Efficient monolithic solution of FE\(^2\) problems

Nils Lange\(^1,\)*, Geraldf Hütter\(^1\), Martin Abendroth\(^1\), and Bjoern Kiefer\(^1\)

\(^1\) TU Bergakademie Freiberg, Institut für Mechanik und Fluidynamik, D-09599 Freiberg

The concurrent FE\(^2\)-method is a very powerful and flexible computational tool for multi-scale problems. However the computational effort is very high. The conventional, staggered (“nested Newton”) solution scheme solves the microscopic problem iteratively within each macroscopic Newton-Raphson (NR) iteration, although the macroscopic deformation gradients as boundary conditions at the micro scale are only estimates. In this contribution a monolithic FE\(^2\) scheme is proposed, solving the displacements of both scales in a common NR loop, which proved being faster by saving expansive micro-scale iterations.

1 Introduction

The macroscopic response of a structure is highly influenced by its micro-structure. If there exists a clear separation between the characteristic length of the macro- and micro-structure, a description through a multi-scale model is an elegant way of avoiding the formulation of a complex effective constitutive law. In the past much effort has been done in the field of micro mechanics, for example to find suitable representative volume elements (RVE), or to determine the effective elastic properties through analytical and numerical procedures. If the micro structure shows a nonlinear or irreversible response, often methods have to be applied, where the RVE is computed concurrently alongside with the structural model.

In terms of numerical methods the FE\(^2\) method is the most common multi-scale method and was introduced by Feyel [1] in 1999. A comprehensive theoretical background of different numerical multi-scale approaches can be found e.g. in [2] or [3].

2 Monolithic solution method

The main drawback of the FE\(^2\) method is it’s high computational cost, resulting from FE simulations behind every macroscopic Gauss point \(\alpha\) (GP). A considerable speedup can therefore be reached by lowering the number of costly microscopic iterations. In contrast to the staggered solution method as shown in Figure 1 on the left hand side, a monolithic algorithm is proposed, where the macro and micro problems are solved in a common NR loop as shown in Figure 1 on the right hand side.

![Flowcharts of the staggered (left) and the monolithic (right) algorithm](image)

As the overall system of equation is huge, it is necessarily be solved by static condensation. This leads to equation (1) where the tangent stiffness \(C_{\alpha}^{\ell}\) is the same as in the staggered scheme, but an algorithmic consistent stress \(\Sigma_{\alpha}^{alg}\) arises.

\[
\sum_{\alpha=1}^{N_{\alpha}} w_{\alpha} \cdot B_{\alpha}^{T} \cdot \left[ \frac{\partial \Sigma_{\alpha}}{\partial H_{\alpha}} \right]^{-1} \cdot \frac{\partial F_{\alpha}}{\partial u_{\alpha}} \cdot \Delta \dot{u}_{\alpha} = - \sum_{\alpha=1}^{N_{\alpha}} w_{\alpha} \cdot B_{\alpha}^{T} \cdot \left[ \frac{\partial \Sigma_{\alpha}}{\partial H_{\alpha}} \right]^{-1} \cdot \left[ \frac{\partial F_{\alpha}}{\partial H_{\alpha}} \right] \cdot \Delta \dot{H}_{\alpha} \tag{1}
\]

After solving the global system (1) for \(\Delta \dot{u}_{\alpha}\), the increment \(\Delta \dot{u}_{\alpha}\) is computed for all GPs \(\alpha\) as shown in equation (2).

\[
\Delta \dot{u}_{\alpha} = - \left[ \frac{\partial F_{\alpha}}{\partial H_{\alpha}} \right]^{-1} \cdot \left[ \frac{\partial F_{\alpha}}{\partial H_{\alpha}} \right] \cdot \Delta \dot{H}_{\alpha} \tag{2}
\]

Further details on the algorithm, the concrete program flow and the Abaqus implementation can be found in [4]. It is noteworthy that recently a similar approach called “Direct FE\(^2\)” has been proposed by Tan et. al [5].

* Corresponding author: e-mail Nils.Lange@imfd.tu-freiberg.de, phone +49 3731 39-3371
3 Numerical example

Foam filters as shown in Figure 2 a) are used e.g. in metal filtration processes to filter out non-metallic inclusions. This leads to a high mechanical loading, especially when the melt impacts the filter at the beginning of the casting process. The ceramic bulk material loaded at very high temperatures leads together with the foam morphology to a complex, irreversible material behavior. This problem is solved with a FE² model which is shown in Figure 2 b) and meshed with linear, rectangular, axisymmetric elements. The RVE of the foam is a single Kelvin cell as shown in Figure 2 c). The material responds elastic-plastic with Youngs modulus $E$, initial yield stress $\sigma_0$ and linear hardening with hardening parameter $h$.

Fig. 2: a) filter setting, b) filter FE-model, c) Kelvin foam RVE

Fig. 3: macroscopic displacement over prescribed pressure

The macroscopic displacement over the prescribed pressure is illustrated in Figure 3, showing nonlinearities due to the contact and elastoplastic material response. Both algorithms yield virtually the same result, whereby the differences only emerge from differing iteration patterns. The monolithic simulation showed to be faster and fewer time increment cutbacks occurred.

A speedup of the FE² method can also be obtained by parallelization, which is shown in Figure 4 (taken from [4]).

4 Summary

The FE² method is a flexible and accurate but computationally expensive multi-scale simulation tool. Therefore an efficient monolithic algorithm is proposed and implemented¹, which showed to be capable of saving costly microscopic iterations.

Acknowledgements The financial support by the Deutsche Forschungsgemeinschaft within the collaborative research center SFB 920 ‘Multi-functional Filter for Metal Melt Filtration — A Contribution towards Zero Defect Materials’ (contract DFG-169148856) is gratefully acknowledged. Open access funding enabled and organized by Projekt DEAL.

References

[1] F. Feyel, Multiscale FE² elastoviscoplastic analysis of composite structures, Comput. Mater. Sci. 16 (1-4) (1999) 344-354.
[2] M.G.D, Geers, V.G., Kouznetsova, K., Matouš, Y., Julien, Homogenization Methods and Multiscale Modeling: Nonlinear Problems, in: Encyclopedia of Computational Mechanics, Vol. 2, second edition, John Wiley & Sons, 2017, pp. 1-34.
[3] J. Schröder, A numerical two-scale homogenization scheme: the FE²-method, in J. Schröder, K. Hackl, Plasticity and Beyond: Micrrostructures, Crystal-Plasticity and Phase Transitions, Springer, 2014, pp. 1-64.
[4] N. Lange, G. Hütter, K. Bjoern, An efficient monolithic solution scheme for FE² problems, Comput. Methods Appl. Mech. Engrg. 382 (2021) 113886.
[5] V.B.C. Tan, K. Raju, H.P. Lee, Direct FE² for concurrent multilevel modelling of heterogeneous structures, Comput. Methods Appl. Mech. Engrg. 360 (2020) 112694.

¹ the program is available open source at: https://tu-freiberg.de/MonolithFE2