Notes on Supersymmetric Gauge Theories in Five and Six Dimensions

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Abstract

We investigate consistency conditions for supersymmetric gauge theories in higher dimensions. First, we give a survey of Seiberg’s necessary conditions for the existence of such theories with simple groups in five and six dimensions. We then make some comments on how theories in different dimensions are related. In particular, we discuss how the Landau pole can be avoided in theories that are not asymptotically free in four dimensions, and the mixing of tensor and vector multiplets in dimensional reduction from six dimensions.

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1 Introduction

One of the most intriguing spin-offs of the recent wave of activity on non-perturbative supersymmetric field and string theories has been the discovery of interacting supersymmetric gauge theories in five and six spacetime dimensions\(^5\)[1-8].

These theories can be constructed from compactification of string theory (including its non-perturbative forms of M and F-theory) on an appropriate manifold [9-19]. As the manifold develops a strong coupling singularity, the set of fields that are not localized on the singularity, decouples from the physics, leaving a theory in flat space time with, possibly, some extra global symmetry \(^4\). The remaining fields provide the gauge and matter content necessary to describe the theory, with the possible addition of tensor multiplets in six dimensions.

Seiberg has found \(^3\)[3, 5] some simple necessary conditions that must be satisfied by the field theory in order to have such a non trivial fixed point. Seiberg’s arguments are purely field theoretical and what is remarkable is that they have a counterpart in the geometry of the compactification, making it quite plausible that the field theoretical requirements are also sufficient.

In this paper we continue this line of analysis by considering two separate issues:

First, we give an explicit survey of all candidate six dimensional theories with simple gauge groups. This is a straightforward exercise in the calculation of anomalies [20-27] and the situation can be summarized as follows. For those groups lacking an independent fourth order Casimir \(^2\) (SU(2), SU(3) and all exceptional groups) the situation is qualitatively similar to the one in five and four dimensions, i.e., there is an “upper bound” on the amount of matter that is allowed in the theory. In particular, pure gauge theories based on these groups are always possible \(^6\). For all other gauge groups, there must be exactly the “right amount” of matter in order to satisfy the consistency condition. For example, the only group in this category for which a pure gauge theory is allowed is Spin(8). As has already been noticed in comparing five to four dimensions \(^3\), the conditions in six dimensions are neither stronger nor weaker than in the other cases.

\(^5\)The theories we discuss in this paper are those with eight supercharges, corresponding to \(N = (1, 0), N = 1\) and \(N = 2\) supersymmetries in six, five and four spacetime dimensions respectively.

\(^6\)See however Note Added.
Second, we study some basic phenomena of “dimensional crossover” as we compactify these theories in five and four dimensions. (These arguments have a potentially wider range of applicability than the setting of this paper.) Given the “mismatch” between the acceptable matter contents in various dimensions, the issue arises on how these theories are related by compactification on a circle. For instance, it is well known that an $SU(2)$ theory can have up to $n_f = 4$ matter hypermultiplets in the fundamental representation in four dimensions, whereas Seiberg \[3\] has shown that one can have up to $n_f = 7$ in five dimensions\[4\]. This means that, by adding an extra compact dimension, (no matter how small), to the four dimensional theory, it is possible to avoid the Landau pole for a certain range of coupling constants. On the contrary, for an $SU(3)$ gauge theory with one hypermultiplet in the symmetric representation $s = 6$ an extra dimension would be disastrous, as the theory is asymptotically free in four dimensions but ill-defined in five \[8\].

Going from five to six dimensions is even more challenging because of the further restrictions on the prepotential and the existence of a new multiplet in six dimensions: the tensor multiplet. We already know that it is possible for a tensor multiplet to turn into a gauge field in the Cartan subalgebra of a particular gauge group (other then $U(1)$!) upon compactification \[6\]. The natural question that arises here is whether it is possible for this field to mix with the dimensional reduction of other gauge fields already present in six dimensions yielding an enlarged gauge symmetry in five dimensions\[5\]. We present evidence against this phenomenon although this might require further studies. Some other technical difficulties in the dimensional reduction of tensor multiplets were discussed in \[29\] in relation to effective actions for $p$-branes \[30\].

