Quasi-perfect absorption achieved throughout low frequency range via acoustic meta-surface

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In this work, we propose a novel design of acoustic meta-surface with coplanar coiled channels and double apertures. Tunable-broadband (30–300 Hz) and quasi-perfect absorption (> 0.9) throughout low frequency range can be realized through active control of the channel length and other structural parameters while the whole structural thickness is only $1/170$ of the operating wavelength. A heat source is introduced in right position of the channel to extend the direct bandwidth of quasi-perfect absorption (> 0.9). By appropriately tuning the channel length, the direct quasi-perfect absorption band (> 0.9) can be extended by as large as 3 times. The analytical model and the theoretical simulation results are also shown in this paper. Since our design strategy is indeed active control and the heat source is easily applicable, our realization should have a high impact on low frequency noise elimination. © 2019 The Japan Society of Applied Physics

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Noise pollution in low frequency range (50–250 Hz) can cause severe damages to aircraft, infrastructures and human health.1–3 To efficiently diminish or eliminate the noise, a lot of methods have been put forwarded, like conventional porous and fibrous materials.4–6 They show some efficiency, but the application is greatly limited by their bulky configurations.7–10 Previous studies in past two decades show that acoustic meta-materials,11–14 and meta-surfaces15–24 with sub-wavelength components are promising candidates in low frequency noise elimination. They usually utilize the locally resonant units which show highly enhanced low frequency energy inside to realize high absorption efficiency. The typical designs amongst them are the membrane-based perfect absorbers,25–28 meta-surfaces with coiled up space channels (including coiled air chamber,29–31 coiled Fabry–Perot channel,32,33 and the coiled channels with embedded apertures34)). They all successfully demonstrate quasi-perfect or even perfect absorption in low frequency range, but only with a single sharp peak (narrow absorption band) and their structures are all fixed once formed to be specimens and the coplanar channels length can not be tuned.35 To enlarge the absorption bandwidth, the strategy of active control36 has been floated. However, the extra controlling device either thickening the structure or enormously increasing the cost restricts the possible applications.

Here we report a novel design of acoustic meta-surface composed of coplanarcoiled channels and double apertures. Based on this structure, quasi-perfect absorption can be achieved throughout the whole of the low frequency range. The overall configuration of the meta-surface is shown in Fig. 1. In part (a), the red plate and the red solid spiral beams (thickness = 1 mm) represent bottom wall and inner walls, respectively. And the blue plate (thickness = 10 mm) indicates the upper coverage. The channel consists of totally 100 unit cells (denoted with brown cubic borders), each with a 10 mm × 10 mm × 10 mm volume and segregated by a lamina (denoted with white plate) with a thickness of 1 mm. The channel is spiral, and its length is able to be manually tuned, thus active control. For a better understanding, a detailed top view of the structural elements from aperture 1 to aperture 2 is shown in part (b). Notice that the distance between each two elements has been manually enlarged for illustration. And the upper coverage is removed. The empty segregations here are outlined with black borders which are white plates in part (a). In Part (c), the segregation can be empty or manually inserted with a plate (denoted with black color) whose size is slightly smaller than the segregation to tune the length of the channel. And thus all walls and the inserted plates are sound hard boundaries. The first aperture (diameter $d_1 = 1.5$ mm, length $l_1 = 10$ mm) is the inlet of sound and is embedded in center of the solid coverage and perpendicular to the channel. The second aperture (diameter $d_2 = 1.5$ mm, length $l_2 = 20$ mm) is embedded in a solid block whose width and depth are the same with that of the channel but its length is 20 mm (denoted by yellow block, under the solid coverage) and positioned parallel to the channel to separate the channel into two sub-channels. Therefore, the total length ($L_s$) of the first sub-channel (from the end of the first aperture to the beginning of the second aperture) is 80 mm while that of the second one ($L_2$) (from the end of the second aperture to the end of the initial channel) is 880 mm. Note that the total thickness of the structure is only 20 mm.

