The Shock Response Prediction of Spacecraft Structure Based on Hybrid FE-SEA Method

Xiong Wang 1, Wei Liu 1,*, Xiao Li 2 and Yi Sun 1, *

Abstract: An improved method based on the Hybrid Finite Element-Statistical Energy Analysis (FE-SEA) method and quasi-steady state theory is proposed to predict the response of spacecraft structure during the process of pyrotechnics separation. Firstly, the amplitude–frequency value of shock load is obtained by using time-frequency conversion technology. Then, according to the frequency response characteristics of each part of the spacecraft structure, a more accurate hybrid FE-SEA model is established. The piecewise loading method is used to predict the response of the hybrid model. Finally, the time domain response results are reconstructed, and the shock response spectrum (SRS) is calculated. Based on the test system of simulating pyroshock, the shock experiment of spacecraft structure is conducted. The high frequency and high velocity character of pyroshock could be effectively simulated, and an accurate shock force function could be obtained through the experiment. This indicates that the numerical results are in line with the ones of the experiment. The SRS obtained from experiments and calculations have similar trends and amplitudes. This conclusion verifies the rationality and sufficient accuracy of the novel method in this paper. The novel method presented in this paper greatly improves the computational efficiency. At the same time, it provides theoretical guidance for shock response prediction of spacecraft structure by steady-state methods.

Keywords: shock response prediction; time-frequency domain transformation; hybrid FE-SEA method; shock response spectrum

1. Introduction

The shock environment exists widely in aerospace engineering. A high-frequency transient shock load will occur during the initiating process of a pyrotechnics device. The response of spacecraft under this shock load is defined as the shock environment [1]. During the process of explosive separation, the explosion and separation can produce a complex shock environment with a short duration, high frequency and high amplitude, which may cause the damage or failure of the satellite system (especially the electronic equipment), resulting in failure of the satellite. Therefore, the transient response analysis of the spacecraft structure under the explosive separation and shock load is an important aspect of satellite development [2]. A reasonable prediction method of the shock response is urgently needed in the process of spacecraft development and the early stage of system level tests so as to reduce the scientific research cost and shorten the development cycle. Because of the complexity of the structure and material properties of spacecraft, the modal density of each part of the spacecraft varies greatly. Due to the large area mass and dense modes of the structure, such as the stellar panel, it is very sensitive to high frequency excitation. Therefore, a single finite element or statistical energy analysis method cannot solve the problem well. The hybrid FE-SEA method based on the wavenumber can effectively solve the problem of a complex spacecraft response [3].

Due to the complexity of the shock environment, the shock signal has obvious high frequency or broadband characteristics. The traditional finite element analysis and other
dynamic analysis methods cannot accurately predict the high frequency response caused by the shock load [4,5]. In engineering practice, the magnitude of the shock load is estimated based on the engineering experience combined with the data of a similar test [6]. Lacher [7] studied a shock test method. A pendulum was used to shock the plate to achieve different shock environments. Statistical energy analysis was used in the numerical simulation [8]. Tagarielli et al. predicted the dynamic response of a composite sandwich beam under shock load. The shock responses of fully clamped monolithic and sandwich beams with elastic panels and compressible elastoplastic cores were analyzed [9]. The FE-SEA method [10] was used to calculate the high frequency or middle high frequency response of the structure. The transient statistical energy analysis (TSEA), the finite element method, the local phase information reconstruction (LMPR), and the virtual mode synthesis (VMSS) were utilized to calculate the transient shock response of the structure. George [11] investigated a method of transient response prediction. The shock load was applied to the structure. The results of the numerical analysis were verified by experiments. Jalal Akbari [12] focused on dynamic response estimation against short-time loading with an updated finite element model, using frequency response functions and the particle swarm optimization technique. Pinnington [13] calculated the exact transient energy response of a two-degree-of-freedom system under impulse, and the accurate transient energy response of the two-oscillator system was studied by using TSEA. At the same time, the two methods were simulated. Bodin [14] synthesized the correlation phase based on the pseudo modal phase change. Combined with the influence of the high frequency filter output and low frequency finite element calculation, the time history response of the equipment in the whole frequency range was predicted. Caresta [15] developed a method to predict the transient response of the shock source under structural shock loading. According to the relationship between the impulse response and contact force, Caresta calculated the structural displacement and contact force. Borello [16] supplemented the missing phase information of the statistical energy analysis by supplementing the substructure modal information. The transfer function of the phase reconfiguration was obtained. Then, the time-dependent structural response was calculated by Fourier inverse transform. This method was called local modal phase information reconstruction. The VMSS method was proposed by E. C. Dalton et al. [17]. The principle of statistical energy analysis was introduced into the classical modal analysis. E. C. Dalton et al. expounded the principles and basic formulas of VMSS. It integrated the method into a general computing program. Zhao et al. [18] proposed an improved VMSS to study the shock response of the structure. Experiments were carried out for verification. For the complex structure and the shock signal containing many kinds of information, the applicability of the above method was reduced. For instance, the VMSS ignores the low mode structure, and the TSEA and LMPR cannot deal with complex structures. The calculation accuracy is also difficult to guarantee.

