Black Hole Thermodynamics and Statistical Mechanics

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Abstract We have known for more than thirty years that black holes behave as thermodynamic systems, radiating as black bodies with characteristic temperatures and entropies. This behavior is not only interesting in its own right; it could also, through a statistical mechanical description, cast light on some of the deep problems of quantizing gravity. In these lectures, I review what we currently know about black hole thermodynamics and statistical mechanics, suggest a rather speculative “universal” characterization of the underlying states, and describe some key open questions.

1 Introduction

Black holes are black bodies.

Since the seminal work of Hawking [1] and Bekenstein [2], we have understood that black holes behave as thermodynamic objects, with characteristic temperatures and entropies. Hawking radiation has not yet been directly observed, of course; a typical stellar mass black hole has a Hawking temperature of well under a microkelvin, far lower than that of the cosmic microwave background. But the thermodynamic properties of black holes are well understood, having been confirmed by a great many independent methods that all yield the same quantitative results: a temperature

$$kT_{\text{Hawking}} = \frac{\hbar}{2\pi}$$

and an entropy

$$S_{\text{BH}} = \frac{A_{\text{horizon}}}{4\hbar G},$$

(2)
where \( A_{\text{horizon}} \) is the horizon area and \( \kappa \) is the surface gravity.

In a typical thermodynamic system, thermal properties are the macroscopic echoes of microscopic physics. Temperature is a measure of the average energy of microscopic constituents; entropy counts the number of microstates. It is natural to ask whether the same is true for the black hole. This is an important question: the Bekenstein-Hawking entropy depends on both Planck’s and Newton’s constants, and a statistical mechanical description of black hole thermodynamics might tell us something profound about quantum gravity. Until about ten years ago, virtually nothing was known about black hole statistical mechanics. Today, in contrast, we suffer an embarrassment of riches: we have many competing microscopic pictures, describing different states and different dynamics but all predicting the same thermodynamic behavior.

In these lectures, I will review what is currently know—and not known—about black hole thermodynamics and statistical mechanics. This is a large subject, and I will have to skip many interesting aspects. In particular, I will not discuss stability analysis, the peculiarities of negative heat capacity, or the complicated question of black hole phase transitions, and I will only lightly touch upon the profound issues of information loss and holography.

Even so, my approach will necessarily be sketchy and idiosyncratic, though I will also try to suggest further references with different emphases and different degrees of detail. I will aim for a broad overview, rather than focusing on the fine points of any one particular approach. Some books and review articles that I have found helpful include [3–7]. In an appendix, I discuss basic black hole properties and explain my notation.

## 2 Black Hole Thermodynamics

I will begin with two somewhat intuitive routes to black hole thermodynamics. One of these is based on the second law of thermodynamics, the other on the four laws of black hole mechanics. Neither route is completely convincing, but together they provide a good foundation for some of the harder quantitative approaches that I shall discuss later.

### 2.1 Entropy and the second law

Imagine dropping a small box of hot gas into a black hole. The initial state includes the gas and the black hole; the final state consists solely of a slightly larger black hole. The initial state certainly has nonzero entropy, in the form of the entropy of the gas. If the second law of thermodynamics is to hold, the final state must have nonzero entropy as well: the larger black hole must gain enough entropy to compensate for the entropy lost when the gas disappears behind the horizon.
We can make this argument somewhat more quantitative [8]. Suppose the box of gas has linear size $L$, mass $m$, and temperature $T$, and that the black hole has mass $M$ and horizon radius $R = 2GM$ (and thus horizon area $A = 16\pi G^2 M^2$). The box of gas will merge with the black hole when its proper distance $\rho$ from the horizon is of order $L$, at which point the disappearance of the gas will lead to a loss of entropy

$$\Delta S \sim -\frac{m}{T}.$$ 

For a Schwarzschild black hole, the proper distance from the horizon is

$$\rho = \int_{2GM}^{2GM+\delta r} \frac{dr}{\sqrt{1-2GM/r}} \sim \sqrt{GM\delta r},$$

so $\rho \sim L$ when $\delta r \sim L^2/GM$. The gas initially has mass $m$, but its energy as seen from infinity is red shifted as the box falls toward the black hole; when the box reaches $r = 2GM + \delta r$, the black hole will gain a mass

$$\Delta M \sim m\sqrt{1-\frac{2GM}{2GM+\delta r}} \sim \frac{mL}{GM}.$$ 

If we now suppose that the box must be as large as the thermal wavelength of the gas, $L \sim \hbar/T$, we see that

$$\Delta S \sim -\frac{mL}{\hbar} \sim -\frac{GM\Delta M}{\hbar} \sim -\frac{\Delta A}{\hbar G}.$$ 

To preserve the second law of thermodynamics, the black hole must gain an entropy of at least order $\Delta A/\hbar G$.

One can perform a similar analysis for a single particle falling into a Kerr black hole (assuming the particle contains at least one bit of entropy) [2], a box containing a simple harmonic oscillator [2], and, using a more sophisticated analysis, a much more general system falling through a horizon [3,9–11]. In each case, a “generalized second law” holds, provided one includes a change of entropy of order $\Delta A/\hbar G$ for the black hole. Such reasoning led Bekenstein to suggest in 1972 that a black hole should itself be attributed an entropy of order $A/\hbar G$ [2].

At the time, there seemed to be a compelling argument against such a hypothesis. Classical black holes are, after all, black: when placed in contact with a heat bath they will absorb energy while emitting none, thus behaving as if they have a temperature of zero [12]. Two years later, Hawking showed that this problem was cured by quantum theory. I shall return to this result below, but let us first consider another classical argument for black hole thermodynamics.
2.2 The four laws of black hole mechanics

In four spacetime dimensions, a stationary asymptotically flat black hole is uniquely characterized by its mass $M$, angular momentum $J$, and charge $Q$. (In the presence of nonabelian gauge fields or certain exotic scalar fields, other kinds of black hole “hair” can occur [13], but this does not change the basic argument.) In the early 1970s, a set of relations among neighboring solutions were found, culminating in Bardeen, Carter, and Hawking’s “four laws of black hole mechanics” [7, 12]. These take a form strikingly similar to the four laws of thermodynamics:

0. The surface gravity $\kappa$ is constant over the event horizon.
1. For any two stationary black holes differing only by small variations in the parameters $M$, $J$, and $Q$,

$$\delta M = \frac{\kappa}{8\pi G} \delta A + \Omega_H \delta J + \Phi_H \delta Q,$$

where $\Omega_H$ is the angular velocity and $\Phi_H$ is the electric potential at the horizon.
2. The area of the event horizon of a black hole never decreases,

$$\delta A \geq 0.$$

3. It is impossible by any procedure to reduce the surface gravity $\kappa$ to zero in a finite number of steps.

As in ordinary thermodynamics, there are a number of formulations of the third law, which are not strictly equivalent; for a proof of the version given here, which is analogous to the Nernst form of the third law of thermodynamics, see [14]. These laws can be generalized beyond the particular four-dimensional “electrovac” setting in which they were first formulated; the first law, in particular, holds for arbitrary isolated horizons [15], and for much more general gravitational actions, for which the entropy can be understood as a Noether charge [16].

Bardeen, Carter, and Hawking noted that these laws closely parallel the ordinary laws of thermodynamics, with the horizon area playing the role of entropy and the surface gravity playing the role of temperature. But they added, “It should however be emphasized that $\kappa/8\pi$ and $A$ are distinct from the temperature and entropy of the black hole. In fact the effective temperature of a black hole is absolute zero... In this sense a black hole can be said to transcend the second law of thermodynamics.”

2.3 Black holes radiate

The first suggestion that black holes might emit radiation was made by Zel’dovich [17], but his argument was qualitative, and applied only to superradiant modes of

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1 See [12], p. 168.
rotating black holes. In 1974, though, Hawking demonstrated that all black holes emit blackbody radiation [1, 18]. The result was startling, and according to Page [19], Hawking himself did not initially believe it. In hindsight, though, one can give a somewhat intuitive description of the effect [20].

Such a description has two main ingredients. The first is that the quantum mechanical vacuum is filled with virtual particle-antiparticle pairs that fluctuate briefly into and out of existence. Energy is conserved, so one member of each pair must have negative energy. (To avoid a common confusion, note that either the particle or the antiparticle can be the negative-energy partner.) Normally, negative energy is forbidden—in a stable quantum field theory, the vacuum must be the lowest energy state—but energy has a quantum mechanical uncertainty of order $\hbar / t$, so a virtual pair of energy $\pm E$ can exist for a time of order $\hbar / E$. The existence of such virtual pairs is experimentally well-tested: for example, virtual pairs of charged particles make the vacuum a polarizable medium, and vacuum polarization is observed in such phenomena as the Lamb shift and in energy levels of muonic atoms [21].

