Analytical model of compact star in low-mass X-ray binary with de Sitter spacetime

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Abstract In this article, we suggest a Sitter model in favor of compact stars in low-mass X-ray binaries. Here, we have considered the presence of a cosmological constant (on a small scale) to investigate the stellar structure. We conclude that this approach is very suitable for the familiar physical model of compact stars in low-mass X-ray binaries. We calculate the probable radius, compactness ($u$) and surface redshift ($Z_s$) of six compact stars in low-mass X-ray binaries, namely Cyg X-2, V395 Carinae/2S 0921–630, XTE J2123–058, X1822–371 (V691 CrA), 4U 1820–30 and GR Mus (XB 1254–690). We also offer a possible equation of state (EOS) for the stellar objects.

Key words: stars: low-mass — stars: mass function — equation of state

1 INTRODUCTION

As compact stars (neutron stars/strange stars) play a crucial role in relating astrophysics, nuclear physics and particle physics, they have motivated many scientists to research the behaviour of highly dense stars. Common neutron stars are built almost entirely of neutrons whereas strange stars are supposed to be composed entirely of strange quark matter (SQM), or the associated conversion (up-down quarks to strange quarks) may be confined to the core of a neutron star (Haensel et al. 1986; Drago et al. 2014). It is well known that neutron stars are bound by gravitational attraction and strange stars are bound by strong interactions as well as gravitational attraction. Therefore, for lower mass neutron stars, the gravitational bound becomes much weaker than for strange stars. Hence, neutron stars become larger in size in comparison to strange stars with the same mass. All the present equations of state (EOSs) of neutron stars have zero surface matter density, whereas the available EOSs of strange stars produce a sharp change in surface density (Farhi & Jaffe 1984; Haensel et al. 1986; Alcock et al. 1986; Dey et al. 1998). A few seconds after the birth of a neutron star, its temperature decreases to less than the Fermi energy, hence for a given EOS the mass and radius of the star depend solely on central density and also it is very hard to find the mass and radius of a neutron star simultaneously. For a detailed study, we suggest the review work of Lattimer & Prakash (2007). Theoretical predictions of masses and radii of spherically-symmetric non-rotating compact stars are based on analytical or numerical solutions of the Tolman-Oppenheimer-Volkov equation, i.e., the TOV equation. From an observational point of view, some promising areas for measuring the mass and radius of compact stars (neutron stars/strange stars) are thermal emission from cooling stars, pulsar timing, surface explosions and gravitational emissions. Experimental scientists face the recent challenges using giant dipole resonances, heavy-ion collisions and parity-violating electron scattering techniques to dependably measure the density of nuclear matter. Actually, the most challenging task is to determine the proper EOS which describes the internal structure of neutron stars (Özel 2006; Özel et al. 2009; Özel & Psaltis 2009; Özel et al. 2010; Güver et al. 2010a, Güver et al. 2010b)). Though masses of a few dozen compact stars have been determined very exactly (to some extent) in binaries (Heap & Corcoran 1992; van Kerkwijk et al. 1995; Stickland et al. 1997; Orosz & Kuulkers 1999; Lattimer & Prakash 2005, 2007), no information on radius is available for these systems. Therefore, the theoretical study of stellar structure is essential to support the correct direction for newly observed masses and radii. Here, some specialized work on compact stars (Lobo 2006;
Electromagnetic waves propagate in vacuum with speed of light $c$. By using high-resolution spectroscopy and it was found to be $1.71 \pm 0.21 \, M_\odot$. In another work, Steeghs & Jonker (2007) measured the mass of the neutron star in $V395$ Carinae/2S 0921–630 with the help of the MIKE echelle spectrograph on the Magellan-Clay telescope by using high-resolution optical spectroscopy and it is $1.47 \pm 0.10 \, M_\odot$. On the other hand, Gelino et al. (2002) measured the mass of the compact star in XTE J2123–058 as $1.53^{+0.30}_{-0.42} \, M_\odot$. Muñoz-Muñoz-Darias et al. (2005) measured the mass of the neutron star in the low-mass X-ray binary (LMXB) X1822–371 (V691 CrA) by studying the K-correction for the case of emission lines formed in the X-ray illuminated atmosphere of a Roche lobe filling star and that was found to be $1.61 \, M_\odot \leq M_{\text{NS}} \leq 2.32 \, M_\odot$. The team of Güver et al. (2010b) measured the mass of the compact star in 4U 1820–30 by using time resolved X-ray spectroscopy of the thermonuclear burst of 4U 1820–30 which was $1.58 \pm 0.06 \, M_\odot$. Barnes et al. (2007) determined the mass of the compact object in GR Mus (XB 1254–690) to be $1.20 \, M_\odot \leq M_{\text{NS}} \leq 2.64 \, M_\odot$.

