THE NUCLEON-NUCLEON POTENTIAL
IN THE CHROMO-DIELECTRIC SOLITON MODEL: STATICS

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Abstract

We study the nucleon-nucleon interaction in the framework of the chromo-dielectric soliton model (CDM). Here, the long-range parts of the nonabelian gluon self-interactions are assumed to give rise to a color-dielectric function which is parameterized in terms of an effective scalar background field. The six-quark system is confined in a deformed mean field through an effective non-linear interaction between the quarks and the scalar field. The CDM is covariant, respects chiral invariance, leads to absolute color confinement and is free of the spurious long range Van der Waals forces which trouble non-relativistic investigations employing a confining potential. Six-quark molecular-type configurations are generated as a function of deformation and their energies are evaluated in a coupled channel analysis. By using molecular states instead of cluster model wave functions, all important six-quark configurations are properly taken into account. The corresponding Hamiltonian includes the effective interaction between the quarks and the scalar background field and quark-quark interactions generated through one gluon exchange treated in
Coulomb gauge. When evaluating the gluonic propagators, the inhomogeneity and deformation of the dielectric medium are taken into account. Results for the adiabatic nucleon-nucleon potential are presented, and the various contributions are discussed. Finally, an outlook is given on how, in the next stage of our investigation, the dynamical effects will be incorporated by employing the generator coordinate method.

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I. INTRODUCTION

The $N-N$ interaction is one of the most basic problems of nuclear physics. There exists extensive experimental information – from $N-N$ scattering data and the properties of the deuteron – but no single theoretical picture seems to be able to describe the relevant physics for all internuclear distances. In $N-N$ phenomenology, both relativistic and non-relativistic, one describes the nucleons as elementary particles interacting through a two-body potential which is either local or includes some nonlocality through momentum-dependence in the interaction. The general features of that potential, i.e. the short-distance core and the long range attraction, have been known for over forty years.

Already in 1935, Yukawa suggested that the finite range attraction was due to the exchange of an intermediate mass, strongly-interacting particle, the subsequently discovered pion. This led to the development of meson field-theoretic models which today form the most accurate phenomenological description of the $N-N$ interaction (see ref. [2] for an excellent overview). In these models, one treats the nucleons as elementary particles with an empirical form factor, and their interactions are usually carried through one boson exchange (OBE) plus two pion exchange (TPE), where the TPE is frequently simulated by a (fictitious) sigma meson.

Within these descriptions, the long range ($r \gtrsim 1.5$ fm) part of the $N-N$ interaction is controlled by one pion exchange, while the intermediate range ($0.5$ fm $\lesssim r \lesssim 1.5$ fm) attraction is dominated by OBE and TPE. The short range ($r \lesssim 0.5$ fm) repulsion is the “mystery” region in such descriptions. It has been described by hard or soft cores, or form factors, both of the order of 0.5-0.8 fm, or by the exchange of vector mesons which have a range of $1/m_\omega \approx 0.2$ fm, a clear inconsistency.

The advent of QCD and quark models has lifted the veil of mystery from the short range $N-N$ problem exposing a new level of simplicity. However, the system is no longer just a two-body, but at least a six-body, entity and more properly a field theoretical problem. The quark core of nucleons is of the order of 0.7 fm (an rms radius of 0.5 fm) and one expects the quark substructure to be effective within a range of $N-N$ separations of up to about 1 fm.

A description of the nucleon-nucleon interaction within the framework of quark degrees
of freedom has been the subject of much research. The ideal venture would be a lattice gauge theory calculation [3], but we are quite far from that stage, and therefore we have to rely on modeling. We mention, non-exhaustively, several different avenues which have been explored in that context: non-relativistic constituent quark (potential) models [4], relativistic current quark models as, for example, the MIT bag model [5] and various soliton models [6], string models and the topological Skyrme model [7,8]. There are many varieties under each category, and we will not attempt to review them all here, but rather recommend the reader to the articles by Oka and Yazaki [9], Myhrer and Wroldsen [10] or Shimizu [11].

In addition to the various Ansätze which have been employed to model quark-quark interactions, one has to further distinguish between static and dynamical calculations. In the static calculations, a local $N-N$ potential is obtained in Born-Oppenheimer [12] (adiabatic) approximation from the energy difference of a deformed six quark bag and two separated nucleons. Non-adiabatic calculations yield a non-local $N-N$ interaction through a consideration of the dynamics involved. In the latter category, usually the resonating group [13] or the generator coordinate method [14] are applied.

Quark models hold promise to give a good description for short and intermediate range, but beyond, say, 1 fm the interaction, although in principle describable in terms of quarks, is much more easily represented by mesonic models, with the nucleonic substructure giving rise to form factors for the meson-nucleon couplings.

The ultimate object of our study is not only to reproduce the two-body data, such as $N-N$ phase shifts and bound state properties of the deuteron, but also to quantify the quark substructure of nuclei. With respect to the latter, we will describe the collision process as an act of fusion followed by a separation into three-quark clusters, and this will be used in conjunction with the Independent Pair Model of nuclei to obtain quark structure functions. The main aspects of our current project can be described as follows:

1) We employ the chromo-dielectric soliton model [15,16], which respects covariance, contains absolute color confinement and is free of the color Van der Waals problem [17] (which is inherent to most non-relativistic calculations). In addition, one gluon exchange is evaluated with a confined gluonic propagator.

2) The six-quark wave function is expanded in terms of “molecular” states [18], including
all configurations based on the two lowest spatial single particle states. This allows for inclusion of basis states normally omitted in cluster model calculations, and which have been demonstrated to be important in decreasing the upper bound in variational calculations [19,20].

3) Dynamics will be handled through the generator coordinate method [21], which leads to a set of coupled integral equations. It has been shown [3] that a significant part of the short-range repulsion is due to dynamics, and the absence of a repulsive core in some early calculations is now seen as an artifact of the adiabatic approximation [22,23]. In addition, the effective interaction is non-local in terms of the $N-N$ separation parameter.

4) In order to reproduce two-body properties, we will attach the interaction we derive to a phenomenological local OBE potential beyond a certain internuclear distance. We could, however, also consider extending our calculation more deeply into the OBE intermediate range region by including quantum surface fluctuations and introducing configurations of the form $q^7\bar{q}$ in addition to our $q^6$ basis states.

In this first of a series of papers, we are mostly concerned with the introduction of the model and a presentation of the formalism we use. Therefore, we restrict ourselves to an adiabatic, or static, approximation. We calculate $<\alpha|H|\alpha>$, where $\alpha$ is the separation or deformation parameter, including diagonalization with respect to the various six-quark configurations. We defer steps (3) and (4) to subsequent papers in this series [24].

The outline of this work is as follows. In Sec. II we review the chromo-dielectric soliton model, which in Sec. III is used to generate single quark wave functions by means of a constrained mean field calculation. Sec. IV is devoted to the “molecular” states which form the basis for the six quark configurations we consider. Sec. V describes the treatment of one gluon exchange and in Sec. VI we present the results of our numerical calculations. Finally, we summarize, conclude and give an outlook on our future work in Sec. VII.

II. THE MODEL

The chromo-dielectric model [15,16,25] is an evolution of the Friedberg-Lee non-topological soliton model [26]. The Lagrangian is the same as the fundamental QCD Lagrangian, supplemented by a scalar field $\sigma$ which is assumed to simulate the gluonic condensate and
other scalar structures which inhabit the complicated physical vacuum. It is assumed that
the scalar field, which has a non-vanishing vacuum expectation value $\sigma_v$, parameterizes the
bulk of the non-perturbative effects which arise due to the non-linearity of QCD. It fur-
thermore governs the chromo-dielectric properties of the medium. The model Lagrangian is
covariant and, for massless quarks, satisfies chiral symmetry. It differs in that respect from
most effective quark models – such as the MIT [27], Friedberg-Lee, or Nielsen-Patkos [28]
models – which explicitly violate chiral symmetry through the interaction of the quarks with
some scalar field. Although the model has its basis in QCD, we regard it as phenomenolog-
ical.

The extra degrees of freedom introduced by the scalar field are redundant. In order to
avoid double counting we do not include diagrams which correspond to structures with the
quantum numbers of the $\sigma$ field. Since the model parameters are readjusted at each level of
approximation to fit key physical data, one might hope that as the level of sophistication of
the calculations is increased, one would find a decoupling of the $\sigma$ degrees of freedom and
would thus be left with pure QCD. But, we are currently far from that stage of sophistication.

We treat the gluons in the Abelian approximation, which is consistent with one gluon
exchange. The primary role of the $\sigma$ field is then to mediate the chromo-dielectric func-
tion, $\kappa(\sigma)$, which, in turn, is designed to guarantee absolute color confinement. The scalar
field, therefore, not only simulates the non-linear and non-perturbative effects, but it also
parameterizes the color-dielectric properties of the space-time continuum.

