We consider the generalized shift operator defined by 

\[(Sh_utf)(g) = \int_G f(tut^{-1}g)\,dt\]

on a compact group \(G\), and by using this operator, we define "spherical" modulus of smoothness. So, we prove Stechkin and Jackson-type theorems.

2000 Mathematics Subject Classification: 42C10, 43A77, 43A90.

1. Introduction. In this paper, we prove some theorems on absolutely convergent Fourier series in the metric space \(L^2(G)\), where \(G\) is a compact group. The algebra of absolutely convergent Fourier series is a subject matter about which a good deal, although far from everything, is known (see [5, page 328]). Like many branches of harmonic analysis on \(T\) and \(R\), the theory of absolutely convergent Fourier series is a fruitful source of questions about the corresponding entity for compact groups. By using some absolute convergence theorems of the classical Fourier series (see [1, 11]), a generalized form of Stechkin [6] and Szasz theorem [1, 11] of the Fourier series on compact groups is obtained. Thus, we solve open problems formulated in [5, page 366] (see also [3, Chapter I, page 9]).

2. Preliminaries and notation. Now, we explain some of the notation and terminologies used throughout the paper.

Let \(G\) be a compact group with a dual space \(\hat{G}\), \(dg\) denote the Haar measure on \(G\) normalized by the condition \(\int_G dg = 1\), and \(\int_G f(g)\,dg\) denote the Haar integral of a function \(f\) on \(G\). Let \(U_\alpha, \alpha \in \hat{G}\) denotes the irreducible unitary representation of \(G\) in the finite dimensional Hilbert space \(V_\alpha\). We reserve the symbol \(d_\alpha\) for the dimension of \(U_\alpha\). Thus, \(d_\alpha\) is a positive integer. Also, we denote by \(\chi_\alpha\) and \(t_{ij}^\alpha (i, j = 1, 2, \ldots, d_\alpha), \alpha \in \hat{G}\) the character and matrix elements (coordinate functions) of \(U_\alpha\), respectively.

Let \(L^p(G)\) be the space of all functions \(f\) equipped with the norm

\[\|f\|_p = \left\{\int_G |f(g)|^p\,dg\right\}^{1/p} \quad (2.1)\]

We write \(\|\cdot\|_p\) instead of \(\|\cdot\|_{L^p(G)}\), and \(L^\infty = C\) is the corresponding space of continuous functions, and \(\|f\| = \max\{|f(g)| : g \in G\}\). As it is known (see [4]...
or \([10, \text{page 99}]\), the space \(L_2(G)\) can be decomposed into the sum
\[
L_2(G) = \sum_{\alpha \in \hat{G}} \Phi H_\alpha,
\] (2.2)
where
\[
H_\alpha = \{ f \in C(G) : f(g) = \text{tr} \left( U_\alpha(g) C \right), \ C = \text{Hom}(V_\alpha, V_\alpha) \}.
\] (2.3)
This theorem is one of the most important results of the harmonic analysis on compact groups. The orthogonal projection \(Y_\alpha : L_2(G) \to H_\alpha\) is given by
\[
(Y_\alpha f)(g) = d_\alpha \int_G f(h) \chi_\alpha(gh^{-1}) dh,
\] (2.4)
where \((Y_\alpha f)(g)\) does not depend on the choice of a basis in \(L_2\). Carrying out this construction for every space \(H_\alpha, \alpha \in \hat{G}\), we obtain an orthonormal basis in \(L_2\) consisting of the functions \(\sqrt{d_\alpha} t_{ij}^{\alpha}(g), \alpha \in \hat{G}, 1 \leq i, j \leq d_\alpha\). Any function \(f \in L_2(G)\) can be expanded into a Fourier series with respect to this basis
\[
f(g) = \sum_{\alpha \in \hat{G}} d_\alpha \sum_{i,j=1}^{d_\alpha} a_{ij}^\alpha t_{ij}^{\alpha}(g),
\] (2.5)
where the Fourier coefficients \(a_{ij}^\alpha\) are defined by the following relations:
\[
a_{ij}^\alpha = d_\alpha \int_G f(g) \overline{t_{ij}^{\alpha}(g)} dg,
\] (2.6)
such that \(\overline{t_{ij}^{\alpha}(g)} = t_{ij}^{\alpha}(g^{-1})\), where \(g^{-1}\) is the inverse of \(g\). Note that (2.5) is a convergent series in the mean and that the Parseval's equality
\[
\int_G |f(g)|^2 dg = \sum_{\alpha \in \hat{G}} \frac{1}{d_\alpha} \sum_{i,j=1}^{d_\alpha} |a_{ij}^\alpha|^2
\] (2.7)
holds. The aforementioned result of harmonic analysis on a compact group can be found, for example, in \([4, 5, 7, 10]\).

