NOMA for Multiple Access Channel in Indoor Visible Light Communications

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Abstract—In this letter, we consider a scenario of indoor visible light communications (VLC) where multiple transmitters (Txs) are communicating to a single receiver and each Tx uses a single light emitting diode (LED). For the considered multiple access channel scenario, we propose a generalized scheme based on non-orthogonal multiple access (NOMA) that can be applied for any number of Txs and any desired spectral efficiency for each of the Txs. We evaluate the performance of the proposed scheme using successive interference cancellation (SIC) based decoding, joint maximum likelihood (JML) decoding, and a combination of SIC and JML decoding. The numerical results indicate superior performance of the proposed NOMA-VLC scheme as compared to the state-of-the-art orthogonal multiple access schemes for VLC.

Index Terms—Multiple access channel, non-orthogonal multiple access, successive interference cancellation, visible light communications.

I. INTRODUCTION

LIGHT emitting diode (LED) based indoor visible light communication (VLC) transmits data by modulating the light intensity, which is typically referred to as intensity modulation. Several modulation and coding schemes have been proposed for multiple input VLC systems for the scenarios where multiple LEDs transmit data simultaneously to a single receiver (Rx) [1], [2]. Recently, non-orthogonal multiple access (NOMA) technique has been proposed for VLC [3], [4]. In NOMA, to decode the data at Rx, successive interference cancellation (SIC) is performed on the received power domain superposed signal [4]. In [5], the authors have presented pulse amplitude modulation scheme for a two user NOMA system by overlapping the constellation points, where, each user can have their own desired spectral efficiency. However, a generalized design considering arbitrary number of transmitters/users has not been discussed. In [3], [4], an on-off keying based implementation has been considered. However, higher order modulation schemes have not been discussed.

Orthogonal frequency division multiplexing (OFDM) based schemes have been proposed for VLC namely direct current (DC)-biased optical OFDM (DCO-OFDM) [6], [7] and asymmetrically clipped optical OFDM (ACO-OFDM) [8]. However, these techniques require Hermitian symmetry plus inverse fast Fourier transform to convert the complex symbols to real domain. Then, DC biasing or clipping the negative part of the signal is required for DCO-OFDM and ACO-OFDM, respectively. Hence, the complexity involved in the implementation of these techniques is higher and also the spectral efficiency is lower. It is also observed that spectral efficiency of DCO-OFDM or ACO-OFDM is half that of radio frequency (RF) NOMA (RF-NOMA) due to the Hermitian symmetry used to convert the complex symbols to real domain.

In NOMA, power domain multiplexing of the symbols is done considering the power allocation coefficients. Here, the symbol multiplexing is done with the actual symbols normalized to unit power. In VLC, the modulation symbols can only be non-negative real values under the typical consideration that the symbols are the LED intensity levels. Due to this non-negativity constraint on the modulation symbols and the information being embedded in the intensity levels, the NOMA schemes proposed for RF mobile communication cannot be directly applied to VLC. Therefore, in this letter, we propose a scheme that assigns a set of power allocation coefficients to each Tx based on the required spectral efficiency for that Tx unlike a single power allocation coefficient for each of the Txs in NOMA-RF. This is done in such a way that these coefficients themselves act as the modulation symbol set. The proposed scheme directly generates non-negative real symbols which avoids the Hermitian symmetry usage, the DC biasing or the clipping of negative portion of the signal which avoids reduction in spectral efficiency compared to RF-NOMA and also makes the system implementation simple.

We consider a scenario of indoor VLC, multiple access channel (MAC) where multiple transmitters (Txs) are communicating to a single user, with each Tx using a single LED and receiver using a photo diode (PD). For MAC, we propose a generalized coding scheme that supports arbitrary number of Txs with desired spectral efficiency for each of the Txs. In terms of bit error rate (BER) and computational complexity, we compare the SIC based decoding, joint maximum likelihood (JML) decoding, and $M$ JML + $(L - M)$ SIC decoding, where, the data from $M$ Txs is decoded using JML decoding and the data from $(L - M)$ Txs is decoded using SIC, given a total of $L$ Txs ($M \geq 2$ and $M < L$). We also discuss the application of the proposed scheme to broadcast channel (BC) and evaluate the BER performance with SIC based decoding.

