Polarons in Alkaline-earth-like Atoms with Multi Background Fermi Surfaces

Jin-Ge Chen,1 Yue-Ran Shi,1 Xiang Zhang,1† and Wei Zhang1,2,∗

1 Department of Physics, Renmin University of China, Beijing 100872, China
2 Beijing Key Laboratory of Opto-electronic Functional Materials and Micro-nano Devices, Renmin University of China, Beijing 100872, China

We study the impurity problem in a Fermi gas of $^{173}$Yb atoms near an orbital Feshbach resonance, where a single moving particle in the $^3P_0$ state interacting with two background Fermi seas of particles in different nuclear states of the ground $^1S_0$ manifold. By employing wave function ansatzs for molecule and polaron states, we investigate various properties of the molecule, the attractive polaron, and the repulsive polaron states. We find that in comparison to the case of only one Fermi sea is populated, the presence of an additional Fermi sea acts as an energy shift between the two channels of the orbital Feshbach resonance. Besides, the fluctuation around the Fermi level would also bring sizable effects to the attractive and repulsive polaron states.

I. INTRODUCTION

Since the orbital Feshbach resonance (OFR) was first proposed theoretically [1] and realized experimentally [2, 3] in $^{173}$Yb atoms, many researches have been performed in alkaline-earth(-like) atoms which greatly enriched the scope of quantum simulation in these systems [4–15]. Among these works, of particular interest is the study of polaron and molecule states in a system of an impurity fermion immersed atop a Fermi sea consisted of particles of another species [13–15]. For alkali atoms near a magnetic Feshbach resonance, such an impurity problem has caught great attention in the past decade [16–23], and is considered to be closely related to itinerant ferromagnetism [24–30] in Fermi systems with large polarization. In the context of alkaline-earth(-like) atoms near an OFR, recent studies consider the configuration of a single impurity in the excited orbital state interacting with a majority Fermi sea in one of the ground orbital states, and discuss various properties of the polaron and molecule states [13–15]. From these works, it is understood that the polaron and molecule states in such a system are drastically different from those in alkali atoms because the OFR is a narrow resonance atop a Fermi sea of particles of another species [13–15].

In this manuscript, we fully employ the multi-channel nature of an OFR and consider a generalized impurity problem, where a single atom in the excited orbital state interacts with two background Fermi seas in the ground orbital states with different spin indices. Taking the atoms of $^{173}$Yb as a concrete example, the ground (denoted as $|g⟩$) and excited ($|e⟩$) orbital states correspond to the two clock-state manifolds $^1S_0$ and $^3P_0$, respectively. As the total electronic angular momentum $J = 0$, the nuclear and the electronic spin degrees of freedom are decoupled, such that the states with different nuclear magnetic numbers $I$ can be labeled as pseudospins $|↑⟩$ and $|↓⟩$, which are Zeeman shifted in the presence of an external magnetic field. Without loss of generality, we may refer the scattering channel consisting of $|g^↓⟩$ and $|e^↑⟩$ states as the open channel, and the one of $|g^↑⟩$ and $|e^↓⟩$ as the closed channel. Due to the differential Zeeman shift between the two channels, an OFR would occur when a two-body bound state within one channel becomes degenerate with the threshold of the other, leading to a crossover from the Bardeen-Cooper-Schrieffer (BCS) regime to the Bose-Einstein condensate (BEC) regime.

The system discussed in this manuscript is illustrated in Fig. 1 where a moving atom in the $|e^↑⟩$ state is interacting with the two ground state Fermi seas of $|g^↓⟩$ and $|g^↑⟩$ atoms. Using the Chevy-like ansatzs for the polaron state with one particle-hole fluctuation and for the bare molecule state without particle-hole fluctuation [16, 17], we characterize various properties of the attractive polaron and the molecule states, including the eigen energy, the wave function, and the effective mass. Specifically, we demonstrate that there exists a molecule-attractive polaron transition across the OFR, with the transition point varying with the Fermi levels. We further focus on the quasiparticle excitation called the repulsive polaron, and investigate the corresponding spectral function, eigen energy, wave function, quasiparticle residue, effective mass, and decay rate. From these results, we conclude that the presence of an extra Fermi sea acts mainly as an energy offset of atomic states, which can be understood as an effective shift of the inter-channel detuning. Meanwhile, the fluctuation around the Fermi level induced by interaction can also blur the resonant-scattering processes and smooth the kink structure in various quantities of the repulsive polaron.

