Multidimensional analysis of particle size fractal characteristics in a farmland soil profile

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Abstract: Soil particle size distribution (PSD) is a basic soil property, closely related to soil erosion, soil moisture, and so on. The multidimensional fractal method has been proved to be a better description of soil PSD. By choosing the four fractal parameters of $\Delta a$, $\Delta a_L/\Delta a_H$, $D_1/D_0$ and $\Delta f(\alpha)/\Delta f(\alpha)_L$, and using the multifractal spectrum, this paper analyzed one- and two-dimensional fractal characteristics of the soil profile (A, B, and C layers from top to bottom) PSD and their relationship on a field scale. The results showed that the PSD characteristics of different soil profiles were more heterogeneous in the east and west (WE) than the north and south (NS) in the study area, and the A layer soil PSD was the most uneven, with a larger degree of dispersion. With the soil layer depth, the soil PSD tended to be uniform. Regardless of one-dimensional or two-dimensional characteristics, the proportion of the low value of soil particle mass percent was relatively larger. The results of this study will not only provide a reference method for analyses of the fractal characteristics of soil PSD on a field scale, but also provide guidance in further understanding the soil structure and soil forming process.

1. Introduction

Soil is composed of particles with irregular shapes and self-similar structures, which form a porous medium. Soil particle size distribution (PSD) has fractal properties\cite{1-5}. PSD is closely related to soil erosion, organic matter content, and soil moisture content. A comprehensive and accurate understanding of the soil PSD fractal characteristics is crucial to understanding soil structures and the process of soil formation. Fractal theory and analysis can effectively describe soil texture \cite{6}. Multifractal analysis can effectively capture the essence of change in the measured values \cite{7}. The effectiveness of utilizing fractal parameters to estimate and analyze soil texture exceeds that of the traditional soil texture triangle model \cite{8-11}. Soil scientists have already utilized fractal principles and methods to analyze geological characteristics, which cannot be easily obtained using a traditional geology method\cite{12-13}. PSD analysis requires a large amount of sampled data. If the overall PSD can be reconstructed from a determined percentage of the soil particle mass, it will have a significant
impact on the mathematical modeling of soil moisture migration, soil erosion, and the soil formation process. Thus, it is necessary to achieve a clear understanding of the soil PSD structure, including the spatial variability in a given direction and the cross-sectional soil structure. Multifractal methods based on soil PSD studies already exist, such as the study of PSD characteristics under different soil management strategies [14-16], and the influence of vegetation on PSD[17-18]. However, there are a number of limitations in the current research methods, including the following: (1) Single dimension studies and concentrated research content. Most studies only utilized the two-dimensional surface area information to analyze different influential conditions (such as land use, soil texture, and vegetation) on soil PSD. It is difficult to fully explore the different PSD fractal characteristics given these limitations. (2) The fractal parameters employed are simple and lack representation. Most studies used a fractal dimension method to analyze soil PSD fractal characteristics. Multifractal parameters have rarely been applied; therefore, it is difficult to give a multidimensional description of the soil PSD characteristics. There have been very few reports of multidimensional, multidirectional, and multifractal analyses of farmland soil profiles. Thus, this area could benefit from further exploration.

A multifractal program based on the MATLAB platform was utilized to calculate four fractal parameters, $\Delta a$, $\Delta a_R/\Delta a_L$, $D_1/D_0$, and $\Delta f(a)_R/\Delta f(a)_L$, and to carry out multidimensional profiling. This paper describes an in-depth study of the farmland soil PSD’s one-dimensional and two-dimensional fractal characteristics and their relationship with different dimensions. Therefore, this paper proposes an analytical method for determining farmland soil PSD characteristics. In addition, this study also serves as a guide for further understanding soil structure and the soil formation process.

