Constraining the geometry of millisecond pulsars through simultaneous radio and gamma-ray light-curve fitting

O. Benli1, J. Pétri1, Dipanjan Mitra2,3

1 Université de Strasbourg, CNRS, Observatoire astronomique de Strasbourg, UMR 7550, F-67000 Strasbourg, France.
2 National Centre for Radio Astrophysics, Tata Institute for Fundamental Research, Post Bag 3, Ganeshkhind, Pune 411007, India.
3 Janusz Gil Institute of Astronomy, University of Zielona Góra, ul. Szafrana 2, 65-516 Zielona Góra, Poland.

2020

ABSTRACT

Context. Since the launch of the Fermi Gamma-Ray Space Telescope, several hundred of gamma-ray pulsars have been discovered, some being radio-loud and some radio-quiet with time-aligned radio and gamma-ray light-curves. In the 2PC, 117 new gamma-ray pulsars have been reported by using three years of data collected by the Large Area Telescope (LAT) on the Fermi satellite, providing a wealth of information such as the peak separation $\Delta$ of the gamma-ray pulsations and the radio lag $\delta$ between the gamma-ray and radio pulses.

Aim. We selected several radio-loud millisecond gamma-ray pulsars with period $P$ in the range 2-6 ms and showing double peak in their gamma-ray profiles. We attempted to constrain the geometry of their magnetosphere, namely the magnetic axis and line of sight inclination angles for each of these systems.

Method. We applied a force-free dipole magnetosphere from the stellar surface up to the striped wind region, well outside the light cylinder, to fit the observed pulse profiles in gamma-rays, consistently with its phase alignment with the radio profile. In deciding whether a fitted curve is reasonable or not, we employed a least-square method to compare the observed gamma-ray intensity with that found from our model, emphasizing the amplitude of the gamma-ray peaks, their separation and the phase lag between radio and gamma-ray peaks.

Results. We obtained the best fits and reasonable parameters in agreement with observations for ten millisecond pulsars. Eventually, we constrained the geometry of each pulsar described by the magnetic inclination $\alpha$ and the light of sight inclination $\zeta$. We found that both angles are larger than approximately 45°.

Key words. stars: pulsars: millisecond – methods: numerical – magnetosphere – plasma

1. Introduction

The launch of the Fermi Gamma-Ray Space Telescope in June 2008 revolutionized the physics of pulsars by discovering hundreds of new gamma-ray pulsars, among them many millisecond ones (Abdo et al. 2013), a surprising result at that time. The second gamma-ray pulsar catalogue (2PC) contains 117 pulsars. Nowadays, the population enriched to almost 300 pulsars, some being radio-loud and some radio-quiet with time-aligned radio and gamma-ray light-curves. In the 2PC, 117 new gamma-ray pulsars have been reported by using three years of data collected by the Large Area Telescope (LAT) on the Fermi satellite, providing a wealth of information such as the peak separation $\Delta$ of the gamma-ray pulsations and the radio lag $\delta$ between the gamma-ray and radio pulses.

Another complication while modelling pulsar electrodynamics stems from the magnetic obliquity with respect to the spin axis of the pulsar. Initial attempts to understand the physics underlying the pulsar magnetosphere commonly focused on the aligned rotator assumption. The reason was that finding a solution for the aligned case is much easier than the more realistic oblique case. Contopoulos et al. (1999) could achieve numerical simulations to encounter FFE for the first time for the aligned rotator. Several authors constructed a force-free pulsar magnetosphere by considering time-dependent simulations for the oblique case (Spitkovsky 2006; Kalapotharakos & Contopoulos 2009; Pétri 2012b). Because the simulations otherwise would be very time-consuming, in these previous works, a large ratio of $R_*/r_{LC} = 0.2-0.3$ was used corresponding to an unreal-
istically fast pulsar with periods ~ 1 ms. Some authors proposed models including huge vacuum gaps in which a differentially rotating plasma flowing around the equatorial plane and named, separated from the oppositely charges domes by huge gaps and name electrosphere (Krause-Polstorff & Michel 1985; Pétri et al. 2002; McDonald & Shearer 2009).

For the first time, Watters et al. (2009) performed a detailed study of pulsar light curves with an individual assessment of polar cap, outer gap and slot gap models. Since then Venter et al. (2009), Pétri (2011) and Pierbattista et al. (2015) contributed to the goal of producing a complete atlas of pulsar light curves for different obliquities $\alpha$ and inclination angles $\varepsilon$.

The estimation of the radio/gamma-ray time lag $\delta$ and gamma-ray peak separation $\Delta$ for those pulsars showing double-peaked light curves and application to the observed light curves of several gamma-ray pulsars, provided in the 1$^{st}$ Fermi Catalogue Abdo et al. (2010a), has been done in Pétri (2011) by using an oblique split monopole solution (Bogovalov 1999) for the field structure and a simple polar cap geometry for the radio counterpart. In this work, we aim to understand the characteristics of the gamma-ray and radio light curves by employing the FFE prescription where we also take into account the real effect of particle flow within the dipolar magnetosphere, onto the field structure. Kalapotharakos et al. (2012) focused only to the orthogonal rotator case by taking into account a large range of conductivity in dissipative pulsar magnetospheres. In Kalapotharakos et al. (2014), the orthogonal condition was relaxed to the whole range of magnetic obliquity to produce $\delta$ and $\Delta$ measured by the observations. By comparing the observed $\Delta - \delta$ diagram with their estimations from the model, these authors favoured the cases with high conductivities. As the conductivity increases the system converges to the FFE limit.

