A NOVEL DESIGN OF THE ROOTS BLOWER

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Received: 13 September 2018; Accepted for publication: 13 February 2019

Abstract. This paper reports a novel curve developed from non-circular gearing theory, which can be applied in rotor profile design of the two-lobe Roots blower. The formulas for calculating the volumetric efficiency and specific flow rate of the blower have also been established. To evaluate this type of the Roots blower, the volumetric efficiency and specific flow rate are being compared with those parameters of the two traditional designs and one recent variant. The results show that with the new design, the specific flow rate significantly increases for 20% to 37%, and the transverse dimension decreases for 4% to 15%. All these changes confirm usefulness and advantages of this new design.

Keywords: hydraulic machine, Roots blower, profile, flow.

Classification numbers: 5.5.1, 5.6.1, 5.10.1.

1. INTRODUCTION

Roots blower belongs to the non-contact hydraulic machines with external mating gears [1]. This kind of blowers with the rotor profile generated by circular arches was invented by the Roots brothers yet in 1860 [2]. In 1875, Palmer and Knox applied cycloidal curves in profile designing process, in which the addendum rotor was epicycloidal and the dedendum rotor was hypocycloidal [3]. Litvin in [4] simplified the rotor profile with the circular addendum and the dedendum created by the curve meshing with the circular addendum in order to guarantee continuous matching process. During the history of 150 years of research and development of the Roots blower, this type of pump has improved from the structure and design aspect, as well as been applied in more industrial fields. One of the improvement methods is to redesign the rotor profile, which can increase the blower flow without enlarging the blower size. In 2008, Hsieh and Hwang developed the blower presented in [3] by changing a trochoid rate to increase the inlet chamber and outlet chamber volumes, and by that to increase the blower flow [5]. In 2015, Hsieh proposed a novel rotor profile generated by the locus of the fixed point of ellipse rolling on the pitch circle [6]. In this paper, the optimal ratio $\lambda$ between the semi-minor and semi-major axes of the rolling ellipse was proven to equal 0.6. In [7], Cai et al. used a conjugated curve in designing three-lobe rotor, in which he combined circular dedendum and cycloidal addendum (2016). In the same year, Shinde also proposed another profile with involute addendum and circular dedendum [8]. With those conjugated profiles, the specifications of the...
blower, especially the flow, are distinctly improved. All these works showed us the importance of finding new and better profile in order to achieve larger flow of the blowers. In the current paper, with the purpose of increasing volumes of the inlet and outlet chambers, we propose a novel design of the Roots blower with the rotor profile \( \{ \Gamma \} \) generated by non-circular gearing theory. Figure 1 describes this method of profile generation: i) the rotor addendum \( \{ \Gamma^+ d \} \) is a curve generated by point M fixed on the circle \( \{ \Sigma^s \} \) when \( \{ \Sigma^s \} \) rolling outside on the elliptical pitch \( \{ \Sigma^e \} \); ii) the rotor dedendum \( \{ \Gamma^- c \} \) is a curve generated by point M fixed on the circle \( \{ \Sigma^s \} \) when \( \{ \Sigma^s \} \) rolling inside on the elliptical pitch \( \{ \Sigma^e \} \).

![Figure 1](image1.png)  
**Figure 1.** Principle of the generation of rotor profile.

2. MATHEMATICAL MODEL OF A NEW ROTOR FOR BLOWER

2.1. Mathematical model of the addendum rotor

To establish the equation of addendum rotor following the method of profile generation presented in section 1, in Figure 2 we have:

- \( \delta/O_0 x_0 y_0 \) : the fixed reference coordinate system with origin \( O_0 \) of \( \{ \Sigma^E \} \).

- \( \delta/O_1 x_1 y_1 \) : the local coordinate system with origin \( O_1 = P_i \) (contact point between \( \{ \Sigma^s \} \) and \( \{ \Sigma^E \} \)) and axis \( O_1 x_1 = O_0 P_i \).

- \( \delta/O_2 x_2 y_2 \) : the local coordinate system connected to the rolling circle \( \{ \Sigma^s \} \), and with origin located at the centre of \( \{ \Sigma^s \} \).

- \( \varphi \) : rotation angle of \( \delta_2 \) in relation to \( \delta_1 \) during relative motion of \( \{ \Sigma^s \} \) when rolling on \( \{ \Sigma^E \} \).