Notation: We use $[.]$, $[.]$, $Z^+$, $[.]$, $[.]$, and $[.]^T$ for ceil operation, floor operation, set of positive integers excluding zero, absolute value, Frobenius norm, and transpose operation, respectively.

II. SYSTEM MODEL

The system model to implement the proposed codes is shown in Fig. 1. It consists of a single Rx receiving data from
transmit power values are given as a set \( \{ \}

These transmit power values are treated as constellation points in the power domain. Given this, the received signal, \( x \), values assigned to \( x \) are treated as constellation points. In this case, the received signal, \( x \), is given as

\[
x = P_1 h_1 + P_2 h_2 + \ldots + P_L h_L + n,
\]

where \( n \) is the real valued additive white Gaussian noise (AWGN) with 0 mean and \( \sigma^2 \) variance, as in (v). At the Rx, for decoding the data transmitted by \( l^{th} \) Tx, the power received from remaining Txs is treated as interference. Similarly, for decoding the data transmitted by any other Tx, the effect of higher power assigned Txs is removed using SIC and the power received from remaining Txs is treated as interference.

For the proposed scheme, we assume the distance between the consecutive constellation points assigned for individual Txs is same, and the constellation points are in the increasing order which is given as follows

\[
P_1 < P_2 < \ldots < P_L, \quad \text{s.t., } |P_k - P_{k+1}| = \lambda_k, \quad (2)
\]

where \( \lambda_k \) is the distance between the consecutive constellation points assigned to \( x^{th} \) Tx and it’s value can vary across Txs.

We define ideal conditions as zero noise variance (\( \sigma^2 = 0 \)) along with availability of perfect channel state information (CSI) at all the Txs and Rx. Consider an example scenario shown in Fig. 2 for two Txs, where the possibility of non-zero BER can occur even in ideal conditions. From Fig. 2 in case \( P_1 h_1 + P_2 h_2 \) is transmitted and \( P_2 h_2 > \delta \), where, \( \delta \) is the mid-point of any two consecutive constellation points as shown in Fig. 2. Then at the Rx, \( P_2 h_2 \) is decoded which is incorrect. Here, \( P_2 h_2 \) is chosen, as from (2), this is the maximum value of the constellation points of second Tx. The condition for zero BER for any number of Txs in ideal conditions is given as follows

\[
P_1^q h_1 + \sum_{r=1}^{L-1} \max_{q_r \in Q^r_r} \{P_{q_r} h_{x+r}\} < \frac{P_2^q h_2 + P_2^q h_2}{2}, \quad (3)
\]

From (3), \( \max_{q_r \in Q^r_r} \{P_{q_r} h_{x+r}\} = P_2^q h_{x+r} \). Hence, (3) can be written as follows

\[
P_2^q h_2 + \sum_{r=1}^{L-1} P_{2^q} h_{x+r} < \frac{P_2^q h_2 + P_2^q h_2}{2}, \quad (4)
\]
On further simplification, (4) becomes

\[ P_x^{q_x+1} = \frac{2}{h_x} \sum_{r=1}^{L} P_{x+r}^{q_x+r} h_{r+x} + P_x^{q_x}. \]

Let

\[ \Delta_x^{q_x+1} = \frac{2}{h_x} \sum_{r=1}^{L} P_{x+r}^{q_x+r} h_{r+x} + P_x^{q_x}, \]

then,

\[ P_x^{q_x+1} = \begin{cases} \frac{\Delta_x^{q_x+1} + 1}{\Delta_x^{q_x+1}} & \text{if } \Delta_x^{q_x+1} \in \mathbb{Z}^+, \\ 1 & \text{otherwise}. \end{cases} \]

Alternatively,

\[ P_x^{q_x+1} = \left[ \Delta_x^{q_x+1} + 1 \right]. \]

Next, we consider Algo. 1 to obtain constellation points for \( L \) Txs with desired spectral efficiency for each of the Txs. The spectral efficiency of the VLC system with \( L \) Txs with the proposed scheme is \( \eta_1 + \eta_2 + \ldots + \eta_L = \sum_{i=1}^{L} \eta_i \) bpcu.

