**Analysis and Optimization of an Intelligent Reflecting Surface-assisted System with Interference**

**Yuhang Jia**
Shanghai Jiao Tong University

**Chenceng Ye**
Shanghai Jiao Tong University

**Ying Cui**
Shanghai Jiao Tong University

---

**Abstract**—In this paper, we study an intelligent reflecting surface (IRS)-assisted system where a multi-antenna base station (BS) serves a single-antenna user with the help of a multi-element IRS in the presence of interference generated by a multi-antenna BS serving its own single-antenna user. The signal and interference links via the IRS are modeled with Rician fading. To reduce phase adjustment cost, we adopt quasi-static phase shift design where the phase shifts do not change with the instantaneous channel state information (CSI). Maximum Ratio Transmission (MRT) are adopted at the two BSs to enhance the receive signals at their own users. First, we obtain a tractable expression of the ergodic rate, leading to a non-convex optimization problem. We obtain a globally optimal solution under certain system parameters, and propose an iterative algorithm based on parallel coordinate descent (PCD), to obtain a stationary point under arbitrary system parameters. Finally, we numerically verify the analytical results and demonstrate the notable gains of the proposal solutions. To the best of our knowledge, this is the first work that studies the analysis and optimization of the ergodic rate of an IRS-assisted system in the presence of interference.

**Index Terms**—Intelligent reflecting surface (IRS), multi-antenna, interference, ergodic rate, phase shift optimization.

---

**I. INTRODUCTION**

Recently, intelligent reflecting surface (IRS), consisting of nearly passive, low-cost, reflecting elements with reconfigurable parameters, is envisioned to serve as a promising solution for improving spectrum and energy efficiency. Most existing works consider IRS-assisted systems where one multi-antenna base station (BS) serves one or multiple users with the help of one multi-element IRS [1–9], or multiple multi-element IRSs [10], and investigate the joint optimization of the beamformer at the BS and the phase shifts at the IRS to maximally improve system performance. According to whether the phase shifts are adaptive to instantaneous channel state information (CSI) or not, these works [1–10] can be classified into two categories, which are illustrated below.

In one category [1–8], the phase shifts are adjusted based on instantaneous CSI which is assumed to be known. For instance, in [1–8], the authors consider the maximization of the sum rate [1], weighted sum rate [2], or energy efficiency [3–7], and the minimization of the transmission power [8]. The authors propose iterative algorithms to obtain locally optimal solutions or nearly optimal solutions of the non-convex problems in [1–8]. In the other category [9], [10], the phase shifts of the IRS are determined by statistics of CSI and do not change with instantaneous CSI, under the assumption that the IRS-assisted links follow Rician fading. In [9], the authors consider fast varying non-line-of-sight (NLoS) components, and maximize the ergodic rate. In contrast, in [10], the authors consider slowly varying NLoS components, and minimize the outage probability. By analyzing problem structures, closed-form optimal phase shifts are obtained for the non-convex problems in [9, 10]. The quasi-static phase shift design in the second one has lower implementation cost, owing to less frequent phase shift adjustment.

Note that all the aforementioned works [1–10] ignore interference from other transmitters, when investigating IRS-assisted communications. However, in practical wireless networks, interference usually has a severe impact, especially in dense networks or for cell-edge users. It is thus critical to take into account the role of interference in designing IRS-assisted systems. In [11], the authors optimize instantaneous CSI-adaptive phase shift design and BS beamforming to maximize the weighted sum rate of an IRS-assisted system in the presence of an interference BS. An algorithm is proposed to obtain a stationary point of an approximate problem of the original non-convex problem. As the instantaneous CSI-adaptive design in [11] has higher implementation cost, it is highly desirable to obtain cost-efficient quasi-static design for IRS-assisted systems with interference.

In this paper, we shall shed some light on the aforementioned issues. We consider an IRS-assisted system where a multi-antenna BS serves a single-antenna user with the help of a multi-element IRS, in the presence of interference generated by a multi-antenna BS serving its own single-antenna user. The antennas at the two BSs and the reflecting elements at the IRS are arranged in uniform rectangular arrays (URAs). The signal and interference links via the IRS are modeled with Rayleigh fading. As in [9], we assume that the line-of-sight (LoS) components do not change (during the considered time duration) and the NLoS components vary fast. To reduce phase adjustment cost, we adopt quasi-static phase shift design, where the phase shifts do not change with instantaneous CSI, but adapt to only CSI statistics. MRT is adopted at the two BSs to enhance the receive signals at their own users. First, we obtain a tractable expression of the ergodic rate. Then, we optimize the phase shifts to maximize the ergodic rate, leading to a non-convex optimization problem. Under certain system parameters, we obtain a globally...
optimal solution. Under the general system parameters, we propose an iterative algorithm based on parallel coordinate descent (PCD), to obtain a stationary point of the non-convex problem. The proposed PCD algorithm is particularly suitable for systems with large-scale IRS and multi-core processors which support parallel computing, compared with the state-of-the-art algorithms, i.e., the block coordinate descent (BCD) algorithm and minorization maximization (MM) algorithm in [8]. Finally, by numerical results, we verify analytical results and demonstrate notable gains of the proposed solutions over existing schemes. We also reveal the specific value of the PCD algorithm for large-scale IRS.

