A logic-based resampling with matching approach to multiple imputation of missing data

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ABSTRACT

Researchers often use model-based multiple imputation to handle missing at random data to minimize bias while making the best use of all available data. However, there are contexts where it is very difficult to fit a model due to constraints amongst variables, and using a generic regression imputation model may result in implausible values. We explore the advantages of employing a logic-based resampling with matching (RWM) approach for multiple imputation. This approach is similar to random hot deck imputation, and allows for more plausible imputations than model-based approaches. We illustrate a RWM approach for multiply imputing missing pain, activity frequency, and sport data using The Childhood Health, Activity, and Motor Performance School Study Denmark (CHAMPS-DK). We match records with missing data to several observed records, generate probabilities for matched records using observed data, and sample from these records based on the probability of each occurring. Because imputed values are generated randomly, multiple complete datasets can be created. They are then analyzed and averaged in the same way as model-based multiple imputation. This approach can be extended to other datasets as an alternative to model-based approaches, particularly where there are time-dependent ordered categorical variables or other constraints between variables.

KEYWORDS

Multiple imputation, missing data, missing at random, hot deck imputation
Missing data are present in most epidemiologic studies, especially those of observational nature (1). There are many options for handling missing data, each with their own assumptions. Choosing an appropriate option is critical, as results will be biased when assumptions about the nature of missingness are not met (2).

Missing data fall into three categories: missing completely at random (MCAR), missing at random (MAR), and missing not at random (MNAR). Data are MCAR when missingness is independent of both observed and unobserved variables. Data are MAR when the pattern of missing data is only associated with observed variables (3). Data are MNAR when the pattern of missing data is associated with observed and unobserved variables.

MCAR data require the least sophisticated methods to address, but the strongest assumption. When data are MCAR, complete case analysis and common imputation methods such as mean or regression imputation, will be unbiased (4). However, MCAR is generally not a reasonable assumption (5), as there are usually factors associated with both missingness and the outcome. For instance, with an outcome of death, data collection might be more difficult in participants who are sicker and more likely to die. All the above-mentioned methods will be biased in this context.

For these reasons, researchers typically work under the weaker assumption that data are MAR. There are established methods to handle MAR data without bias (6,7). MNAR data require advanced methods and complex assumptions dependent on the specific context, and are beyond the scope of this article (3).
In addition to bias, researchers should account for imputed values having increased levels of uncertainty compared to observed values (8). In single imputation, one value is imputed for each missing value, resulting in one complete dataset for analysis. Uncertainty is not accounted for by analyses of the single imputed dataset (7). Single imputation using naïve analytic approaches tends to underestimate standard errors, resulting in confidence intervals that are too narrow. In contrast, in multiple imputation, several complete datasets are created by imputing with random variation. Each dataset is analyzed and results are averaged (7). Multiple imputation typically provides standard errors and confidence intervals that are wider and more appropriate than what are obtained by single imputation (8).

Multiple completed datasets are typically obtained by generating random predictions for missing values from a regression model (9). However, there are contexts where it is difficult to fit a generic regression model that encompasses all hypothesized patterns of missingness, particularly in cases where relationships amongst variables constrain the set of possible values.

In this article, we explore the advantages in such scenarios of employing a logic-based resampling with matching (RWM) approach to multiple imputation, similar to random hot deck imputation (10). We illustrate the process using the CHAMPS-DK study (11) that measured weekly physical activity for Danish schoolchildren over a 5.5 year period.
A COMPARISON OF MODEL-BASED AND LOGIC-BASED IMPUTATION METHODS

Model-based imputation

Model-based imputation requires fitting a model with observed data to predict missing values, which are then imputed. A simple example is mean imputation (12), where the imputed value is the mean of all observed values for an individual or subgroup of individuals (3). Mean imputation is a deterministic single imputation approach, as the same value will be generated each time. Another example is regression imputation (12), which includes additional variables in the prediction model, but a single, non-random quantity is imputed.

To perform multiple imputation, models must generate multiple random values to replace missing values. This is often done by randomly sampling regression parameters or imputed values from an underlying distribution (13).

Most common statistical software can perform model-based multiple imputation (14). While generally straightforward for continuous data, fitting a model is not always obvious when there are constraints between variables and time-dependencies. We describe a particular situation using the CHAMPS-DK study in the section “Example using CHAMPS-DK data”.

Logic-based imputation

Logic-based imputation encompasses methods that use logical rules rather than models to predict missing values. Logic-based methods may use information only from the missing variable of interest, or incorporate information from other variables.
A simple deterministic example is last observation carried forward (LOCF), where missing values are replaced with the previous observed value for the individual. LOCF can lead to illogical or impossible values in certain contexts, and often creates bias (15).

