Cosmological constraints from H II starburst galaxy, quasar angular size, and other measurements

Shulei Cao,1⋆ Joseph Ryan,2† Bharat Ratra1‡

1Department of Physics, Kansas State University, 116 Cardwell Hall, Manhattan, KS 66502, USA
2Department of Physics, Southern Methodist University, Dallas, TX 75275, USA

Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT
We compare the constraints from two (2019 and 2021) compilations of H II starburst galaxy (H II G) data and test the model-independence of quasar angular size (QSO) data using six spatially flat and non-flat cosmological models. We find that the new 2021 compilation of H II G data generally provides tighter constraints and prefers lower values of cosmological parameters than those from the 2019 H II G data. QSO data by themselves give relatively model-independent constraints on the characteristic linear size, l_m, of the QSOs within the sample. We also use Hubble parameter (H(z)), baryon acoustic oscillation (BAO), Pantheon Type Ia supernova (SN Ia) apparent magnitude (SN-Pantheon), and DES-3yr binned SN Ia apparent magnitude (SN-DES) measurements to perform joint analyses with H II G and QSO angular size data, since their constraints are not mutually inconsistent within the six cosmological models we study. A joint analysis of H(z), BAO, SN-Pantheon, SN-DES, QSO, and the newest compilation of H II G data provides almost model-independent summary estimates of the Hubble constant, H_0 = 69.7 ± 1.2 km s^{-1} Mpc^{-1}, the non-relativistic matter density parameter, Ω_m = 0.293 ± 0.021, and l_m = 10.93 ± 0.25 pc.

Key words: cosmological parameters – dark energy – cosmology: observations

1 INTRODUCTION
Many observations indicate that the Universe is currently in a phase of accelerated expansion, however, the theory behind this is not yet well-established. Although the spatially flat ΛCDM model (Peebles 1984) is consistent with most observations (see e.g. Farooq et al. 2017; Scoclin et al. 2018; Planck Collaboration 2020; eBOSS Collaboration 2021), some potential observational discrepancies and theoretical puzzles (see e.g. Di Valentino et al. 2021a; Perivolaropoulos & Skara 2021) suggest that there still is room for other cosmological models, including, for example, non-flat ΛCDM (the Planck Collaboration 2020 cosmic microwave background (CMB) anisotropy TT,TE,EE+lowE+lensing data favor positive spatial curvature) as well as dynamical dark energy. These discrepancies and puzzles motivate us to also study dynamical dark energy models and spatially non-flat models in this paper.

In this paper we use the new González-Morán et al. (2021) H II starburst galaxy (H II G) measured fluxes and inferred absolute luminosities (from their correlation with their measured ionized gas velocity dispersions) as standard candles to constrain cosmological models.2 These new H II G data reach to a slightly higher redshift z ~ 2.5, somewhat higher than the baryon acoustic oscillation (BAO) standard ruler data that reach to z ~ 2.3, that we also use in this paper. In order to determine the expansion rate and geometry of the Universe, it is vital to measure distances using either standard candles or standard rules. More data sets probing wider redshift regions would provide more information and make more contributions to a better understanding of our Universe, so it is worthwhile to seek additional stan-
data should yield tighter constraints that have the potential to soon usefully probe the largely unexplored $2 \lesssim z \lesssim 8$ part of cosmological redshift space.

Our comparisons here between the constraints from the new González-Morán et al. (2021) data and the old González-Morán et al. (2019) data show that the new data provide more restrictive constraints on most cosmological parameters. As noted above, QSO angular size data provide relatively cosmological model-independent estimates of $l_m$. We find that the cosmological constraints from $H(z)$, BAO, SN Ia, QSO, and the new $H$ measurements are not mutually inconsistent, thus we combine them to provide more restrictive constraints on the cosmological and nuisance parameters. The almost model-independent summary constraints from this data combination are measurements of the Hubble constant, $H_0 = 69.7 \pm 1.2$ km s$^{-1}$ Mpc$^{-1}$, the non-relativistic matter density parameter, $\Omega_{m0} = 0.293 \pm 0.021$, and the QSO characteristic linear size, $l_m = 10.93 \pm 0.25$ pc. The estimate of $H_0$ is in better agreement with the median statistics estimate of Chen & Ratra (2011) ($H_0 = 68 \pm 2.8$ km s$^{-1}$ Mpc$^{-1}$) than with the measurements of Planck Collaboration (2020) ($H_0 = 67.4 \pm 0.5$ km s$^{-1}$ Mpc$^{-1}$) and Riess et al. (2021) ($H_0 = 73.2 \pm 1.3$ km s$^{-1}$ Mpc$^{-1}$). Although the most-favored model is the spatially-flat $\Lambda$CDM model, there is room for some mild dark energy dynamics and a little non-zero spatial curvature energy density. We also find that currently accelerating cosmological expansion is favored by most of the data combinations we study (except for QSO data alone).

This paper is organized as follows. The models we study are briefly described in Section 2. The data we used are introduced in Section 3 with the data analysis method presented in Section 4. We summarize our results and conclusions in Sections 5 and 6.

## 2 COSMOLOGICAL MODELS

We use various combinations of observational data to constrain the cosmological parameters of six spatially-flat and non-flat $\Lambda$CDM, XCDM, and $\phi$CDM models and study the goodness of fit. The main features of the models we use are summarized below. We assume a minimal neutrino sector, with three massless neutrino species, with the effective number of relativistic neutrino species $N_{\text{eff}} = 3.046$. We neglect the late-time contribution of non-relativistic neutrinos and treat the baryonic ($\Omega_b h^2$) and cold dark matter ($\Omega_c h^2$) energy density parameters as free cosmological parameters to be determined from the data. The non-relativistic matter density parameter $\Omega_{m0} = (\Omega_b h^2 + \Omega_c h^2)/h^2$ is a derived parameter.

In the $\Lambda$CDM models, the expansion rate function

---

3 However the current QSO compilation is standardizable up to only $z \sim 1.5-1.7$ (Khadka & Ratra 2021a,a).

4 Only a smaller sample of 118 GRBs is reliable enough to be used for cosmological purposes, but include GRBs that probe to $z \sim 8.2$ (Khadka & Ratra 2020c; Khadka et al. 2021a).

