Vertical Load Generated by Walking Person – Comparative Analysis of Selected Load Models

Marek Pantak¹, Kinga Marecik¹

¹Cracow University of Technology, Institute of Building Materials and Engineering Structures, Warszawska 24, Cracow 31-155, Poland

mpantak@pk.edu.pl

Abstract. The natural vibration frequencies of footbridges with a span length greater than 30 m are very often less than 5.00 Hz. This increases the risk of excitation of vibrations of these footbridges by their users and entails the need to perform their dynamic analyses with the use of appropriate models of the dynamic loads generated by pedestrians. The accurateness of estimation of the value of dynamic loads acting on the structure is one of the important issues from the point of view of the accuracy of dynamic analyses. In the paper, the results of comparative analyses of five selected mathematical models of vertical ground reaction forces (VGRFs) were presented. The modelled forces were compared with the VGRFs acquired during the laboratory measurements of the forces generated by walking people. Moreover, the dynamic analyses of single degree of freedom system (SDOF) were carried out to assess the differences between the dynamic responses caused by analysed load models. Performed analyses showed a good agreement between the time courses of the estimated and measured VGRFs. Nevertheless, because of the simplifications adopted in the individual models some of them allow predicting the lower limit and the other upper limit of the dynamic response of the structure.

1. Introduction

Human body movement is a complex process which is influenced by various factors: body weight, height, sex, velocity of motion etc. [1]. One of the important effects of this movement are the forces transmitted to the ground (Ground Reaction Forces (GRFs)). The GRFs arises as a result of the force of gravity and the movement of the human body in different directions: vertical, horizontal (medial-lateral) and longitudinal (anterior-posterior). The vertical component of GRFs reaches the biggest value; nevertheless, the horizontal (medial-lateral) component can also play a significant role in the case of the slender structures exposed to the dynamic influence of the crowd. The proper identification of the values of the GRFs is crucial for the accurate estimation of the dynamic response of the structure subjected to the dynamic load generated by users.

2. Selected load models of Vertical Ground Reaction Forces (VGRFs)

The GRFs values can be determined in various ways: using Newton’s second law of motion (taking into account the fact that movement of the body centre of mass is curvilinear alternately accelerated and decelerated motion) or using inverted pendulum spring-mass model [2-5]. However, these methods require a detailed knowledge of biomechanical data describing the human body and the human movement which go beyond the problems of civil engineering. The easiest and most accurate way to determine the GRFs values seems to be to measure them with the force plates. From the point...
of view of civil engineering purposes, it is important to develop an accurate and the simplest possible mathematical model of the \( GRFs \). Knowledge of the accuracy of estimation of the \( GRFs \) values and simplifications assumed in load models is essential for correct verification of the requirements of the serviceability limit state of vibration. In the further part of the chapter, the characteristics of selected mathematical models of the vertical component of the ground reaction force (\( VGRFs \)) generated during walking are presented.

2.1. Sinusoidal load model
The first of the analysed load model of the \( VGRF \) is the model presented in standard BS 5400 [6]:

\[
F(t) = 180 \sin(2\pi f_v t)
\] (1)

where:
- \( f_v \) – walking frequency,
- \( t \) – time step.

It is worth to notice that equation (1) describes only the pulsating part of the walking force [7]. To have the whole view of the load acting on the ground, the weight of a walking person has to be added to the formula (1). In figure 1 the results of \( VGRFs \) arising from equation (1) in comparison to \( VGRFs \) acquired during the laboratory measurements of the forces generated by walking person were presented. Both functions were normalized with the use of the body weight \( G \). The walking frequency was assumed \( f_v = 1.80 \text{ Hz} \) [8].

