ACCEPTANCE DEPENDENCE OF FLUCTUATION IN PARTICLE MULTIPLICITY

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The effect of limiting the acceptance in rapidity on event-by-event multiplicity fluctuations in nucleus-nucleus collisions has been investigated. Our analysis shows that the multiplicity fluctuations decrease when the rapidity acceptance is decreased. We explain this trend by assuming that the probability distribution of the particles in the smaller acceptance window follows binomial distribution. Following a simple statistical analysis we conclude that the event-by-event multiplicity fluctuations for full acceptance are likely to be larger than those observed in the experiments, since the experiments usually have detectors with limited acceptance. We discuss the application of our model to simulated data generated using VENUS, a widely used event generator in heavy-ion collisions. We also discuss the results from our calculations in presence of dynamical fluctuations and possible observation of these in the actual data.

1. Introduction

The analysis of individual ultra-relativistic nucleus-nucleus collision events has now become feasible as the number of particles produced in such events is large [1,2]. In last few years, the subject of event-by-event analysis in general and fluctuations in various observables in particular has attracted significant attention [3]. This attention has partly been motivated by the fact that, if the evolution of the nuclear matter during the collision passes close to the expected tri-critical point in the nuclear phase diagram, the fluctuations in physical observables will be affected. One of the quantities studied in details is the the fluctuation in the numbers of charged particles and photons. Particularly, the quantity of interest is

\[ W = \frac{\langle \Delta N^2 \rangle}{\langle N \rangle} \]

It was observed that this quantity is close to 2 for charged particles as well as photons and it is also found to depend on the centrality (or impact parameter) of collision. If one assumes that the particle production is purely a statistical process, determined by a Poisson distribution, \( W \) is expected to be unity. Thus the departure of \( W \) from unity was attributed to the dynamics of the collision and attempts were made to explain it from different models. As it turns out, two extreme models, the thermal model [7] and the initial-state interaction model (the so-called wounded nucleon model) are able to explain this number equally well [9]. This may mean that \( W \) is not the right quantity to distinguish between such diverse models or that the origin of this departure is not in the dynamics but somewhere else.

One must note that the experimental data on charge particle or photon multi-

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plicities is available in the restricted region of the total phase space available for the reaction. For example, the experimental data in WA98 experiment is restricted to the pseudo-rapidity between $2.9 < \eta < 4.2$ for photons and $2.35 < \eta < 3.75$ for charged particles. So far, the variation of $W$ as a function of the phase space has not been investigated. Of course, a priori, one does not expect a strong variation of $W$ with the phase space. In fact, the models, such as the thermal models, imply phase space independence of $W$. On the other hand, a naive argument would imply increase in the fluctuations when the phase space is reduced. Thus, the investigation of the dependence of $W$ on the phase space region chosen for the detection is of importance since this may affect the conclusions drawn regarding the physical processes taking place during the reaction. In fact, preliminary analysis of WA98 photon and charge particle data indicates that $W$ decreases when the $\eta$-acceptance is reduced. A similar trend is also observed in simulations (see later for the details). Such a behavior of $W$ needs to be explained. Another possibility is that the interesting physics, such as QGP formation, may be restricted to a limited region of the phase space. That is, in a nucleus-nucleus collision, instead of the whole system, a part of it may undergo phase transition. In that case, the nature of fluctuations may depend on the size of the phase space in which the interesting physics takes place. It would therefore be useful to investigate the phase space dependence of the experimental data. However a recent theoretical calculation in a very different context to what is presented here, has emphasized the importance of studying acceptance dependence. They have studied the effect of acceptance on conserved charged fluctuations.

In the present work we investigate the aspects mentioned in the preceding paragraphs. In particular, we study the variation of $W$ as the rapidity acceptance is changed. As a concrete example, consider that the experimental measurement consists of, say, the number of charged particles detected in a given range of rapidity $[\eta_1, \eta_2]$. Further we select the events having certain range of of transverse energy $\Delta E_T$, which in some sense corresponds to a certain range of impact parameter ($\Delta b$). In several experiments it has been established that the distribution of the number of charged particles (or photons) in these events follow Gaussian distribution closely. For these data, one can calculate $W$ defined earlier. The calculations yield a value close to 2 and a number of physical explanations have been offered for this value. These explanations do not depend on the range of the rapidity window and therefore one would expect that the measured $W$ should be independent of it. Below we shall show that this expectation does not hold and it is important to carefully analyze the acceptance dependence of fluctuations before concluding on presence or absence of dynamical fluctuation in the experimental data.

