ADT-based adaptive back-stepping control for the switched non-affine nonlinear system with uncertain parameters

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1 Introduction

As an efficient recursive methodology, the back-stepping control method has been extensively utilized for the nonlinear strict-feedback systems over the past few decades [1,2]. It is worth mentioning that control inputs appearing in these systems are affine. Nevertheless, the plants in many engineering applications are non-affine systems, such as hypersonic vehicles [3,4], rigid and flexible joint manipulators [5,6], and ultrasonic motors [7], and so on. Because the system state cannot be regarded as the virtual control, the non-affine property makes it difficult to apply the back-stepping method straightforwardly. Namely, the control input \( u \) is difficult to separate and use directly to compensate the negative effects, caused by unknown parameters or external disturbances. Therefore, the final actual control input cannot be obtained following the way of dealing with nonlinear strict-feedback system [8]. Consequently, it is still an enormous challenge for the nonlinear strict-feedback system with non-affine properties in the control field.

Recently, the control designs based on the back-stepping scheme for the nonlinear systems subject to non-affine properties have attracted considerable attention by the worldwide researchers. In [9], the virtual and
practical control laws are developed based on implicit function methods, which are constructed by the mean value theorem and neural networks. The idea in [9] is then extended to various classes of non-affine systems including normal form with zero dynamics in [10], perturbed pure-feedback form with dead zero in [11], pure-feedback form with uncertainties in [12] and applied to helicopters in vertical flight in [13]. Furthermore, since the inputs of neural network approximation generally contain the control variable, the “circular design problem” widely exists in these works. To address this issue, several approaches are developed from different aspects. The dynamic surface control (DSC) techniques are introduced to eliminate approximations of control inputs in [14], where the repeated differentiations of virtual controls are not required. Similarly, combined with continuous functions and DSC technique, the circular construction problem during the controller design process is removed in [15]. To facilitate DSC design, an affine state variable is constructed at each step in [16] with a filtered version of control signal. The authors in [17] and [18] adopt coordinate transformation to obtain an augmented term in the normal form, and the goal is then converted into designing an output feedback controller. Considering the parametric uncertainties, an adaptive back-stepping control method is proposed in [19]. In particular, non-affine structures are exploited in [20] and [21] to represent the virtual controls as a whole and reduce the conservativeness of designed controller, while the usefulness of these structures is neglected by the aforementioned methods.

On the other hand, switched system, constituted by a series of continuous/discrete subsystems with specific rules orchestrating the switching among these subsystems [22,23], provides a powerful tool of modeling and controller design for many physical control systems including near space vehicles [24], networked control systems [25] and electro-hydraulic servo systems [26] and have attracted great attention in the recent few decades. In particular, switched nonlinear systems with affine appearance of states and control inputs have been extensively addressed by the back-stepping methodology, see [27–30]. The existing approaches on the stability analysis or stabilization for the switched nonlinear system could be usually classified into two main categories, i.e., common Lyapunov function and multiple Lyapunov functions. The stability of switched systems can be ensured by the common Lyapunov function method for any arbitrary switching signals [31,32]. However, it would be extremely difficult or even inconceivable to share a common Lyapunov function for all subsystems. Therefore, the multiple Lyapunov functions method is proposed, which can reduce the conservativeness of controller with restrictions on switching signals just like dwell time (AD), average dwell time (ADT), mode-dependent average dwell time (MDADT), and so on [33,34]. However, a restrictive inequation condition $V_p \leq \mu V_q$ with a positive constant $\mu$ is required to satisfy for any two subsystems in these studies, which is so difficult to be ensured for the nonlinear controller design. Fortunately, the recent research in [35] extends the multiple Lyapunov functions method to a less restrictive case with a bounded constant $\Delta$, i.e., $V_p \leq \mu V_q + \Delta$. However, the switched systems in [35] belong to the strict-feedback form; there is little related research on switched nonlinear systems subject to non-affine dynamics.

Motivated by previous discussions, the ADT-based adaptive back-stepping control issue for the switched non-affine nonlinear system with uncertain parameters would be addressed in this paper. The original switched non-affine nonlinear systems are first augmented by an integrator to facilitate the controller design. Then, the non-affine structures are employed as a whole to devise the virtual controls, where DSC methods are integrated to obviate “explosion of complexity” problems of back-stepping methods. Moreover, the switched unknown parameters are estimated by a continuous variable, and the parameter estimation errors constitute the main difference of the Lyapunov function for each subsystem. For the switching signal with certain ADT, the proposed multiple Lyapunov functions control method could ensure the uniform ultimate boundedness of the switched non-affine nonlinear system, and the convergence of tracking errors is also guaranteed even existing switching uncertain parameters. Finally, simulation results illustrate the correctness of proposed methods.

