Fermatean fuzzy soft aggregation operators and their application in symptomatic treatment of COVID-19 (case study of patients identification)

Aurang Zeb1,2 · Asghar Khan2 · Muhammad Juniad2 · Muhammad Izhar3

Received: 19 April 2021 / Accepted: 19 January 2022 / Published online: 22 February 2022
© The Author(s), under exclusive licence to Springer-Verlag GmbH Germany, part of Springer Nature 2022

Abstract
The main focus of this paper is the application of aggregation operators (AOs) in the environment of Fermatean fuzzy soft sets (FFSS). The unique feature of the work is its application in the symptomatic treatment of the COVID-19 disease. For this purpose, the idea of FFSS is introduced which is based on the Senapati and Yagar’s Fermatean fuzzy set. Next we have defined Fermatean fuzzy soft aggregation operators (FFSAOs) like, Fermatean fuzzy soft weighted averaging (FFSWA) operator, Fermatean fuzzy soft ordered weighted averaging (FFSOWA) operator, Fermatean fuzzy soft weighted geometric (FFSWG) operator and Fermatean fuzzy soft ordered weighted geometric (FFSOWG). The prominent properties of these operators are given in details. We have also developed some approaches to solve multi-criteria decision making (MCDM) problems in Fermatean fuzzy soft (FFS) information. An introduction to the novel pandemic, safety measures, and then its possible symptomatic treatment is also provided. The developed operators are utilized in the symptomatic treatment of COVID-19 disease in order to show the practical applications and importance of these AOs as well as Fermatean fuzzy soft information. The stability of the proposed work is also proved by the comparative analysis.

Keywords COVID-19 · Fermatean fuzzy soft set · Operational laws · Fermatean fuzzy soft aggregation operators · Multiple attribute decision making problems

Mathematics Subject Classification 03B52 · 90B50

1 Introduction

Decision making (DM) assumes an imperative part in real life experiences of people, it alludes to a cycle that spreads out all the choices according to the appraisal information of the makers and then chooses the brilliant one, generally occurring in regular day to day existences of ours. In the early time of social advancement, leaders utilized the genuine numbers if all else fails to offer their evaluation information. As the multi attribute decision-making...
(MADM) issues are getting intricate, the specialists can't
give genuine numbers to evaluate the other options.
The imprecision and ambiguities of man kind decisions featured
the insufficiency of the fresh set theory. Consequently,
Zadeh (1965) established the set up of the fuzzy set theory
for uncertain information. A fuzzy set is characterized by
a membership function only and so, the concept of fuzzy
set was extended to intuitionistic fuzzy sets (IFS) by Atan-
essov (1986). IFS consists of two functions known as the
membership ($\mu$) and non-membership ($\nu$) functions satisf-
ying the condition that $0 \leq \mu + \nu \leq 1$. Since the IFS was
proposed, it has received a lot of attention in many fields,
such as pattern recognition, medical diagnosis, and so on.
(see e.g. Dengfeng and Chuntian 2002; Liu et al. 2017; Xu
and Yager 2006). Since there may occur situations when
decision-makers independently evaluate the degree of mem-
bership and non-membership and the sum may be greater
than 1. To handle this problem in Yager (2013), the notion
of Pythagorean fuzzy set (PFS) was proposed in which the
quadricum sum of membership and non-membership degree
is less then 1 i.e., $0 \leq \mu^2 + (0.8)^2 \leq 1$, allowing decision
makers to easily infer that the PFS is more useful than IFS in
depicting fuzzy information. Although the PFS generalizes
the IFS, it cannot describe the following decision informa-
tion. A panel of experts were invited to give their opinions
about the feasibility of an investment plan, and they were
divided into two independent groups to make a decision.
One group considered the degree of the feasibility of the
investment plan as 0.8, while the other group considered
the non-membership degree as 0.78. It was clearly seen that
$0.8 + 0.78 > 1$, $(0.8)^2 + (0.78)^2 > 1$ and thus the situation
could not be described by IFS and PFS. To describe such
evaluation information, Senapati and Yager (2020) proposed
the Fermatean fuzzy set (FFS). FFS gives more freedom
to decision makers in situation when IFS and PFS fails to
support data containing uncertainty. Compared to IFS and
PFS, the FFS gains a stronger ability to describe uncertain
information by expanding the spatial scope of membership and
non-membership. Based on FFS, Wang et al. (2019) developed
a hesitant Fermatean fuzzy multicriteria decision-
making method using Archimedean Bonferroni mean opera-
tors. Senapati and Yager (2019a) proposed Fermatean fuzzy
information weighted aggregation operators, and Liu et al.
(2019b) developed a distance measure method for Fermatean
fuzzy linguistic term sets. Furthermore, Liu et al. (2019a)
developed a new concept of Fermatean fuzzy linguistic set and
some new operations between Fermatean fuzzy numbers
(FFNs) were developed in Senapati and Yager (2019b).

Just like IFS and PFS, almost all fuzzy set extensions have some sorts of limitations. As an effective mathematical
tool, Molodtsov (1999) initiated the concept of soft set the-
ory which is free of limitations and has been demonstrated
as super smart tool to deal with problems encompassing
uncertainties or inexact data. Old-fashioned tools such as
fuzzy set, rough set (Pawlak 1982), vague set (Chen and
Tan 1994) etc., cannot be cast-off effectively because one
of the root problems with these models is the absence of
a sufficient number of expressive parameters to deal with
uncertainty. In order to add a reasonable number of expres-
sive parameters, Molodtsov has shown that soft set theory
has a rich potential to exercise in various fields of Mathemat-
ics. Works on soft set theory are growing very rapidly with
all its potentiality and are being cast-off in different areas of
Mathematics (see e.g. Herawan and Deris 2011; Xiao et al.
2009). In case of the soft set, the parametrization is done
with the assistance of words, sentences, functions etc. Due
to the parametrization property of soft set, researchers have
used soft set with different extensions of fuzzy sets like,
intuitionistic fuzzy soft set (Maji et al. 2001b) and Pythago-
rean fuzzy soft set (Kirişci 2019). Fuzzy soft sets (Maji et al.
2001a), rough soft sets ( Feng et al. (2011)), vague soft sets
(Xu et al. 2010), neutrosophic soft sets (Maji 2013), Fuzzy
bi-polar soft sets (Zeb et al. 2021) etc, have been introduced
with the passage of time and still research is in progress in
the field of soft set theory. Considering, (i) the property of
parametrization of soft set, and (ii) the stronger ability of
Fermatean fuzzy set to describe uncertain information by
expanding the spatial scope of membership and non-mem-
bership that allows more freedom in DM problems, we are
going to define the Fermatean fuzzy soft set (FFSS). We also
define some aggregation operators in the environment of
FFSS. These AOs are utilized in a decision making process
of investigating most serious patient among some patients
with common symptoms of COVID-19. The rest of the paper
is arranged as follows: In Sect. 2, basic concept related to
FFSS are reviewed. The novel aggregation operators and
their properties are studied in Sect. 3 and its subsections. A
decision-making approach has been elaborated in Sect. 4 and
its practical illustration has been provided in Sect. 5. In order

to show the stability of the proposed work, a comparative
analysis has been made in Sect. 6. Finally, conclusion of the
presented work is given in Sect. 7.

2 Preliminaries

Some basic definitions are given here that will help in the
subsequent discussion.

Definition 1 (Atanassov 1986) Let $U$ be a universal
set. An intuitionistic fuzzy set (IFS) $A$ of $U$ is defined as
$A = \{x_i, \mu_A(x_i), \nu_A(x_i) | x_i \in U\}$ where $\mu_A(x_i)$ and $\nu_A(x_i)$ are
respectively denoting the membership and non-membership
grades of $x_i$ to the set $A$ such that $0 \leq \mu_A(x_i), \nu_A(x_i) \leq 1$ and
$0 \leq \mu_A(x_i) + \nu_A(x_i) \leq 1$. The degree of indeterminacy of $x_i$
in the IFS $A$ is calculated by
\[ \pi_A(x_i) = 1 - \mu_A(x_i) - v_A(x_i). \]

**Definition 2** (Yager 2013) Let \( U \) be a universal set. A Pythagorean fuzzy set (PFS) \( P \) of \( U \) is defined as \( P = \{ x_i, \mu_P(x_i), v_P(x_i) \mid x_i \in U \} \) where \( \mu_P(x_i) \) and \( v_P(x_i) \) are respectively denoting the membership and non-membership grades of \( x_i \) to the set \( P \) such that \( 0 \leq \mu_P(x_i), v_P(x_i) \leq 1 \) and \( 0 \leq (\mu_P(x_i))^2 + (v_P(x_i))^2 \leq 1 \). The degree of indeterminacy of \( x_i \) in the PFS \( P \) is calculated by

\[ \pi_P(x_i) = \sqrt{1 - (\mu_P(x_i))^2 - (v_P(x_i))^2}. \]

**Definition 3** (Senapati and Yager 2020) Let \( U \) be a universal set. A Fermatean fuzzy set (FFS) \( F \) of \( U \) is defined as \( F = \{ x_i, \mu_F(x_i), v_F(x_i) \mid x_i \in U \} \) where \( \mu_F(x_i) \) and \( v_F(x_i) \) are respectively denoting the membership and non-membership grades of \( x_i \) to the set \( F \) such that \( 0 \leq \mu_F(x_i), v_F(x_i) \leq 1 \) and \( 0 \leq (\mu_F(x_i))^3 + (v_F(x_i))^3 \leq 1 \) for all \( x_i \) in \( U \). Also, the degree of indeterminacy of \( x_i \) in the FFS \( F \) is calculated by,

\[ \pi_F(x_i) = \sqrt[3]{1 - (\mu_F(x_i))^3 - (v_F(x_i))^3}. \]

The stronger ability of Fermatean fuzzy set to describe uncertain information by expanding the spatial scope of membership and nonmembership that allows more freedom in DM problems is illustrated in Fig. 1 below.