![Figure 2](image2.png)  
**Figure 2.** Principle of the generation of rotor addendum profile.
\( \theta \) : rotation angle of \( \mathcal{G}_1 \) in relation to \( \mathcal{G}_0 \) when \( \{\Sigma^S\} \) rolling from \( P_0 \) (initial position) to \( P_i \) (random position).

\( \gamma \) : rotation angle of \( \mathcal{G}_2 \) in relation to \( \mathcal{G}_0 \) during relative motion.

\( r \) : radius of \( \{\Sigma^S\} \).

\( a, b \) : semi-major and semi-minor axes of \( \{\Sigma^E\} \), respectively.

\( \psi \) : angle parameter of \( \{\Sigma^S\} \).

Let \( M \) be the point fixed on \( \{\Sigma^S\} \), when \( \{\Sigma^S\} \) is rolling outside \( \{\Sigma^E\} \), from figure 2 we have:

\[ \vec{r}_M = \vec{r}_r + \vec{r}_{O_2} + \vec{r}_{O,M} \]  \hspace{1cm} (1)

By transforming equation (1) into the algebraic form, we have:

\[ \begin{align*}
0^0 \mathcal{L}_M^d (\theta, \gamma, \psi) &= 0^0 \mathcal{L}_r (\theta) + 0^1 \mathcal{L}_0 \mathcal{R}_r (z, \theta) + 0^2 \mathcal{L}_2 \mathcal{R}_2 (z, \gamma) \mathcal{L}_M (\psi) \end{align*} \]  \hspace{1cm} (2)

where: \( 0^0 \mathcal{L}_r (\theta), 0^1 \mathcal{L}_0, 0^2 \mathcal{L}_2 \mathcal{R}_2 (z, \gamma) \) are the vectors determining positions of \( P_1, O_2, M \) in the coordinate systems \( \mathcal{G}_0, \mathcal{G}_1, \mathcal{G}_2 \); \( 0^0 \mathcal{R}_r (z, \theta) \) and \( 0^2 \mathcal{R}_2 (z, \gamma) \) are the rotation matrix presenting counterclockwise revolution around \( z \) axis the angles \( \theta, \gamma \). Developing equation (2), we have:

\[ \begin{align*}
\{ \Gamma^d \} : 0^0 \mathcal{L}_M^d (\theta, \xi, \gamma) &= \begin{bmatrix}
x^d_M (\theta, \xi, \gamma) \\
y^d_M (\theta, \xi, \gamma)
\end{bmatrix} = \begin{bmatrix}
\cos \gamma + r \cos \xi + x_p (\theta) \\
\sin \gamma + r \sin \xi + y_p (\theta)
\end{bmatrix}
\end{align*} \]  \hspace{1cm} (3)

equation (3) is presenting profile of the rotor addendum. This equation contains three parameters \( \gamma, \xi, \theta \), and it is necessary to find the relation between them.

- **Determining** \( \xi = \xi (\theta) \)

  From figure 2, it is clearly shown that \( \xi \) is the angle between the common normal of \( \{\Sigma^E\} \) and \( \{\Sigma^S\} \) on the contact point \( P_i \) and the axis \( O_{y_0} \). Therefore \( \xi \) is given by:

\[ \xi (\theta) = \tan^{-1} \left( \frac{-\hat{c}_{x_p} (\theta)}{\hat{c}_{y_p} (\theta)} \right) \]  \hspace{1cm} (4)

where:

\[ \begin{bmatrix}
\hat{c}_{x_p} (\theta) \\
\hat{c}_{y_p} (\theta)
\end{bmatrix} = \begin{bmatrix}
-\rho_p (\theta) \sin \theta \\
-\rho_p (\theta) \cos \theta
\end{bmatrix} \]  \hspace{1cm} (5)

with \( \rho_p (\theta) \) is a distance from arbitrary point \( P_i \) on \( \{\Sigma^E\} \) to the origin \( O_0 \) of \( \mathcal{G}_0 \). According to [9], \( \rho_p (\theta) \) is expressed by:

\[ \rho_p (\theta) = \frac{2ab}{a + b - (a - b) \cos (2\theta)} \]  \hspace{1cm} (6)

