Achieving arbitrary polarization control using complex birefringent meta-materials

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We demonstrate that the key to realizing arbitrary control over pairs of polarization states of light, i.e., transforming an arbitrarily polarized pair of input states to an arbitrarily polarized pair of output states, is the ability to generate pairs of states with orthogonal polarizations from non-orthogonal pairs of initial states. We then develop a new class of non-Hermitian meta-materials, termed complex birefringent meta-materials, which are able to do exactly this. Such materials could facilitate the detection of small polarization changes in scattering experiments, as well as enable new polarization multiplexing schemes in communications networks.

Polarization is one of the fundamental properties of light, and control over the polarization is paramount in many optical communications and imaging applications. In general, the effect of propagation through any media on the polarization of an incident electromagnetic signal can be described as \( |\beta\rangle = S(z)|\alpha\rangle \), where \( |\alpha\rangle \) and \( |\beta\rangle \) are the input and output polarization states, respectively, and \( S(z) \) is a \( 2 \times 2 \) matrix that depends on the properties of the medium, as well as on the propagation distance \( z \). Conventionally, the polarization of a signal is manipulated through the use of birefringent materials \([1,7]\). For lossless birefringent media, with proper choice of material parameters and propagation distance, it is always possible to convert an input polarization \( |\alpha_1\rangle \) to an arbitrary output polarization \( |\beta_1\rangle \). However, once the response to \( |\alpha_1\rangle \) is determined, the output polarization \( |\beta_2\rangle = S|\alpha_2\rangle \) is no longer arbitrary for any other input polarization \( |\alpha_2\rangle \). This is because \( S \) is unitary in lossless media, and thus \( \langle \beta_2|\beta_1\rangle = \langle \alpha_2|\alpha_1\rangle \).

In this Letter, we seek to overcome the limitation of conventional birefringent media by developing a class of meta-materials which enable arbitrary control over pairs of polarization states. By arbitrary control, we demand that for a pair of arbitrary input polarizations \( |\alpha_1\rangle \) and \( |\alpha_2\rangle \), one can generate an arbitrary pair of output polarizations \( |\beta_1\rangle \) and \( |\beta_2\rangle \). Achieving such polarization control has significant implications for a wide range of technologies. For example, with this capability one can map two polarizations that are close to each other into two orthogonal polarizations, which may facilitate the detection of small polarization changes, such as those arising from the imaging of biological tissues \([8,9]\) and thin films \([10]\). Likewise, the ability to completely separate non-orthogonal polarization states could enable new multiplexing schemes in optical communications networks beyond what is currently possible \([11,12]\).

We first show that the key step for achieving arbitrary control over pairs of polarization states is to develop a class of meta-materials which are capable of performing the following polarization transformation as denoted by \( S_\theta \),

\[
\begin{align*}
|1, 1\rangle &= S_\theta|\theta\rangle, \\
|1, -1\rangle &= S_\theta|-\theta\rangle.
\end{align*}
\]

Here we assume propagation along the \( z \)-axis, and label the polarization states in terms of the electric field components in the \( xy \)-plane as \( |E_x, E_y\rangle \). \( |\pm \theta\rangle \) denote the two polarization states that lie on the great circle of the Poincaré sphere passing through \( |1, i\rangle \) and \( |1, 1\rangle \), and are symmetrically placed away from \( |1, i\rangle \), subtending an angle of \( \pm \theta \) with respect to \( |1, i\rangle \). Suppose we can construct a class of materials which can provide \( S_\theta \) for an arbitrary \( \theta \). For an arbitrary pair of input states \( |\alpha_1\rangle \) and \( |\alpha_2\rangle \), using conventional lossless birefringent materials, one can achieve the transformation \([4]\)

\[
\begin{align*}
|\theta_\alpha\rangle &= U_\alpha|\alpha_1\rangle, \\
|\theta_\alpha\rangle &= U_\alpha|\alpha_2\rangle,
\end{align*}
\]

