Constraints on QSO emissivity using H I and He II Lyman α forest

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ABSTRACT

The spectrum of cosmic ultraviolet background radiation at He II ionizing energies \( E \geq 4 \) Ryd is important to study the He II reionization, thermal history of the intergalactic medium (IGM) and metal lines observed in quasi-stellar object (QSO) absorption spectra. It is determined by the emissivity of QSOs at \( E \geq 4 \) Ryd obtained from their observed luminosity functions and the mean spectral energy distribution (SED). The SED is approximated as a power law at energies \( E \geq 1 \) Ryd, \( f_E \propto E^\alpha \), where the existing observations constrain the power-law index \( \alpha \) only up to \( \sim 2.3 \) Ryd. Here, we constrain \( \alpha \) for \( E \geq 4 \) Ryd using recently measured He II Lyman α effective optical depths (\( \tau_\alpha^{\text{He II}} \)), H I photoionization rates and updated H I distribution in the IGM. We find that \( -1.6 < \alpha > -2 \) is required to reproduce the \( \tau_\alpha^{\text{He II}} \) measurements when we use the QSO emissivity obtained from their luminosity function using optical surveys. We also find that the models where QSOs can alone reionize H I cannot reproduce the \( \tau_\alpha^{\text{He II}} \) measurements. These models need modifications, such as a break in mean QSO SED at energies greater than 4 Ryd. Even after such modifications, the predicted He II reionization history, showing that the He II is highly ionized even at \( z \sim 5 \), is significantly different from the standard models. Therefore, the thermal history of the IGM will be crucial to distinguish these models. We also provide the He II photoionization rates obtained from binned \( \tau_\alpha^{\text{He II}} \) measurements.

Key words: galaxies: evolution – intergalactic medium – quasars: general – diffuse radiation.

1 INTRODUCTION

The observed ionization state of the intergalactic medium (IGM) at \( z \leq 6 \) (Gunn & Peterson 1965; Fan et al. 2006; Becker & Bolton 2013) is maintained by cosmic ultraviolet background (UVB) radiation emanating from quasi-stellar objects (QSOs) and galaxies (Miralda-Escude & Ostriker 1990; Shapiro, Giroux & Babul 1994; Haardt & Madau 1996; Shull et al. 1999). Apart from being the main driver of the hydrogen and helium reionization, the UVB maintains the ionization state of metals in the IGM and in the circumgalactic environments of galaxies. Therefore, the spectrum of UVB is important to study the cosmic metal mass density and the metal enrichment of the IGM (see e.g. Songaila & Cowie 1996; Songaila 2001; Bergeron et al. 2002; Carswell, Schaye & Kim 2002; Simcoe, Sargent & Rauch 2004; Peebles et al. 2014; Shull, Danforth & Tilton 2014; Hussain et al. 2017) by relating the observed ionic abundances to metal abundances.

Spectrum of the UVB depends on the spectral energy distribution (SED) of the sources that are contributing to it, mainly QSOs and star-forming galaxies. If we divide the UVB naively into hydrogen ionizing part (1 Ryd < \( E < 4 \) Ryd) and helium ionizing part (\( E \geq 4 \) Ryd), the former is contributed by both galaxies and QSOs but latter is predominantly contributed by only QSOs. The relative contribution by QSOs and galaxies to the hydrogen ionizing part of the UVB depends on average escape fraction \( f_{\text{esc}} \), a parameter that quantifies the amount of hydrogen ionizing photons escaping from galaxies. The \( f_{\text{esc}}(z) \) can be obtained using the measurements of hydrogen photoionization rates (\( \Gamma_{\text{HI}} \)) for a given QSO emissivity and star formation history of galaxies (see Inoue, Iwata & Deharveng 2006; Khaire et al. 2016). On the other hand, for the measured \( \Gamma_{\text{HI}}(z) \) and the H I distribution in the IGM, the helium ionizing part of the UVB depends only on the QSO emissivity at \( E \geq 4 \) Ryd. This emissivity is estimated through the QSO luminosity functions and the mean SED of QSOs. The SED is usually approximated as a power law, \( f_E \propto \nu^\alpha \) at \( E \geq 1 \) Ryd (\( \lambda < 912 \) Å) from the observed composite QSO spectra (Zheng et al. 1997; Telfer et al. 2002; Scott et al. 2004; Stevans et al. 2014; Lusso et al. 2015). Although the existing observations have probed mean QSO SED only up to \( E < 2.3 \) Ryd (\( \lambda < 400 \) Å), it is usually extrapolated up to 35 Ryd (\( \lambda \sim 25 \) Å) to calculate the He II ionizing emissivity and the UVB. The reported values of the power-law index \( \alpha \) show large variation from \(-0.56 \) to \(-1.96 \). Moreover, the number of QSOs where SED at high energies can be directly probed is very small (see e.g. Tilton et al. 2016). The existing measurements of \( \alpha \) over the last two decades are summarized in Table 1. Using different \( \alpha \) in UVB models gives significantly different UVB spectrum especially for \( E \geq 4 \) Ryd. Also, the He II ionizing emissivities obtained
using different $\alpha$ provide different histories of the He II reionization. Like hydrogen ionizing part of the UVB, we need measurements of He II photoionization rates ($\Gamma_{\text{HeII}}$) that can be used to constrain the He II ionizing emissivity. The accurate estimate of UVB spectrum, especially at $E \geq 4$ Ryd ($\lambda \leq 228$ Å), is important for studying the ionization mechanism for high ionization systems such as O vi (see e.g. Danforth & Shull 2005; Tripp et al. 2008; Muzahid et al. 2012; Puchat et al. 2016) and Ne viii (see e.g. Savage et al. 2005, 2011; Narayanan, Savage & Wakker 2012; Meiring et al. 2013; Hussain et al. 2015, 2017) which are believed to trace the warm–hot phase of the IGM. It is also important for studying the thermal history of the IGM (Bolton et al. 2010, 2012; Lidz et al. 2010; Becker et al. 2011; Khrykin, Hennawi & McQuinn 2017) and the process of He II reionization (Faucher-Giguère et al. 2009; McQuinn et al. 2009; Compostella, Cantalupo & Porciani 2013; La Plante & Trac 2016). The above-mentioned importance of $\alpha$ and the issues with its measurements motivate us to theoretically constrain $\alpha$ at $E \geq 4$ Ryd. For that, we use the observations of H I and He II Lyman $\alpha$ forest.

The He II Lyman $\alpha$ forest has been observed for few QSOs at $z > 2.5$ with UV spectrographs on space telescopes such as Far Ultraviolet Spectroscopic Explorer (FUSE; Kriss et al. 2001; Shull et al. 2004; Fechner et al. 2006) and Cosmic Origin Spectrograph (COS) onboard Hubble Space Telescope (HST; Syphers et al. 2011; Worseck et al. 2016). With such observations, the Lyman $\alpha$ effective optical depths of He II ($\tau_{\text{HeII}}^\alpha$; Shull et al. 2010; Worseck et al. 2011; Syphers & Shull 2013) and the ratio of He II to H I in the IGM absorbers (Zheng et al. 2004; Muzahid, Srianand & Petitjean 2011; McQuinn & Worseck 2014) have been measured. The recent measurements of $\tau_{\text{HeII}}^\alpha$ by Worseck et al. (2016) at $2.3 < z < 3.5$ can be used to constrain the He II ionizing emissivity and the properties of QSO SED such as the spectral index $\alpha$. This is what we explore in our analysis.

