Kinetic temperature gradient driven modes in inhomogeneous plasmas

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Abstract

New unstable temperature gradient driven modes in an inhomogeneous plasma are identified. These modes represent transient $\omega \simeq k_{\parallel} v_{th}^{(e,i)}$ sound oscillations in magnetized plasma that are kinetically destabilized via Landau interactions. Electron and ion sound branches are unstable for large values of the Larmor radius parameter $k_{\perp} \rho_{e,i} \gg 1$, respectively. The instability occurs due to a specific plasma response that significantly deviates from Boltzmann distribution in the region $k_{\perp} \rho_{e,i} \gg 1$. Pacs: 52.35 Kt, 52.35 Qz

Small scale instabilities driven by temperature gradients are believed to be responsible for particle and energy transport in a tokamak. The ion temperature gradient driven turbulence
has been identified [1] as a primary cause of ion thermal transport. The electron temperature
gradient driven modes are believed to be largely responsible for the electron transport [2].
Both types of modes, the ion temperature gradient (ITG) and electron temperature gradient
(ETG), have been extensively studied over last decades [3, 4]. In basic slab geometry these
modes are essentially sound waves destabilized by coupling to pressure fluctuations [3, 4].
In toroidal geometry the modes are driven by the unfavourable magnetic curvature rather
than by acoustic oscillations, though the mode may remain essentially slab-like in the region
of the weak/negative shear [19, 20]. In this work we report on a new mechanism of the
destabilization of the temperature gradient driven modes. The mechanism is essentially
related to the wave-particle Landau interaction. However, the most important element is
a newly found regime of plasma response for large values of the Larmor radius parameter,
\( k_\perp^2 \rho_\alpha^2 \gg 1 \), \( \alpha = (e, i) \). In this regime plasma response in essential way deviates from usually
assumed Boltzmann distribution for \( k_\perp^2 \rho_\alpha^2 \gg 1 \). As is shown below modified plasma response
occurs for transient (acoustic type) modes whose eigen-frequency \( \omega \simeq k_\parallel v_{th\alpha} \) does not grow
with increase of \( k_\perp^2 \rho_\alpha^2 \), but rather remains constant. As a result two new branches of the
temperature gradient driven modes occur for \( k_\perp^2 \rho_i^2 \gg 1 \) and \( k_\perp^2 \rho_e^2 \gg 1 \). We call them ion
and electron kinetic temperature gradient (KTG) modes, respectively.

We consider shearless dispersion equation in a local approximation. Within the local
theory the parallel velocity and density perturbations for each species are given by standard
expressions (see e.g. [12, 17])

\[
\frac{\bar{v}_{\alpha||}}{v_{th\alpha}} = -\frac{e_\alpha \hat{\phi}}{T_\alpha} s_\alpha D_\alpha, \tag{1}
\]

\[
\frac{n_\alpha}{n_0} = -\frac{e_\alpha \phi}{T_\alpha} l_\alpha - \frac{e_\alpha \hat{\phi}}{T_\alpha} D_\alpha. \tag{2}
\]

Here

\[
D_\alpha = \left(1 - \frac{\omega_{n\alpha}}{\omega}\right) (1 + s_\alpha Z(s_\alpha)) \Gamma_0(b_\alpha) + \frac{\omega_{T\alpha}}{\omega} s_\alpha \left(\frac{1}{2} Z(s_\alpha) - s_\alpha - s_\alpha^2 Z(s_\alpha)\right) \Gamma_0(b_\alpha)
+ \frac{\omega_{T\alpha}}{\omega} (1 + s_\alpha Z(s_\alpha)) \left( \Gamma_0(b_\alpha) - \Gamma_1(b_\alpha) \right) b_\alpha. \tag{3}
\]
and

\[ l_\alpha = 1 - \left(1 - \frac{\omega_{n\alpha}}{\omega}\right) \Gamma_0(b_\alpha) - \frac{\omega T_\alpha}{\omega} \left(\Gamma_0(b_\alpha) - \Gamma_1(b_\alpha)\right) b_\alpha. \] (4)

