Engineering a controllable coupling between two quantized cavity modes via an ensemble of four-level atoms in the diamond configuration

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We describe how an ensemble of four-level atoms in the diamond-type configuration can be applied to create a fully controllable effective coupling between two cavity modes. The diamond-type configuration allows one to use a bimodal cavity that supports modes of different frequencies or different circular polarizations, because each mode is coupled only to its own transition. This system can be used for mapping a quantum state of one cavity mode onto the other mode on demand. Additionally, it can serve as a fast opening high-Q cavity system that can be easily and coherently controlled with laser fields.

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I. INTRODUCTION

Systems composed of a cavity and atoms, which are trapped inside this cavity, provide a versatile environment for engineering complex non-classical states of light [1–10]. The main reason why these systems are so practical is that one can easily control the evolution of their quantum state just by illuminating atoms with a laser [11–15]. Researchers achieve such high level of control over the evolution of quantum states employing atoms, which can be modeled by few special level schemes. The simplest and frequently considered schemes are three level atoms in Λ and V configurations [16–23]. The main advantage of these atoms is the possibility of working with the two-photon Raman transition involving an intermediate level, which is populated only virtually during the whole evolution. Since atoms are driven by a classical laser field, the Raman transition takes place only if the laser is turned on. The same idea allows for full control of the system evolution in many other level schemes. Therefore researchers have used and studied intensively many different types of atoms coupled to the cavity mode [24–54]. There is, however, one important atomic level scheme, which is almost ignored by researchers in the context of atom-cavity systems — a four-level atom in the diamond configuration (a ◦-type atom, also known as a double-ladder four-level atom). Despite the fact that this level scheme is rich in quantum interference and coherence features [55] and has many other applications [56–59], to the best of our knowledge there are only few articles about the ◦-type atom coupled to the quantized field modes [44–55].

In this paper we study a ◦-type atom interacting with two quantized cavity modes and two classical laser fields. The quantized field modes are coupled to lower atomic transitions while the classical laser fields are coupled to upper atomic transitions, as depicted in Fig. 1. Here, we show that under certain conditions the evolution of this system can be described by a simple effective Hamiltonian, and can be easily controlled just by switching the lasers on and off. We also present two applications of this system. First of them is the transfer of an arbitrary state of light from one mode to the other. Second application is a device that plays the role of an effective cavity, in which we can change the effective Q factor on demand just by turning the lasers on and off. This device is based on the scheme proposed by Tufarelli et al. [56] but it employs ◦-type atoms instead of two-level atoms.

This paper is organized as follows: in Sec. [II] we derive the effective Hamiltonian of simple form that governs the evolution of the analyzed complex quantum system. In Sec. [III] we show using this effective Hamiltonian that the system can perform a state-mapping operation between two modes of the cavity. In Sec. [IV] we describe how to

FIG. 1: Energy levels of an atom in the diamond configuration. Lower atomic transitions are coupled to quantized field modes with frequencies ω and ω'. Upper transitions are driven by classical laser fields with frequencies ν and ν'.

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we assume in the following that $\Omega, \Omega'$ laser fields of frequencies $\nu, \nu'$ are detuned from the corresponding transition frequencies by $\Delta = (E_1 - E_0)/\hbar - \omega' = (E_2 - E_0)/\hbar - \omega$. The upper transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ are driven by coherent laser fields of frequencies $\nu'$ and $\nu$, respectively. The coupling strengths between these atomic transitions and the laser fields are denoted by $\Omega'$ and $\Omega$. Both laser fields are detuned from the corresponding transition frequencies by $\Delta$. Simultaneously, the atom is coupled to all the modes of the EM field, which are assumed to be in the vacuum state.

The atom provides an effective coupling between both the modes. Of course, the effective coupling strength depends on the number of atoms $n$. The higher the number of atoms $n$, the stronger the coupling becomes. We assume that there are $n \geq 1$ identical $\otimes$-type four-level atoms trapped inside the cavity. The evolution of this composite quantum system is governed by the Hamiltonian, which in the rotating frame is given by

$$H = \sum_{k=1}^{n} \{ \Delta \sigma^{(k)}_{11} + \Delta \sigma^{(k)}_{22} + 2\Delta \sigma^{(k)}_{33} + (\Omega \sigma^{(k)}_{23})^2 + \Omega' \sigma^{(k)}_{13} + g a^\dagger \sigma^{(k)}_{02} + g' b^\dagger \sigma^{(k)}_{01} + \text{h.c.} \},$$

where $\hbar = 1$ and $\sigma_{ij} = |i\rangle \langle j|$ denotes the atomic flip operator between states $|i\rangle$ and $|j\rangle$. For the sake of simplicity, we assume in the following that $\Omega$, $\Omega'$, $g$ and $g'$ are real, non-negative numbers.

Similar four-level scheme has been proposed in Ref. [26]. The diamond configuration, however, has the advantage that it allows to use (contrary to level scheme of [26]) atomic transitions with the highest values of the dipole moment. Of course, the higher the dipole moment, the stronger the effective coupling between the modes is.

A. Adiabatic elimination of excited atomic states

1. Reiter-Sørensen method

Initially, all atoms are prepared in the ground state. An atom can be found in one of the excited states, only if it absorbs a single photon. We want to achieve an effective coupling between field modes and no coupling between the modes and atoms. Therefore, the atomic excited states have to be populated only virtually. In this case, we can adiabatically eliminate the atomic excited states and use in calculations an effective Hamiltonian for the ground state subspace. To this end, we use the effective operator formalism for open quantum systems described in Ref. [57]. Let us consider the single atom case first. The Hamiltonian describing a single atom can be easily obtained by simplifying Eq. (1) and it reads

$$H = \Delta \sigma_{11} + \Delta \sigma_{22} + 2\Delta \sigma_{33} + (\Omega \sigma_{23})^2 + \Omega' \sigma_{13} + g a^\dagger \sigma_{02} + g' b^\dagger \sigma_{01} + \text{h.c.}. \quad (2)$$

The Lindblad operators representing spontaneous transitions from the atomic excited states are given by

$$L_1 = \sqrt{\gamma} \sigma_{01}, \quad L_2 = \sqrt{\gamma} \sigma_{02}, \quad L_3 = \sqrt{\gamma_3} \sigma_{13}, \quad L_4 = \sqrt{\gamma_4} \sigma_{13}, \quad (3)$$

where $\gamma, \gamma', \gamma_3$ and $\gamma_4'$ are spontaneous emission rates for the respective transitions. The master equation of Kossakowski-Lindblad form describing the evolution of this system is then given by

$$\dot{\rho} = -i[H, \rho] + \sum_{j=1}^{4} \left[ L_j \rho L_j^\dagger - \frac{1}{2}(L_j^\dagger L_j \rho + \rho L_j^\dagger L_j) \right] \quad (4)$$

The effective-operator formalism for open quantum systems [57] reduces Eq. (4) to an effective master equation, where the dynamics is restricted to the atomic ground state only. In order to apply the effective-operator formalism, we need to provide: the Lindblad operators, the Hamiltonian in the exited-state manifold $H_e$, the ground-state Hamiltonian $H_g$, and the perturbative (de-)excitations of the system $V_+ (V_-)$. These are given by

$$H_e = \Delta \sigma_{11} + \Delta \sigma_{22} + 2\Delta \sigma_{33} + (\Omega \sigma_{23})^2 + \Omega' \sigma_{13} + \text{h.c.}, \quad H_g = 0, \quad (5)$$

$$V_- = ga^\dagger \sigma_{02} + gb^\dagger \sigma_{01}, \quad V_+ = gag a^\dagger \sigma_{02} + gb^\dagger \sigma_{01}. \quad (5)$$

The effective Hamiltonian and collapse operators can be derived using formulas [57]

$$H_{\text{eff}} = -\frac{1}{2} V_- [H_{\text{NH}}^{-1} + (H_{\text{NH}}^{-1})^\dagger] V_+ + H_g, \quad (6)$$

$$L_{\text{eff}}^{(j)} = L_j H_{\text{NH}}^{-1} V_+, \quad (7)$$
where
\[ H_{\text{NI}} = H_0 - \frac{i}{2} \sum_j L_j^\dagger L_j. \] (8)

