Team Selection For Prediction Tasks

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Abstract: Given a random variable $O \in \mathbb{R}$ and a set of experts $E$, we describe a method for finding a subset of experts $S \subseteq E$ whose aggregated opinion best predicts the outcome of $O$. Therefore, the problem can be regarded as a team formation for performing a prediction task. We show that in case of aggregating experts’ opinions by simple averaging, finding the best team (the team with the lowest total error during past $k$ turns) can be modeled with an integer quadratic programming and we prove its NP-hardness whereas its relaxation is solvable in polynomial time. Finally, we do an experimental comparison between different rounding and greedy heuristics and show that our suggested tabu search works effectively.

Keywords: Team Selection, Information Aggregation, Opinion Pooling, Quadratic Programming, NP-Hard

1 Introduction

Predicting the outcome of a random variable is an essential part of many decision making processes [1]. For example, companies need to forecast future customer demands or changes in market regulations to better plan their production [2]. In many cases, lack of sufficient information (like statistical data), compels companies to seek advice from experts [3, 4]. In order to have better informed decisions, it is rationale to integrate opinions of several experts. Studies show that in this way, more accurate predictions can be obtained [5].

In this paper, we consider a situation in which a set of experts are available, each with certain level of expertise and we want to predict the outcome of a continuous variable $O$ using their opinions. For each prediction task, we gather experts’ opinions and aggregate them with a simple linear opinion pooling. As studies show, the arithmetic average of experts’ opinions is an efficient and robust aggregation method [4, 6]. We have prediction profile of each of these experts for $k$ previous prediction tasks. Our goal is to find a subset of experts with best performance, i.e. a subset whose aggregated opinion has the least error regarding the actual outcome of $O$.

To formalize the problem, define $E = \{e_1, \cdots e_n\}$ to be the set of experts. The $e_i$’s prediction and the actual value of $O$ in the $d$-th round are respectively profiled by $x_d$ and $y_{id}$. In order to compare different subsets such as $S$ and $S'$, we use the Sum of Squared Error (SSE) measure over the past $k$ rounds:

$$f(S) = \sum_{d=1}^{k} \left( \frac{\sum_{e_i \in S} y_{id}}{|S|} - x_d \right)^2.$$  

(1)

In the Team Selection problem, our goal is to find a subset $S$ with minimum $f(S)$. In this paper, we first consider the relaxed version of this problem where we just want to assign weights to experts and choose them fractionally. We show that this problem can be easily converted to

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a simple quadratic programming and therefore is polynomially solvable. Then, we show that the integer quadratic programming is NP-Hard. Finally, we propose different heuristics and local search methods for tackling the problem and compare their precision experimentally on different artificial datasets. We show that an augmented algorithm of the tabu-search which is used for solving the clique problem, can give a solution to the Team Selection problem with a very small error.

1.1 Related works

Approaches for forecasting have been studied extensively. They can all be categorized into statistical and non-statistical methods. Statistical approaches require historical data to extract value patterns, whereas non-statistical approaches are based on experts’ judgments and their aggregation [4, 10]. Methods for experts’ judgments aggregation include information markets, opinion pooling, Bayesian and behavioral approaches [5, 7]. For information markets, scoring and compensation rules have been introduced to induce truthful forecasts and ensure participation of experts [8, 9, 11, 12]. Moreover, decision rules are used to exploit aggregated judgments to make a decision [9, 10]. Opinion pooling and Bayesian approaches are mathematical methods for aggregating judgments to obtain accurate probability assessment for an event [7, 11, 13, 14, 15, 16, 17, 18]. Bayesian approach has been widely used in aggregating probability distributions with or without taking the dependence between experts into account [17, 19, 20, 21].

Selecting a subset of experts who provide us with information about the outcome of an event can be regarded as forming a team of advisors. Recently, team formation, as a more general concept has received much attention. For instance, Lappas et al, take the cost of communication among individuals into account and present two approaches for forming a team with minimum communication cost and capable of dealing with a defined task, based on two different communication cost functions [22]. As another example, Chhabra et al, propose a greedy approximation to find an optimal matching between people and some interrelated tasks by taking into account the social network structure as an indicator of synergies between members [23]. Kargar et al, also, suggest approximation algorithms for finding a team with minimum communication and personnel costs [24].

