Wave attenuation mechanism in an acoustic metamaterial with negative effective mass density

H H Huang and C T Sun
School of Aeronautics and Astronautics, Purdue University, W Lafayette, IN 47907, USA
E-mail: sun@purdue.edu

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Abstract. The wave attenuation and energy transfer mechanisms of a metamaterial having a negative effective mass density are studied. The metamaterial considered is represented by a lattice system consisting of mass-in-mass units. The attenuation of wave amplitude for frequencies in the stop band is studied from the energy transfer point of view. It is found that most of the work done by the external force on the lattice system is stored by the internal mass if the forcing frequency is close to the local resonance frequency. However, the energy stored in the internal mass is only temporary; it is taken out by the external force in the form of negative work in a cyclic manner. This behavior is utilized to design metamaterials for preventing stress waves from passing them.
1. Introduction

Metamaterials are generally regarded as materials with manmade microstructures that exhibit unusual responses not readily observed in natural materials. They were first introduced in the field of electromagnetic (EM) wave propagation [1]–[4]. Recently, researchers have begun to look into the acoustic counterpart of (EM) metamaterials. Attention has mainly focused on metamaterials with double negativity [5], and acoustic cloaking metamaterials [6, 7]. Most of the research in acoustic metamaterials is, however, theoretical in nature except for metamaterials with negative effective mass or mass density, which can be realized with manmade microstructures [8]. One type of acoustic metamaterial with negative mass or mass density is composite materials consisting of special microstructures that display local resonances [8]–[11]. If the locally resonant microstructures are periodically placed in the matrix material, the resulting metamaterial also forms a phononic lattice material that creates a stop band, which forbids elastic wave propagation within the bandgap frequency range [12]–[15].

For conventional phononic lattice structures, gigantic structures are required for environmental low-frequency sound and vibration shielding. One possible solution was first provided by Liu et al [8], who fabricated sonic crystals displaying microstructures with local resonances based on the idea from electromagnetic materials with an optical resonance that makes the wave attenuate exponentially. They showed that, at the low frequency gap, the lattice constant of the crystal could be much smaller than the longitudinal wavelength of the wave in epoxy. Since then, a number of studies on acoustic metamaterials with a negative effective mass resulting from local resonances have been published [8]–[11]. Inspired by Sheng et al [9] and Liu et al [10], Chan et al [13] as well as Milton and Willis [16] employed a one-dimensional model to introduce the negative effective mass effect. In addition, based on a mechanical model, which is basically same as those in [13, 16], constructed by Vincent [12], Lazarov and Jensen [14] on studied the gap effect on the model with nonlinear local oscillators. They showed that the position of the gap can be shifted due to the nonlinear effect. Using a simple mass-in-mass lattice model similar to Vincent’s model, Huang et al [15] showed that, if the original lattice is modeled as an equivalent system of single mass units, the effective mass of the equivalent lattice would become negative for frequencies near the local resonance frequency. While preparing the present manuscript, we came across the paper by Yao et al [17] who have...
recently published the result of an experimental study using a one-dimensional mass–spring system and examined the existence of negative effective mass. From their experimental results, a significant transmission reduction in the frequency range that corresponds to Milton and Willis’s negative effective mass was found [16].

The purpose of the present paper is to investigate the mechanism associated with local resonances that causes wave attenuation in acoustic metamaterials when the effective mass density becomes negative. Attention is focused on the mechanism that prevents harmonic waves from propagating in the metamaterial when the wave frequency is near the local resonance frequency. To aid the understanding of this phenomenon, the work done on the lattice by an external force and the subsequent energy flow in the mass-in-mass lattice system are examined, especially at forcing frequencies that are near the local resonance frequency. Numerical results are generated to illustrate the characteristic dynamic behavior of an acoustic metamaterial.

2. Metamaterial and equivalent medium with negative effective mass density

In this section, we briefly review the results presented in [15]. The main focus of the work presented in [15] was to present different continuum models that are used to represent a mass-in-mass system and showed that in certain frequency ranges, the classical continuum model must have negative effective mass density in order to accurately represent the original mass-in-mass lattice system. On the other hand, if a Cosserat-type [18] continuum model is employed to represent the mass-in-mass lattice system, then the negative effective mass is absent from the formulation.

