Energy dissipation in wave propagation in general relativistic plasma

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Abstract

Based on a recent communication by the present authors the question of energy dissipation in magneto hydrodynamical waves in an inflating background in general relativity is examined. It is found that the expanding background introduces a sort of dragging force on the propagating wave such that unlike the Newtonnian case energy gets dissipated as it progresses. This loss in energy having no special relativistic analogue is, however, not mechanical in nature as in elastic wave. It is also found that the energy loss is model dependent and also depends on the number of dimensions.

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1. Introduction

Interaction of electromagnetic waves with plasma, especially reflection and attenuation of electromagnetic waves propagating in plasma layers has attracted a great deal of attention in recent years due to their potential applications in reflecting or absorbing e.m wave energy, broadcasting signals in microwave frequencies etc [1]. Moreover, radiation and energy loss in cosmic plasma via Cerenkov radiation [2] as also Heavy quark energy loss in a weakly coupled QCD plasma [3] were studied in the past. But here we discuss a situation where energy dissipation in the propagation of an electromagnetic wave through a cosmic plasma medium is not due to any scattering or friction but due to the expanding background. In two recent communications [4, 5] we have investigated the propagation of an electromagnetic wave through plasma in an expanding background in the framework of higher dimensional spacetime(HD). The present work is the continuation of the earlier two and possibly the last one in the series where we specifically study the question of energy dissipation of the electromagnetic wave in its interaction with the plasma field. While great strides have been made by general relativists to address the issues coming out of the recent observations in the field of astrophysics and cosmology and despite the fact that more than 90 percent of the cosmic stuff in stellar interior and intergalactic spaces is made up of matter in plasma state the much sought-after union between the plasma dynamics and general relativity still remains elusive. One inhibiting factor against the reunion is possibly the fact that both Einstein’s field equations and plasma equations are highly nonlinear such that a combination of the...
two makes exact analytical solutions very difficult to get forcing people the alternative route to either numerical analysis or a linearized approximation of the plasma equations. However, following the well known (3+1) formulation of general relativity by Arnowitt, Deser and Misner (ADM) and its subsequent development and applications for a covariant formulation of the equations of magnetohydrodynamics (MHD) in general relativity by Thorne and Macdonald [6] there has been some spurt in activities in general relativistic plasma [7, 8, 9, 10]. While classical MHD is rather well developed, not a great deal is known about the GRMHD and partly because of the usual subtleties in defining physically meaningful frame of reference in GR one must be wary of applying the classical results with intense gravitational fields. In his electrodynamics of moving bodies Minkowski has given a covariant decomposition of the electromagnetic fields, which was later extended by Pham Man Quan [11] and most extensively by Lichnerowicz [12] in GRMHD. In a series of articles listed earlier some of us studied the interaction of electromagnetic waves with plasma, propagation of Alfven wave and also found out the dispersion relations and other related properties in an expanding background, for which as a test case we have chosen spatially flat Friedmann-Robertson-Walker (FRW) cosmology generalised to higher dimensions. This is the simplest background, yet it still illustrates how the curvature as well as the nonstaticity of spacetime can affect simple MHD results. For this simple metric it is possible to split the ordinary electromagnetic field tensor, $F^{\mu \nu}$ into the ordinary electric and magnetic fields E and B in terms of which the field equations are more familiar. For our present work the consequences of this simple decomposition is nontrivial. This allows us to make use of the intuition from the known results of flat MHD to be applied to its general relativistic analogue. As the split formalism has been extensively discussed in the literature we shall not, for brevity, restate those things here but refer the readers to reference [4]. We have attempted the analysis in the framework of HD spacetime which is an active area of activity in its attempt to unify gravity with all other forces of nature. It also finds increasing applications in brane inspired cosmology as also in STM theory [13]. Moreover, it may also provide an alternative physical explanation of the current accelerating era of the universe without bringing in any hypothetical quintessential type of scalar field [14] by hand. For our case it assumes particular significance because it is in the realm of early universe that both plasma physics and HD are specially relevant. In fact it can be shown that Einstein’s equations generalized to higher dimensions admit solutions where as the 3D space expands with time the extra dimensions shrink in size as to be currently invisible with the present day experimental technique. It is further conjectured that some stabilising mechanism (quantum gravity may be a possible candidate) stabilises the extra space to the planckian scale. So the universe we see today appears manifestly four dimensional.