2 NP-Hardness

We can model the Team Selection problem by the following integer programming:

\[
\begin{align*}
\text{minimize } g(w) &= \sum_{t=1}^{k} \left( \sum_{i=1}^{n} w_i y_{it} - x_t \right)^2 \\
\text{subject to } \forall i, w_i \in \{0, 1\}
\end{align*}
\]

The relaxed version of this problem, where \( \forall i, 0 \leq w_i \leq 1 \), can be interpreted as weight assignment to each expert to indicate how much we should weigh his opinion. In this section, we first show that the Weight Assignment problem is polynomially solvable by a simple quadratic programming, while its integer programming version (the Team Selection problem) is equivalent to an NP-Hard problem.

Define \( z_{it} = y_{it} - x_t \) for \( 1 \leq i \leq n \) and \( 1 \leq t \leq k \). \( z_{it} \) is the error of the \( i \)-th expert’s forecast.
in the $t$-th turn. So $y_{it} = z_{it} + x_t$, we have

$$g(w) = \sum_{t=1}^{k} \left( \left( \sum_{i=1}^{n} w_i (z_{it} + x_t) \right) - x_t \right)^2$$

$$= \sum_{t=1}^{k} \left( \left( \sum_{i=1}^{n} w_i z_{it} \right) + \left( \sum_{i=1}^{n} w_i x_t \right) - x_t \right)^2$$

$$= k \sum_{t=1}^{k} \left( \sum_{i=1}^{n} w_i z_{it} \right)^2$$

(3)

The term inside the summation can be expanded as

$$\left( \sum_{i=1}^{n} w_i z_{it} \right)^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i z_{it} z_{jt} w_j$$

(4)

Replacing (4) in (3) we get

$$g(w) = \sum_{t=1}^{k} \sum_{i=1}^{n} \sum_{j=1}^{n} w_i z_{it} z_{jt} w_j$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} k \sum_{t=1}^{k} w_i z_{it} z_{jt} w_j$$

$$= \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{t=1}^{k} w_i (2 \sum_{t=1}^{k} z_{it} z_{jt}) w_j.$$

(5)

So the weight assignment problem can be stated as a quadratic programming

$$\text{minimize } \frac{1}{2} w^T Q w$$

subject to $i^T w = 1,$

(6)

where $i$ is the all-one vector and $Q$ is defined as

$$q_{ij} = 2 \sum_{t=1}^{k} z_{it} z_{jt}.$$

Clearly, $Q$ is symmetric and hence the above quadratic programming is valid. We should show that $Q$ is positive-semidefinite i.e. for every non-zero vector $u$ we have $u^T Q u \geq 0.$ Assume that $\sum_{i=1}^{n} u_i = c$, thus with respect to the definition of $Q$ in (4) and (5), the term $u^T Q u$ is exactly $2c^2$ times the error function $g(u)$ which is clearly non-negative.

We know that a quadratic programming with positive-semidefinite matrix can be solved in polynomial time and hence the weight assignment problem is polynomially solvable.

The main result of this section is to show the NP-Hardness of the Team Selection problem.

**Theorem 1.** The Team Selection problem is NP-Hard.

**Proof.** First consider the proposed QCQP (6) for the Weight Assignment problem. Adding constraints $\forall i, w_i \in \{0, \frac{1}{m}\}$ to this QCQP will lead to the following mathematical programming
which is equivalent to the Team Selection problem (when it is solved for $m = 1, 2, \cdots, n$).

$$\begin{align*}
\text{minimize} & \quad \frac{1}{2} w^T Q w \\
\text{subject to} & \quad i^T w = 1 \\
& \quad \forall_i w_i \in \{0, \frac{1}{m}\}
\end{align*}$$

(7)

where

$$q_{ij} = 2 \sum_{t=1}^{k} z_{it} z_{jt}.$$  

Non-zero weight assigned to an expert means he is a member of the resulting solution. We show that this mathematical programming cannot be solved in polynomial time. In order to prove its NP-hardness, we shall reduce the maximum independent set problem in $d$-regular graphs to this problem. Assume that $V(G)$ is the set of vertices of the graph $G$ ($V(G) = \{v_1, v_2, \ldots, v_n\}$ and $deg_G(v_i)$ denotes the $v_i$'s degree in $G$. In the maximum independent problem, the goal is to find an empty subgraph with maximum number of vertices. We will show that every instance of the independent set problem can be transformed to an instance of the following mathematical problem which can then be reduced to the Team Selection problem:

$$\begin{align*}
\text{minimize} & \quad \frac{1}{2} x^T A' x \\
\text{subject to} & \quad i^T x = m \\
& \quad \forall_i x_i \in \{0, 1\}
\end{align*}$$

(8)

where $A' = A + D$, $A$ is the adjacency matrix of $G$ and $D$ is a diagonal matrix with $D_{ii} = deg_G(v_i)$.

