Magnetization reversal by tuning Rashba spin-orbit interaction and Josephson phase in a ferromagnetic Josephson junction

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We theoretically study the magnetization inside a normal metal containing the Rashba spin-orbit interaction (RSOI) induced by the proximity effect in an s-wave superconductor/normal metal/ferromagnetic metal/s-wave superconductor (S/N/F/S) Josephson junction. By solving linearized Usadel equation taking into account of the RSOI, we show that the magnetization appears inside the N due to the proximity effect. The source of magnetization induced by the proximity effect is the odd-frequency spin-triplet Cooper pair (STC) inside the N. This result shows that the magnetization induced by the proximity effect is a good fingerprint to detect spins of STC. We find that the magnetization inside the N shows a damped oscillatory behavior as a function of the thickness of N for finite RSOI. The period of oscillation in the magnetization depends on the magnitude of RSOI. Due to the damped oscillatory behavior and the dependence of RSOI of the magnetization, the direction of the magnetization can be reversed by tuning the RSOI. Moreover, we also find that the direction of the magnetization inside the N can be reversed by changing superconducting phase difference, i.e., Josephson phase. From these results, it is expected that the magnetization depending on the RSOI and Josephson phase can be applicable to superconducting spintronics.

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I. INTRODUCTION

It is well known that the Cooper pair penetrates into a normal metal due to the proximity effect in the s-wave superconductor/normal metal (S/N) hybrid junctions.[1, 2]. One of important phenomena generated by the proximity effect is Josephson effect, which is characterized DC current flowing without a voltage-drop between two separated by N[1, 2]. The Josephson effect is a macroscopic quantum-mechanical phenomenon to preserve superconducting phase (Josephson phase) coherence between two superconductors. Such a phase coherence monotonically decreases with the thickness of N[2].

In the s-wave superconductor/ferromagnetic metal (S/F) junctions, there are interesting phenomena which are not observed in S/N hybrid junctions and thus has been actively studied both theoretically and experimentally[3–21]. Spin-singlet Cooper pairs (SSCs) penetrate into the F due to the proximity effect in an S/F junction. Due to the exchange splitting between up- and down-spin bands in the F, the SSCs in the F have a finite center-of-mass momentum. As a result, the pair amplitude of SSCs shows a damped oscillatory behavior as a function of the thickness of the F[19, 21]. One of the interesting phenomena resulting from the damped oscillatory behavior of the pair amplitude is a π-state in an S/F/S junction, where the current-phase relation in the Josephson current is shifted by π from that of the ordinary S/I/S and S/N/S junctions[3–21].

Another notable phenomenon in the S/F junctions is the appearance of odd-frequency spin-triplet Cooper pairs (STCs) induced by the proximity effect[21, 22]. Where the anomalous Green’s function of the odd-frequency spin-triplet Cooper pair becomes odd function with respect to the fermion Matsubara frequency ωn. In an S/F junction having a uniform magnetization, the STC composed of opposite spin electrons (i.e., total spin projection on z axis being S\textsubscript{z} = 0) and SSCs penetrates into the F due to the proximity effect[21, 22]. The penetration length of STC with S\textsubscript{z} = 0 and SSC into the F is very short and the amplitude of STC inside the F exhibits a damped oscillatory behavior as a function of the thickness of F. The characteristic penetration length is determined by ξF = \sqrt{ℏD_F/hex}, which is typically a order of few nanometers[3–21]. Where, D_F and hex are the diffusion coefficient and the exchange field in the F, respectively.

The STC formed by electrons of equal spin (|S\textsubscript{z}| = 1) can be also induced inside the F due to the proximity effect when a magnetization in the F is non-uniform in S/F junctions. The feature of this STC is approximately 2 orders of magnitude larger than the penetration length of the SSC[24–54]. The penetration length of STC with spin |S\textsubscript{z}| = 1 is determined by ξT = \sqrt{ℏD_F/2πκbk_BT} in Fs (T is temperature). Thus, the proximity effect of the STC with |S\textsubscript{z}| = 1 is referred to as the long-range proximity effect (LRPE).

The STC with |S\textsubscript{z}| = 1 induced by the proximity effect can be detected by the observation of Josephson current in ferromagnetic Josephson junctions (FJJJs). The Josephson current carried by the STC with |S\textsubscript{z}| = 1 shows monotonically decrease as a function of the thickness of F and its decay length of STC is about determined by ξT. The long-range Josephson current flowing through the F has been observed and established experimentally in FJJJs[55–61]. The detection of long-range Josephson current is a piece of evidence for the presence of STC.

Another candidate to evidence the presence of the STC is to measure the spin dependent transport of the STC in S/F junctions, since such a transport directly mea-
sures the spin of the STC \cite{62-69}. For this purpose, recently \( S/F/N \) and \( S/F/N/F/S \) junctions containing the Rashba spin-orbit interaction (RSOI) has been of considerable interest in recent years, since the proximity effect coupled with the RSOI in \( S/F/N \) and \( S/F/N/F/S \) junctions exhibits many fascinating phenomena which are not observed in \( S/F/N \) and \( S/F/N/F/S \) junctions without the RSOI \cite{70-76}. In the \( S/F/N \) junctions, it has been theoretically reported that the pair amplitude of the STC penetrating into \( N \) with the RSOI modulates as functions of the thickness of \( N \) and the magnitude of the RSOI \cite{71,72}. By utilizing the modulation of the pair amplitude of STC, it is possible to freely control \( 0 \)- and \( \pi \)-states by tuning the RSOI in Josephson junctions \cite{70,72}. Moreover, some authors have theoretically predicted the direct evidence of the STC by detecting the spin Hall effect and magnetoelectric effect induced by the STC in Josephson junctions containing F and the RSOI \cite{72,74,76}. The RSOI has an advantage to study spin-dependent transport phenomena, since this interaction can be freely controlled by an external electric field \cite{80}. However, understanding of spin transport of STC with the RSOI has been insufficient, and thus how to manipulate and detect the spin of the STC has the space to study. It is expected that well understanding of spin transport of STC gives us the credence of STC and expedites the development of superconducting spintronics \cite{77,79}.

