Near-, mesoscopic and far-field regimes of a subwavelength Young’s double-slit

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Abstract. The coupling distance between two narrow resonating slits made in a thick metal screen strongly modulates the optical spectral features of their scattering resonances. We show that these non-trivial modulations result from dipolar-type interactions between the slits. The radiation damping, frequency shift and local light enhancement of these modes vary with the coupling distance, and we derive analytical expressions for these physical quantities. We also explain how transmission and antenna-like radiation pattern can be tuned with specific incidence angles.
1. Introduction

Dragila et al [1] were the first to observe in 1985 that thin metallic films, in principle completely opaque, can be rendered highly transparent at specific resonant frequencies. As they clearly demonstrated, the phenomenon takes its origin in the excitation and the coupling of surface plasmon (SP) modes between both sides of the film. Later, the observation by Ebbesen et al [2, 3] of the same kind of resonant optical transmission [4] through metallic films pierced by a periodic nanohole array, strongly revived the interest of the community in subwavelength-scale structured surfaces, especially resonant filters relatively similar to the metal meshes studied in the far infrared [5–7]. Other related systems, such as narrow rectangular slit gratings, were also intensively studied [8–16]. As noted previously, the coupling between SPs on both sides indeed results in a possible channel for the light transmission. However, in the particular case of slits, another channel enters the game: the Fabry–Perot (FP)-type waveguide resonance, which does not exist in the case of subwavelength holes, but which enhances considerably the transmission with respect to the SP channel [11]. Moreover, it was shown that the use of prisms in an attenuated total reflection (ATR) configuration to illuminate the slits also renders the FP transmission channel very efficient, highly selective in frequencies and even tunable without changing the geometrical parameters of the gratings [17, 18].

Instead of a periodic array, we may only consider two slits made on a metallic screen in the manner sketched in figure 1(a). We recognize the famous Thomas Young double-slit device. The only difference from Young is that we specifically consider slits with width w much smaller than the incident light wavelength \( \lambda_0 \), so that the transmission can only occur via a resonant mechanism of the type of SP or FP (or even both together). It was Schouten et al [19] who first examined the problem of the influence of a distance separation of approximately several wavelengths between two subwavelength slits on the resulting diffraction pattern and transmitted power. Using a device designed for avoiding the FP channel, i.e. with a height slit sufficiently different from that of the FP resonant one (see details later in the text), they were able to prove that the total transmission of transverse magnetic (TM) polarized light was strongly modulated as a function of the slit distance. This modulation is due to SP interferences at the mouth of the slits instead of at the detector in the far-field region. Later, Gordon [20] considered the FP channel for the same two-slit device and showed numerically that it may
Figure 1. (a) Sketch of the slit doublet and parameters. Throughout this paper, we take $h = 1.5 \, \mu m$ and $w = 0.3 \, \mu m$. The coupling distance is $d = x_2 - x_1$. Incoming light is TM-polarized. (b) Normalized-to-area transmission of the doublet as a function of $d$, around the fundamental FP resonance wavelength of the single slit $\lambda = 3.9 \, \mu m$: perfect metal, at $\theta = 0^\circ$ (red curve) and $80^\circ$ (green curve). Diamonds give the transmission level of a single slit at normal incidence. (c) The same as (b) but in a real metal case (the surface impedance model \[8, 27\] at the horizontal interfaces).

also produce modulation in the total transmitted power. Aigouy et al \[21\] also provided a slit-doublet experiment at optical wavelengths showing the coexistence in the near-field (under some conditions) of a surface plasmon polariton (SPP) wave together with another wave that has a free-space character (i.e. a lightwave).