We denote by \(Sh_u\) the generalized translation operator on compact group \(G\) defined by
\[
(Sh_u f)(g) = \int_G f(tu^{-1}g) dt,
\] (2.8)
\[
(\Delta_u f)(g) = f(g) - (Sh_u f)(g) = (E - Sh_u) f,
\]
where \( u, g \in G \) and \( E \) is the identity operator. We set

\[
\triangle^k u f = \triangle u \left( \triangle^{k-1} u f \right) = (E - Sh_u)^k f = \sum_{i=0}^{k} (-1)^{k+i} C_k^i Sh_u^i f,
\]

(2.9)

in which \( Sh_0^i f = f \) and \( Sh_u \left( Sh_u^{i-1} f \right) = Sh_u^i f, \ i = 1, 2, \ldots, k \) and \( k \in \mathbb{N} \).

We note that \( \alpha \) is a complicated index. Since \( \hat{G} \) is a countable set, there are only countably many \( \alpha \in \hat{G} \) for which \( \alpha^\alpha_{ij} \neq 0 \) for some \( i \) and \( j \); enumerate them as \( \{ \alpha_0, \alpha_1, \ldots, \alpha_n, \ldots \} \). So, \( d_{\alpha_0} < d_{\alpha_1} < d_{\alpha_2} < \cdots < d_{\alpha_n} < \cdots \). Because of that, the symbol \( \alpha < n \) is interpreted as \( \{ \alpha_0, \alpha_1, \ldots, \alpha_{n-1} \} \subset \hat{G} \), and \( \alpha \geq n \) denotes the set \( \hat{G} \setminus (\alpha < n) \). Let \( d_{\alpha} \), as usual, be the dimension of \( U_{\alpha} \). For typographical convenience, we write \( d_n \) for the dimension of the representation \( U_{\alpha_n}, n = 1, 2, \ldots \). (See [5, page 458].)

We denote by \( E_n(f) \) the approximation of the function \( f \in L_p(G) \) by “Spherical” polynomials of degree not greater than \( n \):

\[
E_n(f) = \inf \left\{ \| f - T_n \|_p : T_n \in \bigoplus_{\alpha < n, \alpha \in \hat{G}} H_{\alpha} \right\}.
\]

(2.10)

The sequence of best approximations \( \{E_n(f)\} \) is a constructive characteristic of the function \( f \). In the capacity of structural characteristic of the function \( f \) on a compact group \( G \), we define its Spherical modulus of smoothness of order \( k \) by

\[
\omega_k(f; \tau) = \sup \left\{ \| (E - Sh_u)^k f \|_p : u \in W_{\tau} \right\},
\]

(2.11)

where \( W_{\tau} \) is a neighborhood of \( e \) in \( G \). In other words,

\[
W_{\tau} = \{ u : \rho(u, e) < \tau, u \in G \},
\]

(2.12)

where \( \rho \) is a pseudometric on \( G \) and \( \tau \) is any positive real number. It is easy to show the following properties of \( \omega_k(f, \tau) \):

(a) \( \lim_{\tau \to 0} \omega_k(f, \tau) = 0 \);

(b) \( \omega_k(f, \tau) \) is a continuous monotonically increasing function with respect to \( \tau \);

(c) \( \omega_k(f_1 + f_2, \tau) \leq \omega_k(f_1, \tau) + \omega_k(f_2, \tau) \);

(d) \( \omega_{k+l}(f, \tau) \leq 2^l \omega_k(f, \tau), l = 1, 2, \ldots \).

