One mystery of the North Atlantic multidecadal variability. An attempt of simple explanation

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Abstract. Forming of the North Atlantic multidecadal variability (MDV) remains quite enigmatic. Some studies connect the long-term North Atlantic oceanic variability to transform of the stochastic atmospheric forcing. On the other hand, the intense heat fluxes directed from ocean to atmosphere precede the large-scale positive sea surface temperature (SST) anomalies in the region (and vice versa). The last phenomenon puts some doubts on the stochastic theory and let to suggest that surface heat fluxes play just a passive role as a response to ocean dynamical processes. Analyzing a toy box model and CMIP5 control experiments we have demonstrated that observed phase shifts between SST and surface heat fluxes do not contradict a stochastic theory. The North Atlantic long-term variability can be induced via a transform of the atmospheric random forcing. However, the role of the ocean circulation processes remains crucial for the MDV forming. Specifically, the stochastic excitation of the meridional overturning circulation reproduces observed and model generated MDV. Direct atmospheric impact on SST cannot induce correctly the phase shift between input and output signals.

1. Introduction

Multidecadal variability of the North Atlantic (NA) climate system is one of the most intriguing climate theory problems. MDV impacts the thermal and precipitation conditions in the vast regions of the northern hemisphere [1-4] and complicates quantifying the rate of modern anthropogenic warming. Mechanisms of MDV forming are still under debate. The atmospheric aerosol content primarily of volcanic origin was considered a long-term external forcing provoking the MDV [5, 6]. However, Zhang et al, 2013 [7] demonstrated later that the role of aerosols in the SST evolution was overestimated. Nonlinear dynamics of the ocean-atmosphere system can be responsible for the low-frequency variability formation in a wide range of time and spatial scales [8]. One of the possible MDV mechanisms suggests the transition of thermohaline circulation between different quasi-stable states [9, 10]. Another widely accepted hypothesis connects the development of low-frequency variability in the NA climate system to stochastic atmospheric forcing [11-14]. Most important source of the forcing is due to the North Atlantic Oscillation (NAO) [14, 15, 16]. The NAO intensity can be directly responsible for the deep oceanic waters forming, especially in the Labrador Sea [17].

The low-frequency variability of the North Atlantic climate system became recently the subject of lively discussion. Clement et al, 2015 [18] connect the root cause of such variability, manifested primarily in AMO (Atlantic Multidecadal Oscillation), to random atmospheric forcing. Supporters of the alternative concept (Zhang et al, 2016) [19] underline the crucial importance of the ocean internal dynamics. Clement et al., 2015 have found that the space-time variability, reflecting the important features of the AMO, is reproduced in both the Coupled General Circulation Models (CGCMs) and the Slab Ocean Models (SOMs). Objections [19] were built mainly on the cross-correlation analysis of the AMO index and the surface heat fluxes. Statistics of CGCMs and SOMs experiments differ radically. SOMs experiments display that the positive phase of AMO is preceded by the heat influx directed from the atmosphere to the ocean. CGCMs long-term runs exhibit the practically opposite picture - in the circumpolar North Atlantic the positive SST anomalies are preceded by an energy flux from the ocean toward the atmosphere. The largest correlation is achieved when surface heat fluxes lead the
AMO for about four years. The development of event looks paradoxical; the intensive cooling from the sea surface precedes the anomalously warm state of the ocean upper layer. Zhang et al., 2016 [19] speculated the phenomenon suggested that heat losses from the ocean surface are almost completely compensated by the advective oceanic energy transport. Thus, the development of AMO is mainly determined by the large-scale Atlantic circulation processes, primarily by the AMOC (Atlantic Meridional Overturning Circulation) and SPG (Subpolar Gyre) intensity. As a result, Zhang et al, 2016 [19] suggested a secondary role of heat fluxes on the air-sea interface in the formation of low-frequency SST variability. They have stressed that the main AMO driver is oceanic dynamics and that the energy transfer fluctuations on the surface just remain the passive response to that.

