Research on weak fault signal detection technology based on Iv system

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Abstract: The detection of weak fault signals is realized by applying non-resonant parametric excitation to Lu chaotic system. Periodic signals far larger than the characteristic frequency of the system are taken as input, the sharp change of the system attractor is observed, and the existence of weak signals is detected. The conditions that the parameters of the detection system should meet are deduced by using the averaging method and Lyapunov method. Numerical research shows that the detection system can quickly reach a stable state. When the amplitude of the detected weak signals changes slightly, the attractor state changes abruptly, and the system is sensitive to the input. Through the "butterfly effect" of the system, feature signals that are difficult to extract by traditional methods can be detected. And the system has strong anti-interference performance.

1. Introduction

Weak signal detection is a hot and difficult problem in modern information theory. It mainly uses electronics, probability theory, mathematical statistics, information theory and other methods to detect the received interfered signal, determine whether there is a useful signal, and obtain relevant parameters. At the same time it can analyze the law of noise generation, study the characteristics of the measured signal and improve the detection accuracy [1]. Traditional methods for detecting weak signals such as narrowband filtering, synchronous superposition, and correlation analysis can extract weak signals by effectively suppressing noise, but the equipment such as measuring instruments and sensors will generate noise, so the measured signal will be drowned by a variety of strong noise signals. Even if it plays a certain role in suppressing strong noise, weak signals are also suppressed while suppressing noise, making it difficult to achieve the purpose of detection. The detection method of chaotic weak signals is based on a nonlinear system, which makes full use of the initial value sensitivity, noise immunity and nonlinear amplification characteristics of the chaotic system to realize signal detection, and can detect weak signals with low SNR. Since this method was proposed in the 1990s, many scholars at home and abroad have carried out in-depth research [2]. Wang Guanyu has studied the weak signal detection of Duffing System under the interference of white noise, and made a lot of theoretical analysis on it, but lacked experimental verification [3]. Lin Hongbo, Qi Fang and et al. proposed a GRNN detection method to extract weak harmonic signal under the background of chaotic noise. Taking the chaotic time sequence generated by Duffing system as the chaotic background, the simulation verified that the harmonic can be detected when the signal-to-noise ratio is -36dB [4]. Li Yue et al. put forward the chaos detection method of weak square wave signal annihilated in the background of color noise, and analyzed the detection of square wave in the background of Gaussian white noise, Gaussian color noise and non-Gaussian color noise. The signal-to-noise ratio of detected
signal in the background of Gaussian color noise can reach -10.845db [5]. Chen Long exerted the non resonant parameter excitation to Chen’s chaotic system, and used the periodic signal far larger than the characteristic frequency of the system as the control input to realize the non feedback chaos control. Through numerical research, it was proved that the obtained control parameters can make the controlled system quickly reach a stable state and have strong anti-interference ability [6]. Wang Huiwu and others proposed a new method of signal detection and parameter estimation based on Duffing system, which only uses a single Duffing oscillator to complete the detection and frequency estimation of sine signal with unknown frequency [7]. Ren Xueping and others proposed a new method for detecting weak signs with improving Duffing oscillator, which can detect weak signal with larger natural frequency, and it is easier to enter large-scale periodic state when disturbed by fault [8]. Wang Mengjiao and others could accurately detect the weak harmonic signal in the background of strong noise by using the controlled Chen system [9]. Utilizing the initial value sensitivity of Lorenz chaotic system and the synchronization of chaotic system, Li Guozheng proposed a new method to measure the frequency value of weak signal in the background of strong noise based on the chaotic synchronization system [10]. In this paper, the weak signal detection problem is studied deeply by using Lu system, especially in determining the sensitive parameter range of the system, which is realized by analytical method. Consequently, a more general calculation method of detection parameters is given.

