Inference of weak nuclear collective from atomic masses

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We explore weakly-collective singly-closed shell nuclei with high-j shells where active valence neutrons and particle-particle correlations may be the dominant collective degree of freedom. The combination of large and close-lying proton and neutron pairing gaps extracted from experimental masses seems to characterize the origin of the weak collective observed in Ni and Sn superfluids with \( N \approx Z \). The trend of \( E2 \) transition strengths, i.e., \( B(E2; 2^-_1 \rightarrow 0^+_1) \) values, in these nuclei is predicted from proton and neutron pairing-gap information. The agreement with the Ni isotopes is excellent and recent experimental results support the trend in the Sn isotopes. This work emphasizes the importance of atomic masses in elucidating nuclear-structure properties. In particular, it indicates that many-body microscopic properties such as nuclear collectivity could be directly inferred from more macroscopic average properties such as atomic masses.

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Nuclear collectivity is controlled by the interplay of particle-hole (\( ph \)) and particle-particle (\( pp \)) excitations. Particle-hole correlations produce deformation through the proton-neutron (\( pn \)) interaction and give rise to nuclear rotations [1]. In this work, we search for weakly-collective low-lying structures in \( N \approx Z \) nuclei where \( pp \) correlations may, a priori, be the dominant degree of freedom. Large separation energies between single-particle orbits due to the spin-orbit interaction [3], together with the attractive short-range pairing interaction [4], should lead to specially stable and spherical nuclei. Singly-closed shell Ni (\( Z = 28 \)) and Sn (\( Z = 50 \)) isotopes with \( N \gtrsim 28 \) and \( N \gtrsim 50 \) are characterized by weakly-collective reduced transition probabilities, i.e., \( B(E2; 2^-_1 \rightarrow 0^+_1) \) values. Moreover, small quadrupole moments of \( Q_S(2^-_1) \approx 0.05 \) eb have been determined for \( ^{60}Ni \) and \( ^{112}Sn \) from reorientation-effect measurements [5]. Quadrupole collectivity cannot solely arise from valence neutrons and proton-core excitations are needed to account for such weakly-collective systems. Proton-core excitations are supported by the positive quadrupole factors of \( 2^-_1 \) states in the even-mass \( ^{58−64}Ni \) isotopes [6] and the enhancement of \( B(E2; 2^-_1 \rightarrow 0^+_1) \) values observed in the neutron-deficient Sn isotopes as the \( N = 50 \) shell closure is approached [7,8].

The latter unveils one of the major conflicts encountered by the nuclear shell model (\( SM \)). Plainly, large-scale \( SM \) calculations predict an inverse parabolic trend of \( B(E2; 2^-_1 \rightarrow 0^+_1) \) values peaking at midshell and cannot reproduce the enhancement of \( E2 \) strengths determined in the \( ^{106−112}Sn \) isotopes using \( ^{88}Sr, ^{90}Zr \) or \( ^{100}Sn \) cores [7,8]. The former cores provide better results and support proton-core excitations. A similarly baffling scenario has recently been revealed by Jungclaus and collaborators at midshell of the tin isotopic chain [13].

High-statistics Coulomb-excitation measurements in inverse kinematics and fits to lineshapes have provided very accurate lifetimes for the \( 2^-_1 \) states in the \( ^{112,114,116}Sn \) isotopes. Longer lifetimes from the accepted values in the nuclear data evaluation [14] yield \( B(E2; 2^-_1 \rightarrow 0^+_1) \) values which clearly deviate from the inverse parabolic trend at midshell and, instead, propose a conspicuous minimum at \( ^{114}Sn \); in agreement with \( N = 66 \) being a subshell closure.

