Cosmological birefringence due to CPT-even Chern-Simons-like term with Kalb-Ramond and scalar fields

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Abstract

We study the CPT-even dimension-six Chern-Simons-like term by including dynamical Kalb-Ramond and scalar fields to examine the cosmological birefringence. We show that the combined effect of neutrino current and Kalb-Ramond field could induce a sizable rotation polarization angle in the cosmic microwave background radiation polarization.

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I. INTRODUCTION

The Lorentz and CPT invariance are foundations of particle physics. Testing the validity of these two invariance principles has been the hottest topic in the field. One of the tests is to use the cosmological birefringence [1, 2], which is an additional rotation of synchrotron radiation from the distant radio galaxies and quasars. Since it is wavelength-independent, it is different from Faraday rotation. The first indication of the cosmological birefringence was claimed by Nodland and Ralston [3]. Unfortunately, it has been shown that there is no statistically significant signal [4, 5]. Nevertheless, this provides a new way to search for new physics in cosmology. In recent years, there are many groups using combined data to constrain this small violation effect. In particular, the analysis by Feng et al. gives $\Delta \alpha = -6.0 \pm 4.0 \text{ deg}$ [2], while the Wilkinson Microwave Anisotropy Probe (WMAP) group $\Delta \alpha = -1.7 \pm 2.1 \text{ deg}$ with five year data [6]. In addition, the Combined WMAP five year data with the BOOMERanG data leads to $\Delta \alpha = -2.6 \pm 1.9 \text{ deg}$ [7, 8], the improved result by the QUaD Collaboration is $\Delta \alpha = 0.64 \pm 0.5 \pm 0.5 \text{ deg}$ [9], and the combined QUaD, WMAP7, B03 and BICEP data indicates $\Delta \alpha = -0.04 \pm 0.35 \text{ deg}$ [10]. It has pointed out that the Planck Surveyor [11] will reach a sensitivity of $\Delta \alpha$ at levels of $10^{-2} - 10^{-3}$ [12], while a dedicated future experiment on the cosmic microwave background radiation polarization would reach $10^{-5} - 10^{-6} \Delta \alpha$-sensitivity [12].

It is known that this phenomenon can be used to test the Einstein equivalence principle as was first pointed out by Ni [13, 14]. Another theoretical origin of the birefringence was developed by Carroll et al. [1, 4]. They modified the Maxwell Lagrangian by adding an CPT violating Chern-Simons term [1], which results in numerous subsequent woks [15]. In Ref. [16], an CPT-even dimension-six Chern-Simons-like term was considered, in which the four-vector $p_\nu$ is related to a neutrino current [16] and a Kalb-Ramond field as an auxiliary field to maintain general gauge invariance. It is clear that an observation of the cosmological birefringence may not imply CPT violation but parity violation.

In this paper, we extend the study in Ref. [16] by considering the dynamics of a Kalb-Ramond field and a scalar field. We consider the flat Friedmann-Lemaitre-Robertson-Walker (FLRW) space-time with the metric: $ds^2 = -dt^2 + a^2(t)dx^2$, where $a(t)$ is the scale factor. We use the convention signature of the metric tensor $g = \text{diag}(−, +, +, +)$ and $\epsilon^{\mu\nu\alpha\beta} = (1/\sqrt{g}) e^{\mu\nu\alpha\beta}$, where $e^{\mu\nu\alpha\beta}$ is the Levi-Civita tensor normalized by $e^{0123} = +1$. We also use
units of \( k_B = c = \hbar = 1 \).

The paper is organized as follows. In Sec. II, we explain the model and derive the equations of motion. We explore the cosmological birefringence in Sec. III. Finally, conclusions are given in Sec. IV.