Another example is hot deck imputation, which can be deterministic or random. Most commonly used in surveys, hot deck imputation involves identifying a pool of “donors” that have similar characteristics to the record with missing data but have observed values (10). The deterministic approach identifies a single donor, typically using nearest neighbour matching. The random approach identifies multiple possible donors, randomly selects a donor, and imputes their observed value (10). Because there is random variation, random hot deck imputation can be used to create multiple data sets for multiple imputation (10).

Logic-based multiple imputation using resampling with matching (RWM)

We propose a resampling with matching (RWM) approach to multiple imputation based on hot deck imputation, with extension to longitudinal studies. We present several worked examples in the following section. In brief, we generate a pool of donors by matching the record with missing data to several observed records either between or within individuals, generating probabilities for these records, and sampling from them based on the probability of each occurring. Because imputed values are selected by probability, multiple different datasets can be created, allowing uncertainty with imputation to be accounted for. Datasets are analyzed and averaged the same way as model-based multiple imputation.
EXAMPLE USING CHAMPS-DK DATA

The relationships between variables in the CHAMPS-DK study on physical activity in Danish schoolchildren (11) includes several constraints that make model-based imputation difficult. Any model-based approach would have to incorporate these logical constraints to avoid imputing implausible values.

Overview of data

We focus on weekly data collected via SMS messages on children’s pain, frequency of recreational activity, and the sports they participated in. If no response was received for a question, the next question was not asked. As SMS messages were sent in a free-text field, responses did not always contain clear answers for the variable of interest. Where possible, entries were corrected by deduction, or else coded as missing.

Pain. Parents received an automated message asking whether their child experienced pain in their upper extremity, lower extremity, and trunk in the past week. Pain that was continuing from a previous injury was coded with an asterisk. These responses were converted into separate variables for each body location (no pain, new pain, old pain), and a fourth composite variable (no pain, new pain in at least one body location, old pain in at least one body location and no new pain).

Activity frequency. After responding about pain, parents were asked to indicate the number of organized leisure time sport activity sessions the child partook in outside of school that week from 1 to 8, with 8 referring to 8 or more sessions.
**Types of sports.** After responding about frequency, parents were asked to indicate the sports the children participated in that week with 1-9 representing different sports, and 10 representing “Other”. We refer to the number of times a child did each sport in a week as the sport count. Some parents would provide a separate number for each session their child participated in, while others would only provide a single number. For example, if a child played football (code 1) three times and handball (code 2) once, some parents would answer 1112, but others would only answer 12, leading to missing data on sport counts.

Rationale for a logic-based approach

We have missing data for pain, frequency, sport, and sport counts. There are multiple constraints amongst variables that present challenges for standard model-based imputation. The frequency of activity must be equal to or greater than the number of sports; each sport performed must have a sport count greater than 0; each sport not performed must have a sport count of 0; and the sum of sport counts must total the frequency.

Developing a model that incorporates all constraints is unlikely to be straightforward in standard regression-based imputation software. Therefore, we use a logic-based approach with rules to handle each constraint. In brief, the observed data naturally obeys the constraints of the data set. We decide which observed records we are willing to borrow information from to impute missing data, creating a pool of matching records. Randomly sampling observations from the pool into multiple datasets satisfies the constraints demanded by the data, and incorporates uncertainty in the imputed values.
In the next sections, we describe our logic-based RWM approach for imputing missing data. For each variable, we decide what information to borrow from to create the pool, including whether to borrow from within or between individuals. We derive sampling probabilities for replacement values from records within the pool, and sample accordingly.

Imputing pain

Pain is recorded for three body locations (trunk, upper extremity, and lower extremity), each of which has no pain, new pain, or old pain. There are 27 ($3^3$) possible values for pain. We believe pain in a particular week is correlated with pain in previous weeks, and also increases likelihood of pain in subsequent weeks. Although we believe that pain in one individual is independent of pain in other individuals, we believe that individuals undergo similar transitions in pain. Therefore, we derive sampling probabilities for possible values of pain, conditioning on gender, using observed patterns over weeks from the study sample.

Consider the female in Figure 1A with one missing pain entry. We focus on one body part at a time. The previous week, the female had no trunk pain. The next week, she also had no trunk pain. We retrieve all triplets of complete pain records for females that had no trunk pain in the first record or third record, forming the pool of potential matches. We calculate the proportion of entries with no, new, and old trunk pain among the middle records. The sampling probabilities used to impute pain are simply the calculated proportions. This is equivalent to randomly selecting one observation for the middle week from the set of all observed individuals which matched on gender and pain levels in the week before and after the missing event. Since most entries in this context had no trunk pain, the probability of sampling a record with no pain
was large (~99%), while the probabilities for sampling new or old pain were small. This process is repeated for each body location until all pains are imputed.

In our logic, we assume that the only variable that affects pain in one body location is pain in nearby weeks at that location within an individual, and that pain patterns over weeks from other individuals of the same gender predict patterns for the individual with missing data. An alternative approach would be to include pain in other body locations, an individual’s activity frequency in nearby weeks, or restrict the sampling pool to observed records from the individual with missing data. However, increasing complexity reduces the number of available records for sampling, which is similar to “overfitting” or “overmatching” the observed data in a statistical model. This could result in inappropriately narrow confidence intervals when the results are averaged across datasets.