3. Main results. We need the following simple but useful lemma.
**Lemma 3.1.** The following equality holds for all $u, g \in G$:

$$
(\text{Sh}_u t_{ij}^\alpha)(g) = \frac{X_\alpha(u)}{d_\alpha} t_{ij}^\alpha(g).
$$  \hfill (3.1)

**Proof.** Using the orthogonality relations and other formulas for matrix elements $t_{ij}^\alpha(g)$ (see [7, page 189]), we have

$$
\int_G t_{ij}^\alpha(tut^{-1}g)dt = \frac{d_\alpha}{d_\alpha} \sum_{p=1}^{d_\alpha} \sum_{q=1}^{d_\alpha} t_{qp}^\alpha(u)t_{ij}^\alpha(g) \int_G t_{iq}^\alpha(t)t_{pq}^\alpha(t)dt
$$

$$
= \frac{1}{d_\alpha} \sum_{p=1}^{d_\alpha} t_{pp}^\alpha(u) t_{ij}^\alpha(g) = \frac{1}{d_\alpha} \chi_\alpha(u)t_{ij}^\alpha(g).
$$  \hfill (3.2)

This proves the lemma.

The following formula is the particular event of the above lemma:

$$
\int_G \chi_\alpha(tut^{-1}g)dt = \frac{X_\alpha(u)\chi_\alpha(g)}{d_\alpha}.
$$  \hfill (3.3)

It can be called a Weyl formula.

We note that the expansion (2.5) is connected with the expansion

$$
f(g) = \sum_{\alpha \in \hat{G}} Y_\alpha(f)(g), \quad Y_\alpha \in H_\alpha,
$$  \hfill (3.4)

which is defined by (2.4), that is, by the equality

$$
Y_\alpha(f)(g) = \sum_{i,j=1}^{d_\alpha} a_{ij}^\alpha t_{ij}^\alpha(g).
$$  \hfill (3.5)

Thus, the coefficients $a_{ij}^\alpha$ are defined by (2.6). Using Lemma 3.1 and the definition of $Y_\alpha$, we obtain

$$
Y_\alpha(\text{Sh}_u f)(g) = \sum_{i,j=1}^{d_\alpha} a_{ij}^\alpha \int_G t_{ij}^\alpha(tut^{-1}g)dt
$$

$$
= \sum_{i,j=1}^{d_\alpha} a_{ij}^\alpha \frac{X_\alpha(u)}{d_\alpha} t_{ij}^\alpha(g)
$$

$$
= \frac{X_\alpha(u)}{d_\alpha} Y_\alpha(f)(g).
$$  \hfill (3.6)

The following are simple facts with frequent usage: if $f \in L_p$, then

1. $\|\text{Sh}_u f\|_p \leq \|f\|_p$;
2. $\|f - \text{Sh}_u f\|_p \rightarrow 0$ as $u \rightarrow e$;
3. $(Y_\alpha(\text{Sh}_u f))(g) = (\chi_\alpha(u)/\chi_\alpha(e))(Y_\alpha f)(g)$ for all $\alpha \in \hat{G}$.

We note that $\chi_\alpha(e) = d_\alpha$. 

