ON THE USE OF QUALITY DIVERSITY ALGORITHMS FOR THE TRAVELING THIEF PROBLEM

Adel Nikfarjam
Optimisation and Logistics
School of Computer and Mathematical Sciences
The University of Adelaide
adel.nikfarjam@adelaide.edu.au

Aneta Neumann
Optimisation and Logistics
School of Computer and Mathematical Sciences
The University of Adelaide
aneta.neumann@adelaide.edu.au

Frank Neumann
Optimisation and Logistics
School of Computer and Mathematical Sciences
The University of Adelaide
frank.neumann@adelaide.edu.au

ABSTRACT

In real-world optimisation, it is common to face several sub-problems interacting and forming the main problem. There is an inter-dependency between the sub-problems, making it impossible to solve such a problem by focusing on only one component. The traveling thief problem (TTP) belongs to this category and is formed by the integration of the traveling salesperson problem (TSP) and the knapsack problem (KP). In this paper, we investigate the inter-dependency of the TSP and the KP by means of quality diversity (QD) approaches. QD algorithms provide a powerful tool not only to obtain high-quality solutions but also to illustrate the distribution of high-performing solutions in the behavioural space. We introduce a MAP-Elite based evolutionary algorithm using well-known TSP and KP search operators, taking the TSP and KP score as the behavioural descriptor. Afterwards, we conduct comprehensive experimental studies that show the usefulness of using the QD approach applied to the TTP. First, we provide insights regarding high-quality TTP solutions in the TSP/KP behavioural space. Afterwards, we show that better solutions for the TTP can be obtained by using our QD approach and it can improve the best-known solution for a number of TTP instances used for benchmarking in the literature.

Keywords Quality Diversity, Traveling Thief Problem, Map-Elites

1 Introduction

In many real-world optimisation problems, several NP-hard problems interact with each other. Such optimisation problems are complex due to the inter-dependencies between the sub-problems. The inter-dependencies make each sub-problem affect the quality and even the feasibility of solutions of the others. This complicates the decision-making process [Bonyadi et al. 2019]. Vehicle routing problems, the traveling thief problem, and patient admission problems are examples of multi-component optimisation problems.

TTP was introduced in 2013 by [Bonyadi et al. 2013]. TTP is the combination of the classical TSP and the KP. Both TSP and KP are well-known, well-studied combinatorial problems. In a nutshell, they integrate the TSP and the KP so that the traveling cost between two cities depends not only on the distance between the cities but also on the weight of the items collected so far. In recent years, several solution approaches have been introduced to TTP. This includes algorithms based on co-evolutionary strategies (Bonyadi et al. 2014, Yafrani and Ahiod 2015), local search heuristics (Polyakovskiy et al. 2014, Maity and Das 2020), profit-based heuristic that modifies tours based on the items’ value (Namazi et al. 2019), simulated annealing (Yafrani and Ahiod 2018), swarm intelligence approaches (Wagner 2016, Zouari et al. 2019). Furthermore, An adaptive surrogate model was proposed in [Namazi et al.]
Our contribution

We employ the concept of QD for solving the TTP. By this means, we scrutinise the distribution of high-performing TTP solutions in the behavioural space of the TSP and the KP and compute very high-quality solutions. We introduce a bi-level MAP-elite based evolutionary algorithm called BMBEA. The algorithm generates new solutions in a two-stage procedure. First, it generates new high-quality TSP tours from old ones by the well established EAX crossover operator \cite{Nagata2013} for the TSP, or as an alternative by 2-OPT \cite{Croes1958}. Second, it utilises dynamic programming \cite{Neumann2018b} or an \((1 + 1)\) evolutionary algorithm to compute an optimal (or near-optimal) packing list for the given TSP tour. Having generated a new solution, BMBEA applies a MAP-Elites based survival selection to achieve a diverse set of high-quality TTP solutions. To achieve diversity, MAP-Elites is applied with respect to the two-dimensional space given by the TSP and KP quality of the TTP solutions. We conduct a comprehensive experimental investigation to analyse and visualise the distribution of high-quality TTP solutions for different TTP instances. Furthermore, we show the capability of BMBEA to generate high-performing TTP solutions. The algorithm results in very high TTP values and improves the best-known TTP solution for some benchmark instances. These contributions are also included in the conference version that was published in the proceeding of GECCO 2022 \cite{Nikfarjam2022a}.
This article extends its conference version (Nikfarjam et al. [2022b]) as follows. We propose a method that eliminates two influential input parameters that need to be tuned for each instance individually in Section 3.5. Using the method requires a higher number of generations to converge compared to the previous method with the considered input values. However, it improves the results, especially for the larger instances. The experimental investigations can be found in Sections 4.1.2 and 3.2.3. We also investigate the impact of Map-Elite based survival selection. We show that the survival selection brings about a diverse set of solutions that prevents premature convergence in 3.4, 4.1.1, and 4.2.2. We also correct some incorrect experimental results from the conference version which were due to an implementation error.

The remainder of the paper is structured as follows. In Section 2, we formally define the TTP problem. We introduce the MAP-Elites based approach for TTP and the BMBEA algorithm in Section 3. We also propose a baseline algorithm to investigate the impact of MAP-Elites in Section 3. We examine the high-quality TTP solutions in terms of their TSP score and report on our results using BMBEA for solving the TTP are shown in Section 4. Finally, we finish with some concluding remarks.

## 2 The Traveling Thief Problem

The traveling thief problem (TTP) is formed by the integration of the traveling salesperson problem (TSP) and the knapsack problem (KP). The TSP problem can be defined on a complete directed graph \( G = (V, E) \) where \( V \) is a set of nodes (cities) of size \( n = |V| \) and \( E \) is a set of pairwise edges between the nodes. There is a non-negative distance \( d(e) \) associated with each edge \( e = (u, v) \in E \). The goal is to find a permutation (tour) \( x : V \to V \) that minimises the following cost function:

\[
f(x) = d(x(n), x(1)) + \sum_{i=1}^{n-1} d(x(i), x(i+1)).
\]

The KP is defined on a set of items \( I \), where \( m = |I| \). Each item \( j \) has a profit \( p_j \) and a weight \( w_j \). In KP, the objective is to find a selection of items \( y = (y_1, \ldots, y_m) \) (where \( y_j \) is equal to 1 if item \( j \) is picked and otherwise, it is equal to 0) that maximises the profit subject to the weight of the selected items not exceeding the capacity of the knapsack (\( W \)). Formally, the goal is to maximise

\[
g(y) = \sum_{j=1}^{m} p_j y_j
\]

subject to

\[
\sum_{j=1}^{m} w_j y_j \leq W.
\]

The TTP is defined on the graph \( G \) same as TSP and a set of items \( I \) where items are scattered on the cities equally. Formally, every city \( i \) except the first one contains a set of items \( M_i \) (a subset of \( I \)). Same as KP, each item \( k \) located in the city \( i \) is associated with a profit \( p_{ik} \) and a weight \( w_{ik} \). To ease the presentation, we do not use the double subscripts for the profits and weights in the following but refer directly to the items at one particular city when required.