The remainder of this paper is organized as follows. In Sec. [1] we present the Hamiltonian of the system under consideration, and introduce the formalism of wave function ansatzs. The properties of the molecule and attractive polaron states throughout the resonance region are discussed in Sec. [11]. Then we focus on the repulsive polaron state and discuss its properties in Sec. [14]. Finally, we summarize in Sec. [15].
and the closed channels are formed by combinations like atoms across an OFR. The system is composed of interaction. An impurity \(|\uparrow\rangle\) is immersed in a two-component \((|\downarrow\rangle \text{ and } |\uparrow\rangle\)) Fermi gas of alkaline-earth(-like) atoms. The impurity can interact with one atom in the \(|\downarrow\rangle\) state, and the two particles in the open-channel can be scattered into the closed channel \((|\uparrow\rangle\)) and \(|\downarrow\rangle\) states through a spin-exchange interaction.

**II. FORMALISM**

We consider the problem of a single impurity immersed in a two-component Fermi gas of alkaline-earth-like atoms across an OFR. The system is composed of two electronic orbital states labeled by \((g, e)\) and two nuclear spin states by \((\uparrow, \downarrow)\), where the open and the closed channels are formed by combinations \(|o\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)\) and \(|e\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)\), respectively. The Hamiltonian of the system can be written as

\[
\hat{H} = \sum_k \alpha_k (a_{k\uparrow}^\dagger a_{k\downarrow} + a_{k\downarrow}^\dagger a_{k\uparrow}) + \sum_k (\epsilon_k + \delta_g) (a_{k\uparrow\uparrow}^\dagger a_{k\downarrow\downarrow}^\dagger a_{k\downarrow\uparrow} a_{k\uparrow\downarrow}) + \sum_k \epsilon_k (\hat{a}_{k\uparrow\uparrow}^\dagger \hat{a}_{k\downarrow\downarrow}^\dagger \hat{a}_{k\downarrow\uparrow} \hat{a}_{k\uparrow\downarrow} + \hat{H}_{\text{int}})
\]

Here, \(a_{k\sigma\sigma}^\dagger\) and \(a_{k\sigma\sigma}\) are creation and annihilation operators associated with fermions with three-dimensional linear momentum \(k\), electronic orbital state \(p = (e, g)\) and nuclear spin (pseudo-spin) state \(\sigma = (\uparrow, \downarrow)\). Due to the differential Zeeman shift, the single-particle dispersion \(\epsilon_k\) in the open and closed channels are shifted by detunings \(\delta_e\) and \(\delta_g\) respectively, as illustrated in Fig. 1. An OFR occurs as the bound state energy within the closed channel becomes degenerate with the scattering threshold of the open channel, or vice versa, by tuning \(\delta_e - \delta_g\).

The interaction Hamiltonian can be expressed in the basis of the orbital symmetric state \(|+\rangle = \frac{1}{\sqrt{2}}(|ge\rangle + |eg\rangle) \otimes (|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle)\) and the orbital antisymmetric state \(|-\rangle = \frac{1}{\sqrt{2}}(|ge\rangle - |eg\rangle) \otimes (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)\), and takes the following form [13]

\[
\hat{H}_{\text{int}} = \sum_q \left[ \frac{g+}{2} \hat{A}_{q,+}^\dagger \hat{A}_{q,+} + \frac{g-}{2} \left( \hat{A}_{q,-}^\dagger \hat{A}_{q,-} + \hat{A}_{q,+}^\dagger \hat{A}_{q,+} \right) \right]
\]

with the operators defined as

\[
\begin{align*}
\hat{A}_{q,+} &= \sum_k (a_{-k+q,g} a_{k+q,e} - a_{-k-q,e} a_{k+q,g}), \\
\hat{A}_{q,-} &= \sum_k (a_{-k+q,g} a_{k+q,e} + a_{-k-q,e} a_{k+q,g}), \\
\hat{A}_{q,+} &= \sum_k (a_{-k+q,g} a_{k+q,e} + a_{-k-q,e} a_{k+q,g}), \\
\hat{A}_{q,+} &= \sum_k (a_{-k-q,g} a_{k+q,e} + a_{-k+q,e} a_{k+q,g}).
\end{align*}
\]

The interaction strengths \(g\) are related to the corresponding s-wave scattering lengths \(a\) via the standard renormalization relation \(1/g = m/(4\pi \hbar^2 a) - \sum_k 1/(2\epsilon_k)\) with \(m\) the atomic mass. Throughout the paper, we consider the atom of \(^{173}\text{Yb}\) as a particular example, where the scattering lengths \(a = 1900\alpha_0\) and \(a_- = 200\alpha_0\) with \(\alpha_0\) the Bohr radius [3, 8].