In the present paper, we give a new physical interpretation and understanding of the two-slit problem with respect to previous works \[19–23\]. Indeed, we prove that the two-slit system behaves in the FP regime as two interacting oscillating dipoles. That point was missing until now in the literature. However, it greatly clarifies the physics. We examine the problem of the slit-distance influence on diffraction in the FP regime (without any SP excitation). Based on a previous study that we carried out on a two-slit bottom-closed system \[24\], we develop both a numerical and an analytical model allowing us to completely understand the underlying physics.
As is known, the field induced by a single dipole contains three different parts: a near-field one that falls as $1/r^3$, a mesoscopic one falling as $1/\lambda r$ and a radiated far-field one falling as $1/\lambda^2 r$, with $r$ being the distance to the dipole and $\lambda$ the exciting light wavelength. Owing to their coupling, the FP dipole modes of the slits couple and split into two new composite modes: the $(\pm)$ symmetric and the $(-)$ antisymmetric one. We give analytical expressions for the fields in each of the cavities as well as the radiation damping and frequency shifts of the two composite modes. This analytical treatment associated with numerical calculations allow us to understand how these two composite modes are modified as a function of slit distance from the near-field coupling up to the far-field one. In the near-field regime of interaction, the symmetric mode is essentially dipolar, whereas the antisymmetric mode is highly quadripolar. This difference is lost in the far-field regime, where the coupling between the two modes is much weaker. We show that the best transmission is obtained in the intermediate mesoscopic field regime. Eventually, we also explore the electromagnetic behavior for any incident light angle (in contrast with other studies that consider only normal incidence) for different scales of coupling distance and we show that the symmetry of the far-field radiation pattern can be two-way switched depending on judiciously tuned excitation parameters. This understanding of the system allows control of its optical properties.

2. Numerical observations and analytical model

2.1. Preliminary observations

All the calculations in this paper are based on the well-known modal approach developed earlier for the present kind of geometry (rectangular slits). This method can give exact results [25, 26], but in a certain range of parameters of materials and of optical frequencies, it may be successfully simplified and approximated by the use of impedance boundary conditions [8, 27]. This method has shown an ability to give excellent agreement with experiments [11, 28, 29], but only for infrared light sent on sub-micrometric structured devices, such as the one we are considering here. When $|\varepsilon| \to \infty$, the surface impedance $Z \sim |\varepsilon|^{-1/2} \to 0$ and the real metal becomes a ‘perfect’ reflector. We know from many previous studies that in the FP regime for micro-cavities under infrared light, the difference in the results obtained between a real and a perfect metal is very small and all the physics—except the excitation of an SP wave—is already contained in the perfect metal case. Since we restrict ourselves in this work to the study of such structures in the infrared, we will analyze the phenomenon in the framework of the perfect metal case throughout this paper.

Our slit doublet is sketched in figure 1(a), and we send on it an incident plane wave TM-polarized, i.e. with the magnetic field parallel to the slits. Let us begin with some numerical observations. The subwavelength cavities are designed such that they specifically present a fundamental FP resonance in the studied spectral range (in the paper, we always take $\lambda = 3.9 \, \mu m$ as an example). Figure 1(b) shows the behavior of the normalized-to-area transmission of the system (perfect metal is considered here) as a function of the coupling distance $d = x_2 - x_1$ for two extreme incidence cases: the normal incidence ($\theta = 0^\circ$) and a grazing angle ($\theta = 80^\circ$). For comparison, the transmission level of a single slit at the resonance wavelength is also displayed. As strong modulation of the far-field signal is observed in the two-slit system, with an apparently faster modulation at large angle. In the latter case, the transmitted power surprisingly increases when the channels are near-field coupled. For completeness,
Figure 2. (a) Spectra of the normalized-to-area transmission of the slit doublet (perfect metal) with $d = 1 \, \mu\text{m}$, at $\theta = 0^\circ$ (red curve) and $80^\circ$ (green curve). Blue diamonds correspond to the response of a single slit at normal incidence. (b) Transmitted radiation pattern of the system corresponding to each maximum of the resonances (polar representation).

Figure 1(c) shows the responses obtained taking into account the skin depth effects in the metal at the horizontal interfaces (surface impedance boundary condition [8, 12, 17, 27] with the gold permittivity). As stated before, whereas the transmission maxima are slightly inferior due to absorption loss, modulations are very weakly perturbed by the non-zero surface impedance. Accordingly, they cannot be fundamentally attributed to SP waves launched from the slit apertures [30] as is the case in Schouten’s experiment [19].

