1. Introduction

The general approach to durability design is given by the standard [1] but the form of the procedure is not appropriate for design practice. The shortage of information related to the transformation processes of the environmental changes on the environmental actions, which is e.g. corrosion of steel, is the basic reason of this. On the other hand, the corrosion process has random character depending on many random variable parameters, therefore, its mathematical interpretation is somewhat complicated.

Generally, two approaches could be applied to verify durability of structures. The first approach is based on the concept of the design structural service life \( t_s \) [2]. The second one is based on the limit state concept design considering effects of environmental actions in the following form

\[
P_l(t) = P\{R(t) - E(t) \leq 0\} \leq P_{th}.
\]

where \( E(t) \) represents the random variable time-dependent action effects and \( R(t) \) is the random variable time-dependent structural member resistance. \( E(t) \) and \( R(t) \) are functions of random variables \( X_i \), which are also time-dependent functions in relation to the models of material degradations.

2. Corrosion of structural steel and reinforcement

The mathematical description of the theoretical background of corrosion processes by means of differential equations is very complicated. Therefore, the development of semi-empirical models was preferred to apply for practical utilisation respecting the appropriate sophisticated level of approximation. This concept introduces need for systematic research and obtaining data related to the corrosion effects. Therefore, the most corrosion models are based on certain assumptions or results of experimental measurements. If the degradation influences of the corrosion process are known, the possibility of modelling corrosion effects using extrapolation of experimentally gained data exists and a mathematical model can be calibrated by means of them.

Even though several long-term experimental investigations were performed in the past and many factors influencing corrosion process are known, the models of time-dependent prediction of corrosion loss are more or less simplified. Regarding random character of many parameters entering the corrosion process, the mathematical statistics and probability theory are the most appropriate approaches to describe the corrosion effects. At present, several corrosion models are known, which should be used to analyse corrosion effects from the viewpoint of structural reliability. The application in bridge engineering is not sufficiently verified, because the developed corrosion models based on laboratory tests are dependent on the modelling of actual environment by means of laboratory equipment. The review
of the most applied structural steel corrosion models describing the time-dependent material loss is presented in Table 1.

Examples of application of those corrosion models were introduced e.g. in [2, 7, 8 and 9].

In the case of the reinforcement corrosion, two models are usually used to describe the change of the reinforcement diameter in time due to corrosion taking into account uniform type of corrosion [10 and 11]. The process of degradation due to corrosion consists of two stages - passive stage and active stage. The passive stage means time $t_p$ from the beginning of the bridge operation, when degradation agents penetrate through the concrete cover up to the level of reinforcement, but reinforcement does not corrode. After that time, the concrete loses its passivation protection and the reinforcement corrosion starts - so called active stage ($t_t$). The process of time-dependent change of reinforcement diameter can be described using following equations

$$
\phi(t) = \phi, \text{for time } t \leq t_p, \tag{2a}
$$

$$
\phi(t) = \phi - 0.0232 \cdot (t - t_p) \cdot i_{corr}, \text{for time } t > t_p, \tag{2b}
$$

$$
\phi(t) = \phi - (t - t_t) \cdot r_{corr}, \text{for time } t > t_t, \tag{2c}
$$

where $r_{corr}$ = corrosion rate [μm/year]; $i_{corr}$ = corrosion current density [μA/cm²] (1 μA/cm² is equal to 11.6 μm/year of corrosion); $t$ = time [years]; $t_p$ = length of time of passive stage for longitudinal main reinforcement in years.

Formula (2b) was assumed according to [10] and formula (2c) was taken over from [11]. The length of the passive stage $t_p$ could be calculated according to the simplified model described in many references, e.g. [12 and 13]

$$
t_p = \frac{e^2}{2 \cdot D} \tag{3}
$$

where $D$ = material constant [mm².s⁻¹].

### 3. Moment resistance of steel cross-section considering corrosion

To determine the effect of corrosion on the moment cross-sectional resistance, the parametric study of six welded beams of the I-shape was performed. As random variables, the cross-sectional dimensions $b_f, t_f, h_w, t_w$ were considered together with random variable steel yield strength $f_y$.