2. Materials and Methods

2.1. Description of sites and soil sampling

The study area was located in Dongbeiwangxiang, Haidian District, Beijing. It was on an piedmont alluvial plain and the soil type was meadow cinnamon soil. The 60 m × 55 m study area was divided into 156 grids based on sample units of 5 m × 5 m. Sampling was carried out with a 1 m soil auger in the selected 100 grid points (Figure 1). In accordance with the soil profile layers, sample aliquots were extracted from each layer and the depth of each layer was measured and recorded. A total of 300 soil samples were taken. Soil particle composition was determined using the traditional pipette method.

![Figure 1. The distribution map of sampling points.](image)

2.2. Data preparation and processing

In order to ensure that one-dimensional north-south (NS) and west-east (WE) data were comparable and representative, the original sampling point data was processed as follows: The average value of soil particles mass percent was obtained in the direction of west to east from samples from rows 1-4 on the same vertical axis. The obtained value was denoted as the new NS sample point 1. Next, a new NS
sample point 1 was obtained. The same method as outlined for rows 1-4 was repeated for samples from rows 2-5 to obtain the new NS sample point 2. This process was repeated until nine new NS sample points were obtained as required for one-dimensional analysis of the soil PSD. Likewise, from south to north, samples from points 1-5 on the same horizontal axis were used to obtain the average value of soil particles mass percent. The obtained value was denoted as the new sample value, which allowed for the establishment of a new WE sample point 1. The mean value from rows 2-6 was used as the new WE sample point 2. The same method was used to obtain nine new WE sample points for analysing PSD fractal characteristics of the WE soil. MATLAB-based software was employed to calculate the multifractal parameters of the different soil particle compositions in the A, B, and C layers, while accounting for the WE and NS direction of the particles in each layer. The subsequent multifractal spectrum was also drawn using MATLAB.

2.3. Theory and methods

The multifractal method was used to describe multidimensional soil PSD fractal characteristics and their correlations. The most commonly applied multifractal parameters are multifractal spectrum width (Δa), multifractal spectrum left-side width (ΔaL), multifractal spectrum right-side width (ΔaR), asymmetry index (ΔaR/ΔaL), generalized fractal dimension (ΔD(q)), dispersion index (Dv/D0), right-side branch length (Δf(a)R), left-side branch length (Δf(a)L), and the ratio of the right-side to the left-side branch length (Δf(a)R/Δf(a)L). In order to select representative parameters that could comprehensively describe multidimensional PSD fractal characteristics, the fractal parameters of soil sand particles from different dimensions were used as analytic target, and the Pearson correlation analysis was carried out to determine the correlation between the different multifractal parameters.

| Parameters          | Δa | ΔaR | ΔaL | ΔaR/ΔaL | ΔD(q) | Dv/D0 | Δf(a)R | Δf(a)L | Δf(a)R/Δf(a)L |
|---------------------|----|-----|-----|---------|-------|-------|--------|--------|---------------|
| Δa                  | 1  | 0.968** | 0.803** | -0.053 | 0.932** | 0.983** | 0.802** | 0.452** | -0.109        |
| ΔaR                 | 1  | 0.627** | 0.099 | 0.235 | 0.396** | 0.930** | 0.877** | 0.238 | 0.038        |
| ΔaL                 | 1  | -0.396** | 0.125 | 0.098 | 0.093 | 0.125 | 0.839** | 0.408** | 0.832** | -0.426**       |
| ΔaR/ΔaL             | 1  | 0.098 | 0.102 | 0.249 | 0.125 | 0.098 | 0.102 | 0.249 | 0.513** | 0.663**       |
| ΔD(q)               | 1  | 0.643** | 0.723** | 0.403** | 0.093 |
| Dv/D0               | 1  | -0.726** | 0.474** | 0.159 |
| Δf(a)R              | 1  | 0.018 | 0.289* | -0.624** |
| Δf(a)L              | 1  | -0.624** |

*Correlation is significant at the 0.05 level (2-tailed).
**Correlation is significant at the 0.01 level (2-tailed).

