The field state dissipative dynamics of two-photon Jaynes-Cummings model with Stark shift in dispersive approximation

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Abstract:

We present the field state dissipative dynamics of two-photon Jaynes-Cummings model (JCM) with Stark shift in dispersive approximation and investigate the influence of dissipation on entanglement. We show the coherence properties of the field is also affected by the cavity when nonlinear two-photon process is involved.

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1 Introduction

Jaynes-Cummings Model (JCM) [1] has been recognized as the simplest and most effective model to describes a two-level atom interaction with the electromagnetic field. It presents an extremely rich and nontrivial dynamics. In addition to its exact solvability within the rotating-wave approximation, the most interesting aspect of its dynamics is the entanglement between the atom and the field. Much attention has been focused on the entanglement of the field and the atom in JCM[2-6]. Recently, entanglement as a physical resource has been used in quantum information science such as quantum teleportation [7], superdense coding [8] and quantum cryptography [9]. In spite of the success achieved by quantum theory in what concerns the prediction of experiments in general, there has been a lot of debate about some of its fundamental

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aspects, one of which is directly related to the entanglement, which presents its most famous illustrations in Einstein-Podolsky-Rosen paradox. People have gain some satisfactory answer about Schrödinger’s cat paradox and why the entanglement phenomenon does not occur in the classical world. Among those the one that stress the role played by the environment, which is represented by a thermal reservoir. Decoherence due to the irreversible coupling of the observed system to the outside world reservoir eventually turn correlated state into a statistical mixture.

The details of the entanglement between two subsystem in the presence of such an environment is worth to study. In Ref. [10] dispersive atomic evolution in a dissipative-driven cavity were studied. The influence of driving field on the quantized driven field and on atom properties in both the dissipative and the lossless cases were obtained. And in Ref.[11], the authors employed JCM in the dispersive approximation in a dissipative cavity at zero temperature to study the entanglement between the atom and the field as well as the decoherence induced by the cavity. They had shown the cavity has practically no influence in the coherence properties of the field from the qualitative point of view but the atom’s coherence properties are strongly influenced by dissipation both qualitatively and quantitatively, although it is not directly coupled to the cavity. The purpose of this paper is to research the dynamics of a two-level atom with Stark shift interaction with the field by two-photon process in a dissipative cavity and to study whether the cavity has practically no influence in the coherence properties of the field when two-photon process is involved. We also plane to research the entanglement influenced by the dissipation. We show that the coherence properties of the field is also affected by the cavity when nonlinear two-photon process is involved. We also observe the entanglement is influenced by dissipation and make the amplitude of oscillation suppress. In two-photon process the relation the coherence loss of the field with the intensity of the field is also given.

2 Two-photon Jaynes-Cummings Model with Stark shift in dispersive approximation

The Hamiltonian of the two-photon JCM including Stark shift with rotating-wave approximation [4,12] is given by

\[ \hat{H} = \omega \hat{a}^{\dagger} \hat{a} + \frac{\omega_0}{2} \hat{S}_z + \hat{a}^{\dagger} \hat{a} (\beta_2 |e\rangle \langle e| + \beta_1 |g\rangle \langle g|) + \lambda (\hat{a}^{\dagger} \hat{S}_- + \hat{a} \hat{S}_+), \]

where \( \omega \) is the field frequency, \( \omega_0 \) is the frequency between the two-level (denoted by \( e \) and \( g \)) of the atom, represented here by the well-know Pauli matrices \( \hat{S}_i \), \( \beta_1 \) and \( \beta_2 \) are effective Stark
shift coefficients, which related to $\lambda_1$ and $\lambda_2$ and $\Delta$ (detuning) as follows:

$$\beta_i = \frac{\lambda_i^2}{\Delta}, \quad i = 1, 2; \lambda = \frac{\lambda_1\lambda_2}{\Delta},$$

(2)

where $\lambda_1$ and $\lambda_2$ denote intermediate state $|i\rangle$ coupling to $|e\rangle$ and $|g\rangle$ with strengths, $\lambda$ measures the two-photon atom-field coupling. In an invariant subspace spanned by $|e\rangle \otimes |n\rangle$ and $|g\rangle \otimes |n + 2\rangle$, the Hamiltonian takes the following form

$$\hat{H} = \begin{bmatrix}
\omega n + \frac{\omega_0}{2} + \frac{\beta_2 n}{2} & \frac{\lambda\sqrt{(n+1)(n+2)}}{\omega n + \frac{\omega_0}{2} + \delta + \beta_1(n+2)} \\
\lambda\sqrt{(n+1)(n+2)} & \omega n + \frac{\omega_0}{2} + \delta + \beta_1(n+2)
\end{bmatrix},$$