![Figure 1](image)

**Figure 1.** Comparison of normalized \( VGRFs \) obtained during laboratory measurement and \( VGRF \) determined using equation (1) (walking frequency \( f_v = 1.80 \text{ Hz} \))

2.2. NA BS EN-1991-2 load model
In the National Annex of BS EN 1991-2 [9] as the vertical load model of the walking person, more complex function is used. The course of the function depends not only on natural vibration frequency of the footbridge, but also on bridge class, span length, considered vibration form and damping value. The way of determination of the function components’ values is described in [9]. The discussed function has the following formula:

\[
F(t) = 280 \cdot k(f_v) \cdot \sqrt{1 + \gamma \cdot (N - 1)} \sin(2\pi f_v \cdot t)
\] (2)

where:
- \( f_v \) – walking frequency,
- \( k(f_v) \) – coefficient dependent on the frequency of walking,
- \( \gamma \) – reduction factor dependent on structural damping and effective span length, determined taking into account the form of the mode shape of the structure,
- \( N \) – a number of pedestrians in the group,
- \( t \) – time step.

In equation (2), as well as in equation (1), only the pulsating part of \( VGRFs \) is defined. To get a full description of the force acting on the ground, the values coming from equation (2) need to be
increased by body weight. The \textit{VGRFs} derived from the use of equation (2) compared to the results of laboratory tests measurement are shown in the normalized form in figure 2.

![Figure 2. Comparison of normalized VGRFs obtained during laboratory measurement and VGRF determined using equation (2) (walking frequency $f_v = 1.80$ Hz)](image)

To generate \textit{VGRFs} shown in figure 2, the following assumptions were made: $N = 1.0$ (what simplify the eq. (2) and eliminate the $\gamma$ factor), $k(f_v) = 1.0$ (for $f_v = 1.80$ Hz).

### 2.3. Fourier series load model

One of the most commonly known models of \textit{VGRFs} is a model based on Fourier series [8], [10]. It is given by the formula:

$$F(t) = G + \sum_{i=1}^{n} G \alpha_i \sin(2\pi i f_v t - \varphi_i)$$

where: $G$ – the weight of the person walking, $f_v$ – walking frequency, $t$ – time step, $\alpha_i$ – Fourier’s coefficients of $i$-th harmonic component, $\varphi_i$ – phase angel of $i$-th harmonic component with respect to the first harmonic, $i$ – the harmonic component number, $n$ – a number of harmonic components included in analysis.

To use formula (3) it is necessary to know the values of $\alpha_i$ and $\varphi_i$. These coefficients, according to the recommendations [8], [10], can take different values. The chart of normalized \textit{VGRF} determined using equation (3) together with values of \textit{VGRFs} from laboratory test measurement was shown in figure 3. The values of $\alpha_i$ and $\varphi_i$, for $i = 1, \ldots, 4$ were assumed as proposed in recommendation [10] (walking frequency $f_v = 1.80$ Hz).

![Figure 3. Comparison of normalized VGRFs obtained during laboratory measurement and VGRF determined using equation (3) (walking frequency $f_v = 1.80$ Hz)](image)
In work [11] modified form of equation (3) describing VGRFs were presented. The proposed function is strictly dependent on walking frequency and the body weight of the person walking. It is given by the formula:

$$F(t) = G \cdot \{1 + A \cdot [\sin(2\pi f_v t) + 0.25 \sin(4\pi f_v t + \pi) + 0.25 \sin(6\pi f_v + \pi)]\}$$  (4)

where: $G$ – the weight of the person walking, $f_v$ – walking frequency, $t$ – time step, $A$ - dynamic load factor calculated according to the empirical formula: $A = 0.4 f_v + 0.6 G - 0.84$ (where $G$ in [kN], assumed $G = 0.70$ kN).

In figure 4 the normalized VGRF determined using equation (4) in comparison to the laboratory test result were shown (walking frequency $f_v = 1.80$ Hz).

2.4. Single foot load model
The load models shown in subsections 2.1, 2.2 and 2.3 represent the sum of the forces generated during walking by the right and left foot (cumulative values of the forces generated during walking cycle over single stance and double stance phases). The load model describing the VGRF generated by single foot can be found in [12] and is given by the formula:

$$F(t) = G \cdot \sum_{i=1}^{5} A_i \cdot \sin\left(\frac{\pi \cdot i \cdot t}{t_c}\right), \quad 0 \leq t \leq t_c$$  (5)

where: $G$ – the weight of the person walking, $f_v$ – walking frequency, $t$ – time step, $A_i$ – Fourier’s coefficients of $i$-th harmonic component, $t_c$ – contact time, $i$ – the harmonic component number.

