Blurred path-spin entanglement in Stern-Gerlach apparatus: interplay between magnetic inhomogeneity and Larmor precession

Nirupam Dutta$^1$ and Ansuman Dey$^2$

$^1$Theoretical Physics Division, Variable Energy Cyclotron Centre, 1/AF, Bidhan Nagar, Kolkata 700 064, India
$^2$S. N. Bose National Centre For basic Sciences, Salitelle, Kolkata-700098, India

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We argue that the non-adiabatic evolution of spin states in Stern-Gerlach apparatus can blur the manifestation of path spin entanglement. This fact introduces a contradiction to the usual perception of spin measurement even in a formally ideal situation. Through quantitative calculation, we have pointed out the specific reason behind the breakdown of adiabatic evolution to be the spatial inhomogeneity of applied magnetic field. The angle $\theta$ between the $z$ component of magnetic moment of the particle and applied magnetic field is also a determining factor for the category of evolution of spin states. Adiabaticity always can be restored by choosing a sufficiently small value of $\theta$. Considering azimuthal inhomogeneity, we have examined the nature of path spin entanglement in the context of Stern-Gerlach experiment.

I. INTRODUCTION

The Stern-Gerlach experiment (SGE) \cite{ref1} plays a fundamental role in quantum mechanics because of its conceptual relevance. SGE first witnessed the quantum nature of intrinsic spin of a particle. In general, different aspects of SGE are complicated to understand and are still being studied by present day researchers. In the experiment, a beam of particles is passed through a Stern-Gerlach (S-G) apparatus which consists of a magnet providing an inhomogeneous magnetic field, in a certain direction (in general). When a screen is put at a distance from where the particles are emerging off the S-G apparatus, two distinct peaks are observed in positive and negative $z$ direction corresponding to two different spin values along $z$ axis. This clearly demonstrates a path spin entanglement. More precisely, the eigenstates of $S_z$ of a charge neutral particle entering the S-G apparatus being in one of the eigenstate of $S_x$ or $S_y$ are entangled with its spatial degree of freedom. The observed peaks on the screen always lead to the detection of up spin in the upward ($z > 0$) direction and down spin in the downward ($z < 0$) direction. This statement is not only true for the measurement of spin along $z$ axis, but is also valid when the spin is measured along any arbitrary direction $\hat{u}$ along which the magnetic field is applied. The magnetic field spatially separates two different spin states $|+\rangle_u$ and $|-\rangle_u$ and they evolve further staying in the same instantaneous states along with the spatial part of the wave function. The final state becomes $|\psi\rangle = \alpha|\phi_+\rangle \otimes |+\rangle_u + \beta|\phi_-\rangle \otimes |-\rangle_u$ with $\alpha$, $\beta$ complex constants. Here, $\phi_+$, $\phi_-$ are the spatial parts in up and down directions while $|+\rangle_u$, $|-\rangle_u$ are the eigenstates of $\hat{\vec{S}}_u$. This, so far, assumes the adiabatic evolution of spin states.

Most studies on conceptual and experimental aspects of SGE have been conducted within the confines of adiabatic regime. There also are instances in extant literature \cite{ref3,ref4} where a non-adiabatic evolution of spin states has been addressed for explicitly time varying magnetic fields. Naturally, such theoretical investigations rest on various simplifying assumptions \cite{ref2}. However, a very close look at the nature of spin state evolution in S-G apparatus reveals that the spatial variation of inhomogeneous magnetic field can play a crucial role in the measurement of spin through path spin entanglement. Spatial inhomogeneity of the magnetic field lends an implicit time dependence to the interaction Hamiltonian which becomes explicit in the particle’s rest frame. A rapid change of interaction Hamiltonian in that case may cause a non adiabatic evolution for spin states.

