Theoretical upper bound on the mass of the LSP in the MNSSM

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Abstract

We study the neutralino sector of the Minimal Non-minimal Supersymmetric Standard Model (MNSSM) where the $\mu$ problem of the Minimal Supersymmetric Standard Model (MSSM) is solved without accompanying problems related with the appearance of domain walls. In the MNSSM as in the MSSM the lightest neutralino can be the absolutely stable lightest supersymmetric particle (LSP) providing a good candidate for the cold dark matter component of the Universe. In contrast with the MSSM the allowed range of the mass of the lightest neutralino in the MNSSM is limited. We establish the theoretical upper bound on the lightest neutralino mass in the framework of this model and obtain an approximate solution for this mass.
1. Introduction

The analysis of fluctuations in the cosmic microwave background (CMB) using recent WMAP satellite data [1] and other precise measurements [2] indicate that about 22%-25% of the energy density of the Universe exists in the form of stable non–baryonic, non–luminous matter, so called dark matter [3]. Although the microscopic composition of dark matter remains a mystery it is clear that it can not consist of any elementary particles which have been discovered so far. Thus the existence of dark matter is the strongest piece of evidence for physics beyond the Standard Model (SM) of electroweak interactions.

The minimal supersymmetric (SUSY) standard model (MSSM) is the best motivated extension of the SM nowadays. Within the MSSM the quadratic divergences, which destabilise the scale hierarchy, are cancelled [4] and the gauge coupling unification can be naturally achieved [5]. If R–parity is conserved the lightest supersymmetric particle (LSP) in the MSSM is absolutely stable and can play the role of dark matter [6]. In most supersymmetric scenarios the LSP is the lightest neutralino. Since neutralinos are heavy weakly interacting particles they explain well the large scale structure of the Universe [7] and can provide the correct relic abundance of dark matter if their masses are of the order of the electroweak (EW) scale [6].

Despite these successes the MSSM suffers from the so-called $\mu$–problem. Namely, the MSSM superpotential contains only one bilinear term $\mu(\hat{H}_d\epsilon\hat{H}_u)$. In order to get the correct pattern of electroweak symmetry breaking, the parameter $\mu$ is required to be of the order of the electroweak scale. While the corresponding coupling is stable under quantum corrections, it is rather difficult (although possible [8]) to explain within Grand Unified theories (GUTs) or supergravity (SUGRA) why the dimensionful parameter $\mu$ should be so much smaller than the Planck or Grand Unification scale.

In the Next–to–Minimal Supersymmetric Standard Model (NMSSM) [9]–[10], which contains an additional SM singlet superfield $\hat{S}$, a $Z_3$ symmetry forbids any bilinear terms in the superpotential allowing the interaction of $\hat{S}$ with the Higgs doublets $\hat{H}_u$ and $\hat{H}_d$: $\lambda\hat{S}(\hat{H}_d\epsilon\hat{H}_u)$. At the EW scale the superfield $\hat{S}$ gets a non-zero vacuum expectation value ($\langle S \rangle = s/\sqrt{2}$) generating automatically an effective $\mu$-term ($\mu_{\text{eff}} = \lambda s/\sqrt{2}$) of the required size. There is a number of phenomenological reasons which make the NMSSM and its modifications quite attractive. First of all fine tuning which is needed to evade the LEP II Higgs mass bounds is less severe within SUSY models with an extra singlet field as compared with the MSSM [11]. The upper bound on the lightest Higgs boson mass in the singlet extensions of the MSSM was studied recently in [12]. The spectrum of Higgs bosons in the considered models depends on how strongly the Peccei–Quinn symmetry..
is broken in these models \cite{13}–\cite{14}. Another nice feature is related with the electroweak baryogenesis which is easier to achieve in SUSY models with an extra singlet field than in the MSSM due to additional terms in the tree–level potential \cite{15}–\cite{16}. Recently SUSY models with extra singlet fields including their implications for dark matter and neutralino collider searches \cite{17} and neutrino physics \cite{18} have been studied.

