Acceleration of particles by black hole with gravitomagnetic charge immersed in magnetic field

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Abstract

The collision of test charged particles in the vicinity of an event horizon of a non-rotating black hole with gravitomagnetic charge immersed in external magnetic field has been studied. The presence of the external magnetic field decreases the innermost stable circular orbits (ISCO) radii of charged particles. The opposite mechanism occurs when the gravitomagnetic charge of a black hole is nonvanishing. For a collision of charged particle moving at ISCO and the neutral particle falling from infinity the maximal collision energy can be decreased by gravitomagnetic charge in the presence of external asymptotically uniform magnetic field.

Keywords

Particle motion \quad Acceleration mechanism \quad NUT spacetime

1 Introduction

At present there is no any observational evidence for the existence of gravitomagnetic monopole, i.e. so-called NUT (Newman et al. 1963) parameter or magnetic mass. Therefore study of the motion of the test particles and particle acceleration mechanisms in NUT spacetime may provide new tool for studying new important general relativistic effects which are associated with nondiagonal components of the metric tensor and have no Newtonian analogues (See, e.g. Nouri-Zonoz and Lynden-Bell (1997); Lynden-Bell and Nouri-Zonoz (1998); Nouri-Zonoz et. al (1999); Nouri-Zonoz (2004); Kagrananova et al. (2008, 2010); Morozova and Ahmedov (2009) where solutions for electromagnetic waves and interferometry in spacetime with NUT parameter have been studied.). Kerr-Taub-NUT spacetime with Maxwell and dilation fields is recently investigated by authors of the paper Aliev et al. (2008). In our preceding papers Morozova et al. (2008); Abdujabbarov et al. (2008) we have studied the plasma magnetosphere around a rotating, magnetized neutron star and charged particle motion around compact objects immersed in external magnetic field in the presence of the NUT parameter. The Penrose process in the spacetime of rotating black hole with nonvanishing gravitomagnetic charge has been considered in Abdujabbarov et al. (2011). The electromagnetic field of the relativistic star with nonvanishing gravitomagnetic charge has been considered by Ahmedov et. al (2012).

Astrophysical processes which may produce high energy radiation near a rotating black hole horizon attract more attention in recent publications. The processes which are related to the effect of Penrose (Penrose 1963) have been properly considered in Piran et. al. (1975); Piran and Shaham (1977a,b). Recently Banados et al. (2009) (BSW) pointed out that the collisions of particles near extremely rotating black hole can produce particles of high center-of-mass energy. The results of Banados et al. (2009) have been commented by Berti et al. (2009) where authors concluded that astrophysical limitations on the maximal spin, back-reaction effects and sensitivity to the initial conditions impose severe limits on the likelihood of such accelerations. The acceleration of particles, circular
geodesics, accretion disk, and high-energy collisions in the Janis-Newman-Winicour spacetime have been considered by [Patil and Joshi 2012; Chowdhury et al. 2012]. The above mentioned discussions forces us to study the particle acceleration in the spacetime of a black hole with nonvanishing gravitomagnetic charge.

Our aim in the present paper is to examine an above mentioned effect of particle acceleration in the presence of gravitomagnetic charge for the case when the collision of particles occurs in the vicinity of nonrotating black hole embedded in magnetic field. It is very interesting to study the electromagnetic fields and particle motion in NUT space with the aim to get new tool for studying new important general relativistic effects. Moreover this demonstration can be interesting because of the existence of both theoretical and experimental evidences that a magnetic field must be present in the vicinity of black holes. Note, that hereafter we use the weak magnetic field approximation in such sense that the energy-momentum of this field does not change the background geometry of black hole. For a black hole with mass \(M\) this condition means that the strength of magnetic field satisfies to the following condition [Piotrovich et.al 2011; Frolov 2012]

\[
B < B_{\text{max}} = \frac{c^4}{G^{3/2}M_\odot} \left( \frac{M_\odot}{M} \right) \sim 10^{19} \frac{M_\odot}{M} \text{Gauss}. \tag{1}
\]

Following to [Frolov 2012], let us call these black holes as weakly magnetized. One can say that this condition is quite general and satisfies for both stellar mass and supermassive black holes.