The conclusion of Zhang et al, 2016 [19] seems consistent with the results of Gulev et al, 2013 [20]. They investigated the SST and H + LE (H - sensible and LE - latent heat fluxes on the surface, positive from the ocean) anomalies obtained from the North Atlantic region observations. Two characteristics are almost synchronized at the low-frequency band and negatively correlated in the high-frequency region of the spectrum. Some doubts arise, however. Strong long-time variations are clearly expressed for the AMO index. If the atmosphere reacts just passively to the long-term oscillations of oceanic circulation, why the multi-decadal atmospheric variability over the region is relatively weak? The NAO index temporal spectrum is close to the white noise [21-23] and not similar to the AMO spectrum characterized by a pronounced predominance of low-frequency variability.

Moreover, correlation, regression, and any other statistical analysis can be used to test a particular physical hypothesis but does not provide sufficient backgrounds for its formulation. The presence or absence of correlation between shifted time series cannot yet serve as a basis for construction physically conditioned cause-effect relationships.

In this study, we show the "paradoxical" phase shift between the external atmospheric forcing and the ocean response is reproducible within the framework of a linear damped oscillator impacted by the stochastic input signal. In the second section, we described shortly the results of the CMIP5 experiments analyses. Section three is devoted to the description and analysis of the conceptual linear model of an oceanic oscillator. The last section contains the discussion and conclusions.

2. CMIP5 data analysis

We have collected area averaged results of the control long-term Coupled Model Intercomparison Project 5 (CMIP5) experiments [24]. Analyzed datasets include SST, surface downwelling (DLF) and upwelling (ULF) longwave and solar (DSF and USF) radiation fluxes, surface sensible and latent heat fluxes. All characteristics were averaged for the North Atlantic region limited 20° W - 70° W and 35° N - 70° N. Totally data for 30 CMIP5 models were analyzed. The region is characterized by relatively homogeneous spatial response to the NAO variations. For the aims of the research, we have used annually averaged data.

The primary goal of the CMIP5 control experiments investigation was the cross-spectral analyses of the spatially averaged SST and the net atmospheric forcing $F_{\text{net}}$ embracing all surface heat fluxes,

$$ F_{\text{net}} = DLF - ULF + DSF - USF - H - LE. $$

The calculations were provided on the base of inverse Fourier transform of the cross-correlation functions. Maximum time lag, $M = 15$.

Estimations of the spectral properties of the spatially averaged SST and the heat fluxes $F_{\text{net}}$ exhibit the behavior typical for the damped stochastically forced oscillator (Figure 1). Spectral density of the net atmospheric forcing is very close to the white noise. However, SST response is characterized by the prevailing of the low-frequency spectral components.
Figure 1. The spectral density estimations, $\hat{S}_T(\omega)$ and $\hat{S}_{F_{\text{net}}}(\omega)$, of the annual mean SST and $F_{\text{net}}$ anomalies, respectively. (Totally 26 model were used). Note the logarithmic scale for the y-axis. $\hat{S}_T(\omega)$ is red-colored. $\hat{S}_{F_{\text{net}}}(\omega)$ is blue-colored. Dots are individual CMIP5 model outputs, solid lines are ensemble averaged values.

Figure 2. Cross-correlation (cross-spectral) estimations of the CMIP5 control experiments.

a) Estimations of the SST and $F_{\text{net}}$ correlation coefficients for different time lags. The net heat flux $F_{\text{net}}$ leads. Blue dots - individual CMIP5 model outputs. Thick red dots - ensemble averaged values.

b) Estimations of the phase shift between SST and $F_{\text{net}}$ anomalies. Gray dots correspond to the individual CMIP5 model outputs. Solid black line corresponds to the ensemble averaged values.

Despite the typical spectral properties, the estimations of $F_{\text{net}}$ and $T$ lagged cross-correlation functions on the first glance seemed not to be supporting the idea of a forced oscillator (Figure 2. a).
The results resemble the findings of Zhang et al [19], which have demonstrated that the strong positive anomalies of SST in the North Atlantic are preceded by intensive heat fluxes directed from the ocean to atmosphere. Our estimations point that this paradoxical effect is most pronounced for the time shifts between 0 and 4 years.

The phase shift between the SST and the net surface heat flux has changed approximately from $-\pi$ at zero frequency to 0 in the high-frequency domain (Figure 2. b), in close agreement with the results of [19, 20]. In the vast majority of CMIP5 models (26 from 30), SST anomalies are out of phase to anomalies of energy fluxes in the low-frequency domain. The remaining four models, IPSL-CM5B-LR, MIROC-ESM-chem, MRI_CGCM3, and NORESM1-ME display different dependences of the phase shift from frequency. This can be due to the relatively short periods of control experiments or perhaps not completely clear simulations problems [25].