The fermion impurity problem, by definition, is to impose a minority impurity atoms against a majority Fermi sea formed by fermions of another type. In this paper, we take the advantage of the multi-state nature of the OFR and consider a generalized configuration of a single impurity in the \(|\uparrow\rangle\) state immersed on two Fermi seas of \(N_f\) atoms in the \(|\downarrow\rangle\) state (open channel) and \(N_i\) atoms in the \(|\uparrow\rangle\) state (closed channel), as illustrated in Fig. 1. We study the molecule and polaron states within such a system using the Chevy-like ansatz [16, 17], which is equivalent with the summation of particle-particle ladder diagrams for the vertex in a non-self-consistent T-matrix approach [17].

In a molecule state, the \(|\uparrow\rangle\) impurity scatters one atom out of the \(|\downarrow\rangle\) Fermi sea and the two atoms form a bound state; or they can be scattered into the closed channel through the spin-flipping processes of the interaction. We then assume a trial wave function as

\[
|M\rangle = \sum_{|k| > k_iF} a_k a_k^\dagger \hat{A}_{q,+}^\dagger a_{-k+q,g} a_{k+q,e} |\downarrow\rangle_{N_f-1} |\uparrow\rangle_{N_f} + \sum_{|k| > k_iF} a_k a_k^\dagger \hat{A}_{q,+}^\dagger a_{-k-q,e} a_{k+q,g} |\downarrow\rangle_{N_f-1} |\uparrow\rangle_{N_f},
\]

where \(Q\) denotes the center-of-mass momentum, and \(a_k^\dagger\) and \(a_k\) are the coefficients of the open and closed channels, respectively. Notice that the summations over linear momentum \(k\) run over states above the corresponding Fermi seas with Fermi momenta \(k_iF\) and \(k_iF\).

By evaluating the energy expectation \(\hat{E}_M(Q) = \langle Q | H | M \rangle_Q\) and taking the variations of \(a_k^\dagger\) and \(a_k\), we can derive the equation for the energy of molecule state [13]

\[
\frac{1}{g^+ g^-} + \frac{1}{2} \left( \frac{1}{g^+} + \frac{1}{g^-} \right) (\Theta_Q + \Theta_Q^* - 2\Lambda_c) \right) + (\Theta_Q - \Lambda_c)(\Theta_Q^* - \Lambda_c) = 0,
\]

where
where
\[
\Theta_Q' = \sum_{|k|>k_{\text{F}}} \frac{1}{\epsilon_k + \epsilon_{Q-k} + \delta_c - E_M}
\]
\[
\Theta_Q = \sum_{|k|<k_{\text{F}}} \frac{1}{\epsilon_k + \epsilon_{Q-k} + \delta_g - E_M},
\]
and \( \Lambda_c = \sum_k 1/(2\epsilon_k) \). Notice that in the expressions above, we shift the energy reference \( E_M = E_M - \sum_{|k|<k_{\text{F}}} \epsilon_k = -\sum_{|k|<k_{\text{F}}} (\epsilon_k + \delta_g) \). With such a reference, the threshold energy \( E_{\text{th}} = \delta_c \) for a noninteracting zero-momentum impurity \(|c\uparrow\rangle\) on the two Fermi surfaces of \(|g\downarrow\rangle_{N_{\uparrow}}\) and \(|g\uparrow\rangle_{N_{\uparrow}}\).

