Inertial blob-hole symmetry breaking in magnetised plasma filaments

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Abstract
Symmetry breaking between the propagation velocities of magnetised plasma filaments with large positive (blob) and negative (hole) amplitudes, as implied by a dimensional analysis scaling, is studied with global (‘full-n’) non-Boussinesq gyrofluid computations, which include finite inertia effects through nonlinear polarisation. Interchange blobs on a flat density background have higher inertia and propagate more slowly than holes. In the presence of a large enough density gradient, the effect is reversed: blobs accelerate down the gradient and holes are slowed in their propagation up the gradient. Drift wave blobs spread their initial vorticity rapidly into a fully developed turbulent state, whereas primary holes can remain coherent for many eddy turnover times. The results bear implications for plasma edge zonal flow evolution and tokamak scrape-off-layer transport.

Keywords: scrape-off layer transport, blob and hole filaments, drift wave turbulence

(Some figures may appear in colour only in the online journal)
been making use of the delta-n or Boussinesq approximation, which assumes small fluctuation amplitudes. The inertial and nonlinear polarisation effects on drift wave turbulence and blob propagation significantly modify the picture of edge and SOL fluctuations. Recent results for global blob propagation obtained with non-Boussinesq codes and models [14–18] demonstrate the relevance of full-n modelling in the edge for more realistic SOL blob transport scalings: large inertial blobs are slowed on flat background profiles, but accelerate strongly down pressure gradients.

In light of these results on large inertial blobs it appears not at all any more evident that large amplitude blobs and holes should be CP symmetric. The following work numerically studies asymmetries between inertial interchange blobs and holes (2-d) and between large amplitude drift vortices (3-d) with initially different polarities. The computational implementation is based on an isothermal reduction of the full-n gyrofluid model by Madsen [19] and reduces to the delta-n model (GEM3) by Scott [20, 27] in the limit of small fluctuation amplitudes and to the full-n model by Wiesenberger [14] in the two-dimensional limit.

2. Model and numerical methods

The full-n 3-d gyrofluid model by Madsen [19] consists of a set of 6-moment equations and of the field equations for the potentials, completed by a first order finite Larmor radius closure. In the following, an isothermal plasma is assumed, where temperature variations in space and time are neglected. A normalised energetically consistent set of 3-d full-n isothermal gyrofluid equations for electrons and ions (species $s \in \{e, i\}$ for the first two moments (corresponding to equations 22 and 23 in [19]), which are the gyrocenter densities $n_s$ and parallel velocities $v_{\parallel s}$, is:

\begin{align}
\frac{\partial \hat{n}_s}{\partial t} &= \frac{1}{B} \left[ \hat{n}_s, \phi_s \right] - \frac{B}{n_s} \frac{\partial}{\partial \nu} \left( \frac{n_s v_{\parallel s}}{B} \right) + \kappa(n_s) \\
\frac{\partial v_{\parallel s}}{\partial t} &= \frac{\mu_s}{B} \left[ v_{\parallel s}, \phi_s \right] - \nabla \cdot \left( \frac{n_s \nabla \phi_s}{n_s} - C \frac{J_s}{n_s} \right) \\
&+ \mu_s \tau_v \kappa(n_s) + 2 \mu_s \tau_p \kappa(n_s)
\end{align}

with $\alpha_s \equiv (\beta_0 \mu_s + \mu_s v_{\parallel s})$ and $n_s \equiv (\phi_s + \tau_p \hat{n}_s)$. Triplet nonlinear terms including the parallel velocity are here neglected. The nonlinear polarisation equation

\begin{equation}
\sum_s Z_s e^2 \Gamma_{1s} n_s \nabla \cdot \left( n_s \frac{\mu_s}{B^2} \nabla \phi \right) = 0.
\end{equation}

determines the electrostatic potential $\phi$. The gyro-screened potential is given by $\phi_s = \Gamma_{1s} \phi - (\mu_s/2B) (\nabla \phi)^2$. Parallel velocities and current are coupled to the vector potential $A_{\parallel}$ via Ampere’s law $\nabla \times A_{\parallel} = -(1/(2\mu_0)) \left( \frac{\partial}{\partial \tau_v} \nabla L \right)$. The gyro-averaging operator in Padé approximation is defined by $\Gamma_{1s} = (1 + (1/2) b_v)^{-1}$ with $b_v = \tau_p \mu_s \nabla^2$. The mass ratio is given by $\mu_s = m_s/(Z_s m_i)$ and the (constant) temperature ratio by $\tau_s = T_s/(Z_s T_i)$.

