Resistances of Metal I-Sections under Bending and Torsion

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Abstract. Interaction formulae for the resistance of I sections under the combination of the internal forces of the bending and the torsion. The resistances of I- and H-sections under combination of the bending and the torsion are investigated for the limit states: (i) elastic and (ii) plastic without strengthening. Simple interaction formulae for I- and H-sections convenient for the standard purposes are presented. There are investigated interactions: (i) the bending moment and the bimoment, (ii) the shear force and the St. Venant torsional moment. Results of the large parametrical study of the shear area I- and H-sections are presented too. The shear area and the shear torsion constant formulae are convenient for any shape of the open or closed cross-section. Metal (steel and aluminium) sections are investigated. The proposals are presented for the new generation of Eurocodes EN 1993-1-1 Design of steel structures and EN 1999-1-1 Design of aluminium structures. The formulae for shear area used in EN 1993-1-1: 2005 may give especially for small sizes HEA 100, HEB 100 and HEM 100 sections the values of the relative shear area up to 1.9. This is not acceptable. The relative shear area is defined as the ratio of shear area to web area $h_{tw}$. The experiments shown that the shear area is restricted by the value $\eta h_{tw}$, where $1.0 \leq \eta \leq 1.2$. The proposal how to improve incorrect formula given in the final draft prEN 1993-1-1, 2017 relating to the interaction of the bending moment and the bimoment is presented. The generalised formula valid for the all section Classes was accepted for the final draft prEN 1999-1-1 and it is presented too. The all analytical results and the formulae of the authors were verified and confirmed by the computer programs QST-TSV-3Blech (RU Bochum) and DUENQ (Dlubal Software). Comparisons of the elastic and 5 plastic resistances of the H-section under the combination of shear force and the St. Venant torsional moment, including one proposed by authors for prEN 1999-1-1.

1. Introduction

The paper describes scientific backgrounds of some topics investigated for the purpose of the new generation of Eurocodes which will be available for technical public in the year 2021. The first author is a member of 5 working groups of the technical committee CEN/TC 250 Structural Eurocodes: a) WG EN 1993-1-1 Design of steel structures, Part 1-1 General rules and rules for buildings, b) WG EN 1993-1-3 Design of steel structures, Part 1-3 General rules - Supplementary rules for cold-formed members and sheeting guidelines, c) WG EN 1993-1-5 Design of steel structures, Part 1-5 Plated structural elements, d) WG EN 1993-2 Design of steel structures, Part 2 Steel bridges, e) WG EN 1999-1-1 Design of aluminium structures, Part 1-1 General structural rules.
2. **Web area, shear area and shear torsion constant**

According to EN 1993-1-1 the shear area for I and H sections may be taken as follows:

a) rolled I and H sections, load parallel to web:

\[
A_{EC3,z} = \max(A_{Vz,EN}, \eta h_w t_w)
\]

where

\[A_{Vz,EN}\] may be written in two different forms

\[A_{Vz,EN} = A - 2bt_f + (t_w + 2r)t_f
\]

\[A_{Vz,EN} = h_w t_w + (4-\pi)r^2 + (t_w + 2r)t_f
\]

factor \(1.0 \leq \eta \leq 1.2\) depends on decision of National Annexes of different countries,

\(A\) is the cross-sectional area,

\(b, t_f\) are the width and thickness of the flange,

\(h_w, t_w\) are the width and thickness of the web,

\(r\) is the root radius,

\(h_f = h_w + t_f\) or \(h_f = h - t_f\), where \(h\) is the overall depth of the I and H section.

b) rolled I and H sections, load parallel to flanges:

\[A_{EC3,y} = 2bt_f\]

Note: this may be find only in the draft of EN 1993-1-1 for the new Eurocode generation. In the current EN 1993-1-1 this information is missing.

c) welded I and H sections, load parallel to web

\[A_{EC3,y} = \eta h_w t_w\]

d) welded I and H sections, load parallel to flanges:

\[A_{EC3,y} = A - h_w t_w = 2bt_f\]

One would expect the value \(A_{EC3,y} = 2bt_f \div 6\). See \(A_{Vz}\) in table 1.

