Information Spreading on Two-Layered Multiplex Networks With Limited Contact

XIULI YU1, QIWEN YANG2, KAILIANG AI3, XUZHEN ZHU2, AND WEI WANG4

1School of Automation, Beijing University of Posts and Telecommunications, Beijing 100876, China
2State Key Laboratory of Networking and Switching Technology, Beijing University of Posts and Telecommunications, Beijing 100876, China
3Zhengzhou Branch, China United Network Communication Company Ltd., Zhengzhou 450000, China
4Cybersecurity Research Institute, Sichuan University, Chengdu 610065, China

Corresponding author: Wei Wang (wwzqbx@hotmail.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 61602048 and Grant 61773148, in part by the China Postdoctoral Science Special Foundation under Grant 2019T120829, in part by the China Postdoctoral Science Foundation (CPSF) under Grant 2018M631073, in part by the Fundamental Research Funds for the Central Universities, and in part by the Sichuan Science and Technology Program under Grant 20YYJC4001.

ABSTRACT Information spreading on multiplex complex networks has been widely concerned. In reality, the constraints of inelastic resources, such as energy, time, money, and an individual’s restrict contact capacities, can affect the information spreading process. In this paper, we propose an information spreading model with limited contact capacity on the two-layered multiplex network. In order to quantitatively study the information spreading, we adopt an edge-based compartmental theory. We find that increasing the contact capacity facilitates information spreading. If the multiplex networks with strong heterogeneous degree distributions, the information outbreak size grows continuously with the transmission probability. However, when the information spreads on multiplex networks with homogeneous or heterogeneous degree distributions, there exists a crossover phenomenon, in which the information outbreak size versus the transmission probability can grow continuously or discontinuously. Our theory agrees well with the numerical simulations.

INDEX TERMS Information spreading, limited contact, threshold model, compartmental theory.

I. INTRODUCTION

Information spreading on complex networks is an important subject in studying the spreading dynamics on complex networks, which aims at investigating the mechanism and laws of information diffusion [1], innovation adoption [2], healthy behavior [3]–[5] and financial behavior [6]. Researchers have explored many potential factors that may affect information spreading on complex networks, including node’s attributes [7]–[10], edge’s attributes [11], [12], community structure [13]–[16], and coreness [17].

Limited contacts and network multiplicity are two crucial factors that markedly affect the information spreading. Existing studies revealed that individuals are always limited by some inelastic resources, such as energy, time, and funds, which restrict their social contact capacities [18]–[21]. Individuals can only communicate with a finite number of neighbors during a short period. Golder et al. found that users only communicate with a few friends at each time in social software, such as Weibo, Facebook, and Skype, even though they have many friends [22]. The scientists usually only coauthored papers with a few partners at short notice [23], [24]. Wang et al. [25] proposed a non-Markovian model to understand the effects of contact capacity on information spreading, and found that enlarging the contact capacity makes the network more fragile to behavior spreading. Therefore, it is necessary to study the influence of limited contact capacity on information spreading.

Besides, people have different connective relations and degree distribution in different networks because of the diversity of social networks. In the past, people have done extensive researches on the modeling of a single-layer network. People may have different friends and social circles in different social networks. For example, friends in real life and online community are not the same. Therefore, the two-layered network model considers more complex information interaction, and has greater research significance. Since the heterogeneity of degree distribution
between multi-layered networks also affects information transmission, considering the models on multilayered networks is more significant [26]–[34]. Shu et al. [35] proposed a piece of non-Markov information spreading model and conducted a numerical study of information spreading in an interdependent spatial network composed of two identical networks. Wang et al. [36] studied the influence of interlayer infection and recovery on information spreading dynamics. Zhu et al. [37] proposed an edge-weight compartmental approach and studied the effect of heterogeneous weight distribution on the weighted two-layered network.