The thief should visit all the cities exactly once, pick some items into the knapsack, and return to the first city. A rent \( R \) should be paid for the knapsack per time unit. The thief’s speed non-linearly depends on the weight of the knapsack. In TTP, we aim to find a solution \( t = (x, y) \) consisting of a tour \( x \) and a KP solution \( y \) (called a packing list in the context of TTP) that maximises

\[
z(x, y) = g(y) - R \left( \frac{d_{x_n x_1}}{\nu_{\max} - \nu W_{x_n}} + \sum_{i=1}^{n-1} \frac{d_{x_i x_{i+1}}}{\nu_{\max} - \nu W_{x_i}} \right)
\]

subject to

\[
\sum_{j=1}^{m} w_j y_j \leq W.
\]

Here, \( \nu_{\max} \) and \( \nu_{\min} \) are the maximal and minimal traveling speed, \( \nu = \frac{\nu_{\max} - \nu_{\min}}{\nu_{\max} - \nu_{\min}} \) is a constant, and \( W_{x_i} \) is the cumulative weight of the items collected from the start of the tour up to city \( x_i \).

In this study, \( z \) serves as the fitness function, and \( f \) and \( g \) serve as the behavioural descriptor (BD). Generally, the fitness function indicates how well a solution solves the given problem, while the BD shows how it solves the problem.
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Figure 1: The representation of an empty map. There are $\delta_1 \times \delta_2$ cells within the map.

and behaves in terms of the features. In this case, the BD presents the length of the tour ($f$) and the value of items collected ($g$), whereby the fitness function returns the overall profit ($z$). Here, we aim to compute a diverse set of high-quality solutions differing in the BD. By this means, we can look into the distribution of high-performing TTP solutions over the 2D space of TSP and KP.

3 Bi-level Map-Elites-based Evolutionary Algorithm

Map-Elites is an evolutionary computation approach where solutions compete with each other to survive. However, competition is only among solutions with a similar BD value in order to maintain diversity. We require a hyperparameter to define the similarity and the tolerance of acceptable differences between two descriptors. In the MAP-Elites algorithms, the BD space is discretised into a grid, where each cell is associated with one BD type. It means each solution belongs to at most one cell in the behavioural space (the map). Map-Elite algorithms typically keep only the best solution in each cell. When a solution is generated, it is assessed and potentially added to the cell with the associated BD. If the cell is empty, the solution occupies the cell; otherwise, the best solution is kept in the cell. The map aids in understanding and visualising the distribution of high-quality TTP solutions. For instance, how much we should move away from the optimal TSP tour and the optimal KP solution to generate high-performing TTP solutions?

Generally, the behavioural space can be extremely large. Thus, it is rational to limit the map to a promising part of the space; otherwise, either the number or the size of the cells increases severely, and as a result the performance and efficiency of the algorithms is undermined. As mentioned, a TTP solution consists of a tour and a packing list that belong to the TSP and the KP components of the problem. Although solving each sub-problems separately does not necessarily result in a high-quality TTP solution, a TTP solution should score fairly good in both features in order to gain high profits. Thus, we focus on solutions within $\alpha_1$ and $\alpha_2$ percents gap to the optimal TSP value ($f^*$) and the optimal KP value ($g^*$), respectively. In this study $\alpha_1$ and $\alpha_2$ are set to 5 and 20, respectively, based on initial experimental investigations. Figure 1 depicts an empty map. There are $\delta_1 \times \delta_2$ cells. Cell $(i,j)$, $1 \leq i \leq \delta_1$, $1 \leq j \leq \delta_2$ contains the best found solution with TSP score in $\left[ f^* + (i - 1) \left( \frac{\alpha_1 f^*}{\delta_1} \right), f^* + (i) \left( \frac{\alpha_1 f^*}{\delta_1} \right) \right]$ and KP score in $\left[ (1 - \alpha_2)g^* + (j - 1) \left( \frac{\alpha_2 g^*}{\delta_2} \right), (1 - \alpha_2)g^* + (j) \left( \frac{\alpha_2 g^*}{\delta_2} \right) \right]$, the corresponding BD. The cell $(1, \delta_2)$ consists of TTP solutions with TSP and KP values closest to the optimums. In this study, we require to know $f^*$ and $g^*$. For this purpose, we can use EAX in [Nagata and Kobayashi 2013] and dynamic programming (DP) in [Toth 1980] to compute $f^*$ and $g^*$ for the TTP instances, respectively.
Algorithm 1 The MAP-Elites-Based Evolutionary Algorithm

1: Find the optimal/near-optimal values of the TSP and the KP by algorithms in [1980], respectively.
2: Generate an empty map and populate it with the initialising procedure.
3: while termination criterion is not met do
4: Generate an offspring and calculate the TSP and the KP scores.
5: if The TSP and the KP scores are within $\alpha_1 \%$, and $\alpha_2 \%$ gaps to the optimal values of BD. then
6: Find the corresponding cell to the TSP and the KP scores.
7: if The cell is empty then
8: Store the offspring in the cell.
9: else
10: Compare the offspring and the individual occupying the cell and store the best individual in terms of TTP score in the cell.

Algorithm 1 describes the BMBEA. The initialising procedure and the operators to generate a new TTP solution will be discussed later. Having generated an empty map, we populate it with an initialising procedure. After generating offspring, we calculate the TSP score and the KP score of the offspring. If the TSP and the KP scores are within $\alpha_1 \%$ and $\alpha_2 \%$ gap of the optimal values, respectively, we find the cell corresponding to those scores; otherwise, the offspring is discarded. If the corresponding cell is empty, the offspring is kept in the cell; otherwise, we compare the offspring and the individual in the cell and keep the individual with the highest TTP score. We repeat steps 3 to 10 until a termination criterion is met.

Evolutionary algorithms require some operators to generate new solutions (offspring) from old ones (parents); BMBEA is no exception. One can see the generating of TTP solutions as a bi-level process. First, new tours can be generated by mutation or crossovers; then, we can compute a suitable packing list for the new tours to have complete TTP solutions.