The second ingredient is the observation that in general relativity, energy—and, in particular, the sign of energy—can be frame-dependent. The easiest way to see this is to note that the Hamiltonian is the generator of time translations, and thus depends on one’s choice of a time coordinate. One must therefore be careful about what one means by positive and negative energy for a virtual pair.

In particular, consider the Schwarzschild metric,

$$ds^2 = \left(1 - \frac{2GM}{r}\right) dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 - r^2 d\Omega^2. \quad (4)$$

Outside the event horizon, $t$ is the usual time coordinate, measuring the proper time of an observer at infinity. Inside the horizon, though, components of the metric change sign, and $r$ becomes a time coordinate, while $t$ becomes a spatial coordinate: an observer moving forward in time is one moving in the direction of decreasing $r$, and not necessarily increasing $t$. Hence an ingoing virtual particle that has negative energy relative to an external observer may have positive energy relative to an observer inside the horizon. The uncertainty principle can thus be circumvented: if the negative-energy member of a virtual pair crosses the horizon, it need no longer vanish in a time $\hbar / E$, and its positive-energy partner may escape to infinity.

We can again make this argument a bit more quantitative. Consider a virtual pair momentarily at rest at a coordinate distance $\delta r$ from the horizon. As in section 2.1, the proper time for one member of the pair to reach the horizon will be

$$\tau \sim \sqrt{GM\delta r}.$$ 

Setting this equal to the lifetime $\hbar / E$ of the pair, we find that

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2 Strictly speaking, the coordinates labeled $r$ and $t$ for $r > 2GM$ are different from those with the same labels for $r < 2GM$, since the Schwarzschild coordinate system is only defined in nonoverlapping patches inside and outside the horizon. But one can rephrase the argument in terms of proper time of infalling observers in a way that dodges this mathematical subtlety [20].
which should also be the energy of the escaping positive-energy partner. This is the energy at $2GM + \delta r$, though; the energy at infinity will be red shifted to

$$E_{\infty} \sim \frac{\hbar}{\sqrt{GM\delta r}} \sqrt{1 - \frac{2GM}{2GM + \delta r}} \sim \frac{\hbar}{GM},$$

independent of the initial position $\delta r$. We might thus expect a black hole to radiate with a characteristic temperature $kT \sim \hbar/GM$. In fact, the precise computations I shall describe below yield a temperature $kT_{\text{Hawking}} = \hbar\kappa/2\pi$, which for a Schwarzschild black hole is $\hbar/8\pi GM$.

Inserting the Hawking temperature (1) into the first law of black hole mechanics (3), we see that black holes can indeed be viewed as thermal objects, with an entropy (2). This result is fundamentally quantum mechanical—the Hawking temperature depends explicitly on $\hbar$—and in some sense quantum gravitational, since the Bekenstein-Hawking entropy depends on $G$ as well.

### 2.4 Can Hawking radiation be observed?

I will return to the more precise and detailed derivations of Hawking radiation below. But let us first address the question of whether this effect can be observed.

For a black hole of mass $M$, the Hawking temperature (5) is

$$T_{\text{Hawking}} \sim 6 \times 10^{-8} \left( \frac{M_{\odot}}{M} \right) K,$$

some eight orders of magnitude smaller than the cosmic microwave background temperature for a stellar mass black hole and far smaller for a supermassive black hole. While there is a chance that we could see Hawking radiation from the final stages of evaporation of primordial black holes [22, 23], such events are expected to be rare and difficult to identify.

Another highly speculative possibility for the detection of Hawking radiation comes from models of TeV-scale gravity. In such models—which typically arise from “brane world” scenarios in which our four-dimensional universe is a submanifold of a higher-dimensional spacetime—gravity may become strong at energies far below the Planck scale. If this is the case, black holes might be produced copiously at accelerators such as the LHC, and their quantum properties could be studied in detail [24, 25].

A third, less direct, route is to look for analogs of Hawking radiation in condensed matter systems. As Unruh first pointed out [26], one can create a sonic event horizon in a fluid flow by allowing the flow to become supersonic beyond some boundary. The same analysis that predicts Hawking radiation from a black hole leads to a pre-
dication of phonon radiation from the sonic horizon of such a “dumb hole.” Similar phenomena can occur in a variety of condensed matter systems, from Bose-Einstein condensates to “slow light” to superfluid quasiparticles, and a number of experimental efforts are underway; for reviews, see [27, 28]. It is worth emphasizing that while such experiments could provide strong evidence for Hawking radiation, which is essentially a kinematical property, they would not test the Bekenstein-Hawking entropy, which depends critically on the dynamics of general relativity [29].

2.5 The many derivations of Hawking radiation

In the absence of direct experimental evidence, how confident should we be about Hawking radiation and black hole thermodynamics? Although Hawking’s derivation involves only standard quantum field theory, we can see from the arguments of section 2.3 that the radiation involves modes with arbitrarily high energies: while the asymptotic energies (5) may be small, they come from red-shifted quanta with much higher energies near the horizon. This has led some to suggest that the derivations might involve an extrapolation of quantum field theory beyond the range it can be trusted [26, 30, 31].

I will return this issue below, but for now let me suggest a partial answer. If only one derivation of Hawking radiation existed, we would clearly need to look very carefully for hidden assumptions and unjustified extrapolations. In fact, though, we have a rather large number of different derivations, which involve very different assumptions and extrapolations and nevertheless all agree. Some of these derivations look at eternal black holes, others at black holes formed from collapse; some involve explicit, detailed computations in particular field theories, others use general properties of axiomatic quantum field theory; some involve Planck-scale fluctuations, others cut off energies well below the Planck scale; some predict only the Hawking temperature, others also allow a computation of the Bekenstein-Hawking entropy. While it is still possible that these derivations all share a common flawed assumption, it seems unlikely that so many methods would converge on the same answer if that answer were wrong. None of this vitiates the need for observational tests—after all, the entire general relativistic description of black holes could be wrong—but it suggests that a failure of black hole thermodynamics would have to be either very subtle or very radical.

I will describe some of these derivations below. Given the nature of these lectures, I will not attempt a full description of any one method; my aim is to give a broad overview, with references that will allow the reader to delve into individual approaches in more detail.
2.5.1 Bogoliubov transformations and inequivalent vacua

As noted above, a crucial ingredient in understanding Hawking radiation is the fact that energy—and, in particular, “positive” and “negative” energy—is frame-dependent. Consider, for simplicity, a free real scalar field $\phi$. Recall that in ordinary quantum field theory in flat spacetime, we quantize $\phi$ by first decomposing the field into Fourier modes,

$$\phi = \sum_k \left( a_k u_k(t, x) + a_k^\dagger u_k^*(t, x) \right) \quad \text{with} \quad u_k = e^{i k \cdot x - i \omega_k t}, \quad \omega_k = \left( |k|^2 + m^2 \right)^{1/2},$$

and then interpret the $a_k$ as annihilation operators and the $a_k^\dagger$ as creation operators. The Fourier modes $u_k$ can be understood as a set of orthonormal functions satisfying

$$\left( \Box + m^2 \right) u_k(t, x) = 0, \quad \partial_t u_k(t, x) = -i \omega_k u_k(t, x),$$

where the second condition determines what we mean by positive and negative frequency, and thus allows us to distinguish creation and annihilation operators. The vacuum is then defined as the state annihilated by all of the $a_k$,

$$a_k |0\rangle = 0.$$

In a curved spacetime, or a noninertial coordinate system in flat spacetime, standard Fourier modes are no longer available. With a choice of time coordinate $t$, though, one can still find modes of the form (7) and perform a decomposition (6) to obtain creation and annihilation operators. Given two different reference frames with time coordinates $t$ and $\bar{t}$, two such decompositions exist:

$$\phi = \sum_i \left( a_i u_i + a_i^\dagger u_i^* \right) = \sum_i \left( \bar{a}_i \bar{u}_i + \bar{a}_i^\dagger \bar{u}_i^* \right),$$

and since the $(u_i, u_i^*)$ are a complete set of functions, we can write

$$\bar{u}_j = \sum_i (\alpha_{ji} u_i + \beta_{ji} u_i^*).$$

This relation is known as a Bogoliubov transformation, and the coefficients $\alpha_{ji}$ and $\beta_{ji}$ are Bogoliubov coefficients [32].

We now have two vacuum states, one annihilated by the $a_i$ and one by the $\bar{a}_i$, and two number operators $N_i = a_i^\dagger a_i$ and $\bar{N}_i = \bar{a}_i^\dagger \bar{a}_i$. Using the orthonormality of the mode functions, it is straightforward to show that

$$\langle \bar{0} | N_i | \bar{0} \rangle = \sum_j |\beta_{ji}|^2.$$

Thus if the coefficients $\beta_{ji}$ are not all zero, the “barred” vacuum will have a nonvanishing “unbarred” particle content.
In [1] and [18], Hawking considered a mass collapsing to form a black hole, and computed the Bogoliubov coefficients connecting an initial vacuum far outside the collapsing matter to a final vacuum after the black hole formed. He found that the “barred” observer at future null infinity will observe a thermal distribution of particles, with a temperature $T = \frac{\kappa}{2\pi}$, $\kappa$ surface gravity. I will not go into details here; three very nice reviews can be found in [4, 29, 33]. The essential physical feature is that ingoing vacuum modes “pile up” at the horizon, giving an exponential relationship between ingoing and outgoing surfaces of constant phase; the integrals that determine the Bogoliubov coefficients $\beta_{ji}$ take the form

$$\int d\omega e^{i\omega_0} e^{-i\frac{\omega}{2} \ln v},$$

yielding gamma functions of complex arguments whose absolute squares give the exponential behavior of a thermal distribution.