II. SYSTEM MODEL

As shown in Fig. 1, one single-antenna user, say user $U$, is served by a BS with the help of an IRS. The BS is referred to as the signal BS of user $U$ or BS $S$. The IRS is installed on the wall of a high-rise building. Another BS serving its own single-antenna user, say user $U'$, causes interference to user $U$, and hence is referred to as the interference BS of user $U$ or BS $I$. The signal BS and the IRS are far from user $U$'. The signal BS and interference BS are equipped with URAs of $M_S \times N_S$ antennas and $M_I \times N_I$ antennas, respectively. The IRS is equipped with a URA of $M_R \times N_R$ reflector elements. Without loss of generality, we assume $M_r \leq N_c$, where $c = S, I$. For notation simplicity, define $\mathcal{M}_c \triangleq \{1, 2, ..., M_c\}, N_c \triangleq \{1, 2, ..., N_c\}$, where $c = S, I, R$. Suppose that the two users do not move during a certain time period. In this paper, we would like to investigate how the signal BS serves user $U$ with the help of the IRS in the presence of interference.

As scattering is often rich near the ground, we adopt the Rayleigh model for the channels between the BSs and the users. Let $\mathbf{h}_{SU}^H \in \mathbb{C}^{1 \times M_S N_S}$, $\mathbf{h}_{UI}^H \in \mathbb{C}^{1 \times M_I N_I}$ and $\mathbf{h}_{IU}^H \in \mathbb{C}^{1 \times M_I N_I}$ denote the channel vectors for the channel between the signal BS and user $U$, the channel between the interference BS and user $U$, and the channel between the interference BS and user $U'$, respectively. Specifically, $\mathbf{h}_{SU}^H = \sqrt{\sigma_{SU}} \mathbf{h}_{SU}^H$, and $\mathbf{h}_{IU}^H = \sqrt{\alpha_{UI}} \mathbf{h}_{IU}^H$, where $c = S, I$, $\alpha_{SU}, \alpha_{UI} > 0$ represent the distance-dependent path losses, and the elements of $\mathbf{h}_{SU}^H$, $\mathbf{h}_{IU}^H$ are independent and identically distributed according to $CN(0,1)$.

As scattering is much weaker far from the ground, we adopt the Rician fading model for the channels between the BSs and the IRS, and the channel between user $U$ and the IRS. Let $\mathbf{H}_{SR} \in \mathbb{C}^{M_R N_R \times M_S N_S}$, $\mathbf{H}_{IR} \in \mathbb{C}^{M_R N_R \times M_I N_I}$ and $\mathbf{H}_{RU}^H \in \mathbb{C}^{1 \times M_R N_R}$ denote the channel matrices for the channel between the signal BS and the IRS, the channel between the interference BS and the IRS and the channel between the IRS and user $U$, respectively. Specifically,

$$
\mathbf{H}_{Cr} = \sqrt{\alpha_{Cr}} \left( \frac{K_{Cr}}{K_{Cr} + 1} \mathbf{H}_{Cr} + \frac{1}{K_{Cr} + 1} \mathbf{H}_{cR} \right),
$$

$$
\mathbf{h}_{RU}^H = \sqrt{\alpha_{RU}} \left( \frac{K_{RU}}{K_{RU} + 1} \mathbf{h}_{RU}^H + \frac{1}{K_{RU} + 1} \mathbf{h}_{LR}^H \right),
$$

where $\alpha_{Cr}, \alpha_{RU} > 0$ represent the distance-dependent path losses, $K_{Cr}, K_{RU} \geq 0$ denote the Rician factors, $\mathbf{H}_{Cr} \in \mathbb{C}^{M_R N_R \times M_S N_S}$ and $\mathbf{h}_{RU}^H \in \mathbb{C}^{1 \times M_R N_R}$ represent the normalized NLoS components, with elements independently and identically distributed according to $CN(0,1)$, and $\mathbf{H}_{cR} \in \mathbb{C}^{M_R N_R \times M_I N_I}$, $\mathbf{h}_{RU}^H \in \mathbb{C}^{1 \times M_R N_R}$ represent the deterministic normalized LoS components, with unit-modulus elements. Note that $\mathbf{H}_{cR}$ and $\mathbf{h}_{RU}^H$ do not change during the considered time period, as the location of user $U$ is assumed to be invariant.

