Microscopic description of the ground state properties of recently reported new isotopes

H. M. Devaraja¹, Y. K. Gambhir¹,², A. Bhagwat³, M. Gupta¹, S. Heinz⁴,⁵, G. Münzenberg¹,⁴
¹ Manipal Centre for Natural Sciences, Manipal University, Manipal 576014, Karnataka, India
² Department of Physics, IIT Bombay, Powai, Mumbai 400076, India
³ UM-DAE Centre for Excellence in Basic Sciences, Mumbai 400 098, India
⁴ GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany
⁵ Justus-Liebig-Universität Giessen, II, Physikalisches Institut, 35392 Giessen, Germany

(Dated: March 28, 2018)

Microscopic investigations for the observed properties of the recently reported five unstable new isotopes are carried out. The ground state properties are calculated in the relativistic mean field (RMF) framework and the results reproduce the experiment well as expected. The α - decay lifetimes are calculated in the double folding model using WKB approximation which requires the relevant Q values of α - decay and the α - daughter (Vα,D) potential. The latter is obtained by folding the effective M3Y nucleon nucleon potential with the RMF nucleon density distributions for the daughter nucleus and that of the α particle which is assumed to be of Gaussian shape. The corresponding decay half - lives are calculated in the double folding model using available phenomenological expression are also presented, discussed and compared. It is observed that the use of accurate Q values ( very close to the experimental values) is crucial for the reliable description of the experimental α - decay half-lives in the WKB framework.

PACS numbers: 21.60.-n,23.60.+e,25.85.Ca,27.90.+b

I. INTRODUCTION

Recently, five new neutron deficient isotopes with Z ≥ 92 have been observed in multi-nucleon transfer reactions [1]. The decay energies (Eα) and half-lives of the respective α - decay chains of these new isotopes, 216U, 223Am, 219Np, 229Am and 233Bk have been measured. The corresponding Q (also denoted by Qα) values are trivially extracted using Eα. The nuclei appearing in these decay chains have odd nuclear mass number A, except for the nuclei appearing in the α - decay chain of 216U. The new isotopes were the products of multi-nucleon transfer reactions in collisions of 48Ca + 248Cm carried out at the velocity filter SHIP [2] at GSI. These decay chains are shown in Fig. 1.

In this brief communication we are mainly concerned with the calculation of α - decay half-lives for the nuclei appearing in α - decay chains of these new isotopes. The calculation requires the relevant Q values of α - decay and in addition the α - daughter potential for the WKB calculations. For completeness, we first calculate the ground state properties of these new isotopes along with the nuclei appearing in their α-decay chains using the relativistic mean field (RMF) framework [3,7] which is akin to the energy density functional formalism. The calculated total binding energies, radii and sizes, the deformation parameter and Q values of the α - decay chains are briefly discussed. The α - decay half-lives are then calculated in the WKB framework employing V4A−D potential obtained using double folding model (tpd approximation). The results are discussed in detail and are compared with the corresponding experimental values and those obtained by using the recent phenomenological expression.

The details of the calculation are briefly described in Section 2. The results are discussed in Section 3, while the summary and conclusion are reserved for Section 4.

II. DETAILS OF THE CALCULATION

The RMF [3,7] starts with a Lagrangian describing Dirac spinor nucleons interacting via the meson and the electromagnetic photon fields. In the mean field approximation, along with time reversal invariance, for the static case one ends up with a nonlinear set of coupled equations - the Dirac type equations with potential terms involving the meson and photon fields describing the nucleon dynamics and Klein - Gordon type equations for mesons with sources involving the nucleon current and densities. This set of equations is to be solved self-consistently. The pairing correlations are incorporated either through the conventional Bardeen-Cooper-Schrieffer (BCS) type procedure (constant gap approximation) for the calculation of the occupation probabilities, which gives rise to a set of equations known as the relativistic mean field (RMF) equations or self consistently resulting into a set of well known relativistic Hartree-Bogoliubov (RHB) equations. The pairing field Δ appearing in the RHB equations becomes diagonal in the constant gap approximation and so decouples into a set of diagonal matrices resulting in the BCS type expressions for the occupation probabilities. As a result the RHB equations reduce to the corresponding RMF equations for the occupation probabilities with constant pairing gap Δ.

