Effective temperature in driven vortex lattices with random pinning

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We study numerically correlation and response functions in non-equilibrium driven vortex lattices with random pinning. From a generalized fluctuation-dissipation relation we calculate an effective transverse temperature in the fluid moving phase. We find that the effective temperature decreases with increasing driving force and becomes equal to the equilibrium melting temperature when the dynamic transverse freezing occurs. We also discuss how the effective temperature can be measured experimentally from a generalized Kubo formula.

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Whether and how can one extend thermodynamic concepts to nonequilibrium systems is a very important challenge in theoretical physics. Many definitions of nonequilibrium temperatures have been proposed in different contexts but it has been rarely checked if they conform with the expected properties of a temperature.

Cugliandolo, Kurchan and Peliti have introduced the notion of time-scale dependent “effective temperatures” $T_{\text{eff}}$ from a modification of the fluctuation-dissipation theorem (FDT) in slowly evolving out of equilibrium systems. $T_{\text{eff}}$ is defined from the slope of the parametric plot of the integrated response against the correlation function of a given pair of observables when the latter is bounded or, equivalently, it is the inverse of twice the slope of the parametric plot of the integrated response against the displacement when the correlation is unbounded. This definition yields a bona fide temperature in the thermodynamic sense since it can be measured with a thermometer, it controls the direction of heat flow for a given time scale and it satisfies a zero-th law within each time scale. $T_{\text{eff}}$ was found analytically in mean-field glassy models and it was successfully studied in structural and spin glasses, both numerically and experimentally, in granular matter and in weakly driven shear flows.

In their study of driven vortex lattices in type II superconductors, Koshelev and Vinokur have defined a “shaking” temperature $T_{\text{sh}}$ from the fluctuating force felt by a vortex configuration moving in a random pinning potential landscape. This lead to the prediction of a dynamic phase transition between a liquid-like phase of vortices moving at low driving forces and a crystalline vortex lattice moving at large forces, when $T_{\text{sh}}$ equals the equilibrium melting temperature of the vortex system. However, later work has shown that the perturbation theory used in breaks down and that the vortex phase at large velocities can be an anisotropic transverse glass instead of a crystal. In spite of this, the shaking temperature introduced in has been a useful qualitative concept, at least phenomenologically. Indeed, the dynamic transitions and moving vortex phases discussed in and in numerical simulations have been observed experimentally and in numerical simulations.

In this Letter we apply the definition of $T_{\text{eff}}$ based on the modifications of the FDT to driven vortex lattices above the critical force and within the fluid moving phase. We compare our results with the shaking temperature of and discuss how to obtain $T_{\text{eff}}$ experimentally from measurements of transverse voltage noise and transverse resistance.

The equation of motion of a vortex in position $R_i$ is:

$$\eta \frac{dR_i}{dt} = -\sum_{j \neq i} \nabla_i U_v(R_{ij}) - \sum_p \nabla_i U_p(R_{ip}) + F + \zeta_i(t),$$

where $R_{ij} = |R_i - R_j|$ is the distance between vortices $i, j$, $R_{ip} = |R_i - R_p|$ is the distance between the vortex $i$ and a pinning site at $R_p$, $\eta$ is the Bardeen-Stephen friction, and $F = \frac{d}{dx} J \times z$ is the driving force due to a uniform current density $J$. The effect of a thermal bath at temperature $T$ is given by the stochastic force $\zeta_i(t)$, satisfying $\langle \zeta_i(t) \rangle = 0$ and $\langle \zeta_i(t) \zeta_j(t') \rangle = 2\eta k_B T \delta(t - t') \delta_{ij}$, where $\langle \ldots \rangle$ denotes average over the ensemble of $\zeta_i$.

We model a 2D thin film superconductor of thickness $d$ and size $L$ by considering a logarithmic vortex-vortex interaction potential: $U_v(r) = -A_v \ln(r/\Lambda)$, with $A_v = \Phi_0^2/8\pi\Lambda$ and $\Lambda = 2\lambda^2/d > L$. The vortices interact with a random distribution of attractive pinning centers with $U_p(r) = -A_p e^{-r/r_p}$. Length is normalized by $r_p$, energy by $A_v$, and time by $\tau = \eta r_p^2 / A_v$. We consider $N_v$ vortices and $N_p$ pinning centers in a rectangular box of size $L_x \times L_y$. Moving vortices induce an electric field $E = \frac{\partial}{\partial t} V \times z$, with $V = \frac{1}{N_v} \sum_i dR_i / dt$.

