Abstract

Proceeding from the main principles of the non-unitary quantum theory of relativistic bi-Hamiltonian systems, a system of Lagrangian fields characterized by a certain dispersion law (mass spectrum of particles), interactions between them and their coupling constants are constructed. In this article the mass spectrum formula for “bare” fundamental hadrons is introduced, and an a priori normalization of particle fields is found as well. Numerical values of some parameters of the present theory are determined.

1 Introduction

Any physical theory has to be expressed in an adequate mathematical representation. Eventually the most of temporary physical theories (even if they have been honored with the Nobel prize) will be rejected because they are semiempirical. But the present theory will survive until a mathematically thinking humanity exists, because it is an adequate mathematical theory. Mathematics do not die, they are always applicable in a proper place, and are only perfected.

Apparently, to construct an adequate theory of elementary particles, it will be necessary to agree with a radical break-up of our former representations about the nature of particles, that are, to a large extent, based on geometrical concepts and differential topology (in this relation the changeover in particle theory should be certainly not less radical than formerly in atomic theory). First of all this remark is to be placed in the context of the field concept, which,
of course, loses force in the subject area of physical processes on supersmall distances.

Like it was necessary in the construction of the consistent atomic theory to completely reject the concept of a geometrical trajectory of an electron and to assume another geometrical image (field or de Broglie waves), now the very moment has come to reject the field concept as functions on the space-time continuum (see [1]).

It is important to emphasize that the transition from trajectory to wave function is, as opposed to the transition from point particle to vibrating string or membrane, not just a generalization of the theory outgoing from a mechanistic analogy with optics, but also it demonstrated a structure like a differentiable manifold in space; in these terms others do not exist [1]. The transition from classical mechanics to quantum mechanics is, in fact, the transition from Newtonian equations for trajectories to the Schrödinger equations for wave functions. Both theories are geometrical in character and are based on the Newtonian concept of space as a differentiable manifold: in the first case the theory is formulated in terms of the space variables (coordinates $x$); in the second case in terms of the vector fiber bundle being built over the space (of bundle sections or fields $\psi(x)$, see [1]). In the development of quantum mechanics, verifying the sense of fields $\psi(x)$ as waves of probability, the accuracy of the information that followed directly from the mathematical apparatus of the theory proved to be a refinement compared to the predictions of classical theory. Based on this example a progress in fundamental physics is supposed to be connected with the development and application of a new and more perfect mathematical theory.\footnote{Let us add to this that before the creation of quantum theory, attempts to geometrize matter (relating the existence of matter to the energy-momentum tensor[2]) did not seem to be doomed. At present, many people consider this approach to be outdated or even erroneous. It follows that instead attempts should be made to derive space from matter.}

The Heisenberg-Schrödinger quantum theory has a wide field of applications forming a set of subareas, which are in some sense enclosed in each other due to the characterizations by different energy and/or distance levels (solids, molecules, atoms, nuclei, elementary particles). To continue this ladder further by assuming (without any solid basis) that particles consist of quantum objects (quarks, gluons), that in turn consist of preons, is a questionable way of thinking. The dubiousness of that assumption is underlined by the fact that these objects are unobservable in space (also, the phenomenon of quark confinement is inconsistent with the concept of Lagrangian or Hamiltonian quantum systems). We think that quarks having small masses may not exist inside protons (like electrons may not exist inside nuclei).
1 INTRODUCTION

In exactly this place radical changes seem to be required in representations of the nature of particles: we should reject composite models of particles, being inadequate for the description of the particle nature, and switch to another paradigm: a new mechanistic concept, which is described by a non-standard dynamical system coming to light on supersmall distances, where, as research shows [1], space-time can be regarded as a discontinuum instead of a continuum. It is essential that this statement is not an axiom, but a theorem: then the theoretical generalization has forced a well-founded possibility. We consider on supersmall distances the field concept to be invalid, so that it makes no sense to speak about particles having masses \( \sim 10^{15} \) GeV.

The transition to the new mechanics (dynamics of relativistic bi-Hamiltonian system [1]) goes together with a giving up of differential equations in space-time. The new mechanics are algebraic in character.

If before an action quantum \( h \) was a fundamental value needed for the transition from trajectories to fields, then now a third constant \( k \) with the dimension \( cm^{-1} \) is the fundamental value for a transition from continuum to discontinuum and from a Lagrangian field system (elementary particles) to the relativistic bi-Hamiltonian system.

In the present article it will be shown that one of main assignments of the new dynamical system is to create fundamental particle fields. In this relation the considered system (by nature a two-level system) is analogous to an atom, in which transitions generate photons. Particles, too, arise as a result of quantum transitions in the above mentioned dynamical system with an upper level (the upper half of light cone) and a lower level (the lower half of light cone, see the main text of the article). Thus, where the composite models tried to answer the question (of what do fundamental particles consist), the present theory answers another question: how to arise particles. Having formulated the subject in such a way, the topic is specific for dynamical theory (therefore for the sake of brevity we shall call the proposed theory a dynamical one).

The purpose of this paper is to formulate the main laws responsible for creating particles. In this perspective it is reminded that Heisenberg already formulated the conditions an adequate particle theory has to satisfy, in the form of main questions that should be answered [3]. Above all the theory should explain: 1) the real observable mass spectrum of particles; 2) their symmetry properties; 3) the kinds and constants of interactions (particle charges).

1) In the given work first of all we determine explicit states of the relativistic bi-Hamiltonian system — semispinor fields \( f(x), \dot{f}(\dot{x}) \) called quanta \( f \) and \( \dot{f} \) in another way. At the same time the mathematical apparatus of the new quantum theory, based on the extended Fock representation of the Heisenberg
algebra $h_{8}^{(*)}$ advanced in [4], plays an important role. This part of the theory, following Neumann’s terminology [5], will be called the non-unitary quantum theory I.

Then we investigate amplitudes of the transition $f \rightarrow \dot{f}$ (this part of the theory will be named the quantum theory II), and derive equations which they satisfy. The relation of fundamental particle fields $\psi^{\Sigma}(X,Y)$ with amplitudes of the transition $O^{\Sigma}(X,Y)$ is established, and the mass spectrum formulae for “bare” fundamental hadrons and leptons are defined. In the present theory the particle fields represent bilocal fields (the bilocality is a direct consequence of the semispinor structure of particle fields or the bi-Hamiltonian property of the considered dynamical system), which describe non-point (smeared (non-infinitesimal)) objects: 1) four coordinates $X_{\mu}$ of the affine space $A_{3,1}$ describe the motion of particles in our space-time; b) additional four coordinates $Y_{\mu}$ describe a particle’s interior as an object in the second (internal) space-time $R_{3,1}$ not depending on our (external) space-time. In the latter any movements are possible only with speeds smaller than the velocity of light $c$. Variables $Y_{\mu}$ are called hidden coordinates (as opposed to open ones $X_{\mu}$).

It is important to emphasize that the quantum transition $f \rightarrow \dot{f}$, irreversible by nature (since the structure $\langle \dot{f}, f \rangle$ is non-Hermitian[1], it should be noted that such structures are not suitable for calculations of probabilities), occurs when quanta $f$ (that might be identified with Feynman’s partons, providing an alternative to quarks!) are in a hardly compressed state. Under these circumstances the quanta $f$ form an ensemble, which is codetermined by the statistical properties of the given ensemble (together with the ensemble of quanta $\dot{f}$ that they pass into), that are described by the distribution function in the relativistic Juttner form [6]. Here parameters (calculable in the theory) appear such as the temperature of quanta $f$ $T_f$, the temperature of quanta $\dot{f}$ $T_{\dot{f}}$ as well as their product — the parameter $\mu^2 = T_f T_{\dot{f}}$ (the theory of ensembles will be called the quantum theory III). The issue regarding the mass spectrum of fundamental particles is closely related to the semispinor structure of their fields, and in the hadron case it is to the Gibbs distribution function of quanta $f$.

Fields of fundamental hadrons (in particular, their a priori normalization) and their mass formulae (that exist of two branches: baryonic and mesonic) are essentially defined by the statistics of quanta $f$. To create leptons (in which case the system passes from the upper half of light cone into the vertex of cone, therefore $T_{\dot{f}} = 0$ and $\mu^2 = 0$) the statistical properties of quanta $f$ are not important. As a consequence the lepton spectrum is much narrower than the hadron spectrum. In this way, the existence of two families of particles — hadrons and leptons — is connected with two topological different transitions.
of the dynamical system: from the upper half of the light cone either into the lower half (hadrons) or into the vertex of the cone (leptons).

Another limit case $\mu^2 \to \infty$, having only sense for fermions, results in objects characterized by a large imaginary mass. Such objects have no wave function, therefore they do not exist as particles in our spacetime (coordinate $X$). They represent particles in the second spacetime (coordinates $Y$), and show themselves only on supersmall distances as virtons playing an essential role in the description of weak interactions (see [7]).

2) For nearly all performed calculations about particles, the relativistic bi-Hamiltonian system was used in a model based on the algebra $h_8^{(s)}$. In this model the isotopic symmetry is represented by the group $U(1)$, all particles here are electrically neutral, therefore the given model is not realistic. In the realistic model based on the algebra $h_{16}^{(s)}$, the isotopic symmetry is represented by the group $U(2)$. The Lorentz symmetry of particles in both models is identical. It is connected with the symmetry of the manifold of coordinates $X$ representing the affine space $\mathbb{A}_{3,1}$. Thus, in the present theory, a space-time continuum in the form of $\mathbb{A}_{3,1}$ is formed after the appearance of particle fields $\psi(X,Y)$ and switching to interactions. If before first a space was mathematically defined, wherein thereafter functions were considered, then now in the present theory this is reversed: we come to the idea of a space after (to start from consideration of transition matrix elements $\langle \hat{f}(\hat{x}), f(x) \rangle$) a certain set of functions $\psi(X,Y)$ has been constructed where $X = \frac{1}{2}(x + \dot{x})$, $Y = \frac{1}{2}(x - \dot{x})$ (for the explanations of these formulae see the main text of the article).

Results for the mass spectrum formulae and the a priori normalization of the wave function of a particle are given for both the realistic and the nonrealistic model.

3) Particles arising from the quantum transition $f \to \hat{f}$ are called fundamental. When considering a continuous degeneration of states $\hat{f}$, which is described in the model $h_{16}^{(s)}$ by the degeneration group $U_i(2) \otimes U_\ell(1) \otimes \hat{T}_{3,1}$, this results in couplings between fields of fundamental particles — interactions are realized by fields corresponding to parameters of the degeneration group (in the way of the Yang-Mills theory [7]). In accordance with three kinds of degeneration there are three kinds of interactions and three kinds of quanta of degeneration fields: strong ($U_i(2)$, first of all these are $\eta$ and $\pi$-mesons), electromagnetic ($U_\ell(1)$, photons) and gravitational ($\hat{T}_{3,1}$, gravitons) ones. In the present theory the weak interactions are by nature distinguished from the above mentioned ones, and, being given by correlations between various fibers of the fiber bundle, of a completely different non-gauge kind [7]. Other kinds of particles (composite ones) are connected with the latter interaction. The association of $U_i(2)$-multiplets
of fundamental particles with composite particles results in higher symmetries (without quarks): — $SU(3), SU(4)$ (etc.)-multiplets.

We would find the wrong (Planck) constant of electromagnetic interactions, if we would determine the Heisenberg algebra $h_{16}^{(*)}$ by the commutation relations

$$[\phi_{\alpha k}, \bar{\phi}_{\beta m}] = \delta_{\alpha\beta} \delta_{km}, \quad [\phi_{\alpha k}, \phi_{\beta m}] = [\bar{\phi}_{\alpha k}, \bar{\phi}_{\beta m}] = 0 \quad (1)$$

More general relations are written in the form

$$[\phi_{\alpha k}, \bar{\phi}_{\beta m}] = \Lambda \delta_{\alpha\beta} \delta_{km} \quad (2)$$

(others are zero, as well as in (1)) where $\Lambda$ is a dimensionless constant (analogous to the Planck constant) which can impossibly be any number, and is connected with the Lie algebra dimension of dynamical variables of our system, equal to 136, by the formula $\Lambda = \sqrt{136}$. In the theory the "bare" electromagnetic charge of a particle is equal to $e = \sqrt{\frac{hc}{136}}$. Radiative corrections to it are considered in [8].

In the brief exposition given below it is assumed that readers will spare no efforts to reconstruct the omitted calculations, filling in blanks in the reasoning. For ourselves we have performed the following task: to state the main results of the theory as coherently as possible in the given volume, and to show that there is a class of indisputable mathematical trues deserving attention. Probably the appendices will help to fill in some blanks.

2 States of relativistic bi-Hamiltonian system $f$ and $\dot{f}$ (Non-unitary quantum theory I)

All symmetry and dynamical properties of particles (including the symmetry properties of space-time) are determined completely by the defined properties of our dynamical system, in particular, by the properties of its states $f(x)$ and $\dot{f}(\dot{x})$ satisfying equations (see [1]):

$$-i \frac{\partial}{\partial x_\mu} f(x) = p_\mu f(x), \quad -i \frac{\partial}{\partial \dot{x}_\mu} \dot{f}(\dot{x}) = \dot{p}_\mu \dot{f}(\dot{x}). \quad (3)$$

Here $x_\mu$ and $\dot{x}_\mu$ are coordinates of the translation group $T_{3,1}$ and $\dot{T}_{3,1}$. The generators of that group are the operators $p_\mu$ and $\dot{p}_\mu$, written in the extended Fock representation in the form (considering the model $h_{8}^{(*)}$, see [4]):

$$P_\mu = kh \bar{\varphi} \sigma_\mu \varphi, \quad \dot{p}_\mu = -kh \frac{\partial}{\partial \varphi} \sigma_\mu \frac{\partial}{\partial \bar{\varphi}} \quad (4)$$
(now the variables $\zeta_\alpha$, $\tilde{z}_\alpha$, $\alpha = 1, 2$, used in [4], are denoted by $\varphi_\alpha$, $\tilde{\varphi}_\alpha$, and additional variables by $\varphi = \tilde{\varphi}_2$, $\tilde{\varphi} = \tilde{\varphi}_2$). $f(x)$ and $\dot{f}(x)$ represent fields not defined on space-time (hence, these are not Lagrangian fields), but on the groups $T_{3,1}$ and $\tilde{T}_{3,1}$ respectively, which act in a fiber ($x_\mu$, $\dot{x}_\mu$ are coordinates in fiber, see [1]). Since 4-vectors $p_\mu$ and $\dot{p}_\mu$ are isotropic: $p_\mu^2 = \dot{p}_\mu^2 = 0$, the fields $f(x)$, $\dot{f}(x)$ describe objects with zero mass. We will call them simply quanta $f$ and $\dot{f}$. The solutions of equations (3) are written in the form $f(x) = e^{i\pi x} f_0$, $\dot{f}(x) = e^{i\pi x} \dot{f}_0$.

1) The states $\dot{f}_0$ are defined as solutions of stationary equations

$$\dot{p}_\mu \dot{f}_0 = \rho_\mu^{(0)} \dot{f}_0$$

and belong (as states of the physical vacuum [4]) to the space of additional variables $F_0$. Such solutions are written in the form

$$\dot{f}_0 = \dot{f}_z = C e^{i\varphi - \tilde{\varphi} z} = C e^{2i\Im z \tilde{\varphi}}$$

where $C$ is a normalization constant and $z$ is a numerical (complex) parameter by means of which eigenvalues $\rho_\mu^0$ in (5) are written in the form $\rho_\mu^0 = \bar{z} \sigma_\mu$ $z = |z|^2 (0, 0, -1, -i)$ where $z = \left( \frac{0}{z} \right)$ (therefore we chose to use the unit system $c = h = k = 1$). It follows from this that $\rho_\mu^{02} = 0$, and $\rho_0^{0} \leq 0$ (the latter condition is a consequence of the boundary condition put on $\dot{f}_z$, because of the limitation $\dot{f}_z$ for $|\varphi| \to \infty$), so that a 4-momentum of quanta $f$ belongs to the lower half of the light cone. Since $\varphi$, $\tilde{\varphi}$ are Lorentzian scalars, $\dot{f}_z$ is a relativistic invariant value. Such states show hardly any degeneration of energy-momentum $\rho_\mu$. In fact, performing the transformation $T(v)$ on equation (5), where $v \in SL(2, \mathbb{C})$, we come to the equations $\dot{p}_\mu \dot{f}_0 = \rho_\mu \dot{f}_0$ in which $\rho_\mu = (L^{-1}(v))_{\mu \nu} \rho_\nu^{(0)}$ can accept arbitrary values from the lower half of light cone: $\rho \in N_\omega$. In this case the vertex of the cone $\rho = 0$ (with the parameter $z = 0$) forms an invariant subset in respect to $L(v)$, so that the set of all values of the 4-momentum $\rho_\mu$ decomposes in two invariant subsets: open $N_\omega$ and closed $\{ 0 \}$ ones.

The solution (6) can be written in the form

$$\dot{f}_z = C e^{i\sqrt{-\mathcal{P}^2} \sin \nu}$$

where $-\mathcal{P}^2 = 2\pi_\mu \rho_\mu$, and $\nu = \arg \varphi - \arg z$. Really, since by definition $\pi_\mu = \tilde{\varphi} \tilde{\sigma}_\mu \varphi$, $\pi_\mu \rho_\mu = \tilde{\varphi} \tilde{\sigma}_\mu \varphi \bar{z} \tilde{\sigma}_\mu z = 2|\bar{z} \varphi|^2$ where $\varphi = \varphi_2$.

As a Lorentz-invariant measure for $N_\omega$ (someway normalized), we have

$$d\mu_f = \frac{2}{\pi} \theta(-\rho_0) \delta(\rho^2) d^4 \rho,$$
and on \{0\} the measure is

\[ d\mu_0 = \delta^4(\rho) \, d^4\rho. \tag{9} \]

d\mu_f and d\mu_0 are called the measures of final states \dot{f}.