Let $\lambda$ and $d (\leq \frac{\lambda}{2})$ denote the wavelength of transmission signals and the distance between adjacent elements or antennas in each row and each column of the URAs. Define $f(\theta^h, \theta^v, m, n) \triangleq 2\pi \frac{d}{\lambda} \sin(\theta^v)((m-1)\cos(\theta^h) + (n-1)\sin(\theta^h))$, $A_{m,n}(\theta^h, \theta^v, M, N) \triangleq \left( e^{j\pi(\theta^h, \theta^v, m, n)} \right)^{m=1,...,M, n=1,...,N}$, and $a(\theta^h, \theta^v, M, N) \triangleq \text{rvec} \left( A_{m,n}(\theta^h, \theta^v, M, N) \right)$, where rvec(·) denotes the row vectorization of a matrix. Then, $\mathbf{H}_{cR}$ and $\mathbf{h}_{RU}^H$ are modeled as [12]:

$$
\mathbf{H}_{cR} = a^H(\theta^h_{cR}, \theta^v_{cR}, M_{cR}, N_{cR})a(\phi^h_{cR}, \phi^v_{cR}, M_{c}, N_{c}),
$$

$$
\mathbf{h}_{RU}^H = a^H(\theta^h_{RU}, \theta^v_{RU}, M_{R}, N_{R}),
$$

where $c = S, I$. Here, $\left( \frac{\phi^h_{ij}}{\phi^v_{ij}} \right)$ represents the corresponding azimuth (elevation) angle of arrival and $\left( \frac{\phi^h_{ij}}{\phi^v_{ij}} \right)$ represents the corresponding azimuth (elevation) angle of departure. Please refer to [13] for details.

Let $\Phi \triangleq (\phi_{m,n})_{m \in \mathcal{M}_R, n \in \mathcal{N}_R} \in \mathbb{C}^{M_R \times N_R}$ represent the constant phase shifts of the IRS with $\phi_{m,n}$ being the phase shift of the $(m, n)$-th element of the IRS, where

$$
\phi_{m,n} \in [0, 2\pi), \quad m \in \mathcal{M}_R, n \in \mathcal{N}_R.
$$

For convenience, define $\Phi(\phi) \triangleq \text{diag} \left( \text{rvec} \left( e^{j\phi_{m,n}} \right)_{m \in \mathcal{M}_R, n \in \mathcal{N}_R} \right) \in \mathbb{C}^{M_R N_R \times M_R N_R}$, where diag(·) denotes a square diagonal matrix with the elements of a vector on the main diagonal. The channel of the indirect signal link between the signal BS and user $U$ via the IRS is given by $\mathbf{h}_{RU}^H \Phi(\phi) \mathbf{H}_{SR}$, and the channel of the indirect interference link between the interference BS and user $U$ via the IRS is given by $\mathbf{h}_{RU}^H \Phi(\phi) \mathbf{H}_{IR}$. We consider

$$
1^\text{If } K_{SR} = 0, K_{IR} = 0, \text{ or } K_{RU} = 0, \text{ the corresponding Rician fading reduces down to Rayleigh fading. If } K_{SR} \to \infty, K_{IR} \to \infty, \text{ or } K_{RU} \to \infty, \text{ only the LoS component exists.}$$

Fig. 1: System Model.
linear beamforming at the signal BS and interference BS for serving user $U$ and user $U'$, respectively. Let $w_S \in \mathbb{C}^{M_S N_S \times 1}$ and $w_I \in \mathbb{C}^{M_I N_I \times 1}$ denote the corresponding normalized beamforming vectors, where $\|w_S\|_2^2 = 1$ and $\|w_I\|_2^2 = 1$. Thus, the signal received at user $U$ is expressed as:

$$Y \triangleq \sqrt{P_S} (h_{RU}^H \Phi(\phi) H_{SR} + h_{SU}^H) w_S X_S + \sqrt{P_I} \left( h_{RU}^H \Phi(\phi) H_{IR} + h_{IU}^H \right) w_I X_I + Z,$$

where $P_S$ and $P_I$ are the transmit powers of the signal BS and interference BS, respectively. $X_S$ and $X_I$ are the information symbols for user $U$ and user $U'$, respectively, with $\mathbb{E} \left[ |X_S|^2 \right] = 1$ and $\mathbb{E} \left[ |X_I|^2 \right] = 1$, and $Z \sim \mathcal{CN}(0, \sigma^2)$ is the additive white Gaussian noise (AWGN).

Assume that the CSI of the equivalent channel between the signal BS and user $U$, i.e., $h_{RU}^H \Phi(\phi) H_{SR} + h_{SU}^H$, is known at the signal BS, and the CSI of the channel between the interference BS and user $U$, i.e., $h_{RU}^H$, is known at the interference BS. Note that for any given $\phi$, $h_{RU}^H \Phi(\phi) H_{SR} + h_{SU}^H$ and $h_{IU}^H$ can be estimated by the signal BS via a pilot sent by user $U$, and $h_{RI}^H$ can be estimated by the interference BS via a pilot sent by user $U'$ [3]. To enhance the signals received at user $U$ and user $U'$, respectively, we consider the instantaneous CSI-adaptive MRT at the signal BS and interference BS, respectively:

$$w_S = \frac{(h_{RU}^H \Phi(\phi) H_{SR} + h_{SU}^H)^H}{\|h_{RU}^H \Phi(\phi) H_{SR} + h_{SU}^H\|_2} \in \mathbb{C}^{M_S N_S \times 1},$$

$$w_I = \frac{h_{IU}^H}{\|h_{RU}^H\|_2} \in \mathbb{C}^{M_I N_I \times 1}.$$

Thus, the signal to interference plus noise ratio (SINR) at user $U$ is given by:

$$\gamma(\phi) \triangleq \frac{P_S \left\| h_{RU}^H \Phi(\phi) H_{SR} + h_{SU}^H \right\|^2_2}{P_I \left( \left( h_{RU}^H \Phi(\phi) H_{IR} + h_{IU}^H \right) + \frac{h_{IU}^H}{\|h_{RU}^H\|_2} \right)^2 + \sigma^2}.$$  

