A novel plasma equilibrium in the high-β, Hall regime that produces centrally-peaked, high Mach number Couette flow is described. Flow is driven using a weak, uniform magnetic field and large, cross field currents. Large magnetic field amplification (factor 20) due to the Hall effect is observed when electrons are flowing radially inward, and near perfect field expulsion is observed when the flow is reversed. A dynamic equilibrium is reached between the amplified (removed) field and extended density gradients.

Fluid flow between two concentric cylinders, Taylor-Couette flow [1], has been a cornerstone of pure and applied fluid mechanics for more than 300 years [2]. Starting with Newton himself, this simple geometry has served as theoretical and experimental platform for hydrodynamics. Couette flow was considered by Stokes when constructing the ubiquitous Navier-Stokes equations of fluid motion. It served as basis for the design of the earliest viscometers, where the name Couette comes from [3, 4]. Couette flow has also been a major tool in modern studies of fluid turbulence, particularly the pioneering work of Taylor [5]. Extending beyond conventional fluids, Couette flow has been used to characterize more complex fluids such as visco-elastic polymers [6, 7] and magnetofluids such as liquid metals, where the flowing fluid is subject to electromagnetic forces in addition to pressure and viscosity. Chandrasekhar and Velikhov simultaneously described the stability of MHD Couette flow in the presence of weak magnetic fields [8, 9], showing dynamics beyond the instabilities encountered in Couette flow of conventional fluids. Most recently, Couette flow of unmagnetized plasma has been realized in the lab and provided for measurements of plasma viscosity [10], opening up access to even more exotic phenomena associated with kinetic effects and compressibility.

Due to the similarity to Keplerian flow \( V_0 \propto r^{-1/2} \), Couette flow has been proposed as model system for laboratory astrophysics. For example, there have been numerous partially successful attempts to experimentally study the magnetorotational instability (MRI) in liquid metal experiments [11–17], but has often been met with complications caused by the appearance of parasitic modes from the boundaries [18–20]. Beyond the simple point that real astrophysical systems are composed of plasmas, plasma Couette experiments open up kinetic physics, Hall effects and mixed charged-neutral systems. In the case of the MRI, plasma experiments highlight issues that are import in hot, dense disks [21–23] and partially-magnetized Hall effects in protostellar systems [24–28]. Other astrophysical systems could benefit from a detailed understanding of two-fluid effects as well, including the dynamo [29, 30].
Hall plasmas result in a massive amplification of the initial field by the Hall effect and a hollowing out of the density profile with centrally peaked flows—radically altering the equilibrium state expected from the MHD model. After presenting a description of the experimental setup, we show equilibrium measurements of plasma VFD in two configurations: outwardly and inwardly directed current. The outwardly directed current case shows strong field amplification, hollow density profiles and Couette flow, while the opposite case shows strong field expulsion and solid-body flow profiles. We then compare these measurements to extended MHD simulations in order to develop a simple two-fluid model of this equilibrium.

The experiments presented here were carried out in two very similar devices, the Big Red Ball (BRB) and the Plasma Couette Experiment (PCX), both operated at the Wisconsin Plasma Physics Laboratory (WiPPL) [10, 36, 37]. Plasma creation and flow drive are achieved by injecting current from hot, emissive lanthanum hexaboride cathodes (LaB$_6$) across a weak, externally applied magnetic field [38, 39]. Due to the multi-cusp confinement scheme for both devices, extremely high-$\beta$ can be achieved with ion inertial lengths ($d_i = c/\Omega_{pi}$) on the order of 1 m, which places these devices firmly in the Hall regime. Argon plasmas are produced by injecting 30-300 A of current from the LaB$_6$ cathodes with a constant neutral fill of approximately 10$^{-5}$ torr. These discharges reach densities on the order of 10$^{17}$-10$^{18}$ m$^{-3}$, electron temperatures of 3-5 eV, and ion temperatures of 0.5-1.5 eV. With the weak applied fields in the range of 0.3-8 G, the electrons are able to execute many gyro-orbits between collisions, while the ion gyroradius is on the order of the device size.