The model Lagrangian density is given by

$$\mathcal{L} = \mathcal{L}_q + \mathcal{L}_\sigma + \mathcal{L}_G,$$  \hspace{1cm} (1)

with

$$\mathcal{L}_q = \overline{\psi} \left( i\gamma^\mu D_\mu - m_q \right) \psi,$$  \hspace{1cm} (2a)

$$\mathcal{L}_\sigma = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma),$$  \hspace{1cm} (2b)

$$\mathcal{L}_G = -\frac{1}{4} \kappa(\sigma) F_{\mu\nu}^c F^{\mu\nu c}.$$  \hspace{1cm} (2c)

Here, $m_q$ is the current quark mass matrix and, since there is no direct coupling between
the quarks and the $\sigma$-field, the Lagrangian is chirally invariant for massless quarks. For the
rest of this investigation we thus set $m_q \equiv 0$. The gauge field tensor is given by
\[ F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu , \]  

where the \( f^{abc} \) are the SU(3) structure constants. \( U(\sigma) \) is the self-interaction energy of the scalar field and is taken to be of the form

\[ U(\sigma) = \frac{a}{2!} \sigma^2 + \frac{b}{3!} \sigma^3 + \frac{c}{4!} \sigma^4 + B . \]  

The quartic form of \( U(\sigma) \) would assure renormalizability \textit{if} \( \kappa \) \textit{were} a constant. Although this is convenient, it is not demanded since we are already dealing with an effective theory. The bag pressure \( B \) (which corresponds to the “bag constant” of the MIT model) is a function of the parameters \( a, b \) and \( c \) and is chosen such that \( U(\sigma) \) has a minimum and vanishes at the value \( \sigma = \sigma_v \), i.e. \( U(\sigma_v) = U'(\sigma_v) = 0 \). We define

\[ U''(\sigma_v) \equiv m_{GB}^2 , \]  

where \( m_{GB} \) is identified with the mass of the lowest \( 0^{++} \) glueball state. In order to guarantee absolute color confinement and a regular behavior as \( \sigma \rightarrow \sigma_v \), the dielectric function must further satisfy

\[ \kappa(\sigma_v) = \kappa'(\sigma_v) = \kappa'(0) = 0 \quad \text{and} \quad \kappa(0) = 1 . \]  

We choose the form (with \( x = \sigma/\sigma_v \))

\[ \kappa(\sigma) = 1 + \theta(x)x^n(nx - (n + 1)) , \]  

and set \( n = 2 \) for our present investigation.

Although the quarks are massless and there is no direct quark-sigma coupling, they still acquire a self-energy – and hence an effective mass – through their interactions with the gluon field. Furthermore, the gluonic propagator depends on \( \sigma \) through \( \kappa(\sigma) \) and is thus also “confined”.

This confinement mechanism has been studied for two particular cases: (1) a uniform dielectric function \[16\] and (2) a cavity model in which \( \kappa \) is unity at the center and goes to zero outside the bag \[25\].

In the preliminary discussion \[15\] employing a uniform dielectric, confinement was exhibited explicitly from a calculation of the quark self-energy in the limit of very heavy, i.e.
fixed, quarks. In calculations with zero mass quarks [16], it was shown that the quarks acquire a “dynamic” mass below some critical \( \kappa_c \). The corresponding self-energy displays an asymptotic form which increases as \( \kappa \) decreases and becomes infinite as \( \kappa \rightarrow 0 \). This corresponds to a realization of spatial confinement, since this mass is “color blind.” It was shown that a massless Goldstone pion arises as a direct consequence of spatial confinement, and that the successful cloudy bag model emerges quite naturally in this formalism. The resulting quark propagator exhibits momentum dependence and hence non-locality.

Color confinement, on the other hand, arises through the enclosure of the quark cavity by the physical vacuum where the dielectric function goes to zero. The gluon field energy is thus infinite if the quark structure in the cavity is not in a color singlet state. Note that \( \kappa \rightarrow 0 \) also ensures that there are no spurious color Van der Waals forces [17] since then the stress-energy tensor vanishes in the vacuum. In a study [25] employing a cavity model, the non-local quark propagator was calculated by solving the Schwinger-Dyson equation.

We utilize the results of these studies and introduce an effective quark mass which is designed to simulate spatial confinement. We thus add an effective coupling between the quarks and the scalar field. This destroys chiral invariance (which, in turn, is restored by the emergence of the Goldstone pion). The form of the added term is

\[
\mathcal{L}_{q\sigma} = - g_{\text{eff}}(\sigma) \bar{\psi} \psi ,
\]

with

\[
g_{\text{eff}}(\sigma) = g_0 \sigma_v \left( \frac{1}{\kappa(\sigma)} - 1 \right)
\]

motivated by comparison with the results of ref. [16]. It turns out, however, that the exact form of that coupling term does not seem to be too important.

Color confinement is realized by incorporating one gluon exchange (OGE) matrix elements both for the quark self-energy and for the mutual interaction terms. Since we work in the Coulomb (or transverse) gauge, where \( \nabla \cdot (\kappa \mathbf{A}) = 0 \), the time component \( A_0 \), which is responsible for color confinement, is instantaneous and frequency-independent. The effective Hamiltonian can be written as

\[
H = \int d^3 \mathbf{r} \mathcal{H}(\mathbf{r}) ,
\]
where
\[ \mathcal{H}(\mathbf{r}) = \psi^\dagger(\mathbf{r}) \left\{ \alpha \cdot \left[ \mathbf{p} - \frac{4}{3} g_s \mathbf{X}^c(\mathbf{r}) \right] + \beta g_{eff}(\sigma(\mathbf{r})) + \frac{4}{3} g_s \mathbf{A}^c_0(\mathbf{r}) \right\} \psi(\mathbf{r}) \]
\[ + \frac{4}{\pi} (\dot{\sigma}(\mathbf{r}))^2 + \frac{1}{2} |\nabla \sigma(\mathbf{r})|^2 + U(\sigma), \]
and where the $\mathbf{X}^c$ are Gell-Mann’s color $SU(3)$ matrices normalized so that $\sum_c \mathbf{X}^c \cdot \mathbf{X}^c = 16/3$. This must be supplemented by the field equations for $\mathbf{A}^c_0$ and $\mathbf{A}^c$, to be given in Sec. V.

In order to fit the five parameters of the chromo-dielectric soliton model, $a$, $b$ and $c$ in $U(\sigma)$, $g_0$ in $g_{eff}$ and the strong coupling constant $\alpha_s = g_s^2/4\pi$, we construct self-consistent solutions for the nucleon. Employing the coherent state approximation, we treat the scalar field classically. In addition, gluonic terms are dropped when determining the quark wave function, $\psi(\mathbf{r})$, or the scalar field, $\sigma(\mathbf{r})$. Making the Ansatz (see ref. [29] for more details) that each quark is in an $s$-state, we have
\[ \psi_m(\mathbf{r}) = \left( \begin{array}{c} \chi_m \\ \sigma \cdot \mathbf{r} \end{array} \right) \]
for the quark wave function, where $\chi_m$ is a Pauli spinor. This gives the mean field equations of motion
\[ \frac{du}{dr} = - (g_{eff}(\sigma) + \epsilon) \cdot v, \]
\[ \frac{dv}{dr} = - 2v - (g_{eff}(\sigma) - \epsilon) \cdot u, \]
\[ \frac{d^2 \sigma}{dr^2} + 2 \frac{d\sigma}{rdr} = \frac{dU(\sigma)}{d\sigma} + n_q \frac{dg_{eff}(\sigma)}{d\sigma}(u^2 - v^2), \]
which are solved self-consistently, with $n_q = 3$ for the nucleon. This yields the solitonic quark wave function and the corresponding scalar field, from which then the quantities of physical interest are calculated while incorporating certain approximate recoil corrections [30]; the latter can be compared with methods using projection and boost [31]. Of the five parameters involved – $a$, $b$, $c$, $g_0$ and $\alpha_s$ – three are fixed by fitting the nucleon mass, the $\Delta$ mass and the proton rms charge radius (0.83 fm). This leaves two free parameters, for which we choose the dimensionless quantities $f = b^2/ac$ and $c$.

In Fig. 1 we show the self-consistently determined scalar field, $\sigma/\sigma_v$, and the corresponding self-interaction potential, $U(\sigma)$, for three different parameter sets. The solid line
corresponds to \( f = 3 \), for which the bag pressure \( B \) vanishes and which creates hard bags with a thin surface. The dashed line corresponds to \( f = \infty \), for which the quadratic term in \( U(\sigma) \) disappears and \( \sigma = 0 \) turns from a second minimum to an inflection point and which generates soft bags with a thick surface. The dot-dashed line corresponds to \( f = 3.2 \). For all sets, \( c \) is kept at a value of 10000, which yields a reasonable figure for the glueball mass \((1.5 \text{ GeV} \lesssim m_{GB} \lesssim 2.0 \text{ GeV})\). In general, for increasing \( f \) or \( c \), the bag gets softer, the glueball mass and the bag pressure increase, the agreement in the axial vector coupling \( g_A \) – which is inherently too small by about 10 percent – improves, but the proton’s magnetic moment \( \mu_p \) – which is also consistently underestimated – grows to differ more from its experimental value. Better agreement with experiment can be achieved by using projection and boost \[^{[31]}\]. Table I gives an overview over the corresponding quantities for the various parameter sets under consideration.