Chen et al. (2020) have recently suggested a force-free magnetospheric model and presented radio, X-ray and gamma-ray light curves in agreement with the observations of the millisecond pulsar PSR J0030+0451 with an estimation of the magnetic inclination of the pulsar ~ 80°. More recently, Kalapotharakos et al. (2020) proposed the FFE model by taking into account an off-centred dipole field and quadrupole components, the Kerr metric for the ray-tracing of photons from a distant observer to the hot spots on the surface of the star.

All these recent studies are very promising to constrain the geometry of the pulsar and are the subject of this paper, of which the plan is as follows. In Section 2 we briefly mention about the radio emission phenomenology of pulsars and draw attention to the complications in the radio pulse profiles of MSPs. In Section 3, we summarize the gamma-ray observations of Fermi/LAT as from 2PC. In Section 4, we describe the force-free model for the magnetosphere and the striped wind where gamma-ray pulses are assumed to be produced in our model. We explain our fitting method in Section 5. Remaining consistent with radio and gamma-ray time-alignment, we reproduced the observed gamma-ray pulse profiles for the sample of MSPs and attempted to determine the geometry of each pulsar. We present our results in Section 6. A broader discussion on our method is discussed in Section 7. We draw conclusions in Section 8.

### 2. Radio Observations and Phenomenology

We aim to use radio and gamma-ray observations to find the dipolar magnetic field emission geometry of the pulsar with respect to the observer. The role of the radio observations enters the problem in the following manner. Firstly one needs to assume that the radio emission arises deeper in the magnetosphere closer to the surface of the neutron star and secondly the emission needs to arise from regions where the magnetic field geometry can be closely approximated with a static dipole. These two assumptions can then be used to treat the radio emission as a proxy for identifying the fiducial dipolar magnetic field line plane, with respect to which the geometry of the gamma-ray emission can be solved. The assumptions hold good for normal period pulse (i.e. pulsar with periods longer than about 100 ms), where detailed phenomenological profile and polarization studies suggest that the radio pulsar emission beam comprises of a central core emission surrounded by two nested conal emissions (see Rankin (1983, 1990, 1993); Mitra & Deshpande (1999)). The emission arises from regions of open dipolar magnetic field lines below 10% of the light cylinder radius, see Mitra & Li (2004) and Mitra (2017). These conclusions were arrived by demonstrating that the observed opening angle of normal pulsars follow the $P^{0.5}$ (where $P$ is the pulsar period) behaviour (see e.g. Skrzyczek et al. (2018)) which is as expected from emission arising from open dipolar magnetic field line region. Secondly, the polarization position angle across the pulse profile follow the rotating vector model, which is an indication of emission arising from regions of diverging magnetic field line geometry. Further, the effect of aberration and retardation in the linear approximation has been observed in normal radio pulsar, see Blaskiewicz et al. (1991) Pétri & Mitra (2020), whereby a shift between the centre of the total intensity profile and the steepest gradient point of the position angle curve is observed. This shift is proportional to $r_{sl}/r_L$ and is used to the radio emission heights.

However, in millisecond pulsars, the normal period pulsar phenomenology cannot be applied. Radio pulse profile does have any orderly behaviour, and the PPA traverse is significantly more complex and cannot be modelled using the RVM. Only recently Rankin et al. (2017) tried to model a few MSP’s in terms of the core cone model and suggested that the radio emission might arise from few tens of kilometres above the neutron star surface. However, these estimates are highly uncertain, since in none of the cases the RVM can be fitted to the PPA traverse. Thus for MSPs observationally, the location of the radio emission region is not constrained. In this study, we acknowledge this drawback and assume that the emission arises from regions anchored on open dipolar magnetic field lines of about 10% of light cylinder radius $r_L$.