II. THE MODEL

We start with the action

\[
S_0 = \int d^4x \sqrt{\mathcal{g}} \left[ -\frac{1}{2}\epsilon \phi^2 R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right. \\
- \frac{\xi_1}{6\phi^2} H_{\mu\nu\alpha} H^{\mu\nu\alpha} + \frac{\xi_2}{\phi^2} j_\mu \left( A_\nu \tilde{F}^{\mu\nu} + \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} \partial_\nu B_{\alpha\beta} \right) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right],
\]

(1)

where \( \phi \) is the scalar field with the potential \( V(\phi) \), \( j_\mu = \bar{f} \gamma_\mu f \equiv (j^0, \vec{j}) \) is the fermion current, \( H_{\mu\nu\alpha} = \partial_{[\mu} B_{\nu\alpha]} \) is the Kalb-Ramond field strength, \( F_{\mu\nu} = \partial_{[\mu} A_{\nu]} \) and \( \tilde{F}^{\mu\nu} = (1/2) \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \) with the electromagnetic vector field \( A_\mu \), and the parameters \( \epsilon, \xi_1 \) and \( \xi_2 \) are unknown constants. It is well-known that Eq. (1) is not gauge invariant under a gauge transformation because of the interaction \( \frac{\xi_2}{\phi^2} j_\mu \left( A_\nu \tilde{F}^{\mu\nu} + \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} \partial_\nu B_{\alpha\beta} \right) \). We note that under the gauge transformation \( A_\mu \to A_\mu + \partial_\mu \theta \), one obtains

\[
\frac{\xi_2}{\phi^2} j_\mu \left( A_\nu \tilde{F}^{\mu\nu} + \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} \partial_\nu B_{\alpha\beta} \right) \to \frac{\xi_2}{\phi^2} j_\mu \left( A_\nu \tilde{F}^{\mu\nu} + \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} \partial_\nu B_{\alpha\beta} \right) \\
+ \frac{1}{2} j_\mu \epsilon^{\mu\nu\alpha\beta} \left[ (\partial_\nu \theta) F_{\alpha\beta} + \partial_\nu \delta B_{\alpha\beta} \right].
\]

(2)

The extra term in Eq. (2) from the gauge transformation should be zero, i.e.,

\[
\frac{1}{2} j_\mu \epsilon^{\mu\nu\alpha\beta} \left[ (\partial_\nu \theta) F_{\alpha\beta} + \partial_\nu \delta B_{\alpha\beta} \right] \\
= \frac{1}{2} j_\mu \epsilon^{\mu\nu\alpha\beta} \left[ \partial_\nu (\theta) F_{\alpha\beta} + \partial_\nu \delta B_{\alpha\beta} \right] = 0,
\]

(3)

which leads to \( \delta B_{\alpha\beta} = -\theta F_{\alpha\beta} \). Therefore, we have to modify the field strength tensor of \( B_{\mu\nu} \) as

\[
\tilde{H}_{\mu\nu\alpha} \equiv H_{\mu\nu\alpha} + A_{[\mu} F_{\nu\alpha]}.
\]

(4)

As a consequence, the gauge invariant action becomes

\[
S_0 = \int d^4 x \sqrt{\mathcal{g}} \left[ -\frac{1}{2}\epsilon \phi^2 R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right. \\
- \frac{\xi_1}{6\phi^2} \tilde{H}_{\mu\nu\alpha} \tilde{H}^{\mu\nu\alpha} + \frac{\xi_2}{\phi^2} j_\mu \left( A_\nu \tilde{F}^{\mu\nu} + \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} \partial_\nu B_{\alpha\beta} \right) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right].
\]

(5)
By varying the action with respect to $\phi$, $g_{\mu\nu}$, $B_{\mu\nu}$ and $A_\mu$, we can have a set of equations of motion as follows:

$$\epsilon \phi R = D_\mu \partial^\mu \phi - \frac{\partial V}{\partial \phi} + \frac{\xi_1}{3 \phi^3} \tilde{H}^2 - 2 \frac{\xi_2}{\phi^2} j_\mu \left( A_\nu \tilde{F}^\mu_\nu + \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} \partial_\nu B_{\alpha\beta} \right),$$  