In this paper, we theoretically propose an another setup and way to detect the STC by using a simple \( S1/N/F/S2 \) junction differently from Ref\cite{73,74,76}. The simple \( S1/N/F/S2 \) junction can be easily achieved by using recent device fabrication techniques. Based on the linearized Usadel equation including the RSOI, we formulate the magnetization induced by the proximity effect. It is shown that the magnetization in the \( N \) only appears when the product of anomalous Green’s functions of the spin-triplet odd-frequency Cooper pair and spin-singlet even-frequency Cooper pair in the \( N \) has a finite value. It is found that the magnetization shows damped oscillatory behavior as a function of the thickness of \( N \). It is also found that the direction of magnetization can be controlled by tuning the magnitude of RSOI. Moreover, we examine the Josephson phase \( \theta \) dependence of magnetization. As a result, the period of oscillation of magnetization is changed by tuning \( \theta \). This result clearly shows that the direction of magnetization can be also controlled by \( \theta \) as well as the magnitude of RSOI. We expect that these results can be applied to the research field of superconducting spintronics.

The rest of this paper is organized as follows. In Sec. [II] we introduce an \( S1/N/F/S2 \) Josephson junction and formulate the magnetization induced by the proximity effect in the \( N \) containing the RSOI of this junction by solving Usadel equation. In Sec. [III] the numerical results of magnetization are given. The thickness and the RSOI dependences of magnetization are discussed. Moreover, we present the Josephson phase \( \theta \) dependence of magnetization. Finally, \( \theta \)-magnetization relation is discussed and the magnetization induced by the proximity effect is estimated for a typical set of realistic parameters in Sec. [IV]. The summary of this paper is given in Sec. [V].

**FIG. 1:** (Color online) Schematic illustration of the \( S1/N/F/S2 \) junction studied, where \( N \) is a normal metal with the Rashba spin-orbit interaction(RSOI), \( F \) is a ferromagnetic metal, and \( S1(2) \) is an s-wave superconductor. Arrow in \( F \) indicates the direction of ferromagnetic magnetization. While the magnetization in \( F \) is fixed along the \( z \) direction. \( d_S \), \( d_F \) and \( d \) are the thicknesses of \( S \), \( F \), and \( N \), respectively, with \( L = d + d_F \). We assume that the magnetization is uniform in \( F \) layer, and that \( d_S \) is much larger than \( \xi_S \).

## II. MAGNETIZATION IN NORMAL METAL INDUCED BY PROXIMITY EFFECT IN AN \( S1/N/F/S2 \) JUNCTION WITH RASHBA SPIN-ORBIT INTERACTION

### A. Set up of junction and anomalous Green’s functions

We consider the Josephson junction composed of s-wave superconductors (\( S \)) separated by normal metal/ferromagnetic metal (\( N/F \)) junction as depicted in Fig. 1. Here, we include the Rashba spin-orbit interaction (RSOI) in the \( N \) and assume the uniform magnetization in \( F \) in the \( S1/N/F/S2 \) junction. We assume that the width of the junction is smaller than \( \xi_T \). In this situation, one dimensional (1D) model may be good approximation. Therefore, we adopt the 1D model to analyze the magnetization induced by the proximity effect in the \( S1/N/F/S2 \) junction.

In the diffusive transport limit, the magnetization inside the \( N \) with RSOI induced by the proximity effect is evaluated by solving the linearized Usadel equation including SU(2) gauge field in each region \( j = N,F \) \cite{71}

\[
\frac{i\hbar D_j}{2}\partial_x \hat{f}^\dagger(\vec{r}^\prime) - i2\hbar \omega_n \hat{f}^\dagger(\vec{r}^\prime) - \text{sgn}(\omega_n) \hbar \nu_{xx}(x) \left[ \hat{\tau}_x, \hat{f}^\dagger(\vec{r}^\prime) \right] = 0, \\
\partial_x = \begin{cases} \partial_x - i\hbar \left[ \hat{A}_x, \bullet \right], & 0 < x < d \\ \partial_x \bullet, & \text{other} \end{cases}
\]

where \( \vec{r}^\prime = (x, \omega_n) \) and \( \hat{A}_x \) is the SU(2) gauge field, which describes the RSOI and \( \alpha_R \) is the RSOI constant.
boundary condition $\frac{\sigma_{\text{F(N)}}}{\sigma_S} \ll \frac{\xi_{\text{F(N)}}}{\xi_S}$, where $\sigma_S$ is the conductivity of $S$ in the normal state and $\xi_S = \sqrt{\hbar D_F/\xi_{\text{F}}}$.