For the polaron state, we consider the following ansatz wave function
\[
|P\rangle_Q = \left[ \gamma a_{Q\uparrow}^\dagger + \sum_{|k|>k_{\text{F}}} \alpha_{kq} a_{Q+q-k,c_{\uparrow}}^\dagger a_{q,c_{\downarrow}} a_{q,g_{\uparrow}} \right] |g\downarrow\rangle_{N_{\uparrow}} |g\uparrow\rangle_{N_{\uparrow}}.
\]
In this expression, the first term corresponds to a bare impurity in the \(|c\uparrow\rangle\) state and two unperturbed Fermi seas, the second term represents a state with one pair of particle-hole excitation atop the \(|g\downarrow\rangle_{N_{\uparrow}}\) Fermi sea, and the third term corresponds to a state where the fermion created above \(|g\downarrow\rangle_{N_{\uparrow}}\) Fermi sea interacts with the \(|c\uparrow\rangle\) impurity and both are scattered into the closed channel.

Using the same method as for the molecule state, we can derive the energy equation for the polaron state, leading to
\[
E_P - \epsilon_Q - \delta_c = \sum_{|q|<k_{\text{F}}} \left\{ \frac{1}{2} \left( \frac{1}{g_{\uparrow}} + \frac{1}{g_{\downarrow}} \right) + \Gamma_{Qq} - \Lambda_c \right\}^{-1},
\]
where
\[
\Gamma_{Qq} = \sum_{|k|>k_{\text{F}}} \frac{1}{\epsilon_k - \epsilon_q + \epsilon_{Q+q-k} + \delta_c - E_P},
\]
\[
\Gamma_{Qq} = \sum_{|k|<k_{\text{F}}} \frac{1}{\epsilon_k - \epsilon_q + \epsilon_{Q+q-k} + \delta_g - E_P}.
\]
Notice that one could write down another polaron state wave function ansatz \(|P'\rangle_Q\) by interchanging the spin indices \(\uparrow\leftrightarrow\downarrow\) from Eq. (7). As the spin-exchange interaction \(H_{\text{int}}\) conserves the number of particles in each spin states, the two polaron states \(|P\rangle\) and \(|P'\rangle\) thus belong to different Hilbert spaces. As a result, the equation for the eigen energy \(E_{P'}\) is completely separated from Eq. (8).

### III. MOLECULE AND ATTRACTIVE POLARON STATES

We first study the case of zero center-of-mass momentum \(Q = 0\), and solve Eqs. (5) and (6) to obtain the energies of the molecule and polaron states. Notice that for polarons, there could exist two branches of solutions with energies below and above the threshold energy \(E_{\text{th}}\), which correspond to the attractive and repulsive polaron states, respectively. In this section, we focus on the attractive polaron branch and discuss its transition to the molecule state, and leave the study of repulsive polaron to Sec. IV.

In Fig. 2 we show the energies of the molecule and attractive polaron states with \(Q = 0\) by varying the Fermi level of \(k_{\text{F}}\). From Figs. 2(a)–2(c), we find that for all values of \(k_{\text{F}}\), the ground state is always the attractive polaron state when \(\delta_c\) is far negative, and turns into the molecule state as \(\delta_c\) increases beyond a transition point \(\delta_c^*\). The transition points for the three cases illustrated in Fig. 2 are all within the BEC side of the OPR, with (a) \(\delta_c^* = -2.28\), \(1/(k_{\text{F}}\delta_c) \approx 0.81\) for \(k_{\text{F}} = 0\); (b)
For both the molecule and attractive polaron states, the wave functions are dominated by the open channel fraction in the negative $\delta_c$ region, while in the positive $\delta_c$ region the closed channel fraction becomes significantly populated. Besides, we also notice that for the attractive polaron state, the bare polaron population $|\gamma|^2$ becomes vanishingly small as the open channel is positively large detuned. These observations are qualitatively consistent with the results for the impurity problem with only one Fermi surface [13].

Next, we discuss the general situation of $Q \neq 0$. For momentum not very far from $Q = 0$, we can perform a series expansion of $Q$ in Eqs. (6) and (8) to calculate the effective masses of molecules and attractive polarons. In the region of large negative $\delta_c$ where the attractive polaron is the ground state, the effective mass for the polaron state is positive and tends to the limiting value of $m_F^L \rightarrow 1/2$ in the BCS limit of $\delta_c \rightarrow -\infty$, where the system reduces to a noninteracting impurity atom of mass $1/2$ atop the two unperturbed Fermi seas. When $\delta_c$ goes beyond the transition point $\delta_c^*$, the molecule becomes the ground state with positive effective mass. Note that as the OPR can not be tuned into the BEC limit with $a_s \rightarrow 0^+$, the molecule effective mass remains $m_F^L > 1$ in this region of $\delta_c$. By further increasing $\delta_c$ to large positive values, the effective masses for both the molecule and attractive polaron states present a diverging behavior and become negative, indicating that the ground state of the system would acquire a finite center-of-mass momentum [13]. In fact, as the open channel is largely detuned above the closed channel with large positive $\delta_c$, the