The basic statistical parameters of those random variables (mean $\mu$ and standard deviation $\sigma$) are presented in Table 2 and designations are defined in Fig. 1.

Standard deviations of those cross-sections were determined using standard tolerances $\delta$ according to standard [14] valid for tolerances of the web height and flange width and in accordance with standard [15] for tolerances of web and flange thickness under assumption that 95% of values of all realisations of the random variables normally distributed occurs in the interval $\sigma_{tt}$

$$
\langle \mu, -\alpha; \mu, +\alpha \rangle \tag{4}
$$

Then the standard deviation $\sigma$ is approximately possible to determine as $2\sigma$. According to above mentioned procedure, the standard deviations of the heights and widths of the investigated cross-sections were estimated.

$h \leq 900 \text{ mm} ... \Delta h = \pm 3 \text{ mm} ... \sigma_{hh} = 1.5 \text{ mm}$

$h \leq 300 \text{ mm} ... \Delta b = \pm 3 \text{ mm} ... \sigma_{wb} = 1.5 \text{ mm}$

| Author                      | Mean value [mm] | Standard deviation [mm] | Distribution |
|-----------------------------|-----------------|-------------------------|--------------|
| Southwell-Melchers [3]      | 0.0844[²/11]    | 0.0564[²/11]            | Normal       |
| Frangopol [4]               | 0.03207[²/5]    | 0.00289[²/4]            | Normal       |
| Qin-Cui [5]                 | 1.67[1 - exp(-t/9.15)] | 0.0674[1 - exp(-t/0.181)] | Normal       |
| Guedes-Soares [6]           | $d_{corr}(t) = 1.5(1 - e^{\frac{-t}{200}})$ and $t$ is time in years | - |
Standard deviations of web and flange thickness are presented in Table 3.

The numerical calculation of the time-dependent moment cross-sectional resistance was processed using software Matlab, where the effect of the random variable cross-sectional dimensions and steel yield strength were taken into account. The normally distributed random variables were generated by means of Latin Hypercube Sampling (LHS) for 10 000 samples.

The courses of the moment resistances were calculated using all above mentioned corrosion models (see Table 1) for lifetime of 100 years in dependence on the time $t$. Corrosion loss $d_{corr}$ according to individual models from Table 1 was implemented into the calculation by means of flange thickness reduction using formula $t_{red} = t_f - d_{corr}$, where $t_f$ is the flange thickness without corrosion loss. To determine the moment cross-sectional resistance considering the corrosion degradation due to flange corrosion, two approaches were applied.

For plastic time-dependent moment resistance $M_{pl}(t)$, the following equation was used

$$M_{pl} = \left[ t_f \cdot h_f \cdot h_t \cdot \left( \frac{1}{4} + b_1 \left( h_v + t_f - d_{corr} \right) \left( t_f - d_{corr} \right) \right) \right] f_t. \quad (5)$$

while the elastic time-dependent moment resistance was expressed by the equation $M_{el}(t)$

$$M_{el} = \left[ \frac{\left( t_f \cdot h_f \cdot h_t \cdot \left( \frac{1}{12} + b_1 \left( t_f - d_{corr} \right) \left( t_f - d_{corr} \right) \left( h_v + t_f - d_{corr} \right) \right) / 6 + b_1 \left( t_f - d_{corr} \right) \left( h_v + t_f - d_{corr} \right) / 2 \right)}{h_f / 2 + t_f - d_{corr}} \right] f_t. \quad (6)$$

Time-dependent course of the means of the plastic and elastic moment resistances of the cross-section 1 respecting the corrosion models according Table 1 are presented in Figs. 2 and 3.
Concurrently, it was found that the value of the coefficient \( b \) was constant for every corrosion model and the values of coefficients \( a, c \) were changing while those coefficients were nearly independent on the cross-sectional classes defining the application of the plastic or elastic moment resistance.

