Impact on the $r$-process from the nuclear mass and lifetime in covariant density functional theory

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Abstract. The covariant density functional theory with a few number of parameters allows a very successful description of the ground-state and excited-state properties for the nuclei all over the nuclear chart. The recent progresses on the application of the covariant density functional theory for the nuclear mass and $\beta$-decay half-life, as well as their influences on the astrophysical rapid neutron-capture process calculation are reviewed.

1. Introduction

In the past years, unstable nuclear beams have extended our knowledge of nuclear physics from the stable nuclei to the exotic nuclei far away from the stability. The study of exotic nuclei has attracted broad attention due to their large isospin values and interesting properties, such as the halo phenomenon [1, 2]. Furthermore, the properties of exotic nuclei, e.g. mass and $\beta$-decay half-life, are essential in understanding the nucleosynthesis via rapid neutron capture ($r$-process) which is responsible for roughly half of the enrichment of elements heavier than iron in the universe [3]. Nevertheless, although great efforts have been dedicated world-wide to experimental investigation, most of the neutron-rich nuclei of relevance to the $r$-process are still beyond the reach of experiments in the foreseeable future and therefore, can only rely on the nuclear theoretical models.

The density functional theory (DFT) with a small number of parameters is successful in describing the ground-state and excited-state properties of the nuclei all over the nuclear chart. In particular, the covariant density functional theory (CDFT) takes the Lorentz symmetry into account in a self-consistent way, and has received wide attention due to its successful description of lots of nuclear phenomenon [4, 5, 6, 7, 8].

There exist a number of attractive features in the CDFT. The most obvious one is the natural inclusion of the nucleon spin degree of freedom and resulting nuclear spin-orbit potential that emerges automatically with the empirical strength in a covariant way. Moreover, it can reproduce well the measurements of the isotopic shifts in the Pb region [9], and give more naturally the origin of the pseudospin symmetry [10, 11] as a relativistic symmetry [12, 13, 14, 15, 16, 17] and the spin symmetry in the anti-nucleon spectrum [18, 19]. It can also include the...
nuclear magnetism [20], i.e., a consistent description of currents and time-odd fields, which plays an important role in the nuclear magnetic moments [21, 22, 23, 24, 25] and nuclear rotations [26, 27, 28, 29]. Altogether, CDFT is a reliable and effective model for nuclear structure, and therefore, it is natural to describe the nuclei based on CDFT.

In this contribution, recent progress on the application of covariant density functional theory for nuclear masses and $\beta$-decay half-lives, as well as their influences on the astrophysical $r$-process calculation are reviewed.

2. Nuclear mass and $\beta$-decay half-life in CDFT

Taking the CDFT with the point-coupling interaction as an example, the starting point is an effective Lagrangian written as a power series in $\bar{\psi}\hat{\Omega}\psi$ and their derivatives,

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu\partial^\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F_{\mu\nu} - e\frac{1}{2}\gamma_5\bar{\psi}\gamma_\mu\gamma_5\gamma_\mu\psi - \frac{1}{2}\alpha_S(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma_\mu\psi) - \frac{1}{2}\alpha_V(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma_\mu\psi) - \frac{1}{2}\delta_S\partial_\nu(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma_\mu\psi) - \frac{1}{2}\delta_V\partial_\nu(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma_\mu\psi) - \frac{1}{2}\delta TV\partial_\nu(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma_\mu\psi),$$

(1)

where all symbols have the usual meaning as in Ref. [8].

The mean-field approximation leads to the replacement of the operators $\bar{\psi}(\hat{\Omega})_i\psi$ by the expectation values which become bilinear forms of the nucleon Dirac spinor $\psi_i$,

$$\bar{\psi}(\hat{\Omega})_i\psi \rightarrow \langle \Phi|\bar{\psi}(\hat{\Omega})_i\psi|\Phi \rangle = \sum_k v_k^2\bar{\psi}_k(\hat{\Omega})_i\psi_k,$$

(2)

where $i$ indicates $S$, $V$, and $TV$. The sum $\sum$ runs over only positive energy states with the occupation probabilities $v_k^2$, i.e., the “no-sea” approximation. Based on these approximations, one finds the energy density functional for a nuclear system

$$E_{DF}[\tau, \rho_S, \rho_V, \rho_T, A_\mu] = \int d^3r \mathcal{E}(r)$$

(3)

with the energy density

$$\mathcal{E}(r) = \mathcal{E}^{\text{kin}}(r) + \mathcal{E}^{\text{int}}(r) + \mathcal{E}^{\text{em}}(r),$$

(4)

which is composed of a kinetic part

$$\mathcal{E}^{\text{kin}}(r) = \sum_k v_k^2\bar{\psi}_k^\dagger(r)(\alpha \cdot \mathbf{p} + \beta m)\psi_k(r),$$

