Pseudo-gap as a signature of inhomogeneous superconductivity in oxide interfaces

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Abstract

We present an explanation for recent tunneling experiments in LaAlO$_3$/SrTiO$_3$ interfaces which is based on the strongly inhomogeneous character of these interfaces. The measurements report signatures of superconductivity in the tunneling spectra while the global resistance of the sample is finite, i.e., a pseudo-gap state. In addition, even when the global resistance vanishes the zero-bias conductance remains finite. We show that these observations can be described by a model of superconducting (SC) islands embedded in a metallic background. The local critical temperatures of the SC island are randomly distributed, some of them necessarily exceeding the critical temperature for global percolation to the zero resistance state. Consequently, tunneling spectra display a suppression of the density of states and coherence peaks already well above the percolative transition. The temperature dependence of the spectra suggests that a sizable fraction of the metallic background becomes SC by proximity effect when the temperature is lowered.

Keywords: inhomogeneous superconductivity, oxide interfaces, tunneling spectra

(Some figures may appear in colour only in the online journal)

1. Introduction

The observation of a two-dimensional (2D) metallic state at the interface of two insulating oxides [1–4], and the subsequent demonstration of its gate-tunable metal-to-superconductor transition [5–8], have attracted much attention in the last decade. Numerous experiments indicate that the 2D electron gas (EG) is inhomogeneous: Transport measurements reveal a large width of the superconducting (SC) transition which suggests charge inhomogeneity [9–13]. At lower carrier density, the measurements show a saturation to a plateau with finite resistance, which is a clear signature of the percolating character of the metal-to-superconductor transition. Magnetometry [14–19], tunneling [20], and piezo-force spectroscopy [21] experiments report submicrometric inhomogeneities. It seems likely that inhomogeneities at nanometric scales [22] coexist with structural inhomogeneities at micrometric scales [23]. Into this picture arguably enter the recent superconductor–insulator–metal tunnel spectroscopy measurements [24], which detect a state with finite resistance, but SC-like density of states (DOS). The experiments are carried out in LaAlO$_3$/SrTiO$_3$ (LAO/STO) interfaces, by depositing a metallic Au electrode on the insulating LAO layer, and then driving a tunnel current $I$ (by means of a bias voltage $V$) between the electrode and the 2DEG (see figure 1 for an illustration). The size of the electrode measures several hundreds $\mu$m (orders of magnitude larger than the inhomogeneities [22]). The carrier density of the 2DEG is tuned by means of a back-gating voltage $V_{G}$ across the STO slab. At very low temperature, $T = 30$ mK, the measurements reveal a gap in the DOS at the Fermi energy ($E_F$) over the whole range $V_{G} \in [-300, 300]$ V, accompanied by more or less pronounced coherence peaks above the gap, signaling SC coherence and pairing as the origin of DOS suppression.
In the carrier depleted regime ($V_G < 0$), the suppression occurs even when global superconductivity is absent down to the lowest accessible temperatures, thereby highlighting the inhomogeneous character of the state formed by regions with SC pairing embedded in the non-SC matrix; upon decreasing $V_G$, the overall DOS at $E_F$ is diminished and the coherence peaks are broadened, suppressed, and shifted to slightly higher energies. At very low carrier concentration, $V_G \approx -300$ V, the coherence peaks have practically vanished, but a substantial gap is still present as a signature of (incoherent) pairing. To avoid any possible confusion, in this work we name pseudo-gap state a state which has non-zero resistance and yet exhibits a suppression of the DOS around $E_F$ and more or less pronounced coherence peaks. The presence of a (pseudo-)gap without clearly distinguishable coherence peaks might be due to some phase with translational symmetry breaking. However, the dip in the DOS spectra is always centered at zero bias, a distinctive feature of particle-particle pairing. Moreover, despite the strong suppression of coherence peaks, there is a generic continuity upon varying $V_G$, with a continuous growth of the coherence peaks when $V_G$ is increased. Therefore we interpret the (pseudo-)gap without coherence peaks as due to the presence of regions with particle-particle pairing without phase coherence. The temperature dependence of the DOS indicates a peculiar pseudo-gap behavior with a suppression persisting above the critical temperature at which the global resistance vanishes (if any). Specifically, at high carrier density $V_G = 200$ V, the 2DEG displays a SC gap and coherence peaks that decrease with temperature and vanish around 300 mK (which agrees with the critical temperature of bulk STO reported in [25]). The dependence grows more complex at $V_G = 0$, and even more so at negative $V_G$, where the gap and the coherence peaks vanish, respectively, at $T \approx 400$ mK and $T \approx 600$ mK, remarkably, well above the respective global $T_c$.

In the present work, we aim to explain these observations in a simple and coherent framework, based on the inhomogeneous character of the 2DEG. Since the measurements are taken over several hundreds \( \mu m \), we propose that the measured pseudo-gap is the result of an average over gapped SC regions and metallic regions with constant DOS.

