EFFECTIVE LAGRANGIANS INDUCED BY THE ANOMALOUS WESS-ZUMINO ACTION AND $I^G(J^{PC}) = 1^−(1^{++})$ EXOTIC STATES

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Abstract

A simple dynamical model for the exotic waves with $I^G(J^{PC}) = 1^−(1^{++})$ in the reactions $\rho\pi \rightarrow \rho\pi$, $\rho\pi \rightarrow \eta\pi$, $\rho\pi \rightarrow \eta'\pi$, $\rho\pi \rightarrow (K^{*}\bar{K} + \bar{K}^{*}K)$, and in the related ones, is constructed beyond the scope of the quark-gluon approach. The model satisfies unitarity and analyticity and uses as a “priming” the anomalous non-diagonal $VPPP$ interaction which couples together the four channels $\rho\pi$, $\eta\pi$, $\eta'\pi$, and $K^{*}\bar{K} + \bar{K}^{*}K$. The possibility of the resonance-like behavior of the $I^G(J^{PC}) = 1^−(1^{++})$ amplitudes belonging to the $\{10\} - \{\overline{10}\}$ and $\{8\}$ representations of $SU(3)$ as well as their mixing is demonstrated explicitly in the 1.3–1.6 GeV mass range which, according to the current experiments, is really rich in exotics.

INTRODUCTION

Phantoms of manifestly exotic $\pi_1$ states with $I^G(J^{PC}) = 1^−(1^{++})$ have more and more agitated the experimental and theoretical communities [1-3]. They were discovered in the 1.3–1.6 GeV mass range in the $\eta\pi$, $\eta'\pi$, $\rho\pi$, $b_1\pi$, and $f_1\pi$ systems produced in $\pi^-p$ collisions at high energies and in $N\bar{N}$ annihilation at rest in the GAMS, KEK, VES, CB, and BNL experiments [1-3].

The first evidence for the possible existence of an exotic $1^{−+}$ state coupled to the $\eta\pi$ and $\rho\pi$ channels and belonging to the icosuplet representation of $SU(3)$ was obtained by J. Schechter and S. Okubo about 37 years ago with the bootstrap technique [4].

Recently theoretical considerations concerning the mass spectra and decay properties of exotic hadrons have been based, in the main, on the MIT-bag model, constituent gluon model, flux-tube model, QCD sum rules, lattice calculations, and various selection rules.

Current algebra and effective Lagrangians are also important sources of theoretical information on exotic partial waves. It is sufficient to remember the prediction obtained within the framework of these approaches for the $\pi\pi$ $S$-wave scattering length with isospin $I = 2$. There also exist a good many of the model constructions which show that the low-energy contributions calculated within the effective chiral Lagrangians framework may in principle transform with increasing energy into resonances with the experimentally

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established parameters. The important ingredient of all these models is the successfully selected unitarization scheme for the original chiral amplitudes which is used to match the low-energy and resonance regions. Such models are well known, for example, for the \( \pi\pi \) scattering channels involving the \( \sigma \) and \( \rho \) resonances (see, for example, Ref. [5]). In the present work we continue in this way and construct

**A MODEL FOR THE \( I^G(J^{PC}) = 1^- (1^{-+}) \) WAVES IN THE REACTIONS**

\( VP \to PP, \ PP \to PP, \) AND \( VP \to VP \)

using, as the starting point, the following anomalous effective Lagrangian for point-like \( VPPP \) interaction of the vector \( (V) \) and pseudoscalar \( (P) \) mesons

\[
L(VPPP) = ih \epsilon_{\mu
u\tau\kappa} Tr(\hat{V}^\mu \partial^\nu \hat{P} \partial^\tau \hat{P}) + i\sqrt{1/3} h' \epsilon_{\mu
u\tau\kappa} Tr(\hat{V}^\mu \partial^\nu \hat{P} \partial^\tau \hat{P}) \partial^\kappa \eta_0,
\]