Assuming that all spontaneous emission rates are negligibly small compared with \(\Omega, \Omega\gamma\) and \(\Delta\) we can approximate \(H_{\text{NI}}\) by
\[ H_{\text{NI}}^{-1} \approx \alpha_1\sigma_{11} + \alpha_2\sigma_{22} + \alpha_3(\sigma_{12} + \sigma_{21}) + \alpha_4\sigma_{33} + \alpha_5(\sigma_{13} + \sigma_{31}) + \alpha_6(\sigma_{23} + \sigma_{32}), \] (9)
where \(\alpha_1 = \xi(\Omega^2 - 2\Delta^2), \alpha_2 = \xi(\Omega^2 - 2\Delta^4), \alpha_3 = -\xi\Omega\gamma\), \(\alpha_4 = -\xi^2\Delta^2, \alpha_5 = \xi\Delta\Omega\), \(\alpha_6 = \xi\Delta\Omega\) with \(\xi = 1/(\Delta[\Omega^2 + \Omega^2 - 2\Delta^2]).\) From the combined Eq. (9) and Eq. (6) we derive the effective Hamiltonian
\[ H_{\text{eff}} = \delta_0(a^\dagger a + b^\dagger b) + \delta_1 b^\dagger b + \delta_2(a^\dagger a + b^\dagger b), \] (10)
where \(\delta_0 = -g^2\alpha_2, \delta_1 = g^2\alpha_2 - g^2\alpha_1\) and \(\delta_2 = -gg'\alpha_3.\) By inserting Eq. (9) into Eq. (7) we obtain the effective Lindblad operators
\[ L_{\text{eff}}^{(1)} = \sqrt{\gamma_0}[\alpha_3 ga + \alpha_1 g'b], \quad L_{\text{eff}}^{(2)} = \sqrt{\gamma_0}[\alpha_2 ga + \alpha_3 g'b]. \] (11)

Unfortunately, deriving the expressions for operators \(L_{\text{eff}}^{(1)}\) and \(L_{\text{eff}}^{(2)}\) is more challenging than deriving \(L_{\text{eff}}^{(1)}\) and \(L_{\text{eff}}^{(2)}\). First of all, the action of the operators \(L_3\) and \(L_4\) takes the system state to one of the excited states \([1]\) or \([2]\), while all the excited states should be populated only virtually. In the single atom case after spontaneous emission from the excited state \([3]\) it is necessary to reset the device, otherwise it will not work properly. Second, the effective operator formalism assumes that the excited states decay to the ground states only. We circumvent these obstacles by choosing such values of parameters that probabilities of occurrence of collapses described by \(L_3\) and \(L_4\) are negligibly small. We will give later conditions for the parameters, which allow us to neglect \(L_3\) and \(L_4\). Using this approximation, we can write the effective master equation as
\[ \dot{\rho} = -i[H_{\text{eff}}, \rho] + \sum_{j=1}^{2} \left[ L_{\text{eff}}^{(j)} \rho (L_{\text{eff}}^{(j)})^\dagger \right. \\
\left. \frac{1}{2} [(L_{\text{eff}}^{(j)})^\dagger L_{\text{eff}}^{(j)} \rho + \rho (L_{\text{eff}}^{(j)})^\dagger L_{\text{eff}}^{(j)}], \right] \] (12)

It is easy to generalize this result to the \(n\) atom case. In this more general case the Hamiltonian is still given by Eq. (10), but with
\[ \delta_0 = -ng^2\alpha_2, \delta_2 = -ngg'\alpha_3, \text{ and} \]
\[ \delta_1 = n(g^2\alpha_2 - g^2\alpha_1). \] (13)

For the sake of simplicity, we have assumed here that coupling strengths \(g\) and \(g'\) are the same for each atom in the ensemble. Note, however, that every atom in a Bose-Einstein condensate indeed experiences an identical coupling to the cavity mode [55]. In a frame rotating at \(\delta_0\) the Hamiltonian takes the form
\[ H_{\text{eff}} = \delta_1 b^\dagger b + \delta_2(a^\dagger a + b^\dagger b), \] (14)
The effective Lindblad operators are now given by
\[ L_{\text{eff}}^{(1)} = \sqrt{n\gamma_0}[\alpha_3 ga + \alpha_1 g'b], \quad L_{\text{eff}}^{(2)} = \sqrt{n\gamma_0}[\alpha_2 ga + \alpha_3 g'b]. \] (15)

2. Alexanian-Bose method

Using the effective operator formalism and assuming that the upper level \([3]\) can be neglected, we have obtained all needed formulas. Unfortunately we still do not know the limits in which these approximations are valid. In order to determine the limits we derive the effective Hamiltonian [10] using another method — a perturbative unitary transformation [58]. An incidental bonus is that this method provides new insights into the dynamics of the four-level atom in the diamond configuration. Let us start by decomposing the Hamiltonian [2] into two parts
\[ H = H_0 + H_1, \] (16)
where
\[ H_0 = \Delta \sigma_{11} + \Delta \sigma_{22} + 2\Delta \sigma_{33} + (\Omega \sigma_{23} + \Omega' \sigma_{13} + \text{h.c.}). \] (17)

Diagonalizing \(H_0\) in the basis \([1], [2], [3]\) leads to the dressed states energies
\[ \Delta, \ (3\Delta - \Omega_\text{R})/2, \ (3\Delta + \Omega_\text{R})/2, \] (18)
and the semiclassical dressed states
\[ |\mu\rangle = N_\mu(-\Omega(1) + \Omega')|2\rangle, \quad |\phi\rangle = N_\phi(2\Omega|1\rangle + 2\Omega|2\rangle + (\Delta - \Omega_\text{R})|3\rangle), \quad |\psi\rangle = N_\psi(2\Omega|1\rangle + 2\Omega|2\rangle + (\Delta + \Omega_\text{R})|3\rangle, \] (19)
where \(\Omega_\text{R} = (\Delta^2 + 4\Omega^2 + 4\Omega'^2)^{1/2}, N_\mu = (\Omega^2 + \Omega'^2)^{-1/2}, N_\phi = (2\Omega_\text{R}(\Omega_\text{R} - \Delta))^{-1/2} \text{ and } N_\psi = (2\Omega_\text{R}(\Omega_\text{R} + \Delta))^{-1/2}.\) Now, using the new basis \([0], [\mu], [\phi], [\psi]\), we re-express the Hamiltonian [2] as
\[ H = \Delta \sigma_{\mu\mu} + (3\Delta - \Omega_\text{R})/2|\phi\rangle + (3\Delta + \Omega_\text{R})/2|\psi\rangle + (N_\mu g\Omega|a\rangle|\sigma_0\rangle - 2N_\psi g\Omega|a\rangle|\sigma_0\rangle + 2N_\phi g\Omega|\sigma_0\rangle + N_\psi g\Omega|b\rangle|\sigma_0\rangle + 2N_\phi g\Omega|b\rangle|\sigma_0\rangle + \text{h.c.})]. \] (20)

Now we can eliminate atomic excited states \([\mu], [\phi]\) and \([\psi]\). To this end, we introduce a unitary transformation [58]
\[ U = \exp(S), \] (21)
where
\[
S = \lambda_1(\sigma_{a0}^\dagger - a^\dagger \sigma_{0a}) + \lambda_2(\sigma_{b0}^\dagger - b^\dagger \sigma_{0b}) + \lambda_3(\sigma_{0a}^\dagger b - b^\dagger \sigma_{0b}) + \lambda_4(b^\dagger \sigma_{0a}^\dagger - \sigma_{0b}^\dagger a^\dagger)
\]
and \(\lambda_i\) are dimensionless parameters such that \(\lambda_i \sqrt{\langle a^\dagger a \rangle}\) and \(\lambda_i \sqrt{\langle b^\dagger b \rangle}\) are very small compared to 1. These parameters will play the role of expansion parameters associated with respective excited states. For example, \(\lambda_1\) and \(\lambda_4\) are associated with the state \(|\mu\rangle\) (see Eq. (22)).

