An Equivalent Magnetic Circuit Model of PMSM Demagnetization Fault

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Abstract. The PMSM (permanent magnet synchronous motor) demagnetization fault simulation model and the equivalent magnetic circuit model under normal operation were established, and the calculation method of each parameter on the equivalent magnetic circuit model was analyzed. Then the demagnetization factor that represents the demagnetization severity was introduced, and the calculation method of parameter under the fault condition of the PMSM was deduced, and the air gap flux densities of the PMSM under different working conditions were calculated. At last, the validity of this model was verified by comparing the results of the partial demagnetization simulation and the equivalent magnetic circuit model.

1. Introduction

PMSM electromagnetic field research can take advantage of the equivalent magnet circuit circuit model of lumped parameter. In the literature [1], the different parts of the air gap magnetic flux density of PMSM are replaced by different reluctance. The permanent magnet is equivalent to a parallel model of constant magnetic flux source and reluctance. The magnetic field quantity represented by reluctance coefficient and magnetic flux leakage coefficient is deduced; literature [2] deduces the equivalent magnetic circuit model of interior permanent magnet synchronous motor in multilayer U-shaped structure and verifies its validity through model simulation; literature [3] analyzes the analytical expression of Lorentz force by the coil of permanent magnet brushless DC motor. Literature [4] effectively designs interior permanent magnet synchronous wind driven motor based on equivalent presence network. Literature [5] takes advantage of equivalent magnetic circuit model to determine the leakage flux coefficient and quiescent operation point of permanent magnet synchronous motor which provides effective data for the design of permanent magnet synchronous motor. For the research on equivalent magnetic circuit model, the only literature [6] which relates the equivalent magnetic circuit model of PMSM demagnetization fault just points out the adoption of equivalent magnetic circuit method in design and focuses on the analysis of simulation performance of local demagnetization fault. Based on this, based on the existing results, this paper introduces the parameter characterizing the demagnetization fault, and corrects the equivalent magnetic circuit model.
of the PMSM in normal operation, and obtains the equivalent magnetic circuit model after the demagnetization fault.

2. The Parameters of the PMSM and the method of the demagnetization fault simulation

The simulation PMSM is 270WS002 provided by Shanghai Automobile Co., Ltd which adopts the typical short-chord winding with the pitch of 5 and the pole pitch of 6 in order that greatly weakens the fifth and seventh harmonic wave, improves the waveform of electromotive force, shortens end connection and saves copper. The main parameters of the PMSM are shown in Table 1. There are 8 magnetic poles, the magnet distributes is a built-in V. Stator winding is a double-layer winding structure, power source is a frequency changer and stator slot is a trapezoidal slot. The structural model of motor is shown in Fig. 1.

Table 1. Parameter of integer slot PMSM

| Main parameter         | Numerical value | Unit |
|------------------------|-----------------|------|
| Rated power            | 42              | kW   |
| Rated speed            | 4000            | r/min|
| Rated efficiency       | 97.1            | %    |
| Rated frequency        | 200             | Hz   |
| Rated torque           | 100             | N·m  |
| Rated voltage          | 380             | V    |
| Permanent magnet material | Ne-Fe-B      |
| Pair of poles number   | 4               |
| Number of stator slots | 48              |

![Figure 1. Simulation model of the PMSM for electric vehicles](image)

The parts in red circle in Fig. 1 are the magnetic poles of PMSM which is demagnetized. Normal, 25% demagnetization, 50% demagnetization and 75% demagnetization fault can be simulated by changing parameters $H_s$ and $B_r$ of permanent magnet material on Ansoft software platform.

3. Equivalent magnetic circuit model of the PMSM

The PMSM normal operation magnetic circuit passes by permanent magnet, rotor yoke, air gap, stator tooth and yoke. Considering the influences of magnetic bridge magnetic flux leakage among magnetic pole, magnetic end and magnet, the equivalent magnetic circuit model of PMSM is obtained according to Ohm law of magnetic circuit [2], as shown in Fig. 2.
Where, $R_g$ is air gap reluctance; $\Phi_g$ is the magnetic flux corresponding to $R_g$; $R_s$ is the rotor core reluctance; $R_r$ is the stator core reluctance; $R_{mo}$ is the internal reluctance in magnetic flux source; $\Phi_{mo}$ is the magnetic flux corresponding to $R_{mo}$; $R_{1m}$ is the end leakage reluctance of the permanent magnet; $R_{2m}$ is the leakage reluctance between two adjacent permanent magnets; $R_{mb}$ is the magnetic bridge reluctance between two adjacent magnetic poles; $\Phi_r$ is the equivalent magnetic flux source of permanent magnet. In equivalent magnetic circuit model, it is assumed that silicon steel sheet is in unsaturated state and the magnetic field strength produced by armature current in stator winding is ignorable. Based on the above assumptions, reluctance of iron core of stator $s_R$ and reluctance of rotor $r_R$ can be ignored. Therefore, Fig. 2 can be reduced to Fig. 3 to obtain the simplified magnetic circuit model which provides convenience for the calculation of density of air gap flux.

\[ g = r_4 - r_1 \]

Where, $r_1$ is the outer radius of the rotor; $r_4$ is the inner radius of the stator.

