An insured person who needs money because he/she is suffering from a terminal illness that requires costly diagnosis and treatment has two possibilities of obtaining money from life insurance. He/she can lapse the policy, and the insurer is obliged to pay the cash surrender value of the policy (CSV). Another possibility for the insured is to sell the policy on the secondary market of life insurance (the viatical market). In this situation, he/she obtains the viatical settlement payment (VSP) from the investor, which is greater than the cash surrender value and less than the death benefit \( c \). Then the viatical company takes a fee for the insurance premiums, and in the case of the death of the insured receives the death benefit (see Figure 1).

A significant issue of the financial analysis of the viatical contract is the analysis of the investor’s expected profit. The relevant elements of the set of the contract terms (VSP, insurance premiums, type of fatal illness) allow determining the range of profit, which is taken into account in designing the viatical contract. The aim was to study the influence of the moment of falling ill (compared to the age and sex of the insured and duration of insurance) on the range of the expected profit of the viatical company.

The authors assumed that \( x \)-years old person bought the whole-life insurance and the insurance period is \( \omega – x \), where \( \omega \) denotes the limit age of the insured. The policyholder is obliged to pay annual premiums \( (p) \) at the beginning of the first \( m \) years of the insurance period \( (m \leq \omega – x) \).
The authors assumed that the insured becomes terminally ill in the $k$-th year of the insurance period ($k = 1, 2, ..., \omega - x$). This means that at the age of $x + k$, he/she could resell the whole or a part of the insurance policy. Note that the insured can resell the partial rights to the death benefit, thus $\alpha \in [0, 1]$ denotes the so-called resale parameter. If $\alpha = 0$, then the viatical contract has not been completed. If $0 < \alpha < 1$, the partial resale of rights to the death benefit are realized. For $\alpha = 1$, the investor bought the rights to the whole death benefit. To model viatical contracts, the paper used the multiple state model presented in [Dębicka, Heilpern 2017].

Let us start from the valuation of viatical settlement payment because the investor's profit depends on the $VSP(k)$ which he/she is willing to pay the policyholder for the repurchase of rights to the death benefits. Note that the cash surrender value in the whole insurance period $CSV(k)$ is defined as the percentage of mathematical net premium reserve created for future benefits $V(k)$. We assume that the surrender value is equal to the net premium reserve (i.e. $CSV(k) = (k)$). In practice $CSV(k) < V(k)$ which allows to estimate the minimum viatical settlement payments, therefore $V(k) < VSP(k)$ (the insured condition). Let $EB_\alpha(k) = aEB(k)$ denote the investor’s expected benefit at the beginning of $k$-th year if the policyholder at that moment resells $\alpha$ percentage of his/her rights to the death benefit. Moreover, let $investor's expected cost of premiums EC_\alpha(k) = aEC(k)$ be the expected value of premiums accumulated at moment $k$, which the investor will pay on condition that the insured is alive. Taking into consideration the investor’s costs and benefits, one can estimate the maximum of VSP as $VSP(k) < EB(k) + EC(k)$ (the investor condition), thus obtaining.

**Fig. 1.** A diagram of the primary and secondary insurance market

Source: based on [Kim 2009].
The investor’s expected profit \( EP_\alpha(k) \) at the beginning of \( k \)-th year is the difference between the expected benefits, and the sum of the expected cost of premiums and the amount to be paid for the repurchase of the rights to the death benefit or part of them

\[
EP_\alpha(k) = EB_\alpha(k) + EC_\alpha(k) - \alpha VSP(k).
\]  (2)

Applying (1) to (2) we obtain the range of investor’s expected profits

\[
0 < E_\alpha P(k) < \alpha \left( EB(k) + EC(k) - V(k) \right).
\]  (3)

Note that actuarial values for insurance contract (premiums and reserves) are calculated for different interest rates and life tables, then investor’s costs and benefits for viatical contracts.

**Example.** Let us consider the whole insurance contract for a 20-year old man (\( x = 20 \)). Assuming that \( \omega = 100 \) (the insurance period is equal to 80 years), we take into consideration three scenarios of paying premiums: the premium is paid over the whole insurance period (\( m = 80 \)), up to 80 years of the insured age (\( m = 60 \)) and up to the approximate age of retirement (\( m = 40 \)). We assume that death benefit is equal to 1 (\( c = 1 \)). For each scenario, the premiums determined based on Polish Life Tables 2008 and calculated under the assumption that the interest rate is constant and equals 1% are as follows

\[
p = \begin{cases} 
0.014771 & \text{for } m = 80 \\
0.015275 & \text{for } m = 60 \\
0.018902 & \text{for } m = 40 
\end{cases}.
\]

Based on the multiple state model presented in [Dębicka, Zmyślona 2018] and [Dębicka, Zmyślona 2019], the authors chose lung cancer as a terminal illness that could be the cause of reselling the insurance. To calculate the investor’s costs and benefits, the paper used the multiple increment-decrement tables presented in [Dębicka, Zmyślona 2016]. It was assumed that the investor’s interest rate is constant and equals 2% and insured would like to resell the whole rights to the death benefit (\( \alpha = 1 \)). Under these assumptions, the range of viatical settlement payment (c.f. (1)) and the corresponding range of the investor’s expected profit (c.f. (3)) are presented in Figure 2.

Note that the moment of becoming ill is always random, and the value of viatical settlement payment depends mostly on the duration of the insurance contract at the moment of being affected by the serious illness. In particular, at the lower limit of the range the viatical settlement payment...
Fig. 2. Ranges of viatical settlement payment and the investor's expected profit for the whole life insurance

Source: [Dębicka, Heilpern 2017].

is higher if the period of payment of contributions is shorter, but the upper limit increases when all the required insurance premiums have been paid. The size of VSP directly affects the investor’s expected profit, and if the investor is forced to offer a higher amount to the insured, it reduces the expected profits that the investor can achieve. In practice, the shorter the insurance contract lasts, the smaller the amount of money to repurchase the right to death benefits may be offered by the investor, which means the higher expected profits for him/her.

Detailed research on the influence of the moment of falling ill (compared to the age and sex of the insured and duration of insurance) on the investor’s expected profit are given in [Dębicka, Heilpern 2017].
The presented results were also used to optimize the profitability of resale rights to death benefits from the point of view of the insured [Dębicka, Heilpern 2020].

Acknowledgements

The support of the grant scheme NON-STANDARD MULTILIFE INSURANCE PRODUCTS WITH DEPENDENCE BETWEEN INSURED 2013/09/B/HS4/00490 is gratefully acknowledged.

References

Dębicka J., Heilpern S. (2017), Investor’s expected profit from viatical settlement. 20th AMSE. Applications of Mathematics in Economics. International Scientific Conference: International Scientific Conference: Szklarska Poręba, Poland, 30 Aug-3 Sept 2017. Conference Proceedings.

Dębicka J., Heilpern S. (2020), The optimization of insurance contracts on the viatical market – in press.

Dębicka J., Zmyślona B. (2016), Construction of multi-state life tables for critical illness insurance – influence of age and sex on the incidence of health inequalities. Silesian Statistical Review, no. 14, pp. 41-63.

Dębicka J., Zmyślona B. (2018), Premium valuation for a multiple state model containing manifold premium-paid states. CEJEME, 10, 27-52.

Dębicka J., Zmyślona B. (2019), Modelling of lung cancer survival data for critical illness insurances. Statistical Methods and Applications, vol. 28, no. 4, pp. 723-747.

Kim H.S. 2009, Life settlement in the US, Research report in KIDI.

Keywords: viatical market, life insurance, multistate model.