Interior point algorithm-based power flow optimisation of a combined AC and DC multi-terminal grid

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Abstract: The high cost of power electronic equipment, lower reliability and poor power handling capacity of the semiconductor devices had stalled the deployment of systems based on DC (multi-terminal direct current system (MTDC)) networks. The introduction of voltage source converters (VSCs) for transmission has renewed the interest in the development of large interconnected grids based on both alternate current (AC) and DC transmission networks. Such a grid platform also realises the added advantage of integrating the renewable energy sources into the grid. Thus a grid based on DC MTDC network is a possible solution to improve energy security and check the increasing supply demand gap. An optimal power solution for combined AC and DC grids obtained by the solution of the interior point algorithm is proposed in this study. Multi-terminal HVDC grids lie at the heart of various suggested transmission capacity increases. A significant difference is observed when MTDC grids are solved for power flows in place of conventional AC grids. This study deals with the power flow problem of a combined MTDC and an AC grid. The AC side is modelled with the full power flow equations and the VSCs are modelled using a connecting line, two generators and an AC node. The VSC and the DC losses are also considered. The optimisation focuses on several different goals. Three different scenarios are presented in an arbitrary grid network with ten AC nodes and five converter stations.

1 Introduction

Electric transmission networks of present day power grids are mostly based on AC technology, which was predominantly the only mode of electricity transmission from generation sites to load centres about a 100 years ago. However, over the past few years this predominance is being questioned [1]. The establishment of the present electric power system dates back over a 100 years when the sole purpose of the power system was to evacuate power from generating stations employing the use of coal to residential consumers that needed the electricity mostly for lighting purposes. Power generation was localised and was built around communities.

With the advancement of technology and industry, the needs of the people grew and leading to the generation sites to grow to supply the needed power. However, with the increasing size of the electrical grid, optimisation and complete control cannot be satisfied by merely increasing the number of generation sites. Today the electrical power system delivers power to agriculture industry, commercial and residential consumers, trying miserably to cope up with the ever growing demand. Systematically the hazards accompanied with relying on an overburdened grid grow in size, scope and convolution with every moment elapsed. The present limited one way interaction makes it difficult for the grid to respond to the ever changing and rising energy demands of the 21st century. The existing transmission systems in Europe and North America as well as India have touched their limits and further increase in the bulk power transmission is expected. This can be observed from the expected increase in the amount of renewable energy to be integrated into the grid galvanised from research in power electronic equipment that enables renewable energy producers to generate greater amounts of power. Moreover, a significant amount of increase is expected in the consumption of electrical energy at load centres very far from production centres.

A number of proposals for such an enhancement of the already present grid can be imagined from the need of higher transmission capabilities and a significant improvement in high voltage direct current (HVDC) technology. One possible arena is to dwell into the increase in number of multiple HVDC point-to-point links in order to directly boost the transmission capacity. This possible solution calls for more interconnections. Multi-terminal DC system (MTDC) grids have also been proposed and deployed all over. Zhu and Booth [2] discuss the general HVDC case. The European case has been examined in [3] and the North American perspective in [4]. MTDC systems could also directly affect the lower voltage grid levels in a positive way [5]. VSCs have many significant advantages as compared with the line commutated converter (LCC) arrangements and hence are used quite ubiquitously all over for the proposed projects. Only VSCs are considered in this paper for the study case and a detailed discussion for this choice of VSC over LCC has been included in Section 2 and in [3].

