A MODEL FOR THE CHEMICAL EVOLUTION OF THE GALAXY
WITH REFUSES

H. J. ROCHA-PINTO$^{1,2}$, L. I. ARANY-PRADO$^{1,2}$
W. J. MACIEL$^3$

1: Observatório do Valongo, UFRJ, Rio de Janeiro, Brazil

2: Observatório Nacional, CNPq, Rio de Janeiro, Brazil

3: Instituto Astronômico e Geofísico, USP, São Paulo, Brazil

Send proofs to:

Dr. Walter J. Maciel
Instituto Astronômico e Geofísico da USP
Av. Miguel Stefano 4200
04301-904 São Paulo SP
Brazil
Abstract. A model is presented for the chemical evolution of the solar neighbourhood which takes into account three families of galactic objects, according to their condensation states: stars, refuses and gas. Stars are defined as every condensed objects with masses greater than or equal to the minimum mass which ignites hydrogen and which will give rise to an evolutionary track on the HR diagram to the left of Hayashi’s limit; refuses include the remnants, which are compact objects resulting from stellar deaths, and the residues, which have masses not large enough to ignite hydrogen; gas is defined as the mass which can be condensed to form stars and/or residues. We have developed equations for the mass evolution of each family, and have studied the gas metallicity distribution within the framework of the instantaneous recycling approximation, adopting different initial conditions. In order to constrain the model parameters we have also used preliminary evaluations of comet cloud masses to investigate the role of the residues as sinks of heavy elements in the Galaxy.
1. Introduction

Models for the chemical evolution of galaxies usually include only two classes of objects, namely *stars* and *gas* (cf. Tinsley, 1980). This classification has as main argument the simplification of the equations, and generally does not cause significant difficulties in the derivation and interpretation of the quantities which are effectively compared with the observational data.

On the other hand, more complete formulations already appeared in some of the early works on chemical evolution (Schmidt, 1959) and also in more recent treatments of the evolution of our Galaxy (Tinsley, 1981, Rana and Wilkinson, 1986) and other spiral galaxies (cf. Ferrini et al., 1992).

The main characteristic of these treatments is the inclusion of non-stellar objects, as our present knowledge makes us sure that an important quantity of metals is probably locked up in some galactic objects such as planets, comets, etc. (cf. Bailey, 1988). Comets are particularly interesting in this respect, as they seem to be a very common phenomenon associated with star formation out of a protostellar cloud (Vanysek, 1987a,b). Similar to the case of interstellar grains, comet formation has probably a small effect in the mass balance of the Galaxy (cf. Meusinger, 1992). On the other hand, these objects may affect some observational properties such as the extinction and polarization of visible light (cf. Greenberg, 1974), and can in principle have an influence on the chemical evolution of the solar neighbourhood as metal sinks, as suggested by Tinsley and Cameron (1974), Vanyšek (1987a,b), and Stern and Shull (1990).

In the present paper, we have introduced a consistent treatment of the evolutionary histories of the different families of galactic objects, taking into account the following condensation states: *stars*, *refuses*, and *gas*. We then study the derived metallicity distribution for the one-zone model of the solar neighbourhood considering a set of initial conditions, and compare our results with observed data from stars. Finally, we make some preliminary calculations of comet cloud masses, in order to investigate the role of comets as heavy element sinks in the galactic disk.

2. Basic Equations

The adopted families of stars, refuses and gas are associated with the following classes of objects, respectively:
**Stars**, which are defined as every condensed object formed with masses \( m > m_l \), where \( m_l \) is the lower limit for the stellar masses, or the minimum mass which produces hydrogen ignition and which will give rise to a track on the HR diagram to the left side of Hayashi’s limit.

**Refuses**, which include **remnants**, or compact objects resulting from stellar deaths, and **residues** of star formation, which are objects condensed from the gas, with masses in the interval \( m_g < m < m_l \), where \( m_g \) is the maximum assumed mass for the gas (e.g. grains).