The stator of the PMSM is an open slot, actual air gap is nonuniform because of the slot effect influence, and the equivalent length of air gap can be calculated according to formula (2).

\[ g_e = g - \frac{\omega_b + \omega_f}{\omega_b + \omega_f - K_ng} \]
Where, $\omega_0$ represents the width of stator slot, $\omega_f$ represents the span of two adjacent stator slots on the stator surface and $K_s$ represents Carter’s coefficient subject to the influences of length of air gap, width of stator slot and other factors. Carter’s coefficient in the paper adopts the method in literature [8] with the calculation method as follows:

$$K_s = \frac{4}{\pi} \left[ \frac{\omega_0}{2g} \tan^{-1} \left( \frac{\omega_0}{2g} \right) - \ln \left( \frac{\sqrt{\omega_0^2 + (2g)^2}}{2g} \right) \right]$$

(3)

Then the air gap area is obtained as follows:

$$A_g = \alpha_p \frac{2\pi(r_e - \frac{g}{2})}{N_p} L$$

(4)

Where, $L$ is length of the linear part of the winding of the PMSM (the length of the iron core); $\alpha_p$ is the polar arc coefficient.

Without regard to cogging effect, PMSM gap flux density waveform under no-load condition can be equalized to rectangular wave. Pole-arc coefficient $\alpha_p$ is a coefficient used to determine the largest magnetic field density under each pole with its physical significance shown in Fig. 4 and it is the distribution of radial component of air gap flux density in polar distance $\tau$ [9].

![Figure 4](image)

**Figure 4.** The distribution of the radial component of the air gap flux density in a polar distance

For the convenience of calculation, the radial component of air gap distributing unequally along periphery is equivalent to equally distributed rectangular wave with the height of $B_h$ and the width of $\alpha_p \tau$. According to the principle of same magnetic flux after and before conversion, then

$$\alpha_p B_h \tau = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} B_h(x) dx$$

(5)

Thus the polar arc coefficient can be represented as: It can be seen in Fig. 4, the calculation of pole arc coefficient is in order to simplify the magnetic circuit calculation. The equivalent air gap magnetic field along the circumference of the uneven distribution is the rectangular wave of uniform distribution, and evenly distribute in the calculation of pole arc length in the range of $b_p$. Its size is equal to the maximum value of magnetic flux density.
5

\[ \alpha_p = \frac{1}{\tau} \int_{a_p}^{b_p} B_\delta(x) \, dx \]

(6)

In this paper, the calculation of pole arc coefficient is the ratio of the calculation of pole arc length \( b_p \) and polar distance \( \tau \) can be represented as:

\[ \alpha_p = \alpha_i = \frac{b_p}{\tau} \]

(7)

The different shapes of magnetic pole and structures of magnetic circuit of PMSM lead to the different influencing factors on waveform of air gap field and calculation pole-arc coefficient of motor. Calculation pole arc and the ratio between length of air gap and polar distance are the main factors determining calculation pole-arc coefficient for the PMSM.

4.2. Calculation of reluctance of each part

The magnetic field line distribution at the end portion of the permanent magnet, between the adjacent permanent magnets and between the flux barriers was shown in Fig. 5.

(a) (b) (c)

Figure 5. Distribution diagram of magnetic field line in each part of permanent magnet

(a) End of permanent magnet; (b) Between two adjacent magnetic poles; (c) Between magnetic bridges

The reluctance and air gap reluctance corresponding to the magnetic field line distribution can be calculated from the distribution diagram, as shown in formula (8) ~ (11).

\[ R_{mm} = \frac{h_{1m1}}{\mu_0 \mu_r A_m} = \frac{h_{2m1}}{\mu_0 \mu_r w_{m1} L} \]

(8)

\[ R_{ml} = \frac{h_{1ml}}{\mu_0 w_{ml} L} \]

(9)

\[ R_{2ml} = \frac{h_{2ml}}{\mu_0 w_{2m} L} \]

(10)
\[ R_e = \frac{\rho_e}{\mu_0 A_e} \]  

Where, \( h_{l1}, h_{l2}, h_{m1}, h_{m2} \) and \( w_{h1}, w_{h2} \) are corresponding to the length and width of the magnet, the end of the magnet, and the adjacent two magnets in the magnetizing direction, \( \mu_0 \) is the permeability of vacuum, and \( \mu_r \) is the relative permeability of the magnet. The magnetic field line distribution of the flux barrier between the magnetic poles is shown as (Fig. 5c). \( \Lambda_{mb} \) represents the magnitude of the permeance. To calculate its value, the arc straight-line permeance model method is setup the air gap flux flow model, as shown in Fig. 6.

