Uses and limitations of relativistic jet proper motions: lessons from galactic microquasars

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ABSTRACT

It is shown that the two-sided jet proper motions observed from the galactic microquasars GRS 1915+105 and GRO J1655-40 in practice only allow us to place lower limits on the Lorentz factors of the outflows. As a consequence, it is not possible to rule out the possibility that jets from X-ray binaries are just as relativistic as those from active galactic nuclei (AGN). This results from the fact that distance estimates place the sources, within uncertainties, at the maximum distance \(d_{\text{max}}\) which corresponds to an intrinsic velocity \(v = c\). The general case is explored, for a range of intrinsic Lorentz factors and angles to the line of sight, and it is shown that a source of significantly relativistic jets will nearly always be observed close to \(d_{\text{max}}\) and as a result it is unlikely that we will ever be able to measure with any accuracy the Lorentz factor of a jet from two-sided proper motions. We will generally not be able to do more than place a lower limit on the Lorentz factor of the flow, and this limit is naturally even lower in the cases where we only observe the approaching jet. On the other hand, under the assumption that any two-sided jets we see are intrinsically relativistic, we can confidently place the source at a distance \(d \sim d_{\text{max}}\). As a result, observations of two-sided proper motions in relativistic jets from AGN would be extremely important for calibration of the cosmological distance scale. While the proper motions do not allow us the measure the Doppler shifts associated with the jets, the ratio of proper motions will correspond to the ratio of frequencies of any emission lines emitted by both jets, which will aid in searching for such lines. Furthermore, it is shown that if the jet is precessing, the product of the proper motions as a function of angle to the line of sight may be used to determine if the jet is only mildly relativistic.

Key words:

- binaries: close
- radio continuum: stars
- ISM: jets and outflows
- stars: neutron
- black hole physics

INTRODUCTION

Relativistic jets, outflows of matter from regions close to accreting black holes and neutron stars, remain amongst the most spectacular yet poorly explained phenomena in high-energy astrophysics. They are ubiquitous amongst Active Galactic Nuclei (AGN) powered by supermassive black holes, as well in spectrally hard and transient outbursting states of stellar mass black holes and neutron stars in X-ray binary systems (XRBs) – see e.g. Hughes (1991); Mirabel & Rodriguez (1999).

One of the key questions in the study of these jets is ‘how relativistic are they’ – i.e. what is the Lorentz factor \((\Gamma = (1 - \beta^2)^{-1/2})\) of the flow? In AGN, the highest inferred Lorentz factors are \(\sim 30h^{-1}\) \((h = H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}, \text{where} \ H_0 \text{ is the Hubble constant})\) – see e.g. Vermeulen & Cohen (1994); Jorstad et al. (2001). Multiple recent detections of arcsec-scale X-ray jets from AGN with Chandra (e.g. Harris & Krawczynski 2002) indicate that relativistic flow velocities are maintained over large distances from the jet base.

In two XRBs we have a clear advantage over studies of AGN, in that we can observe the proper motions and flux ratios of both approaching and receding radio knots (a.k.a. ‘blobs’, ‘plasmons’ etc.) associated with the same ejection event. Observations of ejections from these two sources, GRS 1915+105 (Mirabel & Rodriguez 1994; Fender et al. 1999; Rodriguez & Mirabel 1999; Fender et al. 2002) and GRO J1655-40 (Tingay et al. 1995; Hjellming & Rupen 1995), provide us with unique diagnostics of the jet geometry. How-
ever, as shall be discussed in this paper, and contrary to widespread misconception, they have not allowed us to measure the Lorentz factor of the flow in either case.

**XRB JET PROPER MOTIONS**

In the following discussion we will consider relativistic jets in which proper motions associated with approaching ($\mu_{\text{app}}$) and receding ($\mu_{\text{rec}}$) components can be measured (assuming that both sides of the jet have been correctly associated with the same ejection event, the higher proper motion of the two always corresponds to $\mu_{\text{app}}$). A key point in the following discussion is the assumption of symmetry in ejection velocity for both sides of the jet; possible exceptions to this will be discussed at the end.

