Cooperative Binning for Semi-deterministic Channels with Non-causal State Information

Ido B. Gattegno, Student Member, IEEE, Haim H. Permuter, Senior Member, IEEE, Shlomo Shamai (Shitz), Fellow, IEEE and Ayfer Özgür, Member, IEEE

Abstract

The capacity of the semi-deterministic relay channel (SD-RC) with non-causal channel state information (CSI) only at the encoder and decoder is characterized. The capacity is achieved by a scheme based on cooperative-bin-forward. This scheme allows cooperation between the transmitter and the relay without the need to decode a part of the message by the relay. The transmission is divided into blocks and each deterministic output of the channel (observed by the relay) is mapped to a bin. The bin index is used by the encoder and the relay to choose the cooperation codeword in the next transmission block. In causal settings the cooperation is independent of the state. In non-causal settings dependency between the relay’s transmission and the state can increase the transmission rates. The encoder implicitly conveys partial state information to the relay. In particular, it uses the states of the next block and selects a cooperation codeword accordingly and the relay transmission depends on the cooperation codeword and therefore also on the states. We also consider the multiple access channel with partial cribbing as a semi-deterministic channel. The capacity region of this channel with non-causal CSI is achieved by the new scheme. Examining the result in several cases, we introduce a new problem of a point-to-point (PTP) channel where the state is provided to the transmitter by a state encoder. Interestingly, even though the CSI is also available at the receiver, we provide an example which shows that the capacity with non-causal CSI at the state encoder is strictly larger than the capacity with causal CSI.

Index Terms

Cooperative-bin-forward, cooperation, cribbing, multiple-access channel, non-causal state information, random binning, relay channel, semi-deterministic channel, state encoder, wireless networks.

I. INTRODUCTION

Semi-deterministic models describe a variety of communication problems in which there exists a deterministic link between a transmitter and a receiver. This work focus on the semi-deterministic relay channel (SD-RC) and the multiple access channel (MAC) with partial cribbing encoders and non-causal channel state information (CSI) only at the encoder and decoder. The state of a channel may be governed by physical phenomena or by an interfering transmission over the channel, and the deterministic link may also be a function of this state.

The work of Ido B. Gattegno, Haim H. Permuter and Shlomo Shamai was supported by the Heron consortium via the minister of economy and science, and, by the ERC (European Research Council). The work of A. Özgür was supported in part by NSF grant #1514538.
The capacity of the relay channel was first studied by van der Muelen [1]. In the relay channel, an encoder receives a message, denoted by $M$, and sends it to a decoder over a channel with two outputs. A relay observes one of the channel outputs, denoted by $Z$, and uses past observations in order to help the encoder deliver the message. The decoder observes the other output, denoted by $Y$, and uses it to decode the message that was sent by the encoder. Cover and El-Gamal [2] established achievable rates for the general relay channel, using a partial-decode-forward scheme. If the channel is semi-deterministic (i.e. the output to the relay is a function of the channel inputs), El-Gamal and Aref [3] showed that this scheme achieves the capacity. Partial-decode-forward operates as follows: first, the transmission is divided into $B$ blocks, each of length $n$; in each block $b$ we send a message $M^{(b)}$, at rate $R$, that is independent of the messages in the other blocks. The message is split; after each transmission block, the relay decodes a part of the message and forwards it to the decoder in the next block using its transmission sequence. Since the encoder also knows the message, it can cooperate with the relay in the next block. The capacity of the SD-RC is given by maximizing $\min\{I(X, X_r; Y), H(Z|X_r) + I(X; Y|X_r, Z)\}$ over the joint probability mass function (PMF) $p_{X,X_r}$, where $X$ is the input from the encoder and $X_r$ is the input from the relay. The cooperation is expressed in the joint PMF, in which $X$ and $X_r$ are dependent. However, when the channel depends on a state that is unknown to the relay, the partial-decode-forward scheme is suboptimal [4], i.e., it does not achieve the capacity. The partial-decoding procedure at the relay is too restrictive since the relay is not aware of the channel state.

Focusing on state-dependent SD-RC, depicted in Fig. 1, we consider two situations: when the CSI is available in a causal or a non-causal manner. State-dependent relay channels were studied in [4]–[12]; Kolte et al. [4] derived the capacity of state-dependent SD-RC with causal CSI and introduced a cooperative-bin-forward coding scheme. It differs from partial-decode-forward as follows: the relay does not have to explicitly recover the message bits; instead, the encoder and relay agree on a map from the deterministic outputs space $Z^n$ to a bin index. This index is used by the relay to choose the next transmission sequence. Note that this cooperative-binning is independent of the state and, therefore, can be used by the relay. The encoder is also aware of this index (since the output is deterministic) and coordinates with the relay in the next block, despite the lack of state information at the relay. The capacity of this channel is given by maximizing $\min\{I(X, X_r; Y|S), H(Z|X_r, S) + I(X; Y|X_r, Z, S)\}$ over $p_{X,X_r}p_{X|r}$. Note that $X$ and $X_r$ are dependent, but $X_r$ and $S$ are not. When the state is known causally, a dependency between $X_r$ and $S$ is not feasible. At each time $i$, the encoder can send to the relay information about the states up to time $i$. The relay can use only strictly causal observations $Z^{i-1}$, which may contain information...
on $S_{i-1}$ but not on $S_i$. Furthermore, since the states are distributed independently, the past state at the relay does not help to increase the achievable rate.

The main contribution of this paper is to develop a variation of the cooperative-bin-forward scheme that accounts for non-causal CSI. While the former scheme allows cooperation, the new scheme also allows dependency between the relay’s transmission and the state. When the CSI is available in a non-causal manner, knowledge of the state at the relay is feasible and may increase the transmission rate. The encoder can perform a look-ahead operation and transmit to the relay information about the upcoming states. The relay can still agree with the encoder on a map, and in each transmission the encoder can choose carefully which index it causes the relay to see. The encoder chooses an index such that it reveals compressed state information to the relay, using an auxiliary cooperation codeword. Incorporating look-ahead operations with cooperative-binning increases the transmission rate and achieves capacity. This scheme can be used in other semi-deterministic models, such as the multiple access channel (MAC) with strictly causal partial cribbing and non-causal CSI.

The main contribution of this paper is to develop a variation of the cooperative-bin-forward scheme that accounts for non-causal CSI. While the former scheme allows cooperation, the new scheme also allows dependency between the relay’s transmission and the state. When the CSI is available in a non-causal manner, knowledge of the state at the relay is feasible and may increase the transmission rate. The encoder can perform a look-ahead operation and transmit to the relay information about the upcoming states. The relay can still agree with the encoder on a map, and in each transmission the encoder can choose carefully which index it causes the relay to see. The encoder chooses an index such that it reveals compressed state information to the relay, using an auxiliary cooperation codeword. Incorporating look-ahead operations with cooperative-binning increases the transmission rate and achieves capacity. This scheme can be used in other semi-deterministic models, such as the multiple access channel (MAC) with strictly causal partial cribbing and non-causal CSI.

The MAC with cooperation can also be viewed as a semi-deterministic model, due the deterministic cooperation link. MAC with conferencing, introduced by Willems in [13], consists of a rate-limited private link between two encoders. Permuter et al. [14] showed that for state-dependent MAC with conferencing, the capacity can be achieved by superposition coding and rate-splitting. The cribbing is a different type of cooperation, also introduced by Willems [15], in which one transmitter has access to (is cribbing) the transmission of the other. In [16], Simeone et al. considered cooperative wireless cellular systems and analyzed their performance with cribbing (referred to as In-Band cooperation). The results show how cribbing potentially increases the capacity. A generalization of the cribbing is partial and controlled cribbing, introduced by Asnani and Permuter in [17], when one encoder has limited access to the transmission sequence of the other. The cribbed information is a deterministic function of the transmission sequence. Kopetz et al. [18] characterized the capacity region of combined partial cribbing and conferencing MAC without states. When states are known causally at the first encoder (while the other is cribbing), Kolte et al. [4] derived the capacity, which is achieved by cooperative-bin-forward. We show that the variation of the cooperative-bin-forward scheme achieves the capacity when the states are known non-causally.

The results are examined for several special cases; the first is a point-to-point (PTP) channel where the CSI is available to the transmitter through a state encoder, and to the receiver. Former work on limited CSI was done by Rosenzweig et al. [19], where the link from the state encoder to the transmitter is rate-limited. Steinberg [20] derived the capacity of rate-limited state information at the receiver. In our setting, the link between the state encoder and the transmitter is not is not a rate-limited bit pipe, but a communication channel where the transmitter can observe the output of the state encoder in a causal fashion. We provide an example which illustrates that in this setting the capacity with non-causal CSI available at the state encoder is strictly larger than the capacity with causal CSI at the state encoder, even-though the receiver also has channel state information. This is somewhat surprising given that in a PTP channel the CSI at both the transmitter and receiver, causal and non-causal state information lead to the same capacity.

The remainder of the paper is organized follows. Problem definitions and capacity theorems are given in Section
\[ \text{Encoder} \xrightarrow{x_i(M, S^n)} \text{Relay} \xrightarrow{x_{r,i}(Z^{i-1})} \text{Decoder} \]

Fig. 2: SD-RC with non-causal CSI at encoder and decoder.

**II. Special cases are given in Section III and the new state-encoder problem and the example are given in Section IV. Proofs for theorems are given in Sections V, VI and VII. In Section IX we offer conclusions and final remarks.**

A. **Notation**

We use the following notation. Calligraphic letters denote discrete sets, e.g., \( X \). Lowercase letters, e.g., \( x \), represent variables. A vector of \( n \) variables \((x_1, \ldots, x_n)\) is denoted by \( x^n \). A substring of \( x^n \) is denoted by \( x_i^j \), and includes variables \((x_i, \ldots, x_j)\). Whenever the dimensions are clear from the context, the subscript is omitted. Let \((\Omega, \mathcal{F}, \mathbb{P})\) denote a probability space where \( \Omega \) is the sample space, \( \mathcal{F} \) is the \( \sigma \)-algebra and \( \mathbb{P} \) is the probability measure. Roman face letters denote events in the \( \sigma \)-algebra, e.g., \( A \in \mathcal{F} \). \( \mathbb{P}[A] \) is the probability assigned to \( A \), and \( \mathbb{I}[A] \) is the indicator function, i.e., indicates if event \( A \) has occurred. Random variables are denoted by uppercase letters, e.g., \( X \), and similar conventions apply for vectors. The probability mass function (PMF) of a random variable, \( X \), is denoted by \( p_X \). If \( x \in X \), then \( p_X(x) = \mathbb{P}[X = x] \). Whenever the random variable is clear from the context, we drop the subscript. Similarly, a joint distribution of \( X \) and \( Y \) is denoted by \( p_{X,Y} \) and a conditional PMF by \( p_{Y|X} \).

Whenever \( Y \) is a deterministic function of \( X \), we denote \( Y = f(X) \) and the conditional PMF by \( p_{Y|X} \). If \( X \) and \( Y \) are independent, we denote this as \( X \perp \perp Y \) which implies that \( p_{X,Y} = p_X p_Y \), and a Markov chain is denoted as \( X \leftrightarrow Y \leftrightarrow Z \) and implies that \( p_{X,Y,Z} = p_{X,Y} p_Z \).

An empirical mass function (EMF) is denoted by \( \nu(a|x^n) = \frac{1}{n} \sum_{i=1}^{n} \mathbb{I}[x_i = a] \). Sets of typical sequences are denoted by \( \mathcal{A}_t^{(n)}(p_X) \), which is a \( \epsilon \)-strongly typical set with respect to PMF \( p_X \), and defined by

\[
\mathcal{A}_t^{(n)}(p_X) \triangleq \left\{ x^n : |\nu(a|x^n) - p_X(a)| < \epsilon p_X(a), \forall a \in X \right\}.
\]  

Jointly typical sets satisfy the same definition with respect to (w.r.t.) the joint distribution and are denoted by \( \mathcal{A}_t^{(n)}(p_{X,Y}) \). Conditional typical sets are defined as

\[
\mathcal{A}_c^{(n)}(p_{X,Y}|y^n) \triangleq \left\{ x^n : (x^n, y^n) \in \mathcal{A}_t^{(n)}(p_{X,Y}) \right\}.
\]  

B. **Semi-Deterministic Relay Channel**

We begin with a state dependent SD-RC, depicted in Fig. 2. This channel depends on a state \( S_i \in S \), which is known non-causally to the encoder and decoder, but not to the relay. An encoder sends a message \( M \) to the
decoder through a channel with two outputs. The relay observes an output \( Z^n \) of the channel, which at time \( i \) is a deterministic function of the channel inputs, \( X_i \) and \( X_{r,i} \), and the state (i.e., \( Z_i = z(X_i, X_{r,i}, S_i) \)). Based on past observations \( Z^{i-1} \) the relay transmits \( X_{r,i} \) in order to assist the encoder. The decoder uses the state information and the channel output \( Y^n \) in order to estimate \( \hat{M} \). The channel is memoryless and characterized by the joint PMF 
\[
p_{Y,Z|X,X_r,S} = p_{Z|X,X_r,S} \cdot p_{Y|Z,X,X_r,S}
\]

**Definition 1 (Code for SD-RC)** A \((R, n)\) code \( C_n \) for the SD-RC is defined by
\[
x^n : \left[1:2^{nR}\right] \times \mathcal{S}^n \rightarrow \mathcal{X}^n
\]
\[
x_{r,i} : Z^{i-1} \rightarrow \mathcal{X}_r \quad 1 \leq i \leq n
\]
\[
\hat{m} : Y^n \times \mathcal{S}^n \rightarrow \left[1:2^{nR}\right]
\]

**Definition 2 (Achievable rate)** A rate \( R \) is said to be achievable if there exists \((R, n)\) such that
\[
P_e(C_n) \triangleq \mathbb{P}_{C_n}[\hat{m}(Y^n, S^n) \neq M] \leq \epsilon
\]
for any \( \epsilon > 0 \) and some sufficiently large \( n \).

The capacity is defined to be the supremum of all achievable rates.

**Theorem 1** The capacity of the SD-RC with non-causal CSI, depicted in Figure 2, is given by
\[
C = \max \min \left\{ I(X, X_r; Y|S), I(X; Y|X_r, Z, S, U) + H(Z|X_r, S, U) - I(U; S) \right\}
\]

where the maximum is over \( p_{U|S} p_{X_r|U} p_{X_r|X_r,U,S} \) such that \( I(U; S) \leq H(Z|X_r, S, U) \), where \( Z = z(X, X_r, S) \) and \( |U| \leq \min\{|S||X_r|, |S||Y| + 1\} \).

The proof for the theorem is given in Section V. Let us first investigate the capacity and the role of the auxiliary random variable \( U \). Here, the random variable \( U \) is used to create empirical coordination between the encoder, the relay and the states, i.e., with high probability \((S^n, U^n, X_r^n, X^n)\) are jointly typical w.r.t. \( p_{S,U,X_r,X} \). Note that the PMF factorizes as \( p_{U|S} p_{X_r|U} p_{X_r|X_r,U,S} \); the random variable \( X_r \), which represents the relay, depends on \( S \) through the random variable \( U \). This dependency represents the state knowledge at the relay, using an auxiliary codeword \( U^n \).

**C. Multiple Access Channel with Partial Cribbing**

Consider a MAC with partial cribbing and non-causal state information, as depicted in Figure 3 This channel depends on the state \((S_1, S_2)\) sequence that is known to the decoder, and each encoder \( w \in \{1, 2\} \) has non-causal access to one state component \( S_w \in S_w \). Each encoder \( w \) sends a message \( M_w \) over the channel. Encoder 2 is cribbing Encoder 1; the cribbing is strictly causal, partial and controlled by \( S_1 \). Namely, the cribbed signal at time \( i \), denoted by \( Z_i \), is a deterministic function of \( X_{1,i} \) and \( S_{1,i} \). The cribbed information is used by Encoder 2 to assist Encoder 1.
Fig. 3: State dependent MAC with two state components and one side cribbing. The cribbing is strictly causal – $X_2 = x_{2,i}(M_2, S^n_{2i}, Z_i^{i-1})$.

Fig. 4: State dependent MAC with two state components and one side cribbing. The cribbing is causal – $X_2 = x_{2,i}(M_2, S^n_{2i}, Z_i^i)$.

**Definition 3 (Code for MAC)** A $(R_1, R_2, n)$ code $C_n$ for the state-dependent MAC with strictly causal partial cribbing and two state components is defined by

\[
\begin{align*}
x_1^n &: [1 : 2^{nR_1}] \times S^n_1 \rightarrow X_1^n \\
x_{2,i} &: [1 : 2^{nR_2}] \times S^n_2 \times Z_i^{i-1} \rightarrow X_2 \\
m_1 &: Y^n \times S^n_1 \times S^n_2 \rightarrow [1 : 2^{nR_1}] \\
m_2 &: Y^n \times S^n_1 \times S^n_2 \rightarrow [1 : 2^{nR_2}]
\end{align*}
\]

for any $\epsilon > 0$ and some sufficiently large $n$.

**Definition 4 (Achievable rate-pair)** A rate-pair $(R_1, R_2)$ is achievable if there exists a code $C_n$ such that

\[
P_{\text{e}}(C_n) \triangleq P_{C_n}[\hat{m}_1(Y^n, S^n_1, S^n_2), \hat{m}_2(Y^n, S^n_1, S^n_2)] \neq (M_1, M_2) \leq \epsilon
\]

for any $\epsilon > 0$ and some sufficiently large $n$. 
The capacity region of this channel is defined to be the union of all achievable rate-pairs. We note here that a setup with causal cribbing, depicted in Fig. 4, satisfy a similar definition with $x_2 : [1 : 2^nR_2] \times S'_2 \times Z' \rightarrow X_2$.

**Theorem 2** The capacity region for discrete memoryless MAC with non-causal CSI and strictly causal cribbing in Fig. 3 is given by the set of rate pairs $(R_1, R_2)$ that satisfy

\[
 R_1 \leq I(X_1; Y | X_2, Z, S_1, S_2, U) + H(Z | S_1, U) - I(U; S_1 | S_2) \tag{5a}
\]

\[
 R_2 \leq I(X_2; Y | X_1, S_1, S_2, U) \tag{5b}
\]

\[
 R_1 + R_2 \leq I(X_1, X_2; Y | Z, S_1, S_2, U) + H(Z | S_1, U) - I(U; S_1 | S_2) \tag{5c}
\]

\[
 R_1 + R_2 \leq I(X_1, X_2; Y | S_1, S_2) \tag{5d}
\]

for PMFs of the form $p_{X_1, U | S_1} p_{X_2 | U, S_2}$, with $Z = z(X_1, S_1)$, that satisfies

\[
 I(U; S_1 | S_2) \leq H(Z | S_1, U), \tag{5e}
\]

and $|U| \leq \min \{ |S_2| |S_1| |X_1| |X_2| + 2, |S_1| |S_2| |Y| + 3 \}$.