As already remarked, we work in the one gluon exchange approximation. Since for both the nucleon and the \( \Delta \), all of the quarks are in the same spatial state and the entire system is a color singlet, the total – mutual plus self – color-electrostatic energy is zero. The color-magnetic interaction, on the other hand, is responsible for the \( N-\Delta \) mass splitting. In general, part of this energy difference should be attributed to the different pion dressing of the nucleon and the \( \Delta \), but since at our present level of approximation the soliton does not contain any pionic effects, we disregard this contribution. As usual \[^{[32]}\], the magnetic self-energy contribution is neglected here and no intermediate excitations of the quarks into higher spatial orbitals are taken into account. The evaluation of the \( N-\Delta \) mass splitting allows the adjustment of the strong coupling constant, \( \alpha_s \). Hereby, a “confined” gluonic propagator is used, i.e. the explicit dependence of the gluonic field equations on the dielectric function \( \kappa(\sigma) \) is taken into account. For details see Sec. V and ref. \[^{[32]}\]. The results are given in table I. For comparison, we also show the values of \( \alpha_s \) which we obtain by using a free propagator, i.e. by setting \( \kappa \equiv 1 \).

Another quantity which depends on the gluonic interactions is the string tension \( \theta \). Phenomenologically, it is obtained from fits to heavy quarkonium spectra as the coefficient of the linear term in non-relativistic \( q\bar{q} \) potentials \((\theta_{\exp} \approx 913 \text{ MeV} \[^{[33]}\])\). In the soliton bag model, it can be calculated by considering a flux tube, which is a cylindrical structure of scalar and
gluon fields generated by a quark and an antiquark pair of color charges at infinitely large separation. In particular, by minimizing the energy of such a system, we obtain the spatial form of the scalar and color-electric fields in the tube, and the string tension. Alternatively, we can adjust the parameters of the model to obtain the “experimental” string tension. Results for $\alpha_s$ obtained in this way are also given in Table I. For further details we refer the reader to ref. [32].

III. CONSTRAINED MEAN FIELD APPROXIMATION

The starting point of any evaluation of the multi-dimensional potential energy surface and the input to any calculation employing the generator coordinate method [21] is a wave function which is characterized by a set of deformation parameters, which in the following will be denoted collectively as $\alpha$. In our case, the $\alpha$ describe the static configuration of a system of six quarks and the corresponding deformed scalar $\sigma$-field, which is treated quantum mechanically through the coherent state approach. Consideration of various possible six-quark configurations (see Sec. IV for more details) allows for each deformation the construction of a complete basis, which is indicated by a set of state vectors $|\alpha,n>$. In general, these state vectors are generated by means of a constrained mean field calculation, i.e. by extremizing the expectation value of the total Hamiltonian, as given by $<\alpha|H|\alpha>$, with respect to a variational wave function for the quarks and a coherent state for the scalar field, subject to the constraints

$$<\alpha|Q|\alpha>=Q_0,$$  \hspace{1cm} (14)

where the $Q$ are some moments of the quark distribution as defined through

$$Q = \int \bar{\psi}(r) q(r) \psi(r) d^3r,$$  \hspace{1cm} (15)

for some chosen set $q(r)$. In the above, we have dropped the label “$n$” for simplicity, and we here limit ourselves to a one-dimensional parameter space, i.e. we consider zero-impact trajectories (or central collisions) only. The constrained mean field equations then assume the form

11
\[
\begin{align*}
\{ & \alpha \cdot p + \beta [g_{\text{eff}}(\sigma) - \lambda q(r)] - \epsilon_\mu \} \psi_\mu = 0, \\
-\nabla^2 \sigma + & \frac{dU(\sigma)}{d\sigma} + \frac{dg_{\text{eff}}(\sigma)}{d\sigma} < \overline{\psi} \psi > = 0.
\end{align*}
\]

where \(< \overline{\psi} \psi >\) is the six-quark scalar density and \(\lambda\) is a Lagrange multiplier imposing the subsidiary condition. All gluonic terms have been dropped from the above equations, and the label \(\mu\) identifies the different single-particle quark states.

Instead of specifying the constraint function \(q(r)\) explicitly and solving the above pair of equations simultaneously and self-consistently, it is easier and actually more physical to specify the function

\[
[g_{\text{eff}}(\sigma(r)) - \lambda q(r)] \equiv V_\alpha(r) \equiv g_{\text{eff}}(\sigma_\alpha(r))
\]

for each value of the collective deformation parameters \(\alpha\). \(V_\alpha(r)\) plays the role of an external potential generating the wave function for the quarks. It is expressed in terms of a function of some scalar field with a prescribed deformation, \(\sigma_\alpha(r)\). In employing this scalar field with a particular deformation, we explicitly give up self-consistency between the quark wave functions and the scalar field. However, this lack of self-consistency is necessarily inherent to any constrained calculation.

We construct the field \(\sigma_\alpha(r)\) by folding a Yukawa shaped smoothing function with the union (for \(\alpha > 0\)) or the intersection (for \(\alpha < 0\)) of two spheres \([\mathbb{F}]\), whose centers are separated by a distance \(|\alpha|\), i.e.

\[
\sigma_\alpha(r) = \sigma_v - \sigma_0 \int T_\alpha(r') f(|r - r'|) d^3r',
\]

\[
T_\alpha(r) = \begin{cases} 
\theta(R(\alpha) - |r - \hat{z}\alpha/2|) & \text{for } z \geq 0, \\
\theta(R(\alpha) - |r + \hat{z}\alpha/2|) & \text{for } z < 0,
\end{cases}
\]

\[
f(r) = \frac{\Gamma^2}{4\pi} \frac{e^{-\Gamma r}}{r},
\]

where \(\sigma_v\) is the vacuum expectation value of the scalar field and where \(\alpha > 0\) corresponds to prolate deformations and \(\alpha < 0\) to oblate deformations.

The geometrical parameters, \(R, \Gamma\) and \(\sigma_0\), are determined from the scalar field \(\sigma_N(r)\) of a self-consistent solution for the nucleon, as discussed in the last section, such that the corresponding solitonic scalar field of two free nucleons is well approximated at asymptotic deformations, i.e.
\[ \sigma_{\alpha \to \infty}(r) \rightarrow \sigma_N(r - \hat{z} \alpha/2) + \sigma_N(r + \hat{z} \alpha/2) - \sigma_v. \] (22)

In order to select a definite path in configuration space, as a first approximation the field strength \( \sigma_0 \) and the surface parameter \( \Gamma \) are kept constant and the radius \( R = R(\alpha) \) is varied in such a way that the volume of the scalar field of the six-quark cavity is independent of the deformation parameter and remains fixed at the value of two nucleonic volumes \([1]\). In Fig. 2, we show the field \( \sigma_\alpha(r) \) obtained in that manner for four different \( \alpha \)'s.

Selecting this particular path in the geometrical configuration space spanned by \( R, \Gamma \) and \( \sigma_0 \) is equivalent to treating the scalar \( \sigma \)-field as an incompressible liquid. In order to check this approximation, we constructed self-consistent stationary eigen-states of the total Hamiltonian for a spherically symmetric scalar field, which corresponds to united bags, i.e. to \( \alpha = 0 \). Our findings, which will be discussed in Sec. VI, show that the quality of the “constant volume” approximation is quite remarkable.

The potential \( V_\alpha(r) \) serves to generate a set of single-particle quark states, \( \psi_\alpha^n(r) \), which are determined from the eigenvalue equation (16). Here, we limit ourselves to the lowest states of positive and negative parity, denoted by \( |\sigma> \) and \( |\pi> \) respectively, and to values for the single-particle magnetic quantum numbers of \( m = \pm 1/2 \). The corresponding eigenenergies, \( \epsilon_\sigma \) and \( \epsilon_\pi \), are shown in Fig. 3 for two particular parameter sets \((c = 10000; f = 3 \text{ and } f = \infty)\) adjusted to the standard properties of the nucleon. This figure depicts the increasing binding of the positive parity state, \( |\sigma> \), for small \( \alpha \) as well as the convergence of both levels for separating bags. As \( \alpha \to \infty \), the two states become linear combinations of \( R \) and \( L \), corresponding to an \( s \)-state in each bag. As \( \alpha \to 0 \), the single-particle states evolve to \( \sigma \to s_{1/2} \) and \( \pi \to p_{3/2}, m = \pm 1/2 \).

**IV. QUARK MOLECULAR BASIS STATES**

The classification and construction of antisymmetric six-quark basis states is a central part of any study of the \( N-N \) system in terms of quark degrees of freedom. In principle, the choice of a particular basis is irrelevant, as long as a sufficiently large number of configurations is included. However, in practice we have to content ourselves with a relatively small subset of states, and therefore need to make sure that these include the configurations
which supposedly dominate the exact ground state of the corresponding Hamiltonian.

