Improved constraint on primordial gravitational waves in light of 
the Hubble tension and BICEP/Keck

Gen Ye\textsuperscript{1}\textsuperscript{*} and Yun-Song Piao\textsuperscript{1,2,3,4}\textsuperscript{†}

\textsuperscript{1} School of Physics, University of Chinese Academy of Sciences, Beijing 100049, China
\textsuperscript{2} School of Fundamental Physics and Mathematical Sciences,
Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, China
\textsuperscript{3} International Center for Theoretical Physics Asia-Pacific, Beijing/Hangzhou, China and
\textsuperscript{4} Institute of Theoretical Physics, Chinese Academy of Sciences,
P.O. Box 2735, Beijing 100190, China

Abstract

The Hubble tension that the standard ΛCDM model is suffering from can be resolved with 
pre-recombination early dark energy. We present the first constraint on the tensor-to-scalar ratio 
$r$ in corresponding Hubble-tension-free cosmologies using the most recent BICEP/Keck cosmic 
microwave background (CMB) B-mode polarization data. We find, combining BICEP/Keck with 
Planck18 CMB and baryon acoustic oscillation data, that the models with larger Hubble constant 
$H_0$ will have tighter upper bound on $r$, and resolution $H_0 \sim 73 \text{ km/s/Mpc}$ of the Hubble tension 
tightens the upper bound to $r < 0.028$ (95\%C.L.), 25\% tighter than the ΛCDM constraint $r < 0.036$. We clarify the origin of this tightening bound.

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\textsuperscript{*} yegen14@mails.ucas.ac.cn
\textsuperscript{†} yspiao@ucas.ac.cn
I. INTRODUCTION

Inflation is the current paradigm of early universe [1–4]. It predicts nearly scale-invariant scalar perturbation, which is consistent with the cosmic microwave background (CMB) observations, as well as the constraint on primordial gravitational waves (GWs). The discovery of primordial GWs will solidify our confidence that inflation has ever happened. The primordial GWs will source B-mode polarization in the CMB [5–7], which is currently the most promising way to search for the primordial GWs.

Based on the standard ΛCDM model, combining Planck18 and BICEP/Keck15 data the Planck collaboration has put the constraint on the tensor-to-scalar ratio, \( r < 0.066 \) (95% C.L.) [8]. Recently, combining Planck18, baryon acoustic oscillations (BAO) and latest BICEP/Keck18 data the BICEP/Keck collaboration has lowered the upper bound to \( r < 0.036 \) (95% C.L.) [9]. However, it is well-known that the ΛCDM model suffers the Hubble tension, i.e. the locally measured value of current expansion rate reported by the SH0ES collaboration is \( H_0 \sim 73.04 \pm 1.04 \text{km/s/Mpc} [11] \), in stark (\( \sim 5\sigma \)) tension with \( H_0 = 67.37 \pm 0.54 \text{km/s/Mpc} [8] \) inferred by the Planck collaboration assuming ΛCDM.

Though the possibility of some unknown systematics in data causing this tension [12] cannot be ruled out, the Hubble tension is actually becoming a pointer to new physics beyond ΛCDM, see e.g.[13, 14] for reviews. The inclusion of early dark energy (EDE) [15], see also [16–24], has proved to be a promising route of resolving the Hubble tension. In Hubble-tension-free EDE cosmologies, the bestfit values of cosmological parameters have shifted notably in correlation with the increment in \( H_0 [25] \), see also [26–29]. The parameter shifts can serve as tests of corresponding EDE models, which have been confronted with large scale structure data [26, 30, 31] and high-\( l \) CMB data from ground based experiments [32–37], respectively.