---

5 For recent observational constraints on spatial curvature see Farooq et al. (2015), Chen et al. (2016), Rana et al. (2017), Ooba et al. (2018a,c), Yu et al. (2018), Park & Ratra (2019a,c), Wei (2018), DES Collaboration (2019a), Li et al. (2020), Handley (2019), Efstathiou & Gratton (2020), Di Valentino et al. (2021b), Velasquez-Toribio & Fabris (2020), Vagnozzi et al. (2021a,b), KIDS Collaboration (2021), Arjona & Nesseris (2021), DHawan et al. (2021), and references therein.
where
\[ \Omega_{\Lambda} = 1 - \Omega_m - \Omega_k, \]
with \( \Omega_k \) being the curvature energy density parameter. There are four free parameters: \( h, \Omega_m h^2, \Omega_c h^2, \) and \( \Omega_k \) in the non-flat LCDM case and three in the flat case where \( \Omega_k = 0 \).

In the XCDM parametrizations, the expansion rate function is
\[ E(z) = \sqrt{\Omega_m (1 + z)^3 + \Omega_k (1 + z)^2 + \Omega_X (1 + z)^3(1+w_X)}, \]
where \( w_X \) is the equation of state parameter of the X-fluid, and
\[ \Omega_X = 1 - \Omega_m - \Omega_k. \]
These consist of 120 measurements of the angular size
\[ \theta(z) = \frac{l_m}{D_A(z)}, \]
where \( l_m \) is the characteristic linear size of QSOs in the sample and \( D_A \) (defined below) is the angular size distance. Here we improve on the approach of Cao et al. (2017), Ryan et al. (2019), and Cao et al. (2020, 2021a,b), by using \( l_m \) as a nuisance parameter to be determined from these measurements so that these QSO data are independent of \( H(z) \) data.

The correlation between H"iI/G luminosity \( (L) \) and velocity dispersion \( (\sigma) \) is
\[ \log L = \beta \log \sigma + \gamma, \]
where \( \beta \) and \( \gamma \) are the slope and intercept. \( \log = \log_{10} \) is implied everywhere. Both H"iI/G data sets are corrected for extinction by using the Gordon et al. (2003) extinction law, with
\[ \beta = 5.022 \pm 0.058, \]
and
\[ \gamma = 33.268 \pm 0.083. \]
A detailed description of how to use H"iI/G data can be found in Cao et al. (2020). Note that the systematic uncertainties

data to those from earlier H"iI/G data. We also use these H"iI-G-2021 data and BAO, \( H(z) \), SN Ia, and QSO angular size measurements to constrain cosmological parameters in the models we study.

The SN-Pantheon data we use consist of 1048 SN Ia measurements, spanning the redshift range \( 0.01 < z < 2.3 \), compiled in Scolnic et al. (2018). The SN-DES data we use consist of 20 binned measurements (of 207 SN Ia measurements), spanning the redshift range \( 0.015 \leq z \leq 0.7026 \), compiled in DES Collaboration (2019c). See Cao et al. (2021b) for a description of how we use these SN Ia data.

The QSO data we use, that span the redshift range \( 0.462 \leq z \leq 2.73 \), are listed in Table 1 of Cao et al. (2017). These consist of 120 measurements of the angular size
\[ \theta(z) = \frac{l_m}{D_A(z)}, \]
where \( l_m \) is the characteristic linear size of QSOs in the sample and \( D_A \) (defined below) is the angular size distance. Here we improve on the approach of Cao et al. (2017), Ryan et al. (2019), and Cao et al. (2020, 2021a,b), by using \( l_m \) as a nuisance parameter to be determined from these measurements so that these QSO data are independent of \( H(z) \) data.

The old H"iI/G data (which we dub "H"iI-G-2019") consist of 107 low redshift measurements that span \( 0.0088 \leq z \leq 0.16417 \), used in Chávez et al. (2014) (recalibrated by González-Morán et al. 2019), and 46 high redshift measurements that span \( 0.636427 \leq z \leq 2.42935 \). The new H"iI/G-2021 data, comprising the original 107 low redshift measurements and 74 updated high redshift measurements (that now span \( 0.636427 \leq z \leq 2.545 \)), are listed in Table A3 of González-Morán et al. (2021).

The correlation between H"iI/G luminosity \( (L) \) and velocity dispersion \( (\sigma) \) is
\[ \log L = \beta \log \sigma + \gamma, \]
where \( \beta \) and \( \gamma \) are the slope and intercept. \( \log = \log_{10} \) is implied everywhere. Both H"iI/G data sets are corrected for extinction by using the Gordon et al. (2003) extinction law, with
\[ \beta = 5.022 \pm 0.058, \]
and
\[ \gamma = 33.268 \pm 0.083. \]
A detailed description of how to use H"iI/G data can be found in Cao et al. (2020). Note that the systematic uncertainties

\[ E(z) \equiv \frac{H(z)}{H_0} \text{as a function of redshift } z \text{ is} \]
\[ E(z) = \sqrt{\Omega_m (1 + z)^3 + \Omega_k (1 + z)^2 + \Omega_X (1 + z)^3(1+w_X)}. \]

where
\[ \Omega_X = 1 - \Omega_m - \Omega_k. \]
These equations are a scale factor and an overdot denotes a time derivative, the prime denotes a derivative with respect to the argument, \( m_p \) is the Planck mass, the parameter \( \alpha \geq 0 \), and the constant \( \kappa \) can be treated as a shooting parameter which is determined by the shooting method implemented in the Cosmic Linear Anisotropy Solving System (CLASS) code (Blas et al. 2011).

There are five free parameters: \( h, \Omega_m h^2, \Omega_c h^2, \Omega_k \), and \( \alpha \) in the non-flat LCDM case and four in the flat case where \( \Omega_k = 0 \).

3 DATA

In this paper our main focus is on a new set of H"iI/G data (González-Morán et al. 2021), which we dub “H"iI-G-2021”). We compare cosmological constraints from these H"iI/G-2021

6 For recent observational constraints on the LCDM model see Avsajianishvili et al. (2015), Solá Peracaula et al. (2018, 2019), Zhai et al. (2017), Ooba et al. (2018b, 2019), Park & Ratra (2018, 2019b, 2020), Sangwan et al. (2018), Singh et al. (2019), Ureña-López & Roy (2020), Sinha & Banerjee (2021), and references therein.

7 These measurements were taken from Simon et al. (2005), Stern et al. (2010), Moresco et al. (2012), Zhang et al. (2014), Moresco (2015), Moresco et al. (2016), and Ratsimbazafy et al. (2017).

8 These measurements were taken from Alam et al. (2017), Ata et al. (2018), Carter et al. (2018), DES Collaboration (2019b), and du Mas des Bourboux et al. (2020).
of both H iG and QSO data are not considered so that the reduced \( \chi^2 \) is relatively large.