For a given QSO emissivity at 1 Ryd and a mean SED of QSOs, using our cosmological radiative transfer code (Khaire & Srianand 2013, 2015a,b), we estimate the He II ionizing UVB, photoionization rates of He II and $\tau_{\text{HeII}}^\alpha$. We also calculate the corresponding He II reionization history. By comparing these values with the $\tau_{\text{HeII}}^\alpha$ measurements, we constrain the mean SED of QSOs. We use two models of QSO emissivity, one obtained from the compilation of optically selected QSOs (Khaire & Srianand 2015a) and the other where QSOs can alone reionize H I when extrapolated to $z > 6$ (Madau & Haardt 2015; Khaire et al. 2016). The latter uses the QSO luminosity function of Giallongo et al. (2015) that claimed to detect large number density of low-luminosity QSOs at $z > 4$. Using $\tau_{\text{HeII}}^\alpha$ and $\Gamma_{\text{HeII}}$, measurements, we also estimate the $\Gamma_{\text{HeII}}$ values that depend only on the H I distribution of the IGM and independent of the UVB models.

The paper is organized as follows. In Section 2, we discuss the basic theory to calculate $\tau_{\text{HeII}}^\alpha$ using H I distribution of the IGM and $\Gamma_{\text{HeII}}$ using $\tau_{\text{HeII}}^\alpha$ measurements. In Section 3, we explain the basic theory and assumptions to calculate the He II ionizing emissivity, the UVB and the He II reionization history. In Section 4, we discuss our results for different models of QSO emissivity and uncertainties. We present the summary in Section 5. Throughout this paper, we use cosmology parameters $\Omega_m = 0.7$, $\Omega_b = 0.3$ and $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$ consistent with that from Planck Collaboration XIII (2016).

2 Hit Optical Depths and Photoionization Rates

2.1 Basic theory: Lyman $\alpha$ effective optical depths

The Lyman $\alpha$ effective optical depth for H I ($\tau_{\text{HI}}^\alpha$) and He II ($\tau_{\text{HeII}}^\alpha$) at redshift $z$ is obtained by (Paresce, McKee & Bowyer 1980; Madau & Meiksin 1994),

$$\tau_{\text{HI}}^\alpha(z) = \frac{1 + \frac{z}{\lambda_0}}{\alpha_0} \int_0^{N_{\text{min}}} dN_{\text{HI}} \frac{\partial^2 N}{\partial N_{\text{HI}} \partial \zeta} W_\alpha,$$  \hspace{1cm} (1)

Here, $\alpha$ denotes the species H I or He II, $\lambda_0$ in the rest-frame Lyman $\alpha$ line wavelength of species $x$ (i.e. 1215.67 Å for H I and 303.78 Å for He II), $N_{\text{min}}^\alpha$ is the minimum column density of $x$ used in the integral and $\frac{\partial^2 N}{\partial N_{\text{HI}} \partial \zeta} = f(N_{\text{HI}}, \zeta)$ is the column density distribution of H I. Here, $W_\alpha$ is the equivalent width of the Lyman $\alpha$ line expressed in wavelength units for species $x$ as given by

$$W_\alpha = \int_0^{N_{\text{HI}}} d\lambda \frac{1}{\alpha_0} \phi(\lambda, x),$$  \hspace{1cm} (2)

where, $\phi(\lambda, x)$ is the Voigt profile function for species $x$, $y = N_{\text{HI}}$ when $x = \text{H I}$ and $y = 2\eta \times N_{\text{HI}}$ when $x = \text{He II}$ where $\eta = N_{\text{HeII}}/N_{\text{HI}}$.

The calculation of $\tau_{\text{HI}}^\alpha$ depends on the observed $f(N_{\text{HI}}, \zeta)$. In the absence of the column density distribution of H I, the calculation of $\tau_{\text{HI}}^\alpha$ relies on the estimate of the parameter $\eta$. The $\eta$ determines the amount of $N_{\text{HeII}}$ in intergalactic absorber having H I column density $N_{\text{HI}}$. It is estimated under the assumption that the IGM is in photoionization equilibrium maintained by the UVB. The $\eta$ is independent of $N_{\text{HI}}$ for the absorbers that are optically thin to He II.
Here, $\eta$ fraction of helium. Using equation (5) reduces to

$$
\eta = \frac{n_{\text{He}} \, \alpha^{\text{He}}_x(T) \, \Gamma_{\text{He}}(z)}{n_{\text{H}} \, \alpha^{\text{H}}_x(T) \, \Gamma_{\text{H}}(z)}. \tag{3}
$$

Here, $\alpha^{\text{A}}_x(T)$ and $\Gamma_x$ are the case A recombination rate coefficient (that depends on the gas temperature $T$) and the photoionization rate for species $x$, respectively, whilst $n_{\text{H}}$ and $n_{\text{He}}$ are the number density of total hydrogen and helium in the IGM, respectively. The ratio $n_{\text{He}}/n_{\text{H}} = y_p/(4 - 4y_p)$, where $y_p$ is the primordial mass fraction of helium. Using $y_p = 0.025$ from Planck Collaboration XIII (2016) and the expressions for recombination rate coefficients,\(^1\) equation (3) can be approximated as

$$
\eta_{\text{thin}}(z) = \frac{0.45}{0.06} \left( \frac{T}{10^{3} \text{K}} \right) ^{0.06} \frac{\Gamma_{\text{H}}(z)}{\Gamma_{\text{He}}(z)}. \tag{4}
$$

The above equation shows that $\eta_{\text{thin}}$ weakly depends on the temperature and it is mainly decided by the ratio of $\Gamma_{\text{H}}$, to $\Gamma_{\text{He}}$. Under photoionization equilibrium, $\eta$ at all $N_{\text{H}}$, obtained from radiative transfer simulations can be approximated by the following quadratic equation (Fardal, Giroux & Shull 1998; Faucher-Giguère et al. 2009; Schaye 2001). With the same setup, we also verify these values and fixed line-of-sight length equal to the Jeans length following Becker et al. (2013). Inoue et al. (2014) has used $N_{\text{H}}(z)$ and the expressions for recombination rate coefficients, 1 The case A recombination rate coefficients for H I and He II in units of $\text{cm}^3 \text{s}^{-1}$ are given by $\alpha^{\text{H}}_x(T) = 2.51 \times 10^{-13} T^{0.76}$ and $\alpha^{\text{He}}_x(T) = 1.36 \times 10^{-12} T^{0.70}$, where $T = 10^3 T_{1.3} \text{K}$. 

In the following subsection, we calculate $\eta_{\text{thin}}$ from the $\tau^\text{Hn}_{\text{thin}}$ measurements and estimate the corresponding He II photoionization rates.

2.2 He II photoionization rates

In equations (1) and (2), the value of $\eta_{\text{thin}}$ can be varied to obtain the desired value of $\tau^\text{He II}_{\text{thin}}$. By this method, one can obtain the values of $\eta_{\text{thin}}$ for measured values of $\tau^\text{He II}_{\text{thin}}$. This $\eta_{\text{thin}}$ along with the measurements of $\Gamma_{\text{H}}$ provides $\Gamma_{\text{He}}$ using equation (4). Here, we estimate $\eta_{\text{thin}}$ using recent measurements of $\tau^\text{He II}_{\text{thin}}$ from Worseck et al. (2016). Then, we calculate $\Gamma_{\text{He}}$ using this $\eta_{\text{thin}}$ and the $\Gamma_{\text{H}}$; $\Gamma_{\text{He}}$ values are consistent with the values obtained in the right-hand panel of Fig. 1. The error-bars on $\eta_{\text{thin}}$ arise from 1σ errors on $\tau^\text{He II}_{\text{thin}}$. We need $\eta_{\text{thin}}$ to have values in the range of 40–320 to reproduce the observed distribution of $\tau^\text{He II}_{\text{thin}}$. Note that the $\eta_{\text{thin}}$ calculated in this way ignores the differences in the $\tau^\text{H1}_{\text{thin}}$ one expects for different lines of sight. Although, the line-of-sight average $\tau^\text{H1}_{\text{thin}}$ at the regions where $\tau^\text{He II}_{\text{thin}}$ was measured shows very good agreement with the mean $\tau^\text{H1}_{\text{thin}}$ (Faucher-Giguère et al. 2008; Becker et al. 2013), the same mean $\tau^\text{H1}_{\text{thin}}$ that has been used to obtain $f(N_{\text{H1}}, z)$ by Inoue et al. (2014), significant variations in $\tau^\text{H1}_{\text{thin}}$ occur on the $\Delta z = 0.04$ scales (see fig. 8 of Worseck et al. 2016).

