Abstract

Hypermultiplets are considered in the five-dimensional interval where all fields are continuous and the boundary conditions are dynamically obtained from the action principle. The orbifold boundary conditions are obtained as particular cases. We can interpret the Scherk-Schwarz supersymmetry breaking as a misalignment of boundary conditions while a new source of supersymmetry breaking corresponding to a mismatch of different boundary parameters is identified. The latter can be viewed as coming from boundary supersymmetry breaking masses for hyperscalars and the nature of the corresponding supersymmetry breaking parameter is analyzed. For some regions of the parameter space where supersymmetry is broken (either by Scherk-Schwarz boundary conditions or by boundary hyperscalar masses) electroweak symmetry breaking can be triggered at the tree level.
1 Introduction

The existence of extra dimensions is a general prediction of fundamental (string) theories that aim to unify all interactions, including gravity, and provide a consistent quantum description of them. If the radii of these extra dimensions is as large as the $1$/TeV scale $[1]$, matter can propagate in the bulk and the very existence of extra dimensions can provide new mechanisms for supersymmetry and electroweak breaking $[2]$. In five and six-dimensional theories gauge bosons are located in vector multiplets and matter and Higgs bosons in hypermultiplets. While supersymmetry breaking should be felt primarily by $SU(2)_R$ doublets (fermions in vector multiplets and bosons in hypermultiplets), electroweak breaking by the conventional Higgs mechanism concerns the bosons of the Higgs hypermultiplet. Therefore for a better understanding of supersymmetry and electroweak breaking we should consider mainly the system of Higgs hypermultiplets propagating in the bulk. In this paper we will study propagation of hypermultiplets in five dimensions (5D).

Propagation of matter in the bulk of the fifth dimension has been widely considered in the past $[3, 4]$. The 5D space-time, with coordinates $(x^\mu, y)$, is often constructed as the orbifold $S^1/Z_2$, where points on the circle of radius $R$ $^4$ related by the reflection of the fifth coordinate $y \rightarrow -y$ are identified. In this approach ("upstairs" approach) fields are classified according to the $Z_2$ parity and their boundary conditions (BC’s) at the fixed points imposed. In the orbifold approach the space is singular at the orbifold fixed points $y = 0, \pi$, it has no boundaries and the fields satisfy the circle periodicity and the orbifold parity. In this approach mass terms localized at the fixed points can trigger supersymmetry breaking and make the fermionic wave functions to make discontinuous jumps at them $[5, 6]$.

An alternative approach is working in the fundamental region of the orbifold $[0, \pi]$ and giving up the rigid orbifold BC’s $[7]$. In this approach ("downstairs" approach) fields have no defined parity and BC’s are dynamically determined by the action principle $[8]$. The space is not singular but has boundaries at $y = 0, \pi$ and wave functions are continuous even in the presence of boundary mass terms. In particular the propagation of gauge fermions in the interval has been considered in Ref. $[8]$ where the Scherk-Schwarz (SS) supersymmetry breaking $[9]$ was interpreted as misalignment of BC’s at the two boundaries that departure from supersymmetry in boundary parameters.

In this paper we will consider propagation of hypermultiplets in the interval. We will

\footnote{We will work from here on, unless explicitly stated, in units where $R \equiv 1$.}
construct a globally supersymmetric action and identify possible sources of supersymmetry
breaking corresponding to departure from supersymmetry of boundary parameters. New
patterns of supersymmetry breaking will arise that can trigger electroweak breaking at the
tree-level. The structure of the paper is as follows. In section 2 the general supersymmetric
formalism for hypermultiplets in the interval will be worked out. The equations of motion
for hyperscalars and hyperfermions will be solved in section 3 where mass eigenvalues and
eigenfunctions are explicitly obtained. In particular the conditions for supersymmetry will
be established while supersymmetry breaking by boundary conditions will be considered in
detail in section 4. A comparison with the orbifold approach will be done in section 5. The
nature of supersymmetry breaking by boundary hyperscalar masses is clarified in section 6
where a simple toy model is constructed based on a $U(1)$ gauge theory under which hyper-
multiplets transform. The problem of embedding $SU(2)_L \otimes U(1)_Y$ in the interval will be
considered in section 7 where the interface between supersymmetry and electroweak break-
ing will be studied, including the tree-level prediction for the Higgs mass. Finally section 8
contains our conclusions and some (technical) useful identities are presented in appendix A.

2 Hypermultiplets in the Interval

In this section we will consider the formalism for a single hypermultiplet propagating in
the interval. There are two equivalent approaches. One is to consider the fields in the
hypermultiplet as complex and unconstrained fields: it is the so-called complex hypermul-
tiplet [10]. The second approach is to introduce an $SU(2)_H$ index on the hypermultiplet fields
\[ H^\alpha = (\Phi_i, \Psi_i, F_i)^\alpha, \]
that transforms as a doublet, and to introduce on the fields the reality constraint [11]
\[ \bar{\Phi}^\alpha_i \equiv (\Phi^\alpha_i)^* = \epsilon^{ij} \epsilon_{\alpha\beta} \Phi^\beta_j, \]
\[ \bar{\Psi}^\alpha_i \equiv \Psi^\alpha_i \gamma^0 = \epsilon_{\alpha\beta} (\Psi^\beta) \gamma^C, \]
\[ (2.1) \]
\[ (2.2) \]
The auxiliary fields obey the same constraint, while the hyperfermions now obey a sym-
plectic Majorana constraint with respect to the new $SU(2)_H$.

\[ \bar{\Psi}_i \equiv (\Psi_i)^\dagger \gamma^0 = \epsilon_{\alpha\beta} (\Psi^\beta)^\dagger \gamma^C, \]
\[ (2.3) \]

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5The subscript $i$ transforms as a doublet under the group $SU(2)_R$.
6The convention is such that $\epsilon^{12} = \epsilon_{12} = +1$. 

2
where \( C \) is the 5D charge conjugation matrix. In the following we will use real hypermultiplets and conventions and notations are those of Ref. \[12\]. In principle we always could explicitly solve the reality constraints to obtain the standard complex hypermultiplet, but we will find it useful to express our results in terms of the real fields. It is important to realize that the doublet of real fields describes the same degrees of freedom as one complex hypermultiplet.

We will consider the total action \( S = S_{bk} + S_{bd} \) as the sum of a bulk (\( S_{bk} = S_{bk}^0 + S_{bk}^m \)) and a boundary (\( S_{bd} \)) term, as

\[
S_{bk}^0 = \int_{\mathcal{M}} \left( -\frac{1}{2} \Phi \partial^2 \Phi + \frac{i}{2} \bar{\Psi}^{\gamma} \gamma^M \partial_M \Psi + 2 \bar{F} F \right) \tag{2.4}
\]

\[
S_{bk}^m = \int_{\mathcal{M}} \left( 2i F M \Phi + \frac{1}{2} \bar{\Psi} M \Psi \right) \tag{2.5}
\]

\[
S_{bd} = \int_{\partial \mathcal{M}} \left( \frac{1}{4} \bar{\Psi} S \Psi + \frac{1}{4} (\bar{\Phi} R \Phi)' + \frac{1}{4} \Phi N (-1 + R) \Phi \right) \tag{2.6}
\]

where \( S_{bk}^m \) in (2.5) is a supersymmetric bulk mass action and for simplicity we are using an indexless notation. \( S \) and \( \mathcal{M} \) are matrices in the \( SU(2)_H \) indices, \( R \) has matrix indices in both \( SU(2)_H \) and \( SU(2)_R \) and \( N \) is a real number \(^7\). \( S \) and \( R \) are dimensionless matrices and \( N \) and \( \mathcal{M} \) have dimension of mass. We will show in subsection 2.2 that this action is supersymmetric. In the next section we will first derive the BC’s resulting from the action (2.4), (2.5) and (2.6).

