Differential Transfer Relations of Physical Flux Density Between Time Domains of the Flux Source and Observer

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Differential transfer relations of flux density, general physics quantities' and its corresponding energy's, between time domains of source and observer are derived from conservative rule of various physical quantities and time function between the two domains. In addition, integrated form of the relations are inferred and examples of potential application are illustrated and discussed.

I. INTRODUCTION

Observing whole physical transportation process of a specified micro flux increment from being emitted to detected during a pair of corresponding time intervals on time domains of source and observer respectively, a fundamental fact can be realized that both variations of physical quantity due to possible loss or energy exchange in the process and transportational time for different flux signals within the micro flow element can contribute detected flux density at observing location significantly. Physical quantity, here divided into two main categories — general physics quantity such as phase of waves, mass, charge and particles’ number and micro system’s energy carried by the micro flow element consisted of a kind of or a compound of general physics quantities like a micro multi-charged particle system, contributes the detectable flux density in two ways; for negative contribution, some quantity of the micro flow which originated from source might be lost in the transportational process and fails to reach the observer; for energy exchange related contribution to energy flux density at observer, an amount of energy originated in external system joins or adds to the micro system’s energy for the system’s energy gain till to reach the observer, whereas it is converse for the system’s energy release. Meanwhile a variation of the flux signal’s transportation time will result in a variation of time interval in which the flux element is received by observer with respect to the time interval of the flux element emission at source, so that it affects detecting flux density with time factor. In addition, a variation of longitudinal length of the micro flux element along its moving direction, as a particular physics quantity, exists in the process and accompanied with their physical quantity’s differential transfer relation simultaneously. By combining conservative rule of physical quantity with a knowledge of time function between time domains of signal source and observer [1], we will set up the three differential transfer relations which are accompanied or combined for any kind of physics flux between time domains of flux source and observer, and present integral form of the differential transfer relations, their basic properties and their potentially important application in a few cases.

II. DERIVATION OF THE DIFFERENTIAL TRANSFER RELATIONS, BASIC PROPERTIES AND INFERENCE

A. Derivation of the Three Differential Transfer Relations

For any micro continuous flow consisted of a kind of physical quantity from source to observer, there are three essential differential transfer relations accompanied with the transportation process simultaneously; they are the micro flux increment themselves, corresponding longitudinal length of the micro flow and energy flux, therefore the three transfer relations must share same time related characteristics or time relations.

Specifically there is the transportation process as following:

A micro flux increment \( \Delta Q(t) \) is released from instants \( t - \Delta t \) to \( t \), then pass through the space between source and observer during transfer time \( T(t - \Delta t) \) and \( T(t) \) for head and tail of the micro flow element respectively, and finally are detected at observing location from instant \( t' - \Delta t' = t - \Delta t + T(t - \Delta t) \) to instant \( t' = t + T(t) \) [1], here \( t \in [t, t + T(t)] \). Consequently the differential transfer relations can be derived below.
1. General Physics Flux Density’s Differential Transfer Relation

From factual relation experienced by the micro flux increment \( \Delta Q(t) \) in transportation process, where \( \Delta Q_l(t) \) denotes the loss amount of physical quantity from \( \Delta Q(t) \),

\[
\frac{d\Delta Q(i)}{dt} = -\frac{d\Delta Q_l(i)}{dt} \quad i \in [t, t']
\]  

(1)

\[
\int_t^{t'} \frac{d\Delta Q(i)}{dt} dt = -\int_t^{t'} \frac{d\Delta Q_l(i)}{dt} dt
\]

or

\[
\int_{\Delta Q(t')}^{\Delta Q(t)} d\Delta Q(i) = -\int_{\Delta Q(t')}^{0} d\Delta Q_l(i)
\]

(1')

there

\[
\Delta Q(t') + \Delta Q_l(t') = \Delta Q(t)
\]

(2)

to multiply Eq. 2 with \( \frac{1}{\Delta t} \) or \( \frac{1}{\Delta t'} \), then take a limit as \( \Delta t \) and \( \Delta t' \to 0 \)

\[
\frac{dQ(t')}{dt'} + \frac{dQ_l(t')}{dt'} = \frac{dQ(t)}{dt} \frac{dt}{dt'}
\]

or

\[
J(t') + J_l(t') = J(t) \frac{dt}{dt'}
\]