Integration of wind energy is another core issue that is being discussed for such deployments and poses a huge challenge to realise it in the model. The amount and size of renewable energy sources results in a limiting factor for such a connection. HVDC point-to-point links can provide a possible solution to this problem for connecting the offshore wind parks directly to the load centres [6]. An MTDC grid structure happens to realise the connection between the different wind parks and between the wind park and the HVAC grid [7-9]. These structures can have different topologies; beginning from a link with many terminals, leading to radial connections and ending up in fully meshed grids. The connection between the HVAC grid and the HVAC lines results in new power flows. Even though there is a rich documentation regarding the general VSC technology and its usage in the grids, there happens to be a dearth of publications dealing with the load flow of a combination of a direct current (DC) and an alternate current (AC) grid. Pizano-Martinez et al. [10] show an approach to model a VSC terminal even though it neglects the terminal losses. Gengyin et al. [11] present an algorithm to calculate the losses in VSC terminals for a combined DC and AC power flow model. Terminal losses and losses in lines have been taken into account in this solution but the power settings of the terminals have been kept constant. Therefore optimisation of power flows is not possible with this model. A steady-state model has been formulated in this paper in order to calculate the optimal power flow in a combined AC and DC grid.
This paper also considers the losses in the VSC terminals and in the AC and DC lines. Full power flow equations are used to model the AC grid power flow. The non-linear mathematical optimisation problem is formulated and solved using the interior point algorithm; however, the choice of such an algorithm in itself is not a problem this paper hopes to solve.

2 Technical background

2.1 State of technology

Owing to some key advancements in the electricity transmission technology, certain key concepts have evolved for the construction of the overlay grids that can provide the energy transfer corridors:

1. Three-phase AC technology 50 Hz (AC grids) with voltages >400 kV (750, 1000 kV).
2. Three-phase AC technology with reduced frequency (AC grids 16 2/3 Hz) with voltages >400 kV.
3. HVDC with network controlled converters (LCC-HVDC, HVDC classic).
4. HVDC with self-commutated converters (VSC-HVDC).

Constant research into the development of three-phase AC technology as a result of an increasing requirement on transmitted power and distance has led to the introduction of increased voltage levels [8].

Overhead lines, cables as well as gas insulated lines are now available as AC transport medium. Owing to sophisticated technology as well as lower investment costs, overhead lines provide a standard solution for high transmission voltages. Line conductor monitoring, high temperature conductors as well as considerable improvement of towers in respect with space and field strength distribution are some of the developments in overhead line technology.

Reduced frequency three-phase systems as an alternative between AC and DC grids was introduced for discussion. A reduced frequency of 16 2/3 Hz was conceptualised which is also used in certain countries as traction power supply. Larger distances can be bridged because of reduction in the line impedances. Additionally specific research needs to be done on such systems so as to introduce this technology with regard to high voltages. There is a strong criticism about these technologies as equipments like transformers for reduced frequency AC systems having larger dimensions. A considerable converter expense also needs to be planned which is larger than with DC grids.

The main aim of the introduction of HVDC systems technology is to provide a highly efficient and a flexible transmission system. With the increase in the number and power flow between energy corridors as well as an increasing need of the integration of renewable energy sources into the grid such as wind and solar energies, the need for the presence of a transmission system that could provide the required flexibility was satisfied by the HVDC transmission system. The HVDC system provides the platform to interconnect two AC power systems that are not synchronised as well as transfer of electric power between two distant nodal points through overhead transmission or submarine cables. HVDC systems are more cost effective than AC systems via overhead lines as the costs of transformer stations are not considerable. However, the critical length is reached between 800 and 1200 km. Even though, up to this length, the AC overhead lines are more economical today, HVDC has definite advantages with longer cable connections. For the supergrid deployment, the HVDC transmission system provides superior working conditions, a better power flow control and a definite platform for future additions on the supply side which can be from other conventional generation plants or renewable energy parks. The supergrid would require a substantial control on the transferred power. The HVDC systems in parallel with the AC systems provide such a characteristic which can affect the controllability and flexibility of the bulk power system.

The converter stations from the backbone of the working of an efficient HVDC transmission system. Currently, two kinds of converter technologies, consisting of the line commutated current source converters (CSC’s) and self-commutated VSC’s are mostly used in the HVDC transmission systems.

HVDC systems based on the principle of conventional CSC’s require a substantially large generation source with a very high level of short-circuit ratio in order to operate satisfactorily. In other words, there is a need for the transference of reactive power from the AC system at the point of contacts to the converter so to accomplish the conversion process which amounts to nearly about 50% of the total active power through the converter. Moreover, based on the CSC technology principle, power flow direction can be reversed only by reversing the DC voltage polarity. This characteristic needs a highly complicated switching technique in case the CSC system is used for building an MTDC.