We transform each operator in Eq. (20) using the Baker–Hausdorf lemma
\[
X' = e^S X e^{-S} = X + [S, X] + (1/2!) [S, [S, X]] + \ldots
\]
(23)

If we choose \(\lambda_1 = \mathcal{N}_g g \Omega' / \Delta\), \(\lambda_2 = 4\mathcal{N}_g g \Omega (3\Delta - \Omega_R)\), \(\lambda_3 = 4\mathcal{N}_g g \Omega (3\Delta + \Omega_R)\), \(\lambda_4 = \mathcal{N}_g g \Omega / \Delta\), \(\lambda_5 = 4\mathcal{N}_g g \Omega / (3\Delta - \Omega_R)\), \(\lambda_6 = 4\mathcal{N}_g g \Omega / (3\Delta + \Omega_R)\) then terms which are linear in the field operators vanish in the transformed Hamiltonian. If we moreover drop all terms much smaller than \(\lambda_i^2 \Delta\) then we obtain the effective Hamiltonian (10). Note that both methods, i.e. Reiter-Sørensen method and Alexanian-Bose method, give exactly the same formula for the effective Hamiltonian, despite the fact that both are just approximations.

It is also worth to note that the parameters \(\lambda_i\) are given by the ratios of the effective coupling constants to the dressed state energies. The dressed state energies play the role of detunings in this dressed-state approach. So, \(\lambda_i \ll 1\) means that the corresponding excited state is very far off resonance from the ground atomic state \(|0\rangle\), and thus, its population is small. For instance, the smaller \(\lambda_1\) and \(\lambda_4\) are, the smaller is the population of the state \(|\mu\rangle\).

3. Limits of validity

The effective Hamiltonian (10) works properly if all conditions \(\lambda_i \sqrt{\langle a^\dagger a \rangle} \ll 1\) and \(\lambda_i \sqrt{\langle b^\dagger b \rangle} \ll 1\) are satisfied, that is, if populations of all excited states \(|\mu\rangle\), \(|\phi\rangle\) and \(|\psi\rangle\) are small. However, the effective master equation (12) requires more restrictive conditions to work properly, because we have neglected it the Lindblad operators \(L_2\) and \(L_4\), which describe spontaneous emissions from the excited bare state \(|3\rangle\). Since the excited dressed states \(|\phi\rangle\) and \(|\psi\rangle\) contain contributions of the unwanted bare state \(|3\rangle\), their populations have to be much smaller than the population of the excited state \(|\mu\rangle\). Therefore, we can conclude that the effective master equation (12) works properly if conditions \(\lambda_3 \sqrt{\langle a^\dagger a \rangle} \ll 1\), \(\lambda_4 \sqrt{\langle b^\dagger b \rangle} \ll 1\), \(\lambda_5 \sqrt{\langle a^\dagger a \rangle} \ll 1\), \(\lambda_6 \sqrt{\langle b^\dagger b \rangle} \ll 1\) and \(\max(\lambda_2, \lambda_3, \lambda_5, \lambda_6) \ll \min(\lambda_1, \lambda_4)\) are satisfied. In these cases it is possible to express all the conditions in more compact form: \(|\Delta| \gg g_{\min} \sqrt{\langle a^\dagger a \rangle}\) and \(|\Delta| \gg g_{\min} \sqrt{\langle b^\dagger b \rangle}\), where \(g_{\min} = \min(g, g')\), and \(\max(\lambda_2, \lambda_3, \lambda_5, \lambda_6) \ll \min(\lambda_1, \lambda_4)\). The last condition is fulfilled if we set large enough \(\Omega\) and \(\Omega'\).

In the next sections we are going to use the effective master equation only. Therefore, from now on, we assume that the detuning is large enough, which in many atom case means that
\[
|\Delta| \gg g_{\min} \sqrt{n \langle a^\dagger a \rangle} \quad \text{and} \quad |\Delta| \gg g_{\min} \sqrt{\langle b^\dagger b \rangle}.
\]
(24)

We also assume that \(\Omega\) and \(\Omega'\) are large enough to satisfy the condition
\[
\max(\lambda_2, \lambda_3, \lambda_5, \lambda_6) \ll \min(\lambda_1, \lambda_4).
\]
(25)

B. The limit of high-intensity classical fields

Now we can consider the dynamics of the four-level atom in the diamond configuration in the limit of high-intensity classical fields. In this dressed-state approach there is one ground atomic state \(|0\rangle\) and three excited states \(|\mu\rangle\), \(|\phi\rangle\) and \(|\psi\rangle\). Here, there are three allowed transitions: \(|0\rangle \leftrightarrow |\mu\rangle\), \(|0\rangle \leftrightarrow |\phi\rangle\) and \(|0\rangle \leftrightarrow |\psi\rangle\), each of which is coupled to both cavity modes (see Eq. (20)). When intensities of classical fields tend to infinity, then expansion parameters \(\lambda_2\), \(\lambda_3\), \(\lambda_5\), \(\lambda_6\) tend to zero. It means that only two atomic levels, i.e. \(|0\rangle\) and \(|\mu\rangle\), are enough to describe the evolution of the system — the four-level atom in the diamond configuration effectively works exactly in the same way as the detuned two-level atom in this regime.

C. The effective Hamiltonian for lasers turned off

When the lasers are turned off then the evolution of the system is still governed by the Hamiltonian (1) but with \(\Omega = \Omega' = 0\). The formulas for the effective Hamiltonian given by Eq. (10) and the effective Lindblad operators (15) are also valid in this case. They now take the form
\[
H_{\text{eff}} = -(ng^2 / \Delta) a^\dagger a - (ng^2 / \Delta) b^\dagger b,
\]
\[
L_{\text{eff}}^{(1)} = \sqrt{n \gamma} (g / \Delta) b,
\]
\[
L_{\text{eff}}^{(2)} = \sqrt{n \gamma} (g / \Delta) a.
\]
(26)

In this simple case we can easily get more precise formulas if we perform the adiabatic elimination of excited atomic states assuming from the start that \(\Omega = \Omega' = 0\). In this way we derive
\[
H_{\text{eff}} = -\frac{ng^2 \Delta}{\Delta^2 + \gamma^2 / 4} a^\dagger a - \frac{ng^2 \Delta}{\Delta^2 + \gamma^2 / 4} b^\dagger b,
\]
and
\[
L_{\text{eff}}^{(1)} = \sqrt{\frac{n \gamma}{\Delta - i \gamma / 2}} b, \quad L_{\text{eff}}^{(2)} = \sqrt{\frac{n \gamma}{\Delta - i \gamma / 2}} a.
\]
(27)
It is important to note that there is no coupling between the two cavity modes, and therefore, there is no photon transfer when the lasers are turned off. We will refer to this working mode of the system as to the closed mode.

III. QUANTUM-STATE MAPPING BETWEEN TWO CAVITY MODES

In the previous section we have seen that such complicated system as an ensemble of four-level diamond-type atoms interacting with two quantized field modes can be effectively described by a quite simple Hamiltonian (see Eq. (14)) under certain conditions. We have also seen that we can easily control the evolution of the system just by switching the lasers on and off. Let us now show that we can use this system to transfer an interesting quantum state of one mode (for example a qudit or the Schrödinger’s cat states) to the other mode on demand. It has shown that in special cases, i.e., for coherent states and for qubit states, the Hamiltonian of the form (10) can swap the states of the two modes (21) [23]. Here, we show that it is possible to transfer an arbitrary photonic state.

