Is it possible to estimate the Higgs Mass from the CMB Power Spectrum?

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General Relativity and Standard Model are considered as a theory of dynamical scale symmetry with definite initial data compatible with the accepted Higgs mechanism. In this theory the Early Universe behaves like a factory of electroweak bosons and Higgs scalars, and it gives a possibility to identify three peaks in the Cosmic Microwave Background power spectrum with the contributions of photonic decays and annihilation processes of primordial Higgs, W, and Z bosons in agreement with the QED coupling constant, Weinberg’s angle, and Higgs’ particle mass of about 118 GeV.

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I. INTRODUCTION

The observational data 1 on the Cosmic Microwave Background (CMB) power spectrum show several clear peaks at the orbital momenta $\ell_1 \simeq 220$, $\ell_2 \simeq 546$, $\ell_3 \simeq 800$. These phenomena are explained in the $\Lambda$CDM model 2 by acoustic inhomogeneities of the scalar metric component treated as a dynamical variable. By adjusting parameters of the equations for the acoustic excitations one can provide a good fit of the observed peaks and predict other peaks with higher $\ell$ values, which can be found in future observations. Recall that the $\Lambda$CDM model requires the acoustic explanation of the CMB power spectrum by introduction of a dynamical scalar metric component that is absent in the Wigner classification of relativistic states 3. The dynamical scalar metric component is introduced by the $\Lambda$CDM model without any substantial motivation and clear discussion of the reasons for introducing new
concepts. Moreover, this ΛCDM explanation contradicts to the vacuum postulate. Since the CMB power spectrum is one of the highlights of the present-day Cosmology with far-reaching implications and more precise observations are planned for near future [1], the detailed investigation of any possible flaw of the standard theory deserves an attention and a public discussion.

In this paper we try to describe the CMB power spectrum in accord with the well-established Wigner’s theory of the relativistic state classification, where any relativistic particle in quantum field theory can be associated with a unitary irreducible representation of the Poincaré group given in a definite frame with a positive energy.

The cosmological scale factor, its local excitations used for description of the CMB power spectrum, and Poincaré group transformations can be naturally included in the Wigner classification, if General Relativity is considered as the theory of the joint non-linear realization of the affine and conformal symmetries with the Poincaré group of the vacuum stability [4], where the scale invariance of laws of Nature [5, 6] is realized dynamically by means of the dilaton Goldstone field.

The dynamical scale symmetry plays a role of the principle of a choice of variables in the accepted General Relativity (GR) and the Standard Model (SM) [7]. The dilaton Goldstone field compensates all scale transformations of fields including the cosmological scale factor describing expansion of the Universe lengths in the Standard Cosmology [2]. Nevertheless, the cosmological dynamics can be introduced by help of Einstein’s cosmological principle [8] that means averaging all scalar characteristics including the dilaton field over a constant Universe volume. This cosmological dynamics of the zeroth dilaton mode explains the redshift by a permanent increase of all masses in the Universe and leads to the Conformal Cosmology [9, 10, 11, 12, 13, 14, 15, 16], where all measurable quantities are identified with the conformal ones (conformal time, coordinate distance, and constant conformal temperature).

General Relativity considered as the theory dynamical scale symmetry [4, 6] changes the numerical analysis of supernovae type Ia data [17, 18] and shows the dominance of the scalar field kinetic energy in all epochs of the Universe evolution including the chemical evolution, recombination, and SN explosions.

In the paper we try to describe the CMB power spectrum [19] in GR as the theory of dynamical scale symmetry in accord with the classification of relativistic states [3].
II. DILATONIC VARIABLES IN GENERAL RELATIVITY

Let us consider the accepted General Relativity supplemented by the Standard Model and an additional scalar field \( Q \) governing the Universe evolution

\[
S_U[g, F] = \int d^4x \sqrt{-g} \left[ -\frac{R(g)}{6} + \mathcal{L}_{SM}(F) + \partial_\mu Q \partial^\mu Q \right],
\]

where units \( h = c = M_{\text{Planck}} \sqrt{3/(8\pi)} = 1 \) are used throughout the paper. This action depends on a set of scalar, spinor, vector, and tensor fields \( F_{(n)} = \phi, s, V_\mu, g_{\mu\nu} \) with their conformal weights \( n = -1, -3/2, 0, 2 \), respectively.