For electrons, thus $\tau_e = -1$ and finite Larmor radius (FLR) effects are neglected ($b_v \equiv 0$). The electron contribution to the polarisation in equation (3) is also neglected, with $[\mu_s] \ll [\mu_i]$. The gyrocenter densities $n_s$ are normalised to a constant reference density $n_0$, so that the magnitude of the plasma density $n_s \equiv n_s/n_0$ is of order one. Equations (1) and (2) have been divided by the specific variable densities $n_s$ and logarithmic densities $\hat{n}_s \equiv \ln n_s$ are introduced to ensure positivity, with both $\hat{n}_s$ and $n_s$ appearing in the equations.

The spatial derivative operators are normalised as $\nabla \leftarrow \rho \nabla \phi$ to the drift scale $\rho_s = (c_e B_0)^{-1} q_m T_e$, where $m_i$ is the mass of the main ion species, $T_e$ is a constant reference electron temperature and $B_0$ is a static reference background magnetic field strength. Parallel derivatives are further scaled as $\nabla_{\parallel} \leftarrow (L_{\parallel}/L_{\perp}) \nabla_{\parallel}$ with the connection length $L_{\parallel}$, which for toroidal geometry is given by $L_{\parallel} = 2\pi q R$ with inverse rotational transform $q$ and major torus radius $R$. The drift parameter $\delta = \rho_s/L_{\parallel}$ is used to set the perpendicular length scale $L_{\perp}$. For blob simulations often $L_{\parallel} = \rho_s$ is used (so that $\delta = 1$) and for gradient driven turbulence usually $L_{\perp} \equiv L_{\rho}$ is set as the density gradient length scale $L_{\rho}$. In order to apply the same normalisation length for all presented simulations (including those on drift wave vortices), a normalisation to a typical edge gradient length with $\delta = 0.01$ is used.

The time scale is normalised as $\partial_t \leftarrow (\rho_s/c_e) \partial_t$ and parallel velocities $v_{\parallel s} \leftarrow v_{\parallel s}/c_e$ are normalised by the sound speed $c_s = \sqrt{T_e/m_i}$. Further, $\phi \leftarrow (e \phi/T_e)$, $B \leftarrow B/B_0$, $J_{\parallel} \leftarrow J_{\parallel}/(e n_0 c_e)$, $\delta_{\parallel} \equiv (A_{\parallel}/\delta_{0B} B_0) (L_{\parallel}/q R)$ for a reference electron beta given by $\delta_{0B} = 4\pi n_0 T_e B_0^2$. The collisionality parameter is given by $C = (L_{\parallel}/c_e \rho_s B_0) \eta$ with $\eta = 0.51 (m_i c_e^2)/(n_0 e^2)$. The main plasma parameters are $\hat{n}_s = \mu_e \hat{\beta} = (n_0 T_e/B_0^2)$ and $\hat{C} = (m_i c_e^2)/\rho_s c_e$ with $\hat{e} \equiv (q R/L_{\rho})^2$. In the following only electrostatic blobs and vortices with $\hat{\beta} = 0$ are discussed (while electromagnetic effects are of more relevance for fully developed turbulence).

