Spontaneous topological transitions in polariton condensates due to spin bifurcations

H. Sigurdsson, 1 Y. S. Krivosenko, 2 I. V. Iorsh, 2 I. A. Shelykh, 3, 2 and A. V. Nalitov 3, 2

1 School of Physics and Astronomy, University of Southampton, SO17 1BJ, Southampton, United Kingdom
2 ITMO University, St. Petersburg 197101, Russia
3 Science Institute, University of Iceland, Dunhagi 3, IS-107, Reykjavik, Iceland

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We theoretically explore nonresonantly pumped polaritonic graphene, a system consisting of a honeycomb lattice of micropillars in the regime of strong light-matter coupling. We demonstrate that, depending on the parameters of the structure, such as intensity of the pump and coupling strength between the pillars, the system shows rich variety of macroscopic ordering, including analogs of ferromagnetic, antiferromagnetic, and resonant valence bond phases. Transitions between these phases are associated with dramatic reshaping of the spectrum of the system connected with spontaneous appearance of topological order.

Introduction. Recent decades have witnessed a shift in attention from both the condensed matter and optical community in investigation on the properties of bulk of materials to instead the properties of their interfaces. It is now well known that there exists a particular class of materials with inverted structure of the bands, which possess protected states propagating on the system surface, referred to as topological insulators [1–3]. The energy of these edge states lies within the bandgap, and thus they are protected with respect to scattering into the bulk. Depending on the dimensionality of a system, one should distinguish between three-dimensional (3D) topological insulators where topological phases appear on the two-dimensional (2D) surface boundary of the bulk [4–10], and 2D topological insulators where chiral 1D channels form on the system boundary [11, 12]. In the field of condensed matter physics, the classical example of the latter case is presented by a heterostructure formed by a layer of HgTe sandwiched between CdTe bulk.

It has been recently shown that optical analogs of topologically nontrivial phases, in 2D systems, may arise in purely photonic structures [13, 14] or when driven into the strong light-matter coupling regime, where hybrid quasiparticles known as cavity polaritons are formed [15–20]. Polaritons combine the advantages of photons, such as extremely low effective mass and long coherence length, with those of excitons, namely the possibility of control by external electric and magnetic fields together with strong nonlinear response stemming from interparticle interactions. The spin structure of the exciton (or rather, the polariton) is then directly related to the circular polarization degree of the cavity photonic mode. Such a union leads to a rich interplay between nonlinear and topological properties [21–25].

The vast majority of the current proposals on 2D polariton topological insulators are based on the following requirements: Polaritons should be placed into a 2D lattice of a particular symmetry allowing the appearance of Dirac points in the Brillouin zone where the bands touch each other. Examples are honeycomb [15, 20] and Kagome [18, 23, 24] lattices which can be obtained either by controllable etching of a planar microcavity or by using spatial light modulator to control the profile of the external optical pump. The band inversion and opening of the topological gap is then achieved by cumulative action of the TE-TM splitting of the photonic mode and Zeeman splitting of the excitonic mode induced by the application of an external magnetic field [15]. However, in conventional semiconductor materials excitonic g-factors are extremely small, and one needs magnetic fields of tens of Tesla to open the topological bandgap of at least several meV. The situation can be potentially improved by using diluted magnetic microcavities [26–28]. However, the technology of producing a high quality patterned semimagnetic cavity is still only in its initial stages.

In the present letter we develop an alternative approach for the realization of a 2D polariton Z-topological insulator without application of any external magnetic fields. Our idea is based on the concept of the spontaneous spin bifurcation in a system of localized interacting polariton condensates, first proposed in the Ref. [29] and developed further in the works [30–33]. Here we consider a honeycomb polariton condensate lattice (polariton graphene) under nonresonant pumping. We demonstrate that, depending on the pump intensity and coupling strength between the nodes (condensates) of the lattice, the spin bifurcation mechanism can result in spontaneous formation of distinct spin-ordered lattice phases analogous to ferromagnetic (FM), antiferromagnetic (AFM), and resonance valence bond states. Transition between different phases is associated with cardinal reshaping of the spectrum of the excitations of the system and spontaneous appearance/disappearance of topological order.