Several other observations can be made. Figure 2(a) displays the transmission spectrum through a single slit in the FP resonance regime, together with that of a doublet at a coupling distance $d = 1 \, \mu\text{m}$, both for a normal and an oblique incidence. We remark that the resonance maxima of the two-slit system do not occur at the same wavelength. There is a frequency shift and different quality factors. Moreover, if we study the angular distribution of the far-field transmission, we observe different radiation patterns: at $\theta = 0^\circ$, the scattering lobe is mainly directed in the normal direction $\phi = 0$ and exhibits a dipolar character, whereas at $\theta = 80^\circ$ we have two asymmetric scattering lobes with a quadrupolar character. The isotropic radiation...
pattern of a single slit is also displayed for comparison. How to explain such rich behavior? We will show that particular interferences of the fields scattered by each of the resonant cavities are responsible for the transmission modulations and for the change in symmetry of the radiation pattern. The slit doublet actually exhibits two resonance modes that are generally mixed, but that we can totally control once we understand their physical origin.

2.2. Properties of one slit

We recall that a subwavelength slit always supports a fundamental waveguide mode in TM polarization, which is associated with a surface charge density having opposite signs on both sides of the slit. It explains the occurrence of an effective dipolar momentum at the mouth of the aperture when a cavity resonance occurs \([24, 31, 32]\). In the perfect metal case, the magnetic field scattered above and below the slit is approximately a zero-order Hankel function of the first kind (cylindrical wave). By coupling two or several slits (or effective dipoles), we can perform a particular interference of their scattering lobes by adjusting the separation distance. Light trapping within the channels and the subsequent far-field transmission are then expected to be enhanced or reduced. The modal method allows an easy microscopic analysis of the electromagnetic phenomena that we have shown in figures 1 and 2. First, the magnetic field inside a narrow slit of height \(h\) is governed by the waveguide mode \([8, 12, 13, 33, 34]\) and is expressed as follows,

\[
H_{II}^i(x, y) = [A \cos(ky) + B \sin(ky)] \cap_{w}(x),
\]

for a perfectly reflecting metal. \(\cap_{w}(x) = 1\) when \(|x| < w/2\) and 0 otherwise. \(A\) and \(B\) are unknown complex amplitudes and \(k = 2\pi/\lambda\). The origin of \(y\) is taken at the mid-height of the metallic screen. Under illumination by a plane wave, the reflected and transmitted fields can be, respectively, written as Fourier transforms:

\[
H^I(x, y) = e^{ik(x \sin \theta - y \cos \theta)} + \int_{-\infty}^{\infty} R(u)e^{ik(ux+vy)}du, \tag{1}
\]

\[
H^{III}(x, y) = \int_{-\infty}^{\infty} T(u)e^{ik(ux-xy)}du, \tag{2}
\]

with \(v = \sqrt{1-u^2}\). In the framework of this very general formalism, we can directly apply the energy conservation law. For an arbitrary number \(N\) of identical slits, we obtain

\[
\int_{-\pi/2}^{\pi/2} \frac{\lambda}{Nw} [|\tilde{R}(\sin \phi)|^2 + |T(\sin \phi)|^2] \cos^2 \phi \, d\phi = \sigma, \tag{3}
\]

where we put

\[
\sigma = -\frac{2\lambda}{Nw} \Re[\tilde{R}(u = \sin \theta) \cos(\theta)]. \tag{4}
\]

It is worth noting that the definition of a transmittance normalized to the slit area is here welcome to make relevant comparisons between different cases, as this area is kept constant throughout the present paper. A high transmittance can actually correspond to a negligible quantity of transmitted energy if this area becomes very small, hence this careful remark.
\( \sigma \) can be physically interpreted as the total cross-section of the \( N \) slits in units of \( w \). \( \sigma = 1 \) when the slits are not resonating. Equation (3) means that the part of the energy we do not find in the specular reflected ray is exactly the energy scattered (radiated) by the slits in all the other directions. For a single slit, continuity conditions between each region lead to the explicit amplitudes

\[
A = \frac{\sec(\beta \sin \theta)}{\cos(k h) - G_0 \sin(k h/2)} \quad \text{and} \quad B = \frac{\sec(\beta \sin \theta)}{\sin(k h/2) + G_0 \cos(k h/2)},
\]

where \( \beta = k w/2 < 1 \), and

\[
G_0 = i \beta \int_{-\infty}^{\infty} \sec^2(\beta u) \frac{\sec^2(\beta u)}{\pi \sqrt{1 - u^2}} du \approx i \beta + \frac{2 \beta}{\pi} \left[ \frac{3}{2} - E - \ln(\beta) \right],
\]

