Varying alpha, thresholds and extra dimensions

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Abstract

We consider variations of coupling strengths and mass ratios in and beyond the Standard Model, in the light of various mechanisms of mass generation. In four-dimensional unified models, heavy quark and superparticle thresholds and the electron mass can completely alter the (testable) relation between $\Delta \ln \alpha$ and $\Delta \ln \mu$, where $\mu \equiv m_p/m_e$. In extra-dimensional models where a compactification scale below the fundamental scale is varying, definite predictions may result even without unification; we examine some models with Scherk-Schwarz supersymmetry-breaking.

1 Introduction

If the recently measured cosmological variation in $\alpha$ [1] is to be explained within a unified model, i.e. one in which all couplings are constrained by a relation at a certain energy scale, definite predictions should ensue for the variations of other quantities accessible to astrophysical measurement, in particular $\mu \equiv m_p/m_e$ [3] and $g_p$ (the gyromagnetic ratio of the proton) [4, 2]. The “theory of everything” which would be required to make such predictions does not at present exist, although there are candidates for large parts of it. So far, results have been somewhat discouraging for unification, even considering the amount of theoretical uncertainty attached [12, 15, 8, 16, 34].

To summarize the situation very roughly, the mechanism(s) of mass generation in $d = 4$ field theory, including dimensional transmutation in QCD as well as various ways of generating a Higgs v.e.v. and fermion mass hierarchies, may be very sensitive to changing couplings at high scale, since they can involve slow logarithmic running into the strong coupling regime. But the running of the U(1) and SU(2) gauge couplings is weak and, crucially, $\alpha$ does not depend exponentially on a gauge coupling at high scale, but only linearly (to first approximation).

Thus the fractional variations in the mass ratios which determine $\mu$ (and may affect $g_p$) obtain contributions many times the size of the fractional variation of $\alpha$. However experimental bounds on variations in $\mu$ and $g_p$ are of a similar order to the signal in $\alpha$, namely a few times $10^{-5}$. Hence the number of theories

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that could possibly be consistent with experiment appears drastically reduced and may indicate a fine-tuning problem. Such a conclusion turns out to hold also if the parameters of the Higgs and flavour sectors are held fixed (relative to the fundamental scale), since the variation of the SU(3) coupling at high scale is almost always at least as large as that of the U(1) and SU(2) couplings (whether the variation arises from a varying unified coupling constant, or a varying GUT scale relative to the fundamental scale, or some combination) and the exponential amplification due to strong running of QCD is always present [12, 15, 8, 16, 34].

Our aim is not to cure this apparent problem, since it is obviously desirable that theories be ruled out by experiment. Instead we will take more fully into account effects associated with thresholds in RG running and mass generation, and show that they may significantly alter theoretical expectations, and also point out that predictions can be made in the case of a varying compactification radius even in models without unification.

Important thresholds arise already in the SM, for example from the $W^\pm$ and heavy quarks; theories capable of addressing the hierarchy problem are expected to possess additional thresholds, and imply particular relations for variations in the mass spectrum (which, of course, directly influences $\mu$). To make predictions, one must identify a single varying degree of freedom in the model as the source of the observed effect: given the complexity of realistic unified theories there may be more than one candidate for the source of the variation. Nevertheless certain possibilities are obviously simpler and better-motivated than others.

For a varying GUT gauge coupling, given the SUSY-GUT value $M_G \simeq 2 \times 10^{16}$ GeV and without considering variation of mass ratios in the electroweak and flavour sectors [12, 15, 8], earlier calculations gave $R \equiv \Delta \ln \mu / \Delta \ln \alpha = 36$ with an uncertainty of a few due to higher-order effects. Our estimates, including varying mass ratios $^1$, and with an improved treatment of the proton mass $^2$ give $R = -13 \pm 7$ in a theory with softly-broken supersymmetry at the weak scale and $R = 4 \pm 5$ without.

1.1 New variations and twists from extra dimensions

Various types of model with extra dimensions [32, 33, 40, 41, 43, 44, 45] may have interestingly different predictions since first, gauge and matter fields may propagate in different submanifolds in extra dimensions; second, the RG scale at which the variation of couplings originates is not tied to any unification scale; and lastly, the mechanisms of breaking SU(2) × U(1), supersymmetry or GUT groups in some theories, have a distinctive dependence on the radii of compact extra dimensions.

On the first point, traditional Kaluza-Klein-like theories have a universal dependence on the size of the extra dimension, since all fields live on the same manifold; however, “brane-world” theories restrict some fields to a submanifold.

\(^1\)In the case when electroweak symmetry-breaking occurs in a hidden sector with a gauge coupling varying with the unified coupling

\(^2\)See also [22, 30]
The submanifold may indeed only be $R^3 \times S^1$, in which case couplings involving localized fields are not affected when the size of transverse dimensions varies. Of course, if all fields live on the same brane, there is no new behaviour of varying couplings (except for gravity) resulting from the presence of extra dimensions.

On the second point, the couplings need no longer be determined (by, say, a dilaton v.e.v.) at GUT or string energy scales and run through 14 or more orders of magnitude to reach experimentally-accessible energy scales. Both the fundamental cutoff scale, and compactification scales, can be much lower, thus RG running may be quantitatively different: the scale at which the variation in couplings originates is different. This possibility includes the case of extra-dimensional GUTs with Kaluza-Klein thresholds [33, 34]. In fact, even non-GUT theories may have well-defined predictions, in the case where the variation is due to a cosmologically varying radius. We only need to know the physics at and below the energy scale where the variation is transmitted to the SM fields: in general couplings at this scale will not be unified. (Compare [21], where the unification relations are irrelevant for the effect discussed).

On the third point, Scherk-Schwarz (S-S) symmetry breaking [35, 32, 40, 41] generates a mass scale inversely proportional to the radius of compactification, while perturbative couplings in the effective $D = 4$ theory vary as a power of the radius (depending on which fields propagate in extra dimensions). Thus for a varying radius, mass ratios and coupling strengths (defined at the scale of the inverse radius) will both vary as power laws with small exponents. This contrasts with the exponential dependence of mass scales on the high-energy gauge coupling in the case of mass generation by dimensional transmutation in $D = 4$.  

1.2 Other issues

There are many aspects of varying couplings that we will not discuss in detail. One outstanding question is what form the time- (or space-) dependence takes and what dynamics generate it. Following on from this question, if the variation originates from a light scalar, long-range forces and apparent violations of the weak equivalence principle will likely ensue [28, 30, 29]: see also Section 3.2.

If the spacetime dependence could be predicted, then bounds from the Oklo phenomenon [8, 5] and from the early Universe (in particular nucleosynthesis and CMB) might well be the most stringent. We will take the opposite approach and concentrate on the range of redshifts at which a nonzero variation in alpha is claimed: direct constraints arise from astrophysical observations at the same epoch. See [15, 13, 16, 34] for similar approaches.

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3Classically, the 4d coupling constant of a gauge field $\alpha_i^{(4)}$ propagating in extra dimensions varies simply as $R_i^{-1}$, where $R_i$ is the radius associated with a one-cycle of a compactification manifold, which in the simplest case has the metric of a torus.

4One cannot alter the exponential dependence of $\Lambda_\epsilon$, the confinement scale of strong interactions, on the asymptotic SU(3) coupling constant without radically altering phenomenology.

5These bounds, which depend on nuclear or gravitational physics, do not directly limit $\alpha$, rather they probe some combination of $\alpha$ and other dimensionless couplings: thus one can imagine degeneracies, enhancements or even cancellations depending on how other couplings may vary.
The case of spatial variation is less straightforward. Binning points by redshift is then not useful, and the dependence of alpha on the source (actually, on the absorbing medium) could be highly model-dependent. It could be that couplings measured on Earth are different from those in the gas clouds which are under observation simply by virtue of the different astrophysical environment—possibly due to the different gravitational potentials.

Whatever the dynamics producing the variation, one can investigate correlations between (variations in) the quantities that determine the spectral lines, $\alpha$, $g_p$, and $\mu$. That these quantities are not measured simultaneously for the same sources or absorbers may be of concern. In models with variation over cosmological time, we have to assume that the variation is smooth and relatively slow. In models with spatial variation, we might expect a correlation between the variation of $\alpha$ in one source or absorber, relative to on Earth, and variations of $g_p$ and $\mu$ in another; however in the case of strongly environment-dependent variation we would need much more data on the latter quantities, which are currently bounded by a small handful of systems. If the spatial variation is long-range, rather than environment-dependent, one can say little without more detailed work on the spatial distribution of the sources. Spatial variation might be diagnosed by measurements at low redshift, or by analysis of spatial correlations (requiring a much larger data set than at present).