Here, we employ the well established basis expansion method for the solution of the RMF/RHB equations. We choose spherical (axially deformed) Harmonic Oscillator
These parameters are determined through a fit to the isoscalar - scalar sigma meson is assumed to move. Additional parameters like the non-linear terms in which the meson fields in this basis. The e.m. photon field is expanded the nucleon spinors and separately also (HO) basis for the spherical (deformed) nuclei. Accordingly, we expand the nucleon spinors and separately also the meson fields in this basis. The e.m. photon field is treated in the conventional manner. The details are given in Ref. [4]. Explicit calculations require the parameters appearing in the Lagrangian, namely the nucleon and meson masses and their coupling constants together with the additional parameters like the non-linear terms in which the isoscalar - scalar sigma meson is assumed to move. These parameters are determined through a \( \chi^2 \) fit to the observed ground state properties of a few selected spherical nuclei. This parameter set is then frozen and is used for all the nuclei spread over the entire nuclear chart including super heavy elements (SHE). The parameter set so obtained is not unique and depends upon the detailed ground state properties included in the fit. Several such parameter sets are available in the literature (see, for example, Ref. [5]). Most of the recently reported parameter sets are mainly used for the calculation of the even - even (\( e - e \)) nuclei. The calculated results for the nuclear ground state properties obtained with these different parameter sets are qualitatively similar (e.g. see [8] [9]), some differences do appear at a finer level. Specifically, the use of recently reported parameter sets does improve the calculated binding energies and the \( Q_\alpha \) values significantly. However, the resulting nuclear radii (sizes) and the density distributions remain almost identical.

We employ here the Lagrangian parameter set NL3\(*[10] \) (the improved version of the most widely used in the past, the set NL3 [11]) in our illustrative calculations. For the calculation of partial occupation probabilities arising due to important pairing correlations, we use here the finite range Gogny-D1S interaction [12] [13] which is known to have the right content of pairing, while solving the RHB equations in the spherical H.O. basis. To determine the proton and neutron pairing gaps, we calculate the respective pairing energies in the spherical RMF such that the corresponding pairing energies obtained in the RHB with Gogny-D1S pairing interaction are reproduced. These gaps are then also used in the RMF code for deformed nuclei.

The nuclei under consideration in the present work, have odd mass number \( A \) except for the nuclei appearing in the alpha decay chain of \( ^{219}\text{U} \). It is to be mentioned that for odd/odd-odd deformed nuclei the time reversal invariance is violated. As a result the time odd terms also appear. The RMF calculations incorporating correctly this violation of time invariance are involved [14] and are beyond the scope of the present work which is mainly geared to the calculations of \( \alpha \) - decay half lives of these new isotopes. To overcome this difficulty we use the "tagging" prescription. The level (levels) to be tagged for the last odd (odd-odd) nucleon are guided by the corresponding experimental data or by the results of the deformed RMF calculations for neighbouring nuclei. The "tagging" here means assigning a fixed occupancy to the tagged level(s) through out the iterative RMF calculations. The left over even number of neutrons and even number of protons then preserve the time reversal invariance and the calculation then proceeds in the conventional manner. The details can be found in Refs. [4] [7].
The α- decay half-lives are calculated in the WKB approximation which requires the α - daughter nucleus potential (V_n,D) in addition to its Q - value. The former is calculated using the double folding procedure, by folding the effective nucleon nucleon potential (M3Y interaction with an extra delta function pseudo potential to incorporate the exchange effects) [15–21] with RMF density distributions of the daughter nucleus and that of the α particle. It is known that the calculated α - decay half-lives are very sensitive to its Q - value. Even a few hundred keV difference in Q value can change the calculated half-lives by a few orders of magnitude. Therefore, accurate (close to the experimental) Q - values must be used in the calculation of decay half lives.

