Research Article

Effect of Optimal Subsidy Rate and Strategic Behaviour of Supply Chain Members under Competition on Green Product Retailing

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This paper investigates the impact of the subsidy and horizontal strategic cooperation on a green supply chain where two competing manufacturers distribute substitutable green products through exclusive retailers. Models are formulated in three-stage game structures in five different scenarios, where the government organization determines optimal subsidy by pursuing social welfare maximization. Both manufacturers invest in improving green quality levels of products. The study aims to explore the advantage of vertical integration and strategic collusion from the perspective of green supply chain practice in the presence of subsidy. The key contributions from the present study indicate that under competition, members of both supply chains are able to receive higher profits through horizontal collusion, but green quality levels of the product remain suboptimal. If upstream manufacturers cooperate, government subsidy does not necessarily improve product quality level, and the amount of government expenditure increased substantially. By comparing outcomes where members are vertically integrated with scenarios where members make strategic collusion, we found that the former might outperform by later. Cross-price sensitivity appears as a significant parameter affecting supply chain members’ performance and the amount of government expenditure. Cooperation between members at the horizontal level is a more robust strategic measure than vertical integration if consumers are highly price-sensitive.

1. Introduction

In the past few decades, with increasing environmental awareness, issues on investment in green quality improvement have been regarded as one of the significant solutions to sustainability issues [1, 2]. A recent global survey by Accenture reported that 83% of consumers consider product greenness when making purchasing decisions [3]. Spurred by this market force, an increasing number of manufacturers make green technology investments in their production process to fulfill the demands of environmentally concerned consumers, gain competitiveness, and strengthen their reputations. However, the cost of investment in green technology is usually substantial, which is viewed as one of the main barriers to green production. Manufacturers need to consider an explicit trade-off between the pros and cons of several issues associated with an investment in green technology. Particularly, investment decisions become more complicated while trading with substitutable products, and in such a situation, a manufacturer’s decision will be further affected by the rival manufacturer’s decisions and even the strategic decision of downstream retailers. In this study, we analyze the equilibrium of two competing supply chains (SCs), each of which consists of a single manufacturer, selling its products exclusively through a single retailer. Upstream manufacturers determine wholesale prices and investment in improving green quality levels, and downstream retailers determine market prices. The proposed SC structures fit numerous industries such as gasoline, soft drink, garments, footwear, cars, and electronics accessories, where a manufacturer trades with an exclusive retailer. The demand for substitute products has a negative correlation and creates a rivalry between two competing SCs. Therefore, the first research question addressed by the study is as
follows: how does the investment efficiency under competition affect the downstream retailers and consumers?

In pragmatic environment, “collusion” between two competing manufacturers is not rare [4]. For example, stable collaborative relationships between Apple and Samsung [5, 6] justify this possibility. Upstream manufacturers cooperate for many reasons, such as to increase joint market size, to develop products with new features to protect the present and future share of the market, and to conquer a larger share of the market [7]. In the existing literature, researchers have explored empirically identified scenarios where manufacturers may collude [8, 9]. Nocke and White [10] examined the condition when the collusive effect between upstream manufacturers improves the utility of the overall distribution channel. Piccolo and Reisinger [11] compared the equilibrium price of two competing SCs where two colluding manufacturers maximize joint profits and found that exclusive territories sometime favor collusion. Huang [12] studied the effect of downstream collusion and reported that it can cause a detrimental effect on the performance of the upstream manufacturers. In some recent studies, the strategic aspect of collusion between upstream manufacturers [13] or downstream collusion between retailers [14] is also documented. Therefore, the second research question addressed by the study is as follows: can strategic collusion between manufacturer-manufacturer and retailer-retailer enhance the overall performance of each SC and sustainability? And if so, is it beneficial between upstream members or downstream members?

In green supply chain management, government organizations play an important role by providing subsidy to supply chain members [15–26]. However, the impact of government subsidy on manufacturers’ green technology investment decisions and strategic collusion between horizontal members under chain-to-chain competition have not been well understood in the literature. Therefore, the third intriguing research question arises in that consequence is as follows: does the government expenditure increase in the presence of strategic collusion?

Our work complements the literature on subsidizing manufacturers selling substitutable products under competition in different strategic settings. We consider five scenarios to investigate the characteristics of optimal decisions under horizontal collusion and vertical integration on each firm’s profit, green quality levels, and social welfare. In Scenario UDLD, two upstream manufacturers make wholesale pricing and green quality decisions and two retailers set their respective retail prices by maximizing their respective profit functions [26]. We consider the scenario as a benchmark. In Scenario UCLD, two upstream manufacturers make wholesale pricing and green quality by maximizing total upstream profits, not individual profits, and then two retailers set their respective retail prices by maximizing their individual profits. This game structure is similar to the “collusion” game as discussed by Bian et al. [4]. In Scenario UDLC, two upstream manufacturers make their decision by maximizing their respective profits, but two retailers set retail prices by maximizing the sum of downstream profits. Finally, in Scenario UCLC, both two upstream manufactures and downstream retailers make their decision by maximizing the sum of upstream and downstream profits, respectively. Therefore, Scenarios UCLD, UDLC, and UCLC represent all possible options of horizontal collusion under competition. Finally, Scenario CC is considered where the manufacturer and retailer in each SC are vertically integrated [27]. This will assist us in finding the answer to our third research question: do the outcomes under horizontal collusion outperform the decision attained under vertical integration?

The main insights of our research are summarized as follows: first, to some extent, a dominant equilibrium strategy is for both manufacturers to make upstream collusion; however, they can encounter a prisoner’s dilemma. Increasing consumer cross-price elasticity can intensify the competition to a point where eventually both manufacturers are worse off. While both manufacturers invest to improve green quality levels and make collusion, they try to upsurge the wholesale prices to compensate investment costs, which in turn raises retail prices and worsens double marginalization. Second, higher government subsidy does not always ensure a higher green quality level. Due to higher price-setting power, profits for each firm and government expenditure increase under collusion, but social welfare and green quality levels will be always less. Finally, under SC competition, researchers largely highlighted the benefits of vertical integration between SC members under competition, but we prove that horizontal collusion can be a useful strategic option for competing SC members to improve their respective profits.

1.1. Literature Review. Our study is closely related to three different streams of research such as (i) decision under supply chain competition, (ii) supply chain decision under price and green quality level-sensitive demand, and (iii) government subsidy on green technology investment.

Early seminal work in decisions under SC competition is done by McGuire and Staelin [28], Choi [29], and Moorthy [30], where pricing behaviours of two competing SCs are explored where there are two manufacturers, each sells substitutable products through an independent retailer. However, in the last decades, this research stream is gaining priority form the research community. In this direction, one can categorize the number of publishing articles into two groups. In the first groups, researchers mainly focused on the effect the information asymmetry in SC competition [31], Ai et al. [32], Bian et al. [33], Lee [34], and others. The authors explored optimal decisions mostly in a single game structure under price-dependent demand and explored the effect of information asymmetry in decentralized and centralized settings. In contrast, a group of researchers studied optimal decisions in various game models under symmetric information, and our work is closely related to this stream of research [35]. For example, Wu and Mallik [36] discussed the optimal decisions of competing SCs where members separately maximize their respective profits non-cooperatively, members of one SC imply decentralized decision, and others imply centralized decision, and compared
optimal decision by benchmarking the centralized decision. Zhou and Cao [27] studied the optimal decision of two competing SCs under the price and display quantity dependent demand. The authors derived optimal decisions in three decision-making scenarios, (i) two members in each decentralized SC set decision under the “manufacturer-Stackelberg” game, (ii) manufacturer and corresponding exclusive retailer in each SC are agreeing to form an integrated firm by negotiation, and (iii) one of the SCs implies integrated decision and others strict with the decentralized decision. By comparing profits, the authors conclude that the relatively fierce price competition between two SCs may eliminate the negative effects of double marginalization on the overall SC profit. Similar to Zhou and Cao [27], Li and Li [37] explored optimal decisions for a sustainable SC in three different game structures. By comparing equilibrium, the authors found that the vertical integration of two competing SCs can be beneficial when the product quality competition degree is low and decentralized SC decision can be more preferable if competition degree is more fierce. In this direction, Wang et al. [38] also compared the equilibriums of two competing SCs in three game structures, where the manufacturer acts as a Stackelberg leader, the retailer act as a Stackelberg leader, and both the manufacturer and the retailer act as Bertrand–Nash competitors. However, the work by Zhu and He [39] is different, and instead of comparing optimal decisions under different games, the authors derive optimal decisions in single retailer-single manufacturer, single manufacturer-two retailers, and two manufacturers-two retailers settings. The authors reported that price competition between two retailers could increase the equilibrium product qualities. Seyedhosseini et al. [40] analyzed optimal decision under four game structures, namely, Stackelberg-Cournot, Stackelberg-Collusion, Nash-Cournot, and Nash-Collusion. The authors found that the Nash-Collusion game structure can yield maximum supply chain profits. Bian et al. [4] also studied a model with a different context, where each manufacturer can distribute their products through either a single retailer or both retailers. The authors found that SC members prefer a single distribution channel if the products are substitutable to a sufficient extent. However, the aforementioned studies do not take into consideration of SC members’ option collusion formation and government subsidy. Our study contributes to this stream of research as we investigate horizontal and vertical cooperation in competing SCs.

Another stream of research related to the study is the study where the authors studied the characteristics of supply chain equilibrium where the demand function is influenced by the green degree of the products and retail price [15, 41–48]. This group of researchers mainly focused on the equilibrium decision under the various game structures and on the way to improve the performance of supply chain members through various coordination mechanisms under a single manufacturer-single retailer setting. However, we study the optimal decision under SC competition. In this direction, our work is closely related to Li and Li [37] and Yang et al. [49], where the authors explored the characteristics of optimal decisions under competition. He et al. [50] studied the impact of subsidy in a dual-channel closed-loop supply chain and found that higher subsidy can benefit consumers but not necessarily improve environmental performance. Li and He [51] investigated the pricing and information disclosure strategies in a green supply chain and found that a manufacturer can receive higher benefit by disclosing information. Sana [52] pointed out the strategic advantage of selling green products in the presence of subsidy and reported subsidy is important to encourage manufacturers to trade with green products. However, the authors ignored the combined effect of retail prices, collusion behavior of SC members, and government subsidy in a two manufacturers-two retailers supply chain model. Therefore, our perspective differs from the existing literature.