Various plasma parameters are defined as follows: \( \omega_{n\alpha} = -k_y c T_\alpha/e_\alpha B_0 L_n \), \( \omega T_\alpha = -k_y c T_\alpha/e_\alpha B_0 L_T \), \( L_n^{-1} = -n_0^{-1} \partial n_0/\partial x \), \( L_T = -T_\alpha^{-1} \partial T_\alpha/\partial x \), \( s_\alpha = \omega/k_\parallel v_{\text{th} \alpha} \), \( b_\alpha = k_\perp^2 \rho_\alpha^2/2 \), \( c^2 = 2 T_\alpha/m_\alpha \), \( \rho_\alpha = v_{\text{th} \alpha} m_\alpha c/(e_\alpha B_0) \); \( \Gamma_0, \Gamma_1(b) = I_0, I_1 \exp(-b) \), and \( Z(s) \) is the standard plasma dispersion function. We have introduced auxiliary potential \( \hat{\phi} = \phi + \omega/(k_\parallel c) A \), where \( \phi \) is the electrostatic potential and \( A \) is the magnetic vector potential.

The vector potential \( A \) can be found from the Ampere's law

\[ J_\parallel = en(v_\parallel - v_e) = -\frac{c}{4\pi} \nabla_\perp^2 A. \] (5)

By using (4) and (5) one obtains

\[ A = -\frac{2s_e^2 (D_e \tau + D_i)}{k_\perp^2 \delta^2 - 2s_e^2 (D_e \tau + D_i)} \frac{k_\parallel c \phi}{\omega}, \] (6)

and

\[ \hat{\phi} = \phi - \frac{k_\perp^2 c^2/\omega_{pe}^2}{k_\perp^2 \delta^2 - 2s_e^2 (D_e \tau + D_i)}, \] (7)

for the parallel potential \( \hat{\phi} \). Here \( \tau = T_e/T_i \), and \( \delta^2 = c^2/\omega_{pe}^2 \).

By using the Poisson equation,

\[ -\nabla_\perp^2 \phi = 4\pi e(n_i - n_e), \] (8)

and expressions (2) and (7) we obtain general local dispersion relation

\[ k_\perp^2 \delta^2 \left(\tau_i + \tau_e + D_i \tau + D_e\right) - 2s_e^2 (D_i \tau + D_e) (\tau_i + \tau_e) \]

\[ = -k_\perp^2 \lambda_D^2 \left(k_\perp^2 \delta^2 - 2s_e^2 (D_i \tau + D_e)\right), \] (9)

where \( \lambda_D^2 \) is the Debye length, \( \lambda_D^2 = T_e/(4\pi n_0 e^2) \). This general dispersion equation describes both ITG and ETG modes as well as new kinetic temperature gradient driven modes.

To illustrate existence of a new unstable mode we plot a solution of the dispersion equation (9) as a function of the normalized ion Larmor radius parameter \( \rho_i/L_n \). The following
parameters are fixed $k_y \rho_i = \sqrt{2} \times 0.3$, $k_x \rho_i = \sqrt{2} \times 0.1$, $\beta = 2 \times 10^{-4}$, $\rho_i/L_{T_i} = \sqrt{2} \times 0.1$, $\rho_i/L_{T_e} = \sqrt{2} \times 0.01$, $k_\parallel \rho_i = \sqrt{2} \times 0.002$, $\tau = 1$, We have chosen the above plasma parameters to be exactly the same as in Ref. 17. The plasma pressure parameter $\beta$ is defined as $\beta = 8 \pi n_0 T_i / B_0^2$. [Note that our definition of $v_{th \alpha}$ and $\beta$ differ from those in Ref. 17 by a factor of 2.] Figures 1a and 1b show the real $\omega_r$ and imaginary $\omega_i$ parts of the eigen frequency, respectively. It is already evident from Figs 1a and 1b that there are exist two distinct eigenmodes: in Fig. 1b the left curve is the unstable ITG branch and the right curve is the unstable drift mode branch \[17\] (in terminology of Ref. 17). Our results in Figs 1a and 1b are in complete agreement with Ref. 17.

