Altitude Control of Quadcopters with Absolute Stability Analysis

Binh-Minh Nguyen*, a) Member, Tetsuya Kobayashi* Non-member, Kazuhiro Sekitani* Non-member, Michihiro Kawanishi* Member, and Tatsuo Narikiyo* Non-member

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This paper examines the vertical dynamics of a quadcopter, and presents an approach to achieve better altitude control. The control system includes a multi-sensor-based state observer for estimating vertical speed, a disturbance observer for improving system robustness, and a position controller for tracking the vehicle’s altitude with respect to a reference value. This paper clarifies that it is essential to address the model of the propeller actuators to properly guarantee system stability. It was found that the altitude control system is a multi-input multi-output system. By showing the rank-1 interaction between the actuators, this paper derives the condition for the controller that ensures absolute stability of the control system. The condition can be checked conveniently using a graphical test with the Nyquist plot, thereby alleviating the complexity of system design and analysis. The effectiveness of the proposed approach is evaluated by both numerical simulations and real-time experiments. This approach can be easily extended to the motion control of other general multirotor vehicles.

Keywords: absolute stability, altitude control, disturbance observer, flying vehicle, sector-bounded nonlinearity.

1. Introduction

1.1 Multirotor as a promising multi-agent system In this decade, multirotor flying vehicles (MFVs) have been increasingly utilized in many aspects of human society. Typical examples are aerial photography, environmental monitoring, agricultural spraying, and power line inspection. Under the pandemic situation of COVID-19, MFV could be an innovative solution for delivery and collection of packages, documents, and medicines. Recently, much attentions have been paid to MFV prototypes that can carry passengers through the air. Such prototypes have been developed by many companies, such as Airbus, Rolls-Royce, NEC, Lilium, and EHang. At the forefront of this trend, Japan has brought together 21 domestic companies to propose a roadmap towards flying vehicle society in 2030s.

MFV operates in three-dimensional Euclidean space. Therefore, it is commonly provided with an integrated motion control system, which includes position controllers and attitude (roll, pitch, yaw) controllers. As shown in Fig. 1, MFV is actually a special type of multi-agent system, in which each local agent is a propeller actuator. The agents interact with each other to generate the global motion of the vehicle body. For instance, the roll (pitch/yaw) moment acting on the vehicle body is the results of thrust difference between the agents. Note that, the thrust force has a nonlinear relationship with the rotational speed of the propeller.

1.2 Unsolved issue in position control of MFV To the best of our knowledge, almost the existing works in MFV motion control neglected the propeller actuators and their interaction. For years, the MFV has been merely treated as a rigid body moving in the inertial frame. To clarify this issue, this paper investigated position control of MFV, which can be categorized by (I) Nonlinear controller, and (II) Linear controller, as listed in Table 1.

Nonlinear controller can be designed based on fuzzy logic, sliding mode, passivity based damping assignment, and nonlinear disturbance observer. However, the nonlinear thrust model was not considered to obtain theoretical results. For instance, Guerrero-Sánchez only analyzed the passivity of the vehicle body dynamics. In other words, the mechanism from the thrust command to the actual thrust was completely neglected. The nonlinearity of the vehicle body dynamics can be expressed analytically based on Newton law with rotation matrix. In contrast, the thrust can only be approximated as polynomial function of propeller speed. Thus, attempting to apply nonlinear control theories to the full model of MFV, which includes both vehicle body dynamics and thrust characteristics, one would face high level of uncertainty and complexity. Recently, Nguyen et al proposed a hybrid control approach, which includes outer nonlinear position control and inner linear speed control. This study analyzed system stability without addressing the thrust model and the speed estimation. Since the speed sensor is not available, it is essential to notice that speed estimation should be implemented for MFV motion control system.

