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Boosted PTS Method with Mu-Law Companding Techniques for PAPR Reduction in OFDM Systems

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Abstract: Perpendicular rate of recurrence splitting up a group of numeral television or radio channels that are mixed together for broadcast Orthogonal Frequency Division Multiplexing which can be a potential diffusion method for elevating the transmission capacity of the communication systems. In spite of the significance of OFDM, the primary issue of the peak-to-average power ratio (PAPR) which augments communication system complications, reduces the effectiveness of the communication system, resulting in low performance of bit-error-rate (BER), and making OFDM perceptible toward non-linear distortion within a broadcast. Various techniques were projected for treating PAPR issues, inclusive of partial transmit sequence (PTS) which captivated great interest. Thus, this paper proposed a hybrid method inclusive of a boosted PTS scheme with Mu-law compressing and expanding approach. The PTS approach was boosted through boosting its sub-block partitioning scheme, the place where the aggrandized partitioning scheme consolidated a conventional interleaved partitioning into an adjacent partitioning scheme. The present merger concerning Mu-Law characteristic in time domain for PAPR reduction in OFDM fundamentally boosts PAPR diminution performance. Accordingly, though the simulated pseudorandom sub-block partition method improved PAPR diminution supplementary further than other sub-block partition schemes appertaining to conventional PTS, while maintaining low computational complexity. The findings show that the boosted PTS scheme with Mu-law expanding approach, whilst upholding low computational complexity, achieves considerably superior to the pseudorandom partitioning PTS with regard to various type of modulation format and subcarriers.

Keywords: Sub-block partitioning, Rotation factors, Mu-Law companding, OFDM

I. INTRODUCTION

The OFDM structure has many advantages including high spectral efficiency, parrying interference, as well as sturdiness against signal disappearance [1]. It is habitually utilized in numerical acoustic plus record dissemination, and portable interactive media [2, 3]. Conversely, OFDM is apt large peak-to-average power ratio (PAPR), which result in nonlinear distortion of the power amplifiers and weaken the performance of the OFDM system; this is common in Wavelength Division Multiplexing (WDM) [4, 5] and Mode Division Multiplexing [6, 7]. Several approaches has been taking into consideration to reduce PAPR, including cutting [8], classification [9], clipping and refining [10], tone injection (TI) [11], dynamic constellation expansion [12], tenor preservation [13], and manifold signal depiction schemes such as selected mapping (SLM) [14] and interlacing [15], along with partial transmit
sequence cord (PTS) [16]. Of the current schemes, PTS is considered unique most likely to improve the PAPR of an OFDM signal because of its Longitudinal Method and zero sign deformation [17, 18]. This is important as PTS categories of two methods that characterized as partitioning the original OFDM into a set of disjoint sub-blocks; the other one is to render a group of aspirant signs via adding phase rotated sub-blocks to select the minimum PAPR for broadcast [19]. All of these techniques have advantages and disadvantages in terms of their PAPR reduction capabilities, the cost of BER performance degradation, increased computational complexity, and loss of data rate. However [20-22], PTS is one of the most promising techniques, attractive for its high-quality PAPR reduction. Its main drawback is the high computational cost in finding optimum phase factors as well in finding uncorrelated partitions. In this paper, we offer a boosted PTS method with improvements in two parts, namely, partitioning and phase rotation. For partitioning, we incorporated the AP scheme into the IP scheme resulting in a new partition scheme performing better than either AP and IP. For phase rotation, an optimal set of rotation vectors is derived according to the correlation properties of candidate signals based on the work of [23]. The performance of the boosted PTS method is further improved by applying a nonlinear companding technique, Mu-Law, to the output of the boosted PTS. The application of Mu-Law in the time domain of the OFDM signal significantly improved the PAPR reduction performance.

This paper proceed as follows, the analytical model is presented in Section II. Section III present the boosted PTS approach. IV presents the results and discussion. Finally, the paper is concluded in Section V.

