A coherently prepared asymmetric double semiconductor quantum well (QW) is proposed to realize parity-time (PT) symmetry. By appropriately tuning the laser fields and the pertinent QW parameters, PT-symmetric optical potentials are obtained by three different methods. Such a coherent QW system is reconfigurable and controllable, and it can generate new approaches of theoretically and experimentally studying PT-symmetry phenomena.

The non-Hermitian parity-time (PT)-symmetric Hamiltonians, which were firstly proposed by Bender and Boettcher in 1998, have attracted great attention. Because of the isomorphism between quantum Schrödinger equations and paraxial-wave equations, optical system becomes an ideal bed for experimentally studying PT symmetry. By balancing gain and loss, optical PT symmetry has been realized in different coupled structures, such as waveguides, lattices, micro-cavities, and can be used in the fields of unidirectional propagating, perfect absorbers, photon lasers, phonon lasers and sensors. In addition, several interesting phenomena, such as optical solitons, Bloch oscillations and topological insulators have also been investigated in optical PT-symmetric systems. However, most of the work, particularly in experiment, was based on solid-state materials. Once the structures are fabricated, the properties such as the threshold of the system are unable to be changed.

The susceptibility of host semiconductor quantum wells (QWs) can be controlled via electrical field, carrier density and laser field. The optical response of QWs is significantly enhanced when the frequency of light field is near resonance with the intersubband transition which has large dipole matrix element. As a result, one can obtain a dramatic change in the complex dielectric constant. Combined with electromagnetically induced transparency (EIT), both the refractive index and the absorptive coefficient of QWs can be effectively manipulated via atomic coherence and quantum interference.

In this work, we propose to use a coherently prepared asymmetric double semiconductor QWs to obtain PT symmetry. Such QW systems have been proved to have the possibility to realize quantum coherence and interference. We demonstrate that PT-symmetric optical potentials, such as coupled optical waveguides, one-dimensional (1D) and two-dimensional (2D) PT-symmetric optical lattices can be realized by different methods. Besides, it is possible to control the PT-symmetric properties by tuning the laser fields and the QW parameters. The realization of PT symmetry in coherent QW systems by laser fields have several advantages. Firstly, compared with solid-state materials with micro-structures or nano-structures, PT-symmetric properties in this system can be established by different methods, and can be effectively controlled by various parameters. Secondly, PT symmetry has been constructed theoretically and experimentally in different atomic systems. Compared with these atomic systems, semiconductor QW systems have designable and flexible of energy levels, and they are easy to be integrated and stable for practical application. Thirdly, in such systems large nonlinearity can be realized assisted by EIT, which makes it possible to observe traveling effects of lights in non-Hermitian nonlinear optical systems.

Models and Equations
We consider asymmetric double semiconductor QWs, which consist of a 51-monolayer (145 Å)-thick wide well (WW) and 35-monolayer (100 Å)-thick narrow well (NW). Between them there is a thickness of a 9-monolayer (25 Å)-thick Al0.2Ga0.8As barrier, as shown in Fig. 1(a). There are ten pairs of QWs (each pair consists of one WW, one NW, and one thick barrier), which are isolated from each other by 200-Å-wide Al0.2Ga0.8As buffer layers. All these pairs are sandwiched between nominally undoped 3500-Å-thick Al0.2Ga0.8As layers.

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Such asymmetric double semiconductor QWs can be treated as a four-level N configuration, as shown in Fig. 1(b). Here, levels $|1\rangle$ and $|2\rangle$ are localized hole states in a valence band, while levels $|3\rangle$ and $|4\rangle$ are delocalized bonding and antibonding states in a conduction band, arising from the tunneling effect between the WW and NW via the thin barrier, respectively. The probe field $E_p$ with frequency $\omega_p$ probes the transition $|1\rangle \leftrightarrow |3\rangle$, while the coupling field $E_c$ with frequency $\omega_c$ and the pump field $E_d$ with frequency $\omega_d$ act on transitions $|2\rangle \leftrightarrow |3\rangle$ and $|1\rangle \leftrightarrow |4\rangle$ respectively. The Rabi frequency of the probe, coupling, and pump fields are $\Omega_p = \mu_{13} E_p / 2\hbar$, $\Omega_c = \mu_{23} E_c / 2\hbar$, and $\Omega_d = \mu_{14} E_d / 2\hbar$, respectively, where $\mu_{ij}$ is the associated dipole transition matrix element, and the detuning of the probe, coupling, and pump fields are $\Delta_p = \omega_p - \omega_{31}$, $\Delta_c = \omega_c - \omega_{32}$, and $\Delta_d = \omega_d - \omega_{42}$, respectively, where $\omega_{ij}$ is the transition frequency between levels $|i\rangle$ and $|j\rangle$.