The amplitude \( A_s \), tilt \( n_s \) of primordial scalar perturbations and the tensor-to-scalar ratio \( r \) set the initial condition of CMB. They are converted into the observed CMB anisotropies through perturbation evolution described by a certain cosmological model. Thus it is to be expected that any constraints on the relevant parameters acquired assuming ΛCDM will get modified in the new cosmologies. It has been found in [25] that the shift of primordial scalar

\[ \text{A slightly tighter bound } r < 0.032 \text{ is obtained using the new Planck PR4 data [10].} \]
FIG. 1: 68% and 95% C.L. contour plot of the tensor-to-scalar ratio $r$ versus the primordial scalar spectrum tilt $n_s$, with a color coding for $H_0$. All EDE results include the dataset P18+BK18+BAO+SN, while axiEDE+$M_B$ additionally includes the SH0ES result as a Gaussian prior on the absolute magnitude calibration $M_B$ [38, 39]. The ΛCDM result is from Ref.[9] with the dataset P18+BK18+BAO. Generally, the EDE models with larger $H_0$ have larger $n_s$ and tighter upper bound on $r$. Resolution of the Hubble tension ($H_0 \sim 73$km/s/Mpc) roughly corresponds to $n_s \approx 1$ Ref.[25] and a 25% tighter $r$ upper bound, see (2).

The spectral index scales as

$$\delta n_s \simeq 0.4 \frac{\delta H_0}{H_0},$$

and so the Hubble-tension-free cosmologies actually suggests a scale-invariant Harrison-Zeldovich primordial scalar spectrum, i.e. $n_s = 1$ for $H_0 \sim 73$km/s/Mpc. $n_s = 1$ have profound implication on inflation and primordial Universe, see e.g. recent Refs.[40, 41].

In view of the inevitability of Hubble tension in ΛCDM [42], it is significant and also imperative to constrain $r$ in Hubble-tension-free cosmologies, e.g.[15, 21], using the most recent BICEP/Keck B-mode polarization data. In this Letter, we will present the first constraint on $r$ in corresponding cosmologies. We find that the models predicting larger $H_0$
will have larger $n_s$ and tighter upper bound on $r$, and resolution $H_0 \sim 73 \text{km/s/Mpc}$ of the Hubble tension tightens the upper bound on $r$ to

$$r < 0.028 \ (95\% \text{C.L.}),$$

(2)

25% tighter than the ΛCDM constraint $r < 0.036$ reported by the BICEP/Keck collaboration [9], see Fig.1. We clarify the origin of this tightening bound\(^2\), and comment on the possibility to relax it.

II. MODEL AND DATA

As concrete examples of Hubble-tension-free cosmologies, we limit ourself to the EDE, which must be non-negligible only for a short epoch decades before recombination and can be implemented as a canonical scalar field $\phi$ with a potential $V(\phi)$. The EDE models we consider will be: axion-like EDE [15] with an oscillating potential $V(\phi) = m^2 f_a^2 (1 - \cos(\phi/f_a))^n$ and $n = 3$ (denoted as axiEDE for simplicity), and AdS-EDE [21] with a rolling potential $V(\phi) = V_0 (\phi/\phi_{fr})^4 - V_{ads}$ glued to a cosmological constant $V(\phi) = \text{const.} > 0$ at $\phi = (V_{ads}/V_0)^{1/4} M_p$, where $V_{ads}$ is the depth of anti-de Sitter (AdS) well. The evolution of Universe after recombination is ΛCDM-like, see also e.g.[44].

We use modified versions\(^3\) of CLASS [45, 46] to compute cosmology and the MontePython-3.4 sampler [47, 48] to perform Monte Carlo Markov Chain (MCMC) analysis. In addition to the six ΛCDM parameters $\{\omega_b, \omega_{cdm}, H_0, \ln 10^{10} A_s, n_s, \tau_{reion}\}$, we vary two additional MCMC parameters $\{\ln(1 + z_c), f_{ede}\}$, with $z_c$ being the redshift at which the field $\phi$ starts rolling and $f_{ede}$ the energy fraction of EDE at $z_c$, for both EDE models. The axiEDE model varies yet one more MCMC parameter $\Theta_i \equiv \phi_i/f_a$, the initial position of the field. We set $n_T = 0$ following BK18 [9].