with \( E = 0.5771\ldots \) being Euler’s constant. A similar but incomplete expression for the scattering integral \( G_0 \) is given in [34]. Since \( |G_0| < 1 \), we find the fundamental slit resonance when \( k = k_0 \approx \pi/h - O(w/h) \). At \( k_0 \), the cavity field is governed by

\[
A_0 \approx \frac{2/h}{(k_0 - k) - i \gamma_0/2},
\]

at normal incidence. The damping \( \gamma_0 = 2 k_0 w/h \). The field enhanced inside a single slit at the resonance is then easily calculated: \( |A_0^{\text{max}}| \approx \lambda_0/\pi w \), and the normalized-to-area transmission

\[
\text{Tr} = \beta \left| A_0 \sin \left( \frac{k_0 h}{2} \right) \right|^2
\]

with \( \text{Tr} \approx \sigma/2 \). For the single slit of figure 1, we find that \( \sigma = 8.3 \). The weight of the radiated lightwaves \( (|u| \leq 1 \text{ in } G_0) \) and that of the evanescent ones \( (|u| > 1) \) are intrinsically linked. The spectral width \( \gamma_0 \) comes from radiation (in the absence of any absorption loss), whereas the spectral shift of the eigenwavenumber \( k_0 \) compared to the ideal FP one \( (\pi/h) \) is due to scattered surface waves launched from the apertures (reactive energy).

### 2.3. Two slits

Now, when the metallic surface has two identical slits, indexed by \( n = \{1, 2\} \), separated by an arbitrary distance \( d \), the cavities couple through their scattered fields, which yields

\[
\begin{align*}
A_{n=1,2} &= \sec(\beta \sin \theta) \left[ \frac{\cos(kd \sin \theta/2)}{\cos(k h/2) - G_+ \sin(k h/2)} + i(-1)^n \frac{\sin(kd \sin \theta/2)}{\cos(k h/2) - G_- \sin(k h/2)} \right], \\
B_{n=1,2} &= \sec(\beta \sin \theta) \left[ \frac{\cos(kd \sin \theta/2)}{\sin(k h/2) + G_+ \cos(k h/2)} + i(-1)^n \frac{\sin(kd \sin \theta/2)}{\sin(k h/2) + G_- \cos(k h/2)} \right],
\end{align*}
\]

with

\[
G_{\pm} = i \beta \int_{-\infty}^{\infty} \frac{\sec^2(\beta u)}{\pi \sqrt{1 - u^2}} [1 \pm \cos(k du)] du.
\]
Let us focus again on the fundamental FP mode. Around the resonance \( k_0 \), the respective amplitudes \( A_1 \) and \( A_2 \) of the two-slit system are expressed by

\[
A_n \approx \frac{2}{\hbar} \left[ \frac{\cos(kd \sin \theta/2)}{k_+ - k - i\gamma_+/2} + (-1)^n \frac{i\sin(kd \sin \theta/2)}{k_- - k - i\gamma_-/2} \right],
\]

where we define \( k_\pm = k_0 \mp \Delta \) and \( \gamma_\pm = \gamma_0 \pm \Gamma \) with

\[
\frac{\Delta}{\gamma_0} = \frac{1}{\pi} \int_1^\infty \frac{\sec^2(k_0wu/2)}{\sqrt{u^2 - 1}} \cos(k_0du)du \approx -\frac{Y_0(k_0d)}{2},
\]

\[
\frac{\Gamma}{\gamma_0} = \frac{2}{\pi} \int_0^1 \frac{\sec^2(k_0wu/2)}{\sqrt{1 - u^2}} \cos(k_0du)du \approx J_0(k_0d).
\]

\( J_0 \) and \( Y_0 \) are the zero-order Bessel functions of the first and second kind, respectively. They give a very good approximation of the integrals when \( kw \ll 1 \).

