Mathematical Modeling of Thermal Lensing effect on Nd-Yag Laser Rod in Trans-Receiver Optoelectronics Avionics Assembly and its Interferometric Measurement

Lakshman Singh¹ and Sachin Singh²

¹Project Manager-Russian, Hindustan Aeronautics Limited, Avionics Division-Korwa, Amethi-227412, UP, India; E-mail: laksh_jk@rediffmail.com, sinlakshman@gmail.com

²Operation Manager-Russian, Hindustan Aeronautics Limited, Avionics Division- Korwa, Amethi-227412, UP, India; E-mail: ec.singh@gmail.com

Abstract:

The Nd-Yag laser Rod commonly used Laser Generation element in Optical Detection & Ranging System of Military Aircraft. This research paper points out different issues of Nd-Yag Laser rod during Laboratory test. The paper discusses the thermal lensing phenomenon in the Nd-Yag Laser Rod which causes a thermal fracture in a rod, variation in laser power, measurement error and commutative low accuracy effect in the higher optical assembly. The research find out major unavoidable reasons in the optical assembly which cause thermal lensing phenomenon and formulated a generic mathematical model for the same. This research analyzed the lab observation of thermal lensing by Interferometric measurement and recommended the best suited optical assembly-tuning-alignment procedure for laser- based avionics system based on the research findings.

Keywords: Optoelectronics; Nd-Yag Laser; Trans-Receiver; Thermal Lensing; Interferometric Fringes;

1. Introduction:

The development of fighter aircraft is mainly for armament deployment at the enemy area and for the proper aiming to the target, there is a different type of weapon guidance system available in fighter aircraft. The infrared & LASER is most common technology which is used in weapon guidance system for getting the target co-ordinate. And the Infrared & Laser base weapon aiming & weapon guidance system of fighter aircraft are mainly made of optoelectronics technology. In such type of optoelectronics avionics system, the optical assembly plays the main role for alignment, focusing, laser firing & receiving. In this regard the accurate & quality assembly of optics becomes crucial, and it becomes necessary when it is going to use in fighter aircraft.

For the military application of optical assembly, the special precautions have to be taken during the assembly process to meet the airworthiness standard of different sub-assemblies and final higher assembly. Here author is going to discuss Mathematical Modeling of Thermal Lensing effect on Nd-Yag Laser Rod in Trans-Receiver Optoelectronics Avionics Assembly and its Interferometric Measurement and derived the reason of optical disturbance which results low ranging, de-focusing, range measurement error etc., in Laser Trans-Receiver assembly. The laboratory observation are collected for the performance of Nd-Yag Laser rod during thermal
lensing phenomenon and recommended the precise technique for optical assembly to reduce the optical disturbance effect in higher assembly.

2. Literature Review:

R. M. Waxler, G. W. Cleek, I. H. Malitson, M. J. Dodge, and T. A. Hahn (1971)” Optical and Mechanical Doped Properties of Some Laser Glasses” evaluate the thermo-optic and piezo-optic properties of five laser glasses. The data on change of refractive index due to temperature variation, by stress and its effect on optical properties is discussed. The measurements of the different parameter and discussed the self-focusing of laser light.

Claude A. Klein(1991) “Optical Distortion Coefficients of High-power laser window” research paper concerns the problem of describing and evaluating thermal lensing phenomena that occurs as a result of the absorption of laser light in the solid window. The aberration function expansion method is applied for deriving the two optical distortion coefficients that characterize the degradation in light intensity at the Gaussian focus of an initially diffraction-limited laser beam passing through a weakly absorbing stress-birefringent window.

Andrew Bachmann, Dr. John Arnold and Nicole Langer (2001) “ Controlling Stress In Bonded Optics,” discuss the different type of optical assembly technique and stress generation reason. The paper formulated a mathematical equation for different properties of optics, values of the various parameter of the optical assembly and concluded with the generation stress generated birefringent.