3. Simple North Atlantic stochastic model

A simple box model describing the forming of the low-frequency climate variability in the North Atlantic region was proposed by Legatt et al, 2012 [26]. The model concept is based on the Marshall et al, 2001 hypothesis [11] connecting generation of the long-term oceanic variations to the stochastic but spatially coherent atmospheric forcing. Atmospheric forcing, in turn, is determined mostly by the NAO dynamics. Legatt at al, 2012 [26] suggests that the main factors forming multidecadal North Atlantic variability involve anomalies of meridional $\psi_m$ and horizontal $\psi_g$ stream function, and spatially averaged SST, $T$. The meridional component of stream function intensity can serve as an analog of AMOC index, horizontal component characterizes the SPG. We will consider a slightly simplified version suggesting the relatively weak SPG impact, $\psi_g \equiv 0$. In this case, the dimensionless model equations take the form,

$$\frac{dT}{dt} = m\psi_m - \lambda T + F_T(t), \quad (2)$$

$$\frac{d\psi_m}{dt} = -sT - \alpha \psi_m + F_m(t), \quad (3)$$

where $F_T(t)$ is atmospheric forcing of SST determined by the net heat surface fluxes, $F_m(t)$ is forcing responsible for the generation of deep water formation. External forcing is considered a linear function of a stationary Gaussian random process $X(t) = \frac{\partial W}{\partial t}$ (the derivative of the standard Wiener process, $W$), $F_T(t) = \sigma_{F_T} X(t)$ and $F_m(t) = -\sigma_{F_m} X(t)$, where $\sigma_{F_T}^2$ and $\sigma_{F_m}^2$ are the variances of corresponding fluxes. Sign minus in the last relationship is determined by the fact that heating of the North Atlantic SST leads to prevention of deep water formation and vice versa. We will consider the two types of forcing separately trying to get most simplified solutions. Note also that in our model heat fluxes are considered positive when energy is gained by the ocean (in contrast to Gulev et al, 2013 [20]).

Coefficients $\lambda$ and $\alpha$ describe the heat and impulse dissipation, respectively. Parameters $m, s$ are responsible for linear feedback in the system. Unit of the dimensionless time corresponds to $\sim 4$ years of physical time, product $ms \approx 3$ [26].

The complete correlation and spectral analysis of Legatt et al, 2012 [26] model was implemented by Bekryaev, 2016 [27]. In event of direct forcing of meridional thermohaline circulation, i.e., $F_m(t) \neq 0, F_T(t) = 0$, the evolutionary equation for SST takes the form of linear damping oscillator.
\[ \frac{d^2T}{dt^2} + 2a \frac{dT}{dt} + bT = m F_m(t) = -m \sigma_{F_m} X(t), \quad (4) \]

A solution of (4) can be written as a Duhamel integral (if \( b > a^2 \))

\[ T(t) = \sigma_{F_m} \int_0^\infty h_m(u) X(t - u) du. \quad (5) \]

where \( h_m(u) \) is weight function,

\[ h_m(u) = \frac{-m}{\beta} \exp(-au) \sin(\beta u). \quad (6) \]

Coefficients are defined as follows: \( 2a = \alpha + \lambda; \ b = ms + \alpha \lambda; \ \beta = \sqrt{b - a^2}. \)

The correlation function of the Wiener process derivative owns the delta function properties. It means (see Appendix 1, expression A.1) that the cross-correlation function of the external forcing \( F_m(t) \) and SST output \( T(t) \) can be written in a simple way,

\[ R_{F_m,T}(\tau) = \begin{cases} 
\sigma_{F_m}^2 h_m(\tau), & \text{if } \tau > 0 \\
\frac{\sigma_{F_m}^2 h_m(0)}{2}, & \text{if } \tau = 0 \\
0, & \text{if } \tau < 0 
\end{cases} \quad (7) \]

On the chance of the atmospheric stochastic excitation of SST, i.e. \( F_T(t) \neq 0, \ F_m(t) \equiv 0 \) dynamics of \( T(t) \) slightly differs from the standard linear oscillator. Duhamel form of a solution can be represented as [27]

\[ T(t) = \sigma_{F_T} \int_0^\infty h_T(u) X(t - u) du. \quad (8) \]

