ON MAGNETOHYDRODYNAMIC UNSTEADY FLOWS WITH INDUCED FIELD OVER A STRETCHING SURFACE

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Abstract
The problem of unsteady, incompressible, laminar electrically conducting flow past a continuously stretching surface is investigated based on a time dependent length scale. Similarity conditions for the stretching surface flow velocity and induced magnetic field functions are denied. The governing partial differential equations are first transformed to ordinary ones using similarity transformation. The governing system of equation includes the continuity equation, magnetic continuity equation, Maxwell’s equation, momentum equation and magnetic equations. The resulting similarity equation is then obtained through the use of Maple software. Effects of the unsteadiness parameterould be remarked that the present analysis is more general than any previous analysis.

Model Problem:
Let us consider an unsteady, laminar, incompressible, viscous electrically conducting fluid over a linearly stretching surface. The stretching surface is assumed to be electrically non-conducting. The surface is at rest in an unbounded quiescent fluid. The surface is suddenly stretched with velocity \( U = ax \) along the distance (x) of the surface. The magnetic field \( H_2 \) is applied perpendicular to the stretching surface and the effect of the induced magnetic field is taken into account. The magnetic Reynolds number is taken large enough. So that we can consider induced magnetic field effects. Following the induced magnetic field vector \( \mathbf{H} = (H_1, H_2) \). The normal component of the induced magnetic field, \( H_2 \) vanishes at the surface with the components, \( H_1 \) approaching \( H_1 \) imposed magnetic field value at the edge of the boundary condition.

The governing equations for flow in the stretching surface with the momentum equation and magnetic equation are given as:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0; \quad (1)
\]

Keywords: Boundary layer, similarity transformation, Magnetic media, Maple solution.

1 INTRODUCTION
The unsteady laminar boundary layer flow over a continuously stretching plate was first studied by crane [1], who obtained an exact solution to the Navier–Stokes equations. The development of the boundary layer due to a stretching permeable sheet was studied by Gupta and Gupta [2], who reported and exact solution for the flow field and a solution in incomplete gamma functions for the thermal field. Ali [3] studied the general case when the sheet is stretched with stretching velocity.

The boundary layer flow past a stretching plane surface in the presence of a uniform magnetic field, which has practical relevance in polymer processing, was studied by Pavlov [4]. Andersson [5] then demonstrated that the similarity solution derived by Pavlov is not only a solution to the boundary layer equations, but also represents an exact solution to the complete Navier-Stokes equations. Boundary layer flows on a moving surface occur in several engineering applications. Aerodynamic extrusion of a plastic sheet, the cooling of an infinite metallic plate in a cooling both, condensation processes and a polymer sheet or filament extruded continuously from a dye, or a long thread travelling between a feed roll and wind-up roll, are examples of practical applications. Some literature surveys and reviews of pertinent work in this field are documented by Sakiadis [6] who has studied the boundary layers on a continuous semi-infinite sheet moving unsteadily through a quiescent fluid environment. The unsteady laminar incompressible flow of a viscous electrically conducting fluid with constant properties past a semi-infinite flat plate with aligned magnetic field, without heat transfer, has been studied by Greenspan and Carrier [7], Giauert [8] Gribben [9] and Na [10]. The unsteady boundary layer flow over a stationary semi-infinite flat plate in the presence of magnetic field has been studied by Das [11], Ingham [12], and Goyal and Bansal [13] when the free stream velocity is changed impulsively.

In this paper, we have investigated the unsteady fluid flow in the presence of an aligned magnetic field. The governing ordinary differential equations have been solved by using Maple-13 [14]. The velocity and induced magnetic field profiles in the flow are calculated. It may be remarked that the present analysis is more general than any previous analysis.