2) Next, since \( \pi_\mu = \bar{\varphi}^\mu \varphi, \pi_\mu^2 = 0 \), and \( \pi_0 = \bar{\varphi}_\alpha \varphi_\alpha > 0 \) so that \( \pi \in N_+ \) is the upper half of the light cone. By means of variables \( \pi_\mu \), a Lorentz-invariant measure for the Lagrangian plane \( \Pi_{\alpha=1,2} \frac{d}{4\pi} d\varphi_\alpha \wedge d\bar{\varphi}_\alpha \) can be written in the form \( \theta(\pi_0) \delta(\pi^2) \, d^4\pi \, d\nu \). The measure

\[ d\mu_f = \frac{1}{(2\pi)^{3/2}} \theta(\pi_0) \delta(\pi^2) \, d^4\pi \, \frac{d\nu}{2\pi} \tag{10} \]

centered on the upper half of the light cone \( N_+ \), normalized in a reasonable way, will be called the measure of initial states \( f \). We now notice that states \( f_0 \) are not in the least defined by equations (3).

In defining \( f_0 \), the operator \( \hat{M}^2 = 2\hat{p}_\mu \hat{p}_\mu \) (operator of square mass) and its stationary group \( G^\wedge = GL_\ell(2, \mathbb{C}) \otimes U_i(1) \otimes H_i(1) \) play an important role. Here \( GL_\ell(2, \mathbb{C}) = SL_\ell(2, \mathbb{C}) \otimes U_\ell(1) \otimes H_\ell(1) \). In respect to the separation of phase transformations \( U(1) \) and \( H(1) \) on \( U_\ell(1), H_\ell(1) \) and \( U_i(1), H_i(1) \) see [3].

The states \( f_0 \) are defined as eigenfunctions of the operator \( \hat{M}^2 \):

\[ \hat{M}^2 f_0^\Sigma = F^0_\Sigma f_0^\Sigma, \tag{11} \]

forming a finite-dimensional multiplet \( \Sigma \) of the group \( G^\wedge_{\hat{M}^2} \) (in this case the group \( G^\wedge_{\hat{M}^2} \) plays the role of degeneration group for the “Hamiltonian” \( \hat{M}^2 \)). Here \( \Sigma \) is a set of quantum numbers \( \Sigma = (s,F,D;Y,N) \) which is defined as a finite-dimensional irreducible representation of the group \( G^\wedge_{\hat{M}^2} \) (\( s \) is the Dirac spin connected with the group \( SL_\ell(2, \mathbb{C}) \), \( F \) and \( D \) are the fermion charge and dilatation connected with \( U_\ell(1) \) and \( H_\ell(1) \), and \( Y \) and \( N \) are the hypercharge and isotonic quantum number connected with \( U_i(1) \) and \( H_i(1) \)). Such states, obviously, are written in the form \( f_0^\Sigma = O^\Sigma(\varphi_\alpha, \bar{\varphi}_\beta; \varphi, \bar{\varphi}) \) of an homogeneous polynomial of degree \( N \) in the variables \( \varphi_\alpha, \bar{\varphi}_\beta, \varphi, \bar{\varphi} \) and one of degree \( D \) in the variables \( \varphi_\alpha, \bar{\varphi}_\beta \). In this case the number \( Y \) is the difference between the number of variables \( \varphi_\alpha, \varphi \) and \( \bar{\varphi}_\beta, \bar{\varphi} \). The number \( F \) is the difference between the number of variables \( \varphi_\alpha \) and \( \bar{\varphi}_\beta \), that have equal values for the whole multiplet. The number \( S = Y - F \) is called strangeness (in this perspective the additional variables \( \varphi, \bar{\varphi} \) can possibly also be named quanta of strangeness; the name is justified by an adherence of a variable \( \bar{\varphi} \) to a normal spinor \( \varphi_\alpha \), and subsequently comparing the strange spinor \( \varphi_\alpha \bar{\varphi} \), with spurions in Heisenberg’s
theory [3]). For states $O^\Sigma(\varphi)$ eigenvalues $F^0_\Sigma$ of the operator $M^2$ are equal (see Appendix 1) to

$$F^0_\Sigma = - (N^2 - Y^2 + 8N + 16) = - [(N + 4)^2 - Y^2].$$  \hspace{1cm} (12)

Polynomials $O^\Sigma(\varphi)$ will be called skeletons of particles.

From the solutions of $f^\Sigma_0(x) = e^{i\pi x} O^\Sigma(\varphi)$ it is possible to construct coherent states — fields

$$O^\Sigma(x) = \int f^\Sigma_0(x) d\mu_f = \frac{1}{(2\pi)^{3/2}} \int e^{i\pi x} \theta(\pi_0) \delta(\pi^2) d^4\pi O^\Sigma(\pi)$$ \hspace{1cm} (13)

where

$$O^\Sigma(\pi) = \frac{1}{2\pi} \int d\nu O^\Sigma(\varphi).$$ \hspace{1cm} (14)

These (massless) fields exist on the group $T_{3,1}$, and should not be mixed up with Lagrangian fields given on space-time.

$f^\Sigma_0(x)$ or $O^\Sigma(x)$ are states of an isolated quantum $f^\Sigma$. In case a quantum $f$ forms an ensemble with quanta $f$ (in other fibers) it is necessary to take into account statistical properties of the ensemble. Usual statistical reasonings (see, for example, [9]) result in consideration of the distribution function of quanta $f$ with energy $\pi_0 = \bar{\varphi}_\alpha \varphi_\alpha$ (in the frame of reference connected with the space $\mathcal{F}_0$, it will be $\bar{\varphi}\varphi$), that, representing the Gibbs distribution function, is written in the form

$$w_f = \exp \left( - \frac{\bar{\varphi}\varphi}{T_f} \right)$$ \hspace{1cm} (15)

where $T_f$ is the temperature of the ensemble of quanta $f$. By reasons outlined in [1], it follows that quanta $f$ (which henceforth will be identified with Feynman’s partons) arise from a (generally speaking, reversible) phase transition “particles $\Leftrightarrow$ quanta $f$”, that occurs at superhigh densities of particles (for example, at the Universe collapse or in collisions of particles with superhigh energies), being in thermal balance with particles (at the temperature $T_f$). In this sense they are similar to photons (being in thermal balance with the walls of a black body) and are characterized by minimum of free energy $F_f$, i.e. $(\partial F_f/\partial N_f)_{T_f,V} = 0$ where $N_f$ is the number of quanta $f$ in the ensemble which takes a volume $V$. This condition, as is known [9], entails the vanishing chemical potential of quanta $f$ so that in the case of quanta $f$ the free energy $F$ coincides with the thermodynamical potential $\Omega$.

Clearly, the ensemble of quanta $f$ can be also considered as a gas. Since in each fiber there is not more than one copy of our dynamical system, and since the number of fibers is denumerable, in contrast to the nonenumerably many
points that form the desintegrated space (so that, at the phase transition “continuum → discontinuum” the majority of points are empty, hence, occupation averages are $n_f \ll 1$), the Gibbs distribution coincides in effect with the Boltzmann distribution (and in the strongly degenerate limit, which is never reached for reasons indicated below, it coincides with the Fermi or Bose distributions depending on statistics of the skeleton $O^\Sigma(\varphi)$).

With regard to the statistical properties a field of quantum $f$ in the ensemble is written in the form $f^\Sigma(x) = w_f f_0^\Sigma$.

At the moment of the quantum transition $f \to \dot{f}$, which will be considered below, the function $w_f$ will be written in the relativistic invariant form, i.e. in the Juttner distribution function form (see [6])

$$w_f = \exp \left(-\frac{\pi \rho}{2\mu^2}\right) = \exp \left(-\frac{\mathcal{P}^2}{4\mu^2}\right)$$  \hspace{1cm} (16)

where by definition (which we already mentioned) $-\mathcal{P}^2 = 2\pi \rho$, and the parameter $\mu^2 = 3T_f T_j$ where $T_j = \frac{1}{3}|z|^2$ denoted the temperature of quanta $\dot{f}$. Really, since $\pi \rho$ is relativistic invariant, and since in the special frame of reference connected with the space $\mathcal{F}_0$ in which $\rho_\mu = \rho_\mu^0$ and $\varphi_\alpha = (0)^\phi$, we have $\pi \rho = 2|z|^2 \varphi \varphi$. For $\varphi \varphi$ we obtain $\varphi \varphi = \frac{\pi \rho}{2|z|}$. Thus, in (16) a 4-momentum of quanta $\dot{f}$ $\rho_\mu$ takes the part of 4-velocity. It should also be noticed that $w_f$ is a homogeneous function on the group $T_{3,1}$, and therefore it depends neither on any angular moment $\pi_\mu x_\nu - \pi_\nu x_\mu$ nor on $x_\mu$.

As a result it is possible to introduce the fields of quanta $f$ in the ensemble so

$$f^\Sigma = w_f f_0^\Sigma = e^{-\frac{\mathcal{P}^2}{4\mu^2}} O^\Sigma(\varphi).$$  \hspace{1cm} (17)

Similarly, statistical properties of the quantum ensemble $\dot{f}$, being described by the distribution function

$$w_{\dot{f}} = \exp \left(-\frac{\rho_0 + \mu_{\dot{f}}}{T_j}\right) \simeq \exp \left(-\frac{\mu_{\dot{f}}}{T_j}\right) = \frac{1}{Z}$$  \hspace{1cm} (18)

in which $\mu_{\dot{f}}$ is a chemical potential of quanta $\dot{f}$ (non-zero, since the quanta $\dot{f}$ are accumulated as a result of the transition $f \to \dot{f}$, see below), are taken into account, and $T_j$ are their temperatures as defined above. In (18) we consider $\rho_0 \ll \mu_{\dot{f}}$. As a result a field of quanta $\dot{f}$ in the ensemble is written in the form

$$\dot{f}_z = \frac{C}{Z} e^{i \sqrt{-\mathcal{P}^2} \sin \nu}.$$  \hspace{1cm} (19)
3 Amplitudes of the transition $f \rightarrow \dot{f}$ (Non-unitary quantum theory II)

The amplitude of transition is understood to be a transition matrix element averaging

$$
\langle \dot{f}(\dot{x}), f^\Sigma(x) \rangle = \int \overline{\dot{f}(\dot{x})} f^\Sigma(x) \, d\mu_f
$$

with measure of final states $d\mu_f$ or $d\mu_0$, i.e. the value

$$
O^\Sigma(X;Y) = \int d\mu_{f,0} \langle \dot{f}(X-Y), f^\Sigma(X+Y) \rangle = \langle \langle \dot{f}(X-Y), f^\Sigma(X+Y) \rangle \rangle,
$$

where we denoted $X = \frac{1}{2}(x + \dot{x})$, $Y = \frac{1}{2}(x - \dot{x})$. Thus, there are two similar but topological different transitions: the transition from the upper half of the light cone (the measure $d\mu_f$) to the lower (the measure $d\mu_f$), that will be named hadronic, and the transition from the upper half to the vertex of the cone (the measure $d\mu_0$), that will be named leptonic. By the first transition heavy particles are created, by the second low-mass particles.

It is plain to see that the amplitudes of lepton transitions differ only from the coherent fields (13) by the factor $1/Z$, and are only non-zero if skeletons $O^\Sigma(\varphi) = P(\pi)$ or $O^\Sigma(\varphi) = \varphi_\alpha \bar{\varphi} P(\pi)$ where $P(\pi)$ are polynomials about $\pi_\mu = \bar{\varphi} \frac{i}{2} \bar{\sigma}_\mu \varphi$. Amplitudes appropriated for such skeletons are written in the form

$$
\psi^\Sigma(x) = P \left( -i \frac{\partial}{\partial x} \right) X^0(x), \quad \psi^\Sigma(x) = P \left( -i \frac{\partial}{\partial x} \right) \nu(x),
$$

where

$$
X^0(x) = \frac{1}{(2\pi)^{3/2}} \frac{1}{Z} \int e^{i\pi x \theta(\pi_0)} \delta(\pi^2) \, d^4 \pi,
$$

$$
\nu_\alpha(x) = \frac{1}{(2\pi)^{3/2}} \frac{1}{Z} \int e^{i\pi x \theta(\pi_0)} \delta(\pi^2) \nu_\alpha(\pi) \, d^4 \pi,
$$

and $\nu_\alpha(\pi) = \frac{1}{2}(\pi_1 - i\pi_2) \pi = \bar{\pi} a$. Here $\bar{\pi} = \bar{\sigma}_\mu \pi_\mu = \pi_0 + \bar{\sigma}\pi$, and $a = \frac{1}{2}(\rho^{(1)})$ is a constant spinor. (22) are special bilocal fields representing gradient waves from fields $X^0(x), \nu_\alpha(x)$. From this it follows that only two kinds of particles arise from leptonic transitions, in the case of the model $h_8^{(s)}$ this is a massless boson $X^0$ with spin 0 and a massless fermion $\nu(x)$ with spin 1/2. The fields of particles satisfy equations: $\Box X^0 = 0$, $\frac{i}{2} \frac{\partial}{\partial x_\mu} \nu = 0$. The bilocal fields (23) result from a translation on the 4-vector $Y_\mu$ of a local field $\psi^\Sigma(X)$ (and herein they are specific): $\psi^\Sigma(X + Y) = e^Y \frac{\partial}{\partial x} \psi^\Sigma(X)$.
Now we discuss hadron transitions. If to denote \( \mathcal{P} = \pi - \rho, \mathcal{Q} = \pi + \rho \), amplitudes of hadron transitions can be represented in the form

\[
O^\Sigma(X; Y) = \frac{C}{(2\pi)^{3/2}} \int e^{i\mathcal{P}X + i\mathcal{Q}Y} \theta(\mathcal{P}_0 + \mathcal{Q}_0) \theta(\mathcal{P}_0 - \mathcal{Q}_0) \times \\
\times \delta(\mathcal{Q}^2 + \mathcal{P}^2) \delta(\mathcal{P} \mathcal{Q}) O^\Sigma(\mathcal{P}; \mathcal{Q}) d^4\mathcal{P} d^4\mathcal{Q} \frac{1}{2\pi},
\]

where

\[
O^\Sigma(\mathcal{P}; \mathcal{Q}) = \frac{1}{Z} \int e^{-i\sqrt{-\mathcal{P}^2} \sin \nu - \frac{-\mathcal{P}^2}{4\mu^2}} O^\Sigma(\varphi) \frac{d\nu}{2\pi}
\]

(24)

(25)

(26)

where \( \overset{\wedge}{\mathcal{M}}^2 = 2\hat{p}_\mu p_\mu \) is the operator introduced into consideration above. Denote \( \overset{\wedge}{\mathcal{M}}^2, e^{i\rho x}e^{-i\rho x} = \overset{\wedge}{\mathcal{S}}(x) \). By definition we have \( \overset{\wedge}{\mathcal{S}}(0) = 0 \). This condition allows specifically to isolate a homogeneous part in the transition amplitude equation.

We denote

\[
\overset{\wedge}{\mathcal{S}}^\Sigma(X; Y) = \langle \langle \hat{f}(X - Y), \overset{\wedge}{\mathcal{S}}(X + Y) f^\Sigma(X + Y) \rangle \rangle.
\]

(27)

It is not difficult to make sure that

\[
\overset{\wedge}{\mathcal{M}}^2 f^\Sigma = f^\Sigma(-\mathcal{P}^2)
\]

(28)

(29)

and \( F^0_\Sigma \) is defined by formula (12). In accepted notations the equation (26) is written in the form of the inhomogeneous equation

\[
(\Box_X - F_\Sigma(\Box_X)) O^\Sigma(X; Y) = \mathcal{S}^\Sigma(X; Y).
\]

(30)

The latter can be written in the form of the Bopp equation (the left-hand part in (30) is a polynomial of second degree regarding \( \Box_X \))

\[
\mathcal{D}(\Box_X)O^\Sigma(X; Y) = \left(\Box_X - M^2_{\Sigma B}\right) \times \\
\times \left(\Box_X - M^2_{\Sigma M}\right) O^\Sigma(X; Y) = 4\mu^4 \mathcal{S}^\Sigma(X; Y)
\]

(31)
in which \( \Im^\Sigma(X; Y) \) (27) takes the part of a source for transition amplitudes \( O^\Sigma(X; Y) \). In (31)

\[
M_{\Sigma B}^2 = 2\mu^2 \left\{ N + 5 - \mu^2 + \sqrt{\mu^4 - 2\mu^2(N + 5) + Y^2 + 2N + 9} \right\},
\]

and

\[
M_{\Sigma M}^2 = 2\mu^2 \left\{ N + 5 - \mu^2 - \sqrt{\mu^4 - 2\mu^2(N + 5) + Y^2 + 2N + 9} \right\}.
\]

### 4 Fields of particles

As usual, a solution of the inhomogeneous equation (31) can be written in the form of a sum \( O^\Sigma = \psi^\Sigma + G^\Sigma \) of the partial solution \( G^\Sigma \) of the inhomogeneous equation (31) and the general solution \( \psi^\Sigma \) of the homogeneous equation

\[
\left( \Box_X - M_{\Sigma B}^2 \right) \left( \Box_X - M_{\Sigma M}^2 \right) \psi^\Sigma(X; Y) = 0.
\]

Solutions of equation (34) have physical sense, since they allow

\[
\psi^\Sigma(X; Y) = \frac{1}{(2\pi)^{3/2}} \int e^{iP_X} \psi^\Sigma(P; Y) \delta(P^2 + M_{\Sigma B}^2) d^4P.
\]

In the asymptotics \(|X| \gg |Y|\) the solutions (35) pass to local fields observable in \( \mathbb{A}_{3,1} \)

\[
\psi^\Sigma(X; 0) = \psi^\Sigma(X) = \frac{1}{(2\pi)^{3/2}} \int e^{iP_X} \theta(P_0) \psi^\Sigma(P) \delta(P^2 + M_{\Sigma}^2) d^4P
\]

where \( \theta(P_0)\psi^\Sigma(P) = \psi^\Sigma(P; 0) \), which are characterized by the certain dispersion law \( P^2 = -M_{\Sigma}^2 \). In quantum field theory such fields are interpreted as point particles. As for \( \psi^\Sigma(X; Y) \), the more general fields are bilocal. Such fields are interpreted as non-point (smeared) objects, their interior is described by coordinates \( Y_\mu \) (comparing \( X_\mu \) it can be said that these are usual coordinates of space-time). It is important to emphasize that a particle’s interior is not at all part of our space-time \( \mathbb{A}_{3,1} \), it owes a special space. Note also that the signature and connectivity on manifold \( \mathbb{A}_{3,1}(\exists X_\mu) \) is completely defined by dynamical symmetry of our system, i.e. by the symmetry of the algebra \( h^{(s)}_8 \).