II. ANALYTICAL MODEL
A. PAPR in OFDM System
The OFDM consists of $N$ subcarriers with the same bandwidth. The OFDM inverse fast Fourier transforms the IFFT and forms the transmitted signals. A block of transmitted signal, $X_k = [X_0, X_1, \ldots, X_{N-1}]$ can be represented as:

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi kn}{N}}, 0 \leq n \leq N-1$$

(1)

where the number of subcarriers is denoted as $N$ and the domain frequency is indicated as $X$.

The CCDF is defined as [24, 25]

$$CCDF(PAPR_0) = P_r (PAPR > PAPR_0)$$

(2)

B. Partial Transmitted Sequences (PTS)
The PTS is shown in Figure 1. The sub-block of the partitioned scheme is pooled to demote the PAPR, and the occupied subcarriers’ positions are set to zero [26], expressed as:

$$X = \sum_{m=1}^{M} X_m$$

(3)

The IFFT is distinctly realized for every sub-block to transform the sub-block partitioning from frequency domain to time domain given as:
Figure 1 shows a sample schematic diagram of an ordinary PTS system with 64 subcarriers.

\[ x_m = \sum_{m=1}^{M} \text{IFFT} \{ X_m \} \]  
\[ x = \sum_{m=1}^{M} b_m x_m \]

Figure 2 compares the performance of the three sub-blocks partition method.

Figure 2. Comparison of PAPR reduction performance with different sub-block partitions
C. Mu-Law Companding Technique

Mu-Law is a potent compression method which can be utilized to reduce PAPR [27]. The companding techniques for PAPR reduction are reviewed in [28, 29]. Figure 3 shows the OFDM Mu-Law companding technique.

\[ S_u(t) = \frac{\ln \left(1 + \mu \frac{|S(t)|}{S_{\text{max}}(t)}\right)}{\ln(1 + \mu)} S_{\text{max}}(t) \cdot \text{Sgn}(S(t)) \]  

(6)

where \( S(t) \) is instantaneous magnitude of the input, \( S_{\text{max}}(t) \) is the peak magnitude of \( S(t) \), \( \text{Sgn} \) is a sign function, and \( \mu \) is the Mu-Law compand parameter.

III. BOOSTED PTS APPROACH

This section explains the boosted PTS which merges two important steps, partitioning and phase rotation, both of which need further improvement by incorporating a boosted PTS method with nonlinear companding. This can be done by applying Mu-Law characteristics in the time domain for PAPR reduction in OFDM. The PAPR reduction performance will be improved and at the same time the computational complexity will need to be reduced when comparing the performance of these algorithms with ordinary PTS schemes.

1. Boosted Partitioning Method

The present segment reveals the planned crossbred modus operandi which merges a boosted PTS method in the company of Muu- Rule compressing and expanding entrée like the exposed in shape 4. The improved PTS plan, that integrates interlaced apportionment within neighboring apportionment method, has been suggested [30]. likewise, in favor of neighboring apportionment, the boosted PTS method begins by way of an information contribution framework partitioned into \( V \) neighboring lumps. Then, lumps being split up \( S \) size secondary-Masses. Ultimately, interleaved partitions \( P_i \) are constructed by assigning the secondary-Masses into the partitions as:

\[ P_i \left( \frac{q}{r} \right) = sb_i(q) \]  