The paper is organized as follows. Section II is devoted to study of electromagnetic field and charged particle motion in the magnetized black holes with NUT parameter, with the main focus on the properties of their ISCOs. Particles collisions in the vicinity of a weakly magnetized black hole with nonvanishing gravitomagnetic charge have been studied in Section III. The concluding remarks and discussions are presented in Section IV.

Throughout the paper, we use a space-like signature \((-, +, +, +)\) and a system of units in which \(G = 1 = c\) (However, for those expressions with an astrophysical application we have written the speed of light explicitly). Greek indices are taken to run from 0 to 3 and Latin indices from 1 to 3; covariant derivatives are denoted with a semi-colon and partial derivatives with a comma.

2 Charged particle motion around black hole with nonvanishing NUT charge

Here we will consider a charged particle motion in the vicinity of a black hole of mass \(M\) with gravitomagnetic charge in the presence of an external axisymmetric and uniform at the spatial infinity magnetic field. The appropriate spacetime metric has the following form (see [Dadhich and Turukulov 2002; Bini et al. 2003]):

\[
ds^2 = -\frac{\Delta}{\Sigma} \, dt^2 + 4\frac{\Delta}{\Sigma} \, l \cos \theta dt d\varphi + \frac{1}{\Sigma} \left( \Sigma^2 \sin^2 \theta - 4\Delta l^2 \cos^2 \theta \right) d\varphi^2 + \frac{\Delta}{\Sigma} \, dr^2 + \Sigma d\theta^2, \tag{2}
\]

where parameters \(\Sigma\) and \(\Delta\) are defined as

\[
\Sigma = r^2 + l^2, \quad \Delta = r^2 - 2Mr - l^2,
\]

where \(l\) is the gravitomagnetic monopole momentum. Here we will exploit the existence in this spacetime of a timelike Killing vector \(\xi^\alpha(t) = \partial x^\alpha/\partial t\) and spacelike one \(\xi^\alpha(\varphi) = \partial x^\alpha/\partial \varphi\) being responsible for stationarity and axial symmetry of geometry, such that they satisfy the Killing equations

\[
\xi_{\alpha;\beta} + \xi_{\beta;\alpha} = 0 \tag{3}
\]

which gives a right to write the solution of vacuum Maxwell equations \(\Box A^\mu = 0\) for the vector potential \(A_\mu\) of the electromagnetic field in the Lorentz gauge in the simple form

\[
A^\alpha = C_1 \xi^\alpha(t) + C_2 \xi^\alpha(\varphi). \tag{4}
\]

The constant \(C_2 = B/2\), where gravitational source is immersed in the uniform magnetic field \(B\) being parallel to its axis of rotation. The value of the remaining constant \(C_1\) can be easily calculated from the asymptotic properties of spacetime [2] at the infinity. Indeed in order to find the remaining constant one can use the electrical neutrality of the black hole

\[
4\pi Q = \frac{1}{2} \oint F^{\alpha \beta} \cdot dS_{\alpha \beta} = C_1 \oint \Gamma^\alpha_{\beta \gamma} \tau_\alpha m^\beta \xi^\gamma(t)(\tau k) dS + \frac{B}{2} \oint \Gamma^\alpha_{\beta \gamma} \tau_\alpha m^\beta \xi^\gamma(\varphi)(\tau k) dS = 0, \tag{5}
\]

evaluating the value of the integral through the spherical surface at the asymptotic infinity. Here the equality \(\xi_{\beta;\alpha} = -\xi_{\alpha;\beta} = -\Gamma^\gamma_{\alpha \beta} \xi_\gamma\) following from the Killing equation was used, and element of an arbitrary 2-surface \(dS^{\alpha \beta}\) is represented in the form (see

