Hidden Symmetry of a Fluid Dynamical Model

C. Neves\textsuperscript{2} and C. Wotzasek\textsuperscript{1}

\textsuperscript{1}Instituto de Física
Universidade Federal do Rio de Janeiro
21945-970, Rio de Janeiro, Brazil

\textsuperscript{2}Departamento de Física, ICE, Universidade Federal de Juiz de Fora,
36036-330, Juiz de Fora, MG, Brasil,

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A connection between solutions of the relativistic d-brane system in (d+1) dimensions with the solutions of a Galileo invariant fluid in d-dimensions is by now well established. However, the physical nature of the light-cone gauge description of a relativistic membrane changes after the reduction to the fluid dynamical model since the gauge symmetry is lost. In this work we argue that the original gauge symmetry present in a relativistic d-brane system can be recovered after the reduction process to a d-dimensional fluid model. To this end we propose, without introducing Wess-Zumino fields, a gauge invariant theory of isentropic fluid dynamics and show that this symmetry corresponds to the invariance under local translation of the velocity potential in the fluid dynamics picture. We show that different but equivalent choices of the sympletic sector lead to distinct representations of the embedded gauge algebra.

I. INTRODUCTION

Some years ago, Bordemann and Hoppe \cite{1} demonstrated that the relativistic theory of membranes are integrable systems by reducing the problem to a 2-dimensional fluid dynamics. In that paper the authors simplify the light-cone gauge description of a relativistic membrane \cite{2} moving in a Minkowski space by changing the independent to dependent variables. This allowed them to find explicit solution to all constraints and to reduce the original system of field equations, with four functions, to a system with only two functions. The dynamics is governed by a Hamiltonian reduced to a SO(1, 3) invariant (2+1)-dimensional theory of isentropic fluid dynamics, where the pressure is inversely proportional to the mass-density. Unfortunately, this procedure does not preserve the gauge symmetry exhibit initially by the relativistic theory of membranes, which can call into doubt the reduction procedure. Afterward, the study of the non-relativistic isentropic fluid mechanics model \cite{3} has attracted much attention \cite{4, 5}. This subject is of broader interest since it also offers connections with the hydrodynamical description of quantum mechanics \cite{10, 11}, parton model \cite{4}, black-hole cosmology \cite{12}, hydrodynamics of superfluid systems \cite{13}. Most of these investigations are dedicated to find the solutions of this Galileo invariant system in d-dimensions in connection with the solutions of the relativistic d-brane system in (d+1)-dimensions \cite{5, 6}, which is of direct interest to theoretical particle physics. In particular, this last point was responsible for the recent spate of interest in clarifying the presence of a hidden dynamical Poincare symmetry of this non-relativistic model realized by field dependent diffeomorphism: in terms of the canonical variables one can compute the Poisson algebra and reproduces the Poincare algebra for a system (membrane) in one dimension higher \cite{6}.

In this paper we argue that the U(1) gauge symmetry present on the light-cone description of a relativistic membrane can be preserved on the 2-dimensional theory of isentropic fluid dynamics obtained after the reduction via field-dependent change of variables \cite{4} or dimensional reduction of a scalar relativistic field theory \cite{4}. We will ventilate that this reduced system indeed possess a U(1) gauge symmetry very much like the paradigmatic Maxwell theory, although it is a constrained system with a second-class description. Due to this second-class character, the presence of this local symmetry in this model was not suspected. Indeed, it has been pointed out in \cite{13} that the classical hydrodynamics description of an isentropic fluid is invariant under global translation of the velocity potential. Our goal in this paper is to recover the local U(1) gauge symmetry fixed by the reduction process, that was not disclosed in previous investigations, and consequently to find the dynamically equivalent gauge formulation of the noninvariant isentropic fluid by reverting the nature of the constraints.

In this regard it is worth mentioning that there has been an intense research in second-class constraints conversion recently. The basic idea behind the conversion process is to identify a first-class subset of the constraints. This can be done by means of two distinct concepts. One path follows the proposal of Faddeev-Shatashvili \cite{14} of enlarging the phase-space with Wess-Zumino variables (WZ) \cite{15, 16}. This has been shown to be possible even for nonlinear second-class constraints of arbitrary geometries \cite{17, 18}. Another path of conversion process is confined into the original phase-space from the outset \cite{20, 21}. In this approach half of the constraints are seen as gauge-fixing conditions for the first-class subset. An appropriate projector operator is then constructed that maps all observable to the invariant
sector of the theory. We shall follow this line in this work.

