Off-shell persistence of composite pions and kaons

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In order for a Sullivan-like process to provide reliable access to a meson target as t becomes spacelike, the pole associated with that meson should remain the dominant feature of the quark-antiquark scattering matrix and the wave function describing the related correlation must evolve slowly and smoothly. Using continuum methods for the strong-interaction bound-state problem, we explore and delineate the circumstances under which these conditions are satisfied: for the pion, this requires \(-t \lesssim 0.6\,\text{GeV}^2\), whereas \(-t \lesssim 0.9\,\text{GeV}^2\) will suffice for the kaon. These results should prove useful in planning and evaluating the potential of numerous experiments at existing and proposed facilities.

1. Introduction. The notion that a nucleon possesses a meson cloud is not new [1]. In effect, this feature is kindred to the dressing of an electron by virtual photons in quantum electrodynamics [2] or the existence of dressed quarks with a running mass generated by a cloud of gluons in quantum chromodynamics (QCD) [3–7]. Naturally, any statement that each nucleon is accompanied by a meson cloud is only meaningful if observable consequences can be derived therefrom. A first such suggestion is canvassed in Ref. [8], which indicates, e.g. that a calculable fraction of the nucleon's anti-quark distribution is generated by its meson cloud. Mirroring this effect, one may argue that a nucleon's meson cloud can be exploited as a target and thus, for instance, the so-called Sullivan processes can provide a means by which to gain access to the pion’s elastic electromagnetic form factor [9–13], Fig. 1(a), and also its valence-quark parton distribution functions (PDFs) [14–16], Fig. 1(b).

One issue in using the Sullivan process as a tool for accessing a “pion target” is that the mesons in a nucleon’s cloud are virtual (off-shell) particles. This concept is readily understood when such particles are elementary fields, e.g. photons, quarks, gluons. However, providing a unique definition of an off-shell bound-state in quantum field theory is problematic.

Physically, for both form factor and PDF extractions, \(t < 0\) in Figs. 1, so the total momentum of the \(\pi^*\) is spacelike.\(^1\) Therefore, in order to maximise the true-pion content in any measurement, kinematic configurations are chosen in order to minimise \(|-t|\). This is necessary but not sufficient to ensure the data obtained thereby are representative of the physical pion. Additional procedures are needed in order to suppress non-resonant (non-pion) background contributions; and modern experiments and proposals make excellent use of, e.g. longitudinal-transverse cross-section separation and low-momentum tagging of the outgoing nucleon.

Notwithstanding their ingenuity, such experimental techniques cannot directly address the following question: supposing it is sensible to speak of an off-shell pion with total-momentum \(P\), where \(P^2 = (v - 1)m^2_\pi\), \(m_\pi \approx 0.14\,\text{GeV}\), so that \(v \geq 0\) defines the pion’s virtuality, then how do the qualities of this system depend on \(v\)? If the sensitivity is weak, then \(\pi^*(v)\) is a good surrogate for the physical pion; but if the distributions of, e.g. charge or partons, change significantly with \(v\), then the processes in Figs. 1 can reveal little about the physical pion. Instead, they express features of the entire compound reaction. Since there is no unique definition of an off-shell bound-state, the question we have posed does not have a precise answer. However, as will become clear, that does not mean there is no rational response.

2. Pions: on- and off-shell. All correlations with pion-like quantum numbers, both resonant and continuum, are accessible via the inhomogeneous pseudoscalar Bethe-