Spin bifurcations and phase transitions in polariton graphene. A lattice of driven-dissipative connected polariton condensates is conventionally modeled with a set of generalized Gross-Pitaevskii equations for the spinor order parameters $\Psi_n = (\psi_{n+}, \psi_{n-})^T$, corresponding to...
spin-up and spin-down polaritons at the \( n \)-th site:

\[
\begin{align*}
    i \frac{d \Psi_n}{dt} &= \left[ -\frac{i}{2} g(S_n) - \epsilon + i \gamma \dot{\sigma}_x + \frac{1}{2} \left( \alpha S_n + \alpha S_n^z \sigma_z \right) \right] \Psi_n \\
    &- \frac{1}{2} \sum_{\langle nm \rangle} \left( J + \delta J \left( \cos(2\varphi_m) \sigma_x + \sin(2\varphi_m) \sigma_y \right) \right) \Psi_m, \quad (1)
\end{align*}
\]

where the summation is taken over the nearest neighbors, \( \varphi_m \) are the angles of links connecting the neighboring sites \( n \) and \( m \) of the honeycomb lattice. We can define the \( n \)-th node particle population \( S_n \) and \( z \)-component of the condensate pseudospin \( S_n^z \):

\[
S_n = \frac{|\psi_{n+}^z|^2 + |\psi_{n-}^z|^2}{2}, \quad S_n^z = \frac{|\psi_{n+}^z|^2 - |\psi_{n-}^z|^2}{2}. \quad (2)
\]

We also defined effective decay rate \( g(S_n) = \eta S_n + \Gamma - W \) with \( \Gamma \) being the polariton decay rate, \( W \) the replenishment rate of the condensate non-polarized incoherent pump, and \( \eta \) is the gain-saturation nonlinearity. The constants \( \epsilon \) and \( \gamma \) define the splitting of the \( XY \)-polarized states in both energy and decay respectively due to the inherent cavity birefringence, and \( \alpha = \alpha_1 + \alpha_2 \) and \( \alpha = \alpha_1 - \alpha_2 \) are spin-anisotropic interaction parameters. Finally, \( J > \delta J \) are spin conserving and non-conserving tunneling rates (coupling strength) of polaritons between nodes respectively.

The condensation threshold of the system is defined as the point where an eigenvalue of the linearized Eq. (1) obtains a positive imaginary component due to increase of the laser power \( W \) leading to the triggering of the stimulated bosonic scattering into the condensed state at \( W_{\text{cond}} = \Gamma - \gamma \). Due to the splitting \( \gamma \) in the lifetimes of the linear polarized states the condensate first forms an in-phase, \( Y \)-polarized state, i.e., \( \Psi_n = \Psi_{n+1} \propto (1, -1)^T \) (white area in Fig. 1a). This \( Y \)-polarized state however becomes unstable at higher pumping powers and undergoes a bifurcation into one of two states with high degree of the circular polarization at individual nodes [29, 32].

We begin our consideration by presenting a class of stationary solutions which minimize the spin bifurcation threshold [33],

\[
\Psi_n = \begin{cases} 
\Psi_{n+1}, & \text{if } S_n^z = S_{n+1}^z, \\
-\sigma_y \Psi_{n+1}, & \text{if } S_n^z = -S_{n+1}^z.
\end{cases} \quad (3)
\]

The ansatz above describes in-phase FM bonds and anti-phase AFM bonds between nearest neighbors respectively. Plugging Eq. (3) into Eq. (1), and setting the condition that all nodes have the same number of co- and counter-polarized nearest neighbours (equivalence criteria), the coupled set of the equations of motion reduces to a single equation with a bifurcation threshold,

\[
W_{\text{bif}} = \Gamma - \gamma + \eta \frac{(\epsilon - n_{\uparrow \downarrow} J)^2 + \gamma^2}{\alpha (\epsilon - n_{\uparrow \downarrow} J)}, \quad (4)
\]

where \( n_{\uparrow \downarrow} \) denotes the number of AFM neighbors.

In Fig. 1a we plot the minimum of Eq. (4) as a function of coupling strength \( J \) (red curve) neglecting TE-TM splitting. The cusps in the red curve indicate that the lowest bifurcation point is shared between two distinct spin phases. The four spin phases of interest, obtained by direct numerical integration of Eq. (1), are shown in the Fig. 1b(i-iv) and are labeled \( \text{AFM}, \text{Dipole}, \text{Stripe}, \) and \( \text{FM} \) phases respectively.

We point out that the bifurcation threshold for AFM and FM phases is invariant of \( \delta J \) whereas for dipole and stripe phases, strictly speaking, this is not the case as for them the ansatz given by the Eq. (3) should be modified in the presence of TE-TM splitting. However, given that \( \delta J / J \ll 1 \) (which is usually the case in standard semiconductor microcavities), it is reasonable to infer that \( W_{\text{bif}} \) is only weakly affected by \( \delta J \) and that the calculated red curve shown in the Fig. 1a serves as a good indicator for the bifurcation threshold of these nontrivial states in the presence of TE-TM splitting. We have performed numerical calculations of Eq. (1) that verify that this is indeed
the case.

