Molecule model for deeply bound and broad kaonic nuclear clusters

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(Dated: January 12, 2013)

A molecule model is proposed for the description of the properties of the kaonic nuclear cluster (KNC) $K\Lambda N$ with the structure $N \otimes (K\Lambda)_{I=0}$ and quantum numbers $I(J^P) = 1/2(0^-)$, the large binding energy $B_{K\Lambda N}^{(\text{exp})} = 103(6)$ MeV and the width $\Gamma_{K\Lambda N}^{(\text{exp})} = 118(13)$ MeV, observed recently by the DISTO Collaboration. The theoretical values of the binding energy $B_{K\Lambda N}^{(\text{th})} = 118$ MeV, the width $\Gamma_{K\Lambda N}^{(\text{th})} = 142$ MeV of the KNC $KNN$ and the density $n_{KNN} = 2.71 a_0$, where $a_0 = 0.17$ fm$^{-3}$ is the normal nuclear density, reproduce well the large experimental values. They are calculated with the trial harmonic oscillator wave functions by using chiral Lagrangians, accounting for all self-energy terms, contributing to the masses of the kaonic nuclear clusters $(K\Lambda)_{I=0}$ and $KNN$. In addition the high $p\Lambda^*$ sticking probability in the $pp$ reaction at the kinetic energy $T_p = 2.85$ GeV of the incident proton is explained.

PACS: 11.10 Ef, 13.75 Jz, 24.10 Jv, 25.80 Nv

1. Introduction

Recently \cite{1} we have proposed a molecule model for kaonic nuclear clusters (KNCs) $K\Lambda N$ and $KNN$, which we denote as $k^1_3H$ and $k^2_3H$, with the structure $(K\Lambda N)_{I=0}$ and $N \otimes (K\Lambda N)_{I=0}$, respectively. The binding energies of the KNCs $k^1_3H$ and $k^2_3H$ have been calculated in the tree–approximation and defined by the Weinberg–Tomozawa (WT) interactions $(K\Lambda N)_{I=0} \rightarrow (K\Lambda N)_{I=0}$ and $(K\Lambda N)_{I=1} \rightarrow (K\Lambda N)_{I=1}$ only as $B_{K\Lambda H} = 40.2$ MeV. The WT–interaction $(K\Lambda N)_{I=1} \rightarrow (K\Lambda N)_{I=1}$ contributed to the binding energy of the KNC $k^2_3H$ only due to the $NN$ exchange interaction.

We have shown that in the tree–approximation the molecule model of the KNCs reproduces well the binding energy $B_{K\Lambda H} = 48$ MeV and width of the KNC $k^2_3H$, obtained in the potential model by Akaishi and Yamazaki \cite{2,7}. The binding energy and the width of the KNC $k^2_3H$, calculated in the molecule model of KNCs in the tree–approximation, agree qualitatively also with the results, obtained in other theoretical approaches \cite{3,4,5,6}, but disagree with the $SU(3)$ coupled–channel approach with chiral dynamics, predicting rather shallow bindings \cite{11,12}. In this approach two resonances exist with the same quantum numbers and different masses instead of the $\Lambda(1405)$ (or the $\Lambda^*$) resonance with strangeness $S = -1$, quantum numbers $I(J^P) = 0(1^-)$ and mass $m_{\Lambda^*} = 1405$ MeV. One of these resonances with mass $m_{\Lambda^*_1} = 1420$ MeV, treated as a quasi-bound $(K\Lambda N)_{I=0}$ state, leads to the shallow binding of about 20 MeV of the $KNN$ state. This is unlike the potential model approach, developed in \cite{2,7}, and our molecule model, dealing with the $\Lambda^*$ resonance with mass $m_{\Lambda^*} = 1405$ MeV only, treated as a quasi-bound $(K\Lambda N)_{I=0}$ state.

For the calculation of the binding energies and widths of the KNCs $k^1_3H$ and $k^2_3H$ we use trial harmonic oscillator wave functions, parameterised by two frequencies $\Omega_{\Lambda^*}$ and $\Omega_{\Lambda\Lambda}$, describing $K\Lambda N$ and $N(K\Lambda N)$ correlations, respectively, and chiral Lagrangians with derivative meson–baryon couplings invariant under chiral $SU(3) \times SU(3)$ symmetry, used for the analysis of low–energy strong interactions \cite{13,14}. The frequencies $\Omega_{\Lambda^*}$ and $\Omega_{\Lambda\Lambda}$ take also into account their different strengths in contrast to \cite{1}. Since the angular momenta of the KNCs $k^1_3H$ and $k^2_3H$ are equal to zero, their states are defined by the vibrational degrees of freedom, which are described by the trial harmonic oscillator wave functions \cite{12}. The use of the trial harmonic oscillator wave functions is also supported by the short–range character of forces, producing deeply bound KNCs.

