Three-Dimensional Simulation of Tsunami Run Up Around Conical Island Using Smoothed Particle Hydrodynamics

Sergey K. Buruchenko 1

1 South Ural State University, Chelyabinsk, Russia

E-mail: sergey.k.buruchenko@gmail.com

Abstract. The large-scale laboratory experiments were performed in a 30 m-wide, 25 m-long, and 60 cm-deep wave basin. Waves were realistically created in the tank by a horizontal wave generator with 60 different paddles each 46 cm-wide and moving independently. These experiments provided run-up observations for validating numerical models and supplemented comparisons with analytical results. Smoothed particle hydrodynamics (SPH) is a popular meshfree, Lagrangian method with attractive features in modelling fluid dynamics. The SPH method is capable of dealing with problems with free surface, deformable boundary, moving interface, wave propagation and solid simulation. A weakly incompressible fluid flow SPH model was employed in this paper to investigate the run-up heights of nearshore tsunamis in the vicinity of a circular island. The predicted numerical results have been verified by comparing to available laboratory measurements. A good agreement has been observed.

1. Introduction
Tsunamis are large water waves set in motion either by landslides, submarine volcanic explosions, or sea-bottom deformations associated with large submarine earthquakes. During the last decades several devastating tsunamis have occurred around the Indian and Pacific Ocean area. These tsunamis not only killed many human beings but also caused serious property damages. Especially, near shore tsunamis could cause severe coastal flooding and huge property damage; because it takes a few minutes to reach a coastline and the time to seek refuge from a tsunami is not enough.

On December 26 2004 Samek et al. [1], a major earthquake measuring 9.0 occurred in the Indian Ocean off the coast of Sumatra, Indonesia. The quake occurred in the highly seismic zone of the Burma Microplate in an area known as the Sundra Trench where the India plate begins its subduction under the Sunda plate. The quake created a very large displacement of ocean water resulting in a Tsunami that impacted coastal areas throughout the Indian Ocean. A series of devastating tsunamis killed 230,000 people in 14 countries, and inundated coastal communities with waves up to 30 metres (100 ft) high. Indonesia was the hardest-hit country, followed by Sri Lanka, India, and Thailand.

The Great East Japan Earthquake on 11 March 2011 IAEA Report [2], with a magnitude of 9, generated a series of large tsunami waves that struck the east coast of Japan, the highest being 38.9 m at Aneyoshi, Miyako.

The earthquake and tsunami waves caused widespread devastation across a large part of Japan, with 15 391 lives lost. In addition to this, 8 171 people remain missing, while many more being displaced from their homes as towns and villages were destroyed or swept away. Many aspects of Japan's infrastructure have been impaired by this devastation and loss.
Although all off-site power was lost when the earthquake occurred, the automatic systems at Fukushima Dai-ichi successfully inserted all the control rods into its three operational reactors upon detection of the earthquake, and all available emergency diesel generator power systems were in operation, as designed. The first of a series of large tsunami waves reached the Fukushima Dai-ichi site about 46 minutes after the earthquake.

These tsunami waves overwhelmed the defense of the Fukushima Dai-ichi facility, which were only designed to withstand tsunami waves of a maximum of 5.7 m high. The larger waves that impacted this facility on that day were estimated to be over 14 m high. The tsunami waves reached areas deep within the units, causing the loss of all power sources except for one emergency diesel generator (6B), with no other significant power source available on or off the site, and little hope of outside assistance. In this paper, Smoothed Particle Hydrodynamics GPU-Code will be employed to investigate the run-up heights of near shore tsunamis around a conical island. The numerical results validate with the conical island experiments, as presented in Briggs et al. [3], to ensure reliable computations of tsunami inundation on larger scale. This is a necessary and important step before more complex and non-linear processes are modelled and also validated. A brief description of laboratory facility will firstly be introduced for completeness in the paper. The governing equations will be discretized with a Smoothed Particle Hydrodynamics method. Finally, careful discussion on the numerical solutions and concluding remarks will be described.

2. Smoothed Particle Hydrodynamics

The description of the SPH formulation is beyond the aim of this paper; for a complete review about the main features of this technique the reader is referred to Monaghan [4]. In the SPH formalism, the fluid domain is represented by a set of points (particles) scattered in a non-uniform arrangement which is modified each time step according to the governing dynamics. Thus, the physical properties of particles (mass, density, pressure, position, velocity) can change throughout the simulation due to the interaction of neighbouring points. This interaction depends on a weighting function, herein referred to as the smoothing kernel. SPH is treated as a Lagrangian method by many physicists and is frequently used to model free-surface flows. Originally invented for astrophysics during 1970s Lucy [5], Gingold Monaghan [6], it has been applied to many different fields of fluid dynamics and solid mechanics. Instead of using a mesh, the SPH method uses a set of interpolation nodes placed arbitrarily within the fluid. This gives several advantages in comparison to mesh based methods when simulating nonlinear flow phenomena. The method uses discrete approximations to interpolation integrals to transform differential equations of fluid dynamics into particle summations.

