Implications of partially degenerate neutrinos at a high scale in the light of KamLAND and WMAP

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Abstract

Electroweak radiative corrections can generate the neutrino (mass)$^2$ difference required for the large mixing angle solution (LMA) to the solar neutrino problem if two of the neutrinos are assumed degenerate at high energy. We test this possibility with the existing experimental knowledge of the low energy neutrino mass and mixing parameters. We derive restrictions on ranges of the high scale mixing matrix elements and obtain predictions for the low energy parameters required in order to get the LMA solution of the solar neutrino problem picked out by KamLAND. We find that in the case of standard model this is achieved only when the (degenerate) neutrino masses lie in the range $(0.7 - 2)$ eV which is at odds with the cosmological limit $m_\nu < 0.23 \text{eV}$ (at 95$\%$C.L) established recently using WMAP results. Thus SM radiative corrections cannot easily generate the LMA solution in this scenario. However, the LMA solution is possible in case of the MSSM electroweak corrections with (almost) degenerate spectrum or with inverted mass hierarchy for limited ranges in the high scale parameters.
Introduction: Results from solar and atmospheric neutrinos [1] have greatly helped in establishing patterns for neutrino masses and mixings particularly after the report of the positive evidence of neutrino oscillations seen at KamLand [2]. The allowed possibilities are quite constrained. One needs hierarchical differences in the neutrino \((\text{mass})^2\) and two large and one small \((\leq 0.2)\) mixing angle to fit the observations.

The phenomenological determination of neutrino masses and mixing raises several theoretical questions two of which are \((i)\) why two of the six physical fermionic mixing angles are large and \((ii)\) what is the cause of hierarchy in the solar \((\Delta_S)\) and the atmospheric \((\Delta_{\text{atm}})\) mass scales. There have been number of answers to these questions in variety of frameworks [1]. One possibility is to invoke radiative corrections to understand smallness of \(\frac{\Delta_S}{\Delta_{\text{atm}}}\). These corrections could be weak corrections to the lepton number violating neutrino mass operators [3] or could also come from physics beyond standard model [4, 5]. Possibility of the electroweak corrections generating solar scale has recently been analyzed in [6, 7, 8]. It is assumed that two of the neutrinos are degenerate at some high scale and electroweak corrections result in generation of the solar \((\text{mass})^2\) difference. This possibility was shown to be quite constrained. It leads to a definite prediction for the solar scale namely,

\[
\Delta_S \cos 2\theta_S = 4\delta_\tau \sin^2 \theta_A |m_{ee}|^2 + \mathcal{O}(\delta_\tau^2).
\] (1)

Here \(\Delta_S\) is the mass-squared difference responsible for the solar neutrino oscillations, and the angles \(\theta_S\) and \(\theta_A\) respectively denote the solar and the atmospheric mixing angles at a low scale. \(m_{ee}\) is the effective neutrino mass probed in the 0νββ decay and \(\delta_\tau\) specifies the size of the radiative corrections induced by the Yukawa coupling of the \(\tau\):

\[
\delta_\tau \approx c \left( \frac{m_\tau}{4\pi v} \right)^2 \ln \frac{M_X}{M_Z}.
\] (2)

\(c = \frac{3}{2}, -\frac{1}{\cos^2 \beta}\) in case of the standard model (SM) and the minimal supersymmetric standard model (MSSM) respectively [9, 10, 11].

Additional assumption made in deriving eq.(1) was that the mixing element \(U_{e3}\) of the leptonic mixing matrix \(U\) was zero at high scale. This assumption was motivated by the observed smallness of \(U_{e3}\) at low energy. Given this assumption, eq.(1) makes very strong predictions analyzed in detail in [7, 8]. Two of the major consequences being that the MSSM radiative corrections
cannot generate the large mixing angle (LMA) solution to the solar neutrino problem and in the case of SM one needs $m_{ee}$ close to the present experimental limit [12].