In Figure 1B, the same female had a new upper extremity pain the week before the missing week, and old upper extremity pain the week after. In our data, the sampling for old pain increased, while the probability for no pain decreased.

In longitudinal studies, data may be missing for more than one sequential entry. When multiple entries are missing in a row, we derive probabilities for the first missing entry using the observed pain from the previous week and the next observed pain, accounting for the number of weeks in between. In Figure 2, the participant had new trunk pain, two weeks of missing data, and then no trunk pain. We retrieve all observed records in females where individuals had new trunk pain the week prior, observed or missing pain a week later, and no trunk pain two
weeks later to create our pool. We follow the same procedure as above, calculating the probability of no, new and old trunk pain among these records (Figure 2A).

Imagine we sampled no trunk pain for the first missing record (Figure 2B). For the next missing week, we must treat this imputed no trunk pain as observed to maintain dependence between imputed entries. Accounting for such dependencies is an important challenge for model-based methods. Figure 2C now represents the context of Figure 1 with no trunk pain for the weeks before and after the missing data. The sampling probabilities are therefore the same as Figure 1. For longer runs, we follow the same process until all missing weeks in the run are imputed.

An alternative strategy for imputing multiple sequential missing entries would be to sample a sequence of pains, matching on the previous and next observed pains. In the example above, one might create a pool using all groups of four consecutive observed records in females where new trunk pain and no trunk pain are separated by two weeks with observed values. One could then sample from this pool and impute pain values for both weeks at the same time. However, this might result in overmatching in cases where there are many missing entries in a row.

Imputing frequency

Activity levels tend to change with season and age. Therefore, one’s activity frequency in a particular week is likely to be more similar to their frequency in nearby weeks than weeks further away. Further, children in the same class are exposed to similar factors that may affect exposure to recreational activity such as socioeconomic status, weather, school events (e.g. sports tournaments), or cancellation of regular extracurricular activity. To account for their similar experience, we first calculate the median activity frequency for each week of all
individuals in the same class and of the same gender as the individual with missing data (Figure 3A, Median Class Frequency). Second, we calculate the difference between the individual’s frequency and their class median frequency for each week as a measure of how much activity they tend to do relative to their peers (“Freq-Median Class Freq”).

Our next step is to match within individuals on pain in nearby weeks (we chose 7 weeks before and after the missing entry) (Figure 3A-B). We used our composite pain variable with 3 levels, instead of considering 27 possibilities for pain which would greatly restrict the pool size. We assumed that pain in different body locations does not have different effects on activity frequency. Others might prefer to match on all 27 possible values of pain, but this greatly limits the number of matches within a time period.

Next, we randomly sample one week from the pool of possible weeks that match on pain (Figure 3C). We first impute the individual’s activity relative to their class and gender (Freq-Median-Class Freq). Then, the imputed frequency for the missing week is simply the sum of the imputed median class frequency and the imputed difference between the activity for the class and individual (Figure 3D).

When no matches were available in the 7 weeks before and after, we extended the sampling period to 12 weeks before and after (6 month period), 25 weeks before and after (1 year period), or the entire study. Others may choose different time windows depending on the context.

Our procedure is similar to random generation of values from a fixed effects model. We estimate the fixed effect for activity frequency from the data we have (the median activity
frequency for individuals in the same class of the same gender, whom we assume come from
the same distribution of relevant background characteristics). Then, we sample a residual (the
difference in the median activity frequency between the individual and their class) from the
pool of potential matches. The imputed value is then the sum of the fixed effect and the
residual.

We borrowed the median class and gender frequency from the week in question because we
believed that external factors such as holidays or extracurricular schedules are likely to result in
individuals of the same class and gender being systematically more active in certain weeks than
others. Sampling from the individual’s nearby frequencies would not be able to account for the
effect of the week itself.

An additional limitation unrelated to our method occurs because data were only collected
weekly. We could not identify whether pain reduced activity, or increased activity caused pain,
which might affect our underlying logic.

Imputing sport

We expect that while individuals are likely to participate in similar sports in nearby weeks,
these sports may change over time and season. If an individual never participates in a particular
sport, we would not expect them to have done that sport in a missing week. We also expect
that individuals might do multiple types of sports when they have a high activity frequency, and
fewer types of sports when they have a low activity frequency. Therefore, we match within
individuals on frequency to generate our pool of potential matches.
Similar to our approach for frequency, we match on closest frequency to the missing week within the 7 weeks before and after the missing record for the individual (Figure 4A and B). We randomly sample one of these weeks (Figure 4C), and impute the sports from the sampled week (Figure 4D).

When we did not have an exact match for frequency within the specified time period, we matched on closest frequency within nearby weeks. We did not extend the range of weeks as we did for missing frequency because the type of sports may depend greatly on the time of year.