For the construction of the six quark states, incorporating all possible degrees of freedom, namely color (C), orbital motion (O), spin (S) and isospin (T), we use a classification scheme based on $SU(4)$ spin-isospin symmetry, as introduced by Harvey [34]. Here, one first combines the color singlet $SU(3)$ function, as denoted by the partition $[222]_C$, with an orbital function of a specific symmetry, say $[f]_O$. This leads to a state of combined symmetry $[f]_{OC}$. The latter is then coupled to a spin-isospin $SU(4)$ function with the dual symmetry, $[f']_{TS}$. This yields a configuration which is totally antisymmetric with respect to the interchange of any pair of particles, i.e. the has Young symmetry $[1^6]$. We thus arrive at the notation

$$
\psi_6 = \left( ([f]_O [222]_C) [f]_{OC} [f']_{TS} \right)_{[1^6]}.
$$

(23)

Using “fractional parentage coefficients,” wave functions like (23) can be written as linear combinations of products of antisymmetric states of the first $n-1$ particles and the last particle or, alternatively, of the first $n-2$ particles and the last pair [34]. This helps to reduce the six-body matrix elements of the one- and two-body Hamiltonian to linear combinations of one- and two-body matrix elements, by taking the orthogonality of the $n-1$ and $n-2$ particle states of distinct symmetries into account.

The only sectors which are compatible with $L = 0$ $N$-$N$ partial waves are $T=0$, $S=1$ and $T=1$, $S=0$ [35]. The relevant orbital symmetries are then $[f]_O = [6]$ and [42] if one assumes that each nucleon is asymptotically in an orbital [3] state. For the spin-isospin function, on the other hand, quite a few $SU(4)$ representations, $[f']_{TS}$, would in principle be available, and they can, for example, be found in Harvey’s work [34]. Yet among them only the $[f']_{TS} = [6]$, [51], [42] and [33] – which are labeled with “asterisks” in Harvey’s article – evolve into asymptotic di-baryon states for large inter-nucleon separations. As all other $[f']_{TS}$ states – the “non-asterisked” ones – couple very weakly to the latter [19,20], they can safely be disregarded for the $N$-$N$ problem.

The novelty with respect to Harvey’s scheme – and similar other studies – lays in the choice of the orbital share of the wave function. In most previous calculations, the “cluster model” has been used – see ref. [10] for a review – which describes the orbital degrees of freedom in terms of two separate three-quark clusters centered at the locations of the two respective nucleons, denoted in the following as $|R>$ and $|L>$. In this investigation, on
the other hand, we use “molecular orbitals” [18] where the spatial single-particle states are wave functions of a static single-particle Hamiltonian, such as obtained from constrained Hartree-Fock or soliton mean field theories. In our case, the respective single-quark basis states are the two lowest orbitals of either parity, $|\sigma\rangle$ and $|\pi\rangle$, discussed in the last section. It is obvious that the molecular states are orthogonal at any separation $\alpha$, whereas the cluster model states are not. The latter even overlap completely for vanishing internucleon separation, $<R|L>|_{\alpha=0} = 1$. Also, the molecular states are natural to a mean field description as, for example, the one employed in this work. Note also that in the cluster model, the limit $\alpha \to 0$ requires special care in the normalization of the various symmetry configurations [20]. Otherwise some contributions are mistakenly left out, as was the case e.g. in refs. [22] and [34].

For large separations between the nucleonic bags, $|\sigma\rangle$ and $|\pi\rangle$ become degenerate and turn into orthogonal combinations representing the lowest $s$-state located in either bag. For the study of the $N$-$N$ scattering problem, it is convenient to construct orthogonal, pseudo-right and pseudo-left orbitals, $|r\rangle$ and $|\ell\rangle$, respectively as

$$
|r\rangle, \ell\rangle = \frac{|\sigma\rangle \pm |\pi\rangle}{\sqrt{2}}.
$$

The $|r,\ell\rangle$ molecular states recover the $|R, L\rangle$ cluster model states at large separation, but at finite deformation their behavior is obviously very different from the cluster model states. This proves to be important for the short range part of the $N$-$N$ interaction [19,20].

The transformation from the $|r,\ell\rangle$ to the $|\sigma, \pi\rangle$ scheme for the relevant six quark basis states is given in table 1 of ref. [18]. This table also suggests that, due to the complex structure of those configurations, in all practical calculations it is much simpler to work with the $|r,\ell\rangle$ basis states instead. One can then eventually return to the $|\sigma, \pi\rangle$ configurations at the level of one- or two-body matrix elements. In refs. [19] and [20], results for the united six-quark bag obtained with cluster model wave functions were compared with corresponding calculations employing a molecular basis and, in particular, the constituent quark model and the MIT bag model were investigated in that context. In both cases, the authors found that at zero separation the ground state energies were substantially lowered through the use of the molecular orbitals. The reason for this is that configurations of the type $|\sigma^n \pi^{6-n} (n \neq 3)\rangle$,
which are missing in a cluster model basis, proved to be quite important. For the relativistic
current quark model \[20\], the most relevant states for the isospin-spin channels \((TS) = (01)\)
or \((10)\) turned out to be:

\[
\begin{align*}
|1> &= |NN> , \\
|2> &= |\Delta\Delta> , \\
|3> &= |CC> , \\
|4> &= |42^+[6]_O \{33\}_TS>, \\
|5> &= |42^+[42]_O \{33\}_TS>, \\
|6> &= |42^+[42]_O \{51\}_TS>, \\
|7> &= |51^+[6]_O \{33\}_TS>,
\end{align*}
\]

where the notation of ref. \[18\] was used. The first three form the “physical” basis in Harvey’s
investigation of the \(N-N\) interaction \[34\], and they contain solely configurations of the type
\(|r^3\ell^3>\). The other four are of the form \(|r^4\ell^2 + r^2\ell^4>\) or \(|r^5\ell + r\ell^5>\), denoted as \(42^+\) and
\(51^+\), respectively. Configurations of that type (with \(R\) and \(L\) replacing \(r\) and \(\ell\)) do not occur
in a cluster model basis. In accordance with the findings of ref. \[20\], we choose the seven
states of Eq. (25) to be the basis of our truncated Hilbert space in which the Hamiltonian,
as described in Sec. III (one-body part) and Sec. V (two-body part), has been diagonalized.

V. ONE GLUON EXCHANGE

We treat quark-gluonic interactions in the one gluon exchange approximation. At this
level, we do not encounter the problem of double-counting, since colorless structures which
are already represented by the scalar field begin with two-gluon exchange or the excitation
of \(q\bar{q}\) pairs. Higher order effects which arise due to the non-Abelian character of QCD are
also assumed to be simulated by the scalar field.

In addition, the non-Abelian terms in the QCD gauge field tensor, \(F^a_{\mu\nu}\) of Eq. (3), have
been neglected. The field equations therefore linearize and become identical to Maxwell’s
equations in an inhomogeneous medium, with the exception, however, that now all the field
operators and currents contain Gell-Mann’s color \(SU(3)\) matrices. In this approximation we
find from Eqs. (2a), (2c) and (3)

\[ \partial^{\mu} \left( \kappa(\sigma) \left( \partial_{\mu} A^{c}_{\nu} - \partial_{\nu} A^{c}_{\mu} \right) \right) = J^{c}_{\nu}, \]  

(26)

where the total quark color-current operator is

\[ J^{c}_{\nu} = \frac{g_{s}}{2} \overline{\psi} \gamma_{\nu} \lambda \psi, \]  

(27)

with \( g_{s} = \sqrt{4\pi\alpha_{s}} \). The gluonic fields are explicitly affected by the scalar field through \( \kappa(\sigma) \).

As the dielectric is constructed in such a way as to ensure absolute color confinement – i.e. it vanishes in the vacuum – the resulting gluonic propagators will also be “confined” and there will be no gluons propagating outside the solitonic bags.

In order to solve the field equations, we choose the Coulomb, or transverse, gauge,

\[ \nabla \cdot (\kappa A) = 0, \]  

(28)

which decouples \( A_{0} \) in (26) through

\[ - \nabla \cdot \kappa \nabla A_{0} = J_{0}. \]  

(29)

The field equation for the space components of \( A_{\mu} \) reads

\[ \kappa \partial^{2}_{t} A - \nabla^{2} A + \nabla \times (A \times \nabla \kappa) = J_{t}, \]  

(30)

where the transverse current, which satisfies \( \nabla \cdot J_{t} = 0 \), is defined by means of

\[ J_{t} = J - \kappa \nabla \partial_{t} A_{0}. \]  

(31)

We note that due to the scalar nature of the medium, the field equations are diagonal in the color indices, which have hence been omitted in Eqs. (28) to (31). From these equations we can furthermore deduce the mutual and self-interaction energies between the quarks which arise due to the OGE through

\[ H_{OGE} = \frac{1}{2} \int d^{3}r \ J^{c}_{\mu} A^{\mu c}, \]  

(32)

and finally evaluate their contributions to the one-body (self-interactions) and two-body (mutual interactions) parts of the effective Hamiltonian.

