A direct methodology for the calibration of ductile damage models from a simple multiaxial test

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Abstract. In the present work a straightforward calibration procedure of ductile damage models is proposed. The direct methodology involves the use of a simple multiaxial specimen, to be tested with a universal testing machine, capable to reproduce different stress states in the material. The specimen geometry was the one proposed by Driemeier et al. [1]. In addition, a numerical-analytical procedure was devised for the identification of material strains to fracture and corresponding stress states, directly from experimental tests. This allowed to overcome the use of Finite Element Analysis and inverse methods usually adopted to retrieve the local parameters representative of the material ductility.

1. Introduction
There are many cases where a correct description of the structural performance of the material and an accurate prediction of its ultimate strength is crucial. For instance, this is of major importance in all forming processes, where plastic formability limits and post-manufacturing residual stresses must be determined with great accuracy. Also, the knowledge of the material ultimate resistance is crucial in understanding how a component will withstand potential accidental overloads without reaching failure. For this purpose many damage models for ductile materials have been developed over the years. A ductile damage model is a mathematical model describing how the material gets damage with deformation and enables us to predict the onset of material failure and to know how much the material is damaged when subjected to a load. There are different classes of damage models based on different criteria. In particular, the empirical criteria-based models assume the damage accumulates with the equivalent plastic strain growth weighted on a function of the stress state [2, 3, 4]. These damage models need to be calibrated for the specific material and the calibration procedure is not trivial. It consists of two phases: the first is an experimental campaign in which several tests are carried out to induce different stress states into the material investigated. At this stage the global quantities, such as force and displacement at failure, are collected. The second phase involves numerical finite element simulation of the experimental tests to retrieve local quantities descriptive of the stress state and not directly measurable.

In particular, to perform an accurate calibration it is often necessary to induce several complex stress states in the material, especially with non-linear formulations [5, 6], which require a dedicated multi-axis machines and a custom specimens that are difficult to manufacture. Because of this complexity, these models are not yet used if not marginally by the industries, which often does not have
the know-how or the right equipment to perform these types of tests or manage the subsequent post-processing stage.

In this context, the aim of this work regards the development of a straightforward calibration procedure for ductile damage models in order to simplify and promote the use of damage criteria also on an industrial scale. The new procedure is based on the use of a simple multiaxial specimen, to be tested with a common tensile machine, which is able to generate different stress states, such as pure shear or combined tensile-shear states. The specimen geometry was the one proposed by Driemeier et al [1]. Moreover, to overcome the demanding phase of Finite Element Analysis, a numerical-analytical model was developed for the identification of the local quantities that describe the ductility of the material, directly from the experimental tests.

2. Theoretical background

It is well known that a wide class of ductile damage models consider the accumulated damage depending on the increase of plastic strain, weighted on a function of the stress state, as reported in equation (1).

The function of the stress state can be described using two scalar parameters: the triaxiality $T$ and the Lode parameter $X$. In turn these two quantities are function of the invariant of the stress tensor as stated in equation (2) and (3):

$$D = \int_0^{\varepsilon_f} f(T, X)d\varepsilon_p$$

$$T = \frac{1}{3} \frac{I_1}{\sqrt{3} J_2}$$

$$X = \frac{27}{2} \frac{J_3}{(\sqrt{3} J_2)^3}$$

where $I_1$ is the first stress invariant, $J_2$ and $J_3$ are the second and the third deviatoric stress invariant, respectively. The fracture onset occurs when $D$ reaches a critical value conventionally set equal to 1. Under the assumption of proportional loading $T$ and $X$ are constant and posing $D$ equal to 1, the equation (1) can be inverted obtaining the equation (4).

$$\varepsilon_f = f^{-1}(T, X)$$

This formula describes a surface called “Fracture Locus” in the space $(\varepsilon_p, T, X)$ for which each pair of $(T, X)$ is coupled to a unique value of plastic strain at incipient failure. Usually, $T$ and $X$ and are not perfectly constant, exhibiting limited variation, therefore it is commonly accepted to employ the average value of the two parameters as shown in equations (5) and (6):

$$T_{\text{avg}} = \frac{1}{\varepsilon_f} \int_0^{\varepsilon_f} T(\varepsilon)d\varepsilon_p$$

$$X_{\text{avg}} = \frac{1}{\varepsilon_f} \int_0^{\varepsilon_f} X(\varepsilon)d\varepsilon_p$$