To clarify the nature of the unstable branch we solve the dispersion equation as a function of the $k_y \rho_i$ parameter for fixed $\beta = 2 \times 10^{-4}$, $\rho_i/L_n = \sqrt{2} \times 10^{-2}$, $\rho_i/L_{T_i} = \sqrt{2} \times 10^{-1}$, $\rho_i/L_{T_e} = \sqrt{2} \times 10^{-1}$, $k_x \rho_i = \sqrt{2} \times 10^{-1}$, $\tau = 1$. We also take $\lambda_D = 0$ for this case. The mode growth rate and mode frequency normalized to $k_\parallel v_{th e}$ are shown in Figs. 2a and 2b, respectively. Two new unstable branches exist in the regions $k_y \rho_i \geq 1$ and $k_y \rho_e \geq 1$. The standard ITG mode has a peak growth rate around $k_y \rho_i \simeq 1$, and the standard ETG mode has a growth rate peaked at $k_y \rho_i \simeq 40$ (corresponding to $k_y \rho_e \simeq 1$). A new ion mode (ion KTG) has a maximum growth rate around $k_y \rho_i \simeq 5$, and a new electron mode (electron KTG) saturates to a constant growth rate quickly for large $k_y \rho_i > 200$ ($k_y \rho_e > 5$). It is essential to note that new modes exist in the regions of large Larmor radius parameters (respectively, $k_y \rho_i$ for the ion KTG, and $k_y \rho_e$ for the electron ETG). As we show below, kinetic temperature gradient modes occur due to a peculiar behaviour of the plasma response in the region of large $k_y \rho_\alpha$. Let us consider this response in more details.

A standard notion is that for large values of the Larmor radius parameter, $k_y \rho_\alpha > 1$, the density response of the respective plasma component is Boltzmann due to decaying asymptotics of $\Gamma_{0,1}(b_\alpha) \sim 1/\sqrt{b_\alpha}$ for large $b_\alpha$, so that we have from (3-4) $D_\alpha \ll 1, l_\alpha \simeq 1$. This, in fact, implicitly assumes that the ratio of $\omega_{n\alpha}/\omega$ is finite for large $k_y \rho_\alpha$. In turn, this requires that the mode eigen-frequency increases with $k_y \rho_\alpha$ (linearly or faster). The
latter is definitely true for drift wave type modes, where $\omega \sim \omega_{\alpha}$. However, temperature gradient driven modes, are basically sound waves whose frequency is of the order of the transient (or sound) frequency, $k_{\parallel}v_{\text{th}}$. If the parameter $k_{\parallel}v_{\text{th}}$ remains constant (so $\omega$ is approximately constant), the ratio $\omega_{\alpha}/\omega$ will be increasing with $k_{\parallel}v_{\text{th}}$ that may compensate for the decaying $1/\sqrt{b_{\alpha}}$ factor from $\Gamma_{0,1}(b_{\alpha})$. Then, for large $k_{\parallel}v_{\text{th}} \gg 1$ we may write in the leading order

$$\frac{\omega_{\alpha}}{\omega} \Gamma_0(b_{\alpha}) = \frac{1}{2\sqrt{\pi}} \frac{e_{\alpha}}{v_{\text{th}}} v_{\text{th}},$$

and

$$\frac{\omega_{\alpha}}{\omega} \left( \Gamma_0(b_{\alpha}) - \Gamma_1(b_{\alpha}) \right) b_{\alpha} = \frac{1}{4\sqrt{\pi}} \frac{e_{\alpha}}{v_{\text{th}}} v_{\text{th}}.$$  