The transverse comoving distance \( D_M(z) \), the luminosity
distance \( D_L(z) \), and the angular size distance \( D_A(z) \) are
related through \( D_M(z) = D_L(z)/(1 + z) = (1 + z)D_A(z) \).

\[
D_M(z) = \begin{cases} 
\frac{D_L(z)}{H_0} \sinh \left[ \sqrt{\Omega_b} H_0 D_L(z)/c \right] & \text{if } \Omega_b = 0, \\
\frac{D_L(z)}{H_0} \sin \left[ \sqrt{\Omega_b} H_0 D_L(z)/c \right] & \text{if } \Omega_b > 0, \\
0 & \text{if } \Omega_b < 0,
\end{cases}
\]

where

\[ D_L(z) \equiv c \int_0^z \frac{dz'}{H(z')} \]

with \( c \) being the speed of light (Hogg 1999).

4 DATA ANALYSIS METHODOLOGY

In this paper we use the \textsc{class} code to compute cosmo-
ological model predictions as a function of the cosmological
model and other parameters. These predictions are com-
pared to observational data using the Markov chain Monte
Carlo (MCMC) code \textsc{MontePython} (Audren et al. 2013)
to maximize the likelihood function, \( L \), and thereby
determine the best-fitting values of the free parameters. The pri-
ors on the cosmological parameters are flat and nonzero over the
same ranges as used in Cao et al. (2021b), except that now \( \Omega_b h^2 \in [0.00499, 0.03993] \). The prior range of the QSO
nuisance parameter \( l_{in} \) is not bounded.

The computation of the likelihood functions of \( H(z) \),
BAO, H iG, and QSO data are described in Cao et al. (2020)
and Cao et al. (2021a), whereas of that of the likelihood func-
tions of SN Ia measurements can be found in Cao et al.
(2021b). One can also find the definitions of the Akaike In-
formation Criterion (\( AIC \)) and the Bayesian Information
Criterion (\( BIC \)) in those papers.

5 RESULTS

The posterior one-dimensional (1D) probability distribu-
tions and two-dimensional (2D) confidence regions of the
cosmological and nuisance parameters for the six flat and
non-flat models are shown in Figs. 1–6, in gray (QSO), pink
(H iG-2019), green (H iG-2021), blue (H(z) + BAO), red
(H(z) + BAO + SN-Pantheon + SN-DES, H iBSNPD), and
purple (H(z) + BAO + SN-Pantheon + SN-DES + QSO
+ H iG-2021, H iBSNPDQH). We list the un marginalized
best-fitting parameter values, as well as the corresponding
\( \chi^2 \), \( AIC \), \( BIC \), degrees of freedom \( \nu (\nu \equiv N - n) \), reduced
\( \chi^2 (\chi^2/\nu) \), \( \Delta \chi^2 \), \( \Delta AIC \), and \( \Delta BIC \) for all models and data

9 The value of primordial Helium abundance \( Y_p \) is set using a
standard big-bang nucleosynthesis prediction by interpolation on
a grid of values computed using version 1.2 of the \textsc{PardiEnoPE}
BBN code for a neutron lifetime of 880.2 s. Since we choose the ef-
effective number of relativistic neutrino species \( N_{eff} = 3.046 \), \( \Omega_b h^2 \)
is therefore limited to the range of [0.00499, 0.03993] by the cor-
related predictions of \( Y_p \).

10 We use the \textsc{python} package \textsc{getdist} (Lewis 2019) to deter-
mine the posterior means and uncertainties and to generate the
marginalized likelihood contours.
Table 1: Unmarginalized best-fitting parameter values for all models from various combinations of data.

| Model          | H(z) + BAO | H(z) + BAO + SN-Pantheon | H(z) + BAO + SN-DES | H(z) + BAO + SN-Pantheon + SN-DES + QSO + HIC-2021 |
|---------------|------------|--------------------------|---------------------|-----------------------------------------------------|
| Flat ΛCDM     | H(z) + BAO | H(z) + BAO + SN-Pantheon | H(z) + BAO + SN-DES | H(z) + BAO + SN-Pantheon + SN-DES + QSO + HIC-2021 |
| HIC-2021      | H(z) + BAO | H(z) + BAO + SN-Pantheon | H(z) + BAO + SN-DES | H(z) + BAO + SN-Pantheon + SN-DES + QSO + HIC-2021 |
| HIC-2021      | H(z) + BAO | H(z) + BAO + SN-Pantheon | H(z) + BAO + SN-DES | H(z) + BAO + SN-Pantheon + SN-DES + QSO + HIC-2021 |
| HIC-2021      | H(z) + BAO | H(z) + BAO + SN-Pantheon | H(z) + BAO + SN-DES | H(z) + BAO + SN-Pantheon + SN-DES + QSO + HIC-2021 |
| HIC-2021      | H(z) + BAO | H(z) + BAO + SN-Pantheon | H(z) + BAO + SN-DES | H(z) + BAO + SN-Pantheon + SN-DES + QSO + HIC-2021 |
| HIC-2021      | H(z) + BAO | H(z) + BAO + SN-Pantheon | H(z) + BAO + SN-DES | H(z) + BAO + SN-Pantheon + SN-DES + QSO + HIC-2021 |
| HIC-2021      | H(z) + BAO | H(z) + BAO + SN-Pantheon | H(z) + BAO + SN-DES | H(z) + BAO + SN-Pantheon + SN-DES + QSO + HIC-2021 |
| HIC-2021      | H(z) + BAO | H(z) + BAO + SN-Pantheon | H(z) + BAO + SN-DES | H(z) + BAO + SN-Pantheon + SN-DES + QSO + HIC-2021 |
| HIC-2021      | H(z) + BAO | H(z) + BAO + SN-Pantheon | H(z) + BAO + SN-DES | H(z) + BAO + SN-Pantheon + SN-DES + QSO + HIC-2021 |
| HIC-2021      | H(z) + BAO | H(z) + BAO + SN-Pantheon | H(z) + BAO + SN-DES | H(z) + BAO + SN-Pantheon + SN-DES + QSO + HIC-2021 |
| HIC-2021      | H(z) + BAO | H(z) + BAO + SN-Pantheon | H(z) + BAO + SN-DES | H(z) + BAO + SN-Pantheon + SN-DES + QSO + HIC-2021 |
| HIC-2021      | H(z) + BAO | H(z) + BAO + SN-Pantheon | H(z) + BAO + SN-DES | H(z) + BAO + SN-Pantheon + SN-DES + QSO + HIC-2021 |
| HIC-2021      | H(z) + BAO | H(z) + BAO + SN-Pantheon | H(z) + BAO + SN-DES | H(z) + BAO + SN-Pantheon + SN-DES + QSO + HIC-2021 |
| HIC-2021      | H(z) + BAO | H(z) + BAO + SN-Pantheon | H(z) + BAO + SN-DES | H(z) + BAO + SN-Pantheon + SN-DES + QSO + HIC-2021 |
km s$^{-1}$ Mpc$^{-1}$ (Chen & Ratra 2011), and 0.21σ and 0.33σ lower than the local Hubble constant measurement of $H_0 = 73.2 \pm 1.3$ km s$^{-1}$ Mpc$^{-1}$ (Riess et al. 2021). The lowest $H_0$ estimates are in the non-flat φCDM model and are 70.49 ± 1.81 km s$^{-1}$ Mpc$^{-1}$ and 70.53 ± 1.79 km s$^{-1}$ Mpc$^{-1}$, which are 0.75σ and 0.76σ higher than the median statistics estimate of $H_0 = 68 \pm 2.8$ km s$^{-1}$ Mpc$^{-1}$, and 1.22σ and 1.21σ lower than the local Hubble constant measurement of $H_0 = 73.2 \pm 1.3$ km s$^{-1}$ Mpc$^{-1}$.