Over the years, several forms of the function $f^{-1}(T, X)$ have been proposed in the literature. For instance, equation (7) shows the analytical expression of the damage model proposed by Coppola, Cortese and Folgarait (CCF) [3].

$$\varepsilon_f = \frac{1}{C_1} e^{-C_2 T} \left( \frac{\cos \left[ \frac{\pi}{6} \frac{1}{3} \arccos(y) \right] }{\cos \left[ \frac{\pi}{6} \frac{1}{3} \arccos(y) X \right] } \right)^{\frac{1}{n}}$$
Where $C_1, C_2, \beta$ and $\gamma$ are material parameters to be tuned through a calibration procedure and $n$ is the hardening exponent of the Hollomon’s power law. Instead, the equation (8) represents the damage model devised by Bai and Wierzbicki [4].

$$
\varepsilon_f = \left( \frac{A}{C_2} \left[ \left( \frac{1+C_2^2}{3} \right)^{\frac{1}{2}} \cos \left( \frac{\pi}{6} - \theta \right) + C_1 \left( T + \frac{1}{3} \sin \left( \frac{\pi}{6} - \theta \right) \right) \right] \right)^{-\frac{1}{n}} \tag{8}
$$

Where $C_1$ and $C_2$ are the parameters to be identified when the Von Mises yielding condition is adopted and $n$ is the hardening exponent. Testing different specimen’s geometries permits the identification of the four points in the $\varepsilon_p, T, X$ space. An example of fracture locus is illustrated in figure 1 for a titanium alloy (SLM Ti6Al4V) [7].

3. The “Driemeier” specimen
The starting point of this work was to define the design criteria that led to the choice of specimen. They can be summarised as follows: the easiness of fabrication, the capability of generating both a pure shear state and a multiaxial stress states like tensile-shear with uniaxial tensile machines, the capacity to focus the maximum plastic strain in the test area and the ability to maintain the parameters $T$ and $X$ as constant as possible to satisfy the quasi-proportional loading condition.

After the definition of the design criteria, a literature study was carried out, which for the sake of brevity is not reported, in which several specimens available in the literature designed for shear or tensile-shear tests were analysed. The geometry that most closely meets the design criteria is the one proposed by L. Driemeier et al. [1, 8] shown in figure 2A, a specimen used to perform shear tests with which is possible to obtain information of stress-triaxiality dependence on the material behaviour in the low triaxiality regime. The specimen, hereafter referred to as the “Driemeier”, was considered the most suitable because it concentrates the maximum plastic strain value in its critical point in the gauge centre, it can be produced with conventional machine tools and it is possible to obtain both pure shear and combined tensile-shear, using a universal standard machine by simply varying the position of the two central holes. This is achieved by rotating the centres of the two holes by an alpha angle ($\alpha$) on a circumference centred on the specimen, as illustrated in the figure 2B. In this way, a range of different couple of $T_{avg}$ and $X_{avg}$ can be achieved in a low triaxiality regime.
Figure 2. (A) Original geometry of the “Driemeier” specimen; (B) Representation of alpha angle (α).

The only limitation of the specimen is the constancy of the parameters $T$ and $X$ for high level of strain. In fact, if an excessive displacement is applied, the test area tends to rotate aligning itself with the machine’s axis of pull, turning the test into a tensile test for high plastic strains. To reduce excessive rotation and hence comply with the quasi-proportional loading conditions, a sensitivity analysis was carried out to optimize the geometry.

The final specimen dimensions are shown in figure 3, and they were scaled to be machined from metal bars with a diameter of 20 mm. In the present work, three different configurations of the central holes were chosen, corresponding to three different alpha angles (α) of -10°, +10° and +30°. The -10° angle was chosen to further reduce the rotation of the test area in order to generate a stress state close to pure shear, i.e. $T_{avg}$ and $X_{avg}$ close to zero. Instead, the other two angles were chosen to generate combined tensile-shear stress states and to achieve different values of the Lode parameter at low triaxiality.

Figure 3. Technical drawing of the optimised geometry for the three configurations: -10°, +10°, +30°.
All the analyses carried out in this study were performed through finite element simulations using the commercial software ANSYS 2019. The models were meshed with brick elements and the function “large displacement” of the solver was activated. The displacement was applied to the upper part of the specimen, while the lower part was fixed. The stainless steel 17-4PH was chosen as benchmark material, since it was fully characterised in a study proposed by Nalli et al. [7]. Its constitutive behaviour was modelled using both the multilinear and bilinear stress-strain constitutive law, as shown in figure 4.