### III. RESULTS AND DISCUSSION

The calculated total nuclear binding energies (BE) along with the corresponding point \( r_{p} \) proton \( r_{p} \) radius of the nuclear density distributions, neutron skin \( (r_{n} - r_{p}) \) and the quadrupole deformation parameter \( \beta \) are listed in Table I labeled as NL3*. Recently, extensive Relativistic Continuum Hartree - Bogoliubov (RCHB) calculations using the successful relativistic energy density functional PC-PK1 [22], have been reported and the results are presented in [24]. These spherical calculations properly treat the time odd terms which appear for odd/odd-odd nuclei and use blocking procedure for the relevant tagged state(s) in the calculation. The RCHB equations are solved in the coordinate space (for details see [22, 24]). The relevant results are also presented in the same table labelled ad PC-PK1 for comparison and discussion. Also, the binding energies BE taken from the recent compilation by Wang et al. [24] labeled as AU along with the corresponding values obtained by using the mass formula of Bhagwat [25] denoted by AB are also shown in the same table. It is to be noted that the AU mostly quotes the available experimental values which in fact are almost identical (the maximum deviations are \( \sim 0.5 \text{ MeV} \) to the corresponding AB values. The AB masses of nuclei which are yet to be measured can also be obtained.

The inspection of Table I reveals that the calculated values of BE are in good agreement with the experiment as expected. The PC-PK1 values are considerably in better agreement with the experiment. The average deviation for NL3* (PC-PK1) is \( \sim 0.6\% \) (0.1%).

Both NL3* and PC-PK1 calculated point \( r_{m} \) proton \( r_{m} \) radius of the nuclear density distributions and the neutron skin \( (r_{n} - r_{p}) \) are very close to each other, the differences appear only at second decimal place of fermi. For example the maximum deviation for point \( r_{m} \) proton \( r_{m} \) is \( \sim 0.1 \) fermi while for neutron skin \( (r_{n} - r_{p}) \)
the maximum deviation is \( \sim 0.3 \) fermi.

The calculated NL3* quadrupole deformation parameter \( \beta \) values are very similar to the corresponding values of Möller and Nix \[? \] for most of the nuclei, except for the nucleus \(^{229}\text{Am}\). Further, most of the calculated NL3* \( \beta \) values are small except for \(^{233}\text{Bk}\) where its value is 0.218, indicating that most of these nuclei are close to spherical.

The experimental charge radii are available \[20\] for the following four nuclei in this region and are listed in Table II. Clearly, both NL3* and PC-PK1 calculation (RMF) reproduces the corresponding experimental values very well. The deviations appear only in third decimal place of fermi.

### TABLE II. The calculated (RMF) charge radii along with the corresponding experimental values (Expt.) taken from [26].

| Nucleus | \( r_c \) (fm) | NL3* | PC-PK1 | Expt. |
|---------|----------------|------|--------|-------|
| \(^{91}\text{Fr}\)\(^{126}\) | 5.595 | 5.601 | 5.598 |
| \(^{95}\text{Am}\)\(^{134}\) | 5.577 | 5.580 | 5.573 |
| \(^{93}\text{Np}\)\(^{132}\) | 5.573 | 5.576 | 5.572 |
| \(^{91}\text{Pa}\)\(^{130}\) | 5.590 | 5.592 | 5.585 |

