Large eddy simulation of wind turbine wakes using adaptative mesh refinement

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Abstract. The development of turbulent vortical wakes released downstream of wind turbines is a key physical phenomenon as it presents many technological implications for windfarm design and exploitation. The numerical prediction of these wakes constitutes a challenging problem as they involve the shedding of fine vortical structures, their instabilities, and interactions with an ambient turbulent flow. The capture of these complex, three dimensional, unsteady flow phenomena calls for a Large Eddy Simulation (LES) approach. Yet, the computational cost of a scale resolved LES can be huge and the mesh generation process is not obvious when the zones of interest are not known a-priori. Adaptive mesh refinement (AMR) allows generating Eulerian elements only in the regions of interest of the flow, where an action takes place. The AMR strategy proposed here uses the MMG3D library coupled with the YALES2 unstructured finite volume solver. The method is successfully demonstrated on two test cases, the NTNU blind test case for which experimental data exist and the reference NREL 5MW under dynamic yaw conditions.

1. Introduction

The numerical prediction of wind turbine wakes constitutes a challenging problem as these wakes involve the shedding of fine vortical structures, their instabilities, and interactions with an ambient turbulent flow. The capture of these complex, three dimensional, unsteady flow phenomena calls for a Large Eddy Simulation (LES) approach which is computationally expensive. Indeed, the large disparity of spatial scales leads to huge grids. Yet, many flows of interest in external aerodynamics present both rotational and irrotational regions where no relevant action takes place. This led to the development of naturally adaptive numerical methods that use computational elements (e.g. Lagrangian vortex particles, see [1]) only where the flow is rotational. These methods are very efficient to capture wind turbine wakes very accurately, yet they may prove to be inadequate to handle complex boundaries. In an Eulerian framework, AMR allows adapting the resolution dynamically during the computation. This technique features the following advantages compared to a-priori refined static meshes:

- The adaptive approach requires less a-priori knowledge of the solution.
- It requires significantly less meshing human efforts.
• It allows to save computational elements and generate lighter meshes in the zones where there are no turbulent small scales generation. The computation is theoretically thus less expensive. However, generating a new grid periodically and interpolating the solution presents a significant cost.

AMR is widely used in CFD to simulate a broad range of physical phenomena where a local refinement is required such as in shock waves [2], flames front [3], bluff body wakes [4], two phase flows [5].

To the authors’ best knowledge, only scarce literature exists concerning AMR methods applied to wind turbine problems. Most of the literature includes strategies based on cartesian grids where the mesh is locally refined based on the global truncation error solution as in [6, 7, 8]. Kirby et al. [9] propose a mesh composed of two parts where the adaptation is only performed on the cartesian off-body mesh region while the region close to the bodies is fully unstructured. In contrast to these investigations, our strategy relies on the use of fully unstructured conformal tetrahedral LES grids to cope with both geometrical effects and unsteady effects (e.g. terrain, buildings, wind turbine movement...). The adaptation is based on an objective metric computed from physical parameters. When statistically steady problems are considered, the refinement criteria are based on averaged quantities, for instance, here we use a criterion called $Q_{C1}$ based on metric derivatives which is quite similar to what is proposed in Deskos and Piggot [10] within a URANS framework. We also use criteria based on instantaneous quantities (e.g. cell Reynolds number) to simulate truly unsteady problems, for instance here a yawing turbine.

The AMR approach proposed here allows starting from a coarse grid to identify regions where a high resolution is required. This will allow to capture flow features much smaller than the overall scale of the problem providing adequate higher spatial and temporal resolution where needed.

The methodology is assessed on two test cases: one with a complex geometry and one with an unsteady yawing rotor. It is demonstrated that AMR allows to solve these problems on optimally refined meshes.

2. Methodology

The YALES2 [11] flow solver is used here. It is a massively-parallel finite-volume solver, which is specifically tailored for Large-Eddy Simulation, and relies on a central numerical scheme for spatial discretization that is 4th order on cartesian grids. This order is not formally maintained on unstructured grids considered in this study. However, a similar accuracy has been observed in [12] on the results obtained with structured and unstructured grids with this applied in the context of wind turbine flows. The solver also performs better compared to classical finite volume schemes.

A 4th-order Runge-Kutta method is used for the time integration. This code solves the Navier–Stokes equations for incompressible turbulent flows on both structured and unstructured meshes. As a consequence, it allows to simulate complex geometrical configurations. It also features an actuator line model [13] as described and validated in [12]. The sub-grid scale model used here is that of [14] which allows to avoid subgrid scale dissipation in the vortex cores. The mesh adaptation is performed using the MMG [15, 16] remeshing library which is embedded in YALES2.

