1. Introduction

There exists now large interest to high energy processes induced by both SU(2)×U(1) weak instantons [1-2] and SU(3) strong instantons [3-4] because of their important role in HEP.

As it is known violation of baryon and lepton numbers conservation law [5] due to quantum anomaly [6] can be induced by SU(2)×U(1) weak instantons [7] which represent tunnelling processes, associated with the highly degenerated vacuum structure. Possibility of the baryon and lepton number nonconservation at high energy is connected with the problem of the baryon and antibaryon asymmetry in the observable part of the Universe [8]. At low energies (when energy of the process is less than barrier between different degenerated vacuum stages) cross section $\sigma_{I_{tot}} \approx 10^{-78}$ [5]. In high energy particle collisions (in multi TeV regime) the cross section can increase exponentially. It is associated with multi $W$, $Z^0$ and $H$-bosons production in the instanton field [1].

On the other hand for strong SU(3) instantons in QCD such phenomenon can exist at hundreds Mev [2] and be important in deep inelastic $e\nu$-scattering for decreasing Bjorken variable $X_{Bj}$ and high photon virtuality $Q^2$ [8]. Search of QCD-instantons has started already in $e\nu$-collisions at HERA(H1). The processes have some features: instanton contribution to structure function $F_2(X_{Bj},Q^2)$ rises strongly with decreasing $X_{Bj}$; $\sigma_{I_{tot}}$ strongly peaks with decreasing of $X_{Bj}$; hadronic band emission of semi-hard partons is isotropic in the instanton rest system; current quark jet and characteristic flavour, strangenes-$K^0$, charm and muon flow take place [4,9,10].
Moreover instanton-induced processes manifest new mechanism of multi-particle production and can contribute to intermittency exponent [11].

In this paper we study properties of the second correlation function as signature of SU(3) instanton-induced multi-gluon state for classical instanton with the first quantum correction in QCD.

It should be noted, for this effect the problem of taking into account the hadronization exists. Here it can be solved by the use of the local parton-hadron duality [12].

2. Exclusive Distribution on Gluon Rapidities for Instanton-Induced Multiparticle Production Processes with the First Quantum Correction

The following instanton solution is used [7]:

\[
A_a^{\mu(x)} = \frac{2\eta_a^{\mu\nu}(x-z)_\nu}{g((x-z)^2 + \rho^2)},
\]

where \(A_a^{\mu(x)}\) are gluon fields; \(\eta_a^{\mu\nu}\) is a 't Hooft symbol [5]; \(\rho\) and \(z_\nu\) are size and position of the instanton correspondingly; \(g\) is a constant of strong interaction; greek indexes \(\mu, \nu\) and latin indexes \(a, b\) are SU(3)-group indexes; subscript “cl” denotes quasiclassical approximation.

It is convenient to use reduction formula [13] for the calculation of the amplitude of \(n\) gluon production in instanton field

\[
T_{\mu_1\mu_2...\mu_n}(k_1,k_2,...,k_n) = \int \prod_{j=1}^n dy_j e^{ik_jy_j}(k_j^2 + m^2) \times \\
\times \int [DA]e^{-S[A]}A_{\mu_1}^{\alpha_1}(x)...A_{\mu_n}^{\alpha_n}(x) = \\
= \int \prod_{j=1}^n dy_j e^{ik_jy_j}(k_j^2 + m^2) \int [DA]e^{-S[A]}(A_{\mu_1}^{\alpha_1}(x) + A_{\mu_1}^{\alpha_1}(qu)(x))... \times \\
\times (A_{\mu_n}^{\alpha_n}(cl)(x) + A_{\mu_n}^{\alpha_n}(qu)(x)) = \int \prod_{j=1}^n dy_j e^{ik_jy_j}(k_j^2 + m^2) \times \\
\times \int [DA]e^{-S[A]}A_{\mu_1}^{\alpha_1}(cl)(x)...A_{\mu_n}^{\alpha_n}(cl)(x) + \int \prod_{j=1}^n dy_j e^{ik_jy_j}(k_j^2 + m^2) \times \\
\times \int [DA]e^{-S[A]}A_{\mu_1}^{\alpha_1}(qu)(x)A_{\mu_2}^{\alpha_2}(qu)(x)A_{\mu_3}^{\alpha_3}(cl)(x)...A_{\mu_n}^{\alpha_n}(cl)(x) + ... .
\]