Our datasets include:

- **P18**: Planck 2018 high-$l$ TTTEEE, low-$l$ TT and EEBB likelihoods as well as Planck lensing [49].

\(^2\) Using the BICEP/Keck15 data [43], Ref.[25] found the upper bounds on $r$ in EDE to be similar to that in ΛCDM. However, the situation is different with sufficiently precise B-mode data.

\(^3\) The corresponding cosmological codes are available at: axiEDE (https://github.com/PoulinV/AxiCLASS) and AdS-EDE (https://github.com/genye00/class_multiscf).
• **BK18**: The most recent CMB B-mode polarization data from BICEP/Keck 2018 [9].

• **BAO**: Post-reconstructed BAO measurements from 6dF [50], MGS [51] and BOSS DR12 [52].

• **SN**: the Pantheon dataset, with a single nuisance parameter $M_B$, calibrating the absolute magnitude of the supernovas [53].

In the following we also include the standard ΛCDM model for reference. However, we do not redo the MCMC analysis for the ΛCDM model but directly use the ΛCDM chains from BK18 4 [9] (P18+BK18+BAO).

### III. RESULTS AND DISCUSSION

The MCMC posterior results of $\{H_0, n_s, r\}$ for the ΛCDM and EDE models are presented in Table.I and Fig.2, see appendix-A for the results of all relevant parameters. Here, what we intend to discuss is the tighter upper bound on the tensor-to-scalar ratio $r$. In Fig.2, we see that in EDE models the upper bound on $r$ becomes tighter as $H_0$ increases, and with AdS-EDE ($H_0 = 72.36^{+0.49}_{-0.56}$ km/s/Mpc) we have the tightest bound $r < 0.028$, roughly 25% lower than the ΛCDM bound. The origin of this result is quite complicated, which we will clarify step by step.

We plot the bestfit total B-mode power spectra $C_{l,tot}^{BB}$, the separate contribution from lensing B-mode $C_{l,lensing}^{BB}$ and tensor $C_{l,tensor}^{BB}$, as well as binned data points in Fig.3. The difference in $r$ is obvious and the ΛCDM (dashed black) and AdS-EDE lines for $C_{l,tot}^{BB}$ are easily distinguishable in Fig.3, despite both fit the BK18 data equally well according to Table.II. Reducing $n_s$ in AdS-EDE to its ΛCDM bestfit value (solid cyan) yields nearly identical results to the AdS-EDE bestfit (dotted green) in Fig.3, thus $n_s$ is not directly relevant to the change in the upper bound of $r$.

The difference in upper bound is related to the different bestfit values of $r$ in ΛCDM and EDE. In Fig.4, we plot the response of total BK18 $\chi^2$ to the variation in $r$ in ΛCDM and AdS-EDE, respectively. It is clear that the $\chi^2_{BK}$ responds to $r$ above the bestfit point very similarly in both models, thus the difference in their bestfit values are carried

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4 Available at [http://bicepkeck.org/bk18_2021_release.html](http://bicepkeck.org/bk18_2021_release.html)
FIG. 2: 68% and 95% posterior distribution of the ΛCDM, axiEDE(w/ and w/o $M_B$ prior) and AdS-EDE models. The ΛCDM contours are produced using the publicly available BK18 chains. Gray bands represent the 1σ and 2σ regions of the SH0ES measurement $H_0 = 73.04 \pm 1.04$ km/s/Mpc. $n_s = 1$ is marked by dotted lines.

over to the derived 95% C.L. upper bounds, i.e. in Table.I the difference in $r$ upper bounds between ΛCDM and EDE is approximately equal to the difference in the corresponding bestfit values for both EDE models.

