Planck–Scale Traces from the Interference Pattern of two Bose–Einstein Condensates

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In the present report we analyze the possible effects arising from Planck scale regime upon the interference pattern of two non–interacting Bose–Einstein condensates. We start with the analysis of the free expansion of a condensate, taken into account the effects produced by a deformed dispersion relation, suggested in several quantum–gravity models. The analysis of the condensate free expansion, in particular, the modified free velocity expansion, suggests in a natural way, a modified uncertainty principle that could lead to new phenomenological implications related to the quantum structure of space time. Additionally, we analyze the corresponding separation between the interference fringes after two condensates overlap. Finally, we probe that a large expansion time together with a small initial separation between the condensates are required, in order to improve the sensitivity of the system to possible effects caused by quantum–structure of space–time, upon the corresponding interference pattern.

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I. INTRODUCTION

Recently, the use of many–body systems as theoretical tools in searching some possible Planck scale manifestations has become a very interesting line of research [1–6]. In particular, due to its quantum properties, and also to its high experimental precision, Bose–Einstein condensates become an excellent tool in the search of traces from Planck–scale physics, and has produced several interesting works in this direction [4–11], and references therein.

First of all, in Refs. [1, 2], for instance, it was argued that a modified uncertainty principle, could be used to explore some properties of the center of mass motion of macroscopic bodies, which could lead to observable manifestations of Planck scale physics in low energy earth–based–experiments. However, in Ref. [3], it was suggested that the extrapolation of Planck scale quantization to macroscopic bodies is incorrect, due to the fact that these possible manifestations, would be more weakly for macroscopic bodies than for its constituents. This last conclusion comes from the fact that the corrections caused by the quantum structure of space–time, on the properties associated with the center of mass motion of the macroscopic body, are suppressed by the number of particles (N), composing the system. In other words, as it was argued in Ref. [3], this simple analysis suggests that the possible signals arising from Planck scale quantization, are more weakly for macroscopic bodies than for its own constituents.

Nevertheless, the argument exposed in Ref. [3], seems to be not a generic criterion, at least for some properties associated with Bose–Einstein condensates. For instance, in Refs. [5, 6] it was demonstrated that the corrections arising from the quantum structure of space–time, characterized by a deformed dispersion relation, on some relevant properties associated with a Bose–Einstein condensate scales as a non–trivial function of the number of particles.

As mentioned above, the use of Bose–Einstein condensates opens an alternative scenario in searching some possible Planck scale signals, through a deformed dispersion relation in low–energy earth–based experiments. In fact, the analysis of some relevant properties associated with a homogeneous condensate, i.e., a condensate in a box, for instance, the corresponding ground state energy, and consequently the pressure and the speed of sound [4], present corrections caused by the quantum structure of space–time, which scales as a non–trivial function of the number of particles. Additionally, it is quiet remarkable that the inclusion of a trapping potential improves the sensitivity to Planck scale signals, compared to a condensate in a box [6]. These facts suggest that the properties associated with many–body systems, in particular some properties associated with a Bose–Einstein conden-
sate could be used, in principle, to obtain representative bounds on the deformation parameters \([4, 8–10]\) or to explore the sensitivity for these systems to Planck scale signals \([8, 9]\). Thus, it is quite interesting to explore the sensitivity to Planck scale signals on certain properties of the condensate, in which the corrections caused by the quantum structure of space-time can be amplified, instead of being suppressed.

On the other hand, it is generally accepted that the dispersion relation between the energy \(\epsilon\) and the modulus of momentum \(p\) of microscopic particles, should be modified due to the quantum structure of space–time \([12–15]\). Such a deformed dispersion relation in the non–relativistic limit can be generically expressed in ordinary units as follows \([14, 15]\)

\[
\epsilon \approx mc^2 + \frac{p^2}{2m} + \frac{1}{2M_p} \left( \xi_1 mc \frac{p^3}{m} + \xi_2 p^2 + \xi_3 \frac{p^3}{mc} \right),
\]

where \(c\) is the speed of light, and \(M_p \approx 2.18 \times 10^{-8} \text{Kg}\) is the Planck mass. The three parameters \(\xi_1, \xi_2, \text{ and } \xi_3\) are model dependent \([13, 14]\), and should take positive or negative values close to 1. There are some evidence within the formalism of Loop quantum gravity \([14–17]\) that indicates a non–zero values for the three parameters, \(\xi_1, \xi_2, \text{ and } \xi_3\), and particularly \([16, 18]\) that produces a linear–momentum term in the non–relativistic limit. Unfortunately, as is usual in a possible quantum gravity phenomenology, the possible bounds associated with the deformation parameters, open a wide range of possible magnitudes, which is translated to a significant challenge.