Christina Leidel, Joe Gleason, and David H. Reitze (2003) “Methods of compensation for thermal lensing effects in gravitational wave detectors” address the increasing of the sensitivity of Laser Interferometer Gravitational Wave Detector (LIGO) causes thermal lensing phenomenon due to extremely high heat intensity of high power laser. Author points out distortions in optical components and change the mode quality of the optical beam, severely impairing our ability to mode-match the beam to the resonating cavities.

Kazuma Mawatari, Hisashi Shimizu & Takehiko Kitamori(2004) “Thermal Lensing, Detection,” points out the thermal lensing is one example of the photothermal phenomenon. The heat generation induces a temperature distribution which is similar to a Gaussian profile due to the Gaussian intensity distribution of excitation laser & heat diffusion.

Andrew Clements (2006) “Selection of Optical Adhesives” discuss the issues involved with selection of an the optomechanical adhesive. Paper point out of the important material properties that are used to characterize and distinguish the optomechanical adhesives. The failure mechanisms, wrong processing and recommended adhesives with good chemical and solvent resistance.

James Champagne (2010),"Effects of Temperature Changes in Opto-Mechanical Systems,” has made research to find out the effects of temperature changes on optomechanical systems which is important to understand in order to design stable systems. The temperature changes, stress, deformation in optics properties, dimension change in the optomechnical assembly is discussed in the paper and suggested techniques to prevent by passive and active thermal effect.

Ming-Ying Hsu, Shenq-Tsong Chang & Ting-Ming Huang( 2012) “Thermal optical path difference analysis of the telescope correct lens assembly,” address the effect of correct lens thermal optical path difference (OPD) on the optical performance of the Casse grain telescope system. The correct lens assembly includes several components such as a set of proper lenses,
lens mount, spacer, mount barrel, and retainer. The heat transfer from the surrounding environment to the appropriate lens barrel will cause optical system aberration. The lens barrel heat transfers mathematical analysis; the thermal distortion and stress are solved by the Finite Element Method (FEM) software.

Lakshman Singh (2016) “Double Refraction/ Stress Birefringence in Optical Assembly of Optoelectronics Avionics System of Fighter Aircraft” point out the stress generated double refraction phenomenon in the optical assembly of avionics system.

Lakshman Singh, Prof. Alpana Srivastava and A.J.Sarkate (2017) “Thermal Gradient Effect on Focus Shift of Laser & Infrared Optical Assembly & Thermal Lensing by Nd-Yag Laser Rod in Laser Assembly of Optical Detection & Ranging System of Fighter Aircraft,” find the effect of temperature gradient on refractive index of optics, which causes self-focusing & de-focusing phenomenon in IR & Laser assembly and resulting focus error and derived the thermal lensing phenomenon in Nd-Yag Laser rod.

3. Nd-Yag Laser Rod:

The laser medium of a laser is the site where laser light originates. The laser medium of a ND-YAG laser is a synthetic single-crystal: Yttrium Aluminum Garnet (YAG), in which a small fraction of the yttrium ions are substituted by neodymium ions.

**Doping:** The concentration of Nd ions in the YAG crystal (=doping) is between 1% and 1.5%. If the percentage of Nd ions in the YAG crystal is low, the laser effect will be week. On the another hand, if many Nd ions are present in the YAG crystal, the Nd ions will be crowded in the crystal. Beam quality will suffer.

**Wavelength:** Nd-YAG lasers emit invisible light at a wavelength of 1.06µm. This wavelength is the near infra-red, very close to the visible wavelength range.

**Pump Sources Lamp:** A laser must be supplied with energy before it can emit light. The energy source of a laser is termed “pumps” energy into the system. Nd-YAG laser are optically pumped, i.e., with light, Krypton arc lamps or halogen lamps are employed for CW (continuous wave) mode. On Nd:YAG lasers operating solely in pulsing mode, xenon flash lamps serve as the pump source.

Typical Arrangement of Nd-Yag Laser Rod in Chamber Assembly( Resonator) of Trans-Receiver

1. Nd-Yag Laser Rod
2. Flash Lamp
3. Resonator Mirror
4. Lamp Power Supply
5. Reflective Elliptical Cavity
The Nd-YAG rod and the pumping lamp (a neon lamp) is located parallel in a highly reflective enclosure (the “cavity”). This enclosure can be shaped, for example, like an elliptical cylinder mirror. A laser rod and pump lamp aligned along the two focal axes of the enclosure is ideal for harnessing the pump energy into the Nd-YAG rod.

