Non-Fermi Liquid Behavior Induced by Resonant Diquark-pair Scattering in Heated Quark Matter

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We show how the quasiparticle picture of quarks changes near but above the critical temperature $T_c$ of the color-superconducting phase transition in the heated quark matter. We demonstrate that a non-Fermi liquid behavior of the matter develops drastically when the diquark coupling constant is increased owing to the coupling of the quark with the pairing soft mode: We clarify that the depression and eventually the appearance of a gap structure in the spectral function as well as the anomalous quark dispersion relation of the quark can be understood in terms of the resonant scattering between the incident quark and a particle near the Fermi surface to make the pairing soft mode.

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I. INTRODUCTION

The recent data in the RHIC experiment suggest that the matter created by the RHIC seems to be a strongly coupled system with possible quasi-bound hadrons contained. Furthermore some lattice calculations, though in the quenched approximation, are consistent with the possible view that heavy-quark bound systems such as $J/\psi$ survive the deconfinement transition at finite temperature $T$. These developments have a possibility to change drastically the simple picture of the QGP phase that the system is composed of almost free quasi-particles, although such nontrivial properties of the QGP phase that it may contain quasi-hadronic excitations had been suggested earlier.

In the present Letter, we shall argue that the dense quark matter at relatively low temperature can have other non-trivial properties, i.e., a non-Fermi liquid ones, if the system is close to the critical temperature $T_c$ of the color-superconducting phase transition on a rather generic ground.

First we notice that the low energy effective models of QCD show that the diquark gap at zero temperature may become as large as $\Delta \sim 100\,\text{MeV}$ at lower densities such as those corresponding to $\mu \sim 400\,\text{MeV}$ with $\mu$ being the baryon-number chemical potential of the quark. Accordingly, the ratio $r_\xi \equiv \Delta/\mu$, which is a measure of the ratio of the inter-particle distance to the pair coherence length, may become as large as 0.2 to 0.3. This value is much larger than those in the metal superconductors, i.e., $r_\xi \simeq 0.001$, which account for the validity of the mean-field approximation à la BCS theory for the electric superconductors. Thus one sees that large fluctuations of the diquark-pair are expected, which may invalidate the mean-field approximation, in the color superconductors.

In fact, our previous work with the fixed pairing coupling showed that there exists the precursory soft mode composed of the diquark-pair field even well above $T_c$. One of the remarkable points there was that the coupling of the quark with the soft mode not only modifies the quark dispersion relation but also causes a depression of the quark spectral function around the Fermi energy; the depression was so large that a “pseudogap” is formed in the density of states (DOS) of the quark even with the small diquark coupling that was employed.

The above finding is interesting because they suggest that the properties of quarks at $T$ above but close to $T_c$ are altered from the typical Fermi-liquid to the non-Fermi liquid ones owing to the coupling to the precursory fluctuations of the diquark-pair field. The mechanism to cause such a non-Fermi liquid behavior, however, might not have been clarified enough in the previous work. The purpose of the present Letter is to elucidate the mechanism to realize such a non-Fermi liquid behavior and argue that it is a generic feature of the heated quark matter in the vicinity of $T_c$ of the color superconductivity.

For this purpose, we examine what happens in the quark properties in detail if the pairing fluctuations become stronger by increasing the diquark coupling constant $G_C$; we remark that the limiting case can often clarify physics. Incidentally it may be noticed that $G_C$ indeed could have other values than that in the previous work without any contradiction with other principles and phenomenology. We calculate the dispersion relation and the spectral function of the quark in the T-matrix approximation.

We shall demonstrate the following for the first time: (1) The quark dispersion relation is so largely modified with the increasing $G_C$ that it becomes seemingly multiple-valued around the Fermi surface near but above $T_c$ in the strong coupling regime. (2) The quark spectral function gets to have a developed gap-like structure rather than a depression near but above $T_c$ for the larger $G_C$. We shall clarify that the above non-Fermi liquid behaviors can be nicely understood in terms of the resonant scattering between the quark and a particle near the Fermi surface to make the pairing soft mode.
II. BRIEF SUMMARY OF FORMULATION

