67P/Churyumov-Gerasimenko – potential target for the Rosetta mission

Małgorzata Królikowska
Space Research Centre of the Polish Academy of Sciences
Bartycka 18A, 00-716, Warsaw, Poland

March 20, 2022

e-mail: mkr@cbk.waw.pl
Appeared in Acta Astronomica 53, pp 195-209 (2003)

Abstract
An influence of the non-gravitational effects on the motion of short-period comet 67P/Churyumov-Gerasimenko is investigated. It was found that the normal component of the non-gravitational force exceeds the transverse one and a model of the motion including $A_1$, $A_2$, $A_3$ better fits the observations than model neglecting $A_3$. Assuming asymmetry in $g(r)$ with respect to the perihelion the large value of displacement $\tau$ was derived (about 34 days), and very small negative value of transverse component $A_2$ was obtained. The models of rotating non-spherical nucleus also suggest the large shift of light curve with respect to perihelion ($\tau \geq 30$). The forced precession model of 67P with $\tau = 34$ days gives a prolate spheroidal shape of the rotating nucleus with axial ratio $R_b/R_a = 1.16$, rotational period to equatorial radius $P_{rot}/R_a = 4.6 \pm 1.4$ hrs/km, and torque factor $f_{tor} = 3 \cdot 10^5$ day/AU. The much larger $\tau = 54$ days gives distinctly prolate shape of nucleus with axial ratio $R_b/R_a = 1.71$. The orientation of spin axis of the nucleus and its evolution are presented. The past and the future dynamical evolution of comet 67P is also widely discussed.

1 Introduction
Comet 67P/Churyumov-Gerasimenko was discovered by Klim Churyumov on photographs of 32P/Comas Sola taken by Svetlana Gerasimenko on September 1969 at Alma Ata. From that moment 67P has been detected during its all six returns (Marsden and Williams 2001, Rocher 2003). Now it is still extensively observed during its sixth apparition and it will be potentially observable up to beginning of 2004.

67P/Churyumov-Gerasimenko is unusually active for a short-period comet with the period of 6.6 yr. In the current apparition the comet passed the perihelion on 18 August 2002 and peaked at around magnitude 12, although an outburst of approximately 2 magnitudes at perihelion has been reported. Similar phenomenon was seen in the previous return but with a slightly lower amplitude. On both occasions the rise in the light curve was rapid; the light-curves are presented by Yoshida (2003) at WEB pages. It appears that the 1996 outburst came a few days before perihelion passage, whereas the 2002 event was centered exactly on perihelion. Besides, in the last perihelion passage the tail has
been extended over 10 arcminutes, and seven months after perihelion it still was very well
developed.

Contrary to such photometric activity no major orbital changes occurred since its
discovery in 1969. However, the comet has rather unusual history. During 19th and 20th
centuries prior to 1959 its perihelion distance varied from about 2.5 AU to 2.9 AU, and
this is the reason why the comet was unobservable from the Earth at that time. In
February 1959 the close approach of the comet to Jupiter to within 0.052 AU occurred.
This event caused considerable orbital changes: the perihelion distance has been reduced
from 2.74 AU to 1.28 AU, the eccentricity increased from 0.36 to 0.63 and orbital period
shortened from 8.97 yrs to 6.55 yrs. As a result the comet was discovered in its second
return to perihelion after the close encounter with Jupiter. Hence, the unusual comet
activity detected in the last apparitions could be a result of this remarkably reduction of
perihelion distance taking place not long ago.

67P/Churyumov-Gerasimenko has just been selected as the new target for the Rosetta
mission after the failure to launch the probe for an encounter with 46P/Wirtanen. This
inspired me to investigate the non-gravitational effects in the motion of this comet.

2 Observational material and the method of calculations

The present investigations are based on the archive observations available at the Mi-
nor Planet Center (Cambridge, USA). The whole observational material contains 1207
observations covering the time period from 1969 September 9 to 2003 March 13. The
observations were selected according to the objective criteria elaborated by Bielicki and
Sitarski (1991) for each of six apparitions separately. Finally, 2338 residuals were used for
the orbit improvement.

