Axial Current, Killing Vector and Newtonian Gravity

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Starting from the multiplicative torsion approach of gravity and assuming a Killing vector to be proportional to the axial-vector matter current, here we derive Newton’s law of gravity where the logarithm of the proportionality factor has been found to be the potential function.

PACS numbers: 04.20.-q, 04.20.Cv
Keywords: Entropic Gravity, Axial Anomaly.

1. FROM AXIAL CURRENT TO NEWTONIAN GRAVITY

All we know that the fermions as main building blocks of matter are described by spinor fields. In contrast, the interactions are mediated by bosons. Any realistic spinor theory has therefore to account for the bosons as to be composed of an even number of fermions. It is generally believed that there is a Fermi sea of Neutrinos in the universe and the magnitude of the Fermi energy $E_F$ is related to various cosmological theories\[1\]. Hence it is quite plausible to consider that spinors, massive or massless, are omnipresent at each space-time point of the universe.

By Geroch’s theorem\[2\] we know that - the existence of the spinor structure is equivalent to the existence of a global field of orthonormal tetrads on the space and time orientable manifold. This requires spinors to be soldered with the tetrads at each space-time point. The idea of gravity emerging from spinors is not new and fairly obvious, as one can construct a spin-2 particle as the direct product of spinors\[3, 4\]. The first idea of this type is due to Bjorken\[5\], who attempted to formulate the photon and graviton as a composite state. Another successful attempt is due to Hebecker and Wetterich\[6\]. Their theory can be regarded as a reformulation of gravity in terms of spinors.

In multiplicative torsion approach of gravity\[7–13\] one gets axial vector current 1-form for spinor field $\Psi$, $J_5 = \bar{\Psi} \gamma_5 \gamma \Psi$, to be an exact 1-form, given by the equation

$$d(\mathcal{R} - \beta \phi^2 - \frac{1}{2} \kappa m \bar{\Psi} \Psi) = -\frac{g}{4} \bar{\Psi} \gamma_5 \gamma \Psi,$$

where, $\gamma = \gamma_\mu dx^\mu = \gamma_\alpha e_\alpha dx^\mu \equiv \gamma_\alpha e^\alpha$, here Latin and Greek indices signify local tangent space and external coordinates of the four dimensional space-time manifold, respectively. Here the local flat-space metric is given by $\eta_{ab} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. $\mathcal{R}$ is the Ricci scalar and $\phi$ is a variable parameter of dimension $(\text{length})^{-1}$ and Weyl weight $(-1)$, such that $\phi e^a$, where $e^a$ is the local frame field, has the correct dimension and conformal weight of the de Sitter boost part of the $SO(4,1)$ gauge connection\[8, 9\]. $\phi$ may be linked with the dark matter where $\sqrt{\frac{2}{\kappa}}$ is the mass of the dark field\[9\]. Recently it has been shown that, from $J_5$ it is possible to construct an axial vector current 3-form which is conserved according to Noether’s theorem\[11\]. If we neglect mass terms, then (1) reduces to

$$d\mathcal{R} = -\frac{g}{4} J_5$$

We know that any Killing vector $\xi$ satisfies\[14\]

$$\xi \wedge^* d\mathcal{R} = 0$$

From (2) & (3) we can write

$$\xi \wedge^* J_5 = 0$$

In absence of mass, $J_5$ may be assumed to be a vector on the light cone. Then (4) implies that $\xi \propto J_5$ or,

$$\xi = \chi J_5,$$

where $\chi$ is a dimensional scalar factor. Here we define the surface gravity $\kappa_\xi$ for the Killing vector $\xi$ given by

$$\xi^\mu \nabla_\mu \xi^\nu = \kappa_\xi \xi^\nu$$

or, $\xi \wedge^* d\xi = 2\kappa_\xi^* \xi$, (6)
where $\nabla$ is the torsion-free covariant derivative. Now $\xi$ being a null vector, after using (5) & (6), we can write

$$\kappa_\xi = -\xi^\mu \nabla_\mu V$$

where, $V = -\frac{1}{2} \ln \chi$ \hspace{1cm} (7)