Wilkinson Microwave Anisotropy Probe (WMAP) measurements indicate that nearly 73% of the total mass-energy of the Universe is dark energy (Perlmutter et al. 1998; Riess et al. 2004) and this dark energy is based on the cosmological constant. To obtain a stable cosmological model, Einstein in 1917 introduced the idea of a cosmological constant. Later Zel’dovich (Zel’Dovich 1967), Zel’dovich (1968)) described this repulsion pressure as a vacuum energy arising from quantum fluctuation. However, for the viability of the present-day accelerating Universe, the earlier cosmological constant $\Lambda$ is commonly assumed to be time-dependent in the cosmological domain (Perlmutter et al. 1998; Riess et al. 2004). At the same time, space-dependent $\Lambda$ has a desired outcome from an astrophysical point of view as argued by many researchers (Chen & Wu 1990; Narlikar et al. 1991; Ray & Ray 1993) with respect to the behavior of local massive object such as galaxies. In the present model of compact stars, however, we take the cosmological constant $\Lambda$ as an absolutely constant quantity. This constancy of $\Lambda$ is unable to be ruled out for a system with very small dimensions like compact star systems or elsewhere with various physical conditions (Mak et al. 2000; Dymnikova 2002, 2003; Böhmer & Harko 2005a).

Based on the above knowledge, we examine the presence of a cosmological constant on a small scale to study the structure of compact stars in LMXBs, namely Cyg X-2, V395 Carinae/2S 0921–630, XTE J2123–058, X1822–371 (V691 CrA), 4U 1820–30 and GR Mus (XB 1254–690) and conclude that incorporation of $\Lambda$ describes the compact stars well.

## 2 INTERIOR SPACETIME

We consider stars as static and spherically symmetric bodies whose interior spacetime is

$$ds^2 = -e^{\nu(r)} dt^2 + e^{\lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) .$$

(1)

According to Heintzmann (1969),

$$e^\nu = A^2 (1 + ar^2)^3$$

and

$$e^{-\lambda} = \left[ 1 - \frac{3ar^2}{2} \left( 1 + C (1 + 4ar^2)^{-\frac{3}{2}} \right) \right] ,$$

where $A$, $C$ and $a$ are metric constants. We assume that the energy-momentum tensor for the interior of the compact object has the standard form of

$$T_{ij} = \text{diag}(-\rho, p, p, p) ,$$

where $\rho$ and $p$ are energy density and isotropic pressure respectively.

Einstein’s field equations for metric Equation (1) in the presence of $\Lambda$ are then obtained as (taking $G = 1$ and $c = 1$)

$$8\pi \rho + \Lambda = e^{-\lambda} \left( \frac{\lambda'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2} ,$$

(2)

$$8\pi p - \Lambda = e^{-\lambda} \left( \frac{\nu'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} .$$

(3)

Now, from metric Equation (1) and Einstein’s field Equations (2) and (3), we obtain the energy density ($\rho$) and pressure ($p$) as

$$\rho = \frac{3a \left( \sqrt{1 + 4ar^2} \left( 3 + 13ar^2 + 4a^2 r^4 \right) + C \left( 3 + 9a^2 \right) \right)}{16\pi (1 + ar^2)^{\frac{5}{2}} (1 + 4ar^2)^{\frac{3}{2}}} - \frac{\Lambda}{8\pi} ,$$

(4)
Fig. 1 Variation of matter density ($\rho$) – radius ($r$) (top) and pressure ($p$) – radius ($r$) (bottom) in the stellar interior (taking $a = 0.0016 \text{km}^{-2}$, $C = 1.133$).