Consider a one-dimensional lattice consisting of identical mass-in-mass units as shown in figure 1(a). The equations of motion for the $j$th unit cell are

$$m_1^{(j)} \frac{d^2 u_1^{(j)}}{dt^2} + k_1 (2u_1^{(j)} - u_1^{(j-1)} - u_1^{(j+1)}) + k_2 (u_1^{(j)} - u_2^{(j)}) = 0,$$

$$m_2^{(j)} \frac{d^2 u_2^{(j)}}{dt^2} + k_2 (u_2^{(j)} - u_1^{(j)}) = 0,$$

where $u_\gamma^{(j)}$ ($\gamma = 1$ and 2), denoting the displacement of mass $'\gamma'$ in the $j$th cell, are defined with respect to their equilibrium position. In addition, consider a conventional monatomic lattice with a single mass (see figure 1(b)) which is to be employed to represent the mass-in-mass lattice system, then the negative effective mass is absent from the formulation.

The dispersion equations for the mass-in-mass lattice and the monatomic lattice model have been obtained as

$$\cos q L = 1 - \frac{\delta}{2\theta} \frac{(\omega/\omega_0)^2}{(\omega/\omega_0)^2 - 1} \left[ (\omega/\omega_0)^2 - (1 + \theta) \right]$$

and

$$\cos q L = 1 - \frac{\delta}{2\theta} \frac{(1 + \theta) m_{\text{eff}}}{m_{\text{st}}} \left( \frac{\omega}{\omega_0} \right)^2,$$

respectively, where $\theta = m_2/m_1$, $\delta = k_2/k_1$, $\omega_0^2 = k_2/m_2$ and $m_{\text{st}} = m_1 + m_2$ [15]. In order to make the two lattice systems dynamically equivalent, the effective mass of the monatomic lattice is chosen so that the dispersion curves of the two systems match exactly. In other words, the
equality of equations (3) and (4) requires the effective mass $m_{\text{eff}}$ in the monatomic lattice to satisfy the relation

$$\frac{m_{\text{eff}}}{m_{\text{st}}} = 1 + \frac{\theta}{1 + \theta} \left( \frac{\omega}{\omega_0} \right)^2. \tag{5}$$

Similarly, if a classical elastic solid with a unit cross-sectional area is used to represent the mass-in-mass lattice system, then the equation of motion of the one-dimensional elastic solid is

$$E_{\text{eff}} \frac{\partial^2 u}{\partial x^2} = \rho_{\text{eff}} \frac{\partial^2 u}{\partial t^2}, \tag{6}$$

in which the effective Young’s modulus $E_{\text{eff}}$ is determined by taking the static stress–strain relation of a unit cell, i.e. $E_{\text{eff}} = L k_1$, and $\rho_{\text{eff}}$ is the effective mass density. To establish the dynamical equivalence of the continuum representation to the original mass-in-mass lattice system so that they both produce the same dispersion equation, the effective mass density $\rho_{\text{eff}}$ must be given by

$$\frac{\rho_{\text{eff}}}{\rho_{\text{st}}} = \frac{\theta}{\delta (1 + \theta) \left( \frac{\omega}{\omega_0} \right)^2} \left\{ \cos^{-1} \left\{ 1 - \frac{\delta}{2 \theta} \left( \frac{\omega}{\omega_0} \right)^2 \left[ \left( \frac{\omega}{\omega_0} \right)^2 - (1 + \theta) \right] \right\} \right\}^2 \tag{7}$$

with the static mass density $\rho_{\text{st}} = (m_1 + m_2)/L$.

For the following material constants

$$\theta = \frac{m_2}{m_1} = 9, \quad \delta = \frac{k_2}{k_1} = 0.1 \quad \text{and} \quad \omega_0 = \sqrt{k_2/m_2} = 149.07 \text{ rad s}^{-1} (\approx 23.73 \text{ Hz}) \quad \tag{8}$$

the dimensionless effective mass $m_{\text{eff}}/m_{\text{st}}$ and the dimensionless effective mass density $\rho_{\text{eff}}/\rho_{\text{st}}$ associated with these two equivalent models, respectively, are plotted in figure 2(a) as a function of $\omega/\omega_0$. The two curves in the figure are nearly identical for the selected material constants. They, however, may deviate more if different sets of material constants are chosen.