The problem of fine-tuning the mass of any scalar field responsible for the variation against radiative corrections is an acute one, as noted in (in the context of quintessence) and : we have no further insights into this. The possibility noted in of a rather heavier scalar $m \sim 10^{-3}$ eV in a new weakly-coupled sector alleviates such a problem, but the gravitational energy of the scalar must be dynamically cancelled if the model is to be cosmologically viable.

Finally, the possibility of degeneracy in the interpretation of measurements in terms of atomic physics has been raised: the changes in spectral lines interpreted as varying alpha could in principle be due to varying $\mu$ or $g_p$ instead. A full evaluation for the many-multiplet technique could be complicated, as one needs to know the dependence of many spectral lines on all three parameters; recall that numerical calculations were required to find the expected variations in frequency due to alpha alone. We will, for the moment, take the data on $\alpha$, $\mu$ and $g_p$ at face value.

### 1.3 Note added

After the first version of this paper appeared, Murphy et al. released a preprint with a revised value of $\Delta \alpha/\alpha$, with the change being due to the identification of an additional source of random scatter in some of the systems studied, and a corresponding increase in the statistical errors assigned. This new value is used in the discussion in Section.
2 Mass generation and thresholds

The importance of the Higgs v. e. v. (or the ratio $v_H/M_P$) was realised in [9], which found that its dependence on the SUSY-breaking masses and the top Yukawa was dominant in determining the outcome of nucleosynthesis. We will find that the variation of $v_H$ (and superpartner masses) can also be decisive for the spectral data, since they depend crucially on mass ratios, particularly the ratios of quark and lepton masses to the scale $\Lambda_c$ of QCD. After a general discussion of how thresholds enter the low-energy observables, we give examples of how the inclusion of thresholds, formally a higher-loop effect, can completely alter predictions from unified theories.

We fix notation by quoting the (solution of the) one-loop RG equation for gauge couplings:

$$\alpha_{i}^{-1}(\mu^-) = \alpha_{i}^{-1}(\mu^+) - \frac{b_{i}}{2\pi} \ln \left(\frac{\mu^-}{\mu^+}\right), \quad (1)$$

thus $\beta_{i}$ is negative for asymptotically free groups. Now given a charged field whose decoupling mass is $m_{\text{th}}$ ($m_{\text{th}}$) (in a convenient mass-independent renormalisation scheme), we have

$$\alpha_{i}^{-1}(\mu^-) = \alpha_{i}^{-1}(\mu^+) - \frac{b_{i}^-}{2\pi} \ln \left(\frac{\mu^-}{\mu^+}\right) - \frac{b_{\text{th}}^i}{2\pi} \ln \left(\frac{m_{\text{th}}}{\mu^+}\right) \quad (2)$$

where $b_{\text{th}}^i \equiv b_{i}^+ - b_{i}^-$, the beta-function coefficient being $b_{i}^+$ above the threshold and $b_{i}^-$ below, with tree-level matching at $m_{\text{th}}$. In the case of multiple thresholds one sums the corrections to $\alpha_{i}^{-1}$. For the QCD invariant scale $\Lambda_c \equiv M e^{-2\pi/9\alpha_3(M)}$ where $m_s < M < m_c$, we find for non-supersymmetric theories

$$\frac{\Lambda_c}{\mu^+} = e^{-2\pi/9\alpha_3(\mu^+)} \left(\frac{m_c m_b m_t}{\mu^+}\right)^{2/27} \quad (3)$$

where $\mu^+ > m_t$, and for superpartners (squarks of geometric average mass $m_{\tilde{q}}$ and gluinos of mass $m_{\tilde{g}}$)

$$\frac{\Lambda_c}{\mu^+} = e^{-2\pi/9\alpha_3(\mu^+)} \left(\frac{m_c m_b m_t}{\mu^+}\right)^{2/27} \left(\frac{m_{\tilde{q}} m_{\tilde{g}}}{\mu^+}\right)^{2/9} \quad (4)$$

where $\mu^+ > m_{\tilde{q}} m_{\tilde{g}}$ and the powers $2/27$ and $2/9$ come from ratios of beta-function coefficients, specifically $-b_{3}^\text{th}/b_{3}^\text{th(<m_c)} = b_{3}^\text{th}/9$. Thus the variation in $\Lambda_c$ is

$$\frac{\mu^+}{\Lambda_c} \frac{\Delta \Lambda_c}{\mu^+} = \frac{2\pi}{9\alpha_3(\mu^+)} \frac{\Delta \alpha_3(\mu^+)}{\alpha_3(\mu^+)} + \frac{2}{27} \sum_{q=c,b,t} \frac{\mu^+}{m_q} \Delta m_q + \frac{2}{9} \left(\frac{\mu^+}{m_{\tilde{q}}} \Delta m_{\tilde{q}} + \frac{\mu^+}{m_{\tilde{g}}} \Delta m_{\tilde{g}}\right) \quad (5)$$

where terms in brackets are to be ignored for the non-supersymmetric case. The heavy quark masses to be used are strictly the decoupling masses $m_q(m_q)$, but we showed in [8] that the difference between using this definition and the
“invariant quark mass” $\hat{m}_q$defined such that it has no scale dependence, is negligible.

We see immediately that the threshold terms are of higher order in $\alpha_3$, thus formally they ought to be grouped with the power-law correction to $\Lambda_c$ from two-loop running. However, when the Higgs v. e. v. is varying rapidly compared to $\alpha$, as is generic in theories unified at high scale [15, 8], the threshold terms can become dominant, in contrast to the two-loop term, which is a model-independent infrared effect.

2.1 Electroweak symmetry-breaking

One case where thresholds are important is when electroweak symmetry-breaking (EWSB) is triggered by nonperturbative gauge theory effects and the gauge group giving rise to such effects is unified with the SM gauge couplings at high scale. This case encompasses gravity- and gauge-mediated SUSY-breaking and technicolor or composite theories, under the assumptions that all gauge groups be unified, and all mass scales lower than the fundamental scale $M_X$ (which may be the GUT scale, the string scale related to $\alpha'$ or the Planck scale of whatever underlying theory we are considering) are generated dynamically. The generic dependence of $v_H$ is

$$\frac{v_H}{M_X} = k\alpha_X^n e^{-2\pi m/b_h\alpha_X}$$

where $k$ is a numerical constant and $n$ parameterises a possible power-law dependence, for example through the Higgs quartic coupling which may (as in the MSSM) be related to a gauge coupling varying with $\alpha_X$, or through the gravitational constant in the case of gravity-mediated SUSY-breaking. The value of $m$ is model-dependent, however the product $m/b_h\alpha_X$ can be estimated more reliably. In the case of theories without fundamental Higgs, this equation simply parameterises the v. e. v. of whatever condensate breaks $SU(2) \times U(1)$. Thus,

$$\frac{M_X}{v_H} \Delta \frac{v_H}{M_X} \simeq \left( n + \frac{2\pi m}{b_h\alpha_X} \right) \frac{\Delta \alpha_X}{\alpha_X}$$

Now under the assumption that $k$ and $n$ are order 1, we neglect (the logarithm of) their contribution to Eq. (6) and find

$$\frac{2\pi m}{b_h\alpha_X} \simeq \ln[M_X/v_H] \simeq (2 \times 10^{16})/(2 \times 10^2)] \simeq 32$$

where we identified $M_X$ with the SUSY-GUT value. 6 Thus in Eq. (6) the RHS becomes approximately $(n + 32)\Delta \alpha_X/\alpha_X$, which is consistent with neglecting $n$ of order 1. If instead we take the heterotic string scale $M_X \sim 4 \times 10^{17}$ GeV we evaluate the RHS as $(n + 35)$, thus the uncertainty introduced by a small integer value of $n$ can be neglected. For illustrative purposes we will take

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6In the non-supersymmetric case, single-step unification is of course inconsistent with data. With the nominal values $M_G \simeq 10^{15}$ GeV and $\alpha^{-1}_G \simeq 42$ we obtain 29 rather than 32 for this equation. Non-SUSY unification is of course possible with intermediate breaking scales, however such theories lose predictivity. In what follows the “non-supersymmetric” case will just mean the result when superpartner thresholds are ignored.
\[ \Delta \ln \left( \frac{v_H}{M_X} \right) = (34 \pm 2) \Delta \ln \alpha_X. \] From standard GUT relations, neglecting the effect of thresholds on the running of SU(2) and U(1) one finds \cite{12, 15} \[ \Delta \ln \alpha \equiv \Delta \ln \alpha_{\text{em}}(m_e) \simeq 0.5 \Delta \ln \alpha_X, \] thus we may also write \[ \Delta \ln \left( \frac{v_H}{M_X} \right) = (68 \pm 4) \Delta \ln \alpha. \] Similarly, we may parameterise SUSY-breaking masses as

\[ \frac{\tilde{m}}{M_X} = k' \alpha_X e^{-2\pi m/b} \alpha_X \] (8)

where \( m_{\tilde{q}}, m_{\tilde{g}} \text{ etc.} \) vary as \( \tilde{m} \), thus their variation can also be estimated as \( \Delta \ln (\tilde{m}/M_X) = (68 \pm 4) \Delta \ln \alpha \) given that superpartners are around the electroweak scale.

