The obstacles of geometric problem-solving on solid with vector and triangle approach

F Alghadari¹,²,a and T Herman¹

¹Departement of Mathematics Education, Universitas Pendidikan Indonesia, Jalan Dr. Setiabudi No. 229, Bandung 40154, Indonesia
²Departement of Mathematics Education, STKIP Kusuma Negara, Jalan Raya Bogor No. 24, Jakarta 13770, Indonesia

E-mail: fikialghadari@student.upi.edu

Abstract. The objective of this study is to analyse the obstacles of geometry problem-solving. The case focused on a male student. The object of the study was the answer of Geometry problem on solid and interview. The finding of the research is the obstacles completion categorized in representative, visual-spatial, and conceptual. Solving Geometric problem between two skew lines and triangle concept is misconception on perpendicular concept. Meanwhile, the failure of interpreting the relationship between known information and used concept with solved problem happens in vector concept. Solving Geometry problem of the distance between point and plane, object obstacle is the failure to hook between concepts, meanwhile using vector concept is conception on concept toward solving problem. In whole completion, obstacle happening is the conceptual obstacle.

1. Introduction

National based senior high school in Indonesia has science program which learns two mathematics field namely basic mathematics and additional mathematics. Students learn the same topic but the discussion is the advanced level from another, so the difficulty of materials is different, such as geometry on solid. The reality shows that learning geometry tends to use technology approach recently. By technology, the visualization of solid is used easily and dynamically so it helps students with difficulty to understand spatial properties [1, 2, 3]. Yet, the difficulty of learning is not only on spatial and visualization problem but also students with low and medium ability cannot solve geometry problem because there is knowledge limitation in completion [4]. Furthermore, students cannot solve the problem because of the complexity of understanding basic concept such as Pythagoras, comparison, congruence, and triangle concepts that the combination among the concepts influences completion procedure [1].

Visualization actually has a role in Mathematics and its learning. It is to understand and simplify problem. Yet, solving geometry needs analytical steps tending to use counting ability [5]. Besides, students who has obstacle usually show their limited knowledge in counting of completion [4], or some try to count but there are some conceptual errors [1]. So, the obstacle in solving geometry problem shows that the factor of its cause is on understanding basic geometry concept. Learning Mathematics is not only for students with high ability, considering some obstacles, but also they need to understand some ways in completion with valid and effective geometry representation and counting concept to avoid students’ obstacle. Some ways in solving problem have their strategies. Completion
strategy is the step to improve mathematical ability and its implication on students’ achievement [6, 7]. Yet, solving geometry problem at school focuses on axiomatic triangle concept recently both based conventional and technology. It is possible becoming one of factors causing an error because students must master related concepts [7, 8, 9, 10].

This study is part of higher research that aims to improve students’ mathematical ability with geometry concept obstacle. This study involves two ways in solving geometry problem by different approaches. One of them is triangle approach. Triangle approach is defined as geometry counting which has benefits on triangle concepts such as Pythagoras theorem, trigonometry, or triangle area. Moreover, this study analyzes solving geometry problem with vector concept like cross-product and orthogonal projection, so it is defined as vector approach. So, the starting point of error causing students’ failure in finding solution is the obstacle in solving geometry problem.

2. Objective
The objective of this study is to analyze the obstacles of geometry problem-solving, and whether its obstacle in the categories of representation, spatial-visual, or conceptual types.

3. Methodology
This qualitative research focuses on individual. The data are collected from the students’ answer on their test and interview. The test is on Geometry problem on solid such as cube $ABCD\ EFGH$, length of edge is 4 centimetres (cm), distance counting between: (a) from line $AC$ to $BH$, (b) from point $D$ to plane $BEG$. The common steps to solve its problem is: make a figure of cube; make a representation of point, line, or plane on cube; make an illustration of the distance in spatial dimension; counting its distance by mathematical concepts. So, the stages to solve its problem is: representation, visual-spatial, and conceptual. This stages likes in objective learning of geometry on [11], use visualization, spatial reasoning, and geometric modeling to solve problems.

Then, the interview is done by some questions such as (a) where the location stating the distance on the questions is, (b) why stating the distance is as the distance questioned, (c) what, why, and how the concept is used for completion. Answers of the test and the result of interview are analysed for categorizing the subject’s obstacle in completion by the obstacles characteristics adapted from some mathematical ability indicators presented on the table.

| Type of Obstacle | Criteria |
|------------------|----------|
| Representation   | Showing the inappropriate figure with the information; it is not appropriate in integrating the information on the questions; the error transforms mathematical idea into the simple form [12]. |
| Visual-Spatial   | Showing the inappropriate spatial relationship between objects; it does not figure out the relationship between the part and position in interval; showing untrue representative in dimensionless interval [3, 13, 14]. |
| Conceptual       | The incorrect use of the procedure to solve the problem [7]; misconception and the associated errors against the correct answers; the errors due to nonconceptual understanding of the concept [15]. |

4. Result
Based on the answers of test done by subject using triangle and vector approach, subject shows that he does not solve Geometry problem of distance between skew lines and distance between point and plane by the approaches.