The increase of power in $C_{l,lensing}^{BB}$ (see the dotted green and dashed black lines
In Fig.3) is a major contributing factor to the difference in the bestfit values of \( r \) between EDE and ΛCDM. It remains to understand why such distinct values of \( r \), i.e. \( r = 0.015 \) for ΛCDM and \( r = 0.008 \) for AdS-EDE, can fit the BK18 data equally well. To this end, we perform a crude data cut to BK18 - using only part of BK18’s nine \( l \)-bins (corresponding to the nine data points in Fig.3) to calculate \( \chi^2 \). The results are plotted in Fig.5. Overall it is the first three bins (\( l = 37 - 120 \)) that are most sensitive to the variation in \( r \) since they contribute most of the change in \( \Delta \chi^2_{BK} \). The first bin favors (i.e. smallest \( \Delta \chi^2_{BK} \)) non-zero \( r \). On the other hand, large \( r \) lines peak at the second bin while the line with the smallest \( r = 0.001 \) (blue) displays the deepest dip there, showing a preference for \( r \lesssim 0.001 \). Bin #3 on the other hand prefers \( r \gtrsim 0.04 \).

Bin #3 (\( l = 91 - 120 \)) is much less sensitive (i.e. smaller increment in \( \Delta \chi^2_{BK} \)) for \( r < 0.001 \).
FIG. 3: $C_{l,tot}^{BB}$, $C_{l,lensing}^{BB}$ and $C_{l,tensor}^{BB}$ with different color and line style for the bestfit ΛCDM (dashed black), bestfit AdS-EDE (dotted green) and bestfit AdS-EDE but with $A_s$ (solid orange) or $n_s$ (solid cyan) set to their ΛCDM bestfit values. Points with error bars are binned BK18 [9], SPT [54] and ACT [55] data points. The $n_s$-reduced AdS-EDE lines are nearly identical to those of AdS-EDE bestfit thus the dotted green lines overlap with the cyan ones and are barely visible.

in Fig.5) to small $r$ in EDE\(^5\), which suggests that a much smaller $r$ is allowed to fit bin #2 better without significantly worsening the fit at bin #3. $C_{l,tot}^{BB}$ in EDE and ΛCDM are actually well constrained by BK18 data near bin #3, see the zoomed-in region of Fig.3, both bestfit lines very close to each other and intersecting. In the bestfit ΛCDM, $C_{l,lensing}^{BB}$ contributes roughly 80% power of the total amplitude near $l = 100$, thus the 10% increase, see Fig.6, in $C_{l,lensing}^{BB}$ from ΛCDM to EDE needs a 50% reduction in $C_{l,tensor}^{BB}$ to keep $C_{l,tot}^{BB} = C_{l,tensor}^{BB} + C_{l,lensing}^{BB}$ approximately unchanged, corresponding to a 50% reduction in $r$ bestfit, i.e. $\delta r \sim 0.007$. It is surprising that this rather crude estimate explains the difference in bestfit values of $r$ in Table.I pretty well.

\(^5\) The same applies to bin #1, but the constraining power on $r$ comes mostly from bin #3 due to its higher data quality.
FIG. 4: Response of total BK18 $\chi^2$ to the variation in $r$ in $\Lambda$CDM and AdS-EDE. $\chi^2$ at each point is calculated by varying $r$ with all other parameters, including nuisance, fixed to their bestfit values. The $y$-axis plots $\Delta \chi^2 = \chi^2 - \chi^2_{bestfit}$ for the BK18 dataset.