Luo et al. [19] predicted the vibration of typical beam structures based on the hybrid FE-SEA method. It was found that the hybrid method was more efficient than other methods. Yan et al. [20] researched the hybrid method of aircraft model. The structural noise response of aircraft model under acoustic vibration coupling was studied by the hybrid FE-SEA method. In addition, some other papers have applied the hybrid FE-SEA method to different structures and problems [21,22]. At present, the hybrid FE-SEA method is mainly used to solve the problem of steady-state response. For the prediction of the structural shock response, the application of the hybrid FE-SEA method is rarely studied.

In this paper, the basic theory of FE-SEA hybrid method is derived. This method is extended to shock response prediction. When the shock load is acted on the structure, the response time is very short, and the phase of the response of the structure is very close to the phase of the loaded signal. The amplitude of the response is mainly determined by the frequency response function. A more accurate and applicable prediction method of the shock response based on the hybrid FE-SEA method is presented in this paper. The time domain response results are reconstructed by using the phase information of the loaded signal. We can calculate the SRS needed in engineering based on the time domain response.
A shock experiment for the typical spacecraft structure to the rationality and sufficient accuracy of the novel method is carried in this paper. At the same time, compared with the finite element method and other transient response prediction methods, the complex structures are reasonably calculated, and the computational efficiency is greatly improved by our method. The research in this paper can better predict the shock response of system level spacecraft structure and shorten the development cycle of the spacecraft. It provides a basis for the application of the steady-state method to shock problems.

2. The Basic Theory of Shock Response Prediction

A new prediction method of shock response based on the hybrid FE-SEA method is presented in this paper. The flow chart for the shock response predictions process in this article is illustrated in Figure 1. It is known that the amplitude of the response is mainly determined by the frequency response function, and the typical spacecraft structure is a small damping vibration system. Therefore, we start with the analysis of the loading signal. The quasi-steady state loading method is adopted. Firstly, based on short-time Fourier transform (STFT) technology, the time–frequency domain transform of the shock load signal is studied. The amplitude–frequency value of the shock load and the acceleration phase–frequency characteristics of the shock loading in a single degree of freedom system are obtained. Then, the hybrid FE-SEA model of the structure is established. By using the force load input method, the model is loaded. The acceleration frequency response of each frequency band is calculated separately. Finally, the time domain response results are reconstructed based on the inverse short-time Fourier transform (ISTFT) technique by using a series of acceleration frequency response and the corrected phase information. At the same time, the SRS are calculated based on the shock response spectrum theory.

### Figure 1. Flow chart for the shock response predictions process.

The shock load force function acting on the structure is defined as $x(t)$. STFT is used for the time–frequency analysis of the shock time domain signal. It has superior performance in non-linear and non-stationary signal processing. The basic idea of STFT is to divide the original signal into a number of stationary or approximate stationary segments...
by a window function, which is shifted with time. Then, the frequency spectrum of each time period is calculated and determined. For the convenience of numerical calculation, STFT is discretized in the time domain and frequency domain. After deduction and analysis, the basic form of discrete STFT is written as follows:

$$S(n, \omega r) = STFT_x(n, r) = \sum_{m=-\infty}^{\infty} x(t)w(n-t)e^{-j\frac{2\pi}{N}rt}$$ (1)

in which \(S(n, \omega r)\) contains the results of STFT related to frequency and time, \(n\) is the discrete time, \(r\) is the angular frequency, \(\omega r = \frac{2\pi}{N}r\), \(x(t)\) symbolizes the original signal, and \(w(n-t)\) symbolizes a window function.

According to the STFT theory, the result of the time–frequency analysis is calculated by using MATLAB tools for programming. From the above Equation (1), we can obtain the following:

$$F(\omega)_n = \text{abs}(S(n, \omega r))$$ (2)

$$Ph(\omega)_n = \text{angle}(S(n, \omega r))$$ (3)

where \(F(\omega)_n\) symbolizes the amplitude–frequency value of the shock load, and \(P(\omega)_n\) symbolizes the phase–frequency characteristics.