**Band structure and topological states.** In the following we discuss the excitation spectra of the stable spin bifurcated condensate configurations. We employ the effective field model, treating the effect of spin-polarized lattice nodes by introduction of local z-directed (out of the cavity plane) magnetic fields instead of considering the full Bogoliubov diagonalization. This approximation is valid in the high energy limit $E > U$ with $U$ being characteristic interaction energy of the condensates which can be estimated as: $U \approx \alpha_1 |\psi_n|^{2}$. The effective model allows clear analytical investigation of the topological properties of the lowest band gap opening at $E \sim J$ as long as $U \ll J$. Consideration of the effects appearing due to the small, birefringence induced, complex in-plane magnetic field, governed by the parameters $\epsilon$ and $\gamma$, is left for future investigation.

Firstly, to examine the band structures of the AFM (i) and FM (iv) configurations shown in the Fig. 1b, we scrutinize the following $4 \times 4$ tight-binding Hamiltonian:

$$H_{k} = -\frac{1}{2} \begin{pmatrix} 0 & \hat{J}_{k}^{\dagger} \\ \hat{J}_{k} & 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} \mu_1 \Delta \sigma_z & 0 \\ 0 & \mu_2 \Delta \sigma_z \end{pmatrix}. \tag{5}$$

The above Hamiltonian is written in the basis of the bispinor $|A, + \rangle$ inner-cell states, where $A$ and $B$ indicate the graphene sublattices, and “+(-)” specifies right (left) circular polarization (i.e., the spin of the polaritons). The total Hamiltonian in momentum space is then written $\hat{H} = \sum_{k} |k\rangle \langle k| \otimes H_{k}$. The first term in (5) corresponds to the polaritonic graphene with TE-TM splitting. The $2 \times 2$ operator $\hat{J}_{k}$ dependent on the quasi-wavevector $k = (k_1, k_2)$ is written,

$$\hat{J}_{k} = \hat{J}_{1} + \hat{J}_{2} e^{-ik_1} + \hat{J}_{3} e^{-ik_2}, \tag{6}$$

with

$$\hat{J}_{m} = \begin{pmatrix} J & \delta J e^{-2i\phi_m} \\ \delta J e^{2i\phi_m} & -J \end{pmatrix}, \quad m = 1, 2, 3. \tag{7}$$

and $J$ and $\delta J$ as in (1). Both $k_1$ and $k_2$ can be chosen to vary from $-\pi$ to $\pi$ and to cover the whole Brillouin zone. In the second term of the Hamiltonian (5), the on-diagonal blocks $\mu_1(2)\Delta \sigma_z$ serve to effectively account for the excitations in the magnetic patterns depicted in Fig. 1b. The magnitude of the Zeeman splitting induced by the polarized condensate is written $\Delta = \alpha |S^z|$, where $\sigma_z$ is the z-Pauli matrix, and the coefficients $\mu_1$ and $\mu_2$ define FM ($\mu_1 = \mu_2 = 1$) and AFM ($\mu_1 = -\mu_2 = 1$) phases. The translation basis vectors $(a_{1,2})$ in real space are chosen to be conventional for the graphene lattice.

Since FM phase corresponds to the case of a uniform, out-of-plane, external magnetic field, a gap opens (see Fig. 2d) between the bands, characterized by different Chern numbers, and bridged by chiral edge states [15]. Fig. 2d(i) displays the band structure of this phase, which is numerically obtained for $\delta J = 2/3$, $\Delta = 5/3$, with energy counted in units of $J$. We used the convention for which $K$ and $K'$ points in the first Brillouin zone are positioned at $(k_1, k_2)$ equal to $(2\pi/3, -2\pi/3)$, and vice versa, $\Gamma$ is placed at the origin, and $M$ at $(\pi, \pi)$. Fig. 2d(ii) shows a slice of the band structure along $\Gamma \rightarrow M \rightarrow K \rightarrow \Gamma$ pathway shown by the green solid

![Fig. 2](image-url)

(i) The band structure of the whole Brillouin zone for the four spin phases in the honeycomb lattice of spin bifurcated polariton condensates. Here, $J = 1$ is taken as the unit of energy. (A, D): all the four bands are displayed, (B, C): only two bands above and two bands under the midgap are presented. Bottom panels show (ii) the eigenenergies along $\Gamma \rightarrow M \rightarrow K \rightarrow \Gamma$ pathway, and (iii) band structure of the ribbon with zigzag edges, the edge states are depicted by red and green dots. Parameters for the AFM: $\delta J = 0.1J$, $\Delta = 1.3J$. Stripe: $\delta J = 0.1J$, $\Delta = 2.2J$. Dipole: $\delta J = 0.1J$, $\Delta = 3J$. FM: $\delta J = 2J/3$, $\Delta = 5J/3$. The band structures of the bulk were calculated on a 200×200 mesh grid in $k$-space. The size of ribbons was 30 (width) by 100 (length) unit cells.