Until the current days, linear control approaches are preferable, thanks to their simplicity and convenient in real-time implementation. For instance, proportional integral derivative (PID) control with optimal gains was proposed by Pan et al, and robust analysis of PID control was investigated by Miranda-Colorado et al. Linear matrix inequality (LMI) was utilized to investigate the robustness of the position controller under parameter uncertainties. Lyu et al applied speed-based disturbance observer to improve the hovering performance of quadrotor vehicles. However, this study did not include vehicle speed...
It is nontrivial to address the thrust characteristics for linear controller design. One might empirically approximate the thrust $F_t$ as a polynomial function of the propeller speed $\omega_p$. This function can be linearized about an operating point. To discuss the robustness, the traditional way is to treat actuator interaction are not considered. Linearization is required, but thrust model and actuator interaction are not considered. Speed observer is not considered for DOB implementation and analysis. Lyapunov stability condition. Formulation on nonlinear manifold $\text{SE}(3)$ with Lyapunov stability analysis. Thrust model was not addressed for stability analysis. Thrust model was not considered for Lyapunov stability analysis. Lack of the experimental evaluation. Lack of theoretical analysis and experiment. Thrust model was not considered for theoretical analysis and control gain tuning process. Control gain optimization using cuckoo search algorithm. Thrust model is not considered for theoretical analysis. It is required to simplified the quadrotor dynamics by linearized model. Improvement of hovering performance. Linearization is required, but thrust model and actuator interaction are not considered. Speed observer is not considered for DOB implementation and analysis. Improvement of trajectory tracking in three-dimensional space. Linearization is required, but thrust model and actuator interaction are not considered. A robust stability analysis using state space model is provided. Lack of experimental evaluation. Linearization is required, but thrust model and actuator interaction are not considered. Control parameter optimization based on fuzzy logic, neural-network, or swarm optimization. Linear model identification is required, but thrust model and actuator interaction are not considered.

Table 1. Literature review on the control methods for altitude control and position control of MFVs.

| Method | Merits | Issues to be considered | Ref. No. |
|--------|--------|-------------------------|----------|
| (I) Nonlinear controller | Improvement of uncertainty handling in comparison with type-1 fuzzy control. | Lack of the theoretical analysis. | (6) |
| | Lyapunov stability analysis is provided. | Only simulation demonstration. | |
| | Experimental evaluation. | Thrust model was not addressed for stability analysis. | (7) |
| Double-layer control with outer nonlinear position control and inner speed control. | Simple to be implemented. | Thrust model and speed estimator were not considered for stability analysis. | (8) |
| Sliding mode control. | Neural network algorithm to achieve time-varying sliding surface. | Lack of the experimental evaluation. | (9) |
| Interconnection and damping assignment passivity-based control | Robustness enhancement by integrating with disturbance rejection using extended state observer. | Neglection of the thrust model when performing passivity analysis. | (10) |
| Nonlinear disturbance observer. | Enhancement of the robustness of the input-output feedback linearization controller. | Lack of theoretical analysis and experiment. | (11) |
| | Formulation on nonlinear manifold $\text{SE}(3)$ with Lyapunov stability condition. | Thrust model was not considered for stability analysis. | (12) |
| (II) Linear controller | Integration with Kalman filter for estimating target trajectory. | Lack of theoretical analysis. | (13) |
| Optimal PID control using Nelder-Mead numerical method. | Simple to be implemented. | Thrust model is not considered for stability analysis and control gain tuning process. | (14) |
| | Simple to be implemented. | Control gain optimization using cuckoo search algorithm. | |
| Robust PID control using power reduction methodology. | Rigorous proof of ultimate uniform boundedness of the overall system. | Thrust model was not considered for theoretical analysis. | (15) |
| | Simple to be implemented. | It is required to simplified the quadrotor dynamics by linearized model. | (16) |
| Double-layer control which assumes approximate knowledge of quadrotor parameters. | Optimal control gains can be obtained considering the uncertainty of parameters. | Improvement of hovering performance. | (17) |
| Robust guaranteed cost control via LMI technique and convex optimization. | Linearization is required. | Linearization is required. | (18) |
| Robust control using speed-based disturbance observer. | Speed observer is not considered for DOB implementation and analysis. | Thrust model and actuator interaction are not considered. | |
| Robust control using position-based disturbance observer. | Improvement of trajectory tracking in three-dimensional space. | Linearization is required, but thrust model and actuator interaction are not considered. | |
| Robust control using linear quadratic regulator-based disturbance observer. | A robust stability analysis using state space model is provided. | Lack of experimental evaluation. | (19) |
| Strictly negative imagine control. | Control parameter optimization based on fuzzy logic, neural-network, or swarm optimization. | Linear model identification is required, but thrust model and actuator interaction are not considered. | (20), (21), (22), (23) |

estimation in the $H_\infty$ norm analysis. Recently, negative imaginary theory has been introduced to position control of quadcopter (20). This approach requires the nominal transfer functions of vehicle dynamics obtained via parametric identification using Matlab toolbox (20). The identified transfer function, unfortunately, does not represent the real dynamics of the MFV, which is originally nonlinear. In summary, all the methods listed in group (II) neglects the propeller actuator model and the nonlinear thrust characteristics to simplify theoretical analysis.
signal; and \(y\) represent the control objectives. The popular scheme of \(\mu\)-synthesis can be applied to the diagram in Fig. 2 \((29)\). However, the complexity of system design and analysis certainly increases together with the number of propeller actuators.