For this newly designed meta-surface, we first deduce its analytical model for sound absorption with consideration of both viscous and thermal losses by using the acoustic impedance analysis. According to Crandall’s theory37 and the subsequent derivation of Li34 the acoustic impedance per unit length of a narrow aperture can be written as

$$Z_A = \frac{A}{S_A} \left( \rho_A c_A - \frac{2j \sin \left( \frac{ka}{2} \right)}{\sqrt{(\gamma - (\gamma - 1) \psi_A \psi_b)}} \right),$$

under the assumption that $\lambda \gg d_s$, $d$s with $d_s = \sqrt{2\eta/\rho_0 c_p}$ and $d_b = \sqrt{2k/\rho_0 c_p}$. Where $Z_0 = \rho_0 c_0$ is the characteristic.
acoustic impedance of the medium, $l_0$ is the thickness of the aperture, and $j = \sqrt{-1}$. $A$ is the cross sectional area of the whole structure and $S_A$ is the cross sectional area of aperture. $\psi_v$ and $\psi_h$ are the function of viscous and thermal fields, respectively, derived as

$$
\psi_v = \frac{J_2(k_v d_a/2)}{J_0(k_v d_a/2)}
$$

and

$$
\psi_h = \frac{J_2(k_h d_a/2)}{J_0(k_h d_a/2)},
$$

where $J_n$ is the Bessel function of the first kind and order $n$, and $d_a$ indicates the diameter of the aperture of HR. $k_v$ and $k_h$ are the viscous wave number and thermal wave number, respectively, derived as

$$
k_v^2 = -j\omega \frac{\rho_0}{\eta}
$$

and

$$
k_h^2 = -j\omega \frac{\rho_0 C_p}{\kappa},
$$

where $\eta$ is the dynamic viscosity, $\omega$ is the angular frequency, $C_p$ is the specific heat at constant pressure, and $\kappa$ is the fluid thermal conductivity. And finally, the complex wave number and complex air density can be calculated as

$$
k_v^2 = k_0^2 \left( \gamma - (\gamma - 1) \frac{\psi_h}{\psi_v} \right)
$$

and

$$
\rho_v = \frac{\rho_0}{\psi_v}
$$

The cavity or the channel, as mentioned above, consists of two sub-channels. The acoustic impedance of each sub-channel can be calculated with Kinsler’s theory, which is

$$
Z_D = \rho_0 C_0 \frac{A - \alpha L_{eff} \sin(kL_{eff}) \sin(kL_{eff})}{A \sin^2(kL_{eff}) + (\alpha L_{eff})^2 \cos^2(kL_{eff})},
$$

where $A_c$ is the cross sectional area of the channel, under the assumption that $\alpha / \kappa \ll 1$ and $\alpha L \ll 1$. $L_{eff}$ indicates the effective length of the channel with consideration of the radiation impedance at the cross of channel. And it is given by the following equation

$$
L_{eff} = \frac{C_0}{\omega} \cot^{-1} \left( -\frac{1}{1 - r} \right),
$$

with $r$ indicating the complex reflection coefficient. $\alpha$ refers to the total absorption factor within the fluid and at the walls.
of channel.\textsuperscript{39} And it is written as
\[
\alpha = \frac{\omega^2}{2 \rho_0 c_0^3} \left( \frac{4}{3} \eta + \frac{(\gamma - 1) \kappa}{C_p} \right) + \frac{a + b}{ab} \frac{1}{c_0} \frac{\eta \omega}{2 \rho_0} \left( 1 + \frac{\gamma - 1}{\sqrt{\Pr}} \right),
\]
with \(a\) and \(b\) refer to the width and height of the channel, respectively. And \(\Pr\) is the Prandtl number, \(\Pr = \eta C_p / \kappa\). Another acoustic impedance should be concerned is the end correction from wave radiation and the frictional losses owing to air flow along the walls of aperture.\textsuperscript{40,41} It can be derived as
\[
Z_c = \frac{A}{S_h} \left( 2 \sqrt{2 \rho_0 \eta} + j \omega \rho_0 \delta_i \right),
\]
where
\[
\delta_i = \left( 1 + \left( 1 - 1.25 \frac{d_a}{\sqrt{a^2 + b^2}} \right)^4 \frac{d_a}{3\pi} \right).
\]

Referring to the whole configuration of the meta-surface, one can notice that the relationship between each acoustic element is parallel connection. Thus, we first simplify it to an equivalent electrical circuit, as shown in Fig. 2.