As mentioned earlier we have in the past rather extensively discussed the propagation of electromagnetic waves through different types of perfectly conducting plasma medium. For the very simple, conformally metric form chosen for our test case we find that most of the wellknown the general relativistic results mimic the analogous newtonnian plasma mechanics, except that all the field variables are now non static and share the background expansion of the FRW metric. In this work we turn our attention to another important aspect of wave propagation i.e., energy dissipation.
In classical MHD one finds that no power is dissipated during propagation for a perfectly conducting medium. But the situation drastically changes for an expanding background where energy dissipation does occur. It is observed that the expanding background introduces a sort of dragging force to effect the dissipation, which is not to be confused with the usual mechanical drag. In the present work we have calculated the energy dissipation using our solutions from the previous works. Although we have mainly carried out the exercise in a higher dimensional (HD) background we believe that most of our findings, barring some qualitative differences, apply to the four dimensions also.

2. Mathematical formalism

a. Newtonian Mechanics

Before embarking on the question of dissipation of electromagnetic wave through a plasma medium in general relativity we try to recapitulate, very briefly, the analogous situation in flat space. From any standard textbook on Plasma physics we know that in newtonian mechanics the dielectric constant is hermitian which implies no damping. This can be easily shown as follows: When propagating through a plasma medium the electromagnetic variables, for example, are given by

\[
E = \text{Re} \ E_0 e^{i(k;x - \omega t)}
\]

\[
J = \text{Re} \ J_0 e^{i(k;x - \omega t)}
\]

Here \(E\) and \(J\) may depend on \(\omega\) and \(k\) but not on time such that the average power dissipation in a cycle is given by

\[
P = E_i J_i = \frac{1}{4} \left[ E_0 J_0 e^{2i\phi} + E_0 J_0^* + E_0^* J_0 + E_0^* J_0^* e^{-2i\phi} \right]
\]

where \(\phi = k.x - \omega t\) Again \(J = \sigma E\). Now for homogeneous spacetime the conductivity, \(\sigma\) behaves like a scalar. But for anisotropic spacetime (say an external magnetic field in a particular direction) it is, in general, a tensor of rank two because, as is well-known, the linearized particle velocity

\[
v = \frac{e}{i\omega m} (E + \frac{v}{c} \times B_0)
\]

introduces an anisotropic velocity field such that the constitutive relation reduces to, \(J_i = \sigma_{ij} E_j\) and we get for average dissipation over a complete cycle

\[
<P> = \frac{1}{4} \left[ E_0 \sigma_{ij}^* E_{0j}^* + E_0^* \sigma_{ij} E_{0j} \right]
\]

Now, for any arbitrary vectors and matrix \(A.M.B = B.M^T.A\) where \(T\) refers to transposition. So the last equation implies that

\[
<P> = \frac{1}{4} E_0^* ((\sigma_{ij}^T + \sigma_{ij}) E_{0j})
\]
We also know from elementary plasma mechanics that the dielectric tensor of the plasma medium is related to the conductivity tensor as

$$\varepsilon_{ij} = \delta_{ij} + \frac{1}{-i\omega\varepsilon_0}\sigma_{ij} \quad (7)$$

Now the dielectric tensor for cold plasma (anisotropic in general) is hermitian, as can be checked in any standard text book which necessitates that the conductivity tensor should be anti hermitian i.e. $\sigma_{ab} = -\sigma_{ba}^T$. So there is no energy dissipation in classical plasma dynamics when an electromagnetic wave moves in a plasma medium with a static background.