In this work, we will shed light on the fact that the adiabaticity condition is not trivially respected in S-G experiments (even for a static magnetic field). This gives rise to the possibility of mixing of both up and down spins in either direction. Hence, one cannot then infer a definite spin of the particle by observing its deflection although the split in the distribution pattern will still be there (due to the force on the particles caused by the magnetic field). Intriguingly enough, a finite probability of detecting both up and down spins in each direction remains even when successive S-G apparatuses are placed in the same direction as the first. Thus, though the observed state on the screen is, in general, a path spin entangled one, a specific path does not necessarily correspond to a particular spin. This, we call blurred path spin entanglement. In fact, under certain conditions, instead of down spin the up spin can couple with $\phi_-$ and vice versa. This is absolutely against the conventional wisdom of spin measurement in S-G apparatus. It is important to keep in mind that the mixing of spin states discussed here occurs even in a formally ideal SG experimental setup, unlike that in \cite{ref5}.
II. ADIABATICITY AND PATH SPIN ENTANGLEMENT

Suppose, we have prepared the initial state of a spin half system as,

\[ |\psi(0)\rangle = (c_1|+\rangle_u + c_2|-\rangle_u) \otimes |\phi(0)\rangle, \quad (1) \]

Here \(|\phi(0)\rangle\) is the spatial part of the initial wave function of the spin-half system which is not entangled with the spin state. Now, we allow this state to pass through a Stern-Gerlach apparatus providing an inhomogeneous magnetic field along the direction \(\hat{u}\). The evolved state at any point inside the apparatus at some instant \(\tau\) can be determined from the interaction Hamiltonian \(H(x, y, z)\). It is given by,

\[ |\psi(\tau)\rangle = c_1\phi_+(\tau) \otimes |+\rangle_u + c_2\phi_- (\tau) \otimes |-\rangle_u. \quad (2) \]

when the direction of the magnetic field inside the apparatus is not changing spatially. But, Maxwell’s equations, do not allow this \([6, 7]\). As a way out, the authors in \([6, 8]\) have adopted a time averaged description of Pauli equation thereby obtaining an effective inhomogeneous magnetic field with a fixed direction. This assumption, really, is based on the consideration of very strong magnetic fields \([6, 9]\) observed over time scales much larger than the characteristic time scale of the spin half system. Obviously, it is not impossible to design such a circumstance with control over laboratory conditions. But the same cannot be said of general practical scenarios. The Stern-Gerlach model is widely used to study the path spin entanglement in varied magnetic environments where change in the direction of magnetic field can not be avoided. Well! This is not a concept far removed from the usual understanding. One can always think of a collection of S-G apparatus set in the series to mimic the effect of a spatially changing magnetic field. It is even intuitively imaginable that the evolved state can be expressed as,

\[ |\psi(\tau)\rangle = \alpha(\tau)\phi_+(\tau) \otimes |+\rangle_u + \beta(\tau)\phi_- (\tau) \otimes |-\rangle_u, \quad (3) \]

\(\alpha(\tau), \beta(\tau)\) are two complex coefficients. One can anticpate from the evolution of the system that the specific spin state (up or down) associated with \(\phi_+\) or \(\phi_-\) evolves adiabatically whereas the corresponding space part evolves according to the spatial part of the propagator. The above state carries the signature of path spin entanglement. The speciality of such specific kind of path-spin entanglement is that we can precisely infer spin (up or down along \(\hat{u}(\tau)\)) by detecting the particle in a particular spatial direction as long as the up and down part of \(\phi\) (with respect to \(\hat{u}(\tau)\)) are separated enough to obey formal idealness \([5]\).

At this point, it is worth mentioning that the interaction Hamiltonian \(H\) changes as the particle propagates in an inhomogeneous magnetic field. Due to the inhomogeneity of magnetic field, there is an implicit dependence of \(H\) on time. Therefore, one has to evaluate the evolved states for the interaction Hamiltonian which is changing with the spatial degrees of freedom. Inside an S-G apparatus, the initial superposed state is spatially separated and both the parts undergo an adiabatic evolution if the Hamiltonian changes sufficiently slowly. The adiabatic approximation is valid for an inhomogeneous magnetic field if the following condition \([10, 11]\) is satisfied:

\[ \frac{|\langle \psi_m | \dot{\psi}_n \rangle|}{E_n - E_m} = \frac{|\langle \psi_m | \hat{\nabla} \psi_n \rangle|}{E_n - E_m} \ll 1; \forall m \neq n \quad (4) \]

Here \(|\psi_m\rangle\) and \(|\psi_n\rangle\) are the \(m\)th and \(n\)th instantaneous energy eigenstates with energy \(E_m\) and \(E_n\) respectively while \(\hat{\nabla}\) is the velocity of the charge neutral particle inside the apparatus. Equation (3) is the direct consequence of adiabatic approximation (4) which clearly is not trivially respected.

