Visual contact for two satellites orbits under $J_2$-gravity

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Abstract In this paper, general analytical and computational technique for satellite-to-satellite visibility will be established firstly under the keplerian force, secondary under the effect of earth’s gravitational field (oblateness). The development is generally in the sense that the visibility conditions can be used whatever the types of the satellite orbits may be. Many data are taken to illustrate our technique.

Keywords Satellites orbits; Keplerian force; Earth’s gravitational field

1. Introduction

A satellite under the influence of an inverse square gravitational law has truly constant orbital elements, that is, the set $[a, e, M, i, Q, \omega]$ is composed of constants devoid of explicit time dependency. For many practical problems, the approximation of two-body motion is sufficient, especially if two closely points on a trajectory are under investigation. There are situations in which the cumulative effect of the gradual shift or variation of elements from true epoch values according to perturbative forces cannot be ignored (Brouwer and Clemence, 1961 and Brouwer, 1959).

Mutually visible satellites are defined as two satellites that can maintain direct line of sight between each other for a certain length of time. We primarily concerned with the rise and set time of a given satellite with respect to another, that is, the time of loss or gain of direct line of sight (Noton, 1998 and Maini and Agrawal, 2007).

2. Rise-set function

2.1. Relative rise-set geometry

Consider the geometry defined in Fig. 1. As illustrated, satellites (1) and (2) are in a state of relative rise or set. Indeed, if the vector $S$, which emanates from the dynamical center of the Earth, had magnitude equal to or less than the radius of the Earth and if it were perpendicular to $C$, the chord length vector between the satellites, it is evident that the satellites would not have direct line-of-sight communication. Owing to atmospheric interference, however, a realistic analysis would let the magnitude of $S$ be slightly larger than $a_e$, the radius of the Earth. Letting $\Delta$ be the thickness of the atmosphere or suitable bias factor, it follows that

$$S^2 = \hat{S} \cdot \hat{S} = \left(a_e + \Delta\right)^2. \quad (1.1)$$
2.2. Analytical expression of the relative rise-set function

Examination of Fig. 1 allows the two fundamental vector closure equations to be written as

\[
\begin{align*}
\vec{r}_2 &= \vec{S} + \vec{Z}_2, \quad (2.2) \\
\vec{r}_1 &= \vec{S} + \vec{Z}_1, \quad (2.3)
\end{align*}
\]

where \( \vec{r}_i; i = 1, 2 \) are the position vectors of the satellites and \( \vec{Z}_i; i = 1, 2 \) are two unknown vectors. At relative rise and set of satellite (1) with respect to satellite (2), we have

\[
\vec{S} \cdot \vec{Z}_1 = \vec{S} \cdot \vec{Z}_2 = 0. \quad (2.4)
\]

Then, from the figure

\[
C = \sqrt{(\vec{r}_2 - \vec{r}_1) \cdot (\vec{r}_2 - \vec{r}_1)} = \sqrt{r_2^2 + r_1^2 - 2x}, \quad (2.5)
\]

where \( x = \vec{r}_1 \cdot \vec{r}_2 \).

It is then possible to obtain an analytical expression of the rise and set function, from the planer triangle, (Escobal, 1965), as

\[
R = (\vec{r}_1 \cdot \vec{r}_2)^2 - r_2^2 r_1^2 + (r_2^2 + r_1^2)S^2 - 2S^2 (\vec{r}_1 \cdot \vec{r}_2), \quad (2.6)
\]

where \( S \) is obtained from Eqs. (2.2) and (2.3).

The rise-set function defined by Eq. (2.6) can be used to predict explicitly whether or not satellites are visible to one another. The sign of \( R \) associated with visibility can be obtained by constructing a case in which direct line-of-sight visibility is impossibility as shown in Fig. 2; consequently we can get the rule that

1. Negative value of \( R \rightarrow \) direct line-of-sight communication.
2. Positive value of \( R \rightarrow \) non-visibility.

