ROBUST SOLUTION FOR A MINIMAX REGRET HUB LOCATION PROBLEM IN A FUZZY-STOCHASTIC ENVIRONMENT

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Abstract. In the present paper, a robust approach is used to locate hub facilities considering network risks. An additional objective function, minimax regret, is added to the classical objective function in the hub location problem. In the proposed model, risk factors such as availability, security, delay time, environmental guidelines and regional air pollution are considered using triangular fuzzy-stochastic numbers. Then an equivalent crisp single objective model is proposed and solved by the Benders decomposition method. Finally, the results of both Benders decomposition and commercial optimization software are compared for different instances. Numerical instances were developed based on the well-known Civil Aeronautics Board (CAB) data set, considering different levels of uncertainty in parameters. The results show that the proposed model is capable of selecting nodes as sustainable hubs. Also, the results confirm that using Benders decomposition is more efficient than using classical solution methods for large-scale problems.

1. Introduction. A type of facility that functions as a switching node is called a hub. Using hubs reduces total network costs by reducing the number of connections between pairs of nodes. Nowadays hub facilities are used increasingly because of their economic and traffic reduction benefits. The hub location problem (HLP) is affected by uncertainty of parameters, as well as other problems. For example, due to uncontrollable factors such as equipment breakdown, congestion and volcanic eruption, some network parameters like demand, cost and time may not be known precisely. In this regard, modelling the problem in a deterministic environment seems to be unrealistic, and the occurrence of such events should be considered when simulating real-world situations.

In the real world, uncertain parameters can be classified into fuzzy and random parameters. A fuzzy set approach is used for problems with vague parameters, while
a stochastic approach is associated with obvious parameters where sufficient information is available to estimate the probability distribution function. The trend of variation of seasonal demand can be fitted by a distribution function, while extreme weather events are vague and usually cannot be recognized by a certain value. In the probabilistic approach, random variability is manipulated by a probability distribution function, while membership functions are applied to describe the vagueness in human thoughts in a fuzzy set approach (Chen and Pham [13]). So based on the uncertainty nature of problems, a stochastic or fuzzy approach is considered. For a thorough comparison of the two approaches, see Li et al. [32].

Hub location studies that focus only on fuzzy set or stochastic approaches alone may be incapable of describing the characteristics of real-world situations. Consider designing a hub network that includes safety and traffic, along with other parameters. Safety and traffic seem to be best described as a fuzzy set and a stochastic variable, respectively. Therefore, in such cases, a fuzzy-stochastic approach seems to describe the situation more realistically.

The benefits of both stochastic and fuzzy logic approaches are good reasons for research focusing on fuzzy-stochastic models. Other advantages of the incorporation of both approaches are simple procedures for combining human/expert knowledge (Lee et al. [31]).

Figure 1 depicts the impact of network risk to demonstrate the difference between failure and reliable design. Square and unnumbered nodes illustrate hubs and demand nodes, respectively. Figure 1a is related to the HLP in a regular situation without consideration of risk; Figure 1b is related to a network for which risk is considered. The level of risk is shown by the thickness of the hub nodes; thicker hub nodes have less failure risk.

The existing design difference between the two figures shows the impact of uncertain real-world situations in the resulting networks, i.e., consideration of risk can provide a reliable network that has a different design where reliable facilities are located.

To the best of the authors’ knowledge, a systematic model of a probabilistic fuzzy approach that directly considers hub network risks has not been addressed in the
literature. The capability of the ordinary HLP in combination with fuzzy-stochastic concepts may help to bring the model closer to real-world situations.

This paper proposes a sustainable hub location model in a fuzzy-stochastic environment. It is assumed that network transportation costs are stochastic, while network risk factors are fuzzy-stochastic because of both their random nature and the vagueness of their values. In contrast to previous studies, this approach has the following features:

• Direct consideration of network risks.
• Achieving a model that considers different sources of uncertainty that affect the HLP.
• Using minimax regret as a robust approach.
• Defining a fuzzy-stochastic risk objective function.
• Modeling of the hub network as a multi-objective model.
• Using the Benders decomposition (BD) method as a solution approach.

It is expected that modeling of the problem, including the above features and different sources of uncertainty, can lead to a reliable design for a hub network.

The rest of this paper is organized as follows: Section 2 is a review of previous studies of fuzzy and stochastic HLPs with risk aspects. A two-stage stochastic programming model is suggested in Section 3. In Section 4, the BD method is applied to solve the proposed model. Numerical results are given in Section 5, and the last section is dedicated to final considerations and conclusions.

2. Literature review. Based on the characteristics of the proposed model, which will be explained in more detail in this section, risk studies of problems in hub location, stochastic hub location, fuzzy hub location, multi-objective hub location, and the BD method as a solution approach are briefly reviewed.

2.1. Risk in hub location problems. In real-world situations, hub networks can be affected by uncontrolled events that can lead to disruptions in transshipment, such as tsunamis in the Asian region, offshore piracy attacks in Somalia, and a volcanic ash cloud in Europe. These factors are frequently unknown and unpredictable. Moreover, definition and quantification of risk is difficult; a close look at previous studies shows that there are different definitions of risk, and a general definition is hard to find. In a review of the literature on supply chain risk, Heckmann et al. [27] surveyed different approaches to definition of supply chain risk and risk measures. In their conclusions, they noted the vagueness of definitions of risk and the difficulties of controlling and monitoring risk factors.

To the best of the authors’ knowledge, there is no general definition of risk in the literature, but it can be defined as a chance of damage or loss. However, some researchers present different definitions. Mitchell [36] remarked that if $P_{\text{loss}}$ and $V_{\text{loss}}$ are considered as the probability and value of loss effect respectively, then the value of risk can be calculated as follows:

$$\text{Risk} = P_{\text{loss}} \times V_{\text{loss}}$$

Many different types of risk can have an effect on the design of transportation networks. Some are shown in Figure 2.

Chen et al. [12] provided an analytic network process (ANP) model for environmental risk assessment under natural disasters in international airport projects. Factors like earthquakes, typhoons, lightning, floods, droughts, sandstorms and subsidence were considered as environmental risks to evaluate location decisions.
Pishvae et al. [39] proposed a multi-objective probabilistic model to design a sustainable medical supply chain network. Economic, environmental and social criteria were defined as objective functions. The model was solved by an accelerated BD method.

Some authors have studied disruption as risks in supply chain networks, such as Snyder et al. [49], Jabbarzadeh et al. [29], Atoei et al. [3] and Garcia-Herreros et al. [22], but there has been little focus on risk management in HLPs.

2.2. Stochastic hub location problems. Uncertainty and non-deterministic data are real-world, inherent characteristics. In most HLPs, the flow between nodes is exactly known, while in some applications, such as SMS centres or telecommunication, this assumption is violated.

Location problems can be divided into deterministic and uncertain categories. Uncertainty may arise in demand and transportation costs and time. The concept of uncertainty in network problems is classified into single-stage and two-stage approaches by some researchers.

2.2.1. Single-stage hub location problems. Single-stage problems deal with decision-making with no subsequent recourse. Vasconcelos et al. [53] designed a hub network by using a decentralized management approach. In their model, the companies were chosen on the basis of the stochastic link cost of effective factors such as time and traffic. The HLP in the study by Marianov et al. [33] focused on airports as hubs in air transportation networks. They modeled the problem as an M/D/c queuing system. A chance constraint was defined for the number of airports in the queuing system and a tabu search algorithm (TS) was used for solving the model. Sim
et al. [47] studied a stochastic p-hub center problem with the objective function as maximum travel time. They assumed that travel time was a random variable with a normal distribution function, which was formulated as a chance constraint. Sheppard [46] modeled a location problem to minimize total cost. In another study, a stochastic single HLP was considered as a queuing system by de Camargo et al. [7]. The problem was modeled in an uncertain environment and solved by a hybrid BD method and outer approximations (OA).

2.2.2. Two-stage hub location problems. In many cases, there are both strategic and tactical decisions, and the strategic decisions should be made before a scenario occurs, while the tactical decisions are made after a scenario occurs. In such cases, a two-stage framework can be properly applied. In this approach, the strategic decisions are made by considering possible scenarios, and they are more important than the tactical decision. There are two main approaches to dealing with two-stage models: the first involves the expected value of the second-stage function; the second uses the sampling approach. Zhai et al. [57] developed a P-model to minimize risk by assuming that demands had a probability distribution function, considering the expected value approach. The P-model is a stochastic model with a probability objective function that is frequently applied to situations in which risk exists. The model was solved by a branch and bound (B&B) algorithm.

In some cases, solving the second stage may be time-consuming, so a sampling approach can be efficiently employed to approximate the recourse function. Snyder et al. [48] introduced a stochastic supply chain location problem considering risk pooling. Cost and allocation decisions were determined based on discrete scenarios. The model was solved by a Lagrange relaxation method. Yang [54] proposed a stochastic model for an aerial network HLP. In this model, demand values were changed in different seasons by considering three scenarios. Alumur et al. [2] modeled an HLP for minimizing the maximum regret. Demand and cost parameters were defined based on scenarios for single and multiple allocations. Contreras et al. [15] studied an HLP with uncertain demands and costs. In this model, transportation costs were based on different scenarios. A sample average approximation scheme was employed and the problem was solved by the BD method.

However, the features of the problem considered in the present paper, the proposed model is formulated in the form of a two-stage stochastic problem. Location and allocation decisions are defined as first- and second-stage variables, respectively.

2.3. Fuzzy hub location problems. In the real world, information on network problems is vague in nature. There are two main approaches to modeling location problems in uncertain environments: stochastic and fuzzy set approaches.

Some examples of fuzzy location problems are:

1. Locating emergency facilities for natural disasters in a geographical area, when the location and severity of disasters are often unpredictable;
2. Locating a telecommunication system in which some parameters, such as the reliability of flow between nodes, are uncertain. This uncertainty is due to time and costs, which lead to reliability that is not a random variable, but a fuzzy number. Thus, the reliability can be described by using ambiguous language.

