CP Test in the W Pair Production
via Photon Fusion at NLC

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Abstract:
We study the possibility to test CP invariance in the $W^+W^-$ production via photon fusion at NLC. The predictions of the CP violation effects are made within two Higgs doublet extensions of the minimal standard model, where CP violation is introduced by a neutral Higgs exchange in s channel in our case. The width effect in the Higgs propagator on the CP violation effects is studied in detail. The CP violation effects can be measured in some parameter region of the extensions.
1. Introduction

The $W$ pair production at $e^+e^-$ colliders is an important process, in which the interactions between the weak gauge bosons in the Standard Model can be studied. But, as the c.m.s. energy of an $e^+e^-$ collider increases, the cross section for $e^+ + e^- \rightarrow W^+ + W^-$ decreases as expected from gauge invariance. At NLC with $\sqrt{s} = 500$GeV, the cross section $\sigma(e^+e^- \rightarrow W^+W^-)$ is only about 7pb. The idea of using Compton(back) scattering of laser light to obtain $\gamma\gamma$ collisions at NLC with approximately the same energies and luminosities[1], brings new possibilities to the study of couplings between the gauge bosons, because at higher energy more $W$ pairs are produced via photon fusion than via $e^+e^-$ collision. This has recently attracted attention to the study of CP conserving anomalous couplings between the gauge bosons in $\gamma + \gamma \rightarrow W^+ + W^-$[2,3]. In this work we study the possibility of testing CP invariance in $W$ pair production via photon fusion at NLC.

As is well known, CP violation was found 30 years ago, but it is still not known which form of interaction is responsible for CP violation. The KM matrix[4] provides one possibility. The fact that CP violation has been observed only in the $K$ meson system makes it difficult to determine the form of the CP violating interactions. $W$ pair production in $\gamma\gamma$ collisions gives a new possibility of testing CP invariance, and thus to get more information about the interaction responsible for CP violation.

In Sect. 2 we will discuss how to make CP tests in the process $\gamma + \gamma \rightarrow W^+ + W^-$ at NLC, where the polarization of the initial photons is not known. We take account of the experimental situation and our CP odd observables are constructed in such a way that they are directly measurable in experiment without reconstruction of the W rest frames and the centre of momentum frame of the initial pair of photons. In Sect. 3 we will consider the particular case of two Higgs doublet models, in which CP violation is caused by neutral Higgs exchange. In Sect. 4 we give numerical results for the observables we proposed in Sect. 2 and a summary.
2. CP Constraints and CP odd Observables

We consider the following process in the c.m.s. of the initial state:

$$\gamma(p_1) + \gamma(p_2) \rightarrow W^+(k_1) + W^-(k_2)$$

(2.1)

and the amplitude for the process (2.1) is:

$$T_{fi} = \varepsilon_{\nu_1}(p_1)\varepsilon_{\nu_2}(p_2)\varepsilon^*_{\mu_1}(k_1)\varepsilon^*_{\mu_2}(k_2)A^{\mu_1\nu_2\mu_2\nu_2}(p_1, p_2, k_1, k_2)$$

(2.2)

We assume that the polarization of the initial photons is not known. Then, a CP test in (2.1) is possible only if the polarizations of the $W^+$ or $W^-$ are observed. To obtain information about the polarizations we consider the leptonic decay of the $W$ bosons. That means, we actually consider the process:

$$\gamma(p_1) + \gamma(p_2) \rightarrow W^+(k_1) + W^-(k_2) \rightarrow \ell^+(q_1) + \ell^-(q_2) + \text{neutrinos}$$

(2.3)

For the decay process $W^+(k_1) \rightarrow \ell^+(q_1) + \nu$ with a moving $W^+$ a covariant decay matrix $\rho_{\mu\nu}^+(k_1, q_1)$ can be defined and is normalized as:

$$\frac{1}{4\pi} k_1^0 \int \frac{d\Omega_1}{(1 - \beta \hat{k}_1 \cdot \hat{q}_1)^2} \rho_{\mu\nu}^+(k_1, q_1) = (-g_{\mu\nu} + \frac{k_1^\mu k_1^\nu}{M_W^2})$$

(2.4)

where

$$\hat{k}_1 = \frac{k_1}{|k_1|}, \quad \hat{q}_1 = \frac{q_1}{|q_1|}, \quad \beta = \frac{|k_1^0|}{k_1^0}$$

