Analysis of nonsinusoidal electromagnetic fields in induction units

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Abstract. The paper contains the analytical analysis of nonsinusoidal electromagnetic field in induction units of metallurgical designation. It considers a pancake coil on the surface of a conductive body and magnetohydrodynamic (MHD) stirrer of liquid metals. The solution is obtained in the form of a Fourier series in a comprehensive way. We have obtained differential and integral properties and the relations of electromagnetic parameters to the value of time constant in the inductor winding. The reconciliation analysis has been completed for the results obtained for the electromagnetic characteristics of the units at supplying them with sinusoidal current.

1. Introduction
The modern industry widely employs materials processing technologies based on the use of variable electromagnetic fields [1]. The most common technologies include induction heating [2], induction melting [3], and electromagnetic stirring of liquid metals [4]. Normally, the source of alternating electromagnetic field is a sinusoidal current of low, industrial or high frequency. However, currently, the development of power conversion technology made possible the use of nonsinusoidal currents [5-8].

The number of studies has been up to date completed on the effectiveness in the application of the pulsed electromagnetic fields generated by the sinusoidal current interruption in the units intended for liquid metal stirring [9-11]. The works have shown a specific advantage of using the method of liquid metal exposure to electromagnetic field. In [12], it is proposed to use periodic nonsinusoidal voltage that represents the rectangular alternating-sign pulses as a power source of the electromagnetic liquid metals stirrer. This paper shows that liquid metals mixing can be more efficient under certain parameters and operating modes of the electromagnetic stirrer with nonsinusoidal periodic currents.

Despite the featured advantages of using nonsinusoidal periodic currents, the issue remains understudied in respect of the nature of spatial and time-frequency distributions of electromagnetic forces in the melt, and their impact on the nature of MHD-processes in a liquid metal. This paper presents the results of the solutions.
2. Pancake coil with nonsinusoidal current on the surface of conductive medium

2.1. Statement of the task and solutions of electromagnetic field equations

To explore the nature of electromagnetic field and power density distribution in the conductive body, at the initial stage it is reasonable to consider the task on dissemination of a plane electromagnetic wave, where vectors \( E \) and \( H \) vary according the nonsinusoidal periodic law. The case of a wave is just for many types of induction units either with a flat or cylindrical configuration.

A plane electromagnetic wave can be generated by a pancake coil located near the conducting space, as shown in Figure 1 [13]. Conductive medium is isotropic and has unlimited dimensions in the direction of \( 0x \), \( 0y \), and \( 0z \). In the model shown in Figure 1, the gradients of magnetic and electric field have components

\[
H = e_x H_x, \quad E = e_z E_z.
\]

where \( e_x \) and \( e_z \) are unit vectors.

According to the full-load current, the instantaneous value of the magnetic field gradient in the gap between the coil and the body is determined by the expression [14]

\[
H(t) = j(t) = \frac{W \cdot i(t)}{l}, \tag{1}
\]

where \( j(t) \) is a linear current density in the inductor winding, \( A \cdot m^{-1} \), \( i(t) \) is the instantaneous value of the current in the inductor, \( A \); \( t \) is time, \( s \); \( W \) is the number of coil turns; \( l \) is the length of the inductor, \( m \).

When feeding to the inductor a rectangular periodic voltage with the amplitude \( U_m \) and period \( T \), with the account of the law of commutation, the linear density in the inductor current will vary according to the law (Figure 2) [14]

\[
j(t) = \begin{cases} 
J_m - 2J_m \exp \left( -\frac{t}{\tau} \right), & 0 \leq t < \frac{T}{2}, \\
-J_m + 2J_m \exp \left( -\frac{t}{\tau} + \frac{T}{2\tau} \right), & \frac{T}{2} \leq t < T.
\end{cases} \tag{2}
\]

where \( \tau = L/R \) is the time constant of the inductor, \( s \); \( L \) is the inductance, \( H \); \( R \) is inductor winding resistance, Ohm; \( J_m \) is the amplitude of linear current load, \( A \cdot m^{-1} \).

Figure 1. Pancake coil near infinite conductive body.

Figure 2. Graphs of instantaneous value of voltage and current linear density.