To describe a system at relatively low $T$ and $\rho$, it is appropriate to adopt a low-energy effective theory of QCD [17,18]. Here, we consider the two-flavor quark matter in the chiral limit with the four-Fermi quark-quark interaction [13]

$$\mathcal{L}_C = G_C \sum_A \left( \bar{\psi} i \gamma_5 \tau_2 \lambda_A \psi \right) \left( \psi^T C i \gamma_5 \tau_2 \lambda_A \psi \right),$$

where $C = i \gamma_2 \gamma_0$ is the charge conjugation operator, and $\tau_2$ and $\lambda_A$ denote the antisymmetric flavor $SU(2)_f$ and color $SU(3)_c$ matrices, respectively. The quark chemical potentials $\mu$ of each flavor and color are taken to be the same [13]. We introduce the three momentum cutoff $\Lambda = 650 \text{MeV}$ [18]. As for the diquark coupling constant $G_C$, we vary it in the range $3.11 \text{GeV}^{-2} < G_C < 4.35 \text{GeV}^{-2}$, which is the similar range used in the references [8, 9, 17, 18].

In order to see the quasiparticle picture of the quarks, we examine the spectral function $\rho_0$ and the dispersion relation of the quark $\omega = \omega_{\pi}(k)$. The following formulation is essentially a recapitulation of that given in [12].

The spectral function $\rho_0$ is given by the imaginary part of the retarded Green function $G^R(k, \omega)$

$$\rho_0(k, \omega) = -\frac{1}{i\pi} \text{Tr}[\gamma^0 \text{Im} G^R(k, \omega)].$$

The Green function $G^R(k, \omega)$ is decomposed into the quark and anti-quark parts;

$$G^R(k, \omega) = (G^R_-(k, \omega) \Lambda_-(k) + G^R_+(k, \omega) \Lambda_+(k)) \gamma^0
\frac{\Lambda_-(k) \gamma^0}{R_-(k, \omega) + i\eta} + \frac{\Lambda_+(k) \gamma^0}{R_+(k, \omega) + i\eta} \tag{3}$$

with the projection operators $\Lambda_\pm(k) = (1 \pm \gamma^0 \gamma \cdot \mathbf{k})/2$. The dispersion relations of the quarks and anti-quarks $\omega = \omega_{\pi}(k)$ are defined by the equations

$$\text{Re} R_{\pi}(k, \omega) = 0,$$

respectively. A remark is in order here: Because $\omega_{\pi}(k)$ is merely the solution of the real part but not the whole part of the inverse of the Green function $R_{\pi}(k, \omega_{\pi})$, $\omega_{\pi}(k)$ may not correspond to the peak position of the spectral function and hence also may not represent physical excitations when the imaginary part of the Green function is large.

Our point in the calculation of the quark Green function $G^R(k, \omega)$ lies in incorporating the diquark-pair fluctuations in the quark self-energy in the T-matrix approximation [13,13,14] as is diagrammatically shown in Fig. 1 all the possible anomalous behaviors of the results will originate from the fact that the pair fluctuations actually form the soft mode of the color-superconducting phase transition.

![Feynman diagrams representing the quark Green function in the T-matrix approximation. The thin lines represent the free propagator $G_0$, while the bold ones the full propagator $G$.](image)

In the Matsubara formalism, the self-energy of quarks in the imaginary time $\Sigma(k, \omega_n)$ is given by

$$\tilde{\Sigma}(p, \omega_n) = -G_C \frac{1}{1 + G_C \tilde{Q}(k, \nu_n)},$$

with $\omega_n = (2n + 1)\pi T$ being the Matsubara frequency for fermions and $\tilde{Q}(k, \omega_n) = [(i\omega_n + \mu)\gamma^0 - k \cdot \gamma]^{-1}$ being the free quark propagator. The T-matrix for the quark-quark scattering $\tilde{\Xi}(k, \nu_n)$ reads

$$\tilde{\Xi}(k, \nu_n) = -G_C \frac{1}{1 + G_C \tilde{Q}(k, \nu_n)},$$

with the lowest order polarization function $\tilde{Q}(k, \nu_n)$ [13] and $\nu_n = 2n\pi T$. The T-matrix in the real time $\tilde{\Xi}(k, \omega) \equiv \tilde{\Xi}(k, \nu_n)|_{\nu_n \rightarrow \omega + i\eta}$ has all the information about properties of the pair fluctuations. In particular, the dynamical structure factor

$$S(k, \omega) = -\frac{\text{Im} \tilde{\Xi}(k, \omega)}{2\pi G_C^2(1 - e^{-\omega/T})} \tag{7}$$

represents the excitation probability of the pair field for each $\omega$ and $k$ at finite $T$. It has a prominent peak at the origin near $T_c$ according to the softening of the pair fluctuations.

