Dark Side of the Standard Model: Dormant New Physics Awaken

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We find that the nonperturbative physics of the standard-model Higgs Lagrangian provides a dark matter candidate, “dormant skyrmion in the standard model”, the same type of the skyrmion, a soliton, as in the hadron physics. It is stabilized by another nonperturbative object in the standard model, the dynamical gauge boson of the hidden local symmetry, which is also an analogue of the rho meson.

I. INTRODUCTION

The standard model (SM) has been so successful that it is rather difficult to identify the clue of the new physics beyond the SM, despite the most fundamental problem of the origin of mass, which might require physics beyond the SM #1. Moreover there is some concrete tension between the SM and the reality: apparent absence of the dark matter candidate, θ vacuum parameters due to instantons (strong CP problem, etc.), absence of the first order phase transition for finite temperature and large enough CP violation required by the baryogenesis, etc. These possible failures of the SM may not exclude the SM but may only indicate our ignorance of the nonperturbative physics of the SM itself, although alternatively they could be solved in the context of the new physics beyond the SM.

Among those concrete problems, the dark matter is a central mystery of the particle physics and the astrophysics today.

In this paper we demonstrate a possible resolution of the dark matter without explicit recourse to the physics beyond SM, namely, that the nonperturbative physics of the SM Higgs Lagrangian provides a dormant dark matter candidate, the “dormant skyrmion in the SM (DSSM)”, the same type of the soliton as the hadronic skyrmion which already exists in the nonperturbative dynamics of the nonlinear sigma model (with Skyrme term) without recourse to the underlying theory, QCD.

The hadronic skyrmion is known to be stabilized by the rho meson as the gauge boson of the Hidden Local Symmetry (HLS) without ad hoc Skyrme term. Such an HLS exists in any nonlinear sigma model [2]. The SM Higgs Lagrangian is actually rewritten in the form of nonlinear sigma model and hence has the HLS [3]. We show that the HLS gauge boson, “SM-rho meson (SMρ)”, acquires kinetic term by the nonperturbative dynamics of the SM and hence stabilizes the DSSM. It provides a novel view of SM: the dark matter candidate exists already inside the nonperturbative SM, though not the perturbative SM, without explicit recourse to beyond the SM.

The SM Higgs Lagrangian is customarily written in the linear sigma model which is convenient for the perturbation theory. However, pSM is not a whole story of the SM, since there already exist sphaleron and instanton even for the weak coupling, which are well-known nonperturbative objects not to be described by the pSM. Also the ’t Hooft-Polyakov monopole in the perturbatively renormalizable Georgi-Glashow model similar to the SM Higgs Lagrangian does exist even in the vanishing quartic coupling (BPS limit). Moreover, even in the perturbation the Higgs coupling grows indefinitely to hit the Landau pole in the ultraviolet region thus invalidating the perturbation itself. Thus in the SM as a full quantum theory the nonperturbative effects such as the bound states could emerge if not a narrow resonance before reaching the Landau pole without affecting the successful pSM at low energy. The nonperturbative quantum physics can often be better described by a different parameterization of the same Lagrangian at the classical level (e.g., see footnote #2), the nonlinear sigma model through the polar decomposition in the case at hand.

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#1 For dynamical approach to this problem, see e.g., Ref. [4] and references cited therein
In fact it was shown \(^3\) that the SM Higgs Lagrangian on the broken vacuum can be straightforwardly cast into a scale-invariant version of the nonlinear sigma model based on the manifold \(G/H = SU(2)_L \times SU(2)_R/SU(2)_V\), where both the scale symmetry and chiral symmetry \(G\) are realized nonlinearly, with the SM Higgs being nothing but a (pseudo) dilaton, a Nambu-Goldstone (NG) boson of the spontaneously broken scale symmetry, together with the NG bosons of the spontaneous breaking of \(G\) down to the subgroup \(H\). (Though in that case the corresponding action is scale-invariant, just for convenience we shall hereafter call it the scale-invariant Lagrangian.) Once written in the form of nonlinear sigma model, one readily sees \(^3\) that it has the HLS, since it is known \(^2\) that any nonlinear sigma model is gauge equivalent to another model (HLS Lagrangian) having a symmetry \(G_{\text{global}} \times H_{\text{local}}\), where \(H_{\text{local}}\) is a gauge symmetry (HLS), and the HLS gauge boson, \(SM\rho\), is an auxiliary field as a static composite of the NG bosons.

Here we show that it actually develops kinetic term through nonperturbative dynamics of the NG bosons (longitudinal component of \(W, Z\) bosons when the electroweak gauge switched on) in the SM itself, in a manner similar to the \(CP^{N-1}\) model \(^\#2\) as discussed long time ago \(^2\). Once the HLS gauge boson acquired the kinetic term, it is well known \(^8\) to stabilize the skyrmion, soliton in the nonlinear sigma model, without adding artificial Skyrme term by hand. It is also known \(^3, 10\) that inclusion of the scalar meson, SM Higgs in our case, does not invalidate the stabilization.

Thus we find that the nonperturbative SM provides a dark matter candidate, DSSM, with \(I = J = 0, 1/2, \cdots\) (for isospin \(I\) and spin \(J\)), living dormant inside the SM but not beyond it. Here we consider a complex-scale bosonic-skyrmion, \(X_s\), with \(I = J = 0\). The potential term of the SM Higgs Lagrangian, a small explicit breaking of the scale symmetry, which yields the mass of the SM Higgs as a pseudo dilaton, does not affect the essential features of the above skyrmion physics.

The salient feature of the DSSM is that its zero-momentum coupling to the SM Higgs as a pseudo-dilaton is uniquely determined by the low energy theorem of the spontaneously broken scale symmetry for the dilaton-matter couplings as known for a long time in a different context \(^11\). It yields a unique constraint on its mass from the direct search experiments, \(M_{X_s} \lesssim 13\text{ GeV}\) from the LUX2016 \(^12\,13\). For such a low mass the SM Higgs can decay to \(X_sX_s\) and hence places the constraint \(M_{X_s} \lesssim 18\text{ GeV}\) from the invisible decay of the SM Higgs. The relic abundance is subject to the soliton size of \(DSSM\), which we calculate in a benchmark parameter choice of HLS. The result for \(M_{X_s} = \mathcal{O}(10)\text{ GeV}\) is \(\Omega_{X_s}h^2 \approx 0.1\), consistently with the present data. We discuss it can be discovered/ruled out in the future experiments.