\(^1\) We use a Euclidean metric: \(\{\gamma_{\mu},\gamma_{\nu}\} = 2\delta_{\mu\nu}; \gamma_\gamma = \gamma^4\gamma^1\gamma^2\gamma^3, \quad tr[\gamma_{\mu}\gamma_{\nu}\gamma_{\rho}\gamma_{\sigma}] = -4\varepsilon_{\mu\nu\rho\sigma}; \quad \sigma_{\mu\nu} = (i/2)[\gamma_{\mu},\gamma_{\nu}]; \quad a \cdot b = \sum_{i=1}^{4} a_i b_i; \quad \text{and} \quad P^\mu_{\mu} \text{ spacelike} \Rightarrow P^2 > 0.\)
S主意 Bethe-Salpeter equation:
\[ \Gamma_5(k; P) = Z_4\gamma_5 + \int_{dq}^{\Lambda} [\chi_5(q; P)]_{sr} K^{rs}_4(q; k; P), \]
(1)
where \( \chi_5(q; P) = S(q_0)\Gamma_5(q; P)S(q_0), \) \( q_0 = q + \eta P, \)
\( q = q - (1 - \eta)P, \) \( P \) is the total quark-antiquark momentum;
\( \int_{dq}^{\Lambda} \) represents a Poincaré invariant regularisation
of the four-dimensional integral, with \( \Lambda \) the regularisation
mass-scale; and \( Z_4(\zeta^2, \Lambda^2) \) is the mass renormalisation
constant, with \( \zeta \) the renormalisation point. In addition,
\( S \) is the dressed-propagator for a u- or d-quark (we assume
isospin symmetry throughout), \( K \) is the quark-
antiquark scattering kernel, and the indices \( r,s,t,u \) denote
the matrix structure of the elements in the equation.

The physical \( (\nu = 0) \) pion appears as a pole in the
pseudoscalar vertex, \( v_is. \) [17]
\[ \Gamma_5(k; P) \rho^2 + m^2_\pi = 0 \Rightarrow \frac{\rho^2_\pi}{\rho^2 + m^2_\pi} \Gamma_\pi(k; P) + \text{reg.}, \]
(2)
where “reg.” denotes terms analytic on \( \nu m^2_\pi \simeq 0, \)
\[ \Gamma_\pi(k; P) = \gamma_5 [iE_\pi(k; P) + \gamma \cdot P F_\pi(k; P)]
+ \gamma \cdot k \cdot P G_\pi(k; P) + \sigma_{\mu\nu} k_\mu P_\nu H_\pi(k; P). \]
(3)
is the pion’s Bethe-Salpeter amplitude and \( \rho_\pi \) measures
the ratio of the in-pion condensate and the pion’s leptonic
decay constant [18].

In proposing reactions like those in Fig. 1 as paths to
real-pion targets, one is naively thought to assume that
for some nonzero and sizeable \( v_is, \) the pion pole remains
the dominant feature of the pseudoscalar vertex and the
pion’s wave function is “frozen”:
\[ \Gamma_5(k; P) \rho^2 + m^2_\pi \simeq 0 \Rightarrow \frac{\rho^2_\pi}{\rho^2 + m^2_\pi} \Gamma_\pi(k; P). \]
(4)

With modern methods of experiment and analysis, however,
the reactions in Figs. 1 provide sound realisations of a pion target under softer assumptions; namely, the pole associated with the ground-state pion remains the
dominant feature of the vertex (equivalently, the quark-
antiquark scattering matrix) and the Bethe-Salpeter-like amplitude
describing the related correlation evolves slowly and smoothly with virtuality. Under these conditions,
then \( \forall \nu < v_is \) a judicious extrapolation of a cross-
section to \( \nu = 0 \) will yield a valid estimate of the desired
on-shell result. The question posed in the Introduction
may now be translated into the challenge of determining
the value of \( v_is \) for which these conditions are satisfied.

To address this issue, we consider the following modified
Bethe-Salpeter equation [19]:
\[ \Gamma_5(k; P) = Z_4\gamma_5 + \lambda(\nu) \int_{dq}^{\Lambda} [\chi_5(q; P)]_{sr} K^{rs}_4(q; k; P), \]
(5)
because the quantity \( \delta(\nu) := [\lambda(\nu) - 1] \) can rigorously be
said to measure deviations induced by nonzero pion virtuality. Namely, given any value of \( P^2 = (\nu - 1)m^2_\pi, \)
there is a unique value \( \lambda(\nu) \) for which Eq. (5) exhibits an (off-shell) pion pole at \( (\nu - 1)m^2_\pi. \) Subsequently, a comparison
between the Bethe-Salpeter amplitude obtained at that pole and the \( \nu = 0 \) amplitude will reveal the nature of (any) changes in the internal structure of the associated correlation.\(^2\) The value of \( v_is \) is the boundary of the \( \nu-domain \) for which any such modifications are modest. (Here, “modest” means that all quantitative measures of structural change evolve slowly and smoothly with \( \nu. \)

Notably, since the equation describing the pole’s residue,
\( \text{i.e.} \) the related homogeneous Bethe-Salpeter equation, is
the same in any channel that possesses overlap with the
pion, then for the purpose of elucidating the character of
an off-shell pion, it suffices completely to consider Eq. (5).