where \( U_\alpha \) is unitary and \( \langle \theta_\alpha|\theta\rangle = \langle \alpha_1|\alpha_2\rangle \). For the pair of arbitrary output states \( |\beta_1\rangle \) and \( |\beta_2\rangle \), one can obtain a similar unitary transformation \( U_\beta \) that transforms them to \( |\pm \theta_\beta\rangle \). Therefore, the transformation \( S \) from the input states \( |\alpha_1\rangle \) and \( |\alpha_2\rangle \) to the output states \( |\beta_1\rangle \) and \( |\beta_2\rangle \) is then

\[
S = U_\beta S_{\theta_\beta}^{-1} S_{\theta_\alpha} U_\alpha.
\]

To achieve the transformation as described by Eqs. \([11,12]\) in general requires a non-Hermitian meta-material. We now proceed to show that such a transformation can be realized in a class of complex symmetric meta-materials with its dielectric tensor having the form

\[
\bar{\varepsilon} = \begin{pmatrix}
\varepsilon_{xx} & \varepsilon_{xy} & 0 \\
\varepsilon_{yx} & \varepsilon_{yy} & 0 \\
0 & 0 & \varepsilon_{zz}
\end{pmatrix} = \begin{pmatrix}
0 & 0 & 0 \\
0 & \varepsilon_{rr} & i\varepsilon_{r\theta} \\
0 & i\varepsilon_{r\theta} & \varepsilon_{rr}
\end{pmatrix},
\]

in which

\[
\begin{align*}
\varepsilon_{xx} &= \varepsilon_{yy} = \varepsilon_r - i\varepsilon_i, \\
\varepsilon_{xy} &= \varepsilon_{yx},
\end{align*}
\]
where \( \varepsilon_r, \varepsilon_i, \varepsilon_{xy} \in \mathbb{R} \). Here, an equal amount of gain and loss has been added to the \( x \) and \( y \) axes of a conventional birefringent material, a choice inspired by recent developments in optical media with spatially distributed regions containing equal amounts of gain and loss \([13–28]\). Henceforth, we refer to materials which obey Eqs. \([7\) and \([8]\) as complex birefringent meta-materials.

For light propagating along the \( z \)-axis of such a medium, the allowed wavevectors, \( k_{\pm} \), of a monochromatic signal with frequency \( \omega \), can be found by solving the right eigenvalue equation \([28, 33]\),

\[
\omega^2 \mu \varepsilon_{\perp} |E^R_{\pm}\rangle = k^2 |E^R_{\pm}\rangle,
\]

in which \( \mu \) is the scalar magnetic permeability. As such, the allowed wavevectors and right eigen-polarizations of complex birefringent meta-materials are

\[
k^2 = \frac{\varepsilon_r \pm \varepsilon_{xy} \sqrt{1 - \tau^2}}{\omega^2 \mu},
\]

\[
|E^R_{\pm}\rangle = \frac{1}{N_{\pm}} \left| 1, i\tau \pm \sqrt{1 - \tau^2} \right>,
\]

in which \( \tau = \varepsilon_i / \varepsilon_{xy} \) represents a normalized measure of the strength of the gain and loss in the system, and

\[
N^2_\pm = 2(1 - \tau^2) \pm 2i\tau \sqrt{1 - \tau^2},
\]

is the normalization of the eigenstates. The matrix \( \omega^2 \mu \varepsilon_{\perp} \) also has left eigen-polarizations, which are solutions to \( \langle E^L_{\pm} | \omega^2 \mu \varepsilon_{\perp} = k^2 |E^L_{\pm}\rangle \). Together, the left and right eigen-polarizations form a bi-orthogonal basis, and can be normalized such that \( \langle E^L_{\pm} | E^R_{\pm} \rangle = \delta_{mn} \), with the choice of \( N_{\pm} \) in Eq. \([12]\). In conventional lossless birefringent media \( \varepsilon_{\perp} \) is Hermitian, and so \( \langle E^L_{\pm} | E^R_{\pm} \rangle \neq 0 \). However, for complex birefringent materials, \( \omega^2 \mu \varepsilon_{\perp} \) is complex-symmetric, and the left and right eigen-polarizations are related by \( \langle E^L_{\pm} | E^R_{\pm} \rangle = |E^R_{\pm}|^T \).