Hawking’s derivation was based on a particular choice of vacuum state, but generalizations are possible. For example, one may compare the vacuum of a freely falling observer near the horizon to the vacuum of an observer at future null infinity [34]. One can also look beyond the expectation value of the number operator, and express the full final state in terms of initial modes; one finds that it is exactly thermal [35, 36]. Generalizations to spinor and gauge fields are straightforward, and yield the correct fermionic and bosonic distribution functions.

It is also possible to simplify the problem, by looking at the easier model of an accelerated observer in flat spacetime. Such an observer is naturally described in Rindler coordinates [37]

$$ds^2 = e^{2a\xi} (d\eta^2 - d\xi^2),$$

in which the exponential relationship between the unaccelerated and accelerated modes is easy to verify. A straightforward calculation of Bogoliubov coefficients shows that the accelerated observer will see a thermal bath of “Unruh radiation” with a temperature $kT = \frac{\hbar a}{2\pi}$, where $a$ is proper acceleration [34]. By the principle of equivalence, an observer at rest near the horizon of a black hole should experience the same effect, with the acceleration $a$ replaced by the appropriately blue shifted surface gravity $\kappa$, the acceleration necessary to hold the observer at rest.

As I noted in the preceding section, the exponential relationship between “barred” and “unbarred” modes may be a cause of concern. The modes observed as Hawking radiation by an observer far from the black hole are red shifted from Planck-scale modes near the horizon, and it seems that one has extrapolated quantum field theory far beyond the range in which it is known to be valid. To address this question, a number of authors have looked at the effect of modifying the dispersion relations in a way that removes very high energy modes (see, for example, [26, 38–40]). For example [41], one can replace the standard expression for the energy of a massless field, $\omega_k = |k|$, with

$$\omega_k = \omega_k^2 - \frac{\omega_k^2}{4\pi^2} + \text{glossary entry},$$

The final distribution is actually not quite thermal, but contains a “greybody factor” that reflects the backscattering of some of the emitted radiation into the black hole.
\[ \omega^2 = |\mathbf{k}|^2 - \frac{|\mathbf{k}|^4}{k_0^2}, \]

eliminating modes with trans-Planckian energies. Both numerical and analytical computations show that despite these drastic changes in the high-frequency behavior, thermal Hawking radiation persists. We now have strong evidence that a few simple assumptions—a vacuum near the horizon as seen in a freely falling frame, fluctuations that start in the ground state, and adiabatic evolution of the modes—are sufficient to guarantee thermal radiation [42].

### 2.5.2 Particle detectors in a black hole background

The definitions of vacuum and particle number in the preceding section were taken from ordinary quantum field theory. But finding observables in quantum gravity is notoriously difficult, and one might worry about the applicability of these definitions in a highly curved spacetime. To address this issue, Unruh [34] and DeWitt [43] considered the response of a particle detector in a black hole background, and showed that such a detector sees thermal radiation at the Hawking temperature. Similarly, a static atom outside a black hole will be excited as one would expect in a thermal bath [44].

### 2.5.3 The stress-energy tensor

One can obtain further invariant information about black hole radiation by evaluating the expectation value of the stress-energy tensor of a quantum field in a black hole background. This is a large subject; good introductions can be found in the books [6] and [45]. For these lectures, the most relevant result is that an ingoing negative energy flux at the horizon balances the outgoing flux of Hawking radiation observed at infinity, leading to a back-reaction in which the black hole’s mass decreases (as expected from the intuitive argument of section 2.3) and ensuring energy conservation.

The computation of \( \langle T_{\mu\nu} \rangle \) in a black hole background is generally very difficult (see, for example, [46] or chapter 11 of [6]). In the special case of a massless scalar field—or more generally, a conformally invariant field—in two dimensions, the calculation drastically simplifies [47]. The key difference is that in two dimensions, conservation of the stress-energy tensor is sufficient to determine the full expectation value in terms of the trace anomaly \( \langle T^{H}_{\mu\nu} \rangle \), which, in turn, depends only on characteristics of the field in a flat background. The resulting expectation values are thermal, and the total flux can be used to determine the temperature, which matches the Hawking temperature (1).

Quite recently, Robinson and Wilczek have shown how to extend this result to more than two dimensions, by dimensionally reducing an arbitrary field to two dimensions (or equivalently looking at a partial wave expansion) and trading the trace anomaly for a chiral anomaly [48]. Their method, with some variations (for exam-
ple, [49]), has been quickly extended to a wide variety of black holes. In a beautiful piece of work, Iso, Morita, and Umetsu have further shown that by looking at higher order correlators, one can use similar techniques to obtain not just the total flux, but the full blackbody spectrum of Hawking radiation [50, 51].

### 2.5.4 Tunneling through the horizon

For many physical systems, we know that classically forbidden processes can occur through quantum tunneling. This is the case for Hawking radiation. The idea of a tunneling description dates back to at least 1975 [52], but the nicest form is more recent, coming from Parikh and Wilczek’s insight that one can think of the horizon tunneling past the emitted radiation rather than vice versa [53, 54].

Consider a spherically symmetric system of mass $M$ consisting of a Schwarzschild black hole of mass $M - \omega$ emitting a shell of radiation of mass $\omega \ll M$. In Painlevé-Gullstrand coordinates, chosen because they are stationary and nonsingular at the horizon, the shell moves in a spacetime with metric

$$ds^2 = \left(1 - \frac{2G(M - \omega)}{r}\right)dt^2 - 2\sqrt{\frac{2G(M - \omega)}{r}}dt\,dr - r^2d\Omega^2,$$

and outgoing radial null geodesics satisfy

$$\dot{r} = 1 - \sqrt{\frac{2G(M - \omega)}{r}}.$$ 

Now consider the imaginary part of the action for an outgoing positive energy shell—to be interpreted as an s-wave particle—crossing the horizon from $r_{in}$ to $r_{out}$:

$$\text{Im} I = \text{Im} \int_{r_{in}}^{r_{out}} p_r dr = \text{Im} \int_{r_{in}}^{r_{out}} \int_0^{p_r} d p_r dr = \text{Im} \int_M^{M-\omega} \int_{r_{in}}^{r_{out}} \frac{dr}{\dot{r}} dH,$$ (11)

where I have used Hamilton’s equations of motion to write $dp_r = dH/\dot{r}$ and noted that the horizon moves inward from $GM$ to $G(M - \omega)$ as the particle is emitted. Setting $H = M - \omega$ and inserting the value of $\dot{r}$ obtained from the null geodesic equation, one can perform the integral easily through a contour deformation, obtaining

$$\text{Im} I = 4\pi \omega G \left(M - \frac{\omega}{2}\right)$$ (12)

with $r_{in} > r_{out}$. Again, the physical picture is that the horizon tunnels inward as the black hole’s mass decreases.

By standard quantum mechanics, the tunneling rate in the WKB approximation is then

$$\Gamma = e^{-2\text{Im} I/\hbar} = e^{-8\pi \omega G(M - \frac{\omega}{2})/\hbar} = e^{\Delta S_{BH}}$$ (13)
where $\Delta S_{BH}$ is the change in the Bekenstein-Hawking entropy \((2)\). By the first law of black hole mechanics, this is $\hbar \omega / T_H$, and we recover thermal Hawking radiation.

The tunneling derivation may be easily extended to other classes of black holes, and consistently reproduces the standard results. Its relationship to Hawking’s original derivation is not obvious, but Parikh and Wilczek note that the same analysis can describe a negative-energy particle tunneling into the black hole, thus offering a similar physical picture.

### 2.5.5 Periodic Greens functions

Consider the two-point function of a scalar field $\phi$ in a thermal ensemble of inverse temperature $\beta$:

$$
G_\beta(x,0;x',t) = \text{Tr} \left( e^{-\beta H} \phi(x,0) \phi(x',t) \right) = \text{Tr} \left( \phi(x,0) e^{-\beta H} e^{\beta H} \phi(x',t) e^{-\beta H} \right) = \text{Tr} \left( \phi(x,0) e^{-\beta H} \phi(x',t+\beta) \right) = G_\beta(x',t+\beta;x,0),
$$

(14)

where I have used cyclicity of the trace and the fact that the Hamiltonian generates time translations, so $e^{\beta H} \phi(x',t) e^{-\beta H} = \phi(x',t+\beta)$. In particular, (14) implies that if a thermal Greens function is symmetric in its arguments, it must be periodic in time with period $i\beta$. This argument may be run backwards, and such periodicity in imaginary time may be taken as the definition of a thermal Greens function; in axiomatic quantum field theory, this is formalized as the KMS condition \([55–57]\).

