LEP, TOP and the STANDARD MODEL

V.NOVIKOV, LOKUN, M.VYS OTSKY
ITEP, Moscow, 117259, Russia

A.ROZANOV
ITEP and CPPM, Marseille, France

A simple way of deriving and analyzing electroweak radiative corrections to the $Z$ boson decays is presented in the framework of the Standard Model. The talk is based on a review article by the authors "Electroweak radiative corrections in $Z$ decays" published in Uspekhi Fizicheskikh Nauk, 166 (1996) 539-574, in the May issue dedicated to 75th birthday of A.D.Sakharov. The English translation of the article may be found on WWW (hep-ph 96 06 253).

1 LEP I and SLC

LEP I (CERN) and SLC (Stanford) electron-positron colliders had started their operation in the fall 1989 a few months before A.D.Sakharov passed away. The sum of energies of $e^+ + e^-$ was chosen to be equal to the $Z$ boson mass. LEP I was terminated in the fall of 1995 in order to give place to LEP II which already operates at energy 135 GeV and will finally reach 192 GeV. SLC continues at energy close to 91 GeV.

The reactions, which has been studied at LEP I (detectors: ALEPH, DELPHI, L3, OPAL) and SLC (detector SLD) may be presented in form:

$$e^+e^- \rightarrow Z \rightarrow f\bar{f},$$

where

$$f\bar{f} = \nu\bar{\nu}(\nu_e\bar{\nu}_e, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau) \text{ – invisible,}$$

$$ll(e\bar{e}, \mu\bar{\mu}, \tau\bar{\tau}) \text{ – charged leptons,}$$

$$qq(u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}, b\bar{b}) \rightarrow \text{hadrons.}$$

About 20,000,000 $Z$ bosons has been detected at LEP and about 100,000 at SLC (but here electrons are polarized). Fantastic precision has been reached in the measurement of the $Z$ boson properties:

$$M_Z = 91.188.4 \pm 2.2\text{MeV}, \quad \Gamma_Z = 2,496.3 \pm 3.2\text{MeV},$$

$$\Gamma_h \equiv \Gamma_{\text{hadrons}} = 1,744.8 \pm 3.0\text{MeV}, \quad R_l = \Gamma_h/\Gamma_l = 20.788 \pm 0.032,$$

\(^{a}\)Plenary talk at the Second Sakharov Conference. Lebedev Physical Institute, Moscow, May 20-24, 1996.
\[ \Gamma_{\text{invisible}} = 499.9 \pm 2.5 \text{MeV}. \]

By comparing the last number with theoretical predictions for neutrino decays it was established that the number of neutrinos which interact with \( Z \) boson is 3: \( N_\nu = 2.990 \pm 0.016 \). This is a result of fundamental importance.

More than 2,000 experimentalists and engineers and hundreds of theorists participate in this unique collective quest for truth!

2 **Theoretical analysis: the fundamental parameters.**

It is instructive to compare the electroweak theory with the quantum electrodynamics (QED). In the latter there are two fundamental parameters: mass of the electron, \( m_e \), and its charge, \( e \), or fine structure constant \( \alpha = e^2/4\pi \). Both are not only fundamental, but also known with high precision. Every observable in QED can be expressed in terms of \( m_e \) and \( \alpha \) (and, of course, of energies and momenta of particles participating in a given process).

In the electroweak theory the situation is more complex for several reasons:

1. There are more fundamental charges and masses.
2. They are not independent of each other.
3. Not all of them are known with high accuracy.
4. Thus, as a basic parameter of the theory a quantity is used, which is known with highest accuracy, but which is not fundamental, the four-fermion coupling of the muon decay, \( G_\mu \).

The fundamental masses of the electroweak theory are masses of \( W \) and \( Z \) bosons, \( m_W \) and \( m_Z \). Among the masses of fermions, the most important for the \( Z \)-decay is the mass of the top-quark, \( m_t \).

The fundamental couplings of the electroweak theory are \( e, f, g \), or \( \alpha = e^2/4\pi, \alpha_Z = f^2/4\pi, \alpha_W = g^2/4\pi \): \( e \) is the coupling of photons to electrically charged particles, \( f \) is the coupling of \( Z \) bosons to weak neutral current, e.g. \( \bar{\nu}\nu \), \( g \) is the coupling of \( W \) bosons to weak charged current, e.g. \( \bar{e}\nu \).