A non-zero and relatively large low scale $U_{e3} \sim 0.2$ could change some of the qualitative aspects of predictions based on eq.(1). More importantly, the mixing angle at high scale could even be larger than 0.2 and can still give rise to an experimentally acceptable $U_{e3}$ at the low scale. In this paper we study the general implications of two degenerate neutrinos at high scale without assuming a zero $U_{e3}$. The general numerical analysis carried out in this paper leads to very strong predictions for three of the yet unknown observables namely, $U_{e3}, m_{ee}$ and the absolute neutrino mass $m_{\nu e}$ probed in beta decay [13]. One of the conclusions of our analysis is that a non-zero high scale $U_{e3}$ makes the MSSM viable. In the SM however the LMA mass difference can be generated radiatively only when the the degenerate neutrino mass is in the range of $(0.7 - 2) \, eV$. This requirement is ruled out by the WMAP result that $\Omega_{\nu} h^2 < 0.0076$ (at 95\% C.L.) [14] which for the degenerate neutrino spectrum implies that $m_{\nu} < 0.23$.

**Formalism:** Consider a CP conserving theory specified by a general $3 \times 3$ real symmetric neutrino mass matrix $M_{\nu 0}$ specified at a high scale $M_X$. We require that the solar scale vanishes at $M_X$ and consequently two of the eigenvalues of $M_{\nu 0}$ are degenerate, i.e., we assume $^1 m_{\nu 0i} = (m, -m, m')$ for the neutrino masses at $M_X$. The atmospheric neutrino oscillations are induced by $\Delta_{A0} \equiv |m'^2 - m^2|.$

Neutrino mixing matrix has the following general form under the assumption of CP conservation.

$$U_0 = R_{23}(\theta_2)R_{13}(\theta_3)R_{12}(\theta_1),$$

where $R_{ij}(\theta)$ denotes a rotation in the $ij^{th}$ plane by an angle $\theta$. The $\theta_{1,2}$ are assumed to vary between 0 and $\pi/2$ while $s_3 = \sin \theta_3$ varies over the full range. The neutrino mass matrix at $M_X$ is given by

$$\mathcal{M}_{0\nu} = U_0 \text{Diag.} (m, -m, m') U_0^T.$$  

The matrix $\mathcal{M}_{0\nu}$ determined by eq.(4) is modified by the radiative corrections.

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1The solar angle can be rotated away in case of the other viable possibility $m_{\nu 0i} = (m, m, m')$ which cannot reproduce the observed pattern.
[9]. We assume the RG equations corresponding to the SM or the MSSM. The modified neutrino mass matrix is given [10] in this case by

\[ M_0 \nu \rightarrow M_\nu \approx I_g I_t \left( I U_0 \text{diag.}(m, -m, m') U_0^T I \right) \tag{5} \]

where \( I_{g,t} \) are calculable numbers depending on the gauge and top quark Yukawa couplings. \( I \) is a flavour dependent matrix given by

\[ I \approx \text{diag.}(1 + \delta_e, 1 + \delta_\mu, 1 + \delta_\tau) \].

\( \delta_{e,\mu} \) are obtained from eq.(2) by replacing the tau mass by the electron and the muon masses respectively. The physical neutrino masses and mixing are obtained by diagonalizing the above matrix.

The eigenvalues of \( M_\nu \) can be approximately determined in the limit of vanishing \( m_{e,\mu} \). We find,

\[
\begin{align*}
    m_{\nu_1} &\approx m(1 + 2\delta_\tau(c_1c_2s_3 - s_1s_2)^2) + O(\delta_\tau^2), \\
    m_{\nu_2} &\approx -m(1 + 2\delta_\tau(s_1c_2s_3 + c_1s_2)^2) + O(\delta_\tau^2), \\
    m_{\nu_3} &\approx m'(1 + 2\delta_\tau c_2^2c_3^2) + O(\delta_\tau^2). \tag{6}
\end{align*}
\]