Let \( P(U) \) be the power set of universal set \( U \) and \( E \) be the set of parameters. Let \( A \subseteq E \) then,

**Definition 4** (Mołodtsov 1999) A pair \( (F, A) \) is called soft set over \( U \) where \( F \) is a mapping from \( A \) into the set \( P(U) \), i.e., \( F : A \rightarrow P(U) \). Soft set is a parameterized family of subsets of the set \( U \). Every set \( F(e) \) where \( e \in A \), from this family may be considered as the set of elements of the soft set \( (F, A) = \{ F_e \mid e \in A \} \) where each \( F_e \) is some subset of \( U \).

**Definition 5** (Maji et al. 2001a) Suppose \( FP(U) \) be the collection of all fuzzy subsets of universal set \( U \). A pair \( (F, A) \) is called fuzzy soft set over \( U \) where \( F \) is mapping from \( A \) into the set \( FP(U) \) i.e., \( F : A \rightarrow FP(U) \) and is given by,

\[ (F, A) = \{ F_e \mid e \in A \} \]

\[ F_e = \{ (x, \mu(x), v(x)) \mid x \in U \} \] with \( 0 \leq \mu(x) + v(x) \leq 1 \)

**Definition 6** (Arora and Garg 2018) Suppose \( IFP(U) \) be the collection of all intuitionistic fuzzy subsets of universal set \( U \). A pair \( (F, A) \) is called intuitionistic fuzzy soft set over \( U \) where \( F \) is a mapping from \( A \) into the set \( IFP(U) \) i.e., \( F : A \rightarrow IFP(U) \) and is given by, \( (F, A) = \{ F_e \mid e \in A \} \) where

\[ F_e = \{ (x, \mu(x), v(x)) \mid x \in U \} \] with \( 0 \leq \mu(x) + v(x) \leq 1 \)

**Definition 7** (Kirişçi 2019) Suppose \( PFP(U) \) be the collection of all Pythagorean fuzzy subsets of universal set \( U \). A pair \( (F, A) \) is called Pythagorean fuzzy soft set where \( F \) is a mapping from \( A \) into the set \( PFP(U) \) i.e., \( F : A \rightarrow PFP(U) \) and is given by, \( (F, A) = \{ F_e \mid e \in A \} \) where

\[ F_e = \{ (x, \mu(x), v(x)) \mid x \in U \} \] with \( 0 \leq (\mu(x))^2 + (v(x))^2 \leq 1 \)

**Definition 8** Suppose \( FFP(U) \) be the collection of all Fermatean fuzzy subsets of universal set \( U \). A pair \( (F, A) \) is called Fermatean fuzzy soft set where \( F \) is a mapping from \( A \) into the set \( FFP(U) \) i.e., \( F : A \rightarrow FFP(U) \) and is given by, \( (F, A) = \{ F_e \mid e \in A \} \) where

\[ F_e = \{ (x, \mu(x), v(x)) \mid x \in U \} \] with \( 0 \leq (\mu(x))^3 + (v(x))^3 \leq 1 \)

**Example 1** Let \( U = \{ q_1, q_2, q_3, q_4 \} \) be the set of four medicines that are used for the treatment of a single disease and \( E = \{ e_1, e_2, e_3, e_4 \} \) where \( e_1 \equiv \text{cheap}, e_2 \equiv \text{no side effects}, e_3 \equiv \text{availability in market}, e_4 \equiv \text{expiration period}. \) Then,

(i) A soft set \( (F, A) \) where \( A = \{ e_1, e_3 \} \) can be,

\[ F(x) = \begin{cases} 
\{q_1, q_3\} & \text{if } x = e_1 \\
\{q_1, q_2\} & \text{if } x = e_3 
\end{cases} \]

(ii) A FSS \( (F, A) \) where \( A = \{ e_1, e_2, e_3 \} \) describing the characteristics of a medicine can be,
\( F(x) = \begin{cases} 
\{(q_2, 0.4), (q_4, 0.9)\} & \text{if } x = e_1 \\
\{(q_1, 0.6), (q_3, 0.3)\} & \text{if } x = e_2 \\
\{(q_1, 0.1), (q_2, 0.3), (q_4, 0.8)\} & \text{if } x = e_3 
\end{cases} \)

(iii) An IFSS \((F, A)\) where \(A = \{e_2, e_4\}\), describing the characteristics of a medicine can be,

\[ F(x) = \begin{cases} 
\{(q_1, 0.6, 0.5), (q_2, 0.3, 4, 1)\} & \text{if } x = e_2 \\
\{(q_3, 0.7, 0.3), (q_4, 0.5, 0.1)\} & \text{if } x = e_4 
\end{cases} \]

(iv) A PFSS \((F, A)\) where \(A = \{e_3, e_4\}\), describing the characteristics of a medicine can be,

\[ F(x) = \begin{cases} 
\{(q_2, 0.7, 0.8), (q_4, 0.5, 0.9)\} & \text{if } x = e_1 \\
\{(q_2, 0.6, 0.8), (q_2, 0.9, 0.3)\} & \text{if } x = e_2 \\
\{(q_2, 0.4, 0.8), (q_4, 0.6, 0.7), (q_4, 0.9, 0.8)\} & \text{if } x = e_3 \\
\{(q_1, 0.4, 0.6), (q_2, 0.3, 0.7), (q_4, 0.9, 0.5)\} & \text{if } x = e_3 
\end{cases} \]

It is important to note that, throughout this work, we will denote any Fermatean fuzzy soft number (FFSN) \(F_{eij} = \{\langle \mu_{ij}, v_{ij} \rangle \} \mid x_j \in U, e_j \in A\) of an element \(x_j\) corresponding to a parameter \(e_j\) by \(F_{eij} = \{\langle \mu_{ij}, v_{ij} \rangle \} \). For practical application the ranking of alternatives is done on the basis of their score values, thus we define the score and accuracy functions for FFSNs.

**Definition 9** Let \(F_{eij}\) be a FFSN, the score of \(F_{eij}\) is \(S(F_{eij}) = (\mu_{ij})^3 - (v_{ij})^3\). Clearly, \(S(F_{eij}) \in [-1, 1]\) and if two FFSNs have same scores then, we calculate the accuracy of the FFSNs by \(H(F_{eij}) = \mu_{ij}^3 + v_{ij}^3\) which implies that \(H(F_{eij}) \in [0, 1]\). We use the score function and accuracy function for ranking of two FFSNs, \(F_{eij}\) and \(F_{eipq}\) according to the following.

(i) if \(S(F_{eij}) > S(F_{eipq})\), then \(F_{eij} > F_{eipq}\);
(ii) if \(S(F_{eij}) = S(F_{eipq})\), then
(a) if \(H(F_{eij}) > H(F_{eipq})\), then \(F_{eij} > F_{eipq}\);
(b) if \(H(F_{eij}) = H(F_{eipq})\), then \(F_{eij} = F_{eipq}\).

**Definition 10** Let \(F_{eij} = \{\langle \mu_{ij}, v_{ij} \rangle\}, F_{eipq} = \{\langle \mu_{ipq}, v_{ipq} \rangle\}\) be two FFSNs and \(\lambda > 0 \in \mathbb{R}\), we have:

(i) \(F_{eij} \oplus F_{eipq} = \left\langle \mu_{ij}^3 + \mu_{pq}^3 - \mu_{ipq}\mu_{ipq}^3, vl_{ipq}^3 \right\rangle\),
(ii) \(F_{eij} \otimes F_{eipq} = \left\langle \mu_{ij}\mu_{pq}, \mu_{ij}^3 + v_{pq}^3 - v_{pq}^3 \right\rangle\),
(iii) \(\lambda F_{eij} = \left\langle \lambda \mu_{ij}^3, vl_{ij}^3 \right\rangle\),
(iv) \(F_{eij}^\lambda = \left\langle \mu_{ij}, (1 - (1 - v_{ij}^3)^\lambda) \right\rangle\),
(v) \(F_{eij}^c = \langle v_{ij}, \mu_{ij} \rangle\).

### 3 Aggregation operators for Fermatean fuzzy soft numbers (FFSNs)

Here we introduce aggregation operators in the environment of FFS such as, Fermatean fuzzy soft weighted averaging (FFSWA) operator, Fermatean fuzzy soft ordered weighted averaging (FFSOWA) operator, Fermatean fuzzy soft weighted geometric (FFSWG) operator and Fermatean fuzzy soft ordered weighted geometric (FFSOWG) operator.

