Non-Equilibrium Magnetization in a Ballistic Quantum Dot

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Abstract

We show that Aharonov-Bohm (AB) oscillations in the magnetic moment of an integrable ballistic quantum dot can be destroyed by a time dependent magnetic flux. The effect is due to a nonequilibrium population of perfectly coherent electronic states. For real ballistic systems the equilibrization process, which involves a special type of inelastic electron backscattering, can be so ineffective, that AB oscillations are suppressed when the flux varies with frequency $\omega \sim 10^7$-$10^8$ s$^{-1}$. The effect can be used to measure relaxation times for inelastic backscattering.

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The magnetic moment (and the associated persistent current) induced by a magnetic flux in small conductors is a dramatic manifestation of mesoscopic behavior. While originally predicted to appear in clean one-dimensional (1D) metallic rings [1], much of the recent discussion about persistent currents has focused on metallic rings containing impurities [2,3]. An important effect of the inevitable impurity scattering in such systems is that ‘forbidden gaps’ appear in the spectrum of quantized electron energy levels when plotted as a function of magnetic flux. These gaps lead to an oscillatory dependence of the single electron energies on flux and consequently to a periodic magnetization of small rings and dots [4].

The response to a magnetic flux that does not vary slowly in time may differ from the quasi-static magnetization because of Landau-Zener macroscopic tunneling through the disorder-induced gaps [5]. In sufficiently ‘clean’ systems such tunneling permits the energy levels to take their free-electron values. For integrable ballistic 2D structures formed in gated semiconductor heterostructures, the required minimum rate of change of flux, $\omega_{\text{min}}$, can be low enough to be experimentally accessible [6]. Recent experiments convincingly show Aharonov-Bohm (AB) oscillations in a 2D ballistic ring [7] and the magnetization of ballistic squares has also recently been measured [8]. Even without any gaps in the energy spectrum a varying magnetic field may result in a magnetization that oscillates with flux as observed. This is due to a completely different mechanism involving a redistribution of electrons between levels which are shifted up or down in energy by the changing flux. The redistribution depends on inelastic relaxation processes that cannot be associated with conventional one-phonon scattering. This is because the relaxation of the induced magnetic moment requires simultaneously a large momentum- and a small energy transfer. In this Letter we point out that, as a result, the effective relaxation time $\tau$ is large and one can expect strong non-equilibrium effects in the magnetization. The degree of non-equilibrium depends on the rate of change of flux, $\omega$. If $\omega_{\text{min}} \ll \omega \ll 1/\tau$ we find that the response of a pure quantum dot is quasi-static; i.e. the magnetization takes on values corresponding to equilibrium at the instantaneous magnitude of the flux. The variation — both in magnitude and sign — of the equilibrium magnetic moment with magnetic flux $\Phi$ is characterized by two
distinctive scales. In addition to a flux variation on the Aharonov-Bohm scale of $\Phi_0 = \hbar c/e$, the flux quantum, there are oscillations on a smaller scale $\Phi_0/k_Fa$ that gives a fine structure to the flux dependence of the induced magnetic moment $\Psi$ ($k_F$ is the Fermi wave vector, $a$ is the dot radius).

A time dependent magnetic flux tends to drive the system out of equilibrium. Two distinct types of non-equilibrium effects appear. Even not very far from equilibrium, $\omega\tau \lesssim 1/(k_Fa)$, the fact that relaxation is not instantaneous has the effect of blurring the fine-structure in the flux dependence of the magnetic moment. However, the oscillations of the Aharonov-Bohm type on the larger scale of $\Phi_0$ remain intact. Strong non-equilibrium behavior appears when $\omega\tau$ becomes of order unity or bigger; then the AB oscillations are also washed out and a non-equilibrium diamagnetic moment develops. If magnetic flux increases (from zero) linearly with time, so does the moment — until time $\tau$ when inelastic backscattering is strong enough to stabilize the diamagnetic response.