From a multi-agent point of view, MFV shares a certain level of similarity with the electric vehicle driven by in-wheel-motors (IWM-EV). It has been shown that, the global stability and good performance of the IWM-EV cannot be guaranteed by simply neglecting the IWM actuator dynamics and their interaction \((25)\) \((26)\). Consequently, the following questions should be carefully investigated: (i) Is this possible to neglect the “agent” dynamics when designing the motion control system of MFV? (ii) With respect to the “agent” dynamics, how should we reduce the complexity of system design for practical application?

### 1.3 Research problem and contributions

Motion control of IWM-EV can be studied by firstly investigating the vehicle in a given direction, such as longitudinal and lateral motions, then combining all the subfields to establish an integrated control system \((27)\). The similar strategy can be applied to the study of MFV motion control. Hence, this paper only focuses on the vertical dynamics of MFV. To answer the aforementioned questions, this paper examines one of the fundamental motion control objectives, namely, altitude control. To conduct the study, this paper utilizes a small-scale quadcopter. Due to their simplicity and convenient for practical application, linear control is of interest in this paper.

Unlike the existing methods listed in Table 1, this paper properly addresses the propeller actuator with the thrust model. It turns out that the altitude control system with linear controller is equivalently expressed as a nonlinear multi-input multi-output (MIMO) system. Fortunately, this MIMO system has two important properties. First, the interaction between the actuator belongs to a class of rank-1 matrix. Second, the relationship between the thrust force and the propeller speed belongs to a class of sector-bounded nonlinearity. Thanks to such properties, this paper derives a practical condition for the controller to guarantee absolute stability \((28)\). The condition can be checked graphically with the Nyquist plot, thereby alleviating the complexity of system design and analysis. To stabilize the control system, there is no requirement to approximate the thrust model by empirical polynomial. Furthermore, it is not necessary to linearize the thrust model about an operating point with norm-bounded perturbation.

The original idea of this paper was presented at the 30th IEEE-ISIE \((29)\). This conference paper shows that, by neglecting the thrust model, disturbance observer-based altitude control system might not be stabilized, even if the controller satisfies the small gain theorem. However, the vehicle speed was simply calculated from the vehicle position using a high pass filter in the ISIE paper. In this study, a multi-sensor-based state observer combines the measurement of the visual positioning system and the on-board acceleration sensor to provide a more accurate estimation of the vehicle speed. The estimated speed and the total thrust command are used as inputs of a disturbance observer (DOB), which compensates the disturbances and improves the control performance. Unlike the works of Nguyen et al \((19)\) and Lyu et al \((17)\), the speed observer is included in the stability analysis. The proposed altitude control system, which includes state observer, DOB, and a position controller, is quite complex. Luckily, it can be analyzed conveniently based on the proposed stability condition. Moreover, the proposed approach can be easily extended to other motion control purposes of the general MFVs.

### 1.4 The rest of this paper

Section II describes the experimental system of the quadcopter under study and its vertical dynamics, followed by a motivating example that preliminary answers to the aforementioned question (i). Section III presents the altitude control system, and establishes the absolute stability condition. The proposed approach was evaluated in Section IV through various test scenarios, followed by a discussion on the extension of the proposed approach. Finally, the conclusions and future study are stated in Section V.

### 2. Vehicle model and motivating example

#### 2.1 Experimental system

A small-scale quadcopter is used for this study (Fig. 3). Each propeller of the quadcopter is rotated by a Cobra 2100Kv motor, which is powered by the LiPo 3S 3300mAh battery. The vehicle is equipped with several on-board sensors, including gyroscope, accelerometer, magnetometer, and barometer. Besides, a camera system produced by OptiTrack \((30)\) is used for providing accurate quadcopter position in three-dimensional space. The position measurement is sent to the vehicle via wireless communication. The vehicle has an on-board Intel\textsuperscript{®} Aero Computer with Quad-core 64bit, 2.56GHz processor, 4GB LPDDR3-1600 RAM. The computer serves as the main control board, which processes sensing signals, generates the control signals to the local motor drives, and stores the experimental results.

#### 2.2 Quadcopter model

Focusing on the vertical dynamics, the model from the total thrust command \(F_z^*\) to the vehicle altitude \(z\) can be described as in Fig. 4, where \(I_4\) is the 4 × 4 identity matrix, \(1_4\) is the all-one-column vector of size 4. \(F_z\) is the thrust force generated by the \(i\)th propeller (the index \(i\) is from 1 to 4); \(\theta_p\) represents the combined influence of the gravity and other disturbances; \(a_z\) is the vertical acceleration. The key parameters of the quadcopter model are summarized in Table 2.