On the contrary, VSC’s utilising the insulated gate bipolar transistor (IGBT) valves as well as pulse width modulation (PWM) techniques can lead to the production of a near sinusoidal AC voltage which is fully controllable with respect to magnitude and phase of the AC wave. Unlike the CSC systems, VSCs have no reactive power demand and can as well exchange the reactive power with the AC grid.

VSCs can rapidly control the active power exchange by controlling the phase angle of the produced voltage as well as control the reactive power at each terminal by controlling the magnitude of VSC voltage independent of the DC power transmission. Owing to this property, VSCs can be installed anywhere in the AC grid irrespective of the short-circuit current capacity. Moreover, to change the direction of the power flow its DC-link, VSC does not need to reverse the voltage polarity. This power reversal is observed by changing the direction of the current. Many attempts have already been made to conceptualise the formation of the meshed grids using classic HVDC or CSC technology. However, because of the high amount of complexity involved, the projects were thereby limited to a maximum of three nodes. On the other hand, the VSC-HVDC provides the most suitable conditions for a multi-terminal system, which is the basis for the deployment of a supergrid because of the fact that the number of nodes and the kind of grid topology utilised does not have any limit with regard to VSC-HVDC [11–15].

2.2 VSC-HVDC functional principle

The working of a VSC converter is based on the synchronous functioning of a 6-pulse bridge circuit of IGBT’s (power transistors) controlled by a clocked control signal generating pulses in the range of kilohertz frequencies. Provision has to be provided for the serial switching of the multiple semiconductors in order to account for the limited reverse voltage capacity of the power electronic elements. Intelligent control techniques can introduce a very high flexibility in the output voltage control in order to obtain the desired active and reactive powers [9].

AC voltage is formed by the use of PWM modulation in case of PWM VSC converters and DC voltage is smoothed by the use of DC capacitors. There is a higher precision of synchronism if a higher clock frequency is used. PWM technology has been in use for nearly a decade now and two and three level VSC converters are now available.

Constant research into upgrading the ratings and the frequency of VSC converters has led to the use of a modular construction based on the use of multi-level technology. Submodules consisting of half bridges having two valves and a module capacitor are at the heart of submodule architectures. Partial voltages of the submodules combine to the complete voltage of the branches and thereby branches act as controllable voltage sources.

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The active and reactive powers over the AC lines are calculated by using the standard full flow equations. Owing to its spread, the AC grid consists of more components such as the buses and the lines while as the DC grid consists of less number of components mostly the resistance of lines.

3.1 HVAC grid

According to Fig. 2, the real power $P$ and reactive power $Q$ from node $k$ to node $m$ is given by

$$P_{km} = U_k^2 G_{km} - U_k U_m G_{km} \cos(\theta_k - \theta_m) - U_k U_m B_{km} \sin(\theta_k - \theta_m)$$

$$Q_{km} = -U_k^2 (B_{km} + B_{sh}) + U_k U_m B_{km} \cos(\theta_k - \theta_m) - U_k U_m G_{km} \sin(\theta_k - \theta_m)$$

Consequently from (1) and (2) we have

$$P_k = U_k \sum_{m \in \mathcal{N}} U_m [G_{km} \cos(\theta_k - \theta_m) + B_{km} \sin(\theta_k - \theta_m)]$$

$$Q_k = U_k \sum_{m \in \mathcal{N}} U_m [-B_{km} \cos(\theta_k - \theta_m) + G_{km} \sin(\theta_k - \theta_m)]$$

where $N$ is the set of buses adjacent to $k$ including $k$.

3.2 Converters

A model of the VSC converter station is shown in Fig. 2. The model consists of an additional AC node (AC, c) linked with a short AC line to the consequent AC node $k$ which locates the position of the converter. A transformer is represented by a line placed between the terminal and the node. The load flow is of prior interest in this study and the voltage transformation between (AC, c) and the node $k$ is not of significance. New grid variables $U_c$, $\theta_c$ have been used to control the active and reactive power flow through terminals. The calculation methodology is the same as AC power line flows

$$P_{AC,c} = U_c U_k [G_{ck} \cos(\theta_k - \theta_c) + B_{ck} \sin(\theta_k - \theta_c)]$$

$$Q_{AC,c} = U_c U_k [-B_{ck} \cos(\theta_k - \theta_c) + G_{ck} \sin(\theta_k - \theta_c)]$$