The remainders of this paper would be arranged as follows. The adaptive back-stepping control issue is formulated in Section II. Section III presents the back-stepping control design process and related stability analysis. The simulation results are given in Section IV, and the final conclusion is drawn in Section V.
2 Problem formulation

Consider the following switched non-affine nonlinear system with uncertain parameters,

$$
\begin{align*}
\dot{x}_1 &= \theta_{\sigma(t)}^T \varphi_1 (\bar{x}_1) + f_1 (\bar{x}_1, x_2), \\
\vdots & \quad \vdots \\
\dot{x}_i &= \theta_{\sigma(t)}^T \varphi_i (\bar{x}_i) + f_i (\bar{x}_i, x_{i+1}), \quad i = 2, \ldots, n-1, \\
\dot{x}_n &= \theta_{\sigma(t)}^T \varphi_n (\bar{x}_n) + f_n (\bar{x}_n, u), \\
y &= x_1.
\end{align*}
$$

where $$\bar{x}_i = [x_1, x_2, \cdots, x_i]^T \in \mathbb{R}^i$$ and $$x = \bar{x}_n \in \mathbb{R}^n$$ denote the system states, $$u \in \mathbb{R}$$ and $$y \in \mathbb{R}$$ are control input and system output, respectively. $$\varphi_i(\cdot) : \mathbb{R}^i \to \mathbb{R}^i$$ and $$f_i(\cdot) : \mathbb{R}^{i+1} \to \mathbb{R}$$ represent known smooth nonlinear functions. $$\theta_{\sigma(t)} \in \mathbb{R}^l$$ is the unknown parameter, where the switching signal $$\sigma(t) \rightarrow \Omega = \{1, 2, \ldots, m\}$$ is the piecewise right continuous function, and $$m$$ denotes the total number of subsystems.

The main objective of this paper is to design an adaptive back-stepping control scheme for the switched non-affine nonlinear system (1), which could guarantee the system states are stable and the system output $$y$$ could track the reference trajectory $$y_r$$ with high precision even existing switching uncertainty parameters or switching occurring.

Assumption 1 The $$y_r$$, $$\dot{y}_r$$ and $$\ddot{y}_r$$ are known and bounded.

Assumption 2 For the nonlinear functions $$f_i$$, the following condition can be satisfied,

$$f_0 \leq \left| \frac{\partial f_i (\bar{x}_i, x_{i+1})}{\partial x_{i+1}} \right| < +\infty, \quad i = 1, 2, \ldots, n$$

where $$x_{n+1} = u$$ and $$f_0 > 0$$

Remark 1 Assumption 1 is commonly employed in extant literature on back-stepping control; the identical assumption can be found in [20, 21, 35]. Assumption 2 is on the controllability of system (1), which is a basic condition [20, 21, 32].

Definition 1 [22]. For $$t \in [t, T]$$, define $$N_\sigma(T, t)$$ as the total number of switches, if there exist two constants $$N_0 > 0$$ and $$\tau_a > 0$$ such that

$$N_\sigma(T, t) - N_0 \leq \frac{T - t}{\tau_a}, \quad t \in [t, T]$$

where the $$\tau_a$$ is called ADT of the switching law $$\sigma(t)$$.

Remark 2 For the switched system, the switching signal is a key issue, which would influence the controller design and stability analysis. Generally speaking, the switching signal can be divided into arbitrary switching signal or designed one [31, 32]. The common Lyapunov function approach is needed for the arbitrary signal to stable the switched system, but it is so difficult to find a common Lyapunov function for the switched nonlinear system with so many sub-systems. In order to relax the restrain of common Lyapunov function method, the multiple Lyapunov functions are developed for the switched system with restriction switching signals, just like dwell time (AD), average dwell time (ADT), mode-dependent average dwell time (MDADT), and so on [31, 32, 33]. In this paper, the switching signal is designed based on the average dwell time (ADT) condition, not arbitrary and the ADT-based adaptive back-stepping control for the switched non-affine nonlinear system with uncertain parameters is investigated in this note.

3 Main results

The adaptive back-stepping controller would be first designed for the nonlinear non-affine switched system, and stability analysis is also accomplished in this section. To facilitate the controller design by back-stepping method, define a “extended state” $$x_{n+1} = u$$ and introduce an integrator in system (1) to obtain the augmented switched non-affine nonlinear systems as

$$
\begin{align*}
\dot{x}_1 &= \theta_{\sigma(t)}^T \varphi_1 (\bar{x}_1) + f_1 (\bar{x}_1, x_2), \\
\vdots & \quad \vdots \\
\dot{x}_i &= \theta_{\sigma(t)}^T \varphi_i (\bar{x}_i) + f_i (\bar{x}_i, x_{i+1}), \quad i = 2, \ldots, n-1, \\
\dot{x}_n &= \theta_{\sigma(t)}^T \varphi_n (\bar{x}_n) + f_n (\bar{x}_n, u), \\
y = u
\end{align*}
$$

The controller design for (1) is then transformed to develop an auxiliary control law for (4), where $$v$$ is regarded as the new control input.

3.1 Controller design

The controller design follows the back-stepping design process, which can be described as follows.

Step 1. Define $$z_1 = x_1 - y_r$$ as the tracking error variable, and taking the time derivative of $$z_1$$ yields

$$
\dot{z}_1 = \dot{x}_1 - \dot{y}_r = \theta_{\sigma(t)}^T \varphi_1 (\bar{x}_1) + f_1 (\bar{x}_1, x_2) - \dot{y}_r
$$
Regard $f_1$ as a virtual control input, and corresponding virtual control laws can be designed as follows

$$\alpha_1 = -k_1 z_1 - \hat{\theta}^T \varphi_1 (\tilde{x}_1) + \tilde{y}_r$$  \hspace{1cm} (6)

where the $\hat{\theta}$ denotes the estimation of the switching uncertain parameter $\theta_{\sigma(i)}$ and $k_1 > 0$. The following modified first-order filter is applied to solve the “explosion of complexity” problem [19, 21],

$$\tau_1 \dot{\alpha}_1 + \alpha_1 = \alpha_1 - \tau_1 z_1, \quad \alpha_1 (0) = \alpha_1 (0)$$  \hspace{1cm} (7)

where $\tau_1 > 0$ and $\alpha_1$ is a new intermediate state variable.