**Assumption 1:** The quadcopter is assumed to be close to the hovering condition. It is also assumed that the roll motion, pitch motion, and yaw motion are slight.

![Experimental system of the quadcopter under study.](image-url)
The thrust vector is defined as \( \vec{F} \). The thrust model \( F_z \) via the force-to-PWM gain \( \eta \) and the distribution vector \( \mathbf{1}_N / 4 \).

### 2.3 Motivating example by simulation

Is this possible to neglect the propeller actuator and the thrust model? In the IEEE-ISIE paper, this question was answered by showing a fail example of the DOB designed using small gain theorem (20). This paper answered the question by investigating the traditional linear controller design scheme that neglects the propeller actuator dynamics. A typical example is PID controller, which is popularly designed using the diagram shown in Fig. 6(a).

The transfer functions of the PID controller and the close loop system are

\[
P_z(s) = \frac{C_1(s)P(s)}{1+C_1(s)P(s)} \quad \text{(5)}
\]

In (6), \( P_z(s) \) is calculated with the nominal mass. Selecting the time constant \( \tau_d = 0.005 \) and placing the stable poles \((-193.97, -2.45, -1.79 \pm 4.92j\)} for \( P_2(s) \), the control parameters are obtained as in Table 2. To evaluate the above design scheme, two simulation tests are conducted using Matlab-Simulink.

**Test 1**: The simulation model is the same as the block diagram shown in Fig. 6(a). This means the motion of the quadcopter is only described by the transfer function (1) without addressing the thrust model given by the function \( \phi \).

**Test 2**: Unlike Test 1, the quadcopter is described by Fig. 4, in which the thrust model is represented by a lookup-table obtained from the experimental data in Fig. 5. This means the quadcopter model in Test 2 almost imitates the actual behavior of the real quadcopter.

The altitude reference is \( z^* = 0.5 \) for both tests, and the simulation results are summarized in Fig. 6(b). Although two tests used the same controller, they have completely different results. A stable motion is realized in Test 1. However, instability is shown in Test 2. The disagreement between two tests points out a real problem of the traditional motion control design scheme. By neglecting the actuator dynamics, the traditional scheme might fail to stabilize the overall system and cannot predict the actual behavior of the MFV. In the following Section, this paper presents a new scheme that allows the acknowledgement of the actuator dynamics.

### 3. Proposed approach

**3.1 Description of altitude control system**

### Table 2. Quadcopter model and control parameters

| Quadcopter model                                      |
|-------------------------------------------------------|
| Vehicle mass                                          | 1.21 [kg] |
| Motor’s amplifying gain                               | 2100 [rpm/V] |
| Motor’s time constant                                 | 0.004 [s] |
| Maximum motor speed                                   | 16000 [rpm] |
| Battery voltage                                        | 11.7 [V] |
| Thrust sector (lower)                                  | 8.1 x 10^{-3} [N/rpm] |
| Thrust sector (upper)                                  | 37.1 x 10^{-6} [N/rpm] |
| Force-to-PWM gain                                     | 0.1957 [N/rpm^-1] |
| Positioning’s time delay                              | 4 [ms] |

**Control parameters used in motivating example**

| PID gains                                              |
|-------------------------------------------------------|
| \( K_p = 42.47, K_i = 78.95, K_d = 7.08 \)          |
| D gain’s time constant                                 | \( \tau_d = 0.005 \) [s] |

**Control parameters used in proposed system**

| State observer’s poles                                 |
|-------------------------------------------------------|
| \( \lambda_{1,2} = \{-50, -100\} \)                    |
| DOB’s time constant                                    | \( \tau_d = 0.01 \) [s] |
| DOB’s gain                                             | \( \eta = 0.80 \) |
| PIV’s gains                                            | \( \mu_p = 16.94, \mu_i = 12.10, \mu_d = 7.86 \) |
| D gain’s time constant                                 | \( \tau_d = 0.005 \) [s] |

According to **Assumption 1**, the vertical motion of the vehicle body is described by the transfer function:

\[
P_z(s) = \frac{1}{ms^2} \quad \text{(1)}
\]