In the second term of the Hamiltonian (5), the effective field model allows clear analytical investigation of the topological properties of the lowest band gap opening at $E \sim J$ as long as $U \ll J$. Consideration of the effects appearing due to the small, birefringence induced, complex in-plane magnetic field, governed by the parameters $\epsilon$ and $\gamma$, is left for future investigation.

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The solid orange line separates the gapless (solid dark blue) and gapped (linearly fading blue) domains. The dashed orange lines are positive-integer-$\delta J$ contours. (E) The band structures of FM-type ribbons for $\alpha$: $\delta J=0.2$, $\Delta=0.4$. (\(\beta\)): $\delta J=2/3$, $\Delta=5/3$, and (\(\gamma\)): $\Delta J=0.5$, $\Delta=3.5$. The bands Chern numbers are displayed in light yellow boxes. Here $J=1$ is taken as a unit of measurement.

Note, that a unit cell in these two cases differs from that corresponding to AFM and FM phases and should be constructed as a pair of graphene unit cells taken successively, with the translational vectors being $a_1$ and $2a_2$. The first term in Eq. (8) represents polaritonic graphene with TE-TM splitting, the second term is responsible for the magnetic patterns: for the dipole phase one sets $\mu_1 = \mu_4 = 1$, $\mu_2 = \mu_3 = -1$, for the stripe phase $\mu_1 = \mu_2 = 1$, $\mu_3 = \mu_4 = -1$. Both phases are characterized by topologically trivial band structure and edge states localized within the bulk (see Fig. 2b,c). Figure 3(b,c) shows phase diagram for dipole and stripe phases with points B and C corresponding to the Fig. 2b,c. In the trivial case $\delta J = 0$ one arrives to the gap opening condition $\Delta > J$ and $\Delta > \sqrt{3}J$ for dipole and stripe spin phases respectively which is plotted in Fig. 1a indicating that gap opening only takes place at higher condensate densities (i.e., higher excitation powers).

Conclusions. We have proposed an experimentally friendly geometry for realization of optical $Z$-topological insulator based on polaritonic graphene in the spin bifurcation regime. Differently from previous works, our proposal does not require application of an external magnetic field and the topological order appears spontaneously through many-particle interactions in the regime of non-resonant and non-polarized pumps.