**Step 2.** Denote the second error variable by $z_2 = f_1 (\tilde{x}_1, x_2) - \tilde{\alpha}_1$, and we can attain that

$$\dot{z}_2 = \frac{\partial f_1 (\tilde{x}_1, x_2)}{\partial x_1} x_1 + \frac{\partial f_1 (\tilde{x}_1, x_2)}{\partial x_2} x_2 - \dot{\alpha}_1$$

$$= \frac{\partial f_1 (\tilde{x}_1, x_2)}{\partial x_1} \left( \theta_{\sigma(i)}^T \varphi_1 (\tilde{x}_1) + f_1 (\tilde{x}_1, x_2) \right) + \frac{\partial f_1 (\tilde{x}_1, x_2)}{\partial x_2} \left( \theta_{\sigma(i)}^T \varphi_2 (\tilde{x}_2) + f_2 (\tilde{x}_2, x_3) \right) - \frac{\alpha_1 - \tilde{\alpha}_1}{\tau_1} + z_1$$  \hspace{1cm} (8)

Then, the virtual control law of (8) is proposed as follows.

$$\alpha_2 = \frac{1}{\frac{\partial f_1 (\tilde{x}_1, x_2)}{\partial x_2}} \left[ - \frac{\partial f_1 (\tilde{x}_1, x_2)}{\partial x_1} \hat{\theta}^T \varphi_1 (\tilde{x}_1) - \frac{\partial f_1 (\tilde{x}_1, x_2)}{\partial x_2} \tilde{\alpha}_2 \right]$$

$$= \frac{\partial f_1 (\tilde{x}_1, x_2)}{\partial x_1} \left( \theta_{\sigma(i)}^T \varphi_1 (\tilde{x}_1) + f_1 (\tilde{x}_1, x_2) \right) + \frac{\partial f_1 (\tilde{x}_1, x_2)}{\partial x_2} \left( \theta_{\sigma(i)}^T \varphi_2 (\tilde{x}_2) + f_2 (\tilde{x}_2, x_3) \right) - \frac{\alpha_1 - \tilde{\alpha}_1}{\tau_1} + z_1$$  \hspace{1cm} (9)

where $k_2 > 0$. Similarly, $\tilde{\alpha}_2$ can be acquired by,

$$\tau_2 \dot{\alpha}_2 + \tilde{\alpha}_2 = -k_2 z_2$$

$$= \alpha_2 - \tau_2 \frac{\partial f_1 (\tilde{x}_1, x_2)}{\partial x_2} z_2, \quad \tilde{\alpha}_2 (0) = \alpha_2 (0)$$  \hspace{1cm} (10)

with $\tau_2 > 0$.

**Step i (i = 3, ..., n).** Define $z_i = f_{i-1} (\tilde{x}_{i-1}, x_i) - \tilde{\alpha}_{i-1}$, and it can be obtained that

$$\dot{z}_i = \sum_{j=1}^{i-1} \frac{\partial f_{i-1} (\tilde{x}_{i-1}, x_i)}{\partial x_j} \dot{x}_j + \frac{\partial f_{i-1} (\tilde{x}_{i-1}, x_i)}{\partial x_i} \dot{x}_i - \dot{\alpha}_{i-1}$$

$$= \sum_{j=1}^{i-1} \frac{\partial f_{i-1} (\tilde{x}_{i-1}, x_i)}{\partial x_j} \left( \theta_{\sigma(i)}^T \varphi_j (\tilde{x}_j) + f_j (\tilde{x}_j, x_{j+1}) \right) + \frac{\partial f_{i-1} (\tilde{x}_{i-1}, x_i)}{\partial x_i} \left( \theta_{\sigma(i)}^T \varphi_i (\tilde{x}_i) + f_i (\tilde{x}_i, x_{i+1}) \right) - \frac{\alpha_{i-1} - \tilde{\alpha}_{i-1}}{\tau_{i-1}} + \frac{\partial f_{i-2} (\tilde{x}_{i-2}, x_{i-1})}{\partial x_{i-1}} z_{i-1}$$

The related virtual control law is expressed as

$$\alpha_i = \frac{1}{\frac{\partial f_{i-1} (\tilde{x}_{i-1}, x_i)}{\partial x_{i-1}}} \left[ -k_{i-1} z_i - 2 \frac{\partial f_{i-2} (\tilde{x}_{i-2}, x_{i-1})}{\partial x_{i-1}} z_{i-1} \right]$$

$$= -\hat{\theta}^T \sum_{j=1}^{i-1} \frac{\partial f_{i-1} (\tilde{x}_{i-1}, x_i)}{\partial x_j} \varphi_j (\tilde{x}_j)$$

$$- \sum_{j=1}^{i-1} \frac{\partial f_{i-1} (\tilde{x}_{i-1}, x_i)}{\partial x_j} f_j (\tilde{x}_j, x_{j+1})$$

$$+ \frac{\alpha_{i-1} - \tilde{\alpha}_{i-1}}{\tau_{i-1}}$$  \hspace{1cm} (12)

where $k_i > 0$, and then the intermediate variable $\tilde{\alpha}_i$ can be generated as follows:

$$\tau_i \dot{\tilde{\alpha}}_i + \tilde{\alpha}_i = \alpha_i - \tau_i \frac{\partial f_{i-1} (\tilde{x}_{i-1}, x_i)}{\partial x_i} z_i, \quad \tilde{\alpha}_i (0) = \alpha_i (0)$$  \hspace{1cm} (13)

where the time constant $\tau_i > 0$.

