JOINT OPTIMIZATION OF SAMPLING PATTERNS AND DEEP PRIORS FOR IMPROVED PARALLEL MRI

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ABSTRACT

Multichannel imaging techniques are widely used in MRI to reduce the scan time. These schemes typically perform undersampled acquisition and utilize compressed-sensing based regularized reconstruction algorithms. Model-based deep learning (MoDL) frameworks are now emerging as powerful alternatives to compressed sensing, with significantly improved image quality. In this work, we investigate the impact of sampling patterns on the quality of the image recovered using the MoDL algorithm. We introduce a scheme to jointly optimize the sampling pattern and the reconstruction network parameters in MoDL for parallel MRI. The improved decoupling of the network parameters from the sampling patterns offered by the MoDL scheme translates to improved optimization and thus improved performance. Preliminary experimental results demonstrate that the proposed joint optimization framework significantly improves the image quality.

Index Terms— sampling, parallel MRI, deep learning

1. INTRODUCTION

Modern MRI schemes rely on parallel imaging hardware to accelerate the acquisition. Specifically, the spatial diversity of receive coil sensitivities is used to reduce the number of k-space samples that need to be acquired. Parallel MRI widely utilize regularization algorithms that exploit various image properties (e.g., sparsity, low-rank). These regularized algorithms inject prior information of the images into the recovery process to make the reconstruction from undersampled data well-posed.

Direct inversion convolutional neural networks (CNN) [1, 2] and model-based deep learning algorithms [3–5] are now emerging as powerful alternatives for compressed sensing. Direct inversion schemes rely on a convolutional neural network (CNN) to recover the images from the undersampled data directly. By contrast, model-based methods formulate the recovery as a regularized optimization scheme, where the prior is defined using a non-linear CNN. The iterative algorithm for minimizing the cost function is unrolled, where the parameters of the CNN are learned from exemplary data in an end-to-end fashion. The non-linear convolutional neural networks (CNN) used in these schemes are far more efficient in capturing the non-linear redundancies that exist in images compared to hand-crafted priors (e.g., sparsity in a wavelet domain). The end-to-end training of unrolled architectures in [3–5] facilitates the learning of priors that are complementary to the information obtained from the acquisition scheme.

It is clear that the quality of the reconstructions heavily depends on the specific sampling pattern used to acquire the data, the regularization priors, as well as the hyper-parameters, including the regularization parameter. Early parallel MRI hardware [6] was designed to eliminate the need to sample adjacent k-space samples, making uniform undersampling of k-space a desirable approach. By contrast, compressed sensing schemes that rely on image sparsity advocate for the use of sampling patterns that are incoherent with the specific transform. These contradictory demands imposed by the acquisition scheme and the image priors, as well as a large number of coil elements and the geometry of modern coil arrays, often make it challenging to design the sampling pattern manually. Early empirical studies suggest that incoherent sampling patterns may not be needed for good reconstruction performance of such CNN based reconstruction schemes [4].

The computational optimization of sampling patterns has been explored in MRI, with two broad class of solutions—algorithm-dependent and algorithm-independent. For instance, the approaches [7–10] assume a specific image property (e.g. image support, sparsity) and optimize the sampling patterns to improve the measurement diversity for that specific class; this strategy is completely independent of any specific image reconstruction algorithm or its hyper-parameters. By contrast, the approaches in [11, 12] optimize the sampling pattern, assuming specific reconstruction algorithms (e.g., TV or wavelet sparsity). They utilize a subset of discrete sampling locations using greedy or sparse optimization strategies so as to minimize the reconstruction error using a specific algorithm. The main challenge with these methods is the slow reconstruction algorithm, which restricts the optimization of the pattern to a large class of images. Further, these methods assume fixed reconstruction algorithm and its hyper-parameters during the optimization process.

