Stability analysis of wide area power system under the influence of interval time-varying delay

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ABSTRACT

In view of the problem that time delay always existing in wide area power system can cause severe effects on the operation performance of the whole system, this paper studies the stability of the wide area power system with interval time-varying delays. Firstly, the model of wide area power system with interval time-varying delay is established, based on that, a new augmented vector and new Lyapunov-Krasovskii functional (LKF) are constructed. Then, the delay-partitioning approach, Wirtinger integral inequality, free-matrix-based inequality and convex combination approach are used to estimate the derivative of the functional, and as a result, a less conservative stability criterion for the delayed power system is obtained. Finally, numerical simulations of the typical second-order system, the single machine system and two-area four-generator power system are given to illustrate that the proposed method in this paper expands the stability margin of the system effectively.

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1. Introduction

With the expansion of the scale of modern power system and the interconnection of power grids, the dynamic process of the power system becomes more and more complex. The traditional local control method was unable to meet the requirements of security and stability in the current wide area power system. In recent years, the wide area measurement system (WAMS) based on phasor measurement unit (PMU) has rapidly developed and widely used in power system, which promoted the development of the wide area control in power system (Hadidi & Jeyasurya, 2013; Manousakis, Korres, & Georgilakis, 2012; Yan, Govindarasu, Liu, Ming, & Vaidya, 2015). In the wide area environment, time delay exists in the process of signal transmission and processing, especially in long distance transmission. It has been shown that even small time delay can cause serious negative effects to the stable operation of the power system (Zhang, Zhan, Wei, Shi, & Xie, 2016). So, it is of great practical significance to study the stability of power system under the influence of time delay (Hailati & Wang, 2014; Yang & Sun, 2014).

There are two main methods for analyzing the stability of power systems with time delay: frequency domain method and time domain method. The frequency domain method is mainly based on the transformation of the characteristic equation and the distribution of eigenvalues to determine the stability of the system (Hua, Jian, & Liu, 2013; Li, 2015), the necessary and sufficient conditions for the stability of the system can be obtained by this method, but the calculation process is so complicated that it is difficult to be applied when the operation state of power system jumps or contains time-varying parameters. Compared with the frequency domain method, the time domain method has obvious advantages (Ma, Li, Li, Zhu, & Wang, 2015; Liu, Ding, Wang, & Zhou, 2011), and it is the main method for the stability analysis of time delay power systems. The LKF method based on the Lyapunov stability theory is used most widely in time domain method. This method gives the sufficient conditions for the stability of the system, which leads to a certain conservatism. Therefore, how to reduce the conservatism to expand the stable operation area of the system becomes a hot issue in recent years, and different research methods have been proposed by many researchers. In aspect of LKF construction, by constructing one-integral LKF and double integral LKF (Chen & Cai, 2009; Sun et al., 2015), augmented LKF (Li, Sun, & Wei, 2017), the stability and controller design of power systems with constant time delay and time-varying delay are studied. In aspect of estimating functional derivatives, many new methods are proposed such as free matrices method (Jia, An, & Yu, 2010), the Jesen integral inequality (Dong, Jia, & Jiang, 2015), the free-weighting matrix approach (Huang, Guo, & Sun, 2014), the generalized eigenvalue
method (Ma, Li, Gao, & Wang, 2014), the convex combination approach (Qian & Gao, 2015), the Wirtinger integral inequality method (Qian, Jiang, & Che, 2016), to study the stability analysis and control of the wide area time delay power system. Although the above literatures reduce the conservatism of stability criterion for the time delay power system, they still have some shortcomings, such as the simple LKF, the limitations of the analytic method in reducing conservatism and so on, all of which cause the conservativeness of the stability criteria.

Motivated by the discussion mentioned above, the main purpose of this paper is to study the stability of the wide area power system with interval time-varying delays. By establishing the model of wide area power system with interval time-varying delay, constructing new augmented vector and a new LKF with triple integral terms, dividing the delay interval into two parts, using Wirtinger integral inequality, free-matrix-based inequality and convex combination approach to estimate the derivative of the functional, the less conservative stability conditions are proposed. The numerical examples are also given to show that the proposed method expands the stable operating area of the system effectively.

2. Model of power system with time delay

In this section, based on the traditional power system model, by introducing time-varying delay to describe, the model of power system with time-varying delay is established.