To study the fluctuation-dissipation relation (FDR), we proceed in a similar way as in previous simulations of...
structural glasses \cite{13} and consider the observables:

$$A_\mu(t) = \frac{1}{N_v} \sum_{i=1}^{N_v} s_i r_i^\mu(t) ; \quad B_\mu(t) = \frac{1}{N_v} \sum_{i=1}^{N_v} s_i r_i^\mu(t) ,$$  \hspace{1cm} (1)

where \(s_i = -1,1\) are random numbers with \(\overline{s_i} = 0\) and \(\overline{s_is_j} = 0\), and \(r_i^\mu = R_i^\mu - R_i^\mu_{cm}\) with \(\mu = x,y\) and \(R_i^\mu\) being the center of mass coordinate. Taking \(\mathbf{F} = F\mathbf{y}\) we study separately the FDR in the transverse and parallel directions with respect to \(\mathbf{F}\). The time correlation function between the observables \(A_\mu\) and \(B_\mu\) is

$$C_\mu(t, t_0) = \langle A_\mu(t) B_\mu(t_0) \rangle = \frac{1}{N_v} \sum_{i=1}^{N_v} \langle r_i^x(t_0) r_i^\mu(t) \rangle ,$$  \hspace{1cm} (2)

since the \(r_i^\mu\) are independent of the \(s_i\). The integrated response function \(\chi_\mu\) for the observable \(A_\mu\) is by applying a perturbative force \(\mathbf{f}_0^\mu = \epsilon s_i \hat{\mathbf{\mu}}\) (where \(\hat{\mathbf{\mu}} = \hat{x}, \hat{y}\)) at time \(t_0\) and keeping it constant for all subsequent times on each vortex:

$$\chi_\mu(t, t_0) = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \left[ \langle A_\mu(t) \rangle_\epsilon - \langle A_\mu(t) \rangle_{\epsilon=0} \right] .$$  \hspace{1cm} (3)

We then define a function, \(T_{\text{eff}}^\mu(t, t_0)\), by the relation,

$$\chi_\mu(t, t_0) = \frac{1}{2k_B T_{\text{eff}}^\mu(t, t_0)} \Delta_\mu(t, t_0) ,$$  \hspace{1cm} (4)

where \(\Delta_\mu(t, t_0) = \frac{1}{N_v} \sum_{i=1}^{N_v} \langle (r_i^\mu(t) - r_i^\mu(t_0))^2 \rangle = C_\mu(t, t_0) + C_{\mu}(t_0, t_0) - 2C_\mu(t_0, t_0)\) is the quadratic mean displacement in the direction of \(\hat{\mathbf{\mu}}\). For a system in equilibrium at temperature \(T\) the FDT requires that \(T_{\text{eff}} = T = T_{\text{eff}}^y = T_{\text{eff}}^x\). In a nonequilibrium system, like the driven vortex lattice with pinning, the FDT does not apply. Since we are interested in the stationary states reached by the driven vortex lattice, where aging effects are stopped \cite{13} \cite{14} \cite{15}, then all observables depend only on the difference \(t - t_0\), if we choose \(t_0\) long enough to ensure stationarity. From the parametric plot of \(\chi_\mu(t)\) against \(\Delta_\mu(t)\) one defines the effective temperature \(T_{\text{eff}}^\mu(t)\) using Eq. (4), provided \(T_{\text{eff}}^\mu(t)\) is a constant in each time-scale \cite{13}.

We study the transverse and longitudinal FDR for the moving vortex lattice as a function of driving force, \(F\), solving the dynamic equations for different values of \(A_p\), \(n_v\), and \(T\). The simulations are performed with pinning density \(n_p = N_p r^2_p / L_x L_y = 0.14\) in a box with \(L_x / L_y = \sqrt{3}/2\) and \(N_v = 256\). We consider \(A_p / A_v = 0.35, 0.2, 0.25, 0.1\), \(n_v = N_v r^2_p / L_x L_y = 0.05, 0.07, 0.01\), and \(T \leq 0.01\). We impose periodic boundary conditions with the algorithm of Ref. \cite{13}. Averages are evaluated during 80000 steps of \(\Delta t = 0.1\tau\) after 65536 steps for reaching stationarity. To calculate the response function \(\chi_\mu(t)\), given by Eq. (3), we simulate two replicas of the system, with the perturbative force \(\mathbf{f}_0^\mu = \epsilon s_i \hat{\mathbf{\mu}}\) applied to one of them. Starting from the same initial condition we let the perturbed and unperturbed system evolve for 5000 time steps and calculate \(A_\mu(t)\) and \(A_\mu(t)\) respectively. The replicas then evolve again after changing the realization of the random factors, \(s_i\), and taking the final configuration of the unperturbed system as the new common initial condition. From this we get \(\langle A_\mu(t) \rangle_{\epsilon}\), both for \(\epsilon = 0\) and \(\epsilon \neq 0\), and thereby the response function, \(\chi_\mu(t)\), is determined. We choose \(\epsilon\) small enough in each case to assure a linear response of \(\langle A_\mu(t) \rangle_{\epsilon}\).