Under the condition of low QW carrier intensity, many-body effects are attributed to electron–electron interactions can be neglected. In the interaction picture and under the rotating wave approximation, the Hamiltonian of the QW system can be written as ($\hbar = 1$)

$$H = (\Delta_c - \Delta_p) |2\rangle \langle 2| - \Delta_p |3\rangle \langle 3| - \Delta_d |4\rangle \langle 4| + \{\Omega_p |1\rangle \langle 3| + \Omega_c |2\rangle \langle 3| + \Omega_d |1\rangle \langle 4| + \text{H.c.}\}.$$ (1)

Here, H.c. is the Hamiltonian complex conjugate.

The equation of motion for the density matrix of the system under the relaxation process is

Figure 1. (a) Schematic of one pair of asymmetric double QWs with buffer layers. (b) Band diagram of the asymmetric double QWs, and $z$ represent the wafer-growth direction.
\[ \rho = -i [H, \rho] + \frac{1}{2} [\gamma, \rho], \]  

(2)

where \( \gamma \) is the dissipation matrix. Substituting Eq. (1) into Eq. (2), the density matrix for each element can be obtained:

\[ \dot{\rho}_{22} = i\Omega_c (\rho_{32} - \rho_{23}) + \Gamma_{42}\rho_{44} + \Gamma_{32}\rho_{33} - \Gamma_{22}\rho_{22}, \]  

(3a)

\[ \dot{\rho}_{33} = i\Omega_p (\rho_{23} - \rho_{32}) + i\Omega_d (\rho_{31} - \rho_{13}) - \Gamma_3\rho_{33} + \Gamma_{43}\rho_{44}, \]  

(3b)
\[ \rho_{44} = i \Omega_d (\rho_{14} - \rho_{44}) - \Gamma_4 \rho_{44}, \]  
\[ \dot{\rho}_{12} = -i \Omega_c \rho_{12} + i \Omega_d \rho_{32} + i \Omega_p \rho_{42} - \gamma_{12} \rho_{12}, \]  
\[ \dot{\rho}_{13} = -i \Omega_c \rho_{13} + i \Omega_d \rho_{33} + i \Omega_p \rho_{43} - \gamma_{13} \rho_{13}, \]  
\[ \dot{\rho}_{14} = i \Omega_p \rho_{14} + i \Omega_d \rho_{34} - \gamma_{14} \rho_{14}, \]  
\[ \dot{\rho}_{23} = -i \Omega_p \rho_{23} + i \Omega_d \rho_{33} - \gamma_{23} \rho_{23}, \]  
\[ \dot{\rho}_{24} = -i \Omega_d \rho_{24} + i \Omega_p \rho_{34} - \gamma_{24} \rho_{24}, \]  
\[ \dot{\rho}_{34} = i \Omega_p \rho_{34} - i \Omega_d \rho_{31} + i \Omega_c \rho_{24} - \gamma_{34} \rho_{34}. \]  