The implicit time dependence of the Hamiltonian becomes explicit if one describes the situation from the rest frame of the particle. Obviously, the particle frame is non inertial but constraining the particle to undergo only small changes in velocity during the course of its traversal through the apparatus helps neglect the non inertial contribution. Practically this can be achieved by applying a low intensity magnetic field. Though one can solve the problem by staying in the lab frame, we chose to address the issue from the particle’s rest frame as it avoids computational complexities. Three important sources of implicit time dependence of Hamiltonian in

\[ \text{1 Here the spatial up and down parts correspond to the instantaneous direction } \hat{u}(\tau) \]
this context are,

- 1. The spatial inhomogeneity of the applied magnetic field.
- 2. Edge effect of magnet due to the finite extension of apparatus.
- 3. Sudden change of the magnetic field as particle leaves the apparatus.

### III. INTERPLAY BETWEEN LARMOR PRECESSION AND AZIMUTHAL INHOMOGENEITY OF MAGNETIC FIELD

We have mentioned that there are three important reasons for which one has to consider the Hamiltonian as an explicit function of time \( H(t) \) while describing the situation from particle frame. In our article, we are considering the effect of inhomogeneity of magnetic field to understand the evolution of spin states in S-G apparatus. In the references \cite{8} \cite{9}, it has been argued that the effect of azimuthal component of magnetic field is not important as the average force on the magnetic dipole turns out to be zero due to the rapid precession (Larmor precession) of the magnetic moment. On the other hand, due to the inhomogeneous magnetic field the instantaneous spin states change with time, an effect overlooked by them. Well, the consideration of instantaneous states leads to no change in the result mentioned in equation \cite{3} if the Larmor precession can cope up with the changing magnetic field (a quantitative description is presented later in this section). We will work three component magnetic field to avoid those assumptions made in \cite{9} \cite{8}. The interplay between Larmor precession and inhomogeneity of the magnetic field can be crucial in that case.

In the comoving frame of the particle, the magnetic field appears to change its magnitude and direction with time. Hence, by considering such a magnetic field \( \vec{B}(t) \) interacting with the magnetic dipole moment \( \mu \), the explicit time dependence of the Hamiltonian can be expressed as,

\[
H(t) = -\vec{\mu} \cdot \vec{B}(t) = -(\vec{\mu} \cdot \vec{u}(t)) B = \omega_0 \hat{S}_u(t)
\]

\[
= \frac{\omega_0}{2} \left( \sigma_x \sin \theta \cos \Phi(t) + \sigma_y \sin \theta \sin \Phi(t) + \sigma_z \cos \theta \right).
\]

We have considered here the azimuthal inhomogeneity which makes the Hamiltonian a time dependent quantity through the time dependence of azimuthal angle \( \Phi \). The quantity \( \omega_0 \) is the Larmor frequency of the spin half particle which we have considered as a constant quantity by assuming that the magnitude of the magnetic field is not changing much. \( \sigma_x, \sigma_y, \sigma_z \) are the Pauli spin matrices. \( \theta \) is the angle which the magnetic field makes with \( z \) axis.

Let us consider that a state \( c_+ |+\rangle_u + c_- |−\rangle_u \) enters an S-G apparatus. The initial state \( |\Psi(0)\rangle \) inside the apparatus is a path-spin entangled one as the entanglement is established quite immediately after the particle enters into the apparatus.

\[
|\Psi(0)\rangle = \alpha(0) |+\rangle_u(0) \otimes \phi_+(0) + \beta(0) |−\rangle_u(0) \otimes \phi_-(0).
\]

(6)