Here we like to consider (5) & (7) as two postulates to be true for any axial current $J_5$ of arbitrary spinors (massive or massless) in any space-time point. Using exactness of $J_5$ and taking exterior derivatives of (4), we get

$$d\xi = -2dV \wedge \xi,$$

or, $d^*(d\xi) = -2d^*(dV \wedge \xi)$

or, $-\bar{R}^{ab}_\mu \xi^b = -\Box V \xi^\mu + \partial_\mu \kappa_\xi,$ \hspace{1cm} (9)

Here $\Box V \equiv \bar{\nabla}^a \bar{\nabla}^a V$ and $\bar{R}^{ab}_\mu$ is the torsion-free Ricci tensor.

Using properties of Killing vectors, we may write,

$$0 = d^* \xi = d\chi \wedge \ast J_5 + \chi d^* J_5$$

or, $\chi d^* J_5 = -2\xi \wedge \ast dV = 2\kappa_\xi \eta,$ \hspace{1cm} (10)

here $\eta = \frac{1}{4!} e^a \wedge e^b \wedge e^c \wedge e^d$ is the invariant volume 4-form. This last equation implies that axial anomaly is proportional to the surface gravity $\kappa_\xi$. We know that, in theories like superstring theories and lattice gauge theories, anomaly cancellation takes place[15]. Again we know that $\kappa_\xi$ must be a constant when $\xi$ is along a null geodesic generator of any Killing horizon[16]. So from last equation, for the time being, we may assume $\kappa_\xi = \text{constant}$ and putting it in (9) we get, w.r.t. local indices,

$$\Box V \xi^a = \bar{R}^a_b \xi^b$$ \hspace{1cm} (11)

This equation implies that $\Box V$ is an eigen value of the matrix $\bar{R}^a_b$ with $\xi^b$ being the corresponding eigen vector in the local Minkowski space.

For massive spinors, we may consider, the vector current $J$ to be time-like and the axial vector current $J_5$ to be space-like. Here we may compare the pair $(J, J_5)$ with the pair $(p, s)$ where $p$ is the timelike momentum vector and $s$ is the spacelike spin vector of an one particle state, s.t. $p^a s_a = 0$. This implies that $\xi$ is also a space-like killing vector and is associated with the spin degree of freedom of matter[17].

1. At first we consider the case where the killing vector is space-like.

   (a) For a region dominated by a pressureless isotropic matter with rest mass density $\rho$, we may take,

   $$\xi = \vartheta(0, l, m, n), \quad l^2 + m^2 + n^2 = 1$$

   and $\vartheta$ is a nonzero scalar,

   $$\bar{R}^a_b = 4\pi G \rho \begin{pmatrix}
   -1 & 0 & 0 & 0 \\
   0 & 1 & 0 & 0 \\
   0 & 0 & 1 & 0 \\
   0 & 0 & 0 & 1
   \end{pmatrix}, \quad G \text{ is Newton’s constant},$$ \hspace{1cm} (13)

   $$\bar{G}^a_b = 4\pi G \rho \begin{pmatrix}
   -2 & 0 & 0 & 0 \\
   0 & 0 & 0 & 0 \\
   0 & 0 & 0 & 0 \\
   0 & 0 & 0 & 0
   \end{pmatrix}.$$ \hspace{1cm} (14)

   With this form of the matrix $\bar{R}^a_b$, (11) reduces to

   $$\Box V = 4\pi G \rho$$ \hspace{1cm} (15)

   Equations (15) may be interpreted as the generalisation of the Poisson’s equation of Newtonian gravity in curved space-time. In the case of weak field approximation of a static mass distribution this equation reduces to Poisson’s, given by

   $$\bar{\nabla}_\mu \bar{\nabla}^\mu V \approx \bar{\nabla} \cdot \bar{\nabla} V = 4\pi G \rho$$

   where, $\bar{\nabla} \cdot \bar{\nabla} \equiv \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.
(b) For a region dominated by a radiation fluid with energy density \( \rho \), we may take,

\[
\bar{R}^a_b = \bar{G}^a_b = 8\pi G\rho \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & l^2 & lm & ln \\
0 & lm & m^2 & mn \\
0 & ln & mn & n^2
\end{pmatrix}
\]  

(17)

With this form of the matrix \( \bar{R}^a_b \), (11) reduces to

\[
\Box V = 8\pi G\rho 
\]  

(18)

Equations (15) and (18) justify that radiation produces a gravitational potential which is twice as strong as that of a material particle with the same energy density[14].