We analytically see that the fundamental FP mode splits into two FP ones: in the (+) configuration at \( k = k_+ \), both slits resonate in phase \((A_1 \approx A_2)\), whereas in the (−) configuration \((k = k_-)\), they resonate with opposite phases \((A_1 \approx -A_2)\) provided that \( \theta \neq 0 \). \( \Delta \) represents here the frequency shift between both splitted FP modes (\( \Delta \) can be negative), i.e. to more or less evanescent waves scattered on the horizontal surfaces. Here, we can naturally introduce the normal variables \( A^+ = A_1 + A_2 \) and \( A^- = (A_1 - A_2)/i \) describing two fictitious resonators (in-phase and anti-phase ones). As we will see, the eigenwavenumbers \( k_+ \) and \( k_- \) remain close to \( k_0 \), but the spectral widths \( \gamma_+ \) and \( \gamma_- \) can be very different from each other. Generally, both modes perturb each other in the resonance spectral range, which results in complicated optical responses. However, this perturbation can be overcome. Indeed, the numerator of \( A_n \) is an oscillator strength. Then, to excite a pure symmetric mode \((A^- = 0)\), normal incidence always works and the resonance occurs when the denominator of \( A^+ \) is minimum. For a pure antisymmetric resonance \((A^+ = 0)\), oblique incidence is required and \( \theta \) has to be tuned such that

\[
kd \sin \theta = (2n + 1)\pi
\]

except when \( kd \) is very small, i.e. in the strong near-field coupling case where a grazing angle gives the best excitation. Thus, we may control the phase distribution over both cavities. Moreover, we control the radiation pattern depending on the chosen excited mode. Indeed, the Fourier coefficient of the transmitted field is

\[
T(u) = i \left[ \cos \left( \frac{kdu}{2} \right) \tilde{A}^+ + i \sin \left( \frac{kdu}{2} \right) \tilde{A}^- \right] \frac{2\beta \sec(\beta u)}{\pi v}
\]

with \( \tilde{A}^\pm = \sin(k_0h/2)A^\pm \). For each pure mode, the normalized-to-area transmission of the two-slit system becomes

\[
\text{Tr}(\theta = 0) = \frac{2\beta}{\pi} |\tilde{A}^+|^2 \int_{-\pi/2}^{\pi/2} \cos^2 \left( \frac{kd \sin \phi}{2} \right) d\phi,
\]

\[
\text{Tr}(\theta = \theta_c) = \frac{2\beta}{\pi} |\tilde{A}^-|^2 \int_{-\pi/2}^{\pi/2} \sin^2 \left( \frac{kd \sin \phi}{2} \right) d\phi,
\]

with \( \theta_c \) obeying (14) and \( k \sim k_0 \). The term \( \sec(\beta u) \) in (15) has been approximated to unity. Both the above integrands give the expressions for the antenna-like radiation patterns, in polar
Figure 3. Radiation dampings $\gamma_{\pm}$ (top) and spectral shift $\Delta$ (bottom) of the (+) (red curve) and (−) (green curve) modes around the resonance wavenumber $k_0$, as a function of the coupling distance $d$ (see equation (13)). Quantities are normalized to the radiation damping $\gamma_0$ of the single slit ($w = 0.3 \, \mu m$ and $h = 1.5 \, \mu m$, $\lambda_0 = 2\pi / k_0 = 3.9 \, \mu m$). Cases 1 and 2 correspond to the two cases of distances illustrated in figures 4 and 5, respectively.

coordinates. One then immediately sees that the patterns of the in-phase and anti-phase modes have different symmetry properties. For instance, the normal direction ($\phi = 0$) is always a nodal plane for the (−) mode.

3. Results

We have now all the essential elements to interpret the spectral features of the scattering phenomena in the resonance regime. They directly depend on the strength of the (+) and (−) FP modes. In particular, we can analytically search the opto-geometrical parameters corresponding to the most intensive resonances and transmissions, i.e. the parameters that minimize the radiation dampings.

We thus analyze the behaviour of $\gamma_+$ and $\gamma_-$. Given (13), the coupling distances that minimize the damping are approximately given by the roots of $J_0'(x) = J_1(x)$ where $x = k_0d$, plus the special case where $kd \ll 1$. Due to the difference in sign between $\gamma_+$ and $\gamma_-$, a minimum for one damping is obviously a maximum for the other, so that one mode intrinsically dominates the other depending on the choice of the coupling distance (and provided the mode is correctly excited). Figure 3 shows the evolution of the shift $\Delta$ and that of the spectral widths $\gamma_\pm$ as
a function of $k_0 d$. Their oscillating behaviour is very similar to that obtained in the analogous problem of two coupled oscillating dipoles [35]. We see that $\Delta$ can sometimes cancel (corresponding to the roots of $Y_0(k_0 d)$), which shows that both modes do not necessarily repulse each other. Nevertheless, the eigenmodes cannot have the same spectral characteristics (non-degeneracy): if they have the same eigenfrequency ($\Delta = 0$), they do not have the same damping ($\gamma_+ \neq \gamma_-$). Conversely, when $\gamma_+ = \gamma_-$, the spectral shift $\Delta$ reaches a maximum.

