Fast Near-Field Beam Training for Extremely Large-Scale Array

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Abstract—In this letter, we study efficient near-field beam training design for the extremely large-scale array (XL-array) communication systems. Compared with the conventional far-field beam training method that searches for the best beam direction only, the near-field beam training is more challenging since it requires a beam search over both the angular and distance domains due to the spherical wavefront propagation model. To reduce the near-field beam-training overhead based on the two-dimensional exhaustive search, we propose in this letter a new two-phase beam training method that decomposes the two-dimensional search into two sequential phases. Specifically, in the first phase, the candidate angles of the user are determined by a new method based on the conventional far-field codebook and angle-domain beam sweeping. Then, a customized polar-domain codebook is employed in the second phase to find the best effective distance of the user given the shortlisted candidate angles. Numerical results show that our proposed two-phase beam training method significantly reduces the training overhead of the exhaustive search and yet achieves comparable beamforming performance for data transmission.

Index Terms—Extremely large-scale array (XL-array), near-field communication, beam training.

I. INTRODUCTION

EXTREMELY large-scale array/surface (XL-array/surface) with active or passive elements has emerged as a promising technology to significantly enhance the spectral efficiency and spatial resolution in future sixth-generation (6G) wireless network [1], [2], [3], [4]. Compared with the massive multiple-input-multiple-output (MIMO) for fifth-generation (5G), XL-array for 6G introduces several new channel characterizations. Specifically, with the significant increase of the antenna number and carrier frequency, the users are more likely to locate in the near-field region with spherical wavefront propagation, due to the smaller user/link distance than the enlarged Rayleigh distance. As such, the conventional transceiver designs based on the far-field channel model with the planar wavefront assumption may not be applicable, hence calling for new design approaches dedicated to the near-field communication [5], [6], [7], [8], [9].

In particular, to reap the prominent beamforming gain brought by the XL-array, it is indispensable for the XL-array to perform the near-field beam training for establishing high signal-to-noise ratio (SNR) initial links before efficient channel estimation and data transmission [7], [10]. In contrast to the conventional far-field beam training methods that aim to find the best beam direction based on a predefined angular-domain beam codebook, the near-field beam training requires new codebook designs and efficient search for a proper beam to match the specific channel angle and distance according to the spherical wavefront model. As such, directly applying the conventional far-field beam training methods [10], [11] to the near-field communication systems will result in considerable performance degradation, especially when the user is very close to the XL-array.

To address the above issues, several new beam training methods dedicated to the near-field communication have been recently proposed in the literature [7], [8], [9]. Specifically, the authors in [7] proposed a hierarchical near-field codebook and the corresponding hierarchical near-field beam training scheme, where different levels of sub-codebooks are searched in turn with reduced codebook size. To further reduce the size of the near-field codebook, a novel polar-domain codebook dedicated to the near field was proposed in [8], wherein the angular domain is uniformly sampled while the distance domain is sampled non-uniformly to reduce the codebook size. In addition, by exploiting the near-field beam squint effect, a fast wideband beam training scheme was proposed in [9]. Specifically, by operating beams at different frequencies to be focused on different locations, the optimal beamforming vector is rapidly searched, thus greatly reducing the training overhead. However, the existing near-field training methods for the narrow-band require a two-dimensional exhaustive search for all possible beam directions and distances, thus leading to prohibitively high training overhead.

Motivated by the above, we consider an XL-array communication system and propose a new two-phase fast beam training method to significantly reduce the near-field beam training overhead based on the exhaustive search. To this end, we first show that the actual user spatial direction approximately locates in the middle of a newly defined dominant-angle region. Based on this key observation, we propose to decompose the two-dimensional search into two sequential phases. Specifically, the first phase determines the candidate angles of the user based on the far-field codebook and the angle-domain beam sweeping. Then, the second phase employs a customized polar-domain codebook to find the best effective distance of the user given the shortlisted candidate angles instead of the full angular domain as in the exhaustive search, thus effectively reducing the training overhead. Finally, numerical results show that the proposed fast beam training scheme greatly reduces the training overhead of the exhaustive search, yet without compromising much the XL-array beamforming performance for data transmission.

Manuscript received 4 September 2022; accepted 28 September 2022. Date of publication 6 October 2022; date of current version 9 December 2022. This work was supported by the National Natural Science Foundation of China under Grant 62201242. The associate editor coordinating the review of this article and approving it for publication was G. Brante. (Corresponding author: Changsheng You.)

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Digital Object Identifier 10.1109/LWC.2022.3212344
II. SYSTEM MODEL

As shown in Fig. 1, we consider the downlink beam training for a narrow-band XL-array communication system, where a base station (BS) equipped with N-antenna uniform linear array (ULA) communicates with a single-antenna user.