This adaptive mesh refinement method can be used for two different purposes. First, to construct an adequate mesh in order to perform a well resolved LES on a light mesh. As showed in [16], two different criteria can be used to determine the quality of the mesh. A first criterion $Q_{C1}$ that ensures the correct discretization of the mean flow gradients

$$Q_{C1} = h^2 \max_{i,j=1,2,3} \left\{ \left| \frac{\partial^2 u_j^*}{\partial x_i^2} \right| \right\}$$

(1)
It represents the upper bound of the interpolation error for the velocity field in a discrete space with a size mesh $h$. A second criterion $Qc_2$ is defined as

$$Qc_2 = \frac{E_{sgs}}{E_{sgs} + E_R} \leq 0.2.$$  

(2)

This criterion defined by [17] ensure that the mesh is fine enough to capture 80% of the turbulent kinetic energy. The adaptation strategy followed in this study can be summarized as:

(i) Provide a mesh with a sufficient resolution to capture the geometric details of the case and the initial flow condition.

(ii) Run the simulation with the initial mesh and gather statistics over a through flow.

(iii) Adapt the mesh using MMG to insure a homogeneous interpolation error by applying a constant value of $Qc_1$ over the full computational domain (the target value of $Qc_1$ is determined to control the total number of cells), and to enforce $Qc_2 < 0.2$

(iv) Run the simulation and gather statistics over a through flow and compare to those obtained before adaptation.

(v) If converged, keep the adapted mesh and if not increase the number of cells, reset the statistics and go back to step 3.

This adaptive mesh refinement method allows to progressively refine the mesh and end up with an optimized mesh based on objective parameters. The advantage of this method is to limit the number of adaptation steps which can highly degrade the computing performances of the code. However, this methodology can only be applied when the wake reaches a statistical convergence (i.e. for a non moving rotor). Indeed, $Qc_1$ and $Qc_2$ are evaluated from time averaged velocity field. In order to handle moving conditions (e.g. yaw), a criterion based on an instantaneous quantity has to be defined. It is proposed here to consider a dimensionless criterion which is here based on the vorticity cell Reynolds number, $Re_\omega^h = \omega h^2/\nu$, where $h$ is the local mesh size. This strategy allows to refine the grid only where an action is taking place, i.e. where vorticity is generated. The region where adaptation is required is determined based on a given vorticity cell Reynolds number. The size of the mesh cells are then modified using the target vorticity cell Reynolds number.

3. NTNU blind test

3.1. Case description
The test case investigated here was described in detail in the report by Krogstad et al. [18]. This report provides geometrical parameters, wind tunnel inlet condition and CAD files for the blades and the nacelle. The main focus of the proposed blind test was on the wake development behind the turbine. The wind turbine model considered here consists in a three bladed rotor ($D = 2R = 0.894 \text{ m}$) connected to a nacelle and a tower placed in a wind tunnel with $U_\infty = 10.0 \text{ m/s}$. The blade profile is a NREL S826 airfoil. The dimensions of the wind tunnel are such that $L \times W \times H = 13.42 \times 3.35 \times 2.24 \text{ D}$. The chord length $c(r)$ and pitch angle distribution $\beta(r)$ are given in [12]. A sketch of the flow at an early time is displayed in Fig. 3. As the Reynolds number is high, and the walls of the wind tunnel are far from the zone of interest, a slip wall boundary condition is applied to the wind tunnel walls while a log-law wall model is applied on the mast and the nacelle.

3.2. Results
Fig. 1 displays the different meshes and the turbulent wakes captured on each mesh obtained with the refinement sequence based on criterion $Qc_1$ and $Qc_2$, as described in previous section.
Figure 1. Meshes (vertical cut) obtained using the $Q_{c1}$ and $Q_{c2}$ refinement criteria (left). The finest mesh includes $27.2 \times 10^6$ cells. The turbulent wakes captured on each mesh are also displayed using an iso-contour of the $Q$ criterion of [19] in the Right part.

Figure 2. Mesh (vertical cut) obtained at the last stage of the refinement process using the $Re_{h}$ criterion. The finest mesh includes $10.5 \times 10^6$ cells.

The starting mesh includes $1.2 \times 10^6$ cells, the others include respectively $2.9 \times 10^6$, $6.3 \times 10^6$, $13.1 \times 10^6$ and $27.2 \times 10^6$ cells.