3.1 Search Operators for TSP

We consider EAX crossover ([2013]) to generate new TSP tours. EAX is a highly performing TSP crossover known as one of the state-of-the-art operators in solving TSP. The use of EAX has also been shown to lead to high-quality solutions for the TTP in [2019]. EAX has several variants; we incorporate the EAX-1AB due to its simplicity and efficiency. The EAX consists of three steps. Figure 2 depicts the three steps to implement the EAX-1Ab.

- **AB-cycle:** Generating one AB-cycle from the two parents by alternatively choosing edges from the first and second parents until a cycle is formed (Fig 2.2).
- **Intermediate Solution:** Copying all edges of the first parent to the offspring; then removing the Ab-cycle’s edges that belong to the first parent from the offspring, and adding the other edges of the AB-cycle to it (Fig 2.3).
- **The Complete Tour:** Connecting all sub-tours of the intermediate solution to form a complete tour (Fig 2.4).

To connect two sub-tours, we require to discard one edge from each sub-tour, and add two new edges to connect each end of the deleted edges. For Step 3, we start with the sub-tour $(r)$ with the minimum edge number. Then, we select the 4-tuples of edges such that $\{e_1, e_2, e_3, e_4\} = \arg\min\{−d(e_1)−d(e_2)+d(e_3)+d(e_4)\}$ where $e_1 \in E(r)$ and $e_2 \in E(t) \setminus E(r)$. $E(t)$ and $E(r)$ represent the set of edges formed the intermediate solution $t$ and sub-tour $r$, respectively. We refer the interested readers to [2013] for more details about the process of generating a new tour by the EAX. Alternatively, 2-OPT can be used to generate the TSP tours. 2-OPT is a random neighbourhood search, where two elements of a permutation are selected uniformly at random. Having these elements swapped, we reorder the elements in between in a backward direction.

3.2 Search Operators for KP

In the second phase, we optimise the packing list to match the TSP tour and form a good TTP solution. To this mean, inner algorithms are required to optimise the packing list. When the tour is fixed, and the packing list is optimised, the problem is referred to as Packing While Traveling (PWT) in the literature. [2014] introduced a DP algorithm to solve the PWT problem to optimality.
Figure 2: The representation of the steps to implement EAX.

3.2.1 Dynamic Programming

DP is a classical approach in solving the KP. Here, we employ the DP introduced in Neumann et al. [2018b] to solve the PWT problem. The DP includes a table $\beta$ consisting of $W$ rows and $m$ columns. In the DP, items are processed in the order that their corresponding node appears in the tour. For example, $I_i$ is processed sooner than $I_j$, if the node to which $I_i$ belongs is visited sooner than the node of $I_j$. If two items belong to the same node, they are processed according to their indices. The entry $\beta_{i,j}$ represents the maximal profit that the thief can obtain among all combinations of items $I_k$ with $I_k \preceq I_i$ bringing about the weight exactly equal to $j$. If no combinations lead to the weight $j$, $\beta_{i,j}$ is set to $-\infty$.

Let denote the profit of the empty set by $B(\emptyset)$, that is equal to traveling cost with an empty knapsack. Moreover, we denote the profit by $B(I_i)$ when only item $I_i$ is collected. Thus, for the first item $I_i$ (the first row of the table $\beta$) based on the order aforementioned, we have:

$$\beta_{i,0} = B(\emptyset), \quad \beta_{i,w_i} = B(I_i), \quad \beta_{i,j} = -\infty, \forall j \notin \{0, w_i\}$$

Let show the predecessor of $I_i$ by $I_k$. For the rest of the table, each entry $\beta_{i,j}$ can be computed from $\max(\beta_{k,j}, T)$, where

$$T = \beta_{k,j} - w_i - R \sum_{l=1}^{n} d_l \left( \frac{1}{\nu_{\max} - \nu_j} - \frac{1}{\nu_{\max} - \nu_j - w_i} \right)$$

The $\max_j \beta_{m,j}$ is reported as the optimal profit that the thief can gain from the given tour. Although DP can provide us with the optimal packing list for a given tour, the run-time is quite long. Considering that we compute the packing list in the second level of a bi-level optimisation, it can affect the time efficiency of the BMBEA. Therefore, we propose an EA as an alternative. The interested readers are referred to Neumann et al. [2018b], which analysed the run-time of the DP.

3.2.2 $(1 + 1)$ Evolutionary Algorithm

The $(1 + 1)$ EA is a well-known simple EA that converges fast since it only keeps the best-found solution. First, the new tour generated by the TSP operators inherits its parent’s packing list. Next, a new packing list is generated by mutation. If the new packing list results in a higher TTP score, the new packing list is replaced with the old
one. We continue these steps until a termination criterion is met. For mutation, the bit-flip is used, where each bit is independently flipped by mutation rate $\frac{1}{m}$.

The mutation can result in packing lists violating the knapsack’s capacity. We incorporate a repair function into the (1 + 1) EA to avoid the violation. After the offspring is mutated, the repair function fixes the offspring’s violation. The repair function removes collected items uniformly at random one by one until the packing list complies with the capacity constraint.

3.3 Initialisation

One may notice that it is doubtful to populate the map with random solutions. This is because, the map only accepts TTP individuals with fairly good TSP and KP scores. Therefore, a heuristic approach is required to populate the map initially. We can use the EAX-based algorithm in [Nagata and Kobayashi 2013] to find the optimal/near optimal TSP tours in terms of length. Having extracted the tours, we can compute a good quality packing list for each tour by one of the KP operators mentioned in section 3.2. This results in TTP solutions with high TSP and KP scores, let denote the set of solutions by $P_0$. This initial population enables us to populate the map at the beginning of the BMBEA. In this study, we use a target length ($f^*$) as a termination criterion for the EAX-based algorithm in Nagata and Kobayashi [2013], so as to increase the time-efficiency and diversity of tours. If we do not have the target values it is important to tune the running time of the algorithm for each instance individually. Note that in case of using (1 + 1)EA as the KP operator, it will boost the performance of BMBEA, if we start with the optimal packing plan obtained by Toth [1980].

3.4 A More Relaxed Map

Treating $\alpha_1$ and $\alpha_2$ as input required intensive tuning and preliminary experimental investigations. Randomly selecting values for $\alpha_1$ and $\alpha_2$ not only affects the algorithm’s efficiency, but it may also bring about an infeasible map that is impossible to fill. As a result, the algorithm cannot produce any solution in such cases. For this purpose, we propose another method in which we set the thresholds after the initialization. Having computed $P_0$ as mentioned above, we set the $(1 + \alpha_1)f^*$ to $\arg\max_{x \in P_0} f(x)$ and $(1 - \alpha_2)g^*$ to $\arg\min_{y \in P_0} g(y)$.