In the non-flat ΛCDM model, H iIG-2021 and H iIG-2019 data favor closed spatial hypersurfaces, while in the non-flat ΧCDM parametrization and the non-flat φCDM model, they favor open spatial hypersurfaces. Only in the non-flat φCDM model, however, is $\Omega_{m0}$ more than 1σ away from spatial flatness. Dark energy dynamics is favored by both data sets, but dark energy being a cosmological constant is not disfavored (it is within 1σ or just a little bit more away).

5.2 QSO constraints alone and in comparison to those from other probes

In this subsection we discuss the constraints we obtain solely from the QSO data. As mentioned in Sec. 3, in this paper we improve on earlier analyses of the QSO angular size data by now treating $l_m$, the characteristic linear size of QSOs, as a nuisance parameter to be determined from the observational data. From QSO data alone, in Table 2, $l_m$ ranges from a low of 10.26$^{+2.22}_{-1.42}$ pc for the non-flat φCDM model to a high of 11.90$^{+1.17}_{-1.52}$ pc for the flat ΧCDM parametrization, differing by just 0.38σ. These values are consistent from model to model, largely justifying the use of QSOs as standard rulers, with $l_m = 11.03$ pc, the value we used in our previous studies (taken from Cao et al. 2017). However, ignoring the dependence on cosmological model and the $l_m$ errors, as we and others have previously done, results in mildly biased and somewhat more restrictive ΩφCDM angular size constraints than is warranted by data. These deficiencies are corrected in our improved analyses here. Additionally, we note that in Table 1, the best-fitting values of $H_0$ in flat ΛCDM and flat ΧCDM appear to be unreasonably low. This strange behavior is caused by the large values of $l_m$, which push the $H_0$ values lower to obtain locally minimized $\chi^2$ values. Specifically, from the form of the model-predicted angular size of a quasar,

$$\theta(z) \propto \frac{l_m H_0}{d_S(z)}$$

where $d_S(z) := H_0 D_S(z)$ and suppressing irrelevant parameters, we can see that a large value of $l_m$ requires a small value of $H_0$ in order to keep $\theta(z)$ constant. Since $l_m$ has an unbounded prior range, it can roam over a larger region of parameter space than $H_0$. It therefore has the freedom to move into regions of parameter space where its value is unusually large; if this happens, then $H_0$ must be made small to compensate. This is only a partial answer, since it does not account for the variation of $\Omega_{m0}$ (the effect of which is more complex, as $\Omega_{m0}$ is coupled to the redshift $z$ through the function $d_S(z)$), and so does not fully capture the behavior of $\theta(z)$ across all models, but it does give some insight into the apparently anomalously low values of $H_0$ that appear in some cases.

From the results listed in Table 2, we can draw the following conclusions. First, QSO data alone can only constrain the values of $\Omega_{m0}h^2$ in the flat and non-flat φCDM models. Second, QSO data alone prefer higher values of $\Omega_{m0}$, which are consistent with almost all other probes except for the non-flat φCDM H iIG-2021 case (the posterior mean values being 1.1σ away from each other in this case). Furthermore, QSO data alone do not give tight constraints on $H_0$ or $\Omega_{m0}$. Although in each non-flat model open geometry is favored, given the large error bars, flat geometry is within 1σ.

QSO data favor higher central values of $\Omega_{m0}h^2$ and $\Omega_{m0}$, in both flat and non-flat ΛCDM, compared to the central values favored by the other probes (although QSO constraints have wider error bars than the other constraints). QSO data only very weakly constrain the value of $H_0$ in the flat ΛCDM model, while the fit of QSO data to the non-flat ΛCDM model produces a tighter constraint whose central value is closer to that of the H iIG data and the local value favored by Riess et al. (2021) (with wide error bars, however). In both the flat and non-flat cases, the marginalized values of $l_m$ are close to the value obtained by Cao et al. (2017), with the central value in the flat ΛCDM model here being only 0.02 pc away from that of Cao et al. (2017) (with wider error bars than what they found). QSO data do not provide strong evidence for non-zero spatial curvature in the non-flat ΛCDM model, as the marginalized posterior mean value of $\Omega_{m0}$ is consistent with $\Omega_{m0} = 0$ to within 1σ.

When we look at the flat and non-flat ΧCDM parametrizations, we find that QSO data again favor somewhat large values of $\Omega_{m0}h^2$ and $\Omega_{m0}$ (but, as with flat and non-flat ΛCDM, these have wide error bars) and weak constraints on $H_0$. The central value of $H_0$ in the non-flat case is more consistent with Riess et al. (2021) and with the values derived from the H iIG data. In both cases we find that the marginalized values of $l_m$ are consistent with that of Cao et al. (2017). We also find that QSO data favor values of $\Omega_{m0}$ that are in the phantom regime (consistent with the findings from the H iIG data). In the non-flat case, QSO data favor a relatively large and positive central value (0.170) for $\Omega_{m0}$, corresponding to a spatially open universe, but the error bars are wide enough that this result is still consistent with spatial flatness.

Both the flat and non-flat φCDM models have central

11 When QSO data are combined with other probes, as in the H iBSNPDQH combination, the model-independence of $l_m$ is evident and the determination here is consistent with $l_m = 11.03 \pm 0.25$ pc found by Cao et al. (2017).