4.1. The problem of distance between lines.
Here is the solving Geometry problem of distance between line $AC$ and line $BH$ on cube $ABCD\ EFGH$ 4 cm by triangle approach.
Subject first completes by making cube followed by drawing line \( AC \) and \( BH \). From 2D perspective, the two lines are intersection but there is spatial orientation so subject decides that distance between the lines marked by the length of line segment \( OP \). This condition shows that subject knows the spatial relationship by perceptual spatial ability [13], subject also figures out the relationship between the real segment and position on the space, so it can be concluded that subject does not have spatial obstacle. Some research also state that the men’s spatial skill is better [2, 13]. Here are the subject’s statements on the interview related to his ways in determining distance between skew lines.

Researcher : Why do you state that distance between line \( AC \) and \( BH \) is the length of segment \( OP \)?
Subject : It is to get the right angle on point \( O \) where the point is representing line \( AC \) so the length of segment \( OP \) is the result of the representative lines.

Based on the figure and excerpts above, subject fails in solving problem of distance from line \( AC \) to line \( BH \) because something loses in his sight namely the distance concept between two skew lines which are bounded by perpendicular concept. Conceptually, the distance is the closest distance between the lines. The distance is obtained from the length of line segment connecting the skew lines by perpendicular [2]. The subject does not figure out the whole perpendicular concept. He assumes the perpendicular concept is only on line \( AC \). So, subject determines the length of line segment \( OP \) as the distance between two skew lines. Subject should determine that line segment correlate between line \( AC \) and \( BH \) that has perpendicular with the lines. In other words, the line segment has perpendicular not only on line \( AC \) but also on line \( BH \).

It is not appropriate that the subject determines distance on Geometry correlated with perpendicular concept. Subject has obstacle for his completion because of the inappropriateness. This is the meaning of inappropriate concept with the real concept. It is the reason that subject has obstacle and cannot find the right answers. The error on interpreting meaning of recent concept is misconception [16]. By this condition, subject has misconception of the perpendicular concept meaning because the concept is defined incorrectly. This is the example of concepts error. Concepts error are the type of error categories [17]. So, subject has conceptual obstacle.

In another condition, the subject completes by vector concept. Subject cannot find the right answers. Here is the figure that subject makes.
On figure 2 (b), subject tries to count the distance questioned by using vector $DB(4, -4, 0)$ and vector $BH(-4, 4, 4)$. Subject uses vector $DB$ based on figure 2 (a) that the vector intersects perpendicularly with vector $AC$ on the coordinate point $O(2, 2, 0)$. Then, subject tries to count distance by orthogonal vector projection concept. The trial is shown on figure 2 (a) yet the steps cannot be synchronized with the other steps trying to make perpendicular as distance between two skew lines questioned. Whereas the steps in making line segment from point $O$ to vector $HB$ are the steps directing to the solution. Here are the subject’s statements on the completion.

Researcher : Where is the position stating distance? How is the way to count that and which concept is used?
Subject : It is from midpoint of line $AC$ to line $BH$. I count that by orthogonal vector projection, then I add two vectors.
Researcher : The step is correct, but the completion is not the same as $2/3 \sqrt{6}$. Look at the projected vector, that is not related to distance questioned. Why do you project vector $DB$?
Subject : Oh... I should project vector $BO$.

Subject shows the correct count operation, presents well, but it is less in choosing the representative vector. Based on figure 2 (b), the vector projected is not vector $DB$ on vector $BH$, but it is vector $BO$ on $BH$ so the result of projection utilized to count distance of Geometry using length vector concept as presented by subject. Although the steps direct to the correct completion, but subject still has problem. The problem is that subject fails in using vector representation projected, however subject figure out how to project vector using orthogonal vector projection concept and continues to count by length vector concept. The failure is the error on principles [17].

Based on the situation, researcher reveals that the obstacle experienced by the subject is conception. Whereas it is not the conception of concept and usage to the solving problem, because subject can figure out the projection concept is one of vector concepts used to count distance between skew lines. It is not also the conception obstacle on problem, because subject has shown correct interpretation between known element and strategy in completion and concept used to count distance. The phenomena occur commonly because subject figures out the concept, definition, or theorem but he cannot correlate the information and concept used for completion. The obstacle is conception obstacle of relationship between problem and concept. In other word, misconception and the associated errors are against the correct answers. This condition is called as conceptual obstacle [15].

