Aero-thermal-elastic Coupled Simulations of An Air-cooled Turbine with an FDM solver

Zhaoyuan Guo\textsuperscript{1,2,*}, Qiang Wang\textsuperscript{3,*}, Ping Dong\textsuperscript{1,4}, and Yuting Jiang\textsuperscript{1,5}

\textsuperscript{1}Harbin Engineering University, Heilongjiang, Harbin, China
\textsuperscript{2}The 705th Research Institute, China Shipbuilding Industry Corporation, Xi’an, China
\textsuperscript{3}North University of China, Taiyuan, China

*Corresponding author e-mail: qwang@nuc.edu.cn, *gzyuan1980@163.com, \
\textsuperscript{b}stevendong@hrbeu.edu.cn, \textsuperscript{c}jiangyuting07314@126.com

Abstract. The aero-thermal-elastic coupled simulations of an air-cooled turbine vane were carried out by a developed coupled solver, HIT3D, employing the finite difference method. The pressure on the vane surface and the temperature in the solid vane were obtained by the coupled heat transfer simulation, then the single-way aero-elastic and thermal-elastic analysis on the turbine vane were performed. It shows that the predicted profile temperature employing the transition model agrees well with the measured one, the blade deformation caused by aerodynamic force is negligible, and that the greater thermal deformation and thermal stress locate at the blade leading edge pressure side and at the blade trailing edge, as the result of the strong temperature gradient and the constraint at the vane endwalls.

1. Introduction

Due to the high operating parameters, including temperature and pressure of the gas, there are rather complex physical process in the air-cooled turbines. Such process, are consist of gas flow through the passage and cooling channels, heat transfer between gas and solid blade, and blade elastic problems. The interaction among the flow, thermal and elastic fields are so strong that the coupling of these physical processes should be considered in the design and researches of the turbines.

The common used method for the aero-thermal-elastic coupled simulations of the air-cooled turbines is implemented by two steps as follows: one obtains the gas flow field and blade temperature distributions with two-way coupled or conjugate heat transfer simulations \cite{1-4} at first, then carries out one-way coupled simulations to study the interaction between the gas flow field and blade elastic field, and that between the thermal and elastic fields of the solid turbine blade. Almost all of the coupled solvers employed in researches mentioned above are consisted of two solvers, a CFD (Computational Fluid Dynamics) one and a CSM (computational solid mechanics) one, and coupling boundary conditions. Generally, the CFD solver is based on the finite difference or finite volume method, while the CSM solver is based on the finite element method. The finite element method, which can be applied to the elastic problem simulations of solid blade with rather complex shapes, is challenged by solving problems containing discontinuous in the field. Otherwise the finite difference method has strong advantages in solving nonlinear material elasticity problems and problems with complex boundary conditions \cite{5}.
Based on the preliminary work done by the group [6,7], a coupled solver (HIT3D) of the aero-thermal-elastic coupling problems was developed[8]. The coupled solver, solving the partial differential governing equations, consisted of three different modules for coupled heat transfer, fluid-elastic coupling, and thermal-elastic coupling problems separately. And it was applied to predict multi-field coupling problems of an air-cooled turbines in the study.

2. Numerical Methodology
The mathematical method of the aero-thermal-elastic coupling problems of the air-cooled turbine includes the governing equations for gas flow, heat transfer and elastic problems, boundary conditions and numerical schemes employed.

The dimensionless Reynolds Average Navier-Stokes (RANS) equation of the conservative variables for gas flow in arbitrary, body-fitted coordinates is

\[
\frac{\partial \tilde{U}}{\partial t} + \frac{\partial \tilde{E}}{\partial \xi} + \frac{\partial \tilde{F}}{\partial \eta} + \frac{\partial \tilde{G}}{\partial \zeta} = \frac{\tilde{f}_1}{r} + \frac{1}{\text{Re}} \left( \frac{\tilde{f}_2}{r} + \frac{\partial \tilde{Q}}{\partial \xi} + \frac{\partial \tilde{R}}{\partial \eta} + \frac{\partial \tilde{S}}{\partial \zeta} \right)
\]

where \( \tilde{U} = J \cdot (\rho \rho u \rho v \rho w \rho e) \) is conservative variable, \( J \) the Jacobin determinent, \( \tilde{E}, \tilde{F}, \tilde{G} \) convective terms, \( \tilde{Q}, \tilde{R}, \tilde{S} \) viscous terms and \( \tilde{f}_1, \tilde{f}_2 \) source terms. The closure of the gas flow governing equations are provided by two kinds of eddy viscosity turbulence models, which are B-L algebraic turbulence model [9] and \( q - \omega \) low-Re two-equation turbulence model [10]. The laminar-turbulence transition flow on the turbine blade surface is predicted by AGS transition model [11].

