Landau gauge ghost propagator and running coupling in SU(2) lattice gauge theory

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(Dated: February 22, 2013)

We study finite (physical) volume and scaling violation effects of Landau gauge ghost propagator as well as of the running coupling $\alpha_s(p)$ in the SU(2) lattice gauge theory. We consider lattices with physical sizes between 3 and 7 fm and values of spacing $a$ between 0.2 and 0.07 fm. To fix the gauge we apply an efficient gauge fixing method aimed at finding extrema as close as possible to the global maximum of the gauge functional. We show that finite volume effects turn out to be rather small for momenta $|p| \sim 0.6$ GeV. The relative scaling violations for $\alpha_s(p)$ do not exceed 10% within the bare coupling range $\beta = 4/g^2_0 \in [2.30, 2.55]$ for the same momentum region and decreases quickly with increasing $|p|$. Our results testify in favor of the decoupling behavior.

PACS numbers: 11.15.Ha, 12.38.Gc, 12.38.Aw

Keywords: Lattice gauge theory, ghost propagator, scaling behavior, finite-size effects, gauge fixing, simulated annealing

I. INTRODUCTION

The infrared (IR) behavior of Landau gauge gluon and ghost propagators is believed to be closely related to gluon and quark confinement scenarios. For example, the celebrated Gribov–Zwanziger/Kugo–Ojima (GZKO) color confinement scenario [1–5] prescribes that the gluon propagator $D(p)$ should vanish in the IR limit $p \to 0$ (so-called infrared suppression), while the ghost dressing function is expected to become singular (infrared enhancement). In a very recent paper D. Zwanziger has derived a strict bound $\lim_{p \to 0} p^{d-2} D(p) = 0$ (where $d$ denotes the space-time dimension for the Landau gauge case) allowing also $D(0) \neq 0$ for $d > 2$ [6].

The search for gluon and ghost propagator solutions of Dyson-Schwinger (DS) and functional renormalization group (FRG) equations showed the existence of infrared solutions exhibiting a power-like scaling behavior [7–15] as well as regular so-called decoupling solutions providing an IR-finite limit of both the gluon propagator and the ghost dressing function [16–20]. Both kinds of solutions can be realized by different IR boundary conditions for the ghost dressing function as it has been argued in [21], and both of them can support quark confinement [22]. The decoupling solution does not agree with the GZKO scenario. However, it is in agreement with the refined Gribov-Zwanziger formalism [23, 24].

From the phenomenological point of view the propagators can serve as input to bound state equations as there are Bethe-Salpeter or Faddeev equations for hadron phenomenology [8, 25, 26]. In the ultraviolet limit they allow a determination of phenomenologically relevant parameters such as $\Lambda_{\overline{MS}}$ or condensates $\langle \overline{\psi}\psi \rangle, \langle A^2 \rangle, \ldots$ e.g. by fitting ab-initio lattice data to continuum expressions (see e.g. [27, 28] and citations therein) obtained from operator product expansion and perturbation theory [29, 30].

What concerns the solution of DS and/or FRG equations it is well-known that the system of those equations has to be truncated. The details of truncation influence the behavior of the Green functions especially in the non-perturbative momentum range around 1 GeV, where the Landau gauge gluon dressing function exhibits a pronounced maximum. Therefore, reliable results from ab-initio lattice computations to compare with or even used as an input for DS or FRG equations are highly welcome.

On the lattice, over almost twenty years extensive
studies of the Landau gauge ghost propagators have been carried out (see, e.g., [42–46]. A serious problem in these calculations represent the Gribov copy influence [39, 45]. Numerical lattice results clearly support the decoupling-type of solutions in the IR limit and the lack of IR enhancement of the ghost propagator [35, 42, 43]. However, most of the lattice computations dealing with the IR limit were relying on rather coarse lattices in order to reach large enough volumes. A systematic investigation of lattice discretization artifacts or scaling violations was missing for long time.

