Light-Front model of transition form-factors in heavy meson decay

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Electroweak transition form factors of heavy meson decays are important ingredients in the extraction of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements from experimental data. In this work, within a light-front framework, we calculate electroweak transition form factor for the semileptonic decay of D mesons into a pion or a kaon. The model results underestimate in both cases the new data of CLEO for the larger momentum transfers accessible in the experiment. We discuss possible reasons for that in order to improve the model.

1. Introduction

Charmed mesons inaugurated a new era in particle physics. The discovery of the first heavy meson was an evidence in favor of the standard model as giving the correct description of low energy particle physics. The new quark was confirmed experimentally (see [1]) in a meson quark-antiquark state, i.e., in a heavy-light quark system. Since then, the study of semileptonic decays gained importance, mainly in the last years, as these decays constitute a laboratory to test the standard model predictions and to get information about the CKM [2] matrix elements. These matrix elements, $V_{cq}$, can be extracted from the experimental data of D decay. However, theoretical inputs are necessary in order to interpret the data from the decay experiments as obtained by BES [3], Babar [4], Belle [5], and CLEO [6] collaborations.

The QCD nonperturbative effects in the semileptonic decay are modelled with transitions form factors that have been calculated with lattice QCD [6], QCD sum rules [9] and light-front constituent quark models [10,11,12]. This motivates us to study the semileptonic decay of D mesons within a light-front covariant model (LFCM) proposed in ref. [13] and applied to calculate the pion, kaon and D electromagnetic form factors.

In general, light-front models of hadrons make possible the calculation of many observables, with the advantage of working at the amplitude level obtained directly from matrix elements taken between hadron light-front wave functions. The valence component of the wave function allows a direct interpretation of the relevant model parameters, being also an useful concept when analytical forms of the hadron Bethe-Salpeter amplitude are the starting point to calculate the physical quantities. Applications of light-front models have been performed in many cases like in the calculation of pseudoscalar meson decay constants [11], electromagnetic form factors of the $\rho$-meson [15], pion [16,17] and kaon [18]. In particular constituent light-front quark models (CLFQM), are successfully in describing the pion electromagnetic form factor compared with the new experimental data [19] (see e.g. [20] and references therein).

One important theoretical aspect for the calculation of electroweak form factors performed within the light-front approach, is the inclusion of the non-valence contribution besides the valence one when the Drell-Yan condition is not
respected, as in the case of the computation of time-like from elastic and transition factors (see a discussion in \[20\]). The nonvalence contribution to the electroweak matrix elements of the current are associated with Fock-components of the wave function beyond the valence, that can be as well attributed to two-body currents acting on the valence wave function \[21,22\]. Notably, in some particular light-front models, the non-valence contribution should be included in the calculation of electromagnetic current matrix elements to satisfy the requirements of covariance in the Drell-Yan frame \[16\]. Moreover, in calculations of electroweak observables within light-front frameworks, apart from the frame used, it is important to note that the contribution of nonvalence terms depends on which component of current is employed to extract the form factors (see \[10,23\] for details).

Recently, we have proposed an analytical covariant model of the Bethe-Salpeter amplitude of mesons \[13\] and we have used a Wick rotation of the light-front energy in the Mandelstam formula to calculate electromagnetic form factors (LFCM). The Wick rotation applied to the minus component of the loop-momentum allows a direct four-dimensional integration avoiding the singularities of the Bethe-Salpeter amplitudes in Minkowski space in the calculation of the observables. The computation takes care implicitly, all possible zero-modes or nonvalence contributions to the matrix elements of the electromagnetic current. We have used the above model and calculational strategy to obtain electromagnetic form factors in the space-like regime and also decay constants of light and heavy pseudoscalar mesons.

Once elastic form factors of pseudoscalars have been calculated in the model, it is worthwhile to pursue the investigation of transition form factors in heavy meson decay. The semileptonic decays of the heavy mesons offer an useful laboratory to test theoretical models of the corresponding transition form factors in the weak sector of the standard model. Here, the covariant Bethe-Salpeter vertex model and computational strategy using Wick-rotation on the light-front are applied to calculate the electroweak transition form factors, \(f_+(q^2)\) and \(f_-(q^2)\) for \(D\) meson decays into a pion or a kaon. In sect. 2, we describe the model and give their parameters. In sect. 3, the form factors and kinematics for the decay are presented. The numerical results and conclusions are presented in sect. 4.