The total thrust force that acting on the vehicle body is

\[
F_z = I_z^2F \quad \text{(2)}
\]

where the thrust vector is defined as \( F = [F_1 \cdots F_z]^T \). The thrust force has a nonlinear relationship with the propeller speed \( \omega_i \). Based on the experimental data, the thrust characteristics of the quadcopter under study is obtained as in Fig. 5. Let \( F_z = \phi(\omega_i) \), the nonlinear function \( \phi \) satisfies the following property

\[
\left\{ \phi(\omega_i) - \kappa_1 \omega_i \right\} \left\{ \phi(\omega_i) - \kappa_2 \omega_i \right\} \leq 0, \quad \forall \omega_i \leq \omega_{\text{max}} \quad \text{(3)}
\]

where \( \kappa_1 = 8.1 \times 10^{-5} [\text{N/rpm}] \), \( \kappa_2 = 37.1 \times 10^{-5} [\text{N/rpm}] \). This means \( \phi \) belongs to the sector \([\kappa_1, \kappa_2]\). Next, the transfer function of the motor-propeller is

\[
P_m(s) = \frac{K_m}{\tau_m s + 1} \quad \text{(4)}
\]

where \( K_m \) and \( \tau_m \) are calculated from the motor-propeller parameters. The voltage vector and the speed vectors are defined as \( \mathbf{V} = [V_1 \cdots V_N]^T \) and \( \omega = [\omega_1 \cdots \omega_N]^T \), respectively. The voltage applied to the motor is \( V_i = u_i V_p \), where \( u_i \) is the pulse width modulation (PWM) command to the electronics speed control (ESC). The PWM command vector is \( \mathbf{u} = [u_1 \cdots u_N]^T \). Note that, \( \mathbf{u} \) is mapped from total thrust command \( F_z \) via the force-to-PWM gain \( \eta \) and the distribution vector \( \mathbf{1}_N / 4 \).
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in this paper. To improve the performance of position control, a speed-based DOB is to compensate the model uncertainties and the external disturbances. The DOB includes a nominal model \( R_m(s) \), which represents the nominal model from the thrust command to the vehicle speed. Besides, \( Q_p(s) \) is a low-pass filter. Literature review shows that this type of linear DOB has been utilized for the vehicle speed. Besides, which represents the nominal model from the thrust command to external disturbances. The DOB includes a nominal model in this paper. To improve the performance of position control, a DOB is designed as

\[
\hat{x}(s) = (A_L - A_\alpha + L_c C_\alpha)^{-1}(B_{\alpha} a_{\alpha}(s) + L_\alpha z_{\alpha}(s)) \tag{10}
\]

As shown in Fig. 4:

\[
z(s) = P(s) F(s), \quad a_{\alpha}(s) = s^2 P_{\alpha}(s) F(s) \tag{12}
\]

From (9), (10), (11), and (12), \( P_{ob}(s) \) is finally derived as

\[
P_{ob}(s) = P_{c} s^2 P_{\alpha}(s) + l_1 s P_{\alpha}(s) + l_2 P_{\alpha}(s) \tag{13}
\]

The system in Fig. 7 is equivalently expressed as in Fig. 8(a) with the transfer function

\[
H(s) = \eta P_{o}(s) \frac{Q(s) P_{ob}(s) + P_{ob}(s) P_{ob}(s) P_{ob}(s) C_{s}(s)}{1 - Q(s) P_{ob}(s)} \tag{14}
\]

and the interconnection matrix calculated as

\[
\Gamma = \frac{1 \mathbf{I}_4}{4} \tag{15}
\]

\[\tilde{v}_z = R \hat{x}_z, \quad R = [0 \ 1] \]

\[\text{Notice 1: In Fig. 7, the positioning system is represented by the transfer function } P_{ps}(s). \text{ The acceleration sensor is represented by the transfer function } P_{ac}(s). \quad P_{ac}(s) = 1 \text{ if the sensor } \# \text{ can be treated as an ideal sensor (} \# = ps \text{ or } ac).\]

3.2 System representation for stability analysis In order to investigate system stability, it is possible to neglect the disturbance terms and equivalently expressing the original system in Fig. 7 as the feedback connection of a linear part and a nonlinear part. To this end, this paper firstly obtains the transfer function \( P_{ob}(s) \) from \( F_z \) to \( \tilde{v}_z \) via the vehicle body, sensors, and the state observer. From (8), it can be show that

\[
\dot{x}(s) = (A_L - A_\alpha + L_c C_\alpha)^{-1}(B_{\alpha} a_{\alpha}(s) + L_\alpha z_{\alpha}(s)) \tag{10}
\]

As shown in Fig. 4:

\[
z(s) = P(s) F(s), \quad a_{\alpha}(s) = s^2 P_{\alpha}(s) F(s) \tag{12}
\]