3.5 ($\mu + 1$) EA

We require a similar algorithm with a conventional survival selection to investigate the impact of QD and MAP-Elitism on the results. We consider ($\mu + 1$)EA since it has the same offspring size as the introduced algorithm. Here, we generate an initial population $P_0$, same as Section 3.3. The parents are selected uniformly at random; then, an offspring is generated as described in Section 3.1 and 3.2. After adding the offspring to the population, we remove one individual with the worst TTP score. Algorithm 2 outlines steps required for ($\mu + 1$)EA.

Algorithm 2 ($\mu + 1$) Evolutionary Algorithm

1: Generate an initial population as explained in Section.
2: while termination criterion is not met do
3: Generate an offspring and add it to the population $P$.
4: Discard one individual $p$ from $P$, where $\arg\min_{p \in P} z(p)$.

4 Experimental Investigation

In this section, we use the BMBEA to compute a set of solutions for several TTP instances; then, we plot the map to illuminate the distribution of the solutions over the space of $f$ and $g$. Moreover, we comprehensively compare different search operators and their effects on the distributions and the final maps. We consider the EAX and the 2-OPT for generating tours and the DP and the (1 + 1)EA for computing the packing lists. Employing the operators alternatively, we have four different operator settings. The algorithms are terminated when they reach either of 10000 iterations or 72 hours CPU time. Here, iteration is referred to as the main loop of the BMBEA. We use the TTP instances developed in Polyakovskiy et al. [2014]. Table 1 presents the names of the instances used in the paper. Please note that we select the first instance of each sub-group except for the dsj1000. The renting price ($R$) is set to zero in those instances; the issue makes the TTP instances turn to KPs. We separate the instances into two categories, small and medium. Since DP’s time efficiency is correlated with the number of items, we select instances where $m \leq 500$ for small instances.
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Figure 3: The distribution of TTP solutions of the four competitors over the behaviour space on instance eil51_n250_uncorr-similar-weights_01 (top), pr152_n453_uncorr_01 (middle), and a280_n279_bounded-strongly-corr_01 (bottom). The cells are coloured based on the average TTP scores of the solutions in the cell over ten independent runs.

Figure 4: The frequency of cells housing a TTP solution over 10 independent runs on instance eil51_n250_uncorr-similar-weights_01 (top), pr152_n453_uncorr_01 (middle), and a280_n279_bounded-strongly-corr_01 (bottom).
Table 1: The names of the TTP instances are used in the paper.

| No. | Original Name                      | No.  | Original Name                      |
|-----|------------------------------------|------|------------------------------------|
| 1   | eil51_n50_bounded-strongly-corr_01 | 18   | a280_n279_uncorr_01                |
| 2   | eil51_n150_bounded-strongly-corr_01| 19   | rat575_n574_bounded-strongly-corr_01|
| 3   | eil51_n250_bounded-strongly-corr_01| 20   | rat575_n574_uncorr-similar-weights_01|
| 4   | eil51_n50_uncorr-similar-weights_01| 21   | rat575_n574_uncorr_01               |
| 5   | eil51_n150_uncorr-similar-weights_01| 22   | ds1000_n999_bounded-strongly-corr_02|
| 6   | eil51_n250_uncorr-similar-weights_01| 23   | ds1000_n999_uncorr-similar-weights_06|
| 7   | eil51_n50_uncorr_01                 | 24   | ds1000_n999_uncorr_04               |
| 8   | eil51_n150_uncorr_01                | 25   | u2152_n2151_bounded-strongly-corr_01|
| 9   | eil51_n250_uncorr_01                | 26   | u2152_n2151_uncorr-similar-weights_01|
| 10  | pr152_n151_bounded-strongly-corr_01 | 27   | u2152_n2151_uncorr_01               |
| 11  | pr152_n453_bounded-strongly-corr_01 | 28   | fnl4461_n4460_bounded-strongly-corr_01|
| 12  | pr152_n151_uncorr-similar-weights_01| 29   | fnl4461_n4460_uncorr-similar-weights_01|
| 13  | pr152_n453_uncorr-similar-weights_01| 30   | fnl4461_n4460_uncorr_01             |
| 14  | pr152_n151_uncorr_01                | 31   | ds1000_n999_uncorr_02               |
| 15  | pr152_n453_uncorr_01                | 32   | ds1000_n999_uncorr_03               |
| 16  | a280_n279_bounded-strongly-corr_01  | 33   | ds1000_n999_uncorr-similar-weights_03|
| 17  | a280_n279_uncorr-similar-weights_01 | 34   | ds1000_n999_uncorr-similar-weights_04|

4.1 Analysis of the maps

This section visualises and scrutinises the final map obtained from the BMBEA using different search operators, namely EAX, 2-OPT, DP, and (1 + 1)EA. Figure 3 visualises the final maps obtained from the four competitors in instances 6, 15, and 16. The TSP value increases when we move in the direction of $x$ axis, while moving in the $y$ axis results in a rise in the KP score. Since the TSP is a minimisation and the KP is a maximisation problem, Cell (1,20) consists of the solution with a BD closest to $f^*$ and $g^*$. The maps’ cells are coloured based on the average TTP score of the solution within the cells over 10 independent runs; the hotter colour, the higher the TTP score. We can observe that the west part of the maps tends to contain better TTP solutions. In 8 out of 9 cases, the best solutions are located in a BD of $(1, 1.005)f^*$ and $(0.9, 0.95)g^*$. Moreover, the figure depicts that the maps obtained from BMBEAs using EAX have more hot-coloured cells than the ones with 2-OPT have, which shows the consistency of EAX in generating high-quality solutions. Turning to the comparison between DP and (1 + 1)EA, the latter can populate a larger part of the map.

Figure 4 illustrates the frequency of cells containing a solution over ten independent runs. The instances are the same as Figure 3. Here, a hotter colour indicates a higher frequency. The figure depicts that the algorithms cannot populate the cells close to the optimal KP. Because the algorithms compute the packing list as the second level of a bi-level optimisation procedure. Thus, the KP values are constrained by the given tour. Interestingly, most cells corresponding to the KP values close to $(1 - \alpha_2)g^*$ also remain empty for the same reason, especially when DP is used. Moreover, one may notice that the most red-coloured cells in Figure 3 are coloured red here as well. It illustrates a proportional relationship between the quality of solutions and the frequency. Furthermore, the cells associated with low TSP values (left) of maps are more likely to be empty than the other side. As the TSP value increases, so does the number of tours resulting in such a TSP value rise. This results in a more diverse set of tours and eventually a more diverse set of packing lists and a broader range of the KP score.