2. The vertex model

The vertex model of the Bethe-Salpeter amplitude for the meson-\(q-\bar{q}\) proposed in reference \[13\] is given by the following expression:

\[
\Lambda_M(k, p) = \frac{(k^2 - m_q^2)((p - k)^2 - m_Q^2)}{\Gamma_M} \times \frac{(k^2 - \lambda_M^2 + \mu)^n}{((p - k)^2 - \lambda_M^2 + \mu)^n}.
\]

Here, \(\lambda_M\) is the scale associated with the meson light-front valence wave function and \(n\) is the power of the regulator. The masses of the constituent quarks within the meson state are \(m_q\) and \(m_Q\). In the vertex function, the factors in the numerator avoid the cut due \(q\bar{q}\) scattering if \(m_q + m_Q < M_h\). In order to confine the quarks, the regulator scale \(\lambda_M\) is chosen such that \(\lambda_M > M_h/2\), where \(M_h\) is the meson mass. This constraint does not allow the meson decay into particles of mass \(\lambda_M\).

The general matrix elements for the electromagnetic current, are written as a tree-point function, using the Mandelstam formula, given in the equation below:

\[
\langle p' \mid J^\mu \left( q^2 \right) \mid p \rangle = \frac{N_c}{(2\pi)^2} \int d^4k \ Tr [\mathcal{O}] F(k, p, p'). \tag{2}
\]

where the operator \(\mathcal{O}\) is

\[
\mathcal{O} = \left[ S_F(k - p') \gamma^\mu S_F(k - p) S_F(k) \right].
\]

\(N_c S_F(p)\) is the Feynman propagator for constituent quarks and

\[
F(k, p, p') = \Lambda_M (k, p', p') \Lambda_M (k, p).
\]

The electromagnetic form factor for pseudoscalar mesons is obtained from the matrix element of the conserved vector current as:

\[
\langle p' \mid J^\mu \left( q^2 \right) \mid p \rangle = (p + p')^\mu F_{PS}^{\gamma\mu}(q^2), \tag{3}
\]

with the choice of plus component of the current, \(J^+ = J^0 + J^3\), that has the minimal contribu-
Table 1

|     | Model | \( r_{ps} \) fm | \( \lambda_M \) MeV | \( m_q \) MeV | \( m_Q \) MeV |
|-----|-------|------------------|-----------------|-------------|-------------|
| \( \pi^\pm \) | P3    | 0.296            | 3564            | 210         | 210         |
|      | P4    | 0.350            | 2995            | 215         | 215         |
|      | P5    | 0.392            | 2666            | 220         | 220         |
| \( K^\pm \) | K1    | 0.344            | 3056            | 203         | 327         |
|      | K2    | 0.349            | 3002            | 204         | 328         |
|      | K3    | 0.380            | 2744            | 210         | 334         |
|      | K4    | 0.400            | 2593            | 215         | 339         |
|      | K5    | 0.417            | 2478            | 220         | 344         |
| \( D^\pm \) | D1    | 0.341            | 2163            | 203         | 1442        |
|      | D2    | 0.343            | 2162            | 204         | 1443        |
|      | D3    | 0.344            | 2153            | 210         | 1449        |
|      | D4    | 0.345            | 2146            | 215         | 1454        |
|      | D5    | 0.346            | 2138            | 220         | 1459        |

The plus component of the axial current is adopted as discussed above for the electromagnetic current. The numerical integration in the Euclidean minus momentum allows one to use, equally well, any other component.

The vertex model parameters for the pion, kaon and \( D^+ \) are obtained from the decay constants for given constituent quark masses as presented in table 1. Our choice of constituent quark masses follows the suggestion that the chiral symmetry breaking mechanism produces the light constituent quark mass, and the other quarks like \( s \) and \( c \) have in addition the current quark mass. In our case we use the current quark masses of 124 MeV and 1239 MeV for \( s \) and \( c \), respectively.

We follow the reasonings presented in [24], and the current quark masses are within the accepted range of values (see [1]).

3. Transition Form Factors

The transition form factors of the weak vector current investigated here appears in the semileptonic processes \( D \rightarrow \pi l \nu \) and \( D_s \rightarrow K l \nu \) decays. In particular, the leptonic decay of \( D_s \) allows to extract the decay constant. In the
the final state. The matrix element of the weak transition from lattice and dispersion relation calculations [26].