The phase information of the response of the \(i^{th}\) subsystem corrected on the basis of original signal could be written as follows:

$$Ph(\omega)_j = F(Ph(\omega)) + K_i$$ (4)

where \(K_i\) is the phase delay factor of the shock load passing through the \(i^{th}\) subsystem. \(K_i\) is related to the damping of the \(i^{th}\) subsystem and the distance between the \(i^{th}\) subsystem and the shock source. The sine shock load signal is loaded into the single degree of freedom system with a different damping ratio. The acceleration response analysis of the system is shown in Figure 2. Figure 2a,b shows the acceleration responses of the system with a large damping ratio (0.5) and small damping ratio (0.02), respectively.

It is found that the phase of the acceleration response is almost the same as that of the loading signal in the case of small damping. The above study reveals that when the shock load acts on the structure piecewise and the response time is very short, the phase change is very small in the whole loading time domain. The damping coefficient of the system is generally less than 0.1. The phase of the structural response is very close to that of the loading signal. Therefore, the effect of \(K_i\) on the response prediction is usually ignored.

The complex spacecraft structure contains a variety of structural forms. According to the hybrid FE-SEA theory, the structure can be divided into different subsystems, according to the modal characteristics of each part. The acceleration response of the subsystem with higher modal density has statistical characteristics when it is exposed to the shock load. The statistic characteristics can be processed by the concept of statistical average, while the subsystem with sparse modal density cannot be processed accurately. Therefore, the average value of the statistical energy of all subsystems of the overall spacecraft structure cannot be inferred by the modal density in the frequency range alone, nor can the overall reliable response result be obtained by the finite element method alone. In this case, the most reasonable method is to carry out hybrid FE-SEA modeling for complex structures. The subsystem with higher modal density is the SEA system, and the subsystem with sparse modal density is the FE system. In this paper, the hybrid FE-SEA model of spacecraft structure is established. Based on the FE-SEA model, the acceleration frequency response of the structure in each frequency band is calculated.

In the hybrid FE-SEA method, diffuse reflection means the reverberant energy field after multiple reflections. The direct field only satisfies the output displacement field on the deterministic boundary, and the energy can be radiated out through the direct field subsystem. The function of reverberation field is to superimpose the boundary conditions
of reverberation field and direct field linearly. The deterministic and stochastic boundary conditions are satisfied simultaneously [23,24].

![Figure 2: Acceleration response analysis of a single-degree-of-freedom system under sinusoidal shock loading.](image)

The damping ratio is 0.02.

The damping ratio is 0.5.

Figure 2. Acceleration response analysis of a single-degree-of-freedom system under sinusoidal shock loading.

The dynamic equation of subsystem boundary can be written as follows:

$$D^{(b)}_{dir}q^{(b)} = f^{(b)}_{ext} + f^{(b)}_{rev}$$

where $D^{(b)}_{dir}$ is the dynamic stiffness matrix of direct field, which is used to solve the displacement on the definite boundary, $f^{(b)}_{rev}$ is the reverberation force generated by reverberation field energy, and $f^{(b)}_{ext}$ is the external load. After coupling, only considering the effect of the
external force of the deterministic subsystem on the deterministic subsystem, the dynamic
equation of the deterministic subsystem can be shown as follows:

\[ D_{\text{tot}}q = f_{\text{ext}} + \sum_{m} f_{\text{rev}}^{(b)} \]  

(6)

where \( D_{\text{tot}} = D_d + \sum_{m} D_{\text{dir}}^{(b)} \), \( D_d \) is the dynamic stiffness matrix of the deterministic sub-
system, \( q \) is a deterministic generalized coordinate, and \( f_{\text{ext}} \) is the sum of the external
loads. The set average of Equation (6) is solved. The response cross-spectrum can be given
as follows:

\[ \langle S_{ij} \rangle = \langle S_{i,\text{ext}} \rangle + \sum_{p} \langle S_{j,\text{rev}}^{(b)} \rangle \]  

(7)

where \( S_{i,\text{ext}} \) is the cross-spectrum of external load, and \( S_{j,\text{rev}}^{(b)} \) is the cross-spectrum of
blocked diffuse reverberant force. The set average related to the blocking force \( S_{j,\text{rev}}^{(b)} \) in the
reverberation field can be obtained from the reciprocity between the direct field and the
reverberation field. In a diffuse reverberation field, because of the relationship between the
blocking force of the stochastic subsystem in the reverberation field and the uncertainty of
the system, the set related to \( S_{j,\text{rev}}^{(b)} \) tends to the following values:

\[ S_{j,\text{rev}}^{(b)} = \alpha_b \text{Im}(D_{\text{dir}}^{(b)}) \]  

(8)

where \( \alpha_b = \frac{4E_b}{\omega \eta_{\text{ff},\text{rev}}} \) is the proportional constant related to the reverberation field amplitude,
and \( E_b \) is the diffuse reverberant energy. The energy of each stochastic subsystem can be
calculated by the power balance among subsystems.