We have organized this paper as follows. In Section 2, a review of the fluid dynamics model will be presented, paying particular attention to the complete set of symmetries. In section 3, the original second-class description of the fluid dynamics model will be reformulated as a first-class model, emphasizing that half of the set of constraints is assumed as gauge symmetry generator while the remaining constraints are considered as gauge-fixing terms. The corresponding gauge transformations are explicitly computed. We pay particular attention on the dependence of our choice of the original symplectic structure, which is deceptively trivial, leading to dramatic differences in the final results. The last section will be reserved to stress our conclusion and final discussions. In an appendix, we will sketch the gauge unfixing Hamiltonian method [20,21].

II. THE FLUID DYNAMICS MODEL

After the original reductive procedure of [1], the problem of 2-dimensional flow [1–3] has been investigated intensively [4–9]. An important contribution to this topic was given in [3,4], where two more conserved quantities and the associate symmetries were found. In this section we briefly review these results.

There are many ways to derive the Lagrangian that determine the fluid dynamics of interest here. We start with the Schrödinger Lagrangian [10],

\[
L_S = \int d^d r \left\{ i \psi^* \dot{\psi} - \frac{1}{2} (\nabla \psi^*) \cdot (\nabla \psi) - \bar{V} (\psi^* \psi) \right\},
\]

with \( \bar{V} \) determining any nonlinear interaction. Introducing the representation in terms of mass density and velocity potential, as usual [13],

\[
\psi = \rho^{1/2} e^{i \theta},
\]

into the Schrödinger Lagrangian, we obtain,

\[
\mathcal{L} = \int \! d^d r \left( -\rho \dot{\theta} - \frac{1}{2} \rho (\nabla \theta)^2 - V(\rho) \right),
\]

with

\[
V(\rho) = \bar{V}(\rho) + \frac{1}{8} (\nabla \rho)^2, \quad \rho
\]

which is the hydrodynamical form of the Schrödinger theory [10,11]. Notice that there is a nontrivial interaction, even in the absence of \( \bar{V} \). This result was obtained from a gauge-fixed formulation of a membrane in Minkowski space [1], through a field-dependent change of variables, for the special case \( d = 2 \) and a potential of a strength \( g \) given by,

\[
V(\rho) = g \frac{\rho}{\rho}.
\]

The same result was also obtained from a dimensional reduction of a local relativistic field theory [5].

From this derivation, there is no doubt that the model described by Lagrangian (5), with some restrictions on \( V(\rho) \), possess Galileo symmetry. For completeness, we list the manifest symmetries and the corresponding generators:

- Invariance under space and time translations,

  - Energy

  \[
  H = \int \! d^d r \, \mathcal{E}, \quad \mathcal{E} = \frac{1}{2} \rho \nabla \theta \cdot \nabla \theta + V(\rho)
  \]

  - Momentum

  \[
  \mathbf{P} = \int \! d^d r \, \mathcal{P}, \quad \mathcal{P} = \rho \nabla \theta = \mathbf{j}
  \]
• Invariance under rotation
  – Angular momentum
  \[ J^{ij} = \int d^3r (r^i p^j - r^j p^i) \] (8)

• Galileo boost
  – Boost generator
  \[ B = tP - \int d^3r r\rho \] (9)

• Invariance under global translations of the velocity potential, \( \theta \to \theta + \alpha \), with \( \alpha \) a constant.
  – Charge
  \[ M = \int d^3r \rho \] (10)

Physically, the conservation of the charge \( M = \int d^3x \rho \) (total mass) means that the center of mass of the fluid, \( X = \int d^3x x\rho/M \) moves with constant velocity,
\[ M \frac{dX}{dt} = \int d^3x p, \] (11)
where \( p \) denotes the total momentum of the fluid.