In these cases, the fuzzy set approach is often used to model various HLPs. The fuzzy numbers introduced by Zadeh [56] can represent the data, so the fuzzy decision-making method is needed.
Yang et al. [55] investigated a $P$-hub center problem where fuzzy travel time was minimized as the objective function. A maximum coverage location problem was examined by Davari et al. [17], in which travel time was assumed to be in fuzzy numbers. Hybrid fuzzy simulation and a simulated annealing algorithm were used to solve the problem. Bashiri et al. [4] introduced a $P$-hub center problem to minimize the maximum fuzzy travel time. In their study, the fuzzy VIKOR method was used. Taghipourian et al. [50] defined a hub network in a fuzzy environment as well. Their model included a set of dynamic virtual hubs, which was used to supply the demand points in emergency conditions.

2.4. Multi-objective hub location problems. Generally, HLPs are divided into single- and multi-objective categories. Researchers have focused more on single-objective HLPs, but a few have worked on multi-objective HLPs. Costa et al. [26] defined a multi-objective capacitated HLP. Service time was included in the model as an additional objective function. Mohammadi et al. [37] modelled a multi-objective hub covering problem that minimized network costs and waiting times in a queuing system. A metaheuristic algorithm, namely an imperialist competitive algorithm, was used to solve the model. Another multi-objective model was proposed by Mohammadi et al. [38] for a hub covering problem with uncertain travel time. Minimax travel time was considered as the second objective function. Eghbali et al. [19] introduced a multi-objective reliable hub covering location problem that considered customer convenience. The model minimized the total cost and total number of intermediate links between origins and destinations. Then a non-dominated sorting genetic algorithm-II (NSGA-II) was used to solve the model. Tajbakhsh et al. [51] applied a multi-objective model to single-allocation HLPs as well. They proposed a multi-objective model for minimizing uncovered demands and solved it by a GA.

2.5. Benders decomposition in hub location problems. Various solution methods have been applied to network design problems in previous studies. Some are purely heuristic, while others are based on exact approaches. The BD method is an exact solution method that is used in the presence of complicated variables (Benders,[5]). There have been several studies in which HLPs were solved by the BD method (de Sa et al., [41]; de Sa et al., [42]; Meraen and Yaman, [34]; de Camargo et al., [9]; Gelareh and Nickel, [23]; de Camargo et al., [8]; de Camargo et al., [10]; Contreras et al., [14]; Contreras et al., [16]; de Sa et al., [43]), as well as other problems (Geoffrion, [25]; Gendron, [24]). However, most of these studies of hub location problems that use the BD method were modelled in deterministic environments.

Previous research on the subject is summarized in Table 1 to show a comparison with contributions of the present paper.

In real-world situations, some risk factors, such as natural disasters, exchange rates, congestion, equipment breakdowns, and man-made crises, are the main sources of risks in HLPs. These risks may arise in the form of stochastic variables as well.

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1. Multi-objective decision making
2. Weighted sum method
3. Weighted Chebyshev distance
4. Simulated annealing algorithm
5. Particle swarm optimization algorithm
6. Multi-objective imperialist competitive algorithm
TABLE 1. A brief review on recent researches of multi-objective HLP considering uncertainty.

| Reference                | Year | Model                  | Number of objectives | MOB approach | Risk assessment approach | Risk factors | Objective functions | Uncertainty | Solution approach |
|--------------------------|------|------------------------|----------------------|--------------|--------------------------|--------------|---------------------|-------------|------------------|
| Aye et al. [14]          | 2005 | Single-source network  | Single               |             | Expected value           | Cost         | Cost                |             | Scenario-based    |
| Jahangordi et al. [17]   | 2012 | Single-source network  | Single               |             | Expected value           | Cost         | Cost                |             | Lagrangian relaxation, GA |
| Asgey et al. [2]         | 2013 | Single-source network  | Multiple             | Complementary | Expected value | Cost/Reliability | Cost/Reliability | SORs derived from BD |
| Gouris Bhattacharya et al. [3] | 2014 | Single-source network  | Single               |             | Expected value           | Cost/Reliability | Cost/Reliability | SORs derived from BD |
| Palaneev et al. [18]     | 2011 | Multiple                | Fuzzy                |             | Environmental risks      | Cost         | Cost                |             | 
| Hanssens et al. [4]     | 2001 | HLP                     | Single               |             | Cost                     | Cost         | Cost                |             | NSGA-II           |
| Snyder et al. [49]       | 2007 | Location model          | Single               |             | Cost                     | Cost         | Cost                |             | Lagrangian relaxation, GA |
| Czys et al. [5]         | 2006 | HLP                     | Single               |             | Cost                     | Cost         | Cost                |             | NSGA-II           |
| Jabbarzadeh et al. [29]  | 2012 | Supply chain network    | Single               |             | Cost                     | Cost         | Cost                |             | Scenario-based Lagrangian relaxation, GA |
| Atoei et al. [3]         | 2013 | Supply chain network    | Multiple             | ε-constrained | Expected value | Environmental, macro and supply risks | Cost/Reliability | Reliability/Capacity | Accelerated BD |
| Garcia-Herreros et al. [22] | 2014 | Supply chain network    | Single               |             | Operational costs        | Cost         | The availability of DC |             | 
| Pishvaee et al. [39]    | 2014 | Supply chain network    | Multiple             | WSM          | Environmental, macro and security risks | Cost/Reliability | Reliability/Capacity | Environmental, macro and security risks | 
| Marianov et al. [33]    | 2003 | HLP                     | Single               |             | Cost                     | Cost         | Cost                |             | 
| Snyder et al. [49]       | 2007 | Location model          | Single               |             | Cost                     | Cost         | Cost                |             | Lagrangian relaxation, GA |
| Camargo et al. [9]     | 2008 | HLP                     | Single               |             | Cost                     | Cost         | Cost                |             | 
| Sim et al. [47]         | 2009 | P-hub center problem   | Single               |             | Time                     | Time         | Time                | Chance constraint | Radial heuristic & Local heuristic |
| Zhai et al. [57]        | 2012 | HLP                     | Single               |             | Probability function     | Macro risks  | Services level      |             | 
| Yang [23]               | 2007 | HLP                     | Single               |             | Cost                     | Cost         | Cost                |             | 
| Yang et al. [1]         | 2018 | HLP                     | Single               |             | Cost                     | Cost         | Cost                |             | 
| Costa et al. [26]       | 2008 | Multiple                | WSM/WCD              |             | Cost/Time                | Cost/Time    | Cost                |             | Accelerated BD |
| Yang et al. [55]        | 2013 | P-hub center problem   | Single               |             | Optimisation of waiting time | Time         | Time                |             | 
| Mahammadi et al. [38]  | 2013 | Hub covering problem   | Multiple             | MOICA        | Environmental, operational risks | Cost/Max time | Reliability of path | Time Probability | MOICA |
| Eghbali et al. [19]     | 2013 | Hub covering problem   | Multiple             |             | Cost/Intermediate links  | Reliability of path | Reliability of path | Reliability | NSGA-II           |
| Propose model            | 2018 | HLP                     | Multiple             | WSM          | Expected value           | Environmental, macro and supply risks | Cost/Reliability | Reliability/Capacity | Scenario-based BD |

as fuzzy sets. Most researchers who have considered risks in HLPs have focused on either fuzzy or stochastic variables. It is also worth noting that most have considered only disruption risks, while there are other types of risks. To the best of the authors’ knowledge, there has been no research dealing with fuzzy-stochastic HLPs in risk environments.

As can be seen in Table 1, there have been few papers that considered different types of risks in HLPs. So a robust model is required to consider fuzzy-stochastic HLPs. The present paper considers a model with random fuzzy criteria and minimax regret multiple objectives. To create a more realistic network design, risk factors such as availability, regional air pollution and emergency conditions are considered.

In summary, the proposed model has several features:

- Improving a systematic HLP using a probabilistic fuzzy approach that directly considers both stochastic and fuzzy parameters.
- Using the regret criterion as a robust approach in the model.
- Applying the BD method as a solution approach.

Hence, the model quantifies risk concepts, in contrast with most previous research.

2.6. Motivation for this study. In the real world, information on transportation network characteristics, such as link failures, safety, extreme weather conditions and congestion, is mostly vague and unpredictable, and complete information is usually unavailable. On the other hand, optimization of several conflicting factors is often in hand, where it can be considered in the form of a multi-objective function.

The present study of fuzzy-stochastic HLPs was motivated by the existence of several real-world examples of network design:

1. The volcanic eruption of Mount Kelud in Indonesia in 2014. Because of ash, a great number of flights were canceled and seven airports
were closed because of ash (see http://www.bigstory.ap.org/article/indonesias-mount-kelud-java-island-erupts).

2. The eruption of Eyjafjallajkull in Iceland on May 16, 2010. This event had several worldwide effects, and the European air industry suffered many problems. In particular, the problems worsened due to the closing of two international hub airports in Paris and Frankfurt. Overall, 95,000 flights were cancelled (see http://en.wikipedia.org/wiki/2010_eruptions_of_Eyjafjallaj%C3%B6kull)

3. In the design of the Tehran subway system in 2011, some stations were located without considering all the factors related to geographic location, leading to damage to some monuments.

In these events, some uncontrolled factors, such as extreme weather conditions, regional air pollution, link failures and safety were not considered in network design, which led to unreliable networks. In addition, cases such as the offshore piracy attacks in Somalia, tsunamis and Hurricane Katrina have shown that more characteristics of stochastic environments should be considered. When such events occur, system accuracy can be affected, and unpredicted costs imposed on the system.