While the charged current is a purely V-A current of the form \( = \gamma_\alpha(1+\gamma_5) \), the ratio \( R_f \) between the vector and axial vector neutral currents depends on the third projection of the isotopic spin of the fermion \( f, T^3_f \), and on its electric charge \( Q^f \). The decay amplitude of the \( Z \) boson decay may be written in the form:

\[ M(Z \to f\bar{f}) = \frac{1}{2} f \bar{\psi}_f (g v_f \gamma_\alpha + g A_f \gamma_\alpha \gamma_5) \psi_f Z^\alpha, \]
where $\psi_f$ is the wave function of emitted fermion, $\bar{\psi}_f$ corresponds to the emitted antifermion (or absorbed fermion), $Z^\alpha$ is the wave function of the $Z$ boson. At the tree level

$$g_A f = T_3^f, \quad g_V f = T_3^f - 2Q^f s^2_W.$$

Here

$$T_3^f = +1/2 \text{ for } f = \nu, u, c; \quad T_3^f = -1/2 \text{ for } f = l, d, s, b.$$

Thus

$$R_f = g_V f / g_A f = 1 - 4|Q|^f s^2_W.$$

In the above expressions $s_W \equiv \sin \theta_W$, where $\theta_W$ is the so called weak angle.

At the tree level (no loops): $e / g = s_W \cdot g / f = c_W \cdot m_W / m_Z = c_W$, where $c_W \equiv \cos \theta_W$.

The four-fermion coupling constant $G_\mu$ is extracted from the life-time of the muon, $\tau_\mu$, after taking into account well-known electromagnetic corrections:

$$\frac{1}{\tau_\mu} = \frac{\Gamma_\mu}{\Gamma_\mu} = \frac{G_\mu^2 m_\mu^5}{192\pi^3 \left(1 + \text{well known corrections } \sim \frac{m_e}{m_\mu}, \alpha\right)}.$$

$$G_\mu = (1.16639 \pm 0.00002) \cdot 10^{-5} \text{GeV}^{-2}.$$

In the tree approximation the four-fermion coupling constant $G_\mu$ can be expressed in terms of $W$ boson coupling constant $g$ and its mass $m_W$:

$$G_\mu = \frac{g^2}{4\sqrt{2}m_W^2} = \frac{\pi\alpha}{\sqrt{2}m_W^2 s_W^2} = \frac{\pi\alpha}{\sqrt{2}m_Z^2 s_W^2 c_W^2}.$$

(The last two expressions are derived by using the relations $e / g = s_W$, $\alpha = e^2 / 4\pi$, $m_W / m_Z = c_W$).

3 **Theoretical analysis: the running $\alpha(q^2)$**.

It is well known since 1950’s that electric charge $e$ and hence $\alpha$ logarithmically depend on the square of the four-momentum of the photon, $q^2$. For a real photon $q^2 = 0$, for a virtual one $q^2 \neq 0$. This phenomenon is usually referred to as “the running of $\alpha$”. It is caused by vacuum polarization, by loops of virtual charged particles: charged leptons, $\bar{l}l$, and quarks, $\bar{q}q$, inserted into the propagator of a photon. As a result $\alpha(q^2 = m_Z^2)$ is approximately by 6% larger than $\alpha \equiv \alpha(0)$.
The relation between $\bar{\alpha}$ and $\alpha$ is obtained by summing up an infinite series of insertions: $\bar{\alpha} = \alpha/(1 - \delta \alpha)$; $\delta \alpha = \delta \alpha_l + \delta \alpha_h$, where $\delta \alpha_l$ is the one-loop contribution of leptons, while $\delta \alpha_h$ – is that of quarks (hadrons). The leptonic contribution can be predicted with very high accuracy. The hadronic contribution is obtained on the basis of dispersion relations and low-energy experimental data on $e^+e^-$-annihilation into hadrons.

The value of $\alpha(0)$ is known with very high accuracy: $\alpha(0) = 1/137.035985(61)$; $\alpha$ is very important for QED, but irrelevant to electroweak physics.