2. Now we consider the case where the killing vector is a null vector.

(a) For a region dominated by pressureless isotropic matter with rest mass density \( \rho \), we may take,

\[
\xi^a = \vartheta(1, l, m, n).
\]  

(19)

\[
\bar{R}^a_b = 4\pi G\rho \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}.
\]  

(20)

\[
\bar{G}^a_b = 4\pi G\rho \begin{pmatrix}
-2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}.
\]  

(21)

With this form of the matrix \( \bar{R}^a_b \), (11) reduces to

\[
\Box V = 0
\]  

(22)

(b) For a region dominated by isotropic radiation fluid with energy density \( \rho \), we may take,

In this case we may take

\[
\bar{R}^a_b = \bar{G}^a_b = 8\pi G\rho \begin{pmatrix}
-1 & l & m & n \\
0 & l^2 & lm & ln \\
0 & lm & m^2 & mn \\
0 & ln & mn & n^2
\end{pmatrix}.
\]  

(23)

With this form of the matrix \( \bar{R}^a_b \), (11) reduces to equation (22). Here we see that, for \( \xi \) being a null vector, the forms of \( \bar{G}^a_b \) fail to produce the standard results for \( V \) to be a gravitational potential of Newtonian gravity, i.e. equations (15) and (18).

In view of space isotropy, we may consider \( \bar{G}^a_b \) to be of the general form

\[
\bar{G}^a_b = F \begin{pmatrix}
-1 & dl & dm & dn \\
-dl & bl^2 + c & blm & bmn \\
-dm & blm & bm^2 + c & bmn \\
-dn & bmn & bmn & bn^2 + c
\end{pmatrix}; \quad b, c, d \text{ are scalars and } F = 8\pi G\rho
\]  

(24)

Then

\[
R = F(1 - b - 3c),
\]  

(25)

\[
\bar{R}^a_b = F \begin{pmatrix}
-1 - b + 3c & dl & dm & dn \\
-dl & bl^2 + \frac{1-b-c}{2} & blm & bmn \\
-dm & blm & bm^2 + \frac{1-b-c}{2} & bmn \\
-dn & bmn & bmn & bn^2 + \frac{1-b-c}{2}
\end{pmatrix}.
\]  

(26)
If we take average over the two dimensional unit sphere $l^2 + m^2 + n^2 = 1$ then (24) reduces to its isotropic form, given by

\[
< G^a_b > = F \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & \frac{b}{3} + c & 0 & 0 \\
0 & 0 & \frac{b}{3} + c & 0 \\
0 & 0 & 0 & \frac{b}{3} + c
\end{pmatrix}, \text{where } \int l = \int m = \int n = 0,
\]

\[
\int l^2 = \int m^2 = \int n^2 = \frac{4\pi}{3} \text{ and } \int ln = \int mn = 0.
\]

(27)

This form of $< G^a_b >$ represents an ideal radiation fluid, ideal pressureless fluid or dark energy according as $b + 3c = 1$, 0 or $-3$.

Now we consider the solution of equation (11) in the general case where $R = -2AF$, s.t. $b + 3c = 1 + 2A$,

1. With $\xi = \vartheta(1, l, m, n)$, we have the following solution

\[
\tilde{G}^a_b = F \begin{pmatrix}
-1 & (1 + A + \theta) & (1 + A + \theta) & (1 + A + \theta) \\
-(1 + A + \theta)l & (1 + 2A + 3\theta)l^2 - \theta & (1 + 2A + 3\theta)lm & (1 + 2A + 3\theta)ln \\
-(1 + A + \theta)m & (1 + 2A + 3\theta)lm & (1 + 2A + 3\theta)mn - \theta & (1 + 2A + 3\theta)mn \\
-(1 + A + \theta)n & (1 + 2A + 3\theta)ln & (1 + 2A + 3\theta)mn & (1 + 2A + 3\theta)n^2 - \theta
\end{pmatrix},
\]

\[
= \tilde{R}^a_b + AF\delta^a_b
\]

where,

\[
\frac{b - 1 - 2A}{3} = -c = d - 1 - A = \theta;
\]

s.t. $\Box V = 8\pi G\rho \theta$ and $< \tilde{G}^a_b > = 8\pi G\rho \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & \omega & 0 & 0 \\
0 & 0 & \omega & 0 \\
0 & 0 & 0 & \omega
\end{pmatrix}$ where $\omega = \frac{1 + 2A}{3}$.