To our best knowledge, there are no studies about the effects of limited contacts and network multiplexity on the information spreading. To this end, we propose information spreading model on a two-layer network. Each individual can only transmit information to a limited number of neighbors in the model. We use a threshold model to simulate the reinforcement of nonredundant information on information spreading [38], and propose a generalized edge-based compartmental theory to analyze the mechanism of information spreading quantitatively. The paper is arranged as follows. In Sec. II, we construct an information transmission based on a double-layer network with limited contact capacity. In Sec. III, we account for an edge-based compartmental theory to analyze the information transmission mechanism on the two-layer network. In Sec. IV, we show the numerical simulation method and parameter settings. In Sec. V, we compare the theoretical predictions with the numerical simulation results and explore the influence of finite contact capacity and degree distribution heterogeneity on the information spreading mechanism. In Sec. VI, we draw conclusions.

II. MODEL DESCRIPTIONS

To explore information spreading with limited contact capacity, we construct a model based on a double-layer network with \( N \) nodes and degree distribution \( P(k) \). Two layers \( A \) and \( B \) represent two different social networks, and the edges between nodes stand for their relationships. The information spreading on a double-layer network is described as a generalized Susceptible-Adopted-Recovered (SAR) model. At each time step, every node can occupy only one of the three possible states: susceptible (S), adopted (A), or recovery (R). Nodes in the susceptible state do not adopt the information but can receive information from their neighbor nodes. In the adopted state, nodes have accepted the information and can transmit the information to partial susceptible neighbors. Moreover, the nodes in the recovered state lose interest in the information and no longer participate in the later propagation process.

In consideration of the limited contact capacity, we use \( f(k_j) \) to indicate the contact capacity of an A-state node \( j \), where \( k_j \) is the degree of \( j \). \( f(k_j) \) is equivalent to the number of its neighbors that an A-state node can transmit information. If \( f(k_j) \geq k_j \), the A-state node \( j \) can transmit information to all neighbors. If \( f(k_j) < k_j \), the A-state node \( j \) can only select \( f(k_j) \) of its own neighbor nodes randomly for information spreading, and for the selected neighbor nodes, the probability of adopting information is the transmission probability \( \lambda \). When a neighbor \( i \) of the adopted node \( j \) is in the susceptible state, it will be infected with probability \( \frac{\lambda f(k_j)}{k_j} (X \in \{A, B\}) \).

The well-known ER network model [39] and SF network model [40] are adopted as the physic infrastructure in the experiments. We use \( m_A \) and \( m_B \) to accumulate the number of pieces of information that the node received in layers \( A \) and \( B \). Every time a node \( i \) successfully receives information from a neighbor node in layer \( A \) or \( B \), \( m_A^i \) or \( m_B^i \) will increase one. The adoption thresholds of layers \( A \) and \( B \) are \( T_A \) and \( T_B \). When \( m_A^i \geq T_A \) and \( m_B^i \geq T_B \), the S-state node \( i \) can adopt the information and enter into A-state. The state of a node in a two-layer network is synchronous. That is to say, a node keeps the same state in the two-layer network. Since we considered the influence of non-redundant information on propagations, the information dissemination process is a non-Markovian process.

The information spreading process in the model follows. Initially, a fraction of \( \rho_0 \) nodes are selected as the A-state nodes, and other nodes are in the S-state. For individuals who have not received any behavioral information, the number of the cumulatively received pieces of information \( m_A = m_B = 0 \). At every time step, each adopted node \( j \) first randomly chooses \( f(k_j) \) neighbors and transmits the information to its selected sensitive neighbors with the probability \( \lambda_A(\lambda_B) \) in layer \( A(B) \). Therefore, node \( i \) will be infected with probability \( \frac{\lambda f(k_j)}{k_j} \). If the adopted state node successfully transmits the information to its neighbor node \( i \) in layer \( A(B) \), the number of pieces of information \( m_A(m_B) \) will increase one. Once the transmission is successful, node \( j \) can no longer transmit this information to node \( i \) in layer \( A(B) \). When \( m_A \geq T_A \) and \( m_B \geq T_B \) in layers \( A \) and \( B \), the S-state node \( i \) can enter into A-state. Otherwise, it remains the S-state. The state of the same node in layers \( A \) and \( B \) is synchronous. After the information transmission, the A-state node \( j \) can enter into the R-state with probability \( \gamma \) and cannot participate in the later spreading process in layers \( A \) and \( B \). Finally, the process will terminate when there are no A-state nodes.