Unexpectedly, extra symmetries were found in Refs. [4–6] and demonstrated that they are present on the model with a specific potential \( (V(\rho) = g/\rho) \) as well as for the free case. These extra symmetries are:

• Invariance under time rescaling \( t \to e^\omega t \),
  – Time dilatation
  \[ D = tH - \int d^3r \rho \theta, \] (12)
  with the fields transforming as
  \[ \rho(t, r) \to \rho_\omega(t, r) = e^{-\omega} \rho(e^\omega t, r) \]
  \[ \theta(t, r) \to \theta_\omega(t, r) = e^\omega \theta(e^\omega t, r). \] (13)

• Galileo antiboost,
  – Antiboost generator
  \[ G = \int d^3r (rE - \frac{1}{2} \rho \nabla \theta^2) \] (14)
  leading to
  \[ t \to T(t, r) = t + \frac{1}{2} \omega \cdot (r + R(t, r)) \]
  \[ r \to R(t, r) = r + \omega \theta(T, R) \] (15)
  where
  \[ \theta(T, R) = \theta(t, r - \omega t) + \omega \cdot r - \frac{1}{2} \omega^2 t. \] (16)

In Refs. [5,6] it was shown that only under the very specific density-dependent potential \( (V(\rho) = g/\rho) \), the connection between the Galileo invariant system presented in this section, defined either in d=2 or d-space dimensions, and the relativistic membrane and its generalization to the d-brane system in d=3 or (d+1)-space dimensions, appears. In view of this, it is remarkable to notice that the additional symmetries present on the Galileo invariant system in \( d \geq 1 \) space dimensions with the interacting potential \( V(\rho) = g/\rho \) are also present on the relativistic membrane and its generalization to the d-brane system in \( d \geq 2 \) space dimensions.
III. GAUGE INVARIANT IDEAL FLUID MECHANICS

To describe the gauge invariant hydrodynamics of an isentropic fluid, in d-space dimensions, we follow the presentation already discussed in the Introduction and the technique given in the appendix. Let us start with the Lagrangian given in (3) with an arbitrary potential $V(\rho)$. The canonical Hamiltonian is expressed as

$$\mathcal{H}_0 = \frac{1}{2} \rho (\nabla \theta)^2 + V(\rho).$$

(17)

This is a (second-class) constrained theory but the primary constraints depend on our choice of the symplectic structure in (3), namely,

$$\mathcal{L}^{(1)} = \int d^d r \left( \dot{\rho} \theta - \frac{1}{2} \rho (\nabla \theta)^2 - V(\rho) \right),$$

(18a)

$$\mathcal{L}^{(2)} = \int d^d r \left( -\dot{\rho} \theta - \frac{1}{2} \rho (\nabla \theta)^2 - V(\rho) \right),$$

(18b)

which is deceptively trivial. Due to this difference, we have the following set of primary constraints,

$$\varphi^{(1)}_\alpha = \left\{ \begin{array}{c}
\varphi^{(1)}_1 = \pi_\rho - \theta \\
\varphi^{(1)}_2 = \pi_\theta
\end{array} \right.$$  \hspace{1cm} (19a)

$$\varphi^{(2)}_\alpha = \left\{ \begin{array}{c}
\varphi^{(2)}_1 = \pi_\theta + \rho \\
\varphi^{(2)}_2 = \pi_\rho
\end{array} \right.$$  \hspace{1cm} (19b)

where the superscript indices denote the distinct symplectic structures and the subscript indices denote the constraint family numbers. At this point, it is important to notice that only one constraint, for each choice of sympletic structure $(i)$ in the set of constraints $(\varphi^{(i)}_\alpha)$, can be lifted as a gauge symmetry generator. These constraints satisfy the following (second-class) Poisson algebra,

$$\{ \varphi^{(1)}_1(x), \varphi^{(1)}_2(y) \} = -\delta^{(d)}(x - y),$$

(20a)

$$\{ \varphi^{(2)}_1(x), \varphi^{(2)}_2(y) \} = +\delta^{(d)}(x - y).$$

(20b)

There are no more constraints in either cases. The Dirac Hamiltonian reads,

$$\mathcal{H}^{(i)} = \mathcal{H}_0 + \lambda^{(i)}_1 \varphi^{(i)}_1 + \lambda^{(i)}_2 \varphi^{(i)}_2 ; \ i = 1, 2 \ (\text{no sum})$$

(21)