Finally, we study the effect of government subsidy on green supply chain practices. To support green product manufacturing and promote consumption, government organizations in different countries design various subsidy and tax policies [45, 46, 53, 54]. In the existing literature, researchers studied the impact of several forms of subsidy, such as direct subsidy to manufacturers [15–17, 55], to retailer [55, 56], to consumers [18, 48, 55, 57, 58], to both retailers and manufacturers [20], to manufacturers and consumers [59], and in others way to improve environmental sustainability. However, the literature on the influence of subsidy in SC competition is sparse. In this study, we consider two competitive SCs, each of which consists of one manufacturer and one retailer and discuss the scenarios where two upstream manufacturers receive subsidies based on green technology investment. The optimal subsidy rate is determined by maximizing social welfare function in each of the five scenarios [56]. Comparative analysis among optimal decision in five scenarios can help policymakers to understand how strategic cooperation affects the green quality of the product and explore the trade-off between expenditure and social welfare optimization goal.

2. Model Settings

We consider two ex-ante symmetric supply chains, indexed by \( i = 1, 2 \), and each consists of one manufacturer \( M_i \) and corresponding one exclusive retailer \( R_i \). The two manufacturers sell substitutable products and compete in the end-customer market under price and green quality level-sensitive demand. We consider five different scenarios as presented in Figure 1.

First, Scenario UDLD is considered as benchmark, where members in both SCs take decentralized decision [27, 37, 60]. The next three Scenarios UCLD, UDLC, and UCLC are considered to analyze the effect of horizontal collusion. In Scenario UCLD, two manufacturers optimize the sum of upstream profits, but two downstream retailers optimize their respective profits [4]. In Scenario UDLC, two manufacturers optimize their respective profits, but downstream retailers set retail prices by optimizing total downstream profits. In Scenario UCLC, horizontal and vertical members make their respective decision by maximizing the sum of upstream and downstream profits.
Finally, we consider Scenario CC where each SC is vertically integrated [27].

To characterize the demand (\(D^i\)) for product \(i\), we adopt the following functional forms:

\[
D^i_i(p^i, \theta^i) = a - p^i + \beta p^j + \gamma \theta^j - \delta \theta^j, \quad i = 1, 2, j = 3 - i,
\]

where \(p^i\) and \(\theta^i\) represent market price and green quality level, respectively. Therefore, \(D^i\) has positive correlations with \(\theta^i\) and \(p^j\) and negative correlations with \(\theta^j\) and \(p^i\). Note that without the influence of prices, the demand function is similar to Li and Li [37] or without the impact of green quality level, it is similar to Bian et al. [4].

Similar to [45], we adopt a quadratic form function \(\eta \theta^i\) to represent the green investment efficiency for \(M_i\). To enable fair comparison among the optimal decisions for different scenarios and for analytical simplicity, we assume \(\eta_1 = \eta_2 = \eta\) [56, 61]. For parsimony, we further assume that operational costs for each SC are constant and normalized to zero. We assume that all the parameters are deterministic and evaluate the equilibrium of the five scenarios under symmetric information [16].

To encourage green product manufacturing, the government provides a subsidy on total investment in green technology adaptation, and the government decides the subsidy rate by maximizing social welfare. Therefore, we employ a three-stage game structure. Under competition, the decision about retail prices is taken by retailers, and decisions about wholesale prices and investment to improve product green quality levels are taken by manufacturers. Finally, the government determines the subsidy rate by optimizing the social welfare (SW) function.

For the rest of the paper, we assume that

\[
\eta > \max\left\{ \frac{(28 - 24 \beta^2 + 5 \beta^4)(y - \delta)^2}{2(2 - \beta)(4 - \beta + 2 \beta^2)^2}, \frac{(7 - 5 \beta)(y - \delta)^2}{8(2 - \beta)(1 - \beta)}, \frac{(7 - \beta)(y - \delta)^2}{32(1 - \beta)} \right\}.
\]

Not only does the condition ensure that there must be a threshold for the efficiency limit of the manufacturers’ green technology investment, but also the condition is sufficient for many subsequent analytical results related with the existence of optimal decision in five scenarios. Moreover, under the above assumptions, equilibrium wholesale prices, green quality levels, retail prices, and profits are always positive.

Based on the demand function in equation (1) and assumptions, the profit functions for manufacturers, retailers, and SW function for government organization are given as

\[
\Pi^i_m = (p^i - w^i)D^i_i, \quad i = 1, 2,
\]

\[
\Pi^i_m = w^i D^i_i - \eta (1 - \alpha^i) (\theta^i)^2, \quad i = 1, 2,
\]

\[
SW^s = \sum_{i=1}^{2} \Pi^i_m + \sum_{i=1}^{2} \Pi^i_m + CS - \sum_{i=1}^{2} \eta \alpha^i (\theta^i)^2.
\]

Similar to the existing study, we consider the impact of profits of both SCs, the members’ and consumer surplus (CS), and total government expenditure on the social welfare function [62]. For each scenario, we determined subsidy rate by optimizing SW. We summarize notations used to distinguish optimal decisions under five scenarios in Table 1.

### 3. Model and Decision-Making

In this section, we first characterize optimal decisions in four scenarios and then explore the impact of collusion and subsidy.

#### 3.1. Optimal Decisions in Scenarios UDLD and UCLD

In these two scenarios, two downstream retailers take their decision independently, but two upstream manufacturers take their decision independently in Scenario UDLD, and
increase of wholesale prices of both products. Because prices, retail price of any product would increase with the prices of both products have a positive correlation with retail price. Therefore, we observe that retail price increased with the green quality level in Scenario UCLD.

Stage 2. Two upstream manufacturers quote wholesale prices \( w_i \) and green quality levels \( \theta_i \) by maximizing their respective profits in Scenario UCLD. However, two manufacturers quote wholesale prices \( w_i^{\text{add}} \) and green qualities \( \theta_i^{\text{add}} \) by maximizing sum of upstream profits in Scenario UCLD.

Stage 3. Two downstream retailers choose market prices \( p_i^{\text{add}} \) and \( p_j^{\text{add}} \) in Scenario UCLD and UCLD, respectively. Therefore, the optimization problem in Scenario UCLD is as follows:

\[
\{\max_{p_1^{\text{add}}, p_2^{\text{add}}} \text{SW}^{\text{add}}, \quad \text{max}_{p_i^{\text{add}}, p_j^{\text{add}}} (\Pi_i^{\text{add}} + \Pi_j^{\text{add}}), \quad \max_{p_1^{\text{add}}, p_2^{\text{add}}} (\Pi_1^{\text{add}} + \Pi_2^{\text{add}}). \}
\]

First, two retailers’ responses for retail prices are determined by assuming decision variables of upstream members are given. Solving the first-order conditions \( (d\Pi_i^{\text{add}}/dp_i^{\text{add}}) = 0, i = 1, 2 \), simultaneously, optimal responses on retail prices are obtained as follows:

\[
p_i^{\text{add}} = \frac{a(2 + \beta) + 2w_i^{\text{add}} + w_j^{\text{add}} + (2\gamma - \beta\delta)\theta_i^{\text{add}} + (\beta\gamma - 2\delta)\theta_j^{\text{add}}}{4 - \beta^2}, \quad i = 1, 2, j = 3 - i.
\]

From the above expression, we observe that the wholesale prices of both products have a positive correlation with retail prices. The retail price of any product would increase with the increase of wholesale prices of both products. Because \( 2\gamma > \beta\delta \), we observe that retail price increased with the green quality level of that product. Moreover, if \( \beta\gamma > 2\delta \), then retail price increases with the green quality levels of both products. Because \( (d^2\Pi_i^{\text{add}}/dp_i^{\text{add}}) = -2 < 0 \), i.e., the optimality is ensured.

Plugging response for two retailers in equation (4), profit functions for two manufacturers are obtained as follows:
\[ \Pi_{mi}^{\text{alld}} = \frac{w_i^{\text{alld}}(a(2 + \beta) + w_j^{\text{alld}}\beta - \omega_i^{\text{alld}}(2 - \beta^2) + (2\gamma - \beta\delta)\theta_i^{\text{alld}} + (\beta\gamma - 2\delta)\theta_j^{\text{alld}})}{4 - \beta^2} - (1 - \alpha^{\text{alld}})\eta\theta_i^{\text{alld}}, \quad i = 1, 2, j = 3 - i. \]  

(8)

To determine optimal response for two manufacturers, we solve the first-order conditions \(\partial \Pi_{mi}^{\text{alld}}/\partial w_i^{\text{alld}} = 0\) and \(\partial \Pi_{mi}^{\text{alld}}/\partial \theta_i^{\text{alld}} = 0\), simultaneously. On simplification,

\[ w_i^{\text{alld}} = \frac{2a(1 - \alpha^{\text{alld}})(4 - \beta^2)\eta}{2(1 - \alpha^{\text{alld}})(2 - \beta)(4 - \beta - 2\beta^2)\eta - (\gamma - \delta)(2\gamma - \beta\delta)}, \]  

(9)

\[ \theta_i^{\text{alld}} = \frac{a(2\gamma - \beta\delta)}{2(1 - \alpha^{\text{alld}})(2 - \beta)(4 - \beta - 2\beta^2)\eta - (\gamma - \delta)(2\gamma - \beta\delta)}, \quad i = 1, 2. \]