#### 3.1 Fermatean fuzzy soft weighted averaging (FFSWA) operator

**Definition 11** Let \(Y^{n \times m}\) be matrix of order \(n \times m\) in which entries are from the collection \(\{F_{eij} = \{\langle \mu_{ij}, v_{ij} \rangle\}, (i = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots, m)\}\) of FFSNs and \(\tau = (\tau_1, \tau_2, \ldots, \tau_m)^T, \xi = (\xi_1, \xi_2, \ldots, \xi_n)^T\) be the weighted vectors expressing importance of each parameter \(e_j\) and importance of opinion of experts \(x_i\), respectively such that \(\tau_j > 0, \xi_i > 0\) and \(\sum \tau_j = 1, \sum \xi_i = 1\) then FFSWA operator is a mapping \(FFSWA : Y^{n \times m} \to \mathbb{R}\) defined as

\[ FFSWA(F_{eij}, F_{e11}, F_{e12}, \ldots, F_{e1n}, F_{e21}, F_{e22}, \ldots, F_{e2n}, \ldots, F_{en1}, F_{en2}, \ldots) = \sum_{j=1}^{m} \tau_j \left( \sum_{i=1}^{n} \xi_i F_{eij} \right) \]

**Theorem 1** Let \(F_{eij} = \{\langle \mu_{ij}, v_{ij} \rangle\} (i = 1, 2, \ldots, n; j = 1, 2, \ldots, m)\) be any collection of FFSNs, then the aggregated value by the FFSWA operator is also a FFSN and

\[ FFSWA(F_{eij}, F_{e11}, F_{e12}, \ldots, F_{e1n}, F_{e21}, F_{e22}, \ldots, F_{en1}, F_{en2}, \ldots) = \left\langle \left( \prod_{j=1}^{m} \left( \prod_{i=1}^{n} (1 - \mu_{ij}^3) \right)^{\tau_j} \right)^{\frac{1}{\tau_j}}, \left( \prod_{j=1}^{m} \left( \prod_{i=1}^{n} v_{ij}^3 \right)^{\tau_j} \right)^{\frac{1}{\tau_j}} \right\rangle \]

**Proof** By mathematical induction, for \(n = 1\), we have \(\sum_{i=1}^{n} \xi_i = 1\) so by operations laws in Definition 10,
Fermatean fuzzy soft aggregation operators and their application in symptomatic treatment...

\[ FFSWA(F_{e_1}, F_{e_2}, \ldots, F_{e_m}) = \bigoplus_{j=1}^{m} \tau_j(F_{e_j}) = \left\langle \sqrt[\sum_{j=1}^{m} \left( 1 - \prod_{i=1}^{j} \left( 1 - \mu_{ij}^3 \right) \phi_i \right) \prod_{j=1}^{m} (\psi_j)^{\zeta_j} \right] \right\rangle \]

Similarly, for \( m = 1 \), we have \( \sum_{j=1}^{n} \tau_j = 1 \). So,

\[ FFSWA(F_{e_1}, F_{e_2}, \ldots, F_{e_1}) = \bigoplus_{i=1}^{n} \xi_i(F_{e_i}) = \left\langle \sqrt[\sum_{i=1}^{n} \left( 1 - \prod_{j=1}^{i} \left( 1 - \mu_{ij}^3 \right) \phi_i \right) \prod_{i=1}^{n} (\psi_i)^{\zeta_i} \right] \right\rangle \]

Thus, the result is true for \( n = m = 1 \). Suppose, the result holds for \( m = k_1 + 1, n = k_2 \) and \( m = k_1, n = k_2 + 1 \)

\[ \bigoplus_{j=1}^{k_1+1} \tau_j \left( \bigoplus_{i=1}^{k_2} \xi_i F_{e_i} \right) \]

\[ = \bigoplus_{j=1}^{k_1+1} \tau_j \left( \bigoplus_{i=1}^{k_2} \xi_i F_{e_i} \oplus \xi_{k_2+1} F_{e_{(k_2+1)y}} \right) \]

\[ = \bigoplus_{j=1}^{k_1+1} \tau_j \xi_i F_{e_i} \bigoplus_{j=1}^{k_1+1} \tau_j \xi_{k_2+1} F_{e_{(k_2+1)y}} \]

\[ = \left\langle \sqrt[\sum_{j=1}^{k_1+1} \left( 1 - \prod_{i=1}^{j} \left( 1 - \mu_{ij}^3 \right) \phi_i \right) \prod_{j=1}^{k_1+1} (\psi_j)^{\zeta_j} \right] \right\rangle \]

Now for \( m = k_1 + 1, n = k_2 + 1 \) we get,
Thus it is true for \( m = k_1 + 1 \) and \( n = k_2 + 1 \) and hence, by induction, the result holds for all \( m, n \geq 1 \) since

\[
0 \leq \mu_g \leq 1 \iff 0 \leq \left( \frac{m}{\prod_{j=1}^{n} \left( 1 - \mu_j^3 \right)} \right)^{\alpha_g} \leq 1
\]

\[
0 \leq \nu_i \leq 1 \iff 0 \leq \left( \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \mu_j^3 \right) \right)^{\nu_i} \right) \leq 1
\]

And so, \( 0 \leq \sqrt{\frac{1}{\prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \mu_j^3 \right) \right)^{\nu_i}} - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \mu_j^3 \right) \right)^{\nu_i}} \leq 1 \) Also,

\[
0 \leq \nu_i \leq 1 \iff 0 \leq \left( \prod_{j=1}^{n} \left( 1 - \mu_j^3 \right) \right)^{\nu_i} \leq 1
\]

Finally,

\[
\sqrt{\frac{1}{\prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \mu_j^3 \right) \right)^{\nu_i}} + \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \mu_j^3 \right) \right)^{\nu_i}} \leq 1.
\]

This completes the proof. \( \square \)

**Example 2** Consider the situation of Example 1. Suppose the rating values of experts about five medicines in terms FFSNs are,

By \( d_1 \)

\[
F(e_1) = \{ q_1/(0.7, 0.8), q_2/(0.6, 0.8), q_3/(0.9, 0.4), q_4/(0.9, 0.6) \}
\]

\[
F(e_2) = \{ q_1/(0.9, 0.5), q_2/(0.7, 0.8), q_3/(0.5, 0.4), q_4/(0.8, 0.7) \}
\]

\[
F(e_3) = \{ q_1/(0.7, 0.4), q_2/(0.6, 0.5), q_3/(0.7, 0.4), q_4/(0.7, 0.3) \}
\]

\[
F(e_4) = \{ q_1/(0.8, 0.5), q_2/(0.6, 0.3), q_3/(0.4, 0.3), q_4/(0.9, 0.7) \}
\]

By \( d_3 \)

\[
F(e_1) = \{ q_1/(0.5, 0.4), q_2/(0.7, 0.8), q_3/(0.9, 0.3), q_4/(0.7, 0.6) \}
\]

\[
F(e_2) = \{ q_1/(0.9, 0.5), q_2/(0.8, 0.3), q_3/(0.5, 0.4), q_4/(0.6, 0.2) \}
\]

\[
F(e_3) = \{ q_1/(0.6, 0.4), q_2/(0.8, 0.5), q_3/(0.5, 0.4), q_4/(0.7, 0.3) \}
\]

\[
F(e_4) = \{ q_1/(0.7, 0.5), q_2/(0.8, 0.3), q_3/(0.7, 0.3), q_4/(0.5, 0.7) \}
\]

By \( d_3 \)

\[
F(e_1) = \{ q_1/(0.4, 0.6), q_2/(0.7, 0.5), q_3/(0.8, 0.4), q_4/(0.8, 0.7) \}
\]

\[
F(e_2) = \{ q_1/(0.6, 0.5), q_2/(0.7, 0.4), q_3/(0.5, 0.4), q_4/(0.6, 0.2) \}
\]

\[
F(e_3) = \{ q_1/(0.8, 0.4), q_2/(0.6, 0.5), q_3/(0.6, 0.4), q_4/(0.7, 0.1) \}
\]

\[
F(e_4) = \{ q_1/(0.7, 0.5), q_2/(0.6, 0.3), q_3/(0.4, 0.3), q_4/(0.5, 0.3) \}
\]

In matrix from these information are summarized as,

Fermatean Fuzzy soft matrix for \( q_1 \)

\[
\begin{bmatrix}
   e_1 & e_2 & e_3 & e_4 \\
   d_1 & (0.7, 0.8) & (0.9, 0.5) & (0.7, 0.4) & (0.8, 0.5) \\
   d_2 & (0.5, 0.4) & (0.9, 0.5) & (0.6, 0.4) & (0.7, 0.5) \\
   d_3 & (0.4, 0.6) & (0.6, 0.5) & (0.8, 0.4) & (0.7, 0.5)
\end{bmatrix}
\]