In this paper we present an investigation for the ghost dressing function and – employing previous gluon propagator results [47] – obtain the running coupling within the so-called minimal MOM scheme [48]. We shall use the same lattice field configurations as in [47] which were gauge fixed with an improved method taking into account many copies over all Z(2) Polyakov loop sectors and applying simulated annealing with subsequent overrelaxation. We separately discuss the case of fixed lattice spacing and varying volume (from 3 fm to 7 fm) and the case of fixed physical volume and varying lattice spacing (between 0.21 fm and 0.07 fm). In the range of achieved IR momenta finite-size effects are shown to be negligibly small. But relative finite-discretization effects in the infrared (for a renormalization scale chosen at $\mu = 2.2$ GeV) turn out to be more sizable and can be quantified to reach a 10 percent variation level at $p \simeq 0.4$ GeV in the approximate scaling region explored between $\beta = 2.3$ and $\beta = 2.55$. A similar conclusion will be drawn for the running coupling.

In Section II we introduce the lattice Landau gauge and the corresponding Faddeev-Popov operator and the ghost propagator. In Section II some details of the simulation and of the improved gauge fixing are repeatedly given. In Section V we present our numerical results for the ghost propagator and the running coupling. Conclusions will be drawn in Section V.

II. LATTICE LANDAU GAUGE AND THE GHOST PROPAGATOR

Let us briefly recall how the $SU(2)$ gauge field configurations used in Ref. [47] for measuring the gluon propagator have been created and gauge fixed.

The non-gauge-fixed $SU(2)$ gauge field configurations were generated with a standard Monte Carlo routine using the standard plaquette Wilson action

$$ S = \frac{1}{\beta} \sum_x \sum_{\mu>\nu} \left[ 1 - \frac{1}{2} \text{Tr} \left( U_{x\mu} U_{x+\mu,\nu} U_{x+\nu,\mu}^\dagger U_{x\nu}^\dagger \right) \right], $$

$$ \beta = 4/g_0^2, $$

where $g_0$ denotes the bare coupling constant. The link variables $U_{x\mu} \in SU(2)$ transform under local gauge transformations $g_x$ as follows

$$ U_{x\mu} \rightarrow g_x U_{x\mu} g_x^\dagger = g_{x+\mu} U_{x\mu} g_{x+\mu}, \quad g_x \in SU(2). $$

The standard (linear) definition [49] for the dimensionless lattice vector potential $A_{x+\mu/2,\mu}$ is

$$ A_{x+\mu/2,\mu} = \frac{1}{2\Lambda} \left( U_{x\mu} - U_{x\mu}^\dagger \right) \equiv A_{x+\mu/2,\mu}^a \sigma_a^g/2. $$

This definition, which is not unique, can influence the propagator results in the IR region, where the continuum limit is harder to control.

In lattice gauge theory the most natural choice of the Landau gauge condition is by transversality [49]

$$ (\partial \mathcal{A})_x = \sum_{\mu=1}^{4} (\mathcal{A}_{x+\mu/2,\mu} - \mathcal{A}_{x-\mu/2,\mu}) = 0, $$

which is equivalent to finding a local extremum of the gauge functional

$$ F_U(g) = \frac{1}{4V} \sum_{x\mu} \frac{1}{2} \text{Tr} \left( U_{x\mu}^g \right), $$

with respect to gauge transformations $g_x$. $V = L^4$ denotes the 4d lattice size. The Gribov ambiguity is reflected by the existence of multiple local maxima. The manifold consisting of Gribov copies providing local maxima of the functional [4] and a semi-positive Faddeev-Popov operator (see below) is called the Gribov region $\Omega$, while the global maxima form what is called the fundamental modular region (FMR) $\Lambda \subset \Omega$. Our gauge fixing procedure is aimed to approach $\Lambda$ by finding higher and higher maxima. This is achieved by use of the effective optimization algorithm and finding a large number of local maxima of which the highest is picked up. For recent alternative approaches see [50, 51].

The lattice expression of the Faddeev-Popov operator $M^{ab}$ corresponding to $D^{ab}_{\mu}$ in the continuum theory (where $D^{ab}_{\mu}$ is the covariant derivative in the adjoint representation) is given by

$$ M^{ab}_{xy} = \sum_\mu \left\{ \left( \bar{S}_{x\mu}^{ab} + \bar{S}_{x-\mu}^{ab} \right) \delta_{xy} - \left( \bar{S}_{x+\mu}^{ab} - \bar{A}_{x\mu}^{ab} \right) \delta_{y,x+\mu}^a - \left( \bar{S}_{x-\mu}^{ab} + \bar{A}_{x-\mu}^{ab} \right) \delta_{y,x-\mu}^a \right\} $$

where

$$ \bar{S}_{x\mu}^{ab} = \delta^{ab} \frac{1}{2} \text{Tr} \left( U_{x\mu}^a \right), \quad \bar{A}_{x\mu}^{ab} = -\frac{1}{2} \epsilon^{abc} A_{x+\mu/2,\mu}^c. $$