The paper is structured as follows: in section 2 we present the model; in sections 3 and 4 we compare our theoretical results with the experimental tunneling spectra at low temperature and upon varying the temperature respectively. Section 5 contains our concluding remarks.

2. Model

The equation relating the differential conductance $dI/dV$ to the DOS $\rho(E)$ reads [26]:

$$\frac{dI}{dV}(V) = G_0 - G_1 \int_{-\infty}^{\infty} \rho(E) \frac{df(E + eV)}{dE} dE,$$

where $f(E) = (1 + e^{E/T})^{-1}$ is the Fermi function, $e$ is the electron charge, the constant $G_0 > 0$ customarily accounts for measurement errors such as leakage currents, and $G_1$ is a dimensional constant, which we hereafter reabsorb in the definition of the DOS. In this way the DOS will have the dimension of a conductance. In our model, the 2DEG consists of a metallic background embedding islands in which SC pairing occurs below a local critical temperature $T_c$, randomly distributed with a probability distribution $P(T_c)$. While for SC islands charging effects might be sizable when they are embedded in an insulating matrix, we recall that in our case the islands are embedded in a metallic background which, having the same chemical potential as the islands, acts as a charge reservoir. Consequently, charging effects are expected to be negligible.

Note that the inhomogeneous nature of the interface enters our model in two different ways, both crucial to our description: (1) an inhomogeneity at the nano-scale, which entails the occurrence of two entirely different types of regions, some metallic (down to $T = 0$ K or, at least, down to the lowest measured temperatures) and some metallic at high temperature but SC below a certain local $T_c$, and (2) the SC regions may differ from one another, possibly because of fluctuations of the microscopic disorder, yielding a distribution of critical temperatures. The possibility that also the metallic part is by itself inhomogeneous is not considered here, as it is not crucial to the description of the tunneling spectra, and would require the introduction of further fitting parameters.

In appendix A we report the results for the low-temperature spectra obtained for different probability distributions. It is shown that upon changing from one distribution to another, the resulting DOS do not differ strongly, provided that the mean and width of the distributions are comparable. This holds especially at high $T$, where the fine details of the distribution are smeared by thermal broadening.
The DOS of the 2DEG can be subdivided into three contributions:

$$\rho(E) = (1 - w) N_0 + w \tilde{N}_0 \int_T^{-\infty} dT_e \, P(T_e) + w \int_T^{\infty} dT_e \, P(T_e) \rho_{\Delta(T_e)}(E),$$

where $w \in [0, 1]$ denotes the fraction of the sample occupied by the islands. The first two terms correspond, respectively, to the metallic background and to islands where pairing has not taken place yet, and give an additional constant contribution to the differential conductivity. As shown below, this contribution fully accounts for the background in the tunneling spectra, allowing us to discard $G_0$ in equation (1). If the $T_e$ distribution is so broad to be non-zero also for $T_e < 0$, the second term stays finite even at $T = 0$. More explicitly, the regions having formally a negative $T_e$ should simply be interpreted as regions which stay metallic (i.e. non-SC) down to $T = 0$. In this case, the total fraction of the system that can display pairing down to $T = 0$, $w_{\text{pair}}$, is smaller than $w$. The third term corresponds to islands which have a finite pairing gap $\Delta$. We take the DOS of these islands to be of the form:

$$\rho_{\Delta}(E) = \tilde{N}_0 \left[ (1 - x) - \frac{|E|}{\sqrt{E^2 - \Delta^2}} + x \right] \Theta(|E| - \Delta),$$

$\Theta(E)$ being the Heaviside function. The first term is the standard BCS expression, and describes coherent pairing occurring in a $(1 - x)$ fraction of the whole gapped part. The second term describes islands which have a finite gap, but are too small to exhibit well-established phase coherence. Within our simplified model, the DOS in these small islands vanishes inside the gap while the spectral weight is spread over a very broad energy range. A more realistic description could be implemented introducing an intragap background and a finite energy range for the spectral weight recovery, at the expense of introducing additional fitting parameters. The introduction of this term is motivated by the experimental conductance curves which, in the regime $V_G \ll 0$, exhibit well-formed gaps, but practically no coherence peaks (see figure 3(a) in [24]). A similar feature of a gapped paired state without coherence peaks is observed in Sn nanoparticles (see figure 2(e) in [27]). We define a coherently paired SC fraction $w_{\text{coh}} = (1 - x) w_{\text{pair}}$ and an incoherently paired fraction $w_{\text{inc}} = x w_{\text{pair}}$.