The value of $\bar{\alpha}$ is less accurate: $\bar{\alpha} = 1/128.896(90)$, but $\bar{\alpha}$ is pivotal for electroweak physics. Let us stress that the running of $\alpha$ is a purely electromagnetic effect, caused by electromagnetic loops of light fermions. Contributions of $t\bar{t}$ and $W^+W^-$ are negligibly small and may be taken into account together with purely electroweak loops.

Unlike $\alpha(q^2)$, two other electroweak couplings $\alpha_Z(q^2)$ and $\alpha_W(q^2)$ are not running but "crawling" in the interval $0 \leq q^2 \leq m^2$:

$\alpha_Z(m_Z^2) = 1/22.91$, $\alpha_Z(0) = 1/23.10$;

$\alpha_W(m_Z^2) = 1/28.74$, $\alpha_W(0) = 1/29.01$.

The natural scale for $Z$-physics is $q^2 = m_Z^2$. Therefore it is evident that $\bar{\alpha} \equiv \alpha(m_Z^2)$, not $\alpha \equiv \alpha(0)$ is the relevant parameter. In fact, in all computer codes, dealing with $Z$-physics, $\bar{\alpha}$ enters at a certain stage and substitutes $\alpha$. But this occurs inside the "black box" of the code, while $\alpha$ formally plays the role of an input parameter. In these codes the running of $\alpha$ is considered as (the largest) electroweak correction. We consider this running as purely electromagnetic one and define our Born approximation in terms of $\bar{\alpha}$, $G_\mu$ and $m_Z$.

Instead of angle $\theta_W$, we define angle $\theta(s \equiv \sin \theta, \ c \equiv \cos \theta)$ by relation:

$G_\mu = \frac{g^2(q^2 = 0)}{4 \sqrt{2} m_Z^2} \simeq \frac{e^2(m_Z^2)}{4 \sqrt{2} s^2 m_W^2} = \frac{\pi \bar{\alpha}}{\sqrt{2} s^2 c^2 m_Z^2}$,

where the second equality is based on the "crawling" of $g(q^2)$: $g(0) \simeq g(m_Z^2)$. Thus,

$\sin^2 2\theta = \frac{\pi \bar{\alpha}}{\sqrt{2} G_\mu m_Z^2} = 0.71078(50)$,

$s^2 = 0.23110(23), \ c^2 = 0.76890(23), \ c = 0.87687(13)$.

Our Born approximation starts with the most accurately known observables: $G_\mu$, $m_Z$, $\bar{\alpha}$ (or $s^2$).
The traditional parametrization of electroweak theory in terms of $G_\mu$, $\alpha$, and $s_W^2 \equiv 1 - m_W^2/m_Z^2$ is less convenient ($s_W$ has poor accuracy: $\Delta m_W = \pm 160$ MeV; running of $\alpha$ is not separated from electroweak corrections and overshadows them.)

4 Theoretical analysis: one-loop electroweak corrections.

For the sake of brevity let us choose two observables:

$$s_W^2 \equiv 1 - \frac{m_W^2}{m_Z^2}, \quad s_l^2 \equiv \frac{1}{4} \left(1 - \frac{g V_l}{g_A t}\right) \equiv \frac{1}{4} (1 - R_l).$$

In the Born approximation $s_W^2 = s_l^2 = s^2$. From UA2 and CDF experiments:

$$s_W^2 = 0.2253(31), \quad 2\sigma \text{ away from } s^2 = 0.23110(23).$$

From LEP and SLC:

$$s_l^2 = 0.23141(28), \quad 1\sigma \text{ away from } s^2.$$

(Note the high experimental accuracy of $s_l^2$ compared to that of $s_W^2$.)

In the one-loop approximation

$$s_l^2 = s^2 - \frac{3}{16\pi} \frac{\bar{\alpha}}{c^2 - s^2} V_R(t, m_H),$$

where $c^2 - s^2 = 0.5378$ and the radiative correction depends on the masses of the top quark and higgs. These masses enter via loops containing virtual top quark, or higgs. The coefficient in front of $V_R$ is chosen in such a way that $V_R(t, m_H) \approx t \equiv (m_t/m_Z)^2$. The same asymptotic normalization is used for the radiative corrections to other electroweak observables. The good agreement, within $1 \div 2\sigma$, between experimental values of $s_W^2$, $s_l^2$ and their Born values means that electroweak radiative corrections are anomalously small. The unexpected smallness of $V_R$ is the result of cancellation between large and positive contribution from the $t$-quark loops and large and negative contribution from loops of other virtual particles. This cancellation, which looks like a conspiracy, occurs when $m_{top}$ is around 160 GeV, if higgs is light ($m_H \leq 100$ GeV). If higgs is heavy ($m_H = 1000$ GeV) it occurs when $m_{top}$ is around 210 GeV. Thus, vanishing electroweak radiative corrections tell us that top is heavy.
5 LEPTOP and the general fit.