12 The relatively higher values of $H_0$ seen in the φCDM models pose an apparent challenge to this explanation, but here the best-fitting values of $\Omega_{m0}h^2$ and $\Omega_{m0}h^2$ need to be taken into account. In comparing, for example, the flat ΛCDM model to the flat φCDM model (both of which have nearly identical best-fitting values of $\Omega_{m0}$), we can see that the flat φCDM model has larger best-fitting values of both $\Omega_{m0}h^2$ and $\Omega_{m0}h^2$. From the defining relationship $\Omega_{m0} = (\Omega_{m0}h^2 + \Omega_{m0}h^2)/H_0^2$, keeping $\Omega_{m0}$ constant requires $\Omega_{m0}h^2 + \Omega_{m0}h^2$ and $H_0$ to vary in tandem. If $\Omega_{m0}h^2 + \Omega_{m0}h^2$ both increase, as they do in going from flat ΛCDM to flat φCDM, then $H_0$ must also increase. This then has the effect of lowering $l_m$ (all other parameters being held fixed).
values of $\Omega_0 h^2$ from QSO data that are similar to earlier findings (specifically, they are close to the values of $\Omega_0 h^2$ obtained for the flat $\Lambda$CDM and $\phi$CDM models by Park & Ratra 2018, 2019c). Both flat and non-flat $\phi$CDM have relatively high central values of $\Omega_0 h^2$ and $\Omega_m$ (compared to the other probes), both favor similar large values of $\alpha$ (consistent with $\alpha = 0$, however, to within 1.15$\sigma$ and 1.29$\sigma$ in the flat and non-flat cases, respectively), and both show weak constraints on $H_0$. Both flat and non-flat $\phi$CDM favor posterior mean values of $l_m$ that are consistent to within 1$\sigma$ with the central value obtained by Cao et al. (2017). Like non-flat XCDM, non-flat $\phi$CDM favors a relatively large and positive value of $\Omega_m$, that is nevertheless consistent with spatial flatness to within 1$\sigma$.

5.3 Joint analyses results

Since the constraints derived from $H(z)$, BAO, SN-Pantheon, SN-DES, QSO, H0I-G-2019, and H0I-G-2021 data are not mutually inconsistent, we jointly analyze combinations of these data and summarize these results in this subsection.

The $H(z) +$ BAO and HzBSNPD results are different from, but consistent with, what we obtained in Cao et al. (2021b). The differences arise from the different codes that we used to analyze the data; in Cao et al. (2021b) we used emcee, whereas here we used class and MontePython. It is worth recalling here that, as mentioned above, class constraints $\Omega_0 h^2$ in the range 0.00499 $\leq \Omega_0 h^2 \leq$ 0.03993. Therefore the parameter constraints differ more when the model and data prefer higher values of $\Omega_0 h^2$; this is especially true of the $\phi$CDM model when it is fitted to the $H(z) +$ BAO data combination. As a result, the present constraints on $\Omega_m$ and $\sigma$ in $\phi$CDM with $H(z) +$ BAO data are higher and lower than the ones given in Cao et al. (2021b). The HzBSNPD results are, however, consistent.

The fit to the HzBSNPDQHQ data combination produces, for all models, the most interesting results. By adding QSO and H0I-G-2021 data to HzBSNPD combination, the constraints are slightly tightened. Although the posterior means of $\Omega_0 h^2$ and $\Omega_m h^2$ are relatively higher, those of $\Omega_m$ are lower than the constraints from HzBSNPD. The $\Omega_m$ constraints range from a low of 0.282 $\pm$ 0.016 (flat $\phi$CDM) to a high of 0.298 $\pm$ 0.013 (flat $\Lambda$CDM), a difference of only 0.78$\sigma$.

The constraints on $H_0$ are between $H_0 = 69.54 \pm 1.17$ km s$^{-1}$ Mpc$^{-1}$ (flat $\phi$CDM) and $H_0 = 69.95 \pm 1.18$ km s$^{-1}$ Mpc$^{-1}$ (flat $\Lambda$CDM) — a difference of only 0.25$\sigma$ — which are 0.64$\sigma$ (flat $\Lambda$CDM) and 0.51$\sigma$ (flat $\phi$CDM) higher than the median statistics estimate of $H_0 = 68 \pm 2.8$ km s$^{-1}$ Mpc$^{-1}$ (Chen & Ratra 2011), and 1.85$\sigma$ (flat $\Lambda$CDM) and 2.09$\sigma$ (flat $\phi$CDM) lower than the local Hubble constant measurement of $H_0 = 73.2 \pm 1.3$ km s$^{-1}$ Mpc$^{-1}$ (Riess et al. 2021).\(^{13}\)

For non-flat $\Lambda$CDM, non-flat XCDM, and non-flat $\phi$CDM, we find $\Omega_m = 0.011 \pm 0.067$, $\Omega_0 = -0.054 \pm 0.096$, and $\Omega_{m0} = -0.072^{+0.074}_{-0.073}$, respectively. The non-flat XCDM and $\phi$CDM models favor closed geometry, while the non-flat $\Lambda$CDM model favors open geometry. Note, however, that these results are all consistent with spatially flat hypersurfaces to within 1$\sigma$.

Our results show a slight preference for dark energy dynamics. For flat (non-flat) XCDM, $w_X = -0.950 \pm 0.062$ ($w_X = -0.926^{+0.091}_{-0.066}$), with central values being 0.81$\sigma$ (1.19$\sigma$) away from $w_X = -1$; and for flat (non-flat) $\phi$CDM, $\alpha = 0.288^{+0.252}_{-0.252}$ ($\alpha = 0.405^{+0.165}_{-0.304}$), with central values being 1.14$\sigma$ (1.33$\sigma$) away from $\alpha = 0$.

Th constraints on the nuisance parameter $l_m$ are between $l_m = 10.87 \pm 0.26$ pc (non-flat $\phi$CDM) and $l_m = 10.96 \pm 0.26$ pc (flat $\Lambda$CDM), which differ by 0.2$\sigma$ and so are effectively model-independent, and consistent with $l_m = 11.03 \pm 0.25$ pc (Cao et al. 2017).