- **Determining** \( \gamma = \gamma (\theta) \)

  From 2, using relation between the angles of triangle, therefore \( \gamma (\theta) \) is presented by:

\[ \gamma (\theta) = \xi (\theta) + \psi (\theta) \]  \hspace{1cm} (7)
from equation (7), it is clear that we need to find \( \psi(\theta) \) before calculating \( \gamma = \gamma(\theta) \). When \( \{\Sigma^S\} \) is rolling outside \( \{\Sigma^E\} \), the length of elliptical arc on \( \{\Sigma^E\} \) need to be equal to the length of the circular arc on \( \{\Sigma^S\} \), therefore:

\[
\psi(\theta) = \frac{1}{r} \int_0^\theta \left( \left( \frac{\partial x_p(\theta)}{\partial \theta} \right)^2 + \left( \frac{\partial y_p(\theta)}{\partial \theta} \right)^2 \right)^{\frac{1}{2}} d\theta
\]  

(8)

2.2. Mathematical model of the dedendum rotor

With the same nomenclature already stated in section 2.1, but let \( \{\Sigma^S\} \) is only rolling clockwise inside \( \{\Sigma^E\} \) as described in Figure 3. Similarly, we have the equation of the rotor dedendum profile:

\[
\{\Gamma\}: \begin{align*}
\xi_0(\theta, \xi, \gamma) &= \begin{bmatrix} x_0(\theta, \xi, \gamma) \\ y_0(\theta, \xi, \gamma) \end{bmatrix} \\
&= \begin{bmatrix} r \cos \gamma + r \cos \xi + x_p(\theta) \\ -r \sin \gamma + r \sin \xi + y_p(\theta) \end{bmatrix}
\end{align*}
\]  

(9)

Merging equations (3) and (9), the rotor profile equation \( \{\Gamma\} \) is presented in the universal form as below:

\[
\{\Gamma\}: \begin{align*}
\xi_i(\theta, \xi, \gamma) &= \begin{bmatrix} x_i(\theta, \xi, \gamma) \\ y_i(\theta, \xi, \gamma) \end{bmatrix} \\
&= \begin{bmatrix} r \cos \gamma + r \cos \xi + x_p(\theta) \\ \pm r \sin \gamma + r \sin \xi + y_p(\theta) \end{bmatrix}
\end{align*}
\]  

(10)

Since now, the symbol \( \Gamma \) is used to denote the rotor profile. On the other hand, in equation (10) sign “±” is used as follows:

Using “+” when the angle \( \theta \in \left\{ \theta_E, -\theta_E \right\} \cup \left( \pi + \theta_E \right) \cup \left( 2\pi - \theta_E \right) \).

Using “−” when the angle \( \theta \in \left\{ \theta_E, \pi - \theta_E \right\} \cup \left( \pi + \theta_E \right) \cup \left( 2\pi - \theta_E \right) \).

with \( \theta_E \) is the angle determining limits of the addendum rotor and dedendum rotor (Figure 1), and \( \theta_E \) is given by:

\[
\theta_E = \frac{1}{2} \cos^{-1}\left( \frac{a-b}{a+b} \right).
\]  

(11)

2.3. Designing dimensional parameters of the blower

As presented in section 1, the blower designed by the non-circular gears matching theory in general, and with elliptical gear especially, will have the analytical profile formed in sections 2.1 and 2.2. Based on continuous mating process of this elliptical gear pair, formation of the inlet chamber and outlet chamber is described in Figure 4.
Let: \( E, R, A, d \) be distance of the shafts, radial dimension, transverse dimension of the blower, axial dimension of the inlet and outlet chamber, from figure 4 we have:

\[
E = a + b
\]

\[
R = a + 2r
\]

\[
A = E + 2R
\]

Therefore, parameters \( E, R, A, d \) are the design parameters of the blower, and \( a, b, r \) are the characteristic design parameters of the rotor as well as the blower itself.