To the power system, the dynamic model of generator is described as:

\[
\begin{align*}
\frac{d\omega}{dt} &= \omega - \omega_0 \\
\frac{d\Delta P_e}{dt} &= (P_m - P_e)\omega_0 - D(\omega - \omega_0) \\
\frac{dE_q}{dt} &= E_t - E' - (X_d - X'd)Id \\
U_d &= X_qI_q, U_q = E' - X'dId
\end{align*}
\]

where

\[
\begin{align*}
I_d &= \frac{E' - V\cos\delta}{X_d + X_e} \\
V_g &= \sqrt{\left(\frac{X_eE' + X'dV\cos\delta}{X_q + X_e}\right)^2 + \left(\frac{X_eE' + X'dV\cos\delta}{X_q + X_e}\right)^2}
\end{align*}
\]

The meanings of parameters in the differential equations are given in (Li, 2015). In order to ensure the reliability of the power system, AVR excitation control method is used. Considering the time delay existing in the system, the dynamic equations of the excitation system can be expressed as follows:

\[
T \frac{dE}{dt} = -K[V(t - \tau(t)) - V] - (E - E) \tag{2}
\]

According to (1) and (2), the model of time delay power system can be expressed as follows:

\[
\begin{align*}
\frac{d\omega}{dt} &= \omega - \omega_0 \\
\frac{d\Delta P_e}{dt} &= (P_m - P_e)\omega_0 - D(\omega - \omega_0) \\
\frac{dE_q}{dt} &= E_t - E' - (X_d - X'd)Id \\
\frac{dE_i}{dt} &= -\frac{k}{\tau_s}[V_g(t - \tau(t)) - V_{g0} - \frac{1}{\tau_s}(E_t - E_{f0})]
\end{align*}
\]

Linearizing the equation (3) at the equilibrium point can obtain:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + A_1x(t - h(t)) & t \geq 0 \\
x(t) &= \phi(t) & t \in [-h, 0]
\end{align*}
\]

where \(x(t) \in \mathbb{R}^n\) is the state vector of power system, the initial condition \(\phi(t)\) is a continuously differentiable vector-valued function in \([-h, 0]\), \(A, A_1 \in \mathbb{R}^{n \times n}\) are known constant matrices, \(h(t)\) is the time-varying delay and satisfying \(0 \leq h(t) \leq h, \mu_1 \leq \dot{h}(t) \leq \mu_2 < 1\), where \(h, \mu_1, \mu_2\) are constants.

Considering the disturbance in the system, the system (4) should be expressed as:

\[
\begin{align*}
\dot{x}(t) &= (A + \Delta A)x(t) \\
&\quad + (A_1 + \Delta A_1)x(t - h(t)) \\
x(t) &= \phi(t) & t \in [-h, 0]
\end{align*}
\]

where \(\Delta A, \Delta A_1\) are disturbance terms satisfying \([\Delta A, \Delta A_1] = HF_0[E_a, E_b], H, E_a, E_b\) are known constant matrices, \(F_0\) is free matrix satisfying \(F_0^T F_0 \leq I\).

In order to obtain the main result, the following lemmas are needed.

**Lemma 1:** (Seuret & Gouaisbaut, 2013) For any constant symmetric matrices \(M = M^T > 0, real\ scalars\ a, b\) which satisfying \(a > b, and\ vector\-valued\ function\ \psi : [b \ a] \rightarrow \mathbb{R}^n,\) then the following inequality holds

\[
\int_b^a \psi(s)^T M \psi(s) ds \geq \frac{1}{a - b} \left(\int_b^a \psi(s) ds\right)^T M \left(\int_b^a \psi(s) ds\right) + \frac{3}{a - b} \Phi^T M \Phi,
\]

where \(\Phi = \psi(a) + \psi(b) - \frac{2}{a - b} \int_b^a \psi(s) ds\).

**Lemma 2:** (Park, Kwon, Park, Lee, & Cha, 2015) For a given symmetric matrix \(M = M^T > 0, scalars\ a, b\) which satisfying
\(a > b,\) and vector-valued function \(\varphi : [b, a] \rightarrow \mathbb{R}^n,\) then the following inequality holds
\[
\frac{(a - b)^2}{2} \int_b^a \int_s^a \varphi^T(u)M\varphi(u)du \leq \left( \int_b^a \int_s^a \varphi(u)du \right)^T M \left( \int_b^a \int_s^a \varphi^T(u)du \right) + 2\Phi_d^T M \Phi_d,
\]
where \(\Phi_d = -\int_b^a \int_s^a \varphi(u)du + \frac{3}{a-b} \int_b^a \int_s^a \int_u^a \varphi(v)dv du du.