Following the foundation of the GR as a dynamical scale symmetry \([4, 6]\) we define all observable fields \( \tilde{F}_{(n)} \) as scale-invariants quantities using the following scale transformations of these fields \( F_{(n)} \) in action (1) including the metric components \( g_{\mu\nu} \):

\[
\tilde{F}_{(n)} = \exp\{nD\} F_{(n)}, \quad \tilde{g}_{\mu\nu} = \exp\{2D\} g_{\mu\nu},
\]

where \( D \) is the dilaton compensating scale transformations of all these fields. Any concrete choice of the dilaton as a metric functional \( D[g] \) means a gauge fixing. In \([12, 20, 21]\) this functional is chosen in the form of \( D[g] = -\log |g^{(3)}|/6 \) in accord with the accepted definition of transverse and traceless graviton physical variables given in a definite frame distinguishing the spatial metric components \( g_{ij}^{(3)} \). Therefore, one can remove any scale factor from the spatial metric components in the Dirac-ADM parameterization \([20]\) in terms of the simplex components \( \tilde{\omega}_{(0)}, \tilde{\omega}_{(b)} \) in the Minkowskian tangent space-time

\[
\tilde{ds}^2 = \tilde{\omega}_{(0)}^2 - \tilde{\omega}_{(b)}^2, \quad \tilde{\omega}_{(0)} = e^{-2D} N_d dx^0, \quad \tilde{\omega}_{(b)} = e_{(b)j} (dx^j + N^j dx^0),
\]

where \( e_{(b)j} \) are the triads \([21]\) with the unit spatial metric determinant \( |e_{(b)i}| = 1 \), \( N_d \) is the Dirac lapse function, and \( N^j \) are the shift vector components. In phenomenological applications, one can identify this choice with the CMB co-moving reference frame. In terms of the dilaton variables, the GR action takes the form

\[
S_{GR} = -\int d^4x \sqrt{-g} \frac{R(g)}{6} = \int d^4x \left[ -\frac{v_D^2}{N_d} + \frac{v_{(ab)}^2}{24N_d} - \frac{N_d e^{-4D} R^{(3)}(e) + 8e^{D/2} \Delta e^{-D/2}}{6} \right],
\]

where \( R^{(3)}(e) \) is a curvature, \( \Delta = \partial_i \left[ e_{(a)i}^j \partial_j \right] \) is the Laplace operator, and \( v_D = [\tilde{\partial}_D + \partial_i N^i/3], \ v_{(ab)} = e_{(a)i} v_{(b)i}^i + e_{(b)i} v_{(a)i}^i, \ v_{(a)i} = [\tilde{\partial}_0 e_{(a)i} + e_{(a)i} \partial_i N^i - e_{(a)i} \partial_i N^i / 3] \), are velocities of the metric components, \( \tilde{\partial}_0 = (\partial_0 - N^i \partial_i) \).
Simplex (3) as an object of frame transformations from the Earth frame to the CMB one moving to Leo with the measurable velocity 368 km/s separates the latter from the unmeasurable diffeomorphisms \( x^0 \rightarrow \tilde{x}^0 = \tilde{x}^0(x^0), x^k \rightarrow \tilde{x}^k = \tilde{x}^0(x^0, x^k) \). The principle of diffeo(d)-invariance of observables, \( D(x^0) = D(\tilde{x}^0) \), is at heart of GR. One can see that variables (2) and interval (3) define a d-invariant finite coordinate volume 

\[
\int V_0 d^3 x = V_0 < \infty,
\]

\( d \)-invariant evolution parameter in the field space of events, and a \( d \)-invariant time-interval \( N_0 dx^0 = d\tau \) by Einstein’s cosmological principle \([8]\) as averaging of the dilaton \( D \) and the inverse Dirac lapse function \( N^{-1}_d \)

\[
\langle D \rangle \overline{|_{D=\langle D \rangle := \langle D \rangle + \tilde{S}_U[D, \bar{F}]}} = S_z[\langle D \rangle] + \tilde{S}_U[D, \bar{F}],
\]

where

\[
S_z[\langle D \rangle] = V_0 \int \tau_0 \tau_0 d\tau [-(\partial_\tau \langle D \rangle)^2 + (\partial_\tau \langle \phi \rangle)^2 + (\partial_\tau \langle Q \rangle)^2]_{d\tau = N_0 dx^0}
\]

is the zeroth mode action and the second term \( \tilde{S}_U \) repeats actions \([1]\) and \([4]\) for nonzero harmonics associated with local excitations.

