Partially Averaged Navier-Stokes method based on $k$-$\omega$ model for simulating unsteady cavitating flows

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Abstract. The turbulence closure is significant to unsteady cavitating flow computations as the flow is frequently time-dependent accompanied with multiple scales of vortex. A turbulence bridging model named as PANS (Partially-averaged Navier-Stokes) purported for any filter-width is developed recently. The model filter width is controlled through two parameters: the unresolved-to-total ratios of kinetic energy $f_k$ and dissipation rate $f_\omega$. In the present paper, the PANS method based on $k$-$\omega$ model is used to simulate unsteady cavitating flows over a Clark-y hydrofoil. The main objective of this work is to present the characteristics of PANS $k$-$\omega$ model and evaluate it depending on experimental data. The PANS $k$-$\omega$ model is implemented with various filter parameters ($f_k=0.2$–$1$, $f_\omega=1/f_k$). The comparisons with the experimental data show that with the decrease of the filter parameter $f_k$, the PANS model can reasonably predict the time evolution process of cavity shapes and lift force fluctuating in time. As the PANS model with smaller $f_k$ can overcome the over-prediction of turbulent kinetic energy with original $k$-$\omega$ model, the time-averaged eddy viscosity at the rear of attached cavity decreases and more levels of physical turbulent fluctuations are resolved. What’s more, it is found that the value of $\omega$ in the free stream significantly affects the numerical results such as time-averaged cavity and fluctuations of the lift coefficient. With decreasing $f_k$, the sensitivity of $\omega$-equation on free stream becomes much weaker.

1. Introduction
Cavitation typically occurs when the static pressure decreases below its vapor pressure.[1] The turbulent cavitating flows strongly affect the performances of blades, the propeller, the pump, et al. in engineering applications. Due to the complex, unsteady flow structures associated with turbulence and cavitation dynamics, there are significant computational issues in regard to stability, efficiency, and robustness of the numerical algorithm for turbulent unsteady cavitating flows.[2]

The turbulence model has a significant effect on the simulation of unsteady cavitating flows. The Reynolds-averaged Navier–Stokes (RANS) model fails to predict the vortex shedding behavior and excessively restrain the cavitation instabilities due to the high eddy viscosity.[3] Recently, several hybrid modeling approaches to improve the traditional RANS models have been proposed and developed, such as, detached eddy simulation (DES)[4], a filter-based model (FBM)[5], PANS $k$-$\varepsilon$ model [6] et al. These hybrid models are applied to simulating the cavitating flows and obtain expected results, see ref.[7][8][9][10].

Inspired by their work, the present paper is devoted to investigate the recent PANS $k$-$\omega$ model based on the unsteady cavitating flow over a Clark-y hydrofoil and evaluate it by comparisons with experimental data. We mainly focus on the influence of the filter with $f_k$ on the simulation and the sensitivity of the freestream $\omega$. 

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2. Theoretical formulation

2.1 Favre-averaged continuity and momentum equations

The set of governing equations comprises the conservative form of the incompressible Favre-averaged Navier-Stokes equations, coupling with cavitation model (Zwart cavitation model[11]) and turbulence closure. The mass continuity, momentum equations are given below.

\[
\frac{\partial \rho_m}{\partial t} + \frac{\partial (\rho_m u_j)}{\partial x_j} = 0 \tag{1}
\]

\[
\frac{\partial (\rho_m u_j)}{\partial t} + \frac{\partial (\rho_m u_j u_i)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ (\mu_k + \mu_r) \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \right) \right] \tag{2}
\]

The mixture property, \( \phi_m \), can be expressed as

\[
\phi_m = \phi \alpha_i + \phi_i (1 - \alpha_i) \tag{3}
\]

where \( \alpha_i \) is the liquid volume fraction, \( \phi \) can be density, viscosity, and so on.