The 2-d advection terms are expressed through Poisson brackets $[f, g] = (\partial_f / \partial x) (\partial g / \partial y) - (\partial f / \partial y) (\partial g / \partial x)$ for locally perpendicular coordinates $x$ and $y$. Normal and geodesic components of the magnetic curvature enter the compressional effect due to field inhomogeneity by $\kappa = k_\parallel \partial_x + k_\perp \partial_y$, where the curvature components in toroidal geometry are functions of the poloidal angle $\theta$ mapped onto the parallel coordinate $z$. For a circular torus $k_\parallel \equiv k_\theta \cos(z)$ and $k_\perp \equiv k_\theta \sin(z)$ when $z = 0$ is defined at the outboard midplane. An Arakawa–Karıdaniak numerical scheme [21–23] is used for the computation of equations (1) and (2). The generalised Poisson type equation (3) is solved by a Chebyshev accelerated 4th order red-black SOR scheme [24–26]. For numerical stability, a small perpendicular viscosity term $s_{\parallel} = -\nu_{\parallel} \nabla^2 \hat{n}_s$ is added on the right hand side of equation (1) and in 3-d computations parallel viscous terms $\nu_{\parallel} \nabla^2 \hat{n}_s$ are added to equations (1) and (2), respectively. Boundary conditions in $y$ direction are periodic for 2-d simulations and quasi-periodic (shear-shifted flux tube) for 3-d simulations. The total density is allowed to evolve freely, although for the present short blob propagation times the initial background profiles do not evolve visibly. To avoid
degradation and flows at the radial boundaries, fixed mixed (von Neumann/Dirichlet) vorticity free ($n_i = \Gamma_1 n_i$) boundary conditions are applied in $x$. For longer turbulence simulations with free profile evolution, sources and sinks would rather have to be specified at the radial boundaries.

The delta-n isothermal electromagnetic gyrofluid model [27, 28] is regained by splitting $n_s = n_{s0} + \delta n_s$ into a static constant background density $n_{s0}$ and the perturbed density $\delta n_s$. When $\delta n_s/n_{s0} \ll 1$, the right hand sides of equations (1) and (2) can be linearised by approximating $n_s \approx n_{s0}$ so that

$$\hat{\mathbf{n}}_i \approx \hat{\mathbf{n}}_0 + (\hat{\mathbf{n}}_i/n_{s0})$$

and neglecting all nonlinear terms except the Poisson bracket:

$$\partial_t \hat{n}_i = \frac{1}{B} [\hat{n}_i, \hat{\phi}] - BN_i(\hat{\phi}/B) + \kappa(h_i)$$

(4)

$$\partial_t \alpha_s = \frac{\mu_s}{B} [\hat{\psi}_s, \hat{\phi}] - V_i h_s + 2\mu_s r_s \hat{k}(\hat{\psi}_s) - CJ$$

(5)

The consistent delta-n polarisation equation in the high-$k$ limit is $\sum_s a_s \Gamma_i \hat{n}_i + (1/\tau_s) (\Gamma_0 - 1) \hat{\phi}_s = 0$ with $\Gamma_0 = (1 + b_s)^{-1}$. Linearisation of the low-$k$ equation (3) actually does not include the gyro-screening on the potential and results in $\sum_s a_s \Gamma_i \hat{n}_i = V_i^2 \hat{\phi}$. The velocities and current are again coupled to the parallel component of the fluctuating vector potential by Ampere’s equation $V_i^2 \hat{A}_i = \hat{J}_i = \sum_s a_s \hat{\psi}_s$. The parameter $a_s = Z_s n_{s0}/n_{e0}$ describes the ratio of species reference densities $n_{s0}$ to $n_{e0}$.

3. Large inertial 2-d interchange blobs and holes

In the following, large amplitude blob and hole propagation is compared for the full-n and delta-n models. To separate 2-d interchange and 3-d drift wave effects, at first the computations are restricted to 2-d by neglecting the parallel velocity and parallel derivatives. In this limit the equations correspond to the 2-d full-n model by Wiesenberger [14]. Blobs and holes are initialised as Gaussian density perturbations with width $r = 10\rho_s$ and amplitude $\Delta n = \pm 0.75$ for $n_b = 1$. In the full-n model $n_b$ corresponds to the actual background plasma density, whereas in the delta-n model this can be regarded as a dummy parameter on which the solution does not depend, as the model already implies a large underlying background $n_0 \gg \Delta n$. Dimensional analysis roughly estimates the delta-n and full-n blob propagation speed scalings [14, 17] as $V_{\text{delta}}/c_s \sim \sqrt{\Delta n}$ and $V_{\text{full}}/c_s \sim \sqrt{\Delta n/(n_0 + \Delta n)}$. On this basis inertial blobs could be expected to propagate more slowly than holes (with reverse direction) in the full-n model.