(7)
Where, $P_i\left(\frac{q}{r}\right)$ symbolizes the $q^{th}$ that is the secondary mass components $r$ in $P_i$, like the splitting up, $Sh_i(q)$ point to the $q^{th}$ component of the $i$ secondary mass within the block $r$ of the original data. The interspersed splitting up of the lumps comprises of a number of secondary-Masses $v_s$ being the bulk of the secondary-mass. After that every one of the barren interspersed splitting up comprises of $v_s,v$ components. Concerning the unique average of recurrence of the rate of recurrence domain data $X_i$, the IDFT of each of the divisions is seized autonomously. consequently, IDFT yield for the division $P_i$ is specified via:

$$x_n^{(i)} = \sum_{q=0}^{n-1} \sum_{r=0}^{r-1} P_i\left(\frac{q}{r}\right) e^{j2\pi(i+iv+q)N/N}$$  \hspace{1cm} (8)$$

wherever $x_n^{(i)}$ symbolizes $n^{th}$ the PTS succession model which stands for division $P_i$, the amount of supplementary transporter is offered as $N$, while $i$ is the lump’s amount $(l = N/v)$. $r$ being the indicator of the secondary mass in the division, while $q$ is the secondary mass index $x_n^{(i)}$ is the PTS successions which have stage revolution in the company of a feature $w_i$, exclusive of the initial succession $x_n^{(0)}$ which stays steady $w_i=1$. The stage features $w_i$ is articulated like:

$$w_i = e^{j\varphi}, \quad i = 0, 1, \ldots, (z-1)$$  \hspace{1cm} (9)$$

wherever $\varphi$, haphazardly chooses figures amid $0 \leq \varphi \leq 2\pi$, $z$ is interwoven division lump’s amount, $\tilde{x}_n^{(i)} = w_i \cdot x_n^{(i)}$ is the revolved successions which are employed to be united so as to produce the contender’s broadcast sign $\tilde{x}_n$ that have analogous data in the stage feature.

$$\tilde{x}_n = \sum_{i=0}^{n-1} \tilde{x}_n^{(i)}$$  \hspace{1cm} (10)$$

Accordingly, in the course of the appliance of Muu- Rule compressing and expanding entrée as set by Eq. (7) on the broadcast sign contender $\tilde{x}_n$, the fresh sign is attained in this way:

$$x_n(t) = \frac{\ln \left[\frac{1+u}{\max(\tilde{x}_n)} \right]}{\ln(1+u)} \cdot \max(\tilde{x}_n) \cdot \text{Sgn}(\tilde{x}_n)$$  \hspace{1cm} (11)$$

The procedure is frequented by several periods ($\tau$), every period in the company of a variety of number of stage revolution standards. In each recurrence, PAPR of the contender precursor and the contender precursor in particular are calculated and the matching number of the stage features are kept. Following $\tau$ repetitions, the OFDM sign in the company of the lowly PAPR is broadcasted.
2. Phase Rotation Method

The important issues of PTS must be solved, therefore the method proceeds to optimize the candidates to decrease the complexity. A method of selecting the candidate rotation vector is adopted from [23] and, combined with the partitioning approach, results in what we call the boosted PTS method. The core idea of this method is taking the candidate with the minimum PAPR.

This method is based on analysis of the relationship among phase factors and the correlation of candidate signals. It is used to pre-compute a set of different rotation vectors that result in a set of candidate signals with the lowest correlation. This pre-computation drastically reduces the real-time computational complexity. The candidate phase factors construction problem is based on combination theory. A combination is a method of selecting members from a group. Suppose we have a collection of \( n \) objects. Then the number of combinations of these \( n \) objects taken \( r \) at a time without repetition is actually the number of selections of \( r \) objects out of \( n \) where the order is not counted. It is denoted by:

\[
^n C_r = \frac{n!}{r!(n-r)!} \tag{12}
\]

We constructed a candidate phase factors table through solving the minimization problem in Eq. (14). It is basically a combinatorial problem in a combinatorial space of \(^n C_r\) elements where \( n \) represents the total number of possible rotation vectors and \( r \) is the number of candidate signals.

As already mentioned, in order to reduce the searching complexity, the selection of the phase factors has been limited to a set of a finite number of elements. Therefore, it can be confirmed that the influence of different sub-block partitions brought to a candidate signal’s correlation value satisfies \( b_i^v b_i^{v^*} = 1, -1, i, -i \) when \( v = 1, 2, 3, 4 \).