2.2 Turbulence Model

The PANS model is proposed by Lakshmipathy and Girimaji[12] based on the standard \( k-\omega \) RANS model. The filter width in PANS \( k-\omega \) model is controlled by specifying the ratios of unresolved-to-total kinetic energy, \( f_k \) and unresolved-to-total turbulent frequency, \( f_\omega \). The two parameters are defined as:

\[
f_k = \frac{k_u}{k}, \quad f_\omega = \frac{\omega_u}{\omega} \tag{4}
\]

The \( k \) and \( \omega \) are the total turbulent kinetic energy and turbulent frequency, respectively. Throughout, subscript \( u \) indicates PANS unresolved statistics. The smaller the \( f_k \), the finer is the filter. The value of \( f_k \) can vary from 0 to 1: \( f_k =1 \) represents RANS and \( f_k =0 \) indicates DNS. Here, the Reynolds number considered in the flow fields is considerably high, so \( f_\omega = 1 \) \( f_k \). The two-equation PANS \( k-\omega \) model can be summarized as

\[
\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \nu + \frac{\nu}{\sigma_{uu}} \right) \frac{\partial k}{\partial x_j} \right] + P_u - \beta' k \omega 
\]

\[
\frac{\partial \omega}{\partial t} + U_j \frac{\partial \omega}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \nu + \frac{\nu}{\sigma_{uu}} \right) \frac{\partial \omega}{\partial x_j} \right] + \alpha \nu \omega - \beta' \omega^2 
\]

\[
\beta' = \alpha \beta^* - \frac{\alpha \beta^*}{f_\omega} + \frac{\beta}{f_\omega} \tag{7}
\]

\[
\sigma_{uu} = \sigma \frac{f_k}{f_\omega}, \quad \sigma_{uu} = \sigma \frac{f^2_k}{f_\omega} \tag{8}
\]

\[
\frac{\nu}{\sigma_{uu}} \frac{k}{\omega} = \frac{f_k}{f_\omega} \tag{9}
\]

The values for various coefficients are:

\[
\beta^* = 0.09, \quad \alpha = \frac{5}{9}, \quad \beta = 0.075, \quad \sigma_i = 2.0, \quad \sigma_\omega = 2.0 \tag{10}
\]
3. Results and discussions

Figure 1 shows the computational domain and boundary conditions which are given according to the experimental set-up[13]. The Clark-y hydrofoil with the chord length of 0.07m is located with the angle of attack of 8°. The two important dimensionless parameters, the Reynolds number $Re$ and the cavitation number $\sigma$, are defined as

$$ Re = \frac{U_c c}{\nu} , \quad \sigma = \frac{p - p_v}{\rho U_c^2 / 2} \tag{11} $$

where $U_c$ the inlet velocity, $p$ is the outlet pressure, $p_v$ is the saturated vapor pressure, $c$ is the hydrofoil chord, $\nu$ is the kinetic viscosity and $\rho$ is the water density. Computations are performed for cloud cavitation condition ($\sigma=0.8$). Here, $U_c=10$m/s is imposed at the inlet, with the corresponding Reynolds number of $Re = 7 \times 10^5$. The saturated pressure of water at 25 °C is $p_v=3169$ Pa.

Figure 1. Boundary conditions for Clark-Y hydrofoil, $c$ is the hydrofoil chord

Figure 2 shows the contours of time-averaged water vapor volume fraction and eddy viscosity predicted with different $f_k$, respectively. Wang[14] found that the cloud cavity structure consists of two parts, the attached front portion and the detached rear region. From figure 2, the PANS with higher $f_k$, such as $f_k=1.0 \sim 0.6$, underestimates the detached part and obtains the whole cavity of smaller scale, compared with that of lower $f_k$. As is reported that the poor prediction of cavity shedding may be due to over-prediction of the turbulent viscosity in the rear part of the cavity. [15] The differences of the cavity shape at the rear region predicted by PANS with different $f_k$ imply that the eddy viscosity distributions are handled differently. In figure 2, the eddy viscosity at the rear of hydrofoil decreases with smaller $f_k$ and a dramatic decrease is conducted by the PANS with $f_k=0.2$. Mathematically, to some extent, the eddy viscosity in PANS is proportional to the square of $f_k$ value, as is shown in Equation (9).

| $f_k$  | Water vapor vf. | Eddy viscosity |
|-------|-----------------|----------------|
| 1.0   |                 |                |
| 0.8   |                 |                |
| 0.6   |                 |                |
| 0.4   |                 |                |
| 0.2   |                 |                |

Figure 2. Time-averaged water vapor volume fraction and eddy viscosity contours
To further investigate the PANS model, figure 3 shows the time-evolutions of the computational and experimentally observed cavity structures. It demonstrates that the PANS model with various $f_k$ performs differently in simulating the cloud cavitating flow. Compared with the experimental results, it is found that the PANS with $f_k=0.2$ is capable of capturing the cavity inception at the time of $t_0$, growth toward the trailing edge at $t_0+38\%T$, attached cavity break up at $t_0+58\%T$ and finally large scale cavity shedding at $t_0+84\%T$, in accordance with the qualitative features observed experimentally. In contrast, the cases of $f_k=1.0$ and $f_k=0.6$ present the failure of predicting the attached cavity in larger size and the shedding process of the detached cavity. This is probably because the higher eddy viscosity restrains the cavity growing and induces unexpected unsteady behaviors.

