can be quantized by means of the standard methods.

4. Quantization of the momentum of coupled guided 1D vector modes

We will analyze the modal coupling by introducing a dielectric perturbation of the original refractive index \( n^2(\xi, \gamma) \). This perturbation is described, as usual, by a material polarization which is a new operator \( \hat{P} = (\Delta \hat{\varepsilon}) \hat{e} \), where \( (\Delta \hat{\varepsilon}) \) denotes a perturbation of the electrical permittivity which can present isotropy, anisotropy, non-linearity and so on, and \( \hat{e} \) denotes the total optical field operator in the guide. Moreover, it is assumed that the polarization operator can be expressed as a function of the non-perturbed field operators, in a similar way to the classical case \([25]\). Therefore, the momentum operator for guided 1D vector modes coupled by means of a polarization \( \hat{P} \) can be written as

\[
\hat{M} = \hat{M}_O + \frac{1}{2} \int \int \int_0^T \hat{P} \hat{e} \, d\xi \, dy \, dt = \hat{M}_O + \frac{1}{2} \int \int \int_0^T \hat{P}_i \hat{\varepsilon}_i \, d\xi \, dy \, dt,
\]

where each component \( i \) \((=\xi, \gamma, \eta)\) of the polarization operator can be written, in a formal way, as follows:

\[
\hat{P}_i = \{\varepsilon_{o} \chi_{ijklm} \cdots \hat{\varepsilon}_k \hat{\varepsilon}_l \hat{\varepsilon}_m \cdots \} \hat{e}_j, \quad k, l, m, \ldots, \quad j = (\xi, \gamma, \eta).
\]
However, it is usual that the device presents several kinds of perturbations simultaneously; in particular, let us consider that the integrated optical device is perturbed by both a linear and a nonlinear polarization, that is, \( \hat{P} = \hat{P}_L + \hat{P}_{NL} \), therefore the total momentum is written as

\[
\hat{M} = \hat{M}_0 + \frac{1}{2} \int_0^T \int \int \hat{P}_L \hat{\epsilon} \, d\xi \, d\gamma \, cdt + \frac{1}{2} \int_0^T \int \int \hat{P}_{NL} \hat{\epsilon} \, d\xi \, d\gamma \, cdt. \tag{27}
\]

We will analyze these two perturbations in separate ways: the linear perturbation was analyzed for 2D vector modes and now it will be generalized to 1D vector modes, and next, for expositional convenience, a particular nonlinear perturbation will be considered.

### 4.1. Quantization of the linear momentum of coupled guided 1D vector modes

We start with the quantization of the linear polarization corresponding to an isotropic and inhomogeneous perturbation, that is, \( \hat{P}_L = \Delta \epsilon \, \hat{\epsilon} \) \( (\hat{\epsilon}_i + \hat{\epsilon}_i) \); therefore, the corresponding contribution to the momentum operator will be

\[
\hat{M}_L = \frac{1}{2} \int_0^T \int \Delta \epsilon \, \hat{\epsilon} \, d\xi \, d\gamma \, cdt = \frac{1}{2} \int_0^T \Delta \epsilon \sum_{\rho \sigma} \hat{\epsilon}_{\rho \sigma} \sum_{\rho' \sigma'} \hat{\epsilon}_{\rho' \sigma'} \, d\xi \, d\gamma \, cdt. \tag{28}
\]

Now, attention should be paid to the particular way in which longitudinal components are handled under a linear perturbation with longitudinal component \( \hat{P}_L = \Delta \epsilon \, \hat{\epsilon}_i \); which is necessary because only the transverse components are orthogonal and the mode expansion can be only applied to these components. Therefore, by taking into account the quantized complex Maxwell–Ampere law for the perturbed problem with field operators with positive frequency, the mentioned mode expansion with the transverse components of the 1D vector mode operators with positive frequency such as that given by equation (8b), and again the complex Maxwell–Ampere law for each mode operator, then we obtain the following expression for the longitudinal component of the optical field operator with positive frequency, \( \hat{\epsilon}_1^{(+)} \):

\[
(\epsilon + \Delta \epsilon) \hat{\epsilon}_1^{(+)} = \frac{i}{\omega} \nabla \hat{1} \wedge \hat{\gamma}^{(+)} = \epsilon \frac{\sqrt{\hbar}}{\sqrt{cT}} \sum_{\rho \sigma} \left\{ \frac{1}{\|E_{\rho \sigma}\|} \hat{a}_{\rho \sigma} E_{\rho \sigma} e^{-i\omega t} \right\}. \tag{29}
\]