3. Reduction of rise-set function to a two-parameter function

In terms of the orbital eccentricity \( e \), semi-parameter \( p \) and true anomaly \( f \), the equation of each orbit can be expressed by the relation

\[
r_i = \frac{p_i}{1 + e_i \cos(f_i)}, \quad i = 1, 2.
\]

Also we have

\[
\vec{r}_i = \xi_i \vec{P}_i + \eta_i \vec{Q}_i, \quad i = 1, 2.
\]

where

\[
\begin{align*}
\xi_i &= r_i \cos(f_i), \\
\eta_i &= r_i \sin(f_i).
\end{align*}
\]

The standard orientation vectors \( \vec{P} \) and \( \vec{Q} \), where \( \vec{P} \) is a unit vector from the dynamical center which points at perigee of the orbit and \( \vec{Q} \) is advanced to \( \vec{P} \) by a right angle in the plane and direction of motion that is

\[
\begin{align*}
P_{\omega_i} &= \cos(\omega_i) \cos(\Omega_i) - \sin(\omega_i) \sin(\Omega_i) \cos(I_i), \quad (3.1.1) \\
P_{\varpi_i} &= \cos(\omega_i) \sin(\Omega_i) + \sin(\omega_i) \cos(\Omega_i) \cos(I_i), \quad (3.1.2) \\
P_{\iota_i} &= \sin(\omega_i) \sin(I_i), \quad (3.1.3) \\
Q_{\omega_i} &= -\sin(\omega_i) \cos(\Omega_i) + \cos(\omega_i) \sin(\Omega_i) \cos(I_i), \quad (3.1.4) \\
Q_{\varpi_i} &= -\sin(\omega_i) \sin(\Omega_i) + \cos(\omega_i) \cos(\Omega_i) \cos(I_i), \quad (3.1.5) \\
Q_{\iota_i} &= \cos(\omega_i) \sin(I_i), \quad (3.1.6)
\end{align*}
\]

where \( \omega_i \) is the argument of perigee, \( \Omega_i \) is longitude of the ascending node and \( I_i \) is the orbital inclination (Escobal, 1965 and Kozai, 1959).

Now

\[
\begin{align*}
\vec{r}_1 \cdot \vec{r}_2 &= (\xi_1 P_1 + \eta_1 Q_1) \cdot (\xi_2 P_2 + \eta_2 Q_2) \\
&= A_1 \xi_1 \xi_2 + A_2 \eta_1 \eta_2 + A_3 \eta_1 \xi_2 + A_4 \xi_1 \eta_2, \quad (3.2)
\end{align*}
\]

where

\[
A_1 = \frac{P_1 \cdot P_2}{P_1^2}, \quad A_2 = \frac{Q_1 \cdot P_2}{P_1^2}, \quad A_3 = \frac{Q_1 \cdot Q_2}{Q_1^2}, \quad A_4 = \frac{Q_2 \cdot P_2}{Q_2^2}. \quad (3.3)
\]

Using Eqs. (3.1) and (3.3) into Eq. (3.2) we get

\[
\begin{align*}
\vec{r}_1 \cdot \vec{r}_2 &= p_1 p_2 [D_1 \cos(f_2) \cos(g_1 - f_1) + D_2 \sin(f_2) \cos(\Psi_1 - f_2)] \\
&= \frac{1}{[1 + e_1 \cos(f_1)][1 + e_2 \cos(f_2)]}, \quad (3.4)
\end{align*}
\]

where

\[
\begin{align*}
\sin(g_1) &= \frac{A_2 \sqrt{A_1^2 + A_2^2}}{A_1}; \\
\cos(g_1) &= \frac{A_1}{\sqrt{A_1^2 + A_2^2}};
\end{align*}
\]

\[
\begin{align*}
\sin(\Psi_1) &= \frac{A_4}{\sqrt{A_1^2 + A_4^2}}; \\
\cos(\Psi_1) &= \frac{A_3}{\sqrt{A_1^2 + A_4^2}};
\end{align*}
\]

and

\[
D_1 = \sqrt{A_1^2 + A_2^2}, \quad D_2 = \sqrt{A_3^2 + A_4^2}. \quad (3.5)
\]

