Semi-Markov modeling for assessing reliability of road construction machines in the process of their operation

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Abstract. The article provides a theoretical description of the processes of changing the technical state of road-building machines to assess their reliability at the stage of operation based on semi-Markov models. Reliability is the most important property of machines and characterizes their ability to perform their functions during normal operation. Reliability covers all stages of the life cycle of equipment-its design, manufacture, and operation. The operation stage is the longest in time and the costliest to ensure reliability. This explains the relevance of developing a mathematical model of operational reliability of road-building machines. During the operation of technical objects, their technical condition changes under the influence of objective and subjective factors. As a result, quantitative indicators that characterize the technical condition of machines vary widely and are random in nature. Road-building machines may be in one of the following technical states during operation: perfect, imperfect, upstate, down state. At the same time, a perfect machine is always upstate, and an imperfect one can be either upstate or down state. The transition from one technical state to another is random and can be considered instantaneous with a sufficient degree of accuracy. The probability of any technical state in the future depends only on the state of the process in the present and does not depend on when and how the process was in this state. This makes it possible to use the theory of semi-Markov processes when formalizing the process and developing a mathematical model. Based on semi-Markov modeling, the article shows the sequence of constructing a graph of states and transitions from one state to another for a road construction machine during operation; matrices of transitions and probabilities of transitions from one state to another; systems of differential equations for determining the limiting probabilities of finding a machine in a particular technical state during operation. The developed method of semi-Markov modeling allows estimating the operational reliability of equipment with the reliability allowed by probability theory.

1. Introduction

The complex property of reliability [1] characterizes the ability of machines to maintain their functional properties during normal operation. It is laid down in the design of equipment, ensured during its production and supported during operation. Thus, reliability covers all stages of the machine life cycle.

The longest stage of the life cycle is the stage of operation, including the use of machines for their intended purpose (main purpose), transportation, storage, installation and adjustment (if necessary), as well as maintenance and repair (M&R) [2]. The operating stage is not only the longest in time, but also the costliest to ensure dependability. So, over the entire period of operation, the costs of M&R are several times higher than the cost of a new machine: for cars and construction machines - 5-6 times, for aircraft - 4-5 times, for machine tools - 3 times.
Therefore, the development of a mathematical model for assessing the reliability of road construction machines at the operation stage is an urgent task. The solution to this problem allows us to formalize the processes of changing the technical state of machines, to develop organizational and technical measures to maintain their operability and to ensure the development of proposals for improving dependability in the design and manufacture of equipment.

2. Materials and methods

During operation, the technical condition of machines changes under the influence of objective and subjective factors [3]. Objective factors include: environmental impact (temperature, humidity, dustiness), natural wear and tear, corrosion and aging of structural materials, their durability etc. Subjective factors are the qualifications of drivers, the choice of operating modes etc. As a result of the impact of a combination of factors, the quantitative parameters characterizing the technical condition of a machine change over a wide range, and these changes are random.

Road construction machines belong to the class of recoverable technical objects, and during operation they can be in one of the following technical states [1] – perfect \( S_p \), imperfect \( S_{ip} \), up state \( S_{us} \), down state \( S_{ds} \).

The perfect state is a state of an object in which it meets all the requirements established in the documentation for it. The imperfect state means that the object does not meet at least one of the requirements established in the regulatory and technical documentation. The up state is the kind of state, in which the machine is capable of performing its functions. Being down state, the machine is not capable of performing at least one of the required functions.

Thus, the set of technical conditions \( S \) can be divided into two subsets \( S_p \) and \( S_{ip} \), \( S_{us} \) and \( S_{ds} \). The following relations exist between these subsets: \( S_p \subset S_{us} \), \( S_{ds} \subset S_{ip} \), i.e. the subsets \( S_p \) and \( S_{ds} \) are included into the subsets \( S_{us} \) and \( S_{ip} \).

The subsets \( S_{us} \) and \( S_p \) intersect (the machine may be up state, but imperfect, for example, with a slight oil leak from the unit): \( S_{us} \cap S_{ip} \), and they have a common part. The subsets \( S_{us} \) and \( S_{ds} \), \( S_p \) and \( S_{ip} \) are incompatible-orthogonal. That is, a perfect machine is always up state and an imperfect machine is always down state. However, an imperfect machine can be both up state and down state.

In practical operation, it is enough to talk about the up state of the object \( S_{us} \). It is it that determines the ability of the machine to perform the required functions. This allows, with a sufficient degree of error, not to take into account the technical condition \( S_{ds} \) separately in the future, by including it in the subset of states \( S_{us} \). In this case, a diagram of possible technical states of a road construction machine during operation and transitions from one state to another can be represented in the following form (Fig. 1).