THEOREM 3.2. If \( f \in L_2 \) and \( f \) is not constant, then
\[
E_n(f)_2 \leq \sqrt{\frac{d_n}{d_n-2k}} \omega_k \left( f; \frac{1}{n} \right)_2, \quad n = 1,2,\ldots
\] (3.7)

PROOF. Let \( f \in L_2 \) and \( S_n(f,g) \) denote the \( n \)th partial sum of the Fourier series (2.5), that is,
\[
S_n(f,g) = \sum_{\alpha < n} d_{\alpha}^\alpha \sum_{i,j} a_{ij}^\alpha \cdot t_{ij}(g).
\] (3.8)

Using Parseval’s equality for the compact group \( G \), we have
\[
E_n^2(f)_2 = \|f - S_n(f)_2\|^2 = \sum_{\alpha > n} \frac{d_{\alpha}}{d_{\alpha}} \sum_{i,j} \left| a_{ij}^\alpha \right|^2.
\] (3.9)

Using (3), it is not hard to see that
\[
(Y_{\alpha}(\triangle^k f))(g) = \left( 1 - \frac{X_{\alpha}(u)}{d_{\alpha}} \right)^k (Y_{\alpha}f)(g), \quad \alpha \in \hat{G}.
\] (3.10)

Consequently, \((\triangle^k f)(g) = \sum_{\alpha \in \hat{G}} (1 - \chi_{\alpha}(u)/d_{\alpha})^k a_{ij}^\alpha \cdot t_{ij}(g).\) By another application of Parseval’s equality, we obtain
\[
\|\triangle^k u f\|^2 = \sum_{\alpha > n} \frac{d_{\alpha}}{d_{\alpha}} \sum_{i,j} \left| 1 - \frac{X_{\alpha}(u)}{d_{\alpha}} \right|^{2k} \left| a_{ij}^\alpha \right|^2 \geq \sum_{\alpha > n} \frac{1}{d_{\alpha}} \sum_{i,j} \left| 1 - \frac{X_{\alpha}(u)}{d_{\alpha}} \right|^{2k} \left| a_{ij}^\alpha \right|^2
\] (3.11)

Now, using Bernolly’s inequality \((1+x)^k \geq 1+kx\) for \( x \geq -1 \), we obtain
\[
\|\triangle^k u f\|^2 \geq \sum_{\alpha > n} \frac{1}{d_{\alpha}} \sum_{i,j} \left( 1 - \frac{2k \chi_{\alpha}(u)}{d_{\alpha}} + \frac{k \chi_{\alpha}(u)}{d_{\alpha}}^2 \right)^k \left| a_{ij}^\alpha \right|^2.
\] (3.12)

Consequently,
\[
\|\triangle^k u f\|^2 \geq \sum_{\alpha > n} \frac{1}{d_{\alpha}} \sum_{i,j} \left| a_{ij}^\alpha \right|^2 - \sum_{\alpha > n} \frac{1}{d_{\alpha}} \sum_{i,j} \left( \frac{2k \chi_{\alpha}(u)}{d_{\alpha}} \right)^k \left| a_{ij}^\alpha \right|^2;
\] (3.13)

therefore,
\[
E_n^2(f)_2 \leq \|\triangle^k u f\|^2 + 2k \sum_{\alpha > n} \frac{1}{d_{\alpha}} \sum_{i,j} \left\{ \frac{\chi_{\alpha}(u)}{d_{\alpha}} \right\} \left| a_{ij}^\alpha \right|^2.
\] (3.14)
Let $\Phi_{W_\tau}$ be a nonnegative integrable function vanishing outside $W_\tau$ and satisfying the condition $\int_G \Phi_{W_\tau}(g) \, dg = 1$. For example, we can take $\Phi_{W_\tau} = \xi_{W_\tau} / \mu(W_\tau)$, where $\mu(W_\tau)$ is the Haar measure of $W_\tau$ and $\xi_{W_\tau}$ is the characteristic function of $W_\tau$. Multiplying both sides of (3.14) by $\Phi_{W_1/n}$ and integrating with respect to $u$ on $G$, and using the equality $\int_G |\alpha| \, |\Phi_{W_1/n}(u) \, du |^2 = 1$ (see [7, page 195]), we obtain