Then the Eq. (2.6) become

\[
\begin{align*}
R &= p_1^2 p_2^2 [D_1 \cos(f_2) \cos(g_1 - f_1) + D_2 \sin(f_2) \cos(\Psi_1 - f_2)]^2 \\
&- p_1^2 p_2^2 + S^2 (p_1^2 [1 + e_2 \cos(f_2)]^2 + p_2^2 [1 + e_1 \cos(f_1)]^2) \\
&- 2S^2 p_1 p_2 [D_1 \cos(f_2) \cos(g_1 - f_1) + D_2 \sin(f_2) \cos(\Psi_1 - f_1)] \\
&\times [1 + e_1 \cos(f_1)][1 + e_2 \cos(f_2)]. \quad (3.6)
\end{align*}
\]

If the two satellites in the same orbital plane we have

\[
\begin{align*}
P_1 &= P_2, \quad Q_1 = Q_2 \Rightarrow A_1 = 1, \quad A_2 = 0, \quad A_3 = 0, \quad A_4 = 1 \Rightarrow D_1 = 1, \quad D_2 = 1, \quad g_1 = 0, \quad \Psi_1 = 90^\circ.
\end{align*}
\]

4. Variation of the orbital’s elements

The expansion of a set of elements \([a, e, M, i, \Omega, \omega] \) about some epoch time \( t_0 \) can be attempted by Taylor expansions as
The potential of the Earth is given (Escobal, 1965), by

\[ V = \frac{k^2m}{r} \left[ 1 + \frac{J_2}{2r^2}(1 - 3 \sin^2 \delta) + \frac{J_3}{2r^3}(3 - 5 \sin^2 \delta) \sin \delta \right. \\
\left. - \frac{J_4}{8r^4}(3 - 30 \sin^2 \delta - 35 \sin^4 \delta) \\
- \frac{J_5}{8r^5}(15 - 70 \sin^2 \delta + 63 \sin^4 \delta) \sin \delta + \cdots \right] \]

where

\( m \): the mass of the Earth, \( k \): the gravitational constant, \( J_i \): coefficient of the \( i \)th harmonic.

Since the equation of a conic is

\[ r = a(1 - e^2) \left( 1 + e \cos f \right) \]

and we have the relation

\[ \sin(\delta) = \sin(i) \sin(f + \omega), \]

then perturbative function will be in the following form to the order of \( J_2 \) (Escobal, 1965)

\[ \tilde{R} = k^2 m \left[ \frac{3}{2} \frac{J_2}{a^3} \left( \frac{a}{r} \right) \left( \frac{1}{3} - \frac{1}{2} \sin^2 i \right) + \frac{1}{4} \frac{J_3}{a^3} \left( \frac{a}{r} \right) \sin 2(f + \omega) \right. \]

\[ \left. - \frac{1}{8} \frac{J_4}{a^3} \left( \frac{a}{r} \right) \left( \frac{15}{8} \sin^2 i - \frac{3}{2} \right) \sin(f + \omega) - \frac{3}{8} \frac{J_5}{a^3} \left( \frac{a}{r} \right) \sin 3(f + \omega) \right] \]

\[ \sin i - \frac{35}{8} \frac{J_6}{a^3} \left( \frac{a}{r} \right) \sin^3 i \]

\[ + \sin^3 i \left( \frac{3}{7} - \frac{3}{8} \sin^2 i \right) \cos(f + \omega) + \frac{1}{8} \frac{J_8}{a^3} \sin^4 i \cos 4(f + \omega) \]

where

\( a \): is the semi-major axis of the orbit, \( e \): is the orbital eccentricity, \( f \): is the true anomaly, \( i \): is the orbit inclination, \( \omega \): is the argument of perigee.

We are interested with the secular variation, so we omitted the short and long period terms from the perturbative function, which will be written as

\[ \tilde{R} = k^2 m \left[ \frac{3}{2} \frac{J_2}{a^3} \left( \frac{a}{r} \right) \left( \frac{1}{3} - \frac{1}{2} \sin^2 i \right) \right] \]

Or for only \( J_2 \)-gravity

\[ \tilde{R} \approx k^2 m \left[ \frac{3}{2} \frac{J_2}{a^3} \left( \frac{a}{r} \right) \left( \frac{1}{3} - \frac{1}{2} \sin^2 i \right) \right] \]

Since we are not interested with the periodic variation of the elements, the previous equation may be averaged over the given revolution as

\[ \left( \frac{a}{r} \right)^2 = \frac{1}{2\pi} \int_0^{2\pi} \left( \frac{a}{r} \right)^2 dM = \frac{1}{2\pi} \int_0^{2\pi} \frac{k^2m}{r} \left( \frac{a}{r} \right)^2 \frac{1}{\sqrt{1 - e^2}} df \]

\[ = (1 - e^2)^{3/2}. \]

