Dynamic Phases, Stratification, Laning, and Pattern Formation for Driven Bidisperse Disk Systems in the Presence of Quenched Disorder

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Using numerical simulations, we examine the dynamics of driven two-dimensional bidisperse disks flowing over quenched disorder. The system exhibits a series of distinct dynamical phases as a function of applied driving force and packing fraction including a phase separated state as well as a smectic state with liquid like or polycrystalline features. At low driving forces, we find a clogged phase with an isotropic density distribution, while at intermediate driving forces the disks separate into bands of high and low density with either liquid like or polycrystalline structure in the high density bands. In addition to the density phase separation, we find that in some cases there is a fractionation of the disk species, particularly when the disk size ratio is large. The species phase separated regimes form a variety of patterns such as large disks separated by chains of smaller disks. Our results show that the formation of laning states can be enhanced by tuning the ratio of disk radius of the two species, due to the clumping of small disks in the interstitial regions between the large disks.

I. INTRODUCTION

A large class of systems can be effectively described as a collection of interacting particles moving over a random pinning landscape, where a variety of distinct dynamical phases appear as a function of driving force\textsuperscript{[1]}. Well studied examples of such systems include vortices in type-II superconductors\textsuperscript{[2,3]}, driven Wigner crystals\textsuperscript{[4,5]}, skyrmions undergoing current-induced motion\textsuperscript{[6,7]}, sliding pattern forming assemblies coupled to random landscapes\textsuperscript{[8-11]}, colloids on disordered substrates\textsuperscript{[12,13]}, and active matter moving in complex environments\textsuperscript{[14-19]}. These systems often exhibit multiple nonequilibrium phase transitions, such as a transition from a pinned to a sliding phase followed by transitions to different types of sliding phases. Such transitions are associated with clearly observable changes in the velocity-force curves, fluctuation spectra, and spatial reordering of the particles.

Previous work on dynamical phase transitions in driven systems has primarily focused on long or intermediate range particle-particle interactions that tend to favor a uniform particle density, such as that found in magnetic or charged systems. When particles of this type are placed on quenched disorder composed of randomly placed strong pinning sites, three nonequilibrium phases emerge: a pinned disordered state, a plastic flow state in which the particle positions are disordered and the particle exchange neighbors as they move, and a dynamically reordered anisotropic crystal or moving smectic state that appears at high drives when the effectiveness of the pinning is reduced\textsuperscript{[1]}.

There are numerous examples of systems in which the particle-particle interactions are short ranged or steric, including many types of colloidal suspensions, emulsions, bubbles, and granular matter. Although it might be natural to assume that the short-range interactions would produce simpler behavior than the longer-range interactions when the particles are driven over quenched disorder, it was recently shown that monodisperse hard disks moving over a random pinning landscape exhibit a remarkably rich variety of dynamical phases, including clogging, disordered plastic flow, segregated flow, laning flow, and moving crystal\textsuperscript{[20-23]}. The disk system can form moving density segregated states containing high density bands coexisting with low density regions. In some cases, the dense bands form close packed hexagonal lattices even when the overall density of the system is well below the crystallization density. At higher drives, the crystalline bands break up to form dense one-dimensional chains, while at higher densities the disks form a moving crystalline sol\textsuperscript{[23]}. Density separated phases cost no energy in systems with contact interactions, since the energy remains small even when the particles accumulate in one region and are depleted from another region. In contrast, when the interactions are longer range, the system can minimize its energy by destabilizing and dispersing any locally dense regions.

In this work, we consider bidisperse disks driven over quenched disorder consisting of randomly placed pinning sites. In the absence of driving or pinning, the disks form a jammed solid at densities well below the crystallization density \( \phi = 0.9 \) of pin-free undriven monodisperse disks\textsuperscript{[24,25]}. Both monodisperse and bidisperse disks can exhibit a density segregation into dense and depleted re-
gions, but the bidisperse disks can also undergo species segregation of the two disk sizes. Numerous studies have demonstrated species segregation under nonequilibrium conditions for short range repulsive bidisperse systems including granular matter20–22 and colloids23–25 where the degree of segregation depends on the ratio of particle sizes and the type of driving force applied. There are, however, few studies examining the impact of quenched disorder on size segregation. An understanding of segregation effects in flowing bidisperse disks coupled to quenched disorder not only offers new insights on depinning and sliding phenomena, but also could be used to develop new methods for separating or mixing bidisperse or multidisperse systems of particles. For example, some geological systems can be described in terms of multidisperse disks moving through random pinning, and such systems could undergo dynamic segregation.

This paper is organized as follows. We describe our simulation technique for the bidisperse disks driven over random pinning in Section II. In Section III we show the dynamic patterns that form for a system in which 50% of the disks are large and the radius ratio of the large to small disks is 1.4. In Section IV we consider large disks that are twice as big as the smaller disks while maintaining the fraction of large disks at 50%. In Section V we show that by reducing the fraction of large disks to 10%, we can enhance the segregation and stratification effects. We examine the scaling of the velocity-force curves near depinning in Section VI and we summarize our results in Section VII.

II. SIMULATION

We consider a two dimensional (2D) system of size $L \times L$ with periodic boundary conditions in the $x$ and $y$ directions. The sample contains $N_d = N_s + N_l$ disks, where $N_s$ disks have a small radius of $r_s$ and $N_l$ disks have a large radius of $r_l$. The disk dynamics are governed by the following overdamped equation of motion:

$$\eta \frac{d\mathbf{R}_i}{dt} = \mathbf{F}_{dd} + \mathbf{F}_p + \mathbf{F}_D.$$  \hfill (1)

Here $\eta$ is the damping constant and $\mathbf{R}_i$ is the location of disk $i$. The disk-disk interaction force is $\mathbf{F}_{dd} = \sum_{i \neq j} k (r_{ij} - R_{ij}) \Theta(r_{ij} - R_{ij}) \mathbf{R}_{ij}$, where $r_{ij} = r_i + r_j$, $R_{ij}$ is the radius of disk $i(j)$, $R_{ij} = |\mathbf{R}_i - \mathbf{R}_j|$, $\Theta$ is the Heaviside step function, and the spring constant $k = 50$ is large enough to prevent the disks from overlapping by more than 1% of their radii. The pinning force $\mathbf{F}_p$ is produced by $N_p$ pinning sites modeled as randomly placed non-overlapping parabolic wells cut off at a radius of $r_p = r_l$ that can each capture at most one disk with a maximum pinning force of $F_p = 1.0$. The density $\phi$ of the system is given by the area covered by the disks, $\phi = \pi (N_s r_s^2 + N_l r_l^2) / L^2$, where $L = 60$ and $r_s = 0.5$. We vary $r_l$ and set the radius ratio $\Psi = r_l / r_s$ to $\Psi = 1.4$ in Sec. III and $\Psi = 2.0$ in Sec. IV.