(3)

here \( J(t') > 0, J(t) > 0 \). \( J(t') \) and \( J(t) \) are instantaneous flux or rate of physical quantity’s flow corresponding to instant \( t' \) at observing location and instant \( t \) at flux source respectively; \( J_l(t') \) is rate of a total accumulated flux loss at instant \( t' \) at detecting position caused by the possible loss amount of physical quantity during the transfer time \( t' - t = T(t) = T(t') \); and

\[
J_l(t') = \frac{dQ_l(t')}{dt'} = \int_t^{t'} \frac{d^2Q_l(i)}{dt^2} di = \int_0^{\Delta Q(i)} \frac{dQ_l(i)}{dt} d[\frac{dQ_l(i)}{dt}] \geq 0
\]

(3')

2. Transfer Relation of Longitudinal Length of the micro flow along moving direction

As a particular physics flux, the length can be either compressed or elongated. So except that the micro length \( \Delta \ell(t') \) could be increased or lengthened whereas the general micro increment \( \Delta Q(i) \) can only be decreased in general due to \( \Delta Q_l(t) \), it has same transfer relations as general physics flux has.

\[
\frac{d\Delta \ell(t)}{dt} = -\frac{d\Delta \ell_l(t)}{dt} \quad i \in [t, t + T(t)]
\]  

(4)

\[
\int_t^{t'} \frac{d\Delta \ell(t)}{dt} dt = -\int_t^{t'} \frac{d\Delta \ell_l(t)}{dt} dt
\]

or

\[
\int_{\Delta \ell(t)}^{\Delta \ell(t')} d\Delta \ell(t) = -\int_{\Delta \ell_l(t)}^{0} d\Delta \ell_l(t)
\]

(4')

\[
\Delta \ell(t') + \Delta \ell_l(t') = \Delta \ell(t)
\]

(5)
\[
\frac{d\ell(t')}{dt'} + \frac{d\ell_c(t')}{dt'} = \frac{d\ell(t)}{dt} \frac{dt}{dt'}
\]

\[
v_{io}(t') + v_c(t') = v_{sis}(t) \frac{dt}{dt'}
\]  \hspace{1cm} (6)

Where \(\Delta \ell_c(t)\) denotes the compressed length in transportation process whereas \(\Delta \ell(t') = \int_0^{t'} \frac{d\ell_c(t)}{dt} dt\) denotes the total accumulated length of compression during whole process. \(T = t' - t\); \(v_c(t')\) is a rate of total accumulated compressed length passed through observing point; when \(\Delta \ell_c(t), \Delta \ell(t)\) and \(v_c(t')\) takes positive values they reflect compression related amount respectively, whereas negative values express the opposites’ of compression or elongation related amount.

With a variation of the length, have a change of longitudinal density of the flux quantity and relevant consequence been involved.

\[
\frac{\rho(t')}{\rho(t)} = \frac{\Delta Q(t')/\Delta \ell(t')}{\Delta Q(t)/\Delta \ell(t)} = \frac{\Delta Q(t')/\Delta \ell(t')}{\Delta Q(t)/\Delta \ell(t)}
\]

\[
= \frac{\Delta Q(t) - \Delta Q(t')}{\Delta Q(t)} v_{sis}(t) dt = \frac{\Delta Q(t) - \Delta Q(t')}{\Delta Q(t)} v_{io}(t') dt'
\]

\[
= \left[1 - \frac{\Delta Q(t')}{\Delta Q(t)}\right] \left[1 + \frac{\Delta \ell_c(t')}{\Delta \ell(t')}\right]
\]  \hspace{1cm} (7)

This relates to interaction between the \(\Delta Q(t)\) and external system and to energy exchange in the transportation process.