From (9), (10), (11), and (12), \( P_{ob}(s) \) is finally derived as

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The system in Fig. 7 is equivalently expressed as in Fig. 8(a) with the transfer function

\[
H(s) = \eta P_{o}(s) \frac{Q(s) P_{ob}(s) + P_{ob}(s) P_{ob}(s) P_{ob}(s) C_{s}(s)}{1 - Q(s) P_{ob}(s)} \tag{14}
\]

and the interconnection matrix calculated as

\[
\Gamma = \frac{1 \mathbf{I}_4}{4} \tag{15}
\]

3.3 Absolute stability analysis The four-by-four matrix \( \Gamma \), which is given by (15), represents the interaction between the actuators in the control system. It is interesting to note that \( \Gamma \) is a rank-1 matrix with three zero eigenvalue, and only one non-zero eigenvalue that equals to 1. Moreover, there exists a four-by-four real unitary matrix \( \Lambda \) that diagonalizes \( \Gamma \).

With respect to Assumption 1, the rotational speeds of the propellers are almost the same. Thus, by using the aforementioned matrix \( \Lambda \), the system in Fig. 8(a) can be equivalently transformed into the system in Fig. 8(b). In order to analyze the stability of the system in Fig. 8(b), it is possible to neglect the zero eigenvalues, and only consider the single-input single-output feedback
connection of the non-zero eigenvalue in Fig. 8(c). It is reasonable to assume that \( P_{ps}(s) \) and \( P_{ac}(s) \) are stable transfer functions. By properly selecting the low-pass filter \( Q(s) \), and choosing the state observer gains such that all the eigenvalues of matrix \( A_0 - L_0C_0 \) are in the left half plane, the transfer function \( H_2(s) \) has no pole with positive real part. As discussed in the previous Section, the nonlinear function \( \varphi \) belongs to the sector \([\kappa_1, \kappa_2]\), which defines in Fig. 9 a disk \( \mathcal{D} \). Following the Circle criterion \((30)\), the system in Fig. 8(c) is absolutely stable if the following transfer function is strictly positive real:

\[
F(s) = [1 + \kappa_2H(s)][1 + \kappa_1H(s)]^{-1} \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdOTS
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Substituting the control parameters of the motivating example into (20), the Nyquist plots of $H_1(j\omega)$ are shown in Fig. 13(a). When varying the vehicle mass between 0.8$m_n$ and 1.2$m_n$, it turns out that the Nyquist plot of $H_1(j\omega)$ always intersects with disk $D$. This means the selection of the PID controller in the motivating example fails to satisfy the absolute stability condition. Then, the instability of the motivating example is demonstrated by

![Fig. 7](image_url)

(a) Fig. 7. Propeller speed response with $z^* = 0.5$ [m].

![Fig. 8(c)](image_url)

(b) Fig. 8(c).

![Fig. 12](image_url)

(a) Fig. 7. Propeller thrust response with $z^* = 0.5$ [m].

(b) Fig. 8(c).

Proposition 1

$H_1(s) = \eta P_m(s)P_a(s)P_e(s)C(s)$

(20)

where $m_n$ is the nominal mass, and the tuning gain $K_p$ is between 0 and 1. As the accelerometer is an on-board sensor, it can be assumed that $P_m(s) = 1$. In the experiments, the vehicle position is obtained via a vision system, and sent to the on-board controller via wireless communication. Considering the signal processing time and communication time, a certain delay exists in the position measurement. This time delay is approximately $T_d = 4$ [ms] in the experimental system. By third-order Padé approximation, the transfer function of $P_m(s)$ is approximated as

$$P_m(s) = e^{-T_d s} \approx \frac{1.875 \times 10^3 - 3.75 \times 10^2 s + 3000 s^2 - s^3}{1.875 \times 10^3 + 3.75 \times 10^2 s + 3000 s^2 + s^3}$$

(19)

A simulation model, which captures the real nonlinearities and saturations of the quadcopter under study, is established using Matlab/Simulink. This model is used to perform a fine-tuning process with the constraint given by the Proposition 1. Finally, a set of control parameters were obtained for the proposed system, as listed in Table 1.

The block diagram in Fig. 8(c) is validated by numerical simulation as follows. Let the reference altitude $z^* = 0.5$ [m], this paper compares propeller speeds and thrust forces of the original system in Fig. 7 with that of the system in Fig. 8 (c). As shown in Fig. 11, the propeller speed responses of two diagrams matches with each other. Also, Fig. 12 shows that the thrust force responses of two diagrams are the same.