In the absence of a microscopic model and for the sake of keeping the number of fitting parameters as small as possible, we choose to attribute the same BCS-like temperature dependence of the gap to both the large coherently and small incoherently paired regions

$$\Delta(T_c, T) = 1.76 T_c \tanh \left( \frac{\pi}{1.76} \frac{T_c}{T} - 1 \right).$$

The values of $N_0$ and $\tilde{N}_0$ should be fixed by the experimental DOS measured at high-bias, $N_0$. This value corresponds to an average over the DOS in the purely metallic regions ($N_0$) and the DOS in the regions where pairing can occur ($\tilde{N}_0$):

$$N_0 = N_0(1 - w) + w(N_0 + \delta N_0) = N_0 + w \delta N_0,$$

where $\delta N_0 = \tilde{N}_0 - N_0$. Here, we face the difficulty of having three unknowns, $N_0$, $w$, and $\delta N_0$, yet only one known, $N_0$. Renouncing to treat $N_0$ and $\delta N_0$ as fitting parameters, we are led to make some (very) simplifying assumptions; in the positive gating regime one may assume that the relative difference between the DOS of the two phases is small, allowing us to take $\delta N_0 \sim 0$. On the other hand, in the $V_G < 0$ regime $\delta N_0$ should be rather large but $w_{\text{pair}}$ rapidly decreasing, $w_{\text{pair}} \sim 0$. These assumptions then allow us to simplify the different DOS to

$$N_0 \equiv N_0 = \tilde{N}_0.$$

In this way we are left with four fitting parameters: $T_c$, $\sigma$, $w$, and $x$. A detailed description of the fitting procedures is given in appendix B.

3. Low temperature behavior in $V_G$

The choice of a Gaussian probability distribution proved rather successful in previous phenomenological analyzes of transport (resistance [10–12] and Hall conductance [13]). This choice is also supported by the detailed analysis of the low-temperature spectra obtained for different probability distributions reported in appendix A. Therefore, having established the adequacy of the Gaussian distribution, we now use it to analyze the low temperature spectra by calculating the best fits of the tunneling spectra at $T = 30 \text{ mK}$. Figure 2 shows that the fits agree remarkably well with the experimental data over the whole range of gating; the model reproduces all the characteristics of the low-temperature tunneling spectra, from the suppression of the gap minimum and the occurrence of coherence peaks in the higher carrier density regime, to the broadening and eventual suppression of the peaks at lower carrier density. The corresponding fitting parameters of figure 2, $T_c$, $\sigma$, $w$, and $x$, are reported in figure 3, while the

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**Figure 2.** Fits (black curves) of the experimental tunneling data of [24] (colored curves) with a Gaussian distribution of $T_c$. The fitting parameters are reported in figure 3. The curves at positive (negative) gating have been shifted vertically by $+0.3 \mu S$ ($-0.3 \mu S$) for a better view.
corresponding Gaussian distributions of critical temperature are displayed in figure 4. We begin with the discussion of the average critical temperature $T_c$. Two features stick out: Firstly, we notice that an increase in $V_G$ (i.e., the carrier density) steadily reduces $T_c$. The second striking feature is the value of $T_c$ itself, which for a large interval of (negative) gating is larger than the SC critical temperature usually reported for bulk doped STO (about 300 mK) [25, 28, 29].

The aim of this work is to show how the pseudo-gap phase can be properly accounted for by the inhomogeneous nature of the oxide interface; we do not intend to give an exhaustive explanation of the SC mechanism. Let us just make some comments. The decrease of $T_c$ with increasing $V_G$ suggests that the SC physics in these interfaces cannot be ruled by disorder (namely quenched impurities) alone: increasing the carrier density ($i.e., E_F$) effectively reduces the disorder parameter $(\tau E_F)^{-1}$ ($\tau$ being the scattering time) and, consequently, $T_c$ should increase [30]. Therefore, one is led to attribute the decrease of $T_c$ with $V_G$ to a decrease of the effective pairing potential $\lambda \equiv \gamma_0 \tilde{\gamma}$, where $\gamma$ is the BCS coupling. In a weak coupling BCS regime a change in the DOS $\tilde{\gamma}$ of 10% can easily yield a change in $T_c$ of 50%. It is thus conceivable that the increase of $T_c$ with decreasing gating is due to effects in the DOS of the SC islands. These effects may for example arise from the interfacial confinement (perpendicular to the interface), which leads to quantized levels and a structured DOS, or from different van Hove singularities in the 2D DOS due to the presence of a substantial Rashba coupling (see [31] and the supplemental material of [32]). Further, finite-size correction to the BCS theory arising from confinement effects (as could be reasonable to consider at low gating where the SC islands become smaller and smaller) have been shown to yield an increase in $T_c$ of up to 60% in SC tin nanoparticles [27]. Other than DOS effects, the increase of $T_c$ with decreasing gating could be due to an enhancement of the attractive interaction $g$ as a consequence of specific occurrences like, e.g., the proximity to some form of criticality (as proposed in [22] and [33]).

Noticeably, the width $\sigma$ of the $T_c$ distribution stays nearly constant (see figure 3), showing that this distribution is an intrinsic structural property of the sample, likely related to the local random distribution of impurities and defects, which rules the (local) transition temperature, as commonly occurs in homogeneously disordered superconductors [30].