3. Energy Transfer and Exchange Relations\(vc\) Respect to the \(\Delta Q(t)\)

From factual relations and defined restricted condition or terms in below

\[
\frac{d\Delta E(t)}{dt} = \frac{d\Delta E_{exc}(t)}{dt} - \frac{d\Delta E_i(t)}{dt} \quad t \in [t, t + T(t)]
\]  \hspace{1cm} (7)

\[
\int_t^{t'} \frac{d\Delta E(t)}{dt} dt = \int_t^{t'} \frac{d\Delta E_{exc}(t)}{dt} dt - \int_t^{t'} \frac{d\Delta E_i(t)}{dt} dt
\]
or

\[
\int_{\Delta E(t)}^{\Delta E(t')} d\Delta E(t) = \int_{0}^{\Delta E(t')} d\Delta E_{exc}(t) - \int_{0}^{\Delta E_i(t')=\Delta E(t)-\Delta E_{exc}(t)} d\Delta E_i(t)
\]  \hspace{1cm} (7)

there

\[\Delta E(t') - \Delta E_{exc}(t') + \Delta E_i(t') = \Delta E(t)\]  \hspace{1cm} (8)

and

\[
\frac{dE(t')}{dt'} - \frac{dE_{exc}(t')}{dt'} + \frac{dE_i(t')}{dt'} = \frac{dE(t)}{dt} \frac{dt}{dt'}
\]
or

\[J(t') - J_{exc}(t') + J_i(t') = J(t) \frac{dt}{dt'}\]
or

\[P(t') - J_{exc}(t') + J_i(t') = P(t) \frac{dt}{dt'}\]  \hspace{1cm} (9)
Where \( \Delta E(t) \) and \( \Delta E(t') \) are the corresponding total energy carried by the micro flux increment \( \Delta Q(t) \) and \( \Delta Q(t') \) respectively; \( \Delta E_{\text{se}}(t) \) is a fractional part of \( \Delta E(t) \) and it is carried by \( \Delta Q(t) \) from the flux source to detecting section as a contributed part of energy flux source to observational energy flux at detecting position.

So exchange energy \( \Delta E_{\text{exc}}(t) \) could have positive values for \( \Delta Q(t) \)'s energy gain or negative values for \( \Delta Q(t) \)'s energy loss.

\[ \mathcal{J}(t'), \mathcal{J}(t) \] are energy flux which correspond to \( J(t') = \frac{dE(t')}{dt} \) and \( J(t) = \frac{dE(t)}{dt} \) respectively; here \( \mathcal{J}(t') > 0, \mathcal{J}(t) > 0, \) and

\[ \mathcal{J}(t') = \frac{dE(t')}{dt} = \int_t^{t'} \frac{d^2E(t')}{dt^2} \, dt \]

\[ \mathcal{J}_{\text{exc}}(t') = \frac{dE_{\text{exc}}(t')}{dt} = \int_t^{t'} \frac{d^2E_{\text{exc}}(t')}{dt^2} \, dt \]

\[ \begin{cases} 
> 0 & \text{energy gain of } \Delta Q(t') \\
< 0 & \text{energy loss of } \Delta Q(t') \\
= 0 & \text{no energy exchange of } \Delta Q(t') \text{ with outside} 
\end{cases} \]

B. Basic Properties — Simultaneity, Independent Conservation and Dependent on Rate of Signal Transfer Time

1. Simultaneity of the Accompanied Variation of \( \Delta Q(t) \), \( \Delta t(t) \), \( \Delta E(t) \), the Three Essential Transfer Relations and Time Function Factor

Any change on the physical quantity in the micro increment, longitudinal length, energy carried by \( \Delta Q(t') \) and physical or differential state inside of the micro flux element occurs simultaneously and is inner related.

2. Independent Conservations of Physical Quantity and Energy

Physical quantity and carried energy of a micro flux increment are independently conserved in their transportation process (Ref. Eqs. 1 and 7) and their differential transfer relations between time domains of source and observers; From equation 11

\[ \int_t^{t+T(t)} \frac{d\Delta Q(t)}{dt} \, dt = - \int_t^{t+T(t)} \frac{d\Delta Q(t)}{dt} \, dt \]

there its derivative for \( t' \)

\[ \frac{d\Delta Q(t')}{dt'} + \frac{d\Delta Q(t')}{dt'} = \frac{d\Delta Q(t)}{dt} \left[ 1 - \frac{dT(t')}{dt'} \right] \]