3. The proposed model for the robust hub location problem in a fuzzy-stochastic environment.

3.1. Model description. Real-world situations often include unpredictable events such as congestion, extreme weather conditions and equipment breakdowns, which shows the necessity of consideration risk in hub network design and the inefficiency of deterministic problems. These risk factors may arise in the context of fuzzy sets as well as stochastic variables. In this regard, the proposed robust fuzzy-stochastic HLP is expected to provide for more realistic network design, in which a minimax regret criterion is used as a robust approach to considering stochastic network costs. Because the parameters are uncertain, not all of the strategic and tactical decisions can be determined simultaneously, so the proposed model is formulated based on two-stage stochastic programming. Location decisions are determined based on all possible scenarios, but allocation variables are defined for each scenario. The location of hub facilities is determined in the first stage (here-and-now). The optimal routes for the commodities are fixed as recourse decisions in the second stage for each scenario $s \in S$ (wait-and-see). The parameters are considered for different sets of scenarios according to different levels of network risk. The proposed model includes two objective functions. The first is related to minimax regret, which minimizes the maximum differences between total network costs and the best objective function value for each scenario. The second minimizes expected location fuzzy risks. It is worth noting that network risk factors are considered by using triangular fuzzy-stochastic numbers. The details of the model are described below.

Indices

\begin{itemize}
\item $i, j$ Potential hub nodes ($i, j \in H$).
\item $o$ Origin nodes ($o \in N$).
\item $d$ Destination nodes ($d \in N$).
\item $s$ Scenarios ($s \in S$).
\item $k$ Commodities ($k \in K$).
\end{itemize}
Parameters

- $f_i$: Fixed cost for locating a hub facility at node $i \in H$.
- $c_{ki}^s$: Stochastic transportation cost for commodity $k \in K$ from origin $o \in N$ to hub $i \in H$ based on scenario $s \in S$.
- $c_{ij}^k$: Stochastic transportation cost for commodity $k \in K$ from hub link $i \in H$ to $j \in H$ based on scenario $s \in S$.
- $c_{jd}^k$: Stochastic transportation cost for commodity $k \in K$ from hub $j \in H$ to destination $d \in N$ based on scenario $s \in S$.
- $u_{od}^k$: Stochastic flow for commodity $k \in K$ from origin $o \in N$ to destination $d \in N - \{o\}$ based on scenario $s \in S$.
- $F_{oijd}^k$: Stochastic unit transportation cost for commodity $k \in K$ from origin $o \in N$ to destination $d \in N - \{o\}$ via hubs $i \in H$ and $j \in H$ based on scenario $s \in S$.
- $p_s$: Probability of occurrence of scenario $s \in S$ ($\sum_{s \in S} p_s = 1$).
- $\pi_i^s$: Fuzzy-stochastic risk factor for potential hub node $i \in H$ to be selected as a hub based on scenario $s \in S$.
- $\varphi$: Discount factor to hub links ($0 \leq \varphi \leq 1$).
- $\gamma$: Collection discount factor ($\gamma \leq \varphi$).
- $\tau$: Distribution discount factor ($\tau \leq \varphi$).

Decision variables

- $X_{ki}^s$: Is equal to 1, if commodity $k \in K$ flows from origin $o \in N$ to hub $i \in H$ based on scenario $s \in S$.
- $X_{ij}^k$: Is equal to 1, if commodity $k \in K$ flows from hub $i \in H$ to hub $j \in H$ based on scenario $s \in S$.
- $X_{jd}^k$: Is equal to 1, if commodity $k \in K$ flows from hub $j \in H$ to destination $d \in N$ based on scenario $s \in S$.
- $Y_{oijd}^k$: Is equal to 1, if commodity $k \in K$ flows from origin $o \in N$ to destination $d \in N - \{o\}$ via hubs $i \in H$ and $j \in H$ based on scenario $s \in S$.
- $Z_i$: Is equal to 1, if a hub facility is located in node $i \in H$.

Using discount factors, the value of $F_{oijd}^k = \gamma c_{oi}^k + \varphi c_{ij}^k + \tau c_{jd}^k$ is defined as the stochastic unit transportation cost for commodity $k \in K$ from origin $o \in N$ to destination $d \in N - \{o\}$ via hubs $i \in H$ and $i \in H$ based on scenario $s \in S$.

The assumptions of the model are as follows:

- One or two hub facilities can be located in each path; i.e., direct connection between non-hub nodes is not allowed.
- A non-hub node can be allocated to more than one hub node; i.e., a multi-allocation structure is used in the model.
- The structure of the model is assumed to be multi-commodity.
- Some features, like easy interpretation, computational efficiency and easy analysis, commonly lead to the use of the pattern of triangular fuzzy numbers.
- Therefore, risk factors are defined as triangular fuzzy-stochastic numbers.
- The set of potential hub nodes is separate from the set of demand nodes.

The sequence of modeling consist of the following steps:

**Step 1:** Calculating the integrated fuzzy-stochastic risk factors using the Fuzzy TOPSIS\(^7\) method.

\(^7\)Technique for order preference by similarity to ideal solution
Step 2: Modeling the HLP in the form of a two-stage multi-objective fuzzy-stochastic HLP.

Step 3: Defining the equivalent crisp model using the TH$^8$ method (Torabi and Hassini, [52]).

Step 4: Defining the equivalent integrated single-objective model using the WSM method.

Fuzzy-stochastic risk factors as model parameters should be defined before optimization, so they are calculated according to the related different fuzzy characteristics.

TOPSIS is a practical multi-criteria decision-making (MCDM) method for ranking a number of alternatives with respect to various criteria. The preferred alternatives are selected based on the shortest distance from the positive ideal solution (PIS) and the longest distance from the negative ideal solution (NIS) (for more details, see Hwang and Yoon, [28]).

Definition 3.1. The rectilinear distance between two triangular fuzzy number \( \tilde{a}_1 = (l_1, m_1, h_1) \) and \( \tilde{a}_2 = (l_2, m_2, h_2) \) is calculated as:

\[
\tilde{d}(\tilde{a}_1, \tilde{a}_2) = (|l_2 - h_1|, |m_2 - m_1|, |h_2 - l_1|)
\]

The potential hub nodes and risk factors are defined as alternatives and criteria, respectively. The aim is to integrate risk factors such as safety, availability and weather conditions into a single index for each potential hub node. The fuzzy TOPSIS method can be applied efficiently when the weights of the criteria and the ratings of most of the alternatives are imprecise. Linguistic variables, the form of triangular fuzzy numbers, are utilized to assess the weights of the criteria and the ratings of the alternatives. The following steps are applied to compute integrated fuzzy-stochastic risk factors using the fuzzy TOPSIS method:

Step 1: Identify the appropriate linguistic rating \( \tilde{\pi}_{L_{ks}il} = (\pi_{L_{ks}i1}, \pi_{L_{ks}i2}, \pi_{L_{ks}in}) \) for each potential hub node \( i \in H \) with respect to the risk factor \( l = 1, \ldots, n \) for commodity \( k \in K \) based on scenario \( s \in S \).

Step 2: Construct a decision matrix after transforming linguistic variables to fuzzy values for each commodity \( k \in K \) under scenario \( s \in S \) as follows:

\[
\tilde{D}^s = \begin{pmatrix}
A_1^s & R_{k1}^s & R_{k2}^s & \cdots & R_{kn}^s \\
& \tilde{\pi}_{L_{k1}i1} & \tilde{\pi}_{L_{k1}i2} & \cdots & \tilde{\pi}_{L_{kn}i1} \\
& \tilde{\pi}_{L_{k1}i2} & \tilde{\pi}_{L_{k2}i2} & \cdots & \tilde{\pi}_{L_{kn}i2} \\
& \vdots & \vdots & \ddots & \vdots \\
& \tilde{\pi}_{L_{km}i1} & \tilde{\pi}_{L_{km}i2} & \cdots & \tilde{\pi}_{L_{km}in}
\end{pmatrix}
\]

Where \( A_i^s \) are potential hub nodes as alternatives and \( R_{kl}^ks \) represents the risk factors as criteria for commodity \( k \in K \) in each scenario \( s \in S \).

Step 3: Normalize the fuzzy decision matrix:

\[
\tilde{Q}^{ks} = \left[ \tilde{q}_{il}^{ks} \right]_{m \times n} \quad i \in H, \ l = 1, \ldots, n , k \in K, \ s \in S
\]

In which normalization process is implemented as:

\[
\tilde{q}_{il}^{ks} = \begin{pmatrix}
\pi_{L_{k1l}i1} & \pi_{L_{k2l}i2} & \pi_{L_{knl}in}
\end{pmatrix}
\]

\[ l \in L_1 \]

\[ ^8 \text{Torabi and Hassini} \]
\[ \tilde{q}_{il}^{ks} = \left( \frac{\pi_{1il}^{k-} - \pi_{1il}^{k+}}{\pi_{1il}^{k+} - \pi_{1il}^{k-}}, \frac{\pi_{2il}^{k-} - \pi_{2il}^{k+}}{\pi_{2il}^{k+} - \pi_{2il}^{k-}}, \frac{\pi_{3il}^{k-} - \pi_{3il}^{k+}}{\pi_{3il}^{k+} - \pi_{3il}^{k-}} \right) \quad l \in L_2 \]  

Where

\[ \pi_{il}^{k+} = \max_i \pi_{il}^{k+} \quad l \in L_1 \]  

\[ \pi_{il}^{k-} = \min_i \pi_{il}^{k+} \quad l \in L_2 \]

Call \( L_1 \) and \( L_2 \) sets of benefit-related and cost-related criteria, respectively.