The power flow balance relation of stochastic \( i^{th} \) subsystem can be established as follows:

\[ p_{\text{in},\text{dir}}^i + p_{\text{in},1}^i = p_{\text{out},\text{rev}}^i + p_{\text{diss}}^i + p_{\text{ij}}^i \]  

(9)

where \( p_{\text{in},\text{dir}}^i \) is the input power of the direct field to the random subsystem through the
deterministic boundary, \( p_{\text{in},1}^i \) is the input power directly applied to the random subsystem,
\( p_{\text{out},\text{rev}}^i \) is the output power through the blocked hybrid force with deterministic boundary
conditions, \( p_{\text{diss}}^i \) is the power dissipated with internal loss factor, and \( p_{\text{ij}}^i \) is the power
transferred to other statistical energy subsystems.

Considering the energy equations of each stochastic subsystem, the energy equations
of the \( i^{th} \) stochastic subsystem can be obtained as follows:

\[ \omega (\eta_i + \eta_{\text{d},i}) E_i + \sum_{e} \omega \eta_e n_e E_i / n_i - \sum_{e} \omega \eta_e n_e E_e / n_e = p_{\text{in}}^i + p_{\text{in},\text{ext}}^i \]  

(10)

where \( \eta_i \) is the internal loss factor, \( \eta_{\text{d},i}, \eta_{\text{e},i} \) is the coupling loss factor, \( E_i \) is the energy of the
\( i^{th} \) system, \( p_{\text{in}}^i \) is the input power applied to the \( i^{th} \) system, \( p_{\text{in},\text{ext}}^i \) is the power input by
other systems, and \( \eta_{\text{d},i}, \eta_{\text{e},i}, p_{\text{in},\text{ext}}^i \) can be represented by \( D_{\text{dir}} \) and \( S_{j,\text{rev}}^{(b)} \).

The analysis process based on the hybrid method is as follows: Firstly, the system
is divided into a deterministic subsystem and stochastic subsystem, and the dynamic
stiffness matrix of the joint is obtained. Then, the parameters of the subsystem are analyzed.
Furthermore, the power balance equation of stochastic subsystem is established to solve
the energy of stochastic subsystem. Finally, the deterministic subsystem is solved.

According to the relationship between average velocity and energy response, the
average velocity of the \( i^{th} \) subsystem can be expressed as follows:

\[ v(\omega)_i = \sqrt{\frac{E(\omega)_i}{M_i}} \]  

(11)
where $E(\omega)_i$ is the energy response, $\dot{v}(\omega)_i$ is the velocity response, and $M_i$ is the mass.

According to Equation (11), the acceleration response amplitude of the $i$th subsystem can be obtained as follows:

$$a(\omega)_i = \dot{v}(\omega)_i = 2\pi f \sqrt{\frac{E(\omega)_i}{M_i}}$$  \hspace{1cm} (12)

The inverse short time Fourier transform is discretized in the time domain and frequency domain. After derivation, the basic form of discrete ISTFT is represented as follows:

$$\hat{A}(n) = \frac{1}{Nw(0)} \sum_{k=0}^{N-1} S_x(n,k)e^{i\frac{2\pi}{N}kn}$$  \hspace{1cm} (13)

According to the STFT theory, the inverse short time Fourier transform of signal is realized by using MATLAB tools for programming. So far, we have obtained the results of acceleration frequency response ($a(\omega)_i$) and the phase information ($\text{Ph}(\omega)_i$) corrected on the basis of the original signal. Combined with the two calculation data, the time history response of the $i$th subsystem under shock loading is reconstructed. Then, the shock response spectrum is calculated, based on the time history response.

3. Shock Experiment and Simulation Study of Spacecraft Structure

This section introduces the shock experiment of a typical spacecraft structure. The shock force load function curve and the structure’s acceleration shock response results are obtained through experiments. In addition, the time–frequency results of the shock load signal are analyzed based on STFT. A hybrid model of the spacecraft structure is established. Through the FE-SEA method, the acceleration shock response of the structure is calculated by the quasi-steady state loading method. Comparing experiments and calculations, the accuracy of the impulse response prediction method is verified and discussed.

3.1. Shock Experiment of the Spacecraft Structure

The shock experiment should provide information for subsequent numerical simulation verification in three aspects. First, the shock force function acting on the structure is measured in the test. It is used as the input signal of numerical analysis. Second, the hybrid model is established, according to the structure of spacecraft. Third, the response results of the structure under shock load are obtained. It is used as a reference value for numerical analysis results.

We designed a high frequency and high efficiency shock test platform based on the light-gas gun device. Shock tests of typical spacecraft structures are carried out with this platform. In the tests, a typical structure of the spacecraft is assembled by employing aluminum honeycomb sandwich panels and beams. The four aluminum honeycomb sandwich panels have the same structure; the size is 100 cm × 120 cm, and the thickness is 3 cm. The plates are assembled into the spacecraft cabin by the beam structure.