Then, based on this equivalent electrical circuit, we can find out the total acoustic impedance \(Z_T\) of the meta-surface from Maa’s conclusion.\textsuperscript{42} It can be expressed as
\[
Z_T = Z_{A1} + Z_{C1} + \frac{Z_{D1}(Z_{A2} + Z_{C2} + Z_{D2})}{Z_{D1} + (Z_{A2} + Z_{C2} + Z_{D2})},
\]
And finally, the absorption coefficient of the meta-surface\textsuperscript{39} can be calculated by
\[
\alpha_a = 1 - \left| \frac{Z_T - \rho_0 c_0}{Z_T + \rho_0 c_0} \right|.
\]

With this analytical model, we calculated the absorption coefficient of the above meta-surface structure with \(L_1 = 80\) mm, \(L_2 = 880\) mm (all segregations are empty, without any inserted plates), air density \(\rho_0 = 1.21 \) kg m\(^{-3}\), sound speed \(c_0 = 343\) m s\(^{-1}\), dynamic viscosity of air \(\eta = 1.56 \times 10^{-5}\) Pa s, thermal conductivity of air \(\kappa = 0.02\) W m\(^{-1}\) K\(^{-1}\), and heat capacity at constant pressure of air \(C_p = 1004\) J Kg\(^{-1}\) K\(^{-1}\). And then made comparison with the simulation result, as shown in Fig. 3. Note that the Simulation was conducted with COMSOL Multiphysics\textsuperscript{TM} Version 5.3 with preset Acoustic-Thermoacoustic interaction module. The computational model was configured as the same with that presented in Fig. 1. All needed input parameters were the same with that used in analytical computation. A normally incident plane wave with unit magnitude (1 Pa) impinged along +\(z\) direction. And Sound hard boundaries were imposed on the interfaces between air and solid due to their huge impedance mismatch.

Imagine that an incident wave along +\(z\) direction normally impinges the system with a unit magnitude (1 Pa) and then penetrates the first aperture to the first sub-channel. And subsequently penetrates the second aperture to the second sub-channel. The apertures and sub-channels actually compose three Helmholtz Resonators with the cavities being the whole channel, the first sub-channel, and the second sub-channel, respectively. The sound energy could be highly absorbed at the resonant frequency resulting from the energy dissipation which attributes to the viscous and thermal losses at the walls. And thus presents some peaks in the absorption curve, as demonstrated in Fig. 3. There are totally three peaks. The first peak, corresponding to the lowest resonant frequency, results from the resonance in the whole channel (the first sub-channel plus the second sub-channel) because the length is the largest. The second peak and the third peak are somewhat complicated, but they can be mainly ascribed to resonance in the second sub-channel and the first sub-channel, respectively. It is clear that the two curves’ shapes and the peak positions match perfectly with less than 3\% discrepancy. It demonstrates that our analytical model is correct. This discrepancy, as we concluded, is resulted from some approximation we made and the end correction that applied in the analytical model while not in the simulation computation. Note that here we only present one sample, but it does not conceal the fact that all samples we calculated with the second sub-channel length \(L_2\) varying from 10 to 880 mm show the similar discrepancies (<3\%). Therefore, it is convinced that the analytical model is robust. And furthermore, the values of all three peaks are larger than 0.9 (reaching a quasi-perfect absorption), with noticeable bandwidths (the first peak locates at 29 Hz, the quasi-perfect absorption bandwidth is 13 Hz; the second peak locates at 204 Hz, the quasi-perfect absorption bandwidth is 3 Hz; the third peak locates at 279 Hz, the quasi-perfect absorption bandwidth is 30 Hz). And the frequency span between the first peak and the third peak covers the low frequency range (50–250 Hz).\textsuperscript{35} By decreasing the length of the second sub-channel, we find that all peaks blue-shift, as shown in Fig. 4.

