SELECTION RULES FOR $^{48}\text{Cr}$

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Abstract

In the single j shell ($f_7/2$) $^{48}\text{Cr}$ is the first even-even nucleus for which there are $T=0$ (isoscalar) $J=1^+$ states and $J=0^+$ $T=1$ states. These states are studied here. This nucleus, in the same model space, is midshell for both protons and neutrons. We can assign a new quantum number to all these states which involves the seniority of the protons and of the neutrons despite the fact that the seniority itself is not a good quantum number. This then leads to selection rules for electromagnetic transitions, e.g. $B(M1)$, $B(E2)$ and to vanishings of static electric moments.

1 Introduction.

In this work, we examine a nucleus which, in the single j shell model space is at midshell. The nucleus is $^{48}\text{Cr}$, which can be viewed as having 4 protons and 4 neutrons in the $f_7/2$ shell or as 4 proton holes and 4 neutron holes in that same shell. We will show that there is a quantum number associated with all the states in this model space, one which involves the seniority of the protons and of the neutrons - this despite the fact that seniority itself is not conserved. The good quantum number leads to selection rules for electromagnetic transitions and static moments rules. In particular, we focus on $B(M1)$’s, $B(E2)$’s and quadrupole moments of excited states. A second point of interest is the fact that $^{48}\text{Cr}$ is the first even-even nucleus for which new states appear in a single j shell calculation - these are $J=1^+ T=0$ and $J=0^+ T=1$. We also perform large space calculations to see how things evolve.

Some of the points have been briefly presented in ref [1] but we here add some additional observations e.g. of new $J=0^+ T=1$ states, vanishing quadruple moments, fluctuating quantum numbers along the yrast band etc. Most important we now consider many large space calculations to be compared with the simpler ones.

2 New States in $^{48}\text{Cr}$ in the single j shell model space.

The nucleus $^{48}\text{Cr}$ is of interest for a variety of reasons. Note that in the single j shell ($f_7/2$) there are no $J=1^+ T=0$ in $^{44}\text{Ti}$. There are such states in the odd-odd nucleus $^{46}\text{V}$. However, the first even-even nucleus for which there are isoscalar $J=1^+$ states in the single j shell configuration is $^{48}\text{Cr}$. Likewise this is the first even-even nucleus for which there are $J=0^+ T=1$ states in the single j shell. We wish to study such states in this section both in single j and the complete f-p space.

Another point of interest is that in the single j shell we are at midshell and this leads to selection rules which will be discussed in the next section.
3 Selection Rules for any N=Z Nucleus.

Before discussing selection rules specific to the midshell nucleus \(^{48}\text{Cr}\) let us consider such rules common to any N=Z nucleus. Note that in this single j model space all \(B(M1)\)'s from \(J = 1^+\) \(T = 0\) states to \(T = 0\) states vanish for all \(J_f\) (0, 1, 2). This can be explained by the fact that in the limited model space (\(\tau_{7/2}\)) the isoscalar M1 operator is proportional to \((J_p + J_n)\), total angular momentum operator. This operator, acting on a state \(|\alpha Jm\rangle\) will create a state in the same \(|\alpha J\rangle\) multiplet and thus will not induce M1 transitions to different multiplets. Also in this model space all transitions from \(1^+\) \(T = 1\) states to other \(T = 1\) states \((J_f = 0, 1, 2)\) also vanish. This is a known result which can be related to the vanishing, in an \(N = Z\) nucleus, of the Clebsch-Gordan coefficient \((1\ 1\ 0\ 0\ |10)\).

4 Selection Rules in for Midshell Nuclei: Calculations of \(B(M1)'s\) and \(B(E2)'s\) in \(^{48}\text{Cr}\).

Some of the zeros, however, are specific to \(^{48}\text{Cr}\). In the single j shell we are at midshell. The 4 protons and 4 neutrons can also be regarded as 4 proton holes and 4 neutron holes. As first noted by Escuderos, Zamick [1] and Bayman [2] and shown analytically by Neergaard [3], the quantity \(S = (-1)^{v_p + v_n}/2\) is a good quantum number, where \(v_p\) and \(v_n\) are the seniorities of the protons and neutrons respectively. With the MBZE interaction [2] the \(J = 0^+\) ground state has \(S = +1\). With the same interaction the yrast states of even \(J\) have \(S = (-1)^{J/2}\) i.e. \(S = +1\) for \(J = 0_1\), \(S = -1\) for \(J = 2_1\), and \(S = +1\) for \(J = 4_1\) etc. Along the yrast chain the \(B(E2)'s\) are large and for these we have \(S_f = -S_i\). In the single j model space the \(B(E2)'s\) for transitions in which \(S_f = S_i\) will vanish.

