Refined are the known descriptions of particle behavior with the help of Hamilton function in the phase space of coordinates and their multiple derivatives. This entails existing of circumstances when at closer distances gravitational effects can prove considerably more strong than in case of this situation being calculated with the help of Hamilton function in the phase space of coordinates and their first derivatives. For example, this may be the case if the gravitational potential is described as a power series in $1/r$. At short distances the space metrics fluctuations may also be described by a divergent power series; henceforth, these fluctuations at smaller distances also constitute a power series, i.e. they are functions of $1/r$. For such functions, the average of the coordinate equals zero if the frame of reference coincides with the point of origin.

Keywords: generalized Hamilton function, multiple coordinates derivatives, generalized phase space.

1. Introduction.

We can suppose that the kinematic formula

$$s = s_0 + v \Delta t + \frac{a \Delta t^2}{2}$$

is follow by Taylor’s decomposition

$$s = s_0 + v \Delta t + \frac{a \Delta t^2}{2} + \frac{1}{3!} \ddot{a} \Delta t^3 + \frac{1}{3!} \dddot{a} \Delta t^4 + ...$$

if the acceleration equal to constant.

Newtonian Physics is used Lagrangian $L = L(q, p)$. Here we consider Non-Newtonian case when Lagrangian is depend of coordinates and their multiple derivatives $L = L(q, \dot{q}, \ddot{q}, \ldots, \dddot{q}, \ldots)$. The trajectory $r = r(t)$ of the classical particle can be representative by Taylor’s decomposition in $t_0$.
\[ r(t - t_0) = r(t_0) + r(t_0)(t - t_0) + \frac{1}{2} r(t_0)(t - t_0)^2 + \frac{1}{3!} r(t_0)(t - t_0)^3 + \ldots \]

Than

\[ r(t - t_0) = \sum_{n=0}^{N} \frac{1}{n!} r(t_0)(t - t_0)^n. \]

The first three quantitates from Taylor’s decomposition is ordinary used in kinematic. If to consider the other quantitative than it is very difficult to find the all solutions. Because form Newton’s pioneer’s physical works it is limited three quantitates only. On that time in general cases there are specific movement of the particles where impossible to use \( \dddot{r} = \ddot{r} = \ldots = \dot{r} = 0. \)

Lets accept the denote of the generalized derivation

\[
\begin{align*}
\nabla^{(n)} &= \left( \frac{\partial}{\partial r^{(n)}} \right), \\
\nabla_{\beta}^{(n)} &= \left( \frac{\partial}{\partial r_{\beta}^{(n)}} \right).
\end{align*}
\]

The Generalized Momentum \( p \) is

\[ p_{\alpha} = \sum_{n=0}^{N} (-1)^{n-\alpha} \frac{d^{n-\alpha}}{dt^{n-\alpha}} \left( \frac{\partial L}{\partial r^{(n)}} \right) = \sum_{n=0}^{N} (-1)^{n-\alpha} \frac{d^{n-\alpha}}{dt^{n-\alpha}} \nabla_{\alpha} L, \]

where the Lagrangian is the function of coordinates and their multiple derivatives \( L = L(r, \dot{r}, \ddot{r}, \ldots, \nabla^{(n)} r) \)

The Generalized Energy for the general case is the function of multiple derivatives

\[ E = E_1 + E_2 + E_3 + \ldots = (ar^2 + br^2) + (cr^2 + dr^2) + (g \dot{r}^2 + h \ddot{r}^2) + \ldots \]

here \( a, b, c, d, \ldots \) is coefficients and \( E_1, E_2, E_3, \ldots \) is energy of first rang, second rang, third rang and so on.

The first quantitative is named the potential energy. The second - the kinetic energy. The others is possible name as the energy of the second derivation, the energy of the third derivation and so on.