Fermatean Fuzzy soft matrix for \( q_2 \)

\[
\begin{bmatrix}
   e_1 & e_2 & e_3 & e_4 \\
   d_1 & (0.6, 0.8) & (0.7, 0.8) & (0.6, 0.5) & (0.6, 0.3) \\
   d_2 & (0.7, 0.8) & (0.8, 0.3) & (0.8, 0.5) & (0.8, 0.3) \\
   d_3 & (0.7, 0.5) & (0.7, 0.4) & (0.6, 0.5) & (0.6, 0.3)
\end{bmatrix}
\]

Fermatean Fuzzy soft matrix for \( q_3 \)

\[
\begin{bmatrix}
   e_1 & e_2 & e_3 & e_4 \\
   d_1 & (0.9, 0.4) & (0.5, 0.4) & (0.7, 0.4) & (0.4, 0.3) \\
   d_2 & (0.9, 0.3) & (0.5, 0.4) & (0.5, 0.4) & (0.7, 0.3) \\
   d_3 & (0.8, 0.4) & (0.5, 0.4) & (0.6, 0.4) & (0.4, 0.3)
\end{bmatrix}
\]

Fermatean Fuzzy soft matrix for \( q_4 \)

\[
\begin{bmatrix}
   e_1 & e_2 & e_3 & e_4 \\
   d_1 & (0.9, 0.6) & (0.8, 0.7) & (0.7, 0.3) & (0.9, 0.7) \\
   d_2 & (0.7, 0.6) & (0.6, 0.2) & (0.7, 0.3) & (0.5, 0.7) \\
   d_3 & (0.8, 0.7) & (0.6, 0.2) & (0.7, 0.1) & (0.5, 0.3)
\end{bmatrix}
\]

Let \( \tau = (0.3, 0.2, 0.4, 0.1)^T \) and \( \xi = (0.5, 0.2, 0.3)^T \) be the weight vectors of the parameters and experts respectively. Here we are considering only the Fermatean fuzzy soft matrix for \( q_1 \). By FFSWA operator,

\[
FFSWA(F_{e_1}, F_{e_2}, \ldots, F_{e_m}, F_{e_1}, F_{e_2}, \ldots, F_{e_m}, F_{e_1}, F_{e_2}, \ldots, F_{e_m}) = \left( \left( 1 - \left( 1 - \left( (0.8)^0 (0.4)^2 (0.6)^0 \right)^{0.3} \right)^{0.3} \right)^{0.3} \right)^{0.3}
\]

Lemma 1 If \( e_1 \) is the only parameter then, FFSWA operator reduces to Fermatean fuzzy weighted FFWA operator (Senapati and Yager 2019a).

Proof If \( e_1 \) is the only parameter then, \( m = 1 \) thus Eq. 6 becomes,
\[ \text{FFSWA}(e_{11}, e_{12}, e_{13}, \ldots, e_{nm}) = \left( \sum_{j=1}^{m} \frac{\prod_{i=1}^{n} (1 - \mu_{ij}^{3})^{\xi_i}}{\prod_{i=1}^{n} (v_{ij})^{\xi_i}} \right) \]

which is weighted averaging aggregation operator in the environment of Fermatean fuzzy information. \[ \square \]

### 3.2 Properties of FFSWA operator

The FFSWA operator has the following properties which are stated without proof.

**Property 3.2.1 (Idempotency)** If \( e_{ij} = F_e = (\mu, v) \) \( \forall i, j \) then

\[ \text{FFSWA}(e_{11}, e_{12}, e_{13}, \ldots, e_{1n}, e_{21}, e_{22}, \ldots, e_{mn}) = F_e. \]

**Property 3.2.2 (Shift-Invariance)** If \( e_{ij} = (\mu, v) \), is any other FFSN, then

\[ \text{FFSWA}(e_{11}, e_{12}, e_{13}, \ldots, e_{1n}, e_{21}, e_{22}, \ldots, e_{mn}) = \text{FFSWA}(e_{11}, e_{12}, e_{13}, \ldots, e_{1n}, e_{21}, e_{22}, \ldots, e_{mn}) \oplus F_e. \]

**Property 3.2.3 (Homogeneity)** For any real number \( \lambda > 0 \) we have

\[ \text{FFSWA}(\lambda e_{11}, \lambda e_{12}, \ldots, \lambda e_{1n}, \lambda e_{21}, \lambda e_{22}, \ldots, \lambda e_{mn}) = \lambda \text{FFSWA}(e_{11}, e_{12}, e_{13}, \ldots, e_{1n}, e_{21}, e_{22}, \ldots, e_{mn}). \]

**Property 3.2.4 (Boundedness)** Let

\[ e_{ij}^{\text{min}} = \min_{j} \{ \mu_{ij} \}, \quad e_{ij}^{\text{max}} = \max_{j} \{ v_{ij} \} \]

and \( \lambda e_{ij}^{\text{min}}, \lambda e_{ij}^{\text{max}} \) then,

\[ e_{ij}^{\text{min}} \leq \text{FFSWA}(e_{11}, e_{12}, e_{13}, \ldots, e_{1n}, e_{21}, e_{22}, \ldots, e_{mn}) \leq e_{ij}^{\text{max}}. \]

### 3.3 Fermatean fuzzy soft ordered weighted averaging (FFSOWA) operator

**Definition 12** Let \( \gamma^{n \times m} \) be matrix of order \( n \times m \) in which entries are from the collection \( \{ e_{ij} = (\mu_{ij}, v_{ij}), (i = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots, m) \} \) of FFSNs and \( \eta = (\tau_1, \tau_2, \ldots, \tau_m)^T, \xi = (\xi_1, \xi_2, \ldots, \xi_n)^T \) be the weighted vectors expressing importance of each parameter \( e_{ij} \) and importance of opinion of experts \( x \), respectively such that \( \tau_j > 0, \xi_j > 0 \) and \( \sum_{j=1}^{m} \tau_j = 1, \sum_{i=1}^{n} \xi_i = 1 \) then FFSOWA operator is a mapping \( \text{FFSOWA} : \gamma^{n \times m} \rightarrow \gamma \) defined as

\[ \text{FFSOWA}(F_{e_{11}}, F_{e_{12}}, \ldots, F_{e_{1n}}, F_{e_{21}}, F_{e_{22}}, \ldots, F_{e_{mn}}) = \bigoplus_{j=1}^{m} \left( \prod_{i=1}^{n} e_{ij}^{\xi_j} \right) \]

where \( (\sigma_{12}, \sigma_{13}, \ldots, \sigma_{nm}) \) is a permutation of \((1, 2, \ldots, n : j = 1, 2, \ldots, m)\), such that \( F_{e_{\sigma_{ij}}} \geq F_{e_{\sigma_{ij}}} \) for all \( i = 2, 3, \ldots, n \) and \( j = 2, 3, \ldots, m \).

**Theorem 2** Let \( F_{e_{ij}} = (\mu_{ij}, v_{ij}) \), \( (i = 1, 2, \ldots : j = 1, 2, \ldots, m) \) be any FFSNs, then the aggregated value by the FFSOWA operator is a FFSN and is given by,

\[ \text{FFSOWA}(F_{e_{11}}, F_{e_{12}}, \ldots, F_{e_{1n}}, F_{e_{21}}, F_{e_{22}}, \ldots, F_{e_{mn}}) = \left( \sum_{j=1}^{m} \prod_{i=1}^{n} \left( 1 - \mu_{ij}^{\lambda_{ij}} \right)^{\xi_j} \right)^{\eta_j} \]

**Proof** Follows from Theorem 1 \( \square \)

### 3.4 Properties of FFSOWA operator

We state some properties of the FFSOWA operator without proof.

**Property 3.4.1 (Idempotency)** If \( F_{e_{ij}} = F_e = (\mu, v) \) \( \forall i, j \) then

\[ \text{FFSOWA}(F_{e_{11}}, F_{e_{12}}, \ldots, F_{e_{1n}}, F_{e_{21}}, F_{e_{22}}, \ldots, F_{e_{mn}}) = F_e. \]

**Property 3.4.2 (Shift-Invariance)** If \( F_e = (\mu, v) \), is any other FFSN, then

\[ \text{FFSOWA}(F_{e_{11}}, F_{e_{12}}, \ldots, F_{e_{1n}}, F_{e_{21}}, F_{e_{22}}, \ldots, F_{e_{mn}}) = F_e. \]
**Property 3.4.3** (Homogeneity) For any real number \( \lambda > 0 \) we have,

\[
F_{\text{FSOWA}}(\lambda F_{e_{i1}}, \lambda F_{e_{i2}}, \ldots, \lambda F_{e_{i_n}}, \lambda F_{e_{j1}}, \lambda F_{e_{j2}}, \ldots, \lambda F_{e_{jn}}) = \lambda \{F_{\text{FSOWA}}(F_{e_{i1}}, F_{e_{i2}}, \ldots, F_{e_{i_n}}, F_{e_{j1}}, F_{e_{j2}}, \ldots, F_{e_{jn}})\}
\]