$$\int_G E_n^2(f) \, 2 \, \Phi_{W_1/n}(u) \, du \leq \int_G \left\| \Delta_u f \right\|^2 \, \Phi_{W_1/n} \, du$$

$$+ 2k \sum_{\alpha \geq n} \frac{d_{\alpha}}{d_{\alpha}} \sum_{i,j=1}^{d_{\alpha}} \left| a_{\alpha}^{ij} \right|^2 \int_G |\chi_{\alpha}(u) \, | \Phi_{W_1/n}(u) \, du$$

$$\leq \sup \left\| \Delta_u f \right\|^2 + \frac{2k}{d_n} \sum_{\alpha \geq n} \frac{1}{d_{\alpha}} \sum_{i,j=1}^{d_{\alpha}} \left| a_{ij} \right|^2.$$  (3.15)

Therefore, it is not hard to see that

$$E_n^2(f) \leq \omega^2_k \left( f, \frac{1}{n} \right) + \frac{2k}{d_n} E_n^2(f).$$  (3.16)

Finally, we obtain

$$E_n(f) \leq \sqrt{\frac{d_n}{d_n - 2k}} \, \omega_k \left( f, \frac{1}{n} \right),$$  (3.17)

which proves the theorem.

This theorem is given without proof in [8] for the case where $k = 1$.

We note that the matrix elements of unitary representations $t_\alpha^{ij}(g)$ satisfy the relations

$$\sum_{j=1}^{d_{\alpha}} t_\alpha^{ij}(g) t_\alpha^{jk}(g) = \sum_{j=1}^{d_{\alpha}} t_\alpha^{ij}(g) t_\alpha^{jk}(g) = \begin{cases} 0 & \text{if } i \neq k, \\ 1 & \text{if } i = k. \end{cases}$$  (3.18)

In particular, we have

$$\sum_{j=1}^{d_{\alpha}} \left| t_\alpha^{ij} \right|^2 = 1 \implies \left| t_\alpha^{ij}(g) \right| \leq 1$$  (3.19)

for all $\alpha \in \hat{G}$ and $i,j = 1,2,\ldots,d_{\alpha}$. Furthermore, it is obvious that $|a_{\alpha}^{ij} t_\alpha^{ij}(g)| \leq |a_{ij}^\alpha|$; therefore, according to the sufficient condition for absolutely convergent Fourier series on the group $G$, the series $\sum_{\alpha \in \hat{G}} \sum_{i,j=1}^{d_{\alpha}} |a_{ij}^\alpha|\left| t_\alpha^{ij}(g) \right|$ is convergent. Let $A(G) := \{ f : \sum_{\alpha \in \hat{G}} \sum_{i,j=1}^{d_{\alpha}} |a_{ij}^\alpha| < +\infty \}$. Using Theorem 3.2, and repeating the proof of analogous theorems (see [1, Chapter IX] or [6, Chapter II]) with some changes, we obtain the following theorems.
**Theorem 3.3.** If \( f(g) \in L_2(G) \), then

\[
\sum_{n=1}^{\infty} \frac{\omega_k(f,1/n)^2}{\sqrt{n}} < +\infty \Rightarrow f(g) \in A(G).
\]

(3.20)

This theorem is analogous to the Szasz theorem of the classical Fourier series in the case where \( k = 1 \) and \( G = T \).

**Theorem 3.4.** If \( f(g) \in L_2(G) \), then

\[
\sum_{n=1}^{\infty} \frac{E_n(f)}{\sqrt{n}} < +\infty \Rightarrow f(g) \in A(G).
\]

(3.21)

This theorem is also analogous to a theorem in trigonometric case proved by Stechkin [9].