Here \( \rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1 \) and \( \rho_{ij} = \rho_{ji} \). \( \Gamma_{ij} \) is the natural decay rate between levels \( |i\rangle \) and \( |j\rangle \). We assume that the decay rates from levels \( |3\rangle \) and \( |4\rangle \) in the conduction band to levels \( |1\rangle \) and \( |2\rangle \) in the valence band are identical, there are, \( \Gamma_{31} = \Gamma_{32} \) and \( \Gamma_{41} = \Gamma_{42} \). There is also no decay in the valence band or conduction band, so we conclude that \( \Gamma_{23} = \Gamma_{24} = 0 \). \( \Gamma_i = \sum_j \Gamma_{ij} \) denotes the total decay rate of level \( |i\rangle \). We define \( \gamma_{12} = \gamma_{12} + i(\Delta_p - \Delta_c) \), \( \gamma_{13} = \gamma_{13} + i\Delta_p \), \( \gamma_{14} = \gamma_{14} + i\Delta calculates the condition of non-\( PT \) symmetry, respectively. (d–f) are filed mode of the probe laser beam according to (a–c), (g–i) are propagation properties of the probe laser beam according to (a–c). The other parameters are the same as in Fig. 3.

Figure 4. Real part \( n_R \) and imaginary part \( n_I \) of the refractive index as a function of position \( x \) for different cases, (a) \( \Delta_c = -2.350 \) meV (loss waveguide) and \( \Delta_c = -2.300 \) meV (gain waveguide), representing the condition of below threshold, (b) \( \Delta_c = -2.386 \) meV (loss waveguide) and \( \Delta_c = -2.266 \) meV (gain waveguide), representing the condition of above threshold, (c) \( \Delta_c = -2.325 \) meV (both waveguides), representing the condition of non-\( PT \) symmetry, respectively. (d–f) are filed mode of the probe laser beam according to (a–c). (g–i) are propagation properties of the probe laser beam according to (a–c). The other parameters are the same as in Fig. 3.
In such QW systems, $\Gamma_1 = \Gamma_{d1} + \Gamma_{dph}$ and $\Gamma_2 = \Gamma_{d2} + \Gamma_{dph}$, where $\Gamma$ and $\gamma$ are the population decay rates of subbands (3) and (4) respectively, resulting from longitudinal–optical phonon emission events at low temperature, and $\Gamma_{dph}$ and $\Gamma_{dph}$ are the dephasing decay rates of quantum coherence due to electron–electron scattering, phonon scattering processes; and elastic interface roughness. Based on the previous studies, we assume that $\Gamma_1$ can be equal to $\Gamma_{d1}$ and $\Gamma_2$ can be equal to $\Gamma_{d2}$. Therefore, $\Gamma_3 = \Gamma_{d3} = \Gamma_4 = \Gamma_5$ can be presented. In addition, we assume there is no interference or dephasing between levels (3) and (4), which can be realized based on the appropriate reduction of the temperature.\(^{44}\)

The susceptibility of the QW medium can be obtained through the expression

$$\chi = N\mu_3/\varepsilon_c \cdot \rho_{31} = N\mu_3^2/2\varepsilon_c \hbar \Omega \cdot \rho_{31},$$

where $N$ is the electron density of the QWs, and $\rho_{31}$ can be obtained by Eq. (3). $\chi_3 = \chi_R + i\chi_i$, where $\chi_3$ describes the dispersion properties of the probe field, while $\chi_i$ describes the absorption properties of the probe field with $\chi_i > 0$ ($\chi_i < 0$) indicating loss (gain).

The refractive index of the QW medium can be obtained as $n = \sqrt{\varepsilon_c + \chi}$. Here $\varepsilon_c = n_c^2$ corresponding to the complex dielectric constant of the host QW medium, and $n_c$ is its refractive index when light is far detuned from the resonance. $\chi$ denotes the change in the susceptibility, which results from the coherent contract via the coupling and pump fields near the resonance. Generally, $\chi = \varepsilon_c$, then we have $n \approx \sqrt{\varepsilon_c + \chi/2\varepsilon_c} = n_c + \chi/2n_c = n_c + \chi_R/n_c + i\chi_i/n_c$. We define the real and imaginary parts of the refractive index as $n_R$ and $n_I$, with $n_c$ being the background index of the system. Thus, $n = n_c + n_R + i n_I$, where $n_R = \chi_R/2n_c$ and $n_I = \chi_i/2n_c$. To achieve PT symmetry, the condition of $n_R(r) = n_R(-r)$ and $n_I(r) = -n_I(-r)$ must be satisfied. And for simplicity, we use the unit $N\mu_3^2/4\varepsilon_c \hbar n_c$ in the following calculations.