or

\[ \frac{d\Delta Q(t')}{dt'} + \frac{d\Delta Q(t')}{dt'} = \frac{d\Delta Q(t)}{dt} \frac{dt}{dt'} \]

then

\[ \int_{t_1}^{t_2} \left[ \frac{d\Delta Q(t)}{dt} + \frac{d\Delta Q(t)}{dt} \right] \, dt = \int_{t_1}^{t_2} \frac{d\Delta Q(t)}{dt} \, dt \]

\[ \Delta Q(t_2') + \Delta Q(t_2') - \Delta Q(t_2) = \Delta Q(t_1') + \Delta Q(t_1') - \Delta Q(t_1) \]

thus

\[ \Delta Q(t') + \Delta Q(t') - \Delta Q(t) \equiv 0 \]

Similarly, from equation 11

\[ \int_{v-T(v)}^{v'} \frac{d\Delta E(t)}{dt} \, dt = \int_{v-T(v)}^{v'} \frac{d\Delta E_{\text{exc}}(t)}{dt} \, dt \]

\[ - \int_{v-T(v)}^{v'} \frac{d\Delta E(t)}{dt} \, dt \]
take its derivative for \( t' \), then integration on corresponding \([t'_1, t'_2]\) and \([t_1, t_2]\), there

\[
\Delta E(t') - \Delta E_{exc}(t') + \Delta E_i(t') - \Delta E(t) \equiv 0
\] (12)

3. Flux’s Dependence on the Time Function Factor — Rate of Signal Transfer Time

Differential transfer relations of general physics flux and energy flux are time functions’ derivative dependent (Ref. Eqs. \[3\] and \[4\]), here (Ref. \[1\])

\[
t'(t) = t + T(t), \quad \frac{dt'}{dt} = 1 + \frac{dT(t)}{dt}; \quad T(t) = T(t')
\]

or

\[
t(t') = t' - T(t'), \quad \frac{dt}{dt'} = 1 - \frac{dT(t')}{dt'}
\]

as the time function is reversible and longitudinal length and relative velocity spread transfer relations

\[
\Delta \ell(t') - \Delta \ell(t) = \Delta \ell(t)\big|_{t'} - \Delta \ell(t)\big|_t = \int_t^{t'} \left[ v_{is}(i, t - \Delta t) - v_{is}(i, t) \right] dt
\] (13)

there

\[
v_{io}(t')\frac{dt}{dt'} - v_{sis}(t) = \int_t^{t'} \frac{\partial v_{is}(i, r_{is})}{\partial r_{is}} v_{is}(i, t) \frac{dt}{dt'} dt
\] (14)

or

\[
\frac{dt(t')}{dt(t)} = \frac{v_{io}(t')dt'}{v_{sis}(t)dt} = 1 + \frac{1}{v_{sis}(t)} \int_t^{t'} \frac{\partial v_{is}(i, r_{is})}{\partial r_{is}} v_{is}(i, t) \frac{dt}{dt'} dt
\] (15)

with to multiply equations \[13\] and \[14\], then take limitation as \( \Delta t_j(\dot{i}) \) and \( \Delta t' \to 0 \)

\[
\frac{d\Delta \ell(\dot{i})}{dt}\big|_{t'} - \frac{d\Delta \ell(\dot{i})}{dt}\big|_t = \int_t^{t'} \left[ a_{is}(i, t - \Delta t) - a_{is}(i, t) \right] dt
\] (15)

and

\[
\frac{\partial v_{is}(t', r_{is})}{\partial r_{is}} v_{io}(t') \frac{dt}{dt'} - \frac{\partial v_{is}(t, r_{is})}{\partial r_{is}} v_{sis}(t) = \int_t^{t'} \frac{\partial a_{is}(i, r_{is})}{\partial r_{is}} v_{is}(i, t) \frac{dt}{dt'} dt
\] (16)

or

\[
\frac{d\Delta \ell(\dot{i})}{\Delta t dt}\big|_{t'} = \frac{\partial v_{is}(t', r_{is})}{\partial r_{is}} v_{io}(t')\frac{dt'}{dt} - \frac{\partial v_{is}(t, r_{is})}{\partial r_{is}} v_{sis}(t) dt = 1 + \frac{1}{a_{is}(t) + a_{is}(t, t)} \int_t^{t'} \frac{\partial a_{is}(i, r_{is})}{\partial r_{is}} v_{is}(i, t) \frac{dt}{dt'} dt
\] (16)