At \(T = 0\), there are three different dynamical regimes \cite{10} when increasing \(F\) above the critical depinning force \(F_c\): two fluid phases with plastic flow for \(F_c < F < F_p\) and smectic flow for \(F_p < F < F_t\), and a transverse solid for \(F > F_t\). For fixed pinning density \(n_p\), the characteristic forces \(F_c, F_p\) and \(F_t\) depend on the disorder strength \(A_p\) and vortex density \(n_v\). We start by analyzing the FDR in the smectic flow regime for different values of \(A_p, n_v\) and \(T\). In Fig. 1(a) we show the typical transverse quadratic mean displacements, \(\Delta_x(t)\), and in Fig. 1(b) the integrated transverse response, \(\chi_x(t)\), for this dynamical regime. In Fig. 1(c) we show the FDR parametric plot of \(\chi_x(t)\) against \(\Delta_x(t)\). We see that the equilibrium FDT does not apply in general but two approximate linear relations exist for \(\Delta_x(t) < 0.05 r^2_p\) and for \(\Delta_x(t) > 0.2 r^2_p\), with a non-linear crossover between them. Following Eq. (4), we find that the short displacements region corresponds to the bath temperature \(T = 0.01\), and therefore the equilibrium FDT applies in the trans-
verse direction only for short times, \( t \ll r_p / v \). For the large displacements region we get an effective transverse temperature \( T^x_{\text{eff}}(T) = 0.045 > T \). In the inset of Fig. 1(a) we show the FDR for \( T = 0 \). Comparing the results for different \( T \), we find \( T^x_{\text{eff}}(T) \approx T^x_{\text{eff}}(0) + T \).

In Fig. 2 we analyze the FDR for the longitudinal direction: Fig. 2(a) shows the quadratic mean displacement, \( \Delta_y \), and Fig. 2(b) shows the response, \( \chi_y \). In Fig. 2(c) we obtain the corresponding FDR. We observe that the equilibrium FDT applies for \( \Delta_x(t) < 0.05 r_p^2 \) at the bath temperature \( T \). The plot does not have a constant slope for larger displacements. Similar results are obtained in the plastic flow regime.

In Fig. 3 we show the calculated transverse effective temperature, \( T^x_{\text{eff}} \), for \( T = 0 \) as a function of voltage (i.e., average velocity, \( V \)). We observe that above the critical force, \( T^x_{\text{eff}} \), is a decreasing function of \( V \) that reaches a value close to the equilibrium melting temperature of the unpinned system, \( T_m \approx 0.007 \) \[2\], when the system approaches the transverse freezing transition at \( F = F_t \) (obtained from the vanishing of the transverse diffusion \( D_x \), shown in the inset). It becomes very difficult to compute numerically \( T^x_{\text{eff}} \) for driving forces \( F > F_t \), since \( \Delta_x \) and \( \chi_x \) are bounded at \( T = 0 \) while for finite \( T \) there are very long relaxation times involved. Therefore, we leave the interesting case of obtaining \( T^x_{\text{eff}}(T) \) for \( F > F_t \) for future study. The opposite limit \( F \to 0 \) is also interesting. We shall report on FDT measurements when \( F = 0 \), in the Bragg and vortex glasses, elsewhere.