**Gas**, which is the mass that can be condensed to form stars and/or residues.

Adopting a model for the chemical evolution of the solar neighbourhood with no infall, the total mass of the system is constant and given by

\[
M = M_g + M_s + M_r
\]  

(1)

where \( M_g, M_s, \) and \( M_r \) are the total masses in gaseous, stellar and refuse condensation states, respectively. The gas (\( \mu \)) and refuse (\( \kappa \)) mass fractions are defined by

\[
\mu \equiv \frac{M_g}{M}
\]  

(2)

and

\[
\kappa \equiv \frac{M_r}{M}
\]  

(3)

so that

\[
M_s = (1 - \mu - \kappa)M.
\]  

(4)

We will adopt the usual sudden mass loss approximation, where the stars undergo the entire process of mass loss after a well-defined lifetime. Let \( w_m \) be the remnant mass of a star with initial mass \( m \) and lifetime \( \tau_m \). The rate of mass locked up in the remnants, due to the death of the stars which were born at instants given by \( (t - \tau_m) \) is obtained by

\[
L(t) = \int_{m_l}^{m_u} w_m(m)\Psi(t - \tau_m)\Phi(m)dm
\]  

(5)

and the total ejection rate due the death of these stars is

\[
E(t) = \int_{m_l}^{m_u} [m - w_m(m)]\Psi(t - \tau_m)\Phi(m)dm,
\]  

(6)
where \( m_t \) is an appropriately chosen turnoff mass; \( m_u \) is the upper limit to the stellar mass, or the maximum mass admitted to stars; \( \Psi \) is a generalized formation rate, defined as the total mass condensed into galactic objects per unit time, and \( \Phi \) is a generalized initial mass function, normalized as

\[
\sum_{i=1}^{4} \int_{x_i}^{x_{i+1}} m\Phi(m)dm = \rho + \gamma + \zeta + \varepsilon = 1
\]  

(7)

where \( x_1 = 0, x_2 = m_g, x_3 = m_t, x_4 = m_t, x_5 = m_u \), and \( \rho, \gamma, \zeta \) and \( \varepsilon \) are, respectively, the first, the second, the third and the fourth terms in the sum. Each of these four terms, multiplied by the generalized formation rate will give respectively: the formation rate of objects with masses \( m < m_g \), considered as gas, \( \rho\Psi(t) \); the formation rate of the residues, \( \gamma\Psi(t) \); and the stellar formation rates, \( \zeta\Psi(t) \) and \( \varepsilon\Psi(t) \), relative to stars with masses in the intervals \( m_l < m < m_t \) and \( m_t < m < m_u \), respectively.

It is worth noting that the generalized initial mass function is not necessarily continuous in the above intervals. However, we have assumed its continuity and will take \( m_g \approx 0 \), so that \( \rho \approx 0 \). We will assume that every residue is gravitationally tied with a star and that the formation of stars with mass \( m > m_t \) is not accompanied by formation of residues (Stern and Shull, 1990).

Some residues may undergo evaporation of their H and He. If we assume that \( \hat{\gamma} \) is the fraction of the generalized formation rate that will initially produce such residues, the gas will be replenished by a mass per unit time \((1 - Z)\hat{\gamma}\Psi\) due the evaporation of H and He, where \( Z \) is the metallicity of the gas and \( Z\hat{\gamma}\Psi \) is the mass of metals which go into these residues per unit time. We will assume an instantaneous evaporation. Based on Tinsley and Cameron (1974) and Vanýsek (1987), we set \( \hat{\gamma} \gg \gamma - \hat{\gamma} \), so that as a first approximation \( \hat{\gamma} \approx \gamma \).

Adopting the instantaneous recycling approximation (IRA), \( \tau_m \approx 0 \). The ejection rate to the interstellar medium and the rate of mass locked up in remnants can be simplified as

\[
E(t) = R\Psi(t)
\]  

(8)

where \( R \) is the returned fraction to the interstellar gas,

\[
R = \int_{m_t}^{m_u} [m - w_m(m)]\Phi(m)dm
\]  

(9)

and

\[
L(t) = (\varepsilon - R)\Psi(t).
\]  

(10)
Recalling that $\rho \approx 0$ in equation (7), that $\dot{\gamma} \approx \gamma$, and assuming further that $Z \ll 1$, we can write for the mass rates

$$\frac{d}{dt} M_g(t) = -(\zeta + \varepsilon - R)\Psi(t) \quad (11a)$$

$$\frac{d}{dt} M_s(t) = \zeta\Psi(t) \quad (11b)$$

$$\frac{d}{dt} M_r(t) = (\varepsilon - R)\Psi(t) \quad (11c)$$

Equation (11b) can also be obtained assuming a constant formation rate. In the framework of the IRA, it can be easily interpreted: the rate of change of total stellar mass is only due to the stars that live forever, which have masses in the range $m_l < m < m_t$.