Moreover, although the two right eigen-polarizations are linearly independent for complex birefringent materials with \( |\tau| \neq 1 \), it can be readily seen that they are not orthogonal, \( \langle E^R_+ | E^R_- \rangle \neq 0 \) except for when \( \tau = 0 \) and the system reverts to a conventional birefringent material, or when \( |\tau| \to \infty \) and the system becomes a conventional dichroic material.

The evolution of the polarization of light propagating within a complex birefringent material with \( |\tau| \neq 1 \) can be expressed in terms of the right eigen-polarizations \([11]\), as

\[
|E(z)| = e^{i k_{\pm} z} \left( e^{i \Delta_k z} A_+ |E^R_+\rangle + A_- |E^R_-\rangle \right),
\]

in which \( \Delta_k = k_+ - k_- \) is the additional phase accumulated by \( |E^R_+\rangle \) relative to \( |E^R_-\rangle \) per unit length, and the initial amplitudes, \( A_\pm \), are defined in terms of the left eigen-polarizations as \( A_\pm = \langle E^L_\pm | E(0) \rangle \). The resulting polarization dynamics of a complex birefringent material can be visualized by plotting the output polarization as a function of \( z \) on the Poincaré sphere. The example shown in Fig. \([1a]\) illustrates the polarization dynamics when \( \tau < 1 \), for which both allowed wavevectors are real, \( k_{\pm} \in \mathbb{R} \). This is analogous to the exact phase in parity-time symmetric systems \([13–17]\). For the sake of comparison, we also plot the polarization dynamics for a conventional Hermitian birefringent material in Fig. \([1b]\), as described by setting \( \varepsilon_i = 0 \) in Eq. \([6]\). For both types of materials as shown in Fig. \([1]\) the eigen-polarizations define the fixed point of the dynamics. So long as the incident polarization is not parallel to one of these eigen-polarizations, the polarization of the initial signal forms a closed trajectory around the eigen-polarizations as \( z \) is varied.

When \( \tau = 0 \), which describes a conventional lossless birefringent material, the two eigen-polarizations are located at \( |1,1\rangle \) and \( |1,-1\rangle \), corresponding to two linearly polarized states. The two eigenstates are orthogonal to each other, and the polarization trajectories form circles around the axis formed by the two eigenstates, shown in Fig. \([1b]\). (On the Poincaré sphere, orthogonal states are represented by antipodal points.) As \( \tau \) is increased, so that \( 0 < \tau < 1 \), the two eigen-polarizations remain on the great circle connecting \( |1,1\rangle \) and \( |1,-i\rangle \), but are tilted away from the states \( |1, \pm 1\rangle \) towards \( |1,i\rangle \), which is a manifestation of the non-orthogonality of these two eigenstates. For \(-1 < \tau < 0 \), the eigenstates instead tilt away from \( |1, \pm 1\rangle \) towards \( |1,-i\rangle \). As \( k_{\pm} \in \mathbb{R} \), the polarization trajectories are still closed, but are no longer
centered on the axis formed by the eigenstates, as shown in Fig. 1(a).