To approximate $\xi_{\text{F}}$, we use $\xi_{\text{F}} \approx \xi_{\text{F(N)}}/\xi_S$ and assume $\sigma_{\text{F(N)}} \ll \sigma_S$, the anomalous Green’s function in the $S$ attached to $N$ and $F$ can be approximately given as

$$\hat{f}_s^{\text{S1}}(\vec{r}) = -\hat{\gamma}_s \frac{\Delta_{\text{L(R)}}}{\sqrt{h\omega} + |\Delta_{\text{L(R)}}|^2} \equiv F^{\text{S1}(2)}, \quad (10)$$

where $\Delta_{\text{L(R)}} = \Delta e^{i\theta_{\text{L(R)}}} \ (\Delta: \text{real})$ and $\theta_{\text{L(R)}}$ is the superconducting phase in the left (right) side of $S$s (see Fig. $\text{F}$). The s-wave superconducting gap $\Delta(x)$ is finite only in the $S$ and assume to be constant as follows,

$$\Delta(x) = \left\{ \begin{array}{ll} 0 & -d_S < x < 0 \\ -\Delta_L & 0 < x < L \\ -\Delta_R & L < x < L_S \\ 0 & \text{other} \end{array} \right., \quad (11)$$

From Eqs. $\text{(10)}$ and $\text{(11)}$, it is immediately that the left-hand side of Eqs. $\text{(7)}$ and $\text{(9)}$ becomes zero, since the rigid boundary condition is assumed in the present calculation $\text{[20]}$.

Assuming $d_S/\xi_{\text{F}} \ll 1$, we can perform the Taylor expansion with $x$ for $F(\vec{r})$ as follows $\text{[33]}$ $\text{[51]},$

$$\hat{f}_s^{\text{F}}(\vec{r}) \approx F(\vec{r}, \omega_n) + (x - d) \partial_x F(\vec{r}) \bigg|_{x=d} + \left( \frac{x - d}{2} \right)^2 \partial_x^2 F(\vec{r}) \bigg|_{x=d}. \quad (12)$$

Applying the boundary conditions of Eqs. $\text{(5)}$ and $\text{(8)}$ to Eq. $\text{(12)}$ and substituting Eq. $\text{(12)}$ into Eq. $\text{(11)}$, we can approximately obtain the anomalous Green’s function of $F(\vec{r})$ as

$$\hat{f}_s^{\text{F}}(\vec{r}) \approx -\frac{d_S}{\gamma_F} \partial_x F(\vec{r}) \bigg|_{x=d} + \left( \frac{x - d}{2} \right) \partial_x F(\vec{r}) \bigg|_{x=d} + \hat{F}^{\text{S2}1} + isgn(\omega_n) \frac{h_{\text{ex}} d_S}{2\hbar D_F} \left[ \hat{\gamma}_s, \hat{F}^{\text{S2}} \right]. \quad (13)$$

The general solutions of Eq. $\text{(11)}$ in the $N$ are given by
where $k_\alpha = \sqrt{3\alpha_R^2 + k_N^2}$ and $\sqrt{2|\omega_n|/D_N}$. Here, we assume that $\alpha_R \neq 0$ to obtain Eq. (13). Applying the boundary conditions given in Eqs. (4), (6), (7), and (9) to Eq. (13), and also using the result in Eq. (12), we can obtain the anomalous Green’s functions in the $N$ as

$$f_N^N(\vec{r}) = -i \frac{\Delta_R}{E_{\omega_n}} \left( 1 - k_N d_F \right) \sinh[k_N(x-d)] + i \frac{\Delta_R}{E_{\omega_n}} \sinh(k_N x) Q_{\omega_n}(d), \quad (15)$$

$$f_N^N(\vec{r}) = i f_N^N(\vec{r}), \quad (16)$$

$$f_N^{t_N}(\vec{r}) = \text{sgn}(\omega_n) \frac{\hbar_e d_F^2}{D_F} \Phi_{\omega_n}(d) t_{\omega_n}(x,d), \quad (17)$$

$$Q_{\omega_n}^{-1}(d) = \sinh(k_N d) + \frac{k_N d_F}{\gamma_F} \cosh(k_N d), \quad (18)$$

$$\Phi_{\omega_n}^{-1}(d) = \left( i\omega_R + k_\alpha \right) \frac{d_F}{\gamma_F} \left( e^{i(\omega_R + k_\alpha)L} + e^{-(i\omega_R + k_\alpha)L} \right) C_{\omega_n}^{01}(d)$$

$$+ \frac{d_F}{\gamma_F} \left( i\omega_R - k_\alpha \right) e^{i(\omega_R - k_\alpha)L} \left( i\omega_R + k_\alpha \right) e^{-(i\omega_R + k_\alpha)L} C_{\omega_n}^{32}(d)$$

$$- \frac{d_F}{\gamma_F} \left( i\omega_R - k_\alpha \right) e^{-i(\omega_R + k_\alpha)L} \left( i\omega_R + k_\alpha \right) e^{-(i\omega_R + k_\alpha)L} C_{\omega_n}^{33}(d), \quad (19)$$

$$t_{\omega_n}(x,d) = i2 \left[ C_{\omega_n}^{21}(d) + 2C_{\omega_n}^{31}(d) \right]$$

$$\times \left[ \cos(\omega_R d) \sin(k_\alpha x) + i \sin(\omega_R d) \cos(k_\alpha x) \right]$$

$$- 2 \left[ C_{\omega_n}^{22}(d) + 2C_{\omega_n}^{32}(d) \right] \sin(\omega_R x) e^{-k_\alpha x}$$

$$+ i2 \left[ C_{\omega_n}^{23}(d) + 2C_{\omega_n}^{33}(d) \right] \sin(k_\alpha x) e^{-i\omega_R x}. \quad (20)$$