It has been pointed out \cite{10, 9} that radiative EWSB may depend sensitively on a combination of the top Yukawa coupling and ratios of SUSY-breaking soft mass terms. Hence, any relation between \( v_H/M_X \) and \( \tilde{m}/M_X \) may be significantly more complicated. Variations in \( y_t \) and soft term ratios are exceedingly model-dependent, to the extent that it is futile to look at detailed models. The best we can do is to parameterise our ignorance (somewhat as in \cite{15}) by allowing the Higgs v.e.v. to have a different fractional variation \( \Delta \ln (v_H/M_X)/\Delta \ln \alpha_X \) compared to that of the average superpartner mass Eq. (8). We put

\[ \beta_v = \frac{\Delta \ln (v_H/M_X)}{\Delta \ln \alpha_X}, \quad \beta_S = \frac{\Delta \ln (\tilde{m}/M_X)}{\Delta \ln \alpha_X} \]

where \( \beta_v \) is defined for all models whether SUSY or non-SUSY and \( \beta_S \) only in the supersymmetric case. Thus, from the above discussion, if there is high-scale unification and EWSB is generated by dimensional transmutation, we expect \( \beta_v \simeq \ln (M_X/v_H) \), which also holds for the SUSY case in the absence of strong dependence on \( y_t \) and soft mass ratios; if SUSY-breaking is triggered by nonperturbative effects in a hidden sector we have \( \beta_S \simeq \ln (M_X/\tilde{m}) \).

If the hidden sector group is not unified with the GUT group, in the sense of having the same coupling constant at \( M_X \), these expectations may be altered. In some heterotic string models one obtains gauge kinetic functions \( S + \varepsilon T \) for the visible sector and \( S - \varepsilon T \) for the hidden sector, where \( S \) is the (four-dimensional) dilaton and \( T \) a volume modulus, and \( 1 > \varepsilon > 0 \). If we identify \( S \) as the varying d.o.f. the model behaves similarly to the case of a “unified hidden sector”; if we identify \( T \) then the values of \( \beta_S \) and \( \beta_v \) will be large and negative! Hence, we keep \( \beta_S \) and \( \beta_v \) as free parameters until the final stage of calculation.

### 2.2 Initial estimate of \( \mu \)

In the first approximation we neglect variations in Yukawa couplings at high scale, and obtain

\[ \Delta \ln \left( \frac{\Lambda}{M_X} \right) \equiv \beta_A \Delta \ln \alpha_3(M_X) = \left( \frac{2\pi}{9\alpha_3(M_X)} + \frac{2}{9} \beta_v + \frac{4}{9} \beta_S \right) \Delta \ln \alpha_3(M_X) = (24 \pm \text{few}) \frac{\Delta \alpha_X}{\alpha_X} \text{[non-SUSY]}, \quad (39 \pm \text{few}) \frac{\Delta \alpha_X}{\alpha_X} \text{[SUSY]} \] (9)

\footnote{See section \ref{sec:thresholds} for a treatment including thresholds.}
using $1/\alpha_X \simeq 24$, where we estimate “few” as about 3 and set $\beta_S = 0$ for a theory without superpartners. If the proton mass is well-approximated by a constant times $\Lambda_c$, we can estimate

$$\Delta\mu/\mu = (-10 \pm \text{few})\frac{\Delta\alpha_X}{\alpha_X} \text{ [non-SUSY]}, \quad (5 \pm \text{few})\frac{\Delta\alpha_X}{\alpha_X} \text{ [SUSY]}.$$ (10)

The dependence of the electron Yukawa coupling may also come into the estimate, possibly at the level of changing this value to, say, $-13[+2]$ if we imagine $y_e \propto \alpha_3^X$ due to some dynamics of flavour structure. This expectation contrasts with the cases where the Higgs v. e. v. and superpartner masses are fixed relative to $M_X$, giving $\Delta\mu/\mu \simeq 17\Delta\alpha_X/\alpha_X$, or where $v_H$ varies as in (7) but the resulting effects on and of QCD thresholds are neglected, giving $\Delta\mu/\mu \simeq -17\Delta\alpha_X/\alpha_X$. Thus the mechanisms of mass generation and the presence of thresholds, especially superpartner thresholds, can completely alter expectations for the relations between varying $\mu$ and $\alpha_X$. In the next section we look at the observables $\alpha$ and $\mu$ in more detail and obtain preciser estimates taking mass generation and threshold effects fully into account.

### 2.3 More detailed treatment of $\alpha$ and $m_p$

#### 2.3.1 $\alpha$ and thresholds

At first sight, “varying the unification scale” and “varying the electron mass” while keeping other parameters “constant” should produce equivalent effects on $\alpha$: only the ratio of two masses is physical. However, the behaviour of intermediate thresholds is most important to determine the result. Hence, we include all known charged thresholds to determine what effect they may have on naive GUT predictions.

“Varying $M_G$” is interpreted as all low-energy threshold masses (that originate from EWSB) varying by the same fraction with respect to $M_G$: thus $m_\mu/M_G, m_\tau/M_G, m_W/M_G, \ldots$ vary by the same fraction as $m_e/M_G$. $^8$ “Varying $m_e$” is interpreted as holding all threshold masses fixed in units of $M_G$ except for $m_e/M_G$ itself. The beta-function coefficient per Dirac fermion of charge $Q$ is $4Q^2/3$, thus “varying $m_e$” leads to the relation

$$\alpha_i^{-1} = \alpha_i^{-1}(\mu^+) - \frac{(4/3)\pi}{2\pi} \ln \left(\frac{m_e}{\mu^+}\right), \quad \frac{\Delta\alpha}{\alpha} = \frac{2\alpha}{6\pi} \Delta \ln \frac{m_e}{\mu^+} \approx (7.7 \times 10^{-4}) \Delta \ln \frac{m_e}{\mu^+}$$

where $m_e < \mu^+$ and we take $\alpha_i^{-1}(\mu^+)$ to be fixed.

Since we have $\alpha^{-1} = \alpha_1^{-1} + \alpha_2^{-1}$ at the weak scale and $\text{Tr} Q^2 = \text{Tr} T_3^2 + Y^2 \text{Tr} 1$ over an SU (2) irrep, we easily see that the effect on $\alpha$ of a certain fractional variation in the masses of a SU (2) multiplet with mass below $M_W$ is no different from the same fractional variation in a multiplet lying above $M_W$. We have in general

$$\frac{\Delta\alpha}{\alpha} = \sum_{i=1,2} \frac{\alpha}{\alpha_i(\mu^+)} \Delta\alpha_i(\mu^+) + \alpha \sum_{\text{th}} \frac{Q^{th} Q^{th}}{2\pi} \frac{\Delta \ln m^{th}}{\mu^+} \Delta \ln \frac{m^{th}}{\mu^+}$$ (11)