III. EDGE-BASED COMPARTMENTAL THEORY ON WEIGHTED TWO-LAYERED NETWORK

Based on references [41], [42], we present an edge-based compartmental approach to analyze the model proposed in Section. III. By comparing the densities of three state nodes in two-layer networks, we theoretically analyze the information transmission mechanism of two-layer networks with limited contact capacity.

Assuming that an individual \( i \) is in the cavity state [43], which shows that it can only receive the information from its...
neighbors and cannot transmit the information. \( \theta_{k_j}^X(t)(X \in \{A, B\}) \) denotes the probability of node \( j \) with degree \( k_j \) does not transfer information to \( i \) in time \( t \). The probability that an individual \( i \) does not receive the information by time \( t \) can be denoted by

\[
\theta_A(t) = \sum_{k_j = 0} k_j^A P(k_j^A) \langle k_j \rangle \theta_{k_j}^A(t),
\]

and

\[
\theta_B(t) = \sum_{k_j = 0} k_j^B P(k_j^B) \langle k_j \rangle \theta_{k_j}^B(t).
\]

where \( \frac{k_j^X P(k_j^X)}{\langle k_j \rangle} (X \in \{A, B\}) \) denotes the probability of \( i \) connecting to a node with a degree of \( k_j^X \) in the \( X \) layer.

By time \( t \), the probability of an individual \( i \) with degree \( k_i^X \) receiving \( m_X \) pieces of information cumulatively in layer \( X \) is

\[
\phi_{m_A}(k_i^A, t) = \left( \frac{k_i^A}{m_A} \right) \theta_A(t) k_i^A m_A (1 - \theta_A(t))^{m_A},
\]

and

\[
\phi_{m_B}(k_i^B, t) = \left( \frac{k_i^B}{m_B} \right) \theta_B(t) k_i^B m_B (1 - \theta_B(t))^{m_B},
\]

respectively.

The probability that the node with the degree of \( \tilde{k}_i = (k_i^A, k_i^B) \) is still in susceptible state after accumulating \( m_A \) and \( m_B \) pieces of information by time \( t \) can be expressed as

\[
s(\tilde{k}, t) = (1 - \rho_0) [1 - (1 - \sum_{m_A = 0}^{T_A - 1} \phi_{m_A}(k_i^A, t))] \\
\times (1 - \sum_{m_B = 0}^{T_B - 1} \phi_{m_B}(k_i^B, t))].
\]

Thus the fraction of susceptible individuals in the network at time \( t \) is

\[
S(t) = \sum_{\tilde{k}} P(\tilde{k}) s(\tilde{k}, t).
\]

Because the nodes in the network are in one of the three states, \( \theta_{k_j}^X(t) \) can be expressed as

\[
\theta_{k_j}^X(t) = \xi_{S,k_j}^X(t) + \xi_{A,k_j}^X(t) + \xi_{R,k_j}^X(t),
\]

where \( \xi_{S,k_j}^X(t) \), \( \xi_{A,k_j}^X(t) \) and \( \xi_{R,k_j}^X(t) \), respectively represent the probability that the individual with a degree of \( k_j^X \) is in the susceptible state, the adopted state and the recovered state and has not transmitted information to its neighbors.

Since the individual \( i \) is in the cavity state, it cannot disseminate information to other neighbor nodes. Therefore, the susceptible node \( j \) in layer \( X \) can only obtain information from the remaining \( k_j^X - 1 \) neighbors in the same layer. At the end of time \( t \), the probability of individual \( j \) maintaining the susceptible state in layer \( X \) is

\[
\xi_{S,k_j}^X(t) = (1 - \rho_0) [1 - (1 - \sum_{n_A = 0}^{T_A - 1} \phi_{n_A}(k_j^A, t))] \\
\times (1 - \sum_{n_B = 0}^{T_B - 1} \phi_{n_B}(k_j^B, t)),
\]

and

\[
\xi_{R,k_j}^X(t) = (1 - \rho_0) [1 - (1 - \sum_{n_A = 0}^{T_A - 1} \phi_{n_A}(k_j^A, t))]
\times (1 - \sum_{n_B = 0}^{T_B - 1} \phi_{n_B}(k_j^B - 1, t)).
\]