The weight function,

\[ h_T(u) = \exp(-\lambda u) + \frac{ms(-\exp(-\lambda u)\beta + \exp(-au)\beta \cos(\beta u) + (a-\lambda) \sin(\beta u))}{\beta((a-\lambda)^2 + \beta^2)}. \quad (9) \]

Similar to the equation (7) we can express the cross-correlation of \( F_T(t) \) and \( T(t) \)

\[ R_{F_T,T}(\tau) = \begin{cases} 
\sigma_{F_T}^2 h_T(\tau), & \text{if } \tau > 0 \\
\sigma_{F_T}^2 h_T(0), & \text{if } \tau = 0 \\
0, & \text{if } \tau < 0 
\end{cases} \quad (10) \]
The phase shift between external forcing and SST might be represented as (see Appendix 1)

$$\Delta \varphi(\omega) = \arctan \left( \frac{-\int_{0}^{\infty} h(u) \sin(\omega u) du}{\int_{0}^{\infty} h(u) \cos(\omega u) du} \right)$$  \hspace{1cm} (11)

3.1 Stochastic forcing of SST

On the chance of the random forcing on the SST ($F_T \neq 0$, $F_m \equiv 0$), cross-correlation function $R_{F_T}(\tau)$ reaches a maximum at zero time lag (Figure 3.a). Note that positive extremum at the lag $\tau = 0$ does not agree to the SMIP5 control experiments analysis provided on the Figure 2.a.

The phase shift between $F_T(t)$ and $T(t)$ is determined by the expression

$$\Delta \varphi_{F_T,SST} = \varphi_T - \varphi_{F_T} = \arctan \left( \frac{-\omega(-b + \alpha(\alpha + \lambda) + \omega^2)}{b\alpha + \lambda\omega^2} \right) = \arctan \left( \frac{-\omega(\omega^2 - ms + \alpha^2)}{(ms + \alpha\lambda)\alpha + \lambda\omega^2} \right).$$  \hspace{1cm} (12)

$\Delta \varphi_{F_T,SST} = 0$ on zero frequency. If $ms > \alpha^2$ it is also equal to zero on the frequency described by $\omega^2 = b - \alpha(\alpha + \lambda) = ms - \alpha^2$. The phase shift reduces monotonously if $ms \leq \alpha^2$. In the asymptotic case, $\omega \to \infty$, $\Delta \varphi_{F_T,SST} = -\frac{\pi}{2}$ (Figure 3.b). The high-frequency asymptotic limit is identical to Hasselmann (1976) model [28] linked with the classical Langevin equation

$$\frac{dT}{dt} = -\lambda T + F_T(t),$$  \hspace{1cm} (13)

The phase shift between input and output signals in (13) is determined by $\Delta \varphi = \arctan \left( \frac{-\omega}{\lambda} \right)$.

It is important to note that within the low-frequency range $\omega \in [0, \sqrt{ms - \alpha^2}]$ phase shift is positive so that formally output signal “leads” the input signal. In such a way heat fluxes on the sea surface can be mistakenly interpreted as a passive response to the ocean circulation. This peculiarity can create an illusory impression of the leading ocean circulation role in the forming of the AMO phenomenon. At least in the model (2-3) it definitely does not take place.

What is the origin of the phenomenon? Common sense based interpretation suggests that the cause precedes the effect. The inverse situation looks physically impossible. This, of course, is true. However, within the framework of mutual correlation or spectral analysis, the absence of an adequate model can lead to an inaccurate interpretation of the cause-effect relationships. As a result, the erroneous conclusions about the physical nature of the underlying processes could be made. A close cause-effect problem has been found by Muryashev et al, 2016 [29] in the study of lead-lag relationships between global mean temperature and CO2 content.