### 2.1 The boundary conditions

We take \( S, \mathcal{M} \) and \( R \) to be hermitian. In order for the action (2.4), (2.5) and (2.6) to be real, the reality constraints imply that \( S, \mathcal{M} \) and \( R \) satisfy:

\[
S^\alpha_\beta = \epsilon_{\alpha \gamma} \epsilon^{\beta \delta} S^\gamma_\delta \tag{2.7}
\]

\[
\mathcal{M}^\alpha_\beta = \epsilon_{\alpha \gamma} \epsilon^{\beta \delta} \mathcal{M}^\gamma_\delta \tag{2.8}
\]

\[
R_{\alpha \delta} = \epsilon_{ik} \epsilon^{jl} \epsilon_{\alpha \gamma} \epsilon^{\beta \delta} R_{k \delta}^{l \gamma} \tag{2.9}
\]

\(^7\)It is understood that \( S, R \) and \( N \) can take on different values at the two branes, i.e. the usual index \( f = 0, \pi \) is suppressed here.
With this choice, all terms in the above action are real without partial integration because of the reality constraints. We now make the ansatz \( R = T \otimes S \) where \( T \) acts on \( SU(2)_R \) only, i.e. \( T^i_j \). Then all conditions on \( R, S \) and \( M \) can be formulated in matrix notation as

\[
\mathcal{M}^\dagger = \mathcal{M}, \quad \mathcal{M}^T = -\sigma^2 \mathcal{M} \sigma^2
\]  \hspace{1cm} (2.10)

\[
S^\dagger = S, \quad S^T = -\sigma^2 S \sigma^2
\]  \hspace{1cm} (2.11)

\[
T^\dagger = T, \quad T^T = -\sigma^2 T \sigma^2
\]  \hspace{1cm} (2.12)

The solution to these constraints are

\[
\mathcal{M} = M \vec{p} \cdot \vec{\sigma}, \quad S = \vec{s} \cdot \vec{\sigma}, \quad T = \vec{t} \cdot \vec{\sigma}
\]  \hspace{1cm} (2.13)

where \( \vec{s}, \vec{p} \) and \( \vec{t} \) are real dimensionless vectors, \( M \) is a mass parameter and \( \vec{p} \) a unit vector. All calculations can now be performed without writing explicit indices by use of the identities presented in the Appendix.

Variation of the action gives a bulk and a boundary term \( \delta S = \delta_{\text{bk}} S + \delta_{\text{bd}} S \) the latter coming from the partial integration of the variation of the bulk action and from the variation of the boundary action:

\[
\delta_{\text{bd}} S = \frac{1}{2} \int_{\partial M} \left[ \delta \bar{\Psi} (i \gamma^5 + S) \Psi + \delta \bar{\Phi} (-1 + R) \Phi' + \delta \Phi' (1 + R) \Phi + \delta \bar{\Phi} N (-1 + R) \Phi \right].
\]  \hspace{1cm} (2.14)

The BC’s resulting from this are

\[
(1 + i \gamma^5 S) \Psi = 0, \quad \bar{\Psi} (1 - i \gamma^5 S) = 0
\]  \hspace{1cm} (2.15)

\[
(1 + R) \Phi = 0, \quad \bar{\Phi} (1 + R) = 0
\]  \hspace{1cm} (2.16)

\[
(-1 + R) [\Phi' + N \Phi] = 0, \quad [\bar{\Phi}' + N \bar{\Phi}] (-1 + R) = 0
\]  \hspace{1cm} (2.17)

\(^8\)The choice \( R = T \otimes S \) is motivated by the fact that the supersymmetry transformation laws (see subsection 2.2) make sense. The boundary conditions we will find are the same on both sides of the transformation laws provided \( \epsilon \) fulfills \( (1 + i \gamma^5 T) \epsilon = 0 \).
We find that for consistent fermionic BC’s $\vec{s}$ has to be a unit vector $[8]$. Furthermore, Eqs. (2.16)–(2.17) give rise to eight real BC’s at each brane, whereas we need only four. Thus the bosonic system is clearly overdetermined unless the $8 \times 8$ matrix

$$
\begin{pmatrix}
0 & 1 + R \\
-1 + R & N(-1 + R)
\end{pmatrix}^4
$$

is singular. Its determinant is given by $(1 - s^2 t^2)^4$, which vanishes if $\vec{t}$ is a unit vector. In fact, it is easy to see that in this case $(1 + R)/2$ and $(1 - R)/2$ form mutually orthogonal projectors on two-dimensional subspaces, and hence we reduce the number of independent BC’s on each brane down to four.

Standard orbifold BC’s are obtained as particular cases by taking $S_f = s_f \sigma_3$, $T_f = t_f \sigma_3$, $N = 0$. The (independent) parity eigenstates are then given by $\varphi = \Phi^1_1$, $\varphi^c = \Phi^2_1$, $\psi_L = \Psi^1_L$, $\psi_R = \Psi^1_R$ and their parities are

$$
\begin{align*}
\varphi(y_f + y) &= -s_f t_f \varphi(y_f - y), \\
\varphi^c(y_f + y) &= +s_f t_f \varphi^c(y_f - y), \\
\psi_L(y_f + y) &= -s_f \psi_L(y_f - y), \\
\psi_R(y_f + y) &= +s_f \psi_R(y_f - y).
\end{align*}
$$

However, our formalism allows for more general BC’s. As we will see, it can produce SS-twists in both $SU(2)_R$ as well as $SU(2)_H$ space with SS parameter given by the angle between the vectors $\vec{t}_0$, $\vec{t}_\pi$ and $\vec{s}_0$, $\vec{s}_\pi$ respectively. Furthermore, we can have mixed BC’s for bosons parametrized by the masses $N_f$. A more detailed and general comparison with the orbifold approach will be done in section 5.

### 2.2 Supersymmetry of the action

We now want to show that the action $[2.4]$–$[2.6]$ is indeed supersymmetric. The transformation laws are given by

$$
\begin{align*}
\delta \Phi^\alpha_i &= i \bar{\epsilon}_i \Psi^\alpha, \\
\delta \Psi^\alpha &= -\gamma^M \bar{\epsilon}_i \partial_M \Phi^\alpha_i + 2 \bar{\epsilon}_i F^\alpha_i, \\
\delta F^\alpha_i &= -\frac{i}{2} \bar{\epsilon}_i \gamma^M \partial_M \Psi^\alpha.
\end{align*}
$$

(2.19)
First consider the bulk part, Eqs. (2.4)-(2.5). Under supersymmetry the Lagrangian varies into a total derivative which leaves a brane variation given by

$$\delta_\epsilon S_{bk} = \int_{\partial M} \left( -\bar{\epsilon} \epsilon (i\gamma^5)\Psi + \frac{1}{2}\bar{\Phi} \gamma^\mu \partial_\mu (i\gamma^5)\Psi + \frac{i}{2} \bar{\Phi} \epsilon \Psi' - \bar{\epsilon} \Phi \gamma^5 \Psi \right)$$  (2.20)

Now consider the variation of the boundary action, Eq. (2.6).

$$\delta_\epsilon S_{bd} = \int_{\partial M} \left( -\frac{1}{2} \bar{\epsilon} \gamma^\mu \partial_\mu \Phi \Psi - \frac{1}{2} \bar{\epsilon} \gamma^5 \Phi' \Psi \right.$$  
$$+ \bar{\epsilon} F S \Psi + \frac{i}{2} \bar{\Phi}' R \epsilon \Psi + \frac{i}{2} \bar{\Phi} R \epsilon \Psi' + \frac{i}{2} N \bar{\Phi} (-1 + R) \epsilon \Psi \left) \right.$$  (2.21)

Discarding a total 4D derivative we can rewrite the sum of (2.20) and (2.21) as

$$\delta_\epsilon S = \int_{\partial M} \left( \frac{i}{2} \bar{\epsilon} \Phi (1 + R) \Psi' + \frac{i}{2} (\Phi' + N \Phi)(-1 + R) \epsilon \Psi$$  
$$- \frac{1}{2} \bar{\epsilon} \gamma^5 \Phi \mathcal{M} (1 + i\gamma^5 S) \Psi - i\bar{\epsilon} \gamma^5 \left( F - \frac{i}{2} \bar{\Phi} \mathcal{M} \right) (1 - i\gamma^5 S) \Psi \right)$$  (2.22)

Using the BC's (2.15), (2.16) and (2.17) the first three terms vanish. Finally we use the EOM for $F$

$$F = -\frac{i}{2} \mathcal{M} \Phi, \quad \bar{F} = \frac{i}{2} \bar{\mathcal{M}} \phi$$  (2.23)

to deduce that the whole variation is zero.

3 EQUATIONS OF MOTION AND THE SPECTRUM

In this section we will solve the equations of motion (EOM) in the bulk for the bosonic and fermionic sectors of the hypermultiplet and impose on the solutions the corresponding BC's on both boundaries. We will obtain as a result the mass eigenfunctions and eigenvalues for the different modes.

