Universal behavior of internal friction in glasses below $T_g$: anharmonicity vs relaxation

J. Pelous and C. Levelut
Laboratoire des Colloïdes, Verres et Nanomatériaux, CNRS/UMR5587, Université Montpellier II, cc 69, 34095 Montpellier cedex, France
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Comparison of the internal friction $Q^{-1}$ at hypersonic frequencies between a few K and the glass transition temperature $T_g$ for various glasses brings out general features. At low temperature, $Q^{-1}$ is only weakly dependent on the material. At high temperature but still below $T_g$ the internal friction for strong glasses shows a $T$-independent plateau in a very wide domain of temperature; in contrast, for fragile glass, a nearly linear variation of $Q^{-1}$ with $T$ is observed. Anharmonicity appears dominant over thermally activated relaxational processes at high temperature.

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I. INTRODUCTION

Physical properties at low temperature, the mechanism of the glass transition and the structure and vibrations at the nanometer scale account for many experimental and theoretical contributions in glasses.\textsuperscript{1,2,3,4,5,6,7,8} Search for universal behavior or correlations between these properties are the mains fields explored. For this purpose, it has been useful to classify glasses using the concept of fragility\textsuperscript{9} depending on how the viscosity (or structural relaxation) variation versus temperature deviates from Arrhenius behavior. One of the most debated questions arises from the nature and the origin of the fast relaxation\textsuperscript{10} responsible for a broad quasi elastic contribution in neutron, X-ray and light scattering and also for damping in sound waves and dielectric loss.\textsuperscript{11,12,13,14,15,16} From analysis of susceptibilities, a proportionality between quasi-elastic neutron or light scattering, infrared absorption and internal friction or sound attenuation has been established and experimentally verified in a number of glasses.\textsuperscript{11,12,13,14,15,16}

On the one hand, at very low temperature and for a large frequency range, many physical properties (among those the internal friction $Q^{-1}$) can be quantitatively described within the framework of the “tunneling model”\textsuperscript{17} assuming a phenomenological potential with two asymmetric wells with a distribution of barriers and asymmetries. To improve the description of data above a few K the so called “soft potential model”\textsuperscript{18} extends and generalizes the previous model including distributed harmonic oscillators. At higher temperature, a phenomenological description of coupling of acoustic waves with unspecified thermally activated defects gives a quantitatively correct description of the acoustical attenuation, or equivalently internal friction, at ultrasonic frequencies assuming distributions of energy barriers\textsuperscript{18,19}. Recently\textsuperscript{20} this model has been extended to describe data for a large frequency range within the same formalism and to deduce the distribution of the energy barriers. Moreover some authors claim that inelastic light scattering results can be accounted for by this description for many order of magnitude in frequency in silica glass\textsuperscript{21} as well as in various fragile glass.\textsuperscript{22} In contrast, theoretical calculations explain acoustical damping in amorphous silicon and fused silica by anharmonicity for frequencies in the 10–100 GHz range.\textsuperscript{23} Furthermore, analysis and discussion of data obtained using picosecond optical techniques in the same frequency range conclude that classical relaxation theory cannot explain the frequency and temperature variation observed.\textsuperscript{24} Indeed, a description of the sound velocity (or elastic constants) versus temperature, even at low frequencies, cannot be accounted for assuming this relaxational process alone.\textsuperscript{25,26,27,28}