Below we will consider a ‘clean’ quantum dot in a 2D electron gas, for which the electron time-of-flight around the circumference, $a/v_F$, is the shortest time of the problem, $a/v_F \ll \omega^{-1}, \tau \ll \omega^{-1}_{\text{min}}$. Consequently, the electronic eigenstates remain well defined in a varying external field and the system stays coherent. The only result of a time dependent magnetic flux is a shift of the electronic energy levels and hence a shifted electron distribution in momentum space that stays as sharp as the original one; no broadening whatsoever appears due to the time variation of the magnetic field. The non-equilibrium behavior is determined by the kinetics of the energy level population, which depends on the interplay between the driving magnetic field (time-scale $\omega^{-1}$) and relaxation processes ($\tau$). The unusual effect of a lack of scattering is that quantum oscillations in the magnetization are destroyed. Mainly because the levels remain sharp, there is a possibility for the oscillations to be restored by inelastic scattering.

As an illustration it is useful to first discuss electrons confined to a 1D ring. The eigenenergies $E_m = E_0(m - \alpha)^2$ [$E_0 = h^2/2m^*a^2$ and $\alpha \equiv \Phi/\Phi_0$] of a ring are sensitive to flux and can be labelled by a magnetic quantum number $m = 0, \pm 1, \pm 2, \ldots$. In zero flux the pop-
ulation of $\pm m$-states is symmetric, as in Fig. 1a, and there is no net magnetic moment. Increasing the flux $\alpha$ shifts the states with negative (positive) quantum numbers to higher (lower) energies as shown in Fig. 1b, and a net magnetic moment appears. At the same time, a relaxation of the electron system becomes possible through transitions between high energy $-m$ and low energy $+m$-states (Fig. 1b). The interplay between the flux-driven shift of energy levels and the ‘backflow’ due to relaxation determines the kinetics of the system.

Fig. 1 explicitly illustrates the main features of the relaxation processes that can lead to an equilibration of the momentum distribution. First, they have to be inelastic to compensate for the energy mismatch that is a result of the quantization of energy levels. Secondly, it is necessary that they can provide the large momentum required to reverse the direction of the azimuthal component of the electron momentum ($-m \rightarrow +m$). We emphasize that this type of ‘inelastic backscattering’ does not play a role in ordinary transport problems; it is a special feature of our system. We will discuss possible mechanisms for inelastic backscattering later.

The above qualitative discussion is relevant also for a quantum dot as only the azimuthal motion of the electron important for generation and relaxation of a magnetic moment. In the following we will discuss the quantum dot in a weak magnetic field, where the quantized free electron energies can be expressed as

$$E_{m,n} = E_0 \left[ \gamma_{m,n} + 2m\alpha + \frac{\alpha^2}{3} \left( 1 + \frac{2(m^2-1)}{\gamma_{m,n}^2} \right) \right].$$

(1)

Here $\gamma_{m,n}$ is the $n$:th root of the $m$:th Bessel function, i.e. $J_m(\gamma_{m,n}) = 0$, $n = 1, 2, \ldots$ and $m = 0, \pm 1, \pm 2, \ldots$. We can identify two different energy scales in this spectrum. The smaller scale, $E_0$, corresponds to the average spacing between levels in zero flux; the larger scale, $\Delta E = E_0 m \sim E_0(k_Fa)$ is set by the characteristic shift of the energy levels when the flux is increased by $\Phi_0$. The temperature dependence of the fluctuations in magnetization is related to these energy scales; the fine structure oscillations start to disappear at temperatures of about $T_1 = E_F/k_B N$ (here $N \sim (k_Fa)^2$ is the number of particles in the dot and $E_F$ is the Fermi energy) while the large scale oscillations only begin to vanish at the higher
temperature $T_2 = E_F/k_B\sqrt{N}$.