Concerning SC coherence, we find that in the regime $V_G < 0$ a large fraction $w_{\text{coh}}$ of the islands with paired electrons lacks phase coherence. This incoherent fraction can be related to paired regions of size $L$ smaller than the SC coherence length $\xi_0 \sim 60$ nm [29], and decreases when the carrier density is increased, but is never less than 10%. On the other hand, the coherent SC fraction, $w_{\text{coh}}$, increases and reaches a maximum of 0.4 between $V_G = 0$ and $V_G = 100$ V. Surprisingly, further increasing the carrier density at $V_G > 200$ V leads to a decrease of the coherent fraction (and, possibly, a small increase of the incoherent but gapped fraction). A uniform 2D system percolates as soon as half of the system is SC [9], while space correlations may reduce the fraction required for percolation [10]. The fact that $w_{\text{coh}}$ stays below 0.5 despite a complete percolation of the system, which displays vanishing global resistance for $V_G > -150$ V, therefore strongly suggests that the distribution of coherent SC regions is spatially correlated, as already found in [10] and [11]. The overall paired fraction, $w_{\text{pair}} = w_{\text{coh}} + w_{\text{inc}}$, increases with $V_G$, when $V_G < 0$, in accordance with the idea that the SC fraction increases with the carrier density, but saturates above $V_G \approx 0$ V, and even slightly decreases at large positive $V_G$. The explanation for this unexpected behavior may rest upon the mechanism leading to the inhomogeneous state.

In order to highlight the importance of the parameters $w$ and $x$ we report in appendix B the best fits obtained constraining the interface to be homogeneous (in the sense that the entire interface is covered by SC islands with randomly distributed $T_c$ ($w = 1$)). We also consider what happens if all the paired regions exhibit coherence peaks ($x = 0$), while allowing $w$ to vary. As expected, the resulting fits are much less satisfactory than the ones with the inhomogeneous interface ($w < 1$) or the interface with incoherently paired islands ($x > 0$) given in figure 2.
which, according to equation (7), shows that our model captures the overall effect of temperature, although the fits are less convincing than those in figure 2.

This is certainly due to the very strong constraint of temperature-independent fitting parameters, which is not strictly supported by the data. For instance, at $V_G = 200$ V, the depth and width of the conductance curve at low $T$ suggests a substantial gap $\Delta \approx 60 \mu$eV which, according to equation (7), corresponds to a critical temperature $T_c \approx 340$ mK. However, already at $T \approx 250$ mK, the experimental curves display only a minor DOS suppression, while, according to BCS theory, the regions with sizable low-temperature gaps (i.e., with large $T_c$s) are still SC and yield too large a DOS suppression. This calls for an improvement of the model to account for substantial low-temperature gaps, which however seem to vanish at temperatures lower than expected by standard BCS theory.

4. Temperature dependence of the spectra

Next, we turn to the evolution of the spectra as a function of temperature. We point out that the temperature dependence was measured in different experimental runs, months after the low-temperature spectra of figure 2 were measured [24], so that some aging of the sample cannot be excluded. Figure 5 shows that our model captures the overall effect of temperature, although the fits are less convincing than those in figure 2.

This is certainly due to the very strong constraint of temperature-independent fitting parameters, which is not strictly supported by the data. For instance, at $V_G = 200$ V, the depth and width of the conductance curve at low $T$ suggests a substantial gap $\Delta \approx 60 \mu$eV which, according to equation (7), corresponds to a critical temperature $T_c \approx 340$ mK. However, already at $T \approx 250$ mK, the experimental curves display only a minor DOS suppression, while, according to BCS theory, the regions with sizable low-temperature gaps (i.e., with large $T_c$s) are still SC and yield too large a DOS suppression. This calls for an improvement of the model to account for substantial low-temperature gaps, which however seem to vanish at temperatures lower than expected by standard BCS theory.

4.1. Temperature-dependent weights

A natural possibility to reconcile the low and high-temperature spectra is to assume that the overall paired fraction is not fully established at high $T$ by structural properties alone. It is instead conceivable that, upon lowering the temperature, the proliferation of SC regions influences by proximity effect the metallic background, thereby causing an increase in $w_{\text{pair}}$. To investigate this possibility, we relaxed the constraint of a $T$-independent paired fraction, allowing both $w_{\text{coh}}$ and $w_{\text{inc}}$ (i.e., $w$ and $x$) to be adjusted at each temperature, while the parameters of the $T_c$ distribution are kept fixed. The resulting fits, reported in figure 6, are now in very good agreement with the experimental data. The improvement is obtained by letting $w_{\text{pair}}$, to augment up to 40% when $T$ is lowered, mostly due to an increase of $w_{\text{coh}}$. Below 250 mK, for $V_G = 0$ V and $V_G = -200$ V, or below 150 mK, for $V_G = 200$ V, the paired fractions saturate to nearly constant values.