From the expressions of green qualities, it is observed that both are decreased as the investment efficiency of two manufacturers \(\eta\) decreased and reverse trend is observed with the subsidy rate \(\alpha^{\text{alld}}\) (please find Appendix A). Therefore, the green quality levels will be least if \(\alpha^{\text{alld}} = 0\) and consumers get benefited from the presence of subsidy. Note that in the absence of a subsidy, there is no need to execute the first stage. We represent the optimal solution in wholesale prices and green quality levels are obtained as follows:

We can see that all the diagonal elements of the above the Hessian matrix \(H_{mi}^{\text{alld}}\) are negative; therefore, profit function for each manufacturer is concave if \(\Theta_1 = 4(1 - \alpha^{\text{alld}})(8 - 6\beta^2 + \beta^4)\eta - (2\gamma - \beta\delta)^2 > 0.\)

Using the response for the manufacturers and retailers in equation (5), the social welfare function \(\text{SW}^{\text{alld}}\) is obtained as follows:

\[ \text{SW}^{\text{alld}} = \frac{2a^2\eta\left[2(1 - \alpha^{\text{alld}})^2\Phi_1\eta - (2\gamma - \beta\delta)^2\right]}{\left[2(1 - \alpha^{\text{alld}})(2 - \beta)(4 - \beta - 2\beta^2)\eta - (\gamma - \delta)(2\gamma - \beta\delta)\right]^2}, \]  

(11)

where \(\Phi_1 = 28 - 24\beta^2 + 5\beta^4\). Therefore, the optimal subsidy rate is obtained by solving \(d\text{SW}^{\text{alld}}/d\alpha^{\text{alld}} = 0\). On simplification,

\[ \alpha^{\text{alld}} = \frac{12 + \beta(2 - \beta)(6 - 6\beta - 5\beta^2))\gamma - (28 - \beta(8 + 3\beta(6 - \beta^2)))\delta)}{\Phi_1(\gamma - \delta)}. \]  

(12)
Table 2: Optimal decisions in the absence of subsidy.

| Scenario   | UDLDN       | UCLDN       | UDLCN       | UCLCN       |
|------------|-------------|-------------|-------------|-------------|
| $u^*_j$    | $(2a(4-\beta^2)\eta/\Delta_{0})$ | $(2a(2-\beta)\eta/\Delta_{0})$ | $(4a\eta/\Delta_{0})$ | $(4a\eta/\Delta_{0})$ |
| $\theta^*_j$ | $(a(2\gamma-\beta\delta)/\Delta_{0})$ | $(a(y-\delta)/\Delta_{0})$ | $(a(y-\delta)/\Delta_{0})$ | $(a(y-\delta)/\Delta_{0})$ |
| $P^*_j$    | $(4a\eta(3-\beta^2)/\Delta_{0})$ | $(2a(3-2\beta)\eta/\Delta_{0})$ | $(2a(3-2\beta)\eta(1-\beta)/\Delta_{0})$ | $(6a\eta/\Delta_{0})$ |
| $Q^*_j$    | $(2a\eta(2-\beta^2)/\Delta_{0})$ | $(2a(1-\beta)\eta/\Delta_{0})$ | $(2a(1-\beta)\eta/\Delta_{0})$ | $(2a(1-\beta)\eta/\Delta_{0})$ |
| $\Pi^*_j$  | $(a\eta^2(4a+\beta^2-2\beta\delta))/\Delta_{0}^2$ | $(a\eta^2(1-\beta)\eta)/\Delta_{0}^2$ | $(a\eta^2(1-\beta)\eta)/\Delta_{0}^2$ | $(a\eta^2(1-\beta)\eta)/\Delta_{0}^2$ |

where $\Delta_{0} = (2(\beta^2-\beta(4-\beta^2))^\eta-(\gamma-\delta))$, $\Delta_{0} = 4(2-\beta)\eta-(\gamma-\delta)$, and $\Delta_{0} = 8(1-\beta)\eta-(\gamma-\delta)^2$.

From the above expression of the subsidy rate, we can observe that the subsidy rate is independent of intrinsic market demand. Note that the $SW_{\text{add}}$ function is also concave with respect to $a_{\text{add}}$, if $\Delta_{1} > 0$ because

$$\frac{d^2SW_{\text{add}}}{da_{\text{add}}^2} = \frac{8a^2\Phi_1^4(y-\delta)^3\eta^2}{(2y-\beta\delta)^4\Delta_{1}^4} < 0,$$

where $\Delta_{1} = 2(2-\beta)(4-\beta^2)^2\eta$, $\Phi_1(y-\delta)^2$. In addition,

$$\Theta_1_{\text{add}} = \frac{(2\gamma-\beta\delta)\Delta_{11}}{(14-5\beta^2)(y-\delta)},$$

$$\frac{\alpha_{\text{add}}}{\Delta_{1}} = \frac{12 + \beta(2-\beta)(6-6\beta-5\beta^2))y - (28 - \beta(8 + 3\beta(6 - \beta - \beta^2)))\delta}{\Phi_1(y-\delta)}$$

$$\frac{u_{\text{add}}}{\Delta_{1}} = \frac{2a(2-\beta)^2(2+\beta)(4-\beta^2)\eta}{\Delta_{1}}$$

$$\frac{r_{\text{add}}}{\Delta_{1}} = \frac{a\Phi_1(y-\delta)}{\Delta_{1}}$$

$$\frac{P_{\text{add}}}{\Delta_{1}} = \frac{4a(2-\beta)(3-\beta^2)(4-\beta^2)^2\eta}{\Delta_{1}}$$

$$\frac{\Pi_{\text{mi}}^{\text{add}}}{\Delta_{1}} = \frac{a^2(2-\beta)\Delta_{11}^2}{(14-5\beta^2)(y-\delta)(2y-\beta\delta)}$$

$$\frac{Q_{\text{add}}}{\Delta_{1}} = \frac{2a(2-\beta)(2-\beta^2)(4-\beta^2)^2\eta}{\Delta_{1}}$$

$$\frac{SW_{\text{add}}}{\Delta_{1}} = \frac{2a^2\Phi_1\eta}{\Delta_{1}}$$

$$\frac{T_{S_{\text{add}}}}{\Delta_{1}} = \frac{2a^2\Phi_1(y-\delta)((12 + (2-\beta)(6-6\beta-5\beta^2))y - (28 - \beta(8 + 3\beta(6 - \beta - \beta^2))\delta)\eta}{\Delta_{1}}$$

$$\frac{\Pi_{\text{c}}^{\text{add}}}{\Delta_{1}} = \frac{2a^2(2-\beta)(2-\beta^2)(4-\beta^2)^2\eta(8(2-\beta)(3-\beta^2)(4-\beta^2)^2\eta - (14-5\beta^2)(y-\delta)(2y-\beta\delta))}{\Delta_{1}}$$

Proposition 1. Optimal decision in Scenario UDLD is obtained as follows:
Next, we present optimization problem in Scenario UDLC as follows:

\[
\begin{align*}
\max & \quad \alpha_{\text{ucldSW}}^{}, \\
\max & \quad w_{\text{ucld}}^{}(\theta_{\text{ucld}}^{}) \Pi_{\text{m1}}^{\text{ucld}} + \Pi_{\text{m2}}^{\text{ucld}}, \\
\max & \quad p_{\text{ucld}}^{}(\Pi_{\text{r1}}^{\text{ucld}} + \max p_{\text{ucld}}^{} \Pi_{\text{r2}}^{\text{ucld}}).
\end{align*}
\]

(16)

Therefore, upstream manufacturers optimize their sum of profits, instead of individual profits. We present the detailed derivations in Appendix A for the simplicity of the presentation. The equilibrium decision in Scenario is summarized in Proposition 2.

**Proposition 2.** Optimal decision in Scenario UCLD is obtained as follows:

\[
\begin{align*}
\alpha_{\text{ucld}}^{} &= \frac{3(1 - \beta)}{7 - 5\beta}, \\
w_{\text{ucld}}^{}(\theta_{\text{ucld}}^{}) &= \frac{4a(2 - \beta)^2\eta}{\Delta_2}, \\
\theta_{\text{ucld}}^{} &= \frac{a(y - \delta)(7 - 5\beta)}{\Delta_2}, \\
p_{\text{ucld}}^{} &= \frac{4a(2 - \beta)(3 - 2\beta)\eta}{\Delta_2}, \\
\Pi_{\text{r1}}^{\text{ucld}} &= \frac{16a^2(2 - 3\beta + \beta^2)^2\eta^2}{\Delta_2^2}, \\
\Pi_{\text{m1}}^{\text{ucld}} &= \frac{2a^2(2 - \beta)\eta}{\Delta_2}, \\
Q_{\text{ucld}}^{} &= \frac{4a(2 - \beta)(1 - \beta)\eta}{\Delta_2}, \\
SW_{\text{ucld}}^{} &= \frac{2a^2(7 - 5\beta)\eta}{\Delta_2}, \\
TSG_{\text{ucld}}^{} &= \frac{6a^2(1 - \beta)(7 - 5\beta)(y - \delta)^2\eta}{\Delta_2}, \\
\Pi_{\text{c}}^{\text{ucld}} &= \frac{4a^2(2 - \beta)\eta(8(2 - \beta)(1 - \beta)(3 - 2\beta)\eta - (7 - 5\beta)(y - \delta)^2)}{\Delta_2^2}.
\end{align*}
\]

(17)

By comparing results in Propositions 1 and 2, we identify one of the key results of the study which is presented in Proposition 3.

**Proposition 3.** In between Scenarios UDLD and UCLD,

1. wholesale and retail prices are higher in Scenario UCLD
2. product green quality levels and SW are higher in Scenario UDLD if $\beta < 0.8477$
3. sales volume is always higher in Scenario UDLD

We refer to Appendix C for the proof. According to Proposition 3, in the presence of upstream collusion, competing manufacturer members reduce the quality of products. Predictably, the upstream collusion may harm the downstream members because manufacturers have higher price-setting power. The results also reflect that fact. Noticeably, cross-price sensitivity $(\beta)$ between two SCs is the major parameter affecting the variation. We present comparative analysis, indicating how the optimal decisions behave to changes in the four key parameter values, one at a time. We offer the results formally below in Table 3.