When the closest frequency is less than the frequency of the missing week, any sampled week will necessarily have fewer sports than the frequency of the missing week. These records are then similar to records where parents entered 3 for frequency but only 2 sports. We explain imputation for the number of times each sport was performed in section “Imputing sport counts”.

When the closest frequency is greater than the frequency of the missing week, the sampled week might have a greater number of sports than the frequency of the missing week, which is impossible. In Figure 5A, the missing week has a frequency of 2, but nearby weeks only have frequencies of 3 or more. The randomly sampled week (Figure 5B) has three sports. To select which two of the three sports should be imputed, we calculated the probability of a sport being selected according to its relative proportion during nearby weeks (Figure 5C and 5D). We then select two sports without replacement from the pool, dropping one sport. If there are no matches on closest frequency (i.e. if an individual does not have any frequency information in
nearby weeks or all frequencies are 0), we extend the time period to 12 weeks before and after (6 month period), then 25 weeks (1 year period), then the entire study.

Our specific approach assumes that the types of sports an individual does are only related to their frequency for that week and sports in nearby weeks. More complex matching constraints could include the presence of any pain, or pain related to a specific body location. More constraints will reduce the number of matching records within the chosen time frame.

Imputing sport counts

In this study, parents typically indicated which sports were done but not how many times each sport was done (e.g. Figure 6A, frequency “3”, sport “Basketball, Football”). We need to impute sport counts in cases where the total frequency (observed or imputed) is greater than the number of sports.

For each week with missing sport counts, the individual did at least one session of each sport listed. Therefore, the number of counts we need to impute is the total frequency minus the number of sports listed (Figure 6B). Because sports that were not performed that week must have a zero probability for the remaining counts, they are not included in the sampling pool. Probabilities are derived by dividing the total counts for each sport in nearby weeks (7 weeks before and after) by the total frequency of all sports done in nearby weeks, restricting to sports that were done in the missing week (Figure 6C). In this example, we know the child played basketball once and football once. In nearby weeks, they played basketball 18 times and football 8 times (69% basketball; 31% football). We use these proportions to sample the remaining sport count so that the total sport count matches the frequency (Figure 6D).
Sometimes, there were missing sport counts for other weeks within the time period chosen (Figure 7A). For these weeks, we divide the frequency by the number of sports to obtain average counts in the missing weeks that we are not imputing (Figure 7B). These counts are not imputed into the main dataset; rather, they are temporarily used to determine the probabilities for the week of interest. The counts from observed weeks and average counts for missing weeks are then summed up as above and divided by the total frequency to obtain the sampling probabilities (Figure 7C-D). Once data are imputed for the first missing week, we proceed to the next missing week.

This approach allows for constraints between frequency and sport because we have randomly generated imputations that respect the logic underlying the data structure. However, we still assumed that the probability of doing a particular sport is independent of the other sports. Alternatively, we could use more complex logic that assumes some sports are more likely to be done together, and adjust our sampling scheme accordingly.

**APPLICABILITY OF RESAMPLING WITH MATCHING IMPUTATION TO OTHER DATASETS**

While our dataset dealt with pain and activity data, our general approach can be applied to many different types of data. Our approach for imputing pain can be extended to time-dependent ordered categorical variables, which cannot easily be handled in typical multiple imputation software such as MICE (16). For instance, studies on homelessness often collect living information. Individuals may cycle through homelessness, living in different institutions, and living in a home (17), and there is an implicit ordering to these different conditions. Missing
living information can similarly be imputed by probability based on previous and next observed records.

Another example applies to pharmacoepidemiology, where patients may alternate between analgesics acetaminophen, ibuprofen, naproxen or other drugs. One might collect information on a patient’s total number of over-the-counter medication as well as drug types. The number of drug types cannot exceed the total number of medications within a specified time period. One might also collect information on patients’ side effects. Probabilities of various side effects differ between drugs, and some side effects may never occur for certain drugs. These constraints can also be easily handled with our RWM approach. We emphasize that any specific approach will depend on the context, and that researchers should always consider what values are plausible, what information should be borrowed, and whether to borrow between and/or within individuals.

CONCLUSION

Having an appropriate strategy to handle missing data is crucial to avoid bias. While researchers often use model-based multiple imputation to minimize bias while making the best use of available data, these approaches may result in implausible values where there are constraints between variables. A logic-based resampling with matching approach respects constraints within the data, and can be used for multiple imputation by creating a pool of matching records from observed data, deriving probabilities for these records, and randomly sampling an imputed value. This approach may be particularly useful in observational studies where data are usually not MCAR. Even though many assumptions are required with the RWM approach,
any model-based approach would have to include the same assumptions or risk generating implausible values.
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FIGURE LEGENDS

Figure 1. Imputation of pain for a single missing entry. A. Sampling probabilities for females with no trunk pain in the weeks before and after the missing week. In the observed data, 99% of entries for females with no trunk pain the week before and no trunk pain the week after have no trunk pain in the middle week. We use these proportions as our sampling probabilities. B. Sampling probabilities for females with new trunk pain the week before and no pain the week after the missing week. In the observed data, 79% of entries for females with new trunk pain the week before and no trunk pain the week after have no trunk pain in the middle week. We use these proportions as our sampling probabilities.

