Single-electron excitations and interactions in integer quantum Hall systems at $\nu = 2$

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Abstract. We study the interplay between few-electron excitations generated by voltage pulses and interactions in a quantum Hall system at integer filling factor $\nu = 2$. Electron-electron interactions strongly affect the dynamics of the generated pulses, leading to their fractionalization. In particular, on the innermost edge channel two oppositely charged excitations emerge, which we analyze through their Wigner function. Moreover, we show that interactions provide a signature in the noise generated when these excitations are partitioned by a quantum point contact connecting opposite edges. We compare different shapes of the external drive and we discuss the most convenient in order to extract information about the so-called mixing angle, encoding the strength of interactions between edge channels.

1. Introduction
Coherent ballistic propagation of few-electron wavepackets has been the subject of an intense research activity over the past twenty years, reaching the new paradigm of electron quantum optics (EQO) [1–4]. All systems hosting topologically protected ballistic channels [5–7] are, in principle, suitable for EQO. Among them the quantum Hall (QH) state [8, 9] has been widely studied in the context of EQO, both in the integer [10–16] and fractional [17–21] regimes. The exciting physics of quantum optic-like setups in condensed matter can be enriched by the presence of interactions, often very relevant in one-dimensional systems [22–24] and responsible for exotic phenomena, such as fractionalization [25–28]. Several studies indicated that the role of interactions can be important even in integer QH systems. It is an interesting question to investigate how interactions affect typical observables addressed in EQO experiments.

In this paper we will investigate the interplay between voltage driven excitations and screened Coulomb interactions in an integer QH system at filling factor $\nu = 2$. We consider a Hanbury-Brown Twiss [29] geometry, where pulses injected from one terminal are partitioned in the presence of a tunable quantum point contact (QPC) acting as a beamsplitter. Due to interactions, wavepackets arriving at the beamsplitter are fractionalized excitations, whose charge depend both on the amplitude of the drive and the strength of interactions. By focusing on the inner channel, we first analyze the particle-hole (p-h) content of fractional pulses by means of the Wigner function [30], which gives information on both the time evolution and energy content of a wavepacket. As a second step, we evaluate the photoassisted shot noise due to a...
periodic drive and construct a measurable quantity from which the mixing angle characterizing interactions can be recovered, thanks to the particular symmetry of inner channel excitations. We compare different drives and find that a rectangular wave with small width-to-period ratio is a good tool to determine the mixing angle characterizing the interaction strength.

2. Model

We consider the edge states of a QH system at filling factors \( \nu = 2 \). In this regime, two copropagating chiral ballistic channels are present at each edge of the system. The low energy properties of the system are well described by a chiral Luttinger theory [22, 23, 31] with Hamiltonian \( (\hbar = 1) \)

\[
H_0 = \sum_{r=R,L} \sum_{\alpha=1,2} v_\alpha \int dx \, \bar{\Psi}_{r\alpha}(x)(-i\partial_x)\Psi_{r\alpha}(x) + 2\pi u \sum_{r=R,L} \int dx \, \bar{n}_{r1}(x)n_{r2}(x) .
\]

Here, index \( \alpha = 1, 2 \) labels outer and inner channels, respectively, while \( r = R, L \) labels the upper (right-moving) and lower (left-moving) edge, with \( \vartheta_{R/L} = \pm 1 \). Fermionic operators \( \Psi_{r\alpha}(x), \Psi^{\dagger}_{r\alpha}(x') \) satisfy \( \{ \Psi_{r\alpha}(x), \Psi^{\dagger}_{r'\alpha'}(x') \} = \delta_{rr'}\delta_{\alpha\alpha'}\delta(x-x') \) and \( n_{r\alpha}(x) = \Psi^{\dagger}_{r\alpha}(x)\Psi_{r\alpha}(x) \) are particle density operators. The first term in the Hamiltonian (1) is the free kinetic part, \( v_\alpha \) being the propagation velocity along the edges; the second one describes short distance repulsive electron-electron (e-e) interactions, with coupling strength \( u > 0 \). The system is driven by a time-dependent voltage \( V(t) \) which is spatially uniform and applied in the region \( x < -d \), with \( d > 0 \), in order to simulate a contact in the left part of the edge. This coupling is described by the Hamiltonian

\[
H_g = -e \int dx \, \Theta(-x - d)V(t)n_{R1}(x) ,
\]

with \( \Theta(x) \) the Heaviside step function and \( -e \) the electron charge. Notice that the drive only couples to the outer channel charge density, so that excitations are initially created only on the outer edge channel. Yet, interactions lead to the fractionalization of the generated pulse, so that excitations will also be present on the inner channel. A possible implementation of the coupling (2) could be to drive both channels and then to use a gate [32] or a properly polarized QPC [33] immediately after the driven contact as a filter to transmit only outer channel excitations.