$^8$Since the GUT scale is defined through the masses of superheavy fields that trigger the breaking to $G_{SM}$, their threshold masses are constants in GUT units. However, see $^{[6]}$ and the discussion preceding Section 3.
where $\alpha_1 = g'^2/4\pi$, the second sum is over all charged fields, $f^{\text{th}}$ is 2/3 per chiral (or Majorana) fermion, 1/3 per complex scalar and 11/3 per vector boson. All logarithms $\ln(m_{f^{\text{th}}}/\mu^+)$ are numerically comparable if $\mu^+$ is very large, however in the case of low-scale models the difference between $\ln(m_e/M_X)$ and $\ln(m_W/M_X)$ may be significant. For the light quarks, the confinement scale $\Lambda_c$ provides a dynamical cutoff. We have, setting aside the first term in Eq. (11),

\[
\frac{\Delta \alpha}{\alpha}|_{\text{th}} = \frac{1}{137 \cdot 2\pi} \left( \frac{8}{3} \beta_{\Lambda} \Delta \ln \alpha_X + \frac{4}{3} \Delta \ln \frac{m_c m_l}{M_X^2} + \frac{1}{3} \Delta \ln \frac{m_b}{M_X} + 3 \Delta \ln \frac{m_l}{M_X} \right)
- \frac{21}{3} \Delta \ln \frac{M_W}{M_X} + 8 \Delta \ln \frac{\tilde{m}}{M_X} + \frac{1}{3} \Delta \ln \frac{m_H}{M_X} \right) \tag{12}
\]

\[
\frac{\Delta \alpha}{\alpha}|_{\text{th}} \simeq (0.11 \pm 0.01) \frac{\Delta \alpha_{\text{X}}}{\alpha_X} [\text{non-SUSY}], \ (0.49 \pm 0.03) \frac{\Delta \alpha_{\text{X}}}{\alpha_X} [\text{SUSY}]
\]

which is a non-negligible correction to the direct contribution from varying $\alpha_{1,2}(M_X)$, $(\Delta \alpha/\alpha)_{\text{direct}} = (8\alpha/3\alpha_X) \Delta \alpha_{\text{X}}/\alpha_X \simeq 0.47 \Delta \alpha_{\text{X}}/\alpha_X$. (With the the non-SUSY SU(5) estimate $\alpha_{\text{X}} \simeq 1/42$, this ratio is 0.82.)

\section*{2.3.2 $m_p$ contributions from light quarks and electromagnetism}

The proton mass receives several contributions: studies in chiral perturbation theory\cite{48} indicate a dominant contribution from nonperturbative effects in the chiral limit, which can only be proportional to $\Lambda_c$, and subleading terms depending on $u, d,$ and $s$ quark masses. Heavy quarks are assumed to be well-described by their threshold effects on $\Lambda_c$. There is also an electromagnetic contribution\cite{46, 49}.

The effect of varying light quark masses can be found by applying the Feynman-Hellmann theorem:

\[
\frac{m_q \Delta m_p}{m_p} \approx \frac{m_q \partial m_p}{m_p \partial m_q} = m_q \langle p | \bar{q}q | p \rangle
\]

where the matrix element is essentially the so-called $\sigma$-term. Quark masses appear in this equation such that any multiplicative change of normalization of the $q$ operators cancels out: physical quantities do not depend on a particular definition of quark masses. Hence, as for the heavy quarks, we use asymptotic or “invariant” quark masses directly proportional to the Higgs v.e.v. times a Yukawa coupling.
In the limit of isospin symmetry we have
\[
\Delta \ln \frac{m_p}{\hat{m}} = m_p^{-1} \sigma_{\pi N}(0) = 0.048 \pm 0.01
\]
where \( \hat{m} = (m_u + m_d)/2 \), using \( \sigma_{\pi N}(0) = 45 \pm 10 \text{ MeV} \) \[47, 48\]. Isospin-violating effects can be shown to be small by considering pion-nucleon scattering \[50\] or \( m_p - m_n \), hence we use the coefficient 0.048 for both \( u \) and \( d \) quarks.

The strange contribution is related to the strangeness content of the nucleon \( y \) as
\[
\frac{m_s \partial m_p}{m_p \partial m_s} = \frac{y \sigma_{\pi N}(0)}{2 \hat{m}} \frac{2}{m_p} \langle p|\bar{s}s|p\rangle \langle p|\bar{u}u + \bar{d}d|p\rangle.
\]
An estimate \( \partial \ln m_p/\partial \ln m_s \simeq 0.2 \) was made in \[22\] based on the lattice result \( \langle p|\bar{s}s|p\rangle \simeq 1.5 \) with a sea-quark mass of 0.154 MeV \[51\], corresponding to \( y = 0.36 \pm 0.03 \). However this large value was contested in \[52\] where negative \( y \) consistent with zero was obtained (negative values are unphysical). A safe but imprecise estimate is thus to take \( \partial \ln m_p/\partial \ln m_s = 0.12 \pm 0.12 \), corresponding to \( y = 0.2 \pm 0.2 \) \[48\].

The electromagnetic contribution, composed of the direct electromagnetic self-energy and a part originating from nondegeneracy of pion masses \[49\], gives \( m_p^{-1}\partial m_p/\partial \alpha \sim 1.8 \times 10^{-3} \). Except in the case of nucleosynthesis, isospin-violating and electromagnetic effects can be neglected.\(^9\) For consistency we must put
\[
\frac{\Lambda}{m_p} \frac{\partial m_p}{\partial \Lambda} = 1 - \sum_{q=u,d,s} \frac{\partial \ln m_p}{\partial \ln m_q} = 0.78 \pm 0.1.
\]

Putting it all together, we have
\[
\Delta \ln \frac{m_p}{M} \approx 0.78 \left( \frac{2\pi}{9\alpha_3(M)} \frac{\Delta \alpha_3(M)}{\alpha_3(M)} + \frac{27}{4} \sum_{c,b,t} \frac{\Delta \ln m_q}{M} + \frac{4}{9} \frac{\Delta \ln \hat{m}}{M} \right)
\]
\[
+ \ (0.12 \pm 0.1) \Delta \ln \frac{m_s}{M} + 0.048 \sum_{u,d} \Delta \ln \frac{m_q}{M} + \cdots
\]
\[
= \frac{0.54}{\alpha_3(M)} \Delta \ln \alpha_3(M) + 0.058 \sum_{c,b,t} \Delta \ln \frac{m_q}{M} + 0.35 \Delta \ln \frac{\hat{m}}{M}
\]
\[
+ \ (0.12 \pm 0.1) \Delta \ln \frac{m_s}{M} + 0.048 \sum_{u,d} \Delta \ln \frac{m_q}{M} + \cdots
\]
(14)

where apart from the strange term the individual coefficients have errors of order 10\%. Now as before we can take the simplest unification scenario in which all superpartner and quark masses vary approximately with \( v_H/M_X \) to obtain
\[
\Delta \ln \frac{m_p}{M} \approx \left( \frac{0.54}{\alpha_X} + (0.39 \pm 0.12)\beta_v + 0.35\beta_S \right) \Delta \ln \alpha_X
\]
\[
\Rightarrow \ (13 + (13 \pm 4) + (12 \pm 1)) \Delta \ln \alpha_X
\]
(15)

\(^9\)For a consistent unit-free treatment of \( m_n - m_p \) in nucleosynthesis allowing both \( \alpha \) and QCD parameters to vary, see \[15\]; see also \[30\] on implications for composition-dependent forces.
where the last term on the R.H.S. is to be discarded in the absence of superpartners. Thus the inclusion of light quarks and SUSY thresholds and consequent reduction in the $\Lambda_c$ contribution has a non-negligible effect. Using 

$$\frac{\Delta \ln \mu}{\Delta \ln \alpha_X} = \frac{(\Delta \ln(m_p/M_X) - \beta_v \Delta \ln \alpha_X)/\Delta \ln \alpha_X}{\Delta \ln \alpha}$$

we come to the result

$$\bar{R} \approx 0.54 \alpha^{-1} + (-0.61 \pm 0.12)\beta_v + 0.35\beta_S$$

Without variation in the weak and superpartner scales (setting $\beta_v = \beta_S = 0$) we arrive at the rather model-independent expectation $\bar{R} = 25 \pm (\text{few})$. This differs somewhat from the expectation $\bar{R} \approx 36$ based on $m_p = \text{const.} \times \Lambda_c$ and ignoring the effect on $\alpha$ of varying $\Lambda_c$ through light quark thresholds. (However we will see that this scenario is still ruled out.)

If we allow the weak scale to vary but ignore SUSY thresholds, setting $\beta_v \approx 34, \alpha_X = 1/24$ and $\beta_S = 0$, we obtain $\bar{R} = -13 \pm 7$. The change here is mainly due to the variation in $m_e/M_X$. If we now change $\beta_S$ to $34 \pm 2$ we arrive at $\bar{R} = 4 \pm 5$. Here, $\Delta \ln \mu$ turns positive and $\Delta \ln \alpha$ becomes somewhat larger through the variation of SUSY thresholds.

The “varying $M_G$” scenario of [13] for which $\alpha_G$ is constant can be recovered by taking the limit $\beta_0 \to \infty$. In the presence of one should send $\beta_S$ to infinity and also enforce $\beta_v = \beta_S$. The expectation in this case is for $\bar{R} = -330 \pm 65$ without superpartner contributions, or $\bar{R} = -20 \pm 9$ with. Again, these differ from previous estimates [13] due to our more detailed treatment of $m_p$ and of light quark thresholds. The enormous value of $\bar{R}$ in the “non-SUSY” case is due to a cancellation in $\Delta \alpha$ in the absence of the “direct” contribution.