Figure 2. Imputation of pain for two missing entries in a row. A. There are two weeks in a row where pain is missing. Sampling probabilities for females with new trunk pain the week before and no trunk pain two weeks after the missing week are shown. In the observed data, 76% of entries for females with no trunk pain the week before and no trunk pain the week after have no trunk pain in the second week of the four-week period. We use these proportions as our sampling probabilities. B. No pain is randomly imputed for the first missing week. C. Now, there is a single week where pain is missing. Sampling probabilities for females with no trunk pain the week before or after the missing week are shown. In the observed data, 99% of entries for females with no trunk pain the week before and no trunk pain the week after have no trunk pain in the middle week. D. No pain is randomly imputed for the second missing week.

Figure 3. Imputation of activity frequency. A. There is one week where frequency is missing (black row). Pain is coded as no pain in any location (No pain), new pain in at least one location
(New pain), and old pain in at least one location but no new pain (Old pain). The individual had no pain in this week. The median frequency for the individual’s class and gender is calculated for the missing and surrounding weeks (Median Class Frequency). We also calculate the difference between the individual’s frequency and the median frequency for all observed weeks (Freq-Median Class Freq). B. We match on nearby weeks with the same level of pain (gray rows). The sampling pool is comprised of eight weeks where the individual also experienced no pain. C. One of the weeks in the sampling pool is randomly selected (outlined in black). The difference between the individual’s frequency and the median class frequency for the sampled week is 1. This difference is imputed for the missing week. D. The imputed frequency for the missing week is the sum of the median class frequency for the missing week and the imputed difference between the individual and median class frequency. In this example, the imputed difference of 1 is added to the median class frequency of 2 to obtain an imputed frequency of 3.

**Figure 4. Imputation of sport. A.** There is one week where the sports performed are missing (black row). The individual had a total activity frequency of 2 in this week. B. We match on closest frequency in the nearby weeks. The sampling pool is comprised of weeks where the individual also had frequencies of 2 (gray rows). C. One of these weeks is randomly sampled with equal probability (outlined in black). D. The sports from the sampled week are imputed for the missing week.

**Figure 5. Imputation of sport where the number of sports is greater than the frequency. A.** There is one week where the sports performed are missing (black row). The individual had a
total activity frequency of 2 in this week. B. The sampling pool is comprised of nearby weeks with the closest frequency to the missing week. Since there are no weeks with a frequency of 2, we match on weeks with frequencies of 3 (gray rows). One of these weeks is randomly sampled. C. The sampled week has 3 sports, while the missing week only has a frequency of 2. The number of times in nearby weeks that the individual participated in each sport is determined. For weeks where the frequency is greater than the number of sports, the frequency is divided equally. The relative amount that the individual participated in each sport in nearby weeks is used as the sampling probability. Since the individual did basketball 10.5 times, football 7.5 times, and swimming 1 time, the probabilities are 55% (10.5/19), 40% (7.5/19), and 5% (1/19) respectively. D. Sports are randomly sampled using the sampling probabilities and imputed for the missing week. Basketball and football are randomly imputed.

Figure 6. Imputation of sport counts where a single week is missing. A. There is one week where the total frequency is greater than the number of sports performed (black row). We would like to impute individual counts for each sport that was done. B. The individual participated in at least one session of basketball and one session of football. As the total frequency for the missing week is 3, we still need to impute a single count that is either basketball or football. C. The relative proportion of each sport in the sampling pool (i.e. the sports that were done in the missing week; basketball and football) is calculated for the nearby weeks and used as the sampling probabilities. As basketball was done 9 times and football was done 5 times, the probabilities are 64% (9/14) and 36% (5/14) respectively. D. Basketball is randomly sampled. Sport counts are imputed as two sessions of basketball and one session of football.
Figure 7. Imputation of sport counts where multiple weeks are missing. A. There are two weeks where the total frequency is greater than the number of sports (black rows). We would like to impute individual counts for each sport that was done for both weeks. We focus on imputing sport counts for the first missing week. B. In the missing week, the individual had a frequency of 3 and participated in basketball and football. They must have participated in one session each of basketball and football. We must therefore impute a single count. Sport counts are calculated for the nearby weeks. When the frequency is greater than the number of sports (i.e. for the week with frequency 4), the counts are evenly distributed (assigning 2 counts to basketball and 2 counts to football). C. The relative proportion of each sport in the sampling pool (i.e. matching the sports that were done in the missing week) is calculated for the nearby weeks and used as the sampling probabilities. Since basketball was done 10 times and football 7 times, the sampling probabilities are 59% (10/17) and 41% (7/17) respectively. D. Basketball is randomly sampled. Sport counts are imputed as 2 sessions of basketball and 1 session of football.
FIGURE 1