Where the explicit formulae of the functions $C_{\omega_n}^{ij}(d)$ ($i,j = 1,2,3$) are presented in APPENDIX A. From Eqs. (15) – (17), it is immediately found that $f_N^N(\vec{r})$ representing the SSC is an even function with $\omega_n$, whereas $f_N^{t_N}(\vec{r})$ representing the STC is an odd function with $\omega_n$ since $f_N^{t_N}(\vec{r})$ is proportional to $\text{sgn}(\omega_n)$. Therefore, $f_N^{t_N}(\vec{r})$ describes the odd-frequency STC. It should be noticed that $f_N^{t_N}(\vec{r}) = 0$ since we assume that the present junction is the 1D model and the magnetization in the $F$ has only $z$ component. It should be noticed that $f_N^{t_N}(\vec{r})$ is exactly zero when $h_{ex} = 0$, which corresponds to no magnetic layer in the present junction studied here. This result represents that the RSOI does not induce the STC in the $N$, and thus the magnetic layer is needed to
produce the STC. The interplay of RSOI is to induce a finite \( f^N_{iN}(\vec{r}) \) in the \( N \) of present junction\(^{82}\).

**B. Magnetization induced by proximity effect in normal metal with RSOI**

Based on the quasiclassical Green’s function theory, the magnetization \( \vec{M}(d, \theta) \) induced by the proximity effect is given by\(^{25, 62}\)

\[
\vec{M}(d, \theta) = (M_x(d, \theta), M_y(d, \theta), M_z(d, \theta)) = \frac{A}{V} \int_{0}^{d} \tilde{m}(x, \theta) dx ,
\]

where \( \theta = \theta_R - \theta_L \) is the Josephson phase in the junction and

\[
\tilde{m}(x, \theta) = (m_x(x, \theta), m_y(x, \theta), m_z(x, \theta)) = -g \mu_B \pi N_F k_B T \sum_{\omega_n} \text{sgn}(\omega_n) \text{Im} \left[ \tilde{f}_N^N(\vec{r}) \tilde{f}^{N*}_N(\vec{r}) \right]
\]

with

\[
\tilde{f}_N^N(\vec{r}) = (f_{1x}^N(\vec{r}), -f_{1y}^N(\vec{r}), f_{1z}^N(\vec{r})).
\]

\( \tilde{m}(x, \theta) \) is the local magnetization density in the \( N \), \( g \) is the \( g \) factor of electron, and \( \mu_B \) is the Bohr magneton. \( A \) and \( V = Ad \) are the cross-section area of junction and the volume of \( N \), respectively. The density of states \( N_F \) per unit volume and per electron spin at the Fermi energy is assumed to be approximately the same for up and down electrons of the \( F \) in the quasiclassical Green’s function theory\(^{19, 21}\).

It is obvious in Eq. (22) that \( f_N^N(x) \) and \( \tilde{f}_N^N(x) \) are both required to be nonzero to induce finite \( \tilde{m}(x, \theta) \). However, as described in Sec. (1A) nonzero \( \tilde{f}_N^N(x) \) occurs only when \( F \) layer is involved in the junction and \( \tilde{f}_N^N(x) = 0 \) whenever \( f_N^N(x) = 0 \) for \( \Delta_L = \Delta_R = 0 \). Therefore, the origin of the magnetization in the \( N \) is considered to be due to the STCs induced by the proximity effect\(^{21, 62, 65, 68}\). Because of \( f_N^N(x) = 0 \) (see Sec. (1A), \( m_y(x, \theta) \)) and thus \( M_y(d, \theta) \) are always zero. It is also noticeable that \( M_{z(1)}(d, \theta) \) and \( M_{z(2)}(d, \theta) \) are only induced in the \( N \) when the RSOI and the Josephson coupling are finite in the \( S/N/F \) junction. This result is sharply contrast to Josephson junctions composed of metallic multilayer systems without the RSOI\(^{63, 68, 69}\). Therefore, in what follows, we only consider the \( x \) and \( z \) components of \( \vec{M}(d, \theta) \).

Substituting Eq. (23) and the complex conjugate of Eq. (23) into Eq. (22), and integrating Eq. (22) with \( x \) from 0 to \( d \), we can obtain the \( x \) and \( z \) components of magnetization given by Eq. (21). The \( x \) component \( M_x(d, \theta) \) is decomposed into three parts,

\[
M_x(d, \theta) = M_x^{(1)}(d) + M_x^{(2)}(d, \theta) + M_x^{(3)}(d, \theta),
\]

where

\[
M_x^{(1)}(d) = -g \mu_B \pi N_F k_B T \frac{h_{ex}d^2}{hD_d} \times \sum_{\omega_n} \frac{\Delta^2}{E_{\omega_n}^2} Q_{\omega_n}(d) \text{Im} \left[ \Phi_{\omega_n}(d) u_{\omega_n}(d) \right],
\]

\[
M_x^{(2)}(d, \theta) = g \mu_B \pi N_F k_B T \frac{h_{ex}d^2}{hD_d} \sum_{\omega_n} \frac{\Delta^2}{E_{\omega_n}^2} \left( 1 - \frac{k_N d_F}{\gamma_F} \right)
\times Q_{\omega_n}(d) \text{Im} \left[ \Phi_{\omega_n}(d) u_{\omega_n}(d) \right] \cos \theta,
\]

and

\[
M_x^{(3)}(d, \theta) = g \mu_B \pi N_F k_B T \frac{h_{ex}d^2}{hD_d} \times \sum_{\omega_n} \frac{\Delta^2}{E_{\omega_n}^2} \left( 1 - \frac{k_N d_F}{\gamma_F} \right)
\times Q_{\omega_n}(d) \text{Re} \left[ \Phi_{\omega_n}(d) u_{\omega_n}(d) \right] \sin \theta.
\]