II. SM HIGGS LAGRANGIAN AS A SCALE-INARIANT HLS LAGRANGIAN

The SM Higgs Lagrangian takes the form of the \(SU(2)_L \times SU(2)_R\) linear sigma model:

\[
\mathcal{L}_{\text{SM}} = |\partial_\mu h|^2 - \mu^2 |h|^2 - \lambda |h|^4 = \frac{1}{2} \text{tr} (\partial_\mu M \partial^\mu M^\dagger) - \frac{\mu^2}{2} \text{tr} (MM^\dagger) - \frac{\lambda}{4} \left(\text{tr} (MM^\dagger)\right)^2 ,
\]

\(M \equiv \frac{1}{\sqrt{2}}(i\tau_2 h^* , h) = \frac{1}{\sqrt{2}}(\sigma \cdot 1_{2 \times 2} + 2i\tilde{\pi})\), \((\tilde{\pi} \equiv \tilde{\pi}_a \tau_a / 2)\). Eq.(1) is straightforwardly rewritten into the form \(^3\):

\[
\mathcal{L}_{\text{SM}} = \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{\sigma^2}{4} \text{tr} (\partial_\mu U \partial^\mu U^\dagger) - V(\phi) = \chi^2 \cdot \left[\frac{1}{2} (\partial_\mu \psi)^2 + \frac{1}{4} \text{tr} (\partial_\mu U \partial^\mu U^\dagger)\right] - V(\phi) ,
\]

\[
V(\phi) \equiv \frac{\mu^2}{2} \text{tr} (MM^\dagger) + \frac{\lambda}{4} \left(\text{tr} (MM^\dagger)\right)^2 = \frac{\mu^2}{2} \sigma^2 + \frac{\lambda}{4} \sigma^4 = \frac{\lambda}{4} \phi^4 \left(\chi^2 - 1\right)^2 - 1 ,
\]

\(^\#2\) The \(CP^{N-1}\) is a nonlinear sigma model minimally written in terms of \(2N-2\) NG bosons, but usually parameterized including redundancy: one constraint with Lagrange multiplier and the symmetry \(SU(N)_{\text{global}} \times U(1)_{\text{local}}\), where the redundant \(U(1)_{\text{local}}\) is nothing but the HLS whose gauge boson is an auxiliary field to be solved away at the bare/perturbative theory. It is well established \(^2\,4\,5\) that in the large \(N\) limit, the nonperturbative dynamics change the classical theory as if in the broken phase into the unbroken phase in such a way that the HLS gauge boson necessarily acquires the kinetic term, becoming the true massless gauge boson, and the bare/perturbative NG bosons are no longer the NG bosons but have a mass, an extra free parameter given by the Lagrange multiplier. In \(d=2\) dimensions only the unbroken phase exists in conformity with the Mermin-Wagner-Coleman theorem, while both phases exist in \(2<d<4\) where the HLS gauge boson in the broken phase also acquires the kinetic term, and in addition a mass through the Higgs mechanism, precisely in the same way as the SM in the present paper. The \(d=4\) model has a cutoff acting as a Landau pole where the induced kinetic term of HLS gauge boson vanishes, similarly to our present case \(^2\,3\) (See also \(^3\) for different formulation in \(d=4\)). Note that minimal parameterization of the classical Lagrangian without redundancy is ill-defined at quantum level \(^2\), thus the parameterization is crucial to the nonperturbative quantum physics where the emergence of extra free parameters is mandatory.
where we have used the standard polar decomposition \(^\#3\) of the \(2 \times 2\) complex matrix \(M\) into a positive Hermitian matrix \(H\) times a unitary matrix \(U:\)

\[
M(x) = H(x) \cdot U(x),
\]

\[
H(x) = \frac{1_{2 \times 2}}{\sqrt{2}} \cdot \sigma(x), \quad \left(\sigma(x) = \sqrt{\sigma^2 + \pi^2} = v \cdot \exp \left(\frac{\phi(x)}{v}\right) = v \cdot \chi(x)\right),
\]

\[
U(x) = \exp \left(\frac{2i\pi(x)}{v}\right), \quad \pi \equiv \frac{\pi_a \tau_a}{2},
\]

with \(<\sigma(x)> = v = \sqrt{\frac{\mu^2}{2}} \neq 0 (\chi(x) = 1, \phi(x) = 0).\

Thus the SM Higgs Lagrangian Eq. \((1)\) is trivially identical to Eq. \((2)\). Note that the kinetic term of the latter, \(\chi^2 \cdot \left[\frac{1}{2} (\partial_{\mu} \phi)^2 + \frac{\mu^2}{v^4} \text{tr} (\partial_{\mu} U \partial^{\mu} U^\dagger)\right]\) contains the usual nonlinear sigma model \(\frac{\mu^2}{v^4} \text{tr} (\partial_{\mu} U \partial^{\mu} U^\dagger)\) which transforms as dimension 2 to make the action not scale-invariant. However, the extra dilaton factor \(\chi^2(x) = e^{2\phi(x)/v}\), transforming as dimension 2, makes the whole kinetic term to be dimension 4. Hence the action becomes scale-invariant as it should, since it is just a rewriting of the original kinetic term in Eq. \((1)\) which is scale-invariant (dimension 4).

Actually the kinetic term coincides with the scale-invariant nonlinear chiral Lagrangian based on the coset \(G/H = SU(2)_L \times SU(2)_R/SU(2)_{L+R}\), with the scale symmetry as well as the chiral symmetry being realized nonlinearly. Thus: the SM Higgs \(\phi\) is nothing but a pseudo dilaton, with the explicit breaking of the scale symmetry from the potential term \(V(\phi)\) characterized by

\[
\lambda = \frac{M_\phi^2}{2v^2} \simeq \frac{(125 \text{ GeV})^2}{2 \times (246 \text{ GeV})^2} \simeq \frac{1}{8} \ll 1,
\]

which is very close to the “conformal limit”, \(\lambda \to 0\) with \(v = \text{fixed}\) \((V(\phi) \to 0)\) \(^\#4\), the limit corresponding to the “Bogomol’nyi-Prasad-Sommerfield (BPS) limit” of ’t Hooft-Polyakov monopole in the Georgi-Glashow model \([1]\), similarly to the SUSY flat direction \(^\#5\).

Now that the SM Higgs Lagrangian Eq. \((1)\) is rewritten in the form of the nonlinear sigma model, Eq. \((2)\), we can further rewrite Eq. \((2)\) into another gauge equivalent form having redundant gauge symmetry in terms of the gauge fields as auxiliary fields \(^\#3\): it is well-known \([2, 4, 5]\) that nonlinear sigma model on the manifold \(G/H\) is gauge equivalent to the Hidden Local Symmetry (HLS) model having the internal symmetry \(G_{\text{global}} \times H_{\text{local}}\) with \(H_{\text{local}}\) being the HLS.