It is important to emphasize that equations (34) are no evolutionary equations for fields \( \psi^\Sigma \) (and neither are the Klein-Gordon equations), they are only dispersion equations (see below).

As distinguished from \( \psi^\Sigma \) the solutions \( G^\Sigma \) have not any immediate physical interpretation as they are not known in the standard quantum field theory. In
this perspective we require the non-physical solutions $G^\Sigma$ to be orthogonal to the physical fields $\psi^\Sigma$ in the sense of the Stueckelberg scalar product

$$\int \psi^\Sigma(X;Y) G^\Sigma(X;Y') d^4X = 0.$$  \hspace{1cm} (37)

This condition permits to express the function $\psi^\Sigma(\mathcal{P};Y)$ in terms of the general solution (35) (in the case of hadrons). As $G^\Sigma = O^\Sigma - \psi^\Sigma$, we have

$$\int \psi^\Sigma(X;Y) \overline{\psi^\Sigma(X;Y')} d^4X = \int \psi^\Sigma(X;Y) O^\Sigma(X;Y') d^4X.$$  

Applying the Fourier transformation on $\psi^\Sigma (35)$ and on $O^\Sigma (24)$ and integrating over $d^4X$, we come to a relation that implies that the equality

$$\psi^\Sigma(\mathcal{P};Y) = \int e^{iQY} \theta(\mathcal{P}_0 + Q_0) \theta(\mathcal{P}_0 - Q_0) \times$$

$$\times \delta(\mathcal{P}^2 + Q_0^2) \delta(\mathcal{P}Q) O^\Sigma(\mathcal{P};Q) \frac{d^4Q}{2\pi}$$  \hspace{1cm} (38)

must be valid, and for that purpose the normalized constant $C$ in (19) or in (24) must be equal to

$$C = \delta(0).$$  \hspace{1cm} (39)

As can be seen, the value of $C$ is infinite, and consequently all values containing it as $\hat{f}^z(x), O^\Sigma(X;Y), \Im^\Sigma(X;Y), G^\Sigma(X;Y)$ are infinite, and hence there is no direct physical sense. This result is very important for a consistent physical interpretation of the theory, particularly because it permits to answer the question whether final states (in case of hadron transitions) of our system $\dot{f}$ are observable. Regarding these non-physical states the answer is that they are not observable (after the transition $f \rightarrow \hat{f}$ and creation of particle fields $\psi^\Sigma$ the quanta $\hat{f}$ remain in the orthogonal adjunct to the field $\psi^\Sigma$). Particle fields $\psi^\Sigma(X;Y)$ and fields $f^\Sigma(x)$ of quanta $\hat{f}$ (partons) are only finite (that consequently have physical sense).

As it is seen, the present theory is broader than quantum field theory of particles, containing non-observable (in terms of measure of our space-time) values on principles such as $\hat{f}^z(x), O^\Sigma(X;Y), \Im^\Sigma(X;Y), G^\Sigma(X;Y)$ going beyond the class of generalized functions (see [1]).

Next, focussing the attention on the structure of bilocal fields $\psi^\Sigma(X;Y)$, it is possible to see that it admits a representation in the form of a smeared local field $\psi^\Sigma(X)$, i.e.

$$\psi^\Sigma(X;Y) = F_\Sigma \left( Y; -i \frac{\partial}{\partial X} \right) \psi^\Sigma(X)$$  \hspace{1cm} (40)

where $F_\Sigma \left( Y; -i \frac{\partial}{\partial X} \right)$ is a smearing (differential) operator. This operator depends on the kind of a particle $\Sigma$, in particular, on spin of a particle $s$ as well as on a
spin projection (as $O^\Sigma(\mathcal{P}; \mathcal{Q})$ depends on $Q$). However in certain calculations it satisfies to use an approximate expression for $F_\Sigma$, approximately equal for particles of any kind $\Sigma$, namely

$$F(Y; \mathcal{P}) = \frac{1}{2\pi} \int e^{i\mathcal{Q}Y} \delta(Q^2 + P^2) \delta(\mathcal{PQ}) d^4Q$$

(therefore being valid).

Since for a freefalling particle $\mathcal{P}_\mu$ is a time-like 4-vector, $Q_\mu$, as it follows from (41), is a space-like 4-vector, orthogonal to $\mathcal{P}_\mu$. Hence, “waves” described by coordinates $Y_\mu$ (internal waves in particles, waves in the second space) are spread with a velocity greater than the velocity of light $c$. Thus, such a non-point particle is an original alloy of a usual particle (variables $\mathcal{P}_\mu$) and a tachion (variables $Q_\mu$).

Observe a number of features of the suggested mechanism for creating particle fields. First of all it may be noted that $c$-numerical fields, resulting from the transition in the relativistic bi-Hamiltonian system from state $f$ to state $\dot{f}$, are $L$-spiral fields, since skeletons of particles $O^\Sigma(\varphi)$ are constructed from the only $L$-spinors $\varphi_\alpha$ and their complex conjugates $\bar{\varphi}_\alpha$ (among canonical variables of the system are no $R$-spinors!). Thus $P$ and $C$ symmetries in the given theory are broken right from the start. However, as will be shown in the future, for massive particles, as a result of evolution of $L$-fields (which will be called fundamental), there will be other spiral fields in space-time, in particular $R$-fields according to the first-order differential equations that satisfy particle fields

$$\left(\Gamma^\Sigma_\mu \frac{\partial}{\partial X_\mu} + M_\Sigma \right) \psi^\Sigma(X) = 0.$$ 

These are not postulated here, but are obtained as a simple consequence of symmetry properties of space-time, in particular, of properties evolving from Lorentz transformations of fields $\psi^\Sigma(X; Y)$ (compare with [10] where similar arguments yield Dirac equations). Next, due to theoretical assumption of one hundred per cent fermion-antifermion asymmetry (the values $\varphi_\alpha$ can only be free, all complex conjugate ones $\bar{\varphi}_\alpha$ and $\bar{\varphi}$ are connected with $\varphi_\alpha$ or $\varphi$, see [1]) only the positive-frequent part (see formula (36) containing the $\theta(\mathcal{P}_0)$-function) of fermion fields appear. Negative-frequent parts of fields as well as antifermion fields will follow from switching to interactions, the subject that will be discussed in the future.

5 Problem of mass spectrum of particles

The mass spectrum of “bare” fundamental hadrons — baryons and mesons (particles arising from the quantum transition $f \rightarrow \dot{f}$) — is determined by
formulae (32) and (33). The mass spectrum formula can be written in the form

\[
M^2 = 2\mu^2 \left( \frac{kh}{c} \right)^2 \left\{ N + 5 - \mu^2 + \frac{N}{N + 5} - \frac{\mu^2}{N + 5} + \left[ \frac{N}{2} - \mu^2 \right]^2 \right\} + (-1)^{F+1} \sqrt{\mu^4 - 2\mu^2(N + 5) + Y^2 + 2N + 9}
\]

in which the baryon (fermion) branch has \( F = 1 \), and the meson (boson) branch has \( F = 0 \). Thus, in the present theory the topic regarding the mass spectrum of fundamental particles is connected with the problem of finding eigenfunctions of the operator \( \hat{M}^2 \) (the operator of square mass) for states \( \psi(x) \) of quanta \( f \). Moreover, the existence of two branches — baryons and mesons — is a direct consequence of the contribution of statistical properties to the ensemble of quanta \( f \), in the form of the function \( w_f \approx n_f \), which at the limit of strong degeneration, not being reached for above mentioned reason, is divided into two distribution functions: one for fermions \( n_f^F \) (\( F = 1 \)) and one for bosons \( n_f^B \) (\( F = 0 \)), described by the uniform formula

\[
n_f = \frac{1}{\exp \left( \frac{x\rho}{2\mu^2} \right) + (-1)^{F+1}}.
\]

For large \( \frac{x\rho}{2\mu^2} \) these formulae have the same Boltzmann limit described by formula (16). The appearance of the factor \((-1)^{F+1}\) in (42) is caused by the explicit form of function (43) and follows from the more exact dispersion law:

\[
D_{\Sigma}(X) = (1 + (-1)^F n_f) \left( 1 + 2(-1)^F n_f \right) X^2 + 4\mu^2 \left( \mu^2 - (N + 5)(1 + (-1)^F n_f) \right) X + 4\mu^4 \left( -Y^2 + (N + 4)^2 \right) = 0
\]

where \( X = 2\pi\rho, \) and \( n_f(X) \) is (43), corresponding to states \( f^\Sigma = n_f O^\Sigma(\varphi) \). Roughly speaking, for the same values of quantum numbers \( N, Y \) a greater root of equation (44) corresponds to fermions\(^2\) because of the fermion skeletons being pushed aside. The “forces” are that help to break off the continuum on

\(^2\)Comparing roots of equations (44) or (34) with fermion and boson masses the number \( \text{sgn} D_{\Sigma}'(X) (\text{here } X = \frac{M^2}{4\mu^2}) \) called a dispersion curve index \( D_{\Sigma}'(X) \) plays an important role. The value of the index in points of crossing the curve \( D_{\Sigma}(X) \) with the straight line \( 4\mu^2X \) is equal to \( \text{sgn} D_{\Sigma}' \left( \frac{M^2}{4\mu^2} \right) = 1 \) (normal dispersion) and \( \text{sgn} D_{\Sigma}' \left( \frac{M^2}{4\mu^2} \right) = -1 \) (anomalous dispersion), i.e. \( \text{sgn} D_{\Sigma}' \left( \frac{M^2}{4\mu^2} \right) = (-1)^F \). This theorem (describing the connection between roots of equation (30) and fermion charges) is a consequence of an average energy \( \bar{\pi}_0^{(F)} \) of the fermion skeleton \( O^\Sigma \) at any temperature \( T \) being greater than an average energy \( \bar{\pi}_0^{(B)} \) which falls into the boson skeleton. The ratio \( \bar{\pi}_0^{(F)} / \bar{\pi}_0^{(B)} \) is a monotone function \( T \), equal to \( \infty \) at \( T = 0 \) and \( \frac{9}{8} \) at \( T = \infty \) \[9\]. In our case \( T = T_f \ll 1 \) (see below).
separate points and transform it into a discontinuum. Forces, sticking together separate points of the discontinuum transforming it into a continuum, are an usual interactions (strong, electromagnetic, gravitational) between particles (see below). Formula (31) may be obtained when \( n_f^F \ll 1 \).

The appearance of leptons is connected with a transition of the relativistic bi-Hamiltonian system in the vertex of the cone. In this case a 4-momentum of quanta \( \dot{f} \) is \( \rho_\mu = 0 \) (for \( C = 1 \) and \( \dot{f} = 1/Z \)) is equivalent to \( \mu^2 = 0 \) in formula (42). Hence we obtain that the primeval masses of all leptons \( M_{\Sigma}^2 = 0 \) are a trivial result which the Twistor Program has met [11] (because of that it has not gone).

It is meaningful to consider another limit case of formula (42) namely when \( \mu^2 \to \infty \). As it follows from (42), it exists only for the fermion branch \((F = 1)\) and will be considered henceforth within the framework of the realistic model \( h_{16}^{(*)} \). So now we move over to the description of the latter model.

## 6 Model of the relativistic bi-Hamiltonian system \( h_{16}^{(*)} \)

So far we have considered the most general basic questions of particle field theory by a reconstruction in the model \( h_8^{(*)} \), in which isotopic symmetry is represented by the group \( U(1) \) (generator \( \phi_\phi \)), in which for that reason no charged particles exist.

The realistic isotopic symmetry described by the group \( U(2) \) (for the same Lorentz symmetry) is contained in the model of the relativistic bi-Hamiltonian system based on the Heisenberg algebra \( h_{16}^{(*)} \). The generators, being canonical variables of the system denoted by \( \phi_{\alpha k}, \bar{\phi}_{\beta m} \) (here \( \alpha, \beta = 1, 2, 3, 4 \) are Lorentz or Dirac indices, and \( k, m = 1, 2 \) are isotopic) obey more general commutation relations (2).

1) At first we briefly describe the structure of this system given by relations (1). Dynamical variables of the system represent every possible bilinear form of canonical variables: \( \phi_{\alpha k}\phi_{\beta m}, \phi_{\alpha k}\bar{\phi}_{\beta m}, \bar{\phi}_{\alpha k}\bar{\phi}_{\beta m} \). They form a Lie algebra of dynamical variables, denoted by \( d \), that is isomorphic to the Cartan algebra \( sp^{(*)}(8, \mathbb{C}) \) of dimension 136. Real variables \( \bar{\phi}_{\alpha k}\phi_{\beta m} \) which conveniently written down using of Dirac matrices \( \gamma_N \) and isotopic Pauli matrices \( \tau_k \) in the form \( \bar{\phi}\gamma_N\tau_k\phi \) or

\[
\begin{align*}
p_\mu &= i\bar{\phi}\gamma_\mu P_+\phi, \quad \dot{p}_\mu = -i\bar{\phi}\gamma_\mu P_-\phi, \quad I_{\mu\nu} = \bar{\phi}\Sigma_{\mu\nu}\phi, \\
A &= -\bar{\phi}_\phi - 4, \quad B = -\bar{\phi}\gamma_5\phi - 4, \\
\bar{\iota} &= \frac{1}{2}\bar{\phi}\tau_N\phi, \quad \bar{k} = \frac{1}{2}\bar{\phi}\gamma_5\tau_3\phi, \quad Q = \frac{1}{2}\bar{\phi}(1 + \tau_3)\phi
\end{align*}
\]

(45)
as well as  

\[ \vec{p}_{\mu}^{\pm} = \pm i \bar{\phi} \gamma_{\mu} P_{\pm} \phi \]

and  

\[ \vec{I}_{\mu} = \bar{\phi} \Sigma_{\mu\nu} \tau_{\phi} \]

play the most important role. The physical sense of these variables follows from the commutation relations which they satisfy and is analogous to corresponding variables in the model \( h_{8}^{(s)} \); in (45) \( Q \) is the operator of electric charge.

The extended Fock representation is given by operators

\[ \phi = \left( \frac{\partial}{\partial \varphi_{ak}}, \varphi_{ak} \right), \quad \bar{\phi} = \left( \bar{\varphi}_{ak}, \frac{\partial}{\partial \bar{\varphi}_{ak}} \right), \quad \alpha, k = 1, 2 \]  

(46)

and is constructed in the dual pair of topological vector spaces \( (\mathcal{F}, \mathcal{F}) \) where \( \mathcal{F} \) (and \( \mathcal{F} \)) has the already known structure \( \mathcal{F} = \mathcal{F}_{F} \otimes \mathcal{F}_{0} \). Here \( \mathcal{F}_{F} \) is a space of functions depending on variables \( \varphi_{ak}, \bar{\varphi}_{ak} \) (coordinates on the Lagrangian plane \( L \subset h_{16}^{(s)} \)), and \( \mathcal{F}_{0} \) is a space of functions depending on additional variables (see previous remark) \( \varphi_{k} = \varphi_{2k}, \bar{\varphi}_{2k} \) (Lorentzian scalars, i.e. scalars of the group \( GL_{\ell}(2, \mathbb{C}) \) are generators \( I_{\mu\nu}, A_{\ell}, B_{\ell} \) representing isospinors (spinors of the group \( GL_{i}(2, \mathbb{C}) \) are generators \( \vec{1}, \vec{k}, A_{i}, B_{i} \); concerning a separation of operators \( A \) and \( B \) see [4]). The states of the system satisfy equations (3). But if formerly from (3) it followed the equations  

\[ p_{\mu} \frac{\partial}{\partial x_{\mu}} f(x) = 0, \quad \bar{p}_{\mu} \frac{\partial}{\partial \bar{x}_{\mu}} \bar{f}(\bar{x}) = 0 \]

(as \( p_{\mu}^{2} = \bar{p}_{\mu}^{2} = 0 \)), then now (as \( p_{\mu} = \pi_{\mu}^{(1)} + \pi_{\mu}^{(2)} \) where \( \pi_{\mu}^{(k)} = \bar{\varphi}_{k} \sigma_{\mu} \varphi_{k} \) we have  

\[ p_{\mu}^{2} = 2 \pi_{\mu}^{(1)} \pi_{\mu}^{(2)} = -4 | \det \varphi_{ak}|^{2} = -\kappa \]

(48)

(49)

(47)

(48)

(49)

We cannot consider \( \det \varphi_{ak} = 0 \), since all \( \varphi_{ak} \) are independent so that in the model \( h_{16}^{(s)} \), as opposed to \( h_{8}^{(s)} \), there is no strong relation \( p_{\mu}^{2} = 0 \)). Using Dirac’s terminology and notations [12], we say that equations (47) define a weak coupling \( \det \varphi_{ak} \approx 0 \) that only acts on states \( f \) or on the measures of initial states \( d\mu_{f} \) (such states will be called quanta \( f \)).