\[
\delta_e^c = -1.50, \quad 1/(k_{1F}a_c) \approx 1.01 \text{ for } k_{1F} = 1; \quad (c) \delta_e^c = 2.02, \quad 1/(k_{1F}a_c) \approx 0.87 \text{ for } k_{1F} = 2. \quad \text{Here, the critical scattering length } a_c \text{ is obtained from the relation [8]}
\]

\[
a_c = -a_{s0} + \sqrt{m|\delta_e^c|/\hbar^2 (a_{s0}^2 - a_{s1}^2)}/\hbar^2 - 1, \quad (10)
\]

where $a_{s0} = (a_- + a_+)/2$ and $a_{s1} = (a_- - a_+)/2$.

To further illustrate the effect induced by the additional Fermi surface in the closed channel, we plot in Fig. 3(d) the transition point $\delta_e^c$ as a function of Fermi momentum $k_{1F}$. Notice that the transition point increases monotonically with the Fermi level, reflecting the fact that the presence of the $\{\gamma\}$ Fermi sea blocks the states below the Fermi energy, hence provides an effective offset of the closed channel energy $\delta_g$. As a consequence, the open channel energy $\delta_e$ also needs to elevate for a same amount to compensate such a shift. In fact, the transition point can be well approximated by the Fermi energy $E_{1F}$ for $k_{1F}$ not too large, as depicted in Fig. 3(d). The deviation should be caused by the fluctuation around the Fermi level induced by interaction.

We also calculate the fractions of wave functions in the open and closed channels for the molecule and attractive polaron states, as illustrated in Fig. 4. By comparing with results of $k_{1F} = 1$ and $k_{1F} = 2$, we find that the overall structure of the wave functions are very similar, except a shift of energy offset. In particular, for both the molecule and the attractive polaron states, the wave functions are dominated by the open channel fraction in the negative $\delta_c$ region, while in the positive $\delta_c$ region the closed channel fraction becomes significantly populated. Besides, we also notice that for the attractive polaron state, the bare polaron population $|\gamma|^2$ becomes vanishingly small as the open channel is positively large detuned. These observations are qualitatively consistent with the results for the impurity problem with only one Fermi surface [13].

Next, we discuss the general situation of $Q \neq 0$. For momentum not very far from $Q = 0$, we can perform a series expansion of $Q$ in Eqs. (6) and (8) to calculate the effective masses of molecules and attractive polarons. In the region of large negative $\delta_c$ where the attractive polaron is the ground state, the effective mass for the polaron state is positive and tends to the limiting value of $m_F^L \rightarrow 1/2$ in the BCS limit of $\delta_c \rightarrow -\infty$, where the system reduces to a noninteracting impurity atom of mass $1/2$ atop the two unperturbed Fermi seas. When $\delta_c$ goes beyond the transition point $\delta_c^*$, the molecule becomes the ground state with positive effective mass. Note that as the OPR can not be tuned into the BEC limit with $a_s \rightarrow 0^+$, the molecule effective mass remains $m_F^L > 1$ in this region of $\delta_c$. By further increasing $\delta_c$ to large positive values, the effective masses for both the molecule and attractive polaron states present a diverging behavior and become negative, indicating that the ground state of the system would acquire a finite center-of-mass momentum [13]. In fact, as the open channel is largely detuned above the closed channel with large positive $\delta_c$, the
where the functions $\Gamma_{Q\eta}$ and $\Gamma_{Q\bar{\eta}}$ are defined in Eq. (6).

The spectral function thus takes the following form

$$A(Q, E_P) = -2\text{Im} \frac{1}{E_P + i0^+ - (\epsilon_Q + \delta_e) - \Sigma(Q, E_P)}.$$  \hspace{1cm} (12)

where $\epsilon_Q + \delta_e$ is the energy of a bare impurity with momentum $Q$.