In a previous study of the jamming of bidisperse disks using this model with $\Psi = 1.4$, the jamming density in a pin free sample is $\phi_j \approx 0.84$23. We set $N_p = 1440$, giving a fixed pinning site density of $\phi_p = N_p \pi r_p^2 / L^2 = 0.31$. Previous studies have shown that increasing $\phi_p$ does not alter the behavior, but only shifts the driving forces at which the dynamical transitions occur23. The driving force $\mathbf{F}_D = \mathbf{F}_D \mathbf{\hat{x}}$ is applied uniformly to all disks and is incremented in intervals of $\Delta F_D = 0.05$, where we wait at least $5 \times 10^5$ simulation time steps between increments to ensure that the flow has reached a steady state. On each drive increment, we measure the species-dependent disk velocities, $\langle V_s^x \rangle = N_s^{-1} \sum_{i=1}^{N_s} (\mathbf{v}_i \cdot \mathbf{\hat{x}}) \delta(r_i - r_s)$ and $\langle V_l^x \rangle = N_l^{-1} \sum_{i=1}^{N_l} (\mathbf{v}_i \cdot \mathbf{\hat{x}}) \delta(r_i - r_l)$, where $\mathbf{v}_i$ is the instantaneous velocity of disk $i$. We generate species-dependent histograms of $P(v_x)$, the distribution of velocities $v_x$ of the individual disks in the direction of applied drive, by first allowing the system to reach a steady state and then sampling the velocities every $\Delta t = 5 \times 10^5$ simulation time steps. The corresponding $P(v_y)$ is Gaussian distributed about $v_y = 0$ since the motion of the disks perpendicular to the driving force is unbiased. We also characterize the dynamic phases and phase transitions using velocity-force curves, the transverse root mean square displacements, and other measures of the particle spacing and density.

III. MINIMALLY PHASE SEPARATING SYSTEM WITH $N_l = N_d/2$

We first consider samples with $N_s = N_l$ and a disk diameter ratio of $\Psi = 1.4$. By varying the disk density from $\phi = 0.23$ to $\phi = 0.81$, we obtain a ratio of pinning sites to disks in the range $N_p/N_d = 2.0$ to 0.53. At $\phi = 0.46$ there is one disk for every pin, $N_p/N_d = 1.0$. In Fig. I(a) we plot $\langle V_s^x \rangle$ and $\langle V_l^x \rangle$ versus $F_D/F_p$ for $\phi = 0.23$ to 0.87, and we show the corresponding $d\langle V_s^x \rangle/dF_D$ and $d\langle V_l^x \rangle/dF_D$ versus $F_D/F_p$ curves in Fig. I(b). For $F_D/F_p \geq 1.5$, the velocities increase linearly with drive for all values of $\phi$. In the inset of Fig. I(b) we plot the critical depinning force $F_c$ versus $\phi$. When $\phi$ is low, $F_c \approx F_p$ since each disk can be captured independently by a pinning site. As the disk density increases, $F_c$ drops when the disks begin to interact with each other. Since each pin can capture at most one disk, if an unpinned disk comes into contact with a pinned disk, the driving force on both disks is offset by the pinning force on only one disk, lowering the depinning threshold. The number of disks in contact with each other increases with increasing $\phi$, causing $F_c$ to decrease monotonically. We find no species dependence of $F_c$ at any value of $\phi$. Figure I(c) shows $\Delta \langle V_s^x \rangle = \langle V_s^x \rangle - \langle V_l^x \rangle$ the difference in net velocity between the two disk species. This difference is largest in magnitude near the depinning transition.

At a small disk density of $\phi = 0.23$ in Fig. I both $\langle V_s^x \rangle$ and $\langle V_l^x \rangle$ show relatively sharp depinning transitions, as also indicated by the large single peak at de-
A. Intermediate Disk Densities

Disk-disk interactions become important at $\phi = 0.35$, where Fig. 1(b) shows that a two peak structure emerges in $d(V_x^s)/dF_D$, with one peak at $F_D/F_p = 0.9$ and a smaller second peak at $F_D/F_p = 1.05$. We also find that $d(V_x^l)/dF_D$ has a small peak at $F_D/F_p = 0.7$ and a larger peak at $F_D/F_p = 1.05$. A positive peak in $\Delta(V_x)$ extends over the range $0.8 < F_D/F_p < 1.05$ and is larger in magnitude than what we observe at other values of $\phi$.

In the left panel of Fig. 2(a) we illustrate the disk positions in the pinned state for $\phi = 0.35$ at $F_D/F_p = 0.3$. Here, small numbers of unpinned disks have accumulated behind pinned disks, giving a heterogeneous disk density and reducing the depinning threshold to $F_c/F_p = 0.7$. In some regions, short chains of disks composed preferentially of large disks are stabilized at an angle to the driving direction. In the center panel of Fig. 2(a) we plot the local number density $n_{loc}$ and $n_{loc}$ of large and small disks, respectively, obtained by taking slices of width $w = 4r_s$ through the sample at a fixed value of $y$ and dividing the number of disks of each type in that slice by the slice area. Thus, $n_{loc}^s(y) = (4r_sL)^{-1} \sum_{i} \Theta(|R_y^s - y| - 2r_s)\delta(r_i - r_s)$ and $n_{loc}^l(y) = (4r_sL)^{-1} \sum_{i} \Theta(|R_y^l - y| - 2r_s)\delta(r_i - r_l)$. The difference in local number density, $\Delta n_{loc} = n_{loc}^s - n_{loc}^l$, is shown as a function of $y$ in the rightmost panel of Fig. 2(a). Below the depinning transition, both disk species are distributed uniformly throughout the sample.

Figure 1(c) shows that for $\phi = 0.35$ at $F_D/F_p = 0.9$, the velocity of the small disks is larger than that of the large disks, giving $\Delta(V_x) \approx 0.24$. At this drive the sample develops a horizontal band containing a high local density of small disks moving through a homogeneous distribution of large disks, as illustrated in Fig. 1(b). The peak in $d(V_x^s)/dF_D$ at $F_D/F_p = 0.9$ coincides with the emergence of the dense band of small disks in the region $10 < y < 45$. At $y = 50$ the value of $n_{loc}^s$ is nearly zero, but in the rest of the sample $n_{loc}^l$ is roughly constant. The small disks flow continuously while the large disks undergo stick-slip motion that is enhanced in the vicinity of the band of small disks, as shown in the supplementary video.