In the case at hand, based on the manifold \(G/H = SU(2)_L \times SU(2)_R/SU(2)_{L+R}\), the HLS \(H_{\text{local}} = [SU(2)_L \times SU(2)_R]_{\text{local}}\) was introduced as a redundancy of dividing \(U\) into two parts: \(U = e^{2i\pi/v} = \xi_L^\dagger \xi_R\) such that \(\xi_{L/R}\) transform as \(\xi_{L/R}(x) \to h(x) \cdot \xi_{L/R}(x) \cdot \hat{g}_{L/R}\) and so do the covariant derivatives \(D_{\mu} \xi_{L/R}(x) \equiv (\partial_{\mu} - i \rho_{\mu}(x)) \xi_{L/R}(x)\), where \(h(x) \in H_{\text{local}}\) and \(\hat{g}_{L/R} \in G_{\text{global}}, \rho_{\mu}(x)\) being the gauge boson of \(H_{\text{local}}\). We may parametrize \(\xi_{L/R} = e^{i\beta/F_v} \cdot e^{i\pi x/v}\), with \(\beta(x)\) being the fictitious NG boson to be absorbed into \(\rho_{\mu}(x)\), and \(F_v\) its decay constant (identical to the conventional coupling of \(\rho_{\mu}\) to the vector current). When we fix the gauge of HLS as \(\xi_L^\dagger = \xi_R = \xi = e^{i\pi x/v}\) (unitary gauge \(\hat{\rho}(x) = 0\)) such that \(U = e^{\frac{i}{v}x F_v} \cdot e^{i\pi x/v}\), \(H_{\text{local}}\) and \(H_{\text{global}} \subset G_{\text{global}}\) get simultaneously broken spontaneously (Higgs mechanism), leaving the diagonal subgroup \(H = H_{\text{local}} + H_{\text{global}}\), which is nothing but the subgroup \(H\) of the original

\(^{\#3}\) See e.g., section 5.3 in Ref. \([2]\). Note that the Cartesian coordinate \((\hat{\sigma}, \hat{\pi})\) are chiral non-singlet transforming into each other under the chiral transformation \(g_{L/R} \in SU(2)_{L/R}\). In the polar decomposition, on the other hand, the chiral transformation of \(M, M \to g_{L/R} M g_{L/R}^\dagger\) is inherited by \(U\) consisting of the genuine angular coordinates \(\pi\) (NG bosons, becoming totally the electroweak gauge parameters to be absorbed into \(W, Z\) bosons), \(U \to g_{L/R} U g_{L/R}^\dagger\), while the genuine radial component \(H\) is a chiral singlet (electroweak gauge singlet), \(H \to H\), and so is the physical mode, the SM Higgs field \(\phi\), which is electroweak-gauge invariant as it should be. The physical SM Higgs field \(\phi\) is actually the dilaton transforming as \(\delta_{D} \phi(x) = x + \pi^\mu \partial_{\mu} \phi(x)\) under the scale transformation. Note also that the chiral singlet potential term \(V(\phi)\), having only \(\phi\) and no NG bosons \(\pi\), breaks the scale symmetry explicitly by the amount \(\lambda\).

\(^{\#4}\) The opposite limit, \(\lambda \to 0\) with \(v = \text{fixed}\), leads to the ordinary nonlinear sigma model, where we also have \(V(\phi) \to 0\) but with \(\chi(x) \equiv 1\), so that the scale symmetry compensated by \(\chi^2\) factor in Eq. \((2)\) is completely lost. See Ref. \([1]\). Either limit has no \(\lambda\) coupling, but has derivative couplings instead, which are “weak” in the low energy \(\lambda^2/(4\pi v^2) \ll 1\), so that the perturbation according to the derivative expansion (“chiral perturbation theory”) makes sense.

\(^{\#5}\) Even if we take such a conformal/BPS limit, the theory is still an interesting theory with derivative coupling as in the usual chiral Lagrangian, and thus the quantum corrections will produce the trace anomaly of dimension 4, \(\sim v^4 \chi^4 \ln \chi\), as a new source of the SM Higgs mass as a pseudo-dilaton, which, however, would do not affect the basic nature of the DSSM property discussed here, similarly to the tiny explicit scale-symmetry breaking in the tree-level potential \(V(\phi)\) with \(\lambda \simeq 1/8 \ll 1\).
where the term $\chi(\alpha_{\mu \rho \lambda})$ to identify the scale $\Lambda$ with the Landau pole.

We here denote Maurer-Cartan 1-forms $\alpha_{\mu \rho \lambda} \equiv \frac{1}{2} \partial_{\mu} \xi_{\rho \lambda} \cdot \xi_{\rho \lambda}^{\dagger}$ and the covariantized ones $\hat{\alpha}_{\mu \rho \lambda} \equiv \frac{1}{2} D_{\mu} \xi_{\rho \lambda} \cdot \xi_{\rho \lambda}^{\dagger} = \alpha_{\mu \rho \lambda} - \rho_\mu$, which transform as $\hat{\alpha}_{\mu \rho \lambda} \to h(x) \hat{\alpha}_{\mu \rho \lambda} h(x)$. We further take the linear combinations $\hat{\alpha}_{\mu \parallel} = \frac{1}{2} (\hat{\alpha}_{\mu \rho} + \hat{\alpha}_{\rho \mu}) = \frac{1}{2} (\alpha_{\mu \rho} + \alpha_{\rho \mu}) - \rho_\mu = \frac{1}{2} \left( \frac{F_\rho}{\partial_\mu \rho_\mu} - \frac{1}{2} \rho_\mu [\rho_\mu, \pi] + \cdots \right) - \rho_\mu$ and $\hat{\alpha}_{\mu \perp} = \frac{1}{2} (\hat{\alpha}_{\mu \rho} - \hat{\alpha}_{\rho \mu}) = \frac{1}{2} (\alpha_{\mu \rho} - \alpha_{\rho \mu}) = \frac{1}{2 N} \xi_{\mu \parallel} \cdot \partial_\mu U \cdot \xi_{\mu \parallel}^{\dagger}$. Thus the HLS Lagrangian consists of the two $G_{\text{global}} \times H_{\text{local}}$-invariants:

$$L_{\text{HLS}} = v^2 \left[ \hat{\alpha}_{\mu \perp}^{\dagger} + a v^2 \left[ \hat{\alpha}_{\mu \parallel}^{\dagger} \right] \right] ,$$