### b. General Relativistic Case

The situation changes drastically when a similar analysis is carried out in a nonflat expanding background of arbitrary dimensions. As mentioned in the introduction the authors of this report studied [4, 5] the propagation of electromagnetic waves in an expanding plasma background in the framework of Einstein’s field equations both in four and higher dimensions taking spatially flat Friedmann- Robertson -Walker metric for simplicity. Using the well known 3+1 decomposition formalism of ADM we get the interesting results that the field variables mimic the classical special relativistic results except that the sinusoidal vibrations need to be replaced by Hankel functions and the field parameters are no longer constants but share the background velocity of the embedded metric. Secondly the close resemblance to the special relativistic results may be due to the very simple metrics form we have considered - the conformally flat FRW line element. With more complicated background metric, we suppose, the 3+1 split would yield significantly different results. We shall not go into the details of our earlier works here but to make the present work more tractable and transparent we need to digress time to time to one of our recent papers[4] very briefly and refer to the relevant equations only as and when absolutely necessary.

For our background space we take the (d+1) dimensional generalized FRW space time as

$$ds^2 = dt^2 - A^2 (dx^2 + dy^2 + dz^2 + d\psi_n^2) \quad (8)$$

where $A \equiv A(t)$ is the scale function.

In an earlier work [15] one of us extensively discussed the (d+1) dimensional isotropic and homogeneous space time and assuming an equation of state, $p = \gamma\rho$ found the scale factor as ($p = \text{pressure}, \rho = \text{energy density}$)

$$A \sim t^{\frac{2}{3\gamma(1+\gamma)}} \quad (9)$$

We then wrote down the Maxwell’s equations (for more details see Mcdonald et
al for 3+1 split) for this metric. Before proceeding further let us ask the pertinent question - why is it not possible to formulate the equations of electrodynamics in manifestly covariant form using electric and magnetic four vectors? This is at variance with the case of spin in an external electromagnetic field where one can define a spin 4-vector $s^\mu$ whose spatial part reduces to $s$ in the proper frame of the particle and so subject to the constraint that $s^\mu u_\mu = 0$ with $u_\mu$ the 4-velocity of it \[10\]. A possible answer to this question comes immediately to mind- the electric and magnetic fields are not the spatial component of any four vector. It is only a very particular combination of their components which form a fully covariant object, the electromagnetic field tensor, $F_{\mu\nu}$. Only using this tensor the manifestly covariant form of the Maxwell’s equations can be achieved. However, if one chooses a preferred coordinate system it is indeed possible to use the electric and magnetic fields as $(d+1)$-vector (we are here considering a $(d+1)$ dimensional spacetime) and then finally write down the Maxwell’s equations in a fully covariant form \[17\].

As mentioned earlier the present work investigates plasma physics in curved spacetime. To make use of the intuition gained from the conventional plasma dynamics it is preferable to split the electromagnetic tensor, $F_{\mu\nu}$ into electric and magnetic fields $E$ and $B$ in terms of which equations are more familiar. This requires choosing a particular set of fiducial with respect to which $E$ and $B$ and other physical quantities are measured. In what follows we shall presently see that for our simple background metric the electromagnetic field tensor via $(3+1)$ decomposition does decouple as $d$-dim. electric and magnetic field and for the privileged fiducial observers (FIDOs) one may write

$$F_{\mu\nu} = E_{\nu} u^\mu - E^\mu u_\nu + \epsilon_{\mu\nu\gamma\delta} u_\gamma B_\delta \quad (10)$$

$$J^\mu = \rho e u^\mu + j^\mu \quad (11)$$

$$\Phi^\mu = \phi u^\mu + A^\mu \quad (12)$$

$$F^{*\mu\nu} = B_{\nu} u^\mu - B^\mu u_\nu + \epsilon_{\gamma\nu\mu} E_\gamma \quad (13)$$

where $u^\mu$ is the fiducial $(d+1)$-velocity and $\epsilon_{\mu\nu\gamma\delta}$ is the $(d+1)$ dimensional Levi-Civita tensor.