The string-inspired scenario of [16] is more complex since it posits a string scale $M_{st}$ at which the gauge coupling is invariant, as well as an effective GUT scale $M_G$ varying relative to $M_{st}$. SUSY-breaking and EWSB are taken to be independent of $M_G/M_{st}$. There are two ways to implement the scenario: firstly, one may identify $M_X$ with $M_{st}$ and set $\beta_v = \beta_S = 0 = \alpha_X$; however one has to rederive Eqs. (3–5), (12–13), etc., explicitly including heavy GUT thresholds whose contributions $\propto b_i^{th} \Delta \ln(M_G/M_{st})$ can be found given the beta-function of the unified group.

The second way is to identify $M_X \equiv M_G$ and then impose $\beta_v = \beta_S = -\Delta \ln(M_G/M_{st})/\Delta \ln \alpha_G$ (since the weak and supersymmetry scales are invariant relative to $M_{st}$). The variation in $\alpha_G$ is found by RG running down from $M_{st}$ to $M_G$. The results would differ slightly from those of [16], again due to different treatment of $m_p$ and light quark effects, but the overall conclusion that (even in the absence of rapidly-varying weak and SUSY scales) the great majority of possible GUTs result in values $|\bar{R}| > 10$, should remain.

### 3 Varying radii of extra dimensions

#### 3.1 Rationale of a varying radius

In a model which is described by $(4 + \delta)$-dimensional field theory over some range of energies, there is a hierarchy between the (inverse) compactification
radius or radii $R^{-1}_i$ and the ultraviolet cutoff of the higher-dimensional theory $\Lambda_D$, where $D = 4 + \delta$. This makes it a well-defined problem to compute the effect of varying radius on the 4D low-energy theory, since one can take physics above the scale $R_i^{-1}$ to be unchanged by such variation (apart from the change in masses of Kaluza-Klein modes), consistent with decoupling. It is conceivable that physics above this energy scale is also varying, but such variation would likely not be calculable since it might involve physics at energies equal to or above the cutoff.

The superstring dilaton and moduli are a priori candidates for cosmologically varying scalar fields and may be light enough to survive at low energies, yet they parameterize inverse radii which may be in the range $10^{16} - 18$ GeV. In principle they might come into our analysis, except that the theory above the inverse radius scale may be “stringy” to a significant extent: the cutoff is at the string scale (or near the 11-dimensional Planck scale for M-theory), hence if the inverse radius also approaches this scale there is no range of energy over which extra-dimensional field theory makes sense. In perturbative string theory the dilaton- and moduli-dependence of low-energy quantities are extractable, but we face the usual problem that string models which are realistic enough to contain a reasonable approximation to the SM spectrum, are too complicated for detailed calculations to be practicable. One can however make simplifying assumptions based on the behaviour of less realistic models, leading to results such as those discussed in [9, 8, 16].

3.2 Are long-range forces inevitable?

Naively, one might expect that a varying radius which was the source of varying alpha would mediate observable long-range forces. The reasoning is that, given that the scalar field appearing in the four-dimensional effective action (“radial modulus”) is evolving in a potential on timescales of order the inverse Hubble rate, its mass must be comparable to the Hubble rate, thus it mediates a force over very large distances. The strength of the force (i.e. its coupling to matter) in Kaluza-Klein type theories is found at tree level to be comparable to gravity (see e.g. [24]) thus it is likely ruled out by Solar System tests of deviations from general relativity and tests of the Weak Equivalence Principle [25].\(^\text{10}\) (Of course one would not observe any deviation from the inverse square law in the laboratory.)

However, this reasoning is based on a number of assumptions which are not universally valid: therefore although it appears difficult for a varying radius to evade these bounds, it cannot be dismissed immediately.

The first assumption is that the apparent measured variation is due to a smooth evolution over billions of years. However, as we mentioned in Section 1.2 the variation may be spatial and long-range or short-range; the variation may even operate on distance scales as small as a galactic halo. Hence the mass of a varying scalar may be some orders of magnitude above the inverse

\(^{10}\)Even if one is allowed to adjust by hand the scalar kinetic term and couplings to matter, if one specifies that it should be the origin of the alpha variation, then apparent violations of the WEP should be not far below current bounds [28, 40].
Hubble radius. This does not much help to evade observational bounds, but it is worthwhile to refute an unwarranted assumption.

The second is that the radial modulus is identified with the light scalar that is slowly rolling in a non-varying potential. However, compactified radii may vary without the modulus being light. There is an analogous argument for two scalar degrees of freedom in the SM, the sigma meson corresponding to excitations of the chiral condensate and the Higgs. In the previous parts of this paper we saw that these scalar v.e.v.’s are likely varying (in Planck units) in a way correlated with alpha. By the same argument one would also conclude that the sigma and the Higgs should have masses of order the inverse Hubble length. In fact, the scalars are massive and always at the minima of their potentials: their variation is induced by a varying potential. There is no reason why a variation of the radius should not similarly be induced by a varying stabilization potential, with the mass of the modulus being $10^{-3}$ eV (the current limit of sensitivity of fifth-force experiments) or greater. We do not specify the source of such a varying potential, except that it should not directly affect the coupling constants of SM fields. Thus the radius may mediate the variation to the observable sector.

The third is that the scalar couplings to matter are constant. However, even in the simplest case of unification, this is not the case, since any variation in the SU(3) coupling at high scale results in nonperturbative dynamics at low energy which determines the masses of nucleons as an exponential function. When one combines this fact with the presence of Scherk-Schwarz SUSY-breaking, electroweak symmetry-breaking, varying Yukawa couplings and other low-energy phenomena it becomes clear that the effective couplings to matter will be field-dependent, possibly in a complicated way. It was shown in [26] that, in the case of field-dependent couplings to matter, if there is a value of the scalar where the coupling is small, this value is an attractor for the evolution of the scalar. Hence the deviations from GR may be suppressed. This mechanism was discussed in a model with varying radius in [27], although the details of this model are different from those I discuss below.

The fourth is that the tree-level action for the radial modulus is a good approximation. However, it is possible that there are large corrections to the kinetic term and couplings. In [22] radiative corrections to the action were mentioned as a possible way to evade bounds on a light scalar.

Violations of the WEP and other deviations from GR are very strong bounds on all models which seek to explain the dynamics of varying alpha, not only models with varying radii. We do not have any definite solution to the problem: clearly the question is to be addressed by looking in detail at different models. In considering correlations between observables at a given epoch or in similar environments, the details of the radial modulus or other light scalar dynamics are not directly relevant.

### 3.3 Scherk-Schwarz revisited

As mentioned in the Introduction, Scherk-Schwarz symmetry breaking, in combination with varying radii, appears to lead to a relation between perturbative
gauge couplings and the mass scale of symmetry-breaking very different from radiative or nonperturbative sources of SSB.

However, there are subtleties in the analysis which need to be addressed. It has recently been shown that Scherk-Schwarz breaking in a wide variety of situations can be reformulated as spontaneous breaking with v.e.v.'s being non-trivial functions of position in the extra dimension(s) (see e.g. [36]). This may blur the distinction between Scherk-Schwarz breaking and breaking by radiative or nonperturbative means. Various connections have been made between Scherk-Schwarz supersymmetry-breaking in the fifth dimension and nonzero v.e.v.'s of auxiliary fields which might arise dynamically, both in heterotic M-theory [37] and in simpler orbifold contexts [38].

Such a connection might justify the choice of an extremely small dimensionless parameter $\hat{\alpha} \sim 10^{-13}$ in [15] to characterize the “twisting” of the boundary conditions. Thus what originally appeared as a geometric and explicit breaking induced by a nondynamical choice of boundary conditions, turns out to be a dynamical spontaneous breaking whose size may be determined by nonperturbative gauge theory effects. (In both cases the breaking is soft.)

Hence, it is crucial to trace the mechanisms of symmetry-breaking back to their source, since consistent predictions for varying couplings depend on knowing how the mass scales and v.e.v.'s of the model are generated. In the case just mentioned, since gaugino condensation would take place at a scale much lower than that of the inverse radius, we must take account of the variation in the condensate resulting from a variation of the radius. Thus the assumption that the S-S parameter $\hat{\alpha}$ is constant, which would imply that the SUSY-breaking masses vary strictly with $1/R$, is likely incorrect. However, if a S-S breaking parameter is a number of order 1 or quantized, we take it to be constant since it does not require a separate dynamical explanation. (See [39] for a discussion of dynamics that could fix the value of such a parameter).