The explicit formulae of the functions \( u_{\omega_n}(d) \) and \( w_{\omega_n}(d) \) in Eqs. (25) – (31) are given by APPENDIX B. From Eqs. (25) – (27) and Eqs. (29) – (31), it is immediately found that the magnetizations of \( x \) and \( z \) components are exactly zero when the exchange field \( h_{ex} \) is zero. Therefore, the \( F \) is indeed required to induce the magnetization inside the \( N \). Here, it should be noticed that \( M_x(d, \theta) \) is always zero without the RSOI as mentioned above.

One of the \( \theta \)-independent part of the magnetization, i.e., \( M_x^{(1)}(d) \), is due to the proximity effect induced STCs common in the \( S/F \) and \( S/F \) multilayer systems\(^{21, 65, 68, 69}\). The other \( \theta \)-independent part of the magnetization, i.e., \( M_x^{(2)}(d, \theta) \), appears due to not only the proximity effect but also the presence of RSOI, since the magnetization of the \( F \) points to \( z \) axis in the present junction\(^{21}\). The \( \theta \)-dependent part of \( M_x^{(2)}(d, \theta) \) is induced by the coupling between two \( S \) only when \( S/F \) multilayer systems compose the Josephson junction\(^{68, 69}\). The
\(\theta\)-dependent part of \(M_z^{(2)}(d, \theta)\) is induced by the coupling between two Ss and the finite RSOI. \(M_x^{(3)}(d, \theta)\) and \(M_z^{(3)}(d, \theta)\) appear when the RSOI in \(N\), the exchange field in the \(F\), and the Josephson coupling are finite. The expressions of the magnetizations \(M_x(d, \theta)\) and \(M_z(d, \theta)\) given in Eq. (24) \- (31) are rather complicate. Therefore, we will present numerical results of magnetizations calculated here in the next section.

III. RESULTS

In this section, we numerically evaluate the magnetizations in the \(S/N/F/S\) junction. In order to perform the numerical calculation of \(M_x(d, \theta)\) and \(M_z(d, \theta)\), the temperature dependence of \(\Delta\) is assumed to be \(\Delta = \Delta_0 \tanh(1.74\sqrt{T_C/T - 1})\), where \(\Delta_0\) is the superconducting gap at zero temperature and \(T_C\) is the superconducting transition temperature. The thickness of \(N\) and \(F\) is normalized by \(\xi_D = \sqrt{\hbar D_N/2\pi k_B T}C\) and the magnetizations of \(x\) and \(z\) components are normalized by \(M_0 = (g\mu_B N_F \Delta_0)\).

![FIG. 2](Color online) (a) The \(x\) component \(M_x(d, \theta)\) and (b) the \(z\) component \(M_z(d, \theta)\) of magnetization in the \(N\) as a function of \(\bar{d}\) for \(d/\xi_D = 1.3\). Here we set \(T/T_C = 0.5\), \(\gamma_F = 0.1\), \(d_F/\xi_D = 0.01\), and \(h_{ex} = 30\). \(\bar{d} = \alpha_R \xi_D\) and \(\xi_D = \sqrt{\hbar D_N/2\pi k_B T_C}\). It is clearly found that magnetizations are reversed (a) from negative to positive values and positive to negative values (b) from positive to negative values and negative to positive values by increasing \(\bar{d}\).

A. Thickness dependence of magnetizations induced by the proximity effect

Figure 2 shows the \(x\) and \(z\) components of magnetization induced by the proximity effect inside the \(N\) as a function of \(d\). The solid (black) and dashed (red) lines are magnetizations for \(\bar{d} = 0.2\) and 0.5, respectively. In Fig. 2 the denotation P.R. and N.R. are the abbreviated expressions of positive and negative regions, respectively. The denotation A.L. is the auxiliary line to separate positive and negative regions of magnetization in Fig. 2. We find that \(M_x(d, \theta)\) and \(M_z(d, \theta)\) exhibit damped oscillatory behavior as a function of \(d\). From Fig. 2 it is found that the magnetizations can be reversed by changing the thickness of \(N\). Moreover, it is also clearly found that the period of oscillation of \(M_x(d, \theta)\) and \(M_z(d, \theta)\) becomes short with increasing \(\alpha_R\). Therefore, by setting \(d\) near the thickness of \(M_{z}(z)(d, \theta) \approx 0\), the magnetizations can be easily reversed by tuning \(\alpha_R\).