To estimate $\tau^\text{He II}_{\text{thin}}$, we need $\eta_{\text{thin}}$ value in the same redshift range as the $\Gamma_{\text{H1}}$ measurement. Therefore, we take median of the $\tau^\text{He II}_{\text{thin}}$ measurements in three redshift bins that are $z = 2.3–2.6$, $z = 2.6–3.0$ and $z = 3.0–3.5$. These bins match closely with the redshift bins used for $\Gamma_{\text{H1}}$ measurements by Becker & Bolton (2013). Here, instead of using mean redshift for bins, we use the median redshift since the distribution of $\tau^\text{He II}_{\text{thin}}$ in each bin is not uniform. The median $\tau^\text{He II}_{\text{thin}}$ values in these bins are shown in the left-hand panel of Fig. 1 and provided in Table 2. The error-bars are the 95th percentile values of the distribution of errors in each bin. Since, the highest redshift bin contains most of the lower limits on $\tau^\text{He II}_{\text{thin}}$ measurements, the median $\tau^\text{He II}_{\text{thin}}$ in this bin is also a lower limit. The $\eta_{\text{thin}}$ values required to obtain these binned $\tau^\text{He II}_{\text{thin}}$ measurements are shown in the right-hand panel of Fig. 1 and provided in Table 2. Error-bars on $\eta_{\text{thin}}$ are obtained from the error-bars on median $\tau^\text{He II}_{\text{thin}}$ as shown in the left-hand panel of Fig. 1. The median $\tau^\text{He II}_{\text{thin}}$ and $\eta_{\text{thin}}$ show clear increasing trend with redshift. We obtain $\Gamma_{\text{He}}$ for these $\eta_{\text{thin}}$ values (from equation 4) using the $\Gamma_{\text{H1}}$ measurements of Becker & Bolton (2013) in the corresponding redshift bins. Table 2 summarizes our estimated $\Gamma_{\text{He}}$ values as well as the $\Gamma_{\text{H1}}$ measurements that are used for obtaining them. The errors on $\Gamma_{\text{He}}$ also account for the errors on $\Gamma_{\text{H1}}$ measurements. Note that the $\Gamma_{\text{He}}$ calculated in this way depends only on the $f(N_{\text{H1}}, z)$ and does not depend on the UVB models. Our $\Gamma_{\text{He}}$ values are consistent with the values obtained by Worseck et al. (2016) using their semi-analytic model for post-reionization $\tau^\text{He II}_{\text{thin}}$. We have also calculated the mean free path for $\text{He II}$.
We are interested in computing the He II ionizing UVB to obtain the He II ionizing emissivity. In this section, we explain the basic theory to calculate the He II ionizing UVB, the assumptions involved in estimating He II ionizing emissivity and theory for calculating He II reionization history.

3 HELIUM IONIZING UVB

We are interested in computing the He II ionizing UVB to obtain the He II ionizing emissivity and the He II reionization history to constrain the He II ionizing QSO emissivity. In this section, we explain the basic theory to calculate the He II ionizing UVB, the assumptions involved in estimating He II ionizing emissivity and theory for calculating He II reionization history.

Table 2: $\Gamma_{\text{He II}}$ and $\lambda_{\text{mp}}$ estimates.

| Median $z$ | 2.52 | 2.8 | 3.2 |
|------------|------|-----|-----|
| $\tau_{\text{He II}}$ range | 2.3–2.6 | 2.6–3.0 | 3.0–3.5 |
| $\tau_{\text{He II}}$ (median) | 1.42–0.40 | 2.33–0.57 | 5.26–0.73 |
| $\eta_{\text{thin}}$ | 67.4–4.59 | 90.4–28.7 | 170.8–30.6 |
| $\Gamma_{\text{He II}}$ in $10^{-12}\text{ s}^{-1}$ | 1.035±0.37 | 0.86±0.30 | 0.79±0.28 |
| $\Gamma_{\text{He II}}$ in $10^{-15}\text{ s}^{-1}$ | 6.91±3.32 | 4.26±3.42 | 2.08±1.83 |
| $\lambda_{\text{mp}}$ in pMpc | 32.9±10.7 | 18.7±5.0 | 7.5±1.0 |

Notes. *Errors on the mean $\tau_{\text{He II}}$ correspond to 95th percentile of the distribution of errors on $\tau_{\text{He II}}$ measurements in the redshift bin. $\Gamma_{\text{He II}}$ measurements from Becker & Bolton (2013).

3.1 The UVB

The photoionization rate, $\Gamma_{\nu}(z)$, at redshift $z$ for species $\nu$ is obtained by following integral:

$$\Gamma_{\nu}(z) = \int_{z_0}^{\infty} \frac{4\pi J_0(\nu, z)}{h\nu} \sigma_{\nu}(v).$$

(6)

Here, $\nu$ and $\sigma_{\nu}$ are the ionization threshold frequency and photoionization cross-section for the species $\nu$, respectively, $h$ is Planck constant and $J_0(\nu, z)$, in units of erg cm$^{-2}$ s$^{-1}$ Hz$^{-1}$ sr$^{-1}$, is the angle-averaged specific intensity of the UVB radiation at frequency $\nu$ and redshift $z$. $J_0(\nu, z_0)$ is obtained by solving the cosmological radiative transfer equation (see Peebles 1993; Haardt & Madau 1996),

$$J_0(\nu, z_0) = \frac{c}{4\pi} \int_{z_0}^{\infty} dz' \frac{(1+z')^3}{(1+z)H(z)} \epsilon_{\nu}(z)(1 - e^{-\tau_{\text{eff}}(\nu, z_0, z')}).$$

(7)

Here, $c$ is the speed of light, $H(z) = H_0 \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$ is the Hubble parameter, $\nu$ is related to $v_0$ by $\nu = v_0/(1+z)/(1+z_0)$ and $\epsilon_{\nu}(z)$ is the comoving emissivity of the sources. $\tau_{\text{eff}}(v_0, z_0, z)$ is an effective optical depth encountered by a photon observed at $z_0$ having frequency $v_0$ while travelling from its emission redshift $z$ to $z_0$. Assuming that the IGM clouds along any line of sight are Poisson distributed, the $\tau_{\text{eff}}$ is given by (see Paresece et al. 1980; Padmanabhan 2002),

$$\tau_{\text{eff}}(v_0, z_0, z) = \int_{z_0}^{z} dz' \int_{\nu_0}^{\infty} dN_{\text{HII}} f(N_{\text{HII}}, z')(1 - e^{-\tau_{\nu'}}).$$

(8)

Here, $\tau_{\nu'}$ is the continuum optical depth encountered by photons emitted at frequency $\nu'$ while travelling from their emission redshift $z'$ to $z_0$. It is given by

$$\tau_{\nu'} = N_{\text{HII}} \sigma_{\nu'}(v') + N_{\text{He II}} \sigma_{\nu\text{He II}}(v') + N_{\text{H I}} \sigma_{\nu\text{H I}}(v'),$$

(9)

where, $v' = v_0/(1+z)/(1+z_0)$. In the redshift range of our interest ($z < 4$) He I has negligible contribution to $\tau_{\nu'}$ (see also...
Faucher-Giguère et al. 2009; Haardt & Madau 2012). Therefore, we approximate \( \tau_{\nu} \) as

\[
\tau_{\nu} = N_{\text{He}} \sigma_{\text{He}}(\nu') + \eta \sigma_{\text{He}}(\nu'),
\]

(10)

Note that, here the \( \tau_{\text{eff}} \) depends on \( \eta(N_{\text{He}}) \) and not just on \( \eta_{\text{He}} \). The UVB is obtained by iteratively solving equations (5)–(10) for an assumed ionizing emissivity \( \epsilon_{\nu}(z) \).