The species-dependent velocity distributions $P(v_x)$ in Fig. 1(a) show that $v_x$ is bimodal for each species, with peaks at $v_x = 0$ and $v_x = 0.9$ arising from the alternating pinned and freely flowing motion of each disk. The $v_x = 0.9$ peak is higher for the small disks than for the large disks since the small disks are more likely to move freely due to their separation into a dense band, and similarly the peak at $v_x = 0$ is highest for the large disks, which are more likely to fall into a pinning site due to their greater radius. Strong interactions with the pinning sites are required to produce the $v_x = 0$ peak. Although $P(v_x)$ falls off rapidly above $v_x = F_D = 0.9$, there is still a tail with finite weight at $v_x > F_D$ produced by disks that undergo brief rapid motion just after escaping from a pinning site.
FIG. 2: (a, b, c, d) Left panels: Large disk (blue circles) and small disk (red circles) positions for the system in Fig. 1 with \( \Psi = 1.4 \) and \( N_s = N_l \) at \( \phi = 0.35 \). Center panels: \( n_{loc}^l \) (blue) and \( n_{loc}^s \) (red), the local number density of large and small disks, respectively, averaged over the \( x \) direction for each \( y \) position. Right panels: \( \Delta n_{loc} = n_{loc}^s - n_{loc}^l \) plotted at each \( y \) position. (a) The pinned state at \( F_D/F_p = 0.3 \), where unpinned disks pile up behind pinned disks. (b) Just above depinning at \( F_D/F_p = 0.9 \), where the sample contains a dense liquid-like region in the center surrounded by a gas-like region. (c) \( F_D/F_p = 1.1 \), where the small and large disks become further segregated and the disks from the gas-like region collapse into chains with smectic ordering. (d) \( F_D/F_p = 2.0 \), where the entire sample develops a smectic structure. (e) Detail showing large disk (blue circles), small disk (red circles), and pinning site (gray circles) locations in a portion of the sample from Fig. 2(c) containing both the dense band of large disks and the smectic chains. The disk species are not segregated within the chains, and since the pinning force and driving force are nearly equal, the disks do not experience much transverse displacement as they traverse the pinning sites. In the smectic state, \( P(v_x) \) has a single peak at \( v_x = 1.1 \) with equal weight for both species, as shown in Fig. 3(b). Interactions of the disks with the pins in the lower density portions of the sample produce a broad plateau in \( P(v_x) \) over the range \( 0.1 < v_x < 1.1 \). Since \( F_D > F_p \), the pinning sites can only slow the disks but cannot trap them, so there is no longer a peak at \( v_x = 0 \).

At higher drives for \( \phi = 0.35 \), the smectic ordering spreads throughout the entire sample, as shown in Figs. 2(d) at \( F_D/F_p = 2.0 \). The detailed view of the sample in Fig. 2(e) illustrates that the long chains of disks have greater species separation and reduced fluctuations surrounding the original band of small disks, while the lower density portion of the sample develops smectic ordering consisting of chains of mixed disk sizes that are oriented with the driving direction. We find in Fig. 2(c) that \( \langle V_x^s \rangle \) is slightly larger than \( \langle V_x^l \rangle \) at this drive since the higher density band of small disks is able to move more efficiently over the pinning sites, as illustrated in the supplemental video. Figure 2(e) shows a more detailed plot of the disk positions along with the pinning site locations in a portion of the sample from Fig. 2(c) containing both the dense band of large disks and the smectic chains. The disk species are not segregated within the chains, and since the pinning force and driving force are nearly equal, the disks do not experience much transverse displacement as they traverse the pinning sites. In the smectic state, \( P(v_x) \) has a single peak at \( v_x = 1.1 \) with equal weight for both species, as shown in Fig. 3(b). Interactions of the disks with the pins in the lower density portions of the sample produce a broad plateau in \( P(v_x) \) over the range \( 0.1 < v_x < 1.1 \). Since \( F_D > F_p \), the pinning sites can only slow the disks but cannot trap them, so there is no longer a peak at \( v_x = 0 \).

In Fig. 2(c) at \( F_D/F_p = 1.1 \), the band of small disks in the \( \phi = 0.35 \) system becomes more diffuse. Simultaneously, the large disks segregate into dense bands surrounding the original band of small disks, while the lower density portion of the sample develops smectic ordering consisting of chains of mixed disk sizes that are oriented with the driving direction. We find in Fig. 2(c) that \( \langle V_x^s \rangle \) is slightly larger than \( \langle V_x^l \rangle \) at this drive since the higher density band of small disks is able to move more efficiently over the pinning sites, as illustrated in the supplemental video. Figure 2(e) shows a more detailed plot of the disk positions along with the pinning site locations in a portion of the sample from Fig. 2(c) containing both the dense band of large disks and the smectic chains. The disk species are not segregated within the chains, and since the pinning force and driving force are nearly equal, the disks do not experience much transverse displacement as they traverse the pinning sites. In the smectic state, \( P(v_x) \) has a single peak at \( v_x = 1.1 \) with equal weight for both species, as shown in Fig. 3(b). Interactions of the disks with the pins in the lower density portions of the sample produce a broad plateau in \( P(v_x) \) over the range \( 0.1 < v_x < 1.1 \). Since \( F_D > F_p \), the pinning sites can only slow the disks but cannot trap them, so there is no longer a peak at \( v_x = 0 \).

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in the $y$ direction compared to the chains which form at lower $F_D$. The dynamics of this state are illustrated in the supplemental movie. Similar lane formation was observed for a low density of monodisperse disks driven over quenched disorder, and is due in part to the fact that strong density modulations incur no energy penalty in systems with short range interactions. Although on average $\Delta n_{\infty} \approx 0$, indicating that the large scale species segregation found at lower drives is lost, we find that individual chains can be preferentially composed of a single species of disk. The velocity distributions $P(v_x)$ are similar to those shown in Fig. 3(b) but have a sharper peak at $v_x = F_D$.

The moving smectic state we observe differs from those predicted by theory and observed in simulation and experiment to occur in driven systems with quenched disorder such as vortices in type-II superconductors confined to two dimensions. The short-range nature of the disk-disk interactions permits the emergence of extreme chaining behavior in which the disks are nearly in contact along the driving direction but are well-spaced in the transverse direction. In contrast, superconducting vortices strongly repel one another at short distances, and thus have a more even spacing in the directions parallel and transverse to the drive. Adjacent vortex rows in the smectic state contain dislocations that can glide along the driving direction and permit the rows to slide past one another. For the disk system, adjacent rows are noninteracting and can move completely independently of each other.