1) Decoding Mechanism: Decoding for the proposed scheme can be performed using SIC decoding or JML decoding. In SIC based decoding, decoding of the signal from the \( x^{th} \) Tx happens after performing SIC of the signals received from \( 1^{st} \) to \( (x-1)^{th} \) Tx and the received signals from \( (x+1)^{th} \) to \( L^{th} \) Tx is treated as interference as given in Algo. 2.

We assume that the constellation points corresponding to all the Txs are known at the Rx. In Algo. 2, \( P_x \) denotes the power estimate of \( x^{th} \) Tx. In JML decoding, we perform ML decoding over all possible combinations of constellation points to decode as

\[ [\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_L] = \min_{q_x} \epsilon_{Q_x} \{ \| y - [h_1 h_2 \ldots h_L] [p_1^{q_1} p_2^{q_2} \ldots p_L^{q_L}]^T \| \}. \]

In \( M \) JML + \( (L-M) \) SIC decoding, the data from \( M \) Txs is decoded using JML decoding and the data from \( (L-M) \) Txs is decoded using SIC decoding for \( M \geq 2 \) and \( M < L \). Note that when \( M = 1 \), the \( M \) JML + \( (L-M) \) SIC decoding is same as SIC decoding, and when \( M = L \), it becomes JML decoding. The number of computations with JML decoding, SIC based decoding, and \( M \) JML + \( (L-M) \) SIC decoding is given in Table. 1. It can be observed that as the value of \( L \) and \( \eta_L \) increase, the computations involved in JML decoding will be significantly higher as compared to SIC based decoding. Further, for \( M \) JML + \( (L-M) \) SIC decoding, the number of computations also depends on the value of \( M \). Note that, for the \( M \) Txs data that is being decoded using JML decoding, the condition to perform SIC in (4) can be relaxed. However, it is to be noted that for those \( M \) Txs, where, JML decoding is used at the Rx, the number of computations increases. The average number of ML computations involved in orthogonal multiple access (OMA) are atleast equal to that of the computations involved in NOMA JML decoding. This can be shown as follows with the consideration of same average spectral efficiency between the schemes. The average number of computations involved in JML decoding is \( L 2^{L-1} \eta \) and that in OMA is \( \sum_{i=1}^{L} 2^i \eta \) such that

\[ L 2^{L-1} \eta \leq \sum_{i=1}^{L} 2^i \eta_i. \]

On simplifying (6), we get

\[ \frac{1}{L} \sum_{i=1}^{L} (2^{L-i} - \sum_{i=1}^{L} \eta_i) \geq 1. \]

From \( \log(\theta) < \theta \forall \theta > 0 \), (7) is written as follows

\[ \frac{1}{L} \sum_{i=1}^{L} (L \eta_i - \sum_{j=1}^{L} \eta_j) \geq 1, \]

Finally,

\[ \sum_{i=1}^{L} \eta_i \geq \frac{1}{L} \sum_{j=1}^{L} \eta_j \geq 1 \forall L \geq 2. \]

From (8), it is observed that the average number of ML computations involved in OMA are atleast equal to that of the computations involved in NOMA JML decoding.

A. Application of the Proposed Scheme to Broadcast Channel

The proposed design can also be used for BC, where, the data of multiple users is multiplexed and transmitted through a single LED Tx. Without loss of generality, here we assume each user is equipped with a single PD Rx. The data is decoded at the users' end by employing SIC, where, the received signal at \( \alpha^{th} \) user is given as follows

\[ y_\alpha = S_\alpha + n_\alpha, \]
where, $n_{aw}$ is the 0 mean real valued AWGN noise at the $\alpha^{th}$ user with $\sigma^2_{\alpha}$ variance and $S_{\alpha}$ is the received signal at the $\alpha^{th}$ user and is given as $S_{\alpha} = \left( U_{1}^{\alpha} + U_{2}^{\alpha} + \ldots + U_{K}^{\alpha} \right) g_{\alpha}$, where, $g_{\alpha}$ is the channel gain between Tx and $\alpha^{th}$ user. Here, $U_{r}^{\alpha}$ denotes the transmit power corresponding to $\alpha^{th}$ Rx for $\alpha \in \{1, 2, \ldots, K\}$, where, $K$ is the total number of users and $r \in \{1, 2, \ldots, 2^{n_r}\}$. The constellation points of user are identical to that of the constellation points of the Txs in MAC. The number of ML computations in decoding $\alpha^{th}$ user is $\sum_{i=1}^{2^{n_r}}$. Note that this value includes the ML computations involved in SIC process of $\alpha - 1$ users while decoding $\alpha^{th}$ user’s data.