Then

\[ \tilde{R} = k^2 m \left[ \frac{3}{2} \frac{J_2}{a^3} (1 - e^2)^{3/2} \left( \frac{1}{3} - \frac{1}{2} \sin^2 i \right) \right]. \] (4.1)

4.1. Rate of change of the elements

It is possible to verify that the perturbative function, the elements of the orbit and time are related by Lagrange’s planetary equations.
\[ \frac{da}{dt} = 2 \frac{\partial \tilde{R}}{\partial M} \]

\[ \frac{de}{dt} = \frac{1 - e^2}{na^2 e} \frac{\partial \tilde{R}}{\partial M} - \frac{\sqrt{1 - e^2}}{na^2 e} \frac{\partial \tilde{R}}{\partial \omega} \]

\[ \frac{d\omega}{dt} = - \frac{\cos I}{na^2 \sin \sqrt{1 - e^2}} \frac{\partial \tilde{R}}{\partial M} - \frac{\sqrt{1 - e^2}}{na^2 \sin \sqrt{1 - e^2}} \frac{\partial \tilde{R}}{\partial e}, \quad (4.2.1) \]

From the Eq. (4.1) of \( \tilde{R} \) and by the previous equations we find that parameters \( a, e, I, \) and \( \omega \) experience no secular variations (Escobal, 1965; Sterne, 1960 and Brouwer, 1959).

Mathematically, the mean anomaly \( M \) is defined as

\[ M = n(t - T), \]

where

\( T \): the time of perifocal passage, \( n \): the unperturbed mean motion.

To calculate the variations in the parameters which experience the secular variations, we find that

\[ \frac{dM}{dt} = n \left[ 1 + \frac{3}{2} J_2 \frac{1 - e^2}{p^2} \left( 1 - \frac{3}{2} \sin^2 I \right) \right] \equiv \bar{n}, \]

and

\[ \int \frac{d\Omega}{\cos I} dt = - \frac{3}{2} J_2 \cos I \int \bar{n} dt \]

Or

\[ \Omega = \Omega_0 + \left( \frac{3}{2} \frac{J_2}{p^2} \cos I \right) \bar{n}(t - t_0). \]

By same way we find that

\[ \omega = \omega_0 + \frac{3}{2} J_2 \left( 2 - \frac{5}{2} \sin^2 I \right) \bar{n}(t - t_0), \]

and

\[ M = M_0 + \left( \frac{3}{2} J_2 \frac{1 - e^2}{p^2} \left( 2 - \frac{5}{2} \sin^2 I \right) \right) \bar{n}(t - t_0) + n_0(t - t_0). \]

5. Computational algorithm

In what follows computational algorithm of the mutual visibility between two Earth Satellites will be established whatever the types of their orbits may be.

**Purpose:** Mutual visibility between two Earth satellites.

**Input:**

\( a_i, q_i, e_i, I_i, \omega_i, \Omega_i, T_i; \) \( i = 1, 2, S, \Delta, t, k, \mu. \)