(3)

where detuning $\delta = 2\omega - \omega_0$ measures how off resonance the two system are. The eigenvalues are

$$E_+ = \omega n + \frac{\omega_0}{2} + \frac{\beta_1(n+2)}{2} + \frac{\beta_2 n}{2} - \frac{\delta}{2} + \frac{1}{2}(|\beta_1(n+2)| - |\beta_2(n+1)| + \beta_2 - \delta^2 + 4\lambda^2(n+1)(n+2))^{\frac{1}{2}}.$$

(4)

The dispersive limit of two-photon process with Stark shift is obtained when $\hat{H}_I$ can be considered as a small perturbation in the following sense:

$$\frac{\beta_1(n+2)}{|\delta - \beta_2|} \ll 1, \quad \frac{\beta_2(n+1)}{|\delta - \beta_2|} \ll 1$$

(5)

for any relevant $n$.

$$E_+ = \omega n + \frac{\omega_0}{2} + \beta_2 n + \frac{\beta_1\beta_2}{|\delta - \beta_2|}(n+1)(n+2),$$

$$E_- = \omega n + \frac{\omega_0}{2} - \delta + \beta_1(n+2) - \frac{\beta_1\beta_2}{|\delta - \beta_2|}(n+1)(n+2).$$

(6)

If the condition Eq.(5) is fulfilled for all $n$ value, we can work with the effective Hamiltonian

$$\hat{H}_{eff} = \omega \hat{a}^+ \hat{a} + \frac{\omega_0}{2} \hat{S}_z + \hat{a}^+ \hat{a} (\beta_2 |e\rangle \langle e| + \beta_1 |g\rangle \langle g|)$$

$$+ \Omega [(\hat{a}^+ \hat{a} + 1)(\hat{a}^+ \hat{a} + 2)|e\rangle \langle e| - \hat{a}^+ \hat{a} (\hat{a}^+ \hat{a} - 1)|g\rangle \langle g|],$$

(7)

where for simplicity we let $\frac{\beta_1\beta_2}{\delta - \beta_2} = \Omega$. 

3
3 Time evolution of the initial field state

We assume that there is a reservoir coupled to the field in the usual way. In the dispersive approximation, a two-level atom interacting with a quantum field in a dissipative cavity has a standard form ($\hbar = 1$)

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}_{eff}, \hat{\rho}] + \mathcal{L}\hat{\rho}. \hspace{1cm} (8)$$

The losses in the cavity are phenomenologically represented by the superoperator $\mathcal{L}$. At the zero temperature, we have

$$\mathcal{L}\hat{\rho} = \kappa(2\hat{a}\hat{\rho}\hat{a}^+ - \hat{a}^+\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^+\hat{a}), \hspace{1cm} (9)$$

where $\kappa$ is the damping constant. In the interaction picture, the master equation takes the form

$$\frac{d\hat{\rho}}{dt} = -i[\hat{a}^+(\beta_2|e\rangle\langle e| + \beta_1|g\rangle\langle g|) + \Omega((\hat{a}^+\hat{a} + 1)(\hat{a}^+\hat{a} + 2)|e\rangle\langle e|)
- \hat{a}^+(\hat{a}^+ - 1)|g\rangle\langle g|, \hat{\rho}] + \kappa(2\hat{a}\hat{\rho}\hat{a}^+ - \hat{a}^+\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^+\hat{a}). \hspace{1cm} (10)$$

Two-photon process is a nonlinear process. Completely solving equation (10) is not an easy task. Here we just plan to consider the subsystem field property. We write the reduced field operator as

$$\hat{\rho}_F(t) = Tr_A\hat{\rho}(t) = \hat{\rho}_{gg}(t) + \hat{\rho}_{ee}(t). \hspace{1cm} (11)$$

Liouvillians corresponding to the matrix elements $\hat{\rho}_{gg}$ and $\hat{\rho}_{ee}$ have the form

$$\mathcal{L}_{gg} = 2\kappa F + 2i\Omega(\mathcal{M}^2 - \mathcal{P}^2) - (\kappa + i\Omega_g)\mathcal{M} - (\kappa - i\Omega_g)\mathcal{P}, \hspace{1cm} (12)$$

$$\mathcal{L}_{ee} = 2\kappa F - 2i\Omega(\mathcal{M}^2 - \mathcal{P}^2) - (\kappa + i\Omega_e)\mathcal{M} - (\kappa - i\Omega_e)\mathcal{P}, \hspace{1cm} (13)$$

where $\Omega_g = \beta_1 + \Omega, \Omega_e = \beta_2 + 3\Omega$. The superoperators in Eq.(12) and (13) are defined as $F\hat{\rho} = \hat{a}\hat{\rho}\hat{a}^+, \mathcal{M}\hat{\rho} = \hat{a}^+\hat{a}\hat{\rho}, \mathcal{P}\hat{\rho} = \hat{\rho}\hat{a}^+\hat{a}$. They satisfy the commutation relation [10]

$$[F, \mathcal{M}] = F, \hspace{1cm} [F, \mathcal{P}] = F, \hspace{1cm} [\mathcal{M}, \mathcal{P}] = 0. \hspace{1cm} (14)$$

