COLD DARK MATTER DETECTION VIA THE LSP-NUCLEUS ELASTIC SCATTERING

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Abstract

The momentum transfer dependence of the LSP-nucleus elastic scattering cross sections is studied. New input SUSY parameters obtained in a phenomenologically allowed parameter space are used to calculate the coherent rate for various nuclear systems and the spin matrix elements for the proposed $^{207}\text{Pb}$ target. The results are compared to those obtained from other cold dark matter detection targets.
1 Introduction

Recently, the possibility to directly detect the lightest supersymmetric particle (LSP), a candidate for cold dark matter, via the recoiling of a nucleus in the elastic scattering process:

$$\chi + (A, Z) \rightarrow \chi + (A, Z)^*$$

(1)

($\chi$ denotes the LSP) has been proposed. In the present study we proceed as follows:

1) We write down the effective Lagrangian at the elementary particle (quark) level obtained in the framework of supersymmetry as described in refs. [1, 2]

2) We go from the quark to the nucleon level using an appropriate quark model for the nucleon. Special attention in this step is paid to the scalar couplings, which dominate the coherent part of the cross section and the isoscalar axial currents which depend on the assumed quark model.

3) We compute the relevant nuclear matrix elements using as reliable as possible many body nuclear wave function hopping that, by putting as accurate nuclear physics input as possible, we will be able to constrain the SUSY parameters as much as possible.

4) We calculate the modulation of the cross sections due to the earth’s revolution around the sun and the motion of the sun around the center of the galactic disc by a folding procedure. Our goal is to calculate LSP-nucleus event rates for $^{207}$Pb nucleus by using new input SUSY parameters [3, 4] obtained in a phenomenologically allowed parameter space. We focus on the spin matrix elements of $^{207}$Pb, since this target, in addition to its experimental qualifications, has the advantage of a rather simple nuclear structure. Furthermore, its spin matrix element, especially the isoscalar one, does not exhibit large quenching as that of the light and up to now much studied $^{29}$Si and $^{73}$Ge nuclei. We compare our results to those given from other cold dark matter detection targets.

The total cross section of the LSP-nucleus reaction (1) can be written as [4]

$$\sigma = \sigma_0 \left( \frac{m_1}{m_p} \right)^2 \frac{1}{(1 + \eta)^2} \left\{ A^2 \left[ \beta^2 (f_V^0 - f_V^1) \frac{A - 2Z}{A} \right]^2 + (f_S^0 - f_S^1) \frac{A - 2Z}{A} I_0(u) - \frac{\beta^2}{2} \frac{2\eta + 1}{(1 + \eta)^2} (f_V^0 - f_V^1) \frac{A - 2Z}{A} I_1(u) \right\} + (f_A^0 \Omega_0(0))^2 I_{00}(u) + 2f_A^0 f_A^1 \Omega_0(0) \Omega_1(0) I_{01}(u) + (f_A^1 \Omega_1(0))^2 I_{11}(u) \}$$

(2)

where $m_1$, $m_p$ is the LSP, proton masses, $\eta = m_1/m_p A$ and $\sigma_0 \approx 0.77 \times 10^{-38} \text{cm}^2$. The parameters $f_S^0$, $f_V^0$, $f_A^0$, with $\tau = 0, 1$ an isospin index, describe the scalar, vector and axial vector couplings and are determined in various SUSY models [3, 4] (see ref. [2]). The momentum transfer enters via $u$ as

$$u = \frac{1}{2} \left( 2\beta m_1 c^2 b (1 + \eta) \right)^2, \quad \beta = \frac{v}{c} \approx 10^{-3}$$

(3)

($b$ is the harmonic oscillator parameter). In Eq. (2), the terms in the square brackets describe the coherent cross section and involve the integrals $I_\rho(u)$ ($\rho = 0, 1$) while the other terms...
Figure 1: (a) Plot of the integrals $I_{11}(u)$ and $I_0(u)$ for Pb, where $u$ is given by Eq. (3). We see that $I_{11}$ is quite a bit less retarded compared to $I_0$. (b) $K_0^l$ integrals (for $l=0$ and $l=1$) entering the dominant scalar part of the event rate.

describe the spin dependence of the cross section by means of the integrals $I_{\rho\rho'}(u)$ ($\rho, \rho' = 0, 1$, isospin indices) which are normalized so as $I_{\rho}(0) = 1$ and $I_{\rho\rho'}(0) = 1$. For their definitions see ref. [2] where they are calculated by using realistic nuclear form factors.

In Fig. 1(a) we show the variation of $I_0(u)$ (the others behave similarly). We see that $u$ can be quite big for large mass of the LSP and heavy nuclei even though the energy transfer is small ($\leq 100$ KeV). The total cross section can in such instances be reduced by a factor of about five.