As early as 1976, Bisognano and Wichmann showed that the Greens function for a uniformly accelerated observer obeys the KMS condition \([58]\). By the equivalence principle, the same should hold for an observer at rest near the horizon of a black hole. This is indeed the case, as shown by Gibbons and Perry \([59, 60]\), who further demonstrated that the periodicity corresponds exactly to the expected Hawking temperature \([4]\).

### 2.5.6 Gravitational instantons

The periodicity of Greens functions described above suggests that it might be worthwhile to consider the analytic continuation of black hole spacetimes to “imaginary time.” Near the horizon $r = r_+$, a stationary black hole metric takes the approximate form

$$
ds^2 = 2\kappa(r-r_+)dt^2 - \frac{1}{2\kappa(r-r_+)}dr^2 - r_+^2d\Omega^2.
$$

Continuing to imaginary time $t = i\tau$ and replacing $r$ by the proper distance

$$
\rho = \frac{1}{\kappa} \sqrt{2\kappa(r-r_+)}
$$

to the horizon, we obtain the “Euclidean black hole” metric
\[ ds^2 = d\rho^2 + \kappa^2 \rho^2 d\tau^2 + r^2 d\Omega^2. \]  

The \( \rho - \tau \) portion of this metric may be recognized as that of a flat two-plane in polar coordinates, with imaginary time \( \tau \) serving as the angular coordinate. The horizon \( \rho = 0 \) has shrunk to a point. To avoid a conical singularity at the origin, we must require that \( \kappa \tau \) have period \( 2\pi \), i.e., that \( \tau \) have period \( 2\pi / \kappa = 1 / kT_{\text{Hawking}} \).

This result provides a simple way to understand the periodicity of the Lorentzian Greens functions in imaginary time. But it does more: it allows a steepest descent (“instanton”) approximation to the gravitational path integral and a semiclassical derivation of the Bekenstein-Hawking entropy [61]. The key ingredient is the observation that on a manifold with boundary, the ordinary Einstein-Hilbert action must be supplemented by a boundary term, without which it may have no extrema [61, 62]. At an extremum, the “bulk” contribution to the action,

\[
\frac{1}{16\pi G} \int d^4x \sqrt{|g|} R,
\]

vanishes, but the boundary term can give a nonzero contribution. In the original work in this field, the boundary term was taken at infinity [61, 63], but it may more intuitively be placed at the origin of the Euclidean black hole, that is, at the horizon [64–66]. This boundary term may be evaluated in a number of ways—a particularly elegant approach involves dimensional reduction to a disk in the \( \rho - \tau \) plane [64]—

and yields an extremal action

\[
\tilde{I}_{\text{Euc}} = \frac{A_{\text{horizon}}}{4\hbar G} - \beta (M + \Omega J + \Phi Q). \tag{16}
\]

This Euclidean saddle point contributes \( e^{\tilde{I}_{\text{Euc}}} \) to the partition function, and from (16), we can recognize the result as the grand canonical partition function for a system with entropy \( S_{\text{BH}} = A_{\text{horizon}} / 4\hbar G \).

These results can be extended to much more general stationary configurations containing horizons [67]. The essential ingredient is a Killing vector with zeros, which become boundaries upon continuation to Euclidean signature. One can also obtain an equivalent result by canonically quantizing the system while including the boundary terms; the boundary term at the horizon gives rise to a new term in the Wheeler-DeWitt equation, from which one can again recover the Bekenstein-Hawking entropy [68].

### 2.5.7 Black hole pair creation

A further path integral derivation of black hole entropy comes from studying the spontaneous pair creation rate for black holes in a background magnetic field [69], electric field [70], de Sitter space [71], or more complicated combinations of external fields [72]. One consistently finds that the production rate is enhanced by a factor
of $e^{S_{BH}}$, exactly the phase space factor one would expect for a system in which the Bekenstein-Hawking entropy gives the logarithm of the number of states.

### 2.5.8 Quantum field theory and the eternal black hole

Yet another derivation of Hawking radiation comes from considering quantum field theory on an eternal black hole background. Recall that in Kruskal coordinates, a black hole spacetime splits into four regions, as shown in figure [Fig. 1](#).

Consider a state defined on a Cauchy surface $\Sigma$ that passes through the bifurcation sphere. Region $II$ is invisible to an observer living in region $I$, so such an observer should trace over the degrees of freedom in that region. Even if the initial state is pure, such a trace will lead to a density matrix describing the physics in region $I$. This makes it plausible that the region $I$ observer will see thermal behavior, and detailed calculations show that this is indeed the case.

In particular, for a free quantum field there is at most one quantum state, the Hartle-Hawking vacuum state, that is regular everywhere on the horizon [5, 73]. For a scalar field, a direct computation shows that the density matrix obtained by tracing over region $II$ is thermal, with a temperature $T_{\text{Hawking}}$ [74]. For more general fields, the same can be shown by means of fairly sophisticated quantum field theory [5, 73], or by general path integral arguments [75].

### 2.5.9 Quantum gravity in 2+1 dimensions

Most standard derivations of black hole thermodynamics hold in an arbitrary number of dimensions, with changes only in the greybody factors for Hawking radiation.
In three spacetime dimensions, though, many approaches become much simpler. The BTZ solution \([76,77]\) is a vacuum solution of the Einstein field equations in 2+1 dimensions with a negative cosmological constant. It has all of the standard features of a rotating black hole—an event horizon, an inner Cauchy horizon, the same causal structure as that of a (3+1)-dimensional asymptotically anti-de Sitter black hole—but is, at the same time, a space of constant negative curvature. This latter feature greatly simplifies many derivations: for example, Greens functions can be computed exactly and their periodicity in imaginary time exhibited explicitly (see \([77]\) for a review). As was first suggested in \([78]\), it might even be possible to use the relationship between three-dimensional general relativity and two-dimensional conformal field theory \([79]\) to find an exact description of the quantum states of the BTZ black hole; the present status of this conjecture is discussed in \([80]\).

The simplicity of the (2+1)-dimensional setting also permits an approach that is not readily available in higher dimensions. The methods I have described so far are based on properties of quantum fields in a classical, or at best semiclassical, black hole background. In three dimensions, one can work in the opposite direction, starting with a quantum black hole coupled to a classical source. As I shall discuss further in section 4.3, three-dimensional gravity with a negative cosmological constant is closely related to a two-dimensional field theory living at the “boundary” of asymptotically anti-de Sitter space. Emparan and Sachs have shown how to couple this two-dimensional field theory to a classical scalar field, allowing the computation of transition rates among black hole states due to emission and absorption of the classical field \([81]\). By using detailed balance arguments, they recover standard Hawking radiation, including the correct greybody factors, from this fundamentally quantum gravitational picture.

### 2.5.10 Other microscopic approaches

The derivations I have described so far are essentially “thermodynamic,” based on macroscopic properties of black holes. As I shall discuss in the following sections, we now also have a large number of “statistical mechanical” derivations, based on analyses of the microscopic states of the black hole. These microscopic approaches are not complete—string theory derivations, for example, are most reliable for extremal and near-extremal black holes, while loop quantum gravity derivations contain an order one parameter that, so far, must be adjusted by hand—but they seem to work well within their ranges of validity. When combined with the macroscopic approaches above, they provide strong evidence for the reality of black hole thermodynamics.
3 Black Hole Statistical Mechanics

In ordinary thermodynamic systems, thermal properties are macroscopic reflections of the underlying microscopic physics. Temperature is a measure of the average energy of the constituents of a system, for instance, while entropy is essentially the logarithm of the number of states with specified macroscopic properties. The connection between the microscopic and macroscopic properties, given by statistical mechanics, has been remarkably successful across physics.

Given the thermodynamic properties of black holes, it is natural to ask whether these, too, have a statistical mechanical interpretation. Such an explanation would almost certainly involve quantum gravity—the Bekenstein-Hawking entropy involves both Planck’s constant $\hbar$ and Newton’s constant $G$—and we might hope to learn something about the deep mysteries of quantum gravity.

To find such a statistical mechanical description, one should, in principle, carry out a number of steps:

1. find a candidate quantum theory of gravity (not an easy task);
2. identify black holes in the theory (also not easy);
3. identify observables such as horizon area (surprisingly hard—finding physical observables in a quantum theory of gravity is notoriously difficult [82]);
4. count the microstates for a black hole configuration (perhaps easier, but still not trivial);
5. compare to the Bekenstein-Hawking entropy (perhaps relatively easier);
6. compute interactions with external fields, evaluate Hawking radiation, etc. (not at all easy);
7. try to identify new quantum gravitational effects (the horizon area spectrum? evaporation remnants? higher order corrections to the Bekenstein-Hawking entropy? correlations across the horizon?).

