Improved error of electromagnetic shielding problems by a two-process coupling subproblem technique

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ABSTRACT

Introduction: The direct application of the classical finite element method for dealing with magneto dynamic problems consisting of thin regions is extremely difficult or even not possible. Many authors have been recently developed a thin shell model to overcome this drawback. However, this development generally neglects inaccuracies around edges and corners of the thin shell, which leads to inaccuracies of the magnetic fields, eddy currents, and Joule power losses, especially increasing with the thickness. Methods: In this article, we propose a two-process coupling subproblem technique for improving the errors that overcome thin shell assumptions. This technique is based on the subproblem method to couple SPs in two-processes. The first scenario is an initial problem solved with coils/stranded inductors together with thin region models. The obtained solutions are then considered as volume sources for the second scenario, including actual volume improvements that scope with the thin shell assumptions. The final solution is, to sum up, the subproblem solutions achieved from both scenarios. The extended method is approached for the \( h \)-conformal magnetic formulation. Results: The obtained results of the method are checked/compared to be close to the reference solutions computed from the classical finite element method and the measured results. This can be pointed out in a very good agreement. Conclusion: The extended method has also been successfully applied to the practical problem (TEAM workshop problem 21, model B). Key words: Magnetic flux density, eddy current losses, Joule power losses, thin shells, finite element method, subproblem method (SPM)

INTRODUCTION

The direct application of the finite element method (FEM)\(^1\) for dealing with magneto dynamic problems consisting of thin regions is extremely difficult or even not possible. Many authors\(^2\) have been recently developed a thin shell (TS) model in order to overcome this drawback. However, this development generally neglects inaccuracies around edges and corners of TS, which leads to inaccuracies of the local fields (magnetic fields, eddy currents, and Joule power losses...). The aim of this study is to propose a two-process coupling subproblem (SP) technique for improving the errors appearing from the TS models that were developed\(^2\). The technique is herein based on the subproblem method (SPM) presented by many authors,\(^3-7\). The technique allows to couple SPs in two-processes. The first scenario is an initial problem solved with coils/stranded inductors and thin region models; the obtained solutions are then considered as volume sources (VSs) (express as of permeability and conductivity material in conducting regions) for the second scenario including actual volume improvements that scope with the TS assumptions.\(^2\) The final solution is, to sum up, the SP solutions achieved from both the scenarios. The extended method is implemented for the magnetic field density formulation and applied to a practical problem (TEAM workshop Problem 21, model B).\(^8\)

COUPLING SUBPROBLEM TECHNIQUE

In the strategy SP, a canonical magneto dynamic problem \( i \), to be solved at procedure \( i \), is solved in a domain \( \Omega_i \) with boundary \( \partial\Omega_i = \Gamma_i = \Gamma_{h,i} \cup \Gamma_{b,i} \). The eddy current belongs to the conducting part \( \Omega_{c,i} (\Omega_{c,i} \subset \Omega_i) \), whereas the stranded inductors are the non-conducting \( \Omega_{s,i} \), with \( \Omega_{c,i} \cup \Omega_{s,i} = \Omega \). The Maxwell's equations together with the following constitutive relations\(^3-7\).

\[
\begin{align*}
\text{curl } b &= j, \quad \text{div } b = 0, \quad (1a-\text{b}) \\
\text{curl } e &= -\partial_i b_i \\
\end{align*}
\]

\[
\begin{align*}
\mu_i b_i + b_i &= j_i, \\
\sigma_i^{-1} j_i + e_s &= j_s, \quad (2a-\text{b}) \\
\n\times e_{s\gamma} &= j_{f\gamma} \quad (3)
\end{align*}
\]

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where \( \mathbf{b}_i \) is the magnetic flux density, \( \mathbf{h}_i \) is the magnetic field, \( \mathbf{e}_i \) is the electric field, \( j_i \) is the electric current density, \( \mu_i \) is the magnetic permeability, \( \sigma_i \) is the electric conductivity and \( n \) is the unit normal exterior to \( \Omega_i \). The surface field \( j_{s,i} \) in (2 c) is a surface source (SS) expressed as changes of interface conditions (ICs) and is generally defined as a zero for classical homogeneous boundary conditions (BCs). If nonzero, it can consider as SS that account for particular phenomena presenting at the idealized thin regions between the positive and negative sides of \( \Gamma_i \) (\( \Gamma_i^+ \) and \( \Gamma_i^- \)).