Since for a static electric moment, e.g. \(Q(2^+)\), the "transition" matrix element is from a state to itself there is no change of \(S\) and so this moment vanishes. However, with configuration mixing one gets a static quadrupole moment which is close to the rotational value. In ref [4] the calculated values are \(Q = -35.42\ \text{e fm}^4\) and \(B(E2,2\rightarrow0) = 312.37\ \text{e}^2\text{fm}^4\) This is consistent with the formulae of the a simple rotor model [5]:

\[
Q = |(3K^2-J(J+1))/((J+1)((2J+3))|Q_0|| B(E2,K J_1\rightarrow K J_2) = 5/(16\pi)e^2Q_0^2 <J_12K 0|J_2K|^2
\]

The respective values of \(Q_0\) are 123.97 and 125.30.

We show a brief example of the selection rules in Table I. We consider \(B(E2)'S\) from the \(J = 0_1^+\) \(S = +1\) state first two \(J = 2^+\) states. The \(2^+_1\) state has \(S = -1\) whilst the \(2^+_2\) state has, just like the \(J = 0_1^+\) ground state. We use the Shell model Code NUSHELLX of B. A. Brown and W.D.M. Rae [6].

| \(J\) | Large \(Q\) | Single \(j\) \(Q\) |
|---|---|---|
| \(2_1\) | 1225.5 | 452.6 |
| \(2_2\) | 2.367 | 0 |

With regards to magnetic dipole transitions it is easy to show that for \(B(M1)\) not to vanish a necessary condition is that \(S_f = S_i\) i.e. \(\Delta S = 0\). This can be easily shown by examining the wave functions of MBZE. [2]. When the M1 operator acts on a basis state \([J_p,J_n]\) it creates a state with the same \([J_p,J_n]\) (including any internal quantum numbers). We then overlap with the final state. In the latter only the component with the same \([J_p,J_n]\) will contribute. If \(D(J_p,J_n)|\) for the initial state is non-zero then the corresponding coefficient for the final state will be non zero only if \(S_f = S_i\).

We next take a casual look at magnetic dipole transitions and look for selection rules for \(B(M1)\) values. With the MBZE [2] interaction, the \(S\) values for the first 3 \(J = 0^+\) \(T = 0\) states are +1,-1 and +1 respectively whilst the only 2 \(J = 0^+\) \(T = 1\) states both have \(S = -1\). For \(J = 0^+\) \(T = 2\) all 3 states have \(S = +1\). Hence the first and third \(J = 1^+\) \(T = 1\) states will connect with all three but the second will not connect with any \(J = 0^+\) \(T = 2\) states. For the first 3 \(J = 1^+\) \(T = 0\) states the \(S\) values are -1,+1 and -1; for \(J = 1^+\) \(T = 1\) they are +1,-1, and +1; for \(J = 2^+\) \(T = 0\) they are -1,+1,+1 and finally , for \(J = 2^+\) they are -1, +1, -1.
For the lowest "special" $J = 1^+ T = 0$ state which has $S = -1$ there will be no transitions in the single j shell model space to any $T = 0$ states, and there will be non-zero $B(M1)'s$ only to the $2 J = 0^+ T = 1$ states, to the second $J = 1^+ T = 1$ state and to the first and third $J = 2^+ T = 1$ states.

The above selection rules can be obtained as easy generalizations of results for particles of one kind, as described e.g. in R.D. Lawson’s book [7] and based on early work by G. Racah [8]. See especially eq. 3.59 and the discussions that follow. It is there shown that the matrix element of the $O^\lambda$ operator for particles is related to that for holes by a phase factor $(-1)^{1+\lambda+(v-v')/2}$. In that work $v$ refers to the seniority of particles of one kind. We simply replace $v$ by $(v_p + v_n)$ and $v'$ by $(v'_p + v'_n)$. In order to get a non-vanishing matrix element, the phase factor must be positive. For $B(M1) \lambda = 1$ and for $B(E2) \lambda = 2$. This explains the selection rules in a more formal way.