The spin matrix element of heavy nuclei like $^{207}$Pb has not been previously evaluated, since one expects the relative importance of the spin versus the coherent mode to be more pronounced on light nuclei. However, the spin matrix element in the light isotopes is quenched, while that of $^{207}$Pb does not show large quenching. For this feature, we recently proposed $^{207}$Pb nucleus as an important candidate in the LSP detection. Furthermore, this nucleus has some additional advantages as: i) it is believed to have simple structure (one $2p1/2$ neutron hole outside the doubly magic nucleus $^{208}$Pb) and ii) it has low angular momentum and therefore only two multipoles $\lambda = 0$ and $\lambda = 2$ with a $J$-rank of $\kappa = 1$ can contribute even at large momentum transfers.

The dependence of the cross section on the momentum transfer for $^{207}$Pb has been extensively studied in ref. [2]. In Fig. 1(a) the variation of the integral $I_{11}(u)$, which describe the spin part of $^{207}$Pb, is shown.

2 Folding of the cross section with a Maxwellian distribution

Due to the revolution of the earth around the sun, the event rate becomes modulated. This effect is studied by folding the total cross section Eq. (2) with an appropriate distribution. If
we assume a Maxwell type distribution, which is consistent with the velocity distribution of LSP into the galactic halo, the counting rate for a target with mass $m$ takes the form [2]

\[
\langle \frac{dN}{dt} \rangle = \rho(0) \frac{m}{M_m} \sqrt{<v^2> <\Sigma>}
\]

where $\rho(0) = 0.3 GeV/cm^3$, the LSP density in our vicinity and $<\Sigma>$ is given by

\[
<\Sigma> = (\frac{m_1}{m_p})^2 \frac{\sigma_0}{(1+\eta)^2} \left\{ A^2 \left[ <\beta^2> \right. \right.
\]

\[
\times \left( f_V^0 - f_V^1 \frac{A-2Z}{A} \right)^2 \left( J_0 - \frac{2\eta + 1}{2(1+\eta)^2} J_1 \right) + \left( f_S^0 - f_S^1 \frac{A-2Z}{A} \right)^2 J_0 
\]

\[
+ \left( f_A^0 A_0(0) \right)^2 J_{00} + 2f_A^0 f_A^1 A_0(0) A_1(0) J_{01} + \left( f_A^1 A_1(0) \right)^2 J_{11} \}
\]

The parameters $J_0$, $J_\rho$, $J_\rho^0$ describe the scalar, vector and spin part of the velocity averaged counting rate, respectively. They are functions of $\beta_0 = v_0/c$, $\delta = 2v_1/v_0$ and $u_0$, where $v_0$ is the velocity of the sun around the galaxy, $v_1$ the velocity of the Earth around the sun and $u_0$ is given by an expression like that of Eq. (3) with $\beta \rightarrow \beta_0$. Since $\delta \approx 0.27 << 1$, we can expand the J-integrals in powers of $\delta$ and retain terms up to linear in $\delta$. Thus, for each mechanism (vector, scalar, spin) we obtain two integrals associated with $l = 0$ and $l = 1$. The most important $K^2$ integrals are shown in Fig. 1(b). For some others see ref. [2] By exploiting the above expansion of K-integrals, the counting rate can be written in the form

\[
\langle \frac{dN}{dt} \rangle = \langle \frac{dN}{dt} \rangle_0 (1 + h \cos \alpha)
\]

where $\alpha$ is the phase of the earth’s orbital motion and $\langle dN/dt \rangle_0$ is the rate obtained from the $l = 0$ multipole. $h$ represent the amplitude of the oscillation, i.e. the ratio of the component of the multipole $l = 1$ to that of the multipole $l = 0$.

### 3 Results and discussion

The three basic ingredients of our calculation were the input SUSY parameters, a quark model for the nucleon and the structure of the nuclei involved. The input SUSY parameters used have been calculated in a phenomenologically allowed parameter space (cases #1, #2, #3) as explained in ref. [4]

In Tables 1, we compare the spin matrix elements at $q = 0$ for the most popular targets considered for LSP detection $^{207}Pb$, $^{73}Ge$ and $^{29}Si$. We see that, the spin matrix elements $\Omega^2$ of $^{208}Pb$ are even a factor of three smaller than those for $^{73}Ge$ obtained in ref. [3]

The spin contribution, arising from the axial current, was computed in the case of $^{207}Pb$ system (see Tables 2, 3 and 4). For the isovector axial coupling the transition from the quark to the nucleon level is trivial (a factor of $g_A = 1.25$). For the isoscalar axial current we considered two possibilities depending on the portion of the nucleon spin which is attributed to the quarks,
Table 1: Comparison of the static spin matrix elements for three typical nuclei.

| Component | $^{207}Pb_{1/2}^-$ | $^{78}Ge_{9/2}^+$ | $^{29}Si_{1/2}^+$ |
|-----------|--------------------|-------------------|-------------------|
| $\Omega_1^0(0)$ | 0.231              | 1.005             | 0.204             |
| $\Omega_1^0(0)\Omega_0(0)$ | -0.266             | -1.078            | -0.202            |
| $\Omega_2^0(0)$ | 0.305              | 1.157             | 0.201             |

Table 2: The spin contribution in the $LSP - ^{207}Pb$ scattering for two cases: EMC data and NQM Model. The LSP mass is $m_1 = 126, 27, 102 GeV$ for #1, #2, #3 respectively.