3. Computed properties of an off-shell pion. Hitherto,
there are neither ambiguities nor model assumptions,
and the character of an off-shell pion can be assessed
by any nonperturbative approach that provides
access to the solution of Eq. (5). We choose to approach
the problem using methods developed for the continuum
bound-state problem [28–31].

The kernel of Eq. (5) involves the dressed light-quark propagators, so it is coupled with the light-quark gap equation. The problem can therefore be analysed by using a symmetry-preserving truncation of this pair of equations. A systematic scheme is described in Refs. [32–34]; and the leading-order term is the widely-used rainbow-ladder (RL) truncation. It is known to be capable of delivering a good description of \( \pi- \) and \( K-mesons \) [28–31], for example, because corrections in these
can be largely negligible owing to the preservation of relevant
Ward-Green-Takahashi identities.

A more realistic description is provided by the class of
symmetry-preserving DB kernels [35], \( \text{i.e.} \) dynamical chiral
symmetry breaking (DcsB) improved kernels, which shrink the gap between nonperturbative continuum-QCD
and the \textit{ab initio} prediction of bound-state properties
[36–38]. A basic difference between the two is that DB
kernels produce a smoother transition between the weak-
and strong-coupling domains of QCD, something that is
expressed in mesons, \( \text{e.g.} \) via softer leading-twist parton
distribution amplitudes (PDAs) [39–41]. Having made
the distinctions clear, we now note that the RL truncation
is adequate herein because we aim to explore con-
trasts between bound-state properties off- and on-shell,
and differences between RL and DB results will largely
cancel in such ratios.

In RL truncation, the relevant gap- and Bethe-Salpeter equations are \( (\rho = k \cdot q, T_{\mu\nu}(p) = \delta_{\mu\nu} - p_\mu p_\nu/p^2) \) [42–44]:
where $Z_2$ is the quark wave function renormalisation; and

$$\Gamma_5(k; P) = Z_4 \gamma_5 - \lambda(v) Z_2^2 \int d^3 q \overline{G}(p^2) T_{\mu \nu}(p) \frac{\lambda^a}{2} \gamma_\mu \chi_5(q; P) \frac{\lambda^a}{2} \gamma_\nu.$$  

Eqs. (6), (7) are complete once the process-independent running interaction is specified; and we use [44, 45]

$$\overline{G}(s) = \frac{8\pi^2}{\omega^5} \zeta^3 e^{-s/\omega^2} + \frac{8\pi^2 \gamma_m}{\ln[\pi + (1 + s/\Lambda_{QCD}^2)]^2},$$  

where $\gamma_m = 12/25$, $\Lambda_{QCD} = 0.234 \text{ GeV}$; $\tau = e^2 - 1$ $\mathcal{F}(s) = \{1 - \exp(-s/[4m_t^2])\}/s$, $m_t = 0.5 \text{ GeV}$; $\zeta = 0.8 \text{ GeV}$, $\omega = m_t$; and a renormalisation scale $\zeta = \Xi_{19} = 19 \text{ GeV}$ [42]. The connection between Eq. (8) and QCD’s gauge sector is canvassed elsewhere [36–38]. Here we only note that Eq. (8) has the correct shape but is too large in the infrared, for reasons that are well understood. Notwithstanding this, used judiciously in RL truncation, Eq. (8) serves as a valuable tool for hadron physics phenomenology. (Notably, for a wide range of observables, Eq. (8) produces results that are practically equivalent to those computed using earlier parametrisations [42, 46].)