$$v^2 \left[ \hat{\alpha}_{\mu \perp}^{\dagger} + a v^2 \left[ \hat{\alpha}_{\mu \parallel}^{\dagger} \right] \right] = \frac{v^2}{4} \left( \partial_\mu U \partial^\mu U^\dagger \right) ,$$

$$a v^2 \left[ \hat{\alpha}_{\mu \parallel}^{\dagger} \right] = a v^2 \left[ \rho_\mu - \alpha_{\mu \parallel} \right] = F_\rho^2 \left( \rho_\mu - \frac{1}{F_\rho} \partial_\mu \rho + i \frac{1}{2 \rho^2} [\rho_\mu, \pi] + \cdots \right)^2 ,$$

where $F_\rho^2 = a v^2$ to normalize the kinetic term of the fictitious NG boson $\bar{\rho}(x)$. The first term is identical to the original nonlinear sigma model, while the second term is the HLS-invariant mass term of the $\rho_\mu$ as obvious in the unitary gauge $\bar{\rho} = 0$, which also contains the $\rho \rho \pi \pi$ coupling.

Without kinetic term of the auxiliary field $\rho_\mu$, the second term in Eq.[6] vanishes: $a v^2 \left[ \rho_\mu - \alpha_{\mu \parallel} \right] = 0$, when equation of motion $\rho_\mu = \alpha_{\mu \parallel}$ is used. Then Eq.[6] is simply reduced back to the original nonlinear sigma model as it should be for the auxiliary field $\rho_\mu$.

It was further shown [2] that the SM Higgs Lagrangian Eq.[1] (hence Eq.[1] as well) is gauge equivalent to the scale-invariant version [13] of the HLS Lagrangian Eq.[6], up to the scale-violating potential $V(\phi)$, for the internal symmetry $G_{\text{global}} \times H_{\text{local}} = [SU(2)_L \times SU(2)_R]_{\text{global}} \times [SU(2)_{L+R}]_{\text{hidden}}$. Having extra dilaton kinetic term and the dilaton factor $\chi^2 = e^{20}\sqrt{v}$ responsible for the theory to be scale-invariant (up to $V(\phi)$), it reads in the unitary gauge:

$$L_{\text{SM/HLS}} = \chi^2 \cdot \left( \frac{1}{2} (\partial_\mu \phi)^2 + v^2 \left[ \hat{\alpha}_{\mu \perp}^{\dagger} + a v^2 \left[ \hat{\alpha}_{\mu \parallel}^{\dagger} \right] \right] \right) - V(\phi) ,$$

$$= e^{20} \left[ \frac{1}{2} (\partial_\mu \phi)^2 + \frac{v^2}{4} \left( \partial_\mu U \partial^\mu U^\dagger \right) + a v^2 \left[ \rho_\mu - \frac{i}{2 \rho^2} [\rho_\mu, \pi] + \cdots \right]^2 \right] - V(\phi) ,$$

where the term $\chi^2 a v^2 \left[ \hat{\alpha}_{\mu \parallel}^{\dagger} \right] = \chi^2 a v^2 \left[ \rho_\mu - \alpha_{\mu \parallel} \right]^2$ is the G-invariant scale-invariant mass term of the SM-higgs meson (SMH). Now we discuss that the kinetic term of SMH is generated dynamically by the quantum loop [2, 5], similarly to the CP$^N$ model (See footnote #2), in the same sense as the dynamical generation of the kinetic term (and the quartic coupling as well) of the composite Higgs in the Nambu-Jona-Lasinio model, which is an auxiliary field at the tree level or at composite scale (Landau pole of the pSM) [16]. In order to discuss the off-shell $\rho_\mu$ (in space-like momentum) relevant to the skyrmion stabilization [6], we adopt the background field gauge as in Ref. [5]. Integrating out high frequency modes from the composite scale $\Lambda$ to the scale $\mu$ in the Wilsonian sense at one-loop, the kinetic term is generated with the gauge coupling $g$ [5]:

$$- \frac{1}{2 g^2} \text{tr}[\rho^2_{\mu \mu}] , \quad \frac{1}{g^2} = \frac{C_2(G)}{(4\pi)^2} \left( \frac{a^2}{24} + \left( - \frac{11}{3} + \frac{1}{24} \right) \right) \ln \frac{\Lambda^2}{\mu^2} ,$$

with $C_2(G) = N_f$ and the number of chiral flavors $N_f = 2$, where in the second equation the first term is the loop contribution of $\pi$ (longitudinal W/Z when the electroweak gauging switched on) with $g_{\rho \rho \pi} = a/2$, the second the loop of the dynamically generated SM rho, with the usual factor $-11/3$ and the last one is from the loop of the would-be NG boson (the longitudinal SM rho) having the $\rho$ coupling $1/2$, which ends up with $\frac{1}{g^2} = \frac{N_f}{16\pi^2} \frac{a^2}{24} \ln \frac{\Lambda^2}{\mu^2}$.

For $a > \sqrt{87}$ and/or $\mu^2 > M^2_\rho$ (second and third terms are decoupled), we find $1/g^2 \to 0$ for $\mu \to \Lambda$, which allows us to identify the scale $\Lambda$ with the Landau pole.

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#6 The $M^2_\rho$ would develop (potentially large) imaginary parts in the time-like region for decaying to the WW, WZ if $M_\rho > 2M_{W/Z}$. However, this would not affect the skyrmion physics which is relevant to the space-like $\rho$. 


In the present paper, therefore, we confine ourselves only to the case $a \gg \sqrt{\Lambda}$ \#7. Rescaling the kinetic term to the canonical one, we have the (off-shell) mass $M^2_{\pi}(\mu) = a(\mu) g^2(\mu) v^2 \sqrt{\mu / \Lambda}$, as $a(\Lambda) = a$. (As shown in Ref. \[9\], the parameter $a$ also gets corrected by the HLS gauge loop contributions, to have the renormalization-group running when it scales down from the Landau pole $\Lambda$ to the infrared scale $\mu$. More explicit discussion based on the renormalization group analysis will be presented in another publication.)

Note that the one-loop contribution to the kinetic term of the auxiliary field is not literally one-loop perturbation but actually corresponds to an infinite geometric summation of the one-loop diagram (“bubble sum”) in the explicit nonperturbative treatment without auxiliary field in $1/N$ expansion \[10\]. (Also see the large $N$ description for the $CP^N-1$ model in the footnote \#2. In the present SM case, the number of the chiral flavors $N_f (= 2)$ would play the role of the $N$ for the $CP^N-1$ model. More explicit derivation is to be supplied in another publication.)