We next consider what happens in the complete $f$-$p$ shell model space. In Table II we show the large space results from various $J = 1 T = 0$ states to lowest and second $J = 0^+$ and $J = 2^+$ states and likewise from various $J = 1^+ T = 1$ states. All $B(M1)'s$ in the upper half are $T = 0$ to $T = 0$ transitions and indeed they would have vanished in the single j-shell calculations. In the lower half of Table II we have $T = 1$ to $T = 0$ transitions and indeed the $B(M1)'s$ are on the whole much larger. Let is focus on the lowest $J=1 T=1$ to the lowest $J=0 T=0$ transition. The value of $B(M1)$ is 1.101. The orbital value is 0.3046 and the spin value is 0.2475. The amplitudes add constructively to give the total $B(M1)$. When considered in reverse i.e. from 0 to 1 the $B(M1)$ is 3 times as large and is often compared to the idealized purely orbital scissors mode.

Table II: $B(M1)'s$ in a Large Space ($\mu_\lambda^2$)

| $J = 1^+ T = 0$ | Lowest $J = 0^+$ | Second $J = 0^+$ | Lowest $J = 2^+$ | Second $J = 2^+$ |
|-----------------|------------------|------------------|------------------|------------------|
| $n = 1$         | 0.2003 E-3       | 0.2124 E-4       | 0.1095 E-4       | 0.6845E-4       |
| 2               | 0.1343 E-1       | 0.1334 E-3       | 0.5432 E-2       | 0.1288 E-3       |
| 3               | 0.5903 E-3       | 0.9063 E-5       | 0.5268 E-4       | 0.7698 E-4       |
| 4               | 0.5461 E-4       | 0.2338 E-2       | 0.4631 E-4       | 0.3361 E-2       |
| 5               | 0.7451E-5        | 0.1677 E-5       | 0.2297 E-3       | 0.1098 E-2       |

| $J = 1^+ T = 1$ | Lowest $J = 0^+$ | Second $J = 0^+$ | Lowest $J = 2^+$ | Second $J = 2^+$ |
|-----------------|------------------|------------------|------------------|------------------|
| $n = 1$         | 0.1101 E+1       | 0.1221 E-1       | 0.4665 E0        | 0.5316 E-1       |
| 2               | 0.6551 E0        | 0.1813 E0        | 0.3838 E0        | 0.4569 E-1       |
| 3               | 0.1570 E0        | 0.2142 E0        | 0.9775E-1        | 0.6353 E0        |
| 4               | 0.2353 E0        | 0.1709 E0        | 0.1768 E-2       | 0.4243 E0        |
| 5               | 0.5526 E-1       | 0.2574 E0        | 0.4835 E-2       | 0.2511 E0        |

In Table III we show $B(E2)'s$ from various $J = 1^+ T = 0$ states to the 2 lowest $J = 2^+$ and likewise from various $J = 1^+ T = 1$ states. The largest $B(E2)$ in Table III is 25.32 e$^2$fm$^4$. This is considerably smaller than value 1225.5 e$^2$fm$^4$ for the collective $J = 0^+ \rightarrow J = 2^+$ transition shown in Table I.
Table III: B(E2)'s in a Large Space (e²fm⁴).

| J = 1⁺, T = 0 → | Lowest J = 2⁺ State | Second J = 2⁺ State |
|-----------------|----------------------|---------------------|
| n = 1           | 0.1897 E2            | 0.4232 E-2          |
| 2               | 0.5243 E1            | 0.5807 E0           |
| 3               | 0.8518 E-1           | 0.2532 E2           |
| 4               | 0.2944 E0            | 0.1551 E-1          |
| 5               | 0.8348 E-3           | 0.4874 E-1          |

| J = 1⁺, T = 1 → | Lowest J = 2⁺ State | Second J = 2⁺ State |
|-----------------|----------------------|---------------------|
| n = 1           | 0.6694 E1            | 0.1909 E-1          |
| 2               | 0.4376 E1            | 0.2359 E-1          |
| 3               | 0.3735 E0            | 0.2125 E1           |
| 4               | 0.1676 E-2           | 0.7543 E0           |
| 5               | 0.1546 E0            | 0.5553 E-1          |

5 B(M1) Transitions Involving New States

In Table IV we show M1 transitions between new states - that is states which do not appear in the single j shell of even-even nuclei lighter than 48Cr. We first note that the lowest J=0 T=1 state is at a rather high excitation energy 8.89 MeV. The lowest J=1 T=0 state is 4.81 MeV. There are a few significant B(M1)'s e.g. from J=1T=0 at 4.184 MeV to three J=0 T=1 states at 10.20, 10.53 and 10.78 with B(M1) values 0.235, 0.495 and 0.379.