One of the sources of the enhanced $C_{l,\text{lensing}}^{BB}$ in EDE is an excess of power in $C_{l}^{\phi\phi}$ at $l = 200 - 800$, away from its primary peak. We compare the $C_{l,\text{lensing}}^{BB}$ brought by different $C_{l}^{\phi\phi}$ from different unlensed $C_{l}^{EE}$ in Fig.6. We set $A_L = 1$ throughout this paper. It is clear from Fig.6(a) that the enhancement in $C_{l,\text{lensing}}^{BB}$ nearly entirely comes from the difference in $C_{l}^{\phi\phi}$. This is understandable since $C_{l}^{EE}$ (lensed) in both bestfit models are tightly constrained by the Planck data, and so nearly the same. The major contribution to the difference in $C_{l,\text{lensing}}^{BB}$ between $\Lambda$CDM and AdS-EDE comes from $C_{l}^{\phi\phi}(200 < l < 800)$, see Fig.6(b). Interestingly, the peak of $C_{l}^{\phi\phi}$ (related to the peak of the matter power spectrum $P_k$, corresponding to scales entering horizon near matter-radiation equality) is excluded from this multiple range. It also explains why in Fig.3 changing $n_s$ has nearly no effect on the lensing B-mode, since $200 < l < 800$ is near the pivot scale $k_{\text{pivot}} = 0.05\text{Mpc}^{-1}$ which is
FIG. 5: $\Delta \chi^2 = \chi^2 - \chi^2_{\text{best fit}}$ for BK18 with data cuts. The $x$-axis represents that only the first $x$ bins (counting from low-$l$) out of the total nine $l$-bins of BK18 are used in the calculation of $\chi^2$ and $y$-axis for the difference in BK18 $\chi^2$ compared with the bestfit model with the same data cuts. Complete BK18 dataset corresponds to bin #=9 on the horizontal axis.

FIG. 6: The lensing B-mode calculated using different $C^{\phi\phi}_l$ and $C^{EE}_l$. Left panel: Relative difference in $C^{BB}_{l,\text{lensing}}$ compared with $\Lambda$CDM bestfit. Results are calculated using different combination of unlensed $C^{EE}_l$ and $C^{\phi\phi}_l$ from the $\Lambda$CDM and AdS-EDE bestfit model. Right panel: $C^{BB}_{l,\text{lensing}}$ calculated from bestfit AdS-EDE unlensed $C^{EE}_l$, using different portion of the $C^{\phi\phi}_l$ from the same model, compared with the full lensing $C^{BB}_l$. 
FIG. 7: The lensing convergence spectrum $C_{\kappa\kappa} = l^2(l + 1)^2C_{\phi\phi}/4$ in the bestfit $\Lambda$CDM and AdS-EDE models. AdS-EDE models with $A_s$ and/or $n_s$ set to their $\Lambda$CDM bestfit values are also plotted. The vertical dotted lines mark the position of $l = 200$ and $l = 800$. The gray shaded regions are the band power constraints from Planck18 lensing reconstruction [56]. Comparing the solid blue and green dashed dotted lines, changing $n_s$ affects the peak (away from pivot) but not the region $200 < l < 800$ (near the pivot).

much more sensitive to $A_s$ rather than $n_s$.

We attribute the additional power in $C_{\phi\phi}^E(200 < l < 800)$ to the increment in $\omega_{cdm}$ and $A_s$, which enhances amplitude of the matter power spectrum $P_k$ on all scales smaller than $k_{eq}$, the peak of $P_k$, so $C_{\phi\phi}^E(200 < l < 800)$. On the other hand, for EDE, the increment in $\omega_{cdm}$ in fact has minor effect on the peak (located near $l \sim 60$) height of $C_{\phi\phi}^E$, see appendix-B for details. The shift (1) of $n_s$ suppresses power at $l \sim 60$ by $1 - (l_{\text{peak}}/l_{\text{pivot}})^{\delta n_s} \sim 1 - (60/500)^{0.03} \approx 6\%$ but does not obviously affect $200 < l < 800$ which is near $l_{\text{pivot}} \approx 500$. AdS-EDE brings nearly identical $C_{\phi\phi}^E$ near $l \lesssim 100$ as $\Lambda$CDM does under the same initial condition. To compensate for the power deficit at the lensing peak $l \sim 60$ caused by increasing $n_s$, the amplitude $A_s$ has been slightly increased in AdS-
FIG. 8: \( r - n_s \) plot highlighting the \( \Lambda \)CDM and AdS-EDE plus ultra light axion (ULA) models. The upper bound on \( r \) in AdS-EDE+ULA is relaxed to be comparable with \( \Lambda \)CDM.