After solving the mathematical programming (8), all the vertices with $x_i = 1$ make a subgraph $S$. Let $i(S)$ for $S \subseteq V(G)$ denotes the number of $G$’s edges which reside in $S$. That is to say,

$$i(S) = |\{e = (x, y) \in E(G) | x, y \in S\}|.$$

First notice that

$$x^T A' x = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j A'_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j A_{ij} + \sum_{i=1}^{n} x_i^2 D_{ii}.$$

It is easy to show that

$$\sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j A_{ij} = 2i(S),$$

and

$$\sum_{i=1}^{n} x_i^2 D_{ii} = dm$$

Thus

$$x^T A' x = 2i(S) + dm.$$  

Minimizing $x^T A' x$ with constraint $\sum_{i=1}^{m} x_i = m$ leads to a $m$-vertex subgraph with minimum number of edges. To reduce the maximum independent set problem to the mathematical program (8), it is sufficient to solve (8) for all $1 \leq m \leq n$ and report the maximum $m$ for which the solution is equal to $dm$. 

\[4\]
Finally, we reduce the problem (8) to the mathematical programming (7). It is enough to choose $z_i$s in such a way that $Q = A' \cdot$. Recall that

$$q_{ij} = 2 \sum_{d=1}^{k} z_{it} z_{jt} = 2Z_i \cdot Z_j$$

where $Z_i$ is a $k$-element vector composed of $z_{id}s$. We need $q_{ij} = A'_{ij}$. In other words, when there exists an edge between $v_i$ and $v_j$ in $G$, we should have $Z_i \cdot Z_j = \frac{1}{2}$, when $i = j$, it should be $\text{deg}_G(v_i) = d$ and otherwise it should be 0. To do this, first set $k = |E(G)|$, thus $Z_i$ has a component for each edge of $G$. We set $Z_i$’s $l$’th component to $\frac{1}{\sqrt{2}}$ if $v_i$ is connected to the $l$’th edge and otherwise we set it to 0.

3 Tabu Search

In the previous section, we showed that the Team Selection problem is NP-Hard while its relaxed version i.e. the Weight Assignment problem is solvable in polynomial time. In this section, we propose a tabu search algorithm to solve the Team Selection problem.

Tabu Search has proved high performance in finding sets with specific characteristics. Different variations of this method have been used for approximating the best solution for similar problems like the Maximum Clique, Maximum Independent Set, Graph Coloring and Minimum Vertex Cover [25, 26, 27]. We choose the algorithm introduced in [25] for solving the Maximum Clique problem as a basis and transform it to an algorithm for the Team Selection problem. Tabu Search starts from an initial solution and iteratively replaces it with one of its neighbors in order to get closer to the optimal solution. In each iteration, a local search is done for finding a group whose collective prediction has the least error. If there is no such neighbor, current solution is regarded as a local minimum. To escape from local minimums, Tabu Search allows the least worse neighbor to be selected. Wu & Hao use Probabilistic Move Selection Rule (PMSR) when no improving solution is found in neighborhood. This strategy helps to move to other neighbors when the quality of the local minimum is much less than that of the optimal solution [25]. We use a similar strategy in our proposed algorithm. For preventing previous solutions from being revisited, Tabu Search uses a tabu list which records the duration of each element being kept from moving into or out of current solution.

Algorithm 1 shows the pseudo code of our proposed tabu search. The first line shows the initialization of the first set (team), which then goes through improvements in the main loop. As the initial set can play an important role in Tabu Search performance [25], we suggest the initial set to be equal to the set of $m$ experts who are given the largest weights in an optimum solution for the Weight Assignment problem (this is shown by $\text{MaxWeightsAssignedTo}(E, m)$).

In each iteration of the loop, the amount of improvement gained by each possible swap is calculated simply by subtracting SSE of the team resulting from swapping two experts (one in the current set with another out of that) from the SSE of the current team. If the best possible swap results in a better solution (lower SSE), then the current set is updated with the new solution. Otherwise, a random set is selected as the current solution with probability $P$. In another word, $P$ is the probability of escaping from a local minimum. Like various kinds of Tabu Search, we use tabu list to prevent producing repeated sets. Therefore, after substituting a member with another expert out of the current set, tabu list is updated with regard to tabu tenure values calculated for both selected experts. This implies that for some time these experts are not allowed to move in or out of the current set in next iterations.