4.1.1 MAP-Elitism vs. $(\mu + 1)$EA

In this section, we compare the sets of solutions obtained by BMBEA and $(\mu + 1)$EA. Both algorithms have identical initialisation, parent selection, and generating offspring. Therefore, the difference between these algorithms is limited to survival selection. The aim is to investigate the impact of MAP-Elitism.

Figure 5 compares MAP-Elitism and elitism in survival selection. The first three columns belong to $(\mu + 1)$EA that show the initial population, all solutions generated during the search, and the final population, respectively. The fourth column illustrates the population obtained by BMBEA. One can observe that $(\mu + 1)$EA converges to a single solution. Since the algorithm uses EAX crossover and DP as operators, it’s impossible for the algorithm to find any other solutions from this point. On the other hand, the BMBEA’s final population consists of a vast number of solutions with different properties. Thus, it can potentially find better-quality solutions if we continue the search. The other difference we can observe in Figure 5 is that diversity of solutions decreases during the search $(\mu + 1)$EA. On the contrary, the diversity increases using MAP-Elitism survival selection.
4.2.1 Maps With The Relaxed Approach

The maps obtained from the relaxed approach are similar to the prefixed method. The best quality solutions come from areas with similar distances to TSP and KP optimal values. However, $\alpha_1$ and $\alpha_2$ can differ in each run. Figure 6 illustrates the $\alpha_1$ and $\alpha_2$ in 10 independent runs on instances 1 to 18. Here, after using EAX (Nagata and Kobayashi [2013]) to generate high-quality tours, we compute an optimal packing list for the tours by DP. Note that we used TSP optimal value as the termination criterion to boost time efficiency and diversity in tours. Figure 6 shows $\alpha_1$ and $\alpha_2$ belongs to [0.04 0.14] and [0.11 0.33], respectively. Since the TSP sub-problem is identical on instances 1 to 9, we can observe a similar trend for $\alpha_1$ on those instances. This argument is also true for instances 10 to 15 and 16 to 18. On the other hand, KP sub-problems are unique for all cases. Therefore, $\alpha_2$ is different in each instance. These observations can support our claim that $\alpha_1$ and $\alpha_2$ should be tuned for each instance separately in the prefixed method.

4.2 Best found TTP Solutions

In this section, we analyse the performance of BMBEA in solving TTP. First, we look into the operators and draw a comparison in alternatively incorporation of the proposed operators into BMBEA. Then, we scrutinise the algorithm’s survival selection by comparing BMBEA to $(\mu + 1)EA$. Last, we study the method proposed for relaxing $\alpha_1$ and $\alpha_2$ and compared it to BMBEA with fixed $\alpha_1$ and $\alpha_2$.

4.2.1 Operators

We now compare the search operators, EAX, 2OPT, DP, and $(1 + 1)EA$, in terms of the best-found TTP solution in this section. We consider instances in a range of 51 to 280 cities, and 50 to 453 items from Polyakovskiy et al. [2014]. Table 1 shows the average and the best TTP solutions and the average CPU time in ten independent runs for the four competitors and the best-known TTP values. The best-known values are obtained from Chagas and Wagner [2022], and Wuijts and Thierens [2019], both these papers compared their results to those of 21 algorithms analysed in Wagner et al. [2018]. Wagner et al. [2018] reported their results on all instances from Table 11 while Wuijts and Thierens [2019] and Chagas and Wagner [2022] used some of the instances in their studies. The best-known values

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Figure 5: The illustrations of solutions obtained by $(\mu + 1)EA$ and Map-elitism in the behavioral space on instance el51.n250_uncorr-similar-weights_01 (top), pr152.n453_uncorr_01 (middle), and a280.n279.bounded-strongly-corr_01 (bottom). The cells are coloured based on the TTP scores of the solutions in the cell over one single run.
Instances

Figure 6: The values of $\alpha_1$ and $\alpha_2$ in the relaxed method

include Chagas and Wagner [2022] in instances 1 to 18, 22 to 24, and 28 to 34, and Wuijts and Thierens [2019] in instances 1 to 7, and 15 to 18. Note that our termination criterion differs from 10 minutes CPU time in Wagner et al. [2018] and 2500 local searches in Wuijts and Thierens [2019]. The results indicate that EAX outperforms 2-OPT in terms of TTP score in most cases. The observations are confirmed by a Kruskal-Wallis test at significance level 5% and Bonferroni correction. Turning to the comparison of the KP operators, $(1 + 1)$EA yields very decent objective values and can compete with DP, which results in the optimal packing list. In general, an increase in the size of instances severely affects the run time of the MBBEA using DP. On the other hand, the run times of $(1 + 1)$EA are significantly shorter. For example, the EAX-EA averagely finishes the 10000 iterations in 240.9 seconds on instance 3. The figure is about 18990.2 seconds for EAX-DP. This is while the algorithm’s run time employing $(1 + 1)$EA remains reasonable. More Interestingly, Table 2 also indicates that all variants of MBBEA result in very decent TTP scores. In instances 10 and 13, the introduced algorithms beat the best TTP scores and can hit the best known on instances 1 and 4. Since the DP is not time-efficient in larger instances, we consider the $(1 + 1)$EA for computing the packing list. Table 8 shows the results on 12 instances from 575 to 4661 cities and 574 to 4460 items. As one can observe, EAX dominates 2-OPT in these instances. Moreover, the algorithm using EAX improved the best-found solution in 8 out of 12 cases. For example, the TTP score significantly increased from 893 to 1137.5 in instance 4. One can notice that the TTP score of the algorithm using 2-OPT is negative in this instance. Polyakovskiy et al. [2014] balanced the instances in the TSP and the KP, but the TSP sub-problem is more dominating in some of the dsj1000 sub-group. The traveling cost is high in these particular instances, and the items do not compensate for the high cost. Having the TSP sub-problem more dominating, it is not surprising that the EAX outperforms the 2-OPT. Moreover, The domination is even stronger in four other instances of the dsj1000 sub-group in a way that the best-known values are negative. We investigate the four instances separated from the others due to the dominance of the TSP sub-problem over the KP. It means that the high-quality TTP solutions are closer to the TSP optimal value and more away from the KP optimal values. The current $\alpha_1$ and $\alpha_2$ are set for the balanced instances. Thus, we need to reset the $\alpha_1$ and $\alpha_2$ to populate the map. Based on initial experimental investigations, we set $\alpha_1$ and $\alpha_2$ to 2 and 60 percent, respectively. Table 4 summarises the
Table 2: Comparison of the search operators in terms of the TTP score and CPU time on the small size instances. In columns Stat the notation $X^+$ means the median of the measure is better than the one for variant $X$, $X^-$ means it is worse, and $X^*$ indicates no significant difference. Stat shows the results of Kruskal-Wallis statistical test at significance level 5% and Bonferroni correction.