Lemma 3: (Park, Ko, & Jeong, 2011). For given positive integers \(m, n,\) variable \(\alpha \in (0, 1),\) for given matrices \(Z \in \mathbb{R}^{m \times n} > 0,\) \(K_1 \in \mathbb{R}^{n \times m},\) \(K_2 \in \mathbb{R}^{m \times n},\) the function
\[
\Upsilon = \frac{1}{\alpha} \xi^T K_1^T Z K_1 \xi + \frac{1}{1 - \alpha} \xi^T K_2^T Z K_2 \xi.
\]
if there exists a matrix \(Y \in \mathbb{R}^{m \times n}\) and satisfying
\[
\begin{bmatrix} Z & Y \\ \ast & Z \end{bmatrix} > 0,
\]
then the following inequality holds
\[
\min_{\alpha \in (0, 1)} \Upsilon \geq \begin{bmatrix} K_1 \xi \\ K_2 \xi \end{bmatrix}^T \begin{bmatrix} Z & Y \\ \ast & Z \end{bmatrix} \begin{bmatrix} K_1 \xi \\ K_2 \xi \end{bmatrix}.
\]

Lemma 4: (Zeng, He, & Wu, 2015). Let \(x(s)\) be a differentiable function \(x(s) \mid s \in [a, b]\), for symmetric matrices \(F \in \mathbb{R}^{n \times n}\) and \(B, L \in \mathbb{R}^{n \times 3n}\), any matrices \(G \in \mathbb{R}^{3n \times 3n}\) and \(C, N \in \mathbb{R}^{3n \times n}\) satisfying \(\begin{bmatrix} B & G \\ L & C \\ \ast & \ast \end{bmatrix} \geq 0,\) then the following inequality holds:
\[
- \int_a^b \dot{x}(s)F\dot{x}(s)ds \
\leq \theta^T [(b-a)B + \frac{b-a}{3} L + He(N\Psi_1 + C\Psi_2)] \theta
\]
where \(\Psi_1 = [l, -l, 0], \Psi_2 = [-l, l, 2l], \theta = \begin{bmatrix} x^T(b), x^T(a) \end{bmatrix}^T.\)

3. Stability analysis of time varying delay power system

In this section, based on the established model of time varying delay power system, by constructing new augmented terms and new Lyapunov-Krasoskii functional, applying less conservative methods to dealing with the derivatives of the functional, the developed stability criteria of the wide area power system with time varying delays is obtained.

Firstly, let:
\[
\tilde{x}^T(t) = \begin{bmatrix} x^T(t), \int_{t-h(t)}^t x^T(s)ds, \int_{t-h(t)}^t \int_{t-h(s)}^t x^T(u)du \\ \int_{t-h(t)}^t \int_{t-h(s)}^t \int_{t-h(u)}^u x^T(v)dv du \\ \int_{t-h(t)}^t \int_{t-h(s)}^t \int_{t-h(u)}^u \int_{t-h(v)}^v x^T(w)dw dw \\
\end{bmatrix}
\]

and the proposed LKF contains triple integral term \(V_5(t),\) so, more information of the time delay is employed, which play a key role in the further reduction of conservation.
Taking the time derivatives of $V(t)$ along the trajectory of system (4) yield:

$$
\dot{V}_1(t) = 2\tilde{Z}(t)\tilde{p}(t) 
$$

(11)

$$
\dot{V}_2(t) = \dot{h}(t)\eta_1^T(t)W\eta_1(t) + 2h(t)\eta_1^T(t)W\eta_1(t)
- h(t)\eta_2^T(t)M\eta_2(t) + 2(h - h(t))\eta_2^T(t)M\eta_2(t)
$$

(12)

$$
\dot{V}_3(t) = \dot{x}^T(t)Qx(t) - (1 - \dot{h}(t))x^T(t - h(t))Qx(t - h(t))
+ (1 - \dot{h}(t))x^T(t - h(t))Rx(t - h(t))
- x^T(t - h)Rx(t - h)
$$

(13)

$$
\dot{V}_4(t) = h^2\dot{x}^T(t)Zx(t) - h\int_{t-h}^{t} Zx(s)Zx(s)ds 
$$

(14)

Deviding the integral interval $[t - h, t]$ into subintervals, compared with the existing works (Ma et al., 2014; Qian et al., 2016; Sun et al., 2015), more information of the time delay is employed, which efficiently reduces the conservatism of the proposed approach. Then, Wirtinger integral inequality is applied to tackling with the integral terms on two subintervals, compared with the methods in (Park, Kwon, Park, & Lee, 2011; Qian & Gao, 2015; Seuret & Gouaisbaut, 2013) which use Jensen integral inequalities, this method is beneficial to expand the stable operating area of the system.