Therefore, the given model of the prediction of corrosion loss should be applied from the viewpoint of environment type using the “degradation coefficient” \( b \) and “shape coefficients” \( a, c \) by means of which the time-dependent course of the degradation function of the cross-sectional resistance could be calculated for the designed cross-section and its lifetime.

Considering that degradation coefficient \( b \) represents time-dependent corrosion process depending on relevant corrosion model and when for coefficients \( a \) and \( c \) the relation \( a + c = 1 \) is valid, then the shape coefficient \( a \) is only necessary to determine according to following equation valid for Frangopol or Melchers corrosion models

\[
a = \frac{b_f (h_c + t_f - b_o t^a) (t_f - b_o t^a) - b_f (h_c + t_f) t_f}{W_{p,0} (e^t - 1)}
\]

where coefficients \( b_o, b_f \) are considered in accordance with the appropriate Frangopol or Melchers corrosion models (0.03207, 0.5 or 0.084, 0.823). The resulting value of the coefficient \( a \) is the minimum value in time interval within the element lifetime. The approach presented above is relatively complicated for practical utilisation. Therefore, it would be appropriate to process the value of the coefficient \( a \) in tabular form.

The Qin-Cui and Guedes-Soares corrosion models enable a more simple calculation of the coefficient \( a \). Its value could be directly obtained when the corrosion loss determined according to the appropriate model in time \( t = \infty \) is taken away from the relation (9). Then, the relation (9) should be adjusted in the following form

\[
a = \frac{b_f (h_c + t_f - d_{corr, \infty}) (t_f - d_{corr, \infty}) - b_f (h_c + t_f) t_f}{W_{p,0} (e^t - 1)}
\]

The value of \( d_{corr, \infty} = 1.67 \text{mm} \) for Qin-Cui model and \( d_{corr, \infty} = 1.50 \text{mm} \) for Guedes-Soares corrosion model should be substituted into the equation (10). To prove the correctness of the assumptions related to the coefficients \( a, b \) and derivation of the general solution, the new parametric study was worked up for 2500 beams having welded I-shaped cross-section, whose cross-sectional characteristics and yield strength were considered as random variables.

It follows from the new parametric study that the degradation coefficient \( b \) has the constant value for given corrosion model and values of coefficients \( a, c \) were only changing. Degradation coefficient \( b \) is dependent on the relevant corrosion model and also on the environmental condition. It is also clear that calculation of the shape coefficient \( a \) could be simplified using the following relation

\[
a = \frac{b_f (2t_f + h_c)}{W_{p,0}} \alpha_{corr, model}, \tag{11}
\]

where \( \alpha_{corr, model} \) introduces the corrosion coefficient given in Table 4 according to the appropriate corrosion model.

Final values of the degradation and corrosion coefficients \( b \) and \( \alpha_{corr, model} \) according to Table 4

| Corrosion model      | Coefficient | \( \alpha_{corr, model} \) |
|----------------------|-------------|---------------------------|
| Frangopol            | 0.0194      | 3.278E-04                 |
| Melchers             | 0.0055      | 8.460E-03                 |
| Qin-Cui              | 0.2158      | 1.655E-03                 |
| Guedes-Soares        | 0.1003      | 1.490E-03                 |

Fig. 4 Comparison of numerically obtained time-dependent course of plastic moment resistance to the resistance calculated according to proposed analytical relation (12)

Finally, the time-dependent moment resistance of I-shaped cross-section with degradation of the both flanges could be determined according to following equation

\[
M_{tdl} (t) = M_{tdl, t=0} \left[ 1 + a (e^{-at} - 1) \right], \tag{12}
\]

where value of the coefficient \( a \) should be calculated using equation (11) and degradation coefficients \( b, \alpha_{corr, model} \) should be considered in accordance with Table 4. The comparison of the numerically determined courses of the plastic moment resistance of the cross-section 1 to calculations according to relation (12) for individual corrosion models is introduced in Fig. 4. The very good compliance of the analytical model with numerical analyses can be identified.
4. Moment resistance of reinforced concrete cross-section considering reinforcement corrosion

Concrete structures shall be adequately safe against failure and must also exhibit satisfactory performance in service. The internal forces as bending moments \( M \) and shear forces \( V \) are considered as the relevant structural response to actions. Vertical bending seems to be the main action effect determining the cross-sectional dimensions and arrangement of the longitudinal reinforcement, which are firstly contemplated to provide the necessary moment resistance.