| In. | EAX-DP Average (1) | Stat | Best | CPU time | EAX-EA Average (2) | Stat | Best | CPU time | Best-known value |
|-----|--------------------|------|------|----------|--------------------|------|------|----------|-----------------|
| 1   | 4267.1 $2^\star 3^4$ | 4269.4 | 86.5 |          | 4243.1 $1^\star 3^4$ | 4269.4 | 29.4 |          | 4269.4          |
| 2   | 7236 $2^\star 3^4$  | 7252.8 | 2275 |          | 7086.6 $1^\star 3^4$ | 7216.4 | 112.6 |          | 7532            |
| 3   | 11713 $2^\star 3^4$ | 11733.9 | 18990.2 |      | 11621.1 $1^\star 3^4$ | 11674.5 | 240.9 |          | 12804           |
| 4   | 1449.8 $2^\star 3^4$ | 1440 | 31.1 |          | 1443.6 $1^\star 3^4$ | 1460 | 26.9 |          | 1460            |
| 5   | 4250.1 $2^\star 3^4$ | 4269.6 | 259.2 |          | 4235.1 $1^\star 3^4$ | 4248.3 | 112.2 |          | 4365            |
| 6   | 5739.1 $2^\star 3^4$ | 5792.2 | 884.3 |          | 5712.1 $1^\star 3^4$ | 5792.2 | 219.7 |          | 6359            |
| 7   | 2808 $2^\star 3^4$  | 2854.5 | 40.2 |          | 2781.2 $1^\star 3^4$ | 2848.1 | 30.5 |          | 2871.1          |
| 8   | 6838.8 $2^\star 3^4$ | 6884.4 | 456 |          | 6834.8 $1^\star 3^4$ | 6884.4 | 98.7 |          | 7037            |
| 9   | 11753.2 $2^\star 3^4$ | 11753.2 | 2185 |          | 11748.2 $1^\star 3^4$ | 11753.2 | 214.7 |          | 12478           |
| 10  | 11140.5 $2^\star 3^4$ | 11140.5 | 3639.7 |      | 11113.3 $1^\star 3^4$ | 11134.8 | 119.7 |          | 11117.4         |
| 11  | 25507.5 $2^\star 3^4$ | 25525.5 | 224885.8 |     | 25148.2 $1^\star 3^4$ | 25405.3 | 552.8 |          | 25664.4         |
| 12  | 3540.2 $2^\star 3^4$ | 3669 | 253.6 |          | 3520.6 $1^\star 3^4$ | 3665.4 | 112.8 |          | 3791.9          |
| 13  | 13374.8 $2^\star 3^4$ | 13628.3 | 5234.4 |      | 13210.6 $1^\star 3^4$ | 13345.5 | 521.9 |          | 13556.9         |
| 14  | 5398.3 $2^\star 3^4$ | 5398.3 | 386.8 |          | 5398.3 $1^\star 3^4$ | 5398.3 | 99.5 |          | 5615            |
| 15  | 20456.8 $2^\star 3^4$ | 20456.8 | 16047.8 |     | 20455.4 $1^\star 3^4$ | 20456.8 | 476.2 |          | 20705.8         |
| 16  | 18449.6 $2^\star 3^4$ | 18595.5 | 39024.7 |     | 18190.3 $1^\star 3^4$ | 18244.5 | 254.5 |          | 19499           |
| 17  | 19163.3 $2^\star 3^4$ | 9201.1 | 1496.5 |          | 9122 $1^\star 3^4$ | 9176.8 | 241.9 |          | 9998            |
| 18  | 19419.2 $2^\star 3^4$ | 19493.4 | 3588.3 |          | 19375.8 $1^\star 3^4$ | 19495.4 | 215.8 |          | 20491           |

results on the four instances. The EAX, as expected, outperforms the 2-OPT in all four cases. More importantly, the EAX-based algorithm improved the TTP values for instances 31, 32 by 1.1 and 37.1, respectively. Note that the results presented in Tables 1 and 3 are different from the ones presented in the conference version (Nikfarjam et al., 2022b) which are incorrect due to an implementation mistake.

4.2.2 $(\mu + 1)$EA vs. BMBEA

We now compare the BMBEA to $(\mu + 1)$EA in solving the problem. As aforementioned, The difference between the two algorithms is in survival selection, where BMBEA select the next generation based on MAP-Elitism and $(\mu + 1)$EA takes the most elite solutions. Table 5 summarises the results of these algorithms where EAX and DP are considered as the operators. The table shows that the BMBEA has a higher average in 9 cases out of 18. The statistical test confirms a meaningful difference in the results in two instances favouring BMBEA. Both competitors result in the same average TTP score on four instances, and $(\mu + 1)$EA brings about a higher mean on four instances. However, the statistical investigation finds no meaningful difference in those instances. Each algorithm outperforms the other in

\footnote{The TTP solutions can be accessed at https://github.com/NikfarjamAdel/Traveling-Thief-Problem}
Table 3: Performance of the MAP-Elite based approach in terms of the TTP score. The notations are in line with Table 2.

| In. | EAX-EA (1) Average Stat Best CPU time | 2-OPT-EA Average (2) Stat Best CPU time | Best-known value |
|-----|--------------------------------------|----------------------------------------|------------------|
| 19  | 32625.3 2+ 33092 835 29687.5 1+30065.8 728 | 32993.1 |
| 20  | 18975.9 2+ 19188.4 708 17622.2 1+17803.8 685 | 19379.7 |
| 21  | 35175.8 2+ 35512.2 696 33456.6 1+34455.2 674 | 35015.2 |
| 22  | 642.3 2+ 1137.5 1625 -4337 1-2693.5 1696 | 893.4 |
| 23  | 51988.6 2+ 52651.8 1624 48382.8 1+49830.4 1685 | 51303.4 |
| 24  | 29201.5 2+ 32072.9 1627 25214.6 1+25618.4 1620 | 28304 |
| 25  | 104549.9 2+ 105434.5 7937 95300.4 1+96468.9 7975 | 105908.1 |
| 26  | 71829.9 2+ 73152.8 6914 67954.6 1+69060.6 7285 | 72308.7 |
| 27  | 107975.3 2+ 109395.4 6848 104852.4 1+106735 7091 | 108236.1 |
| 28  | 258901.5 2+ 260839.7 38669 238212.3 1+240916 45390 | 263040.2 |
| 29  | 129168.4 2+ 131072 36670 122606.8 1+123626.8 39520 | 131486.2 |
| 30  | 230888.9 2+ 237097.6 32136 225694.2 1+227466.4 31796 | 233343 |