However, the NMSSM suffers from a domain wall problem in the early Universe which can be avoided in the Minimal Non–minimal Supersymmetric Standard Model (MNSSM) as will be discussed in section 2.1. In this letter we consider the neutralino sector of the MNSSM. We concentrate on the mass of the lightest neutralino because it can be absolutely stable and therefore may play the role of the cold dark matter. We establish a theoretical upper bound on the lightest neutralino mass in the MNSSM which depends rather strongly on the parameters of the considered model. In the allowed part of the parameter space the mass of the lightest neutralino does not exceed 80 – 85 GeV. This permits to distinguish the MNSSM from the MSSM and other SUSY models with an extra singlet superfield at future colliders. We also find an approximate solution for the lightest neutralino mass. It will enable us to estimate the mass of this particle if charginos and Higgs bosons are discovered in the nearest future. The article is organised as follows. In the next section we define the MNSSM in more detail. In section 3 we examine the allowed range of the lightest neutralino mass in the MNSSM and in section 4 we obtain an approximate solution for its mass. Our results are summarised in section 5.

2. The MNSSM

2.1 Superpotential

As already mentioned the NMSSM itself is not without problems. The vacuum expectation values of the Higgs fields break the $Z_3$ symmetry in the NMSSM. This leads to the formation of domain walls in the early Universe \cite{19} which create unacceptably large anisotropies in the cosmic microwave background radiation \cite{20}. In an attempt to break the $Z_3$ symmetry operators suppressed by powers of the Planck scale could be introduced. But these operators give rise to a quadratically divergent tadpole contribution, which destabilises the mass hierarchy \cite{21}. Dangerous operators can be eliminated if an invariance under $Z_2^R$ or $Z_5^R$ symmetries is imposed \cite{22}–\cite{23}. The linear term $\Lambda$ in the superpotential which is induced in this case by high order operators is too small to upset the mass hierarchy but large enough to prevent the appearance of domain walls. The corresponding simplest extension of the MSSM is the Minimal Non–minimal Supersymmetric Standard Model (MNSSM) \cite{13}, \cite{16}, \cite{23}–\cite{24}. The superpotential of the MNSSM
can be written as

\[ W_{\text{MNSSM}} = \lambda \hat{S}(\hat{H}_d\hat{H}_u) + \xi \hat{S} + W_{\text{MSSM}}(\mu = 0). \]

### 2.2 Neutralino and chargino sectors

The neutralino sector in SUSY models is formed by the superpartners of the neutral gauge and Higgs bosons. Since the sector responsible for electroweak symmetry breaking in the MNSSM contains an extra singlet field the neutralino sector of this model includes one extra component besides the four MSSM ones. This is an additional Higgsino \( \tilde{S} \) (singlino) which is the fermion component of the singlet superfield \( \hat{S} \). After the breakdown of gauge symmetry the Higgsino mass terms in the MNSSM Lagrangian are induced by the trilinear interaction \( \lambda \hat{S}(\hat{H}_d\hat{H}_u) \) in the superpotential (1). As a result their values are determined by the coupling \( \lambda \) and the vacuum expectation values of Higgs fields. The gaugino masses are set by \( M_1 \) and \( M_2 \) which are the \( SU(2) \) and \( U(1)_Y \) soft gaugino mass parameters that break global supersymmetry. In supergravity models with uniform gaugino masses at the Grand Unification scale the renormalisation group flow yields a relationship between \( M_1 \) and \( M_2 \) at the EW scale, i.e. \( M_1 \simeq 0.5M_2 \). The mixing between gauginos and Higgsinos is proportional to the corresponding gauge coupling and the vacuum expectation value of the scalar partner of the considered Higgsino. Thus after the electroweak symmetry breaking the superpartners of the electromagnetically neutral components of the Higgsino doublets \( \tilde{H}_d^0 \) and \( \tilde{H}_u^0 \), of the singlino \( S \) as well as the electromagnetically neutral \( SU(2) \) and \( U(1)_Y \) gauginos (\( \tilde{W}_3 \) and \( \tilde{B} \)) mix forming a 5 \times 5 neutralino mass matrix which in the interaction basis \( (\tilde{B}, \tilde{W}_3, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S}) \) reads