### 3. Gamma-ray Observations

The LAT instrument onboard the Fermi Satellite, launched in June 2008, detects gamma-rays with energies between 20 MeV and ~ 300 GeV. The full three-years data used to compile the 2PC in Abdo et al. (2013) has been collected via the observations from 2008 August 4 to 2011 August 4. The gamma-ray pulsar search by using the known rotation ephemerides of radio or X-ray pulsars has led to the discovery of 61 gamma-ray pulsars in the 2PC. Phase alignment of gamma-ray pulses with radio pulses provides vital information about the geometry of the different emission regions. Another method used in the 2PC for discovering gamma-ray pulsars is the blind periodicity search which can provide ephemeris of pulsars relying only on the LAT gamma-ray data. In the catalogue, 36 new gamma-ray pulsars were discovered with this method. Besides, radio searches of several hundred LAT sources by the Fermi Pulsar Search Consortium provided 47 new pulsar discovery, including 43 MSPs and 4 young or middle-aged pulsars (Ray et al. 2012). One of the most prominent difference of the 2PC from the 1PC is the much higher ratio of MSPs to the normal pulsar population.
Spectra of gamma-ray pulsars are relatively insensitive to the neutron star period, peaking around cut-off energy of several GeV for all of them. Moreover, the subexponential decrease of the spectra above this cut-off ruled out the polar cap scenario for gamma-ray emission because of the strong magnetic opacity at these energies (Abdo et al. 2009). Slots gaps and outer gaps have since then been favoured site of high-energy photon production. While there is no independent observational constraint on the location of the gamma-ray emission, however, because the radio time lag \( \delta \) and peak separation \( \Delta \) values found in 2PC is supported by extensive force-free and dissipative magnetosphere simulations, these regions are shifted more and more towards the light-cylinder and even outside in the striped wind (Péri 2011). Thus, in this study, we use a magnetosphere model (see section 4) where the gamma-ray emission starts from the light cylinder into the striped wind.

4. Magnetosphere and emission model

As the neutron star is surrounded by a plasma made of electron-positron pairs producing the observed radiation, we need an accurate solution for the neutron star magnetosphere. Therefore the radio and gamma-ray light-curve computations rely on the electromagnetic field structure extracted from the force-free pulsar magnetosphere obtained by time-dependent numerical simulations. Such calculations have been performed by Péri (2020) using his time-dependent pseudo-spectral code detailed in Péri (2012b). For concreteness, the neutron star radius is set to \( R/n_L = 0.2 \) corresponding to a 1.2 ms period and the artificial outer boundary is put at \( r = 7n_L \) where the light-cylinder radius is \( n_L = c/\Omega \) and \( \Omega = 2\pi/P \). The numerical solution of the electromagnetic field is therefore expressed as a series in Fourier-Chebyshev polynomials and can be accurately evaluated at any arbitrary point within the simulation box. The striped wind starting from the light-cylinder is also self-consistently computed on almost one wavelength \( \lambda_e = 2\pi n_L \).

The two emission sites, radio and high-energy, are described as follows. Radio photons are expected to emanate from the open field line regions above the polar caps whereas the gamma-ray photons are produced in the current sheet of the striped wind, outside the light-cylinder at \( r \geq n_L \). The emissivity of the striped wind drops sharply with distance, so we only considered emission in the shell \( r \in [1,2]n_L \). We extended our previous analysis made by Péri (2011) where a split monopole wind was assumed to be directly connected to the stellar surface. Now this approximation which actually does not hold inside the light-cylinder is replaced by a realistic force-free dipole magnetosphere smoothly linking the quasi-static zone inside the light-cylinder to the wind zone outside the light-cylinder. In Fig. 1, we show the sky-maps produced by our striped wind model for six different magnetic inclinations angles \( \alpha \). In each panel, the phase of prominent radio peak is set to zero, and gamma-ray light curves are plotted accordingly by a time-alignment shift of the phase for synchronization with the radio pulse profile.

The gamma-ray pulse width is a combination between the relativistic beaming effect and the depth of the line of sight crossing the current sheet. For infinitely thin current sheet, this width \( W_p \) scales as the inverse of the wind Lorentz factor \( \Gamma \) and \( W_p \propto 1/\Gamma \) as shown by Kirk et al. (2002). If the thickness is finite, it will reach a lower limit reflecting the current sheet thickness scaled to the wind wavelength \( \lambda_{LC} \) for ultra-relativistic speeds \( \Gamma \gg 1 \). This has been observed in the Geminga pulsar as reported by Abdo et al. (2010b). The gamma-ray peak intensity for each pulse also depends on the length of the line of sight of crossing the current sheet in each pulse. Contrary to the split monopole case, the striped wind geometry is less symmetric because of the distorted dipole due to rotation and plasma effects. This loss of symmetry imprints the associated pulsed profile, which is also less symmetric, showing discrepancies in the peak intensities, compared to the split monopole emission.

The sky maps are constructed as follows. First, we determine the polar cap rims by identifying the last closed field lines grazing the light-cylinder. These field lines also draw the base of the current sheet in the striped wind. More precisely, radio emission is assumed to occur over the whole polar cap area at \( r = R \) (therefore at zero altitudes in our simulation box). As we are not interested in the exact radio pulse profiles, which can often have a complex shape with multiple components, but only in the location of the maximum intensity taken as phase zero for time-alignment purposes, we modulate the polar cap emission with a Gaussian profile \( w \) peaking at the magnetic axis such that

\[
w = e^{-\frac{1}{2}((\mu \cdot \mathbf{n})^2 - 1)}
\]

where \( \mu \) is the magnetic moment vector and \( \mathbf{n} \) the unit vector pointing into the line of sight direction. This profile is not intended to reproduce exact pulse profiles but simply to easily locate the maximum of the radio peak that sets the phase zero for the radio and gamma-ray light-curves. Emission in the striped wind is located around the current which is identified by the location outside the light-cylinder where the radial magnetic field component reverses sign. In an ideal picture, this current sheet is infinitely thin, but due to dissipation, it will spread to a certain thickness depending on the plasma condition.

