Cyclic Period in the CBE Model

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Abstract

In a cyclic entropy model in which the extroverse is jettisoned at turnaround with a Come Back Empty (CBE) assumption, we address matching of the contraction scale factor $\dot{a}(t) = f(t_T)a(t)$ to the expansion scale factor $a(t)$, where $f(t_T)$ is the ratio at turnaround of the introverse to extroverse radii. Such matching is necessary for infinite cyclicity and fixes the CBE period at $\sim 2.6T_y$. 

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1 Introduction to the CBE model

In physics there exist simple-to-state questions which are difficult to answer. For example, here is a plausible examination question: Show how to construct an infinitely cyclic cosmological model which is consistent with the second law of thermodynamics.

This has been studied since 1931 when a no-go theorem [1] of Tolman stated that, subject to certain assumptions, there cannot exist any solution to creating the cosmological sequence

\[ \text{Expansion} \rightarrow \text{Turnaround} \rightarrow \text{Contraction} \rightarrow \text{Bounce} \rightarrow \text{etc.} \]  

where the entropy of the universe obeys the second law of thermodynamics. Fortunately in this case, one can be guided by a more recent discovery about Nature.

The most important questionable assumption implicit in the no-go theorem [1] of 1931 was pointed out not by a physicist but by Nature herself in 1998 when observers discovered [2,3] that the expansion rate of the universe is accelerating. To my knowledge, nobody had questioned prior to 1998 that the expansion rate was decelerating. Once one knows that it is accelerating it is very enlightening with respect to the second law of thermodynamics because the superluminal accelerated expansion creates, starting at the onset of dark energy domination, the extroverse into which entropy built up from irreversible processes during expansion can be jettisoned with impunity from a retained introverse. The CBE (Comes Back Empty) assumption is that the retained introverse contains energy of radiation, dark energy and curvature but no matter, luminous or dark, including no black holes.

The superluminal accelerated expansion is surely the most important discovery in observational cosmology since Hubble [4] established the expansion of the universe. In terms of entropy it made it possible to distinguish the two parts of the universe after the dark energy domination began at \( t_{DE} \sim 9.8 \text{Gy} \). Those two parts which play a crucial role in the CBE model are the introverse and extroverse which we shall now define. Note that although the CBE assumption was first introduced in [5,6] the only CBE model discussed in the present paper is the recently improved CBE model [7] which eschews the use of phantom dark energy.

The introverse is the same as the visible universe, or particle horizon, whose radius \( R_{IV}(t) \) is given by

\[ R_{IV}(t) = c \int_0^t \frac{dt}{a(t)}, \]  

where \( a(t) \) characterizes the expansion history of the universe, being the scale factor in the FRLW metric which assumes homogeneity and isotropy

\[ ds^2 = dt^2 - a(t)^2 \left[ \frac{dr^2}{1 - k(t)r^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \]  

\[ \frac{dr^2}{1 - k(t)r^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \]
where \( k(t) \) is the curvature.

Inserting the well-known expansion history and normalizing the scale factor to \( a(t_0) = 1 \) at the present time \( t_0 = 13.8 Gy \) (all times are measured relative to the would-have-been big bang) one finds that at the end of the radiation-dominated era \( a(t_m = 47 ky) = 2.1 \times 10^{-1} \), a value which will be important in the CBE matching condition to be discussed in this paper. At the commencement of the superluminal accelerated expansion at \( t_{DE} = 9.8 Gy \) one finds \( a(t_{DE}) = 0.75 \). After this time the scale factor \( (t \geq 9.8 Gy) \) is

\[
a(t) = 0.75 \exp[H_0(t - t_{DE})],
\]

(4)

where the observed value of the Hubble constant is \( H_0 \simeq (13.8 Gy)^{-1} \).

Substituting Eq.(4) in Eq.(2) one finds that the radius of the introverse \( R_{IV}(t) \) grows from an initial value at \( t = t_{DE} \)

\[
R_{IV}(t_{DE}) = 39Gly,
\]

(5)

to its present value

\[
R_{IV}(t_0) = 44Gly,
\]

(6)

and reaches an asymptotic value at \( t \sim 50 Gy \) so that

\[
R_{IV}(50 Gy \leq t < \infty) \simeq 58Gly.
\]

(7)

The extroverse starts to form after \( t = t_{DE} \) and its radius \( R_{EV}(t) \) is defined initially as equal to the introverse radius

\[
R_{EV}(t_{DE}) = R_{IV}(t_{DE}) = 39Gly,
\]

(8)

and thereafter for \( t_{DE} \leq t \) is

\[
R_{EV}(t) = \frac{a(t)}{a(t_{DE})} R_{EV}(t_{DE}).
\]

(9)

This leads to the present value of the extroverse radius

\[
R_{EV}(t_0) = 52Gly,
\]

(10)

which is significantly above \( R_{IV}(t_0) \) given by Eq.(6), as discussed in [7].