We thus have

\[
\Delta_{21} \equiv m_{\nu_2}^2 - m_{\nu_1}^2 \approx 4\delta_\tau m^2 \left( \cos 2\theta_1(s_2^2 - s_3^2c_2^2) + s_3 \sin 2\theta_1 \sin 2\theta_2 \right) + O(\delta_\tau^2). \tag{7}
\]

It is conventional to order masses in a way that makes the solar scale \( \Delta_S \) positive. The phenomenological analysis then restricts \( \theta_S \) to be < \( \pi/4 \). The \( \Delta_{21} \) as defined above can have either sign. For positive \( \Delta_{21} \), \( \Delta_S = \Delta_{21} \) and \( \cos 2\theta_S = \cos 2\theta_1 \). One needs to interchange first two eigenvalues and eigenvector of \( M_\nu \) in the opposite case with \( \Delta_{21} < 0 \) giving us \( \Delta_S = -\Delta_{21} \) and \( \cos 2\theta_S = -\cos 2\theta_1 \). In either situation, one needs \( \Delta_{21} \cos 2\theta_1 > 0 \) in order to obtain the LMA solution. For \( s_3 = 0 \), the sign of \( \delta_\tau \) determines the sign of \( \Delta_{21} \cos 2\theta_1 \). As a result one cannot reproduce the LMA solution in the case of MSSM as already remarked [7, 8]. The introduction of a non-zero \( s_3 \) changes this behavior. As follows from eq.(7) one can now obtain positive \( \Delta_{21} \cos 2\theta_1 \) in case of the MSSM also for some range in parameter space corresponding to \( s_3 \cos 2\theta_1 < 0 \). This range is limited since \( s_3 \) cannot be very large without conflicting with the observed bound on \( U_{e3} \).

Realization of the LMA solution in this framework would require some restrictions on the initial high scale parameters and would also restrict the values...
of the low energy observables. These observables include $m_{ee}, \Delta S, U_{e3}$ and the electron neutrino mass $m_\nu_e$. We study these restrictions in detail below through the following procedure:

**Input parameters:** We randomly vary the input mixing angles $\theta_i$ and the mass $m$ over the range $0 < \theta_{1,2} < \pi/2$, $-\pi/2 < \theta_3 < \pi/2$ and $0 < m < 2$ eV. Since the atmospheric mass scale does not significantly change by the radiative corrections, we fix the third mass $m'$ by requiring $\Delta_{\text{atm}} \equiv |m'^2 - m^2| = (0.05)^2$. $m'$ can have either sign relative to $m$ and could be heavier or lighter than $m$. We will consider $|m'| > |m|$ and $m' \ll m$. The MSSM radiative corrections involve an additional parameter $\cos \beta$ which is also randomly varied in the range (0-1).

**Output values of observables:** Eq. (5) is numerically diagonalized for each choice of the randomly chosen input variables. This gives us the output values of the solar and atmospheric masses and mixing angles, $U_{e3}$ as well as $m_{ee}$ and $m_{\nu_e}$. The known output parameters are required to lie in the range $[1, 15]$: 

$$
|U_{e3}| \leq 0.2 \\
4 \cdot 10^{-5} \text{eV}^2 \leq \Delta_S \leq 2.8 \cdot 10^{-4} \text{eV}^2 ; \\
0.2 \leq \tan^2 \theta_S \leq 0.8 , \\
1.2 \cdot 10^{-3} \text{eV}^2 \leq \Delta_{\text{atm}} \leq 5.0 \cdot 10^{-3} \text{eV}^2 ; \\
0.8 \leq \sin^2 2\theta_A \leq 1.0 .
$$

The ranges quoted for the solar parameters is $3\sigma$ level. The KamLand results on the anti neutrino oscillations do not allow the full range of the LMA solution quoted above but the allowed values at 90% CL lie in two different ranges which are subsets of the above range. Consistency of the results with KamLand can be explicitly seen from the figures to be presented.