**Step 4:** Construct a weighted normalized fuzzy decision matrix by using \( \tilde{w}_{il}^{ks} \) as the weight vector of risk factors and the normalized fuzzy decision matrix as follows:

\[ \tilde{V}_{il}^{ks} = \left[ \tilde{v}_{il}^{ks} \right]_{m \times n} \quad i \in H, l = 1, \ldots, n, k \in K, s \in S \]  

\[ \tilde{v}_{il}^{ks} = \tilde{q}_{il}^{ks} \odot \tilde{w}_{il}^{ks} = (v_{il}^{ks \cdot 1}, v_{il}^{ks \cdot 2}, v_{il}^{ks \cdot 3}) \] (9)

**Step 5:** Determine the fuzzy positive and negative ideal solutions.

To address the weighted normalized fuzzy decision matrix, it is obvious that the range of the elements \( \tilde{v}_{il}^{ks} \) belongs to the closed interval \([0, 1]\). So the PIS and NIS can be defined as:

\[ \tilde{A}_{il}^{+ks} = (\tilde{v}_{il}^{+k1}, \tilde{v}_{il}^{+k2}, \ldots, \tilde{v}_{il}^{+kn}) \quad k \in K, s \in S \] (10)

\[ \tilde{A}_{il}^{-ks} = (\tilde{v}_{il}^{-k1}, \tilde{v}_{il}^{-k2}, \ldots, \tilde{v}_{il}^{-kn}) \quad k \in K, s \in S \] (11)

So that

\[ \tilde{v}_{il}^{+k} = (1, 1, 1) \quad k \in K, s \in S \] (12)

\[ \tilde{v}_{il}^{-k} = (0, 0, 0) \quad k \in K, s \in S \] (13)

**Step 6:** Calculate the fuzzy distance values from the positive and negative ideal solutions of node \( i \in H \) under scenario \( s \in S \) for commodity \( k \in K \), as follows.

\[ \tilde{d}_{il}^{+ks} = \sum_{k=1}^{n} \tilde{d} (\tilde{v}_{il}^{ks} \cdot \tilde{v}_{il}^{+k}) \quad i \in H, k \in K, s \in S \] (14)

\[ \tilde{d}_{il}^{-ks} = \sum_{k=1}^{n} \tilde{d} (\tilde{v}_{il}^{ks} \cdot \tilde{v}_{il}^{-k}) \quad i \in H, k \in K, s \in S \] (15)

Where \( \tilde{d} (\tilde{v}_{il}^{ks} \cdot \tilde{v}_{il}^{+ks}) \) denotes the rectilinear distance between two fuzzy numbers and is computed by equation (1).

**Step 7:** Aggregated fuzzy-stochastic risk factors for potential hub node \( i \in H \) under scenario \( s \in S \) can be calculated according to the following equation:

\[ \tilde{\pi}_{il}^{s} = \sum_{k \in K} \frac{\tilde{d}_{il}^{-ks}}{\tilde{d}_{il}^{+ks} + \tilde{d}_{il}^{-ks}} \quad i \in H, s \in S \] (16)

It is worth noting that the aggregated risk factors are obtained as \( \tilde{\pi}_{il}^{s} = (\pi_{il}^{s \cdot 1}, \pi_{il}^{s \cdot 2}, \pi_{il}^{s \cdot 3}) \) at the end of the algorithm. These integrated risk factors will be used as model inputs. Then, the proposed robust HLP in a fuzzy-stochastic environment, which is called the RHLFSE, is formulated as follows:

\[ \min_{s \in S} \max_{s \in S} \partial_s \] (17)
min \( \Gamma \approx \sum_{i \in H} \sum_{s \in S} p^s \pi^s_i Z_i \) \hspace{1cm} (18)

Subject to:

\[
\begin{align*}
\sum_{i \in H} \sum_{j \in H} X_{ij}^s & = 1 & & o \in N, \ d \in N \setminus \{o\}, k \in K, s \in S \hspace{1cm} (19) \\
\sum_{j \in H} Y_{ij}^s & \leq Z_i & & o \in N, \ d \in N \setminus \{o\}, i \in H, k \in K, s \in S \hspace{1cm} (20) \\
\sum_{i \in H} Y_{ij}^s & \leq Z_j & & o \in N, \ d \in N \setminus \{o\}, j \in H, k \in K, s \in S \hspace{1cm} (21) \\
X_{oi}^s & \leq Z_i & & o \in N, i \in H, k \in K, s \in S \hspace{1cm} (22) \\
X_{ij}^s & \leq Z_i & & i, j \in H, k \in K, s \in S \hspace{1cm} (23) \\
X_{ij}^s & \leq Z_j & & i, j \in H, k \in K, s \in S \hspace{1cm} (24) \\
Y_{oi}^s & \leq X_{oi}^s & & o \in N, d \in N \setminus \{o\}, i \in H, k \in K, s \in S \hspace{1cm} (25) \\
Y_{ij}^s & \leq X_{ij}^s & & o \in N, d \in N \setminus \{o\}, i, j \in H, k \in K, s \in S \hspace{1cm} (26) \\
Y_{ij}^s & \leq X_{ij}^s & & o \in N, d \in N \setminus \{o\}, i, j \in H, k \in K, s \in S \hspace{1cm} (27) \\
X_{oi}^s & \leq \sum_{j \in H \in N \setminus \{o\}} \sum_{d \in N} Y_{oijd}^s & & o \in N, i \in H, k \in K, s \in S \hspace{1cm} (29) \\
X_{ij}^s & \leq \sum_{o \in N} \sum_{j \in H \in N \setminus \{o\}} Y_{oijd}^s & & i, j \in H, k \in K, s \in S \hspace{1cm} (30) \\
X_{ij}^s & \leq \sum_{o \in N \setminus \{d\}} \sum_{i \in H} Y_{oijd}^s & & d \in N, j \in H, k \in K, s \in S \hspace{1cm} (31) \\
\partial_s = \sum_{i \in H} f_i Z_i + \sum_{o \in N} \sum_{i \in H} \sum_{j \in H} \sum_{d \in N \setminus \{o\}} \sum_{k \in K} \sum_{s \in S} \sum_{i \in H} w_{od}^s F_{oijd}^s Y_{oijd}^s - \Psi_s^* & & s \in S \\
\hspace{1cm} (32)
\end{align*}
\]

Where the symbol ‘\( \approx \)’ denotes the fuzzified version of ‘\( = \)’. Also, \( \Psi_s^* \) is the optimal value of the following model and \( \partial_s \) is the regret value related to the cost objective for each scenario \( s \in S \).

\[
\begin{align*}
\min_{s \in S} & \sum_{i \in H} f_i Z_i + \sum_{o \in N} \sum_{i \in H} \sum_{j \in H} \sum_{d \in N \setminus \{o\}} \sum_{k \in K} \sum_{s \in S} \sum_{i \in H} w_{od}^s F_{oijd}^s Y_{oijd}^s - \Psi_s^* & & s \in S \\
\hspace{1cm} (34)
\end{align*}
\]

Subject to: (3)-(15)

\[
X_{oi}^s, X_{ij}^s, X_{ij}^s, Y_{ij}^s \geq 0, Z_i \in \mathbb{R}^{[H]} \hspace{1cm} o \in N, \ d \in N \setminus \{o\}, i, j \in H, k \in K, s \in S \hspace{1cm} (33)
\]

Equations (17)-(18) present objective functions. The first minimizes the maximum differences between total network costs and the best objective function value of each scenario, and the second minimizes expected location fuzzy risks. Constraint (19) shows that there is only one path from each origin \( o \in N \) to destination \( d \in N \setminus \{o\} \). Constraint (20) guarantees that until a hub facility is not located in node \( i \in H \), no distribution can occur from the mentioned hub node. Constraint (21) confirms that if a hub facility is located in node \( j \in H \), then collection can occur from other nodes to the mentioned hub node. Constraints (22)-(23) guarantees that a link from a hub node can be allocated if an origin hub facility is located in node \( i \in H \). Similarly, constraints (24)-(25) guarantee that a link can be allocated only
for an active destination hub node \( j \in H \). Constraints (26)-(31) show that a path from demand node \( o \in N \) to destination node \( d \in N \) through hub nodes \( i \in H \) and \( j \in H \) can exist if related links are constructed. Constraint (32) defines regret for each scenario, i.e., the difference between the cost objective and the optimal cost objective value is calculated for each scenario \( s \in S \). And finally, constraint (33) defines the variables. Variables \( X^k_{oi}^s, X^k_{ij}^s, X^k_{jd}^s, Y^k_{oijd}^s \) are binary in nature, but for simplification of calculations they are considered in the form of continuous variables.

3.1.1. Extracting the equivalent crisp model. According to the literature, the LH\(^9\), MW\(^10\) and LZL\(^11\) methods are the most popular multi-objective solution approaches. The LZL method frequently generates solution which domain the solutions of LH algorithm. But the two-phase form of the LZL method requires more computational effort than other single-phase approaches such as LH and MW, for which reason it seems inefficient. The MW method may yield low-quality, unbalanced solutions that are not acceptable. Consequently, in order to improve the efficiency of these approaches, a hybrid single-phase method based on the MW and LH methods, called the TH method, is applied in the proposed model to define an equivalent crisp model (for more details see Torabi and Hassini, 2008).

Assume that \( \tilde{CO} = (a_1, a_2, a_3) \) is a triangular possibility distributed objective function. Based on the TH method, minimizing the mentioned fuzzy objective function requires minimization of \( a_2 \), maximization of \( (a_2 - a_1) \), and minimization of \( (a_3 - a_2) \). According to the above descriptions, the multi-objective fuzzy-stochastic model is first converted into an equivalent auxiliary crisp multi-objective stochastic model as follows:

\[
\begin{align*}
\min & \quad \max_{s \in S} \partial_s \\
\max & \quad \Gamma_1 = \sum_{i \in H} \sum_{s \in S} p^s (\pi^s_{2i} - \pi^s_{1i}) Z_i \\
\min & \quad \Gamma_2 = \sum_{i \in H} \sum_{s \in S} p^s \pi^s_{2i} Z_i \\
\min & \quad \Gamma_3 = \sum_{i \in H} \sum_{s \in S} p^s (\pi^s_{3i} - \pi^s_{2i}) Z_i
\end{align*}
\]

Subject to: (19)-(31)

\[X^k_{oi}^s, X^k_{ij}^s, X^k_{jd}^s, Y^k_{oijd}^s \geq 0, \quad Z_i \in \mathbb{B}^{|H|}, \quad o \in N, \quad d \in N \setminus \{o\}, \quad i, j \in H, k \in K, s \in S\]

3.1.2. Multi-objective optimization methodology. At this stage, what is desired is an equivalent integrated single-objective model to deal with the multi-objectivity. In the beginning, a linear composite objective function, the WSM, is employed (Deb \([18]\)). Other authors have used the WSM before (Caballero et al. \([6]\); Alumar and Kara \([1]\); and Erkut and Alp \([20]\)). Since there are different units in objective functions, they must be scalarized. The ideal and nadir values of the objective functions need to be defined. The ideal values of the objective functions are given by \( \partial^\ast, \Gamma_1^\ast, \Gamma_2^\ast \) and \( \Gamma_3^\ast \). Also \( \partial^{\text{max}}, \Gamma_1^{\text{min}}, \Gamma_2^{\text{max}} \) and \( \Gamma_3^{\text{max}} \) are considered as

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\(^9\)Lai and Hwang
\(^10\)Modified version of Werner’s approach
\(^11\)Li and Zhang and Li
the nadir values. Moreover, the weights of the objective functions are denoted by \( \theta_n \), \( \sum_n \theta_n = 1, \theta_n > 0 \), \( n = 1, \ldots, 4 \).