From the form (7) it follows that a trivial zero eigenvalue is always present, such that at the Gribov horizon $\partial \Omega$ the first non-trivial zero eigenvalue appears.
For configurations with a constant field, with \( b_0 = \bar{b}_0 \) and \( b_{x\mu} = \bar{b}_{x\mu} \) independent of \( x \), there exist eigenmodes of \( M \) with a vanishing eigenvalue. Thus, if the Landau gauge is properly implemented, \( M[U] \) is a symmetric and semi-positive definite matrix.

The ghost propagator \( G^{ab}(x,y) \) is defined as [31, 52]

\[
G^{ab}(x,y) = \delta^{ab} G(x-y) \equiv \langle \left( M^{-1} \right)^{b}_{a} x y | U \rangle , \tag{8}
\]

where \( M[U] \) is the Faddeev-Popov operator, on the sector orthogonal to the strict zero modes. Note that the ghost propagator becomes translationally invariant (i.e., dependent only on \( x - y \)) and diagonal in color space only in the result of averaging over the ensemble of gauge-fixed representants of the original gauge-unfixed Monte Carlo gauge ensemble. The ghost propagator in momentum space can be written as

\[
G(p) = \frac{1}{3V} \sum_{x,y} e^{-2\pi i p \cdot (x-y)} \langle \left( M^{-1} \right)^{a}_{x} y | U \rangle , \tag{9}
\]

where the coefficient \( \frac{1}{3V} \) is taken for a full normalization, including the indicated color average over \( a = 1, \ldots, 3 \). In what follows we will denote the (bare) ghost dressing function as

\[
J(p) \equiv p^2 G(p) . \tag{10}
\]

III. DETAILS OF THE COMPUTATION

The Monte Carlo (MC) simulations had been carried out at several \( \beta \)-values between \( \beta = 2.2 \) and \( \beta = 2.55 \) for various lattice sizes \( L \). Consecutive configurations (considered to be statistically independent) were separated by 100 sweeps, each sweep consisting of one local heatbath update followed by \( L/2 \) microcanonical updates. In Table II we provide the full information about the field ensembles used in this investigation. The corresponding information concerning the gluon propagator has been communicated in [47].

For gauge fixing we employ the \( Z(2) \) flip operation as discussed in [53, 58]. For completeness we recall the main features. The method consists in flipping all link variables \( U_{x\mu} \) attached and orthogonal to a selected 3d plane by multiplying them with \(-1 \in Z(2)\). Such global flips are equivalent to non-periodic gauge transformations. They represent an exact symmetry of the pure gauge action. The Polyakov loops in the direction of the chosen links and averaged over the orthogonal 3d plane obviously change their sign. Therefore, the flip operations combine the \( 2^4 \) distinct gauge orbits (or Polyakov loop sectors) related to strictly periodic gauge transformations into a single large gauge orbit.

| \( \beta \) | \( a^{-1} [\text{GeV}] \) | \( a [\text{fm}] \) | \( L \) | \( \alpha L [\text{fm}] \) | \( N_{\text{meas}} \) | \( N_{\text{copy}} \) |
|---|---|---|---|---|---|---|
| 2.20 | 0.938 | 0.210 | 14 | 2.94 | 400 | 48 |
| 2.30 | 1.192 | 0.165 | 18 | 2.97 | 200 | 48 |
| 2.40 | 1.654 | 0.119 | 26 | 3.09 | 200 | 48 |
| 2.50 | 2.310 | 0.085 | 36 | 3.06 | 400 | 80 |
| 2.55 | 2.767 | 0.071 | 42 | 2.98 | 200 | 80 |
| 2.20 | 0.938 | 0.210 | 24 | 5.04 | 400 | 48 |
| 2.30 | 1.192 | 0.165 | 30 | 4.95 | 400 | 48 |
| 2.40 | 1.654 | 0.119 | 42 | 5.00 | 200 | 80 |
| 2.30 | 1.192 | 0.165 | 44 | 7.26 | 200 | 80 |

TABLE I: Values of \( \beta \), lattice sizes, number of measurements and number of gauge copies used throughout Ref. [47] and this paper. The lattice spacing was fixed to its physical value using the string tension \( \sqrt{\sigma} = 440 \text{ MeV} \) (see [53, 54]).