The Lorentz-invariant measure \( d\mu \) of the Lagrangian plane \( L \) is of the form

\[ d\mu = \prod_{\alpha,k=1,2} \frac{i}{4\pi} d\varphi_{ak} \wedge d\bar{\varphi}_{ak} \]  

(48)

This measure can be expressed in terms of variables \( \pi_{\mu}^{(k)} \) by the following:

\[ d\mu = \frac{1}{16} \prod_{k=1,2} d\omega_{k} \delta(\pi^{(k)2}) \theta(\bar{\pi}^{(k)}) d^{4} \bar{\pi}^{(k)} = \]

\[ = \theta(\bar{\pi}^{(0)}) \theta(-\pi^{2}) \delta(\pi^{2} + \kappa) d^{4} \pi d^{4} \nu d\kappa \]  

(49)

(49)
where \( \omega_k = \arg \varphi_{2k} \), and

\[
d^4 \tilde{v} = \frac{1}{16} d\omega_1 d\omega_2 \delta(\Pi^2) \delta(\pi^2 - 2\pi\Pi) d^4\Pi \frac{\theta(\Pi_0)\theta(\pi_0 - \Pi_0)}{\theta(\pi_0)}
\] (50)

(here denote \( p_\mu = \pi_\mu, \pi^{(2)}_\mu = \Pi_\mu \)). According to (47) the measure \( d\mu_f \) represents a contraction of the measure \( d\mu \) on the “light” cone \( \kappa = 0 \) and is explicitly written (the weak coupling \( \det \varphi_{\alpha k} \approx 0 \))

\[
d\mu_f = \frac{1}{(2\pi)^{3/2}} \theta(\pi_0) \delta(\pi^2) d^4\pi d^4\nu
\] (51)

where \( d^4\nu = \left(\frac{2}{\pi}\right)^3 d^4 \tilde{v} \). The measure \( d\mu_f \) is normalized so that formula (13) is valid for coherent states, thus the measure \( d^4\nu \) is normalized by the condition \( f d^4\nu = 1 \) (see Appendix 2). In the model \( h^{(s)}_{16} \) by \( O^\Sigma(\pi) \) in (13) we understand the integral

\[
O^\Sigma(\pi) = \int d^4\nu O^\Sigma(\varphi)
\] (52)

where \( O^\Sigma(\varphi) \) are skeletons of particles in the model \( h^{(s)}_{16} \).

The differences between the coherent fields and their secondary quantized Lagrangian analogues are 1) not being quantized, 2) under compression (collaps) being desintegrated into separate Fourier-components — non-Lagrangian fields \( f^\Sigma(x) \), i.e. quanta \( f \) (or partons) representing solutions of equations (3). In [1] this process is referred to as the phase transition “particles \( \Leftrightarrow \) quanta \( f \)”. It is rather essential to remark that in the model \( h^{(s)}_{16} \) all coherent states are electroneutral, since integration (see (52)) of charged skeletons or skeletons with hypercharge \( Y \neq 0 \) such as \( \bar{\varphi}\bar{\tau}\varphi_\alpha \) or \( \varphi_\alpha \) gives zero. This conclusion is rather essential for cosmology: before the first Big Bang (total transition \( f \rightarrow \dot{f} \)) the Universe consisted of a mix of two neutral gases — bosons \( X^0 \) and fermions \( \nu \) (see (23)).

In the model \( h^{(s)}_{16} \) the solutions of equations for \( \dot{f} \) are written in a form similar to (6):

\[
\dot{f}_z = \frac{C}{Z} \exp(z_k\varphi_k - \bar{\varphi}_kz_k)
\] (53)

where \( z_k = (z_2, z_3) \) are complex parameters representing an isospinor. From (53) it follows that \( \rho^0_\mu = -\bar{z}z(0, 0, 1, i) \) so that there is a weak coupling \( \rho^2_\mu \approx 0 \) (in the model \( h^{(s)}_8 \) there was a strong coupling). Thus, as well as in the model \( h^{(s)}_8 \), \( \rho_\mu \) takes on values from the lower half of light cone \( N_- \) which has the invariant measure \( \frac{2}{\pi} \theta(-\rho_0) \delta(\rho^2) d^4\rho = d\mu_f \) (the measure of final states or quanta \( \dot{f} \)).

To consider amplitudes of hadron transitions as an integral over the product \( \dot{f}_z(x) f^\Sigma(x) \) with respect to the measure of initial and final states (see (20),
(21)), one can obtain inhomogeneous equations, similar to (30), for them and the mass spectrum formula for “bare” hadrons (analogue of formula (42), see Appendix 1)

\[
M_\Sigma^2 = 2\mu^2 \left( \frac{kh}{c} \right)^2 \left\{ N + 6 - \mu^2 + \right.
+(-1)^{F+1} \sqrt{\mu^4 - 2\mu^2(N + 6) + 4i(i + 1) + 2N + 4} \right. 
\]

(54)

where \( i \) and \( N \) are spin and the isotonic quantum number of a skeleton \( O_\Sigma(\varphi) \), and \( \mu^2 = 3T_fT_f \) where now \( T_f = \frac{1}{3}z_kz_k \).

Formula (54) results in a number of verified predictions. So according to this formula the isovector \( \Sigma \) -hyperon mass \( M_\Sigma \) is more than the isoscalar \( \Lambda \)-hyperon mass \( M_\Lambda \) (\( M_\Sigma > M_\Lambda \)), and the isovector \( \rho \)-meson mass \( M_\rho \) is less than the isoscalar \( \omega \)-meson mass (\( M_\rho < M_\omega \)) that is in overall agreement with experimental results (see Appendix 4) (for these particles \( N = 2 \)). Moreover, the ratio \( M_\Lambda M_\omega / M_\Sigma M_\rho \) depends neither on the parameter \( \mu^2 \) nor on fundamental constants \( c, h, k \), but is equal to the number \( \sqrt{7} \approx 1,08 \). The experimental value of this ratio is \( 1,16 \pm 0,76 \) (the inaccuracy is caused by the large width of \( \rho \)-mesons).

In the model \( h_{16}(s) \), as well as in the model \( h_{8}(s) \), the hadron fields are presented in the form of (35), (38) where

\[
O^\Sigma(\mathcal{P}, \mathcal{Q}) = \frac{1}{Z} \int d^4\nu O^\Sigma(\varphi) e^{-\frac{\nu^2}{4\mu^2} - 2i \Re m \tilde{z}_k \varphi_k}, \quad (55)
\]

and \( O^\Sigma(\varphi) \) are skeletons of particles in the given model. Calculations (see Appendix 2) give the following factorized expression for \( O^\Sigma(\mathcal{P}, \mathcal{Q}) \):

\[
O^\Sigma(\mathcal{P}, \mathcal{Q}) = \frac{1}{Z} N_\Sigma(X) O^{(i)}_\Sigma(z) O^{(f)}_\Sigma(\mathcal{P}, \mathcal{Q}), \quad X = \sqrt{-\mathcal{P}^2} \quad (56)
\]

where \( O^{(f)}_\Sigma \) and \( O^{(i)}_\Sigma \) are Lorentzian and isotopic wave-functions. In particular, for the baryon octet the skeletons are

\[
O^N(\varphi) = \varphi_{ak}, \quad O^\Lambda(\varphi) = \varphi_{ak} \tilde{\varphi}_k, \quad O^\Sigma(\varphi) = \tilde{\varphi}^\tau \varphi_\alpha, \\
O^\Xi(\varphi) = \frac{2}{\sqrt{-\mathcal{P}^2}} (\tilde{\varphi} \varphi_\alpha)(\varphi_k \overset{+}{\rho} \varphi_m) \tilde{\varphi}_m
\]

(57)

(The quantum number \( N \) for nucleons, \( \Lambda \)-, \( \Sigma \)- and \( \Xi \)-generators are equal to 1, 2, 2, 5 respectively), the isotopic factors are of the form

\[
O^{(i)}_N = z_k, \quad O^{(i)}_\Lambda = 1, \quad O^{(i)}_\Sigma = \frac{\tilde{z}^\tau z}{\tilde{z} \tilde{z}}, \quad O^{(i)}_\Xi = \frac{z_k}{\tilde{z} \tilde{z}}. \quad (58)
\]
Lorentzian factors for these particles (spin 1/2) are equal in value, namely \( O^{(\ell)}_\Sigma = \bar{\pi}a \) (\( a \) is a constant spinor, see above). For the factor \( N_\Sigma \) in these cases we have

\[
N_N = \left( \frac{2}{M_N} \right)^2 J_2(M_N) e^{-\frac{M_N^2}{4\mu^2}}, \quad N_\Lambda = \frac{2}{M_\Lambda} J_1(M_\Lambda) e^{-\frac{M_\Lambda^2}{4\mu^2}},
\]

\[
N_\Sigma = -\frac{2}{M_\Sigma} J_3(M_\Sigma) e^{-\frac{M_\Sigma^2}{4\mu^2}}, \quad N_\Xi = J_2(M_\Xi) e^{-\frac{M_\Xi^2}{4\mu^2}},
\]

where \( J_n \) is the Bessel function.

Similar expressions can be obtained for the vector octet of mesons as well as for other multiplets.

The factor \( O^{(i)}_\Sigma N_\Sigma(M_\Sigma) \) contains very important information about the mechanism of particle creation: it defines a priori the creation probability of a hadron \( \Sigma \) in the quantum transition \( f^\Sigma \to \dot{f}_z \) (in the non-unitary theory by definition the state \( f^\Sigma \) has a slope in relation to \( \dot{f}_z \), and the angle between these states defines the given probability). The mentioned probability \( W_\Sigma \) is defined as \( |O^{(i)}_\Sigma N_\Sigma|^2 \). And it is clear that the total probability as a sum of partial probabilities \( W_\Sigma \) must be equal to one

\[
\sum_\Sigma W_\Sigma = 1. \tag{60}
\]

In fact, condition (60) results from the normalization condition of skeletons \( O^\Sigma(\varphi) \). Due to the one hundred per cent baryon-antibaryon asymmetry (see [4]), the skeletons are constructed (in the Lorentzian system connected with the space \( F_0 \)) exclusively from additional variables \( \varphi_k \). To represent an irreducible finite-dimensional representation basis of the group \( G_{_{M^2}} = GL_\ell(2, \mathbb{C}) \otimes U_i(2) \otimes H_i(1) \) (see paragraph 2; on the space \( F_0 \) this group is \( U_i(2) \otimes H_i(1) \), since \( \varphi \) are scalars), they are written in the form [13]

\[
O^\Sigma(\varphi) = \frac{\varphi_1^m \varphi_2^{2i-m}}{\sqrt{m!(2i-m)!}} = f_m^{(i)} \tag{61}
\]

where \( i \) is isospin, and \( m \) is its projection so that

\[
\sum_{m=0}^{2i} |f_m^{(i)}|^2 = \frac{(|\varphi_1|^2 + |\varphi_2|^2)^{2i}}{(2i)!} = \frac{(\varphi \varphi)^{2i}}{(2i)!} = w_i.
\]

As can be seen, for a fixed \( i \) the value \( m \) satisfies the Bernoulli distribution. The isospin \( i \) satisfies the Poisson distribution since

\[
\sum_{i=0, \frac{1}{2}, 1, \ldots} w_i e^{-\bar{\varphi}_\varphi} = 1. \tag{62}
\]
As \( \varphi \) accepts small values (small oscillations, see below), it is possible to put
\[
\sum_i w_i = 1. \tag{63}
\]

One remark. If a bi-Hamiltonian fiber is imagined as a sea of coupled ad-
ditional variables \( \bar{\varphi} \varphi \), the configuration \( (\bar{\varphi} \varphi)^n \) arises in it under (62) with the probability given by the Poisson distribution
\[
P(n) = \frac{(\bar{\varphi} \varphi)^n}{n!} e^{-\bar{\varphi} \varphi}.
\]

It is natural to consider functions \( u_m^{(i)} = \exp \left( -\frac{\bar{\varphi} \varphi}{2} \right) f_m^{(i)} \) forming an orthonormal system of functions in the space \( \mathcal{F}_0 \) in relation with the scalar product
\[
(f, g) = \int_{\mathbb{C}^2} \bar{f}(\varphi) g(\varphi) d\mu(\varphi)
\]
with measure
\[
d\mu(\varphi) = \prod_{k=1,2} \frac{i}{4\pi} d\varphi_k \wedge d\bar{\varphi}_k
\]
in fact \( (u_m^{(i)}, u_{m'}^{(i')}) = \delta_{ii'} \delta_{mm'} \) for which the condition
\[
\sum_{i=0,1,2, \ldots} \sum_{m=0}^{2i} |u_m^{(i)}|^2 = \sum_i e^{-\bar{\varphi} \varphi} w_i = 1
\]
is satisfied.

Let us return to condition (63) which took place before the transition \( f \to \dot{f} \). After the transition \( f \to \dot{f} \) and the creation of fundamental hadron fields this normalization condition of skeletons passes to condition (60).

We now show that in (62) the exponent can be put equal to 1, so \( |\varphi| \) is a small value. Both the Bernoulli and Poisson distribution are realized in each fiber. Another distribution used by us — the Gibbs distribution \( \exp \left( \frac{-\bar{\varphi} \varphi}{T_f} \right) \) — describes the statistics of fibers. It follows from this that values \( \bar{\varphi} \varphi \sim T_f \) as well as \( |\varphi| \sim \sqrt{T_f} \). We now see that \( T_f \sim 10^{-6} \) so that the value \( |\varphi| \sim 10^{-3} \) is really small.

Dimensionless parameters of the theory, such as \( T_f, z_k \) (or the sum \( T_j = \frac{1}{2} (|z_1|^2 + |z_2|^2) \) and ratio \( \varepsilon = |z_1/z_2| \)) and \( 1/Z \), are necessary to perform calculations in the framework of the present theory. The factor \( 1/Z \) was calculated in [7]. Here parameters \( T_f, T_j \) and \( \varepsilon \) will be found.

At first we define the parameter \( \mu^2 = 3 T_f T_j \). It is used in formula (54) defining the mass spectrum of “bare” (non-interacting) hadrons and is found from conditions (generally named the minimum principle) put on factors of the quadratic form \( P_{\Sigma}(X) = (X - M^2_{\Sigma B})(X - M^2_{\Sigma M}) \). Denote by
$P_{\Sigma_0}(X)$ the form for which a baryon root $M_{\Sigma B}$ has the least value. This value corresponds to the point $i_0 = -\frac{1}{2}$ and $N_0 = -1$ and is equal to $M_{\Sigma_0 B}^2 = 2\mu^2 \{ 5 - \mu^2 + \sqrt{\mu^4 - 10\mu^2 + 1} \}$. The form $P_{\Sigma_0}(X)$ accepts the least value at $X = \frac{1}{2} \left( M_{\Sigma_0 B}^2 + M_{\Sigma_0 M}^2 \right)$, equal to $-\frac{1}{4} \left( M_{\Sigma_0 B}^2 - M_{\Sigma_0 M}^2 \right)^2 = -4\mu^4 \left( \mu^4 - 10\mu^2 + 1 \right)$. The latter expression as a function of the parameter $\mu^2$ has a minimum at $\mu^2 = \frac{15}{4} \left( 1 - \sqrt{1 - \frac{8}{225}} \right) \approx 0,067$ (determined by the equation $2\mu^4 - 15\mu^2 + 1 = 0$).

The other parameter $z_k z_k$ is used in condition (60) in the same way as the parameter $\mu^2$. Since proton and neutron take on the greatest probability in the sum (60) (herein it is not difficult to be convinced looking at formulae (58), (59)), condition (60) with large accuracy is written in the form

$$\bar{z} z \left( \frac{2}{M_N} \right)^4 J_2^2(M_N) e^{-\frac{M_N^2}{2\mu^2}} \approx 1.$$  

If $M_N$ takes the value $M_N = 1,14$ given by formula (54) at $\mu^2 = 0,067$ ($N = 1, \ i = \frac{1}{2}$), for $\bar{z} z$ we obtain $\bar{z} z = 0,86 \cdot 10^5$. Thus $T_f = 0,3 \cdot 10^5$. In the same time for $T_f = \frac{4}{3} T_f$ we have $T_f = 0,78 \cdot 10^{-6}$. Another dimensionless parameter $\eta = \frac{3 T_f}{T_f}$, playing an important role in cosmology, is equal to $\eta = 10^{11}$. The parameter $\varepsilon$ will be determined below.

2) Representations of algebra (2). In this case the coordinates on the Lagrangian plane $L \subset h_{16}^{(s)}$ are still denoted by $\varphi_{ak} \bar{\varphi}_{ak}$. The representation of algebra (2) given by operators

$$\phi^H = \left( \Lambda \partial / \partial \varphi_{ak} \right), \quad \bar{\phi}^H = \left( \bar{\varphi}_{ak}, -\Lambda \partial / \partial \bar{\varphi}_{ak} \right)$$  

(64) is called the $H$-representation (or hadronic). In this representation dynamical variables of the system (quadratic Hamiltonians) form the algebra $d = \{ d_-, d_0, d_+ \}$ graduated by powers of $\Lambda$ in which

$$d_+ : \quad \Lambda \partial_{\varphi m}, \quad \Lambda \bar{\partial}_{\bar{\varphi} \beta m}, \quad \Lambda \partial_{\varphi \beta m}, \quad \Lambda \bar{\partial}_{\bar{\phi} \beta m}$$

$$d_- : \quad 1 \Lambda \varphi_{ak} \varphi_{\beta m}, \quad 1 \Lambda \bar{\varphi}_{ak} \bar{\varphi}_{\beta m}, \quad 1 \Lambda \bar{\varphi}_{ak} \varphi_{\beta m}, \quad 1 \Lambda \varphi_{ak} \bar{\varphi}_{\beta m}$$

$$d_0 : \quad \varphi_{ak} \partial_{\beta m}, \quad \bar{\varphi}_{ak} \bar{\partial}_{\bar{\beta} m}, \quad \varphi_{ak} \bar{\partial}_{\bar{\beta} m}, \quad \bar{\varphi}_{ak} \partial_{\beta m}$$

(here $\partial_{\varphi m} = \partial / \partial \varphi_{ak}, \bar{\partial}_{\bar{\beta} m} = \partial / \partial \bar{\varphi}_{ak}$). Obviously we have

$$[d_0, d_0] \subset d_0, \quad [d_-, d_0] \subset d_0, \quad [d_+, d_+] = [d_-, d_-] = 0, \quad [d_+, d_-] \subset d_0.$$

Another representation, the compressed one (or long-drawn one, if $\Lambda > 1$) in relation to (64) given by operators

$$\phi^L = \left( \partial / \partial \varphi_{ak} \right), \quad \bar{\phi}^L = \left( \Lambda \varphi_{ak}', -\partial / \partial \bar{\varphi}_{ak} \right),$$  

(65)
where \( \varphi'_{\alpha k} \) is connected with \( \varphi_{\alpha k} \) by a (generally speaking) purely isotopic transformation \( V : \varphi'_{\alpha k} = V_{km} \varphi_{\alpha m}, (V \in SL_i(2, \mathbb{C})) \), is called the \( L \)-representation (or leptonic one).