4.2. The problems of distance between point and plane.
By triangle concept, here is the counting of Geometric distance between from point $D$ to plane $BEG$ on cube $ABCD EFGH$ and the length of edge 4 units.

![Figure 3. The completion of distance from point to plane by triangle approach.](image)

Figure 3 (b) shows that subject tells that the length of line segment $BK$ is a half of $BM$, point $K$ on altitude of plane $BEG$ attracted form point $B$ and intersecting line $EG$ on point $M$. On that counting, subject believes that the length of line segment $BK$ is a half of $BM$. Subject’s assumption is the point of error. The error is on unknown information but the subject did not check the truth. The error is the
sample that there is interactive process in solving problem namely the interaction between the known and unknown information [10, 16], and the unknown information is the mathematical concept that has been learned [8, 9, 10], so subject needs knowledge and understanding on the concept to solve the problem to support the process of finding solution.

Basically, subject has illustrated the distance questioned so he has good ability for the completion. Evaluating subject’s skill is based on his spatial skill in showing figure 3 (a) as correct illustration. The measurements of spatial skill are showing the correct spatial relationship between objects, and understanding the relationship between real part and position on space, showing correct representation on dimensional space [3, 13]. However, the inappropriate assumption directs the inappropriate completion. Whereas the subject can check his assumption through triangle BDM by revealing the size of triangle sides, so he can figure out after verification that it is isosceles triangle which has two bases with equivalent side length that is length of side DM is same as length of side BM. The base that has equivalent length from triangle BDM is side DM and BM, if the altitude is attracted from point D and side of perpendicular BM, so the altitude does not divide side BM into two equivalent segments. Here are subject’s statements on the interview.

Researcher : Do you know the size and type of BDM triangle?
Subject : Yes, the length of line segment BD, DM, and BM is respectively 4√2, 2√6, and 2√6, so triangle BDM is isosceles triangle.
Researcher : Is line DK perpendicular and does it divide line segment BM into length equivalent?
Subject : I am mistaken, because I did not check the size of triangle.

It cannot be categorized into spatial obstacle because figure 3(a) shows that the subject can illustrate the distance to find the length. One factor causing the obstacle is because subject has error in using the procedure for solving problem. This obstacle does not come from the subject’s comprehension on basic concept application. Based on the interview, it does not come from the abstract basic concept, yet it is not corrected in detail. In other words, the subject does not detail all abstractions needed for solving the problem. This is not the characteristic of representation obstacle because the subject is correct in integrating the information on the question. Subject also shows the figure which is appropriate with the information. However, the starting point that the obstacle appear is the effect of failure in correlating triangle altitude and triangle type, so the obstacle can be categorized into conceptual obstacle.

It is different from the condition that the subject finishes by vector approach. Calculating the distance between a point to a plane, here is the result.

Based on the figure above, vector (4, 0, 4) is the representation of DG line. Then, subject tries to complete by orthogonal vector projection. The vector chosen by subject to apply is vector (2, 2, 2), but it is not in plane and is the point coordinate D so the subject chooses vector without based concept. In other words, the vector is used by subject without conception as basic of use. The subject does not identify it well so there is no skill of reasoning to think logically or no step in the action. Solving problem needs reasoning skill [7]. The obstacle happens because students likely do not use the cross-product concept, or do not know the concept which is the step in applicable completion. Subject obstacle is the conception in concept toward solving problem. This is conceptual obstacle.
5. Conclusion
Based on the result and finding, it can be concluded that there are several obstacles that are faced by the subject on problem in solving Geometry problem on solid: (a) Distance between two lines, the subject’s obstacle in solving the problem of distance using triangle concept is that there is misconception on meaning and using perpendicular, meanwhile the subject’s obstacle in solving problem using vector concept is the failure in interpreting the relationship between known information and used concept with solved problem, (b) The distance between point and plane, the subject’s obstacle in solving problem using triangle concept is the failure to hook between concepts, meanwhile using vector concept is the conception of concept toward solving problem. The conclusion from the completion is that subject has conceptual obstacle.

There are some limitations for revealing obstacles faced by subject in this study. It is because the plan used to solve Geometry problem identified is on two ways namely triangle and vector concept approaches. Besides, the subject not only uses the combination of two approaches in solving problem but also the result of study concludes that the obstacle of solving Geometry problem on solid not only focuses on visualization to help in technology based learning but also the obstacle conceptual that is not known and realized its benefit and usage.

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