Therefore, the ergodic rate for the IRS-assisted transmission with interference is given by:

$$C(\phi) \triangleq \mathbb{E} \left[ \log_2 \left( 1 + \gamma(\phi) \right) \right]. \quad (2)$$

Note that when $P_I = 0$, $M_S = 1$ or $N_S = 1$, and $M_R = 1$ or $N_R = 1$, $C(\phi)$ in [2] reduces to the ergodic rate for the IRS-assisted transmission without interference and with ULA at the signal BS and IRS in [9]. Later, we shall see that our general analysis and optimization results recover those in [9].

III. ANALYSIS OF ERGODIC RATE

In this section, we analyze the ergodic rate of the IRS-assisted system in the presence of interference. Define $r_c \triangleq \frac{K_c K_{RU}}{(K_c+1)(K_{RU}+1)}$, $\theta_{c,m,n} \triangleq f \left( \gamma(\phi)^{\frac{1}{2}}, \gamma(\phi), m, n \right) - f \left( \gamma(\phi)^{\frac{1}{2}}, \gamma(\phi), m, n \right)$, $m \in M_R, n \in N_R$, and

$$y_c(\phi) \triangleq \left\| \sum_{m=1}^{M_R} \sum_{n=1}^{N_R} e^{j \theta_{c,m,n} + j \phi_{m,n}} \right\|^2 \in [0, M_R^2 N_R^2], \quad c = S, I.$$

Note that $\left\| h_{RU}^H \Phi(\phi) H_{SR} \right\|^2_2 = M_c N_c y_c(\phi)$, i.e., $M_c N_c y_c(\phi)$ represents the sum power of the LoS components for the channel of the indirect signal link between the signal BS $c$ and user $U$ via the IRS. The exact expression of $C_{ab}(\phi)$ is not tractable. As in [9], using Jensen’s inequality, we can obtain its analytical upper bound [1].

Theorem 1 (Upper Bound of Ergodic Rate):

$$C(\phi) \leq \log_2 \left( 1 + \gamma_{ub}(\phi) \right) \triangleq C_{ub}(\phi), \quad (4)$$

where $\gamma_{ub}(\phi)$ is given by (3), as shown at the top of the next page.

Later in Section [2], we shall show that $C_{ab}(\phi)$ is a good approximation of $C(\phi)$, and can facilitate its evaluation and optimization. From Theorem 1, we can draw the following conclusions. $C_{ab}(\phi)$ increases with $P_S$, $M_S$, $N_S$, $\alpha_{SR}$ and $\alpha_{SU}$, and decreases with $P_I$, $\alpha_{IR}$, $\alpha_{IU}$ and $\sigma^2$.

IV. OPTIMIZATION OF ERGODIC RATE

In this section, we maximize the ergodic rate of the IRS-assisted system in the presence of interference. Specifically, we would like to maximize the upper bound $C_{ub}(\phi)$ of the ergodic rate $C(\phi)$, or equivalently maximize $\gamma_{ub}(\phi)$ by optimizing the phase shifts $\phi$ subject to the constraints in (1).

Problem 1 (Ergodic Rate Maximization):

$$\gamma_{ub}^* \triangleq \max_{\phi} \gamma_{ub}(\phi), \quad s.t. \ (1)$$

where $\gamma_{ub}(\phi)$ is given by (3). Let $\phi^*$ denote an optimal solution.

Note that Problem 1 is a challenging non-convex problem. In the following, we tackle Problem 1 in some special cases (with certain system parameters) and the general case (with arbitrary system parameters), respectively.

A. Globally Optimal Solutions in Special Cases

Define $\eta = P_I \alpha_{IR} \alpha_{SU} M_R N_R (\tau_S - \tau_I) + \alpha_{SR} \alpha_{UTS} - \alpha_{IR} \alpha_{SU} \tau_I + \sigma^2 \alpha_{SRTS}$. Note that $\eta \in \mathbb{R}$ and $(x) \in [0, 2\pi)$ as $\frac{x}{2\pi} = \frac{x}{2\pi} - \left\lfloor \frac{x}{2\pi} \right\rfloor \in [0, 1)$ for all

\[2\text{later, we shall see that } \phi \text{ can be determined based on some system parameters.}\]

\[3\text{we omit all proofs due to page limitation. Please refer to [13] for details.}\]
\[
A_{S,m,n}^{(t)} \triangleq P_S M_S N_S \left( \alpha_{SR} \alpha_{RU} \left( \tau_S \left( 1 + \sum_{k \neq m, l \neq n} e^{j\left( \phi_{k,l}^{(t)} + \theta_{S,k,l} \right)} \right)^2 + M_R N_R (1 - \tau_S) \right) + \alpha_{SU} \right),
\]
\[
A_{I,m,n}^{(t)} \triangleq P_I \left( \alpha_{IR} \alpha_{RU} \left( \tau_I \left( 1 + \sum_{k \neq m, l \neq n} e^{j\left( \phi_{k,l}^{(t)} + \theta_{I,k,l} \right)} \right)^2 + M_R N_R (1 - \tau_I) \right) + \alpha_{IU} \right) + \sigma^2.
\]