The analytical formulas for all electroweak observables have been incorporated in our computer code which we dubbed LEPTOP. The fit of all electroweak data by LEPTOP gives:

\[ m_t = 180 \pm 7^{+7}_{-21} \text{ GeV}. \]

The central value \((180 \pm 7)\) corresponds to \(m_H = 300 \) GeV; the shifts \((+18, -21)\) – to \(m_H = 1000\) and \(60\) GeV, respectively. This prediction is in perfect agreement with the recent (spring 1996) data on the direct measurements of the top mass by two collaborations at FNAL:

\[ m_t = 175.6 \pm 5.7 \pm 7.4 \text{ GeV} \quad \text{(CDF)} , \quad m_t = 170 \pm 15 \pm 10 \text{ GeV} \quad \text{(D0)}. \]

(Here the first uncertainty is statistical, the second – systematic one.)

Electroweak radiative corrections depend on \(\ln m_H/m_Z\) and give at present unreliable limits on \(m_H\).

Hadronic decays of \(Z\) are sensitive to the value of the gluonic coupling \(\alpha_s\):

\[ \Gamma_q \equiv \Gamma(Z \to q\bar{q}) = 12\Gamma_0[\hat{g}_{Aq}^2 R_{Aq} + \hat{g}_{Vq}^2 R_{Vq}], \]

where

\[ \Gamma_0 = \frac{G_F m_Z^3}{24\sqrt{2}\pi} = 82.944(6) \text{ MeV}, \]

and the "radiators" \(R_{Aq}\) and \(R_{Vq}\) contain QCD corrections caused by emission and exchange of gluons. In the first approximation \(R_{Vq} = R_{Aq} = 1 + \frac{\hat{\alpha}_s}{\pi}\), where \(\hat{\alpha}_s \equiv \alpha_s(m_Z^2)\) in the \(\overline{\text{MS}}\) scheme. The LEPTOP fit of all electroweak data gives: \(\hat{\alpha}_s = 0.124(4)^{-2}\); here the central value \((0.124 \pm 0.004)\) corresponds to \(m_H = 300 \) GeV; the shifts +0.002 and -0.002 – to \(m_H = 1000 \) GeV and 60 GeV, respectively.

Let us note, that low energy processes (deep inelastic scattering, \(\Upsilon\)-spectroscopy) give much smaller values of \(\hat{\alpha}_s\), around 0.110, when extrapolated to \(q^2 = m_Z^2\). There are different opinions on the seriousness of this discrepancy.

Another problem is connected with the experimental value of the width of the decay \(Z \to b\bar{b}\). Theoretically the ratio \(R_b = \Gamma_b/\Gamma_h\) is not sensitive to \(\hat{\alpha}_s\), \(m_t\) and \(m_H\); the theory predicts: \(R_b = 0.2155(3)^{-2}\); here again the central value \((0.2155 \pm 0.0003)\) corresponds to \(m_H = 300 \) GeV, and shifted by -0.0007 for \(m_H = 1000 \) GeV and by +0.0007 for \(m_H = 60 \) GeV. Experimentally \(R_b = 0.2219(17)\), which is more than 3\(\sigma\) larger than the theoretical prediction based on the Standard Model.
Both problems (of $\hat{\alpha}_s$ and of $R_b$) may be solved by new physics. Theorists try to change their predictions by considering: a) the existence of light supersymmetric particles (squarks, winos, gluinos), which contribute to electroweak loops; b) the existence of another $Z$ boson – $Z'$, which is more strongly coupled to $b\bar{b}$, than to $e\bar{e}$ ("beautyphilic and leptophobic"). But maybe, experimentalists, can also change their numbers?