The second ingredient is the simulated annealing (SA) method, which has been investigated independently and found computationally more efficient than the exclusive use of standard overrelaxation (OR) [50, 58]. The SA algorithm generates gauge transformations \( g(x) \) by MC iterations with a statistical weight proportional to \( \exp (4V F_U[g]/T) \). The “temperature” \( T \) is an auxiliary parameter which is gradually decreased in order to guide the gauge functional \( F_U[g] \) towards a maximum, despite its fluctuations. In the beginning, \( T \) has to be chosen sufficiently large in order to allow rapidly traversing the configuration space of \( g(x) \) fields in large steps. As in Ref. [52] we have chosen \( T_{\text{init}} = 1.5 \). After each quasi-equilibrium sweep (that includes both heatbath and microcanonical updates) \( T \) has been decreased in equidistant steps. The final SA temperature has been fixed according to the requirement that during the following execution of the OR algorithm the violation of the transversality condition

\[
\max_{x,\mu} \left| \sum_{\mu} \left( A_{x+\mu/2;\mu} - A_{x-\mu/2;\mu} \right) \right| < \epsilon_{\text{tor}} \tag{11}
\]

decreases in a monotonous manner for the majority of gauge fixing trials, until finally the transversality condition (11) becomes uniformly satisfied with an \( \epsilon_{\text{tor}} = 10^{-7} \). A monotonous OR behavior is reasonably satisfied for a final lower SA temperature value \( T_{\text{final}} = 0.01 \) [57]. The number of temperature steps interpolating between \( T_{\text{init}} \) and \( T_{\text{final}} \) has been chosen to be 1000 for the smaller lattice sizes and increased to 2000 for the lattice sizes 30 and bigger. The finalizing OR algorithm using the Los Alamos type overrelaxation with the overrelaxation parameter value \( \omega = 1.7 \) requires typically a number of iterations varying from \( O(10^2) \) to \( O(10^3) \).
In what follows we call the combined algorithm employing SA (with finalizing OR) and $Z(2)$ flips the ‘FSA’ algorithm. By repeated starts of the FSA algorithm we explore each $Z(2)$ Polyakov loop sector several times in order to find there the best (“be”) copy. The total number of copies per configuration $N_{\text{copy}}$, generated and inspected for each $\beta$-value and lattice size is indicated in Table I.

Some more details suitable to speed up the gauge fixing procedure are described in [43].

In order to suppress lattice artifacts from the beginning we followed Ref. [33] and selected the allowed lattice momenta as surviving the cylinder cut

$$
\sum_{\mu} k_{\mu}^2 - \frac{1}{4} (\sum_{\mu} k_{\mu})^2 \leq 1.
$$

Moreover, we have applied the “a-cut” $p_{\mu} \leq (2/a)\alpha$ for every component, in order to keep close to a linear behavior of the lattice momenta $p_{\mu} = (2\pi k_{\mu})/(aL)$, $k_{\mu} \in (-L/2, L/2)$. We have chosen $\alpha = 0.5$. Obviously, this cut influences large momenta only.

We define the renormalized ghost dressing function according to momentum subtraction schemes (MOM) by

$$
J_{\text{ren}}(p, \mu) = Z(\mu, 1/a) J(p, 1/a) \quad (13)
$$

$$
J_{\text{ren}}(p = \mu) = 1. \quad (14)
$$

In practice, we have fitted the bare dressing function $J(p, 1/a)$ with an appropriate function (see Eq. (15) below) and then used the fits for renormalizing $J$. Assuming that lattice artifacts are sufficiently suppressed it has to be seen, whether multiplicative renormalizability really holds in the non-perturbative regime. For this it is sufficient to prove that ratios of the renormalized (or unrenormalized) propagators obtained from different cutoff values $1/a(\beta)$ will not depend on $p$ at least within a certain momentum interval $[p_{\text{min}}, p_{\text{max}}]$, where $p_{\text{max}}$ should be the maximal momentum surviving all the cuts applied.