Due to the limited contact capacity, an A-state node with degree \( k_j^X \) selects a neighbour to transmit information with probability \( \frac{f(k_j^X)}{k_j^X} \). And the information can transmit through this edge with probability \( \lambda \). So the spreading probability of individual \( j \) through an edge is \( \frac{\lambda f(k_j^X)}{k_j^X} \). The evolution of \( \theta_{k_j}^X(t) \) can be calculated as

\[
\frac{d\theta_{k_j}^X(t)}{dt} = -\frac{\lambda f(k_j^X)}{k_j^X} \xi_{A,k_j}^X(t).
\]

The probability that the A-state nodes turn into the R-state is \( \gamma \), and the evolution of \( \xi_{R,k_j}^X(t) \) can be expressed as

\[
\frac{d\xi_{R,k_j}^X(t)}{dt} = \gamma \xi_{A,k_j}^X(t)(1 - \frac{\lambda f(k_j^X)}{k_j^X}).
\]

Considering the initial conditions \( \theta_{k_j}^X(0) = 1 \) and \( \xi_{R,k_j}^X(0) = 0 \), we can obtain

\[
\xi_{R,k_j}^X(t) = \gamma [1 - \theta_{k_j}^X(t)]\frac{k_j^X}{\lambda f(k_j^X)} - 1].
\]

Substituting Eq.(12) into the evolution equation of \( \xi_{R,k_j}^X(t) \), it can be converted into

\[
\frac{d\theta_{k_j}^X(t)}{dt} = -\frac{\lambda f(k_j^X)}{k_j^X} [\theta_{k_j}^X(t) - \xi_{S,k_j}^X(t)] \\
+ \gamma [1 - \theta_{k_j}^X(t)](1 - \frac{\lambda f(k_j^X)}{k_j^X}].
\]

When \( t \to \infty \), we can get \( \theta_{k_j}^X(\infty) \) from Eq.(13), that is

\[
\theta_{k_j}^X(\infty) = \frac{\xi_{S,k_j}^X(\infty) + \gamma}{1 + \gamma [\frac{k_j^X}{\lambda f(k_j^X)} - 1]}.
\]
Substituting $\theta_k^X(\infty)$ into Eq. (1) and (2), we can obtain

$$\theta_\gamma(\infty) = \sum_{k_\gamma=0}^{k_\gamma} \frac{\mathcal{P}(k_\gamma^X)}{\langle k_\gamma^X \rangle} \theta_k^X(\infty),$$

$$= \frac{f_\gamma(\theta_A(\infty), \theta_B(\infty))}{\theta_A(\infty)}.$$  

(15)

For simplicity, we rewrite $\theta_\gamma(\infty)$ as

$$\theta_A = F_A(\theta_B),$$

and $\theta_B(\infty)$ as

$$\theta_B = F_B(\theta_A).$$

(17)

There exists a discontinuous growth pattern [43] when Eq. (16) is tangent to Eq. (17) with $\theta_A < 1$ and $\theta_B < 1$. We can derive the critical conditions from the following equation

$$\frac{\partial f_A(\theta_A(\infty), \theta_B(\infty))}{\partial \theta_B(\infty)} = \frac{\partial f_B(\theta_A(\infty), \theta_B(\infty))}{\partial \theta_A(\infty)} = 1.$$  

(18)

In order to facilitate the analysis, we use the special cases of $T_A = T_B = 1$ to illustrate the general theory. Eqs. (8) and (9) are simplified to

$$\xi_{A_k^\gamma}(t) = \theta_A(t)^{k_\gamma^A} - \theta_B(t)^{A_k^\gamma} - \theta_B(t)^{B_k^\gamma},$$

(19)

and

$$\xi_{A_k^\gamma}(t) = \theta_A(t)^{k_\gamma^A} + \theta_A(t)^{k_\gamma^A} - \theta_B(t)^{k_\gamma^B} - \theta_B(t)^{k_\gamma^B}. $$