In Fig. 4 we illustrate the time-dependent behavior of the $0 < \phi < 1.5$. Figure 4(a) shows the instantaneous values of $V_x^s$ and $V_x^l$ versus time at driving forces ranging from $F_D/F_p = 0.70$ to 1.15. In Fig. 4(b), we show the corresponding ratio $R = V_x^s/V_x^l$ versus time. At $F_D \leq 0.70$, the disks are pinned, and $V_x^s = V_x^l = 0$ except for a brief sharp decay at very early times from a nonzero value. At intermediate $F_D$ values of 0.75, 0.8, and 0.85, we find large fluctuations in both $V_x^s$ and $V_x^l$, and although the velocities of the two disk species are initially identical, as the system evolves the velocities separate so that at long times $V_x^s > V_x^l$. At $F_D/F_p = 0.9$, where the small disks first segregate into a band, we can fit the velocity of the small disks to a stretched exponential form, as shown in Fig. 4(c) where we find $V_x^s \propto e^{-t/\tau}$ with $\tau = 1.22 \times 10^7$. For comparison, we show a fit to $V_x^s \propto t^{\alpha}$ with $\alpha = 0.26 \pm 0.01$, which gives a poorer fit. We find a similar stretched exponential behavior at $F_D/F_p = 0.95$, and we show in Sec. III.C that this behavior is associated with enhanced transverse diffusion. The stretched exponential time response suggests that the formation of the segregated band of small disks is similar to an absorbing phase transition of the type found in clogging systems.

For $F_D/F_p = 1.0$, 1.05, and 1.10, a stretched exponential fit gives a large time constant $\tau$, and we show in Sec. III.C that these drives produce superdiffusion in the transverse direction. At higher driving forces $F_D > 1.10$, the sample quickly reaches a steady flow state with constant $V_x^s$ and $V_x^l$.
B. High Disk Density

When $\phi = 0.46$, the effect of interstitial or unpinned disks on the depinning process becomes more important, and the depinning threshold drops to $F_c/F_p = 0.5$, as shown in Fig. 4(a). The peak in $(V_x^s)$ and $(V_y^s)$ at depinning is diminished in size, and we find that $\Delta(V_x) \approx 0.04$ over the range $0.5 < F_D/F_p < 1.0$. At $F_D/F_p = 0.5$, illustrated in Fig. 5(a), $\Delta(V_x) \approx 0$ and both types of disks are in a gas-like state containing small regions of higher disk density in the form of clumps and chains. For this drive, the plots of $n^{loc}_l$ and $n^{loc}_s$ in Fig. 6 show that each disk species is uniformly distributed across the sample. The corresponding velocity histogram $P(v_x)$ in Fig. 6(a) shows a bimodal distribution produced by the stick-slip motion of the disks, which are interacting strongly with the pinning sites. The $v_x = 0$ peak is higher than the $v_x = F_D$ peak, indicating that the disks spend more time sticking and less time slipping, giving a low value of $\Delta(V_x)$ in Fig. 4(a). At $F_D/F_p = 1.3$ in Fig. 5(d), where we again have $\Delta(V_x) \approx 0$, the disks phase segregate into a liquid region surrounding a smectic region, which extends from $40 < y < 55$. The smectic state is characterized by strongly asymmetric spacing of the disks, which are much closer together parallel to the drive than perpendicular to the drive. In this case, the smectic region contains mostly small disks and is of relatively low density. The density of the liquid region varies as a function of $y$, and the liquid is composed mainly of large disks separated by horizontal gaps for $10 < y < 30$, while a densely packed liquid containing nearly equal numbers of small and large disks appears for $y < 10$. The large disks are almost completely depleted in the regions $y = 30$ and $40 < y < 50$ but have a nearly uniform density in the rest of the sample, as shown by the plot of $n^{loc}_l$ in Fig. 6(d). In Fig. 6(b), $P(v_x)$ has a single peak at $v_x = F_D = 1.3$ and a broad distribution of velocities in the range $0.3 \leq v_x \leq 2.3$, including a low velocity plateau.

For higher disk densities of $\phi = 0.58$ to 0.87, $F_c$ continues to decrease with increasing $\phi$ while $\Delta(V_x)$ becomes small. The increased disk-disk interactions that occur at the higher densities not only diminish the depinning force, but also equalize the velocities of each disk species due to the higher frequency of disk-disk collisions. In Fig. 5(b), we show a $\phi = 0.70$ sample at $F_D/F_p = 0.5$, where the disks are in a liquid state containing some small localized clumps and chains. There is some species segregation, with the small disks preferentially located at the top of the sample and the large disks preferentially residing at the bottom of the sample, as indicated by the plots of $n^{loc}_l$ and $n^{loc}_s$ in Fig. 5(b). We find a bimodal distribution of $P(v_x)$ as shown in Fig. 6(c), but the two peaks are barely higher than the background plateau since the increased disk-disk interactions reduce the effectiveness of the pinning sites. The same sample at $F_D/F_p = 1.3$ develops polycrystalline structure in which the disks form wide species separated bands, as illustrated in Fig. 5(c). The polycrystalline clusters tend to be aligned in the driving direction. Figure 6(d) shows a single peak in $P(v_x)$ at $v_x = F_D$ along with a broad distribution of velocities over the range $0.4 \leq v_x \leq 2.4$. The plateau at low $v_x$ has vanished since all of the disks are always moving at this drive, and it is replaced by a rapid decrease in $P(v_x)$ with decreasing $v_x$.

At $\phi = 0.81$, Fig. 5(c) shows that when $F_D/F_p = 0.5$, the disks have a combination of liquidlike and polycrystalline structure. Although the plot of $n^{loc}_l$ indicates that there is a local increase of small disk density near $y \approx 55$, the disks are nearly jammed, and as a result further species segregation is suppressed. In Fig. 6(e), $P(v_x)$ has lost its distinct peaks and has a much more Gaussian shape, since the strong interactions between the disks prevent individual disks from being trapped by the pins. At $F_D/F_p = 1.3$ for the same sample in Fig. 5(f), the disk structure is nearly the same except that any slight tendency for segregation into a band has been destroyed. The plot of $P(v_x)$ in Fig. 6(c) shows a spread of velocities about $v_x = F_D$ due to the tightly packed motion of the disks.

For densities of $\phi = 0.81$ and above, the disks have a glassy arrangement at both low and high drives, and the high packing fraction inhibits rearrangements of the disks, preventing both species segregation and the realignment of the polycrystalline regions with the driving direction. We have tested the system for finite size effects using a larger sample with $L = 200$, where we found structures similar to those illustrated in Figs. 2 and 5. The only difference is that the large system can accommodate multiple layers of segregated bands along the $y$ direction.