**Theorem 3** The capacity region for discrete memoryless MAC with non-causal CSI and causal cribbing in Fig. 7 is given by the set of rate pairs $(R_1, R_2)$ that satisfy the equations in 5 for PMFs of the form $p_{X_1, U | S_1} p_{X_2 | Z, U, S_2}$.

We note here that when $S_2$ is degenerated, i.e., there is only one state component, the capacity region in both theorems is given by degenerating $S_2$. Note that the difference between Theorems 2 and 3 is conditioning on $Z$ in the PMF $p_{X_2 | Z, U, S_2}$. Here, the auxiliary random variable $U$ plays a double role. The first role is similar to the role in the SD-RC; it creates dependency between $X_2$ and $S_1$. This is done using a cooperation codeword $U^n$; Encoder 1 selects a codeword that is coordinated with the states. Encoder 2 uses this codeword in order to cooperate. Since the codeword depends on the state, so does $X_2^n$. When there are two state components, the second component is used by Encoder 2 to select the cooperation codeword from a collection. The second role is to generate a common message between the encoders.

In Section VI we provide proof for Theorem 2 when there is only one state component. The proof for the general case is given in Section VII and is based on the case with a single state component. The proof for Theorem 3 is given in Section VIII. In the following section we examine the results in cases which emphasize the role of $U$.

III. Special Cases

A. Cases of State-Dependent SD-RC

**Case 1: SD-RC without states:** When there is no state to the channel, i.e., the channel is fixed throughout the transmission, the capacity of SD-RC is given by Cover and El-Gamal as

\[
 \max_{p_{X_r, X}} \min \{ I(X, X_r; Y), I(X; Y | X_r, Z) + H(Z | X_r) \}. \tag{6}
\]
This case is captured by degenerating $S$. Then, $S$ can be omitted from the information terms in Theorem 1 and the joint PMF is $p_{U|X_1,Y|X_2}p_{X_1,X_2,Y|X_1,Y}1_{X_1,X_2,Y}$. Choosing $U = X_r$ recovers the capacity. Therefore, we see that here, $U$ plays the role of a common message between $X_r$ and $X$.

**Case 2: SD-RC with causal states** Consider a similar configuration to that in Fig. 2 and assume that the states are known to the encoder in a causal manner. Although this is not a special case of the non-causal configuration, it emphasizes the role of $U$ further. The capacity for this channel was characterized by Kolte et al [4, Theorem 2] by

$$
C = \max_{P_{X_r}, P_{X_r|S}} \min \{ I(X, X_r; Y|S), I(X; Y|X_r, Z, S) + H(Z|X_r, S) \} 
$$

(7)

where $Z = z(X, X_r, S)$. Let us compare this capacity to the one with non-causal states. In the causal case, we see that $X$ and $X_r$ are dependent, but $X_r$ and $S$ are not. In the non-causal case (eq. (4)), $X_r$ and $S$ are dependent. The random variable $U$ generates empirical coordination w.r.t. $P_{U|S}$, and then uses it as common side information at the encoder, relay and decoder. When the state is known causally, such dependency cannot be achieved since the states are drawn i.i.d. and the relay observes only past outputs of the channel. The capacity of the causal case is directly achievable by Theorem 1 by substituting $U = X_r$ and $X_r \perp \perp S$.

**B. Cases of State-Dependent MAC with Partial Cribbing**

Let us investigate the role of the auxiliary random variable $U$ in the MAC configuration via special cases of Theorem 2. We consider here the naive case of one state component, i.e., $S_2$ is degenerated. We denote $S \equiv S_1$ to emphasize this. Proofs for these cases are given in Appendix B.

**Case A: Multiple Access Channel with states (without cribbing):** Consider the case of a multiple access channel with CSI at Encoder 1 and the decoder, depicted in Fig. 5. It is a special case without cribbing (i.e., $z = \text{constant}$).

The capacity region, characterized by Jafar [21], is defined by all $(R_1, R_2)$ pairs that satisfy

$$
R_1 \leq I(X_1; Y|X_2, S) 
$$

(8a)

$$
R_2 \leq I(X_2; Y|X_1, S) 
$$

(8b)

$$
R_1 + R_2 \leq I(X_1, X_2; Y|S) 
$$

(8c)
with PMFs that factorize as $p_{X_1|S}p_{X_2}$.

**Case B: Multiple Access Channel with Conferencing:** Consider a case of MAC with conferencing, as depicted in Fig. 6. In this case, the channel depends only on part of $x_1$, which we denote by $x_{1c}$. The other part of $x_1$, denoted by $x_{1p}$, is known in a strictly causal manner to Encoder 2.

This setting is different from previous works, which considered a rate-limited cooperation. Here we use a sequence with noiseless communication and a fixed alphabet $X_1$. It turns out that the capacity region of the channel is the same for both a strictly causal and a non-causal cooperation link. The capacity of both cases when $X_{2,i} = x_{2,i}(M_2, X_1^{i-1})$ and $X_{2,i} = x_{2,i}(M_2, X_1^n)$ is

$$R_1 \leq I(X_{1c}; Y|U, S) + R_{12} - I(U; S)$$  \hspace{1cm} (9a)

$$R_2 \leq I(X_2; Y|X_{1c}, U, S)$$ \hspace{1cm} (9b)

$$R_1 + R_2 \leq \min \{I(X_{1c}, X_2; Y), I(X_{1c}, X_2; Y|U) + R_{12} - I(U; S)\}$$ \hspace{1cm} (9c)

$$R_{12} = \log_2 |X_1|$$ \hspace{1cm} (9d)

for $p_{U,X_{1c}S|Y}p_{X_2|U}p_{Y|X_{1c},X_2,S}$.

**Case C: Point-to-point with non-causal CSI:** Consider a configuration of a PTP channel with non-causal CSI, depicted in Fig. 7. This is a special case of the MAC when $R_2 = 0$ and $p_{Y_2|X_1,X_2,S} = p_{Y_2|X_1,S}$. The capacity of this channel was given by Wolfowitz [22, Theorem 4.6.1] as

$$C = \max_{p_{X_1|S}} I(X_1; Y|S).$$ \hspace{1cm} (10)
IV. POINT-TO-POINT WITH STATE ENCODER AND CAUSALITY CONSTRAINT

A. The State Encoder with a Causality Constraint

We introduce a new setting, depicted in Fig 8, of a PTP channel with a state encoder (SE) and a causality constraint. The SE has non-causal access to CSI and assists the encoder to increase the transmission rate. The causality constraint enforces the encoder to depend on past observations of the SE. This setting is attractive since it is a special case of the MAC, and similar settings may be special cases of more complicated models.

The setting is defined for two cases; one with non-causal CSI and the other with causal CSI. Explicitly, the setting with non-causal CSI is defined by a state encoder \( E_1 \) \( x_{1,i} : S^n \rightarrow X_1 \), an encoder \( E_2 \) \( x_{2,i} : [1 : 2^{nR_2}] \times X_1^{i-1} \rightarrow X_2 \) and a decoder (D). Note that the encoder depends on strictly causal information from the state encoder. The second setting, however, is defined slightly different. First, the state encoder depends on causal CSI, i.e., \( x_{1,i} : S_i \rightarrow X_1 \). Secondly, the encoder can use causal information from the state encoder and not strictly causal. Namely, \( x_{2,i} : [1 : 2^{nR_2}] \times X_1^i \rightarrow X_2 \). We will first discuss on the inclusion of the non-causal case in the MAC setting.

To apply the MAC with partial cribbing to this case, consider the following situation with only one state component. Encoder 1 has no access to the channel, i.e., \( p_{Y|X_1,X_2,S} = p_{Y|X_2,S} \), and no message to send (\( R_1 = 0 \)). Its only job is to assist Encoder 2 by compressing the CSI and sending it via a private link. The private link is the partial cribbing with \( z(x_1, s) = x_1 \). When the link between the encoders is non-causal, i.e., when \( x_{2,i} = f(M_2, X_1^n) \), using the characterization of Rosenzweig [19] with a rate limit of \( R_s = \log |X_1| \) yields

\[
C = \max_{p_U | S \in P_{X_2(U)}} I(X_2; Y | U, S). \tag{11}
\]

When there is a causality constraint, the transmission at time \( i \) can only depend on the strictly causal output of state encoder, i.e., \( x_{2,i} = f(M_2, X_1^{i-1}) \); nonetheless, the capacity remains.

Briefly explained, the capacity is achieved as follows. The transmission is divided to blocks (block-Markov coding). In each block, Encoder 1, which serves as the state encoder, sends a compressed version of the states of the next block. After each transmission block, Encoder 2 has a compressed version of the state of the current transmission block and uses it for coherent transmission.

---

Fig. 8: Comparison between causal and non-causal CSI.
B. An Example - Non-causal CSI Increases Capacity

The non-causal CSI in the MAC configuration does increase the capacity region in the general case. The following example proves this claim. Consider a model where the channel states are coded, as depicted in Fig. 8. Case (a) is a non-causal case, and (b) is causal. As we previously discussed, the channel in Fig. 8(a) is a special case of the non-causal state dependent MAC with partial cribbing. Similarly, Fig. 8(b) is a special case of causal state dependent MAC with partial cribbing [4].

Since this is a point-to-point configuration, it is a bit surprising that the non-causal CSI increases capacity; when the states are perfectly provided to the encoder, the capacity with causal CSI and with non-causal CSI coincide. As we will next show, in the causal case, the size of $X_1$ can enforce lossy quantization on the state, while in the non-causal case, the states can be losslessly compressed.

![Fig. 9: Example of a state dependent channel.](image)

For every channel $p_{Y|X_2,S}$ and states distribution $p_S$,

$$C_{nc} = \max_{p_U|S, p_{X_2|U}} I(X_2; Y|S, U), \quad C_c = \max_{1_{X_1|S, p_{X_2|X_1}}} I(X_2; Y|S, X_1)$$  \hspace{1cm} (12)

where $C_{nc}$ and $C_c$ are the capacity of non-causal and causal CSI configurations, respectively. Assume that the states distribution is

$$p_S(s) = \begin{cases} \frac{2}{3} & \text{if } s = 0, 1 \\ 1 - p & \text{if } s = 2. \end{cases}$$  \hspace{1cm} (13)

For each state there is a different channel; these channels are depicted in Fig. 9. A Z-channel for $s = 0$, an S-channel for $s = 1$, where both share the same parameter $\alpha$, and a noiseless channel for $s = 2$.

The idea is that when the CSI is known non-causally we can compress $S$ while in a causal case we cannot. Assume that $X_1$ is binary, and $p$ is small enough, for instance $p = 0.2$, such that

$$H(S) < \log_2 |X_1| = 1.$$  \hspace{1cm} (14)

Therefore, taking $U = S$ satisfies $I(U; S) = H(S) \leq 1$ and results in the non-causal capacity

$$C_{nc} = \frac{p}{2} (C_{Z-channel}(\alpha) + C_{S-channel}(\alpha)) + (1 - p)$$  \hspace{1cm} (15)

where

$$C_{Z-channel}(\alpha) = C_{S-channel}(\alpha) = H_b\left(\frac{2H_b(\alpha)/\bar{\alpha}}{1 + 2H_b(\alpha)/\bar{\alpha}}\right) - \frac{H_b(\alpha)/\bar{\alpha}}{1 + 2H_b(\alpha)/\bar{\alpha}}.$$  \hspace{1cm} (16)
TABLE I: Capacity of PTP with coded CSI - numerical evaluations for $p = 0.2$.

| $\alpha$ | No-CSI | Causal CSI | Non-causal CSI |
|----------|--------|------------|----------------|
| 0        | 1      | 1          | 1              |
| 0.5      | 0.8623 | 0.8633     | 0.8644         |
| 1        | 0.8    | 0.8        | 0.8            |

On the other hand, the capacity for causal CSI is

$$C_c = \max_\beta \left[ \frac{p}{2} C_{Z,\text{channel}}(\alpha) + \frac{p}{2} \left( H_b \left( \beta + \beta \alpha \right) - \beta H_b(\alpha) \right) + (1-p)H_b(\beta) \right].$$

(17)

The capacity can be achieve by one of several deterministic functions $x_1(S)$. Each function, maps both $S = 2$ and $S = 1/0$ to one letter, and $S = 0/1$ to the other letter, respectively. Note that this operation causes a lossy quantization of the CSI. For comparison, we also provide the capacity when there is no CSI at the encoder, which is

$$C_{\text{no-CSI}} = p \left( H_b \left( \frac{1 + \alpha}{2} \right) - 0.5H_b(\alpha) \right) + (1-p).$$

(18)

The capacity of the channels (non-causal, causal, no CSI) for $p = 0.2$ are summarized in Table I. There are two points where the three channels results in the same capacity. The first is when $\alpha = 0$; in this case, the channel is noiseless for $s = 0, 1, 2$ and the capacity is 1. There is no need for CSI at the encoder and, therefore, the capacity is the same (among the three cases). The second point is when $\alpha = 1$; the channel is stuck at 0 and stuck at 1 for $s = 0$ and $s = 1$, respectively, and noiseless for $s = 2$. In this case we can set $P_{X_1}(1) = 0.5$ for every $s$ and achieve the capacity. Therefore, the encoder does not use the CSI in those cases. However, for every $\alpha \in (0, 1)$, the capacity of the non-causal case is strictly larger than of the others, which confirms that non-causal CSI indeed increases the capacity region.

V. PROOF FOR THEOREM I

A. Direct

Before proving the achievability part, let us investigate important properties of the cooperative-bin-forward scheme. This scheme was derived by Kolte et al. [4] and is based on mapping the discrete finite space $Z^n$ to a range of indexes $L = [1 : 2^{nR_B}]$. We refer this function as cooperative-binning for two reasons: 1) it randomly maps $Z^n$ into $2^{nR_B}$ bins, and 2) the random binning is independent of all other random variables, which make 'suitable’ for cooperation. For instance, a sequence $z^n \in Z^n$ can be drawn given $v^n$, but its bin index is drawn uniformly, i.e., $\text{bin}(z^n) \sim \text{Unif}[1 : 2^{nR_B}]$, and is not a function of $v^n$. Thus, if we observe $z^n$ we can find $\text{bin}(z^n)$ without knowing $v^n$. This index is used to create cooperation between the encoder and a relay.
Lemma 1 (Indirect covering lemma) Let \( \{Z^n(k)\}_{k \in [1:2^nR]} \) be a collection of sequences, each sequence is drawn i.i.d according to \( \prod_{i=1}^n p_{Z|V}(z_i|v_i) \). For every \( z^n \in Z^n \), let Bin\((z^n) \) \( \sim \) Unif\([1:2^nR_B]\). For any \( \delta_1, \delta_2 > 0 \), if

\[
R < H(Z|V) - \delta_1
\]

\[
R < R_B - \delta_2,
\]

then,

\[
\lim_{n \to \infty} \mathbb{P}\left[ |\{l : \exists k \text{ s.t. } \text{Bin}(Z^n(k)) = l\}| < 2^{n(R-\delta_n)}|V^n = u^n \right] = 0
\]

where \( \delta_n \to 0 \) as \( n \to \infty \).

The proof for this lemma is given in Appendix A. Lemma 1 states that by choosing \( R < H(Z|V) - \delta_1 \) and \( R_B > R + \delta_2 \), we can guarantee (with high probability) that we will see approximately \( 2^{n(R-\Delta_n)} \) different bins indexes. Having these indexes allow us to assign to each one of them a sequence or threat them as bins (i.e. use the index to create a list). For instance, if we assign each index \( l \in [1:2^nR_B] \) a sequence \( u^n(l) \sim \prod_{i=1}^n p_U(u_i(l)) \), we can perform covering [23 Lemma 3.3] in order to create coordination with another sequence \( s^n \), by choosing \( R > I(U;S) + \delta \).

The coding scheme works as follows. Divide the transmission to \( B \) block and choose a distribution \( p_{X,U|S}p_{X,|S} \). Draw a codebook for each block \( b \) which consist of the followings. A cooperative-binning function (a map from \( Z^n \) to \( [1:2^nR_B] \), drawn uniformly), a collection of \( 2^{n(I(U;S)+\delta)} \) codewords \( z^{n(b)}(m^{(b)},k) \) for each \( m^{(b)} \in [1:2^nR'] \) indexed by \( k \in [1:2^n\tilde{R}] \), a sequence \( x^{n(b)}(m^{n(b)}) \) for each \( m^{n(b)} \in [1:2^nR''] \), cooperation codeword \( u^n(b)(l) \) and a relay codeword \( x^{n(b)}(m^{n(b)}(l)) \) for each \( l \in 2^nR_B \).

To send a message \( m^{(b)} \), recall that the link from the encoder to the relay is deterministic. Therefore, the Encoder can dictate which sequence the relay will observe during the block. Thus, it look at the collection of \( z^{n(b)} \) sequences and search for \( k \) s.t. \( z^{n(b)}(m^{(b)},k) \) points toward a cooperation codeword \( u^n \) that is coordinated (typical) with
The decoding procedure is done forward using a sliding window technique, derived by Carleial [24]. At each block \( b \), the decoder imitates the encoder procedure for every possible \( m^{(b)} \) and finds \( \hat{k}^{(b)}(m^{(b)}) \) and \( \hat{l}^{(b)}(m^{(b)}) \). To ensure that the mapping from \( (m^{(b)}, \hat{k}^{(b)}) \) to \( \hat{l}^{(b)} \) is unique, we take \( R' + \hat{R} < R_B \) and \( R' + \hat{R} < H(Z|X_r, U, S) \). Then, the decoder looks for \( (\hat{m}^{(b)}, \hat{m}'^{(b)}) \) such that: 1) all sequences at the current block are coordinated, and 2) \( (s^{(b+1)}_n, u^{(n)}(\hat{l}^{(b)}(\hat{m}^{(b)})), x^{(n)}_r(u^{(n)}(\hat{l}^{(b)}(\hat{m}^{(b)})))) \) are coordinated. Setting \( R'' < I(X;Y|Z, X_r, U, S) \) and \( R < I(X, X_r; Y|S) \) ensures reliability in the decoding procedure.

We will now give a formal proof for the achievability part. Fix a PMF \( p_{U|S|X_r,U|X_r,U,S} \) and let \( p_{Z|X_r,U,S} \) be such that \( p_{Z|X_r,U,S|X_r,U,S} = p_{X_r,U,S}1_{Z|X_r,U,S} \). We use block-Markov coding as follows. Divide the transmission into \( B \) blocks, each of length \( n \). At each communication block \( b \), we transmit a message \( M^{(b)} \) at rate \( R \). Each message \( M^{(b)} \) is divided to \( M''^{(b)} \) and \( M'''^{(b)} \), with corresponding rates \( R' \) and \( R'' \), respectively.