(5)

and an interaction part

$$\mathcal{E}^{\text{int}}(r) = \frac{\alpha_S}{2}\rho_S^2 + \frac{\beta_S}{3}\rho_S^3 + \frac{\gamma_{\text{S}}}{4}\rho_S^4 + \frac{\delta_S}{2}\rho_S\Delta\rho_S + \frac{\alpha_V}{2}\rho_V\rho_T + \frac{\gamma_V}{4}(j_{\mu}j^{\mu})^2 + \frac{\delta_V}{2}j_{\mu}\Delta j^{\mu} + \frac{\alpha_{TV}}{2}\bar{\gamma}^{\mu}(\vec{j}_{TV})_\mu + \frac{\delta_{TV}}{2}\bar{\gamma}^{\mu}(\vec{j}_{TV})_\mu + \Delta(\vec{j}_{TV}),$$

(6)

with the local densities and currents

$$\rho_S(r) = \sum_k v_k^2\bar{\psi}_k(r)\psi_k(r),$$

(7a)

$$j^{\mu}_{\text{V}}(r) = \sum_k v_k^2\bar{\psi}_k(r)\gamma^{\mu}\psi_k(r),$$

(7b)

$$\vec{j}_{\text{TV}}^{\mu}(r) = \sum_k v_k^2\bar{\psi}_k(r)\bar{\gamma}^{\mu}\psi_k(r),$$

(7c)
and an electromagnetic part

\[ E_{\text{em}}(r) = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - F_0^\mu \partial_0 A_\mu + e A_\mu j_\mu. \]  

(8)

Minimizing the energy density functional Eq. (3) with respect to \( \bar{\psi}_k \), one obtains the Dirac equation for the single nucleon

\[ [\gamma_\mu (i \partial_\mu - V_\mu) - (m + S)] \psi_k = 0. \]  

(9)

The single-particle effective Hamiltonian contains local scalar \( S(r) \) and vector \( V_\mu(r) \) potentials,

\[ S(r) = \Sigma S, \quad V_\mu(r) = \Sigma V + \vec{\tau} \cdot \vec{\Sigma}_T V, \]  

(10)

where

\[ \Sigma S = \alpha S \rho_S + \beta S \rho_S^2 + \gamma S \rho_S^3 + \delta S \Delta \rho_S, \]  

(11a)

\[ \Sigma V = \alpha V j_\nu^V + \gamma V (j_\nu^V)^3 + \delta V \Delta j_\nu^V + e A_\mu, \]  

(11b)

\[ \vec{\Sigma}_T V = \alpha TV \vec{j}_TV + \delta TV \Delta \vec{j}_TV. \]  

(11c)

As the translation symmetry is broken in the mean-field approximation, proper treatment of center-of-mass (c.m.) motion is very important. The c.m. correction energy can be taken into account in the oscillator approximation or in a microscopic way [30, 31, 32].

Based on the nuclear ground state, one can perform the random phase approximation (RPA) calculation for the low-lying excited states and giant resonances, as furthermore, for the \( \beta \)-decay rates of neutron-rich nuclei. With the individual transition strengths \( B_m \), the \( \beta \)-decay half-life of an even-even nucleus can be calculated in the allowed Gamow-Teller approximation,

\[ T_{1/2} = \frac{D}{g_A^2 \sum_m B_m f(Z, E_m)}, \]  

(12)

where \( D = 6163.4 \pm 3.8 \) s and \( g_A = 1 \). The sum runs over all final states with an excitation energy smaller than the \( Q_\beta \) value. The intergrated phase volume \( f(Z, E_m) \) is

\[ f(Z, E_m) = \int_{E_m}^{E_{m_e}} p_e E_e (E_m - E_e)^2 F_0(Z, E_e) dE_e, \]  

(13)

where \( p_e, E_e, \) and \( F_0(Z, E_e) \) denote the emitted electron momentum, energy, and Fermi function, respectively [33]. The \( \beta \)-decay transition energy \( E_m \) corresponds to the energy difference between the initial and final state calculated using the RPA

\[ E_m = \Delta_{np} - E_{\text{RPA}}, \]  

(14)

where \( E_{\text{RPA}} \) is the RPA energy with respect to the ground-state of the parent nucleus and corrected by the difference of the neutron and proton Fermi energies in the parent nucleus [34], and \( \Delta_{np} \) is the neutron-proton mass difference.

3. Influence of nuclear mass on astrophysical \( r \)-process calculation

Nuclear mass is one of the key nuclear inputs for astrophysical \( r \)-process calculation, from which one can directly determine the neutron separation energy, shell gap and also the \( \beta \)-decay energy. However, the majority of neutron-rich nuclei related to the astrophysical \( r \)-process are still out of the reach of present experimental capabilities, and therefore reliable predictions of theoretical
**Figure 1.** Features of the r-process calculated using the CDFT mass table and available data. The solid line denotes the border of nuclei with known masses in both neutron-deficient and neutron-rich sides. The yellow squares show the r-process path. The observed and calculated solar r-process abundance curves are plotted versus the mass number $A$ in the inset, whose x-axis is curved slightly to follow the r-process path. Taken from Ref. [36].