### III. COSMOLOGICAL DYNAMICS OF THE ZEROTH DILATON MODE

Let us consider the Early Universe when one can neglect all these local excitations \( \tilde{S}_U \approx 0 \) (complete expressions of action \([6]\) see in Appendix A). In this case, the cosmological evolution of the Empty Universe arises in the form of a conformal mechanics of zeroth harmonics of all scalar fields \( F = D, \phi, Q \) with equations \( \partial_\tau^2 \langle F \rangle = 0 \) and the initial data

\[
\langle \phi \rangle_I = M_W / (g\sqrt{2}), \quad \partial_\tau \langle \phi \rangle_I = 0; \quad \langle Q \rangle_I = Q_0, \quad \partial_\tau \langle Q \rangle_I = H_0
\]
defined so that the mechanism of spontaneous electroweak symmetry breaking does not differ from the accepted one in SM. Recall, that the accepted spontaneous symmetry breaking mechanism is based on the Coleman–Weinberg potential equation $dV(⟨φ⟩I)/d⟨φ⟩I = 0$ in the perturbation theory restricted by the constraint $∂τ⟨φ⟩ = 0$. In our perturbation theory, loop diagrams also lead to the effective potential with the same equation $dV(⟨φ⟩I)/d⟨φ⟩I = 0$ treated as a constraint that keeps the vacuum equation $∂^2τ⟨φ⟩ = 0$.

Using as example this potential free model of the Empty Universe, one can see that the Standard Cosmology observable quantities are connected with the conformal ones by relation (2):

$$F_{(n)SC} = e^{-n⟨D⟩}F_{(n)CC}. \quad (9)$$

This relation determines the scale factor

$$e^{-⟨D⟩} = a(z) = (1 + z)^{-1}, \quad (10)$$

conformal masses and time

$$\bar{m} = a(z)m_0, \quad d\eta = dτa^2(τ), \quad (11)$$

and the horizon $\tilde{H} = H_0a^{-2}$. In this case, the dilaton solution of the motion equation $∂^2⟨D⟩ = 0$ takes the form

$$⟨D⟩ = ⟨D⟩_0 + H_0(τ - τ_0). \quad (12)$$

In terms of the effective cosmological factor (10) and conformal time (11) this solution becomes

$$a(η) = a_0\sqrt{1 + 2H_0(η - η_0)}. \quad (13)$$

The cosmological dynamics of the Conformal Cosmology (CC) strongly differs from the heuristic phenomenology of the accepted Standard Cosmology (SC) including the ΛCDM model by a constant measurable volume defined by Eqs. (3), running masses, conformal time (11), and the constant CMB conformal temperature $T_{CC} = T_{SC}a(z) = 2.725$ K during the cosmological evolution process. The dilaton variables (2) and (3) explain redshift by the permanent increase of all masses in the Universe. The corresponding luminosity-distance – redshift relation $H_0\bar{ℓ}(z) = z + z^2/2$ does not contradict the recent
SN data \[17\] analyzed in the framework of the Conformal Cosmology \[9, 11\], where the redshift is explained by running masses \[11\], in the case of the conformal mechanics \[8\] leading to the rigid state dominance $H_0 \gg \langle \sqrt{T_d} \rangle$.

Calculation of the primordial helium abundance \[10, 28\] takes into account weak interactions, the Boltzmann factor, $(n/p) e^{\Delta m/T} \sim 1/6$, where $\Delta m$ is the neutron-proton mass difference, which is the same for both SC and CC, $\Delta m_{SC}/T_{SC} = \Delta m_{CC}/T_{CC} = (1+z)^{-1}m_0/T_0$, and the square root dependence of the $z$-factor on the measurable time-interval defined in Eqs. (11) and (13) $(1 + z)^{-1} \sim \sqrt{1 + 2H_0(\eta - \eta_0)}$ explained by the dominant rigid state. In SC, where the measurable time-interval is identified with the Friedmann time, this square root dependence of the $z$-factor is explained by the radiation dominance.

Quantization of the theory constructs a vacuum state with minimal energy defined as $E_U = P(D) = 2V_0 \partial_\tau \langle D \rangle$ with the conservation law $\partial_\tau E_U = 0$ \[13, 14, 15, 16\] (see Appendix B).

IV. THE EARLY UNIVERSE AS FACTORY OF HIGGS PARTICLES

It was shown in Ref. \[12\] that the Empty Universe acts as a factory of longitudinal vector bosons and Higgs particles ($h$) distinguished by their direct interaction with the dilaton. In particular, the field equations for creation and annihilation operators take the form

$$\partial_\eta \tilde{F}^\pm(k, \eta) = \pm i \sqrt{k^2 + \tilde{m}_F^2} \tilde{F}^\pm(k, \eta) + i \partial_\eta \langle D \rangle(\eta) \tilde{F}^\mp(k, \eta) + i[H_{\text{int}}, \tilde{F}^\pm(k, \eta)].$$

The third term leads to collisions and the Boltzmann-type distribution \[22\]

$$B(k, \tilde{T}_F) = \left\{ \exp \left[ \sqrt{\frac{k^2 + \tilde{m}_F^2}{k_B \tilde{T}_F}} - 1 \right] \right\}^{-1}. \quad (14)$$