![Figure 1. Scheme of possible technical conditions of a road construction machine](image-url)

Markov processes are widely used in reliability theory. A random process is called a Markov process if the probability of any state in the future depends only on its state in the present and does not depend on when and how the process ended up in this state [4]. If in this case the time spent in any of the states is described by an arbitrary (except for exponential) distribution function, then such a process is defined as a semi-Markov - SM-process. SM-processes are the development and generalization of Markov processes [5].
3. Semi-Markov modeling of the operational reliability of a road construction machine

An SM-process is given both by the matrix function \( Q(t) = \{Q_{ij}(t)\}_{m \times m} \), the elements of which are the probabilities of transition from the state \( i \) to the state \( j \) during the time \( t \), and by the initial distribution vector \( P(0) \).

The graph of states and transitions from one technical state to another for the operation of a road construction machine, taking into account Fig. 1, will have the following form (Fig.2).

\[
\begin{align*}
P_1 & \quad Q_{12} \quad \lambda_{12} \quad P_2 \quad Q_{23} \quad \lambda_{23} \quad P_3 \\
& \quad \lambda_{32} \quad Q_{31} \quad \lambda_{31}
\end{align*}
\]

**Figure 2.** State and transition graph:

- \( P_1 \) – probability of perfect state;
- \( P_2 \) – probability of up state;
- \( P_3 \) – probability of down state;
- \( Q_{12} \) (\( \lambda_{12} \)) – probability (intensity) of damage;
- \( Q_{23} \) (\( \lambda_{23} \)) - probability (intensity) of failure;
- \( Q_{32} \) (\( \lambda_{32} \)) - probability (intensity) of recovery;
- \( Q_{31} \) (\( \lambda_{31} \)) - probability (intensity) of repair.

The evolution of the SM-process with respect to the transition times is described by the following equation of state

\[
P_{i+t} = Q_{ij}(t) \times P_j \quad (1)
\]

The moment in time corresponding to the beginning of a certain state is denoted as \( t \), to the end - \( t_{i+1} \). Obviously, the time interval \( t \) is a characteristic of the state, since it determines its duration. The random variable \( t_i \) can be characterized by the mathematical expectation of the time spent in the state \( M[t_i] \).

The process of changing the technical state of road construction machines is characterized not only by the set of states – \( S \) - and by the set of random variables of the time of stay in the states - \( t \). These states arise in the process under consideration continuously and sequentially, i.e. at a certain moment of time \( t_1, t_2, t_3 \) are "stitched together" and form a process. At these moments, the machine makes transitions from one technical state to another. Assuming that in SM-processes such transitions are carried out instantaneously, one can consider as a quantitative characteristic of transitions not the time of transition to an adjacent state, but the probabilities or intensities (relative frequencies) of transitions \( \lambda_{ij} \) from the \( i \)-th to the \( j \)-th state.

\[
Q_{ij} = \frac{n_{ij}}{\sum_{i=1}^{N} n_{ij}}, \quad \lambda_{ij} = \frac{n_{ij}}{\sum_{i=1}^{N} n_{ij}}, \quad (2)
\]

where \( n_{ij} \) is the statistical number of direct transitions from the \( i \)-th to the \( j \)-th state at a sufficiently large fixed time interval for observing the object – \( T_0 \).

\( N \) is the total number of states observed in the interval \( T_0 \).
The matrices of transitions and transition probabilities, taking into account Fig. 2, are presented in Tables 1, 2.

**Table 1. Transition matrix**

| Technical state S | Perfect S<sub>p</sub> | Up state S<sub>us</sub> | Down state S<sub>ds</sub> | \( \sum_{i=1}^{N} n_{ij} \) |
|-------------------|----------------------|------------------------|--------------------------|---------------------|
| Perfect S<sub>p</sub> | 0                    | \( n_{ij} \)           | \( n_{ij} \)             | \( \sum n_{ij} \)   |
| Up state S<sub>us</sub> | 0                    | \( n_{ij} \)           | 0                        | \( \sum n_{ij} \)   |
| Down state S<sub>ds</sub> | \( n_{ij} \)         | \( n_{ij} \)           | 0                        | \( \sum n_{ij} \)   |

**Table 2. Transition probability matrix** \( Q_{ij}(t) \)

| Technical state S | Perfect S<sub>p</sub> | Up state S<sub>us</sub> | Down state S<sub>ds</sub> |
|-------------------|----------------------|------------------------|--------------------------|
| Perfect S<sub>p</sub> | 0                    | \( Q_{ij} \)          | \( Q_{ij} \)             |
| Up state S<sub>us</sub> | 0                    | \( Q_{ij} \)          | 0                        |
| Down state S<sub>ds</sub> | \( Q_{ij} \)        | \( Q_{ij} \)          | 0                        |

The matrix \( Q_{ij}(t)_{3x3} \) (Table 2) determines the structure of the local process of transitions of a road construction machine from one technical state to another. In order for this matrix to determine the structure of all transitions to the technical state, it is necessary to show the ergodicity of the process in the observation time \( T_0 \) section.

