Load and Stress Distribution of Thread Pair and Analysis of Influence Factors

Shikun Lu1,2*, Dengxin Hua1*, Yan Li1*, Fang yuan Cui1, Pengyang Li1

1Faculty of Mechanical and Precision Instrument Engineering, Xi'an University of Technology, Xi'an 710048
2Laiwu Vocational and Technical College, Laiwu 271100

Prof. Dengxin Hua: xauthdx@163.com
Prof. Yan Li: ly-jxy@xaut.edu.cn
Mrs. Fangyuan Cui: cfyxaut@126.com
Dr. Peng-yang Li: lipengyang@xaut.edu.cn

*Corresponding Author: Shi-kun Lu; email: lushikun@163.com; phone:18263458369.
Corresponding author: Li Yan, Ph.D, Professor and PhD supervisor.

Abstract: Analyzing the axial load(axial force) distribution and the stress distribution of the root of the thread is helpful for accurately predicting the thread failure and optimizing the design of the structure of the nut and the bolt. The existing research mainly analyzes the axial load distribution and stress distribution of the thread by using the two-dimensional(2D) finite element model(FEM), and they have no theoretical support. In this paper, a three-dimensional(3D) finite element model was established, and a theoretical model of the stress calculation of the root of the thread was established. The theoretical calculation results of this paper were compared with the results of the 3D finite element model analysis. The results show that the two are in good agreement with each other, which verifies the consistency between the theoretical model and the 3D finite element model. Then, the stress of the root of the bolt and nut is investigated under the action of axial force. And the factors, such as Young’s modulus E, axial total load F, friction coefficient μ, radial thickness H, engaging threads number N, thread defect which affect the load distribution are systematically studied.

1. Introduction
Threaded connections are used to connect parts into a whole [1–2], used to transmit force, torque, or movement, etc. [3] Bolt connections are widely used in various engineering fields, such as aviation, aerospace, machine tools, and precision instruments, and the accuracy of the bolt connection directly affects the quality of equipment assembly. Thread connection stiffness [4-5] also directly affects equipment assembly stiffness, and a high-quality threaded connection can effectively improve equipment grades. For these reasons, it is important to study force distribution and the factors that influence the design of a durable and high-quality thread connection.
In order to analyze the thread stress, Yang Qiang et al. [6] established a 2D model, and performed FE analysis of the thread. On this basis, the shape of the nut thread was improved. Zhou Guanghui [7] and others also used the 2D FE analysis method to summarize the influence of the nut structure and the thread design parameters on the axial load distribution of the transmission bolt. Yin YiHui [8] and others used the FE numerical simulation method to consider the elastic-plastic nature of bolts. Kenny and Patterson [9] described a method of measuring thread strain and stress. Sopwith[10] theoretically analyzed the uneven distribution of nut load distribution and discussed a method of improving the uneven distribution of load. Ibrahim Alkan [11] analyzed the effect of preloaded dental implant screw pressure distribution. Kenny [12] reviewed studies of the distribution of load and stress in fastening threads and Miller [13] established a mathematical thread stress analysis spring model, and compared it with the FE and experimented to verify the correctness of the mathematical model. Wang and Marshek [14] proposed an improved thread analysis spring model to predict the load distribution of the thread part of the connector, compared the load distribution of the elastic and yield threaded joints, and discussed the influence of the yield curve on load distribution. In order to study the load distribution, CHEN Haiping [15] discussed the analytical, photo elasticity and 2D FE methods, and the factors, which affect the load distribution are systematically studied, and the main factors were investigated by regression analysis.

2D thread model [15] for FE analysis, is a 3D model of simplification, which ignores a lot of geometric parameters. It is clear that this analysis method can not reflect the actual.

The above study did not establish a theoretical model of the root stress of thread, and they do not also analyze, or fully analyze, the effect of the thread defect, nut diameter, nut force position, joint surface friction coefficient, thread number, and other factors on the distribution of thread load and the stress distribution. In this paper, a theoretical model of the stress calculation of the root of the thread is established, and the 3D FE model is used to study the influence of the thread defect, nut radial dimension, friction coefficient, and the influence of the number of circles on the load distribution of the bolt thread. So the research of this paper is helpful for accurately predicting the thread failure and optimizing the design of the structure of the nut and the bolt.