**Step n+1.** Define $z_{n+1} = f_n (\tilde{x}_n, x_{n+1}) - \tilde{\alpha}_n$ and the new control input $v$ is designed in this step. Since $x_{n+1} = u$, we have

$$\dot{z}_{n+1} = \sum_{j=1}^{n} \frac{\partial f_n (\tilde{x}_n, x_{n+1})}{\partial x_j} \dot{x}_j + \frac{\partial f_n (\tilde{x}_n, x_{n+1})}{\partial u} - \dot{\alpha}_n$$

$$= \sum_{j=1}^{n} \frac{\partial f_n (\tilde{x}_n, x_{n+1})}{\partial x_j} \dot{x}_j + \frac{\partial f_n (\tilde{x}_n, x_{n+1})}{\partial u}$$

$$+ \frac{\partial f_{n-1} (\tilde{x}_{n-1}, x_n)}{\partial x_n} z_n$$  \hspace{1cm} (14)

The final control law of (14) is chosen as

$$v = \frac{1}{\frac{\partial f_n (\tilde{x}_n, x_{n+1})}{\partial x_{n+1}}} \left[ -k_{n+1} z_{n+1} - 2 \frac{\partial f_{n-1} (\tilde{x}_{n-1}, x_n)}{\partial x_n} z_n \right]$$

$$- \hat{\theta}^T \sum_{j=1}^{n} \frac{\partial f_n (\tilde{x}_n, x_{n+1})}{\partial x_j} \varphi_j (\tilde{x}_j)$$

$$- \sum_{j=1}^{n} \frac{\partial f_n (\tilde{x}_n, x_{n+1})}{\partial x_j} f_j (\tilde{x}_j, x_{j+1}) + \frac{\alpha_n - \tilde{\alpha}_n}{\tau_n}$$  \hspace{1cm} (15)

For Eqs. (5), (8), (11) and (14), the adaptive laws $\hat{\theta}$ are proposed as

$$\hat{\theta} = k \sum_{i=1}^{n+1} \sum_{j=1}^{i} \frac{\partial f_{i-1} (\tilde{x}_{i-1}, x_i)}{\partial x_j} \varphi_j (\tilde{x}_j) - k_0 \hat{\theta}$$  \hspace{1cm} (16)

where $f_0 (\cdot) = x_1$ and $\varphi_{n+1} (\cdot) = 0$, $K$ and $k_0$ are positive constants.
Remark 2 In this paper, the switched non-affine nonlinear systems with unknown parameters are investigated, i.e., \( \dot{x}_n = \theta^T_{\tau(t)} \sigma_n (\tilde{x}_n) + f_n (\tilde{x}_n, u) \). Because of the non-affine properties, the control input \( u \) is difficult to separate and use directly to compensate the negative effects, caused by unknown parameters or switching instant. Combined with the idea of “extended state”, a new virtual extended state \( x_{n+1} \) is designed, and define as \( x_{n+1} = u \), and \( x_{n+1} = \dot{u} = v \). Then, the new augmented switched non-affine nonlinear system can be obtained and the controller design in Step n+1, and the similar way is adopted in [36].

