On Large-Scale Dynamic Topic Modeling with Nonnegative CP Tensor Decomposition*

Miju Ahn, Nicole Eikmeier, Jamie Haddock, Lara Kassab, Alona Kryshchenko, Kathryn Leonard, Deanna Needell, R. W. M. A. Madushani, Elena Sizikova, Chuntian Wang

Abstract There is currently an unprecedented demand for large-scale temporal data analysis due to the explosive growth of data. Dynamic topic modeling has been widely used in social and data sciences with the goal of learning latent topics that emerge, evolve, and fade over time. Previous work on dynamic topic modeling primarily employ the method of nonnegative matrix factorization (NMF), where slices of the data tensor are each factorized into the product of lower-dimensional nonnegative matrices. With this approach, however, information contained in the temporal dimension of the data is often neglected or underutilized. To overcome this issue, we propose instead adopting the method of nonnegative CANDECOMP/PARAPAC (CP) tensor decomposition (NNCPD), where the data tensor is directly decomposed into a minimal sum of outer products of nonnegative vectors, thereby preserving the temporal information. The viability of NNCPD is demonstrated through application to both synthetic and real data, where significantly improved results are obtained compared to those of typical NMF-based methods. The advantages of NNCPD over such approaches are studied and discussed. To the best of our knowledge, this is the first time that NNCPD has been utilized for the purpose of dynamic topic modeling, and our findings will be transformative for both applications and further developments.

Miju Ahn  
Department of Engineering Management, Information, and Systems, Southern Methodist University, Dallas, TX, 75205, U.S.A, e-mail: mijua@smu.edu

Nicole Eikmeier  
Department of Computer Science, Grinnell College, Grinnell, IA, 50112, U.S.A, e-mail: eikmeier@grinnell.edu

Jamie Haddock  
Department of Mathematics, University of California, Los Angeles, CA, 90095-1555, U.S.A, e-mail: jhaddock@math.ucla.edu

Lara Kassab  
Department of Mathematics, Colorado State University, Fort Collins, CO, 80523, U.S.A, e-mail: kassab@math.colostate.edu

Alona Kryshchenko  
Department of Mathematics, California State University, Channel Islands, Camarillo, CA, 93012, U.S.A, e-mail: alona.kryshchenko@csuci.edu

Kathryn Leonard  
Department of Computer Science, Occidental College, Los Angeles, CA, 90041, U.S.A, e-mail: kleonard.ci@gmail.com

Deanna Needell  
Department of Mathematics, University of California, Los Angeles, CA, 90095, U.S.A, e-mail: deanna@math.ucla.edu

R. W. M. A. Madushani  
Section of Infectious Diseases, Boston Medical Center, MA, 02118, U.S.A, e-mail: madushani.rajapaksha@bmc.org

Elena Sizikova  
Center for Data Science, New York University, New York, NY, 10011, U.S.A e-mail: es5223@nyu.edu

Chuntian Wang  
The Department of Mathematics, University of Alabama, Tuscaloosa, AL, 35487, U.S.A e-mail: cwang27@ua.edu

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1 Introduction

In today’s society, there is an unprecedented demand for efficient, quantitative, and interpretable methods to study large-scale data in various fields such as finance, economy, social media, psychology, and political sciences (see, e.g., [DE09, ACD+18, SS12, BF18, YCS09, CZA07]). The area of study on which we focus is known as topic modeling, which investigates ways to reveal latent themes and topics in a dataset. Dynamic topic modeling (see, e.g., [CZA07, TBR18, SS12]) investigates how topics emerge (i.e., new topics are formed), evolve (i.e., topics gradually change meaning), and fade (i.e., topics disappear in importance), and is of particular interest for the analysis of data with a temporal component.

Data obtained in the applications mentioned above is often of high dimension, including one or more temporal dimensions, and is well-represented by tensors, a common algebraic representation for high-dimensional arrays (see [KB09] for a tutorial). The crucial step of (dynamic) topic modeling is to decompose high-dimensional tensors into interpretable representations with attention to the temporal information. In addition, one may also be interested in finding such decompositions with some additional structure, such as nonnegativity, which allows for interpretability of topics as opposed to traditional approaches like principal component analysis (PCA) where factors often cancel.

In previous works, the typical methods for such nonnegative tensor decompositions are mainly based on nonnegative matrix factorization (NMF) where a matricized version of the tensor sliced along the temporal dimension is factorized using NMF. There are two basic approaches for the NMF-based nonnegative tensor decomposition: (i) NMF applied directly to tensor slices (Direct NMF), where a tensor is broken into slices along the temporal dimension and each time slice is decomposed independently using NMF [LS71, PT94, APTJ95, LS99]; and (ii) Fixed-factor Nonnegative Matrix Factorization (Fixed NMF), where the slices along the temporal dimension are concatenated together and decomposed with one of the factors being fixed [CZA07]. More advanced NMF-based approaches based on these two basic ones have been developed, e.g., a windowing technique, where multiple temporal slices are considered at once, which forces factorizations of nearby slices in the temporal dimension to be similar [SS12, CZAO+15].

A significant drawback of NMF-based nonnegative tensor decompositions, however, is their failure to respect the temporal mode. For Direct NMF, the data are treated as “independent” across time. For Fixed NMF, the data are assumed to share the same latent topics over time. Neither assumption holds true in many application domains. Moreover, it is often such changes in the topic structure and information that is relevant to the application, and it is those changes that need to be identified. Therefore, it is imperative to find a tensor decomposition method that captures the full information of the topics evolving over time. To this end, we propose to adopt the Nonnegative CANDECOMP/PARAPAC tensor decomposition (NNCPD) [CC70, H+70] for dynamic topic modeling, where the tensor is directly decomposed into sums of outer products of (one-dimensional) vectors. Although there have been many applications of NNCPD to various areas of data analysis [KB09], to the best of our knowledge, it has not been applied to dynamic topic modeling.

Contribution. To compare the method of NNCPD to the traditional NMF-based methods, we consider dynamic topic modeling of synthetically generated datasets, as well as those drawn from the 20 Newsgroup Data [Lan95, Ren08] which is a large-scale benchmark dataset used in testing topic modeling methods. The numerical experiments demonstrate that the NNCPD outperforms both Direct NMF and Fixed NMF in two key ways:

1. NNCPD incorporates the temporal dimension in its factors, regardless of which mode captures temporal information, and detects the dynamics of the topics through time, i.e., the emergence, evolution, and fading of topic. These phenomena are clearly visualized by NNCPD, while the other methods may obscure these phenomena.
2. NNCPD is more robust to the approximation of the number of topics and noise in the data than the other methods. When the true number of topics may be slightly overestimated, the other methods often fit topics to the noise. In contrast, the performance of NNCPD does not significantly deteriorate for data with a reasonable signal-to-noise ratio.