Table 4: Performance of the MAP-Elite based approach on the unbalanced instances. The notations are in line with Table 2.

| In. | EAX-EA (1) Average Stat Best CPU time | 2-OPT-EA Average (2) Stat Best CPU time | Best-known value |
|-----|--------------------------------------|----------------------------------------|------------------|
| 31  | -49282 2+ 468622.8 1555 -51704.2 1-51635 1671 | -49149.9 |
| 32  | -7241 2+ 4855 1549 -10493 1-9906.8 1709 | -7714.6 |
| 33  | -63137 2+ 61797.3 1560 -66602 1-63939 1709 | -61709.1 |
| 34  | -24508 2+ 24263.5 1631 -27573 1-26654 1684 | -19215.2 |

three cases regarding the best TTP score out of 10 runs. At the same time, they performed equally in the rest of the instances.

Figure 5 shows that ($\mu + 1$)EA converges to a single solution; therefore, there is no hope of finding better solutions by increasing the number of iterations. On the other hand, there is a good chance that an increase in the number of iterations results in better solutions for BMEEA. We investigate this by comparing the results in 10000 and 100000 iterations. Since DP can be time-consuming, we used $(\mu + 1)\text{EA}$ as the KP operator for this round of experiments.

Table 5: The comparison of $(\mu + 1)\text{EA}$ and Map-elite in solving TTP with using DP as KP operator. The notations are in line with Table 2.

| In. | Map-Elitism Average (1) Stat Best | $(\mu + 1)\text{EA}$ Average (2) Stat Best |
|-----|----------------------------------|-----------------------------------------|
| 1   | 4267.1 2+ 4269.4 4264.9 1+4269.4 | |
| 2   | 7236 2+ 7252.8 7193.4 1+7252.8 | |
| 3   | 11713 2+ 11733.9 11693.5 1+11733.9 | |
| 4   | 1449.8 2+ 1460 1441.6 1+1460 | |
| 5   | 4250.1 2+ 4269.6 4243.8 1+4260.7 | |
| 6   | 5739.1 2+ 5792.2 5761 1+5792.2 | |
| 7   | 2808 2+ 2854.5 2785 1+2833.6 | |
| 8   | 6838.8 2+ 6884.4 6830.3 1+6884.4 | |
| 9   | 11753.2 2+ 11753.2 11753.2 1+11753.2 | |
| 10  | 11140.5 2+ 11140.5 11140.5 1+11140.5 | |
| 11  | 25507.5 2+ 25525.5 25520.8 1+25525.5 | |
| 12  | 3540.2 2+ 3669 3549.4 1+3687.1 | |
| 13  | 13374.8 2+ 13628.3 13243.4 1+13345.5 | |
| 14  | 5398.3 2+ 5398.3 5398.3 1+5398.3 | |
| 15  | 20456.8 2+ 20456.8 20456.8 1+20456.8 | |
| 16  | 18449.6 2+ 18595.5 18418.2 1+18444.1 | |
| 17  | 9163.3 2+ 9201.1 9164.5 1+9277.7 | |
| 18  | 19419.2 2+ 19493.4 19447.7 1+19507 | |
Table 6: The comparison of \((\mu + 1)\)\ EA and and Map-elite in solving TTP with using \((1 + 1)\)\ EA as KP operator. The notations are in line with Table 2.

| In. | \(10^3\) iterations | \(10^5\) iterations |
|-----|----------------------|----------------------|
|     | BMBEA (1) | \((\mu + 1)\) (2) | BMBEA (1) | \((\mu + 1)\) (2) |
|   1 |         |         | 4243.1 | 4269.4 |
| 2   |         |         | 7086.6 | 7216.4 |
| 3   |         |         | 11621.1 | 11647.5 |
| 4   |         |         | 1443.6 | 1460.7 |
| 5   |         |         | 4353.1 | 4248.3 |
| 6   |         |         | 5712.1 | 5792.2 |
| 7   |         |         | 2781.2 | 2848.1 |
| 8   |         |         | 6834.8 | 6884.4 |
| 9   |         |         | 11748.2 | 11753.2 |
| 10  |         |         | 11113.3 | 11134.8 |
| 11  |         |         | 25148.2 | 25605.3 |
| 12  |         |         | 3520.6 | 3665.4 |
| 13  |         |         | 13210.6 | 13345.5 |
| 14  |         |         | 5398.3 | 5398.3 |
| 15  |         |         | 20454.5 | 20456.8 |
| 16  |         |         | 18190.3 | 18244.5 |
| 17  |         |         | 9122.2 | 9176.8 |
| 18  |         |         | 19375.8 | 19495.4 |

Table 6 indicates results for BMBEA and \((\mu + 1)\)\ EA when the KP operator is altered to the \((1 + 1)\)\ EA, and the number of iterations is equal to \(10^3\) and \(10^5\). The table shows that \((\mu + 1)\)\ EA bring about a higher mean of TTP score in 10 cases compared to BMBEA when the termination criterion is set to \(10^3\). The figure stands at 8 for BMBEA. The statistical investigation finds a meaningful difference in two instances in favour of \((\mu + 1)\)\ EA. One can notice the contradiction of this part of the table to Table 5. This is because the algorithms need more iterations to converge when we use \((1 + 1)\)\ EA as the KP operator instead of DP. Giving more time to the algorithms can bring about similar results to Table 5. The other part of Table 6 sheds light on this matter.

We can see that BMBEA outperforms \((\mu + 1)\) on 16 out of 18 instances in terms of average TTP score when the number of iterations is set to \(10^5\). Also, a meaningful difference can be found in 12 instances, all favouring BMBEA. Increasing the number of iterations has no effect on the results of \((\mu + 1)\)\ EA since the algorithm converges within \(10^3\) iterations. On the other hand, it considerably improves the BMBEA performance. This shows the efficiency of MAP-Elitism in preserving diversity and preventing premature convergence.