2. Comparisons of 3D FE Method and Analytic Method

2.1 Analytic method [10] [15-16]

As shown in the Fig. 1, the upper surface of the nut as a starting point to establish the coordinate system, the axial load \( F_x \) distribution along the \( x \)-axis is expressed(SOPWITH method)[10] as

\[
F_x = F \frac{\sinh\left(\frac{x}{L}\theta_1\right)}{\sinh(\theta_1)}
\]

(1)

\( F \) is the axial total load, and where \( \theta_1 \) can be calculated from the [10].

YAMATOTO [16] also derives the formula of axial load \( F_x \) along the axial direction of any cross section of the nut.(YAMATOTO's method)

\[
F_x = F \frac{\sinh(x\lambda)}{\sinh(L\lambda)}
\]

(2)
where $\lambda$ can be calculated from the Appendix.

When the value of $\theta_1$ and $\lambda$ are get, the distribution of the axial force $F_x$ can be calculated by taking them into formulas (1) and (2) respectively. When axial force $F_x$ can be calculated, the axial stress distribution can be calculated. The average stress distribution in the axial direction of the thread is given by the following[17]

$$\bar{\sigma}_{A-stress} (x) = \frac{F_x}{A_{rcsa}}$$  \hspace{1cm} (3)

Where, $A_{rcsa}$ is the radial root cross-sectional area (effective cross-sectional area). So, the average axial stress distribution of bolts and nuts can be expressed as

$$\bar{\sigma}_{b-a-stress} (x) = \frac{F_b}{A_{bcs-area}}$$  \hspace{1cm} (4)

$$\bar{\sigma}_{n-a-stress} (x) = \frac{F_n}{A_{ncs-area}}$$

where, $A_{bcs-area}$ is the radial cross-sectional area of the root of the bolt (effective cross-sectional area), and $A_{ncs-area}$ is the root radial cross-sectional area (effective cross-sectional area) of the nut.

Suppose the stress concentration of the maximum stress $\sigma_{max}$, and the reference stress $\sigma_n$ ratio is defined as the stress concentration factor, that is, when the stress is tensile stress, compressive stress, bending stress, the stress concentration factor $K_\sigma$ can be expressed[17-19]

$$K_\sigma = \frac{\sigma_{max}}{\sigma_n}$$  \hspace{1cm} (5)

Therefore, the bolt, nut thread root stress can be expressed as

$$\sigma_{b\sigma} (x) = K_\sigma \sigma_{b-a-stress} (x) = \frac{K_\sigma F_b}{A_{bcs-area}}$$

$$\sigma_{n\sigma} (x) = K_\sigma \sigma_{n-a-stress} (x) = \frac{K_\sigma F_n}{A_{ncs-area}}$$  \hspace{1cm} (6)

Where, $K_\sigma$ is the thread stress concentration factor.

2.2 FE model

In this paper, ANSYS Workbench is used to establish 3D model for thread connection, and mesh subdivision of thread contact surface, the FE model of screw pair is shown in the following Fig. 3. Here, a triangle thread connection of M4.00×0.80 mm is established. The axial load $F$ is 100N and the number $N$ of thread engagement is 8. The load diagram of thread joint surface and total load acting surface is shown as Fig. 2, then mesh the model (see Fig. 3) the FE analysis is carried out (see Fig. 4).