Figures 3(a) and (b) show \(x\) and \(z\) components of magnetization as a function of \(\alpha_R\). Here, we set the thickness of \(N\) as \(d/\xi_D = 1.3\). From Fig. 3(a), it is found that the sign of \(M_x(d, \theta)\) is changed from negative to positive with increasing \(\alpha_R\) within \(0.4\xi_D \lesssim \alpha_R \lesssim 0.6\xi_D\). The sign of \(M_x(d, \theta)\) is changed from positive to negative with further increasing \(\alpha_R\). From Fig. 3(b), it is found that the sign of \(M_z(d, \theta)\) is changed from negative to positive with further increasing \(\alpha_R\).
increasing $\alpha_R$ within $0.4\xi_D \lesssim \alpha_R \lesssim 0.6\xi_D$. By further increasing $\alpha_R$, the sign of $M_z(d, \theta)$ is changed from positive to negative. From these results, it is clearly found that the direction of $x$ and $z$ components of magnetization can be reversed by tuning $\alpha_R$.

**B. RSOI dependence of magnetization and Magnetization-phase relation**

Figure 4 shows the magnetization as a function of $d$. Figures 4 (a) and (b) are the $x$ and $z$ component of magnetization, respectively. In Fig. 4, the solid (black) and dashed (red) lines are magnetizations for $\theta = 0$ and $\pi/2$, respectively. $\bar{\alpha}$ is set to be 0.5. From Fig. 4 it is found that the period of oscillation of $M_x(d, \theta)$ and $M_z(d, \theta)$ change by tuning $\theta$. It should be noticed that the variation of the oscillation period of the magnetizations becomes large when $d/\xi_D > 1$ as shown in Fig. 4. By setting $d/\xi_D$ near third minimum of $M_x(d, \theta)$, we can perform magnetization reversal by changing $\theta$ as well as $\alpha_R$ (see Figs. 3 and 4).

Figure 4 shows the magnetization as a function of $\theta$, i.e., magnetization-phase relation. Figures 4 (a) and (b) are the $x$ and $z$ component of magnetization, respectively. The thickness of $N$ is set to be $d/\xi_D = 1.45$. The behavior of $M_x(d, \theta)$ with $\theta$ is a cosine function as shown in Fig. 4 (a). On the other hand, the behavior of $M_z(d, \theta)$ with $\theta$ is a sine function as shown in Fig. 4 (b). From Fig. 5 it is immediately found that $M_x(d, \theta)$ and $M_z(d, \theta)$ vary from positive to negative values and vice versa by changing $\theta$. This result indicates that the magnetizations can be reversed by changing $\theta$ when the thickness of $N$ is appropriately set as mentioned above.

**IV. DISCUSSION**

Here, we discuss why the magnetization-phase relation of $M_x(d, \theta)$ is shifted by $\pi/2$ compared with that of $M_z(d, \theta)$. From Eq. (22), the $x$ component of magnetization is proportional to $\text{Im}[f_x(\vec{r})f_{zx}(\vec{r})]$. From Eq. (10), i.e., $f_{zx}(\vec{r}) = if_{xz}(\vec{r})$, $\text{Im}[f_x(\vec{r})f_{xz}(\vec{r})] = \text{Im}[f_x(\vec{r})f_{xz}(\vec{r})e^{i\pi/2}]$. Therefore, the magnetization–phase relation of $M_x(d, \theta)$ is shifted by $\pi/2$ compared with that of $M_z(d, \theta)$.

We approximately estimate the amplitude of magnetization induced by the proximity effect. As shown in Figs. 2, 4, the magnetization in the $N$ has a finite value in the length scale of $\xi_D$. In the dirty limit, $\xi_D$ is a range of several dozen to several hundred nanometers. Therefore, the magnetization induced by the proximity
can be controlled by tuning amplitude and the oscillation period of the magnetizations depend on the Josephson phase (θ). We have found that the magnetizations induced in the α can be controlled by tuning increasing α function of the thickness of N. We have shown that the magnetization inside the junction is induced due to the proximity effect, the exchange field in the F, and the RSOI in the N. On the other hand, the θ-dependent parts \( M^{(2)}_z(d, θ) \) and \( M^{(3)}_z(d, θ) \) results from the finite Josephson coupling between two Ss, the exchange field in the F, and the RSOI in the N in the S/N/F/S junction. (ii) \( M^{(1)}_z(d) \), which always appears in the S/F junctions is induced due to the proximity effect. The θ-dependent part \( M^{(2)}_z(d, θ) \) results from the finite Josephson coupling between two Ss. \( M^{(3)}_z(d, θ) \) appears only when the Josephson coupling, the exchange field, and the RSOI are finite. For the θ-dependence of magnetizations, we have found that \( M^{(2)}_z(x(z), d, θ) \) is the cosine function with θ and \( M^{(3)}_z(x(z), d, θ) \) sinusoidal with θ.

We have also discussed that the magnetization induced by the proximity effect can be easily reversed by tuning \( θ \) and \( θ \) is a good fingerprint to observe the spin of STC.