The paper is organized as follows. In Section two we review some of the results on the five dimensional theories \[8\] for further reference; in Section three we discuss the consistency requirements for six dimensional theories and finally, in Section four we discuss various aspects of dimensional crossover.

\[7\] $n_f = 8$ is a borderline case that does not have a strong coupling limit but must also be taken into account under certain circumstances.

\[8\] This question was also raised in \[8\].
2 Review of five dimensional theories

In this section we briefly review some of the results for five dimensional
supersymmetric gauge theories with eight supercharges and simple gauge
group $G$. This analysis was started in [3] and extended in [17, 6, 8]. There
are two possible multiplets: the vector multiplet, whose real scalar com-
ponent we denote by $\phi^a$, $a = 1, \cdots, \dim G$, and a set of hypermultiplets transforming
under a generic representation $R_1 \oplus R_2 \cdots$ of the gauge group. Throughout
the paper, we denote the number of flavors in a certain represent ation $R$ by
$n_R$; if $R$ is pseudoreal and no global anomaly is present, one is allowed to
couple half a hypermultiplet to the gauge field, i.e., $n_R$ can be a half-integer.

The Coulomb branch is parametrized by the scalars $\phi^i$ in the vector mul-
tiplet belonging to the Cartan subalgebra of $G$, $i = 1, \cdots, \text{rank } G$, and it is
topologically a wedge given by modding out the Cartan subalgebra by the
action of the Weyl group. In [8] it is shown that the necessary condition for
the existence of a non trivial UV fixed point is that, in the limit $g_0 = \infty$, the
quantum prepotential

$$F = \frac{1}{2g_0^2} \delta_{ij} \phi^i \phi^j + \frac{c_{\text{class}}}{6} d_{ijk} \phi^i \phi^j \phi^k + \frac{1}{12} \left( \sum_{\alpha} |\alpha_i \phi^i|^3 - \sum_{R} n_R \sum_{w \in R} |w_i \phi^i + m_R|^3 \right)$$

must be a convex function throughout the Coulomb branch [9]. ($\alpha$ denotes all
the roots of $G$, $w$ all the weights of $R$ and $d_{ijk}$ is the third rank symmetric
invariant tensor that exists only for the groups $SU(N)$ with $N \geq 3$; $c_{\text{class}}$ is
quantized by considering the global anomaly associated with $\pi_5(SU(N)) = Z$.) The analysis of [8] yields the following results:

- For $SU(N)$ only the fundamental representation $\mathbf{f}$ and the antisymmet-
riv two tensor $\mathbf{a}$ are allowed and the quantization condition is

$$c_{\text{class}} + (n_\mathbf{f} + Nn_\mathbf{a})/2 \in \mathbb{Z}. \tag{2}$$

For $N > 8$ only $n_\mathbf{a} = 0$ and $n_\mathbf{f} + 2|c_{\text{class}}| \leq 2N$ are allowed. For $5 \leq N \leq 8$, $n_\mathbf{a} = 1$ and $n_\mathbf{f} + 2|c_{\text{class}}| \leq 8 - N$ is also allowed and, for $N = 4$, $n_\mathbf{a} \equiv n_\mathbf{f} = 2$ and $n_\mathbf{f} \equiv n_4 = c_{\text{class}} = 0$ is also allowed. In the degenerate cases of $SU(2)$ and
$SU(3)$, where the antisymmetric representation is either trivial or conjugate
to the fundamental, one finds $n_\mathbf{f} \equiv n_2 \leq 7$ and $n_\mathbf{f} \equiv n_3 \leq 6$ respectively.

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9Excluding the case of $F$ identically zero in this limit; i.e. excluding the presence of
adjoint matter.
• For $Sp(N)$ ($N \geq 2$ being $Sp(1) = SU(2)$), one again finds that the only possible representations are $f$ and $a$ with the requirements $n_a = 0$, $n_f \leq 2N + 4$ or $n_a = 1$, $n_f \leq 7$. The second case reduces the system to a direct product of $SU(2)$ groups as already familiar from four dimensions [32-35], where a non-renormalization theorem on the Higgs branch can be used.

• For $Spin(N)$ ($N \geq 7$ being $Spin(6) = SU(4)$ etc...) the only representations allowed are the vector $v$ and the spinor $d$. One always has $n_v \leq N - 4$ and, for $N = 7, 8, 9, 10, 11, 12$, one can also have $n_d \leq 4, 2, 2, 1, 1$ in the same order.

• The exceptional groups have only one representation “f” smaller than the adjoint, except for $E_8$ that has none. The bounds are $n_f \leq 4, 3, 4, 3$ for $G_2$, $F_4$, $E_6$ and $E_7$. In the case of $E_7$ one can have an odd number of half-hypermultiplets. $E_8$ cannot be coupled to matter in this way.