Near-field channel model: Generally speaking, the electromagnetic radiation field can be divided into the far-field and near-field regions, leading to different channel characterizations [12]. Specifically, the user is assumed to locate in the far-field or near-field region when the BS-user distance is larger or smaller than the well-known Rayleigh distance, denoted by $D = \frac{2\lambda}{\pi}$ with $\lambda$ denoting the antenna array aperture and the carrier wavelength, respectively. In particular, for XL-array communication systems in high-frequency bands (e.g., mmWave/Terahertz), say $D = 0.4$ m and $f = 100$ GHz, the Rayleigh distance is about 107 m and thus the user usually locates in the near field, whose channel is modeled as follows.

Without loss of generality, the N-antenna BS is placed at the y-axis with the coordinate of the $n$-th antenna given by $(0, \delta_n d)$, where $\delta_n = \frac{2n-N-1}{2}$ with $n = 1, 2, \ldots, N$, and $d = \frac{\lambda}{2}$ is the antenna spacing. Then, based on the spherical wavefront propagation model, the near-field line-of-sight (LoS) channel from the BS to the user, denoted by $h_{\text{near}}^H$, can be modeled as [8]

$$h_{\text{near}}^H = \sqrt{N} h b^H(\theta, r),$$  

where $h = \frac{\sqrt{2}}{\lambda} e^{-2\pi j x/\lambda}$ is the complex-valued channel gain with $\beta$ and $r$ denoting the reference channel gain at a distance of 1 m and the distance between the BS center and the user, respectively. $b^H(\theta, r)$ denotes the near-field steering vector, given by

$$b^H(\theta, r) = \frac{1}{\sqrt{N}} \left[ e^{-j2\pi (r(0) - r)/\lambda}, \ldots, e^{-j2\pi (r(N-1) - r)/\lambda} \right],$$

where $\theta$ denotes the spatial angle at the BS, given by $\theta = 2d \cos(\phi)/\lambda$ with $\phi$ denoting the physical angle-of-departure (AoD) from the BS center to the user; $r(n) = \sqrt{r^2 + \delta_n^2 d^2 - 2r\theta \delta_n d}$ represents the distance between the $n$-th antenna at the BS (i.e., $(0, \delta_n d)$) and the user.

III. BENCHMARK SCHEMES

In this section, we introduce two benchmark beam-training schemes and point out their main issues.

A. Angle-Domain Far-Field Beam Training Based on Exhaustive Search

First, consider the conventional far-field beam training method and its issues when directly applied in the near-field beam training. Based on the planar wavefront assumption, the far-field channel model $h_{\text{far}}^H$ can be characterized as

$$h_{\text{far}}^H = \sqrt{N} h a^H(\theta),$$

where $a^H(\theta) = \frac{1}{\sqrt{N}} \left[ 1, e^{-j\pi \theta}, \ldots, e^{-j\pi (N-1)\theta} \right].$ 

Similar to (3), the received signal at the user can be characterized as

$$y_{\text{far}} = h_{\text{far}}^H v x + z_0 = \sqrt{N} h a^H(\theta) v x + z_0.$$

For the classical far-field beam training, the entire spatial domain $[-1, 1]$ is divided into $N$ equal-size sectors, represented by their central directions specified by $\theta_n = \frac{2n-N-1}{N}, n \in N \triangleq \{1, 2, \ldots, N\}$. As such, the far-field codebook consisting of $N$ codewords can be constructed as $W = \{w_1, w_2, \ldots, w_N\}$, each codeword $w_n$ corresponding to a transmit beam pointing to the central direction $\theta_n$, i.e., $w_n = a(\theta_n)$ [10]. Then, the BS sequentially selects its $N$ codewords as its transmit beamforming vector over the training symbols. Last, the user measures its received signal power over time and feeds back the best codeword index that leads to the maximum received signal power.