Indeed, the most difficult aspect in searching experimental hints relevant for the quantum–gravity problem is the smallness of the involved effects \([19, 20]\). If this kind of deformations are characterized by some Planck scale, then the quantum gravity effects become very small for a single particle \([13, 14]\). It is precisely in this direction that some many–body properties associated with Bose–Einstein condensates, could be helpfully to improve the sensitivity of possible effects caused by the quantum structure of space–time.

Here it is noteworthy to mention that one of the more interesting phenomena related to Bose–Einstein condensates, is the interference pattern when two condensates overlap \([21–22]\). The interference pattern is a manifestation of the wave (quantum) nature of these many–body systems, and could be produced even when the two condensates are initially completely decoupled. Then, after switching off the corresponding traps, this allow the systems expand, overlap, and eventually produce interference fringes. Such an interference pattern was observed in the experiment \([22]\), among others, where interference fringes with a period of \(\sim 15 \times 10^{-6} \text{ meters}\) were observed after switching off the trapping potential and letting the condensates expand for 40 milliseconds and overlap. Indeed, several experiments associated with the interference pattern of condensates in different situations has been made, see for instance \([23, 25]\) and references therein. Let us remark that when the trapping potential is turned off, the free velocity expansion of the cloud corresponds, approximately, to the velocity predicted by the Heisenberg’s uncertainty principle \([21–22]\).

In this aim, we explore the free velocity expansion of the condensate and consequently, the corresponding interference pattern when two of these systems overlap, assuming that the single particle energy spectrum is given by Eq. (4), taken into account only the leading order deformation, \(i.e.,\) setting \(\xi_2 = \xi_3 = 0\). Additionally, we are not interested here in the relative phase between the two condensates, which is a non–trivial topic and also deserves deeper analysis. Thus, we restrict ourselves on the analysis of the free expansion of the condensate together with the separation of the interference fringes when two of these systems overlap.

**II. ANOMALOUS DISPERSION RELATION AND FREE EXPANSION OF THE CONDENSATE**

In order to explore the properties of the condensate under free expansion, let us propose the following modified energy associated with the system

\[
E(\psi) = \int dr \left[ \frac{\hbar^2}{2m} |\nabla \psi(r)|^2 + V(r) |\psi(r)|^2 + \frac{1}{2} U_0 |\psi(r)|^4 + \hbar \alpha |\psi(r)| |\nabla |\psi(r)| | \right],
\]

where \(\psi\) is the wave function of the condensate or the so–called order parameter, \(V(r) = m \omega_0^2 r^2 / 2\) is the external potential, that we will assume for simplicity as an isotropic harmonic oscillator. The term \(U_0 = \Delta \alpha^2 a\), depicts the interatomic potential, being \(a\) the s–wave scattering length \(i.e.,\) only two-body interactions are taken into account. Notice also that we have introduced the contributions due to the deformation parameter \(\alpha = \xi_1 mc \frac{2a}{\pi} \), assuming, as mentioned above that \(\xi_2 = \xi_3 = 0\). If we set \(\alpha = 0\), we recover the usual expression associated with the total energy of the cloud \([21]\).

An accurate expression for the total energy of the cloud can be obtained employing, as usual, an anzats of the form \([21]\)

\[
\psi(r) = \frac{N^{1/2}}{\pi^{3/4} R^{3/2}} \exp(-r^2 / 2R^2) \exp(i\phi(r)),
\]

where \(N\) is the corresponding number of particles and \(R\) a characteristic length, that is interpreted as the radius of the system.

Notice that Eq. (5) corresponds to the solution of the Schrödinger equation associated with non–interacting systems, where the phase \(\phi\) can be associated with particle currents \([21]\). Thus, by inserting the anzats \([3]\) in
the energy functional \(E\) we are able to obtain the corresponding energy

\[
E = E_F + E_R,
\]

where \(E_F\) is the kinetic energy associated with particle currents

\[
E_F = \frac{\hbar^2}{2m} \int d^3r \left| \frac{d\psi(r)}{dr} \right|^2. \tag{5}
\]

