Quark-Model Baryon-Baryon Interaction Applied to Neutron-Deuteron Scattering. III

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The low-energy breakup differential cross sections of the neutron-deuteron (nd) scattering are studied employing the energy-independent version of the quark-model baryon-baryon interaction fss2. This interaction reproduces almost all the breakup differential cross sections predicted by the meson-exchange potentials for neutron incident energies \( E_n \leq 65 \) MeV. The space star anomaly of 13 MeV nd scattering is not improved even in our model. Some overestimation of the breakup differential cross sections at \( E_n = 22.7 - 65 \) MeV implies that systematic studies of various breakup configurations are necessary both experimentally and theoretically.

Subject Index: 200, 205

§1. Introduction

The three-nucleon (3N) system is appropriate to study the underlying nucleon-nucleon (NN) interaction, since many techniques for solving the system exactly are well developed nowadays. Ample experimental data have already been accumulated especially for the low-energy neutron-deuteron (nd) and proton-deuteron (pd) scattering, and extensive studies to detect the 3N force have been carried out on the basis of modern meson-exchange potentials, and more recently, of the chiral effective field theory. Most research studies in such a direction are concerned with energies higher than 100 MeV for the nucleon incident energy \( E_n \) in a laboratory (lab) system, since the 3N force effect is expected to be revealed prominently at high energies. On the other hand, the discrepancies in various 3N observables between the theory and the experiment in the \( E_n \leq 65 \) MeV region are not resolved even with the recent accurate treatment of the Coulomb force. This is particularly true for nucleon-induced deuteron breakup processes. It is therefore worthwhile to reexamine the NN interaction itself if the present-day realistic force is the most appropriate one to start with.

In previous studies, referred to as I and II hereafter, we have applied the quark-model (QM) baryon-baryon interaction fss2 to the problems of neutron-deuteron (nd) elastic scattering. This interaction model fss2 describes available NN data in a comparable accuracy with the modern meson-exchange potentials. By eliminating the inherent energy dependence of the resonating-group kernel, fss2 was found to yield a nearly correct triton binding energy, S-wave nd scattering lengths and low-energy eigenphase shifts without introducing the 3N force. The predicted elastic differential cross sections have sufficiently large cross section...
minima at $E_n = 35 - 65$ MeV and $\theta_{cm} = 130^\circ - 135^\circ$, in contrast to the predictions made by standard meson-exchange potentials.\textsuperscript{10} The so-called $A_y$ puzzle at low energies $E_n \leq 25$ MeV is largely improved in this model.\textsuperscript{11} In this research, we continue these studies by examining $3N$ breakup processes with various decaying kinematics in the energy range of $E_n \leq 65$ MeV. Our main motivation is to determine if the quite different off-shell properties, originating from the strong nonlocality of the QM baryon-baryon interaction, affect the $3N$ breakup differential cross sections. In contrast to the elastic scattering amplitude, the breakup amplitude covers a wide momentum region of the three-body phase space. It is found unfortunately that fss2 gives predictions similar to the meson-exchange potentials and does not improve much the discrepancies between the theoretical and experimental results.

This paper is organized as follows. In §2.1, the formulation of the breakup differential cross sections is given in terms of the direct breakup amplitude. Various kinematical configurations for the three-body decay are introduced in §2.2. A minimal description of the $3N$ breakup kinematics is given in Appendix A. The isospin factors for the breakup amplitudes are derived in Appendix B. The comparison with the experimental data is presented in §3 for energies $E_n = 8, 10, 10.3, 13, 16, 19, 22.7$ and 65 MeV. The difference of our results from the predictions made by meson-exchange potentials is also discussed in detail. We close this paper with a summary of this series of investigations in §4.

§2. Formulation

2.1. Breakup differential cross sections

Following the notation of Refs. 1), I and II, the three-body breakup amplitude is given by

$$ U_0 |\phi) = (1 + P) T |\phi) = (1 + P) t \hat{Q} |\phi) . $$

To derive the breakup differential cross sections, we start from Fermi’s golden rule

$$ dN = \frac{2\pi}{\hbar} |\langle pq | U_0 |\phi) |^2 \int_0^\infty p^2 dp \delta(E - E_{pq}) q^2 dq d\hat{p} d\hat{q} , $$

and divide it by the incident flux $j = (3\hbar q_0/2M)/(2\pi)^3$. Here, $E_{pq} = (h^2/M)(p^2 + 3q^2/4)$, $M$ is the nucleon mass, and $q_0$ is the incident momentum related to the energy $E = (3h^2/4M)q_0^2 + \varepsilon_d$ in the center-of-mass (cm) system. We obtain

$$ \frac{d^5 \sigma}{d\hat{p} d\hat{q} dq dp} = \frac{1}{j} \frac{dN}{d\hat{p} d\hat{q} dq dp} = (2\pi)^4 \frac{2M}{3h^2} \frac{1}{q_0} \int_0^\infty p^2 dp \delta(E - E_{pq}) q^2 |\langle pq | U_0 |\phi) |^2 $$

$$ = (2\pi)^4 \left( \frac{2M}{3h^2} \right)^2 \frac{3p_0q_0^2}{4q_0} \sum_{\Gamma} |\langle pq \Gamma | U_0 |\phi) |^2 , $$

where Eq.I (2.88)\textsuperscript{*} is used to perform the $p$-integral. In Eq. (2.3), $\Gamma = \Gamma_\sigma \Gamma_\tau$ is the spin-isospin quantum numbers in the $LS$-coupling scheme and the subscript 0

\textsuperscript{*} In the following, we cite the equations in I (or II), while adding I (or II) in front of the equation numbers.
in the matrix element implies the on-shell condition $|p| = p_0 = \sqrt{(3/4)(qM^2 - q^2)}$ with $q_M = \sqrt{q_0^2 - \kappa_d^2}$. Here, $\kappa_d$ is related to the deuteron binding energy $|\varepsilon_d|$ through $|\varepsilon_d| = (3h^2/4M)\kappa_d^2$. In this study, we use the notations $\Gamma_\sigma = (s_2^2)SS_z$ and $\Gamma_\tau = (t_2^2)T_z$ to specify the quantum numbers in the $LS$-coupling scheme, i.e.,

$$\langle \gamma | \hat{p}, \hat{q}; 123 \rangle = \sum_\gamma |p, q, \gamma \rangle \langle \gamma | \hat{p}, \hat{q}; 123 \rangle,$$

$$\langle \hat{p}, \hat{q}; 123 | \gamma \rangle = [Y_{(\lambda\ell)}(\hat{p}, \hat{q}) \xi_{\Gamma_\sigma}(12, 3)]_{JJ_z} \eta_{\Gamma_\tau}(12, 3), \quad (2.4)$$

with $\gamma = [(\lambda\ell)L\Gamma \sigma]_J J J_z$; $\Gamma_\sigma$, and $\xi_{\Gamma_\sigma}$ and $\eta_{\Gamma_\tau}$ being the three-particle spin and isospin wave functions, respectively. In Eq. (2.4), $Y_{(\lambda\ell)}(\hat{p}, \hat{q}) = [Y_{\lambda}(\hat{p})Y_{\ell}(\hat{q})]_{LM}$ are the angular functions. In the initial state, we use channel-spin representation as in the elastic scattering. We take the sum of Eq. (2.3) over all the spin and isospin quantum numbers and divide it by the initial spin multiplicity 6. The selection of the detected particles in the final state is controlled by the isospin projection operator $O_\tau$, the explicit form of which will be specified later. The breakup differential cross sections of the $nd$ scattering are therefore calculated from

$$\frac{d^5 \sigma}{d \hat{p} \, d \hat{q} \, dq \, dp} = (2\pi)^4 \left(\frac{2M}{3h^2}\right)^2 \frac{3}{4} \frac{p_0 q^2}{q_0^2} \frac{1}{6} \sum_\Gamma \sum_{S_c S_{cz}} |\langle pq \Gamma | O_\tau (1 + P) | \phi_{0}; S_c S_{cz} \rangle |^2.$$

(2.5)