A)  

| No trunk pain | -1 week | No pain | Old pain | New pain |
|---------------|---------|---------|----------|----------|
|               |         | 0.9919  | 0.0017   | 0.0064   |
| X             |         |         |          |          |
| No trunk pain | +1 week |         |          |          |

B)  

| New trunk pain | -1 week | No pain | Old pain | New pain |
|----------------|---------|---------|----------|----------|
|               |         | 0.9919  | 0.0017   | 0.0064   |
| X             |         |         |          |          |
| No trunk pain | +1 week |         |          |          |
FIGURE 2

A) New trunk pain
   X
   X
   No trunk pain

-1 week
+2 weeks

| No pain | Old pain | New pain |
|---------|----------|----------|
| 0.7567  | 0.2425   | 0.0007   |

B) New trunk pain
   No trunk pain
   X
   No trunk pain

-1 week
+2 weeks

| No pain | Old pain | New pain |
|---------|----------|----------|
| 0.7567  | 0.2425   | 0.0007   |

C) New trunk pain
   No trunk pain
   X
   No trunk pain

-1 week
+1 week

| No pain | Old pain | New pain |
|---------|----------|----------|
| 0.9919  | 0.0017   | 0.0064   |

D) New trunk pain
   No trunk pain
   No trunk pain
   No trunk pain

-1 week
+1 week

| No pain | Old pain | New pain |
|---------|----------|----------|
| 0.9919  | 0.0017   | 0.0064   |
### FIGURE 3

**A**

| Pain      | Frequency | Median Class Frequency | Freq - Median Class Freq |
|-----------|-----------|------------------------|-------------------------|
| No pain   | 2         | 1                      | 1                       |
| New pain  | 3         | 2                      | 1                       |
| Old pain  | 0         | 2                      | -2                      |
| Old pain  | 1         | 1                      | 0                       |
| Old pain  | 0         | 1                      | -1                      |
| No pain   | 1         | 0                      | 1                       |
| No pain   | 1         | 1                      | 0                       |
| No pain   | 2         |                        |                         |
| No pain   | 2         | 1                      | 1                       |
| Old pain  | 1         | 1                      | 0                       |
| No pain   | 0         | 0                      | 0                       |
| No pain   | 2         | 1                      | 1                       |
| New pain  | 2         | 1                      | 1                       |
| No pain   | 3         | 2                      | 1                       |
| No pain   | 3         | 2                      | 1                       |

**B**

| Pain      | Frequency | Median Class Frequency | Freq - Median Class Freq |
|-----------|-----------|------------------------|-------------------------|
| No pain   | 2         | 1                      | 1                       |
| New pain  | 3         | 2                      | 1                       |
| Old pain  | 0         | 2                      | -2                      |
| Old pain  | 1         | 1                      | 0                       |
| Old pain  | 0         | 1                      | -1                      |
| No pain   | 1         | 0                      | 1                       |
| No pain   | 1         | 1                      | 0                       |
| No pain   | 2         |                        |                         |
| No pain   | 2         | 1                      | 1                       |
| Old pain  | 1         | 1                      | 0                       |
| No pain   | 0         | 0                      | 0                       |
| No pain   | 2         | 1                      | 1                       |
| New pain  | 2         | 1                      | 1                       |
| No pain   | 3         | 2                      | 1                       |
| No pain   | 3         | 2                      | 1                       |

**C**

| Pain      | Frequency | Median Class Frequency | Freq - Median Class Freq |
|-----------|-----------|------------------------|-------------------------|
| No pain   | 2         | 1                      | 1                       |
| New pain  | 3         | 2                      | 1                       |
| Old pain  | 0         | 2                      | -2                      |
| Old pain  | 1         | 1                      | 0                       |
| Old pain  | 0         | 1                      | -1                      |
| No pain   | 1         | 0                      | 1                       |
| No pain   | 1         | 1                      | 0                       |
| No pain   | 2         |                        |                         |
| No pain   | 2         | 1                      | 1                       |
| Old pain  | 1         | 1                      | 0                       |
| No pain   | 0         | 0                      | 0                       |
| No pain   | 2         | 1                      | 1                       |
| New pain  | 2         | 1                      | 1                       |
| No pain   | 3         | 2                      | 1                       |
| No pain   | 3         | 2                      | 1                       |

