CONVECTIVE MOTION
IN A VIBRATED GRANULAR LAYER

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Abstract

Experimental results are presented for a vertically shaken granular layer. In the range of accelerations explored, the layer develops a convective motion in the form of one or more rolls. The velocity of the grains near the wall has been measured. It grows linearly with the acceleration, then the growing rate slows down. A rescaling with the amplitude of the wall velocity and the height of the granular layer makes all data collapse in a single curve. This can provide insights on the mechanism driving the motion.

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Granular materials display different behaviors depending on the circumstances: they can be assimilated to solids, liquids or gases [1]. Yet the features and the origin of their behavior are far from understood and are the subject of current research. The focus of this work is on the motion that arises in a layer of noncohesive granular material when submitted to a vertical vibration. Such layers are known to develop large scale collective motions [1–3]. Once a certain threshold in the acceleration is reached (close to $g$, the acceleration of gravity) grains begin to move orderly. This movement takes a form in many respects similar to that obtained in convecting fluids heated from below (the Rayleigh-Bénard instability [4]). Other instabilities, such as parametric wave patterns and the formation of a heap, have also been reported for granular layers [5, 6]. Here we are dealing only with the closed flow of granular material, at velocities much smaller than the maximum speed of the vibrating container.

This convective flow is usually downward near the wall, and the grains rise at the center of the container [3]. For a given frequency, if the acceleration is increased there is a bifurcation towards a pattern of rolls: the velocity oscillates spatially along one preferred direction [7]. This symmetry-breaking bifurcation is known to display strong hysteresis. Further augment of the acceleration leads to a disordered state.

Some mechanisms have been proposed to explain this phenomenon. The most obvious is the friction of the grains with the walls, as suggested by Knight et al. [3]. Other effect concerns the Reynolds dilatancy notion: a tightly packed granular material must increase its volume if it has to change shape. During each cycle, the material undergoes a process involving a compaction and a dilatation that has been proposed to be at the origin of the motion [8]. And yet another analysis — based on the assumption that a “granular temperature” drives the flow, as in thermal convection — has been recently proposed [9]. The role of the surrounding gas has also been described in the case of the formation of a heap [10]: it acts as a lubricant; but it is generally admitted that it is not at the origin of the motion. Notice that all these explanations are not mutually exclusive. While they may all contribute to the convection, it is difficult to ascertain their relative significance in a particular situation.
A measurement of the velocity can clearly help to understand the underlying physics. But carrying out such a measurement in a non-invasive way is complicated. The use of trace particles is possible but painful and time-consuming. Magnetic resonance imaging has been employed to obtain velocity profiles along a cross section of the container, but the setup is entangled by the fact that an electromagnetic shaker cannot be placed inside the machine. We have arranged an experiment that allows us to obtain the local velocities at the wall of the container. While the nature of the flow is three-dimensional, the velocity at the boundaries provides a good way to define it.

The experimental setup consists of a cylindrical box with the lateral wall and top made of polycarbonate, and the base made of aluminum. The diameter of the box is 52 mm. Care was taken to avoid electrostatic effects by sprinkling the box with an antistatic spray and changing the beads whenever they are seen to stick to the wall. The box is attached to the shaft of a shaker. The electromagnetic shaker (TiraVib 52110) is able to deliver a sinusoidal vibration to the box with a distortion smaller than 1%, and its stiffness in the transversal direction is such that the residual acceleration is less than 0.05 $g$ typically. The shaker is driven by an amplifier that is in turn commanded by a function generator, from where the frequency and amplitude are set. Two accelerometers are attached to the box to measure acceleration in the vertical and one horizontal direction. The acceleration is acquired by a digital oscilloscope. Both the function generator and the oscilloscope are linked to a PC computer. An image acquisition system is also employed. It consists of a standard CCD camera with a macro lens. Images are recorded on a commercial VCR for subsequent treatment.

The procedure to measure the velocities involves the acquisition of the bead traces under proper illumination. A spatiotemporal diagram is obtained by registering a vertical line of pixels at regular time intervals and stacking them. The angle of the traces yields the bead velocities. In order to measure the average angle we perform a Fourier transform of the image and find the position of the peaks. All errors combined yield to an uncertainty of about 10% at most.
We have observed that the speed of the beads near the walls depends on height. They move faster in the upper zone and their velocity decreases as they approach the bottom, where the grains slow down and are entrained into the bulk —the velocity has a radial component to close the flow. Therefore we have taken our measurements in the upper third of the granular layer. We checked that the bead velocity there does not depend on height, their radial velocity being negligible. In that zone it is possible to track a bead and to obtain a measurement of the downward velocity. Nevertheless, unavoidably the beads suffer small displacements in the azimuthal direction due to the rearrangements of the layer, and so the traces have a small dispersion. The source of error coming from the dispersion of the velocities adds to the fact that near the bottom the beads leap downward when another bead beneath is absorbed into the bulk.