The dipolar polar cap size has a half opening angle \( \theta_{pc} \approx \sin \theta_{pc} = \frac{\sqrt{R/n_{LC}}}{c} \). This corresponds to a radio pulse profile width of \( W \approx 3/2 \theta_{pc} \) and which takes into account the magnetic field line curvature, assuming a static dipole which is a good approximation at the surface as long as \( R \ll n_{LC} \). For our 1.2 ms pulsar computed from the simulations, it corresponds to a width of \( W \approx 38 \) deg, but for the sample of millisecond pulsars we have fitted below, with on average a 3 ms period, this width shrinks to \( W \approx 25 \) deg. Therefore, when the observer line of sight \( \zeta \) lies within the radio emission beam \( (\zeta - W \leq \zeta \leq \zeta + W) \), one will still detect a radio pulse. We added this constrain in the fitting procedure explained in the next section.

5. Fitting Method

The radio and gamma-ray light curves of the pulsars are produced by numerical calculations assuming a force-free magnetosphere. To produce a complete sky-map, we trace the inclination angle between the magnetic dipole moment and the rotation axis from \( \alpha = 0^\circ \) to 90\(^\circ\) with 5\(^\circ\) resolution, for the inclination of the line of sight with respect to the rotational axis \( (\zeta) \) varying from \( 0^\circ \) to 180\(^\circ\) with 2\(^\circ\) resolution. The maximum intensities for radio and gamma-rays are independently normalized to unity. We do not model the particle distribution within the magnetosphere with a self-consistent physical approach. In that sense, our model is purely geometric. The gamma-ray intensity is a free parameter of the model while the form of the light curves is obtained by solving the equations numerically for the force-free electrodynamics (see Section 4). We are mainly interested in the time lag \( \delta \) between the radio and the first gamma-ray peak, the gamma-ray peak separation \( \Delta \) and the ratio of the peak amplitude. These are geometric constraints, and hence we cannot model the exact pulse profile shapes.
In our analyses, we take into account the radio pulse profiles provided in the 2PC to reproduce gamma-ray pulse profiles calculated by our model with the consistent time lags between gamma-ray and radio peaks. The radio observations of PSR J0030+0451, PSR J0614–3329, PSR J1614–2230, PSR J2017+0603 and PSR J2302+4442 by the Nançay Radio Observatory (1.4 GHz band), of PSR J0102+4839 (1.5 GHz band) and PSR J1124–3653 (0.8 GHz band) by the Green Bank Telescope, of PSR J0437–4715 and PSR J1514 by the Parkes Radio Telescope (1.4 GHz band) and of PSR J2043+1711 by the Arecibo Observatory (1.4 GHz band), were used in the 2PC, see Abdo et al. 2013 and the ATNF Pulsar Catalogue\(^1\) (Manchester et al. 2005).

We set the time of maximum radio intensity to phase zero and shifted the gamma-ray profile accordingly. In our model, there is a lag \(\delta\) between the gamma and radio peaks up to 0.5 in phase. For \(\zeta\) and \(\alpha\) values producing double-peaked gamma-ray light-curves towards the direction to the observer, we measure the phase difference of these peaks (peak separation, \(\Delta\)). The expected peak separation measured from our model for a given set \(\zeta - \alpha\) can be seen in Fig 2. The maximum peak separation is 0.5 in units of the pulsar period \(P\). Note that the distribution of the peak separation in the force-free model is compatible with that found from the asymptotic split-monopole solutions (Pétri 2011).

For each source, in order to determine the goodness of fits to the observed gamma-ray light curve, we utilize a simple least square procedure as follows and try to pin down the reasonable \(\alpha\) and \(\zeta\) values producing the best-fitting model curves. The reduced-\(\chi^2\) is defined by

\[
\chi^2 = \frac{1}{\nu} \sum \frac{(I_{\text{obs}}^i - I_{\text{model}}^i)^2}{\sigma_i^2} \tag{2}
\]

where \(\nu\) is the degree of freedom which equals the number of data points minus the number of fitted parameters. \(I_{\text{obs}}^i\) and \(\sigma_i^2\) are the observed intensity and associated error of gamma-rays for the \(i\)th phase bin. \(I_{\text{model}}^i\) is the model prediction at the observational phase bin. Since the phase space in the model outputs is evenly distributed from 0 to 1 with 0.01 intervals, we interpolate the model light curve and find the intensity at the phase of observation to be able to apply the least square anal-

\(^1\) http://www.atnf.csiro.au/research/pulsar/psrcat

---

Fig. 1: Sky-maps for \(\alpha = 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ\) and \(90^\circ\). The x-y axes denote the rotation phase (in the unit of \(100 \times P\)) and \(\zeta\) (in the unit of \(2^\circ\)) for each panel while the intensity of gamma-rays (in arbitrary units) are indicated by the colour bar on the right of each panel.
ysis to the intensities. Regarding the observation data, we take the lower bounds for each bin in the light curve as the time of observation of the associated intensity. Since the difference of lower and upper bounds for each bin is marginal enough, e.g. 0.01 for gamma-ray and smaller for the radio light curves, using the minimum phase of each bin instead of the mean phase does not change the fit quality significantly. Also, we subtract the nominal value for the background (reported in the 100 MeV - 100 GeV energy band for each source) from the total intensity to remove background noise effect.