We now distinguish three different coupling regimes: (i) a far-field coupling regime that we call the interferential regime ($d > \lambda$), (ii) a near-field coupling regime ($d < \lambda/2$) and (iii) a mesoscopic coupling regime that we call the transient one ($d \sim \lambda/2$).

### 3.1. The interferential regime

Let us begin with the interferential regime. One simply sees in figure 3 that the in-phase and anti-phase modes can exchange their roles by progressively increasing the distance $d$. According to (16) and (17), the number of scattering lobes (i.e. the far-field interference fringes) of the system naturally increases with the coupling distance. In the limit $d \to \infty$, $\Delta \to 0$ and $\gamma_\pm \to \gamma_0$: the cavity coupling becomes negligible and we retrieve the response of two non-interacting isolated slits. We have, in particular, illustrated two different cases for $d \sim \lambda$, already indicated by dotted lines in figure 3: a case where the coupling distance implies $\Delta = 0$ (figure 4) and the other where the coupling distance implies $\gamma_+ = \gamma_-$ (figure 5). In the first case, the $(-)$ and $+$ resonances have the same eigenwavenumber (i.e. $k_0$), but the $(-)$ mode is prevalent since $\gamma_- < \gamma_+$. Depending solely on the incidence angle, we can choose to excite one mode or the other and choose precisely the radiation pattern, as shown in figure 4. Scattering lobes of the $(-)$ resonance obviously have a greater amplitude. Conversely, if one tries to have the same radiation damping and the same transmission power for both modes, we cannot get it at the same wavenumber. Indeed in that case, the shift $\Delta$ automatically meets a maximum. This is what is shown in figure 5. Here again, the symmetry of the radiation pattern can be chosen. For greater coupling distances (at many wavelengths), a real metal should be considered to precisely study the interference phenomena modulated by the surface wave excitations as in [30, 36–38]. Indeed, in that case the total magnetic field scattered at the surface, far from the resonant cavity, would be the sum of two contributions. The first one is a photonic wave due to the scattered lightfield which behaves as $e^{ikx}/\sqrt{kx}$ for a perfect (or nearly perfect) metal. This is the contribution that carries the interaction between the two slits in our case. The second one is an SP wave that behaves as $e^{ik_{sp} x}/\sqrt{\varepsilon}$ (where $k_{sp} = \sqrt{\varepsilon/(\varepsilon + 1)}$ and $\varepsilon$ is the metal permittivity). At many wavelengths and in the optical (visible light) regime, the SP wave has a small spatial damping factor and would be responsible for the long-range energy transport (except if $|\varepsilon| \gg 1$). Finally, at short distance, the scattered lightwaves rather predominate in the vicinity of the (subwavelength) cavity apertures [37]; that is why they are strongly involved in the near-field coupling case.

### 3.2. The near-field regime

New electromagnetic behaviour is observed in the near-field coupling regime and has been partly studied for two closed grooves [24]. We have maximum values of $\gamma_+$ whereas $\gamma_-$ vanishes: $\gamma_+ \approx 2\gamma_0$ and $\gamma_-/\gamma_0 \approx (k_0 d/2)^2$ when $k_0 d \to 0$ (perfect metal). Besides, $\Delta \approx -[\ln(k_0 d/2) + E]/2$. The prevalence of the $(-)$ mode is conserved even when the shift is
Figure 4. (a) Transmission spectra of the doublet for a coupling distance \( d = 4.55 \mu \text{m} \) (case 1). Normal incidence excites the pure symmetric mode, whereas \( \theta = \theta_c = 25.4^\circ \) (see equation (14)) excites the pure antisymmetric one. \( d \) is such that the spectral shift \( \Delta \) is zero and such that the spectral width of the \((-)\) mode meets a minimum (high transmission), contrary to the \((+)\) mode (low transmission). (b) Radiation pattern corresponding to each mode, at the maximum of the resonances (polar representation).