\[
M_{\tilde{\chi}^0} = \begin{pmatrix}
M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta & 0 \\
0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta & 0 \\
-M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu_{\text{eff}} & -\frac{\lambda v}{\sqrt{2}} s_\beta \\
M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu_{\text{eff}} & 0 & -\frac{\lambda v}{\sqrt{2}} c_\beta \\
0 & 0 & -\frac{\lambda v}{\sqrt{2}} s_\beta & -\frac{\lambda v}{\sqrt{2}} c_\beta & 0
\end{pmatrix},
\]

where \( s_W = \sin \theta_W, c_W = \cos \theta_W, s_\beta = \sin \beta, c_\beta = \cos \beta \) and \( \mu_{\text{eff}} = \frac{\lambda s}{\sqrt{2}} \). Here we introduce \( \tan \beta = v_2/v_1 \) and \( v = \sqrt{v_1^2 + v_2^2} = 246 \text{ GeV} \), where \( s, v_1 \) and \( v_2 \) are the vacuum expectation values of \( S, H_d \) and \( H_u \), respectively.

The top–left 4 \times 4 block of the mass matrix (2) contains the neutralino mass matrix of the MSSM where the parameter \( \mu \) is replaced by \( \mu_{\text{eff}} \). From Eq. (2) one can easily see
that the neutralino spectrum in the MNSSM may be parametrised in terms of

\[ \lambda, \quad \mu_{\text{eff}}, \quad \tan \beta, \quad M_1, \quad M_2. \quad (3) \]

The mass parameters \( M_2 \) and \( \mu_{\text{eff}} \) also define the masses of the charginos, the superpartners of the charged gauge and Higgs bosons. Since the SM singlet superfield \( \hat{S} \) is in the MSSM, namely

\[ \text{spectrum. Consequently the chargino mass matrix and its eigenvalues remain the same as in the MSSM, namely} \]

\[ m_{\chi_{1,2}}^2 = \frac{1}{2} \left[ M_2^2 + \mu_{\text{eff}}^2 + 2M_W^2 \pm \sqrt{(M_2^2 + \mu_{\text{eff}}^2 + 2M_W^2)^2 - 4(M_2\mu_{\text{eff}} - M_W^2 \sin 2\beta)^2} \right]. \quad (4) \]

Unsuccessful LEP searches for SUSY particles including data collected at 90 GeV and 209 GeV set a 95% CL lower limit on the chargino mass of about 100 GeV \[25\]. This lower bound constrains the parameter space of the MNSSM restricting the absolute values of the effective \( \mu \)-term and \( M_2 \) from below, i.e. \( |M_2|, |\mu_{\text{eff}}| \geq 90 - 100 \text{GeV.} \]

3. Upper bound on the mass of lightest neutralino

Theoretical restrictions on the masses of the neutralinos cannot be established using directly the neutralino mass matrix because its eigenvalues can in general be complex. In order to find appropriate bounds on these masses it is much more convenient to consider the matrix \( M_{\chi^0_0}M_{\chi^0_0}^\dagger \) whose eigenvalues are positive definite and equal to the absolute values of the neutralino masses squared. In the field basis \( (\tilde{B}, \tilde{W}_3, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S}) \) the hermitian matrix \( M_{\chi^0_0}M_{\chi^0_0}^\dagger \) takes the form:

\[
\begin{pmatrix}
|M_1|^2 + M_2^2 s_W^2 & -M_2^2 c_W s_W & -M_Z s_W A^* & M_Z s_W B^* & 0 \\
-M_2^2 c_W s_W & |M_2|^2 + M_2^2 c_W^2 & M_Z c_W C^* & -M_Z c_W D^* & 0 \\
-M_Z s_W A & M_Z c_W C & |\mu_{\text{eff}}|^2 + \rho^2 & (\nu^2 - M_2^2) c_\beta s_\beta & \nu^* \mu_{\text{eff}} c_\beta \\
M_Z s_W B & -M_Z c_W D & (\nu^2 - M_2^2) c_\beta s_\beta & |\mu_{\text{eff}}|^2 + \sigma^2 & \nu^* \mu_{\text{eff}} s_\beta \\
0 & 0 & \nu^* \mu_{\text{eff}} c_\beta & \nu^* \mu_{\text{eff}} s_\beta & |\nu|^2
\end{pmatrix},
\]