We mainly focus on the sources with clearly measured gamma-ray and radio light curves with high signal-to-noise ratios. Of the numerous sources observed both in gamma-rays and radio bands and reported in the 2nd Fermi Catalogue (Abdo et al. 2013), we pick-up pulsars with (1) double-peak observed in gamma-rays for the complete phase of the pulsar, (2) well-described prominent peak in the radio profile and clear peak locations, (3) gamma-ray peak detection time which is not very close to that of the prominent radio peak, to obtain their gamma-ray peak phases (localized accordingly to their radio peaks). We should note that the gamma-ray peak properties of pulsars, which do not suit the item (3) given above, could not be reproduced by our model. Some freedom in the emission height could add retardation effects shifting the gamma-ray light-curves in the right direction. However, this is at the expense of adding a new free parameter in our model that we want to avoid in a first attempt to constrain the geometry.

As mentioned earlier, to obtain the geometry for our sample pulsars, we fit the observed gamma-ray light curves with the model light curves by $\chi^2$ minimization process given by equation (2). Note that in our model there are three factors that influence the geometry i.e. (1) height ratio of the gamma-ray peaks (amplitude), (2) gamma-ray peak separation, $\Delta$, (3) phase lag between radio and associated gamma-ray peaks, $\delta$. However bridge emission between the gamma-ray peaks, as often seen in so many observed gamma-ray light curves is not predicted by the model. Thus, while finding best-fit and calculating $\chi^2$ value, although we take into account off-peak gamma-ray data together with bridge emission between gamma-ray peaks, we aim to retrieve the light curves best suited to the three above conditions, $\Delta$, $\delta$ and peak amplitude ratio. Therefore, we still regard $\chi^2$ values for the best fit of some pulsars as to be good enough even though they are not close to unity. We also examined a weighted fit for the peaks, such as calculating $\chi^2_{\nu}$ only around peaks or giving more weight to the data around peaks which do not lead to the estimation of different geometry. We, therefore, apply the un-weighted fitting method for all pulsars here with one exception (see Table 1).

6. Millisecond pulsars with time-aligned radio and gamma-ray pulse profiles

In the framework of force-free electrodynamics, we aimed to fit the observed gamma-ray light-curves (peak properties) and tried to constrain reasonably the values of the angles $\alpha$ and $\zeta$ for millisecond pulsars with high signal-to-noise ratios. The best fits of ten pulsars are retrieved by minimizing the square of the differences between the gamma-ray intensity observed and obtained from the model (2), integrated with all phase bins for each pulsar. We show and discuss our results below for each source, separately.

In our model, we assume that radio emission heights to be close to the stellar surface of the simulated magnetosphere and the gamma-ray pulsations to start at $n_{LC}$ up to $2n_{LC}$ for the sake of simplicity. Radio emission height and the region where exactly gamma-ray pulsations are produced possess some uncertainties giving us some freedom of shifting the gamma-ray light curves with respect to the radio peak, denoted by an additional phase shift $\phi$ accounting for a time of flight effect reflecting the variation in emission height, less than approximately 10% in the leading or trailing direction. For each pulsar, we set the offset $\phi$ to the value which gives the best fit to the pulsars gamma-ray light curve and plotted reduced chi-squared statistics for different angles of the line of sight and obliquity by adopting this $\phi$. Although the least $\chi^2_{\nu}$ can be found with any $\alpha - \zeta$, some angles can not be excepted as good solutions. The expectable solutions are restricted by the geometrical conditions, (1) $\alpha - W \leq \zeta \leq \alpha + W$ (we adopt $W \sim 20^\circ$ for all pulsars in this work) to be able to observe radio pulsation and (2) $\zeta \approx \alpha \geq \pi/4$ to see simultaneous radio and gamma-ray pulsations. We assume $\theta_{\nu_c} = 20^\circ$ for all MSPs investigated in this paper. Also, the retrieved gamma-ray light curves show symmetry in $\zeta$ around 90°.

6.1. PSR J0030+0451:

PSR J0030+0451 is a MSP with $P = 4.87$ ms which show double-pulses in gamma-rays and has relatively clear light curve profiles in gamma-ray energy bands. The primary radio peak of the source shows a multi-peak structure rather than a smooth single peak. The interpulse observed with much smaller amplitude compared to the main pulse implies a geometry close to an orthogonal rotator while how close $\alpha$ to $90^\circ$ depends on the details of the radio emission procedure. As can be seen in Fig. 3a, we could obtain the best fit to the gamma-ray light curve of PSR J0030+0451 by fixing $\alpha = 70^\circ$ and $\zeta = 60^\circ$ giving a reduced chi-squared $\chi^2_{\nu} = 7.2$. We do not need to shift the model curve along the phase to be able to find a plausible fit. We found that $\alpha = (55^\circ - 75^\circ)$ and $\zeta = (52^\circ - 74^\circ)$ gives fits with $7 \leq \chi^2_{\nu} \leq 12$ (see Fig. 3b).