The non-gravitational equations of cometary motion have been integrated numerically
using recurrent power series method (Sitarski 1989, 2002) taking into account the per-
turbations by all the nine planets. All numerical calculations presented here are based
on the Warsaw numerical ephemeris DE405/WAW of the Solar System, consistent with
high accuracy with the JPL ephemeris DE405 (Sitarski 2002). The standard epoch of
2003 Dec. 27 was accepted in all the calculations as the starting epoch of integration.

3 Models with constant non-gravitational parameters

To estimate the non-gravitational force acting on the rotating cometary nucleus with
sublimating water from its surface the standard Marsden method (Marsden et al. 1973)
was used. This formalism assumes that the three components of a non-gravitational
acceleration have a form:

\[ F_i = A_i \cdot g(r), \quad A_i = \text{const} \quad \text{for} \quad i = 1, 2, 3, \]  

where \( F_1, F_2, F_3 \) represent the radial, transverse and normal components of the non-
gravitational acceleration, respectively. The function \( g(r) \) simulates the ice sublimation
rate as a function of the heliocentric distance \( r \):

\[ g(r) = \alpha (r/r_o)^{-m} \left[ 1 + (r/r_o)^n \right]^{-k}, \]
Table 1: Non-gravitational parameters and orbital elements for the 67P/Churyumov-Gerasimenko derived from all positional observations (six apparitions). Non-gravitational parameters $A_1, A_2, A_3$ are given in units of $10^{-8}$AU·day$^{-2}$. Angular elements $\omega, \Omega, i$ are referred to Equinox J2000.0 (Epoch: 20031227). Numbers in parentheses denote uncertainties: $0.63175088(7) \equiv 0.63175088 \pm 0.00000007$

|               | Model Ia                  | Model Ib                  |
|---------------|---------------------------|----------------------------|
| $A_1$         | 0.054440±0.002665         | 0.088327±0.003598          |
| $A_2$         | 0.0098084±0.0000173       | -0.0013637 ± 0.0009429    |
| $A_3$         | 0.030187±0.002189         | 0.033855±0.002152         |
| $\tau$       |                           | 34.314±2.128              |
| $T$           | 20020818.28695(7)         | 20020818.28685(6)         |
| $q$           | 1.29064789(25)           | 1.29064249(15)           |
| $e$           | 0.63175088(7)            | 0.63175242(4)            |
| $\omega$      | 11°40976(8)              | 11°40848(7)              |
| $\Omega$      | 50°92865(7)              | 50°92965(7)              |
| $i$           | 7°12415(1)               | 7°12413(1)               |
| $rms$         | 1′16                     | 1′12                     |

where the exponential coefficients $m, n$ and $k$ are equal to 2.15, 5.093, and 4.6142, respectively. The normalization constant $\alpha = 0.1113$ gives $g(1 \text{ AU}) = 1$; the scale distance $r_0 = 2.808 \text{ AU}$.

To generalize Eqs. 1-2 of the non-gravitational effects to asymmetric case in respect to perihelion we simply substitute $g(r')$ instead of $g(r)$, where $r' = r(t - \tau)$, and the time shift $\tau$ represents the time displacement of the maximum of the function $g(r)$ with respect to the perihelion.

For whole time interval the constant values of $A_1, A_2$ and $A_3$ (and eventually $\tau$ in the asymmetric case) were calculated along with the six corrections to the orbital elements. The results are given in Table 1 as Model Ia and Model Ib, for the symmetric and asymmetric case, respectively. The solution of using the asymmetric non-gravitational acceleration model do not significantly decreases the $rms$ in comparison to symmetric model (see also Yeomans and Chodas (1989)), however the photometric observations give arguments for clear asymmetry in the light curves of 67P. The derived time shift $\tau = 34.3$ is in an excellent agreement with the visual light curve of 67P obtained by Morris (Hanner et al. 1985). He show that the light curve of the comet during its 1982-83 apparition reached a brightness maximum approximately 35 days after perihelion. The result of 34.3 days for the time shift is also in a good agreement with the light curves of the comet presented in WEB page by Kidger (2003). It seems that photometric and positional observations of 67P independently provide consistent determination of $\tau$. I repeated the calculations for shorter arc of 1982 May 31 – 2003 Mar. 13, i.e. only for last four apparitions. Surprisingly, quite the same time shift of $\tau = 34.9$ days was obtained.