The Hamiltonian \( H_0 \) is diagonalized as follows. First of all, fermionic operators can be expressed in terms of bosonic ones through the bosonization identity [34]

\[
\Psi_{r\alpha}(x) = \frac{F_{r\alpha}}{\sqrt{2\pi a}} e^{-i\sqrt{2\pi}F_{r\alpha}(x)} ,
\]

where \( F_{r\alpha} \) are Klein factors and \( a \) is a short-distance cutoff. The commutation relations of bosonic fields read \( [\Phi_{r\alpha}(x), \partial_x \Phi_{r'\alpha'}(x')] = -i\vartheta_{r'}\delta_{r\alpha} \delta(x-x') \). By using the identity (3) and the relation \( \sqrt{2\pi}n_{r\alpha}(x) = -\partial_x \Phi_{r\alpha}(x) \), together with the rotation

\[
\begin{pmatrix}
\Phi_{r1} \\
\Phi_{r2}
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\Phi_{r+} \\
\Phi_{r-}
\end{pmatrix} , \quad \tan 2\theta = \frac{2u}{v_1 - v_2} ,
\]

\( H_0 \) is recast in diagonal form

\[
H_0 = \sum_{r=R,L} \sum_{\eta=\pm} \frac{v_\eta}{2} \int dx \, [\partial_x \Phi_{r\eta}(x)]^2 ,
\]

with \( v_\pm = (v_1 + v_2)/2 \pm u/\sin 2\theta \) [31, 35]. The parameter \( \theta \in [0, \pi/2] \) is generally referred to as the mixing angle and quantifies how much the two channels on a given edge are “mixed” due
to interactions. It is then possible to write down the equations of motion for fields \( \Phi_{r \eta} \), in the presence of the external drive (2). It is easy to show that for \( x > -d \) they are solved by [36, 37]

\[
\Phi_{r \pm}(x, t) = \phi_{r \pm}(x - v \pm t, 0) + \frac{\xi_{r \pm}}{\sqrt{2\pi}} \int_{-\infty}^{t} \int_{-d}^{d} V(t') dt',
\]

where \( \phi_{r \pm}(x, t) \) is the field without any applied voltage and \( \xi_{R \pm} = -e \cos \theta \), \( \xi_{R -} = e \sin \theta \) and \( \xi_{L \pm} = 0 \). As expected, the only fields acquiring a non-trivial evolution are those of the upper edge (\( r = R \)), since the lower one is not coupled to the external voltage.

The last ingredient is a term responsible for tunneling of electrons between the two edges:

\[
H_\lambda = \lambda \Psi_{L2}^\dagger(0) \Psi_{R2}(0) + \text{H.c.}
\]

This describes a QPC at \( x = 0 \), polarized in such a way that tunneling only occurs between inner channels on opposite edges, while outer channels are fully transmitted and their dynamics is therefore unaffected by the presence of the QPC.

3. Charge fractionalization and Wigner function

We start our analysis by considering the time evolution of the charge density due to the presence of the time dependent drive \( V(t) \), in the case when the tunneling is not yet activated (\( \lambda = 0 \)). Only the upper edge (\( r = R \)) will be then considered in this section.

It is well known that interactions in one-dimensional systems lead to fractionalization [22, 23, 26, 38]. This behaviour is also present in our system: the combined effect of external drive and interactions generates four different wavepackets. On both the outer and the inner channel, two excitations are present: a fast one, propagating at velocity \( v_+ \), and a slow one, propagating at velocity \( v_- \). The charge of these excitations can be conveniently expressed in terms of the parameter

\[
q = -e \int_{-\infty}^{+\infty} V(t) dt,
\]

representing the charge (in units of \(-e\)) transferred to the system by the voltage drive. By denoting with \(-eq_\alpha\) the charge of the fast (\( \eta = + \)) and slow (\( \eta = - \)) wavepacket on channel \( \alpha \), then \( q_+ = q \cos^2 \theta \), \( q_- = q \sin^2 \theta \) and \( q_\pm = \pm q \sin \theta \cos \theta \equiv \pm Q \). Notice that outer channel excitations have a different charge, but with the same sign, while those on the inner channel are identical but oppositely charged. In the following, we will only focus on inner channel excitations, which turn out to be extremely useful to extract information about the mixing angle.