Table IV: 48Cr B(M1)'s from J=1, T=0 States to J=0, T=1 States (transitions from columns to rows)

| Excitation Energy | 4.8143 | 6.5689 | 7.3052 | 7.975 |
|-------------------|--------|--------|--------|-------|
| 8.8901            | 0.106  | 0.1914 | 0.000748 | 0.01852 |
| 9.044             | 0.003637 | 0.09062 | 0.000575 | 0.1085 |
| 9.4744            | 0.0168 | 0.02171 | 0.004442 | 0.1682 |
| 10.2023           | 0.2356 | 0.004279 | 0.1618 | 1.74E-06 |
| 10.5339           | 0.4952 | 0.03943 | 0.01063 | 0.005871 |
| 10.7829           | 0.3787 | 0.0461 | 0.04746 | 1.4242 |
| 11.1399           | 0.01044 | 0.1266 | 0.02773 | 0.06704 |
| 11.2304           | 0.142 | 0.04477 | 2.45E-07 | 0.01947 |
| 11.7178           | 0.0617 | 0.1182 | 0.01249 | 0.1123 |
| 11.89             | 0.1114 | 0.3067 | 0.01444 | 0.003353 |

6 Closing Remarks: Relations to Other Nuclei

Before closing we wish to make some comparisons other special nuclei in the single j shell model space. In general the wave functions [2] can be written as ΣD(J_p,J_n)(J_p,J_n) where D(J_p,J_n) is the probability amplitude that in a state of total angular momentum I and isospin T the protons couple to J_p and the neutrons to J_n. We first consider other N=Z nuclei such as 44 Ti. We note that the wave functions have the property D(J_n,J_p) = (-1)^(I+T)D(J_p,J_n). Thus, for example we can see visually what states have isospin one.

We next consider states with the same number of neutron holes as protons e.g. 48Ti with 2 protons and 2 neutron holes. It was noted by McCullen, Bayman and Zamick [9] and can be seen visually [2] that D(J_p,J_n) = (-1)^s D(J_n,J_p) where the authors called "s" the signature quantum number. Unlike the case of the N=Z nuclei states of different signature ca have the same isospin.
Finally we come to midshell $^{48}$Cr. We here emphasize the visual aspects. As shown in the appendix one can see a lot of zeros in the wave functions that one does not see in other nuclei. Further analysis leads us to the fact that $(-1)^{(v_n+v_p)/2}$ is a good quantum number. This is all the more remarkable since neither $v_p$ or $v_n$ or $v$ are good quantum numbers.

For future experiments, we mention in ref [10] calculations of the scissors mode excitations in $^{48}$Cr. There should be large M1 excitations of a $1^+$ $T=1$ state in $^{48}$Cr. Since $^{48}$Cr is unstable one cannot directly excite this state with electrons. However, recent experiments involving deexcitations, albeit from a different nucleus from the scissors mode [10] to several branches could be promising as applied to this nucleus.

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Appendix

As an example we show the wave functions of the first 2 $J=0^+ T=0$ states in $^{48}$Cr from Ref [2].

Table V: Selected single j-shell wave functions with the MBZE interaction [2] in $^{48}$Cr.

| Energy (MeV) | $J=0^+$ | $J=0^+$ |
|--------------|---------|---------|
| $J_pJ_n$     |         |         |
| 00           | 0.7494  | 0       |
| 22           | 0.5445  | 0       |
| 22*          | 0       | 0.6738  |
| 2*2          | 0       | 0.6738  |
| 2*2*         | 0.1243  | 0       |
| 44           | 0.1951  | 0       |
| 44*          | 0       | 0.2144  |
| 4*4          | 0       | 0.2144  |
| 4*4*         | 0.2521  | 0       |
| 5*5          | 0.0932  | 0       |
| 66           | 0.1231  | 0       |
| 8*8*         | 0.0393  | 0       |

In Table V the * designates a seniority 4 basis state. The wave functions of MBZE are written as follows:

$$\Psi = \sum D^{\alpha J}(J_p, J_n)[J_p, J_n]^J$$ (1)

The value e.g. 0.5445 is the probability amplitude that in the lowest $J=0^+$ state the 2 protons couple to angular momentum 2 and likewise the 2 neutrons. The lowest $J=0^+$ state has $S=+1$ and the next one has $S=-1$. From ref [2] we see that there are 4 $J=0^+ T=0 S=+1$ states and 2 $J=0^+ T=0 S=-1$ states. This is true for any charge independant interaction. For $J=1^+ T=0$ all states have $S=-1$. 

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