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The corresponding matrix elements are evaluated by first determining the “gluon propagator in medium”. We follow here Bickeboeller et al. [36], as corrected subsequently by Tang and Wilets [37], where, however, the corrections in ref. [37] do not affect the matrix elements needed here. In those papers, the respective Green’s functions of the differential equations (29) and (30) were calculated by making an expansion in either spherical harmonics (for the scalar Green’s function \( G(\mathbf{r}, \mathbf{r'}) \)) or vector spherical harmonics (for the tensor Green’s function \( G^{ij}(\mathbf{r}, \mathbf{r'}, \omega) \)). The resulting coupled differential equations were then solved numerically in an angular momentum representation. Since Eq. (29) for the time component \( A_0 \) contains no explicit time derivatives, the corresponding Green’s function, \( G(\mathbf{r}, \mathbf{r'}) \), is frequency independent and the field \( A_0 \) is consequently instantaneous. For further details the reader may consult refs. [36] and [37].

The part of the OGE interactions that arises from the time component of the gluonic field \( A^c_0 \) and which is mediated through the scalar Green’s function is responsible for the realization of color confinement, and the part of the OGE interactions that stems from the spatial components of the gluonic field \( A^c \) as cast by the tensor Green’s function \( G^{ij}(\mathbf{r}, \mathbf{r'}, \omega) \) generates the color-magnetic hyperfine interaction which, in turn, produces the \( N-\Delta \) mass splitting. As usual, the self-interaction terms have been included in the time part of the OGE and have been neglected for the spatial contributions. This is in accordance with the minimal self-energy prescription of the MIT bag model [38]. Only when taking the color-electrostatic self-energy diagrams arising from the time component of the gluonic field into account, the color-electrostatic interaction between two well separated nucleonic color singlets vanishes, as is required by color neutrality [5].

In addition, the OGE matrix elements which arise from the tensor Green’s function, i.e. from the color-magnetic hyperfine interaction, are somewhat smaller than the ones which are generated by the color-electrostatic interaction. The reason for this reduction is that the latter always involve the “small” lower components of the relativistic quark spinors. Inasmuch as the magnetic interaction is not directly involved in color confinement, we simplify our calculations by using a free (\( \kappa = 1 \)) tensor propagator, \( G^{ij}_f(\mathbf{r}, \mathbf{r'}, \omega) \), and an effective \( \alpha_s \rightarrow \alpha_s^{\text{free}} \) adjusted to yield the experimental \( N-\Delta \) splitting.

To leading order in the strong coupling constant, the gluonic field energy contribution
to the effective Hamiltonian can be written as a sum of terms each involving a c-number, configuration-independent energy and an operator which depends on color, spin, flavor and orbital quantum numbers [5]. We find

$$<H_{OGE}> = V_{pqrs}^0 <b^+_p \lambda^c b_q \lambda^c b_s> + V_{pqrs}^z <b^+_p \sigma z^c b_q \lambda^c b_s>$$

$$+ V_{pqrs}^{pqqp} <b^+_p \lambda^c \lambda^c b_q \lambda^c b_s> + \sum_q V_{pqrs}^{pqqp} <b^+_p \lambda^c \lambda^c b_q \lambda^c b_s>$$

$$+ \sum_q V_{pqrs}^{z} <b^+_p \lambda^c \sigma z^c b_q \lambda^c b_s> + \sum_q V_{pqrs}^{\perp} <b^+_p \lambda^c \sigma z^c \lambda^c \lambda^c b_q \lambda^c b_s>, \quad (33)$$

where the first three terms on the right hand side correspond to the two-body mutual interactions, depicted in Fig. 4a, and the last three correspond to the one-body self-interactions, as shown in Fig. 4b. The b’s (b’’s) are annihilation (creation) operators for the single-quark states |σ> and |π> which were introduced in Sec. III. The configuration-dependent matrix elements, <b+...b>, are evaluated with the method of “fractional parentage coefficients”, which was outlined in the last section. They generate an explicit mixing between the various six-quark configurations under consideration.

The V_{pqrs}, on the other hand, are configuration independent interaction energies,

$$V_{pqrs}^0 = \sum_{m,m'} \mathcal{V}_{pmqm}^{r_m's_m'}$$

$$V_{pqrs}^z = \sum_{m,m'} (-)^{m-m'} \mathcal{V}_{pmqm}^{r_m's_m'}$$

$$V_{pqrs}^\perp = \sum_{m \neq m'} \mathcal{V}_{pmqm'}^{r_m's_m'}, \quad (34)$$

where each individual summand

$$\mathcal{V}_{pmqm}^{r_m's_m} = \frac{g_s^2}{32} \int dr \int dr' \left[ \rho_{pmqm}(r) G(r, r') \rho_{r_m's_m}(r') \right.$$

$$- j_{pmqm}^{k}(r) G^{kl}(r, r', |\epsilon_p - \epsilon_q|) j_{r_m's_m}^{l}(r') \left. \right], \quad (35)$$

is expressed in terms of the scalar and tensor Green’s functions, G(r, r’) and G^{kl}(r, r’, ω), and the various components of the time-independent single-quark four-vector current

$$\overline{\psi}_{\mu pm_p}(r) \gamma^\mu \psi^{qm_q}(r) \equiv \left( \rho_{pm_pqm_q}(r), j_{pm_pqm_q}^k(r) \right). \quad (36)$$

The first term under the integral in Eq. (35) stems from the color-electrostatic interaction, which is responsible for color confinement, and the second term arises due to the color-magnetic hyperfine interaction, which yields the N-∆ splitting.
As we restrict ourselves to values for the magnetic single-particle quantum numbers of $m_p, m_q, \ldots \in \{\pm 1/2\}$, we can couple the $\hat{z}$-components of the angular momenta of the two “incoming” (or “outgoing”) quarks to two-body “pseudospin” states, $|SM>$, where $|SM> \in \{|00>,|10>,|1 \pm 1>\}$. The corresponding formalism was outlined in detail in ref. [20]. This allows the separation of the electric one-body and two-body contributions to the effective Hamiltonian into matrix elements of three different spin-operators, $O_0, O_z$ and $O_\perp$, as was carried out in Eq. (33).

VI. RESULTS AND DISCUSSION

In the following, we will present our results for the adiabatic, local $N$-$N$ potential,

$$V_{ad}^{NN} = <H_{bag}^1> + <H_{OGE}> - 2E_N,$$  

obtained in Born-Oppenheimer approximation [12] from the energy difference of a deformed six-quark bag and two well separated non-interacting nucleons. The underlying effective Hamiltonian can be separated into various contributions. There is the one-body term,

$$<H_{bag}^1> = \epsilon_\sigma <b_\sigma^+ b_\sigma> + \epsilon_\pi <b_\pi^+ b_\pi>$$

$$- \frac{1}{2} \int d^3r \left( \frac{dg_{eff}(\sigma)}{d\sigma} \sigma <\bar{\psi}\psi> + \frac{dU(\sigma)}{d\sigma} \sigma - 2U(\sigma) \right)$$

$$+ \int d^3r (g_{eff}(\sigma) - g_{eff}(\sigma_\alpha)) <\bar{\psi}\psi>$$

with the scalar quark density

$$<\bar{\psi}\psi> \equiv \epsilon_\sigma <b_\sigma^+ b_\sigma> \bar{\psi}_{\sigma m} \psi_{\sigma m} + \epsilon_\pi <b_\pi^+ b_\pi> \bar{\psi}_{\pi m} \psi_{\pi m}.$$  

$<H_{bag}^1>$ arises from the single-particle energies, $\epsilon_\sigma$ and $\epsilon_\pi$ of Eq. (16) and depicted in Fig. 4, the scalar field $\sigma$ from Eq. (17) as well as the external potential generating the quark wave functions, $g_{eff}(\sigma_\alpha)$ of Eq. (18). Note the distinction between the scalar field $\sigma$, which is an explicit dynamical degree of freedom, and the auxiliary quantity $\sigma_\alpha$, which, as outlined in Sec. III, is only used for generating single-quark wave functions with a certain deformation.