5.4 Model comparison

The values of the reduced $\chi^2$ ($\chi^2/\nu$, $\Delta \chi^2$, $\Delta AIC$, and $\Delta BIC$) are reported in Table 1, where $\Delta \chi^2$, $\Delta AIC$, and $\Delta BIC$ are the differences between the values of the $\chi^2$, $AIC$, and $BIC$ for a given model and the ones for flat $\Lambda$CDM. Here a negative (positive) value of $\Delta \chi^2$, $\Delta AIC$, or $\Delta BIC$ means that the given statistic favors (disfavors) the model under consideration relative to flat $\Lambda$CDM. We find that, except for a few of the $H(z) +$ BAO and H0I-G-2019 cases, the flat $\Lambda$CDM model is the most favored model among all six models we study. The $AIC$ does not show strong evidence against any of the models.\(^{14}\) However, we find that some data combinations show strong evidence against the models we study, when these models are analyzed using the $BIC$, as follows. First, the HzBSNPD combination strongly disfavors non-flat $\Lambda$CDM and very strongly disfavors non-flat XCDM and non-flat $\phi$CDM. Second, the HzBSNPDQHQ combination strongly disfavors non-flat $\Lambda$CDM, flat XCDM, and flat $\phi$CDM, and very strongly disfavors non-flat XCDM and non-flat $\phi$CDM. Furthermore, strong evidence against non-flat XCDM as well as non-flat $\phi$CDM are provided by the H0I-G-2021 and QSO data.

6 CONCLUSION

We find that the new H0I-G-2021 data provide more restrictive cosmological parameter constraints and also prefer lower values of $\Omega_m$, $w_X$, and $\Omega_0$ than those favored by the H0I-G-2019 data.

nations (Gott et al. 2001; Calabrese et al. 2012) as well as with other recent $H_0$ measurements (Chen et al. 2017; DES Collaboration 2018; Gómez-Valent & Amendola 2018; Planck Collaboration 2020; Dominguez et al. 2019; Cuceu et al. 2019; Zeng & Yan 2019; Schöneberg et al. 2019; Blum et al. 2020; Lun et al. 2020; Philcox et al. 2020; Birrer et al. 2020; Demzel et al. 2021; Pogosian et al. 2020; Boruah et al. 2021; Kim et al. 2020; Harvey 2020; Zhang & Huang 2021; Lin & Ishak 2021).\(^{14}\) There is weak evidence for the reference model when $\Delta AIC(BIC) \in [0, 2]$, positive evidence when $\Delta AIC(BIC) \in (2, 6]$, strong evidence when $\Delta AIC(BIC) \in (6, 10]$, and very strong evidence when $\Delta AIC(BIC) > 10$ (Kass & Raftery 1995).

\(^{13}\) Other local expansion rate $H_0$ measurements result in slightly lower central values with slightly larger error bars (Riess et al. 2015; Zhang et al. 2017; Dhawan et al. 2018; Fernández Areñas et al. 2018; Breuval et al. 2020; Efstathiou 2020; Khetan et al. 2021; Rameez & Sarkar 2021; Freedman 2021). Our $H_0$ estimates are consistent with earlier median statistics determini-
Table 2: One-dimensional posterior mean parameter values and uncertainties (±1σ error bars or 2σ limits) for all models from various combinations of data.

| Model         | Data set         | $\Omega_{\text{b}}h^2$ | $\Omega_{\text{m}}h^2$ | $\Omega_{\text{b}}$ | $w_\text{x}$ | $\alpha$ | $H_0$ | $t_m$ |
|---------------|------------------|-------------------------|-------------------------|---------------------|-------------|--------|-------|-------|
| Flat CDM      | $H(z) + \text{BAO}$ | 0.0241 ± 0.0029 | 0.1193 ± 0.0092 | 0.299 ± 0.017 | -- | -- | 69.29 ± 1.84 | -- |
|               | HtG-2019         | -- | 0.1258 ± 0.0287 | 0.289 ± 0.073 | -- | -- | 71.80 ± 1.94 | -- |
|               | HtG-2021         | 0.0225 ± 0.0108 | 0.1023 ± 0.0292 | 0.243 ± 0.051 | -- | -- | 71.91 ± 1.92 | -- |
|               | QSO              | 0.387 ± 0.0992 | 0.1874 ± 0.1955 | > 38.09 | -- | -- | 11.05 ± 1.85 | -- |
|               | HzBSNPDF         | 0.0237 ± 0.0028 | 0.1208 ± 0.0074 | 0.301 ± 0.013 | -- | -- | 69.10 ± 1.80 | -- |
|               | HzBSNPQH        | 0.0250 ± 0.0021 | 0.1208 ± 0.0064 | 0.298 ± 0.013 | -- | -- | 69.95 ± 1.18 | 10.96 ± 0.26 |
| Non-flat CDM  | $H(z) + \text{BAO}$ | 0.0253 ± 0.0040 | 0.1134 ± 0.0196 | 0.293 ± 0.025 | 0.40±0.102 | -- | 68.75 ± 2.45 | -- |
|               | HtG-2019         | 0.0224 ± 0.0108 | 0.1245 ± 0.0413 | 0.285 ± 0.077 | -0.05±0.295 | -- | 71.95 ± 2.04 | -- |
|               | HtG-2021         | 0.0225 ± 0.0108 | 0.1035 ± 0.0019 | 0.243 ± 0.055 | -1.00±0.238 | -- | 72.15 ± 2.07 | -- |
|               | QSO              | 0.387 ± 0.0992 | 0.1874 ± 0.1955 | > 38.09 | -- | -- | 11.05 ± 1.85 | -- |
|               | HzBSNPDF         | 0.0248 ± 0.0036 | 0.1157 ± 0.0164 | 0.296 ± 0.023 | 0.027 ± 0.074 | -- | 68.82 ± 2.02 | -- |
|               | HzBSNPQH        | 0.0256 ± 0.0042 | 0.1188 ± 0.0138 | 0.295 ± 0.021 | 0.011 ± 0.067 | -- | 69.90 ± 1.18 | 10.96 ± 0.25 |
| Flat XCDM     | $H(z) + \text{BAO}$ | 0.0297 ± 0.0046 | 0.0934 ± 0.0195 | 0.283 ± 0.023 | -- | -- | 65.85 ± 2.38 | -- |
|               | HtG-2019         | 0.0224 ± 0.0099 | 0.1419 ± 0.0287 | 0.327 ± 0.108 | -- | -- | 72.37 ± 2.18 | -- |
|               | HtG-2021         | 0.0223 ± 0.0108 | 0.1297 ± 0.0493 | 0.288 ± 0.087 | -- | -- | 72.66 ± 2.19 | -- |
|               | QSO              | 0.387 ± 0.0992 | 0.1874 ± 0.1955 | > 38.09 | -- | -- | 11.05 ± 1.85 | -- |
|               | HzBSNPDF         | 0.0256 ± 0.0033 | 0.1121 ± 0.0108 | 0.293 ± 0.016 | -- | -- | 68.57 ± 1.74 | -- |
|               | HzBSNPQH        | 0.0267 ± 0.0032 | 0.1142 ± 0.0103 | 0.290 ± 0.016 | -- | -- | 69.69 ± 1.20 | 10.94 ± 0.25 |
| Non-flat XCDM | $H(z) + \text{BAO}$ | 0.0288 ± 0.0048 | 0.0977 ± 0.0179 | 0.294 ± 0.027 | -1.12±0.136 | -- | 66.01 ± 2.43 | -- |
|               | HtG-2019         | 0.0223 ± 0.0107 | 0.1288 ± 0.0438 | 0.291 ± 0.109 | 0.088±0.484 | -- | 71.96 ± 2.06 | -- |
|               | HtG-2021         | 0.0224 ± 0.0107 | 0.1109 ± 0.0154 | 0.255 ± 0.090 | 0.078±0.322 | -- | 72.23 ± 2.08 | -- |
|               | QSO              | 0.387 ± 0.0992 | 0.1874 ± 0.1955 | > 38.09 | -- | -- | 11.05 ± 1.85 | -- |
|               | HzBSNPDF         | 0.0245 ± 0.0036 | 0.1193 ± 0.0170 | 0.302 ± 0.024 | -0.69±0.119 | -- | 68.95 ± 1.96 | -- |
|               | HzBSNPQH        | 0.0255 ± 0.0043 | 0.1189 ± 0.0135 | 0.297 ± 0.020 | -0.54±0.096 | -- | 69.73 ± 1.20 | 10.89 ± 0.26 |
| Flat φCDM     | $H(z) + \text{BAO}$ | 0.0323 ± 0.0060 | 0.0810 ± 0.0188 | 0.267 ± 0.025 | -- | -- | 65.09 ± 2.24 | -- |
|               | HtG-2019         | 0.0214 ± 0.0178 | 0.0561 ± 0.0154 | 0.157±0.047 | -- | -- | 70.97 ± 1.91 | -- |
|               | HtG-2021         | 0.0213 ± 0.0172 | 0.0468 ± 0.0131 | 0.135±0.043 | -- | -- | 71.16 ± 1.99 | -- |
|               | QSO              | 0.387 ± 0.0992 | 0.1874 ± 0.1955 | > 38.09 | -- | -- | 71.15 ± 1.98 | -- |
|               | HzBSNPDF         | 0.0273 ± 0.0032 | 0.1051 ± 0.0222 | 0.284 ± 0.017 | -- | -- | 68.33 ± 1.81 | -- |
|               | HzBSNPQH        | 0.0284 ± 0.0031 | 0.1078 ± 0.0112 | 0.282 ± 0.016 | -- | -- | 69.54 ± 1.17 | 10.92 ± 0.25 |