### 4.2.3 Relaxed Method

Tuning \(\alpha_1\) and \(\alpha_2\) requires a lot of computational effort. It should be done for each instance separately to make sure the area of focus in the feature space is not infeasible and promising. As we observed in the previous section, the initial values \((\alpha_1 = 0.05, \alpha_2 = 0.2)\) result in an infeasible focused area for instances 31 to 34. For this purpose, we propose the relax method where \(\alpha_1\) and \(\alpha_2\) are set based on the initial solutions. Here, we investigate the impact of this method on the performance of BMBEA. To do so, we compare the BMBEA using the relaxed method with the prefixed \(\alpha_1\) and \(\alpha_2\).

Table 7 summarises the experimental investigation. EAX and \((1 + 1)\)\ EA are considered for the operators. When the termination criterion is set to \(10^3\) iterations, the prefixed method performs statistically better in 4 out of 27 cases. A meaningful difference can be found in one instance in favour of the relaxed method. At the same time, there is no statistically significant difference in the rest of the cases. However, the average TTP score of the relaxed method is higher in four instances. We can observe that this method performs better in experiments with longer runs. It has a higher average TTP score in 19 cases, while the prefixed approach outperforms it in 5 instances. Both algorithms perform equally in three cases. Statistical tests confirm significant differences in the two instances for the relaxed approach. We can conclude that the relaxed method can result in a decent TTP score. At the same time, it eliminates the need to fine two influential parameters. It is noteworthy that both algorithms can beat the best-known TTP values in instances 20 and 25 in the longer runs.
| In | 10^3 iterations |  | 10^5 iterations |  |
|---|---|---|---|---|
|  | Prefixed (1) | Relaxed (2) | Prefixed (1) | Relaxed (2) |
| Average | Stat | Best | Average | Stat | Best | Average | Stat | Best |
| 1 | 4243.1 | 2° | 4269.4 | 4246.6 | 1° | 4269.4 | 4269.4 | 1° | 4269.4 |
| 2 | 7086.6 | 2° | 7216.4 | 7116.7 | 1° | 7231.7 | 7202.5 | 1° | 7252.8 |
| 3 | 11621.1 | 2° | 11647.5 | 11619.5 | 1° | 11697.5 | 11651.9 | 1° | 11733.9 |
| 4 | 1443.6 | 2° | 1460 | 1437.1 | 1° | 1445.8 | 1458.9 | 1° | 1460 |
| 5 | 4235.1 | 2° | 4248.3 | 4235.8 | 1° | 4260. | 4255.2 | 1° | 4274.5 |
| 6 | 5712.1 | 2° | 5792.2 | 5655 | 1° | 5729. | 5776.6 | 1° | 5792.2 |
| 7 | 2781.2 | 2° | 2848.1 | 2799.1 | 1° | 2840 | 2804.7 | 1° | 2854.5 |
| 8 | 6834.8 | 2° | 6884.4 | 6833.8 | 1° | 6884.4 | 6858.8 | 1° | 6884.4 |
| 9 | 11748.2 | 2° | 11753.2 | 11744.7 | 1° | 11753.2 | 11749 | 1° | 11753.2 |
| 10 | 11113.3 | 2° | 11134.8 | 11103 | 1° | 11135.4 | 11137.1 | 1° | 11140.4 |
| 11 | 25148.2 | 2° | 25405.3 | 25036.9 | 1° | 25332.2 | 25966.9 | 2° | 25524.3 |
| 12 | 3520.6 | 2° | 3665.4 | 3487.4 | 1° | 3547.6 | 3528.3 | 1° | 3687.1 |
| 13 | 13210.6 | 2° | 13345.5 | 13270 | 1° | 13345.5 | 13326.6 | 1° | 13345.5 |
| 14 | 5398.3 | 2° | 5398.3 | 5387.6 | 1° | 5398.3 | 5398.3 | 1° | 5398.3 |
| 15 | 20456.4 | 2° | 20456.8 | 20421.6 | 1° | 20456.8 | 20456.8 | 1° | 20456.8 |
| 16 | 18190.3 | 2° | 18244.5 | 18187.2 | 1° | 18300.5 | 18381 | 1° | 18438.6 |
| 17 | 9122 | 2° | 9176.8 | 9074.2 | 1° | 9143.1 | 9249.3 | 1° | 9233.1 |
| 18 | 19375.8 | 2° | 19495.4 | 19247 | 1° | 19432.1 | 19559 | 1° | 19738.3 |
| 19 | 32625.3 | 2° | 33092 | 32354.2 | 1° | 33080 | 33358.4 | 1° | 33759 |
| 20 | 18975.9 | 2° | 19188.4 | 18965.4 | 1° | 19092.8 | 19279.8 | 1° | 19538.3 |
| 21 | 35175.8 | 2° | 35512.2 | 35101.1 | 1° | 35429.5 | 35720.2 | 1° | 36021.4 |
| 22 | 642.3 | 2° | 1137.5 | 541.5 | 1° | 879.3 | 1535.5 | 1° | 2621.1 |
| 23 | 51988.6 | 2° | 52651.8 | 51885.5 | 1° | 52345.7 | 52292.8 | 1° | 52973.6 |
| 24 | 29201.5 | 2° | 32072.9 | 28711.7 | 1° | 29299.3 | 29492.5 | 1° | 32085.1 |
| 25 | 104549.9 | 2° | 105434.5 | 103863.4 | 1° | 105025 | 105442.9 | 1° | 106465.7 |
| 26 | 71829.9 | 2° | 73152.8 | 71615.1 | 1° | 73141 | 72110.7 | 1° | 72494.7 |
| 27 | 107975.3 | 2° | 109395.1 | 108885.5 | 1° | 109929 | 109402.2 | 2° | 110346 |

5 Conclusion

In this study, we incorporated the concept of QD into solving the TTP. To the best of our knowledge, this is the first time the QD concept has been used to solve a combinatorial problem. The behaviour descriptor for our approach is defined by comparing it to a simple (µ + 1) EA. Moreover, we investigated the impact of Map-elitism on the final solutions by comparing it to a simple (µ + 1)EA. The results indicated that Map-elitism boosts both the diversity and quality of solutions.

It would be interesting to incorporate more complex MAP-Elite approaches such as CVT-MAP-Elites [Vassiliades et al., 2018] into the introduced algorithm. Using such an approach can discretise the behavioural space more intelligently. Moreover, several multi-component combinatorial optimisation problems can be found in literature where QD is highly beneficial to understanding the inter-dependencies of components and the distribution of solutions in the behavioural space.

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On the Use of Quality Diversity Algorithms for The Traveling Thief Problem
Adel Nikfarjam, Aneta Neumann, and Frank Neumann
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