As described in Mirabel & Rodriguez (1994), measurement of $\mu_{\text{app}}$ and $\mu_{\text{rec}}$ allows a determination of the following product:

$$\beta \cos \theta = \frac{(\mu_{\text{app}} - \mu_{\text{rec}})}{(\mu_{\text{app}} + \mu_{\text{rec}})}$$

where $\theta$ is the angle of the ejection to the line of sight and $\mu_{\text{app}}, \mu_{\text{rec}}$ are the approaching and receding proper motions respectively (see also Rees 1966; Blandford, McKee & Rees 1977).

Once the proper motions are measured, the angle of ejection, $\theta$, and consequently the intrinsic velocity, $\beta$, are uniquely determined for every distance since

$$\tan \theta = \frac{2d}{c} \left( \frac{\mu_{\text{app}} \mu_{\text{rec}}}{\mu_{\text{app}} - \mu_{\text{rec}}} \right)$$

and the product $\beta \cos \theta$ is already known.

The variation of $\beta$ and $\theta$ as a function of distance for GRS 1915+105 was presented in Fender et al. (1999). There is a maximum distance to the source corresponding to $\beta = 1$ (i.e. $\Gamma = \infty$):

$$d_{\text{max}} = \frac{c}{\sqrt{(\mu_{\text{app}} \mu_{\text{rec}})}}$$

At this upper limit to the distance you also find the maximum angle of the jet to the line of sight,

$$\theta_{\text{max}} = \cos^{-1} \left( \frac{\mu_{\text{app}} - \mu_{\text{rec}}}{\mu_{\text{app}} + \mu_{\text{rec}}} \right)$$

**MICROQUASAR MEASUREMENTS**

For two galactic X-ray binary systems, GRS 1915+105 and GRO J1655-40, $\mu_{\text{app}}$ and $\mu_{\text{rec}}$ have been measured for multiple ejection events. The data are summarised in table 1.
Figure 2. Indications of the uncertainties in derived Lorentz factors for jets with different intrinsic velocities and inclinations. Each panel shows the Lorentz factor which would be derived for the jets, given a distance estimate expressed as a fraction of the real distance, for a range of jet angles. The four panels show the solutions for four different intrinsic Lorentz factors. The points at which the functions reach the top of the panel correspond to $d_{\text{max}}$ for that angle and Lorentz factor. For all solutions except those with the lowest velocities and smallest angles, the true distance lies very close to $d_{\text{max}}$, indicating (a) it will be practically impossible to constrain $\Gamma$, (b) sources so observed will in reality lie close to $d_{\text{max}}$, allowing a distance estimate based on the proper motions alone (see also Fig 3). Note that the lower two panels have different ordinates than the upper two panels.
Both sources have fairly accurate independent distance estimates. For GRS 1915+105, Mirabel & Rodríguez (1994) estimate a distance of 12.5 ± 1.5 kpc based on HI measurements. Dhawan, Goss & Rodríguez (2000) revise this distance estimate to 12 ± 1 kpc. The large X-ray column and optical extinction to the source are in agreement with this relatively large distance.

For GRO J1655-40, McKay & Kesteven (1994) estimated a distance of 3.5 kpc, and Tingay et al. (1995) estimated a distance of 3–5 kpc. The kinematic model fit performed by Hjellming and Rupen (1995) resulted in a distance estimated of 3.2 ± 0.2 kpc. Most recently, Greene, Bailyn & Orosz (2001) derive \( d = 3.79 ± 0.69 \) kpc based on modelling of optical data.

Comparison of these distance estimates with the values for \( d_{\text{max}} \) listed in table 1 reveals immediately that all the distance estimates place the sources very close to (or even beyond!) \( d_{\text{max}} \). The result of this is that from such observations we can only place a lower limit on the Lorentz factor of the jets.