(29)

2. With $\xi = \vartheta(0, l, m, n)$, we have the following solution

\[
\tilde{G}^a_b = F \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & \frac{A + 3\theta - 1}{4} l^2 + \frac{1 + A - \theta}{2} & \frac{A + 3\theta - 1}{2} l m & \frac{A + 3\theta - 1}{2} l n \\
0 & \frac{A + 3\theta - 1}{4} m^2 + \frac{1 + A - \theta}{2} & \frac{A + 3\theta - 1}{2} m n & \frac{A + 3\theta - 1}{2} m n \\
0 & 0 & \frac{A + 3\theta - 1}{4} n^2 + \frac{1 + A - \theta}{2} & \frac{A + 3\theta - 1}{2} n^2
\end{pmatrix},
\]

(30)

\[
= \tilde{R}^a_b + AF\delta^a_b
\]

where,

\[
\frac{2b + 1 - A}{3} = 1 + A - 2c = \theta, d = 0, \omega = \frac{1 + 2A}{3};
\]

s.t. $\Box V = 8\pi G\rho \theta$ and $< \tilde{G}^a_b > = 8\pi G\rho \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & \omega & 0 & 0 \\
0 & 0 & \omega & 0 \\
0 & 0 & 0 & \omega
\end{pmatrix}$

(31)

It is to be noted that role of $\theta$ is significant in the field equation of the potential function $V$ but $\theta$ disappears from the isotropic average expression of the energy-momentum tensor. Hence isotropic form of the matter and $\theta$ are hitherto unrelated.

Considering standard results of General Relativity[14], equation (24) encompasses Newtonian Gravity for the following values of the parameters $\omega$ and $\theta$:

- Radiation $\Rightarrow \omega = \frac{1}{3}, \theta = 1$;
- Mass $\Rightarrow \omega = 0, \theta = \frac{1}{2}$;
- Dark Energy $\Rightarrow \omega = -1, \theta = -1$,

These values of $\omega$ and $\theta$ give us the relation

\[
\theta = \frac{3\omega + 1}{2}(1 + \omega(\omega - \frac{1}{3})(\omega + 1)\epsilon(\omega)),
\]

(32)

where $\epsilon(\omega)$ is an arbitrary function of $\omega$. 
• As a special case we consider $\epsilon(\omega) = 0$, s.t. $\theta = \frac{3\omega+1}{2}$, having $T_{ab} =$ diagonal $(\rho, p, p, p)$ and $p = \omega \rho$ is the equation of state of the isotropic matter. Using this relation in equation (29), we get the standard FRW result

$$\Box V = 4\pi G \rho(1 + 3\omega) \quad \text{and} \quad <\bar{G}^a_b> = 8\pi G \rho \frac{1}{2} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \omega & 0 & 0 \\ 0 & 0 & \omega & 0 \\ 0 & 0 & 0 & \omega \end{pmatrix}.$$ (33)

It is well known that the curvature energy density corresponding to the spatial hypersurfaces of the Friedmann universe does not act as a source of gravitational potential. Here the case is given by $\omega = -\frac{1}{3}$, i.e. $p = -\frac{1}{3} \rho$. In this case, for both null or spacelike $\xi^a$,

$$\bar{G}^a_b = R^a_b - F\delta^a_b = F \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & -\frac{1}{3} \end{pmatrix}.$$ (34)

s.t. $\Box V = 0$ and $<\bar{G}^a_b> = 8\pi G \rho \frac{1}{2} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 0 & 0 & -\frac{1}{3} \end{pmatrix}.$ (35)

2. FROM NEWTONIAN GRAVITY TO HOLOGRAPHIC PRINCIPLE

Recently, Verlinde[18] has proposed a remarkable new idea linking classical gravity to entropic force, which attracted much interest[19]. He has derived Newton’s second law and Einstein’s equation from the relation between the entropy of a holographic screen and the mass inside the screen. Padmanabh[20], earlier than Verlinde, has also proposed that classical gravity can be derived from the equipartition energy of horizons. Let us try to understand, in brief, this holographic nature of gravity.