The shock experiment of the structure is shown in Figure 3. The structure is suspended on the test support by using elastic ropes. The power unit of shock loading device is the light-gas gun which is shown in Figure 4a. By changing the pressure value of the pressure chamber of the light-gas gun, the shock load is adjusted. Bullets with various geometries and vibrating rods of different materials and sizes are fabricated. By selecting the bullet and the vibrating rod, the waveform of the loading signal can be adjusted, and the shock load of our concerned frequency and magnitude can be obtained. A shock force sensor and a vibration rod are arranged at the shock position of the structure, which is shown in Figure 4b. In the experiment, the function of the shock force sensor is to measure the electric signal of the shock force function of the bullet acting on the steel plate. The measuring range of the force sensor is 80,000 N. The measured signal can be directly converted into the pressure signal by using the piezoresistive relationship obtained by calibrating the sensor in advance. It provides input force for numerical calculation.
Figure 3. Shock experiment of the typical spacecraft structure.

Subsequently, the inclusion curves of SRS results of all acceleration sensor data on a board are used as the test results of the board. The acceleration sensors are arranged on four plates in the principle of random distribution. Four acceleration sensors are arranged on each board. The measuring range of the acceleration sensor is 20,000 Hz and 50,000 g. Through the signal acquisition system, the acceleration response of each position of the structure is collected. The measurement and control system of the experiment can be seen in Figure 5. Based on the measurement and control system, the results are collected, analyzed and processed. Finally, the acceleration shock response spectrum of the structure can be calculated, according to the shock response results. According to the acceleration signal of each measuring point, the average acceleration response and SRS of the structure in space is calculated. The test experiment is used to provide reference values for numerical analysis.

3.2. The Time–Frequency Analysis of Shock Load

The input force load measured from the experiment is used as the input load of the numerical analysis. This high frequency shock load signal is shown in Figure 6. The explosion shock signal has the characteristics of short duration and fast mutation. The peak value of shock signal in Figure 6 is obvious with high frequency, and the maximum value reaches 30,000 N. The experimental result is consistent with the characteristics of the shock signal from the explosion.

According to the STFT theory in the previous chapter, the time–frequency analysis of the typical high-frequency shock load signal in Figure 6 is calculated. The shock load signal is transformed by STFT, and the results are shown in Figure 7. The STFT results include both the temporal and frequency characteristics of the signal. It can be found that each time period contains a frequency value, and the frequency energy is more concentrated between 1 ms and 2 ms, which is related to the characteristics of the time domain signal.
Figure 3. Shock experiment of the typical spacecraft structure.

(a) Shock loading device based on the light-gas gun.

Figure 4. Shock loading device and measurement method of force signal.

(b) Measurement method of force signal.

Figure 5. The measurement and control system of the experiment.
3.2. The Time–Frequency Analysis of Shock Load

The input force load measured from the experiment is used as the input load of the numerical analysis. This high frequency shock load signal is shown in Figure 6. The explosion shock signal has the characteristics of short duration and fast mutation. The peak value of shock signal in Figure 6 is obvious with high frequency, and the maximum value reaches 30,000 N. The experimental result is consistent with the characteristics of the shock signal from the explosion.

According to the **STFT** theory in the previous chapter, the time–frequency analysis of the typical high-frequency shock load signal in Figure 6 is calculated. The shock load signal is transformed by **STFT**, and the results are shown in Figure 7. The **STFT** results include both the temporal and frequency characteristics of the signal. It can be found that each time period contains a frequency value, and the frequency energy is more concentrated between 1 ms and 2 ms, which is related to the characteristics of the time domain signal.

**Figure 6.** The high-frequency shock load signal from experiment.

**Figure 5.** The measurement and control system of the experiment.
3.3. Hybrid FE-SEA Model and Solution

The typical spacecraft structure used in the experiment is shown in Figure 3. The hybrid FE-SEA model of the structure is established based on FE-SEA theory, as shown in Figure 8.
In the typical structure, the aluminum honeycomb sandwich panels behave with low stiffness but high modal density. This accords with the characteristics of SEA. So, the aluminum honeycomb sandwich panels are built as SEA model. The modal density of the connecting beam is low. It is built as a FE model. The process of model establishment meets the accuracy requirements of simplification and equivalence. The beam structure is made of cylindrical aluminum alloy, with a length of 110 cm and a density of 2750 kg/m$^3$. The finite element model of the beam structure is established, according to the beam structure form in Figure 3, with a mass of 4.3 kg. The plate structure is an aluminum honeycomb sandwich plate structure; the size is 100 cm $\times$ 120 cm, and the thickness is 3 cm. The thickness of the upper and lower aluminum plates is 0.1 cm, and the density is 2700 kg/m$^3$. The thickness of honeycomb sandwich structure is 2.8 cm, and the density is 48 kg/m$^3$. The statistical energy analysis model of sandwich structure is established according to the actual structure, and the mass is 8.2 kg.