In this method, a function \( C \) is considered to estimate the average value of correlation of the candidate signals as follows:

\[
C = \frac{1}{MV^2} \sum_{1 \leq i < l \leq U, U} \left| \sum_{v=1}^{V} b_i^v b_l^{v^*} \right|^2 \tag{13}
\]

where \( C \) is representing the average value of the correlation, and \( M \) denotes all combinations of \( i \) and \( l \) which satisfy \( 1 \leq i < l \leq U, U \) the number of candidate signals, \( V \) the number of data sub-blocks, and \( M = U(U-1)/2 \). \( b_i^v b_l^{v^*} \) represents correlation between the \( v^{th} \) sub-blocks of the \( i^{th} \) and \( l^{th} \) candidates.

The objective is to find a set of candidate signals corresponding to minimum correlation, \( C_{\text{min}} \). The phase factors which can be summarized as
where $b^*$ and $b^{**}$ represent phase factors.

Let us illustrate the problem of selecting lowest correlation candidate signals in the case where the number of phases is 2 and the number of partitions is 4. Therefore, the number of possible phase vectors is $2^4 = 16$. If the phases are -1 and +1, the resulting 16 phase vectors are as shown in Figure 4.

If the number of candidate signals is 7 then the 7 corresponding phase vectors can be chosen in $16 \choose 7 = 11,440$ different ways. That is, there are 11,440 possible sets of phase vectors with each set containing 7 elements. Out of these 11,440 different sets, the one with the lowest correlation can be determined through solving the minimization problem (Eq. 21). The solution (7 candidate phase vectors resulting in lowest correlation signals) is shown in Table 1, where the phase index identifies one of 11,440 different sets of phase vectors.

![Figure 4: Phase vectors for 2 phase levels and 4 partitions](image)

| Phase index | Phase 0 | Phase 1 | Phase 2 | Phase 3 |
|-------------|---------|---------|---------|---------|
| 0           | -1      | -1      | -1      | 1       |
| 1           | -1      | -1      | 1       | -1      |
| 2           | -1      | 1       | 1       | -1      |
| 3           | 1       | -1      | -1      | 1       |
| 4           | 1       | -1      | -1      | 1       |
| 5           | 1       | -1      | 1       | -1      |
| 6           | 1       | -1      | 1       | 1       |

Table 1: Results of the best candidate with lowest correlation signals
The candidate phase vectors found by this phase rotation approach, as described in the previous partitioning approach, are applied to the time domain signal given by Eq. (16) which was obtained through partitioning and IFFT transformation. The procedure to obtain candidate signals is similar to the one shown in the partitioning approach, except that in the present case phase rotation vectors computed using the phase rotation approach were used instead of randomly chosen phase factors. Finally, the candidate signal with minimum PAPR is selected for transmission.

The phase rotation approach selects phase rotation factors from the table of candidate rotation vectors with the lowest correlation, and this table is computed only once. Thus, in our boosted PTS scheme, it is not required to perform any research in the space of phase vectors as done in the ordinary PTS method. As the candidate phase vectors in this approach are generated by the correlation minimization technique, the computational complexity is significantly reduced compared with the ordinary PTS method.

Finally, by combining the Mu-Law companding technique in Eq. (7) with the transmit signal candidate \( \tilde{x}_n \), the new signal is expressed as:

\[
\tilde{x}_n(t) = \frac{\ln \left( 1 + u \frac{|\tilde{x}_n|}{\max(\tilde{x}_n)} \right)}{\ln(1 + u)} \max(\tilde{x}_n) \cdot \text{Sgn}(\tilde{x}_n)
\]

The PAPR transmitted signal is computed and verified with its corresponding set of phase factors then stored. Finally, the lowest PAPR is transmitted after \( \tau \) iterations.