Here the RHS terms are measured by the fiducial observers in the usual manner of flat spacetime, and which therefore have the usual physical interpretations and are orthogonal to $u^\mu$ whereas the LHS terms are the reconstructed charge-current $(n+1)$-vector, $J^\mu$, $(n+1)$-vector potential, $\Phi^\mu$ etc.

Moreover the electric current, $J^\mu$ is now the sum of the two terms corresponding to the convection current and the conduction current, $j^\mu$ respectively and $j^\mu u_\mu = 0$.

One can invert these relations to get

$$\rho e = -J^\mu u_\mu \quad (14)$$

$$j^\mu = \gamma^{\mu\nu} J_\nu \quad (15)$$

$$E^\mu = F_{\mu\nu} u_\nu \quad (16)$$

$$B^\mu = -\frac{1}{2} \epsilon_{\mu\nu\gamma\delta} u_\nu F_{\gamma\delta} \quad (17)$$
\[ \phi = -\Phi^\mu u_\mu \]  
\[ A^\mu = \gamma^{\mu
u} u_\nu \]  

We are now in a position to formulate the general relativistic Maxwell’s equations for our simple FRW metric to get (see ref. 2):

\[ \nabla . E = 4\pi \rho_e \]  
\[ \nabla . B = 0 \]  
\[ \frac{\partial E}{\partial t} = KE + cA^{-1} \nabla \times B - 4\pi J \]  
\[ \frac{\partial B}{\partial t} = KB - cA^{-1} \nabla \times E \]  
\[ \frac{\partial \rho_e}{\partial t} = K \rho_e - \nabla . J \] (charge conservation)

and finally the particle equation of motion in \((d + 1)\) dimensions as

\[ \frac{D A^{d-1} p}{D\tau} = A^{d-1} q \left( E + A^{-1} v \times B \right) \]  

or

\[ \frac{D p}{D\tau} = \frac{d - 1}{d} K p + q \left( E + A^{-1} v \times B \right) \]

where

\[ \frac{D}{D\tau} = \frac{1}{\alpha} \left( \partial_t + v . \nabla \right) \]

is the convective derivative and the \(d\)-momentum

\[ p = m_e \Gamma v \]  

\((m_e\) is the rest mass, \(\Gamma\) is the boost factor, and \(v\), the \(d\)-velocity).

Here \(\nabla\) and \(\nabla \times\) are the ordinary Minkowskian divergence and curl in Cartesian co-ordinates. Thus we see that at least for the very simple type of metric chosen the electromagnetic field tensor of general relativity is split up and gets decomposed as ordinary flat space electric and magnetic field.

In what follows we shall consider, for simplicity, the small amplitude linear theory such that the convective derivative simply reduces to ordinary derivative, \(\frac{d}{dt}\).

We see that the equations (20, 21) have the form familiar from flat-spacetime, Lorentz frame electrodynamics. They permit one to characterise \(E\) and \(B\) by electric and magnetic field lines while the rest have a slightly different form with some additional inputs from curved geometry (e.g., \(A\) and \(K\) terms).

In this section we investigate the situation where a plasma in thermodynamic equilibrium is slightly disturbed through the passage of an electromagnetic wave. We assume that an external ambient magnetic field is also present. We, however, assume the plasma medium to be cold so that the pressure can be neglected when considering the particle equation of motion. In stellar systems one often encounters situations where relaxation times are much larger than the age of the universe so that collisions (hence pressure) may be neglected. The effect of an electric field is not
generally seriously considered because of the well known Debye shielding effect. The general problem of an electromagnetic wave propagating along an arbitrary direction with the external magnetic field is given by Appleton and Hartee in the Newtonian case when studying the propagation of radio waves in ionosphere. Considering the fact that a general solution with arbitrary $\theta$ is very difficult to tackle in an expanding background with arbitrary number of dimensions we shall restrict ourselves to the cases when the electromagnetic wave propagates parallel and perpendicular to the magnetic field. However the topic is of great importance in astrophysics and space science where electromagnetic wave propagation in magnetized plasma is very relevant.

