Silver ion release from Ag/PET hollow fibers: Mathematical model and its application to food packing

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Abstract
A food packing system requires an antimicrobial environment with high air permeability to guarantee absolutely both food safety and long and reliable durability. A hollow fiber membrane containing silver ions is the best candidate for this purpose. However, a safe and controllable release process has become a pressing issue in the practical application. Here, we use the predator–prey model to predict the antimicrobial activity of silver ions released from a hollow fiber, and Staphylococcus aureus was applied to the antibacterial experiment. Both theoretical and experimental results show that there are an optimal concentration of the silver ions and an optimal release time. This article sheds a bright light on the design of a new kind of food packing systems using the hollow fiber membrane.

Keywords
Hollow fiber, predator–prey model, food packing, fractal, bio-inspired hollow fibers

Introduction
A food packing system should guarantee absolutely both food safety and long and reliable durability. To this end, the system should have, first, high air permeability to ensure a fresh air in the system; second, it should prevent microbial resources from entering into the system; third, the material of system should be environmentally friendly and harmless to humans. Hollow fibers with silver ions have good antibacterial property and high air permeability, and the natural fibers have also excellent antimicrobial property, and their geometrical properties contribute to their bio-functional properties.¹⁻⁶ It is interesting to note that the fractal dimensions of many nature fibers are closed to the golden mean (1.618), implying an optimal geometric structure. Polar bear hairs are hollow fibers which have an excellent thermal insulation property,¹⁻³ cocoons with hierarchical structures have excellent air permeability,⁴ fractal-like wool fibers have high heat release rate from its body,⁵ and geckos enable smart adhesion property to their foot hairs.⁶ It can be concluded that natural hollow fibers containing silver ions are the best candidate for the food packing system, a packing system with hollow fiber membranes has high air permeability and good vibration resistance.⁷,⁸ This article is to

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study the bio-inspired hollow fibers and to study the silver ion release from the artificial fibers. However, a safe and controllable release process has become a pressing issue in practical applications.

Silver ion release from hollow fibers has been caught much attention, and its antibacterial ratio plays an important role in practical applications, especially in food packing systems. Recently, an intelligent packaging system was designed successfully by cellulose–polypyrrole–ZnO films, which sheds a new light on new packing systems by advanced nanotechnology.

Polyethylene terephthalate (PET) hollow fiber membranes containing silver ions have a significant antimicrobial activity and high air permeability, making them the best candidate for food packings. Nanoscale hollow fibers can now be produced by the bubble electrospinning. Figure 1 shows the cross section of the Ag/PET hollow fiber. The silver particles are attached to the inner wall of the fibers and distributed evenly, when the hollow fibers’ free ends are immersed into water or meet water moisture; the released ions have a good antibacterial property, because silver ions can cause a fast diffusion of proteins in live bacteria; and additionally silver nanoparticles have high surface energy (geometric potential) and also have good antibacterial activity. TiO₂-Ag nanoparticles and zinc oxide nanoparticles can be also used for this purpose.

Polletti et al. gave a systematical review on the active packaging using nanotechnology; Kurek et al. proposed a mathematical model for the antimicrobial agent release from a bio-based film; and Esmaeili et al. studied numerically ellipsoidal particles deposition; however, the mathematical models for ion release were rare and very preliminary; in this article, we will use the well-known predator–prey model to study the antibacterial ratio, so that the pressing issue of a safe and controllable release process can be solved.

An antimicrobial model of silver ions

The Lotka–Volterra model or named by the predator–prey model is used to describe the relationship between predators and prey in an ecosystem. It is similar to the silver-bacterium process. In the process, silver ions are viewed as predators that feed on bacteria, and bacteria are referred as prey. According to our previous results, the antimicrobial model can be expressed as

\[
\frac{dN(t)}{dt} = (\gamma - \lambda y)N(t) \tag{1}
\]

\[
\frac{dy}{dt} = (-b + \mu N)y(t) \tag{2}
\]

where \(N(t)\) is the biomass density of the prey-bacteria, \(y\) is the mass density of the predator-silver ions, \(\gamma\) is the growth rate of the prey, \(b\) is the death rate of the predator, \(\lambda\) is the rate of change in the prey, \(\mu\) is the rate of change in the predator.