Where \( \Delta t_j(\dot{i}) \) is time interval during which the \( \Delta \ell(\dot{i}) \) pass through a specific point, that has a relative distance \( r_{is} \) with respect to signal source; \( v_{sis}(t) \) is initial relative velocity of head of the length with respect to signal source, \( v_{io}(t') \) is a relative velocity of the length’s head with respect to observing point; \( a_{is}(t) \) and \( a_{is}(t') \) are corresponding accelerations respectively.
C. Major Inference From Eq. (3) and (9)

1. General Physics Flux Transfer Function between Two Time Domains

\[ Q(t') + Q_l(t') = Q(t_0) + \int_{t_0}^{t'-T(t')} J(\dot{t})d\dot{t} \]  

(17)

or

\[ \int_{t_0}^{t'} [J(\dot{t}) + J_l(\dot{t})]d\dot{t} = \int_{t_0}^{t} J(\dot{t})d\dot{t} \]  

(17)

2. Energy Flux Transfer Function between The Time Domains

\[ \int_{t_0}^{t'} [\mathcal{J}(\dot{t}) - \mathcal{J}_{exc}(\dot{t}) + \mathcal{J}_l(\dot{t})]d\dot{t} = \int_{t_0}^{t} \mathcal{J}(\dot{t})d\dot{t} \]  

(18)

or

\[ E(t') - E_{exc}(t') + E_l(t') = E(t_0) + \int_{t_0}^{t'-T(t')} \mathcal{J}(\dot{t})d\dot{t} \]  

(18)

Both of the transfer functions are ever increasing function.

III. ILLUSTRATION AND DISCUSSION

A. Examples of the Relations’ Use

1. Phase Flux Density and Consequent Energy Flux Density

For wave phase flux, \( \frac{d\Delta \varphi_l(\dot{t})}{d\dot{t}} \equiv 0, \frac{d\Delta \varphi_l(t')}{dt'} \equiv 0 \), from equation (3) there

\[ \omega'(t') = \omega(t)[1 - \frac{dT(t')}{dt'}] \]

or

\[ \omega'(t') \left[ 1 + \frac{dT(t')}{dt'} \right] = \omega(t) \]  

(19)

or

\[ \int_{t_0}^{t'} \omega'(\dot{t})d\dot{t} = \int_{t_0}^{t} \omega(\dot{t})d\dot{t} \]  

(20)

and from equation (17), the phase transfer function

\[ \varphi(t') = \varphi(t_0) + \int_{t_0}^{t} \omega(\dot{t})d\dot{t} \]  

(20)

\[ \omega'(t') = \frac{d\varphi(t')}{dt'} \] is observer’s measuring angular frequency; \( \omega(t) = \frac{d\varphi(t)}{dt} \) is emitting angular velocity or frequency at phase flux source for the same micro phase increment.
From equation 19 and suppose \( \mathcal{J}_{exc}(t') = \frac{dE_{exc}(t')}{dt} = 0 \) in non-absorption media,

\[
\mathcal{J}(t') = \mathcal{J}(t) \left[ 1 - \frac{dT(t')}{dt'} \right] - \int_{t' - T(t')}^{t'} \frac{d\mathcal{J}(t)}{dt} dt \\
= \mathcal{J}(t) \left[ 1 - \frac{dT(t')}{dt'} \right] - T(t') \frac{d\mathcal{J}(t)}{dt} \xi \\
\xi \in \{ t' - T(t'), t' \}
\]

or \( P(t') = P(t) \left[ 1 - \frac{dT(t')}{dt'} \right] - T(t') \frac{d\mathcal{J}(t)}{dt} \xi \) (21)

According to the electromagnetic wave energy flux relation, observer’s measuring or measured optical intensity not only has been determined by optical energy’s spatial diminishing factor related with \( T(t) \), but also depend upon time function’s factor which is determined by a rate of signal transfer time. From equation 19, it’s known that time function’s effect must be considered in frequency-shift relevant measures and data analysis.