$$
\dot{V}_5(t) = \frac{h^4}{4}\dot{x}^T(t)F\dot{x}(t) - \frac{h^2}{2}\int_{t-h}^{t} \dot{x}^T(u)Fx(u)du 
$$

(16)

Deviding the integral into two parts in $\dot{V}_5$, we can get:

$$
- \frac{h^2}{2}\int_{t-h}^{t} \dot{x}^T(u)Fx(u)du 
\leq - \frac{h^2}{2}\left[\int_{t-h}^{t} \dot{x}^T(u)Fx(u)du + \int_{t-h}^{t} \dot{x}^T(u)Fx(u)du\right]
$$

Using the lemma 2 to deal with $\int_{t-h}^{t} \dot{x}^T(u)Fx(u)du$ and $\int_{t-h}^{t} \dot{x}^T(u)Fx(u)du$, we can obtain:

$$
\int_{t-h}^{t} \int_{t-h}^{t} \dot{x}^T(u)Fx(u)du 
$$

(17)

where $\tilde{Z} = \text{diag}(Z, Z)$, $\alpha = h(t)/h$, $\Gamma_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $\Gamma_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $\Gamma_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $\Gamma_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$.
Using the lemma 4 to deal with \( \int_{t-h(t)}^{t} x^T(s)F x(s) \, ds \) can be obtained:

\[
\int_{t-h(t)}^{t} x^T(s)F x(s) \, ds \leq \theta^T \left[ h(t) \left( B + \frac{L}{3} \right) + He \left( N \left[ I \quad -I \quad 0 \right] \right) + C \left[ -I \quad -I \quad 2I \right] \right] \theta \tag{20}
\]

where \( \theta = \left[ x^T(t), x^T(t-h(t)), \frac{1}{h(t)} \int_{t-h(t)}^{t} x^T(s) \, ds \right]^T \).

**Remark 3:** Similar to the method dealing with \( \dot{V}_4 \), the delay interval of the functional \( \dot{V}_5 \) is also decomposed into two subintervals. To the double integral terms, double Wirtinger integral inequality and Free-matrix-based integral equality are used to reduce the estimating error, so that the obtained result is much less conservative than those in (Chaibi & Tissir, 2012; Seuret & Gouaisbaut, 2013; Zeng et al., 2015).

Combining (11)–(20), it can be obtained:

\[
\dot{V}(t) = \sum_{i=1}^{5} \dot{V}_i(t) = \xi^T \Pi \xi \tag{21}
\]

where

\[
\Pi = [E_{ij}]_{i,j=1,2,\ldots,7}
\]

\[
\xi^T = \left[ x^T(t), x^T(t-h(t)), x^T(t-h) \frac{1}{h(t)} \int_{t-h(t)}^{t} x^T(s) \, ds, \frac{1}{h(t)} \int_{t-h(t)}^{t} x^T(s) \, ds, \frac{1}{h(t)} \int_{t-h(t)}^{t} x^T(s) \, ds \right]
\]

By applying Lyapunov Stability Theorem, when \( \dot{V} = \xi^T \Pi \xi < 0 \), system (4) is asymptotically stable.

Based on the above proof, the main result for the asymptotic stability of the system (4) is given as follows:

**Theorem 1:** For the given scalar \( h, \mu_1, \mu_2 \), system (4) is asymptotically stable if there exist definite symmetric matrices \( P \in R^{5n \times 5n}, W \in R^{2n \times 2n}, M \in R^{2n \times 2n}, Q \in R^{n \times n}, R \in R^{n \times n}, Z \in R^{n \times n}, F \in R^{n \times n}, G \in R^{n \times n}, N \in R^{n \times n}, \) and matrices \( B \in R^{3n \times 3n}, L \in R^{3n \times 3n}, C \in R^{3n \times n}, \) satisfying that:

\[
\begin{bmatrix} B & G & N \\ L & C & * \\ * & F & * \end{bmatrix} \geq 0, \quad \Pi = [E_{ij}] < 0, \quad (i,j=1,2,\ldots,7) \tag{22}
\]

where

\[
E_{11} = P_{11}A + A^T P_{11} + P_{12} + P_{13}^T + P_{14} + P_{14}^T + \hat{h}(t)W_{11} + h(t)W_{11}A + h(t)A^T W_{11} + W_{12} + W_{12} - \hat{h}(t)M_{11} + (h - h(t))M_{11}A + (h - h(t))A^T M_{11} + 0.5 h^2 A^T ZA + \frac{h^2}{4} A^T FA - 4Z - \frac{3}{2} h^2(h - h(t)) \times \hat{h}(t) \left( B_{11} + \frac{1}{3} L_{11} \right) + N_{11} + N_{11}^T - C_{11} - C_{11}^T
\]