Additionally, \(E_R\) can be interpreted as the energy associated with an effective potential, which is equal to the total energy of the condensate when the phase \(\phi\) does not vary in space. The term \(E_R\) contains the contributions of the ground state energy \((E_0)\), the harmonic oscillator potential \((E_P)\), and the contributions due to the interactions among the particles within the condensate \((E_I)\). Notice that we have inserted also the contribution \(E_{\alpha}\) caused by the deformation parameter \(\alpha\)

\[
E_R = E_0 + E_P + E_I + E_{\alpha}, \tag{6}
\]

where

\[
E_0 = \frac{\hbar^2}{2m} \int d^3r \left| \frac{d\psi(r)}{dr} \right|^2, \tag{7}
\]

\[
E_P = \frac{1}{2} m\omega_0^2 \int d^3rr^2 \left| \psi(r) \right|^2, \tag{8}
\]

\[
E_I = \frac{1}{2} U_0 \int d^3r \left| \psi(r) \right|^4, \tag{9}
\]

\[
E_{\alpha} = \hbar \alpha \int d^3r \left| \frac{d^2\psi(r)}{dr^2} \right|. \tag{10}
\]

Consequently, \(E_R\) can be written as follows

\[
E_R = \frac{3}{4} \frac{\hbar^2}{mR^2} N + \frac{3}{4} m\omega_0^2 R^2 N
+ \frac{U_0}{2(2\pi)^{3/2} R^3} N^2 - \frac{2\hbar}{\sqrt{\pi} R} N, \tag{11}
\]

where we have used the trial function \(\psi\) together with Eqs. (7)–(10) in order to obtain the above expression.

The equilibrium radius of the system \(R_0\), can be obtained by minimizing the total energy \((E)\). Additionally, the contribution of the kinetic energy \((E_F)\) is positive definite, and is zero when the phase \(\phi\) is constant \([21]\).

However, when the radius \(R\) differs from its equilibrium condition, after the external potential \(V(r) = m\omega_0^2 r^2/2\) is turned off at, let say \(t = 0\), there is a force that change \(R\) and produces an expansion of the cloud. In order to determine an equation for the dynamics of the system, we must deduce the corresponding kinetic energy \(E_F\) in function of time, through its dependence on the radius \(R\). Changing \(R\) from its initial value to a new value \(\bar{R}\) amounts to a uniform dilatation of the system, since the new density distribution \(\left| \psi(r) \right|^2 = n(r)\) may be obtained from the old one by changing the radial coordinate of each atom by a factor \(\bar{R}/R\), see Ref. [21] for details. Thus, the velocity of a particle can be expressed as follows

\[
v(r) = \frac{\bar{R}}{R}. \tag{12}
\]

Consequently, the kinetic energy \((E_F)\) is given by

\[
E_F = \frac{mN}{2R^2} \int d^3rn(r) v^2, \tag{13}
\]

where the ratio between the integrals is a mean–square radius of the condensate \([21]\).

Then, it is quite easy to obtain the kinetic energy \((E_F)\) by using the anzats Eq. (3), with the result \(E_F = 3R^2 N m/4\). Moreover, assuming that the energy is conserved at any time, we obtain the following energy conservation condition associated with our system

\[
\frac{3m\dot{R}^2}{4} + \frac{3\hbar^2}{4mR^2} + \frac{U_0}{2(2\pi)^{3/2} R^3} N - \alpha \frac{2\hbar}{\sqrt{\pi} R} = 0 \tag{14}
\]

where \(R_0\) is the radius of the condensate at time \(t = 0\), which is approximately equal to the oscillator length \(a_{ho} = (\hbar/m\omega_0)^{1/2}\) and \(R\) is function of time and corresponds to the radius at time \(t\). Eq. (14) must be solved numerically, even in the case \(\alpha = 0\). However, if we neglect inter–particle interactions, i.e., setting \(U_0 = 0\) then, we are able to obtain an analytical solution for the above equation, with the result

\[
\frac{1}{\beta^2} \sqrt{\beta^2 R^2 + \frac{2\hbar \alpha}{\sqrt{\pi} R}} = \frac{3\hbar^2}{4m} \tag{15}
\]

\[
- \frac{\hbar \alpha}{\sqrt{\pi} \beta} \ln \left[ \frac{\beta^2 R + \hbar \alpha}{\beta^2 R_0 + \hbar \alpha} + \left( \frac{\beta^2 R + \hbar \alpha}{\beta^2 R_0 + \hbar \alpha} \right)^2 - 1 \right]^{1/2}
\]

\[
= \sqrt{\frac{4}{3m} t},
\]

where we have defined

\[
\beta^2 = \frac{3\hbar^2}{4mR_0^2} - \frac{2\hbar \alpha}{\sqrt{\pi} R_0}. \tag{16}
\]