Until recently, these steps seemed far beyond reach. In 1996, though, Strominger and Vafa published a remarkable paper in which they explicitly computed the entropy of a class of extremal black holes in string theory from the microscopic quantum theory [83]. Since then, a flood of new microscopic derivations of black hole thermodynamics has appeared. The new puzzle—the “problem of universality”—is that although these derivations seem to be using very different methods to count very different states, they all obtain the same thermodynamic properties.

3.1 The many faces of black hole statistical mechanics

In this section, I will briefly review some of the statistical mechanical approaches to black hole thermodynamics, and in particular the Bekenstein-Hawking entropy. As in section 2.5 I will not go into detail, but will instead try to provide an overall flavor of the work, along with references for further study.
3.1.1 String theory: weakly coupled strings and branes

The first breakthrough in the counting of black hole microstates came with the work of Strominger and Vafa on extremal black holes in string theory [83]. Their approach can be summarized as follows.

The effective low-energy field theory coming from string theory contains a number of gauge fields, each of which can give a charge to a black hole. An extremal supersymmetric (BPS) black hole is uniquely characterized by its charges; in particular, its horizon area can be expressed in terms of these charges. Given such a black hole, one can imaging tuning down the couplings, weakening gravity until the black hole “dissolves” into a gas of weakly coupled strings and branes. In this weakly coupled system, the charges can be expressed in terms of the number of strings and branes and the quantized momentum carried by strings. Furthermore, the states—the excitations of the string-brane system—can be explicitly counted [84]. We can therefore write the number of states in terms of the numbers of strings and branes, and thus the charges. Comparing this number to the horizon area, we recover the standard Bekenstein-Hawking entropy as the logarithm of the number of states.

One might worry that the number of states might not be the same in the weakly coupled system as in the strongly coupled black hole. For the supersymmetric case, though, this number is protected by nonrenormalization theorems. For black holes far from extremality, on the other hand, the computations are much more difficult; there are qualitative arguments that give an entropy proportional to the horizon area, but the exact proportionality factor of $1/4$ is difficult to obtain [85, 86].

It was quickly realized that the Strominger-Vafa results could be extended to a wide variety of extremal and near-extremal black holes, and through duality relations to a number of nonextremal black holes as well. Nice reviews can be found in [87] and [88]; for recent progress on the four-dimensional Kerr black hole, see [89].

This string theory approach has been remarkably successful, determining not only the Bekenstein-Hawking entropy for extremal and near-extremal black holes, but also describing their interactions with other fields and their emission of Hawking radiation. The method has one peculiarity, though, to which I will return below. Suppose you ask me for the entropy of a three-charge black hole in five dimensions. I will compute the horizon area in the strongly coupled theory in terms of the charges, compute the number of states in the weakly coupled theory in terms of the charges, compare the two, and reply that the entropy is one-fourth of the horizon area. If you now ask me for the entropy of a four-charge black hole, or a black hole in six dimensions, I cannot simply tell you that it is one-fourth of the horizon area; I must recompute the horizon area and the number of states in terms of the new parameters and compare the answers again. Each new black hole requires a new calculation: the theory tells us that the number of microstates of a black hole matches the Bekenstein-Hawking entropy $S_{BH}$, but it tells us so one black hole at a time.
3.1.2 String theory: “fuzzballs”

One can run the argument in the preceding section backwards: given a particular excitation of the weakly coupled string and brane system, one can ask exactly what geometry results at strong coupling. The result is a “fuzzball” picture, in which particular black hole states correspond to complicated geometries that have no horizon or singularity, but that look very much like black hole geometries outside the would-be horizon [90, 91]. In special cases, one can count the number of such “fuzzball” geometries and reproduce the Bekenstein-Hawking entropy, and it seems likely that this result can be extended to more general black holes, although it is an open question whether simple geometric descriptions will always suffice [92]. Samir Mathur has discussed this approach extensively in his lectures, to which I refer the reader [84].

3.1.3 String theory: the AdS/CFT correspondence

Yet another string theory approach to black hole statistical mechanics is based on Maldacena’s celebrated AdS/CFT correspondence [93, 94]. This very well-supported conjecture asserts a duality between string theory in \( d \)-dimensional asymptotically anti-de Sitter spacetime and a conformal field theory in a flat \((d-1)\)-dimensional space that can, in a sense, be viewed as the boundary of the AdS spacetime. This correspondence is naturally “holographic” (see section 5.2), describing the black hole in terms of a lower-dimensional theory and thus offering a framework for understanding the dependence of entropy on area rather than volume.

For asymptotically anti-de Sitter black holes, this correspondence makes it possible to compute entropy by counting states in a (nongravitational) dual conformal field theory. The simplest case is the (2+1)-dimensional BTZ black hole discussed in section 2.5.9, whose dual is a two-dimensional conformal field theory. As I shall discuss in section 4.1, the density of states in such a theory has an asymptotic behavior controlled by a single parameter, the central charge \( c \). For asymptotically anti-de Sitter gravity in 2+1 dimensions, this central charge is dominated by a classical contribution, which was discovered some time ago by Brown and Henneaux [95]. Strominger [96] and Birmingham, Sachs, and Sen [97] independently realized that this result could be used to compute the BTZ black hole entropy, reproducing the Bekenstein-Hawking expression.

While this result applies directly only to the special case of three-dimensional spacetime, it has an important generalization. Many of the higher dimensional near-extremal black holes of string theory—including black holes that are not themselves asymptotically anti-de Sitter—have a near-horizon geometry of the general form \( BTZ \times \text{trivial} \), where the “trivial” part merely renormalizes constants in the calculation of entropy. As a consequence, the BTZ results can be used to find the entropy of a large class of stringy black holes, including most of the black holes whose states can be counted in the weak coupling approach of section 3.1.1 [98].
3.1.4 Loop quantum gravity

In the quest for quantum gravity, the leading alternative to string theory is loop quantum gravity [99]. The fundamental “position” variable in this theory is a three-dimensional SU(2) connection; a state is a complex-valued function of (generalized) connections. A useful basis of states consists of spin networks, graphs with edges labeled by SU(2) representations (“spins”) and vertices labeled by intertwiners. A spin network state can be evaluated on a given connection to give a complex number by computing the holonomies along the edges in the specified representations and combining them with the intertwiners at the vertices.

Given a surface \( \Sigma \), one can define an area operator \( \hat{A}_\Sigma \) that acts on loop quantum gravity states. It may be shown that spin networks are eigenfunctions of these operators, with eigenvalues of the form

\[
A_\Sigma = 8\pi\gamma G \sum_j \sqrt{j(j+1)},
\]

where the sum is over the spins \( j \) of edges of the spin network that cross \( \Sigma \). The parameter \( \gamma \), the Barbero-Immirzi parameter, represents a quantization ambiguity, and its physical significance is poorly understood; theories with different values of \( \gamma \) may be inequivalent, but it has been suggested that \( \gamma \) may not appear in properly renormalized observables [100] or in a slightly different approach to quantization [101].

Given this structure, a natural first attempt to count black hole states is to enumerate inequivalent spin networks crossing the horizon that yield a specified area [102, 103]. The more careful variation of this idea [104, 105] takes into account the fact that when one restricts to a black hole spacetime, one must place “boundary conditions” on the horizon to ensure that it is, in fact, a horizon. These conditions, in turn, require the addition of boundary terms to the Einstein-Hilbert action, which induce a three-dimensional Chern-Simons action on the horizon. The number of states of this Chern-Simons theory is closely related to the number of spin networks that induce the correct horizon area, but with slightly more subtle combinatorics. The ultimate result is that the black hole entropy takes the form [106, 107]

\[
S = \frac{\gamma_M}{\gamma} \frac{A_{\text{horizon}}}{4\hbar G},
\]

where \( \gamma \) is the Barbero-Immirzi parameter and

\[
\gamma_M \approx .23753
\]

is a numerical constant determined as the solution of a particular combinatoric problem. If one chooses \( \gamma = \gamma_M \), one thus recovers the standard Bekenstein-Hawking entropy.

The physical significance of this rather peculiar value of the Barbero-Immirzi parameter is not understood, and it may reflect an inadequacy in the quantization
procedure or the definition of the area operator [101]. Note, though, that $\gamma$ only appears in the combination $G\gamma$, so this choice may be viewed as a finite renormalization of Newton’s constant. If the same shift occurs in the attraction between two masses, its interpretation becomes straightforward. Unfortunately, the Newtonian limit of loop quantum gravity is not yet well enough understood to see whether this is the case.

In any case, though, once $\gamma$ is fixed for one type of black hole—the static Schwarzschild solution, say—the loop quantum gravity computations give the correct entropy for a wide variety of others, including charged black holes, rotating black holes, black holes with dilaton couplings, black holes with higher genus horizons, and black holes with arbitrarily distorted horizons [108, 109]. In particular, there is no need to restrict oneself to near-extremal black holes. Hawking radiation, on the other hand, is not yet very well understood in this approach, although there has been some progress [110, 111].