**Computational sequence:**

1. If \( e_i > 1 \) then \( n_i = k \sqrt{\frac{a_i}{e_i}} \) and \( q_i = a_i (1 - e_i). \)
2. If \( e_i = 1 \) then \( n_i = k \sqrt{\frac{a_i}{e_i}} \) and \( q_i = a_i (1 - e_i). \)
3. If \( e_i < 1 \) then \( n_i = k \sqrt{\frac{a_i}{e_i}} \) and \( q_i = a_i (1 - e_i). \)
4. \( M_i = n_i(t - T_i). \)
5. If \( e_i > 1 \) then solve \( F_i \) from Kepler equation of hyperbolic orbit using Newton method and then \( f_i \) as follows
   (a) Let \( (F_i)_{0} = 6 M_i. \)
   (b) \( (F_i)_{n+1} = (F_i)_{n} + \left( \frac{M_i - e_i \sin F \cos F - (F_i)}{n_i \cos F_{n+1} - \frac{1}{n_i}} \right) \)
   (c) If \( |F_i|_{n+1} - (F_i)|_n > 0.00000001 \) go to b else \( F_i = (F_i)_{n+1}. \)
   (d) \( f_i = \tan^{-1} \left( -\sin F_i \sqrt{\frac{a_i}{e_i}} \cos F_i \right) \) and end.
6. If \( e_i < 1 \) then solve \( f_i \) from Barkar's equation as follows
   (a) Let \( A_i = \frac{3}{2} M_i. \)
   (b) \( B_i = \left( \frac{A_i}{I_i} + 1 \right)^{1/3}. \)
   (c) \( C_i = B_i - \frac{2}{3}. \)
   (d) \( f_i = 2 \tan^{-1}(C_i). \)
7. If \( e_i < 1 \) then solve for \( E_i \) from Kepler's equation using Newton method and then \( f_i \) as follows
   (a) Let \( E_i = M_i. \)
   (b) \( (E_i)_{n+1} = (E_i)_{n} + \frac{E_i - e_i \sin E_i - (E_i)}{e_i \cos E_i - \frac{1}{e_i}} \)
   (c) If \( |E_i|_{n+1} - (E_i)|_n > 0.00000001 \) go to b else \( E_i = (E_i)_{n+1}. \)
   (d) \( f_i = \tan^{-1} \left( \frac{\sin E_i \sqrt{\frac{a_i}{e_i}}}{\cos E_i - e_i} \right) \) and end.
8. \( r_i = \frac{(1 + e_i) h_i}{1 + e_i \cos F_i} \)
9. \( \bar{P} \) and \( \bar{Q} \) from Eq. (3).
10. \( \bar{z}_i = r_i \cos(f_i) \) and \( \eta_i = r_i \sin(f_i). \)
11. \( r_i = \bar{z}_i + \eta_i \bar{Q}_i. \)
12. Compute the mutual visibility function \( R \) from Eq. (2.6). Whenever this value is negative, the satellites can see each other at the given time \( t. \)
13. The algorithm is completed.

6. Results and conclusion

6.1. Test orbits

We will take as an example the following seven satellites. Satellite_1, Satellite_2, Satellite_3 and Satellite_4 are nearly circular, SatellitPe_5 is elliptical orbit, but Satellite_6 is parabolic orbit and finally Satellite_7 is hyperbolic orbit (http://celestrak.com). The three-line elements of seven satellites are:
6.2. Numerical results

We apply the above algorithm on some of the seven satellites to get the time and date of what satellite observes the other. Tables 1–7 give these results.

| Satellite_1: | EGYPTSAT 1 |
|-------------|------------|
| 1 13117U 07012A 08142.74302347 .00000033 00000-0 13654-4 0 2585 |
| 2 31117 098.0526 218.7638 0007144 061.2019 298.9894 14.69887657 58828 |

| Satellite_2: | TRMM |
|-------------|------|
| 1 25063U 97074A 08141.84184490 .00000000 00000-0 41919-4 0 7792 |
| 2 25063 034.9668 053.5885 0001034 271.1427 088.9226 15.558752 63184 |

| Satellite_3: | GOES 3 |
|-------------|-------|
| 1 10953U 78062A 08140.64132336 .00000110 00000-0 10000-3 0 1137 |
| 2 10953 014.2164 003.1968 0001795 336.4858 023.4617 01.00280027 62724 |

| Satellite_4: | NOAA 3 |
|-------------|-------|
| 1 06920U 73086A 08141.92603915 .00000000 00000-0 +10000-3 0 00067 |
| 2 06920 101.7584 171.9430 0006223 187.3360 172.7614 12.4028935563642 |

| Satellite_5: | NAVSTAR 46 |
|-------------|-----------|
| 1 25933U 99055A 08142.141232352 .00000000 00000-0 +10000-3 0 00126 |
| 2 25933 051.0650 222.9459 0007944 032.8625 327.6958 02.00868102 63184 |

| Satellite_6: | Parabola |
|-------------|---------|
| 1 00000U 00000A 08141.33960007 .00000000 00000-0 0 000001 |
| 2 00000 035.3423 067.8765 001.0000 253.7654 138.0967 02.65768444 63184 |

| Satellite_7: | Hyp_1 |
|-------------|-------|
| 1 00000U 00000A 08141.89332000 .00000000 00000-0 0 000001 |
| 2 00000 072.8721 105.6746 001.164 065.8757 221.4654 02.00868102 63184 |