where

\[
\rho^2 = M_2^2 c_\beta^2 + \nu^2 s_\beta^2, \quad \sigma^2 = M_2^2 s_\beta^2 + \nu^2 c_\beta^2, \quad \nu = \frac{\lambda v}{\sqrt{2}},
\]

\[
A = M_1^* c_\beta + \mu_{\text{eff}} s_\beta, \quad C = M_2^* c_\beta + \mu_{\text{eff}} s_\beta, \\
B = M_1^* s_\beta + \mu_{\text{eff}} c_\beta, \quad D = M_2^* s_\beta + \mu_{\text{eff}} c_\beta.
\]
Since the minimal eigenvalue of any hermitian matrix is less than its smallest diagonal element at least one neutralino in the MNSSM is always light, because the mass of the lightest neutralino is limited from above by the bottom–right diagonal entry of matrix (5), i.e. $|m_{\tilde{\chi}_1^0}| \lesssim |\nu|$. Therefore in contrast to the MSSM the lightest neutralino in the MNSSM remains light even when the SUSY breaking scale tends to infinity.

However, the obtained theoretical bound on the lightest neutralino mass can be improved significantly. In order to get a more stringent limit on $|m_{\tilde{\chi}_1^0}|$ one can perform an unitary transformation of matrix (5) so that $M_{\tilde{\chi}^0}M_{\tilde{\chi}^0}^T \rightarrow U M_{\tilde{\chi}^0}M_{\tilde{\chi}^0}^T U^T$, where

$$U = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & -s_\beta & c_\beta & 0 \\
0 & 0 & c_\beta & s_\beta & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}.$$  \quad (6)

As a result we get

$$\begin{pmatrix}
|M_1|^2 + M_Z^2 s_W^2 & -M_Z c_W s_W & M_Z c_W \tilde{A}^* & -M_Z c_W \tilde{B}^* & 0 \\
-M_Z c_W s_W & |M_2|^2 + M_Z^2 c_W^2 & -M_Z c_W \tilde{C}^* & M_Z c_W \tilde{D}^* & 0 \\
M_Z s_W \tilde{A} & -M_Z c_W \tilde{C} & |\mu_{\text{eff}}|^2 + \tilde{\rho}^2 & \frac{(\nu^2 - M_2^2)}{2} \sin 4\beta & 0 \\
-M_Z s_W \tilde{B} & M_Z c_W \tilde{D} & \frac{(\nu^2 - M_2^2)}{2} \sin 4\beta & |\mu_{\text{eff}}|^2 + \tilde{\sigma}^2 & \nu^* \mu_{\text{eff}} \\
0 & 0 & 0 & 0 & |\nu|^2
\end{pmatrix}, \quad (7)
$$

where

$$\tilde{\rho}^2 = M_Z^2 \sin^2 2\beta + |\nu|^2 \cos 2\beta, \quad \tilde{\sigma}^2 = M_Z^2 \cos 2\beta + |\nu|^2 \sin^2 2\beta,$$

$$\tilde{A} = M_1 \sin 2\beta + \mu_{\text{eff}}, \quad \tilde{B} = M_1^* \cos 2\beta,$$

$$\tilde{C} = M_2 \sin 2\beta + \mu_{\text{eff}}, \quad \tilde{D} = M_2^* \cos 2\beta.$$

Since we can always choose the field basis in such a way that the bottom-right $2 \times 2$ block of the mass matrix (7) becomes diagonal its two eigenvalues also restrict the mass interval of the lightest neutralino. In particular, the absolute value of the lightest neutralino mass squared has to be always less than or equal to the minimal eigenvalue $\mu_0^2$ of this submatrix, i.e.

$$|m_{\tilde{\chi}_1^0}|^2 \lesssim \mu_0^2 = \frac{1}{2} \left[ |\mu_{\text{eff}}|^2 + \tilde{\sigma}^2 + |\nu|^2 - \sqrt{\left( |\mu_{\text{eff}}|^2 + \tilde{\sigma}^2 + |\nu|^2 \right)^2 - 4|\nu|^2 \tilde{\sigma}^2} \right]. \quad (8)$$

The value of $\mu_0$ decreases with increasing $|\mu_{\text{eff}}|$, hence reaching its maximum value, i.e. $\mu_0^2 = \min\{\tilde{\sigma}^2, |\nu|^2\}$, when $\mu_{\text{eff}} \to 0$. Taking the LEP bound on $\mu_{\text{eff}}$ into account and also
the theoretical upper bound on the Yukawa coupling $\lambda$ which is caused by the requirement that the perturbation theory is valid up to the Grand Unification scale, requiring $\lambda < 0.7$, we find that $\mu_0^2 < 0.8 M_Z^2$, i.e. $|m_{\chi_1^0}| < 80 - 85 \text{ GeV}$.