This is illustrated in Figs 1(a),(b). In these figures the solutions for \( \beta \) and \( \theta \), based on the observed proper motions, are plotted as a function of distance to the sources. Also indicated are the best distance estimates, as well as the Lorentz and relativistic Doppler factors resulting from the solutions to \( \beta \) and \( \theta \). What is clear, for both sources, is that the distance estimates – already fairly accurate – cannot do more than place a lower limit of 2–3 on the Lorentz factors of the ejections. No upper limit is possible as the range of possible distances includes \( d_{\text{max}} \). Consequently we can only place upper limits on the Doppler shifts associated with the jets.

These figures clearly illustrate that for these two celebrated sources, the measured proper motions combined with the distance uncertainties do not allow us to measure how relativistic the jets are. In this paper we shall show that this will almost always be the case.

### CAN WE LIMIT \( \Gamma \) USING THE FLUX RATIOS?

A further misunderstanding propagating in the literature is that the flux ratio observed between the approaching and receding knot is somehow an independent confirmation of any distance or velocity measurement already derived from the proper motions. This asymmetry in brightness between the approaching and receding knots is due to a combination of classical Doppler and relativistic aberration effects, both contained in the relativistic Doppler factor

\[
\delta = \frac{1}{\Gamma(1 - \beta \cos \theta)}
\]

An object moving at angle \( \theta \) to the line of sight with velocity \( \beta \) (and resultant Lorentz factor \( \Gamma \)) will have an observed surface brightness \( \delta \) brighter, where \( 2 < k < 3 \) \((k = 2 \) corresponds to the average of multiple events in e.g. a continuous jet, \( k = 3 \) corresponds to emission dominated by a singularly evolving event\). Therefore the ratio of flux densities from approaching and receding knots – measured at the same angular separation from the core, so as to sample the knots at the same age in their evolution – will be given by:

\[
\frac{S_{\text{app}}}{S_{\text{rec}}} = \left( \frac{\delta_{\text{app}}}{\delta_{\text{rec}}} \right)^{k - \alpha}
\]

where \( \alpha \) is the spectral index of the emitting region, defined such that \( S_{\nu} \propto \nu^\alpha \). This additional term compensates for the Doppler shifted spectrum when observing at a single frequency. The ratio of the proper motions is simply the ratio of the Doppler factors, so

\[
\frac{S_{\text{app}}}{S_{\text{rec}}} = \left( \frac{\mu_{\text{app}}}{\mu_{\text{rec}}} \right)^{k - \alpha}
\]

Thus once \( \mu_{\text{app}} \) and \( \mu_{\text{rec}} \) have been measured, the only additional information obtained by measuring the flux ratio between approaching and receding jet relates to the parameter \( k \). Although it may seem counter-intuitive that the flux ratio should remain constant as \( \beta \) increases, this is because at the same time \( \theta \) is also increasing. The meaning of \( k \) will not be explored in detail here; however a small point is worth making: in observations in which we can be fairly confident that we have resolved a single radio knot, then \( k \) should have the value 3. If we measure a value less than this it may indicate that the bulk velocity of the flow is significantly less than the pattern velocity which we are observing (for further discussion see e.g. Blandford et al. 1977).

### THE GENERAL CASE

It is straightforward to calculate the proper motions for jets of a given \( \beta \) and \( \theta \), and compare them to the values we would derive using the method outlined above, for varying estimates of the distance to the source.

In Figs 2(a–d) we plot the inferred Lorentz factor as a fraction of the intrinsic Lorentz factor of the jet, as a function of the distance estimated to the source expressed as a fraction of the true distance. In each figure the different curves indicate different intrinsic angles to the line of sight, and each of the four panels represents a different intrinsic Lorentz factor \((2, 5, 10, 50)\). The points at which the curves intersect with the upper abscissa corresponds to \( d_{\text{max}} \) for the particular combination of proper motions observed.