To figure out the origin of abnormal behavior of the phase shift in the model (1-2) let us consider an input signal being a differentiable function of time. In this case, the SST evolution can be described by a linear differential equation of the second order,
\[ \frac{d^2T}{dt^2} + 2a \frac{dT}{dt} + bT = \frac{dF_T}{dt} + \alpha F_T. \]  

(14)

In event of the periodical external force, \( F_T = \cos(\alpha t) \), the expression (14) can be integrated and rewritten in the form,

\[ T(t) = \frac{(b\alpha + \lambda \omega^2)\cos(\alpha t) + \omega(\alpha s + \alpha^2 + \omega^2)\sin(\alpha t)}{b^2 + (\alpha + \lambda)^2 \omega^2 - 2b\omega^2 + \omega^4}. \]  

(15)

The phase shift between \( T \) and \( F_T \) is determined by the same equation (12) as in the random forcing variant. In the asymptotic case \( \alpha \to 0 \) right-hand side (14) is proportional to the term \( -\sin(\alpha t) \). The negative sine function is responsible for apparent, but in the matter of fact, an illusory leading behavior of SST.

Figure 3. The analytical cross-correlation (cross-spectral) analysis of the stochastic model (2-3). 

a) The cross-correlation function of the external forcing \( F(t) \) and SST output, \( T(t) \).

b) The phase shift between the stochastic input and SST output of the system.

Red line – the thermal random forcing, \( F_T(t) \neq 0 \). Blue line - the stochastic atmospheric excitation of thermohaline circulation, \( F_m(t) \neq 0 \). Green line corresponds to the Langevin equation. Parameters: \( m = 3, s = 1, \alpha = \lambda = 0.5, \sigma_{F_T}^2 = 1, \sigma_{F_m}^2 = 1 \). The numerical values for \( \alpha \) and \( \lambda \) parameters correspond to the e-folding time of 8 years.

3.2. Stochastic excitation of meridional overturning circulation

Forcing of the meridional overturning circulation (\( F_m(t) \neq 0, F_T(t) = 0 \)) in the model (2-3) led to a completely different behavior of the cross-correlation function \( F_{m,T}(\tau) \) (Figure 3.a). At the zero time lag, the correlation is negative and relatively small. The nearest extremum time of the cross-correlation function \( \tau_{\text{ext}} = \frac{1}{\beta} \arctan \left( \frac{\beta}{a} \right) \approx 0.75 \). That corresponds to \( \sim 3 \) years and resembles well the estimations built on the CMIP5 control experiments (Figure 1.a).
The phase shift between input and output signals can be represented as

$$\Delta \varphi_{F_n,T} = \varphi_T - \varphi_{F_n} = \arctan \left( \frac{-(\alpha + \lambda)\omega}{-(ms + \alpha \lambda - \omega^2)} \right)$$

(16)

At zero frequency $\Delta \varphi_{F_n,T}(0) = -\pi$ and asymptotically $\Delta \varphi_{F_n,T} \to 0$, as $\omega \to \infty$.

Figure 3b demonstrates examples of phase shift as a function of frequency for different kinds of stochastic atmospheric forcing in the model (2-3) and for Langevin equation (13). Comparison of the results presented in Figures 2 and 3 allows suggesting that the direct thermal SST excitation cannot reproduce the estimates built on CMIP5 models. Contrary, stochastic influence on meridional overturning circulation resembles the CMIP5 experiments phase shifts quite well.

Equation (16) allows estimating the frequency (temporal) ranges with the positive and negative correlation between SST and net surface heat fluxes. Correlations are positive if the phase shift,

$$\Delta \varphi_{F_n,T} = \varphi_T - \varphi_{F_n} \in \left[ -\frac{\pi}{2}, 0 \right],$$

and negative if $\Delta \varphi_{F_n,T} = \varphi_T - \varphi_{F_n} \in \left[ -\pi, -\frac{\pi}{2} \right]$. The critical value of frequency $\omega$, which corresponds $\Delta \varphi_{F_n,T} = -\frac{\pi}{2}$, is determined by the relation $\omega_{cr} = \sqrt{ms + \alpha \lambda}$.

Taking into account that typical $ms \approx 3$ and that unit dimensionless time $T$ corresponds to 4 years of physical time [26] we can estimate the characteristic time of correlation sign change, $t_{cr} = 4T_{cr} = \frac{8\pi}{\omega_{cr} \sqrt{ms + \alpha \lambda}} \approx 14\text{ years}$. The last result is very close to the estimations of the correlation sign change time (12-14 years) obtained by Gulev et al, 2013 [20].