The graphical proof against results in Table 3 is presented in a supplementary document (8) (available here). For graphical validation, we use the following parameters: $a = 300$, $\beta = 0.3$, $\eta = 0.5$, $y = 0.5$, and $\delta = 0.2$. According to Table 3, one can find that green quality levels, sales volumes, SW, and subsidy rate increase with $y$ and $\beta$ and decrease with $\delta$ and $\eta$. It is sensible that the green quality levels decrease if
The manufacturer is not efficient in investment, and consequently, overall performance decreased. On the other hand, in a green-sensitive market, if γ increases, the demand for the product increases. A notable result is that the consumers’ cross-price elasticity or sensitivity with the green quality level becomes negligible (β = 0), then only retail prices of both products will increase with green quality levels of both products. The sum of downstream profit functions is concave because the value of the determinant of the Hessian matrix ($H^i_{p_{udlc}}$) is obtained as follows:

$$H^i_{p_{udlc}} = \begin{vmatrix} \frac{\partial^2 n^i_{udlc}}{\partial p_{1i}^2} & \frac{\partial^2 n^i_{udlc}}{\partial p_{1i} \partial p_{2i}} \\ \frac{\partial^2 n^i_{udlc}}{\partial p_{1i} \partial p_{2i}} & \frac{\partial^2 n^i_{udlc}}{\partial p_{2i}^2} \end{vmatrix} = \begin{vmatrix} -2 & 2\beta \\ 2\beta & -2 \end{vmatrix} = 4(1-\beta^2) > 0.$$
Moreover, diagonal elements are also negative. Substituting optimal responses for two retailers, profit functions for two manufacturers are obtained as follows:

\[
\Pi_{mi}^{\text{udlc}} = \frac{w_i^{\text{udlc}}(a - w_i^{\text{udlc}} + \omega_j^{\text{udlc}}\beta + \gamma \theta_i^{\text{udlc}} - \delta_{j}^{\text{udlc}})}{2} - \left(1 - \alpha^{\text{udlc}}\right)\eta \beta_{i}^{\text{udlc}}^2, \quad i = 1, 2, \quad j = 3 - i.
\]  

(21)

Therefore, the optimal response for two manufacturers is obtained by solving \((\partial \Pi_{mi}^{\text{udlc}}/\partial \omega_i^{\text{udlc}}) = 0\) and \((\partial \Pi_{mi}^{\text{udlc}}/\partial \theta_i^{\text{udlc}}) = 0\), respectively. After simplification, the optimal responses for two manufacturers are obtained as follows:

\[
w_i^{\text{udlc}} = \frac{4a(1 - \alpha^{\text{udlc}})\eta}{4(1 - \alpha^{\text{udlc}})(2 - \beta)\eta - \gamma(y - \delta)}
\]
\[
\theta_i^{\text{udlc}} = \frac{a(y - \delta)}{4(1 - \alpha^{\text{udlc}})(2 - \beta)\eta - \gamma(y - \delta)}.
\]

(22)

To verify concavity, we determine the value of determinant of the Hessian matrix \(H_{mi}^{\text{udlc}}\) for the profit function of each manufacturer as follows:

\[
H_{mi}^{\text{udlc}} = \begin{vmatrix}
\frac{\partial^2 \Pi_{mi}^{\text{udlc}}}{\partial w_i^{\text{udlc}} \partial w_i^{\text{udlc}}} & \frac{\partial^2 \Pi_{mi}^{\text{udlc}}}{\partial w_i^{\text{udlc}} \partial \theta_i^{\text{udlc}}} \\
\frac{\partial^2 \Pi_{mi}^{\text{udlc}}}{\partial w_i^{\text{udlc}} \partial \theta_i^{\text{udlc}}} & \frac{\partial^2 \Pi_{mi}^{\text{udlc}}}{\partial \theta_i^{\text{udlc}} \partial \theta_i^{\text{udlc}}}
\end{vmatrix} = \begin{vmatrix}
-1 & \frac{y}{2} \\
\frac{\gamma}{2} & -2(1 - \alpha^{\text{udlc}})\eta
\end{vmatrix} = \Theta_i = \frac{4}{\Delta_2}.
\]

(23)

Because the values of diagonal elements are negative, therefore, the profit function for the manufacturer is also concave if \(\Theta_i = 8(1 - \alpha^{\text{udlc}})\eta - y^2\). Substituting optimal responses for both manufacturers and retailers, the SW\text{udlc} function is obtained as

\[
SW^{\text{udlc}} = \frac{2a^{\text{udlc}}(1 - \alpha^{\text{udlc}})^2(7 - 5\beta)\eta - (1 - \beta)y^2}{(1 - \beta)(4(1 - \alpha^{\text{udlc}})(2 - \beta)\eta - y^2 + \gamma\delta)^2}.
\]

(24)

Therefore, the optimal subsidy rate is obtained by solving \((\partial SW^{\text{udlc}}/\partial \alpha^{\text{udlc}}) = 0\). On simplification, the subsidy rate is obtained as \(\alpha^{\text{udlc}} = 1 - (2(2 - \beta)(1 - \beta)\eta(7 - 5\beta)(y - \delta))\). Note that the SW\text{udlc} function is concave with respect to \(\alpha^{\text{udlc}}\) because

\[
\frac{\partial^2 SW^{\text{udlc}}}{\partial \alpha^{\text{udlc}} \partial \alpha^{\text{udlc}}} = \frac{-8a^{\text{udlc}}(7 - 5\beta)^4(y - \delta)\eta^2}{(1 - \beta)y^2(\Delta_2)^3} < 0.
\]

(25)

Moreover, \(\Theta_i^{\text{udlc}} = (\gamma \Delta_3)(4(7 - 5\beta)(y - \delta)) > 0\), if \(\Delta_3 = 16(2 - \beta)(1 - \beta)\eta - (7 - 5\beta)(y - \delta) > 0\). Therefore, optimal solution always exists if \(\Delta_3 > 0\) and \(\Delta_3 > 0\). By using back substitution, we obtain simplified values of optimal decision as presented in Proposition 4.

**Proposition 4.** Optimal decision in Scenario UDLC is obtained as follows:

\[
\alpha^{\text{udlc}} = 1 - \frac{2(2(2 - \beta)(1 - \beta)\eta(7 - 5\beta)(y - \delta))}{6(7 - 5\beta)(y - \delta)}.
\]

\[
\beta_i^{\text{udlc}} = \frac{a(7 - 5\beta)(\gamma - \delta)}{\Delta_2},
\]

\[
\omega_i^{\text{udlc}} = \frac{8a(2 - \beta)(1 - \beta)\eta}{\Delta_2},
\]

\[
\rho_i^{\text{udlc}} = \frac{4a(2 - \beta)(3 - 2\beta)\eta}{\Delta_2},
\]

\[
\Pi_{mi}^{\text{udlc}} = \frac{16a^{2}(2 - \beta)^2(1 - \beta)\eta^2}{\Delta_2},
\]

\[
\Pi_{mi}^{\text{udlcu}} = \frac{2a^{2}(2 - \beta)(1 - \beta)\eta(12(2 - \beta)(1 - \beta)\eta - (7 - 5\beta)(y - \delta))}{\Delta_2},
\]

\[
Q_i^{\text{udlc}} = \frac{4a(2 - \beta)(3 - 2\beta)\eta}{\Delta_2},
\]

\[
SW^{\text{udlc}} = \frac{2a^{2}(7 - 5\beta)\eta}{\Delta_2},
\]

\[
TS^{\text{udlc}} = \frac{2a^{2}(7 - 5\beta)(y - \delta)((1 + 1)(1 + 2)(2 - \beta)\eta(7 - 5\beta)(y - \delta))}{\Delta_2},
\]

\[
\Pi_{mi}^{\text{udlc}} = \frac{4a^{2}(2 - \beta)(1 - \beta)\eta(2(2 - \beta)(3 - 2\beta)\eta - (7 - 5\beta)(y - \delta))}{\Delta_2}.
\]

(26)

Next, we present optimization problem in Scenario UCLC as follows:

\[
\begin{align*}
\max & \quad SW^{\text{udlc}}, \\
\text{s.t.} & \quad (\alpha^{\text{udlc}}, \rho_i^{\text{udlc}}, \omega_i^{\text{udlc}}, \Pi_{mi}^{\text{udlc}} + \Pi_{mi}^{\text{udlc}}, (\Pi_{mi}^{\text{udlc}}) + \Pi_{mi}^{\text{udlc}}).
\end{align*}
\]

(27)

Therefore, upstream manufacturers optimize their sum of profits, instead of individual profits. We presented the detailed derivations in Appendix B for the simplicity of the presentation. The equilibrium decision in Scenario UCLC is summarized in 5.

**Proposition 5.** Optimal decision in Scenario UCLC is obtained as follows:
$\alpha_{uclc} = \frac{3 - \beta}{7 - \beta}$

$\omega_{uclc}^i = \frac{16a\eta}{\Delta_2}$

$\theta_{uclc}^i = \frac{a(\gamma - \delta)}{\Delta_3}$

$P_{i}^{uclc} = \frac{24a\eta}{\Delta_3}$

$\Pi_{ri}^{uclc} = \frac{64a^2(1 - \beta)\eta^2}{\Delta_3}$

$\Pi_{mi}^{uclc} = \frac{4a^2\eta}{\Delta_3}$

$Q_{i}^{uclc} = \frac{8a(1 - \beta)\eta}{\Delta_3}$

$SW_{uclc} = \frac{2a^2(7 - \beta)\eta}{\Delta_3}$

$TG_{uclc} = \frac{2a^2(7 - \beta)(3 - \beta)(\gamma - \delta)^2\eta}{\Delta_3}$

$\Pi_{cri}^{uclc} = \frac{8a^2\eta(48(1 - \beta)\eta - (7 - \beta)(\gamma - \delta)^2)}{\Delta_3}$

Proposition 6. In between Scenarios UDLC and UCLC,

1) wholesale and retail prices are higher in Scenario UCLC

2) product green quality levels, SW, and sales volumes are always higher in Scenario UDLC

We refer to Appendix D for the proof of Proposition 6, and we observe that results are similar to Proposition 3. Combining results in the above six propositions, we conclude that the strategic collusion can reduce the green quality levels of products under competition. We present a comparative analysis of decision variables to four key parameters in Scenarios UDLC and UCLC in Table 4.