The source fields \( b_{s,i} \) and \( e_{s,i} \) in (2 a-b) are VSs. In the SPM, the changes of materials from the TS region (\( i = 1 \), \( \mu_1 \) and \( \sigma_1 \)) to the volume improvement (\( i = 2 \), \( \mu_2 \) and \( \sigma_2 \)) can be defined via VSs 4-7:

\[
\begin{align*}
\mathbf{h}_{s,2} &= (\mu_2 - \mu_1) \mathbf{h}_1, \\
\mathbf{e}_{s,2} &= (\sigma_2^{-1} - \sigma_1^{-1}) \mathbf{j}_1
\end{align*}
\] (4 a-b)

The total fields can be defined via a superposition method 1, i.e.

\[
\begin{align*}
\mathbf{b} &= \mathbf{b}_1 + \mathbf{b}_2 = \mu_2(\mathbf{h}_1 + \mathbf{h}_2) \\
\mathbf{e} &= \mathbf{e}_1 + \mathbf{e}_2 = \sigma_2^{-1}(\mathbf{j}_1 + \mathbf{j}_2)
\end{align*}
\] (5 a-b) (6 a-b)

**FINITE ELEMENT WEAK FORMULATION**

**Magnetic field intensity formulation**

By starting from the Ampere’s law (1c), the weak conform magnetic field formulation of SP i (\( i \equiv 1, 2 \)) can be written as 3-7:

\[
\begin{align*}
\nabla \times (\mu_i \mathbf{h}_i, \mathbf{h}_i^*)_{\Omega_i} + (\sigma_i^{-1} \nabla \mathbf{h}_i, \nabla \mathbf{h}_i^*)_{\Omega_i} \\
-\nabla (\mathbf{b}_{s,i}, \mathbf{h}_i^*)_{\Omega_i} + (\mathbf{e}_{s,i}, \mathbf{h}_i^*)_{\Omega_i} \\
+ \mathbf{n} \times \mathbf{e}_i, \mathbf{h}_i^*_{\Gamma_i^-} = 0, \forall \mathbf{h}_i \in H^1(\nabla \cdot, \Omega_i).
\end{align*}
\] (7)

The magnetic field \( \mathbf{h}_i \) in (7) is decomposed into parts, \( \mathbf{h}_i = \mathbf{h}_{s,i} + \mathbf{h}_{r,i} \), where \( \mathbf{h}_{s,i} \) is the source magnetic field defined via an imposed electric current density in the stranded inductors \( \Omega_{e,i} \), that is

\[
\begin{align*}
\nabla \times (\mu_i \mathbf{h}_{s,i}, \mathbf{h}_{s,i}^*)_{\Omega_{e,i}} = (j_{s,i}, \nabla \mathbf{h}_{s,i}^*)_{\Omega_{e,i}}, \\
\forall \mathbf{h}_{s,i} \in H^1(\nabla \cdot, \Omega_{e,i}).
\end{align*}
\] (8)

and \( \mathbf{h}_{r,i} \) is the associated reaction magnetic field, which we have to define, i.e.

\[
\begin{align*}
\text{curl } \mathbf{h}_{r,i} &= j_{s,i} \quad \text{in } \Omega_{e,i} \\
\text{curl } \mathbf{h}_{r,i} &= 0 \quad \text{in } \Omega_{c,i} - \Omega_{e,i}
\end{align*}
\] (9)

In the non-conducting regions \( \Omega_{c,i} \), the reaction field \( \mathbf{h}_{r,i} \) is thus defined via a scalar potential 7.

The function space \( H^1(\nabla \cdot, \Omega_i) \) in (7) and (8) is a curl-conform containing the basis functions for \( \mathbf{h}_i \) and \( \mathbf{h}_{r,i} \) as well as for the test function \( \mathbf{h}_i^* \) and \( \mathbf{h}_{r,i}^* \) (at the discrete level, this space is defined by finite edge elements); notations (\( \cdot, \cdot \)) and \(< \cdot, \cdot >\) are respectively a volume integral in and a surface integral of the product of their vector field arguments. The integral surface term \(<\mathbf{n} \times \mathbf{e}_i, \mathbf{h}_i^*_{\Gamma_i^-} >_{\Gamma_i} \) in (7) is defined as a homogeneous Neumann BC, e.g., imposing a symmetry condition of “zero magnetic flux”, i.e.

\[
n \times \mathbf{e}_i_{\Gamma_i^-} = 0 \Rightarrow n \cdot \mathbf{b}_{s,i} = 0.
\] (10)