### 2.3. Condition of the characteristic design parameters for generation profile of the blower

There is a problem that not every set of the characteristic design parameters \( a, b, r \) can help to generate the rotor profile. Practically, the rotor profile can not be generated by some sets of parameters, and interference between profiles or undercutting phenomenon can happen with the other sets. It raises the need to set condition for those parameters. Let \( C^E, C^S \) be circumference of \( \Sigma^E \) và \( \Sigma^S \), respectively:

\[
C^E = 2\pi \left[ \left( \frac{\partial y_p(\theta)}{\partial \theta} \right)^2 + \left( \frac{\partial y_p(\theta)}{\partial \theta} \right)^2 \right]^{1/2} d\theta
\]

\[
C^S = 2\pi r
\]

following the principle of profile generation presented in section 2, \( \Sigma^S \) is only rolling on \( \Sigma^E \). The symmetricity of the rotor profile is also taken into consideration, we have:

\[
C^E = 4C^S \quad \text{or} \quad 2\pi \left[ \left( \frac{\partial y_p(\theta)}{\partial \theta} \right)^2 + \left( \frac{\partial y_p(\theta)}{\partial \theta} \right)^2 \right]^{1/2} d\theta = 8\pi
\]
on the other hand, from figures 1 and 4, if the profile is symmetrical and satisfies (17), then radius of the rolling circle $\{\Sigma^5\}$ need to fulfil following condition:

$$r < \frac{b}{2}$$

(18)

Let $\lambda = \frac{b}{a}$ be the characteristic design parameter of the blower. By substituting $\lambda$ into (17) and (18), we have:

$$0.5 \leq \lambda \leq 1$$

(19)

Formulas (18) and (19) are boundary condition of the characteristic design parameters for generating the rotor profile of the blower.

3. DESIGN AND EVALUATION

3.1. Theoretical specific flow rate

Specific flow rate of blower is defined by the volume amount discharged out while the driving shaft accomplish one revolution. According to [10], the specific flow rate $Q_r$ is given by:

$$Q_r = 2Zsd$$

(20)

where: $d$ is the radial dimension of the blower; $Z$ is the number of the rotor teeth; $s$ is the area of perpendicular cross section (see Figure 5). From Figure 5, we have:

$$S = \frac{1}{2} \left[ R^2 - S_{r_2}^2 \right]$$

(21)

with $S_{r_1}$ is the area of the rotor cross section perpendicular to the blower shaft, and $S_{r_2}$ given by:

$$S_{r_1} = 4(S_d + S_c)$$

(22)

where $S_d$ and $S_c$ are the areas of the sub cross-section bordered by the profiles of addendum rotor and dedendum rotor in relation to the origin $O_2$ (figure 5). $S_d^d$ and $S_c^c$ are calculated as follows:

$$S_d^d = \frac{\theta_1}{\theta_2} \left[ \frac{\partial y_1}{\partial \theta}(\theta, \xi, \gamma) \right]_{\theta_1}^{\theta_2} (\theta, \xi, \gamma) d\theta$$

(23)

$$S_c^c = \frac{\pi^2}{\theta_2} \left[ \frac{\partial y_1}{\partial \theta}(\theta, \xi, \gamma) \right]_{\theta_1}^{\theta_2} (\theta, \xi, \gamma) d\theta$$

(24)

Case study

Given are the set of characteristic design parameters: radius of $\{\Sigma^5\} r = 11.6704 \text{mm}$; rolling ellipse $\{\Sigma^5\}$ with major semiaxis $a = 56.6591 \text{mm}$, minor semiaxis $b = 33.9955 \text{mm}$; radial
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dimension \(d = 130\, \text{mm}\); number of rotor teeth \(Z = 2\). The specific flow rate of the blower will be equal \(Q_c = 3.37 \times 10^6 \, \text{mm}^3/\text{rev}\).

3.2. Volumetric efficiency of the blower

According to [6], to access the theoretical flow of the blower, the volumetric efficiency \(\eta\) is being used. This parameter is defined by the ratio between the volume blown from inlet to outlet of the blower in one revolution of the driving shaft and the total volume measured inside of the blower stator. From Figure 5, we have:

\[
\eta = \frac{2ZS}{S_{\text{stator}}} \times 100\%
\]  
(25)

where: \(S_{\text{stator}}\) is the area of the inside blower chamber (See figure 4), and \(S_{\text{stator}}\) is given by:

\[
S_{\text{stator}} = (a+2r) \sqrt{(a+2r)+2(a+b)}.
\]  
(26)