Here the conformal boson temperature $\tilde{T}_F \sim T_0$ is determined by the collision integral kinetic equation $\tilde{n}(\tilde{T}) = [\tilde{\sigma}_F \text{ scat} r_F]^{-1}$, where $\tilde{n}(\tilde{T}_F)$ is the particle number density and $\tilde{\sigma}_F \text{ scat}$ is the cross section, if the free length $r_F$ is identified with the horizon $\tilde{d}(z) = a(z)^2 H_0^{-1}$ in CC \[12, 23\]

$$r_F = [\tilde{n}(\tilde{T}_F) \tilde{\sigma}_F \text{ scat}]^{-1} \sim \tilde{d}(z) = a(z)^2 H_0^{-1}. \quad (15)$$

Creation of these primordial particles started at the moment $a_{W1}$ when their wavelengths coincided with the horizon length $\tilde{M}_W^{-1} = [a_{W1} M_{W0}]^{-1} \sim \tilde{H}_W^{-1} = a_{W1}^2 (H_0)^{-1}$, as it follows from the uncertainty principle. This gives the instance of creation of primordial particles,
in particular, $W$-bosons

$$a_{W1}^3 \sim \frac{H_0}{M_{W0}} \simeq 19 \cdot 10^{-45} \rightarrow a_{W1} \simeq 2.7 \cdot 10^{-15}. \tag{16}$$

The conformal photon temperature value $\tilde{T}_\gamma = T_\gamma(z)a(z) = T_\gamma(0)$ can be estimated from the kinetic equation (15). If $\tilde{n}(\tilde{T}_F) \sim \tilde{T}_\gamma^3$, one can see that this temperature value $T_\gamma(0) \approx (\tilde{M}_{W1}^2 \tilde{H}_{W1})^{1/3} = (M_{W0}^2 H_0)^{1/3} = 2.3K$ is astonishingly close to the observed temperature $T_0 = 2.725 \text{ K}$ of the cosmic microwave background radiation. The latter can be treated as the final decay product of the primordial bosons that inherits their temperature.

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V. THE CMB TEMPERATURE ANISOTROPY

The collision integral equation (15) generalized to anisotropic decays \( T_0 \rightarrow T_0 + \Delta T \)
\( \sigma \rightarrow \sigma + \sigma_{\gamma\gamma} \) gives us the formula \( |\Delta T/T_0| \simeq (2/3)|\sigma_{\gamma\gamma}/\sigma| \) and a possibility to estimate the magnitude of the CMB anisotropy. Its observational value about \( 10^{-5} \sim \alpha^2 \) [19] testifies to the dominance of the two photon processes. Therefore, the CMB anisotropy revealed in the region of the three peaks \( \ell_1 \simeq 220, \ell_2 \simeq 546, \) and \( \ell_3 \simeq 800 \) can reflect parameters of the primordial bosons and their decay processes, in particular \( h \rightarrow \gamma\gamma, W^+W^- \rightarrow \gamma\gamma, \) and \( ZZ \rightarrow \gamma\gamma. \) The values of multipole momenta at the peaks can be obtained by a simple dimensional analysis using the accepted formula \[ \ell_{Pd} = a_{Pd}^3 H_0 H_0 = \frac{a_{Pd}^3 H_0^2}{a_{Pd}^3}, \] where \( a_{Pd}^3 = a_{Pd}^2 H_0^{-1} = a_{Pd}^{-1} d_{Pd} \) is the conformal horizon \( (15) \) at the instances of the processes \( (P) h \rightarrow \gamma\gamma, W^+W^- \rightarrow \gamma\gamma, \) and \( ZZ \rightarrow \gamma\gamma \) marked by the corresponding cosmological scale factor \( a_{Pd}, \) and \( \tilde{M}_{Pd} = M_{P0} a_{Pd} \) is the conformal mass of emitters in the given process.

One can see that the more horizon-length the more number of emitters covered by the horizon, and the more values of multipole momenta. The substitutions of \( \tilde{d}_{Pd} = a_{Pd}^2 H_0^{-1} \) and \( \tilde{M}_{Pd} = M_{P0} a_{Pd} \) into the Eq. (19) give us the multipole momenta

\[ \ell_{Pd} = a_{Pd}^3 M_{P0} = \frac{a_{Pd}^3 M_{P0}}{a_{Pd}^3}, \] where the initial data \( M_{P0}/H_0 = 1/a_{Pd}^3 \) given by Eq. (16) is taken into account, and \( \ell_{hd} = \ell_1, \ell_{Wd} = \ell_2, \ell_{Zd} = \ell_3. \)