(2.5)

and $\Omega_1$ is the solid angle of $q_1$. At tree-level in the SM the decay matrix $\rho_{\mu\nu}^+(k_1, q_1)$ takes the form:

$$\rho_{\mu\nu}^+(k_1, q_1) = \frac{3}{2} (-M_W^2 g_{\mu\nu} + 2(k_1^\mu q_1^\nu + k_1^\nu q_1^\mu) - 4q_1^\mu q_1^\nu + 2\varepsilon_{\mu\nu\alpha\beta} k_1^\alpha q_1^\beta)$$

(2.6)

Similarly, one can also obtain the covariant decay matrix $\rho_{\mu\nu}^-(k_2, q_2)$ for $W^-(k_2) \rightarrow \ell^-(q_2) + \bar{\nu}$. The probability for the process (2.3) may be written in terms of the decay matrices as:

$$R(p_1, k_1, q_1, q_2) = \frac{1}{4} A^{\nu_1\nu_2\mu_1\mu_2}(p_1, p_2, k_1, k_2) A^{*\nu_1\nu_2\mu_1\mu_2}(p_1, p_2, k_1, k_2) \cdot \rho_{\mu_1\mu_1'}^+(k_1, q_1) \rho_{\mu_2\mu_2'}^-(k_2, q_2)$$

(2.7)
If CP invariance holds, the following relation holds for $R$

$$R(p_1, k_1, q_1, q_2) = R(p_1, k_1, -q_2, -q_1). \tag{2.8}$$

The expectation value of any observable $O$, which is a function of $p_1, k_1, q_1$ and $q_2$ can be obtained:

$$< O > = \frac{1}{N} \int f_\gamma(x_1) f_\gamma(x_2) \frac{dx_1 dx_2}{x_1 x_2} \frac{\beta}{2(4\pi)^2} \int d\Omega \cdot \frac{1}{4\pi} (k_1^0)^{-2} \int \frac{d\Omega_1}{(1 - \beta k_1 \cdot \hat{q}_1)^2} \frac{1}{4\pi} (k_1^0)^{-2} \int \frac{d\Omega_2}{(1 + \beta \hat{k}_1 \cdot \hat{q}_2)^2} \cdot O \cdot R(p_1, k_1, q_1, q_2) \tag{2.9}$$

Here, $d\Omega(d\Omega_2)$ is the solid angle of $k_1(q_2)$. The normalization factor $N$ is defined so that $< 1 > = 1$. The distribution function $f_\gamma(x)$ gives the proportion of the photons with the fraction $x$ of the energy carried by the electron or positron. In the expression (2.9) any experimental cuts can be included, but for our purpose they must be CP blind.

The momenta $p_1, k_1, q_1$ and $q_2$ are not directly measurable. Due to the missing neutrinos and the lack of knowledge about the c.m.s. of the initial state they can never be known completely in the experiment. To construct CP odd observables, we will use the lepton momenta, which are directly measured in experiment and are related to $q_1$ and $q_2$ through a Lorentz boost. We denote these momenta with $q_+ = (E_+, q_+)$ and $q_- = (E_-, q_-)$ for the lepton $\ell^+$ and $\ell^-$ repectively. We construct the following CP odd observables:

$$O_1 = \frac{E_+ - E_-}{M_W}$$

$$O_2 = (\hat{p} \cdot \hat{q}_+)^2 - (\hat{p} \cdot \hat{q}_-)^2 \tag{2.10}$$

$$O_3 = \hat{p} \cdot (\hat{q}_+ - \hat{q}_-) \hat{p} \cdot (\hat{q}_+ \times \hat{q}_-)$$

with

$$\hat{q}_+ = \frac{q_+}{|q_+|}, \hat{q}_- = \frac{q_-}{|q_-|} \tag{2.11}$$

and the vector $\hat{p}$ is the direction of motion of the electron or positron. Because of the Bose symmetry of the two photon initial state the expectation value of any observable which
is odd in $\hat{p}$, is zero. With these observables one can also define the corresponding CP asymmetries:

$$A_i = \frac{N(O_i > 0) - N(O_i < 0)}{N(O_i > 0) + N(O_i < 0)} \quad (i = 1, 2, 3) \quad (2.12)$$

Where $N(O_i > 0)(N(O_i < 0))$ denotes the number of events with $O_i > 0(\ O_i < 0)$. Any nonzero $< O_i >$ or any nonzero $A_i$ indicates CP violation. Further, the observables $O_1$ and $O_2$ are CPT odd, the expectation values of them and the corresponding asymmetries can be nonzero only if an absorbtive part of the amplitude for (2.1) and CP violation exist.