Examining the polarization dynamics of Fig. 1(a), we note that there are two states lying on the great circle connecting $|1, 1\rangle$ and $|1, i\rangle$, indicated by the purple crosses, which can be transformed to the two states $|1, 1\rangle$ and $|1, -1\rangle$, indicated by purple circles, with a proper choice of the propagation distance $l$. Therefore, complex birefringent meta-materials with $0 < |\tau| < 1$ indeed provide the key non-trivial step required for achieving arbitrary control over pairs of polarization states, which is to realize $S_\theta$ as defined in Eqs. (11) and (2). Mathematically, for a given pair of input states $|\pm\rangle$, the complex birefringent meta-material which transforms these two states to $|1, \pm 1\rangle$ satisfies

$$e^{i\Delta k l} = \frac{\langle E_k^1 \pm \theta | E_k^1 | 1, \pm 1 \rangle}{\langle E_k^1 \pm \theta | E_k^1 | 1, \pm 1 \rangle}.$$  \hspace{1cm} (14)

Here, as $\tau$ appears in both $\Delta k$ and the left eigenstates $\langle E_k^1 |$, Eq. (14) represents a complex transcendental equation which can be solved for $\tau$ and $l$. One can show that for any given value of $0 < |\tau| < 1$, one can achieve orthogonal output states for any choice of $\theta$ with the proper choice of propagation distance $l$. In general, as the choice of initial states become parallel ($\theta \to 0$), stronger gain and loss ($|\tau| \to 1$) along with longer propagation distances are required to achieve orthogonal output states.

Up to this point we have shown that, in the regime where $|\tau| < 1$, complex birefringent materials, as described in Eqs. (10), (11), exhibit polarization dynamics that can be used to achieve arbitrary control over pairs of polarization states. This class of metamaterials also exhibits interesting polarization dynamics with $|\tau| \geq 1$. When $|\tau| > 1$, both eigenvalues $k_\pm$ become complex, with $k_+ = k_-^*$. This is analogous to the broken phase in parity-time symmetric systems. In this regime the eigen-polarizations reside along the great circle on the Poincaré sphere, as shown in Fig. 2(a), and the material provides polarization-dependent attenuation and amplification. As $|\tau| \to \infty$, the system becomes a conventional dichroic material, with orthogonal eigen-polarizations $|1, 0\rangle$ and $|0, 1\rangle$.

When $|\tau| = 1$, complex birefringent materials possess an exceptional point [31, 32] where both the eigenvalues and eigenvectors coalesce, and the eigenvectors become self-orthogonal, $\langle E_k^1 | E_k^R \rangle = 0$. This yields two unique properties. First, the expression for the evolution of the electric field [13] is no longer valid as $\omega^2 \varepsilon \omega_z$ has a non-trivial Jordan normal form. Instead, the evolution of the field must be expressed in terms of the single remaining eigenvector, $|E_R\rangle$ and its associated Jordan vector, $|J^R\rangle$.

![FIG. 2. (a) Flow lines (yellow) depict the polarization dynamics on the Poincaré sphere when light travels along the $z$-direction through a complex birefringent material as described by Eqs. (10)-(13) with $\tau = 1.5$. The eigenvectors of the dielectric tensor are shown in red, with the arrow indicating the direction in which the polarization flows for $\varepsilon_{xy} > 0$. (b) Flow lines (yellow) depict the polarization dynamics when light travels along the $z$-direction through a complex birefringent material as described by Eqs. (14)-(15) with $\tau = 1$. The single eigen-polarization of the system is $|1, i\rangle$ (red).](image-url)
FIG. 3. (a) Schematic of a complex birefringent meta-material, consisting of layers of a conventional birefringent material (gray), and layers of a material containing gain (red) and loss (blue), forming a comb. The patterning of the structure is assumed to be fine enough relative to the wavelength of the light so as to be in the effective medium limit. (b) Transformation of the polarization on the Poincaré sphere of two signals through 59μm of a complex birefringent meta-material, as shown schematically in (a). Here, for an incident light with wavelength 1.55μm, we have used calcium carbonate whose fast and slow axes are rotated 7.47° with respect to the lab frame, yielding a dielectric tensor with ε_{xx} = 2.66, ε_{yy} = 2.19, and ε_{aa} = 0.063. The isotropic gain has ε = 4 − 0.1i, and the isotropic loss has ε = 9.74 + 0.63i. By forming a comb consisting of 77% gain regions and 23% loss regions, the effective dielectric tensor is anisotropic in this layer, with ε_{∥} = 5.34 + 0.07i and ε_{⊥} = 4.64 − 0.07i. By using 30nm layers of calcium carbonate, and 20nm layers of the gain and loss, the total system constitutes a complex birefringent metamaterial with ε_{xx} = 3.454 − 0.028i, ε_{yy} = 3.453 + 0.028i, and ε_{xy} = 0.038, which corresponds to τ ≈ 0.75. The initial signal polarizations are separated by 16.2° (cyan circles), while the final polarization states are nearly orthogonal (cyan triangles). The surrounding medium is index matched to the complex birefringent meta-material, with ε = 3.46.