**Codebook:** For each block \( b \in [1 : B] \), a codebook \( C^{(b)} \) is generated as follows:

- **Binning:** Partition the set \( Z^n \) into \( 2^{nR_B} \) bins, by choosing uniformly and independently an index \( i^{(b)}(z^n) \sim U [1 : 2^{nR_B}] \).

- **Cooperation codewords:** Generate \( 2^{nR_B} \) \( u \)-codewords
  
  \[
  u^n(l^{(b-1)}) \sim \prod_{i=1}^{n} p_U(u_i), \quad l^{(b-1)} \in [1 : 2^{nR_B}] \tag{22a}
  \]

- **Relay codewords:** For each \( u^n \in U^n \) generate \( x_r^n \)-codeword \( x^n_r(u^n) \sim \prod_{i=1}^{n} p_{X_r|U}(x_r,i|u_i) \).

- **z-codewords:** For each \( u^n \in U^n \), \( x^n_r \in X^n_r \) and \( s^n \in S^n \), generate \( 2^{n(R' + \hat{R})} \) \( z \)-codewords

  \[
  z^n(m^{(b)}, k^{(b)}|x^n_r, u^n, s^n) \sim \prod_{i=1}^{n} p_{Z|X_r,U,S}(z_i, x_r,i, s_i), \quad m^{(b)} \in [1 : 2^{nR'}], \quad k^{(b)} \in [1 : 2^{n\hat{R}}] \tag{22b}
  \]

- **Transmission codewords:** For each \( z^n \in Z^n \), \( u^n \in U^n \), \( x^n_r \in X^n_r \) and \( s^n \in S^n \) draw \( 2^{nR''} \) \( x \)-codewords

  \[
  x^n(m''^{(b)}|z^n, x^n_r, u^n, s^n) \sim \prod_{i=1}^{n} p_{X|Z,X_r,U,S}(x_r|z_i, x_r,i, u_i, s_i), \quad m''^{(b)} \in [1 : 2^{nR''}] \tag{22c}
  \]

The block-codebook \( C^{(b)} \) consist of all the sequences that was generated for this block. Note that by this construction, all block-codebooks are independent of each other.

**Encoder:** Let \( l^{(0)} = m^{(1)} = m''^{(1)} = m^{(B)} = m'''^{(B)} = k^{(B)} = 1 \). This block prefix is done in order to begin the transmission with coordinated cooperation sequence, which is not yet known at the relay. Assume that \( l^{(b-1)} \) is known due to former operations at the encoder, and denote

\[
  z^n(m^{(b)}, k^{(b)}|l^{(b-1)}, s^{(b)}) = z^n(m^{(b)}, k^{(b)}|x^n_r(u^n(l^{(b-1)})), u^n(l^{(b-1)}), s^{(b)}), \tag{23}
  \]

\(^1\)For each \( z^n \) there is a bin index, and for each bin index there is an \( u^n \) sequence. Therefore, the covering is called indirect.
First, the encoder finds \( k^{(b)} \) such that
\[
\left( u^n(\text{bin}(z^n(m^{(b)})), s^{n(b+1)}) \right) \in \mathcal{A}^{(n)}(p_{S,U})
\] (24)
and sets \( l^{(b)} = \text{bin}(z^n(m^{(b)}, k^{(b)}|l^{(b-1)}, s^{n(b)})) \). Then, it sends
\[
x^n\left( m^{(b)}; z^n(m^{(b)}, k^{(b)}|l^{(b-1)}, s^{n(b)}), u^n(l^{(b-1)}), s^{n(b)} \right)
\]. (25)

We abbreviate the notation by \( x^n(m^{(b)}; m^{(b)}, k^{(b)}|l^{(b-1)}, s^{n(b)}) \).

**Relay:** Assume \( l^{(b-1)} \) is known. At block \( b \), send \( x^n(l^{(b-1)}) \). Denote this sequence by \( x^n(l^{(b-1)}) \). After the relay observes \( z^n(b) \), it determines \( l^{(b)} = \text{bin}(z^n(b)) \).

**Decoder:** We perform decoding using a sliding window; this is a decoding procedure that decodes from block 1 to \( B - 1 \), and therefore reduces the delay for recovering message bits at the decoder\(^2\). We start at block 2, since the first cooperation sequence is not necessarily typical with the states at that block. Moreover, since the first message is fixed, the decoder can imitate the encoder's operation and find \( l^{(1)} \).

Assume \( l^{(b-1)} \) is known due to previous decoding operations. At block \( b \), the decoder performs:

1. For each \( m^{(b)} \), look for \( \hat{k}^{(b)}(m^{(b)}, l^{(b-1)}, s^{n(b+1)}) \) and \( \hat{l}^{(b)}(m^{(b)}, l^{(b-1)}, s^{n(b)}, s^{n(b+1)}) \) the same way that the encoder does. We denote these indexes by \( \hat{k}^{(b)}(m^{(b)}) \) and \( \hat{l}^{(b)}(m^{(b)}) \).
2. Look for unique \( \hat{m}^{(b)}, \hat{m}^{(b)} \) such that (26) are satisfied.

**Analysis of error probability:** The code \( C_n \) is defined by the block-codebooks and the encoders and decoder functions. We bound the average probability of an error at block \( b \), conditioned on successful decoding in blocks

\[ \left( s^{n(b)}, u^n(l^{(b-1)}), x^n(l^{(b-1)}), z^n(\hat{m}^{(b)}, \hat{k}(\hat{m}^{(b)}|l^{(b-1)}, s^{n(b)}), x^n(\hat{m}^{(b)}|l^{(b-1)}, s^{n(b)}), y^n(b) \right) \] (26a)

\[ \in \mathcal{A}^{(n)}(ps.u.x., x.z.y) \]

\[ \left( s^{n(b+1)}, u^n(\hat{l}^{(b)}(m^{(b)})), x^n(\hat{l}^{(b)}(m^{(b)})), y^n(b+1) \right) \in \mathcal{A}^{(n)}(p_{S,U,X}, Y) \] (26b)
[1 : b − 1]. Without loss of generality we assume that \( M^{(b)} = 1 \) for each \( b \in [1 : B] \). Define the events

\[
E_1(b) = \left\{ \forall k^{(b)} : \left( U^n(B) (Z^n(1, k^{(b)} | L^{(b-1)}), S^{n(b+1)}) \notin A^{(n)} (P_{S,U}) \right) \right\}
\]

(27a)

\[
E_2(b) = \left\{ \exists m^{(b)} \neq 1 : B^{(b)} (Z^n(m^{(b)}, k^{(b)} | L^{(b-1)}, S^{n(b)}) = L^{(b)}, \text{ for some } k^{(b)} \right\}
\]

(27b)

\[
E_3(b) = \left\{ \text{Condition (27c) is not satisfied by } \left( \hat{m}^{(b)}, \hat{m}''^{(b)} \right) = (1, 1) \right\}
\]

(27c)

\[
E_4(b) = \left\{ \text{Condition (27d) is not satisfied by } \left( \hat{m}^{(b)}, \hat{m}''^{(b)} \right) \neq (1, 1) \right\}
\]

(27d)

\[
E_5(b) = \left\{ \hat{L}^{(b)} = L^{(b)} \right\}
\]

(27e)

\[
\hat{E}(b) = \bigcup_{j=1}^{b} \{ E_1(j) \cup E_2(j) \cup E_3(j) \cup E_4(j) \cup E_5(j) \}
\]

(27f)

(27g)

The average probability of an error is upper bounded by

\[
P_e^n = \mathbb{E}_{C_n} \left[ P_e^n (C_n) \right]
\]

(28a)

\[
\leq \mathbb{P} \left[ \hat{E}(B) \right]
\]

(28b)

\[
\leq \sum_{b=1}^{B} \left[ \mathbb{P} \left[ E_1(b) \mid E_5(b-1) \right] + \mathbb{P} \left[ E_2(b) \mid E_5(b-1) \right] + \mathbb{P} \left[ E_3(b) \mid E_5(b-1), E_5^c(b) \right] \right.
\]

(28c)

\[
\left. + \mathbb{P} \left[ E_4(b) \mid E_5(b-1), E_5^c(b), E_5^c(b) \right] + \mathbb{P} \left[ E_5(b) \mid E_5(b-1), E_5^c(b), E_5^c(b) \right] \right]
\]

(28d)

where the second inequality follows from union bound and conditioning. We will now investigate the probability of each event.

- Event \([E_1(b) \mid E_5(b-1)]\): By lemma[1] the probability of seeing less than \( 2^n (R - \Delta_u) \) different bins (indexed by \( l \)) goes to 0 if \( R < H(Z \mid S, U) - \delta_1 \) and \( R_B > R + \delta_2 \). Denote

\[
A = \left\{ \text{there are less than } 2^n (R - \Delta_u) \text{ different bin indexes} \right\}
\]

(29a)

\[
\mathcal{D} = \{ l : \exists k \text{ such that } \text{Bin}(Z^n(k)) = l \}
\]

(29b)

Assume that \( A \) and \( B \) are two events. Then \( \mathbb{P} [A \cup B] \leq \mathbb{P} [A] + \mathbb{P} [B | A^c] \).
Therefore,
\[ P[E_1(b)|E_5(b - 1)] \leq P[E_1(b)|E_5(b - 1), A^c] + P[A|E_5(b - 1)] \]
\[ = P\left[ \forall k, (U^n(Bin(Z^n(k))), S^n) \in A_e^{(n)}(P_{U,S}) \bigg| E_5(b - 1), A^c \right] + \epsilon'_n \]
\[ \leq P\left[ \bigcap_{l \in D} (U^n(l, S^n) \in A_{e}^{(n)}(P_{U,S}) \bigg| E_5(b - 1), A^c \right] + \epsilon'_n \]
\[ \leq \left( 1 - 2^{-n(I(U;S) + \delta' (\epsilon))} \right)^{2^{n(R - \Delta_n)}} + \epsilon'_n \]
\[ \leq \exp \left\{ -2^n(R - I(U;S) - \Delta_n - \delta'(\epsilon)) \right\} + \epsilon'_n \]
which tend to 0 when \( n \to \infty \) if \( \tilde{R} > I(U;S) + \Delta_n + \delta'(\epsilon) \).

- Event \([E_2(b)|E_5(b - 1)]: \) Denote \( Z^n(m^n(b), k(b)) = Z^n(m^n(b), k(b)|L^{b-1}, S^n(b)) \).

Consider
\[ P[E_2(b)|E_5(b - 1)] = P\left[ \exists m^n(b) \neq 1 : Bin(b)(Z^n(m^n(b), k(b))) = L^{(b)}, \text{ for some } k(b) \right] \]
\[ = P\left[ \exists m^n(b) \neq 1 : Bin(b)(Z^n(m^n(b), k(b))) = Bin(b)(Z^n(1, K^{(b)})), \text{ for some } k(b) \right] \]
\[ \overset{(a)}{\leq} \sum_{m^n(b) > 1, k(b)} P\left[ Bin(b)(Z^n(m^n(b), k(b))) = Bin(b)(Z^n(1, K^{(b)})) \right] \]
\[ \leq \sum_{m^n(b) > 1, k(b)} P\left[ Bin(b)(Z^n(m^n(b), k(b))) = Bin(b)(Z^n(1, K^{(b)})), Z^n(m^n(b), k(b)) = Z^n(1, K^{(b)}) \right] \]
\[ + \sum_{m^n(b) > 1, k(b)} P\left[ Bin(b)(Z^n(m^n(b), k(b))) = Bin(b)(Z^n(1, K^{(b)})), Z^n(m^n(b), k(b)) \neq Z^n(1, K^{(b)}) \right] \]
\[ \leq \sum_{m^n(b) > 1, k(b)} Z^n(m^n(b), k(b)) = Z^n(1, K^{(b)}) \]
\[ + \sum_{m^n(b) > 1, k(b)} P\left[ Bin(b)(Z^n(m^n(b), k(b))) = Bin(b)(Z^n(1, K^{(b)})) \bigg| Z^n(m^n(b), k(b)) \neq Z^n(1, K^{(b)}) \right] \]
\[ \leq 2^{n(R' + \tilde{R})} 2^{-n(H(Z|X_r, U, S) - \delta_1(\epsilon))} + 2^{n(R' + \tilde{R})} 2^{-nR_B} \]

where (a) follows by union bound. Therefore, this probability goes to zero if
\[ R' + \tilde{R} < R_B \]
\[ R' + \tilde{R} < H(Z|X_r, U, S) - \delta_1(\epsilon) \]

- Event \([E_3(b)|E_5(b - 1), E_1'(b)]: \) Recall that \((U^n(\tilde{L}^{(b)}(1)), S^n(b)) \in A_{e}^{(n)}(p_{S,U}) \). Therefore, by the conditional typicality lemma \[23 \] Chapter 2.5], the probability of this event goes to zero as \( n \) goes to infinity.

- Event \([E_4(b)|E_5(b - 1), E_2(b), E_1'(b)]: \) We need to distinct between the events in block \( b \) and \( b + 1 \). Note that conditioning on \( E_5 \) ensures us that for \( m^n(b) \neq 1 \) we have \( \tilde{l}^{(b)}(m^n(b)) \neq L^{(b)} \). Therefore, at block \( b + 1 \),
for each \( m'(b) \neq 1 \) the tuple \( (S^{n(b+1)}(i''), Y^{n(b)}(i'')), X_n(i''(m'(b))), Y_n(i''(m'(b))) \) is independent of \( Y^{n(b+1)} \) given \( S^{n(b+1)} \). Therefore,

\[
P \left[ \left( S^{n(b+1)}(i''), Y^{n(b)}(i''), X_n(i''(m'(b))), Y_n(i''(m'(b)))) \in A^n_{\epsilon}\left(p_{S,U,X,Y}\right) \right] \leq 2^{-n(I(X_r,U|Y,S)-\delta_4(\epsilon))} \tag{37} \]

At block \( b \), \( (U^{n}(i^{(b-1)}), X_n^{n}(i^{(b-1)}), S^{n}(b), Y^{n}(b)) \in A^n_{\epsilon}(p_{S,U,X,Y}) \) with high probability. Therefore, we consider two cases:

1) \((\hat{m}'(b), \hat{m}''(b)) = (1,1)\)

2) \((\hat{m}'(b), \hat{m}''(b)) = (1,*)\)

The statistical relations between the chosen sequences (by \( m'(b) \) and \( m''(b) \)) are summarized in Table II. A standard application of the packing lemma [23, Lemma 3.1] derives with the following bounds:

\[
R < I(Z; Y) - H(X_r, U, S) - \delta_3(\epsilon) + I(X_r, U; Y|S) - \delta_4(\epsilon) \tag{38a} \\
R'' < I(X; Y) - I(Z; X_r, U, S) - \delta_5(\epsilon) \tag{38b} 
\]

Note that \( I(Z; Y|X_r, U, S) + I(X_r, U; Y|S) = I(X_r, X; Y|S) \) since \( U \leftrightarrow (Z, X_r, X) \leftrightarrow Y \) form a Markov chain and \( Z \) is a function of \( (X, X_r, S) \).

Following this derivation, the probability of an error goes to zero if

\[
R' + \hat{R} < R_B \quad \text{(39a)} \\
R' + \hat{R} < H(Z|X_r, U, S) - \delta_1(\epsilon) \quad \text{(39b)} \\
\hat{R} > I(U; S) + \Delta_n + \delta(\epsilon) \quad \text{(39c)} \\
R < I(X_r, U; Y|S) - \delta_3(\epsilon) - \delta_4(\epsilon) \quad \text{(39d)} \\
R'' < I(X; Y|Z, X_r, U, S) - \delta_5(\epsilon) \quad \text{(39e)}
\]

Performing Fourier-Motzkin elimination (can be done using [25]) on the rates in (39) yields

\[
R \leq I(X, X_r; Y|S) \tag{40a} \\
R \leq I(X; Y|X_r, Z, S, U) + H(Z|X_r, S, U) - I(U; S) \tag{40b}
\]

Cardinality bounds on the auxiliary random variable \( U \) are obtained by performing Convex Cover Method [23, Appendix C].
B. Converse

Assume that the rate \( R \) is achievable,

\[
P_e(C_n) \leq \epsilon. \quad (41)
\]

By Fano’s inequality, we have

\[
H(M|Y^n, S^n) \leq H(P_e(C_n)) + P_e(C_n) \log (|\mathcal{M}| - 1) = n\epsilon_n \quad (42)
\]

where

\[
\epsilon_n = \frac{1}{n} H(P_e(C_n)) + P_e(C_n) R. \quad (43)
\]

Note that \( \epsilon_n \to 0 \) when \( \epsilon \to 0 \). Consider

\[
nR = H(M) \quad (44a)
\]

\[
\overset{(a)}{=} H(M|S^n) \quad (44b)
\]

\[
\overset{(b)}{\leq} I(M; Y^n|S^n) + n\epsilon_n \quad (44c)
\]

\[
= \sum_{i=1}^{n} I(M; Y_i|S_i) + n\epsilon_n \quad (44d)
\]

\[
= \sum_{i=1}^{n} I(M; X^{i-1}; Y_i|S_i) + n\epsilon_n \quad (44e)
\]

\[
= \sum_{i=1}^{n} I(M; X^{i-1}; Y_i|S_i) + n\epsilon_n \quad (44f)
\]

\[
\overset{(c)}{\leq} \sum_{i=1}^{n} I(M; X^{i-1}; X_{r,i}, Y^{i-1}|S^n) + n\epsilon_n \quad (44g)
\]

\[
\overset{(d)}{=} \sum_{i=1}^{n} I(X_{i+1}, X_{r,i}, Y_i|S_i) + n\epsilon_n \quad (44h)
\]

where:

(a) - since \( M \perp \perp S^n \),

(b) - follows by Fano’s inequality,

(c) - since \((M, X^{i-1}, Y^{i-1}, S^{n|\cdot}) \leftrightarrow (X_i, X_{r,i}, S_i) \leftrightarrow Y_i\) is a Markov chain,

(d) - by setting \( Q \sim U[1 : n] \) to be a time-sharing random variable,

(e) - since \( Q \leftrightarrow (X_Q, X_{r,Q}, S_Q) \leftrightarrow Y_Q \) is a Markov chain.