In formula (2) we consider \(A_{\mu}^{\alpha}(x) = A_{\mu}^{\alpha}(cl)(x) + A_{\mu}^{\alpha}(qu)(x)\), where \(A_{\mu}^{\alpha}(qu)(x)\) is small fluctuations near classical field \(A_{\mu}^{\alpha}(cl)(x)\); subscript "qu" denotes quantum correction [14]. The first term in formula (2) corresponds to the main (quasiclassical) approximation and is given by the following expression [15]:
where \( k_1, k_2, \ldots, k_n \) are the 4-momenta of the produced gluons, \( k_j = (\vec{k}_j, iE_j) \), \( C_1 = 4\pi^2\rho^2/g \), \( m \) is effective gluon mass.

In the formula (3) we as usual do not write \( \delta \)-function, which is connected with law of conservation of 4-momentum [1]. For the correct normalizing we must rewrite formula (3) in the following way:

\[
T_{\mu_1\mu_2\ldots\mu_n}^{(cl)}(k_1, k_2, \ldots, k_n) = (C_1)^n \prod_{j=1}^{n} \eta_{\mu_j\nu_j}^{k_j} \Theta(n_{\text{max}} - n),
\]  

(3')

where \( n_{\text{max}} = \sqrt{s}/m \).

Then the second term in (2) is the first quantum correction to the classical amplitude. It is calculated on the basis of gluon propagation function in instanton field [16] and is given by the following formula [2] (for two produced gluons):

\[
T_{\mu\nu}^{(qu)}(k_1, k_2) = (C_2)^2 \eta_{\mu\alpha}^{d} \eta_{\nu\beta}^{d} k_1^\alpha k_2^\beta \varepsilon^{\alpha\beta\gamma\delta} \eta_{\gamma\delta}^{c} \frac{k_1^c k_2^c}{(k_1, k_2)^2},
\]

(4)

where \( C_2 = 2\pi i\rho; \ (k_1, k_2) = (\vec{k}_1\vec{k}_2) - E_1E_2 \).

We make the following natural assumption (in laboratory subsystem in \( ep \)-collision) [17]:

\[
(k_i^L)^2 \gg (\vec{k}_i^T)^2 \gg m^2, \quad k_i^T \equiv |\vec{k}_i^T| = k_i^T, \\
E_i \approx k_i^T chy_i, \quad k_i^L \approx k_i^T shy_i.
\]

(5)

Let us write the expressions for the probabilities of the gluon production processes going through instanton mechanism in dependence on rapidity variables:

\[
P_n(y_1, \ldots, y_n) = |T_n^{(cl)}(y_1, \ldots, y_n) + T_n^{(qu)}(y_1, \ldots, y_n)|^2 \approx \\
\approx T_n^{(cl)}(y_1, \ldots, y_n) [T_n^{(cl)}(y_1, \ldots, y_n)]^* + T_n^{(qu)}(y_1, \ldots, y_n) [T_n^{(cl)}(y_1, \ldots, y_n)]^* + \\
+ T_n^{(cl)}(y_1, \ldots, y_n) [T_n^{(qu)}(y_1, \ldots, y_n)]^* = A^n \prod_{j=1}^{n} ch2y_j \Theta(n_{\text{max}} - n) - \\
-\alpha A^{n-2} \left[ \Pi(y_1, y_2) ch2y_3 \ldots ch2y_n + \ldots + \Pi(y_{n-1}, y_n) ch2y_1 \ldots ch2y_{n-2} \right] \times \\
x \Theta(n_{\text{max}} - n);
\]

where \( A = 3(k^T)^2(C_1)^2 \), \( \alpha = 8(C_1)^2(C_2)^2(k^T)^2 \), \( \Pi(y_1, y_2) = sh(y_1 - y_2) \times \times th(y_1 - y_2) \). In the formula (6) the first term corresponds to the classical approximation and the others are the first quantum correction contribution.
3. Inclusive Distribution and Second Correlation Function for Instanton-Induced Multigluon Production Processes with the First Quantum Correction