semileptonic decay of a heavy pseudoscalar meson to a light one, the nonperturbative dynamics of QCD appears through the matrix elements of the weak vector current parameterized in terms of two transitions form factors, $f_+(q^2)$ and $f_-(q^2)$:

$$\langle \pi(K)|V^\mu|D(D_s)\rangle = (P' + P)^\mu f_+(q^2) + f_-(q^2)q^\mu,$$

where $V^\mu = \bar{q}_\gamma^\mu c$, with $q = (d, s)$, and $q^\mu = (P' - P)^\mu$. $P^\mu$ stands for the four-momentum of the D meson and $P'^\mu$ for the light pseudoscalar in the final state. The matrix element of the weak vector current can be written also as:

$$\langle \pi(K)|V^\mu|D(D_s)\rangle =$$

$$f_+(q^2)\left[(P' + P)^\mu - \frac{M^2_{D(D_s)} - M^2_{\pi(K)}}{q^2}q^\mu\right] + f_0(q^2)\frac{M^2_{D(D_s)} - M^2_{\pi(K)}}{q^2}q^\mu,$$

when the momentum transfer vanishes $f_+(0) = f_0(0)$. In the calculation of transition form factors, we use the plus component of the weak vector current.

The reference frame used in the case of the elastic electromagnetic form factors is defined by [20]: $P^+ = P'^+ + q^+$, $q^+ \geq 0$, $P_\perp = \overline{P}_\perp = 0$ and $P'^2 = P^2 = M^2_h$. For the decay process, we use the following reference frame: $P^+ = P'^+ - q^+$, $q^+ < 0$ and $P_\perp = \overline{P}_\perp = 0$. The numerical integration of the matrix elements is done with the Wick rotation of the minus momentum as formulated in [13].

4. Results and Conclusion

In the LFCM models used here $n = 10$, we have chosen the range parameters $\lambda_M$ by fitting the experimental pseudoscalar decay constants, the are presented in the table 1, together with the charge radius. The experimental pion charge radius is 0.672 fm [19] much larger than our results, that comes as a consequence of the regulator power $n = 10$. The same is observed for the kaon which has an experimental charge radius of 0.56 fm [19].

| Source      | $f^{\pi \to K}_{L}(q^2)$ | $f^{\pi \to K}_{T}(q^2)$ |
|-------------|---------------------------|---------------------------|
| BES [3]     | 0.780±                    | 0.730±                    |
| Belle [4]   | 0.695±0.023               | 0.624±0.036               |
| BaBar [5]   | 0.728±                    | -                         |
| CLEO [6]    | 0.739±0.009               | 0.665±0.019               |
| CLEO-c [7]  | 0.763±                    | 0.628±                    |
| LQCD [8]    | 0.733±                    | 0.643±                    |
| Model       | $f^{\pi \to K}_{L}(q^2)$  | $f^{\pi \to K}_{T}(q^2)$  |
| (K1,D1, - ) | 0.729                    | -                         |
| (K2,D2, - ) | 0.772                    | -                         |
| (K3,D3,P3)  | 0.913                    | 0.491                     |
| (K4,D4,P4)  | 1.020                    | 0.739                     |
| (K5,D5,P5)  | 1.100                    | 0.936                     |

Table 2

Experimental, lattice and model transition form factor results for $q^2 = 0$.

We should observe that smaller power in the vertex function $\Lambda_M(k, p)$ give a good fit for low energy observables calculated with this model [13] and describe well the electromagnetic form factors of the pion and kaon [13]. But, in the case of the transition electromagnetic form factors we need the higher power $n$, in order to reproduce the experimental transition form factor $f_+(q^2)$ in the low momentum region (see figures 1 and 2). While the slope near $q^2 = 0$ seems reasonable, the calculation underestimate the experimental data for large momentum transfers for the range
of constituent quark masses used. The effect of a light quark mass variation of 10% is seen mainly at large momentum transfers.

In table 2 we show transition form factors $f_+(0)$ for $D$ and $D_s$ compared to the experimental data and to lattice calculation. The sensitivity to the constituent quark mass is strong, and no simultaneous fit of $D$ and $D_s$ transition form factor at zero momentum transfers was achieved by the model. The small sizes of the pion and kaon are presumably responsible for the missing curvature demanded at large momentum transfers by the data. The large power implied in a high value of the scale $\lambda_M$ of about 3 GeV for the light mesons, to compensate for the power $n=10$ while fitting the decay constant. This hard scale as seen in table 1 shrinks both pion and kaon, as well as the D meson. The overlap between the wave function represented by $f^+(0)$ and the slope of the transition form factor seems not that sensitive to these hard scales but to the physical decay constants. Therefore, the vertex has to have a much softer dependence in momentum, and a smaller power should be required for a better fitting of these observables. Indeed the expected behavior of the vertex function with momentum should have a power about $n=2$, as expected from the one-gluon exchange and the monopole decay of the pion electromagnetic form factor. Therefore, it is expected that even the $D$ meson should be somewhat larger than the present calculations indicate of a charge radius of 0.34 fm, in order to accommodate a reasonable fit to the transition form factors.

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