Since the honeycomb panel and the beam structure are connected by screws, a mixed point connection is established between the FE subsystem and the SEA subsystem. In VAone, corresponding material attributes are generated, according to material type settings. In the early stage, we conducted the parameter test of a single-plate, double-plate and four-plate structure. The internal loss factor, coupling loss factor and modal density of SEA model are obtained through typical structural parameter measurement experiments. As shown in Figure 8, the plate shocked by the load and the three response plates are defined as plate 1, plate 2, plate 3 and plate 4, respectively.

For the statistical energy analysis subsystem, only the modal characteristics of the structure need to be concerned. In the process of modeling, the form of the structure is simplified. After the SEA model is established, the modal density of the subsystem in each frequency band is detected based on the VAone intermediate frequency band modal number analysis. Based on hybrid connection detection, the connection mode between subsystems is modified. The modal number of SEA subsystem in each frequency band is greater than five. The SEA model meets the calculation standard. The parameters of hybrid connection are corrected by a parameter test. The convergence and stability of the finite element subsystem are analyzed. The overall modal density of the finite element subsystem is low. So, the number of grids meets the requirements of calculation accuracy. The vibration mode of finite element structure is consistent with that of an actual beam structure. Therefore, the model based on hybrid method can effectively characterize the actual structure. The prediction of structural shock response based on the model is reasonable and effective. Taking the time–frequency analysis results as the input load, the quasi-steady state loading mode is used to calculate the coupled plate structure. The loading mode of the force spectrum is used, and the result of the structural response is predicted based on the hybrid FE-SEA method. By using the phase information and a series of frequency response, the time domain response is reconstructed by utilizing the ISTFT technique. The results of the overall time domain response are reconstructed. The SRS of typical spacecraft structure can be calculated based on the time domain response results. So far, we obtain the final shock response of the structure based on the hybrid FE-SEA method.

Figure 9 illustrates the SRS results of the typical spacecraft structure, which are obtained based on the hybrid FE-SEA method. Obviously, the four SRS curves have similar trends with the change of frequency. The value shows an increasing trend from 100 Hz to 2500 Hz, and reaches the inflection point and the maximum at about 2500 Hz. It starts to decrease from 2500 Hz and tends to a stable value. For the same frequency bandwidth, the SRS results show a decreasing trend from plate 1 to plate 4. However, the SRS results of plate 2 and plate 3 are almost the same. This result conforms to the actual transmission law of the impulse response. Plate 1 is the closest to the source of the shock load, and plate 4 is the farthest from the source of the shock load. The farther away from the shock source, the smaller the shock response. Plate 3 and plate 2 are symmetrical to the shock source, so their SRS are the same, in theory. However, due to the simplification of the model and
the errors in the reconstruction of the response results, there are very slight differences between their SRS.

Figure 9. The SRS result of spacecraft structures by hybrid FE-SEA method.

3.4. Results and Discussion

In this chapter, the shock response of the loading plate and the response plate of the spacecraft structure are obtained by the hybrid FE-SEA method. The SRS of calculation and experiment for the plate 1 are plotted in Figure 10. The SRS of the calculation and experiment for the other three response plates are shown in Figures 11–13, respectively. With the change of frequency, the SRS curves of the four plate test results have a similar trend. The value shows an upward trend form, reaching the inflection point and the maximum at about 2500 Hz. At the same frequency band width, the values from plate 1 to plate 4 show a downward trend. The test results are consistent with the physical law of shock load transmission in the structure. By comparing the numerical and experimental data of the spacecraft structure, the rationality and sufficient accuracy of the proposed method in this paper are discussed.

From Figures 10–13, it is shown that the numerical results are in good agreement with the experimental results. The maximum and inflection points of the two results are close. The evolution trend of the two curves with the increase of frequency are the same. In most frequency bands, the calculated results are within 6 dB of the experimental results. However, it is obvious that in some cases, the error of the calculation results is large, especially in some low-frequency bands.

The finite element model of the structural cabin is established, and the traditional finite element is used to analyze the response of the structural cabin. In order to ensure the calculation accuracy of high frequency band, the grid size should meet the requirements of the modal analysis. The structure needs to be divided into at least 170,000 units. It is calculated every 10 µs step, and it takes more than 24 h to calculate 10,000 µs. The high-frequency band of the calculation results will be inaccurate, due to the high modal density. The new method is used to predict the shock response of the spacecraft cabin, which avoids
the meshing of the plate structure. The same computer is used for the calculation, according to the frequency band based on the hybrid model. The above calculation results can be obtained in only a few minutes, which greatly improves the calculation efficiency. The calculation accuracy of high frequency band is also guaranteed.