C. Transverse Diffusion and Topological Order

To further distinguish the phase behavior of each disk species, we measure the disk displacements in the direction transverse to the applied drive,

$$
\langle \delta y^2_{s(i)} \rangle = \frac{1}{N_s} \sum_{i=1}^{N_s} [y_i(t) - y_i(t_0)]^2,
$$

for the small and large disks, respectively. In Fig. 7, we plot $\langle \delta y^2_{s(i)} \rangle$ and $\langle \delta y^2_{l(i)} \rangle$ obtained over the time interval $1 \times 10^7$ to $5 \times 10^7$ simulation time steps versus $F_D/F_p$ for samples with $\phi = 0.35$, 0.58, and 0.7. We also show the corresponding diffusive exponents $\alpha_s$ and $\alpha_l$ obtained from long-time fits to $\langle \delta y^2_{s(i)} \rangle \propto t^{\alpha_s(i)}$. At all densities, $\langle \delta y^2_{s(i)} \rangle = 0$ and $\alpha_s(i) = 0$ for $F_D < F_c$ when the disks are motionless. Previous studies of monodisperse disks showed superdiffusive transverse flow with $\alpha > 1$ in regimes where density phase separation occurred, since the increased frequency of disk-disk interactions in the high density region produces a greater amount of disk motion transverse to the driving direction. The bidisperse disks have a more complex behavior since a wider variety of phase separated states occur that extend down...
to lower densities. In particular, the large and small disks generally exhibit different transverse diffusive behavior in the species separated regimes.

In Fig. 7(a) at $\phi = 0.35$, both disk species undergo subdiffusive transverse motion with $\alpha_s < 1$ when $F_D > F_c$. Transverse movement is suppressed at low disk density due to the infrequency of disk-disk collisions. Near $F_D/F_p = 1.0$, we find large fluctuations of $\alpha_s$ and $\alpha_l$ due to the gradual emergence of the dense species separated bands illustrated in Fig. 2(b,c). At $F_D/F_p = 0.9$ and 1.0, the dense liquid band of small disks is surrounded by a homogeneous low density gas of large disks, and we find subdiffusive behavior with $\alpha_s(l) < 1$. Superdiffusive behavior with $\alpha_s(l) > 1$ appears at $F_D/F_p = 0.95$ where the small disks have more fully segregated into a distinct horizontal band, and also at $F_D/F_p = 1.05$ and 1.1 where the small disks form a smectic low density state containing horizontal chains. Similar fluctuations in $\alpha_s(l)$

FIG. 5: Left panels: Large disk (blue circles) and small disk (red circles) positions for the system in Fig. 1 with $\Psi = 1.4$ and $N_s = N_l$. Center panels: $n_{s,loc}^l$ (blue) and $n_{s,loc}^s$ (red) as a function of $y$ position. Right panels: $\Delta n_{s,loc}$ as a function of $y$ position. (a) $\phi = 0.58$ and $F_D/F_p = 0.5$, where there is a driven homogeneous phase. (b) $\phi = 0.70$ and $F_D/F_p = 0.5$, showing a segregated liquid. (c) $\phi = 0.81$ and $F_D/F_p = 0.5$, where we find an isotropic polycrystalline phase. (d) $\phi = 0.58$ and $F_D/F_p = 1.3$, where the system fractionates into a liquid and smectic phase. (e) $\phi = 0.70$ and $F_D/F_p = 1.3$, where the system is liquid throughout but forms distinct horizontal bands. (f) $\phi = 0.81$ and $F_D/F_p = 1.3$, which shows an isotropic polycrystalline state similar to that found at lower drives.

FIG. 6: $P(v_x)$ for the small disks (red) and large disks (blue) for the system in Fig. 1 at (a) $\phi = 0.58$ and $F_D/F_p = 0.5$; (b) $\phi = 0.58$ and $F_D/F_p = 1.3$; (c) $\phi = 0.7$ and $F_D/F_p = 0.5$; (d) $\phi = 0.7$ and $F_D/F_p = 1.3$; (e) $\phi = 1.3$ and $F_D/F_p = 0.5$; (f) $\phi = 1.3$ and $F_D/F_p = 1.3$. 
FIG. 7: The transverse displacements $\langle \delta y_s^2 \rangle$ (red dashed line) and $\langle \delta y_l^2 \rangle$ (blue dashed line) for the small and large disks obtained after $1 \times 10^7$ simulation time steps vs $F_D/F_p$ and the corresponding diffusive exponent $\alpha_s$ (red squares) and $\alpha_l$ (blue squares) for the system in Fig. 1 at (a) $\phi = 0.35$, (b) 0.58, and (c) 0.70.

Appearance near $F_D/F_p = 1$ for $0.35 < \phi < 0.5$, where some samples reach a steady phase segregated, particle separated state within $\Delta t = 5 \times 10^7$ time steps while others do not.

In Fig. 7(b) at $\phi = 0.58$ we find diffusive transverse motion with $\alpha_{s(l)} \approx 1$ whenever the disk density is homogeneous, including near depinning and for driving forces at which densely packed polycrystalline regions appear. For drives just above depinning, both types of disk undergo superdiffusive transverse motion as the species separation illustrated in Fig. 5(a) occurs. The large disks transition to diffusive behavior at $F_D/F_p = 0.75$, while the small disks remain superdiffusive until $F_D/F_p = 1.3$. Above $F_D/F_p = 1.3$, the driving force dominates the disk motion and the transverse displacements are subdiffusive for both species. In Fig. 7(c) at $\phi = 0.70$, the transverse motion is diffusive at depinning when $F_D = F_c$. The large disks are superdiffusive in the range $0.3 < F_D/F_p < 1.0$, and become diffusive at higher drives. The small disks are diffuse for $0.3 < F_D < 0.5$, superdiffusive for $0.5 < F_D < 1.5$, and diffusive above $F_D = 1.5$. A similar intermediate superdiffusive phase was observed in Ref. 22. When the disk density is high, we find a transition from diffusive to subdiffusive behavior coinciding with the emergence of a locked polycrystalline phase. For example, at $\phi = 0.814$, $\alpha_{s(l)} \approx 1$ for all $F_D > F_c$. At $\phi = 0.87$, $\alpha_{s(l)} \approx 0$ since the disks are kinetically trapped.