$$p = \frac{-3a (3\sqrt{1+4ar^2} (-1 + ar^2) + C + 7acr^2)}{16\pi (1 + ar^2)^2 (1 + 4ar^2)^{3/2}} + \frac{\Lambda}{8\pi}. \tag{5}$$

From Equation (4) and Equation (5) we get the central density ($\rho_0$) and central pressure ($p_0$) of the star respectively

$$\rho_0 = \rho(r = 0) = \frac{3a (3 + 3C)}{16\pi} - \frac{\Lambda}{8\pi},$$

$$p_0 = p(r = 0) = \frac{3a (3 - C)}{16\pi} + \frac{\Lambda}{8\pi}.$$

It is known that $\Lambda > 0$ suggests space is open. In order to explain the accelerating state of the Universe, it is supposed that energy in a vacuum is responsible for this expansion. As a consequence, vacuum energy has some gravitational influence on stellar structure. It is recommended that the cosmological constant is responsible for that energy of the vacuum. The value of the cosmological constant $\Lambda$ has not been consistent with various scenarios. Though from a cosmological point of view, its order of magnitude is $10^{-52} \text{m}^{-2}$, on a local scale (for example near black holes, neutron stars and various massive objects) it is not essential to follow the large scale fine tuning values of $\Lambda$ (Bordbar et al. 2016).

In this section we have studied the following features of our model presuming the value of $\Lambda = 0.00111 \text{km}^{-2}$ (nearer to the value of Böhmer & Harko 2005b; Bordbar et al. 2016). We have considered this value for mathematical consistency and stability of the compact star. As “$a$” and “$C$” specify the central density of the configurations, we calculate it and use it in our model as we know that core properties of the compact star depend on the central density.

Also, we observe (Fig. 1) that both matter density and pressure are maximum at the center and decrease monotonically to the boundary. Interestingly, pressure falls to zero at the boundary, though density does not. Therefore, it may be justified to regard these compact stars as strange stars where the surface density remains finite rather than neutron stars for which the surface density vanishes at the boundary (Farhi & Jaffe 1984; Haensel et al. 1986; Alcock et al. 1986; Dey et al. 1998). It should be mentioned here that we fix the values of constants $a = 0.0016 \text{km}^{-2}$ and $C = 1.133$, so that pressure falls from its maximum value (at the center) to zero at the boundary.

3 EXPLORATION OF PHYSICAL PROPERTIES

In this section we study the following properties of the compact star in an LMXB.

3.1 Energy Conditions

In our model we observe that all the energy conditions, namely null energy condition (NEC), weak energy condition (WEC), strong energy condition (SEC) and dominant energy condition (DEC), are satisfied at the center ($r = 0$) of the star. From Figure 1, we observe that all the energy conditions obey:

(i) NEC: $p_0 + \rho_0 \geq 0$,
(ii) WEC: $p_0 + \rho_0 \geq 0$, $\rho_0 \geq 0$,
(iii) SEC: $p_0 + \rho_0 \geq 0$, $3p_0 + \rho_0 \geq 0$,
(iv) DEC: $\rho_0 > |p_0|$.

See Table 1 for numerical justification of energy conditions satisfied in our model.

| $\rho_0$ (km$^{-2}$) | $p_0$ (km$^{-2}$) | $\rho_0 + p_0$ (km$^{-2}$) | $3\rho_0 + p_0$ (km$^{-2}$) |
|---------------------|------------------|--------------------------|---------------------------|
| 0.000566894         | 0.000222451      | 0.000789345              | 0.00123425                |
3.2 TOV Equation

In our stellar model we observe that static equilibrium configurations are present due to the availability of gravitational ($F_g$) and hydrostatic ($F_h$) forces.

$$F_h + F_g = 0,$$

where,

$$F_g = \frac{1}{2} \nu' (\rho + p),$$

$$F_h = \frac{d}{dr} \left( p - \frac{\Lambda}{8\pi} \right).$$

Figure 2 shows the equilibrium state of the compact object under gravitational and hydrostatic forces in our stellar model.

3.3 Stability

Now, we examine the stability of the model. For a stable stellar model it is always required that the speed of sound should be less than the speed of light ($c = 1$) everywhere within the stellar object, i.e. $0 \leq v^2 = \left( \frac{dp}{d\rho} \right) \leq 1$ (Herrera 1992; Abreu et al. 2007). For this purpose we plot the sound speed in Figure 3 (top) and observe that it satisfies the inequalities $0 \leq v^2 \leq 1$ well. Therefore our stellar model is stable.