### TABLE III. The \( Q \) - values for \( \alpha \) - decay chains (for details see text).

| Nucleus | NL3* | PC-PK1 | AB | Expt. |
|---------|------|--------|----|-------|
| \(^{223}\text{Bk}\)\(^{136}\) | 8.56 | 11.07 | 8.44 | 7.90 |
| \(^{229}\text{Am}\)\(^{134}\) | 9.27 | 11.29 | 8.51 | 8.14 |
| \(^{225}\text{Np}\)\(^{132}\) | 9.82 | 11.21 | 8.74 | 8.79* |
| \(^{223}\text{Pa}\)\(^{130}\) | 9.31 | 10.34 | 9.10 | 9.25* |
| \(^{217}\text{Ac}\)\(^{128}\) | 9.73 | 11.06 | 9.53 | 9.83* |
| \(^{212}\text{Ra}\)\(^{126}\) | 4.31 | 4.56 | 6.75 | 6.88 |
| \(^{223}\text{Am}\)\(^{128}\) | 11.58 | 13.62 | 10.83 | |
| \(^{219}\text{Np}\)\(^{126}\) | 8.35 | 8.12 | 9.10 | 8.98* |
| \(^{215}\text{Pa}\)\(^{124}\) | 6.34 | 5.67 | 8.35 | 8.23 |
| \(^{211}\text{Ac}\)\(^{122}\) | 5.47 | 5.13 | 7.61 | 7.62* |
| \(^{207}\text{Fr}\)\(^{120}\) | 5.21 | 5.16 | 6.90 | 6.89* |
| \(^{215}\text{U}\)\(^{124}\) | 6.31 | 6.10 | 8.79 | 8.49 |
| \(^{212}\text{Th}\)\(^{122}\) | 6.21 | 5.40 | 8.07 | 7.98 |
| \(^{208}\text{Fr}\)\(^{121}\) | 4.78 | 4.70 | 6.82 | 6.74 |

The calculated (NL3*) \( Q \) values of \( \alpha \) -decay of the new isotopes and their daughter nuclei are listed in Table III. The corresponding PK-PC1 values taken from

[FIG. 2. Calculated RMF Nucleon density distribution using N3* set of parameters.]

and with that obtained by using the mass formula of Bhagwat [25] labeled as AB along with the corresponding available experimental values (Expt.) \[1\] are listed in the same table for comparison. The experimental \( Q \) value of the new isotope \(^{223}\text{Am}\) is not available. The corresponding values of the rest of the nuclei taken from the live chart \[28\] marked by superscript (*) are also shown in the same table for completeness. It is clear from the table that all the three (NL3*, PC-PK1 and AB) \( Q \) values are similar and reasonably agree with the experiment. Quantitatively, the deviations rise even up to 3 MeV in some cases. This type of agreement is considered to be quite acceptable in view of the fact that the \( Q \) value is the difference between the binding energies (BE) of the parent nucleus and the daughter nucleus plus that of \( \alpha \) particle. The BE themselves are very large. A small error in one may easily upset the quality of the obtained agreement. At the finer level it is observed that AB values are closer to the Expt. The NL3* \( Q \) values though are similar to those of AB and the Expt. values but differ substantially (>1 MeV) in several cases (e.g. for \(^{229}\text{Am}, 225\text{Np}, 213\text{Fr}, 215\text{Pa}, 211\text{Ac}, 207\text{Fr}, 216\text{U}, 212\text{Th}\), and \(^{208}\text{Fr}\)). Similar deviations, for the PC-PK1 case are relatively larger as compared to that of NL3*. As we shall see, these differences play a very crucial role in the calculation of the corresponding \( \alpha \) - decay half-lives. We end this discussion by restating that the Relativistic Mean Field (Relativistic energy density functional) theories provide accurate and reliable description of nuclear ground state properties for nuclei spread over the entire periodic table.

As stated before, the \( \alpha \) - decay half-lives are calculated in the WKB approximation. The required \( \alpha \) - daughter nucleus potential (\( V_{\alpha D} \)) is calculated by folding the effective nucleon nucleon potential (M3Y interaction + pseudo potential to incorporate the exchange effects)
with RMF density distributions of the daughter nucleus and that of the $\alpha$-particle.

It is known (e.g. see [4]) that the calculated RMF nucleon density distributions give good account of the experiment. The calculated RMF nucleon (both proton and neutron) density distributions obtained by using NL3* Lagrangian parameters are shown in Fig. 2 for $^{229}$Am. Very small oscillations appearing in some of the calculated density distributions at short distances (see Fig. 2) smear out during the folding process, resulting into a smooth $\alpha$ - daughter nucleus potential ($V_{\alpha D}$). As an illustration the calculated $\alpha$ - $^{229}$Am - potential is shown in figure 3. It is expected that such calculated $V_{\alpha D}$ potentials are reliable and can be used with confidence in the WKB framework for the calculation of $\alpha$ - decay half lives.

