Letter to the Editor

Correlation of velocity and density contributions to spectroscopic channel maps: Reality check on Kalberla et.al (2022)

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ABSTRACT

Context. The existence of magnetized turbulence in the interstellar neutral hydrogen (HI) is well accepted. On the basis of theory, a number of techniques to obtain turbulence spectrum and magnetic field direction and strength have been developed and successfully applied to HI spectroscopic data. To better separate the imprints of density and velocity fluctuations to the channel maps, a new theory-based technique, the Velocity Decomposition Algorithm (VDA, Yuen et.al 2021), has been created. The technique demonstrates that the intensity fluctuations are separated into a component \( p_v \) that mostly arise from velocity fluctuations and \( p_d \) that mostly arise from density fluctuations. The VDA helps to clarify the nature of the filamentary structure observed in channel maps.

Aims. A recent publication (Kalberla et.al 2022) claims that the application of VDA to HI4PI data provides a negative correlation of \( p_v \) and \( p_d \), which according to the authors invalidates the technique since it requires that of \( p_v \) and \( p_d \) have zero correlation. However, the quantities \( p_v \) and \( p_d \) given by VDA are naturally orthogonal which can be trivially checked analytically or numerically. That means the correct application of the VDA to any data must provide zero correlation. This is the point that we clarify in this paper and search for the cause of the mistake in the application of the VDA in Kalberla et.al (2022) that resulted in the erroneous conclusion.

Methods. We prove analytically that by construction \( p_v \) and \( p_d \) are not correlated. We identify the likely mistake in the VDA expression that Kalberla et.al (2022) used and reproduce their figures with the incorrect expression.

Conclusions. We conclude that the detrimental mistake that Kalberla et.al (2022) made at their analysis completely invalidate their scientific claim that Yuen et.al (2021) is not compatible to observation.

Key words. ISM: clouds – ISM: structure – dust, extinction – turbulence – ISM: magnetic fields – magnetohydrodynamics (MHD)

1. Introduction

The atomic neutral hydrogen (HI) is well known to be filamentary and to extend over multiple scales. HI observations became the peril of astrophysical research in the 21st century. The physical nature of HI is needed to understand why HI is filamentary and to extend over multiple scales. HI observations became the basis technique, the Velocity Decomposition Algorithm (VDA, Yuen et.al 2021), has been created. The technique demonstrates that the intensity fluctuations are separated into a component \( p_v \) that mostly arise from velocity fluctuations and \( p_d \) that mostly arise from density fluctuations. The VDA helps to clarify the nature of the filamentary structure observed in channel maps.

A proposed physical understanding of the nature of HI is that some, not all nor none, of the features exhibited in spectroscopic position-position-velocity (PPV) channel maps are formed due to the imprint of turbulence motions along the line of sight. This effect, which is called velocity caustics (Lazarian & Pogosyan 2000), has been studied theoretically and numerically (Yuen et.al 2021). Originally caustics were believed to be highly theorized objects that were not detectable in channel maps yet contributed to their observed intensity. In Yuen et.al (2021) we provided a way for separating the fluctuations of intensity \( p = p(X, v) \) at some POS position \( X \) and velocity \( v \) into velocity fluctuations \( p_v(X, v) \) and density fluctuations \( p_d(X, v) \). By construction, \( p(X, v) = p_v + p_d \) and \( \langle p_v p_d \rangle = 0 \) Our numerical study in Yuen et.al (2021) provided a framework to systematically show that \( p_v \neq 0 \) in simulations and observational data, and to show that \( p_v \) can be more present than \( p_d \) in channel maps, contrary to the widely circulated claim that velocity caustics are not measurable in HI data (Clark et.al 2019; Kalberla et.al 2020).

The goal of this paper is to rebut the central claim of Kalberla et.al (2022). First of all, by construction of Yuen et.al (2021), \( \langle p_v p_d \rangle = 0 \), regardless of any actual physics and whether velocity or density are actually correlated in PPV space. Any opposite results simply mean that the VDA equations are used incorrectly. In what follows, in §2 we identify a mistake in VDA expressions that Kalberla et.al (2022) use and provide a new analysis of the same data with the correct VDA expressions. Naturally, we get \( \langle p_v p_d \rangle = 0 \). In §3 we offer some background to the history of the controversy and our views on velocity caustics. Our conclusions are presented in §4.