Figure 1 shows a diagram of the flow scheme for both devices. The BRB is a spherical device, roughly 3 m in diameter, while PCX is cylindrical and roughly 1 m diameter, 1 m tall. For the BRB, current is driven radially outward using a set of 6 cathodes and two large ring anodes placed near the poles. In PCX, the current is driven radially inward with a single cathode on axis and 4 anodes located near the edge. In terms of the flow rotation vector, $\Omega$, BRB operates with $B \perp \Omega$ (antiparallel), while PCX has $B \parallel \Omega$. This is true regardless of the direction of the applied magnetic field, since the rotation is set by the $J \times B$ torque. By having both orientations, we are able to compare large qualitative differences in the resulting equilibria.

The magnetic field is measured by a calibrated 15-position, 3-axis Hall probe array with a resolution of approximately 0.1 G [40]. Density, electron temperature, and flow are measured by a single position combination Mach and Langmuir probe using standard analysis techniques. Both Hall and electrostatic probes are spatially scanned over the areas indicated in Fig.1 over the course of many shots, with fixed electrostatic probes used to determine shot-to-shot reproducibility. In addition to probes, PCX is equipped with a unique, high-resolution Fabry-Perot spectrometer, which is able to measure chord integrated ion temperature and flow to better than 0.1 eV and 50 m/s precision [41]. By taking chord data at different locations, profiles of these quantities are constructed.

Figure 2 shows data collected from the BRB where the outward directed current drives large magnetic field amplification. A bias is applied at $t = 0.25$ s between the LaB$_6$ cathodes and the polar anodes that creates and sustains a second-long steady plasma. Immediately after the plasma is created and current begins to flow between the electrodes, massive field amplification is observed. Fig-
MRI requirement however no characteristics of criteria for hydrodynamic instability as well as the ideal file has enough shear to meet the Rayleigh circulation. In the no dissipation limit, the centrally peaked flow profile. The bulk viscosity for these plasmas is significant viscosity couples this flow from the partially magnetized center, allowing the Lorentz force to drive only flux lines around the same radius as the anode location. With the amplified field, the ions become partially magnetized near the center, allowing the Lorentz force to drive flow. Like edge-driven flow experiments, viscosity couples this flow from the partially magnetized region outward to the unmagnetized portion of the profile. The bulk viscosity for these plasmas is significant (~40 m/s²), allowing for strong momentum transport. In the no dissipation limit, the centrally peaked flow profile has enough shear to meet the Rayleigh circulation criteria for hydrodynamic instability as well as the ideal MRI requirement, however no characteristics of instability are observed due to the large viscosity.

When the current direction is reversed using PCX, the magnetic field is removed from the plasma. Figure 3(lower) shows linear profiles from the reversed current case on PCX. The roughly 7 G initial magnetic field is completely removed from the central region of the plasma. Along with the field removal, an elongated density gradient is seen that extends from the plasma edge well into the bulk volume. This gradient is significantly longer than the typical one seen from the multi-cusp confinement (~10 cm). For cases with larger field amplification (1 and 4 in Fig. 2(c)) shows the magnetic field relative to the applied field and flux lines for four separate cases. In the MHD case, Ekman circulation develops and drives a radial outward flow which is modified by the applied field and flux lines. The Hall term is included, the field is frozen into the electron fluid, where the direction of radial flow is determined by the applied current. When the Hall term is included, the field is frozen into the electron fluid, where the direction of radial flow is determined by the applied current. The simulations confirm that volumetric flow drive only amplifies field with the extended Ohm’s law terms included and outwardly directed radial current, corresponding to \( B \parallel \Omega \). In Hall runs with the opposite current direction, the field is mostly removed from the bulk of the plasma volume, matching the observations made on PCX.

The Hall effect mechanism responsible for the field amplification or removal can be easily seen by considering
the extended Ohm’s law,

\[ \mathbf{E} - \mathbf{V} \times \mathbf{B} = \eta \mathbf{J} + \frac{1}{ne} (\mathbf{J} \times \mathbf{B} - \nabla P_e) \tag{1} \]

where \( \eta \) is the plasma resistivity and \( P_e \equiv nkT_e \) is the electron pressure. In the high-\( \beta \) Hall limit, the \( \mathbf{J} \times \mathbf{B} \) and electron pressure terms dominate in balancing the applied \( E_r \). By considering the toroidal component of Ohm’s law and setting the non-equilibrium inductive electric field to zero, a relationship between the radial and toroidal currents is found,

\[ E_\phi = 0 = \eta J_\phi - \frac{1}{ne} J_r B_z \rightarrow J_\phi = \frac{\Omega_{ce}}{\nu_e} J_r \tag{2} \]

where the Spitzer form of resistivity has been used to relate the resistivity to the electron collision frequency, \( \nu_e \). In cases where the electrons are well magnetized relative to their collisions, a large toroidal current can be formed from cross-field current. Applying Ampere’s law, when \( J_r > 0 \) (as on the BRB) the induced \( J_\phi \) will always act to enforce the magnetic field, while when \( J_r < 0 \) (as on PCX), the toroidal current will act against the existing magnetic field.