The meshes are obtained using successive adaptations. For each adaptation, the new mesh is built based on (i) The target value of $Q_{c1}$ applied over the computational domain is automatically computed such that the element number is roughly doubled in comparison with the previous mesh, and (ii) the constraint that $Q_{c2} < 0.2$ (see [16] for the details).

One observes that the cell density increases progressively in the rotor plane, in the cylindrical zone including vortical wake structures emanating from blade tips as well as in the wake of the
The first mesh doesn’t allow to capture the blade tip vortical structures, while the successive refinement levels lead to a mesh that is able to capture a broad range of turbulence scales ranging from the helical vortical structures produced by the blade tip to the fine scale structures in the wake of the mast and the nacelle as well as in the downstream region of the wake. These structure have a significant impact on the results as demonstrated in [12]. Once a converged mesh is obtained, the computations are relaunched to gather turbulence statistics without the dynamic adaptation. This technique avoid to dynamically adapt the mesh in the wake region while this part of the mesh is statistically stable. The computational cost is thus reduced as the mesh adaptation steps are highly time consuming. However, the dynamic adaptation method has also been applied here for the sake of comparison. The target vorticity cell Reynolds is fixed at \( Re^h_\omega = 200 \). Fig. 2 shows the mesh obtained with the dynamic strategy. The simulation was stopped after the refinement zone has reached few diameters due to the high computational cost. Turbulence statistics were then collected on this mesh without the dynamic adaptation. As the refinement zone extends up to \( x \approx 3D \), the results for \( x > 3D \) can be considered as the product of a very large eddy simulation (VLES).

When the adaptation is triggered, 96% of the wall clock time of the time step is dedicated to mesh adaptation. The remaining 4% are used for the flow computation on the newly generated mesh. This phase is computationally intensive but replaces the generation of a static a-priori refined mesh by the user. For this case, the grid obtained after this refinement step is displayed in Fig. 2. The flow statistics are then gathered on this new mesh without adaptation at a much lower computational cost. Indeed, the mesh obtained is usually much lighter compared to an a-priori refined static mesh (e.g. 10M cells vs. 114M cells).

Fig. 3, displays the topology of the wake at an early time. It shows that the vortex tubes are fairly well captured over a distance of \( \approx 2D \). One also observe that the method allows to capture large scale vortical structures emanating from the mast.

The results obtained with the AMR framework are compared with a static refined mesh
solution including approximately 114M of cells. The velocity profiles (see Fig. 4) taken at different stations in the streamwise direction are consistent with those of the experiment. The two different mesh refinement strategies also provide results in good agreement with each other. The results are also consistent with those provided in [12]. Compared to the static refined mesh solution, the AMR results seem to be better in the blade tip vortex region, indicating that the tip vortex cores are better resolved. However, the results in the wake center region are less convincing. The TKE profiles (see Fig. 5) exhibit a shape that is consistent with that of the experimental results. The TKE is underestimated in the vicinity of the peaks but the values are higher compared to those obtained with the static refined mesh for \(x/D = 1\). For \(x/D = 3\) and \(x/D = 5\), the values are lower than for static refined mesh but are still consistent. The improvements observed in the tip blade vortex area for adapted meshes can be explained considering the size of cells. Indeed, even if the static mesh includes four times more cells, the cell size in this region is finer for the optimized mesh based on \(Q_{c1}\) and \(Q_{c2}\) \((h/D = 0.01\) vs. \(h/D = 0.17\)). In contrast, the cell size in the center of the wake is twice coarser compared to the static mesh. The \(Q_{c1}\) and \(Q_{c2}\) criteria could be adapted to obtain a better resolution in this region, yet it is demonstrated here that a reasonable agreement with the reference results can be obtained at a lower computational cost.

4. NREL-5MW
The relevance of the proposed approach is also demonstrated on a challenging test case involving a rotor under dynamic yaw conditions. This test case is not representative of the nominal operation of a real yaw controlled wind turbine. However, it can model the case for which the wind is changing its direction with a faulty controller or the case for which a turbine is considered as a flow agent and deviate itself its wake so that the following turbine encounters...
lower dynamic loads. Recall that the aim pursued here is to prove that the mesh will adapt itself when the wake is moving. To this end, this test case can be considered as challenging as one cannot build an optimal mesh that will stay optimal throughout the simulation. A mesh which is homogeneously refined in a zone bounding the whole wake movement, would indeed represent a huge computational cost.