In the quasi-static limit the induced moment can be obtained directly from the thermodynamic potential of the system. One finds

$$M_{eq} = \frac{c}{S} \sum_{m,n} n_F(E_{m,n}) M_{m,n},$$  \hspace{1cm} (2)

where $M_{m,n}$ is the magnetic moment of a single quantum state $(m,n)$,

$$M_{m,n} = -M_0 \left[ 2m + \frac{2\alpha}{\gamma_{m,n}} \left( 1 + \frac{2(m^2 - 1)}{\gamma_{m,n}^2} \right) \right].$$ \hspace{1cm} (3)

Here $M_0 = \frac{\pi a^2 E_0}{\Phi_0}$, $n_F(E) = \left[ 1 + e^{(E-\mu)/k_B T} \right]^{-1}$ is the Fermi distribution function. Results at different temperatures for the magnetic moment as a function of magnetic flux for the two cases of constant chemical potential (dot connected to reservoir[s]) and constant number of particles \[12\] (isolated dot) are given in Fig. 2. We note that the flux dependence of the induced moment appears qualitatively quite similar for the dot and a metallic ring. An important quantitative difference is that the amplitude of the moment fluctuations in the ‘clean’ dot is of order $\pi a e v_F/c$, rather than $\pi e v_F \ell/c$ as in a ‘dirty’ metallic ring when $\ell \ll a$. Induced moments on this scale was recently observed in a ballistic ring system \[7\].

If the time variation of the magnetic flux is not slow, essential differences appear; a nonequilibrium distribution function, $f_{m,n}(t)$, has to replace the Fermi function $n_F[E_{m,n}(\alpha(t))]$ when the moment is calculated from Eq. (2). To get explicit results we restrict ourselves to the simplest case for which the relaxation time approximation applies. Here

$$\frac{\partial f_{\gamma}}{\partial t} = -\frac{f_{\gamma}(t) - n_F[E_{\gamma}(\alpha(t))]}{\tau_{\gamma}(t)}, \hspace{1cm} \gamma \equiv m, n,$$ \hspace{1cm} (4)

This approximation is valid at low temperatures when electrons decay spontaneously with a lifetime $\tau_{\gamma}(t)$. The latter depends on the configuration of electronic levels $E_{\gamma} [\alpha(t)]$ and is therefore time dependent.

The induced moment for the special case that the magnetic flux increases linearly with time, $\alpha(t) = \omega t$, is shown in Fig. 3. For simplicity, we have assumed the relaxation time
τ to be time independent. This is the case when the main source of relaxation is due to an exchange of electrons between the quantum dot and its surrounding. We believe that this approximation gives a qualitatively correct picture also in general. Two non-equilibrium effects can be seen; small deviations from equilibrium result in a smearing of the fine structure in the flux-dependence of the moment. This happens when $\omega \tau \sim 1/k_F a$ (left inset of Fig. 3). Large-scale oscillations of the induced moment, which correspond to the ‘usual’ Aharonov-Bohm effect with a period of the order of the flux quantum, are not affected. Drastic changes appear when the relaxation time is so large that $\omega \tau \sim 1$ or smaller. In this case the AB oscillations disappear in favor of a diamagnetic current – linearly dependent on flux (time) – until it saturates at some large flux (time). The value of the saturation moment can be readily obtained as $M_{\text{sat}} \sim N M_0 \omega \tau$. This result tells us that the saturation moment is limited only by a finite relaxation time $\tau$ and can be very large in systems where relaxation is weak. Another interesting feature is the transient processes appearing in the case of step-like changes in the flux-value. An example of such a behavior is shown in the right inset of Fig. 3. Here the flux is (instantaneously) changed from one value to another, for which the equilibrium induced moment is not very different. Nevertheless, the initial diamagnetic response is quite large and decays to the new equilibrium value on the time scale $\tau$.

A number of mechanisms can be expected to relax a nonequilibrium momentum distribution in the quantum dot. If connected to a reservoir of particles (fixed chemical potential), the reservoir acts as a sink for energetic dot-electrons and a source of thermal electrons. This particle-exchange mechanism can be characterized by a relaxation time $\tau_e$.

Completely different relaxation mechanisms, associated with inelastic backscattering, come into play if the quantum dot is isolated (fixed number of particles). A possible mechanism for the required small energy- and large momentum transfer is simultaneous scattering by impurities and phonons [13]. Using a Green’s function formalism [14], one finds an order-of-magnitude estimate for the scattering rate as $\tau_s^{-1} \approx \hbar/E_F \tau_{\text{imp}} \tau_{\text{ph}}$. Here $\tau_{\text{imp}}$ is the impurity- and $\tau_{\text{ph}} \approx \Delta^3/\hbar^3 \omega_D^2$ [14] the phonon relaxation time, $\omega_D$ is the Debye frequency, while $\Delta$ is the transferred energy. This energy is proportional to the magnetic flux; for a
transition between the states \( n, m_1 \rightarrow n, m_2 \) one has \( \Delta \approx 2E_0 \alpha |m_1 - m_2| \). Using these estimates and typical parameters for semiconductor nanostructures one gets \( \tau_s^{-1} \approx (10^7 - 10^8) \alpha^3 \).