**Figure 6.** Arc straight-line permeance model

The margin magnetic permeance \( \Lambda_{mb} \) is the differential of the width of the magnetic permeance, then the sum of the infinity is calculated, each length is \( w_{mb} + \pi x \), and the calculation equation is therefore obtained, as shown in formula (12).

\[ \Lambda_{mb} = \sum \frac{\mu_0 L}{w_{mb} + \pi x} dx \]  

Convert the differentiation into integration for calculation, as shown in formula (13).

\[ \Lambda_{mb} = \int_0^{\infty} \frac{\mu_0 L}{w_{mb} + \pi x} dx = \frac{\mu_0 L}{\pi} \ln\left(1 + \frac{\pi g_{mb}}{w_{mb}}\right) \]  

Due to the inverse relationship between magnetic permeance and reluctance, as shown in formula.

\[ R_{mb} = \frac{1}{\Lambda_{mb}} \]  

The reluctance of the flux barrier can be obtained by solving (13) and (14).

4.3 Calculation of air gap flux density

The equivalent magnetic flux source of permanent magnet was calculated as shown in formula (15).

\[ \Phi_i = B_r A_n = B_r w_{fl1} L \]  

Where \( b \) is the width between the adjacent magnetic poles; \( B_r \) is the remanence of the magnet whose magnitude is determined by the size of the permanent magnet material.

In Fig. 3, according to the Kirchhoff magnetic circuit law, the air gap flux is:
\[
\Phi_e = \frac{\Phi_r / R_s}{1/R_{m0} + 2/R_{ab} + 2/R_{1nl} + 2/R_{2nl} + 1/R_s} \quad (16)
\]

Therefore, the air gap flux density:
\[
B_e = \frac{\Phi_e}{A_e} = \frac{\Phi_r / R_s}{(1/R_{m0} + 2/R_{ab} + 2/R_{1nl} + 2/R_{2nl} + 1/R_s)A_e} \quad (17)
\]

5. The PMSM parameters calculation state under the demagnetization fault condition

For the magnetic poles that were demagnetized, their magnetic flux source and magnetic flux source internal impedance from the equivalent magnetic circuit modeling parameters will change. \( K \) represents the demagnetization ratio factor, which is the percentage of the part that can’t return to the original value divided by the original value, thus:
\[
\mu_r = K\mu_r, B'_r = KB_r \quad (18)
\]

The total equivalent flux density is the sum of the normal part of magnetic pole and the demagnetized part of magnetic pole, when the permanent magnet is demagnetized, the remanence density of the normal part is \( B_r \), while the remanence density of the demagnetized part changes to \( K B_r \), the equivalent remanence density of which is:
\[
B'_r = \frac{3}{4} B_r + K \frac{1}{4} B_r = \frac{3 + K}{4} B_r = \lambda B_r \quad (19)
\]

Where, \( \lambda = \frac{3 + K}{4} \).

In the simplified magnetic circuit model shown in Fig. 3, the equivalent resistance of the permanent magnet is \( 4R_{m0} \), which means the reluctance of each pole under normal circumstances is \( R_{m0} \). Therefore, when the magnetic circuit parameters were changed, the internal resistance of the magnetic flux source of demagnetized pole is:
\[
R'_{m0} = \frac{h_{ml}}{\mu_0 \mu_r w_{ml} L} = \frac{R_{m0}}{K} \quad (20)
\]

Combined with the equivalent flux density formula (19), the equivalent internal resistance after demagnetization fault is:
\[
4R'_{m0} = 3R_{m0} + \frac{R_{m0}}{K} \quad (21)
\]

And then it can be deduced:
\[
R'_{m0} = \frac{3K + 1}{4K} R_{m0} = \alpha R_{m0} \quad (22)
\]

Where, \( \alpha = \frac{3K + 1}{4K} \).
According to the parallel principle of magnetic circuit, parameter $\gamma$ was introduced to simplify the calculation, its value is:

$$\gamma = \frac{1/R_g}{1/R_{mo} + 2/R_{mb} + 2/R_{md} + 2/R_{md} + 1/R_g} \quad (23)$$