An alternate approach to black hole entropy also exists within the framework of loop quantum gravity [112]. Here, one again looks at a horizon area determined by edges of a spin network, but instead of counting states in an induced boundary theory, one merely asks the number of ways the spins can be joined to a single interior vertex. This amounts, in essence, to completely coarse-graining the interior state of the black hole, and is comparable in spirit to the thermodynamic derivation of section 2.5.8. One again obtains an entropy proportional to the horizon area, although with a different value of the Barbero-Immirzi parameter.

3.1.5 Induced gravity

In 1967, Sakharov suggested that the Einstein-Hilbert action for gravity might not be fundamental [113]. If one starts with a theory of fields propagating in a curved spacetime, counterterms from renormalization will automatically induce a gravitational action, which will almost always include an Einstein-Hilbert term at lowest order [114]. Gravitational dynamics would then be, in Sakharov’s terms, a sort of “metric elasticity” induced by quantum fluctuations.

One can write down an explicit set of “heavy” fields that can be integrated out in the path integral to induce the Einstein-Hilbert action. By including nonminimally coupled scalar fields, one can obtain finite values of Newton’s constant and the cosmological constant. It is then possible to go back and count states of the heavy fields in a black hole background [115]. The nonminimal couplings lead to some subtleties in the definition of entropy, but in the end the computation reproduces the standard Bekenstein-Hawking value. Furthermore, the reduction to a two-dimensional conformally invariant system near the horizon, in the spirit of the thermodynamic approach of section 2.5.3, allows a counting of states by standard methods of conformal field theory [116]. We thus obtain a new, and apparently quite different, view of the microstates of a black hole as those of the ordinary quantum fields responsible for inducing the gravitational action.
3.1.6 Entanglement entropy

As discussed in section 2.5.8, one way to obtain the thermodynamic properties of a black hole is to trace out the degrees of freedom behind the horizon, generating a density matrix for the external observer from a globally pure state. This process also produces a quantum mechanical “entanglement entropy,” which can be thought of as a measure of the loss of information about correlations across the horizon. The suggestion that this entanglement entropy might account for the Bekenstein-Hawking entropy is an old one [117, 118], and it is not hard to show that for many (although not all [119]) states, the entanglement entropy is proportional to the horizon area: the main contribution comes from correlations among degrees of freedom very close to the horizon, and does not involve “bulk” degrees of freedom. The coefficient of this entropy, on the other hand, is infinite, and must be cut off [120], leading to an expression that depends strongly on both the nongravitational content of the theory (the number and species of “entangled” fields contributing to the entropy) and the value of the cutoff.

The same modes that cause the entanglement entropy to diverge also give divergent contributions to the renormalization of Newton’s constant, and it has been suggested that the two divergences may compensate [121]. This notion has recently gained new life with a proposal by Ryu and Takayanagi for a “holographic” description of entanglement entropy [122, 123], in which the $d$-dimensional spacetime containing a black hole is embedded at the asymptotic boundary of $(d+1)$-dimensional anti-de Sitter space. The idea is inspired by the string theory AdS/CFT correspondence, and can be largely proven to work in situations in which such a correspondence exists [124]; the bulk anti-de Sitter metric provides a natural cutoff, yielding finite contributions to both $S$ and $G$. When applied to a black hole, the proposal correctly reproduces the standard Bekenstein-Hawking entropy [125], providing yet another physical picture of the relevant microstates.

3.1.7 Other approaches

A variety of other microscopic descriptions of black hole thermodynamics have also been proposed. In the causal set formulation of quantum gravity—in which a continuous spacetime is replaced by a discrete set of points with prescribed causal relations—there is evidence that the Bekenstein-Hawking entropy is given by the number of points in the future domain of dependence of a spatial cross-section of the horizon [126]. York has estimated the entropy obtained by quantizing the quasi-normal modes [127] of the Schwarzschild black hole, finding a result that lies within a few percent of the Bekenstein-Hawking value [128]. Black hole entropy can be related to the Kolmogorov-Sinai entropy of a string spreading out on the black hole horizon [129]. A number of mini- and midisuperspace models—models in which most of the degrees of freedom of the gravitational field are frozen out—have also been proposed to explain black hole statistical mechanics [130–132], though none is yet very convincing.
One can also build “phenomenological” models of black hole microstates, in which the horizon area is simply assumed to be quantized [133–136]. Such models do not, of course, tell us why area is quantized, and thus do not address the fundamental physical questions of black hole statistical mechanics, but they can suggest useful directions for further research.

Suppose, for example, that the black hole area spectrum is discrete and equally spaced, and that the exponential of the entropy \(2\) gives an exact count of the number of states at a given horizon area. Then the difference between two adjacent values must be an integer; that is,

\[
\Delta A = 4\hbar G \ln k
\]

(18)

for some integer \(k\). Hod has pointed out [137] that for the Schwarzschild black hole, the most highly damped quasinormal modes [127]—the damped “ringing modes” of an excited black hole—have frequencies whose real part approaches

\[
\text{Re } \omega = \ln 3/8\pi GM
\]

(a numerical result later verified analytically [138]). If one applies the Bohr correspondence principle and argues that area eigenstates of the black hole should change by emission of quanta of energy \(\hbar \omega\), one obtains

\[
\Delta A = 32\pi G^2 M \Delta M = 4\hbar G \ln 3,
\]

matching (18) with \(k = 3\). It is not yet clear whether this result has deep significance. It seems to extend to general single-horizon black holes [139] and in a more complicated way to many “stringy” black holes [140], but results for charged and rotating black holes are unclear (for an optimistic view, see [141]).

One can also describe the Bekenstein-Hawking entropy as a count of the number of distinct ways that a black hole with specified macroscopic properties can be made from collapsing matter [10]. Like the phenomenological models of area quantization, this result does not really describe the microscopic degrees of freedom of the black hole itself (except perhaps in the “membrane paradigm” [142]), but it strongly suggests that if the formation of a black hole is a unitary process, such degrees of freedom must exist.

### 4 The problem of universality

One of the main lessons of the preceding section is that a great many different models of black hole microphysics yield the same thermodynamic properties. Some of these models are clearly ad hoc, but others are carefully worked out consequences of serious approaches to quantum gravity. So the new question is why everyone is getting the same answer.
To some extent, this “problem of universality” is a selection issue: there are undoubtedly computations that gave the “wrong” answer for black hole entropy and were discarded without being published. But as noted in section 3.1.1 even within a particular well-motivated and successful string theory model we do not yet understand the universality of the entropy-area relationship. And regardless of what one may think about any one particular approach, one must still explain why any microscopic model reproduces the results of Hawking’s original thermodynamic computation, a computation that seems to require no information about quantum gravity at all.

There are other situations, of course, in which thermodynamic properties do not depend too delicately on an underlying quantum theory. For example, for a large range of parameters the entropy of a box of gas depends only very weakly on whether the molecules are fermions or bosons. But in cases like this, we have a classical microscopic description, and the correspondence principle guarantees that the quantum theory will give a good approximation for the classical results. For a black hole, things are different: the only classical description we have is one in which black holes have no hair—no phase space volume—and thus no entropy. We need something new, some new principle that determines the quantum mechanical density of states in terms of the classical characteristics of a black hole.

I do not know the ultimate explanation for this universal behavior, but in the remainder of this section, I will make a tentative suggestion and offer some evidence that it may be correct.

**4.1 The Cardy formula**

I only know of one well-understood case in which universality of the sort we see in black hole statistical mechanics appears elsewhere in physics. Consider a two-dimensional conformal field theory, that is, a theory in two spacetime dimensions that is invariant under diffeomorphisms (“generally covariant”) and Weyl transformations (“locally scale invariant”). If we choose complex coordinates \( z \) and \( \bar{z} \), the basic symmetries of such a theory are the holomorphic and antiholomorphic diffeomorphisms \( z \to f(z), \bar{z} \to f(\bar{z}) \). These are canonically generated by “Virasoro generators” \( L[\xi] \) and \( \bar{L}[^{\bar{\xi}}] \) [143]. Such a theory has two conserved charges, \( L_0 = L[\xi_0] \) and \( \bar{L}_0 = \bar{L}[^{\bar{\xi}}_0] \), which can be thought of as “energies” with respect to constant holomorphic and antiholomorphic transformations, or alternatively as linear combinations of energy and angular momentum.