Let us first consider the spin-isospin sum $I = \sum_\Gamma |\langle pq \Gamma | (1 + P) \phi \rangle|^2$ by neglecting the initial spin quantum numbers for the time being. The effect of the permutation $P_{(123)}^{\alpha}$ in $(1 + P) = \sum_{\alpha=1}^{3} P_{(123)}^{\alpha}$ is defined as

$$\langle pq | P_{(123)}^{\alpha} f \rangle \equiv P_{(123)}^{\alpha} f(p, q) = f(p_\alpha, q_\alpha), \quad \text{(2.6)}$$

if the function $f(p, q)$ contains no spin-isospin degree of freedom. In fact, we should use

$$\langle pq \Gamma | P_{(123)}^{\alpha} f \rangle = \langle \Gamma | P_{(123)}^{(\sigma \tau)}^{\alpha} f(p_\alpha, q_\alpha) \rangle \quad \text{(2.7)}$$

where $P_{(123)}^{(\sigma \tau)}$ is the permutation operator in the spin-isospin space and the bra-ket notation is used for the spin-isospin degree of freedom. Using these notations and the completeness relationship in the spin-isospin space, $\sum_\Gamma |\langle \Gamma | \rangle |^2 = 1$, we find

$$I = \sum_{\alpha, \beta=1}^{3} \langle f(p_\alpha, q_\alpha) | P_{(123)}^{(\sigma \tau)}^{3-\alpha} P_{(123)}^{(\sigma \tau) \beta} f(p_\beta, q_\beta) \rangle \quad \text{(2.8)}$$

Here, we separate the $\alpha$ and $\beta$ sum into the diagonal part ($\alpha = \beta$) and off-diagonal part ($\alpha \neq \beta$). In the off-diagonal part, we specify $\alpha$ and $\beta$ by the cyclic permutations of (123) ($(\alpha \beta \gamma) = (123)$-cyclic). For these terms, the $\alpha-\beta$ and $\beta-\alpha$ terms are complex conjugate to each other. Thus, we obtain

$$I = \sum_{\alpha=1}^{3} \langle f(p_\alpha, q_\alpha) | f(p_\alpha, q_\alpha) \rangle + 2 \sum_{(\alpha \beta \gamma)} \text{Re} \langle f(p_\alpha, q_\alpha) | P_{(123)}^{(\sigma \tau)} f(p_\beta, q_\beta) \rangle$$

(2.9)
where $\sum'$ implies the sum over the three cyclic permutations of $(\alpha\beta\gamma) = (123)$.

The extension to $I = \sum_{\Gamma} |\langle pq\Gamma|O_{\tau}(1 + P)f\rangle|^2$, incorporating the isospin projection operator $O_{\tau}$, is rather easy. Here, $O_{\tau}$ is specified as

$$
O^{\alpha\beta} = \left(P_{(123)}^{\tau}\right)^{3-\alpha}O_{\tau}\left(P_{(123)}^{\tau}\right)^{\beta} (= O_{\tau}^{\beta\alpha\dagger}),
$$

we obtain

$$I = \sum_{\alpha=1}^{3} \sum_{\Gamma} f^*_f(p_\alpha, q_\alpha) \langle \tilde{\Gamma}|O_{\tau}^{\alpha\alpha}|\Gamma\rangle f_f(p_\alpha, q_\alpha)$$

$$+ 2 \sum_{(\alpha\beta\gamma)} \sum_{\tilde{\Gamma},\Gamma} \text{Re} \left\{ f^*_f(p_\alpha, q_\alpha) \langle \tilde{\Gamma}|P_{(123)}^{\tau}O_{\tau}^{\alpha\beta}|\Gamma\rangle f_f(p_\beta, q_\beta) \right\}.
$$

The spin-isospin factors in Eq. (2.12) are calculated by separating the spin-isospin state $|\Gamma\rangle$ into the spin and isospin parts, $|\Gamma\rangle = |\Gamma_\sigma\rangle|\Gamma_{\tau}\rangle$. We find

$$
\langle \tilde{\Gamma}|O_{\tau}^{\alpha\alpha}|\Gamma\rangle = \delta_{\tilde{\Gamma}_{\tau}\tilde{S}_{\tau}} \delta_{\tilde{S}_{\tau}s} \langle \tilde{\Gamma}_{\tau}|O_{\tau}^{\alpha\alpha}|\Gamma_{\tau}\rangle,
$$

$$
\langle \tilde{\Gamma}|P_{(123)}^{\tau}O_{\tau}^{\alpha\beta}|\Gamma\rangle = \delta_{\tilde{\Gamma}_{\tau}\tilde{S}_{\tau}} (-1)^{1+s} X_{\tilde{S}_{\tau}s}^{\tilde{S}_{\tau}S} \langle \tilde{\Gamma}_{\tau}|O_{\tau}^{\alpha\beta}|\Gamma_{\tau}\rangle,
$$

where $\tilde{\Gamma}_{\tau} = (\tilde{S}_{\tau})^4 \tilde{S}_{\tau} \tilde{S}_{\tau}$ and $\tilde{\Gamma}_{\tau} = (\tilde{S}_{\tau})^2 \tilde{T}_{\tau}$, and Eq. (B.1) is used for the spin part. We also extend the definition in Eq. (B.1) for the spin part to the isospin part as in Eq. (B.2). Using the definition of $X_{l\ell,t}^{\tau(\alpha\beta)}$ in Eq. (B.2), we can write the matrix elements in Eq. (2.13) as

$$
\langle \tilde{\Gamma}|O_{\tau}^{\alpha\alpha}|\Gamma\rangle = \delta_{\tilde{\Gamma}_{\tau}\tilde{S}_{\tau}} \delta_{\tilde{S}_{\tau}s} X_{l\ell,t}^{\tau(\alpha\alpha)},
$$

$$
\langle \tilde{\Gamma}|P_{(123)}^{\tau}O_{\tau}^{\alpha\beta}|\Gamma\rangle = \delta_{\tilde{\Gamma}_{\tau}\tilde{S}_{\tau}} (-1)^{1+s} X_{\tilde{S}_{\tau}s}^{\tilde{S}_{\tau}S} (-1)^{1+t} X_{l\ell,t}^{\tau(\alpha\beta)},
$$

for $(\alpha\beta\gamma)$ = a cyclic permutation of $(123)$. Thus, we find

$$I = \sum_{\alpha=1}^{3} \sum_{\Gamma} \delta_{\tilde{\Gamma}_{\tau}\tilde{S}_{\tau}} \delta_{\tilde{S}_{\tau}s} X_{l\ell,t}^{\tau(\alpha\alpha)} f^*_f(p_\alpha, q_\alpha) f_f(p_\alpha, q_\alpha)$$

$$+ \sum_{(\alpha\beta\gamma)} \sum_{\tilde{\Gamma},\Gamma} \delta_{\tilde{\Gamma}_{\tau}\tilde{S}_{\tau}} (-2) X_{\tilde{S}_{\tau}s}^{\tilde{S}_{\tau}S} X_{l\ell,t}^{\tau(\alpha\beta)} \text{Re} \left\{ f^*_f(p_\alpha, q_\alpha) f_f(-p_\beta, q_\beta) \right\},
$$
where the generalized Pauli principle \((-1)^{s+t+\lambda} = -1\) is used for the two-nucleon part of \(f_{\Gamma}(p_{\beta}, q_{\beta})\). The isospin factors \(X_{t,t}^{\tau(\alpha\beta)}\) are explicitly given in Appendix B.

We assign the direct breakup amplitude to \(f_{\Gamma}(p, q)\) in Eq. (2.15) through

\[
\begin{align*}
\hat{f}_{\Gamma,S_{c}S_{c\ell}}(p, q) &= -(2\pi)^2 \left( \frac{2M}{3\hbar^2} \right) \langle pq\Gamma|T|\phi_{q_0}; S_{c}S_{c\ell}\rangle_0 ,
\end{align*}
\]

with \(T = t\hat{Q}\). The partial wave decomposition is given by

\[
\begin{align*}
\hat{f}_{\Gamma,S_{c}S_{c\ell}}(p, q) &= (4\pi)^\frac{t}{2} \sum_{\gamma, \ell', J_{\ell'}} f^{(\text{db})J}_{\gamma, (\ell'S_{c})}(q) \sum_{\ell M SS_{z}} \langle \ell M SS_{z}| J_{J_{z}} \rangle Y(\gamma)L_{J_{z}}\hat{p}(\hat{Q}) \\
&\quad \times \sum_{m'} \langle \ell' m'S_{c}S_{c\ell}| J_{J_{z}} \rangle Y_{\ell' m'}^*(\hat{q}_0) ,
\end{align*}
\]

where the prime on the sum implies that we take all the orbital angular momentum sum for the \(LS\) coupling scheme of \(\gamma\), i.e., the sum over only \((\lambda\ell)L\) with \(\gamma = [(\lambda\ell)LI_{\gamma}] J_{J_{z}} \Gamma_{\gamma}\). (Note the extra \((4\pi)\) factor for the scattering amplitude.) It is convenient to define

\[
\hat{Q}_{i\mu\gamma}^{(\ell'S_{\ell})} = \sum_{(\ell'S_{\ell})} \hat{Q}_{i\mu\gamma}^{(\ell'S_{\ell})} f^{J}_{(\ell'S_{\ell})}(\ell S_{c})
\]