2. Charged Particles Flux and Energy Flux in Linac

For charged particles flux in linear accelerator there exists the particles’ number relation of a micro increment

\[
\Delta N(t') + \Delta N_i(t') = \Delta N(t) \quad \text{and} \quad \Delta N(t') \leq \Delta N(t) \quad (22)
\]

consequently

\[
\Delta N(t')q + \Delta N_i(t')q = \Delta N(t)q \quad \Delta N(t')q \leq \Delta N(t)q
\]

\[
\Delta N(t')m_q + \Delta N_i(t')m_q = \Delta N(t)m_q \quad \Delta N(t')m_q \leq \Delta N(t)m_q
\]

This means that the number of particles in the micro element will diminish as they are accelerated. However, according to the number’s flux differential transfer relation

\[
\frac{dN(t')}{dt'} = \frac{dN(t)}{dt} \left[ 1 - \frac{dT(t')}{dt'} \right] - \frac{dN_i(t')}{dt'} \quad (23)
\]

at observing location instantaneous number flux can exceed the emitting number flux due to time function related factor when \( \frac{dT(t')}{dt'} < 0 \).

Furthermore, as to rf accelerator \( T(t) \) or \( T(t') \) is periodic function with a designated \( \tau^* \), \( T(t + \tau^*) = T(t) \) and \( T(t' + \tau^*) = T(t') \); hence on different time domains along higher energy of the micro flux increment, there will be ever increasing high frequency fractional factor in a Fourier analysis on number flux, charge flux, mass flux and energy flux, although number of the particles, and corresponding mass and charge of the specific micro increment will decrease continuously in the transportation process.

According to equation 18 the carried energy of the micro increment, \( \Delta E(t) \), can be progressively increased by positive \( \Delta E_{exc}(t) \) values; then

\[
\Delta E(t') = \Delta E(t) + \Delta E_{exc}(t') - \Delta E_i(t') \geq \Delta E(t)
\]

(24)

Compare equation 22

\[
\Delta M(t') = \Delta M(t) - \Delta M_i(t') \leq \Delta M(t)
\]

with equation 21, we can make an inference that the energy gain \( \Delta E(t') - \Delta E(t) \) can not be stored as or converted into any mass of the micro element for \( \Delta M(t') - \Delta M(t) \leq 0 \). Then where has been the energy gain stored in the micro flux increment? We will find it stored as internal electrical potential energy in the micro increment, and partially converted to the mass center’s kinetic energy of the micro increment 2.
B. Potential Application

A. By means of phase transfer function (Ref. Eqs. 19, 20), the phase function on observer time domain at measuring location of wave interference region can be transformed to phase source time domain on which the angular frequency or velocity is known. Then phase difference on observer time domain is expressed by term of time function and known or designated parameter of phase source [3].

B. Energy flux analysis on frequency-shift astronomical measurement [4].

C. As an analytic approach one longitudinal properties, energy exchange and flux of charged particles in acceleration [2].

IV. CONCLUSION

The differential transfer relations of general physics flux between time domains of source and observer, accompanied with a longitudinal length’s transfer relation, and corresponding energy flux occur or exist simultaneously; and they reflect different aspect of a specified flux increment’s transportation process between source and observer. Time function factor could significantly affect the values of corresponding fluxes on two time domains and itself or rate of it is a necessary fraction of the transfer relations. The differential transfer relations are set up on conversation of various physics quantities independently and shared time transfer relation. Based on the differential transfer relation of flux, its integral form — physical quantity transfer function between the time domains is inferred.

[1] J. Luo, *Time function of signal transportation between time domains of signal source and observer*, (In Preparation).
[2] J. Luo and C. Zhang, *Energy differential structure and exchange of specified micro flux increment of charged particles in longitudinal acceleration*.
[3] J. Luo, *Phase difference function and unified equations of steady and non-steady interference*.
[4] J. Luo, *Distance related to red shift caused by dispersion medium and characteristic optical source*, (In Preparation).