3.3. Evaluation

To prove the advantages of this design, the authors have carried out comparison of the volumetric efficiency, specific flow rate and blower dimension between the proposed design and two traditional variants - type 1 [3], type 2 [4], which were presented in [6, 11, 12] (Those two traditional designs had already been applied in manufacturing real product). A new design of Hsieh [6] - called type 3 is also added to comparing process.

a) Type 1: Traditional design (Palmer and Knox [3] in 1875)

- **Profile equation**: According to [3], the rotor profile \(\{\Gamma^r\}\) consist of design epicycloidal addendum \(\{\Gamma^d\}\) and hypocycloidal dedendum \(\{\Gamma^c\}\):

\[
\{\Gamma\}: r_1 = \left\{ \begin{array}{l}
\pm r_1 \cos \frac{R_L + r_1 \cos \theta + (R_L \pm r_1 \cos \theta)}{r_1} \\
- r_1 \sin \frac{R_L + r_1 \cos \theta + (R_L \pm r_1 \sin \theta)}{r_1}
\end{array} \right.
\]  
(27)

where: \(R_L, r, \theta\) are radius of the circle pitch \(\{\Sigma^L\}\), radius of the rolling circle \(\{\Sigma^s\}\) and parametric angle \(\{\Sigma^\theta\}\), respectively (Figure 6). The signs “±” and “±” are chosen by the following rules: the upper sign for \(\{\Gamma^d\}\), the lower sign for \(\{\Gamma^c\}\).

- **Dimensional design parameters**: from figure 6, those parameters are given by:

\[
\begin{align*}
E &= 2R_L \\
R &= R_L + 2r \\
A &= E + 2R
\end{align*}
\]  
(28)

- **Condition for profile generation**: to generate the profile, according to [10], the characteristic design parameters need to satisfy:

\[
R_L = 2Zr
\]  
(29)
b) Type 2: Traditional design (Litvin [4] in 1956)
- Profile equation: In [4], the rotor addendum profile \( \Gamma^d \) are drawn by circular arch (Figure 7), with equation:
\[
\{\Gamma^d\}: r_c^d = \left[ \frac{\rho \cos \theta + c}{\rho \sin \theta} \right] \tag{30}
\]
The rotor dedendum profile \( \Gamma^e \) is the meshing curve with \( \{\Gamma^d\} \) (Figure 7) presented by following equation:
\[
\{\Gamma^e\}: r_c^e = \left[ \frac{\rho \cos(\theta - 2\alpha) + c \cos 2\alpha - 2R_c \cos \alpha}{\rho \sin(\theta - 2\alpha) - c \sin 2\alpha + 2R_c \sin \alpha} \right] \tag{31}
\]
where: \( R_c, c, \rho \) are radius of \( \{\Sigma^L\} \), distance from origin \( O_1 \) to the centre of the rotor circular arch, radius of the rotor addendum \( \{\Gamma^d\} \), respectively. And \( \theta, \alpha \) are the parametric angle of \( \{\Sigma^L\} \) and rotation angle of the driving shaft.
- Dimensional design parameters: from Figure 7, the design parameters of the blower are:
\[
\begin{align*}
E &= 2R_c \\
R &= c + \rho \\
A &= E + 2R
\end{align*}
\] (32)
- Condition for profile generation: According to [4], \( \lambda = c/R_c \) is the characteristic design parameter of the rotor profile. Condition for profile generation with no singularities is:
\[
0.5 \leq \lambda < 0.9288 \tag{33}
\]