In Fig. 5 we plot the spectral function as a function of $\delta_e$ and energy $E$ for $Q = 0$ and $k_{F1} = 1$. There exist two branches where the spectral function is strongly peaked. The lower branch at energy $E_{P+} - E_{th} < 0$ corresponds to the attractive polaron discussed in the previous section. When $\delta_e/k_{1F} \lesssim 1.5$, the attractive polaron is a stable ground state. As $\delta_e/k_{1F}$ goes beyond $-1.5$, the attractive branch becomes unstable towards decay into a molecule and a hole, or a molecule, two holes and a fermion, or a molecule and another higher particle-hole excitations, which is referred as the molecule-hole continuum and illustrated by the light-yellow area above the attractive polaron branch. The upper branch at $E_{P+} - E_{th} > 0$ corresponds to the repulsive polaron. For $\delta_e/k_{1F} \gtrsim 4.0$, the repulsive polaron branch merges into the molecule-hole continuum. For $-4.0 \leq \delta_e/k_{1F} \lesssim 0$, the repulsive polaron is a well-defined quasiparticle with strongly peaked spectral function. As $\delta_e/k_{1F} > 0$, the repulsive polaron peak is also blurred due to the coupling between the repulsive polaron and the closed-channel scattering states \cite{19}. These two branches of polaron state can also be obtained from the relation

$$E_P - \delta_e = \text{Re}\Sigma(Q = 0, E_P). $$  \hspace{1cm} (13)

In fact, the solutions of the equation above are consistent with the peaks of the spectral function, as depicted in Fig. 5.

We now characterize the repulsive polaron state by calculating its populations of wave function in different channels, the quasiparticle residue, and the effective mass. The residue of a polaron state is defined as \cite{33}

$$Z_{\pm} = \left\{ 1 - \text{Re}\left[ \frac{\partial\Sigma(Q = 0, E_P)}{\partial E_P} \right]_{E_p=\pm} \right\}^{-1},$$  \hspace{1cm} (14)

and the effective mass as \cite{33}

$$m_{p_{\pm}} = \frac{m}{Z_{\pm}} \left\{ 1 + \text{Re}\left[ \frac{\partial\Sigma(Q, E_P)}{\partial Q^2} \right]_{Q=0, E_P=\pm} \right\}^{-1}. $$  \hspace{1cm} (15)

From Figs. 6(a) and 6(b), we find that the repulsive polaron is mainly composed by a bare impurity for $\delta_e$ not so negative, where the wave function acquires a dominant term of $|\gamma|^2$. This is consistent with the behaviors of quasiparticle residue [Fig. 6(c)] and effective mass [Fig. 6(d)], which both indicate that the repulsive polaron tends to reduce to a bare impurity with $Z_+ \rightarrow 1$ and $m_{p+} \rightarrow 1/2$, which is weakly interacted with the majority background Fermi seas in the limit of large positive $\delta_e$. \cite{24}. In addition, we also note that the kink structure present in the case of a single Fermi surface with $k_{F1} = 0$ is blurred and eventually smoothed out as imposing an additional Fermi sea. The occurrence of these kinks is rooted from a resonance-like behavior, where the
Figure 6. (Color online) (a), (b) The populations of repulsive polaron wave function in different channels for (a) $k_{TF}=0$ and (b) $k_{TF}=1$ with zero center-of-mass momentum. Note that the bare impurity channel $|\gamma|^2$ becomes dominating with increasing $\delta_e$, indicating that the interaction effect between the bare impurity and the background Fermi sea is small, as can also be implied from (c) the quasiparticle residue $Z_e$ and (d) the effective mass $m_{P+}$. The wave function, residue and effective mass all have an obvious kink structure near $\delta_e/k_{TF} = -0.87$ for $k_{TF} = 0$, as a result of resonant-like scattering between the two channels [15]. In the presence of the second Fermi sea, the kinks are shifted to large $\delta_e$ to accommodate the energy offset induced by the Fermi level $E_{TF}$, and are smoothed out due to the fluctuations around the Fermi sea induced by interaction. (c) Inset: residues for attractive polarons.