Therefore, for the perturbed problem the longitudinal component of the optical field operator must be rewritten as follows:

\[
\hat{\epsilon}_1 = \frac{\epsilon}{\epsilon + \Delta \epsilon} \frac{\sqrt{\hbar}}{\sqrt{cT}} \sum_{\rho \sigma} \left\{ \frac{1}{\|E_{\rho \sigma}\|} \left[ \hat{a}_{\rho \sigma} E_{\rho \sigma} e^{-i\omega t} + \hat{a}^+_{\rho \sigma} E^*_{\rho \sigma} e^{i\omega t} \right] \right\}. \tag{30}
\]

Now, by inserting both the transverse components of the field operators given by equation (8a) and the longitudinal component given by equation (30) into equation (28), by performing temporal integration and by taking into account that the creation and annihilation operators commute for different modes, we obtain non-crossing terms \( (\rho \sigma = \rho' \sigma') \) and crossing terms \( (\rho \sigma \neq \rho' \sigma') \) different from zero. Therefore, the linear momentum operator of coupled modes is given by the following expression:

\[
\hat{M}_L = \sum_{\rho \sigma} \hbar \text{sgn}(\rho \sigma) \kappa_{\rho \sigma, \rho' \sigma'} \hat{a}^+_{\rho \sigma} \hat{a}_{\rho' \sigma'} + \sum_{\rho \sigma < \rho' \sigma'} \left\{ \hbar \kappa_{\rho \sigma, \rho' \sigma'} \hat{a}_{\rho \sigma} \hat{a}^+_{\rho' \sigma'} + \text{h.c.} \right\}, \tag{31}
\]

where, as in the free case, commutation rules given by equation (13b) have been used in the calculation of non-crossing terms in such a way that positive contribution of the vacuum
fluctuations were chosen. Moreover, the self-coupling coefficient $\kappa_{\rho\sigma, \rho\sigma}$ and the cross-coupling coefficients $\kappa_{\rho\sigma, \rho'\sigma'}$ are given by the expressions

$$
\kappa_{\rho\sigma, \rho\sigma} = \frac{\omega}{2} \int \Delta \epsilon \mathbf{E}_{\rho\sigma} \mathbf{E}_{\rho\sigma}^* d\xi d\psi + \frac{\omega}{2} \int \Delta \epsilon F(\Delta \epsilon) \mathbf{E}_{\rho\sigma} \mathbf{E}_{\rho\sigma}^* d\xi d\psi,
$$

$$
\kappa_{\rho\sigma, \rho'\sigma'} = \frac{\omega}{2} \int \Delta \epsilon \mathbf{E}_{\rho\sigma} \mathbf{E}_{\rho'\sigma'}^* d\xi d\psi + \frac{\omega}{2} \int \Delta \epsilon F(\Delta \epsilon) \mathbf{E}_{\rho\sigma} \mathbf{E}_{\rho'\sigma'}^* d\xi d\psi,
$$

(32a)

(32b)

where $F(\Delta \epsilon) = \epsilon/(\epsilon + \Delta \epsilon)$; note that for weakly coupled modes, that is, $\Delta \epsilon/\epsilon \ll 1$, the following approximation can be used $F(\Delta \epsilon) \approx 1$, which simplifies equations (32a) and (32b).