According to EN 1999-1-1 the shear area for I sections and for rectangle may be taken as follows:

e) unwelded I sections without holes, load parallel to web

\[A_{EC9,z} = h_w t_w\]

f) rectangle without welds and holes

\[A_{EC9,z} = 0.8h_w t_w\]

Theoretically the shear areas and shear torsion constant (not given in Eurocodes) may be calculated according to table 1, where there are formulae valid for any shapes of sections loaded parallel to \(z\) and \(y\) axes. The formulae valid for I section and rectangle are given in table 1 too. They enabled to perform the large parametrical study showing comparisons of different formulae valid for IPE, HEA, HEB and HEM sections of various sizes. The results of parametrical study are given in Figures 1-15. The results in the figures are given in dimensionless form to achieve general validity. On the horizontal axis there are ratios of continuously changing flange thickness to the flange thickness of the section under investigation \(t_f\). The vertical line going through the value 1,0 relates to the flange thickness of the section under investigation. The range of ratio thicknesses on the horizontal axis is from 0 (rectangle \(h_w t_w\)) till 2 (the flange thickness is 2-times greater than the thickness of the section under investigation). On the vertical axis there are different shear areas divided by the reference value
One expects that the minimum value of this ratio will be $5/6$ valid for a rectangle (I section web without flanges).

**Table 1.** Cross-section properties with taking into account shear influence. Analogy between bending and torsion.

| Bending theory of the 2nd order | Mixed torsion of the 1st order |
|---------------------------------|-------------------------------|
| **Differential equations**      | **Differential equations**    |
| $M_y''(x) - \mu \frac{N}{E_\gamma} = -\mu q_x(x)$ | $M_z''(x) - \mu \frac{N}{E_\gamma} = -\mu q_y(x)$ |
| **Reduction factor $V_x < 1.0$** | **Reduction factor $V_y < 1.0$** |
| $V_x = \frac{I_y^2}{A} \int_a^b \left[ \frac{S_y(s)}{t(s)} \right]^2 \, dA$ | $V_y = \frac{I_y^2}{A} \int_a^b \left[ \frac{S_y(s)}{t(s)} \right]^2 \, dA$ |
| **Shear area $A_{\gamma y}$**     | **Shear area $A_{\gamma y}$**     |
| $A_{\gamma y} = V_x \cdot A = \int_a^b \left[ \frac{S_y(s)}{t(s)} \right]^2 \, dA$ | $A_{\gamma y} = V_y \cdot A = \int_a^b \left[ \frac{S_y(s)}{t(s)} \right]^2 \, dA$ |
| **Factor**          | **Factor**          |
| $\mu_z = \frac{1}{1 + \frac{N}{G A_{\gamma y}}}$ | $\mu_z = \frac{1}{1 + \frac{N}{G A_{\gamma y}}}$ |
| **For rectangular** | **For rectangular** |
| $A_{\gamma y} = \frac{b t_w}{h_t} \left( 1 + \frac{c_b}{b} \right) \left( 1 + \frac{c_h}{b} \right)$ | $A_{\gamma y} = \frac{b t_w}{h_t} \left( 1 + \frac{c_b}{b} \right) \left( 1 + \frac{c_h}{b} \right)$ |

For warped (deplanation) measure

$0 \leq \mu_o = \frac{1}{1 + \mu_z} = \frac{1}{1 + \frac{N}{G A_{\gamma y}}} < 1$

For non-warping sections $\mu = 0$
For closed sections $\mu < 1.0 \rightarrow 0$
For open sections $\mu > 1.0$.

For I-section

$\mu_o = \frac{c_1}{c_1 + c_2}$

Characteristic length (torsion-bending constant)

$L_{o,\mu} = \sqrt{\frac{E_{\gamma y}}{\mu_N}} \left[ m \right]$

Mixed torsion coefficient and mixed torsion parameter

$\kappa_0 = \frac{\mu_o G_{L_0}}{E_{\gamma y}} \left[ m^{-1} \right]$. $\kappa_0 L \left[ \right]$
**Figure 1.** IPE 80. Shear areas comparisons