Organization. The paper is organized as follows. In Sect. 2, we give a brief overview on NMF and the tensor decompositions considered in the article. We then illustrate NNCPD as a tool for topic detection for dynamic topic modeling on three-way synthetic data (Sect. 3.1) and the 20 Newsgroup data (Sect. 3.2), and perform numerical experiments to test robustness of NNCPD (Sect. 3.3). The output of our NNCPD experiments are compared to those...
of Direct NMF and Fixed NMF, and the advantages of NNCPD over these typical NMF-based methods are analyzed and discussed. We conclude with discussion and remarks in Sect. 4.

2 Overview and Notations

In this section, we introduce the basic notions of NMF-based nonnegative tensor decompositions and NNCPD. Formally, a tensor is a multidimensional array and the order of a tensor is the number of dimensions, also known as ways or modes. A first-order tensor is a vector, a second-order tensor is a matrix, and tensors of order three or higher are called higher-order tensors [KB09]. In what follows, to distinguish between decompositions for tensors and matrices, we will use the term factorization only in reference to matrix factorization (including matrix factorization subroutines in some tensor analysis methods) and the term decomposition only in reference to tensor decomposition.

2.1 NMF-based Nonnegative Tensor Decompositions

In this section, we introduce two typical NMF-based nonnegative tensor decompositions: Direct NMF [LS71, PT94, APTJ95, LS99] and Fixed NMF [CZA07]. We start with a general introduction of NMF for matrices.

2.1.1 NMF for Matrices

Nonnegative matrix factorization (NMF) seeks to find an approximate factorization of a nonnegative data matrix $X \in \mathbb{R}_{\geq 0}^{n_1 \times n_2}$ into a nonnegative features matrix $A$ and a nonnegative coefficients matrix $B$

$$X \approx AB, \quad A \in \mathbb{R}_{\geq 0}^{n_1 \times r}, \quad B \in \mathbb{R}_{\geq 0}^{r \times n_2},$$

(1)

where $r \in \mathbb{N}$ corresponds to the number of latent topics in the data, and is typically much smaller than $n_1$ and $n_2$. We note that the outer product representation of matrix multiplication lets us rewrite the product $AB$ as

$$X \approx AB = \sum_{i=1}^{r} a_i \otimes b_i,$$

(2)

where $a_i \in \mathbb{R}_{\geq 0}^{n_1}$ is a column of $A$ and $b_i \in \mathbb{R}_{\geq 0}^{n_2}$ is a row of $B$. Generally, the factorization is computed by approximately minimizing the reconstruction error

$$E(X; A, B) = ||X - AB||_F,$$

(3)

where $|| \cdot ||_F$ denotes the Frobenius norm of matrices. When the minimum of reconstruction error vanishes we say an exact NMF is obtained.

In an application where $X$ is a data matrix whose columns represent documents and whose rows represent words, the matrix $A$ gives a topic representation for each word, and $B$ a topic representation for each document. Furthermore, one might hope that the topics detected by NMF truly correspond to a set of topics that describe the set of documents well, and this is often the case in application of NMF to data. See Figure 1 for a visualization of NMF as in [1].

2.1.2 Direct NMF and Fixed NMF

Datasets considered in the context of dynamic modeling are often represented as higher-order tensors, for example, a data tensor $X \in \mathbb{R}_{\geq 0}^{n_1 \times n_2 \times n_3}$, whose first, second, and third modes represent documents, words, and time, respectively. A
Fig. 1 A visualization of the factor matrices in NMF of $X \approx AB$, where $X \in \mathbb{R}_{\geq 0}^{n_1 \times n_2}$, $A \in \mathbb{R}_{\geq 0}^{n_1 \times r}$ and $B \in \mathbb{R}_{\geq 0}^{r \times n_2}$. The edges of the matrix visualized in blue and red represent the modes of the matrix with dimension $n_1$ and $n_2$, respectively.

A natural way to decompose this third-order tensor is to perform NMF on temporal mode slices (or collections thereof) of the tensor. There are two basic approaches of NMF-based nonnegative tensor decomposition: Direct NMF and Fixed NMF.

Direct NMF on tensor slices performs NMF independently on each slice of the tensor $[LS71, PT94, APTJ95, LS99]$. Given $X \in \mathbb{R}_{\geq 0}^{n_1 \times n_2 \times n_3}$, slicing along the third mode gives nonnegative matrices $X_i \in \mathbb{R}_{\geq 0}^{n_1 \times n_2}$ for $i = 1, 2, \ldots, n_3$, each of which is factored into nonnegative matrices

$$X_i \approx A_i S_i, \quad A_i \in \mathbb{R}_{\geq 0}^{n_1 \times r}, S_i \in \mathbb{R}_{\geq 0}^{r \times n_2}, \quad i = 1, \ldots, n_3, \quad (4)$$

where the $A_i$'s will be referred to as the Direct NMF $A$ factors, and the $S_i$'s the Direct NMF $S$ factors. This form of nonnegative tensor decomposition fails to capture inherent structures within the tensor along the time dimension. Stacking the products of the Direct NMF $A$ and $S$ matrices forms an approximation to $X$, which will be referred to as the Direct NMF reconstruction. The reconstruction error for Direct NMF is defined as:

$$E(X; A_i, S_i, i = 1, \ldots, n_3) = \|X - \hat{X}\|_F, \quad (5)$$

where $\hat{X}$ denotes the Direct NMF reconstruction.

In [CZA07], the authors define an alternative nonnegative tensor decomposition, which we refer to as Fixed NMF. This decomposition performs NMF simultaneously on the $n_3$ slices along mode three, $X_i, i = 1, \ldots, n_3$, with the same $A$. They consider a sequence of nonnegative matrix factorizations $(A, S_1), (A, S_2), \ldots, (A, S_{n_3})$ such that

$$X_i \approx AS_i, \quad A \in \mathbb{R}_{\geq 0}^{n_1 \times r}, S_i \in \mathbb{R}_{\geq 0}^{r \times n_2}, \quad i = 1, \ldots, n_3, \quad (6)$$

where $A$ will be referred to as the Fixed NMF common $A$ factor, and the $S_i$'s the Fixed NMF $S$ factors. In other words, Fixed NMF fixes a single dictionary matrix $A$ and searches for the representations $S_i$ for each of the slices $X_i$. Stacking the products of the Fixed NMF $A$ matrix and $S$ matrices forms an approximation to $X$, which will be referred to as the Fixed NMF reconstruction. The Fixed NMF reconstruction error is defined as:

$$E(X; A, S_i, i = 1, \ldots, n_3) = \|X - \hat{X}\|_F, \quad (7)$$

where $\hat{X}$ denotes the the Fixed NMF reconstruction.