By using these asymptotics for the electron KTG mode in the region $k_{\parallel}v_{\text{th}} \gg 1$ and $k_{\parallel}v_{\text{th}} \gg 1$ we obtain the following response functions

$$l_i = 1 - \frac{1}{2\sqrt{\pi}} \frac{v_{ti}}{\omega L_n} + \frac{1}{4\sqrt{\pi}} \frac{v_{ti}}{\omega L_{Ti}},$$

$$l_e = 1 + \frac{1}{2\sqrt{\pi}} \frac{v_{te}}{\omega L_n} - \frac{1}{4\sqrt{\pi}} \frac{v_{te}}{\omega L_{Te}},$$

$$D_i = -\frac{1}{4\sqrt{\pi}} \frac{v_{ti}}{\omega L_n} \frac{1}{s_i^2} + \frac{1}{4\sqrt{\pi}} \frac{v_{ti}}{\omega L_{Ti}} \frac{1}{s_i^2},$$

$$D_e = -\frac{1}{2\sqrt{\pi}} \frac{v_{te}}{\omega L_n} \left( 1 + s_e Z(s_e) \right) + \frac{1}{2\sqrt{\pi}} \frac{v_{te}}{\omega L_{Te}} \left( \frac{1}{2} Z(s_e) - s_e - s_e^2 Z(s_e) \right)$$

$$+ \frac{1}{4\sqrt{\pi}} \frac{v_{te}}{\omega L_{Te}} \left( 1 + s_e Z(s_e) \right).$$

The electron KTG mode is a destabilized electron sound wave with $\omega \simeq k_{\parallel}v_{\text{th}}$ so we retain a full plasma dispersion function in $D_e$ to account for the electron Landau interactions. In this region $\omega/k_{\parallel}v_{\text{th}} \gg 1$ so we can simplify $Z(s_i)$ functions. Solution of dispersion equation (9) with simplified response functions (12-15) and $\lambda_D = 0$ is shown in Fig. 2 by squares. We note here, that in fact plasma response is not given exactly by expressions (1-2) but contains
additional terms of the order of $\rho_i/L_n$ [18]. These additional terms do not affect asymptotic expressions (12-15) as long as $\rho_i/L_n < 1$.

A similar expansion can be made in the region $k_y\rho_i \gg 1$. Then the ion response for the ion KTG branch becomes

$$l_i = 1 - \frac{1}{2\sqrt{\pi}} \frac{v_{ti}}{\omega L_n} + \frac{1}{4\sqrt{\pi}} \frac{v_{ti}}{\omega L_{Ti}},$$

$$D_i = \frac{1}{2\sqrt{\pi}} \frac{v_{ti}}{\omega L_n} (1 + s_i Z(s_i)) - \frac{1}{4\sqrt{\pi}} \frac{v_{ti}}{\omega L_{Ti}} s_i \left( \frac{1}{2} Z(s_i) - s_i - s_i^2 Z(s_i) \right)$$

$$- \frac{1}{2\sqrt{\pi}} \frac{v_{ti}}{\omega L_{Ti}} (1 + s_i Z(s_i)).$$

The electron response functions can be simplified in this region by explicitly using $k_\perp \rho_e < 1$ and $\omega/k_\parallel v_{te} < 1$. Then one obtains

$$l_e = 1 - \left( 1 - \frac{\omega_{ne}}{\omega} \right),$$

$$D_e = \left( 1 - \frac{\omega_{ne}}{\omega} \right) \left( 1 - 2s_e^2 + is_e \sqrt{\pi} \right) \left( 1 + \frac{\omega_{Te}}{\omega} s_e \left( -2s_e + i \frac{\sqrt{\pi}}{2} \right) \right),$$

$$l_e = 1 - \left( 1 - \frac{\omega_{ne}}{\omega} \right) \left( 1 - b_e \right) - \frac{\omega_{Te}}{\omega} b_e,$$

$$D_e = \left( 1 - \frac{\omega_{ne}}{\omega} \right) \left( 1 - 2s_e^2 + is_e \sqrt{\pi} \right) \left( 1 - b_e \right) + \frac{\omega_{Te}}{\omega} s_e \left( -2s_e + i \frac{\sqrt{\pi}}{2} \right) (1 - b_e)$$

$$+ \frac{\omega_{Te}}{\omega} \left( 1 - 2s_e^2 + is_e \sqrt{\pi} \right) b_e.$$  