First, we need the formula for the average photon number in the mode represented by the annihilation operator b, assuming that initially this mode is empty, while the mode represented by a is prepared in the Fock state |n_{ph}\rangle. This formula will help us investigate the photon transfer process. We can derive it introducing the superposition bosonic operator of both field modes

\[ C = \sqrt{1 - \epsilon a - \sqrt{\epsilon} b}. \]

We choose such \( \epsilon \) that the Hamiltonian (10) can be expressed in the form \( H_{\text{eff}} = -\delta_r C^\dagger C \), where \( \delta_r = (4\delta_2^2 + \frac{g^2}{4})^{1/2} \). Using this form of the Hamiltonian one can derive the formula for the average photon number

\[ \langle b^\dagger b \rangle = n_{ph} \left( 1 - \frac{\delta_2}{\delta_r} \right) \sin^2 (\delta_r t/2). \]

In Fig. 2 we plot the average photon number as a function of time. This figure shows that all \( n_{ph} = 2 \) photons can be transferred from the first mode to the second mode. However, this is possible only if \( \delta_1 = 0 \). We want the state mapping to be perfect, and therefore, we restrict ourselves to this case only. We can make \( \delta_1 \approx 0 \) by choosing values of \( \Omega \) and \( \Omega' \), which are much greater than \( \Delta \) and satisfy condition \( \Omega' g \approx \Omega g' \). If one wants \( \delta_1 = 0 \) then values of \( \Omega \) and \( \Omega' \) have to be chosen more precisely

\[ \Omega' = \sqrt{(\Omega^2 - 2\Delta^2)g^2/g^2 + 2\Delta^2}. \]

For reference, we also calculate numerically the average photon number using the non-Hermitian Hamiltonian

\[ \tilde{H} = (\Delta - i\gamma/2)\sigma_{11} + (\Delta - i\gamma/2)\sigma_{22} + (2\Delta - i\gamma'^2/2)\sigma_{33} + i\Omega\sigma_{23} + i\Omega'\sigma_{13} + ga\sigma_{12} + gb\sigma_{30} + \text{h.c.}, \]

which governs the evolution of this open system during the time intervals when no collapse occurs [59, 60]. We have obtained the Hamiltonian (32) by inserting Eqs. (2) and (3) into

\[ \tilde{H} = H - i\frac{1}{2} \sum_j L_j^\dagger L_j. \]

As one can see from Fig. 2, the analytical results are in a remarkable agreement with the numerical solution even for quite considerable values of \( \gamma, \gamma' \) and \( \gamma'' \) (where \( \gamma'' = \gamma_3 + \gamma_4 \)) as long as parameter regime justifies adiabatic elimination. The effect of \( \gamma \) and \( \gamma' \) on the state mapping is described in more detail in the last paragraph of this section.

From Eq. (30) we can infer that the \( \pi \) pulse time is given by the formula

\[ t_\pi = \pi/\delta_r, \]

from which one can observe one more important feature of the Hamiltonian. It is evident that the time of such \( \pi \) pulse is independent of \( n_{ph} \), and thus, we are able to perform the state-mapping operation defined by \( |n_{ph}\rangle_A \otimes |0\rangle_B \rightarrow |0\rangle_A \otimes |n_{ph}\rangle_B \).

Let us move into the rotating frame, in which the Hamiltonian takes the form

\[ H_{\text{eff}} = -\delta_2 (a^\dagger b + b^\dagger a) \]

and let us assume that the first mode is initially prepared in some interesting quantum state \( |\Psi_0\rangle = \sum_k c_k |k\rangle_A \), while the second mode is empty. Then, by switching the lasers on for \( t_\pi \), one can map this interesting state onto the second mode

\[ \left( \sum_k c_k |k\rangle_A \right) \otimes |0\rangle_B \rightarrow |0\rangle_A \otimes \left( \sum_k c_k |k\rangle_B \right). \]
FIG. 3: Deviation of \( t_x(n_{ph}) \) from \( t_x(1) \) (in percent) for \( (g', \Delta, \Omega, \Omega')/g = (1, 10, 33, 33) \) (open squares) and for \( (g', \Delta, \Omega, \Omega')/g = (1, 30, 100, 100) \) (solid circles). The second parameter regime justifies adiabatic elimination for \( n_{ph} = 9 \), whereas the first one only for \( n_{ph} = 1 \).

In a frame rotating at different frequency, in which the Hamiltonian takes the form

\[
H_{\text{eff}} = \delta_1 (a^\dagger a + b^\dagger b) + \delta_2 (a^\dagger b + b^\dagger a),
\]

phase factors appear and the \( \pi \) pulse changes the initial state according to

\[
(\sum_k c_k |k\rangle_A) \otimes |0\rangle_B \rightarrow |0\rangle_A \otimes \left( \sum_k c_k e^{i\phi_n(k)} |k\rangle_B \right),
\]

where \( \phi_n(n_{ph}) = -n_{ph} \pi (\delta_2 + \delta_c)/(2\delta_2) \). Note that for the parameters values used in Fig. 3, \( \delta_0 = -2.74 \) and \( \delta_2 = 2.98 \), so \( \delta_0 \approx -\delta_2 \). For the Hamiltonian \( \Omega_0 \), \( \delta_1 = 0 \) and large \( \Omega \) there are no phase factors, because \( \delta_0 \) tends to \(-\delta_2 \) for large \( \Omega \) and, thus, the Hamiltonian \( \Omega_0 \) tends to the form given by Eq. (35).

The independence of \( t_x \) from \( n_{ph} \) is crucial for the state-mapping operation. Unfortunately, \( t_x \) is independent of \( n_{ph} \), only in the approximated model \( \tilde{\Omega_0} \), in which we adiabatically eliminated all atomic excited levels. Numerical calculations show that \( t_x \) increases with \( n_{ph} \) in the more general model of the system given by the Hamiltonian \( \tilde{\Omega_0} \). However, as long as the adiabatic elimination is justified, we can neglect the dependence \( t_x \) on \( n_{ph} \), as is seen in Fig. 3.

It is seen from Fig. 3 that there are jumps of the value of \( t_x \). These jumps come from the fact that populations of atomic excited levels oscillate with high frequencies \( 61[63] \). Thus, there are many local closely-spaced maxima of the population of the desired final state \( |0\rangle_A \otimes |n_{ph}\rangle_B \). Therefore, the global maximum \( (t_x) \) changes sometimes discontinuously with increasing of \( n_{ph} \) — from one local minimum to the next one. We can neglect these jumps as long as the adiabatic elimination is justified.

Let us now investigate the effect of \( \gamma \) and \( \gamma' \) on the state-mapping operation. To this end we need non-Hermitian Hamiltonian, which we obtain by inserting Eqs. (10) and (15) into Eq. (34). Assuming that \( \Omega, \Omega' \ll \Delta \) and \( \delta_1 = 0 \), this Hamiltonian can be quite well approximated by

\[
\tilde{H} = -2\delta_2 C^\dagger C - \frac{i}{2} \gamma_{\text{eff}} C^\dagger C,
\]

where the effective dissipation rate is given by

\[
\gamma_{\text{eff}} = \frac{2ng^2(g^2\gamma' + g^2\gamma)}{\Delta^2(g^2 + g^2)^2}.
\]

It is clear that the fidelity of the state mapping \( F \) and the probability that no collapse occurs during this operation \( P \) are close to one only if the effective dissipation rate \( \gamma_{\text{eff}} \) is much less than the effective coupling strength \( \delta_2 \). For \( g = g' \) and \( \gamma = \gamma' \), the expression for the effective dissipation rate takes the simpler form \( \gamma_{\text{tot}} = n\gamma g^2/\Delta^2 \).

In this special case, \( F \) and \( P \) depend on the ratio \( \gamma/\Delta \). Let us now check this result numerically using the non-Hermitian Hamiltonian

\[
\tilde{H} = \sum_{k=1}^n \left\{ (\Delta - i\gamma'/2)\sigma_{11}^{(k)} + (\Delta - i\gamma/2)\sigma_{22}^{(k)} \right. \\
+ (2\Delta - i\gamma''/2)\sigma_{33}^{(k)} + (\Omega_{23}^{(k)} + \Omega'_{23}^{(k)}) \right. \\
+ g a^\dagger \sigma_{02}^{(k)} + g'b^\dagger \sigma_{01}^{(k)} + \text{h.c.} \left. \right\}.
\]

First, we have to choose specific values of parameters. The choice of the atom-cavity system determines \( g, g', \gamma, \gamma' \) and \( \gamma'' \). For macroscopic cavities \( g/2\pi \) is typically of the order of 10 MHz and \( \gamma \) ranges from about 0.2g to \( g/12 \) \([54]\). Let us set \( g' = g = 2\pi \cdot 10 \text{ MHz}, \gamma' = \gamma = 2g \) and \( \gamma'' = g \). The choice of the initial state determines the Fock state \( |n_{ph}\rangle \), to which the state mapping has to be faithful. Let the initial state of the atom be \( |\psi_0\rangle = (|0\rangle_A + |1\rangle_A + |2\rangle_A + |3\rangle_A)/2 \). If there are four atoms trapped in the cavity, then the detuning has to satisfy \( \Delta \gg g\sqrt{4 \cdot 3} \). We set \( \Delta = 35g \). Finally, we choose the value of \( \Omega \) and calculate \( \Omega' \) using Eq. (31). These values have to be large enough to satisfy the condition \( \Omega \gg \Omega' \). It is easy to check that for \( \Omega = \Omega' = 175g \) this condition is fulfilled, and therefore, adiabatic elimination is justified.