**Results**

**PT-symmetric optical waveguides.** In order to realize PT-symmetric optical waveguides, we use a pair of coupling laser beams to form two coupled waveguides. Such pair of beams propagate in QWs along $y$ direction. Then, a pump and a probe laser beams with wider laser dimension propagate in the same direction as those of the coupling beams. The schematic diagram is shown in Fig. 2. The pair of coupling laser beams have an identical Gaussian intensity profile so that the total spatial intensity distribution of them varying in $x$ direction is

$$I(x) = A e^{-\frac{(x-a)^2}{2\sigma^2}} + A e^{-\frac{(x+a)^2}{2\sigma^2}},$$

where $A$ and $a$ are the constant and the half separation between the two waveguides respectively. $2\sqrt{2\ln2}\sigma$ can describe the full width at half maximum (FWHM) of the waveguides. By choosing two different detuning of the coupling fields, gain can be introduced to one waveguide and loss can be introduced to the other, even though other parameters are identical. In such consideration, the refractive index in each waveguide spatially varies only with the intensity of the coupling.

To realize gain and absorption in two waveguides simultaneously, different detuning of the coupling fields need to be found. For sake of it, we calculate the real (dispersion) $\chi_R$ and imaginary (gain or absorption) $\chi_I$ parts of the susceptibility as a function of $\Delta_n$, and show the results in Fig. 3(a,b), respectively. The intensity of the
The coupling field in each waveguide is corresponding to Gaussian profile, so $\chi_I$ needs to get larger with increasing coupling intensity. For this reason, we only consider a negative value of the coupling detuning. It can be seen from the figures that for different value of $\Omega_c$, $\chi_R$ reaches to the maximum value, and $\chi_I$ is close to zero at $\Delta_c = -2.325 \Gamma$. At the vicinity of the zero point, gain is obtained on the left side, and absorption is abstained on the right side. This property called refractive index enhancement with vanishing absorption, has been demonstrated in the early papers.

In order to realize gain and loss in two waveguides simultaneously by using two different coupling detuning and maintain other parameters identical, the relation between the susceptibility and the coupling intensity needs to be established. Therefore, we choose $\Delta_c = -2.360 \Gamma$ and $\Delta_c = -2.278 \Gamma$ in two waveguides, and draw $\chi_R$ and $\chi_I$ as a function of $\Omega_c$ shown in Fig. 3(c,d). It can be seen that with selected coupling detuning, the curves of $\chi_R$ are identical, and the curves of $\chi_I$ are matched with slight difference, respectively. It should be noted that, in

![Figure 6](https://doi.org/10.1038/s41598-019-39085-6)

**Figure 6.** (a) Real part $\chi_R$ and (b) imaginary part $\chi_I$ of the susceptibility as a function of probe detuning $\Delta_p$ for different pump Rabi frequency, $\Omega_d = 0.8 \Gamma$ (red dotted line), $\Omega_d = 1 \Gamma$ (blue dashed line), and $\Omega_d = 1.2 \Gamma$ (black solid line). The parameters are $\Omega_c = \Gamma$ and $\Delta_c = \Delta_d = 0$. The other parameters are the same as in Fig. 3.

![Figure 7](https://doi.org/10.1038/s41598-019-39085-6)

**Figure 7.** Real part $n_R$ and imaginary part $n_I$ of the refractive index as a function of position $x$ for unchanged absolute value and different signs of $\Delta_p$ and $\delta \Omega_d$, respectively. (a) $\Delta_p = 2.592 \text{ meV}$ and $\delta \Omega_d = 0.1 \Gamma$, (b) $\Delta_p = 2.592 \text{ meV}$ and $\delta \Omega_d = -0.1 \Gamma$, (c) $\Delta_p = -2.592 \text{ meV}$ and $\delta \Omega_d = 0.1 \Gamma$, (d) $\Delta_p = -2.592 \text{ meV}$ and $\delta \Omega_d = -0.1 \Gamma$. The parameters are $\Omega_d0 = \Gamma$. The other parameters are the same as in Fig. 6.
Fig. 3(d) we flipped the curve of the gain one by multiplying a minus sign to compare the two curves of gain and loss directly.

