Error Evaluation of the Crown Profile of Logarithmic
Generatrix Roller

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Abstract. In order to realize the accurate assessment of the profile error of the crown of the bearing roller, according to the geometric characteristics and the definition of the shape error of the contour element of the logarithmic bus-type roller, based on the principle of least squares, the least squares piece-wise fitting and error evaluation method of the logarithmic bus-type (combined form of the two-segment logarithmic curve) bearing roller crown profile are studied. Firstly, the three-point method is used to determine the curvature of the discrete data points of the convexity curve, and the boundary point of the two logarithmic curves is determined by the curvature difference method. Secondly, the measuring points adjacent to both sides of the dividing point are selected as auxiliary dividing points; the auxiliary dividing points and the corresponding logarithmic curve segment measuring points are fitted together to fit a series of least squares curves and calculate the corresponding errors. Through comparison and judgment, the least square error of the logarithmic bus-type roller profile is finally determined. Finally, through comparison and judgment, the least square error of the logarithmic bus-type roller profile is finally determined. The example results show that the total error of the logarithmic bus-type roller crown profile curve is 0.0071mm. The method in this paper can effectively realize the fitting and error evaluation of the bearing crown profile error, and it also verifies that the logarithmic prime line can reduce the stress concentration and improve the service life of the bearing.

1. Introduction
During the working process, stress concentration occurs due to fatigue pitting on both ends of the bearing roller due to uneven force. In order to reduce the stress concentration generated by the edge of the roller, the roller or raceway is modified by super finishing, that is, the convexity design is carried out, so that the generatrix of the roller or raceway has a slightly convex curve shape. It can ensure the uniform distribution of the load on the rollers or raceways, avoid premature fatigue and damage to the bearing rollers, thereby increasing the service life of the bearing. At present, the roller modification curves that are widely used in engineering include full arc convex, arc modified convex, logarithmic curve convex. The full-arc convex type and the circular-arc modified convex type have certain stress concentration. A large number of studies have shown that the logarithmic curve convex type can evenly distribute the load and effectively reduce the stress concentration, so it is widely used in engineering. However, the slight convexity contour error of the modified roller bus bar will cause a strong difference in its contact stress and the hydrodynamic lubricant film, which will have an important impact on the performance of the bearing. Therefore, the study of the overall fitting and error evaluation algorithm of the logarithmic curve crown profile of bearing rollers is of great significance to ensure the quality and accuracy of the rollers.
2. Mathematical model of logarithmic bus-type roller crown

The logarithmic curve roller convexity generatrix is composed of two mutually symmetrical logarithmic curves. As shown in Figure 1.

![Figure 1. Composition of roller convexity bus](image)

The mathematical model of the logarithmic convexity generatrix of cylindrical roller is established by formula (1).

\[
\begin{align*}
  y &= a_1 + b_1 \ln x (1 \leq x \leq \frac{c}{2}) \\
  y &= a_2 + b_2 \ln(c - x) (\frac{c}{2} \leq x \leq c - 1)
\end{align*}
\]

3. Least squares algorithm evaluation steps

3.1. Determine the dividing point of two logarithmic curves

3.1.1. Curvature calculation of intermediate data points

The logarithmic convexity bus is composed of two symmetrical logarithmic curves. The intersection of the two logarithmic curves is the dividing point of the logarithmic convex. The arc fitting method is used to calculate the curvature of the discrete points of the logarithmic curve profile (the data points at the first and the end are not considered), and the position of the boundary point is judged according to the curvature characteristics.[11]

Let \( P_i(x_i, y_i) (i = 1, 2, ..., N) \) be the N measuring points on the measured logarithmic convexity curve. The curvature of each measurement point is calculated as follows.

![Figure 2. Curvature calculation](image)

\[
K_i = \frac{2S}{L_i Q_i L_{i+1}}
\]

In the formula
\[
L_i = \|P_i - P_{i-1}\| = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}
\]
\[
Q_i = \|P_{i+1} - P_i\| = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}
\]
\[ L_{i+1} = \|P_{i+1} - P_i\| = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \]

\[ S = \pm \frac{1}{4} \sqrt{(L_i + L_{i+1} + Q_i)(L_{i+1} + Q_i - L_i) \times (L_i - L_{i+1} + Q_i)(L_{i+1} + L_i - Q_i)} \]

When \( P_{i+1} P_i \times P_{i+1} P_i > 0 \), the triangle area \( S \) takes a positive value, otherwise, \( S \) takes a negative value.