4. Results
The present section reports the results of the simulations used to verify the stress state at low triaxiality regime generated by the three different configurations introduced above -10°, +10°, +30°. Figure 5 shows the history of actual values of $T$ and $X$, as a function of accumulated plastic strain. It is evident that suitably different pairs of $T$ and $X$ values were obtained, and this allows a proper characterisation of the fracture locus at low triaxiality regime. However, these values can only be considered approximately constant for plastic strain smaller than about 0.65, complying with the quasi-proportional loading hypothesis. Above this threshold, there is an increase of $T$ and $X$ due to the excessive rotation of the section.

Figure 5. History of the instantaneous values of the Triaxiality and Lode parameters.
Considering a value of plastic strain of about 0.63, within the threshold, and using the formulae (5-6), it was possible to derive $T_{avg}$ and $X_{avg}$ for the three configurations as reported in table 1. In addition, the value of the displacement, applied to the specimen in the finite element simulation, required to obtain a deformation of 0.63 is reported.

| Configuration                  | $d$ [mm] | $\varepsilon_p$ [mm/mm] | $T_{avg}$ [-] | $X_{avg}$ [-] |
|-------------------------------|----------|--------------------------|---------------|---------------|
| $\alpha = -10^\circ$ Pure shear | 2.4      | 0.63                     | 0.05          | 0.15          |
| $\alpha = +10^\circ$ Combined shear-tension | 1.87     | 0.63                     | 0.11          | 0.38          |
| $\alpha = +30^\circ$ Combined shear-tension | 1.5      | 0.63                     | 0.18          | 0.60          |

It is clear that the configuration at -10°, characterised by a couple of $T$ and $X$ with values close to zero, generates a stress state nearly equal to that of pure shear ($T = 0$, $X = 0$) as anticipated above. The other two couples represent a good combination of values from which to derive information of stress state at low triaxiality for the calibration procedure, and thus improve the prediction accuracy of the damage models. Another important result, that fulfils one of the design requirements, is the presence of the maximum plastic strain value in its critical point in the gauge centre. A proof of this result is illustrated in figure 6A where only the contour plot of the -10° configuration is shown. Moreover, figure 6B and figure 6C display the contour plots of $T_{avg}$ and $X_{avg}$. These two quantities, and therefore the corresponding graphs, are not present in the standard set of available outputs, but ANSYS allows the implementation of a custom script, i.e. textual commands in the "ANSYS Parametric Design Language" (APDL), with which to numerically derive and graphically represent an user-defined result. Therefore, the contour plots in figures 6B and 6C were obtained through the implementation of a custom APDL script devised to represent the behaviour of $T_{avg}$ and $X_{avg}$.

Figure 6. Driemeier -10°, contour plot of: (A) Plastic Strain, (B) Triaxiality, (C) Lode parameters.
In addition to the significant results obtained with the Driemeier test specimen, a mathematical model was created to derive the local parameters describing the stress state directly from the experimental tests without using the Finite Element Method (FEM). Namely, a set of formulae was developed for each type of experimental test to derive \( \varepsilon_p \), \( T_{avg} \) and \( X_{avg} \) at the most critical point, requiring only the knowledge of the displacement at failure \( d_f \) from the experimental tests and the knowledge of the bilinear constitutive law of the material, i.e. yield point \( \sigma_y \) and tangent modulus \( E_t \). These formulae were developed as a result of an extensive simulation campaign performed via FEM with the aim of analysing the behaviour of \( \varepsilon_p \), \( T_{avg} \) and \( X_{avg} \) as a function of a general displacement \( d \) applied to the specimen and as function of the material represented by \( \sigma_y \) and \( E_t \).

The procedure used to derive these analytical expressions is the following. Once the three Driemeier specimen configurations were selected, extensive ranges of \( \sigma_y \), \( E_t \) and \( d \) were chosen within which to perform the study to consider a wide spectrum of ductile materials, namely \( \sigma_y \) 200-1250 [MPa], \( E_t \) 100-1200 [MPa] and \( d \) 0.15-4 [mm] respectively. Subsequently, many finite element simulations were carried out for each configuration and for each combination of \( \sigma_y \) and \( E_t \) for different values of displacement \( d \). In particular the effect of \( \sigma_y \) and \( E_t \) was studied individually, i.e. \( \sigma_y \) was studied keeping \( E_t \) fixed for a certain value of \( d \) and vice versa. From now on, for the sake of brevity, the only focus will be on the Driemeier configuration -10°, being the procedure the same for the other geometries. For example, according to figure 7, the functions that best fit the effects of \( \sigma_y \) and \( E_t \) on plastic strain are the logarithmic function and the decreasing exponential function respectively.