Moreover, by denoting $\tilde{\beta}_{\rho\sigma} = \beta_{\rho\sigma} + \kappa_{\rho\sigma, \rho\sigma}$, the sum of free and linear momenta given by equations (14) and (31) can be written as follows:

$$
\hat{M}_{OL} = \hat{M}_O + \hat{M}_L = \sum_{\rho\sigma} \hbar \text{sgn}(\rho\sigma) \tilde{\beta}_{\rho\sigma} \hat{a}_{\rho\sigma}^\dagger \hat{a}_{\rho\sigma} + \sum_{\rho\sigma < \rho'\sigma'} \{ \hbar \kappa_{\rho\sigma, \rho'\sigma'} \hat{a}_{\rho\sigma} \hat{a}_{\rho'\sigma'}^\dagger + \text{h.c.} \},
$$

(33)

which is the quantum momentum operator for coupled guided 1D vector modes under a linear coupling polarization; modes can be forward or backward. Interesting quantum analysis has been recently made for counterpropagating modes [35], whose linear coupling has been analyzed by quantization by means of the standard correspondence principle. It is easy to obtain, from both equation (33) and the commutation rules (13b), the Heisenberg equations for the forward and backward annihilation operators $\hat{a}_{\rho\sigma > 0}$ and $\hat{a}_{\rho\sigma < 0}$, that is

$$
-i\hbar \partial_t \hat{a}_{\rho\sigma} = [\hat{a}_{\rho\sigma}, \hat{M}_{OL}] = \hbar \tilde{\beta}_{\rho\sigma} \hat{a}_{\rho\sigma} + \text{sgn}(\rho'\sigma') \sum_{\rho'\sigma' \neq \rho\sigma} \hbar \kappa_{\rho\sigma, \rho'\sigma'} \hat{a}_{\rho\sigma} \hat{a}_{\rho'\sigma'}^\dagger.
$$

(34)

Note that the function $\text{sgn}(\rho'\sigma')$ in the coupling terms appears in a natural way by starting from: the momentum operator, commutation rules and Heisenberg equations. This momentum operator can describe quantum modal linear coupling in directional couplers, in integrated gratings, integrated junctions and so on. Obviously, for the free case the spatial evolution of the annihilation operator is the one corresponding to a classical mode, that is, $\hat{a}_{\rho\sigma}(\eta) = \hat{a}_{\rho\sigma}(0) \exp[i \beta_{\rho\sigma} \eta]$, therefore, $\hat{a}_{\rho\sigma}^\dagger(\eta) = \hat{a}_{\rho\sigma}^\dagger(0) \exp[-i \beta_{\rho\sigma} \eta]$; this suggests the use of the well-known interaction picture, that is, by substituting the change: $\hat{a}_{\rho\sigma}(\eta) = \hat{A}_{\rho\sigma}(\eta) \exp[i \beta_{\rho\sigma} \eta]$ into equation (33), or into equation (34), thus, the first term in the momentum operator is a constant, which cannot be considered, and therefore the momentum can be rewritten in the interaction picture (I) as follows:

$$
\hat{M}_{OL}^I = \sum_{\rho\sigma < \rho'\sigma'} \{ \hbar \kappa_{\rho\sigma, \rho'\sigma'} \hat{A}_{\rho\sigma} \hat{A}_{\rho'\sigma'}^\dagger \exp[i (\beta_{\rho\sigma} - \beta_{\rho'\sigma'}) \eta] + \text{h.c.} \}.
$$

(35)

This ends the derivation of the linear momentum operator for weakly or strongly coupled modes in a photonic waveguide structure or integrated photonic device.

4.2. Quantization of the nonlinear momentum of coupled guided 1D vector modes

In this section, we will extend the present analysis to a nonlinear perturbation. Obviously, a great deal of nonlinear problems can be presented and their general treatment would be very complex, therefore, by expositional convenience we will center our attention on a particular problem, for example, a directional coupler formed by $N = 3$ guides weakly coupled of which one of them

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is nonlinear with a $\chi^2$ nonlinearity generating a second-harmonic mode in a potassium titanyl phosphate (KTP) material. This problem was proposed and solved in a classical way by Assanto et al. [33] for the case of $N = 2$ (a linear waveguide and another nonlinear one). The quantum treatment was made later by Peřina Jr and Peřina [36] starting from a momentum operator proposed in a heuristic way. Our aim will be to obtain ab initio the above nonlinear momentum operator by using the theoretical formulation given in this work. For that, let us consider, for expositional convenience, $N = 3$ (that is, a $3 \times 3$ coupler) where there are two linear waveguides and one nonlinear central waveguide in such a way that it illustrates the general procedure for deriving any other nonlinear momentum operator in a waveguide structure.