3.1 HYPERSCALARS

We first make a general mode decomposition of bosonic fields as

$$\Phi_\alpha^a(x,y) = \sum_n f^n_{i,n}(y) \phi_n(x)$$  (3.1)
where $\phi_n(x)$ is the real 4D mass eigenstate corresponding to the mass $m_n$. The solution to the EOM arising from the bulk action (2.4) and (2.5) is given by

$$ f(y) = \cos(\Omega y) a + \sin(\Omega y) b $$

where $a_i^\alpha$ and $b_i^\alpha$ are constant matrices and $\Omega_n = \sqrt{m_n^2 - M^2}$.

The BC’s at $y = 0$, Eqs. (2.16) and (2.17) yield

$$ a = (1 - R_0) \varphi $$
$$ b = \left[ 1 + R_0 - \frac{N_0}{\Omega} (1 - R_0) \right] \varphi $$

where $\varphi$ is a constant unconstrained matrix. Imposing the BC’s at $y = \pi$ determines $\varphi$ and gives the discrete eigenvalue spectrum as a function of $\vec{s}_f$ and $\vec{t}_f$. Defining the angles $\omega$ and $\tilde{\omega}$ by

$$ \vec{s}_0 \cdot \vec{s}_\pi = \cos(2\pi \tilde{\omega}) $$
$$ \vec{t}_0 \cdot \vec{t}_\pi = \cos(2\pi \omega) $$

the bosonic mass eigenvalues are given as the solutions of the equation

$$ A(\omega + \tilde{\omega}, N_0, N_\pi) A(\omega - \tilde{\omega}, N_0, N_\pi) = 0 $$
$$ A(\varphi, N_0, N_\pi) = \sin^2(\pi \varphi) - \frac{N_0 - N_\pi}{\Omega} \tan(\Omega \pi) - \left[ \cos^2(\pi \varphi) + \frac{N_0 N_\pi}{\Omega^2} \right] \tan^2(\Omega \pi) $$

Notice that the parameter $\omega$ describes a Scherk-Schwarz twist and thus it corresponds to supersymmetry breaking. The parameter $\tilde{\omega}$ is a twist in the global $SU(2)_H$ symmetry and amounts to a supersymmetric mass, as we will see in the next section. As for the mass parameters $N_f$ they can conserve or break supersymmetry depending on their relation with $M \vec{p} \cdot \vec{s}_f$ as we will see. For special values of $(\omega, \tilde{\omega})$ the mass formula becomes a perfect square, indicating a degeneracy in the spectrum. The values where this happens are given by $(\omega, 0)$, $(\omega, \frac{1}{2})$, $(0, \tilde{\omega})$ and $(\frac{1}{2}, \tilde{\omega})$.

We want to close this section by noticing that the condition for the existence of an exactly massless mode is given by the equation

$$ (n_0 - \tau^{-1})(n_\pi + \tau^{-1}) = \cos^2 \pi (\omega \pm \tilde{\omega})(1 - \tau^{-2}) $$

From here on and for notational simplicity we will drop, unless explicitly stated, the subscript $n$ as well as we will use the compact notation where the $i$ and $\alpha$ indices are omitted.
where
\[ n_f = \frac{N_f}{M} \]  
and \( \tau \equiv \tanh(M\pi) \). This defines a hyperbola which divides the \((n_0, n_\pi)\) plane in regions where the lightest mode is tachyonic or physical respectively. This will be discussed in more detail in section 4.

### 3.2 Hyperfermions

We will define the Dirac fermion in the 5D action as
\[
\Psi^\alpha = \begin{pmatrix} \chi^\alpha \\ \bar{\psi}^\alpha \end{pmatrix}
\]  
(3.8)
where \( \chi^\alpha \) and \( \bar{\psi}^\alpha \) are Weyl fermions subject to the reality condition \( \psi_\alpha = \epsilon_{\alpha\beta}\chi^\beta \). We will make the mode decomposition
\[
\chi^\alpha(x, y) = \sum_n f_n^\alpha(y)\chi_n(x)
\]
\[
\bar{\psi}^\alpha(x, y) = \sum_n g_n^\alpha(y)\bar{\chi}_n(x)
\]
(3.9)
where \((\chi_n(x), \bar{\chi}_n(x))^T\) is the 4D Majorana spinor with mass eigenvalue \(m_n\). We now define the vector
\[
h(y) = \begin{pmatrix} f(y) \\ g(y) \end{pmatrix}
\]  
(3.10)
where we have dropped the indices \( \alpha, n \). The bulk EOM corresponding to the bulk action (2.4) and (2.5) has the solution
\[
h(y) = \mathcal{U}(y)\, h(0); \quad \mathcal{U}(y) = \cos(\Omega y) + (im\sigma^2 + M\sigma^3) \frac{\sin(\Omega y)}{\Omega}
\]
(3.11)
where \(\sigma^{2,3}\) are acting on the space of Eq. (3.10) and \(\mathcal{M}\) is acting on \(SU(2)_H\) indices.

We now apply the BC’s (2.15) at the two boundaries. In particular the BC’s at \(y = 0\) imply that
\[
h(0) = (1 - \sigma^3 S_0) \, \tilde{h}
\]  
(3.12)
and at \(y = \pi\)
\[
V(\pi) \, \tilde{h} \equiv (1 + \sigma^3 S_\pi) \mathcal{U}(\pi) \,(1 - \sigma^3 S_0) \, \tilde{h} = 0
\]
(3.13)
The 4 × 4 matrix \( V(\pi) \) has rank \( r \leq 2 \) because it is proportional to the projector \((1 + \sigma^3 S_\pi)/2\). The existence of a non-trivial solution requires \( r = 1 \) which provides the constraint satisfied by the mass eigenvalues. The result can be expressed in terms of the angles \( \tilde{\omega} \) defined in Eq. (3.4) and \( \alpha_f \) defined by

\[
\vec{p} \cdot \vec{s}_f = \cos(2\pi \alpha_f) \equiv c_f
\]  

(3.14)

The fermion mass eigenvalues satisfy then the equation

\[
1 - \tilde{c} - 2 (c_0 - c_\pi) \frac{M}{\Omega} \tan(\Omega \pi) - \left[ 1 + \tilde{c} + 2 c_0 c_\pi \frac{M^2}{\Omega^2} \right] \tan^2(\Omega \pi) = 0
\]  

(3.15)

where the quantity \( \tilde{c} = \cos(2\pi \tilde{\omega}) \). Note that the quantities \( c_0 \) and \( c_\pi \) are not completely independent but are bound to lie inside an elliptical disk

\[
\frac{(c_0 + c_\pi)^2}{\cos^2 \pi \tilde{\omega}} + \frac{(c_0 - c_\pi)^2}{\sin^2 \pi \tilde{\omega}} \leq 4.
\]  

(3.16)

This condition stems from the fact that the three angles between the three vectors \( \vec{s}_f, \vec{p} \) are not independent but rather constrained by triangle inequalities. For instance, if \( \tilde{c} = 1 \) it is clear that \( 0 \leq c_0 = c_\pi \leq 1 \) (in this case the ellipse actually shrinks to a line). It will also be convenient to express the condition Eq. (3.15) in terms of the function \( A \) defined in Eq. (3.5):

\[
A(\tilde{\omega}, c_0 M, c_\pi M) = 0.
\]  

(3.17)

The bosonic, Eq. (3.5), and fermionic, Eq. (3.17), spectra can easily encompass the cases already studied in the literature. For instance for the particularly simple case where \( N_f = M = 0 \) the bosonic spectrum provided by Eq. (3.5) is given by \( m_n = n \pm \omega \pm \tilde{\omega} \) while the fermionic one, provided by Eq. (3.15), is given by \( m_n = n \pm \tilde{\omega} \) in agreement with the results of the model studied in Ref. 3.