\( \phi_+(0) \) and \( \phi_-(0) \) are the up and down components of spatial wave function with respect to the plane whose normal makes an angle \( \theta \) with \( z \) axis. For \( \theta = 0 \), the up component directs along \( +z \) and the down along \( −z \). Whether the evolution of the state will be adiabatic or non adiabatic is determined through the ratio\cite{8} obtained by making use of instantaneous energy eigenstates of \( H \),

\[
|−(t)\rangle_u = \begin{pmatrix} e^{-i\frac{\omega_0}{2} t \sin \theta} \\ -e^{\frac{\omega_0}{2} t \cos \theta} \end{pmatrix},
\]

\[
|+(t)\rangle_u = \begin{pmatrix} e^{-i\frac{\omega_0}{2} t \cos \theta} \\ e^{\frac{\omega_0}{2} t \sin \theta} \end{pmatrix}
\]

(7)

corresponding to the energy eigenvalues \( E_- \) and \( E_+ \) respectively. Henceforth, we will denote \( u(t) \) by the slightly more compact notation \( u_t \) to express instantaneous states.

\[
\left| u_t \right| \left( \frac{\frac{\partial}{\partial t} |+\rangle_u + |−\rangle_u \right) = \frac{\Phi}{2\omega_0} \sin \theta.
\]

(8)

The above equation shows that \( \theta, \Phi, \omega_0 \) are the determining factors for the adiabatic evolution of the spin states. For non zero values of \( \sin \theta \), \( \omega_0 \) should be much larger than \( \Phi \) in order to satisfy the adiabatic condition. It implies that the Larmor precession should be rapid enough to cope up with the instantaneous direction of the magnetic field.

In order to know the time evolution of spin states, one needs to solve the Schrödinger equation for the time dependent Hamiltonian \( H(t) \) with a given initial state. The final state could be written as,

\[
|\Psi(t)\rangle = |a(t)\rangle_u + |b(t)\rangle_u.
\]

(9)

Plugging this into Schrödinger equation we get the following set of linear differential equations,

\[
\dot{a}(t) + \frac{i}{2} \left[ \omega_0 - \Phi(t) \cos \theta \right] a(t) - \frac{i}{2} \Phi(t) b(t) \sin \theta = 0
\]

\[
\dot{b}(t) - \frac{i}{2} \left[ \omega_0 - \Phi(t) \cos \theta \right] b(t) - \frac{i}{2} \Phi(t) a(t) \sin \theta = 0
\]

(10)

For any quantitative prediction we need to know the time dependence of \( \Phi(t) \). To proceed further, we present a solution by considering terms up to 1st order in the Taylor series expansion of \( \Phi(t) \). A general solution is always possible but we have considered this simple case to show the drastic role played by the azimuthal inhomogeneity.

\[
\Phi(t) = \Phi_0 + \frac{d\Phi(t)}{dt}|_{t=0} + \mathcal{O}(2)
\]

(11)
This is actually equivalent to assuming a small change in velocity of the particle as it travels through the apparatus (mentioned in the previous section). The instantaneous states look (by setting $\Phi_0 = 0$ without any loss of generality),

\[
|-(t)\rangle_{u_t} = \begin{pmatrix} e^{-i \hat{\omega} t} \sin \frac{\theta}{2} \\ -e^{i \hat{\omega} t} \cos \frac{\theta}{2} \end{pmatrix},
\]

\[
|+(t)\rangle_{u_t} = \begin{pmatrix} e^{-i \hat{\omega} t} \cos \frac{\theta}{2} \\ e^{i \hat{\omega} t} \sin \frac{\theta}{2} \end{pmatrix}.
\] (12)

The adiabatic condition becomes $\omega = \frac{d\Phi(t)}{dt} |_{t=0}$. The spin part $|+(0)\rangle_{u(0)}$ in $\Phi$ coupled to the spatial part $\phi_+$ evolves to the final state

\[
|t\rangle_+ = \alpha_+(t) |+(t)\rangle_{u_t} + \alpha_- (t) |-(t)\rangle_{u_t},
\] (13)

at some instant $t$. The coefficients $\alpha_+$ and $\alpha_-$ can be evaluated by solving the differential equation $10$ with the initial condition $\alpha_+(0) = 1$ and $\alpha_- (0) = 0$. They are

\[
\alpha_+ = \cos \frac{\hat{\omega} t}{2} - i \frac{\omega_0 - \omega \cos \theta}{\omega} \sin \frac{\hat{\omega} t}{2},
\]

\[
\alpha_- = i \frac{\omega \sin \theta}{\omega} \sin \frac{\hat{\omega} t}{2}.
\] (14)

$\hat{\omega}$ is a combination of $\omega$ and $\omega_0$:

\[
\hat{\omega} = \sqrt{\omega_0^2 + \omega^2 - 2\omega_0 \omega \cos \theta}.
\] (15)

Therefore, the usual perception that the up spin will be separated from the down spin can not be valid in general. Rather, in both the directions, there will be mixture of up and down spin states.

