Superradiance in transport through ensemble of double quantum dots

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Abstract. We theoretically study the transport through an ensemble of double quantum dots (DQDs) fabricated in a microwave cavity. The DQDs are coupled to a common field of photon by coupling constant $\lambda$, whereas the photons escape from the cavity with the rate of $\Gamma_{ph}/\hbar$. When $\Gamma_{ph} \gg \lambda$, a photon emission from the DQDs creates an entanglement among electrons in the DQDs, which enhances the rate of the subsequent emission (superradiance). We propose a transport experiment to observe the superradiance using a sequence of voltage pulses.

1. Introduction
A double quantum dot (DQD) fabricated on semiconductors has a high potential for the application to a charge qubit for the quantum information processing. When an electron is trapped in the DQD, its state is given by a superposition between $|L\rangle$ and $|R\rangle$, where $|L\rangle$ and $|R\rangle$ are localized states in the left and right dots, respectively, representing the computational basis states of “0” and “1”. The manipulation of a single charge qubit was performed by the transport experiment using a sequence of voltage pulses [1]: The coherent oscillation between $|L\rangle$ and $|R\rangle$ was observed, which reflects the formation of $(|L\rangle \pm |R\rangle)/\sqrt{2}$. The manipulation of double qubits was also reported [2], in which the generation of entangled states between the DQDs is required.

In the present study, we theoretically study the entangled states in an ensemble of DQDs fabricated in a microwave cavity. The DQDs are coupled to a common field of photon in the cavity, whereas the Coulomb interaction between DQDs is neglected. We pay attention to the entanglement generation by the “superradiance.” Generally, the superradiance takes place when an ensemble of two-level systems is coupled to a common bosonic field. Suppose that the upper level ($|\uparrow\rangle$) in a system transits to the lower level ($|\downarrow\rangle$) by the boson emission. The Hamiltonian is given by

$$H_I = \lambda \sum_{j=1}^{N} \left( S^+_j b + S^-_j b^\dagger \right),$$

where $S^+_j$ and $S^-_j$ are the raising and lowering operators for the $j$th system, whereas $b^\dagger$ and $b$ are the creation and annihilation operators for a boson. If all the systems are initially in the upper level, $|\uparrow\uparrow\ldots\uparrow\rangle \equiv |S_{tot} = N/2, S_{tot,z} = N/2\rangle$, the final state is $(|\downarrow\uparrow\ldots\uparrow\rangle + |\downarrow\downarrow\ldots\uparrow\rangle + \ldots)/\sqrt{N} = |N/2, (N/2 - 1)\rangle$ after a boson is emitted. This entangled state markedly enhances the rate of the subsequent emission of bosons (Dickes’ superradiance) [3, 4]: Electronic state $|N/2, m\rangle$ changes
We consider the superradiance.

The superradiance was reported in an ensemble of quantum dots in a cavity [5].

We propose a transport experiment to observe the superradiance from DQDs located in a microwave cavity. By the pulse experiment, all the left dots are occupied by electrons at $t = 0$ (initialization). After the time evolution without the connection to the leads for the period of $t$ (manipulation), the occupation number in the right dots, $N_R(t)$, is read out by the current to the drain lead (measurement).

We restrict ourselves to a resonant situation that the level spacing in the DQDs, $\Delta \varepsilon$, is tuned to be close to the energy of a photon $\hbar \omega$. The state of electrons and photons evolves within the DQDs.

In the measurement, the occupation number in the right dots, $N_R(t)$, is read out by the current to the drain lead. We show that the current as a function of $t$ directly indicates the superradiance.

2. Model and calculation method

We consider $N$ identical DQDs in a cavity with a single mode of photon. The Hamiltonian is

$$H = H_{\text{DQD}} + \hbar \omega b^\dagger b + H_1,$$

where

$$H_{\text{DQD}} = \sum_{j=1}^{2N} \left[ \varepsilon_L a_{Lj}^\dagger a_{Lj} + \varepsilon_R a_{Rj}^\dagger a_{Rj} + T \left( a_{Lj}^\dagger a_{Rj} + a_{Rj}^\dagger a_{Lj} \right) \right],$$

$$H_1 = \sum_{j=1}^{2N} \left( \langle L | e E_0 \cdot r | L \rangle a_{Lj}^\dagger a_{Lj} + \langle R | e E_0 \cdot r | R \rangle a_{Rj}^\dagger a_{Rj} \right) \left( b + b^\dagger \right).$$

$a_{Lj}^\dagger$ and $a_{Rj}^\dagger$ (a$Lj$ and a$Rj$) are creation (annihilation) operators for an electron in state $|L\rangle$ and $|R\rangle$, respectively, in DQD $j$. $b^\dagger$ and $b$ are those for a photon in the cavity. In a DQD, the energy levels are denoted by $\varepsilon_L$ and $\varepsilon_R$ ($\varepsilon_L > \varepsilon_R$), and tunnel coupling is by $T$. The energy of a photon is $\hbar \omega$. In the electron-photon interaction, $H_1$, $E_0 = (\hbar \omega/2\epsilon)^{1/2}\alpha(r)$, where $\alpha(r)$ is the normalized mode function for the cavity photon, and $\epsilon$ is the permittivity of medium [6]. We assume that $\alpha(r)$ is uniform in space since DQDs are located within the photon wavelength.