From the randomly varied set of 100,000 points, we collect the acceptable choice of the (high scale) input variables which lead to the low energy parameters lying in the ranges in eq.(8). This procedure also gives us the predicted values of the output observables like $m_{ee}, m_{\nu_e}$ corresponding to the acceptable choices of the input parameters. Results of our analysis are presented in case of the SM as well as MSSM in Figs.(1-5).

Fig.(1) shows the allowed values of the input parameters $s_1, s_3$ consistent with random variations of these and other parameters. The effect of the radiative corrections on mixing angles is not pronounced in case of the SM and the input ranges for $s_3$ and $\tan^2 \theta_1$ coincide approximately with the allowed ranges in $U_{e3}$ and $\tan^2 \theta_S$. Radiative corrections can be appreciable in case of the MSSM and an $s_3$ as large as 0.4 can lead to $U_{e3} \leq 0.2$ at the low scale. But values of $s_3$ larger than this cannot reproduce the correct low energy pa-
rameters. Two different patches in the figure correspond to $s_3 > 0, \cos 2\theta_1 < 0$ and $s_3 < 0, \cos 2\theta_1 > 0$ both of which can lead to the correct solution as argued above.

In Fig.(2), we display output values of the solar scale $\Delta S$ and $m_{ee}$ consistent with the random variation of the input parameters. The allowed points span the entire range in $\Delta S$ and thus the two sub regions corresponding to the KamLand results are easily obtained. The predicted values of $m_{ee}$ are quite restricted. Typical lower bound in case of the SM is around $0.05 - 0.1$ and most points crowd in the range $0.4 - 1.0$ eV in case of the SM. Typical range preferred by MSSM is $m_{ee} \sim 0.2 - 0.4$ eV.

We show the values of $|U_{e3}|$ realized in the random analysis in Fig.(3) as a function of $m_{ee}$. In large number of cases, $|U_{e3}|$ is seen to lie close to the experimental limit in case of the SM. The MSSM also predicts larger values but allows smaller values $|U_{e3}| \sim 0.02$ also.

Fig.(4) is prediction for the mass $m$ which corresponds to the electron neutrino mass probed in the tritium beta decay. One sees a clear preference for $m \sim 0.7 - 2$ eV. In fact $3\sigma$ lower bound on $m$ following from approximate eq.(7) and realized in the figure is $m > 0.7$ eV. MSSM also prefers similar values but it can still allow $m$ as low as $\sim 0.1$ eV due to the presence of an additional parameter $\tan \beta$.

We had initially chosen 100,000 random points in both cases but the allowed set of input points is much larger in case of the SM. This clearly shows that radiative corrections in SM can more easily reproduce the low energy neutrino spectrum than in MSSM. This was to be expected since when $s_3$ is zero, MSSM cannot lead to the LMA solution at all [8]. Introduction of $s_3$ now allows MSSM but the allowed values of $s_3$ are constrained by the observed bound on $U_{e3}$ and as a result one gets correct solution in case of MSSM for much smaller number of input points. In contrast, the SM can reproduce correct spectrum even for $s_3 = 0$ as clearly seen in Fig.(1).

We assumed three neutrinos to be almost degenerate in the above analysis. Alternative possibility corresponds to the inverted mass hierarchy in which only the solar pair has non-zero mass mass $m$, the third mass $m'$ being zero or much smaller. It was argued [7, 8] that in all these models the weak radiative corrections are unable to give the LMA solution if $s_3 = 0$. This conclusion
arises because the mass $m$ in this case is required to be close to the atmospheric scale. The resulting $\Delta_S$ is at least an order of magnitude smaller than required by LMA in case of the SM. MSSM has additional parameter $\tan \beta$ which can overcome this suppression but $\tan^2 \theta_S$ is predicted to be greater than 1 when $s_3 = 0$. Non-zero $s_3$ changes this conclusion. As already discussed, one can obtain $\tan^2 \theta_S < 1$ even in case of MSSM if $s_3 \cos 2 \theta_1 < 0$. In this case one can get a $\Delta S$ corresponding to the LMA solution even for $m^2 \sim \Delta_{\text{atm}}$ by choosing a large $\tan \beta$. This is demonstrated in Fig. (5) which shows allowed values of $|s_3|$ and $m$ resulting from the random variations of input parameters. Unlike in earlier figures, the mass $m$ is now varied only near the atmospheric range specifically in the interval $0.01\text{–}0.1 \text{ eV}$ and $\cos \beta$ is varied in the range $0.001\text{–}0.2$ since only these ranges are expected to give the correct $\Delta_S$ and $\Delta_{\text{atm}}$. As seen in the figure, we do get the correct solutions although for limited number of output values starting with 100,000 input points as before.