Mode growth rate and real frequency obtained with (18-21) and (9) with $\lambda_D = 0$ are shown in Fig. 2 by circles.

Recently, it has been emphasised that the ETG modes can be significantly modified by a finite value of the Debye length parameter [19]. Electron KTG mode is strongly stabilized in a high temperature plasma by the finite Debye length as shown in Fig. 3. Note that effect of the Debye length screening may be different for the negative shear plasma [19].

Though the modified ion response for large ion Larmor radius is critical for the ion KTG mode, the mode itself is destabilized by the electron temperature gradient as demonstrated
in Fig 4. Ion KTG mode exists for large gradients of the electron temperature (dashed line, $\eta_e = 5$). The mode growth rate is significantly reduced for smaller values of $\eta_e$ (solid line, $\eta_e = 0.5$). Thus we may conclude that ion KTG mode is in fact a hybrid mode, in which both ion and electron response are essential. Electromagnetic effects are expected to be important for this mode existing in the intermediate region between standard ITG and ETG modes. Though for larger $\eta_e$ we observe a standard finite $\beta$ suppression (curve labeled by diamonds in Fig. 5), for low $\eta_e = 0.5$, finite plasma pressure effectively leads to the extension of the ITG instability into the shorter wavelengths (curve labeled by squares in Fig. 5). It is interesting to note that for $\beta = 0.1$ taken for this case, the skin depth size $c/\omega_{pe} \simeq 0.1 \rho_i$, which makes $k_y c/\omega_{pe} \leq 1$ in the region of the unstable ion KTG.

In summary, we have identified two new branches of the temperature gradient driven modes. These modes are closely related to acoustic type modes that are destabilized by the temperature gradient effects. A modified plasma response in the region of $k_y \rho_\alpha \gg 1$ is essential for new instability. There is certain similarity of the KTG modes to the universal instability of drift waves that is driven by density gradient and wave-particle interaction [21]. The KTG modes considered in this work will be affected by a finite value of the magnetic shear but noting a special role of the temperature gradients and the modified plasma response for $k_y \rho_\alpha \gg 1$ it remains unclear whether these modes can be stabilized by the shear similarly to the universal instability [22,23]. One can expect that KTG modes can persist in the region of weak and/or negative magnetic shear where the mode structure is essentially that of a shearless slab [20]. Toroidal effects will further modify the KTG modes but are not expected to cause their stabilization [24]. Generally, investigation of a general case requires nonlocal integral analysis [13,15,19] that is left for future work.

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FIGURES

FIG.1. Normalized (to the ion cyclotron frequency) frequency (a) and growth rate (b) of the ITG and unstable drift wave modes versus the normalized ion Larmor radius \( \rho_i/L \). The solid line corresponds to the ITG branch, while the dashed line is that of the destabilized drift wave branch. Parameters are the same as those in Ref. \( \beta = 2 \times 10^{-4}, \rho_i/L_{Ti} = \sqrt{2} \times 10^{-1}, \rho_i/L_{Te} = \sqrt{2} \times 10^{-2}, k_x \rho_i = \sqrt{2} \times 10^{-1}, k_y \rho_i = 3\sqrt{2} \times 10^{-1}, k_{\parallel} \rho_i = 2\sqrt{2} \times 10^{-3} \).