Let, at time $t$, $P$ be a test particle at a distance $r$ from a mass $m$ at O. Draw a spherical holographic screen through $P$ having centre at O. Information of the mass $m$ takes a time $\Delta t = \frac{r}{c}$, $c$ is the velocity of light, to reach the spherical boundary. Therefore at any time $t$ only the whole of the holographic screen carries the net information of $m$ at O which originates from a past time $t - \Delta t$. Total number of bits available for carrying this information of mass $m$ at time $t$ is

$$N = \frac{A}{4\pi r^2 c^3} = \frac{4\pi r^2 c^3}{G h}, \quad G = \text{Newton’s Constant.}$$ (36)

From definition of temperature, using equipartition rule[21], we have

$$E = m c^2 = \frac{1}{2} N k_B T$$ (37)

and then identifying $T$ with Unruh temperature[21]

$$T = \frac{\hbar a}{2\pi k_B c},$$ (38)

we get Newton’s law for acceleration

$$a = \frac{G m}{r^2}.$$ (39)

This derivation of Newton’s law of gravitation is more than a ‘action at a distance’ in nature. The holographic view emerges from the non-instantaneous ability of signals $\Box V = 0$ in equation (35), carrying information of the mass $m$, to reach the spherical boundary. This holographic origin of gravity claims it (gravity) to be an entropic force!

We see that (16) is Poisson’s equation for a static mass density $\rho$. This equation also implies Newtonian gravity, i.e., acceleration $a$ of a test particle, due to a point mass $m$ at a distance $r$, is given by (39), where $\kappa = \frac{8\pi G}{c^2}$.

Starting from (39), defining Unruh temperature by (38) and then using thermodynamic relation (37) we get (36) in the reverse order. Thus the holographic principle, i.e., the maximal storage space, or total number of bits, is
proportional to the area $A$, is a consequence of Newtonian form of gravity. It is to be noted that in this approach the role of Newton’s constant in the holographic principle as the minimal unit of surface area is not by an ad-hoc prescription. In multiplicative torsion approach of gravity the emergence of Newton’s constant is through field equations\textsuperscript{7–9}. $\kappa$ has topological origin, it is inversely proportional to the topological Nieh-Yan density. It is to be noted that, in GR, one gets Newtonian Gravity only when one applies weak field approximation on the metric’s time component. But in the present formalism the metric has no such direct role. Even, after deriving equations (15) & (22), in the Minkowskian limit 

$$g_{\mu\nu} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

these equations reduce to standard equations of special theory of relativity. Here (22) implies that, in a place having zero mass distribution, $V$ propagates with velocity of light and in static case, having some non-zero mass distribution, (15) reduces to Poisson’s equation. By this way the holographic principle doesn’t contradict ‘special theory of relativity’ but it is likely not to be valid in curved space time, at least in the case of strong gravity!

3. DISCUSSION

Taken as a whole, our model and Verlinde's approach may be seen as playing complementary roles. In Verlinde’s approach Newton’s universal law of gravitation is a consequence of certain thermal and entropic properties of the constituents of spacetime, whereas in our model these properties appear in a reverse consequence. First we consider that spinors (massive or massless) are everywhere in the space-time and axial currents are proportional to killing vectors and then the gravitational potential $V$ is nothing but the logarithm of the proportionality factor. In static case together with weak field approximation the development of $V$ is given by the Poisson’s equation of Newtonian Gravity. Then moving in reverse order to that of Verlinde’s approach we get holographic principle as a logical consequence such that Newton’s constant plays the role of minimal unit of surface area. It appears that, though not ‘action at a distance’, the holographic principle is valid only in the weak field case!

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