\[
E_{12} = P_{11}A_1 - (1 - \hat{h}(t))P_{12} + (1 - \hat{h}(t))P_{13} + (1 - h(t))P_{15} + h(t)W_{11}A_1 - (1 - h(t))W_{12} + (h - h(t))M_{11}A_1 + (h - h(t))M_{12} + h^2 A^T ZA_1 + h^2 A^T FA_1 - 2Z - Y_{11} - Y_{21} - Y_{12} - Y_{22} + \frac{h^2}{2}(h - h(t)) \left[ \hat{h}(t) \left( B_{12} + \frac{1}{3} L_{12} \right) - N_{11} - C_{11} + N_{21}^T - C_{21}^T \right] - E_{13} = -P_{13} - M_{12} + Y_{11} + Y_{21} - Y_{12} - Y_{22}
\]

\[
E_{14} = (\hat{h}(t) - 1)P_{14} + h(t)P_{21}A + h(t)P_{22} + h(t)P_{24} + h(t)W_{21}A + W_{22} + 6Z + h^2 \left( h - h(t) \right) \times \hat{h}(t) \left( B_{13} + \frac{1}{3} L_{13} \right) - 2C_{11} + N_{31}^T - C_{31}^T
\]

\[
E_{15} = -P_{15} + (h - h(t))(P_{31}A + P_{32} + P_{34} + M_{21}A) + 2Y_{12} + 2Y_{22}
\]

\[
E_{16} = -\hat{h}(t)P_{14} + h(t)(P_{41}A + P_{42} + P_{44}) + 3h^2 F, \quad E_{17} = h(t)P_{15} + (h - h(t))(P_{51}A + P_{52} + P_{54})
\]

\[
E_{22} = (h(t) - 1)Q + (1 - \hat{h}(t))R + h^2 A^T ZA_1 + h^2 A^T FA_1 - 8Z + 2Y_{11} + 2Y_{12} - 2Y_{21} - 2Y_{22} - \frac{3}{2} h^2 F + \frac{h^2}{2}(h - h(t)) \left[ \hat{h}(t) \left( B_{22} + \frac{1}{3} L_{22} \right) - N_{21} - N_{21}^T - C_{21} - C_{21}^T \right] - E_{23} = -Y_{11} + Y_{21} + Y_{12} - Y_{22} - 2Z
\]

\[
E_{24} = h(t)P_{21}A_1 - h(t)(1 - \hat{h}(t))(P_{22} + P_{23} + P_{25}) + h(t)W_{21}A_1 - (1 - \hat{h}(t))W_{22} + 6Z + 2Y_{21}
\]
+ 2Y_{22} + \frac{h^2}{2}(h - h(t)) \left[ h(t) \left( B_{23} + \frac{1}{3} L_{23} \right) 
+ 2C_{21} - N_{31}^T - C_{31}^T \right]

E_{25} = (h - h(t)) (P_{31} A_1 + M_{21} A_1)
\quad - (h - h(t))(1 - h(t)) (P_{32} + P_{33} + P_{35})
\quad + (1 - h(t)) M_{22} - 2Y_{12} + 2Y_{22} + 6Z
E_{26} = h(t) P_{31} A_1 - h(t)(1 - h(t))(P_{42} + P_{43} + P_{45})
E_{27} = (h - h(t)) P_{31} A_1 + (h - h(t))((1 - h(t)))
\quad \times (P_{52} + P_{53} + P_{55}) + 3h_2^2 F
E_{33} = - R - 4Z, E_{34} = - h(t) P_{23} - 2Y_{21} + 2Y_{22},
E_{35} = (h(t) - h) P_{33} - M_{22} + 6Z
E_{36} = - h(t) P_{43}, E_{37} = (h(t) - h) P_{33}
E_{44} = - h(t)(1 - h(t)) P_{24} - \dot{h}(t) W_{22} - 12Z - 3h^2 F
\quad + \frac{h^2}{2}(h - h(t)) \left[ h(t) \left( B_{33} + \frac{1}{3} L_{33} \right) + 2C_{31} + 2C_{31}^T \right]
E_{45} = - h(t) P_{25} - (h - h(t))(1 - h(t)) P_{34} - 4Y_{22},
E_{46} = - h(t) \dot{h}(t) P_{24} - h(t)(1 - h(t)) P_{44} + 6h^2 F
E_{47} = h(t) \dot{h}(t) P_{25} - (h - h(t))(1 - h(t)) P_{54},
E_{55} = (h(t) - h) P_{35} - 12Z - 3h^2 F
E_{56} = (h(t) - h) \dot{h}(t) P_{34} - (h - h(t)) P_{45},
E_{57} = (h - h(t)) \dot{h}(t) P_{35} - (h - h(t)) P_{55} + 6h^2 F
E_{66} = - h(t) \dot{h}(t) P_{44} - 18h^2 F,
E_{67} = h(t) \dot{h}(t) P_{45} - (h - h(t)) \dot{h}(t) P_{54},
E_{77} = (h - h(t)) \dot{h}(t) P_{55} - 18h^2 F