**Property 3.4.4** (Boundedness) Let

\[
F_{e_0} = \left\{ \min_{i,j} \{\mu_{ij}\}, \max_{i,j} \{\nu_{ij}\} \right\}
\]

\[
F_{e_0}^+ = \left\{ \max_{i,j} \{\mu_{ij}\}, \min_{i,j} \{\nu_{ij}\} \right\}
\]

then,

\[
F_{e_0} \leq F_{\text{FSOWA}}(F_{e_{i1}}, F_{e_{i2}}, F_{e_{i3}}, \ldots, F_{e_{i_n}}, F_{e_{j1}}, F_{e_{j2}}, \ldots, F_{e_{jn}}) \leq F_{e_0}^+
\]

### 3.5 Fermatean fuzzy soft weighted geometric (FFSWG) operator

**Definition 13** Let \( \Upsilon^{n \times m} \) be matrix of order \( n \times m \) in which entries are from the collection \( \{F_{e_i} = (\mu_{ij}, \nu_{ij}), (i = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots, m)\} \) of FFSNs and \( \tau = (\tau_1, \tau_2, \ldots, \tau_m)^T, \xi = (\xi_1, \xi_2, \ldots, \xi_n)^T \) be the weighted vectors expressing importance of each parameter \( e_i \) and importance of opinion of experts \( x \) respectively such that \( \tau_j > 0, \xi_i > 0 \) and \( \sum_{j=1}^{m} \tau_j = 1, \sum_{i=1}^{n} \xi_i = 1 \) then FFSWG operator is a mapping \( \text{FFSWG} : \Upsilon^{n \times m} \rightarrow \Upsilon \) defined as

\[
\text{FFSWG}(F_{e_{i1}}, F_{e_{i2}}, \ldots, F_{e_{i_n}}, F_{e_{j1}}, F_{e_{j2}}, \ldots, F_{e_{jn}})
\]

\[
= \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \xi_i F_{e_i} \right)^{\tau_j}
\]

**Theorem 3** Let \( F_{e_i} = (\mu_{ij}, \nu_{ij}) \) \( (i = 1, 2, \ldots, n; j = 1, 2, \ldots, m) \) be any collection of FFSNs, then the aggregated value by the FFSWG operator is also a FFSN and is given by

\[
\text{FFSWG}(F_{e_{i1}}, F_{e_{i2}}, \ldots, F_{e_{i_n}}, F_{e_{j1}}, F_{e_{j2}}, \ldots, F_{e_{jn}})
\]

\[
= \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \mu_{ij} \right)^{\tau_j}, \sqrt{1 - \prod_{j=1}^{m} \left( \prod_{i=1}^{n} \left( 1 - \nu_{ij} \right)^{\xi_i} \right)^{\tau_j}}
\]
Now for \( m = k_1 + 1, n = k_2 + 1 \), we get,

\[
\left( \bigotimes_{j=1}^{k_1+1} e_{ij} \right)^{F_{i-j}} = \left( \bigotimes_{j=1}^{k_1} e_{ij} \right)^{F_{i-j}} \left( \bigotimes_{j=1}^{k_2+1} e_{ij} \right)^{F_{i-j}}.
\]

Thus it is true for \( m = k_1 + 1 \) and \( n = k_2 + 1 \) and by induction, the result holds for all \( m, n \geq 1 \).

Since,

\[
0 \leq \nu_j^3 \leq 1 \iff 0 \leq \sqrt[n]{\prod_{j=1}^{n} (1 - \nu_j^3)^{\xi_j}} \leq 1
\]

\[
\iff 0 \leq \sqrt[m]{\prod_{j=1}^{m} (1 - \nu_j^3)^{\xi_j}} \leq 1
\]

Thus the aggregated value obtained by \( FFSWG \) operator is again a \( FFSN \).
Example 3 Take Fermatean fuzzy soft matrix for \( q_2 \) from Example 2,
\[
\begin{bmatrix}
    e_1 & e_2 & e_3 & e_4 \\
    d_1 & (0.6,0.8) & (0.7,0.8) & (0.6,0.5) \\
    d_2 & (0.7,0.8) & (0.8,0.3) & (0.8,0.5) \\
    d_3 & (0.7,0.5) & (0.7,0.4) & (0.6,0.5) \\
\end{bmatrix}
\]

Using FFSWG operator,
\[
\text{FFSWG}(F_{e_1}, F_{e_2}, \ldots, F_{e_m}, F_{e_2}, \ldots) = \left( \prod_{i=1}^{m} \prod_{j=1}^{n} \left( \mu_j \right)^{\xi_i} \right)^{\frac{1}{\xi_i}} \left( 1 - \prod_{i=1}^{m} \prod_{j=1}^{n} (1 - \nu_j)^{\xi_i} \right)^{\frac{1}{\xi_i}}
\]

\[
\text{FFSWG}(F_{e_1}, F_{e_2}, \ldots, F_{e_m}, F_{e_2}, \ldots) = \left( \prod_{i=1}^{m} \prod_{j=1}^{n} \left( \mu_j \right)^{\xi_i} \right)^{\frac{1}{\xi_i}} \left( 1 - \prod_{i=1}^{m} \prod_{j=1}^{n} (1 - \nu_j)^{\xi_i} \right)^{\frac{1}{\xi_i}}
\]

Lemma 2 If \( e_i \) is the only parameter then, FFSWG operator reduces to Fermatean fuzzy weighted FFWG operator (Senapati and Yager 2019a).

Proof If \( e_i \) is the only parameter then, \( m = 1 \) thus Eq. 8 becomes,
\[
\text{FFSWA}(F_{e_1}, F_{e_2}, \ldots, F_{e_m}) = \left( \prod_{i=1}^{m} \left( \mu_j \right)^{\xi_i} \right)^{\frac{1}{\xi_i}} \left( 1 - \prod_{i=1}^{m} (1 - \nu_j)^{\xi_i} \right)^{\frac{1}{\xi_i}},
\]

which is weighted geometric aggregation operator in the environment of Fermatean fuzzy information. □

3.6 Properties of the FFSWG operator

Property 3.6.1 (Idempotent) If \( F_{e_i} = F_e = (\mu, v) \) \( \forall i,j \) then
\[
\text{FFSWG}(F_{e_1}, F_{e_2}, \ldots, F_{e_m}, F_{e_2}, \ldots) = F_{e_2}, \ldots, F_{e_m}, F_{e_2}, \ldots, F_{e_2} = F_e.
\]

Property 3.6.2 (Shift-Invariance) If \( F_e = (\mu, v) \), is any other FFSN, then
\[
\text{FFSWG}(F_{e_1}, F_{e_2}, \ldots, F_{e_m}, F_{e_2}, \ldots) = \left( \prod_{j=1}^{m} \sum_{j=1}^{n} \xi_j F_{e_{i_j}} \right)^{\frac{1}{\xi_j}}
\]

where \( (\sigma_{12}, \sigma_{13}, \ldots, \sigma_{nm}) \) is a permutation of \( (1,2,\ldots,n) \) such that \( F_{e_{i_j}} \leq F_{e_{i_{j+1}}} \) for all \( i = 2,3,\ldots,n \) and \( j = 2,3,\ldots,m \).

Theorem 4 Let \( F_{e_i} = (\mu_j, v_j) \), \( (i = 1,2,\ldots,n \mid j = 1,2,\ldots,m) \) be any FFSNs, then the aggregated value by the FFSWG operator is a FFSN and is given by,
FFSOWG\left( F_{e_{11}}, F_{e_{12}}, \ldots, F_{e_{1m}}, F_{e_{21}}, F_{e_{22}}, \ldots, F_{e_{2n}}, \ldots, F_{e_{mn}} \right)
= \left( \prod_{i=1}^{m} \left( \prod_{j=1}^{n} \left( \mu_{ij} \right) \right) \right) \sqrt[3]{1 - \prod_{j=1}^{n} \left( \left( 1 - \nu_{ij} \right) \right)} \tag{9}

Proof Follow from Theorem 3. □

3.8 Properties of FFSOWG operator

Some properties of FFSOWG operator are stated without proof.

Property 3.8.1 [Idempotency] If $F_{e_i} = F_e = (\mu, \nu) \forall i, j$ then

$\text{FFSOWG}(F_{e_{11}}, F_{e_{12}}, \ldots, F_{e_{1m}}, F_{e_{21}}, F_{e_{22}}, \ldots, F_{e_{2n}}, \ldots, F_{e_{mn}}) = F_e$.

Property 3.8.2 (Shift-Invariance) If $F_{e_i} = (\mu, \nu)$ is any other FFSN then

$\text{FFSAW}(F_{e_{11}}, F_{e_{12}}, \ldots, F_{e_{1m}}, F_{e_{21}}, F_{e_{22}}, \ldots, F_{e_{2n}}, \ldots, F_{e_{mn}}) = \lambda \{ \text{FFSOWG}(F_{e_{11}}, F_{e_{12}}, \ldots, F_{e_{1m}}, F_{e_{21}}, F_{e_{22}}, \ldots, F_{e_{2n}}, \ldots, F_{e_{mn}}) \}$.