![FIG. 1. (Color online) Two-neutron separation energies for the \( ^{102−117}Sn \) isotopes. Small deviations from the smooth trend arise at \( ^{110}Sn \) and \( ^{115}Sn \). A cubic-spline interpolation has been used for visual purposes.](http://www.107.256.82.26:8080/au/arXiv:1007.2658v2)
mation at \( N = 60 \) for \(^{100}\text{Zr} \) and \(^{98}\text{Sr} \), as compared with much weaker deformations in more neutron-deficient isotopes, is characterized by a sudden drop in the binding energies. Shape coexistence has been observed and strong rotational bands built on the ground state and low-lying \( 0^+_2 \) excitations in \(^{100}\text{Zr} \) and \(^{98}\text{Sr} \). Further high-precision mass measurements of neutron-rich Sn isotopes advocates for a restoration of the \( N = 82 \) shell closure \(^{20, 21}\). Figure \( 1 \) shows \( S_{2N} \) values from \(^{102}\text{Sn} \) to \(^{117}\text{Sn} \). Small deviations from the smooth trend at \(^{110}\text{Sn} \) and \(^{115}\text{Sn} \) may suggest structural changes. The latter points at the \( N = 66 \) subshell gap as supported by the minimum in the collective trend at \(^{118}\text{Sn} \). The origin of the small deviation at \(^{110}\text{Sn} \) is more obscure. This work attempts at elucidating whether weakly-collective \( B(E2; 2_1^+ \rightarrow 0_1^+) \) values in the Ni and Sn isotopes and atomic masses are related in a comprehensive manner.

Within the BCS pairing model \(^{13, 22–24}\), the \( 2_1^+ \) excitation is created by breaking one Cooper pair, and is interpreted as a two quasiparticle which lies at least twice the pairing energy, \( 2\Delta \). For \( N \approx Z \) nuclei, proton and neutron Fermi surfaces lie close to each other, henceforth we assume that the interplay of both proton and neutron pairing gaps may contribute to the overall oscillation of the Fermi surface and the collective origin of the \( 2_1^+ \) state \(^{22, 23–24}\). Intuitively, we introduce the relative pairing gap, \( \Delta_r \), defined by,

\[
\Delta_r^2 \equiv |(\Delta_p - \Delta_n)(\Delta_p + \Delta_n)| = |\Delta_p^2 - \Delta_n^2|,
\]

where the first term \( (\Delta_p - \Delta_n) \) is the resonant factor, which accounts for the proximity of proton and neutron pairing-gaps. That is, the smaller the energy difference between both pairing gaps, the larger the overlap of proton and neutron pairing fields. The second term \( (\Delta_p + \Delta_n) \) is the energy factor, and accounts for the energy that can be provided to the nuclear system before breaking Cooper pairs, i.e., a quantity that enhances the possibility of having spherical nuclei, where vibrations may occur.

The magnitude of the neutron, \( \Delta_n \), and proton, \( \Delta_p \), pairing gaps can be determined from experimental odd-even mass differences \(^{25}\) derived from the Taylor expansion of the nuclear mass in nucleon-number differences \(^{15}\). These prescriptions assume that pairing is the only non-smooth contribution to nuclear masses. We extract \( \Delta_n \) and \( \Delta_p \) from the symmetric five-point difference \(^{15}\), which accounts better for blocking effects in odd-mass nuclei between shell gaps \(^{27}\).

\[
\Delta_n^{(5)} = -\frac{1}{8}[M(Z, N + 2) - 4M(Z, N + 1) + 6M(Z, N) - 4M(Z, N - 1) + M(Z, N - 2)] \tag{2}
\]

\[
\Delta_p^{(5)} = -\frac{1}{8}[M(Z + 2, N) - 4M(Z + 1, N) + 6M(Z, N) - 4M(Z - 1, N) + M(Z - 2, N)]. \tag{3}
\]

Here, we make the strong assumption of a valid \( \Delta_p \) in the region of study, although the kink of binding energies at shell closures would, a priori, not allow a Taylor expansion. This assumption is supported by the \(^{56}\text{Ni} \) and \(^{100}\text{Sn} \) soft cores and Ref. \(^{27}\). Doubly magic \(^{56}\text{Ni} \) and \(^{100}\text{Sn} \) are not included in the pairing-gap systematics since in these cases both magic-number and Wigner cusps span singularities in the mass surface \(^{28}\). The top panel of Fig. \( 2 \) shows \( \Delta_n \) and \( \Delta_p \) in the even-mass Ni (left panel) and Sn (right panel) isotopes. As expected, \( \Delta_n \) lies lower than the corresponding \( \Delta_p \).

