Numerical simulation of turbulent Rayleigh-Bénard mercury convection in a circular cylinder with introducing small deviations from the axisymmetric formulation

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Abstract. Results of DNS are reported for turbulent Rayleigh-Bénard mercury convection (Pr = 0.025) in a cylindrical container with the aspect ratio of unity at the Rayleigh number of Ra = 10^6. The main goal of the study is to estimate the influence of small deviations from the axisymmetric formulation on the behavior of large-scale circulation (LSC) developing in the container. The deviations examined are a small tilt of the cylinder or prescribed sinusoidal disturbances of the bottom wall temperature. The effects of the computational grid structure in the central region of the flow are investigated as well. It has been shown that the azimuthal low-frequency reorientations of the LSC, which are typical for the axisymmetric problem setting, can be fully suppressed by imposing relatively small non-axisymmetric disturbances of various kinds. Time-averaged 3D velocity and temperature fields, as well as spatial non-uniformities of local heat transfer are analyzed for a “locked” LSC case.

1. Introduction
Rayleigh-Bénard (RB) convection is an important phenomenon that occurs in numerous natural processes (convection in the oceans and in the Earth’s outer core) and in industrial applications (electric circuit cooling, semiconductor crystal growth technology, fast reactors, etc.). A typical RB setup consists of a vertically oriented cylindrical container, filled with fluid heated from below and cooled from above.

As a result of numerous experimental studies performed for fluids with significantly different Prandtl numbers, it has been found out that a dominate feature of the turbulent RB convection developing in the cylinder is a large-scale circulation (LSC) (see, for instance, the review paper [1]). In case of a confined cylinder with an aspect ratio of unity, the LSC is characterized by large convection roll covering the entire domain with background turbulence [1-4].

For a setup with a strict vertical orientation of the cylinder axis and ideally axisymmetric boundary conditions there is no preferential azimuthal position of the convection roll, and, accordingly, nothing impedes its azimuthal rotation over time. Experimental observations dealing with the idealized configurations show that in most cases the roll changes its orientation slowly in time with irregular behavior [2-4]. For a particular configuration, azimuthal behavior of the LSC is determined by relatively small, hardly controlled deviations from axial symmetry that unavoidably takes place in laboratory models. The above mentioned features of the LSC were revealed also in a number of computational...
studies devoted to numerical simulation of transitional and turbulent RB convection in cylinders [1, 5-8]. For numerical simulations, possible deviations from the strict axisymmetry of the computational model may be attributed to a nonideal computational grid structure and/or to features of the used numerical algorithm.

The occasional azimuthal reorientations of the convection roll complicate the process of obtaining time-averaged 3D flow fields and quantities characterizing the background turbulence in a circular container. To fix the roll at a certain position, one can artificially introduce a proper stabilizing factor, for example, a slight tilt of the container. This approach is widely used in experimental studies [2, 9, 10]. Another way for intentional introduction of disturbances, which break the problem axisymmetry, is to impose small controlled deviations from the heating uniformity.

In this paper, the effect of small deviations from the axisymmetric problem setting on turbulent RB convection is studied numerically, based on the direct numerical simulation (DNS) approach. The case of the low-Pr fluid (mercury) convection in a cylindrical container with an aspect ratio of 1 is considered. It is shown that low-frequency azimuthal oscillations of the LSC can be fully suppressed by the tilt of the container at a small angle or by means of prescribing a proper non-uniform temperature distribution on the horizontal walls.

2. Problem setting
DNS of turbulent RB convection is performed for the case of a cylindrical container with the diameter-to-height, $D/H$, aspect ratio of 1 (figure 1a). Fluid motion is described by a set of governing equations, which includes the Navier-Stokes equations written in the Boussinesq approximation and the energy transport equation. No-slip conditions are imposed on all the boundaries. Prescribed temperature distributions, uniform or non-uniform, are set on the lower and upper walls. The lateral cylindrical surface is assumed adiabatic.