In addition, there is the one gluon exchange contribution
\begin{equation}
\langle H_{OGE} \rangle = \langle H_1^\sigma \rangle + \langle H_1^\pi \rangle + \langle H_2^{\sigma\sigma \sigma} \rangle + \langle H_2^{\pi\pi \pi} \rangle + \langle H_2^{\sigma\pi \sigma \pi} \rangle + \langle H_2^{\sigma \pi \sigma \pi} \rangle + \langle H_2^{\sigma \sigma \pi \pi} \rangle + \langle H_2^{\sigma \pi \pi \sigma} \rangle + \langle H_2^{\sigma \sigma \sigma \pi} \rangle + \langle H_2^{\pi \pi \pi \sigma} \rangle + \langle H_2^{\pi \sigma \sigma \pi} \rangle + \langle H_2^{\pi \sigma \pi \sigma} \rangle + \langle H_2^{\pi \pi \sigma \sigma} \rangle,
\end{equation}

with the one-body self-energy terms, \( \langle H_1^\sigma \rangle \) and \( \langle H_1^\pi \rangle \), and the various two-body contributions \( \langle H_2^{pqrs} \rangle \) allowed by parity conservation. The relevant expressions can be derived from Eqs. (33) through (36), and the corresponding diagrams are shown in Fig. 5. It is important to note that in the OGE self-energy terms only the color-electrostatic interaction was taken into account, and that the \( \sigma - \pi - \sigma \) and \( \pi - \sigma - \pi \) “off-diagonal” self-energy terms are essential in providing for the confinement of the color-electric flux \[3\].

The adiabatic potential of Eq. (37) will be shown as a function of the deformation parameter \( \alpha \), which was introduced in Sec. III (see Eqs. (19) to (21)). For large prolate deformations, \( \alpha \) coincides with the true nucleon-nucleon separation. For smaller deformations, however, only a dynamical calculation employing e.g. the generator coordinate method would yield the transformation to the exact inter-nucleon separation \[3\]. In Fig. 6 we show the dependence of the inter-nucleon separation \( r \) on the deformation parameter \( \alpha \) as taken from ref. \[3\] where the \( N-N \) interaction was investigated in terms of quark degrees of freedom within the Friedberg-Lee soliton model and not including gluonic effects. We note that the “spherical” configuration, \( \alpha = 0 \), corresponds to a still finite inter-nucleon separation, and that \( r \to 0 \) is approached for oblate deformations, i.e. for \( \alpha < 0 \). Although the exact form of the transformation \( r(\alpha) \) depends on the details of the Hamiltonian and can thus only be established by a consideration of the dynamics involved, which we leave to subsequent work \[24\], the general behavior will still be similar to the one depicted in Fig. 6.

Approximate recoil corrections \[30\], momentum projection and boost \[31\] are not incorporated into the present six-quark calculations. For consistency, the potentials we calculate here are therefore normalized with respect to the energy and not the mass of two non-interacting nucleons. We find this quantity from a self-consistent calculation of the nucleon, as outlined in Sec. II. The latter also allows the adjustment of the parameters of the model, which are fit to the experimental proton rms charge radius (0.83 fm), the \( N-\Delta \) mass splitting (293 MeV) and the nucleon mass (939 MeV),

\[ M_N = \sqrt{E_N^2 - \langle P^2 \rangle} , \]

(41)
where the approximate recoil corrections \( < P^2 > \) stem from the quark wave functions and from the scalar \( \sigma \)-field. Results of these calculations are summarized in Table I, and the two parameter sets for which the \( N-N \) interaction was evaluated are given in Table II. The quantity \( E_N \) varies between 1145 MeV for the set with \( f = \infty \) and 1212 MeV for \( f = 3 \).

Throughout this investigation, the strong coupling constant \( \alpha_s \) we use for evaluating the gluonic share of the effective Hamiltonian is obtained by fitting the experimental \( N-\Delta \) splitting employing either a “free” or a “confined” gluonic propagator. The corresponding values for \( \alpha_s^{free} \) and \( \alpha_s^{conf} \) for the two parameter sets under consideration are listed in the sixth and seventh column of Table II, respectively. Thus, when evaluating the color-magnetic hyperfine interaction, where, as pointed out in the last section, a “free” gluonic propagator is used, \( \alpha_s^{free} \) is substituted for the strong coupling constant. Correspondingly, \( \alpha_s^{conf} \) is used for the color-electrostatic part of the interaction where, on the other hand, “confined” propagators are employed.

In addition, in all actual numerical calculations an infrared regularization [15] of the dielectric function, \( \kappa(\sigma) \) of Eq. (7), was introduced in order to handle the infinities in the one gluon exchange diagrams associated with a vanishing dielectric constant. We replace \( \kappa(\sigma) \) with

\[
\kappa(\sigma) \rightarrow \kappa(\sigma) \left(1 - \kappa_v\right) + \kappa_v \tag{42}
\]

and use \( \kappa_v = 0.1 \) throughout this investigation after having convinced ourselves that our final results are stable with respect to variations in the regularization parameter \( \kappa_v \). Note that “intermediate quantities” – as e.g. the energies associated with individual diagrams or the strong coupling constant \( \alpha_s \) – will, however, well depend on \( \kappa_v \).

In Figs. 7 and 8, we show the adiabatic \( N-N \) potential obtained from a diagonalization of the effective Hamiltonian in the Hilbert space spanned by the six-quark configurations listed in Eq. (25). Results are depicted for the isospin-spin channels \( (TS) = (01) \) (Fig. 7) and \( (TS) = (10) \) (Fig. 8). For \( (TS) = (01) \), the potential can furthermore be split into a central and a tensor part, where

\[
V_{cent}^{(TS)=(01)} = \frac{1}{3} \left( 2V_{M=\pm 1}^{(TS)=(01)} + V_{M=0}^{(TS)=(01)} \right), \tag{43a}
\]

\[
V_{tens}^{(TS)=(01)} = \frac{1}{6} \left( V_{M=\pm 1}^{(TS)=(01)} - V_{M=0}^{(TS)=(01)} \right). \tag{43b}
\]
The central interaction we find is purely repulsive with a “soft” core whose maximum varies between 200 MeV ($f = 3$, solid line) and 350 MeV ($f = \infty$, dot-dashed line) for the two parameter sets under consideration. As outlined in Sec. II, for $f = 3$ the bag pressure vanishes which leads to hard bags with a thin surface, and for $f = \infty$ $U(\sigma = 0)$ turns from a second minimum to an inflection point which, in turn, generates soft bags with a thick surface. This leads to a very different behavior of the surface energy associated with the scalar $\sigma$-field which extends to the variations observed in the interaction. Also, the cusp in the central potential around $\alpha \approx 2.3$ fm for $f = 3$ originates in those particular surface dynamics. At that deformation the scalar field turns abruptly from forming two separated bags to spanning just one united cavity.

An intermediate range attraction, as observed in earlier calculations of that type (see e.g. ref. [5]) is not at all visible in our results. De Tar [5] attributes the latter to the strong color-electrostatic attraction within the quark triplets. Although the color-electrostatic one gluon exchange diagrams are entirely attractive, in our investigation their effects are actually more than cancelled by the repulsive self-energy diagrams, $\langle H_1^\sigma \rangle$ and $\langle H_1^\pi \rangle$. Note that the color-magnetic self-energies were left out altogether in this investigation. On the other hand, the long and medium range $N$-$N$ attraction should actually be attributed to the meson exchange, and to get a good description of this in a quark model presumably requires the “sea” quarks to be taken into account explicitly [39], which are not accounted for in our investigation at this stage. To cure that shortcoming, we plan [40] to include an explicit pion exchange between the quarks [41] which will then lead to an effective pionic dressing of the individual nucleons along the lines proposed by Miller et al. [42].

To determine the relevance of the so-called “hidden-color” [34] states ($|3\rangle$ through $|7\rangle$ in Eq. (25)), which asymptotically fission into color non-singlets, in Fig. 9 we show their relative admixture to the six-quark ground state of the effective Hamiltonian as a function of the deformation of the bag. We observe that their contributions become significant as soon as the nucleonic bags overlap considerably, and that up to 50 percent of the ground state wave function can actually be made up of “hidden-color” states for small inter-nucleon separations, i.e. oblate deformations. This proves the importance of channel coupling in that realm, and is consistent with the findings of ref. [20] as corrected in ref. [43].
The different contributions to the adiabatic $N-N$ potential are analyzed in Figs. 10 and 11 and in Table III. In Fig. 10, we show the various potentials we obtain when employing different approximations for the one gluon exchange. Results are depicted for the isospin-spin channel $(TS) = (01)$ and for a two-nucleon state with the spins aligned antiparallel along the separation axis, i.e. for $M = 0$.

The dashed line corresponds to a calculation where the OGE was left out altogether, and in agreement with an earlier investigation [6] where the Friedberg-Lee soliton model was applied to $N-N$ scattering we find a strongly attractive adiabatic potential. The dotted line shows the results of a calculation where only the color-magnetic hyperfine interaction was included, which in the literature is quoted as being responsible for the short-range repulsive core [14]. In contrast to that belief, the spin-spin interaction reduces the attraction but does not yield any repulsion. The dot-dashed and the solid line correspond to calculations where, in addition, the color-electrostatic OGE was included, and they indeed yield a repulsive core. In contrast to the solid line, which shows the results of a calculation employing a “confined” scalar Green’s function, $G(r, r')$, the dot-dashed line corresponds to the use of a free gluonic propagator, $G_f(r, r')$, as outlined in Sec. V. The differences between both calculations prove to be rather minute, which gives us confidence, that the uncertainties we encounter from also evaluating the color-magnetic hyperfine interaction with a free tensor propagator are as well quite small.