### 3.1.2. Determination of the difference in curvature of the measuring point and the demarcation point

The curvature difference \( E_i \) of the measuring point is calculated as follows:

\[ E_i = K_i - K_{i-1} \quad (3) \]

Since the curvature of the two logarithmic curves changes significantly at the boundary point, the curvature difference should be a local maximum or minimum. Therefore, how the curvature difference satisfies:

\[ |E_i| \leq \varepsilon \quad (4) \]

\( \varepsilon \) is a threshold.

### 3.2. Least Square Fitting of Two Logarithmic Curves and Error Calculation

#### 3.2.1. Constructing auxiliary demarcation points

The demarcation point initially determined based on the curvature difference method of data points is not necessarily the demarcation point when the errors of the two curves are both minimum. Therefore, \( t \) auxiliary measurement points need to be selected at the left and right ends of the dividing point \( P_j(x_j, y_j) \) for further judgment. Denoted as \( P_{j-1}(x_{j-1}, y_{j-1}) \) and \( P_{j+1}(x_{j+1}, y_{j+1}) \) respectively. Then, the demarcation point and the auxiliary demarcation point constitute a total of \( 2t+1 \) data points.

#### 3.2.2. Piece wise fitting and error calculation

The auxiliary dividing point is used to determine the data of the two logarithmic curves, and the fitting is performed according to the principle of least squares.

For the logarithmic curve at the left end (fitting data: \( P_1(x_1, y_1) \sim P_{j-1}(x_{j-1}, y_{j-1}) \); \( P_1(x_1, y_1) \sim P_{j-1}(x_{j-1}, y_{j-1}) \); \( P_1(x_1, y_1) \sim P_{j-1}(x_{j-1}, y_{j-1}) \); \( P_1(x_1, y_1) \sim P_{j+1}(x_{j+1}, y_{j+1}) \), \( 2t+1 \) left least square logarithmic curve equations can be obtained. Since the left end of the logarithmic curve has \( 2t+1 \) fitting situations, \( 2t+1 \) errors of the left end of the logarithmic curve can be obtained.

In the same way, since the logarithmic curve at the right end also has \( 2t+1 \) fitting situations, \( 2t+1 \) logarithmic curve errors at the right end can be obtained.

### 3.3. Total error of logarithmic crown profile

According to the definition of contour error, it can be inferred that when the left end logarithmic curve and the right end logarithmic curve are respectively fitted at the segment points, the maximum error of the two segments can contain the entire convexity profile. When the two logarithmic curves are respectively fitted to calculate the error through the above calculation steps, there are a total of \( (2t+1) \) cases, then \( (2t+1) \) maximum error values can be obtained, which is recorded as \( \Delta_{(2t+1)} \).

The area constructed by the minimum of the \( (2t+1) \) maximum error values is the total error of the logarithmic crown profile:

\[ Q = \min \left\{ \Delta_{(2t+1)} \right\} \quad (5) \]
4. Instance verification

In order to verify the correctness of the least squares fitting and error evaluation algorithm of the logarithmic bus-type roller crown profile. According to the geometric characteristics of the logarithmic convexity curve. Two symmetrically arranged logarithmic curves on the standard position of the two-dimensional plane coordinate axis are selected as the mathematical model for the evaluation of the bearing crown profile error. Discrete the data points of the selected standard curve and add random errors in the normal direction to form a simulated actual measurement data for simulation. After the above algorithm processing, the demarcation point is calculated.

As shown in Table 1, select 49 points on the set logarithmic convexity curve (Figure 3) formula (Equation 6) and add random errors in the normal direction to construct actual measurement data.

\[
\begin{align*}
y &= 10 + 2 \ln x (1 \leq x \leq 25) \\
y &= 10 + 2 \ln(50 - x) (x \geq 25)
\end{align*}
\]

![Figure 3. The mathematical model of the standard logarithmic bus profile](image)