Simulation parameters here are $\kappa = 0.05$, $\delta = 0.01$ and $\tau_s = 0$. The computational grid is $n_x \times n_y = 512 \times 256$ with resolution $(192 \times 96)\rho_s$. Figure 1 (top) shows the symmetric evolution of blobs and holes in the delta-n model (top) for times $t = 0$, $t = 12.5$ and $t = 25$. For simultaneous reversal of

![Figure 1. Top (delta-n): large amplitude blob/hole evolution in the delta-n model at $t = 0$ (red bold), $t = 12.5$ (orange dashed) and $t = 25$ (black thin line). The contours of delta-n blob and hole coincide for simultaneous reversal of amplitude and $x$-direction. Bottom (full-n): different states of inertial blob (left) and hole (right: $x$-direction mirrored) at $t = 25$.](https://example.com/figure1.png)
direction and sign of the amplitude, delta-n blob and
hole coincide: the density contours are identical for blobs and
(reversed) holes. The bottom figures show the different states at
t = 25 for an inertial full-n blob (left) and full-n hole (right). The
inertial blob has a more coherent head and propagates slower
than for the delta-n case, whereas the inertial hole fragments
more strongly and propagates faster, as predicted by the inertial
scaling. Figure 2 (left) shows the corresponding time evolu-
tion of the x-coordinates of the center of mass (bottom lines)
and the propagation fronts (upper lines) for the delta-n case
(black dashed lines), the inertial blob (thin red lines) and iner-
tial hole (bold blue lines, mirrored in x direction). The radial
center of mass position is determined by

$$x_{\text{center}} = \frac{\sum_{i,j} x_i (n_e(x_i, y_j) - n_b(x_i))}{\sum_{i,j} (n_e(x_i, y_j) - n_b(x_i))}$$

and the center of mass velocity by

$$v_{\text{center}} = \Delta x_{\text{center}} / \Delta t.$$

The blob front position is here simply determined as the furthest outward x position where the
density deviates more than 10% from the initial background
profile. The acceleration occurs mostly in the initial quasi-linear
phase (compare center-of-mass velocity plots in right figure),
while at later times the center-of-mass velocities drop and the
front velocities saturate nearly equally for blobs and holes. The
maximum center of mass velocity in general depends on the
initial blob amplitude and width, which are here kept fixed. For
given width and amplitude, the maximum velocities are found
to be similar, with a slightly reduced maximum velocity for the
inertial (full-n) blob compared to the delta-n blob and a slightly
increased velocity for the inertial hole. This observation is con-
sistent with results for large amplitude blobs presented in [14].

So far a constant background density has been assumed. Now a linearly decreasing background density profile

$$n_b(x) = 2(1 - x/x_{\text{max}})$$

is considered. Blobs thus propagate into regions of lower background density and holes into higher
density. In a delta-n model the blob/hole velocity would be
unchanged. Inertial blobs with $V_{\text{inert}} = \pm \sqrt{\Delta n/(n_b(x) + \Delta n)}$
however can be expected to accelerate and holes to be slowed
down. For large enough gradients the inertial effects on blob/
hole velocities found for flat profiles can even be reversed.

This is demonstrated in computations with $\Delta n(x) = \pm 0.85$ and resolution $(128 \times 256)\rho_s$, with the initial blob/hole located
in the middle of the domain, for otherwise identical param-
eters, in figure 3. The top figure shows a delta-n blob propa-
gating down a density gradient at $t = 15$. In the bottom density
contour plots (at $n = 1$) of the same delta-n blob (thin black line) and its anti-symmetric delta-n hole (dashed black line)
are shown, together with a full-n inertial blob (bold red line) that has accelerated further down the gradient (i.e. to the right
side) and a full-n hole (bold blue line) which is slowed during
propagation into denser regions.