Ahmedov and Rakhmatov 2003)

\[
dS^{\alpha \beta} = -\tau^\alpha \wedge m^\beta(\tau k) dS + \eta^{\alpha \beta \mu \nu} \tau_\mu n_\nu \sqrt{1 + (\tau k)^2} dS,
\]
and the following couples

\[ m_{\alpha} = \frac{\eta_{\alpha\mu} \tau^\lambda n^\nu k^\sigma}{\sqrt{1 + (\tau k)^2}}, \]

\[ n_{\alpha} = \frac{\eta_{\alpha\mu} \tau^\lambda k^\mu n^\nu}{\sqrt{1 + (\tau k)^2}}, \]

\[ k^\alpha = -(\tau k)^\alpha + \sqrt{1 + (\tau k)^2} \eta^\mu_{\alpha\nu} \tau_{\mu} m_{\rho} n_{\nu}. \]

are established between the triple \{k, m, n\} of vectors, \( n^\alpha \) is normal to 2-surface, space-like vector \( m^\alpha \) belongs to the given 2-surface and is orthogonal to the four-velocity of observer \( \tau^\alpha \), a unit spacelike four-vector \( \alpha \) to the given 2-surface and is orthogonal to \( m^\alpha \), \( dS \) is invariant element of surface, \( \wedge \) denotes the wedge product, \( \ast \) is for the dual element, \( \eta_{\alpha\beta\gamma\delta} \) is the pseudotensorial expression for the Levi-Civita symbol \( \epsilon_{\alpha\beta\gamma\delta} \).

Then one can insert \( \tau_0 = -(1 - M/r) \), \( m^1 = (1 - M/r) \), and asymptotic values for the Christoffel symbols \( \Gamma^1_{00} = M/r^2 \) and \( \Gamma^0_{11} = -l(1 - 2M/r) \cos \theta/r \) in the flux expression (11) and get the value of constant \( C_1 = 0 \). Parameter \( l \) does not affect on constant \( C_1 \) because the integral \( \int_0^{\pi} \cos \theta \sin \theta d\theta = 0 \) vanishes.

Thus the 4-vector potential \( A_\mu \) of the electromagnetic field will take the following form

\[ A_0 = -\frac{\Delta}{\Sigma} Br \cos \theta \]

\[ A_3 = \frac{1}{2} \Sigma^2 \sin^2 \theta - 2 \Delta l^2 \cos^2 \theta \]  

The orthonormal components of the electromagnetic fields measured by zero angular momentum observers (ZAMO) with four velocity components

\[ (\tau^\alpha)_{\text{ZAMO}} \equiv \left( \frac{\sqrt{\Sigma}}{\Delta \Sigma \sin^2 \theta}, 0, 0, \frac{2 \Delta l \cos \theta}{\sqrt{\Delta \Sigma \Sigma \sin^2 \theta}} \right) \]  

\[ (\tau_\alpha)_{\text{ZAMO}} \equiv \left( \frac{\Delta \Sigma \sin^2 \theta}{\Sigma}, 0, 0, 0 \right) \]  

are given by expressions

\[ E^r = -\frac{Br l}{\sqrt{\Sigma}} \left( 1 - \frac{M}{r} \right) \sin 2\theta, \]  

\[ E^\phi = \frac{Bl}{\Sigma^2} \left( \frac{\Delta}{\Sigma \sin \theta} \right) \left[ \Sigma^2 + (\Sigma^2 - 2 \Delta l^2 \cos \theta) \frac{2 \cos \theta}{\sin^2 \theta} \right] \sin^2 \theta, \]  

\[ B^r = \frac{B \tan \theta}{\Sigma \sqrt{\Sigma}} \left( R - \Sigma^2 \right), \]  

\[ B^\phi = \frac{Br}{\Sigma^2} \left( \frac{\Delta}{\Sigma} \cos^2 \theta \right) \]  

\[ \times \left\{ \left[ \Delta - \Sigma \left( 1 - \frac{M}{r} \right) \right] 4 l^2 + \Sigma^2 \tan^2 \theta \right\}, \]  

where the following notation has been used:

\[ R = \Sigma^2 \sin^2 \theta - 4 \Delta l^2 \cos^2 \theta. \]