Several different granular materials have been used. The results we are reporting here have been obtained with glass beads of 0.5 ± 0.1 mm in diameter. Nevertheless, the same phenomena are also observed in sand, for example, with grains approximately that large but much more variable in size and form. Several layer depths were explored in the range 20 < H < 70, where H is the layer depth normalized by the bead diameter. In addition, the measurements shown here have been carried out at a frequency f = 110 Hz, although in a wide range (from about 70 Hz to 120 Hz) the behavior was qualitatively similar. The non-dimensional acceleration $\Gamma = A\omega^2/g$ (where $A$ is the amplitude and $\omega = 2\pi f$ the angular frequency) for which measurements were made ranges from about 0.5 to 7. For $\Gamma > 7$ the movement becomes rather disordered to make meaningful measurements.

Let us show the sequence of events for $H = 20$, in the understanding that a similar picture emerges for other layer depths. When $\Gamma$ is small, the beads begin to move uniformly down the wall. Usually a small heap (much shorter than the height of the layer) is formed. If the box is well leveled (we do it to within ± 0.5 mrad) the heap appears at the center of the layer. As $\Gamma$ is increased, the heap decreases in height and by $\Gamma \geq 3$ the layer surface is almost flat; it just curves down near the wall. At the same time, the velocity grows larger and larger. At this stage, the value of the velocity near the wall does not depend on the
azimuthal position along the wall.

A further increase of acceleration produces a bifurcation towards another pattern: two rolls appear, breaking the circular symmetry of the previous planform (see Fig. 1). The acceleration threshold shows little —if any— dependence on the frequency in the range explored; it is about \( \Gamma = 6 \). This transition, as remarked by Aoki and coworkers [7], presents hysteresis: once the rolls have formed, if acceleration is decreased they revert to the previous pattern at a lower value of \( \Gamma \). The orientation of the rolls in the circular box from one run to another is seemingly random.

A profile of the velocities along the azimuthal direction is shown in Fig. 2 before and after the bifurcation. The spatial modulation of the velocity field is clearly noticeable. The projection of the velocities onto the vertical cross section of the cylinder is well fitted by a sine. The resemblance of these two patterns with those found in small aspect ratio Bénard convection is striking. When a fluid layer open to the atmosphere heated from below begins to move, fluid goes up at the center and descends near the wall. If the temperature at the bottom is increased, a pair of rolls appear forming a pattern quite similar to the one found in granular materials [11]. The spatial patterns obtained bear a remarkable similarity. This strongly suggest that the convective motion in a shaken granular layer may be governed, at least in principle, by the symmetry restrictions imposed by the container. In order to provide the complete boundary conditions, the velocity field at the bottom of the container is needed. Work is in progress to obtain it following the same method.

In the range of frequencies and heights explored, we have always observed this transition from a circular pattern to two rolls. Further increase of the acceleration (explored only for some values of \( H \)) leads to an increase in the number of rolls, as described in previous experiments [7]. The rolls are superimposed to the downward flow at the wall (see Fig. 2 for \( \Gamma = 6.10 \); note that the mean velocity is not zero).

In order to investigate the dependence of the velocity \( v \) near the wall with the acceleration, we will focus in the first pattern, the one that appears for small \( \Gamma \). This planform is concordant with the cross-section profiles obtained by Knight et al. [1]. The velocity field
takes the form of a torus, the grains rising at the center and going down near the walls. As it is not dependent of the azimuthal coordinate, a single value of the velocity determines the field. The dependence of $v$ on the acceleration is shown in Fig. 3a for two layer depths: $H = 20$ and $H = 40$. The velocity is larger for thicker layers, and it grows monotonously with $\Gamma$. The relationship between $v$ and $\Gamma$ near the threshold hints at an imperfect transcritical bifurcation (the growth of $v$ is almost linear, as seen in Fig. 3b). The numerical results of Ramírez et al., who propose a “thermal convection” mechanism, predict a supercritical bifurcation towards their first convective state. Nevertheless, the first convective state they found is banned in the experiment for symmetry reasons, and besides their prediction is made for mean values of the velocity.