The equivalent magnetic flux source after the demagnetization fault is obtained with formula (15):

$$\Phi' = B_r A_{r1} L = B_r w_{r1} L = \frac{3 + K}{4} \Phi_r = \lambda \Phi_r \quad (24)$$

The air gap flux density is obtained with formula (17), (23) ~ (24):

$$B_g' = \frac{\Phi_g'}{A_g} = \lambda \gamma B_g \quad (25)$$

On the basis of the above, the parameters of the PMSM are calculated under different operating conditions, the results are shown in Table2.

| Parameter | 100% demagnetization | 75% demagnetization | 50% demagnetization | 25% demagnetization | Normal |
|-----------|----------------------|---------------------|---------------------|---------------------|--------|
| $\alpha$  | $\infty$             | 1.75                | 1.25                | 1.083               | 1      |
| $\lambda$ | 0.75                 | 0.8125              | 0.875               | 0.9375              | 1      |
| $\gamma$  | 0.978                | 0.9175              | 0.8952              | 0.8836              | 0.86   |
| $R_{mo}$ (k$\Omega$) | $\infty$ | 1905.75             | 1361.25             | 1179.4             | 1089   |
| $B_r$ (T)  | 0.9225               | 0.999               | 1.076               | 1.153               | 1.23   |
| $\Phi_r$($\times 10^{-3}$Wb) | 2.94           | 3.185               | 3.43                | 3.675               | 3.92   |
| $B_g$ (T)  | 0.3726               | 0.3787              | 0.4079              | 0.4318              | 0.5182 |

The data in Table.3 shows that:

1. After the demagnetization fault, the internal resistance of the flux source of the demagnetized part is increased from the initial 1089 k$\Omega$ all the way to a large value ($K$ is close to 0 at this point), thus, when the complete demagnetization of the poles, the equivalent magnetic circuit of this branch is equal to an open circuit status.

2. According to the formula of magnetic flux source, the magnitude of flux source is gradually decreasing by $\lambda$ times after the demagnetization fault.

3. The calculated air gap flux density presented a decreasing trend, the air gap flux density of 75% demagnetization and 100% demagnetization didn’t have much difference, the simulation results also verified that this phenomenon, which is probably because the field-loss was too severe in these two conditions, and the overall air gap flux density had a high distortion.

6. Comparison of calculation results and simulation results

After performing the no-load simulation to the partial demagnetization fault model built up in Fig. 1 and carrying out the FFT analyses to the air gap flux density, the fundamental component was obtained, then the calculated magnetic circuit simulation results were compared as shown in Fig. 7.
Figure 7. Comparison of air gap magnetic density simulation results and calculation results at various operation conditions of the PMSM

According to Fig. 7, the simulation results is in good with the calculated results of the magnetic circuit model, the simulation results were a little bit higher, but the max error was within 5% range, which is allowed in the engineering scope, the error decreased along with the increasing of demagnetization extent, the causes of error may come from the following aspects:

1. In the equivalent magnetic circuit model, the unsaturated silicon steel is assumed, and the magnetic field generated by the armature current passing through the stator winding was neglected, but in fact, the stator current will produce a magnetic field and affect the magnetic field of the rotor, which is the so-called armature reaction.

2. When the magnetic bridge reluctance between two adjacent magnetic poles was calculated, the solving concept of surface-mount type of permanent magnet structure was used, however in fact, the magnetic leakage of built-in permanent magnet is relatively high, the effect of the flux leakage of flux-insulation material on this part of reluctance was neglected and the effect of air gap is considered.

3. After the demagnetization fault, the internal impedance of the flux source in the equivalent magnetic circuit model overlay the demagnetized part and the normal part of the equivalent permanent magnet, while the simulation analysis was considered as a whole, these two methods have different principles, which resulted in certain error.

4. In the equivalent magnetic circuit model, the reluctance of the stator and rotor core was neglected to simplify the calculation, while the simulation model in which some factors were neglected was a relatively idealized model, but the physical and geometrical characteristics of the PMSM were taked into account.

7. Conclusion
The equivalent magnetic circuit model of the demagnetization fault of PMSM was presented in this paper, and the calculation methods of each parameter in the model were analyzed specifically. Then by using the nature of the magnetic circuit, the formula of the air gap flux density of the PMSM after the demagnetization fault was obtained, and the fundamental component of the air gap flux density of PMSM was calculated under different degrees of partial demagnetization. The calculated results and the simulation results of the equivalent magnetic circuit model of air gap flux density after the demagnetization fault were compared, the results verified that within the error of 5% of the engineering permit, the model is correct and then the causes of the errors were analyzed.

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