Astrophysically it is interesting to know the limiting cases of expressions (11) - (14), for example in either linear or quadratic approximation \( O(a^2/r^2, l^2/r^2) \) in order to give physical interpretation of possible physical processes near the slowly rotating relativistic compact objects, where they take the following form:

\[ E^r = \frac{2Bl \cos \theta}{r} \left( 1 - \frac{3M}{r} \right), \]  

\[ E^\phi = \frac{Bl \sin \theta}{r} \left( 1 + \frac{2 \cos \theta}{\sin^2 \theta} \right), \]  

\[ B^r = B \cos \theta \left[ 1 + \frac{2 l^2}{r^2 \sin^2 \theta} \right], \]  

\[ B^\phi = B \sin \theta \left[ 1 - \frac{M}{r} + \frac{1}{16 r^2 \sin^2 \theta} \right. \]  

\[ \times \left( 4 l^2 - 4 M^2 + 4 (7 l^2 + M^2) \cos 2\theta \right) \]  

\[ \left\} \right., \]  

In the limit of flat spacetime, i.e. \( M/r \rightarrow 0 \) and \( l^2/r^2 \rightarrow 0 \), expressions (15)-(18) give

\[ E^r \rightarrow 0, \]  

\[ B^r \rightarrow B \cos \theta, \quad B^\phi \rightarrow B \sin \theta. \]  

As expected, expressions (19)- (20) coincide with the solutions for the homogeneous magnetic field in Newtonian spacetime.

The dynamical equation for a charged particle motion can be written as

\[ m \frac{dv^\mu}{d\tau} = q F^\mu_\nu u^\nu, \]  

(21)
where $\tau$ is the proper time, $u^\mu$ is the 4-velocity of a charged particle, $u^\mu u_\mu = -1$, $q$ and $m$ are its charge and mass, respectively. $F_{\alpha\beta} = A_{\beta,\alpha} - A_{\alpha,\beta}$ is the antisymmetric tensor of the electromagnetic field, which has the following four independent components

\[
F_{01} = \frac{B}{\Sigma} 2 [(r - M) - \Delta r] \cos \theta , \\
F_{02} = \frac{B}{\Sigma} \Delta l \sin \theta , \\
F_{13} = -\frac{B}{\Sigma^2} 8 l^2 [(r - M) - r \Delta r + B r \sin^2 \theta , \\
F_{23} = \frac{B}{\Sigma} (4 l^2 \Delta + \frac{1}{2} \Sigma^2) \sin 2 \theta .
\]

While considering the charged particle motion around black hole immersed in the magnetic field it is easy to use two conserved quantities associated with the Killing vectors: the energy $E > 0$ and the generalized azimuthal angular momentum $L \in (-\infty, +\infty)$:

\[
E = -\xi_{(t)}^\mu P_\mu = \frac{m \Delta}{\Sigma} \left( \frac{dt}{d\tau} + 4 l \cos \theta \frac{d\varphi}{d\tau} + \frac{q}{m} B l \cos \theta \right),
\]

\[
L = \xi_{(\varphi)}^\mu P_\mu = \frac{q B}{m} l \cos \theta \frac{dt}{d\tau} + (\Sigma \sin^2 \theta - 4 l^2 \Delta \sin^2 \theta) \left( \frac{d\varphi}{d\tau} + \frac{q B}{m} \right).
\]  

(23)