Based on the above analysis, firstly, we demonstrate the possibility of realizing spatial modulation of the refractive index. Making use of Eq. (4), and choosing the FWHM \( \sigma = 7 \mu m \) and the separation between the two waveguides as 20 \( \mu m \), we plot the real \( n_R \) and the imaginary \( n_I \) parts of the refractive index as a function of \( x \) shown in Fig. 4(a–c). It can be seen from the figures that the real part of the refractive index \( n_R \) is an even function of \( x \), while that of imaginary part \( n_I \) is an odd function of \( x \). By changing the electron intensity of QW, the absolute values of \( n_R \) and \( n_I \) can be modified with equal scale simultaneously with the changes of the electron intensity of QW.

Figure 8. (a) Real part \( n_R \) and (b) imaginary part \( n_I \) of the refractive index as functions of position \( x \) and probe detuning \( \Delta \). The parameters are \( \delta \Omega = 0.1 \Gamma \). (c) Real part \( n_R \) and (d) imaginary part \( n_I \) of the refractive index as functions of position \( x \) and modulation intensity \( \delta \Omega_d \). The parameters are \( \Delta_p = 2.592 \) meV. The other parameters are the same as in Fig. 7.

Figure 9. (a) Real part \( n_R \) and (b) imaginary part \( n_I \) of the refractive index as functions of position \( x \) and position \( y \). The parameters are \( \Delta_p = 2.592 \) meV and \( \delta \Omega = 0.1 \Gamma \). The other parameters are the same as in Fig. 7.
By controlling the ratio of the real and imaginary parts of the refractive index, we can adjust the condition of the system below or above the PT-symmetric threshold. It can be seen from Fig. 3(a,b) that $\chi_R$ changes less than $\chi_I$ at the vicinity of zero point ($\Delta_c = -2.325 \Gamma$). Therefore, we can alter the coupling detuning in the waveguides to control the ratio. For instance, with different values of the coupling detuning, the ratio of the real and imaginary parts is 12 and 80, and the system can be operated below (Fig. 4(a)) or above (Fig. 4(b)) the PT-symmetric threshold respectively. In addition, when the coupling detuning used in both waveguides is $-2.325 \Gamma$, the system even becomes a non-symmetric one with much larger ratio (Fig. 4(c)).

Secondly, we present the field modes in the waveguides in Fig. 4(d–f) corresponding to Fig. 4(a–c), respectively. When the condition of the system is below the threshold (Fig. 4(a)), the PT symmetry condition is satisfied, the eigenvalues should be real, and the field modes should be symmetric. However, because of the imperfect PT symmetry condition, the eigenvalues have a very small imaginary part, and the two field modes are slightly asymmetric, as shown in Fig. 4(d). For the case of above the threshold (Fig. 4(b)), that is, the PT symmetry is broken, the eigenvalues become complex, where the imaginary part of the refractive index represents the gain or loss for each filed mode. Figure 4(e) illustrates that, the light modes become strongly asymmetric. For the non-symmetric one (Fig. 4(c)), the light modes are perfectly symmetric, which is shown in Fig. 4(f).

Thirdly, we show in Fig. 4(g–i) the propagation characteristics of the probe beam according to the condition of Fig. 4(a–c) respectively. When the condition of the system is below threshold, the probe beam oscillates periodically between the two waveguides, as show in Fig. 4(g). When the case is above the threshold, it can be seen from Fig. 4(h) that an exponentially growing mode occurs, which signifies the onset of PT symmetry breaking. For the passive waveguides, the probe beam also oscillates periodically (Fig. 4(i)), which is similar to the case of below threshold. However, the oscillation period in Fig. 4(i) is shorter than that of in Fig. 4(g).

In a practical system, it is very difficult to realize perfect PT-symmetric condition. So, we evaluate the degrees of asymmetry of the system by analyzing the asymmetry function of $\Delta(x) = n(x) - n^*(-x)$, and find that the degree of asymmetry is very small. Therefore, it is manageable and has little impact on practical experiments.