We assume that the releasing process of silver ions is not affected by the number of bacteria in the antibacterial process. The nonlinear system is given in equations (1) and (2) by Taylor method, the variational method, the variational iteration method, and others. The approximate solution reads

\[
y(t) = y_s(1 - e^{-kt}) \tag{3}
\]

\[
N(t) = N(0) \cdot e^{(\gamma - \lambda y)t} \tag{4}
\]

where \(y_s\) and \(k\) are constants.

The antibacterial rate can be calculated as

\[
\eta = \frac{N(0) \cdot e^{\gamma t} - N(0) \cdot e^{(\gamma - \lambda y)t}}{N(0) \cdot e^{\gamma t}} \tag{5}
\]

Experimental verification

The PET-based hollow fiber with silver particles (Ag/PET hollow fiber) as an antibacterial material was bought from Shenzhen Yang Qian Material Application Technology Co, Ltd, China. Silver nitrate and nitric acid were bought from Sinopharm Group Chemical Reagent Co, Ltd, China. Nutrient agar medium, nutrient broth medium, 0.9% saline, disodium hydrogen phosphate, phosphoric acid II hydrogen potassium, and deionized water were bought from Ningbo Aobo Scientific Instrument Co, Ltd, China. Staphylococcus aureus (No. ATCC 6538) was bought
from Guangdong Huankai Microbial Technology Co, Ltd, China.

The inductively coupled plasma atomic emission spectrometer (ICP-OES) was used to determine the amount of silver ions. The scanning electron microscope (Hitachi 3-4800) was used to observe the cross section of the Ag/PET hollow fiber.

*S. aureus* was used as a sample bacteria. A single colony was taken on a petri dish, and then a nutritious broth medium was covered on it for overnight culturing. The concentration of the bacteria was diluted to $10^8$ CFU (colony-forming unit)/mL. Polyethylene terephthalate (PBS) solution of 200 mL was added into 1 mL bacteria solution, and the resultant solution was termed as a control group. PBS solution of 1 mL with 0.1 g of Ag/PET hollow fiber was termed as a test group. Bacterial suspension of 1 mL from the control group and the test group at various periods (0, 1, 2, 4, 6, 8, 12, and 16 h) was taken to count the bacteria number and to make the curve of the bacterial growth. Thus, we can calculate the antibacterial rate and draw the antibacterial curve based on the control group and test group. The fixed bacterial samples were sputter-coated with a gold layer and observed under a scanning electron microscope.

We investigated the morphology of *S. aureus* bacteria after incubation with Ag/PET hollow fiber by scanning electron microscopy (SEM). As shown in Figure 2(a), images exhibited no change in the control test, and the average size of *S. aureus* cell is 1–2 µm. Conversely, there was a remarkable alteration when treated with Ag/PET hollow fiber (Figure 2(b)). The bacterial cell’s membrane was expanded and became rough, which indicated that the Ag/PET hollow fiber had the ability to detect the presence of bacterial in the text sample.

The bacteria will grow at the natural rate of growth, and the growth curve of the bacteria is shown in Figure 3. The fitting equation of the above test data results in

$$N(t) = 741.5 e^{0.27t}, \gamma = 0.27$$  \hspace{1cm} (6)

The growth of the bacteria is inhibited by the silver ions, when the Ag/PET hollow fiber is added. At this time, the relationship of the antibacterial rate of the Ag/PET hollow fibers and the contact time is shown in Figure 4. The fitting equation of the above test data is

$$\eta = 1 - e^{-0.45(1-e^{-0.95})}, \lambda = 0.95$$  \hspace{1cm} (7)

When $\eta$ is equal to the ratio of $\gamma$ to $\lambda$, the number of *S. aureus* does not increase over times, and the corresponding amount of silver ions reaches the threshold concentration. The antibacterial properties of Ag/PET hollow fiber depend on the silver ions released from the fiber.