For the case \( \tilde{\omega} = 0, c_0 = c_\pi = 1 \) (i.e. the three vectors \( \vec{s}_f, \vec{p} \) aligned) and \( N_f = M \) the bosonic spectrum from Eq. (3.5) is given by the solution of

\[
\sin^2(\pi \omega) = \frac{\Omega^2 + M^2}{\Omega^2} \sin^2(\Omega \pi)
\]  

(3.18)

while the fermionic spectrum is given by \( m_n^2 = n^2 + M^2(1 - \delta_{n0}) \) in agreement with the results in Ref. 13. While other cases can be easily studied using the general equations we will next concentrate in particularly interesting cases for physics purposes. In particular, we will examine how supersymmetry can be broken and how vectorlike fermions can arise.
4 Supersymmetry breaking

The bosonic spectrum described as the solution of Eq. (3.5) depends on four dimensionless parameters: $\omega$, $\tilde{\omega}$ and $n_f$. Similarly the fermionic spectrum described as the solution of Eq. (3.15) depends on three parameters: $\tilde{\omega}$ and $\alpha_f$. The parameter $\omega$ is a genuine Scherk-Schwarz supersymmetry breaking parameter while a particular relation between $n_f$ and $\alpha_f$ can conserve/break supersymmetry as we will see in this section. Finally $\tilde{\omega}$ affects to both bosons and fermions and can play the role of a supersymmetric mass if the only source of supersymmetry breaking is the parameter $\omega$ as we have seen in the simple example described at the end of the previous section.

Comparison between (3.5) and (3.17) dictates the supersymmetric relation between $n_f$ and $c_f = \cos(2\pi \alpha_f)$. Indeed this is given by

$$n_f = c_f$$  (4.1)

If $n_f$ does not satisfy the relations (4.1) supersymmetry is broken. In fact even if the Lagrangian is (on-shell) supersymmetric the spectrum is not. This source of supersymmetry breaking can also be understood as follows. The BC’s Eqs. (2.15)–(2.17) are generally not stable under the supersymmetry transformations Eq. (2.19); in other words the variation of the fields does not fulfill the BC’s. As can easily be shown, the BC’s are stable if and only if $n_f = c_f$. In summary supersymmetry breaking arises from two different sources: one is the non-alignment of the vectors $\vec{i}_f$ (or equivalently $\omega \neq 0$); another one is the departure from zero of $n_f - c_f$. In both cases the supersymmetric limit is continuously connected which suggests that in the locally supersymmetric extension of the action local supersymmetry might be spontaneously broken. This point is extremely important and deserves a detailed investigation.

4.1 Boundary supersymmetry-breaking hyperscalar masses

In order to discriminate between the Scherk-Schwarz mechanism and other sources of supersymmetry breaking, we will first fix $\omega = 0$.

As mentioned earlier, the massless bosonic modes lie on a hyperbola in the $(n_0, n_\pi)$ plane, Eq. (3.6). It is clear that the mass squared of one eigenmode changes sign when one crosses this curve. As can be explicitely checked from Eq. (3.5), at $n_0 = n_\pi = 0$ there are no tachyonic modes, while at $n_0 = n_\pi \gg 1$ there is one (complex) tachyon with $m^2 = -N_0^2$. 

10
and at $n_0 = -n_\pi \gg 1$ there are two degenerate tachyons with $m^2 = -N_0^2$. It is then easy to identify three distinct regions with different number of tachyonic eigenvalues. For the case $\omega = 0$, we illustrate this situation in Fig. 1 by showing a plot in the $(n_0, n_\pi)$ plane. In

![Figure 1: The hyperbola of massless modes for $\pi MR = 1.5$, $\omega = 0$ and $\tilde{\omega} = 0.15$ (left panel) and $\tilde{\omega} = 0.44$ (right panel) in the plane $(n_0, n_\pi)$. The clear region to the upper left has no tachyonic modes, the darkly shaded (blue) region between to the two branches has one tachyonic eigenvalue, and the region to the lower right has two tachyonic modes. The ellipse corresponds to the allowed points in the plane $(c_0, c_\pi)$. The two dots mark the points where the fermions are massless.](image)

the same plot we also provide the allowed values of $c_{0,\pi}$, defined by Eq. (3.16). The interior of the ellipse corresponds to the allowed values in the plane $(c_0, c_\pi)$. The supersymmetric points $n_f = c_f$ are thus also limited to the inside of the ellipse. Note that for $\omega = 0$ the ellipse cannot overlap with the shaded region, for this would mean the fermions to acquire tachyonic masses. However, for certain values of $M$ and $\tilde{\omega}$, there are two points where the ellipse is tangent to the hyperbola, the intersection points corresponding to exactly massless supersymmetric spectra. These points are given by

$$
\begin{pmatrix}
c_0 \\
c_\pi
\end{pmatrix} = \tau^{-1} \begin{pmatrix}
\sin^2 \pi \tilde{\omega} \pm \cos \pi \tilde{\omega} \sqrt{\tau^2 - \sin^2 \pi \tilde{\omega}} \\
-\sin^2 \pi \tilde{\omega} \pm \cos \pi \tilde{\omega} \sqrt{\tau^2 - \sin^2 \pi \tilde{\omega}}
\end{pmatrix}
$$

(4.2)

and are obviously constrained to

$$
\tau^2 \geq \sin^2 \pi \tilde{\omega}.
$$

(4.3)

Some tachyonic spectra will be investigated in subsection 4.2 and used for electroweak symmetry breaking in section 7.
For other values of $\tau$ and $\tilde{\omega}$ there is no intersection of the ellipse with the hyperbola and hence there are no massless fermions. This is actually the case in the right panel of Fig. 1. Notice that there all points $n_f = c_f$ now correspond to supersymmetric but massive spectra.

The case $\tilde{\omega} = \frac{1}{2}$ is special. In fact if the hypermultiplet $H$ transforms non-trivially under the gauge group the 4D theory might be anomalous if the fermion modes are not paired to get a Dirac mass. This happens for instance if the hypermultiplet scalar zero mode is identified with the Higgs field doublet $H$ in the Standard Model. A quick glance at Eq. (3.15) shows that a sufficient condition for this to happen is $\tilde{\omega} = 1/2$, i.e. $\tilde{c} = -1$. In that case the ellipse degenerates to the line $c_0 = -c_\pi$ and the spectrum becomes vectorlike, as Eq. (3.15) for fermions (Higgsinos) becomes a perfect square

$$\left\{ 1 - c_0 \frac{M}{\Omega} \tan(\Omega \pi) \right\}^2 = 0 \quad (4.4)$$

For instance, when $c_0 = 0$, then the fermionic spectrum is given by

$$\Omega_n = n + \frac{1}{2} \quad \Rightarrow \quad m_n^2 = M^2 + \left( n + \frac{1}{2} \right)^2 \quad (4.5)$$

For $c_0 = -c_\pi = -1$ the spectrum can be calculated in the large $M$ limit and is given by

$$\Omega_n = n + \mathcal{O}(M^{-1}) \quad \Rightarrow \quad m_n^2 = M^2 + n^2 + \mathcal{O}(M^{-1}), \quad n \geq 1 \quad (4.6)$$

Finally, for the case $c_0 = -c_\pi = 1$ there is an exactly massless Dirac fermion in the limit $\tau = 1 \ (M \rightarrow \infty)$. In fact, it can be shown that for $MR \gtrsim 1$ there is a light Dirac fermion with mass

$$m = 2M \exp(-\pi M). \quad (4.7)$$

Interestingly enough the wavefunctions of the two chirality degrees of freedom localize towards different branes.

### 4.2 Scherk-Schwarz Supersymmetry Breaking

For $\omega \neq 0$ the situation changes. Even for $c_f = n_f$ the theory does provide different spectra for fermions and bosons as supersymmetry is now broken by the Scherk-Schwarz mechanism. In Fig. 2 we show the situation in the case $\omega = \frac{1}{2}$. In the left panel ($\tilde{\omega} = 0.15$) one can...
Figure 2: Same as Fig. 1 but with $\omega = \frac{1}{2}$ and $\bar{\omega} = 0.15$ (left panel) and $\bar{\omega} = \frac{1}{2}$ (right panel).

see that for $n_f = c_f$ bosons always have strictly positive mass-squared, even at the points where the fermions are massless\(^{12}\). In order to have massless or tachyonic scalars, one has to move away from points $n_f = c_f$, thereby introducing the new source of supersymmetry breaking discussed above.