However, the conventional far-field beam training method cannot be applied to the considered near-field communication. This is because the far-field channel model defined in (4) mainly depends on the spatial angle $\theta$, while the near-field steering vector in (2) is jointly determined by both the distance and spatial angle. Thus, straightforwardly applying the far-field beam training method to the near-field communication scenario will result in significant rate performance loss due to the channel-model mismatch (see Section V).
B. Polar-Domain Near-Field Beam Training Based on Exhaustive Search

For the near-field beam training, we consider the typical polar-domain near-field codebook proposed in [8], which is given by \( F = \{ F_1, F_2, \ldots, F_N \} \) with \( F_n = \{ f_{n,0}, f_{n,1}, \ldots, f_{n,S_n-1} \} \), \( \forall n \in N \), wherein each codeword \( f_{n,\theta_n} \), \( \theta_n \in S_n \triangleq \{ 0, 1, \ldots, S_n - 1 \} \) corresponds to a beam pointing to a specific direction-and-distance pair (i.e., \( (\theta_n, r_{n,\theta_n}) \)), which is given by \( f_{n,\theta_n} = b(\theta_n, r_{n,\theta_n}) \). Specifically, the spatial domain is equally divided into \( N \) sectors represented by its central angle; while in the distance domain, for each angle \( \theta_n \), the distance is divided into \( S_n \) (which may not be the same for different \( \theta_n \)) non-uniform regions where the distance sampling interval gets larger as the distance increases. This is expected since the distance effect on the beamforming design diminishes with the distance and finally disappears in the far-field region (i.e., only determined by the spatial angle). Based on the proposed polar-domain codebook [8], the exhaustive-search based beam training method can be applied to find the best beam codeword for the user. However, this exhaustive-search method requires a total of \( \sum_{n=1}^{N} S_n \) training symbols, which is prohibitively high when \( N \) is large and hence results in insufficient/less time for data transmission.

IV. PROPOSED TWO-PHASE NEAR-FIELD BEAM TRAINING METHOD

To reduce the huge training overhead of the near-field beam training method based on the exhaustive search, in this section, we propose a new two-phase beam training method and show its appealing advantages as compared to the benchmark schemes.

First, we make the following key definitions.

**Definition 1:** Define \( A(u^H, w) \) as the normalized beam gain of the beamforming vector \( w \) along the channel steering vector \( u^H \), which is given by

\[
A(u^H, w) = |u^H w|.
\]

**Definition 2:** Define \( w(\Omega) \) as the far-field beamforming vector function for the spatial angle \( \Omega \in [-1, 1] \). Then, let \( C(b^H(\theta, r), w) \) denote the dominant-angle region for \( b^H(\theta, r) \) that characterizes the spatial angle region where using the far-field beamforming vector \( w \) can lead to sufficiently high beam power for \( b^H(\theta, r) \). Mathematically, \( C(b^H(\theta, r), w) \) is given by

\[
C(b^H(\theta, r), w) = \{ \Omega | A(b^H(\theta, r), w(\Omega)) > \rho \max_w A(b^H(\theta, r), w) \},
\]

where \( \rho \in (0, 1) \) is the beam gain threshold for \( b^H(\theta, r) \). In particular, \( \rho \) can be set as \( 1/\sqrt{2} \), corresponding to the 3 dB dominant-angle region [11].

Then, we show in Fig. 2 the normalized beam gains of the far-field beamforming vector \( w \) versus the spatial angle under both the far- and near-field channel models, which are given by \( A(b^H_{\text{far}}, w) = |a^H(\theta)w| \) and \( A(b^H_{\text{near}}, w) = |b^H(\theta, r)w| \), respectively. One key observation is highlighted as follows.

**Observation 1 (Angle-in-the-Middle):** In Fig. 2, the true spatial angle \( \theta \) approximately locates in the middle of the dominant-angle region based on the far-field beamforming vector \( w \). Mathematically, we have \( \theta \approx \text{Med}(C(b^H(\theta, r), w)) \).

**Observation 2** indicates that the true spatial angle for the near-field user can be approximately located by using the conventional far-field beam training method. This thus motivates the proposed design of a new two-phase near-field beam training method. The key idea is to firstly estimate the candidate spatial angle based on the far-field codebook, following by the effective distance estimation in the second phase based on a customized polar-domain codebook. The detailed procedures for the two phases are elaborated as follows.

- **First phase (Candidate angle estimation):** Motivated by **Observation 1**, we aim to estimate the spatial angle of the user in the first phase based on the far-field beam training method. Specifically, an angle-domain codebook \( W \) is constructed with each codeword pointing to one spatial direction.