Here it is worth to notice that at large values of the effective $\mu$–term the theoretical restriction on $|m_{\chi_1^0}|$ (3) tends to zero independently of the value of $\lambda$. Indeed, for $|\mu_{\text{eff}}|^2 \gg M_Z^2$ we have

$$|m_{\chi_1^0}|^2 \lesssim \frac{|\nu|^2 \tilde{\sigma}^2}{(|\mu_{\text{eff}}|^2 + \tilde{\sigma}^2 + |\nu|^2)}.$$  \hspace{1cm} (9)

Thus in the considered limit the lightest neutralino mass is significantly smaller than $M_Z$ even for large values of $\lambda \sim 0.7$.

4. Approximate solution

4.1 Characteristic equation

The masses of the lightest neutralino can be computed numerically by solving the characteristic equation $\det (M_{\chi^0} - \kappa I) = 0$. In the MNSSM the corresponding characteristic polynomial has degree 5 because the neutralino spectrum is described by a $5 \times 5$ mass matrix. After a few simple algebraic transformations we get

$$\det (M_{\chi^0} - \kappa I) = \left( M_1 M_2 - (M_1 + M_2) \kappa + \kappa^2 \right) \left( \kappa^3 - (\mu_{\text{eff}}^2 + \nu^2) \kappa + \nu^2 \mu_{\text{eff}} \sin 2\beta \right) + M_Z^2 \left( \tilde{M} - \kappa \right) \left( \kappa^2 + \mu_{\text{eff}} \sin 2\beta \kappa - \nu^2 \right) = 0,$$  \hspace{1cm} (10)

where $\tilde{M} = M_1 c_W^2 + M_2 s_W^2$. Although one can find a numerical solution of Eq. (10) for each set of the parameters (3) it is worth to derive either an exact or approximate solution of the characteristic equation (10) to explore the dependence of the lightest neutralino mass on these parameters. Unfortunately, in the general case the exact solution of this equation is very complicated. But in the limit when one of the eigenvalues of the mass matrix (2) goes to zero one can obtain an approximate solution of Eq. (10). Indeed, if $\kappa \to 0$ we can ignore all higher order terms with respect to $\kappa$ in the characteristic equation keeping only the term which is proportional to $\kappa$ and the $\kappa$–independent one. The application of this method is justified in the MNSSM because the mass of the lightest neutralino is limited from above and the upper bound on $|m_{\chi_1^0}|$ tends to zero with increasing $|\mu_{\text{eff}}|$ or decreasing $\lambda$, as argued in the previous section. Actually one can easily check that for a reasonable choice of the parameters ($\mu_{\text{eff}}, M_2 \gtrsim 200 \text{ GeV}, \lambda = 0.1 - 0.7, \tan \beta = 3 - 20$ and $M_2 \sim 0.5M_1$) the lightest neutralino mass is always significantly less than the mass of the second lightest one. Therefore, we can expect that the approximate solution obtained in
this way would describe the exact one with high accuracy in a large part of the parameter space.

However, if we proceed in that way it would mean that we would allow only one neutralino to be light. Then the lightest neutralino mass will be consistently described if the four other neutralino states are considerably heavier than $M_Z$. One can expect that at least three neutralino states which correspond to the superpartners of the neutral components of the Higgs doublets and of the neutral $SU(2)$ gauge boson satisfy this requirement, because $|M_2| < M_Z$ and $|\mu_{eff}| < M_Z$ are ruled out by chargino searches at LEP. But the mass of the neutralino state which is predominantly the superpartner of the $U(1)_Y$ gauge boson is set by $M_1$ which may have a value below $M_Z$. If there are two light states in the neutralino spectrum then the coefficient in front of the linear term with respect to $\kappa$ in Eq. (10) may be relatively small. In this case the term which is proportional to $\kappa^2$ should be taken into account as well in order to obtain a suitable approximate solution for the mass of the lightest and second lightest neutralino. The inclusion of the quadratic term improves the agreement between the numerical and approximate solutions even when the second lightest neutralino is heavier than $M_Z$. Omitting all higher order terms involving $\kappa^n$ with $n > 2$ in the characteristic equation we find