As an excited quasiparticle, the repulsive polaron has a finite width in the spectral function, and can decay into low-lying states. For alkali atoms, it has been shown experimentally that the dominating decay channel is the coupling to the bare impurity state in the attractive-polaron branch as the interaction is not in the deep-BEC regime [33]. For the present case of orbital Feshbach resonance, it is natural to assume that the scenario is similar in consideration of the fact that the repulsive polaron wave function is mainly consisted of the bare impurity, as shown in Figs. 6(a) and 6(b). Such a decay rate can be calculated as [15] [33]

$$\Gamma_{PF} = -2Z_e\text{Im}[\tilde{\Sigma}(0, E_{P+})],$$

where $Z_e$ is the residue for the repulsive polaron, and

$$\tilde{\Sigma}(Q, E_{P+}) = \sum_{\Delta Q < k_{TF}} \left[ \frac{1}{2} \left( \frac{1}{g_e} + \frac{1}{g_{P+}} \right) + (1 - Z_e) (\Gamma Q_q - \Lambda) \right] \cdot \frac{1}{\left( \frac{1}{g_e} - \frac{1}{g_{P+}} \right)^2} \cdot \frac{1}{\left( \frac{1}{g_e} - \frac{1}{g_{P+}} \right) + \Gamma Q_q - \Lambda}^{-1}. \quad (17)$$

Note that in the expression of $\tilde{\Sigma}$, we have replaced the free-fermion propagator $(\Gamma Q_q - \Lambda)$ by $(1 - Z_e) (\Gamma Q_q - \Lambda)$ in the self-energy $\Sigma$ of Eq. (11), in order to take into account the fact that the final state of the decay channel, i.e., a bare impurity in the attractive-polaron branch, exists with a probability approximately given by $(1 - Z_e)$.

The results of $\Gamma_{PF}$ calculated from Eq. (16) are shown in Fig. 7. The overall behavior of $\Gamma_{PF}$ as a function of $\delta_e$ is qualitatively similar for different values of $k_{TF}$. Specifically, in the regime of small negative and positive $\delta_e$ where the open-channel is detuned above the closed channel, the decay of the repulsive polaron is dominated by the closed channel. The decay rate increases as the system is tuned further towards large positive $\delta_e$. On the other hand, in the regime of small positive and large negative $\delta_e$ where the open channel is energetically favorable, the decay rate is open-channel dominated and increases as further decreasing $\delta_e$. The competition between the two channels thus gives rise to a non-monotonic behavior of $\Gamma_{PF}$, which reaches a minimum when the two channels are nearly degenerate. By moving further towards the BCS limit of large negative $\delta_e$, the decay rate first starts to drop due to the decreasing quasiparticle residue $Z_e$, as shown in Fig. 6(c), and finally becomes undefined as the repulsive polaron branch eventually merges into the molecule-hole continuum as illustrated in Fig. 5 where the repulsive polaron is no longer a well-defined quasi-
particle.

V. CONCLUSION

We study the impurity problem of Fermi gases near an orbital Feshbach resonance, where an atom in the \( |e↑\rangle \) state is immersed on top of two Fermi seas filled by the two \( |g⟩ \) states. By calculating the energies of the molecule and attractive polaron with zero center-of-mass momentum \( Q = 0 \), we find a polaron-to-molecule transition by crossing an orbital Feshbach resonance, whose exact location is shifted by the presence of Fermi level \( E_{F} \) in the closed channel. We further study the properties of the attractive polaron and molecule states by characterizing the wave function and the effective mass. For the repulsive polaron state, we introduce the retarded self-energy \( \Sigma \) to calculate the spectral function, the quasiparticle residue, the wave function distribution, the effective mass, and the decay rate. From these results, we conclude that the presence of an additional Fermi sea in the closed channel acts as an energy offset of the closed channel and consequently shifts the polaron-molecule transition and other key characteristics to higher values of detuning \( \delta_{c} \). The fluctuation around the Fermi level induced by the spin-exchange interaction would also blur the resonant-like behavior and smooth out the kink structure in various properties of the repulsive polaron. Our results can be studied experimentally using the experimental techniques in alkaline-earth(-like) atoms.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China (Grant Nos. 11434011, 11522436, 11704408, 11774425), and the Research Funds of Renmin University of China (Grant No. 16XNLQ03). X.Z. acknowledges support from the National Postdoctoral Program for Innovative Talents (Grant No. BX201601908) and the China Postdoctoral Science Foundation (Grant No. 2017M620991).