Magnetic field gradient in the conducting half-space area will satisfy the differential equation (index "x" is further omitted) [10]

\[
\frac{\partial^2 H(t,y)}{\partial y^2} - \mu_0 \gamma \frac{\partial H(t,y)}{\partial t} = 0, \tag{3}
\]
and boundary conditions

\[ H(\infty,t) = 0; \quad H(0,t) = j(t). \]  

Applying the Fourier integral transformation in the integrated form \[15\], we obtain the solution for magnetic and electric field gradient vector

\[ H(\hat{y},\hat{t}) = 2J m \sum_{n=1}^{\infty} |\tilde{\Psi}_n| \exp\left(-\sqrt{n}\hat{y}\right) \cos(\alpha_n - \sqrt{n}\hat{y} + 2n\pi\hat{t}), \]

\[ E(\hat{y},\hat{t}) = \frac{2J m}{\gamma^2} \sum_{n=1}^{\infty} \sqrt{2n} |\tilde{\Phi}_n| \exp\left(-\sqrt{n}\hat{y}\right) \cos(\alpha_n - \sqrt{n}\hat{y} + 2n\pi\hat{t} + \pi/4). \]

Here \( \tilde{\Psi}_n = \Psi_n / J_m \); \( \hat{y} = y / \delta \); \( \hat{t} = t / T \); \( \alpha_n = \text{arg}(\Psi_n) \); \( \Psi_n = \int_0^T j(t) \exp(-ik_n t) \hat{t} \); \( \delta = \left( \frac{T}{m_0^2 T} \right)^{1/2} \).

The active electromagnetic power coming into the conductive area through a single surface

\[ P_{aw} = \frac{2J_m^2}{\gamma^2} \sum_{n=1}^{\infty} \sqrt{2n} |\tilde{\Psi}_n| \cdot N_n, \]

where \( N_n = \int_0^T \left[ \cos(k_n t + \alpha_n + \pi/4) \cdot \hat{t} \right] \hat{t} \).

2.2. Analysis of electromagnetic characteristics

Figure 3 illustrates the relation of a relative surface active power on the surface of a conducting half-space to the relative time constant of the inductor \( \tau / T \). As seen from the graph, with the decrease of the inductor time constant the surface power density increases.

Figure 4 illustrates the graph of the relation of an active power released in a layer of penetration depth \( \delta \), related to active power on the surface of conductive body, to \( \tau / T \). It is obvious that the closer the shape of the magnetic field is to rectangular, the more active power is released in the \( \delta \) layer (up to 90% of the total active power). With the increase in the inductor time constant the active power value lowers and ceases to change at values \( \tau / T > 0.15 \). In addition, within the whole range \( \tau / T \) range the active power there is distinguished a bigger amount of active power, compared to a plane wave of a sinusoidal field.

**Figure 3.** The relation diagram of a relative flow of active power on the surface of the half-space to value \( \tau / T \). \( P_b = J^2 / 2\gamma \delta \).

**Figure 4.** The relation of the released depth layer \( \delta \) referred to the active power on the surface of the half-space to the value \( \tau / T \).

It is worth noting that such properties of a nonsinusoidal periodic magnetic field can be useful, for example, in the field of the induction surface hardening, where within a short period of time it is
required to heat the steel product in a thin surface layer and at the same time not to heat the inner layers of the product.

3. Electromagnetic stirrer of liquid metal with nonsinusoidal periodic current

3.1. Statement of the task and solutions of electromagnetic field equations

It is common practice with the liquid metals electromagnetic stirrers to use moving (rotating or traveling) magnetic fields [16]. To generate a moving magnetic field in the inductor it is required to install two or more windings that are supplied from the source with the same shape, frequency and voltages, phase-shifted relative to each other. The most efficient and easiest in the design is the electromagnetic stirrer of a traveling magnetic field with crossover windings [17].

Taking the model from [18] as the basis and adding similar assumptions we obtain the design model shown in Figure 5.

![Figure 5](image)

**Figure 5.** Design model of MHD stirrer (ρ is the number of poles pairs; τ_{ind} is pole division in the inductor, m; j_m is the linear current load in mth slot, A m^{-1}; X is the period on the x axis, m; x_m is the coordinate of a middle point in the mth slot, m; 2\Delta_m is a slot width, m, \delta' is the value of an air gap, m).

Linear current load in the mth slot is determined by the law

\[
j_m(t) = \begin{cases} 
J_m - 2J_m \exp\left[\left(-t + t_m + \frac{T}{2}\right)\frac{1}{\tau}\right]; & t_m - \frac{T}{2} < t < t_m \\
-J_m + 2J_m \exp\left[\left(-t + t_m\right)\frac{1}{\tau}\right]; & t_m < t < t_m + \frac{T}{2}
\end{cases}
\]

(9)

where \( t_m = \frac{T}{2} + \frac{T}{M}(m - 1) \); \( J_m \) is an amplitude of linear current density in mth slot, A m^{-1}; \( M \) is the number of slots.