| Serial number | Set data point | Normal error | Construct measurement data |
|---------------|----------------|--------------|---------------------------|
|               | x              | y            | \(\delta\)               | \(x_i\) | \(y_i\) |
| 1             | 1.0000         | 10.0000      | 0.0051                    | 0.9949  | 10.0003 |
| 2             | 2.0000         | 11.3863      | -0.0032                   | 2.0032  | 11.386  |
| 3             | 3.0000         | 12.1972      | 0.004                     | 2.996   | 12.1978 |
| 4             | 4.0000         | 12.7726      | -0.0037                   | 4.0036  | 12.7719 |
| 5             | 5.0000         | 13.2189      | 0.012                     | 4.9988  | 13.2192 |
| 6             | 6.0000         | 13.5835      | -0.01                     | 6.0096  | 13.5806 |
| 7             | 7.0000         | 13.8918      | 0.0061                    | 6.9942  | 13.8938 |
| 8             | 8.0000         | 14.1589      | -0.0072                   | 8.0067  | 14.1562 |
| 9             | 9.0000         | 14.3944      | 0.0016                    | 8.9985  | 14.3951 |
| 10            | 10.0000        | 14.6052      | -0.0065                   | 10.0058 | 14.6023 |
| 11            | 11.0000        | 14.7958      | 0.01                      | 10.9912 | 14.8006 |
| 12            | 12.0000        | 14.9698      | -0.0043                   | 12.0037 | 14.9676 |
| 13            | 13.0000        | 15.1299      | 0.0026                    | 12.9978 | 15.1313 |
| 14            | 14.0000        | 15.2781      | -0.0034                   | 14.0028 | 15.2762 |
| 15            | 15.0000        | 15.4161      | 0.0048                    | 14.9962 | 15.419  |
| 16            | 16.0000        | 15.5452      | -0.0052                   | 16.0041 | 15.5419 |
| 17            | 17.0000        | 15.6664      | 0.0061                    | 16.9954 | 15.6704 |
| 18            | 18.0000        | 15.7807      | -0.0016                   | 18.0012 | 15.7797 |
| 19            | 19.0000        | 15.8889      | 0.0042                    | 18.997  | 15.8918 |
| 20            | 20.0000        | 15.9915      | -0.0031                   | 20.0022 | 15.9893 |
| 21            | 21.0000        | 16.089       | 0.0053                    | 20.9963 | 16.0929 |
| 22            | 22.0000        | 16.1821      | -0.0021                   | 22.0014 | 16.1805 |
| 23            | 23.0000        | 16.271       | 0.0044                    | 22.9971 | 16.2743 |
| 24            | 24.0000        | 16.3561      | -0.0034                   | 24.0022 | 16.3535 |
| 25            | 25.0000        | 16.4378      | 0.0029                    | 24.9982 | 16.44   |
| 26            | 26.0000        | 16.5361      | -0.0041                   | 26.0025 | 16.3529 |
| 27            | 27.0000        | 16.271       | 0.0052                    | 26.9969 | 16.2752 |
Use 3.1.1 to simulate and analyze the data points on the logarithmic curve set in Table 1 and the measurement data points constructed after adding random errors. We can get the change rule of the curvature value and the curvature difference of the corresponding data point, as shown in Figure 4.

|   |   |   |   |   |
|---|---|---|---|---|
| 28 | 28.0000 | 16.1821 | -0.0039 | 28.0023 |
| 29 | 29.0000 | 16.089 | 0.0021 | 28.9988 |
| 30 | 30.0000 | 15.9915 | -0.0034 | 30.0019 |
| 31 | 31.0000 | 15.8889 | 0.01 | 30.9946 |
| 32 | 32.0000 | 15.7807 | -0.0072 | 32.0038 |
| 33 | 33.0000 | 15.6664 | 0.0032 | 32.9983 |
| 34 | 34.0000 | 15.5452 | -0.0016 | 34.0008 |
| 35 | 35.0000 | 15.4161 | 0.0048 | 34.9976 |
| 36 | 36.0000 | 15.2781 | -0.0037 | 36.0018 |
| 37 | 37.0000 | 15.1299 | 0.0044 | 36.9979 |
| 38 | 38.0000 | 14.9698 | -0.01 | 38.0047 |
| 39 | 39.0000 | 14.7958 | 0.0071 | 38.9968 |
| 40 | 40.0000 | 14.6052 | -0.0021 | 40.0009 |
| 41 | 41.0000 | 14.3944 | 0.0031 | 40.9986 |
| 42 | 42.0000 | 14.1589 | -0.0034 | 42.0015 |
| 43 | 43.0000 | 13.8918 | 0.0019 | 42.9992 |
| 44 | 44.0000 | 13.5835 | -0.0027 | 44.0011 |
| 45 | 45.0000 | 13.2189 | 0.0036 | 44.9985 |
| 46 | 46.0000 | 12.7726 | -0.0044 | 46.0018 |
| 47 | 47.0000 | 12.1972 | 0.0051 | 46.998 |
| 48 | 48.0000 | 11.3863 | -0.0018 | 48.0007 |
| 49 | 49.0000 | 10.0000 | 0.0021 | 48.9992 |

(a) Curvature graph of the set logarithmic curve data points
Figure 4. Pattern of variation Curvature and curvature difference of data points

(b) Curvature difference graph of the set logarithmic curve data points

(c) Curvature diagram of the data points of the error logarithm curve

(d) Curvature difference graph of the data points of the error logarithm curve
It can be seen from Figure 4 that the curvature map (a) of the data on the set curve is basically consistent with the curvature map (c) of the simulation data generated by adding random errors. The curvature difference map (b) of the data on the set curve is basically consistent with the curvature difference map (d) of the simulation data generated by adding random errors. Explain that this method determines the correctness of the demarcation point. Using 3.1.2 to calculate the measurement point \( P_i(x_i, y_i) \), the maximum value of the curvature difference can be obtained as 0.0766, and the threshold value is set to 0.05, and the boundary point is obtained as \( P_{25}(x_{25}, y_{25}) \).