Here $P_\mu = m u_\mu + q A_\mu$ is the generalized 4-momentum of a charged particle. It was first shown by Zimmerman and Shukla (1980) for spherical symmetric case (NUT spacetime) and later by Bini et al. (2003) for axial symmetric case (Kerr-Taub-NUT spacetime) that the orbits of the test particles are confined to a cone with the opening angle $\theta$ given by $\cos \theta = 2E \Sigma / \Delta$. It also follows that in this case the equations of motion on the cone depend on $l$ only via $l^2$ (Bini et al. 2003; Abdujabbarov et al. 2003). The main point is that the small value for the upper limit for gravitomagnetic moment has been obtained by comparing theoretical results with experimental data as (i) $l \leq 10^{-24}$ from the gravitational microlensing (Rahvar and Habibi 2004), (ii) $l \leq 1.5 - 10^{-18}$ from the interferometry experiments on ultra-cold atoms (Morozova and Ahmedov 2009), (iii) and similar limit has been obtained from the experiments on Mach-Zehnder interferometer (Kagramanova et al. 2003). Due to the smallness of the gravitomagnetic charge let us consider the motion in the quasi-equatorial plane when the motion in $\theta$ direction changes as $\theta = \pi / 2 + \delta \theta(t)$, where $\delta \theta(t)$ is the term of first order in $l$, then it is easy to expand the trigonometric functions as $\sin \theta = 1 - \delta \theta^2(t)/2 + O(\delta \theta^3(t))$ and $\cos \theta = \delta \theta(t) - O(\delta \theta^3(t))$. Neglecting the small terms $O(\delta \theta^2(t))$, one can easily obtain the geodesic equation in the following form

\[
\frac{dt}{d\tau} = \frac{E \Sigma}{m \Delta} ,
\]

\[
\frac{d\varphi}{d\tau} = \frac{L}{m \Sigma} - \frac{q B}{2 m} ,
\]

\[
\frac{dr}{d\tau} = \left( \frac{E}{m} - U \right) ,
\]

where $U$ denotes the effective potential as

\[
U = \Delta \Sigma \left( 1 + \Sigma \chi^2 \right) ,
\]  

(27)

and

\[
\chi = \frac{L}{m \Sigma} - \frac{q B}{2 m} .
\]  

(28)

In the expressions (23)–(26) the terms being proportional to the second and higher orders of the small parameter $\delta \theta$ are neglected. In the Fig. 1 the radial dependence of the effective potential of the radial motion of the charged particle is presented for the different values of dimensionless gravitomagnetic parameter $l/M$ and magnetic parameter $b = qBM/m$.

From the Fig. 1 one can obtain the behavior of the charged particle motion in the presence of both the gravitomagnetic charge and magnetic field. In the presence of the gravitomagnetic charge the minimum of the effective potential shifts towards the observer at the infinity which means that the orbits of the charged particles may become unstable. The minimum value of the radius of the stable circular orbits increases. The influence of the magnetic field has the opposite effect: the presence of the magnetic field decreases the minimum value of the circular orbits radius and charged particles may come much closer to the central object.

In order to study innermost stable circular orbits (ISCO) we use its first and second derivatives of the $U$ and equalize them to zero:

\[
U' = \frac{-2 \Delta r (1 + 2E \Sigma \chi)}{\Sigma^2} + \frac{1}{r} \left( 1 + \frac{\Delta}{\Sigma} \right) (1 + \Sigma \chi^2) = 0
\]  

(29)

\[
U'' = \frac{8 \Delta L^2 \Sigma^2 r^2}{\Sigma^4} + \frac{2}{\Sigma} \left( \frac{4 \Delta r^2 - 3 \Delta}{\Sigma^2} - 2 \right) \times (1 + 2E \Sigma) + \frac{2}{\Sigma} (1 + \Sigma \chi^2) = 0
\]  

(30)

Now we have two equations with three unknown quantities $L$, $r$ and $\chi$. Solving the equation (29) we derive $L$ in terms $r$ and $\chi$

\[
L = -\frac{2 \Delta r^2 + \Delta \Sigma + \Sigma^2 + \Delta \Sigma^2 \chi^2 + \Sigma^3 \chi^2}{4 \Delta r^2 \Sigma^2} .
\]  