Except for the subsidy rate, the trends of the optimal decisions remain similar in four scenarios. The cross-price elasticity is the major parameter mostly responsible for the variation of government subsidy rate. In all four scenarios, the subsidy rate in independent from investment efficiency for the manufacturers. Note that an increase in $\beta$ or $\delta$ increases the competitive gap between two manufacturers. Because an increase in a manufacturers’ capability compared to the competitor usually prompts a relative variation in the wholesale price or green quality level in a reverse way, the result also reflects that fact.

As noted earlier, the Table 2 below summarizes optimal decision in the absence of subsidy.

In the next section, we use the results to evaluate the effect of subsidy.

4. Model Analysis

4.1. Nature of Retail Prices and Green Quality Levels. In the section, we focus mainly on market prices and green quality levels of the products; that is, we analyze optimal decisions to pinpoint the scenario which is favorable from the perspective of consumers. Previously, we prove that $\theta_{i}^{udd} > \theta_{i}^{udc}$ and $\theta_{i}^{udd} > \theta_{i}^{dcl}$. From Propositions 2 and 3, we observe $\theta_{i}^{udd} = \theta_{i}^{udc}$ and Propositions 1 and 5,

\[
\theta_{i}^{udd} - \theta_{i}^{udc} = \frac{2a\beta(2 + \beta)(3 - 2\beta)(48 - 58\beta - 11\beta^2 + 21\beta^3 - 2\beta^4)(\gamma - \delta)\eta}{\Delta_3\Delta_1} > 0.
\]  

(29)

if $\beta < 0.9282$. Similarly, comparing market prices, we obtain

\[
P_{i}^{udd} - P_{i}^{udc} = \frac{4a\beta(2 + \beta)(3 - 2\beta)\eta(4 - \beta - 2\beta)^{\gamma - \delta)^2\eta - (25 - \beta(3 + 9\beta - \beta^3)(\gamma - \delta)^2\eta}{\Delta_1\Delta_3} > 0,
\]  

(30)

\[
P_{i}^{udd} - P_{i}^{udc} = 0.
\]  

(31)
Table 4: Characteristics of optimal decision in Scenarios UDLC and UCLC.

| α UDLC | u i | p i | q i | η i | δ i | β i | α UDLC | u i | p i | q i | η i | δ i | β i | SW i |
|--------|-----|-----|----|-----|-----|----|--------|-----|-----|----|-----|-----|----|-----|
|        |     |     |    |     |     |    |        |     |     |    |     |     |    |     |
| α UDLC | u i | p i | q i | η i | δ i | β i | α UDLC | u i | p i | q i | η i | δ i | β i | SW i |
|        |     |     |    |     |     |    |        |     |     |    |     |     |    |     |

Therefore, using the results in Propositions 3 and 6, we propose the following proposition.

Proposition 7

(1) Optimal GLs satisfy \( q_i^{udlc} < q_i^{udld} = q_i^{ucld} < q_i^{ucld} \) if \( \beta < 0.8477 \)

(2) Optimal retail prices satisfy \( p_i^{udld} < p_i^{udld} = p_i^{ucld} < p_i^{ucld} \)

From Proposition 7, we can observe that the quality level of the products is always higher in Scenario UDLN and lowest in Scenario UCLC. However, the reverse trend is observed for retail prices. Therefore, if members form collusion, then consumers may need to pay more for lower quality products. Note that members form collusion to achieve greater control for their decision, consequently, they have more price-setting power. The downstream retailers are able to set higher market prices, and upstream manufacturers reduce the investment to gain higher profits, subsequently, consumers suffer, and the results reflect that fact. To obtain more detailed insights, we draw the following figure that represents prices in different scenarios and the ratios \( (\theta_i^b/p_i^b) \), which reflects a comparative view about how much consumers need to pay for green quality.

Some notable insights from Figure 2 are as follows: (i) although the quality of product is always higher in the presence of subsidy, consumers also need to pay more. Therefore, under SC competition, government subsidy might not keep market prices. (ii) The ratio of \( (\theta_i^b/p_i^b) \) is higher in Scenario UDLN, consequently, without collusion always favorable for consumers. (iii) If upstream and downstream members form collusion, then government subsidy may have the least impact. As we have seen, the ratio in Scenario UCLC may be lower compared to the Scenarios UDLN or UCLN, when the members do not receive any subsidy. Overall, a government subsidy to the manufacturers becomes less effective in the perspective of consumers if members form collusion.

4.2. Nature of Profits for Manufacturers and Retailers.

Before we look into individual profits for members in each SC, first we compare the sales volumes. From Propositions 2 and 4, we can observe that \( Q_i^{udld} - Q_i^{udlc} = 0.0 \). Moreover,
Consequently, both retailers receive lower profits if upstream manufacturers maximize their joint profits. The graphical representation of profits for retailers and manufacturers and the total amount of government expenditure in four different scenarios are presented in the figure below.

Figure 3 demonstrates the following key outcomes: (i) as expected, both manufacturers prefer competition at the downstream level and gain higher benefit in Scenario UCLD or UDLD. The above figure reflects that fact. (ii) Noticeably, both retailers prefer competition at upstream level and receive higher profits in Scenario UDLC or UCLD. (iii) Scenario UCLC remains dominated by other three, which is in line with Propositions 7 and 8. (iv) Most importantly, higher government expenditure might not yield a higher green product. It can be observed that manufacturers receive a higher amount of subsidy in Scenario UCLD, but product quality always remains at the highest level in Scenario UDLD.

Therefore, members in the competing SCs face prisoner’s dilemma. Through collusion, they can achieve profit maximization objective, but they need to trade lower quality product. Till, a region exists that represents unique preference, i.e., Scenario UDLD, where all the members can receive higher profits if cross-price elasticity and green level sensitivity are higher. However, this occurs due to the presence of this subsidy. Note that under the value of the same parameter, two manufacturers always receive higher profits in Scenario UCLDN and retailers receive higher profits in Scenario UDLCN or UCLDN as presented in Figure 4.

From the above Figure 4, we can observe that competing SC members need to change their strategic collaboration decision with their competitors in the presence of subsidy. Downstream retailers can prefer strategic collusion between upstream members in the absence of subsidy, which is not true previously. Therefore, whether to make collusion with rivals at the horizontal level is influenced by government intervention. Next, we examine the optimal decision from the perspective of government organizations.

4.3. Nature of Social Welfare and Government Subsidy.

First, we identify the scenario where SW reaches at higher level. From Propositions 2 and 4, we can observe that $SW_{uclc} - SW_{udlc} = 0$. Moreover,

$$SW_{uclld} - SW_{udld} = \frac{4a^2\beta\eta^2 (2 + \beta)(3 - 2\beta)(48 - 58\beta - 11\beta^2 + 21\beta^3 - 2\beta^4)}{\Delta_1\Delta_3} > 0.$$
Figure 3: (a) Profits for manufacturers, (b) profits for retailers, and (c) total government subsidies (blue: UDLD ≥ max{UCLD, UDLC, UCLC}; red: UCLC ≥ max{UCLD, UDLC, UDLD}; black: UCLD ≥ max{UDLD, UDLC, UCLC}; gray: UDLC ≥ max{UCLC, UCLD, UDLD}; white: no feasible region for $\beta \in (0, 0.8)$ and $\gamma \in (0, 0.9)$.

Figure 4: (a) Profits for manufacturers and (b) profits for retailers (blue: UDLDN ≥ max{UCLDN, UDLCN, UCLCN}; red: UCLCN ≥ max{UCLDN, UDLCN, UDLDN}; black: UCLDN ≥ max{UDLDN, UDLCN, UCLCN}; gray: UDLCN ≥ max{UCLCN, UCLDN, UDLDN}; white: no feasible region for $\beta \in (0, 0.8)$ and $\gamma \in (0, 0.9)$. 
If $\beta < 0.9281$, therefore, using the results in Propositions 3 and 6, we propose the following proposition.

**Proposition 9.** Optimal SWs satisfy $SW_{uclc} < SW_{ucld} = SW_{adlc} < SW_{adld}$ if $\beta < 0.8477$.

The result makes sense, and green quality level, sales volume, and total subsidy are maximum in Scenario UCLD. Consequently, if members make collusion, then the impact of subsidy reduces. Figure 5 demonstrates the increment of sales volume and green quality level and total subsidy in different scenarios.

From Figure 5, one can find that government subsidy might improve both product consumption and quality, which is anticipated. Noticeably, all three figures demonstrate a clear difference of four scenarios based on the nature of strategic collusion. In Scenarios UCLC and UCLD, product consumption and quality are higher for the lower value of $\beta$; however, as $\beta$ increases, those are higher in ULDL and UDLC. From the perspective of the total expenditure, the figure exhibits a similar trend.

**5. An Extension: Two Integrated Supply Chains (CC)**

So far, we have analyzed four scenarios to study the impact of horizontal cooperation. To analyze the consequence of vertical cooperation, we derive the optimal decision where members of both SCs implement the vertically integrated decision. In this scenario, both the manufacturers and retailers in each SC are willing to form “an integrated firm” and jointly determine the retail price and green quality level which maximizes the total profit for each SC, and they belong to [27, 37]. In this scenario, profit functions for two competing SCs ($\Pi_{ti}^{cc}$, $i = 1, 2$) are obtained as follows:

$$\Pi_{ti}^{cc} = p_i^{cc}D_i^{cc} - \eta(1 - a^{cc})(\theta_i^{cc})^2, \quad i = 1, 2. \tag{36}$$

Note that if both SCs are integrated, wholesale prices become irrelevant. The derivation of the optimal decision in this scenario remains similar; hence, we omitted the proof. The simplified expressions of decision variables are presented below.