![Figure 2](image1.png)

**Figure 2.** SEM images of *S. aureus* on a food packing system (a) without and (b) with silver ions. The scale bar is 1 mm.

![Figure 3](image2.png)

**Figure 3.** The growth of *S. aureus* unloaded Ag/PET hollow fibers.

The threshold time for antibacterial silver ions is named as $T$. The threshold concentration of the silver ions is calculated as follows
The time required to reach the threshold concentration of silver ions is calculated as

\[ T = \frac{\ln \left( \frac{1 - 0.33}{0.46} \right)}{-0.11} = 8.2 \text{ h} \]  

(9)

**Discussion**

This article proposed a self-contained mathematical model for ion release process and antimicrobial activity of silver ions simultaneously, and the theoretical analysis is of critical importance for the design of a packing system. The present food packing system might not be harmful to the human physiological system; however, an experimental verification is needed to validate our prediction.

This model can predict the optimal ion concentration and the optimal release time, and the present experiment can be used as a paradigm for real applications. Although the mathematical model seems to be rigorous, the assumptions can be made to fix the real conditions, and a fractal modification might be very much needed. Recently, a fractal model was suggested to study the release property of ions from a hollow fiber, and the fractal calculus and fractional calculus can predict more accurately the antibacterial activity.

**Conclusion**

This article shows that the silver ions released from hollow fibers have effective antibacterial activity. We use the well-known predator–prey model to predict the antibacterial activity, and the experimental results show the theoretical prediction is reliable and can be used for optimization of hollow fibers.

In this article, *S. aureus* was applied to the antibacterial experiments for the model, and the results showed that the optimal concentration of the silver ions is 0.28 mg/L and the optimal release time is 8.2 h for our experiment.

**Declaration of conflicting interests**

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**References**

1. He JH, Li ZB and Wang QL. A new fractional derivative and its application to explanation of polar bear hairs. *J King Saud Univ Sci* 2016; 28(2): 190–192.
2. Wang QL, Shi XY, He JH, et al. Fractal calculus and its application to explanation of biomechanism of polar bear hairs. *Fractals* 2019; 27(5): 1992001.
3. Wang QL, Shi XY, He JH, et al. Fractal calculus and its application to explanation of biomechanism of polar bear hairs. *Fractals* 2018; 26(6): 1850086.
4. Liu FJ, Zhang XJ and Li X. Silkworm (Bombyx Mori) cocoon vs. wild cocoon multi-layer structure and performance characterization. *Therm Sci* 2019; 23(4): 2135–2142.
5. Fan J, Yang X and Liu Y. Fractal calculus for analysis of wool fiber: mathematical insight of its biomechanism. *J Eng Fiber Fabr* 2019; 14: 1–4.
6. Li XX and He JH. Nanoscale adhesion and attachment oscillation under the geometric potential Part 1: the formation mechanism of nanofiber membrane in the electrospinning. *Result Phys* 2019; 12: 1405–1410.
7. Kuang W, Wang J, Huang C, et al. Homotopy perturbation method with an auxiliary term for the optimal design of a tangent nonlinear packaging system. *J Low Freq Noise Vibr Act Control* 2019; 38(3–4): 1075–1080.
8. Song HY. A modification of homotopy perturbation method for a hyperbolic tangent oscillator arising in nonlinear packaging system. *J Low Freq Noise Vibr Act Control* 2019; 38: 914–917.
9. Lin L, Gong WZ and Wang SY. Hollow PET fibers containing silver particles as antibacterial materials. *J Text Inst* 2011; 102(5): 419–423.
10. Lin L, Gong WZ, Wang X, et al. Preparation and characterizations of antibacterial PET-based hollow fibers containing silver particles. *Mater Lett* 2011; 65(9): 1375–1377.
11. Lin L and Yao SW. Release oscillation in a hollow fiber—part 1: mathematical model and fast estimation of its frequency. *J Low Freq Noise Vibr Act Control* 2019; 38(3–4): 1703–1707.
12. Lin L and Yao SW. Fractal diffusion of silver ions in hollow cylinders with unsmooth inner surface. *J Eng Fiber Fabr* 2019; 14: 1–5.