There are two terminating conditions for this algorithm. For the first, the main loop terminates by not finding any better set after $\text{maxIter}$ successive iterations. Second condition of termination is when the current solution is equal to the solution of the Weight Assignment problem. That is to say, there is no other set with less SSE.
Algorithm 1 Tabu Search For Team Selection Problem

Require: A Set of experts \((E)\), Expert’s sequence of past predictions, integer \(MaxIter\) (Maximum number of successive tries which fail to find better solution), \(m\) (size of the team)

Ensure: A team with minimum SSE if found

1: \(S \leftarrow \text{MaxWeightsAssignedTo}(E, m)\)
2: \(\text{lowerBoundOfSSE} \leftarrow g(w)\) \(\{ w \text{ contains weights assigned to } E \} \)
3: \(i \leftarrow 0\) \{ number of iterations \}
4: \(\text{bestSet} \leftarrow S\) \{ Records the best solution found so far \}
5: while \(i < MaxIter\) do
6: \(S' \leftarrow S \cup \{v\} \setminus \{u\}\) with minimum SSE among all \(u, v\) pairs not in tabu list
7: if \(f(S') < f(S)\) then
8: \(S \leftarrow S'\)
9: else
10: \(S \leftarrow S'\) with probability \(1 - P\)
11: or a random neighbor with probability \(P\)
12: Update the tabu list \{ List of all \(u, v\) pairs which are tried in iterations \}
13: if \(f(S) = \text{lowerBoundOfSSE}\) then
14: \(\text{return } S\)
15: if \(f(S) < f(\text{bestSet})\) then
16: \(\text{bestSet} \leftarrow S\)
17: \(i \leftarrow 0\)
18: else
19: \(i \leftarrow i + 1\)
20: \(\text{return } \text{bestSet}\)

4 Comparison

In this section, inspired from algorithms proposed for similar problems, we propose different heuristics for the Team Selection problem and compare their efficiency with the tabu search proposed in Section 3.

4.1 Heuristics

Random Rounding: Random rounding defines a threshold \((T)\) and selects experts with weights higher than \(T\) with probability \(P\) and the others with probability \(1 - P\). This process will continue until \(m\) experts are selected. Our experiments show that lower amounts of \(P\) yields better results.

Max-Weights: This rounding algorithm takes the \(m\) experts with largest weights as members of the team.

Min-Effect: In each round, this algorithm tries to find a member who has the minimum effect on the SSE of \(E\). According to the equation (3), the effect of each person on the SSE function can be calculated as the following:

\[
\left(2w_i \sum_{j \neq i} w_j \sum_{d=1}^{k} z_{id}z_{jd}\right) - w_i^2 \sum_{d=1}^{k} z_{id}^2
\]  

\(9\)

Best Pairs: Despite the fact that experts with high prediction error are not desirable, aggregated opinions of two or more of them may be. This is due to the bracketing concept [3]. Thus, in this algorithm we allow pairs whose aggregated opinion has the minimum absolute error to be selected. The algorithm computes sum of the absolute errors of the aggregated opinions of all pairs over past \(k\) days, then report \([\frac{m}{2}]\) of pairs with smallest calculated values. For odd values of \(m\), last person would be the one among remained experts with minimum sum of absolute errors.
**Remove Least Weights**: This algorithm runs the Weight-Assignment problem’s algorithm iteratively and removes one with the least weight each time. The process continues until \( m \) experts are gathered.

### 4.2 Comparison of Algorithms

In this section, we evaluate the tabu search and other heuristics for solving the Team Selection problem. We consider four different simulation scenarios with 15 experts, each with known distribution for their predictions and tested the algorithms for team sizes from 1 to 10. These scenarios are based on two measures for evaluating quality of expert’s distribution (calibration and informativeness introduced by Hammitt & Zhang in [3]) and are described as follows:

- **Normal1**: In this case, random variable \( O \) and experts’ beliefs have normal distribution with \( \mu = 10 \), thus, experts’ information are calibrated. Standard deviation of each expert’s distribution is randomly selected from \([1, 2]\).

- **Normal2**: This case models calibrated but less informative experts. Therefore, like the previous case, all distributions are normal with \( \mu = 10 \), but this time, standard deviations of experts’ predictions are between 1 and 7 (\( \sigma_i \) is randomly selected from \([1, 7]\)).