and \( S^{n|\cdot} = (S^{i-1}, S_{i+1}) \).
Next, let \( U_i \triangleq (S_i^{i-1}, Z_i^{i-1}) \). Consider the identity

\[
H(Z^n, X^n_r | S^n) = H(Z^n, X^n_r, S^n) - H(S^n) \tag{45a}
\]

\[
\overset{(f)}{=} \sum_{i=1}^{n} \left( H(Z_i, X_{r,i}, S_i | Z_{i-1}^i, X_{r,i-1}^i, S_{i-1}^i) - H(S_i) \right) \tag{45b}
\]

\[
= \sum_{i=1}^{n} \left( H(Z_i | X_{r,i}, S_i, Z_{i-1}^i, X_{r,i-1}^i, S_{i-1}^i) + H(X_{r,i}, S_i | Z_{i-1}^i, X_{r,i-1}^i, S_{i-1}^i) - H(S_i) \right) \tag{45c}
\]

\[
\overset{(g)}{=} \sum_{i=1}^{n} \left( H(Z_i | X_{r,i}, S_i, Z_{i-1}^i, S_{i-1}^i) + H(S_i | Z_{i-1}^i, S_{i-1}^i) - H(S_i) \right) \tag{45d}
\]

\[
= \sum_{i=1}^{n} \left( H(Z_i | X_{r,i}, S_i, U_i) + H(S_i | U_i) - H(S_i) \right) \tag{45e}
\]

\[
= \sum_{i=1}^{n} \left( H(Z_i | X_{r,i}, S_i, U_i) - I(U_i; S_i) \right) \tag{45f}
\]

\[
\overset{(h)}{=} n \left( H(Z_Q | X_{r,Q}, S_Q, U_Q, Q) - I(U_Q; S_Q | Q) \right) \tag{45g}
\]

\[
\overset{(i)}{=} n \left( H(Z_Q | X_{r,Q}, S_Q, U_Q, Q) - I(U_Q, Q; S_Q) \right) \tag{45h}
\]

where:

(e) - since \( S^n \) is i.i.d.,

(g) - since \( X_i^r \) is a function of \( Z_{i-1}^i \),

(h) - by definition of \( Q \) as a time-sharing random variable,

(i) - since \( Q \perp\!\!\!\perp S_Q \). Therefore, the following hold:

\[
I(U_Q, Q; S_Q) \leq H(Z_Q | X_{r,Q}, S_Q, U_Q, Q) \tag{46a}
\]

\[
H(Z^n, X^n_r | S^n) = n \left( H(Z_Q | X_{r,Q}, S_Q, U_Q, Q) - I(U_Q, Q; S_Q) \right) \tag{46b}
\]

The second bound is obtained by

\[
nR = H(M) \tag{47a}
\]

\[
= H(M | S^n) \tag{47b}
\]

\[
= H(M, Z^n, X^n_r | S^n) \tag{47c}
\]

\[
= H(Z^n, X^n_r | S^n) + H(M | Z^n, X^n_r, S^n) \tag{47d}
\]
The second term is upper bounded by

\[ H(M|Z^n, X^n_r, S^n) \leq I(M; Y^n|Z^n, X^n_r, S^n) + n\epsilon_n \]  
\[ = \sum_{i=1}^{n} I(M; Y_i|Y^{i-1}, Z^n, X^n_r, S^n) + n\epsilon_n \]  
\[ \leq \sum_{i=1}^{n} I(M, Y_{i-1}, Z^n_{i+1}, X^n_r, S^n_{i+1}, X_i; Y_i|Z_i, X_{r,i}, S_i, U_i) + n\epsilon_n \]  
\[ (j) = \sum_{i=1}^{n} I(X_i; Y_i|Z_i, X_{r,i}, S_i, U_i) + n\epsilon_n \]

\[ = n (I(X_Q; Y_Q|Z_Q, X_Q, S_Q, U_Q, Q) + H(Z_Q|X_Q, S_Q, U_Q) - I(Q, U_Q; S_Q) + \epsilon_n) \]

therefore,

\[ nR \leq n (I(X_Q; Y_Q|Z_Q, X_Q, S_Q, U_Q, Q) + H(Z_Q|X_Q, S_Q, U_Q) - I(Q, U_Q; S_Q) + \epsilon_n) \]

where (j) follows since \((M, Y^{i-1}, Z^n_{i+1}, X^n_r, S^n_{i+1}) \leftrightarrow (X_i, S_i, U_i, Z_i, X_{r,i}) \leftrightarrow Y_i\) is a Markov chain.

We need to show that the following conditions hold:

- \(Q\) is independent of \(S_Q\).
- The following Markov chains hold
  \[
  (M, X^{i-1}, Y^{i-1}, S^n_{i-1}) \leftrightarrow (X_i, X_{r,i}, S_i) \leftrightarrow Y_i \]
  \[
  (M, Y^{i-1}, Z^n_{i+1}, X^n_r, S^n_{i+1}) \leftrightarrow (X_i, S_i, U_i, Z_i, X_{r,i}) \leftrightarrow Y_i
  \]
- \(p_{Y_Q|X_Q, X_{r,Q}, Z_Q, S_Q, U_Q, Q}(y|x, x_r, z, s, u, q) = p_{Y|X, X_r, Z, S}(y|x, x_r, z, s)\)
- \(Z_Q = z(X_Q, X_{r,Q}, S_Q)\)

The first condition holds due to the i.i.d. distribution of the states sequence \(S^n\). The distribution on the random variables is

\[
p(m, s^n, x^n_r, z^n_s, y^n) = p(m) \prod_{i=1}^{n} p(s_i) \prod_{i=1}^{n} (x_i|m, s^n)1(x_{r,i}|z^{i-1})1(z_i|x_i, x_{r,i}, s_i)p_{Y|X, X_r, Z, S}(y|x_i, x_{r,i}, z_i, s_i).
\]

The Markov chains in the second condition can be readily seen from this distribution. Moreover, for each \(i\), \(Z_i = z(Z_i, Z_{r,i}, S_i)\) and the third condition also holds. By defining \(U = (U_Q, Q)\), \(X = X_Q, X_r = X_{r,Q}, S = S_Q\) and \(Z = Z_Q\), we derive with the following bound:

\[ R \leq I(X, X_r; Y|S) + \epsilon_n \]
\[ R \leq I(X; Y|X_r, Z, U, S) + H(Z|X_r, S, U) - I(U; S) + \epsilon_n \]

with PMF that factorizes as

\[ p_{U|S}P_{X_r|U}P_{X|X_r,U,S} \]

that satisfies \(I(U; S) \leq H(Z|X_r, S, U)\). This completes the proof for the converse part. \(\square\)
VI. PROOF FOR MAC WITH ONE STATE COMPONENT

A. Direct

We first discuss the achievability scheme for the case where $S_2$ is degenerate. To ease the notation we use $S = S_1$. The capacity region for this case is given by

$$R_1 \leq I(X_1; Y|X_2, Z, S, U) + H(Z|S, U) - I(U; S)$$  \hspace{1cm} (53a)

$$R_2 \leq I(X_2; Y|X_1, S, U)$$  \hspace{1cm} (53b)

$$R_1 + R_2 \leq I(X_1, X_2; Y|Z, S, U) + H(Z|S, U) - I(U; S)$$  \hspace{1cm} (53c)

$$R_1 + R_2 \leq I(X_1, X_2; Y|S)$$  \hspace{1cm} (53d)

for PMFs of the form $p_{U|S}p_{X_1|S, U}p_{X_2|U}$, with $Z = z(X_1, S)$, that satisfies

$$I(U; S) \leq H(Z|S, U),$$  \hspace{1cm} (53e)

and $|U| \leq \min\{|S||X_1||X_2| + 2, |S||Y| + 1\}$.

The coding scheme for this case gives the key steps for the general case (in Theorem 2). Note that Encoder 2 plays here a double role: First, it helps Encoder 1 to deliver his message $M_1$ by cribbing $Z^{i-1}$ at each time $i$. This is done using the cooperative-bin-forward scheme as in the SD-RC in section V. Second, it delivers its own message $M_2$ to the decoder using the same transmission sequence $X_2^n$. To do so, a superposition code is built on the shared common information which is represented by the sequence $U^n$. This common information also coordinated with the states sequence $S^n$. The decoding procedure however is done backward, which is called backward decoding.

We will now give a detailed proof for the achievable rates.

Fix a PMF $p_{U|S}p_{X_1|U,S}p_{X_2|U}$ and $\epsilon > 0$. Divide the transmission to $B$ blocks, each of length $n$. At each communication block $b$, we transmit at rates $R_1$ and $R_2$. We perform rate-splitting for $R_1$: at each block $b \in [1: B]$, split the rate $R_1 = R_1' + R_1''$, with the message $M_1^{(b)} = (M_1^{(b)}, M_1'^{(b)})$ accordingly.

**Codebook:** The codebook $C_n$ is defined to be collection of block-codebooks, $\{C_n^{(b)}\}_{b \in [1:B]}$. For each block $b \in [1:B]$, a codebook $C_n^{(b)}$ is generated as follows:

- **Binning:** Partition the set $Z^n$ into $2^{nR_1}$ bins, by drawing uniformly and independently an index

$$\text{bin}^{(b)}(z^n) \sim U\left[1:2^{nR_1}\right] \quad \forall z^n \in Z^n$$  \hspace{1cm} (54a)

- **Cooperation codewords:** Generate $2^{nR_B}$ $u$-codewords

$$u^n(l^{(b-1)}) \sim \prod_{i=1}^{n} p_{U}(u_i(l^{(b-1)})), \quad l^{(b-1)} \in \left[1:2^{nR_B}\right]$$  \hspace{1cm} (54b)

- **Cribbed codewords:** For each $u^n \in U^n$ and $s^n \in S^n$, generate $2^{n(R_1'+R_1)}$ $z$-codewords,

$$z^n(m^{(b)}_1, k^{(b)}|u^n, s^n) \sim \prod_{i=1}^{n} p_{Z|U,S}(z_i|u_i, s_i), \quad m^{(b)}_1 \in \left[1:2^{nR_1}\right], \quad k^{(b)} \in \left[1:2^{nR_1}\right].$$  \hspace{1cm} (54c)

- **Transmission codewords at Encoder 1:** For each $z^n \in Z^n$, $u^n \in U^n$ and $s^n \in S^n$ generate $2^{nR_1}$ $x_1$-codewords,

$$x^n_1(m^{(b)}_1|z^n, u^n, s^n) \sim \prod_{i=1}^{n} p_{X_1|Z,U,S}(x_{1,i}|z_i, u_i, s_i), \quad m^{(b)}_1 \in \left[1:2^{nR_1}\right].$$  \hspace{1cm} (54d)
The transmitted codeword is denoted by $x$, the average rates can be made close to $R$ and all block-codebooks are independent of each other. One can show that for every fixed $l^{(1)}$, the prefixes and suffix blocks, the average rates are $\bar{u}$ at blocks $n$. Assuming that $\hat{B} = B^2$ and $l^{(1)}$ be equal to one. Namely, at blocks 1 and $B$ the encoders don’t send any message, and hence these blocks are prefix and suffix for the transmission. Here, in addition to the suffix block that is used in block Markov coding schemes, the prefix block is used for the encoders to agree on the second cooperation codeword that is typical with $s^{(2)}$. However, for $l^{(0)}$ the corresponding cooperation sequence $u^n(l^{(0)})$ is not necessarily typical with the states in the first block. Due to prefix and suffix blocks, the average rates are $\bar{R}_1 = \frac{B-2}{B} R_1$ and $\bar{R}_2 = \frac{B-2}{B} R_2$. By choosing $B$ sufficiently large, the average rates can be made close to $R_1$ and $R_2$ as desired.

**Encoder 1:** Denote by $s^{(b)}$ the states sequence from block $b$, and

$$z^n(m^{(b)}_1, k^{(b)} | i^{(b-1)}, s^{(b)}) \triangleq z^n(m^{(b)}_1, k^{(b)} | u^n(l^{(b-1)}), s^{(b)})$$

(55)

the cribbed codewords from codebook $C^{(b)}$. At block $b$, the encoder looks for $k^{(b)}$ such that

$$\left(u^n(\text{bin}^{(b)}(z^n(m^{(b)}_1, k^{(b)} | u^n(l^{(b-1)}), s^{(b)}))), s^{n(b+1)}\right) \in A^{(n)}_c(p_{S,U}).$$

(56)

If such $k^{(b)}$ cannot be found, choose $k^{(b)}$ uniformly. If more than one was found, choose the first. Set

$$l^{(b)} = \text{bin}^{(b)}(z^n(m^{(b)}_1, k^{(b)} | u^n(l^{(b-1)}), s^{(b)}))$$

(57)

and transmits the codeword

$$x^{n}_1(m^{(b)}_1 | z^n(m^{(b)}_1, k^{(b)} | u^n(l^{(b-1)}), s^{(b)}), u^n(l^{(b-1)}), s^{(b)})$$

(58)

The transmitted codeword is denoted by $x^{n}_1(m^{(b)}_1 | m^{(b)}_1, k^{(b)}, l^{(b-1)}, s^{(b)}).

**Encoder 2:** Assuming that $l^{(b-1)}$ is known from previous encoding operations, at block $b$ Encoder 2 transmits $x_2^n(m^{(b)}_2 | u^n(l^{(b-1)}))$. We denote this codeword as $x_2^n(m^{(b)}_2 | l^{(b-1)})$. At the end of the block, this encoder observes $z^n(m^{(b)}_1, k^{(b)} | l^{(b-1)}, s^{(b)})$, and sets $l^{(b)} = \text{bin}^{(b)}(z^n(m^{(b)}_1, k^{(b)} | l^{(b-1)}, s^{(b)})).$

One can show that for every fixed $B$, we can take $n$ to be large enough to make the probability of an error small as desired.
Decoding: The decoding procedure is done backwards, Laneman and Kramer. We start decoding from block B to block 2. Assuming \( l^{(b)} \) is known by decoding former blocks, the decoder performs:

1) For each \( l^{(b-1)} \), find \( \hat{k}^{(b)}(l^{(b-1)}, l^{(b)}, s^{n(b+1)}) \) and \( \hat{m}_1^{(b)}(l^{(b-1)}, l^{(b)}, s^{n(b+1)}) \), abbreviated as \( \hat{m}_1^{(b)}(l^{(b-1)}) \) and \( \hat{k}^{(b)}(l^{(b-1)}) \), such that

\[
\bin(s^n(\hat{m}_1^{(b)}(l^{(b-1)}), \hat{k}^{(b)}(l^{(b-1)})) | l^{(b)}, s^{n(b)}(l^{(b-1)})) = l^{(b)}
\]  

(60)

2) Denote the channel’s output at blocks \( b \) by \( y^{n(b)} \). Find a unique tuple \( (\hat{l}^{(b-1)}, \hat{m}_1^{(b)}, \hat{m}_2^{(b)}) \) such that \( 59 \) is satisfied. If such \( (\hat{l}^{(b-1)}, \hat{m}_1^{(b)}, \hat{m}_2^{(b)}) \) cannot be found, choose each uniformly. Recall that \( m_1^{(B)} = k^{(B)} = m_2^{(B)} = k^{(B)} = 1 \). To initialize the decoding procedure, in block \( B \) (first decoding block) find \( l^{(B-1)} \) using condition \( 59 \).

Analysis of error probability: The code \( C_n \) is defined by the block-codebooks \( \{c^{(b)}_n\}_{b=1}^B \), the encoders and decoder functions. We bound the average probability of an error at block \( b \); encoding error events are conditioned on successfully encoding blocks \([1 : b - 1]\), and decoding error events are conditioned on successfully decoding blocks \([b + 1 : B]\). Without loss of generality we assume that \( M_1^{(b)} = M_1^{(b)} = M_2^{(b)} = 1 \) for each \( b \in [1 : B] \).

Define the events

\[
E_1(b) = \{ \forall k^{(b)} : (U^n(\text{Bin}^{(b)}(Z^n(1, k^{(b)} | L^{(b-1)}, S^{n(b)}))), S^{n(b+1)}) \notin \mathcal{A}_b^{(n)}(p_{S,U}) \} \\
E_2(b) = \{ \exists m_1^{(b)} 
eq 1 : \text{Bin}^{(b)}(Z^n(\hat{m}_1^{(b)}, k^{(b)} | L^{(b-1)}, S^{n(b)})) = L^{(b)}, \text{ for some } k^{(b)} \} \\
E_3(b) = \{ \text{Condition } 59 \text{ is not satisfied by } (\hat{l}^{(b-1)}, \hat{m}_1^{(b)}, \hat{m}_2^{(b)}) = (L^{(b-1)}, 1, 1) \} \\
E_4(b) = \{ \text{Condition } 59 \text{ is satisfied by some } (\hat{l}^{(b-1)}, \hat{m}_1^{(b)}, \hat{m}_2^{(b)}) \neq (L^{(b-1)}, 1, 1) \} \\
E_5(b) = \{ \hat{l}^{(b)} = L^{(b)} \} \\
\hat{E}_1(b) = \bigcup_{j=1}^b \{ E_1(j) \cup E_2(j) \} \\
\hat{E}_2(b) = \bigcup_{j=b}^B \{ E_3(j) \cup E_4(j) \cup E_5(j) \}
\]

(61a)

(61b)

(61c)

(61d)

(61e)

(61f)

(61g)

The average probability of an error is upper bounded by the union of these events in all blocks,

\[
P_e^n = \mathbb{E}_{C_n} \left[ P_e^n (C_n(C_n)) \right] \\
\leq \mathbb{P} \left[ \hat{E}_1(B) \cup \hat{E}_2(1) \right] \\
\leq \mathbb{P} \left[ \hat{E}_1(B) \right] + \mathbb{P} \left[ \hat{E}_2(1), \hat{E}_1(B) \right] \\
\leq \sum_{b=1}^B \left( \mathbb{P} \left[ E_1(b) \right] + \mathbb{P} \left[ E_2(b) \right] + \mathbb{P} \left[ E_3(b) | E_5(b + 1), \hat{E}_1(B) \right] \\
+ \mathbb{P} \left[ E_4(b) | E_5(b + 1), \hat{E}_1(B) \right] + \mathbb{P} \left[ E_5(b) | E_5(b), E_4(b), E_4(b + 1), \hat{E}_1(B) \right] \right)
\]

(62a)

(62b)

(62c)

(62d)

(62e)

In \[59\] showed that for the MAC (in contrary to SD-RC) sliding window decoding is sometimes inferior to backward decoding in terms of achievable rates.
– Event \([E_1(b)]\): We need to satisfy that the probability of seeing \(U^n(l^{(b)})\) that is jointly typical with \(S^{n,(b+1)}\) go to 1 as \(n\) goes to infinity. According to Lemma 1\(\square\) we can ensure that \(P[E_1(b)|\hat{E}_1(b-1)]\xrightarrow{n\to\infty} 0\) by taking \(R_B > \hat{R}_1 + \delta_1(\epsilon)\), \(\hat{R}_1 > I(U;S) + \Delta_n\) and \(\hat{R}_1 < H(Z|U,S) - \delta_2(\epsilon)\).