(20)

Substituting Eqs. (19) and (20) into Eq. (14), we can obtain

$$\theta_k^A(\infty) = \frac{1 - \rho_0[H_A^0(\theta_A(\infty)) + H_B^0(\theta_B(\infty))]}{1 + \gamma[\frac{k_\gamma^{\lambda}}{\lambda_\gamma(k_\gamma^{\lambda})} - 1]} + \frac{\gamma - H_A^1(\theta_A(\infty), \theta_B(\infty))}{1 + \gamma[\frac{k_\gamma^{\lambda}}{\lambda_\gamma(k_\gamma^{\lambda})} - 1]},$$

(21)

and

$$\theta_k^B(\infty) = \frac{1 - \rho_0[H_A^0(\theta_A(\infty)) + H_B^0(\theta_B(\infty))]}{1 + \gamma[\frac{k_\gamma^{\lambda}}{\lambda_\gamma(k_\gamma^{\lambda})} - 1]} + \frac{\gamma - H_A^1(\theta_A(\infty), \theta_B(\infty))}{1 + \gamma[\frac{k_\gamma^{\lambda}}{\lambda_\gamma(k_\gamma^{\lambda})} - 1]},$$

(22)

where

$$H_A^0(x) = x^k, $$

(23)

$$H_A^{1,0}(x, y) = \frac{1}{k_\gamma^A} \frac{\partial H_A^{0,0}(x, y)}{\partial x},$$

(24)

and

$$H_A^{1,0}(x, y) = \frac{1}{k_\gamma^B} \frac{\partial H_A^{0,0}(x, y)}{\partial y}. $$

(25)

(26)

Subsequently, substituting Eqs. (21) and (22) into Eq. (18) can obtain the critical conditions.

We can get the solutions of the Eq.(14) by graphic method in Fig.1. When $c = 1$, the equation has only one solution regardless the value of $\lambda$. Therefore, $\theta(\infty)$ decreases continuously with $\lambda$ and leads to a continuous increase of $R(\infty)$. When $c = 8$, the number of the equation roots changes with the increase of $\lambda$. There is saddle point bifurcation since the number of the roots is either 1 or 2 or 3. The equation has only one root when $\lambda$ is smaller (for example, $\lambda = 0.3$). When $\lambda$ is larger (for example, $\lambda = 0.4$), the equation has three roots. When the number of roots is greater than 1, only the largest stable root has physical significance. As shown in Fig.1, there
exists a discontinuous critical point $\lambda_c = 0.423$, where the root of the equation is the tangent point. When $\lambda$ is larger than $\lambda_c$, the root of Eq.(14) rapidly decreases to a small value, which causes $\theta(\infty)$ decrease discontinuously and leads to a discontinuous increase of $R(\infty)$.

IV. NUMERICAL METHOD
In order to ensure the accuracy of the simulations, we average at least $10^3$ dynamical realizations. We set the network size $N = 10^4$ and mean-degree $\langle k_A \rangle = \langle k_B \rangle = \langle k \rangle = 10$. All individuals on the same network have the same contact capacity $f(k) = c$. We explore the effect of the limited contact capacity on information spreading mechanism on the two-layered ER-ER network and SF-SF network. For the ER-ER network, layer $X (X \in \{A, B\})$ is an ER network with Poisson degree distribution $p_X(k_X) = e^{-\langle k_X \rangle} \langle k_X \rangle^\langle k_X \rangle / \langle k_X \rangle!$. For the SF-SF network, layer $X$ with power-law degree distribution $p_X(k_X) = \zeta_X k_X^{-\nu_X}$, where $\zeta_X = 1 / \sum_{k_X} k_X^{-\nu_X}$ and parameter $\nu$ denotes the degree exponent of layers $A$ and $B$. The heterogeneity of network degree distribution is negatively correlated with degree distribution exponent $\nu$. When $\nu$ is small, there are a few large-degree nodes and
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V. SIMULATION RESULTS AND ANALYSIS