3 Consistency conditions in six dimensions

In six dimensions, Seiberg’s necessary condition [5] is that, after gravity has decoupled, the gauge anomaly can be canceled by the introduction of at least one tensor multiplet. The contribution to the anomaly eight-form from the gauge multiplet and the hypermultiplets is

$$I_8 = \text{tr}_{\text{Adj}} F^4 - \sum_R n_R \text{tr}_R F^4 \equiv \alpha \text{tr}_F F^4 + c (\text{tr}_F F^2)^2. \quad (3)$$

To be able to cancel this part of the anomaly without introducing gravity and with at least one tensor multiplet we need $\alpha = 0$ and $c > 0$ [28]. This requirement is very easy to investigate.

The simple groups can be divided into two classes. The first class consists of those groups that do not have an independent fourth order Casimir [28]. They are $SU(2), SU(3)$ and all the exceptional groups $G_2$, $F_4$, $E_6$, $E_7$ and $E_8$. For these particular groups $\text{tr}_{\text{Adj}} F^4$ and $\text{tr}_R F^4$ can always be expressed in terms of $\text{tr}_F (F^2)^2$ and $\alpha = 0$. The anomaly condition $c > 0$ becomes an upper bound on the number of matter hypermultiplets, and, hence, closer in spirit to the results in four and five dimensions. All other groups possess a fourth order Casimir, and the requirement $\alpha = 0$ implies that one must have just the right amount of matter.

10 The case $c = 0$ need also be considered in certain cases.
We first consider groups in the first category and investigate the possible matter content. We need only consider representations whose dimension is smaller than the dimension of the adjoint. This restricts the possibilities to the fundamental representations, except for $SU(3)$, where we also may have the symmetric $s = 6$. The groups $SU(2)$ and $E_7$ have pseudoreal fundamental representations which allows for a half-integer number of hypermultiplets.

Using the tables of [25] we can calculate the anomaly polynomial and find that $c$ is proportional to

$$
16 - n_2 \quad \text{for} \quad SU(2)
$$
$$
18 - n_3 - 17n_6 \quad \text{for} \quad SU(3)
$$
$$
10 - n_7 \quad \text{for} \quad G_2
$$
$$
5 - n_{26} \quad \text{for} \quad F_4
$$
$$
6 - n_{27} \quad \text{for} \quad E_6
$$
$$
4 - n_{56} \quad \text{for} \quad E_7,
$$

from which one immediately reads off the required bounds. Note that the $6$ of $SU(3)$, that is allowed by asymptotic freedom in four dimensions and forbidden in five, has reappeared in six dimensions, where one can have $n_3 = 0$ and $n_6 = 1$.\footnote{This particular example, and a few others based on the groups $SU(2)$, $SU(3)$ and $G_2$, have been shown to be affected by a global anomaly, see Note Added.}

Let us then turn to the second class of simple groups for which there is an independent fourth order Casimir. In this case we have to solve both equations $\alpha = 0$ and $c > 0$. The result is no longer a bound for the maximum number of matter multiplets, but the matter content must exactly compensate the contributions of the gauge sector. There is no global anomaly as $\pi_6(G)$ is trivial in all these cases.

• For $SU(N)$, $N > 3$, we need only consider the case of $n_f$ multiplets in the fundamental representation, $n_s$ in the second rank symmetric, $n_a$ in the second rank antisymmetric and $n_t$ in the third rank antisymmetric. For $N \geq 8$, we can have the three combinations:

$$
(n_f, n_a, n_s, n_t) = (2N, 0, 0, 0), (N + 8, 1, 0, 0), (N - 8, 0, 1, 0).
$$

(5)

For $N = 7$ we find only:

$$
(n_f, n_a, n_s, n_t) \equiv (n_7, n_{21}, n_{28}, n_{35}) = (14, 0, 0, 0), (15, 1, 0, 0).
$$

(6)
For $N = 6$ one can finally have\footnote{This is the only case where $t$ is allowed. Also, in this case, the anomaly cancels completely for $(n_f, n_a, n_t) = (16, 1, 0)$ for any $N \geq 2$, and $(n_f, n_a, n_t) \equiv (n_{6}, n_{15}, n_{20}) = (0, 1, 1, 0), (16, 2, 0, 0), (17, 1, 0, 1/2), (18, 0, 0, 1)$ and $(1, 0, 1, 1/2)$.}

\[(n_f, n_a, n_s, n_t) \equiv (n_6, n_{15}, n_{21}, n_{20}) = (12, 0, 0, 0), (14, 1, 0, 0), (15, 0, 0, \frac{1}{2}). \tag{7}\]