1 Where, for the sake of simplifying the notations, we do not show the dependence of \( p_v \) and \( p_d \) on PPV coordinates \((X, v)\)
2. Mistake in VDA equation in Kalberla et al. (2022)

2.1. Algebraic proof of \( \langle p_v p_d \rangle = 0 \)

The main idea in Yuen et al. (2021) is that, for a given spectroscopic cube \( p(X,v) \) and its integrated intensity map \( I = \int dv_p \), the density fluctuations \( p_d \) and velocity fluctuations \( p_v \) can be given by:

\[
p_v = p - (\langle p \rangle - \langle p(I) \rangle) \frac{I - \langle I \rangle}{\sigma_I^2}
\]

\[
p_d = (\langle p \rangle - \langle p(I) \rangle) \frac{I - \langle I \rangle}{\sigma_I^2}
\]

where \( \sigma_I^2 = (I - \langle I \rangle)^2 \). Readers can refer to §3 of Yuen et al. (2021) for the construction and justification of the technique.

Notice that the use of Eq. (1) will mathematically guarantee that \( \langle p_d p_v \rangle = 0 \):

\[
\langle p_d p_v \rangle = \left( \langle p \rangle - \langle p(I) \rangle \right) \frac{I - \langle I \rangle}{\sigma_I^2} \left( p - (\langle p \rangle - \langle p(I) \rangle) \frac{I - \langle I \rangle}{\sigma_I^2} \right) = \left( \langle p \rangle - \langle p(I) \rangle \right)^2 \frac{(I - \langle I \rangle)^2}{\sigma_I^2} - \left( \langle p \rangle - \langle p(I) \rangle \right) \frac{I - \langle I \rangle}{\sigma_I^2} = 0
\]

where we utilize the fact that \( \sigma_I^2 = (I - \langle I \rangle)^2 \) at the 2nd term of line 3. We can see from Eq. (2) that \( \langle p_d p_v \rangle = 0 \) follows from equation Eq. (1). Thus if one uses the correct form of Eq. (2) the orthogonality of \( \langle p_d p_v \rangle \) must be present.

That also means, if any data with or without density and velocity correlations does not reproduce this result, this must be a mistake in the use of the original VDA equations employed, independent of the type of physical models that HI has on the sky. Notice that all the criticism of the VDA in Kalberla et al. (2022) is based on the fact that in their analysis \( \langle p_d p_v \rangle \neq 0 \) is invalid. Therefore it is apparent that \( \langle p_d p_v \rangle \neq 0 \) cannot be obtained with any data.

2.2. Tracking the source of the mistake via reverse engineering

It is an interesting question how Kalberla et al. (2022) got their results while using the VDA. Below we attempt to identify the source of their mistake.

We suspect that the error in Kalberla et al. (2022) is due to the use of incorrect equations:

\[
p_v, \text{Kalberla22} = p - (\langle p \rangle - \langle p(I) \rangle) \frac{I - \langle I \rangle}{\sigma_I}
\]

\[
p_d, \text{Kalberla22} = (\langle p \rangle - \langle p(I) \rangle) \frac{I - \langle I \rangle}{\sigma_I^2}
\]

Our guess was based on the fact that Fig.1 of Kalberla et al. (2022) has weird color bar ranges. Usually for \( p_v \) the color bar is symmetric, i.e. it should range from \(-A\) to \(A\) for some value of \(A\) as \( \langle p_v \rangle = 0 \). However, we spot that the value of \( p_v \) is abnormally high and that of \( p_d \) is abnormally low in Fig.1 of Kalberla et al. (2022), suggesting that there are over-subtraction in the term \( \langle p \rangle - \langle p(I) \rangle \). We tried a few combinations, e.g. \( \langle pp \rangle - \langle p(I) \rangle \) and \( \langle p \rangle - \langle I \rangle \) and finally realize that Eq. (3) is the best in reproducing Fig.1 of Kalberla et al. (2022). See https://www.github.com/kyuen2/kalberla_2022 for the illustration.

To test whether our identification of mistakes in Kalberla et al. (2022) is right we apply Eq. (3) to the same regions used in Kalberla et al. (2022), namely (1) full sky HI4PI and the three examples Yuen et al. (2021) have used: (2) the \( 8\times8^\prime \) GALFA data centered at RA=4.00 and DEC=10.35; (3) the \( 8^\prime \times 8^\prime \) GALFA data centered at RA = 228.00° and DEC = 18.35°; (4) and the region considered in Clark et al. (2015), i.e. 195° < RA < 265°, 19.10° < DEC < 38.7°.

2.3. Reproduction of Figures in Kalberla et al. (2022) with erroneous formulae

To start with, we use the full sky HI4PI data cube. The public data from HI4PI website that we use has a channel width of \( \Delta v = 1.29km/s \), so we are constrained to select the channel \( v_{\text{fsr}} \approx 1.39km/s \pm 0.65 \) to reproduce their results. In Fig. 1 we show the actual \( p_d, p_v \) using the correct VDA formula given by Eq. (1) and given by Eq. (2). When we use the incorrect formula given by Eq. (3), our results are very similar to Kalberla et al. (2022)’s Figure 1. The results with Eq. (1) differs significantly from Kalberla et al. (2022)’s Figure 1.