The dimensionless parameters determining the convection are the Prandtl number, $Pr = \nu/\chi$, and the Rayleigh number, $Ra = Pr \cdot (V_b H/\nu)^2$, where $V_b = (g \beta \Delta T H)^{0.5}$ is the characteristic flow velocity, $\nu$ is the kinematic viscosity, $\chi$ is the thermal diffusivity coefficient, $g$ is the gravity acceleration, $\beta$ is the thermal expansion coefficient, and $\Delta T = T_h - T_c$ is the characteristic temperature difference between the hot and the cold walls. The present computations have been carried out at $Ra = 10^6$ and $Pr = 0.025$.

It is assumed that the container can be tilted at a small angle, $\phi$, with respect to the gravity vector, as illustrated in figure 1b. Two variants of the container tilt are considered in the present work: $\phi = 1^\circ$ and $2^\circ$. In both the cases, the lower and the upper walls are kept at constant temperatures: $T_h$ and $T_c$ correspondingly. In case of the strict vertical orientation of the cylinder, $\phi = 0$, the bottom temperature is assumed either constant, $T_h$, or distributed according to the following expression: $T = T_h + \Delta \cdot \sin(\omega x)$, where $\Delta$ is the amplitude of the imposed sinusoidal perturbation, $x$ is the horizontal coordinate, and $\omega$ is a parameter ensuring a half period of temperature change along the axis. The perturbation amplitude of the bottom temperature is $0.035\Delta T$ or $0.1\Delta T$. The upper wall temperature is always constant.

3. Computational aspects
The governing equations were solved using the fractional-step method as implemented in the in-house finite-volume unstructured-grid CFD-code SINF/Flag-S developed at the SPbPU [11]. Time-advancing was performed with the second-order Crank-Nicholson scheme. The third-order QUICK scheme was used for spatial discretization of the convective fluxes. The diffusion terms were approximated by the second order central-difference scheme.
Computational grids used in the present work consisted of hexagonal elements. Most of the computations were performed with Grid 1, which had a central unstructured (asymmetric) area with a diameter of 0.8D (figure 2a). Additional computations were carried out using Grid 2. In this case, the unstructured-region diameter was reduced to 0.3D. Both grids contained approximately $5 \times 10^5$ cells; the vertical size of the first step near the horizontal walls was about $10^{-4}H$.

All the computations started from the zero initial velocity fields. The average of the upper and lower wall temperatures was taken as the initial temperature field. The time step was less than one thousandth of the characteristic convection time, $t_b = H/V_b$, and the local Courant number did not exceed unity. The samples used for averaging, started after a long transient period, amounted up to 2000 dimensionless time units.

In order to analyze behavior of the LSC, a number of monitoring points were used. Seven of them were positioned at half height of the container at a distance of 0.15D from the side wall, with the angular spacing of 30°, as shown in figure 2b.

### 4. Results and discussion

Figure 3 illustrates the LSC dynamics for three computational cases. Namely, time variations of the vertical velocity component at monitoring points 1-7 are shown for the original (axisymmetric) problem formulation (figure 3a), for the case of the container tilted by $2^\circ$ (figure 3b), and for the case of imposed sinusoidal temperature variations with $\Delta = 0.1\Delta T$ (figure 3c). All these data have been obtained using Grid 1. From the presented sample of 2000$t_b$ one can clearly see that in case of the axisymmetric formulation, low-frequency changes of convection direction take place at all the monitoring points. These changes are due to the LSC azimuthal reorientations. A slight tilt of the container (figure 3b) or small temperature non-uniformities on the bottom wall (figure 3c) provide “locking” of the convection
roll, and the mid vertical plane of the roll, averaged over several decades of \( t_b \), becomes aligned either with the tilt plane or with the direction of the bottom temperature non-uniformity.