V. RESULTS AND DISCUSSION

The performance evaluation of the boosted PTS with the traditional PTS algorithm of three different types of sub-blocks partition schemes (adjacent, interleaved and pseudorandom) is carried out using a numerical simulation. The simulations were conducted to assess the performance of the PTS techniques for PAPR reduction, with input data blocks of length 64 (\( N = 64 \)) are random partitioned into 4 sub-blocks (\( V = 4 \)), the number of alternative phase factors is \( W = 4 \) are selected from \( \{\pm1, \pm j\} \), candidate signals \( U = 8 \), and different types of modulation format including QPSK, 8PSK, 16QAM and 64QAM, respectively, the CCDF of the PAPR for the boosted PTS method and the traditional PTS schemes are highlighted in CCDF threshold=10^{-3}.

The QPSK modulation format of CCDF versus PAPR is shown in Figure 5. The PAPR of the original OFDM (signal without PTS) was 10.2 dB, IP-PTS was 8.4 dB, AP-PTS was 8.1 dB, PRP-PTS was 6.8 dB, partitioning PTS method (improvement only in the partitioning step) 5.8 dB and boosted method was 4.1 dB, respectively. The IP has better performance than the original OFDM by 1.8 dB. As well as, the AP, PRP, partitioning PTS method and boosted PTS method reduced the PAPR related to IP around 0.3 dB, 1.6 dB, 2.6 dB and 4.3 dB, respectively. The above statistical
numerical result, it is evident that boosted PTS method gives far better PAPR reduction than partitioning PTS method and ordinary PTS schemes.

Figures 6, 7 and 8 show the performance of boosted PTS compared to the partitioning PTS method and the ordinary PTS schemes (IP AP and PRP) with the original OFDM when the modulation formats are 8PSK, 16QAM and 64QAM, respectively. As observed in Table 2, the PAPR reduction for the boosted PTS method degrades slightly compared to the partitioning PTS, ordinary PTS and VL-AP PTS schemes.

| Modulation | CCDF | PAPR of Boosted PTS method (dB) | PAPR of Partitioning PTS method (dB) | PAPR of PRP-PTS (dB) | PAPR of AP-PTS (dB) | PAPR of IP-PTS (dB) | PAPR of Original OFDM (dB) |
|------------|------|---------------------------------|-------------------------------------|---------------------|-------------------|-------------------|--------------------------|
| QPSK       | $10^3$ | 4.3                             | 5.8                                 | 6.8                 | 8.1               | 8.5               | 10.9                     |
| 8PSK       | $10^3$ | 4.1                             | 5.8                                 | 6.8                 | 8.1               | 8.4               | 10.2                     |
| 16QAM      | $10^3$ | 3.9                             | 5.9                                 | 6.8                 | 7.9               | 8.2               | 10.1                     |
| 64QAM      | $10^3$ | 3.7                             | 5.8                                 | 7                   | 8.2               | 8.7               | 9.9                      |

Therefore, from these figures we can deduce that the PAPR reduction performance for the boosted PTS in conjunction with Mu-Law is better than the ordinary partitioning PTS method.
Figure 6. Boosted PTS scheme combined with MU-Law technique compared to other PTS techniques for 8QPSK modulation format

Figure 7. Boosted PTS scheme combined with MU-Law technique compared to other PTS techniques for 16 QAM modulation format
VI. CONCLUSION

There is a real need to design a phase factor on the average value of correlation in order to achieve better PAPR reduction and overcome the exponential complexity that results from the use of conventional PTS which requires a comprehensive search of all candidates to obtain the ideal phase weight and the weight vector. On the other hand, another improvement can be realized by utilizing the augmented companding technique of Mu-Law incorporation with PTS. This study has described boosted PTS with the enhancement in three different approaches, namely partitioning, phase rotation, and augmentation with companding. The study shows that the PAPR performance of the boosted PTS with the Mu-Law companding technique is better than the PRP-PTS in different modulation formats. The boosted PTS as the advantage of achieving a significant reduction of PAPR while maintaining a low computational complexity.

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