2. Description of chaos detection system
The mathematical model of Lv system is:

\[
\begin{align*}
x' &= a(y - x) \\
y' &= -xz + cy \\
z' &= xy - bz
\end{align*}
\]  

(1)

when \( a = 35, b = 3, c = 28 \), the system is in a chaotic state, its phase diagram is shown in Figure 1. Each state variable is regarded as a harmonic signal with continuous phase and random envelope changes. The characteristic frequency of the system is defined as the average change rate of signal phase [11], it can be written as:

\[
\omega_0 = \lim_{T \to \infty} \frac{2\pi N(t)}{T},
\]

where \( N \) is the number of wave peaks or troughs of the signal in time \( T \). For the system (1), the characteristic frequency can be calculated as \( \omega_0 \approx 10.68 \text{rad/s} \). Since the change of parameter \( C \) has a great influence on the system behavior, the parametric excitation non-feedback control system can be designed as:

\[
\begin{align*}
x' &= a(y - x) \\
y' &= -xz + c[1 + k \cos(\omega t)]y \\
z' &= xy - bz
\end{align*}
\]  

(2)

where \( k \) is the amplitude of the detected weak signal, and the frequency of the excitation signal \( \omega \) is much higher than the system characteristic frequency \( \omega_0 \). By adjusting the parameter \( k \), the response of the Lv system changes dramatically, so as to judge the weak signal detection.

3. Determination of system sensitive parameters
The average method [13] is used to analyze the dynamic characteristics of the detection system (2). The average method is an approximate analytical method for solving nonlinear vibration equations. The basic idea is to divide the solution of nonlinear vibration system into two time scales of fast change and slow change for analysis. In the fast change period, it can be regarded as a simple harmonic vibration with constant amplitude and initial phase angle, but its amplitude and initial phase angle will change slowly in a long time, that is to say, the solution of the system is a simple harmonic vibration of envelope slowly changing. Drawing lessons from the basic idea for \( \dot{x} \), and referring to the variable processing methods in references [11,14], the state variables in the controlled system (2) can be divided into slow variables varying according to the characteristic frequency \( \omega_0 \) of the system and fast variables varying according to the excitation frequency \( \omega \) of the external parameters. Slow variables
describe the long-term trend of the system evolution, while fast variables represent the local oscillation form of the system variables, so as long as the slow variable system is globally stable, the control goal can be achieved. Now the system variable is expressed as:

\[
\begin{align*}
\dot{x} &= x_s + x_q \\
\dot{y} &= y_s + y_q \\
\dot{z} &= z_s + z_q
\end{align*}
\]

(3)

the subscripts s and q represent slow variable and fast variable respectively. In order to obtain the evolution equation of slow variable, the following assumptions are made according to the research results in [11,14]: the average value of fast variable in the period \( T = 2\pi/\omega \) is set as zero, and its amplitude is far smaller than that of slow variable. Let operator \( \langle \cdot \rangle \) represent \( \frac{1}{T} \int_{t}^{t+T} (\cdot) dt \), substitute in equation (3) and treat each slow variable as a constant in period \( T \), and use average method to integrate system variables to obtain the evolution equation of slow variable.

\[
\begin{align*}
\dot{x}_s &= a(y_s - x_s) \\
\dot{y}_s &= -x_s z_s + c y_s + c k \langle y_q \cos(\omega t) \rangle \\
\dot{z}_s &= x_s y_s - b z_s
\end{align*}
\]

(4)

in the above formula, the highe-order small quantities \( x_q z_q \) and \( x_q y_q \) which are not zero after integration are omitted. By comparing the detection system (2) with the equation (4) after integration, the evolution equation of fast variable \( y_q \) can be obtained:

\[
\dot{y}_q = c y_q + c k \cos(\omega t) y_s
\]

(5)

The solution of formula (5) is:

\[
y_q = \frac{(k \omega \sin(\omega t) - c \cos(\omega t))}{\omega^2 + c^2} y_s
\]

(6)

Substitute formula (6) into system (4), and use the following integral relation:

\[
\langle \cos^2(\omega t) \rangle = \frac{1}{2} \\
\langle \cos(\omega t) \sin(\omega t) \rangle = 0
\]

The equation of slowly varying system is obtained as:

\[
\begin{align*}
\dot{x}_s &= a(y_s - x_s) \\
\dot{y}_s &= -x_s z_s + c e y_s \\
\dot{z}_s &= x_s y_s - b z_s
\end{align*}
\]

(7)

where the parameter is \( c_e = c - c^3 k^2 / 2(\omega^2 + c^2) \).