where \( h \) and \( h' \) are the coupling constants, \( \hat{P} = \sum_{a=1}^{8} \lambda_a P_a / \sqrt{2}, \ \hat{V}^\mu = \sum_{a=0}^{8} \lambda_a V_a^\mu / \sqrt{2}, \) and \( \lambda_a \) are the Gell-Mann matrices. This Lagrangian induced by the anomalous Wess-Zumino action and generates the tree exotic amplitudes with \( I^G(J^{PC}) = 1^- (1^{-+}) \) for the inelastic reactions \( \rho \pi \to \eta_8 \pi, \ K^* K \to \eta_8 \pi, \ K^* K \to \eta_8 \pi \) belonging to the \( \{10\} - \{1\bar{0}\} \) representation of \( SU(3) \) (in this case, there are, at least, the \( qq\bar{q} \bar{q} \) states in the \( s \)-channel) and the tree amplitudes for the reactions \( \rho \pi \to \eta_0 \pi, \ K^* K \to \eta_0 \pi, \ K^* K \to \eta_0 \pi \) belonging to the \( \{8\} \) representation of \( SU(3) \) (in this case, there are both \( qq\bar{q} \bar{q} \) and \( q\bar{q}g \) states in the \( s \)-channel). In the next orders, these tree amplitudes induce as well the exotic ones for the elastic processes \( \rho \pi \to \rho \pi, \ \eta \pi \to \eta \pi, \) and so on. In this connection it is of interest to consider the following \( 4 \times 4 \) system of scattering amplitudes for the coupled exotic channels of the reactions \( VP \to VP, \ VP \leftrightarrow PP, \) and \( PP \to PP \):

\[
T_{ij} = \begin{bmatrix}
T(\rho \pi \to \rho \pi) & T(\rho \pi \to \eta \pi) & T(\rho \pi \to \eta' \pi) & T(\rho \pi \to K^* K) \\
T(\eta \pi \to \rho \pi) & T(\eta \pi \to \eta \pi) & T(\eta \pi \to \eta' \pi) & T(\eta \pi \to K^* K) \\
T(\eta' \pi \to \rho \pi) & T(\eta' \pi \to \eta \pi) & T(\eta' \pi \to \eta' \pi) & T(\eta' \pi \to K^* K) \\
T(K^* K \to \rho \pi) & T(K^* K \to \eta \pi) & T(K^* K \to \eta \pi) & T(K^* K \to K^* K)
\end{bmatrix}.
\]

The subscripts \( i, j = 1, 2, 3, 4 \) are the labels of the \( \rho \pi, \ \eta \pi, \ \eta' \pi, \) and \( K^* K \) channels, respectively (the abbreviation \( K^* K \) implies just the \( K^* K \) and \( K^* K \) channels).

We consider three natural limiting (in the sense of \( SU(3) \) symmetry) cases: (i) \( h' = 0, \) i.e., when all exotic amplitudes belong to the \( \{10\} - \{1\bar{0}\} \) representation of \( SU(3); \) (ii) \( h = 0, \) i.e., when all exotic amplitudes belong to the octet representation of \( SU(3); \) and (iii) \( h' = h, \) when the original \( VPPP \) interaction possesses neton symmetry with respect to the \( 0^- \) mesons.

To obtain the unitarized amplitudes in coupled channels, we sum up all the possible chains of the \( s \)-channel loop diagrams the typical examples of which are given below.

\[\begin{array}{cccccccc}
\rho^0 & \eta & \rho^0 & \eta & \rho & \eta & \rho^0 & \eta
\end{array}\]

\[\begin{array}{cc}
\pi^- & \pi^- & \pi^- & \pi^- & \pi^- & \pi^- & K & \pi^-
\end{array}\]

It is the well known field theory way of the unitarization. The relevant summation can

\[1\]We use the pseudoscalar octet-singlet (\( \eta_8 - \eta_0 \)) mixing angle \( \theta_\rho \approx -20^\circ. \]
Figure 1: The cross sections of the reactions $\rho^0\pi^- \rightarrow \rho^0\pi^-$, $\rho^0\pi^- \rightarrow \eta\pi^-$, $\rho^0\pi^- \rightarrow \eta'\pi^-$, and $\rho^0\pi^- \rightarrow K^*0K^-$ and the phases of the $\rho\pi \rightarrow \rho\pi$ and $\rho\pi \rightarrow \eta\pi$ amplitudes. The correspondence between the curve numbers and the reaction channels is shown just in the figure. $\tilde{h} = 0.10746$, $\tilde{h}' = 0$, $C_{11} = 0.17 \text{ GeV}^2$, $C_{12} = 1.25 \text{ GeV}^2$ in case (i), $\tilde{h} = 0.10746$, $\tilde{h} = 0$, $C_{11} = 0.34 \text{ GeV}^2$, $C_{12} = 0.67 \text{ GeV}^2$ in case (ii), and $\tilde{h} = \tilde{h}' = 0.10746$, $C_{11} = 0.49 \text{ GeV}^2$, $C_{12} = 0.5 \text{ GeV}^2$ in case (iii).
be easily carried out by using the matrix equation $\tilde{T}_{ij} = h_{ij} + h_{im} \Pi_{mn} \tilde{T}_{nj}$ for the auxiliary invariant amplitudes $\tilde{T}_{ij}$, the solution of which has the form: $\tilde{T}_{ij} = [(1 - \hat{h} \hat{\Pi})^{-1}]_{im} h_{mj}$. Here $h_{ij}$ is the matrix of the coupling constants generated by the Lagrangian and $\Pi_{ij} = \delta_{ij} \Pi_j$ is the diagonal matrix of the loops, $\Pi_j = s F_j/(6\pi)$ for the $VP$ loops ($i=1,4$) and $\Pi_i = F_i/(24\pi)$ for the $PP$ loops ($i=2,3$), the functions $F_i$ are defined by the doubly subtracted dispersion integrals