The heat conduction equation considering the thermoelastic coupling is as follows.

\[
\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (\nabla \lambda T) - T_0 \beta \frac{\partial e}{\partial t} - \nabla \cdot q + \phi
\]

where \( k \) is thermal conductivity, \( c \) specific heat capacity, \( \rho \) density, \( T_0 \) thermal deformation temperature, \( q \) heat flux density, and \( \phi \) thermal source term. The parameters \( e \) and \( \beta \) in formula (2) are calculated by following algebraic

\[
e = \frac{1}{3 \lambda + 2G} \Theta + 3 \alpha_0 \Delta T, \quad \beta = (3 \lambda_0 + 2G) \alpha_0
\]

where \( \alpha_0 \) is linear expansion coefficient, \( \lambda \) and \( G \) are Lame constants.

The governing equation for elastic problems is as follows.

\[
(\lambda + \mu) \nabla^2 \theta + G \nabla^2 u_n + F_n - \frac{\alpha_2 E}{1 - 2v} \frac{\partial T^n}{\partial y} = \rho \frac{\partial^2 u_n}{\partial t^2}
\]

where \( u \) is displacement, \( F \) body force, \( T \) temperature, and \( \theta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \). Lame constants \( \lambda \) and \( G \) are provided by \( \lambda = \frac{E v}{(1 + v)(1 - 2v)} \) and \( G = \frac{E}{2(1 + v)} \), where \( E \) and \( v \) are elastic modulus and Poisson's ratio separately.

The interactions among the gas flow, solid thermal and elastic fields are predicted by weak coupling method. For the coupled heat transfer simulations, the direct coupling method is utilized for thermal
data exchange, temperature and heat flux density, at flow-solid interfaces. For the single-way thermal-elastic simulations, the temperature distributions at the blade surfaces, calculated by coupled heat transfer simulations, are set as boundary conditions, and the thermal field in the solid blade are obtained by solving formula (2). For the single-way aero-elastic coupling simulations, set the pressure distributions at the surfaces as the boundary conditions, and the deformation and stress of the solid blade are obtained by solving formula (3). Ref. 8 provides the detailed numerical schemes employed in the coupled solver.

3. Aero-thermal-elastic coupled simulations of NASA-MARKII guide vane

The aero-thermal-elastic coupled simulations are carried out with the developed coupled solver, HIT3D. The 5411 operating conditions of NASA-MARKII guide vane [12] is served as the test case, and the boundary conditions are listed in table 1 and table 2. The geometrical parameters of the vane, having 10 cooling air channels inside, is shown in figure 1. And the computational grids are provided in figure 2. The multiblock structure grids are generated by ICEM-CFD, the y+ of the adjacent grid is limited less than 5, which is fine enough for the mesh independence solution of gas flow. The solid blade is also discretized by the structure mesh.

The coupled heat transfer simulations are carried out firstly, and one can obtain the pressure distributions on the vane surface and the thermal field in the solid vane, which are set as the boundary conditions of the following coupled simulations. Then the single-way aero-elastic and thermal-elastic

![Figure 1. Geometrical parameters for NASA-MARKII guide vane](image-url)
Figure 2. Computational Grids