**Figure 3.** Dynamics of the vertical velocity component at (top-down) monitoring points 1-7: (a) in case of axisymmetric problem formulation; (b) in case of the container tilt by \( 2^\circ \); (c) in case of non-uniform temperature distribution on the bottom wall, \( \Delta = 0.1\Delta T \).
High-frequency velocity oscillations at all the monitoring points are associated with background turbulence. In cases of the locked LSC, velocity time variations at point 4 also include contributions of moderate frequency oscillations, with a period of the order of $100t_b$. The origin of these oscillations has been clarified in [9, 10] and related works.

Figure 4 illustrates instantaneous and time-averaged flow fields computed with Grid 2 for the case of the container tilted by $2^\circ$. Due to absence of the low-frequency azimuthal reorientations of the LSC, the computed sample of $2000t_b$ is sufficient to get reliable statistics for the simulated RB convection. Remarkably, that in the mid vertical section of the LSC the fluid motion consists of a global vortex and four smaller vortices in each corner.

Figure 4. (a) Instant vertical velocity isosurfaces (the blue/red colors correspond to the downward/upward flow, $|V_y| = 0.26V_b$); (b) time-averaged temperature and velocity distributions in the mid vertical LSC section; (c) time-averaged vertical velocity field in the mid horizontal plane.

In figure 5, values of the time-averaged vertical velocities obtained for monitoring points 1-7 in seven computational runs are shown versus the azimuthal position. In all the cases, samples of $2000t_b$ were used for averaging; open/filled symbols correspond to the Grid1/Grid2 data. One can see that, in consistence with the previous experimental findings [3, 9], even a small tilt of the container (by only $1^\circ$) is practically sufficient for "locking" the LSC. Note here that for point 4 ($\alpha = 90^\circ$) one should expect zero values of the time-averaged vertical velocity, if the LSC is locked. However, computations with Grid 1 have resulted in small non-zero values, both at $\varphi = 1^\circ$ and $2^\circ$. It is caused by specifics of the central unstructured region of Grid 1 that does not possess a full symmetry with respect to the tilt plane. Due to this deficiency of Grid 1, the LSC mid plane is slightly turned with respect to the tilt plane. It is remarkable that computations with Grid 2 possessing a much better symmetry have given the expected (zero) value of the averaged vertical velocity. A proper level of temperature non-uniformities on the bottom wall also leads to locking the convection roll. The above-given remarks concerning deficiencies of Grid 1 are applicable to this case as well.

Figure 6 presents distributions of the time-averaged Nusselt number on the bottom wall obtained with the axisymmetric problem formulation and for the case of the container tilt of $2^\circ$. As previously, samples of $2000t_b$ were used for averaging in both cases. One can see that in the tilted container case, zones with a dramatically increased (decreased) heat fluxes appear in the flow regions with descending (ascending) motion. For the original (axisymmetric) formulation, averaging over a rather long sample gives a more uniform Nusselt number distribution due to smoothing action of the low-frequency azimuthal reorientations of the LSC. However, the area-averaged values of the Nusselt number for these cases are very close; they are between 5.6 and 5.7 that is in accordance with results of our previous computations [11].
Figure 5. Time-averaged vertical velocity values vs azimuthal position (at points 1–7).

Figure 6. Time-averaged Nusselt number distributions on the bottom wall for the axisymmetric formulation (left) and with the container tilt of $\varphi = 2^\circ$ (right).

Conclusions
In consistence with experimental findings, the present DNS has shown that a small tilt of the container (by $1^\circ$-2°) is sufficient to suppress azimuthal low-frequency LSC reorientations that are typical for the axisymmetric RB convection problem setting. Similar effects are produced by bottom temperature non-uniformities in the form of sinusoidal perturbations. Deviations from the symmetry in distribution of computational grid cells may lead to a small turn of the mid vertical section of the locked time-averaged LSC with respect to the plane of the container tilt. The averaged large-scale circulation consists of a global vortex and four smaller vortices in each corner. In case of the locked LSC, the time-averaged Nusselt number distributions on the lower and upper walls are strongly non-uniform.

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