Cooling: As the laser material heats up during optical pumping; a system must be provided for heat dissipation. This is usually achieved by cooling of water circulating around the Nd-YAG rod in so-called “Flow.”

4. Experimental Set-Up for Checking Laser Assembly (Nd-Yag) and Measurement of Fringes:

Figure-2: (a) Laboratory Set-up for Interferometer, (b) Ray Diagram of Lab Set-up, (c) Laser Assembly

Setup Brief: The experimental set-up includes Laser Source and Collimator to align the laser beam to fire at one surface of Nd-Yag rod. One transparent coupler in between collimator and laser rod, where interferometric fringes are made. The analog camera is placed in front of the coupler to observe the fringes shape.
5. Experiment Process:

**Step1:** Clean the Surface of Active Element (Nd Doped Glass) with Mixture of Spirit and Absolute Alcohol and dry it before putting on experimental set-up. First, check the fringes shape in open condition of the laser rod.

**Step2:** Now check the fringes shape in the assembled condition of laser rod in a resonator, which should be \( n + 1 \) only. The assembly should be leak proof for circulation of coolant inside the resonator. The sealing of Laser rod is made by applying the force of 260-320Kgf to press the combination of Indium-Teflon-Indium gasket through Flange with Ring as shown and leave the force for 10 min. The leak test carried out for conformity of the assembly quality.

The temperature profile in the end-pumped laser cylindrical crystal, seen from the pumped end. The highest temperatures occur around the beam axis. Due to cooling of the outside faces, there is a heat flow and thus inevitably a temperature gradient in the radial direction. (Figure Taken from Book Literature)

---

**Figure-3:** (a) Temp. Profile of Laser Rod Surface, (b) Fringes Pattern by surface of Laser Rod

---

**Before assembly of Nd-Yag Laser Rod**

Actual Figure of Interferometric Fringes shape in open condition of Nd-Yag Laser Rod

The uniformity of red color shows that there is no pressure on the surface of the Rod and there is no path difference between incident laser & reflected laser from another end of the rod surface.

**After assembly of Nd-Yag Laser Rod**

Actual figure of Interferometric Fringes shape in assembled condition of Nd-Yag Laser Rod

The Black spot of the circumference of the fringes shown that there is pressure on the end surface of the laser rod. There is path difference between incident laser & reflected laser from another end of the rod surface due to pressure on surface and this results black pattern on the circumference.

---

**Figure-4:** Actual Figure of Interferometric Fringes in Laser Rod Surface by Laboratory Observation
6. Thermal Lensing and Stress Birefringes by Laser Rod:

The research work finds the effect of Thermal Lensing and stress in Nd-Yag Laser rod of Trans-Receiver assembly of avionics articles. The lab finding of the optical assembly analyzes and found that laser rod behaves as per specify performance parameter but its performance degraded in elevated or lowered temperature about Laser Energy, as an Aircraft may fly in varying condition. Sometimes it is observed that Laser energy is fine under normal temperature but during cold/hot it varies and some time goes below tolerance, hence leading to rejection of assemblies at a higher level. This variation occurs because of formation of thermal lensing and stress in the optical assembly which is not avoided, but its effect can be reduced by the special assembly technique. The results of the thermal lensing and stress effect is observed by Interferometer set-up.