1) **Angle-domain beam sweeping:** The BS sequentially sends \( N \) training symbols, while it dynamically tunes its beam direction (specified by the codeword) according to the predefined angle-domain codebook \( W = \{ w_1, w_2, \ldots, w_N \} \). For each codeword \( w_n \), the received signal at the user is given by

\[
y(w_n) = \sqrt{Nh^H(\theta, r)a(\theta_n)x + z_n}.
\]

2) **Candidate angle estimation:** After the beam sweeping, based on **Definition 2**, the index set of the codewords for which the user receives significantly high power (i.e., dominant-angle region), denoted by \( \Psi \), is given by \( \Psi = \{ n | |y(w_n)|^2 > \rho^2 \max_w |y(w)|^2 \} \). Note that the median angle of the obtained dominant-angle region may not be accurate enough due to the power fluctuation, the existence of received noise and the approximation in **Observation 1**. To address this issue, we thus propose a new **middle-K angle selection scheme** that selects \( K \) candidate spatial angles in the middle of the quantized dominant-angle region rather than selecting one spatial angle only. Specifically, let \( \Xi \) denote the **candidate angle index set**, which is given by \( \Xi = \{ |\text{Med}(\Psi)| - \lfloor \frac{\Xi}{2} \rfloor, \ldots, |\text{Med}(\Psi)|, \ldots, |\text{Med}(\Psi)| + \lfloor \frac{\Xi}{2} \rfloor \} \) with \( K \) denoting the total number of candidate spatial angles.

![Normalized beam gains of w(Ω) along the far- (marked by dotted lines) versus near-field (marked by solid lines) steering vectors, i.e., a^H(·) and b^H(·), under four angles (marked by black dashed lines), where the BS antenna number is 256 and the carrier frequency is 100 GHz. The BS-user distances of the near-field and far-field are set as 1 m and 100 m, respectively.](image-url)
Proposed Two-Phase Beam Training Method

1. Phase 1: Candidate angle estimation
   1. Use the far-field codebook \( W \) for the angle-domain beam sweeping and obtain the dominant-angle region \( \Xi \).
   2. Obtain \( \Xi \) according to the proposed middle-\( K \) angle selection scheme.

2. Phase 2: Effective distance estimation
   1. For \( k \in \Xi \) do
   2. Obtain the dominant-angle region \( \Xi_k \).
   3. Obtain the best polar-domain codeword \( \hat{f}_{k,s_k} \).
   4. Distance-domain beam sweeping: The BS sends \( \hat{f}_{k,s_k} \) to the user.
   5. Effective distance estimation:
      - Distance-domain beam sweeping: The BS sends \( \hat{f}_{k,s_k} \) to the user.
      - Obtained effective distance estimation based on the non-uniform distance sampling method \[ (10) \]. Note that the effective user distance is defined as the best sampled distance for the user in the polar-domain codebook, which may not be the true BS-user distance due to the limited number of sampling. Specifically, the polar-domain codebook \( F \) is adopted which consists of \( \sum_{n=1}^{N} S_n \) sets of codewords, each corresponding to one specific angle-and-distance pair.
      1. Distance-domain beam sweeping: The BS sends \( K \) sets of pilot symbols to the user. For each candidate angle \( \theta_k, k \in \Xi \), the sampled distances are given by
         \[
         r_k,s_k = \frac{1}{s_k} Z_\Delta (1 - \theta_k^2), \quad s_k = 0, 1, 2, 3, \ldots, \tag{10}
         \]
         where \( Z_\Delta = \frac{D^2}{2 \alpha \Delta} \) is the threshold distance defined to limit the column coherence between two near-field steering vectors [8]. The corresponding non-uniform distance index set, denoted by \( \mathcal{R}_k \), is given by
         \[
         \mathcal{R}_k \triangleq \{0, \ldots, s_k, \ldots, S_k - 1\}. \tag{11}
         \]
         Then, given the polar-domain codeword \( f_{k,s_k} \), the received signal at the user is given by
         \[
         y(f_{k,s_k}) = \sqrt{N} h b^H(\theta_k, r_k) b(\theta_k, r_k, s_k) x + z_0. \tag{12}
         \]
      2. Beam determination: After the distance-domain beam sweeping in the second phase, the user determines its best polar-domain beam codeword, denoted by \( f_{k^*,s_{k^*}} \), based on its received signal powers/SNRs. Mathematically, the best polar-domain codeword index is given by
         \[
         (k^*, s_{k^*}) = \arg \max_{k,s_k \in \mathcal{R}_k} |y(f_{k,s_k})|^2. \tag{13}
         \]
         The best beam direction-and-distance is thus given by
         \[
         (\theta_{k^*}, r_{k^*}, s_{k^*}). \tag{14}
         \]
         The detailed steps for the proposed two-phase beam training method are summarized in Algorithm 1.

Remark 1 (Accuracy of Effective Distance Estimation): Based on the non-uniform distance sampling method in (10), the distance domain is more frequently sampled in the short-distance region and less sampled otherwise. This indicates that the effective distance estimation is more accurate when the user is closer to the BS, while it may incur a large error in the long-distance region. The error is expected since the effective distance is resolved from the best beam-training vector, while the distance has the negligible effect in the far-field region. Fortunately, it is worth noting that the possible inaccurate distance estimation due to the non-uniform distance sampling will not affect too much the beamforming performance of the proposed beam training method, which is the ultimate goal of the near-field beam training and will be numerically validated in Section V.