In this figure, the first peak blue-shifts with a nearly constant quasi-perfect absorption bandwidth. The second peak and the third peak switch the domination at about 650 mm. The coupling between the second mode and the third mode should account for the disappearance of the third peak. It should be stressed that the minimum frequency of the first peak that can be achieved is 29 Hz, and the maximum frequency of the second peak is 303 Hz. In addition, their frequency spans connect at about 200 Hz. That means we can get a frequency range (30–300 Hz) whose corresponding absorption coefficient is larger than 0.9 by manually tuning the length of the second sub-channel. Although this is not directly realized by a single peak (direct bandwidth) and not conventionally broadband as well (we call it as “tunable broadband”), the connection of the first and the second peak could ensure a feasible application. Referring to the overall configuration of the designed meta-surface (Fig. 1), one can
manually change the sub-channel length to get a desired absorption frequency and bandwidth according to their actual request, even after the meta-surface has been applied in device. By this strategy, we have realized a quasi-perfect absorption throughout the low frequency range. As far as we know, quasi-perfect absorption in such low frequency with “tunable broadband” has never been reported before.

However, we still want to get a direct and continuous broadband to extend its application, despite achieving such a “tunable broadband”. To explore a feasible method, we intensively investigated the fundamental equations in pressure acoustics, and then our attention was drawn to the extra sources. By introducing an extra heat source (a preset element in Pressure Acoustics Module) in sub-channel two, as shown in Fig. 5, we found that the trough in between two peaks can be unfolded, as shown in Fig. 6.

The heat source occupies full volume of the second sub-channel. Actually, there are several options for its position, for instance, the first sub-channel, the second aperture, the second sub-channel, and their combination. We finally chose sub-channel two and rejected all other options after we conducted simulations based on the best performance on raising the absorption coefficient in between two peaks. And then the length of the heat source was determined to be the same with that of the second sub-channel to decrease the needed heat quantity. The simulation result is shown in Fig. 6.

In this figure, it is clear that there are totally two peaks initially (in black curve), peak one (peak value = 108 Hz, direct quasi-perfect absorption bandwidth = 20 Hz) and peak two (peak value = 291 Hz, direct quasi-perfect absorption bandwidth = 30 Hz). When adding the heat source and gradually increasing the heat quantity, both peaks shift closer to each other and the trough is raised up, leading to a continuous quasi-perfect absorption (>0.9) with a broad bandwidth (142 Hz) at heat quantity $= 6500 \text{ W m}^{-3}$. Compared to the result without heat source, the direct bandwidth has been increased by nearly 3 times (142 Hz compared to 20 + 30 Hz) which is quite effective and astonishing. The peak shift may be attributed to the parametric change (like sound speed, air density, etc.) resulted from temperature variation by the heat source. On the other
hand, the heat source also imposed extra heat in sub-channel two that may enhance the thermal loss at the walls and the apertures by the heat and acoustic coupling. And thus the sound absorption was increased. What matters most is that the quasi-perfect bandwidth is now continuous, not two discrete peaks. Although the minimum frequency of the quasi-perfect absorption band is 141 Hz which is still somewhat higher than the lower limit of the low frequency range, it is actually able to be dragged down below 50 Hz if we keep extending the length of the second sub-channel. However, this length should not be too large because the meta-surface length $d_u$ could be also extremely large, resulting in the boost of directly reflected wave.

The extra heat source we added here actually has two parameters, heat quantity $Q$ and temperature $T$. Heat quantity raises up the absorption coefficient while high temperature lowers it. In most working conditions of device, high temperature is a very common factor that deteriorates the performance of sound absorption meta-surface. Because most of the key parameters that determine the absorption coefficient are directly concerned with temperature. Fortunately, we found that the deterioration can be compensated by heat quantity. That is to say, we can enlarge the heat quantity to counteract the influence of high temperature. And this is validated by our simulation (referring to the supplementary information Fig. 1, available online at stacks.iop.org/JJAP/58/120904/mmedia).

In conclusion, we designed a novel acoustic meta-surface based on two apertures and coplanar coiled channels and correctly built up the analytical model. With this meta-surface, we obtained a "tunable broadband" (30–300 Hz) and quasi-perfect absorption (>0.9) by manually tuning the sub-channel length while the overall thickness of the structure is only 1/170 of the operating wavelength (acoustic frequency taken as 100 Hz). And furthermore, we achieved a continuous quasi-perfect absorption broadband (142 Hz) by
innovatively introducing a heat source on the designed structure. Since the sub-channel length can be easily manually tuned even after applied in device without need of extra controlling device and heat quantity compensates the deterioration of high temperature, our designed meta-surface has wide application in high temperature conditions.

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