### III. Emergence of the DSSM from the SM Higgs Lagrangian

Now that the SM-rho meson has been dynamically generated, the Lagrangian of the SM with the HLS, Eq.\([7]\), thus takes the form \[8\]:

$$\mathcal{L} = \chi^2 \cdot \left( \frac{1}{2} (\partial_\mu \phi)^2 + \frac{v^2}{4} \text{tr}(\partial_\mu U^\dagger \partial^\mu U) \right) + \chi^2 \cdot a^2 v^2 \text{tr} \left( g \cdot \rho_\mu - \frac{i}{2} \rho_\mu \left[ \partial_\mu \pi, \pi \right] + \cdots \right)^2 - \frac{1}{2} \text{tr} \rho^2_{\mu\nu} + \cdots, \tag{9}$$

where the SM\(\rho\) kinetic term is re-scaled to the canonical one, i.e., $\rho_\mu(x) \to g \cdot \rho_\mu(x)$, and the last “+ \cdots” includes the scale-symmetry violating potential term $V(\phi)$ and quantum mechanically induced higher terms. This is our basic Lagrangian for the SM Higgs including the nonperturbative quantum effects. Near the Landau pole, such that $g \gg 1$ with $a = \text{fixed}$, this is reduced to the original SM Higgs Lagrangian at classical level, Eq.\([2]\) (equivalently Eq.\([11]\)).

The soliton energy $E$ thus takes the form similar to that analyzed in Ref.\([\tilde{9}]\) based on the scale-invariant version of the HLS Lagrangian for QCD hadrons such as $\pi, \rho, \omega, \sigma$ (except for the potential term as well as the pion mass and $\omega$ meson terms which are missing in our case):

$$E = E_\pi + E_{\pi\rho} + E_\rho + E_\chi,$$

$$E_\pi = 4\pi \int_0^\infty \frac{d r \, v^2 \chi(r)}{\sqrt{2}} \left( \frac{F'(r)^2}{2} + \sin^2 \frac{F(r)}{r} \right),$$

$$E_{\pi\rho} = 4\pi \int_0^\infty \frac{d r \, v^2 \chi(r)}{\sqrt{2}} \left( G'(r)^2 + \frac{G(r)G(r+2)^2}{2r^2} \right),$$

$$E_\rho = 4\pi \int_0^\infty \frac{d r \, v^2 \chi(r)}{\sqrt{2}} \left( \frac{F'(r)^2}{2} + \frac{M^2_{\phi}}{4} (\chi(r)^2 - 1)^2 \right), \tag{10}$$

where we took the usual hedgehog ansatz for the fields as done in Ref.\([\tilde{9}]\): $\chi = e^{\phi(r)}, \rho_{\mu=i} = e^{i k a \rho \frac{G(r)}{g r}}, \rho_{\mu=i} = 0, U = e^{i \hat{r} \cdot F(r)}$ with $\hat{r}$ being unit normalized vector. In the above expressions the prime attached on the fields denotes the derivative with respect to $r$. The equations of motion for $F(r), \phi(r)$, and $G(r)$ are obtained by minimizing the soliton energy:

$$F''(r) = - \left( \frac{2}{r} + \phi'(r) \right) F'(r) + \frac{1}{r^2} \left( 2a(G(r) + 1) \sin F(r) + (1 - a) \sin 2F(r) \right),$$

$$G''(r) = a(gv)^2 (G(r) + 1 - \cos F(r)) + \frac{G(r)G(r+1)(G(r) + 2)}{r^2},$$

$$\phi''(r) = -\phi'(r)^2 - \frac{2}{r} \phi'(r) + \frac{M^2_{\phi}}{2} (\chi(r)^2 - 1) + F'(r)^2 + \frac{2}{r^2} \sin^2 F(r). \tag{11}$$

\#7 If the SM is regarded as an effective theory of some underlying theory to provide the bare $\rho$ kinetic term already at $\Lambda$, the parameter $a$ could be much smaller as the QCD rho meson with $a \approx 2$. 
To obtain a solution for the topological number $N_{X_s} = 1$, we take $F(0) = \pi$. From the equation of motion one can see the boundary condition for $G(0)$ as $G(0) = -2$. The profile functions should also satisfy the boundary conditions at infinity as

$$F(\infty) = 0, \quad G(\infty) = 0, \quad \chi(\infty) = 1.$$  \hfill (12)

It is also well known \cite{8, 9} that this system without the Skyrme term has a stabilized skyrmion, irrespectively of the factor $\chi^2$ which makes the rho and the skyrmion as well as the Higgs to be scale-invariant nonlinearly (up to small effects of the term $+ \chi^2 M$ in Eq.(13)).

In fact the SM\$ kinetic term is reduced to the Skyrme term in the limit $a \to \infty$ with $g$ fixed \cite{8} (infinite mass limit $M^2_0 = a g^2 v^2 \to \infty$), since the SM\$ field $\rho_\mu$ can be integrated out via the equation of motion, $g_\mu \rho_\mu \approx a_\mu||$ in such a way that $g^2 \rho_\mu \rho_\nu \approx \delta(\alpha_{\mu, \perp} \alpha_{\nu, \perp}) = \delta(\alpha_{\mu, \perp} \alpha_{\nu, \perp})$, which is plugged back into the Lagrangian Eq.(14). In this limit the SM\$ effects other than the DSSM are invisible in the collider physics, so that all the successful results of the pSM are intact. In fact the perturbation theory of the SM Higgs in Eq.(1) is independent of the parameterization \cite{17}, and is exactly the same as Eq.(2) since the auxiliary field has no effect in the pSM. The SM\$ kinetic term now reads the Skyrme term \cite{18}:

$$-\frac{1}{2} \text{tr}[\rho^2_{\mu \nu}] \bigg|_{g=\text{fixed}} \xrightarrow{a \to \infty} -\frac{1}{2 g^2} \text{tr}[[\alpha_{\mu, \perp}, \alpha_{\nu, \perp}]]^2 = \frac{1}{32 g^2} \text{tr}[[\partial_\mu U U^\dagger, \partial_\nu U U^\dagger]^2],$$  \hfill (13)

where we used $\alpha_{\mu, \perp} = \xi_L(\partial_\mu U)\xi_L^\dagger/(2i)$. Note that the “Skyrme term limit”, $a \to \infty$ with $g$ = fixed, is different from the “classical limit”, $g \to \infty$ with $a$ = fixed, which we already discussed is the limit going back to the original SM Higgs Lagrangian at classical level, Eq.(11) and/or Eq.(2).