Identifying all photon energies in these processes with the mean photon one \( k_{eff} \) in the CMB, multiplied by the corresponding \( z \)-factor \( a_{Pd}/a_{Pd}, \) we can obtain the Gamov-type relation for the spectrum of photon energy \( E_{P\gamma} = M_{P0} = m_{h0}/2, M_{W0}, M_{Z0} \) in the processes \( h \rightarrow \gamma\gamma, W^+W^- \rightarrow \gamma\gamma, \) and \( ZZ \rightarrow \gamma\gamma, \) respectively, with the peaks in the CMB spectrum

\[ M_{P0} = k_{eff} \frac{a_{Pd}}{a_{Pd}} = k_{eff} \ell_{Pd}^{\frac{1}{3}}, \]

where \( k_{eff} \simeq 9.8\text{GeV} \) is defined by the boson masses: \( \ell_{Wd}^{\frac{1}{3}} = M_{W0}/k_{eff}. \) The empiric formula (21) \( M_{P0} = M_{W,Z} = k_{eff} \ell_{Pd}^{\frac{1}{3}} \) describes the ratio of \( W \) and \( Z \) masses (i.e. the Weinberg angle)
 \[
\frac{M_Z}{M_W} = 1.134 \approx (\frac{800}{546})^{\frac{1}{3}} = 1.136 \rightarrow \sin^2 \theta_W \approx 0.225.
\]
The value of Higgs particle mass is estimated as
\[
\frac{m_h}{2} = M_W \left( \frac{\ell_1}{\ell_2} \right)^{1/3} = M_W \left( \frac{220}{546} \right)^{1/3} \simeq 59 \text{ GeV},
\] (23)
if one takes into account that in the process \( h \to 2\gamma \) the photon energy is the half of the Higgs boson mass. This value of the Higgs boson mass
\[
m_h = 118 \text{ GeV}
\] (24)
is close to the present fit of the LEP experimental data supporting rather low values just above the experimental limit \( m_h > 114.4 \text{ GeV} \). 

VI. SUMMARY

We describe the CMB power spectrum in accord with the well-established Wigner classification of relativistic states treating General Relativity as the theory of the dynamical scale symmetry with the Poincaré group of the vacuum stability [4]. In particular, the CMB moving with the velocity 368 km/s to Leo is considered as an object of Poincaré group transformations in order to pass in the CMB comoving frame [3]. In this frame, the cosmological dynamics can be introduced by help of Einstein’s cosmological principle [5] that means averaging all scalar characteristics including the dilaton field over a constant Universe volume. This cosmological dynamics based on the first principles of general relativity and relativistic invariance gives us a possibility to describe the SN Ia data by the ordinary free kinetic motion [7] of all scalar fields (dilaton, Higgs, and \( Q \)) with the initial data [8] and the positive energy and vacuum postulate. In this case, the CMB arises as final decay product of the primordial vector bosons and Higgs particles created from the vacuum in agreement with the value of the CMB temperature and baryon number density. The CMB power spectrum can be explained by two photon decays of these primordial particles [21] that lead to a value of the Higgs mass [23] about 120 GeV in agreement with the Weinberg angle and QED coupling constant.

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Appendix A: Dynamics of the dilaton

The variation of action (6) with respect to the lapse function $N_d$ leads to the energy constraint

$$N_d \frac{\delta S_U}{\delta N_d} = 0 \rightarrow \frac{[\partial_0\langle D \rangle]^2 - [\partial_0\langle \phi \rangle]^2 - [\partial_0\langle Q \rangle]^2}{N_d} - N_d T_d = 0,$$  \hspace{1cm} (A.1)

where

$$T_d = -\frac{\delta S_U}{\delta N_d} = \frac{4}{3} e^{-7D/2} \Delta e^{-D/2} + \sum_{J=0,2,3,4} e^{-JD} T_J \langle \tilde{F} \rangle$$  \hspace{1cm} (A.2)

is the local energy density as the sum of energy densities $T_J = \langle T_J \rangle + \bar{T}_J$ in terms of conformal fields (2) repeating cosmological regimes of the rigid state $J = 0$, radiation $J = 2$, mass $J = 3$, curvature $J = 4$, in the SC, where $\langle T_J \rangle = H_0^2 \Omega_J$.

Averaging the energy constraint (A.1) over the volume $V_0$ leads to the global constraint

$$[\partial_\tau\langle D \rangle]^2 = [\partial_\tau\langle \phi \rangle]^2 + [\partial_\tau\langle Q \rangle]^2 + \langle \sqrt{T_d} \rangle^2$$  \hspace{1cm} (A.3)

and determines the diffeo-invariant lapse function

$$N' = \frac{N_d}{N_0} = \frac{\langle \sqrt{T_d} \rangle}{\sqrt{T}_d}$$  \hspace{1cm} (A.4)

and the diffeo-invariant interval $d\tau = N_0 dx^0$ through the energy density (A.2).