From equations (5) and (7), it is evident that negative mass and mass density would occur near the local resonance frequency $\omega_0$. According to these equations, negative effective mass and mass density occur in the range

$$1 < \left( \frac{\omega}{\omega_0} \right)^2 < 1 + \theta. \tag{9}$$
Figure 2. (a) Dimensionless effective mass $m_{\text{eff}}/m_1$ and the mass density $\rho_{\text{eff}}/\rho_1$ as a function of dimensionless frequency $\omega/\omega_0$; (b) complex dimensionless wavenumber $qL = \alpha + i\beta$ as a function of $\omega/\omega_0$. In this plot the solid line, $\beta$ represents the attenuation factor.

Any frequency in this range, if substituted in the dispersion equation (3), yields a complex dimensionless wavenumber $qL$. This gives a stop band, where no real solution for wavenumber $q$ exists. By denoting the complex solution as $qL = \alpha + i\beta$, the numerical results for $\alpha$ and $\beta$ for the material constants given by equation (8) are plotted in figure 2(b). It is seen that the wave attenuation factor $\beta$ increases as frequency approaches the local resonance frequency $\omega_0$.

From equation (9), it is easy to see that the frequency range in which the effective mass is negative can be tuned easily by varying the ratio $\theta$ of the two masses. One extreme case is when $\theta = m_2/m_1 = 0$, in other words, $m_2 \to 0$ or $m_1 \to \infty$, the gap disappears as the diatomic mass-in-mass lattice model would reduce to a monatomic lattice. The other extreme case with $m_1 \to 0$ or $m_2 \to \infty$ would lead to negative effective mass for nearly all frequencies above the local resonance frequency $\omega_0$. Mathematically, as $m_1 \to 0$, dispersion equation (3) becomes

$$\cos qL = 1 - \frac{\delta}{2} \left[ \frac{\left(\omega/\omega_0\right)^2}{1 - \left(\omega/\omega_0\right)^2} \right].$$

Only the acoustic mode is preserved for this special case. In fact, this reduced model represents a monatomic lattice of mass 2 discretely attached to a continuous spring $k_1$ with spring $k_2$. No harmonic wave can propagate in this model with a frequency above the local resonance frequency.

3. Mechanism of energy transfer and wave attenuation

In the equivalent monatomic lattice of infinite extent, a negative effective mass indicates spatial attenuation of wave amplitude. Since the original lattice system is purely elastic, the energy carried by the wave cannot be dissipated. A similar observation has also been obtained by Lazarov and Jensen [14] when they studied a nonlinear lattice model. Based on the principle of conservation of energy, the energy transferred through an elastic material must be conserved. Consequently, wave attenuation in a perfectly elastic medium implies that the energy must be transferred to and stored somewhere instead of propagating along the lattice system.

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explanation can be found by considering the energy flow and transport velocity in the system. When no energy absorption is present, the velocity of energy transport is equal to the group velocity \( v_g \). From the dispersion equation (3) the group velocity is obtained as

\[
v_g = \frac{d\omega}{dq}.
\] (11)

It is easy to see from the dispersion curves shown in figure 3 that \( v_g \to 0 \) as the wave frequency approaches the stop band for either the acoustical mode or the optical mode. This also implies that the energy distribution in the system becomes stationary when the frequency approaches the stop band.

In order to understand the mechanism that causes the spatial attenuation of wave amplitude within the bandgap, a lattice model with a finite number of mass-in-mass units is considered to facilitate the use of numerical methods. At one end of the finite lattice model, a forcing agent is applied providing continuous energy input to the lattice in the form of propagating waves.

3.1. Single mass-in-mass unit

3.1.1. Energy taken out by the external excitation agent. To understand the work done by an external force on a lattice system, we first consider a single undamped mass-in-mass oscillator shown in figure 4. The oscillator is connected to a rigid wall by a linear spring with spring constant \( k_1 \) and is subjected to a prescribed displacement function \( U(t) \). Without loss of generality, we consider the transient response of the oscillator to the prescribed displacement \( U(t) = u_0 \sin(\omega t) \) \( H(t) \) applied to mass 1. The unit-step function \( H(t) \) is defined as

\[
H(t) = \begin{cases} 
1, & t \geq 0, \\
0, & t < 0,
\end{cases}
\] (12)

The equations of motion for the two-degree-of-freedom oscillator are

\[
m_1 \frac{\partial^2 u_1}{\partial t^2} = F(t) - k_1 u_1(t) - k_2 [u_1(t) - u_2(t)],
\] (13)
where \( u_1 \) and \( u_2 \) are the displacements of \( m_1 \) and \( m_2 \), with respect to their equilibrium positions. \( F(t) \) is the external force associated with the applied displacement excitation and needs to be determined.