2.2 CANDECOMP/PARAFAC (CP) Decomposition and NNCPD

In this section, we introduce the nonnegative CANDECOMP/PARAFAC (CP) decomposition (NNCPD), which generalizes NMF for matrices to higher-order tensors [CC70, H+70].
2.2.1 Methodology of CP Decomposition and NNCPD

Unlike the NMF-based nonnegative tensor decompositions, CP decompositions treat the tensor as a whole. The CP decomposition and NNCPD factorize a tensor into a sum of component rank-one tensors without slicing it along the temporal mode. For example, given a third-order tensor \(X \in \mathbb{R}^{n_1 \times n_2 \times n_3}\), an exact rank-\(r\) CP decomposition of \(X\) can be written as

\[
X = \sum_{\ell=1}^{r} a_{\ell} \otimes b_{\ell} \otimes c_{\ell},
\]

where \(\otimes\) denotes the outer product and \(a_{\ell} \in \mathbb{R}^{n_1}, b_{\ell} \in \mathbb{R}^{n_2}, c_{\ell} \in \mathbb{R}^{n_3}\), \(\ell = 1, \ldots, r\). Further, we can explicitly write out the entries of \(X\) as follows,

\[
x_{ijk} = \sum_{\ell=1}^{r} a_{\ell i} b_{\ell j} c_{\ell k},
\]

where \(i = 1, \ldots, n_1\), \(j = 1, \ldots, n_2\), and \(k = 1, \ldots, n_3\). The factor matrices of CP decomposition refer to the combination of the vectors from the rank-one components, i.e., \(A = [a_1 \ a_2 \ \cdots \ a_r]\), and likewise for \(B\) and \(C\). The rank of the tensor \(X\), denoted \(\text{rank}(X)\), is the smallest integer \(r\) so that \(X\) may be expressed as the sum of exactly \(r\) rank-one tensors. Similar to NMF, an approximate CP decomposition may be computed. Fix an \(r\) and approximately minimize the reconstruction error

\[
E(X; A, B, C) = \|X - \hat{X}\|_F.
\]

The solution \(\hat{X} = \sum_{\ell=1}^{r} a_{\ell} \otimes b_{\ell} \otimes c_{\ell}\) will be referred to as a rank-\(r\) CP reconstruction of \(X\). When the reconstruction error vanishes, an exact CP decomposition as in (8) is obtained. See Figure 2 for a visualization of the rank-\(r\) CP decomposition.

Fig. 2 A visualization of the factor matrices in a CP decomposition. The edges of the tensor visualized in blue, red, and green represent the modes of the tensor with dimension \(n_1, n_2\), and \(n_3\), respectively.

Note that NMF specializes matrix factorization to factorizing a nonnegative data matrix into the product of two (lower-dimensional) nonnegative factor matrices. In the same way, NNCPD specializes the CP decomposition to decomposing a nonnegative data tensor into the sum of rank-one tensors which are the outer product of nonnegative vectors. Nonnegativity is necessary when we desire to preserve inherent properties of the original tensor data. For example, a tensor of images will have entries representing pixel values that must be nonnegative. Specifically, given a third-order tensor \(X \in \mathbb{R}_{\geq 0}^{n_1 \times n_2 \times n_3}\) and a fixed integer \(r\), the approximate NNCPD of \(X\) seeks \(A \in \mathbb{R}_{\geq 0}^{n_1 \times r}, B \in \mathbb{R}_{\geq 0}^{n_2 \times r}, C \in \mathbb{R}_{\geq 0}^{n_3 \times r}\) so that

\[
X \approx \sum_{\ell=1}^{r} a_{\ell} \otimes b_{\ell} \otimes c_{\ell},
\]
where \( \otimes \) denotes the outer product and \( a_\ell, b_\ell, \) and \( c_\ell \) are the columns of \( A, B, \) and \( C, \) respectively. \( A, B, \) and \( C \) will be referred to as the NNCPD factors. A nonnegative approximation with fixed \( r \) is obtained by approximately minimizing the reconstruction error between \( X \) and the NNCPD reconstruction \( \hat{X} = \sum_{\ell=1}^r a_\ell \otimes b_\ell \otimes c_\ell \) among all the nonnegative vectors. When the reconstruction error vanishes we say that an exact rank-\( r \) NNCPD is obtained. The nonnegative rank, denoted as \( \text{rank}_+ (X) \), is the minimum integer \( r^* \) so that there exists an exact rank-\( r^* \) NNCPD of \( X \). In what follows, unless otherwise stated, when we refer to rank of a tensor we are referring to nonnegative rank.

### 2.2.2 Existence and Uniqueness of Rank-\( r \) NNCPD

In [Qi18], the authors explore rank-\( r \) NNCPD of tensors whose rank is unknown, raising the question of when rank-\( r \) approximations exist and are unique. The existence question is answered by the following proposition, but the uniqueness question has not yet been completely settled.

**Proposition 1 ([Qi18])** Let \( X \) be a generic nonnegative tensor with \( r < \text{rank}_+ (X) \), and \( Y = \arg \min_{\text{rank}_+(Y) \leq r} ||X - Y|| \). Then \( Y \) is unique and has \( \text{rank}_+(Y) = r \).

What is not known is whether or not the resulting rank-\( r \) approximation \( Y \) itself has a unique rank-\( r \) decomposition. A series of results gives only partial answers. Kruskal’s theorem [Kru77] provides a test for 3-tensors based on the \( k \)-rank of the factor matrices of an NNCPD of the tensor. Generic results on spaces of tensors [Qi18, DDL14] give partial results for spaces with specific conditions on tensor dimensions. The strongest of these restricts the product of the tensor dimensions to no more than 15,000 which is too small for most realistic cases, and imposes additional restrictions on tensor dimensions. Another strong result proves uniqueness for NNCPD for tensors of ranks of 2 or 3 and dimensions of at least 3. In what follows, we require existence but not uniqueness.

### 3 Comparison of NNCPD and NMF-based Nonnegative Tensor Decompositions

In this section, we perform numerical experiments showcasing how one might interpret the factors of NNCPD for dynamic topic modeling. Each of these experiments highlights different features of NNCPD for topic modeling. In all the experiments, NNCPD outperforms Direct NMF and Fixed-factor NMF, as NNCPD provides more comprehensive analysis of the topic evolution while the other methods fail to detect key changes. In our experiments, we use the tensorlab package [VDS+16] with MATLAB.

#### 3.1 Synthetic Dataset Numerical Experiments

In our first experiment, we consider numerical experiments on synthetic datasets where topic evolution is simple, and only one topic changes (Sect. 3.1.1). In our second experiment, we consider more complex data with repeated topic emergence, topic fading, and shifting topics (Sect. 3.1.2).