13. Lin L, Li HG and Yao SW. Experimental verification of the fractional model for silver ion release from hollow fibers. *J Low Freq Noise Vibr Act Control* 2019; 38(4): 1041–1044.

14. Pirsa S and Shamusi T. Intelligent and active packaging of chicken thigh meat by conducting nano structure cellulose-polypyrrole-ZnO film. *Mater Sci Eng C* 2019; 102: 798–809.

15. Liu YQ, Feng JW, Zhang CC, et al. Air permeability of nanofiber membrane with hierarchical structure. *Therm Sci* 2018; 22(4): 1637–1643.

16. Zhang X, Fan YS, Tian G-J, et al. Influence of fiber diameter on filtration performance of polyester fibers. *Therm Sci* 2019; 23(4): 2291–2296.

17. Yang ZP. Filtration efficiency of a cigarette filter with X- or Y-shaped fibers. *Therm Sci* 2019; 23(4): 2517–2522.

18. Cheng T, Xu L and Wang M. Effect of surface active agent on bubble-electrospin polyacrylonitrile nanofibers. *Therm Sci* 2019; 23(4): 2481–2487.

19. Li Y and He JH. Fabrication and characterization of ZrO2 nanofibers by critical bubble electrospinning for high-temperature-resistant adhesion and separation. *Adsorp Sci Technol* 2019; 37(5–6): 425–437.

20. Yu DN, Tian D, Zhou CJ, et al. Wetting and supercontraction properties of spider-based nanofibers. *Therm Sci* 2019; 23(4): 2189–2193.

21. Tian D, Zhou CJ and He JH. Sea-silk based nanofibers and their diameter prediction. *Therm Sci* 2019; 23(4): 2253–2256.

22. Zhou CJ, Li Y, Yao SW, et al. Silkworm-based silk fibers by electrospinning. *Result Phys* 2019; 15: 102646.

23. He CH, He JH and Sedighi HM. Fangzhu (方诸): an ancient Chinese nanotechnology for water collection from air: history, mathematical insight, promises and challenges. *Math Method Appl Sci*. Epub ahead of print 6 April 2020. DOI: 10.1002/mma.6384.

24. Ji FY, He CH, Zhang JJ, et al. A fractal Boussinesq equation for nonlinear transverse vibration of a nanofiber-reinforced concrete pillar. *Appl Math Model* 2020; 82: 437–448.

25. He JH. On the height of Taylor cone in electrospinning. *Result Phys* 2020; 17: 103096.

26. Sadoon AA, Prabhat K and Jack F. Silver ions caused faster diffusive dynamics of histone-like nucleoid-structuring proteins in live bacteria. *Appl Environ Microbiol* 2020; 86(6): e02479–19.

27. Yang ZP, Dou F and Yu T. On the cross-section of shaped fibers in the dry spinning process: physical explanation by the geometric potential theory. *Result Phys* 2019; 14: 102347.

28. Fan J, Zhang YR and Liu Y. Explanation of the cell orientation in a nanofiber membrane by the geometric potential theory. *Result Phys* 2019; 15: 102537.

29. Wang CX, Xu L and Liu GL. Smart adhesion by surface treatment experimental and theoretical insights. *Therm Sci* 2019; 23(4): 2355–2363.

30. Liu Y, Hussain M, Memon H, et al. Solar irradiation and Nageia nagi extract assisted rapid synthesis of silver nanoparticles and their antibacterial activity. *Digest J Nanomater Biostruct* 2015; 10(3): 1019–1024.

31. Yu L, Memon H, Bhavsar P, et al. Fabrication of alginate fibers loaded with silver nanoparticles biosynthesized via Dolcetto grape leaves (Vitis vinifera cv.): morphological, antimicrobial characterization and in vitro release studies. *Mater Focus* 2016; 5: 216–221.

32. Memon H, Wang H, Yasin S, et al. Influence of incorporating silver nanoparticles in protease treatment on fiber friction, antistatic, and antibacterial properties of wool fibers. *J Chem* 2018; 2018: 4845687.