We have theoretically shown that the magnetizations are decomposed of three parts, i.e., \( M_x(z)(d, θ) = M^{(1)}_{x(z)}(d) + M^{(2)}_{x(z)}(d, θ) + M^{(3)}_{x(z)}(d, θ) \). (i) Appearance of \( M^{(1)}_{x(z)}(d) \), which is the θ-independent part of magnetization is due to the proximity effect, the exchange field in the F, and the RSOI in the N. On the other hand, the θ-dependent parts \( M^{(2)}_{x(z)}(d, θ) \) and \( M^{(3)}_{x(z)}(d, θ) \) results from the finite Josephson coupling between the two Ss, the exchange field in the F, and the RSOI in the N in the S/N/F/S junction. (ii) \( M^{(1)}_z(d) \), which always appears in the S/F junctions is induced due to the proximity effect. The θ-dependent part \( M^{(2)}_z(d, θ) \) results from the finite Josephson coupling between two Ss. \( M^{(3)}_z(d, θ) \) appears only when the Josephson coupling, the exchange field, and the RSOI are finite. For the θ-dependence of magnetizations, we have found that \( M^{(2)}_{x(z)}(d, θ) \) is the cosine function with θ and \( M^{(3)}_{x(z)}(d, θ) \) sinusoidal with θ.

We have also discussed that the magnetization induced by the proximity effect can be large enough to be detected in typical experiments. Therefore, it is expected that a Josephson junction including the F and the RSOI in the N such as the one studied here can have a potential for low Joule heating spintronics devices, since the direction of the magnetization inside the N can be easily controlled by changing \( θ \) and \( θ \).

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**Appendix A: Coefficients** \( C_{ij}^{(α)}(d) \)

In Eqs. (19) and (20), the coefficients \( C_{ij}^{(α)}(d) \) are given by
In this APPENDIX, we will first provide the analytical form of the local magnetization density in the $N$. Within quasiclassical Green’s function theory, the local magnetization density $\bar{m}(x, \theta)$ in the $N$ is obtained by substituting Eqs. (15) – (17) into Eq. (22). The $x$ component $m_x(x, \theta)$ of the local magnetization density can be decompose into $\theta$-independent and $\theta$-dependent parts

$$ m_x(x, \theta) = m_x^{(1)}(x) + m_x^{(2)}(x, \theta) + m_x^{(3)}(x, \theta). \quad (B1) $$

where

$$ m_x^{(1)}(x, \theta) = -g\mu_N N_F k_B T h_{ex} \sum \frac{\Delta^2}{E_{\omega_n}^2} Q_{\omega_n}(d) \times \sinh(k_N x) \Im[\Phi_{\omega_n}(d) t_{\omega_n}(x, d)], \quad (B2) $$

$$ m_x^{(2)}(x, \theta) = g\mu_N N_F k_B T h_{ex} \sum \frac{\Delta^2}{E_{\omega_n}^2} \left( 1 - \frac{k_F d_F}{\gamma_F} \right) \times Q_{\omega_n}(d) \sinh[k_N (x - d)] \times \Im[\Phi_{\omega_n}(d) t_{\omega_n}(x, d)] \cos \theta, \quad (B3) $$
and

\[
m^{(3)}_x(x, \theta) = g \mu_B \pi N_F k_B T \frac{h_{D_F}}{v_{D_F}} \sum_{i \omega_n} \Delta^2 \left(1 - \frac{k_F d_F}{\gamma_F}\right) \\
	imes Q_{\omega_n}(d) \sinh[k_N(x - d)] \\
	imes \Re[\Phi_{\omega_n}(d)t_{\omega_n}(x, d)] \sin \theta, \quad (B4)
\]

where the functions \(Q_{\omega_n}(d), \Phi_{\omega_n}(d),\) and \(t_{\omega_n}(x, d)\) are given in Eqs. (18) – (20). Substituting Eqs. (B2) – (B4) into Eq. (21) and performing the integration with \(x\), we can obtain the \(x\) component of magnetization,

\[
M_x(d, \theta) = \frac{1}{T} \int_0^d m_x(x, \theta) \, dx = \frac{1}{T} \int_0^d m^{(1)}_x(x) \, dx + \frac{1}{T} \int_0^d m^{(2)}_x(x, \theta) \, dx + \frac{1}{T} \int_0^d m^{(3)}_x(x, \theta) \, dx \\
= -g \mu_B \pi N_F k_B T \frac{1}{T} \sum_{i \omega_n} \frac{h_{D_F}}{v_{D_F}} \frac{\Delta^2}{E_{\omega_n}^2} Q_{\omega_n}(d) \int_0^d \sinh(k_N x) \Im[\Phi_{\omega_n}(d)t_{\omega_n}(x, d)] \, dx \\
+ g \mu_B \pi N_F k_B T \frac{1}{T} \sum_{i \omega_n} \frac{h_{D_F}}{v_{D_F}} \frac{\Delta^2}{E_{\omega_n}^2} \left(1 - \frac{k_N d}{\gamma_F}\right) Q_{\omega_n}(d) \int_0^d \sinh[k_N(x - d)] \Im[\Phi_{\omega_n}(d)t_{\omega_n}(x, d)] \, dx \cos \theta \\
+ g \mu_B \pi N_F k_B T \frac{1}{T} \sum_{i \omega_n} \frac{h_{D_F}}{v_{D_F}} \frac{\Delta^2}{E_{\omega_n}^2} \left(1 - \frac{k_N d}{\gamma_F}\right) Q_{\omega_n}(d) \int_0^d \sinh[k_N(x - d)] \Re[\Phi_{\omega_n}(d)t_{\omega_n}(x, d)] \, dx \sin \theta \\
= M^{(1)}_x(d, \theta) + M^{(2)}_x(d, \theta) + M^{(3)}_x(d, \theta),
\]