- **Normal3**: In the third case, we simulate a situation in which some of the experts are not calibrated. For doing this, experts’ beliefs have normal distribution with random means that are selected uniformly from \([8, 12]\). Like Normal1, standard deviations are chosen randomly between 1 and 2.

- **Exp**: For the final case, we simulate both the reality and the experts’ predictions with exponential distributions with \( \mu = 10 \).

As presented in Figures 3 to 6 and Table 1, Tabu Search produces better results in all cases. It also can be seen that the result of this algorithm is very near to the best possible algorithm which tries all the feasible solutions and return the best one. This means that our suggested algorithm is less sensitive to the distribution of event \( O \). Thus, Tabu Search is more reliable than other proposed heuristics. Best Pairs’s efficiency for normal distributions is comparable with Tabu search. Moreover, the average execution time of Best Pairs is around 0.02 of Tabu Search (Table 2). Therefore, it would be an acceptable method for quickly forming a team. However, in the case of exponential distributions, Best Pairs performance for small teams is even worse than Min-Effect which is due to the increase in diversity of the forecasts. Hence, the probability of neutralizing an expert’s error by another, decreases.

**Table 1**: Absolute Error Average Of Algorithms and Heuristics

| Algorithm     | Normal1 | Normal2 | Normal3 | Exponential |
|---------------|---------|---------|---------|-------------|
| RandomRounding| 10.348  | 128.937 | 17.702  | 1102.74     |
| Max-Weights   | 2.153   | 38.329  | 4.831   | 121.021     |
| Min-Effect    | 4.26    | 84.39   | 7.84    | 297.97      |
| Tabu Search   | 0.145   | 2.186   | 0.18    | 14.833      |
| BestPairs     | 1.788   | 8.507   | 2.26    | 178.273     |
| RemoveLW     | 1.897   | 38.895  | 4.188   | 105.951     |
4.3 Other Experiments

The effect of the team size: As the number of hired experts determines the cost incurred, we would like to know the effect of the team size on the accuracy of the aggregated opinion of its members. Therefore, in our simulations we capture the accuracy for different team sizes and depict the results in Figure 5. This figure shows the optimal solution for different sizes of $E$. The results show that increasing the number of experts first reduces but then increases SSE again. Therefore, we can conclude large values for $m$ is neither cost effective nor efficient.

The effect of the prediction profile size: It is trivial that having more information about the experts’ past predictions, improve the quality of the final result. The question here is how much is enough. We observe that for large values of the size of the experts’ prediction profile, the decrease in SSE will finally stop. Therefore, the first point with minimum value would be the optimal number of past records. The results of this experiment can be seen in Figure 6.

Table 2: Comparison of execution time average. The algorithms were executed for finding best 8 experts among 15 experts in case of Normal2.

| Name of Algorithm | Average Exe.Time |
|-------------------|------------------|
| BestGroup         | 27.3742          |
| RemoveLW         | 0.0461           |
| Tabu Search       | 0.0345           |
| Min-Effect        | 0.0036           |
| Max-Weights       | 0.0035           |
| RandomRounding    | 0.0013           |
| BestPairs         | 0.0009           |
5 Conclusion

In this paper, we addressed the Team Selection problem in which we wanted to form a team of experts with minimum error for performing a prediction task. To simplify the problem, we first studied the relaxed version of the problem (the Weight Assignment problem) in which our goal was to find the best weights for linear opinion pooling. We proved that this problem can be solved with a simple quadratic programming in polynomial time. Then we proved that the Team Selection problem is NP-hard. In the rest of the paper, we proposed a tabu search algorithm for solving the problem. Our experiments show the superior accuracy of this algorithm compared to other proposed algorithms. It is also shown that the accuracy of this algorithm is comparable to the best possible algorithm.

References

[1] T. Sprenger, P. Bolster, A. Venkateswaran, Conditional prediction markets as corporate decision support systems—an experimental comparison with group deliberations, The Journal of Prediction Markets 1 (3) (2012) 189–208.

[2] I. Attarzadeh, S. H. Ow, Software development cost and time forecasting using a high performance artificial neural network model, in: Intelligent Computing and Information Science, Springer, 2011, pp. 18–26.

[3] J. K. Hammitt, Y. Zhang, Combining experts’ judgments: Comparison of algorithmic methods using synthetic data, Risk Analysis 33 (1) (2013) 109–120.