The dilaton field $D = \langle D \rangle + \bar{D}$ is defined by equation

$$\frac{\delta S_U}{\delta D} = 0 \rightarrow 2\partial_\tau^2\langle D \rangle = \langle T_D \rangle,$$  \hspace{1cm} (A.5)

$$\langle \partial_0 - N^l \partial_l \rangle P_{\bar{D}} = T_D - \langle T_D \rangle,$$  \hspace{1cm} (A.6)

where

$$T_D = -\frac{\delta S_U}{\delta D} = \frac{2}{3} \left\{7N e^{7D/2} \Delta e^{D/2} + e^{D/2} \Delta \left[ Ne^{7D/2} \right] \right\} + N \sum_{J=0,2,3,4} J e^{JD} T_J,$$  \hspace{1cm} (A.7)

and $P_{\bar{D}} = 2\sqrt{T_d} = 2 \left[ (\partial_0 - N^l \partial_l) \bar{D} + \partial_l N^l / 3 \right] / N_d$ is the dilaton momentum.

Eqs. (A.3), (A.5) can be treated as the exact analogy of the Friedmann equation in the tangent space-time defined by simplex components

$$\tilde{\omega}_{(0)} = e^{-2D} N d\tau = e^{-2\bar{D}} \frac{\langle \sqrt{T_d} \rangle}{\sqrt{T}_d} d\eta, \quad \tilde{\omega}_{(b)} = e_{(b)k} dx^k + N_{(b)} d\eta.$$  \hspace{1cm} (A.8)

The Hamiltonian approach to the theory (6) was considered in [13] in the Dirac gauge [20]

$P_{\bar{D}} = 0, \partial_k e_{(b)k} = 0$. 

Appendix B: The Newton law status in cosmological perturbation theory

The investigation of the large-scale structure in the Early Universe is one of the highlights of the present-day Cosmology with far-reaching implications. In particular, the comparison of the cosmological perturbation theory in the \( \Lambda \)CDM model with the Hamiltonian approach to the same cosmological perturbation theory \([13]\) reveals essential differences of these approaches and their physical consequences.

In order to demonstrate these consequences, we consider the case of integrable diffeom- invariant spacial coordinates, when the simplex components in interval \([3]\) \( e_{(b)} dx^i = \omega^{(3)}_b = dx_{(b)} \) are total differentials. The latter means that the coefficients of the spin-connection are equal to zero \( \sigma_{(a)(b)(c)} = e_{(a)j} \left[ \partial_{(b)} e^j_{(c)} - \partial_{(c)} e^j_{(b)} \right] = 0 \) together with the three-dimensional curvature \( R^{(3)} = 0 \) in accord with observational data \([19]\). In this case, the transverse components of the shift vector can be defined by

\[
T_{(0)(a)} = -e^i_{(b)} \frac{\delta S_U}{\delta N_i} = -\partial_{(b)} p_{(b)(a)} + \sum_{f=0,3,F} p_f \partial_{(a)} f = 0, \quad (B.1)
\]

\[
p_{(b)(a)} = \frac{1}{3} v_{(ab)} = \frac{1}{6N} \left( 2\delta_{(a)(b)} \partial_{(c)} N_{(c)} - \partial_{(a)} N_{(b)} - \partial_{(b)} N_{(a)} \right). \quad (B.2)
\]

While the shift vector longitudinal component is given by the Dirac constraint \( \partial_{(a)} e^{-3D} = \partial_{(b)} \left( e^{-3D} N_{(b)} \right) \). The lapse function and dilaton are determined as solutions of Eqs. \((A.4)\), and \((A.6)\). Solutions of these Eqs., in the first order in the Newton coupling constant, take forms \([13, 14]\)

\[
e^{-D/2} = 1 + \frac{1}{2} \int d^3 y \left[ G_{(+)}(x, y) T^_{(+)\mu}(y) + G_{(-)}(x, y) T^_{(-)\mu}(y) \right], \quad (B.3)
\]

\[
N e^{-\gamma D/2} = 1 - \frac{1}{2} \int d^3 y \left[ G_{(+)}(x, y) T^\mu_{(+)\nu}(y) + G_{(-)}(x, y) T^\mu_{(-)\nu}(y) \right], \quad (B.4)
\]

where \( D_{(\pm)}(x, y) \) are the Green functions satisfying the equations

\[
[\pm m^2_{(\pm)} - \Delta] G_{(\pm)}(x, y) = \delta^3(x - y), \quad (B.5)
\]

\[
m^2_{(\pm)} = \frac{3(1 + z)^2}{4} \left[ 14(\beta \mp 1)\Omega_{(0)}(z) \mp \Omega_{(1)}(z) \right] H_0^2, \quad (B.6)
\]

\[
\beta = \sqrt{1 + \left[ \Omega_{(2)}(z) - 14\Omega_{(1)}(z) \right]/[98\Omega_{(0)}(z)]}, \quad (B.7)
\]

\[
\Omega_{(n)}(z) = \sum_{J=0,2,3,4,6} J^n(1 + z)^{2-J} \Omega_J, \quad \Omega_J = \langle T_J \rangle / H_0^2. \quad (B.8)
\]