Momentum variables in the \( H \)-representation are written in the form
\[
H p_\mu = \frac{1}{\Lambda} \bar{\varphi} \sigma_\mu \varphi, \quad H \dot{p}_\mu = -\Lambda \partial \bar{\varphi} \sigma_\mu \varphi, \quad L p_\mu = \Lambda \bar{\varphi} \sigma_\mu \varphi, \quad L \dot{p}_\mu = -\Lambda \partial \bar{\varphi} \sigma_\mu \varphi.
\]

Obviously, at \( \Lambda \neq 1 \) the mass spectrum formula for hadrons remains the same as (54), leaving also the definition of 4-momenta \( P_\mu \) and \( Q_\mu \) as well as the formula for particle fields and transition amplitudes unchanged. However, by \( X_\mu \) and \( Y_\mu \) now it is necessary to understand that
\[
X_\mu = \frac{1}{2} \left( \frac{1}{\Lambda} x_\mu + \Lambda \dot{x}_\mu \right), \quad Y_\mu = \frac{1}{2} \left( \frac{1}{\Lambda} x_\mu - \Lambda \dot{x}_\mu \right).
\]

In Appendix 3 it will be shown that in (2) \( \Lambda \) can impossibly be a number; \( \Lambda \) turns out to be connected with the dimension \( \dim d \) of the algebra of dynamical variables \( d \) by the formula \( \Lambda = \sqrt{\dim d} \). In the model \( h_{16}^{(*)} \) the dimension \( \dim d = 136 \).

3) Hadron and lepton era. In the present theory the creation of particles occurs in strict order: at first hadrons are generated, and subsequently when the density of quanta \( f \) decreases, leptons will be generated. The creation of hadrons (hadron era) is described by the \( H \)-representation of algebra (2). In this representation a 4-momentum of quanta \( \dot{f} \) is large \( H \dot{p}_\mu = -\Lambda \partial \bar{\varphi} \sigma_\mu \bar{\varphi} \) (factor \( \Lambda \)), the state \( \dot{f}_z \) is of the form (53) where \( z_k = z(\varepsilon) \) (here \( \varepsilon = z_1/z_2, z = z_2 \)), and a 4-momentum of quanta \( f \) is small \( H p_\mu = \frac{1}{\Lambda} \bar{\varphi} \sigma_\mu \varphi \) (factor \( 1/\Lambda \)). Therefore the distribution function \( w_f (16) \) is rather essential. As a consequence of the irreversible (that aspect being emphasized!) quantum transition \( f \rightarrow \dot{f} \) both neutral and charged hadrons are created. It is important to notice that \( H \dot{p}_\mu \) and \( H p_\mu \) are neutral operators in the sense that they commute with the operator of electromagnetic charge \( Q : [H \dot{p}_\mu, Q] = [H p_\mu, Q] = 0 \). However states \( \dot{f}_z \) (at \( \varepsilon \neq 0 \)) are characterized by an uncertain charge not equal to zero \( Q \), since \( Q \dot{f}_z \neq 0 \) (that is why charged hadrons occur). Due to the assumption of complete (one hundred per cent) fermion-antifermion asymmetry (see [4]) only the fermions will be created. The antifermion creation will happen subsequently as a result of switching to interactions. Clearly, in the hadron era a large positive charge, not compensated by anything, arises (before the transition the charge of quanta \( f \) was equal to zero, see above, in the model \( h_{16}^{(*)} \) the coherent fields (13) as a matter of fact are neutral leptons with zero mass), due to the proton component. In fact, probabilities of creating various charged components of the baryon octet (main hadron component) satisfy the condition (they result from
so that negative $\Xi^-$-baryons cannot compensate the positive charge of protons. Hence, the process creating only hadrons would be accompanied by a violation of the electric charge conservation law (for mesons this problem is not present: the charge $\rho^+$ is compensated by the charge $\rho^-$, and the charge $K^*^+$ is by the charge $K^*^-$). To compensate the charge of protons only opposite charged leptons can occur due to the following.

After a creation of hadrons (the transition $N_+ \rightarrow N_-$) the density of quanta $f$ sharply decreases, so that the density of quanta $\dot{f}$ on the lower half of light cone ($\rho \in N_-$) increases. Now basically there are transitions in the vertex of cone $N^\rightarrow \{0\}$ (lepton era). This era is characterized by the density function $w_f = 1$ (as $\rho_\mu = 0$ that is equivalent to the parameter $z = 0$) and is described by the (long-drawn) $L$-representation of algebra (2). In this representation the 4-momentum $Lp_\mu = \Lambda \bar{\varphi}' \bar{\sigma}_\mu \varphi'$ is large (the factor is $\Lambda$), and $L\dot{p}_\mu = \frac{1}{\Lambda} \bar{\sigma}_\mu \bar{\varphi}' \bar{\varphi}'$ is small (the factor is $\frac{1}{\Lambda}$). Graphically speaking, the yoke ($H\dot{p}, \dot{H}p$) can overturn, as a result it can take another position ($L\dot{p}, Lp$) such that when $p$ rises, $\dot{p}$ drops to slide in $\{0\}$ (in the vertex of cone), an additional purely isotopic rotation $\varphi_k \rightarrow V_{km} \varphi_m$ occurs (the process of transition is described by Lorentz-invariant amplitudes, and it occurs in the space $F_0$ of additional variables — Lorentzian scalars $\bar{\varphi} = \bar{\varphi}_2$, $\bar{\varphi} = \bar{\varphi}_2$ where Lorentz transformations are trivial). As a consequence a 4-momentum $\frac{1}{\Lambda} \bar{\varphi} \bar{\sigma}_\mu \varphi$ passes into a 4-momentum $\Lambda \pi'_\mu$:

$$\frac{1}{\Lambda} \bar{\varphi} \bar{\sigma}_\mu \varphi \rightarrow \Lambda \pi'_\mu = \Lambda \bar{\varphi}' \bar{\sigma}_\mu \varphi' = \Lambda \bar{\varphi} \bar{\sigma}_\mu \varphi V^+ V \varphi = \frac{1}{\Lambda} \bar{\varphi} \bar{\sigma}_\mu \bar{V} + \bar{V} \bar{\varphi}$$

where $\varphi'_{\alpha k} = V_{km} \varphi_{\alpha m}, V \in SL_i(2, C)$ (det $V = 1$), and $\bar{V} = \Lambda V \in GL_i(2, C)$ (det $\bar{V} = \Lambda^2$). Since it may be taken $V^+ V = \tau_m A_m$ where $A_m$ is a time-like 4-isovector ($A_m^2 = 1$, since det $V = 1$), it may be written $\pi'_\mu = \pi^A_\mu$ where $\pi^A_\mu = \bar{\varphi} \sigma_\mu A \varphi$, and $\bar{A} = \tau_m A_m$. Being distinct from $\pi_\mu$ the 4-momentum $\pi^A_\mu$ carries an electrical charge, since it does not commute with the charge operator $Q : [\pi^A_\mu, Q] \neq 0$. However the state $\dot{f}_0 = 1/Z$ corresponding to the vertex of cone is neutral: $Q \dot{f}_0 = 0$ (that is why finishing transitions will be in the vertex of cone and that is why an isotopic rotation $V$ becomes possible; at $z \neq 0$ the transformation is not admissible, in this case only dilatations $\varphi \rightarrow \Lambda \varphi$ are allowed). With the transformed 4-momentum the lepton fields are written in the form (compare with (13))

$$\psi^\Sigma(X + Y) = \frac{1}{(2\pi)^{3/2}} \frac{1}{Z} \int e^{i\Lambda \pi'(X + Y)} \theta(\pi_0) \delta(\pi^2) d^4 \pi O^\Sigma(\pi)$$

(68)
where as always
\[ O^\Sigma(\pi) = \int d^4 \nu O^\Sigma(\varphi). \]

In (68) it is convenient to switch from variables \( \varphi \) to variables \( \varphi' = V \varphi \). Since for such transformations the measure \( d\mu_f \) does not change, we come to the formula
\[ \psi^\Sigma(X + Y) = \frac{1}{(2\pi)^{3/2}} \frac{1}{Z} \int e^{i\Lambda \pi(X+Y)} \theta(\pi_0) \delta(\pi^2) \, d^4 \pi \tilde{O}^\Sigma(\pi) \]
where
\[ \tilde{O}^\Sigma(\pi) = \int d^4 \nu O^\Sigma(V^{-1}\varphi). \quad (69) \]

As always, all skeletons with non-zero hypercharge and isospin will result in zero. However, now also charged fields occur.

For an example of the lowest states of fermions, the skeletons are in fact of the form
\[ O^\Sigma(\varphi) = \bar{\varphi}_n \varphi_\alpha, \]
and the integral
\[ O^\Sigma(\pi) = \int d^4 \nu \bar{\varphi}_n \varphi_\alpha = \delta_{n0} \varphi_\alpha(\pi) \]
is only different from zero at \( n = 0 \), i.e. for \( \bar{\varphi}_0 \varphi_\alpha \). This neutral lepton field is a neutrino. Next, as \( O^\Sigma(V^{-1}\varphi) = \bar{\varphi} V^{-1} \tau_n V^{-1} \varphi_\alpha \), and \( V^{-1} \tau_n V^{-1} = L_{nn'}(V^{-1}) \tau_{n'} \), we have
\[ \tilde{O}^\Sigma(\pi) = L_{nn'}(V^{-1}) \int d^4 \nu \bar{\varphi}_n' \varphi_\alpha = L_{n0}(V^{-1}) \varphi_\alpha(\pi) \]
where \( L_{n0} = \frac{1}{2} Sp V^{-1} \tau_n V^{-1} \). Fields with \( L_{00} \) and \( L_{30} \) are neutral, and fields with \( L_{+0} = L_{10} + iL_{20} \) and with \( L_{-0} = L_{10} - iL_{20} \sim \varepsilon \) are charged. As can be seen, charged leptons occur.

It remains to find a special kind of the transformation \( V \). Since the given transformation is only performed with one purpose, namely to compensate any charge, neutral fields (nonzero before the transformation) remain so after the transformation. In particular, this concerns a field with the skeleton \( \bar{\varphi} \tau_3 \varphi_\alpha \) which before the transformation was equal to zero. After the transformation it is \( \sim L_{30} \). Therefore it must be \( L_{30} = 0 \). This means that
\[ Sp V^{-1} \tau_3 V^{-1} = 0. \quad (70) \]

If after the transformation \( \varphi \to \bar{V} \varphi \) the hadrons began to appear, all of them should be neutral. This means that it must be
\[ \bar{Z} \bar{V} = \bar{Z}^0 \]
where \( Z^0 \sim \begin{pmatrix} 1 \\ 1 \end{pmatrix} \) is a neutral isospinor. From the condition
\[
\tilde{z} \tilde{V} \tilde{V}^+ z = \tilde{z} z = |z|^2 \left( 1 + |\varepsilon|^2 \right) \tag{71}
\]
(since for the transformation \( \tilde{V} \) neither energy nor temperature of quanta \( \dot{f} \) should be changed) it follows that \( z^0 = z \sqrt{1 + |\varepsilon|^2} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \). From equations (70), (71) we find
\[
V = \begin{pmatrix}
\frac{\Lambda}{\sqrt{1 + |\varepsilon|^2}} & \frac{\varepsilon}{\Lambda \sqrt{1 + |\varepsilon|^2}} \left( 1 + \frac{\Lambda^2 - 1}{|\varepsilon|^2} \right) \\
-\frac{\Lambda \bar{\varepsilon}}{\sqrt{1 + |\varepsilon|^2}} & \frac{1}{\Lambda \sqrt{1 + |\varepsilon|^2}} \left( 1 - |\varepsilon| \sqrt{\Lambda^4 - 1} \right)
\end{pmatrix} \quad \tag{72}
\]
At last the most important condition should be thought over. If the transformation \( \tilde{V} \) is performed again, it may not lead to a new position of the yoke \((\dot{p}, p)\) or to a new generation of particles. Actually the system has to return to the \( H \)-representation. With reference to \( V \) this means that the twice applied transformation \( V \) must be proportional to an unit transformation: \( V^2 = \alpha \). Hence we obtain \( \det V = \pm \alpha \). As \( \det V = 1 \), \( \alpha = \pm 1 \). For matrices in the form (72) only the condition \( \alpha = -1 \) is acceptable, therefore
\[
V^2 = -1. \tag{73}
\]
From (73) it follows that in (72)
\[
|\varepsilon| = \frac{\sqrt{\Lambda^2 + 1}}{\sqrt{\Lambda^2 - 1}}, \tag{74}
\]
and consequently the matrix \( V \) must be of the form
\[
V = \begin{pmatrix}
\sqrt{\Lambda^2 - 1} \quad \sqrt{\Lambda^2 + 1} e^{i\chi} \\
-\sqrt{\Lambda^2 + 1} e^{-i\chi} \quad -\sqrt{\Lambda^2 - 1} \end{pmatrix} = \begin{pmatrix}
\Lambda^2 - 1 \quad \varepsilon \\
-\bar{\varepsilon} \quad -1
\end{pmatrix}, \tag{75}
\]
where \( \chi = \arg \varepsilon \). Since in the model \( h^{(s)}_{16} \) \( \Lambda^2 = 136, \ |\varepsilon| = \sqrt{137/135} \), and the matrix \( V \) takes the form
\[
V = \frac{1}{\sqrt{2}} \begin{pmatrix}
\sqrt{135} \quad \sqrt{137} e^{i\chi} \\
-\sqrt{137} e^{-i\chi} \quad -\sqrt{135}
\end{pmatrix}.
\]
It is interesting to observe that a priori probabilities of creating proton \( W_p \) and neutron \( W_n \) are not equal (see (58))
\[
\frac{W_p}{W_n} = |\varepsilon|^2 = \frac{137}{135} = 1.015. \tag{76}
\]
Thus, the transition \( f \to \dot{f} \) will generate slightly more protons than neutrons.
From the formula $V^+V = \tau_m A_m$ it is possible to find components of the isovector $A_m$

$$A_0 = \Lambda^2, \quad A_3 = 0, \quad A_1 = \sqrt{\Lambda^4 - 1} \cos \chi, \quad A_2 = \sqrt{\Lambda^4 - 1} \sin \chi,$$

and from the formula $\frac{1}{2} S^m V^+ \tau_n V^{-1} = L_{n0}$ the values $L_{n0}$ are

$$L_{00} = \Lambda^2, \quad L_{30} = 0, \quad L_{+0} = -\sqrt{\Lambda^4 - 1} e^{-i\chi}, \quad L_{-0} = -\sqrt{\Lambda^4 - 1} e^{i\chi}.$$

4) So, hadrons arise from the transition $H_p \rightarrow H_p$, leptons arise from the transition $L_p \rightarrow L_p$. There are two more transition possibilities: $H_p \rightarrow L_p$ and $L_p \rightarrow H_p$.

In the first case both 4-momenta $H_p$ and $L_p$ are small (zero as a matter of fact) so that in this case the system passes from the vertex of the upper half of light cone into the vertex of the lower half of light cone. This is a vacuum transition. Its amplitude is equal to the factor $1/Z$.

In the second case both 4-momenta are large: $L_p = \Lambda \bar{\varphi} \, \sigma_{\mu} \varphi$, $H_p = -\Lambda \partial \bar{\sigma}_{\mu} \partial$ (\Lambda is a factor, the matrix $V = 1$). This is that limiting case when the parameter $\mu^2 \rightarrow \infty$. In this limit, being only meaningful for fermions ($F = 1$), the equation (30) passes into the equation (it is necessary to pay attention on the factor $\Lambda^2$)

$$\left(\square_X - \Lambda^2 F^0_\Sigma\right) O^{\Sigma}(X; Y) = \Im^{\Sigma}(X; Y),$$

and formula (54) gives

$$M^2_{T\Sigma} = -\Lambda^2 \left[(N + 5)^2 + 7 - 4i(i + 1)\right].$$

As can be seen, in our spacetime (coordinates $X_\mu$) the objects arising in this case are characterized by an imaginary mass. In addition they have no wave function, since solutions of the equations

$$\left(\square_X - M^2_{T\Sigma}\right) \psi^{\Sigma}(X; Y) = 0$$

do not represent any fields. Such objects do not exist as particles. However in the second space (coordinates $Y_\mu$) these objects represent usual particles, since $\psi^{\Sigma}(X; Y)$ as functions $Y$ satisfy the Klein-Gordon equation

$$\left(\square_Y + M^2_{T\Sigma}\right) \psi^{\Sigma}(X; Y) = 0$$

and are characterized by real masses

$$M^2_{T\Sigma} = \Lambda^2 \left[(N + 5)^2 + 7 - 4i(i + 1)\right].$$

But the second (internal) space is inaccessible for observations. In our space the objects in question can show themselves only as virtons, being objects which are
described by a distribution function (for the lack of any wave function), compare with [14]. To go out (because of their large mass for an instant) from the second space in our space, they certainly have to return into the second space where they exist as usual particles. In that way these objects stick together different points of the discontinuum to transform it in a continuum [7].

7 At last

God gave the people a mind to comprehend the truth, but He did not give them the criteria of the truth, He preserved it for Himself. That is why people often consider lies as the truth and vice versa, and that is why we depend from God and must not lose contact with Him.

Appendix 1

The determination of the mass spectrum of “bare” fundamental hadrons is connected with the determination of eigenvalues of the operator $M^2 = 2\hat{p}_\mu p_\mu$ on states $f^\Sigma = w O^\Sigma$ of relativistic bi-Hamiltonian system: $M^2 f^\Sigma = F_\Sigma (-P^2) f^\Sigma$. Here we find the expression for $F_\Sigma (-P^2)$.