PT-symmetric optical lattices—Type I. In this part, we first consider an 1D optical lattices of period $\Lambda_x$ along x direction, and in each lattices the electron density of QWs is spatially modulated. The electron density in the $i$th trap can exhibit a Gaussian distribution $N_i(x, y) = N e^{-(x-x_i)^2/\sigma_x^2}$, where $N$ is the electron density of a homogeneous QW sample, $x_i$ is the $i$th trap center, and $\sigma_x$ is the half width. The modulation of electron density of QWs can be achieved by using high order surface grating. We further consider a pump laser field with periodically modulated intensity $\Omega_p = \Omega_0 + \delta \Omega_p \sin [2\pi (x - x_i)/\Lambda_x]$, which can be easily achieved in experiment by using an imperfect standing-wave (SW) field with unequal forward and backward components. We show the schematic diagram of QW system in Fig. 5. The probe field and coupling field propagate along $z$ direction, and the SW pump fields propagate along $x$ direction.

First, we calculate the real $\chi_R$ and imaginary $\chi_I$ parts of the susceptibility as a function of $\Delta_p$ for varying value of $\Omega_d$ by solving Eq. (3) without modulation of pump intensity or the electron density, and the corresponding results are shown in Fig. 6. It can be seen from red dotted line in Fig. 6(b) that when $\Omega_p = 0.8 \Gamma$, a typical EIT with positive value (loss) is realized. With increasing value of $\Omega_p$, at both sides of the EIT windows, $\chi_I$ becomes positive (gain), which is known as coherent Raman gain without population inversion [blue dashed line and black...
solid line in Fig. 6(b)]. Meanwhile, $\chi_R$ changes from positive dispersion to negative dispersion in the vicinity of EIT window, as shown in Fig. 6(a). The figures indicate that it is possible to realize PT symmetry by choosing suitable modulations.

Then, with modulation of the pump intensity and electron density, we calculate the real ($n_R$) and imaginary ($n_I$) parts of the refractive index as a function of $x$ and show the results in Fig. 7. Figure 7(a) shows the condition of $\Delta_p = 2.525$ meV and $\delta \Omega_c = 0.1 \Gamma$. The value of $\Delta_p$ is chosen according to blue dashed line in Fig. 6(b), where $\chi_I$ has zero value around $\Delta_p = 2.592$ meV, and thus it exhibits gain or absorption if $\Delta_p$ is slightly tuned away from

**Figure 11.** Real part $n_R$ and imaginary part $n_I$ of the refractive index as a function of position $x$ for unchanged absolute value and different signs of $\Delta_p$, $\delta \Omega_c$, and $\delta \Omega_d$, respectively, (a) $\Delta_p = 2.525$ meV, $\delta \Omega_c = 0.3 \Gamma$ and $\delta \Omega_d = 0.1 \Gamma$, (b) $\Delta_p = 2.525$ meV, $\delta \Omega_c = 0.3 \Gamma$ and $\delta \Omega_d = -0.1 \Gamma$, (c) $\Delta_p = 2.525$ meV, $\delta \Omega_c = -0.3 \Gamma$ and $\delta \Omega_d = 0.1 \Gamma$, (d) $\Delta_p = 2.525$ meV, $\delta \Omega_c = -0.3 \Gamma$ and $\delta \Omega_d = -0.1 \Gamma$, (e) $\Delta_p = -2.525$ meV, $\delta \Omega_c = 0.3 \Gamma$ and $\delta \Omega_d = 0.1 \Gamma$, (f) $\Delta_p = -2.525$ meV, $\delta \Omega_c = 0.3 \Gamma$ and $\delta \Omega_d = -0.1 \Gamma$, (g) $\Delta_p = -2.525$ meV, $\delta \Omega_c = -0.3 \Gamma$ and $\delta \Omega_d = 0.1 \Gamma$, (h) $\Delta_p = -2.525$ meV, $\delta \Omega_c = -0.3 \Gamma$ and $\delta \Omega_d = -0.1 \Gamma$. The parameters are $\Omega_0 = 3 \Gamma$ and $\Omega_0 = \Gamma$. The other parameters are the same as in Fig. 6.
this point. It can be seen that \( n_R \) is an even function of the lattice position \( x \), while \( n_I \) is an odd function of lattice position \( x \) with balance gain and loss. Such results clearly indicate that PT symmetry is built in QW system by modulating the pump field and the electron density.