Than the Generalized Lagrangian’s equation is
\[
\sum_{n=0}^{N} (-1)^{n+1} \frac{d^n}{dt^n} \left( \frac{\partial L}{\partial \dot{r}^n} \right) = \sum_{n=0}^{N} \frac{d^n}{dt^n} \nabla L = 0
\]

or
\[
\frac{\partial L}{\partial \dot{r}} - \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{r}} \right) + \frac{d^2}{dt^2} \left( \frac{\partial L}{\partial \dot{r}} \right) - \frac{d^3}{dt^3} \left( \frac{\partial L}{\partial \dot{r}} \right) + \frac{d^4}{dt^4} \left( \frac{\partial L}{\partial \dot{r}} \right) + \ldots = 0
\]

or in our case
\[
\sum_{n=0}^{N} (-1)^{n+1} \frac{d^n}{dt^n} = 0
\]

because
\[
\delta S = \delta \int L(\dot{r}, \ddot{r}, \dddot{r}, \ldots, r^{(n)}) \, dt = \\
= \int \sum_{n=0}^{N} (-1)^{n} \frac{d^n}{dt^n} \left( \frac{\partial L}{\partial \dot{r}^n} \right) \delta r^\beta \, dt = \int \left( (-1)^{n} \frac{d^n}{dt^n} \nabla r \cdot \delta r^\beta \right) \, dt = 0
\]

From the Generalized Lagrangian's equation it is follow the Generalized Newtonian's equation
\[
F + F^{(1)} + F^{(2)} + \ldots = \frac{dp}{dt} + \frac{d^2 p^{(1)}}{dt^2} + \frac{d^3 p^{(2)}}{dt^3} + \ldots
\]

or
\[
\sum_{\alpha=0}^{\infty} F^{(\alpha)} = \sum_{\alpha=0}^{\infty} \frac{dp^{(\alpha+1)}}{dt^{\alpha+1}}
\]

if we denote
\[
F = \frac{dp}{dt}, \text{ where } F = \frac{\partial L}{\partial \dot{r}}, p = \frac{\partial L}{\partial \dot{r}} \\
F^{(1)} = \frac{dp^{(1)}}{dt}, \text{ where } F^{(1)} = \frac{\partial L}{\partial \dot{r}}, p^{(1)} = \frac{\partial L}{\partial \dot{r}} \\
F^{(2)} = \frac{dp^{(2)}}{dt}, \text{ where } F^{(2)} = \frac{\partial L}{\partial \dot{r}^2}, p^{(2)} = \frac{\partial L}{\partial \dot{r}^2} \\
F^{(\alpha)} = \frac{dp^{(\alpha)}}{dt}, \text{ where } F^{(\alpha)} = \frac{\partial L}{\partial \dot{r}^{(\alpha)}}, p^{(\alpha)} = \frac{\partial L}{\partial \dot{r}^{(\alpha)}}
\]

The Generalized action function \( S \) with coefficients \( p, q, l, m, \ldots \) is
\[
S = \sum_{\alpha=0}^{\infty} p^{(\alpha)} r^{(\alpha)} = \sum_{\alpha=0}^{\infty} \sum_{n=0}^{N} (-1)^{n} r^{(n)} \frac{d^n}{dt^n} \left( \frac{\partial L}{\partial \dot{r}^n} \right) = \\
= \sum_{\alpha=0}^{\infty} \sum_{n=0}^{N} \frac{d^n}{dt^n} \nabla r^{(n)} = \nabla L
\]
\[ S = L_1 t + L_2 t^2 + ... = (-pr^2 + qr^2) t + (-lr^2 + mr^2) t^2 + ... \]

here \( L_1, L_2, L_3, ... \) is energy of first rang, second rang, third rang and so on.