At first we consider the model $h^{(s)}$. Here 4-momenta of the system are of the form $p_\mu = i\bar{\varphi} \gamma_\mu P_+ \phi = \bar{\varphi}_\beta (\bar{\sigma}_\mu)^\beta\alpha \varphi_\alpha$, $\hat{p}_\mu = -i\bar{\varphi} \gamma_\mu P_- \varphi = -\partial^\beta (\bar{\sigma}_\mu)^{\beta\dot{\alpha}} \partial^{\dot{\alpha}}$ where $\partial^\alpha = \frac{\partial}{\partial \varphi^\alpha}$. Therefore $2\hat{p}_\mu p_\mu = -2\partial \bar{\sigma}_\mu \bar{\varphi} \sigma^\mu \varphi = -(\hat{N} + 4)^2 + Y^2$ (we have used the completeness condition of matrices $\sigma_\mu$: $\Sigma_{\mu=1}^4 (\hat{\sigma}_\mu)^\alpha_{\beta} (\bar{\sigma}_\mu)^{\beta\dot{\alpha}} = 2\delta^\delta_{\alpha} \delta^\gamma_{\dot{\alpha}}$ and the definition $\hat{N} = \varphi_\alpha \partial^\alpha + \bar{\varphi}_{\dot{\alpha}} \bar{\partial}^{\dot{\alpha}}, \hat{Y} = \varphi_\alpha \partial^\alpha - \bar{\varphi}_{\dot{\alpha}} \bar{\partial}^{\dot{\alpha}}$). Since skeletons $O^\Sigma$ are eigenvectors of operators $\hat{N}$ and $\hat{Y}$: $\hat{N} O^\Sigma = N^\Sigma O^\Sigma, \hat{Y} O^\Sigma = Y^\Sigma O^\Sigma$ ($N^\Sigma$ and $Y^\Sigma$ are eigenvalues, here and in the sequel the index $\Sigma$ in eigenvalues is suppressed, and $w = \exp(-\omega^2 \bar{\varphi} \varphi)$ is a distribution function of quanta $f$ on energy $\varepsilon_f = \bar{\varphi} \varphi$ (for a while a temperature of quanta $f$ is denoted as $T_f = 1/\omega^2$) as well as $\hat{N}$ and $\hat{Y}$ are first-order differential operators, $\hat{N} (O^\Sigma w) = (\hat{N} O^\Sigma) w + O^\Sigma (\hat{N} w)$, such that by definition $w$ it follows $\hat{Y} w = 0$. Therefore we have

$M^2 f^\Sigma = [-(N + 4)^2 + Y^2] f^\Sigma - O^\Sigma (\hat{N} w) - 8O^\Sigma (\hat{N} w) - 2NO^\Sigma (\hat{N} w)$.}

Next, since

$$\hat{N} w = (\varphi \partial + \bar{\varphi} \bar{\partial}) e^{-\omega^2 \bar{\varphi} \varphi} = 2\omega^2 \frac{\partial}{\partial \omega^2} w.$$
it is possible to write
\[ M^2 f^\Sigma = F^0_\Sigma f^\Sigma - 4 \left[ (\omega^2)^2 \frac{\partial^2}{(\partial \omega^2)^2} + (N + 5)\omega^2 \frac{\partial}{\partial \omega^2} \right] f^\Sigma \]
where \( F^0_\Sigma = -(N + 4)^2 + Y^2 \) (see (12)). Let us now make use of the identity \( \frac{\partial}{\partial \omega^2} w = -\varphi \varphi w \). We write \( \omega^2 \frac{\partial}{\partial \omega^2} w = -\omega^2 \varphi \varphi w = \frac{P^2}{4\mu^2} w \). The latter equality follows from the possibility to write the Gibbs function representation \( \exp(-\omega^2 \varphi \varphi) \) in the relativistic invariant Juttner form \( \exp(-\frac{P^2}{4\mu^2}) \) where \( -P^2 = 4\pi \rho = 4\varphi \varphi z z \), and \( \mu^2 = z z / \omega^2 = 3 T_f T_j \) (see formula (16)). Therefore \( F^\Sigma \) is written in the form
\[ F^\Sigma(-\mathcal{P}^2) = F^0_\Sigma + \frac{N + 5}{\mu^2} (\mathcal{P}^2) - \frac{(\mathcal{P}^2)^2}{4\mu^4} . \]
In the coordinate representation we have
\[ F^\Sigma(\Box) = F^0_\Sigma + \frac{N + 5}{\mu^2} \Box - \frac{\Box^2}{4\mu^4} \]  
(Appx. 1)
where \( \Box \) is the D’Alembertian with respect to variables \( X_\mu \).

In the model \( h_{16}^{(s)} \) the operator \( \hat{M}^2 \) is other, as here \( p_\mu = \varphi \dot{\sigma}_\mu (\sigma_\mu)^\alpha \beta \varphi_\beta \), \( \dot{p}_\mu = -\partial_\beta (\sigma_\mu)^\alpha \beta \bar{\varphi}_k \partial_\alpha (\sigma_\mu)^\alpha \beta \). Using the completeness condition of matrices \( \sigma_\mu \), we write
\[ \hat{M}^2 = 2\dot{p}_\mu p_\mu = -4\partial_\beta \bar{\varphi}_k \varphi_\beta \varphi_m \varphi_{am} \]
and using the completeness condition of matrices \( \hat{t}_a = (\bar{\tau}, \pm 1) : \Sigma_a \left( \frac{\hat{t}_a}{k_m} \right) \left( \bar{\tau}_a \right)_{np} = 2\delta_{kp} \delta_{nm} \), we shall have (here \( \tau^T \) is a transposed matrix)
\[ \hat{M}^2 = -2\partial \hat{t}_a \varphi \partial \bar{\tau}_a \varphi^T \bar{\varphi} = -2\partial \hat{t}_a \varphi (\bar{\varphi} \bar{\tau}_a \varphi^T + 4\delta_a a) = \]
\[ = -2 \left[ \partial \bar{\tau} \varphi \bar{\varphi} \partial \bar{\tau} + (\partial \varphi)(\bar{\varphi} \partial) + 4\partial \varphi \right] . \]
Using then definitions of the operators \( \hat{N} = \varphi \partial + \varphi \bar{\partial} , \hat{Y} = \varphi \partial - \varphi \bar{\partial} , \hat{t} = -\frac{1}{2} (\varphi \tau^T \partial - \varphi \bar{\tau} \bar{\partial}) , \hat{k} = -\frac{1}{2} (\varphi \tau^T \partial + \varphi \bar{\tau} \bar{\partial}) \), the expression for \( \hat{M}^2 \) may be written in the form
\[ \hat{M}^2 = -4 \left[ \frac{1}{2} (\hat{t}^2 - \hat{t}^2) + \frac{1}{8} (\hat{N}^2 - \hat{Y}^2) + 2 \hat{N} + 8 \right] . \]

Essentially, in this case the operator \( \hat{M}^2 \) can only be expressed in terms of generators of the isotopic group \( SL_2(2, \mathbb{C}) \otimes U_1(1) \otimes H_1(1) \); in so doing, generators
of the Lorentz group \( \text{SL}_\ell(2, \mathbb{C}) \otimes U_\ell(1) \otimes H_\ell(1) \) do not occur. As a consequence of this circumstance, the operator \( \hat{M}^2 \) has the same expression on both the whole space \( \mathbf{F} \) and subspace \( \mathcal{F}_0 \). Therefore, in the future we can consider either \( \hat{M}^2 \) on the space \( \mathcal{F}_0 \) or (that is equivalent) to pass into a Lorentzian system such that \( \varphi_{1k} = 0 \).

The expression for \( \hat{M}^2 \) can be developed into

\[
\hat{M}^2 = 4 \hat{\tau}^2 - N^2 - 10 \hat{N} - 32 - 2 \left( \Delta - \frac{1}{4} Y^2 - \frac{1}{4} N^2 - \hat{N} \right) \quad \text{(Appx. 2)}
\]

where \( \Delta = \hat{\tau}^2 + \hat{k}^2 \) is a Casimir operator of the algebra \( \text{sl}_i(2, \mathbb{C}) \). On \( \mathcal{F}_0 \) we have \( \varphi_k = \varphi_{2k} \) are additional variables, \( \partial_k = \frac{\partial}{\partial \varphi_k} \)

\[
\Delta - \frac{1}{4} Y^2 - \frac{1}{4} N^2 = \frac{1}{2} \left[ (\varphi \tilde{\tau}^T \partial + (\tilde{\varphi} \tilde{\tau} \tilde{\partial})^2 \right] - \frac{1}{2} \left[ (\varphi \partial)^2 + (\tilde{\varphi} \bar{\partial})^2 \right] = \\
= \frac{1}{2} \sum_{a=1}^{4} \left[ \varphi_{\tilde{\tau}^a} \partial \chi^T_{\tilde{\tau}^a} \partial + \varphi_{\tilde{\tau}^a} \bar{\partial} \chi^T_{\tilde{\tau}^a} \bar{\partial} \right].
\]

Using the completeness condition of matrices \( \tilde{\tau}^a \):

\[
\sum_{a=1}^{4} \left( \chi^T_{\tilde{\tau}^a} \right)_{km} \left( \chi^T_{\tilde{\tau}^a} \right)_{np} = 2 \tau_{kn} \tau_{pm}
\]

where \( \tau = \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right) \), we obtain

\[
\Delta - \frac{1}{4} Y^2 - \frac{1}{4} N^2 = (\varphi \tau)_{k} \partial_m \varphi_k (\partial \tau)_m + (\tilde{\varphi} \tilde{\tau})_{k} \tilde{\partial}_m \tilde{\varphi}_k (\tilde{\partial} \tau)_m = \\
= \varphi \tau \varphi \partial \tau \partial + \tilde{\varphi} \tilde{\tau} \tilde{\varphi} \tilde{\partial} \tilde{\partial} + \varphi \partial + \tilde{\varphi} \bar{\partial}.
\]

And as \( \varphi \tau \varphi = 0 \), we have

\[
\Delta - \frac{1}{4} Y^2 - \frac{1}{4} N^2 = \hat{N}.
\]

Hence, the expression in round brackets in (Appx. 2) is equal to zero. So that actually

\[
\hat{M}^2 = 4 \hat{\tau}^2 - N^2 - 10 \hat{N} - 32.
\]

Now it is not difficult to find the expression for \( F_{\Sigma}(-P^2) \) in the equation \( \hat{M}^2 f^\Sigma = F_{\Sigma}(-P^2) F^\Sigma \) where \( f^\Sigma = O^\Sigma w \) (\( O^\Sigma \) is the skeleton, \( w \) is the Gibbs distribution function). In the coordinate representation we have

\[
F_{\Sigma}(\square) = F_{\Sigma}^0 + \frac{N}{\mu^2} \square - \frac{\square^2}{4 \mu^4} \quad \text{(Appx. 3)}
\]
where \( F_\Sigma^0 = -(N + 5)^2 - 7 + 4i(i + 1) \), and \( N \) and \( i \) are the isotonic number and isospin of the skeleton \( O^\Sigma \) respectively.

### Appendix 2

There we find explicit amplitudes \( O^\Sigma(\mathcal{P}, \mathcal{Q}) \) determined by formula (55):

\[
O^\Sigma(\mathcal{P}, \mathcal{Q}) = \int d^4\tilde{\nu} O^\Sigma(\varphi_{\alpha k}, \varphi_k) e^{-\frac{\pi\mu^2}{4\mu^2} - 2i\sin\tilde{z}_k\varphi_k} = \\
= \exp\left(-\frac{P^2}{4\mu^2}\right) O^\Sigma\left(-\frac{\partial}{\partial\tilde{z}}, \frac{\partial}{\partial\tilde{z}}\right) I \quad \text{(Appx. 4)}
\]

where \( O^\Sigma(\varphi) \) are skeletons of particles, and

\[
I(\pi, \rho) = \int d^4\tilde{\nu} \exp(\tilde{\varphi}_{\alpha k}z_{\alpha k} - \tilde{z}_{\alpha k}\varphi_{\alpha k}) \quad \text{(Appx. 5)}
\]

such that \( z_{\alpha k} = \delta_{\alpha 2} z_k \), and the measure \( d^4\tilde{\nu} \) is defined by formula (50).

Consider at first the integral appearing in the model \( h_8^\Sigma(\star) \) (see (25))

\[
\mathcal{J} = \int_0^{2\pi} \frac{d\omega}{2\pi} e^{\tilde{\varphi}_{\alpha} z_{\alpha} - \tilde{z}_{\alpha} \varphi_{\alpha}} \quad z_{\alpha} = z\delta_{\alpha 2}
\]

where \( \omega = \text{arg} \varphi_2 \). From definitions \( \pi_{\mu} = \tilde{\varphi}^+ \sigma_{\mu} \varphi \), \( \rho_{\mu} = \tilde{z}^+ \sigma_{\mu} z \) \((\pi_{\mu}^2 = \rho_{\mu}^2 = 0)\) it follows that \( \pi\rho = 2|\tilde{z}_{\alpha} \varphi_{\alpha}|^2 \). As \( 2\pi\rho = -P^2 \), \( \tilde{z}_{\alpha} \varphi_{\alpha} = \frac{1}{2}\sqrt{-P^2} e^{i\chi} \). Therefore

\[
\mathcal{J} = \frac{1}{2\pi} \int_0^{2\pi} e^{i\sqrt{-P^2}\sin\chi} d\omega.
\]

Since

\[
\varphi_{\alpha} = e^{i\omega} \left( e^{i\nu} |\varphi_1| \right) \quad \text{and} \quad z_{\alpha} = e^{i\nu} \left( e^{i\alpha} |z_1| \right),
\]

we obtain

\[
\tilde{\varphi}_{\alpha} z_{\alpha} - \tilde{z}_{\alpha} \varphi_{\alpha} = 2 \left[ \sin(\omega - \nu + \psi - \alpha) |\varphi_1 z_1| + \sin(\omega - \nu) |\varphi_2 z_2| \right],
\]

\[
\tilde{\varphi}_{\alpha} z_{\alpha} + \tilde{z}_{\alpha} \varphi_{\alpha} = 2 \left[ \cos(\omega - \nu + \psi - \alpha) |\varphi_1 z_1| + \cos(\omega - \nu) |\varphi_2 z_2| \right].
\]

It follows from this that

\[
d\omega \frac{d}{d\omega} \sqrt{-P^2} \sin\chi = \sqrt{-P^2} \cos\chi \frac{d\chi}{d\omega} d\omega = \sqrt{-P^2} \cos\chi d\omega,
\]

i.e. \( d\chi = d\omega \), and consequently

\[
\mathcal{J} = \frac{1}{2\pi} \int_0^{2\pi} e^{i\sqrt{-P^2}\sin\chi} d\chi = J_0(-P^2)
\]
where \( \mathcal{J}_0 \) is the Bessel function.

Integrating now (Appx. 4) with respect to \( \omega_1 \) and \( \omega_2 \), we obtain

\[
I = \int d^4 \nu \, e^{\tilde{\varphi}_a z \, z_{ak} - \bar{z}_a \tilde{\varphi}_a} = \int d^2 \nu \, \mathcal{J}_0(\sqrt{-\mathcal{P}_1^2}) \mathcal{J}_0(\sqrt{-\mathcal{P}_2^2})
\]

where \(-\mathcal{P}_2^2 = 2\rho^{(2)2}\), \(-\mathcal{P}_1^2 = 2\rho^{(1)2}(\pi - \Pi)\), and

\[
d^2 \nu = \frac{2 \theta(\Pi_0) \theta(\pi_0 - \Pi_0)}{\theta(\pi_0)} \delta(\Pi^2) \delta(2\pi\Pi - \pi^2) d^4 \Pi.
\]

Expanding functions \( \mathcal{J}_0 \) in a series, we shall have

\[
I = \sum_{n,m=0}^{\infty} \frac{(-1)^n}{2^n(n!)^2} \frac{(-1)^m}{2^m(m!)^2} \int d^2 \nu \, (\rho^{(2)2})^n (\rho^{(1)2}(\pi - \Pi))^m.
\]

As

\[
(\rho^{(1)2}(\pi - \Pi))^m = \sum_{l=0}^{m} (\rho^{(1)2})^{m-l} (\rho^{(1)2})^l (-1)^l C_l^m,
\]

\( (C_l^m) \) is the number of combinations,

\[
I = \sum_{n,m,l} \frac{(-1)^{n+m+l}}{2^n(n!)^2 2^m(m!)^2} C_l^m (\rho^{(1)2})^{m-l} \int d^2 \nu \, (\rho^{(1)2})^l (\rho^{(2)2})^n.
\]

We need to calculate integrals in the form \( (\pi^2 = 0) \):

\[
\int d^2 \nu \, \Pi_{\alpha} \ldots \Pi_{\omega} = \frac{1}{\pi} \int \Pi_{\alpha} \ldots \Pi_{\omega} \frac{\theta(\Pi_0) \theta(\pi_0 - \Pi_0)}{\theta_0} \delta(\Pi^2) \delta(\pi \Pi) d^4 \Pi.
\]

It is clear that the integration result must be of the form \( A(s) \sum_{\alpha}^{\pi} \Pi_{\alpha} \ldots \Pi_{\omega} \) such that it remains only to find \( A(s) \). For this purpose we pass to the system in which \( \pi_\mu = (0, 0, \pi_3, \pi_0) \) such that \( \pi_3 = \pi_0 \). In it we have

\[
\frac{1}{\pi} \int \frac{\theta(\Pi_0) \theta(\pi_0 - \Pi_0)}{\theta_0} \delta(\Pi_1^2 + \Pi_2^2 + \Pi_3^2 - \Pi_0^2) \delta(\pi(\Pi_3 - \Pi_0)) \times
\]

\[
\times \Pi_{\alpha} \ldots \Pi_{\omega} \, d\Pi_1 \, d\Pi_2 \, d\Pi_3 \, d\Pi_0 = \frac{1}{\pi} \int \delta(\Pi_1^2 + \Pi_2^2) \, d\Pi_1 \, d\Pi_2 \int_0^\pi \frac{d\Pi_0}{\pi_0} \Pi_{\alpha} \ldots \Pi_{\omega}.
\]

As

\[
\frac{1}{\pi} \int \delta(\Pi_1^2 + \Pi_2^2) \, d\Pi_1 \, d\Pi_2 = \frac{1}{\pi} \int_0^{2\pi} d\theta \int_0^\infty \delta(r^2) \, r \, dr = 1
\]

and the integral \( \frac{1}{\pi_0} \int_0^{\pi_0} d\Pi_0 \, \Pi_{\alpha} \ldots \Pi_{\omega} \) is only nonzero for \( \alpha, \ldots, \omega, \) equal to 3 or 0, and \( \Pi_3 = \Pi_0 \),

\[
\frac{1}{\pi_0} \int_0^{\pi_0} d\Pi_0 \, \Pi_{\alpha} \ldots \Pi_{\omega} = \frac{1}{\pi_0} \int_0^{\pi_0} d\Pi_0 \, \Pi_{\alpha} \ldots \Pi_{\omega} = \frac{\pi_0^s}{s + 1}.
\]
In any system the result is written as \( \frac{1}{s+1} \pi_\alpha \cdots \pi_\omega \). Therefore for \( A(s) \) we have

\[
A(s) = \frac{1}{s+1}.
\]