Our stellar model is also dynamically stable in the presence of thermal radiation. This type of stability has been investigated by several authors, namely Chandrasekhar (1964), Bardeen et al. (1966), Knutsen (1988), Mak & Harko (2013).

3.4 Matching Conditions

Here, we match the interior metric of the star with the exterior Schwarzschild-de Sitter metric at the boundary ($r = b$)

$$ds^2 = -\left( 1 - \frac{2M}{r} - \frac{\Lambda r^2}{3} \right) dt^2$$

$$+ \left( 1 - \frac{2M}{r} - \frac{\Lambda r^2}{3} \right)^{-1} dr^2$$

$$+ r^2 (d\theta^2 + \sin^2 \theta d\phi^2).$$

From continuity of the metric function across the boundary, we get the compactification factor as

$$\frac{M}{b} = \frac{1}{2} \left[ \frac{3ab^2 \left( 1 + C(1 + 4ab^2) - \frac{1}{2} \right)}{2 (1 + ab^2)} - \frac{\Lambda b^2}{3} \right].$$
3.5 Mass-Radius Relation and Surface Redshift

For a static spherically symmetric perfect fluid sphere, the maximum allowable mass-radius ratio should be \( \frac{M}{R} < \frac{4}{9} \) (Buchdahl 1959). In our stellar model, we have calculated the gravitational mass \( M \) in the presence of a cosmological constant as

\[
M = 4\pi \int_0^b \rho r^2 \, dr = \frac{3ab^3 \left( 1 + C(1 + 4ab^2)^{-\frac{1}{2}} \right)}{4(1 + ab^2)} - \frac{\Lambda b^3}{6},
\]

where the radius of the star is taken as \( b \). Hence, the compactness \( u \) of the star can be written as

\[
u = \frac{M}{b} = \frac{1}{2} \left[ \frac{3ab^2 \left( 1 + C(1 + 4ab^2)^{-\frac{1}{2}} \right)}{2(1 + ab^2)} - \frac{\Lambda b^3}{6} \right].
\]

The behaviors of the mass function and compactness of the star in our model are shown in Figure 4 and Figure 5 (top) respectively.

The surface redshift \( Z_s \) analogous to the above compactness \( u \) will be

\[
Z_s = [1 - 2u]^{-\frac{1}{2}} - 1.
\]

4 DISCUSSION AND CONCLUDING REMARKS

It should be noted here that the model described by Heint IIa in Heintzmann (1969) is useful to study both neutron and strange stars depending upon the choice of metric parameters \( a \) and \( C \) (Kalam et al. 2016, 2017). In this article, we have investigated whether the same Heint IIa metric is capable of explaining compact stars in LMXBs or not. To this end, we have explored the physical behavior of six compact stars within LMXBs, namely Cyg X-2, V395 Carinae/2S 0921–630, XTE J2123–058, X1822–371 (V691 CrA), 4U 1820–30 and GR Mus (XB 1254–690) by considering the nature of isotropic pressure. Here we have also merged the previous cosmological constant \( \Lambda \) in Einstein’s field equation in favor of studying the stellar structure. Effectively, we obtained an analytical solution for a fluid sphere which is really interesting with respect to many physical properties, which are summarized as follows:

(i) In our model, in the interior of the compact stars, density and pressure are well defined functions (positive definite at the center) (Fig. 1). It can be noted here that pressure and density are both maximum at the origin and interestingly pressure falls to zero (monotonically decreasing) towards the boundary while density does not. Here, we assume the values of constants
Fig. 6 Probable radii of Cyg X-2, 2S 0921–630, XTE J2123–058, X1822–371 (V691 CrA), 4U 1820–30 and GR Mus (XB 1254–690).