The Fourier’s coefficients proposed in [12] are dependent on walking frequencies. The single foot load model can be very useful during numerical modelling of dynamic forces generated by walking pedestrian. The contact time $t_c$ that occurs in equations (5) describes the total contact time of a single foot when it touches the ground. During walking, that time is longer than the period of walking $T$. The phase when both feet are touching the ground in the same time is described by $\Delta t$ and represents the time within which forces generated by the left and right foot are overlapping.

With the use of equation (5), it is possible to determinate the continuous walking force. The representation of the continuous force can be defined in the following way:
\[ F(t) = \begin{cases} G \cdot \sum_{i=1}^{5} A_i \cdot \left[ \sin \left( \frac{\pi i}{t_c} t \right) + \sin \left( \frac{\pi i}{t_c} (t + T) \right) \right], & T \cdot k < t \leq T \cdot k + \Delta t \\ G \cdot \sum_{i=1}^{5} A_i \cdot \sin \left( \frac{\pi i}{t_c} t \right), & T \cdot k + \Delta t < t \leq T \cdot (k + 1) \end{cases} \] 

for \( k \in \mathbb{N} \) (\( \mathbb{N} \) – natural numbers).

In figure 5, the construction of continuous walking force with the use of superposition of forces generated by single foot is presented.

According to [13], the value of contact time \( t_c \) is approximately equal to \( t_c \approx 1.32T \). It gives for frequency \( f_v = 1.80 \text{ Hz} \) the overlapping time \( \Delta t \approx 0.176 \text{ s} \).

In figure 6, the normalized VGRF determined using equation (6) in relation to laboratory tests was presented. The overlapping time was assumed as \( \Delta t \approx 0.176 \text{ s} \).

\[ \begin{align*} \text{Figure 5.} & \text{ Continuous walking force constructed with the use of superposition of the single foot.} \\
& \text{Figure 6.} \quad \text{Comparison of normalized VGRFs obtained during laboratory measurement and VGRF determined using equation (6) (walking frequency} f_v = 1.80 \text{ Hz).} \]
3. Comparative analysis of VGRFs time courses
In figure 7 the time courses of the VGRFs determined using equations (1) – (4) and (6) in relation to VGRF obtained during laboratory test are presented (walking frequency $f_v = 1.80 \text{ Hz}$).

![Comparison of normalized VGRFs determined using equations (1) – (4) and (6) in relation to exemplary VGRF obtained during laboratory test (walking frequency $f_v = 1.80 \text{ Hz}$)](image_url)

In figure 7 the good agreement between VGRF measured during laboratory test and VGRF determined using equations (1) – (4) and (6) can be observed. To compare the VGRF time courses the value of peak amplitude measured from the level of $F_{GRF}(t)/G = 1.0$ (level of normalized body weight) and peak to peak amplitude were used. The VGRF measured during laboratory tests has an asymmetric shape with regard to the level of normalized body weight. The maximum peak amplitude of the VGRF from laboratory tests (measured above the level of normalized body weight) is about 0.40 $G$. The minimum peak amplitude of the VGRF from laboratory tests (measured below the level of normalized body weight) is about 0.25 $G$. The peak to peak amplitude of the VGRF from laboratory tests is 0.60 $G$.

The VGRF determined using equation (1) reaches the peak amplitude equal 0.25 $G$ and peak to peak amplitude equal 0.50 $G$. The VGRF determined using equation (2) is characterized by peak amplitude 0.45 $G$ and peak to peak amplitude equal 0.90 $G$. The peak amplitude of the VGRF determined using equation (3) is 0.32 $G$ and peak to peak amplitude is 0.64 $G$. The peak amplitude of the VGRF determined using equation (4) is 0.39 $G$ and peak to peak amplitude is 0.78 $G$. In the case of VGRF determined by means of equation (6) with asymmetric shape with regard to level of normalized body weight the maximum peak amplitude of the VGRF (measured above the level of normalized body weight) is 0.53 $G$ and minimum peak amplitude of the VGRF (measured below the level of normalized body weight) is about 0.29 $G$. The peak to peak amplitude of the VGRF determined using equation (6) is 0.82 $G$.