Similarly, the final state corresponding to the initial part $|-(0)\rangle_{u(0)}$ will be

\[
|t\rangle_- = \beta_+ |+(t)\rangle_{u_t} + \beta_- |-(t)\rangle_{u_t},
\] (16)

This is again a mixture of up and down spins. The coefficients $\beta_+$ and $\beta_-$ is determined in the same way as done in eq $[14]$ by solving the Schrödinger equation $[12]$.

\[
\beta_- = \cos \frac{\hat{\omega} t}{2} + i \frac{\omega_0 - \omega \cos \theta}{\omega} \sin \frac{\hat{\omega} t}{2},
\]

\[
\beta_+ = i \frac{\omega \sin \theta}{\omega} \sin \frac{\hat{\omega} t}{2}.
\] (17)

This kind of mixing of spin appears as the initially separated up and down spin now goes through a non adiabatic evolution eventually giving birth to the blurred path-spin entanglement. In principle, for a non zero $\theta$, one has to consider such non-adiabatic evolution of states whenever the quantity $\omega_0$ is not very large compared to $\omega$. The final spin state which we observe on the screen is the one at the instant $t$ when the particle just leaves the apparatus. That instant has to be determined from the time taken by the particle to complete the travel through the S-G apparatus. Therefore, the longitudinal dimension of the apparatus is important to explain the observed pattern on the screen.

It is also noticeable that the non adiabatically evolved spin states have a precession frequency which is different from the Larmor frequency $\omega_0$. The modification also depends on $\theta$ and $\omega$. This happens because the instantaneous spin eigenstates are now weighted by a periodic function ($\cos \hat{\omega} t$ or $\sin \hat{\omega} t$).

IV. BLURRED PATH-SPIN ENTANGLEMENT

In the previous section, we have derived the final spin states corresponding to the each component coupled with spatial up and down parts. Now, the general solution would be the combination of both with the initial weight $c_+$ and $c_-$. A non-adiabatic evolution:

\[
|\Psi(t)\rangle = c_+ (\alpha_+ |+(t)\rangle_{u_t} + \alpha_- |-(t)\rangle_{u_t}) \otimes \phi_+(t)
\]

\[
+ c_-(\beta_+ |+(t)\rangle_{u_t} + \beta_- |-(t)\rangle_{u_t}) \otimes \phi_-(t)
\]

\[
= (d_1 \phi_+(t) + d_2 \phi_-(t)) \otimes |+(t)\rangle_{u_t}
\]

\[
+ (k_1 \phi_+(t) + k_2 \phi_-(t)) \otimes |-(t)\rangle_{u_t}
\] (18)

As we describe the evolution from the rest frame of the particle, the up and down components of the spatial wave function remain same as the Hamiltonian does not have explicit spatial dependence any more. They can be considered as up and down components with respect to the plane whose normal is now $\hat{u}(t)$. This is justified as long as the angle $\theta$ is constant and $\Phi(t)$ does not change much ($< \pi/2$) by the time the particle leaves the apparatus. Hence, the final state is also an entangled state as the spatial and spin part of the state is not product separable. Although there is path-spin entanglement, the state can not be utilized to infer the definite spin by detecting them in a specific path of the particle. Rather, there will be a mixture of up and down spins corresponding to each of the spatial directions. This blurs the possibility to measure specific spin by knowing its path. In fact, one can easily check that a breakdown of adiabaticity can not result in disentanglement of the initially path spin entangled state. For disentanglement one needs to satisfy $d_1 = d_2 = \frac{c}{\sqrt{2}}$ and $k_1 = k_2 = \frac{k}{\sqrt{2}}$ which is not allowed in this context.