$^a km s^{-1} Mpc^{-1}$.
$^b$ pc.
$^c H(z) + \text{BAO} + \text{SN-Pantheon} + \text{SN-DES}$. 
$^d H(z) + \text{BAO} + \text{SN-Pantheon} + \text{SN-DES} + \text{QSO} + H \text{tG-2021}$. 

\[ \]
Figure 1. One-dimensional likelihoods and $1\sigma$, $2\sigma$, and $3\sigma$ two-dimensional likelihood confidence contours for flat $\Lambda$CDM. Left panel shows individual data set and $H(z) + BAO$ results and the right panel shows joint data sets constraints. The zero-acceleration lines are shown as black dashed lines in the left panel, which divide the parameter space into regions associated with currently-accelerating (left) and currently-decelerating (right) cosmological expansion, while in the right panel the joint analyses favor currently-accelerating expansion.

Figure 2. Same as Fig. 1 but for non-flat $\Lambda$CDM. The flat $\Lambda$CDM case is shown as the cyan dash-dot lines, with closed spatial hypersurfaces either below or to the left. The black dashed line in the left panel is the zero-acceleration line, which divides the parameter space into regions associated with currently-accelerating (below left) and currently-decelerating (above right) cosmological expansion. In the right panel, the joint analyses favor currently-accelerating expansion.
Figure 3. One-dimensional likelihoods and 1σ, 2σ, and 3σ two-dimensional likelihood confidence contours for flat XCDM. The black dashed line in the left panel is the zero-acceleration line, which divides the parameter space into regions associated with currently-accelerating (below left) and currently-decelerating (above right) cosmological expansion. In the right panel, the joint analyses favor currently-accelerating expansion. The magenta lines represent $w_X = -1$, i.e. the flat $\Lambda$CDM model.

Figure 4. Same as Fig. 3 but for non-flat XCDM, where the black dashed zero-acceleration lines are computed for the third cosmological parameter set to the $H(z) + BAO$ data best-fitting values listed in Table 1, with currently-accelerating cosmological expansion residing below left. The flat XCDM case is denoted as the cyan dash-dot lines, with closed spatial hypersurfaces either below or to the left. The magenta lines represent $w_X = -1$, i.e. the non-flat $\Lambda$CDM model. In all cases except for the QSO only case, almost all of the favored parameter space is associated with currently-accelerating cosmological expansion.
Figure 5. One-dimensional likelihoods and 1σ, 2σ, and 3σ two-dimensional likelihood confidence contours for flat φCDM. The black dashed lines are the zero-acceleration lines, which divides the parameter space into regions associated with currently-accelerating (below left) and currently-decelerating (above right) cosmological expansion. The α = 0 axis is the flat ΛCDM model. In all cases except for the QSO only case, almost all of the favored parameter space is associated with currently-accelerating cosmological expansion.

Figure 6. Same as Fig. 5 but for non-flat φCDM, where the black dashed zero-acceleration lines are computed for the third cosmological parameter set to the H(z) + BAO data best-fitting values listed in Table 1. Currently-accelerating cosmological expansion occurs below left of these lines. The cyan dash-dot lines represent the flat φCDM case, with closed spatial geometry either below or to the left. The α = 0 axis is the non-flat ΛCDM model. In the right panel, the joint analyses favor currently-accelerating expansion.
We find that the QSO characteristic linear size $l_m$ is relatively model-independent, so QSOs can be treated as approximate standard rulers but the uncertainty in $l_m$ must be accounted for in the analysis.