For $SU(4)$ and $SU(5)$ the third rank representation $t$ is conjugate to the fundamental $f$ and to the antisymmetric $a$ respectively, and we also find $n_s = 0$; hence

\[(n_f, n_a) \equiv (n_4, n_6) = (8, 0), (12, 1) \text{ for } SU(4)\]
\[(n_f, n_a) \equiv (n_5, n_{10}) = (10, 0), (13, 1) \text{ for } SU(5). \tag{8}\]

- Of the $Spin(N)$, $N \geq 7$, representations we consider the vector representation $v$, the spin representation $d$, the symmetric second rank representation $s$ and the antisymmetric third rank representation $t$. It turns out that there is a solution only in the absence of the $s$ and $t$ representations. For $Spin(7)$ we get three possibilities $(n_v, n_d) \equiv (n_7, n_8) = (0, 2), (1, 4)$ and $(2, 6)$. For $N = 8, \ldots, 12$ there are four possibilities:

\[
\begin{align*}
    n_v &= N - 8, \quad n_d = 0 \\
    n_v &= N - 7, \quad n_d = 2^5 - [(N + 1)/2] \\
    n_v &= N - 6, \quad n_d = 2^6 - [(N + 1)/2] \\
    n_v &= N - 5, \quad n_d = 3 \cdot 2^5 - [(N + 1)/2],
\end{align*}
\tag{9}\]

where the brackets $[ \ ]$ denote the integer part. Notice in particular that for $Spin(8)$ we find a solution for $c > 0, \alpha = 0$ without introducing any matter. For $Spin(13)$ we get $(n_v, n_d) \equiv (n_{13}, n_{64}) = (5, 0)$ and $(7, 1/2)$. For $N \geq 14$ the spinor representations $d$ are no longer allowed, and the solution is $n_v = N - 8$.

- For $Sp(N)$, $N \geq 2$ we consider matter in the fundamental $f$ and both in the second and in the third rank antisymmetric representations $a$ and $t$. There is a solution only for $2N + 8$ multiplets in the fundamental representation and none in the antisymmetric representations. The anomaly cancels completely for $(n_f, n_a, n_t) = (16, 1, 0)$ for any $N \geq 2$, and $(n_f, n_a, n_t) \equiv (n_6, n_{14}, n_{14'}) = (17, 0, 1/2)$ for $N = 3$. 
4 Dimensional crossovers

4.1 Going from four to five dimensions

As we have seen, the consistency requirements for gauge theories in different dimensions show a very complex behavior. A theory that is admissible in a lower dimension might be ill-defined in a higher dimension and vice versa. It is interesting to study the behavior of such theories as we change the effective number of dimensions. Let us consider compactifying a five dimensional theory on a circle; The dimensionless quantity that determines the effective number of dimensions is $\phi R$, where $R$ is the radius of compactification and $\phi$ the magnitude of the Higgs field on the Coulomb branch.

Let us study a couple of examples in detail. We begin with the case of gauge group $SU(2)$ with $n_f$ fundamental hypermultiplets. When $n_f < 4$, we assume $1/R$ and $\phi$ are both $>> \Lambda$ so that we can ignore instanton effects; The full solution in the IR is, of course, given in Ref. [35, 36], and it is well defined for small $\phi R$ as well.

Using the results of [37], we find that the effective coupling is given by summing up all the one loop contributions coming from the Kaluza-Klein modes on the circle:

$$\frac{1}{g_{eff}^2} = \frac{1}{g_0^2} + 4 \log(\sinh^2(2\pi R\phi)) - n_f \log(\sinh^2(\pi R\phi)).$$

(10)

At small $\phi R$ we recover

$$\frac{1}{g_{eff}^2} = \frac{1}{g_0^2} + 4 \log(2\pi R\phi)^2 - n_f \log(\pi R\phi)^2,$$

(11)

i.e. an effectively four dimensional theory. In this case the theory is asymptotically free for $n_f < 4$. In the infrared, i.e. small $\phi R$, the full solution guarantees that $1/g_{eff}^2$ remains positive. For large $\phi R$ we find

$$\frac{1}{g_{eff}^2} = \frac{1}{g_0^2} + (8 - n_f)2\pi R\phi,$$

(12)

i.e. an effectively five dimensional theory. The dimensionful five dimensional coupling $g_5$ is related to the one above through $1/g_{eff}^2 = 2\pi R/g_5^2$. In this case a sensible UV-theory needs $n_f \leq 8$ in order not to hit a Landau pole. The combined behavior is illustrated in figure 1.
Figure 1: The effective coupling for $SU(2)$ with $n_f$ fundamental hypermultiplets on $\mathbb{R}^4 \times S^1$.