##### 3.1.1 Monotonic Dynamic Topic Modeling Dataset Experiment

We consider a toy example of a \( 10 \times 20 \times 30 \) nonnegative tensor. In an application, it could represent a dataset whose first mode of dimension 10 corresponds to time, second mode of dimension 20 corresponds to survey questions and the third mode of dimension 30 corresponds to users who answer those questions across time.
In Figure 3, we show slices of the tensor across the first mode (time). This construction is a simple example of a situation when the first 15 users are very similar, and the second 15 users are very similar. For example, these two groups could correspond to healthy and sick patients. The healthy group answers all 20 questions similarly across time, which is evident by the correlation between the first 15 columns of each slice. The sick group also always answers the questions similarly, but those answers change drastically between time step 5 and 6 (perhaps after being given some medication, for example). That shift is captured by the change in the appearance of the last 15 columns between slices 5 and 6. Two questions we wish to ask are (i) can we detect this topic change between time 5 and 6? (ii) can we identify that there are two core groups of patients?

To answer the above questions, we perform Direct NMF and Fixed NMF slice by slice, slicing across the first mode (note that we thus do not use any temporal correlation) to compute the $A$ and $S$ factors. We also compute an NNCPD for the entire tensor, to obtain NNCPD factor matrices $A$, $B$, and $C$. Our results for this first example are shown in Figures 3-6. The associated reconstruction errors are presented in the captions of the figures. For this example, it seems that both the Fixed NMF and our NNCPD method are able to highlight the topic shift.

In contrast, when the data is transposed as a tensor with dimensions $10 \times 30 \times 20$ so that there is more variation in the third rather than second mode, this shift becomes harder to detect using the Fixed NMF approach. That is, as in Figure 7, we consider a $10 \times 30 \times 20$ tensor constructed so that for all ten slices, first 15 rows are highly correlated. For the first five slices, the second fifteen rows are also highly correlated, and the same for last five slices. Thus, for a given slice, the first 15 rows are correlated and last 15 rows are correlated. Across slices (time), a topic change has occurred between slice 5 and 6. Figures 8-10 show for the results of the Direct NMF, Fixed NMF and NNCPD performed on the transposed tensor. Therefore, without knowing a priori the structure of the dynamic component (e.g. what mode it lies in), we have no reason to believe these prior approaches will be able to detect such an event. NNCPD on the other hand, clearly and easily highlights such changes along any mode.

For each of these experiments, the reconstruction error listed in the figure captions is for the single trial associated with the included plots. As there is random noise added to the tensor, these errors could vary trial to trial. This variance in the error and its relationship to the size of the noise will be explored further in Sect. 3.3.

![Fig. 3 Nonnegative $10 \times 20 \times 30$ tensor example constructed so that for all ten slices, the first 15 columns are highly correlated. For the first five slices, the second fifteen columns are also highly correlated, and the same for last five slices. Thus, for a given slice, the first 15 columns are correlated and last 15 columns are correlated. Across slices (time), a topic change has occurred between slice 5 and 6.](image-url)

### 3.1.2 Complex Dynamic Topic Modeling Dataset Experiment

In this section we consider synthetic tensor datasets with more complicated topic evolution than those in Sect. 3.1.1 such as emergence, fading, and shifting.

**Topic Emergence and Fading Experiment**

We now consider a synthetic tensor data that models situations where a topic emerges, fades, then re-emerges. We construct a nonnegative tensor $X \in \mathbb{R}_{\geq 0}^{14 \times 30 \times 20}$ as follows,
Fig. 4 Direct NMF performed slice by slice for tensor in Figure 3. Left: Direct NMF factors with $r = 5$ topics. It appears that the first topic corresponds to the topic showcased by the first 15 columns of each tensor slice. Right: Direct NMF $S$ factors with $r = 5$. Clearly in each slice the first and last 15 columns are highly correlated. Direct NMF reconstruction error is 30.1421.

Fig. 5 Fixed NMF performed on matricized tensor for tensor in Figure 3 for $r = 5$. Left: Fixed NMF common $A$ factor for each slice. Right: Fixed NMF $S$ factors for each slice. Clearly in each slice the first and last 15 columns are highly correlated. The topic change is also evident between slices 5 and 6. Fixed NMF reconstruction error is 2.6603.

Fig. 6 NNCPD factors $A$, $B$, and $C$ (Left, Center, and Right, respectively) for tensor from Figure 4. Notice that factor $A$ showcases topic evolution across time (slices). There is a clear event between slice 5 and 6. The $B$ factor showcases, among other things, that the first topic seems to correspond to the first 15 columns in each slice of the tensor. The $C$ factor seems to indicate that although topics may be changing, the first 15 columns are always correlated, as are the last 15 columns. NNCPD reconstruction error is 0.045761.

$$X_{ijk} = \begin{cases} \sin((k - 1)(2\pi/9)) & \text{for } i = 1, \cdots, 14, \quad j = 1, \cdots, 15, \text{ and } k = 1, \cdots, 10 \\ |z_i| & \text{for } i = 1, \cdots, 14, \quad j = 15, \cdots, 30, \text{ and } k = 1, \cdots, 10 \\ |w_i| & \text{for } i = 1, \cdots, 14, \quad j = 1, \cdots, 30, \text{ and } k = 10, \cdots, 20, \end{cases}$$

where $z_i$ and $w_i$, defined for each $i$, are drawn from the standard normal distribution. See Figure 11 where we show slices across the third mode, the temporal dimension. In applications, this dataset might be a situation where symptoms...
Fig. 7 Nonnegative $10 \times 30 \times 20$ tensor example constructed so that for all ten slices, the first 15 rows are highly correlated. For the first five slices, the second fifteen rows are also highly correlated, and the same for the last five slices. Thus, for a given slice, the first 15 rows are correlated and the last 15 rows are correlated. Across slices (time), a topic change has occurred between slice 5 and 6.

Fig. 8 Direct NMF performed slice by slice for tensor in Figure 7. Left: Direct NMF $A$ factors with $r = 5$ topics. Right: Direct NMF $S$ factors with $r = 5$. Clearly in each slice the first and last 15 rows are highly correlated. Direct NMF reconstruction error is $1031.9668$.

Fig. 9 Fixed NMF performed on matricized tensor for tensor in Figure 7 for $r = 5$. Left: Fixed NMF common $A$ factor for each slice. Right: Fixed NMF $S$ factors for each slice. The $A$ factor showcases the correlations between rows of the slices, but it is now harder to detect a topic change has occurred after time slice 5. Fixed NMF reconstruction error is $2.1398$.

Fig. 10 Direct NMF and Fixed NMF by slicing the tensor across mode 3, the temporal dimension, as displayed in Figures 12 and 13 respectively. We then compute a NNCPD for the entire tensor, to obtain NNCPD factors $A$, $B$, and $C$ displayed in Figure 14. The associated reconstruction errors are presented in the captions of the figures.