We then focus on Fig. 2 of Kalberla et al. (2022). The left and middle panels of Fig. 2 shows both \( p_d, p_v \) and NCC(\( p_d, p_v \)) using both sets of equations (Eq. (1), Eq. (3)). We can see that the actual correct formula (Eq. (1)) correctly reproduce the expected 0 for both \( p_d, p_v \) and NCC(\( p_d, p_v \)).

However, if we use Eq. (3) employed in Kalberla et al. (2022), we retrieve the pattern that is shown in Fig 2 of Kalberla et al. (2022), as in the left of Fig 2. We can see that due to the mistake,Kalberla et al. (2022) cannot reproduce the default orthogonality condition for \( p_v \) and \( p_d \). As a result, the corresponding NCC(\( p_d, p_v \)) is also non-zero as we show in Fig 2.

In addition, we compute the NCC curve using both the correct and erroneous formula on data sets (2)-(4) as discussed above. We see in Fig 3 that our VDA formula clearly reproduce \( \langle p_d p_v \rangle = 0 \) as expected by Eq. (2). Again, we found that using the erroneous formula (Eq. (3)) we could reproduce the results of Fig 4 upper, middle, and lower panel of Kalberla et al. (2022).

All the figures beyond Fig 4 of Kalberla et al. (2022) are based on the same erroneous equation used in Fig 1,2 and 4, and are all therefore invalid.

3. Discussions

Our tests and results presented in the previous section are significant enough that we could end the present publication with the following conclusion. It is rather apparent that Kalberla et al. (2022) made a rudimentary mistake when implementing the VDA method, which is the foundation of their entire paper. Our current letter is intended to clarify that the harsh criticism of the VDA in Kalberla et al. (2022) is based on the erroneous use of the VDA.

\footnote{We believe that the channel resolution (\( \Delta v = 1km/s \)) quoted in §4 of Kalberla et al. (2022) is an apparent typo. See Table 1 of HI4PI collaboration for the correct number.}

\footnote{Notice that the amplitude of \( \langle p_d p_v \rangle \) that we obtain using Eq. (3) is off by a factor of 2 compared to Fig. 2 of Kalberla et al. (2022). However, we believe this is due to the fact that we do not have access to their HI data with channel width \( \Delta v = 1km/s \), as the publicly available cube has \( \Delta v = 1.29km/s \). The reasoning is very simple: A lower value of \( p \) will significantly decrease the value of \( \rho_{d,kalberla} \). Naturally, the thickness of the channels cannot affect the default condition \( \langle p_d p_v \rangle = 0 \).

\footnote{Fig. 3 of Kalberla et al. (2022) is the only figure that is irrelevant to VDA as it was meant to explain Fig 2 of Kalberla et al. (2022). However, Fig. 2 of Kalberla et al. (2022) is wrong.}
K.H. Yuen, K.W. Ho & A. Lazarian: Density and velocity correlation in ISM

Fig. 1. A set of figures showing the $p_d, p_v, (p_d p_v)$ through Eq.1 (upper row) and Eq.3 (lower row). Readers should notice that the lower row of the paper is visually very similar to that of Kalberla et.al (2022)’s Figure 1, as we intentionally use very similar colorbar as theirs.

Fig. 2. Two figures showing how the $h_i$ varies as a function of $v_{LSR}$ (in km/s) for the true VDA equation and the suspicious equation used in Kalberla et.al (2022). We can see from the two figures that our VDA method (Eq.1) correctly reproduce the expected orthogonality according to Eq.2. Nevertheless, we obtain something very similar to their results (highly negative $p_d p_v$).

of the VDA equation. Therefore we focus on this central result of Kalberla et.al (2022) and do not comment on other issues, e.g. whether Gaussian decomposition is applicable on top of VDA. We note that Kalberla et.al (2022) is similar to Kalberla et al.(2020) which provided a rather harsh criticism of the LP00 theory and its applications. In Yuen et.al (2021), we answered of the criticism in Kalberla et al. (2020). Here we find that it is necessary and important for the community to understand some of the history and our views concerning velocity caustics.