**D**

| Pain      | Frequency | Median Class Frequency | Freq - Median Class Freq |
|-----------|-----------|------------------------|-------------------------|
| No pain   | 2         | 1                      | 1                       |
| New pain  | 3         | 2                      | 1                       |
| Old pain  | 0         | 2                      | -2                      |
| Old pain  | 1         | 1                      | 0                       |
| Old pain  | 0         | 1                      | -1                      |
| No pain   | 1         | 0                      | 1                       |
| No pain   | 1         | 1                      | 0                       |
| No pain   | 2         |                        |                         |
| No pain   | 1         | 2                      | 1                       |
| No pain   | 2         | 1                      | 1                       |
| Old pain  | 1         | 1                      | 0                       |
| No pain   | 0         | 0                      | 0                       |
| No pain   | 2         | 1                      | 1                       |
| New pain  | 2         | 1                      | 1                       |
| No pain   | 3         | 2                      | 1                       |
| No pain   | 3         | 2                      | 1                       |
FIGURE 4

A)

| Frequency | Sport                  |
|-----------|------------------------|
| 2         | Basketball             |
| 1         | Basketball             |
| 2         | Football               |
| 3         | Basketball, Football, Swimming |
| 2         | Basketball             |
| 2         | Basketball, Football   |
| 1         | Basketball             |
| 2         | Basketball             |
| 0         | N/A                    |
| 0         | N/A                    |

B)

| Frequency | Sport                  |
|-----------|------------------------|
| 2         | Basketball             |
| 1         | Basketball             |
| 2         | Football               |
| 3         | Basketball, Football, Swimming |
| 2         | Basketball             |
| 2         | Basketball, Football   |
| 1         | Basketball             |
| 2         | Basketball             |
| 0         | N/A                    |
| 0         | N/A                    |

C)

| Frequency | Sport                  |
|-----------|------------------------|
| 2         | Basketball             |
| 1         | Basketball             |
| 2         | Football               |
| 3         | Basketball, Football, Swimming |
| 2         | Basketball             |
| 2         | Basketball, Football   |
| 1         | Basketball             |
| 2         | Basketball             |
| 0         | N/A                    |
| 0         | N/A                    |

D)

| Frequency | Sport                  |
|-----------|------------------------|
| 2         | Basketball             |
| 1         | Basketball             |
| 2         | Football               |
| 3         | Basketball, Football, Swimming |
| 2         | Basketball             |
| 2         | Basketball, Football   |
| 1         | Basketball             |
| 2         | Basketball, Football, Swimming |
| 2         | Basketball             |
| 0         | N/A                    |
| 0         | N/A                    |
### FIGURE 5

#### Table A)

| Frequency | Sport              |
|-----------|--------------------|
| 3         | Basketball         |
| 1         | Basketball         |
| 3         | Football           |
| 3         | Basketball, Football, Swimming |
| 3         | Basketball         |
| 3         | Basketball, Football |
| 3         | Basketball, Football, Swimming |
| 1         | Basketball         |

#### Table B)

| Frequency | Sport              |
|-----------|--------------------|
| 3         | Basketball         |
| 1         | Basketball         |
| 3         | Football           |
| 3         | Basketball, Football, Swimming |
| 3         | Basketball         |
| 3         | Basketball, Football |
| 1         | Basketball         |

#### Table C)

| Frequency | Sport              |
|-----------|--------------------|
| 3         | Basketball         |
| 1         | Basketball         |
| 3         | Football           |
| 3         | Basketball, Football, Swimming |
| 3         | Basketball         |
| 3         | Basketball, Football |
| 1         | Basketball         |

#### Table D)

| Frequency | Sport              |
|-----------|--------------------|
| 3         | Basketball         |
| 1         | Basketball         |
| 3         | Football           |
| 3         | Basketball, Football, Swimming |
| 3         | Basketball         |
| 3         | Basketball, Football |
| 1         | Basketball         |

---

Sample 2 sports without replacement.

| Frequency | Sport              |
|-----------|--------------------|
| 3         | Basketball         |
| 1         | Basketball         |

17.5x Basketball – 58%
10.5x Football – 35%
2x Swimming – 7%
**FIGURE 6**

| Frequency | Sport               | Sport Counts     |
|-----------|---------------------|------------------|
| 3         | Basketball          | 3x Basketball    |
| 1         | Basketball          | 1x Basketball    |
| 3         | Football            | 3x Football      |
| 3         | Basketball, Football, Swimming | 1x Basketball, 1x Football, 1x Swimming |
| 3         | Basketball          | 3x Basketball    |
| 2         | Basketball, Football, Swimming | 1x Basketball, 1x Football, 1x Swimming |
| 1         | Basketball          | 1x Basketball    |
| 0         | N/A                 | N/A              |
| 0         | N/A                 | N/A              |

| Frequency | Sport               | Sport Counts     |
|-----------|---------------------|------------------|
| 3         | Basketball          | 3x Basketball    |
| 1         | Basketball          | 1x Basketball    |
| 3         | Football            | 3x Football      |
| 3         | Basketball, Football, Swimming | 1x Basketball, 1x Football, 1x Swimming |
| 3         | Basketball          | 3x Basketball    |
| 2         | Basketball, Football, Swimming | 1x Basketball, 1x Football, 1x Swimming |
| 1         | Basketball          | 1x Basketball    |
| 0         | N/A                 | N/A              |
| 0         | N/A                 | N/A              |
### FIGURE 7