In what follows the subtraction momentum has been chosen to be $\mu = 2.2$ GeV.

IV. RESULTS

A. Finite volume effects

Data for $J_{\text{ren}}(p, \mu)$ for various volumes are presented in Fig. [1] for $\beta = 2.3$ and in Fig. [2] for $\beta = 2.4$. To see the finite volume effects in more detail we fitted the data at $\beta = 2.3$ for $aL \simeq 7$ fm and at $\beta = 2.4$ for $aL \simeq 5$ fm with a fitting function of the form

$$
f_J(p) = \frac{b_1}{(p^2)^c} + \frac{b_2 p^2}{p^2 + m^2_{\text{gh}}} \quad (15)
$$

(see Table II). This ansatz, while describing the data reasonably well within the given momentum range, will not be applicable in the IR limit, when we assume that $J(p)$ exhibits an inflection point and bends to a finite value $J(0)$. The latter was reported for $SU(2)$ [47] at $\beta = 2.20$ on larger lattice sizes ($40^4$) than considered here.

In the right panels of Fig. [1] and Fig. [2] respectively, the relative deviations from the fit function are shown for $\beta = 2.3$ and $\beta = 2.4$, respectively. One can see that for both $\beta$ values finite volume effects for lattices even with $aL \simeq 3$ fm are small (less than 1%) for momenta $|p| \gtrsim 0.6$ GeV.

B. Lattice spacing effects

In Fig. [3] we show the momentum dependence of the renormalized ghost dressing function $J_{\text{ren}}(p)$ for five different lattice spacings but for (approximately) the same physical size $aL \simeq 3$ fm (for the exact values see Table I). Finite-spacing effects for $\beta = 2.2, 2.3, 2.4$ in comparison with $\beta = 2.55$ are evident. To see them in more detail we have fitted also the data at $\beta = 2.55$ with the fit function Eq. (15) (see Table II). Then, in the right panel of Fig. [3] we have plotted the relative deviation of the data from this fitting result for all values of the lattice coupling parameter $\beta$. Note that the deviations of the data points for $\beta = 2.55$ from the zero-constant line just illustrate the quality of the fit.

Related to our choice of the (re)normalization momentum $\mu = 2.2$ GeV and due to the rather small statistical errors for the ghost dressing function we see clear scaling violations especially in the IR region but also for $p > \mu$. At the lowest (here accessible) momenta the violations at $\beta = 2.3 (\beta = 2.4)$ relative to $\beta = 2.55$ are staying below 10% (4%). Thus, in comparison with corresponding estimates for the gluon propagator (see Fig. 13 in [47]) which were more noisy, we can say that the relative scaling violations of the ghost dressing function turn out to be somewhat larger.

Similar to the case $aL \simeq 3$ fm we observe analogous lattice spacing effects on volumes with linear size $aL \simeq 5$ fm. The respective results are depicted in Fig. [4]. In this case the fit for $\beta = 2.4$ was employed (see Table II).
Taking the gluon dressing function results from \cite{47} into account we can compute the minimal MOM scheme running coupling \cite{48} via

\[ \alpha_s(p) = \frac{g_0^2}{4\pi} Z(p) J(p)^2, \]  

where \( Z(p) \) and \( J(p) \) are the bare gluon and ghost dressing functions, respectively.

For the running coupling we use the following fitting function:

\[ f_\alpha(p) = \frac{c_1 p^2}{p^2 + m_\alpha^2} + \frac{c_2 p^2}{(p^2 + m_\alpha^2)^2} + \frac{c_3 p^2}{(p^2 + m_\alpha^2)^3}. \]  

The fit results for the same combinations of values \((\beta, L)\) as for the ghost dressing function (see Table II) are collected in Table III.

Finite volume effects for the running coupling are shown in Fig. 5 for \( \beta = 2.3 \) and in Fig. 6 for \( \beta = 2.4 \). In both cases one can see the finite volume effects to be reasonably small (less than 5%) at a linear physical lattice extension \( aL \simeq 3 \) fm and for momenta \(|p| \gtrsim 0.6 \) GeV.