[4] Y. Chen, C.-H. Chu, T. Mullen, D. M. Pennock, Information markets vs. opinion pools: An empirical comparison, in: Proceedings of the 6th ACM conference on Electronic commerce, ACM, 2005, pp. 58–67.

[5] A. Graefe, J. S. Armstrong, R. J. Jones Jr, A. G. Cuzán, Combining forecasts: An application to elections, International Journal of Forecasting 30 (1) (2014) 43–54.

[6] Y. Chen, C.-H. Chu, T. Mullen, Predicting uncertain outcomes using information markets: trader behavior and information aggregation, New Mathematics and Natural Computation 2 (03) (2006) 281–297.

[7] R. T. Clemen, R. L. Winkler, Aggregating probability distributions, Advances in Decision Analysis (2007) 154–176.

[8] A. Othman, T. Sandholm, Decision rules and decision markets, in: Proceedings of the 9th International Conference on Autonomous Agents and Multiagent Systems: volume 1-Volume 1, International Foundation for Autonomous Agents and Multiagent Systems, 2010, pp. 625–632.
[9] C. Boutilier, Eliciting forecasts from self-interested experts: scoring rules for decision makers, in: Proceedings of the 11th International Conference on Autonomous Agents and Multiagent Systems-Volume 2, International Foundation for Autonomous Agents and Multiagent Systems, 2012, pp. 737–744.

[10] Y. Chen, I. Kash, Information elicitation for decision making.

[11] S. C. Hora, Expert judgment, Encyclopedia of Quantitative Risk Analysis and Assessment.

[12] H. Zhang, E. Horvitz, Y. Chen, D. C. Parkes, Task routing for prediction tasks, in: Proceedings of the 11th International Conference on Autonomous Agents and Multiagent Systems-Volume 2, International Foundation for Autonomous Agents and Multiagent Systems, 2012, pp. 889–896.

[13] C. Genest, J. V. Zidek, Combining probability distributions: A critique and an annotated bibliography, Statistical Science 1 (1) (1986) 114–135.

[14] V. Dani, O. Madani, D. M. Pennock, S. Sanghaj, B. Galebach, An empirical comparison of algorithms for aggregating expert predictions, arXiv preprint arXiv:1206.6814.

[15] R. A. Jacobs, Methods for combining experts’ probability assessments, Neural computation 7 (5) (1995) 867–888.

[16] S. French, Aggregating expert judgement, Revista de la Real Academia de Ciencias Exactas, Fisicas y Naturales. Serie A. Matematicas 105 (1) (2011) 181–206.

[17] P. A. Morris, Decision analysis expert use, Management Science 20 (9) (1974) 1233–1241.

[18] R. Michaeli, L. Simon, An illustration of bayes’ theorem and its use as a decision-making aid for competitive intelligence and marketing analysts, European Journal of Marketing 42 (7/8) (2008) 804–813.

[19] M. Kallen, R. Cooke, Expert aggregation with dependence, in: Probabilistic Safety Assessment and Management, Elsevier Science, 2002, pp. 1287–94.

[20] M. Mostaghimi, Combining ranked mean value forecasts, European journal of operational research 94 (3) (1996) 505–516.

[21] M. Mostaghimi, Bayesian estimation of a decision using information theory, Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on 27 (4) (1997) 506–517.

[22] T. Lappas, K. Liu, E. Terzi, Finding a team of experts in social networks, in: Proceedings of the 15th ACM SIGKDD international conference on Knowledge discovery and data mining, ACM, 2009, pp. 467–476.

[23] M. Chhabra, S. Das, B. Szymanski, Team formation in social networks, in: Computer and Information Sciences III, Springer, 2013, pp. 291–299.

[24] M. Kargar, M. Zihayat, A. An, Affordable and collaborative team formation in an expert network, Department of Computer Science and Engineering, York University, Technical Report CSE-2013 1.

[25] Q. Wu, J.-K. Hao, An adaptive multistart tabu search approach to solve the maximum clique problem, Journal of Combinatorial Optimization 26 (1) (2013) 86–108.

[26] Q. Wu, J.-K. Hao, Coloring large graphs based on independent set extraction, Computers & Operations Research 39 (2) (2012) 283–290.

[27] Q. Wu, J.-K. Hao, An effective heuristic algorithm for sum coloring of graphs, Computers & Operations Research 39 (7) (2012) 1593–1600.