3.2 Stability analysis

Define the error variables of the dynamic surfaces as 
\[
\tilde{a}_i = \tilde{a}_i - a_i, \quad i = 1, 2, \cdots, n,
\]
we have
\[
f_i (\tilde{x}_i, x_{i+1}) - a_i = f_i (\tilde{x}_i, x_{i+1}) - \tilde{a}_i + \tilde{a}_i - a_i = z_{i+1} + \tilde{a}_i, \quad (i = 1, 2, \cdots, n)
\]
A combination of (5) and (6) comes to
\[
\dot{z}_1 = \theta^T_{\sigma(t)} \varphi_1 (\tilde{x}_1) + a_1 - \dot{y}_r + f_i (\tilde{x}_1, x_2) - \alpha_1 = -k_1 z_1 + \theta^T_{\sigma(t)} \varphi_1 (\tilde{x}_1) + z_2 + \tilde{a}_1
\]
where \( \theta_{\sigma(t)} = \theta_{\sigma(t)} - \tilde{\theta} \) denotes the estimation error. Similarly, the time derivatives of \( z_2, \cdots, z_{n+1} \) can be computed as
\[
\dot{z}_2 = -k_2 z_2 - z_1 + \theta^T_{\sigma(t)} \left( \frac{\partial f_1 (\tilde{x}_1, x_2)}{\partial x_1} \varphi_1 (\tilde{x}_1) + \frac{\partial f_1 (\tilde{x}_1, x_2)}{\partial x_2} \varphi_2 (\tilde{x}_2) \right) + \frac{\partial f_1 (\tilde{x}_1, x_2)}{\partial x_2} (z_3 + \tilde{a}_2)
\]
\[
\dot{z}_3 = -k_3 z_3 - \frac{\partial f_{i-2} (\tilde{x}_{i-2}, x_{j-1})}{\partial x_{j-1}} z_{i-1}
\]
\[
\dot{z}_i = -k_i z_i - \frac{\partial \sigma_i (\tilde{x}_{i-1}, x_i)}{\partial x_i} + \theta_{\sigma(t)} \sum_{j=1}^{i} \frac{\partial \sigma_i (\tilde{x}_{i-1}, x_i)}{\partial x_j} \varphi_j (\tilde{x}_j)
\]
\[
\dot{z}_{n+1} = -k_{n+1} z_{n+1} - \frac{\partial f_{n-1} (\tilde{x}_{n-1}, x_n)}{\partial x_n} z_n
\]
\[
\dot{z}_n + \theta^T_{\sigma(t)} \sum_{j=1}^{n} \frac{\partial f_n (\tilde{x}_n, x_{n+1})}{\partial x_j} \varphi_j (\tilde{x}_j)
\]
Recall the time derivatives of \( \tilde{a}_i, \quad (i = 1, 2, \cdots, n) \), which can be obtained as follows.
For \( i = 1 \), it can be verified from (6) and (7) that
\[
\dot{\tilde{a}}_1 = \tilde{a}_1 - \tilde{a}_1 = \frac{\partial a_1}{\partial \tau_1} z_1 - \frac{\partial a_1}{\partial \tau_1} \dot{z}_1 - \frac{\partial a_1}{\partial \tau_1} \dot{y}_r
\]
(22)
where \( B_i (\cdot) \) represents the continuous function subject to \( \alpha_1 \) and \( \tilde{a}_1 \). Moreover, for \( i = 2, \cdots, n \), one obtains
\[
\dot{\tilde{a}}_i = \tilde{a}_i - \tilde{a}_i = \frac{\partial a_i}{\partial \tau_i} z_i
\]
(23)
with \( B_i (\cdot) \) being continuous functions with respect to the time derivatives of \( a_1, \quad i = 2, \cdots, n \).
For any \( p \in M \), the corresponding Lyapunov function is selected as
\[
V_p = \frac{1}{2} \sum_{i=1}^{n+1} z_i^2 + \frac{1}{2} \theta^T_{\sigma(t)} \theta_p + \frac{n}{2} \sum_{i=1}^{n} \tilde{a}_i^2 p \in M
\]
(24)
where \( \theta_p = \theta_{\sigma(t)} - \tilde{\theta} \) denotes the parameter estimation error for the \( p \)-th subsystem. With (16) and (18)-(23), the derivative of \( V_p \) can be obtained as follows,
\[
\dot{V}_p = - \sum_{i=1}^{n} k_i z_i^2 + z_1 \tilde{a}_1 + \sum_{i=2}^{n} \frac{\partial f_{i-1} (\tilde{x}_{i-1}, x_i)}{\partial x_i} z_i \tilde{a}_i + \frac{k_0}{\kappa} \theta^T_{\sigma(t)} \theta
\]
(25)
It can be seen that
\[
\tilde{a}_1 \tilde{a}_1 \leq \frac{1}{\tau_1} \tilde{a}_1^2 - z_1 \tilde{a}_1 + |\tilde{a}_1| B_1 (\cdot)
\]
(26)
\[
\tilde{a}_i \tilde{a}_i \leq \frac{1}{\tau_i} \tilde{a}_i^2 - \frac{\partial f_{i-1} (\tilde{x}_{i-1}, x_i)}{\partial x_i} z_i \tilde{a}_i + \tilde{a}_i B_i (\cdot)
\]
(27)
Substituting (26) and (27) into (25) gives
\[
\dot{V}_p \leq - \sum_{i=1}^{n+1} k_i z_i^2 - \sum_{i=1}^{n} \frac{1}{\tau_i} \tilde{a}_i^2 + \frac{k_0}{\kappa} \theta^T_{\sigma(t)} \theta + \sum_{i=1}^{n} |\tilde{a}_i| B_i (\cdot)
\]
(28)
To show the boundedness of \( B_1(\cdot) \), we need to ensure that the arguments of \( B_1(\cdot) \) are bounded. According to assumption 1, for a given constant \( C_0 > 0 \), the set \( \Pi_0 = \{ y^2 + \tilde{y}^2 + \tilde{y}_r^2 \leq C_0 \} \) is compact in \( \mathbb{R}^3 \). On the other hand, the sets \( \Pi_{ip} = \{ \sum_{j=1}^{n+1} \tilde{\alpha}_j^2 + \frac{1}{2} \tilde{\theta}_p^T \tilde{\theta}_p + \sum_{j=1}^{n} \tilde{\alpha}_j^2 \leq C_{ip} \} \) are also compact in \( \mathbb{R}^{(n+i)+n} \) for given positive constants \( C_{ip} \), \( i = 1, \ldots, n \). Since \( \tilde{\theta} = \theta_p - \tilde{\theta}_p \) and \( \| \tilde{\theta}_p \| \) is bounded, \( \| \tilde{\theta}_p \| \) is also bounded on \( \Pi_{ip} \). Hence, \( x_1, \ldots, x_{n+1}, \tilde{\alpha}_1, \ldots, \tilde{\alpha}_i, \tilde{\theta}_p, y_r, \tilde{y}_r \) are bounded on \( \Pi_0 \times \Pi_{ip} \).

