Calculations of Three-Nucleon Reactions

1 Introduction

Recent progress in the construction of chiral nucleon–nucleon (NN) and three-nucleon forces (3NF) allows to test chiral dynamics in 3N reactions up to the next-to-next-to-next-to-leading-order (N^3LO) of the chiral expansion. It provides also an opportunity to test if consistent two- and three-nucleon forces are able to explain the low-energy A_p puzzle.

The large disagreement between theory and data for the neutron–neutron quasi-free scattering (nn QFS) cross section in low energy neutron–deuteron breakup reaction indicates the possibility that two neutrons can form a bound state when interacting in a 1S_0 state. We discuss consequences of the existence of a dineutron on observables in nd reactions.

The study of nucleon–deuteron (Nd) elastic and breakup processes revealed that at higher energies there are cases where the non-relativistic description based on NN interactions only is insufficient to explain the data.

On 1 August 2012 Walter Glöckle passed away. We dedicate this paper to Walter, who was a great friend and collaborator.
These discrepancies generally increase with energy. Only in some cases does the inclusion of certain types of 3N forces lead to an improvement. We discuss effects of relativity on higher-energy Nd elastic scattering and breakup observables.

2 \( A_y \) Puzzle and the \( N^{3}\text{LO} \) Chiral Three-Nucleon Force

In order to describe the 2N system with the same high precision as provided by standard semi-phenomenological NN potentials one needs to go to \( N^{3}\text{LO} \) in chiral expansion [1, 2]. In the following, results of 3N Faddeev calculations based on five versions of chiral \( N^{3}\text{LO} \) potentials, which use different cut-off’s for the Lippmann–Schwinger equation and spectral function regularization [1] and which equally well describe the 2N system, will be presented. In that order of the chiral expansion six topologies contribute to the 3NF: \( 2\pi \)-exchange, \( 2\pi - 1\pi \)-exchange, ring, \( 1\pi \)-exchange-contact, \( 2\pi \)-exchange-contact and a purely contact term. In addition, there are also leading relativistic corrections. The first three topologies belong to long-range contributions [3], while others are of short-range character [4]. These terms do not involve any unknown low-energy constants and the full \( N^{3}\text{LO} \) 3NF depends on two parameters, D and E, coming with the \( 1\pi \)-exchange-contact and the purely contact term, respectively. A recently developed efficient method of partial wave-decomposition [5] allowed us to apply the \( N^{3}\text{LO} \) 3NF in 3N Faddeev calculations. First results presented in the following were obtained without the short-range \( 2\pi \)-exchange-contact term and leading relativistic corrections in that 3NF. In the left column of Fig. 1 the \( A_y \) puzzle is exemplified for nd data taken at 14.1 MeV. High-precision semi-phenomenological NN potentials (light shaded band) cannot describe the data and including the \( 2\pi \)-exchange Tucson–Melbourne (TM) 3NF (dark shaded band) only partially fills out the discrepancy in the maximum of \( A_y \). Taking the next-to-leading order (NLO) chiral NN potential overestimates the data for \( A_y \) (upper band in the right column of Fig. 1), while next-to-next-to-leading order (\( N^{2}\text{LO} \)) potentials describe the \( A_y \) data quite well (middle band in the right column of Fig. 1). Such behavior can be traced back to the large sensitivity of \( A_y \) to the \( ^3P_j \) NN force components and to a poor description, especially for \( ^3P_2 \), of the experimental phase-shifts by the NLO and \( N^{2}\text{LO} \) chiral potentials [1]. Only with the \( N^{3}\text{LO} \) NN potentials is the \( A_y \) puzzle again regained (lower band in the right column of Fig. 1) and predictions for \( A_y \) become similar to those obtained with semi-phenomenological potentials.

The chiral \( N^{3}\text{LO} \) 3NF is not able to explain the \( A_y \) puzzle (see Fig. 2). The effect of that force is practically negligible and it slightly lowers the \( A_y \) maximum. A resolution of the \( A_y \) puzzle must thus be due to either the \( N^{4}\text{LO} \) chiral 3NF [7] or an incorrect knowledge of the low-energy \( ^3P_j \) NN phase-shifts.