Finally, the composite function can be extracted according to the following equation.

\[
\min \omega = \theta_1 \left( \partial' - \partial'^* \right) + \theta_2 \left( \Gamma_1^* - \sum_{s \in S} \sum_{i \in \mathcal{H}} p^i \left( \pi^s_{2i} - \pi^s_{1i} \right) Z_i \right) \\
+ \theta_3 \left( \sum_{s \in S} \sum_{i \in \mathcal{H}} p^i \pi^s_{2i} Z_i - \Gamma_2^* \right) + \theta_4 \left( \sum_{s \in S} \sum_{i \in \mathcal{H}} p^i \pi^s_{2i} Z_i - \Gamma_3^* \right)
\]

Subject to: (19)-(31)

\[
\partial' \geq \sum_{i \in \mathcal{H}} f_i Z_i + \sum_{o \in \mathcal{N}} \sum_{i \in \mathcal{H}} \sum_{d \in \mathcal{N} \setminus \{o\}} \sum_{k \in \mathcal{K}} \sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{H}} \sum_{j \in \mathcal{H}} \sum_{d \in \mathcal{N}} n^{k}_{oi} X^{h}_{ij} X^{k}_{jd} X^{s}_{oijd} - \Psi^*_s \quad s \in S
\]

(39)

Where \( \omega \) is the optimum value of the equivalent single objective function and \( \partial' \) is an auxiliary variable for linearizing the minimax model.

4. Solution method. The Benders decomposition method is a decomposition approach that is based on partitioning of mixed integer programs (Benders [5]). Because of the variable types used in the problem, the proposed model has a special structure in the constraints (20)-(25), a block structure; i.e., fixing the complicated variables (integer variables) allows the MIP problem to be decomposed into linear and integer programming problems.

4.1. Motivation for applying the BD method on HLPs. It is known that HLPs fall in the NP-hard complexity class so that only small instances can be solved optimally in a reasonable amount of time (Kara [30]). Also, the RHLP-FSE model is an extended version of the classical HLP with more complexity. So the NP-hardness of the RHLP-FSE model can be shown. In the worst case, where \(|i| = |j| = |o| = |d| = n, |k| = k, |s| = s\), the proposed model consists of \( O(n) \) binary variables, \( O(n^4ks) \) continuous variables, and \( O(n^4) \) constraints. The complexity of the RHLP-FSE in comparison with the classical HLP that was introduced by Campbell [11] is demonstrated in Table 2.

| Problem    | No. of constraints | No. of variables |
|------------|--------------------|------------------|
| RHLP-FSE   | \( 3n^4ks - n^4ks + 5n^2ks - nks \) | \( n\) \(, n^4ks \) \(, 3n^2ks \) |
| Classical HLP | \( 2n^4 + n^2 \) | \( n\) \(, n^4 \) |

In this regard, it seems that by decomposing the problem into two smaller problems, larger instances can be solved efficiently. This provides the motivation to apply the BD method to solve the model.
4.2. Benders decomposition algorithm. For given vectors $\hat{Z}_i \in \mathbb{B}_H^k$ the subproblem (SP) in the space variables $X_{oi}^k, X_{ij}^k, X_{jd}^k, Y_{oi}^k, \hat{\theta}$ can be given as follows:

$$\text{min} \quad \Omega = \theta_1 \left( \frac{\partial^*}{\partial \max - \partial^*} \right)$$

Subject to :(19), (26) - (31)

$$\sum_{j \in H} v_{oi}^k s \leq \hat{Z}_i \quad o \in N, \ d \in N\{o\}, \ i \in H, \ k \in K, \ s \in S \quad (41)$$

$$\sum_{i \in H} v_{oi}^k s \leq \hat{Z}_j \quad o \in N, \ d \in N\{o\}, \ j \in H, \ k \in K, \ s \in S \quad (42)$$

$$X_{oi}^k s \leq \hat{Z}_i \quad o \in N, i \in H, k \in K, s \in S \quad (43)$$

$$X_{ij}^k s \leq \hat{Z}_j \quad i, j \in H, k \in K, s \in S \quad (44)$$

$$X_{jd}^k s \leq \hat{Z}_j \quad d \in N, i \in H, k \in K, s \in S \quad (45)$$

$\partial^* \geq \sum_{i \in H} f_i \hat{Z}_i + \sum_{o \in N} \sum_{i \in H} \sum_{j \in H} \sum_{d \in N\{o\}} \sum_{k \in K} \sum_{s \in S} W_{od}^k s F_{ojd}^k \hat{X}_{oi}^k s - \Psi^* s \quad s \in S \quad (47)$

Suppose that $u^q, q = 1, ..., 14$ are the values of the dual variables associated with constraints (19), (26) - (31), and (41) - (47), respectively. The dual subproblem (DS) can be found as follows:

$$\text{max} \quad U = \sum_{o \in N} \sum_{d \in N\{o\}} \sum_{k \in K} \sum_{s \in S} u_{od}^1 s - \sum_{o \in N} \sum_{i \in H} \sum\sum_{d \in N\{o\}} \sum_{k \in K} \sum_{s \in S} u_{oid}^3 s \hat{Z}_i$$

$$- \sum_{o \in N} \sum_{j \in H} \sum\sum_{d \in N\{o\}} \sum_{k \in K} \sum_{s \in S} u_{ijd}^4 s \hat{Z}_j - \sum_{o \in N} \sum_{i \in H} \sum\sum_{k \in K} \sum_{s \in S} u_{oi}^5 s \hat{Z}_i$$

$$- \sum_{i \in H} \sum\sum_{j \in H} \sum\sum_{k \in K} \sum_{s \in S} u_{ij}^7 s \hat{Z}_j - \sum_{i \in H} \sum\sum_{j \in H} \sum\sum_{k \in K} \sum_{s \in S} u_{ij}^9 s \hat{Z}_j$$

$$- \sum_{j \in H} \sum\sum_{d \in K} \sum\sum_{s \in S} u_{jd}^7 s \hat{Z}_j - \sum_{s \in S} u_{jd}^1 s \right) \quad (48)$$

Subject to:

$$u_{od}^1 s - u_{od}^2 s - u_{ijd}^4 s - u_{ijd}^8 s - u_{oid}^9 s - u_{oid}^{10} s + u_{oi}^{11} s + u_{ij}^{12} s + u_{jd}^{13} s$$

$$- u_{od}^2 s \quad F_{ojd}^k s \leq 0 \quad o \in N, \ d \in N\{o\}, \ i, j \in H, k \in K, s \in S \quad (49)$$

$$- u_{oi}^4 s + \sum_{j \in H} \sum\sum_{d \in N} u_{ijd}^8 s - u_{oi}^{11} s \leq 0 \quad o \in N, \ i \in H, k \in K, s \in S \quad (50)$$

$$- u_{ij}^9 s + u_{ij}^{12} s \quad \Psi^* s \quad i, j \in H, k \in K, s \in S \quad (51)$$

$$- u_{jd}^7 s + u_{jd}^{13} s \quad \Psi^* s \quad d \in N, \ j \in H, k \in K, s \in S \quad (52)$$

$$\sum_{s \in S} u_{id}^4 s \leq \frac{\theta_1}{\partial \max - \partial^*} \quad s \in S \quad (53)$$
The Benders master problem (MP) is:

\[
\min \Omega = Z_{\text{lower}} + \theta_1 \left( \frac{-\theta^*}{2 \max_{\theta^*} - \theta^*} \right) + \theta_2 \left( \frac{\Gamma_1^* - \sum_{i \in H} \sum_{s \in S} p^s (\pi_{2i}^* - \pi_{1i}^*) Z_i}{\Gamma_1^* - \Gamma_1^{\min}} \right) \\
+ \theta_3 \left( \frac{\sum_{i \in H} \sum_{s \in S} p^s \pi_{2i}^* Z_i - \Gamma_2^*}{\Gamma_2^{\max} - \Gamma_2^*} \right) \\
+ \theta_4 \left( \frac{\sum_{i \in H} \sum_{s \in S} p^s (\pi_{3i}^* - \pi_{2i}^*) Z_i - \Gamma_3^*}{\Gamma_3^{\max} - \Gamma_3^*} \right)
\]

Subject to:

\[
Z_{\text{lower}} \geq 0 \\
Z_i \in \mathbb{B}^{\lfloor |H| \rfloor} \quad i \in H
\]