4. Applications to compact group \( SU(2) \). The group \( SU(2) \) consists of unimodular unitary matrices of the second order, that is, matrices of the form

\[
\begin{pmatrix}
\alpha & \beta \\
-\beta^* & \alpha^*
\end{pmatrix}, \quad |\alpha|^2 + |\beta|^2 = 1.
\]

(4.1)

Therefore, each element \( u \) of \( SU(2) \) is uniquely determined by a pair of complex numbers \( \alpha \) and \( \beta \) such that \( |\alpha|^2 + |\beta|^2 = 1 \). We have (see [5]) the relation “\((\alpha, \beta) \mapsto (\phi, \theta, \psi)\),” where \( \alpha \beta \neq 0 \), \( |\alpha|^2 + |\beta|^2 = 1 \), and the parameters \( \phi \), \( \theta \), and \( \psi \) are called Euler angles defined by

\[
|\alpha| = \cos \frac{\theta}{2}; \quad \text{Arg} \alpha = \frac{\phi + \psi}{2}; \quad \text{Arg} \beta = \frac{\phi - \psi}{2}.
\]

(4.2)

Let \( \phi \), \( \theta \), and \( \psi \) satisfy the conditions

\[
0 \leq \phi < 2\pi, \quad 0 \leq \theta < \pi, \quad -2\pi \leq \psi < 2\pi.
\]

(4.3)

Also, we know that the dimension of the representation \( T^l \) of \( SU(2) \) is equal to \( 2l + 1 \), where \( l = 0, 1/2, 1, \ldots \) and the matrix elements of \( T^l \) for group \( SU(2) \) are defined by

\[
t^l_{mn}(u) = e^{-(n\psi + m\phi)} p^l_{mn}(\cos \theta) t^{(m-n)}. \quad (4.4)
\]

Expressing \( t^l_{mn}(u) \) in terms of \( p^l_{mn}(\cos \theta) \), we arrive at the following conclusion:

Any function \( f(\phi, \theta, \psi) \), \( 0 \leq \phi < 2\pi, 0 \leq \theta < \pi, \) and \( -2\pi \leq \psi < 2\pi \) belonging to the space \( L^2(SU(2)) \) such that

\[
\int_{-2\pi}^{2\pi} \int_{0}^{2\pi} \int_{0}^{\pi} |f(\phi, \theta, \psi)|^2 \sin \theta \, d\theta \, d\phi \, d\psi < \infty
\]

(4.5)
can be expanded into the mean-convergent series

\[ f(\phi, \theta, \psi) = \sum_{l} \sum_{m=-l}^{l} \sum_{n=-l}^{l} \alpha_{mn}^l e^{-i(m\phi + n\psi)} p_{mn}^l(\cos \theta), \quad (4.6) \]

where

\[ \alpha_{mn}^l = \frac{2l+1}{16\pi^2} \int_{-2\pi}^{2\pi} \int_{0}^{0} \int_{0}^{\pi} \int_{-2\pi}^{2\pi} \int_{0}^{0} f(\phi, \theta, \psi) e^{i(m\phi + n\psi)} p_{mn}^l(\cos \theta) \sin \theta d\theta d\phi d\psi. \quad (4.7) \]

In addition, we obtain from Parseval's equality that

\[ \sum_{l} \sum_{m=-l}^{l} \sum_{n=-l}^{l} \frac{1}{2l+1} |\alpha_{mn}^l|^2 = \frac{1}{16\pi^2} \int_{-2\pi}^{2\pi} \int_{0}^{0} \int_{0}^{\pi} \int_{-2\pi}^{2\pi} \int_{0}^{0} |f(\phi, \theta, \psi)|^2 \sin \theta d\theta d\phi d\psi. \quad (4.8) \]

Using Theorem 3.2, we obtain the following theorem.