This model of electric- and magnetic-fields vector have the following elements:

\[
E = e_x E_x; H = e_y H_{s1,2} + e_z H_{s1,2}
\]

Electric-field vector \( E \) satisfies the functions of equations [18] (index “ y ” is omitted):

in area 1, \( 0 \leq z \leq \delta' \)

\[
\frac{\partial^2 E_x}{\partial z^2} + \frac{\partial^2 E_x}{\partial x^2} = 0
\]

(10)

in area 2
\[
\frac{\partial^2 E_i}{\partial z^2} + \frac{\partial^2 E_i}{\partial x^2} = 0
\]  

(11)

Below are the area boundary conditions that are found true:

\[
\frac{\partial E_i}{\partial z}(0, x, t) = \begin{cases} 
\frac{\partial}{\partial t} \mu_0 j_m, & x_m < x < x_m + A_m \\
0 & \text{else}
\end{cases}

\]

\[
E_i(\delta', x, t) = E_i(\delta', x, t) = \frac{\partial E_i}{\partial z}(\delta', x, t) = \frac{\partial E_i}{\partial z}(\delta', x, t) = E_i(\infty, x, t) = 0
\]

(12)

Applying the Fourier double integral transformation in the integrated form, we obtain the solution for vectors of electric field gradient and magnetic inductance in area 1.

\[
E_i(z, x, t) = -\frac{i \mu J X}{\pi T} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{k G_m(z)}{Q_{in}} N_{in} \exp(i \zeta_n t) \exp(i \xi_n x)
\]

(17)

\[
B_{i1}(z, x, t) = \int \frac{\partial E_i(z, x, t)}{\partial x} dt = \frac{i \mu J}{\pi} \sum_{k=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{Q_{in}} G_m(z) \exp(i \zeta_n t) \exp(i \xi_n x)
\]

(18)

Here

\[ i = \sqrt{-1}; \]
\[ \zeta_k = \frac{2k \pi}{T}; \]
\[ \xi_n = \frac{2n \pi}{X}; \]
\[ G_m(z) = \xi_n \chi_n \phi_n (z - \delta') - \phi_n \chi_n \xi_n (z - \delta'); \]
\[ Q_{in} = \xi_n \chi_n \phi_n \xi_n \chi_n \delta'; \]
\[ \phi_n^2 = \xi_n^2 + i \chi_n \mu_0 \gamma + i \mu_0 \gamma \mu_0 \xi_n; \]
\[ N_{in} = \sum_{m=1}^{M} (I_{km1} + I_{km2}) \sin(\xi_n A_m) \exp(-i \xi_n x); \]
\[ I_{km1} + I_{km2} = \frac{1}{T} \int_0^T \frac{j_m(t)}{J_m} \exp(-i \xi_n t). \]

Instantaneous electromagnetic power and strength are determined by expressions

\[
p(t) = \sum_{m=1}^{M} E_i(0, x_m, t) j_m(t) 2 A_m l
\]

(19)

\[
f(t) = \sum_{m=1}^{M} B_{i1}(0, x_m, t) j_m(t) 2 A_m l
\]

(20)

where \( l \) is the width of the inductor in the y axis direction.
3.2. Analysis of electromagnetic characteristics

Figures 6 and 7 display graphs for instantaneous values of relative linear current density and the gradient of the electric field at the parameters $M = 4$, $\tau = T/20$, $u = 0$, $X = 2\tau_{ind}$ (longitudinal edge effect is not taken into account). The connection layout for a two-phase winding in this case is AYBX. It is seen that the rise and decay points of linear current load cause sharp changes in the electrical field tension. This is explained by the fact that the electrical field gradient is proportional to the derivative of the magnetic field gradient. Therefore, with the decrease in the inductor winding time constant $\tau$, the tension amplitude of the electric field will increase.

Figures 8 and 9 display graphs of the instantaneous values of electromagnetic power and strength at different values of $\tau$. The black solid line in the graphs shows the instantaneous values of the electromagnetic strength and power for the case of sinusoidal current in the inductor, which represent constant values, since symmetrical multi-phase system with sinusoidal currents is balanced [14]. By contract, in the case of nonsinusoidal currents the power and strength change over time and have sharply nonsinusoidal nature. The shape and amplitude of the instantaneous power varies depending on the value of $\tau$.