(6)

$$\epsilon \phi^2 G_{\mu\nu} = \left[ \frac{1}{2} (\partial_\alpha \phi)^2 + V(\phi) \right] g_{\mu\nu} - \partial_\mu \phi \partial_\nu \phi + \frac{\xi_1}{6 \phi^2} \tilde{H}^2 g_{\mu\nu} + \left( \frac{1}{4} F^2 g_{\mu\nu} - F_\mu \alpha F^\alpha_\nu \right)$$

$$+ \epsilon (D_\nu D_\mu \phi^2 - D^\sigma D_\sigma \phi^2 g_{\mu\nu}) - \frac{1}{\phi^2} \tilde{H}_{\mu\beta} \tilde{H}^\alpha_\nu, \tag{7}$$

$$D_\mu \left( \frac{\xi_1}{\phi^2} \tilde{H}^{\mu\alpha} + \frac{\xi_2}{2 \phi^2} \epsilon^{\mu\alpha\beta} j_\beta \right) = 0, \tag{8}$$

$$D_\nu F^\nu_\mu - D_\nu \left( \frac{2 \xi_1}{\phi^2} \tilde{H}^{\nu\alpha} A_\alpha + \frac{\xi_2}{\phi^2} \epsilon^{\nu\alpha\beta} j_\beta A_\alpha \right) = \frac{\xi_1}{\phi^2} \tilde{H}^{\nu\alpha} F_\nu A_\alpha - \frac{\xi_2}{\phi^2} j_\nu \tilde{F}^\nu_\mu. \tag{9}$$

Since $\tilde{H}^{\mu\alpha}$ is a totally antisymmetric tensor, we can write $\tilde{H}^{\mu\alpha} = \epsilon^{\mu\alpha\beta} T_\beta$, where $T_\beta$ is a vector with mass dimension three. Thus, Eq. (8) is rewritten to

$$\epsilon^{\mu\alpha\beta} \partial_\mu \left( \frac{\xi_1}{\phi^2} T_\beta + \frac{\xi_2}{2 \phi^2} j_\beta \right) = 0. \tag{10}$$

Focusing on the space-time manifold with first trivial homology group, any closed one-form is an exact one-form. Therefore, from Eq. (10), we can express the torsion field as

$$\frac{1}{\phi^2} \left( \xi_1 T_\beta + \frac{\xi_2}{2} j_\beta \right) = \partial_\beta \Phi, \tag{11}$$

where $\Phi$ is a dimensionless pseudo-scalar. With the help of Eq. (11), we can further simplify the equations of motion to be

$$\epsilon \phi R = D_\mu \partial^\mu \phi - \frac{\partial V}{\partial \phi} - \frac{2 \phi}{3 \xi_1} (\partial_\mu \Phi)^2 + \frac{\xi_2}{2 \xi_1 \phi^2} (j_\mu)^2, \tag{12}$$

$$\epsilon \phi^2 G_{\mu\nu} = \left[ \frac{1}{2} (\partial_\alpha \phi)^2 + V(\phi) \right] g_{\mu\nu} - \partial_\mu \phi \partial_\nu \phi + \epsilon (D_\nu D_\mu \phi^2 - D^\sigma D_\sigma \phi^2 g_{\mu\nu})$$

$$+ \frac{1}{\xi_1 \phi^2} \left[ \phi^4 (\partial_\alpha \Phi)^2 - \xi_2 \phi^2 j_\alpha \partial^\alpha \Phi + \frac{\xi_2}{4} (j_\alpha)^2 \right] g_{\mu\nu}$$

$$+ \left( \frac{1}{4} F^2 g_{\mu\nu} - F_\mu \alpha F^\alpha_\nu \right) - 2 \frac{\xi_1}{\phi^2} \left( \frac{\phi^2}{\xi_1} \partial_\mu \Phi - \frac{\xi_2}{2 \xi_1} j_\mu \right) \left( \frac{\phi^2}{\xi_1} \partial_\nu \Phi - \frac{\xi_2}{2 \xi_1} j_\nu \right), \tag{13}$$

$$D_\mu F^\mu_\nu = -\frac{1}{4} (\partial_\mu \Phi) \tilde{F}^\mu_\nu. \tag{14}$$
III. COSMOLOGICAL BIREFRINGENCE