**Algorithm 1** PCD Algorithm for Obtaining a Stationary Point in General Case

1. **initialization:** choose any \( \phi^{(0)} \) as the initial point, and set \( t = 0 \).
2. **repeat**
   3. For all \( m \in M_R \) and \( n \in N_R \), compute \( \phi_{m,n}^{(t)} \) according to 3.
   4. Update \( \phi^{(t+1)} \) according to 4.
   5. Set \( t = t + 1 \).
   6. **until** some convergence criteria is met.

**Theorem 2** (Globally Optimal Solutions in Special Cases):
In Special Case (i), any \( \phi^* \) satisfying 1 is optimal, and \( y_S (\phi^*) = y_I (\phi^*) = 1 \). In Special Case (ii), any \( \phi^* \) with \( \phi_{m,n}^* = \Lambda (\alpha - \theta_{I,m,n}) \), \( m \in M_R, n \in N_R \), for all \( \alpha \in \mathbb{R} \), is optimal, and \( y_S (\phi^*) = y_I (\phi^*) = M_R^2 N_R^2 \).

Note that based on Theorem 2, we can obtain a globally optimal solution in Special Case (iii), by solving a system of linear equations. In addition, substituting \( y_S (\phi^*) \) and \( y_I (\phi^*) \) into 1, we can obtain the optimal value of Problem 1, i.e., \( \gamma_{ub}^{(t)} \). Theorem 2 can be further interpreted as follows. Statement (i) of Theorem 2 is for the case of a single-element IRS. In this case, \( y_S (\phi) = y_I (\phi) = 1 \) for all \( \phi \), and hence the phase shift of the single element has no impact on the ergodic rate. Statement (ii) and Statement (iii) of Theorem 2 are for the symmetric arrangement IRS with \( \delta_{SR}^{(t)} = \phi_{IR}^{(t)} \) and \( \delta_{IR}^{(t)} = \phi_{SR}^{(t)} \). Accordingly, \( y_S (\phi) = y_I (\phi) = \gamma_{ub} (\phi) \), and \( \eta \) actually represents the derivative of \( \gamma_{ub} (\phi) \) with respect to \( y (\phi) \). When \( \eta > 0 \), the phase shifts that achieve the maximum sum power of the LoS components for the channels of the indirect signal and interference links, i.e., \( M_R^2 N_R^2 \), also maximize the ergodic rate. When \( \eta \leq 0 \), the phase shifts that achieve the minimum sum power of the LoS components for the channels of the indirect signal and interference links, i.e., 0, also maximize the ergodic rate. Statement (iv) of Theorem 2 is for the case without interference. In this case, the phase shifts that achieve the maximum sum power of the LoS components for the channels of the indirect signal links, i.e., \( M_R^2 N_R^2 \), also maximize the ergodic rate. The optimization result recovers the one under the ULA model for the multi-antenna BS and multi-element IRS in 9.

**B. Stationary Point in General Case**

In this part, we consider the general case. Note that the iterative algorithms based on BCD and MM in 6 can be extended to obtain a stationary point of Problem 1 in the general case. In particular, in the BCD algorithm, all blocks (i.e., \( \phi_{m,n}, m \in M_R, n \in N_R \)) are sequentially updated at each iteration; in the MM algorithm, only an approximate problem is solved in each iteration. In the following, we propose an iterative algorithm based on PCD, where at each iteration, all coordinates are updated in parallel, each according to a closed form expression, to obtain a stationary point of Problem 1. The goal is to improve the computation efficiency when multi-core processors are available, especially for large \( M_R \) and \( N_R \). Let \( \phi^{(t)} \) denote the phase shifts at the \( t \)-th iteration. At each iteration, we first maximize \( \gamma_{ub}^{(t)} (\phi) \) w.r.t. each phase shift \( \phi_{m,n} \) with the other phase shifts being fixed.