The main focus of this work is to jointly optimize the sampling pattern and the deep prior in a MoDL framework with application to parallel MRI. The joint optimization strat-
egy is expected to provide improved performance compared to the classical pseudo-random patterns. We study the utility of the proposed strategy with both direct inversion \[1, 2\] and model-based methods \[3, 4\]. The strong coupling between CNN parameters and the specific sampling pattern in direct inversion schemes \[1, 2\] makes their joint optimization challenging. By contrast, model-based schemes use the information of the sampling pattern within the reconstruction algorithm, thus decoupling the CNN block from changes in sampling pattern. This improved decoupling between the parameters in model-based approaches is expected to offer improved performance.

2. METHOD

2.1. Image Formation & Reconstruction

We consider the recovery of the complex image \(x \in \mathbb{C}^{M \times N}\) from its possibly non-Cartesian Fourier samples:

\[
b[i, j] = \sum_{m \in \mathbb{Z}^2} s_j[m] \cdot x[m] \cdot e^{-jk_i^Tm} + n[i, j], \quad k_i \in \Theta.
\]

(1)

Here, \(\Theta\) is a set of sampling locations and \(n[i, j]\) is the noise process. \(s_j\) corresponds to the sensitivity of the \(j^{th}\) coil. The above mapping can be compactly represented as \(b = A_\Theta(x) + n\). The operator \(A_\Theta\) captures the information about the sampling pattern as well as the receive coil sensitivities. Model based algorithms are widely used for the recovery of images from heavily undersampled measurements such as \[1\]. These schemes pose the reconstruction as an optimization problem of the form:

\[
x_{\Theta, \Phi} = \arg\min_x ||b - A_\Theta(x)||^2 + R_\Phi(x).
\]

(2)

Here, \(R_\Phi\) is a regularization penalty (e.g. transform domain sparsity, when \(R(x) = \lambda ||Tx||_r\)) with \(\Phi\) denoting the parameters of the regularizer and the transform. The notation \(x_{\Theta, \Phi}\) for the solution of (2) denotes its dependence on the regularization parameters as well as sampling pattern.

2.2. Deep learning based image recovery

Direct inversion schemes \[1, 2\] rely on a deep CNN \(M_\Theta\) to recover the images from \(A_\Theta(b)\); the CNN with parameters \(\Phi\) learns to invert the class of images for the specific sampling pattern \(\Theta\). Large networks are often needed in this setting to learn the inverse of \(A_\Theta\) over the image class, which implies large training datasets are often needed to learn the large number of parameters. Another challenge with this scheme is the strong coupling between the CNN parameters and the sampling pattern, which makes the joint optimization of the sampling pattern and the deep network challenging.

Model based deep learning (MoDL) \[3\] rely on a formulation similar to (2), where the hand-crafted image regularization penalties in (2) are replaced with learned priors; image recovery is formulated as:

\[
\hat{x}_{\Theta, \Phi} = \arg\min_x ||b - A_\Theta(x)||^2 + ||x - D_\Phi(x)||^2,
\]

(3)

where \(D_\Phi\) is a residual learning based CNN that is designed to extract the noise and alias terms in \(x\). The optimization problem specified by (3) is solved using an iterative algorithm, which alternates between \(D_\Phi\) and a data-consistency step \(Q_\Theta(z) = (A_{\Theta}^T A_{\Theta} + Z)^{-1} (z + A_{\Theta}^T b)\), which is implemented using a conjugate gradients algorithm. This iterative algorithm is unrolled to obtain a deep network \(M_{\Theta, \Phi}\), where the weights of the CNN blocks and data consistency blocks are shared across iterations as shown in Fig. 1. Specifically, the solution to (3) is given by

\[
\hat{x}_{\Theta, \Phi} = M_{\Theta, \Phi}(A_\Theta(x)).
\]

(4)

Thus, the main distinction between MoDL and direct inversion schemes is the structure of the network \(M_{\Theta, \Phi}\). Since the data consistency block is used within MoDL, considerably smaller CNNs are sufficient in the MoDL setting, which translates to significantly lower amount of training data. The parameters of the network specified by \(\Phi\) are from a set of training images \(x_i; i = 1, ..., N\), so as to minimize the \(\ell_2\) training error

\[
\Phi^* = \arg\min_{\Phi} \sum_{i=1}^{N} ||M_{\Theta, \Phi}(A_\Theta(x_i)) - x_i||^2.
\]