![Fig. 2 Thread joint surface and total load acting surface](image)

Table 1 Thread model parameter

| Parameter                  | Nut length: $L$/mm | Semi-angle of thread: $\beta$ | Pitch of thread: $a$/mm | Depth of fundamental triangle of thread: $b$/mm | Depth of the thread: $d$/mm |
|----------------------------|-------------------|-----------------------------|------------------------|----------------------------------|------------------------------|
| Value                      | 6.38              | 308                         | 0.8                    | 0.7                              | 0.53                         |

| Parameter                  | Mean diameter of thread: $D$/mm | Poisson’s ratio of nut: $\sigma$ | Root diameter of nut: $D_n$/mm | Poisson’s ratio of bolt: $\nu_b$ | Root diameter of nut: $E_{n}$/Pa |
|----------------------------|-------------------------------|-----------------------------|--------------------------------|----------------------------------|--------------------------------|
| Value                      | 3.47                          | 0.3                         | 4.08                           | 0.3                              | $2 \times 10^{11}$               |
Parameter | Equivalent outside diameter of nut: $D_3/\text{mm}$ | Young’s modulus of the bolt: $E_b/\text{Pa}$ | Diameter of hole in bolt: $D_0/\text{mm}$ | Coefficient of friction: $\mu$
---|---|---|---|---
Value | 10.09 | $2 \times 10^{11}$ | 0 | 0.2

Table 2 The results of the extraction of the axial force of each contact surface.

| Which circle? | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Force/N | (F1) | (F2) | (F3) | (F4) | (F5) | (F6) | (F7) | (F8) |
| | 40.688 | 22.064 | 13.244 | 8.2098 | 5.56 | 4.0867 | 3.2956 | 2.8532 |

2.3 Comparison of 3D FEM and Analytic Method

2.3.1 Comparison of Load Distribution

In the SOPWITH’s method, substituting the parameter values of Table 1 into equation that from Eq.(1-2) to Eq.(1-7) (See Appendix), $\theta_1$ is calculated and have a value of 4.2349. The following Fig.5 shows the axial load comparison of the SOPWITH’s method and the FEM.

From Fig. 5 and Fig. 6, it can be seen that the three-dimensional finite element results of the axial load distribution of threads are basically consistent with the two analytical algorithms (SOPWITH algorithm and YAMATOTO algorithm). Therefore, it is considered that the 3D FE model of thread load distribution proposed in this paper is accurate and reliable.

2.3.2 Comparison of the Stress Distribution

After experiments, the stress concentration factor $K_\sigma$ of the standard thread root is 3.85[17-19]. The
total axial load $F$ is 200N. The results of theoretical calculations and the results of FE analysis for comparison, see Fig.7.

![Fig.7 Thread root stress](image)

As can be seen from the Fig.7, the stress deviation of thread root of bolt between FEM and YAMATOTO's method is small. The nut thread root stress deviation between FEM and YAMATOTO's method is a bit big, but the trend is basically the same. Therefore, the FE analysis of the stress results can be considered credible.

3. Stress analyses of nut and bolt
The M4.00×0.80 is established. To improve the efficiency of the analysis, the hexagonal nut is simplified into a round nut, and the outer diameter of the nut is $\phi_H=\phi 4.69$, $\phi 5.7$, $\phi 6.49$, $\phi 10.09$ and $\phi 16.09$ mm (see Table 3) respectively. The friction coefficient of the thread contact surface is 0.2, and the total axial load $F$ at the end of the bolt is 200N.

The modulus of Young’s materials of bolt and nut is $2E+11$ Pa, Poisson’s ratio is 0.3, the compressive yield strength $2.5E+08$ Pa, and the tensile ultimate strength is $4.6E+08$ Pa. The meshing of bolt and nut is that the bolt has 272,544 nodes and 65,736 elements, the nut has 77,882 nodes and 16,300 elements. After that, FE analysis is performed. The stress of the root of the bolt thread and the nut are obtained respectively, as shown in Fig. 8 and Fig. 9.

![Fig. 8 Stress nephogram of threaded connection](image)

(单位/MPa)
According to Fig. 9, when the axial load is the same, the thicker the nut is, the smaller the maximum stress at the root of the bolt and nut thread is. The reason for this phenomenon is that: the nut is thin, thread is easy to slip out (see Fig. 10), the greater the screw deformation, the greater the stress on the first circle is. (see Fig. 9).

Fig. 10 Sketch diagram of thread deformation

Fig. 9 shows that: The stresses at the root of each pair of threaded bolts are smaller than the root stresses of the corresponding bolt threads.