6.2. PSR J0102+4839:

PSR J0102+4839 is another MSP with $P = 2.96$ ms which has a primary peak and a second peak with lower amplitude in
Fig. 3: The best fitting gamma-ray light curves on the left panel and the reduced chi-square distributions on the right panel.
Fig. 4: The best fitting gamma-ray light curves on the left panel and the reduced chi-square distributions on the right panel.
gamma-rays. Its radio profile has a broad single peak featured with broader trailing edge compared to the leading edge of the peak. We obtained the best fit to the gamma-ray profile with $\alpha = 55^\circ$, $\xi = 70^\circ$. We calculated $\chi^2$ for different $\alpha$ and $\xi$ values which indicates that reasonable fits can be obtained with $2.5 \leq \chi^2 \leq 4$ for $\alpha = (50^\circ - 70^\circ)$ and $\xi = (50^\circ - 76^\circ)$ (see Fig. 3b).

6.3. PSR J0437–4715:

We analysed PSR J0437–4715, the slowest MSP in our sample with $P = 5.76$ ms, which is an example of the sources showing radio pulsation and single peak in gamma-rays. Our results imply that the inclination angle is well-constrained, $\alpha = 45^\circ$. The best fit is obtained by $\alpha = 45^\circ$, $\xi = 40^\circ$, $\phi = 0.04$ with $\chi^2 = 1.2$, see Fig. 3e while $\alpha = 45^\circ$ and $\xi = (34^\circ - 42^\circ)$ yields good fits within the range $1.2 \leq \chi^2 \leq 2$ (Fig. 3f).
6.4. PSR J0614–3329:

PSR J0614–3329, with \( P = 3.15 \) ms, has clear double peaks in the gamma-rays light curve. The amplitude of the leading peak is slightly less than the trailing peak. The best fit to the gamma-rays with \( \chi^2 = 10.4 \) is obtained with the parameters \( \alpha = 75^\circ, \zeta = 56^\circ \) and \( \delta = -0.02 \), see Fig. 3g. The fits with \( 10 < \chi^2 < 15 \) imply \( \alpha = (70^\circ - 75^\circ) \) and \( \zeta = (60^\circ - 70^\circ) \) (see Fig. 3h). In spite of high \( \chi^2 \) which is mostly due to the off-pulse emission, there is a strong match between the peaks produced by the model and the observations which is what we are looking for as our main goal.

6.5. PSR J1124–3653:

The gamma-ray profile of PSR J1124–3653 (\( P = 2.41 \) ms) can be interpreted as (1) having a single peak feature with trailing edge gradually decreasing in intensity or (2) through the end of the trailing edge there is a less prominent second peak. We obtained the best fit to the gamma-ray profile with \( \alpha = 65^\circ, \zeta = 46^\circ, \phi = 0.07 \) and \( \chi^2 = 2.4 \) (see Fig. 4a) while reasonable fits can be obtained with \( 2.4 < \chi^2 < 4 \) for \( \alpha = (50^\circ - 65^\circ) \) and \( \zeta = (46^\circ - 66^\circ) \), see Fig. 4b. We note that the amplitude of the second peak is over-estimated in the best fit.

6.6. PSR J1514–4946:

PSR J1514–4946 (\( P = 3.59 \) ms) shows double peaks in gamma-rays with bridge emission in between these peaks. Our model does not address this inter-pulse emission. Trying to find best fits for the entire light curve at any phase did not worked for this pulsar because of bridge emission. We therefore focused only on the radio observations which is what we are looking for as our main goal.

6.7. PSR J1614–2230:

The best fit to the gamma-ray light curve of PSR J1614–2230 (\( P = 3.15 \) ms) is shown in Fig. 4f. It is obtained by choosing \( \alpha = 85^\circ \) and \( \zeta = 86^\circ \) and the phase offset \( \phi = 0.08 \) for the gamma-ray light curve giving a \( \chi^2 = 4.2 \). The reasonable fits with \( 7 < \chi^2 < 12 \) can be retrieved by the ranges \( \alpha = (70^\circ - 90^\circ) \) and \( \zeta = (66^\circ - 90^\circ) \) (Fig. 4f). However, solutions with \( \alpha \approx 90^\circ \) cannot represent the true geometry of the system because interpulse emission in radio was not observed for PSR J1614–2230.

6.8. PSR J2017+0603:

PSR J2017+0603 is a MSP with \( P = 2.90 \) ms which has a similar gamma-ray profile with that of PSR J1514–4946, but with a narrower peak separation, \( \Delta \approx 0.25 \). The best fit is obtained by setting \( \alpha = 55^\circ, \zeta = 48^\circ \) and \( \phi = -0.02 \) with \( \chi^2 \approx 2.7 \), see Fig. 4g. The geometry could be relatively well constrained for this pulsar with \( \alpha = (45^\circ - 55^\circ) \) and \( \zeta = (40^\circ - 52^\circ) \) by the presumption that the model curves with \( 2.7 < \chi^2 < 4 \) are good candidates to represent the observed light curve (Fig. 4h).