– Event \([E_2(b)]\): Denote \(Z^n(m_1^{(b)},k^{(b)}) = Z^n(m_1^{(b)},k^{(b)}|L^{(b-1)},S^{(b)})\). Consider

\[
P[E_2(b)] = P[\exists m_1^{(b)} \neq 1 : Bin(b)(Z^n(m_1^{(b)},k^{(b)})) = L^{(b)}, \text{ for some } k^{(b)}]
\]

\[
= P[\exists m_1^{(b)} \neq 1 : Bin(b)(Z^n(m_1^{(b)},k^{(b)})) = Bin(b)(Z^n(1,K^{(b)})), \text{ for some } k^{(b)}]
\]

\[
\leq \sum_{m_1^{(b)} > 1,k^{(b)}} P[Bin(b)(Z^n(m_1^{(b)},k^{(b)})) = Bin(b)(Z^n(1,K^{(b)}))]
\]

\[
\leq \sum_{m_1^{(b)} > 1,k^{(b)}} P[Bin(b)(Z^n(m_1^{(b)},k^{(b)})) = Bin(b)(Z^n(1,K^{(b)})), Z^n(m_1^{(b)},k^{(b)}) = Z^n(1,K^{(b)})]
\]

\[
+ \sum_{m_1^{(b)} > 1,k^{(b)}} P[Bin(b)(Z^n(m_1^{(b)},k^{(b)})) = Bin(b)(Z^n(1,K^{(b)})), Z^n(m_1^{(b)},k^{(b)}) \neq Z^n(1,K^{(b)})]
\]

\[
\leq \sum_{m_1^{(b)} > 1,k^{(b)}} P[Z^n(m_1^{(b)},k^{(b)}) = Z^n(1,K^{(b)})]
\]

\[
+ \sum_{m_1^{(b)} > 1,k^{(b)}} P[Bin(b)(Z^n(m_1^{(b)},k^{(b)})) = Bin(b)(Z^n(1,K^{(b)})) \bigg| Z^n(m_1^{(b)},k^{(b)}) \neq Z^n(1,K^{(b)})]
\]

\[
\leq 2^n(R_1' + \hat{R}_1 - R_B)2^{-n(H(Z|U,S) - \delta_3(\epsilon))} + 2^n(R_1' + \hat{R}_1)2^{-nR_B}
\]

Therefore, this probability goes to zero as \(n \to \infty\) if \(R'_1 + \hat{R}_1 < H(Z|U,S) - \delta_3(\epsilon)\) and \(R'_1 + \hat{R}_1 < R_B\).

– Event \([E_4(i)|E_6^n(b+1), \hat{E}_1(B)]\): Note that given \(\hat{E}_1(b-1)\), the cooperation codeword \(U^n(L^{(b-1)})\) and \(S^{(b)}\) are jointly typical. Recall that by the codebook generated \(i.i.d\), each block-codebook is independent of each other and the channel is memoryless. Thus, by conditional typicality lemma, the probability of this event goes to 0 when \(n \to \infty\).

– Event \([E_4(i)|E_6^n(b+1), \hat{E}_1(B)]\): There are several cases in which this error can occur:

1) \(\{l^{(b-1)},m_1^{(b)},m_2^{(b)}\} = (\neq L^{(b-1)},*,*)\)

2) \(\{l^{(b-1)},m_1^{(b)},m_2^{(b)}\} = (L^{(b-1)},1,1)\)

3) \(\{l^{(b-1)},m_1^{(b)},m_2^{(b)}\} = (L^{(b-1)},1,1)\)

4) \(\{l^{(b-1)},m_1^{(b)},m_2^{(b)}\} = (L^{(b-1)},1,1)\)

The probability of each case is bounded by standard application of the packing lemma as follows. The statistical relations between the codewords are summarized in Table. Denote by \(\hat{E}_{i,j,k}(b)\) the event that \(59\) is satisfied
### TABLE III: Statistical relations in the decoding procedure for MAC with cribbing

| $l^{(b-1)}$ | $m_1^{(b)}$ | $m_2^{(b)}$ | PMF                  |
|-------------|-------------|-------------|----------------------|
| $\not= L^{(b-1)}$ | *           | *           | $P.S.U,X_1,X_2,ZP|S$          |
| $L^{(b-1)}$    | 1           | 1           | $P.S.U,X_1,X_2,ZP|Z,U,S$ |
| $L^{(b-1)}$    | 1           | 1           | $P.S,U,X_1,X_2,ZP|X_1,Z,U,S$ |
| $L^{(b-1)}$    | 1           | 1           | $P.S,U,X_1,X_2,ZP|X_2,Z,U,S$ |

by $(l^{(b-1)}, m_1^{(b)}, m_2^{(b)}) = (i, j, k)$. By union-bound, case 1 is upper-bounded by

\[
\Pr \left[ \bigcup_{i \not= L^{(b-1)}, j,k} \tilde{E}_{i,j,k}(b) \mid E^c_0(b+1), \tilde{E}_1(B) \right] \leq \sum_{i \not= L^{(b-1)}, j,k} \Pr \left[ \tilde{E}_{i,j,k}(b) \mid E^c_0(b+1), \tilde{E}_1(B) \right] \leq 2^{-n(I(U;S) - \delta_4(\epsilon))} \times \Pr \left[ \tilde{E}_{i,j,k}(b) \mid E^c_0(b+1), \tilde{E}_1(B), (U^n(i), S^n(b)) \in A_{0,5\epsilon}^n (P.S.U) \right] \leq 2^{n(R_0 + R_0' + R_2) - n(I(U;S) - \delta_4(\epsilon))} \times \Pr \left[ \tilde{E}_{i,j,k}(b) \mid E^c_0(b+1), \tilde{E}_1(B), (U^n(i), S^n(b)) \in A_{0,5\epsilon}^n (P.S.U) \right] \leq 2^{n(R_0 + R_0' + R_2) - n(I(U;S) - \delta_4(\epsilon))} \times \Pr \left[ \tilde{E}_{i,j,k}(b) \mid E^c_0(b+1), \tilde{E}_1(B), (U^n(i), S^n(b)) \in A_{0,5\epsilon}^n (P.S.U) \right]
\]

where:

(a) follows since for each $i \not= L^{(b-1)}$, $U^n(i)$ is independent of $S^n(b)$,

(b) follows since for each $i \not= L^{(b-1)}$, the codewords corresponding to $(i, j, k)$ are independent of $Y^n(b)$ given $S^n(b)$.

Similarly, case 2 is upper bounded by

\[
\sum_{j > 1, k > 1} \Pr \left[ \tilde{E}_{L^{(b-1)}, j,k}(b) \mid E^c_0(b+1), \tilde{E}_1(B) \right] \leq 2^{n(R_0' + R_2)} 2^{-n(I(X_1,X_1,Z) - \delta_4(\epsilon))}
\]

since for $j \neq 1, k \neq 1$, the codewords corresponding to $j,k$ are independent of $Y^n(b)$ given $S^n(b)$ and $U^n(L^{(b-1)})$.

Case 3 by

\[
\sum_{k > 1} \Pr \left[ \tilde{E}_{L^{(b-1)},1,k}(b) \mid E^c_0(b+1), \tilde{E}_1(B) \right] \leq 2^{nR_2} 2^{-n(I(X_2,Y|X_1,U,S) - \delta_7(\epsilon))}
\]

because $X^n_2(k) | L^{(b-1)}$ is independent of $Y^n(b)$ given $(S^n(b), U^n(L^{(b-1)}), X^n_1(1|1, K(0), L^{(b-1)}), S^n(b))$, and case 4 by

\[
\sum_{j > 1} \Pr \left[ \tilde{E}_{L^{(b-1)},j,1}(b) \mid E^c_0(b+1), \tilde{E}_1(B) \right] \leq 2^{nR_0'} 2^{-n(I(X_1,Y|X_2,Z) - \delta_8(\epsilon))}
\]
since $X^n_k(1|1, K^{(b)}, L^{(b-1)})$ is independent of $Y^{n(b)}$ given $(S^{n(b)}, U^n(L^{(b-1)}), X^n_2(k|L^{(b-1)}), S^{n(b)})$. Thus, the above probabilities goes to zero as $n \to \infty$ if

\[
R_B + R''_1 + R_2 < I(X_1, X_2; Y|S) - \delta_5(\epsilon) + I(U; S) - \delta_4(\epsilon)
\]  
(68a)

\[
R''_1 + R_2 < I(X_1, X_2; Y|U, S, Z) - \delta_6(\epsilon)
\]  
(68b)

\[
R_2 < I(X_2; Y|U, S, X_1) - \delta_7(\epsilon)
\]  
(68c)

\[
R''_1 < I(X_1; Y|Z, U, S, X_2) - \delta_8(\epsilon)
\]  
(68d)

- Event $\left[ E_5^{(b)} | E_3^{(b)}, E_4^{(b)}(b), E_5^{(b)}(b + 1), \bar{E}_1^{(b)}(B) \right]$:

The probability of this event is zero since $E_5^{(b)} \cap E_4^{(b)} = \phi$.

Henceforth, we derived with the following bounds

\[
\bar{R}_1 < H(Z|U, S) - \delta_2(\epsilon)
\]  
(69a)

\[
R_B > \bar{R}_1 + \delta_1(\epsilon)
\]  
(69b)

\[
\bar{R}_1 > I(U; S) + \Delta_n
\]  
(69c)

\[
R'_1 + \bar{R}_1 < H(Z|U, S) - \delta_3(\epsilon)
\]  
(69d)

\[
R'_1 + \bar{R}_1 < R_B
\]  
(69e)

\[
R_B + R''_1 + R_2 < I(X_1, X_2; Y|S) + I(U; S) - \delta_4(\epsilon) - \delta_5(\epsilon)
\]  
(69f)

\[
R''_1 + R_2 < I(X_1, X_2; Y|U, S, Z) - \delta_6(\epsilon)
\]  
(69g)

\[
R_2 < I(X_2; Y|U, S, X_1) - \delta_7(\epsilon)
\]  
(69h)

\[
R''_1 < I(X_1; Y|Z, U, S, X_2) - \delta_8(\epsilon)
\]  
(69i)

Together with the identity $R_1 = R'_1 + R''_1$ and non-negativity of all rates. Applying Fourier-Motzkin elimination (can be done using (65)) to eliminate $R'_1, R''_1, \bar{R}_1$ and $R_B$, yields

\[
I(U; S) < H(Z|U, S)
\]  
(70a)

\[
R_2 < I(X_2; Y|U, S, X_1)
\]  
(70b)

\[
R_1 < I(X_1; Y|U, S, X_2, Z) + H(Z|U, S) - I(U; S)
\]  
(70c)

\[
R_1 + R_2 < I(X_1, X_2; Y|S)
\]  
(70d)

\[
R_1 + R_2 < I(X_1, X_2; Y|U, S, Z) + H(Z|U, S) - I(U; S)
\]  
(70e)

This closes the proof for the direct part. □

B. Converse

Assuming that the rate pair $(R_1, R_2)$ is achievable,

\[
P_{\epsilon}(C_n) \leq \epsilon.
\]  
(71)
By Fano’s inequality, we have
\[ H(M_1, M_2|Y^n, S^n) \leq H_b(P_e(C_n)) + P_e(C_n) \log (|M_1 \times M_2| - 1) \] (72)
where \( H_b(\cdot) \) is the binary entropy function. Define
\[ \epsilon_n = \frac{1}{n} (H_b(P_e(C_n)) + P_e(C_n) \log (|M_1 \times M_2| - 1)) \] (73a)
\[ \leq \frac{1}{n} H_b(P_e(C_n)) + P_e(C_n) (R_1 + R_2) \] (73b)
Note that \( \epsilon_n \to 0 \) when \( \epsilon \to 0 \). To show that the region in (53) is an outer bound, we first identify the auxiliary random variable \( U_i \triangleq (Z^{i-1}, S^{i-1}) \).

Consider
\[ 0 \leq H(Z^n|S^n) = H(Z^n, S^n) - H(S^n) \] (74a)
\[ \overset{(a)}{=} \sum_{i=1}^{n} H(Z_i, S_i|Z_i^{i-1}, S_i^{i-1}) - H(S_i) \] (74b)
\[ \overset{(b)}{=} \sum_{i=1}^{n} H(Z_i|S_i, U_i) - (H(S_i) - H(S_i|U_i)) \] (74c)
\[ \overset{(c)}{=} \sum_{i=1}^{n} H(Z_i|S_i, U_i) - I(S_i; U_i) \] (74d)
\[ \overset{(d)}{=} n(H(Z_Q|U_Q, S_Q, Q) - I(S_Q; U_Q|Q)) \] (74e)
where:
(a) - since \( S^n \) is i.i.d,
(b) - by definition of \( U_i \),
(c) - by setting \( Q \) to be a time sharing-random variable, \( Q \sim U[1 : n] \).

Therefore, we have shown that
\[ I(U_Q; S_Q|Q) \leq H(Z_Q|U_Q, S_Q, Q) \] (75a)
\[ H(Z^n|S^n) = n (H(Z_Q|U_Q, S_Q, Q) - I(U_Q; S_Q|Q)) \] (75b)
An upper bound on $R_1$ is established as follows

\[ nR_1 = H(M_1) \]

\(\stackrel{(a)}{=} H(M_1|S^n)\) \hspace{1cm} (76a)

\(\stackrel{(b)}{=} H(M_1, Z^n|S^n)\) \hspace{1cm} (76b)

\[ = H(Z^n|S^n) + H(M_1|Z^n, S^n) \]

\(\stackrel{(c)}{=} H(Z^n|S^n) + H(M_1|Z^n, S^n, M_2)\) \hspace{1cm} (76c)

\[ = H(Z^n|S^n) + H(M_1|Z^n, S^n, M_2) - H(M_1|Z^n, S^n, M_2, Y^n) + H(M_1|Z^n, S^n, M_2, Y^n) \]

\(\leq\) \hspace{1cm} (76d)

\[ \leq H(Z^n|S^n) + I(M_1; Y^n|Z^n, S^n, M_2) + n\epsilon_n \]

\[ = H(Z^n|S^n) + \sum_{i=1}^{n} I(M_1; Y_i|Z^n, S^n, Y^{i-1}, M_2) + n\epsilon_n \]

\(\stackrel{(e)}{=} H(Z^n|S^n) + \sum_{i=1}^{n} I(M_1; Y_i|Z^n, S^n, Y^{i-1}, M_2, X_{2,i}) + n\epsilon_n \)

\[ \leq H(Z^n|S^n) + \sum_{i=1}^{n} I(M_1, M_2, X_{1,i}, Y_i^{i-1}, Z^n_{i+1}, S^n_{i+1}; Y_i|Z_i, S_i, X_{2,i}) + n\epsilon_n \]

\(\stackrel{(f)}{=} H(Z^n|S^n) + \sum_{i=1}^{n} I(X_{1,i}; Y_i|Z_i, S_i, X_{2,i}, U_i) + n\epsilon_n \)

\(\stackrel{(g)}{=} n(H(Z_Q|U_Q, S_Q, Q) - I(U_Q; S_Q|Q) + I(X_1; Y_Q|X_2, Q, S_Q, Z_Q, U_Q, Q) + \epsilon_n) \)

\[ (76l) \]

where:

(a) - since $M_1 \perp S^n$,

(b) - since $Z^n$ is a function of $M_1$ and $S^n$,

(c) - since $M_2 \perp (M_1, S^n, Z^n)$,

(d) - by Fano’s inequality,

(e) - since $X_{2,i}$ is a function of $M_2$ and $Z^{i-1}$,

(f) - due to the Markov chain $(M_1, M_2, Z^n_{i+1}, S^n_{i+1}, Y^{i-1}) \leftrightarrow (X_{1,i}, X_{2,i}, S_i, U_i, Z_i) \leftrightarrow Y_i$,

(g) - follows from (75) and time-sharing variable $Q$. 

Applying similar arguments, we get an upper bound for $R_2$

\[ nR_2 = H(M_2) \]  
\[ = H(M_2|S^n, M_1) \]  
\[ = H(M_2|S^n, M_1) - H(M_2|S^n, M_1, Y^n) + H(M_2|S^n, M_1, Y^n) \]  
\[ \leq I(M_2; Y^n|S^n, M_1) + n\epsilon_n \]  
\[ = \sum_{i=1}^{n} I(M_2; Y_i|S^n, M_1, Y^{i-1}) + n\epsilon_n \]  
\[ = \sum_{i=1}^{n} I(M_2, X_{2,i}; Y_i|S^n, M_1, Y^{i-1}, Z^i, X_{1,i}) + n\epsilon_n \]  
\[ \leq \sum_{i=1}^{n} I(M_2, M_1, Y^{i-1}, S^n_{i+1}, X_{2,i}; Y_i|S^i, Z^{i-1}, X_{1,i}) + n\epsilon_n \]  
\[ \overset{(h)}{=} \sum_{i=1}^{n} I(X_{2,i}; Y_i|S_i, X_{1,i}, U_i) + n\epsilon_n \]  
\[ = n(I(X_{2,Q}; Y_Q|S_Q, X_{1,Q}, U_Q, Q) + \epsilon_n) \]  

where:

(h) follows since $(M_2, M_1, Y^{i-1}, S^n_{i+1}) \leftrightarrow (X_{1,i}, X_{2,i}, S_i, U_i) \leftrightarrow Y_i$ is a Markov chain.

The first upper bound for the sum-rate is:

\[ n(R_1 + R_2) = H(M_1) + H(M_2) \]  
\[ = H(M_1, M_2) \]  
\[ = H(M_1, M_2|S^n) \]  
\[ = H(Z^n|S^n) + H(M_1, M_2|S^n, Z^n) \]  
\[ \leq H(Z^n|S^n) + I(M_1, M_2; Y^n|S^n, Z^n) + n\epsilon_n \]  
\[ = H(Z^n|S^n) + \sum_{i=1}^{n} I(M_1, M_2; Y_i|S^n, Z^n, Y^{i-1}) + n\epsilon_n \]  
\[ = H(Z^n|S^n) + \sum_{i=1}^{n} I(M_1, M_2, X_{1,i}, X_{2,i}; Y_i|S^n, Z^n, Y^{i-1}) + n\epsilon_n \]  
\[ \leq H(Z^n|S^n) + \sum_{i=1}^{n} I(M_1, M_2, Y^{i-1}, S^n_{i+1}, Z^n_{i+1}, X_{1,i}, X_{2,i}; Y_i|S^i, Z^i) + n\epsilon_n \]  
\[ = H(Z^n|S^n) + \sum_{i=1}^{n} I(X_{1,i}, X_{2,i}; Y_i|S_i, Z_i, U_i) + n\epsilon_n \]  
\[ = n(H(Z_Q|U_Q, S_Q, Q) - I(U_Q; S_Q|Q) + I(X_{1,Q}, X_{2,Q}; Y_Q|S_Q, U_Q, Z_Q, Q) + \epsilon_n) \]
and the second upper bound by:

\[
H(M_1, M_2) = H(M_1, M_2 | S^n) \leq I(M_1, M_2; Y^n | S^n) + n\epsilon_n
\]

\[
= \sum_{i=1}^{n} I(M_1, M_2; Y_i^{i-1}, S^n) + n\epsilon_n
\]

\[
\leq \sum_{i=1}^{n} I(M_1, M_2; Y_i^{i-1}, S^n, X_{1,i}, X_{2,i}; Y_i | S_i) + n\epsilon_n
\]

\[
= n(I(X_1,Q, X_2,Q; Y_Q | S_Q, Q) + \epsilon_n)
\]

\[
\leq n(I(X_1,Q, X_2,Q; Y_Q | S_Q) + \epsilon_n)
\]

where the last inequality is due to the Markov chain \( Q \leftrightarrow (X_1,Q, X_2,Q, S_Q) \leftrightarrow Y_Q \).