Solving Eq. (6) for the dressed propagator, $S(k) = 1/\gamma \cdot k A(k^2) + B(k^2)$, is now straightforward; and, with the solution in hand, the kernel of Eq. (7) is fully determined. Thus, using $m_\pi = 3.4 \text{ MeV}$, at the on-shell point, $\lambda(v = 0) = 1$, we obtain [45]: $m_\pi = 0.134 \text{ GeV}$, $f_\pi = 0.093 \text{ GeV}$ in fair agreement with experiment [47].

With this foundation, we can begin to explore the persistence of piconic characteristics as one takes the correlation off-shell. To that end, in Fig. 2 (upper panel) we depict the $v$-dependence of the virtuality eigenvalue: the result is linear on $v \lesssim 45$,

$$\lambda(v) = 1 + 0.016 v,$$  

i.e. the change in $\lambda(v)$ is purely kinematic and, hence, the pion pole dominates the quark-antiquark scattering kernel $\forall v < 45$.

The next issue to address is if/how the internal structure of the correlation is modified. A detailed picture of possible rearrangements of the pion’s internal structure can be obtained by studying the impact of $v > 0$ on the scalar functions in Eq. (3). This is illustrated in Fig. 2 (lower panel), which depicts the $k^2$-dependence of the ratio of the leading Chebyshev moment for one of the ultraviolet (UV) dominant amplitudes in Eq. (3), where for any function that leading moment is ($x = k \cdot P/\sqrt{k^2 P^2}$):

$$\mathcal{W}(k^2; P^2) = \frac{2}{\pi} \int_{-1}^{1} dx \sqrt{1 - x^2} \mathcal{W}(k^2, x; P^2).$$  

The evolution pattern of the correlation’s internal structure is more subtle than that of $\lambda(v)$. Notwithstanding that, we find that structural modifications are significant $\forall v > 45$. Moreover, there is a measure of ambiguity in demarcating the domain within which structural changes can be considered modest. We therefore choose conservatively and identify $v_S \approx 31$, since on the domain $v \lesssim v_S$ the pattern exhibited by the ratios in Fig. 2 is both simple and readily interpreted. Namely, on $k^2 \lesssim 1 \text{ GeV}^2$, i.e. at length-scales $\xi \gtrsim 0.2 \text{ fm}$, the impact of $v \neq 0$ on the pion’s internal structure is modest, even at $v = 31$. The domain $k^2 \in [1, 4] \text{ GeV}^2$ is a smooth region of transition into the UV. Then, on $k^2 \gtrsim 4 \text{ GeV}^2$, viz. for $\xi \lesssim 0.1 \text{ fm}$, one observes plateaux, which describe nearly constant shifts in the amplitudes. The magnitude of the shifts grows with $v$ and that growth is linear to within 3.5%.

The UV tail of the pion’s Bethe-Salpeter amplitude maps algebraically into a $v$-dependence of $\rho_\pi^L$ in Eq. (2):

$$i \rho_\pi^L(v) = Z_4 \text{ tr}_{\text{CD}} \int d^3 q \gamma_5 \chi(q^2, q \cdot P; v),$$  

where $\chi_\pi = S(q_0) \Gamma_\pi(q^2, q \cdot P; v) S(q_0)$ and the trace is over colour and spinor indices, because the value of the integral in Eq. (11) is determined by the ultraviolet be-
where it behaves as an asymptotic form. Therefore, we can fit the form factor with the following ansatz:

\[ f_\pi(v) p_\mu = Z_4 \text{tr}_{\text{CD}} \int_{q^2} A(q^2, q \cdot P; v). \quad (12) \]

One can now form the product \( \kappa_q^2(v) := f_\pi(v) r^a_q(v) \), which is a quark-antiquark core density for the correlation, an in-pion condensate \[18\], whose growth with virtuality is depicted in Fig. 3. Unsurprisingly, given the preceding observations, \( \kappa_q^2(v) \) grows approximately linearly with virtuality on \( v \leq v_0 \):

\[ \kappa_q^2(v) \approx \kappa_q^2(0)[1 + 0.032v], \quad \kappa_q^2(0) = (0.28 \text{GeV}^3). \quad (13) \]