In order to calculate inclusive distribution $\rho_n(y_1, ..., y_n)$ we must by standard method take into account all canals of the process. In our case we obtain:

$$
\rho_n(y_1, ..., y_n) = A^n ch2y_1 ... ch2y_n[\Theta(n_{max} - n) + R_0(n)] - \\
-\alpha A^{n-2}[\Pi(y_1, y_2)ch2y_3 ... ch2y_n + ... + \Pi(y_{n-1}, y_n)ch2y_1 ... ch2y_{n-2}] \times \\
\times (\Theta(n_{max} - n) + R_0(n)) - \frac{\alpha}{2} A^n ch2y_1 ... ch2y_n \Pi(Y) R_2(n) - \\
-\alpha A^{n-1}[Q(Y, y_1)ch2y_2 ... ch2y_n + ... + Q(Y, y_n)ch2y_1 ... ch2y_{n-1}] R_1(n), \quad (7)
$$

where we denoted

$$
\Pi(Y) = \int_{-Y}^{Y} \Pi(y_1, y_2) = sh(y_1 - y_2)th(y_1 - y_2)dy_1 dy_2 = \\
= 4(sh(\frac{Y}{2}))^2 - \pi Y + 8 \sum_{k=0}^{\infty} (-1)^k \frac{(2k + 1)}{2k + 1} \exp(-\frac{Y}{2}[2k + 1])sh(\frac{Y}{2}[2k + 1]);
$$

$$
Q(Y, y) = 2shY chy + arcth[schy - Y] - arcth[schy + Y];
$$

$$
R_i(n) = \sum_{m=1}^{\infty} \frac{[Ash2Y]^{m-i}}{(m-i)!} \Theta(n_{max} - n - m),
$$

$$
i = 0, 1, 2, \quad (0 \leq R_2(n) \leq R_1(n) \leq R_0(n) \leq 1). \quad (8)
$$

Two-particle correlation function in dependence on rapidities

$$
C_2(y_1, y_2) = \rho_2(y_1, y_2) - \rho_1(y_1) \rho_1(y_2)
$$

if $n_{max} \gg 1$ and $\Theta(n_{max} - 1) = \Theta(n_{max} - 2) = 1; R_i(1) \approx R_i(2) \approx 1; \quad i = 0, 1, 2$, has the following form:

$$
C_2(y_1, y_2) = [-2A^2 + \frac{3}{2} \alpha A^2 \Pi(Y)]ch2y_1ch2y_2 - 4\alpha A \pi ch(y_1 + y_2)th(y_1 - y_2) - \\
-2\alpha sh(y_1 - y_2)th(y_1 - y_2) + 2\alpha A \pi ch(y_1 + y_2) + ch2y_1] + \\
+ 2\alpha A[arctg(\frac{chy_1}{shY})]ch2y_2 + arctg(\frac{chy_2}{shY})ch2y_1]. \quad (9)
$$

Corresponding curve lies in negative region of the plot $C_2(y_1, 0)$, has maximum at $y_1 = 0$ and minima at $y_1 = +3, 5; -3, 5$ (see fig.1). Central maximum corresponds to the quasiclassical part of (9); two minima are contribution of
the first quantum correction. For the estimation the parameters are taken to have the next values: $\sqrt{s} = 50\, GeV$, $m = 100\, MeV$, $\rho = 1\, GeV^{-1}$, $Y \approx 4$, $k^T = 0, 1\, GeV$.

Fig.1. Correlation function vs. rapidity $y_1$ of one of the particle, when $y_2 = 0$.

Conclusion

Thus, in addition to the known footprints of instanton induced events we have obtained two gluon correlation function which is negative and has specific structure at the rest system of one of the particle. With the help of local parton-hadron duality the result can be used for hadrons and is of interest for HERA experiments [4].

The estimations of contributions of quantum corrections of higher orders and of adequate kinematical restrictions, separation of hadrons from semi-hard quarks and from accompanying gluons to be used in Monte-Carlo analysis are now in progress.

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