For system (7), the Lyapunov function is constructed as

\[
V = \frac{1}{2}(x_s^2 + y_s^2 + z_s^2)
\]

after derivation, we get

\[
\dot{V} = x_s \dot{x}_s + y_s \dot{y}_s + z_s \dot{z}_s
\]

\[
= -ax_s^2 + ax_s y_s + c e y_s^2 - b z_s^2
\]

\[
= -[x_s \ y_s \ z_s] A[x_s \ y_s \ z_s]^T
\]

where

\[
A = \begin{bmatrix}
a & -a/2 & 0 \\
-a/2 & -c e & 0 \\
0 & 0 & b
\end{bmatrix}
\]

It can be solved that when \( c_e < -\frac{a}{4} \), the matrix \( A \) is positive definite and \( V < 0 \), so that system (7) is asymptotically stable at the origin. Therefore, the condition that the system sensitive parameters should satisfy is:
4. Numerical results

When the parameter $a = 3.5, b = 3, c = 2.8$ of the detection system (2), considering the original system frequency $\omega_0 \approx 10.68\text{rad/s}$, take the parameter excitation frequency $\omega = 180\text{rad/s}$. According to formula (8), the condition that the parameter of the detection system should meet in response to the shock is $k > 10.54$. Take the initial value of the detection system is $x(0) = -1, y(0) = -1, z(0) = 1$, and use the fourth-order Runge Kutta method to solve when $k$ is different.

Take the control parameter $k = 20.5$ to get the phase diagram change of the detection system and the time-domain response curve of system variables $x, y, z$ as shown in Figure 1 and Figure 2.

![Phase Diagram and Time-Domain Response](image)

Figure 1. The output phase diagram of the detection system when $k$ is 20.5.
Figure 2. Time history diagram of detection system output when k is 20.5.

When there is no excitation, the time-domain response curve of system variables x, y, z is shown in Figure 3.

By comparing the changes after the application of weak signal excitation, the phase trajectory changes to the center origin, and the system variable z is sensitive to the detected weak signal.

In order to achieve a better control effect, a bifurcation diagram of K value for variable z is made as shown in Figure 4. To analyze the variation trend of variable z, take k in the range of 10-20 to make Figure 4 (a), it is found that the change of z in this section has been in a downward trend. Figure 4 (b) shows that the fluctuation range of z at the threshold of 20.9 is the smallest and the most stable, and a large change will occur after passing the critical threshold.
Figure 4. Bifurcation diagram of system response $k$ with change $z$.

Through Figure 4 (b), it is found that the system after divergence is stable near $k = 21.18$. Fig. 5 and Fig. 6 are the three-dimensional phase diagram and the variable change diagram with time when $k = 21.2$.

Figure 5. The output phase diagram of the detection system when $k$ is 21.2.
Figure 6. The output response curve of the detection system when k is 21.2.

From the above research, it can be found that Lv system is sensitive to weak signal detection, especially at the critical threshold, which can show typical butterfly effect, so that weak signal can be detected effectively, and realize the weak signal detection target in engineering.

5. Conclusion

Based on the sensitivity of chaotic parameters, a weak signal detection model of Lv chaotic system is designed. The average method and Lyapunov method are used to deduce the universal occurrence region of sensitive parameters analytically from the analytical point of view, and the accurate range of detection parameters is obtained according to the specific requirements. The numerical study shows that the theoretical analysis results of the sensitive parameters are correct. By selecting appropriate parameters, the system can be stable at the equilibrium point and periodic orbit, and has strong anti-interference performance. Compared with the parametric resonance perturbation method, this method can give strict theoretical analysis results.

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