### 2. Statistical fluctuation

Let us consider the consequences of limiting the acceptance on the fluctuations. We shall assume that the probability of finding a particle in the rapidity window $\Delta \eta$ is given by binomial or normal distribution. For this we shall consider that the
particles are emitted in a full rapidity range $[\eta_1, \eta_2]$ and in an experiment one actually detects the particles in a smaller rapidity window $\Delta \eta$. We can then compute $W$ for the smaller acceptance and determine its dependence on the rapidity window $\Delta \eta$.

2.1. Binomial Distribution

Consider a situation in which the distribution of particles in a rapidity window $\Delta \eta$ is decided by a binomial distribution. That is, the probability that a particle will be in $\Delta \eta$ is $p$ and the distribution is purely statistical. Then the probability of finding $n$ particles in $\Delta \eta$ out of total $N$ particles in $[\eta_1, \eta_2]$ is given by

$$P(n, N) = \frac{N!}{n!(N - n)!} p^n(1 - p)^{N - n}$$

(1)

For the binomial distribution, the first and second moments are $< n(N) >= pN$ and $< n^2(N) >= Np(1 - p) + N^2p^2$. Then fluctuation in $n$ for a fixed value of $N$ is given as,

$$W(n, N) = \frac{< n^2(N) > - < n(N) >^2}{< n(N) >} = 1 - p : 0 \leq p \leq 1$$

(2)

Let us now assume that the event-by-event distribution of particles detected in $[\eta_1, \eta_2]$ is given according to a normal distribution with average number $N_0$ and variance $\sigma$ (Actually, one can consider any other distribution whose first two moments are $N_0$ and $\sigma$ respectively). Now we need to compute $< n >$ and $< n^2 >$ when $N$’s are distributed according to the distribution defined above. A straightforward calculation results in,

$$< n > = p < N >= pN_0$$

(3)

and

$$< n^2 >= < N > p(1 - p) + < N^2 > p^2.$$  

(4)

With this

$$W = 1 - p + \frac{p\sigma^2}{N_0} = 1 - p + pW_0$$

(5)

Where $W_0 = \sigma^2/N_0$. For Poisson distribution, $W_0$ is unity and as mentioned earlier, its value is close to 2 in experiments.

The expression of $W$ obtained above is interesting. What it tells is that if $W_0$ for the rapidity range $[\eta_1, \eta_2]$ is unity, as in case of Poisson distribution or when $\sigma^2 =$ $< N >$ for normal distribution, $W$ for the rapidity range $\Delta \eta$ is also unity. On the other hand, if $W_0$ is larger than unity, $W$ lies between unity and $W_0$ and $W$ approaches unity in the limit of small $p$. Thus, contrary to the expectations, the fluctuations at the smaller rapidity window become smaller. We can, in fact, turn this argument around. In any present day heavy-ion experiment, one never has complete acceptance of the phase space for a given particle species. Therefore, if the particles falling in the limited acceptance of an experiment are following binomial
distribution, the ‘actual’ value of $W_0$ for the experiment should be larger than the measured $W$ for a limited acceptance $\Delta \eta$. Unfortunately, it is not possible to use this argument to extrapolate and obtain $W_0$ for full acceptance.

2.2. Gaussian Distribution

Now consider a situation in which the probability of observing $n$ particles in $\Delta \eta$ (out of $N$ which are emitted in $[\eta_1, \eta_2]$) is given by Gaussian distribution of the form,

$$P(n) = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{(n-pN)^2}{2\sigma_n^2}}$$

(6)

where $p = n/N$, with $<n(N)> = pN$ and $<n^2(N)> = \sigma_n^2 + p^2N^2$

If we assume that the particles detected in $[\eta_1, \eta_2]$ are distributed according to a Gaussian distribution with average number $N_0$ and variance $\sigma$, we will need to compute $<n>$ and $<n^2>$ when $N$’s are distributed according to the Gaussian distribution. The simple calculation yields,

$<n> = p <N> = pN_0$ and $<n^2> = \sigma_n^2 + p^2 <N^2>$.