Reconsideration of the motivating example

By setting the tuning gain $K_p$ of the DOB to be zero, the control system in Fig. 7 is reduced to be an altitude control system using PID controller. The transfer function $H(s)$ is reduced to be

$H_1(s) = \eta P_m(s)P_a(s)P_e(s)C(s)$

(20)

space, thereby introducing more burdens to the tuning process. With respect to the fast dynamics of the motor, this paper only selected $P_m(s)$ and $Q(s)$ as first order transfer functions:

$$P_m(s) = \frac{1}{m_s}, \quad Q(s) = \frac{K_p}{s + 1}$$

(18)

where $m_n$ is the nominal mass, and the tuning gain $K_p$ is between 0 and 1. As the accelerometer is an on-board sensor, it can be assumed that $P_m(s) = 1$. In the experiments, the vehicle position is obtained via a vision system, and sent to the on-board controller via wireless communication. Considering the signal processing time and communication time, a certain delay exists in the position measurement. This time delay is approximately $T_d = 4$ [ms] in the experimental system. By third-order Padé approximation, the transfer function of $P_m(s)$ is approximated as

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(20)
real-time experiment, and the result is shown in Fig. 13(b). It is very dangerous to conduct this test situation, as the experiment system is located in-door. Hence, we only conducted the test in a short time. The control system was turn-off as the quadcopter reaches the reference altitude for the third time at about 12.2 [s]. After that, the quadcopter fell down under earth gravity.

4.3 Evaluation of the proposed altitude control

As shown in Fig. 7, the proposed control system allows two operating modes as follows:

Mode 1 (PID only): In this mode, the DOB gain \( K_D \) is set to zero. The thrust is only generated by the PID controller with the gains \( K_p = 16.94, K_i = 12.10, K_d = 7.86, \) and \( \tau_d = 0.005 [s] \), as listed in Table 1.

Mode 2 (PID with DOB): The PID in Mode 2 is the same as that of Mode 1. As listed in Table 1, the DOB is selected with \( m_a = 1.21 [kg], K_p = 0.80, \text{ and } \tau = 0.01 [s] \). The eigenvalues of the state observer are placed at \( [-50, -100] \).

Mode 1 and Mode 2 were evaluated by three tests, including a graphical test and three real-time experimental tests, as demonstrated in Fig. 14 and Fig. 15, respectively.

Test 1 (Graphical test): Fig. 14(a) and Fig. 15(a) show the Nyquist plots of two modes with different values of vehicle mass between 0.8\( m_a \) and 1.2\( m_a \). It is transparent that no Nyquist plot intersects with disk \( D \). This theoretically says that the absolute stability is guaranteed in either Mode 1 or Mode 2. It can also be predicted that the system in Mode 2 has a higher level of robustness. In comparison with Mode 1, the Nyquist plots of Mode 2 always maintain bigger distances to disk \( D \).

Test 2-A (Hovering to 0.5 [m]): The setting of the experiment is shown in Fig. 3. A small load is connected to the center of the quadcopter body via a string with a certain level of elasticity. The actual mass of the load is unknown to the controller. As shown in Fig. 14(b) and Fig. 15(b), the quadcopter successfully reaches the reference altitude in either mode. Thanks to the DOB, Mode 2 can improve the control performance by reducing the fluctuation. The root-mean-square deviation (RMSD) of tracking error is 0.055 [m] with Mode 1, and only 0.034 [m] with Mode 2.

Test 2-B (Hovering to 1.0 [m]): The setting of Test 2-B is the same as Test 2-A, but the altitude reference is set as 1.0 [m], as shown in Fig. 14(c) and Fig. 15(c). Again, the tracking performance is improved by using Mode 2. For comparison, the RMSD of the tracking error is 0.079 [m] with Mode 1, and only 0.049 [m] with Mode 2.

Test 3 (Altitude tracking under strong disturbance): At the beginning of the test, the quadcopter lifts to the altitude of 1 [m]. Since it is quite dangerous to use high power fan to generate external force in the vertical direction, a disturbance is generated as a step signal by the on-board computer. Its amplitude is approximate 1.5 times of the gravitational disturbance. This disturbance is introduced to the total thrust command. It is treated as unknown signal to the controller. As shown in Fig. 14(d), by using Mode 1 of the controller, the quadcopter suffers a serious drop. Although the quadcopter tries to return to the desired altitude, it drops again due to the continuity of the disturbance. Thanks to the DOB in Mode 2 of the controller, the quadcopter system is quite robust to prevent the influence of the disturbance. The quadcopter only drops approximately 0.4 [m]. Then, it takes less than 3 seconds to successfully return to the desired altitude. The control performance of the quadcopter, therefore, is remarkably improved by utilizing DOB.