\( \Omega_{J=0,2,3,4,6} \) are partial density of states: rigid, radiation, matter, curvature, \( \Lambda \)-term, respec-
tively; \( \Omega_{(0)}(0) = 1 \), and \( H_0 \) is Hubble parameter,

\[
\begin{align*}
\mathcal{T}^{(\mu)}_{(\pm)} &= \mathcal{T}_{(0)} \mp 7\beta[7\mathcal{T}_{(0)} - \mathcal{T}_{(1)}], \\
\mathcal{T}^{(\nu)}_{(\pm)} &= [7\mathcal{T}_{(0)} - \mathcal{T}_{(1)}] \pm (14\beta)^{-1} \mathcal{T}_{(0)}
\end{align*}
\]  

(B.9) (B.10)

are the local currents.

In the case of point mass distribution in a finite volume \( V_0 \) with the zeroth pressure and the density \( T^{(0)}(x) = \frac{3}{4a^2} M \left[ \delta^3(x - y) - \frac{1}{V_0} \right] \),

(B.11)
solutions (B.3), (B.4) take the Schwarzschild-type form

\[
\begin{align*}
\mathcal{N}e^{-\sqrt{D}/2} &= 1 - \frac{r_g}{4r} \left[ \frac{14\beta + 1}{28\beta} e^{-m_{(+)}(z)r} + \frac{14\beta - 1}{28\beta} \cos m_{(-)}(z)r \right]_{H_0 = 0} = 1 - \frac{r_g}{4r}, \\
\mathcal{N}e^{-\sqrt{D}/2} &= 1 - \frac{r_g}{4r} \left[ \frac{1 + 7\beta}{2} e^{-m_{(+)}(z)r} + \frac{1 - 7\beta}{2} \cos m_{(-)}(z)r \right]_{H_0 = 0} = 1 + \frac{r_g}{4r},
\end{align*}
\]

where \( \beta = \sqrt{25/49} \approx 1.01/\sqrt{2} \), \( m_{(+) - 3m_{(--)}, m_{(-)} = H_0 \sqrt{(1 + z)\Omega_M} 3/2} \). These solutions have spatial oscillations and the nonzero shift of the coordinate origin.

One can see that in the infinite volume limit \( H_0 = 0, \ a = 1 \) these solutions coincide with the isotropic version of the Schwarzschild solutions: \( e^{-\sqrt{D}/2} = 1 + \frac{r_g}{4r}, \mathcal{N}e^{-\sqrt{D}/2} = 1 - \frac{r_g}{4r}, N^k = 0 \). However, any nonzero cosmological density \( \langle T_{d}^{1/2} \rangle > 0 \) forbids negative values of the lapse function \( \mathcal{N} = \langle T_{d}^{1/2} \rangle / T_{d}^{1/2} > 0 \) \[13, 14, 16]\.

Appendix C: Baryon-antibaryon asymmetry

In SM, in each of the three generations of leptons (e, \( \mu, \tau \)) and color quarks, we have four fermion doublets – in all there are \( n_L = 12 \) of them. Each of 12 fermion doublets interacts with the triplet of non-Abelian fields \( A^1 = (W^{-} + W^{(+)})/\sqrt{2}, A^2 = i(W^{-} - W^{(+)})/\sqrt{2}, \) and \( A^3 = Z/\cos \theta_W \), the corresponding coupling constant being \( g = e/\sin \theta_W \). It is well known that, because of a triangle anomaly, W- and Z- boson interaction with lefthanded fermion doublets \( \psi_L^{(i)}, i = 1, 2, ..., n_L \), \( \psi_L^{(i)} \), leads to nonconservation of the number of fermions of each type \( i \) \[24\],

\[
\partial_{\mu}J^{(i)}_{L\mu} = \frac{1}{32\pi^2} \text{Tr} \tilde{F}_{\mu\nu} \tilde{F}^*_{\mu\nu},
\]

(C.1)
where $\hat{F}_{\mu\nu} = -i F_{\mu\nu} g_W \tau_a / 2$ is the strength of the vector fields, $F_{\mu\nu}^a = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g \epsilon^{abc} A^b_\mu A^c_\nu$.