The picture that emerges, therefore, is an off-shell pion whose internal structure is essentially unaltered at length-scales \( \ell_\pi \geq 0.1 \text{ fm} \). On the other hand, at the core (\( \ell_\pi \lesssim 0.1 \text{ fm} \)) the quark-antiquark density increases slowly with virtuality, reaching a value at \( v = 31 \) which is roughly twice that of the on-shell pion, in line with expectations based upon the plateau in Fig. 2. (A linear fit to \( \kappa_q^2(v) \) on \( v \in [0, 55] \) is a poor representation of the result: the rms-difference is greater than 10% and it underestimates \( \kappa_q^2(0) \) by 40%.)

As evident in Fig. 1, only one pion is off-shell when using the Sullivan process to generate a hadron target. Consequently, the modest structural changes described above enter linearly in the scattering amplitudes. Their impact is illustrated in Fig. 4, which depicts the \( \pi^\star(v) + \gamma \rightarrow \pi \) transition form factor, \( F_\pi(Q^2, v) \). Using the “brute force” algorithm employed in Ref. [49] (to compute the propagators, Bethe-Salpeter amplitudes, photon-quark vertex, and scattering amplitude) yields the curves drawn in the upper panel of Fig. 4. Those curves terminate at \( Q^2 = 4 \text{ GeV}^2 \) because the algorithm is unreliable at larger momenta.

To complete the calculation of \( F_\pi(Q^2, v) \) directly at arbitrarily large spacelike \( Q^2 \), it would be necessary to use the method introduced in Ref. [50], i.e., develop a new perturbation theory integral representation for the Bethe-Salpeter amplitude at each required value of the virtuality. That is straightforward but time consuming, so we employ a simpler expedient. Namely, we capitalise on the analysis in Ref. [50], which shows that the computed elastic pion form factor can accurately be interpolated by a monopole multiplied by a simple factor that restores the correct QCD anomalous dimension. We therefore write

\[
F_\pi(Q^2, v) = \frac{1}{1 + Q^2/m_0^2} A(Q^2, v) \quad (14a)
\]

\[
A(Q^2, v) = \frac{1 + Q^2 a_0(v)}{1 + Q^2 [a_0^2(v) / b_0(v)] \ln(1 + Q^2 / \Lambda_{QCD}^2)} \quad (14b)
\]

where \( m_0 = 0.72 \text{ GeV} \) (i.e., the \( \rho \)-meson mass computed using this framework [44]) is fixed by the elastic pion form factor, and \( a_0(v), b_0(v) \) are fitted to the behaviour of \( F_\pi(Q^2, v) \) on \( Q^2 \in [0, 4] \text{ GeV}^2 \):

\[
a_0(v) = 0.29(1 + 0.028 v), \quad (15a)
\]

\[
b_0(v) = 2.3(1 + 0.017 v). \quad (15b)
\]

The lower panel depicts a collection of such constrained extrapolations. Pointwise comparison with Fig. 2 in Ref. [50] demonstrates the veracity of Eq. (14) for \( v = 0 \).

An important feature of the transition form factor is highlighted by the lower panel of Fig. 4, viz. once again,
In the region of low momenta, the form factor is dominated by a monopole, while in the region of high momenta, it behaves as an asymptotic form. Therefore, we can fit the form factor with the following ansatz:

$$Q^2 F_\pi(Q^2) \approx N_{\text{QCD}}^2 \frac{\pi}{16 \pi \alpha_s(Q^2)} f_\pi^2 w_\pi^2,$$