7. Analysis:

7.1 Thermal Effect:

The center beam axis of Nd-Yag Laser Rod has a higher temperature and outer region low temp due to coolant flow. It crates temperature gradient refractive index variation. Most of the optical material undergoes a change in refractive index “n” with temperature, at the rate of change \( \frac{\delta n}{\delta T} \). The absolute \( \frac{\delta n}{\delta T} \) of optical material -

\[
\left( \frac{\delta n}{\delta T} \right)_{\text{abs}} = n_{\text{air}} \left( \frac{\delta n}{\delta T} \right)_{\text{air}} + n \left( \frac{\delta n}{\delta T} \right)_{\text{air}}
\]

Let us assume some notation-

\[\Phi = \text{power of lens}\]
\[f = \text{focal length of lens}\]
\[T = \text{temperature of optical material & media (homogenous temp. distribution)}\]
\[C = \text{surface curvature of lens}\]
\[\alpha_L = \text{lens thermal expansion coefficient}\]
\[\alpha_H = \text{housing thermal expansion coefficient}\]
\[t = \text{thickness of lens}\]
\[n = \text{refractive index of lens}\]
\[r = \text{radii of lens curvature}\]
\[\gamma = \text{thermal glass constant}\]

so,
\[f = \frac{1}{\Phi} \text{ and } n_{\text{temp}} = n + \left( \frac{\delta n}{\delta T} \right) \Delta T \]

\[
\frac{\delta f}{\delta T} = - \frac{1}{\Phi} \left( \frac{\delta f}{\delta T} \right) \text{ and } \frac{\delta \Phi}{\delta T} = \Phi \left\{ \left( \frac{\delta n}{\delta T} \right) - \alpha_L \right\}
\]

so,
\[\frac{\delta f}{\delta T} = -f \left\{ \left( \frac{\delta n}{\delta T} \right) - \alpha_L \right\} \div \left( \frac{n-1}{(n-1)} \right) \text{ (1)}
\]

Hence, the focus disturbance results due to temperature variation.

\[n_{\text{temp}} = n + \left( \frac{\delta n}{\delta T} \right) \Delta T \]

and \[n = n_0 + \frac{1}{2} n_2 E_0^2\]

For a point close to axis
\[ T(r) = T(0) + r \left( \frac{dT}{dr}(r=0) \right) + \frac{r^2}{2} \left( \frac{d^2T}{dr^2}(r=0) \right) \]

So, \[ T(r) - T(0) = -\left( \frac{(\alpha P_0) / (4\pi n_0^2)}{k r^2} \right) \exp(-\alpha z) \] \[ r^2 \]

As above equation shows that temperature variation is parabolic, so refractive index variation is also parabolic which cause dispersion of coherent intense laser beam after passing through the optical media.

**7.2 Stress Effect:**

The mechanical stress on the optical surface leads the formation of the virtual lens due to bulging which de-focus the laser beam and double refraction phenomenon which make diffraction of the laser beam. If we apply Excess pressure to seal the active element, it may be possible that some bulges are created after thermal Cycling of Assembly as shown in fig:

![Formation of the Virtual Lens due to excess pressure on the surface of the Laser Rod](image)

Figure-5: Formation of the Virtual Lens due to excess pressure on the surface of the Laser Rod (a) Pictorial view of excess pressure, (b) Diversion of Laser Beam due to the formation of the virtual Lens.

Now this lens will have its focus leading to the introduction of optical Path difference inside the active Element and thus forming the fringes. Hence, the laser beam path gets diverted to its original path.

**7.3 Double Refraction Effect:**

The optics used in the assembly of optoelectronics avionics system are of high quality & satisfy military airworthiness specification. They are isotropic and have no natural briefings. But the isotropic material can become anisotropic by application of stress. There is stress generated in optics of optical assemblies due to its mounting, gluing, change in temperature, vibration, shock and residual stress is always present in glass/optics due to the annealing and fabrication process. This stress in optics introduces double refraction into the optics or in extreme case may damage the optics. The stress generated double refraction in optics cause optical path difference per unit path length of light (nm/cm) and polarization of incident electromagnetic wave, which results suffer from transmission loss from optics. Stress can make an isotropic material birefringent, creating a pattern form which double refraction occurs. In double refraction, the unpolarised light beam splits up into two linearly polarized beams. The beam which travels un-deviated is known as ordinary ray (abbreviated as the o-ray) and obeys Snell’s law of refraction. On the other hand, the second beam which does not obey Snell’s law is known as the extra-ordinary ray (abbreviated as e-ray).
Stress in optics introduces double refraction into the optics and split light into o-ray and e-ray.