Where \( Z_{\text{lower}} \) is the total network cost. Also, \( \pi^q \in E(p_D) \) and \( \pi^q \in R(p_D) \), \( q = 1, \ldots, 14 \) define extreme points and extreme rays of the DS, respectively. Using these variables, the relaxed master problem (RMP) is defined as follows:

\[
\min \Omega = Z_{\text{lower}} + \theta_1 \left( \frac{-\theta^*}{2 \max_{\theta^*} - \theta^*} \right) + \theta_2 \left( \frac{\Gamma_1^* - \sum_{i \in H} \sum_{s \in S} p^s (\pi_{2i}^* - \pi_{1i}^*) Z_i}{\Gamma_1^* - \Gamma_1^{\min}} \right) \\
+ \theta_3 \left( \frac{\sum_{i \in H} \sum_{s \in S} p^s \pi_{2i}^* Z_i - \Gamma_2^*}{\Gamma_2^{\max} - \Gamma_2^*} \right) \\
+ \theta_4 \left( \frac{\sum_{i \in H} \sum_{s \in S} p^s (\pi_{3i}^* - \pi_{2i}^*) Z_i - \Gamma_3^*}{\Gamma_3^{\max} - \Gamma_3^*} \right)
\]

Subject to:

\[
Z_{\text{lower}} \geq \sum_{o \in N} \sum_{d \in N \setminus \{o\}} \sum_{k \in K} \sum_{s \in S} \pi^1_{od ks} Z_i - \sum_{o \in N} \sum_{i \in H} \sum_{d \in N \setminus \{o\}} \sum_{k \in K} \sum_{s \in S} \pi^2_{od ks} Z_i - \sum_{i \in N} \sum_{j \in H} \sum_{d \in N \setminus \{o\}} \sum_{k \in K} \sum_{s \in S} \pi^3_{ijd ks} Z_j - \sum_{o \in N} \sum_{i \in H} \sum_{k \in K} \sum_{s \in S} \pi^4_{oi ks} Z_i \\
- \sum_{i \in N} \sum_{j \in H} \sum_{d \in N \setminus \{o\}} \sum_{k \in K} \sum_{s \in S} \pi^5_{ijd ks} Z_j - \sum_{i \in H} \sum_{j \in H} \sum_{k \in K} \sum_{s \in S} \pi^6_{ij ks} Z_i \\
- \sum_{j \in H} \sum_{d \in N} \sum_{k \in K} \sum_{s \in S} \pi^1_{jd ks} Z_j - \sum_{s \in S} \psi^1_s \left( \sum_{i \in H} f^s_i Z_i \right) \quad \forall \pi^q \in E(p_D)
\]

\[
0 \geq \sum_{o \in N} \sum_{d \in N \setminus \{o\}} \sum_{k \in K} \sum_{s \in S} \pi^1_{od ks} - \sum_{o \in N} \sum_{i \in H} \sum_{d \in N \setminus \{o\}} \sum_{k \in K} \sum_{s \in S} \pi^2_{od ks} Z_i - \sum_{i \in N} \sum_{j \in H} \sum_{d \in N \setminus \{o\}} \sum_{k \in K} \sum_{s \in S} \pi^3_{ijd ks} Z_j - \sum_{o \in N} \sum_{i \in H} \sum_{k \in K} \sum_{s \in S} \pi^4_{oi ks} Z_i \\
- \sum_{i \in N} \sum_{j \in H} \sum_{d \in N \setminus \{o\}} \sum_{k \in K} \sum_{s \in S} \pi^5_{ijd ks} Z_j - \sum_{i \in H} \sum_{j \in H} \sum_{k \in K} \sum_{s \in S} \pi^6_{ij ks} Z_i \\
- \sum_{j \in H} \sum_{d \in N} \sum_{k \in K} \sum_{s \in S} \pi^1_{jd ks} Z_j - \sum_{s \in S} \psi^1_s \left( \sum_{i \in H} f^s_i Z_i \right) \quad \forall \pi^q \in R(p_D)
\]
Note that a B& B framework is used to solve master problems. UB is introduced as the upper bound of the optimal solution value. Moreover, t denotes the iteration number. Algorithm 1 shows the pseudocode of the BD algorithm.

**Algorithm 1** Benders decomposition algorithm for RHLP-FSE.

```plaintext
set \( LB = -\infty \), \( UB = \infty \), \( \hat{Z}^0 = 0 \)
while \( \frac{UB-LB}{UB} > \varepsilon \) do
  set \( \hat{Z}^t \) on initial value for \( t=1 \)
  solve \( DS(\hat{Z}^t) \) to obtain dual objective value \( (U^*) \)
  if \( DS(\hat{Z}^t) \) is bounded with extreme points \( \hat{\pi}^t_q \)
    add optimality cut (57) to RMP
    \( UB = U^*(\hat{Z}^t) + \Omega^*(\hat{\pi}^t_q) \)
  else
    if \( DS(\hat{Z}^t) \) is unbounded then obtain extreme rays \( \hat{\pi}^t_q \)
      add feasibility cut (58) to RMP
  end if
  solve RMP to obtain \( Z^t \) \( t > 1 \)
  \( LB = \Omega^*(\hat{\pi}^t_q) + U^*(\hat{Z}^t) \)
end while
\( t \rightarrow t + 1 \)
End
```

Note that the maximum number of iteration \((Iter_{max})\) and CPU time \((Time_{max})\) are considered as stopping criteria for the algorithm.

5. **Numerical results.** First the solution approach is discussed by comparing it to the results of commercial software, and then the necessity of modeling in the fuzzy environment is investigated. Finally, the effect of risk-related parameters in designing the network is considered. All numerical instances are solved on an Asus Studio PC with an Intel Core i7 CPU at 1.73 GHz and 4 GB of RAM. Moreover, CPLEX 12.5 is used as a solver for the mixed integer linear programming.

Numerical instances are based on the well-known CAB data set with minor modifications. Link discount factors are assumed to be \( \varphi = 0.1 \), \( \gamma = 0.8 \) and \( \tau = 0.8 \) for different link types. The objective function weights are set to 0.25. Uncertain unit transportation costs in different scenarios are generated from the interval \([0.05,1]\)\ast(distance) and the distances are assumed to be the same as the values of the CAB data set. Also, the flow between nodes \( o \in N \) and \( d \in N \) is assumed to be stochastic and distributed randomly in \((0.001,0.005) \ast w(i,j)\) while \( w(i,j) \) is the flow reported in the CAB data set. Moreover the fixed cost of hub establishment is assumed to be uniformly distributed in \((10000 \ast \log O_1(i), 20000 \ast \log O_1(i))\), where, \( O_1(i) = \sum_j w(i,j) \). Also, we set \( Iter_{max} = 20 \) and \( Time_{max} = 2000(\text{sec}) \). The usage risk factors are calculated based on the value of scores as mentioned before. The present study considered two possible risk factors, which are called class I and class II; the second contains higher risk values. Finally, five scenarios are randomly generated with two scenarios in class I and three in class II.

Symbols in the tables are as follows:
• $N$: The number of total nodes.
• $H$: The number of potential hub nodes.
• $\Omega^*$: The optimal integrated objective value.
• $Z^*$: The optimal location decision.
• $\%Gap$: The percent deviation between the upper and lower bounds of the BD algorithm, when the optimal solution cannot be found. That is $\%Gap = 100(UB - LB)/UB$, where $UB$ and $LB$ are the upper and lower bounds.
• $Time(s)$: The total time in seconds.
• N.a.: Not available.

5.1. Consideration of the BD method as a solution approach. The proposed solution method was evaluated by comparing the solution of the model by GAMS software and BD for different sizes of the problem, as well as risk factor classes.

Table 3. Results of different-sized instances for class I risk factors

| $|N|$ | $|H|$ | No of variables | No of constraints | B&B solution | BD solution | No of iterations | $\%Gap$ |
|-----|------|-----------------|------------------|--------------|-------------|-----------------|--------|
|     |      |                 |                  | $\Omega_{\text{B&B}}$ | $z_{\text{B&B}}$ | $\Omega_{\text{BD}}$ | $z_{\text{BD}}$ | Time (s) |
| 4   | 9    | 245             | 65520            | 0.3214       | 4           | 92.9            | 0.3214       | 4           | 153.2    | 2        | 0 |
| 6   | 7    | 545             | 91665            | 0.3255       | 6           | 98.3            | 0.3255       | 6           | 194.5    | 7        | 0 |
| 6   | 9    | 540             | 147630           | 0.3410       | 3.4         | 108.9           | 0.3410       | 3.4         | 178.7    | 16       | 0 |
| 8   | 7    | 965             | 161032           | 0.3299       | 6           | 265.2           | 0.3299       | 6           | 234.3    | 8        | 0 |
| 10  | 7    | 1505            | 247905           | n.a.         | n.a.        | -1              | 0.2979       | 2.4         | 478.3    | 11       | 0 |
| 12  | 7    | 2160            | 355010           | n.a.         | n.a.        | -1              | 0.2390       | 3.5         | 763.2    | 11       | $10^{-6}$ |
| 15  | 7    | 169261          | 551880           | n.a.         | n.a.        | -2              | 0.2881       | 2.5         | 1101.1   | 8        | $10^{-5}$ |
| 16  | 5    | 3540            | 332330           | n.a.         | n.a.        | -2              | 0.2671       | 5           | 1371.6   | 7        | $10^{-4}$ |
| 18  | 5    | 124575          | 419630           | n.a.         | n.a.        | -2              | 0.2648       | 5           | 1427.4   | 10       | $10^{-4}$ |
| 20  | 5    | 298936          | 976605           | n.a.         | n.a.        | -2              | 0.2754       | 4           | 1872.2   | 9        | $10^{-3}$ |

B&B: B&B result, BD: Benders decomposition result, -1*: Time limitation, -2*: Lack of memory