The analytic continuation of $\tilde{\Sigma}(k, \omega_n)$ to the real axis from the upper-half complex-energy plane gives the self-energy in the real time $\Sigma^R(k, \omega) = \Sigma(k, \omega_n)|_{\omega_n \sim \omega + i\eta}$. Using the projection operators $\Lambda_\pm(k)$, the self-energy is decomposed into the quark and anti-quark parts

$$\Sigma_\pm(k, \omega) = \text{Tr}[\Sigma^R(k, \omega) \Lambda_\pm(k) \gamma^0]/2,$$

and $R_{\pi}(k, \omega)$ in Eq. 3 is now found to be $R_{\pi}(k, \omega) = \omega + \mu + k - \Sigma_\pi(k, \omega)$. Notice that $\rho_0(k, \omega)$ around the Fermi energy is given almost solely by the quark part $G^R_-$, and hence the corresponding part of the self-energy $\Sigma_-(-k, \omega)$ is responsible for the quasiparticle properties of the quark near the Fermi surface. For later convenience, we rewrite the explicit form of $\text{Im} \Sigma_-(k, \omega)$ given in [13] slightly using $S(k, \omega)$:

$$\text{Im} \Sigma_-(k, \omega) = \frac{\pi G_C^2}{2} \int \frac{d^3q}{(2\pi)^3} S(k + q, \omega + E_q - \mu) \left\{ 1 - \frac{k \cdot q}{E_q} \right\}$$

FIG. 1: Feynman diagrams representing the quark Green function in the T-matrix approximation. The thin lines represent the free propagator $G_0$, while the bold ones the full propagator $G$. 

The behavior of the quark self-energy \( \Sigma^{-}(k, \omega) \) is important for the enhancement of \( \Sigma^{-}_{\omega}(k, \omega) \) because of the factor \( G_{C}^{2} \) in Eq. (8). We notice that the behavior of \( \Re \Sigma_{-}(k, \omega) \) can be understood by the growth of \( |\Im \Sigma_{-}(k, \omega)| \) and the Kramers-Kronig relation

\[
\Re \Sigma_{-}(k, \omega) = -\frac{1}{\pi} P \int d\omega' \Im \Sigma_{-}(k, \omega')/(\omega - \omega').
\]

The peculiar behavior of \( \omega_{-}(k) \) can be understood as follows. First, recall that \( \omega_{-}(k) \) at \( k = k_{F} \) is the solution of \( \Re R_{-}(k_{F}, \omega) = \omega - \Re \Sigma_{-}(k_{F}, \omega) = 0 \). Accordingly, the solutions \( \omega_{-}(k_{F}) \) are given graphically by the crossing points of \( \Re \Sigma_{-}(k_{F}, \omega) \) and \( \omega \) denoted by the straight dash-dotted line in the lower panel of Fig. 3. One sees how there eventually appear the three solutions of \( \omega_{-}(k_{F}) \) for large \( G_{C} \), as mentioned before. A remark is in order here: The group velocity as defined by \( v_{g} = d\omega_{-}(k)/dk \) is seemingly larger than the speed of light near \( \omega_{-}(k) = 0 \). However, it corresponds to the peak position of the imaginary part of the self-energy \( |\Im \Sigma_{-}(k, \omega)| \), which means that the excitations around the origin have a quite large damping rate, and hence there does not appear a peak in the spectral function. The solution near \( \omega = 0 \) thus does not represent a physical excitation spectrum of the quasiparticle.