Several analytical (phenomenological expressions involving number of adjustable parameters) for the calculation of $\alpha$ - decay half lives, mostly based on the original viola-Seaborg formula ([29]) are also available in the literature. For completeness, we have used one of the recent expression reported by Dasgupta - Schubert and Reyes (31) is given below for $\alpha$ - decay half lives $T_{1/2}$ in Seconds:

$$\log_{10}(T_{1/2}) = a + bA^{1/6}Z^{1/2} + cZQ^{-1/2}$$ (1)

with parameters:

\begin{align*}
a &= -25.68, b = -1.1423, c = 1.5920 \text{ for odd- } Z, \text{ odd- } N \\
a &= -29.48, b = -1.1130, c = 1.6970 \text{ for odd- } Z, \text{ odd- } N
\end{align*}

We have calculated the relevant $\alpha$ - decay half lives using this expression (Eq.1), for comparison and discussion.

It is known that the calculated WKB $\alpha$ - decay half lives are very, sensitive to its respective $Q$ - values. Even a few hundred keV difference in $Q$ - value can change cal-

\begin{table}[h]
\centering
\caption{The estimates of the $\alpha$ - decay half-lives (in seconds) along with the corresponding experimental values. For details see text.}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
\textbf{Nucleus} & \textbf{NL3*} & \textbf{PC-PK1} & \textbf{AB} & \textbf{Pheno} & \textbf{Expt} & \textbf{Pheno} & \textbf{Expt} \\
\hline
$^{97}$Bk & $1.15 \times 10^{-1}$ & 1.08 & 0.28 & 19.0 & 22.58 & 21$^{18.17}$
\hline
$^{209}$Am & $1.93 \times 10^{-4}$ & 1.14 & 0.03 & 0.49 & 0.62 & 3.7$^{1.04}$
\hline
$^{225}$Ra & $1.58 \times 10^{-6}$ & 68.10 & 1.20$\times 10^{-3}$ & 8.63$\times 10^{-4}$ & 1.23$\times 10^{-3}$ & 3.6$^{2.2.2}$
\hline
$^{221}$Au & $6.53 \times 10^{-6}$ & 4.15$\times 10^{-9}$ & 2.43$\times 10^{-5}$ & 9.68$\times 10^{-6}$ & 1.45$\times 10^{-5}$ & 5.9$^{1.7.7}$
\hline
$^{217}$Ac & $5.86 \times 10^{-7}$ & 4.83$\times 10^{-8}$ & 4.20$\times 10^{-7}$ & 8.09$\times 10^{-8}$ & 1.23$\times 10^{-7}$ & 6.9$^{0.4}$
\hline
$^{213}$Fr & $2.81 \times 10^{-15}$ & 14.70 & 38.20 & 11.20 & 12.20 & 63$^{1.78}$
\hline
$^{223}$Am & $1.58 \times 10^{-9}$ & 4.08$\times 10^{-9}$ & 5.32$\times 10^{-8}$ & 3.39$\times 10^{-8}$ & 7.31$\times 10^{-8}$ & 5.2$^{12.0}$
\hline
$^{219}$Np & $2.82 \times 10^{-2}$ & 2.32$\times 10^{-3}$ & 1.79$\times 10^{-4}$ & 1.16$\times 10^{-4}$ & 1.47$\times 10^{-4}$ &
\hline
$^{215}$Pa & $1.98 \times 10^{-5}$ & 0.14 & 5.32$\times 10^{-3}$ & 1.21$\times 10^{-2}$ & 1.31$\times 10^{-2}$ & 6.0$^{13.8}$
\hline
$^{211}$Ac & $6.90 \times 10^{-8}$ & 0.22 & 0.23 & 0.20 & 0.23 & 7.8$^{0.64}$
\hline
$^{207}$Fr & $2.04 \times 10^{-9}$ & 6.43$\times 10^{-2}$ & 13.80 & 14.20 & 14.46 & 1.7$^{3.9}$
\hline
$^{215}$Cm & $9.44 \times 10^{5}$ & 3.85$\times 10^{-2}$ & 6.59$\times 10^{-4}$ & 4.85$\times 10^{-3}$ & 2.68$\times 10^{-3}$ & 3.8$^{8.8}$
\hline
$^{221}$Th & $2.30 \times 10^{5}$ & 0.04 & 0.02 & 0.03 & 0.23$\times 10^{-2}$ & 0.17$^{0.43}$
\hline
$^{208}$Fr & $1.17 \times 10^{12}$ & 1.06$\times 10^{2}$ & 26.10 & 55.00 & 85.09 & 51$^{117}$
\hline
\end{tabular}
\end{table}
culated half-lives by a few orders of magnitude (see Ref. [19]). The results are displayed in Table IV. The symbols NL3*, PC-PK1, AB and Expt. correspond to the calculated WKB \( \alpha \)-decay half-lives using their respective \( Q \)-values (listed in Table III).