by the solutions \(\hat{Q}_{i\mu\gamma} = p_i^2 \omega_i \mu_i \sqrt{\omega_i} (p_i, q_i, \gamma) \hat{Q} |\psi\rangle\) of the basic AGS equation in Eq. I (2-59) and the elastic scattering amplitude \(f^{J}_{(\ell'S_{\ell})}(\ell S_{c})\) in Eq. I (2-92). The partial-wave amplitude for the direct breakup, \(f_{\gamma, (\ell'S_{c})}(q)\), is expressed as

\[
\begin{align*}
f^{(\text{db})J}_{\gamma, (\ell'S_{c})}(q) &= \sum_{i} \langle p_0|t_{\gamma}(h^2 p_0^2/M)|p_i\rangle \sum_{\mu} S_{\mu}(q) \frac{1}{q_0 \sqrt{\omega}} \hat{Q}_{i\mu\gamma}^{(\ell'S_{\ell})},
\end{align*}
\]

if we use the the spline interpolation for a particular \(q\). For practical calculations, it is convenient to adopt a particular coordinate system with \(\hat{q}_0 = e_z\) in Eq. (2.17). Then the basic direct breakup amplitude in the spin-isospin space is calculated from

\[
\begin{align*}
\hat{f}_{\Gamma,S_{c}S_{c\ell}}(p, q) &= \sqrt{4\pi} \sum_{\gamma, \ell', J_{\ell'}} f^{(\text{db})J}_{\gamma, (\ell'S_{c})}(q) \langle L (S_{c\ell} - S_{\ell}) SS_{z}| JS_{c}\rangle \\
&\quad \times \ell' \langle \ell' m'S_{c}S_{c\ell}| JS_{c}\rangle Y(\lambda\ell)L_{J_{z}}(S_{c\ell} - S_{\ell})\hat{p}(\hat{q}) ,
\end{align*}
\]

with \(\vec{\ell}' = \sqrt{2\ell' + 1}\). The differential cross sections in the cm system are given by

\[
\frac{d^5 \sigma}{d\hat{P} d\hat{Q} dQ dq} = \frac{3}{4} \frac{p_0^2 q_0^2}{1} \sum_{S_{c}, S_{c\ell}} \left[ \sum_{a=1}^{3} \sum_{\Gamma, \Gamma'} \delta_{S_{c}, S_{c\ell}} \delta_{S_{c}, S_{c\ell}} X_{t,t}^{\tau(\alpha\beta)} f^{*}_{\Gamma, S_{c}S_{c\ell}}(p_{\alpha}, q_{\alpha}) f_{\Gamma, S_{c}S_{c\ell}}(p_{\alpha}, q_{\alpha}) \right] \\
+ \sum_{(\alpha\beta\gamma)} \sum_{\Gamma, \Gamma'} \delta_{S_{c}, S_{c\ell}} (-2) X_{S_{c}, S_{c\ell}}^{\tau(\alpha\beta)} \text{Re} \left\{ f^{*}_{\Gamma, S_{c}S_{c\ell}}(p_{\alpha}, q_{\alpha}) f_{\Gamma, S_{c}S_{c\ell}}(-p_{\beta}, q_{\beta}) \right\} .
\]

\(^{\text{4)}\) This corresponds to the direct term of \(f_{\gamma, (\ell'S_{c})}^{(\text{db})}(q)\) in Eq. I (2.92).
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The breakup differential cross sections in the lab system, \((d^5 \sigma/d \mathbf{k}_1 d \mathbf{k}_2 d S)\), are specified by the two directions \(\mathbf{\hat{k}}_1\) and \(\mathbf{\hat{k}}_2\), and the energy \(S\) measured along the locus of the \(E_1-E_2\) energy plane. They are obtained from the right-hand side of Eq. (2.21) by simply changing the phase space factor\(^1\)

\[
\rho_{cm} = \frac{3}{4} \frac{p_0 q^2}{q_0}, \quad \rightarrow \quad \rho_{lab} = \frac{M}{\hbar^2} \frac{3k_1 k_2}{2q_0} \left[ \left( 2 - \frac{k_{lab}}{k_2} \cos \theta_2 + \frac{k_1}{k_2} \cos \theta_{12} \right)^2 + \left( 2 - \frac{k_{lab}}{k_1} \cos \theta_1 + \frac{k_2}{k_1} \cos \theta_{12} \right) \right]^{-1/2}.
\]

(2.22)

The details of the three-body kinematics are summarized in Appendix A.

2.2. Three-nucleon breakup kinematics

Assuming that we detect two outgoing particles 1 and 2, the breakup differential cross sections are specified by the two polar angles \(\theta_1\) and \(\theta_2\), and the difference of the azimuthal angles \(\phi_{12} = \phi_1 - \phi_2\), in addition to the energy \(S\) determined from the kinematical curve (\(S\)-curve) in the \(E_1-E_2\) energy plane. The starting arc length \(S = 0\) is quite arbitrary and we follow the convention for the experimental setup. In Appendix A, we have parametrized the locus in the \(k_1-k_2\) plane with an angle \(\theta\), and the starting point \(S = 0\) is uniquely determined by specifying \(\theta_{st}\). We also assume that the beam direction of the incoming particle is the \(z\)-axis and set \(\phi_1 = \pi, 15)\) which determines the \(x\)-axis.

It is customary to classify the three-body breakup kinematics into the following six categories on the basis of the classical (or geometrical) argument:\(^1, ^4\)

1. Quasi-free scattering (QFS): one of the nucleons in the final state is at rest in the lab system.
2. Final-state interaction (FSI): the relative momentum of the two outgoing nucleons is zero.
3. Collinear configuration (COLL): one of the outgoing nucleons is at rest in the cm system, and the other two have back-to-back momenta.
4. Symmetric space star configuration (SST): the three nucleons emerge from the reaction point in the cm system, keeping equal momenta with 120\(^\circ\) relative to each other and perpendicular to the beam direction (on the \(x-y\) plane in the cm system).
5. Coplanar star configuration (CST): the same as the symmetric space star configuration, but with the three momenta lying on the reaction plane.
6. Nonstandard configuration (NS): other nonspecific configurations.

These are mathematically distinguished by particular values of the lab momentum \(k_\alpha\), the relative momenta \(p_\alpha\) and \(q_\alpha\), etc., and provide a rough guidance to detect which portion of the two-nucleon \(t\)-matrix is responsible at the final stage of the reaction, according to the structure of the direct breakup amplitudes in Eq. (2.19). For example, Ref. 1) argues that the first Born term of the QFS with \(k_\alpha = 0\) is approximately a product of an on-shell two-nucleon \(t\)-matrix and the deuteron wave
function at zero momentum. It is known that the 3N force effect is rather small under the QFS condition. On the other hand, the collinear configurations with \( q_\alpha = 0 \) are expected to be sensitive to the 3N force intuitively. The experimental study by Correll et al.\cite{16} was carried out to study the effect of the 3N force intensively in the reaction \( H(\vec{d}, 2p)n \) around these configurations at a deuteron incident energy \( E_d = 16 \) MeV. Furthermore, the FSI is characterized by \( p_\alpha = 0 \), for which the half off-shell \( t \)-matrix in Eq. (2.19) generates a large peak corresponding to the \( 1S_0 \) positive-energy bound state near the zero-energy threshold. Although the height of the peak is influenced by the background amplitude \( \hat{Q}_{i\mu\gamma} \), the FSI peak is usually well reproduced. The disagreement with the \( nd \) data is reported at the early stage for the SST configuration, which is still an unsolved problem called the space star anomaly.\cite{17} Note, however, that the disagreement between the theory and the experiment is also seen in some other coplanar star and nonstandard configurations, for which the off-shell properties of the two-nucleon \( t \)-matrix are expected to play a role in different ways. We will examine these on a case-by-case basis in the next section.

§3. Results and discussion

3.1. \( H(\vec{d}, 2p)n \) reaction at \( E_d = 16 \) MeV

It is important to take a sufficient number of discretization points and partial waves to get well converged results, especially for the breakup differential cross sections. In this study, we take \( n_1-n_2-n_3=6-6-5 \) in the notation introduced in § 3.1 of I, unless otherwise specified. This means that the three intervals \([0, \frac{1}{2} q_M], [\frac{1}{2} q_M, \frac{\sqrt{3}}{2} q_M] \) and \([\frac{\sqrt{3}}{2} q_M, q_M] \) are discretized by the six-point Gauss Legendre quadrature for each, and the total number of the discretization points for \( q \) is \( 35 (= 6 \times 3 + 6 \times 2 + 5) \). The three-body model space is truncated by the two-nucleon angular momentum \( I_{\text{max}} \), which depends on the incident energy of the neutron. We find that \( I_{\text{max}} = 3 \) is large enough for \( E_n \leq 19 \) MeV. The Coulomb force is entirely neglected in the present calculations.