A rough approximation for the modified width of the packet which is valid for large expansion times and \(\alpha \ll 1\), renders the following solution

\[
R^2(t) = R_0^2 + \left[ \frac{\hbar^2}{m^2 R_0^2} - \frac{\alpha}{m \sqrt{3\pi} m R_0} \right] t^2 + ..., \tag{17}
\]

which is also equivalent when \(R_0 \gg R_0\) for \(\alpha \ll 1\). If we set \(\alpha = 0\) then, we recover the usual solution \([21]\)

\[
R^2(t) = R_0^2 + \left( \frac{\hbar}{m R_0} \right)^2 t^2. \tag{18}
\]
Notice that in the usual case, $\alpha = 0$, $v_0 = \frac{h}{m\lambda}$ is defined as the velocity expansion of the condensate, corresponding to the velocity predicted by the Heisenberg’s uncertainty principle for a particle confined a distance $R_0$ [21]. Thus, in the usual case $\alpha = 0$, the width of the cloud at time $t$ can be written in its usual form $R^2 = R_0^2 + (v_0 t)^2$ [21].

On the other hand, from Eq. (17), we are able to define the square modified velocity expansion ($v_0^{\alpha}$) as follows

$$\langle v_0^{\alpha} \rangle^2 = \frac{\hbar^2}{m^2 R_0^2} - \frac{8}{3 \sqrt{\pi}} \frac{\hbar}{m R_0}$$

which is well defined, since the deformation parameter $\alpha$ has dimensions of velocity. The modification caused by $\alpha$ is quite small, then the following expansion is justified

$$\langle v_0^{\alpha} \rangle = \frac{\hbar}{m R_0} - \frac{4}{3 \sqrt{\pi}} \alpha + O(\alpha^2).$$

Here, let us remark that the presence of the deformation parameter $\alpha$ suggests a modification to the Heisenberg’s uncertainty principle, which appears in a natural way, just by looking up to the predicted modified velocity ($v_0^{\alpha}$). If we define a new deformation parameter $\alpha' = \alpha \frac{4}{3\sqrt{\pi}}$, together with $R_0 = x$ then, the resulting modified uncertainty principle suggests to be

$$\Delta x \Delta p \geq \frac{\hbar}{2} - \alpha' x + O(\alpha^2).$$

Notice that the leading order modification obtained from the analysis of the free expansion of the condensate, is apparently linear in the position which, as far we know, has been not reported in the literature, see for instance Refs. [26–28] and references therein. If so, this fact would open some new phenomenological implications concerning to the quantum–structure of space time, which is a non–trivial topic and deserves deeper investigation that will be presented elsewhere.

On the other hand, the quantity $\hbar/m$, can be measured by comparing the de Broglie wavelength and velocity of a particle (in fact is the velocity predicted by the Heisenberg uncertainty principle), as demonstrated in Ref. [29] in measurements using neutrons. Indeed, the quantity $\hbar/m$ is also related to the velocity $v_0$ by de Broglie equation

$$\frac{\hbar}{m} = \lambda v_0.$$  

where $\lambda$ is the corresponding wavelength. The velocity $v_0$ of the neutrons is measured using a very precise time–of–flight method, leading to $\hbar/m = 3.956033332(290) \times 10^{-7} m^2/s$ and in consequence a precise determination of the fine–structure constant of order $137.03601062(503) \times 10^{-8}$ was obtained [22]. Both measurements with a relative uncertainty of a few parts in 10–8

These ideas were also extended in Ref. [30], trough measurements of the kinetic energy of an atom recoiling due to absorption of a photon using an interferometric technique called ”contrast interferometry”, in a sodium Bose–Einstein condensate. The quantity $\hbar/m$ can be extracted from a measurement of the photon recoil frequency ($\omega_r$) defined as follows [31]

$$\omega_r = \frac{\hbar}{2m} k^2,$$

where $k$ is the wavevector of the photon absorbed by the atom, whose value is accurately accessible [31]. There, a measurement of the photon recoil frequency leads to $\omega_r = 2\pi \times 24.9973kHZ(1 \pm 6.7 \times 10^{-6})$.

Finally, let us add that the form of the energy dispersion relation [11], was constrained by using high precision atom–recoil frequency measurements [14, 15]. In such scenario, bounds for the deformation parameters of order $\xi_1 \sim 1.8 \pm 2.1$ and $|\xi_2| \sim 10^6$ were obtained.