Bending limit state comes into being when the limit strain is achieved at least in one of the materials (concrete in compression or reinforcement in tension) [16 and 17]. In the case of girder bridges, the bridge deck is always connected with beams, therefore, the cross-section is considered as flanged beam (cross-section of the T shape). The time-dependent moment resistance \( M_d(t) \) of the reinforced concrete flanged beam is given in accordance with standard [18] by formula

\[
M_d(t) = F(t) \cdot z(t) = A_{sl}(t) \cdot f_{sd} \cdot z(t) = A_i(t) \cdot f_{sd} \left( \left[ h - c - \frac{\phi(t)}{2} \right] - A_i(t) \cdot f_{sd} \right)
\]

(13)

where \( F(t) \) is the time-dependent force in tensioned reinforcement changed due to corrosion, \( z(t) \) is the time-dependent lever of internal forces, \( f_{sd} \) is the design compressive strength of concrete, \( h \) is the cross-sectional height, \( c \) is the height of slab (beam flange), \( b \) is the beam cross-sectional width, \( b_{ef} \) is the effective cross-sectional width, \( n \) is the number of longitudinal reinforcements in cross-section, \( A_i(t) \) is the time-dependent reinforcement cross-sectional area changed due to reinforcement corrosion and \( \phi(t) \) is the longitudinal reinforcement diameter (see Fig. 5).

As it was presented in [19 and 20], equation (13) could be modified into its final form for moment resistance depending only on time

\[
M_d(t) = M_{d0}(0) + k_1 \cdot (t - t_0) + k_2 \cdot (t - t_0)^2 + k_3 \cdot (t - t_0)^3 + k_4 \cdot (t - t_0)^4
\]

(14)

where \( k_1, k_2, k_3, k_4 \) are the parameters depending on material and geometrical characteristics with their physical significance

\[
k_1 = A \cdot 0.0232^3 \cdot i_{corr}^2, \quad k_2 = A \cdot r_{corr}^4
\]

(15a,b)

or \( k_3 = (4 \cdot A \cdot \phi + B) \cdot 0.0232^3 \cdot i_{corr}^2, \quad k_4 = (4 \cdot A \cdot \phi + B) \cdot r_{corr}^4.\) (16a,b)

\[
k_2 = (6 \cdot A \cdot \phi^3 + 3 \cdot B \cdot \phi + C) \cdot 0.0232^2 \cdot i_{corr}, \quad k_3 = (6 \cdot A \cdot \phi^3 + 3 \cdot B \cdot \phi^2 + 2 \cdot C \cdot \phi) \cdot r_{corr}^3
\]

(17a,b)

\[
k_1 = (4 \cdot A \cdot \phi^3 + 3 \cdot B \cdot \phi^2 + 2 \cdot C \cdot \phi) \cdot 0.0232 \cdot i_{corr}, \quad k_4 = (4 \cdot A \cdot \phi^3 + 3 \cdot B \cdot \phi^2 + 2 \cdot C \cdot \phi) \cdot r_{corr}
\]

(18a,b)

where \( A, B, C \) are the time-independent parameters

\[
A = -\frac{\pi^2 \cdot n^2 \cdot f_{sd}^2}{32 \cdot b_{ef} \cdot f_{sd}}
\]

(19)

\[
B = -\frac{\pi \cdot n \cdot f_{sd}}{8}
\]

(20)

\[
C = \frac{\pi \cdot n \cdot f_{sd} \cdot (h - c)}{4}
\]

(21)

The parametric study was performed in accordance with the approach described above.