It is noticeable that before the transition from one to several rolls (for $\Gamma < 6$) two different regimes exist with two distinct slopes (Fig. 3b). The changing of slope is more evident for the series corresponding to $H = 20$. This behavior is more clearly displayed if the velocity $v$ is normalized –dividing it by the amplitude of the velocity of the container wall $v_w$. In doing this we are assuming that somehow the movement of the wall is driving the flow. The normalized velocity $v/v_w$ is shown in Fig. 4 as a function of $\Gamma$. Clearly all series now give the same behavior: a first stage where $v/v_w$ grows with $\Gamma$ and a second one where it remains almost constant. This saturation strongly suggests an effect related to friction, a force that depends on the velocity. The friction could either be internal (among the grains themselves) or external (between grains and wall). It is difficult to make an educated guess to decide which is the case; additional work is needed to further clarify this point. The mechanisms invoked by Knight et al. [3] and by Ramírez et al. [9] are both compatible in principle with such a fact.

An additional rescaling can be done dividing the normalized velocity by $H$. After doing this, all the curves collapse in a single one (Fig. 5). This dependency with $H$ is revealing, but again it is difficult to decide whether it is the friction with walls or the “granular temperature” the driving force. The first one should yield a scaling with $H$ for a fixed cross section, as it is the case. The second one could also give a scaling with $H$, although the
reason why it should scale \textit{linearly} is —to our knowledge— unexplained.

In summary, we have presented the results of an experiment in which the velocity of the grains near the walls has been measured locally. This allows a quantitative description of the convecting patterns. Moreover, the behavior of the velocity shows two different behaviors as the acceleration $\Gamma$ is increased; this strongly suggests that a mechanism associated with friction is involved. The scaling of the velocity with $v_w$ and $H$ favors the case of a driving force associated with wall friction, although “thermal convection” cannot be excluded.

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Figure Captions

Figure 1 The surface of the convecting granular material under vertical vibration \((\Gamma = 6.10, f = 110Hz, H = 20)\). The lateral illumination reveals the prominences of the relief, showing a planform of two rolls that protrude slightly at their axes. The surface of the pattern consisting of a single toroidal roll is almost featureless, except for a small heap at small accelerations.

Figure 2 Velocity profiles near the wall as a function of the azimuthal angle \(\theta\): circles, \(\Gamma = 2.01\); triangles, \(\Gamma = 3.07\); squares, \(\Gamma = 6.10\). (This last series is noisy but it has the virtue that by chance the maximum and minimum of \(v\) are seen). Note that only a portion of the complete circumference is accessible (about 160°). All the velocities are downward. It can be seen that \(v\) is constant for small \(\Gamma\) –the convective pattern is a single toroidal roll– while for larger \(\Gamma\) an spatial oscillation is revealed –two rolls are formed, as shown in Fig. 1. The measurements were taken at \(f = 110Hz\) and \(H = 20\).

Figure 3 The dependence of \(v\) on \(\Gamma\) hints to an imperfect transcritical bifurcation (a), where a linear growth is clearly apparent for small acceleration (b). For a larger range of accelerations (but still corresponding only to the first pattern, i.e. the toroidal convective roll, for which \(v\) does not depend on the azimuthal coordinate) two linear regimes are seen. In the second one, the slope is smaller. Squares correspond to \(H = 40\) and triangles to \(H = 20\). The data shown correspond to several runs, some of them increasing \(\Gamma\) and others decreasing it: no hysteresis is found in \(v\). Measurements were made at \(f = 110Hz\). The fits are just guides for the eye.

Figure 4 The normalized velocity \(v/v_w\), where \(v_w\) is the maximum velocity of the wall, as a function of \(\Gamma\). Squares: \(H = 20\); circles: \(H = 80/3\); down triangles: \(H = 100/3\); up triangles: \(H = 30\); diamonds: \(H = 140/3\). The fits (logistic functions) are guides for the eye.
Figure 5 The rescaled velocity $\frac{v}{v_H}$ as a function of $\Gamma$. Data are the same than in Fig. 4, all combined. The logistic fit is a guide for the eye.
Figure 1
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Figure 2  A. Garcimartín et. al.
Figure 3(a,b) A. Garcimartín et. al.
Figure 5
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