**Figure 2.** The impact of nuclear $\beta$-decay half-lives on the calculated r-process abundance. The solid (dashed) curves correspond to r-process abundance calculated with the RHFB+QRPA (FRDM+QRPA) $\beta$-decay half-lives in comparison with the data denoted by the points. Panels (a)-(d) correspond to the process times $\tau_r = 1.5$, 2.0, 2.5 and 3.0 s, respectively. Taken from Ref.[48].

models have to be used [35, 36, 37, 38, 39, 40, 41]. By employing the state-dependent BCS method with a $\delta$ force, the first systematic study of the ground-state properties for over 7000 nuclei was performed by using CDFT [42], where the rms deviation of neutron separation energy $S_n$ with respect to the known experimental data is 0.7 MeV.

Initiated by the development of nuclear masses, the r-process calculation based on the classical r-process approach is performed [43, 36]. The best simulation using the CDFT mass table and
the available data [44] is presented in Figure 1. The $\beta$-decay properties are taken from the calculation results of the finite-droplet model (FRDM) plus quasiparticle RPA [45]. In general, the solar $r$-process abundance peaks can be well reproduced. However, there are still large troughs before the second and third abundance peaks for the calculated $r$-process abundance distribution. These deficiencies in reproducing the solar $r$-process abundances can be mainly traced back to two aspects from the nuclear physics point of view: the shell effect and the location of nuclear shape transition. More detailed investigations on the nuclei with neutron numbers around magic numbers are demanded in microscopic nuclear physics models, which indicate that serious effort is needed to improve the formulation of the CDFT. Recently, a new parametrization PC-PK1 for CDFT with nonlinear point-coupling interaction is proposed, which can provide very good descriptions for the nuclear masses as well as its isospin dependence [46, 47]. Therefore, a new CDFT mass table with PC-PK1 is expected, and it will be interesting to investigate its influence on the astrophysical $r$-process calculation.

4. Influence of nuclear lifetime on astrophysical $r$-process calculation

Another key nuclear input for astrophysical $r$-process calculation is $\beta$-decay half-life, which can set the timescale of the astrophysical $r$-process and hence determine the production of heavy elements in the universe. Recently, fully self-consistent proton-neutron QRPA based on the relativistic Hartree-Fock-Bogoliubov (RHFB) theory is developed [48]. With an isospin-dependent proton-neutron pairing interaction in the isoscalar channel, the systematical calculation of $\beta$-decay half-lives for neutron-rich nuclei with $20 \geq Z \geq 50$ is performed. The results well reproduce the experimental $\beta$-decay half-lives, especially for nuclei with half-lives less than one second.

Based on the predicted $\beta$-decay half-lives, a classical $r$-process calculation is performed [48]. The available nuclear masses are taken from Ref. [49], otherwise the predictions of CDFT [42] are employed. Compared with the FRDM+QRPA calculation, the half-lives calculated with RHFB+QRPA model produce a faster $r$-matter flow at the $N = 82$ region, and thus yield higher $r$-process abundances of elements with $A \geq 140$, which is an important result for the estimate of the duration of the $r$-process, and hence the origin of heavy elements in the universe. In addition, the abundance at $A \sim 80$ is higher using the RHFB+QRPA result can be easily understood, because it takes more time to pass the $r$-path nuclei $^{78}$Ni and $^{80}$Zn and, as a result, higher abundances are accumulated.

5. Summary

Recent progress on the nuclear masses and $\beta$-decay half-lives in CDFT, as well as their influences on the $r$-process calculation are reviewed. The solar $r$-process abundance distribution can be generally well reproduced by simulations using the CDFT, where the large troughs before the second and third abundance peaks for the calculated $r$-process abundance distribution can be mainly traced back to the shell effect and the location of nuclear shape transition. Based on the $\beta$-decay half-lives of neutron-rich nuclei with $20 \geq Z \geq 50$ calculated by RHFB+QRPA model, a remarkable speeding up of $r$-matter flow is predicted which leads to enhanced $r$-process abundances of elements with $A \geq 140$, an important result for the understanding of the origin of heavy elements in the universe. In the future, with the new parametrization PC-PK1, the systematic calculation for nuclear masses and $\beta$-decay half-lives based on CDFT as well as astrophysical $r$-process calculation with these nuclear inputs will be performed.

Acknowledgments

We would like to thank H. Z. Liang, W. H. Long, T. Nikšić, Y. F. Niu, D. Vretenar for their contributions to the investigations presented in this proceeding. This work was supported in
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