Table 4. Results of different-sized instances for class II risk factors

| $|N|$ | $|H|$ | No of variables | No of constraints | B&B solution | BD solution | No of iterations | $\%Gap$ |
|-----|------|-----------------|------------------|--------------|-------------|-----------------|--------|
|     |      |                 |                  | $\Omega_{\text{B&B}}$ | $z_{\text{B&B}}$ | $\Omega_{\text{BD}}$ | $z_{\text{BD}}$ | Time (s) |
| 4   | 9    | 245             | 65520            | 0.2670       | 4           | 87.4            | 0.2670       | 4           | 133.4    | 2        | 0 |
| 6   | 7    | 545             | 91665            | 0.2487       | 1           | 201.4           | 0.2487       | 1           | 198.9    | 4        | 0 |
| 6   | 9    | 540             | 147630           | 0.2500       | 6           | 214.4           | 0.2500       | 6           | 220.0    | 5        | 0 |
| 8   | 7    | 965             | 161032           | 0.2384       | 6           | 264.2           | 0.2384       | 6           | 248.3    | 3        | 0 |
| 10  | 7    | 1505            | 247905           | n.a.         | n.a.        | -1              | 0.2425       | 5           | 348.5    | 5        | 0 |
| 12  | 7    | 2160            | 355010           | n.a.         | n.a.        | -1              | 0.3131       | 4.7         | 654.2    | 7        | $10^{-6}$ |
| 15  | 7    | 169261          | 551880           | n.a.         | n.a.        | -2              | 0.2657       | 2.5         | 985.3    | 6        | $10^{-5}$ |
| 16  | 5    | 3540            | 332330           | n.a.         | n.a.        | -2              | 0.3117       | 5           | 1321.6   | 6        | $10^{-4}$ |
| 18  | 5    | 124575          | 419630           | n.a.         | n.a.        | -2              | 0.3736       | 2           | 1563.2   | 12       | $10^{-4}$ |
| 20  | 5    | 298936          | 976605           | n.a.         | n.a.        | -2              | 0.2576       | 20          | 1983.1   | 7        | $10^{-3}$ |

The results shown in tables 3 and 4 confirm the efficiency and robustness of the BD algorithm. The results also show the failure of CPLEX to solve test problems due to time and memory limitations. The results obtained show that the algorithm can solve the model for different sizes in both classes, while the CPLEX solver is only able to solve the model with small sizes due to time and memory limitation. The results clearly indicate that the BD method outperforms the CPLEX solver for large-scale instances, and the limitations of using a general purpose solver are obvious.
Also, the numerical results show that a lower number of potential hub nodes are selected as hub locations in class I. This confirms that increasing the values of risk factors leads to decreasing the number of hubs, so considering risk factors in stochastic location problems seems to be effective. Furthermore, it is observed that in most cases, potential hub nodes 2, 4, 5 and 6 are selected as hub nodes. In other words, they are robust for selection in different conditions. To evaluate the performance of the algorithm, the fixed costs of the mentioned nodes and their associated risk values were increased and the model was resolved once again. It is observed that the mentioned nodes were not selected as the location of hub facilities. This confirms the effective performance of the presented BD algorithm.

Moreover, the convergence of the applied BD algorithm is analysed for one of the instances with 10 potential hub nodes. The trend of convergence is shown in Figure 3. According to the trend of convergence, the BD algorithm is converged in iteration 11 and the accuracy of this algorithm can be confirmed.

The capability of the BD method to reach optimal solutions for the mentioned instances is illustrated in Figure 4. Figure 4 makes it clear that in more than 60% of instances, both the BD method and the CPLEX solver are able to reach optimal solutions. In addition, in just 10% of the instances, the optimality gap is $10^{-3}\%$. This confirms that the BD method performs efficiently in the RHLP-FSE problem.

5.2. The effects of considering the probabilistic and fuzzy logic approaches in the hub location model. To consider the effectiveness of the proposed approach, its performance is compared with other previous possible designs. A comparison between fuzzy-stochastic, fuzzy and stochastic model results may help to show the usefulness of integrating the fuzzy and stochastic approaches. Other researchers have compared pairs of the mentioned approaches in other problems (Salman and Yucel [44]; Eydi and Mirakhorli [21]; Mirakhorli [35]; Santoso et al., [45]; Roy and Maiti [40]). To assess the performance of the applied fuzzy-stochastic approach, the hub network is designed for three cases: the stochastic, fuzzy and fuzzy-stochastic models, which are represented in Figure 5a, Figure 5b and Figure
ent. As illustrated in figure

The results are demonstrated in Figure 5. For simplicity, all of networks are considered as a single commodity model. The overall performance of the BD method is confirmed by Figure 4. The highest possible value is replaced by stochastic and fuzzy parameters for the stochastic and fuzzy versions of the model. For simplicity, all of networks are considered as a single commodity model. The results are demonstrated in Figure 5.

Figure 4. The overall performance of the BD method

Figure 5. Comparison of output for stochastic, fuzzy and fuzzy-stochastic problems
An instance is tested using some modifications of information from the CAB dataset, especially with select spread nodes. The set of hubs for the fuzzy-stochastic model becomes \{Dallas, Chicago, Boston\} while the corresponding sets for the stochastic and fuzzy models become \{Los Angeles, Memphis, Chicago\} and \{Minneapolis\}, respectively. The discrepancy between outputs is confirmed by Figure 5. This comparison also confirms the necessity of considering the proposed model when faced with stochastic elements in a fuzzy environment. As illustrated in Figure 5, only one hub is established in case 2, while by the proposed model three nodes are selected to work as hubs. This shows that the proposed model considers the failure possibility and suggests increasing the service level by establishing more hubs.

5.3. The effects of considering network risk in the hub location model. A comparison experiment is implemented to demonstrate the necessity of considering risk in a risky environment. The solution of the proposed model for different sizes is compared with a classic model (CM) in Table 5. Parts of the proposed model, including risk parameters, are not considered in classic model.

| \( |N| \) | \( |H| \) | Continues Binary Equality Inequality | \( z^*_{CM} \) | No of constraints | No of variables | CM solution | RHLP-FSE solution | Lost flows | Lost flows |
|---|---|---|---|---|---|---|---|---|---|---|
| 4 | 5 | 6976 | 5 | 245 | 22725 | 1.3, 4 | 7543 | 3.4 | 6060 |
| 4 | 7 | 13336 | 7 | 245 | 42525 | 1.2, 3, 4 | 12110 | 4 | 9599 |
| 6 | 5 | 14776 | 5 | 545 | 48825 | 1.3, 4 | 3714 | 2 | 4892 |
| 6 | 7 | 28456 | 7 | 545 | 91665 | 1.3, 4, 6 | 8825 | 2.4 | 5982 |
| 8 | 5 | 25576 | 5 | 965 | 85125 | 3.5 | 9691 | 6 | 508 |
| 8 | 7 | 49456 | 7 | 965 | 160125 | 1.3, 4, 6, 7 | 9687 | 6.7 | 1446 |

Since the solutions of the RHLP-FSE and CM models are different and the number of located facilities decreases in the presence of risks, it can be concluded that risk is an important factor in HLPs and must be considered in hub network design. Furthermore, 1,000 failure scenarios were simulated for predetermined O-D flows considering each node failure probability, and lost flows were calculated for both CM and RHLP-FSE networks. The calculated lost flows are considered as a comparison measure. The obtained results show that the sum of lost flows in the CM network is over 51,570 commodities, while it decreases to 28,577 commodities for the RHLP-FSE network (45% less). The design of the hub network by the proposed approach is more reasonable and has more probable benefits in the case of possible failure of existing hubs.

The analysis of the relationship between the value of the risk sensitivity factor and the number of opened hubs is shown in Figure 6. Risk states are treated as a linguistic variable by categorizing them into “very low,” “low,” “medium,” “high” and “very high.” This analysis shows that the number of established hubs decreases as the risk values increase.

6. Conclusion. Tsunamis in the Asian region, offshore piracy attacks in Somalia, Hurricane Katrina, and European ash clouds show that transshipment can be interrupted. Therefore, in order to bring network design closer to real-world situations,
it is worthwhile to consider unpredictable events such as delay times in transportation systems, link failures in railways, and flight delays caused by air pollution and cargo exhaustion. In the present study, a minimax regret robust HLP is presented with a fuzzy-stochastic approach. In an attempt to design a robust hub network, risk factors such as congestion, security and regional air pollution are considered. Then, the proposed model is solved by a BD algorithm. Finally, to validate this method, the results of the BD algorithm are compared with the solution by the CPLEX solver for different instances. The numerical results confirm the efficient performance of the BD algorithm. Sensitivity analysis shows that the number of established hubs is decreased by increasing risk values, and validates the proposed model and solution approach. Future research could apply the sample average approximation method to consider efficient scenarios in cases involving unlimited possible scenarios. Another approach could be using an accelerated BD algorithm for solving the model more efficiently. Finally, another area for further research could involve considering a heuristic algorithm to solve the master problem of the BD to improve efficiency of the solution method efficiency.

REFERENCES

[1] S. Alumur and B. Y. Kara, A new model for the hazardous waste location-routing problem, Comp. Oper. Res., 34 (2007), 1406–1423.
[2] S. A. Alumur, S. Nickel and F. Saldanha-da-Gama, Hub location under uncertainty, Trans. Res. Part B: Method., 46 (2012), 529–543.
[3] F. Atoei, E. Teimory and A. Amiri, Designing reliable supply chain network with disruption risk, Int. J. Indust. Eng. Comp., 4 (2013), 111–126.
[4] M. Bashiri, M. Mirzaei and M. Randall, Modeling fuzzy capacitated p-hub center problem and a genetic algorithm solution, Appl. Math. Model., 37 (2013), 3513–3525.
[5] J. F. Benders, Partitioning procedures for solving mixed-variables programming problems, Numer. Math., 4 (1962), 238–252.
[6] R. Caballero, M. González, F. M. Guerrero, J. Molina and C. Paralera, Solving a multiobjective location routing problem with a metaheuristic based on tabu search. Application to a real case in Andalusia, European Journal of Operational Research, 177 (2007), 1751–1763.
[7] R. S. de Camargo, G. de Miranda, Jr. and R. P. M. Ferreira, A hybrid outer-approximation/benders decomposition algorithm for the single allocation hub location problem under congestion, *Oper. Res. Lett.*, **39** (2011), 329–337.