Now let us discuss the mechanism to realize the gap-like structure in the quark spectrum in Fig. 2. Due to the softening of the diquark-pair fluctuations near \( T_{c} \), a particle near the Fermi energy is scattered by the soft mode and creates a hole, while a hole can create a particle by absorbing the soft mode as shown in Fig. 3. We remark that the hole must be also created near the Fermi surface because of the energy conservation. More intuitively, the incident particle and a particle near the Fermi surface, which may be within the Fermi sea or thermally excited one, make a resonant scattering to form the pairing soft mode and vice versa. This resonating processes which are only effective around the Fermi surface induce a virtual mixing between the particles and holes, i.e., a (virtual) Bogoliubov transformation! In other words, because the particle and hole energies \( \omega = k - \mu \) and \( \omega = \mu - k \) cross at the Fermi energy as shown in Fig. 5, the mixing between the particles and holes leads to the level repulsion of the energy spectrum of the particle and hole, which makes the gap-like structure as shown in the right panels of Fig. 4. This mechanism which induces the gap-like structure owing to the resonant scattering to make the softening pairing mode is also known in the condensed matter physics.

We shall next show how the behavior of the quark spectrum can be nicely reproduced by quantifying the notion of the resonant scattering. As was seen above, the softening of the pair fluctuations near \( T_{c} \) is responsible for the resonant scattering to become effective. Since the softening is described as the enhancement of \( S(k, \omega) \) around the origin in the \( k-\omega \) plane, let us approximate the dy-
The meaning of the choice of the pre-factor containing a $\Delta$ in front of the delta functions will be clear later; $\Delta$ is to be related with the imaginary part of the $T$ matrix $\Xi$ and the boson distribution function [15, 16]. Needless to say, the dynamical structure factor above $T_c$ in reality never vanishes in the whole $\omega-k$ plane nor takes the form of Eq. (10) even at $T = T_c$; the fluctuations with finite $\omega$ and $k$ make a width of the quasiparticles. However,
that the peculiar behavior of the quark spectrum can be
this simple treatment will be found to clearly illustrate
that the precursory pairing soft mode modifies the quasi-
particle picture near the Fermi energy to make the pairing soft mode.

Although the present work is based on a model calculation, the mechanism proposed here for realizing a non-
Fermi liquid behavior of fermion systems can be model-
indeed. This is because the essential ingredient
here is the scattering of a particle near Fermi surface to
make the soft mode which is inherent to any second-order
or weak first-order phase transition. It would be thus
intriguing to explore the quasiparticle picture near the
critical temperature of other phase transitions of QCD
crystal, including the chiral phase transition at finite
temperature [20].

We should notice that there is another mechanism
which causes a non-Fermi liquid behavior of ungapped
quark matter: A resummed perturbation theory shows
[21] that the coupling of the quark with unscreened long-
range gauge (gluonic) fields gives rise to vanishing of both
the residues of the quasiparticle excitations and the group
velocity \( \omega = dE/dp \) at the Fermi surface at small
temperature without recourse to any phase transition. Notice
that the vanishing group velocity implies the infinite
density of states of quasiparticles, which is highly in con-
trast with our result, i.e., the pseudogap formation as a
precursory phenomenon of the CSC at finite \( T \). Although
the perturbation theory adopted in [21] is only applicable
for extremely high-density quark matter, it would be inter-
esting to see how the non-Fermi liquid behaviors due
to unscreened gluonic fields survive and compete with
those owing to the soft mode of CSC at moderate den-
sity and temperature.

In this work, we have employed the non-selfconsistent
T-matrix approximation which is essentially a linear ap-
proximation for the fluctuations. Nevertheless the results
obtained in this approximation can be close to reality so
long as \( \varepsilon \equiv (T - T_c)/T_c \) is not so small as noted in [12, 13]; see [15]. In the present work, the strong fluctuations of
the pair-field were induced with the varied \( G_C \) for fixed
\( \varepsilon = 0.01 \). Similar results may be obtained for smaller \( \varepsilon \)
but with a fixed \( G_C \) since the fluctuations diverge as \( \varepsilon \)
go to zero.

The situation, however, might not become so simple
when the system may enter the Ginzburg region where
the nonlinear fluctuation effects play an essential role and
somewhat moderate behavior may be realized for the
quark properties. A renormalization-group treatment,
for instance, would be necessary to incorporate the non-
linear effects of the fluctuations, which is beyond the
scope of the present work and left for future investiga-
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[22] The dynamical structure factor $S(k, \omega)$ around the origin for $\varepsilon = 0.01$ is not very sensitive to $G_C$ in the range of $G_C$ employed in this work, while the fluctuations for larger momentum become more significant as $G_C$ is increased.