The history of the displacement of mass 1 is readily obtained as

\[
\begin{align*}
  u_1(t) &= U(t) = u_0 \sin \omega t H(t). 
\end{align*}
\]

Using the solution given by equation (15) together with the quiescence initial condition for mass 2, we obtain the transient solution for equation (13) as

\[
\begin{align*}
  u_2(t) &= \frac{u_0}{1 - (\omega/\omega_0)^2} \left[ \sin(\omega t) - \frac{\omega}{\omega_0} \sin(\omega_0 t) \right] H(t). 
\end{align*}
\]

Substituting equations (15) and (16) into (13) and using the same definition of \( \theta \) and \( \delta \) yields the force function

\[
F(t) = m_1 u_0 \omega_0^2 \left[ \left( \frac{\theta}{\delta} - \left( \frac{\omega}{\omega_0} \right)^2 \right) \sin(\omega t) + \frac{\theta (\omega/\omega_0)^2}{1 - (\omega/\omega_0)^2} \sin(\omega_0 t) \right] H(t). 
\]

Based on the force and displacement functions above, the instantaneous power input, \( P(t) \), can be calculated as

\[
P(t) = F \frac{du_1}{dt}. 
\]

Or explicitly,

\[
P(t) = m_1 u_0^2 \omega_0^2 \omega \left[ \left( \frac{\theta}{\delta} - \left( \frac{\omega}{\omega_0} \right)^2 \right) \sin(\omega t) \right.
\]

\[
+ \frac{\theta (\omega/\omega_0)^2}{1 - (\omega/\omega_0)^2} \sin(\omega_0 t) \left. \right] \cos(\omega t) H(t). 
\]

We further compute the total work done by the external excitation up to time \( t \) by integrating the power input, i.e.

\[
W_e = \int_0^t P(t) dt, 
\]

which yields

\[
W_e = m_1 u_0^2 \omega_0^2 \left\{ \left( \frac{\theta}{\delta} - \left( \frac{\omega}{\omega_0} \right)^2 \right) \Gamma_1 + \frac{\theta (\omega/\omega_0)^2}{1 - (\omega/\omega_0)^2} \Gamma_2 \right\}. 
\]
Figure 5. The total external work done in time domain by the external excitation for the single mass-in-mass model, (a) $\omega = 1.006 \omega_0$ and (b) $\omega = 1.080 \omega_0$. Material constants $\theta = m_2/m_1 = 9$, $\delta = k_2/k_1 = 0.1$ and $\omega_0 = \sqrt{k_2/m_2} = 149.07 \text{ rad s}^{-1}$ are used.

where

\[
\Gamma_1 = \frac{1}{4} [1 - \cos 2\omega t],
\]

\[
\Gamma_2 = \frac{1}{2} \left[ \frac{1}{1 + (\omega/\omega_0)} (1 - \cos(\omega_0 + \omega)t) + \frac{1}{1 - (\omega/\omega_0)} (1 - \cos(\omega_0 - \omega)t) \right].
\]

Note that the work done is given in terms of two cosine functions with two different arguments and vanishes over any integer number of the period $T_{\text{max}} = 2\pi/\omega_{\text{min}}$, in which

\[
\omega_{\text{min}} = \{ |\omega - \omega_0| , |\omega + \omega_0| , 2\omega \}.
\]

This means that the energy input in the system is later taken out within a cycle. In other words, there is actually no net work done by the external force to the oscillator over a period of $T_{\text{max}}$. Evidently it is expected since the system is lossless. The time histories of work done for two forcing frequencies near the local resonance frequency are shown in figure 5. Apparently negative work is done (i.e. energy taken out of the system) during half of the excitation cycle. The behavior of negative work done here will be further investigated in the finite lattice model later to explain the bandgap effect on wave propagation. In addition, we note that in figure 5(a) the maximum amount of energy stored and the cyclic period are much greater than those in figure 5(b). These phenomena are further examined in the next subsection.

3.1.2. Energy stored temporarily by the internal mass. If the frequency of the external excitation in equation (17) is near the local resonance, it would result in a great amount of energy going in and out of the system in a cyclic manner as shown in figure 5. It is of interest to find the distribution of this stored energy between the two masses. For this purpose, we consider the steady-state solution with

\[
u_1(t) = \hat{u}_1 \sin(\omega t) = u_0 \sin(\omega t),
\]

\[
u_2(t) = \hat{u}_2 \sin(\omega t) = \frac{u_0}{1 - (\omega/\omega_0)^2} \sin(\omega t).
\]
Figure 6. The energy distribution rate of the mass-spring model ($\theta = m_2/m_1$).