We now turn to the boundedness of \( x_1, \ldots, x_{n+1} \). Notice that \( x_1 = z_1 + y_r \), then \( x_1 \) is bounded. Moreover, since \( f_1 (\tilde{x}_1, x_2) = z_2 + \tilde{\alpha}_1 = z_2 + \alpha_1 + \tilde{\alpha}_1 \) and \( \alpha_1 \) is a continuous function of \( z_1, \tilde{\theta}, x_1 \) and \( \tilde{y}_r \), \( f_1 (\tilde{x}_1, x_2) \) takes a bounded value. Applying mean value theorem to \( f_1 (\tilde{x}_1, x_2) \) yields

\[
f_1 (\tilde{x}_1, x_2) = f_1 (\tilde{x}_1, 0) + \frac{\partial f_1 (\tilde{x}_1, x_2)}{\partial x_2} \bigg|_{x_2 = \xi} x_2 \tag{29}
\]

where \( \xi \in (\min (0, x_2), \max (0, x_2)) \) and by Assumption 2, we have

\[
\left| \frac{\partial f_1 (\tilde{x}_1, x_2)}{\partial x_2} \right|_{x_2 = \xi} \geq f_0 > 0 \tag{30}
\]

A combination of (29) and (30) leads to the boundedness of \( x_2 \). By the same token, we can deduce that \( x_3, \ldots, x_{n+1} \) are bounded. As a result, there exist positive constants \( D_{\tilde{\theta}_p} \) such that \( |B_1(\cdot)| \leq D_{\tilde{\theta}_p} \) on \( \Pi_0 \times \Pi_{ip} \).

By the Young’s inequality, we obtain

\[
|\tilde{\alpha}_i B_1(\cdot)| \leq \frac{1}{2} \tilde{\alpha}_i^2 + \frac{1}{2} D_{\tilde{\theta}_p}^2 \tag{31}
\]

\[
\tilde{\theta}_p^T \tilde{\theta}_p = \tilde{\theta}_p^T (\theta_p - \tilde{\theta}_p) \leq -\frac{1}{2} \tilde{\theta}_p^T \tilde{\theta}_p + \frac{1}{2} D_{\tilde{\theta}_p}^2 \theta_p \tag{32}
\]

Using (31) and (32) in (28) results in

\[
\dot{V}_p \leq -\sum_{i=1}^{n+1} k_i \tilde{z}_i^2 - \frac{k_0}{2\kappa} \tilde{\theta}_p^T \tilde{\theta}_p - \sum_{i=1}^{n} \left( \frac{1}{\tau_1} - \frac{1}{2} \right) \tilde{\alpha}_i^2 + \frac{1}{2} \theta_p^T \theta_p + \frac{1}{2} D_{\tilde{\theta}_p}^2 \theta_p \leq -a V_p + b \tag{33}
\]

where \( a = \min \left\{ 2k_1, \ldots, 2k_{n+1}, k_0, \frac{3}{\tau_1} - 1, \ldots, \frac{3}{\tau_1} - 1 \right\}, b = \max_{\rho \in M} \left\{ \frac{1}{2} \left( |\theta_p| + \sum_{i=1}^{n} D_{\rho, ip}^2 \right) \right\}. \)

For \( \forall p, q \in M \), it is noticed that the only difference between the corresponding Lyapunov functions \( V_p \) and \( V_q \) lies in \( \frac{1}{\kappa} \tilde{\theta}_p^T \tilde{\theta}_p \) and \( \frac{1}{\kappa} \tilde{\theta}_q^T \tilde{\theta}_q \). Define \( \mu = 2 \), and \( \mu Y = \frac{1}{\kappa} \Theta^2 \), where \( \Theta = \max_{p, q \in M} \{ \| \theta_p - \theta_q \| \} \), then one has

\[
\frac{1}{2} \sum_{i=1}^{n+1} \tilde{z}_i^2 - \frac{\mu}{2} \sum_{i=1}^{n+1} \tilde{z}_i^2 \leq 0 \tag{34}
\]

\[
\frac{1}{2} \sum_{i=1}^{n} \tilde{\alpha}_i^2 - \frac{\mu}{2} \sum_{i=1}^{n} \tilde{\alpha}_i^2 \leq 0 \tag{35}
\]

Note that \( \tilde{\theta}_p = \tilde{\theta}_q + \theta_p - \theta_q \), it can then be verified that

\[
\frac{1}{2\kappa} \tilde{\theta}_p^T \tilde{\theta}_p - \frac{\mu}{2\kappa} \tilde{\theta}_q^T \tilde{\theta}_q = \frac{1}{2\kappa} \tilde{\theta}_q^T \tilde{\theta}_q + \frac{1}{\kappa} \left( \theta_p - \theta_q \right)^T \left( \theta_p - \theta_q \right)
+ \frac{1}{2\kappa} \left( \theta_p - \theta_q \right)^T \left( \theta_p - \theta_q \right)
- \frac{\mu}{2\kappa} \tilde{\theta}_q^T \tilde{\theta}_q
\leq \frac{1}{\kappa} \left( \theta_p - \theta_q \right)^T \left( \theta_p - \theta_q \right) - \frac{\mu}{2\kappa} \tilde{\theta}_q^T \tilde{\theta}_q \leq 0 \tag{36}
\]

A combination of (34)–(36) implies

\[
V_p \leq \mu V_q + Y \quad \forall p, q \in M \tag{37}
\]

With the help of above analysis and back-stepping scheme design, the following theorem can be obtained.