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Table 1  EGYPTSAT_1 and TRMM are visible during the times.

| Date | Time               | to     | Date | Time               |
|------|--------------------|--------|------|--------------------|
|      | Year   | Month | Day  | Hour | Minute | Seconds | Year   | Month | Day  | Hour | Minute | Seconds |
| 2008 | 5      | 22    |      | 12   | 22     | 25.99    | To     | 2008 | 5    | 22   | 12     | 29     | 50.99    |
|      | 13     | 10    | 39.98| 13   | 16     | 49.98    |        | 14   | 3    | 41.96|        |        |         |
|      | 14     | 58    | 56.96| 14   | 50     | 24.94    |        | 22   | 46   | 30.79|        |        |         |
|      | 22     | 43    | 18.79| 22   | 46     | 30.79    |        | 23   | 34   | 54.77|        |        |         |
| 2008 | 5      | 23    |      | 0    | 16     | 56.77    | To     | 2008 | 5    | 23   | 0     | 23     | 15.76    |
|      | 1      | 3     | 58.74| 1     | 11     | 25.74    |        | 1    | 59   | 35.73|        |        |         |
|      | 2      | 38    | 07.71| 2     | 47     | 37.71    |        | 3    | 35   | 40.70|        |        |         |
|      | 3      | 25    | 15.70| 4     | 23     | 36.68    |        | 5    | 11   | 31.66|        |        |         |
|      | 4      | 59    | 45.67| 6     | 47     | 59.67    |        | 7    | 34   | 50.62|        |        |         |
|      | 5      | 47    | 06.65| 6     | 47     | 59.67    |        | 7    | 34   | 50.62|        |        |         |
|      | 6      | 34    | 28.64| 5     | 11     | 22.64    |        | 8    | 15   | 16.60|        |        |         |
|      | 7      | 21    | 57.62| 8     | 15     | 16.60    |        | 9    | 2    | 52.99|        |        |         |
|      | 8      | 9     | 26.61| 9     | 10     | 45.61    |        | 10   | 57   | 36.57|        |        |         |
|      | 9      | 5     | 47.58| 11   | 32     | 23.58    |        | 12   | 08   | 28.54|        |        |         |

Table 2  EGYPTSAT_1 and GOES_3 are visible during the times.

| Date | Time               | to     | Date | Time               |
|------|--------------------|--------|------|--------------------|
|      | Year   | Month | Day  | Hour | Minute | Seconds | Year   | Month | Day  | Hour | Minute | Seconds |
| 2008 | 05     | 22    |      | 12   | 14     | 01.00    | To     | 2008 | 05   | 22   | 13     | 17     | 55.97    |
|      | 13     | 47    | 46.96| 14   | 09     | 51.96    |        | 15   | 39   | 18.93|        |        |         |
|      | 14     | 28    | 06.95| 16   | 21     | 13.91    |        | 17   | 13   | 25.90|        |        |         |
|      | 15     | 23    | 50.93| 18   | 19     | 30.87    |        | 19   | 32   | 47.85|        |        |         |
|      | 16     | 06    | 46.92| 20   | 27     | 18.83    |        | 21   | 08   | 59.82|        |        |         |
|      | 17     | 00    | 01.90| 21   | 44     | 05.79    |        | 22   | 44   | 05.79|        |        |         |
|      | 17     | 43    | 17.89| 23   | 08     | 51.76    |        | 2008 | 05   | 23   | 00     | 08     | 51.76    |
|      | 18     | 35    | 44.87| 2008 | 05    | 23       |        | 01   | 30   | 38.74|        |        |         |
|      | 19     | 18    | 05.86| 02   | 22     | 33.72    |        | 03   | 01   | 10.71|        |        |         |
|      | 20     | 10    | 12.84| 04   | 35     | 24.68    |        | 05   | 27   | 48.66|        |        |         |
|      | 21     | 41    | 22.81| 05   | 27     | 48.66    |        | 06   | 11   | 02.64|        |        |         |
|      | 22     | 39.57| 07.62| 07   | 47     | 11.61    |        | 08   | 42   | 26.60|        |        |         |
|      | 23     | 04    | 17.78| 09   | 23     | 18.58    |        | 09   | 57   | 40.55|        |        |         |
|      | 24     | 25.59| 14.57| 10   | 57   | 40.55    |        | 11   | 15   | 36.55|        |        |         |
|      | 25     | 14    | 36.55| 12   | 00     | 00.00    |        | 13   | 00   | 00.00|        |        |         |
Table 3  Hyp_1 and Parabola are visible during the times.