3. The generalized formation rate and initial mass function

The slow rate of growth of the abundances of the heavy elements produced by the metal sink effect due to the refuses (Tinsley and Cameron 1974; Vanýsek 1987a,b) is built in our model, and can be obtained by appropriately chosen fractions of the generalized formation rate. In order to determine this rate, we will follow Tinsley and Cameron (1974) and assume that the mass of metals which go into comets per unit time is at least equal in magnitude to the mass of metals which go into the associated star. Since we have assumed $\dot{\gamma} \approx \gamma$, it follows that $\gamma \sim \zeta$. In order to estimate the fractions of the formation rate, we have used the stellar IMF from Miller and Scalo (1979) for $m \geq m_l$. We have assumed that the generalized initial mass function ($\Phi$) for residues is proportional to $m^{-x}$, and re-normalized $\Phi$ in the interval $(m_g, m_u)$ assuming $\gamma \lesssim \zeta$. We have taken $m_l = 0.1 M_\odot$ (Larson, 1992), $m_t = 1 M_\odot$ and $m_u = 100 M_\odot$ (Tinsley, 1980). It can be shown that if $\gamma \lesssim \zeta$, then the slope $x$ of the generalized initial mass function for residues should be $\lesssim 1.8$.

The fractions of the generalized formation rate can then be computed and we have obtained $\gamma \approx 0.3$, and $\zeta \approx 0.3$, so that $\varepsilon \approx 0.4$. The returned fraction can be computed from equation (9) as $R \approx 0.24$, where we have taken $w_m = 0.7 M_\odot$ for $m \leq 4 M_\odot$, and $w_m = 1.4 M_\odot$ for $m > 4 M_\odot$ (Tinsley, 1980).
4. Metallicity Distribution

Following Tinsley (1980), the metallicity of the gas for the conditions adopted here can be obtained from the equation

\[ \frac{d}{dt} Z(t)M_g(t) = -Z(1 - R)\Psi + y(\varepsilon + \zeta - R)\Psi \]  

(12)

where we have kept the definition of the heavy element yield \( y \) as the total mass of new ejected metals relative to the mass locked up in stars and remnants,

\[ y = \frac{1}{\zeta + \varepsilon - R} \int_{m_{1}}^{m_{u}} mpZ_m(m)\Phi(m)dm \]  

(13)

where \( pZ_m \) is the so-called stellar yield, or the mass fraction of a star with mass \( m \) that is converted to metals and ejected.

From equations (11a) and (12) the metallicity can be integrated as

\[ Z(t) = \frac{y(1 - \gamma - R)}{\gamma} \left[ 1 - \left( \frac{\mu}{\mu_0} \right)^{\frac{\gamma}{1 - \gamma - R}} \right] + Z_0 \left( \frac{\mu}{\mu_0} \right)^{\frac{\gamma}{1 - \gamma - R}}. \]  

(14)

where \( \mu = \mu(t) \), so that \( \mu = \mu_0 \) and \( Z = Z_0 \) for \( t = 0 \). We set \( t = 0 \) in that instant when the disk reached its final mass \( M \).

In order to show the dependence of the heavy element abundance with \( \gamma \), we will take \( y = 0.01 \) (Tinsley and Cameron, 1974; Maciel, 1992) and \( R = 0.24 \) as discussed in section 3. We have also adopted an initially unenriched gas, so that \( Z_0 \approx 0 \). Figure 1 shows \( Z \) as a function of the ratio \( \mu/\mu_0 \) for some representative values of \( \gamma \). We see that the smaller is the value for \( \gamma \), the greater is \( Z(\mu \rightarrow 0) \). Tentative limits for \( \gamma \) are provided, assuming that \( Z(\mu \rightarrow 0) \approx Z_\odot = 0.02 \pm 0.01 \) in equation (14). On the basis of the assumed error, we can see in figure 2 that values for \( \gamma \) in the range 0.19-0.38 are preferred. It can be seen that the residue mass fraction obtained in section 2 lies approximately in the middle of this interval.