### 3.4 Units and definitions

For varying extra-dimensional radii, the relevant dimensionless quantities are the radius relative to the cutoff of the extra-dimensional field theory $R \Lambda_D$, and relative to the extra-dimensional Planck length $R M_{P}^{(D)}$. Since by assumption $(4 + \delta)$-dimensional physics above $R^{-1}$ is invariant, we expect $\Lambda_D / M_{P}^{(D)}$ to be unchanged, hence the two ratios carry the same information. We will not be dealing with gravitational effects, hence we take $R \Lambda_D$ as the “order parameter”.

In contrast to the usual designation of $M_P$ as a constant, resulting in “Planck units” if we have $M_P = 1$ (other conventions import factors of $\sqrt{8\pi}$, etc.), we could choose to work in “radius units” in which $R$ is constant and $\Lambda_D$ and $M_P$ vary. Thus “varying $R$”, in addition to affecting the classical relation between couplings in $4 + \delta$ dimensions and $D = 4$, implies RG evolution in the extra-dimensional theory, since we are changing the ratio of the UV scale to the IR scale(s). Since there are a large number of thresholds in such theories, with masses determined primarily by $R$, radius units may be more convenient.
3.5 RG “running” in $4 + \delta$ dimensions

As is well known \[33\], the behaviour of coupling constants under change of scale is radically altered above the compactification scale if fields propagate in extra dimensions. The usual procedure is to decompose extra-dimensional operators into K-K modes whose masses are fixed multiples of $R^{-1}$ and treat the massive modes as thresholds at which the effective four-dimensional beta function changes. With a large number of such modes, one finds an “averaged-out” behaviour of power-law running of the effective 4d coupling. Obviously, the running depends on which fields are allowed to propagate “in the bulk”, and proceeds as normal in 4d if all fields coupled by a particular operator are “on the brane”.

A cutoff is required since gauge and Yukawa couplings quickly become non-perturbative with increasing energy, being irrelevant operators in higher dimensions \[11\]. For boundary conditions imposed at a scale $\Lambda_D$ to be meaningful, the theory should be perturbative at this scale. By assumption, the coupling strengths of all operators in the $(4 + \delta)$-dimensional theory at and above $\Lambda_D$ do not vary.

We quote the relevant formula from \[33\]

$$\alpha_i^{-1}(\mu^-) = \alpha_i^{-1}(\Lambda_D) - \frac{b_i}{2\pi} \ln \frac{\mu^-}{\Lambda_D} + \frac{\tilde{b}_i}{2\pi} \ln \frac{M_\delta}{\Lambda_D} + \frac{\tilde{b}_i X_\delta}{2\pi \delta_i} \left( \left( \frac{\Lambda_D}{M_\delta} \right)^{\delta_i} - 1 \right)$$  \hspace{1cm} (17)

where $M_\delta = R^{-1}$ is the mass of the first Kaluza-Klein mode. Thus

$$\frac{\Delta \ln \alpha_i(\mu^-)}{\alpha_i(\mu^-)} = \left( \delta_i \alpha_i^{-1}(\Lambda_D) - \frac{\tilde{b}_i}{2\pi} - \frac{X \tilde{b}_i}{2\pi} \left( \frac{M_\delta}{\Lambda_D} \right)^{-\delta_i} \right) \Delta \ln \frac{M_\delta}{\Lambda_D} + \frac{b_i}{2\pi} \Delta \ln \frac{\mu^-}{\Lambda_D},$$  \hspace{1cm} (18)

recalling that $\alpha_i(\Lambda_D) \propto (M_\delta/\Lambda_D)^{\delta_i}$, where the $i$’th gauge group propagates over $\delta_i$ extra dimensions. The first term inside brackets arises from this classical dependence, the second from logarithmic K-K contributions and the third from the power-law correction; the last term being the usual $D = 4$ scale dependence. Formally, the second and third terms inside brackets are suppressed by a loop factor relative to the first, however the power-law term may have a large enhancement, essentially due to a large number of K-K modes. However, we can choose the cutoff such that the classical term dominates, as follows.

If couplings are perturbative at the cutoff $\Lambda_D$, we can choose a lower effective cutoff $\Lambda'$, integrating out all modes between the two. For $\Lambda' > M_\delta$ the form of Eq. \[17\] will remain the same and the extra-dimensional gauge coupling at $\Lambda'$ can equally be taken constant. Thus we lower $\Lambda'$ to just above $M_\delta$, making the power-law contribution of K-K modes as small as possible and, in practice, negligible. The quantum terms in the brackets can then be neglected as formally of higher order, since we do not expect $\tilde{b}$ to be very large. We similarly neglect (the variation in) the difference between $\ln(\mu^-/\Lambda')$ and $\ln(\mu/M_\delta)$, and

\[11\] For energies far enough above $R^{-1}$, the behaviour of the compactified theory should reproduce that in uncompactified $D = 4 + \delta$ field theory, and the Kaluza-Klein modes can be approximated by a continuum.
set }\alpha_i(\Lambda') \approx \alpha_i(M_\delta)\text{ in all static quantities. Whether } (b_i/2\pi)\Delta \ln(\mu^-/M_\delta)\text{ is also of higher order, will depend on how the low energy scale is dynamically generated: if } \mu^- \text{ varied with the QCD scale } \Lambda_c \text{ it could compete with the first term.}

Similar considerations for Yukawa couplings imply that the simple classical formulae

\[ \Delta \ln \alpha_i(M_\delta) = \delta_i \Delta \ln \frac{M_\delta}{\Lambda'}, \quad \Delta \ln y(M_\delta) = \left(\frac{p_y}{2} + c\right) \Delta \ln \frac{M_\delta}{\Lambda'} \]

are good approximations in the perturbative regime, where the Yukawa coupling \( y \) couples \( p_y = 0, 1, 2, 3 \) fields (either fermionic or scalar) propagating in extra dimensions and \( (3-p_y) \) fields localized “on the brane”. The integer \( c \) takes the value 0 for Yukawa couplings localized in extra dimensions, as in some models based on \( N = 2 \) supersymmetry in \( D = 5 \), and 1 for a bulk coupling in the case where all fields propagate in the extra dimension. For a supersymmetric Higgs mass term \( \mu_S H_1 H_2 \) localized on the brane with Higgs in the bulk, \( \mu_S \) varies directly proportional to \( M_\delta, \mu_S/M_\delta \text{ constant; for other configurations (i.e. universal extra dimensions or localized Higgses)} \mu_S \text{ arises directly from a mass term in the } D = 5 \text{ theory and thus varies as } \mu_S/M_\delta \propto (M_\delta/\Lambda')^{-1}.

We take one varying dimension, the generalization to more than one is trivial. Substituting for \( \alpha_i(M_\delta) \) and neglecting the variation in low-energy thresholds, including \( \mu^- \), relative to \( M_\delta, \text{ we find} \)

\[ \Delta \ln \alpha_i(\mu^-) = \frac{\alpha_i(\mu^-)}{\alpha_i(M_\delta)} \delta_i \Delta \ln \alpha_i(M_\delta) = \left(1 + \frac{\alpha_i(\mu^-)b_i}{2\pi} \ln \frac{\mu^-}{M_\delta}\right) \delta_i \Delta \ln \frac{M_\delta}{\Lambda'}. \]

Then using the naive formulae \( \Delta \alpha^{-1} = \Delta \alpha_Y^{-1} + \Delta \alpha_Z^{-1} \) and \( \Delta \ln(m_Y/m_e) = (2\pi/9\alpha_3(M_\delta)) \Delta \ln \alpha_3(M_\delta), \text{ and neglecting running of } \alpha \text{ below the electroweak scale and possible variation in } m_e/M_\delta, \text{ we find} \)

\[ \Delta \ln \alpha = \left(\delta_Y \cos^2 \theta_W + \delta_2\sin^2 \theta_W + \frac{\alpha(M_Z)}{2\pi}(\delta_Y b_Y + \delta_2 b_2) \ln \frac{M_Z}{M_\delta}\right) \Delta \ln \frac{M_\delta}{\Lambda'}, \]

\[ \Delta \ln \mu = \frac{2\pi \delta_3}{9} \left(\alpha_3^{-1}(M_Z) + \frac{b_3}{2\pi} \ln \frac{M_Z}{M_\delta}\right) \Delta \ln \frac{M_\delta}{\Lambda'}, \]

thus without superpartners we have

\[ \bar{R} \approx \delta_3 \left(\frac{5.9 - 7}{9} \ln \frac{M_Z}{M_\delta}\right) \left(\delta_Y \cos^2 \theta_W + \delta_2\sin^2 \theta_W + \frac{1}{804} \left(\frac{41}{6} \delta_Y - \frac{19}{6} \delta_2\right) \ln \frac{M_Z}{M_\delta}\right)^{-1} \]

and with superpartners (in the case \( M_\delta \gg \bar{m} \))

\[ \bar{R} \approx \delta_3 \left(\frac{5.9 - 3}{9} \ln \frac{M_Z}{M_\delta}\right) \left(\delta_Y \cos^2 \theta_W + \delta_2\sin^2 \theta_W + \frac{1}{804} (11\delta_Y + \delta_2) \ln \frac{M_Z}{M_\delta}\right)^{-1}. \]