Hence, the master equation can be solved by applying the dynamical symmetry method proposed in Ref. [13] and we have
\[ \hat{\rho}_{gg}(t) = e^{L_{gg}t} \hat{\rho}_{gg}(0) \]
\[ = \exp[i\Omega_t(M^2 - P^2) - (\kappa + i\Omega_g)tM - (\kappa - i\Omega_g)tP] \times \exp[(1 - e^{-2\kappa t + 2i\Omega t(M - P)}) \frac{\kappa F}{\kappa - i\Omega(M - P)} \hat{\rho}_{gg}(0)], \tag{15} \]
\[ \hat{\rho}_{ee}(t) = e^{L_{ee}t} \hat{\rho}_{ee}(0) \]
\[ = \exp[-i\Omega_t(M^2 - P^2) - (\kappa + i\Omega_e)tM - (\kappa - i\Omega_e)tP] \times \exp[(1 - e^{-2\kappa t - 2i\Omega t(M - P)}) \frac{\kappa F}{\kappa + i\Omega(M - P)} \hat{\rho}_{ee}(0)], \tag{16} \]

We assume the initial state of the system as
\[ |\Psi_{a-f}\rangle = \frac{1}{\sqrt{2}}(|e \rangle + |g \rangle) \otimes |\alpha \rangle, \tag{17} \]

where, as is usual in recent experiments \cite{14}, the atom enters the cavity in a coherence superposition and finds there a coherent field state $|\alpha \rangle$, therefore initially $\hat{\rho}_{gg}(0) = \hat{\rho}_{ee}(0) = \frac{1}{2}|\alpha \rangle\langle\alpha|$, finally we get
\[ \hat{\rho}_{gg}(t) = \frac{1}{2} \exp(-|\alpha|^2) \sum_{m,n} \frac{\alpha^m \alpha^*}{\sqrt{m!n!}} \exp[\Gamma_{mn}(t) + i\Theta_{gmn}(t)]|m\rangle\langle n|, \tag{18} \]
\[ \hat{\rho}_{ee}(t) = \frac{1}{2} \exp(-|\alpha|^2) \sum_{m,n} \frac{\alpha^m \alpha^*}{\sqrt{m!n!}} \exp[\Gamma_{mn}(t) + i\Theta_{emn}(t)]|m\rangle\langle n|, \tag{19} \]

where
\[ \Gamma_{mn} = -\kappa(m + n)t + \frac{|\alpha|^2 \kappa}{\kappa^2 + \Omega^2(m - n)^2} \times \{\kappa - e^{-2\kappa t}[\kappa \cos 2\Omega t(m - n) - \Omega(m - n) \sin 2\Omega t(m - n)]\}, \tag{20} \]

and
\[ \Theta_{imn}(t) = -\Omega_i(m - n)t \pm \Omega(m^2 - n^2)t \pm \frac{|\alpha|^2 \kappa \Omega(m - n)}{\kappa^2 + \Omega^2(m - n)^2} [1 - e^{-2\kappa t} \cos 2\Omega t(m - n)], \tag{21} \]

where $i = e, g$, when $i = g$ last equation chose $+$ and $-$ corresponding to $i = e$. We obtained the $k$th moment of amplitude
\[ < a^n > = \frac{1}{2} \alpha^n \exp[\Gamma_{n0}(t) + |\alpha|^2(e^{-2\kappa t} \cos 2\Omega nt - 1)] \times \{\exp[i|\alpha|^2 e^{-2\kappa t} \sin 2\Omega nt + \Theta_{gmn}(t)] + \exp[i(-|\alpha|^2 e^{-2\kappa t} \sin 2\Omega nt + \Theta_{en0}(t))]. \tag{22} \]
One may check the decay behavior of the system by measuring the \( k \)th moment of amplitude. Here we aim to discuss the coherence loss of the field by means of idempotency defect or linear entropy, which is convenient way to study the coherence properties of the density operator as a function of time. Linear entropy is defined as \[ S_f = 1 - \text{Tr}(\hat{\rho}_f^2). \] (23)