We note that the following conditions must hold:

- \( S_Q \) is independent of \( Q \), and \( p_{S_Q}(s) = p_s(s) \).
- The Markov \((X_1,Q, S_Q) \leftrightarrow (Q, U_Q) \leftrightarrow X_2,Q\) holds.
- \( P_{Y_Q|X_1,Q, X_2,Q, S_Q, U_Q, z_Q,Q}(y|x_1, x_2, s, u, z, q) \) is equal to \( p_{Y|X_1, x_2, S}(y|x_1, x_2, s) \).
- \( Z_Q = z(X_1,Q, S_Q) \).

To prove the second condition, consider

\[
p(z^{i-1}, s^i, x_{1,i}, x_{2,i}) = \sum_{m_1, m_2} p(m_1)p(m_2)p(s^{i-1})p(s_i)p(z^{i-1}|s_i, m_1)p(x_{1,i}|z^{i-1}, s^i, m_1)1(x_{2,i}|m_2, z^{i-1})
\]

\[
= \sum_{m_1, m_2} p(m_1)p(s^{i-1})p(s_i)p(z^{i-1}|s_i, m_1)p(x_{1,i}|z^{i-1}, s^i, m_1)p(m_2)1(x_{2,i}|m_2, z^{i-1})
\]

\[
= \sum_{m_1, m_2} p(s^{i-1})p(s_i)p(z^{i-1}, x_{1,i}, m_1 | s^i)p(x_{2,i}, m_2 | z^{i-1})
\]

\[
= p(s^{i-1})p(s_i) \sum_{(m_1) \in M_1} p(z^{i-1}, x_{1,i}, m_1 | s^i) \sum_{(m_2) \in M_2} p(x_{2,i}, m_2 | z^{i-1})
\]

\[
= p(s^{i-1})p(s_i)p(z^{i-1}, x_{1,i} | s^i)p(x_{2,i} | z^{i-1})
\]
which proves that for each \( i \in [1 : n] \), the Markov \((X_{1,i}, S_i) \leftrightarrow (Z^{i-1}, S^{i-1}) \leftrightarrow X_{2,i}\) holds, and therefore the Markov in the second condition holds. The third condition is due to the memoryless property of the channel and that for random time \( Q \), the channel's input are \((X_1, Q, X_2, S_Q)\). To see this, consider the PMF of the random variables, that is given by

\[
p(m_1, m_2, s^n, x^n_1, z^n, x^n_2, y^n) = \prod_{i=1}^{n} p(m_i)p(m_2) \prod_{i=1}^{n} 1(x_{1,i}|m_1, s^n)1(z_i|x_{1,i}, s_i)1(x_{2,i}|m_2, z^{i-1})p(y_i|x_{1,i}, x_{2,i}, s_i) \quad (82a)
\]

It is easy to verify that the Markov chains in Eq. 80 also hold due to this distribution.

Note that \( I(U_Q; S_Q|Q) = I(U_Q, Q; S_Q) \) due to the first condition. Let \( U = (U_Q, Q), S = S_Q, X_1 = X_{1,Q}, X_2 = X_{2,Q}, Z = Z_Q \) and \( Y = Y_Q \). Thus, the rate-bounds become

\[
R_1 \leq I(X_1; Y|S, Z, U, X_2) + H(Z|U, S) - I(U; S) + \epsilon_n \quad (83a)
\]

\[
R_2 \leq I(X_2; Y|S, U, X_1) + \epsilon_n \quad (83b)
\]

\[
R_1 + R_2 \leq I(X_1, X_2; Y|U, Z, S) + H(Z|U, S) - I(U; S) + \epsilon_n \quad (83c)
\]

\[
R_1 + R_2 \leq I(X_1, X_2; Y|S) + \epsilon_n \quad (83d)
\]

with PMF that factorizes as

\[
p_S(s)p_{U|S}(u|s)p_{X_1|U,S}(x_1|u, s)1[z = z(x, s)] p_{X_2|U}(x_2|u)p_{Y|X_1,X_2,S}(y|x_1, x_2, s) \quad (84)
\]

This completes the proof for the converse part.

\[\square\]

VII. PROOF FOR THEOREM 2

A. Direct

The achievability part of Theorem 2 is based on previous section, with additional operation at Encoder 2. To avoid unnecessary repetitions, we only provide the differences in the achievability part relative to that in the previous section.

Codebook generation: Draw a cooperative-bin function bin\((z^n) \sim \text{Unif}[1 : 2^{nR_2}]\) for all \( z^n \in \mathcal{Z}^n \). Draw \( 2^{n(R_2' + R_2'')} \) sequences \( u^{n(b)}(l', l'') \) for \( l'^{(b-1)} \in [1 : 2^{nR_2'}] \) and \( l'' \in [1 : 2^{nR_2''}] \), each one is distributed according to \( \prod_{i=1}^{n} P_U(u_i(l', l'')) \). For each \( l', l'' \) and \( s^n_i \), draw \( 2^{n(R_2' + R_2'')} \) sequences \( z^n(m'_i, k|l', l'', s^n_i) \) distributed according to \( \prod_{i=1}^{n} P_{Z|U,i}(z_i|u_i(l', l''), s^n_i) \), \( s^n_i \), \( z_i(m'_i, k|l', l'', s^n_i) \) and \( 2^{nR_2} \) codewords \( x_2^n(m_2|l', l'', s^n_2) \) distributed according
Given $m_1(b)$, look for $k(b)$ such that there exist $\tilde{m}(b)$ that satisfies

$$\left(u^{n(b+1)}(\tilde{m}(b), l^{n(b)}), s^{n(b+1)}\right) \in \mathcal{A}^{(n)}(P_{S_1, U}),$$

where $\tilde{m}(b) = \text{bin}\left(z^n(m_1'^{1}, k(b))|l^{n(b-1)}, l^{n(b-1)}, s^{n(b)}\right)$.

This procedure is illustrated in Figure 11. The cooperative bin index is a superbin, that contains several $u^n$ sequences. The selected superbin contains a sequence $u^n$ that is coordinated with the states.

**Encoder 2:** At the end of each block $(b-1)$, the superbin index $l^{b-1}$ is known from the cribbed sequence $z^n(b-1)$. First, look for the first $\tilde{l}^{n(b-1)}$ s.t. $\left(u^{n(b)}(l^{b-1}), l^{n(b-1)}, s_2^{n(b)}\right) \in \mathcal{A}^{(n)}(P_{S_2, U})$. Then, the encoder sends $x^n_2(m_2^{(b)}|l^{n(b-1)}, l^{n(b-1)}, s_2^{n(b)})$.

**Decoder:** The decoding is done backwards. Assume that $l^{(b)} = (l^{n(b)}, l^{n(b)})$ is known from previous decoding operations.

1. For each $l^{n(b)}$, find $\tilde{m}^{n(b)}$ the same way that encoder 2 does. Then, find $\tilde{m}_1^{n(b)}(l^{n(b-1)}, s_2^{n(b)})$ and $\tilde{k}(b)$ s.t. bin $\left(\tilde{m}_1^{n(b)}, \tilde{k}(b)|l^{n(b-1)}, l^{n(b-1)}, s_1^{n(b)}\right) = l^{(b)}$. If there are multiple functions that satisfies the above, choose one uniformly. Note that there are total of $2^{nR'_B}$ tuples of functions, since we choose exactly one tuple for each $l^{n(b)} \in [1 : 2^{nR'_B}]$.

2. Look for $\left(\tilde{l}^{n(b-1)}, \tilde{m}_1^{n(b)}, \tilde{m}_2^{n(b)}\right)$ such that (85) is satisfied. We denote $\tilde{l}^{n(b-1)} = \tilde{l}^{n(b-1)}, l^{n(b-1)}$ for abbreviation.
Note at at block \( B \), the decoder knows the messages and therefore it needs only to find \( l^{(B-1)} \) according to the first operation.

**Error analysis:** Without loss of generality assume all messages \( m_1^{(b)}, m_1^{(b)}, m_2^{(b)} \) are equal to 1 for all \( b \in [1 : B] \).

We begin with the event of encoding error. Recall that according to Lemma 1 we can ensure that we will see approximately \( R''_B + \hat{R} \) different indexes by taking \( \hat{R} \leq H(Z|U, S_1) \) and \( \hat{R} < R'_B \). Thus, the existence of a sequence \( U^{n(b+1)} \) that is coordinated with \( S_1^{n(b+1)} \) is also ensured by taking \( I(U; S_1) < R'_B + \hat{R} \). Moreover, it follows from Markov lemma [23] that \( \left(U^{(b+1)}, S_2^{(b+1)}\right) \in \mathcal{A}_{1}^{(n)} \) with high probability (goes to 1 when \( n \) goes to infinity). Denote the selected superbin of the next block by \( L^{(b)} \) and the selected index in the bin by \( l^{(b)} \). At Encoder 2, we ensure that there exist only one \( l^{(b)} \) such that the sequence \( U^{n}(L^{(b)}, l^{(b)}) \) is jointly typical with \( S_2^{n(b+1)} \); this is done by taking \( R''_B < I(U; S_2) \). At the decoder, an error occurs if equation (88) is satisfied by \( \left(\hat{l}^{(b-1)}, \hat{m}_1^{(b)}, \hat{m}_2^{(b)} \right) \neq (L^{(b-1)}, 1, 1) \). This event is bounded by the union of the following events:

1) There exist \((\hat{m}_1^{(b)}, \hat{k}^{(b)})\) such that bin \( Z_{1}^{n(b)} (\hat{m}_1^{(b)}, \hat{k}^{(b)} | L_{1}^{(b-1)}, S_1^{n(b)}) = L_{1}^{(b)} \).

2) \( \left(\hat{l}^{(b-1)}, \hat{m}_1^{(b)}, \hat{m}_2^{(b)} \right) = (\neq L^{(b-1)}, *, *) \)

3) \( \left(\hat{l}^{(b-1)}, \hat{m}_1^{(b)}, \hat{m}_2^{(b)} \right) = (L^{(b-1)}, > 1, > 1) \)

4) \( \left(\hat{l}^{(b-1)}, \hat{m}_1^{(b)}, \hat{m}_2^{(b)} \right) = (L^{(b-1)}, 1, > 1) \)

5) \( \left(\hat{l}^{(b-1)}, \hat{m}_1^{(b)}, \hat{m}_2^{(b)} \right) = (L^{(b-1)}, > 1, 1) \)

Following similar steps as in Section [VI] a standard application of the packing lemma results in

\[
R'_1 + \hat{R} < H(Z|U, S_1) \tag{87a}
\]

\[
R'_1 + \hat{R} < R'_B \tag{87b}
\]

\[
R'_B + R''_1 + R_2 < I(X_1, X_2; Y|S_1, S_2) + I(U; S_1) \tag{87c}
\]

\[
R''_1 + R_2 < I(X_1, X_2; Y|Z_1, U, S_1, S_2) \tag{87d}
\]

\[
R_2 < I(X_2; Y|X_1, U, S_1, S_2) \tag{87e}
\]

\[
R''_1 < I(X_1; Y|X_2, Z_1, U, S_1, S_2) \tag{87f}
\]

and the encoding constraints are

\[
\hat{R} + R''_B > I(U; S_1) \tag{87g}
\]

\[
\hat{R} < H(Z|S_1, U) \tag{87h}
\]

\[
\hat{R} < R'_B \tag{87i}
\]

\[
R''_B < I(U; S_2), \tag{87j}
\]
Performing FME on (87) yields

\[ R_1 < I(X_1; Y|Z, U, X_2, S_1, S_2) + H(Z|U, S_1) - I(U; S_1|S_2) \]  
\[ R_2 < I(X_2; Y|X_1, U, S_1, S_2) \]  
\[ R_1 + R_2 < I(X_1, X_2; Y|U, Z, S_1, S_2) + H(Z|U, S_1) - I(U; S_1|S_2) \]  
\[ R_1 + R_2 < I(X_1, X_2; Y|S_1, S_2) \]

for all PMFs that factorize as \( P_{X_1,U|S_1}P_{X_2|U,S_2} \) and \( Z = z(X_1, S_1) \). Note that \( I(U; S_1) - I(U; S_2) = I(U; S_1|S_2) \) since \( S_2 \leftrightarrow S_1 \leftrightarrow U \) form a Markov chain.

\[ \square \]

B. Converse

Let \( U_i \triangleq (Z^{i-1}, S_1^{i-1}, S_2^{n,i+1}) \).

\[ H(Z^n|S_1^n, S_2^n) = H(Z^n, S_1^n, S_2^n) - H(S_1^n, S_2^n) \]  
\[ = \sum_{i=1}^{n} [H(Z_i|S_1, S_2, Z_i^{i-1}, S_1^{i-1}, S_2^{i+1}) - H(S_1, S_2)] \]  
\[ = \sum_{i=1}^{n} [H(Z_i|S_1, S_2, U_i) + H(S_1, S_2|U_i) - H(S_1, S_2)] \]  
\[ \leq \sum_{i=1}^{n} [H(Z_i|S_1, S_2, U_i) - I(U_i; S_1, S_2)] \]  
\[ = \sum_{i=1}^{n} [H(Z_i|S_1, S_2, U_i) - I(U_i; S_1|S_2)] \]  
\[ = n [H(Z_Q|S_1, S_2, Q, U_Q, Q) - I(U_Q; S_1, S_2|Q)] \]

where (a) follows since \( S_1^n \) and \( S_2^n \) are drawn i.i.d in pairs, (b) follows by our definition of \( U_i \) and (c) is derived by setting \( Q \sim \text{Unif}[1:n] \) to be a time sharing random variable. Note that the following Markov chains hold:

![Diagram](https://via.placeholder.com/150)

Fig. 12: Proof for Markov chains \( S_{2,i} \leftrightarrow S_{1,i} \leftrightarrow U_i \) and \( S_{2,i} \leftrightarrow (S_{1,i}, U_i) \leftrightarrow Z_i \) using an undirected graphical technique [27]. The undirected graph corresponds the PMF \( P(s_1^n, s_2^n, z^n) = P(s_1^{i-1}, s_2^{i-1})P(s_1, s_2)P(s_1^{i+1}, s_2^{i+1})P(z^n|s_1^n) \). The Markov chains follows since all paths from \( S_{2,i} \) to all other nodes go through \( S_{1,i} \).
Recall that the PMF on \((m_1, m_2, s_1^n, s_2^n, x_{1,i}, z^n, x_{2,i})\) is

\[
P(m_1, m_2, s_1^n, s_2^n, x_{1,i}, z^n, x_{2,i}) = P(m_1)P(m_2) \prod_{i=1}^{n} P(s_{1,i}, s_{2,i}) \]  \(91a\)

Note that \(Z^n\) is a deterministic function of \((M_1, S^n_1)\) since \(X^n_1\) is. Therefore, the Markov chain \((S_{1,i}, X_{1,i}) \leftrightarrow (S_{2,i}, U_i) \leftrightarrow X_{2,i}\) is readily proven from the PMF. As for the other Markovs in \((90)\), we use an undirected graphical technique in Figure \([12]\). It is also straightforward to show that \(S_{2,Q} \leftrightarrow (S_{1,Q}, U_Q, Q) \leftrightarrow Z_Q\) holds. Therefore,

\[
H(Z^n|S^n_1, S^n_2) = n[H(Z_Q|S_{1,Q}, S_{2,Q}, U_Q, Q) - I(U_Q; S_{1,Q}|S_{2,Q}, Q)] \]  \(92\)

\[
= n[H(Z_Q|S_{1,Q}, U_Q, Q) - I(U_Q; S_{1,Q}|S_{2,Q}, Q)] \]  \(93\)

Note that due to this identity, \(I(U_Q; S_{1,Q}|S_{2,Q}, Q) \leq H(Z_Q|S_{1,Q}, U_Q, Q)\). We proceed to bound \(R_1\) and \(R_2\). Note that by Fano’s inequality,

\[
H(M_1, M_2; Y^n, S^n_1, S^n_2) \leq n\epsilon_n \]  \(94\)

where \(\epsilon_n \to 0\) when \(n \to \infty\). Bounding \(R_1\) yields

\[
nR_1 = H(M_1) \]  \(95a\)

\[
(a) \to H(M_1|S^n_1, S^n_2) \]  \(95b\)

\[
(b) \to H(M_1, Z^n|S^n_1, S^n_2) \]  \(95c\)

\[
= H(M_1|Z^n, S^n_1, S^n_2) + H(Z^n|S^n_1, S^n_2) \]  \(95d\)

\[
(c) \to H(M_1|Z^n, S^n_1, S^n_2, M_2) + H(Z^n|S^n_1, S^n_2) \]  \(95e\)

\[
\leq I(M_1; Y^n|Z^n, S^n_1, S^n_2, M_2) + H(Z^n|S^n_1, S^n_2) + n\epsilon_n \]  \(95f\)

where:

(a) - follows since \(M_1 \perp (S^n_1, S^n_2)\)

(b) - follows since \(Z^n = f(M_1, S^n_1)\),
(c) - follows since $M_2 \perp (M_1, Z^n, S^n_1, S^n_2)$. It follows that

$$I(M_1; Y^n | Z^n, S^n_1, S^n_2, M_2) = \sum_{i=1}^{n} I(M_1; Y_i | Y^{i-1}, Z^n, S^n_1, S^n_2, M_2)$$  \hspace{1cm} (96a)

$$= (d) \sum_{i=1}^{n} I(M_1, X_{1,i}; Y_i | X_{2,i}, Y^{i-1}, Z^n, S^n_1, S^n_2, M_2)$$  \hspace{1cm} (96b)

$$\leq (e) \sum_{i=1}^{n} I(X_{1,i}; Y_i | X_{2,i}, Y^{i-1}, Z^n, S^n_1, S^n_2, M_2)$$  \hspace{1cm} (96c)

$$= \sum_{i=1}^{n} I(X_{1,i}; Y_i | X_{2,i}, Z^n, S^n_1, S^n_2, M_2)$$  \hspace{1cm} (96d)

$$= n I(X_{1,Q}; Y_Q | X_{2,Q}, Z_Q, S_{1,Q}, S_{2,Q}, U_Q, Q)$$  \hspace{1cm} (96e)

where (d) follows since $X_{1,i}$ is a function of $(M_1, S^n_1)$ and (e) follows by moving $(M_2, Y^{i-1}, Z^n_{i+1}, S^n_{1,i+1}, S^n_2, M_2)$ from the conditioning to the left hand side of the mutual information; since the channel is memoryless and without feedback, $(M_2, Y^{i-1}, Z^n_{i+1}, S^n_{1,i+1}, S^n_2) \leftrightarrow (X_{1,i}, X_{2,i}, S_{1,i}, S_{2,i}) \leftrightarrow Y_i$ holds.