maximum, which is different from the interferential regime. The antisymmetric mode presents quadrupolar character whereas the symmetric one is strongly dipolar, as initially observed in figure 2(b). A resonance both spectrally and spatially localized is here expected at large incidence angles, leading to important light enhancement and transmission. This is illustrated in figure 5. Taking \( \theta = 80^\circ \) and \( d = 0.6 \mu \text{m} \) \((=0.15\lambda_0)\), we get \( \text{Tr} = 5.7 \) \((\sigma = 11.5)\) and a mid-height spectral width of \(0.38 \mu \text{m} \) only, whereas we have \( \text{Tr} = 2.4 \) and a spectral width of \(1.82 \mu \text{m} \) at normal incidence (figure 6). For comparison, \( \text{Tr} = 4.2 \) and the spectral width is \(1.24 \mu \text{m} \) for a single slit. Moreover, the normalized magnetic field intensity inside the slits reaches \(|H_z/H_{\text{inc}}|^2 = 90 \) \((=20 \text{ for a single slit})\). It should be noted that the residual damping \( \gamma_- \) is due to retardation effects, as in the case of near-field coupled dipoles [24], which is rather counter-intuitive given the very small distance that separates the resonators. In the case of a real metal, the actual damping cannot be smaller than absorption losses, and we should obtain
Figure 5. (a) Transmission spectra for a coupling distance \( d = 3.55 \, \mu\text{m} \) (case 2). Normal incidence excites the pure symmetric mode, whereas \( \theta = \theta_c = 31^\circ \) (see equation (14)) excites the pure antisymmetric one. \( d \) is such that the spectral widths of both modes are equal, whereas the spectral shift \( \Delta \) meets a maximum. (b) Radiation pattern corresponding to each pure mode, at the maximum of resonance (polar representation).

something like

\[
A_n(k = k_-, \theta \neq 0) \propto (-1)^n \frac{\sin(k_0d \sin \theta/2)}{\Gamma' + \gamma_-/2},
\]

where \( \Gamma' \) is mainly due to the damping of the waveguide mode propagating along the vertical walls [39]. Since the excitation coefficient (numerator of \( A_n \)) vanishes when \( k_0d \rightarrow 0 \), the antiphase resonance should collapse under some very subwavelength distance, due to irreducible (even small) absorptions; hence a compromise between a minimal coupling distance and the excitation efficiency of the quadrupolar mode is to be found. On the other hand, the in-phase mode is much less sensitive to metal absorption as \( \gamma_+ \sim \gamma_0 \). In the near-field coupling regime, one can quickly calculate the ratio of the transmission due to the \((-)\) mode to that of the \((+)\) mode. Given (11), (16) and (17), we have

\[
\frac{\text{Tr}^- (\theta \neq 0, k = k_-)}{\text{Tr}^+ (\theta = 0, k = k_+)} = \frac{1}{2} \left( \frac{k_d}{2} \right)^2 \left| \frac{\gamma_+}{\gamma_-} \right|^2 = 2 \sin^2 \theta
\]
Figure 6. (a) Transmission spectra of the doublet in the near-field coupling regime, with $d = 0.6 \, \mu m = 0.15 \lambda_0$ (a perfect metal). Normal incidence excites the strongly damped symmetric mode, whereas a grazing angle excites the antisymmetric one (weak spectral width). (b) The corresponding spectra of the magnetic field intensity enhancement inside both slits.

with $kd < 1$. The original result is that the symmetric resonance scatters less light than the antisymmetric one if $\theta > \pi/4$. Another remark can be made. Figure 5(b) shows that the system presents a high frequency susceptibility at the ($-$) resonance, i.e. drastic variations of the near-field intensity occur inside each slit. This phenomenon has been analyzed for two grooves in [24]. Actually, near the wavenumber $k_0$, it is possible to combine the excitations of both eigenmodes ($+$) and ($-$) by appropriately tuning the incidence angle.