In the right panel of Fig. 2 we choose $\bar{\omega} = \frac{1}{2}$. As opposed to the previous case, it is now possible to have a tachyon at $n_f = c_f$. A particularly interesting choice is $n_0 = c_0 = -n_\pi = -c_\pi = 1$ where (for $MR \gtrsim 1$) there are two light scalars with masses squared

$$m^2 = \pm 4M^2 \exp(-\pi MR)$$

while the vectorlike fermion zero mode is still given by Eq. (4.7). These particular cases were already considered in Refs. \(^{14} \text{[\text{14}],}\text{[\text{15}]}\) in the context of the orbifold approach\(^{13}\). In Fig. 3 we show the numerical solution for these masses for general values of $M$. Notice that for $MR \gg 1$ both bosonic and fermionic masses are exponentially suppressed with the corresponding phenomenological troubles. However for values $MR \ll 1$ the exponential suppression disappears, although the fermion remains lighter than the Higgs boson for $MR \gtrsim 0.2$, and correspondingly the present experimental bounds on charginos can put a lower bound on the Higgs mass in this class of models. The fact that the zero mode boson

\(^{12}\)Notice that the masslessness condition for fermions is still determined by the intersection of the ellipse with the $\omega = 0$ hyperbola, not shown in the plot.

\(^{13}\)The relation to the orbifold approach will be clarified in section 5.
is tachyonic and that supersymmetry is broken by Scherk-Schwarz boundary conditions provides a priori a very promising class of models of electroweak symmetry breaking. In fact supersymmetry breaking is supersoft (finite) as it is due to global effects typical of the Scherk-Schwarz breaking, while electroweak symmetry is accomplished at the tree level and the electroweak breaking scale (Higgs mass) can be decoupled from the inverse radius of compactification, which can help to solve the little hierarchy problem. The wave function of the Higgs is exponentially localized to one of the interval boundaries, which can help in (partially) solving the problem of fermion masses while the radion can be stabilized by the Casimir energy at one or two-loop order as recently proposed. A detailed analysis of these, and other questions (outside the scope of the present paper) will be considered elsewhere.

Figure 3: Light particle spectrum for $\omega = \tilde{\omega} = \frac{1}{2}$, $n_0 = c_0 = 1$, $n_\pi = c_\pi = -1$. The plot shows $|m_R|$ as a function of $M R$. The blue (solid) line corresponds to the bosons, the lower curve being the tachyon. The red (dashed) is the mass of the lightest fermion.

Another particularly interesting supersymmetry breaking case is $\omega = \frac{1}{2}$ and $N_0 = N_\pi \equiv N$ where supersymmetry is broken by both the SS parameter and by the boundary terms $N_f$. The bosonic spectrum is now determined by the equation

$$\left(\Omega^2 + N^2\right) \frac{\tan^2(\Omega \pi)}{\Omega^2} = 0$$  \hspace{1cm} (4.9)

---

\(^{14}\) Of course radiative corrections have to be taken into account. For $M = 0$ this has been done in Ref. and it was found in Ref. that EW symmetry breaking can occur as long as the localizing mass term for the top hypermultiplet is not too big. Of course the situation changes when we slightly localize the Higgs field and allow for a non-vanishing value of the parameter $M$. In that case the tachyonic tree level mass becomes more and more important when $M$ increases, reaching a maximum at around $MR \sim 0.5$, and in this region EW symmetry breaking can occur even with a fully localized top.
with solutions
\[ m_n^2 = n^2 + M^2 \quad (n \neq 0) \]
\[ m_0^2 = M^2 - N^2 \] (4.10)

The spectrum for the case \( M = N = 0 \) coincides with the \( \mathbb{Z}_2 \times \mathbb{Z}_2' \) model of Ref. [4] while the case with \( M = N \neq 0 \) is a generalization but still the bosonic zero mode is massless. The case with \( M \neq N \) is a different generalization where the bosonic zero mode is massive. An interesting possibility is when \( M < N \) in which case the zero mode is a tachyonic state and can play the role of the Higgs doublet of the Standard Model and trigger electroweak (and supersymmetry) breaking at the tree level. In particular the case \( M = 0 \) reproduces the fermionic sector of the model of Ref. [4] while for \( N \neq 0 \) the bosonic sector has a tachyonic zero mode.

The wave function for the bosonic modes is given, from (3.3), as
\[ f_n(y) = \left( \cos(\Omega y)(1 - R_0) + \sin(\Omega y) \frac{\sin(\Omega y)}{\Omega} [1 + R_0 - N_0(1 - R_0)] \right) \varphi \] (4.11)
where \( \varphi \) has to be determined from the BC’s at \( y = \pi \). For the bosonic sector of the previous case where \( R_0 = R_\pi \), \( N_0 = N_\pi = N \), the wave function of the zero mode, satisfying \( \Omega^2 + N^2 = 0 \), is defined by
\[ f_0(y) = e^{-Ny} \varphi \quad \text{with} \quad (1 + R)\varphi = 0 \] (4.12)

Two components of \( \varphi \) are projected away by the last condition in (4.12) while the reality condition implies that only one independent component is kept. The latter one is fixed by the normalization condition. Eq. (4.12) shows that for \( N \neq 0 \) the bosonic zero mode is localized at the \( y = 0 \) (\( y = \pi \)) boundary for \( N > 0 \) (\( N < 0 \)).

In this case we do not expect supersymmetry breaking to be supersoft because, on top of the non-vanishing Scherk-Schwarz parameter we have a departure from the supersymmetric relation (4.1). On the other hand, from the phenomenological point of view the Higgsino masses are much larger than the Higgs mass and present experimental bounds on chargino masses do not constrain at all the present model. In fact the nature of the supersymmetry breaking will be clarified in section [5] while some comments about electroweak breaking for this class of models will be presented in section [6].
5 Comparison with the orbifold approach

In order to compare the previous formalism with the more usual orbifold approach, and to also shed light on the nature of the previously considered supersymmetry breaking, we show in this section that the same physical theory can be obtained if one considers the orbifold $S^1/\mathbb{Z}_2$. We assign the following parities to the fields

$$\Psi(-y) = i\gamma^5 \sigma_3 \Psi(y), \quad \bar{\Psi}(-y) = -\bar{\Psi}(y)i\gamma^5 \sigma_3,$$

$$\Phi(-y) = \sigma_3 \otimes \sigma_3 \Phi(y), \quad \bar{\Phi}(-y) = \bar{\Phi}(y)\sigma_3 \otimes \sigma_3,$$

$$F(-y) = -\sigma_3 \otimes \sigma_3 F(y), \quad \bar{F}(-y) = -\bar{F}(y)\sigma_3 \otimes \sigma_3.$$  

We also could introduce Scherk-Schwarz twists for the $SU(2)_R$ and $SU(2)_H$ symmetries.

However since the presence of an $\tilde{\omega} \neq 0$ parameter amounts to a supersymmetric mass, while the nature and interpretation of a Scherk-Schwarz twist $\omega \neq 0$ has been widely clarified in the literature [5, 6, 8], we will simplify our discussion in this section by assuming $\omega = \tilde{\omega} = 0$. Furthermore, we replace the action given in Eqs. (2.4)–(2.6) by

$$S^0_{bk} = \int \left( -\frac{1}{2} \bar{\Phi} \partial^2 \Phi + \frac{i}{2} \bar{\Psi} \gamma^M \partial_M \Psi + 2 \bar{F} F \right),$$

$$S^m_{bk} = \int \left( 2i \bar{\Phi} \Phi + \frac{1}{2} \bar{\Psi} \gamma^M \Phi \right),$$

$$S_{bd} = \int \left( N_0 \delta(y) - N_\pi \delta(y - \pi) \right) \bar{\Phi} \Phi.$$  

In order to have well-defined parity for the mass terms, we take the vector $\vec{p}$ defined in Eq (2.13) to be

$$p = (p_1, p_2, \epsilon(y)p_3),$$

where $\epsilon(y)$ is the sign-function. Choosing $p_1 = p_2 = 0$ one reproduces the odd mass terms for hypermultiplets previously considered in the literature [17, 18, 14, 13]. The boundary mass terms involving the $N_f$ parameters are similar to the ones encountered in Eq. (2.6). In fact the boundary conditions (5.2) require $R = -\sigma_3 \otimes \sigma_3$, so that by using this in Eq. (2.6) we find Eq. (5.6). The additional factor of 2 comes from the fact that the support of the delta function on the circle is twice the one on the interval, while the relative sign of the
two boundaries reflects our convention of taking the orientation of the boundary at \( y = 0 \) to be negative. Boundary mass terms—which in the interval give rise to boundary conditions on the orbifold generate jumps for the profiles of wave functions across the brane. It is easy to calculate these jumps for the special kind of mass terms of Eq. (5.6). All fields are continuous except the \( \partial_5 \) derivatives of even bosonic fields, which satisfy

\[
(1 + \sigma_3 \otimes \sigma_3)[\Phi'(0^+) + N_0 \Phi(0)] = 0, \tag{5.8}
\]

\[
(1 + \sigma_3 \otimes \sigma_3)[\Phi'(\pi^-) + N_\pi \Phi(\pi)] = 0. \tag{5.9}
\]

Here we write the matrix \( (1 + \sigma_3 \otimes \sigma_3) \) to project on the even fields only. The spectrum can now be directly inferred from subsections 3.1 and 3.2. The bosonic one is given by Eq. (3.5) with \( \omega = \tilde{\omega} = 0 \). For the fermionic one, notice that in order to produce our orbifold boundary conditions, we have to choose \( \vec{s}_0 = \vec{s}_\pi = (0, 0, -1) \) and hence must use \( c_0 = c_\pi = -p_3 \) in Eq. (3.15).