To characterize lane formation, we measure the transverse nearest neighbor distance $\ell_{nn}$, the average perpendicular distance between disks that are in contact, given by

$$\langle \ell_{nn} \rangle = \sqrt{\Theta(r_{dd}^2 - R_{ij})(\hat{R}_{ij} \cdot \hat{y})^2},$$

where $R_{ij} = R_i - R_j$. In Fig. 8 we plot $\langle \ell_{nn} \rangle$ versus $F_D/F_p$ for $\phi = 0.06$ to $\phi = 0.81$. For small disk densities in the range $\phi = 0.06$ to 0.12, almost no disks are in contact with each other and $\langle \ell_{nn} \rangle$ is nearly zero. For higher disk densities, in the pinned state the disks tend to form blockages perpendicular to the drive that become more extensive as $\phi$ increases, giving larger values of $\langle \ell_{nn} \rangle$. As the depinning threshold is approached, these blockages fall apart, so that $\langle \ell_{nn} \rangle$ decreases monotonically over the range $0 < F_D < F_c$. For $\phi = 0.29$ and $\phi = 0.35$, $\langle \ell_{nn} \rangle = 0$ just below depinning where nearly all disk-disk contacts are lost, followed by a peak in $\langle \ell_{nn} \rangle$ near $F_D/F_p = 1$, where phase segregation into low and high density regions occurs. When chain structures form at higher $F_D$, $\langle \ell_{nn} \rangle$ plateaus to a small but finite value. At $\phi = 0.58$ and $\phi = 0.81$, $\langle \ell_{nn} \rangle$ decreases steadily for $F_D > F_c$, where $F_c = 0.4$ and 0.2, respectively. At $\phi = 0.87$, which is near the jamming limit, $\langle \ell_{nn} \rangle \approx 0.6$ for all drives.

In Fig. 9 we show a heightfield plot of the $\Delta(V_x)$ data from Fig. 1(c) as a function of disk density $\phi$ versus driv-
is followed at higher φ. As pins outnumber the disks, the system depins directly into the disordered, liquid/smectic phase V. For intermediate drives, the density phase separated liquid-gas state from Fig. 2(b). Phase II, consisting of homogeneous flow, is illustrated in Fig. 2(c). Phase III is the moving polycrystalline state shown in Fig. 5(c) and the moving banded solid, appears in Fig. 5(e), and phase VII is the polycrystalline state IV. Enhanced Crystallization and Banding with Larger Radius Ratio at $\Psi = 2.0$.

We next increase the radius ratio to $\Psi = 2.0$, a value that is known to produce phase separation for disks packed out of equilibrium. We fix $N_s = N_i$ and consider disk densities in the range $\phi = 0.19$ to $0.88$, corresponding to $N_D/N_p = 0.25$ to 1.125. Here a disk density of $\phi = 0.78$ corresponds to a ratio $N_D/N_p = 1.0$.

The plot of $\langle V_x \rangle$ and $\langle V_z \rangle$ versus $F_D/F_p$, curves in Fig. 10(b), as well as the plot of $\Delta V_x/F_D$ versus $F_D/F_p$ in the inset of Fig. 10(b), are also similar to what was shown in Fig. 1(b). In Fig. 10(c), we find that the large disks have a higher velocity than the small disks, as highlighted in the inset of Fig. 10(c); however, at low driving forces and high $\phi$, we find that the large disks have a higher velocity than the small disks, as indicated in the inset of Fig. 10(c). At the lowest density of $\phi = 0.20$ in Fig. 10, the small and large disks both have the same behavior, and the depinning occurs sharply at $F_D/F_p = 1.0$, with a distinct transition from pinned to elastic flow. Since this system contains fewer disks than the $\Psi = 1.4, \phi = 0.23$ system, the depinning transition is sharper, and the peak in $\langle V_x \rangle/F_D$ and $\langle V_z \rangle/F_D$ at $F_D/F_p = 1.0$ is larger.

At $\phi = 0.29$, we find an enhancement in the velocity of the small disks near depinning since the large disks can easily be pinched by traps and other large disks, while the small disks slip through smaller apertures to form a segregated dense band, as illustrated in Fig. 11. At $F_D/F_p = F_c = 0.95$. Here the large disks are uniformly distributed through the sample, while the small disks are concentrated in a band extending from $45 < y < 60$. This is the same type of segregation found in Fig. 2(b). In Fig. 10(b), $\langle V_z \rangle/F_D$ peaks at $F_D/F_p = 0.95$, whereas $\langle V_x \rangle/F_D$ peaks at $F_D/F_p = 1.0$, indicating that the smaller disks begin to flow freely at lower drives than the larger disks. Above $F_D/F_p = 1$, there is a transition to a liquid of small disks surrounded by a smectic state of large disks, as illustrated in Fig. 11(d) for $F_D/F_p = 1.1$. This is accompanied by a large positive peak in $\Delta \langle V_x \rangle$ over the range $1.05 < F_D/F_p < 1.25$, as shown...
the range $0.8 < F_D/F_P < 1.0$, indicating that the small disks can flow more easily in the liquid smectic region, which they preferentially occupy. At $\phi = 0.49$, there is a pronounced crossover in $\Delta \langle V_2 \rangle$ in Fig. 10(c) from a negative value for $0.6 < F_D/F_P < 0.7$ to a positive value for $0.8 < F_D/F_P < 1.0$, indicating that the large disks are moving faster than the small disks at lower drives but slower at higher drives.

For $\phi = 0.59$, $\Delta \langle V_2 \rangle$ is never positive but has an enhanced negative region at low drives above depinning in the range $0.4 < F_D/F_P < 0.9$, as highlighted in the inset of Fig. 10(c). Species segregation of the disks occurs in the window $0.8 < F_D/F_P < 0.9$. As illustrated in Fig. 11(b) for $F_D/F_P = 0.9$, the large disks form a cluster that spans nearly the entire system, while the small disks are concentrated in a band ranging from $20 < y < 40$. The small disks form relatively few disk-disk contacts, making them less likely to be depinned due to disk-disk interactions, and thus reducing their velocity compared to the large disks. At higher drives, all of the disks depin and the difference in velocity among the two disk species drops to zero. At $F_D = 1.1$, shown in Fig. 11(e), the small disks remain in a single high density band while the large disks form a low density smectic state at $0 < y < 10$ coexisting with a high density liquid state containing polycrystalline regions at $35 < y < 60$. A low density void region appears at $10 < y < 20$. The motion of the particles in this state is illustrated in the Supplementary Material.

When $\phi \geq 0.59$, $d\langle V_2^x \rangle/dF_D$ and $d\langle V_2^y \rangle/dF_D$ have a smooth rather than sharp increase at $F_D = F_c$. There is an extended regime in which the velocity of the large disks is higher than that of the small disks, with $\Delta \langle V_2 \rangle < 0$ over the range $0.2 < F_D/F_P < 1.5$ for the $\phi = 0.79$ system. As shown in Fig. 11(c) for $\phi = 0.79$ at $F_D/F_P = 0.9$, a significant fraction of the large disks form tight polycrystalline packings while the small disks form trapped clusters over specific horizontal windows. The structure remains similar at higher drives, as shown in Fig. 11(e) at $F_D/F_P = 1.1$. For larger systems with $L = 200$ at high $\phi$, we find multiple large polycrystalline regions rather than a single band spanning the system.