Recent experimental data \cite{10}, obtained from the analysis of exclusive $K^+ \pi^-\pi^0$ reaction at the incident proton kinetic energy $T_p = 2.85$ GeV, have shown a signature for the creation of a compact object $X$ with a probable structure $X = KNN$, decaying into the $p\Lambda^0$ state, $X \rightarrow p + \Lambda^0$ \cite{10}. According to \cite{10}, such a compact object possesses an unexpected large $13$ binding energy $B_X = 103(6)$ MeV and the width $\Gamma_X = 118(13)$ MeV in the two–body reaction channel $pp \rightarrow K^+X$ at high momentum transfer. For a deeply bound state $X$ with a structure $KNN$ and the binding energy $B_X = 103(6)$ MeV the three–body pionic decay modes are suppressed \cite{16}. The theoretical analysis of deeply bound states of this kind cannot be carried out within the coupled–channel Faddeev equation approach.
and the $SU(3)$ coupled−channel approach with chiral dynamics \cite{11,12}. These approaches are unable to
describe the two−body reaction channels, enhanced at
high momentum transfer, and the two−body non−pionic
decay modes of the deeply bound $KNN$ state.

In this letter we propose a modified molecule model of
KNCs, going beyond the tree−approximation, and apply
it to the description of the deeply bound $KNN$ state, ob−
served by the DISTO Collaboration \cite{10}. In this model
we treat this $KNN$ state as the KNC $^2_{1H}$ with the struc−
ture $N \otimes (\bar{K}N)_{I=0}$. As has been pointed out in \cite{11}, the
depdeeply bound state $^2_{1H}$ should be analysed together with
a simpler deeply bound state $^1_{1H}$, having a structure
$(\bar{K}N)_{I=0}$, which we identify with the $\Lambda^*$ resonance with
mass $m_{\Lambda^*} = 1405$ MeV \cite{11} (see also \cite{17}−\cite{19}). The im−
portant role of the $\Lambda^*$ resonance in the formation of the
$X = \bar{K}NN$ resonance in the $pp \to XK^+$ reaction has
been investigated recently in \cite{20} for the kinetic energies
$T_p = 2.50$ GeV and $T_p = 2.85$ GeV of the incident pro−
tons. It has been shown that the $\Lambda^*$ resonance is the
doorway state for the formation of the $\bar{K}NN$ state with
a high $p\Lambda^*$ sticking probability at $T_p = 2.85$ GeV.

Since the widths of the KNC $^1_{1H}$ and $^2_{1H}$ have been
calculated in \cite{11}, the analytical expressions of which we
use in this letter, we shall focus on the calculation of the
binding energies and other properties of these states, such
as densities, sizes and formation probabilities in the
$pp$ reaction by taking into account all relevant interaction
channels.

2. Analytical expression and numerical value of the
binding energy of KNC $^1_{1H}$

Since in \cite{11} we have been restricted by the tree−
approximation, the binding energy of the KNC $^1_{1H}$ has
been determined by the Weinberg−Tomozawa (WT)\n$(\bar{K}N)_{I=0} \to (\bar{K}N)_{I=0}$ interaction only. Such a cal−
culation does not take into account the contributions of
other possible WT−interactions, making an impor−
tant influence on the dynamics of the KNC $^1_{1H}$. Here
we calculate the contribution of all interaction channels
$(\bar{K}N)_{I=0} \to (\bar{K}N)_{I=0}$, $(\bar{K}N)_{I=0} \to (\Sigma\pi)_{I=0}$ and
$(\bar{K}N)_{I=0} \to \Lambda\eta$ in terms of the self−energy loop cor−
corrections, where the vertices of Feynman diagrams are
defined by the WT−interactions. These contributions are
required also by a self−consistency of the analysis of the
binding energy of the KNC $^1_{1H}$ with its decay modes.
Indeed, the real parts of these diagrams give the contri−
butions to the mass of the KNC $^1_{1H}$, whereas the imagi−
ary parts define the partial widths of the kinematically
allowed decay channels.