6.9. PSR J2043+1711:

Of the sources examined in this study, PSR J2043+1711 is the fastest spinning pulsar with \( P = 2.38 \) ms. The amplitudes and locations of both pulsations in the gamma-ray are well fitted with our model. We obtained the best fit by setting \( \alpha = 55^\circ, \zeta = 74^\circ \) and \( \phi = 0.06 \) with \( \chi^2 = 5.3 \) while the reasonable model curves could be found by the angles in the range \( \alpha = (50^\circ - 75^\circ) \) and \( \zeta = (46^\circ - 80^\circ) \) for \( 5.3 < \chi^2 < 8 \), see Fig. 5b. We want to point out that we do not show the observed radio light curve of the source because we have not found any radio data in the 2PC files corresponding to this pulsar. However, it is not directly related to our analysis here as we only deal with the time-alignment of gamma-ray and radio not the exact shape of the radio profile.

6.10. PSR J2302+4442:

PSR J2302+4442, with a period of \( P = 5.19 \) ms, shows double peaks in the gamma-rays. The best fit is produced with \( \alpha = 50^\circ, \zeta = 60^\circ \) and an offset \( \phi = 0.04 \) with \( \chi^2 = 11.1 \), see Fig. 5c. The ranges of the angles, provided that \( 11.1 < \chi^2 < 17 \), are found to be \( \alpha = (45^\circ - 60^\circ) \) and \( \zeta = (48^\circ - 62^\circ) \), see Fig. 5d. In the best fit curve, the second peak perfectly matches with the profile of the observed second peak. Although we over-estimate the amplitude of the first peak, the peak locations are in line with the observations.

7. Discussion

In this study, we focused on the relation between the radio time lag, the gamma-ray peak separation, the ratio of the peak amplitude to obtain the geometric parameters of pulsar magnetospheres, obliquity and inclination of line of sight. However, there are some uncertainties in the precise location of the radio and high-energy emission sites. This leads to possible variations in the time lag between radio and gamma-ray. For instance, retardation effects within the light-cylinder due to the finite propagation speed of light amount to an additional phase shift of

\[
\phi_{\text{ret}} = \frac{\Delta r}{2\pi r_L} \leq \frac{1}{2\pi} \approx 0.16.
\]

This shift has been implemented in our approach by adding a phase \( \phi \) that also corrects for the period of MSP being longer than the 1.2 ms used for the force-free simulations and the emission sky maps. An additional unknown comes from the observation side. Indeed, there is no hint for the maximum of the radio peak to represent the centre of the pulse profile. It could well be that one or several cone emission patterns produce the pulses with sub-pulses of different intensities. In our simulations, we exactly know the location of the magnetic axis and the associated polar cap centre. Therefore, we used as phase zero for our simulations the radio peak. Nevertheless, for the observational data, there is no hint of a one to one correspondence between peak intensity and middle of the pulse profile. This effect adds another unconstrained phase shift between radio and gamma-ray. Further, while estimating the radio time lag \( \delta \), we have assumed the radio emission to originate from regions of open dipolar magnetic field lines. However, estimates of \( \delta \) will be affected if the radio emission arises from regions where there is the influence of the strong non-dipolar magnetic field.

On a more fundamental physical side, the microphysics of pair creation, acceleration and their outflow into the striped wind is still ill-understood. The particle density number, its energy distribution function and their radiation spectra are not known.
which renders the PPA analysis useless to constrain $\alpha$ (2004) or the presence of non-dipolar surface magnetic fields, attitude dependent aberration retardation $\eta$ their geometry. The distortion of the PPA can result due to altitude. We utilized such techniques to deduce independent knowledge of integrals of pulsars. Such attempts have been avoided in this study because of too many uncertainties in the model. Our approach, although simple, deal with the least number of parameters but already satisfactorily fits existing light curves. There is no doubt that more extensive works, including a kinetic description of the magnetosphere and its associated radiation mechanisms, will unveil many important fundamental issues in pulsar physics.

8. Conclusion

We numerically solved the equations for force-free pulsar magnetospheres for many obliquities $\alpha$ and computed the corresponding gamma-ray pulsar emission emanating from the striped wind. Through individual analyses of ten millisecond pulsars with spin periods in the range $P \sim (2-6)$ ms, we showed that their gamma-ray pulse profiles could be faithfully reproduced by assuming that radio pulses escape from open field line regions close to the polar caps. In contrast, gamma-ray pulses are produced in the current sheet of the striped wind, outside the light cylinder. In this study, we constrained the range of parameters describing the geometry of each pulsar, namely their magnetic and light of sight inclination angles with respect to the spin axis of the star. In order to retrieve good fits to the observed gamma-ray light curves of each source, we have applied the least-square method to compare the intensities reported and found from the model in each bin of the observation throughout one complete rotation of the star. We presented our estimations together with their statistics in Table 1. We were able to fit all pulsars correctly.

The up-to-date measurement of radio pulse profiles and polarization position angle (PPA) traverse of millisecond pulsars indicates extremely irregular behaviour. Therefore, we could not utilize such techniques to deduce independent knowledge of their geometry. The distortion of the PPA can result due to altitude dependent aberration retardation effects Mitra & Seiradakis (2004) or the presence of non-dipolar surface magnetic fields, which renders the PPA analysis useless to constrain $\alpha$ and $\zeta$.