$$F_i = C_{1i} + s C_{2i} + \frac{s^2}{\pi} \int_{m_{i+}^2}^{\infty} \frac{[P_i(s')]^3 ds'}{\sqrt{s'} s'^2 (s' - s - \varepsilon)},$$

where $\sqrt{s}$ is the invariant mass of two-body systems, $P_i(s')$ and $m_{i+}$ are the particle momentum and the sum of the particle masses in the $i$ intermediate state.

Next, we define $D = \det(\hat{1} - \hat{h} \hat{\Pi})$. It is clear that all the physical amplitudes $T_{ij}$ are proportional to $1/D$. Thus a simplest way to discover “by hand” a possible resonance situation is that to find zero of $\text{Re}(D)$ at fixed values of $h$, $h'$ and $\sqrt{s}$ (for example, at $\sqrt{s} = 1.43$ GeV). Leaving the potentialities of the model almost unchanged, we assume that $C_{11} = C_{14}$ and $C_{21} = C_{24} = 0$ for the $VP$ loops, $C_{12} = C_{13}$ and $C_{22} = C_{23} = 0$ for the $PP$ loops. Thus, in most considered variants, we used as the essential free parameters only two subtraction constants $C_{11}$ and $C_{12}$. As for the coupling constant $h$, one may claim [6] that it is not too large in the scale defined by the combination $2g_{\rho\pi\eta}g_{\omega\rho\pi}/m_{\rho}^2 \approx 284$ GeV$^{-3}$, namely, that $|\tilde{h} = F_3^3 h| \lesssim 0.4$ (where $F_\pi \approx 130$ MeV), and we are guided by the values of $\tilde{h}$ (and $\tilde{h}' = F_3^3 h'$) near 0.1.

Figure 1 shows the typical energy dependences, which occur in our model for cases (i), (ii), and (iii), for the four reaction cross sections $\sigma(\rho^0 \pi^- \rightarrow \rho^0 \pi^-)$, $\sigma(\rho^0 \pi^- \rightarrow \eta \pi^-)$, $\sigma(\rho^0 \pi^- \rightarrow \eta' \pi^-)$, and $\sigma(\rho^0 \pi^- \rightarrow K^+ K^-)$ and for the phases of the $\rho \pi \rightarrow \rho \pi$ and $\rho \pi \rightarrow \eta \pi$ amplitudes. They clearly demonstrate the resonance effects found in the invariant mass region 1.3–1.4 GeV (a similar resonance picture is also obtained for the 1.5–1.6 GeV mass region [7]). Furthermore, the comparison of the obtained cross section values (see Fig. 1) with those of the conventional $a_2(1320)$ resonance production, $\sigma(\rho^0 \pi^- \rightarrow a_2 \rightarrow \rho^0 \pi^-) \approx 5.7$ mb and $\sigma(\rho^0 \pi^- \rightarrow a_2 \rightarrow \eta \pi^-) \approx 2.36$ mb at $\sqrt{s} = m_{a_2} = 1.32$ GeV, indicates conclusively that we are certainly dealing with the resonance-like behavior of the $I^G(JPC) = 1^-(-^+)\pi$ exotic waves, at least, in the $\rho \pi$, $\eta \pi$, and $\eta' \pi$ channels. Summarizing we conclude that our calculation (see Ref. [7] for details) gives a further new reason in favor of the plausibility of the existence of an explicitly exotic $\pi_1$ resonance in the mass range 1.3–1.6 GeV.

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