**Theorem 1** Considering the switched non-affine nonlinear systems (1) subject to uncertain parameters, if the assumptions 1 and 2 are fulfilled, control laws and adaptive laws are proposed as (15) and (16), then it can be obtained that all signals are uniformly ultimately bounded for the switching signal \( \tau_a \), which conforms to ADT conditions, i.e., \( \tau_a > \log \left( \frac{v}{\alpha} \right) \). Besides, the tracking error satisfies the following inequality,

\[
\lim_{t \to \infty} |y(t) - y_r(t)| \leq \sqrt{2 \left( \mu^{1+N_0} b + \frac{e^{\alpha N_0 \tau_a} Y}{1 - e^{-\epsilon \tau_a}} \right)} \tag{38}
\]

where \( \epsilon \in (0, a - \log \mu) \).

**Proof** Define the Lyapunov function as \( W(t) = e^{\alpha t} V_{\sigma(t)}(t) \), and then one has

\[
\dot{W}(t) = \alpha e^{\alpha t} V_{\sigma(t)}(t) + e^{\alpha t} \dot{V}_{\sigma(t)}(t) \leq b e^{\alpha t}, \quad t \in [t_c, t_c+1) \tag{39}
\]
By integration on the time interval \([t_\ell, t_{\ell+1})\), one obtains
\[ W(t_{\ell+1}) \leq W(t_\ell) + \int_{t_\ell}^{t_{\ell+1}} be^{at} dt \quad (40) \]

Using the inequality \(V_\sigma(t_{\ell+1}) \leq \mu V_\sigma(t_{\ell+1}) + Y \) in (40) yields
\[ W(t_{\ell+1}) = e^{a(t_{\ell+1}-t_\ell)} V_\sigma(t_{\ell+1}) + Y \leq a \int_{t_\ell}^{t_{\ell+1}} be^{at} dt + e^{a(t_{\ell+1}-t_\ell)} Y + Y \]

Choosing an arbitrary \(T > t_0 = 0\) and iterating bode sides of (41) over \([0, T]\), we see that
\[ W(T^-) \leq W(t_{N_\sigma(T,0)}) + \int_{t_{N_\sigma(T,0)}}^{T} be^{at} dt \leq \mu W(t_{N_\sigma(T,0)}) - 1 + \mu \int_{t_{N_\sigma(T,0)}}^{T} be^{at} dt + \mu \int_{t_{N_\sigma(T,0)}}^{T} be^{at} dt \leq \cdots \leq \mu^{N_\sigma(T,0)} W(0) \]
\[ \leq \mu^{N_\sigma(T,0)-1} \int_{t_{j-1}}^{t} be^{at} dt + \mu^{N_\sigma(T,0)-1} \int_{t_j}^{T} be^{at} dt \]

Note that \(\tau_\alpha > \log \mu/a\), and for \(\forall \epsilon \in (0, a - (\log \mu/j_\alpha))\), we can obtain that \(\tau_\alpha > \log \mu/(a - \epsilon)\). Besides, it follows that \(N_\sigma(T,0) - j \leq N_\sigma(T, t_{j+1}) + 1\) by the definition of \(N_\sigma(T, t_{j+1})\). Hence,
\[ \leq \mu^{N_\sigma(T,0)-1} \int_{t_{j-1}}^{t} be^{at} dt + \mu^{N_\sigma(T,0)-1} \int_{t_j}^{T} be^{at} dt \]
\[ \leq \mu^{N_\sigma(T,0)-1} \int_{t_{j-1}}^{t} be^{at} dt + \mu^{N_\sigma(T,0)-1} \int_{t_j}^{T} be^{at} dt \]
\[ \leq \mu^{N_\sigma(T,0)-1} \int_{t_{j-1}}^{t} be^{at} dt + \mu^{N_\sigma(T,0)-1} \int_{t_j}^{T} be^{at} dt \]
\[ \leq \mu^{N_\sigma(T,0)-1} \int_{t_{j-1}}^{t} be^{at} dt + \mu^{N_\sigma(T,0)-1} \int_{t_j}^{T} be^{at} dt \]
\[ \leq \mu^{N_\sigma(T,0)-1} \int_{t_{j-1}}^{t} be^{at} dt + \mu^{N_\sigma(T,0)-1} \int_{t_j}^{T} be^{at} dt \]
\[ \leq \mu^{N_\sigma(T,0)-1} \int_{t_{j-1}}^{t} be^{at} dt + \mu^{N_\sigma(T,0)-1} \int_{t_j}^{T} be^{at} dt \]
\[ \leq \mu^{N_\sigma(T,0)-1} \int_{t_{j-1}}^{t} be^{at} dt + \mu^{N_\sigma(T,0)-1} \int_{t_j}^{T} be^{at} dt \]

Substituting (43) and (44) into the right-hand side of (42) and in view of the definition of \(W(t)\), one has
\[ W_\sigma(T^-) \leq e^{N_\sigma(T) \log \mu/(a - \epsilon)} + \mu^{N_\sigma(T) - 1} \int_{t_{N_\sigma(T,0)}}^{T} be^{at} dt \]
\[ \leq \epsilon^{N_\sigma(T) \log \mu/(a - \epsilon)} + \mu^{N_\sigma(T) - 1} \int_{t_{N_\sigma(T,0)}}^{T} be^{at} dt \]

which shows that \(V_\sigma(t)\) bounded, and \(z_1, \cdots, z_{n+1}, z_{n+1}, \alpha_1, \cdots, \alpha_1, \theta, \theta_\sigma(t)\) are all bounded as a result. Similar to the foregoing procedure that uses (29) and (30), the boundedness of \(x_1, \cdots, x_{n+1}\) can then be deduced. Therefore, all signals of the closed-loop system are bounded for the switching law \(\sigma(t)\) corresponding to ADT \(\tau_\alpha > \log \mu/a\). Moreover, notice that \(z_1^2(T^-) \leq 2V_\sigma(T^-)\) and taking \(T \to \infty\) yield (38).