Table 1. Coolant flow conditions

| Channel No. | Diameter (mm) | Mass flow rate (kg/s) | $T_{in}$ (K) |
|-------------|---------------|-----------------------|--------------|
| Channel No. | 1             | 2                     | 3            | 4            | 5            | 6            | 7            | 8            | 9            | 10           |
| 1           | 6.3           | 6.3                   | 6.3          | 6.3          | 6.3          | 6.3          | 3.1          | 3.1          | 1.98         |
| 2           | $2.46 \times 10^{-1}$ | $2.37 \times 10^{-1}$ | $2.38 \times 10^{-1}$ | $2.47 \times 10^{-1}$ | $2.33 \times 10^{-1}$ | $2.28 \times 10^{-1}$ | $2.38 \times 10^{-1}$ | $7.75 \times 10^{-1}$ | $5.11 \times 10^{-1}$ | $3.34 \times 10^{-1}$ |
| 3           | 6.3           | 6.3                   | 6.3          | 6.3          | 6.3          | 6.3          | 3.1          | 3.1          | 1.98         |
| 4           | $2.46 \times 10^{-1}$ | $2.37 \times 10^{-1}$ | $2.38 \times 10^{-1}$ | $2.47 \times 10^{-1}$ | $2.33 \times 10^{-1}$ | $2.28 \times 10^{-1}$ | $2.38 \times 10^{-1}$ | $7.75 \times 10^{-1}$ | $5.11 \times 10^{-1}$ | $3.34 \times 10^{-1}$ |
| 5           | 6.3           | 6.3                   | 6.3          | 6.3          | 6.3          | 6.3          | 3.1          | 3.1          | 1.98         |
| 6           | $2.46 \times 10^{-1}$ | $2.37 \times 10^{-1}$ | $2.38 \times 10^{-1}$ | $2.47 \times 10^{-1}$ | $2.33 \times 10^{-1}$ | $2.28 \times 10^{-1}$ | $2.38 \times 10^{-1}$ | $7.75 \times 10^{-1}$ | $5.11 \times 10^{-1}$ | $3.34 \times 10^{-1}$ |
| 7           | 6.3           | 6.3                   | 6.3          | 6.3          | 6.3          | 6.3          | 3.1          | 3.1          | 1.98         |
| 8           | $2.46 \times 10^{-1}$ | $2.37 \times 10^{-1}$ | $2.38 \times 10^{-1}$ | $2.47 \times 10^{-1}$ | $2.33 \times 10^{-1}$ | $2.28 \times 10^{-1}$ | $2.38 \times 10^{-1}$ | $7.75 \times 10^{-1}$ | $5.11 \times 10^{-1}$ | $3.34 \times 10^{-1}$ |
| 9           | 6.3           | 6.3                   | 6.3          | 6.3          | 6.3          | 6.3          | 3.1          | 3.1          | 1.98         |
| 10          | $2.46 \times 10^{-1}$ | $2.37 \times 10^{-1}$ | $2.38 \times 10^{-1}$ | $2.47 \times 10^{-1}$ | $2.33 \times 10^{-1}$ | $2.28 \times 10^{-1}$ | $2.38 \times 10^{-1}$ | $7.75 \times 10^{-1}$ | $5.11 \times 10^{-1}$ | $3.34 \times 10^{-1}$ |

Table 2. The boundary conditions of simulation

|          | Total pressure/Pa | Total temperature/K | Turbulence intensity/% | Pressure/Pa |
|----------|-------------------|---------------------|------------------------|------------|
| inlet    | 337097            | 788                 | 6.5                    |            |
| outlet   | 175713            |                     |                        |            |

3.1. Coupled heat transfer simulation

The mach number distribution at mid-span is illustrated in figure 3. Due to the high ratio of inlet total pressure to outlet pressure, acceleration gas flow occurs on the suction side, resulting in a strong shock (Ma=1.47) at approximately dimensionless position $x/c=0.45$, where $x$ and $c$ are the axial position and axial chord length, respectively. The acceleration is so strong that a weaker shock (Ma=1.16) takes place in the front of the trailing edge.

As is shown in figure 4, the predicted profile pressure at mid-span by different turbulence models and transition models agree well with the measured one. Owing to the acceleration flow on the vane suction side, the pressure drops rapidly, otherwise there are maximum values of the static pressure at
the same positions as the local shock waves. The pressure on the pressure side drops rather slowly, revealing local weaker acceleration flow than that on the suction side.