33. Farshchi E, Pirsa S, Roufegarinejad L, et al. Photocatalytic/biodegradable film based on carboxymethyl cellulose, modified by gelatin and TiO2-Ag nanoparticles. *Carbohydr Polym* 2019; 216(15): 189–196.

34. Pirsa S, Shamusi T and Kia EM. Smart films based on bacterial cellulose nanofibers modified by conductive polypyrrole and zinc oxide nanoparticles. *J Appl Polym Sci* 2018; 135(34): 46617.

35. Zanetti M, Carimi TK, Dalcanton F, et al. Use of encapsulated natural compounds as antimicrobial additives in food packaging: a brief review. *Trends Food Sci Tech* 2018; 81: 51–60.

36. Kurek M, Laridon Y, Torrieri E, et al. A mathematical model for tailoring antimicrobial packaging material containing encapsulated volatile compounds. *Innov Food Sci Emerg Tech* 2017; 42: 64–72.

37. Esmaeili AR, Sadjadi B and Akbarzadeh M. Numerical simulation of ellipsoidal particles deposition in the human nasal cavity under cyclic inspiratory flow. *J Braz Soc Mech Sci Eng* 2020; 42: 243.

38. Yoshida T, Jones LE, Ellner SP, et al. Rapid evolution drives ecological dynamics in a predator–prey system. *Nature* 2003; 424(6946): 303–306.

39. Liu HY, Li ZM and Yao YJ. A fractional nonlinear system for release oscillation of silver ions from hollow fibers. *J Low Freq Noise Vibr Act Control* 2019; 38(1): 88–92.

40. He JH. Taylor series solution for a third order boundary value problem arising in architectural engineering. * Ain Shams Eng J*. Epub ahead of print 12 March 2020. DOI: 10.1016/j.asej.2020.01.016.

41. He JH. Variational principle and periodic solution of the Kundu–Mukherjee–Naskar equation. *Result Phys* 2020; 17: 103031.

42. He JH and Ain QT. New promises and future challenges of fractal calculus: from two-scale Thermodynamics to fractal variational principle. *Therm Sci* 2020; 24(2A): 659–681.

43. Anjum N and He JH. Laplace transform: making the variational iteration method easier. *Appl Math Lett* 2019; 92: 134–138.

44. Tao ZL, Chen GH and Chen YH. Variational iteration method with matrix Lagrange multiplier for nonlinear oscillators. *J Low Freq Noise Vibr Act Control* 2019; 38(3–4): 984–991.

45. He JH. A short review on analytical methods for to a fully fractional model for silver ion release from hollow fibers. *Result Phys* 2019; 23(4): 2253–2256.
47. Lin L, Chen Y, Wang X, et al. Antibacterial efficiencies and release behaviours between PP-based and PET-based hollow fibers containing silver particles. *Adv Mater Res* 2011; 332–334: 739–742.

48. Ling L, Gong WZ, Chen JW, et al. Structure and antibacterial analysis of silver fiber. In: *Proceedings of the fiber society 2009 spring conference*, vols. I and II, Shanghai, China, 27–29 May 2009, pp. 1021–1023. Shanghai, China: Donghua University.

49. Ling L, Gong WZ, Chen JW, et al. Structural characterization and antibacterial study of silver fiber. In: *Proceedings of the 2008 international symposium on fiber-based scaffolds for tissue engineering*, Shanghai, China, 8–9 December 2008, pp. 48–51. Shanghai, China: Donghua University.

50. Liu HY, Yao SW, Yang HW, et al. A fractal rate model for adsorption kinetics at solid/solution interface. *Therm Sci* 2019; 23(4): 2477–2480.

51. Li XJ and Liu Z. A fractal two-phase flow model for the fiber motion in a polymer filling process. *Fractals*. Epub ahead of print 10 May 2020. DOI: 10.1142/S0218348X20500930.

52. Ain QT and He JH. On two-scale dimension and its applications. *Therm Sci* 2019; 23(3): 1707–1712.

53. He JH. Thermal science for the real world: reality and challenge. *Therm Sci* 2020; 23: 2131–2133.