### Table 2 Evaluated Parameters for Compact Stars

| Star              | Observed Mass ($M_\odot$) | Radius from Model (in km) | Redshift from Model | Compactness from Model |
|-------------------|---------------------------|---------------------------|---------------------|------------------------|
| Cyg X-2           | 1.71 ± 0.21               | 11.55 ± 0.65              | 0.331 ± 0.0396      | 0.2169 ± 0.017          |
| 2S 0921–630       | 1.44 ± 0.10               | 10.75 ± 0.35              | 0.2834 ± 0.0194     | 0.1962 ± 0.0092         |
| XTE J2123–058     | 1.53 ± 0.30               | 10.8 ± 1.1                | 0.2897 ± 0.0614     | 0.1973 ± 0.0287         |
| X1822–371 (V691 CrA) | 1.61 ± 0.42            | 11.2 ± 2.32               | 0.3740 ± 0.0655     | 0.2334 ± 0.0254         |
| 4U 1820–30        | 1.58 ± 0.06               | 11.2 ± 0.2                | 0.3087 ± 0.0117     | 0.208 ± 0.0052          |
| GR Mus (XB 1254–690) | 1.92 ± 0.72             | 12 ± 2.1                  | 0.3734 ± 0.1352     | 0.2285 ± 0.0547         |

(a, C) in the metric and $\Lambda$ so that pressure reduces to zero at the boundary. By assuming constant values $a$, $C$ and $\Lambda$, we calculate the central density $\rho_0$ as $5.67 \times 10^{-6}$ km$^{-2}$ (7.651 $\times 10^{14}$ gm cm$^{-3}$) and central pressure $p_0$ as $2.245.1 \times 10^{-7}$ km$^{-2}$ (5.557 $\times 10^{35}$ dyn cm$^2$) (Table 1). These values satisfy energy conditions, TOV equation and Herrera’s stability condition. They are also stable with regard to infinitesimal radial perturbations. From the mass function (Eq. (8)), all desired interior properties of a compact star are able to be evaluated and satisfy the Buchdahl mass-radius relation ($\frac{2M}{R} < \frac{3}{2}$) (Figs. 4 and 5 (top)). The surface...
redshift with respect to compact stars is found under the standard measure \((Z_s \leq 0.85)\), which is optimal (Fig. 5 (bottom)) (Haensel et al. 2000). We estimate the EOS, which would be \(p = \alpha e^{(-\rho/\beta)} + \eta e^{(-\rho/\delta)} + \xi\) whereinto \(\alpha, \beta, \eta, \delta, \xi\) are constants and their units are \(\text{km}^{-2}\).

Figure 7 indicates that a stiff EOS (Özel 2006; Lai & Xu 2009 and Guo et al. 2014) is appropriate rather than a soft EOS.

![Fig. 7 Possible pressure \((p)\)-density \((\rho)\) relation (EOS) in the stellar interior taking \(a = 0.0016 \text{km}^{-2}\), \(C = 1.133\), where \(\alpha, \beta, \eta, \delta, \xi\) are constants and all are in units of \(\text{km}^{-2}\).

(ii) From our mass function graph in Figure 6, and Equations (9) and (10), we obtain the radii, compactnesses and surface redshift for six compact stars within LMXBs of Cyg X-2, V395 Carinae/2S 0921–630, XTE J2123–058, X1822–371 (V691 CrA), 4U 1820–30 and GR Mus (XB 1254–690). A detailed comparison is shown in Table 2.

It should be mentioned here that we are actually considering the Heint IIa metric with de Sitter spacetime to describe the compact stars within LMXBs where included metric parameters \(a\) and \(C\) are assessed by computing all related modes. When metric parameters values for the EOS are known, additionally the central density is set. In general, the mass-radius curve is considered under a conferred EOS for different values of central density; with a definite value of the central density, the mass and radius of a compact star are defined. However, our model is diverse and theoretically attractive. According to our model, six compact stars within the LMXBs Cyg X-2, V395 Carinae/2S 0921–630, XTE J2123–058, X1822–371 (V691 CrA), 4U 1820–30 and GR Mus (XB 1254–690) have identical values of \(a\) and \(C\), and therefore have an identical central density and identical EOS. A further interesting part of our stellar model is that if we begin out of the center by a particular central density, the structure of a compact star can be determined by restricting any radius wherein the pressure goes to zero.

Therefore, our conclusion is that we may find a useful relativistic model in the case of compact stars within LMXBs by a suitable choice of values of the metric parameters \(a\) and \(C\) in the metric given by Heint IIa (Heintzmann 1969).

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