Here, we are interested in calculating the He II ionizing UVB at \( 2 < z < 4 \). For that we need He II ionizing emissivity (at \( \lambda \leq 228 \) Å) and \( \Gamma_{\text{He}} \) to estimate \( \eta \). Since, we are using the measured values of \( \Gamma_{\text{He}} \) at \( z > 2 \), we do not need to explicitly calculate the H ionizing UVB. However, note that to calculate the He II ionizing UVB at \( z = z_{0} \) we need \( \Gamma_{\text{He}}(z) \) at \( z > z_{0} \). Therefore, in our UVB calculations, along with the \( \Gamma_{\text{He}} \) measurements by Becker & Bolton (2013) at \( 2.4 < z < 4.8 \), we use \( \Gamma_{\text{He}} \) at \( z = 2 \) from Bolton & Haehnelt (2007) and at \( z > 5 \) from Calverley et al. (2011) and Wyithe & Bolton (2011). We also estimate the UVB for \( 1 \) higher and lower values of measured \( \Gamma_{\text{He}}(z) \) to study the uncertainties arising in our results due to the uncertainties in the measured \( \Gamma_{\text{He}} \).

The following subsection explains the usual procedure to estimate the He II ionizing emissivity.

### 3.2 Helium ionizing emissivity

In the absence of Population-III stars at the redshifts of our interest, star-forming galaxies emit a negligible amount of He II ionizing photons. Therefore, the helium ionizing emissivity \( \epsilon_{\nu} \) at \( \lambda \leq 228 \) Å is contributed by QSOs alone. Using the expression for QSO emissivity at \( 912 \) Å \( (\epsilon_{912}^{Q}) \) and the mean SED of QSOs at \( \lambda < 912 \) Å which is usually approximated as a power-law \( f_{\nu} \propto \nu^{\alpha} \), the \( \epsilon_{\nu} \) can be written as

\[
\epsilon_{\nu}(z) = \left( \frac{\nu}{\nu_{912}} \right)^{\alpha} \epsilon_{912}^{Q}(z),
\]

(11)

where \( \nu_{912} = c/912 \) Å Hz.

Helium ionizing emissivity depends on \( \epsilon_{912}^{Q} \) and \( \alpha \). The \( \epsilon_{912}^{Q} \) is obtained from QSO luminosity function along with the mean SED from optical to extreme UV wavelengths (up to \( \sim 912 \) Å) that is well observed. However, at \( \lambda < 912 \) Å, the power-law index \( \alpha \) is measured only up to \( \lambda \sim 425 \) Å (see Table 1). In absence of any observational constraints, this emissivity is usually extrapolated to smaller wavelengths (up to \( \sim 25 \) Å) to estimate the He II ionizing emissivity. Moreover, the values of \( \alpha \) reported in the literature over last two decades are not consistent with each other. Reported values vary from \( -0.56 \) to 1.96 as summarized in the Table 1. The estimates of He II ionizing UVB and the \( \Gamma_{\text{He}} \) are severely affected by the choice of \( \alpha \) in the UVB models. These issues motivate us to constrain the \( \alpha \) at \( \lambda \leq 228 \) Å that is consistent with the \( \Gamma_{\text{He}} \) measurements and \( \Gamma_{\text{He}}^{d} \). For that, we use two models of \( \epsilon_{912}^{Q}(z) \), namely model A and model B, as explained below:

(i) Model A: the model A uses the QSO luminosity functions observed at UV and optical wavebands at all redshifts as compiled in Khaire & Srianand (2015a, see their table 1). To estimate the He II ionizing emissivity and UVB, model A takes \( \alpha \) as a free parameter and \( \epsilon_{912}^{Q}(z) \) in units of erg s\(^{-1}\) Hz\(^{-1}\) Mpc\(^{-3}\) as (Khaire & Srianand 2015a)

\[
\epsilon_{912}^{Q}(z) = 10^{24.6} (1 + z)^{5.9} \frac{\exp(-0.36z)}{\exp(2.2z) + 25.1}.
\]

(12)

This is a simple fit through the compiled \( \epsilon_{912}^{Q} \) values as shown in Fig. 2 (blue solid curve). This model needs additional contribution to H I ionizing photons from star-forming galaxies to reionize H I at \( z > 5.5 \) and to be consistent with the \( \Gamma_{\text{He}} \) measurements at \( z > 3 \) (see Khaire et al. 2016).

(ii) Model B: in addition to the QSO luminosity functions observed at UV and optical wavebands at \( z < 4 \), model B uses the QSO luminosity function from Giallongo et al. (2015) at \( z > 4 \) obtained by selecting QSO candidates based on their X-ray fluxes. In contrast with model A, model B does not require any contribution from star-forming galaxies to reionize H I i.e. QSOs alone reionize H I in this model (e.g. Madau & Haardt 2015, hereafter MH15; Khaire et al. 2016). Therefore, the He II ionizing emissivity obtained through choice of \( \alpha \) and \( \epsilon_{912}^{Q}(z) \) in model B has to simultaneously satisfy the observational constraints on H I reionization (Schumberger et al. 2014; McGreer, Mesinger & D’Odorico 2015; Planck Collaboration XIII 2016) at \( z > 5.5 \), unresolved X-ray background at \( z > 5 \) (Moretti et al. 2012) and \( \epsilon_{912}^{Q}(z) \) obtained by Giallongo et al. (2015) at \( z > 4 \). These constraints provide little room to change \( \alpha \) for a given \( \epsilon_{912}^{Q}(z) \) in model B. It is unlike the model A where the discrepancy in H I ionizing photons due to decreasing value of \( \alpha \) can be resolved by increasing the contribution from star-forming galaxies. Therefore, instead of making \( \alpha \) as a free parameter, for fixed value of \( \alpha \) and corresponding \( \epsilon_{912}^{Q}(z) \) we explore a break in QSO SED at He II ionizing part (\( E \geq 4 \) Ryd) required to satisfy the \( \epsilon_{912}^{Q}(z) \) measurements. In model B, we take two values of \( \alpha \) and the corresponding two forms of \( \epsilon_{912}^{Q}(z) \) which are shown to be consistent with the constraints mentioned above. First, we take \( \alpha = -1.4 \) (consistent with Stevans et al. 2014) and \( \epsilon_{912}^{Q}(z) \) as

\[
\log \epsilon_{912}^{Q}(z) = 25.35 \exp(-0.0047z) - 2.55 \exp(-1.61z).
\]

This is consistent with the model presented in Khaire et al. (2016). We denote this combination of \( \alpha \) and \( \epsilon_{912}^{Q}(z) \) as model B1. Secondly, we take \( \alpha = -1.7 \) (consistent with Lusso et al. 2015) and \( \epsilon_{912}^{Q}(z) \) as

\[
\log \epsilon_{912}^{Q}(z) = 25.15 \exp(-0.0026z) - 1.5 \exp(-1.3z).
\]

This is the model presented in MH15. We denote this combination of \( \alpha \) and \( \epsilon_{912}^{Q}(z) \) as model B2. We show both of them along with the compiled data in Fig. 2.