Temporal stability of the system demands,

$$\{ \varphi^{(i)}_\alpha(x), \mathcal{H}^{(i)} \} = 0 ; \ \alpha = 1, 2$$

(22)

leading to an explicit solution for the multipliers $\lambda^{(i)}_\alpha$ as,

$$\lambda^{(1)}_\alpha = \left\{ \begin{array}{c}
\lambda^{(1)}_1 = \partial_i (\rho \partial_i \theta) \\
\lambda^{(1)}_2 = -\frac{1}{2} (\nabla \theta)^2 - \frac{\partial V(\rho)}{\partial \rho}
\end{array} \right.$$  \hspace{1cm} (23a)

$$\lambda^{(2)}_\alpha = \left\{ \begin{array}{c}
\lambda^{(2)}_1 = -\frac{1}{2} (\nabla \theta)^2 - \frac{\partial V(\rho)}{\partial \rho} \\
\lambda^{(2)}_2 = -\partial_i (\rho \partial_i \theta)
\end{array} \right.$$  \hspace{1cm} (23b)

Substitution of these results into the Dirac Hamiltonian leads to,

$$\mathcal{H}^{(1)} = \frac{1}{2} \rho (\nabla \theta)^2 + V(\rho) + \partial_i (\rho \partial_i \theta) (\pi_\rho - \theta) - \left( \frac{1}{2} (\nabla \theta)^2 + \frac{\partial V(\rho)}{\partial \rho} \right) \pi_\theta,$$

(24a)

$$\mathcal{H}^{(2)} = \frac{1}{2} \rho (\nabla \theta)^2 + V(\rho) - \partial_i (\rho \partial_i \theta) \pi_\rho - \left( \frac{1}{2} (\nabla \theta)^2 + \frac{\partial V(\rho)}{\partial \rho} \right) (\pi_\theta + \rho).$$

(24b)
As expected both descriptions differ by trivial terms. In spite of that the conversion procedure will lead to quite distinct systems with dramatic consequences, reflecting the interrelation between the different choices of symmetry generators for each case and the natural intrinsic symmetries of the potential.

To disclose the hidden symmetry we follow the procedure reviewed in the appendix. As mentioned after Eq. (A7), one of the constraints is chosen as symmetry generator while the other is kept as gauge fixing condition. For the systems above, (19a) and (19b), we will choose, without loss of physical contents, \( \varphi_1^{(i)}(x) \) to play the role of symmetry generator. To choose the inhomogeneous constraints as symmetry generators is just a technical detail. The important consequence is that for the second case \((i=2)\) the lift of the global translation invariance of the velocity potential to the local case is automatically provided by this choice of generator since

\[
\delta \theta = \varepsilon \{ \theta, \varphi_1^{(2)} \} = \varepsilon. \tag{25}
\]

For the first sympletic structure, in opposition to the other case, this is not the natural choice, since \( \varphi_1^{(1)} \) is the mass density translator not the potential translator,

\[
\delta \theta = \varepsilon \{ \theta, \varphi_1^{(1)} \} = 0. \tag{26}
\]

We will however insist in follow this line of action to explore its consequences. As shown below this fact has striking effects over the first-class Hamiltonian structure.

At this point all correction terms present on Eq. (A8), given in appendix, can be explicitly computed just using the general relations

\[
A_1^{(i)} = \{ \varphi_1^{(i)}(x), H^{(i)} \},
A_2^{(i)} = \{ \varphi_1^{(i)}(x), A_1^{(i)} \},
A_3^{(i)} = \{ \varphi_1^{(i)}(x), A_2^{(i)} \},
\vdots
A_n^{(i)} = \{ \varphi_1^{(i)}(x), A_{n-1}^{(i)} \}; \quad n = 1, 2, \ldots,
\]

with \( H^{(i)} = \int_y H^{(i)}(y) dy \). The first sympletic structure \((i = 1)\) requires an infinite number of correction terms to lift the global translation symmetry, given by

\[
A_1^{(1)} = \{ \varphi_1^{(1)}(x), H^{(1)} \} = \frac{\partial V}{\partial \rho^2} \rho \theta,
A_2^{(1)} = \{ \varphi_1^{(1)}(x), A_1^{(1)} \} = \frac{\partial^3 V}{\partial \rho^5} \rho \theta - \frac{\partial^2 V}{\partial \rho^2},
A_3^{(1)} = \{ \varphi_1^{(1)}(x), A_2^{(1)} \} = \frac{\partial^4 V}{\partial \rho^8} \rho \theta + 2 \frac{\partial^3 V}{\partial \rho^3},
A_4^{(1)} = \{ \varphi_1^{(1)}(x), A_3^{(1)} \} = \frac{\partial^5 V}{\partial \rho^9} \rho \theta - 3 \frac{\partial^4 V}{\partial \rho^4},
A_5^{(1)} = \{ \varphi_1^{(1)}(x), A_4^{(1)} \} = \frac{\partial^6 V}{\partial \rho^{10}} \rho \theta + 4 \frac{\partial^5 V}{\partial \rho^5},
\vdots
A_n^{(1)} = (-1)^{n+1} \frac{\partial^{n+1} V}{\partial \rho^{n+1}} \rho \theta + (n - 1) \frac{\partial^n V}{\partial \rho^n}. \tag{27}
\]