**Theorem 4.1.** If \( f(\phi, \theta, \psi) \in L_2(SU(2)) \), then

\[ E_n(f)_2 \leq \sqrt{1 + \frac{2}{n-1}} \omega_k(f, \frac{1}{n})_2, \]

\[ \left\{ \sum_{l \geq n} \sum_{m=-l}^{l} \sum_{n=-l}^{l} \frac{1}{2l+1} |\alpha_{mn}^l|^2 \right\}^{1/2} \leq \sqrt{1 + \frac{2}{n-1}} \omega_k(f, \frac{1}{n})_2. \quad (4.9) \]

Using the relation between the polynomial \( P_n^{(\alpha, \beta)}(z) \) and \( p_{mn}^l(z) \), we conclude that

\[ p_{mn}^l(z) = 2^{-m} \left[ \frac{(l-m)!(l+m)!}{(l-n)!(l+n)!} \right]^{1/2} (1-z)^{(m-n)/2}(1+z)^{(m+n)/2} P_{l-m}^{(m-n,m+n)}(z). \quad (4.10) \]

The Jacobi polynomials obtained here are characterized by the condition that \( \alpha \) and \( \beta \) are integers and \( n + \alpha + \beta \in \mathbb{Z}_+ \).

Now, we consider the following case.

Let \( L_2^{(\alpha, \beta)}[-1,1] \) be the Hilbert space of the functions \( f \) defined on the segment \([-1,1]\) with the scalar product

\[ (f_1, f_2) = \int_{-1}^{1} f_1(x)f_2(x)(1-x)^\alpha(1+x)^\beta dx; \quad (4.11) \]

then, any function \( f \) in this space is expanded into the mean-convergent series

\[ f(x) = \sum_{n=0}^{\infty} \alpha_n \hat{p}_n^{(\alpha, \beta)}(x), \quad (4.12) \]
where the polynomials $\hat{P}_n^{(\alpha,\beta)}(x)$ are given by
\[
\hat{P}_k^{(\alpha,\beta)}(x) = 2^{-(\alpha+\beta+1)/2} \left[ \frac{k!(k+\alpha+\beta)!((\alpha+\beta+2k+1))}{(k+\alpha)!(k+\beta)!} \right]^{1/2} P_k^{(\alpha,\beta)}(x),
\]
(4.13)

\[
\alpha_n = \int_{-1}^{1} f(x) \hat{P}_n^{(\alpha,\beta)}(x)(1-x)^\alpha(1+x)^\beta dx.
\]
(4.14)

The Parseval’s equality
\[
\int_{-1}^{1} |f(x)|^2 (1-x)^\alpha(1+x)^\beta dx = \sum_{n=0}^\infty |\alpha|^2
\]
holds. The formulas (4.12), (4.14), and (4.15) are proved for integral nonnegative values of $\alpha$ and $\beta$. We can show that they are valid for arbitrary real values of $\alpha$ and $\beta$ exceeding $-1$. Finally, we reach the following theorem.

**Theorem 4.2.** If $f(x) \in L_2[-1,1]$, then the following hold for Jacobi series:
\[
E_n(f)_2 \leq \sqrt{1 + \frac{2}{n-1} \omega_k \left( f, \frac{1}{n} \right)_2},
\]
\[
\left\{ \sum_{l=n}^\infty |\alpha_l|^2 \right\}^{1/2} \leq \sqrt{1 + \frac{2}{n-1} \omega_k \left( f, \frac{1}{n} \right)_2}.
\]
(4.16)

**NOTE.** For the ideas similar to this paper we refer to [2] and its references.

**ACKNOWLEDGMENTS.** This research was supported by Tabriz University. We would like to thank the research office of Tabriz University for its support.

**REFERENCES**

[1] W. K. Bari, *Trigonometric Series*, vol. II, Holt, Rinehart and Winston, New York, 1967.

[2] G. Benke, *Bernšte˘ın’s theorem for compact groups*, J. Funct. Anal. 35 (1980), no. 3, 295–303.

[3] R. E. Edwards, *Fourier series. A Modern Introduction. Vol. I*, 2nd ed., Graduate Texts in Mathematics, vol. 64, Springer-Verlag, New York, 1979.

[4] S. Helgason, *Groups and Geometric Analysis*, Pure and Applied Mathematics, vol. 113, Academic Press, Florida, 1984.