Note that, while calculating the He II ionizing UVB, we also take into account the emissivity from diffuse He II Lyman continuum emission by following the prescription given in Haardt & Madau (2012) and Faucher-Giguère et al. (2009). He II ionizing emissivity is important to calculate the He II reionization history. For each of the model emissivities mentioned above, we also estimate the He II reionization history following the standard prescription as mentioned in the next subsection.

### 3.3 Helium reionization

We calculate reionization history of He II by solving following differential equation to estimate the volume-averaged He II fraction \( (Q_{\text{He II}}^{d}) \) Shapiro & Giroux 1987; Madau, Haardt & Rees 1999; Barkana & Loeb 2001

\[
\frac{dQ_{\text{He II}}^{d}}{dt} = \frac{n_{\text{He II}}(T)}{(T_{\text{He II}})} \chi C(n_{\text{He II}}) Q_{\text{He II}}^{d}.
\]

(15)

Here, \( n_{\text{He II}} = 1.87 \times 10^{-7} \nu_{p}^{4}/(4 - 4\nu_{p}) \) cm\(^{-3}\) is the comoving number density of helium, \( n(t) \) is comoving number density of He II ionizing photons per unit time, \( C \) is the clumping factor of He II, \( \chi \) is number of photo-electrons per hydrogen atom, \( a(t) \) is the scale factor.
The process of helium reionization is complete when $Q_{\text{HeH}}(z)$ becomes unity and that $z_{re}$ is called as reionization redshift. We take clumping factor from cosmological hydrodynamical simulations of Finlator et al. (2012) as $C(z) = 9.25 - 7.21 \log (1 + z)$. Note that, if instead we use $C(z)$ from Shull et al. (2012a) then the obtained $z_{re}$ for model A is higher by 0.05. In the He\textsc{III} regions, we take $\chi = 1 + [y_p/(2 - 2y_p)]$ and $T = 200000 \text{ K}$ to solve for $Q_{\text{HeH}}(z)$.

4.1 Model A: constraints on $\alpha$

The He\textsc{ii} ionizing UVB depends not only on the He\textsc{ii} ionizing emissivity from QSOs but also on the $\Gamma_{\text{HI}}(z)$ through the calculations of $\eta$. The $\Gamma_{\text{HI}}(z)$ depends on emissivity from both QSOs and galaxies. Therefore, the $f_{\text{esc}}$ that decides the galaxy contribution to $\Gamma_{\text{HI}}$ also affects the He\textsc{ii} ionizing UVB as shown in Khaire & Srianand (2013). Here, since we directly use the measured values of $\Gamma_{\text{HI}}$ to calculate the He\textsc{ii} ionizing UVB, we do not need to calculate the $f_{\text{esc}}$ explicitly. We refer reader to Khaire et al. (2016) for the required values of $f_{\text{esc}}$ to obtain the $\Gamma_{\text{HI}}$ measurements that are used here.

We first consider the model A for which the emissivity is obtained from the QSO luminosity function from UV and optical surveys, as given in equation (12). With this emissivity, we calculate the He\textsc{ii} ionizing UVB by varying the spectral index $\alpha$.\footnote{Note that the $\epsilon_{912}^O(z)$ given in equation (12) is obtained for $\alpha = -1.4$ at $\lambda \leq 10000 \text{ Å}$. Therefore, when we vary $\alpha$ we multiply $\epsilon_{912}^O(z)$ by a correction factor $k = (1000/912)^{1.4 - \alpha}$.} For each $\alpha$, we also vary $\Gamma_{\text{HI}}(z)$ within its 1$\sigma$ uncertainty. The calculated UVB for each $\alpha$ and $\Gamma_{\text{HI}}$ provides $\Gamma_{\text{HeH}}(z)$ and $\eta(z)$. Using this $\eta(z)$ in equations (1) and (2), we calculate $\tau_{\text{HeH}}^O(z)$. In this way, we generate $\tau_{\text{HeH}}^O(z)$ for UVB models with different $\alpha$ and $\Gamma_{\text{HI}}$. This along with $\tau_{\text{HeH}}^O$ measurements helps us to constrain values of $\alpha$.

To obtain the binned $\tau_{\text{HeH}}^O$ measurements, as given in Table 2, we calculate the required $\alpha$ in the UVB as a function of $\Gamma_{\text{HI}}(z)$ within its measured uncertainty. The results are shown in the left-hand panel of Fig. 3. Regions with vertical and horizontal stripes provide the joint constraints on $\Gamma_{\text{HI}}$ and $\alpha$, which is required to obtain the binned $\tau_{\text{HeH}}^O$ at $z = 2.52$ and $z = 2.8$, respectively. Within 1$\sigma$ range in

4 RESULTS AND DISCUSSION

Following the procedure mentioned above, we calculate the He\textsc{ii} ionizing UVB and the He\textsc{ii} reionization history for the QSO emissivities from model A and B. The results of which are discussed in the following subsections.
measured $\Gamma_{\text{HI}}(z)$, we need UVB with $-2.2 < \alpha < -1.4$ at $z = 2.52$ and with $-2.15 < \alpha < -1.55$ at $z = 2.8$. We do not calculate the required $\alpha$ to satisfy $\tau_{\text{He II}}^{\text{HI}}$ at highest redshift bin that is a lower limit.

The onset of large scatter in $\tau_{\text{He II}}^{\text{HI}}$ measurements seen at $z > 2.7$ suggests that the He II reionization has completed at $z > 2.7$ (Furlanetto & Dixon 2010; Shull et al. 2010; Worseck et al. 2011, 2016). At $z > z_{\text{re}}$, the He II ionizing UVB may not be uniform (see Furlanetto 2009; Davies & Furlanetto 2014), therefore, predicted $\tau_{\text{He II}}^{\text{HI}}$ may not match the measurements. To find $z_{\text{re}}$, we also calculated the reionization history. The obtained $Q_{\text{HI}}(z)$ for models with different $\alpha$ is shown in the right-hand panel of Fig. 3. The redshift of He II reionization depends on He II ionizing emissivity and therefore on $\alpha$. The QSO SED becomes flatter for higher $\alpha$ that gives higher He II ionizing emissivity. Therefore, higher values of $\alpha$ leads to early He II reionization. If we impose an additional constraint on reionization redshift, such as $2.6 < z_{\text{re}} < 3.0$ consistent with the trend in $\tau_{\text{He II}}^{\text{HI}}$ data, we need $-2.0 < \alpha < -1.65$. The range in required $\alpha$ has shown with grey shade in the left-hand panel of Fig. 3. Combining these constraints obtained with the binned $\tau_{\text{He II}}^{\text{HI}}$ and the $z_{\text{re}}$, together, $\alpha$ can have values from $-1.6$ to $-2.0$.

Measurements of $\alpha$ reported in the literature over last two decades are summarized in the Table 1. Let us compare the $-1.6 > \alpha > -2.0$ obtained here with the recent measurements of it. Lusso et al. (2015) obtained $\alpha = -1.7 \pm 0.61$ at $z \sim 2.4$ using 53 QSOs where the smallest wavelength probed by them is 600 Å. Stevans et al. (2014) obtained $\alpha = -1.4 \pm 0.15$ at $z < 1.5$ using 159 QSOs observed from HST-COS where the smallest wavelength probed by them is 475 Å. However, they had fewer than 10 QSOs that probe $\lambda < 600$ Å. Tilton et al. (2016) compiled 11 new QSOs from HST-COS at $1.5 < z < 2.1$ where the smallest wavelength probed by them is $\lambda \sim 425$ Å. They combined these with nine existing QSOs from Stevans et al. (2014) and measured $\alpha = -0.72 \pm 0.26$ in wavelength range 450 < $\lambda$ < 700 Å. The $-1.6 > \alpha > -2.0$ obtained by us is consistent with the measurements of Lusso et al. (2015). It is within $2\sigma$ uncertainty from Stevans et al. (2014). However, it is $4\sigma$ lower than the measurements of Tilton et al. (2016). Note that our inferred value of $\alpha$ is obtained by modelling the UVB at $\lambda < 228$ Å and at $2 < z < 3.5$. Here, we assumed that the QSO SED at $\lambda < 912$ Å follows a single power law and does not change with redshift, same as assumed in other studies. The single power-law assumption may not be true since there are no measurements that probe SED at $\lambda < 400$ Å. Tilton et al. (2016) suggested that a simple power law may not be sufficient to explain the QSO SED, even at $\lambda < 700$ Å. Moreover, the observed QSOs spectra probing $\lambda < 500$ Å are biased towards most luminous QSOs. Therefore, one expects that these measurements can also be biased. Also, the mean QSO SED may have redshift dependence. It is important to study such a redshift dependence of $\alpha$ in the direct observations.