In Fig. 11, the adiabatic potential for the isospin-spin channel $(TS) = (10)$ is split into a gluonic contribution, $< H_{OGE} >$ of Eq. (40), and a part independent of the one gluon exchange, $< H_{bag} >$ of Eq. (38). The non-gluonic contribution, which stems from the single particle energies, the scalar $\sigma$-field and the external potential used to generate the quark wave functions is always attractive, while the OGE part of the Hamiltonian is purely repulsive. Also, the differences between the two parameter sets, which we observed in the central adiabatic potentials shown in Figs. 7 and 8, almost entirely stem from $< H_{bag} >$, with the “hard” set ($f = 3$, the solid line) being much more attractive. As already previously mentioned, the origin of that variations is the very different behavior of the surface energy associated with the scalar $\sigma$-field for the two parameter sets under consideration. In spite of that differences, their gluonic shares of the energy are very similar.
At this point, a more detailed comparison with De Tar’s pioneering study of the adiabatic two-nucleon interaction in the framework of the MIT bag model is in place. As can be seen from Figs. 8 and 9 of ref. [5], for the MIT bag model the non-gluonic contribution, $< H_{1}^{\text{bag}} >$, is repulsive, while the gluonic share of the Hamiltonian, $< H_{\text{OGE}} >$, yields all the attraction. This is just opposite to our findings. The difference concerning the non-gluonic interaction is due to the very different nature of the surface dynamics of the scalar background field for the MIT and the solitonic bag. The differences in the gluonic share of the Hamiltonian, on the other hand, arise most probably both from the approximations De Tar is making in the evaluation of the OGE self-energy terms and from the differences in the color-dielectric constant $\kappa(r)$ and thus also in the gluonic propagators between the two models. Also, the six-quark configuration space we are using is much larger than the one De Tar was employing, and his single-quark states are rather artificial constructions while our “molecular orbitals” are eigen-states of the constrained mean field Hamiltonian.

To get a more detailed understanding of the origins of our results, in Table III we list the various contributions to the adiabatic $N-N$ potential stemming from the individual diagrams shown in Fig. 5. The energies shown are the expectation values of the ground state of the effective Hamiltonian for the isospin-spin channel $(TS) = (10)$. They correspond to the limiting cases of two well separated nucleons ($\alpha = 3.5$ fm) as well as one united spherically symmetric cavity ($\alpha = 0.0$ fm). The individual OGE mutual interaction terms $< H_{2}^{\text{OGE}} >$ are all attractive, while the self-energy diagrams, $< H_{1}^{\sigma} >$ and $< H_{1}^{\pi} >$, are entirely repulsive. Furthermore, only if the color-electrostatic “off-diagonal” self-energy terms $\sigma - \pi - \sigma$ and $\pi - \sigma - \pi$ are taken into account, the color-electric flux is confined [5], and the interaction vanishes at asymptotic inter-nucleon separations, i.e. $< H > \rightarrow 0$ for large $\alpha$. That this can really be observed in our results – see the forth and seventh column in the table – is non-trivial and is a nice confirmation that our numerics are correct. It can also be seen from Table III, that the adiabatic potential at vanishing $N-N$ separation arises from significant cancellations between individual terms that are rather large, with the repulsive one gluon exchange contributions overpowering the attractive non-gluonic share of the effective Hamiltonian.

For all the results reported in this section, the single-quark wave functions were gener-
ated from a scalar potential having a particular shape, as characterized by the geometrical parameters $R(\alpha), \Gamma$ and $\sigma_0$ (see Eqs. (19) to (21)). As outlined in Sec. III, the radius of the six-quark bag $R(\alpha)$ was varied as a function of the deformation such that the volume of the $\sigma$-field cavity remains fixed. In order to test the validity of that method, for $\alpha = 0$ we constructed self-consistent eigen-states of the total Hamiltonian requiring a spherically symmetric scalar field. In Table IV, we compare results for the geometrical shape parameters and the adiabatic potential, which we find from the self-consistent calculation with the corresponding quantities obtained by using the “constant volume” approach. Results are listed for $(TS) = (01)$ and $M = 0$. Although the geometrical parameters of the self-consistently determined $\sigma$-field are quite different from the ones characterizing $\sigma_\alpha$ in the “constant volume” approximation, the resulting adiabatic potentials are very similar. This shows, that the uncertainties we encounter by choosing this particular path in the configuration space spanned by the geometrical parameters $R$, $\Gamma$ and $\sigma_0$ will also be minute and should hence not effect our conclusions.

VII. CONCLUSION AND OUTLOOK

We have evaluated the adiabatic nucleon-nucleon potential in a relativistic quark bag model which yields spatial as well as color confinement and is free of the spurious color Van der Waals forces troubling most non-relativistic calculations in that realm. The six-quark system we investigate is confined in a deformed bag-like mean field through an effective non-linear interaction between the quarks and a scalar field. The shape of this confining field is adjusted to reproduce the corresponding quantity for the asymptotic case of two well separated non-interacting nucleons, and is then varied with deformation treating the scalar field as an incompressible liquid.

Six-quark molecular-type configurations are then generated as a function of deformation and their energies are evaluated in a coupled channel analysis. By using molecular states instead of cluster model wave functions, we can be sure that all important six-quark configurations were properly taken into account, which is a necessary prerequisite for finding reasonable results.

The corresponding effective Hamiltonian includes not only the interaction between the
quarks and the scalar background field but also quark-quark interactions generated through one gluon exchange evaluated in Abelian approximation. Furthermore, when calculating the gluonic propagators mediating that interaction, the inhomogeneity and deformation of the dielectric medium were taken into account and the Coulomb gauge was applied.

Results for the adiabatic local nucleon-nucleon potential have been presented for the different spin-isospin channels which are compatible with $L = 0$ $N-N$ partial waves, and they differ quite considerably from a realistic phenomenological interaction fit to the experimental phase shifts. Although the adiabatic central potentials display a “soft” repulsive core, as is desirable from phenomenology, they totally lack the intermediate range attraction, which was observed in earlier calculations of that type and which was attributed to the strong color-electrostatic attraction [5].

Although the color-electrostatic exchange diagrams are entirely attractive also in our investigation, their effects are actually more than cancelled by the repulsive gluonic self-energy diagrams. A detailed analysis of the different contributions to the effective Hamiltonian unveils that the non-gluonic one-body terms would lead to considerable attraction for vanishing inter-nucleon separation, while the one gluon exchange mutual and self-interaction terms produce all the repulsion. To be more specific, in our case it is the color-electric one gluon exchange which leads to the repulsion at small $N-N$ separations and not the spin-spin color-magnetic hyperfine interaction, which in the literature is quoted as being responsible for the short-range repulsive core [14].

Considering that the long and medium range nucleon-nucleon attraction should actually be attributed to explicit meson exchange and not to quark rearrangement, we are not at all surprised not to get a good description of this part of the interaction in a quark model which does not include the “sea” quarks. We plan to overcome that detriment by either including quantum surface fluctuations, which would introduce configurations of the form $q^7\bar{q}$ in addition to our $q^6$ basis states, or by considering an explicit pion exchange between the individual quarks along the lines followed in the cloudy bag model [42]. The latter mechanism is favorable as it also leads to a restoration of the broken chiral symmetry. Work in that direction is currently in progress [40].

Finally, we also plan to account for the dynamics of the $N-N$ interaction by extending
this work through means of the generator coordinate method [24]. It has been shown that a significant part of the short-range $N-N$ repulsion is due to dynamics [3], and that the absence of a repulsive core in some early calculations was an artifact of the adiabatic approximation [22,23]. In addition, the effective $N-N$ interaction is highly non-local in terms of the separation parameter.

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TABLE I. Results of a self-consistent mean field calculation for the nucleon for various parameter sets as characterized by $f = b^2/ac$ and $c$, and adjusted to yield the same recoil corrected proton rms radius of 0.83 fm and recoil corrected proton mass of 939 MeV. The quantities listed are the $0^{++}$ glueball mass $m_{GB}$, the bag pressure $B$, the nucleon’s axial vector coupling constant $g_A$ and the proton’s magnetic moment $\mu_p$. The strong coupling constant $\alpha_s$ can either be adjusted to yield the mass of the $\Delta$-resonance of $1232$ MeV – employing a free or a “confined” gluonic propagator – or can be obtained from an evaluation of the string tension, $\theta$. Corresponding values for $\alpha_s$ are listed. For further details see ref. [15] and references therein.