Figure 10. Shock responses of experiment and calculation for plate 1.

Figure 11. Shock responses of experiment and calculation for plate 2.
Figure 11. Shock responses of experiment and calculation for plate 2.

Figure 12. Shock responses of experiment and calculation for plate 3.

Figure 13. Shock responses of experiment and calculation for plate 4.

The error reason is analyzed in the following. One is that there are errors in the parameters of the SEA model obtained from the measurement of typical structure. The other is that the modal density of the structure is low in some low frequency bands, which cannot meet the modal requirements for accurate results. Another reason is that four acceleration sensors are randomly arranged on each plate in the experiment, and
the response result of the plate is the average of the five sensors. Since the number of measurement points is limited, the average response of these measurement points will also bring errors.

It can be found that the calculated results of plate 1 are more consistent with the experimental results than the other three plates. The calculated results of the plate 4 are the worst. It can be found that the farther the structure is from the shock source, the less consistent the calculated and experimental results are. In order to describe the above comparison results clearly, the coefficient $E$ is constructed to represent the error, which is defined as follows:

$$E = \frac{\sum_{f=f_0}^{f_N} |SRS_c(f) - SRS_e(f)|}{\sum_{f=f_0}^{f_N} SRS_e(f)}$$

(14)

where $SRS_c(f)$ and $SRS_e(f)$ are the calculated SRS and the experimental SRS, respectively. Obviously, the coefficient $E$ is positive. The greater the coefficient $E$, the greater the error.

By using Equation (14), the coefficient $E$ for the plate in Figures 10–13 can be calculated to be 0.179, 0.604, 0.596 and 0.697, respectively.

This analysis shows that the error increases when the stress wave passes through the connecting structure. When the energy is transmitted from the shock source to the response plate, the more subsystems it passes through, the greater the prediction error of the response plate. The hybrid connection and structural transfer parameters cannot accurately describe the actual energy transfer. We should note that the calculation results are generally larger than the test results, and the calculation results are relatively conservative. The farther away from the shock source, the larger the phase error of the structural response. In the second section, the variable $K_i$ is proposed. The larger the $K_i$ is, the more conservative the calculation result. However, in general, the magnitude of the shock load at a distance from the shock source is relatively small. In future, it will be necessary to increase research on the structural connection setting. In the calculation process, the control variables can be summarized, according to the shock source distance and transmission law so as to improve the calculation accuracy.

4. Conclusions

This study presents a new method of predicting the pyroshock response of a structure based on the hybrid FE-SEA method. The following conclusions can be achieved: Combined with the time–frequency domain transformation of the load shock signal and time history response reconstruction, the hybrid FE-SEA model is applied to the shock response prediction of the structure. This indicates that the numerical results based on the new method are in good agreement with the experimental results. The rationality and sufficient accuracy of the prediction results of the structural shock response based on the hybrid FE-SEA method is verified. The complex spacecraft structure is suitable to be modeled as a hybrid model. By means of this, we can better grasp the frequency response characteristics of the results. The reasonable finite element model of the structure with low modal density can be established. At the same time, the problems caused by units’ division in the high frequency sections are avoided. Our new method can predict the middle- and high-frequency shock response more effectively and reasonably, and the computational efficiency is greatly improved, compared with the traditional finite element method. This paper provides a theoretical basis for predicting the shock response of the spacecraft structure by using more steady-state methods.

**Author Contributions:** Conceptualization, Y.S.; Data curation, X.W.; Investigation, W.L.; Methodology, X.W.; Software, X.L.; Validation, X.W. and Y.S.; Writing—original draft, X.W.; Writing—review & editing, W.L. and X.L. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was funded by the National Defense Science and Technology Enhancement Program of China.

**Institutional Review Board Statement:** Not applicable.
Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare that there is no conflict of interests regarding the publication of this paper.