One can see that normal component of non-gravitational acceleration is significantly greater than transverse component for both models. To compare our analysis of the non-gravitational effects with those published by other authors who neglected the normal component, I have repeated all calculation assuming $A_3 = 0$; the results are summarized in Table 2. This model fits the observations with the $rms$ larger than Model Ia by about
Table 2: Non-gravitational parameters $A_1$ and $A_2$ obtained with assumption that normal component $A_3$ is equal zero

| Arc of observations | $A_1$ | $A_2$ | References |
|---------------------|-------|-------|------------|
| 1969 – 1988 (229 obs.) | $A_1 = +0.069 \pm 0.020$ | $A_2 = +0.010 \pm 0.001$ | Chodas & Yeomans (1989) |
| 4 app; $rms = 1''34$ |       |       |     |
| 1969 – 1997 (474 obs.) | $A_1 = +0.07$ | $A_2 = +0.0099$ | MPC 34423 |
| 5 app; $rms = 1''0$ |       |       |     |
| 1975 – 1997 (424 obs.) | $A_1 = +0.07 \pm 0.007$ | $A_2 = +0.0094 \pm 0.0002$ | Muraoka |
| 4 app; $rms = 0''85$ |       |       | www.aerith.net |
| 1975 09 09 – 2003 01 06 | $A_1 = (+0.05037 \pm 0.00226)$ | $A_2 = (+0.00936 \pm 0.00002)$ | Cometary Notes of Bureau des longitudes (note no 29) |
| 5 app (747 obs.) |       |       |     |
| $rms = 0''70$ |       |       |     |
| 1969 08 08 – 2003 03 13 | $A_1 = (+0.04954 \pm 0.00275)$ | $A_2 = (+0.009786 \pm 0.000018)$ | present |
| 6 app (1207 obs.) |       |       | calculations |
| $rms = 1''20$ |       |       |     |

0''04.

4 Rotating cometary nucleus and the forced precession model

The non-gravitational parameters for the model of the rotating spherical nucleus were also determined. In such model three parameters $A_1, A_2$ and $A_3$ are now the functions of time by relations: $A_i = A \cdot C_i(t), i = 1, 2, 3$, where $C_i(t)$ are direction cosines for the non-gravitational force acting on the rotating cometary nucleus (Królikowska et al. 1998). Three next non-gravitational parameters describing the model of the rotating cometary nucleus are angular parameters: $\eta$ – the lag angle of the maximum outgassing behind subsolar meridian, $I$ – equatorial obliquity and $\phi$ – cometocentric solar longitude at perihelion. The values of four parameters, $A, \eta, I$ and $\phi$, are presented in Table 3 as Model IIa, and IIb for the symmetric and asymmetric cases, respectively. In the Model IIb the value of time shift $\tau$ was taken from Model Ib.

An assumption of the flattened nucleus represents the next step towards the more realistic cometary models. In this case, the forced precession of the spin axis could arise due to a torque if a vector of the jet force does not pass through the center of the nonspherical nucleus. The precession rate is a function of the nucleus orientation, the lag angle $\eta$, the modulus of the reactive force $A$, the nucleus oblateness $s$, and the precession factor $f_p$, which depends of the rotation period and nucleus size. In such model the six parameters are derived: $A, \eta, I, \phi, s$ and $f_p$ (Król likowska et al. 1998). Unfortunately the seventh parameter $\tau$ was impossible to obtained from the observational data. Thus, the
Table 3: Physical parameters for rotating cometary nucleus and orbital elements linking all positional observations (six apparitions) of 67P/Churyumov-Gerasimenko. Angular elements $\omega$, $\Omega$, $i$ are referred to Equinox J2000.0. Non-gravitational parameter $A$ is given in units of $10^{-8}$ AU/day$^2$, the precession factor $f_p$ is in units of $10^6$ day/AU, time shift $\tau$, is in days. Subscript '0' in $I_0$ and $\phi_0$ denotes the values on the starting epoch of integration (Epoch: 20031227)