(5)

2.3. Proposed Joint Optimization Strategy

The main focus of this work is to jointly optimize both \(D_\Phi\) and \(Q_\Theta\) blocks in the MoDL framework with the goal of improving the reconstruction performance. Specifically, we propose to jointly learn the sampling pattern \(\Theta\) and the CNN parameter \(\Phi\) from training data using

\[
\{\Theta^*, \Phi^*\} = \arg\min_{\Theta, \Phi} \sum_{i=1}^{N} ||M_{\Theta, \Phi}(A_\Theta(x_i)) - x_i||^2.
\]

(6)

This proposed J-MoDL framework can be generalized to other error metrics such as perceptual error. Note that the
CNN parameters in MoDL are more decoupled from the sampling pattern, making the training easier than the direct inversion schemes. Note that the CNN parameters in direct inversion schemes are closely coupled with the sampling scheme since the network need to invert the signal.

2.4. Parametrization of the sampling pattern

In this work, we restrict our attention to the optimization of the phase encoding locations in MRI, while the frequency encoding direction is fully sampled. Mathematically, we model the sampling set as the translates of a single pattern $\Gamma$.

$$\Theta = \bigcup_{i=1}^{P} (\Gamma + \theta_i)$$  \hspace{1cm} (7)

Here $\Theta = \{\theta_i; i = 1, \ldots, P\}$ are the $P$ phase encoding locations, while $\Gamma$ is the set of samples on a line. In addition to reducing the parameter space, this approach also simplifies the implementation; the $Q_{\Theta}$ blocks can be implemented analytically in terms of 1-D Fourier transforms. Note that this framework can be generalized to sample arbitrary trajectories (e.g., radial lines with arbitrary angles).

2.5. Network and dataset details

Figure 1 shows the proposed joint model-based deep learning framework (J-MoDL), where we simultaneously optimize for the sampling pattern $\Theta$ as well as the network parameters $\Phi$. The network in Fig. 1 was unrolled for $K=5$ iterations i.e., five iterations of alternating minimization were used to solve Eq. (6). The forward operator $A_{\Theta}$ is implemented as a 1-D discrete Fourier transform to map the spatial locations to the continuous domain Fourier samples specified by $\Theta$, following the weighting by the coil sensitivities as described by [1]. The data-consistency block $Q_{\Theta}$ is implemented using conjugate gradients algorithm. The CNN block $D_{\Phi}$ is implemented as a UNET with four pooling and unpooling layers. The parameters of the blocks $D_{\Phi}$ and $Q_{\Theta}$ are optimized to minimize $\mathcal{E}$. We relied on the automatic-differentiation capability of TensorFlow to evaluate the gradient of the cost function with respect to $\Theta$ and $\Phi$.

Experiments were performed on publically available parallel MRI knee dataset as in [4]. The training data consisted of 381 slices from 10 subjects, whereas test data had 80 slices from 2 subjects. Each slice in the training and test dataset had different coil sensitivity maps that were estimated using the ESPRIT [13] algorithm. For comparison, we also study the optimization of the sampling pattern in the context of direct inversion (i.e., when a UNET is used for image inversion). A UNET with same number of parameters was used in the study.

Table 1. The average PSNR (dB) and SSIM values obtained over the test data of two subjects with total of 80 slices using different optimization strategies.

| Acc. | Optimize | UNET | MoDL | UNET | MoDL |
|------|----------|------|------|------|------|
| 4x   | $\Phi$ alone | 29.95 | 34.21 | 0.83 | 0.91 |
|      | $\Theta$ alone | 28.85 | 37.66 | 0.86 | 0.96 |
|      | $\Theta, \Phi$ Joint | 34.02 | 41.28 | 0.93 | 0.96 |
| 6x   | $\Phi$ alone | 29.24 | 32.40 | 0.82 | 0.89 |
|      | $\Theta$ alone | 24.45 | 33.31 | .078 | 0.93 |
|      | $\Theta, \Phi$ Joint | 29.62 | 35.93 | 0.89 | 0.93 |

Fig. 2. This figure compares a slice of the test data reconstructed using MoDL and J-MoDL approaches at 4x acceleration. The zoomed areas clearly shows that joint learning preserves the fine details.