The measured values reported in [1] and listed in Table IV under the header Expt.[ref-1], have very large errors (mostly statistical). It is to be noted that two values of the measured \( \alpha \)-decay half-lives are quoted in Table I of Ref. [1] for \(^{229}\)Am,\(^{225}\)Np and \(^{213}\)Fr nuclei. Therefore, an average of these two measured values are listed as experimental (Expt.[ref-1]) values in Table IV for each of these nuclei. The values of the half-lives for the known isotopes which have not been measured in [1] taken from the live chart. \(^{233}\) values are also shown with the superscript *. The \( \alpha \)-decay half-life for the nucleus \(^{219}\)Np has not been reported in [1] and also is not available in the live chart. The corresponding \( \alpha \)-decay half-lives, calculated using the phenomenological expression (Eq.1) with the experimental (Expt. listed in Table III) \( Q \)-values are also shown in Table IV under the header Pheno, for comparison and discussion.

It is clear from the Table IV that the calculated half-lives (RMF, MN, AB and Expt.) using their respective \( Q \)-values listed in Table III, differ widely among themselves indicating high sensitivity on the \( Q \)-values used, as expected. It is interesting to note that all the estimated decay half lives (NL3*, PC-PK1, AB and also Expt. and Pheno (where AB-Q values are used)) obtained by using the corresponding \( Q \) values listed in Table III for \(^{223}\)Am are very small \((\sim 10^{-8} \text{ seconds})\) while the corresponding experimental value (Expt.[ref-1]) is comparatively very large \((\sim 10^{-3} \text{ seconds})\). This case was marked “pile up” by the authors of [1]. This then indicates the necessity for repeating their procedure for extracting this value of half life for \(^{223}\)Am.

The calculated (NL3* and PC-PK1) half lives differ considerably, from the corresponding experimental values in most of the cases, either grossly underestimating or overestimating the experimental values. This again is a reflection of the high sensitivity on the \( Q \)-values used in the calculation of half lives. Therefore the calculated values of the half lives for both NL3* and PC-PK1 are not reliable, mainly due to the fact that their respective \( Q \) values differ from the experiment and therefore are not accurate enough to be used in the WKB calculation of half lives.

The results under the header AB where the used \( Q \)-values which well represent the experimental \( Q \)-values, agree well (within a factor of \( \sim 8 \)) with the corresponding experimental values except for \(^{233}\)Bk and \(^{229}\)Am where it considerably underestimates \((\sim 5 \text{ by a factor of } 100)\) the experimental value. On the other hand the corresponding Expt. and Pheno values obtained by using the experimental \( Q \) values almost reproduces the experiment, indicating that the said deficiency can be rectified by using accurate (experimental or very close to the experimental) \( Q \) values. For the rest of the nuclei, the values of half lives presented in Table IV for all the three AB, Pheno and Expt., are very similar to each other and reasonably agree with the corresponding experimental (expt.[ref-1]) values. The calculated AB and Pheno half lives are very close (with in a factor of 4) to each other except for \(^{233}\)Bk and \(^{229}\)Am and also reasonably agree with the experiment. The Expt. values improve this agreement with the experiment further as is evident from the values listed in Table IV.