**Figure 3.** Distribution of mach number at mid-span

**Figure 4.** Comparison of predicted vane surface pressure with the measured one at mid-span

**Figure 5.** Comparison of predicted vane surface temperature with the measured one at mid-span
The comparison between the predicted profile temperature with the measured one, provided in figure 5, shows that the numerical results by the AGS algebraic transition model agree well with the measured one, and that the results by the B-L turbulence model deviates from the measured one to the greatest extent. The simulation employing low-Re two-equation model, involving damping function to damp turbulent kinetic energy production in the boundary layers, predicts the profile temperature with less deviation from the measured one than that employing B-L turbulence model. Based on the numerical results predicted by the simulations with AGS transition model, the single-way aero-elastic and thermal-elastic coupled simulations are carried out.

3.2. Single-way aero-elastic coupled simulation

The boundary conditions of single-way aero-elastic coupled simulation are set as follows. The forces, including surface one and body one, on the vane outer surface and cooling air holes inner surfaces, are set as 0. The thermal displacements at vane endwalls are also set as 0. The vane material is ASTM standard 310 stainless steel selected (0Cr25Ni20), with Passion ratio 0.3, linear expansion coefficient $1.3 \times 10^{-5}$ and elastic modulus $2.0 \times 10^5$ MPa. The temperature in the solid vane is provided by coupled heat transfer simulations.

Figure 6 presents the pressure distributions on the vane surface. There is rather little change of static pressure on the vane surface along the vane height at the same axial position. The static pressure decreases along the vane surface, except of the shock wave positions on the suction side, where the local pressure increases rapidly.

As is shown in figure 7, the total deformation of the vane increases from the vane leading edge to the trailing edge, along with the decreased vane thickness. The maximum of the total deformation exists at the vane trailing edge, which could be attributed to the local structure and the suddenly increased pressure difference on both the suction and the pressure sides. Due to the presence of the cooling holes inside the vane, the ability of the vane to resist external forces changes along the axial direction, and there are local maximums of the total deformation at the location of the cooling hole. Since the displacement at hub and shroud is set to 0, the total deformation of the vane increases first and then decreases from the hub to the shroud of the vane. The largest total deformation locates at the mid-span, and its value is only 0.00035mm, which is neglected compared with the vane height of 76.2 mm. Thus on steady conditions, the influence of the elastic deformation of the aerodynamic force on the flow field is small and negligible.
Figure 7. Total deformation on the vane surface

Figure 8 provides the equivalent stress distribution on the vane surface. It is shown that the equivalent stress in most of the areas the vane outer surface is about 1.2MPa, and that the stress concentration occurs at endwalls, where the value of the equivalent stress reaches up to 4.5MPa, several times larger than that in most areas of the vane outer surface. The equivalent stress value has fluctuations on the vane surface, which may be attributed to several reasons as follows. Firstly the pressure on the vane surface changes gradually, causing fluctuations in equivalent stress. Secondly there are ten cooling air holes inside the vane, and the deformation generated near the cooling hole and away from the cooling hole is different, leading to fluctuating strain and equivalent stress. Thirdly one-side difference scheme is adopted to discrete partial differential term on the vane surface, which also causes numerical error, and such error is more significant at the interfaces of the grid block. And the numerical error is able to be eliminated by utilizing high-order difference scheme for boundary nodes.

Figure 8. Equivalent stress on the surface of the vane

As is analyzed above, the maximum value of the vane stress generated by the aerodynamic force occurs near the endwalls. In the regions away from the endwalls, the value of such stress is relatively low, and the local maximum value occurs near the solid surfaces, including the inner surface of cooling air holes and vane outer surface.

3.3. Single-way thermal-elastic coupled simulation
The single-way thermal-elastic coupled simulation of NASA-MARKII guide vane is carried out. The vane is thermal deformed by the thermal loads, and the thermal deformation of solid vane is shown in figure 9. With the action of thermal load, the volume of the vane expands, leading to the displacement of the vane material. The original vane is a straight one on the normal temperature condition, otherwise the vane stack line for the hub to shroud is not a line any longer on the test conditions, which is induced by free expansion of the solid vane, with zero-deformation condition constrained at endwalls. The total deformation at the leading and trailing edges is larger than that in other regions, due to the local high
temperature and large expansion ratio. With the same reason, there is smaller total deformation on the pressure side than that on the suction side. There are some fluctuations of the total deformation near the cooling air holes, and the value of local total deformation increases while the distance to the cooling air holes decreases. The reasons are as follows. On the one hand, the cooling hole changes the vane structure, and compared to the solid vane without any cooling air holes, the effective area of the vane subjected to the external load is reduced, resulting weakened deformation resistance. On the other hand, the existence of the cooling air leads to higher temperature gradient near the cooling air holes, resulting in larger local thermal stress. Since the total deformation are constrained to 0 at endwalls, its maximum occurs at the mid-span.