For the UVB with different $\alpha$ and the mean value of measured $\Gamma_{\text{HI}}(z)$, the obtained $\tau_{\text{He II}}^{\text{HI}}(z)$ is shown in the left-hand panel of Fig. 4 along with the measurements from Worseck et al. (2016) and binned $\tau_{\text{He II}}^{\text{HI}}$ data from Table 2. It shows that the measured $\tau_{\text{He II}}^{\text{HI}}$ data can be reproduced for $-1.6 > \alpha > -2.0$. To reproduce binned median $\tau_{\text{He II}}^{\text{HI}}$ data from Table 2, the UVB with $\alpha = -1.8$ is preferred. We also mark the redshift of He II reionization, $z_{\text{re}}$, for each $\alpha$. In the post-He II-reionization era, i.e. at $z < z_{\text{re}}$, the UVB models are expected to produce the mean $\tau_{\text{He II}}^{\text{HI}}$ measurements and may not be at $z > z_{\text{re}}$.

In the right-hand panel of Fig. 4, we show $\tau_{\text{He II}}^{\text{HI}}(z)$ for the UVB with $\alpha = -1.8$ obtained using the mean $\Gamma_{\text{HI}}(z)$ as well as $1\sigma$ higher and lower $\Gamma_{\text{HI}}(z)$ measurements. The shaded region shows the range in $\tau_{\text{He II}}^{\text{HI}}$ arising from the uncertainty in $\Gamma_{\text{HI}}$ measurements. Since it covers most of the $\tau_{\text{He II}}^{\text{HI}}$ measurements at the post-He II-reionization era, i.e. at $z < 2.8$, we prefer the UVB with $\alpha = -1.8$. The $\Gamma_{\text{He II}}(z)$ and $\eta_{\text{thin}}(z)$ obtained from this UVB are shown in Fig. 5. Both show good agreement with the values estimated from the binned $\tau_{\text{He II}}^{\text{HI}}$ data (from Table 2) as explained in Section 2.2. The $\Gamma_{\text{He II}}(z)$ and $\eta_{\text{thin}}(z)$ obtained for the UVB with $1\sigma$ higher and lower $\Gamma_{\text{HI}}(z)$ show the spread in these values due to the uncertainty in $\Gamma_{\text{HI}}(z)$. The very good agreement between $\Gamma_{\text{He II}}(z)$ and $\eta_{\text{thin}}(z)$ obtained from the full UVB model and the one estimated using equation (1) to equation (3) (see Section 2) shows the validity of the approximations used in latter.
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**Figure 4.** Left-hand panel: $\tau_{\text{He II}}(z)$ estimated from UVB models obtained for $\epsilon_{912}^Q(z)$ of model A (equation 12) with different spectral index $\alpha$ of the mean QSO SED (for $\lambda \leq 912$ Å). Vertical grey lines with different line-styles mark the redshift of He II reionization (see right-hand panel of Fig. 3). The UVB with $\alpha = -1.8$ reproduce the binned $\tau_{\text{He II}}$ measurements. Here, $\Gamma_{\text{HI}}(z)$ values obtained for all the models are consistent with the mean values from Becker & Bolton (2013). Right-hand panel: $\tau_{\text{He II}}(z)$ estimated from the UVB with $\alpha = -1.8$ with different $\Gamma_{\text{HI}}$ consistent with the 1σ higher and lower values. The shaded region show the range in $\tau_{\text{He II}}(z)$ due to uncertainty in the measured $\Gamma_{\text{HI}}$. In both panels $\tau_{\text{He II}}$ measurements by Worseck et al. (2016) are shown by diamonds and binned $\tau_{\text{He II}}$ data by circles.

**Figure 5.** $\Gamma_{\text{He II}}(z)$ (left-hand panel) and $\eta_{\text{thin}}(z)$ (right-hand panel) obtained from UVB models by using $\epsilon_{912}^Q(z)$ of model A (equation 12) with $\alpha = -1.8$. Solid, dash and dotted curves show results obtained from UVB with mean $\Gamma_{\text{HI}}$, 1σ higher and lower $\Gamma_{\text{HI}}$, respectively. Red circles show our estimates of $\Gamma_{\text{He II}}$ and $\eta_{\text{thin}}$ from binned $\tau_{\text{He II}}$ data, as described in Section 2.2 (Table 2).

All the models mentioned above assume a single power-law SED of QSOs at $\lambda \leq 912$ Å. The SED may not be a single power law; rather it can consist of broken power laws or have breaks at smaller wavelengths. To obtain the same He II ionizing emissivity as obtained for our preferred model with $\alpha = -1.8$ but with different value of $\alpha$, a break in the mean QSO SED at a wavelength $228 \leq \lambda_b \leq 912$ Å can be applied. The value of the break, the number $<1$ that is multiplied to the specific intensity at $\lambda \leq \lambda_b$, can be approximated as $(\lambda_b/912\text{ Å})^{1.8+\alpha}$. For example, when we assume $\alpha = -1.4$ consistent with measurements of Stevans et al. (2014) and Shull, Stevans & Danforth (2012b), we verify that a break in QSO SED at $\lambda_b = 228$ Å by a factor of 0.6 gives the same $\tau_{\text{He II}}(z)$ as obtained for single power-law SED with $\alpha = -1.8$. Although, the break can be applied at $228 \leq \lambda_b \leq 912$ Å, hereafter we consider the break only at $\lambda_b = 228$ Å. A slight decrease in the resultant $\Gamma_{\text{HI}}$ due to such break in QSO SEDs can be compensated by marginally increasing $f_{\text{esc}}$ from galaxies. This SED break can be thought as the escape fraction of He II ionizing photons from QSOs. However, in

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3 The purpose of the SED break is to reduce the He II ionizing emissivity. Therefore, it is effective to have at $\lambda_b \geq 228$ Å.
the absence of any physical models, such a break in QSO SED and its interpretation should be treated with caution.

We have used $\alpha = -1.4$ in Khaire et al. (2016) to estimate the required $f_{esc}$ of $\text{H}^1$ ionizing photons from galaxies to obtain the $\Gamma_{\text{HI}}$ measurements. If we use the $\alpha = -1.6$ to $-2.0$ instead, we need an additional increase in the predicted $f_{esc}$ in Khaire et al. (2016) by less than 20 per cent.

4.2 Model B: break in SED

Now we consider the two combinations of $\alpha$ and $\epsilon_{\text{Q}}^\text{QSO}(z)$ from model B (equations 13 and 14) that include the emissivity from low-luminosity X-ray-selected QSOs of Giallongo et al. (2015) at $z > 4$ and reionize $\text{H}^1$ alone. The model B1 (equation 13) uses $\alpha = -1.4$ and the model B2 (equation 14) uses $\alpha = -1.7$. We calculate the UVB and $\tau_{\text{HeII}}$ for these models. The results are shown in the left-hand panel of Fig. 6. The comparison with the data shows that these models cannot reproduce the $\tau_{\text{HeII}}$ measurements.