For comparison, \( \Delta_r \) values and \( 2_1^+ \) excitation energies in the \(^{58–68}\text{Ni} \) and \(^{104–130}\text{Sn} \) isotopes are plotted in the

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**FIG. 2.** (Color online) The top panel shows proton, \( \Delta_p \), neutron, \( \Delta_n \), and relative \( \Delta_r \), pairing gaps extracted from the 2003 Atomic Mass Evaluation (AME03) \(^{23}\) for the even-mass Ni (left) and Sn (right) isotopes. The bottom panel shows a comparison of \( 2_1^+ \) excitation energies and \( \Delta_r \) values. A cubic-spline interpolation has been used to remark uncertainty effects of \( \Delta_r \) values in the Sn isotopes. Mass uncertainties regarding pairing gaps in the Ni isotopes are considered negligible, although this is not the case for \(^{68}\text{Ni} \).
bottom panels of Fig. 2 (left and right panels, respectively). Excitation energies and $\Delta_r$ values follow a similar trend at $^{60-64}$Ni and differ for $^{58}$Ni and $^{66-68}$Ni. For instance, whereas the energy difference of the $2^+_1$ states in $^{58}$Ni and $^{66}$Ni is only $\sim 30$ keV, there is a sharper energy difference of 150 keV between their $\Delta_r$ values. $\Delta_r = 0.985$ MeV for $^{58}$Ni and it decreases to minimum values of $\Delta_r = 0.635$ and 0.608 MeV for $^{66}$Ni and for $^{62}$Ni, respectively. Protons and neutron pairing gaps begin to diverge at $^{64}$Ni, $\Delta_r = 0.705$ MeV, with $\Delta_r$ becoming much smaller than $\Delta_p$. $\Delta_r = 1.237$ MeV in $^{60}$Ni, and has a maximum value of $\Delta_r = 1.555$ MeV at $^{68}$Ni, where $\Delta_r$ has a maximum energy of $\sim 2.1$ MeV and $\Delta_p - \Delta_n$ has the largest energy difference.

Moreover, $\Delta_r$ values and $2^+_1$ energies in the Sn isotopes follow a similar parabolic trend from $^{116}$Sn to $^{130}$Sn. However, unlike excitation energies, the trend of $\Delta_r$ values between $^{108}$Sn to $^{116}$Sn clearly shows a sharp minimum at $^{110}$Sn with $\Delta_r = 0.498$ keV. For lighter and heavier Sn isotopes, proton and neutron pairing gaps begin to diverge, with $\Delta_r$ values of 0.791 and 0.799 MeV at $^{108}$Sn and $^{112}$Sn, respectively. $\Delta_r$ increases to a maximum value of 1.300 MeV for $^{116}$Sn. From $^{116}$Sn to $^{130}$Sn, $\Delta_r$ values follow a smooth parabolic trend, with $\Delta_r$ being larger than for $^{108-112}$Sn. Larger $\Delta_r$ values of 1.076 and 1.010 MeV are found for $^{104}$Sn and $^{106}$Sn, respectively. A strong correlation with quadrupole collectivity can be inferred from the trends of $\Delta_r$ values in the Ni and Sn isotopes.

From a global fit to available $B(E2;0^+_1 \rightarrow 2^+_1)$ values throughout the nuclear chart, Grodzins deduced an exceptional formula that calculates surprisingly well $B(E2;0^+_1 \rightarrow 2^+_1)$ values from well-known $2^+_1$ energies [29]. Raman improved the fit from a larger data set [30] and the Grodzins-Raman’s empirical formula is given by,

$$B(E2;0^+_1 \rightarrow 2^+_1) = (2.57 \pm 0.45) Z^2 A^{-2/3} E(2^+_1)^{-1} \ (4)$$