Two voltage sources have been used to model the conversion from AC to DC. Generator connections to the AC node (AC, c) are represented as an AC source while as to (DC, c) as DC source. The active power balance on both the sides guarantees the AC and DC side power balance which is differentiated by the losses in the converter

$$P_{AC,c} + P_{loss} + P_{DC,c} = 0$$

Converter losses are calculated by

$$P_{loss} = a + b I_c + c I_c^2$$

$$I_c = \frac{\sqrt{P_c^2 + Q_c^2}}{3U_c}$$

where $a = 11.0331 \times 10^{-3}$; $b = 3.464 \times 10^{-3}$; and $c = 5.5335 \times 10^{-3}$ in per unit (pu).

3.3 HVDC grid

Since steady state has been considered, the DC grid (Fig. 3) is constructed by using only resistive lines. Thus, Ohm’s law and Joules
law can be used to calculate the power flow

\[ P_{\text{DC, \text{kw}}} = \frac{2U_i(U_{i} - U_{m})}{R_{\text{Lin}}} \]  

Equation (10)

\[ P_{\text{DC, \text{kn}}} = 2U_{\text{DC}} \sum_{i \in \mathcal{N}} \frac{U_{\text{DC},c} - U_{m}}{R_{\text{Lin}}} \]  

Equation (11)

\[ P_{\text{DC,c}} = 2U_{\text{DC,c}}J_{\text{DC,c}} \]  

Equation (12)

\( N \) being the set of DC buses adjacent to bus \( c \).

4 Optimisation

For the development of a mathematical model and consequent solution, the model described in the previous sections is used. The problem can be defined as a non-linear one with both equality and not equality constraints. The next section is dedicated to the solution of the problem.

4.1 State vector and control variables

State vector \( x \) consists of all the state variables. A voltage \( U_i \) represents the AC grid nodes. Moreover, the converter side AC bus also has a voltage \( U_{\text{DC,i}} \). \( n \) represents the number of AC nodes and \( p \) represents the number of converters. For each bus \( \theta_i \) and each converter \( \theta_{j,p} \), the state vector gets increased by one more variable. The next state variable involves the active and reactive power generations. A generator bus adds two variables (\( P_i \) and \( Q_i \)) to \( x \) and each static var compensator (SVC) also adds one \( Q_{i} \) to the state vector \( x \).

On the other hand, for the DC grid only the voltage variables \( U_{\text{DC,j}} \) at the DC nodes

\[
x = \begin{bmatrix} U_i \\ U_{\text{DC,i}} \\ \theta_i \\ \theta_{j,p} \\ P_i \\ Q_i \\ \theta_{ref} \\ U_{\text{DC,j}} \end{bmatrix}
\]

Equation (13)

In any case, the length of the state vector is dependent on the AC grid size and the corresponding number of terminals

\[ I_x = 2i + 3p + 2g + h \]  

Equation (14)

where \( p \) is the total number of converters, \( g \) represents the total number of generators in the grid and \( h \) represents the number of SVC devices. Also \( n \) represents the number of AC nodes.

The control variables represent a subset of the state vector \( x \). An AC node voltage is controllable if the node is connected to an SVC device or a generator. Moreover, all DC voltage levels at all converters are also controllable.

4.2 Objective function

Objective function being very important to the definition and consequent solution of an optimisation problem needs to be clearly defined. Several variations are possible, for example

\[ \min_x f(x) = \sum_{i=1}^{n} \alpha_i P_i + \beta_i P_i^2 \]  

Equation (15)

\[ \min_x f(x) = \sum_{i=1}^{n} P_i \]  

Equation (16)

As already introduced, \( P_i \) represents the active power generation at a node \( i \) and \( n \) represents the total number of AC nodes. Equation (15) represents a more general case in which the generation costs associated with the normal working are minimised with a second-order model. However, (16) represents the minimisation of the overall production. Many other variations of the objective function are also possible.