In Fig. 4 we show the dependence of \( T^x_{\text{eff}} \) with pinning amplitude, \( A_p \), and vortex density, \( n_v \). In all the cases we observe that \( T^x_{\text{eff}} \) \( \to T_m \) when \( F \to F_t \); even when \( F_t \) depends on \( A_p, n_v \). The inset of Fig. 4 shows that we can approximately collapse all the curves plotting \( T^x_{\text{eff}} \) vs \( V / A_p^{\alpha} \) with \( \alpha \approx 0.5 \). The shaking temperature for a single vortex is expected to satisfy this scaling with \( \alpha = 0 \) \[3\]. The same result should apply in the limit of non-interacting vortices or incoherent motion \[4\]. In the opposite case of motion of a rigid lattice we can apply the one particle result to a Larkin-Ovchinikov correlation volume where the pinning force summation gives an effective pinning amplitude \( \sqrt{A_p} \), and therefore \( \alpha = 1 \) in this limit. It is noteworthy that the value we find is intermediate between these two limits.
We now show that the same $T^\epsilon_{\text{eff}}$ can be obtained from a different observable. In particular from experimentally accessible quantities like the transverse resistivity and the voltage fluctuations. If $T^\epsilon_{\text{eff}}$ is well defined in a given time scale we can expect a generalized Kubo formula to hold \[\text{(5)}\].

\[
R_x(t) = \frac{1}{k_B T^\epsilon_{\text{eff}}(t)} \int_0^t dt' \langle V_x(t)V_x(t') \rangle,
\]

where $R_x(t) = \langle \frac{dV_x(t)}{dt} \rangle_{t=0} - \langle \frac{dV_x(0)}{dt} \rangle_{t=0}$ is the linear transverse resistance. If we now make a parametric plot of $R_x(t)$ vs. $\int_0^t dt' \langle V_x(t)V_x(t') \rangle$ we again find a linear slope equal to $1/T$ for $t \ll r_p/v$ and a second linear slope of $1/T^\epsilon_{\text{eff}}$ for $t > r_p/v$. In Fig. 2(a) we compare the two effective transverse temperatures obtained using Eqs. (4) and (5). We see that they are similar within the error bars. The shaking temperature, $T_{\text{sh}}$, defined in Eq. (3) of Ref. \[\text{[21]}\] is proportional to the time integral of the correlation function of the pinning force $F_{\text{pin}} = \sum_i f_{ip}(t)$. $T_{\text{sh}}$ can be obtained from Eq. (6) if we replace $R_x(t)$ with the single vortex value $R_0 = 1/\eta$ and the integral of the $V_x(t)$ correlation function is taken for all $t$ [since for the transverse direction $V_x(t) \propto F_x(t)$]. In other words, $T_{\text{sh}}$ of \[\text{[21]}\] corresponds to taking the average slope in the parametric plot of the generalized Kubo formula [or in the parametric plot of the FDR shown in Fig. 1(c)], see also \[\text{[21]}\]. The approach followed here permits defining an effective temperature which takes into account all the information on its time scale dependence that allows for a thermodynamic interpretation of $T^\epsilon_{\text{eff}}$. In this way we see clearly that there is a non-trivial value of the transverse effective temperature $T^\epsilon_{\text{eff}}$ for time scales $t > r_p/v$. Since one drives the system in the longitudinal direction $y$, the condition of slow motion needed to prove the thermodynamic properties in $T^\eta_{\text{eff}}$ is not matched. Thus, it is no surprise that we do not find the same value of $T^\epsilon_{\text{eff}}$ in the longitudinal direction \[\text{[22]}\]. Furthermore, we find that the short-range correlations of the moving fluid are important and give a non-trivial dependence with disorder strength $A_p$. Besides this, we have demonstrated that in the moving fluid phase $T^\epsilon_{\text{eff}}$ satisfies two important results of \[\text{[1]}\]: (i) additivity of temperatures, $T^\epsilon_{\text{eff}}(T) = T^\epsilon_{\text{eff}}(0) + T$ and (ii) dynamic freezing occurs when $T^\epsilon_{\text{eff}}(T) = T_m$. From the analysis of Eq. (6), we note that $\langle V_x(t)V_x(0) \rangle$ can be obtained from transverse voltage noise measurements (from $|V_x(\omega)|^2$) and $R_x(t)$ from time dependent measurements of the transverse resistivity. It will therefore be interesting to have experimental measurements of $T^\epsilon_{\text{eff}}$ to test quantitatively the dynamic freezing transition. Finally, we stress that a complete dynamic theory of the moving vortex system has to capture the features here described.

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\[\text{[22]}\] We note that also in shear flows an effective temperature has been obtained only in the direction perpendicular to the driving force \[\text{[1]}\].