**Figure 2.** IPE 100. Shear areas comparisons

**Figure 3.** IPE 300. Shear areas comparisons

**Figure 4.** IPE 600. Shear areas comparisons

**Figure 5.** HEA 100. Shear areas comparisons

**Figure 6.** HEA 300. Shear areas comparisons
Figure 7. HEA 600. Shear areas comparisons

Figure 8. HEA 1000. Shear areas comparisons

Figure 9. HEB 100. Shear areas comparisons

Figure 10. HEB 300. Shear areas comparisons

Figure 11. HEB 600. Shear areas comparisons

Figure 12. HEB 1000. Shear areas comparisons
Nevertheless, for the sections with big ratio \( \frac{2bt_f}{h_w t_w} = 4 \) this ratio may be smaller than \( \frac{5}{6} = 0.8333 \), approximately 0.7 (figures 5, 6, 9, 10, 13, 14). The minimum values for such not real sections are indicated by the other vertical line. Another vertical line denoted by symbol \( t_w/t_f \) is valid for the I or H section with constant thickness \( (t_f = t_w) \). The dotted line indicates level \( \frac{5}{6} \) valid for rectangle, the dashed line denotes the reference level \( \frac{1}{2} \) (shear area is \( h_w t_w \), \( \eta = 1 \)). The dot and dash line is valid for the maximum possible value \( \eta = 1.2 = 6/5 \). The bottom solid line is valid for shear area \( A_{yz} \) calculated according to table 1. The middle solid line is valid for shear area used in German standard DIN 18800 \( h_w t_w = (h_w + t_w) t_w \). The upper solid line is valid for \( A_{yz,EN} \) which should be not lesser than \( \eta h_w t_w \). The relative shear area \( A_{yz,EN} / (h_w t_w) \) may achieve the big value \( \approx 1.9 \), which were for rolled I and H sections reportedly confirmed by the experiments. The maximum value \( \eta = 1.2 = 6/5 \) has origin in the experiments.
3. Plastic resistance of I-section under bending and torsion

The following formulae may be used for I sections made of steel (with yield strength $f_y$ and safety factor $\gamma_M = \gamma_{M0} = 1.0$) or made of aluminium alloys (with yield strength $f_y = f_o$ and safety factor $\gamma_M = \gamma_{M1} = 1.1$).

The problem was solved in [1, 2, 3, 6], in [6] also for channel section, where it was shown that:

a) Interaction formula for I section of Class 1 and 2 (cross-section plastic resistance may be utilized) under bimoment and bending moment is

$$\left( \frac{M_{y,Ed}}{M_{pl,y,Rd}} \right)^2 + \frac{B_{Ed}}{B_{pl,Rd}} = 1.0$$

which may be written in the form

$$M_{pl,y,B,Rd} = \sqrt{1 - \frac{B_{Ed}}{B_{pl,Rd}}} M_{pl,y,Rd} = \rho_{pl,My,B} M_{pl,y,Rd}$$

where $\rho_{pl,My,B}$ is the reduction factor of bending moment $M_{pl,y,Rd}$ due to bimoment $B_{Ed}$.

$b$) Interaction formula for I section of Class 3 (cross-section elastic resistance, index $el$) and Class 4 (effective cross-section resistance, index $eff$, which becomes $el$ for fully effective cross-section, when $W_{eff} = W_{el}$ and $t_{eff,f} = t_f$)

$$\frac{M_{y,Ed}}{M_{eff,y,Rd}} + \frac{B_{Ed}}{B_{eff,Rd}} = 1.0$$

which may be written in the form

$$M_{eff,y,B,Rd} = 1 - \frac{B_{Ed}}{B_{eff,Rd}} M_{eff,y,Rd} = \rho_{eff,My,B} M_{eff,y,Rd}$$

where $\rho_{eff,My,B}$ is the reduction factor of bending moment $M_{eff,y,Rd}$ due to bimoment $B_{Ed}$.