4.3 Equality constraints

Equations (15) and (16) can be minimised while respecting many constraints with respect to the nodes and the buses. The power balance has to be maintained at every node, that is, the inflow and outflow of power at every node have to be balanced. This has to be respected for AC as well as DC nodes

\[ P_{\text{Gen},i} - P_{\text{Load},i} - P_{\text{Line},i} = 0 \]  

Equation (17)

\[ Q_{\text{Gen},i} - Q_{\text{Load},i} - Q_{\text{Line},i} = 0 \]  

Equation (18)

Values for \( P_{\text{Line},j} \) and \( Q_{\text{Line},j} \) are calculated using the previously defined (5) and (6).

Since the power flow from the AC grid to the DC and vice versa takes place through a converter, the power flow through the converters represents another equality constraint. Converter losses are thus calculated from (8) and (9)

\[ P_{\text{AC},i} = P_{\text{DC},j} - P_{\text{loss}} \]  

Equation (19)

The voltage angle at the slack bus which is chosen from the AC nodes is set to zero that is the reference value

\[ \theta = \theta_{\text{ref}} \]  

Equation (20)

4.4 Inequality constraints

In addition to the equality constraints, in order to fulfil the physical constraints of the system, certain inequality constraints also need to be introduced for the optimisation problem. The line power limit represents the most important inequality constraint.

The thermal stresses of each DC and AC lines are controlled by introducing a limit on the active power flow through a line. The terminals themselves are also protected from overload by using the same technique on any additional AC line

\[ \max(P_{\text{Line}}, P_{\text{DC}}) \leq P_{\text{max,Line}} \]  

Equation (21)

Production of power generators in the system is limited as well which is always kept less than a generators individual limit. The active power is kept within the range of zero to the maximum production of the generating station while as the reactive power ranges from a positive value to a negative value. Owing to the capability curves of the generators, the power limits are coupled to each other

\[ 0 \leq P_i \leq P_{\text{max},i}(Q_i) \]  

Equation (22)

\[ Q_{\text{min},i}(P_i) \leq Q_i \leq Q_{\text{max},i}(P_i) \]  

Equation (23)

For the safety of the equipment, the variance of the voltage levels at every AC and DC node is kept within a minimum and a maximum level

\[ U_{\text{min},i} \leq U_{i} \leq U_{\text{max},i} \]  

Equation (24)

\[ U_{\text{DC, min},j} \leq U_{\text{DC,j}} \leq U_{\text{DC, max},j} \]  

Equation (25)
5 Study case

For the solution of the optimisation problem, a random combined AC-DC grid is used. The topology of the grid is shown in Fig. 4. Detailed analysis has been provided for this case.

5.1 Test grid

A completely new grid for the study purposes was designed as shown in Fig. 4. It consists of ten AC nodes, numbered from 1 to 10. About 13 transmission lines are used to connect the AC nodes. The DC side of the grid consists of five converter stations numbered 11–15. Another five AC lines were added to the line model increasing the total number of AC lines to 18. The DC grid in itself consists of six transmission lines which results in a meshed DC grid topology. Load and production centres are spread throughout the network. The network extensively consists of eight loads and six generators. Two SVCs are also present, one placed at node 10 and the other at node 3. The active power input at these nodes is 0. The state vector for this network has 49 entries according to (14).

To emulate the renewable integration, two special features are also introduced in this network. Node 9 involves an isolated generation system, wind park connected through the HVDC grid to the rest of the system. Another feature is the parallel HVAC and HVDC lines from nodes 2 to 5 and 11 to 13, respectively.

Standard values have been taken for the AC and DC line parameters. The AC line lengths are of the order of 200–600 km, whereas for the DC lines the length is of the order of 360–800 km. AC grid nominal voltage is 380 and 320 kV for the DC grid.