Besides the main difficulty arising from the connection between large low-temperature gaps and the associated high critical temperatures, the spectra for $V_G = 0$, $-200$ V has the peculiarity of slightly decreasing with temperature before they ultimately increase around $T = 150$ mK. In both cases, insisting on standard BCS theory and a $T_c$ distribution fixed in temperature, inconsistencies between low- and high-temperature regime are inevitable. In the light of these experimental features one is bound to conclude that additional effects in temperature are present, other than the ones stemming from the Fermi function. As shown above, the possibility that the coherent ($w_{\text{coh}}$) and incoherent ($w_{\text{inc}}$) fraction may vary with temperature successfully resolves the difficulties raised above. In the following we investigate and critically discuss three alternative solutions.

4.2. Distribution of SC gaps

The first adjustment consists in taking a distribution of SC gaps. The physical idea behind this adjustment is simple. The gap of an island may not be exactly uniform (although the island has a single, well-defined critical temperature), but follows a certain distribution. For instance, one can assume that the electrons of an island fill up several sub-bands, which, as soon as one of them becomes SC, become SC as well. The strengths of intra- and inter-band coupling being different, this may lead to a distribution of gaps. Depending on the distribution chosen, this mimics strong coupling effects since a given $T_c$ also yields gaps larger than the BCS prediction. For the sake of simplicity, we assume that an island has a flat
distribution of gaps $P(\Delta) = (\Delta_{\text{max}} - \Delta_{\text{min}})^{-1} \Theta(\Delta_{\text{max}} - \Delta) \Theta(\Delta - \Delta_{\text{min}})$, which leads to the following density of states:

$$\rho_{\text{SC}}(E, \Delta_{\text{min}}, \Delta_{\text{max}}) = \frac{1}{N_0} \int_{-\infty}^{+\infty} \frac{d\Delta P(\Delta)\Theta(|E| - \Delta)}{\sqrt{E^2 - \Delta^2}}.$$

with $N = \Delta_{\text{max}} - \Delta_{\text{min}}$. We choose $\Delta_{\text{min}} = \Delta_{\text{BCS}}$, $\Delta_{\text{max}} = (1 + X)\Delta_{\text{BCS}}$, while $\Delta_{\text{BCS}}$ is related to $T_c$ via the standard BCS equation. For the sake of definiteness, we present the results for $X = 0.4$. The fits with smaller values of $X$ are barely distinguishable from the fits with $X = 0$, and larger values are hardly justifiable as a slight modification of the BCS theory. The $T_c$ distribution is determined by the best fit at $T = 70$ mK. The results obtained with a Gaussian $P(T_c)$ are reported in figure 5. The best fits are obtained with $P(T_c)$ distributions shifted to temperature lower than their $X = 0$ counterparts (black solid lines). To some extent, this mitigates the discrepancy between the critical temperatures inferred from low-temperature fitting and the actual temperature at which the DOS suppression vanishes. However, at intermediate temperature the fits still require improvement.

### 4.3. Temperature-dependent coupling strength

The straightforward extension of the previous adjustment is to take explicitly strong coupling deviations from BCS that may vary in temperature. To this end we introduce a factor $\alpha(T)$ in the standard BCS expression

$$\Delta(T_c, T) = 1.76 \alpha(T) T_c \text{Tanh} \left( \frac{\pi}{1.76} \sqrt{\frac{1}{T_c} - 1} \right). \quad (7)$$

Taking a Gaussian $P(T_c)$ we determine $T_c$, $\sigma$ and $w$ (fixed in temperature) such as to minimize the overall difference between the theoretical and the experimental curves for all temperatures while the factor $\alpha(T)$ is let free to take values in the interval [0.8, 1.3]. Although values $\alpha(T) < 1$ do not correspond to a strong coupling regime, we allowed for this possibility for the sake of comprehension. The results given in figure 8 show an improvement with respect to $\alpha = 1$. We discuss the consequences of $\alpha(T)$ for the gating $V_G = 200$ V. There, the suppression in the DOS vanishes around $T_c^{\text{max}} \sim 250$ mK, corresponding to a gap $\Delta_{\text{max}} \sim 38 \mu$eV (for $\alpha = 1$). However, the low temperature ($T = 70$ mK) spectra reveals gaps around $60 \mu$eV and slightly smaller ones at
intermediate temperatures \( T \sim 150 \) mK. To obtain gaps of this order some islands with \( T \sim 340 \) mK have to be present in the system and, consequently, the Gaussian fit reported in figure A1 is obtained with parameters yielding non-negligible weight around this temperature \( T_c = 237 \) mK, \( \sigma = 151 \) mK, \( w = 0.67, x = 0.1 \). Clearly, such a distribution causes a too strong suppression for \( T \sim 250 \) mK. Taking \( \alpha(70 \) mK) = 1.3 allows to take a smaller \( P(T_c) \) shifted to lower temperatures \( T_c = 209 \) mK and \( \sigma = 81 \) mK, \( w = 0.59, x = 0.2 \) thereby improving the fits at high temperatures while still yielding good fits at low temperature as well. A similar reasoning applies to \( V_G = 0, -200 \) V. In addition there the high temperature curves \( T = 350 \) mK are best fitted with \( \alpha < 1 \), signaling an attenuation of superconductivity. This attenuation is reminiscent of the decrease in temperature of the coherent weight in figure 6(d).
4.4. Temperature-dependent pair-breaking