c) Type 3: new design in 2015 (Hsieh [6])
- Profile equation: In this type of blower, \( \{\Gamma^d\} \) is the locus of a point fixed on the rolling ellipse \( \{\Sigma^{IS}\} \), when \( \{\Sigma^{ES}\} \) is only rolling outside on \( \{\Sigma^L\} \) of the gear. \( \{\Gamma^e\} \) is the locus of a point fixed on \( \{\Sigma^{IS}\} \), when \( \{\Sigma^{ES}\} \) is only rolling inside on \( \{\Sigma^L\} \) (Figure 8). The equation of \( \{\Gamma\} \) is given by:
\[
\{\Gamma\}: r_c = \left[ \pm a_1(1 - \cos \psi \cos(\xi \pm \theta) \cos(\xi \pm \theta) \pm b_1 \sin(\xi \pm \theta) \sin \psi + R_c \cos \theta \right] \\
- a_1(1 - \cos \psi \sin(\xi \pm \theta) \cos(\xi \pm \theta) \sin \psi + R_c \sin \theta
\] (34)
where: \( R_c, a_1, b_1 \) are radius of \( \{\Sigma^L\} \), major semi-axis of \( \{\Sigma^{ES}\} \) and minor semi-axis of \( \{\Sigma^{IS}\} \), respectively. And \( \theta, \psi \) are the parametric angle of \( \{\Sigma^L\} \) and \( \{\Sigma^{ES}\} \). \( \xi \) is the angle between the common normal of \( \{\Sigma^L\} \) and \( \{\Sigma^{ES}\} \) on the contact point, and the line connecting the centers of the rotor 1 and rotor 2.
- Dimensional design parameters: from Figure 8, those parameters are presented as below:
\[
\begin{align*}
E &= 2R_c \\
R &= R_c + 2a_1 \\
A &= E + 2R
\end{align*}
\] (35)
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- **Condition for profile generation**: As stated in [6], the characteristic design parameter is calculated by formula \( \lambda = a_i / h_i \). Condition for profile generation with no self-intersection:

\[
0.4 \leq \lambda \leq 1. 
\]

\[(36)\]

d) **Analysis and evaluation**

Let take two case studies into consideration:

- **Case study 1**: Determine characteristic design parameters of the variants of the blower, of which the radial dimension \( R = 72 \) mm and the axial dimension \( d = 150 \) mm. The characteristic parameters are shown in Table 1.

Table 1. Characteristic design parameters.

| Blower type                  | Characteristic design parameters | A [mm] | Volumetric efficiency |
|------------------------------|----------------------------------|--------|-----------------------|
| Type 1 (Fig. 6) (1875)       | \( R_L \) [mm] \( r \) [mm] | 48.0000 | 240.0000 | \( \eta = 54.09\% \) |
| Type 2 (Fig. 7) (1956)       | \( R_L \) [mm] \( c \) [mm] \( \rho \) [mm] | 44.0921 | 232.1842 | \( \eta = 63.66\% \) |
| Type 3 (Fig. 8) (2015)       | \( a_1 \) [mm] \( b_1 \) [mm] | 43.6754 | 231.3508 | \( \eta = 64.49\% \) |
| Roots blower with new design (Fig. 5) | \( a \) [mm] \( b \) [mm] \( r \) [mm] | 51.6393 | 221.4587 | \( \eta = 81.21\% \) |

From Table 1 we have:

(i) The volumetric efficiency is increasing, and the transverse dimension is decreasing in the history of blower development, which matches well with our notice made in section 1.

(ii) The volumetric efficiency of the newly proposed blower is distinctly higher than efficiency of the older designs: 27.12 % higher in comparison with type 1, 18.15 % higher than type 2, and 16.72 % higher than type 3. It means that the specific flow rate is the largest one, while dimension of the novel blower is smallest, which leads to consideration that the novel design is approximately optimal. For thorough evaluation we are going to examine case number 2 below.

- **Case study 2**: In order to generally evaluate, we are going to determine the characteristic design parameter of each type of blower based on parameter \( \lambda \) (\( \lambda \) needs to satisfy conditions 19, 29, 33, 36 for each type of blower to generate profile), with constraint of radial dimension \( R = 72 \) mm and axial dimension \( d = 150 \) mm for all of the evaluated blowers. On the other hand, from conditions (19 and 36), when \( \lambda = 1 \), both \( \{S\} \) and \( \{S^{E2}\} \) become a circle, which means that blower of type 3 and actually proposed blower will transform to type 1. Therefore, if the increment \( \Delta \lambda = 0.1 \) to satisfy conditions (19, 29, 33, 36), then \( \lambda \in [0.5 \div 1] \). From the values of \( \lambda \), the relation of the characteristic design parameters for each type will be found. By
substitution to formulas (12, 13, 17, 29, 32, 23), and solve this repeating problem in Matlab, the characteristic design parameters will be listed in the Table 2 below.