Thus, as a result, the equation (9) represents the formula that best approximates the plastic strain behaviour. In particular, the terms \( c_1, c_2, c_3 \) and \( c_4 \) are dependent on the imposed displacement according to the equations (10-13). The same analysis procedure was performed for \( T_{avg} \) and \( X_{avg} \). The equations (9, 14, 15) of \( \varepsilon_p \), \( T_{avg} \) and \( X_{avg} \) graphically represent a 3D surfaces as a function of \( d \), \( \sigma_y \) and \( E_t \). For example, still regarding the configuration -10°, in figure (8) three surfaces that describe \( \varepsilon_p \) are reported at three different displacement values: 1.665, 2.59 and 3.515 mm.

\[
\varepsilon_p = c_1 e^{(-c_2E_t)} + c_3 \ln(\sigma_y) + c_4 \tag{9}
\]

\[
c_1 = 0.3738d - 0.1214 \tag{10}
\]
\[ c_2 = -1.37 \times 10^{-4} d + 2.894 \times 10^{-3} \]  
(11)
\[ c_3 = 0.1163 d - 0.1002 \]  
(12)
\[ c_4 = -0.6778 d + 0.7481 \]  
(13)
\[ T_{avg} = 0.04 \]  
(14)
\[ X_{avg} = -c_1 E_t + c_2 \sigma_y + c_3 \]  
(15)
\[ c_1 = 3.728 \times 10^{-6} e^{0.927d} + 2.341 \times 10^{-6} \]  
(16)
\[ c_2 = 1.849 \times 10^{-5} d + 5.123 \times 10^{-5} \]  
(17)
\[ c_3 = 0.0478 d - 0.0316 \]  
(18)

Notice that, for the configuration \(-10^\circ\), in equation (14), the value of \(T_{avg}\) was assumed equal to 0.04, because the variation with the material and displacement was observed to be minimal, thus it was considered constant. Considering all the simulation with different combinations of \(d\), \(\sigma_y\) and \(E_t\), the value of \(T_{avg}\) varied within the range 0.02-0.08, thus a value of 0.04 was considered a fair compromise. The equations (16-18) represent the parameters of the \(X_{avg}\) taking into account the dependence of displacement.

![Figure 8](image)

**Figure 8.** Three surfaces that describe the plastic strain in the space \((E_t, \sigma_y, \varepsilon_p)\) at three different displacement values.
Regarding the traditional method, the simulation of the experimental test is required and to find the triad \((\varepsilon_p, T_{avg}, X_{avg})\) it is necessary to stop the simulation at the same experimental condition in which the material failure occurs, i.e. at the same value of the displacement at failure. Then \(\varepsilon_p\) and the history of \(T, X\) are exported to calculate \(T_{avg}\) and \(X_{avg}\). On the contrary, the use of the mathematical model allows to overcome the finite element analysis and the inverse method normally adopted to derive the local parameters representative of the material ductility.

A detailed example of how to use the mathematical model is provided below. Consider the Driemeier specimen at alpha \(-10^\circ\) configuration and the corresponding results obtained via FEM for a displacement of 2.4 mm shown in Table 1 and Figure 6. Knowing only the applied displacement and the bilinear constitutive law of the material, it is possible to apply the formulae (9-18) and determine \(\varepsilon_p\), \(T_{avg}\) and \(X_{avg}\). For the 17-4PH material, values of 1035 and 460 MPa for yield strength and tangent modulus respectively were chosen to define the bilinear elastic-plastic constitutive law of the material, Figure 4. Table 2 compares the results obtained through both methods, i.e. via FEM and via Mathematical Model. The values obtained are very close to those obtained via FEM, demonstrating the goodness of the direct method.

**Table 2.** Comparison of values obtained with both FEM and mathematical model for the \(-10^\circ\) configuration applying a displacement of 2.4 mm.

|       | FEM   | Math. Model |
|-------|-------|-------------|
| \(\varepsilon_p\) [mm/mm] | 0.63  | 0.60        |
| \(T_{avg}\) [-]           | 0.05  | 0.04        |
| \(X_{avg}\) [-]           | 0.15  | 0.16        |

For a more general overview of the results, Table 3 provides a comparison of the triad values \((\varepsilon_p, T_{avg}, X_{avg})\) obtained from both the Finite Element Method and the mathematical model for several displacement \(d\). The parameters evaluated through mathematical model are very close to those acquired via FEM, meaning that the validation of the model is positive.