We derive with the bound

$$R_1 \leq n [I(X_{1,Q}; Y_Q | X_{2,Q}, Z_Q, S_{1,Q}, S_{2,Q}, Q) + H(Z_Q | S_{1,Q}, U_Q, Q) - I(U_Q; S_{1,Q} | S_{2,Q}) + \epsilon_n]$$  \hspace{1cm} (97)

Following similar steps, we have

$$nR_2 = H(M_2)$$  \hspace{1cm} (98a)

$$= H(M_2 | S^n_1, S^n_2, M_1)$$  \hspace{1cm} (98b)

$$\leq I(M_2; Y^n | S^n_1, S^n_2, M_1) + n \epsilon_n$$  \hspace{1cm} (98c)

$$= \sum_{i=1}^{n} I(M_2; Y_i | Y^{i-1}, S^n_1, S^n_2, M_1) + n \epsilon_n$$  \hspace{1cm} (98d)

$$= \sum_{i=1}^{n} I(M_2, X_{2,i}; Y_i | Y^{i-1}, S^n_1, S^n_2, M_1, X_{1,i}) + n \epsilon_n$$  \hspace{1cm} (98e)

$$\leq \sum_{i=1}^{n} I(Y^{i-1}, S^n_{1,i+1}, S^n_{2,i+1}, M_1, M_2, X_{2,i}; Y_i | S^n_1, S^n_{2,i}, X_{1,i}) + n \epsilon_n$$  \hspace{1cm} (98f)

$$= \sum_{i=1}^{n} I(X_{2,i}; Y_i | S^n_1, S^n_{2,i}, X_{1,i}) + n \epsilon_n$$  \hspace{1cm} (98g)

$$= n I(X_{2,Q}; Y_Q | X_{1,Q}, S_{1,Q}, S_{2,Q}, Q) + n \epsilon_n$$  \hspace{1cm} (98h)
The sum-rate $R_1 + R_2$ is upper bounded by

$$n(R_1 + R_2) = H(M_1) + H(M_2)$$

$$= H(M_1, M_2)$$

$$= H(M_1, M_2 | S_1^n, S_2^n)$$

$$= H(M_1, M_2, Z^n | S_1^n, S_2^n)$$

$$= H(M_1, M_2 | Z^n, S_1^n, S_2^n) + H(Z^n | S_1^n, S_2^n)$$

$$\leq I(M_1, M_2; Y^n | Z^n, S_1^n, S_2^n) + H(Z^n | S_1^n, S_2^n) + n\epsilon_n$$

where

$$I(M_1, M_2; Y^n | Z^n, S_1^n, S_2^n) = \sum_{i=1}^n I(M_1, M_2; Y_i | Y_i^{i-1}, Z^n, S_1^n, S_2^n)$$

$$\leq \sum_{i=1}^n I(M_1, M_2, S_{i,1}^n, S_{i,2}^{i-1}, Y_i^{i-1}, Z_{i-1}^i, X_{1,i}, X_{2,i}; Y_i | Z^n, S_1^n, S_2^n)$$

$$= nI(X_{1,Q}, X_{2,Q}; Y_Q | Z_Q, S_1^Q, S_2^Q, U_Q, Q)$$

and therefore, it follows from the identity in (92) and the above that

$$n(R_1 + R_2) \leq n\left[I(X_{1,Q}, X_{2,Q}; Y_Q | Z_Q, S_1^Q, S_2^Q, U_Q, Q) + H(Z_Q | S_1^Q, U_Q, Q - I(U_Q; S_1^Q | S_2^Q)) + \epsilon_n\right]$$

and the second upper bound by:

$$n(R_1 + R_2) = H(M_1, M_2)$$

$$= H(M_1, M_2 | S_1^n, S_2^n)$$

$$= H(M_1, M_2, Z^n | S_1^n, S_2^n)$$

$$\leq I(M_1, M_2; Y^n | S_1^n, S_2^n) + n\epsilon_n$$

$$= \sum_{i=1}^n I(M_1, M_2; Y_i | Y_i^{i-1}, S_1^n, S_2^n) + n\epsilon_n$$

$$\leq \sum_{i=1}^n I(M_1, M_2, Y_i^{i-1}, S_1^{i-1}, S_2^{i-1}, X_{1,i}, X_{2,i}; Y_i | S_1^{i-1}, S_2^{i-1}) + n\epsilon_n$$

$$= nI(X_{1,i}, X_{2,i}; Y_i | S_1^{i-1}, S_2^{i-1}) + n\epsilon_n$$

$$= n(I(X_{1,Q}, X_{2,Q}; Y_Q | S_1^Q, S_2^Q, Q) + \epsilon_n)$$

$$\leq n(I(X_{1,Q}, X_{2,Q}; Y_Q | S_1^Q, S_1^Q) + \epsilon_n)$$
where the last inequality is due to the Markov chain $Q \leftrightarrow (X_1, Q, X_2, Q) \leftrightarrow Y_Q$. Thus, we obtained the following region

\[
R_1 < I(X_1; Q|Y)X_2, Z, Q, S_1, Q, S_2, Q) + H(Z_Q|S_1, Q, U, Q) - I(U_Q; S_1, Q|S_2, Q) 
\]

(103a)

\[
R_2 < I(X_2; Q|X_1, Q, S_1, Q, S_2, Q) 
\]

(103b)

\[
R_1 + R_2 < I(X_1, Q, X_2, Q; Y_Q|S_1, Q, S_2, Q, U, Q, Q) + H(Z_Q|S_1, Q, U, Q, Q) - I(U_Q; S_1, Q|S_2, Q) 
\]

(103c)

\[
R_1 + R_2 < I(X_1, Q, X_2, Q; Y_Q|S_1, Q, S_1, Q) 
\]

(103d)

\[
0 < H(Z_Q|S_1, Q, U, Q, Q) - I(U_Q; S_1, Q|S_2, Q, Q) 
\]

(103e)

for PMFs of the form

\[
p(q)p_{S_1, S_2}(s_{1, q}, s_{2, q})p(w_{1, q}, s_{1, q}, q)p(x_{2, q}|w_{1, q}, s_{2, q})p(y|X_1, X_2, s_1, s_2 = y_Q, x_{1, q}, x_{2, q}, s_{1, q}, s_{2, q}). 
\]

(104)

Note that the PMF in (104) regarding $S_1, S_2$ and $Y$ follows since the states are i.i.d. and the channel is memoryless and fixed (per state). The rest of the proof (regarding the removal of the time sharing random variable $Q$) is straightforward using the same steps as in the case of one state component in Appendix VI-B. Therefore, by letting $U = (U_Q, Q), X_1 = X_1, Q, X_2 = X_2, Q, Y = Y_Q, Z = Z_Q, S_1 = S_1, Q$ and $S_2, Q$ we obtain the capacity region in Theorem 2.

\[\square\]

VIII. Proof for Theorem 3

The proof for this theorem heavily relies on the proofs from previous sections. The achievability part builds on cooperative-bin-forward scheme from section VII by combining it with instantaneous relaying (a.k.a Shannon strategies). To avoid unnecessary repetition, we only provide the differences on the achievability part and the proofs for Markov chains in the converse.

**Achievability:** The codebook generation is done similarly as in VII-A with additional conditioning on $Z$ when drawing $x^n_2(m_{2}^{(b)}|l^{(b-1)}, s^n_2)$. Namely, the codebook constructed for Encoder 2 are as follows. For each block $b$, $s^n_2 \in S^n_Z$, $z \in Z$ and $(l^{(b-1)}, l^{(b-1)})$, draw $2^{nR_2}$ codewords

\[
x^n_2(m_{2}^{(b)}|z, u^n(l^{(b-1)}, l^{(b-1)}), s^n_2) \sim \prod_{i=1}^{n} p_{X_2|Z, U, S_2}(x_{2, i}|z, u_i(l^{(b-1)}, l^{(b-1)}), s_{2, i}) 
\]

(105)

In each transmission block, Encoder 1 performs the same operations as before. Encoder 2 also performs the same operation, but at each time $i$ it transmits $x_{2, i}(m_{2}^{(b)}|z_i, u^n(l^{(b-1)}, l^{(b-1)}), s^n_2)$. The decoder performs backward decoding as before w.r.t. the new codebook. All other operations are preserved and the same error analysis holds. The derivation result in the same achievable rate region, under the new PMF factorization $p_{U, X_1|S_1, P_{X_2}|Z, U, S_2}$.

**Converse:** The only difference in the converse compares to that of the previous section is that we need to show the PMF factorization and prove the new Markov chains. The rate bounds on $R_1$ and $R_2$ are the same and obtained
using the exact same arguments. Continuing the derivation from this point, we need to show that the following Markov chains hold

\[ S_{2,i} \leftrightarrow S_{1,i} \leftrightarrow U_i \]  \\
\[ S_{2,i} \leftrightarrow (S_{1,i}, U_i) \leftrightarrow Z_i \]  \\
\[ (S_{1,i}, X_{1,i}) \leftrightarrow (S_{2,i}, U_i, Z_i) \leftrightarrow X_{2,i} \]

(106a) (106b) (106c)

Note that now the PMF of the random variable is

\[ p(m_1, m_2, s_{1,i}^n, s_{2,i}^n, x_{1,i}, z^n, x_{2,i}) = p(m_1)p(m_2) \prod_{i=1}^n p(s_{1,i}, s_{2,i}) \left[ 1(x_{1,i}, z^n|s_{1,i}^n, m_1)1(x_{2,i}|z^i, s_{2,i}^n, m_2) \right] \]

(107)

Now \( x_{2,i} \) is also a function of \( z_i \) and not only \( z^{i-1} \). Therefore, the first two Markov-chains hold due to the same arguments in the previous section. As for the last Markov, consider

\[ p(m_1, m_2, s_{1,i}^n, s_{2,i}^n, x_{1,i}, z^n, x_{2,i}) = p(m_1)p(m_2) \prod_{i=1}^n p(s_{1,i}, s_{2,i}) \left[ 1(x_{1,i}, z^n|s_{1,i}^n, m_1)1(x_{2,i}|z^i, s_{2,i}^n, m_2) \right] \]

\[ = p(s_{1}^{i-1})p(s_{1,i}, s_{2,i})p(s_{2,i+1}^n)1(x_{1,i}, z^n|s_{1,i+1}^n)1(x_{2,i}, z^i|s_{2,i+1}^n)1(x_{2,i}, m_2|z^i, s_{2,i}^n) \]

\[ = p(s_{1}^{i-1})p(s_{1,i}, s_{2,i})p(s_{2,i+1}^n)1(x_{1,i}, z^n|s_{1,i+1}^n) \]

\[ \times p(x_{2,i}, m_2, s_{2,i}^{i-1}|z^i, s_{2,i}^n, s_{1}^{i-1}) \]

(108a) (108b) (108c) (108d)

Summing for \( (m_1, m_2, z_{i+1}^n, s_{2,i}^{i-1}, s_{1,i+1}^n) \) results in

\[ p(s_{1}^{i-1})p(s_{1,i}, s_{2,i})p(s_{2,i+1}^n)p(x_{1,i}, z^i|s_{1}^{i-1}, s_{2,i+1}^n)1(x_{2,i}, z^i|s_{2,i}^{i-1}) \]

(109)

in which \( (S_{1,i}, X_{1,i}) \leftrightarrow (S_{2,i}, Z_{1,i}^{i-1}, S_{2,i+1}^n, Z_i) \leftrightarrow X_{2,i} \) is Markov. All other arguments regarding the memoryless property of the channel and the time-sharing random variable \( Q \) hold. This concludes the proof of Theorem 3.

\[ \Box \]

IX. CONCLUSIONS AND FINAL REMARKS

Using a variation of the cooperative-bin-forward scheme, we have found the capacity of the SD-RC and MAC with partial cribbing, when non-causal CSI is given to the decoder and one of the transmitters. Remarkably in both setups only one auxiliary random variable is used for obtaining the capacity region. The same cooperation codeword is designated to play both the roles of common message and compression of the state sequence. It is evident that in the special case of the MAC the non-causal access to the state endowed states compression and, consequently, increased the capacity region.

Cooperative-bin-forward heavily relies on the fact that the link for the cooperation, i.e., the link from the encoder to the relay (or the cribbed signal in the MAC) is deterministic. Since the transmitter can predict and dictate the observed output (by the relay) it can coordinate with the relay based on the same bin index. However, it is not
known how the cooperative-bin-forward scheme can be generalized to cases where the link between the encoder and the relay is a general noisy link.

APPENDIX A

PROOF FOR INDIRECT COVERING LEMMA

In section V we presented an indirect covering lemma. Although we do not actually perform covering in a traditional manner, we do ask for the number of seen bin indexes. Namely, we want to bound the following probability

\[ P_e^{(n)} \triangleq \mathbb{P} \left[ \left| \left\{ l : \exists k \text{ s.t. Bin} \left( Z^n(k) \right) = l \right\} \right| < 2^{n(R - \delta_n)} \right] \leq \Delta_n \tag{110} \]

and ensure that both \( \delta_n \) and \( \Delta_n \) goes to zero as \( n \) goes to infinity.

Assume \( v^n \in A_{(n)}(pv) \) and recall that according to the random experiment, we have

\[ \mathbb{P} \left[ \left\{ Z^n(k) \right\}_k, \{\text{Bin}(Z^n(k))\}_k \right| V^n = v^n \right] = \prod_{k=1}^{2^nR} p_{Z|V}(z^n(k)|v^n)^{2^{-nR_B}} \tag{111} \]

where \( p_{Z|V}(z^n(k)|v^n) = \prod_{i=1}^{n} p_{Z|V}(z_i(k)|v_i) \).

Define the sets

\[ \mathcal{D}_1 \triangleq \left\{ k : (Z^n(k), v^n) \in A_{(n)}^{(n)} \right\} \tag{112a} \]
\[ \mathcal{D}_2 \triangleq \left\{ k : Z^n(k) \neq Z^n(j) \quad \forall j \neq k \text{ and } k, j \in \mathcal{D}_1 \right\} \tag{112b} \]
\[ \mathcal{D}_3 \triangleq \left\{ k : \text{Bin} \left( Z^n(k) \right) \neq \text{Bin} \left( Z^n(j) \right) \quad \forall j \neq k \text{ and } k, j \in \mathcal{D}_2 \right\} \tag{112c} \]

and the events

\[ E_1 \triangleq |\mathcal{D}_1| < 2^{n(R - \delta_n^{(1)})} \tag{113a} \]
\[ E_2 \triangleq |\mathcal{D}_2| < 2^{n(R - \delta_n^{(2)})} \tag{113b} \]
\[ E_3 \triangleq |\mathcal{D}_3| < 2^{n(R - \delta_n^{(3)})} \tag{113c} \]

By definition of \( E_3 \) and law of total probability, it follows that

\[ P_e^{(n)} \leq \mathbb{P} [E_3 | V^n = v^n] \tag{114a} \]
\[ \leq \mathbb{P} [E_1 | V^n = v^n] + \mathbb{P} [E_2 | E_1^c, V^n = v^n] + \mathbb{P} [E_3 | E_2^c, E_1^c, V^n = v^n] \tag{114b} \]

We will bound each probability separately.
1) Define \( \theta_k = 1 \left[ (Z^n(k), v^n) \in \mathcal{A}_c^{(n)}(\rho_{Z,V}) \right] \), and note that \( \theta_k \) is Bernoulli \( (\rho_n) \), where \( 1 - \delta_n \leq \rho_n \leq 1 \) and \( \delta_n \to 0 \) as \( n \to \infty \). Therefore, for any \( \delta' > 0 \) we have

\[
\mathbb{P} \left[ E_1 | V^n = v^n \right] = \mathbb{P} \left[ |D_1| \leq 2^n(R - \delta_n^{(1)}) | V^n = v^n \right]
\]

(a) - by setting \( \delta_n^{(1)} = -\frac{1}{n} \log_2 \rho_n (1 - \delta') \) \( \to 0 \) as \( n \to \infty \),

(b) - by Chernoff's inequality \( [23, \text{Appendix B}] \),

where:

\[
\delta_n^{(1)} = \Delta_n^{(1)}
\]

where:

(a) - by setting \( \delta_n^{(1)} = -\frac{1}{n} \log_2 (\rho_n (1 - \delta')) \) \( \to 0 \) as \( n \to \infty \),

(b) - by Chernoff's inequality \( [23, \text{Appendix B}] \),

and \( \Delta_n^{(1)} \to 0 \) as \( n \to \infty \).

2) We will now deal with \( E_2 \). First, note that given \( E_1 \), we have with probability one that \( |D_1| > 2^n(R - \delta_n^{(1)}) \).

We are interested in \( |D_2| \), so let us define the normalized amount of bad sequences in \( D_1 \),

\[
C_2 = \frac{1}{|D_1|} \sum_{k \in D_1} \mathbb{1} \left[ \exists j \neq k : Z^n(j) = Z^n(k), j \in D_1 \right]
\]

(116)

By this definition, it follows that \( |D_2| = |D_1|(1 - C_2) \). First, we bound the expected value of \( C_2 \) by

\[
\mathbb{E}[C_2 | V^n = v^n, E_1^c] = \sum_{d_1} \mathbb{P}[D_1 = d_1 | V^n = v^n, E_1^c] \mathbb{E}[C_2 | V^n = v^n, E_1^c, D_1 = d_1]
\]

(117a)

\[
= \sum_{d_1} \mathbb{P}[D_1 = d_1 | V^n = v^n, E_1^c] \frac{1}{|d_1|} \times \sum_{k \in d_1} \mathbb{P}[\exists j \neq k : Z^n(j) = Z^n(k), j \in D_1 | V^n = v^n, E_1^c]
\]

(117b)

\[
\leq \sum_{d_1 : |d_1| > 2^n(R - \delta_n^{(1)})} \mathbb{P}[D_1 = d_1 | V^n = v^n, E_1^c] \frac{1}{|d_1|} \sum_{k \in d_1} 2^{-n(H(Z|V) - \epsilon')} \]

(117c)

\[
= \sum_{d_1 : |d_1| > 2^n(R - \delta_n^{(1)})} \mathbb{P}[D_1 = d_1 | V^n = v^n, E_1^c] |d_1| 2^{-n(H(Z|V) - \epsilon' + \delta_n^{(1)})}
\]

(117d)

\[
\leq 2^n(R - H(Z|V) + \epsilon' + \delta_n^{(1)})
\]

(117e)

Therefore, for any \( \gamma_1 > 0 \) it follows by Markov's inequality that

\[
\mathbb{P} \left[ C_2 > 2^{-n\gamma_1} | E_1^c, V^n = v^n \right] \leq 2^{n(R - H(Z|V) + \delta_n^{(1)} + \epsilon' + \gamma_1)}
\]

(118a)

\[
= \Delta_n^{(2)}
\]

(118b)

where \( \Delta_n^{(2)} \to 0 \) as \( n \to \infty \) if \( R < H(Z|V) - \gamma_1 \) and \( \gamma_1 = \delta_n^{(1)} + \epsilon' + \gamma_1 \). By setting \( \delta_n^{(2)} = \delta_n^{(1)} - \frac{1}{n} \log_2 \left( 1 - 2^{-n\gamma_1} \right) \) we have

\[
\mathbb{P} \left[ E_2 | E_1^c, V^n = v^n \right] \leq \Delta_n^{(2)}.
\]

(119)
and \( \delta_n^{(2)} \to 0 \) as \( n \to \infty \).