**Remark 3** For each subsystem, the Lyapunov function is requested strictly decreasing in the multiple Lyapunov functions method. This constraint is relaxed in [33, 37], but the inequality \(V_\rho \leq \mu V_q\) is needed to be satisfied, which is difficult to be ensured for the nonlinear control design. Fortunately, the research in [35]
extends the multiple Lyapunov functions method to a less restrictive case with a bounded constant $\Delta$, i.e., $V_p \leq \mu V_q + \Delta$. Under such a weaker condition, the global boundedness of all the closed-loop signals is ensured by the extended multiple Lyapunov functions method in [35]. However, it can be found that the non-affine peculiarity is not considered in [35], and there is little related research on switched nonlinear systems subject to non-affine peculiarity. Therefore, inspired by the [35], the less restrictive case with a bounded constant is employed in this work to deal with the subject to non-affine dynamics.

4 Numerical simulation

The following second-order switched non-affine nonlinear systems are considered to illustrate the effectiveness of the proposed adaptive back-stepping control methods,

$$\begin{align*}
\dot{x}_1 &= \theta_1(t)x_1^2 + x_2 + \frac{x_3^2}{5} \\
\dot{x}_2 &= \theta_1(t)\frac{1 - e^{-x_2^2}}{1 + e^{-x_2^2}} + u + \frac{u^3}{7} \\
y &= x_1
\end{align*}$$

where the switching signal $\sigma(t) \in \Omega = \{1, 2\}$, i.e., there are two subsystems. The unknown parameters are chosen as $\theta_1 = 0.4$ and $\theta_2 = 0.1$. The system initial states are selected as $x_1(0) = 1$, $x_2(0) = u(0) = 0$, and the reference trajectory $y_r = \sin(t)$.

For the proposed control parameters, we take $k_1 = 5$, $k_2 = 5$, $k_3 = 5$, $\tau_1 = 0.05$, $\tau_2 = 0.05$, $\kappa = 1$ and $k_0 = 2$. It can be verified that $\alpha = 2$, and the closed-loop system is ensured to be uniformly ultimately bounded for every $\sigma(t)$ with average dwell time $\tau_a > 0.3466 = \log 2 / 2$. The switching signal is shown in Fig 1, and it could be not difficult to see that the dwell time $\tau_a = 2.5 > 0.3466$ with $N_0 = 2$, which is satisfied with the ADT condition.

Applying the proposed ADT-based adaptive back-stepping control scheme, the simulation results are given in Figs. 2, 3, 4, 5 and 6. In addition, the same adaptive back-stepping control problem for the nonlinear system with uncertain parameters is addressed in [19], and the uncertain parameters $\theta(t)$ is also consider and handled by the adaptive estimations, which are also employed in our work. Thus, the reference [19] is used as the comparison simulation, and the results are also
depicted in Figs. 2, 3, 4, 5 and 6 It is obvious that the tracking missions for the uncertain switched system with non-affine could be successfully completed by the two adaptive back-stepping control schemes, but a better tracking performance can be guaranteed by the proposed control method.

Figure 2 shows the responses of system output \( y \), and it can be found that the \( y \) can successfully track the reference signal \( y_r \), even the subsystem switching and in presence of unknown parameters. Noticeably, a higher tracking precision can be guaranteed by the proposed back-stepping control scheme at subsystem switching occurring. The similar conclusion could be obtained in Fig. 3, which denotes the tracking errors \( y - y_r \).

As displayed in Fig. 3, the tracking errors would converge quickly and errors is not exceeding 0.06, whereas the tracking errors is 0.26 by the back-stepping control method in [19]. Figures 4 and 5 indicate the responses of the system state \( x_2 \) and \( \hat{\theta} \), respectively. Furthermore, the control input \( u \) is shown in Fig. 6, and the required control energies are limited, which is reasonable for the engineering applications.

5 Conclusions

The ADT-based adaptive back-stepping control schemes for the switched non-affine nonlinear system with uncertain parameters are proposed in this paper. The application of back-stepping approach is made feasible by augmenting the original systems with an integrator. The virtual controls are developed with direct utilization of the structures in the system dynamics at each step. By employing the DSC technique, the “explosion of complexity” problem is avoided, and a continuous variable is structured to estimate the switched unknown parameters. Combined with multiple Lyapunov functions and ADT, the stability of the switched system is analyzed. The uniformly ultimately bounded of the closed-loop system and tracking errors would be guaranteed by the designed adaptive back-stepping control schemes. The actuator saturation faults are important issues for the trajectory tracking mission, which are not considered in this paper and may be our future work.

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Data availability All data generated or analyzed during this study are included in this article.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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