| Date | Time | to | Date | Time |
|------|------|----|------|------|
| Year | Month | Day | Hour | Minute | Seconds | Year | Month | Day | Hour | Minute | Seconds |
| 2008 | 05 | 22 | 12 | 00 | 00.00 | To | 2008 | 05 | 23 | 12 | 00 | 00.00 |

Table 4  EGYPTSAT_1 and Hyp_1 are visible during the times.

| Date | Time | to | Date | Time |
|------|------|----|------|------|
| Year | Month | Day | Hour | Minute | Seconds | Year | Month | Day | Hour | Minute | Seconds |
| 2008 | 05 | 22 | 12 | 24 | 36.00 | To | 2008 | 05 | 22 | 12 | 46 | 47.98 |
| 13 | 13 | 38.98 | 14 | 24 | 43.95 |
| 14 | 51 | 35.94 | 15 | 13 | 41.94 |
| 15 | 40 | 28.93 | 16 | 02 | 39.92 |
| 16 | 29 | 32.91 | 16 | 51 | 37.91 |
| 17 | 18 | 25.90 | 17 | 40 | 36.89 |
| 18 | 07 | 29.88 | 18 | 29 | 34.87 |
| 18 | 56 | 22.86 | 19 | 18 | 32.86 |
| 19 | 45 | 26.85 | 20 | 07 | 30.84 |
| 20 | 34 | 19.83 | 20 | 56 | 28.83 |
| 21 | 23 | 23.82 | 21 | 45 | 26.81 |
| 22 | 12 | 17.80 | 22 | 34 | 25.79 |
| 23 | 01 | 20.78 | 23 | 23 | 23.78 |
| 23 | 50 | 14.77 | 2008 | 05 | 23 | 00 | 12 | 22.76 |

Table 5  GOES_3 and Parabola are visible in the time.

| Date | Time | to | Date | Time |
|------|------|----|------|------|
| Year | Month | Day | Hour | Minute | Seconds | Year | Month | Day | Hour | Minute | Seconds |
| 2008 | 05 | 22 | 12 | 00 | 00.00 | To | 2008 | 05 | 23 | 12 | 00 | 00.00 |

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### Table 6  
EGYSAT_1 and Parabola are visible in times.

| Year | Month | Day | Hour | Minute | Seconds | to | Year | Month | Day | Hour | Minute | Seconds |
|------|-------|-----|------|--------|---------|-----|------|-------|-----|------|--------|---------|
| 2008 | 05    | 22  | 12   | 00     | 00.00   | to | 2008 | 05    | 22  | 12   | 01     | 50.00   |
|      |       |     | 12   | 29     | 50.99   |    |      |       |     | 12   | 30     | 52.99   |
|      |       |     | 13   | 18     | 51.97   |    |      |       |     | 13   | 19     | 53.97   |
|      |       |     | 14   | 07     | 48.96   |    |      |       |     | 14   | 08     | 51.95   |
|      |       |     | 14   | 56     | 49.94   |    |      |       |     | 15   | 06     | 49.92   |
|      |       |     | 15   | 34     | 46.93   |    |      |       |     | 16   | 15     | 43.90   |
|      |       |     | 17   | 12     | 44.89   |    |      |       |     | 17   | 13     | 47.89   |
|      |       |     | 18   | 12     | 45.88   |    |      |       |     | 18   | 13     | 48.87   |
|      |       |     | 19   | 01     | 42.86   |    |      |       |     | 19   | 02     | 45.86   |
|      |       |     | 19   | 30     | 44.85   |    |      |       |     | 20   | 00     | 39.84   |
|      |       |     | 20   | 39     | 40.83   |    |      |       |     | 21   | 00     | 43.82   |
|      |       |     | 21   | 28     | 42.81   |    |      |       |     | 22   | 01     | 37.81   |
|      |       |     | 22   | 17     | 38.80   |    |      |       |     | 23   | 06     | 40.78   |
|      |       |     | 23   | 55     | 36.77   |    | 2008 | 05    | 23  | 00   | 16     | 39.76   |