Property 3.8.3 (Homogeneity) For any real number $\lambda > 0$ we have

$\text{FFSOWG}(\lambda F_{e_{11}}, \lambda F_{e_{12}}, \ldots, \lambda F_{e_{1m}}, \lambda F_{e_{21}}, \ldots, \lambda F_{e_{2n}}, \ldots, \lambda F_{e_{mn}}) = \lambda \{ \text{FFSOWG}(F_{e_{11}}, F_{e_{12}}, \ldots, F_{e_{1m}}, F_{e_{21}}, F_{e_{22}}, \ldots, F_{e_{2n}}, \ldots, F_{e_{mn}}) \}$.

Property 3.8.4 (Boundedness) Let

$F_{e_0} = \left\{ \min_{j} \{ \mu_{ij} \}, \max_{j} \{ \nu_{ij} \} \right\}$ and $d$

$F_{e_+} = \left\{ \max_{i} \{ \mu_{ij} \}, \min_{i} \{ \nu_{ij} \} \right\}$

then

$F_{e_0} \preceq \text{FFSOWG}(F_{e_{11}}, F_{e_{12}}, \ldots, F_{e_{1m}}, F_{e_{21}}, F_{e_{22}}, \ldots, F_{e_{2n}}, \ldots, F_{e_{mn}}) \preceq F_{e_+}$

4 Decision making approach based upon proposed operators

Here we present MCDM method based on the proposed operators. Let $Q = \{x_1, x_2, \ldots, x_r\}$ be the set of $r$ different alternatives, which are going to be evaluated by $n$ experts $y_1, y_2, \ldots, y_n$ under the constraints of $m$ parameters $E = \{e_{11}, e_{12}, \ldots, e_{1m}\}$. Suppose $\xi = (\xi_1, \xi_2, \ldots, \xi_n)^T$ and $\tau = (\tau_1, \tau_2, \ldots, \tau_m)$ are weighting vectors of experts and parameters respectively for Fermatean fuzzy soft arguments

$F_{e_i} (i = 1, 2, \ldots, n : j = 1, 2, \ldots, m)$ with $\xi_j > 0, \tau_j > 0$ and $\sum_{j=1}^{n} \xi_j = 1, \sum_{i=1}^{m} \tau_j = 1$. These decision makers will give their opinions about the alternatives in terms of FFSNs, $F_{e_i} = \langle \mu_{ij}, \nu_{ij} \rangle$ such that $0 \leq (\mu_{ij})^3 + (\nu_{ij})^3 \leq 1$. These information are then collected in a decision matrix $D = \left( F_{e_i} \right)_{n \times m}$.

Using proposed operators, the aggregated matrix \( FFSN(x_k) \) for the alternatives $x_k$ is obtained. Finally, the score function of the aggregated FFSNs is used to rank the alternatives. Fig. 3 is the pictorial representation of the given approach.

The approach is step-wise given as below:

Step I. Collect the information related to each alternative under different parameters and arrange them in the form of Fermatean fuzzy soft matrix $D_{n \times m} = \langle \mu_{ij}, \nu_{ij} \rangle$.

$$D_{n \times m} = \left( \begin{array}{cccc}
\langle \mu_{11}, \nu_{11} \rangle & \langle \mu_{12}, \nu_{12} \rangle & \ldots & \langle \mu_{1m}, \nu_{1m} \rangle \\
\langle \mu_{21}, \nu_{21} \rangle & \langle \mu_{22}, \nu_{22} \rangle & \ldots & \langle \mu_{2m}, \nu_{2m} \rangle \\
\vdots & \vdots & \ddots & \vdots \\
\langle \mu_{n1}, \nu_{n1} \rangle & \langle \mu_{n2}, \nu_{n2} \rangle & \ldots & \langle \mu_{nm}, \nu_{nm} \rangle 
\end{array} \right)$$

| Total Cases | Deaths | Recovered | Critical |
|-------------|--------|-----------|----------|
| 80,077,514  | 1,748,352 | 55,953,977 | 102,841 |

Fig. 2 COVID-19 cases worldwide
Fig. 3 Clinical representation of a COVID-19 patient

- Fever/Headache
- Hemoptyis
- Cough
- Shortness of breath
- Pneumonia
- Septic shock
- Renal failure
- Myalgia
- Diarrhea
**Symptoms Chart**

| Symptoms        | COVID-19 | Cold       | Flu        | Seasonal Allergies |
|-----------------|----------|------------|------------|--------------------|
| Incubation period | 2-14 days | 1-3 Days   | 1-4 Days   | Varies             |
| Symptom onset   | Gradual  | 7-10 Days  | 3-7 Days   | Varies             |
| Fever           | Common   | Rare       | Common     | Sometimes          |
| Fatigue         | Sometimes| Sometimes  | Common     | Sometimes          |
| Cough           | Common   | Mild       | Common (Usually dry) | Sometimes |
| Sneezing        | No       | Common     | No         | Sometimes          |
| Aches and pains | Sometimes| Common     | No         | No                 |
| Runny or stuffy nose | Rare | Common     | Sometimes  | Common             |
| Sore throat     | Sometimes| Common     | Sometimes for children | No |
| Diarrhea        | Rare     | No         | Sometimes for children | No |
| Headaches       | Sometimes| Rare       | Common     | Sometimes          |
| Shortness of breath | Sometimes | No | No | Sometimes |
| Itchy nose, eyes or roof of the mouth | No | No | No | Common |
| Watery, red or swollen eyes | No | No | No | Common |

Sources: World Health Organization | Center for Disease Control | American College of Allergy, Asthma & Immunology

**Fig. 4** Comparison of symptoms of COVID-19 with cold, flu and seasonal allergies

**Step 2.** Normalize the collective information decision matrix by transforming rating values of cost type parameters into benefit type parameters if any by using normalization formula (Xu and Hu (2010)),

\[
r_{ij} = \begin{cases} 
F_{c_{ij}} & \text{for cost type parameters} \\
F_{b_{ij}} & \text{for benefit type parameters} 
\end{cases}
\]

**Step 3.** Aggregate the FFSNs, \( F_{c_{ij}} \) \((i = 1, 2, \ldots, n; j = 1, 2, \ldots, m)\) for each alternative \( x_{k} \) \((k = 1, 2, \ldots, r)\) into collective decision matrix by any of the proposed operator.

**Step 4.** Find the score values \( S(F_{c_{ij}}) \) of \( F_{c_{ij}} \) for each alternative \( x_{k} \) \((k = 1, 2, \ldots, r)\).

**Step 5.** Rank the alternatives \( x_{k} \), and find out which one is best and which one is the worst, then select the best one.
5 Practical example

Here we present the practical application of our proposed work. We will focus on the investigation of symptomatic treatment of COVID-19 disease by utilizing the presented procedure using Fermatean fuzzy soft operators in the environment of FFS information. But before that, a short background of COVID-19 pandemic is given as under. COVID-19 pandemic: we as a community are fighting against an invisible enemy, the "COVID-19" disease. The disease is caused by sever acute respiratory syndrome coronavirus (SARS-CoV-2) (see e.g. Organization et al. 2020a, b). The first case was reported in Wuhan city of China in December 2019 and spread almost all over the world in a short period of time. So far, more than 80,077,514 cases of COVID-19 and 1,748,352 deaths (up to 27th December 2020) have been reported (Fig. 2).

Due to the alarming situations, World Health Organization (WHO) announced public health emergency of international concern on January 30, 2020. The Emergency Committee on COVID-19 reconvened on 1st August 2020, 4th time and agreed that the outbreak of COVID-19 still

---

**Fig. 5** COVID-19 cases in different areas of Pakistan

**Fig. 6** Plan for isolation of COVID-19 patients in red and green zones
Table 1 Fermatean fuzzy soft matrix for patient $P_1$

|     | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ |
|-----|-------|-------|-------|-------|-------|
| $d_1$ | (0.9, 0.4) | (0.8, 0.5) | (0.7, 0.4) | (0.6, 0.4) | (0.8, 0.5) |
| $d_2$ | (0.8, 0.3) | (0.7, 0.5) | (0.3, 0.1) | (0.6, 0.5) | (0.5, 0.3) |
| $d_3$ | (0.7, 0.3) | (0.6, 0.3) | (0.5, 0.2) | (0.7, 0.5) | (0.6, 0.3) |

Table 2 Fermatean fuzzy soft matrix for patient $P_2$

|     | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ |
|-----|-------|-------|-------|-------|-------|
| $d_1$ | (0.6, 0.3) | (0.7, 0.5) | (0.7, 0.3) | (0.5, 0.3) | (0.9, 0.3) |
| $d_2$ | (0.7, 0.5) | (0.8, 0.5) | (0.4, 0.2) | (0.7, 0.3) | (0.7, 0.5) |
| $d_3$ | (0.5, 0.2) | (0.7, 0.5) | (0.6, 0.5) | (0.8, 0.3) | (0.7, 0.4) |

Table 3 Fermatean fuzzy soft matrix for patient $P_3$

|     | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ |
|-----|-------|-------|-------|-------|-------|
| $d_1$ | (0.9, 0.3) | (0.6, 0.5) | (0.5, 0.4) | (0.9, 0.5) | (0.7, 0.2) |
| $d_2$ | (0.6, 0.4) | (0.9, 0.4) | (0.5, 0.3) | (0.8, 0.4) | (0.9, 0.6) |
| $d_3$ | (0.6, 0.2) | (0.9, 0.4) | (0.7, 0.4) | (0.3, 0.1) | (0.9, 0.6) |

Table 4 Fermatean fuzzy soft matrix for patient $P_4$

|     | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ |
|-----|-------|-------|-------|-------|-------|
| $d_1$ | (0.7, 0.6) | (0.8, 0.7) | (0.8, 0.5) | (0.7, 0.4) | (0.4, 0.1) |
| $d_2$ | (0.7, 0.6) | (0.6, 0.3) | (0.7, 0.4) | (0.9, 0.6) | (0.6, 0.4) |
| $d_3$ | (0.7, 0.3) | (0.8, 0.3) | (0.7, 0.3) | (0.4, 0.3) | (0.5, 0.1) |

constitutes a public health emergency of international concern (WHO). The pandemic has changed the way of living, canceling a lots of sports, schools, religious and political activities.