(31)
Substituting this equation into $\chi = \pm \frac{K}{A} \left( 1 \pm \sqrt{1 - \frac{A \Sigma K^2}{2} \left( \Delta + \Sigma - \frac{2r^2}{\Sigma} \Delta \right)} \right)$, one can easily obtain the equation expressing $\chi$ in terms of $r$

$$\chi_{\pm} = \frac{K}{\Sigma A} \left( 1 \pm \sqrt{1 - \frac{A \Sigma}{K^2} \left( \Delta + \Sigma - \frac{2r^2}{\Sigma} \Delta \right)} \right)^{\frac{1}{2}}, \quad (32)$$

where

$$A = 8\Delta r^2 - 5\Delta^2 \Sigma + 12\Delta r^2 \Sigma - 8\Delta \Sigma^2 - 3\Sigma^3,$$

$$K = -2\Delta^2 r^2 + 2\Delta^2 \Sigma - 4\Delta r^2 \Sigma + 3\Delta \Sigma^2 + \Sigma^3.$$

The signs $\pm$ correspond to co-rotating and counter-rotating particle orbits, respectively. The dependence of the $\chi_{\pm}$ from the ISCO radius are shown in Fig. 4.

The shift of the shape of $\chi_{\pm}$ to the right direction with increasing the gravitomagnetic charge corresponds to increasing of ISCO radius in the presence of NUT charge.

From the Fig. 2, one can see that $\chi_-$ function steadily increase when particle approach to the black hole horizon and go to infinity in the case when gravitomagnetic charge vanishes. In the case of nonvanishing NUT-parameter, the function $\chi_-$ falls in the point of singularity. But by reason of that jumps are located inside the horizon, the effects that take place there are not observable relatively to detached observer and cannot be interpreted as some full physical theory. In other hand the spacetime of nonrotating black hole allows us to find analytic continuations of our theories inside the black hole horizon up to a point of singularity by excepting it.

We concentrate on a circular motion of the charged particle in the presence of NUT charge. At ISCO radius the effective potential has minimum. The 4-momentum of a test charged particle at the circular orbit of radius $r$ is $(\text{Frolov} (2012))$

$$p^\mu = m\gamma (e^\mu_{(t)} + ve^\mu_{(\phi)}), \quad (33)$$

$$e^\mu_{(t)} = (\Sigma/\Delta)^{1/2} \xi^\mu_{(t)} = (\Sigma/\Delta)^{1/2} \xi^\mu_{(t)}, \quad (34)$$

$$e^\mu_{(\phi)} = \Sigma^{-1/2} \xi^\mu_{(\phi)} = \Sigma^{-1/2} \xi^\mu_{(\phi)}. \quad (35)$$

Here $v$ is a velocity of a charged particle with respect to a rest frame. $v$ can be both positive and negative, and $\gamma$ is the Lorentz gamma factor, which is always positive. Using the condition of the normalization for the momentum $p^2 = -m^2$ one has $\gamma = (1 - v^2)^{-1/2}$. For the positive charge $q > 0$ the Lorentz force acting on a charged particle with $v > 0$ is repulsive, i.e. the force is directed outwards the black hole, while for $v < 0$ the Lorentz force is attractive.

Following to $(\text{Frolov} (2012))$, one can use an expression $d\phi/d\tau = v\gamma/r$ and $(25)$ with $\theta = \pi/2$. This implies

$$\frac{\gamma v}{r} = \chi. \quad (36)$$

From this expression one can easily find

$$\gamma = \sqrt{1 + r^2 \chi^2} \quad \text{and} \quad v = \frac{r\chi}{\sqrt{1 + r^2 \chi^2}}. \quad (37)$$

Using $(37)$ one can find the values of the velocity $v_{\pm}$ and $\gamma$-factor $\gamma_{\pm}$. Fig. 5 shows the velocity of a particle at the ISCO as a function of its radius, while Fig. 4 shows the dependence of $\gamma_{\pm}$ from $r_{\text{ISCO}}$.

The Fig. 4 shows the dependence of the velocity of the particle at ISCO. Since there are two values of velocity for the same radius one can interpret them as two values of velocity which correspond for two opposite directions of motion of the particles. Since charged particle and magnetic field interaction depends on the

Fig. 1  The radial dependence of the effective potential of radial motion of the charged particle for the different values of the gravitomagnetic charge (a): $l/M = 0$ (solid line), $l/M = 0.4$ (dot-dashed line), $l/M = 0.8$ (dashed line) and for the different values of the dimensionless magnetic parameter $b = qBM/m$ (b): $b = 0.2$ (solid line), $b = 0.4$ (dot-dashed line), $b = 0.8$ (dashed line).
velocity direction there should be two values $v_{\pm}$ for each $r_{\text{ISCO}}$. Furthermore the absence of external magnetic field (right border of the plots) one can obtain only one solution for the velocity at ISCO. One should mention that in the non-relativistic case one can get the Keplerian velocity profile.