### 3. CP Violation in Two Higgs–Doublet Models

In the process we are studying the effect of CP violation from the minimal standard model is zero up to two loop level at least and is thus too small to be observed. We consider two Higgs–doublet extensions of the minimal standard model. In these extensions CP violation is due to the complex expectation values of the Higgs-doublets. However, in general CP violation occurs together with the presence of flavour changing neutral currents at the tree level. The flavour changing neutral currents can be eliminated by imposing some discrete symmetry on the Lagrangian. This discrete symmetry is softly broken in the Higgs potential[5]. In this way it is possible to construct models with CP violation due to the complex expectation value of the Higgs doublets in absence of the flavour changing neutral currents.

The effect of CP violation from these extensions has been studied in top quark decay[6] and in the $t\bar{t}$ system produced at $pp$ colliders[7] or at $e^+e^-$ [8,9] colliders, where CP asymmetries can be large as $10^{-3}$. CP violation in the interactions between the gauge bosons is also studied in [10] and [11]. The process considered here gives another opportunity to study CP violation in such extensions.

In the extensions mentioned above, CP violation in the process discussed in Sect. 2 can be obtained at one loop level. At this level there is only one type of diagram which
introduces CP violation. This type of diagram is shown in Fig. 1, where the loop is a fermionic loop. CP violation is caused by the couplings between $\bar{f}i\gamma_5f$ and neutral Higgs fields, and the heaviest fermion is dominant. In the following we will take only the top quark into account and employ the notation used in [10] and [11]. In this notation CP violation due to the neutral Higgs exchange is parameterized with a $3 \times 3$ real orthogonal matrix $d$, the nonzero off diagonal matrix elements $d_{3j}$ and $d_{j3} (j = 1, 2)$ indicating CP violation. The couplings involved in Fig. 1 are:

$$L = e \frac{m_t}{2 M_W \sin \theta_W} \text{ctg} \beta d_{3j} \phi_j \bar{t}i\gamma_5t - i \frac{2}{3} e \bar{t} \gamma_{\mu} t A^{\mu}$$

$$+ e \frac{M_W}{\sin \theta_W} d_{1j} \phi_j W_{\mu}^+ W^{-\mu}$$

where $\phi_j (j = 1, 2, 3)$ are the mass eigenstates of the neutral Higgs fields, $\text{ctg} \beta = v_2/v_1$ is the ratio of the absolute expectation values of the two Higgs-doublets. We assume that the $\phi_1$ is the lightest Higgs particle and is dominant in Fig. 1. From (3.1) and Fig. 1 we obtain the CP violating amplitude $T_A$:

$$T_A = \frac{16}{3} \frac{\alpha^2}{\sin \theta_W^2} \text{ctg} \beta d_{11} d_{31} I(\sqrt{\hat{s}}) D_H(\hat{s}, M_H)$$

$\cdot \varepsilon^{\nu_1}(p_1) \varepsilon^{\nu_2}(p_2) \varepsilon^*_{\mu_1}(k_1) \varepsilon^*_{\mu_2}(k_2) g^{\mu_1\mu_2} \varepsilon_{\nu_1} \varepsilon_{\nu_2} \alpha \beta P_1^\alpha P_2^\beta$

with

$$\hat{s} = (p_1 + p_2)^2,$$

$$I(z) = \begin{cases} \frac{1}{2z} \left( \arcsin(z) \right)^2, & \text{for } z \leq 1 \\ \frac{1}{2z} \left( \frac{\pi}{2} + \frac{1}{2} \ln \frac{z + \sqrt{z^2 - 1}}{z - \sqrt{z^2 - 1}} \right)^2, & \text{for } z > 1 \end{cases}$$

Here $M_H$ stands for the mass of $\phi_1$ and $D_H$ is its propagator which is discussed further in Sect. 4. The CP violating part of the quantity $R$ defined in (2.7) is then obtained through the interference between $T_A$ and the amplitude for (2.1) at the tree-level from the standard model.