$$A\kappa^2 - B\kappa + C = 0,$$

where

$$A = 1 + \frac{\nu^2 - M_Z^2}{\mu_{eff}^2 + \nu^2} \frac{\mu_{eff} \sin 2\beta}{M_1 + M_2} + \frac{M_Z^2}{\mu_{eff}^2 + \nu^2} \frac{\tilde{M}}{M_1 + M_2},$$

(12)

$$B = \frac{M_1 M_2}{M_1 + M_2} + \left( \frac{\nu^2}{\mu_{eff}^2 + \nu^2} - \frac{M_Z^2}{\mu_{eff}^2 + \nu^2} \frac{\tilde{M}}{M_1 + M_2} \right) \mu_{eff} \sin 2\beta - \frac{M_Z^2 \nu^2}{(M_1 + M_2)(\mu_{eff}^2 + \nu^2)},$$

(13)

$$C = \frac{\nu^2}{\mu_{eff}^2 + \nu^2} \left( \frac{M_1 M_2}{M_1 + M_2} \mu_{eff} \sin 2\beta - \frac{\tilde{M}}{M_1 + M_2} M_Z^2 \right).$$

(14)

In order to reduce the characteristic equation (10) to Eq. (11) we have divided both parts of this equation by $(\mu_{eff}^2 + \nu^2)(M_1 + M_2)$. One can simplify Eq. (11) even further taking into account that the second and last terms in Eq. (12) can be neglected since they are much smaller than unity in most of the phenomenologically allowed part of the MNSSM parameter space. Then the mass of the lightest neutralino can be approximated by

$$|m_{\chi_1^0}| = \text{Min} \left\{ \frac{1}{2} \left| B - \sqrt{B^2 - 4C} \right|, \frac{1}{2} \left| B + \sqrt{B^2 - 4C} \right| \right\}.$$

(15)
4.2 Numerical results and discussion

In Fig. 1 (a)–(c) we plot the numerical and approximate solutions for the lightest neutralino mass as a function of $\mu_{\text{eff}}$, $M_2$ and $\tan \beta$. For simplicity we assume that all parameters appearing in the neutralino mass matrix are real. We also choose $M_1 = 0.5 M_2$ and $\lambda = 0.7$ which is the largest possible value of $\lambda$ that does not spoil the validity of perturbation theory up to the GUT scale. From Fig. 1 (a)–(b) it becomes clear that $|m_{\chi_1^0}|$ attains its maximum at certain values of $M_2$ and $\mu_{\text{eff}}$. The corresponding maximum value of $|m_{\chi_1^0}|$ is always less than the upper bound on the lightest neutralino mass derived in the previous section.

As follows from Fig. 1 (a)–(c) the approximate solution (15) describes the numerical one with relatively high accuracy even for small $M_2 \simeq \mu_{\text{eff}} \simeq 200$ GeV. One can also see that the mass of the lightest neutralino may be very small or even zero for large values $\lambda \sim 0.7$. This happens because the determinant of the neutralino mass matrix (2) is zero for a certain relation between the parameters (3), namely, when

$$M_1 M_2 \mu_{\text{eff}} \sin 2 \beta = M M^2_Z.$$  \hspace{1cm} (16)

The condition (16) is fulfilled automatically when $M_1 \sim M_2 \to 0$. It means that the lightest neutralino mass always vanishes when $M_1$ and $M_2$ go to zero. At the same time condition (16) can be satisfied at non–zero values of the soft gaugino masses. This can be seen in Fig. 1 (a)–(c). Since in Fig. 1 (a) and (c) we plot $|m_{\chi_1^0}|$ for non–zero values of the soft gaugino masses the lightest neutralino mass vanishes only once. At the same time in Fig. 1 (b) where we examine the dependence of $|m_{\chi_1^0}|$ on $M_2$ the mass of the lightest neutralino vanishes twice: once for $M_2 = 0$ and once for a non–zero value of $M_2$ that obeys Eq. (16). In the approximate solution (15) the vanishing of the mass of the lightest neutralino corresponds to the vanishing of $C$, which is proportional to the determinant of the neutralino mass matrix (2), i.e. $C = \frac{\det M_{\tilde{\chi}^0}}{(\mu_{\text{eff}}^2 + \nu^2)(M_1 + M_2)}$.