| Solution | EMC DATA $<dN/dt>_0 (y^{-1} Kg^{-1})$ | h | NQM MODEL $<dN/dt>_0 (y^{-1} Kg^{-1})$ | h |
|----------|--------------------------------------|---|--------------------------------------|---|
| #1       | $0.285 \times 10^{-2}$               | 0.014 | $0.137 \times 10^{-2}$               | 0.015 |
| #2       | $0.041$                              | 0.046 | $0.384 \times 10^{-2}$               | 0.056 |
| #3       | $0.012$                              | 0.016 | $0.764 \times 10^{-2}$               | 0.017 |

indicated by EMC and NQM. [2] The ground state wave function of $^{208}Pb$ was obtained by diagonalizing the nuclear Hamiltonian in a 2h-1p space which is standard for this doubly magic nucleus.

For the coherent part (scalar and vector) we used realistic nuclear form factors. In ref. [2] we studied three nuclei, representatives of the light, medium and heavy nuclear isotopes ($Ca$, $Ge$ and $Pb$) for three different quark models. In table 4 we present the results for $Pb$ obtained for these models denoted by A (only quarks u and d in the nucleon) and B and C (heavy quarks included) for the LSP masses shown in table 3 (cases #4-#9). We see that the results vary substantially and are sensitive to the presence of quarks other than u and d into the nucleon.

The total cross section is almost the same for LSP masses around 100 GeV. We obtained results in the context of the quark models NQM, EMC, for SUSY models #1-#3 [3] (Tables 2 and 3) and SUSY models #4-#9 [4] (Tables 4).

4 Conclusions

In the present study we found that for heavy LSP and heavy nuclei the results are sensitive to the momentum transfer as well as to the LSP mass and other SUSY parameters. From the Tables 3 and 4 we see that, the results are also sensitive to the quark structure of the nucleon in the following sense: i) The coherent scalar (associated with Higgs exchange) for model A (u and d quarks only) is comparable to the velocity suppressed vector coherent contribution. Both are at present undetectable. ii) For models B and C (heavy quarks in the nucleon) the coherent scalar contribution is dominant. Detectable rates $<dN/dt>_0 \geq 100 y^{-1} Kg^{-1}$ are possible in a number of models with light LSP.

The spin contribution is sensitive to the nuclear structure. It is undetectable if the LSP is primarily a gaugino.

The folding of the event rate with the velocity distribution provides the modulation effect
Table 3: Ratio of spin contribution \((^{207}\text{Pb}/^{73}\text{Ge})\) at the relevant momentum transfer with the kinematical factor \(1/(1 + \eta)^2\), \(\eta = m_1/A m_p\). 

| Solution | #1 | #2 | #3 | #4 | #5 | #6 | #7 | #8 | #9 |
|----------|----|----|----|----|----|----|----|----|----|
| \(m_1 (\text{GeV})\) | 126 | 27 | 102 | 80 | 124 | 58 | 34 | 35 | 50 |
| NQM      | 0.834 | 0.335 | 0.589 | 0.394 | 0.537 | 0.365 | 0.346 | 0.337 | 0.417 |
| EMC      | 0.645 | 0.345 | 0.602 | 0.499 | 0.602 | 0.263 | 0.341 | 0.383 | 0.479 |

Table 4: Event rates for \(^{207}\text{Pb}\). The LSP mass is \(m_1 = 80, 124, 58, 34, 35, 50 \, \text{GeV}\) for the cases \#4 – \#9 respectively. Cases \#8, \#9 are no-scale models. The values of \(<dN/dt>_0\) for Model A and the Vector part must be multiplied by \(\times 10^{-2}\). 

| Scalar Part | Vector Part | Spin Part |
|-------------|-------------|-----------|
| \(\langle dN/dt \rangle_0\) | \(h\) | \(\langle dN/dt \rangle_0\) | \(h\) | \(\langle dN/dt \rangle_0\) | \(h\) |
| A | B | C | EMC | NQM |
| #4 | 0.03 | 22.9 | 8.5 | 0.003 | 0.04 | 0.054 | 0.80 \(-3\) | 0.16 \(-2\) | 0.015 |
| #5 | 0.46 | 1.8 | 1.4 | -0.003 | 0.03 | 0.053 | 0.37 \(-3\) | 0.91 \(-3\) | 0.014 |
| #6 | 0.16 | 5.7 | 4.8 | 0.007 | 0.11 | 0.057 | 0.44 \(-3\) | 0.11 \(-2\) | 0.033 |
| #7 | 4.30 | 110.0 | 135.0 | 0.020 | 0.94 | 0.065 | 0.67 | 0.87 | 0.055 |
| #8 | 2.90 | 73.1 | 79.8 | 0.020 | 0.40 | 0.065 | 0.22 | 0.35 | 0.055 |
| #9 | 2.90 | 1.6 | 1.7 | 0.009 | 0.95 | 0.059 | 0.29 | 0.37 | 0.035 |

\(h\). In all cases it is small, less than \(\pm 5\%\).

References

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