First of all, it is important to remember the nonlinear susceptibility tensor of the KTP crystal and to connect it with the waveguide structure. For that, let us consider the following relationship between the subindices $\{123\}$ to characterize the nonlinear susceptibility of the KTP crystal and the subindices $\{\xi \gamma \eta\}$ characterizing the waveguide structure: $1 = \eta$, $2 = \xi$ and $3 \equiv \gamma$. We also remember that KTP is a crystal belonging to the $mm2$ crystal class, with coefficients of the nonlinear susceptibility tensor, in contracted notation, fullfilling the following relationship order: $d_{33} > d_{34} > d_{31} > d_{15} > d_{32}$.

On the other hand, we assume that guides 1, 2 and 3 of the coupler are monomode, therefore the quasi-TE fundamental modes ($\rho \sigma = 11$) are allowed in the guides, and accordingly, the photons are excited in modes with a dominant $\gamma$-component for the associated field operator, that is, $e_{\gamma 11(1,2,3)}$ (or $e_{\eta 11(1,2,3)}$). Moreover, we assume that linear guide 1 (or linear guides 1 and 3) is (are coherently) excited in the mode $e_{\gamma 11(1)}$ (in the modes $e_{\eta 11(1,3)}$) and by means of directional coupling the energy is transferred to the central guide 2 (nonlinear guide) in the mode $e_{\gamma 11(2)}$ and second harmonic generation (SHG) is produced in this guide in the mode $e_{\gamma 11(4)}$. We also assume that linear coupling between $e_{\gamma 11(4)}$ and $e_{\eta 11(1,3)}$ is negligible because of the different frequencies of these modes. In short, since any general optical field in the coupler will be a quasi-TE field, that is $e_{\gamma}$, it is clear that only the term $d_{33}$ makes an important contribution to the SHG, because it provides a dominant nonlinear polarization component, that is, $P_{NL} \approx 2d_{\gamma \gamma} e_{\gamma}^2 u_{\gamma}$, therefore the nonlinear momentum operator is given by the expression

$$\hat{M}_{NL} = \frac{1}{2} \int \int \int \hat{P}_{NL} \hat{e} \, d\xi \, dy \, d\tau \approx \frac{1}{2} \int \int \int \hat{e}_{\gamma} \hat{e}_{\gamma}^* \, d\xi \, dy \, d\tau. \quad (36)$$

The rest of terms, by taking into account the commutativity of the components of the modal field operator [37], are of the type $\hat{c}_\xi^\dagger \hat{c}_\eta$ and $\hat{c}_\eta^\dagger \hat{c}_\gamma$. However, according to the previous analysis they can be regarded as negligible terms. Now we can calculate the nonlinear momentum operator taking into account the four modes, that is, $\rho \sigma m = 11m$ with $m = 1, 2, 3, 4$, therefore by substituting into equation (36) the following operator field:

$$\hat{e}_{\gamma} = \sum_{11(m)} \frac{\sqrt{\hbar \omega_m}}{\sqrt{c \, T}} \left\{ \frac{1}{||E_{11m}||} \left[ \hat{a}_{11m}^\dagger E_{\gamma 11m} e^{-i \omega_m t} + \hat{a}_{11m} E_{\gamma 11m}^* e^{i \omega_m t} \right] \right\}, \quad (37)$$

where $\omega_4 = 2\omega_1 = 2\omega_2 = 2\omega_3$, by performing integration, and by taking into account that $2d_{\gamma \gamma}$ is localized in the guide (2) (that is, there is a negligible nonlinear coupling between modes $m = 1, 4$ and $m = 3, 4$), we easily obtain

$$\hat{M}_{NL} = \hbar \left\{ \kappa_{112,114} \hat{a}_{112}^\dagger \hat{a}_{114}^\dagger + \text{h.c.} \right\}, \quad (38)$$

where $\kappa_{112,114}$ is the nonlinear coupling coefficient between modes 112 and 114, propagating in the guide (2), obtained after performing the integration indicated in equation (36). Now,
remaking equation (33) for the free and linear coupling of this coupler (with copropagating modes) we obtain

\[ \hat{M}_{OL} = \sum_{1\leq m < n \leq 11} \hbar \beta_{1m} \hat{a}_{1m}^\dagger \hat{a}_{1n} + \sum_{1\leq m < n \leq 11} \left\{ \hbar \kappa_{1m,1n} \hat{a}_{1m} \hat{a}_{1n}^\dagger + \text{h.c.} \right\}. \] (39)