Figure 2 shows the contours of the difference between the exact and the approximate solution (15) in the $\mu_{\text{eff}}$-$M_2$ parameter plane. It can be seen that this difference is smaller than 1 GeV in most of the phenomenologically allowed parameter space, in large regions even smaller than 0.1 GeV.

Finally we would like to add that the two solutions of the reduced form of the characteristic equation (11) describe with good accuracy not only the lightest neutralino mass but also the mass of the second lightest one if the second lightest neutralino is considerably lighter than the other states. Such a pattern of the neutralino spectrum is realised, for example, when $M_1 \ll M_2, \mu_{\text{eff}}$. Although it is rather difficult to find any justification of this scenario within SUSY GUT or string inspired models it is not excluded by either
Figure 1: Mass of the lightest neutralino in the MNSSM (solid), its upper bound according to Eq. (8) (dashed) and its approximate solution according to Eq. (15) (dotted) for $\lambda = 0.7$, $M_1 = 0.5M_2$ and (a) $\tan\beta = 5$, $M_2 = 200$ GeV, (b) $\tan\beta = 5$, $\mu_{\text{eff}} = 200$ GeV, (c) $M_2 = \mu_{\text{eff}} = 200$ GeV.
Figure 2: Contours of the absolute value of the difference between the mass of the lightest neutralino in the MNSSM and its approximate solution (15) in [GeV] for $\lambda = 0.7$, $M_1 = 0.5 M_2$ and (a) $\tan \beta = 5$, (b) $\tan \beta = 30$. In the shaded region is $m_{\tilde{\chi}_1^\pm} < 100$ GeV.

LEP or Tevatron searches. If $\nu, M_\chi \lesssim M_1$ in the considered limit then the mass of the second lightest neutralino can be approximated by

$$|m_{\chi_2^0}| \approx \left| \frac{M_1 M_2}{M_1 + M_2} \right|.$$  

(17)

In this case the lightest and the second lightest neutralino are predominantly singlino and the superpartner of the $U(1)_Y$ gauge boson.

4.3 Approximate solution for decoupling limit

With increasing effective $\mu$–term and soft gaugino masses the lightest neutralino mass decreases (see Fig. 1 (a)–(b)). From Fig. 1 (a)–(c) it becomes clear that the difference between the numerical and approximate solutions reduces when $\mu_{\text{eff}}, M_1$ and $M_2$ grow. If either $\mu_{\text{eff}}$ or $M_1$ and $M_2$ are much larger than $M_Z$, $B^2 \gg C$ and the approximate solution for the lightest neutralino mass can be presented in a more simple form:

$$|m_{\chi_1^0}| \approx \frac{C}{B} \approx \frac{|\mu_{\text{eff}}|^2 \sin 2\beta}{\mu_{\text{eff}}^2 + \nu^2}.$$  

(18)

According to Eq.(18) the mass of the lightest neutralino is inversely proportional to the effective $\mu$–term. It vanishes when $\lambda$ tends to zero. In the limit $\lambda \to 0$ the equations for the extrema of the Higgs boson effective potential which determines the position of the physical vacuum imply that the vacuum expectation value of the singlet field rises as
$M_Z/\lambda$. In other words the correct breakdown of electroweak symmetry breaking requires $\mu_{\text{eff}}$ to remain constant when $\lambda$ goes to zero. As a result from Eq. (18) it follows that the mass of the lightest neutralino is proportional to $\lambda^2$ at small values of $\lambda$. At this point the approximate solution (18) improves the theoretical restriction on the lightest neutralino mass derived in the previous section. This is because at small values of $\lambda$ the upper bound (9) is proportional to $\mu_{\text{eff}}$.