The NNCPD provides superior visualization of topic evolution through time, as demonstrated in the factor matrices of the decomposition in Figure 14, where the NNCPD $A$ factor shows the topic representation for each word, the $B$ factor displays topic representation for each document, and the $C$ factor displays the evolution of the topics through time.
Fig. 10 NNCPD factors A, B, and C (Left, Center, and Right, respectively) for tensor from Figure [7]. Notice that factor A showcases topic evolution across time (slices). There is a clear event between slice 5 and 6. The B factor showcases, among other things, that the second topic (which by A remains constant over time), corresponds to the first 15 rows of the tensor slices. Topic 3 seems to correspond to the emerging topic in the last 15 rows of the tensor slices. NNCPD reconstruction error is 0.046283.

NNCPD alone succeeds in detecting all the events in the topic evolution. The temporal factor C illustrates which topics persist throughout time and which are emerging and fading.

Fig. 11 Tensor example $X \in \mathbb{R}^{14 \times 30 \times 20}_{\geq 0}$ is constructed so that in the first ten slices of the tensor, the first 15 columns are highly correlated and the second 15 columns are highly correlated. Further, in the second ten slices, all 30 columns are highly correlated. Further, across slices, topic changes have occurred between slices 10 and 11.

**Topic Shift Experiment**

We next consider a data tensor that can model a situation where an event happens that shifts the topics discussed by users, such as an election that shifts the topics discussed by political parties before and after. We construct a nonnegative data tensor $X \in \mathbb{R}^{14 \times 30 \times 20}_{\geq 0}$ as follows,

$$ T_{ijk} = \begin{cases} \sin \left( (k - 1)(2\pi/9) \right) & \text{for } i = 1, \ldots, 14, \ j = 1, \ldots, 15, \text{ and } k = 1, \ldots, 10 \\ |z_i| & \text{for } i = 1, \ldots, 14, \ j = 15, \ldots, 30, \text{ and } k = 1, \ldots, 10 \\ \sin \left( (k - 1)(2\pi/9) \right) & \text{for } i = 1, \ldots, 14, \ j = 15, \ldots, 30, \text{ and } k = 10, \ldots, 20 \\ |z_i| & \text{for } i = 1, \ldots, 14, \ j = 1, \ldots, 15, \text{ and } k = 10, \ldots, 20 \end{cases} $$

where $z_i$, defined for each $i$, is drawn from the standard normal distribution. See Figure [15] where we show slices across the third mode, the temporal dimension.
Fig. 12 Direct NMF performed slice by slice for tensor in Figure 11 with $r = 3$ topics. Left: Direct NMF $A$ factors. It appears that the first topic (in the $A$ factor) can be seen in the second 15 columns of the first ten slices of the tensor $T$. Right: Direct NMF $S$ factors aligned with the $A$ factors. In each of the first ten slices, the first and last 15 columns are highly correlated. Direct NMF reconstruction error is 0.021549.

Fig. 13 Fixed NMF performed on matricized tensor for tensor in Figure 11 for $r = 3$. Left: Fixed NMF common $A$ factor for each slice. Right: Fixed NMF $S$ factors for each slice. In the right plot, for each of the first ten slices the first and last 15 columns are highly correlated and the topic change is also evident between slices 10 and 11. Fixed NMF reconstruction error is 0.065896.

We perform Direct NMF and Fixed NMF by slicing the tensor along mode 3, the temporal dimension, as displayed in Figures 16 and 17 respectively. We then compute a NNCPD for the entire tensor, to obtain NNCPD factors $A$, $B$, and $C$ displayed in Figure 18. The associated reconstruction errors are presented in the captions of the figures.

We observe an additional strength of NNCPD for topic modeling. We know a priori that there are two topics in this tensor, although the rank is 4 (an example where the number of topics is different than the rank of the tensor). We observe in Figure 18 how NNCPD can roughly detect and showcase both of these facts through its factor matrices. The NNCPD $A$ factor displays 4 different topics, but the first and third are very similar. The second and fourth are also very similar. NNCPD $B$ factor indicates which columns are associated with the topics. NNCPD showcases that there are four topics, two of which are very similar, and that between slices 10 and 11 the sets of documents associated with the topics change, showing the evolution of the column representation of the topics. Furthermore, the NNCPD factor shows us the topic evolution through time. Therefore, NNCPD gives additional information that the Direct NMF and Fixed NMF are not able to provide.
Fig. 14 NNCPD factors $A$, $B$, and $C$ (Left, Center and Right, respectively) for tensor from Figure [11]. The $A$ factor seems to indicate the topics; for instance, the third topic can be seen in the second 15 columns of the first ten slices of the tensor. The $B$ factor showcases, among other things, that the third topic seems to correspond to the second 15 columns (in each of the first ten slices of the tensor using the factor $C$). Notice that factor $C$ showcases topic evolution across time (mode 3). Further, there is a clear event between slices 10 and 11. NNCPD reconstruction error is 0.0025301.

Fig. 15 Tensor example $X \in \mathbb{R}_{\geq 0}^{14 \times 30 \times 20}$ sliced along mode 3, the temporal dimension. In all slices, the first 15 columns are highly correlated and the second 15 columns are highly correlated. Across time, topic changes have occurred between slice 10 and 11.

3.2 The 20 Newsgroups Dataset Numerical Experiments

The 20 Newsgroups dataset is a collection of approximately 20,000 text documents containing the text of messages from 20 different newsgroups on the distributed discussion system Usenet which functioned similarly to current internet discussion forums. The documents are partitioned nearly evenly across the 20 newsgroups which can be further classified into six supergroups (computers, for sale, sports/recreation, politics, science, religion) [KL008]. We apply NNCPD, Direct NMF, and Fixed NMF to two tensor datasets constructed from this data. We see that NNCPD produces more easily interpretable results than Direct NMF or Fixed NMF.

For the first experiment, we use a fixed number of documents appearing first in the list. We observe that NNCPD is able to identify the latent topics that were used to construct the tensor. However, due to the lack of the normalization within topics of the second experiment, the ability of NNCPD to identify all topics and their emergence in the dataset is hindered in the first experiment. For this reason, for our second experiment, we select documents that are highly correlated within each topic, and apply normalization factors to ensure that all the topics have similar frequency of words and therefore contribute equally to the tensor. With the normalization of the dataset, NNCPD is able to identify latent topics, and to pinpoint emergence of a new topic at the time of its emergence.
Fig. 16 Direct NMF performed slice by slice for tensor in Figure 15 with $r = 4$ topics. Left: Direct NMF $A$ factors. It appears that the first topic corresponds to the topic showcased by the second 15 columns of each of the first 10 tensor slices and consequently in the first 15 columns of each of the second 10 tensor slices. Right: Direct NMF $S$ factors aligned with the $A$ factors. In all of the slices, the first and last 15 columns are clearly highly correlated. Direct NMF reconstruction error is 0.029248.