References
1. Daniel, R. Pyroshock Test Criterial. In NASA Technical Standard; NASA-STD-7003A; NASA: Washington, DC, USA, 2011.
2. Moening, C.J. Pyrotechnic shock flight failures. In Proceedings of the 31st Annual Technical Meeting of the Institute of Environmental Sciences, Las Vegas, NV, USA, January 1985.
3. Ding, J.; Zhao, J.; Han, Z. Research development of spacecraft pyroshock technique. J. Astronaut. 2014, 35, 1339–1349.
4. Girard, A.; Courau, E. Pyroshock database for satellites. In Proceedings of the European Conference on Spacecraft Structure, Materials and Mechanical Testing, Noordwijk, The Nehterland, 29 November 2000.
5. Lee, J.-R.; Chia, C.C.; Kong, C.-W. Review of pyroshock wave measurement and simulation for space systems. Measurement 2012, 45, 631–642. [CrossRef]
6. Himelblau, H.; Kern, D.L.; Manning, J.E.; Piersol, A.G.; Rubin, S. Dynamic Environmental Criteria. In NASA Technical Standard; NASA-HDBK-7005; NASA: Washington, DC, USA, 2001.
7. Lacher, A.; Jüngel, N.; Wagner, U.V.; Bäger, A. Analytical calculation of in-plane response of plates with concentrated masses to shock and application to pyroshock simulation. J. Sound Vib. 2012, 331, 3358–3370. [CrossRef]
8. Ren, K. High frequency vibration energy transfer in a system of three plates connected at discrete points using statistical energy analysis. J. Sound Vib. 2006, 296, 539–553.
9. Tagarielli, V.; Deshpande, V.; Fleck, N. Prediction of the dynamic response of composite sandwich beams under shock loading. Int. J. Impact Eng. 2010, 37, 854–864. [CrossRef]
10. Humphry, I.H.; Langley, R.S. Predicting shock response in uncertain structures using the Hybrid method. J. Phys. Conf. Ser. 2009, 181, 012005. [CrossRef]
11. Bikakis, G.S.E.; Dimou, C.D.; Sideridis, E.P. Ballistic shock response of fiber-metal laminates and monolithic metal plates consisting of different aluminum alloys, AEROSP. Sci. Technol. 2017, 69, 201–208.
12. Pinninngton, R.; Lednik, D. Transient Statistical Energy Analysis of An Impulsively Excited Two Oscillator System. J. Sound Vib. 1996, 189, 249–264. [CrossRef]
13. Akbari, J.; Nazari, L.; Mirzaei, S. Vibration Response Evaluation under Shock-Type Loading with Emphasis on Finite Element Model Updating. Shock Vib. 2020, 2020, 8861827. [CrossRef]
14. Bodin, E.; Bre’vart, B. Pyrotechnic shock response predictions combining statistical energy analysis and local random phase reconstruction. J. Acoust. Soc. Am. 2002, 112, 156–163. [CrossRef] [PubMed]
15. Caresta, M.; Langley, R.S.; Woodhouse, J. Transient response of structures with uncertain properties to nonlinear shock loading. J. Sound Vib. 2013, 332, 5821–5836. [CrossRef]
16. Borello, G.; Courjal, A. Modelling Payloads Using SEA for Vibroacoustic and Shock Prediction; Inter AC-L Union: Milan, France, 2005.
17. Dalton, E.C.; Chambers, B.N. Analysis and validation testing of impulsive load response in complex, multi-compartmented structures. In Proceedings of the AIAA Structures, Structural Dynamics and Materials Conference, New Orleans, LA, USA, 10–23 April 1995. [CrossRef]
18. Zhao, H.; Ding, J.; Zhu, W.; Sun, Y.; Liu, Y. Shock response prediction of the typical structure in spacecraft based on the hybrid modeling techniques. Aerosp. Sci. Technol. 2019, 89, 460–467. [CrossRef]
19. Luo, W.; Cheng, L.; Tong, L.; Yu, W.; Mechefske, C. Prediction and Analysis of Structural Noise from a U-beam Using the FE-SEA Hybrid Method. Promet Traffic Transp. 2018, 30, 333–342. [CrossRef]
20. Yan, Y.; Li, P.; Lin, H. Analysis and experimental validation of the middle-frequency vibro-acoustic coupling property for aircraft structural model based on the wave coupling hybrid FE-SEA method. J. Sound Vib. 2016, 371, 227–236. [CrossRef]
21. Wu, F.; Chen, Y.; Yao, L.; Hu, M. The development of hybrid ES-FE-SEA method for mid-frequency vibration analysis of complex built-up structure. Appl. Math. Model. 2018, 64, 298–319. [CrossRef]
22. Gao, R.; Zhang, Y.; Kennedy, D. A hybrid boundary element-statistical energy analysis for the mid-frequency vibration of vibro-acoustic systems. Comput. Struct. 2018, 203, 34–42. [CrossRef]
23. Shorter, P.J.; Langley, R.S. On the reciprocity relationship between direct field radiation and diffuse reverberant loading. J. Acoust. Soc. Am. 2005, 117, 85–95. [CrossRef] [PubMed]
24. Shorter, P.; Langley, R. Vibro-acoustic analysis of complex systems. J. Sound Vib. 2005, 288, 669–699. [CrossRef]