Now, we consider the simplest $\phi^4$ potential for the scalar with both $V_0$ and $\lambda$ larger than zero

$$V(\phi) = \lambda (\phi^2 - \phi_0^2)^2 + V_0 .$$  \hspace{1cm} (15)

Defining $\phi_0^2 = m^2 / (2 \lambda)$ and taking all the $\phi$ field in the equations to be $\phi_0$, equations of motion become

$$\epsilon \phi R = -2 \frac{\phi_0}{\xi_1} (\partial_\mu \Phi)^2 + \frac{\xi_2^2}{2 \xi_1 \phi_0^2} (j_\mu)^2 ,$$ \hspace{1cm} (16)

$$\epsilon \phi^2 G_{\mu\nu} = V_0 g_{\mu\nu} + \left( \frac{1}{4} F^2 g_{\mu\nu} - F_{\mu\alpha} F^\alpha_{\nu} \right)$$

$$+ \frac{1}{\xi_1 \phi_0^2} \left[ \phi_0^4 (\partial_\alpha \Phi)^2 - \xi_2 \phi_0^2 j_\alpha \partial_\alpha \Phi + \frac{\xi_2^2}{4} (j_\alpha)^2 \right] g_{\mu\nu}$$

$$- 2 \frac{\xi_1}{\phi_0^2} \left( \frac{\phi_0^2}{\xi_1} \partial_\mu \Phi - \frac{\xi_2}{2 \xi_1} j_\mu \right) \left( \frac{\phi_0^2}{\xi_1} \partial_\nu \Phi - \frac{\xi_2}{2 \xi_1} j_\nu \right) .$$ \hspace{1cm} (17)

Taking the trace of Eq. (17), we have

$$- \epsilon \phi^2 R = 4 V_0 + 2 \frac{\phi_0^2}{\xi_1} (\partial_\mu \Phi)^2 - 2 \frac{\xi_2}{\xi_1} j_\mu \partial_\mu \Phi + \frac{\xi_2^2}{2 \xi_1 \phi_0^2} (j_\mu)^2 .$$ \hspace{1cm} (18)

By combining Eqs. (16) and (18), we obtain

$$4 V_0 - 2 \frac{\xi_2}{\xi_1} j_\mu (\partial_\mu \Phi) - \frac{\xi_2^2}{\xi_1 \phi_0^2} (j_\mu)^2 = 0 .$$ \hspace{1cm} (19)

In the FLRW Universe, it is reasonable to assume a homogeneous and isotropic fermion current and torsion field \[16\], i.e., $j_\mu = (j_0(t), \vec{0})$ and $T_\mu = (T_0(t), \vec{0})$. From Eq. (19), we have the evolution equation for the dimensionless pseudo-scalar $\Phi$:

$$4 V_0 + 2 \frac{\xi_2}{\xi_1} j_0 (\partial_0 \Phi) - \frac{\xi_2^2}{\xi_1 \phi_0^2} (j_0)^2 = 0 .$$ \hspace{1cm} (20)

The solution of Eq. (20) can be easily derived as

$$\partial_0 \Phi = -2 \frac{\xi_1 V_0}{\xi_2 j_0} + \frac{\xi_2}{2 \phi_0^2} j_0 .$$ \hspace{1cm} (21)