From equation (25), it is evident that the displacement of the internal mass (mass 2) is theoretically unbounded when subjected to the local resonance frequency $\omega_0$. Since energy dissipation is absent, there are only two forms of energy in the system, i.e. the potential energy stored in the springs and the kinetic energy of the moving masses. Because the steady-state solutions are obtained based on the assumption that the response and the driving force are in phase ($0^\circ$) or in antiphase ($180^\circ$), we can determine the total energy by either considering only the maximum kinetic energy stored in the masses (potential energy equals zero at this moment), or the maximum potential energy stored in the springs (kinetic energy is zero) in a cycle. Based on the kinetic energies of the two masses, we define distribution ratio,

$$R_{DR} = \frac{KE_2}{KE_1 + KE_2} = \frac{\theta}{\left(1 - (\omega/\omega_0)^2\right)^2 + \theta}$$

(26)

to quantify the fraction of energy stored in mass 2. In equation (26), $KE_1 = m_1\dot{u}_1^2/2$ and $KE_2 = m_2\dot{u}_2^2/2$.

The energy distribution ratio is plotted as a function of forcing frequency in figure 6 for $\theta = 1$. It is shown that the ratio is close to unity if the frequency approaches the local resonance. This implies that the work done by the external excitation is totally transferred to and stored in the internal mass when excited at the local resonance. In what follows, the energy input by the external excitation will, in theory, take forever to return back to the excitation agent.

3.2. Mass-in-mass lattice system

Using the finite element method we perform a numerical study of wave propagation in a lattice with 2500 lattice units as shown schematically in figure 7. The lattice spacing is selected to be sufficiently smaller than the longitudinal wavelength. The selected excitation is, again, the prescribed harmonic displacement.

The numerical solutions are obtained using the commercial finite element code ABAQUS Explicit. The material constants in equation (3) together with the amplitude $u_0 = 0.01$ are used...
Figure 7. The finite lattice model with boundary conditions.

Figure 8. Numerical results of the total external work done by the external excitation for the discrete lattice model (a) $\omega = 1.006\omega_0$ and (b) $\omega = 1.080\omega_0$.

in the computation. The histories of the external work done for two frequencies that are close to the local resonance are shown in figure 8. The trend of the external work done is seen to be similar to that in figure 5. It is noted that when the forcing frequency, $\omega = 1.080\omega_0$ (still in the range of negative effective mass, see figure 8(b)), is not so close to the local resonance, the wave attenuation would take a longer distance to complete. As a result, the total input work does not return to zero in the initial cycles indicating that some amount of energy is leaked into the system in the form of attenuated wave propagation and would take a longer time to stop the wave. In summary, two points should be noted. Firstly, while in the bandgap, the propagating wave is attenuated because the energy is taken out by the driving force in the form of negative work. Secondly, when close to the local resonance, large amount of energy is temporarily stored in the internal mass and later still taken out by the driving force.

4. Dynamic response of metamaterials with negative effective mass density

4.1. Metamaterials with tailorable material constants

A possible metamaterial with negative effective mass density may be constructed in the form of a composite material consisting of periodic mass-in-mass microstructures embedded in a matrix material as shown in figure 9. This metacomposite possesses a local resonance frequency, an essential element to the desired metamaterial.

The most obvious characteristic of metamaterials is the local resonance frequency, $\omega_0 = \sqrt{k_2/m_2}$, that can be easily tailored by choosing a softer spring or by using a larger internal
mass. In other words, the range of bandgap frequencies can be shifted at ease, making this metamaterial a potential material for blocking low-frequency sound and vibration.

4.2. Spatial wave attenuation

Consider free harmonic wave propagation in a mass-in-mass lattice of infinite extent shown in figure 1(a). For a harmonic wave motion, the lattice displacements assume the periodic form

\[ u_j^{(j)} = u_j^{(j+N)}, \]

where \( N \) is an integer representing the periodic number of unit cells, and the wave motion of the \((j+n)\)th unit cell \((n \text{ is an integer such that } n < N)\) can be written as

\[ u_j^{(j+n)} = B_\gamma e^{i(nqL-\omega t)}, \]

in which \( B_\gamma \) represents the wave amplitude, \( q \) the wave number, and \( \omega \) the angular frequency. The substitution of equation (28) in equations (1) and (2) yields two coupled homogeneous equations for \( B_1 \) and \( B_2 \), from which the dispersion equation (the relation between \( \omega \) and \( q \)) and the amplitude ratio \( B_2/B_1 \) are obtained. We have

\[ \frac{|B_2|}{|B_1|} = \frac{1}{1 - (\omega/\omega_0)^2}. \]