3) We will follow similar arguments as the previous bound. Define

\[
C_3 = \frac{1}{|D_2|} \sum_{k \in D_2} 1 \left[ \exists j \neq k : \text{Bin}(Z^n(j)) = \text{Bin}(Z^n(k)), j \in D_2 \right] \tag{120}
\]

and recall that the probability of each bin index is independent of the realization of \( \{ Z^n(k) \} \). It follows that

\[
\mathbb{E}[C_3|E_2^c, E_1^c, V^n = v^n] \leq 2^{n(R - R_B + \delta_n^{(2)})}. \tag{121}
\]

By Markov’s inequality, for any \( \gamma_2 > 0 \)

\[
P[E_3|E_2^c, E_1^c, V^n = v^n] = P[|D_3| < 2^{n(R - \delta_n^{(3)})}|E_2^c, E_1^c, V^n = v^n] \tag{122a}
\]

\[
\leq 2^{n(R - R_B + \delta_n^{(2)} + \gamma_2^2)} \tag{122b}
\]

\[
= \Delta_n^{(3)} \tag{122c}
\]

where \( \Delta_n^{(3)} \to 0 \) and \( \delta_n^{(3)} = \delta_n^{(2)} - \frac{1}{n}(1 - 2^{-n\gamma_2^2}) \to 0 \) as \( n \to \infty \), if \( R < H(Z|V) - \gamma_2 \) where \( \gamma_2 = \delta_n^{(2)} + \gamma_2^2 \).

Finally, for any \( \gamma_1, \gamma_2 > 0 \) and \( n \) sufficiently large, if

\[
R < H(Z|V) - \gamma_1 \tag{123a}
\]

\[
R < H(Z|V) - \gamma_2 \tag{123b}
\]

then

\[
P_e^{(n)} \leq \Delta_n^{(1)} + \Delta_n^{(2)} + \Delta_n^{(3)} \tag{124}
\]

where \( \delta_n^{(i)}, \Delta_n^{(i)} \) tends to 0 when \( n \to \infty \) for \( i = 1, 2, 3 \). \( \square \)

**APPENDIX B**

**Proofs for special cases of MAC**

The special cases in section [III] are captured by Theorem 3. We restate here the region as a reference for the following derivations. To simplify the derivations, we consider the region for only one state component \( S \) which is available only at Encoder 1. The capacity region for discrete memoryless MAC with non-causal CSI in Fig. 3 is given by the set of rate pairs \( (R_1, R_2) \) that satisfy

\[
R_1 \leq I(X_1; Y|X_2, Z, S, U) + H(Z|S, U) - I(U; S) \tag{125a}
\]

\[
R_2 \leq I(X_2; Y|X_1, S, U) \tag{125b}
\]

\[
R_1 + R_2 \leq I(X_1, X_2; Y|Z, S, U) + H(Z|S, U) - I(U; S) \tag{125c}
\]

\[
R_1 + R_2 \leq I(X_1, X_2; Y|S) \tag{125d}
\]

for PMFs of the form \( p_U|S|p_{X_1|S,U|p_{X_2}|U} \), with \( Z = z(X_1, S) \), that satisfies

\[
I(U; S) \leq H(Z|U, S), \tag{125e}
\]
Case A: Multiple Access Channel with states (without cribbing): This case is captured by Theorem 2 by setting $z(x_1, s) = 0$, $\forall x_1 \in X_1$, $s \in S$, since in this configuration there is no cribbing between the encoders. The inequality in (125c) results in $I(U; S) \leq 0$, which enforces $U$ to be independent of $S$. Thus, region in (125) becomes

\begin{align}
R_1 &\leq I(X_1; Y | S, U, X_2) \\
R_2 &\leq I(X_2; Y | S, U, X_1) \\
R_1 + R_2 &\leq I(X_1, X_2; Y | S, U) \\
R_1 + R_2 &\leq I(X_1, X_2; Y | S),
\end{align}

with PMF of the form $p_{U|X_1,U,S}p_{X_2|U}$. Note that $U \leftrightarrow (X_1, X_2, S) \leftrightarrow Y$ forms a Markov chain. Therefore, the last inequality is redundant. It also implies that the capacity region in (126) is outer bounded by (8); degenerating $U$ achieves that outer bound.

Case B: State dependent MAC with cooperation: We investigate capacity region for the case of orthogonal cooperation link and channel transmission, as depicted in Fig. 5. The cooperation link here is strictly causal due to the cribbing, i.e., $X_{2,i} = f(M_2, X_{1,p}^i)$. First, note that the region in (5) is an outer bound, since it is the capacity region of non-causal cooperation, i.e., when $X_{2,i} = f(M_2, X_{1,p}^i)$. The strictly causal configuration is captured by the cribbing setup when setting $X_1 = (X_{1c}, X_{1p})$, $Z = X_{1p}$, and the channel transition PMF to $p_{Y|X_{1c},X_{2c},S}$. Then, the region in (125) becomes

\begin{align}
R_1 &\leq I(X_{1c}; Y | S, X_{1p}, U, X_2) + H(X_{1p}|U, S) - I(U; S) \\
R_2 &\leq I(X_2; Y | S, U, X_{1c}, X_{1p}) \\
R_1 + R_2 &\leq I(X_{1c}, X_{2c}; Y | S, U, X_{1p}) + H(X_{1p}|U, S) - I(U; S) \\
R_1 + R_2 &\leq I(X_{1c}, X_{1p}, X_{2c}; Y | S) \\
I(U; S) &\leq H(X_{1c}|U, S),
\end{align}

for PMFs of the form $p_{U|S}p_{X_1|U,S}p_{X_2|U}$. Note that $I(X_{1c}, X_{1p}, X_{2c}; Y | S) = I(X_{1c}, X_{2c}; Y | S)$ because $X_{1p} \leftrightarrow (X_{1c}, X_{2c}, S) \leftrightarrow Y$ is a Markov chain. We identify the rate $H(X_{1p}|U, S)$ as the cooperation rate $R_{12}$. Let $p_{X_1|U,S} = p_{X_{1p}|U,S}p_{X_{1c}|U,S}$, and $p_{X_{1p}|U=S}$ be a uniform distribution for every $(u, s) \in U \times S$. By doing so, $H(X_{1p}|U, S) = \log_2 |X_{1p}|$ and $I(X_1; Y | X_2, U, S, X_{1p}) = I(X_{1c}; Y | X_{2c}, U, S)$. The latter holds since $X_{1p} \leftrightarrow (X_{1c}, X_{2c}, S) \leftrightarrow Y$ is a Markov chain and $X_{1c}$ is independent of $X_{1p}$. By denoting $R_{12} = \log_2 |X_{1p}|$, the regions in (5) and (127) coincide.

Case C: Point-to-point with non-causal CSI: First, note that the channel depends only on $X_1$ and $S$. Encoder 1 sends a message over the channel, and the states are revealed to it non-causally at the beginning of the transmission. Encoder 2, however, has no message to send; in fact, it cannot send anything over the channel since the channel’s
output is not affected by $X_2$ at all. Therefore, the rate $R_2$ is 0. This configuration is captured by the MAC when

$$R_2 = 0$$ (128a)

$$p_Y|X_1, X_2, S = p_Y|X_1, S.$$ (128b)

Inserting (128) into Theorem 2 derives with

$$R_1 \leq I(X_1; Y|S, U, Z, X_2) + H(Z|U, S) - I(U; S)$$ (129a)

$$R_1 \leq I(X_1, X_2; Y|S, U, Z) + H(Z|U, S) - I(U; S)$$ (129b)

$$R_1 \leq I(X_1, X_2; Y|S)$$ (129c)

$$I(U; S) \leq H(Z|U, S)$$ (129d)

with $p.s.u.x_1 1_z|x_1, s p_x_2|u p_y|x_1, s$. Due to the Markov chains $X_2 \leftrightarrow (X_1, S) \leftrightarrow Y$, $X_2 \leftrightarrow (X_1, U, S, Z) \leftrightarrow Y$ and $X_2 \leftrightarrow (U, S, Z) \leftrightarrow Y$, the following identities hold

$$I(X_1, X_2; Y|S) = I(X_1; Y|S)$$ (130a)

$$I(X_1, X_2; Y|S, U, Z) = I(X_1; Y|S, U, Z, X_2)$$ (130b)

$$I(X_1, X_2; Y|S, U, Z) = I(X_1; Y|U, Z, S)$$ (130c)

Therefore, the region in (129) reduces to

$$R_1 \leq I(X_1; Y|S, U, Z) + H(Z|U, S) - I(U; S)$$ (131a)

$$R_1 \leq I(X_1; Y|S)$$ (131b)

$$I(U; S) \leq H(Z|U, S)$$ (131c)

This region is smaller or equal to (10); if we drop the first and last inequalities, we get the expression for capacity. On the other hand, to show that the capacity in (10) is achievable, degenerate $U$ in (131a). The result is that $I(U; S) = 0$, and the last inequality is redundant. Moreover,

$$I(X_1; Y|S, Z) + H(Z|S) = I(X_1, Z; Y|S) - I(Z; Y|S) + H(Z|S)$$ (132a)

$$= I(X_1, Z; Y|S) + H(Z|Y, S),$$ (132b)

so $Z = f(X_1, S)$, thus $I(X_1; Y|S) = I(X_1, Z; Y|S)$. Therefore, the first inequality becomes $R_1 \leq I(X_1; Y|S) + H(Z|S, Y)$, which is also redundant due to the second.

The expressions for the capacity after dropping the constraints are not exactly the same, since the PMF domains are different. However, the capacity coincide, due to the objective and maximization.
Point-to-point with state encoder and output causality constraint: This configuration is captured by the MAC with cribbing, by setting

\[ R_1 = 0 \]  
\[ p_{Y|X_1,X_2,S} = p_{Y|X_2,S} \]  
\[ z(x_1, s) = x_1. \]

The region in (125) reduces to

\[ R_2 \leq I(X_2; Y|S, U, X_1) \]  
\[ R_2 \leq I(X_2; Y|S, U, X_1) + H(X_1|U, S) - I(U; S) \]  
\[ R_2 \leq I(X_1, X_2; Y|S) \]  
\[ I(U; S) \leq H(X_1|U, S). \]

with \( p_{U,X_1|S} p_{X_2|U} p_{Y|X_2,S} \). Notice that \( I(X_2; Y|S, U, X_1) \leq I(X_1, X_2, U; Y|S) \), and both \((U, X_1) \leftrightarrow (X_2, S) \leftrightarrow Y\) and \(X_1 \leftrightarrow (X_2, S) \leftrightarrow Y\) are Markov chains. Therefore, the third inequality is redundant. Moreover, from the constraint \( I(U; S) \leq H(X_1|U, S) \), it follows that \( I(X_2; Y|S, U, X_1) \leq I(X_2; Y|U, X_1, S) + H(X_1|U, S) - I(U; S) \); thus, the second inequality is also redundant. The Markov chains \( Y \leftrightarrow (U, S) \leftrightarrow X_1 \) and \( Y \leftrightarrow (X_2, S, U) \leftrightarrow X_1 \) imply that \( I(X_2; Y|S, U, X_1) = I(X_2; Y|S, U) \). Therefore, the region is further reduced to

\[ R_2 \leq I(X_2; Y|S, U) \]  
\[ I(U; S) \leq H(X_1|U, S). \]

Note that \( H(X_1|U, S) \leq \log_2 |X_1| \), and therefore, this region is upper bounded by the capacity. By taking \( P_{X_1|U,S} \) to be uniform distribution for every \((u, s) \in U \times S\), the conditional entropy \( H(X_1|U, S) \) equals to \( \log_2 |X_1| \) and we achieve the capacity.

References

[1] E. C. V. D. Meulen, “Three-terminal communication channels,” Advances in Applied Probability, vol. 3, no. 1, pp. 120–154, 1971. [Online]. Available: [http://www.jstor.org/stable/1426331](http://www.jstor.org/stable/1426331)

[2] T. Cover and A. E. Gamal, “Capacity theorems for the relay channel,” IEEE Trans. Inf. Theory, vol. 25, no. 5, pp. 572–584, Sep 1979.

[3] A. El Gamal and M. Aref, “The capacity of the semi-deterministic relay channel,” IEEE Trans. Inf. Theory, vol. 28, no. 3, p. 536, May 1982.

[4] R. Kolte, A. Özgür, and H. Permuter, “Cooperative binning for semideterministic channels,” IEEE Trans. Inf. Theory, vol. 62, no. 3, pp. 1231–1249, March 2016.

[5] B. Akhbari, M. Mirmohseni, and M. R. Aref, “Compress-and-forward strategy for relay channel with causal and non-causal channel state information,” IET Communications, vol. 4, no. 10, pp. 1174–1186, July 2010.

[6] M. N. Khormuji, A. E. Gamal, and M. Skoglund, “State-dependent relay channel: Achievable rate and capacity of a semideterministic class,” IEEE Trans. Inf. Theory, vol. 59, no. 5, pp. 2629–2638, May 2013.

[7] M. N. Khormuji and M. Skoglund, “The relay channel with partial causal state information,” in In Proc. Int. Symp. Inf. Theory (ISIT-2008), Dec 2008, pp. 1–6.
[8] Z. Deng, F. Lang, B. Y. Wang, and S. m. Zhao, “Capacity of a class of relay channel with orthogonal components and non-causal channel state,” in *Proc. Int. Symp. Inf. Theory (ISIT-2013)*, July 2013, pp. 2696–2700.

[9] A. Zaidi, S. Shamai, P. Piantanida, and L. Vandendorpe, “Bounds on the capacity of the relay channel with noncausal state at the source,” *IEEE Trans. Inf. Theory*, vol. 59, no. 5, pp. 2639–2672, May 2013.

[10] A. Zaidi, S. P. Kotagiri, J. N. Laneman, and L. Vandendorpe, “Cooperative relaying with state available noncausally at the relay,” *IEEE Trans. Inf. Theory*, vol. 56, no. 5, pp. 2272–2298, May 2010.

[11] Y. H. Kim, “Capacity of a class of deterministic relay channels,” *IEEE Trans. Inf. Theory*, vol. 54, no. 3, pp. 1328–1329, March 2008.

[12] I. E. Aguerri and D. Gündüz, “Capacity of a class of state-dependent orthogonal relay channels,” *IEEE Trans. Inf. Theory*, vol. 62, no. 3, pp. 1280–1295, March 2016.

[13] F. Willems, “The discrete memoryless multiple access channel with partially cooperating encoders (corresp.),” *IEEE Trans. Inf. Theory*, vol. 29, no. 3, pp. 441–445, May 1983.

[14] H. Permuter, S. Shamai, and A. Somekh-Baruch, “Message and state cooperation in multiple access channels,” *IEEE Trans. Inf. Theory*, vol. 57, no. 10, pp. 6379–6396, Oct 2011.

[15] F. Willems and E. van der Meulen, “The discrete memoryless multiple-access channel with cribbing encoders,” *IEEE Trans. Inf. Theory*, vol. 31, no. 3, pp. 313–327, May 1985.

[16] O. Simeone, N. Levy, A. Sanderovich, O. Somekh, B. M. Zaidel, H. V. Poor, S. Shamai *et al.*, “Cooperative wireless cellular systems: An information-theoretic view,” *Foundations and Trends in Communications and Information Theory*, vol. 8, no. 1-2, pp. 1–177, 2012.

[17] H. Asnani and H. H. Permuter, “Multiple-access channel with partial and controlled cribbing encoders,” *IEEE Trans. Inf. Theory*, vol. 59, no. 4, pp. 2252–2266, April 2013.

[18] T. Kopetz, H. Permuter, and S. Shamai, “Multiple Access Channels With Combined Cooperation and Partial Cribbing,” *IEEE Trans. Inf. Theory*, vol. 62, no. 2, pp. 825–848, Feb 2016.

[19] A. Rosenzweig, Y. Steinberg, and S. Shamai, “On channels with partial channel state information at the transmitter,” *Information Theory, IEEE Transactions on*, vol. 51, no. 5, pp. 1817–1830, May 2005.

[20] Y. Steinberg, “Coding for channels with rate-limited side information at the decoder, with applications,” *IEEE Trans. Inf. Theory*, vol. 54, no. 9, pp. 4283–4295, Sept 2008.

[21] S. Jafar, “Capacity with causal and noncausal side information: A unified view,” *IEEE Trans. Inf. Theory*, vol. 52, no. 12, pp. 5468–5474, Dec 2006.

[22] J. Wolfowitz, *Coding Theorems of Information Theory*, ser. Ergebnisse der Mathematik und Ihrer Grenzgebiete 31. Springer Berlin Heidelberg, 1964.

[23] A. El Gamal and Y.-H. Kim, *Network information theory*. Cambridge University Press, 2011.

[24] A. Carleial, “Multiple-access channels with different generalized feedback signals,” *IEEE Trans. Inf. Theory*, vol. 28, no. 6, pp. 841–850, Nov 1982.

[25] I. B. Gattegno, Z. Goldfeld, and H. H. Permuter, “Fourier-motzkin elimination software for information theoretic inequalities,” *IEEE Inf. Theory Soc. Newsletter, arXiv:1610.03990*, vol. 65, no. 3, pp. 25–28, Sep. 2015, available at https://www.ee.bgu.ac.il/~fmeit/.

[26] Laneman, J Nicholas and Kramer, Gerhard, “Window decoding for the multiaccess channel with generalized feedback,” in *Proc. Int. Symp. Inf. Theory (ISIT-204)*, 2004, pp. 281–281.

[27] H. H. Permuter, Y. Steinberg, and T. Weissman, “Two-way source coding with a helper,” *IEEE Trans. Inf. Theory*, vol. 56, no. 6, pp. 2905–2919, June 2010.