3 The Dineutron and Its Influence on nd Observables

Cross sections for the symmetric-space-star (SST) and quasi-free-scattering (QFS) configurations of the nd breakup are extremely stable with respect to the underlying dynamics. Different potentials, alone or combined
with standard 3N forces, provide practically the same SST and QFS cross sections. Also, the chiral N³LO 3NF is no exception and cannot explain the discrepancy with the data found for the SST configuration [8]. At low energies the cross sections in the SST and QFS configurations are dominated by S-waves. For the SST configuration the largest contribution to the cross section comes from ³S₁ while for neutron–neutron (nn) QFS the ¹S₀ dominates. Neglecting rescatterings the QFS configuration resembles free NN scattering. For free, low-energy neutron–proton (np) scattering one expects contributions from ¹S₀ np and ³S₁ forces. For free nn scattering only the ¹S₀ nn is allowed. That implies that QFS nn would be a powerful tool to study the nn interaction.

The measurement of QFS np cross sections have shown good agreement of data with theory [9], confirming thus good knowledge of the np force. For nn QFS it was found that theory underestimates the data by ∼20 % [9]. The large stability of the QFS cross sections to the underlying dynamics, implies that the present day ¹S₀ nn interaction is probably incorrect. Modifications of the ¹S₀ nn force by multiplying its matrix elements by a factor λ lead to large changes of the nn QFS cross sections, leaving the np ones practically unchanged [10–12]. To remove the discrepancy found in experiment for nn QFS one needs to increase λ by about 8 %. Such increased strength of the ¹S₀ nn force leads to a nearly bound ¹S₀ state of two neutrons [11,12]. That raises the question to what extent is the existence of ¹S₀ dineutron compatible with available nd data. It turns out that the total nd cross section data, total nd elastic scattering cross section and total nd breakup cross section seem not to exclude two neutrons being bound with a ∼−100 keV binding energy [12]. The dineutron influences the nd elastic scattering angular distribution only at forward angles, changing the slope of the cross section. No reliable data at these angles are available [12]. The strongest argument against the dineutron is provided by four measured nn final-state interaction (FSI) configurations [13]. Their analysis gave consistent negative values for the nn scattering length. It seems that with a positive scattering length one would get nn scattering length values which are configuration dependent. Changing to positive nn scattering lengths reduces drastically the magnitude of the FSI peak at large proton energy in the spectra of protons from incomplete nd breakup. Integrating the experimental peak provides an angular distribution for n+d→p+dineutron transition. Comparing it to theoretical values excludes binding energies for dineutron larger in magnitude than ∼100 keV (see Fig. 3).

The most favorable conditions to detect the dineutron would exist when two neutrons mostly occupy the ¹S₀ state. Such a situation is provided by the ³H nucleus and the γ(³H, p)nn reaction seems to be advantageous in searching for the dineutron. The spectra of outgoing protons in that reaction are strongly distorted by the existence of the dineutron (see Fig. 4) and in addition to that distortion a peak corresponding to γ + ³H → p + dineutron transition should appear at largest outgoing proton energies. The magnitude of that peak is determined by the angular distribution for that transition.

The existence of the dineutron does not directly explain the discrepancy in the cross sections in SST configuration. Taking λ values for which a dineutron exists even increases that discrepancy [12]. This does not
Fig. 3 (color online) Lab. angular distribution for d(n,p)dineutron reaction at $E = 14$ MeV incoming neutron lab. energy. Different curves correspond to different factor $\lambda$ by which the $^1S_0$ dineutron matrix element of the CD Bonn potential was multiplied in order to produce the $^1S_0$ dineutron. The dotted (red) curve corresponds to $\lambda = 1.18$ and the dineutron binding energy $\epsilon_{nn} = -80$ keV. The dashed (blue) to $\lambda = 1.21$ and $\epsilon_{nn} = -144$ keV, the solid (orange) to $\lambda = 1.3$ and $\epsilon_{nn} = -441$ keV, and the dashed-double-dotted (indigo) to $\lambda = 1.4$ and $\epsilon_{nn} = -939$ keV. The red and violet full circles, squares, and rhombus result from integration of the FSI peak in spectra of outgoing protons from incomplete d(n,p)nn breakup from refs. [14–16], respectively.