Table 2. Characteristic design parameters of each type of blower based on parameter $\lambda$.

| Blower type | Characteristic design parameters | $\lambda = 0.5$ | $\lambda = 0.6$ | $\lambda = 0.7$ | $\lambda = 0.8$ | $\lambda = 0.9$ | $\lambda = 1.0$ |
|-------------|----------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Type 2      | $R_L$ [mm]                       | 63.0654         | 59.3078         | 55.4314         | 51.5470         | 47.7664         | -               |
| Type 2      | $c$ [mm]                         | 31.5327         | 35.5847         | 38.8020         | 41.2376         | 42.9898         | -               |
| Type 2      | $\rho$ [mm]                      | 46.4673         | 42.4153         | 39.1980         | 36.7624         | 35.0102         | -               |
| Type 3      | $R_L$ [mm]                       | 47.3150         | 48.2868         | 49.2524         | 50.1980         | 51.1156         | 52.0000         |
| Type 3      | $a_1$ [mm]                       | 15.3425         | 14.8566         | 14.3738         | 13.9010         | 13.4422         | 13.0000         |
| Type 3      | $b_1$ [mm]                       | 7.6713          | 8.9139          | 10.0617         | 11.1208         | 12.0979         | 13.0000         |
| Roots blower with new design | $a$ [mm]                      | 55.9426         | 55.2426         | 54.4895         | 53.6912         | 52.8578         | 52.0000         |
| Roots blower with new design | $b$ [mm]                      | 27.9713         | 33.1456         | 38.1426         | 42.9529         | 47.5720         | 52.0000         |
| Roots blower with new design | $r$ [mm]                      | 11.0287         | 11.3787         | 11.7552         | 12.1544         | 12.5711         | 13.0000         |

Note: when $\lambda = 1$ blower of type 3 and new design of blower will transform to type 1.

Applying equations (14, 28, 32, 35) for the data calculated in Table 2, we can obtain the graph in Figure 9.
From the graph 9, it is noticeable that the transverse dimension A of the novel design is the smallest. And together with type 3, this dimension will increase and reach maximum value when $\lambda = 1.0$ (become type 1). On the other hand, the blower of type 2 will have decreasing transverse dimension when $\lambda$ increases. By applying the new design, the dimension of blower will be smaller for about 4% to 15% in comparison with the previous types.

To evaluate the flow of the new design, by substitution the characteristic design parameters in Table 2 into the profile equation ($T$) given by formulas (10, 22, 25, 26, 29), and by calculation similarly in section 3.1 with $S$ is the area of the cross sections described in figures 5, 6, 7, and 8, corresponding with each type of blowers, we can obtain graph 10 below.

From Figure 10, when $\lambda$ is increasing, we have:

(i) The specific flow rate of the proposed blower and type 3 decreases when value $\lambda$ rises and reaches minimum value when these two type transform to type 1 ($\lambda = 1.0$). However, when ($\lambda = 0.5$), the specific flow rate of the proposed blower is 20% larger than the flow rate of the blower type 3, and 37% larger in comparison with type 1.

(ii) The theoretical specific flow rate of the blower type 2 increases when $\lambda$ rises from 0.5 to 1, and reaches maximum value with $\lambda = 0.9$. However, it is still lower for 21% than the flow rate of the new design.

Finally, it is clearly shown that with the blowers of type 2, type 3 and of new design, when the transverse dimension $A$ increases, the flow rate will decrease and vice versa. Only with the blowers of type 1, in order to achieve larger flow rate, the dimension of the blower also need to be raised. All of these have confirmed the advantages of the newer types of blower.

4. CONCLUSIONS

This research has proposed the novel curve developed from non-circular gearing theory, i.e. elliptical gears, which can be applied in designing Roots blower. In comparison of the new
designed blower with the previous ones, the volumetric efficiency and theoretical specific flow rate (in one revolution) increase from 20 % to 37 % (*with the same radial and axial dimension*), and transverse dimension decreases from 4 % to 15 %. All of these changes confirm the advantages of the proposed blower, from size and flow aspects. Therefore, this new variant can be useful when designing industrial Roots blower, when only flow rate is taken into consideration. Some applications can be listed as fruit and vegetable driers, oxygen provider for the oven of the thermal power plants etc. But it stills needs to be improved when stability of the flow rate as well as stability of the pressure are required.

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