[5] E. Hewitt and K. A. Ross, *Abstract Harmonic Analysis. Vol. II: Structure and Analysis for Compact Groups. Analysis on Locally Compact Abelian Groups*, Die Grundlehren der Mathematischen Wissenschaften, vol. 152, Springer-Verlag, New York, 1970 (German).

[6] J.-P. Kahane, *Séries de Fourier absolument convergentes*, Ergebnisse der Mathematik und ihrer Grenzgebiete, vol. 50, Springer-Verlag, Berlin, 1970 (French).

[7] M. A. Na ˘ımark and A. I. Štern, *Theory of Group Representations*, Grundlehren der Mathematischen Wissenschaften, vol. 246, Springer-Verlag, New York, 1982 (German).
[8] S. F. Rzaev, *L₂-Approximation on compact groups*, Proc. “Questions on Functional Analysis and Mathematical Physics Conference”, Baku, 1999, pp. 418–419.

[9] S. B. Stechkin, *On absolute convergence of orthogonal series*, Dokl. Akad. Nauk. SSSR 102 (1955), 37–40.

[10] N. Ja. Vilenkin and A. U. Klimyk, *Representation of Lie groups and Special Functions. Vol. 1*, Mathematics and Its Applications, vol. 72, Kluwer Academic Publishers, Dordrecht, 1991.

[11] A. Zygmund, *Trigonometric Series. 2nd ed. Vols. I, II*, Cambridge University Press, New York, 1959.

H. Vaezi: Faculty of Mathematical Sciences, University of Tabriz, Tabriz, Iran
E-mail address: hvaezi@tabrizu.ac.ir

S. F. Rzaev: Institute of Mathematics and Mechanics, Azerbaijan Academy of Sciences, Baku, Azerbaijan
E-mail address: rzseymur@hotmail.com
Special Issue on
Time-Dependent Billiards

Call for Papers

This subject has been extensively studied in the past years for one-, two-, and three-dimensional space. Additionally, such dynamical systems can exhibit a very important and still unexplained phenomenon, called as the Fermi acceleration phenomenon. Basically, the phenomenon of Fermi acceleration (FA) is a process in which a classical particle can acquire unbounded energy from collisions with a heavy moving wall. This phenomenon was originally proposed by Enrico Fermi in 1949 as a possible explanation of the origin of the large energies of the cosmic particles. His original model was then modified and considered under different approaches and using many versions. Moreover, applications of FA have been of a large broad interest in many different fields of science including plasma physics, astrophysics, atomic physics, optics, and time-dependent billiard problems and they are useful for controlling chaos in Engineering and dynamical systems exhibiting chaos (both conservative and dissipative chaos).

We intend to publish in this special issue papers reporting research on time-dependent billiards. The topic includes both conservative and dissipative dynamics. Papers discussing dynamical properties, statistical and mathematical results, stability investigation of the phase space structure, the phenomenon of Fermi acceleration, conditions for having suppression of Fermi acceleration, and computational and numerical methods for exploring these structures and applications are welcome.

To be acceptable for publication in the special issue of Mathematical Problems in Engineering, papers must make significant, original, and correct contributions to one or more of the topics above mentioned. Mathematical papers regarding the topics above are also welcome.

Authors should follow the Mathematical Problems in Engineering manuscript format described at http://www.hindawi.com/journals/mpe/. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http:// mts.hindawi.com/ according to the following timetable:

| Event                        | Date       |
|------------------------------|------------|
| Manuscript Due               | March 1, 2009 |
| First Round of Reviews       | June 1, 2009 |
| Publication Date             | September 1, 2009 |

Guest Editors

Edson Denis Leonel, Department of Statistics, Applied Mathematics and Computing, Institute of Geosciences and Exact Sciences, State University of São Paulo at Rio Claro, Avenida 24A, 1515 Bela Vista, 13506-700 Rio Claro, SP, Brazil; edleonel@rc.unesp.br

Alexander Loskutov, Physics Faculty, Moscow State University, Vorob’evy Gory, Moscow 119992, Russia; loskutov@chaos.phys.msu.ru