We first investigate Correll et al.’s experiment,\cite{16} i.e., the \( H(\vec{d}, 2p)n \) reaction with the deuteron incident energy \( E_d = 16 \) MeV. This corresponds to the nucleon-induced breakup reaction at a nucleon incident energy of 8 MeV. We generate the direct breakup amplitude for the deuteron incident reaction by adding an extra phase factor \((-1)^\ell\) to each term of Eq. (2.20), corresponding to the change from \( q_0 \) to \(-q_0 \) in Eq. (2.17).\footnote{We thank Professor H. Witala for informing us about this phase change.} The decay kinematics for the deuteron incident reaction is discussed in Appendix A. The breakup differential cross sections for the \( d(n, 2n)p \) reaction with \( E_n = 8 \) MeV are compared with Correll et al.’s data in Fig. 1, with respect to two collinear (COLL1 and COLL2) and two nonstandard (NS1 and NS2) configurations. The starting point \( S = 0 \) is chosen as the collinear points or nearest point, as discussed in Appendix A. The dashed curve, solid curve and bold solid curve correspond to the \( S+D \) (i.e., \( 3S_1 + 3D_1 \) and \( 1S_0 \) only), \( I_{\text{max}} = 2 \) and \( I_{\text{max}} = 3 \) cases, respectively. The solid curves almost overlap with the bold curves, and \( I_{\text{max}} = 2 \) is
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Fig. 1. Breakup differential cross sections for the reaction $d(n, 2n)p$ with $E_n = 8$ MeV. The experimental data are the deuteron incident reaction $H(\vec{d}, 2p)n$ with an energy $E_d = 16$ MeV.\textsuperscript{16}

Actually good enough at this energy. We also see that even the restriction to the S+D model space is not too stringent. If we compare our results with the meson-exchange predictions in Ref. 1, we find that they are very similar to each other. The calculated values are somewhat too small especially in COLL1, COLL2 and NS1, although to a less extent for the meson-exchange predictions. There is no exact collinear point in the case of NS2, and the best agreement with the experiment is obtained in this case.

3.2. $d(n, 2n)p$ reaction at $E_n = 10.3$ MeV

The breakup differential cross sections for the $d(n, 2n)p$ reaction at $E_n = 10.3$ MeV are shown in Figs. 2 and 3, together with the experimental data by Gebhardt et al.\textsuperscript{18} The figure numbers, e.g., fig. 5, in the panels correspond to the original figure numbers in Ref. 18). The two large peaks shown in figs. 5–13 are the $np$
Fig. 2. Breakup differential cross sections for the reaction $d(n,2n)p$ with $E_n = 10.3$ MeV. The experimental data are cited from Ref. 18) with the same figure numbers, e.g., fig. 5 etc., in the panels.
Fig. 3. Same as Fig. 2, but for other kinematical configurations.
final state interaction peaks with $p_1 \sim 0$ on the lower-$S$ side and those with $p_2 \sim 0$ on the higher-$S$ side. On the whole, the comparison with the experiment gives fair agreement, but some discrepancies found in Ref. 18) still persist. In Ref. 18), the experimental data are compared with the solutions of the AGS equations in the $W$-method, using a charge-dependent modification of the Paris potential. Their results and ours are strikingly similar to each other, sharing the same problems for the detailed fit to the experiment. The peak heights for the $p_2 \sim 0$ final state interaction peaks are not precisely reproduced in figs. 5, 7, 8, or 10, probably because we did not take into account the charge dependence of the two-nucleon interaction. In fig. 9, our result is worse than the theoretical calculation result in Ref. 18). The collinear point is realized at $S = 4$ MeV in fig. 11, at $S = 7.5$ MeV in fig. 13, at $S = 6$ MeV in fig. 14 and at $S = 5.6$ MeV in fig. 15. In fig. 11, we have obtained a smooth curve around the collinear point, just as the theoretical calculation in Ref. 18). The breakup differential cross sections around the collinear points are well reproduced. In fig. 16, the flat structure between $S = 3 – 7$ MeV is just the same as the theoretical calculation in Ref. 18). The experimental data of Ref. 18) for the symmetric space star configuration are plotted in the first panel of Fig. 6. Here again, we have obtained a result very similar to the theoretical prediction in Ref. 18).

3.3. Quasi-free scattering

We show in Fig. 4 the breakup differential cross sections for the quasi-free scattering at energies $E_{\text{lab}} = 10.5 – 65$ MeV. We find some deviation from the experimental data at the peak position for all the energies. Detailed investigation of the Coulomb effect in Ref. 8) has revealed that this overestimation at the peak position is reduced to some extent. However, the reduction is probably not large enough except for $E_{\text{lab}} = 13$ MeV. Figure 8 in Ref. 8) implies that this reduction is energy-dependent. The large overestimation at $E_{\text{lab}} = 19$ MeV may not be resolved only by the Coulomb effect. The direct incorporation of the Coulomb force is necessary for our QM interaction. In Fig. 4, we can see that the roles of higher partial waves are important at higher energies. The partial waves up to $I_{\text{max}} = 3$ are clearly necessary for $E_{\text{lab}} = 19$ MeV. At energies $E_{\text{lab}} = 22.7$ and 65 MeV, we need more partial waves up to $I_{\text{max}} = 4$.

3.4. Final state interaction

Four examples of the breakup differential cross sections for the $np$ final state interaction are shown in Fig. 5. Almost all the data are for the $d(p, pp)n$ reaction. In the 10.5 and 13 MeV cases, the $pd$ data are shown using open circles, while the $nd$ data are shown with bars, some dots and diamonds. The lower peaks are the $np$ final state interaction peaks with $p_1 \sim 0$, while the upper peaks are those with $p_2 \sim 0$. Here, we find that the higher peaks are slightly too small. The Coulomb correction slightly increases the peak height, and improves the fit to the experiment to some extent. We probably need more careful treatment of the charge dependence of the $NN$ interaction, just as in the previous 10.3 MeV case. We also see that the minimum point at $S = 11 – 12$ MeV for the $E_{\text{lab}} = 16$ MeV reaction is too low. The $3N$ force might be necessary to enlarge the cross sections and get a good fit to the
Fig. 4. Breakup differential cross sections for the quasi-free scattering (QFS). The experimental data for the $d(p, pp)n$ reaction are cited from Ref. 19) for 10.5 MeV, from Ref. 20) for 13 MeV, from Ref. 21) for 19 MeV, from Ref. 22) for 22.7 MeV and from Ref. 23) for 65 MeV. For the reaction $d(n, 2n)p$ with $E_n = 10.5$ MeV, the experimental data shown by bars are cited from Ref. 19).
experiment.\textsuperscript{27)

3.5. **Symmetric space star configurations**

As is well known, a large discrepancy appears in the breakup differential cross sections in the symmetric space star configurations.\textsuperscript{17) This is shown in Fig. 6, where our results in the various model spaces are compared with the \textit{nd} and \textit{pd} data. A peculiar thing is that all theoretical calculations of the \textit{d}(n, nn)p reaction at $E_{\text{lab}} = 13$ MeV deviate largely from the old and new \textit{nd} data,\textsuperscript{17,25,26)} although the deviation is not much for the 10.3, 19 and 65 MeV data. The \textit{pd} data at 13 MeV in Ref. 20) are more than 30% smaller than the \textit{nd} data. A theoretical study of the Coulomb effect for the symmetric space star configuration in Ref. 8) shows that it is generally

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{Breakup differential cross sections for the final state interaction (FSI). The experimental data are cited from Ref. 19) (\textit{pd}: open circles, \textit{nd}: filled circles with bars) for 10.5 MeV, from Refs. 24)–26) (\textit{nd}: bars, filled circles, filled diamonds) and 20) (\textit{pd}: open circles) for 13 MeV, from Ref. 27) (\textit{pd}) for 16 MeV and from Ref. 21) (\textit{pd}) for 19 MeV.}
\end{figure}
Fig. 6. Breakup differential cross sections for the symmetric space star (SST) configurations. The experimental data are cited from Refs. 18), 28) (nd) and 19) (pd: [10.5 MeV]) for 10.3 MeV, from Refs. 17), 25), 26) (nd) and 20) (pd) for 13 MeV, from Ref. 21) (pd) for 19 MeV and from Ref. 29) (pd) for 65 MeV.
very small irrespective of the energy (see Fig. 6 of Ref. 8)). Our results at $E_{\text{lab}} = 13$ MeV, $\theta_1 = \theta_2 = 50.5^\circ$ and $\phi_{12} = 120^\circ$ are located just between the lower $pd$ data and the higher $nd$ data, which is very similar to other predictions made by the meson-exchange potentials. In the other geometrical configurations at 13 MeV, the cross sections in the space star 2 case ($\theta_1 = 25^\circ$, $\theta_2 = 50.5^\circ$, $\phi_{12} = 120^\circ$) are about half of the experimental values and those in the space star 3 case ($\theta_1 = 39^\circ$, $\theta_2 = 50.5^\circ$, $\phi_{12} = 120^\circ$) are almost 30% smaller than the experimental values. The same situation is observed in the Faddeev calculations using the Paris potential (see Figs. 29 and 30 of Ref. 26)). Note that we need sufficient partial waves for the convergence of the symmetric space star configurations in particular, which has already been pointed out in Ref. 1).