We conclude the considerations on the temperature dependence by looking at the effect of pair-breaking. Traditionally, this is done by means of a finite-lifetime-broadened density of states, the so-called Dynes formula:

\[ \rho(E) = \left| \text{Re} \left\{ \frac{E - i\Gamma}{\sqrt{(E - i\Gamma)^2 - \Delta^2}} \right\} \right|, \]  

where \( \Gamma \) is a measure of the pair-breaking rate. As before, we take a Gaussian \( P(T_c) \) with \( T_c, \sigma \), and \( w \) fixed in temperature (and \( x = 0 \)) such as to minimize the overall difference between the theoretical and the experimental curves for all temperatures while \( \Gamma(T) \) may vary with \( T \).

On general grounds, if pair breaking is due to the interaction of Cooper pairs with impurities and/or collective degrees of freedom, one expects it to become stronger with increasing temperature. Furthermore, it is reasonable to expect that the pair-breaking energy scale \( \Gamma \) stays much smaller than the SC gap and becomes eventually comparable with it only at temperatures close to \( T_c \). Figure 9 shows that the fits at low temperatures deviate from these expectations: at zero and negative \( V_G \), \( \Gamma \) is a mildly non-monotonic function of temperature; furthermore, already at \( T = 70 \text{ mK} \), the pair-breaking parameter is substantial fraction of the average gap \( \Delta = \Delta_{BCS}(T_c) \) without \( T \). More precisely, the fits yield \( \Delta = 24, 34, 41 \mu\text{eV} \) for \( V_G = 200, 0, -200 \mu\text{V} \) respectively, while \( \Gamma(T = 70 \text{ mK}) \sim 15 \mu\text{eV} \). These values are comparable to those found in [24].

5. Conclusion

In this paper we phenomenologically assume that the LXO/STO interfaces are strongly inhomogeneous on the nanometric scales, with a random distribution of SC critical temperatures. This assumption has been proven successful in explaining resistivity [9–12], Hall conductance [13], and diamagnetic susceptibility [11–13]. Of course, like in any phenomenological approach, we cannot exclude that other assumptions might work as well, but the direct observation of an inhomogeneous density distribution [21] naturally entails a distribution of local critical temperatures. In this sense, our work ‘demonstrates’ that the recent observation of a pseudogap in LAO/STO is explained by the well established inhomogeneous character of the 2DEG in this system. Specifically, a metallic background embeds regions which display pairing below randomly distributed \( T_c \). The features of the \( T_c \) distribution are structurally determined and therefore do not depend on temperature. We find that a Gaussian distribution is more suited to fit the data. Since in disordered superconductors there is a rather broad range of \( T_c \) linearly decreasing with resistance [30], we can infer a Gaussian distribution of resistances and, consequently, of inverse...
scattering time. This could reflect a normal Gaussian distribution of quenched microscopic impurities.

Another mechanism leading to fluctuations in the gap and local critical temperatures is associated with quantum confinement effects in nanoparticles [27]. This seems particularly important whenever the size of the SC regions is comparable with or smaller than the coherence length (which in STO is around $\xi_0 \sim 60 \text{ nm}$). Inside most of the SC regions, standard BCS coherence is established, while a smaller fraction only displays incoherent pairing, likely associated with the size $L$ of these islands being smaller than the SC coherence length $\xi_0$. This indicates that inhomogeneities in these 2DEG also occur on length scales smaller than 60 nm. This estimate agrees with the analysis of suppression of superconductivity by magnetic fields [22] and with theoretical estimates within a microscopic model [33].

Attempts to fit the temperature dependence of the spectra also lead to the conclusion that some (30–40%) of the paired regions at low temperature are likely formed by proximity effect in the metallic matrix and were not ‘foreseen’ at higher temperature. The importance of proximity for establishing superconductivity in the quantum critical regime at low carrier density in LaTiO$_3$/STO was also assessed in [22]. The introduction of other physical effects (such as deviations from the BCS temperature dependence of the gap, pair-breaking effects, ...) may improve the fits and lead to a reduction of the temperature dependence of $w_{\text{pair}}$. Specifically, we find that adjustments with $P(\Delta)$, $\alpha(T)$ and $\Gamma(T)$ do lead to an overall improvement of the fits in temperature. Compared to the outcomes with $w(T)$ however, the improvements with a non-Dynes-like adjustment ($P(\Delta)$ and $\alpha(T)$) are small. And the downside of taking $\Gamma(T)$, although yielding excellent fits, is the rather atypical behavior of a pronounced pair-breaking strength already at low temperatures. In addition, there might be a conceptual inconsistency: in the absence of a metallic background, the introduction of $\Gamma$ is necessitated by the finite zero-conductance in the experimental curves. However, it has recently been demonstrated that the phenomenological Dynes ansatz of equation (8) is inapplicable when (microscopic) disorder is the main source of broadening [34]. Specifically, it was shown that disorder can lead to a suppression of coherence peaks and the appearance of sub-gap states but it never yields a non-zero conductance at zero-bias. In conclusion, the very recognition of the inhomogeneous character of the LXO/STO interfaces naturally leads to a direct interpretation of the pseudo-gap effects observed in tunneling experiments and to infer interesting physical properties, which should guide the microscopic modeling of these systems.