On the other hand at hypersonic frequencies available from Brillouin scattering experiments, an attempt to describe experimental results for $Q^{-1}$ in silica glass using the soft potential model\textsuperscript{29} did not give satisfactory results when the temperature exceeds 10–20K. Moreover, comparisons of ultrasonic and hypersonic attenuation in glasses have demonstrated that thermally activated relaxations, dominant for temperatures higher than 10 K at low frequencies, cannot explain quantitatively the values observed at higher frequencies, not only in silica glass (Ref. 30,31) but also in other glasses (Ref. 32,33). Taking into account that the amplitude of sound attenuation is not very different at high temperature in glasses and in crystals, anharmonicity has been invoked, by analogy with processes well known in crystals, to explain results at hypersonic frequencies. However, other authors\textsuperscript{34} explain the same $Q^{-1}$ data in silica glass using the formalism of thermally activated processes. More recently careful analysis in silica glass have quantified the relative parts of different processes responsible for the internal friction\textsuperscript{35} and demonstrate that anharmonicity dominates at high temperature. Finally, the frequency dependence of the damping of vibrations up to the THz range yield conflicting interpretations.\textsuperscript{36,37,38,39,40} Various frequency dependences at different temperatures are attributed to different origins such as relaxational, anharmonic or non-dynamic processes. Recently from measurements...
sound scattering were again put forward. So the description in the literature of the origin of the sound attenuation in glasses presents contradictory interpretations and it is not clear if the frequency and temperature variation of the internal friction can be interpreted in a formalism common to all glasses. Furthermore, most of the $Q^{-1}$ experimental determinations concern temperatures lower than room temperature and most of the data below $T_g$ focus on fragile glass, and on silica glass as representative of strong glasses. As silica glass, like other tetrahedrally coordinated glasses, shows a number of specific unexplained anomalies, the question arises if the behavior observed in silica is similar to that in other glasses, as suggested in Ref. [34]. To bring some light to these fields, we have made a comparison of results obtained for internal friction at hypersonic frequencies in a number of glasses. This comparison reveals new characteristics common to all materials at low temperature and in the temperature range below $T_g$.

II. EXPERIMENTS AND RESULTS

The general behavior of hypersonic properties of glasses is illustrated in Fig. 1 which gives results for the sound velocity and hypersonic attenuation, shown through the inverse mean free path, in window glass (72% SiO$_2$, 14% Na$_2$O, 9% CaO, 3% MgO, 1% Al$_2$O$_3$ plus other minor oxides). These data were obtained from Brillouin scattering experiments with experimental conditions common to a series of previously published papers [30,34,46].

The Brillouin frequency shift $\Delta \nu$ is related to the longitudinal sound wave velocity $V_l$, the refractive index $n$ and the scattering angle $\theta$ by

$$\Delta \nu = 2nV_l \sin(\theta/2)/\lambda,$$

where $\lambda$ is the wavelength of the incident light. From the full width of the Brillouin line, $\Gamma$, one can deduce either the mean free path, $L^{-1}$,

$$L^{-1} = \Gamma \times 2\pi/V_l,$$

or the internal friction, $Q^{-1}$, defined as

$$Q^{-1} = \Gamma/\Delta \nu.$$

The accuracy of the experiments is about 0.1% for the determination of sound velocity and 5–10% for the mean free path (or the internal friction).

![FIG. 1: Left axis: Longitudinal sound velocity versus temperature in a window glass (see composition in Tab. I) measured at hypersonic frequencies by Brillouin scattering in the backscattering configuration with $\lambda = 514.5$ nm. Right axis: inverse mean free path versus temperature in the same glass.](image)

Fig. 1 shows results in the full temperature range investigated for the two more standard glass formers, SiO$_2$ [34,46,47] and B$_2$O$_3$ [48,49] together with window glass. At least two characteristics common to these glasses can be pointed out. First, in a large range of temperature, $Q^{-1}$ is temperature independent within the accuracy of the experiments.
The value of the anharmonic contribution in silica deduced by subtraction of the relaxational part is estimated to be equal to $Q^{-1} = 2.8 \times 10^{-3}$.

TABLE I: Comparison of experimental $Q^{-1}$ and calculated anharmonic contribution of $Q^{-1}$. The experimental data are given at 300 K. For silica and boron oxide, the experimental values are identical at $T_g$ and room temperature. Data for SiO$_2$ are from Ref. 46 and 47, data for B$_2$O$_3$ from Ref. 48 and 49, data for PMMA from Ref. 52. Anharmonic contribution $Q^{-1}$, calculated at $T_g$ for SiO$_2$ and at 300K for B$_2$O$_3$ and PMMA. The calculated values are estimated from equation 5. The values of density, Debye and longitudinal sound velocity, specific heat and Grüneisen parameter used for the claculation are also given in the table. We used $\tau_{ph} = 10^{-13}$ s, the Grüneisen parameter $\gamma$ from Ref. 55 and $C_v$ from Ref. 56 for SiO$_2$ and PMMA, and from Ref. 57 for B$_2$O$_3$.