The SPH scheme is implemented in the DualSPHysics code Gómez-Gesteira et al. [7]. The code is implemented using both the C++ and CUDA programming languages. The code can then be executed either on the CPU or on the GPU since all computations have been implemented both in C++ for CPU simulations and in CUDA for the GPU simulations. The philosophy underlying the development of DualSPHysics is that most of the source code is common to CPU and GPU which makes debugging straightforward as well as the code maintenance and new extensions. This allows the code to be run on workstations without a CUDA-enabled GPU, using only the CPU implementation. On the other hand, the resulting codes should be necessarily different since code developers have considered efficient approaches for every processing unit.

Computational runtime increases dramatically with the number of particles in the SPH simulations. Hence, parallelisation methods are essential to run simulations with a huge number of particles in a reasonable execution time. GPUs constitute a suitable hardware for scientific tasks where mathematical calculations are carried out using large sets of data.

3. Laboratory Experiments

On December 12, 1992, a 7.5 magnitude earthquake of Flores Island, Indonesia, caused a tsunami that was responsible for over 750 deaths on the nearby Babi Island Briggs et al. [1]. The catastrophe caused devastating effects to the island and its inhabitants. One of the most unique aspects of the tsunami at
Babi Island was the refraction of the leading wave of the tsunami around the island and its convergence on the backside of the island. The convergence caused an extremely high run-up on the back side of the island due to the combination of two waves with equal amplitudes that were in phase. Motivated by the event, the Coastal Engineering Research Center (CERC) in Vicksburg, Mississippi performed large-scale laboratory experiments in a 30-m wide, 25 m long, and 0.60 m deep wave basin Briggs et al. [8]. The center of a circular island was located at \( x = 15 \) m and \( y = 13 \) m. The island had the shape of a truncated, right circular cone with diameters of 7.2 m at the toe and 2.2 m at the crest. The vertical height of the island was approximately 62.5 cm.

A directional spectral wave generator (DSWG) was installed along the x-axis and was used to generate solitary waves. The total length of the wave maker is 27.432 m and it consists of 60 individual paddles moving parallel to the water surface; each of them is independently and electronically driven. A sketch of the island geometry is also shown in Figure 1.

![Figure 1. A schematic sketch illustrating the experiment. A top view of the conical island experiment showing maximum run-up caused by the two refracted waves converging on the back of the island](image)

According to figure 1 left, gauges had following positions:
- **Gauge 6:** Front of island over toe \( X=9.36 \) m, \( Y=13.8 \) m,
- **Gauge 9:** Front of island at shallowest point \( X=10.36 \) m, \( Y=13.8 \) m,
- **Gauge 16:** Side of island at shallowest point \( X=12.9 \) m, \( Y=11.22 \) m,
- **Gauge 22:** Back of island at shallowest point \( X=15.56 \) m, \( Y=13.80 \) m.

More detailed description on laboratory experiments including facilities can be found in Briggs et al. [8] and Liu et al. [9], hence that is not repeated here again.

### 4. Numerical results

The simulation was done with normalized wave height of 0.181-m, which are normalized with the water depth of 0.32 m. Wave gauges 6, 9, 16 and 22 were used to compare the water elevation and the maximum run-up around the island at four different gauges located at the front and back of the conical island relative to the wavemaker (Figure 2, 3). Gauges 6, 9 and 22 are located in vicinity of the still-water shoreline; whereas gauge 6 is located above the toe of the conical island. Synolakis et al. [10] demanded that mass is conserved, which may be a problem for fixed-mesh methods, such as finite differences; however, the SPH method is a Lagrangian method and since no particle can leave the domain during the simulations, mass is conserved.
The conical island experiment is part of NOAA’s standards and procedures for tsunami inundation models Synolakis et al. [10], [11]. For SPH simulation results, water elevations and run-up can be directly computed from the location of the particles. The comparison between measured and simulated data is good. Also the comparison at the backside of the conical is good, and the model is able to reproduce the amplitude and phase accurately.

Figure 4 shows a sequence of snapshots of velocity distribution around and at the back of the island for time 30, 33 and 34 sec. It is evident that although wave celerity is smaller near the shoreline, the depth-averaged particle velocity is much larger near the shoreline. Two trapped waves collide at the lee side of the island and create the convergence particle velocity at $t = 34$ s (Figure 4).

The simulation was carried out with 16 million particles. The total runtime of simulation is 70 hours.
5. Conclusions
A 3D SPH numerical model has been validated over a laboratory tsunami run-up around conical island. The results show a good reliability of the model in representing the propagation of a solitary wave on a 3D basin, in terms water level. The accuracy and efficiency of three-dimensional weakly-compressible Smoothed Particle Hydrodynamics (WCSPH) numerical model are validated by comparison between a numerical solution and experimental results. Once an acceptable level of confidence is achieved, SPH can be used as an effective tool for the prediction of these waves and can prove to be extremely useful in minimizing the loss of life and property. Further development proposes the allowance for simulation the coupling effects between fluid and structures (fluid-structure interaction and sloshing phenomenon) that may impair the resistance and/or operating capabilities of the offshore structures.

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