|                | Model IIa          | Model IIb          | Model III Forced precession model |
|----------------|-------------------|-------------------|-----------------------------------|
| $A$            | 0.064173 ± 0.003258 | 0.096424 ± 0.003519 | 0.10345 ± 0.00560 |
| $\eta$         | 35°90±3°35         | 27°56±1°90         | 28°29±2°17          |
| $I_0$          | 72°49±0°99         | 90°38±0°91         | 88°19±0°76          |
| $\phi_0$       | 333°98±6°88        | 317°25±5°01        | 315°24±5°63         |
| $f_p$          | —                 | —                 | -1.849 ± 1.360      |
| $s$            | —                 | —                 | -0.1613 ± 0.0685    |
| $\tau$         | —                 | —                 | 34.314              |
| $T$            | 20020818.28696(7)  | 20020818.28686(7)  | 20020818.28668(7)   |
| $q$            | 1.29064840(25)    | 1.29064313(25)    | 1.29064511(25)      |
| $e$            | 0.63175074(7)     | 0.63175225(7)     | 0.63175157(7)       |
| $\omega$       | 11°40'936(8)      | 11°40'783(8)      | 11°40'785(8)        |
| $\Omega$       | 50°92'908(7)      | 50°93'033(7)      | 50°93'037(7)        |
| $i$            | 7°12'413(1)       | 7°12'410(1)       | 7°12'410(1)         |
| rms            | 1'16              | 1'12              | 1'11                |
Figure 1: **Left side:** Temporal variation of angle $I, \phi$ and components $F_1, F_2, F_3$ of the non-gravitational force due to the spin axis precession of the nucleus of 67P/Churyumov-Gerasimenko. Dashed horizontal line on the upper panel divides models with prograde rotation (consistent with the sense of the cometary orbit around the sun, $I < 90^\circ$) from models with retrograde rotation ($I > 90^\circ$); dashed horizontal line on the middle panel indicates changes of configuration of poles in the perihelia. Thick, solid curves show the evolution of $I, \phi, F_1, F_2, F_3$ for Model III represented by black dots in the right-side panel (see also Table 3), and thin, dashed curves give the respective evolution for the model represented by black squares in the right-side panel.

**Right side:** Family of forced precession models parameterized by time shift $\tau$ and all fit the observations with the $rms = 1''11$. From top to bottom: $A$ – the non-gravitational parameter given in units of $10^{-8}$ AU/day$^2$, $\eta$ – the lag angle, $I_0$ – the equatorial obliquity of the spin axis relative to the orbital plane, $\phi_0$ – the cometocentric solar longitude at perihelion, $f_p$ – the precession factor given in units of $10^7$ day/AU, and $s$ – the oblateness of the nucleus. Subscript '0' in $I_0$ and $\phi_0$ denotes the values on the starting epoch of integration (Epoch: 20031227). The dashed parts indicate that models are nonphysical ($f_{tor} = f_p \cdot s < 0$). The black dots show the position of Model III (see Table III) and black squares – the forced precession model with the time shift $\tau = 54.0$. 


right panel in Fig. 1 shows family of forced precession models parameterized by the time shift $\tau$. Each model from the family fits the observational data with the same $rms = 1.11$. Since the precession factor $f_p$ is related to the torque factor, $f_{tor}$ introduced by Sekanina (1984) by $f_p = s \cdot f_{tor}$, $f_p$ and $s$ should have both positive or both negative values. For $\tau < 30$ days the negative values of $f_p$ and positive values of $s$ were obtained. Therefore, the symmetric forced precession models as well as forced precession models with negative or small positive value of $\tau$ were excluded. It turns out that only the models with significant positive time shift are allowed. Table 3 shows the fully consistent forced precession model (Model III) where the value of time shift $\tau$ was taken from Model Ib. The negative value of $s$ found in this forced precession model suggests that nucleus of Comet 67P/Churyumov-Gerasimenko has a prolate spheroidal shape with axial ratio $R_b/R_a = 1 - s = 1.16$, where, $R_b$ and $R_a$ are the polar and equatorial radii, respectively. The motion of the cometary rotation axis represented by angles $I$ and $\phi$ is pointed by thick, solid curves in the left panel of Fig. 1. Qualitative variations of the non-gravitational force components acting on the comet during its successive returns to the sun are also shown. One can see that the significant reduction of the perihelion distance in 1959 caused significant increase of the non-gravitational force. After 1959 the time variations of components of non-gravitational force are rather regular. This model is also attractive since its very moderate values of $f_p$ and $s$ which give small value of torque factor $f_{tor} = 3 \cdot 10^5$ day-AU$^{-1}$. There are also possible models with larger $\tau$ and one of them with $\tau = 54$ days is visualized in Fig. 1 by thin, dotted curve for comparison with Model III. This model (black squares in the left panel of Fig 1) is characterized by distinctly prolate shape of nucleus with axial ratio $R_b/R_a = 1 - s = 1.71$ and torque factor 20 times greater than that in Model III ($f_{tor} = 6.4 \cdot 10^6$ day-AU$^{-1}$).