4.1. Case description
The simulations are performed for an isolated NREL offshore 5-MW baseline wind turbine [20] operating at its optimum tip-speed ratio \((T SR = \lambda = \frac{\omega R}{U \infty} = 7.55)\). The rotor blades are modeled actuator lines (see [12]). The spatial resolution is such that 32 actuator elements are used to model each blade. The computational domain initially extends over \(10D \times 6D \times 6D\). The rotor region is refined based on the spatial resolution of the blades (cells of around 2 m) and cells of 50 m are generated everywhere else. It leads to an initial grid with approximately \(8.4 \times 10^5\) nodes. The wind turbine rotor is located 3\(D\) after the inlet. The time step is adjusted so that the maximum CFL < 0.75 throughout the whole domain. The initial yaw angle is equal to 0\(^o\) and decreases from 0.5 degree per second during the simulation. As in this case, the rotor is moving, one cannot define a steady statistic state. As a consequence it is not possible to use the previous methodology relying on statistical convergence of the wake topology and the mesh. For this reason, the vorticity based cell Reynolds number \(Re^h\) was used to refine the grid dynamically. \(Re^h\) was defined to ensure at least three points per vortex core size in the blade tip vortex.
4.2. Results

The topology of this flow is displayed in Fig. 6 at a time for which the rotor has already turned of a significant amount. The tip vortex spirals are well captured in the vicinity of the rotor. The vortex merging is enhanced since the wake is moving. The wake dynamics is thus different compared to a non moving rotor. It is thus not obvious that the vortex tubes will stay separated and transported unaltered over a long distance as in a non yawing case. Fig. 7 suggest that the helical vortex tubes merge sooner compared to a static case forming an axi-symmetric cylindrical vorticity sheet that is destabilized due to non linear interactions and trigger a Kelvin Helmholtz instability that is well captured thanks to the refinement strategy. The hub vortex is also well captured. The region where the root and tip helical vortical structures are refined while the rest of the mesh is coarsened. It is worth to note that the mesh refinement closely follows both the vortical structures and the global wake movement as observed at a later time (see Fig. 7 at $t^* = 11.5$).

The solution obtained with the AMR framework has been compared to an a-priori refined mesh solution (see Fig. 8). The static refined region of this mesh has a spatial resolution of around 2 m equivalent. The number of cells in the a-priori refined mesh is three times that of the dynamically adapted mesh. Yet, similar results are obtained for the dynamic of the wind turbine flow and induction factor maps. However, the time to solution with adaptation is doubled compared to the static case. Indeed, the initial mesh considered for the adaptation is very crude with low quality elements, asking a lot of work for the adaptation algorithm. The adaptation frequency could also be further investigated to optimize the computational cost.

5. Conclusions and further work

This contribution demonstrates the feasibility of using AMR strategy to simulate wind turbine wakes. Two different strategies have been investigated. The criteria used for the mesh adaptation are either computed from averaged (for $Qc_1$ and $Qc_2$) or instantaneous quantities (for $Re_h$). The first technique features an optimal computational cost as the mesh is only refined in the region where the flow is faster. The second technique is more computationally expensive but provides finer resolution in regions of interest. Further work could include the use of adaptive time stepping to improve the efficiency of the adaptation process.
Figure 7. Instantaneous 2D vorticity contours of the wake generated by the NREL yawing rotor. The grid cells are superimposed to show the accurate following of the vortical structures by the AMR scheme. The right part of the figure focuses on the zone where the flow is rotational.

Figure 8. Instantaneous \(t^* = t U_\infty / D = 8.25\) visualization of the 2D vorticity contours (left) and the induction factor (right) of the wake generated by the NREL yawing rotor using an a-priori static mesh (top) and the AMR scheme (bottom)
interest and one takes benefit of the steadiness of the flow statistics to gather the turbulence statistics on a converged mesh. The second technique proposed here is computationally expensive and should only be considered when the operating conditions are unsteady. The case considered here is a yawing NREL rotor. It is showed that the mesh is refined in regions that closely follows the vortical structures allowing to properly capture the wake dynamics at the expense of a significant amount of computational time spent in the remeshing procedure. Additional work will consist in investigating the AMR strategy to adopt in order to include turbulent inflow modeling (e.g. with a precursor Mann box [21], [22]). This will allow a more accurate representation of atmospheric turbulence.

A smart refinement strategy has to be defined as the flow will include turbulent structures in the whole domain which can induce a huge number of elements if the refinement criterion is not appropriate. In this framework, an implicit “masking strategy” will be used based on a vorticity threshold. The adaptation will only trigger in regions of high vorticity induced by wind turbine structures.

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