As a result, one cavity can be nearly extinguished whereas its neighbor strongly resonates, so that light transmission occurs through only one cavity of the subwavelength doublet or the other. The Young slit experiment may be interestingly reconsidered in this case. Such behavior is also observed when the two-slit system is repeated periodically [39].

3.3. The transient regime

Finally, the best transmission we can obtain through the slit doublet at normal incidence corresponds to the transient regime. For a barely subwavelength coupling distance, say $d = 0.61 \lambda_0$ (first root of $J_1(k_0d)$), we obtain the absolute minimum of $\gamma_+ \approx 0.6 \gamma_0$. Then, it
strongly favors the (+) mode. Such a condition curiously corresponds to the Rayleigh–Abbe resolution criterion. What is more, the shift \( \Delta \approx 0 \), so that we obtain cavities both resonating at their original eigenwavenumber \( k_0 \) but with a quality factor better than that when they are uncoupled. At \( \theta = 0 \), cavities resonate exactly in phase with a field intensity enhancement almost three times \( (\gamma_0/\gamma_s)^2 \) as high as that in the isolated case \( (kd \gg 1) \). Consequently, transmission is enhanced all the more. Such theoretical results are totally consistent with numerical observations performed with the FDTD method in the near-infrared [40]. In our example, taking \( d = 2.4 \, \mu m \), simulations give a maximum transmission \( T_r = 6.8 \) at normal incidence (4.2 for a single slit) and a normalized magnetic field intensity of \( |H_z/H_0|^2 = 53 \) inside both cavities (20 for a single one). It is to be noted that as the number \( N \) of coupled slits increases, we can always find an optimal equidistance for which resonance and transmission are maximum at \( \theta = 0 \) (around the wavenumber \( k_0 \)). When \( N \rightarrow \infty \), i.e. tending towards a grating, this distance \( d_{opt} \) asymptotically tends to the wavelength \( \lambda_0 \) (not shown). The enhanced field also increases and reaches \( |A^{max}| = d_{opt}/w \) [8, 13, 41] within all the cavities: compared to the single slit where \( |A_0^{max}| = \lambda_0/(\pi w) \), we finally gain a \( \pi \) factor in a periodical structure. However, \( d_{opt} \) must stay just lower than \( \lambda_0 \) (the cavities do not resonate exactly in phase); otherwise a new diffraction order (radiation leak) appears.

Eventually, let us comment on the case where slits would have a finite length \( L \) in their longitudinal direction (rectangular hole). In this case, the magnetic field inside each cavity becomes [11, 42]

\[
H^H_z = \sin \left[ \frac{\pi}{L} \left( z + \frac{L}{2} \right) \right] \left[ A \cos(k_y y) + B \sin(k_y y) \right] \cap_w (x) \cap_L (z)
\]

with \( k_y = \sqrt{k^2 - (\pi/L)^2} \). Enhanced transmission occurs near the cut-off wavelength \( \lambda \sim 2L \) of the fundamental waveguide mode [43] and independently of \( h \) (we have some zero-order FP resonators), which allows the use of metallic films thinner than those for infinite channels. The scattering phenomena described above, based on dipolar-type couplings, should remain valid since rectangular holes exhibit a significant dipolar momentum along their vertical walls at the cut-off frequency.

4. Conclusion

In conclusion, we have shown that when varying the coupling distance between two subwavelength metallic slits made in a thick metallic screen, we obtain a concomittant tuning of their eigenfrequencies and transmission powers in the spectral region of the FP mode. Far-field and near-field amplitudes can be strongly reduced or enhanced compared to that of a single slit. We have presented a microscopic and analytical study of the system, enabling fine control of their scattering resonances and antenna-like radiation patterns. The sub- or super-radiance effects [44] explored here are due to dipole–dipole-type interactions between the slits. SPP at the horizontal interfaces are not implied but can perturb (spectral shift) or damp these effects. Physical interpretations were shown in the perfect metal case, but quantitative results should be established with an exact model at frequencies where the metal permittivity is not strongly negative. This will be discussed in our future work. Eventually, some general properties could be transposed to other kinds of subwavelength resonators having a different geometry and generating their own dipole momenta. They could even be used, for example, in the development of new electron waveguides exhibiting optical-like behavior [45]. Our calculations and the
physics described could also find some applications in acoustics where such resonant effects may also be observed [46].

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