A strong increase of the internal friction is observed. This feature is associated with the $\alpha$ relaxational processes coupled to acoustic waves. A signature at $T_g$ is also observed for the sound velocity or other related elastic constants (Fig. 1).

Another important feature common to various glasses, not pointed out before, is displayed in Fig. 3 where results for temperatures lower than 150 K were considered. Below this temperature, the amplitudes of the internal friction tend to a common behavior for oxide glasses as different as silica, boron oxide or window glass. A peak reminiscent of the ultrasonic one, well identified in pure silica, does not appear for any other glass, including the fragile glasses shown in Fig. 4. In this figure, $Q^{-1}$ values are compared for four examples of fragile glass: one electrolytic glass, LiCl-4H$_2$O, one organic polymer (PMMA), and one inorganic chain-like phosphate glass. The amplitude of the internal friction is similar in the strong window glass and in the fragile electrolyte glass, at low temperature, but significant differences between samples can be observed just below $T_g$.

![Comparison of internal friction in strong glasses: silica (Ref. 46 and 47 for the low temperature part and Ref. 47 for the high temperature part), boron oxide (Ref. 48 for the low temperature part and Ref. 49 for the high temperatures) and window glass.](image)

When the temperature increases, the internal friction continues to increase in fragile glasses whereas it levels off in strong glasses. This behavior has also been demonstrated in other polymeric glasses (polystyrene) or in an optical strong glass above room temperature.
III. DISCUSSION

As pointed in the introduction, thermally activated relaxational processes have been invoked to explain mechanical damping in glasses below $T_g$. Those processes are useful to describe peaks observed at ultrasonic frequencies. As the same defects characteristic for the disorder are partly responsible for the very low temperature properties, the distribution function of barriers and asymmetries of the tunneling model useful to describe internal friction and sound velocity below a few K are the starting point to consider thermally activated processes when the temperature is increased. Different hypothesis follow to extend the model and concern the form of the distribution functions. Assuming a flat distribution of asymmetries, $Q^{-1}$ is proportional to the imaginary part of the susceptibility, and can be expressed as a function of $g(V)$ the temperature independent distribution function of barriers $V$ and $\tau = \tau_0 \exp V/kT$ the relaxation time for hopping between adjacent potential wells. At hypersonic frequencies and sufficiently high temperature $\omega \tau_0 << 1$ is satisfied, $\tau_0$ being the fastest relaxation time, and $\omega = 2\pi \Delta \nu$. Then

$$Q^{-1} \approx T g(V).$$

A maximum in $Q^{-1}$ in ultrasonic experiments implies a cut-off $V_{\text{max}}$ in $g(V)$. Different distributions $g(V)$ have been tested in the literature but the general characteristic temperature dependence of $Q^{-1}$ can be discussed without a precise description of $g(V)$.

The rough proportionality to $T$ observed at hypersonic frequencies up to near 100 K for strong glasses or below $T_g$ for other more fragile glasses can be attributed to this process (Eq. 4). The similarity of values obtained in different glasses can be related to the parameters of tunneling models deduced from experiments at very low temperature which show close values for strong and fragile glass. $Q^{-1}$ does not vary much for different glasses, about a factor of 2, as the parameter characteristic for the contribution of tunneling defects to the internal friction, as pointed out by Pohl. One can note the astonishingly low value of $Q^{-1}$ for glycerol. For most glasses, in ultrasonic experiments the $Q^{-1}$ peak is broader and appears at higher temperature than in silica glass, a broadened distribution function with higher potential barriers gives a good fit of the data but calculations at hypersonic frequencies of the contribution of these thermally activated processes using the same parameters predict an amplitude much lower than experimental values or a maximum at a temperature higher than $T_g$, so that another process, anharmonicity, can be put forward at high temperature.

The second process under consideration is anharmonicity. It can be pointed out that the origin of this anharmonicity can be specific to glasses as developed by the authors of Refs. 55, 61, 62 but a quantitative test is not possible in the currently available models. Furthermore, these authors have pointed out that fragility is correlated with anharmonicity, and this contribution should occurs also in fragile glasses; this has been recently confirmed by molecular dynamic simulation.

The similarity between $Q^{-1}(T)$ curves in glasses and crystals at high temperature suggests that the same formalism can be used in both materials.