Symptoms: Research have shown that, the symptoms of COVID-19 in a patient appears from 2.5 to 7 days after infection and the maximum period is around 14 days. Moreover, there are several symptoms of the disease like, fever, headache, cough, sore throat, shortness of breath, hemoptysis, and then pneumonia, septic shock, myalgia etc., in late stages as illustrated in Fig. 3.

Since there is no specific treatment so far, and experts are heavily relying on the symptomatic treatment of the disease. Also, the symptoms are greatly linked to some other infections like Cold, Flu and Seasonal Allergies. Figure 4 shows how the symptoms of these infections are related to each other.

Table 5 Results by the proposed operators

| Operator | $P_1$ | $P_2$ | $P_2$ | $P_4$ |
|----------|-------|-------|-------|-------|
| FFSWA    | (0.7177, 0.3325) | (0.7052, 0.3760) | (0.7972, 0.3713) | (0.7152, 0.3785) |
| FFSOWA   | (0.7244, 0.3466) | (0.7007, 0.3825) | (0.8098, 0.3559) | (0.7067, 0.3351) |
| FFSWG    | (0.6359, 0.4037) | (0.6549, 0.4302) | (0.7004, 0.4239) | (0.6839, 0.4932) |
| FFSOWG   | (0.6641, 0.4045) | (0.6637, 0.4288) | (0.7165, 0.4189) | (0.6674, 0.4598) |

Table 6 Score values using score function

| Patients | FFSWA | FFSOWA | FFSWG | FFSOWG |
|----------|-------|-------|-------|-------|
| $P_1$    | 0.3852 | 0.3385 | 0.2322 | 0.2267 |
| $P_2$    | 0.3292 | 0.2881 | 0.2247 | 0.2135 |
| $P_3$    | 0.4259 | 0.4860 | 0.2765 | 0.2943 |
| $P_4$    | 0.3367 | 0.3153 | 0.1906 | 0.2001 |

Table 7 Final ranking orders

| Operators | Ranking orders |
|-----------|----------------|
| FFSWA     | $P_3 > P_1 > P_4 > P_2$ |
| FFSOWA    | $P_3 > P_1 > P_4 > P_2$ |
| FFSWG     | $P_3 > P_1 > P_4 > P_2$ |
| FFSOWG    | $P_3 > P_1 > P_4 > P_2$ |

That is why, the possibility that an expert may make a wrong decision about a patient can not be ignored. Infect, it has also been observed that, sometimes patients with infections like Cold, Cough and Flu are treated as a case of COVID-19. Therefore, it is very important for experts to investigate any patient with serious care and full attention in order to make their decision more wise and accurate. Pakistan: As all over the world, the novel pandemic has also changed the way of life in Pakistan which is a growing economic state and has less resources to deal with these critical situations (Sarwar et al. 2020). However, local and provincial governments are taking serious action by locking down markets, schools, universities and other public places and raising awareness through social media, T.V channels to reduce the transmission of the pandemic. Up to 27th of December 2020, the total number of confirmed cases in Pakistan are about 473, 309. The government is keen to to control the transmision of the pandemic taking some unusual and hard steps. Due to lockdown (smart lockdown strategy in special) and other precautionary measures, the rate of recovery during the first wave of COVID-19 was incredibly good. Another positive side is that, the death ratio was a lot lower in Pakistan. So far, 423, 892 peoples have recovered while 9929 have lost the run (http://covid-19.gov.pk). The following graph (Fig. 5) shows these details up to 27th of December 2020.

Pakistan currently has the 8th-highest number of cases in Asia and the 28th highest number of confirmed cases in the
To limit and to reduce exposures for other patients and health care personnel, it is imperative to promptly identify and separate active cases by instituting screening system for signs and symptoms of disease along with specific RT-PCR (real time reverse transcription) testing in suspected inpatients and health care personnel (HCP).

**Application:** Keeping social distance is the most important precautionary measure, therefore, to avoid crowds in hospitals/health care centers, it is important to separate patients who have been tested and declared negative for COVID-19 must be sent to the green zone (Area of the hospital reserved for patients declared negative for COVID-19). While those declared positive must be kept in the red zone (Area of the hospital reserved for patients declared positive for COVID-19). In order to provide all necessary health cares, patients in red zone must also be categorized on the basis of severity of the disease as Mild, Moderate, Severe and Critical. In case of,

- **Mild:** Treatment is symptomatic and can be managed at home and does not require inpatient care.
- **Moderate:** Can be managed either at home, or as inpatient at red zone.
- **Severe:** Requires oxygen therapy, has dyspnea, hypoxia, or > 50 percent lung involvement on image within 24–48 h; (In red zone).
- **Critical:** Requires mechanical ventilation, has respiratory failure, shock, or multiorgan dysfunction (isolation in red zone) (Wang et al. 2020). This plan is also explained in Fig. 6.

We consider the situations of four patients $P_i (i = 1, 2, 3, 4)$ from the red zone and will try to find where to keep them on...
the basis of severity of the disease. A panel of three experts (doctors) is going to treat these patients symptomatically. Considering some parameters, experts will give their opinion about each patient in terms of FFSNs. There are several parameters however, the set of parameters under which these patients are to be treated is arranged by these experts as \( E = \{ e_1, e_2, e_3, e_4, e_5 \} \) where \( e_1 \equiv \) Headache, \( e_2 \equiv \) Cough, \( e_3 \equiv \) Shortness of breath, \( e_4 \equiv \) Fever, \( e_5 \equiv \) Sore throat. It is important to note that, if a patient is found to have any of these five symptoms, then it is termed as ‘Case’ and if not then termed as ‘Control’. We assume that, all the patients \( P_i \) are infected thus, each of them represents a case. Let \( \xi = (0.2, 0.3, 0.5)^T \) and \( r = (0.2, 0.3, 0.1, 0.25, 0.15)^T \) be the weighted vectors of experts \( d_i \) and parameters \( e_j \), respectively. The rating values by these experts in terms of FFSNs are listed as below,

Rating values by the experts \( d_i \) for the patients
\[
\begin{align*}
P(e_1) &= \{ P_1/(0.9, 0.4), P_2/(0.6, 0.3), P_3/(0.9, 0.3), P_4/(0.7, 0.6) \} \\
P(e_2) &= \{ P_1/(0.8, 0.5), P_2/(0.7, 0.5), P_3/(0.6, 0.5), P_4/(0.8, 0.7) \} \\
P(e_3) &= \{ P_1/(0.7, 0.4), P_2/(0.7, 0.3), P_3/(0.5, 0.4), P_4/(0.8, 0.5) \} \\
P(e_4) &= \{ P_1/(0.6, 0.4), P_2/(0.5, 0.3), P_3/(0.9, 0.5), P_4/(0.7, 0.4) \} \\
P(e_5) &= \{ P_1/(0.8, 0.5), P_2/(0.9, 0.3), P_3/(0.7, 0.2), P_4/(0.4, 0.1) \}
\end{align*}
\]

Rating values by the experts \( d_i \) for the patients
\[
\begin{align*}
P(e_1) &= \{ P_1/(0.8, 0.3), P_2/(0.7, 0.5), P_3/(0.6, 0.4), P_4/(0.7, 0.6) \} \\
P(e_2) &= \{ P_1/(0.7, 0.5), P_2/(0.8, 0.5), P_3/(0.9, 0.4), P_4/(0.6, 0.3) \} \\
P(e_3) &= \{ P_1/(0.3, 0.1), P_2/(0.4, 0.2), P_3/(0.5, 0.3), P_4/(0.7, 0.4) \} \\
P(e_4) &= \{ P_1/(0.6, 0.5), P_2/(0.7, 0.3), P_3/(0.8, 0.4), P_4/(0.9, 0.6) \} \\
P(e_5) &= \{ P_1/(0.5, 0.3), P_2/(0.7, 0.5), P_3/(0.9, 0.6), P_4/(0.6, 0.4) \}
\end{align*}
\]

Rating values by the experts \( d_i \) for the patients
\[
\begin{align*}
P(e_1) &= \{ P_1/(0.7, 0.3), P_2/(0.5, 0.2), P_3/(0.6, 0.2), P_4/(0.7, 0.3) \} \\
P(e_2) &= \{ P_1/(0.6, 0.3), P_2/(0.7, 0.5), P_3/(0.9, 0.4), P_4/(0.8, 0.2) \} \\
P(e_3) &= \{ P_1/(0.5, 0.2), P_2/(0.6, 0.5), P_3/(0.7, 0.4), P_4/(0.7, 0.3) \} \\
P(e_4) &= \{ P_1/(0.7, 0.5), P_2/(0.8, 0.3), P_3/(0.3, 0.1), P_4/(0.4, 0.3) \} \\
P(e_5) &= \{ P_1/(0.6, 0.3), P_2/(0.7, 0.4), P_3/(0.9, 0.6), P_4/(0.5, 0.1) \}
\end{align*}
\]

Step 1 In matrix from these information are summarized as (Tables 1, 2, 3, 4).