Future values of \( R_{EV}(t) \) can be illustrated by examples. When the introverse radius approaches its asymptotic value, the extroverse radius is already much larger than \( R_{IV}(t = 50 Gy) \) given by Eq.(7) namely

\[
R_{EV}(t = 50 Gy) = 720Gly.
\]

(11)
An interesting later value of $R_{EV}(t)$, extraordinarily large, is

$$R_{EV}(t = 1 Ty) = 5.6 \times 10^{32} Gly,$$

(12)

In the CBE model, at a turnaround time $t = t_T$ to be determined in the next subsection, the scale factor for the contracting universe is $\dot{a}(t) = f(t_T)a(t)$ with the fraction $f(t_T)$ given by

$$f(t_T) = \frac{R_{IV}(t_T)}{R_{EV}(t_T)}.$$  

(13)

As shown in [7], this reduction in size of the adiabatically contracting universe with low entropy explains, without any need for an inflationary era, the flatness observed for the present universe and further predicts that the present flatness is accurate to many decimal places.

2 Matching of the Scale Factor

An important requirement for infinite cyclicity is that the scale factor $a(t)$ for the expansion era be matched correctly to that of the previous contracting era. Recall the the scale factor is redefined as $\dot{a}(t) = f(t_T)a(t)$ at turnaround so that, at first sight, it might appear that an inverse transformation $a(t) = f(t_T)^{-1}\dot{a}(t)$ might be necessary. However, this is not the case because the subluminal decelerating contraction rate is far more gradual than the superluminal accelerating expansion rate.

The contraction is radiation dominated throughout so that the relevant matching condition is at the transition time, $t_m \sim 47ky$, between radiation domination and matter domination of the expansion era, namely

$$\dot{a}(t_m) = a(t_m) = 2.1 \times 10^{-4}.$$  

(14)

This matching condition allow us to fix the turnaround time $t_T$ of the CBE model and hence its cyclic period, as follows.

First note that $t_T$ is necessarily in the asymptotic region of the introverse where

$$R_{IV}(t_T) \simeq 58Gly,$$

(15)

and consequently

$$\dot{a}(t_T) = f(t_T)a(t_T) = \frac{58Gly}{R_{EV}(t_T)}a(t_T).$$  

(16)
We know also that
\[ R_{EV}(t_T) = a(t_T)R_{EV}(t_0) = a(t_T) \times 52Gly, \quad (17) \]
which, when combined with Eq. (16), reveals that
\[ \hat{a}(t_T) = \frac{58Gly}{52Gly} = 1.11, \quad (18) \]
which, independent of the turnaround time \( t_T \) provided that it is in the asymptotic region \( t_T \gtrsim 50Gy \).

The matching condition, Eq. (14) is now straightforward to implement because \( \hat{a}(t) \) contracts with the radiation-dominated behavior
\[ \hat{a}(t) = \hat{a}(t_T) \left( \frac{t}{t_T} \right)^{\frac{3}{2}}, \quad (19) \]
and the matching requirement is therefore
\[ \hat{a}(t_m) = 1.11 \left( \frac{47ky}{t_T} \right)^{\frac{3}{2}} = a(t_m) = 2.1 \times 10^{-4}, \quad (20) \]
which has the unique solution \( t_T = 1.3Ty \).

Only with this choice of turnaround time does the contracting universe match smoothly on to the time-reverse of the expansion radiation dominated era in such a manner that infinite cyclicity is achieved. The total cyclic period of the CBE model is thus
\[ \tau_{CBE} = 2t_T = 2.6Ty. \quad (21) \]

3 Discussion

In the CBE model, gravitational interactions play a role in the overall behavior of the expansion and contraction of the introverse but not in its entropy because the gravitational entropy which is dominated by black holes is jettisoned at turnaround to the extroverse.

The model solves the question of constructing a cyclic cosmological model which respects the second law of thermodynamics and it seems unlikely that such a difficult and highly constrained question can have two really different solutions.
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References

[1] R.C. Tolman, Phys. Rev. 38, 1758 (1931).

[2] S. Perlmutter, et al. (Supernova Cosmology Project). Astrophys. J. 517, 565 (1999).
   [astro-ph/9712212]

[3] A.G. Riess, at al. (High-Z Supernova Search Team). Astron. J. 116, 1009 (1998).
   [astro-ph/9805201]

[4] E. Hubble, Proc. Nat. Acad. Sci. 15, 168 (1929).

[5] L. Baum and P.H. Frampton, Phys. Rev. Lett. 98, 071301 (2007).
   [hep-th/0610213]

[6] P.H. Frampton, Did Time Begin? Will Time End?
   Maybe the Big Bang Never Occurred.
   World Scientific Publishing Company (2009)

[7] P.H. Frampton, Cyclic Entropy: An Alternative to Inflationary Cosmology.
   Int. J. Mod. Phys. A30, 1550129 (2015). arXiv 1501.03054[gr-qc].