More explicitly, from Eq.(14) in the limit $a \to \infty$ with $g$ = fixed, $G(r)$ can be reduced to $G(r)$ = $\cos F(r)$ $-$ 1 in the background profile function, then $E_\rho$ becomes the Skyrmion. (One can easily see that the energy $E_\rho$ with $G(r)$ = $\cos F(r)$ $-$ 1 is equivalent to the one given the Skyrmie term Eq. (13) when the hedgehog ansatz $U = e^{i F(r)} F(r)$ is assumed.) Then under minimization of the new soliton energy the equations of motion reads

$$F''(r) = -\frac{1}{r^2 \chi(r)^2 + 2 (gv^2)^2 \sin^2 F(r)} \left( \chi(r)^2 (-\sin 2 F(r) + 2 r F'(r)) + 2 r^2 \chi(r)^2 \phi'(r) F'(r) + \frac{\sin 2 F(r)}{(gv^2)^2} \left( F'(r)^2 - \frac{\sin^2 F(r)}{r^2} \right) \right),$$

$$\phi''(r) = \phi'(r)^2 - \frac{2}{r} \phi'(r) + \frac{M^2_0}{2} (\chi(r)^2 - 1) + F'(r)^2 + \frac{2}{r^2} \sin^2 F(r).$$  \hfill (14)

This happen to be similar to the soliton equations analyzed in Ref.\cite{10} up to the difference in the potential term (higher terms like $\phi^3, \phi^4 \cdots$). While the Skyrmie term in Ref.\cite{10} was introduced by hand from outside of the SM, in our case the same Skyrme term is generated by the nonperturbative dynamics of the SM itself. Note that the hedgehog profile of the Higgs field takes the form somewhat different between the dilaton in our case and the scale non-invariant shifted field in Ref.\cite{10}, but both cases numerically lead to similar results. Moreover, the crucial difference is that our Higgs and DSSM are both dictated by the scale symmetry, while those of Ref.\cite{10} are completely free parameters. (See the discussions below through sections \cite{14} and \cite{15} to be given before the explicit discussions on the soliton properties of the DSSM in section \cite{16}).

Thus the DSSM emerges as a soliton solution from the Lagrangian Eq.(15) (here we consider $I = J = 0$) to be a topological bosonic matter carrying the topological number, which we call $U(1)_{X_s}$. The $U(1)_{X_s}$ symmetry protects the decay of the DSSM ($X_s$) completely, so the $X_s$ can be a dark matter candidate.

The DSSM is essentially generated by the scale-invariant part of Eq.(15) and hence its coupling is dictated by the nonlinear realization of the scale symmetry. This yields a salient feature of the DSSM which characterizes phenomenological consequences as the constraints from both direct and indirect detection experiments to be discussed in sections \cite{14} and \cite{16}.

In the low energy limit $q^2 \ll v^2 \approx (246 \text{ GeV})^2$ the DSSM ($X_s$) coupling to the SM Higgs $\phi$ is an unambiguously determined by the low-energy approach of the scale symmetry \cite{11} \#8, as described by the scale-invariant form of the lowest derivative effective Lagrangian:

$$\mathcal{L}_{X_s}(q \ll v) = \partial_\mu X_s \partial^\mu X_s - M^2_{X_s} \chi^2 X_s^2 X_s.$$  \hfill (15)

\#8 The low-energy theorem for the dilaton $\phi(q_{\nu})$ coupling to matter $B$ at $q_{\nu} \to 0$ reads $g_{\nu BB} = 2 M_B^2 / v$, $M_B / v$ for complex scalar and spin $1/2$ fermion, respectively \cite{14}, which is consistent with the scale invariance of the mass term; $M_B^2 B^\dagger B = M_B^2 B^\dagger B + (2 M_B^2 / v)$.
FIG. 1: The spin-independent elastic scattering cross section of the DSSM $X_s$ per nucleon as a function of the mass $M_{X_s}$ in unit of $\text{cm}^2$ (solid curve). Also have been shown the most stringent constraint at present from the latest LUX2016 experiment and projected experiments with the xenon target by the end of this decade. The gray domain, surrounded by the dashed curve on the bottom, stands for the atmospheric and astrophysical neutrino background.

From this we can readily read the $\phi - X_s$ interaction:

$$\mathcal{L}_{X_s}^{\text{int}}(q \ll v) = -\frac{2M_{X_s}^2}{v^2} \left(v \cdot \phi X_s^+ X_s + \phi^2 X_s^+ X_s + \cdots\right),$$

the first term of which is relevant to the dark matter detection experiments for weakly-interacting massive-particle (WIMP):

$$\mathcal{L}_{\phi X_s}^{\text{int}}(q \ll v) = -2\lambda_{X_s} v \cdot (\phi X_s^+ X_s), \quad \lambda_{X_s} \equiv \frac{M_{X_s}^2}{v^2}. \tag{17}$$

IV. DSSM IN DIRECT DETECTION EXPERIMENTS

The DSSM $X_s$ can be measured in the direct detection experiments of dark matter such as the LUX and PandaX-II experiments. The spin-independent (SI) elastic scattering cross section of the DSSM $X_s$ per nucleon ($N = p, n$), through the Higgs ($\phi$) exchange at zero-momentum transfer, can be calculated by the standard formula as a function of the DSSM mass $M_{X_s}$:

$$\sigma_{\text{SI}}^{\text{elastic/nucleon}}(X_s N \rightarrow X_s N) = \frac{\lambda_{X_s}^2}{\pi M_{\phi}^2} \left[\frac{Z}{A} \cdot m_s(p, X_s) g_{\phi pp} \frac{m_p}{M_{X_s}} + \frac{A - Z}{A} \cdot m_s(n, X_s) g_{\phi nn} \frac{m_n}{M_{X_s}}\right]^2, \tag{18}$$

with the target nucleus Xe: $(Z = 54, A = 131.293, v = 0.931 \text{ GeV}, m_{p(n)} \simeq 938(940) \text{ MeV}, v \simeq 246 \text{ GeV}$ and $M_{\phi} \simeq 125 \text{ GeV}$, where $m_s(N, X_s) = \frac{M_{X_s} m_N}{M_{X_s} + m_N}$ and $g_{\phi pp(nn)} = \sum_q \sigma_q^{p(n)} / v \simeq 0.248(0.254)$.