The second sympletic structure \((i = 2)\), on the other hand, only requires two correction terms to covariantize the canonical structure,

\[
A_1^{(2)} = \{ \varphi_1^{(2)}(x), H^{(2)} \} = - \partial \int_y \partial[\rho \delta^{(d)}(x - y)] \rho \theta,
A_2^{(2)} = \{ \varphi_1^{(2)}(x), A_1^{(2)} \} = - \partial \int_y \partial[\rho \delta^{(d)}(x - y)],
A_3^{(2)} = \{ \varphi_1^{(2)}(x), A_2^{(2)} \} = 0. \tag{29}
\]
As discussed above, the minimal correction for the second Hamiltonian structure to produce a gauge invariant formulation happens because the constraint \((\varphi_1^{(2)})\) promoted to first-class lifts the global translation symmetry as a local gauge symmetry. This is not true for the other sympletic structure. Bringing these results into the projected Hamiltonian derived in the appendix (Eq. (A8)), the gauge invariant Hamiltonians correspondent to those different sympletic structures are,

\[
\hat{H}^{(1)} = \frac{1}{2} \rho (\nabla \theta)^2 + V(\rho) + \partial_i (\rho \partial_i \theta) (\pi_\rho - \theta) - \frac{1}{2} (\nabla \theta)^2 \pi_\theta + \sum_{n=1}^\infty \frac{(-1)^n}{n!} \partial^n V \pi_\theta^n
\]

\[
\hat{H}^{(2)} = \frac{1}{2} \rho (\nabla \theta)^2 + V(\rho) - \partial_i (\rho \partial_i \theta) \pi_\rho - \left( \frac{1}{2} (\nabla \theta)^2 + \frac{\partial V(\rho)}{\partial \rho} \right) (\pi_\theta + \rho) + \pi_\rho \partial_x \int_y \partial_y [\rho \delta^{(d)}(x - y)] \pi_\rho - \frac{1}{2} \pi_\rho^2 \partial_x \int_y \partial_y [\rho \delta^{(d)}(x - y)].
\]

It is noticeable that the second symplectic structure indeed contains a finite number of corrections while the first one contains infinite correction terms which, however, can be summed up to a closed form, as shown below.

The infinitesimal gauge transformations associated to each of these invariant models are now computed,

\[
\delta^{(i)} \rho = \varepsilon \{\rho, \varphi_1^{(i)}\} = \begin{cases} 
\delta^{(1)} \rho = \varepsilon \\
\delta^{(2)} \rho = 0 
\end{cases}
\]

\[
\delta^{(i)} \theta = \varepsilon \{\theta, \varphi_1^{(i)}\} = \begin{cases} 
\delta^{(1)} \theta = 0 \\
\delta^{(2)} \theta = \varepsilon 
\end{cases}
\]

\[
\delta^{(i)} \pi_\rho = \varepsilon \{\pi_\rho, \varphi_1^{(i)}\} = \begin{cases} 
\delta^{(1)} \pi_\rho = 0 \\
\delta^{(2)} \pi_\rho = -\varepsilon 
\end{cases}
\]

\[
\delta^{(i)} \pi_\theta = \varepsilon \{\pi_\theta, \varphi_1^{(i)}\} = \begin{cases} 
\delta^{(1)} \pi_\theta = \varepsilon \\
\delta^{(2)} \pi_\theta = 0 
\end{cases}
\]

As mentioned, the local translation of the velocity potential \(\theta\) is directly realized with the choice of the second symplectic structure, Eq. (32), while the choice of the first structure, produces a local shift on the mass density, Eq. (31).

It is important to observe that these reformulations of fluid dynamics as gauge theories were obtained here without specifying the dependence of the interacting potential on \(\rho\). Of course this is not a remarkable result for the question put forward by the constraint conversion mechanism, but it might be helpful to shed light on the question related to the connection of the Galileo invariant system with the relativistic membranes and its generalizations to the d-branes system, which only appears under the specific potential \(\tilde{H}\).