As generators of diffeomorphisms, the Virasoro generators have an algebra that is almost unique [144]:
\[
\{L[\xi], L[\eta]\} = L[\eta \xi' - \xi \eta'] + \frac{c}{48\pi} \int d\xi (\eta' \xi'' - \xi' \eta'')
\]
\[
\{L[\xi], \bar{L}[\bar{\eta}]\} = 0
\]
\[
\{L[\xi], \bar{L}[\bar{\eta}]\} = L[\bar{\eta} \xi' - \xi \bar{\eta}'] + \frac{\bar{c}}{48\pi} \int d\bar{\xi} (\bar{\eta}' \xi'' - \xi' \bar{\eta}'').
\] (19)

The central charges \(c\) and \(\bar{c}\) determine the unique central extension of the ordinary algebra of diffeomorphisms. These constants can occur classically, coming, for instance, from boundary terms in the generators [95], or can appear upon quantization.

Now consider a conformal field theory for which the lowest eigenvalues of \(L_0\) and \(\bar{L}_0\) are nonnegative numbers \(\Delta_0\) and \(\bar{\Delta}_0\). In 1986, Cardy discovered a remarkable result [145, 146]: the density of states \(\rho(\Delta, \bar{\Delta})\) at eigenvalues \((\Delta, \bar{\Delta})\) of \(L_0\) and \(\bar{L}_0\) has the simple asymptotic behavior

\[
\ln \rho(\Delta, \bar{\Delta}) \sim 2\pi \left\{ \sqrt{\frac{c_{\text{eff}} \Delta}{6}} + \sqrt{\frac{\bar{c}_{\text{eff}} \bar{\Delta}}{6}} \right\}, \quad \text{with} \quad c_{\text{eff}} = c - 24\Delta_0, \quad \bar{c}_{\text{eff}} = \bar{c} - 24\bar{\Delta}_0.
\] (20)

The entropy is thus determined by the symmetry, independent of any other details—exactly the sort of universality we are looking for.

A typical black hole is neither two-dimensional nor conformally invariant, of course, so this result may at first seem irrelevant. But there is a sense in which black holes become \textit{approximately} two-dimensional and conformal near the horizon. For fields in a black hole background, for instance, excitations in the \(r-t\) plane become so blue shifted relative to transverse excitations and dimensionful quantities that an effective two-dimensional conformal description becomes possible [147–149]. Indeed, as noted in section 2.5.3, the full Hawking radiation spectrum can be derived from such an effective description [50, 51]. Martin, Medved, and Visser have further shown that a generic near-horizon region has a conformal symmetry, in the form of an approximate conformal Killing vector [150, 151].

\section{4.2 Horizons and constraints}

For the special case of the \(2+1\)-dimensional BTZ black hole, the Cardy formula can be used directly to count states. For this solution, the boundary at infinity is geometrically a two-dimensional flat cylinder, and the asymptotic diffeomorphisms that respect boundary conditions satisfy a Virasoro algebra with a classical central charge [95], which can be used in the Cardy formula [96,97]. As described in section 3.1.3, this calculation can be extended to a number of near-extremal black holes whose near-horizon geometry contains a BTZ factor. For more general black holes, though, something new is needed.

One key question, I believe, is how to specify that one is talking about a black hole in quantum gravity. One cannot simply require a fixed metric: the components of the metric do not all commute, and cannot be simultaneously specified in a quan-
tum theory. For the BTZ case, the key element is a set of boundary conditions at infinity, but in general it seems more natural to consider conditions at the horizon. Two approaches to this question are currently under investigation, each leading to an effective two-dimensional conformal description in which the Cardy formula might be applicable.

4.2.1 The horizon as a boundary

The first approach [152, 153] is to introduce “boundary conditions” at the horizon. The horizon is not, of course, a genuine boundary, but it is a place at which we must restrict the value of the metric, precisely to ensure that it is a horizon. As in the BTZ case, such a restriction forces us to add new boundary terms to the canonical generators of diffeomorphisms, changing their algebra. One finds a conformal symmetry in the $r-t$ plane with a classical central charge. For a large variety of black holes, it has been shown that the Cardy formula then yields the correct entropy.\footnote{For this section, see [154] for further references.}

On the other hand, the diffeomorphisms whose algebra yields that central charge, essentially those that leave the lapse function invariant, are generated by vector fields that blow up at the horizon. This is not necessarily a bad thing—from the perspective of an external observer, many physical quantities diverge at the horizon—but the status of these transformations is not clear. In addition, the “horizon as boundary” method has trouble with the two-dimensional black hole, and some normalization issues are not completely sorted out. A related approach is to look for approximate conformal symmetry near the horizon [155, 156]; one again finds a Virasoro algebra with a central charge that seems to lead to the correct entropy, but there are again some normalization ambiguities.

4.2.2 Horizon constraints

A more recent approach [157, 158] is to impose the presence of a horizon by adding “horizon constraints” in the canonical formulation of gravity, that is, introducing new constraints that restrict data on a specified surface to be that of a black hole horizon. In outline, the procedure is this:

1. dimensionally reduce to the two-dimensional $r-t$ plane near the horizon;
2. continue to Euclidean signature, shrinking the horizon to a point as in section \ref{sec:2.5.6} and evolve radially;
3. impose constraints on a small circle around the horizon that force the initial data to be that of a “stretched horizon”;
4. adjust the diffeomorphism constraints on the stretched horizon a la Bergmann and Komar [159–161] to make them commute with the new horizon constraints;
5. find the resulting algebra and central charge.

The Cardy formula again reproduces the correct Bekenstein-Hawking entropy.
4.2.3 Universality again

If either of these approaches is to be an answer to the “problem of universality,” it must be that the horizon conformal symmetries are secretly present in the various other computations of black hole entropy. I do not know whether this is the case; it is a subject of continuing research.

One fairly simple test is to compare the near-horizon Virasoro algebra of section 4.2.2 with the asymptotic Virasoro algebra of the BTZ black hole, which is the key element in the AdS/CFT computations of section 3.1.3. It is shown in [158] that after a suitable matching of coordinate choices, the central charges and conformal weights exactly coincide, providing one piece of evidence for the proposed explanation of universality. There is also an intriguing link to the loop quantum gravity approach of section 3.1.4: the induced horizon Chern-Simons theory in loop quantum gravity is naturally associated with a two-dimensional conformal field theory [162], whose central charge matches the horizon central charge of section 4.2.2. Searches for hidden conformal symmetry in loop quantum gravity, the fuzzball approach, and induced gravity are currently underway.

4.3 What are the states?

In light of the problem of universality, is there anything general we can say about the states responsible for black hole thermodynamics? At first sight, the answer must be “no”: if a universal underlying structure controls the density of states, there should be many different models with different degrees of freedom but with the same thermodynamic properties. Nevertheless, it may still be possible to find an effective description that is valid across models.

To see this, let us first return to the BTZ black hole. In three spacetime dimensions, general relativity has a peculiar feature: it is a topological theory, with no propagating degrees of freedom [163]. Where, then, do the black hole degrees of freedom come from?

The answer to this paradox is at least partially understood [80]. For the (2+1)-dimensional Einstein-Hilbert action to have any black hole extrema, one must impose anti-de Sitter boundary conditions at infinity. Diffeomorphisms that do not respect these boundary conditions are no longer true invariances of the theory, and states one might naively take to be physically equivalent—states that differ only by a diffeomorphism—must be considered distinct if the diffeomorphism connecting them is incompatible with the boundary conditions. New physical degrees of freedom thus appear, which can be labeled by diffeomorphisms that fail to respect the anti-de Sitter boundary conditions. The action for these new degrees of freedom can be extracted explicitly from the Einstein-Hilbert action [164], and the resulting dynamics is that of a Liouville theory, a two-dimensional conformal field theory whose central charge matches the classical value obtained by Brown and Henneaux [95].
Whether one can actually count the states in this theory to reproduce the Bekenstein-Hawking entropy remains an open question [80, 165].

For higher dimensional black holes, the problem is quite a bit more difficult. One possible approach is to start with the Virasoro algebra (19) for the near-horizon conformal algebra of section 4.2.2. In Dirac quantization, the existence of a constraint ordinarily restricts the physical states: we should require that

\[ L[\xi]|\text{phys}\rangle = \bar{L}[\bar{\xi}]|\text{phys}\rangle = 0. \] (21)

But if the central charge \( c \) is nonzero, these conditions are incompatible with the algebra (19). The solution is known in conformal field theory—one can, for instance, require only that the positive frequency parts of the Virasoro generators annihilate physical states [143]—but the result is much the same as for the BTZ black hole: certain states that were originally counted as nonphysical have now become physical. While it is not exactly the same, this phenomenon is reminiscent of the Goldstone mechanism [166], in which a spontaneously broken symmetry leads to massless excitations in the “broken” directions. And like the Goldstone mechanism, it can provide an effective description of degrees of freedom that is independent of their fundamental physical makeup.

One way to see whether this picture makes sense is to examine the path integral measure. The effect of adding a central charge to the Virasoro algebra is to make certain constraints second class [160, 161]. The presence of such second class constraints leads to a new term in the measure, similar to the Faddeev-Popov determinant in quantum field theory [167]. Such a determinant can be interpreted as a contribution to the phase space volume, or the density of states, and might explain the counting of black hole states. For the present case, the relevant determinant is of the form

\[
\det \left| -\frac{c}{12} \frac{d^3}{dx^3} + \frac{d}{dx} L + L \frac{d}{dx} \right|^{1/2}
\]

with \( L = L_0 + L_1 e^{2ix} + L_{-1} e^{-2ix} \).