**Proposition 10.** Optimal decision in Scenario CC is obtained as follows:

$$a^{cc} = \frac{\gamma + \beta\gamma - 3\delta}{3\gamma - 3\delta},$$

$$p_i^{cc} = \frac{2a(2 - \beta)\eta}{\Delta_{cc}},$$

$$\theta_i^{cc} = \frac{3a(\gamma - \delta)}{\Delta_{cc}},$$

$$Q_i^{cc} = \frac{2a(2 - \beta)\eta}{\Delta_{cc}}\Pi_i^{cc} = \frac{a^2(2 - \beta)\eta(4(2 - \beta)\eta - 3\gamma(\gamma - \delta))}{\Delta_{cc}^2}. \tag{37}$$

where $\Delta_{cc} = 2(2 - \beta)^2\eta - 3(\gamma - \delta)^2$.

There is a long debate on the issue of whether the members in two competing SCs should cooperate with the vertical members or with their rival, i.e., horizontal members. As mentioned earlier in the work by Zhao and Cao [27], Li and Li [37], and Fang and Shou [64], the authors emphasized on the issue of vertical cooperation by comparing the outcomes in three different models: (i) members make a decentralized decision, similar to Scenario ULDL, (ii) one integrated SC, and one decentralized SC, and (iii) both SCs are decentralized. However, they ignore the impact of government subsidy. Therefore, we find the answer for the following question: is it profitable for SC members to cooperate with their rival instead of members belong to the same SC? First, we compare the green quality levels in Scenarios ULDL and CC, and we obtain

$$\theta_i^{adld} - \theta_i^{cc} = \frac{2(2 - \beta)^2(2 + \beta)(17\beta + 2\beta^2 - 7\beta^3 - 10)\eta}{\Delta_i\Delta_{cc}} \tag{38}$$

Therefore, one can find that consumers’ cross-price elasticity is the only parameter responsible for the difference, and if $\beta > 0.6525$, then product quality is always less in Scenario CC. Subsequently, we conclude that horizontal cooperation can enhance quality level. Note that the quality level is always higher in ULDL compared to UCLD, ULLC, and UDLC. Similarly, the difference between green quality levels in between Scenarios CC and UCLC is

$$\theta_i^{adlc} - \theta_i^{cc} = \frac{2a\eta(\gamma - \delta)(2 + \beta)(10 - 13\beta + \beta^2)}{\Delta_j\Delta_{cc}} > 0, \tag{39}$$

if $\beta > 0.8211$. Based on the discussion, we proposed the following proposition.

**Proposition 11.** (1) Optimal green quality level is higher in Scenario ULDL compared to Scenario CC if $\beta > 0.6525$.

The graphical representation for the total profits for two competing SCs in five scenarios is presented in Figure 6.

From the above, we can note another important contribution of the study is that members have the option on strategic agreements with their rivals, which can not only ensure higher profits but also can build consumers’ resilience with higher product quality. Figure 6 shows that total profits and green quality level in Scenario CC are always higher compared to Scenario UCLC, but it is not true with if
we see the results with other three scenarios where the competing SC members collaborate with horizontal competitors. From the managerial perspective, in today’s business world, we can find many instances where competition increasingly transformed from confrontation to cooperation in achieving economies of scale and range [65].
Continuing in this direction, this study made an effort to analyze such a possibility of collusion and compared the corresponding equilibrium. Our analysis reveals that such cooperation has the potential to improve the overall SC performance which will be an interesting insight to the managers in handling such a business environment.

6. Discussion

This work is motivated by governments’ push for sustainability via various subsidy programs. For instance, government organizations subsidize firms that produce energy-efficient appliances in countries such as the USA and Canada, China, Germany, India, and others in different modes such as green credit mode, manufacture subsidy mode, and sales subsidy mode. In this context, supply chain members aim to maximize their respective profits, while the government emphasizes on measuring its impact on social welfare, consumer surplus, environmental benefit, and other goals. In the existing literature, researchers also highlighted this issue from various perspectives; however, the literature on strategic cooperation under government subsidy is sparse. Our findings can provide some guidance for the government and supply chain members to comprehend regarding their action. It highlights that the strategic collusion can reduce the green quality levels of products although the government needs to allocate more funding. Essentially, collusion is a strategic measure for competing members to accomplish their pricing decision to increase profits. Consequently, it hurts the consumers, increases government expenditure, and reduces the overall social welfare. The results also demonstrate that the competing manufacturers can set prices that are higher than the competitive prices. As a result, regulators need to monitor the situation that conditions facilitate the formation of cartels and then implement subsidy program to accomplish sustainability goal.

7. Conclusion

In conclusion, we have modeled interaction among government organizations, manufacturers, retailers, and consumers where two manufacturers distribute products through two exclusive retailers. Optimal decisions are derived under five different scenarios to pinpoint government subsidy’s impact to improve the green quality levels under competition. Comparative studies are conducted analytically and numerically to highlight managerial implications for SC members and policymakers on how strategic collusion or vertical integration can affect government expenditure to stimulate environmental performance.

Our study’s key findings are as follows: first, regarding performance for manufacturers and retailers in the perspective of their respective profits, we find that they are benefited more from the collusion. However, product green quality level is less. Both upstream manufacturers permanently welcome more downstream competition whereas downstream retailers welcome upstream competition. Second, in the perspective of green product quality, strategic collusion always leads to suboptimal product quality in the presence of subsidy. Product consumption also reduces. Although the significance of vertical cooperation is studied in the literature, we pointed out that the horizontal collusion can serve as a strategic tool for the competing supply chain members to receive higher profits. Third, in the presence of collusion, government expenditure increased but not product quality. We find that if SC members optimize their respective profits, then government subsidy is higher, and the green quality level is also increased. Fourth, the study indicates that there is an optimal subsidy rate for all five scenarios, more significant levels of those may increase expenditure without bringing potential outcomes. Towards another step ahead, we prove a potential correlation between both strategic pacts among competitors both in upstream and downstream levels and total expenditure. A careful examination is warranted from a government organization’s perspective, and they must take care to identify the possibility of such deals before subsidizing manufacturers.

In terms of future research, the present study can be extended in several directions. We ignore the effect of cross-channel selling. Consequently, it will be interesting to explore the characteristics of the optimal decisions in the presence of another degree of competition. We explore the scenarios where the manufacturers receive the subsidy; therefore, it could be fruitful to examine the characteristics where the subsidy to be received by retailers or customers, or both. We ignored the effect of cost-sharing agreement between the manufacturer and retailer at the vertical level or between two manufacturers at a horizontal level. Therefore, one can study the effect of contract mechanisms such as cost-sharing contract, trade-credit policy, cost-tariff contract [66], and revenue-sharing at vertical level or bargaining contract mechanism horizontal level. Next, in the proposed supply chain strategic structure, one can introduce market uncertainty or limits on government expenditure to assess how it might affect product green quality levels [68, 69].

Appendix

(A). Optimal Decision in Scenario UCLD

Because in Scenarios UDLD and UCLD, both downstream retailers take their respective decisions by optimizing their respective profits. Therefore, in both scenarios, the optimal response for the retailers remains the same, and with their response, profits for two manufacturers are obtained as

$$\Pi_{mi}^{uclld} = \frac{u_i^{uclld} \left( a(2 + \beta) + w_j^{uclld} \beta - w_i^{uclld}(2 - \beta^2) + (2y - \beta \delta) \theta_i^{uclld} + (\beta y - 2 \delta) \theta_j^{uclld} \right)}{4 - \beta^2} - \left(1 - \alpha_i^{uclld}\right) \eta_i^{uclld}, \quad (A.1)$$
where $i = 1, 2$, $j = 3 - i$. In contrast to Scenario UDLD, two manufacturers optimize the sum of total profits as $\pi_{m}^{\text{ucl}} = \Pi_{m1}^{\text{ucl}} + \Pi_{m2}^{\text{ucl}}$. Therefore, the optimal response for two manufacturers is obtained by solving $(\partial \pi_{m}^{\text{ucl}}/\partial w_{m}^{\text{ucl}}) = 0$ and $(\partial \pi_{m}^{\text{ucl}}/\partial \theta_{m}^{\text{ucl}}) = 0$, respectively. After simplification, the optimal responses are obtained as follows:

$$
w_{m}^{\text{ucl}} = \frac{2a(1 - \alpha_{m}^{\text{ucl}})(2 - \beta)\eta}{4(1 - \alpha_{m}^{\text{ucl}})(2 - \beta)(1 - \beta)\eta - (\gamma - \delta)^2}, \tag{A.2}
$$

$$
\theta_{m}^{\text{ucl}} = \frac{a(\gamma - \delta)}{4(1 - \alpha_{m}^{\text{ucl}})(2 - \beta)(1 - \beta)\eta - (\gamma - \delta)^2}.
$$

