Reliability Analysis of a Steel Structure with Potentially Brittle Steel Members

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Abstract. In recent years, more and more researches on structural reliability theory and methods have been carried out, and various methods for calculating reliability have been developed. In this study, a truss structure is taken as an object. Based on the relaxed linear programming bounds method, a mathematical method is established to calculate the reliability when some members of the steel structure could be brittle. The numerical examples have been used to investigate its accuracy and efficiency. Compared with the Monte Carlo simulation, the proposed method is more efficient, and its bounds are Introduction.

1. Introduction
Structural reliability analysis is to study the safety of the structure on the influence of various factors. Actually, it is the probability of the structural system to complete the specified function. There are many ways to calculate the reliability \cite{1-3}. XIE and ZHOU \cite{4-6} studied the generating function method of the structural reliability analysis. Then, CHANG use the generating function to test the accuracy of some theoretical bounds in the relaxed linear programming bounds method (RLP) \cite{7-8}. This paper focuses on the reliability analysis of steel structure. The brittle failure of steel structures could occur which was found in Kobe earthquake in 1995 and Northridge earthquake in 1994. It is a drawback that the brittle failure of the steel is usually ignored in the past research. This paper formulates a new reliability calculation method for structural system. The proposed method can also estimate a steel structure with a lot of members.

2. Research method
2.1. Structural reliability analysis
The structural reliability analysis is calculated by using the probability theory method. Generally, the influencing factors causing structural system damage are collected as random variables, and the random variable is used as the basis for calculating the structural failure probability (e.g., physical parameters of materials, welding of beam and column joints, load effects, etc.). Random variables are denoted by $X=[X_1, X_2, X_3, ..., X_n]^T$, where $X_i$ is the $i$th random variable, and corresponds to an influencing factor in the $n$-dimensional space. Then, the failure mode of a single member in the structure is identified, namely failure of the system with brittle members or without brittle members. Furthermore, the structure can be simplified as a series system or a parallel system or a series-parallel hybrid system according to the structural force form. For example, in a statically determinate structural
truss, if a single member fails, the entire structure fails. Therefore, the structure is simplified as a series system, as shown in Figure 1.

![Simplified tandem system](image)

**Figure 1.** Simplified tandem system.

According to the random variable $X$, the objective function is determined as $F = g(X)$, where $g(X)$ is the function that determines the failure of the structure. The objective function is used to evaluate whether the structure is safe or not. When $g(X) < 0$, the structure is in a failure state; when $g(X) > 0$, the structure is in a reliable state; when $g(X) = 0$, the structure is in a limit state. Assuming that the $X_1, X_2, ..., X_n$ joint probability density function is $f_{X_1, X_2, ..., X_n}(x_1, x_2, ..., x_n) = f_X(x)$, then the failure probability of the structure is $P_F = P[g(X) \leq 0] = \int_{\mathbb{R}^n} f_X(x) dx$.

The final result of the integration is structural reliability.

### 2.2. System reliability analysis of potentially brittle members

In the process of designing steel structures, researchers often only consider the conventional plastic failure [10-12]. However, brittle failures were found at the steel beam-column joints in the Northridge earthquake in 1994 and the Kobe earthquake in 1995. Therefore, in the safety analysis of steel structures, plastic failure and brittle failure all should be considered.

Five conclusions were obtained in the experiments: (a) Low probability of brittle fracture of members; (b) Britteness of members generally is lower than their plastic strength; (c) Simultaneous brittle and plastic failure of individual members is not possible; (d) Correlation between brittle members is small; and (e) Brittle strength and plastic strength of individual members are not related [9].

In the calculation of reliability, the brittle failure and plastic failure of individual members are considered as mutually exclusive events based on the conclusion (c). Therefore, the analysis is as follows:

$$P_s(F) = P_s(F_b) + P_s(F_d)$$

Where $P_s$ is the failure probability of the system; $P_s(F_b)$ is the failure probability of the system with brittle members; $P_s(F_d)$ is the failure probability of the system without brittle members.

If a member is already known as a potentially brittle member in a structural system, in order to facilitate the calculation of the probability, the system is decomposed into two subsystems during the analysis, namely the failure probability of the system with brittle members and the failure probability of the system without brittle members. Then, the calculated results are obtained by formula (1).

When calculating the reliability of the subsystem, the relaxed linear programming bounds method has been used to calculate the probability. When analyzing members in a structural system, sometimes a member is known as a potential brittle member, and sometimes is unknown. Based on this situation, the calculation formula is divided into two discussions:

① There is a potentially brittle member, and its number is known:

$$P_s(F) = P_b \times P_s(F_b) + (1 - P_b) \times P_s(F_d) ; i \in (1, n)$$

② There is a potentially brittle member, and its number is unknown
\[ P_s (F) = \sum_{i=1}^{n_1} (1 - P_{b_i}) \times P_{s_2} (F_d) ; i \in (1, n); n_1 \leq n \]  

In formulation (2) ~ (3), \( P_{b_i} \) is the probability of brittle failure of the \( i \)th member; \( P_{s_1} \) is the failure probability of the system with brittle members; \( P_{s_2} \) is the failure probability of the system without brittle members; \( n_1 \) is the number of potential brittle failure members; \( n \) is the number of all system members; \( i \) is the brittle number of the member.

3. Example

The truss shown in the figure below is given. The truss was used by Song and Der Kiureghian as an example of a steel structure. Because this is a statically determinate structure, the failure of any member leads to failure of the truss. Therefore, the truss is a series system. \( L \) represents the load acting on the truss and \( b \) represents the length of the truss. Neglecting the buckling failure mode, let \( X_i \) (\( i = 1, 2, 3, ..., 7 \)) represent the tensile strength or tensile strength of a member in compression (\( \rho \): Correlation coefficient between members).

Figure 2. Static truss as a series system (t: intensity, c: compression)

Considering different correlation coefficients, the results calculated by the relaxed linear programming bounds method and Monte Carlo (MC) simulation as shown in Figure 3-9. The CPU time is as shown in Figure 10.

Figure 3. Conditional probability \( P_{b_i} = 0.05 \) compared with Monte Carlo.

Figure 4. Conditional probability \( P_{b_i} = 0.1 \) compared with Monte Carlo.
Figure 5. Conditional probability $P_{bi} = 0.2$ compared with Monte Carlo.

Figure 6. Conditional probability $P_{bi} = 0.3$ compared with Monte Carlo.

Figure 7. Correlation coefficient $\rho = 0.1$ compared with Monte Carlo

Figure 8. Correlation coefficient $\rho = 0.5$ compared with Monte Carlo.

Figure 9. Correlation coefficient $\rho = 0.9$ compared with Monte Carlo.

Figure 10. Computation time vs. Monte Carlo.
The results are as follows:

(1) When the conditional probability is the same, the total failure probability of the structure will decrease with the increasing of the correlation coefficient between members as shown in Figure 3-6.

(2) When the correlation coefficients are the same, the probability of total failure of the structure will increase with the increasing of conditional probability as shown in Figure 7-9.

(3) When the same number of members is calculated for the same conditions, the CPU time of relaxed linear programming method is less than the CPU time of Monte Carlo simulation method as shown in Figure 10.

4. Conclusion
A new calculation method is established for the failure analysis of steel structures in this paper. By analyzing the failure probability of each member, the approximate probability of failure probability was obtained. Comparing with Monte Carlo simulation results, the results of the method for steel structures with potential brittle failure are close to the actual engineering results. A large number of members can be calculated with a short CPU time.

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