5 Orbital evolution

Studying the evolution of short-period comets belonging to the jupiter-family comets we should confine to short-term numerical integrations, say up to one thousand years in the past as well as in the future. Accordingly, the dynamical evolution of Comet 67P was followed up to one milenium from the starting epoch of integration (2003 Dec. 27). Calculations were performed for all five models discussed in the previous sections (Tables 1 & 2). Fig. 2 shows the non-gravitational evolution of the perihelion distance, $q$, the eccentricity, $e$, and the inclination, $i$. Comparison between non-gravitational evolution and evolution without non-gravitational effects is presented in Fig. 3 for starting orbital elements taken from Model Ia (solid lines) and the model with the assumed normal non-gravitational parameter $A_3 = 0$ (dotted lines), respectively.

In the case of Comet Churyumov-Gerasimenko the very close approach to Jupiter in 2020 makes future orbital predictions for the longer time than two century uncertain. Before 2020, six encounters with Jupiter will take place (in 2018, 2078, 2125, 2161, 2172, and 2209 for all five models, see Fig. 4), however without any spectacular orbital changes. After close encounter with Jupiter in 2020 the future evolution will be unpredictable, but some evolution similarities are visible. In all five models many approaches to Jupiter are expected between 2220–2500 (Fig. 4) with at least one closer than 0.1 AU. During last five centuries significantly less approaches to Jupiter happened than will occur in the next 500 years. In February 1959 – as was mentioned before – the comet approached Jupiter to within 0.052 AU what caused considerable changes in orbital elements, especially in the perihelion distance: it was reduced from 2.74 AU to 1.28 AU, and, in the consequence, the comet was discovered soon after that. Fig. 3 clearly shows that in the same event the eccentricity increased from 0.26 to 0.63 and the orbital inclination decreased from 23.2 to 7.2. Past encounters with Jupiter occurred in roughly one century time intervals. Up to
Figure 2: Time evolution of the orbital elements $q$, $e$ and $i$ of 67P Churyumov-Gerasimenko. The present orbit of the comet determined from whole observational data (1969 Sept. 9 – 2003 Mar. 13) was integrated for over 500 yrs back and forward (starting point – 2003 Dec 27 – is indicated by black dots) and its non-gravitational evolution is shown with:
solid, thick lines – for evolution correspond to Model Ia
dotted, thick lines – for evolution correspond to Model Ib
dashed lines – for evolution correspond to Model IIa
dotted-dashed lines – for evolution correspond to Model IIb
dotted lines – for evolution correspond to Model III
Figure 3: Time evolution of the orbital elements $q$, $e$ and $i$ of 67P Churyumov-Gerasimenko for Model Ia (thick and thin solid lines) and model with $A_3 = 0$ (thick and thin dotted lines). The thick lines represent the non-gravitational evolution, the thin lines show pure gravitational evolution which starts from the same orbital elements as in the non-gravitational case. In the model with $A_3 = 0$ the value of $A_1 = (+0.04954 \pm 0.00275)$ and $A_2 = (+0.009786 \pm 0.000018)$ in units of $10^{-8}$ AU/day$^2$ are derived from observations (see also Table 2).
Figure 4: Distribution of all close approaches of the comet to Jupiter which appeared during evolution of the starting orbit in the Models Ia (panel (A)), Ib (panel (B)), IIa (panel (C)), IIb (panel (D)), III (panel (E)), respectively. Panel (F) presents cumulative distribution of all close encounters with Jupiter which occurred in evolution of 20 clones of orbit constructed from the Model Ia. The evolution was performed backwards and forwards up to 500 yrs. Starting moment of integrations is shown by dotted vertical line. The y-axis ($d_J$) shows the depths of individual close encounters of the comet with Jupiter (closer than 0.8 AU).

Now many researches conclude that 67P is a comparatively recent visitor to the inner solar system having had $q = 4.0$ AU prior to 1840 and $q > 2.75$ AU until a Jupiter encounter in 1959. However, Figs. 2–3 show that the evolution before 1700 is highly uncertain.