This gives :

$$W = \frac{\sigma_n^2}{pN_0} + pW_0$$

(7)

This shows that depending on the value of $p$ for a given $W_0$ the fluctuation in a smaller acceptance ($W$) can be higher as well as lower than that of $W_0$.

3. Application to simulated data

Ideally, one should test the validity of our proposal with the experimental data. This is being done. In this work we demonstrate the application of the above ideas to the simulated $Pb + Pb$ events at $158 \cdot AGeV$ generated from VENUS 4.12 event generator. 45K VENUS events were generated between impact parameter $0 - 12 \text{ fm}$. In order to minimize the contribution due to fluctuation from impact parameter, the distributions of $\pi^0$, $\pi^-$ and $\pi^+$ were chosen within narrow 5% bins of cross section of the minimum bias transverse energy ($E_T$). It must be mentioned that since impact parameter is not directly measurable in experiment, observable like transverse energy or forward energy can be used to define the centrality of the reaction. A highly central event corresponds to lower impact parameter event and leads to higher transverse energy production. To determine the transverse energy of the reaction, we have done a fast simulation in which we calculate transverse energy from VENUS by taking the resolution factors for the hadronic and electromagnetic energy of a realistic calorimeter as used in WA80 and WA98 experiment into account. The resolution of transverse electromagnetic energy was taken to be $17.9\%/\sqrt{E}$ and that for hadronic energy was $46.1\%/\sqrt{E}$, where $E$ is expressed in GeV.
Event-by-event particle multiplicity distribution is obtained within the rapidity range $[\eta_1, \eta_2] = [2.0, 4.0]$, for different 5% bins in centrality (0 – 5%, 5 – 10% · · · 55 – 60%). These distributions are then fitted to Gaussians and fit parameters mean ($N_0$) and standard deviation ($\sigma$) are obtained. Fluctuation $W_0 = \sigma^2/N_0$ is then calculated. We then select one unit $\Delta \eta$ window, and obtain the multiplicity distributions, the Gaussian fit parameters of these distributions are used to obtain fluctuations $W$. We find that $W < W_0$ for all the transverse energy bins. The centrality dependence is shown in Figure 1.

Using the value of $W_0$ calculated for $[\eta_1, \eta_2]$ in Eqn. 5 we obtain the new $W_{model}$ for 1.0 unit $\Delta \eta$. These are shown in Figure 2, for $\pi^0$ and $\pi^-$, $\pi^+$ shows similar trend as for $\pi^-$. It can be seen that the model calculations (solid lines) match very well with values of fluctuation from VENUS in smaller acceptance (open circles). The statistical errors are shown in the figure. Similar procedure has been followed for the actual experimental data, to verify the above discussed models and distributions. The preliminary results of the calculation for actual data is seen to follow a similar trend.

4. Presence of dynamical fluctuation

An interesting possibility is that an exotic physical process is restricted to a region of phase space. An example is the formation of quark matter in a small volume in the early phase of the collision. One can argue that this volume maps into a certain ($\eta, \phi$) region of the phase space. Let us assume that this region is smaller than the rapidity range $[\eta_1, \eta_2]$ but covers the $\Delta \eta$ region of the phase space and leads to an enhancement in multiplicity compared to a statistical case. Now, the fraction of particles going into the rapidity region $\Delta \eta$ will depend on whether an exotic process occurs in the rapidity region $\Delta \eta$. Let us assume that these probabilities are $p$ and $q$ for exotic process not occurring and occurring in $\Delta \eta$ respectively. Assuming that fraction $\beta$ of the events have exotic process occurring in the rapidity region $\Delta \eta$, the expected value of $W$ can be obtained as follows.

If $P_1(n)$ and $P_2(n)$ are binomial distributions, with probabilities $p$ and $q$ respectively. Further if $\beta$ is the fraction of events out of total events having probability distribution $P_1(n)$ then the Probability distribution of all the events is given as,

$$P(n) = \beta P_1(n) + (1 - \beta)P_2(n)$$

with $<n(N)> = \beta pN + (1 - \beta)qN$

and $<n^2(N)> = \beta p(1 - p)N + (1 - \beta)q(1 - q)N + (\beta pN + (1 - \beta)qN)^2$

The fluctuation can be calculated to be

$$W = 1 - \frac{\beta p^2 + (1 - \beta)q^2}{\beta p + (1 - \beta)q} + W_0(\beta p + (1 - \beta)q)$$