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Appendix A. Distribution of critical temperatures at low temperature

We start the discussion of the fitting procedure by looking at various distributions. To find out what the important features of the distribution should be, we take the following four characteristic distributions: Box, Lorentzian, skewed Lorentzian, and Gaussian

$$\frac{1}{T_{c,\text{max}} - T_{c,\text{min}}} \Theta(T_{c,\text{max}} - T_c) \Theta(T_c - T_{c,\text{min}}),$$  \hspace{0.5cm} (A.1)

$$\frac{1}{\pi} \frac{T_{c,\text{max}} - T_c}{\gamma^2 + (T_{c,\text{max}} - T_c)^2},$$  \hspace{0.5cm} (A.2)

$$\frac{1}{N} \frac{T_{c,\text{max}} - T_c}{\gamma^2 + (T_{c,\text{max}} - T_c)^2} \Theta(T_{c,\text{max}} - T_c) \Theta(T_c),$$  \hspace{0.5cm} (A.3)

$$\frac{1}{\sqrt{2\pi} \sigma} \exp \left( -\frac{(T_c - T)^2}{2\sigma^2} \right).$$  \hspace{0.5cm} (A.4)

Below we report the best fits (left panel) and the corresponding $T_c$ distributions (right panel) at $T = 70 \text{ mK}$ for $V_G = 200 \text{ V}$ (figure A1), $0 \text{ V}$ (figure A2) and $-200 \text{ V}$ (figure A3).

The Box distribution (equation (A.1), yellow curves), being constant in the interval $[T_{c,\text{min}}, T_{c,\text{max}}]$ and zero elsewhere, allows for a precise control over the range of critical temperatures. However, this sharpness is somewhat at odds with the rather broad coherence peaks of the experimental curves. Indeed, inspection of the three figures below reveals that the fits result in too accentuated coherence peaks. Instinctively, one would think that this flaw is cured by taking a larger incoherent part ($w_{\text{inc}} = w \times$). Although this is the case, larger $w_{\text{inc}}$ lead to overall worse fits, because increasing $x$ the fits worsen for $|V| \lesssim 100 \mu\text{V}$.

Fitting the experimental data with a Lorentzian distribution (equation (A.2), green curve) gives rather convincing agreement between experiment and theory. The fits exhibit the interesting feature of $w_{\text{inc}} = 0$, i.e. all the islands with pairing are coherent. This is due to the distribution’s very large width (its variance is even divergent) which spreads the coherence peaks over a broad range of energies. On the down side, this large width corresponds to a (rather unrealistic) situation with small but non-negligible weight up to critical temperatures of 1 K (especially for low gating).

The effect of asymmetry is studied considering a skewed Lorentzian distribution (equation (A.3), blue curve). The fits shown in figures A1–A3 are less convincing compared to those obtained with the previous two distributions. The main reason for the failure is that in order to have enough weight around the main gap (compare for example with the Box distribution) one needs rather large values of $\gamma$ which then give too much weight to low critical temperatures, i.e. around 0 $\mu\text{V}$. At higher energies, the fits exhibit the same
shortcoming of too strong coherence peaks as did the fits with the Box distribution, due to the sharp decrease of \( P(T_c) \) at \( T_c^{\text{max}} \).

The Gaussian distribution (equation (A.4), red curve) yields very convincing fits both for the gapped part as well as the coherence peaks. In some sense, it has the advantages of both the Box distribution and the Lorentzian; while the former has equal weight in a certain interval of \( T_c \), leading to good fits at intermediate energies (\( \sim 50 \mu eV \)), the latter allows for a good description of the center of the spectra (\( \sim 0 \mu eV \)) and the coherence peaks (\( \sim 100 \mu eV \)). We conclude this discussion on the probability distribution with three important remarks. First of all, one might intuitively assume that with four adjustable parameters, any distribution allows to fit the experimental data. As we showed convincingly above, this is not true. But for the Gaussian, all the \( P(T_c) \) distributions considered yield fits with a (more or less pronounced) mismatch compared to the experimental curves. Secondly, the fitting parameters enter in the fitting procedure within a theoretical framework and not within, for example, an interpolating polynomial. Consequently, they admit a physical interpretation and their reasonability can be judged to some extent. Last but not least, the
behavior of the fitting parameters as a function of the gate voltage and the temperature turns out to be meaningful, and this supports our conviction that the physical parameters they represent capture the relevant physics of oxide interfaces at the nanoscale.