Apart from the smallest angles and lowest velocities,

### Table 1. Simultaneous measurements of approaching and receding knot velocities in the jets from two galactic black hole binaries. Refs: MR94 = Mirabel & Rodríguez 1994; F99 = Fender et al. 1999; HR95 = Hjellming & Rupen 1995. HR95 do not provide estimates of their measurement uncertainties.

| Source        | \( \mu_{\text{app}} \) (mas/d) | \( \mu_{\text{rec}} \) (mas/d) | \( \beta \cos \theta \) | \( \theta_{\text{max}} \) (degrees) | \( d_{\text{max}} \) (kpc) | REF |
|---------------|-------------------------------|-------------------------------|--------------------------|----------------------------------|---------------------------|-----|
| GRS 1915+105  | 17.6 ± 0.4                    | 9.0 ± 0.1                     | 0.323 ± 0.016            | 71                               | 13.7                      | MR94|
|               | 23.6 ± 0.5                    | 10.0 ± 0.5                    | 0.41 ± 0.02              | 66                               | 11.2                      | F99 |
| GRO J1655-40  | 54                            | 45                            | 0.09                     | 85                               | 3.5                       | HR95|

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there is for all the curves an extremely rapid variation in the inferred Lorentz factor close to the true distance to the source. The figures demonstrate clearly that it will be effectively impossible to measure the distance accurately enough to constrain the Lorentz factor. A related point is that all significantly relativistic jets will have by necessity lie very close to $d_{\text{max}}$ (Fig 3). This leads to one useful conclusion – if the jets we observe are intrinsically significantly relativistic, which seems to be the case, then measurements of two-sided proper motions will give us an accurate distance estimate. As a result, this means that observations of two-sided jet proper motions in AGN, were they ever to be observed, would be extremely useful for calibrating the cosmological distance scale. Unfortunately, to date most well-studied AGN are significantly Doppler-boosted (so-called ‘Doppler favouritism’), implying small angles of the jets to the line of sight, and resulting in no measurements of two-sided relativistic proper motions so far.

As noted above, there is a glimmer of hope for the lowest velocities and smallest angles, where the swing in the curves around the true distance is not too dramatic. However (a) this discussion is really concentrating on significantly relativistic jets, and (b) the smallest angles to the line of sight will have the largest ratios of proper motions and fluxes between the approaching and receding sides of the jet, making the measurements increasingly hard to make.

This is illustrated in Fig 4, in which the proper motions (scaled to a distance of 1 kpc) and resultant apparent velocity as a fraction of the speed of light are plotted for different intrinsic Lorentz factors as a function of angle to the line of sight. The receding proper motions are very similar for all intrinsic Lorentz factors, but the approaching proper motions are differing functions which peak at progressively smaller angles (the peaks occur at $\theta \sim 1/\Gamma$ radians). Note that for both GRS 1915+105 and GRO J1655-40 the ratio of approaching to receding proper motions has been $<3$, which, as this figure illustrates, indicates immediately that whatever the Lorentz factor, they must be at large angles to the line of sight (and therefore, consulting Fig 3, unless the jets are only mildly relativistic, means that they must both lie at $d \sim d_{\text{max}}$). If we are hoping to measure the Lorentz factor from the proper motions of a jet close to the line of sight then the ratio of proper motions becomes increasingly large – and therefore increasingly hard to measure accurately. The ratio of fluxes is even greater, being the ratio of proper motions raised to some power $k$ (at the same angular separation), and so compared to the approaching component the receding jet will appear to be extremely faint and slow moving. Most likely we will observe only the approaching jet, or jet plus core if activity is still ongoing (as has been the case to date for AGN).

This leads us to consider an alternative approach to at least limiting the Lorentz factor. For a jet of apparent velocity $\beta_{\text{app}}$, the intrinsic Lorentz factor is at least as large as $\beta_{\text{app}}$, corresponding to the solution for $\theta = 1/\Gamma$. In this way observations of one-sided proper motions can allow us to place a lower limit on the Lorentz factor. How accurate is this method compared to two-sided proper motions? In fact it can never place a more constraining lower limit on $\Gamma$ than can be obtained by measurement of two-sided proper motions. This is natural, since the lower limits to the Lorentz factors measured from one-sided proper motions assume the jet is at its optimum angle, resulting in maximum apparent velocity, which will generally not be the case.