The grid is subjected to three different scenarios or situations power flow calculation having broad limits for all variables is referred to as the ‘base’ case. The DC and AC line limits are set at 0.75 pu and the voltage levels can vary from 0.95 to 1.05 pu. Connections to terminals are set at 1 pu. The reactive power

| Node | Base | Wind | Limit |
|------|------|------|-------|
| \(K\) | \(m\) | \(P_{\text{lim}}\) | \(Q_{\text{lim}}\) | \(P_{\text{lim}}\) | \(Q_{\text{lim}}\) |
| 1 | 2 | 0.42 | 0.42 | 0.46 | 0.18 | 0.42 | 0.16 |
| 1 | 3 | 0.28 | 0.28 | 0.29 | 0.12 | 0.27 | 0.13 |
| 2 | 3 | 0.26 | 0.26 | 0.24 | 0.07 | 0.26 | 0 |
| 2 | 5 | –0.22 | –0.22 | –0.24 | 0.09 | –0.23 | 0.07 |
| 5 | 1 | –0.03 | –0.03 | –0.03 | 0 | –0.02 | 0 |
| 6 | 5 | 0.01 | 0.01 | 0.01 | 0 | 0.01 | 0 |
| 7 | 6 | –0.14 | –0.14 | –0.13 | 0.02 | –0.08 | 0 |
| 8 | 7 | 0.25 | 0.25 | 0.21 | 0.04 | 0.12 | 0.03 |
| 8 | 5 | 0 | 0 | 0 | 0 | –0.01 | 0 |
| 9 | 10 | 0.05 | 0.05 | 0.07 | 0.04 | 0.13 | –0.04 |
| 4 | 6 | 0.01 | 0.01 | 0.01 | 0 | –0.01 | 0 |
| 1 | 4 | 0.01 | 0.01 | 0.01 | 0 | 0.01 | 0 |
| 8 | 10 | 0.47 | 0.47 | 0.45 | 0.04 | 0.39 | 0.02 |
| 2 | 11 | –0.64 | –0.64 | –0.56 | –0.27 | –0.64 | –0.28 |
| 4 | 12 | 0.17 | 0.17 | –0.12 | 0.01 | 0.53 | 0 |
| 5 | 13 | 0.46 | 0.46 | 0.53 | 0.01 | 0.4 | 0 |
| 7 | 14 | –0.67 | –0.67 | –0.74 | –0.1 | –0.93 | –0.05 |
| 9 | 15 | 0.75 | 1 | 0 | 0 | 0.75 | 0.75 |

Table 1 AC grid power flows (pu)

Table 2 DC grid power flows (pu)
output of generators is set at 1 pu and the active power output is set at 2 pu. A maximum of 0.5 pu reactive power can be obtained from the converters. Node 9 consists of a wind park generation having a fixed power output at 0.75 pu.

In situation 2, the wind power generation is increased to 1 pu keeping other quantities similar and is referred to as ‘wind’ scenario.

In the third scenario, ‘limit’ puts a limit of 0.39 pu to the AC line from nodes 8 to 10 and a limit of value 0.35 pu to the DC line from 15 to 12 and decreasing the reactive power output of the SVC component at node 3 to 0.3 pu. Moreover, other conditions are similar to the base case.

5.2 Power flow results

The objective function as in (16) is solved and the optimal power flow solution is obtained. This practice results in the minimisation of the overall network losses. For this example, the MATLAB’s built in solve function ‘fmincon’ as well as the interior point algorithm was used to obtain the optimal power flow. Table 1 lists the active power flow solution and Table 2 consists of the reactive power flow.

Fig. 5 shows the resulting voltage level for the AC nodes for each scenario/the possible voltage range is represented by the dashed lines and the division between the normal nodes (1–10) and the additional nodes (11–15) is shown by the vertical black line. Fig. 6 represents the voltage levels of the meshed DC grid.

The objective function was designed to minimise the losses. Consequently, the voltage levels at the nodes in both the grids reached a maximum possible value but inside the given limits. Fig. 7 shows the active power generations. None of the generators in the network reach their capacity limit, except generator 9 which was the wind park connected to the network. Similar results have been obtained for reactive power generation as in Fig. 8.

In scenario 2, voltage nodes at all the nodes except node 15 are lower than the previous case. Thus, there is a higher power flow from node 15 to the rest of the network which is reasonable considering the high supply.