A. Transverse Diffusion and Topological Order

In Fig. 12, we plot the transverse diffusion $\langle \delta y_2^x \rangle$ and $\langle \delta y_2^y \rangle$ along with the exponents $\alpha_s$ and $\alpha_t$ versus $F_D/F_P$ for the $\Psi = 2.0$ system from Fig. 10. At $\phi = 0.59$ in Fig. 12(a), we find homogeneous flow with $\alpha_s \approx \alpha_t \approx 1$ at low driving forces of $0.4 < F_D/F_P < 0.6$, indicating diffusive behavior. At intermediate driving forces, $0.6 < F_D/F_P < 1.2$, the small disks are subdiffusive since they have become confined in a horizontal band, as shown in Fig. 11(b) and (d). The large disks are superdiffusive for $0.6 < F_D/F_P < 1.0$, and become subdiffusive at higher drives once their structure changes from a homogeneous liquid with small voids to a denser liq-
FIG. 11: Left panels: Large disk (blue circles) and small disk (red circles) positions for the system in Fig. 10 with $\Psi = 2.0$ and $N_s = N_l$. Center panels: $n_{l_{\text{loc}}}$ (blue) and $n_{s_{\text{loc}}}$ (red) as a function of $y$ position. Right panels: $\Delta n_{l_{\text{loc}}}$ as a function of $y$ position. (a) $\phi = 0.29$ and $F_D/F_p = 0.95$. (b) $\phi = 0.59$ and $F_D/F_p = 0.9$. (c) $\phi = 0.79$ and $F_D/F_p = 0.9$. (d) $\phi = 0.29$ and $F_D/F_p = 1.1$. (e) $\phi = 0.59$ and $F_D/F_p = 1.1$. (f) $\phi = 0.79$ and $F_D/F_p = 1.1$.

FIG. 12: Transverse displacements $\langle \delta y^2 \rangle$ (red dashed line) and $\langle \delta y_s^2 \rangle$ (blue dashed line) for the small and large disks obtained after $1 \times 10^7$ simulation time steps vs $F_D/F_p$ and the corresponding diffusive exponent $\alpha_s$ (red squares) and $\alpha_l$ (blue squares) for the system in Fig. 10 with $\Psi = 2.0$ for densities $\phi = (a) 0.59$ and (b) 0.79.

FIG. 13: $\langle \theta_{nn} \rangle$ vs $F_D/F_p$ for the system in Fig. 10 with $\Psi = 2.0$ for $\phi = 0.20$ (circles), 0.39 (triangles), 0.59 (squares), and 0.79 (stars).

uid containing a large horizontal gap. The small voids permit a transverse flow of the large disks that is suppressed once a large void opens at higher drives. For $1.2 < F_D/F_p < 2.0$, the driving force dominates the behavior of both disk species, which form chain states that move subdiffusively in the transverse direction. At $\phi = 0.79$ in Fig. 12b, $\alpha_s \approx \alpha_l \approx 1$ at all driving forces above depinning, indicating diffusive transverse flow for both disk species. This is expected in a liquid phase containing polycrystalline regions of homogeneous density.

In Fig. 13, we characterize the lane structure of the disks based on the average angle between disks that are in contact,

$$\langle \theta_{nn} \rangle = \frac{1}{N_d} \sum_i N_d \Theta(r_i^{ij} - R_{ij}) \tan^{-1} \left( \frac{R_{ij} \cdot \hat{y}}{R_{ij} \cdot \hat{x}} \right),$$

sampled every $\Delta t = 5 \times 10^5$ simulation time steps after the system has reached a steady state. This measure is closely related to $\langle l_{nn} \rangle$ from Fig. 8. Figure 13 shows $\langle \theta_{nn} \rangle$ versus $F_D/F_p$ for systems with $\phi = 0.20, 0.39, 0.59,$ and 0.79. For $\phi = 0.20$, $\langle \theta_{nn} \rangle$ is low for all drives due to the smectic structure which favors disk-disk contacts that are aligned with the $x$ direction. We find $\langle \theta_{nn} \rangle \approx 30^\circ$ near...
depinning for $\phi = 0.79$, since the polycrystalline disk arrangements tend to contain crystallites aligned with the $x$ axis that contribute angles of 0° and 60° equally to the sum. As the driving force increases, $\langle \theta_{nn} \rangle$ decreases monotonically due to an increase in the amount of smectic or chainlike ordering in the system. For $\phi = 0.39$ and $\phi = 0.59$, a local maximum in $\langle \theta_{nn} \rangle$ at $F_D/F_p = 1$ is produced by the denser structures that form when the phase separation is maximized for nearly equal pinning and driving strengths. This is followed by a decrease in $\langle \theta_{nn} \rangle$ at higher drives as smectic ordering emerges.

In Fig. 14 we show a heightfield plot of $\Delta(V_x)$ as a function of $\phi$ versus $F_D/F_p$ for the $\Psi = 2.0$ system. Compared to the $\Psi = 1.4$ system in Fig. 9, we find a much larger region in which $\Delta(V_x) < 0$. This indicates that increasing the relative size of the large disks can also increase their velocity relative to the small disks when the driving force is close to the depinning threshold and the total disk density is sufficiently large.

V. LOWER FRACTION OF LARGE DISKS, $N_l = N_d/10$

We next investigate the effect of changing the disk species ratio from $N_s = N_l = 0.5N_d$ to $N_s = 0.9N_d$ and $N_l = 0.1N_d$ for a system with $\Psi = 1.4$. We find the same general phases as described in Sec. III but with a greater tendency for the large disks to move faster than the small disks. In Fig. 15(a), we plot $\langle V_x^s \rangle$ and $\langle V_x^l \rangle$ versus $F_D/F_p$ for the $N_l = 0.1N_d$ system at a disk density of $\phi = 0.48$. We find plastic depinning for both disk species, as indicated by the concave shape of the velocity-force curve, followed by a transition at higher drives to a linear dependence. At $F_D/F_p = 0.9$, illustrated in Fig. 16(a), the system can be divided into three regions: a small disk liquid, a small disk gas, and a mixed gas-like region containing both disk species at an intermediate density. At a higher drive of $F_D/F_p = 1.1$ in Fig. 16(b), the small disk liquid has increased in density and contains a few large disks. A window of large disk liquid containing some small disks runs along one
side of the small disk liquid, while the low density region of the sample contains roughly equal numbers of small and large disks arranged in a smectic structure. Due to the strong species segregation, these phases resemble the states found for monodisperse disks in Ref.23. Over the range 0.2 < $F_D/F_p < 1.6$ where the species separation occurs, $\langle V_s^i \rangle > \langle V_s^l \rangle$, giving $\Delta (V_s) < 0$ (not shown).