To sum up these first results, in a delta-n model 2-d inter-
change blobs and holes evolve identically and regardless of

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure2}
\caption{Left: time evolution of the center of mass (bottom lines) and propagation fronts (upper lines) for a delta-n blob (dashed black), inertial blob (thin red) and inertial hole (bold blue). Right: center of mass speeds. (Time in units $L_\perp/c_s$.)}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure3}
\caption{Top: delta-n blob evolved until $t = 15$ on a density gradient. Bottom: $n = 1$ density contour plots of the delta-n blob (thin black line), anti-symmetric delta-n hole (dashed black line), accelerated full-n inertial blob (bold red line) and decelerated full-n hole (bold blue line).}
\end{figure}
a background density gradient. In the inertial full-n model, on a constant background the blobs move more slowly and coherently and the holes faster and more fragmented. On the other hand, on a background density gradient the inertial full-n holes, which move up the gradient, decelerate, whereas blobs accelerate down the gradient. The relative evolution and propagation of negative and positive perturbations thus strongly depends on the background gradient. Gradient steepening around the separatrix (where blobs and holes are most likely born) accordingly will lead to very different transport behaviour for full-n (or non-Boussinesq) models compared to results obtained in delta-n models.

Next, the inertial evolution for warm ions with $\tau_i = 2$ is considered, which is a typical value for the tokamak SOL. Warm ions primarily enhance the blob propagation speed [18] by contributing to the interchange drive, break the (approximate) up-down symmetry through FLR effects on polarisation and remain more coherent [14, 29]. Although the radial and poloidal propagation is more complicated (the hole head e.g. changes direction twice within the computation time), the major conclusions remain (figure 4): the inertial large blob center-of-mass velocity is larger than for the hole. The front velocity of the hole is initially higher than for the blob, but is reduced after sufficient propagation into the denser region.

4. Large inertial 3-d blobs, holes and drift vortices

3-d field-aligned computations of blobs and holes include different parallel electron and ion dynamics, which introduces charging and polarisation of the pressure perturbation, resulting in Boltzmann spinning of the blob [30].

The charging of a blob, which is initially localised in parallel direction, is a consequence of the higher parallel mobility of the electrons. In equation (2) the acceleration $\partial v_s / \partial t \sim 1/\mu_s$ is inversely proportional to the species mass, so that the resulting parallel current $J_\| \approx n_e e v_e$ is mostly carried by the electrons. In the absence of collisions ($C = 0$), electrons tend, according to the parallel component of equation (2), towards a Boltzmann response with $\nabla_\| \phi = 0$, so that the electrostatic potential spatially aligns with the blob. The resulting $E \times B$ drift leads to a perpendicular spinning vortex around the blob. For finite collisionality $C > 0$ the relative importance of spinning is controlled by the balance between the divergence of the parallel current and the divergence of the diamagnetic and polarisation currents under quasi-neutrality. The Boltzmann charging then is reduced and depending on $C$ the radial interchange drive competes with poloidal drift wave motion.

The present computations show that the spin-up of blob rotation by 3-d drift wave dynamics is strongly dependent on the collisionality parameter: for typical edge pedestal values in the closed-flux-surface region of $\hat{C} = 3.5$ the Boltzmann charging is dominant (see [16, 30]), but for an order of magnitude larger values ($\hat{C} \sim 20–50$), as more appropriate for mid-SOL plasmas, the interchange drive and the typical 2-d like blob plume structure actually prevail. In the presence of a density gradient, drift wave type propagation in the electron diamagnetic direction and instability add to the dynamics.
First, the flat background profile case with $\tau_i = 0$ (as in figure 1) is re-considered by extending the otherwise same computation to $n_z = 16$ planes in the field-aligned direction, including consistent poloidal (parallel) variation of the background magnetic field gradient $\kappa(z)$. The initial background density here is set constant in the parallel direction and the initial electrostatic potential and parallel velocities are zero. The blobs are initially localised in the middle $z$ plane.