Work on evaluating and understanding this expression is in progress.

Perhaps the most important test of this idea would be to couple the effective horizon degrees of freedom to external matter and see if one could reproduce Hawking radiation. In 2+1 dimensions, this can be done [81]. In higher dimensions, it may be possible to take advantage of the conformal description of Hawking radiation discussed in section 2.5.3, but this remains to be seen.

5 Open Questions

Some thirty-five years after the seminal papers of Hawking and Bekenstein, black hole equilibrium thermodynamics is a mature subject. The role of trans-Planckian excitations near the horizon, discussed in section 2.5.1, is not yet fully understood, and questions of possible observational tests remain of great interest, but I will risk
the claim that the macroscopic thermodynamic properties of black holes are largely under control.

The microscopic, statistical mechanical, picture of the black hole, in contrast, is poorly understood, and is the subject of a great deal of research. This is hardly surprising—black hole microstates are almost certainly quantum gravitational, and we are still far from a complete, compelling theory of quantum gravity.

Much of the current research focuses on particular microscopic models of black holes, from string theory, loop quantum gravity, and a number of other perspectives. But there are also some broader open questions. In these lectures, I have emphasized one of these, the problem of universality, mainly because it is a focus of my own research. But I will close by briefly mentioning two other deep questions.

5.1 The information loss paradox

Consider a configuration of matter in a pure state—a spherically symmetric state of a scalar field, for instance—that collapses to form a black hole, which then evaporates by Hawking radiation. If Hawking radiation is exactly thermal, and if the black hole evaporates completely, the ultimate result will be a transition from an initial pure state to a final mixed (thermal) state [168]. Such an evolution is not unitary, and seems to violate the basic principles of quantum mechanics. Similarly, we can imagine a black hole held at equilibrium by the continual ingestion of mass to balance its Hawking radiation; this would seem to allow us to convert an arbitrarily large amount of matter from a pure to a mixed state.

The solution to this paradox is heavily debated [169–171]. If the black hole horizon is fundamental (as it is not in, for instance, the “fuzzball” proposal discussed in Mathur’s lectures [84]), there is wide agreement that any answer must involve a breakdown of locality; see, for example, [172–174]. But there is certainly no consensus as to how such a breakdown might occur. The answer is likely to involve deep problems of quantum gravity, a setting in which nonlocality is both inevitable and very poorly understood [82].

5.2 Holography

As a count of microscopic degrees of freedom, the Bekenstein-Hawking entropy [2] has a peculiar feature: the number of degrees of freedom is determined by the area of a surface rather than the volume it encloses. This is very different from conventional thermodynamics, in which entropy is an extensive quantity, and it implies that the number of degrees of freedom grows much more slowly with size than one would expect in an ordinary thermodynamic system.

This “holographic” behavior [175, 176] seems fundamental to black hole statistical mechanics, and it has been conjectured that it is a general property of quantum
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gravity. It may be that the generalized second law of thermodynamics requires a similar bound for any matter that can be dropped into a black hole; a nice review of such entropy bounds can be found in [177]. The AdS/CFT correspondence discussed in section 3.1.3 is perhaps the cleanest realization of holography in quantum gravity, but it requires specific boundary conditions. A more general formulation proposed by Bousso [178] is supported by classical computations [179], and is currently a very active subject of research, extending far beyond its birthplace in black hole physics to cosmology, string theory, and quantum gravity.

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Appendix: Black Hole Basics

Intuitively, a black hole is a “region of no return,” an area of spacetime from which not even light can escape. For a spacetime that looks asymptotically close enough to Minkowski space, this intuitive picture is formalized by the notion of an event horizon, the boundary of the past of future null infinity, that is, the boundary beyond which no light ray can reach infinity [180]. The event horizon has been extensively studied, and has many interesting global properties: for example, it cannot bifurcate and cannot decrease in area.

Unfortunately, while the event horizon has nice properties, it does not seem to be quite the right object to capture local physics. The problem is that the event horizon is teleological: that is, its definition requires knowledge of the indefinite future. To illustrate this with a thought experiment, imagine that we are at the center of a highly energetic ingoing spherical shell of light, currently two light years from Earth. Suppose this shell is so energetic that it has a Schwarzschild radius of one light year. If I now shine a flashlight into the sky, one year from now the light will have traveled one light year, where it will meet the incoming shell just as the shell reaches its own Schwarzschild radius. At that point, the pulse of light from the flashlight will be trapped at the horizon of an ordinary Schwarzschild black hole, and will be unable to travel any farther outward. In other words, in this scenario we are now at the event horizon of a black hole, even though we will detect no change in our local observations until we are abruptly crushed out of existence two years from now.

Since it seems implausible that Hawking radiation “now” can depend on such future events, the event horizon is probably not quite the right object for the study of black hole thermodynamics. Over the past few years, a number of attempts have been made to suitably “localize” the horizon; a nice review can be found in [181].

\[5\] This is admittedly not very likely, but note that it cannot be ruled out observationally: no signal could propagate faster than such a shell, so we would not know of its existence until it reached us.
In these lectures, I will mainly use the concept of an “isolated horizon” [182], a locally defined surface that seems appropriate for equilibrium black hole thermodynamics. An isolated horizon is essentially a null surface whose area remains constant in time, as the horizon of a stationary black hole does. A thought experiment may again be helpful. Imagine a spherical lattice studded with equally spaced flashbulbs, set to all go off at the same time (as measured in the lattice rest frame). When the bulbs flash, they will emit two spherical shells of light, one ingoing and one outgoing. In ordinary nearly flat spacetime, the area of the outgoing sphere increases with time. At the horizon of a Schwarzschild black hole, on the other hand, it is not hard to check that the area of the outgoing sphere remains constant, while inside the horizon, both spheres decrease in area.

To generalize this example, we first define a nonexpanding horizon $\mathcal{H}$ in a $d$-dimensional spacetime to be a $(d-1)$-dimensional submanifold such that [15, 182]

1. $\mathcal{H}$ is null, with null normal $\ell_a$;
2. the expansion of $\mathcal{H}$ vanishes: $\vartheta(\ell) = q^{ab}\nabla_a \ell_b = 0$, where $q_{ab}$ is the induced metric on $\mathcal{H}$;
3. $-T^{ab} \ell_b$ is future-directed and causal.

These conditions imply the existence of a one-form $\omega_a$ such that

$$\nabla_a \ell^b = \omega_a \ell^b \quad \text{on } \mathcal{H}.$$ 

The surface gravity for the normal $\ell^a$ is then defined as

$$\kappa(\ell) = \ell^a \omega_a. \quad (22)$$

Note, though, that the normal $\ell^a$ is not unique: a null vector has no canonical normalization, so if $\ell^a$ is a null normal to $\mathcal{H}$ and $\varphi$ is an arbitrary function, $e^{\varphi} \ell^a$ is also a null normal to $\mathcal{H}$. We can partially fix this scaling ambiguity by demanding further time independence: we define a weakly isolated horizon by adding the requirement

4. $\mathcal{L}_\omega = 0$ on $\mathcal{H}$,

where $\mathcal{L}$ denotes the Lie derivative. This constraint implies the zeroth law of black hole mechanics, that the surface gravity is constant on the horizon.

Even with this last condition, the null normal $\ell^a$ may be rescaled by an arbitrary constant. Such a rescaling also scales the surface gravity, so the numerical value of $\kappa(\ell)$ remains undetermined. This reflects a genuine physical ambiguity in the choice of time at the horizon. Note that the first law of black hole mechanics requires such an ambiguity: mass is only defined relative to a choice of time, so for consistency, rescaling time must also rescale the surface gravity.

For a stationary black hole, $\ell^a$ can be chosen to coincide with the Killing vector that generates the horizon, whose normalization is fixed at infinity—that is, we can

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6 The outgoing sphere remains outgoing with respect to the lattice, of course; as the lattice collapses, its area decreases even faster than that of the outgoing light sphere.
use the global properties of the solution to adjust clocks at the horizon by comparing them to clocks at infinity. In this case, the isolated horizon coincides with the Killing horizon discussed in Gernot Neugebauer’s lectures [183]. If, on the other hand, we wish to focus on physics only at or very near the horizon, the normalization becomes more problematic. One can use the known properties of exact solutions to write an expression for the surface gravity in terms of other quantities at the horizon, thereby fixing \( \ell \) [15], but so far the procedure seems somewhat artificial.

As noted in section 2.2, weakly isolated horizons obey the four laws of black hole mechanics, the second law in the strong form that the area, by definition, remains constant. Generalization to dynamical, evolving horizons are also possible, and could provide a setting for nonequilibrium black hole thermodynamics; for a recent review, see [184].

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