In Fig. 15(b), we plot the time evolution of $\langle V_x^s \rangle$ and $\langle V_x^l \rangle$ for the same $\phi = 0.48$ system at $F_D$ values ranging from $F_D/F_p = 0.2$ to $F_D/F_p = 2.0$. For $F_D/F_p \leq 0.2$, the system is pinned and $\langle V_x^s \rangle = \langle V_x^l \rangle = 0$. When $F_D/F_p \geq 0.4$, we find $\langle V_x^s \rangle > \langle V_x^l \rangle$, with $\langle V_x^l \rangle$ remaining nearly constant over time while $\langle V_x^s \rangle$ decays. For intermediate driving forces of $0.6 < F_D/F_p < 1.2$, the $\langle V_x^s \rangle$ curves have an exponential shape, $\langle V_x^s \rangle \propto e^{-t/\tau_x} + \langle V_{x0} \rangle$, as shown in Fig. 15(c) for $F_D/F_p = 0.8$, where $\tau_x = 8.46 \times 10^5$. A similar fit of $\langle V_x^s \rangle$ at the same drive gives a time constant $\tau_s = 1.19 \times 10^6$ that is somewhat larger. As $F_D/F_p$ increases above 1.2, the system rapidly reaches a steady state and the difference between the velocity of the small and large disks vanishes. Due to the lengthy transient dynamics at intermediate $F_D/F_p$, we wait a minimum of $2 \times 10^6$ simulation time steps before measuring the velocity-force curves shown in Fig. 15(a).

In Fig. 17 we plot the transverse displacements $\langle \delta y_s^2 \rangle$ and $\langle \delta y_l^2 \rangle$ versus $F_D/F_p$ for the $\phi = 0.48$ sample along with the corresponding exponents $\alpha_s$ and $\alpha_l$. All four quantities increase monotonically between $F_D = F_c$ and $F_D/F_p = 0.4$. At intermediate $F_D$, we find subdiffusive transverse motion of the small disks with $\alpha_s < 1$ accompanied by superdiffusive transverse motion of the large disks with $\alpha_l > 1$. Here the small disks are confined within a dense liquid, while the large disks are in a low density region in which interactions with pinning sites can enhance the transverse diffusion. At large $F_D$ where smectic structures emerge, both disk species have subdiffusive transverse motion.

VI. SCALING NEAR THE DEPINNING TRANSITION

In systems of particles that have long range interactions, the velocity-force relationship scales as $V \propto (F_D - F_c)^{-\beta}$. For elastic depinning in which the structure of the particle lattice remains unchanged, $\beta = 2/3$, while when the depinning transition is plastic, $\beta > 1$. For Coulomb and screened Coulomb interaction potentials, the plastic depinning exponents are $\beta \approx 1.65$ and 2.0, respectively, while simulations of depinning of superconducting vortices with a Bessel function vortex-vortex interaction give $\beta = 1.3$. It is interesting to ask whether similar scaling of the velocity-force curves occurs in the disk system. For monodisperse disks with $N_p/N_d > 0.288$, it was shown in Ref.23 that the velocity-force curves can be fit to a power law with $1.4 < \beta < 1.7$.

In Fig. 18(a,b) we plot $\langle V_x^s \rangle$ and $\langle V_x^l \rangle$ versus $F_D - F_c$ on a log-log scale at densities of $\phi = 0.46$ and 0.58, re-
FIG. 18: \(\langle V_s^x \rangle, \langle V_l^x \rangle\) vs \(F_D - F_c\) on a log-log scale for the sample from Fig. 1 with \(\Psi = 1.4\) and \(N_s = N_l\). We fit the data to \(\langle V_s^x \rangle \propto (F_D - F_c)^{-\beta}\) (pink lines). (a) \(\phi = 0.46\) with \(\beta = 1.0\). (b) \(\phi = 0.58\) with \(\beta = 1.3\).

spectively. By fitting the portion of the curve closest to depinning, we find \(1.0 < \beta < 1.3\). The scaling fit can be performed only for \(\phi > 0.35\) and does not work at low disk densities. We find similar scaling fits for sufficiently large disk densities for the \(\Psi = 2.0\) system and for the \(\Psi = 1.4\) and \(N_l = 0.1 N_d\) system. The depinning is clearly not elastic, but the lower values of \(\beta\) compared to systems with longer range interactions suggest that the type of plastic depinning that occurs may be different for short range interacting systems than for longer range interacting systems.

VII. SUMMARY

We examine the dynamics of bidisperse disks driven over random quenched disorder to explore the dynamical phases of particles with short range interaction forces. At low disk densities, we observe a pinned state that transitions into a strongly chained state where the disks can undergo local demixing but where the overall disk distribution is homogeneous. At intermediate disk densities, the disks depin into a disordered flow state exhibiting stick slip dynamics, followed by a species segregated state in which the small disks form clusters and the large disks remain evenly distributed throughout the sample. For intermediate drives the disks form a partially pinned state exhibiting both species separation and density segregation, while at high drives a mixed laned state emerges. At high disk densities of \(\phi > 0.75\), a rigid polycrystalline state appears that moves as a solid and undergoes no species or density segregation. Both the density and the species segregation effects are the most prominent near \(F_D = F_p\) when the driving force and pinning force directly compete. The anisotropic fluctuations induced by the pinning at high drives favor the formation of laned states. It is also possible to induce mixing between the two species just above the depinning transition. By increasing the radius of the large disks compared to that of the small disks, we find a larger amount of crystallization and banding of the large disks, while the small disks tend to form an interstitial liquid. Lowering the fraction of large disks compared to the fraction of small disks tends to increase the velocity of the large disks compared to that of the small disks, which species separate into a disordered liquid that flows unevenly over the pinning sites. When the disk density is sufficiently large, we find scaling of the velocity-force curves near the plastic depinning transition with an exponent that is slightly smaller than what is observed in systems with longer range interparticle interactions, suggesting that the plastic depinning transition may have distinct features when the interaction range is very short.

Our results could be relevant to multi-species flows of soft matter through random substrates or the flow of granular matter over a disordered background. It would be interesting to explore possible segregation effects for bidisperse systems with long range particle-particle interactions driven over random disorder. In the disk system, the segregation of particles into clumps reduces the number of disk-disk collisions and enhances the disk flow.

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