The results indicate that in two dimensions, there was a significant negative correlation between Δa and Dv/D0 (Table 1). Parameters Δa and Dv/D0 were not correlated with ΔaR/ΔaL. However, ΔaR, ΔaL, ΔD(q), Δf(a)R, and Δf(a)L all had a significantly positive correlation with Δa. One-dimensional particle analysis showed the same correlation characteristics. The properties of different parameters, which were the represented PSD characteristics, were taken into consideration and combined with the results shown in Table 1. This study has selected the parameters Δa, ΔaR/ΔaL, Dv/D0, and Δf(a)R/Δf(a)L.
and utilized a multifractal spectrum to conduct a multidirectional, multidimensional analysis of the fractal characteristics of the farmland soil PSD.

The research area was assumed to be a square with length $d$. In the study area, there were $M$ sampling points, and soil particle composition was measured at each sampling point. An observation scale of $\varepsilon = 1/n$ was set, where $n > 1$, and the entire research area was divided into $n$ rows and $n$ columns, which formed a total of $N = n^2$ small grids, where length $= d/n$. The average of the measured value in the $i$-th grid $a_i(\varepsilon)$ was calculated, and a summation carried out for the entire study area. The probability of soil particle composition in the $i$-th grid can be determined as follows:

$$p_i(\varepsilon) = \frac{a_i(\varepsilon)}{\sum_{i=1}^{N} a_i(\varepsilon)}$$  \hspace{1cm} (1)

where $p_i(\varepsilon)$ is the probability of soil composition in the $i$-th grid. Given real number $q$, the definition of a partition function is as follows:

$$\chi_i(q,\varepsilon) = \sum_{i=1}^{N} p_i(\varepsilon) \varepsilon^q$$  \hspace{1cm} (2)

where $\chi_i(q,\varepsilon)$ is the $q$ order of probability in the $i$-th grid.

$$D_q = \frac{\ln \chi_i(q,\varepsilon)}{(q-1)\ln \varepsilon}$$  \hspace{1cm} (3)

$$a(q) = \lim_{\varepsilon \to 0} \frac{\sum_{i=1}^{N} \chi_i(q,\varepsilon) \ln p_i(\varepsilon)}{\ln \varepsilon}$$  \hspace{1cm} (4)

When $q = 0$ and $1$, the corresponding $D_0$ and $D_1$ denote the capacity dimension and dimension entropy, respectively. Dimension entropy, $D_1$, measures the soil unit aggregation abundance and degree of aggregation. For a particular distribution, $D_0$ is the maximum value of $D_1$. When $D_1 = D_0$, it becomes a single fractal, with minimum distribution heterogeneity. This implies that $D_1/D_0$ can be used as an indicator for soil dispersion.

$$f(a(q)) = \lim_{\varepsilon \to 0} \frac{\sum_{i=1}^{N} \chi_i(q,\varepsilon) \ln p_i(\varepsilon)}{\ln \varepsilon}$$  \hspace{1cm} (5)

After obtaining the values mentioned above, multifractal parameters of soil PSD would be obtained. For more information regarding multifractal modeling, one may refer to the relevant publications [19-23]. And then, a multifractal program was developed based on the MATLAB platform which was used to calculate soil PSD fractal parameters $D_1/D_0$, $\Delta a_R/\Delta a_L$, and $\Delta f(a)_R/\Delta f(a)_L$, and to draw the multifractal spectrum.

3. Results and Discussions

3.1. Soil PSD one-dimensional fractal characteristics
The one-dimensional soil particle mass percent and the multifractal parameters $\Delta a$, $D_1/D_0$, and $\Delta a_R/\Delta a_L$ were compared between WE and NS direction (Figure 2).
Figure 2. Comparison of one-dimensional multifractal parameters of PSD between WE and NS directions. Four icons as a, b, c and d respectively represented Δα, D₁/D₀, Δαᵣ/Δαₗ and particle mass percent.