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[1] M. Z. Hasan and C. L. Kane Rev. Mod. Phys. 82, 3045 (2010).
[2] Xiao-Liang Qi and Shou-Cheng Zhang, Rev. Mod. Phys. 83, 1057 (2011).
[3] A. Bansil, Hsin Lin, and Tammoy Das, Rev. Mod. Phys. 88, 021004 (2016).
[4] C. L. Kane and E. J. Mele, Phys. Rev. Lett. 95, 146802 (2005).
[5] L. Fu, C. L. Kane, and E. J. Mele, Phys. Rev. Lett. 98, 106803 (2007).
[6] J. E. Moore and L. Balents, Phys. Rev. B 75, 121306 (2007).
[7] R. Roy, Phys. Rev. B 79, 195322 (2009).
[8] Y. Xia, D. Qian, D. Hsieh, L. Wray, A. Pal, H. Lin, A. Bansil, D. Grauer, Y. S. Hor, R. J. Cava, and M. Z. Hasan, Nature Phys. 5, 398 (2009).
[9] Y. Zhang, C.-X. Liu, X.-L. Qi, X. Dai, Z. Fang, and S.-C. Zhang, Nature Phys. 5, 438 (2009).
[10] Y. L. Chen, J. G. Analytis, J.-H. Chu, Z. K. Liu, S.-K. Mo, X. L. Qi, H. J. Zhang, D. H. Lu, X. Dai, Z. Fang, S. C. Zhang, I. R. Fisher, Z. Hussain, and Z.-X. Shen, Science 325, 178 (2009).
[11] B. A. Bernevig, T. L. Hughes, and S.-C. Zhang, Science 314, 1757 (2006).
[12] M. Konig, S. Wiedmann, C. Brune, A. Roth, H. Buhmann, L. W. Molenkamp, X.-L. Qi, and S.-C. Zhang, Science 318, 766 (2007).
[13] F. Haldane, and S. Raghu, Phys. Rev. Lett. 100, 013904 (2008).
[14] A. B. Khanikaev, S. H. Mousavi, W.-K. Tse, M. Kargarian, A. H. MacDonald, and G. Shvets, Nat. Mater. 12, 233-239 (2013).
[15] A. V. Nalitov, D. D. Solnyshkov, and G. Malpuech, Phys. Rev. Lett. 114, 116401 (2015).
[16] T. Karzig, C.-E. Bardyn, N. H. Lindner and G. Refael, Phys. Rev. X 5, 031001 (2015).
[17] C.-E. Bardyn, T. Karzig, G. Refael and T. C. H. Liew, Phys. Rev. B 91, 161413 (2015).
[18] D. R. Gulevich, D. Yudin, I. V. Iorsh, and I. A. Shelykh, Phys. Rev. B 94, 115437 (2016).
[19] V. K. Kozin, I. A. Shelykh, A. V. Nalitov, and I. V. Iorsh, Phys. Rev. B 98, 125115 (2018).
[20] S. Klomt, T. H. Harder, O. A. Egorov, K. Winkler, R. Ge, M. A. Bandres, M. Emmerling, L. Worschech, T. C. H. Liew, M. Segev, C. Schneider, and S. Höfling, Nature 562, 552-556 (2018).
[21] O. Bleu, D. D. Solnyshkov, and G. Malpuech, Phys. Rev. B 93, 085438 (2016).
[22] Charles-Édouard Bardyn, Torsten Karzig, Gil Refael, and Timothy C. H. Liew, Phys. Rev. B 93, 020502(R) (2016).
[23] D. R. Gulevich, D. Yudin, D. V. Skryabin, I. V. Iorsh and I. A. Shelykh, Sci. Rep. 7, 1780 (2017).
[24] H. Sigurdsson, G. Li, and T. C. H. Liew, Phys. Rev. B 96, 115453 (2017).
[25] Yaroslav V. Kartashov and Dmitry V. Skryabin, Phys. Rev. Lett. 119, 253004 (2017).
[26] A. Brunetti, M. Vladimirova, D. Scalbert, R. Andr, D. Solnyshkov, G. Malpuech, I. A. Shelykh, and A. V. Kavokin, Phys. Rev. B 73, 205337 (2006).
[27] Mateusz Krol, Rafal Mirek, Katarzyna Lekenta, Jean-Guy Rousset, Michal Nawrocki, Michal Matuszewski, Jacek Szczytko, Wojciech Pauksy, Barbara Pietka, Sci. Rep. 8, 6694 (2018).
[28] M. Krol, R. Mirek, D. Stephan, K. Lekenta, J.-G. Rousset, W. Pauksy, A. V. Kavokin, M. Matuszewski, J. Szczytko, B. Pietka, arXiv:1809.00757.
[29] H. Ohadi, A. Dreismann, Y.G. Rubo, F. Pinsker, Y. del Valle-Inclan Redondo, S.I. Tsintzos, Z. Hatzopoulos, P.G. Savvidis, and J.J. Baumberg, Phys. Rev. X 5, 031002 (2015).
[30] H. Ohadi, Y. del Valle-Inclan Redondo, A. Dreismann, Y.G. Rubo, F. Pinsker, S.I. Tsintzos, Z. Hatzopoulos, P.G. Savvidis, and J.J. Baumberg Phys. Rev. Lett. 116, 106403 (2016).
[31] Alexander Dreismann, Hamid Ohadi, Yago del Valle-Inclan Redondo, Ryan Balili, Yuri G. Rubo, Simeon I. Tsintzos, George Deligeorgis, Zacharias Hatzopoulos, Pavlos G. Savvidis, and Jeremy J. Baumberg, Nature Materials 15, 1074 (2016).
[32] H. Ohadi, A. J. Ramsay, H. Sigurdsson, Y. del Valle-Inclan Redondo, I.A. Shelykh, Y.G. Rubo, P.G. Savvidis, and J.J. Baumberg Phys. Rev. Lett. 119, 067401 (2017).
[33] H. Sigurdsson, A. J. Ramsay, H. Ohadi, Y. G. Rubo, T. C. H. Liew, J. J. Baumberg, and I. A. Shelykh, Phys. Rev. B 96, 155403 (2017).
[34] A. V. Nalitov, G. Malpuech, H. Tercas, and D. D. Solnyshkov, Phys. Rev. Lett. 114, 026803 (2015).