$M_{eff,y,Rd}$ is the bending moment of the elastic cross-section resistance

$$M_{eff,y,Rd} = W_{eff,y} f_y / \gamma_M$$

$B_{eff,Rd}$ is the bimoment of the elastic cross-section resistance

$$B_{eff,Rd} = h_{eff,f} b^2 f_y / (6 \gamma_M)$$
The formulae (10, 14) may be generalised to be valid for Classes 1, 2 (plastic resistance), Classes 3 or 4 (elastic resistance of full or effective section) and elastic-plastic resistance too

\[ M_{y,B,Rd} = 1 - \left(1 - \frac{B_{ed}}{B_{rd}}\right)^{1/\alpha_{o}} M_{y,Rd} = \rho_{my,b} M_{y,Rd} \quad (17) \]

where

\[ \alpha_{ep} = 1 + \frac{W - W_{el}}{W_{pl} - W_{el}} \quad (18) \]

\( W = W_{pl} \) for Classes 1 and 2, \( W = W_{el} \) for Class 3 and \( W = W_{eff} \) for Class 4,

\[ M_{y,Rd} = \alpha W_{el,y,f_y} \gamma_M \quad (19) \]

with \( \alpha \) defined in clause 6.2.61 of EN 1999-1-1 (the interaction between \( W_{el} \) and \( W_{pl} \) may be used)

\[ B_{rd} = h_t j f_{y} b^2 f_{o} / (\alpha_{B} \gamma_{M1}) \quad (20) \]

\[ \alpha_{B} = 6 - 2 \frac{W - W_{el}}{W_{pl} - W_{el}} \quad (21) \]

For the new generation EN 1999-1-1 it was proposed [4]

\[ M_{y,B,Rd} = 1 - \left(1 - \frac{B_{ed}}{B_{rd}}\right)^{1/\gamma_{B}} M_{y,Rd} = \rho_{my,b} M_{y,Rd} \quad (22) \]

where

\[ \gamma_{B} = \alpha_{B}^{2} \quad \text{and} \quad B_{rd} = h_t j f_{y} b^2 f_{o} / (4 \gamma_{M1}) \quad (23) \]

and

\[ V_{pl,z,Ti,Rd} = \sqrt{1 - \frac{\tau_{t,rd}}{\gamma_{M1}}} V_{pl,z,Rd} = \rho_{vz,tr} V_{pl,z,Rd} \quad (24) \]

For I sections, when torsion is present, the reduced design moment resistance \( M_{y,V,B,Rd} \) should be taken as the design resistance of the cross-section calculated using a reduced strength as follows

\[ (1 - \rho) \rho_{my,B,f_o} \text{ for the web area } h_w f_{w} \quad (25) \]

\[ \rho_{my,B,f_o} \text{ for the rest of the cross-section} \quad (26) \]

\[ \rho = (2V_{ed} / V_{pl,z,Ti,Rd} - 1)^{2}, \text{ but } \rho = 0 \text{ if } V_{ed} \leq 0,5 V_{pl,z,Ti,Rd} \quad (27) \]

The design value of the bending moment \( M_{ed} \) and design value of shear force \( V_{ed} \) at each cross-section shall satisfy

\[ \frac{M_{y,Ed}}{M_{y,B,Rd}} \leq 1,0, \quad \frac{V_{z,Ed}}{V_{pl,z,Ti,Rd}} \leq 1,0 \quad (28) \]
The safety factor value $\gamma_{M1} = 1.0$ was used in the calculations and creating of the diagrams on figures 16 and 17.

The figure 16 shows the various resistances of IPE 300 section under combination of the bending moment and bimoment in graphical form:

a) the elastic resistance described by the formula (14),

b) the elastic-plastic resistance defined by the formula (17),

c1) the approximate plastic resistance given by the formula (10),

c2) the exact plastic resistance calculated by the computer program [5],

c3) the exact plastic resistance obtained from the analytical solution derived in [1].

Th was shown that approximate plastic resistance $c1$ is very close to the more exact results $c2$ and $c3$ and therefore is convenient for standard purposes. The authors proposed to replace the incorrect formula which is in the final draft of prEN 1993-1-1 Design of steel structures [7] by the formula (10). The formula (10) which is valid for section Classes 1 and 2 was generalised by Torsten Höglund. His formula (22) is valid for all section Classes 1, 2, 3 and 4. The formula (22) was accepted by the working group WG EN 1999-1-1 and may be already found in the final draft prEN 1999-1-1 Design of structures made of aluminium alloys [8].