4. The influence of axial total load $F$ on the force and stress of bolt thread

Take the M4.00×0.80, and $\varphi H = 5.7$mm, the number of threaded engagement $N$ is 4. Respectively, take the end of the bolt action load $F$ as 100N, 200N, 300N, and 400N. The Young’s modulus of bolt and nut of materials is $2 \times 10^{11}$Pa, Poisson's ratio is 0.3. The friction coefficient of the thread contact surface is 0.2. The structural parameters and other material parameters of bolt and nut are same as Table 1. Then FE analysis is done, the axial load of circle of bolt threads is obtained respectively, see Fig. 11, and the stress of the root of the bolt thread is obtained (see Figure 12).
The number of circles | Percentage of total load
---|---
F=100N | 0.6
F=200N | 0.5
F=300N | 0.4
F=400N | 0.3
F=500N | 0.2

The number of circles | Percentage of total load
---|---
F=500N | 0.6
F=600N | 0.5
F=700N | 0.4
F=800N | 0.3

Fig. 11 The percentage of the total load of each thread

Figure 11 shows the axial load on the threads of each circle when the bolt thread is subjected to different axial total load. It can be seen from Fig. 11 that, when the material is in the elastic range, the bolt end total loads are 100N, 200N, 300N, and 400N, and the percentage of axial load in each circle in the total axial load is almost unchanged. It is shown that the total load has little effect on the distribution of the force of the thread under the elastic range of the material. With the increase of the axial load, the load of each thread increases at the same rate.

The following can be seen from Fig. 12: (1) The stress of the root of the bolt thread increases with the increase of the axial load. (2) The stress of the root of each circle is not equal, the stress increment of the root of the fourth circle is the smallest, and the stress of the first circle is the largest. (3) There is an increase in the axial load, the root of the first circle is damaged first.

5. Influence of the friction coefficient on the force and stress distribution of each circle of the thread

In threaded connections, different lubrication conditions result from the use of different lubricants. What is the influence of different lubrication conditions on the load distribution of the bolt teeth? People are still unknown these. In order to investigate the influence of lubrication on the force distribution of bolt thread contacts, in this paper, a FE analysis of thread contact under different lubrication conditions is performed. The bolt load F is 200N, the Young’s modulus of the bolt same as the nut, it is $E=2E+11Pa$. The value of bolt and nut structure parameters and other material parameters are the same as those mentioned above 4. The friction coefficients of the threads were taken as from 0 to 1, respectively, and the FE analysis is performed. The axial load of bolt threads is obtained, as shown in figure 13, and the maximum stress of the root of the bolt thread is obtained (see Fig 14).
The first circle
The second circle
The third circle
The fourth circle

The force of each circle/N
Frictionless
The friction coefficient is 0.4.
The friction coefficient is 0.3.
The friction coefficient is 0.2.
The friction coefficient is 0.1.

(a)                                   (b)
Fig. 13 The force of each circle of different friction coefficient

The following can be seen from Fig.13: (1) The load on the fourth thread, third and second threads of the bolt is gradually increasing slowly with the increase of friction coefficient. The load on the first thread gradually decreases slowly. The greater the coefficient of friction is, the more the force of each circle tends to be averaged. This shows that the friction coefficient can change the load distribution of the threads. However, because the bolt and nut materials are elastic, no matter the friction coefficient, the load of each thread cannot be equal. (2) It can be seen from the figure that the effect of the friction coefficient on the load distribution of each thread is not obvious. The effect of the friction on the contact surface of thread can be negligible. It can be seen from Figure 14 that the friction coefficient has a little effect on the maximum stress of the thread under the load, as the friction coefficient increases, the maximum stress of the bolt gradually decreases and reaches a fixed value.