Fig. 17 Fixed NMF performed on matricized tensor for tensor in Figure 15 for $r = 4$. Left: Fixed NMF common $A$ factor for each slice. Right: Fixed NMF $S$ factors for each slice. Clearly, in all of the slices the first and last 15 columns are highly correlated. Fixed NMF reconstruction error is 0.045349.

Fig. 18 NNCPD factors $A$, $B$, and $C$ (Left, Center and Right, respectively) for tensor from Figure 15. NNCPD $A$ factor seems to indicate the topics; for instance, the second topic can be seen in the second 15 columns of the first ten slices of the tensor. NNCPD $B$ factor showcases, among other things, that the second topic seems to correspond to the the first 15 columns in each of the first ten slices of the tensor. Notice that NNCPD $C$ factor showcases topic evolution across the temporal dimension, mode 3. There is a clear event between slices 10 and 11. NNCPD reconstruction error is 0.00054029.
3.2.1 Experiments on the 20 Newsgroups Data Without Normalization

Our first experiment is on a dataset constructed from a subset of the full 20 Newsgroups data, using only four of the 20 newsgroups: space, for sale, baseball, and atheism. We select 160 documents from the for sale and baseball newsgroups, 120 documents from the space newsgroup, and 40 documents from the atheism newsgroup. Each of these documents is represented as a word-count vector of length 9850 (we remove misspelled, stop, and unused words from the original dataset). From this, we build a three-mode tensor where the first mode represents documents, the second represents words, and the third represents time.

We stack the documents into a 24 x 9850 x 20 (document by word by time step) tensor as visualized in Figure 19. The documents form the mode two fibers of the tensor. We place the 160 for sale newsgroup documents into the first eight document slices, the 160 baseball newsgroup documents into the next eight document slices, and 80 of the space newsgroup documents into the next four document slices. We then split the remaining four document slices into two sections. In these slices, the first ten time steps are the 40 remaining documents from the space newsgroup and the next ten time steps are the 40 documents from the atheism newsgroup.

We perform Direct NMF and Fixed NMF with $r = 4$ on the mode three slices; the results are presented in Figures 20 and 21. We then compute a NNCPD for the entire tensor; the NNCPD factor matrices $A$, $B$, and $C$ are presented in Figure 22. Finally, we present the keywords for each topic (the ten words with highest magnitude in each of the four topics of NNCPD factor matrix $B$) in Table 1. We present the associated reconstruction error in the captions of the figures. While the keywords suggest that the NNCPD is identifying the four latent topics in the constructed tensor, the NNCPD factor matrices do not elucidate the temporal topic structure as in the previous experiments. We hypothesize that this is due to highly varying norms in the sets of documents from different newsgroups and lack of correlation between documents from the same newsgroup. For this reason, we construct a parallel dataset which we normalize so that topics have approximately equal norm. This dataset and associated experiments are described in the next subsection.

![Fig. 19 Visualization of the construction of the first 20 Newsgroups tensor.](image)

| Topic 1 | Topic 2 | Topic 3 | Topic 4 |
|---------|---------|---------|---------|
| edu     | system  | venus   | jesus   |
| writes  | drive   | space   | matthew |
| don     | unix    | kilometers | disciples |
| com     | misc    | miles   | mark    |
| article | serial  | soviet  | cock    |
| team    | up      | vega    | heaven  |
| like    | software | balloon | john   |
| just    | computers | planet | br   |
| game    | forsale | earth   | unto    |
| up      | steve   | new     | mary    |

Table 1 Topic keywords from NNCPD $B$ factor matrix for first 20 Newsgroups dataset.
Fig. 20 Direct NMF performed slice by slice for tensor visualized in Figure 19 with \( r = 4 \) topics. Left: Direct NMF \( A \) factors. Right: Direct NMF \( S \) factors aligned with the \( A \) factors. It is difficult to identify what the four topics represent due to the dimensionality of the \( S \) factors. Direct NMF reconstruction error is 211.9746.

Fig. 21 Fixed NMF performed on matricized tensor for tensor visualized in Figure 19 for \( r = 4 \). Left: Fixed NMF common \( A \) factor for each slice. Right: Fixed NMF \( S \) factors for each slice. Again, it is difficult to identify what the four topics represent due to the dimensionality of the \( S \) factors. Fixed NMF reconstruction error is 282.6005.

Fig. 22 NNCPD factors \( A \), \( B \), and \( C \) (Left, Center, and Right, respectively) for tensor visualized in Figure 19. The only temporal information clear in from the factor \( C \) is that the first topic (likely baseball according to Table 1) persists through all twenty time slices. Other temporal information is obscured. NNCPD reconstruction error is 309.6122
3.2.2 Experiments on the 20 Newsgroups Data with Normalization

Our second experiment is on another dataset constructed from a subset of the full 20 Newsgroups data with suitable renormalizations. Again, we construct a dataset using only four of the 20 newsgroups: space, for sale, baseball, and atheism. From each of these newsgroups, we randomly select 250 documents and then choose only the ten documents with the highest pairwise linear correlation. We then build a three-mode tensor where the first mode represents documents, the second represents words, and the third represents time.

In this tensor, we use repeated copies of the documents in the sections corresponding to a single topic to ensure high correlation. We place the ten selected for sale documents into the first ten documents of the first time slice. We permute these ten selected documents and place them into the first ten documents of each of the remaining nine time slices. We then place the ten selected space documents into the second ten documents of the first time slice. We permute these documents and place them in the second ten documents of each of the remaining nine time slices. We then place the ten selected atheism documents into the third ten documents of the first time slice. We permute these documents and place them in the third ten documents of each of the next four time slices. We place the ten selected baseball documents into the third ten documents of the sixth time slice. We permute these documents and place them in the third ten documents of each of the last four time slices. This construction is depicted in Figure 23.

Finally, to ensure that the NNCPD detects each of the topics and uncovers clear structure of the tensor, we use normalization weights to make approximately equal norm within each of the topic sections of the tensor so that every topic has similar word frequency values. We normalize so that the for sale topic section, the space topic section, the atheism topic section of the tensor has Frobenius norm 1500, 1500, 1400, and 1800, respectively. We choose these slightly different weights to balance not only the overall norm throughout these sections, but also the largest entries in these sections.

We perform Direct NMF and Fixed NMF with $r = 4$ on the mode three slices; the results are presented in Figures 24 and 25. We then compute a NNCPD for the entire tensor; the NNCPD factor matrices $A$, $B$, and $C$ are presented in Figure 26. Finally, we present the keywords for each topic (the ten words with highest magnitude in each of the four topics of NNCPD $B$ factor) in Table 2. We present the associated reconstruction error in the captions of the figures.