where \(M^{(1)}_x(d), M^{(2)}_x(d, \theta),\) and \(M^{(3)}_x(d, \theta)\) are given in Eqs. (25) – (27). The functions \(u_{\omega_n}(d)\) and \(u_{\omega_n}(d)\) in Eqs. (26) – (27) are given as

\[
\begin{align*}
  u_{\omega_n}(d) &= u^{(1)}_{\omega_n}(d) + u^{(2)}_{\omega_n}(d) + u^{(3)}_{\omega_n}(d), \\
  u^{(1)}_{\omega_n}(d) &= -2 \left[C^{21}_{\omega_n}(d) + 2C^{31}_{\omega_n}(d)\right] k_N \sin[(\alpha_R - i k_n) d] - (\alpha_R - i k_n) \sinh(k_N d) \frac{(\alpha_R - i k_n)^2 + k_N^2}{(\alpha_R - i k_n)^2 + k_N^2}, \\
  u^{(2)}_{\omega_n}(d) &= 4\alpha_R \left[C^{22}_{\omega_n}(d) + 2C^{32}_{\omega_n}(d)\right] e^{-k_n d} \\
  &\times \frac{k_n k_N \cos(\alpha_R d) + k_N \alpha_R \sin(\alpha_R d) - e^{k_n d} [(k_n k_N \cosh(k_N d) - (2\alpha_R^2 + k_N^2) \sinh(k_N d))]}{(\alpha_R^2 + k_N^2)^2 + 2(\alpha_R^2 - k_N^2) k_N^2 + k_N^4}, \\
  u^{(3)}_{\omega_n}(d) &= -i4\alpha_R \left[C^{23}_{\omega_n}(d) + 2C^{33}_{\omega_n}(d)\right] \\
  &\times \frac{ik_n k_N \cosh(k_N d) + k_N \alpha_R \sinh(k_N d) - \alpha_R k_n \left[i k_N \cosh(k_N d) + 2\alpha_R \sinh(k_N d)\right]}{(\alpha_R^2 + (k_n - k_N)^2)(\alpha_R^2 + (k_n + k_N)^2)} e^{-i\alpha_R d}
\end{align*}
\]
\[ w_{\omega_n}(d) = w_{\omega_n}^{(1)}(d) + w_{\omega_n}^{(2)}(d) + w_{\omega_n}^{(3)}(d), \]  
\[ w_{\omega_n}^{(1)}(d) = \frac{e^{-(i\alpha_R + k_\alpha) d}}{\alpha_R - i k_\alpha} \left[ \left( -1 + e^{(i\alpha_R + k_\alpha) 2d} \right) k_N \cosh (k_N d) - i \left( 1 + e^{(i\alpha_R + k_\alpha) 2d} \right) (\alpha_R - i k_\alpha) \sinh (k_\alpha d) \right], \]  
\[ w_{\omega_n}^{(2)}(d) = e^{-k_\alpha d} \left[ a_{\omega_n}^{(1)}(d) + a_{\omega_n}^{(2)}(d) - a_{\omega_n}^{(3)}(d) \right] \left( \alpha_R^2 + k_\alpha^2 \right)^{-\frac{1}{2}} \left( 4\alpha_R^2 - k_\alpha^2 \right)^{-\frac{1}{2}} \left( k_N^2 \right)^{-\frac{1}{2}}, \]  
\[ w_{\omega_n}^{(3)}(d) = \frac{1}{2} e^{-i\alpha_R d} \left[ a_{\omega_n}^{(1)}(d) + b_{\omega_n}^{(2)}(d) + b_{\omega_n}^{(3)}(d) \right], \]  
\[ a_{\omega_n}^{(1)}(d) = 2\alpha_R k_\alpha k_N e^{-k_\alpha d}, \]  
\[ a_{\omega_n}^{(2)}(d) = -2\alpha_R k_N \cosh (k_N d) \left[ k_\alpha \cos (\alpha_R d) + \hat{\alpha} \sin (\alpha_R d) \right], \]  
\[ a_{\omega_n}^{(3)}(d) = 2\alpha_R \left[ (2\alpha_R^2 + k_\alpha^2) \cos (\alpha_R d) + 2k_\alpha \alpha_R \sin (\alpha_R d) \right] \sinh (k_N d), \]  
\[ w_{\omega_n}^{(3)}(d) = \frac{1}{2} e^{-i\alpha_R d} \left[ a_{\omega_n}^{(1)}(d) + b_{\omega_n}^{(2)}(d) + b_{\omega_n}^{(3)}(d) \right], \]  
\[ b_{\omega_n}^{(1)}(d) = e^{i\alpha_R d} \left[ \frac{i\alpha_R}{\alpha_R^2 + (k_\alpha - k_N)^2} - \frac{i\alpha_R}{\alpha_R^2 + (k_\alpha + k_N)^2} \right], \]  
\[ b_{\omega_n}^{(2)}(d) = -i\alpha_R \cosh [(k_\alpha - k_N) d] + \frac{k_\alpha - k_N}{\alpha_R^2 + (k_\alpha - k_N)^2} \sinh [(k_\alpha - k_N) d], \]  
\[ b_{\omega_n}^{(3)}(d) = \frac{i\alpha_R \cosh [(k_\alpha + k_N) d] + (k_\alpha + k_N) \sinh [(k_\alpha + k_N) d]}{\alpha_R^2 + (k_\alpha + k_N)^2}. \]

We can also obtain the z component of magnetization given in Eq. (29) - (31) obeying same procedure to derive the x component of magnetization.

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