5. Experimental application

The influence of longitudinal reinforcement corrosion on moment resistance of the bridge structural element was investigated. The reinforced concrete flanged beam subjected to bending with material and geometrical characteristics given in Table 5 obtained from measurement on the real bridge structure was used.

Material characteristics used in parametric study

| Material characteristics | Notation | Unit | Value |
|--------------------------|----------|------|-------|
| Strength of concrete in compression | \( f_{sd} \) | N.mm\(^2\) | 20.0 |
| Yield strength of reinforcement | \( f_{yd} \) | N.mm\(^2\) | 179.23 |
| Height of cross-section | \( h \) | m | 0.837 |
| Width of cross-section | \( b \) | m | 0.322 |
| Effective width of cross-section | \( b_{ef} \) | m | 1.545 |
| Concrete cover thickness - main | \( c \) | mm | 29.6 |
| Concrete cover thickness - stirrups | \( c_s \) | mm | 15.6 |
| Number of bars (reinforcement) | \( n \) | pcs | 7 |
| Bar diameter - main longitudinal | \( \varphi \) | m | 0.030 |
| Bar diameter - stirrups | \( \varphi_s \) | m | 0.014 |
| Corrosion current density | \( i_{corr} \) | \( \mu A/cm^2 \) | 5.0 |
| Material constant | \( D \) | mm\(^3\) \(s\) \(^{-1}\) | 4.82 \(10^7\) |

![Fig. 5 Flanged beam - denotation](image)
Using formula (22a) and if \( t_0 = 0 \), the reinforcement diameter changes due to corrosion from the basic diameter \( \phi = 30 \text{ mm} \) to the average value \( \phi = 29.37 \text{ mm} \) would be 54.31 years (for \( i_{corr} = 0.50 \mu \text{A/cm}^2 \)), 27.16 years (for \( i_{corr} = 1.0 \mu \text{A/cm}^2 \)), 9.05 years (for \( i_{corr} = 3.0 \mu \text{A/cm}^2 \)), or 5.43 years (for \( i_{corr} = 5.0 \mu \text{A/cm}^2 \)).

The corresponding length of the passive stage \( t_0 \) can be obtained by modification of formulas (22a, 22b), if \( t_0 \) is equal to age of the bridge \( t = 65 \text{ years} \). The shortest length of the passive stage was obtained for \( i_{corr} = 0.50 \mu \text{A/cm}^2 \) and was equal to \( t_0 = 10.69 \text{ years} \). The lengths of time of passive stages are equal to \( t_0 = 37.84 \text{ years} \) for \( i_{corr} = 1.00 \mu \text{A/cm}^2 \), \( t_0 = 55.95 \text{ years} \) for \( i_{corr} = 3.00 \mu \text{A/cm}^2 \) and \( t_0 = 59.57 \text{ years} \) for \( i_{corr} = 5.00 \mu \text{A/cm}^2 \).

Now, it is possible to predict the moment resistance change of reinforced concrete flanged beam up to the end of design lifetime \( T_d = 100 \text{ years} \). The results are shown in Fig. 7.

The paper presents the results of the research concerning the influence of the corrosion of the structural steel or reinforcement on the time-dependent moment cross-sectional resistances of I-shaped cross-section and also on the cross-section of the flanged bridge beam. Two approaches are introduced related to the expressing the effect of corrosion on the moment cross-sectional resistance. The first approach is based on the assumption taking into account the effect of corrosion of structural steel using approximated degradation function \( F(t) \) and multiple it by the moment cross-sectional resistance in time \( t = 0 \text{ year} \) to obtain the actual moment resistance in time \( t \) respecting the known corrosion models. The second approach was derived to allow for effect of reinforcement corrosion on the moment cross-sectional resistance of reinforced concrete flange beam respecting the known corrosion models valid for reinforcement corrosion. It follows from result analysis that the reinforcement corrosion has significant influence on change of resistances in time.
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