![Total Deformation](image1)

(a) Pressure side (b) Suction side

**Figure 9.** Predicted deformation of the vane

There are many reasons for generation of the thermal stress in the turbine vane, such as the thermal expansion or contraction of turbine vane constrained by the endwalls, the temperature gradient in the vane, the combination of materials with different linear expansion coefficients, the impurity inside the vane material, the anisotropy of the blade material, and so on. For the NASA-MARKII vane investigated in the paper, the main reasons for the thermal stress are the constraint of the upper and lower endwalls, and the uneven temperature distribution inside the vane, as is provided in figure 10.

![Temperature Distribution](image2)

**Figure 10.** Predicted temperature distribution on the vane surface

![Von Mises Stress Distribution](image3)

**Figure 11.** Predicted Von Mises Stress distribution on the vane surface
Figure 11 provides the Von Mises stress distribution on the vane surface. There are significantly high equivalent stress at the endwalls, and the local maximum, with the value of about 7000 MPa, occurs at the leading edge on the pressure side and the trailing edge. Attributed to local large thermal deformation and higher temperature gradient, the vane stress on the suction side are higher than that on the pressure side.

The predicted Von Mises stress value is of the order of 103 MPa, exceeding the allowable limit of the vane material, and it is related to the boundary conditions set in the simulations. The too high local thermal stress is the result of the constrained upper and lower endwalls, which is not same as the actual situation. In practice, the upper and lower endwalls of the turbine vanes are not completely fixed, and they would change with the expansion of the shroud and the hub. The belts of the vanes also constrain the vane deformation. Accurate defining the boundary condition of elastic simulations is the key to predict the stress distribution of the vanes. Thermal analysis of combined structures of the complete vane including the shroud and the hub, instead of the single vane, is carried out during the design process.

Figure 12 provides the temperature distribution at the mid-span. As a result of the flow stagnation, the leading edge of the vane, close to the stagnation point, is with the highest temperature of the mid-span. After the leading edge, both the fluid acceleration and the cooling effect rapidly decrease the local temperature of the vane surface. Similarly to that on the suction side, the surface temperature on the pressure side is also, but slowly decreased, which is attributed to the rather slow local acceleration flow. As is shown in figure 13, the value of equivalent stress at the area surrounded by the 1st hole to the 4th hole, where there are good cooling effective, is about 400 MPa. The local minimum of the stress occurs between the 2nd hole and the 3rd hole, where the three principal stress values are relatively small, as is provided in figure 14.
Figure 13. Predicted Von Mises Stress distribution at mid-span

(a) $\sigma_1$

(b) $\sigma_2$

(c) $\sigma_3$

Figure 14. Predicted Principal Stress distributions at mid-span
The maximum equivalent stress of the vane occurs near the 7th cooling hole, and the stress near the pressure side is higher than that near the suction side. Such high equivalent stress refers to large local temperature gradient caused by good cooling effective and small distance from the cooling air hole to vane outer surface. The equivalent stress of the vane near the 8th and 9th cooling holes is less than that near the 7th hole, although the local temperature is higher than the rear one, due to the larger distance of these holes to the wall.

It is noted that the temperature at the trailing edge is higher than that on other regions at mid-span, and that its value is up to 640K. Since the rather thin thickness of the vane at the trailing edge, the local temperature gradient is rather small. Hence the value of the stress is not so large. Three principle stress distributions are presented in fig.13. It is shown that the third principal stress changes with the larger range than the first and the second ones, and that its absolute value is also greater than the other two principal stress. Hence the region with the largest absolute value of the third principal stress should be consistent with that with the largest value of equivalent stress at mid-span. It is found in fig. 13 that the values of the three principal stress near the 10th hole are quite small, hence the local equivalent stress is not large, too.