EDE, which further enhances \( C_{l}^{\phi \phi} \) in the relevant multiple range \( 200 < l < 800 \). The above observation about \( n_s \) and \( A_s \) has been confirmed in Fig.7.

As clarified, it is the enhanced matter power spectrum, so lensing potential \( C_{l,lensing}^{BB} \), at \( l > 200 \) that results in the tightened bound on \( r \). Thus suppressing matter power spectrum on small scales will possibly relax the bound, which might be relevant with resolving \( S_8 \) tension. As a cross check, we confront the AdS-EDE plus ultra light axion model [57] (able to restore cosmological concordance with both \( S_8 \) and \( H_0 \)) with P18+BK18+BAO+SN dataset as well as the Gaussian prior \( S_8 = 0.755^{+0.019}_{-0.021} \) [58], and plot the MCMC results in Fig.8. As expected, the constraints on \( r \) is relaxed to \( r < 0.034 \) (95\%C.L.) (but still \( n_s \approx 1 \)), comparable to the \( \Lambda \)CDM result \( r < 0.035 \) (95\%C.L.).

IV. CONCLUSION

We present the first constraint on primordial GWs, quantified as the tensor-to-scalar ratio \( r \), in Hubble-tension-free EDE cosmologies using the most recent BK18 data. It is found
that the upper bound on $r$ gets tightened in correlation with the increment in $H_0$, and the most stringent bound $r < 0.028$, as opposed to $r < 0.036$ reported by the BICEP/Keck collaboration for $\Lambda$CDM [9], is obtained using the EDE model with the largest Hubble constant, i.e. $H_0 = 72.36^{+0.49}_{-0.56}$ km/s/Mpc in AdS-EDE. We argued that this tightening of bound is a manifestation of the competition between $C_{l,tensor}^{BB}$ and $C_{l,lensing}^{BB}$ with $C_{l,tot}^{BB} = C_{l,tensor}^{BB} + C_{l,lensing}^{BB}$ constrained by the BK18 data. In the EDE models, the increment in $\omega_{cdm}$ and $A_s$ brings more power at intermediate and small scales ($l > 200$) in the matter power spectrum, enhancing $C_{l}^{\phi\phi}(200 < l < 800)$ and thus $C_{l,lensing}^{BB}$. As a consequence, the $C_{l,tensor}^{BB}$ allowed by BK18 must be lowered, so the smaller $r$.

Our results underline that the exact upper bound on $r$ is model-dependent even with the most recent BK18 data. The take-away message for primordial Universe model building is that though the order of magnitude constraint from observation $r \lesssim \mathcal{O}(10^{-2})$ is to be respected, detailed value of the bound might be different in different cosmological models, which will have profound implication to inflation and early Universe physics. Our results also highlight the crucial roles of weak lensing from scales $k = \mathcal{O}(0.01 \sim 0.1\text{Mpc}^{-1})$, probed by galaxy surveys such as DES [59] and Euclid [60], in constraining $r$ using BICEP/Keck data, and of accurate measurements of CMB on small scales, with surveys such as Simons Observatory [61] and CMB-S4 [62], in differentiating the Hubble-tension-free cosmologies. It might also be interesting to restudy $A_L$ with the BK18 data [9] in beyond $\Lambda$CDM cosmologies which modify $C_{l}^{\phi\phi}$.

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Appendix A: More MCMC results

Table I and Fig. 9 show the posterior results for cosmological parameters. Despite using exactly the same MCMC chains as BK18, we obtained a slightly smaller upper bound $r < 0.035$, which we attribute to different analysis configuration in the GetDist package. Table II presents the per experiment bestfit $\chi^2$ for each model.