Figure 2 shows that regardless of the WE or NS direction, the difference between the same types of soil particle percentages were relatively small. However, there was a significant difference between the multifractal parameters Δα, D₁/D₀ and Δαᵣ/Δαₗ. In addition, the same fractal parameter in different layers tended to have similar characteristics. In each layer, the sand, silt, and clay Δα values were greater in the WE direction than in the NS. Whereas D₁/D₀ was the opposite, suggesting that the soil PSD in the WE direction of the research area showed greater heterogeneity and uneven distribution. Δαᵣ and Δαₗ represent the probability of a high value (or a large value, q>>1) and low value (or a small value, q<<1), respectively. Δαᵣ/Δαₗ is the ratio of the right-side spectral width to the left-side spectral width. It is used to measure the symmetry (or asymmetry) of the left and the right sides of the f(α) multispectral spectrum. The closer the value is to 1, the closer the ratio of the larger value to the small value[16],[21]. For the soil profile of each layer, the Δαᵣ/Δαₗ values of WE were greater than 1. For the sand and clay particles in the NS direction, the Δαᵣ/Δαₗ values were also greater than 1. So, it indicates that the proportion of the low value of soil particle mass percent was relatively large.

3.2. Soil PSD two-dimensional fractal characteristics

Equations (1) through (5) are used to calculate the multifractal parameters of the two-dimensional soil particle percentage content in different layers. The parameters calculated include Δα, D₁/D₀, Δαᵣ/Δαₗ and Δf(αᵣ)/Δf(αₗ) (Table 2). Table 2 shows that the two-dimensional sand particle Δα value decreased with each layer (from top to bottom). Silt and clay particles had a minimal Δα value in layer C, which indicated the soil profile PSD heterogeneity was the weakest in the bottom layer. In accordance with the U.S. soil texture classification system and soil particles distribution at each layer, layer C was sandy loam soil, layers A and B were loam soil. This was consistent with the finding that the coarser soil texture, the smaller the fractal dimension[24-26]. D₀ is the capacity dimension or box dimension, which can be used to measure soil particle abundance. D₁ is the dimension entropy or information entropy, which can be used to measure dispersion [27]. D₁/D₀ can be used to measure the degree of uniformity. When D₁/D₀ is close to 1, this indicates a more uniform distribution[16]. The D₁/D₀ value of the sand, silt, and clay
The study showed that the $D_1/D_0$ value was the smallest in layer A (Table 2). The corresponding particle type had a larger value in the B and C layers, and a $D_1/D_0$ value closer to 1. In the study area, the layer A PSD was the most uneven, with a larger degree of dispersion. This was particularly prominent for sand particles, which also confirmed the conclusion demonstrated by the $\Delta a$ value. The sampling location was farmland that might be concluded the human activity as the main reason to cause the uneven soil PSD in layer A. Soil PSD were more homogenous in layers B and C. This was likely due to a stronger influence of the parent material, and weaker human and environmental impacts.

| Soil profile-layer | Particle type | Soil particle mass percent (%) | $\Delta a$ | $\Delta a_{R}/\Delta a_{L}$ | $\Delta f(a)_{R}/\Delta f(a)_{L}$ | $D_1/D_0$ |
|--------------------|---------------|-------------------------------|------------|-----------------------------|---------------------------------|------------|
| A                  | Sand          | 48.59                         | 0.876      | 2.983                       | 1.451                           | 0.994      |
|                    | Silt          | 39.393                        | 0.676      | 0.615                       | 0.641                           | 0.993      |
|                    | Clay          | 12.017                        | 0.683      | 1.116                       | 1.017                           | 0.992      |
|                    | Sand          | 51.313                        | 0.642      | 1.591                       | 1.022                           | 0.997      |
| B                  | Silt          | 35.356                        | 0.876      | 1.686                       | 1.135                           | 0.994      |
|                    | Clay          | 13.331                        | 0.714      | 2.445                       | 1.432                           | 0.994      |
|                    | Sand          | 55.303                        | 0.539      | 6.358                       | 4.01                            | 0.997      |
| C                  | Silt          | 30.666                        | 0.578      | 0.599                       | 0.678                           | 0.993      |
|                    | Clay          | 14.031                        | 0.44       | 1.053                       | 0.841                           | 0.995      |