One can see that in various limiting cases the fluctuation $W$ approaches the pure statistical case discussed in the previous section:
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- Case 1: \( \beta = 1 \Rightarrow W = (1 - p) + pW_0 \)
- Case 2: \( \beta = 0 \Rightarrow W = (1 - q) + qW_0 \)
- Case 3: \( p = q \Rightarrow W = (1 - p) + pW_0 \)

Figure 2 shows the variation of \( W \) with fraction of events having statistical fluctuation, for a \( W_0 = 2 \) in the full coverage and probability \( p = 0.4 \). One can see that, the fluctuation in limited acceptance is reduced to 1.4. However as the percentage of events having dynamical fluctuation \( (1 - \beta) \) increases, for \( q = 0.8 \), the fluctuation in limited acceptance also increases (solid line). The differences are clearly visible for events sets having dynamical fluctuation for more than 10% of events. In actual experiment one does not know which events have dynamical fluctuation in an ensemble of events collected over a period of data taking. So analysis of data will yield a fixed \( W \) following Eqn. 5 as:

\[
W = 1 - \bar{p} + \bar{p} \frac{\sigma^2}{N_0} = 1 - \bar{p} + \bar{p}W_0
\]  

(10)

Where \( \bar{p} = \beta p + (1 - \beta)q \). The variation of this \( W \) with \( \beta \) is also shown as dashed line in Figure 2. It shows a linear increase with decrease in \( \beta \). The differences between the values of \( W \) from Eqn’s 9 and 10 are beyond statistical errors shown in figure, for certain range of \( \beta \). With increased statistics one should in principle observe this difference. If presence of dynamical fluctuations has an impact parameter dependence, i.e. if probability of QGP-type fluctuations increases with increases in centrality of the reaction due to increase in energy density, then one should study the centrality dependence of \( W \). The \( \beta \) values may be replaced by centrality parameter or impact parameter in figure, and a deviation from linear dependence would indicate presence of dynamical fluctuations. It must be mentioned that as the value of \( q \) decreases the difference between the two curves also reduces and they match with each other as \( \beta \) approaches 0 and 1. These set the constraints in detecting the dynamical fluctuations.

Another possibility of detecting dynamical fluctuations through our model from actual experimental data, where one will obtain a single value of fluctuation \( W \), is through construction of mixed events. In order to make a conclusion regarding the presence of dynamical fluctuations, one has to compare this value to what is expected from a pure statistical process. For this a set of events has to be constructed from data, so that they preserve all the inherent detector related fluctuations and broad global features, like total multiplicity in an event but remove the local dynamical fluctuations. Such events are referred to as mixed events or mixed data and techniques for constructing these exist (see for example Ref. 15). A comparative study of fluctuations obtained from data and mixed events constructed from data will help arriving at proper conclusions regarding the presence of dynamical fluctuations as discussed above.

5. Summary
In the present work we have investigated the dependence of the event-by-event multiplicity fluctuations on the acceptance of the detector. Unlike the naive expectations the fluctuations in the data generated from VENUS event generator decrease when the acceptance is decreased. Similar tendency is also observed in the preliminary results from WA98 data. We find that the trend of the simulated data is reproduced by assuming that the probability distribution in the smaller rapidity window follows binomial distribution. From this observation, one may infer that the event-by-event fluctuations in the experiment, when the full rapidity acceptance is included, may be larger than the experimentally observed fluctuations since the experiments are necessarily restricted to smaller rapidity acceptance. One needs to keep this mind when one is attempting to explain the fluctuations from theoretical models. We have also investigated the possibility of detecting the dynamical fluctuations if some special dynamics is restricted to a small phase space.

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Figure 1: Fluctuation for $\pi^0$ and $\pi^-$ obtained from VENUS for 2 unit $\Delta\eta$ coverage (solid points) and for 1.0 unit $\Delta\eta$ coverage (open points). The model calculations are shown as solid line. The statistical errors are within the symbol size. The results for $\pi^+$ are similar to those obtained for $\pi^-$. 

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Figure 2: Fluctuation (W) for smaller acceptance as a function probability of events being statistical type (\(\beta\)), for \(q = 0.8\). The fluctuation in larger acceptance (\(W_0\)) is taken as 2 and \(p = 0.4\).