Similar to the calculation in Ref. \[16\], the change in the position angle of the polarization plane $\Delta \alpha$ at the redshift $z \equiv 1/a - 1$ is given by

$$\Delta \alpha = 2 \int (\partial_0 \Phi) \frac{dt}{a(t)} = 2 \int_0^{1100} \left( -2 \frac{\xi_1 V_0}{\xi_2 j_0} + \frac{\xi_2}{2 \phi_0^2} j_0 \right) \frac{dz}{H_0 (1 + z)^{3/2}} .$$ \hspace{1cm} (22)
where \( H_0 = 2.1 \times 10^{-42} h \) GeV is the Hubble constant with \( h \simeq 0.7 \) at the present and we have assumed our Universe is flat and matter-dominated. To estimate \( \Delta \alpha \) in Eq. (22), we take the zero component of the fermion current \( j_0 \) to be the (lightest) neutrino asymmetry, say, the electron neutrino in our universe,

\[
j_0 = \Delta n_{\nu_e} = \frac{1}{12 \zeta(3)} \left( \frac{T_{\nu_e}}{T_\gamma} \right)^3 \pi^2 \xi_{\nu_e} n_\gamma = \frac{2}{33} \xi_{\nu_e} T_{\gamma_0}^3 (1 + z)^3, \tag{23}
\]

where \( T_{\gamma_0} \) is the CMB temperature at the present, \( \xi_{\nu_e} \) is the degeneracy parameter for the electron neutrino and \( (T_{\nu_e}/T_\gamma)^3 = 4/11 \) is assumed. In Ref. [18] the bound on the degeneracy parameter is \(-0.046 < \xi_{\nu_e} < 0.072 \) for a 2σ range of the baryon asymmetry.

Inserting Eq. (23) into Eq. (22), we have

\[
\Delta \alpha = 2 f(z)^{1100} \tag{24}
\]

where \( f(z) \) is given by

\[
f(z) = \left( \frac{66 \xi_1 V_0}{7 \xi_2 \xi_{\nu_e} T_{\gamma_0}^3 H_0} \right) (1 + z)^{-7/2} + \left( \frac{2 \xi_2 \xi_{\nu_e} T_{\gamma_0}^3}{165 \phi_0^2 H_0} \right) (1 + z)^{5/2}. \tag{25}
\]

Therefore, there is a bound of the function \( |f(z)| \)

\[
|f(z)| \geq 2 \left[ \frac{4 \xi_1 V_0}{35 \phi_0^2 H_0^2 (1 + z)} \right]^{1/2}, \tag{26}
\]

which can be thought of as a bound on the contribution of the effective cosmological constant \( V_0 \). As an illustration, for example, by taking \( \phi_0 = M_{pl} = \sqrt{1/8\pi G} \) the reduced Planck mass (taking \( \epsilon = 1 \) hereafter), \( V_0 \sim 10^{-85} \) (GeV)\(^4\), \( \xi_1 = 1 \), \( \xi_2 = 1 \) and \( \xi_{\nu_e} \sim 10^{-3} \), we get \( \Delta \alpha \sim -9.7 \times 10^{-2} \), which could explain the results in Refs. [2, 6–9].

**IV. CONCLUSIONS**

In the present paper, we have studied the CPT-even dimension-six Chern-Simons-like term in Ref. [16] by including dynamical torsion and scalar fields to explain the cosmological birefringence effect. The combined effect of the Kalb-Ramond field and neutrino current induces a sizable rotation polarization angle in the CMB data provided that there is a non-zero neutrino number asymmetry.

It is interesting to note that the effect induced by the Kalb-Ramond field is the inverse of the one due to the neutrino current, as shown in Eq. (25). In contrast to the model
in Ref. [16], in which a similar dimension-six interaction with an undetermined effective coupling constant was examined, we consider, however, the dynamical scalar field as the coupling constants of the Ricci scalar, Kalb-Ramond field and interaction terms. Namely, the effective coupling constant $\phi_0^2$ is related to $M_{pl}$ in the Einstein-Hilbert action. Because of this limitation, the contribution to the angle $\Delta \alpha$ is highly suppressed to $O(10^{-32})$, and the corresponding $V_0$ has to be around $10^{-85}$ (GeV)$^4$ to match the current observational constraint.

Finally, we remark that there should be other interesting cosmological phenomenology in this model [19], which will be studied elsewhere.

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