At a particular value of $\phi << 1/R$, we have renormalized the value of the coupling to the same fixed value for all values of $n_f$. If the coupling is small enough we find that the curves remain above zero for values $n_f \leq 8$ and for this we would like to give the following physical interpretation.

It is commonly believed that IR-free gauge theories in four dimensions, like $SU(2)$ with more than 4 hypermultiplets in the fundamental representation, are inconsistent in the UV and will hit a Landau pole. As long as the number of hypermultiplets are not greater than 8 we see that these theories are in fact saved if there is a fifth dimension that opens up. In the UV the original four dimensional theory connects smoothly to the IR limit of a five dimensional theory. This, when we go ever deeper into the UV, has a UV-fixed point as described in [3]. If we measure the effective coupling to be $g$ at $\phi \sim \mu$ we can estimate that an $R$ such that

$$R > \frac{1}{\mu} e^{-\frac{1}{2(n_f-4)} \frac{1}{\mu^2}}$$

is needed to avoid a Landau pole.

To conclude, theories with $n_f < 4$ are “saved by instantons” in the IR and theories with $4 < n_f \leq 8$ are “saved by the fifth dimension” in the UV.

There are also examples of theories that are asymptotically free in four
dimensions but ill-defined in five. Such an example is $SU(3)$ with a $6$. In this case, if we consider fixing $R > 0$, there is always a $\phi$ large enough that the theory hits a Landau pole along some direction on the Coulomb branch. The only way to have $SU(3)$ with a $6$ is therefore to have $R \equiv 0$.

4.2 Going from five to six dimensions

What happens to the six dimensional theories that we have studied when dimensionally reduced to five dimensions? The tensor multiplet will clearly give rise to a vector multiplet in five dimensions and one might hope that it will combine with the vector multiplet already present in six dimensions to give interesting five dimensional physics. Let us investigate the possibility for this to happen.

The question can be formulated in the following way; let $N_T$ be the number of tensor multiplets and $G$ the gauge group already present in six dimensions. After dimensional reduction to five dimensions, the Coulomb branch becomes $(N_T + \text{rank } G)$-dimensional and the question is whether the theory can be identified with an acceptable five dimensional theory based on a simple group $\tilde{G}$, s.t., $\text{rank } \tilde{G} = N_T + \text{rank } G$.

Let us take $N_T = 1$ and $G = SU(2)$ for simplicity, compactify on a circle of radius $R$ and go far out in the Coulomb branch, where naive dimensional reduction is allowed. In six dimensions we have, symbolically, an interaction term $T A A$, where $T$ and $A$ are the tensor and vector multiplets. This gives rise to an interaction that is locally of the type $A_1 A_2 A_2$ in five dimensions, where $A_1$ is the vector multiplet arising from the dimensionally reduced tensor multiplet and $A_2$ comes from the vector already present in six dimensions. In addition, we could also have a term $A_2 A_2 A_2$ that would be invisible in six dimensions. Cubic terms containing more than one $A_1$ are forbidden on dimensional grounds.

Therefore, if we consider the prepotential $F = A_1 A_2 A_2 + b A_2 A_2 A_2$ and compute the matrix of coupling constants

$$\tau = \begin{pmatrix} 0 & \phi_2 \\ \phi_2 & \phi_1 + 3b\phi_2 \end{pmatrix},$$

we see that it has always a negative eigenvalue. We are therefore not able to find an acceptable five dimensional theory after dimensional reduction.
Note Added

The correct treatment of global anomalies has subsequently been given in [38], (see also [39, 40]). The groups affected by the global anomaly are $SU(2)$, $SU(3)$ and $G_2$ and the allowed matter content is $n_2 = 4, 10$ for $SU(2)$, $n_3 = 0, 6, 12$ and $n_6 = 0$ for $SU(3)$, and finally $n_7 = 1, 4, 7$ for $G_2$.

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