It may be written the equality

\[
\int d^2 \nu (\rho^{(1)} \Pi)^l (\rho^{(2)} \Pi)^n = \frac{1}{l+n+1} \left( \rho^{(1)} \Pi \right)^l \left( \rho^{(2)} \Pi \right)^n,
\]

so that for \( I \) we obtain

\[
I = \sum_{n,m,l} \frac{(-1)^{n+m+l}}{2^n (n!)^2 2^m (m!)^2} \frac{C_l^m (\pi \rho^{(2)})^n (\pi \rho^{(1)})^m}{l+n+1}.
\]

As

\[
\sum_{m=0}^m \frac{(-1)^l}{m!} \frac{m! n!}{l+n+1} = \frac{m! n!}{(m+n+1)!},
\]

\[
I = \sum_{n,m=0}^\infty \frac{(-\pi \rho^{(1)}/2)^m}{m!} \frac{(-\pi \rho^{(2)}/2)^n}{n! (m+n+1)!} = \sum_{n=0}^\infty \frac{(-\pi \rho^{(1)}/2)^m}{m!} \mathcal{J}_{m+1}(\sqrt{2 \pi \rho^{(2)}}) \left( \frac{2}{\sqrt{2 \pi \rho^{(2)}}} \right)^{m+1}
\]

where \( \mathcal{J}_{m+1} \) is the Bessel function. To take advantage of the formula \([15]\)

\[
\sum_{m=0}^\infty \frac{(-1)^m}{m!} \left( \frac{t(z+t/2)}{z} \right)^m \mathcal{J}_{m+1}(z) = \frac{z}{z+t} \mathcal{J}_1(z+t)
\]

in which it is necessary to put \( z = \sqrt{2 \pi \rho^{(2)}} \), \( \pi \rho^{(1)} = t(\sqrt{2 \pi \rho^{(2)}} + t/2) \), i.e. \( t = \sqrt{2 \pi \rho} - \sqrt{2 \pi \rho^{(2)}} \) where \( \rho = \rho^{(1)} + \rho^{(2)} \), we find for \( I \) the final expression

\[
I(\pi, \rho) = \frac{2}{\sqrt{2 \pi \rho}} \mathcal{J}_1(\sqrt{2 \pi \rho}) = \frac{2}{\sqrt{-\mathcal{P}^2}} \mathcal{J}_1(\sqrt{-\mathcal{P}^2}), \quad -\mathcal{P}^2 = 2 \pi \rho.
\]

At \( \rho = 0 \) (i.e. \( z = 0 \)) it follows from this that

\[
\int d^4 \nu = 1.
\]

It is not difficult to show that definition (Appx. 4) results in the factorized expression for amplitudes \( O^\Sigma(\mathcal{P}, \mathcal{Q}) \) or \( O^\Sigma(\pi, \rho) \):

\[
O^\Sigma(\pi, \rho) = O_{\phi}^{(l)}(\pi, \rho) O_{\phi}^{(i)}(z) N_{\Sigma}(X), \quad X = \sqrt{2 \pi \rho} \quad \text{(Appx. 6)}
\]

where \( O_{\phi}^{(l)} \) and \( O_{\phi}^{(i)} \) are denoted Lorentzian and isotopic factors. Let us find at first all three factors in the model \( h_{\phi}^{(s)} \). In this model \( L \)-skeletons \( O^\Sigma(\varphi) \) are written in the form

\[
O^\Sigma(\varphi) = \varphi_\alpha \cdots \varphi_\chi \varphi_\delta \cdots \varphi_\omega.
\]
Spinor indices equal to 2 answer additional variables. For definiteness (due to the complete fermion-antifermion asymmetry) we hold that \( a \geq b \). Denoting \( a + b = N \) and \( a - b = Y \), we rewrite \( O^\Sigma(\varphi) \) in the form

\[
O^\Sigma(\varphi) = \varphi_{\delta} \cdots \varphi_{\chi} \varphi_\alpha \varphi_{\dot{\alpha}} \cdots \varphi_\omega \varphi_{\dot{\omega}}.
\]

As \( \pi_\mu = \bar{\varphi} \frac{\partial}{\partial x_\mu} \varphi \), \( \varphi_\alpha \varphi_{\dot{\alpha}} = \frac{1}{2} \pi_{\alpha \dot{\alpha}} \) where \( \pi = \pi_\mu \bar{\sigma}_\mu \). Therefore, by definition of amplitudes \( O^\Sigma(\pi, \rho) \) it follows that

\[
O^\Sigma(\pi, \rho) = e^{-\frac{x^2}{4m^2}} O^\Sigma \left( -\frac{\partial}{\partial z_\alpha}, -\frac{\partial}{\partial \bar{z}_{\dot{\alpha}}} \right) J_0(X) =
\]

\[
e^{-\frac{x^2}{4m^2}} \frac{(N-Y)/2}{\pi_{\alpha \dot{\alpha}} \cdots \pi_{\omega \dot{\omega}}} \left( -\frac{\partial}{\partial z_\delta} \right) \cdots \left( -\frac{\partial}{\partial \bar{z}_{\dot{\chi}}} \right) J_0(X).
\]

To write \( \frac{\partial}{\partial z_\delta} = (\bar{\pi} z)_\alpha \frac{\partial}{\partial X} \) and to use the formula [15]

\[
\left( \frac{1}{X} \frac{\partial}{\partial X} \right)^m \frac{1}{X^\nu} J_\nu(X) = (-1)^m \frac{1}{X^{\nu+m}} J_{\nu+m}(X),
\]

we obtain (here \( (\bar{\pi} z)_\delta = \bar{\pi} \delta z \))

\[
O^\Sigma(\pi, \rho) = e^{-\frac{x^2}{4m^2}} (\bar{\pi} z)_\delta \cdots (\bar{\pi} z)_\chi \frac{1}{2} \pi_{\alpha \dot{\alpha}} \cdots \pi_{\omega \dot{\omega}} \frac{1}{X^Y} J_Y(X). \quad \text{(Appx. 7)}
\]

Here \( Y \geq 0 \). In the case of \( b \geq a \) (\( Y \leq 0 \)) this formula would be written in the form

\[
O^\Sigma(\pi, \rho) = e^{-\frac{x^2}{4m^2}} (\bar{\pi} z)_\delta \cdots (\bar{\pi} z)_\chi \frac{1}{2} \pi_{\alpha \dot{\alpha}} \cdots \pi_{\omega \dot{\omega}} \frac{(-1)^Y}{X^{\lvert Y \rvert}} J_{\lvert Y \rvert}(X).
\]

If to distinguish additional variables in \( O^\Sigma(\varphi) \), it is necessary to write

\[
O^\Sigma(\varphi) = \varphi^a \bar{\varphi}^b \varphi_\alpha \bar{\varphi}_{\dot{\alpha}} \varphi_\omega \bar{\varphi}_{\dot{\omega}}.
\]

where \( a' = a - a_s, b' = b - b_s \). Here \( \varphi_\pm \varphi_2, \bar{\varphi}_\pm \bar{\varphi}_2 \) are Lorentzian scalars. From \( O^\Sigma(\varphi) \) it is possible to construct the \( SL(2, \mathbb{C}) \)-multiplet, realizing a finite-dimensional representation \( (s, \bar{s}) = (a'/2, b'/2) \) of the group \( SL(2, \mathbb{C}) \).

A canonical basis of such representations is written in the form

\[
O^\Sigma(\varphi) = \varphi^a \bar{\varphi}^b O^{(s_0, s_1)}_{j,j_3}(\zeta) \quad \text{(Appx. 8)}
\]
where (see [16])

\[
O_{j,j_3}^{(s_0,s_1)}(\zeta) = (-1)^{j-s_0} \sqrt{\frac{(j+s_0)!(j-s_0)!}{(j+j_3)!(j-j_3)!}} \zeta^{s_0+j_3} (1 + |\zeta|)^{s_1-s_0-1} \times \n
\times P_{j-s_0}^{(s_0-j_3,s_0+j_3)} \left( \frac{|\zeta|^2 - 1}{|\zeta|^2 + 1} \right).
\]

Here \( s_0 = s - s = \frac{a'-b'}{2} = \frac{1}{2}F \), \( s_1 = s + s + 1 = \frac{1}{2}(a' + b') + 1 = \frac{1}{2}D + 1 \), \( s_0 \leq j \leq s_1 - 1 \), \( -j \leq j_3 \leq j \), \( \zeta = \varphi_1/\varphi_2 \), and \( P_n^{(a,b)} \) are Jacobi polynomials. The functions \( O_{j,j_3}^{(s_0,s_1)}(\zeta) \) are normalized by the condition

\[
\frac{i}{2\pi} \int O_{j,j_3}^{(s_0,-s_1)}(\zeta) O_{j,j_3}^{(s_0,s_1)}(\zeta) d\zeta \wedge d\bar{\zeta} = \frac{(j-s_0)!}{2j + 1} \delta_{jj'} \delta_{j_3j'_3}
\]

(pay attention that the representation \((s_0,-s_1)\) is conjugate to \((s_0,s_1)\)). Functions (Appx. 6) can developed further so \((Y_s = Y - F)\)

\[
O^\Sigma(\varphi) = (-1)^{j-s_0} \sqrt{\frac{(j+s_0)!(j-s_0)!}{(j+j_3)!(j-j_3)!}} \varphi_1^{s_0+j_3} \varphi_2^{s_0-j_3} (\bar{\varphi}_2 \varphi_2)^{b_3} \times \n
\times (\bar{\varphi} \varphi)^{s_1-s_0-1} P_{j-s_0}^{(s_0-j_3,s_0+j_3)} \left( \frac{\varphi \sigma_3 \varphi}{\varphi \varphi} \right).
\]

At the same time the amplitude \(O^\Sigma(\pi, \rho)\) is written in the form

\[
O^\Sigma(\pi, \rho) = (-1)^{j-s_0} \sqrt{\frac{(j+s_0)!(j-s_0)!}{(j+j_3)!(j-j_3)!}} \left( \frac{\pi_0 - \pi_3}{2} \right)^{b_3} \pi_0^{s_1-s_0-1} \times \n
\times P_{j-s_0}^{(s_0-j_3,s_0+j_3)} \left( \frac{\pi_3}{\pi_0} \right) \left( \pi z \right)^{Y_s-s_0-j_3} e^{-\frac{x^2}{4m^2}} \frac{1}{X Y} J_Y(X)
\]

having obviously a factorized structure.

Now we go over to the model \( h_{16}^{(*)} \). In the system where \( \varphi_{1k} = 0 \) (or in the space \( \mathcal{F}_0 \)) the skeletons of particles are expressed in terms of additional variables \( \varphi_k \bar{=} \varphi_{2k} \), \( \bar{\varphi}_k \bar{=} \bar{\varphi}_{2k} \) (isotopic variables now play a spin role)

\[
O^\Sigma(\varphi) = \overbrace{\varphi_A \ldots \varphi_j}^{A} \overbrace{\bar{\varphi}_j \ldots \bar{\varphi}_z}^{B}
\]

in a way analogous to the model \( h_8^{(*)} \). Representation (Appx. 8) is now rewritten in the form

\[
O^\Sigma(\varphi) = \varphi_2 A \bar{\varphi}_2 B O_{i,j_3}^{(i_0,i_1)}(\zeta), \quad \zeta = \frac{\varphi_1}{\varphi_2}
\]
where \( i_0 = \frac{1}{2}(A - B) = \frac{1}{2}Y, \ i_1 = \frac{1}{2}(A + B) + 1 = \frac{1}{2}N + 1, \ \frac{1}{2}Y \leq i \leq \frac{1}{2}N, \ -i \leq I_3 \leq i \) are weights of a representation of the isotopic group \( SL_4(2, \mathbb{C}) \). In the same way as earlier on we have

\[
O^\Sigma(\varphi) = (-1)^{i-i_0} \frac{(i + i_0)!(i - i_0)!}{(i + i_3)!(i - i_3)!} \varphi_1^{i_0+i_3} \varphi_2^{i_0-i_3} (\bar{\varphi}\varphi)^{i_1-i_0-1} \times \\
\times P_{i-i_0}^{(i_0-i_0,i_0+i_3)} \left( \frac{\bar{\varphi}^3\varphi}{\varphi^3} \right).
\]

Expanding Jacobi polynomials in a series

\[
P_n^{(\alpha,\beta)}(x) = \frac{1}{2^n} \sum_{m=0}^n C_{n+\alpha}^m C_{n-m}^{n+\beta} (x - 1)^{n-m} (x + 1)^m,
\]

we rewrite the previous formula:

\[
O^\Sigma(\varphi) = \omega_{i,i_3}^{(i_1,i_0)} \varphi_1^{i_0+i_3} \varphi_2^{i_0-i_3} (\bar{\varphi}\varphi)^{i_1-i_0-1} \times \\
\times \sum_{m=0}^{i-i_0} (-1)^m C_{m}^{i-i_3} C_{i-i_0-m}^{i+i_3} (\bar{\varphi}_1\varphi_1)^m (\bar{\varphi}_2\varphi_2)^{i-i_0-m}
\]

where \( \omega_{i,i_3}^{(i_1,i_0)} = \frac{(i+i_0)!(i-i_0)!}{(i+i_3)!(i-i_3)!} \). Since calculating \( O^\Sigma(\pi, \rho) \) it is necessary to put \( \varphi_k = -\frac{\partial}{\partial z_k} \) such that \( -\frac{\partial}{\partial z_k} = \frac{\pi_0 - \pi_3}{2} \frac{d}{dX} \), and \( X = \sqrt{2\pi\rho} = \sqrt{2(\pi_0 - \pi_3)}z_kz_k \), it is possible to write

\[
\varphi_1^{i_0+i_3+m} \varphi_2^{i_0-i_3-m} \to \left( \frac{\pi_0 - \pi_3}{2} \right)^{i+i_0} \bar{z}_1^{i_0+i_3+m} \bar{z}_2^{i_0-i_3-m} D^{i+i_0}.
\]

Therefore, as \( \varphi_k \varphi_k = \frac{1}{2}(\pi_0 - \pi_3) \), we have

\[
O^\Sigma(\pi, \rho) = e^{-\frac{X^2}{4\mu^2}} \omega_{i,i_3}^{(i_1,i_0)} (i + i_3)! (i - i_3)! \left( \frac{\pi_0 - \pi_3}{2} \right)^{i+i_0-1} \times \\
\times \sum_{m=0}^{i-i_0} (-1)^m \frac{\partial}{\partial z_1}^m (\partial/\partial z_2)^{i-i_0-m} \varphi_1^{i_0+i_3+m} \varphi_2^{i_0-i_3-m} \bar{z}_1^{i_0+i_3+m} \bar{z}_2^{i_0-i_3-m} D^{i+i_0} I(X).
\]

Denote \( v_k = \frac{\pi_0 - \pi_3}{2} |z_k|^2 D \). The validity of the following formula is verified by the method of induction

\[
\frac{1}{m!} \left( \frac{\partial}{\partial z_m} \right)^m z_k^{\alpha+m} f(X) = z_k^\alpha : L_m^\alpha(v_k) : f(X).
\]

Here \( f(X) \) is any function depending only on \( z_k \) by means of \( X \), and \( L_m^\alpha \) are Laguerre polynomials. A normal product sign : : suggests that the degree \( (v_k)^n \) is understood to be \( (\frac{\pi_0 - \pi_3}{2} |z_k|^2)^n \). Therefore \( O^\Sigma(\pi, \rho) \) may be written as

\[
O^\Sigma(\pi, \rho) = e^{-\frac{X^2}{4\mu^2}} \omega_{i,i_3}^{(i_1,i_0)} (i + i_3)! (i - i_3)! \left( \frac{\pi_0 - \pi_3}{2} \right)^{i+i_0-1} \times
\]
\[ \times z_1^{i_0+i_3} z_2^{i_0-i_3} \sum_{m=0}^{i-i_0} (-1)^m \frac{L_m^{i_0+i_3}(v_1) L_m^{i_0-i_3}(v_2)}{(i_0+i_3+m)!(i-i_3-m)!} D^{i+i_0} I(X). \]

We cannot sum up here the series over any \( v_1 \) and \( v_2 \). But this is unnecessary. Since in the theory the parameters \( z_1 \) and \( z_2 \) are large (see paragraph 6), it suffices to know the result at \( v_1, v_2 \to \infty \). As \( L_n^0(x) \to (\frac{-x}{n})^n/n! \), for large \( v_1 \) and \( v_2 \) we have

\[ (-v_2)^{i-i_0} \sum_{m=0}^{i-i_0} \left( \frac{-v_1}{v_2} \right)^m \frac{m!(i_0+i_3+m)!(i-i_0-m)!(i-i_3-m)!}{(i-i_3)!(i_0+i_3)!} \]

\[ = \frac{(-v_2)^{i-i_0}}{(i-i_0)!!} \binom{v_1+v_2}{i-i_3} (i_0+i_3)! P_{i-i_0}^{(i_0-i_3,i_0+i_3)} \left( \frac{v_1-v_2}{v_1+v_2} \right). \]

Here \( \binom{v_1+v_2}{i-i_3} \) is the hypergeometric function, and \( P_{i-i_0}^{(i_0-i_3,i_0+i_3)} \) are Jacobi polynomials. As \( v_1 + v_2 = \frac{\pi_1 - \pi_3}{2} \frac{2}{x} zD \), and \( \frac{v_1-v_2}{v_1+v_2} = \frac{\pi_3}{zz} \), finally for \( O^\Sigma(\pi, \rho) \) we obtain

\[ O^\Sigma(\pi, \rho) = e^{-\frac{X^2}{4x^2} \omega_{i_1,i_3}^{(i_1,i_0)}} \left( \frac{\pi_0 - \pi_3}{2} \right)^{i_1+i_0-1} z_1^{i_0+i_3} z_2^{i_0-i_3} (zz)^{i-i_0} \]

\[ \times P_{i-i_0}^{(i_0-i_3,i_0+i_3)} \left( \frac{\pi_3}{zz} \right) D^{2i} I(X). \]

