A Computational Model for Quantum Measurement

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Is the dynamical evolution of physical systems objectively a manifestation of information processing by the universe? We find that an affirmative answer has important consequences for the measurement problem. In particular, we calculate the amount of quantum information processing involved in the evolution of physical systems, assuming a finite degree of fine-graining of Hilbert space. This assumption is shown to imply that there is a finite capacity to sustain the immense entanglement that measurement entails. When this capacity is overwhelmed, the system’s unitary evolution becomes computationally unstable and the system suffers an information transition (“collapse”). Classical behavior arises from the rapid cycles of unitary evolution and information transition. Thus, the fine-graining of Hilbert space determines the location of the “Heisenberg cut”, the mesoscopic threshold separating the microscopic, quantum system from the macroscopic, classical environment. The model can be viewed as a probabilistic complement to decoherence, that completes the measurement process by turning decohered improper mixtures of states into proper mixtures. It is shown to provide a natural resolution to the measurement problem and the basis problem.

KEY WORDS: Quantum measurement theory; quantum information processing; entanglement production.

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1. INTRODUCTION

Inspite of the tremendous success of standard quantum mechanics, its interpretational aspects continue to puzzle us, particularly in regard to measurement and its extrapolation to the macroscopic level.\(^1\)\(^2\) Although an overwhelming number of current studies and applications of quantum mechanics do not depend on the philosophical resolutions of these difficulties, they will become important for understanding future experiments where advancing technology will permit one to probe mesoscopic phenomena.\(^3\)

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Further, they can help to resolve the measurement problem, a subtle but arguably important problem in quantum epistemology and foundations. In simple words, it is concerned with understanding how macroscopic phenomena, in specific measurement outcomes, are classical even though the underlying microscopic states exist as quantum mechanical superpositions.\(^{(4)}\) Let a system \(S\) to be measured be in the pure state 
\[
|\psi\rangle = \sum_{i=1}^{n} a_i|\hat{\theta}\rangle,
\]
with \(\sum_{i=1}^{n} |a_i|^2 = 1\) where \(|\hat{\theta}\rangle\) are a complete set of eigenstates of some observable \(\hat{A}\) that can be measured by a measuring apparatus \(M\). Formally, measurement is represented by the action of the projection operator \(\pi_i = |\hat{\theta}\rangle\langle\hat{\theta}|\) on \(|\psi\rangle\), with probability \(|a_i|^2\), and more generally, by the action of arbitrary measurement operators.\(^{(5,6)}\) Under such action, the system \(S\) is said to undergo an irreversible “reduction of the state vector,” “collapse of the wavefunction,” or “quantum leap/jump” to an eigenstate. In contrast, Schrödinger evolution is given by the deterministic action of a unitary operator. This difference in the actions of unitary evolution and measurement is the simplest manifestation of the measurement problem.

To be precise, we need to take into consideration the measuring apparatus. Following von Neumann,\(^{(7)}\) we visualize the measurement as beginning with \(S\) interacting with \(M\) in the “ready” state \(|R\rangle\). During this “pre-measurement” phase, the interaction Hamiltonian entangles \(S\) with \(M\):

Assuming the apparatus can be characterized by a single degree of freedom, represented by the states \(|\xi_i\rangle\) that span the pointer basis, one obtains the state

\[
|\Psi\rangle = \sum_{i} a_i|\hat{\theta}\rangle|\xi_i\rangle,
\]

whereby every state \(|\hat{\theta}\rangle\) gets correlated with a definite macroscopic apparatus state \(|\xi_i\rangle\). Equation (1) implies a superposition of macroscopic configurations of \(M\), contrary to our everyday experience of determinateness of macroscopic objects. Expressed verbally, how/when do the logical AND’s (the \(\oplus\)’s in the summation over \(i\)) in Eq. (1) becomes OR’s? This is the measurement problem. As recently demonstrated in Ref. 8, such a projective operation represents a break-down in the principle of linear superposition for state vectors, irrespective of the details of the measuring apparatus and the measurement process.

The oldest attempt to make sense of this dual dynamics in quantum mechanics is the Copenhagen interpretation, due mostly to Bohr and Heisenberg.\(^{(1)}\) It was not a single interpretation, but rather a collection of somewhat different viewpoints, the main thrust being that physical systems evolve as quantum objects, whereas measuring apparatuses and their outcomes are to be represented classically: if a quantity \(Q\) is measured in system \(S\) at time \(t\) then \(Q\) has a particular value in \(S\) at time \(t\). A difficulty