These models also predict higher redshift for completion of $\text{He}^\text{II}$ reionization, as $z_{\text{HeII}} = 5.2$ for model B1 and $z_{\text{HeII}} = 4.5$ for model B2. It is one of the issues of such high QSO emissivity models. Therefore, these models need modifications. We cannot change values of $\alpha$ since they are already adjusted along with $\epsilon_{\text{Q}}^\text{QSO}(z)$ to reionize $\text{H}^1$ alone without requiring any contribution from galaxies and to satisfy different observational constraints on $\text{H}^1$ reionization. However, we can break the respective SEDs at $\lambda \leq 228$ Å so that the $\text{H}^1$ ionizing emissivity and its prediction for $\text{H}^1$ reionization remains the same but the $\text{He}^\text{II}$ ionizing emissivity reduces.

We estimate the $\tau_{\text{HeII}}$ for the UVB obtained with different SED breaks at $\lambda \leq 228$ Å. We find that for model B1, we need SED break of a factor $\sim 0.4$ at $\lambda \leq 228$ Å to reproduce the $\tau_{\text{HeII}}$ measurements. For model B2, since it already has steeper SED with $\alpha = -1.7$, an SED break of factor $\sim 0.7$ at $\lambda \leq 228$ Å is needed. The $\tau_{\text{HeII}}$ obtained in these models with such modifications are shown in the right-hand panel of the Fig. 6. The values of $\Gamma_{\text{HeII}}$ obtained for these models are shown in Fig. 7. These are in good agreement with the values estimated using binned $\tau_{\text{HeII}}$ data. In the left-hand panel of Fig. 8, we show the $\epsilon_{\text{Q}}^\text{QSO}$ at $z = 3$ for an illustrative purpose from the models B1 and B2 with the SED breaks obtained here. For comparison, we also show the $\epsilon_{\text{Q}}^\text{QSO}$ at $z = 3$ from model A with no SED break. In all three models, although the $\text{H}^1$ ionizing emissivities are different, the respective breaks in models B1 and B2 achieve the similar $\text{He}^\text{II}$ ionizing emissivities as model A. With such modifications, these models also predict lower $\text{He}^\text{II}$ reionization redshift. For model B1, the $z_{\text{HeII}}$ is now 3.4 and for model B2 it is 3.3. The $Q_{\text{HeII}}(z)$ is shown in the right-hand panel of Fig. 8. Note that if we use the clumping...


The $\tau^{\text{He II}}$ at different energies for $z = 3$ (left-hand panel) and $Q_{\text{He II}}(z)$ (right-hand panel) obtained from different models. These models are model B1 (equation 13 and $\alpha = -1.4$; dashed curve) with SED break of 0.4 at $E \geq 4$ Ryd, model B2 (equation 14 and $\alpha = -1.7$; dot-dashed curve) with SED break of 0.7 at $E \geq 4$ Ryd and model A (equation 12 and $\alpha = -1.8$; solid curve) with no break in SED.

The main difference between the model A and model B (both B1 and B2) is the He II reionization history. Even though the models B1 and B2 are modified with the SED breaks to reproduce the $\tau^{\text{He II}}$ measurements, the $Q_{\text{He II}}(z)$ predicted by them differs significantly from model A, as shown in the right-hand panel of Fig. 8. For example, at $z \sim 4$ in model A only 10 (3) per cent of the volume in the Universe is in He II as compared to the 60 (40) per cent in the model B. The He II reionization process is more extended and slower in model B as compared to model A. This difference will show imprints on the thermal history of the IGM (see also D’Aloisio et al. 2016; Mitra, Choudhury & Ferrara 2016) that will be crucial to distinguish these models.

To distinguish model A where galaxies dominate the H I reionization and model B where QSOs alone reionize H I, apart from the thermal history of the IGM the detection of the 21 cm brightness temperature fluctuations will be crucial (Kulkarni et al. 2017). Also, in such models $\epsilon^{\text{QSO}}(z)$ should be higher than the model B2 to reionize H I alone that will require higher emissivity than Giallongo et al. (2015) and it may not be consistent with upper limits on the unresolved X-ray background at high-$z$ (see Haardt & Salvaterra 2015).

The value of $N_{\text{He II}}^{\text{min}}$ is crucial for $\tau^{\text{He II}}$ since the fit to the $f(N_{\text{He II}}; z)$ is very steep at low values of $N_{\text{He II}}$. We took $N_{\text{He II}}^{\text{min}}$ to have minimum equivalent width of 5.2 $\times$ 10$^{-3}$ Å, which reproduce the $\tau^{\text{He II}}$ measurements with $N_{\text{He II}}^{\text{min}} = 10^{12}$ cm$^{-2}$. The $\tau^{\text{He II}}$ does not converge rapidly if we extrapolate the fitting form of the observed $f(N_{\text{He II}}; z)$ to smaller $N_{\text{He II}}$ values. However, note that the Inoue et al. (2014) obtained the fit to $f(N_{\text{He II}}; z)$ at low $N_{\text{He II}}$ values using the measurements from Kim et al. (2013) that probe minimum $N_{\text{He II}} \sim 10^{12.7}$ cm$^{-2}$. For $N_{\text{He II}} < 10^{12.5}$ cm$^{-2}$, the $f(N_{\text{He II}}; z)$ is rather flat and even shows decreasing trend (refer to fig. 7 from D’Odorico et al. 2016). If we assume that $f(N_{\text{He II}}; z)$ is constant or decreasing at $N_{\text{He II}} < 10^{12} or 10^{12.5}$ cm$^{-2}$ then the $\tau^{\text{He II}}$ converges rapidly. When we use a constant $f(N_{\text{He II}}; z)$ at $N_{\text{He II}} < 10^{12}$ cm$^{-2}$ and $N_{\text{He II}}^{\text{min}} = 0$, we find that the maximum increase in $\tau^{\text{He II}}$ at $z < 3.5$ is less than 10 per cent as compared to the value we obtain by assuming $N_{\text{He II}}^{\text{min}} = (16/\eta_{\text{min}}) \times 10^{12}$ cm$^{-2}$ and less than 20 per cent by assuming $N_{\text{He II}}^{\text{min}} = 10^{12}$ cm$^{-2}$. This does not affect our results significantly.

For the measured values of $\Gamma_{\text{He II}}$, values of $\eta$ depend on He II ionizing emissivity. We discussed the constraints on the SED, however, we assumed fixed $\epsilon^{\text{QSO}}_{\text{He II}}(z)$ values in each model. As mentioned earlier, we cannot change $\epsilon^{\text{QSO}}_{\text{He II}}(z)$ without changing $\alpha$ in the models that alone reionize H I, such as the models B1 and B2. However, we can change it in the model A. If we uniformly reduce the $\epsilon^{\text{QSO}}_{\text{He II}}(z)$ in our model A by 10 per cent (20 per cent) at $z > 2$ allowed by the

4.3 Model uncertainties

Here, we discuss the uncertainties in our models and how they affect the results presented in the preceding subsections. The estimates of $\tau^{\text{He II}}$ depend on three quantities, the assumed $b$-parameter, the $N_{\text{min}}^{\text{He II}}$ and the $\eta$ obtained from the UVB.