From the above discussion it may be concluded that the microscopic estimates AB, and Expt. and so also Pheno of the \( \alpha \)-decay half-lives using the corresponding experimental (where available) ground state \( Q \) values represent a fair estimate of the corresponding experimental (measured and yet to be measured) values.

The experimental (Expt.[ref-1]) values of half lives listed in Table IV range from seconds to \( \sim 10^{-6} \text{ seconds} \), besides having large errors, in view of this observation the agreement with in a factor of 10 with the experiment is acceptable in such studies in this nuclear region. Therefore the agreement obtained here between the calculated and the experimental half lives is remarkable indeed.

It is strongly advocated that the use of the experimental ground state \( Q \)-values is essential in the microscopic calculation of the half-lives to obtain good description of the experiment.

Most of the nuclei considered here are unstable and decay through various decay modes like \( \alpha \)-emission, fission, electron capture etc.. Usually, one or two decay modes dominate. A complete microscopic description of complex fission process is still lacking, though some semi microscopic model descriptions do exist (e.g. see [32-35]). The detailed calculations are quite involved and are beyond the scope of the present work.

IV. SUMMARY AND CONCLUSION

The ground state properties of the recently reported new isotopes \(^{216}\)U, \(^{223}\)Am, \(^{219}\)Np, \(^{229}\)Am and \(^{233}\)Bk along with the elements appearing in their respective \( \alpha \)-decay chains are calculated in the frame work of the relativistic mean field (RMF) theory. The calculated binding energy per nucleon (BE/A), the neutron (proton) point radii \( r_n \) \((r_p)\) of the RMF density distributions, the neutron skin \((r_n - r_p)\), the mass and the charge radii, the quadrupole deformation parameter \( \beta \) and the corresponding \( Q \)-values of \( \alpha \)-decay, compare well with the corresponding experimental values (where available). The \( \alpha \)-decay half-lives are calculated using the WKB approximation which requires the corresponding \( Q \)-values and the parent - daughter potential. The latter is obtained in the double folding procedure \( (tpq - \text{approximation}) \). Though the calculated RMF and MN \( Q \)-values are close to the corresponding experimental values, the small differences between them and the experimental values, play a crucial role and may even change the calculated half-lives by
a few orders of magnitude. Therefore, the present RMF and MN Q values are not accurate enough to be used in the WKB calculation of the α - decay half lives.

It is strongly emphasized that the use of the accurate (experimental or very close to the experimental) Q values is crucial in the microscopic as well as phenomenological, calculations of the α - decay half-lives.

ACKNOWLEDGMENTS

AB acknowledges partial financial support from the Department of Science and Technology, Govt. of India and the Swedish National Research Council (VR) (through grant No. DST/INT/SWD/VR/P-04/2014). H.M. Devaraja, acknowledges financial support from the LOEWE program.