4. Discussion and Conclusions

Analyses of the CMIP5 control experiments display that SST and net heat fluxes on the North Atlantic air-sea interface negatively correlate in the low-frequency and positively correlate in the high-frequency ranges. This picture looks quite far from the classical Hasselmann scheme. However, the results can be explained in the framework of a simple stochastic model which takes into account linear feedbacks of SST and Atlantic meridional overturning circulation. The conceptual idea of the stochastically forced linear oscillator for the forming of the North Atlantic MDV has been proposed previously in some studies (e.g. [30, 31]). Moreover, an almost linear relationship has been found between NAO forcing and AMOC response [32, 33]. The stochastically forced damped oscillator concept looks adequate for the explaining some quite intriguing North Atlantic climate phenomenon. Particularly this hypothesis explains the low-frequency out of phase relationship of spatially averaged SST and surface net heat fluxes.

The research of the toy stochastic model reveals that effect of the surface energy fluxes on MDV can be very different and depends on the object of forcing. Impact of the heat fluxes on the SST can lead to the phase shifts that generally are close to the classical Hasselmann scheme except for low-frequency domain. In this domain, SST anomalies formally seem to forerun the atmospheric forcing. It is important, however, this is a quite elusive effect determined by the non-classical form of the oceanic oscillator excitation. We have to note also that the direct forcing of SST cannot lead to a negative correlation between SST and heat fluxes. It seems quite obvious from the phase shift bounded between $-\frac{\pi}{2}$ and $-\frac{\pi}{2}$. The positive correlation of the SST and net energy fluxes in the low-frequency domain contradicts the results of observation analyses and our estimations based on CMIP5 experiments. In the low-frequency domain such correlation has been found significantly negative. Besides external impact on the SST in the stochastic model means the negative correlation of the AMO and AMOC
indexes (Bekryaev, 2016) [27]. This result disagrees to the GCM experiments estimates that point as a rule to the positive regression of the two governing indexes [25, 34].

On the contrary, if the meridional overturning circulation is affected directly by the atmospheric stochastic forcing the picture looks quite different. In this case, strong resemblance has been revealed between the analytical box model solution and the estimations built both on observational data [20] and CMIP5 control experiments. The SST and net heat fluxes are negatively correlated in the low-frequency domain and are in phase in the high-frequency limit. This resemblance allows suggesting the key mechanism of the North Atlantic MDV formation is based on the stochastic surface energy fluxes directly impacting the meridional overturning circulation. Moreover, stochastic model (2-3) under such conditions is characterized by the positive correlation between AMO and AMOC indexes (Bekryaev, 2016 [27]).

The additional support of the hypothesis follows from the cross-correlation functions decay comparison in the model (2-3). The forcing of the meridional overturning circulation leads to the negative values of the cross-correlation function between SST and net surface heat fluxes in the time lag range between zero and approximately eight years. These results are in relative concordance with the CMIP5 model experiments. An analysis of the phase shifts and other cross-correlation characteristics between the North Atlantic SST and net surface heat fluxes localized in the regions of deep water forming could shed some light on the subject.

5. Appendix. The phase shift between stochastic forcing and system output

Cross-correlation function of the input process \( F(t) \) and output process \( T(t) \) can be written in the form of Duhamel integral from the product of weight and correlation functions of the input signal

\[
R_{FT}(\tau) = \int_{0}^{\infty} h(u)R_{F}(\tau-u)du.
\]  

The cross-spectral density of \( F(t) \) and \( T(t) \) is determined by

\[
S_{FT}(\omega) = \int_{-\infty}^{\infty} R_{FT}(\tau)\exp(-i\omega\tau)d\tau.
\]

From (A.1) and (A.2) follows

\[
S_{FT}(\omega) = S_{F}(\omega)\int_{0}^{\infty} h(u)\exp(-i\omega u)du.
\]  

The phase shift \( \Delta \phi \) between input \( F(t) \) and output \( T(t) \) signals is determined [35] as

\[
\Delta \phi(\omega) = \arctan \left( \frac{\text{Im}(S_{FT}(\omega))}{\text{Re}(S_{FT}(\omega))} \right).
\]

Equations (A.3) and (A.4) allow expressing the phase shift using integrals from weight functions,

\[
\Delta \phi(\omega) = \arctan \left( \frac{-\int_{0}^{\infty} h(u)\sin(\omega u)du}{\int_{0}^{\infty} h(u)\cos(\omega u)du} \right).
\]
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