As the next step, following to Frolov (2012) we will study the center-of-mass collision of two particles in the vicinity of a black hole with nonvanishing gravitomagnetic charge, when one of these particles has the mass $m$ and charge $q$ and rotates along the circular orbit. Another particle, which is neutral has a mass $\mu$ and 4-momentum $k$ and freely falls from the rest at spatial infinity. From the conservation of momenta one can write the momentum of the system at the moment of collision as

$$ P = p + k. \quad (38) $$

This implies that the center-of-mass energy $E_{\text{c.m.}}$ of two colliding particles is

$$ E_{\text{c.m.}} = m^2 + \mu^2 - 2(p, k). \quad (39) $$

Using the equation of particle motion around black hole with nonvanishing gravitomagnetic charge (24)–(26) one can obtain the following relation for the center-of-mass energy of two colliding particles for the weak magnetic field approximation:

$$ E_{\text{c.m.}} \simeq 0.3 \sqrt{\frac{96 - l^2}{8 + l^2}} l^{1/4}. \quad (40) $$

In the Table 1 the dependence of the ISCO radius and the center of mass energy of colliding charged particles have been shown. From the results on can conclude that gravitomagnetic charge correction prevents the particle from the infinite acceleration.

From the obtained result one can observe that the presence of the gravitomagnetic charge will decrease the
Table 1 The dependence of the center of mass energy and ISCO radii from the magnetic parameter $b$ for the different values of the specific gravitomagnetic charge $l/M$:

| $l/M$ | 0   | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 4.52 |
|-------|-----|-----|-----|-----|-----|-----|------|
| $r_{\text{ISCO}}$ | $2 + 0.58b^{-1}$ | $2.02 + 0.58b^{-1}$ | $2.08 + 0.59b^{-1}$ | $2.17 + 0.60b^{-1}$ | $2.28 + 0.61b^{-1}$ | $2.41 + 0.62b^{-1}$ | $5.63 + 0.74b^{-1}$ |
| $E_{\text{c.m.}}/m$ | $1.747b^{1/4}$ | $1.745b^{1/4}$ | $1.738b^{1/4}$ | $1.725b^{1/4}$ | $1.708b^{1/4}$ | $1.688b^{1/4}$ | $1.129b^{1/4}$ |

Fig. 3 Velocity of the particles at $r_{\text{ISCO}}$ as a function of its radius for different values of the gravitomagnetic charge: $l/M = 0$ (solid line), $l/M = 0.4$ (dot-dashed line), $l/M = 0.8$ (dashed line).

value of the center of mass energy. The role of the magnetic field in particle accelerating process is to decrease the innermost stable circular orbits radii. As particles come closer to the black hole horizon their energy at infinity is going to increase. The role of the gravitomagnetic charge in this process is opposite: the presence of the gravitomagnetic charge increase the radii of ISCO. Backreaction effects, upper limit for magnetic field, and sensitivity to the initial conditions, there appears to be some upper limit for the center of mass energy of the infalling particles. One of the mechanisms offered in this paper is appearing due to the gravitomagnetic charge correction which prevents the particle from the infinite acceleration.

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3 Conclusion

In this report we have obtained the expressions for the energy and angular momentum as well as ISCO of the charged particle in the vicinity of the black hole in presence of gravitomagnetic charge and exterior magnetic field.

Recently, Banados et al. (2009) underlined that a rotating black hole can, in principle, accelerate the particles falling to the central black hole to arbitrary high energies. Frolov (2012) has shown that the magnetic field could play a role of charged particle accelerator near the nonrotating black hole. Because of some mechanisms such as astrophysical limitations on the maximum spin,
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