Let us next study supersymmetry of this action. The supersymmetry variation of the bulk action is now given by

\[
\delta S^{0}_{\text{bk}} = 0, \quad \delta S^{m}_{\text{bk}} = -2p_3M [\delta(y) - \delta(y - \pi)] \bar{\Phi} \gamma^5 \sigma_3 \Psi, \tag{5.10}
\]

while the boundary piece varies into

\[
\delta S^{\text{bd}} = 2i [N_0 \delta(y) - N_\pi \delta(y - \pi)] \bar{\Phi} \gamma^5 \sigma_3 \Psi. \tag{5.11}
\]

Making use of our parity assignments Eq. (5.1) we conclude that for these two pieces to cancel we must have

\[
n_0 = n_\pi = -p_3. \tag{5.12}
\]

To compare with the interval approach, we note again that there \( c_0 = c_\pi = -p_3 \) and thus we find that for the action to be supersymmetric, relation (4.1) must hold. Therefore departure from the supersymmetric relation (4.1) implies supersymmetry breaking. However in contrast to the interval case where the action itself is supersymmetric for any values of \( N_f \), the breaking here is explicit and can be viewed as coming from localized soft masses for the even hyperscalars. Splitting the masses \( N_f \) into a supersymmetric and a soft piece, \( N_f = -p_3M + M_f \) we can write the localized soft breaking Lagrangian as

\[
S^{\text{hyper}}_{\text{soft}} = \int \left( M_0 \delta(y) - M_\pi \delta(y - \pi) \right) \bar{\Phi} \Phi. \tag{5.13}
\]
Supersymmetry breaking produced by the soft mass terms for even scalars in the action (5.13) bears strong similarities with the usual Scherk-Schwarz supersymmetry breaking by twisted boundary conditions in the gaugino (and gravitino) sector. In fact twisted Scherk-Schwarz boundary conditions for the gauginos $\lambda^i$ ($i = 1, 2$) can be produced by localized gaugino soft masses with an action $[5, 6, 8]$

$$S_{\text{soft}}^{\text{gauge}} = \int \bar{\lambda} (M_0 \delta(y) - M_\pi \delta(y - \pi)) \lambda + \text{h.c.} \quad (5.14)$$

However the nature of supersymmetry breaking by boundary scalar masses is very different that of the Scherk-Schwarz supersymmetry breaking (which provides a supersoft or finite breaking) as we will see in the next section.

6 Supersymmetry breaking by boundary masses

In this section we will study the nature of supersymmetry breaking by localized scalar masses as in Eq. (2.6) for

$$N_f = \vec{p} \cdot \vec{s}_f M + M_f \quad (6.1)$$

with $M_f \neq 0$. For simplicity we will assume the case of vector-like fermions analyzed in subsection 4.2, $\omega = \tilde{\omega} = 1/2$, ($R_0 = R_\pi = R$) when there are no supersymmetric masses, i.e. $M = 0$. This case gives rise to a tachyonic zero mode in the bosonic spectrum for $M_0 = M_\pi$ and it is particularly interesting.

The gauge interactions of the hypermultiplet $\Phi$ depend on the gauge group. For simplicity we will assume in this section a $U(1)$ gauge group with generator $Q$. Consistency with the reality condition (2.2) implies that the generator $Q$ satisfies $[12]$

$$\sigma^2 Q = -Q^* \sigma_2 \quad , \quad (6.2)$$

and we will then make the choice

$$Q = \frac{1}{2} \sigma_3 \quad . \quad (6.3)$$

The quartic Lagrangian comes from the integration of the $U(1)$ auxiliary field $\tilde{X}$ in

$$\mathcal{L}_D = 2\tilde{X}^2 + g_5 \bar{\Phi} \bar{\sigma}_R \cdot \tilde{X} \otimes Q\Phi \quad (6.4)$$
where $g_5$ is the 5D $U(1)$ gauge coupling and $\bar{\sigma}_R$ are the $SU(2)_R$ generators. Integration of $\vec{X}$ yields

$$
\mathcal{L}_D = -\frac{1}{8} g_5^2 (\bar{\Phi} \bar{\sigma}_R \otimes \mathcal{O})^2 .
$$

(6.5)

We will now call the independent components of $\Phi$ as

$$
\Phi_1^1 = H_1, \quad \Phi_1^2 = H_2
$$

(6.6)

and will use the reality conditions (2.2) for the other components,

$$
\Phi_2^2 = \bar{H}_1, \quad \Phi_1^2 = -\bar{H}_2 .
$$

(6.7)

The quartic potential is then given by

$$
V_D = \frac{1}{8} g_5^2 (|H_1|^2 + |H_2|^2)^2 .
$$

(6.8)

Unlike in section 3 we will consider the term in Eq. (2.6) as a perturbation and solve the equations of motion in the absence of it. The mass eigenvalues are then given as $\Omega_n = n$ and the mass eigenstates can be read off from Eq. (3.2) by just putting $N_0 = N_\pi = 0$ there, i.e.

$$
H_1 = \frac{1}{\sqrt{2\pi}} \sum_{n=1}^{\infty} \cos ny \ H_1^{(n)}(x)
$$

$$
H_2 = \frac{1}{\sqrt{\pi}} \sum_{n=1}^{\infty} \sin ny \ H_2^{(n)}(x) .
$$

(6.9)

Now using the mode decomposition (6.9) we can write the boundary Lagrangian (2.6) as

$$
\mathcal{L}_{bd} = \frac{1}{\pi} \sum_{m,n=-\infty}^{\infty} \left[ M_0 - (-1)^{m+n} M_\pi \right] H_1^{(m)} H_1^{(n)} .
$$

(6.10)

The renormalization of the boundary mass parameters $M_f$ is given by loop diagrams induced by the quartic Lagrangian (6.8) with one or more $M_f$-insertions $^{15}$. Since the leading divergence is given by diagrams with one mass insertion, we will concentrate in diagrams as those in Fig. 4.

$^{15}$In this toy model the diagram with zero mass insertions will be quadratically divergent due to the generation of a localized Fayet-Iliopoulos (FI) term $^{19}$. This can be seen as a renormalization of the supersymmetric mass term $M$ and is clearly separable from the renormalization of the soft mass terms $M_f$. Of course one could avoid the generation of such terms by considering a second Higgs which does not interfere with the EW symmetry breaking process (as e.g. a second hypermultiplet with a (large) positive squared mass zero mode).
Figure 4: One-loop diagram renormalizing $M_f$.

The contribution from the diagrams in Fig. 4 is proportional to the factor

$$I = \frac{1}{\pi} \left[ M_0 - (-1)^{m+n} M_\pi \right] g^2 J$$

where $g = g_5/\sqrt{\pi}$ is the 4D gauge coupling, $J$ is given by the Feynman integral

$$J = \sum_{k=-\infty}^{\infty} \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2 + (k + m)^2} \frac{1}{p^2 + (k + n)^2}$$

$$= \sum_{\ell=-\infty}^{\infty} \int \frac{d^4 p \, dz}{(2\pi)^4} \frac{1}{p^2 + (z + m)^2} \frac{1}{p^2 + (z + n)^2} e^{2i\ell z}$$

and we have made use of Poisson resummation.