3.6. Coplanar star configurations

Let us move to the coplanar star configurations in Figs. 7 and 8. The agreement between our calculation and the data is satisfactory in general, but some deviations still exist. For example, in the first panel of 13 MeV, the new $nd$ data$^{17}$ are much closer to the calculational results than the old data,$^{26}$ but some underestimation still exists in the calculation. The underestimation of the cross sections at the minimum point $S = 9$ MeV of CST2 and also that at the $np$ final state interaction peaks at approximately $S = 10$ MeV in CST3 and CST4 are features common to the meson-exchange potentials. See Figs. 11 – 13 in Ref. 26). For 16 MeV reactions, we have given a comparison not only for the coplanar star configuration, but also for the intermediate star (IST) configuration, both of which are very similar to the predictions made by the other models given in Ref. 27). For $E_{\text{lab}} = 22.7$ MeV data, the curves are not plotted as functions of $S$ but of $E_2$ for the second particle. For this and $E_{\text{lab}} = 65$ MeV reactions, we find a large contribution of higher partial waves up to $I_{\text{max}} = 4$. In the symmetric backward plane star configuration of 65 MeV, denoted by CST2, the original experimental data are shifted to the larger side of $S$ by 3.5 MeV, since the starting positions of $S = 0$ do not seem to be the same between our calculation and the experiment.

3.7. Collinear configurations

The comparison for the collinear configurations is shown in Figs. 9 and 10. For these configurations, the comparison with the experiment is generally good. The Coulomb force has an appreciable effect to increase the breakup cross sections at the collinear point, especially on the low-energy side,$^8$ which is much more important than the $3N$ force effect. In the first panel with $E_{\text{lab}} = 10.5$ MeV (COLL1), we find some kinematical mismatch of the final state interaction peak at $S \sim 10$ MeV. For COLL2 – COLL5 with $E_{\text{lab}} = 13$ MeV, the small breakup cross sections around the collinear points (minimum points) shift to a better direction to fit the experimental data as a result of the expected Coulomb effect. This would also be true for the minimum point for $E_{\text{lab}} = 19$ MeV. On the other hand, the breakup cross sections in COLL1 – COLL4 for $E_{\text{lab}} = 65$ MeV seem to be slightly overestimated.
Fig. 7. Breakup differential cross sections for the coplanar star (CST) configurations at energies $E_{\text{lab}} = 13$ and 16 MeV. The result for the intermediate star (IST) configuration is also shown at $E_{\text{lab}} = 16$ MeV. The experimental data are cited from Refs. 17) and 26) (nd) for 13 MeV and from Ref. 27) (pd) for 16 MeV.
3.8. Nonstandard configurations

The comparison of our results with the experimental data for the nonstandard configurations is shown in Figs. 11 and 12. Here, we find a large deviation from the experimental data in some cases, again a feature common to the meson-exchange predictions. These are the NS2 nd scattering of 13 MeV and the pd scattering of 22.7 and 65 MeV. A large number of figures, i.e., NS1 – NS9, for 13 MeV are very similar to the predictions made by the Paris potential in Ref. 26). The huge final state interaction peaks in NS4, NS7 and NS8 are very similar to the results obtained by the Malfliet-Tjon potential. In the 22.7 and 65 MeV cases, the calculated results are completely off the experimental data.
Fig. 9. Breakup differential cross sections for the collinear (COLL) configurations. The experimental data are cited from Ref. 19) (pd and nd) for 10.5 MeV and from Refs. 17) and 26) (nd) for 13 MeV.
Fig. 10. Same as Fig. 9, but for other geometrical configurations and energies. The experimental data are cited from Ref. 26) \((nd)\) for 13 MeV, from Ref. 21) \((pd)\) for 19 MeV and from Ref. 31) \((pd)\) for 65 MeV.
Fig. 11. Breakup differential cross sections for the nonstandard (NS) configurations. The experimental data are cited from Ref. 26 (nd).
Fig. 12. Same as Fig. 11, but for other geometrical configurations and energies. The experimental data are cited from Ref. 26) (nd) for 13 MeV, from Ref. 22) (pd) for 22.7 MeV and from Ref. 1) (pd) for 65 MeV.
§4. Summary

In this study as well as in previous studies,\(^{10,11,14}\) we have applied the quark-model (QM) baryon-baryon interaction fss2 to the neutron-deuteron (\(nd\)) scattering problems in the Faddeev formalism for composite particles. The main motivation is to investigate the nonlocal effect of the short-range \(NN\) interaction in a realistic model, reproducing all the two-nucleon properties and yet based on the naive three-quark structure of nucleons. The calculations are carried out using the 15-point Gaussian nonlocal potential constructed from fss2, which is accurate enough to reproduce the converged triton binding energy of fss2 with an accuracy of 15 keV and the \(NN\) phase shift parameters with a difference of less than 0.1°.\(^{14,32}\) The potential keeps all the nonlocal effects of the original fss2, including the energy-dependent term of the QM resonating-group method (RGM). This energy dependence is eliminated by the standard off-shell transformation utilizing the square root of the normalization kernel for two three-quark clusters. It is extremely important to deal with this energy dependence properly, since an extra nonlocal kernel from this procedure is crucial to reproduce all the elastic scattering observables below \(E_n \leq 65\) MeV.\(^{10,11}\)

In this research, we have studied the neutron-induced deuteron breakup differential cross sections for incident energies \(E_n \leq 65\) MeV, and compared them with available experimental data and the predictions made by meson-exchange potentials. We have found that our calculations reproduce almost all the results for the breakup differential cross sections predicted by the meson-exchange potentials, including the disagreement with the experiment. This feature is probably related to the structure of the direct breakup amplitudes in Eqs. (2.18) and (2.19). First, they are constrained by the elastic scattering amplitudes \(f_{J}(\ell' S'_c)(\ell S_c)\) in the initial stage. In the final stage of reactions, only the half-off shell two-nucleon \(t\)-matrix appears owing to the energy conservation for outgoing nucleons. The effect of the completely off-shell \(t\)-matrix therefore appears only at the stage of solving the basic AGS equations for \(\tilde{Q}_{ij\nu\gamma}(\ell' S'_c)^J\), for which the present investigations imply that the difference between our QM \(NN\) interaction and the meson-exchange potentials is rather minor. On the whole, the agreement with the experimental data is fair, but there exist some discrepancies in certain particular kinematical configurations, which are commonly found for both our predictions and meson-exchange predictions. In particular, the space star anomaly of 13 MeV \(nd\) scattering is not improved even in our model. There are major disagreements in the breakup differential cross sections in some of the nonstandard configurations. In our model, some overestimations of cross sections are found at an energy \(E_n = 65\) MeV. Since these large disagreements can be resolved with neither the Coulomb effect nor the introduction of the \(3N\) force, systematic studies from more basic viewpoints of the \(NN\) interaction are still needed both experimentally and theoretically.

In spite of the apparent disagreement between the theory and the experiment in some of the breakup differential cross sections, our QM baryon-baryon interaction fss2 was still very successful in reproducing almost all other experimental data of the three-nucleon system without reinforcing it with the three-body force. These include:
1) a nearly correct binding energy of the triton, 2) the reproduction of the doublet and quartet S-wave scattering lengths, 2a and 4a, 3) the not too small differential cross sections of the nd elastic scattering up to \( E_n \sim 65 \text{ MeV} \) at the diffraction minimum points, 4) the improved maximum height of the nucleon analyzing power \( A_y(\theta) \) in the low-energy region \( E_n \leq 25 \text{ MeV} \), although insufficient, and 5) the breakup differential cross sections with many kinematical configurations discussed in this paper. Many of these improvements are related to the sufficiently attractive nd interaction in the \( ^2S \) channel, in which the strong distortion effect of the deuteron is very sensitive to the treatment of the short-range repulsion of the \( NN \) interaction. In our QM \( NN \) interaction, this part is described by the quark exchange kernel of the color-magnetic term of the quark-quark interaction. In the strangeness sector involving the \( \Lambda N \) and \( \Sigma N \) interactions, the effect of the Pauli repulsion on the quark level appears in some baryon channels. It is therefore interesting to study \( \Sigma^\pm \)-deuteron scattering in the present framework to find the repulsive effect directly related to the quark degree of freedom. Such an experiment is being planned at the J-PARC facility.