In Figs 7(b) and 6(d), we further show the cases for the same absolute value of \( \Delta p \) and \( \delta \Omega_d \) as that of in Fig. 7(a), but with different sign of them. The results show that the PT-symmetric properties are maintained in all cases. However, the relation between \( n_R \) and \( n_I \) are different. It is found that positive (negative) value of \( \Delta p \) results in the positive (negative) value of \( n_R \) and have no effect on the profile of \( n_I \). On the other hand, the sign of \( \delta \Omega_d \) determines the position of gain and loss in each period, for instance, with positive (negative) value of \( \delta \Omega_d \), \( n_I \) is gain (loss) in one half period \(-0.5 \leq x/\Lambda_x \leq 0\). The understanding of impacts of the parameters on spatial features of the refractive index is important because of its potential application, such as asymmetric light diffraction.

We next check what will happen if the absolute value of \( \Delta p \) and \( \delta \Omega_d \) is changed. To do so, we plot 2D \( n_R \) and \( n_I \) as functions of position \( x \) and probe detuning \( \Delta p \). The parameters are \( \delta \Omega_c = 0.3 \Gamma \) and \( \delta \Omega_d = 0.1 \Gamma \). (a) Real part \( n_R \) and (b) imaginary part \( n_I \) of the refractive index as functions of position \( x \) and modulation intensity \( \delta \Omega_c \). The parameters are \( \Delta p = 2.525 \text{ meV} \) and \( \delta \Omega_c = 0.3 \Gamma \). (c) Real part \( n_R \) and (d) imaginary part \( n_I \) of the refractive index as functions of position \( x \) and probe detuning \( \Delta p \). The parameters are \( \Delta p = 2.525 \text{ meV} \) and \( \delta \Omega_d = 0.1 \Gamma \). The other parameters are the same as in Fig. 11.

**Figure 12.** (a) Real part \( n_R \) and (b) imaginary part \( n_I \) of the refractive index as functions of position \( x \) and probe detuning \( \Delta p \). The parameters are \( \delta \Omega_c = 0.3 \Gamma \) and \( \delta \Omega_d = 0.1 \Gamma \). (c) Real part \( n_R \) and (d) imaginary part \( n_I \) of the refractive index as functions of position \( x \) and modulation intensity \( \delta \Omega_c \). The parameters are \( \Delta p = 2.525 \text{ meV} \) and \( \delta \Omega_d = 0.3 \Gamma \). (e) Real part \( n_R \) and (f) imaginary part \( n_I \) of the refractive index as functions of position \( x \) and modulation intensity \( \delta \Omega_d \). The parameters are \( \Delta p = 2.525 \text{ meV} \) and \( \delta \Omega_d = 0.1 \Gamma \). The other parameters are the same as in Fig. 11.
respectively. The results clearly show that the monotonicity of \( n_R \) probe field propagates along modulating both pump and coupling fields. The schematic diagram of the system is shown in Fig. 10, where the different cases. Figure 11(a) shows the condition of \( n_R \) as functions of \( x \) and \( y \), while \( n_I \) is an odd functions of \( x \) and \( y \), therefore, 2D PT-symmetric optical lattices is realized in QW systems.

\[ \Omega_d = \Omega_{d0} + 0.5 \delta \Omega_d \sin[2\pi(x - x_0)/\Lambda_x] + \sin[2\pi(y - y_0)/\Lambda_y], \]

We plot in Fig. 9(a,b) the top view of \( n_R \) and \( n_I \) as functions of \( x \) and \( y \) using the same parameters in Fig. 7(a), respectively. The results clearly show that \( n_R \) is an even functions of \( x \) and \( y \), while \( n_I \) is an odd functions of \( x \) and \( y \), therefore, 2D PT-symmetric optical lattices is realized in QW systems.