For \( (g', \Delta, \Omega, \Omega', \gamma, \gamma', \gamma'')/g = (1, 35, 175, 175, 2, 2, 1) \) and \( n = 4 \) we have found that \( F = 0.993 \) and \( P = 0.885 \). In the case of one atom trapped in the cavity \( (n = 1) \), for the same parameters, we have found that \( F = 0.995 \) and \( P = 0.886 \). One can see that \( F \) and \( P \) are almost the same in the two cases. The only important difference is the time of the state-mapping operation — \( t_x = 26.5/g \) and \( t_x = 105.6/g \) for \( n = 4 \) and \( n = 1 \), respectively. The time of the state mapping in the one-atom case is almost four times larger than that in the four-atom case. This result is in an agreement with Eq. (54). We can make
IV. FAST OPENING HIGH-Q CAVITY SYSTEM

In this section we show another application of the quantum system — a fast opening high-Q cavity system that can be easily and coherently controlled with classical laser fields. The device is based on similar principles as the setup of Tufarelli et al. [56], but it employs four-level atoms in the diamond configuration instead of two-level atoms. The main idea of both setups is to couple a high-Q cavity mode to a low-Q cavity mode through atoms. Such a device would be very useful, because on the one hand we need a high Q factor to reach the strong coupling regime [12] [14] [64] [65], in which we can generate a complex non-classical state of light trapped inside optical resonator [11] [10]. On the other hand, we need a low Q factor to extract this state from the resonator into a waveguide before it will be distorted by the cavity damping. The device proposed by Tufarelli et al. [56] makes it possible to change the effective Q factor. If atoms are absent, there is no coupling between the two modes and the whole system works as an effective high-Q cavity. If we move atoms into the cavity, then photons leak out of the high-Q mode through the low-Q mode and the whole device works as an effective low-Q cavity. Instead of shifting the atoms out of the cavity we can shift atoms out of resonance using a laser and the dynamic Stark effect. As long as the laser illuminates atoms, there is no coupling between modes. Here, we propose to replace two-level atoms by four-level atoms in the diamond configuration. Our modification allows us to use a bimodal cavity, which supports circularly polarized modes of the same or different polarizations and frequencies. Moreover, it requires intense laser light to illuminate atoms only in short time intervals, when we need the coupling between modes. When the laser is switched off, there is no coupling between modes.

In the following we assume, unless explicitly stated otherwise, that the device is working in the open mode, i.e., both lasers are turned on (\( \Omega, \Omega' \neq 0 \)). We also assume that the quantum state of field was prepared in advance in the mode represented by the operator \( a \). Under these assumptions, let us derive a quantity that describes the quality of the field extracted from the resonator into a waveguide. We will refer to this quantity as to the figure of merit of the proposed device.

\[ t_\pi \text{ smaller in the one-atom case by setting smaller } \Delta \text{ but then the ratio } \gamma/\Delta \text{ increases and the dissipation reduces the fidelity and the success probability. For instance, if we set } (g', \Delta, \Omega, \Omega', \gamma', \gamma'')/g = (1, 17, 85, 85, 2, 2, 1) \text{ in the one-atom case then the time of the state mapping is reduced to } t_\pi = 51.6/g. \text{ Then, however the dissipation reduces the fidelity and the success probability to } F = 0.979 \text{ and } P = 0.795, \text{ respectively.} \]

A. Interaction with an external field

After the adiabatic elimination of atomic excited states we can restrict our considerations to a simplified model, which does not include atomic variables. Such simplified model makes it easy to take into account all photon losses. To this end, we model the device as two cavity modes, which decay emitting the radiation into five traveling modes, as is depicted in Fig. 4. One of these traveling modes is accessible experimentally. This accessible traveling mode can be, for example, a waveguide. Other traveling modes are inaccessible, and thus, provide losses. The two cavity modes interact according to the effective Hamiltonian \( H \). The photon emission from the mode, represented by \( b \), into the waveguide is described by the Lindblad operator \( L_{bb} = \sqrt{\eta}b \). The absorption in the mirrors for this mode is modeled as the photon emission into an inaccessible mode and described by \( L_{bb}' = \sqrt{\eta}b \). The losses in the mirrors for the \( a \) mode are taken into account in the same manner. The photon absorption from the \( a \) mode is described by the operator \( L_{aa} = \sqrt{\eta}a \). Although the simplified model does not include atomic variables, spontaneous emissions from the excited atomic states \( |1 \rangle \) and \( |2 \rangle \) are taken into account by assuming that there are two inaccessible traveling modes, into which photons from both modes can be emitted in the way described by the Lindblad operators \( L_{bb}^{(1)} \) and \( L_{bb}^{(2)} \).

The device working in the open mode has to transfer the state of the \( a \) mode to the waveguide. In order to calculate a quantity, which measures how close the output field into waveguide is to the initial \( a \) mode field, it is necessary to describe the interaction of the quantum system with the accessible traveling modes. To this end, we use the input-output theory [71] [72], because it is perfectly suitable for the scheme illustrated in Fig. 4. We have followed the treatment of [71] to derive the Heisenberg-Langevin equations for this scheme. These equations take
transmittivity $F$

It is worth to note the similarity between Eq. (47) and

$$
\begin{align*}
\dot{a} &= (-\delta_2 + (\sqrt{\zeta_1} + \sqrt{\zeta_2})/2)b \\
&\quad + (\kappa + \zeta_1 + \zeta_2)/2)\sqrt{\zeta_1}c - \sqrt{\zeta_2}d, \\
\dot{b} &= (-\delta_2 + (\sqrt{\zeta_1} + \sqrt{\zeta_2})/2)a \\
&\quad + (-\delta_2 + (\eta_\text{tot} + \theta_1 + \theta_2))/2)b - \sqrt{\eta}b_\eta \\
&\quad - \sqrt{\eta}b_\nu - \sqrt{\eta}c - \sqrt{\eta}d, \\
\end{align*}
$$

where $a_\nu(t), b_\nu(t), c(t)$ and $d(t)$ are output field operators of inaccessible traveling modes, $b_\eta(t)$ is the output field operator of the waveguide mode and $\zeta_1 = n/\zeta_1 g^2$, $\theta_1 = n/\zeta_1 g^2$, $\zeta_2 = n/\zeta_2 g^2$, $\theta_2 = n/\zeta_2 g^2$, $\eta_\text{tot} = \eta \eta + \eta$.

The matrix form of equations (42) is given by

$$
\dot{\nu} = M \nu - \nu_{\text{out}},
$$

with

$$
M = \begin{bmatrix}
\kappa + \zeta_1 + \zeta_2 \\
\sqrt{\zeta_1} \theta_1 + \sqrt{\zeta_2} \theta_2 - \delta_2 \\
\sqrt{\zeta_1} \eta_\text{tot} + \theta_1 + \theta_2 \\
\delta_2 - \delta_2
\end{bmatrix},
$$

where $\nu = [a, b]^T$ and $\nu_{\text{out}} = [\sqrt{\eta}a_\nu + \sqrt{\zeta_1}c + \\
\sqrt{\zeta_2}d, \sqrt{\eta}b_\eta + \sqrt{\eta}b_\nu + \sqrt{\eta}b_\nu + \sqrt{\eta}d]^T$.