An analytical expression for the cumulative distribution of stars of a given metallicity can be derived for the one-zone model with metal retention by refusens. Recalling the definition of the gas and refuse fractionary superficial densities, equations (2) and (3), respectively, the fraction of stars born until the metallicity has reached a value \( Z \) is

\[ S(Z) = \frac{M_s}{M_{s_1}} = \frac{1 - \mu(Z) - \kappa(Z)}{1 - \mu_1 - \kappa_1}, \]  

(15)
where the subscript 1 indicates present values. From equations (11a) and (11c) we can write

\[ \kappa = \kappa_0 + \frac{\varepsilon - R}{1 - \gamma - R} (\mu_0 - \mu), \]  

where the subscript 0 again indicates initial values. From equations (14), (15) and (16), and recalling the definitions (2) and (3), we obtain, after some algebra

\[ S(Z) = \frac{a - b\mu_0 \left[ \frac{(Z/Z_1 - 1) - (Z/Z_1 - Z_0/Z_1)(\mu_1/\mu_0)}{Z_0/Z_1 - 1} \right]^{1 - \gamma - R}}{a - b\mu_1}, \]  

where \( a \) and \( b \) are constants given by

\[ a = 1 - \kappa_0 - \frac{\varepsilon - R}{1 - \gamma - R} \mu_0, \]  

\[ b = \frac{\zeta}{1 - \gamma - R}. \]  

Of course, \( S \) is normalized, so that from (17) we have \( S = 1 \) for \( Z = Z_1 \).

In order to analyze the results of equation (17), we have varied the initial conditions, namely \( \mu_0, \kappa_0 \) and \( Z_0 \). We have chosen eight sets of initial conditions, labeled by letters A to H, which are shown in table 1. To obtain numerical estimates, we have used the mass fractions given in section 3, namely \( \gamma = 0.3, \zeta = 0.3 \), and \( \varepsilon = 0.4 \). We have further assumed \( \mu_1 = 0.1 \) (Tinsley, 1980; Pagel and Patchett, 1975), and \( R = 0.24 \).

Figure 3 shows cases A to D. As a comparison with the observational data, the asterisks in the figure are obtained from the differential metallicity distribution of 132 G dwarfs in a cylinder passing through the Sun and perpendicular to the galactic plane (Pagel, 1989). We have taken \( Z_1 = 1.19Z_\odot \), according to Table 2 of Pagel (1989), which corresponds to the central value of the largest metallicity bin.

Case A will arise in a disk with non-unitary gas fraction, but with no initial refuses. The distribution will predict even greater values than the simple model (Schmidt, 1963). This is due to the fact that when we set \( \mu_0 \neq 1 \) and \( \kappa_0 = 0 \), we are necessarily accepting some primordial stars with low metallicities. Case B is the simple model with metal sink effect. Cases C and D shows “prompt initial enrichment” models in which the burst of star formation will also lead to the formation of refuses.

Figure 4 shows the effect of increasing \( Z_0 \) fixing the initial gas and refuse mass fractions. We can see that the fits to the observational data are much better, especially for the
models with higher initial heavy element abundances (models G, H). This result is particular
ly interesting when we compare models A and E, where the inclusion of refuses and a very small initial heavy element abundance produce a large difference in the cumulative distribution at low metallicities.

5. Comets and residues

Tinsley (1974) has pointed out that two empirical results provide powerful constraints on chemical evolution models, namely the G-dwarf problem and the slow enrichment rate of the ISM. In our model, the assumption that comets are like sinks of metals explains easily this slow enrichment, provided that the slope of the generalized initial mass function for the residues is \( \lesssim 1.8 \). On the other hand, the G-dwarf problem is also explained if we postulate that a generation of primordial massive stars will give to the disk an initial metallicity, as well as some initial remnants. Of course, the initial conditions are connected, so that the values for \( \mu_0 \) and \( \kappa_0 \) are likely to depend on the value for \( Z_0 \). The crucial assumption in our model is that \( \dot{\gamma} \approx \gamma \).