We took $b = 28$ km s$^{-1}$ for H I as well as He II assuming that the turbulence dominates the Doppler broadening. If the thermal broadening dominates the $b$-parameter then the $b$ for He II becomes 14 km s$^{-1}$. This $b$-parameter gives 38 per cent smaller $\tau^{\text{He II}}$ as compared to the one obtained earlier for each UVB model presented here. To match the $\tau^{\text{He II}}$ measurements, this model will require more steep QSO SED (i.e. small $\alpha$) or small value of break in the QSO SED at $\lambda \leq 228$ Å. With this $b$, we find that for QSO emissivity from model A, we need $-1.8 < \alpha > -2.0$ to reproduce the $\tau^{\text{He II}}$ measurements and to obtain $z_{\text{He II}} < 3$. For models B1 and B2, we need a break in QSO SED of factor 0.3 and 0.5 at $\lambda \leq 228$ Å, respectively, to match the $\tau^{\text{He II}}$ measurements.

The parameter $\eta$ affects the $\tau^{\text{He II}}$ since it is a function of $\eta$ and $N_{\text{He II}}$. The $\eta$ values in the models are chosen to be consistent with the measured $\tau^{\text{He II}}$ values. The $\eta$ values are fixed in each model to reproduce the $\tau^{\text{He II}}$ measurements. However, the $\eta$ values are not significantly different from model A at $z > 2$.

The factor for He II from Shull et al. (2012a) then the obtained $z_{\text{He II}}$ is higher by additional 0.2. Therefore, the models with SED steeper than $\alpha = -1.7$ can be consistent with the $\tau^{\text{He II}}$ measurements at $z < 3$ but cannot reproduce the trend in increasing $\tau^{\text{He II}}$ at $z > 3$. Also, in such models $\epsilon^{\text{QSO}}_{\text{He II}}(z)$ should be higher than the model B2 to reionize H I alone that will require higher emissivity than Giallongo et al. (2015) and it may not be consistent with upper limits on the unresolved X-ray background at high-$z$ (see Haardt & Salvaterra 2015).

The factor for He II from Shull et al. (2012a) then the obtained $z_{\text{He II}}$ is higher by additional 0.2. The $\epsilon^{\text{QSO}}_{\text{He II}}(z)$ values taken in these models are not significantly different from model A at $2.3 < z < 3.2$ (see Fig. 2). Therefore, the models with SED steeper than $\alpha = -1.7$ can be consistent with the $\tau^{\text{He II}}$ measurements at $z < 3$ but cannot reproduce the trend in increasing $\tau^{\text{He II}}$ at $z > 3$. Also, in such models $\epsilon^{\text{QSO}}_{\text{He II}}(z)$ should be higher than the model B2 to reionize H I alone that will require higher emissivity than Giallongo et al. (2015) and it may not be consistent with upper limits on the unresolved X-ray background at high-$z$ (see Haardt & Salvaterra 2015).
uncertainties in the QSO luminosity functions, we find that the \( \eta \) increases due to a decrease in He II ionizing emissivity. This leads to higher \( \tau_{\text{He II}} \) by 10–15 per cent (25–40 per cent) over redshift 2–3.5. For such models, we find that \(-1.5 > \alpha > -1.9\) is needed to reproduce the \( \tau_{\text{He II}} \) measurements.

Note that the variation in \( \tau_{\text{He II}} \) arising from all these uncertainties is smaller than the one arising from the uncertainty in the measured \( \Gamma_{\text{HI}} \), itself (see the right-hand panel of Fig. 4). In future, more stringent constraints on the QSO SED can be obtained using accurate measurements of \( \Gamma_{\text{HI}} \) and more observations of \( \tau_{\text{He II}} \) in the post-He II-reionization era \((z < 2.6)\). Currently, there are only two sightlines, HE2347–4342 and HS1700+6416, that probe He II Lyman \( \alpha \) forest at \( z < 2.6 \).

5 SUMMARY

Here, we present a method that constrains the He II ionizing emissivity using \( \tau_{\text{He II}} \) measurement obtained from He II Lyman \( \alpha \) forest and the distribution of H I in the IGM obtained from H I Lyman \( \alpha \) forest. The method uses our cosmological radiative transfer code developed to calculate the UVB by varying the input He II ionizing emissivity to be consistent with \( \tau_{\text{He II}} \) measurements. The He II ionizing emissivity depends on the QSO emissivity obtained from their luminosity functions and the mean QSO SED extrapolated at \( E \geq 4 \) Ryd. The latter has been observationally constrained only up to \( E \sim 2.3 \) Ryd. We constrain the QSO SED at \( E \geq 4 \) Ryd required to satisfy the recent measurements of \( \tau_{\text{He II}} \) (Worseck et al. 2016) using models of updated QSO emissivity at 1 Ryd (Khair & Srianand 2015a) and H I distribution of the IGM (Inoue et al. 2014) in our UVB code. We have also calculated the \( \Gamma_{\text{He II}} \) (provided in Table 2) from the binned \( \tau_{\text{He II}} \) data which depends only on the H I column density distribution at \( N_{\text{HI}} < 10^{16} \text{cm}^{-2} \) and the \( \Gamma_{\text{HI}} \) measurements at \( z > 2.2 \) (Becker & Bolton 2013).

The mean SED obtained from QSO composite spectra is usually approximated as a power-law \( f_{\nu} \propto E_{\nu}^{-\alpha} \) at \( E \geq 1 \) Ryd. For QSO emissivity obtained using their luminosity functions from optical surveys, we find that the \( \tau_{\text{He II}} \) measurements are well reproduced when we use the power-law index \(-1.6 < \alpha < -2.0\). The UVB models with this \( \alpha \) not only reproduce the majority of the \( \tau_{\text{He II}} \) measurements but also reionize He II at \( 2.6 < z_{\text{reio}} < 3.0 \), consistent with the trend seen in the \( \tau_{\text{He II}} \) data. The \(-1.6 < \alpha < -2.0\) constrained here is consistent with the measurements of Luo et al. (2015) and Stevans et al. (2014) but \( 4\sigma \) lower than the measurement by Tilton et al. (2016). We prefer the UVB model with \( \alpha = -1.8 \) because it reproduces the \( \tau_{\text{He II}} \) measurements and our estimated \( \Gamma_{\text{He II}} \) values within the uncertainties in the measured \( \Gamma_{\text{HI}} \).

We also consider models of QSO emissivity that include the luminosity function obtained from low-luminosity X-ray-selected QSOs presented by Giallongo et al. (2015) at \( z > 4 \). These models are constructed such that they can reionize H I without requiring any contribution from galaxies (MH15; Khair et al. 2016) when extrapolated to \( z > 6 \). We find that these models cannot reproduce the \( \tau_{\text{He II}} \) measurements and need modifications to reduce the He II ionizing emissivity. For such a model with \( \alpha = -1.4 \) from Khair et al. (2016), we need a break in mean QSO SED at \( E > 4 \) Ryd of a factor \( \sim 0.4 \). Similarly, for a model with \( \alpha = -1.7 \) from MH15 we need break of a factor \( \sim 0.7 \) (see the left-hand panel of Fig. 8 for illustration of such SED breaks). These modified models give epoch of He II reionization at 3.3–3.4 which is significantly smaller than 4.5–5.2 obtained without such modifications. However, even with such modifications the He II reionization history is significantly different from standard models (see the right-hand panel of Fig. 8) that do not include the luminosity function of Giallongo et al. (2015). The thermal history of the IGM will play crucial role in distinguishing these models.

The method presented here requires better observational constraints on both \( \Gamma_{\text{HI}} \) and H I distribution in the IGM, as well as measurements of \( \tau_{\text{He II}} \) over a large redshift range, to accurately constrain the mean QSO SED together with its redshift dependence. Using different QSO SEDs provides significantly different UVB at He II ionizing wavelengths. Observations of metal line ratios tracing lower and higher energies around He II ionization potential (such as C IV and Si IV) can be considered to test different models of the UVB (see e.g. Fechner 2011). We plan to carry such studies in future.

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