Large differences are seen in the third scenario. The AC voltage has a value of 1 pu reduced from 1.05 pu because of the presence of the SVC device. A direct consequence of this is the increase in the reactive power production at terminal 11. Moreover, a shift in active power generation occurs as a result of the limiting of the AC line from node 8 to node 10. The power flow is shifted from nodes 8 to 7 from node 10. Active power generation at node 4 increases. The flow over the DC line from converters 12 to 14 also increases. Compared with the base case, the limit on the DC line from nodes 12 to 15 leads to a decrease at node 14 and increase of the DC voltage at node 12 compared with the scenario 1. At this point, there is an increase in the flow into converter 14.

6 Conclusion

An optimal power flow problem described in the form of an objective function and various equality and inequality constraints is
presented in this paper. The methodology employed in this paper can be used to solve many other optimisation problems related to power flow in large networks. There is no limit to the number of AC nodes or converter stations and consequently it can be used to obtain detailed power flows of very large networks such as ‘supergrids’ which are formed by interconnecting asynchronous grids across countries.

The AC side is modelled and solved with the full power flow equations. The VSCs are represented with a model based on connecting line, additional node and two generators. The DC line losses and the VSC losses have also been considered.

7 References

[1] Meah K., Ula S.: ‘Comparative evaluation of HVDC and HVAC transmission systems’, June 2007, pp. 1–5
[2] Zhu J., Booth C.: ‘Future multi-terminal HVDC transmission systems using voltage source converters’. 45th Int. Universities Power Engineering Conf. (UPEC), September 2010, pp. 1–6
[3] Van Hertem D., Ghandhari M., Delimar M.: ‘Technical limitations towards a supergrid – a European prospective’. IEEE Int. Energy Conf. and Exhibition (EnergyCon), December 2010, pp. 302–309
[4] Dike D., Momoh O.: ‘An integrated AC/DC super-grid system – a mechanism to solving the North American power crisis’. 39th Southeastern Symp. on System Theory SSST, March 2007, pp. 204–209
[5] Jiang H., Ekström A.: ‘Multiterminal HVDC systems in urban areas of large cities’, IEEE Trans. Power Deliv., 1998, 13, (4), pp. 1278–1284
[6] Weigt H., Jeske T., Leuthold F., von Hirschhausen C.: ‘Take the long way down – integration of large-scale north sea wind using HVDC transmission’, Energy Policy, 2010, 38, (7), pp. 3164–3173
[7] Bell K., Cirio D., Denis A., et al.: ‘Economic and technical criteria for designing future off-shore HVDC grids’. IEEE PES Innovative Smart Grid Technologies Conf. Europe (ISGT Europe), October 2010, pp. 1–8
[8] Xu L., Williams B., Yao L.: ‘Multi-terminal DC transmission systems for connecting large offshore wind farms’, IEEE PES General Meeting – Conversion and Delivery of Electrical Energy in the 21st Century, July 2008, pp. 1–7
[9] Hans-Peter N., Ängquist L.: ‘Perspectives on power electronics and grid solutions for offshore wind farms’, ELFORSK, Elforsk rapport 1096, November 2010
[10] Pizano-Martinez A., Fuerte-Esquivel C., Ambriz-Perez H., Acha E.: ‘Modeling of VSC-based HVDC systems for a Newton-Raphson OPF algorithm’, IEEE Trans. Power Syst., 2007, 22, (4), pp. 1794–1803
[11] Gengyi L., Ming Z., Jie H., Guangkai L., Haifeng L.: ‘Power flow calculation of power systems incorporating VSC-HVDC’. Int. Conf. on Power System Technology PowerCon, November 2004, vol. 2, pp. 1562–1566
[12] Zhang X.-P.: ‘Multiterminal voltage-sourced converter-based HVDC models for power flow analysis’. IEEE Trans. Power Syst., 2004, 19, (4), pp. 1877–1884
[13] Beerten J., Cole S., Belmans R.: ‘A sequential AC/DC power flow algorithm for networks containing multi-terminal VSC HVDC systems’. IEEE PES General Meeting, July 2010, pp. 1–7
[14] University of Washington: ‘Power systems test case archive’. Available at http://www.ee.washington.edu/research/pstca, May 1993
[15] Jovicic D., Lamont L., Abbott K.: ‘Control system design for VSC transmission’, Electr. Power Syst. Res., 2007, 77, (7), pp. 721–729