Step 2 Since all the parameters are of same type, hence there is no need to normalize the data. Step 3 The aggregated rating values of each patient \( P_i (i = 1, 2, 3, 4) \) by the proposed operators are given in Table 5.

Step 4 The score values \( S(F_{e_j}) \) are given in Table 6.

Step 5 Final ranking orders are given in the following Table 7.

From Table 7, it is clear that the ranking orders of the alternatives are same and \( P_3 \) is the patient in the critical stage having respiratory failure, shock, or multiorgan dysfunction and requires mechanical ventilation therefore,

- \( P_3 \) must be isolated in the isolation ward at red zone in the hospital.
- \( P_1 \) is in the severe stage and requires oxygen therapy, having dyspnea, hypoxia, or \( > 50 \) percent lung involvement on image within 24–48 h, thus \( P_1 \) (In red zone at hospital).
- \( P_2 \) and \( P_3 \) are respectively in the moderate and mild stages of the disease or vice versa, however treatment is symptomatic and they can be managed at home and does not require inpatient care both of them can be treated as inpatient. For further assistance one can examine Fig. 7.

Figure 7 shows the comparison between score values obtained by FFSWA, FFSOWA and FFSWG, FFSOWG operators. The red line in the figure is representing the ranking order of alternatives \( P_i (i = 1, 2, 3, 4) \) obtained by FFSWA and FFSWG operator, while the blue line is representing the ranking order of the alternatives obtained by FFSOWA and FFSOWG operator.

6 Comparative analysis

In this final section, we are going to compare our results with results of existing operators. We adopt Fermatean fuzzy soft information from Shahzadi and Akram (2021), where the FFS matrices for four different antivirus masks \( x_i (i = 1, 2, 3, 4) \) are aggregated using Fermatean fuzzy soft Yager average and geometric operators. The final scores and ranking orders corresponding to FFS Yager average (FFSWA) and FFS Yager geometric (FFSWG) operators are given in Table 8. According to their results, the antivirus mask \( x_1 \) is the most suitable mask (best alternative).

By applying the proposed approach using FFSWA and FFSWG operators, we obtained the aggregated matrix about four antivirus masks \( x_i (i = 1, 2, 3, 4) \) as given in Table 9.

This matrix is obtained by aggregating the four matrices given in Tables 4 to 7 in Shahzadi and Akram (2021). From this matrix, a comparative study has been established with the existing work developed in Shahzadi and Akram (2021) which is based on Fermatean fuzzy soft Yager aggregation operators on FFS environment. Table 10 shows the final comparison with existing method, which also shows that the best alternative is \( x_1 \).

Clearly, the ranking orders by the proposed operators are identical with ranking orders of FFS\(_Y\)WA and FFS\(_Y\)WG operators. This proves the stability of our proposed method. The basic advantage of proposed method is that, it is capable to facilitate the description of real world problems with the help of properties like, parameterization, fuzziness and so, the method can be used in decision making problems instead of other existing methods in the environment of Fermatean fuzzy soft set.
Figure 8 is the graphical representation of the comparison of score values by FFS\textsubscript{YWA} and FFS\textsubscript{WSA} operators. The ranking order of the alternatives obtained by FFS\textsubscript{YWA} operator is represented by the bluish cones in front, while the ranking order of alternatives obtained by FFS\textsubscript{WSA} operator is represented by the red cones behind.

7 Conclusion

We have explored the (MADM) problems with Fermatean fuzzy soft information and introduced FFSWA, FFSOWA, FFSWG, and FFSOWG operators in the environment of Fermatean fuzzy soft sets. The four basic properties of these operators are studied. An approach has been developed to solve the Fermatean fuzzy soft MADM problems. Next, the approach has been tested through a case study of searching out the most serious patient with COVID-19 disease. Lastly, the stability of the proposed method is provided by comparing the work with existing work in the environment of FFSS. In future, we shall extend the idea of Fermatean fuzzy soft information to introduce more operators like, Fermatean fuzzy soft Dombi aggregation operators, Fermatean fuzzy soft Einstein hybrid aggregation operators and Fermatean fuzzy soft Hamacher aggregation operators.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

References

Arora R, Garg H (2018) A robust aggregation operators for multicriteria decision-making with intuitionistic fuzzy soft set environment. Sci Iran 25(2):931–942
Atanassov KT (1986) New operations defined over the intuitionistic fuzzy sets. Fuzzy Sets Syst 20(1):87–96
Chen SM, Tan JM (1994) Handling multicriteria fuzzy decision-making problems based on vague set theory. Fuzzy Sets Syst 67(2):163–172
Dengfeng L, Chuntian C (2002) New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions. Pattern Recogn Lett 23(1–3):221–225
Feng F, Liu X, Leoreanu-Fotea V, Jun YB (2011) Soft sets and soft rough sets. Inf Sci 181(6):1125–1137
Herawan T, Deris MM (2011) A soft set approach for association rules mining. Knowl Based Syst 24(1):186–195
Kirisci M (2019) New type pythagorean fuzzy soft set and decision-making application. arXiv:190404064
Liu Y, Bi JW, Fan ZP (2017) Ranking products through online reviews: a method based on sentiment analysis technique and intuitionistic fuzzy set theory. Inf Fusion 36:149–161
Liu D, Liu Y, Chen X (2019) Fermatean fuzzy linguistic set and its application in multicriteria decision making. Int J Intell Syst 34(5):878–894
Liu D, Liu Y, Wang L (2019) Distance measure for fermatean fuzzy linguistic term sets based on linguistic scale function: an illustration of the todim and topsis methods. Int J Intell Syst 34(11):2807–2834
Maji PK (2013) Neutrosophic soft set. Ann Fuzzy Math Inf 5(1):157–168
Maji PK, Biswas R, Roy A (2001) Fuzzy soft sets. Fuzzy Math 9:589–602
Maji PK, Biswas R, Roy AR (2001) Intuitionistic fuzzy soft sets. J Fuzzy Math 9(3):677–692
Molodtsov D (1999) Soft set theory-first results. Comput Math Appl 37(4–5):19–31
Organization WH et al (2020) Considerations for quarantine of individuals in the context of containment for coronavirus disease (Covid-19): interim guidance, 19 March 2020. World Health Organization, Tech. rep
Organization WH et al (2020b) Critical preparedness, readiness and response actions for Covid-19—7 March 2020
Pawlak Z (1982) Rough sets. Int J Comput Inf Sci 11(5):341–356
Senapati T, Yager RR (2019) Fermatean fuzzy weighted averaging/ geometric operators and its application in multi-criteria decision making methods. Eng Appl Artif Intell 85:112–121
Senapati T, Yager RR (2019) Some new operations over fermatean fuzzy numbers and application of fermatean fuzzy wpm in multiple criteria decision making. Informatica 30(2):391–412
Senapati T, Yager RR (2020) Fermatean fuzzy sets. J Ambient Intell Humaniz Comput 11(2):663–674
Shahzadi G, Akram M (2021) Group decision-making for the selection of an antivirus mask under fermatean fuzzy soft information. J Int Fuzzy Syst 40(1):1401–1416
Wang H, Wang X, Wang L (2019) Multicriteria decision making based on archimedean bonferroni mean operators of hesitant fermatean 2-tuple linguistic terms. Complexity 2019(4):1–19
Wang C, Pan R, Wan X, Tan Y, Xu L, Ho CS, Ho RC (2020) Immediate psychological responses and associated factors during the initial stage of the 2019 coronavirus disease (COVID-19) epidemic among the general population in China. Int J Environ Res Public Health 17(5):1729
Xiao Z, Gong K, Zou Y (2009) A combined forecasting approach based on fuzzy soft sets. J Comput Appl Math 228(1):326–333
Xu Z, Hu H (2010) Projection models for intuitionistic fuzzy multiple attribute decision making. Int J Inf Technol Decis Mak 9(02):267–280
Xu Z, Yager RR (2006) Some geometric aggregation operators based on intuitionistic fuzzy sets. Int J Gen Syst 35(4):417–433
Xu W, Ma J, Wang S, Hao G (2010) Vague soft sets and their properties. Comput Math Appl 59(2):787–794
Yager RR (2013) Pythagorean membership grades in multicriteria decision making. IEEE Trans Fuzzy Syst 22(4):958–965
Zadeh LA (1965) Zadeh, fuzzy sets. Inf Control 8:338–353
Zeb A, Khan A, Izhar M, Hila K (2021) Aggregation operators of interest. Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.