The propagators in (6.12) have poles in the complex $z$-plane at locations $z = -n \pm ip$ and $z = -m \pm ip$. In this way for $\ell \neq 0$ the $z$-integrations contour can be closed by an infinite semicircle. Picking the residues of the corresponding poles provides the factor

$$e^{-2\pi |\ell| p}$$

that makes the integrand in the remaining integral to exponentially converge in the limit $p \to \infty$ and the corresponding integral to be finite. However for $\ell = 0$ there appears a linear divergence. In fact one can write

$$J = \int \frac{d^4 p \, dz}{(2\pi)^4} \frac{1}{p^2 + (z + m)^2} \frac{1}{p^2 + (z + n)^2} + \text{finite terms}$$

$$= \frac{1}{64\pi} \Lambda + \text{finite terms}$$

where $\Lambda$ is the ultraviolet (UV) cutoff. One can interpret the result in (6.13) as a linear renormalization of the brane mass terms as

$$N_f = M_f (1 + \Delta), \quad \Delta = \frac{g^2}{64\pi} \Lambda R + \cdots$$
Notice that to leading order the radiative corrections to the boundary mass terms $\Delta$ are boundary independent. Therefore the condition $M_0 = M_\pi$ is not spoiled by the (leading) correction in (6.14).

Now that we have the loop-corrected localized soft masses one can go back to section 3 and compute the bosonic spectrum to all orders in the boundary masses as in subsection 3.1. In fact for the model under consideration ($N_0 = N_\pi = N$) the bosonic zero mode is a tachyon with a mass [see Eq. (4.10)]

$$m_0^2 = -N^2(1 + \Delta)^2. \quad (6.15)$$

A final comment concerning the UV sensitivity of the tachyonic (Higgs) mass will help to clarify the nature of the supersymmetry breaking induced by the boundary bosonic masses. This breaking is soft from the point of view that it does not induces any cubic counterterm in the 5D theory. However the mass term renormalizes linearly on the boundary, which induces in turn a linear renormalization in the Higgs mass. This renormalization can be simply understood from dimensional analysis since the operator $M_f \Phi \Phi$, in terms of 4D fields, has dimension three while the gaugino mass operator in (5.14), $M_f \lambda \lambda$, has dimension four: while the former is linearly sensitive to the cutoff the latter is not. However this sensitivity does not destabilizes the Higgs mass for values of the cutoff $\Lambda R \lesssim 10^2$: in fact considering for simplicity the weak coupling, $g^2/64\pi \sim 2 \times 10^{-3}$ and $\Delta \lesssim 0.2$. Finally, in models with a single Higgs the quadratically divergent FI term is the dominant effect and we would require a lower cutoff ($\Lambda R \lesssim 1$) to keep this effect small.

Finally, we have embedded in this section, and for the sake of analyzing the supersymmetry breaking induced by boundary scalar masses, a $U(1)$ gauge theory in the interval. We will consider in the next section how the whole gauge group $SU(2) \otimes U(1)$ can be similarly embedded.

# 7 ELECTROWEAK BREAKING

We will now consider the case where the hypermultiplet is a doublet under the $SU(2)_L$ gauge symmetry. To this end we must generalize the formalism of the previous sections, where only one hypermultiplet was considered to one where there are two. The reality condition (2.2) is now written as

$$\Phi^i_\alpha = \epsilon^{ij} \rho_{\alpha \beta} \Phi^j_\beta. \quad (7.1)$$
where the tensor $\rho_{\alpha\beta}$ can be written in the form \[11\]
\[\rho = \text{diag}(\epsilon \oplus \epsilon) = 1 \otimes \epsilon \quad \text{or} \quad \rho_{\alpha\beta} = \delta_{\alpha_1\beta_1}\epsilon_{\alpha_2\beta_2} \quad (7.2)\]

In particular the reality condition for hyperscalars $\Phi_i^{\alpha} = \Phi_i^{\alpha_1,\alpha_2}$ is given by

\[\Phi_i^{\alpha_1,2} = (\Phi_1^{\alpha_1,1})^* \equiv \bar{\Phi}_{\alpha_1,1}, \quad \Phi_i^{\alpha_1,1} = -(\Phi_1^{\alpha_1,2})^* \equiv -\bar{\Phi}_{\alpha_1,2} \quad (7.3)\]

It is now easy to see that the generators of the symmetry group that preserve the reality constraint must satisfy

\[\rho T^A = -T^{A*}\rho. \quad (7.4)\]

The largest possible symmetry group is thus generated by

\[\{\sigma^2 \otimes 1, \quad \sigma^1 \otimes \sigma^i, \quad \sigma^3 \otimes \sigma^i, \quad 1 \otimes \sigma^i\} \quad (7.5)\]

which is the spinor representation of $SO(5)$. As we will see the BC’s will however break this to a subgroup and so does a nonzero mass term in the bulk.

The reality constraints for the boundary matrices $S$ and $T$ are given by Eqs. \[2.7\] and \[2.9\] with the substitution $\epsilon_{\alpha\beta} \rightarrow \rho_{\alpha\beta}$ where the operator $\rho$ is defined in Eq. \[7.2\]. We thus find the generalizations

\[S^T\rho = -\rho S \quad M^T\rho = -\rho M \quad (7.6)\]

while the constraint \[2.12\] remains unchanged. We conclude that $S$ and $M$ are $so(5)$ valued. We expect the biggest unbroken subgroup if we choose $S_0 \propto S_\pi \propto M$. In fact all such choices are equivalent and lead to an $SU(2) \otimes U(1)$ subgroup. The most convenient one is to take $S_f \propto 1 \otimes \sigma^3$ which leads to $SU(2) \otimes U(1)$ generated by

\[\{\sigma^2 \otimes 1, \quad \sigma^1 \otimes \sigma^3, \quad \sigma^3 \otimes \sigma^3, \quad 1 \otimes \sigma^3\} \quad (7.7)\]

The formal proof of supersymmetry of the action as well as the solution to the EOM go along similar lines as those followed in previous sections. In particular the mode decomposition for bosons and fermions is that given in Eqs. \[3.1\] and \[3.9\], respectively. In order to have a vectorlike fermion spectrum as well as unbroken $SU(2) \otimes U(1)$, we will fix $S_0 = -S_\pi = 1 \otimes \sigma^3$, i.e. $\omega = 1/2$. The remaining freedom we have for the fermionic parameters is $c_0 = -c_\pi = \pm 1$. With these parameters the Higgsino mass is given by \[4.4\] with $c_0 = \pm 1$ which for large $M$ gives for the lightest mode mass $m = 2M \exp(-\pi M R)$ for
\(c_0 = 1\) and \(m = M\) for \(c_0 = -1\) respectively. Note that for \(MR \gtrsim 1\) the Higgsino becomes too light for \(c_0 = +1\) and one should fix \(c_0 = -1\) instead.

We will now consider (for illustrative purposes) the model where we break supersymmetry by choosing \(\omega = 1/2\), \(N_0 = N_\pi \equiv N\). The mass of the Higgs boson doublet is then given by (4.10). The eigenstate of the (tachyonic) zero mode of the Higgs doublet is

\[
\Phi_{\alpha_1,1}(x,y) = N^{-1} e^{-Ny} H_{\alpha_1}(x), \quad \Phi_{\alpha_2,2}(x,y) = N^{-1} e^{-Ny}[H_{\alpha_1}(x)]^* \tag{7.8}
\]

all other components vanishing. Here \(H(x)\) is the 4D physical Higgs field and \(\Phi\) fulfills the BC’s with \(S_0 = -S_\pi = 1 \otimes \sigma^3\) and \(T_0 = -T_\pi = -\sigma^3\),

\[
\frac{1}{2}(1 + R_f)\Phi(x,y_f) = 0, \quad R_f = -\sigma^3 \otimes 1 \otimes \sigma^3 \tag{7.9}
\]

The normalization factor is determined to be \(N^2 = (1 - e^{-2\pi NR})/2N\). Notice that \(SU(2)_L \otimes U(1)_Y\) acts on the physical Higgs field \(H\) in the standard way, i.e. by the generators \(\frac{1}{2}\sigma^i, \frac{1}{2}\). The effective 4D theory is obtained by integrating over the extra dimension. The mass Lagrangian becomes

\[
\mathcal{L}_m = (N^2 - M^2) |H|^2 \tag{7.10}
\]