The physical meaning of this formula remains unknown. Similarly, given the qualitative agreement between $\Delta_r$ values and $2^+_1$ energies in the even-mass Ni and Sn isotopes, $E2$ collectivity might be estimated using the inverse of $\Delta_r$. For that, the pairing-gap collective strength, $\Delta_c$, is defined as,

$$\Delta_c \equiv \frac{2 \Omega}{\Delta_r} \quad (5)$$

where $2\Omega = (2j + 1)$ is the average particle number, i.e., the total number of proton and neutron Cooper pairs that may contribute to the collective motion. $\Delta_c$ values are given in units of $[E^{-1}]$. In order to examine the interplay of proton-core excitations and $pp$ correlations for Ni and Sn isotopes with $N \approx Z$, only the $f_{7/2}$ and $g_{9/2}$ proton and neutron orbits, respectively, will be included in Eq. 5. These orbits are fully occupied and the large spacial overlap of magnetic substates may enhance pairing correlations. This assumption may not be valid for the very neutron-rich Sn isotopes.

The origin of Eq. 5 lies within the BCS framework. For the special case of a pure pairing force in a single-j shell, the gap equation yields the two-quasiparticle energy [31]

$$E_k + E_{k'} = 2\Delta = G \Omega, \quad (6)$$

where $E_k, E_{k'}$ are the quasiparticle energies at the Fermi surface and $G$ the pairing strength. Given the qualitative agreement between $\Delta_r$ and $2^+_1$ energies, it can be assumed that $2\Delta \approx \Delta_r$ and Eq. 5 can be written as,

$$\Delta_c \approx \frac{2}{G} \quad (7)$$
That is, nuclear collectivity is inversely proportional to the pairing strength.

Finally, Fig. 3 shows the systematics of experimental $B(E2; 2^+ → 0^+)$ values (blue diamonds) as compared with single-particle estimates (1 W.u.) in the even-mass $^{58–68}$Ni and $^{106–130}$Sn isotopes. Strikingly, the trend of $E2$ strengths in these Ni isotopes is in agreement with $\Delta_e$ values (left panel of Fig. 3). In fact, the trend of $\Delta_e$ values provides a better agreement than large-scale $SM$ and $MF$ calculations. For $^{62}$Ni, $\Delta_e$ presents a maximum in the systematics which corresponds to the lowest $2^+_1$ energy and the strongest collectivity predicted by large-scale $SM$ and $MF$ calculations.

The trend of quadrupole collectivity in the Sn isotopes is not as precisely defined as in the Ni isotopes, although the enhancement of collectivity in the neutron-deficient Sn isotopes is well established. The trend of $\Delta_e$ values in the Sn isotopes is plotted in the right panel of Fig. 3 and shows an enhancement of $E2$ strengths in the neutron-deficient Sn isotopes, with a sharp maximum at $^{110}$Sn. This maximum is unlikely since the energy spectrum of $^{108}$Sn shows typical properties of singly closed-shell nuclei that can simply be explained with a $\delta$-function interaction. The $B(E2; 2^+_1 → 0^+_1)$ values for $^{104}$Sn and $^{108}$Sn are, respectively, either unknown or with large uncertainties. The most precise $B(E2; 2^+_1 → 0^+_1)$ in $^{106}$Sn by Ekström and collaborators indicates, however, a decreasing trend with a smaller $B(E2; 2^+_1 → 0^+_1) = 13.1(2.6)$ W.u. as compared with $^{108}$Sn, in agreement with the trend proposed in this work. In addition, Jungclaus and co-workers have recently determined much lower and precise $E2$ strengths for $^{112,114,116}$Sn, with decreasing absolute $B(E2; 2^+_1 → 0^+_1)$ values from $^{112}$Sn to $^{116}$Sn (not plotted in this work). These remarkable results point out at $^{118}$Sn as the new minimum in the collective trend at midshell, in disagreement with large-scale $SM$ calculations, and supporting the $N = 66$ subshell closure and the current work. Summarizing, this work proposes that microscopic many-body properties such as nuclear collectivity could be inferred from atomic masses.

Further transition strengths and mass measurements are needed in the neutron-deficient and mid-shell Sn region to confirm the collective trend proposed in this work. In particular, more accurate experimental data is needed in the key $^{110}$Sn isotope. Curiously, the experimental masses accepted in the 2003 atomic mass evaluation concerning pairing gaps in $^{110}$Sn are either from unpublished private communications or based on $\beta$ end-point measurements.

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