We first investigate the effect of adoption threshold $T_A$ and contact capacity $c$ on information spreading on the double-layer strong heterogeneous networks. We found that the information outbreak size $R(\infty)$ decreases with the increase of $T_A$ and increases with the increase of $c$ in Fig.2(a) and (b). In addition, $R(\infty)$ versus the transmission probability $\lambda$ grows continuous with the change of $T_A$ and $c$. As shown in subgraph(c), the number of nodes in the subcritical state decreases continuously with the increase of $\lambda$, then decreases continuously. The reason is that there exist a larger number of nodes with larger degree in the strong heterogeneous networks. These nodes make the individuals in subcritical state gradually turn into A-state nodes with the increase $\lambda$, so $R(\infty)$ versus $\lambda$ also grows continuously.

Then we explore the effect of adoption threshold $T_B$ and contact capacity $c$ on $R(\infty)$ and the fraction of subcritical individuals on the double-layer weak heterogeneous networks. We can also find that $R(\infty)$ decreases with the increase of $T_B$ and increases with the increase of $c$ in Fig.3(a) and (b). However, unlike the results on strong heterogeneous networks, $R(\infty)$ on the weak heterogeneous networks can grow continuous or discontinuous with the change of $c$. $R(\infty)$ grows continuous when $c$ is small, and $R(\infty)$ grows discontinuous when $c$ is larger. As shown, in Fig.3(c), when $c = 2$, the number of nodes in the subcritical state increases continuously with an increase of $\lambda$. When $c = 8$, firstly, the number of nodes in the subcritical state increases continuously with an increase of $\lambda$. After the number of nodes reaches a maximum, it decreases gradually. Since the degree distributions of the weak heterogeneous networks are more uniform, most individuals have the same degrees and adopt the behavior with the same probability. When $c$ is larger, the increase of $\lambda$ may cause a large number of nodes to adopt the behavior at the same time, which leads to a discontinuous growth pattern.
in the $R(\infty)$. In Fig.3(d), the peak value of relative variance is applied for expressing the critical transmission probability. With the decrease of adoption threshold, the critical transmission probability decreases, but the growth pattern of $R(\infty)$ on $\lambda$ does not change.

In Fig.4, we explore the effect of adoption threshold $T$ and contact capacity $c$ on information spreading on double-layer ER networks. Since the ER network is relatively uniform, we can conclude similar to Fig.3. There is a cross phenomenon in the growth pattern of $R(\infty)$ with $\lambda$: from continuous to discontinuous. Besides, the theoretical predictions agree with the numerical results very well on double-layer SF networks in Fig.2 and Fig.3. But in Fig.4, there is a discrepancy between the two predictions on double-layer ER networks. The reason is that we assume that the probability of each adopted node selecting the neighbor to transmit information is average in the theoretical predictions, which is not strict for the ER network with uniform degree distributions. Large scale nodes in strong heterogeneous networks are more likely to adopt the behavior, which is also more consistent with our assumption in the theoretical part, that is, the probability of node adopting information is $\frac{\sqrt{\langle k^2 \rangle}}{k}$.

Then we explore the effects of $c$ and $\lambda$ on $R(\infty)$ with different $\nu$ on information spreading in Fig.5. Comparing subgraphs (a) with (b), we can find that when $\nu$ is small, the growth pattern of $R(\infty)$ on $\lambda$ is always continuous regardless of the value of $c$. Instead when $\nu$ is larger in subgraph(b), $R(\infty)$ versus $\lambda$ grows discontinuously or continuously. In order to further study the effect of $\nu$ and $\lambda$ on the continuity, we study the influence of $\nu$ and $\lambda$ on $R(\infty)$ with different $c$ in Fig.6.

As shown in Fig.6, when $c = 1$, $R(\infty)$ increases continuously with the increase of $\lambda$, the growth pattern of $R(\infty)$ is independent of the heterogeneity of the degree distribution. When $c = 8$, there exists a cross phenomenon: $R(\infty)$ grows from continuous to discontinuous with the increase of $\lambda$. As shown in the Fig.6, there is a critical degree distribution exponent $\nu_c$. When $\nu$ is larger than $\nu_c$ (below the white line), $R(\infty)$ versus $\lambda$ grows discontinuously. Otherwise, $R(\infty)$ versus $\lambda$ grows continuously (above the white line). In addition, above the white line, we can also get the values of $\lambda_c$ which we mentioned in Fig.1 by number of iterations method [45]. When $\lambda$ is larger than $\lambda_c$, $\theta(\infty)$ suddenly decreases to a very small value, resulting in a discontinuous increase of $R(\infty)$.