For $\hat{C} = 7.5$ the Boltzmann spinning effect indeed is well pronounced, as shown in figure 5: the radial propagation is reduced compared with the 2-d (or a more strongly resistive) case. Holes charge up negatively and blobs positively and obtain opposite Boltzmann spins. As the head is accordingly rotationally advected, the upper arm of the blob and the lower arm of the hole get more pronounced, respectively. It is also observed that the spinning hole shows stronger coherence than the blob.

Finally, the 3-d evolution of large amplitude ($\Delta n = \pm 0.85$) drift wave blobs and holes in a sheared slab geometry with $\hat{s} = 1, \kappa = 0, \hat{C} = 3.5$ and $\hat{\epsilon} = 18000$ on a background edge density gradient $n_b(x) = 1.5 - x/x_{\text{max}}$ is studied. The simulation domain is $(n_x \times n_y \times n_z) = (96 \times 256 \times 16)\rho_s$. Drift wave blobs show a rapid transition into fully developed turbulence. Here only the initial stage is considered. For small amplitudes (or in a delta-n model) the development of nonlinear drift vortices is exactly CP-symmetric for initial blobs compared to holes (up to computing precision): the spatio-temporal contours are identical for reversal of the density gradient direction (P), while density fluctuation, potential and vorticity
amplitudes are also reversed (C). Figure 6 on top shows the vorticity $\Omega = \nabla \times \mathbf{v}$ of blob (left) and hole (right) delta-n drift vortices at $t = 50$. Large drift wave blobs and holes however show different evolution in the consistent full-n model: the primary blob vortex (figure 6 bottom left) has spread and its amplitude is decreased compared to the delta-n case, whereas the hole vortex (bottom right) is compressed radially with a strongly increased vorticity amplitude. The hole actually can be observed as a coherent tripolar vortex for quite some time (multiple eddy turnover times) during the development into a fully turbulent state of the secondary drift wave structures.

As drift wave turbulence in the outer closed-flux-surface edge pedestal region near the separatrix can acquire fluctuation amplitudes in the same order of magnitude as the background, these results also show the relevance of full-n models for edge turbulence (and probably for the understanding of edge transport barriers) in addition to the relevance for modelling of SOL blobs and interchange turbulence.

5. Conclusions

To summarise, symmetry breaking between the evolution of magnetised plasma filaments with large positive (blob) and negative (hole) amplitudes has been found. Interchange blobs on flat density background have higher inertia than holes and propagate more slowly. In the presence of a large enough density gradient, blobs accelerate down the gradient and holes are slowed in their propagation up the gradient. Gradient steepening at the blob/hole birth region (supposedly near the separatrix) can thus lead to enhanced blob velocities and transport into the outer SOL. This mechanism would be consistent with observations at various tokamaks on an effect of core density increase on flattening of the outer SOL profiles [31, 32]. Another implication is that in the presence of a strong background gradient the inward impurity convection across the separatrix by holes can be reduced and alignment of (trace and non-trace) impurities in vortices [33] can be expected to be significantly modified.

Full-n effects on large amplitude edge turbulence vortices, as they were demonstrated in this work, can lead to profound consequences. For example, large inward propagating holes can remain coherent on a turbulent background for significant times. It would be possible for such holes to be trapped on resonant surfaces (where they would not be filled up rapidly by parallel convection) and rotate for longer times with the background plasma. This could explain phenomena like palm tree modes [34].

Most of all, strong effects on the generation and structure of zonal flows (and, supposedly, mean flows) can be expected. As ion temperature fluctuations in the SOL also can achieve large amplitudes [35], both SOL and edge turbulence have to be studied with more complete source-driven full-n gyrofluid models including temperature and heat transport equations (or full-f gyrokinetic equations [36]) and with consistent coupling to the SOL including appropriate sheath boundary conditions. Such models are presently under development. The presented results clearly show the necessity for full-n, non-Boussinesq turbulence and blob transport models for the tokamak edge/SOL region.

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