Here, we see that not only are the keywords in Table 2 meaningfully associated to the latent topics in the dataset, but also the NNCPD $C$ factor in Figure 26 exhibits the temporal topic information, while this information is difficult to glean from Direct NMF and Fixed NMF due to the dimensionality of the factor matrices.

3.3 Noise Dataset Robustness Numerical Experiments

In this section, we perform numerical experiments on noise datasets to test robustness of NNCPD compared to Direct NMF and Fixed NMF.
3.3.1 Construction of the Noise Dataset

Given a nonnegative deterministic tensor $X \in \mathbb{R}_{\geq 0}^{n_1 \times n_2 \times n_3}$ of the form $X = \sum_{i=1}^{r^*} a_i \otimes b_i \otimes c_i$, with given vectors $a_i$, $b_i$ and $c_i$, and $r^*$ denoting the exact rank of $X$, we define a tensor $T$ with noisy measurements,

$$T = X + N,$$  \hspace{1cm} (1)
Table 2 Topic keywords from NNCPD B factor matrix for second 20 Newsgroups dataset.

| Topic 1 | Topic 2 | Topic 3 | Topic 4 |
|---------|---------|---------|---------|
| edu     | idle    | theism  | book    |
| writes  | won     | belief  | chemistry |
| article | lost    | reason  | good    |
| orbit   | mattingly | irrational | edu |
| oort    | don     | say     | ibm     |
| au      | edu     | fanaticism  | udel |
| why     | tigers  | rational | writing |
| hst     | scores  | fanaticism  | chopin |
| cloud   | york    | argument | guide   |
| earth   | beloved | correlated | paperback |

where \( N \) is a noise tensor. We will implement two types of noise tensor \( N \) in the experiments below. First, we consider a noise tensor \( N := |Z| \) for \( Z \in \mathbb{R}^{n_1 \times n_2 \times n_3} \) defined as

\[
Z = \sigma \sum_{i=1}^{r_N} n_{ai} \otimes n_{bi} \otimes n_{ci},
\]

where \( r_N \) denotes the exact rank of \( Z \), \( \sigma > 0 \) is a noise parameter, and the vectors \( n_{ai} \in \mathbb{R}^{n_1} \), \( n_{bi} \in \mathbb{R}^{n_2} \), and \( n_{ci} \in \mathbb{R}^{n_3} \) have entries sampled from \( N(0, 1) \), the standard normal distribution. Second, we consider a noise tensor \( N = \sigma |\tilde{Z}| \), where the entries of the tensor \( \tilde{Z} \) are sampled from the standard normal distribution.

We carry out the robustness experiments by modulating the variance of the noise parameter \( \sigma \) and rank of the noise tensor \( r_N \), and examine the resulting reconstruction error. Here the NNCPD reconstruction error is \( \|\hat{T} - X\|_F \), where \( \hat{T} \) denotes the NNCPD reconstruction of the tensor \( T \), as we do not wish to fit the noise \( N \). The reconstruction error is defined similarly for Direct NMF and Fixed NMF, which is \( \|\tilde{T} - X\|_F \) where \( \tilde{T} \) is the Direct NMF or Fixed NMF reconstruction of the tensor \( T \).

3.3.2 Experiment Output on Noise Dataset

We experiment to test robustness of NNCPD by adding noise to the tensor \( X \) described in the first experiment in Sect. 3.1.2 (see Figure 11). The Frobenius norm of \( X \) is \( \|X\|_F \approx 117.1778 \) and the exact rank of \( X \) is \( r^* = 3 \). We compute reconstruction errors using Direct NMF, Fixed NMF and NNCPD for various ranks \( r \). We report the median\(^2\) of the reconstruction error over 50 runs in Figures 27-29. We let the noise parameter vary, \( \sigma = 10^{-3}, 10^{-2}, 10^{-1}, 1 \) from left to right (starting from the top left plot).

Results in Figure 27 are for a noise tensor \( N \) with rank one \((r_N = 1)\). We compute the norm of \( N \) for each noise parameter \( \sigma \),

\[
\|N\|_F \approx \begin{cases} 
0.0531 & \text{for } \sigma = 10^{-3}, \\
0.8174 & \text{for } \sigma = 10^{-2}, \\
6.5349 & \text{for } \sigma = 10^{-1}, \\
54.2263 & \text{for } \sigma = 1.
\end{cases}
\]

For large values of \( \|N\|_F \) and while \( \|N\|_F < \|X\|_F \), we notice that the NNCPD reconstruction error \( \|\hat{T} - X\|_F \approx \|N\|_F \) for \( r \geq r^* + r_N \) (in this case \( r \geq 4 \)), and remains stable.

\(^2\) We choose not to report the mean because Direct NMF and Fixed NMF are often not stable or robust, and therefore result in arbitrarily large values making it hard to observe the behavior of NNCPD as the rank varies.
Results in Figure 28 are for a noise tensor \(N\) with rank two \((r_N = 2)\). We compute the norm of \(N\) for each noise parameter \(\sigma\),

\[
\|N\|_F \approx \begin{cases} 
0.1554 & \text{for } \sigma = 10^{-3}, \\
1.6394 & \text{for } \sigma = 10^{-2}, \\
12.3065 & \text{for } \sigma = 10^{-1}, \\
129.9499 & \text{for } \sigma = 1.
\end{cases}
\]

For large values of \(\|N\|_F\) and while \(\|N\|_F < \|X\|_F\), we notice that NNCPD reconstruction error \(\|\hat{T} - X\|_F \approx \|N\|_F\) for \(r \geq r^* + r_N\) (in this case \(r \geq 5\)) and remains stable. For \(\sigma = 1\), i.e. when \(\|N\|_F = 129.9499 > 117.1778 = \|X\|_F\), we see that NNCPD does not detect that \(r = 3\) is the rank of the true tensor \(A\). At the same time, the reconstruction error is minimum for \(r = 5\). This suggests that since \(\|N\|_F > \|X\|_F\), NNCPD is fitting to the noise tensor \(N\) and not the true tensor \(X\) because there is better hope minimizing the error fitting to the noise tensor.

Results in Figure 29 are for a noise tensor \(N = \sigma|\tilde{Z}|\), where the entries of the tensor \(\tilde{Z} \in \mathbb{R}^{n_1 \times n_2 \times n_3}\) are sampled from the standard normal distribution.

\[
\|N\|_F \approx \begin{cases} 
0.0911 & \text{for } \sigma = 10^{-3}, \\
0.9226 & \text{for } \sigma = 10^{-2}, \\
9.2329 & \text{for } \sigma = 10^{-1}, \\
91.9537 & \text{for } \sigma = 1.
\end{cases}
\]

For these experiments, \(N\) almost surely has full rank. We notice that NNCPD reconstruction error \(\|\hat{T} - X\|_F < \|N\|_F\) for \(r < 10\); in contrast, the reconstruction error of Fixed NMF and Direct NMF often exceed \(\|N\|_F\) for \(r < 10\). The experiments suggest that the NNCPD reconstruction for overestimates of the rank can tolerate more noise when the noise tensor is of high rank than of low rank (with the same magnitude).