**Figure 3.** Multifractal spectra of soil PSD for different layers (A, B, and C, respectively, represent the different soil; a, b, and c, respectively, represent sand, silt, and clay).
The $f(a)$ value reflects the shape of the curvature and the degree of the symmetry (Figure 3). The shape and the symmetry of the multifractal spectrum are critical in the understanding of soil PSD heterogeneity[11]. The $\Delta a_R/\Delta a_L$ value (ratio of right to left spectral width), is the parameter that describes the symmetry of the multifractal spectrum. Figure 3 shows that in each layer, there was a stronger degree of spectral width asymmetry observed in the sand particles. The width of the left-side branch ($q > 0$) was significantly smaller than the right side ($q < 0$). In the terms of different layers, the difference between the left spectral width ($\Delta a_L = 0.218$) and the right spectral width ($\Delta a_R = 0.652$) of the layer A sand particles was maximum (0.434). Likewise, in layer C sand particles, the difference between the left and the right spectral width was maximum that $\Delta a_R = 0.446$ was 6.358 times larger than $\Delta a_L = 0.073$. In layer B sand particles, the right spectral width was 1.591 times the width of the left. The clay particles in layers A and C showed a relatively symmetrical spectral width. In layer C clay particles, the left and the right spectral widths were very similar ($\Delta a_L= 0.226$ and $\Delta a_R= 0.214$). With the exception of silt particles in layers A and C, the multifractal spectrum of all the different types of particles in the soil profile showed a consistent trend in terms of spectral width. The right spectral widths were greater than the left, which indicates the proportion of low value of soil particle mass percent was relatively large.

### 3.3. Multifractal characteristics of different dimensions

Figure 3 and Table 2 show that in layer A, the two-dimensional sand particle $\Delta a_R/\Delta a_L$ value was 2.983, and the right-side spectral width was large, indicating that the portion of low value of sand particle mass percent was relatively large. The one-dimensional NS $\Delta a_R/\Delta a_L$ value was 3.512, which was closer to two-dimensional than the WE $\Delta a_R/\Delta a_L$ value (1.289). Sand particle $\Delta f(a)_b/\Delta f(a)_l$ was $> 1$. 

![Figure 4. The scatter diagram of different fractal dimensions for soil PSD.](image-url)
(1.451), indicating the right spectral length was longer than the left, and the NS value was closer to two-dimensional. In layer A, the silt and the clay particle $D_1/D_0$, $\Delta a_R/\Delta a_L$, and $\Delta f(a)_R/\Delta f(a)_L$ showed that the one-dimensional NS value was closer to two-dimensional. In layer B, the two-dimensional sand particle $\Delta a_R/\Delta a_L$ and $D_1/D_0$ values were 1.591 and 0.997, respectively, which were closer to the one-dimensional NS values. However, in layer B, the sand particle $\Delta f(a)_R/\Delta f(a)_L$ value (1.022) was closer to that of the one-dimensional WE value (1.350). In layer B, the silt particle $\Delta a_R/\Delta a_L$ value was 1.686, and the $\Delta f(a)_R/\Delta f(a)_L$ value was 1.135, which was closer to the WE $\Delta a_R/\Delta a_L$ value (1.971). However the $D_1/D_0$ value was the opposite. In layer B, the clay particle $\Delta a_R/\Delta a_L$ value (2.445) was relatively large, which was quite different from the one-dimensional level. However, the $\Delta f(a)_R/\Delta f(a)_L$ (1.432) and $D_1/D_0$ (0.994) values were closer to the one-dimensional values. The relationship between the layer C soil particles and one-dimensional values showed a similar trend as observed in layer B. Layers B and C soil PSD was influenced by both NS- and WE-directional values. However, the layer A soil PSD was predominantly affected by the NS direction, which may be related to factors of soil formation. As the study area was on an alluvial plain, layer A was situated on the surface, and more likely to be affected by environmental factors such as wind and water movement. This was consistent with the finding that layer A had greater azimuth attributes as compared with layers B and C. This finding was produced by Gao et al., who applied the Kriging method to analyze the attributes of each layer[28]. However, in the study area, there was not necessarily a link between the fractal degrees for different dimensions. The two-dimensional clay particles of layer B had the maximum multifractal spectrum width of $\Delta a = 0.876$. It exceeded the WE value of $\Delta a = 0.391$, the NS value of $\Delta a = 0.206$, and the sum of these two values. Layers A and C showed the same trend, indicating that the two-dimensional soil PSD fractal strength is not necessarily reflected one-dimensionally. The opposite conclusion is also true.