### B. Figure of merit

Now, we can follow closely the treatment of Tusarelli et al. [58] to get the figure of merit of the scheme. First, we have to define the bosonic operator for the waveguide field traveling away from the device

$$
\begin{align*}
\nu_{\text{out}} = & \int_0^\infty u(\tau)b_\eta(\tau)d\tau,
\end{align*}
$$

with $u(\tau)$ being a temporal profile of the form

$$
\begin{align*}
u(\tau) &= \frac{[e^{-\nu(\tau)}]\,1,2}{\sqrt{\int_0^\infty [e^{-\nu(\tau)}]_{1,2}^2d\tau}}.
\end{align*}
$$

Next, we introduce the bosonic operator $h_{\text{ext}}$ representing all inaccessible traveling modes. We do not need to know the specific form of $h_{\text{ext}}$ in our calculations. Then we can relate the annihilation operator $a$ at the time $t = 0$ to the output modes using the formula

$$
\begin{align*}
a(0) &= \sqrt{F}f_{\text{out}} - \sqrt{1 - F}h_{\text{ext}},
\end{align*}
$$

where

$$
F = \int_0^\infty [e^{-\nu(\tau)}]_{1,2}^2d\tau.
$$

It is worth to note the similarity between Eq. (47) and a unitary transformation representing a beam splitter of transmittivity $F$. This similarity allows us to consider an abstract beam splitter described by relations

$$
\begin{align*}
a(0) &= \sqrt{F}f_{\text{out}} - \sqrt{1 - F}h_{\text{ext}}, \\
a_{\text{vac}}(0) &= \sqrt{1 - F}f_{\text{out}} + \sqrt{F}h_{\text{ext}}.
\end{align*}
$$

The abstract mode $a_{\text{vac}}(0)$ must be empty, because the total excitation number has to be conserved, i.e., the initial number of photons inside the $a$ mode has to be equal to the total number of photons inside outgoing modes $f_{\text{out}}$ and $h_{\text{ext}}$. Using the abstract beam-splitter model of the device it is easy to get formula for $f_{\text{out}}$:

$$
\begin{align*}
f_{\text{out}} &= \sqrt{F}a(0) + \sqrt{1 - F}a_{\text{vac}}(0).
\end{align*}
$$

The parameter $F$ satisfies $0 \leq F \leq 1$ and, as it is easy to see from Eq. (41), it can work as a figure of merit, because as $F$ gets closer to one, the output field $f_{\text{out}}$ gets closer to the initial field $a(0)$. This fact is especially clearly seen in the Schrödinger picture [56].

$$
\begin{align*}
\rho_{\text{out}} &= e^{(1 - F)L}\rho_0, \\
\end{align*}
$$

where $\rho_0$ is the initial state of the $a$ mode, $\rho_{\text{out}}$ is the final state of the $f_{\text{out}}$ mode and the Liouvillian is given by

$$
\begin{align*}
L\rho &= \frac{1}{2}(2a^\dagger a^\dagger - a^\dagger a - a^\dagger a).
\end{align*}
$$

In order to investigate how well the initial quantum state can be extracted from the cavity using the device presented in Fig. 4, we have to express the figure of merit $F$ as a function of parameters of this device. It can be done using the method presented in Ref. [56]. First, we express the figure of merit as

$$
\begin{align*}
F &= \eta\chi_{1,2,1}(M),
\end{align*}
$$

where

$$
\begin{align*}
\chi_{1,2,1}(M) &= \int_0^\infty [e^{-\nu(\tau)}]_{1,2}d\tau,
\end{align*}
$$

is an element of the tensor $\chi$. We can express this tensor in the matrix form as

$$
\chi(M) = \int_0^\infty e^{-\nu(\tau)} \otimes e^{-\nu(\tau)}d\tau,
$$

where $\otimes$ indicates the Kronecker product. Since $\chi(M)$ is the solution to a Sylvester equation, we can obtain all elements of $\chi(M)$ just by solving linear system of equations

$$
(M \otimes I)\chi(M) + \chi(M)(I \otimes M^\dagger) = I \otimes I,
$$

where $I$ indicates the $2 \times 2$ identity matrix. In this way we derive the formula for $\chi_{1,2,1}(M)$, which we insert into Eq. (41). Unfortunately, the obtained expression is too complex to be useful, and thus, it is necessary to resort to further approximations. If we assume that $\eta_\text{tot} \gg \delta_2 \gg \kappa, \zeta_1, \zeta_2, \theta_1, \theta_2$ then the figure of merit can be well approximated by

$$
\begin{align*}
F &= \frac{\eta}{\eta_\text{tot}} \frac{1}{\eta_\text{tot}} [\kappa + \sqrt{\zeta_1} + \sqrt{\zeta_2}]/2 \\
&\quad - \frac{\eta_\text{tot}(\kappa + \sqrt{\zeta_1} + \sqrt{\zeta_2})}{4\delta_2^2 + \eta_\text{tot}(\kappa + \sqrt{\zeta_1} + \sqrt{\zeta_2})}.
\end{align*}
$$
From Eq. (58), one can observe a couple of features of this device. First, the figure of merit can be close to one only under the condition \( 4\delta^2 \gg \eta \text{rot}(\kappa + \zeta_1 + \zeta_2) \). Assuming \( \delta_1 = 0 \), this condition leads to conditions:

\[
\eta \text{rot} \ll n g' / \gamma, \quad \eta \text{rot} \ll n g'' / \gamma', \quad \eta \text{rot} \ll \delta_2 (\delta_2 / \kappa) \quad (59)
\]

It follows that \( \kappa \) has to be at least two orders of magnitude smaller than \( \delta_2 \). For currently available atom-cavity systems all these conditions can be satisfied only for a large number of atoms \( n \).

Second, \( F \) is independent of \( \delta_1 \) in the mentioned regime. Typically, \( g \neq g' \) in concrete realizations of the four-level atom, and therefore, usually \( \delta_1 \neq 0 \). A non-zero value of \( \delta_1 \) decreases \( F \) when dissipative rates are too large. It is possible to make \( \delta_1 = 0 \) by setting appropriate value of \( \Omega', \) i.e., this one given by Eq. (31). From Eq. (58), it is seen that such precise setting of \( \Omega' \) is not necessary in the regime, in which the figure of merit is close to one. This feature makes choosing values of parameters easier. It is also worth to note that in the regime \( \Omega \gg \Delta \) Eq. (31) takes the simpler form \( \Omega' g \approx \Omega g' \).

C. Performance analysis

Let us now investigate the usefulness of the considered device to extract a field state from the \( a \) mode. We assume that there is only one optical cavity. This cavity supports two electromagnetic field modes of different frequencies \( \omega \) and \( \omega' \) (see Fig. 1). The first of them is considered as the \( a \) mode, while the second one as the \( b \) mode. Each cavity mirror is described by its radius of curvature \( r \), transmission coefficients \( T \) and \( T' \) for the \( a \) mode and the \( b \) mode, respectively, and less coefficient \( L \), which is assumed to be the same for both modes. The \( a \) mode requires very low values of \( T \) and \( L \) for both mirrors. To our knowledge, these parameters take the lowest value for the mirror that has been used in the experiment of [31] [73]. We set these values in our calculations, i.e., \( T_{\text{small}} = T_1 = T_2 = T'_1 = 1.8 \) ppm and \( L = 3.15 \) ppm, where the subscripts indicate the mirror. The radius of curvature of both mirrors is 50 mm [31] [73].

Now we can only the cavity length \( l \) and the transmission coefficient \( T'_2 \), and therefore, we want to plot the figure of merit \( F \) as a function of these two quantities. First, we have to choose a concrete realization of the \( \Omega \)-type atom. Let us choose a \(^{87}\text{Rb} \) atom, as in the mentioned above experiment [31] [73], and its levels \( |5S_{1/2}, F = 2, m_F = 2 \rangle, |5P_{3/2}, F = 3, m_F = 3 \rangle, |6P_{3/2}, F = 3, m_F = 3 \rangle \) and \( |6D_{3/2}, F = 3, m_F = 3 \rangle \) to serve as \( |0 \rangle, |1 \rangle, |2 \rangle \) and \( |3 \rangle \), respectively. This choice determines values of modes frequencies to be \( \omega / 2 \pi = 713.28 \) THz and \( \omega' / 2 \pi = 384.23 \) THz [74]. The lifetimes of all used here excited levels can be found in Ref. [75]. It is important that the lifetime of the level \( |3 \rangle \) is longer \( (\tau_3 = 256 \text{ ns}) \) than lifetimes of the other excited levels \( (\tau_1 = 112 \text{ ns for } |1 \rangle \text{ and } \tau_2 = 26.25 \text{ ns for } |2 \rangle) \). So our assumption that spontaneous emissions from the excited level \( |3 \rangle \) can be neglected in calculations is justified not only by small population of this level, but also by \( \tau_3 > \tau_1 \) and \( \tau_3 > \tau_2 \). The spontaneous emissions can take the \(^{87}\text{Rb} \) atom from the states \( |1 \rangle \) and \( |2 \rangle \) only to state \( |0 \rangle \). Hence, it is easy to calculate corresponding spontaneous emission rates: \( \gamma / 2 \pi = 1.42 \) MHz and \( \gamma' / 2 \pi = 6.06 \) MHz.