Among all possible drives, a Lorentzian shaped voltage with quantized area has the property of generating minimal excitations, known as Levitons [39–41]. The are purely electron-like or hole-like and do not contain any additional p-h pair cloud. Let us then consider a Lorentzian drive with typical temporal extension \( \tau \)

\[
V(t) = -q \frac{2\tau}{e \tau^2 + t^2}.
\]

In a noninteracting system, the condition for a Lorentzian pulse to be a Leviton is that \( q \in \mathbb{Z} \): for positive values a \( q \)-electron excitation is generated, while negative values correspond to \( q \)-hole excitations. In our system, it is the number of charges carried by the fractionalized excitations that has to be integer. This, of course, depends on the parameter \( q \), as well as on the mixing angle \( \theta \). The situation in which all fractionalized wavepackets have integer charge is only possible for particular values \( q \) and \( \theta \) [33]. However, when we restrict our attention to the inner channel excitations, the only condition to be fulfilled is that \( Q = q \sin \theta \cos \theta \) be an integer number. This
Figure 1. Zero-temperature excess Wigner functions $\Delta W_{2\pm}$ of fractional excitations on the inner channel, as a function of energy $\omega$ and distance $\Delta x$ from the center of the wavepacket. Panels (a) and (b) represent, respectively, the excess Wigner functions for fast and slow excitations in the case when $Q = 1$, i.e. when they both carry an integer charge. Panels (c) and (d) are the same as the previous ones, but for the case $Q = 1/2$.

An interesting way to visualize these excitations is through their Wigner function [30]. In particular, we are interested in the variation of the Wigner function with respect to the equilibrium situation when no drive is applied to the system. This quantity is called the excess Wigner function and is defined as [42]

$$\Delta W_{\alpha}(x, t; \omega) = \int_{-\infty}^{+\infty} dt' e^{i\omega t'} \left\langle \Psi_{\alpha}^\dagger \left( x, t + \frac{t'}{2} \right) \Psi_{\alpha} \left( x, t + \frac{t'}{2} \right) - \Psi_{\alpha}^\dagger \left( x, t - \frac{t'}{2} \right) \psi_{\alpha} \left( x, t + \frac{t'}{2} \right) \right\rangle$$

where $\psi_{\alpha}(x, t)$ is the fermionic field with no drive applied. The Wigner function gives a mixed space/time-energy representation, which is useful to have information about both the time evolution and the energy content of excitations. When the two fractionalized excitations are well separated spatially, it is possible to write the Wigner function as a sum of the contributions of each excitation individually:

$$W_{\alpha}(x, t; \omega) = W_{\alpha+}(x - v_+ t; \omega) + W_{\alpha-}(x - v_- t; \omega).$$

The condition of spatial separation (i.e. negligible overlap between wavepackets) is achieved at times $t \gg \tau v_+ + \tau v_- - v_+ v_- t$, since the fast/slow pulse is centered around $x_{\pm} = x - v_{\pm} t$ and has spatial extension $v_{\pm} \tau$.

In Fig. 1 we present the excess Wigner functions $\Delta W_{2\pm}$ in this limit, for a Lorentzian pulse and for two different values of $Q$, as a function of energy $\omega$ and the displacement $\Delta x$ from the center of the pulse. Panels (a) and (b) refer to the situation when $Q = 1$; then, the fast (slow) wavepacket is purely a single electron (hole) excitation. This is reflected in the fact that the Wigner function vanishes for negative (positive) energies. On the other hand, panels (c) and (d) are again excess Wigner functions $\Delta W_{2\pm}$, but for $Q = 1/2$. In this situation, since $Q$ is not an integer, the two excitations are not purely electron- or hole-like, but are dressed with a cloud of p-h pairs. Indeed panel (c) shows a non vanishing signal at $\omega < 0$; likewise a non
zero contribution is present in panel (d) at \( \omega > 0 \). A complementary analysis can be carried out by computing nonequilibrium momentum distributions [42–44] that also give direct information about the p-h content of wavepackets.