Enforcing \( \delta_Y = \delta_2 = \delta_3 \) as required by an extra-dimensional GUT we obtain

\[ \bar{R} \approx \frac{5.9 - 7/9 \ln(M_Z/M_\delta)}{1 + \frac{1}{804} \cdot \frac{41}{3} \ln(M_Z/M_\delta)} \text{ [non-SUSY],} \quad \frac{5.9 - 3/9 \ln(M_Z/M_\delta)}{1 + \frac{1}{804} \cdot 12 \ln(M_Z/M_\delta)} \text{ [SUSY].} \]
If we use the more detailed treatment of the proton mass Eq. (14) then the value of $\bar{R}$ is simply reduced by a factor 0.78.

In the small radius limit $M_\delta \to M_G$ where the K-K modes do not influence unification we recover the SUSY-GUT expectation $\bar{R} \approx 33$. In the opposite limit where $M_\delta$ approaches $M_Z$ we would obtain $\bar{R} \gtrsim 6$.

The suppression of $\bar{R}$ compared to its value in 4D GUTs was named $\kappa$ in [34], who found $\kappa \simeq 0.16$ in the limit of “low scale unification”. Our estimate is $\kappa \gtrsim 0.18$ taking the value $\bar{R}_{4DGUT} \approx 33$. Note that the estimate $\bar{R}_{4DGUT} \sim 36$ used in [34] can also be obtained from our formula, after accounting for the running of $\alpha_{em}$ below $M_Z$ through $\Delta \ln \alpha = \left(\alpha/\alpha(M_Z)\right) \Delta \ln \alpha(M_Z)$.

The above expressions for $\bar{R}$ are obtained without knowing any details of the supposed extra-dimensional GUT theory: the identity of the GUT group, the pattern of breaking, the localization of matter, etc. We do not even need to have a unified theory! The only parameter is the ratio $M_Z/M_\delta$. Taking a value $M_\delta = 5$ TeV slightly larger than the minimum, without superpartners, we obtain $\bar{R} \approx 9.2$: the logarithmic variation of $\alpha^{-1}_3(M_\delta)$ with increasing $\alpha^{-1}_3(M_\delta)$ is significant at low compactification scales.

In more general “brane world” models, one has more freedom to choose the integers $\delta_i$ and $p_i$. Clearly, the choice $\delta_3 = 0$ leads to $\Delta \ln \mu = 0$, up to effects (so far set to zero) of thresholds below $M_\delta$ and variation in $m_e/M_\delta$. This is just the obvious scenario in which the strong force does not “feel” the variation because it is not propagating round the varying dimension. Data will turn out to favour small values of $\bar{R}$, thus assigning $\delta_3 = 0$ is an obvious choice, if somewhat arbitrary in the absence of a concrete model.

To assess the influence of thresholds and varying $m_e/M_\delta$ we will now turn to more concrete models of mass generation (which, however, will not have the feature that $\delta_3 = 0$).

### 3.6 Electroweak breaking by boundary conditions

The possibility of electroweak symmetry-breaking triggered by Scherk-Schwarz supersymmetry-breaking on an orbifold [40, 41] leads to a class of predictive scenarios where the variation of all observables, under a variation in radius, can be found with very few assumptions. The variations in gauge and Yukawa couplings (and $\mu_S$) as sketched above can be applied to obtain the behaviour of $v_H$ and fermion masses. The main result will be the variation of $m_e/M_\delta$ and the resulting effect on $\mu$. As we will see, given the assumption that all mass parameters in the electroweak sector originate from compacification, the fractional variation in fermion and vector boson masses relative to $M_\delta$ cannot be more than a few times that in $M_\delta/\Lambda'$ (compared to around thirty times, in the scenarios of high-scale models with EWSB through dimensional transmutation). Thus variations in charged thresholds are indeed small enough to neglect in these models. As for supersymmetric partners, the nature of SUSY-breaking ensures that the leading behaviour of soft masses is a constant times $M_\delta$, thus the variation of their thresholds is also negligible. Since (depending on details

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12 Of course, if unknown intermediate thresholds appear below $M_\delta$, all predictions are off.
of the model) superpartner masses are around the compactification scale, we use the SM beta-functions to run up to \(M_\delta\) (as in Eq. (19)).

As a first estimate, we truncate the Higgs potential to the quadratic and quartic terms and assume that first, the quadratic Higgs mass term is generated solely by the radiative effects of the top and stop loops and thus varies as \(M_\delta^2 y_t^2\), and second, that due to the underlying SUSY structure the quartic Higgs couplings vary as \(\alpha v\), \(\alpha = 2\) for the range of values of \(\mu\) is the dominant effect for the range of values of \(\mu\).

Thus, in the one-Higgs model of [41], in this approximation, the Higgs v.e.v., which varies as \(v^4 = (\text{const.}) (g^2 + g'^2)^{-1} R^{-4}\), the expected dependence on \(y_t\) does not appear, due to a quirk of minimization of the potential, and the dependence on the quartic D-term coupling is different. Hence we have \(\Delta \ln(v_H/M_\delta) \simeq (1/2)\Delta \ln(M_\delta/\Lambda')\) and \(\Delta \ln(m_{q,l}/M_\delta) \simeq \Delta \ln(M_\delta/\Lambda')\).

In the model of [41] the minimization conditions do not give a simple form for the dependence of \(v_H\) on input parameters, due to the presence of two Higgs doublets and radiative corrections. The ratio of the two Higgs v.e.v.’s \(\tan \beta\) is around 37 so we neglect \(1/\tan^2 \beta\) next to 1 and obtain

\[
\Delta \ln \frac{M_\delta}{\Lambda'} = \Delta \ln \left\{ \left( \frac{\mu_S}{M_\delta} \right)^2 + \left( 2\lambda_t + \frac{g^2}{8 \cos^2 \theta_W} \right) \left( \frac{v_H}{M_\delta} \right)^2 \right\} \tag{24}
\]

where \(\lambda_t\) is a quartic coupling proportional to \(y_t^4\) with a weak logarithmic dependence on \(RM_Z \propto M_Z/M_\delta\). Then clearly the dependence \(\mu_S/M_\delta \propto (M_\delta/\Lambda')^{-1}\) is the dominant effect for the range of values of \(\mu_S\) considered in [41]. For Scherk-Schwarz parameter \(\omega = 1/3, \mu_S = 600\text{ GeV}\) and \(R^{-1} = 3\text{ TeV}\) we find \(\Delta \ln(v_H/M_\delta) = 44\Delta \ln(M_\delta/\Lambda')\) while for \(\mu_S = 200\text{ GeV}\) and \(R^{-1} = 3\text{ TeV}\) we find \(\Delta \ln(v_H/M_\delta) = 7\Delta \ln(M_\delta/\Lambda').\) Roughly, \(\Delta \ln(v_H/M_\delta)\) scales with \((\mu_S/M_Z)^2\).

The extreme sensitivity to \(\mu_S\) and the fact that this parameter is put in by hand prevent the model from making any definite prediction. Essentially there is no dynamical explanation of the mu-term, it is a brane mass term in the \(D = 5\) theory which we have to assume is non-varying relative to the extra-dimensional cutoff. It is not correlated with the compactification scale (times some coupling constants), thus, in radius units, it varies inversely compared to the radiative corrections triggering EWSB. If \(\mu_S\) is somewhat larger than \(M_Z\) it completely dominates the variation of \(v_H\) giving a large and arbitrary result.

In general, if there are comparable mass scales with different dynamical origins, the variation of a quantity obtained by adding different mass terms can
be very sensitive to the proportion of different terms, thus the model parameters need to be known rather precisely. If the mu-term were to arise from the compactification scale then the dependence on the actual value of $\mu_S/M_Z$ would be much milder and the scenario could be predictive.