3. EXPERIMENTS AND RESULTS

Table 1 summarizes the reconstruction quality using peak signal to noise ratio (PSNR) and structural similarity index (SSIM). This work evaluates two deep learning paradigms, namely direct inversion based frameworks such as UNET and the model-based framework such as MoDL, at four-fold (4x) and six-fold (6x) acceleration (Acc.) factors. The two deep learning frameworks are compared at three optimization levels while having the same number of trainable parameters. At the first optimization level, denoted as $\Phi$ alone in Table 1, only the reconstruction network parameters are trained without optimizing the sampling mask. The MoDL framework provides around 4 dB and 3 dB improvement in the 4x and 6x acceleration cases respectively. This improved performance can be attributed to additional conjugate-gradient based data consistency step in the MoDL framework, as discussed in [3].

Second, $\Theta$ alone optimization, here only the sampling mask is optimized, while keeping the reconstruction parameters fixed to the values obtained from the first case. In this
Fig. 3. A test data slice reconstructed using J-UNET and J-MoDL approaches at 6x acceleration. Yellow arrows are pointing to a feature that is blurred in the J-UNET whereas J-MoDL correctly retains it. Red arrow points to a feature incorrectly sharpened by the J-UNET. Green arrow is pointing to a feature completely missed by the J-UNET.

In this case, we observe that the performance of the UNET declines. Note that the UNET parameters $\Phi$ need to be tightly coupled to the specific sampling pattern; since we rely on a UNET that is optimized for another pattern, the approach results in lower performance. The performance improvement in the MoDL framework by only optimizing the sampling parameters $\Theta$ indicates that an optimized sampling mask can further improve the reconstruction quality. It also indicates that the reconstruction network parameters $\Phi$ in the MoDL framework are more decoupled from the choice of the sampling pattern.

Lastly, denoted as $\Theta, \Phi$ joint optimization, here we jointly trained both the reconstruction parameters $\Phi$ and the sampling mask $\Theta$. The J-MoDL PSNR values, on average, are 7 dB higher as compare to the J-UNET method. The results demonstrate the benefit of the decoupling of the sampling and CNN parameters offered by MoDL in the joint optimization strategy.

Figure 2 shows an example slice from the test dataset that visually compares the benefit of jointly optimizing both the sampling pattern and the network parameters as compared to the network alone in the model-based deep learning framework. The zoomed image portion shows that joint learning using J-MoDL better preserves the soft tissues in the knee at the four-fold acceleration case.

Figure 3 visually demonstrates the benefit of joint optimization in the model-based framework as compared to joint optimization in direct inversion based method at six-fold acceleration. The utilization of the data-consistency step in a model-based framework helps to reduce the blurring, better preserving the high-frequency details, and reduces hallucination as pointed by arrows in the zoomed area.

Further experiment empirically demonstrates that the unrolled J-MoDL training proceeds relatively smooth despite the problem formulation being highly non-convex. Initially, we trained a MoDL architecture with incoherent sampling patterns. These trained network parameters were used as an initialization for the five independent J-MoDL training processes. Each of the five J-MoDL architectures was initialized with different random sampling masks and trained for 50 epochs. The plot in Fig. 4 shows that all the five models converge to nearly the same local minima values.

Figure 5 shows the initial and learned sampling pattern obtained by the joint optimization in the MoDL framework. It can be noted that the learned mask promotes to capture more samples at low frequency as compared to the initial mask. Further, the learned mask does not necessarily capture all the low-frequency samples as compared to the initial mask.

4. CONCLUSIONS

This work proposed a model-based deep learning framework to optimize for both the sampling and the reconstruction network parameters jointly for parallel MRI. Experimental results demonstrated the benefits of optimizing the sampling mask in the proposed model-based framework. The proposed J-MoDL architecture provides better decoupling between the sampling and reconstruction network parameters. This decoupling makes the reconstruction network more robust to changes in sampling patterns as compared to joint reconstruc-
tion in the direct-inversion based techniques.

5. REFERENCES

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