4. Photoassisted shot noise

An experimentally relevant tool to access p-h content of a given excitation is to consider the low frequency noise [33, 45, 46] generated when the wavepacket is partitioned on a QPC, thus inducing fluctuations in the current. Here, we argue that a measurable quantity built from the low frequency noise \[33, 45, 46\] generated when the wavepacket is partitioned on a QPC, thus

\[ S = 2 \int_0^T dt \int_{-\infty}^{+\infty} dt' \left[ \langle I_{L2}(-d, t + t') I_{L2}(-d, t) \rangle - \langle I_{L2}(-d, t) \rangle^2 \right] , \]

with \( I_{L2}(x, t) = e^{2(2\pi)^{-1/2}} [v_+ \sin \theta \ \partial_x \Phi_{L+}(x, t) + v_- \ \cos \theta \ \partial_x \Phi_{L-}(x, t)] \) the current operator on the lower edge, inner channel. The above quantity can be calculated perturbatively in the tunneling amplitude \( \lambda \) appearing in (7). Up to second order, the result reads \((k_B = 1)\)

\[ S = \frac{S_0 \Omega}{2\pi} \sum_{l=-\infty}^{+\infty} |p_l(Q, t_d)|^2 \coth \left( \frac{\Omega}{2T} \right) , \quad p_l(Q, t_d) = \sum_{m=-\infty}^{+\infty} p_{l+m}(Q)p^*_m(Q)e^{2\pi imt_d/T} , \]

where \( T \) is the temperature, \( t_d = d(v_+^{-1} - v_-^{-1}) \) and \( S_0 = 2e^2|\lambda|^2/v_+^2 \). As discussed in [44], for non-overlapping Lorentzian pulses such that \( Q \in \mathbb{N} \), \( S(Q) = \frac{S_0 \Omega}{\pi} Q \). This is because \( S \) can be linked to the number of electron and hole excitations and when \( Q \in \mathbb{N} \) (and the fractionalized wavepacket do not overlap significantly), then just \( Q \) electrons and \( Q \) holes are generated. This leads to the conclusion that the quantity

\[ X(Q) = 2\pi \frac{2S(Q) - S(2Q)}{S_0 \Omega} \]

should be zero for non-overlapping Lorentzian packets when \( Q \) is integer and this can be exploited so as to extract the angle \( \theta \), by recalling \( Q = q \sin \theta \cos \theta \).

In Fig. 2 we analyze the behavior of \( X(Q) \) at zero temperature for different types of periodic drives. In particular, we consider a Lorentzian train, a sine, a rectangular and a triangular wave. All these drives, except for the sine, are characterized by a width-to-period ratio \( \eta \). For the rectangular wave, this is nothing but the duty cycle of the signal, for the triangular wave \( \eta \) is taken as the full width of the triangle with respect to the period, for Lorentzian pulses \( \eta = \tau/T \), where \( \tau \) is the width of a single pulse, as in (9). Panels (a) and (b) show different values of \( \eta \), as specified in the caption. We immediately observe that in order to obtain \( X(Q) = 0 \) at \( Q \in \mathbb{N} \)
with a Lorentzian drive, one has to go to very small values of \( \eta \), which are not experimentally feasible. This rules out the Lorentzian as the natural candidate to extract the mixing angle from the quantity (14). However, any other drive exhibiting clear features in \( X(Q) \) at particular values of \( Q \) is suitable for this purpose. By looking at Fig. 2, we see that the sine drive does not show any peculiarity, while the rectangular wave has very prominent maxima in correspondance of half-integer values of \( Q \) and secondary maxima for integer values. Therefore, by plotting (14) as a function of the experimentally tunable parameter \( q \) and looking for the values at which \( X(q) \) has principal maxima, the mixing angle can be extracted from the relation \( Q = q \cos \theta \sin \theta \), with \( Q \) half integer. The triangular wave shares the same qualitative behavior as the rectangular one and becomes more and more similar to it when reducing \( \eta \). This is reasonable, since both signals approximate a Dirac comb in the limit \( \eta \to 0 \) [47]. In general, the effect of reducing \( \eta \) is to increase the visibility of the maxima and to improve their localization. A comparison between rectangular and triangular wave shows that maxima are better localized at half integers values of \( Q \) in the former case. The rectangular wave is then a better drive to extract information about the mixing angle.

5. Conclusions

We have analyzed the interplay of an external drive and e-e interactions in an integer QH system at \( \nu = 2 \), focusing on the inner channel. Due to interactions, the excitation generated by the drive fractionalizes into oppositely charged wavepackets, whose p-h content was analyzed by means of their Wigner function. Their charge being the same (up to a sign), it is possible from noise measurement to recover the mixing angle \( \theta \) encoding the strength of interactions between edge channels. For this purpose, the rectangular drive appears to be the best candidate.

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