4. Conclusions

The formulae (2) and (3) used in (EN 1993-1-1, 2005) may give especially for small sizes HEA 100 (figure 5), HEB 100 (figure 9) and HEM 100 (figure 13) sections the value of the relative shear area up to 1.9. The relative shear area is defined as the ratio of shear area to web area $h_w t_w$. The experiments shown that the shear area is restricted by the value $\eta h_w t_w$, where $1.0 \leq \eta \leq 1.2$. This is not acceptable to have 1.9 $h_w t_w$, if there is no scientific background and no evidence based on experiments. The formula (1) should be changed. Such big plastic shear resistance $V_{pl,z,Rd}$ influences in negative way also all interaction formulae, where $V_{pl,z,Rd}$ is used. See the clauses 6.2.7, 6.2.8, 6.2.10 in (EN 1993-1-1, 2005).

The proposal (see formula (10)) how to improve incorrect formula given in the final draft prEN 1993-1-1, 2017 [7]. The generalised formula (22) valid for all section Classes was accepted for the final draft prEN 1999-1-1 [8]. The all above analytical results and the formulae of the authors were
verified and confirmed by the computer programs QST-TSV-3Blech (RU Bochum) [5] and DUENQ (Dlubal Software) [9] too.

The figure 17 shows comparisons of elastic resistance and 5 plastic resistances of HEM 600 section under combination of shear force (vertical axis) and St. Venant torsional moment (horizontal axis). Plastic resistances are as follows:

1) based on the analytical solution described in [1],
2) calculated from the formula (24) for the factor $\alpha = 1.0$,
3) calculated from the formula (24) for the factor $\alpha = 1.25$, which is used in EN 1993-1-1,
4) calculated by the computer program [5]
5) calculated from the formula (24) for the factor $\alpha = 1.5$, which is proposed for prEN 1999-1-1,

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References
[1] I. Baláž, M. Kováč, T. Živner, Y. Koleková, “Resistances of I-Section to Internal Forces Interactions,” Key Engineering Materials, vol. 710, pp.309-314, 2016.
[2] I. Baláž, M. Kováč, T. Živner, Y. Koleková, “Plastic resistance of H-section to interaction of bending moment $M_{y,Ed}$ and bimoment $B_{Ed}$,” Proceedings of the 2nd International Conference on Engineering Sciences and Technologies, CRC Press, Taylor & Francis Group, A Balkema Book, pp. 33-38, 2016.
[3] I. Baláž, Y. Koleková, “Resistances of I- and U-sections Combined bending and torsion internal forces,” Proceedings of EUROSTEEL 2017, Copenhagen, paper No. 13_12_772, pp.1-10, 2017.
[4] I. Baláž, T. Höglund, Amendment AM EC9-1-1-2017 – 43, Internal document of CEN / TC 250 WG EN 1999-1-1, 2017.
[5] Ch. Wolf, J. Frickel, Program QST-TSV-3Blech, Ruhr-Universität Bochum, 2006.
[6] I. Baláž, M. Kováč, T. Živner, Y. Koleková, “Plastic resistance of IPE-section to interaction of bending internal forces $M_{y,Ed}$, $V_{z,Ed}$ and torsion internal forces $B_{Ed}$, $T_{o,Ed}$ and $T_{t,Ed}$,” Proceedings of the 2nd International Conference on Engineering Sciences and Technologies, CRC Press, Taylor & Francis Group, A Balkema Book, pp. 27-32, 2016.
[7] prEN 1993-1-1: 2017, Eurocode 3 - Design of steel structures, Part 1-1: General rules and rules for buildings, CEN/TC 250/SC 3/WG 1, N 229, Final Draft prepared by the Project Team SC3.T1, Date of document: 2017.12.18, Document stage: CEN Enquiry. CEN Brussels.
[8] prEN 1999-1-1:2017, Eurocode 9 - Design of aluminium structures, Part 1-1: General structural rules, CEN/TC 250/SC 9, N 579, prEN 1999-1-1 Final Draft marked, Date of document: 2017-11-02, CEN Brussels.
[9] Dlubal Software GmbH, Programm DUENQ (Shape-Thin) 8.07.03.185540 x32, 2016.