From Eq. (18) one can also see that the mass of the lightest neutralino decreases when $\tan \beta$ grows. The numerical results of our analysis summarised in Fig. 1 (a)–(c) confirm that $|m_{\chi_1^0}|$ becomes smaller when $\tan \beta$ raises from 3 to 10. However, if $\tan \beta \gtrsim \zeta = \frac{2M_1M_2\mu_{\text{eff}}}{M M_Z^2}$, Eq. (18) does not provide an appropriate description of the lightest neutralino mass. Indeed, in accordance with Eq. (18) the mass of the lightest neutralino vanishes at large values of $\tan \beta$ while Fig. 1 (c) demonstrates that $|m_{\chi_1^0}|$ approaches to some constant non–zero value with raising of $\tan \beta$. More accurate consideration of the approximate solution (15) allows to reproduce the asymptotic behaviour of the lightest neutralino mass at $\mu_{\text{eff}}, M_2, M_1 \gg M_Z$ and at very large $\tan \beta \gg \zeta$. It is given by

$$|m_{\chi_1^0}| \to \frac{\nu^2 M_Z^2}{\mu^2 + \nu^2} \left| \frac{\tilde{M}}{M_1 M_2} \right|.$$  

(19)

So once again the approximate solution (15) improves the theoretical restriction on the lightest neutralino mass because the upper limit (8)–(9) on $|m_{\chi_1^0}|$ obtained before depends rather weakly on $\tan \beta$.

5. Conclusions

In this letter we have examined the theoretical restrictions on the lightest neutralino mass within the Minimal Non–minimal Supersymmetric Standard Model. In order to derive the appropriate upper bound we consider the hermitian matrix $M_{\tilde{\chi}^0_1} M_{\tilde{\chi}^0_1}^\dagger$ where $M_{\tilde{\chi}^0_1}$ is the neutralino mass matrix. The eigenvalues of this matrix are the absolute values of the neutralino masses squared. Therefore all eigenvalues of $M_{\tilde{\chi}^0_1} M_{\tilde{\chi}^0_1}^\dagger$ are positive definite. Using the theorem that the smallest diagonal element of a hermitian matrix is always larger than the minimal eigenvalue of this matrix we establish an upper bound on the mass of the lightest neutralino $m_{\chi_1^0}$ in the MNSSM. The direct application of this theorem leads to the conclusion that $|m_{\chi_1^0}|$ has to be always less than $|\lambda|v/\sqrt{2}$. A more stringent limit on the lightest neutralino mass can be obtained by applying an unitary transformation to the matrix $M_{\tilde{\chi}^0_1} M_{\tilde{\chi}^0_1}^\dagger$. As a result we have found that $|m_{\chi_1^0}|$ does not exceed 80 − 85 GeV. The corresponding upper bound depends rather strongly on the effective $\mu$–term $|\mu_{\text{eff}}|$.
which is generated after the electroweak symmetry breaking. At large values of $|\mu_{\text{eff}}|$ the upper limit on $|m_{\chi_1^0}|$ goes to zero so that the mass interval of the lightest neutralino shrinks drastically.

Assuming that $|m_{\chi_1^0}|$ is considerably less than the masses of the other neutralino states we have derived an approximate solution for the lightest neutralino mass. The obtained solution describes the numerical one with high accuracy in a large region of the phenomenologically allowed part of the MNSSM parameter space. Our numerical analysis and analytic considerations show that $m_{\chi_1^0}$ decreases with increasing $\tan \beta$ and decreasing coupling $\lambda$. At small values of $\lambda$ the mass of the lightest neutralino is proportional to $\lambda^2$. The lightest neutralino mass also decreases with increasing $\mu_{\text{eff}}$, $M_1$, and $M_2$. We have argued that at large values of the effective $\mu$–term $m_{\chi_1^0}$ is inversely proportional to $\mu_{\text{eff}}$. In the allowed part of the parameter space the lightest neutralino is predominantly singlino that makes its direct observation at future colliders challenging. In forthcoming publications we plan to consider the potential discovery of such a neutralino at the LHC and ILC.

In summary, the obtained theoretical restriction on the lightest neutralino mass allows to discriminate the MNSSM from other SUSY models where the mass of the lightest neutralino is not limited from above. If no light neutralino is detected at future colliders the MNSSM will be ruled out.

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