6. The influence of the radial thickness of the nut on the force distribution of the bolt threads
Take total load $F$ as 100N. The Young's modulus of the nut and the bolt is $2E +11$ Pa. and the bolt structure parameter selection in accordance with the above 4 and the dimensions of the nut is shown in table 3. The friction coefficient of thread contact surface is 0.2. The FE analysis was performed, and gets the bolt root curve of stress, as shown in Fig. 16, and the axial load of bolt threads is also obtained(see Fig.15).

| Diameter of nuts øH/mm | a   | b   | c   | d   | e   |
|-------------------------|-----|-----|-----|-----|-----|
|                         | 4.69| 5.7 | 6.49| 10.09| 16.09|
The following can be seen from Fig. 15: The thickness of the nut is thinner, and the distribution of the load of each coil is more uneven. Conversely, and the thicker the radial thickness of the nut, the more average the load distribution of the bolt thread. The radial thickness of nut has an influence on the load distribution of bolt thread. However, from the general trend, the diameter of the nut radial thickness increases to a certain value, and the impact on the load distribution of the bolt thread is negligible, and the following can be seen from Fig. 16: (1) when the radial thickness of the nut is different, the stress distribution in the diameter of the bolt is uneven, and the stress at the root of the first circle is the maximum, and then gradually decreases. (2) It can also be seen that the smaller the radial thickness of the nut, the less uniform distribution of the stress in the minor diameter of the bolt. Conversely, the greater of the radial thickness of the nut, the more uniform of the stress distribution in the minor diameter of the bolt thread. (3). When the radial thickness of the nut is increased to a certain extent, the trend of the stress distribution in the minor diameter of the bolt is not obvious.

7. The influence of the number of circles on the load distribution

M4.00×0.80 was selected as the study object. The structure parameters of nuts are shown in table 4. The Young modulus of nuts and bolts is 2E+11 Pa. The friction coefficient of the thread contact surface is 0.2, and then the FE analysis is performed.

| Bolt joints | 1 | 2 | 3 | 4 |
|-------------|---|---|---|---|
| The number of threads in engagement | 3 | 4 | 6 | 8 |

The following can be seen from Fig. 17: (1) The number of circles of bolt thread affects the load distribution of thread. (2) The distribution of the load in each circle decreases gradually. (3) The load on each circle thread is greater than that of other bolt threads when the bolt is bolted in the three circles. When the bolt is screwed in the four circles, the load on each circle is greater than the load of the bolt circles of the six circles and the eight circles thread. When the bolt is bolted in the six circles thread, the load distributed in each circles thread is greater than the load of the corresponding circles.
of the bolt thread of the eight circles. (4) It can be seen that with the increase of the number of circles, the load distribution of each circles thread has been reduced. (5) This shows that when the number of thread circles is increased, the thread load at the front circle can be shared, when the number of the circles increases to a certain extent, the number of circles continues to increase, and the change of the load on the front circles will be less obvious.

8. Conclusions
1) 3D FE analysis results of the axial load distribution of the thread are in roughly agreement with the SOPWITH's method and YAMATOTO's method.
2) 3D FE analysis results of the thread root stress distribution of the thread are in roughly agreement with this paper's method.
3) The analysis shows that, usually, in a pair of threads engaged with each other, the bolt root stress is greater than the nut, so the first damage is the bolt, not the nut.
4) In the elastic range, the size of the bolt axial load has little effect on the load percentage of the threads. With the increase of axial load of bolt, the load of each circle is increased by the same proportion, and the percentage of the load of each circle thread is basically unchanged.
5) The Young's modulus has an effect on the load distribution of the bolt threads. If the shape of the nut is not changed, changing the ratio of Young's modulus of the nut to the bolt has a slight effect on improving the distribution of the thread load.
6) The radial thickness of nuts has an effect on the distribution of the load of the threads. The smaller the thickness of the nuts is, the more uneven the load distribution is. From the overall trend, the radial thickness of the nut increases to a certain extent, and the impact of the bolt thread load distribution will be very small.
7) The number of circles in engagement has an effect on the load distribution of each circle. With the increase of the number of circles, the load of each circle is reduced. The number of circles is increased and the load of the original circles can be shared. When the number of circles is increased to a certain degree, the change of the load quantity of the previous circles is not obvious.
8) The coefficient of friction has a slight effect on the force distribution of the threads, but not obvious, and this effect is almost negligible.

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