Fig. 4 (color online) The energy spectra of outgoing protons from reaction $\gamma(^3H, p)nn$ at $E_\gamma = 15$ MeV. The (red) solid curve is based on AV18 potential and standard meson-exchange currents [17]. The (blue) dashed and (black) dashed-dotted curve result when $^1S_0$ nn force of AV18 potential is multiplied with factor $\lambda = 1.16$ and 1.22, leading to binding energy of dineutron $\epsilon_{nn} = -108$ keV and $-323$ keV, respectively.

exclude, however, the possibility to explain that discrepancy by contributions from secondary reactions, which are possible when the dineutron exists.

4 Relativistic Effects in 3N Continuum

At incoming nucleon energies above $\approx 100$ MeV clear discrepancies between theory and data, e.g. in the Nd elastic scattering angular distribution and the nd total cross section, appear, even when a $2\pi$-exchange 3NF is included in the calculations. To find out if additional 3NF components, which become active at higher energies, are responsible for these discrepancies, the magnitude of relativistic effects in 3N continuum must be determined.
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**Fig. 5** (color online) The elastic nd scattering angular distributions at the incoming neutron lab. energy $E = 250$ MeV. The solid (red) and dotted (blue) lines are results of the non-relativistic Faddeev calculations with the CD Bonn potential alone and combined with the TM 3NF, respectively. The relativistic predictions based on the CD Bonn potential without Wigner spin rotations are shown by the dashed (blue) lines. The dashed-dotted (brown) lines show results of relativistic calculations with the TM 3NF included. The pd data (x-es) are from ref. [22] and nd data (circles) from ref. [23]. The inset and right figure display details of the cross sections in specific angular ranges.

**Fig. 6** (color online) The tensor analyzing powers $A_{yy}$ and $A_{xz}$ in elastic nd scattering at the incoming neutron lab. energy $E = 135$ MeV. For description of lines see Fig. 5. The pd data (open circles) are from ref. [24].

The 3NF Faddeev equation is set up for a breakup operator and solved in momentum space and partial wave projected. In the relativistic case Jacobi momenta are constructed using Lorentz boosts instead of Galilean boosts, the resolvents involve relativistic kinetic energies, the two-body interactions in the three-body problem appear inside of square roots in a manner dictated by S-matrix cluster properties, and the permutation operators include Wigner rotations [18–20].

When the 3NF’s do not act, the effects of relativity are seen in Nd elastic scattering cross section at backward angles only (see Fig. 5). Relativity increases slightly the non-relativistic cross section. For spin observables only small effects of relativity are observed [18] (see Fig. 6). When a 3NF is included the interplay of relativity and 3NF’s leads to a slight increase of cross sections at angles larger than $\theta_{cm} \approx 100^\circ$, again leading only to small effects for spin observables [20] (see Figs. 5, 6).

For the breakup cross section large relativistic effects are localized in specific regions of the phase-space. They lead to a characteristic pattern of relativistic versus non-relativistic cross section and at $E_{lab} = 200$ MeV those changes can be up to $\approx \pm 60\%$ [21]. For breakup spin observables effects of relativity found in calculations with NN forces only are seen with practically the same magnitude when 3NF is added [20].

**5 Summary**

The chiral N$^3$LO 3NF is too weak to explain the low-energy $A_y$ puzzle. It also does not provide an explanation of discrepancies found for cross sections in the nn QFS and SST configurations of the low-energy nd breakup. Existing nd data seem not to exclude the possibility of two neutrons forming bound $^1S_0$ state with binding energy of $\approx -100$ keV. The existence of the dineutron could provide an explanation for the nn QFS cross section discrepancy.
An exactly Poincaré invariant formulation of three-nucleon scattering using realistic interactions leads to significant changes of the breakup cross section at higher energies and in certain regions of phase space. For the elastic scattering cross sections the small changes are restricted to backward angles and practically no effects are seen for spin observables. Therefore the relativity is not responsible for large discrepancies found in elastic Nd scattering. They must originate from 3NF components, which become active at higher energies. Therefore we expect that 3NF's in all their complexity have to be taken into account in 3NF Faddeev calculations.

Acknowledgments  This work was supported by the Polish National Science Center under Grant No. DEC-2011/01/B/ST2/00578. It was also partially supported by the European Community-Research Infrastructure Integrating Activity “Exciting Physics Of Strong Interactions” (acronym WP4 EPOS) under the Seventh Framework Programme of EU. The numerical calculations have been performed on the supercomputer cluster of the JSC, Jülich, Germany and Ohio Supercomputer Centre, USA (Project PAS0680).

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