[8] R. S. de Camargo, G. de Miranda, Jr., R. P. M. Ferreira and H. P. Luna, Multiple allocation hub-and-spoke network design under hub congestion, *Comp. Oper. Res.*, **36** (2009), 3097–3106.

[9] R. S. de Camargo, G. de Miranda, Jr. and H. P. Luna, Benders decomposition for the uncapacitated multiple allocation hub location problem, *Comp. Oper. Res.*, **35** (2008), 1047–1064.

[10] R. S. de Camargo, G. de Miranda, Jr. and H. L. P. Luna, Benders decomposition for hub location problems with economies of scale, *Transportation Science*, **43** (2008), 86–97.

[11] J. F. Campbell, Integer programming formulations of discrete hub location problems, *Eur. J. Oper. Res.*, **72** (1994), 387–405.

[12] Z. Chen, H. Li, H. Ren, Q. Xu and J. Hong, A total environmental risk assessment model for international hub airports, *Int. J. Proj. Man.*, **29** (2011), 856–866.

[13] G. Chen and T. T. Pham, *Introduction to Fuzzy Sets, Fuzzy Logic, and Fuzzy Control Systems*, CRC Press, 2000.

[14] I. Contreras, J.-F. Cordeau and G. Laporte, Benders decomposition for large-scale uncapacitated hub location, *Operations Research*, **59** (2011), 1477–1490.

[15] I. Contreras, J.-F. Cordeau and G. Laporte, Stochastic uncapacitated hub location, *Eur. J. Oper. Res.*, **212** (2011), 518–528.

[16] I. Contreras, J.-F. Cordeau and G. Laporte, Exact solution of large-scale hub location problems with multiple capacity levels, *Transportation Science*, **46** (2012), 439–459.

[17] S. Davari, M. H. Fazel Zarandi and A. Hemmati, Maximal covering location problem (MCLP) with fuzzy travel times, *Expert Systems with Applications*, **38** (2011), 14535–14541.

[18] K. Deb, *Multi-objective Optimization Using Evolutionary Algorithms*, Wiley-Interscience Series in Systems and Optimization, John Wiley & Sons, Ltd., Chichester, 2001.

[19] M. Eghbali, M. Abedzadeh and M. Setak, Multi-objective reliable hub covering location considering customer convenience using NSGA-II, *Int. J. Syst. Assur. Eng. Manag.*, **5** (2014), 450–460.

[20] E. Erkut and O. Alp, Designing a road network for hazardous materials shipments, *Comp. Oper. Res.*, **34** (2007), 1389–1405.

[21] A. Eydi and A. Mirakhorli, An extended model for the uncapacitated single allocation hub covering problem in a fuzzy environment, in *Proceedings of the International Multiconference of Engineers and Computer Scientists*, International Association of Engineers, Hong Kong, Vol II, (2012).

[22] P. Garcia-Herreros, J. M. Wassick and I. E. Grossmann, Design of resilient supply chains with risk of facility disruptions, *Indust. Eng. Chem. Res.*, **53** (2014), 17240–17251.

[23] S. Gelareh and S. Nickel, A benders decomposition for hub location problems arising in public transport, in *Operations Research Proceedings 2007* Part VI, Springer Berlin Heidelberg, 2008, 129–134.

[24] B. Gendron, Decomposition methods for network design, *Procedia-Social and Behavioral Sciences*, **20** (2011), 31–37.

[25] A. M. Geoffrion, Generalized Benders decomposition, *J. Optimization Theory Appl.*, **10** (1972), 237–260.

[26] M. da Graça Costa, M. E. Captivo, and J. Clímaco, Capacitated single allocation hub location problem—A bi-criteria approach, *Comp. Oper. Res.*, **35** (2008), 3671–3695.

[27] I. Heckmann, T. Comes and S. Nickel, A critical review on supply chain risk—Definition, measure and modeling, *Omega*, **52** (2015), 119–132.

[28] C. L. Hwang and K. Yoon, *Multiple Attributes Decision Making. Methods and Applications*, Springer-Verlag, Berlin-New York, 1981.

[29] A. Jabbarzadeh, S. G. Jalali Naini, H. Davoudpour and N. Azad, Designing a supply chain network under the risk of disruptions, *Math. Probl. Eng.*, (2012), Art. ID 234324, 23 pp.

[30] B. Y. Kara, *Modeling and Analysis of Issues in Hub Location Problems*, Doctor of Philosophy Thesis, Bilkent University, Ankara, Turkey, 1999.

[31] H. E. Lee, K. H. Park and Z. Z. Bien, Iterative fuzzy clustering algorithm with supervision to construct probabilistic fuzzy rule base from numerical data, *Fuzzy Systems, IEEE Transactions on*, **16** (2008), 263–277.
[32] J. Li, G. H. Huang, G. Zeng, I. Maqsood and Y. Huang, An integrated fuzzy-stochastic modeling approach for risk assessment of groundwater contamination, Jour. Environ. Manag., 82 (2007), 173–188.
[33] V. Marianov and D. Serra, Location models for airline hubs behaving as M/D/c queues, Comp. Oper. Res., 30 (2003), 983–1003.
[34] M. Merakl and H. Yaman, Robust intermodal hub location under polyhedral demand uncertainty, Transportation Research Part B: Methodological, 86 (2016), 66–85.
[35] A. Mirakhorli, Capacitated single-assignment hub covering location problem under fuzzy environment, in Proceedings of the World Congress on Engineering and Computer Science, 2 (2010), 20–22.
[36] V.-W. Mitchell, Organizational risk perception and reduction: A literature review, British Journal of Management, 6 (1995), 115–133.
[37] M. Mohammadi, R. Tavakkoli-Moghaddam and R. Rostami, A multi-objective imperialist competitive algorithm for a capacitated hub covering location problem, Int. J. Indust. Eng. Comp., 2 (2011), 671–688.
[38] M. Mohammadi, F. Jolai and R. Tavakkoli-Moghaddam, Solving a new stochastic multi-mode p-hub covering location problem considering risk by a novel multi-objective algorithm, Appl. Math. Model., 37 (2013), 10053–10073.
[39] M. S. Pishvaee, J. Razmi and S. A. Torabi, An accelerated Benders decomposition algorithm for sustainable supply chain network design under uncertainty: A case study of medical needle and syringe supply chain, Trans. Res. Part E: Log. Trans. Rev., 67 (2014), 14–38.
[40] T. K. Roy and M. Maiti, A fuzzy EOQ model with demand-dependent unit cost under limited storage capacity, Eur. J. Oper. Res., 99 (1997), 425–432.
[41] E. M. de Sá, R. Morabito and R. S. de Camargo, Benders decomposition applied to a robust multiple allocation incomplete hub location problem, Computers & Operations Research, 89 (2018), 31–50.
[42] E. M. de Sá, R. Morabito and R. S. de Camargo, Efficient Benders decomposition algorithms for the robust multiple allocation incomplete hub location problem with service time requirements, Expert Systems with Applications, 93 (2018), 50–61.
[43] E. M. de Sá, R. S. de Camargo and G. de Miranda, An improved Benders decomposition algorithm for the tree of hubs location problem, European J. Oper. Res., 226 (2013), 185–202.
[44] F. S. Salman and E. Yücel, Emergency facility location under random network damage: Insights from the Istanbul case, Comput. Oper. Res., 62 (2015), 266–281.
[45] T. Santos, S. Ahmed, M. Goetschalckx and A. Shapiro, A stochastic programming approach for supply chain network design under uncertainty, Eur. J. Oper. Res., 167 (2005), 96–115.
[46] E. S. Sheppard, A conceptual framework for dynamic location–Allocation analysis, Environment and Planning A, 6 (1974), 547–564.
[47] T. Sim, T. J. Lowe and B. W. Thomas, The stochastic p-hub center problem with service-level constraints, Comput. Oper. Res., 36 (2009), 3166–3177.
[48] L. V. Snyder, M. S. Daskin and C. P. Teo, The stochastic location model with risk pooling, Eur. J. Oper. Res., 179 (2007), 1221–1238.
[49] L. V. Snyder, M. P. Scaparra, M. S. Daskin and R. L. Church, Planning for disruptions in supply chain networks, Tutorials in Operations Research, (2006), 234–257.
[50] F. Taghipourian, I. Mahdavi, N. Mahdavi-Amiri and A. Makui, A fuzzy programming approach for dynamic virtual hub location problem, Appl. Math. Model., 36 (2012), 3257–3270.
[51] A. Tajbakhsh, H. Haleh and J. Razmi, A multi-objective model to single-allocation ordered hub location problems by genetic algorithm, Int. J. Acad. Res. Bus. Soc. Sci., 3 (2013). Available from: http://hrmars.com/admin/pics/1640.pdf.
[52] S. A. Torabi and E. Hassini, An interactive possibilistic programming approach for multiple objective supply chain master planning, Fuzzy Sets and Systems, 159 (2008), 193–214.
[53] A. D. Vasconcelos, C. D. Nassi and L. A. Lopes, The uncapacitated hub location problem in networks under decentralized management, Comput. Oper. Res., 38 (2011), 1656–1666.
[54] T.-H. Yang, Stochastic air freight hub location and flight routes planning, Appl. Math. Model., 33 (2009), 4424–4430.
[55] K. Yang, Y. Liu and G. Yang, An improved hybrid particle swarm optimization algorithm for fuzzy p-hub center problem, Comput. Industr. Eng., 64 (2013), 133–142.
[56] L. A. Zadeh, Fuzzy sets, Information and Control, 8 (1965), 338–353.
[57] H. Zhai, Y. Liu and W. Chen, Applying minimum-risk criterion to stochastic hub location problems, Procedia Engineering, 29 (2012), 2313–2321.
[58] URL: http://www.bigstory.ap.org/article/indonesia-mount-kelud-java-island-erupts, last visited: September 2014.
[59] URL: http://en.wikipedia.org/wiki/2010_eruptions_of_Eyjafjallajökull, last visited: September 2014.

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