S_{21}^{21} [31], as shown in the Supplementary Information [35]. These conserved quantities stem from the fact that complex birefringent materials are invariant upon the operation that switches the x and y axes of the system, M, and the time-reversal operation, T, thus

\[(MT)S(MT) = S^{-1}\].

(17)

There are many possible experimental realizations of complex birefringent materials. A dielectric response as described by Eqs. [31] − [33] has been previously realized experimentally in a meta-surface structure [26]. However, in order to observe the polarization dynamics effects, and to achieve the capability for arbitrary control over pairs of polarization states, neither of which are considered in [26], it would be interesting to create three-dimensional media where the propagation distance can be varied. Therefore, here instead we discuss the construction of three-dimensional systems with the appropriate dielectric tensor. As an example, one could construct a meta-material consisting of an ordinary birefringent material interspersed with layers containing regions of both gain and loss, as depicted in FIG. 3a. Transfer matrix calculations of this exact structure without using the effective medium approximation in the propagation direction confirm its ability to separate a pair of initial states with similar polarizations to be nearly orthogonal, shown in FIG. 3(b). Alternatively, there are many methods for adding birefringence to optical fibers geometrically, allowing for ε_{xy} ≠ 0. By doping such a birefringent fiber, gain could be added to both ε_{xx} and ε_{yy}. Then, all that is required to realize complex birefringence is the ability to add loss specifically to ε_{yy}. Regardless of the specific realization chosen, the experimental design of complex birefringent materials benefits from the critical feature that the amount of gain and loss, ε_{i}, necessary to observe significant non-trivial polarization dynamics is set by the anisotropy of the system, ε_{xy}, which can be designed to be quite small. Thus, very little gain or loss is necessary to realize arbitrary control over pairs of polarization states in these materials.

Here, we have focused on meta-materials with ε_{xx} = ε^{*}_{yy}, as this choice yields a regime of parameter space, |τ| < 1, where the eigenvalues of the system are real, and as such the change in intensity of an incident signal is bounded. However, many other choices of ε_{xx}, ε_{yy} ∈ C, such as birefringent materials with loss in a single polarization channel, i.e. ε_{xx} = ε_{r} and ε_{yy} = ε_{r} + 2iε_{i} with ε_{xy} ≠ 0, will still yield non-orthogonal eigenvectors, which can allow for nearly arbitrary control over pairs of polarization states as shown in FIG. S2 in the Supplementary Information [35]. Similarly, off-axis propagation in directions which do not conserve M also results in an entirely complex spectrum even for |τ| < 1. Fortunately, the rate at which k_{±} acquires an imaginary component for off-axis propagation is slow relative to the change in propagation angle, as shown in FIG. S3, and thus the off-axis components of a wave-packet traveling through a complex birefringent meta-material will experience similar polarization dynamics to the on-axis component.

In conclusion, we have developed a theory of complex symmetric anisotropic dielectric materials and demonstrated that such systems enable arbitrary control over pairs of polarization states. In particular, such complex birefringent materials may have applications in both splitting signals with adjacent polarizations and nearly combining signals with orthogonal polarizations. Experimentally, these materials benefit from the fact that the scale of the gain and loss required to observe these effects is set by the off-diagonal anisotropy in the system.
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