Thus, in our approach the mass of the KNC $^1_{1H}$ is
determined as

$$M_{^1_{1H}} = m_N + m_K - B_{^1_{1H}}^{WT} + \sum_X \delta M_{^1_{1H}}^{(X)}$$  \hspace{1cm} (1)

where $B_{^1_{1H}}^{WT}$ defines the tree−level contribution, caused by
the $(\bar{K}N)_{I=0} \to (\bar{K}N)_{I=0}$ WT−interaction. The mass−corrections $\delta M_{^1_{1H}}^{(X)}$ are calculated for the complete set of
two−body intermediate states $X = (\bar{K}N)_{I=0}, \Lambda^0\eta$ and $(\Sigma\pi)_{I=0}$. The calculation shows that only the inter−
mediate state $\Lambda^0\eta$ gives a finite contribution. Its Feynman
diagram is shown in Fig. 1. The binding energy of the
KNC $^1_{1H}$ with the mass−correction is

$$B_{^1_{1H}} = B_{^1_{1H}}^{WT} - \frac{27}{512\pi} \frac{m_K}{F^2} \sqrt{m_{\Lambda^*}^2 - m_K^2 \left(\frac{\mu_{\Lambda^*}\Omega_{\Lambda^*}}{\pi}\right)^{3/2}}.$$ \hspace{1cm} (2)

Setting the binding energy $B_{^1_{1H}}$ of the KNC $^1_{1H}$ equal
to $B_{^1_{1H}} = m_N + m_K - m_{\Lambda^*} = 29$ MeV, we get $\Omega_{\Lambda^*} = 59.5$ MeV. It is larger compared with the frequency
$\Omega_{\Lambda^*} = 46.3$ MeV, derived in \cite{11}, thus leading also to a
stronger binding of the KNC $^2_{1H}$.

Using the results, obtained in \cite{11}, we calculate the
width $\Gamma_{^1_{1H}} = 36.4$ MeV of the KNC $^1_{1H}$, which is
defined by the $(\bar{K}N)_{I=0} \to (\Sigma\pi)_{I=0}$ WT−interaction only.
It compares well with $\Gamma_{^1_{1H}} = 40$ MeV, accepted for the
description of the $\Lambda^*$ state in \cite{2}.

3. Analytical expression of the binding energy of
KNC $^2_{1H}$

The contributions to the mass of the KNC $^2_{1H}$ are
given by the Feynman diagrams in Fig. 2. For the binding
energy of the KNC $^2_{1H}$ we obtain the following ex−
pression, including the contributions of the tree−level ap−
proximation and self−energy loop corrections

$$B_{^2_{1H}}^{jj'} = (B_{^2_{1H}}^{WT})^{jj'} - \sum_X \delta M_{^2_{1H}}^{(X)^{jj'}} = (B_{^2_{1H}}^{WT})^{jj'}$$

$$+ \delta^{jj'} \frac{\xi}{\sqrt{2}} \frac{81}{8192\pi^2} \frac{(m_\pi + m_K)^2}{F^4} \frac{m_\pi^2}{m_K^2} \left(\frac{m_\pi^2 - m_K^2}{\mu_{\Lambda^*}\Omega_{\Lambda^*}}\right)^{3/2}$$

$$\times \left(\frac{\mu_{\Lambda^*}\Omega_{\Lambda^*}}{\mu_{\Lambda^*}\Omega_{\Lambda^*}}\right)^{3/2} + \delta^{jj'} \frac{1}{\sqrt{2}} \frac{141}{8192\pi^2} \frac{(m_\pi + m_K)^2}{F^4}$$

$$\frac{m_\pi^2}{m_K^2} \left(\frac{m_\pi^2 - m_K^2}{\mu_{\Lambda^*}\Omega_{\Lambda^*}}\right)^{3/2}.$$ \hspace{1cm} (3)

Here $j$ and $j'$ are nucleon isospin indices, $\Omega_{\Lambda^*\eta}$ is the
frequency of the relative motion of the nucleon $N$ and

![FIG. 1: The Feynman diagram of the main correction to the mass of the KNC $^1_{1H}$.](image-url)
the pair $\bar{K}N$, $\mu_{A^*}$ and $\mu_{A^*N}$ are the reduced masses of the $(\bar{K}N$ and $N(\bar{K}N)$ systems, respectively, and $\xi = 4m_N/(m_N + m_K + 4m_N)$.