4. Numerical simulations

In this section, four typical numerical examples are given to show the less conservatism and the effectiveness of the proposed method in this paper.

Example 1: Consider system (4) with time-varying delay and the parameters as follows:

\[
A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, A_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}
\]

The purpose of this example is to compare the conservatism of the stability conditions by applying different existing methods. When \( h_1 = 0, \mu = 0.5 \), the upper bound of time delay given in (Chen & Cai, 2009) is 2.42, using the method of this paper, the upper bound of time delay is 2.63. When \( \mu = 0, 0.1, 0.5, 0.8 \), by Theorem 1 of this paper, the obtained upper delay bounds are 6.0594, 4.7261, 2.6317, 2.2539 respectively.

Table 1 lists the results of the maximum allowable delay bounds when \( \mu (\mu = -\mu_1 = \mu_2) \) takes different values. From Table 1, it can be clearly seen that the results obtained by Theorem 1 are less conservative than the methods presented in (Ariba & Gouaisbaut, 2009; Park, Kwon, et al., 2011; Qian & Gao, 2015).

Figure 1 gives stability margin of typical second-order system with time delay by different methods, it can be seen that the larger stability margin of typical of the system is obtained in this paper compared with the results in (Ariba & Gouaisbaut, 2009; Park, Kwon, et al., 2011; Qian & Gao, 2015).

Example 2: Consider system (4) with time-varying delay and the parameters as follows:

\[
A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, A_1 = \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}
\]

For different \( \mu \), Table 2 lists the results of the maximum allowable delay bounds obtained by Theorem 1 in this paper and results in (Ariba & Gouaisbaut, 2009; Park, Kwon, et al., 2011; Seuret & Gouaisbaut, 2013). Figure 2
Table 2. Maximal time delay for different $\mu$.

| $\mu$ | 0.1 | 0.2 | 0.5 | 0.8 |
|-------|-----|-----|-----|-----|
| [24]  | 5.901 | 3.839 | 2.003 | 1.404 |
| [25]  | 5.823 | 3.824 | 2.008 | 1.357 |
| [20]  | 6.059 | 3.672 | 1.411 | 1.275 |
| Theorem 1 | 6.3715 | 4.1502 | 2.1344 | 1.6251 |

Table 3. Maximal time delay for different methods.

| Method | [14] | [19] | [11] | [12] | Theorem 1 |
|--------|-----|-----|-----|-----|----------|
| $h$/ms | 65.4 | 65.4 | 65.29 | 61.3 | 71.90 |

Figure 2. Stability margin of typical second-order system with time delay.

Figure 3. Single-machine infinite-bus system.

gives stability margin of typical second-order system with time-varying delay. It is clear that the proposed method in this paper has less conservatism and larger stability margin than those in (Ariba & Gouaisbaut, 2009; Park, Kwon, et al., 2011; Seuret & Gouaisbaut, 2013).

Example 3: The single-machine infinite-bus system is chosen to verify the effectiveness of Theorem 1, and the parameters of the system resource from (Qian et al., 2016) (Figure 3).

where

$$A = \begin{bmatrix} 0 & 376.9911 & 0 & 0 \\ -0.0963 & -0.7000 & -0.0801 & 0 \\ -0.0480 & 0 & -0.1667 & 0.1000 \\ 0 & 0 & 0 & -0.1000 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 38.0187 & 0 & -95.2560 & 0 \end{bmatrix}.$$

Table 3 lists the maximal allowable time delays of the single-machine infinite-bus system by using different methods when there is no disturbance in the system. The upper time-delay bound obtained by Theorem 1 is 71.9 ms, which is larger than the results in (Chen & Cai, 2009; Jia et al., 2010; Qian et al., 2016; Sun et al., 2015).

It is clear that the proposed method in this paper has less conservatism and larger stability margin than those in (Chen & Cai, 2009; Jia et al., 2010; Qian et al., 2016; Sun et al., 2015).