Figure-6: Double Refraction phenomenon (splitting of unpolarised wave into o-ray & e-ray)

The velocity of o-ray is same in all direction, but the velocity of e-ray due to anisotropic behavior of stress optics is different in different direction. The velocity of o-ray & e-ray is same along optic-axis. The velocity of o-ray & e-ray are given

\[ V_{ro} = \frac{c}{n_o} \]

\[ \frac{1}{V_{re}^2} = \frac{\sin^2 \theta}{(c/n_e)^2} + \frac{\cos^2 \theta}{(c/n_o)^2} \quad ; \quad n_o \text{ & } n_e \text{ constant of optics for o-ray & e-ray.} \]

\[ \theta \text{ = angle that e-ray makes with optics axis} \]

Along the optics axis \( \theta = 0; \) \( V_{ro} = V_{re} = \frac{c}{n_o}; \) If \( \theta = \pi/2; \) \( V_{re} (\theta = \pi/2) = \frac{c}{n_e} > \frac{c}{n_o} \rightarrow \) Negative Optics

For positive optics \( n_e < n_o; \) \( V_{re} (\theta = \pi/2) = \frac{c}{n_e} < V_{ro} \)

Now consider the case of negative optics in the optical assembly of avionics system, here e-wave will travel faster than o-wave. This velocity difference cause phase difference between e-ray & o-ray which depend on the thickness of optics. By this phase difference these two rays will generate an interference pattern. Out phase will cause destructive and in phase cause constructive interference, i.e. we get minimum and maximum light pattern on optics. This results uneven output of light intensity and makes the performance of optical assembly poor.

Figure-7: Incidence of un-polarised light beam on optics of thickness “t” & double refraction
Let incident light beam electric vector \( E_0 \) along the x-axis (which is plane of optics). Since optics is stress in optical assembly of avionics due to its assembly mounting, application of adhesive and due to external atmospheric force on optics it cause double refraction phenomenon on incident light. Assume that due to double refraction light split into two beam of o-ray (slow) & e-ray (fast) along y-axis & z-axis respectively.

Amplitude of 
- o-ray: \( E_0 \sin \varphi \)
- e-ray: \( E_0 \cos \varphi \)

since \( n_e \neq n_o \) no the emergent light beam (which is a superposition of this two beam) will be elliptically polarized.

Let \( x = 0 \): plane of optics

Y & Z axis component of incident beam can be written as

\[
\begin{align*}
E_y &= E_0 \sin \varphi \cos (kx - \omega t) \\
E_z &= E_0 \cos \varphi \cos (kx - \omega t)
\end{align*}
\]

\( k = \omega/c \): free space wave number

At \( x = 0 \) (on the phase of optics)

\[
\begin{align*}
E_y (x = 0) &= E_0 \sin \varphi \cos \omega t \\
E_z (x = 0) &= E_0 \cos \varphi \cos \omega t
\end{align*}
\]

At \( x = t \) (thickness of optics – emergent light)

\[
\begin{align*}
E_y &= E_0 \sin \varphi \cos (\omega t - \theta_o) \quad \text{: o-ray} \\
E_z &= E_0 \cos \varphi \cos (\omega t - \theta_e) \quad \text{: e-ray}
\end{align*}
\]

\( \theta_o = n_o kd \)

\( \theta_e = n_e kd \)

\( \Delta \theta = \theta_o - \theta_e \): phase difference of o-ray & e-ray

\[
\Delta \theta = kd (n_o - n_e) = \frac{\omega}{c} (n_o - n_e)d
\]

The thickness of optics determine phase difference between o-ray & e-ray

\( \Delta \theta = \frac{\omega}{c} (n_o - n_e)d \)

Path length difference between o-ray & e-ray introduces by optics of optical assembly in Trans-Receiver system which is proportional to stress value of photo elastic constant for this material and thickness as well as its form. The path difference crate fringes formation which represent optical path disturbance for laser beam in the Trans-Receiver.