With the set of equations split to $(d + 1)$ formalism we are now in a position to attempt applications in varied plasma phenomena.

If as usual we set $k_c c = \omega_i$ (the angular frequency of the wave at some initial time $t = t_i$) then we get from the above Maxwell’s equations (see reference 2 for details)

$$E^\mu = E^\mu_0 i \sqrt{\frac{2k_c c d(1 + \gamma)}{\pi \{d(1 + \gamma) - 2\}}} t^{\frac{2}{d(1 + \gamma)}} e^{-i \omega_d t^{\frac{d(1 + \gamma)}{2}}} e^{i k_i \cdot r}$$

(29)

where

$$\omega_d = \omega_i t^{\frac{-2}{d(1 + \gamma)}}$$

(30)
gives a measure of the red shift of the photon due to background expansion. For radiation dominated era $\gamma = \frac{1}{d}$, $\omega_d = \omega_i t^{\frac{-2}{d}}$, so the rate at which the frequency decreases is maximum in 4D universe. Moreover damping is greater in radiation era.

This finding may have nontrivial implications for astrophysics. In an earlier work [19] one of us showed that the process of nucleosynthesis in higher dimensional space time is markedly different from that in 4D space time. So like the previous classical case the equation (29) may again be written as

$$E^\mu = E^\mu_0 (x, t)e^{i(k_i x - \omega t)}$$

(31)

with the essential difference that here $E_0$ depends both on space and time and all the other physical quantities like $k$ and $\omega$ share the expansion of the universe and

$$\omega = \omega_i \frac{d(1 + \gamma)}{d(1 + \gamma) - 2} t^{\frac{-2}{d(1 + \gamma)}}$$

(32)

Moreover, the exponent $(k_i x - \omega t)$ may be written in a tensorial form as $k^a x^a$, where the 4-vector $k^a = (k_i, -\omega t)$.

Now referring again to our earlier paper we find that for a two component plasma

$$v = \frac{i q t^2}{m_e \Gamma \omega_i} E = -\frac{i e}{m_e \Gamma \omega_d} E$$

(33)

The last equation is very similar to the flat space case except that here, $\omega_d$ is not a constant but shares the background expansion. With $J^\mu = n_0 q v^\mu$ we get

$$J^\mu = J^\mu_0 e^{i(k_i x - \omega t)}$$

(34)
where

\[ J_0 = -\frac{e^2 E_0}{m_0 \omega_i} \sqrt{\frac{2k_i c d(1 + \gamma)}{\pi d(1 + \gamma) - 2}} t^{-\frac{2(1 + d)}{d}} \]  

(35)

This relation via equation (24) simplifies to

\[ J = \frac{ie^2}{m_e \omega_i} t^{-\frac{2}{d+1}} E \]  

(36)

In a recent communication [4] we have shown that when an electromagnetic wave propagates through a plasma medium in a (d+1) dimensional expanding background in the presence of an ambient external magnetic field along the z direction the dielectric tensor comes out to be

\[ \epsilon_{11} = \epsilon_{22} = \epsilon_{44} = \epsilon_{55} = \ldots = \epsilon_{dd} = 1 - \frac{\omega_p^2}{\omega_d^2 - \omega_c^2} = p_1 \text{(say)} \]  

(37)

\[ \epsilon_{12} = \epsilon_{14} = \epsilon_{15} = \ldots = \epsilon_{1d} = \frac{\omega_c}{\omega_d} \frac{\omega_p^2}{\omega_d^2 - \omega_c^2} = p_2 \]  