Further, if there is random disturbance in the system, the excitation amplification factor should be:

$$K'_A = K_A + r$$

where, $K_A$: Excitation amplification factor setting value; $K'_A$: Excitation amplification factor with disturbance; $r$: A scalar that reflects the disturbance to $K'_A$.

Let the matrix $H, E_a, E_b$ as follows:

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r \end{bmatrix}, E_a = 0, E_b = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}.$$

Let $r \in [0, 10]$, Table 4 lists the maximal allowable time delays obtained by different methods when $r = 0.5, 1, 1.5, 2, \ldots, 10$ (taking 0.5 as an interval), and Figure 4 gives stability margin of the single-machine infinite-bus system in this paper and (Jia et al., 2010; Qian et al., 2016; Sun et al., 2015).

It can be seen, with the increase of disturbance term $r$, the stability margin of the system with time-varying delay becomes smaller. It is clear that our results has less conservatism and larger stability margin than those in (Jia et al., 2010; Qian et al., 2016; Sun et al., 2015).

Example 4: In this example, the two-area-four-machine power system shown in Figure 5 is used to verify the effectiveness of the main result in this paper. The detailed parameters of the two-area-four-machine power system are given in (Chen & Cai, 2009). Where

$$A = \begin{bmatrix} 0 & 0 & 0 & 376.9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 376.9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 376.9 \\ -0.073 & 0.065 & 0.004 & -0.730 & 0.272 & 0.076 \\ 0.058 & -0.087 & 0.009 & 1.160 & -0.343 & -0.134 \\ 0.008 & 0.011 & -0.082 & -0.020 & 0.947 & -0.554 \end{bmatrix}.$$
Table 4. Maximal time delay for different $r$.

| $r$ | [14] | [12] | [19] | Theorem 1 | $r$ | [14] | [12] | [19] | Theorem 1 |
|-----|------|------|------|-----------|-----|------|------|------|-----------|
| 0.5 | 0.0570 | 0.0587 | 0.0650 | 0.0713 | 5   | 0.0397 | 0.0457 | 0.0617 | 0.0642   |
| 1   | 0.0534 | 0.0576 | 0.0647 | 0.0702 | 6   | 0.0370 | 0.0444 | 0.0609 | 0.0631   |
| 1.5 | 0.0516 | 0.0545 | 0.0639 | 0.0685 | 7   | 0.0263 | 0.0392 | 0.0593 | 0.0608   |
| 2   | 0.0478 | 0.0515 | 0.0632 | 0.0669 | 8   | 0.0220 | 0.0363 | 0.0586 | 0.0594   |
| 3   | 0.0439 | 0.0479 | 0.0624 | 0.0654 | 9   | 0.0180 | 0.0343 | 0.0578 | 0.0582   |

Figure 4. Comparison of stability margins obtained by different methods.

Figure 5. Two-area-four-machine power system.

Table 5. Maximal time delay for different methods.

| Method | [5] | [17] | [26] | [19] | Theorem 1 |
|--------|-----|------|------|------|-----------|
| $h/s$  | 0.195 | 0.288 | 0.328 | 0.440 | 0.519     |

Table 5 shows the upper bounds of time-delay for the two-area-four-machine power system by different methods, it can be seen that Theorem 1 in this paper has less conservatism than those in (Chaibi & Tissir, 2012; Ma et al., 2014; Yang & Sun, 2014).

5. Conclusion

This paper studies the stability of the wide area power system with interval time-varying delays. By establishing the model of the wide area power system with interval time-varying delay, a novel LKF with augmented vector is constructed. Then, the delay-partitioning approach, wirtinger-based integral inequality, free-matrix-based inequality and convex combination approach are used to estimate the derivative of the functional, and as a results, the new stability criterion with less conservatism is obtained. Finally, the numerical examples are given to show the effectiveness of the proposed method in this paper.

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Disclosure statement

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References

Ariba, Y., & Gouaisbaut, F. (2009). An augmented model for robust stability analysis of time-varying delay systems. *International Journal of Control*, 82(9), 1616–1626.

Chaibi, N., & Tissir, E. H. (2012). Delay dependent robust stability of singular systems with time-varying delay. *International Journal of Control, Automation and Systems*, 10(3), 632–638.

Chen, X., & Cai, P. Z. (2009). Lmi-based method for power system stability analysis with considering time-delay. *Proceedings of the Chinese Society of Universities for Electric Power System & Its Automation*, 21(6), 84–91.

Dong, C. Y., Jia, H. J., & Jiang, Y. L. (2015). Time-delay stability criteria for power system with integral quadratic form. *Automation of Electric Power Systems*, 39(24), 35–40.
Hadidi, R., & Jeyasurya, B. (2013). Reinforcement learning based real-time wide-area stabilizing control agents to enhance power system stability. *IEEE Transactions on Smart Grid*, 4(1), 489–497.