It is interesting at this juncture, in order to put our work in perspective, to relate our results with the gauge formulation obtained in Ref. [19] through the enlargement of the phase space with Wess-Zumino (WZ) variables in the context of BFFT formalism [15,16]. It is worth mentioning that in the BFFT formalism, unlike [20,21], all constraints are covariantized and it is not clear to us at this point how this formalism will produce the distinctic results showed above. In [15], the authors proposed an invariant Lagrangian, given in terms of an infinite number of corrections, that can be written in closed form as,

\[
L = \theta \dot{\rho} + \varphi \dot{\theta} - \frac{1}{2} (\rho - \varphi) (\nabla \theta)^2 - V(\rho - \varphi),
\]

where \(\varphi\) is the WZ variable. The corresponding first-class Hamiltonian is given by

\[
H = \frac{1}{2} (\rho - \varphi) (\nabla \theta)^2 + V(\rho - \varphi)
\]

while the full set of constraints for this theory is given by

\[
\phi_1 = \theta - \pi_\rho, \\
\phi_2 = \pi_\varphi, \\
\phi_3 = \pi_\theta - \varphi.
\]
Although the Dirac matrix is singular, some constraints have nonvanishing Poisson brackets. This is so because the authors of [19] failed to recognize \( \phi^2 \) as one of the primary constraints of the theory. Due to this, the set of constraints (37) can be split into its first and second-class components. Redefining the constraints as,

\[
\hat{\phi}_1 = \theta - \pi_\rho - \pi_\varphi, \\
\hat{\phi}_2 = \pi_\varphi, \\
\hat{\phi}_3 = \pi_\theta - \varphi,
\]

(38)

turns the \( \hat{\phi}_1 \) into first-class while \( \hat{\phi}_2 \) and \( \hat{\phi}_3 \) remain second-class constraints satisfying the canonical Poisson bracket,

\[
\{ \hat{\phi}_2, \hat{\phi}_3 \} = \delta^{(d)}(x - y).
\]

(39)

Taking these constraints equal to zero in a strong way, the Dirac brackets among the phase space variables can be computed. Due to the Maskawa-Nakajima theorem [22], the Dirac brackets are bound to be canonical. In view of this, the gauge invariant Hamiltonian becomes

\[
H = \frac{1}{2}(\rho - \pi_\theta)(\nabla \theta)^2 + V(\rho - \pi_\theta),
\]

(40)

and the remaining (first-class) constraint, namely,

\[
\hat{\phi}_1 = \theta - \pi_\rho,
\]

(41)

becomes the generator of symmetry. This system is equivalent to the solution obtained by us with the choice of the first-symplectic structure following the gauge unfixing scheme [20, 21] after summing up the infinite terms of (30a).

IV. CONCLUSION

The description of a relativistic membrane in terms of a reduced fluid dynamics [1, 4], has intensified the study of this topic over the last years, establishing the connection between this Galileo invariant system in \( d \)-space dimensions and relativistic membranes and its generalization to the \( d \)-branes system in \((d+1)\)-space dimensions. This led to the identification of two extra symmetries but the resulting hydrodynamical system lacks gauge symmetry. Inspired by this result, we argue that the symmetry present on the gauge description of relativistic membrane may be recovered on theory of isentropic fluid dynamics, after the reduction process [1, 4]. In this paper the gauge invariant version for the fluid theory was obtained and the gauge symmetry recovered by using the gauge unfixing technique [20, 21].

The use of this technique has made it clear that the final result is dramatically dependent on the initial choice of the symplectic structure. This is so because, in the present case the symmetry restored is that of translation of the velocity potential. A quite simple procedure is obtained using the constraint that effectively translate that variable (the second symplectic structure in this case) leading to just two correction terms in the covariantization process. This seems natural since this is the constraint that elevates the global symmetry originally defined in the system, i.e., the translation of the velocity potential, Eq. (10). On the other hand, when the first symplectic structure is adopted, the gauge generator involved is not the velocity translator but the mass density translator instead. Since the potential appearing in the original Hamiltonian (34) is only dependent on the density \( \rho \), each application of this generator produces a derivative of this term. Consequently, in order to perform the task of lifting the global symmetry into a local symmetry, an infinite number of terms is required.