While dismissing the VDA, Kalberla et.al (2022) strongly supported the interpretation that HI filaments are entirely density fluctuations (see Clark et al. (2019), Kalberla et al. (2020)). The controversy over the importance of velocity caustics in HI maps started with Clark et al. (2019) argument that there is a strong correlation between the FIR dust emission and the intensity fluctuations of HI channel maps. This argument was supported in a number of publications, e.g. Peek & Clark (2019), Kalberla et al. (2020). The arguments that it is wrong to disregard the velocity caustics were also presented. For example Yuen et al. (2019) points out that the results from Clark et al. (2019) were not properly normalized and the correlation of FIR dust emission and HI channel maps does not imply that there is no correlation between velocity fluctuations and FIR dust emissions. Yuen et al. (2021) provided further evidence of the role of velocity caustics, proving, at the very least, that the claim that all structures on the sky are predominately density fluctuations is wrong. In fact, the relative contribution of density and velocity depends on the region of study. For example, in Yuen et al. (2021) we gave two examples, namely the regions (2, RA=4, DEC=10.35) and (3, RA=228, DEC=18.35) that we captured from GALFA-DR2. The region (2) under our analysis has subdominant density contribution (Fig.22 of Yuen et.al 2021). However, for region (3)

\[ h_i^2 \]

This is a trivial logic. Showing A is similar to B does not exclude the possibility that C can also be similar to B despite A and C being mutually exclusive. Here A is HI channels, B is FIR dust emission and C is caustics. Similarity operator is not associative.

Article number, page 3 of 5
we actually found from our analysis that the density and velocity contributions are in equal portion for that particular region (Fig 23 of [Yuen et al., 2021]). We believe that in accordance with LP00 theory, both density and velocity fluctuations contribute to PPV fluctuations and their relative role could be evaluated by the toolbox in [Yuen et al., 2021]. [Yuen et al., 2021] demonstrates that velocity caustics provide significant contribution to observed filaments in PPV, which is contrary to the claim from [Clark et al., 2019; Kalberla et al., 2020].

It is also important to note that the concept of velocity and density contributions in the channel maps does not necessarily extend to the full sky. For instance, the application of the VDA to a big statistically inhomogeneous region can induce uncertainties. It is a mistake, therefore, to use the VDA on the full sky HI data with a huge statistical inhomogeneity that arises from, e.g. galactic motions.

Kalberla et al. (2022) also questioned the validity of the MHD simulations. For instance, they claim “Our result are in clear contradiction to the MHD simulations and theoretical expectations from uncorrelated velocity and density fields with NCC ~ 0.” (Sec 6.5 of [Kalberla et al., 2022]). Given their critical mistakes in their paper, we abstain from replying to their criticisms, which should be merely regarded as a biased opinion with no solid ground.

4. Summary

In this paper, we show that Kalberla et al. (2022) made a simple yet fatal mistake in terms of the implementation of Eq 1 when analyzing the observational data. This mistake is not a scientific one, but rather a rudimentary programmatic error leading to the erroneous main results of Kalberla et al. (2022) and invalidating all their figures but Figure 3. Their discussions based on their

\[ p_v = \frac{\langle p_d p_v \rangle}{\langle p_d \rangle} \]

wrongly computed \( p_v \), leading to the erroneous "discovery" that \( \langle p_d p_v \rangle \neq 0 \) is also misleading. In particular,

- We showed that \( \langle p_d p_v \rangle = 0 \) is always true if \( p_v \) and \( p_d \) are defined by the VDA equations. Therefore Kalberla et al. (2022) finding that \( \langle p_d p_v \rangle \neq 0 \) is a result of incorrect use of the VDA equations.
- We identified the mistake in equations employed in Kalberla et al. (2022) and reproduced their figures analyzing their data using erroneous equations. In parallel we provided the results obtained with the correct formulae and demonstrated that we can indeed obtain \( \langle p_d p_v \rangle = 0 \) in any selected HI4PI data.
- Furthermore, the authors of Kalberla et al. (2022) mistakenly use VDA on the full sky HI data. This reveals a lack of understanding of the core the VDA technique. The VDA is designed to work for a single turbulent system. It is not meant to apply to full-sky HI data with a huge statistical inhomogeneity that arises from, e.g. galactic motions.

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Notice that the authors of [Kalberla et al., 2022] suggest that there are multiple clouds along the line of sight. The correct way of doing VDA analysis here is to first identify the clouds’ individual Gaussian profile, apply VDA on those individual profile, and cross check with our analysis.

![Fig. 3. Three figures showing the NCC values as a function of \( v_{lsr} \) (in km/s) from the actual Eq 1 and the wrongly used Eq 5 for the three regions: left: 8° × 8° GALFA data centered at RA=4.00 and DEC=10.35; middle: to the 8° × 8° GALFA data centered at RA = 228.00° and DEC = 18.35°; right: for the region 195° < RA < 265°, 19.1° < DEC < 38.7°.](https://www.github.com/kyuenz/LazDDA)