Hailati, G., & Wang, J. (2014). Multiple time delays analysis and coordinated stability control for power system wide area measurement. *Transactions of China Electrotechnical Society*, 6(19), 279–289.

Hua, Y. E., Jian, H., & Liu, Y. (2013). A method for computing eigenvalue of time-delayed power systems based on pade approximation. *Automation of Electric Power Systems*, 37(7), 25–30.

Huang, L. Q., Guo, J. B., & Sun, H. D. (2014). Wide-area anti-delay coordinated control among FACTS controllers. *Electric Power Automation Equipment*, 34(1), 37–42.

Jia, H. J., An, Y. H., & Yu, X. D. (2010). A delay-dependent robust stability criterion for power system and its application. *Automation of Electric Power Systems*, 34(3), 6–11.

Jing, M. A., Yinan, L. I., Junchen, L. I., Zhu, X., & Wang, Z. (2015). A time-delay stability control strategy considering jump characteristic of power system. *Power System Technology*, 11(1), 185–192.

Li, W. X. L. (2015). Stability analysis for stochastic time delay power system. *Power & Energy*, 36(1), 10–15.

Li, N., Sun, Y. H., & Wei, Z. N. (2017). Delay-dependent stability criteria for power system based on wirttinger integral inequality. *Automation of Electric Power Systems*, 41(2), 108–113.

Liu, X., Ding, C., Wang, Z., & Zhou, P. (2011). Direct method to analyze small signal stability of electric power systems. *Electric Power Automation Equipment*, 31(7), 1–4.

Ma, J., Li, J., Gao, X., & Wang, Z. (2014). Research on time-delay stability upper bound of power system wide-area damping controllers based on improved free-weighting matrices and generalized eigenvalue problem. *Power System Protection & Control*, 64(18), 476–482.

Ma, J., Li, Y., Li, J., Zhu, X., & Wang, Z. (2015). A time-delay stability control strategy considering jump characteristic of power system. *Power System Technology*, 39(4), 1033–1038.

Manousakis, N. M., Korres, G. N., & Georgilakis, P. S. (2012). Taxonomy of pmu placement methodologies. *IEEE Transactions on Power Systems*, 27(2), 1070–1077.

Park, P. G., Ko, J. W., & Jeong, C. (2011). Reciprocally convex approach to stability of systems with time-varying delays. *Automatica*, 47(1), 235–238.

Park, M., Kwon, O., Park, J. H., Lee, S., & Cha, E. (2015). Stability of time-delay systems via wirttinger-based double integral inequality. *Automatica*, 55(C), 204–208.

Park, M. J., Kwon, O. M., Park, J. H., & Lee, S. M. (2011). A new augmented lyapunov–krasovskii functional approach for stability of linear systems with time-varying delays. *Applied Mathematics and Computation*, 217(17), 7197–7209.

Qian, W., & Gao, C. (2015). A stability criterion for power system with time-varying delay based on convex combination. *Power System Protection & Control*, 43(19), 37–42.

Qian, W., Jiang, P. C., & Che, K. (2016). Stability analysis for power system with time-delay based on wirttinger inequality. *Power System Protection & Control*, 44(23), 79–85.

Seuret, A., & Gouaisbaut, F. (2013). Wirttinger-based integral inequality: Application to time-delay systems. *Automatica*, 49(9), 2860–2866.

Sun, G., Tu, Y., Sun, Y., Wei, Z., Xu, T., & Wang, S. (2015). An improved robust stability criterion for power systems with time-varying delay. *Automation of Electric Power Systems*, 39(3), 59–62.

Yan, J., Govindarasu, M., Liu, C. C., Ming, N. I., & Vaidya, U. (2015). Risk assessment framework for power control systems with pmu-based intrusion response system. *Journal of Modern Power Systems and Clean Energy*, 3(3), 321–331.

Yang, B., & Sun, Y. Z. (2014). A new wide area damping controller design method considering signal transmission delay to damp interarea oscillations in power system. *Journal of Central South University*, 21(11), 4193–4198.

Zeng, H. B., He, Y., & Wu, M. (2015). Free-matrix-based integral inequality for stability analysis of systems with time-varying delay. *IEEE Transactions on Automatic Control*, 60(10), 2768–2772.

Zhang, L., Zhan, Z. P., Wei, L. P., Shi, B., & Xie, X. (2016). Test and analysis of piecewise delay in wide area measurement system. *Automation of Electric Power Systems*, 40(6), 101–106.