To further explore linkages and the strength of the relationship between different dimensions of fractal characteristics, two-dimensional and different one-dimensional fractal parameter scatter diagrams were generated and fitted to the curve (Figure 4).

Figure 4 illustrates that, $D_1/D_0$ and $\Delta a_R/\Delta a_L$ had a significant linear relationship with the two-dimensional level ($p < 0.01$), indicating the NS direction was closely related to the two-dimensional level. However, no significant relationship was observed in the WE direction. Instead, the $D_1/D_0$ value was negatively correlated with the two-dimensional level. This also confirmed that at the two-dimensional level, NS had some degree of impact on the different particles in the different layers. Figure 4 $\Delta a$ reflects that there was not necessarily a link between the fractal degrees for different dimensions. So, in order to reveal the relationship between the fractal characteristics of soil PSD and soil formers, it is necessary to carry out multidirectional and multidimensional analyses.

4. Conclusions

This paper utilized a multifractal method to analyze one- and two-dimensional fractal characteristics of the soil profile (A, B, C layers, from top to bottom) PSD and their relationship on a field scale. The following conclusions can be drawn:

(1) In the study area, the multifractal parameters of the soil profile from different layers and particle types indicated that the WE-directional $\Delta a$ and $D_1/D_0$ values were always smaller than those of the NS direction. The soil PSD in the WE direction of the research area showed greater heterogeneity and an uneven distribution. In general, the one-dimensional WE and NS $\Delta a_R/\Delta a_L$ values were greater than 1. This indicates that the proportion of low value (small value) of soil particle percentage content was relatively large.

(2) The A layer two-dimensional soil PSD was the most heterogeneous, and with an increase in depth, the soil PSD became more uniform. The soil particle multifractal spectrum showed that the right-side spectral width was generally greater than the left. The lower value (small value) of the soil PSD content was more prominent.

(3) The A layer two-dimensional PSD was influenced by the NS-directional effect. In layers B and C, the soil PSD was influenced by a combination of the NS and WE effects. Overall, the different
parameters between different fractal dimensions were not necessarily linked. Therefore, the soil fractal strengths were not necessarily related. In order to reveal the relationship between the fractal characteristics of soil PSD and soil factors, it is necessary to carry out multidirectional and multidimensional analyses.

(4) The results have shown that the use of multifractal parameters and a multifractal spectrum to conduct multidirectional, multidimensional analyses of the soil PSD results in a comprehensive analysis of farmland soil particle characteristics. This paper has provided a methodological guidance for future research of this type. As the study area was relatively small, located on an alluvial plain, and mostly used for agricultural purposes, the difference observed between the influences of topography, parent material, human activities, and the external environment was relatively small. Therefore, it is difficult to carry out quantitative comparisons. In order to reveal a more comprehensive system of the regional soil formation process, future studies should consider using soil formation factors as fractal parameters in subsequent quantitative analyses.

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