Assuming the peak amplitude and peak to peak amplitude determined for the VGRF measured during laboratory tests as a reference values it can be observed that the VGRF determined using equation (1) has amplitudes lower than amplitudes of VGRF measured during laboratory test. The VGRF determined using equation (2) achieves the peak to peak amplitude much greater than the amplitude of the VGRF measured during laboratory test. The VGRFs determined using equations (3) and (4) reach the amplitudes comparable with the laboratory test result. The amplitudes of the VGRF determined using equation (6) reach the values greater than the amplitudes of the VGRF measured during laboratory test.

4. Comparative analyses of the dynamic responses
In order to check the differences between dynamic responses determined by the analysed load models, the dynamic analyses of simple single degree of freedom system (SDOF, figure 8) were carried out.
Figure 8. SDOF system used in dynamic analyses

The analyses were performed using the VGRF generated by means of equations (1) – (4) and (6) as well as ten different time courses of the VGRF measured during laboratory test. During the analyses, the resonance effect was modelled ($f_v = f = 1.80$ Hz was assumed). The SDOF system was loaded by dynamic force for 60 seconds to achieve the steady-state conditions. The time courses of the VGRFs determined during laboratory measurements and by means of equation (6) was created by adding up forces generated by single foot with adequate time shift in order to generate the resonant loads (see subsection 2.4, figure 5).

In figure 9 and in table 1 the vibration accelerations obtained during dynamic analyses are presented. In figure 9a the minimum (grey chart) and maximum (black chart) values of vibration acceleration determined using ten different time courses of the VGRF from laboratory tests are given.

Figure 9. Vibration acceleration of the SDOF system caused by dynamic forces a) measured during laboratory test, b) determined by equation (1), c) determined by equation (2), d) determined by equation (3), e) determined by equation (4), f) determined by equation (6)
Table 1. Maximum vibration accelerations of the SDOF system caused by different load models

| Load model                  | Acceleration [m/s²] | Percentage change | Comment                                      |
|-----------------------------|---------------------|-------------------|----------------------------------------------|
| Laboratory test force       | 0.68 ± 0.11a        | Reference value   | -                                            |
| Equation (1)                | 0.66                | −3.0%             | Mean value of acceleration                    |
| Equation (2)                | 1.02                | +50.0%            | Upper limit of acceleration (conservative model) |
| Equation (3)                | 0.79                | +16.2%            | Intermediate value                            |
| Equation (4)                | 0.77                | +13.3%            | Intermediate value                            |
| Equation (6)                | 0.90                | +32.5%            | Upper limit of acceleration                   |

a Mean value ± standard deviation (acceleration determined using ten different time courses of the VGRF).

Comparing the obtained values of vibration accelerations, it can be observed that the load model described by the equation (1) allow predicting the mean value of vibration acceleration induced during walking with normal speed ($v \approx 1.34$ m/s, $f_v \approx 1.80$ Hz). The load models described by equations (2) and (6) allow predicting the upper limit of vibration acceleration. It should be noted that equation (2) was calibrated for walking speed $v = 1.70$ m/s (fast walking, $f_v \approx 2.10$ Hz) what lead to higher force amplitudes. This model will be conservative when will be used in dynamic analyses of the structures loaded by people walking slowly or at a normal pace. The load models described by equations (3) and (4) allow estimating the value of the vibration acceleration located between the minimum and maximum acceleration determined using VGRFs measured during laboratory tests (figure 9a).

5. Conclusions

Analysing the amplitudes of the VGRF presented in figure 7 and values of vibration accelerations presented in figure 9 and in table 1, it can be concluded that all the models reproduce the course of the VGRF and dynamic response of the structure with satisfactory accuracy. However, it should be remembered that due to the different simplifications adopted in the individual models, some of them allow predicting the lower limit and the other upper limit of the dynamic response of the structure. Proper use of the analysed load models in the dynamic analyses of the structures allows correct estimating of the value of vibration accelerations caused by pedestrians.

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