In (3.2) the coupling parameters $\text{ctg} \beta$ and $d_{11}d_{13}$ are unknown. From the upper bound of the electric dipole moment of the neutron one can not obtain enough information to constrain $d_{11}d_{13}$. From the fact that the $d$ is a $3 \times 3$ real orthogonal matrix an upper bound
can be derived:

\[ d_{11}d_{13} \leq \frac{1}{2} \]  

(3.4)

As to the ratio \( \text{ctg} \beta \), certain discrete symmetries may lead to the so called ”natural choice”, \( \text{ctg} \beta < 1 \), based on the observation that \( m_t >> m_b \). However, not all discrete symmetries, which can be imposed on the theory to eliminate the flavour changing neutral currents at tree level, lead to \( \text{ctg} \beta < 1 \), for example, the models I and III listed in [8] allow \( \text{ctg} \beta > 1 \). It is possible that the ratio \( \text{ctg} \beta \) may be larger than one.

It should be pointed out that CP violation in these extensions of the SM can also be studied in the single Higgs production via photon fusion, if the polarization of the initials photons is observed and the Higgs is light enough to be produced. A detailed analysis in this case was made in [12].

4. The numerical Results and the Summary

Before presenting our numerical results for the observables we defined above we give a detailed discussion on the Higgs propagator. As is well known, the absorptive part in the propagator may lead to some significant effect, although the absorptive part usually comes from higher order. For \( M_H < 2M_W \) we neglect this part in our calculation since it can be expected that this part is too small to give significant contributions to our observables. Therefore, we take the propagator \( D_H \) to have the form:

\[ D_H(\hat{s}, M_H) = \frac{1}{\hat{s} - M_H^2}, \quad \text{for} \ M_H < 2M_W \]  

(4.1)

In this case, the absorptive(dispersive) part of the amplitude \( T_A \) in (3.2) corresponds to \( \text{Im}I(z)(\text{Re}I(z)) \).

For \( M_H > 2M_W \), the absorptive part in the \( D_H \) does lead to significant effects in our observables. We parametrize the propagator in Breit–Wigner approximation in term of
the total decay width $\Gamma_H$:

$$D_H(\hat{s}, M_H) = \frac{(\hat{s} - M_H^2) - i\Gamma_H M_H}{(\hat{s} - M_H^2)^2 + \Gamma_H^2 M_H^2}, \quad \text{for } M_H > 2M_W \quad (4.2)$$

and the absorptive (dispersive) part of the amplitude $T_A$ is corresponding to $\text{Im}(I(z) \cdot D_H)$ ($\text{Re}(I(z) \cdot D_H)$). Since the $\Gamma_H$ in the two Higgs-doublet extensions depends on the unknown mass $M_H$ as well as the unknown coupling parameters $d_{ij}$ and $\tan \beta$($\cot \beta$), our observables in general have a complicated dependence on the unknown coupling parameters. However, for the small $\Gamma_H$ the expression in (4.2) can be approximated by:

$$D_H(\hat{s}, M_H) = \frac{1}{\hat{s} - M_H^2} - i\pi \delta(\hat{s} - M_H^2) \quad (4.3)$$

We used the both expressions for the $D_H$ in our numerical calculations of our CP odd observables. By varying $\Gamma_H$ from 0 to 40GeV we find that the expression in (4.3) is a good approximation for our observables. However, for $M_H > 2m_t \quad \Gamma_H$ can be very large because of the large mass $M_H$ and the new decay channel. For such large values of $\Gamma_H$, for example, for $\Gamma_H > 100\text{GeV}$, some numerical results of our observables can be changed in order of 50% compared with them by small $\Gamma_H$. Keeping this in mind, we present only our results calculated with the small $\Gamma_H$ and in this case the expectation value of the CP odd observables or the CP asymmetries are proportional to the product of $d_{11}d_{31}$ and $\cot \beta$.