Results for the scaling check of \( \alpha_s(p) \) taking into account four lattice spacings for the same extent of \( aL \simeq 3 \) fm are presented in Fig. 7. We see relative...
FIG. 3: Left: The momentum dependence of the renormalized ghost dressing function $J_{\text{ren}}(p)$ for five different $\beta$-values or lattice spacings. The physical linear box size is $aL \simeq 3$ fm. The fitting curve belongs to the smallest available lattice spacing ($\beta = 2.55$). Right: The relative deviation of the ghost dressing function data $J_{\text{ren}}(p)$ from the fitting result.

deviations for $\beta = 2.3$ in comparison with $\beta = 2.55$ up to a 10%-level within the momentum range explored.

Since the running coupling $\alpha_s$ seems to tend to zero in the IR limit, our results obtained within the framework of Landau gauge fixing as described above are fully compatible with the IR-decoupling scenario discussed in the context of the Dyson-Schwinger and functional renormalization group approach [21, 60].

V. CONCLUSIONS

Completing an earlier work [47] we have computed the Landau gauge ghost dressing function for lattice $SU(2)$ pure gauge theory. Together with the former results for the gluon propagator we also have presented the minimal MOM scheme running coupling. We employed the same gauge-fixed field configurations as in [47] obtained with a gauge fixing method consisting of a combined application of $Z(2)$-flips and repeated simulated annealing with subsequent overrelaxation for the gauge functional. This method was invented in order to get as close as possible to the fundamental modular region i.e. to the global extremum of the gauge functional. It previously was shown to solve the Gribov problem with suppressed finite size effects [54, 58].

Assuming that the Gribov problem is kept under control to the best of our present knowledge we con-

TABLE III: Values of the fit parameters for $\alpha_s$ (Eq. (17)) in physical units and the corresponding $\chi^2_{df}$. 

| $\beta$ | $L$ | $m_a$ | $c_1$ | $c_2$ | $c_3$ | $\chi^2_{df}$ |
|---------|-----|-------|-------|-------|-------|---------------|
| 2.55    | 42  | 1.04(1)| 0.205(3)| 2.47(3)| 13.9(9)| 1.3          |
| 2.40    | 42  | 1.01(1)| 0.16(1)| 2.68(5)| 11.7(7)| 0.81         |
| 2.30    | 44  | 1.03(1)| 0.19(4)| 2.4(2) | 14(2)  | 0.95         |
FIG. 5: Left: The momentum dependence of the running coupling for three different lattice sizes at $\beta = 2.3$. The curve shows a fit with Eq. (17) for $aL \simeq 7$ fm. Right: The relative deviation of the data for the running coupling $\alpha_s(p)$ from the fitting curve.

FIG. 6: Same as in Fig. 5 but for two lattice sizes at $\beta = 2.4$. The curve shows the fit result with Eq. (17) for $aL \simeq 5$ fm.

centrated ourselves on systematic effects like finite size and lattice spacing dependences. While finite size effects were confirmed to be rather small the lattice spacing artifacts turned out to be non-negligible as for the renormalized ghost dressing function as for the running coupling. In both cases for the linear lattice size of approximately 3 fm (and for the ghost dressing function with a subtraction momentum of $\mu = 2.2$ GeV) we saw relative deviations at $\beta = 2.30$ from the results obtained at our largest $\beta = 2.55$ reaching a ten percent level at the lowest accessible momentum values and still around five percent in the non-perturbative region around 1 GeV. This tells us that lattice results for Landau gauge gluon and ghost propagators in this momentum range have still to be taken with some caution what concerns the Gribov problem and the continuum limit.

However, even if the ghost dressing function for not too small momenta has been well fitted by a weakly IR singular behavior (see Eq. (15)), thus hiding a IR finite limit, the result for the running coupling $\alpha_s(p)$ turned out to be robust what concerns the compatibility with the infrared decoupling solution of DS and FRG equations.

Acknowledgments

This investigation has been partly supported by the Heisenberg-Landau program of collaboration between the Bogoliubov Laboratory of Theoretical Physics of the Joint Institute for Nuclear Research Dubna (Russia) and German institutes and partly by the DFG grant Mu 932/7-1. VB is supported by RFBR grant 11-02-01227-a and by grant 8376 of the Russian Ministry of Science and Education.
FIG. 7: Left: The momentum dependence of the running coupling for four different $\beta$-values or lattice spacings at lattice extent $aL \simeq 3$ fm. The curve shows the fit result with Eq. [17] for $\beta = 2.55$. Right: The relative deviation of the data from this fit.

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