**Problem 2** (Block-wise Optimization Problem w.r.t. \( \phi_{m,n} \) at Iteration 1):

\[
\gamma_{ub}^{(t)} \triangleq \arg \max_{\phi_{m,n} \in [0,2\pi]} A_{SRU,m,n}^{(t)} \cos (\phi_{m,n} + A_{I,m,n}^{(t)} + A_{S,m,n}^{(t)}) + A_{I,m,n}^{(t)}
\]

where \( A_{SRU,m,n}^{(t)} \) and \( A_{I,m,n}^{(t)} \) are given by 5 and 6, as shown at the top of this page, and

\[
A_{SRU,m,n}^{(t)} \triangleq 2 P_S M_S N_S \alpha_{SR} \alpha_{RU} \tau_S \left| \sum_{k \neq m,l \neq n} e^{j\left( \phi_{k,l}^{(t)} + \theta_{S,k,l} \right)} \right|
\]
\[
A_{I,m,n}^{(t)} \triangleq 2 P_I \alpha_{IR} \alpha_{RU} \tau_I \left| \sum_{k \neq m,l \neq n} e^{j\left( \phi_{k,l}^{(t)} + \theta_{I,k,l} \right)} \right|
\]

\[
A_{c,m,n}^{(t)} \triangleq \theta_{c,k,l} - \left( \sum_{k \neq m,l \neq n} e^{j\left( \phi_{k,l}^{(t)} + \theta_{c,k,l} \right)} \right), c = S, I.
\]
By taking the derivative of the objective function of Problem 1 w.r.t. $\phi_{m,n}$, and setting it to zero, we obtain the equation in (7), as shown at the top of this page, where

$$A_{1,m,n}^{(t)} \sin(\phi_{k,l}) + A_{2,m,n}^{(t)} \cos(\phi_{k,l}) = A_{SRU,m,n}^{(t)} A_{IRU,m,n}^{(t)} A_{SRU,m,n}^{(t)} \sin(A_{\angle S,m,n}^{(t)} - A_{\angle IRU,m,n}^{(t)}),$$

(7)

$$\phi_{m,n}(t) = \begin{cases} \arctan \frac{A_{1,m,n}^{(t)}}{A_{2,m,n}^{(t)}} - \arccos \frac{A_{SRU,m,n}^{(t)} A_{IRU,m,n}^{(t)} \sin(A_{\angle S,m,n}^{(t)} - A_{\angle IRU,m,n}^{(t)})}{\sqrt{\left(A_{1,m,n}^{(t)}\right)^2 + \left(A_{2,m,n}^{(t)}\right)^2}}, & A_{1,m,n}^{(t)} > 0, \\ \arctan \frac{A_{1,m,n}^{(t)}}{A_{2,m,n}^{(t)}} - \arccos \frac{A_{SRU,m,n}^{(t)} A_{IRU,m,n}^{(t)} \sin(A_{\angle S,m,n}^{(t)} - A_{\angle IRU,m,n}^{(t)})}{\sqrt{\left(A_{1,m,n}^{(t)}\right)^2 + \left(A_{2,m,n}^{(t)}\right)^2}} + \pi, & A_{1,m,n}^{(t)} < 0. \end{cases}$$

(8)

where $\rho(t)$ is a positive diminishing stepsize satisfying:

$$\rho(t) > 0, \quad \lim_{t \to \infty} \rho(t) = 0, \quad \sum_{t=1}^{\infty} \rho(t) = \infty, \quad \sum_{t=1}^{\infty} \left(\rho(t)^2\right) < \infty.$$

The details of the PCD algorithm are summarized in Algorithm 1. By [15], we know that $\phi(t) \to \phi^*$ as $t \to \infty$, where $\phi^*$ is a stationary point of Problem 1.

V. NUMERICAL RESULTS

In this section, we numerically evaluate the performance of the proposed solutions in an IRS-assisted system [11], where the signal BS, the interference BS, user $U$ and the IRS are located at $(0,0)$, $(600,0)$, $(d_{SU}, 0)$, $(d_{IR}, d_{RU})$ (in $m$), respectively, and user $U$ lies on the line between the signal BS and the interference BS, as shown in Fig. 2. In the simulation, we set $d = \frac{1}{2}, M_S = N_S = 4, M_I = N_I = 4, M_{\delta} = N_R = 8, P_S = P_I = 30$dBm, $\sigma^2 = -104$dBm, $\varphi_{SR} = \varphi_{RU} = \pi/3, \varphi_{IR} = \varphi_{IR} = \pi/4, \varphi_{IR} = \varphi_{IR} = \pi/6, d_R = 250m, d_{SU} = 250m, d_{RU} = 20m$, if not specified otherwise. We consider a path loss model similar to those in [3, 8, 11]. Specifically, the distance-dependent path losses $\alpha_{SU}, \alpha_{IU}, \alpha_{SR}, \alpha_{IR}, \alpha_{RU}$ follow $\alpha_i = \frac{10000}{10000 + i}$, $\ i = SU, IU, SR, IR, RU$. Due to extensive obstacles and scatters, we set $\bar{\alpha}_{SU}, \bar{\alpha}_{IU} = 3.5$. As the location of the IRS is usually carefully chosen, we assume that the link between the BSs and the IRS experience free-space path loss, and set $\bar{\alpha}_{SR}, \bar{\alpha}_{IR} = 2$, as in [3]. In addition, we set $\bar{\alpha}_{RU} = 3$, due to few obstacles.

In all cases, we consider two baseline schemes. In particular, Baseline 1 reflects the ergodic rate of the counterpart system without IRS in Section V [8, 10]; Baseline 2 chooses the phase shifts uniformly at random [8, 10], and shows the ergodic rate over 10000 random choices. In the general case, besides the proposed PCD algorithm, we also evaluate the BCD and MM algorithms [6]. We adopt the same convergence criterion, i.e., $\gamma_{ub}(\phi(t+1)) - \gamma_{ub}(\phi(t)) \leq 10^{-6}$, for the PCD, BCD and MM algorithms. For ease of illustration, we refer to the stationary points obtained by the PCD, BCD and MM algorithms as the PCD, BCD and MM solutions, respectively.