We also jointly analyzed a total of 1411 measurements, consisting of 31 $H(z)$, 11 BAO, 1048 SN-Pantheon, 20 SN-DES, 120 QSO, and 181 $H_{iG}$-2021 data points to constrain cosmological and nuisance parameters in six flat and non-flat DES, 120 QSO, and 181 $H_{iG}$-2021 (HzBSNP+DQH) data combination as follows. First, the constraint on $H_0$ from this independent summary features of the constraints obtained from this $H(z) + BAO + SN$-Pantheon + SN-DES + QSO + $H_{iG}$-2021 (HzBSNP+DQH) data combination as follows. Second, the constraint on $\Omega_m$ is $\Omega_m = 0.283 \pm 0.021$, which is in good agreement with many other recent measurements (e.g. $0.315 \pm 0.007$ from TTEEE+lowE+lensing CMB anisotropy data in the flat $\Lambda$CDM model of Planck Collaboration 2020). Third, the determination of $H_0$ is $H_0 = 69.7 \pm 1.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which is in better agreement with the estimate of Chen & Ratra (2011) than with the measurements of Planck Collaboration (2020) and Riess et al. (2021). There is some room for dark energy dynamics or a little spatial curvature energy density, but overall the flat $\Lambda$CDM model is the best candidate model.

ACKNOWLEDGEMENTS

We thank Javier de Cruz Pérez, Ana Luisa González-Morán, Narayan Khadka, and Chan-Gyung Park for useful discussions. This work was partially funded by Department of Energy grant DE-SC0011840. The computing for this project was performed on the Beocat Research Cluster at Kansas State University, which is funded in part by NSF grants CNS-1006860, EPS-1006860, EPS-0919443, ACI-1440548, and NIH P20GM113109.

DATA AVAILABILITY

The $H_{iG}$ data used in this article were provided to us by the authors of González-Morán et al. (2019, 2021). These data will be shared on request to the corresponding author with the permission of the authors of González-Morán et al. (2019, 2021).

REFERENCES

Alam S., et al., 2017, MNRAS, 470, 2617

Amati L., Guidorzi C., Frontera F., Della Valle M., Finelli F., Landi R., Montanari E., 2008, MNRAS, 391, 577

The following summary values are obtained with the same method used in Cao et al. (2021b), where we take the summary central value to be the mean of the two of six central-most values. As for the uncertainty, we call the difference between the two central-most values twice the systematic uncertainty and the average of the two central-most error bars the statistical uncertainty, and compute the summary error bar as the quadrature sum of the two uncertainties.

Amati L., D’Agostino R., Luongo O., Muccino M., Tantalo M., 2019, MNRAS, 486, L46

Arjona R., Nesseris S., 2021, Phys. Rev. D, 103, 103539

Ara M., et al., 2018, MNRAS, 473, 4773

Audren B., Lesgourgues J., Benabed K., Prunet S., 2013, J. Cosmology Astropart. Phys., 2013, 001

Avsajanishvili O., Samushia L., Arkhipova N. A., Kahliaishvili T., 2015, preprint, (arXiv:1511.09317)

Birrer S., et al., 2020, A&A, 643, A165

Blas D., Lesgourgues J., Tram T., 2011, J. Cosmology Astropart. Phys., 2011, 034

Blum K., Castorina E., Simonović M., 2020, ApJ, 892, L27

Borah S. S., Hudson M. J., Lavaux G., 2021, MNRAS, Breuval L., et al., 2020, A&A, 643, A115

Calabrese E., Archidiacono M., Melchiorri A., Ratra B., 2012, Phys. Rev. D, 86, 043520

Cao S., Zheng X., Biesiada M., Qi J., Chen Y., Zhu Z.-H., 2017, A&A, 606, A15

Cao S., Ryan J., Ratra B., 2020, MNRAS, 497, 3191

Cao S., Ryan J., Khadka N., Ratra B., 2021a, MNRAS, 501, 1520

Cao S., Ryan J., Ratra B., 2021b, MNRAS, 504, 300

Carter P., Beurier F., Percival W. J., Blake C., Koda J., Ross A. J., 2018, MNRAS, 481, 2371

Chávez R., Terlevich R., Terlevich E., Bresolin F., Melnick J., Plionis M., Baselakos S., 2014, MNRAS, 442, 3565

Chávez R., Plionis M., Baselakos S., Terlevich R., Terlevich E., Melnick J., Bresolin F., González-Morán A. L., 2016, MNRAS, 462, 2431

Chen G., Ratra B., 2003, ApJ, 582, 586

Chen G., Ratra B., 2011, PASP, 123, 1127

Chen Y., Ratra B., Biesiada M., Li S., Zhu Z.-H., 2016, ApJ, 829, 61

Chen Y., Kumar S., Ratra B., 2017, ApJ, 835, 86

Cuceu A., Farr J., Lemos P., Font-Ribera A., 2019, J. Cosmology Astropart. Phys., 2019, 044

Czerny B., et al., 2021, Acta Physica Polonica A, 139, 389

DES Collaboration 2018, MNRAS, 480, 3879

DES Collaboration 2019a, Phys. Rev. D, 99, 123505

DES Collaboration 2019b, MNRAS, 483, 4866

DES Collaboration 2020b, ApJ, 874, 150

Demianski M., Piedipalumbo E., Sawant D., Amati L., 2021, MNRAS, 506, 903

Denzel P., Coles J. P., Saha P., Williams L. L. R., 2021, MNRAS, 501, 784

Dhawan S., Jha S. W., Leibundgut B., 2018, A&A, 609, A72

Dhawan S., Alsing J., Vagnozzi S., 2021, MNRAS, 506, L1

Di Valentinio E., et al., 2021a, Classical and Quantum Gravity, 38, 153001

Di Valentinio E., Melchiorri A., Silk J., 2021b, ApJ, 908, L9

Domínguez A., et al., 2019, ApJ, 885, 137

du Mas des Bourboux H., et al., 2020, ApJ, 901, 153

eBOSS Collaboration 2021, Phys. Rev. D, 103, 083533

Efstathiou G., 2020, preprint, (arXiv:2007.10716)

Efstathiou G., Gratton S., 2020, MNRAS, 496, L91

Fana Dirirsia F., et al., 2019, ApJ, 887, 13

Farooq O., Mania D., Ratra B., 2015, Ap&SS, 357, 11

Farooq O., Ranjeet Madiyar F., Crandall S., Ratra B., 2017, ApJ, 835, 26

Fernández Arenas D., et al., 2018, MNRAS, 474, 1250

Freedman W. L., 2021, ApJ, 919, 16

Gómez-Valent A., Amendola L., 2018, J. Cosmology Astropart. Phys., 4, 051

González-Morán A. L., et al., 2019, MNRAS, 487, 4669

González-Morán A. L., et al., 2021, MNRAS, 505, 1441

Gordon K. D., Clayton G. C., Misselt K. A., Landolt A. U., Wolff M. J., 2003, ApJ, 594, 279

Gott III J. R., Vogele M. S., Podariu S., Ratra B., 2001, ApJ, 549, 1