**In conclusion, the numerical experiments for robustness of NNCPD suggest that this decomposition is stable and robust to noise. In contrast, Direct NMF and Fixed NMF display unstable behaviour as we overestimate the rank of the tensor. Further, the experiments show that these kinds of tests can potentially estimate the rank (or an upper bound on the number of topics) of the tensor using NNCPD. We also notice a stable behaviour for NNCPD with a cusp or a minimum at \(r^*\) for tolerable noise.**

### 4 Conclusion

In this article, we have proposed a new methodology for large-scale dynamic topic modeling arising from the explosion of data in the information era. Previous works primarily employ NMF-based methods to decompose high dimensional data tensors that have one or more temporal dimensions. These methods seek to factorize a matricized version of the tensor sliced along the temporal dimension. Often a tensor is broken directly into time slices and each slice is decomposed individually using NMF (Direct NMF). Alternatively, authors in [CZA07] decompose the concatenated time slices of a tensor with one of the factors being fixed (Fixed NMF). There is a significant disadvantage of such NMF-based methods, in that the temporal mode of the data is not respected, thereby neglecting or oversimplifying the temporal information. To address this issue, we proposed using the method of nonnegative CP tensor decomposition (NNCPD) where the tensor is directly decomposed into a minimal sum of outer products of nonnegative vectors. In this way, critical temporal information is preserved, and events such as topic evolution, emergence and fading become significantly easier to identify.

In order to compare NMF methods with our NNCPD approach, we performed numerical experiments applied to deterministic synthetic datasets (Sect. 3.1), a real-life news dataset (Sect. 3.2), and on synthetic noisy tensor datasets (Sect. 3.3). We demonstrated how the factors of NNCPD can be interpreted for dynamic topic modeling. For
In these experiments, we add to all of the entries of the tensor $X$ positive noise drawn from the standard normal distribution. The noise is added in the form of a rank 1 tensor $N$. We report the median of the reconstruction error of NNCPD, Direct NMF, and Fixed NMF over 50 runs. We let the noise parameter vary $\sigma = 10^{-3}, 10^{-2}, 10^{-1}$, from left to right (starting from the top left plot).

In addition, for the real-life news dataset (Sect. 3.2), we find that by normalizing sections of the tensor that correspond to topics, and then using NNCPD, we can clearly identify times of topic emergence and fading using the temporal factor matrix, and can clearly determine the document structure of the tensor using the document factor matrix (Figure 26). In contrast, without normalization, this structure is less detectable, as can be seen in Figure 22 due to high word frequencies in some documents that dominate the structure of the decomposition. The reason may be that normalization ensures that all topic sections of the tensor have similar weight so that the NNCPD approximation objective function promotes fitting rank one tensors to each topic. To conclude, we see in our initial experimentation (Sect. 3.2.1 and 3.2.2) that approximate normalization among latent topics in a given dataset is important for naive NNCPD use in dynamic topic modeling. In future work, we will explore regularization methods that will assist datasets without this normalization property.

Finally, for noisy tensor datasets (Sect. 3.3) the experiment output suggests that NNCPD is superior to Direct NMF and Fixed NMF in stability and robustness to noise. Further, we observe that NNCPD is stable to overestimates of the rank, suggesting that the NNCPD-rank of a tensor can be estimated by producing a plot showcasing the reconstruction error of NNCPD with various ranks.
In these experiments, we add to all of the entries of the tensor $X$ positive noise drawn from the standard normal distribution. The noise is added in the form of a rank 2 tensor $N$. We report the median of the reconstruction error of NNCPD, Direct NMF, and Fixed NMF over 50 runs. We let the noise parameter vary $\sigma = 10^{-3}, 10^{-2}, 10^{-1}$, 1 from left to right (starting from the top left plot).

In summary, NNCPD proves to be a powerful tool for dynamic topic modeling, and compares favorably with typical NMF-based methods. We believe that our work provides an introduction to and evidence for the value of further exploration of this new approach of dynamic topic modeling through NNCPD.

References

ACD+18. Angela Ambrosino, Mario Cedrini, John B Davis, Stefano Fiori, Marco Guerzoni, and Massimiliano Nuccio. What topic modeling could reveal about the evolution of economics. Journal of Economic Methodology, 25(4):329–348, 2018.

APTJ95. Pia Anttila, Pentti Paatero, Unto Tapper, and Olli Järvinen. Source identification of bulk wet deposition in finland by positive matrix factorization. Atmospheric Environment, 29(14):1705–1718, 1995.

BF18. André Bittermann and Andreas Fischer. How to identify hot topics in psychology using topic modeling. Zeitschrift für Psychologie, 2018.

CC70. JDouglasCarrollandJih-Jie Chang. Analysis of individual differences in multidimensional scaling via an n-way generalization of “Eckart-Young” decomposition. Psychometrika, 35(3):283–319, 1970.

CZA07. Andrzej Cichocki, Rafal Zdunek, and Shun-ichi Amari. Nonnegative matrix and tensor factorization [lecture notes]. IEEE signal processing magazine, 25(1):142–145, 2007.

CZW+15. Yong Chen, Hui Zhang, Junjie Wu, Xingguang Wang, Rui Liu, and Mengxiang Lin. Modeling emerging, evolving and fading topics using dynamic soft orthogonal nmf with sparse representation. In 2015 IEEE International Conference on Data Mining, pages 61–70. IEEE, 2015.

DDL14. Ignat Domanov and Lieven De Lathauwer. Generic uniqueness conditions for the canonical polyadic decomposition and INDSCAL. arXiv:1405.6238 [math], May 2014. arXiv: 1405.6238.
In these experiments, we add to all of the entries of the tensor $X$ positive noise drawn from the standard normal distribution $Z'$. We report the median of the reconstruction error of NNCDP, Direct NMF, and Fixed NMF over 50 runs. We let the noise parameter vary $\sigma = 10^{-3}, 10^{-2}, 10^{-1}$, 1 from left to right (starting from the top left plot).
YCS09. Tae Yano, William W Cohen, and Noah A Smith. Predicting response to political blog posts with topic models. In Proceedings of Human Language Technologies: The 2009 Annual Conference of the North American Chapter of the Association for Computational Linguistics, pages 477–485. Association for Computational Linguistics, 2009.