Summary and implications for models: We have numerically investigated consequences of having vanishing $\Delta_S$ at a high scale. This assumption can account for the smallness of $\Delta_S$ after inclusion of the weak radiative corrections. The predicted value for the solar scale is linked to observable low scale parameters. This results in stringent predictions which can be tested.

Basic assumption of our analysis is two degenerate neutrinos. This can be realized in number of ways and there are numerous models which predict two [4] or all three [16] neutrinos to be degenerate. If the third neutrino has comparable mass then one gets the almost degenerate scenario. Precise information on the common mass of these degenerate neutrinos has recently been provided by the data on microwave anisotropy [14]. In combination with the information on the galactic structures, this data imply a bound of $m < 0.23 \text{eV}$ at $95\% \text{CL}$ [14, 17]. As our Fig.(4) shows, this mass is required to be in the range $(0.7\text{–}2)$ eV in case of SM but it could be smaller for MSSM. Thus the SM radiative corrections and degenerate spectrum cannot account for the LMA solution at $95\% \text{CL}$. The lower bound is also inconsistent with the $99\% \text{CL}$ limit, $m < 0.3 \text{eV}$ following from analysis by Giunti in [17].

The standard model also fails in generating correct $\Delta_S$ if neutrinos have inverted mass hierarchy. It is possible to obtain the LMA solution in case of the MSSM which allows degenerate mass in the range $0.1 \text{–} 2 \text{eV}$ range. Likewise,
one can also obtain the LMA solution in case of MSSM and the inverted mass hierarchy, see Fig.(5). The LMA solution and value \( m < 0.23 \text{eV} \) occurs in both these cases only for very limited range of parameters.

Another related but testable prediction of the scheme is \( m_{ee} \) probed by \( 0
\nu \beta \beta \) decay results. The present experimental limit on this scale is uncertain due to the unknown nuclear matrix element. The present bound is \([12]\) \( m_{ee} < 0.4 \text{h eV} \) at 95\% CL. The \( h \sim 0.6 - 2.8 \) parameterizes\(^2\) the uncertainty in nuclear matrix element. The present scenario is consistent with this limit (see Figs. 2 and 3) but it can be constrained with improvement on our knowledge of the solar parameters, \( m_{ee} \) and \( U_{e3} \).

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Figure 1: Allowed values of the high scale mixing elements $s_1$ and $s_3$ consistent with the known neutrino oscillation constraints in case of the MSSM and SM. The other input variables are varied randomly as described in the text.
Figure 2: The predicted correlation between the 0νββ decay parameter $m_{ee}$ (in eV) and the solar scale $\Delta S$ (in eV$^2$) resulting after the random variations in input parameters as described in the text. The upper (lower) figure is for MSSM (SM).
Figure 3: The predicted correlation between the $0\nu\beta\beta$ decay parameter $m_{ee}$ (in eV) and $|U_{e3}|$ resulting after the random variations in input parameters as described in the text. The upper (lower) figure is for MSSM (SM).
Figure 4: The predicted correlation between the absolute neutrino mass $m$ (in eV) and the solar scale $\Delta_S$ (in eV$^2$) resulting after the random variations in input parameters as described in the text. The upper (lower) figure is for MSSM (SM).
Figure 5: The predicted correlation between the neutrino mass $m$ (in eV) and the absolute value of $s_3$ in inverted hierarchy models. Input parameters are randomly varied as described in the text.