FIG.2. Normalized wave frequency (a) and growth rate (b) for the ITG, ETG modes and kinetic modes. Plasma parameters: \( \beta = 2 \times 10^{-4}, \rho_i/L_n = \sqrt{2} \times 10^{-2}, \rho_i/L_{Ti} = \sqrt{2} \times 10^{-1}, \rho_i/L_{Te} = \sqrt{2} \times 10^{-2}, k_x \rho_i = \sqrt{2} \times 10^{-1}, k_{\parallel} \rho_i = 2\sqrt{2} \times 10^{-3}, \tau = 1 \). Solid line – standard ITG and ion KTG; dashed line – standard ETG and electron KTG.

FIG.3. Stabilization of the Kinetic Electron Temperature Gradient mode by the Debye screening effect: solid line – \( \lambda_D/\rho_e = 0 \), dotted-dashed line – \( \lambda_D/\rho_e = 0.70 \), dotted line – \( \lambda_D/\rho_e = 2.21 \). Plasma parameters: \( \beta = 2 \times 10^{-4}, \rho_i/L_n = \sqrt{2} \times 10^{-2}, \rho_i/L_{Ti} = \sqrt{2} \times 10^{-1}, \rho_i/L_{Te} = \sqrt{2} \times 10^{-2}, k_x \rho_i = \sqrt{2} \times 10^{-1}, k_{\parallel} \rho_i = 2\sqrt{2} \times 10^{-3}, \tau = 1 \).

FIG.4. Effect of the electron temperature gradient on KTG modes. Solid line – \( \rho_i/L_{Te} = \sqrt{2} \times 10^{-2} \) (\( \eta_e = 0.5 \)), dashed line \( \rho_i/L_{Te} = \sqrt{2} \times 10^{-1} \) (\( \eta_e = 5 \)). Plasma parameters: \( \beta = 2 \times 10^{-4}, \rho_i/L_n = \sqrt{2} \times 10^{-2}, \rho_i/L_{Ti} = \sqrt{2} \times 10^{-1}, k_x \rho_i = \sqrt{2} \times 10^{-1}, k_{\parallel} \rho_i = 2\sqrt{2} \times 10^{-3}, \tau = 1 \).

FIG.5. Effect of a finite plasma pressure on the KTG modes. Triangles – \( \rho_i/L_{Te} = \sqrt{2} \times 10^{-1}, \beta = 2 \times 10^{-4} \); diamonds squares – \( \rho_i/L_{Te} = \sqrt{2} \times 10^{-1}, \beta = 0.1 \); circles \( \rho_i/L_{Te} = \sqrt{2} \times 10^{-2}, \beta = 2 \times 10^{-4}, \rho_i/L_{Te} = \sqrt{2} \times 10^{-2}, \beta = 0.1 \). Other plasma parameters: – \( \rho_i/L_n = \sqrt{2} \times 10^{-2}, \rho_i/L_{Ti} = \sqrt{2} \times 10^{-1}, k_x \rho_i = \sqrt{2} \times 10^{-1}, k_{\parallel} \rho_i = 2\sqrt{2} \times 10^{-3}, \tau = 1 \).
\frac{\omega}{\Omega_i} \text{ vs } \frac{\rho_i}{L_n}
\[ \frac{\omega_i}{\Omega_i} \] vs. \[ \frac{\rho_i}{L_n} \]
The figure shows a plot of $\omega/(k_y v_{the})$ versus $k_y \rho_i$. The graph includes several data points and a smooth curve. The x-axis represents $k_y \rho_i$ on a logarithmic scale, ranging from $10^{-2}$ to $10^3$. The y-axis represents $\omega/(k_y v_{the})$ also on a logarithmic scale, ranging from $-8 \times 10^{-2}$ to $4$. The data points are scattered across the graph, and the smooth curve indicates the trend of the function.
\[ \frac{\omega_i}{(k_{||} v_{th})} \]
\[ \frac{\omega_i}{(k_y \rho_i)} \]