The binding energy $(B_{K,H}^{WT})^{ij}$ is calculated in the tree–approximation and determined by the $NKN \rightarrow N\Lambda^0\eta, N(\Sigma\pi)_{I=0}$ and $NN\bar{K} \rightarrow N\Lambda^0\eta$ interactions, respectively, which correspond to the intermediate states $X = N\Lambda^0\eta, N(\Sigma\pi)_{I=0}$ and $N(\Sigma\pi)_{I=1}$ and $N\Lambda^0\eta$ states in the $\frac{3}{2}^-H \rightarrow \frac{3}{2}^-H$ transition. The coupling constants of these interactions are calculated by using chiral Lagrangians. The contributions of the $NKN \rightarrow N(\Sigma\pi)_{I=1}$ and $NN\bar{K} \rightarrow N\Lambda^0\eta$ interactions appear due to the $NN$ exchange interaction.

4. Numerical values of binding energy, width and density of KNC $\frac{3}{2}^-H$

For the large binding energy of the KNC $\frac{3}{2}^-H$, measured by the DISTO Collaboration [10], the pionic decay modes $N\Sigma\pi$ are suppressed due to a very small phase volume of the final state. As a result the non-pionic decay modes $\frac{3}{2}^-H \rightarrow N\Lambda^0$ and $\frac{3}{2}^-H \rightarrow N\Sigma$ are allowed only. The partial widths of the non-pionic decay modes have been calculated in [11]. Using these results, Eq. (3), and the frequency $\Omega_{A^*} = 59.5$ MeV, fixed from the binding energy of the KNC $\frac{1}{2}^-H$, we obtain the binding energy $B_{K,H} = (74.5)_{WT} + (+5.4)_{NA\eta} + (-12.0)_{N(\Sigma+\Lambda^0)\eta} = 118$ MeV and the width $\Gamma_{K,H} = (97)_{NA\eta} + (45)_{N\Sigma} = 142$ MeV. They are calculated for the frequency $\Omega_{A^*N} = 105.0$ MeV, providing an optimal ratio $B_{K,H}/\Gamma_{K,H} = 0.83$ in comparison with the experimental one $B_{K,H}^{exp}/\Gamma_{K,H}^{exp} = 0.87(11)$ [10].

In addition these frequencies define the high density of the KNC $\frac{3}{2}^-H$ equal to

\[
n_{\frac{3}{2}^-H}(0) = \frac{1}{\pi^{3/2}} \frac{(\mu_{A^*} + \mu_{A^*N})^3}{(\Omega_{A^*} + \Omega_{A^*N})^{3/2}} = 2.71 n_0.
\]

which agrees well with the density $n_{KNN}(0) \sim 3 n_0$, estimated in [10], where $n_0 = 0.17$ fm$^{-3}$ is the normal nuclear density. The root mean square (rms) radius of the KNC $\frac{3}{2}^-H$ is $R_{K,H} = 0.89$ fm and thus is direct evidence for the compactness of the deeply bound KNC $\frac{3}{2}^-H$. As we show below the value of the rms $R_{K,H}$ is $0.89$ fm agrees well with the high $\Lambda^+$ sticking probability, observed in the $pp$ reaction at 2.85 GeV kinetic energy of the incident protons [21]. For the KNC $\frac{3}{2}^-H$ we get a low density $n_{\frac{3}{2}^-H}(0) = (\mu_{A^*} \cdot \Omega_{A^*}/\pi)^{3/2} = 0.37 n_0$ and a large rms radius $R_{K,H} = 1.74$ fm.

The results, obtained in the molecule model, are summarised in Table I and compared with the experimental data by the DISTO Collaboration, which are well reproduced. In addition the widths of all partial decay modes $\frac{3}{2}^-H \rightarrow N\Lambda^0, N\Sigma^+$ and $N\Sigma^0$ are predicted.

| Molecule model | DISTO |
|---------------|-------|
| $B_{K,H}$     | 29.0  |
| $\Gamma_{K,H}$ | 36.4  |
| $n_{\frac{3}{2}^-H}(0)$ | 0.37n$_0$ |
| $B_{\frac{3}{2}^-H}$ | 118  |
| $\Gamma_{\frac{3}{2}^-H \rightarrow N\Lambda^0}$ | 97   |
| $\Gamma_{\frac{3}{2}^-H \rightarrow N\Sigma^+}$ | 15   |
| $\Gamma_{\frac{3}{2}^-H \rightarrow N\Sigma^0}$ | 30   |
| $n_{\frac{3}{2}^-H}(0)$ | 142  |

TABLE I: The binding energies and widths of the KNC $\frac{1}{2}^-H$ and $\frac{3}{2}^-H$, measured in MeV, and their densities in terms of the normal nuclear density $n_0 = 0.17$ fm$^{-3}$.