Appendix B. Low-temperature fits with \( w = 1 \) or \( x = 0 \)

We explain in the main text that the inhomogeneous character of the interface enters our model in two different ways, both crucial to our description of the 2DEG: The first way is as an inhomogeneity at the nano-scale, which entails the occurrence of two entirely different types of regions, some metallic at all (experimentally accessible) temperatures and some metallic at high temperature but SC below a certain local \( T_c \). Associated to this inhomogeneity is the parameter \( w \) denoting the fraction of the 2DEG which may become SC (the purely metallic fraction corresponds then to \( 1 - w \)). Secondly, the SC regions may differ from one another, possibly because of fluctuations of the microscopic disorder, yielding a distribution of critical temperatures \( P(T_c) \). In the case of the Gaussian distribution this yields two parameters, the mean critical temperature \( T_c \) and the width \( \sigma \).

The fourth parameter of our model is \( x \), which denotes the fraction of SC islands that exhibit a finite gap but no coherence peaks, e.g., because the size of the island is too small to support phase coherence. As we mentioned in the main text its introduction is motivated by the experimental data which (especially at very low carrier density) shows a strong suppression in the density of states but practically no coherence peaks.

The fact that all the fits in the main text yield \( w < 1 \) is crucial for our conclusion of an inhomogeneous interface. To substantiate this outcome we confront in figure A4 the best fits with \( w \) variable (i.e., the same as in figure 2) and with \( w = 1 \), i.e., with a homogeneous SC interface (in the sense that the SC islands cover the entire interface). As expected, we find that the experimental data is hardly fittable with \( w = 1 \).

While the outcome \( w < 1 \) is expected from the experimental data, whether the parameter \( x \) is zero or not is less evident. For instance, we have shown in appendix A that the best fits with a Lorentzian distribution are obtained with \( x = 0 \) (although these fits are less convincing than the Gaussian fits with \( x \neq 0 \)). In figure B1 we confront the best fits with a Gaussian distribution, \( w \) variable and \( x \) either variable (i.e., the same as in figure 2) or \( x = 0 \). The mediocrity of the \( x = 0 \) fit compared to that with \( x > 0 \) suggest that the introduction of the parameter \( x \) is not only theoretically sound but also necessary.

Appendix C. Fitting procedure

The outcomes of this work rely on fits of the experimental data. In the following we explain in detail how these fits were obtained in the various cases.

\[ T = 30 \text{ mK}, V_G \in [-300, 300] \text{ V} \]: The data was extracted from figure 3(a) of [24] and normalized by subtracting a straight line such that the \( \frac{dI}{dV} \) values coincide for \( \pm 250 \mu V \). We centered the data by a small horizontal shift (of the order of a few \( \mu V \)). We discretized the curves in steps of \( 1 \mu V \) from \(-250\) to \( 250 \mu V \). For each set of parameters \( T_c, \sigma, w, x \) we calculated the difference

\[
D = \sum_{V=-250}^{250} \left| \frac{dI}{dV}(V) \right|_{\text{exp}} - \left| \frac{dI}{dV}(V) \right|_{\text{th}},
\]

where ‘exp’ and ‘th’ denote the experimental and the
theoretical value of the differential conductivity, respectively. The best fit corresponds to the set \( \sigma_{T, x, w} \) which minimizes \( D \).

\[ T = 70 \text{ mK}, \ V_G = 200, 0, -200 \text{ V}: \] We obtained the raw data for the behavior in temperature directly from the authors of [24]. We normalized the data by first fitting the normal state signal at \( T = 700 \text{ mK} \) with a parabola, which we then subtracted from the low temperature curves. Again, we centered the data by a small horizontal shift (of the order of a few \( \mu \text{V} \)). We calculated the difference as in equation (C1) for \( V_G = 200, 0 \text{ V} \), while for \( V_G = -200 \text{ V} \) we enlarged the interval to \([-300, 300]\) \( \mu \text{V} \). As before, the best fit corresponds to the set \( T, \sigma, w, x \) which minimizes \( D \).

\[ T \geq 70 \text{ mK}, \ V_G = 200, 0, -200 \text{ V}: \] The analysis of \( \alpha(T) \) and \( \Gamma(T) \) were done by calculating \( D \) for a set of parameters \( \sigma_{T, x, w} \) with \( x > 0 \) fixed and \( T, \sigma, w, \Gamma(T) \) with \( x = 0 \), respectively. From all these sets we chose the one which minimized the sum of \( D \) for all temperatures, with the constraint that \( T, \sigma, w \) be the same.

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