In principle, the scheme presented here works properly even with only one trapped atom. In real experiments, however, this scheme will require a much larger number of atoms to achieve the figure of merit that is close to unity. In order to compare our scheme with the original scheme of Tufarelli et al. [56], we set here the same number of atoms as in Ref. [56], i.e., \( n = 1000 \). Trapping 1000 rubidium atoms and preparing them in the \( |5S_{1/2}, F = 2, m_F = 2 \rangle \) state is possible using fiber-based Fabry-Perot cavities [76] [77]. We have chosen the macroscopic cavity in our considerations. A number of atoms trapped inside macroscopic cavities is typically of the order of \( 10^5 \) [78].

Now we can calculate the coupling strength \( g \) using

\[
g = \frac{\sqrt{3\pi c^2 \gamma}}{2 \omega^2 V},
\]

where \( c \) is the speed of light and \( V \) is the cavity mode volume given by

\[
V = \pi c \sqrt{l(2r - l)} / (4\omega).
\]

In order to calculate the coupling strength \( g' \) we have to replace \( \omega \) and \( \gamma \) by \( \omega' \) and \( \gamma' \) in (60) and (61).

The cavity damping constants of the considered here scheme can be calculated using formulas

\[
\kappa = c(1 - R) / (l \sqrt{R}),
\eta = T_2 \eta_{\text{rot}} / N,
\eta' = (2L + T_1 \eta_{\text{rot}} / N,\]

where \( R = 1 - l - T_{\text{small}}, N = 2L + T'_1 + T'_2 \) and

\[
\eta_{\text{rot}} = \frac{1}{l R^{1/4}} \sqrt{(1 - L - T'_2)^{1/4}}.
\]

Finally, we have to fix values of \( \Delta, \Omega \) and \( \Omega' \). It is necessary to choose these values carefully. On the one hand, they should be big enough to make adiabatic elimination justified. On the other hand, they cannot be too big, because \( \delta_2 \) and \( F \) decrease with increasing \( \Delta \). In our computations we set \( \Delta / g = 700 \) and \( \Omega / \Delta = 5 \), which justifies adiabatic elimination for cavity states with \( (a|a|) \ll 10 \). Then \( \Omega' \) is given by (31).

Now we can plot \( F \) as a function of \( l \) and \( T'_2 \). For the parameters given above the formula (58) gives only raw approximation of the figure of merit. Therefore, we have calculated \( F \) numerically using Eq. (48), and we have obtained in this way results presented in Fig. 5.
to 0.97. Unfortunately, the near-concentric configuration of the macroscopic mirrors is extremely sensitive to misalignment, and therefore, it would be difficult or even impossible to achieve such high value of $F$. For the confocal configuration $l = r$, which is the most stable configuration, the figure of merit can be equal to 0.92. This value is still quite high and it is higher than $F$ of the original scheme of Tufarelli et al. [56]. Of course, we can always increase the figure of merit by increasing $n$. To show this we plot also the figure of merit for $n = 8000$. It is seen from Fig. 6 that now $F = 0.97$ even for the confocal configuration.

Let us now check the approximate formula (58) for the parameter regime corresponding to the confocal configuration with 1000 atoms. Setting $l = 50$ mm and $T_2^\prime = 800$ ppm, we get $(g,g',\Delta,\Omega,\Omega',\gamma,\gamma',\eta_{tot},\kappa)/2\pi = (0.1,0.29,72.8,364,989,1.4,6.06,0.6,0.4,4.7\cdot10^{-3})$ MHz. These lead to $(\zeta_1,\theta_1,\zeta_2,\theta_2)/2\pi = (1.3,2.3,1.2,2.5)$ kHz and $\delta_{2}/2\pi = 0.14$ MHz. As mentioned earlier, the formula (58) is valid if the conditions $\delta_{2} \gg \kappa,\zeta_1,\theta_1,\zeta_2,\theta_2$ and $\eta_{tot} \gg \delta_{2}$ are fulfilled. One can see that the first condition is fulfilled. However, the ratio $\eta_{tot}/\delta_{2}$ is only 2.8. Nevertheless, the value of the figure of merit calculated using Eq. (58) ($F = 0.95$) is quite close to the value obtained numerically using Eq. (48) ($F = 0.92$).

Now, we can discuss the experimental feasibility of the scheme. The figure of merit $F = 0.92$ is high enough to consider experimental realization of the fast opening high-$Q$ cavity system. However, one should keep in mind that in the experiment described in Refs. [54, 73] parameters $L = 3.15$ ppm and $T_{\text{small}} = 1.8$ ppm were reached only for $\omega/2\pi = 384.23$ THz. Unfortunately, we have no experimental data for $\omega/2\pi = 713.28$ THz. Thus, we assume that the values of $L$ and $T_{\text{small}}$ are the same for both cavity modes. If these values are larger due to technical limitations, then the figure of merit will be smaller than 0.92. The figure of merit will also be lowered by other imperfections present in real experimental setups such as: scattering, inhomogeneous coupling strengths, imperfect optical pumping, coupling to other levels, dephasing, laser noise, Stark-shift fluctuations at non-zero temperature etc. Therefore, reaching the target figure of merit can be very challenging.

D. The device working in the closed mode

So far, we have investigated the device working in the open mode. Let us now consider this device working in the closed mode. For the device working in the closed mode both lasers are turned off. The effective Hamiltonian derived with $\Omega = \Omega' = 0$ is given by Eq. (27). It is seen that there is no interaction between the $a$ mode and the $b$ mode, and therefore, photons do not leak out of the $a$ mode through the $b$ mode. The only destructive role played by atoms trapped inside the cavity is the increase of photon losses caused by the spontaneous emission from the atomic excited state [2]. The decay of the $a$ mode associated with the atomic spontaneous emission is described by an effective decay rate $\kappa_a \approx n\gamma (g/\Delta)^2$ (see Eq. (26)). We have found out that for the parameter values used in the previous section with $l = 50$ mm, $T_2^\prime = 800$ ppm and $n = 1000$ this effective decay rate $\kappa_a/(2\pi) = 2.9 \cdot 10^{-3}$ MHz is less than the cavity decay rate associated with the absorption in the mirrors $\kappa/(2\pi) = 4.7 \cdot 10^{-3}$ MHz. Knowing $\kappa_a$, we can take atomic spontaneous emissions into account just by making the replacement $\kappa \rightarrow \kappa' = \kappa + \kappa_a$.

V. CONCLUSIONS

We have studied a quantum system composed of $\Diamond$-type atoms and an optical cavity supporting two electromagnetic field modes, in which these atoms are per-
manently trapped. We have considered the case, where lower atomic transitions (see Fig. [1]) are coupled to the field modes and upper atomic transitions are driven by classical laser fields. We have shown that this complex quantum system can be described by an effective Hamiltonian of the simple form given in Eq. (14) if intensities of the lasers fields and the detuning are sufficiently large. We have also shown that the evolution of this system can be easily controlled just by turning lasers on and off. We have presented two examples of applications of the system. The first application is a state transfer from one quantized mode to another. We have shown that the time of the state transfer is independent of the number of photons. Thus, it is possible to map a quantum state of photons. Thus, it is possible to map a quantum state of one mode onto the other mode. As the second application of the system, we have presented a device that plays the role of a switchable cavity, which can work on demand either as a low-Q cavity, or as a high-Q cavity. The ◊-type atoms allow for fast switching between these two working modes just by switching the lasers on and off. Moreover, ◊-type atoms make this device to be especially well suited for a bimodal cavity, which supports circularly polarized modes of the same or different polarizations and frequencies.