4.4 Vertical motion control of general MFV

Considering an MFV driven by \( N \) propeller actuators, its altitude control system or vertical speed control system can be generally expressed as the feedback connection system in Fig. 16 with the interconnection matrix

\[
\Gamma = \frac{1}{N} 
\]

where \( 1_N \) is the all-one column vector of size \( N \).

It can be shown from (21) that \( \Gamma \) is a rank-1 matrix with only one non-zero eigenvalue of 1. Moreover, there exists a real unitary matrix \( A \) that diagonalizes \( \Gamma \). Hence, the proposed approach can be extended to other prototype of MFVs without special difficulty.

Notice 3: Other analysis can be considered to further understanding the performance of the control system. For instance, in Fig. 16, additional perturbations can be included to addressed the uncertainty of the motor parameter and the vehicle mass. Thanks to the diagonalizability of matrix \( \Gamma \), generalized frequency variable approach \(^{(35)} \) can be utilized to discuss the robust stability issue.

5. Conclusions

To properly design and stabilize the altitude control system of MFVs, this paper shows that it is necessary to address the nonlinear relationship between the thrust force and propeller speed, and the interaction between the propeller actuators. This paper presents a practical approach to model the altitude control system and guarantee its absolute stability. The merits of the proposed stability condition are as follows. First, the condition is independent of the number of propeller actuators. Second, it can be checked easily using Nyquist plot. The condition reduces the complexity of system design and analysis. It provides a tool to graphically demonstrate the system stability. Besides the classical way of DOB design using \( \| w \|_\infty \) norm, this paper contributes another way to analyze the DOB-based control system.

The proposed approach is applied to design the altitude control system of a quadcopter, which includes state observer, disturbance observer, and tracking controller. Its effectiveness has been evaluated and discussed by numerical simulations, graphical tests, and real-time experiments. The proposed approach can be straightforwardly applied to the vertical motion control of the general MFV prototypes. In future study, the proposed approach will be further extended to full motion control system of MFV. The authors are also interested in robust stability analysis, time-delay compensation, and actuator fault tolerance for MFVs. DOB with higher-order nominal model would be another interest.

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Altitude Control of Quadcopters with Absolute Stability Analysis (Nguyen et al.)

(a) Test 1: Graphical test.

(b) Test 2-A: Hovering to 0.5 [m].

(c) Test 2-B: Hovering to 1.0 [m].

(d) Test 3: Altitude tracking under strong disturbance.

Fig. 14. Evaluation of Mode 1 (PID only).

(a) Test 1: Graphical test.

(b) Test 2-A: Hovering to 0.5 [m].

(c) Test 2-B: Hovering to 1.0 [m].

(d) Test 3: Altitude tracking under strong disturbance.

Fig. 15. Evaluation of Mode 2 (PID with DOB).
Fig. 16. General expression for motion control of MFV.

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Binh Minh Nguyen (Member) He received his M.S and Ph.D. degrees from the University of Tokyo in 2012 and 2015. He was a researcher at the same university from 2015 to 2020. He is currently a researcher at Toyota Technological Institute. His research interests include global control, passivity control, and their applications in electric vehicles, flying vehicles, and power systems. He is a member of IEEE and IEJE. Since 2020, he has served as an Associate Editor at the IEEE Vehicular Technology Magazine. He is also an Area Editor at the EAI Endorsed Transactions on Industrial Networks and Intelligent Systems.
Tetsuya Kobayashi  (Non-member) He is currently a master course student at Toyota Technological Institute. He is doing research on motion control of drones using robust controller and state observer.

Kazuhiro Sekitani  (Non-member) He is currently a master course student at Toyota Technological Institute. He is interested in safe landing control for drones in emergency situation.

Michihiro Kawanishi  (Member) He received his M.S. and Ph.D. degrees from Kyoto University in 1994 and 1998. He was a Research Associate at Kobe University from 1996 to 2006. In 2006 he became an Associate Professor at Toyota Technological Institute. His research interests include control system design with numerical optimization and its application to mechanical systems. He is a member of IEEE, ISCIE, JSME, and RSJ.

Tatsuo Narikiyo  (Non-member) He received his B.S., M.S., and Ph.D. degrees from Nagoya University, Japan, in 1978, 1980, and 1984, respectively. From 1983 to 1990, he was a Research Scientist with the Government Industrial Research Institute. In 1990, he joined the Department of Advanced Science and Technology, Toyota Technological Institute, where he is currently a Distinguished Professor. He retired in 2018. His research interests include nonlinear control systems theory and robotics.