In order to obtain an order-of-magnitude estimate of the importance of comets as part of the residues, we have estimated the total initial residue mass \( \gamma \Psi \). Using \( 0.19 < \gamma < 0.38 \), and a present value of the generalized formation rate similar to the star formation rate, \( \Psi \approx 10 M_\odot \text{pc}^{-2} \text{Gyr}^{-1} \) (Tinsley, 1980; Miller and Scalo, 1979), we have \( \gamma \Psi \sim 1.9 - 3.8 M_\odot \text{pc}^{-2} \text{Gyr}^{-1} \). The corresponding term for comets can be estimated by

\[
Z\dot{\gamma} \Psi \sim \frac{N_c h_c M_c}{V \tau} \tag{20}
\]

where \( N_c \) is the number of comets, \( h_c \) is the comet galactic scale height, \( M_c \) is the average nuclear mass of a comet, \( V \) is the total volume considered and \( \tau \) is the system lifetime. We have first taken into account the whole solar system, where \( N_c \sim 2.5 \times 10^6 \) (Allen, 1973), \( V \sim 3.1 \times 10^{-11} \text{pc}^3 \) and \( h_c \sim 4.0 \times 10^{-4} \text{pc} \) with a radius of 40 AU (Allen, 1973), \( \tau \sim 5 \text{Gyr} \), and \( M_c \sim 10^{-16} M_\odot \) (Vanýsek, 1987), so that \( Z\dot{\gamma} \Psi \sim 6.5 \times 10^{-4} M_\odot \text{pc}^{-2} \text{Gyr}^{-1} \). Taking the average heavy element abundance during the solar system lifetime \( Z \sim 0.01 \), we have \( \dot{\gamma} \Psi \sim 0.06 M_\odot \text{pc}^{-2} \text{Gyr}^{-1} \), which is much lower than the \( \gamma \Psi \) fraction estimated above. Assuming now the existence of the so-called “Massive Oort Cloud” with \( h_c \sim 0.10 \text{pc} \) for an adopted radius of \( 10^4 \text{AU} \) (Vanýsek 1987), \( V \sim 4.8 \times 10^{-4} \text{pc}^3 \), \( \tau \sim 5 \text{Gyr} \), and \( M_c \sim 10^{-16} M_\odot \), we need \( N_c \sim 10^{13} \) comets to account for most of the residue mass,
in agreement with independent results by Stern and Shull (1990), Vanýsek (1987a,b) and Greenberg (1974).

Acknowledgements

H.J.R.-P. wishes to acknowledge gratitude to Dr. L. da Silva for his support. This work was partially supported by UFRJ, CAPES, CNPq and FAPESP.

References

Bailey, M.E.: 1988, in M.E. Bailey, D.A. Williams, (eds.), Dust in the Universe, Cambridge University Press, Cambridge, p. 113.
Allen, C.W.: 1973, Astrophysical Quantities, Athlone.
Ferrini F., Matteucci F., Pardi C., and Penco, U.: 1992, Astrophys. J. 387, 138.
Greenberg, J.M.: 1974, Astrophys. J. 189, 19.
Larson, R.B.: 1992, Monthly Notices Roy. Astron. Soc. 256, 641.
Maciel, W.J.: 1992, Astrophys. Space Sci. 196, 23.
Meusinger, H.: 1992, Astrophys. Space Sci. 188, 19.
Miller G.E., and Scalo, J.M.: 1979, Astrophys. J. Suppl. 41, 513.
Pagel, B.E.J.: 1989, in J. E. Beckman and B. E. J. Pagel (eds.), Evolutionary Phenomena in Galaxies, Cambridge University Press, Cambridge, p. 201.
Pagel, B.E.J., and Patchett, B.E.: 1975, Monthly Notices Roy. Astron. Soc. 172, 13.
Rana, N.C., and Wilkinson, D.A.: 1986, Monthly Notices Roy. Astron. Soc. 218, 497.
Schmidt, M.: 1959, Astrophys. J. 129 243.
Schmidt, M.: 1963, Astrophys. J. 137 758.
Stern, S.A, and Shull, J.M.: 1990 Astrophys. J. 359, 506.
Tinsley, B.M.: 1974, Astrophys. J. 192, 629.
Tinsley, B.M.: 1980, Fund. Cosm. Phys. 5, 287.
Tinsley, B.M.: 1981, Astrophys. J. 250, 758.
Tinsley, B.M., and Cameron, A.G.W.: 1974, Astrophys. Space Sci. 31, 31.
Truran, J.W., and Cameron, A.G.W.: 1971, Astrophys. Space Sci. 14, 179.

Vanýsek, V.: 1987a, in Symposium on the diversity and similarity of comets, ESA SP-278, Brussels, p. 745.

Vanýsek, V.: 1987b, in J. Palouš (ed.) 10th IAU European Regional Meeting, Praha, p. 279.