As noted in earlier works, without a $H_0(M_B)$ prior, axiEDE with Planck CMB data on
FIG. 9: 68% and 95% posterior distributions of all cosmological parameters in the ΛCDM and EDE models.

its own yields nearly identical results to ΛCDM and $f_{ede}$ is compatible with zero, despite a bestfit point outside the 1σ contours where EDE is non-negligible and $H_0$ is much larger. This is because both the ΛCDM bestfit ($f_{ede} = 0$) and the EDE bestfit ($f_{ede} \neq 0$) are local minima in the full phase space, but other EDE parameters (such as $\Theta_i$ or $z_c$) are essentially free in the $f_{ede} = 0$ case, thus the phase space around the ΛCDM bestfit viable to the MCMC chain is dimensionally larger than that around the EDE bestfit point. And without
TABLE II: Bestfit $\chi^2$ for each likelihood. The $\Lambda$CDM bestfit is taken to be the point with the lowest total $\chi^2$ value in the publicly available chains by BK18.

any $H_0(M_B)$-related prior, the EDE bestfit is only marginally better than the $\Lambda$CDM one, see $\chi^2$ in axiEDE in Table.II, thus the MCMC chain will inevitably center around the $\Lambda$CDM local minima, see also [64] for recent discussion.

The AdS-EDE model resolves the Hubble tension even without a $H_0(M_B)$-related prior partially because the AdS phase masks out the $\Lambda$CDM bestfit point since too small $f_{ede}$ will result in the field confined in the disastrous AdS region and is not favored.

Appendix B: CMB lensing constraints in AdS-EDE

As can be seen in Fig.7, $C_l^{\phi\phi}$ is nearly identical in both AdS-EDE and $\Lambda$CDM around its peak under the same initial conditions (i.e. $A_s$ and $n_s$). This is because in EDE the parameter most relevant to the peak height of $C_l^{\phi\phi}$ is $\Omega_m h^{0.5}$ rather than $\omega_m = \Omega_m h^2$. Using the variable $y = h\chi$, $\chi$ being the angular diameter distance, and the Limber approximation, the lensing convergence spectrum $C_l^{\kappa\kappa} = l^2(l + 1)^2C_l^{\phi\phi}/4$ writes

$$C_l^{\kappa\kappa} = \frac{9}{4} \Omega_m^2 h^2 \int_0^\infty dy a^{-2}(y)\hat{g}_L^2(y)P_m\left(k = \frac{l}{\chi h}, \eta(y)\right)$$

with $\eta$ the conformal time. The lensing kernel $\hat{g}_L$ with the window function $W(y)$ is

$$\hat{g}_L(y) = \int_y^\infty dy' \left(1 - \frac{y}{y'}\right) W(y').$$
The angular location of the matter power spectrum peak ($l_{eq}$) is constrained by CMB data in EDE [15] as well as its peak height. Thus the peak height of $C_\ell^{\kappa\kappa}$ is relevant to the prefactor $\Omega_m h^{0.5}$ in (B1). The Planck lensing reconstruction therefore constrains $\Omega_m h^{0.5} \sim \text{const.}$, which is nearly invariant across both models in Fig. 10, while $\Omega_m$ show some difference. Note this observation does not contradict the result of Ref. [27], in which $\Omega_m \sim \text{const.}$ is the background level CMB+BAO constraint. In the actual analysis, since there is residual freedom in both data constraints, the MCMC results make a compromise between different datasets. Thus the actual degeneracy direction is

$$\Omega_m \sim h^{-\alpha}, \quad 0 < \alpha < 0.5,$$

(B3)
or equivalently \( \omega_m \sim h^\beta \) with \( 1.5 < \beta < 2 \). As an example, numerical principle component analysis of the AdS-EDE chain yields \( \Omega_m h^{0.4} \sim \text{const.} \), so \( \alpha = 0.4 \) and \( \beta = 1.6 \).

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