However, in order to analyze an alternative procedure compared to those used in Refs. [14, 15], i.e., by using the modified free expansion velocity of the condensate Eq. (20), we are able to obtain the following modified de Broglie equation associated with our system

$$\frac{2\pi \hbar}{m} = R_0 \left( v_0 - \alpha \frac{8}{3 \sqrt{\pi}} \right).$$

Consequently, the modified photon recoil frequency $\omega_r^{(\alpha)}$ is given by

$$\omega_r^{(\alpha)} = \frac{R_0}{4\pi} \left( v_0 - \alpha \frac{8}{3 \sqrt{\pi}} \right) k^2.$$  

Where we have assumed that the wave vector $\vec{k}$ of the photon absorbed by an atom is independent of the deformation parameter $\alpha$.

Therefore, the relative shift $\frac{\omega_r^{(\alpha)} - \omega_r}{\omega_r} \equiv \frac{\Delta \omega_r^{(\alpha)}}{\omega_r}$ caused by the deformation parameter $\alpha$ is given by

$$\frac{\Delta \omega_r^{(\alpha)}}{\omega_r} = \frac{4 R_0 m}{\pi^3/2 \hbar}.$$  

The value for $\omega_r = 2\pi \times 24.9973kHZ(1 \pm 6.7 \times 10^{-6})$ obtained in Ref. [30], together with Eq. (20), allows us to obtain a bound for the deformation parameter $\xi_1$, under typical laboratory conditions. In such a case we are able to obtain an upper bound up to $|\xi_1| \sim 1$, by using the relative shift Eq. (20) trough its dependence on the modified velocity expansion Eq. (20), which is compatible with the upper bound reported in Refs. [14, 15].

### III. INTERFERENCE PATTERN OF TWO CONDENSATES AND PLANCK SCALE SIGNALS

Finally, let us analyze the interference pattern of two overlapping Bose–Einstein condensates, in order to explore some possible Planck–scale signals in such phenomenon. If there is coherence between two condensates,
the state may be described by a single condensate wave function, which has the following form

$$\psi_{1,2}(r, t) = \sqrt{N_1} \psi_1(r, t) + \sqrt{N_2} \psi_2(r, t), \quad (27)$$

where $N_1$ and $N_2$ correspond to the number of particles within each cloud. After the free expansion, the two condensates overlap and interfere. If the effects of interactions are neglected in the overlap region, the particle density at any point is given by

$$n_{1,2}(r, t) = |\psi_{1,2}(r, t)|^2 = N_1|\psi_1(r, t)|^2 + N_2|\psi_2(r, t)|^2 \quad (28)$$

$$+ 2\sqrt{N_1 N_2} \text{Re}[\psi_1(r, t) \psi^*_2(r, t)].$$

The third right hand term of expression (28) corresponds to an interference pattern \[21\], caused by the overlap of the two condensates. In order to obtain the corrections caused by the deformation parameter $\alpha$, on the properties of the interference pattern of two condensates, let us appeal as usual, to the following time dependent condensate wave functions \[21\]

$$\psi_1(r, t) = \frac{e^{i\phi_1}}{(\pi R_0^2(t))^{3/4}} \exp \left[ -\frac{(r - d/2)^2(1 - iht/mR_0^2)}{2R_0^2(t)} \right], \quad (29)$$

$$\psi_2(r, t) = \frac{e^{i\phi_2}}{(\pi R_0^2(t))^{3/4}} \exp \left[ -\frac{(r + d/2)^2(1 - iht/mR_0^2)}{2R_0^2(t)} \right], \quad (30)$$

where $\phi_1$ and $\phi_2$ are the initial phases for each condensate, $R_0$ is the initial radius of the cloud, which is approximately equal to the oscillator length $a_{ho} = (\hbar/m\omega_0)^{1/2}$. Additionally, $R_0(t)$ is the the width of a packet at time $t$, given by Eq. (17). If we set $\alpha = 0$ in Eqs. (29) and (30) then, we recover the usual expressions \[21\].