| $f$  | $c$    | $m_{GB}$ (MeV) | $B$ (MeV/fm$^3$) | $g_A$ | $\mu_p$ ($\mu_N$) | $\alpha_s|_{\Delta}$ | $\alpha_s|_{\theta}$  |
|------|--------|----------------|------------------|-------|-----------------|---------------------|------------------------|
| 3.0  | 30000  | 2933           | 0                | 1.24  | 2.21            | 3.36                | 1.32  | 2.93          |
|      | 10000  | 1948           | 0                | 1.21  | 2.29            | 3.36                | 1.42  | 3.78          |
|      | 3000   | 1254           | 0                | 1.18  | 2.34            | 3.34                | 1.57  | 4.78          |
|      | 1000   | 734            | 0                | 1.12  | 2.42            | 2.90                | 2.25  | 13.20         |
| 3.2  | 30000  | 2501           | 40               | 1.21  | 2.28            | 3.49                | 1.43  | 1.83          |
|      | 10000  | 1787           | 32               | 1.20  | 2.31            | 3.48                | 1.46  | 2.08          |
|      | 3000   | 1214           | 22               | 1.19  | 2.34            | 3.45                | 1.56  | 2.53          |
|      | 1000   | 783            | 12               | 1.15  | 2.38            | 3.18                | 2.03  | 4.47          |
| $\infty$ | 30000 | 2355           | 67               | 1.21  | 2.31            | 3.66                | 1.47  | 1.61          |
|      | 10000  | 1755           | 62               | 1.21  | 2.31            | 3.64                | 1.50  | 1.67          |
|      | 3000   | 1243           | 52               | 1.20  | 2.32            | 3.58                | 1.59  | 1.88          |
|      | 1000   | 874            | 38               | 1.18  | 2.35            | 3.49                | 1.79  | 2.43          |
TABLE II. Parameter sets for which the adiabatic $N$-$N$ potential was calculated. The sets are adjusted to yield the proton rms charge radius, the nucleon mass and the $N$-$\Delta$ mass splitting, where the latter quantity was evaluated employing a free as well as a “confined” gluonic propagator. For further details see Sec. II.

| $f$ | $a$ (fm$^{-2}$) | $b$ (fm$^{-1}$) | $c$ | $g_0$ | $\alpha_s|_{N\Delta}$ | $\alpha_s^{\text{free}}$ | $\alpha_s^{\text{conf}}$ |
|-----|-----------------|-----------------|-----|-------|-------------------|-------------------|-------------------|
| 3.0 | 97.45           | -1709.9         | 10000 | 0.81  | 3.36              | 3.64              | 1.42              |
| $\infty$ | 0.00            | -726.1          | 10000 | 1.80  | 3.64              | 1.50              |

TABLE III. The various contributions to the adiabatic $N$-$N$ potential stemming from the individual diagrams shown in Fig. 5 and given in Eqs. (38) and (40). Results are shown for well separated ($\alpha = 3.5$ fm) as well as completely overlapping nucleonic bags ($\alpha = 0$ fm), and also for the adiabatic potential at $\alpha = 0$ fm, which is the difference of the latter two quantities. The energies shown correspond to the isospin-spin channel $(TS) = (10)$ and are obtained using the parameter sets given in Table II.

| $<H_1^{\text{bag}}>$ | $f = 3.0$ | $f = \infty$ |
|----------------------|-----------|--------------|
|                      | $E|_{0.0\text{fm}}$ | $E|_{3.5\text{fm}}$ | $V_{ad}^{NN}|_{0.0\text{fm}}$ | $E|_{0.0\text{fm}}$ | $E|_{3.5\text{fm}}$ | $V_{ad}^{NN}|_{0.0\text{fm}}$ |
| $<H_1^\sigma>$       | 93        | 240          | -147         | 212        | 256          | -44           |
| $<H_1^\pi>$          | 5849      | 3757         | 2092         | 5506       | 4685         | 821           |
| $<H_1^{\sigma\sigma}>$ | 2654 | 3756       | -1102        | 3093       | 4682         | -1589         |
| $<H_2^{\sigma\sigma\sigma}>$ | -3728 | -1100 | -2628 | -3222 | -1507 | -1715 |
| $<H_2^{\pi\pi}>$     | -901      | -1098        | 197          | -1101      | -1504        | 403           |
| $<H_2^{\sigma\pi}>$  | -3403     | -2504        | -899         | -3651      | -3390        | -261          |
| $<H_2^{\pi\sigma}>$  | -92       | -1402        | 1310         | -144       | -1480        | 1336          |
| $<H_2^{\pi\pi\sigma}>$ | -403 | -1649       | 1246        | -518       | -1742        | 1224          |
| $<H>$                | 69        | 0            | 69           | 175        | 0            | 175           |
TABLE IV. The geometrical parameters, $R(\alpha)$, $\Gamma$ and $\sigma_0$, characterizing the field $\sigma_\alpha$ (see Eqs. (19) to (21)) from which then the external scalar potential, $V_\alpha$ of Eq. (18), is obtained. The quantities shown correspond to the isospin-spin channel $(TS) = (01)|_{M=0}$ and to zero deformation, i.e. $\alpha = 0$ fm. Self-consistent solutions constrained to yield a spherically symmetric scalar field are compared with the “constant volume” approach outlined in Sec. III.

| f   | $R$ (fm) | $\Gamma$ (fm$^{-1}$) | $\sigma_0$ (fm$^{-1}$) | $V_{ad}^{NN}$ (MeV) |
|-----|----------|----------------------|------------------------|----------------------|
| 3.0 | constant volume | 1.021 | 4.230 | 0.4050 | 128.8 |
|     | self-consistent | 1.178 | 5.548 | 0.3514 | 124.2 |
| $\infty$ | constant volume | 1.226 | 2.167 | 0.2592 | 264.5 |
|     | self-consistent | 1.257 | 2.032 | 0.2249 | 262.3 |
FIG. 1. The scalar field, $\sigma/\sigma_v$, and the corresponding self-interaction potential, $U(\sigma)$ of Eq. (4), determined from a self-consistent evaluation of the nucleon for three different parameter sets. The solid line corresponds to $f = 3$, the dot-dashed line to $f = 3.2$ and the dashed line to $f = \infty$. For all sets $c$, is kept at a value of 10000.
FIG. 2. The scalar field, $\sigma_\alpha$ of Eq. (19), from which the single-quark wave functions are generated by means of Eqs. (16) and (18) for four different values of the deformation parameter $\alpha$ between 2 fm and −2 fm. The fields correspond to the parameter set with $f = 3$ and $c = 10000$, and are shown with equal increments between adjacent contours.
FIG. 3. The eigenenergies of the lowest single-particle states of positive and negative parity determined from Eq. (16) for values of the magnetic quantum number of $m = \pm 1/2$. Results are shown for two particular parameter sets with $c = 10000$ and either $f = 3$ (solid line) or $f = \infty$ (dot-dashed line).

FIG. 4. Typical one gluon exchange diagrams that contribute to the gluonic share of the effective Hamiltonian, $< H_{OGE} >$ of Eq. (33). The diagrams shown in (a) correspond to the two-body mutual interactions, while the graphs depicted in (b) correspond to one-body self-interactions.
FIG. 5. Diagrammatic representation of the various one-body (< $H^{\text{bag}}_1$ >) and two-body (< $H^{\text{b}}_{1\pi}$ >) contributions to the effective Hamiltonian.

FIG. 6. The inter-nucleon separation $r$ as a function of the deformation parameter $\alpha$. The figure is taken from ref. [6] where the Friedberg-Lee soliton model was applied to $N-N$ scattering. No gluonic effects were taken into account in ref. [6].
FIG. 7. The adiabatic $N-N$ potential for the isospin-spin channel $(TS) = (01)$ split into a central – Eq. (43a) – and a tensor part – Eq. (43b). The solid line corresponds to the parameter set with $f = 3$ and the dot-dashed line to the set with $f = \infty$. Both sets were adjusted to the standard properties of the nucleon and are listed in Table II.
FIG. 8. The adiabatic $N-N$ potential for the isospin-spin channel $(TS) = (10)$. The labeling is the same as in Fig. 7.
FIG. 9. The relative admixture of “hidden color” states, $|3\rangle$ through $|7\rangle$ in Eq. (25), to the ground state of the effective Hamiltonian for two different isospin-spin channels, $(TS) = (01)|M=0$ and $(TS) = (10)$, and for the two parameter sets listed in Table II. The solid line corresponds to $f = 3$ and the dot-dashed line to $f = \infty$. 
FIG. 10. The various adiabatic $N-N$ potentials obtained when employing different approximations for the one gluon exchange. Results are shown for the isospin-spin channel $(T S) = (01)|M=0$ and for the two parameter sets given in Table II. The dashed line corresponds to a calculation where the OGE was left out altogether and the dotted line shows the results of a calculation where only the color-magnetic hyperfine interaction was included. The dot-dashed and the solid line correspond to calculations where, in addition, different versions of the color-electrostatic OGE were taken into account. In detail, the solid line shows the results of a calculation employing a “confined” Green’s function, while the dot-dashed line corresponds to the use of a free gluonic propagator.
FIG. 11. The adiabatic potential for the isospin-spin channel \((TS) = (10)\) is split into a part independent of the one gluon exchange, \(\langle H_1^{\text{bag}} \rangle\) of Eq. (38), and a gluonic contribution, \(\langle H_{OGE} \rangle\) of Eq. (40). The solid line corresponds to \(f = 3\) and the dot-dashed line to \(f = \infty\).