The quartic Lagrangian comes from integrating out the \(SU(2)_L \otimes U(1)_Y\) auxiliary fields \(\vec{X}^A\) where \(A = 1, 2, 3\) labels the generators of \(SU(2)_L, T^A,\) and \(\vec{X}^4\) the generator of \(U(1)_Y, Y\). From the action of the super-Yang-Mills and hypermultiplets

\[
\mathcal{L}_D = 2 \vec{X}^A \cdot \vec{X}^A + g_A \bar{\Phi}_i (\bar{\sigma}_R)^i_j \vec{X}^A (T^A)^\alpha_i \Phi^\alpha_j , \tag{7.11}
\]

where \(g_A\) is the 5D gauge coupling corresponding to the generator \(T^A\). Integration of \(\vec{X}^A\) in (7.11) yields

\[
\mathcal{L}_D = -\frac{1}{8} g_A^2 \left( \bar{\Phi} \bar{\sigma}_R \otimes T^A \Phi \right)^2 . \tag{7.12}
\]

Next we particularize (7.12) to the zero mode Higgs doublet \(^{17}\) of Eq. (7.8). We get the Lagrangian \(^{18}\)

\[
\mathcal{L}_D = -\frac{1}{8} \left( g_5^2 + g_5'^2 \right) |H|^4 e^{-4Ny} \frac{N^2}{N^2} \tag{7.13}
\]

\(^{16}\)We normalize the generators to \(\text{tr}\{T^A T^B\} = \frac{1}{2} \delta^{AB}\).

\(^{17}\)We can assume here that non-zero modes with masses controlled by \(1/R \approx \text{few TeV}\) are much larger than the weak scale and they have been integrated out.

\(^{18}\)For the \(SU(2)_L \otimes U(1)_Y\) group with 5D gauge couplings \(g_5\) and \(g_5'\).
Putting together Eqs. (7.10) and (7.13), expanding the neutral component of the Higgs doublet as $H^0 = h/√2 + iχ^0$ (where $h$ is the normalized Higgs field with a vacuum expectation value $⟨h⟩ = v = 246$ GeV) and integrating over the fifth dimension we obtain for the Higgs field the tree-level potential

$$V = -\frac{1}{2}(N^2 - M^2)h^2 + \frac{1}{32}(g^2 + g'^2)\kappa(\pi NR)h^4$$

(7.14)

where $g$ and $g'$ are the corresponding 4D gauge couplings $^1$ and $κ(\pi NR)$, defined by

$$κ(x) = x \coth(x),$$

(7.15)

comes from the normalization factor of the zero-mode wave function in (7.8). Fixing the minimum of the potential to the physical value $v$ one finds the tree-level Higgs mass as a function of the $Z$-boson mass $m_Z$

$$m_H^2 = κ(\pi NR)m_Z^2,
N^2 - M^2 = \frac{1}{2}m_H^2$$

(7.16)

Some comments about (7.16) are in order here. The Higgs mass in (7.16) is the tree-level mass. Its natural value is $m_Z$ as $κ(x) = 1 + x^2/3 + \ldots$ and for values of $N, M, m_H \simeq m_Z$, $NR \ll 1$ and $κ(\pi NR) \sim 1$, and the second equality in (7.10) is naturally satisfied. On the other hand, as in the minimal supersymmetric standard model (MSSM), to obtain the prediction of the physical Higgs mass radiative corrections should be added: they are controlled by top-quark mass and (logarithmically) by the mass and mixing angle of the third generation squarks. One should meet in this model the large $\tan β$ MSSM prediction for the SM-like Higgs mass. It seems however possible to enhance the Higgs mass with values of $N \gg m_Z$. In fact for $\pi NR ≫ 1$ the relation

$$m_H \simeq m_Z\sqrt{πNR}$$

(7.17)

holds. For instance for $NR \simeq 1$, $m_H \simeq 160$ GeV. Of course the price to pay is that some fine-tuning between $N$ and $M$ is required from (7.16). In general a measure of the fine-tuning $10^{-ε}$ can be given as

$$10^{-ε} \simeq \left[\frac{m_H^2}{N^2}\right] = (πRm_Z)^2 \frac{\coth(πNR)}{(πNR)}$$

(7.18)

$^1$4D and 5D gauge couplings $g_4$ and $g_5$ are related to each other as $g_5^2 = πRg_4^2$.  

24
A plot of $\varepsilon$ as a function of $N$ is presented in Fig. 5, where we have fixed $1/R \sim 4$ TeV in agreement with present bounds from electroweak precision measurements [20]. In particular

$$\varepsilon \sim 1\%$$

and $N \simeq 1$ TeV and a tree-level Higgs mass $m_H \simeq 100$ GeV.

Finally one-loop radiative corrections to the Higgs mass will correct the localized mass $N$ by the factor $1 + \Delta$, as it was described in section 6. However, as pointed out there, for moderate values of the cutoff the corrections should be under control and they will not destabilize the electroweak minimum.

### 8 Conclusions

In this paper we have analyzed the formalism of hypermultiplets propagating in the five-dimensional interval $[0, \pi R]$. We have written down an explicit supersymmetric bulk + brane action where the field boundary conditions are dynamically obtained from the action principle. The orbifold boundary conditions are obtained as particular cases. The theory is characterized by three vectors in $SU(2)_R$ space (two boundary unit vectors $\vec{s}_f$ and $\vec{t}_f$ and one bulk unit vector $\vec{p}$), and one boundary ($n_f$) and one bulk ($M$) scalars. A misalignment of the vectors $\vec{s}_f$ on the two boundaries gives rise to a supersymmetric mass for the hypermultiplet and that of the vectors $\vec{t}_f$ is interpreted as the Scherk-Schwarz supersymmetry breaking. Finally the presence of the boundary scalars $n_f$ is also a potential source of supersymmetry breaking if there is a mismatch between $n_f$ and $\vec{p} \cdot \vec{s}_f$. In fact we can define soft scalar masses $M_f$, as $n_f = \vec{p} \cdot \vec{s}_f + M_f/M$, that can break supersymmetry and electroweak symmetry at the tree level.

While the nature of the Scherk-Schwarz supersymmetry breaking was already clear,
and known to be equivalent to boundary gaugino masses for 5D vector multiplets, that of supersymmetry breaking by boundary hyperscalar masses is clearly an issue. In fact while it is known that the Scherk-Schwarz supersymmetry breaking is one-loop finite, it provides a two-loop linear divergence corresponding to the one-loop renormalization of the gauge coupling \[21\]. We have proven in this paper that localized hyperscalar masses have one-loop linear divergences corresponding to the renormalization of a dimension-three operator on the 4D boundary. As a consequence the electroweak minimum remains stable for values of the cutoff \(\Lambda R \lesssim 10^2\) which means that it does not spoil the little hierarchy.

For the particular example we have worked out in some detail, where supersymmetry and electroweak breaking are triggered at the tree level, the natural tree-level value of the Higgs mass is \(m_Z\) unless a fine tuning of parameters is done in which case it can be raised to somewhat higher values. It is worth investigating in the future the nature and softness of other possible supersymmetry and electroweak breaking patterns as well as the phenomenology of the models presented in this paper.

A Appendix: useful identities

In this appendix we present some useful identities to deal with fields obeying reality constraints. For fermions as in Eq. (2.3), we find the following rules for bilinears:

\[
\bar{\Psi} \sigma_H \gamma \Omega = -\alpha(\sigma_H) \bar{\Omega} \sigma_H \gamma \Psi,
\]

where \(\gamma = \{1, \gamma^M\}\). and \(\alpha(\sigma)\) is defined by \(\alpha(\sigma_0) = +1\) and \(\alpha(\sigma_i) = -1\). The reality properties are given by

\[
(\bar{\Psi} \sigma_H \gamma \Omega)^* = -\alpha(\sigma_H) \bar{\Psi} \sigma_H \gamma \Omega.
\]

For scalars as in Eq. (2.2), the corresponding transposition rule is

\[
\bar{\Phi} \sigma_R \otimes \sigma_H \Sigma = \alpha(\sigma_R) \alpha(\sigma_H) \bar{\Sigma} \sigma_R \otimes \sigma_H \Phi,
\]

while the reality properties read

\[
(\bar{\Phi} \sigma_R \otimes \sigma_H \Sigma)^* = \alpha(\sigma_R) \alpha(\sigma_H) \bar{\Phi} \sigma_R \otimes \sigma_H \Sigma,
\]
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