In the Akhieser regime using the same formalism of network viscosity as in crystals, the dominant process expected in glasses can be expressed as $\tilde{\omega}$.
with \( A = \gamma^2 C_v T V_l / 2 \rho V_d^3 \) where \( C_v \) is the specific heat per unit volume, \( \gamma \) the Grüneisen constant, \( V_l \) the longitudinal sound velocity, \( \rho \) the density and \( V_d \) Debye velocity, \( \tau_{ph} \) is the mean lifetime of thermal phonons. More precise calculations of anharmonicity showed that a plateau is observed at high frequencies.

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Taking \( \tau \) proportional to \( T^{-1} \) (verified for example in various form of silica in Ref. 34) the product \( C_v \tau \) is a constant and can explain the leveling off for the plateau \( Q^{-1}(T) \) observed in oxide glasses. Such a contribution also exist in fragile glass and is superimposed on a relaxational one which determine the observed \( T \) dependence. In order to quantify the relative part of these two contributions, the anharmonic contribution \( Q^{-1} \) deduced from Eq. (5) are calculated and given in Tab. I for the glasses for which the parameters are known or can be estimated. The Grüneisen parameters \( \gamma \) are taken from the article of Novikov, the specific heat per unit volume from Ref. 56, Debye velocity, \( V_d \) is obtained from Pohl or from our own measurements. \( \tau_{ph} \) is taken to be equal to \( 10^{-13} \) s, as often in the literature. This lower limit for \( \tau_{ph} \) provide a lower limit value for \( Q^{-1} \). Due to uncertainties on some parameters (\( \gamma \) and \( \tau_{ph} \)) only a rough estimate can be made but it appears that an important part of the internal friction can be attributed to anharmonic process in all glasses from room temperature to \( T_g \). Concerning the anomaly of glycerol, for temperatures lower than 150 K, the contribution of anharmonicity to internal friction is expected to be negligible (the condition \( \omega \tau_{ph} < 1 \) may not be fulfilled). On the other hand, the parameter characteristics for the tunneling amplitude is the same as for silica so that we do not have an explanation for the astonishingly low values of \( Q^{-1} \) in glycerol.

Finally, to verify the consistency of the results, the velocity variation vs temperature can be considered. This is not possible in silicate glasses as structural anomalies dominate the elasticity. Below \( T_g \) the variations \( V(T) \) in glasses other than silica are linear. From the relative slope \( (1/V_l)(\delta V_l/\delta T) = \beta \gamma \) where \( \beta \) is the thermal expansion, \( \gamma \) is determined for the mode under study and can be different of \( \gamma \) in Eq. (5) where \( Q^{-1} \) is calculated in relation with the thermal bath of phonons. An estimation of the Grüneisen constant \( \gamma \) of the mode can be deduced; we found \( \gamma = 3 \) for both boron oxide and PMMA; these values correlate well with the literature data used for the independent \( Q^{-1} \) calculation (2.7 and 2.5, respectively from Ref. 55).

**IV. CONCLUSION**

Our contribution demonstrates that, at hypersonic frequencies accessible by Brillouin scattering experiments, the internal friction \( Q^{-1} \) in strong oxide glasses is basically \( T \)-independent above room temperature up to the glass transition. Moreover, the relaxation peak observed in vitreous silica or tetrahedrally coordinated glasses is not observed in any other glass. In strong and fragile glasses, \( Q^{-1} \) differs from that of crystals mainly at low temperature and so the same mechanisms
from activated relaxational processes specific to glasses are efficient at low temperature and in more fragile glasses and can explain the rough proportionality to $T$ observed. The small variation with chemical composition of the internal friction in the hypersonic regime below 100 K extend the remark about universality of low temperature properties pointed out in the review paper of Pohl toward high frequencies and toward higher temperatures than expected.

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* Electronic address: claire@lcvn.univ-montp2.fr

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It can also be noted that our results concern only longitudinal waves for which good accuracy can be obtained for the Brillouin line-width determination but it has been pointed before that the behavior is similar for transversal waves in this frequency range. In GeO$_2$, analogue to SiO$_2$ structure, $Q^{-1}$ shows also a very broad Brillouin peak near 300 K; such a peak disappears in chemically multicomponent glasses.