Keeping this in mind let me only speculate about past and future evolution extending over ±5 kyr. The evolutionary calculations were performed to demonstrate how Jupiter controls the evolution of the Comet Churyumov-Gerasimenko. To do this the classification scheme proposed by Horner (2003) was applied for evolutionary non-gravitational calculations according to Model Ia (the left-hand side of Fig. 5) and Model Ib (the right-hand side of Fig.5). The evolution in Model Ia is different in details from the evolution in Model Ib. Starting at -3000 BC the cometary orbits were placed in the JS region in the upper panel of Fig 5 and started to evolve to the left (Model Ia) or firstly to the right to JU region and back to the left (Model Ib). During the dynamical evolution the perihelion distance of 67P being under the Jupiter control ($4.0 < q < 5.0$ AU) kept its value within $5.0 < q < 5.8$ AU, (Model Ia) or slowly evolved from ~4.0 AU to ~5.0 AU (Model Ib). The Tisserand parameter $T_J$ was around the boundary value of 2.8 which divides loosely bound Jupiter-family comets (class III in Horner (2003)) from tightly bound JFC (class...
Figure 5: **Upper panels:** Plot of eccentricity versus semimajor axis for evolution of comet 67P according to Model Ia (the left-hand side) and Model Ib (the right-hand side), respectively. The dashed thin curves marks the boundaries of the aphelion or perihelion zones controlled by Jupiter. Jupiter’s zone of control is taken as a three times Hill radii. In the zone of \(4 \text{ AU} < q < 6.6 \text{ AU}\) to the right of line \(Q > 4 \text{ AU}\) the regions belonging to SP comet’s categories: J (objects for which both perihelion and aphelion are under Jupiter’s control), JS (perihelion is under Jupiter’s and the aphelion under Saturn’s control), and JU (perihelion – as previously, and the aphelion under Uranus’s control) (Horner et al. 2003). The evolution was performed backwards and forwards up to 5 kyr.

**Lower panels:** Changes of Tisserand parameter, \(T_{J, \text{perih}}\), \(T_{J, \text{aph}}\), \(T_{S, \text{aph}}\) during time interval in which the perihelion (aphelion) falls in the Jupiter’s (or Saturn’s) zone of control. The horizontal dashed lines show the boundaries differentiate between SP comets using the Tisserand parameter. After Horner et al. (2003) two of this four-fold division are denoted by III (third class having \(2.5 \text{ AU} \geq T_J > 2.8 \text{ AU}\)) and IV (fourth class having \(T_J \geq 2.8 \text{ AU}\)) correspond to loosely bound Jupiter-family comets and tightly bound Jupiter-family comets, respectively.
Figure 6: Time evolution of the orbital elements $q$, $e$ and $i$ of 20 randomly selected orbits of 67P/Churyumov-Gerasimenko. The evolution was performed backwards and forwards up to 500 yrs from the starting moment of integrations (2003 Dec 27). The nominal, starting orbit of the comet corresponds to Model Ia and its dynamical evolution is given by solid, thick curve.
Table 4: Dispersion of non-gravitational parameters and orbital elements derived for 20 randomly selected orbits of 67P/Churyumov-Gerasimenko (Epoch: 20031227; Equinox: J2000.0)

| $A_1$   | $A_2$   | $A_3$   |
|---------|---------|---------|
| 0.05444 | 0.009808| 0.03019 |
| +0.00530| +0.000022| +0.00254|
| -0.00626| -0.000046| -0.00546|

| $T$       | $q$        | $e$      | $\omega$ | $\Omega$ | $i$   |
|-----------|------------|----------|----------|----------|-------|
| 20020818.28695 | 1.29064789 | 0.63175088| 11.40976 | 50.92865 | 7.12415|
| +0.00011 | +0.00000041| +0.00000012| +0.00012 | +0.00015 | +0.00001|
| -0.00014 | +0.00000043| +0.00000013| +0.00015 | +0.00012 | +0.00003|