To facilitate the calculation, select a data point on the left and right sides of the dividing point as an auxiliary dividing point, and then use 3.2.2 to fit the left end logarithmic curve and calculate the error to obtain 3 left least squares logarithmic curve equations and Error, the calculation results are shown in Table 2:

| data point | Logarithmic curve parameters at the left end | Error |
|------------|---------------------------------------------|-------|
| \( P_1 \sim P_{24} \) | \( a_1 = 10.0020 \), \( b_1 = 1.9992 \) | 0.0070 |
| \( P_1 \sim P_{25} \) | \( a_1 = 10.0018 \), \( b_1 = 1.9994 \) | 0.0071 |
| \( P_1 \sim P_{26} \) | \( a_1 = 10.0014 \), \( b_1 = 1.9914 \) | 0.0096 |

In the same way, select one data point on the left and right sides of the dividing point as the auxiliary dividing point, and then use 3.2.2 to fit the logarithmic curve at the right end and calculate the error to obtain 3 right least squares logarithmic curve equations and errors. The calculation results are shown in Table 3:

| data point | Logarithmic curve parameters at the right end | Error |
|------------|---------------------------------------------|-------|
| \( P_{24} \sim P_{49} \) | \( a_2 = 10.0116 \), \( b_2 = 1.9923 \) | 0.0172 |
| \( P_{25} \sim P_{49} \) | \( a_2 = 9.9987 \), \( b_2 = 2.0008 \) | 0.0046 |
| \( P_{26} \sim P_{49} \) | \( a_2 = 9.9983 \), \( b_2 = 2.0006 \) | 0.0046 |

Calculate the total error of the logarithmic crown profile according to 3.3, see Table 4.

| data point | Two-stage curve error | Total error |
|------------|-----------------------|-------------|
| \( P_1 \sim P_{24}, P_{24} \sim P_{40} \) | 0.0070, 0.0172 | 0.0072 |
| \( P_1 \sim P_{25}, P_{25} \sim P_{40} \) | 0.0071, 0.0046 | 0.0071 |
| \( P_1 \sim P_{26}, P_{26} \sim P_{40} \) | 0.0096, 0.0046 | 0.0096 |

It can be seen from Table 4 that the total error obtained by fitting the logarithmic convexity profile by the curvature difference constructing the boundary point is 0.0071mm, and when the total error is the smallest, the boundary point of the two logarithmic curves is at \( P_{25}(x_{25}, y_{25}) \), and at the same time get the corresponding parameters. The simulation data after adding the random error is compared with the theoretical setting curve data processing result, as shown in Table 5.
Table 5. Comparison of data processing results

|                      | Logarithmic curve parameters at the left end | Logarithmic curve parameters at the right end | error |
|----------------------|---------------------------------------------|----------------------------------------------|-------|
| Set curve            | a_1=10, b_1=2                              | a_2=10, b_2=2                                 | 0     |
| this article         | a_1=10.0018, b_1=1.9994                     | a_2=9.9987, b_2=2.0008                       | 0.0071|

It can be seen from Table 5 that the parameters obtained after fitting the data after adding random errors to the theoretical curve using the principle of least squares are basically consistent with the theoretical logarithmic convex curve parameters. Through the above series of steps, the logarithmic crown profile error is calculated to be 0.0071mm, which is higher than the accuracy of arc-corrected crown profile curve fitting.

5. Conclusion
This paper presents the least square fitting and error evaluation method of the logarithmic bearing roller crown profile generatrix. Analyze the effectiveness of the method through example simulation, and get:

(1) The logarithmic bearing roller crown profile generatrix is selected as the research object, and the mathematical model of the logarithmic bearing roller crown profile is determined according to the shape of the profile.

(2) Using the three-point arc method to study the curvature of the discrete data of the logarithmic bearing roller crown profile, and the curvature of the discrete data after adding random errors on the basis of the mathematical model.

(3) The method of curvature difference is used to find the demarcation points of theoretical data and discrete data, and the auxiliary demarcation points are constructed by using the demarcation points.

(4) Using dividing points and auxiliary dividing points to fit logarithmic bearing roller crown contours in sections. The overall error of the roller crown profile is obtained.

(5) This article provides an effective method for evaluating the profile error of bearing roller crown. This method is simple to program and does not require coordinate conversion.

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