Remark 2 (Universal in Both Near- and Far-Field Communications): It is worth mentioning that the proposed two-phase beam training method can be easily extended to a universal beam training method applicable to both the near- and far-field communications. Specifically, after the first phase (candidate angle estimation), the user can decide whether the received signal power is dispersed in a large angle domain. If the user receives strong signal power only in a small number of training symbols (e.g., only one), it indicates that the user is likely to locate in the far-field region and thus it does not need to execute the second phase. Otherwise, it goes to the second phase for the near-field effective distance estimation.

Remark 3 (Training Overhead): The total number of training symbols of our proposed two-phase beam training method, denoted by \( T^{(2P)} \), is given by

\[
T^{(2P)} = T^{(ae)} + T^{(de)} = N + \sum_{k=1}^{K} S_k, \tag{14}
\]
where \( T^{(ae)} = N \) denotes the training overhead of the first phase (candidate angle estimation) and \( T^{(de)} = \sum_{k=1}^{K} S_k \) denotes the training overhead of the second phase (effective distance estimation). It can be easily verified that \( T^{(2P)} \) is much smaller than the training overhead of the exhaustive-search based near-field beam training method [8], i.e., \( T^{(es)} = \sum_{n=1}^{N} S_n \). For instance, consider the case where \( N = 256, f = 100 \) GHz, \( S_n = S_k = 6, \forall n \in N, k \in \Xi \) [8], and \( K = 3 \). Then, we have \( T^{(2P)} = 274 \), which is much smaller than \( T^{(es)} = 1536 \). Last, it is worth mentioning that there exists a fundamental tradeoff between the beam training performance versus the number of candidate angles, \( K \). Specifically, when \( K \) is larger, it incurs a larger training overhead, while leading to a higher beamforming gain due to the more accurate angle estimation.

V. Numerical Results

In this section, we provide numerical results to validate the effectiveness of our proposed two-phase beam training method. Specifically, we consider an XL-MIMO communication system with \( N = 256, \alpha_\Delta = 1.2 \) [8], \( f = 100 \) GHz, and \( \beta = (\lambda/4\pi)^2 = -72 \) dB. We define the (reference) SNR of the XL-MIMO system as \( \text{SNR} = \frac{P\beta N}{\gamma^2 d^2} \), where the noise power is set as \( \gamma^2 = -70 \) dBm. To characterize the beam determination accuracy, we first define \( (\hat{n}, \hat{s}_{n}) \) as the index of the best polar-domain codeword, which is given by \( (\hat{n}, \hat{s}_{n}) = \arg \max_{f \in F} |h^{H}(\theta, r)|^2 \). As such, we define the (beam-training) success rate as \( P_{\text{succ}} = \frac{M}{M} \sum_{k=1}^{M} \mathbb{I}(\hat{n} = k^*) \mathbb{I}(\hat{s}_{n} = s_{k^*}) \), where \( \mathbb{I}(\cdot) \) stands for the indicator function and \( M \) denotes the number of beam training realizations which is set as \( M = 1000 \). Moreover, the achievable rate is given by \( R = \log_2(1 + \frac{P\beta N|b^{H}(\theta, r)|^2}{\gamma^2 d^2}) \).
success rate and achievable rate in Figs. 3(c) and 3(d). The BS suffers a large estimation error due to low SNR.

beam determination, while the LS channel estimation method beam-training method can still form strong beams for effective than the LS channel estimation method. This is because the beam-training method attains much better rate performance noting that in the challenging low-SNR regime, our proposed training scheme under different SNRs. In addition, it is worth training method significantly outperforms the far-field beam training method for the XL-array communication systems. It was shown that by sequentially performing beam sweeping in the angular and then distance domains, our proposed two-phase beam training method can significantly reduce the training overhead of the existing methods based on the two-dimensional exhaustive search, while achieving high-quality beamforming performance. Moreover, it is worth mentioning that the proposed two-phase beam training method is applicable to both the far- and near-field communication systems.

VI. C ONCLUSION

In this letter, we proposed a novel two-phase near-field beam training method for the XL-array communication systems. It was shown that by sequentially performing beam sweeping in the angular and then distance domains, our proposed two-phase beam training method can significantly reduce the training overhead of the existing methods based on the two-dimensional exhaustive search, while achieving high-quality beamforming performance. Moreover, it is worth mentioning that the proposed two-phase beam training method is applicable to both the far- and near-field communication systems.

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