7.4 Non-Uniform Doping:

The non-uniform doping of impurities in Intrinsic semiconductor leading to the formation of areas with varying refractive index inside the active element (Nd-Yag Laser Rod) and it changes
its path from an original path leading to the optical path difference and this path difference leads to the formation of fringes in optics.

8. Results:

The thermal lensing effect in optical assembly of Nd-Yag Laser rod of Trans-Receiver is determined by the combined effects of the temperature-stress-doping-double refraction which results variation of the refractive index and the distortion of the end-face curvature of the rod. The final result of the mathematical formula is mentioned here for reference purpose from the literature review.

The expression of the total focal length \( f \) of thermal lens is given by

\[
f = \frac{K A}{P_h} \left[ \frac{1}{2dT} \frac{dn}{dT} + \alpha C_{\phi} n_0^3 + \alpha r_0 (n_0-1) \right]^{-1}
\]

Where

- \( K \) = The thermal conductivity;
- \( A \) = cross-sectional rod area;
- \( P_h \) = total heat dissipated in the rod,
- \( C_{\phi} \) = photo-elastic coefficient;
- \( \alpha \) = thermal coefficient of expansion;
- \( r_0 \) = radius of the rod;
- \( l \) = length of the rod.

For the Nd:YAG crystal, a temperature-dependent variation of the refractive index, assembly stress, stress due to its mounting, gluing, change in temperature, vibration, shock and residual stress, non-uniform doping, etc. constitutes the main contribution of the thermal lensing.

Considering the object wave \( O(x,y) \) and reference wave \( R(x,y) \) interfere on the coupler target plane, the intensity can be given by

\[
I(x,y) = |O(x,y)|^2 + |R(x,y)|^2 + R^*(x,y) \cdot O(x,y) + R(x,y) \cdot O^*(x,y)
\]

where the symbol \( \ast \) denotes the complex conjugate operation, and \( x, y \) is the rectangular coordinates on the coupler target plane.

Due to thermal lensing, the change of refractive index variation cause path difference \( \Delta l(x,y) \) and there by introducing interference fringes. Then the phase change \( \Delta \phi(x,y) \) due to the index change is given by

\[
\Delta \phi(x,y) = \frac{2 \pi}{\lambda} \Delta l(x,y) = \frac{2 \pi}{\lambda} \int_0^L [n(x,y,z) - n_0] dz
\]

9. Recommendation:

The research work also carried out to avoid the thermal lensing phenomenon in Nd-Yag Laser by some of the optical assembly technique and same is also validated in the lab. The suggestion recommendation of optical assembly technique can’t remove the thermal lensing error 100%, but its effect can be reduced which result in good performance of laser output by Trans-Receiver.

- The sealing of Nd-YAG laser rod in resonator made by a selective gasket having different dimensions to suit the assembly and choose malleable material for making gasket like Indium but material should not be reactive to the coolant used to cool the active element (Laser Rod).
• If the thermal lensing phenomenon is not avoidable by Interferometric fringes, results the thermal stabilization process may be adopted and repeat the thermal cycle by a cold & hot temperature ranging ± 60 to 90-degree centigrade for 1 to 2 hours. If the fringes counting does not reduce as per required measurement, the full assembly must be dismantled to the lower stage and start again the assembly with precaution and restart again.

10. Conclusion:
In conclusion a mathematical model of Thermal Lensing effect on Nd-Yag Laser Rod in Trans-Receiver Opto-Electronics Avionics is developed. The thermal lensing effect on Nd-Yag laser rod plays a critical role in laser output power, de-focusing and diffraction phenomenon on optics of the assembly. This model determined Thermal Lens on Nd-Yag Laser Rod and validated the model by the experimental set-up of interferometer in the lab. The suggested recommendation is laboratory experimental observation to reduce the effect of Thermal Lensing phenomenon in ND-Yag Laser Rod of Trans-Receiver Assembly of Avionics system.