We stress that any scattering mechanism leads to a time-dependent relaxation rate because of the time dependence of \( \Delta \), the flux-dependent energy difference between initial and final state. Hence, contrary to the particle-exchange mechanism, we expect inelastic backscattering to cause a non-exponential relaxation of the non-equilibrium magnetic moment. The total relaxation rate can be estimated as \( \tau^{-1} = \tau_e^{-1} + \tau_s^{-1} \). We believe that the relative importance of the two mechanisms can be controlled in structures where the coupling between the quantum dot and adjacent reservoirs can be varied by means of a gate voltage. Among other possible mechanisms for inelastic backscattering, one can mention inelastic scattering caused by atomic two-level systems (see [15] for a review) and by electronic two-level systems created by close pairs of filled and empty donors in a doped region of the structure (always present in semiconductor heterostructures).

In conclusion we have shown that in a varying magnetic flux the magnetization of a 2D ballistic quantum dot is very sensitive to the conditions of relaxation in the system. In contrast to the usual destructive role played by inelastic scattering in mesoscopic phenomena, here inelastic scattering restores an Aharonov-Bohm type of quantum oscillations in the magnetization. In the absence of such relaxation, strong non-equilibrium behavior suppresses these oscillations in favor of large diamagnetic moments which are determined by flux-rather than Landau level quantization as in bulk materials. A special type of inelastic backscattering is responsible for relaxation in the case of an isolated dot, and determines the maximum (saturation) value of the non-equilibrium diamagnetic moment in the case of a magnetic flux which increases linearly with time. By monitoring the transient behavior of the induced moment as the magnetic field is switched from one value to the other we propose it might be possible to measure the characteristic time of inelastic backscattering estimated to be of the order of \( 10^{-8} - 10^{-7} \) seconds.

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FIGURES

FIG. 1. Schematic dependence of the energy eigenvalue $E_{m,n}$ on the magnetic quantum number $m$ for fixed $n$ (corresponding to a ring-shaped part of the dot): (a) occupied (filled circles) and unoccupied (empty circles) quantum states at zero magnetic field, (b) energies of the same states at finite field. To reach equilibrium the system has to relax by inelastic backscattering events involving small energy- and large momentum transfer (indicated by arrow; see text).

FIG. 2. (a) Magnetic moment vs. normalized magnetic flux $\alpha = \Phi/\Phi_0$ in a ballistic quantum dot with fixed chemical potential $\mu$ calculated at various temperatures. Units of moment, $M_0 = \pi a^2 E_0/\Phi_0$, and temperature, $E_0/k_B$, contain the quantum unit of flux $\Phi_0$ and the average spacing between energy levels $E_0$. The fine structure in the flux dependence of the moment disappears at $T \sim 1$, whereas the Aharonov-Bohm oscillations of period unity ($\Phi_0$) persist until $T \sim k_F a$ ($a$ is the dot radius). (b) Zero-temperature calculations for fixed $\mu$ and for fixed number of particles give qualitatively similar results.

FIG. 3. Magnetic moment in a ballistic quantum dot vs. a normalized magnetic flux that grows linearly with time, $\alpha(t) = \omega t$, from $t = 0$. The parameter $\omega \tau$, where $\tau$ is the relaxation time, measures the rate of change of flux. The top left panel shows that the fine structure in the flux dependence of the moment is smeared when $\omega \tau \sim 1/k_F a$ ($a$ is the dot radius). For larger values of $\omega \tau$ a large diamagnetic moment is proportional to $\alpha(t) \propto t$ until it saturates at $\alpha \sim \omega \tau$. The top right panel shows the current response to a sudden change of flux (cf. top left panel). By monitoring how the current relaxes towards a new equilibrium value, $\tau$ could be measured.