![Figure 3. Instantaneous contours of water vapor volume fraction](image)

The influence of $f_k$ in PANS on the unsteady characters of cloud cavitating flows can be demonstrated by the time-dependent lift coefficient as shown in figure 4. Obviously, although the lift signals predicted by the PANS with different $f_k$ are fluctuating periodically in time, yet there are substantial differences in the fluctuation detail. The PANS with higher $f_k$, such as $f_k=1.0$ and $f_k=0.8$, can predict a relatively steady lift curve with little fluctuations. With decreasing $f_k$, the PANS can resolve more scales in the field and the lift coefficient curve presents more fluctuations in time which is in good agreement with the experiment.

![Figure 4. Time-evolutions of hydrofoil lift coefficient](image)
Figure 5 gives the time-averaged u-velocity component profiles tracked along y-direction at different specified positions in the chordwise. It is found that the trend of velocity curve predicted by any PANS model is similar to the experimental result at any position. From the figure 5(a)–(c), the time-averaged attached cavity becomes thicker, and accordingly the boundary layer also expands along the y-direction. Although the differences between predictions and experimental data are more substantial at the rear region, such as at x/c=0.8, x/c=1.0 and x/c=1.2, still the agreement is reasonable considering the difficulties in experimental measurement. To draw a comparison between two PANS methods, the results of PANS k-ε model[16] are also shown in figure 5. In contrast, the distributions of the velocity in the boundary layer obtained by PANS k-ε model with $f_k=0.2$ is closer to the experimental results, while the PANS k-ω model with $f_k=0.2$ predicts velocity profiles even further away from the experiments. The reason for the poor performance of the k-ω model may lie in its freestream dependency.

![Figure 5: Time-averaged x-direction velocity at different locations](image)

Figure 5. Time-averaged x-direction velocity at different locations

In order to investigate the sensitivity of the freestream $\omega$ to the PANS k-ω model, figure 6 and figure 7 give the contours of time-averaged water vapor volume fraction and the time-dependent lift coefficient curve predicted by PANS with different $f_k$ and $\omega$, respectively. From figure 6, it is found that the size of the time-averaged cavity increases with decreasing the value of $f_k$ for the both cases of $\omega=1$ and $\omega=60$. When keeping the $f_k$ to be constant, the contour of time-averaged water vapor volume...
fraction presents different distributions with different values of $\omega$. Moreover, in figure 7, the lift coefficient signals fluctuate in the different forms under condition of different values of $\omega$ but the same $f_k$. In contrast, the difference caused by $\omega$ becomes smaller gradually with decreasing $f_k$. Just as seen in Fig7(c), when $f_k=0.2$, the lift coefficient curves predicted by the PANS models with different $\omega$ almost overlap each other.

| $f_k$ | $\omega=1$ | $\omega=60$ |
|------|----------|----------|
| 1.0  | ![Image](image1) | ![Image](image2) |
| 0.6  | ![Image](image3) | ![Image](image4) |
| 0.2  | ![Image](image5) | ![Image](image6) |

Figure 6. Time-averaged water vapor volume fraction

![Graph](graph1)

Figure 7. Time-evolutions of hydrofoil lift coefficient

4. Conclusions
In this paper, the PANS $k-\omega$ model is used for simulating the unsteady cavitating flow over a Clark-y hydrofoil. Based on the experimental data, the PANS model is evaluated. The important conclusions
are the followings:

1) Compared with the experimental data, the PANS model can reasonably predict the time evolution process of cavity shapes and lift coefficient fluctuating with decreasing the filter parameter $f_k$. What’s more, as the PANS model with smaller $f_k$ can overcome the over-prediction of the time-averaged eddy viscosity at the rear of attached cavity decreases and more levels of physical turbulent fluctuations are resolved.

2) The value of $\omega$ in the freestream significantly affects the numerical results such as time-averaged cavity and fluctuations of the lift coefficient. With decreasing $f_k$, the sensitivity of $\omega$-equation on freestream becomes much weaker.

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