We take the photon distribution function given in [1], where we assume that the laser energy is 1.26eV and the $e-\gamma$ conversion factor is one. To simulate experimental cuts, we select for measuring the CP violation effect only these events, in which the lepton energy $E_{+(-)}$ is larger than 10GeV and the angle of the outgoing leptons with respect to the electron or positron beam direction is not smaller than $10^0$. Using $\sqrt{s} = 500\text{GeV}$ and $m_t = 150\text{GeV}$ we have for different Higgs mass $M_H$ the following numerical results:

For $M_H = 100\text{GeV}$:

$$A_1 = 1.36 \times 10^{-4} d_{11} d_{31} \cot \beta, \quad < O_1 > = 1.46 \times 10^{-4} d_{11} d_{31} \cot \beta$$

$$A_3 = 1.68 \times 10^{-4} d_{11} d_{31} \cot \beta, \quad < O_3 > = 6.5 \times 10^{-5} d_{11} d_{31} \cot \beta \quad (4.4)$$
For $M_H = 200\text{GeV}$:

\begin{align}
A_1 &= -2.82 \times 10^{-4} d_{11} d_{31} \cot \beta, \quad < O_1 > = -3.1 \times 10^{-5} d_{11} d_{31} \cot \beta \\
A_3 &= 2.35 \times 10^{-4} d_{11} d_{31} \cot \beta, \quad < O_3 > = 1.05 \times 10^{-4} d_{11} d_{31} \cot \beta
\end{align}

(4.5)

For $M_H = 350\text{GeV}$:

\begin{align}
A_1 &= -7.65 \times 10^{-4} d_{11} d_{31} \cot \beta, \quad < O_1 > = -7.36 \times 10^{-4} d_{11} d_{31} \cot \beta \\
A_3 &= 3.45 \times 10^{-4} d_{11} d_{31} \cot \beta, \quad < O_3 > = 1.17 \times 10^{-4} d_{11} d_{31} \cot \beta
\end{align}

(4.6)

For $M_H = 500\text{GeV}$:

\begin{align}
A_1 &= -1.35 \times 10^{-4} d_{11} d_{31} \cot \beta, \quad < O_1 > = -1.53 \times 10^{-4} d_{11} d_{31} \cot \beta \\
A_3 &= -1.27 \times 10^{-4} d_{11} d_{31} \cot \beta, \quad < O_3 > = -4.48 \times 10^{-5} d_{11} d_{31} \cot \beta
\end{align}

(4.7)

We do not present the results for $< O_2 >$ and $A_2$ because they are one order of magnitude smaller than the results for $< O_1 >$ and $A_1$. To determine the sensitivity of our CP odd observables we also calculated the variances of them and the cross section for the process (2.3) at NLC. Using the standard model at tree level and taking the leptonic branching ratio($\approx 33\%$) of the $W$ decay into account, we have under the conditions mentioned above:

\[ \sigma = 4\text{pb}, \quad < O_2^2 > = 0.57, \quad < O_3^2 > = 0.096 \]

(4.8)

Note that the variance for an asymmetry is identically 1. Assuming the luminosity per year at NLC to be $10\text{fb}$, the number of the available events is about $4 \cdot 10^4$. We obtain then the statistical errors for our observables:

\[ \delta A_1 = \delta A_3 = \sqrt{\frac{1}{N_{\text{event}}}} = 0.5\%, \quad \delta O_1 = \sqrt{\frac{< O_1^2 >}{N_{\text{event}}}} = 0.4\%, \quad \delta O_3 = \sqrt{\frac{< O_3^2 >}{N_{\text{event}}}} = 0.15\% \]

(4.9)

CP violation is detectable only if the $< O_i >$ or $A_i (i = 1, 2, 3)$ are at least larger than their statistical error. Taking $< O_1 >$ at $M_H = 350\text{GeV}$ as an example, the product $d_{11} d_{31} \cot \beta$ should be larger than 5.4.

To summarize: in this work we studied the possibility of detecting CP violation in $\gamma \gamma \to W^+ W^-$ at NLC, our CP odd observables and the corresponding CP asymmetries
are constructed with the directly measured energies and momenta of the leptons from the $W$ decay. For the observables we propose one can detect CP violation without requiring complete knowledge about the c.m.s. of the initial photons and about the rest frame of the $W$ bosons. Therefore, our observables are easy to measure. The prediction of the CP violation effects is worked out for two Higgs doublet models. The effect of the Higgs width is studied and it is significant. If the Higgs sector of these models is not CP invariant, then CP violation can be measured with our CP odd observables in some parameter region.

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Figure Caption

Fig.1. One of the two Feynman graphs for the CP violating amplitude. The other one is to obtain through interchanging the two photons.