Taking the integral of the equality in (C.1) with respect to conformal time and the three-dimensional variable $x$, we can find a relation between the change

$$\int_{\eta_l}^{\eta_0} d\eta \int d^3 x \partial_\mu j^{(i)} L_\mu = F^{(i)}(\eta_0) - F^{(i)}(\eta_l) = \Delta F^{(i)}$$

(C.2)

of the fermion number $F^{(i)} = \int d^3 x j^{(i)}$ and the Chern-Simons functional $F_{\mu\nu}^* \hat{F}_{\mu\nu}$, so that after integration Eq. (C.1) takes the form

$$\Delta F^{(i)} = N_{CS} \neq 0, \quad i = 1, 2, ..., n_L.$$  

(C.3)

The equality in (C.3) is considered as a selection rule – that is, the fermion number changes identically for all fermion types: $N_{CS} = \Delta L^e = \Delta L^\mu = \Delta L^\tau = \Delta B / 3$; at the same time, the change in the baryon charge $B$ and the change in the lepton charge $L = L^e + L^\mu + L^\tau$ are related to each other in such a way that $B - L$ is conserved, while $B + L$ is not invariant. Upon taking the sum of the equalities in (C.3) over all doublets, one can obtain $\Delta (B + L) = 12 N_{CS}$ [24].

We can evaluate the expectation value of the Chern-Simons functional (C.3) (in the lowest order of perturbation theory in the coupling constant) in the Bogoliubov vacuum $b|0 > = 0$. Specifically, we have

$$N_{CS} = N_W = - \frac{1}{32\pi^2} \int_0^{\eta_L} d\eta \int d^3 x \langle 0 | \text{Tr} \hat{F}_{\mu\nu}^W \hat{F}_{\mu\nu}^W | 0 \rangle,$$

(C.4)

where $\eta_L$ is the W-boson lifetime, and $N_W$ is the contribution of the primordial $W$ boson. The integral over the conformal spacetime bounded by three-dimensional hypersurfaces $\eta = 0$ and $\eta = \eta_L$ is given by $N_W = 2 \alpha_W V_0 \int_0^{\eta_L} d\eta \int dk |k|^3 R_W(k, \eta)$, where $\alpha_W = \alpha_{QED} / \sin^2 \theta_W$ and $R_W = \frac{i}{\sqrt{2}} < 0 | b^+ b^+ - b^- b^- | 0 > = - \sinh(2r(\eta_L)) \sin(2\theta(\eta_L))$ is the Bogoliubov condensate [12] that is specified by relevant solutions to the Bogoliubov equations. Upon a numerical calculation of this integral, we can estimate the expectation value of the Chern-Simons functional in the state of primordial bosons.

At the vector-boson-lifetime value in (18), this yields the following result at $n_\gamma = 2, 402 \times T^3 / \pi^2$

$$\frac{N_{CS}}{V_0} = \frac{N_W}{V_0} = 4 \alpha_W T^3 \times 1.44 = 0.8 n_\gamma.$$

(C.5)
where \( n_\gamma \) is the number density of photons forming Cosmic Microwave Background radiation. On this basis, the violation of the fermion-number density in the cosmological model being considered can be estimated as \[ \Delta F^{(i)}/V_0 = N_{CS}/V_0 = 0.8n_\gamma. \]

According to SM, there is the CKM-mixing that leads to CP nonconservation, so that the cosmological evolution and this nonconservation freeze the fermion number at \( \eta = \eta_L \). This leads to the baryon-number density \[ n_b(\eta_L) = X_{CP}\Delta F^{(i)}/V_0 \simeq X_{CP}n_\gamma(\eta_L), \]

where the factor \( X_{CP} \) is determined by the superweak interaction of \( d \) and \( s \) quarks, which is responsible for CP violation experimentally observed in \( K \)-meson decays \[27\].

From the ratio of the number of baryons to the number of photons, one can deduce an estimate of the superweak-interaction coupling constant: \( X_{CP} \sim 10^{-9} \). Thus, the evolution of the Universe, primary vector bosons, and the aforementioned superweak interaction \[27\] lead to baryon-antibaryon asymmetry of the Universe

\[ \frac{n_b(\eta_L)}{n_\gamma(\eta_L)} \simeq X_{CP} = 10^{-9}. \] (C.6)

Thus, the primordial bosons before their decays polarize the Dirac fermion vacuum and give the baryon asymmetry frozen by the CP – violation so that for billion photons there is only one baryon.

The problem is to show that the Universe matter content considered as the final decay product of primordial bosons is in agreement with observational data \[12\].

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