References:
[1]. Riedl M J ,Optical Design Fundamentals for Infrared Systems (SPIE),2001.
[2]. W. Koechner , Thermal Lensing in a Nd:YAG Laser Rod ,OSA Publication, Vol-9, Issue11, pp 2548-2553,1970.
[3] Abramovich, W.E. Althouse, R.W. Drever, Y. G’ursel, S. Kawamura, F.J. Raab, D. Shoemaker, L. Sievers, R.E. Spero, K.S. Thorne, R.E. Vogt, R. Weiss, S.E. Whitcomb, and M.E. Zucker, LIGO: The Laser Interferometer Gravitational-Wave Observatory, Science 256, 325-333 (1992).
[4] Ming-Ying Hsu, Shenq-Tsong Chang & Ting-Ming Huang, “Thermal optical path difference analysis of the telescope correct lens assembly,” National Applied Research Laboratories December 2012 Advanced Optical Technologies 1(6):447-453 · December 2012.
[5] V Letokhov V S and Chebotayev V P, Nonlinear Laser Spectroscopy (Springer Berlin Heidelberg),1997.
[6] P. N. Butcher and D. Cotter, The Elements of Nonlinear Optics (Cambridge University Press, Cambridge, 1991). Crystal Research and Technology, ISBN 0-521-34183-3, 1991.
[7] Claude A. Klein, "Optical distortion coefficients of high-power laser windows," Optical Engineering 29(4), (1 April 1990).
[8] Cheng X, Miao X, Wang H, Qin L, Ye Y, He Q, Ma Z, Zhao L and He S 2014 Advances in Condensed Matter Physics 2014 1-7,2014.
[9] Ri-Qing Lv; Yong Zhao; Jin Li; Ya-Nan Zhang,”Theoretical Research on the Thermal-Lens Effect of Magnetic Fluid by Using Brownian Dynamics Method” IEEE Magnetics Society, 02 September 2016.
[10] Waxler R, Cleek G, Malitson I, Dodge M and Hahn T 1971 Journal of Research of the National Bureau of Standards Section A: Physics and Chemistry 75A 163, 1971.
[11] Mawatari K, Shimizu H and Kitamori T, 2014 Encyclopedia of Microuidics and Nanouidics (Springer US) pp 1-9, 2014.

[12] Singh L, Srivastava A and Sarkate A J, 4th IEEE Uttar Pradesh Section International Conference on Electrical, Computer and Electronics (UPCON) (IEEE), 2017.

[13] Eichler HJ, Haase A, Menzel R and Siemoneit A, Thermal lensing and depolarization in a highly pumped Nd:YAG laser amplifier, Journal of Physics D: Applied Physics, Volume 26, Number 11 (1884), 1993.

[14] Ming-Ying Hsu, Wei-Cheng Lin, Chih-Wen Chen, Ting-Ming Huang, Chia-Yen Chan, Kun-Huan Wu and Shenq-Ysong Chan, Stress Optical Path Difference Analysis of Off-Axis Lens Ray Trace Foot-Print, Conference Paper in Proceedings of SPIE - The International Society for Optical Engineering 8769:87692K · June 2013.

[15] Lohmann A.W., Principles of Optical Computing. In: Martellucci S., Chester A.N. (eds) Nonlinear Optics and Optical Computing. Ettore Majorana International Science Series, vol 49. Springer, Boston, MA, ISBN: 978-1-4612-7900-6, 1990.

[16] J Turley and G Sines, The anisotropy of Young's modulus, shear modulus and Poisson's ratio in cubic materials, Journal of Physics D: Applied Physics, Volume 4, Number 2, 1971.

[17] Champagne James A., Crowther Blake G and Burge James H, Thermo-opto-mechanical analysis of a cubesat lens mount, Proceedings of SPIE - The International Society for Optical Engineering · September 2011.

[18] Rogers, P. “Athermalization of IR Optical Systems, Proceedings Volume 10260, Infrared Optical Design and Fabrication: A Critical Review; 102600F (1991).

[19] Rogers Philip J, Athermalized FLIR optics Proc. SPIE 1354, Athermalized FLIR optics, 0000 (1 January 1991).

[20] Yasuhisa Tamagawa and Toru Tajime, Dual-band optical systems with a projective athermal chart: design, OSA Publication, Applied Optics, Vol 36, Issue-1, pp 297-301, 1997.