From the above expressions, we can see that wholesale prices decrease and product quality increases with $\alpha_{m}^{\text{ucl}}$, but the reverse trend follows for $\eta$. The Hessian matrix $(H_{m}^{\text{ucl}})$ for the sum of the profit function two manufacturers is obtained as

$$
H_{m}^{\text{ucl}} = \begin{bmatrix}
\frac{\partial^2 \pi_{m}^{\text{ucl}}}{\partial w_{1}^{\text{ucl}} \partial w_{1}^{\text{ucl}}} & \frac{\partial^2 \pi_{m}^{\text{ucl}}}{\partial w_{1}^{\text{ucl}} \partial w_{2}^{\text{ucl}}} & \frac{\partial^2 \pi_{m}^{\text{ucl}}}{\partial w_{1}^{\text{ucl}} \partial \theta_{1}^{\text{ucl}}} & \frac{\partial^2 \pi_{m}^{\text{ucl}}}{\partial w_{1}^{\text{ucl}} \partial \theta_{2}^{\text{ucl}}} \\
\frac{\partial^2 \pi_{m}^{\text{ucl}}}{\partial w_{2}^{\text{ucl}} \partial w_{1}^{\text{ucl}}} & \frac{\partial^2 \pi_{m}^{\text{ucl}}}{\partial w_{2}^{\text{ucl}} \partial w_{2}^{\text{ucl}}} & \frac{\partial^2 \pi_{m}^{\text{ucl}}}{\partial w_{2}^{\text{ucl}} \partial \theta_{1}^{\text{ucl}}} & \frac{\partial^2 \pi_{m}^{\text{ucl}}}{\partial w_{2}^{\text{ucl}} \partial \theta_{2}^{\text{ucl}}} \\
\frac{\partial^2 \pi_{m}^{\text{ucl}}}{\partial \theta_{1}^{\text{ucl}} \partial w_{1}^{\text{ucl}}} & \frac{\partial^2 \pi_{m}^{\text{ucl}}}{\partial \theta_{1}^{\text{ucl}} \partial w_{2}^{\text{ucl}}} & \frac{\partial^2 \pi_{m}^{\text{ucl}}}{\partial \theta_{1}^{\text{ucl}} \partial \theta_{1}^{\text{ucl}}} & \frac{\partial^2 \pi_{m}^{\text{ucl}}}{\partial \theta_{1}^{\text{ucl}} \partial \theta_{2}^{\text{ucl}}} \\
\frac{\partial^2 \pi_{m}^{\text{ucl}}}{\partial \theta_{2}^{\text{ucl}} \partial w_{1}^{\text{ucl}}} & \frac{\partial^2 \pi_{m}^{\text{ucl}}}{\partial \theta_{2}^{\text{ucl}} \partial w_{2}^{\text{ucl}}} & \frac{\partial^2 \pi_{m}^{\text{ucl}}}{\partial \theta_{2}^{\text{ucl}} \partial \theta_{1}^{\text{ucl}}} & \frac{\partial^2 \pi_{m}^{\text{ucl}}}{\partial \theta_{2}^{\text{ucl}} \partial \theta_{2}^{\text{ucl}}}
\end{bmatrix}
$$

$$
= \begin{bmatrix}
\frac{-2(2 - \beta^2)}{4 - \beta^2} & \frac{2\beta}{4 - \beta^2} & \frac{2\gamma - \beta\delta}{4 - \beta^2} & \frac{\beta\gamma - \beta\delta}{4 - \beta^2} \\
\frac{2\beta}{4 - \beta^2} & \frac{-2(2 - \beta^2)}{4 - \beta^2} & \frac{\beta\gamma - \beta\delta}{4 - \beta^2} & \frac{2\gamma - \beta\delta}{4 - \beta^2} \\
\frac{2\gamma - \beta\delta}{4 - \beta^2} & \frac{\beta\gamma - \beta\delta}{4 - \beta^2} & \frac{-2(1 - \alpha_{m}^{\text{ucl}})\eta}{4 - \beta^2} & 0 \\
\frac{\beta\gamma - \beta\delta}{4 - \beta^2} & \frac{2\gamma - \beta\delta}{4 - \beta^2} & 0 & \frac{-2(1 - \alpha_{m}^{\text{ucl}})\eta}{4 - \beta^2}
\end{bmatrix}
\tag{A.3}
$$

The values of principal minors of above Hessian matrix $(H_{m}^{\text{ucl}})$ are

$$
H_{m1}^{\text{ucl}} = -\frac{2(2 - \beta^2)}{4 - \beta^2} < 0,
$$

$$
H_{m2}^{\text{ucl}} = \frac{4(1 - \beta^2)}{4 - \beta^2} > 0,
$$

$$
H_{m3}^{\text{ucl}} = -\frac{2\Theta_2}{4 - \beta^2},
$$

$$
H_{m4}^{\text{ucl}} = \frac{\Theta_3}{4 - \beta^2},
$$

respectively. Therefore, the joint profit function for two manufacturers is concave if

$$
\Theta_2 = 4\left(1 - \alpha_{m}^{\text{ucl}}\right)(4 - 5\beta^2 + \beta^4)\eta - (2 + \beta^2)(\gamma^2 + \delta^2) + 6\gamma\delta\eta > 0.
$$

$$
\Theta_3 = (\gamma^2 - \delta^2)^2 - 8(1 - \alpha_{m}^{\text{ucl}})
$$

$$
\left((2 + \beta^2)(\gamma^2 + \delta^2) - 6\gamma\delta\eta\right)\eta + 16(1 - \alpha_{m}^{\text{ucl}})^2(4 - 5\beta^2 + \beta^4)\eta^2 > 0,
$$

for $\alpha_{m}^{\text{ucl}} \in (0, 1)$.

Similar to previous Scenario UDLD, if we substitute $\alpha_{m}^{\text{ucl}} = 0$, we can obtain an optimal decision as presented in Table 2.

Substituting optimal responses for both manufacturers and retailers, the $SW_{\text{ucl}}$ function is obtained as

$$
SW_{\text{ucl}} = \frac{2\alpha^2\eta\left[2\left(1 - \alpha_{m}^{\text{ucl}}\right)^2(1 - \beta)(7 - 5\beta)\eta - (\gamma - \delta)^2\right]}{\left(4\left(1 - \alpha_{m}^{\text{ucl}}\right)(2 - \beta)(1 - \beta)\eta - (\gamma - \delta)^2\right)^2}\tag{A.6}
$$

Therefore, the optimal subsidy rate is obtained by solving $(dSW_{\text{ucl}}/d\alpha_{m}^{\text{ucl}}) = 0$. On simplification, the subsidy rate is obtained as $\alpha_{m}^{\text{ucl}} = (3(1 - \beta)/7 - 5\beta)$. Note that the $SW_{\text{ucl}}$ function is concave with respect to $\alpha_{m}^{\text{ucl}}$ if $\Delta_2 = 8(2 - \beta^2)(1 - \beta)\eta - (7 - 5\beta)(\gamma - \delta)^2 > 0$, because

$$
d^2SW_{\text{ucl}}\frac{dSW_{\text{ucl}}}{d\alpha_{m}^{\text{ucl}}}|_{\alpha_{m}^{\text{ucl}}} = \frac{-8\alpha^2(7 - 5\beta)^4(1 - \beta)(\gamma - \delta)^2\eta^2}{\Delta^{(3/2)}}\tag{A.7}
$$

Moreover,

$$
\Theta_2|_{\alpha_{m}^{\text{ucl}}} = \frac{2\Delta_{21}}{7 - 5\beta},
$$

$$
\Theta_3|_{\alpha_{m}^{\text{ucl}}} = \frac{\Delta_{21}\Delta_{22}}{(7 - 5\beta)^2},
$$

where $\Delta_{21} = 8(2 - \beta)(4 - 5\beta^2 + \beta^4)\eta - (7 - 5\beta)((2 + \beta^2)\gamma^2 - 6\gamma\delta + (2 + \beta^2)\delta^2)$ and $\Delta_{22} = 8(2 - \beta)(1 + \beta)(2 + \beta)\eta - (7 - 5\beta)(\gamma + \delta)^2$; that is, optimal solution exists in Scenario UCLD if $\Delta_2 > 0, \Delta_{21} > 0$, and $\Delta_{22} > 0$. 

The following inequalities ensure that the wholesale price and green quality levels in Scenario UDLD decreased with $\eta$ and increased with $\alpha_{\text{udld}}$.

\[
\begin{align*}
\frac{\partial w_{i}^{\text{udld}}}{\partial \eta} &= \frac{-2a(1 - \alpha_{\text{udld}})(4 - \beta^2)(y - \delta)(2y - \beta\delta)}{(2(1 - \alpha_{\text{udld}})(2 - \beta)(4 - \beta^2)\eta - (y - \delta)(2y - \beta\delta))} < 0, \\
\frac{\partial \theta_{i}^{\text{udld}}}{\partial \eta} &= \frac{-2a(1 - \alpha_{\text{udld}})(2 - \beta)(4 - \beta^2)(2y - \beta\delta)}{(2(1 - \alpha_{\text{udld}})(2 - \beta)(4 - \beta^2)\eta - (y - \delta)(2y - \beta\delta))} < 0, \\
\frac{\partial w_{i}^{\text{udld}}}{\partial \alpha_{\text{udld}}} &= \frac{2a(4 - \beta^2)(2y - \beta\delta)}{(2(1 - \alpha_{\text{udld}})(2 - \beta)(4 - \beta^2)\eta - (y - \delta)(2y - \beta\delta))} > 0, \\
\frac{\partial \theta_{i}^{\text{udld}}}{\partial \alpha_{\text{udld}}} &= \frac{2a(2 - \beta)(4 - \beta^2)(2y + \beta\delta)\eta}{(2(1 - \alpha_{\text{udld}})(2 - \beta)(4 - \beta^2)\eta - (y - \delta)(2y - \beta\delta))} > 0.
\end{align*}
\]

Therefore, optimal is ensured if $\eta$ satisfies the condition in model formulation.