**PT-symmetric optical lattices—Type II.** In this part, we show that PT symmetry can also be achieved by modulating both pump and coupling fields. The schematic diagram of the system is shown in Fig. 10, where the probe field propagates along \( z \) direction, and the SW coupling and SW pump fields propagate along \( x \) direction. Therefore, we have modulated coupling and pump fields, \( \Omega_p = \Omega_{p0} + \delta \Omega_p \cos[2\pi(x - x_0)/\Lambda_x], \delta \Omega_p = \Omega_{p0} + \delta \Omega_p \sin[2\pi(x - x_0)/\Lambda_x] \)

First, we plot the real and imaginary parts of the refractive index \( n_R \) and \( n_I \) as a function of \( x \) in Fig. 11 for different cases. Figure 11(a) shows the condition of \( \Delta_p = 2.592 \) meV and the modulation intensities \( \delta \Omega_p = 0.3 \Gamma \) and \( \delta \Omega_d = 0.1 \Gamma \). It can be seen from figure that the PT symmetry is appears, where \( n_R \) is an even function of \( x \) and \( n_I \) is an odd function of \( x \). The relations of \( n_R \) and \( n_I \) can also be modified by changing the sign of the detuning \( \Delta_p \) and the modulation intensities \( \delta \Omega_p \) and \( \delta \Omega_d \) and the changing of their sign will not destroy the PT symmetry. There are eight kinds of combinations of these three parameters, corresponding to eight kinds of spatial refractive index, and we plot the other seven kinds in Fig. 11(b–h). It can be found that \( \Delta_p, \delta \Omega_p \) or \( \delta \Omega_d \) have different impacts on the features of \( n_R \) and \( n_I \). More specifically, \( \Delta_p \) determines \( n_R \) being positive or negative, and \( \delta \Omega_p \) determines the monotonicity of \( n_R \), and \( \delta \Omega_d \) determines the positions of the gain and loss in one period.

We also check the impact of absolute value of \( \Delta_p, \delta \Omega_p \) or \( \delta \Omega_d \) on the properties of PT-symmetric system. Therefore, we plot 2D \( n_R \) and \( n_I \) as functions of \( x \) and \( \Delta_p \) in Fig. 12(a,b), as functions of \( x \) and \( \delta \Omega_p \) in Fig. 12(c,d), and as functions of \( x \) and \( \delta \Omega_d \) in Fig. 12(e,f), respectively. The figures clearly show that \( n_R \) and \( n_I \) are modulated along \( x \) direction for varying \( \Delta_p, \delta \Omega_p \) or \( \delta \Omega_d \). However, when these parameters are detuned form the value used in Fig. 11(a), the system will loss the PT-symmetric properties.

Last, we show that it is possible to realize 2D PT-symmetric optical lattices by applying 2D modulation of the pump and coupling laser fields, which are

\[ \Omega_c = \Omega_{c0} + 0.5 \delta \Omega_c \left\{ \cos[2\pi(x - x_0)/\Lambda_x] + \cos[2\pi(y - y_0)/\Lambda_y]\right\}, \]

Using the same parameters in Fig. 11(a), we calculate \( n_R \) and \( n_I \) as functions of \( x \) and \( y \), and show the top view of \( n_R \) and \( n_I \) as functions of \( x \) and \( y \) in Fig. 13(a,b), respectively. The results indicate that 2D PT-symmetric optical lattices can be achieved, with \( n_R \) being an even functions of \( x \) and \( y \), and \( n_I \) being an odd functions of \( x \) and \( y \), respectively.
Conclusion
In conclusion, we have demonstrated that coherent asymmetric double semiconductor QWs can be an ideal candidate for studying PT symmetry. We have showed that PT-symmetric optical waveguides can be realized by using two coupling fields with different detuning, 1D and 2D PT-symmetric optical lattices can be realized by spatial modulation of pump laser fields and the electron density of QWs, or by spatial modulation of pump and coupling fields. In addition, the PT-symmetric properties realized in QW systems, such as the relation between the real and the imaginary parts of the complex refractive indices, can be controlled by changing the laser fields and the parameters of QWs.

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**Author Contributions**

S.-C.T. proposed the research and carried out the calculations. R.-G.W. verified the results. All authors discussed the results and reviewed the manuscript.

**Additional Information**

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