(38)

\[ \epsilon_{31} = \epsilon_{32} = \epsilon_{34} = \ldots = \epsilon_{3d} = 0 \]  

(39)

\[ \epsilon_{33} = 1 - \frac{\omega_p^2}{\omega_d^2} = p_3 \]  

(40)

so that in matrix form

\[ \epsilon_{\mu \nu} = \begin{pmatrix} p_1 & ip_2 & 0 & ip_2 & \ldots & ip_d \\ -ip_2 & p_1 & 0 & ip_2 & \ldots & 0 \\ 0 & 0 & p_3 & 0 & \ldots & 0 \\ -ip_2 & -ip_2 & 0 & p_1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ -ip_d & \ldots & \ldots & \ldots & \ldots & p_1 \end{pmatrix} \]  

(41)

Here \( \omega_p \) is the plasma frequency given by

\[ \omega_p^2 = \frac{b_d m_0 e^2}{m_e} \]  

(42)

\[ \omega_c = \frac{eB}{m_e c} t^{\frac{d}{d+1}} \equiv \frac{e \hat{B}}{m_e c} \]  

(43)

where \( \hat{B} \), the orthogonal magnitude of the ambient magnetic field is given by \( \hat{B} = |(B)_z(B)^z|^{1/2} = B t^{\frac{d}{d+1}} \) for our system. Further the constant

\[ b_d = \frac{2^{d/2} \pi^{d/2}}{(d-1)!!} \quad (d \text{ even}) \]  

(44)

\[ b_d = \frac{2^{(d+1)/2} \pi^{(d-1)/2}}{(d-2)!!} \quad (d \text{ odd}) \]  

(45)

Thus the introduction of the magnetic field generates varied modes transforming the dielectric constant scalar \( \epsilon \) in equation (35) to a second rank tensor \( \epsilon_{ij} \). Although
the equations (30) - (40) exactly resemble the analogous expressions in Newtonian theory the fact remains that all the frequencies now depend on time rather than being constant. Further the cyclotron frequency $\omega_c$ decays as $t^{-2(d-1)/(d+1)}$ exactly similar to the orthogonal component of the magnetic field.

A little introspection of the dielectric tensor shows that like the special relativistic analogue it is the anisotropy character of the plasma medium which introduces the tensorial behaviour of conductivity because if the external magnetic field is switched off, $p_2$ vanishes and $p_1 = p_3$ and it becomes a pure scalar. Moreover it is manifestly anti hermitian when $\omega$ is a real. So one would expect that like the previous analysis the average energy dissipation will be nil. But it is definitely not the case as the following analysis shows. Let us once again recall the energy dissipation expression

$$P = E_\mu J^\mu = \frac{1}{4} \left[ E_{0\mu} J_0^{\mu*} + E_{0\mu} J_0^{\mu*} + E_{0\mu} J_0^{\mu*} e^{-2i\phi} \right]$$ (46)

where $\phi = k.x - \omega t$ and $E_{01}$ and $J_{01}$ are both functions of time due to the expansion of the background space time. One may at this stage use the equation (7) to find that

$$\sigma_{\mu\nu} = -i\omega d \frac{b_d}{d}[\epsilon_{\mu\nu} - \delta_{\mu\nu}]$$ (47)

Following the previous argument and using the usual matrix transformation rules it follows that the second and the third terms of the equation (46) give

$$\frac{1}{4} E_{01}^* (\sigma_{ji} + \sigma_{ij}) = 0$$ (48)