Usually, the Hamilton function[1] is expressed as a function in the phase space of coordinates and their first derivatives. However, it can be assumed that there exist such complex types of particle motion when these are to be described by multiple coordinate derivatives. Such motion types can comprise, for instance, fluctuations particle motion. There exists also an Appel[2] definition introducing a so-called acceleration energy, which is a quadratic form in coordinate first derivatives. Therefore, the Hamilton function can be written as a sum of the quadratic form of coordinates, expressing the system potential energy, the quadratic form of first coordinate derivatives, expressing the kinetic energy, and the quadratic form of second coordinate derivatives, which is the Appel acceleration energy. The Generalized Hamilton function shall take on the form

\[
H = \sum_{\alpha=0}^{\infty} p_{\alpha} \frac{\partial L}{\partial r_{\alpha}} = \sum_{\alpha=0}^{\infty} \sum_{n=0}^{N} (-1)^n r^{(n)} \frac{d^n}{dr^n} \left( \frac{\partial L}{\partial r^{(n)}} \right) =
\]

\[
= \sum_{\alpha=0}^{\infty} \sum_{n=0}^{N} (-1)^n r^{(n)} \frac{d^n}{dr^n} \nabla_{\alpha} L
\]

We shall call this function the generalized Hamilton function.

This function shall be studied in the phase space of coordinates and their multiple derivatives.

2. Scalar potential in the phase space of coordinates and their multiple derivatives.

The Generalized Poisson’s equation for the scalar potential \( \phi^{(n)} \) of gravitational field in this case from the sources with density distribution of the source \( \rho \) and factor \( \kappa \) depending on the system of units shall take on the form

\[
\sum_{n=0}^{N} (-1)^n \frac{d^n}{dr^n} \left( \frac{\partial \phi}{\partial r^{(n)}} \right) = \kappa \rho
\]

or, in our case, Generalized Poisson’s equation is
\[ \sum_{n=0}^{N} (-1)^n \frac{d^n}{dt^n} \nabla \varphi = \kappa \rho. \]

Than the solution of Generalized Poisson’s equation is

\[ \varphi = \varphi_0 \exp \frac{k}{r}, \]

named the Generalized Green’s function.

Its solution is

\[ \varphi = \varphi^1 \left( \frac{k}{r-r_0} \right) + \varphi^2 \left( \frac{k^2}{(r-r_0)^2} \right) + \varphi^3 \left( \frac{k^3}{(r-r_0)^3} \right) + \ldots + \varphi^N \left( \frac{k^{N+1}}{(r-r_0)^{N+1}} \right). \]

For the gravitational field the potential is

\[ \varphi = \frac{GM}{r} \exp \frac{k}{r}, \]

where \( \varphi \)- potential, \( G \)- gravitational constant, \( k \)- unknown constant, which may be equal to the gravitational radius \( r_g \), \( M = \int \rho dv \) - mass, \( r = x - x_0 \ll 1 \), \( x \) and \( x_0 \)- coordinates. The constant \( k \) is unknown, but if \( k \) is equal to the Plank constant \( l_p = 10^{-33} \) cm than this potential is always the same as Newtonian potential \( \varphi = GM/r \). If constant \( k \) is equal to the size of nuclear \( k = 10^{-15} \) m than the gravitational force is equal to nuclei forces because at the this distant gravitational forces is change on exponential law and may be stronger than electromagnetic forces. The electromagnetic forces it is possible to express by exponential characters of the interaction too. But constants will be another in this case.

3. Conclusion

From this paper follow, that the phase space of coordinates and there multiple derivative gives the corrected Newton’s formula for gravitational potential \( \varphi \) of two mass \( m \) is

\[ \varphi = Gm \left( a \frac{k}{r} + b \frac{k^2}{r^2} + c \frac{k^3}{r^3} + \ldots \right), \]

where \( a,b,c,\ldots \) - constants.

Here \( k \) is the unknown constant which have the seance of distance. For example, if \( k \sim 10^{-15}m \) and more we have always Newtonian low.

For long distances \( r \gg k \), we have the equation for the gravitational potential \( \varphi = Gm \frac{k}{r^2} \), where \( k = \frac{k}{a} \).
For particle described by the generalized Hamilton function at small distances, i.e. when the series diverges, there shall be much stronger forces acting than it is usually considered in calculations employing the Hamilton function. This theory of short-range interaction explains interaction of bodies at small distances and refines the description of their interaction in case of their increase. It can be supposed that this method can be applied to cases when the force of gravitational attraction of particles described by the Hamilton function, at low distances.

In this approach it is clear that the exponential characters of gravitational and electromagnetic interaction completed the description of dark energy.

References

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