where

$$w_\pi = \frac{1}{3} \int_0^1 dx \frac{1}{x} \varphi_\pi(x),$$

and

$$\varphi_\pi(x)$$ is the pion’s twist-two valence-quark PDA. Contemporary analyses demonstrate that ground-state meson PDAs are well represented by $\varphi(x) = N_e^m [\pi(1-x)]^p$, where $N_e$ ensures $\int_0^1 dx \varphi(x) = 1$. Moreover, when the consistently-computed PDA is used, Eq. (16) underestimates the direct RL calculation by only 15% on $Q^2 \approx 8$ GeV$^2$. One may therefore equate Eq. (16) with 85% of the UV limit of Eq. (14) and infer $p$. This procedure yields $p(v = 0) = 0.29$, to be compared with $p = 0.30$ in Ref. [50], thereby confirming its validity and also the remark following Eqs. (15)\(^3\). For $v > 0$, Eq. (16) receives minor modifications: $f_\pi^2 \to f_\pi f_\pi(v)$ and $w_\pi^2 \to w_\pi w_\pi(v)$, where $\varphi(x; v)$ is a PDA for the off-shell pion. Using the revised formula in the matching procedure and assuming the offset remains at 15%, then $p(v = 31) = 0.105$. This inferred virtuality-dependence of the PDA is depicted in Fig. 5: the dilation grows modestly with increasing $v$. Such a connection between the UV behaviour of the pion’s Bethe-Salpeter amplitude and dilation of the PDA is readily verified using a simple generalisation of the algebraic model introduced in Ref. [39].

At this point, we use generalised parton distributions (GPDs) to translate the behaviour of $F_\pi(Q^2, v)$ into insights regarding the impact of virtuality on extractions of the pion’s valence-quark PDF via the process in Fig. 1(b). In particular, recall that the elastic form factor can be written [58–60]:

$$F_\pi(Q^2) = \int_{-1}^{1} dx H_{\pi^+}^u(x, 0, Q^2),$$

$$u^\pi(x) = H_{\pi^+}^u(x > 0, 0, 0),$$

where $H_{\pi^+}^u(x, 0, Q^2)$ is the pion’s GPD and $u^\pi(x)$ is its valence-quark distribution function. Notably, too, at a typical hadronic scale [61]:

$$H_{\pi^+}^u(x, 0, Q^2) \approx (1-x)^2, \quad \forall Q^2 < \infty.$$ (18)

\(^3\) Direct comparison is meaningful because Ref. [50] neglected evolution of the pion’s Bethe-Salpeter wave function, whose role and importance is discussed in Refs. [56, 57].

Hence, considering a half off-shell generalisation of the GPD, which may be accomplished following Ref. [62], using a matrix element defined with an initial state corresponding to the lowest-mass pole solution of Eq. (5), and given the modest $v$-dependence of $F_\pi^2(Q^2, v)$, Eqs. (17), (18) indicate that $u^\pi(x; v)$ will behave similarly. In particular, the power-law describing its decay on $x \approx 1$ should not depend strongly on $v$.

4. Conclusion. One can define and explore the properties of an off-shell pion by introducing a virtuality eigenvalue, $\lambda(v)$, into the Bethe-Salpeter equations describing the formation of bound-states and correlations in scattering channels that overlap with the pion. The pion pole dominates the scattering matrix so long as $\lambda(v)$ is linear in the virtuality, $v$. Within this linearity domain, alterations of the pion’s internal structure induced by $v > 0$ can be analysed by charting the $v$-dependence of the pointwise behaviour of the Bethe-Salpeter amplitude describing the correlation. Following this procedure, we demonstrated that for $v < v_S = 31$, which corresponds to $-t < 0.6$ GeV$^2$ in the notation of Fig. 1, the off-shell correlation serves as a valid pion target. Namely, on this domain the properties of the off-shell correlation are simply related to those of the on-shell pion and, consequently, a judicious extrapolation to $v = 0$ will deliver reliable results for pion properties.

In the present context it is natural to ask for a similar statement concerning the kaon. We have addressed this issue by repeating the analysis described herein for a fictitious $s + \bar{s}$ pseudoscalar bound-state. Using a $s$-quark current-mass that produces the empirical $\phi$-meson mass [45], we obtain $m_{s\bar{s}0} = 0.7$ GeV and find $v_{s\bar{s}0}^2 = 2.7$ (units of $m_{s\bar{s}0}^2$). Interpolating to the kaon mass, we estimate that an off-shell correlation in this channel can serve as a valid meson target on $-t < 0.9$ GeV$^2$.\(^{15}\)
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