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Appendix A

Three-Nucleon Breakup Kinematics

In this appendix, we will discuss the 3N breakup kinematics used in this study. We choose the standard set of Jacobi coordinates in the momentum space as \( \alpha = 3 \) and set

\[
p = \frac{1}{2}(k_1 - k_2), \quad q = \frac{1}{3}(2k_3 - k_1 - k_2), \quad K = k_1 + k_2 + k_3, \quad (A.1)
\]

with \( k_\alpha \) being the momentum coordinate of the particle \( \alpha \) in the lab system. We also choose the z-axis as the direction of the incident particle and assume that the magnitude of the incident momentum is \( q_0 \) in the cm system. This implies that the cm incident momentum and energy are always \( q_{cm} = q_0 e_z \) and \( E_{cm} = \frac{(3\hbar^2/4M)q_0^2}{2} \) in either case of the nucleon or deuteron incident particle. Here, \( e_z \) is the unit vector.
of the z-axis. In the lab system, the incident momentum \( k_{\text{lab}} \) and the energy \( E_{\text{lab}} \) are given by

\[
\begin{align*}
  k_{\text{lab}} &= \frac{2}{\sqrt{2}} q_0 e_z , \\
  E_{\text{lab}} &= \frac{\hbar^2}{2M} k_{\text{lab}}^2 = \frac{9\hbar^2}{8M} q_0^2 = \frac{3}{2} E_{\text{cm}} , \\
  k_{\text{lab}} &= 3q_0 e_z , \\
  E_{\text{lab}} &= \frac{\hbar^2}{4M} k_{\text{lab}}^2 = \frac{9\hbar^2}{4M} q_0^2 = 3E_{\text{cm}} ,
\end{align*}
\]

for nucleon-incident, \( q_0 \) is determined from the energy conservation in Eq. (A.3) by modifying \( k_{\text{lab}} \) to \( (2/3) k_{\text{lab}} \):

\[
\begin{align*}
  q &= \left[ ((2/3) k_{\text{lab}} - k_1 \cos \theta_1 - k_2 \cos \theta_2)^2 + (k_1 \sin \theta_1)^2 + (k_2 \sin \theta_2)^2 \\
  &+ 2k_1 k_2 \sin \theta_1 \sin \theta_2 \cos \phi_{12} \right]^{1/2} ,
\end{align*}
\]

The two-nucleon momentum \( p = |p| \) is determined from the energy conservation in the cm system:

\[
E = E_{\text{cm}} + \varepsilon_d = \frac{\hbar^2}{M} \left( p^2 + \frac{3}{4} q^2 \right) ,
\]

where \( \varepsilon_d = -(3\hbar^2/4M) k_d^2 \) is the deuteron energy. It is convenient to use the threshold momentum \( q_M = \sqrt{q_0^2 - k_d^2} \) for the deuteron breakup, by which we
find
\[ p = \sqrt{\frac{3}{4}(qM^2 - q^2)} \equiv p_0. \] (A-6)

The angles of \( \hat{p} \) and \( \hat{q} \) are obtained from
\[
\begin{align*}
\cos \theta_p &= \frac{1}{2p} (k_1 \cos \theta_1 - k_2 \cos \theta_2), \\
\cos \phi_p &= \frac{1}{2p \sin \theta_p} (k_1 \sin \theta_1 \cos \phi_1 - k_2 \sin \theta_2 \cos \phi_2), \\
\sin \phi_p &= \frac{1}{2p \sin \theta_p} (k_1 \sin \theta_1 \sin \phi_1 - k_2 \sin \theta_2 \sin \phi_2), \\
\cos \theta_q &= \frac{1}{q} \left( \frac{2}{3} k_{lab} - k_1 \cos \theta_1 - k_2 \cos \theta_2 \right), \\
\cos \phi_q &= -\frac{1}{q \sin \theta_q} (k_1 \sin \theta_1 \cos \phi_1 + k_2 \sin \theta_2 \cos \phi_2), \\
\sin \phi_q &= -\frac{1}{q \sin \theta_q} (k_1 \sin \theta_1 \sin \phi_1 + k_2 \sin \theta_2 \sin \phi_2). \quad (A-7)
\end{align*}
\]

To determine \( k_1 \) and \( k_2 \) from \( S \), we start from the energy conservation in the lab system:
\[ E_{lab} + \varepsilon_d = \frac{\hbar^2}{2M} \left( k_1^2 + k_2^2 + k_3^2 \right). \] (A-8)

We rewrite this as
\[ k_1^2 + k_2^2 + k_1k_2 \cos \theta_{12} - k_{lab}(k_1 \cos \theta_1 + k_2 \cos \theta_2) + \Delta = 0, \] (A-9)
where \( \cos \theta_{12} = (\hat{k}_1 \cdot \hat{k}_2) \), and we define
\[ \Delta = -\frac{M}{\hbar^2} (E_{lab} + \varepsilon_d) + \frac{1}{2} k_{lab}^2 = \begin{cases} \\
\frac{3}{4} \kappa_d^2 & \text{for nucleon-incident} \\
\frac{1}{4} k_{lab}^2 + \frac{3}{4} \kappa_d^2 & \text{for deuteron-incident} 
\end{cases} \] (A-10)

We rotate the \( k_1-k_2 \) plane by \( 45^\circ \) and parametrize the ellipse with an angle \( \theta \). In this process, it is convenient to express \( \theta_{12} \) by \( \theta_0 \), which is defined by
\[ \theta_0 = \frac{1}{2} \text{Arccos} \left( \frac{1}{2} \cos \theta_{12} \right). \] (A-11)

Here, \text{Arccos} implies the principal value of the arccosine function between 0 and \( \pi \). Note that \( \theta_0 \) changes from \( \pi/6 \) to \( \pi/3 \) with a change in \( \theta_{12} \) from 0 to \( \pi \). If we use this \( \theta_0 \), the solution of Eq. (A-9) is parametrized as
\[ k_1 = \frac{k_{lab}}{2\sqrt{2}} \frac{A}{\sin 2\theta_0} C(\theta - \theta_0), \quad k_2 = \frac{k_{lab}}{2\sqrt{2}} \frac{A}{\sin 2\theta_0} S(\theta - \theta_0), \] (A-12)
with
\[ C(\theta) = \cos \theta + C_0, \quad S(\theta) = \sin(\theta + 2\theta_0 - \pi/2) + S_0, \] (A.13)
and
\[ C_0 = \frac{\sqrt{2}}{A} \frac{\cos \theta_1 - \cos 2\theta_0 \cos \theta_2}{\sin 2\theta_0}, \quad S_0 = \frac{\sqrt{2}}{A} \frac{\cos \theta_2 - \cos 2\theta_0 \cos \theta_1}{\sin 2\theta_0}. \] (A.14)

In Eq. (A.12), we have defined
\[ A = \sqrt{2} \left\{ \frac{(\cos \theta_1)^2 + (\cos \theta_2)^2 - 2 \cos 2\theta_0 \cos \theta_1 \cos \theta_2}{(\sin 2\theta_0)^2} - \frac{4\Delta_{k_{\text{lab}}}}{k_{\text{lab}}^2} \right\}^{\frac{1}{2}}. \] (A.15)

We measure the arc length \( S \) in the \( E_1-E_2 \) plane counterclockwise, starting from a certain starting point \( \theta_{\text{st}} \). The expression \( S(\theta) \) is obtained by integrating
\[ dS = \sqrt{(dE_1)^2 + (dE_2)^2}; \]

\[ S(\theta) = \frac{\hbar^2 k_{\text{lab}}^2}{2M} \left( \frac{A}{2 \sin 2\theta_0} \right)^2 \int_{\theta_{\text{st}}}^{\theta} d\theta \ f(\theta - \theta_0), \] (A.16)

with
\[ f(\theta) = \sqrt{(C(\theta) \sin \theta)^2 + (S(\theta) \cos(\theta + 2\theta_0 - \pi/2))^2}. \] (A.17)

Note that \( S(\theta) \) is a monotonically increasing function of \( \theta \), satisfying \( S(\theta_{\text{st}}) = 0 \) and \( S(\theta + 2\pi) = S(2\pi) + S(\theta) \).