The interference term in Eq. (28) thus in given by

$$\text{Re}[\psi_1(r, t) \psi^*_2(r, t)] = \frac{e^{-d^2/(2\pi R_0^2(t))}e^{d^2/2\pi R_0^2(t)}}{\pi R_0^2(t)^{3/2}}$$

$$\times \cos \left( \frac{\hbar}{m} \frac{r \cdot d}{R_0^2(t)} + \phi \right). \quad (31)$$

Notice that the phase shift $\phi = \phi_1 - \phi_2$ is measurable, although the individual phases $\phi_1$ and $\phi_2$ are not \[32\]. Here the pre-factor $\exp(-r^2/R_0^2(t))$ depends slowly on $r$ but the cosine function can give rise to rapid spatial variations. We can notice also from Eq. (31) that planes of constant phase are perpendicular to the vector between the centers of the two clouds. The positions of the maxima depend on the relative phase of the two condensates, and if we take $d$ to lie in the $z$ direction, the distance between maxima is given by

$$z_\alpha = 2\pi \frac{mR_0^2(t)R_0^2}{\hbar d}. \quad (32)$$

If the expansion time is sufficiently large, i.e., the cloud has expanded to a size much greater than $R_0$ then, as mentioned before, $R_0^2(t)$ is given approximately by Eq. (17). Therefore, the distance between maxima associated with the interference fringes is given by the following expression

$$z_\alpha \approx 2\pi \left( \frac{\hbar}{md} - \frac{8\alpha R_0}{3\sqrt{\pi d}} \right). \quad (33)$$

When $\alpha = 0$, we recover the usual result \[21, 22\]. In the usual case, $\alpha = 0$, the separation between maxima is typically of order $10^{-6}$m \[22\]. From relation (33), we are able to obtain the sensitivity of our system to Planck scale signals upon the fringes separation. Under typical laboratory conditions the correction caused by the deformation parameter $\alpha$ can be inferred up to $|\xi_1| \times 10^{-10}$ meters, i.e., four orders of magnitude smaller than the typical distance between the maxima reported in Ref. \[22\], when $|\xi_1| \sim 1$.

On the other hand, the possibility to obtain a measurable correction associated with the deformation parameter $\alpha (\delta z_\alpha)$ requires that, if $\Delta(z)$ is the experimental error, then $\Delta(z) < |\delta z_\alpha|$. Thus, in this case this entails

$$\Delta(z) \leq |\alpha| \frac{16\sqrt{\pi R_0 t}}{3d}. \quad (34)$$

Under typical laboratory conditions, an experimental uncertainty $\Delta(z)$ of order $|\xi_1| \times 10^{-10}$ meters (i.e., very strict conditions), could be tuned, in principle, below Planck-scale induced separation of the maxima interference fringes in typical conditions, i.e., $\omega_0 \sim 10$Hz and a typical mass of order $m \sim 10^{-6}$Kg, $d = 40 \times 10^{-6}$ meters, together with a free expansion time of order $t = 40 \times 10^{-3}$s.

Notice also that in order to obtain a distance between the maxima of order $\sim 10^{-6}$ meters, an initial separation of order $d \approx 10^{-9}$ meters is necessary, i.e., the separation between the two clouds must be three orders of magnitude smaller than typical separations, when $\xi_1 \sim 1$, for typical expansion times of order $40 \times 10^{-3}$s.

Conversely, an expansion time of order 10 sec, could lead to separation's fringes of the same order ($10^{-6}$ meters reported in \[22\]), for typical initial separations of the two clouds of order $40 \times 10^{-6}$ meters.

In other words, small separations between the two condensates, together with large expansion times, could be used to search small traces arising from the quantum structure of the space-time, upon the interference pattern of two Bose–Einstein condensates.

IV. CONCLUSIONS

We have analyzed the free expansion of a condensate, and also the properties when two of these systems overlap, assuming as a fundamental fact a deformed dispersion relation. We have proved that the free velocity expansion is corrected as a consequence of a possible quantum structure of space time. Additionally, the predicted
modified velocity expansion, suggests in a natural way a modification in the Heisenberg’s uncertainty principle, which in principle, open the possibility to explore some phenomenological consequences in other systems and clearly deserves deeper investigation.

Finally, we have explored possible traces arising from Planck scale physics upon the properties associated with the interference fringes when two condensates overlap under typical laboratory conditions. Furthermore, we have proved that small separations between two condensates, together with a large expansion time, could be used to explore possible signals from the quantum structure of space–time. Nevertheless, this scenario must be extended to more realistic situations, in which the contribution caused by the interactions among the constituents of the condensate are representative, and could be useful to improve the results obtained in the present report.

We must add that the possible detection of these corrections, could be out of the current technology. However, it is remarkable that an adequate choice of the initial conditions in the free expansion of the condensates open the possibility of planning specific scenarios that could be used, in principle, to obtain a possible traces caused by the quantum structure of space–time.

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