In the thermal and structure design of the air-cooled turbine, the temperature inside the solid vane should be controlled to a certain range, and the temperature distribution is reasonable so that the gradient cannot be too large. The aim of the researches on the flow and heat transfer near the solid walls is at the accurately predicting the aero and thermal loads of the air-cooled turbines. With these loads, the deformation and the stress distributions of the vane are obtained by the elastic analysis. Then the predicted results are applied to structure design of the air-cooled turbines. Furthermore, the aero-thermal-elastic coupling technique and the optimization methods can be employed to optimize the turbine structure design and to improve the reliability of the turbine.

4. Conclusions

Three turbulence models and the AGS algebraic transition model are employed in the simulations. The predicted pressure with these models agree well with the measured one, otherwise the predicted temperatures differ due to the abilities of the models to predict transition flows. The predicted temperatures with the AGS algebraic transition model are with the smallest deviation from the measured ones.

Compared with the thermal deformation and stress of the vane, the aerodynamic ones are negligible.

The maximum equivalent stress of the vane is at the leading edge near the pressure side and the trailing edge, where it occurs the maximum deformation. Equivalent stress concentration is mainly caused by constraints at the endwalls. Accurate and reasonable boundary conditions for the thermal structural analysis is one of the key issues for accurately predicting the thermal stress distribution of the vane.

Thermal gradient is the key factor for the equivalent stress. At the mid-span, the minimum of the equivalent stress occurs at the leading edge on the suction side due to the local low thermal gradient, and the maximum one near the 7th hole close to the pressure side.

References

[1] J. Wu, Y. Li, Z. Zhao, and et al. Numerical simulation of supercritical carbon dioxide turbine thermal stress based on multiphysics coupling method. IOP Conference Series: Materials Science and Engineering, 631(2019) 032026
[2] W. Ba, X. Li and X. Ren. Aero-thermal coupled through-flow method for cooled turbines with new cooling model. Proceedings of the Institution of Mechanical Engineers, Part A: Journal of Power and Energy, 232(2018) 254-265
[3] L. Song, P. Zhu, J. Li and et al. Effect of purge flow on endwall flow and heat transfer characteristics of a gas turbine blade. Applied Thermal Engineering, 110(2017) 504-520
[4] S. Scholla, T. Verstraetea, F. Duchaine and et al. Conjugate heat transfer of a rib-roughened internal turbine blade cooling channel using large eddy simulation. International Journal of
Heat and Fluid Flow, 61(2016): 650-664

[5] K. Parseh and K. Hejranfar. Development of a high-order compact finite-difference total Lagrangian method for nonlinear structural dynamic analysis. Applied Mathematical Modelling, 63(2018) 179-202

[6] C. Zhou, Q. Wang, Z. Guo and et al. Coupled heat transfer analysis of air cooling gas turbine. Journal of Propulsion Technology, 30(2011) 566-570 [in Chinese]

[7] Q. Wang, Z. Guo, C. Zhou and et al. Coupled heat transfer simulations of an air-cooled turbine with single cooling channel. Journal of Aerospace Power, 26(2011) 78-84 [in Chinese]

[8] Z. Guo, Q. Wang and G. Feng. Calculation model and calculation examples for thermal-flow-elastic conjugate simulations of turbine engine. ACTA Aeronautica Sinica, 30(2009) 213-216 [in Chinese]

[9] B. S. Baldwin and H. Lomax. Thin layer approximation and algebraic model for separated flows. 16th Aerospace Sciences Meeting, Hubtsville, AL U.S.A., 1978

[10] T. J. Coakley. Turbulence modeling method for the compressible Navier-Stokes equations. 16th Fluid and Plasmadynamics Conference, Danvers, MA U.S.A., 1983

[11] B. J. Abu-Ghannam and R. Shaw. Natural transition of boundary layers - the effects of turbulence, pressure gradient, and flow history. Journal of Mechanical Engineering Science, 22(1980) 213-228

[12] L. D. Hylton, M. S. Milhec, E. R. Turner and et al. Analytical and experimental evaluation of the heat transfer distribution over the surface of turbine vanes. Final Report Detroit Diesel Allison, Indianapolis, IN, U.S.A., 1983