Table 7  
TRMM and Hyp_1 are visible during the times.

| Year | Month | Day | Hour | Minute | Seconds | to | Year | Month | Day | Hour | Minute | Seconds |
|------|-------|-----|------|--------|---------|-----|------|-------|-----|------|--------|---------|
| 2008 | 05    | 22  | 12   | 10     | 49.00   | to | 2008 | 05    | 22  | 12   | 20     | 58.99   |
|      |       |     | 12   | 57     | 3.98    |    |      |       |     | 13   | 07     | 14.98   |
|      |       |     | 13   | 43     | 22.97   |    |      |       |     | 14   | 13     | 32.96   |
|      |       |     | 14   | 29     | 37.95   |    |      |       |     | 15   | 15     | 48.95   |
|      |       |     | 15   | 15     | 56.94   |    |      |       |     | 16   | 15     | 06.93   |
|      |       |     | 16   | 02     | 11.92   |    |      |       |     | 16   | 16     | 22.92   |
|      |       |     | 16   | 48     | 30.91   |    |      |       |     | 17   | 16     | 41.90   |
|      |       |     | 17   | 34     | 45.89   |    |      |       |     | 18   | 18     | 56.89   |
|      |       |     | 18   | 21     | 04.88   |    |      |       |     | 19   | 19     | 30.86   |
|      |       |     | 19   | 53     | 38.85   |    |      |       |     | 20   | 20     | 48.84   |
|      |       |     | 20   | 39     | 53.83   |    |      |       |     | 21   | 21     | 04.83   |
|      |       |     | 21   | 26     | 12.82   |    |      |       |     | 22   | 22     | 22.81   |
|      |       |     | 22   | 12     | 27.80   |    |      |       |     | 22   | 23     | 38.80   |
|      |       |     | 22   | 58     | 46.79   |    |      |       |     | 23   | 23     | 56.78   |
|      |       |     | 23   | 45     | 01.77   |    |      |       |     | 12    | 55     | 12.77   |

| Year | Month | Day | Hour | Minute | Seconds | to | Year | Month | Day | Hour | Minute | Seconds |
|------|-------|-----|------|--------|---------|-----|------|-------|-----|------|--------|---------|
| 2008 | 05    | 23  | 00   | 31     | 20.76   | to | 2008 | 05    | 23  | 00   | 41     | 30.75   |
|      |       |     | 01   | 17     | 35.74   |    |      |       |     | 01   | 27     | 46.74   |
|      |       |     | 02   | 03     | 54.73   |    |      |       |     | 02   | 14     | 04.72   |
|      |       |     | 02   | 50     | 09.71   |    |      |       |     | 03   | 00     | 20.71   |
|      |       |     | 03   | 36     | 28.70   |    |      |       |     | 04   | 36     | 38.69   |
|      |       |     | 04   | 22     | 43.68   |    |      |       |     | 05   | 32     | 54.68   |
|      |       |     | 05   | 09     | 01.67   |    |      |       |     | 05   | 19     | 12.66   |
|      |       |     | 05   | 55     | 17.65   |    |      |       |     | 06   | 05     | 27.65   |
|      |       |     | 06   | 41     | 35.63   |    |      |       |     | 06   | 51     | 46.63   |
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### Table 7 (continued)  TRMM and Hyp.1 are visible during the times.

| Date | Time to | Date | Time |
|------|---------|------|------|
|      | Year    | Month | Day  |
|      | 07      | 27    | 51.62|
|      | 08      | 14    | 09.60|
|      | 09      | 00    | 24.59|
|      | 09      | 46    | 43.57|
|      | 10      | 32    | 58.56|
|      | 11      | 19    | 17.54|
|      | 07      | 38    | 01.62|
|      | 08      | 24    | 19.60|
|      | 09      | 10    | 35.59|
|      | 09      | 56    | 53.57|
|      | 10      | 43    | 09.56|
|      | 11      | 29    | 27.54|

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