For practical calculation, we first discretize the integral region of Eq. (A.16) into small intervals by
\[ \theta_\mu = \theta_{\text{st}} + \mu \frac{2\pi}{N}, \] (A.18)

with \( \mu = 1 - N \). The number of discretization points \( N \) is typically \( N = 200 \). We use third-order spline interpolation for \( \theta \),
\[ f(\theta - \theta_0) = \sum_{\nu=1}^{N} S_{\nu}^{(\kappa)}(\theta) f(\theta_\nu - \theta_0), \] (A.19)

with
\[ S_{\nu}^{(\kappa)}(\theta) = \sum_{m=0}^{3} \alpha^{(m)}_{\nu} (\theta - \theta_\kappa)^m \quad \text{for} \quad \theta \in [\theta_{\kappa-1}, \theta_{\kappa}]. \] (A.20)

If we integrate Eq. (A.16) over \( \theta \) from \( \theta_{\text{st}} \) to \( \theta_\mu \) using Eqs. (A.19) and (A.20), we can carry out the \( \theta \)-integral analytically and obtain
\[ S_\mu = S(\theta_\mu) = \frac{\hbar^2 k_{\text{lab}}^2}{2M} \left( \frac{A}{2 \sin 2\theta_0} \right)^2 \times \sum_{m=0}^{3} (-1)^{m+1} \frac{1}{m+1} \left( \frac{2\pi}{N} \right)^{m+1} \sum_{\nu=1}^{N} \sum_{\kappa=1}^{\mu} \alpha^{(m)}_{\nu} f(\theta_\nu - \theta_0). \] (A.21)
To obtain the angle $\theta$ from the arc length $S$, we again use the spline interpolation technique,

$$\theta(S) = \sum_{\mu=1}^{N} S_\mu(S) \theta(S_\mu) .$$  \hspace{1cm} (A.22)

Here, $S_\mu(S)$ is the third-order spline function for the mesh points $\{S_\mu\} = \{S_0, S_1, S_2, \cdots, S_N\}$ with $S_0 = 0$. From Eq. (A.22) with $\theta(S_\mu) = \theta_\mu$, we obtain

$$\theta(S) = \theta_{st} + \frac{2\pi}{N} \sum_{\mu=1}^{N} \mu S_\mu(S) ,$$  \hspace{1cm} (A.23)

where Eq. (A.18) and $\sum_{\mu=1}^{N} S_\mu(S) = 1$ are used. We therefore only need to calculate the sum in Eq. (A.21) and prepare the coefficients of the spline interpolation for the mesh points $\{S_\mu\}$.

The starting angle $\theta_{st}$ is selected as follows. Let us first consider the nucleon-incident reaction. In this case, it is convenient to define the angle $\theta(-)$ through

$$\cos \theta(-) = \frac{2\sqrt{\Delta}}{k_{lab}} = \frac{2}{\sqrt{3}} \frac{k_d}{q_0} .$$  \hspace{1cm} (A.24)

If we assume $k_2 = 0$ in Eq. (A.9), we find that there are two non-negative solutions, i.e.,

$$k_1^{(\pm)} = \frac{k_{lab}}{2} \left[ \cos \theta_1 \pm \sqrt{(\cos \theta_1)^2 - (\cos \theta(-))^2} \right] ,$$  \hspace{1cm} (A.25)

only when $\theta_1 < \theta(-)$. We choose the larger $k_1^{(+)}$ as the starting point to measure $S$. In this case, we can easily find that the corresponding $\theta$ is given by

$$\theta_{st} = \text{Arccos} S_0 - \theta_0 ,$$  \hspace{1cm} (A.26)

where $S_0$ is given in Eq. (A.14). When $\theta_1 > \theta(-)$, we have two cases. In the case of $\theta_2 \leq \theta(-)$, we choose $k_1 = 0$ and

$$k_2^{(-)} = \frac{k_{lab}}{2} \left[ \cos \theta_2 - \sqrt{(\cos \theta_2)^2 - (\cos \theta(-))^2} \right]$$

(A.27)

as the starting point with

$$\theta_{st} = \text{Arccos} C_0 + \theta_0 - \pi .$$

(A.28)

In the case of $\theta_2 > \theta(-)$, the ellipse does not cross over either the $k_1$- or $k_2$-axis. We therefore use the smaller value of $k_1 = k_2$ as the starting point. This condition yields

$$\theta_{st} = \text{Arccos} \left( \frac{1}{A} \frac{\cos \theta_1 - \cos \theta_2}{\sqrt{2 \sin \theta_0}} \right) + \pi ,$$

(A.29)
and

\[
k_1^{(-)} = k_2^{(-)} = \frac{k_{\text{lab}}}{4 \cos \theta_0} \left[ \frac{\cos \theta_1 + \cos \theta_2}{2 \cos \theta_0} - \sqrt{\left( \frac{\cos \theta_1 + \cos \theta_2}{2 \cos \theta_0} \right)^2 - \frac{4\Delta}{k_{\text{lab}}^2}} \right].
\]  

(A·30)

When the deuteron is an incident particle, there is no crossing point across either the \( k_1 \)- or \( k_2 \)-axis. We follow the procedure by Correll et al.\(^{16}\) They discussed the deuteron-incident reaction around collinear configurations, choosing a collinear point as the starting point to measure \( S \). The collinear point \( \theta_c \) is defined as the configuration with \( q = (2/3)k_{\text{lab}} - (k_1 + k_2) = 0 \). To find the corresponding \( \theta_{\text{st}} = \theta_c \), we assume \( \phi_{12} = \pi \left( \phi_1 = \pi, \phi_2 = 0 \right) \) and the \( x-z \) plane as the reaction plane, just as in the experimental setup. Under this assumption, the magnitude \( q^2 \) is expressed as

\[
q^2 = \left( k_1 \cos \theta_1 + k_2 \cos \theta_2 - \frac{2}{3}k_{\text{lab}} \right)^2 + (k_1 \sin \theta_1 - k_2 \sin \theta_2)^2,
\]  

resulting in the two conditions

\[
k_1 \cos \theta_1 + k_2 \cos \theta_2 = \frac{2}{3}k_{\text{lab}} , \quad k_1 \sin \theta_1 = k_2 \sin \theta_2 .
\]  

(A·32)

If \( \theta_1 = \theta_2 \), we assume \( k_1 = k_2 \) and find \( \theta_c = \pi/2 \) with

\[
k_1 = k_2 = \frac{k_{\text{lab}}}{4} \left( \frac{\cos \theta_1}{(\cos \theta_0)^2} + \frac{A}{\sqrt{2} \cos \theta_0} \right).
\]  

(A·33)

This corresponds to the \( k_1^{(+)} = k_2^{(+)} \) case of Eq. (A·30) with the opposite sign for the second term. In the general case, the three conditions of Eq. (A·32) and the energy conservation in Eq. (A·9) are not always simultaneously satisfied. (Note that we only need two conditions to determine \( k_1 \) and \( k_2 \).) We take the following procedure to determine \( \theta_c \). Let us use the notations

\[
a = \frac{\cos \theta_1 - \cos \theta_2}{\sqrt{2} \sin \theta_0} , \quad b = \frac{\cos \theta_1 + \cos \theta_2}{\sqrt{2} \cos \theta_0} ,
\]  

(A·34)

to simplify the expressions. We have \( a^2 + b^2 = A^2 + (8\Delta/k_{\text{lab}}^2) \) and define a new angle \( \alpha \) through\(^1\)

\[
\cos \alpha = \frac{a}{\sqrt{a^2 + b^2}} , \quad \sin \alpha = \frac{b}{\sqrt{a^2 + b^2}} .
\]  

(A·35)

We consider \( I(\theta) = k_1 \cos \theta_1 + k_2 \cos \theta_2 \) as a function of \( \theta \) using Eq. (A·12) and other equations. We find

\[
I(\theta) = k_1 \cos \theta_1 + k_2 \cos \theta_2 = \frac{1}{4}k_{\text{lab}} \sqrt{a^2 + b^2} \left[ A \cos(\theta - \alpha) + \sqrt{a^2 + b^2} \right].
\]  

(A·36)

\(^1\) By using \( \alpha, C_0 \) and \( S_0 \) in Eq. (A·14) are respectively expressed as \( C_0 = \sqrt{1+\varepsilon^2} \cos(\alpha - \theta_0) \) and \( S_0 = \sqrt{1+\varepsilon^2} \sin(\alpha + \theta_0 - \pi/2) \) with \( \varepsilon = (2\sqrt{2\Delta/k_{\text{lab}}A}) \).
The crossing point with $I(\theta) = (2/3)k_{\text{lab}}$ is found only when the condition

$$\frac{1}{A} \left| \frac{8}{3 \sqrt{a^2 + b^2}} - \sqrt{a^2 + b^2} \right| \leq 1 \quad (A.37)$$

is satisfied. The solution $\theta = \theta_c$ is found as

$$\theta_c = \alpha \pm \arccos \frac{1}{A} \left[ \frac{8}{3 \sqrt{a^2 + b^2}} - \sqrt{a^2 + b^2} \right], \quad (A.38)$$

and a unique point is determined when the equality is satisfied in Eq. (A.37). Next, we examine whether or not the condition $k_1 \sin \theta_1 = k_2 \sin \theta_2$ is satisfied for the two solutions of Eq. (A.38). The one satisfying this condition is the collinear point with $q = 0$ from Eq. (A.31). If both solutions satisfy the condition, we choose the smaller one for $\theta_c$. If neither of the solution satisfies the condition, there is no exact collinear point. In this case, we minimize Eq. (A.31) with respect to $\theta$ at the interval bounded by the two solutions of Eq. (A.38).