[1] R. Zhang, Y. Cheng, H. Zhai, and P. Zhang, Phys. Rev. Lett. 115, 135301 (2015).
[2] G. Pagano, M. Mancini, G. Cappellini, L. Livi, C. Sias, J. Catani, M. Inguscio, and L. Fallani, Phys. Rev. Lett. 115, 265301 (2015).
[3] M. Höfer, L. Riegger, F. Scazza, C. Hofrichter, D. R. Fernandes, M. M. Parish, J. Levinsen, I. Bloch, and S. Fölling, Phys. Rev. Lett. 115, 265302 (2015).
[4] Y. Cheng, R. Zhang, and P. Zhang, Phys. Rev. A 93, 042708 (2016).
[5] T.-S. Deng, W. Zhang, and W. Yi, Phys. Rev. A 96, 030701(R) (2017).
[6] M. Iskin, Phys. Rev. A 94, 011604(R) (2016).
[7] M. Iskin, Phys. Rev. A 95, 013618 (2017).
[8] J. Xu, R. Zhang, Y. Cheng, P. Zhang, R. Qi, and H. Zhai, Phys. Rev. A 94, 033609 (2016).
[9] L. He, J. Wang, S.-G. Peng, X.-J. Liu, and H. Hu, Phys. Rev. A 94, 043624 (2016).
[10] Y.-C. Zhang, S. Ding, and S. Zhang, Phys. Rev. A 95, 041603(R) (2017).
[11] S. Wang, J.-S. Pan, X. Cui, W. Zhang, and W. Yi, Phys. Rev. A 95, 044634 (2017).
[12] Y. Cheng, R. Zhang, and P. Zhang, Phys. Rev. A 95, 013624 (2017).
[13] J.-G. Chen, T.-S. Deng, W. Yi, and W. Zhang, Phys. Rev. A 94, 053627 (2016).
[14] J. Xu, and R. Qi, arXiv:1710.00785v1.
[15] T.-S. Deng, Z.-C. Lu, Y.-R. Shi, J.-G. Chen, W. Zhang, and W. Yi, Phys. Rev. A, accepted.
[16] F. Chevy, Phys. Rev. A 74, 063628 (2006).
[17] R. Combescot, A. Recati, C. Lobo, and F. Chevy, Phys. Rev. Lett. 98, 180402 (2009).
[18] M. Punk, P. T. Dumitrescu, and W. Zwerger, Phys. Rev. A 80, 053605 (2009).
[19] S. Zölner, G. M. Bruun, and C. J. Pethick, Phys. Rev. A 83, 021603(R) (2011).
[20] M. Klawun and A. Recati, Phys. Rev. A 84, 033607 (2011).
[21] M. M. Parish, Phys. Rev. A 85, 051603(R) (2011).
[22] C. Trefzger and Y. Castin, Phys. Rev. A 85, 053612 (2012).
[23] M. Koschorreck, D. Pertot, E. Vogt, B. Fröhlich, M. Feld, and M. Köhl, Nature 485, 619 (2012).
[24] P. Massignan and G. M. Bruun, Eur. Phys. J. D 65, 83-89 (2011).
[25] P. Massignan, Phys. Rev. Lett. 110, 230401 (2013).
[26] C. Kohstall, M. Zaccanti, M. Jag, A. Trenkwalder, P. Massignan, G. M. Bruun, F. Schreck, and R. Grimm, Nature 485, 615 (2012).
[27] X. Cui and H. Zhai, Phys. Rev. A 81, 041602(R) (2010).
[28] S. Pilati, G. Bertaina, S. Giorgini, and M. Troyer, Phys. Rev. Lett. 105, 030405 (2010).
[29] M. Cetina, M. Jag, R. S. Lous, I. Fritsche, J. T. M. Waldraven, R. Grimm, J. Levinsen, M. M. Parish, R. Schmidt, M. Knap, E. Demler, Science 354, 96 (2016).
[30] F. Scazza, G. Valtolina, P. Massignan, A. Recati, A. Amico, A. Burchianti, C. Fort, M. Inguscio, M. Zaccanti, G. Roati, Phys. Rev. Lett. 118, 083602 (2017).
[31] S. Mondal, D. Inotani and Y. Ohashi, arXiv:1704.00154v1.
[32] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. 82, 1225 (2010).
[33] F. Scazza, G. Valtolina, P. Massignan, A. Recati, A. Amico, A. Burchianti, C. Fort, M. Inguscio, M. Zaccanti, and G. Roati, Phys. Rev. Lett. 118, 083602 (2017).