5. Concluding discussion

The large total width of the KNC $\frac{3}{2}^-H$, such as calculated above and observed in [10], is a strong indication of the compactness of the $\bar{K}NN$ system as was pointed out already in [10]. Thus, one can assert that a large width $\Gamma_{\frac{1}{2}^-H} = 142$ MeV of the KNC $\frac{3}{2}^-H$ is fully caused by its large density $n_{\frac{3}{2}^-H}(0) = 2.71 n_0$ [10]. In turn, the width $\Gamma_{\frac{1}{2}^-H} = 36.4$ MeV of the KNC $\frac{1}{2}^-H$, which is smaller compared with the width of the KNC $\frac{3}{2}^-H$, should imply a smaller density. This agrees qualitatively with the value $n_{\frac{1}{2}^-H}(0) = 0.37 n_0$.

The main contributions to the binding energy of the KNC $\frac{3}{2}^-H$ come from the $(\bar{K}N)_{I=0} \rightarrow (\bar{K}N)_{I=0}$ and $(\bar{K}N)_{I=1} \rightarrow (\bar{K}N)_{I=1}$ WT–interactions and the effective $NNK \rightarrow N\Lambda^0\eta$ interaction, the coupling constant of which has the same strength as the $(\bar{K}N)_{I=0} \rightarrow (\bar{K}N)_{I=0}$ WT–interaction.

For a confirmation of the compactness of the KNC $\frac{3}{2}^-H$
in the molecule model of KNCs we can estimate the depth $U_{\Lambda^*N}$ of the potential of the $\Lambda^*N$ interaction in the KNC $^2\Lambda H$. We get $U_{\Lambda^*N} = -B_{^2\Lambda H} - \frac{1}{2}\Omega_{\Lambda^*N} = -276$ MeV, which is larger than the value $U_{\Lambda(1405)N(1530)} = -200$ MeV, originally proposed by Yamazaki and Akaishi [3]. This confirms the existence of the KNC $^2\Lambda H$ as a very compact $KNN$ system, caused by a very strong $\Lambda^*N$ interaction.

According to [3] (see also [16]), the KNC $^2\Lambda H$ should appear as a result of a sticking of the $\Lambda^*p$ pair into the KNC $^2\Lambda H$, produced in the $pp \to K^+\Lambda^*p \to K^+\Lambda^0p$ reaction. A high $\Lambda^*p$ sticking probability has been recently observed in [20]. In the molecule model we can estimate the sticking probability of the $\Lambda^*N$ pair in the center of mass frame as the probability of finding the $\Lambda^*N$ pair inside the KNC $^2\Lambda H$. Thus, the sticking of the $\Lambda^*N$ pair into the KNC $^2\Lambda H$ with relative momenta $p \leq q < \infty$ is given by

$$P(\Lambda^*N \to ^2\Lambda H)(p) = \int_{q \geq p} |\Phi_{\Lambda^*N}(\vec{q})|^2 \frac{d^3q}{(2\pi)^3}.$$  \hspace{1cm} (5)

where $\Phi_{\Lambda^*N}(\vec{q})$ is the wave function of a relative motion of the $\Lambda^*N$ pair inside the KNC $^2\Lambda H$. Since the $\Lambda^*N$ pair should be confined inside the KNC $^2\Lambda H$, the confining area is restricted by a finder of order of $D = 2R_{^2\Lambda H} = 1.78$ fm. This defines the minimum relative momentum $p \sim 1/D = 111$ MeV/c for the $\Lambda^*N$ sticking in the $pp$ reaction. The sticking probability $P(\Lambda^*N \to ^2\Lambda H)(p) \sim 0.94$, calculated for $p \sim 1/D = 111$ MeV/c, agrees well with the experimental data on the high sticking probability of the $p\Lambda^*$ pair, observed in the reaction $pp \to K^+p\Lambda^0$ at the kinetic energy $T_p = 2.85$ GeV of the incident proton [20].

This research was partly supported by the DFG cluster of excellence "Origin and Structure of the Universe" of the Technische Universität München and by the Austrian “Fonds zur Förderung der Wissenschaftlichen Forschung” (FWF) under contract P19487-N16.

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