Finally, we explore the effect of the heterogeneity of network degree distribution on information spreading. As shown in Fig.7, we can find that when $\lambda$ is smaller, reducing the degree distribution exponent $\nu$ (increasing the heterogeneity...
of degree distribution) can promote information transmission. Instead, when $\lambda$ is larger, increasing the degree distribution exponent $\nu$ (reducing the heterogeneity of degree distribution) can promote information transmission. The reasons for this phenomenon are as follows [45]: There are some nodes with large degrees and many nodes with small degrees in the strong heterogeneous network. The nodes with large degrees can receive information even if $\lambda$ is very small. However, when $\lambda$ is larger, the weakly heterogeneous network with more average degree distribution is more conducive to information transmission.

In order to verify our conclusion in the real network, we do experiments on the dataset of Email network (Email) [46], containing the network of email communication of University Rovira i Virgili (URV) in Tarragona, Spain. Layer $A$ is the Email network (Email) and layer $B$ is the random reconnection network of layer $A$. The number of nodes in the network is 1133 and $(k_A) = (k_B) = (k) = 9.62$. As shown in Fig.8, there is a cross phenomenon in the growth pattern of $R(\infty)$ with $\lambda$: from continuous to discontinuous. When $c$ is large, the number of critical nodes increases first and then decreases. We can get a conclusion similar to that on the double-layer weak heterogeneous network.

VI. CONCLUSION

Due to the limitation of resources, energy, and other factors, information spreading on social networks will be affected. In this paper, we consider the limited contact transmission process on two-layer networks. We propose an information transmission model with limited contact capacity on the two-layer network and an edge-based compartmental theory to analyze the impact of information transmission from both simulation and theoretical perspectives. Through many experiments, our simulation results and theoretical results can fit very well.

We found that increasing the contact capacity $c$ can facilitate information spreading continuously on the double-layer strong heterogeneous networks. On double-layer strong heterogeneous networks and double-layer ER networks, there exists a crossover phenomenon, in which $R(\infty)$ can grow continuously or discontinuously with the increase of $\lambda$. We also found that there is a critical degree distribution exponent $\nu_c$. Only when $\nu$ is larger than $\nu_c$, we can find the crossover phenomenon by increasing $c$. We used the discontinuous critical point to explain the reasons for this phenomenon. Besides, increasing the heterogeneity of degree distribution can promote information transmission when $\lambda$ is very small while reducing the heterogeneity of degree distribution can promote information transmission when $\lambda$ is larger. In this paper, we study the limited contact information spreading on two-layer networks. Our theoretical analysis process can be applied to the study of other influencing factors on two-layer networks. Besides, we explore the impact of limited communication on information dissemination, which provides a reference for the process research of resource allocation affected by time, space, energy and other factors on social networks.

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KAILIANG AI was born in 1975. He received the bachelor’s degree in accounting from the Henan University of Economics and Law. He currently majors in management of network information at the Zhengzhou Branch of China Unicom.

XUZHEN ZHU was born in 1984. He received the Ph.D. degree in communication and information system from the Beijing University of Posts and Telecommunications (BUPT), Beijing, China, in 2015. He is currently an Associate Professor with the School of Information and Communication Engineering, BUPT. He has participated in national-level scientific research projects, such as projects 863 of the Natural Science Foundation and the National Natural Science Foundation of China. His research interests include data mining and spreading dynamics in large data environment.

WEI WANG received the Ph.D. degree from the University of Electronic Science and Technology of China, Chengdu, China, in 2017. He is currently an Associate Professor with Sichuan University, Chengdu. He has published more than 60 articles in the field of network science and spreading dynamics. His current research interests include investigating the spreading mechanisms of information, epidemic, rumor, and associated critical phenomena in complex networks.

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