IV. Numerical Results

For numerical results, we consider the VLC channel model as in [2]. We assume Tx positions are fixed and the Rx/user is also stationary and generate the channel gains for the scenarios in Fig. 3 using the parameters in Table. II. We normalize the constellation points as follows and these can be scaled to change the brightness levels in indoor environment.

$$P_{x} = \frac{P_{Q_x}}{\sum_{x=1}^{L} \sum_{q=1}^{2^{n_r}} P_{Q_x}^{q} h_{x}^{2}} \quad \forall x \in X, q_x \in Q_x,$$

where, $P_{Q_x}$ is the normalized constellation point. For MAC, the average received signal-to-noise ratio (SNR) is denoted by $\overline{SNR}$ and given as follows

$$\overline{SNR} = \frac{\sum_{x=1}^{L} \sum_{q=1}^{2^{n_r}} P_{Q_x}^{q} h_{x}^{2}}{\sigma^2 L \sum_{x=1}^{L} 2^n_{x}}.$$ 

Similarly, for BC, the received SNR at $\alpha^{th}$ user denoted by $\overline{SNR}_{\alpha}$ is given as follows

$$\overline{SNR}_{\alpha} = \frac{\sum_{i=1}^{2^{n_r}} U_{i}^{\alpha} h_{\alpha}^{2}}{\sigma^2_{\alpha} 2^{n_r}}.$$ 

In Fig. 4 the BER performance of JML decoding and SIC decoding is compared for 2 Txs scenario shown in Fig. 3(a). It is observed that the BER with JML decoding is significantly better as compared to SIC decoding. However, this improved performance with JML decoding comes at the cost of computational complexity. It is also observed that the proposed scheme with JML decoding performs better compared to OMA [10].

In Fig. 5 the BER performance of JML decoding, SIC decoding, and $M$ JML + $(L - M)$ SIC decoding for $M = 2, 3$ with 4 Txs in MAC.

In Fig. 6 the BER performance of the proposed scheme with SIC decoding is compared with OMA [10] and DCO-OFDM NOMA [6] with fixed power allocation (FPA) and gain ratio power allocation (GRPA) [3] for 2 users scenario of BC shown in Fig. 3(c). The DCO-OFDM NOMA uses...
Fig. 7: BER performance comparison of proposed NOMA scheme with OMA and DCO-OFDM NOMA in BC with 2 users with identical channel gains.

16-phase shift keying (16-PSK) to show simulation results for two power allocation factors denoted by $\rho$ equal to 2/3 and 1/9. It is observed that the proposed scheme outperforms the conventional OMA and DCO-OFDM NOMA with both FPA and GRPA in terms of the BER performance. However, if the channel gains of both users shown in Fig. 3 (c) is equal (assuming User 2 position Fig. 3 (c) is changed to make the gains identical), DCO-OFDM NOMA with $\rho = 1/9$ outperforms proposed scheme at high SNRs. Note that when $\rho = 1$ for FPA it is identical to GRPA for the identical channel gains case. The proposed scheme implementation is much simpler compared to DCO-OFDM NOMA and also the latency experienced in DCO-OFDM NOMA could be relatively high due to it’s complex system model involving inverse fast Fourier transform (IFFT) and fast Fourier transform (FFT) at Tx and Rx, respectively.

V. CONCLUSIONS

In this letter, we have proposed a generalized coding scheme for MAC and evaluated it’s performance in terms of BER and computational complexity with SIC based decoding, JML decoding, and $M$ JML + $(L-M)$ SIC decoding. It is observed that the BER performance gain with JML decoding comes at the cost of computations and the performance of $M$ JML + $(L-M)$ SIC decoding depends on $M$. It is observed that the proposed scheme performs better than the state-of-the-art OMA scheme. The application of the proposed scheme for BC is validated and it’s superior performance is observed as compared to the OMA scheme and DCO-OFDM NOMA in most of the scenarios.

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