However, as we assumed, the linear coupling between modes \( m = 1, 4 \) and \( m = 3, 4 \) is negligible because of the different frequencies; likewise, the linear coupling between \( m = 2, 4 \) is also negligible because there is no linear perturbation \( \Delta \epsilon \) between them, and finally the linear coupling between modes \( m = 1, 3 \) is negligible because of the large separation between them, therefore there is only linear coupling between guides 1 and 2, and between guides 2 and 3; now, by using a contracted notation \( \left( 1 m \equiv m \right) \) and taking into account equations (38) and (39) the following total momentum operator is obtained:

\[ \hat{M} = \sum_{m=1}^{4} \hbar \beta_{m} \hat{a}_{m}^\dagger \hat{a}_{m} + \sum_{n=1}^{2} \hbar \left\{ \kappa_{2,2n-1} \hat{a}_{2n-1} \hat{a}_{2n}^\dagger + \text{h.c.} \right\} + \hbar \left\{ \kappa_{2,4} \hat{a}_{2} \hat{a}_{4}^\dagger + \text{h.c.} \right\}. \] (40)

The corresponding Heisenberg equations are calculated as it was done in (34), therefore, by considering that the three guides are equal, that is: \( \beta_{1} = \beta_{2} = \beta_{3} = \beta_{L} \neq \beta_{4} = \beta_{NL} \), \( \kappa_{2,1} = \kappa_{2,3} = \kappa_{L} \) and writing \( \kappa_{2,4} = \kappa_{NL} \), we obtain, after a simple calculation, the following equations:

\[ -i \partial_{\tau} \hat{a}_{1} = \beta_{L} \hat{a}_{1} + \kappa_{L}^{\star} \hat{a}_{2}, \] (41a)

\[ -i \partial_{\tau} \hat{a}_{2} = \beta_{L} \hat{a}_{2} + \kappa_{L} \hat{a}_{1} + \kappa_{NL} \hat{a}_{2} \hat{a}_{4} + 2 \kappa_{NL}^{\star} \hat{a}_{1} \hat{a}_{2} \hat{a}_{4}, \] (41b)

\[ -i \partial_{\tau} \hat{a}_{3} = \beta_{L} \hat{a}_{3} + \kappa_{L}^{\star} \hat{a}_{2}, \] (41c)

\[ -i \partial_{\tau} \hat{a}_{4} = \beta_{NL} \hat{a}_{4} + \kappa_{NL} \hat{a}_{2}^{\dagger}. \] (41d)

This result is clearly identical to the one proposed in [21, 36] if the interaction picture is used and guide 3 is removed; accordingly, equation (41c) for \( \hat{a}_{3} \) has to be removed and it must be taken as \( \hat{a}_{3} = 0 \) in equation (41b). As it was also pointed out in [21] the main novelty of this momentum operator is that the Heisenberg equations are different from those obtained directly from the classical coupling equations by applying the correspondence principle, in particular, the factor 2 in the term \( 2 \kappa_{NL} \hat{a}_{1} \hat{a}_{2} \hat{a}_{4} \) of equation (41b); therefore, the theoretical formulation presented in this work allows to obtain many other momentum operators in such a way that the corresponding Heisenberg equations cannot be obtained by direct quantization of classical equations, which justifies the interest of this formulation about the quantization of the momentum operator. Analogous results will arise with the spin operator in integrated photonics, which will be presented in a following work.

5. Summary

We have derived here, the linear and nonlinear quantum momentum operators for integrated photonic waveguide devices described by coupled mode propagation. Quantization has been obtained by taking into account the modal orthonormalization on a cross section of an integrated photonic waveguide, the modal vector structure and the modal reversal-time symmetry; likewise, a simple canonical quantization for the free momentum operator based on the flow of energy in a photonic integrated waveguide has been presented. Finally, by calculating the Heisenberg equations for a general linear modal coupling problem and a particular nonlinear
directional coupling problem, it has been proved that the quantum results are fully consistent with both the classical descriptions and the usual prescriptions used in quantum optical propagation, especially in counterpropagation problems and nonlinear copropagation problems.

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