The plot is shown in Fig. 1 along with the currently strongest exclusion limit from the latest LUX2016 experiment. The present upper bound on the DSSM mass $M_{X_s}$ is thus read off as

$$M_{X_s} \lesssim 13 \text{ GeV}. \tag{19}$$

$\phi B^1 B + \cdots$ and $M_B \bar{B} X_B = M_B \bar{B} B + (M_B/v) \phi \bar{B} B + \cdots$ for complex scalar and spin 1/2 fermion, respectively. Incidentally, this also applies to the Yukawa coupling of SM Higgs $\phi$ to other matter, the quarks/leptons, $g_{q/l}^{Y} = M_q/l/v$, which is consistent with the SM Higgs being a pseudo-dilaton.
Note that we have the upper bound instead of the lower bound in contrast to conventional WIMP models due to the characteristic dilatonic coupling proportional to $m_{X_s}^2$ as in Eq. (17). In Fig. 1 we have also shown the expected limits from the projected direct detection experiments with the target nucleus of xenon [21] which will be activated by the end of this decade. From this, one can see that the XENON1T experiment has the sensitivity to exclude, or discover the DSSM with the mass up to $\sim 10$ GeV, and it will get lower up to $\sim 9$ GeV for the LUX-ZEPLIN (LZ) experiment.

V. INDIRECT SEARCH LIMIT FROM HIGGS INVISIBLE DECAY

Since $M_{X_s} < M_\phi/2$ as placed by the LUX2016 limit in Eq. (19), the DSSM can be constrained by the Higgs invisible decay searched at collider experiments. As the Higgs $\phi$ acts as a pseudo dilaton, the Higgs-onshell coupling to $X_s\bar{X}_s$, relevant to the invisible decay process, should be the same as that determined by the low-energy theorem, i.e., $g \sim M_\phi \ll v$ in Eq. (17). The partial decay width of the Higgs $\phi$ to the $X_s\bar{X}_s$ is thus unambiguously computed from Eq. (17) to be

$$\Gamma(\phi \rightarrow X_s\bar{X}_s) = \frac{\lambda_{X_s}^2}{4\pi M_\phi^2} \sqrt{1 - \frac{4M_{X_s}^2}{M_\phi^2}}. \quad (20)$$

The branching ratio is then constructed as $\text{Br}[\phi \rightarrow X_s\bar{X}_s] = \Gamma(\phi \rightarrow X_s\bar{X}_s)/\Gamma_\phi = \Gamma(\phi \rightarrow X_s\bar{X}_s)/[\Gamma_\phi^{\text{SM}} + \Gamma(\phi \rightarrow X_s\bar{X}_s)]$, with the total SM Higgs width (without the $X_s\bar{X}_s$ decay mode) $\Gamma_\phi^{\text{SM}} \simeq 4.1$ MeV at the mass of 125 GeV [23]. The currently most stringent upper limit on the Higgs invisible decay has been set by the CMS Collaboration combined with the run II data set with the luminosity of 2.3 fb$^{-1}$ [24]. Figure 2 shows the exclusion limit on the $X_s$ mass at 95% C.L., $\text{Br}_{\text{invisible}} < 0.2$ [24]. From the figure, one reads off the upper limit,

$$M_{X_s} < 18 \text{ GeV}, \quad (21)$$

which is milder than the constraint from the direct detection experiment in Eq. (19). In Fig. 2 the future prospected 95% C.L. limits in the LHC and ILC experiments [25] have also been shown. We thus see that the 14 TeV LHC with the luminosity of 300 fb$^{-1}$ has the potential to exclude, or detect the DSSM with the mass up to $\sim 14$ GeV, which is slightly less sensitive compared to the direct detection experiments. The ILC with higher statistics will be more sensitive to reach the exclusion and discovery sensitivity for the mass up to $\sim 6$ GeV.
VI. DSSM IN THERMAL HISTORICAL

The SM Higgs Lagrangian with the HLS in Eq. (3) is formulated in the vacuum where the electroweak symmetry is broken, hence in the thermal history of the universe the DSSM emerges after the electroweak phase transition at the temperature $T = O(\nu)$. At that time the DSSM was in the thermal (chemical) equilibrium with the photon and other SM particles due to the Higgs portal coupling in Eq. (14). As the universe cooled down to be at the temperature $x = M_{X_s}/T \sim 1$, the DSSM $X_s$ became nonrelativistic and the number density begins to fall down like $\sim e^{-M_{X_s}/T}$. Then the DSSM density gets so diluted due to the expansion of the universe, to make the DSSM cease to interact, and finally freezes out at $x_f = M_{X_s}/T_f (= O(10))$. Below $T = T_f$ ($x > x_f$) the DSSM number density per comoving volume stays to be constant and the DSSM cools down to become a cold dark matter just like WIMPs, with the relic abundance observed in the universe today.

Such a relic abundance can be estimated by the standard procedure, so-called the freeze-out thermal relic [26]:

$$\Omega_{X_s} h^2 = \frac{2 \times (1.07 \times 10^9) x_f}{\sigma_{\gamma}(T_f)^{1/2} m_p GeV(\sigma_{ann} v_{rel})},$$

where $M_{pl}$ stands for the Planck mass scale $\sim 10^{19}$ GeV, $\langle \sigma_{ann} v_{rel} \rangle$ is the thermal average of the annihilation cross section times the relative velocity of $X_s \bar{X}_s$, $v_{rel}$, and $g_*(T_f)$ denotes the effective degrees of freedom for relativistic particles at $T = T_f$. The prefactor 2 comes from counting both $X_s$-particle and $\bar{X}_s$-anti-particle present today. The freeze-out temperature $T_f$ can be determined by $x_f = \ln[2 \times 0.038 \times [g_*(T_f)x_f]^{-1/2}M_{pl}].$ $M_{X_s} \cdot \langle \sigma_{ann} v_{rel} \rangle$.

Since the freeze-out temperature ($T_f \sim M_{X_s}/10 = O(1)$ GeV) is expected to be much smaller than the Higgs mass scale, the Higgs portal coupling formula in Eq. (17), derived from the low-energy theorem for the scale symmetry, is applicable to estimate the freeze-out of the Higgs portal process. The explicit computation actually shows that the Higgs portal process decouples from the thermal equilibrium at $x_f/\text{Higgs portal} \sim 10$ for $M_{X_s} = O(10)$ GeV.