This is like the special relativistic case discussed earlier but the similarity just ends there. Skipping intermediate steps we shall calculate the average value of the rest two terms over a complete cycle in some mathematical details to visualize the differences from the previous case. A long but otherwise straightforward calculation now gives that

$$< P > = \alpha(d) \left< t^{-2(1+2d)/(d+1)} \sin 2(kx - \omega t) \right>$$ (49)

where $\alpha(d) = -\frac{E_0^2 e^2 d(1+\gamma)}{\pi m_e (1+\gamma)d-2}$

The equation (49) decomposes to

$$< P > = \alpha(d) \left[ \sin 2k_1 x \int_0^T t^{-2(1+2d)/(d+1)} \cos 2\omega dt - \cos 2k_1 x \int_0^T t^{-2(1+2d)/(d+1)} \sin 2\omega dt \right]$$ (50)

We have so far analyzed the whole situation in (d+1) dimensional spacetime with a general equation of state $p = \gamma \rho$. But, as pointed out earlier, both higher dimensional cosmology and plasma phenomena are most relevant in the very early phase of the universe when the cosmology was in radiation dominated state with $\gamma = 1/d$. This input will considerably simplify the already cumbersome mathematical expressions although we believe most of our inferences will be valid in a general equation of state also. For the last term in equation (50) we find through equation (24)

$$\frac{1}{w_i} \beta^{d-1} \int_0^T u^{-\frac{(d+1)}{2}} \sin u^2 du$$ (51)
where $\beta t^{(d-1)} = u^2$ Here $\beta = \frac{2m_e e \omega_i}{e^2 E_0^d} \alpha_d$. Also the time period which also is sharing the expansion of the cosmos is given by

$$T(t) = \frac{\pi}{\omega_i} t^{(\frac{d}{d+1})}$$

The equation (51) integrates to

$$u^{\frac{(7d+1)}{d+1}} \int_0^T \sin u^2 du - \frac{-(7d-1)}{(d-1)} \int_0^T u^{\frac{8d}{d+1}} \sin \left[ \frac{\pi}{\omega_i} \left( \frac{u}{\sqrt{\beta}} \right)^{\frac{1}{d-1}} \right] du$$

While dealing with the other expression we get almost similar results. The special integrals we are dealing here are called Fresnel Integrals which admit the following power series expansions that converge for all $x$

$$S(x) = \int_0^x \sin u^2 du = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+3}}{(4n+3)(2n+1)!}$$

$$C(x) = \int_0^x \cos u^2 du = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+1}}{(4n+1)(2n)!}$$

The limits of these functions as $x$ goes to $\infty$ are known, being equal to $\sqrt{\frac{\pi}{8}}$. Also series expansion tells of non zero value of it for any $x$. Thus we can conclude that the average rate of doing work is non zero as an electromagnetic wave passes through a plasma medium. This dissipation is unique and has no special relativistic analogue coming as it does due to the expansion of the background. As commented earlier that with expansion the background the density of the lines of force due to the ambient magnetic field gets thinned out which, in turn, results in the apparent damping of the magnetic field. It may once again be pointed out that this type of damping is not mechanical in nature as one observes in the Axionic dissipation or that due to friction or viscosity or collision common in natural processes but may be termed as expansion inspired damping.

### 3. Discussion:

We have here invoked the well-known 3+1 split formalism of physics to electrodynamics in FRW-like background. Although we have worked out the problem of energy dissipation of a plasma wave in HD spacetime we believe that most of the findings are valid, at least qualitatively, in the 4D spacetime also. To start with the FRW cosmology is chosen for the very simple reason that it is most easy to handle, yet it illustrates how the curvature and nonstaticity of the background can change the plasma MHD results. In the process we have got the interesting result unlike the special relativistic case where no dissipation occurs the curvature and expansion do introduce a new phenomena of dissipation unique in general relativity only without having any classical analogue. This is not caused by the usual mechanical friction-type forces as one frequently encounters in classical mechanics. This may be interpreted as caused due to the thinning out of the magnetic field lines density.
due to background expansion which results in the attenuation of the magnetic field strength and power dissipation. As a future exercise one should consider an inhomogeneous background to check what role inhomogeneity plays in the whole process. Generalization to non linear plasma may also introduce interesting physics.

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