However, at the instant $t = \frac{(2n+1)\pi}{\omega}$ for $\cos \theta = \frac{\omega_0}{\omega}$,

\[
\alpha_+ = \beta_- = 0
\]

Voila! The up spin is now detected in the downward direction whereas the down spin appears in the upward direction, as is clear from the solution in eq [18]. This is absolutely opposite to the conventional wisdom of historical Stern-Gerlach experiment.
Therefore, we can infer that the angle θ plays an important role to determine the final state which leaves the apparatus and evolves freely further up to the screen. The instant t will be determined through the longitudinal dimension of the apparatus. Hence, the length of the apparatus is unavoidable even in the prediction of a transverse SGE.

Furthermore, we can easily derive the final state when the Larmor frequency ω₀ is much larger than ω. Hence,

\[ \alpha_+ = e^{-i \frac{\omega t}{2 \omega_0}}; \quad \alpha_- = 0 \]
\[ \beta_- = e^{i \frac{\omega t}{2 \omega_0}}; \quad \beta_+ = 0, \]

as

\[ \bar{\omega} = \omega_0 \sqrt{1 + \frac{\omega^2}{\omega_0^2} - 2 \frac{\omega \cos \theta}{\omega_0}} \approx \omega_0. \]

Now, plugging this in equation 18, we have

\[ \Psi(t) = c_+ e^{-i \frac{\omega t}{2 \omega_0}} |(0)\rangle_u(0) \otimes \phi_+(t) + c_- e^{i \frac{\omega t}{2 \omega_0}} |-(0)\rangle_u(0) \otimes \phi_-(t) = c_+ |+\rangle_u(0) \otimes \phi_+(0) + c_- |-\rangle_u(0) \otimes \phi_-(0), \]

as \( \phi_+(t) = \phi_+(0) \) and \( \phi_-(t) = \phi_-(0) \). This is equivalent to the solution 2 which is obtained in [6, 8] applicable for a time averaged magnetic field which does not change its’ direction. One can easily realize that the same situation can be originated through the addition of a very high constant magnetic field along the direction \( \bar{u}(0) \) and the result then directly communicate with previously understood results [5, 6, 8]. We have not introduced any averaging concept in Scrödinger equation which makes the solution applicable at arbitrary time scale. The rapid precession does not see the changing direction of magnetic field. The final state observed at a time scale \( t = \frac{2M}{\bar{\omega}_0} \) leads to the solution of Pauli equation given in [6].

\[ \text{V. CONCLUSION} \]

In this article, we specifically have investigated the interplay between Larmor precession and azimuthal inhomogeneity of magnetic field in the context of path-spin entanglement of a spin half system in S-G experiments. The main emphasis of the work is to point out the necessary and sufficient condition for the adiabatic evolution of spin states which is also the basis of spin measurement in S-G apparatus. The effect of inhomogeneous magnetic field previously has been taken care of in literature to study dynamical nature of entanglement. A very specific reference in this context is one by Caldeira. et. al. [8]. They have presented the analysis by considering a magnetic field which is not changing in direction but in magnitude only. It is obvious, that in that context, the instantaneous states do not change with time. By considering the same fact one always can get back to the result in [8] from our findings as well. For simplicity, we have considered only azimuthal inhomogeneity. In a forthcoming paper [13] we will present the evolution of path-spin entanglement for general inhomogeneous magnetic field to study concurrence. We have seen that not only the ratio of \( \Phi(t) \to \omega_0 \) but the angle θ is also a deciding factor of the category of evolution of spin states. For \( \theta = 0 \), we can arrive at the usual text book result for S-G experiment even by describing through instantaneous states. When \( \theta \) is not sufficiently small, the spin states will evolve non-adiabatically. We have pointed out that in case of non adiabatic evolution the entanglement still can persist but that will not be useful to infer any definite spin associated with the specific path. The mixing of different spin was previously realized from the perspective of non-ideal S-G measurement [5] where operational and formal non-ideal behaviour was discussed. Our article does not deal with such sources of spin mixing. Rather, we report that the interplay between inhomogeneity of magnetic field and the Larmor precession is quite generic and was overlooked until now.

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