IV therein). Thus, during past evolution prior to comet’s discovery the perihelion was controlled by Jupiter and aphelion was placed in the Saturn zone of control (Fig. 5). In the Model Ia about 950 AD also the aphelion started to be under Jupiter’s zone of control and prior to $\sim$ 1500 AD both perihelion and aphelion were under Jupiter’s control (J class, see Fig. 5). Then the cometary orbit with $e < 0.2$ and semimajor axis $a \simeq 5$ is placed in the lowest part of upper panel in the Fig. 5. After 1500 AD the aphelion of the comet become under Jupiter control and will be under Jupiter control prior to about 4000 AC. The lower panel of Fig. 5 shows that in almost whole time interval considered in the Figs 2–4 the aphelion of 67P is under Jupiter’s control, e.g. the aphelion distance falls between $4.0 \text{ AU} < Q < 6.6 \text{ AU}$ (Models Iab). Future evolution shows some similarities to the past evolution. Once again the perihelion starts to be in the Jupiter’s zone of control, and perihelion – within Saturn’s zone of control (Model Iab) and next within Uran’s zone of control (Model Ib).

Additionally, the statistical approach was used for the study of the past and future motion of comet 67P. The sample of 20 clones of nominal orbit were constructed according to the method described by Sitarski (1998) for the standard symmetric model (Model Ia) with constant non-gravitational parameters $A_1, A_2$ and $A_3$. Sitarski’s procedure allows to derive the set of randomly selected orbits (clones) which all fit the observations almost with the same $rms$ as nominal orbit. The range of six orbital elements taken as starting orbit to dynamical calculations are given in Table 4. Next, each randomly selected orbit was integrated backwards and forwards up to 1 kyr. The differences in past and future evolution of $q, e$ and $i$ are clearly visible in Fig. 6. The evolution is well defined in the period of [-300; +250] years, outside this time-interval dynamical behaviour starts to be chaotic.

Comet Churyumov-Gerasimenko belongs to so called Near Earth Comets (NEC) (Baalke 2003). In the present dynamical calculations covering the time period 1500–2500 the close encounters with the Earth to within 0.2 AU were analyzed. Such events were found only for the Model IIb among five nominal orbits described in Table 1 & 2, and for 10 of 20 randomly selected orbits in the Model Ia. The first close encounter with the Earth occurs in 2239 (to within 0.045 AU, Model IIb), i.e. after remarkably close encounter with Jupiter in 2220. After that the series of close approaches to the Earth were detected for this nominal orbit (comet runs inside the earth’s orbit). For randomly selected orbit the closest approaches to within 0.019 AU were found among cumulative number of 19
encounters with the Earth to within 0.2 AU.

6 Conclusions

Considering the results obtained from different models of the non-gravitational motion of jupiter-family comet 67P/Churyumov-Gerasimenko, the principal conclusions from this study are following.

1. The non-gravitational effects detected in the comet Churyumov-Gerasimenko motion seem to be small and stable during all six apparitions despite different outbursts observed in the light-curves of this comet.

2. The normal component of the non-gravitational force exceeds the transverse one and the model of motion including $A_1, A_2, A_3$ better fits the observations of 67P than the model with neglecting $A_3$ (Tables 1 & 2).

3. Investigation of the non-gravitational motion of the rotating cometary nucleus of 67P indicates a large value of the time shift $\tau > 30$ days which measures the displacement of maximum of $g(r)$ with respect to perihelion. Thus the value of $\tau = 34$ days independently derived from the model with constant parameters $A_1, A_2, A_3$ is consistent with the forced precession model.

4. The forced precession model of 67P with assumed $\tau = 34$ days gives a prolate spheroidal shape of the rotating nucleus with axial ratio $R_b/R_a = 1.16$, and $P_{rot}/R_a = 4.6 \pm 1.4$ h/km. This value of $P_{rot}/R_a$ lies within the region occupied by comets with known sizes and rotational periods (Fig. 5 in Królikowska et al. (2001)).

5. Dynamical evolution of 67P is well defined to about $\sim 250$–300 years backward and forwards in time.

6. Considering the results of short-time integrations (within the interval of $\pm 1$ kyr) of 20 randomly selected orbits from Model Ia, it seems possible that the comet will evolve into an Earth crossing orbit in the future as well as into an orbit of larger perihelion distance than 3.0 AU.

7. Within the time interval [-1000 BC, 4000 AD] the aphelion of the comet 67P seems to be under Jupiter’s control. Outside this time interval the perihelion predominantly falls within the zone of Jupiter control.

7 Acknowledgements

I am deeply indebted to Professor Grzegorz Sitarski for helpful discussion and constructive comments on these investigations.

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