The most recent and best quality published gamma-ray data of pulsars have been reported in the 2nd Fermi Catalogue in 2013. Since then, almost one decade as passed and more observations have been accumulated, increasing the signal to noise ratio of gamma-ray light curves, with better intensity resolution. It is expected to be published as the 3rd Fermi Pulsar Catalogue in the future, which will undoubtedly increase the number of gamma-ray pulsars as well as provide even more stringent constraints on gamma-ray data to be tested with our model.

Acknowledgements

This work is supported by the CEFIPRA grant IFC/F5904-B/2018. We would like to acknowledge the High Performance Computing center of the University of Strasbourg for supporting this work by providing scientific support and access to computing resources. Part of the computing resources was funded by the Equipex Equip@Mes project (Programme Investissements d’Avenir) and the CPER Alsacalcul/Big Data. DM acknowledges the support of the Department of Atomic Energy, Government of India, under project no. 12-R&D-TFR-5.02-0700.

References

Abdo, A. A., Ackermann, M., Ajello, M., et al. 2010a, ApJS, 187, 460
Abdo, A. A., Ackermann, M., Ajello, M., et al. 2010b, Apj, 720, 272
Abdo, A. A., Ackermann, M., Atwood, W. B., et al. 2009, ApJ, 696, 1084
Abdo, A. A., Ajello, M., Allafort, A., et al. 2013, ApJS, 208, 17
Bai, X.-N. & Spitkovsky, A. 2010, ApJ, 715, 1262
Blaskiewicz, M., Cordes, J. M., & Wasserman, I. 1991, ApJ, 370, 643
Bogovalov, S. V. 1999, Astronomy and Astrophysics, 349, 1017
Cerutti, B., Philippov, A. A., & Spitkovsky, A. 2016, Monthly Notices of the Royal Astronomical Society, 457, 2401
Chen, A. Y., Yuan, Y., & Vasileioupolous, G. 2020, ApJL, 893, L38, publisher: American Astronomical Society

Contopoulos, I., Kazanas, D., & Fendt, C. 1999, ApJ, 511, 351
Coroniti, F. V. 1990, ApJ, 349, 538
Deutsch, A. J. 1955, Annales d’Astrophysique, 18, 1
Kalapotharakos, C. & Contopoulos, I. 2009, A&A, 496, 495
Kalapotharakos, C., Harding, A. K., & Kazanas, D. 2014, ApJ, 793, 97
Kalapotharakos, C., Harding, A. K., Kazanas, D., & Contopoulos, I. 2012, The Astrophysical Journal Letters, 754, L1
Kalapotharakos, C., Wadasingh, Z., Harding, A. K., & Kazanas, D. 2020, arXiv:2009.08567
Kirk, J. G., Skjæraasen, O., & Gallant, Y. A. 2002, Astronomy & Astrophysics, 388, L29
Krause-Polstorff, J. & Michel, F. C. 1985, Monthly Notices of the Royal Astronomical Society, 213, 43P
Manchester, R. N., Hobbs, G. B., Teoh, A., & Hobbs, M. 2005, The Astronomical Journal, 129, 1993
McDonald, J. & Shearer, A. 2009, The Astrophysical Journal, 690, 13
Michel, F. C. 1994, The Astrophysical Journal, 431, 397
Mitra, D. 2017, J Astrophys Astron, 38, 52
Mitra, D. & Deshpande, A. A. 1999, A&A, 7
Mitra, D. & Li, X. H. 2004, A&A, 421, 215
Mitra, D. & Seiradakis, J. H. 2004, in arXiv:astro-ph/0401335, arXiv: astroph/0401335
Pierbattista, M., Harding, A. K., Grenier, I. A., et al. 2015, A&A, 575, A3
Pétrit, J. 2011, MNras, 412, 1870
Pétrit, J. 2012a, MNras, 424, 2023
Pétrit, J. 2012b, MNras, 424, 605
Pétrit, J. 2018, MNras, 477, 1035
Pétrit, J. 2020, Universe, 6, 74
Pétrit, J., Heyvaerts, J., & Bonazzola, S. 2002, A&A, 384, 414
Pétrit, J. & Mitra, D. 2020, MNras, 491, 80
Rankin, J. M. 1983, The Astrophysical Journal, 274, 359
Rankin, J. M. 1990, The Astrophysical Journal, 352, 247
Rankin, J. M. 1993, The Astrophysical Journal, 405, 285
Rankin, J. M., Archibald, A., Hessels, J., et al. 2017, The Astrophysical Journal, 845, 23
Ray, P. S., Abdo, A. A., Parent, D., et al. 2012, arXiv e-prints, arXiv:1205.3089
Skrzypczak, A., Basu, R., Mitra, D., et al. 2018, The Astrophysical Journal, 854, 162
Spitkovsky, A. 2006, ApJ, 648, L51
Venter, C., Harding, A. K., & Guillemot, L. 2009, The Astrophysical Journal, 707, 800
Watters, K. P., Romani, R. W., Weltevrede, P., & Johnston, S. 2009, ApJ, 695, 1289