The following inequalities ensure that the wholesale price and green quality decreased with $\eta$ and increased with $\alpha_{\text{udlc}}$.

\[
\begin{align*}
\frac{\partial w_{i}^{\text{udlc}}}{\partial \eta} &= \frac{4a(1 - \alpha_{\text{udlc}})\gamma(y - \delta)}{(4(1 - \alpha_{\text{udlc}})(2 - \beta)\eta - \gamma(y - \delta))^2} < 0, \\
\frac{\partial \theta_{i}^{\text{udlc}}}{\partial \eta} &= \frac{4a(1 - \alpha_{\text{udlc}})(2 - \beta)\gamma}{(4(1 - \alpha_{\text{udlc}})(2 - \beta)\eta - \gamma(y - \delta))^2} < 0, \\
\frac{\partial w_{i}^{\text{udlc}}}{\partial \alpha_{\text{udlc}}} &= \frac{4a\gamma(\gamma - \delta)\eta}{(4(1 - \alpha_{\text{udlc}})(2 - \beta)\eta - \gamma(y - \delta))^2} > 0, \\
\frac{\partial \theta_{i}^{\text{udlc}}}{\partial \alpha_{\text{udlc}}} &= \frac{4a(2 - \beta)\gamma\eta}{(4(1 - \alpha_{\text{udlc}})(2 - \beta)\eta - \gamma(y - \delta))^2} > 0.
\end{align*}
\]

(B). Optimal Decision in Scenario UCLC

In Scenarios UDLC and UCLC, both downstream retailers take their respective decisions by optimizing the sum of downstream profits. Therefore, in both scenarios, the optimal response for the retailers remains the same. Similar to Scenario UDLC, the optimal response for the retailers remains the same. However, in contrast with the Scenario UDLC, two manufacturers optimize the sum of upstream profits as $\Pi_{m_i}^{\text{udlc}} = \Pi_{m_i}^{\text{udlc}} + \Pi_{m_i}^{\text{udlc}}$. Therefore, the optimal response for two manufacturers is obtained by solving \( (\partial \pi_{\text{udlc}}^{\text{udlc}}/\partial \omega_{\text{udlc}}) = 0, (\partial \pi_{\text{udlc}}^{\text{udlc}}/\partial \theta_{\text{udlc}}) = 0, (\partial \pi_{\text{udlc}}^{\text{udlc}}/\partial \omega_{\text{udlc}}^{\text{udlc}}) = 0, \) and \( (\partial \pi_{\text{udlc}}^{\text{udlc}}/\partial \theta_{\text{udlc}}^{\text{udlc}}) = 0, \) respectively. After simplification, the optimal responses are obtained as follows:

\[
\begin{align*}
\omega_{i}^{\text{udlc}} &= \frac{4a(1 - \alpha_{\text{udlc}})\eta}{8(1 - \alpha_{\text{udlc}})(1 - \beta)\eta - \gamma(y - \delta)^2}, \\
\theta_{i}^{\text{udlc}} &= \frac{a(\gamma - \delta)}{8(1 - \alpha_{\text{udlc}})(1 - \beta)\eta - \gamma(y - \delta)^2}.
\end{align*}
\]
To verify concavity, we compute the Hessian matrix $(H^\text{uclc}_m)$ for the joint profit function for two manufacturers as follows:

$$
H^\text{uclld}_m = \begin{bmatrix}
\frac{\partial^2 \pi^\text{uclc}_m}{\partial w_1^2} & \frac{\partial^2 \pi^\text{uclc}_m}{\partial w_1 \partial w_2} & \frac{\partial^2 \pi^\text{uclc}_m}{\partial w_1 \partial \theta_1} & \frac{\partial^2 \pi^\text{uclc}_m}{\partial w_1 \partial \theta_2} \\
\frac{\partial^2 \pi^\text{uclc}_m}{\partial w_1 \partial w_2} & \frac{\partial^2 \pi^\text{uclc}_m}{\partial w_2^2} & \frac{\partial^2 \pi^\text{uclc}_m}{\partial w_2 \partial \theta_1} & \frac{\partial^2 \pi^\text{uclc}_m}{\partial w_2 \partial \theta_2} \\
\frac{\partial^2 \pi^\text{uclc}_m}{\partial w_1 \partial \theta_1} & \frac{\partial^2 \pi^\text{uclc}_m}{\partial w_2 \partial \theta_1} & \frac{\partial^2 \pi^\text{uclc}_m}{\partial \theta_1^2} & \frac{\partial^2 \pi^\text{uclc}_m}{\partial \theta_1 \partial \theta_2} \\
\frac{\partial^2 \pi^\text{uclc}_m}{\partial w_1 \partial \theta_2} & \frac{\partial^2 \pi^\text{uclc}_m}{\partial w_2 \partial \theta_2} & \frac{\partial^2 \pi^\text{uclc}_m}{\partial \theta_2^2} & \frac{\partial^2 \pi^\text{uclc}_m}{\partial \theta_2^2}
\end{bmatrix}
$$

$$
= \begin{bmatrix}
-1 & \beta & y/2 & -\delta/2 \\
\beta & -1 & -\delta/2 & y/2 \\
y/2 & -\delta/2 & -2(1-\alpha^\text{uclc})\eta & 0 \\
-\delta/2 & y/2 & 0 & -2(1-\alpha^\text{uclc})\eta
\end{bmatrix}
$$

(B.3)

The values of principal minors of the above Hessian matrix are $H^\text{uclld}_m = -1 < 0$; $H^\text{uclld}_m = 1 - \beta^2 > 0$; $H^\text{uclld}_m = -\Theta_3/4$; and $H^\text{uclld}_m = (\Theta_6/16)$, respectively, where $\Theta_3 = (8(1-\alpha^\text{uclc})(1-\beta^2)\eta - y^2 + 2\beta y \delta - \delta^2)$ and $\Theta_6 = (y^2 - \delta^2)^2 - 16(1 - \alpha^\text{uclc})(y^2 - 2\beta y \delta + \delta^2)\eta + 64(1 - \alpha^\text{uclc})^2(1 - \beta^2)\eta^2$. Therefore, joint profit function for two manufacturer is also concave if $\Theta_3 > 0$ and $\Theta_6 > 0$.

Substituting optimal responses for both manufacturers and retailers, the SW$^\text{uclc}$ function is obtained as

$$
SW^\text{uclc} = \frac{2a_2^2\eta\left(2(1-\alpha^\text{uclc})^2(7-\beta)(1-\beta^2)\eta - (y-\delta)^2\right)}{(8(1-\alpha^\text{uclc})(1-\beta)\eta - (y-\delta)^2)^2}.
$$

(B.4)

Therefore, the optimal subsidy rate is obtained by solving $(dSW^\text{uclc}/d\alpha^\text{uclc}) = 0$. On simplification, the subsidy rate is obtained as $\alpha^{\text{uclc}} = (3 - \beta/7 - \beta)$. Note that the SW$^\text{uclc}$ function is concave with respect to $\alpha^\text{uclc}$ if

$$
\frac{d^2SW^\text{uclc}}{d\alpha^\text{uclc}^2} \Big|_{\alpha^\text{uclc}} = -\frac{8a_2^2(7-\beta)^2(1-\beta)(y-\delta)^2\eta^2}{(32(1-\beta)\eta - (7-\beta)(y-\delta))^3} < 0.
$$

(B.5)

In addition,

$$
\Theta_{3|\alpha^\text{uclc}} = \frac{(32(1-\beta)\eta - (7-\beta)(y-\delta)^2)^2}{16(7-\beta)^2}
$$

(B.6)

Therefore, optimal decision exists if $\Delta_2 > 0$.

(C. Proof of Proposition 3)

By comparing and simplifying optimal decision in Scenarios UCLDL and UDLDD, we obtain the following relations:

$$
\omega^\text{uclld}_i - \omega^\text{uclld}_i = \frac{2a(2-\beta)^2\eta\beta(4(2-\beta)^2\beta(4-\beta^2)\eta - (26 - \beta(3 + 11\beta))(y-\delta)^2)}{\Delta_1 \Delta_2} > 0,
$$

$$
P^\text{uclld}_i - P^\text{uclld}_i = \frac{4a\eta\beta(2(2-\beta)(4(2-\beta)^2\eta - \beta(25 - \beta(17 + 9\beta - 6\beta^2))(y-\delta)^2}}{\Delta_1 \Delta_2},
$$

$$
\theta^\text{uclld}_i - \theta^\text{uclld}_i = \frac{2a\eta(y-\delta)(2-\beta)^2\beta(24 - 31\beta - 7\beta^2 + 12\beta^3)}{\Delta_2 \Delta_1} > 0, \quad \text{if } \beta < 0.8477,
$$

$$
Q^\text{uclld}_i - Q^\text{uclld}_i = \frac{-2a(2-\beta)\eta \neq (4(2-\beta)^2(1-\beta)^2\beta(4-\beta^2)\eta - \beta(1 + \beta)(2 - \beta^2)(y-\delta)^2}{\Delta_1 \Delta_2} < 0,
$$

$$
SW^\text{uclld} - SW^\text{uclld} = \frac{4a^2 \eta^2 (2-\beta)^2\beta(24 - 31\beta - 7\beta^2 + 12\beta^3)}{\Delta_2 \Delta_1} > 0, \quad \text{if } \beta < 0.8477.
$$
The above relations ensure the proof.

(D). Proof of Proposition 6

The following inequalities ensure the proof of Proposition 6:

\[ u_i^{ulc} - u_i^{udc} = \frac{8a_1 \eta \beta (16 (2 - \beta) (1 - \beta) \eta - (13 - (10 - \beta) \beta) (\gamma - \delta)^2)}{\Delta_2 \Delta_3} > 0, \]
\[ p_i^{udc} - p_i^{ulc} = \frac{4a_1 \eta \beta (16 (2 - \beta) (1 - \beta) \eta - (25 - 21 \beta + 2 \beta^3) (\gamma - \delta)^2)}{\Delta_2 \Delta_3} > 0, \]
\[ q_i^{ulc} - q_i^{udc} = \frac{8a_1 (1 - \beta) \beta (12 - 11 \beta + \beta^3) (\gamma - \delta) \eta}{\Delta_2 \Delta_3} > 0, \]
\[ SW^{ulc} - SW^{udc} = \frac{16a_1^2 \eta^2 (1 - \beta) \beta (12 - 11 \beta + \beta^3)}{\Delta_2 \Delta_3} > 0. \]

The proposition is proved.

Data Availability

The data used to support the findings of this study are included within the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Supplementary Materials

We presented the graphical representation of sensitivity analysis of wholesale prices, retail prices, green quality levels, sales volumes, profit for the retailers, profits for the manufacturers, total subsidies, and social welfare functions in four scenarios with respect to green quality sensitivity, investment efficiency of manufacturers, cross-green quality sensitivity, and cross-price elasticity in Figures S1, S2, S3, and S4, respectively. (Supplementary Materials)

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