**WHAT CAN WE LEARN?**

We have established above that it will be practically impossible to do more than place a lower limit on the Lorentz factor of a relativistic jet from proper motions, whether one- or two-sided. However, the proper motions themselves can be used to make a distance estimate to the source, more accurate the more relativistic the jet intrinsically is. What else can we learn from the proper motions?

As already stated, the ratio of proper motions is also the ratio of Doppler factors. This may be useful in asso-
Caveats

As stated above, everything calculated in this paper is only strictly valid under the assumption of symmetric ejection events. Observations of jets from the neutron star XRB Sco X-1 (Fomalont et al. 2001a, 2001b) have shown us that the resolved sites of radio emission may in some cases simply be the regions of jet–ISM interaction and may not reflect the underlying bulk velocity of the flow. This is even more dramatically demonstrated by observations of large-scale decelerating jets from the black hole transient XTE J1550-564 (Corbel et al. 2002; Kaaret et al. 2003; Tomášek et al. 2003). As a result the application of the results in this paper, e.g. estimating the distance $\sim d_{\text{max}}$ should, wherever possible, be based upon measurements as early as possible in the flight of the ejecta. Finally, the observed correlation between peak radio and X-ray fluxes from X-ray transients (Fender & Kuulkers 2001) would be destroyed if the Lorentz factor of the radio emitting region were too large (unless the X-ray emission were also beamed, which would however imply a huge selection effect on observations of X-ray binaries) – while current data may be too sparse to constrain this at present, this may be the best approach for limiting the Lorentz factors of jets from X-ray binaries in the future.

CONCLUSIONS

In this paper the uses and limitations of relativistic jet proper motions have been explored, under the assumption of intrinsically symmetric ejection velocities. The main results derived are:

• For the two galactic ‘microquasars’ for which two-sided proper motions have been measured, even the relatively small uncertainties in the distances estimates result in an almost complete inability to constrain the Lorentz and Doppler factors. Measurement of the flux ratios of approaching and receding components does not provide any additional constraints.

• Exploring the general case, it is found that this will nearly always be the situation – i.e. that all relativistic jets will be observed near the distance $d_{\text{max}}$ at which $\beta \sim 1$ and we will be unable to place an upper limit on the Lorentz factor of the flow. Conversely, this means that relativistic jet sources will always be observed close to $d_{\text{max}}$. This means that for AGN, observations of two-sided proper motions will not allow accurate measurement of the Lorentz factor of the jets, but will be extremely important for calibration of the cosmological distance scale, without requiring the observation of Doppler-shifted emission lines.

• It is shown that the product of the approaching and receding proper motions varies significantly with angle to the line of sight for jets which are only mildly relativistic, whereas for highly relativistic jets the product is practically invariant. This opens up the possibility of constraining the Lorentz factor of a precessing jet by measurement of the product around the precession cycle.

Mildly relativistic precessing jets

There is even a possibility to achieve the goal of limiting the Lorentz factor, in the case of a jet whose angle to the line of sight changes, for example due to precession. This can be seen from Fig 2, where at lower Lorentz factors the $d_{\text{max}}$ is quite a strong function of angle, whereas it is not at all for the higher Lorentz factors. This is illustrated in Fig 5, in which the product $\mu_{\text{app}}\mu_{\text{rec}}$ is plotted for varying angles, for different Lorentz factors. Apart from the smallest angles to the line of sight, at which two-sided proper motions are anyway unlikely to be detected, the most relativistic jets have an almost constant value of this product, whereas the slower jets have a significantly varying product. For example, a jet with a mean angle to the line of sight of 60 degrees, precessing with a half-opening angle of 20 degrees would produce a $\sim 25\%$ change in the product $\mu_{\text{app}}\mu_{\text{rec}}$ over its precession period if it had an intrinsic Lorentz factor of 2. If the jet has a Lorentz factor of five or more, the fractional change in the product over the precession cycle is $5\%$ or less. This approach, albeit almost certainly limited to galactic sources (due to the necessity of tracking in time a periodic precession cycle) presents an interesting possibility for limiting the Lorentz factors.
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