Regarding the approximation committed using the mathematical model, in Table 4 the average of the relative error in percentage \((err_{rel}^{avg})\) and the standard deviation \((Std)\) for each test configuration are provided. The error committed is acceptable considering the benefits of the mathematical method: a simplification of the calibration operation and a significant reduction in computational time since the use of FEM is no more required.

**Table 3.** Comparison between the parameters obtained using the mathematical model and via FEM.

| Driemeier \(-10^\circ\); \(d=1.665\) mm | Driemeier \(-10^\circ\); \(d=2.59\) mm | Driemeier \(+10^\circ\); \(d=1.53\) mm | Driemeier \(+10^\circ\); \(d=2.38\) mm | Driemeier \(+30^\circ\); \(d=1.35\) mm | Driemeier \(+30^\circ\); \(d=2.1\) mm |
|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|
| FEM                                   | Math. Model                            | FEM                                   | Math. Model                            | FEM                                   | Math. Model                            |
| \(\varepsilon_p\) [mm/mm]             | \(T_{avg}\) [-]                       | \(X_{avg}\) [-]                       | \(\varepsilon_p\) [mm/mm]             | \(T_{avg}\) [-]                       | \(X_{avg}\) [-]                       |
| 0.438                                 | 0.036                                 | 0.109                                 | 0.679                                 | 0.049                                 | 0.155                                 |
| 0.415                                 | 0.040                                 | 0.123                                 | 0.651                                 | 0.040                                 | 0.175                                 |
| 0.488                                 | 0.090                                 | 0.329                                 | 0.747                                 | 0.116                                 | 0.419                                 |
| 0.488                                 | 0.097                                 | 0.335                                 | 0.732                                 | 0.125                                 | 0.422                                 |
| 0.515                                 | 0.167                                 | 0.556                                 | 0.804                                 | 0.207                                 | 0.669                                 |
| 0.547                                 | 0.173                                 | 0.569                                 | 0.822                                 | 0.213                                 | 0.680                                 |
Table 4. Mean relative error (%) and the standard deviation committed using the mathematical model.

|      | $\text{err}_{\text{avg}}(\varepsilon_{\text{pl}})$ | $\text{err}_{\text{avg}}(T_{\text{avg}})$ | $\text{err}_{\text{avg}}(X_{\text{avg}})$ | $\text{Std}(\varepsilon_{\text{p}})$ | $\text{Std}(T_{\text{avg}})$ | $\text{Std}(X_{\text{avg}})$ |
|------|---------------------------------|---------------------------------|---------------------------------|-----------------|--------------------|--------------------|
| -10° | 6.5%                            | 30.8%                           | 9.6%                            | 0.026           | 0.015              | 0.015              |
| +10° | 2.2%                            | 8.8%                            | 1.6%                            | 0.021           | 0.010              | 0.008              |
| +30° | 6.0%                            | 3.0%                            | 1.9%                            | 0.063           | 0.008              | 0.013              |

5. Conclusion
The aim of this work was to develop a straightforward procedure to simplify the calibration of ductile damage models to make these failure criteria widespread on an industrial scale. The first step in the roadmap was to choose a suitable specimen from literature that met the following requirements: easiness of production, capability to confine the maximum value of plastic strain in the test area, capacity to generate pure shear or shear-tension states through a uniaxial testing machine and the capacity to maintain the quasi-proportional loading conditions. The chosen specimen that met all the requirements is the one proposed by L. Driemeier. The geometry of the specimen was optimised with respect to the original to keep the Triaxiality and Lode parameters as constant as possible. In addition, three different geometry configurations (-10°, +10°, +30°) generating respectively one pure shear stress state and two combined tensile-shear states were verified via FEM. The last step to further simplify the procedure was to develop a mathematical model that permits to derive locally in the critical point the plastic strain, $T$ and $X$ directly from the experimental test knowing the material and the displacement at failure. Since this numerical-analytical procedure only requires the use of a spreadsheet, it is no longer necessary to derive parameters using the finite element method, which means a saving in computational time and a straightforward procedure. The objective of the future work is to produce these specimens and to carry out experimental tests in order to calibrate several damage models for different materials using the new procedure presented in this work.

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