We set $\delta_{SR}^{(h)} = \delta_{SR}^{(v)} = \pi/6, \delta_{IR}^{(h)} = \delta_{IR}^{(v)} = \pi/6$ in Special Case (ii) and Special Case (iii), set $K_{SR} = K_{IR} = K_{RU} = 20$dB in Special Case (ii), and set $K_{SR} = -20$dB, $K_{IR} = K_{RU} = 20$dB in Special Case (iii). From Fig. 3(a) and Fig. 3(b) illustrate the ergodic rate versus $M_R (= N_R)$ and $P_I$, respectively, in Special Case (ii) and Special Case (iii). From these figures, we can make the following observations. The analytical ergodic rate of the optimal solution ($C_{ub}(\phi^*)$) and the ergodic rate of the optimal solution obtained by Monte Carlo simulation ($C(\phi^*)$) are very close to each other, which verifies that $C_{ub}(\phi)$ is a good approximation of $C(\phi)$; the ergodic rates of the proposed optimal solution and PCD solution are very close in the considered cases. From Fig. 3(a) we can observe that the ergodic rates of the proposed solutions and the design with random phase shifts increase with $M_R (= N_R)$, mainly due to the increment of reflecting signal power. From Fig. 3(b) we can see that the ergodic rate of each scheme decreases with $P_I$.

In the general case, we set $\delta_{SR}^{(h)} = \delta_{SR}^{(v)} = \pi/6, \delta_{IR}^{(h)} = \delta_{IR}^{(v)} = \pi/8, K_{SR} = K_{IR} = K_{RU} = 20$dB, $K_{IR} = 10$dB, if not specified otherwise. Fig. 4(a) Fig. 4(b) Fig. 4(c) and Fig. 4(d) illustrate the ergodic rate versus $K_{SR}, K_{RU}, d_{IR}$ and $d_{SU}$, respectively, in the general case. From these figures, we can
we can see that when the number of IRS elements is large, the gain of the proposed PCD algorithm in computation time over the BCD and MM algorithms increases with the number of the cores on a server, due to its parallel computation mechanism. Note that in practical systems with multi-core processors, the value of the PCD algorithm will be prominent, especially for large-scale IRS.

VI. CONCLUSION

In this paper, we considered an IRS-assisted system in the presence of interference. First, we obtained a tractable expression of the ergodic rate. Next, we optimize the phase shifts to maximize the ergodic rate. Then, we proposed the PCD algorithm to obtain a stationary point in the general scenario. Finally, by numerical results, we verified analytical results and demonstrated notable gains of the proposed solutions over existing schemes. The results in this paper provide important insights for designing practical IRS-assisted systems.

REFERENCES

[1] Q.-U.-A. Nadeem, A. Kammoun, A. Chaaban, M. Debbah, and M.-S. Alouini, “Large intelligent surface assisted mimo communications,” arXiv preprint arXiv:1903.08127, 2019.
[2] G. Yang, X. Xu, and Y.-C. Liang, “Intelligent reflecting surface assisted non-orthogonal multiple access,” arXiv preprint arXiv:1907.03133, 2019.
[3] H. Guo, Y.-C. Liang, J. Chen, and E. G. Larsson, “Weighted sum-rate optimization for intelligent reflecting surface enhanced wireless networks,” arXiv preprint arXiv:1905.07920, 2019.
[4] Q. Wu and R. Zhang, “Weighted sum power maximization for intelligent reflecting surface aided wireless communication,” IEEE Commun. Lett., pp. 1–1, 2019.
[5] X. Yu, D. Xu, and R. Schober, “Miso wireless communication systems via intelligent reflecting surfaces : (invited paper),” in 2019 IEEE/CIC ICCC, Aug 2019, pp. 735–740.
[6] X. Yu, D. Xu, and R. Schober, “Enabling secure wireless communications via intelligent reflecting surfaces,” arXiv preprint arXiv:1904.09573, 2019.
[7] C. Huang, A. Zappone, G. C. Alexandropoulos, M. Debbah, and C. Yuen, “Reconfigurable intelligent surfaces for energy efficiency in wireless communication,” IEEE Trans. Wireless Commun., vol. 18, no. 8, pp. 4157–4170, Aug 2019.
[8] Q. Wu and R. Zhang, “Intelligent reflecting surface enhanced wireless network via joint active and passive beamforming,” IEEE Trans. Wireless Commun., pp. 1–1, 2019.
[9] Y. Han, W. Tang, S. Jin, C. Wen, and X. Ma, “Large intelligent surface-assisted wireless communication exploiting statistical CSI,” IEEE Trans. Veh. Technol., vol. 68, no. 8, pp. 8238–8242, Aug 2019.
[10] Z. Zhang, Y. Cui, F. Yang, and L. Ding, “Analysis and optimization of outage probability in multi-intelligent reflecting surface-assisted systems,” arXiv preprint arXiv:1909.02193, 2019.
[11] C. Pan, H. Ren, K. Wang, W. Xu, M. Elkashlan, A. Nallanathan, and L. Hanzo, “Intelligent reflecting surface for multiecell mimo communications,” arXiv preprint arXiv:1907.10664, 2019.
[12] H. L. Van Trees, Optimum array processing: Part IV of detection, estimation, and modulation theory. John Wiley & Sons, 2004.
[13] Y. Jia, C. Ye, and Y. Cui, “Power-efficient multi-quality multicast beamforming based on scalable video coding and superposition coding,” Technical Report, 2019. [Online]. Available: http://wci.sjtu.edu.cn/personal/ysjia/publications.html
[14] Z. He and X. Yuan, “Cascaded channel estimation for large intelligent metasurface assisted massive mimo,” IEEE Commun. Lett., pp. 1–1, 2019.
[15] P. Richtárik and M. Takáč, “Parallel coordinate descent methods for big data optimization,” Mathematical Programming, vol. 156, no. 1, pp. 433–484, Mar. 2016. [Online]. Available: https://doi.org/10.1007/s10107-015-0901-6