In addition to the Higgs portal process, one should note that the DSSM is essentially a soliton, an extended particle with a finite radius. The $X_s \bar{X}_s$ “annihilation” into the $U(1)_{X_s}$ current can be viewed as the classical $X_s \bar{X}_s$ collision with the $U(1)_{X_s}$ charge radius $\langle r_{X_s}^2 \rangle_{X_s}$ and the black disc approximation could be applied, so

$$\langle \sigma_{ann} v_{rel} \rangle_{\text{radius}} \sim 4\pi \cdot \langle r_{X_s}^2 \rangle_{X_s},$$

(Similar observation was made in Ref. [10].) As it will turn out, the freeze-out of this classical collision is actually later than the Higgs portal process, $x_f/\text{radius} \sim 20 > x_f/\text{Higgs portal}$ for $M_{X_s} = O(10)$ GeV, so the relic abundance of the DSSM is determined by this classical collision [9].

To estimate the size of $\langle r_{X_s}^2 \rangle_{X_s}$, as a simple benchmark, we shall take the infinite SM $\rho$ mass limit, $a \rightarrow \infty$ with $g$ = fixed, Eq. (14). Then the skyrmion system is reduced to essentially the same as the one analyzed in Ref. [10] (up to the scale-non-invariant form of the Higgs profile, which we have checked does not affect the soliton solution and the mean radius). The soliton mass $M_{X_s}$ is similar to that already given in Ref. [10], while the scale of the mean radius $\langle r_{X_s}^2 \rangle_{X_s}$ (not shown in Ref. [10]) is calculated by the standard formula [27, 28]:

$$\langle r_{X_s}^2 \rangle_{X_s} = 4\pi r^2 \rho_{X_s}(r), \quad \rho_{X_s}(r) = \frac{v_x}{4\pi}X_s^0(r),$$

where $X_s^0$ is the topological charge current $X_s^0 = \frac{\epsilon^{\alpha\beta\gamma}}{2\pi^2} \text{Tr} [(U^\dagger \partial_\gamma U)(U^\dagger \partial_\beta U)(U^\dagger \partial_\alpha U)].$ Our result for $C_r$ is given in Fig. 8.

Thus in the Skyrme term limit $a \rightarrow \infty$ with $g$ = fixed as our benchmark case, we have

$$\langle r_{X_s}^2 \rangle_{X_s} \simeq \frac{(2.2)^2}{g^2 v^2}, \quad M_{X_s} \simeq \frac{350}{g} \simeq 17 \text{ GeV} \times \left(\frac{500}{g}\right),$$

$$\Omega_{X_s} h^2 \simeq \frac{0.1}{\left(\frac{500}{g}\right)^2},$$

where the estimate of the thermal relic abundance accumulated by the black disc collision was made, and the freeze-out temperature $x_f = M_{X_s}/T_f \approx 20$ is computed to be almost constant in the mass and the radius, and we have used

#9 In the sense of magnitude relation on the reaction rates, the present DSSM-production scenario therefore looks similar to the self-interacting dark matter scenario in which the dark-sector self-interaction gets dominant for the thermal-freeze out relic.
$g_\star(T_f) = 100$ as the effective degree of freedom at the freeze out. The benchmark value $g = 500 (\gg 1)$ corresponds to 17 GeV as a reference value of the upper bound from the direct and indirect searches in the region $M_{X_s} = \mathcal{O}(10)$ GeV. Although we have two parameters ($a, g$) for the generic HLS case Eq. (11), Eq. (24) has only one parameter $g$ in our benchmark case Eq. (14) in the Skyrme term limit $a \to \infty$ with $g$ = fixed. Note that the value $g \sim 500$ from $M_{X_s} = \mathcal{O}(10)$ GeV in our benchmark case trivially satisfies the constraint from the vector boson scattering, $g \gtrsim 0.4$ [10]. Thus we find $\Omega_{X_s} h^2 = \mathcal{O}(0.1)$ for $M_{X_s} = \mathcal{O}(10)$ GeV, which is roughly consistent with the presently observed dark matter relic $\simeq 0.12$ [29].

Since the DSSM has a conserved skyrmion number, there may be an alternative way to asymmetrically generate the current relic abundance of the DSSM through the electroweak sphaleron process together with both the DSSM and the SM Higgs Lagrangian. This possibility will be pursued elsewhere.

VII. CONCLUSION AND DISCUSSION

We have shown a novel possibility that the dark matter candidate exists already within the Standard Model (SM), not beyond it, through nonperturbative dynamics. The SM Higgs Lagrangian is cast into precisely the scale-invariant nonlinear sigma model, with the SM Higgs being a pseudo dilaton. It is further shown to be gauge equivalent to the scale-invariant version of the hidden local symmetry (HLS) Lagrangian whose dynamical gauge boson “SM-rho meson (SM$\rho$)” stabilizes the skyrmion, “Dormant Skyrmion in the SM (DSSM)” $X_s$, a novel candidate for the dark matter without explicit recourse to possible underlying theory. The scale invariance of the whole dynamics is essential, which unambiguously determines the couplings of the DSSM to the SM Higgs as a pseudo-dilaton in terms of the low energy theorem of the spontaneously broken scale invariance. This imposes a definite constraint on the mass $M_{X_s} \lesssim 13$ GeV from the direct detection experiments [13]. With such a mass smaller than half of the SM Higgs mass, we have also found the constraint $M_{X_s} \lesssim 18$ GeV from the SM Higgs invisible decay data [24] definitely by the low-energy theorem.

Based on this salient constraint we have discussed the thermal history of the DSSM, and estimated the thermal relic abundance, in view of the extended size of the soliton, in some benchmark cases including the heavy SM$\rho$ mass limit (scale-invariant Skyrme model limit). It was shown that the estimated present relic density of the DSSM with the mass of $\mathcal{O}(10$ GeV) is roughly consistent with the observed-cold dark matter relic-abundance $\Omega_{\text{cldm}} h^2 \simeq 0.12$.

Although our rough estimate on the relic abundance was made only in the benchmark cases, including the heavy SM$\rho$ mass limit $a \to \infty$ with $g$ = fixed, more interesting would be the lighter SM$\rho$ mass case. There have been many studies on this case in the hadron physics [3, 8]. In the forthcoming paper we will report explicit calculations of the size and mass of the DSSM for the whole parameter space ($g, a$).

We have also discussed discovery/exclusion possibilities in the future experiments. Both the prospected direct detection and indirect detection experiments have the sensitivity enough to discover the DSSM with the mass of $\mathcal{O}(10$ GeV) within a couple of decades (See Figs 1 and 2). If the DSSM signals were not observed, conversely, the DSSM mass could be constrained by future experiments to be lower and lower, so that the relic abundance could get
larger. Then the DSSM might exceed the present cold-dark matter density, which would imply necessity to go beyond the SM. At any rate, the future-prospected experiments will clarify whether or not the DSSM can explain the dark matter presently observed in our universe.

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