The trace discontinuity \(<\mathbf{n} \times \mathbf{e}_i, \mathbf{h}_i^*_{\Gamma_i^-} >_{\Gamma_i} \) appearing in (7) is considered as a TS model and given as 1:

\[
<\mathbf{n} \times \mathbf{e}_i, \mathbf{h}_i^*_{\Gamma_i^-} >_{\Gamma_i} = \mu_i \mathbf{b}_i \nabla \mathbf{h}_i (2\mathbf{h}_{e,i} + \mathbf{h}_{d,i})_i + \mathbf{h}_i^*_{\Gamma_i^-} + \frac{1}{\sigma_1} \mathbf{h}_{d,i}^*_{\Gamma_i^-} \mathbf{h}_i^*_{\Gamma_i^-} >_{\Gamma_i^-} \] (11)

where \( \mathbf{h}_{e,i} \) and \( \mathbf{h}_{d,i} \) are continuous and discontinuous components of \( \mathbf{h}_i \), and \( \beta_i \) is a factor defined as

\[
\beta_i = \gamma_i^{-1} \tanh \left( \frac{d_i \gamma_i}{2} \right),
\gamma_i = 1 + j \frac{d_i}{2}, \delta_i \] (12)

for \( d_i \) and \( \delta_i \) being the local thickness of the TS and skin depth, respectively.

**Projected solutions between thin shell and volume improvement**

The obtained solution \( \mathbf{h}_1 \) in (1) sub-domain of the TS model \( \Omega_1 \) is now considered as a VS in a sub-domain of the volume improvement (current problem) \( \Omega_2 \) (\( i = 2 \)). This means that at the discrete level, the source \( \mathbf{h}_1 \) solved in the mesh of the \( \Omega_1 \) has to be projected in mesh \( \Omega_2 \) via a projection method 9. This can be done via its curl limited to \( \Omega_2 \), i.e.

\[
\begin{align*}
&\{ \text{curl } \mathbf{h}_{1-2}, \text{curl } \mathbf{h}_2 \}_{\Omega_2} = \{ \text{curl } \mathbf{h}_1, \text{curl } \mathbf{h}_2 \}_{\Omega_2}, \\
&\forall \mathbf{h}_2 \in H^1(\text{curl}, \Omega_2).
\end{align*}
\] (13)

Where \( H^1(\text{curl}, \Omega_2) \) is a gauged curl-conform function space for the projected source \( \Omega_1-2 \) and the test function \( \mathbf{h}_2 \).

**Magnetic field intensity formulation with volume improvement**

The solution in (7) with the TS model (solved from the first scenario) is forced as a VS for solving the second problem (that contains an actual volume/volume improvement) through the volume integrals \( \partial_i (\mathbf{b}_{s,i}, \mathbf{h}_i^*)_{\Omega_i} \) and \( (\mathbf{e}_{s,i}, \nabla \mathbf{h}_i^*)_{\Omega_i} \), where \( \mathbf{b}_{s,i}, \mathbf{e}_{s,i} \) [524-527]
At the discrete level, the source fields $h_1$ and $j_1$ defined in the mesh of the TS model ($i = 1$) via (7) are now projected in the mesh of the current SP/volume improvement SP ($i = 2$) via (13) shown in Section Projected solutions between thin shell and volume improvement.

**APPLICATION TEST**

The test problem is a TEAM workshop problem 21 (model B), with two excitation coils and a magnetic steel plate (Figure 1). The thickness of the plate is 10 mm, and the electric conductivity is $\sigma = 6.484$ MS/m, the relative magnetic permeability $\mu = 200$, the frequency $f = 50$ Hz, and the exciting current of 25A. The test problem is solved in a 3-D case.

The distribution of the magnetic flux density $b$ in a cut-plane due to the exciting/imposed current in the coils with a simplified mesh of the TS SP is shown in Figure 2.

The results obtained on the magnetic flux density from the volume improvement are also compared with the measured results pointed out in Figure 5. The maximum and minimum errors between two method are approximately 10.9% and 1.5%, respectively. This is said that there is a very suitable validation of the extended method.

**DISCUSSION AND CONCLUSION**

A two-process coupling subproblem technique with the magnetic field formulation has been successfully extended for improving errors on the local fields of magnetic flux density, eddy current density and Joule power loss density around the edges and corners of the TS approximations proposed in 2. The obtained results of the method are checked to be close to the reference solution in computation of the classical FEM and are also compared to be similar the measured results from a TEAM workshop problem 21 (model, B) proposed by many authors. This
The extension of the method could be also implemented in the time domain and the nonlinear case (proposed in \(^{10}\)) in next study. All the steps of the technique have been validated and applied to international test problem (TEAM workshop problem 21, model B)\(^8\). In particular, the achieved results is a good condition to analyze the influence of the fields to around electrical/electronic devices when taking a shielding plate into account.

**COMPLETING INTERESTS**
The author declares that there is no conflict of interest regarding the publication of this paper.

**AUTHOR’S CONTRIBUTIONS**
All the main contents, source-codes and the computed results of this article have developed by the author.

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