---

**CONCLUSION**

In this paper, we considered an IRS-assisted system in the presence of interference. First, we obtained a tractable expression of the ergodic rate. Next, we optimize the phase shifts to maximize the ergodic rate. Then, we proposed the PCD algorithm to obtain a stationary point in the general scenario. Finally, by numerical results, we verified analytical results and demonstrated notable gains of the proposed solutions over existing schemes. The results in this paper provide important insights for designing practical IRS-assisted systems.

**REFERENCES**

[1] Q.-U.-A. Nadeem, A. Kammoun, A. Chaaban, M. Debbah, and M.-S. Alouini, “Large intelligent surface assisted mimo communications,” arXiv preprint arXiv:1903.08127, 2019.
[2] G. Yang, X. Xu, and Y.-C. Liang, “Intelligent reflecting surface assisted non-orthogonal multiple access,” arXiv preprint arXiv:1907.03133, 2019.
[3] H. Guo, Y.-C. Liang, J. Chen, and E. G. Larsson, “Weighted sum-rate optimization for intelligent reflecting surface enhanced wireless networks,” arXiv preprint arXiv:1905.07920, 2019.
[4] Q. Wu and R. Zhang, “Weighted sum power maximization for intelligent reflecting surface aided wireless communication,” IEEE Commun. Lett., pp. 1–1, 2019.
[5] X. Yu, D. Xu, and R. Schober, “Miso wireless communication systems via intelligent reflecting surfaces : (invited paper),” in 2019 IEEE/CIC ICCC, Aug 2019, pp. 735–740.
[6] X. Yu, D. Xu, and R. Schober, “Enabling secure wireless communications via intelligent reflecting surfaces,” arXiv preprint arXiv:1904.09573, 2019.
[7] C. Huang, A. Zappone, G. C. Alexandropoulos, M. Debbah, and C. Yuen, “Reconfigurable intelligent surfaces for energy efficiency in wireless communication,” IEEE Trans. Wireless Commun., vol. 18, no. 8, pp. 4157–4170, Aug 2019.
[8] Q. Wu and R. Zhang, “Intelligent reflecting surface enhanced wireless network via joint active and passive beamforming,” IEEE Trans. Wireless Commun., pp. 1–1, 2019.
[9] Y. Han, W. Tang, S. Jin, C. Wen, and X. Ma, “Large intelligent surface-assisted wireless communication exploiting statistical CSI,” IEEE Trans. Veh. Technol., vol. 68, no. 8, pp. 8238–8242, Aug 2019.
[10] Z. Zhang, Y. Cui, F. Yang, and L. Ding, “Analysis and optimization of outage probability in multi-intelligent reflecting surface-assisted systems,” arXiv preprint arXiv:1909.02193, 2019.
[11] C. Pan, H. Ren, K. Wang, W. Xu, M. Elkashlan, A. Nallanathan, and L. Hanzo, “Intelligent reflecting surface for multicell mimo communications,” arXiv preprint arXiv:1907.10664, 2019.
[12] H. L. Van Trees, Optimum array processing: Part IV of detection, estimation, and modulation theory. John Wiley & Sons, 2004.
[13] Y. Jia, C. Ye, and Y. Cui, “Power-efficient multi-quality multicast beamforming based on scalable video coding and superposition coding,” Technical Report, 2019. [Online]. Available: http://wci.sjtu.edu.cn/personal/ysjia/publications.html
[14] Z. He and X. Yuan, “Cascaded channel estimation for large intelligent metasurface assisted massive mimo,” IEEE Commun. Lett., pp. 1–1, 2019.
[15] P. Richtárik and M. Takáč, “Parallel coordinate descent methods for big data optimization,” Mathematical Programming, vol. 156, no. 1, pp. 433–484, Mar. 2016. [Online]. Available: https://doi.org/10.1007/s10107-015-0901-6