(29)

Using the complex form of dimensionless wavenumber \( qL = \alpha + i\beta \), we obtain

\[ |u_j^{(j+n)}| = |B_1 e^{i(nqL-\omega t)}| = |B_1| e^{-n\beta}. \]

(30)

Note that \( \alpha = 0 \) or \( 2\pi \) when \( qL \) is complex (i.e. in the stop band). Thus, the dimensionless displacement envelope for mass 1 is obtained as

\[ \frac{u_j^{(j+n)}}{u_j^{(j)}} = e^{-n\beta}. \]

(31)

Equation (31) is plotted in figure 10 for various frequencies within the bandgap. Obviously, when its frequency approaches the local resonance, the propagating wave exhibits significant attenuation as expected.

In the discussion above it is clear that the mass-in-mass microstructure is capable of stopping a wave from propagating into the metamaterial if the wave frequency is near the
local resonance frequency. For a pulse consisting of a spectrum of frequencies, a number of microstructures with different local resonance frequencies are needed. As an example, we consider a metamaterial with four sections of different microstructures (or mass-in-mass units). Each section consists of 500 unit cells with a specifically tuned local resonance. The four local resonance frequencies are artificially selected close to but lower than 100, 200, 300 and 400, respectively. Thus, the total metamaterial is formed by 2000 mass-in-mass units of four different local resonance frequencies.

The excitation is generated by a prescribed displacement given by

$$U(0, t) = 0.01 (\sin 100t + \sin 200t + \sin 300t + \sin 400t) H(t),$$

applied at one end of the lattice system with the other end fixed. It is easy to see that the generated wave is composed of four excitation displacement histories of four distinct frequencies. It is expected that the wave generated by \(\sin 100t H(t)\) would be attenuated within the first section layer and that generated by \(\sin 200t H(t)\) filtered out within the second section, and so on. A snapshot for wave propagation is shown in figure 11. Apparently, the combined wave almost disappears when propagating into the 4th section of the model. By analyzing the frequency spectrum of the wave in each section as shown in figure 12, we can see that the input disturbances are filtered out section by section and nearly disappear in the last section.

4.3. Amplitude of internal masses

Similar to that discussed in the previous subsection for infinite mass-in-mass lattice, the displacement envelope for mass 2 can be easily obtained. Using the complex form of dimensionless wavenumber \(qL = \alpha + i\beta\) together with equation (29), we derive the displacement of the internal mass at the \((j+n)\)th unit cell:

$$|u_2^{(j+n)}| = |B_2e^{i(nqL-\omega t)}| = \frac{|B_1|}{1 - (\omega/\omega_0)^2} e^{-n\beta}.$$
By simply combining equations (30) and (33), the dimensionless displacement envelope for mass 2 is obtained as

$$ \left| \frac{u_2^{(j+n)}}{u_1^{(j)}} \right| = \left| \frac{1}{1 - (\omega/\omega_0)^2} \right| e^{-jn\beta}. $$

(34)

Equation (34) is plotted in figure 13 for various dimensionless frequencies. It is evident that when the input frequency approaches the local resonance, the amplitudes of the first few internal masses are significantly amplified.

5. Conclusion

The wave attenuation mechanisms in a mass-in-mass metamaterial containing locally resonant microstructures were studied. It was shown that this metamaterial with negative effective mass density demonstrates a new bandgap created by the presence of the local resonance. With the local-resonance type bandgap, the metamaterial behaves dynamically like a phononic bandgap material. The main advantage of the present metamaterial is that it is easy to design the stop band location by tailoring the local resonance frequency. It was also shown that the present metamaterial can give rise to a significant wave attenuation effect near the local resonance frequency. This property can be used to block waves from passing this metamaterial. We have
Figure 13. Dimensionless displacement envelope for mass 2 for various dimensionless frequency $\omega/\omega_0$. In the plot, $u_2^{(j)}$ represents the displacement of mass ‘γ’ in the jth cell.

demonstrated that a metamaterial containing multiple microstructures with a spectrum of local resonance frequencies provides the capability of drastically reducing the magnitude of stress waves generated by dynamic sources.

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