An ergodic random process [6] is a special case of a stochastic stationary process. A stationary process is considered ergodic if the normalized correlation function characterizing in this case the transition time tends to 0 (instantaneous transition).

The absolute frequencies or probabilities that the machine will get into one or another technical condition during operation during the observation time \( T_0 \) are determined by the formula with normalization to 1.

\[ P_{ij} = \frac{\sum_{i=1}^{N} n_{ij}}{\sum_{i=1}^{N} \sum_{j=0}^{N} n_{ij}}. \quad (3) \]

The resulting one row table \( P=(P_1; P_2; P_3) \) is called an embedded row vector of state frequencies and is the vector \( P(0) \) of the initial probability distribution.

The set of the matrix \( Q_{ij}(t) \) and the vector \( P(0) \) defines an embedded chain of a semi-Markov process. The set of sets \( \{S\} \), \( \{I\} \), the matrix \( Q_{ij}(t) \) and the vector \( P(0) \) determines the whole process of changing the technical conditions of a road construction machine during operation. This process belongs to SM-processes, since the following conditions are met [7]:

- The transition matrix \( Q_{ij}(t) \) satisfies the conditions of the Markov matrix; the matrix \( Q_{ij}(t) \) is square and has a finite order \( m \times m = 3 \times 3 = 9 \), all elements of the matrix are non-negative;
- The values \( t_i \) are random and characterized by mathematical expectation \( M[t_i] \);
- the vector \( P(0) \) has only stationary states.

In practice, it is advisable to use the transition intensities \( \lambda_{ij} \) as consistent estimates of the probabilities of transitions from one technical state to another. The limiting probabilities of finding a machine in a particular state are determined using the system of differential equations by A.N. Kolmogorov [8].

To construct differential equations, we use the following mnemonic rule. On the left side, the time derivative of the probability of finding the road construction machine in the \( i \)-th technical state at the moment \( t \). The right-hand side contains the products of the transition intensity \( \lambda_{ij} \) by the probability of
the state from which the arrow leaves. The sign of the product is positive if the arrow enters the state under consideration, and negative if the arrow leaves the state under consideration (Fig. 2).

Thus, the number of terms on the right side of the equation is equal to the number of arrows connecting the state under consideration with other states. The system of differential equations is supplemented by the normalization condition $\sum_{i=1}^{N} P_i(t) = 1$.

Then the system of differential equations will have the following form.

For the perfect state $S_p$
\[
\frac{dP_1(t)}{dt} = \lambda_{31} P_3(t) - \lambda_{12} P_2(t). \tag{4}
\]

For the up state $S_{up}$
\[
\frac{dP_2(t)}{dt} = \lambda_{12} P_1(t) - \lambda_{23} P_3(t) + \lambda_{32} P_3(t) \tag{5}
\]

For the down state $S_{ds}$
\[
\frac{dP_3(t)}{dt} = \lambda_{23} P_2(t) - \lambda_{32} P_2(t) + \lambda_{31} P_1(t). \tag{6}
\]

For a stationary steady state operation, all derivatives $P_i(t) = 0$, and the system of differential equations transforms into a system of algebraic equations and with the addition of the normalization condition has the following form:

\[
\lambda_{31} P_3 - \lambda_{12} P_2 = 0; \tag{7}
\]

\[
\lambda_{12} P_1 - \lambda_{23} P_3 + \lambda_{32} P_3 = 0; \tag{8}
\]

\[
\lambda_{23} P_2 - \lambda_{32} P_2 - \lambda_{31} P_1 = 0; \tag{9}
\]

\[
\sum_{i=1}^{3} P_i(t) = 1. \tag{10}
\]

By solving this system of equations, the limiting probabilities of finding a road construction machine in various technical states during operation are determined.

Statistical values for determining the $P_{ij}$ and $\lambda_{ij}$ values can be obtained quite simply by timely and competent processing of the currently existing operational documentation, such as the "Machine maintenance and repair log", "Incident log" for elevator equipment, etc. With the statistical generalization and accumulation of the information obtained, the accuracy of the calculations will increase.

4. Conclusion

The proposed method of semi-Markov modeling allows, with the reliability admitted by the theory of probability and the apparatus of mathematical statistics, to assess the operational reliability of equipment and to determine the limiting probability of finding a road-building machine in serviceable, efficient and faulty technical states, as well as the probability of transition from one state to another throughout the entire machine life cycle operation. In turn, this makes it possible to solve the following tasks: determination of the readiness of equipment to fulfill its functional purpose; planning stocks of human and material resources for maintenance and repair; formation of car parks taking into account reservation; provision of feedback between the operator-manufacturer-designer to increase the reliability of newly created equipment.

References
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