We restrict ourselves to a resonant situation that the level spacing in the DQDs, $\Delta \varepsilon \equiv \varepsilon_L - \varepsilon_R$, is close to the photon energy $\hbar \omega$. When $\Delta \varepsilon$ is much larger than the tunnel coupling $T$, the Wannier functions in a DQD are given by $|\tilde{L}\rangle \approx |L\rangle + (T/\Delta \varepsilon)|R\rangle$ and $|\tilde{R}\rangle \approx |R\rangle - (T/\Delta \varepsilon)|L\rangle$.

Using the field operators $\tilde{a}_{Lj}^\dagger$ and $\tilde{a}_{Rj}^\dagger$ for states $|\tilde{L}\rangle$ and $|\tilde{R}\rangle$ in DQD $j$, $H_{\text{DQD}}$ is rewritten as

$$H_{\text{DQD}} = \sum_{j=1}^{N} \left( \tilde{\varepsilon}_L \tilde{a}_{Lj}^\dagger \tilde{a}_{Lj} + \tilde{\varepsilon}_R \tilde{a}_{Rj}^\dagger \tilde{a}_{Rj} \right),$$

with $\tilde{\varepsilon}_L = \varepsilon_L + T^2/\Delta \varepsilon$ and $\tilde{\varepsilon}_R = \varepsilon_R - T^2/\Delta \varepsilon$. The electron-photon interaction is expressed as

$$H_1 = \sum_{j=1}^{N} \left( \langle \tilde{L} | e E_0 \cdot r | \tilde{L} \rangle \tilde{a}_{Lj}^\dagger \tilde{a}_{Lj} + \langle \tilde{R} | e E_0 \cdot r | \tilde{R} \rangle \tilde{a}_{Rj}^\dagger \tilde{a}_{Rj} \right) + (T/\Delta \varepsilon) \left( \langle \tilde{R} | e E_0 \cdot r | \tilde{L} \rangle - \langle \tilde{L} | e E_0 \cdot r | \tilde{R} \rangle \right).$$

Figure 1. Our model for an ensemble of $N$ DQDs located in a microwave cavity. By the pulse experiment, all the left dots are occupied by electrons at $t = 0$ (initialization). After the time evolution without the connection to the leads for the period of $t$ (manipulation), the occupation number in the right dots, $N_R(t)$, is read out by the current to the drain lead (measurement).
Figure 2. The occupation number in the right dots, \( N_R/N \), as a function of time \( t \). The number of DQDs is \( N = 1 \) (thin solid line), \( 2 \) (dashed line), and \( 3 \) (thick solid line). The escape rate of photons from the cavity is \( \Gamma_{ph} = \bar{\hbar} \) with \( \Gamma_{ph} = 20\lambda \), where \( \lambda \) is the electron-photon coupling constant. Inset: \( N_R \) in a single DQD (\( N = 1 \)) with \( \Gamma_{ph} = 0 \).

Figure 3. The occupation probability of states \( |S_{tot} = N/2, S_{tot,z} = M; 0 \rangle \) in \( N \) DQDs (see text), with (a) \( N = 2 \) and (b) \( 3 \). Thin solid line shows the occupation probability of the other states. The escape rate of photons from the cavity, \( \Gamma_{ph}/\hbar \), is \( 20\lambda/\hbar \), where \( \lambda \) is the electron-photon coupling constant.

\[
\rho_j^L (\hat{a}_{Lj}^\dagger \hat{a}_{Rj}^\dagger + \hat{a}_{Rj}^\dagger \hat{a}_{Lj}) (b + b^\dagger). \text{ Assuming that } |(\hat{L} | e E_0 \cdot r | \hat{L})|, |(\hat{R} | e E_0 \cdot r | \hat{R})| \ll \hbar \omega, \text{ we drop the terms that couple states with a large energy mismatch and obtain}
\]
\[
H_1 = \lambda \sum_{j=1}^{N} \left( \hat{a}_{Lj}^\dagger \hat{a}_{Rj} b + \hat{a}_{Rj}^\dagger \hat{a}_{Lj} b^\dagger \right), \quad (5)
\]

where \( \lambda = (T/\Delta \epsilon) (\hat{R} | e E_0 \cdot r | \hat{R}) - (\hat{L} | e E_0 \cdot r | \hat{L}) \). \( H_1 \) in Eq. (5) is similar to that in Eq. (1).