An ordering for the terms in Eq. 1 for parameters in either device indicates that the flow induction and resistivity terms are negligible compared to the current induction and electron pressure. The resistivity term is kept to arrive at the expression in Eq. 2 because some electron collisions are required for cross-field current. However, for both species, collisions do not play a large role in the radial force balance and have been neglected in this simple model.

While the electrons are well magnetized and drifting to create strong toroidal currents, the ions are mostly unmagnetized and ballistic. However, the ions still play an important role in managing the toroidal current since their force balance sets the density profile. In the absence of the Lorentz force, a radial electric field sets up to balance the centrifugal force from ion flow and the ion pressure gradient,

\[ -nm_i \frac{V_e^2}{r} = neE_r - \frac{\partial P_i}{\partial r} \tag{3} \]

where \( P_i \equiv kT_i n \) is the ion pressure. For these plasmas the terms on the RHS of equation 3 dominate and so the ion pressure gradient is largely balanced by the radial electric field and the ions can be thought of a Boltzmann-like in this equilibrium. When the electric field is outwardly directed (like on BRB), the density profile is hollow, while an inwardly directed electric field (like on PCX) causes the density to peak on axis. This electric field couples the ions and electrons, completing the equilibrium model.

Attributing plasma current entirely to the electrons due to their high mobility and using the electric field from Eq. 3, leads to the standard plasma equilibrium condition in the radial direction,

\[ J_\phi B_z = \frac{\partial}{\partial r} (P_i + P_e) - nm_i \frac{V_e^2}{r} \tag{4} \]

where the last term is a small correction arising from the ion flow. This standard equilibrium coupled with the Hall mechanism in Eq. 2 shows that the generated current necessarily causes extended density gradients and that the direction of the gradient is dependent on the injected current direction (electric field in the ion force balance). The equilibrium described here takes advantage of the well-confined plasmas in multicusp devices, where an ambipolar field in the small cusp region keeps the ions from leaving the plasma. In previous work with similar flux expulsion experiments [47], an ad hoc radial electric field is used to complete the electron force balance. Here however, this electric field is well described by Eq. 3, leading to the standard MHD force balance in Eq. 4. The coupling of the electrons and ions via the electric field is an essential feature of this Hall framework.

In both configurations the total amount of poloidal magnetic flux was not conserved during the progression of the experiments. This apparent creation or annihilation of flux is the result of the Hall effect’s conversion of the flux carried by the injected current into poloidal flux in the plasma. In both cases, external power supplies provide a source of flux beyond what is generated from the external Helmholtz coils. On BRB, the magnetic field is amplified by nearly a factor of 20 at nearly 300 A of injected current.

![FIG. 5. Scan of injected current versus normalized change in magnetic field for the two experiments. Top: The PCX case shows strong diamagnetic field removal, approaching total removal at approximately 80 A of injected current. For the BRB case, the field is amplified by nearly a factor of 20 at nearly 300 A of injected current.](Image)
into poloidal flux by the Hall mechanism in Eq. 2, more field is added or removed from the plasma.

In summary, the present study demonstrates a new type of plasma flow drive, similar to Couette flow, that uses cross-field currents to drive cylindrically symmetric plasmas with sheared flows. We have shown conclusively that a priori unimportant and weak ($\beta \gg 1$) magnetic fields can, in fact, greatly influence the large-scale equilibrium via the Hall effect. Similar conclusions have been noted in the case of magnetic reconnection where Hall effects can control large scale dynamics by influencing the scale where the magnetic field vanishes [48–50]. To our knowledge, however, our results offer the first experimental suggestion that ignoring two-fluid effects may not be justified for a broader range of astrophysical systems.

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