The quantity \( \text{Tr}(\hat{\rho}_f^2) \) can be taken as a measure of the degree of purity of the reduced state; for a pure state \( S_f \) is zero but for \( S_f \simeq 1 \) the state corresponds to a mixture, with information effectively lost. From Eq.(15) through Eq. (23), we have

\[
S_f = 1 - \exp\left(-2|\alpha|^2\right) \sum_{m,n} \frac{|\alpha|^{2(m+n)}}{m!n!} \exp[2\Gamma_{mn}(t)] \cos^2 \left(\frac{\Theta_{gmn}(t) - \Theta_{emn}(t)}{2}\right). \] (24)

The function \( \Gamma_{mn}(t) \) in Eq. (20) embody the effect of reservoir because it vanishes for \( k \to 0 \). Although the complicate expression in Eq. (24) is not analytical, it contain the function \( \Gamma_{mn}(t) \), the coherence property of field should be affected by cavity. We will discuss it in next section.

4 The property of the field state dissipation

The function \( \Gamma_{mn}(t) \) in Eq. (24) presented in exponential factors controls the coherence loss of the field. It is always nonpositive and decrease with time and have the similar form to the usual dissipative JCM [11]. But the situation is not the same. There the \( \Gamma(t) \) function did not appear in the idempotent defect of the field. On other hand, in Ref. [11] the field and the atom disentangle at instant \( t_d = \frac{\pi}{2\omega} \), during disentangle only the field is found in a pure state and \( s_f(t_d) = 0 \). That circumstance is the same as JCM without dissipation, thus the coherence properties of the field have no influence by the cavity in qualitatively. Here due to the two-photon process and the Stark shift the function \( \Gamma_{mn}(t) \) appear in Eq.(24). Except the initial time at no instants the field is in pure state, because during evolution the value of linear entropy \( s_f(t) \neq 0 \) ( Of course, the equilibrium state of the field corresponds to vacuum, the linear entropy of the field is zero). Therefore we can reckon the field is also influenced by the cavity in qualitatively. We will further show the judgment in Fig.(1).

We plot the field’s idempotency defect as a function of time for several \( \kappa \) values in Fig. (1). It is noticed the behavior of \( s_f(t) \) is complicated, presenting local maxima and minima in wave packet trajectory. The local maxima and minima is due to the field interaction with a atom,
corresponding to entanglement and disentanglement. Because of the influence of dissipation on entanglement, the amplitude of local maxima and minima decrease with time. It is exactly as usual JC with dissipation. Due to the repeated period of entanglement and disentanglement the state of the atom and field loss and gain coherence but the coherence recovered by the atom is never that which was lost. The field finally change into pure state (vacuum state) and its coherence lost completely. From Fig. (1) we also observe that the wave packet trajectory is determined by the cavity dissipative. The larger values of $\kappa$ the more rapid is the field’s idempotency defect reach its asymptotic value zero. We can assure that the coherence property of the field is affected by cavity in quantitatively.

The dependence of the idempotency defects of the field with the intensity of the field is shown in Fig. (2). We choose the same value $\kappa$ and obtain the similar wave packet trajectory of linear entropy. With the increase of the intensity, the classicality of the field become obvious and the entanglement between the two subsystem change weaker. The amplitude of local maxima and minima is suppressed much more. Another contribution of intensity is to increase the maxima value of $S_f$. Due to enhance of intensity, the degree of maxima mixture state fortify. We also notice that with the increase of the intensity of the field, nonlinear behavior of the field lost and gain its coherence become obvious. In our calculation, we include the Stark shift but we find that the field coherence loss is affected little by different values of Stark shift coefficients $\beta_1$ and $\beta_2$.

5 Conclusion

We study the dynamics of a two-level atom with Stark shift interaction with the field by two-photon process in a dissipative cavity and solve the complicated Liouvil equation. We obtain idempotency defects of the field and show as follow: (1) the coherence property of the field is affected by cavity not only in qualitatively but also in quantitatively when two-photon process and Stark shift is involved. (2) The influence of dissipation on entanglement make the amplitude of each state suppressed. (3) The larger the intensity of the field is the weaker the entanglement of subsystem and the larger maxima degree of mixture state.

Fig. 1 Idempotency defects of the field as a function of $\Omega t$ for different values of dissipation constant $\kappa$. Where $\alpha = 1, (\beta_2 - \beta_1)/\Omega = 0.02$ and (a): $\kappa/\Omega = 0.02$; (b): $\kappa/\Omega = 0.04$; (c): $\kappa/\Omega = 0.1$.

Fig. 2 Idempotency defects of the field as a function of $\Omega t$ for different values of the intensity of the field. For all plots, we chose $\kappa/\Omega = 0.04, (\beta_2 - \beta_1)/\Omega = 0.02$, where (a): $\pi = 1.0$; (b): $\pi = 2.0$; (c): $\pi = 3.0$.
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