Using the Hamiltonians in Eqs. (4) and (5), we examine the time evolution of our system in a situation of pulse experiment. We set \( \epsilon_L - \epsilon_R = \hbar \omega \). We simply denote \( |\hat{L} \rangle \) and \( |\hat{R} \rangle \) by \( |L \rangle \) and \( |R \rangle \) hereafter. In the initial state, all the left dots are occupied by an electron without photon; \( |LL \cdots L; 0 \rangle \). The time evolution of the density matrix \( \rho \) is calculated using the Liouville equation, \( \hbar \dot{\rho} = -i[H, \rho] + \mathcal{L} \cdot \rho \). The term of \( \mathcal{L} \cdot \rho \) describes the escape of photon from the cavity with the rate of \( \Gamma_{ph}/\hbar \). We evaluate the averaged occupation number in the right dot, \( N_R/N \), as a function of time \( t \).

3. Results

First of all, we examine a single DQD without the escape of photon, \( \Gamma_{ph}/\hbar = 0 \). Since the energy of \( |L; 0 \rangle \) coincides with that of \( |R; 1 \rangle \), the coherent oscillation takes place between them. As a result, \( N_R(t) \) oscillates between 0 and 1 with the period of \( \pi \hbar/\lambda \), as shown in the inset.
in Fig. 2. This reflects the formation of electron-photon complex states, so-called “polaritons”, $(|L;0\rangle \pm |R;1\rangle)/\sqrt{2}$. (For $N \geq 2$ with $\Gamma_{ph}/\hbar = 0$, many-body complex states of electrons and photons are formed. Then $N_R(t)$ shows a complicated coherent oscillation among them [7].)

When the escape rate of photon is finite, the coherent oscillation is damped. If the escape rate $\Gamma_{ph}$ is so large that $\Gamma_{ph} \gg \lambda$, $N_R(t)$ monotonically goes to unity: After the electron tunneling from $|L;0\rangle$ to $|R;1\rangle$, the photon disappears immediately ($|R;1\rangle \rightarrow |R;0\rangle$). Thus an electron in $|R;0\rangle$ hardly tunnels back to the left dot by the absorption of a photon. Here, we focus on the situation of large $\Gamma_{ph}$ and discuss the superradiance.

Figure 2 shows $N_R(t)/N$, when the number of DQDs is $N = 1$, 2, and 3, in the case of $\Gamma_{ph} = 20\lambda$. $N_R(t)/N$ goes to unity for all $N$. The saturation time is smaller for larger $N$, which is ascribable to the superradiance of photon: For $N = 1$, the transition takes place from $|L;0\rangle$ to $|R;0\rangle$ with the rate of $\lambda^2$ except for a numerical factor. For $N = 2$, $|LL;0\rangle \rightarrow (|RL;0\rangle + |LR;0\rangle)/\sqrt{2} \rightarrow |RR;0\rangle$, with the rate of $2\lambda^2$ and $2\lambda^2$, respectively [8]. These states correspond to $|S_{tot} = 1, S_{tot,z} = M\rangle$ with $M = 1, 0$, and $-1$, if states $|L\rangle$ and $|R\rangle$ are assigned to spin-up and -down, respectively. The second transition is enhanced by the superradiance, as discussed in section 1. For $N = 3$, $|LLL;0\rangle \rightarrow (|RLL;0\rangle + |LRL;0\rangle + |LLR;0\rangle)/\sqrt{3} \rightarrow (|RRR;0\rangle + |RLR;0\rangle + |LRR;0\rangle)/\sqrt{3} \rightarrow |RRR;0\rangle$, with the rate of $3\lambda^2$, $4\lambda^2$, and $3\lambda^2$ [8]. These correspond to $|S_{tot} = 3/2, S_{tot,z} = M\rangle$ with $M = 3/2, 1/2, -1/2$, and $-3/2$. The entanglement in the second and third states enlarges the transition rates for the same reason.

To confirm the scenario of superradiance, we plot the occupation probability in the above-mentioned states for $N = 2$ and 3 in Fig. 3. The occupation probability in the other states is negligibly small, as indicated by thin solid lines. This clearly indicates that the entanglement is created in the intermediate states. This entangled state could be used for the manipulation of two charge qubits: For $N = 2$, the state with $M = 0 [(|RL;0\rangle + |LR;0\rangle)/\sqrt{2}]$ is observed with the probability of 40% if the length of voltage pulse $t$ is properly tuned.

4. Conclusions
We have examined the transport through an ensemble of DQDs fabricated in a microwave cavity, where the DQDs are coupled to a common field of photon. When the escape rate of photon is sufficiently large, the superradiance takes place. We have proposed a transport experiment to observe the superradiance using a sequence of voltage pulses.

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[8] If $N$ DQDs are coupled to different photon fields, the electronic states would change as follows: For $N = 2$, $|LL;0\rangle \rightarrow |RL;0\rangle$ or $|LR;0\rangle \rightarrow |RR;0\rangle$, with the rate of $2\lambda^2$ and $\lambda^2$, respectively. For $N = 3$, $|LLL;0\rangle \rightarrow |RLL;0\rangle$, $|LRL;0\rangle$, or $|LLR;0\rangle \rightarrow |RRR;0\rangle$, $|RLR;0\rangle$, or $|LRR;0\rangle \rightarrow |RRR;0\rangle$, with the rate of $3\lambda^2$, $2\lambda^2$, and $\lambda^2$. 