Thus, we again have come to Jacobi polynomials from which we proceeded. Next, as

\[ D^{2i} I = \left( \frac{-2}{X dX} \right)^{2i} \frac{2}{X} J_1(X) = \left( \frac{2}{X} \right)^{2i+1} J_{2i+1}(X), \]

see [15], if to keep in mind that on the space \( F_0 \) the equality \( \frac{\pi_0 - \pi_3}{2} = \frac{X^2}{4x^2} \) is taken place, we shall have

\[ O^\Sigma(\pi, \rho) = e^{-\frac{X^2}{4x^2} \omega_{i_1,i_3}^{(i_1,i_0)}} z_1^{i_0+i_3} z_2^{i_0-i_3} (zz)^{i-i_0} \times \]

\[ \times P_{i-i_0}^{(i_0-i_3,i_0+i_3)} \left( \frac{\pi_3}{zz} \right) \left( \frac{X}{2} \right)^{2i+3} J_{2i+1}(X). \]

This expression takes place when a representation of the algebra \( h_{16}^{(s)} \) is considered on the subspace \( F_0 \), i.e. when \( \varphi_{1k} = 0 \). In the case the additional variables \( \varphi_{2k} \) and spinors \( \varphi_{\alpha,k} \) have no distinctions. It is not difficult to pass to the case when a representation of \( h_{16}^{(s)} \) is considered on the whole space \( F = F_F \otimes F_0 \). It
is only necessary that not all $\frac{\pi_0 - \pi_3}{2}$ are equal to $\frac{X^2}{4zz}$, namely, $O^\Sigma(\pi,\rho)$ is written so:

$$O^\Sigma(\pi,\rho) = \left(\frac{\pi_0 - \pi_3}{2}\right)^F \omega_{i,i_3}^{(i_1,i_0)} \frac{z_1^{i_0+i_3}z_2^{i_0-i_3}}{z z} P_{i-i_0}^{(i_0-i_3,i_0+i_3)} \left(\frac{\bar{z}T_{3\bar{z}}}{\bar{z} z}\right) \times \left(\frac{X}{2}\right)^{2i_1-2F-3} J_{2i+1}(X) e^{-\frac{X^2}{4\mu^2}} \quad \text{(Appx. 9)}$$

where $F$ is the fermion charge of amplitude. The rest $\frac{\pi_0 - \pi_3}{2}$ is the second component of spinor $\varphi_\alpha(\pi) = \frac{1}{2}(\pi_1 + i\pi_2) = \frac{1}{2}\pi_2 (\text{the first component disappears when passing to } F_0 \text{ and occurs in the return transition from } F_0 \text{ to } F)$. Therefore on $F$ the Lorentzian factor $\left(\frac{\pi_0 - \pi_3}{2}\right)^F$ in (Appx. 6) is written in the form

$$O^\Sigma_i = \frac{1}{2} \pi_\omega \ldots \pi_{\omega 2} = \frac{1}{2F} \varphi_\alpha(\pi) \ldots \varphi_\omega(\pi). \quad \text{(Appx. 10)}$$

For isotopic factor we shall have

$$O^\Sigma_i = \omega_{i,i_3}^{(i_1,i_0)} \frac{z_1^{i_0+i_3}z_2^{i_0-i_3}}{z z} P_{i-i_0}^{(i_0-i_3,i_0+i_3)} \left(\frac{\bar{z}T_{3\bar{z}}}{\bar{z} z}\right), \quad \text{(Appx. 11)}$$

and for $N_\Sigma$ we obtain

$$N_\Sigma = \left(\frac{M_\Sigma}{2}\right)^{2i_1-2F-3} J_{2i+1}(M_\Sigma) e^{-\frac{M_\Sigma^2}{4\mu^2}} \quad \text{(Appx. 12)}$$

(let us assume $X^2 = M_\Sigma^2$). As $\frac{\pi_\omega}{z z} = |\eta^2 - 1| = \frac{1}{\lambda^2} = \frac{1}{136}$, in (Appx. 11) it is possible to take $P_{i-i_0}^{(i_0-i_3,i_0+i_3)}(0)$.

Consider the $L$-spinor $\varphi_\alpha(\pi)$. It may be written as

$$\varphi_\alpha(\pi) = \frac{1}{2} \bar{\pi}_\alpha = \bar{\pi} a = \bar{\pi} + \bar{\rho}$$

where $a = \frac{1}{2}(0^1)$ is a constant spinor. Define now a $R$-spinor by the formula

$$\chi_\alpha(\mathcal{P},\mathcal{Q}) = \frac{\bar{\rho} \bar{\pi} a}{X} = \frac{\left(\bar{\rho} - \bar{\pi}\right) \left(\bar{\rho} + \bar{\pi}\right) a}{4X}.$$

If in (38) instead of $O^\Sigma(\mathcal{P},\mathcal{Q})$ we substitute $\psi = \langle \chi_\alpha(\mathcal{P},\mathcal{Q}) \rangle$ and subsequenty we integrate on $\mathcal{Q}$, previously having put $Y_\mu = 0$ (thus linear powers of $\mathcal{Q}$ vanish), we come to a bispinor $\psi = \langle \chi \rangle$ equal to

$$\psi(\mathcal{P}) = \frac{1}{2} \left(\frac{M_\Sigma a}{\mathcal{P} a}\right).$$

As can be seen, a wave function of a Dirac particle in the momentum representation is a Penrose twistor.
Appendix 3

Here it will be shown that the number $\Lambda$ including in the commutation relations (2) cannot be any number, it is connected with the dimension $\dim d$ of the Lie algebra $d$ of the dynamical group by the relation $\Lambda = \sqrt{\dim d}$.

Consider the general case of the algebra $h_{2n};$ the dimension of its automorphism group $Sp(n, C)$ (the dynamical group $D^{(n)}$ for which the Lie algebra is denoted by $d^{(n)}$) is equal, as is known, to $\dim d^{(n)} = N(N+1)$ where $N = 2n$.

First of all it should be noticed that if representations with different $\Lambda$ and $\Lambda'$ are mathematically equivalent, satisfying the condition $\Lambda \Lambda' > 0$ (they are not equivalent to the representation with $\Lambda = 0$), then physically such representations are not equivalent since different electromagnetic charges are connected with them (as will be shown below).

Consider an arbitrary transformation (an excitation of system) from the reductive group of dynamical transformations $D^n \otimes U(1)$ generated by the Lie algebra $d^{(n)} \oplus \frac{A}{4}$ (its dimension is equal to $\frac{N(N+1)}{2}+1$). Since the semisimple group $D^{(n)}$ covers twice the group $Sp(n, C)$ to realize a “spinor” representation of the latter (just consider the subgroup $SL(2, C)$ in $Sp(n, C)$ which is covered twice by an appropriate subgroup in $D^{(n)}$, see [4]; in publications the representation is referred to as a metaplectic one), and quadruply the minimal group $Sp(n, C)/\mathbb{Z}_2$, which is locally isomorphic to the group $Sp(n, C)$ (see [1] where the group $D^{(n)}$ realizes semispinor representations of the rotation group $SO(3)$), it is natural to consider that the reductive group covers quadruply the minimal group locally isomorphic to it. Therefore the group $u(1)$ covers quadruply the group $U(1)$, a generator of which is $\Lambda$, and consequently $\frac{A}{4}$ is a generator of the group $u(1)$.

A transformation from the group $D^n \otimes u(1)$ is written in the form $e^{i\varphi}$ where $\varphi = \Gamma_k \theta_k + \frac{A}{4} \theta$. In the case one can write an eikonal equation:

$$\sum_{k=1}^{n(2n+1)} \left( \frac{\partial \varphi}{\partial \theta_k} \right)^2 + \left( \frac{\partial \varphi}{\partial \theta} \right)^2 = \sum_{k=1}^{n(2n+1)} \Gamma_k^2 + \left( \frac{\Lambda}{4} \right) = 0.$$  \hspace{1cm} (Appx. 13)

Hence we obtain

$$|\Lambda| = \left( -16 \sum_{k=1}^{n(2n+1)} \Gamma_k^2 \right)^{1/2}.$$  \hspace{1cm} (Appx. 14)

Now we shall show that $-16 \sum_{k=1}^{n(2n+1)} \Gamma_k^2 = n(2n + 1)$.

Let a set of $2n \times 2n$-matrices $\{\gamma_k\}, \ k = 1, 2, \ldots, n(2n+1)$ be a basis in $Sp(n)$. These matrices are antisymmetric in the sense that they satisfy the condition $E^{-1} \gamma_k^T E = -\gamma_k$. They obey to Lie brackets $[\gamma_k, \gamma_m] = C_{km}^l \gamma_l$ where $C_{km}^l$ are structural constants of the Lie algebra $sp(n)$. The mapping $\gamma_k \rightarrow \Gamma_k = \frac{1}{2\Lambda} E \phi \gamma_k \phi$
where $\phi_{\alpha}$ are generators of the algebra $h_{2n}$ satisfying the commutation relations

$$[\phi_{\alpha}, \phi_{\beta}] = \Lambda E_{\alpha\beta} \quad \text{(Appx. 15)}$$
defines a homomorphism (in fact an isomorphism) from the algebra $sp(n)$ to the algebra $d^{(n)}$, as

$$[\Gamma_k, \Gamma_m] = \frac{1}{2\Lambda} E[\gamma_k, \gamma_m] \phi = C^l_{km} \Gamma_l. \quad \text{(Appx. 16)}$$

Consider the complete orthonormal system of $2n \times 2n$-matrices $\gamma_k$ acting in $h_{2n}$ as in a linear space. The normalization condition is written in the form

$$(\gamma_k, \gamma_m) = Sp \gamma_k \gamma_m = c \delta_{km}. \quad \text{(Appx. 17)}$$

These matrices can be chosen in such a way that the antisymmetric ones coincide with a basis of the Lie algebra $sp(n)$, $c = \frac{1}{2}$ [17].

From (Appx. 17) it follows that if $\gamma_k \neq 1$, $Sp \gamma_k = 0$. For the unit matrix we fix the index $k = 0$. From (Appx. 17) it follows that $\gamma_0 = \sqrt{\frac{c}{2n}} I$ where $I$ is the $2n \times 2n$-unity matrix. We write now the completeness condition of matrices $\gamma_k$:

$$\sum_{k=0}^{(2n)^2-1} (\gamma_k)_{\alpha\beta} (\gamma_k)_{\gamma\delta} = c \delta_{\alpha\delta} \delta_{\gamma\beta}. \quad \text{(Appx. 18)}$$

This condition is in agreement with the normalization condition (Appx. 17).

Let us contract formula (Appx. 18) with generators $\phi$ of the algebra $h_{2n}$. As a result we obtain

$$\sum_{k=0}^{n(2n+1)} (E\phi \gamma_k \phi) (E\phi \gamma_k \phi) = c E\phi (\phi E\phi) \phi = -c\Lambda^2 n^2, \quad \text{(Appx. 19)}$$
as $\phi E\phi = \Lambda n$ (it follows from (Appx. 15)). In this formula from the full set of $(2n)^2$ matrices $\gamma_k$ the only remainders are $n(2n + 1)$ antisymmetric matrices, forming a basis of the Lie algebra $sp(n)$, and the unit matrix. Other $n(2n-1) - 1$ symmetric matrices (except for the unit one) satisfying the condition $E^{-1} \gamma_k^T E = \gamma_k$ after contracting with $\phi$ have given zero. In the case of matrices $\gamma_0$ we have

$$E\phi \gamma_0 \phi = -\Lambda \sqrt{\frac{cn}{2}}.$$

Since the matrix $\gamma_0$ does not belong to the algebra $sp(n)$, we exclude it from the sum (Appx. 19). As a result we obtain

$$\sum_{k=1}^{n(2n+1)} (E\phi \gamma_k \phi) (E\phi \gamma_k \phi) = -c\Lambda^2 n^2 - c\Lambda^2 n \frac{n}{2} = -\frac{c}{2} \Lambda^2 n(2n + 1). \quad \text{(Appx. 20)}$$
To recall the definition of operators $\Gamma_k$ and assume $c = 1/2$, we come to the relation (attention should be paid to a minus sign in the right part)

$$\sum_{k=1}^{n(2n+1)} \Gamma_k^2 = -\frac{1}{16} n(2n + 1).$$  
(Appx. 21)

From (Appx. 14) we now obtain

$$|\Lambda| = \sqrt{n(2n + 1)} = \sqrt{\frac{N(N+1)}{2}}.$$  
(Appx. 22)

Henceforth we shall see that the primeval value of the Sommerfeld fine structure constant $\alpha = \frac{1}{\Lambda^2}$. In the case of the algebra $h^{(s)}_{16} \ n = 8$, therefore $\Lambda = \sqrt{136}$. This result could also be obtained from the Heisenberg theory [3] (for this purpose all is present there). In our opinion Heisenberg was close to this idea.

We remind also about the Eddington work [18] in which, as a matter of fact, he has predicted formula (Appx. 22) in the specific case $n = 8$, and he has "improved" it having put the right part equal to $\sqrt{\frac{N(N+1)}{2}} + 1$. However, he expressed the doubling of the number $n$, i.e. $N = 2n$ not correctly.
Appendix 4

Table of masses of “bare” fundamental hadrons (in GeV).

| $M_{\Sigma}$ | N: $i = \frac{1}{2}$ | $\Lambda$: $i = 0$ | $\Sigma$: $i = 1$ | $\Delta$: $i = \frac{3}{2}$ |
|--------------|----------------------|---------------------|---------------------|---------------------|
| $N = 2n + 1$ | $Y = 1$              | $Y = 0$             | $Y = 0$             | $Y = 1$             |
| $N = 2n + 2$ | $Y = 0$              |                     |                     |                     |
| $N = 2n + 3$ | $Y = 0$              |                     |                     |                     |

| n | Theor. | Exper. | Theor. | Exper. | Theor. | Exper. | Theor. | Exper. |
|---|--------|--------|--------|--------|--------|--------|--------|--------|
| 0 | 1,14   | 0,94   | 1,15   | 1,12   | 1,22   | 1,19   | 1,34   | 1,23   |
| 1 | 1,27   | —      | 1,31   | 1,33   | 1,36   | —      | 1,44   | —      |
| 2 | 1,39   | 1,39   | 1,43   | 1,40   | 1,47   | 1,38   | 1,55   | 1,55   |
| 3 | 1,49   | 1,47   | 1,53   | 1,52   | 1,57   | 1,58   | 1,64   | 1,65   |
| 4 | 1,60   | 1,65   | 1,63   | 1,60   | 1,67   | 1,67   | 1,73   | 1,69   |
| 5 | 1,69   | 1,67   | 1,72   | 1,69   | 1,76   | 1,76   | 1,91   | 1,90   |
| 6 | 1,78   | 1,78   | 1,81   | 1,82   | 1,84   | 1,84   | 1,89   | 1,89   |
| 7 | 1,86   | 1,81   | 1,89   | 1,87   | 1,92   | 1,92   | 1,97   | 1,95   |
| 8 | 1,94   | 1,99   | 1,97   | 2,01   | 1,99   | 2,00   | 2,04   | —      |
| 9 | 2,02   | 2,00   | 2,05   | 2,08   | 2,06   | 2,03   | 2,11   | —      |
| 10| 2,09   | 2,10   | 2,12   | 2,11   | 2,13   | 2,10   | 2,18   | 2,16   |

| $M_{\Sigma}$ | $\Xi$: $i = \frac{1}{2}$ | $\varepsilon$: $i = 0$ | $\rho$: $i = 1$ | $K^*$: $i = \frac{1}{2}$ |
|--------------|--------------------------|------------------------|-----------------|-----------------|
| $N = 2n + 5$ | $Y = -1$                 | $Y = 0$                | $Y = 0$         | $Y = 1$         |
| $N = 2n$     |                          |                        |                 |                 |
| $N = 2n + 3$ |                          |                        |                 |                 |

| n | Theor. | Exper. | Theor. | Exper. | Theor. | Exper. | Theor. | Exper. |
|---|--------|--------|--------|--------|--------|--------|--------|--------|
| 0 | 1,39   | 1,31   | 0,73   | 0,70   | 0,72   | 0,75   | 0,84   | 0,89   |
| 1 | 1,50   | —      | 0,82   | 0,78   | 0,85   | —      | 0,95   | —      |
| 2 | 1,60   | 1,53   | 0,93   | 0,98   | 0,96   | 0,98   | 1,05   | —      |
| 3 | 1,69   | 1,68   | 1,03   | 1,02   | 1,07   | 1,10   | 1,15   | —      |
| 4 | 1,75   | 1,82   | 1,12   | 1,08   | 1,16   | 1,17   | 1,23   | —      |
| 5 | 1,86   | —      | 1,21   | 1,27   | 1,25   | 1,25   | 1,32   | 1,28   |
| 6 | 1,94   | 1,94   | 1,29   | 1,28   | 1,34   | 1,31   | 1,40   | 1,40   |
| 7 | 2,02   | 2,03   | 1,37   | 1,30   | 1,42   | 1,41   | 1,47   | 1,43   |
| 8 | 2,09   | 2,12   | 1,45   | 1,42   | 1,49   | 1,54   | 1,55   | 1,58   |
| 9 | 2,16   | —      | 1,52   | 1,51   | 1,57   | 1,60   | 1,62   | 1,65   |
| 10| 2,23   | 2,25   | 1,60   | 1,67   | 1,66   | 1,66   | 1,68   | 1,70   |

$\mu^2 = 0,065$, $k^{-1} = 2 \cdot 10^{-14}$ cm, $khc = 1$ GeV,

The experimental mass values are taken from [19].

We have to note that masses of low-lying states (in particular, of nucleons) are appreciably renormalized by switching on interactions, the existence
of which are connected with the degeneration group of the state \( \hat{f}(z) \). It is necessary to keep in mind that quanta of degeneration fields (first of all, \( \pi, \eta, K \)-mesons, photon, graviton) are not fundamental particles, they are a new kind of elementary particles.

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