A further important point emphasized in this paper is that this embedded symmetry does not lie on the WZ sector, as proposed by phase-space enlarging techniques, but it lies on the original phase space. That makes it clear that the gauge symmetry manifest in the relativistic theory of membranes correspond to the local translations symmetry of the velocity potential in the fluid dynamics model, as explained in Section III. With this result we established a complete connection between Galileo invariant system and \( d \)-branes system.

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APPENDIX A: THE GAUGE UNFIXING FORMALISM

In the Hamiltonian reduction of the second-class constrained systems, the constraints are classified as primary and secondary [23]. Secondary constraints are consistency conditions following from the primary constraints. This consistency algorithm must be implemented until all independent constraints are obtained. Consider a Hamiltonian system
in a 2M dimensional phase space \((q,p)\) with an even number of second-class constraints \(\phi_\alpha \approx 0\) \((\alpha = 1, 2, \ldots, 2N,\) where \(N < M\) described as,

\[
H = H_c + u_\alpha \phi_\alpha, \quad \alpha = 1, 2, \ldots, 2N, \tag{A1}
\]

where \(H_c\) represents the canonical Hamiltonian and \(u_\alpha\) are the Lagrange multipliers. For second-class constraints \(\phi_\alpha\), the \(u_\alpha\) can be determined everywhere by demanding,

\[
\{\phi_\beta, H\} = \{\phi_\beta, H_c\} + u_\alpha \{\phi_\beta, \phi_\alpha\} \approx 0,
\]

\[
u_\alpha \approx -\frac{\{\phi_\beta, H_c\}}{\{\phi_\beta, \phi_\alpha\}}, \tag{A2}
\]

for \(\alpha = 1, 2, \ldots, 2N\). Using these results into the total Hamiltonian (A1), we then have the consistency condition for \(\phi_\alpha\),

\[
\{\phi_\alpha, H\} = G_{\alpha\beta} \phi_\beta, \tag{A3}
\]

where \(G_{\alpha\beta}\) are structure constants. This completes Dirac’s consistency algorithm.

The main idea of the gauge unfixing procedure is to consider half of the second-class constraints as gauge fixing conditions over the remaining first-class, gauge generators, constraints. Next the first-class Hamiltonian is obtained in a systematic way using a properly constructed projector operator. Let us consider a system with two second-class constraints, \(\phi_1\) and \(\phi_2\), satisfying the following Poisson brackets algebra,

\[
C = \{\phi_1, \phi_2\}. \tag{A4}
\]

Redefining the constraints as

\[
\xi \equiv C^{-1} \phi_1,
\]

\[
\psi \equiv \phi_2, \tag{A5}
\]

we have

\[
\{\xi, \psi\} = 1 + \{C^{-1}, \psi\} C \xi, \tag{A6}
\]

so that \(\xi\) and \(\psi\) are canonically conjugate on the surface defined by \(\xi = 0\). The total Hamiltonian, following (A1) is

\[
H = H_c + u_1 \xi + u_2 \psi. \tag{A7}
\]

Let us maintain only \(\xi\) as a natural constraint relation. This will allow us to obtain the first-class system relative to it. However notice that at first, \(\{\xi, H\} \neq 0\), so that in principle, \(\xi\) and \(H\) do not satisfy a first-class algebra. The proper first-class Hamiltonian can be expressed by the Poisson projection

\[
\tilde{H} = H - \psi \{\xi, H\} + \frac{1}{2!} \psi^2 \{\xi, \{\xi, H\}\} - \frac{1}{3!} \psi^3 \{\xi, \{\xi, \{\xi, H\}\}\} + \ldots
\]

\[
= : \exp^{-\psi \xi} : H, \tag{A8}
\]

which satisfies the first-class condition

\[
\{\xi, \tilde{H}\} = 0. \tag{A9}
\]

The first-class Hamiltonian \(\tilde{H}\) was rewritten in terms of a projection form with \(\psi\) respecting the ordering rule defined by Poisson bracket operation above.

As discussed, this formalism converts a second-class system into first-class directly into the original phase-space. This result is equivalent to a partial gauge-fixing of the WZ sector when the converting formalism using the phase-space enlargement Faddeev-Slatshevili idea is adopted. In this context, the ambiguity in the choice of the first-class constraint is related to different choices of fixing the WZ sector.
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