Figure 1. Physical model and coordinate

Continuity equation:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0; \quad (1)
\]
Magnetic continuity equation:

\[
\frac{\partial H_1}{\partial x} + \frac{\partial H_2}{\partial y} = 0; \quad (2)
\]

Momentum equation:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial H_1}{\partial x} + \frac{\partial H_2}{\partial y} + \frac{\mu_0}{\rho} [H_1 (H_1)_x + H_2 (H_2)_y];
\]

Magnetic equation:

\[
\frac{\partial H_1}{\partial t} + u (H_1)_x + v (H_1)_y - H_1 u_x - H_2 u_y = \alpha_1 (H_1)_y; \quad (4)
\]

Subject to the boundary conditions:

\[
u = U_w, v = 0, \frac{\partial H_1}{\partial y} = H_2 = 0 \quad \text{at} \ y = 0,
\]

\[
u \rightarrow 0, H_1 \rightarrow H \quad \text{at} \ y \rightarrow \infty \quad (5)
\]

where \( u \) and \( v \) are the components of velocity in the \( x \)-axis and \( y \)-axis respectively; \( H_1, H_2 \) are the components of the induced magnetic field in the \( x \)-axis and \( y \)-axis respectively. \( \mu_0, \rho, \nu \) are kinematic viscosity, density of the fluid and dynamic viscosity respectively. \( \alpha_1 \) is the magnetic diffusivity.

Similarity transformations:

In order to make sure to facilitate the analysis full set of governing equation along with the boundary conditions. We assume that the stretching velocity \( U_w(x,t) \) as follows:

\[ U_w(x,t) = \frac{ax}{1 - ct}; \]

Where \( a \) and \( c \) are constants with \( a > 0, c > 0 \).

The form of \( U_w(x,t) \) has been chosen in order to devise a new similarity transformation, the continuity equation \( (1) \) and \( (2) \) are satisfied by introducing a stream function \( \psi \) such that:

\[
u = \frac{\partial \psi}{\partial y}, \quad \frac{\partial \psi}{\partial x} = - \frac{\partial \psi}{\partial x}, \quad H_1 = \frac{\partial \psi}{\partial y} H_2 = - \frac{\partial \psi}{\partial x};
\]

The momentum and magnetic equations can be transformed into the corresponding ordinary differential equations by the following transformation:

\[
\eta = \left( \frac{U_w}{\nu} \right)^{1/2} y;
\]

\[
f(\eta) = \frac{\psi}{(U_w \nu x)^{1/2}};
\]

\[
\psi = (U_w \nu x)^{1/2} f(\eta);
\]

\[
\phi = H_0 (U_w \nu x)^{1/2} g(\eta);
\]

\[
u = (U_w \nu x)^{1/2} f^\prime \left( \frac{U_w}{\nu x} \right)^{1/2};
\]

\[
u = U_w f^\prime;
\]

\[
V = (U_w \nu x)^{1/2} \left( \frac{\nu}{x^2} f^\prime - f \right);
\]

\[
H_1 = \frac{\partial \phi}{\partial y} = H_0 \left( (U_w \nu x)^{1/2} f^\prime \right)^{1/2} \left( \frac{U_w}{\nu x} \right)^{1/2} = H_0 U_w f^\prime;
\]

\[
H_2 = \frac{\partial \phi}{\partial x} = H_0 (U_w \nu x)^{1/2} g^\prime \frac{\eta}{2x} - H_0 (U_w \nu x)^{1/2} \frac{\eta}{2x} x^{-1/2};
\]

Where,

\[ \eta \] is the dimensionless variable.

\[ \psi \] is the stream function.

\[ \phi \] is the magnetic stream function.

\[ f^\prime \rightarrow \text{Dimensionless velocity.} \]

\[ g^\prime \rightarrow \text{Induced magnetic field function.} \]

Finally we have the transformed ordinary differential equations are:

\[
f^\prime\prime + f f^\prime - f^2 - A f^\prime + \frac{2}{3} f^\prime + \beta \left( g^\prime - g^\prime\prime \right) = 0;
\]

\[
A g^\prime + f g^\prime - \frac{1}{2} f^\prime g = A g^\prime + \eta g^\prime = 0;
\]