3.6.2 More precise estimate of $\bar{R}$

Finally we combine the variation of $v_H$ and fermion masses in the model of [41] with the detailed treatment of the proton mass to obtain the estimate of $\bar{R}$. In this case the variations in $m_f/M_\delta$ is indeed small enough that one can neglect all threshold corrections and quark mass contributions to the variation of $m_p/M_\delta$. Taking account of the variation $\Delta \ln(m_e/M_\delta) \simeq \Delta \ln(M_\delta/\Lambda')$, which simply subtracts 1 from the denominator of Eq. 23, we find

$$\bar{R} = 6.3 \pm 1.$$  

It has been noted that a divergent Fayet-Iliopoulos term might arise in models of this type [42]: the question of whether this could have a considerable effect on $\bar{R}$ is interesting but beyond the scope of this paper.

3.7 GUT breaking by boundary conditions

At the other end of the scale of compactification radii are scenarios where the energy scale of the extra dimension is just below the SUSY-GUT scale, as required if a GUT is to be broken by orbifold boundary conditions [44, 43]. These types of theory do not have dramatic low-energy predictions of detectable K-K modes, however they are claimed to be an improvement over four-dimensional GUTs in terms of predictivity and agreement with current values of $\sin^2\theta_W$ and $\alpha_3(M_Z)$, at least for certain simple models. The ratio of the compactification scale to a fundamental cutoff may also play a role in generating fermion mass hierarchies, thus the variation of fermion masses and $y_t$ with a varying radius may be under control (although a full model explaining the ratio $m_e/m_\tau$ has not appeared).

However, as previously noted the mechanism of EWSB is not fully accounted for in such models: the dynamical origin of the hierarchy $M_W/M_\delta$ is not specified. Since this is also the major cause of uncertainty in four-dimensional GUT models, extra-dimensional GUTs of this type currently do not have predictions distinguishable from 4D GUTs for varying couplings.

4 Comparison with data and conclusion

There are four types of observable that may be considered in the redshift range of interest: $\alpha$, $X \rightarrow \alpha^2 g_p m_e/m_p$, $Y \rightarrow \alpha^2 g_p$ and $\mu \rightarrow m_p/m_e$ where $\rightarrow$ signifies “is a measurement of”. Current values are as follows:

$$\alpha - 1 \Delta \alpha = (-0.543 \pm 0.116) \times 10^{-5}, \ 0.2 < z < 3.7 \ [1]$$  

$$X^{-1} \Delta X = (0.7 \pm 1.1) \times 10^{-5}, \ z = 1.78 \ [4]$$  

19
\[ Y^{-1} \Delta Y = (-0.16 \pm 0.54) \times 10^{-5}, \quad z = 0.68, \quad (27) \]
\[ (-0.20 \pm 0.44) \times 10^{-5}, \quad z = 0.25 \quad (28) \]
\[ \mu^{-1} \Delta \mu = (5.0 \pm 1.8) \times 10^{-5}, \quad z = 3.02 \quad (29) \]

Theory and data can be usefully compared for \( \alpha \) and \( \mu \); a variation in \( g_p \) cannot reliably be related to more fundamental parameters.\(^{13}\)

With three independent types of measurement and four observables, one may in principle check consistency and obtain values of \( g_p \) or \( m_p/m_e \) in two different ways. Given measurements at similar redshifts and ignoring complications due to spatial variation, one could eliminate \( g_p \) as follows:

\[ \frac{Y}{\alpha^2} \rightarrow g_p, \quad \frac{X\mu}{\alpha^2} \rightarrow g_p. \quad (30) \]

If the resulting fractional variation were consistent but large compared to \( \alpha^{-1} \Delta \alpha \), it would indicate a large quark mass contribution to \( g_p \), which would be rather unexpected. Similarly one can compare the “direct” measurement of \( \mu \) to that extracted from

\[ \frac{Y}{X} \rightarrow \frac{m_p}{m_e} \]

which will be consistent if and only if the values for \( g_p \) are. The value of \( \alpha \) itself derives only from the “direct” measurement.

If we pretend that current data can legitimately be combined, then \( Y/\alpha^2 \) would give \( \Delta \ln g_p = (0.93 \pm 0.6) \times 10^{-5} \) whereas \( X\mu/\alpha^2 \) gives \( \Delta \ln g_p = (6.8 \pm 2.1) \times 10^{-5} \). The two values differ by over 2.5 standard deviations, so one hesitates to combine them; the second value also deviates from zero by over 3 sigma. The variation in \( m_p/m_e \) derived from \( Y/X \) would be \( \Delta \ln m = (-0.9 \pm 1.2) \times 10^{-5} \), also differing from the “direct” measurement by over 2.5\( \sigma \). But such combinations of measurements are strictly not correct, since they correspond to widely differing redshifts and environments. Such checks can only be taken seriously if the number and reliability of data increase significantly and the data sets truly overlap.

However, both the measurement of \( \mu \) at \( z = 3.02 \) and the derived value \( Y/X \) at redshift around 1 bound any model where smooth and not too rapid time variations in \( \alpha \) imply variations in \( m_p/m_e \), since a nonzero variation in \( \alpha \) appears to occur at least for redshifts 1 to 3. To stand a chance of agreeing with experiment, \( \Delta \ln \mu \) should be in the range \((-2.9, 5.8) \times 10^{-5}\), thus \( \bar{R} \) should be in the range \((-10.5, 5.5)\) (rounded to the nearest half integer).

It is difficult for many of the models to fit in this window: the “best fit” among 4d unified theories discussed above is the case where \( \beta_v \) and \( \beta_S \) are around 34 (some scenarios discussed in \[16\] may also survive). With some “adjustment” of \( \beta_v \) to take account of a varying top Yukawa in concrete models of radiative EWSB a better fit might be obtained, but this hardly constitutes a motivation for further investigation. Our main result is that in models with

\(^{13}\)It has been claimed that \( g_p \) cannot vary substantially, being essentially a Clebsch-Gordan coefficient \[16\], but also that it may have a substantial but unknown correlation with the strange quark mass \[22\].
high-scale unification the variation of thresholds and fermion masses can be a leading contribution to the variations in $\alpha$ and $\mu$: we presented a detailed discussion of these contributions.

In extra-dimensional models, the variation of a compactified dimension relative to the cutoff can lead to definite predictions, both for coupling constants and mass ratios, even if there are no unification relations. One only needs to know the physics below the compactification scale. Predictions depend mostly on the compactification scale $M_\delta$ and on which fields propagate in extra dimensions. As a special case, we confirm a previous result [34] that the variation of the strong coupling is suppressed relative to the variation of $\alpha$ in extra-dimensional GUTs with low unification scale.

If, as in some “brane-world” models, the gluons of QCD do not propagate round the varying dimension, while one or both of the SU(2)$\times$U(1) groups do, the variation of the RG invariant scale $\Lambda_c$ and the proton mass (relative to $M_\delta$) derive only from thresholds and quark masses and are likely to be small, while the variation in $\alpha$ is not suppressed. Thus one contribution to $\bar{R}$ is small. However, such a scenario also requires a concrete mechanism of EWSB to be self-consistent. If the variations of the electron and quark masses also turn out to be small, one would reach the region of $\bar{R}$ allowed by observation.

Note that the coupling of a light scalar $\phi$, supposed to be the source of the variation, to matter depends on the function $dm/d\phi$, where $m$ is the mass-energy of the body considered in Planck units. This quantity, which is correlated with some of the deviations from GR, is at leading order given by $d(m_N/M_P)/d\phi$. Violations of the WEP due to composition-dependent forces will also include a term varying as $(d/d\phi)(m_p + m_e - m_n)/M_P$. Thus, scenarios in which the variations of $m_p/M_P$ and $v_H/M_P$ are both small are better placed to satisfy bounds on both the variation of $\mu$ and on deviations from GR, than if $\bar{R}$ is small but $m_e/M_P$ and $m_p/M_P$ both have large variations.

In the extra-dimensional models we considered which do include a definite recipe for EWSB, all gauge bosons are in fact required to propagate round the compact dimension. In one model, the variation of $v_H$ relative to $M_\delta$ was extremely sensitive to the undetermined value of the mu-term $\mu_S$, thus we were not able to arrive at a prediction. In another model with fewer parameters, agreement with data is tenuous in that the prediction for $\bar{R}$ is positive and larger than 5, even including the variation in $m_e/M_\delta$. However it is possible that slightly different models of EWSB might result in definite predictions with a larger variation in $m_e/M_\delta$ which could bring $\bar{R}$ within an acceptable range.

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