#### A) Frequency, Sport, Sport Counts

| Frequency | Sport            | Sport Counts       |
|-----------|------------------|--------------------|
| 3         | Basketball       | 3x Basketball      |
| 1         | Basketball       | 1x Basketball      |
| 3         | Football         | 3x Football        |
| 3         | Basketball,      | 1x Basketball      |
|           | Football,        | 1x Football        |
|           | Swimming         | 1x Swimming        |
| 3         | Basketball       | 3x Basketball      |
| 2         | Basketball,      | 1x Basketball      |
|           | Football,        | 1x Football        |
| 1         | Basketball       | 1x Basketball      |
| 3         | Basketball,      | 3x Basketball      |
|           | Football         | 1x Football        |
| 1         | Basketball       | 1x Basketball      |
| 3         | Basketball       | 3x Basketball      |
| 1         | Football         | 1x Football        |
| 3         | Basketball,      | 1x Basketball      |
|           | Football,        | 1x Football        |
|           | Swimming         | 1x Swimming        |
| 3         | Basketball       | 3x Basketball      |
| 4         | Basketball,      | 3x Basketball      |
|           | Football         | 2x Football        |
| 0         | N/A              | N/A                |
| 0         | N/A              | N/A                |

#### B) Frequency, Sport, Sport Counts

| Frequency | Sport            | Sport Counts       |
|-----------|------------------|--------------------|
| 3         | Basketball       | 3x Basketball      |
| 1         | Basketball       | 1x Basketball      |
| 3         | Football         | 3x Football        |
| 3         | Basketball,      | 1x Basketball      |
|           | Football,        | 1x Football        |
|           | Swimming         | 1x Swimming        |
| 3         | Basketball       | 3x Basketball      |
| 2         | Basketball,      | 1x Basketball      |
|           | Football         | 1x Football        |
| 1         | Basketball       | 1x Basketball      |
| 3         | Basketball       | 3x Basketball      |
| 1         | Football         | 1x Football        |
| 3         | Basketball,      | 1x Basketball      |
|           | Football,        | 1x Football        |
|           | Swimming         | 1x Swimming        |
| 3         | Basketball       | 3x Basketball      |
| 4         | Basketball,      | 2x Basketball      |
|           | Football         | 2x Football        |
| 0         | N/A              | N/A                |
| 0         | N/A              | N/A                |

#### C) Frequency, Sport, Sport Counts

| Frequency | Sport            | Sport Counts       |
|-----------|------------------|--------------------|
| 3         | Basketball       | 3x Basketball      |
| 1         | Basketball       | 1x Basketball      |
| 3         | Football         | 3x Football        |
| 3         | Basketball,      | 1x Basketball      |
|           | Football,        | 1x Football        |
|           | Swimming         | 1x Swimming        |
| 3         | Basketball       | 3x Basketball      |
| 2         | Basketball,      | 1x Basketball      |
|           | Football         | 1x Football        |
| 1         | Basketball       | 1x Basketball      |
| 3         | Basketball,      | 3x Basketball      |
|           | Football         | 1x Football        |
| 1         | Basketball       | 1x Basketball      |
| 3         | Basketball       | 3x Basketball      |
| 1         | Football         | 1x Football        |
| 3         | Basketball,      | 1x Basketball      |
|           | Football,        | 1x Football        |
|           | Swimming         | 1x Swimming        |
| 3         | Basketball       | 3x Basketball      |
| 4         | Basketball,      | 2x Basketball      |
|           | Football         | 2x Football        |
| 0         | N/A              | N/A                |
| 0         | N/A              | N/A                |

#### D) Frequency, Sport, Sport Counts

| Frequency | Sport            | Sport Counts       |
|-----------|------------------|--------------------|
| 3         | Basketball       | 3x Basketball      |
| 1         | Basketball       | 1x Basketball      |
| 3         | Football         | 3x Football        |
| 3         | Basketball,      | 1x Basketball      |
|           | Football,        | 1x Football        |
|           | Swimming         | 1x Swimming        |
| 3         | Basketball       | 3x Basketball      |
| 2         | Basketball,      | 1x Basketball      |
|           | Football         | 1x Football        |
| 1         | Basketball       | 1x Basketball      |
| 3         | Basketball       | 3x Basketball      |
| 1         | Football         | 1x Football        |
| 3         | Basketball,      | 1x Basketball      |
|           | Football,        | 1x Football        |
|           | Swimming         | 1x Swimming        |
| 3         | Basketball       | 3x Basketball      |
| 4         | Basketball,      | 2x Basketball      |
|           | Football         | 2x Football        |
| 0         | N/A              | N/A                |
| 0         | N/A              | N/A                |

1x Basketball 1x Football 1x ??

15x Basketball = 68%
9x Football = 32%

1x Basketball 1x Football 1x Basketball

15x Basketball = 68%
9x Football = 32%