### Appendix B

**Isospin Factors for the Breakup Amplitudes**

In this appendix, we extend the definition of the spin factors

$$\langle \tilde{\Gamma}_\sigma | (P_{(123)}^\sigma)^\alpha | \Gamma_\sigma \rangle = \delta_{S,S'} \delta_{S_2,S_2} \left\{ \begin{array}{l} (-1)^{1+s} X^S_{s,s} \\ \text{for } \alpha = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \end{array} \right. \quad (B.1)$$

to the isospin factors and calculate the matrix elements $\langle \tilde{\Gamma}_\tau | O^\alpha \beta | \Gamma_\tau \rangle$ in Eq. (2.13). The most convenient definition of the isospin factors is probably

$$\langle \tilde{\Gamma}_\tau | O^\alpha \beta | \Gamma_\tau \rangle = \langle (\tilde{t}_2 \tau)_{1/2} T_z | (P_{(123)}^\tau)^{3-\alpha} O_\tau (P_{(123)}^\tau)^\beta | (\tilde{t}_2 \tau)_{1/2} T_z \rangle$$

$$= \left\{ \begin{array}{c} (-1)^{1+t} X_{t,t}^{\tau(\alpha \beta)} \\ \text{for } \beta - \alpha = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \end{array} \right., \quad (B.2)$$

which yields the results in Eq. (2.14). If we set $O_\tau = 1$, all the factors $X_{t,t}^{\tau(\alpha \beta)}$ in Eq. (B.2) are reduced to $X_{t,t}^{1/2}$ and $\delta_{t,t} (\alpha = \beta \text{ in mod } 3)$ since $O_\tau^{\alpha \beta} = (P_{(123)}^\tau)^{\beta-\alpha}$. Here, $X_{t,t}^{1/2}$ is the common matrix with the spin factors given by

$$X_{s,s'}^{1/2} = \left( \begin{array}{cc} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{array} \right), \quad X_{s,s'}^{3/2} = \left( \begin{array}{cc} 0 & 0 \\ 0 & 1 \end{array} \right). \quad (B.3)$$

In Eq. (B.3), the upper row (the left-most column) corresponds to $s = 0 (s' = 0)$ and the second row (the right-most column) corresponds to $s = 1 (s' = 1)$. 

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To calculate $X^\tau_{\bar{t},t}^{(\alpha\beta)}$, we only need $\langle \bar{t} | \mathcal{O}_\tau | t \rangle$ with $| t \rangle = | (t_{\bar{t}}/2) T_z \rangle$, since $P^\tau_{(123)}$ does not change the total isospin $T = 1/2$. Furthermore, all $\mathcal{O}_\tau$’s in Eq. (2.10) are expressed by the symmetric isospin operator $t_z = (\tau_z(1) + \tau_z(2))/2$ and the antisymmetric operator $t^a_z = (\tau_z(1) - \tau_z(2))/2$ for the two-particle states $\eta_{\bar{t}}(1,2)$. The former does not change $t = 0$ or 1, while the latter interchanges them. For $pp$ or $nn$, the nonzero matrix elements are only for $\bar{t} = t = 1$, but $np$ and $pn$ contains $t^a_z$. However, we only need to calculate the sum of the $np$ and $pn$ contributions. Furthermore, Eq. (2.5) tells us that $np$ and $pn$ give the same contribution owing to the permutation operator $(1 + P)$. We can therefore assume $\mathcal{O}_\tau$ in Eq. (B.2) as

$$\mathcal{O}^{pp} = \frac{1}{2} t_z(1 + t_z), \quad \mathcal{O}^{nn} = \frac{1}{2} t_z(-1 + t_z), \quad \mathcal{O}^{pm} + \mathcal{O}^{np} = 1 - t^2_z. \quad (B.4)$$

If we decompose $t^2_z$ into the rank 0, 1 and 2 tensors as $t^2_z = (1/3)t^2 + \sqrt{2/3}[t t]^{(2)}$, we immediately find that the rank 2 tensor does not contribute to the matrix elements since the total isospin in our case is $T = 1/2$. We can therefore replace $t^2_z$ in Eq. (B.4) with $(1/3)t^2$, resulting in

$$\langle \bar{t} | \mathcal{O}^{pp} | t \rangle = \langle \bar{t} | \mathcal{O}^{nn} | t \rangle = \delta_{\bar{t},t} \delta_{t,1} \frac{2}{3},$$
$$\langle \bar{t} | \mathcal{O}^{pm} | t \rangle + \langle \bar{t} | \mathcal{O}^{np} | t \rangle = \delta_{\bar{t},t} \left( \delta_{t,0} + \frac{1}{3} \delta_{t,1} \right). \quad (B.5)$$

It is convenient to introduce the isospin projection operators $P_0 = (1 - \tau(1) \cdot \tau(2))/4$ and $P_1 = (3 + \tau(1) \cdot \tau(2))/4$, and define

$$X^\tau_{\bar{t},t}^{(\alpha\beta)} = \begin{cases} 
( -1 )^{1 + t} \langle \bar{t} | (P^\tau_{(123)})^{3-\alpha} P_\tau (P^\tau_{(123)})^{\beta} | t \rangle & \text{for } \beta - \alpha = 1 \\
( -1 )^{1 + t} \langle \bar{t} | (P^\tau_{(123)})^{3-\alpha} P_\tau (P^\tau_{(123)})^{\beta} | t \rangle & \text{for } \beta - \alpha = 2 \text{ in (mod 3)} \\
\langle \bar{t} | (P^\tau_{(123)})^{3-\alpha} P_\tau (P^\tau_{(123)})^{\alpha} | t \rangle & \text{for } \beta - \alpha = 3
\end{cases} \quad (B.6)$$

for $\tau = 0$ and 1. Since $P_\tau$ is expressed as $P_\tau = | \tau \rangle \langle \tau |$ in our model space, the matrix elements $\langle \bar{t} | (P^\tau_{(123)})^{3-\alpha} P_\tau (P^\tau_{(123)})^{\beta} | t \rangle = \langle \bar{t} | (P^\tau_{(123)})^{3-\alpha} | \tau \rangle \langle \tau | (P^\tau_{(123)})^{\beta} | t \rangle$ can be easily calculated using Eq. (B.1). The correspondence

$$\mathcal{O}^{pp}, \mathcal{O}^{nn} \sim \frac{2}{3} P_1, \quad \mathcal{O}^{pm} + \mathcal{O}^{np} \sim P_0 + \frac{1}{3} P_1 \quad (B.7)$$

from Eq. (B.5) yields

$$X^{pp}, X^{nn} = \frac{2}{3} X^{1(\alpha\beta)}, \quad X^{pm} + X^{np} = X^{0(\alpha\beta)} + \frac{1}{3} X^{1(\alpha\beta)} \quad (B.8)$$

in the matrix form. Here, $X^\tau(\alpha\beta)$ with $\tau = 0$ and 1 are given by
\[ (\tau = 0 \text{ factors}) \]
\[ X^{0(11)} = \left( \frac{1}{4}, \frac{\sqrt{3}}{4} \right), \quad X^{0(22)} = \left( \frac{1}{4}, -\frac{\sqrt{3}}{4} \right), \quad X^{0(33)} = \left( 1, 0 \right), \]
\[ X^{0(12)} = X^{0(21)} = \left( -\frac{1}{4}, -\frac{\sqrt{3}}{4} \right), \]
\[ X^{0(23)} = t X^{0(32)} = X^{0(13)} = t X^{0(31)} = \left( \frac{1}{2}, 0 \right). \]  

\[ (\tau = 1 \text{ factors}) \]
\[ X^{1(11)} = \left( \frac{3}{4}, -\frac{\sqrt{3}}{4} \right), \quad X^{1(22)} = \left( \frac{3}{4}, \frac{\sqrt{3}}{4} \right), \quad X^{1(33)} = \left( 0, 0 \right), \]
\[ X^{1(12)} = X^{1(21)} = \left( \frac{3}{4}, -\frac{\sqrt{3}}{4} \right), \]
\[ X^{1(23)} = t X^{1(32)} = X^{1(13)} = t X^{1(31)} = \left( 0, -\frac{\sqrt{3}}{2} \right). \]  

(B.9)

(B.10)

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