![Figure 6](image1.png) **Figure 6.** Instantaneous values of linear current load in relative units in the $m^{th}$ slot.

![Figure 7](image2.png) **Figure 7.** Instantaneous values of electrical field tension in relative units in point $x_m$ for $m^{th}$ slot. $E_b=(2J_{m\tau_{ind}}\mu_0)/T$.

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The magnitude and nature of changes of electromagnetic forces also depend on $\tau$. In addition, it can be seen that the average for the period value of electromagnetic force is higher than in case of sinusoidal currents coming into power inductor winding. It is worth noting that such a character of EM force change makes it possible to generate the impulse mechanical effect in the melt, which allows performing stirring and the melt homogenization at the level of microinhomogeneities.
4. Conclusions

There have been completed the analysis of the electromagnetic properties of induction units connected to a source of rectangular alternating voltage. Below are the conclusions made:

1. Supplying nonsinusoidal current to the flat inductor allows increasing the value of active power released at the depth of sinusoidal current penetration;
2. Connecting a flat inductor to the source of rectangular alternating voltage enables controlling the process of heating the conductive body by changing the time constant of the inductor;
3. Connecting multiphase winding inductor of the liquid metals MHD-stirrer to the source of rectangular alternating voltage enables obtaining various forms of instant power and electromagnetic force depending on the time constant of the inductor winding;
4. The impact effect of electromagnetic force in the preparation of multicomponent homogeneous melts enables eliminating the microinhomogeneity.

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References

[1] Nacke B 2012 Theoretical Background and Aspects of Electrotechnologies: Physical Principles and Realization: Intensive Course Basic I Publishing house of ETU;
[2] Rudnev V 2018 Induction Heating and Heat Treating for Aerospace Applications Adv Mater Process 176(2) 58-61 PubMed PMID: WOS:000426999300018
[3] Pavlovs S et al 2011 LES modelling of turbulent flow, heat exchange and particle transport in industrial induction channel furnaces Magnetohydrodynamics 47(4) 399-412
[4] Fdhila R, Sand U and Eriksson J E et al 2016 A stirring history ABB Review 3 45-48
[5] Branover H et al 2005 Novel potentialities of electromagnetic stirring of melts in metallurgy Proceedings of the joint 31-4
[6] Eckert S, Nikritiyuk P and Rabiger D et al 2008 Efficient melt stirring using pulse sequences of a rotating magnetic field: Part I Flow field in a liquid metal column Metall Mater Trans B
[7] Ma X, Yang Y and Wang B 2009 Effect of pulsed magnetic field on superalloy melt International Journal of Heat and Mass Transfer 52(23-24) 5285-92
[8] Wang X et al 2009 A periodically reversed flow driven by a modulated traveling magnetic field: Part I Experiments with GaInSn Metallurgical and Materials Transactions B 40(1) 82
[9] Kolesnichenko A, Podoltsev, A and Kucheryavaya I 1994 Action of pulse magnetic field on molten metal ISIJ international 34(9) 715-21
[10] Musaeva D, Ilin V and Baake E, et al 2015 Numerical Simulation of the Melt Flow in an Induction Crucible Furnace Driven by a Lorentz Force Pulsed at Low Frequency Magnetohydrodynamics

[11] Guo-Jun C, Yong-Jie Z and Yuan-Sheng Y 2013 Modelling the unsteady melt flow under a pulsed magnetic field *Chinese Physics B* **22**(12) 124703

[12] Timofeev V, Lybzikov G and Khatsayuk M, et al 2015 Magnetohydrodynamic stirrer liquid metal with a non-sinusoidal currents *Engineering & Technologies* 166-177

[13] Lupi S, Forzan M and Aliferov A 2015 Induction and direct resistance heating *Springer*

[14] Neyman L and Demirchan K 1981 Theoretical backgrounds of electrical engineering; in 2 volumes Textbook for high schools 1 edition 3 536

[15] Zill D, Wright W and Cullen M 2011 Advanced engineering mathematics *Jones & Bartlett Learning*

[16] Xiaodong W et al 2015 Flow, heat and mass transfers during solidification under traveling/rotating magnetic field *International Journal of Energy and Environmental Engineering* **6**(4) 367-73

[17] CEng A, CEng J 2011 – 2016 Technology for electromagnetic stirring of aluminum reverberatory furnaces *Light Metals* 1193-8

[18] Timofeev V and Khatsayuk M 2016 Theoretical design fundamentals for mhd stirrers for molten metals *Magnetohydrodynamics* (0024-998X) **52**(4)