Now the boundary conditions \( (5) \) becomes:

\[
f = 0, f^\prime = 1, g^\prime = g = 0 \quad \text{at} \ \eta = 0;\]

\[
f^\prime = 0, g^\prime \rightarrow 1 \quad \text{at} \ \eta = \infty; \quad (6)
\]

2 RESULTS AND DISCUSSIONS:

The solution of the transformed governing equation and boundary conditions \( (6 - 8) \) are accomplished through Maple-13 as described in \[14\] over a range of magnetic force number \( \beta = 0, 2, 4; \) reciprocal magnetic prandtl number \( \lambda = 0.1, 0.5, 5 \) and unsteadiness parameter \( A = 0, 1, 2. \) The velocity profiles \( f^\prime \) and \( g^\prime \) have been shown in figure \( (1-6) \) for different values of the governing parameters.

The effects of magnetic force number \( \beta \) on the dimensionless velocity profiles \( f^\prime(\eta) \) which displayed in figure-2. It is observed that the presence of \( \beta \) causes higher induction to the fluid which enhanced the velocity. It is evident from this simulation that the boundary condition \( f^\prime = 1 \) is satisfied.

![Figure 2. Induced velocity profiles for different values of \( \beta \).](image)

The induced magnetic field profiles \( g^\prime(\eta) \) for various values of magnetic force number \( \beta \) against \( \eta \) and shown in the figure 3. From this figure we see that the \( g^\prime(\eta) \) profiles increases with the increasing values of magnetic force number inducting that the magnetic force accelerates the fluid motion.

![Figure 3. Induced magnetic field profiles \( g^\prime(\eta) \) for different values of \( \beta \).](image)
The velocity profiles for the various values of the unsteadiness parameter $A$ and presented in the figure 4. From this figure we see that the velocity profiles decreases with an increasing $A$. It is interesting features that without unsteadiness the flow has overshoot along the boundary condition than that of with unsteadiness and satisfy the boundary condition $f'(\infty)=1$.

Figure 4. Velocity profiles $f'(\eta)$ for different values of $A$.

Figure 5. presents the effects of the unsteadiness parameter $A$ on induced magnetic function. The profiles for the various values of the unsteadiness parameter $A$ presented in the figure 4. It shows that the magnetic field $g'(\eta)$ decreases with $\eta$ for $\beta=2$ and $\lambda=0.5$. It can be seen that the flow profiles decreases with increasing $A$.

Figure 5. Magnetic field profiles $g'(\eta)$ for different values of $A$.

The velocity profiles for the various values of the reciprocal magnetic prandtl number $\lambda$ and presented in the figure 6. We see that the velocity increases in the boundary region with increasing $\lambda$.

Figure 6. Velocity profiles $f'(\eta)$ for different values of $\lambda$.

The effects of varying the reciprocal magnetic prandtl number $\lambda$ on the induced magnetic functions are depicted in figure 7. We see that flow profiles increases with increasing $\lambda$. At higher $\lambda$, the flow induced more than that of lower $\lambda$. At lower $\lambda$, the flow has reverse overshoot on the boundary due to low Lorentz force.

Figure 7. Magnetic field profiles $g'(\eta)$ for different values of $\lambda$.

3 CONCLUSION

The problem of unsteady MHD boundary layer and magnetic fluid flow due to a continuous stretching plate immersed in an electrically conducting fluid was investigated by using Maple. Our main target was to investigate the velocity profiles and magnetic fields function for governing parameters, unsteadiness parameter $A$, Magnetic prandtl number $\lambda$, magnetic force parameter $\beta$ through similarity transformations, the governing time dependent boundary layer and magnetic equations for momentum and magnetic field are reduced to couple ordinary differential equations which are then solved through the use of Maple-13 [14].

The similarity solution indicate that:

(1). The boundary layer thickness increases as the values of $\beta$ increase.
(2). Velocity and magnetic functions against similarity variable increases as the values of $A$ increases. Also we have seen overshoot on boundary when $A=0$.
(3). The both flow profiles increases as the values of $\lambda$ increase.

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