[1] H. M. Devaraja et al., Phys. Lett. B 748, 199 (2015).
[2] G. Münzenberg et al., Nucl. Instrum. Methods 161, 65 (1979).
[3] P. G. Reinhardt, Rep. Prog. Phys. 52, 439 (1989) and references cited therein.
[4] Y. K. Gambhir, P. Ring and A. Thimet, Ann. Phys. (NY) 198, 132 (1990).
[5] P. Ring, Prog. Part. Nucl. Phys. 37, 193 (1996) and references cited therein.
[6] D. Vretenar, A. V. Afanasjev, G. A. Lalazissis and P. Ring, Phys. Rep. 409, 101 (2005) and references cited therein.
[7] A. Bhagwat and Y. K. Gambhir, Int. J. Mod. Phys. E 20, 1663 (2011).
[8] S. E. Agbemava, A. V. Afanasjev, D. Ray and P. Ring, Phys. Rev. C 89, 054320 (2014).
[9] W. Zhang et al., Nucl. Phys. A 753, 106 (2005).
[10] G. A. Lalazissis, S. Karatzikos, R. Fossion, D. Pena Arteaga, A. V. Afanasjev and P. Ring, Phys. Lett. B671, 36 (2009).
[11] G. A. Lalazissis, J. König and P. Ring, Phys. Rev. C 55, 540 (1997).
[12] J. F. Berger, M. Girod and D. Gogny, Nucl. Phys. A 428, 23 (1984).
[13] T. Gonzalez-Llarena, J. L. Egido, G. A. Lalazissis and P. Ring, Phys. Lett. B 379, 13 (1996).
[14] A. V. Afanasjev and H. Abusara, Phys. Rev. C 81, 014309 (2010).
[15] G. R. Satchler and W. G. Love, Phys. Reports 55, 183 (1979).
[16] A. K. Choudhuri, Nucl. Phys. A 449, 243 (1986).
[17] D. T. Khoa, W. von Oertzen and H. G. Bohlen, Phys. Rev. C 49 1652, (1994).
[18] D. N. Basu, Phys. Rev. C 66, 027601 (2002) and references cited therein.
[19] Y. K. Gambhir, A. Bhagwat and M. Gupta, Ann. Phys. (N.Y.) 320, 429 (2005).
[20] Y. K. Gambhir, A. Bhagwat, M. Gupta and Arun K. Jain, Phys. Rev. C 68, 044316 (2003).
[21] Y. K. Gambhir, A. Bhagwat and M. Gupta, Phys. Rev. C 71, 037301 (2005).
[22] P. W. Zhao, Z. P. Li, M. Yao and J. Meng, Phys. Rev. C 82, 054319 (2010).
[23] X. W. Xia, Y. Lim, P. W. Zhao, H. J. Liang, X. Y. Qu, Y. Chen, H. Lin, L. F. Zhang, S. Q. Zhang, Y. Kim and J. Meng, [arXiv:1704.08906v2[nucl-th], 14 Sept. 2017. Chin. Phys. Lett. 29, 042101 (2012).
[24] M. Wang et al., Chinese Phys. C 36, 1287 (2012), and 36, 1603 (2012) and 41, 03003 (2017).
[25] A. Bhagwat, Phys. Rev. C 90, 064306 (2014).
[26] I. Angeli and K. P. Marinova, At. Data Nucl. Data Tables 99 (2013) 69.
[27] P. Möller, J. R. Nix and K. -L. Kratz, At. Data Nucl. Data Tables 66, 131 (1997).
[28] https://www-nds.iaea.org/relnsd/vchart/html/ VChartHTML.html.
[29] V. E. Viola and G. T. Seaborg, Journal of inorganic and nuclear chemistry 28, 697 1966.
[30] N. Dasgupta - Schubert, M.A. Reyes, At. Data Nucl. Data Tables 93, 90 (2007).
[31] N. Dasgupta - Schubert, M.A. Reyes, At. Data Nucl. Data Tables 93, 90 (2007).
[32] S. G. Nilsson et al., Nucl. Phys. A 131, 1 (1969).
[33] M. Brack et al., Rev. Mod. Phys. 44, 320 (1972).
[34] G. Ripka. Nuclear Self consistent Fields (1975) (North Holland, New York).
[35] A. Sobiczewski et al., Nucl. Phys. A 131, 67 (1969).
[36] R. Smolanczuk, J. Skalski, A. Sobiczewski, Phys. Rev. C 52, 1871 (1995).
[37] I. Mutain, Z. Patyk and A. Sobiczewski, Acta. Phys. Pol. B34, 2141 (2003).
[38] A. Zdeb, M. wardaand K. Pomorski Phys. Scr. 89, 054015 (2014).