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[1] J. Larson, J. Mod. Opt. 53, 1867 (2006).
[2] F. O. Prado, F. S. Luiz, J. M. Villas-Bôas, A. M. Alcalde, E. I. Duzzioni, and L. Sanz, Phys. Rev. A 84, 053839 (2011).
[3] P. Domokos, M. Brune, J. Raimond, J. Raimond, and S. Haroche, EPJ D 1, 1 (1998).
[4] C. M. Savage, S. L. Braunstein, and D. F. Walls, Opt. Lett. 15, 628 (1990).
[5] Y. Rong-Can, L. Gang, L. Jie, and Z. Tian-Cai, Chin. Phys. B 20, 060302 (2011).
[6] G. Nikoghosyan, M. J. Hartmann, and M. B. Plenio, Phys. Rev. Lett. 108, 123603 (2012).
[7] K. Liu, L.-B. Chen, P. Shi, W.-Z. Zhang, and Y.-J. Gu, Quantum Inf Process 12, 3057 (2013).
[8] S. Bouguella and Z. Ficek, Phys. Rev. A 88, 022317 (2013).
[9] X.-Q. Xiao, J. Zhu, G. He, and G. Zeng, Quantum Inf Process 12, 449 (2013).
[10] Q.-G. Liu, Q.-C. Wu, C.-L. Leng, Y. Liang, X. Ji, and S. Zhang, Quantum Inf Process 13, 2801 (2014).
[11] J. McKeever, A. Boca, A. D. Boozer, R. Miller, J. R. Buck, A. Kuzmich, and H. J. Kimble, Science 303, 1992 (2004).
[12] A. D. Boozer, A. Boca, R. Miller, T. E. Northup, and H. J. Kimble, Phys. Rev. Lett. 98, 193601 (2007).
[13] B. Weber, H. P. Specht, T. Müller, J. Bochmann, M. Mücke, D. L. Moehring, and G. Rempe, Phys. Rev. Lett. 102, 030501 (2009).
[14] C. Nölleke, A. Neuzner, A. Reiserer, C. Hahn, G. Rempe, and S. Ritter, Phys. Rev. Lett. 110, 140403 (2013).
[15] B. Hacker, S. Welte, G. Rempe, and S. Ritter, Nature 536, 193 (2016).
[16] A. Imamoglu, D. D. Awschalom, G. Burkard, D. P. DiVincenzo, D. Loss, M. Sherwin, and A. Small, Phys. Rev. Lett. 83, 4204 (1999).
[17] A. Miranowicz, S. K. Özdemir, Y.-x. Liu, M. Koashi, N. Imoto, and Y. Hirayama, Phys. Rev. A 65, 062321 (2002).
[18] A. D. Boozer, A. Boca, R. Miller, T. E. Northup, and H. J. Kimble, Phys. Rev. Lett. 98, 193601 (2007).
[19] K. Härkönen, F. Plastina, and S. Maniscalco, Phys. Rev. A 80, 033841 (2009).
[20] J. Cheng, Y. Han, and L. Zhou, J. Phys. B 45, 015505 (2012).
[21] L.-H. Zhang, M. Yang, and Z.-L. Cao, EPJ D 68, 109 (2014).
[22] B. Casabone, K. Friebe, B. Brandstätter, K. Schüppert, R. Blatt, and T. E. Northup, Phys. Rev. Lett. 114, 023602 (2015).
[23] P. Dong, J. Liu, L.-H. Zhang, and Z.-L. Cao, Physica B 495, 50 (2016).
[24] G.-W. Lin, X.-B. Zou, M.-Y. Ye, X.-M. Lin, and G.-C. Guo, Phys. Rev. A 77, 064301 (2008).
[25] X.-Q. Shao, L. Chen, and S. Zhang, J. Phys. B 41, 245502 (2008).
[26] M. V. Sharypov and B. He, Phys. Rev. A 87, 032323 (2013).
[27] M. S. Everitt and B. M. Garraway, Phys. Rev. A 90, 012335 (2014).
[28] D. D. Yavuz, Phys. Rev. A 71, 053816 (2005).
[29] R. M. Serra, C. J. Villas-Bôas, N. G. de Almeida, and M. H. Y. Moussa, Phys. Rev. A 71, 045802 (2005).
[30] Y.-F. Xiao, X.-B. Zou, Z.-F. Han, and G.-C. Guo, Phys. Rev. A 74, 044303 (2006).
[31] C. Kurz, M. Schug, P. Eich, J. Huwer, P. Müller, and J. Eschner, Nat. Commun. 5, 5527 (2014).
[32] S. Lloyd, M. S. Shabir, J. H. Shapiro, and P. R. Hemmer, Phys. Rev. Lett. 87, 167903 (2001).
[33] S. Clark, A. Peng, M. Gu, and S. Parkins, Phys. Rev. Lett. 91, 177901 (2003).
[34] T. Wilk, S. C. Webster, A. Kuhn, and G. Rempe, Science 317, 488 (2007).
[35] B.-Q. Ou, L.-M. Liang, and C.-Z. Li, Opt. Commun. 282, 2870 (2009).
[36] X. Yan, H. Qiong-Yi, G. C. LaRocca, M. Artoni, X. Ji-Hua, and G. Jin-Yue, Phys. Rev. A 73, 013816 (2006).
[37] S. Kajari-Schröder, G. Morigi, S. Franke-Arnold, and G.-D. Roßnagel, Phys. Rev. A 84, 053819 (2011).
[38] F. E. Becerra, R. T. Willis, S. L. Rolston, and L. A. Orozco, Phys. Rev. A 74, 013816 (2006).
[39] M. C. Stowe, A. Pe'er, and J. Ye, Phys. Rev. Lett. 100, 123603 (2012).
[40] B. Hacker, S. Welte, G. Rempe, and S. Ritter, Phys. Rev. Lett. 110, 140403 (2013).
[41] B. Hacker, S. Welte, G. Rempe, and S. Ritter, Nature 536, 193 (2016).
[42] A. Imamoglu, D. D. Awschalom, G. Burkard, D. P. DiVincenzo, D. Loss, M. Sherwin, and A. Small, Phys. Rev. Lett. 83, 4204 (1999).
[43] A. Miranowicz, S. K. Özdemir, Y.-x. Liu, M. Koashi, N. Imoto, and Y. Hirayama, Phys. Rev. A 65, 062321 (2002).
[44] A. D. Boozer, A. Boca, R. Miller, T. E. Northup, and H. J. Kimble, Phys. Rev. Lett. 98, 193601 (2007).
[45] K. Härkönen, F. Plastina, and S. Maniscalco, Phys. Rev. A 80, 033841 (2009).
[41] A. Kölle, G. Epple, H. Kübler, R. Löw, and T. Pfau, Phys. Rev. A 85, 063821 (2012).
[42] H.-g. Lee, H. Kim, J. Lim, and J. Ahn, Phys. Rev. A 88, 053427 (2013).
[43] M. Parniak and W. Wasilewski, Phys. Rev. A 91, 023418 (2015).
[44] G. Grynberg and P. R. Berman, Phys. Rev. A 41, 2677 (1990).
[45] U. W. Rathe and M. O. Scully, Phys. Rev. A 52, 3193 (1995).
[46] X. ming Hu and J. sheng Peng, Chin. Phys. Lett. 16, 648 (1999).
[47] W.-W. Deng, H.-T. Tan, and G.-X. Li, J. Phys. B 41, 155502 (2008).
[48] S. Qamar, M. Al-Amri, S. Qamar, and M. S. Zubaír, Phys. Rev. A 80, 023418 (2009).
[49] G. Qiang and S. Tong-Qiang, Chin. Phys. B 21, 124202 (2012).
[50] F. Wang, Laser Phys. Lett. 10, 125203 (2013).
[51] H. Baghshahi, M. Tavassoly, and A. Behjat, Commun. Theor. Phys. 62, 430 (2014).
[52] R. F. Offer, J. W. C. Conway, E. Riis, S. Franke-Arnold, and A. S. Arnold, Opt. Lett. 41, 2177 (2016).
[53] H. R. Baghshahi, M. K. Tavassoly, and A. Behjat, EPJ Plus 131, 80 (2016).
[54] T. Tufarelli, A. Ferraro, A. Serafini, S. Bose, and M. S. Kim, Phys. Rev. Lett. 112, 133605 (2014).
[55] F. Reiter and A. S. Sørensen, Phys. Rev. A 85, 032311 (2012).
[56] M. Alexanian and S. K. Bose, Phys. Rev. A 52, 2218 (1995).
[57] M. B. Plenio and P. L. Knight, Rev. Mod. Phys. 70, 101 (1998).
[58] H. J. Carmichael, An Open Systems Approach to Quantum Optics (Springer, Berlin, 1993).