Dynamic modeling and control of adaptive impedance for human-manipulator interaction with uncertainty

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Abstract. Robots have been used widely in every field in nowadays, such as in transport, machining and some other tasks interaction with the human. Impedance control is a useful method to maintain the dynamic relationship between the end-point of the manipulator and the external environment. Considering the uncertainties of the robot parameters and the environment, this paper proposes an adaptive impedance controller, before the controller designed, the dynamic model of the manipulator is established. Then a experimental and simulation study is made to verify the control algorithm, the results show that the adaptive impedance control is valid and easy to implement.

1. Introduction

Robots have been used widely in every field in nowadays, such as transport and machining and some other tasks interaction with the human [1,2]. Under such working conditions, the manipulator needs to interact with the external environment. For the safety of the objects or the efficient of the works, the contact force between the manipulator and the external environment must be precisely controlled. The most commonly used control method is impedance control [3].

Impedance control is to maintain the dynamic relationship between the end of the robot and the external environment [4,5]. This makes the impedance control of the robot very challenging due to the uncertainty of the environment and the manipulator model parameters. Therefore, these difficult problems attracted an increasing number of researchers in recent years [6]. Surdilovic used intelligence adaptive impedance control for motion control [7]. Adaptive impedance control was proposed when the uncertainty of environment was considered [8-11]. In the meantime, the series manipulator dynamics model is highly coupled nonlinear, which also makes the control algorithm become quite complex [12]. In order to control the manipulator in real time, the dynamic model of the manipulator needs to be properly simplified, but the simplified model may be quite different from the actual manipulator, which affects the accuracy and the speed of the manipulator impedance control. In order to facilitate the design of control algorithms, Euler-Lagrange equation are commonly used to establish the dynamic model of the robot [13].

This paper mainly focuses on the dynamic modeling and adaptive impedance control of the serial manipulator interaction with the human. The other parts of the paper are organized as follows. Section 2 presents the dynamic model of the serial manipulator using Lagrange Equations, which is followed by...
the detailed design of adaptive impedance control in section 3. In addition, a simulation of two rigid links planar robot has been worked out to verify the adaptive impedance control theory in Section 4. Finally, Section 5 concludes the paper and provides a further discussion.

![Figure 1: The position description of manipulator](image)

2. Dynamic modeling

In order to establish the dynamic model of the serial manipulator, we should describe the each point in the link in the initial frame, as shown in Figure 1. For a serial manipulator, a floating coordinate system can be defined at each joint to describe the link position, assuming that the coordinate system of the base link is \( \{0\} \), then the \( i \)-th link position coordinates can be expressed in the inertial coordinate system as

\[
r_i^0 = A_i^0 r_i^i
\]  

(1)

where \( A_i^0 = A_i^0 A_{i-1} A_{i-2} \ldots A_1 \), in which \( A_i^{-1} \) represents the transformation matrix between the \( i \)-th coordinate system and the \( i-1 \)-th coordinate system. In planner manipulator, we can obtain

\[
A_i^{-1} = \begin{bmatrix}
\cos \theta_i & -\sin \theta_i & L_i \cos \theta_i \\
\sin \theta_i & \cos \theta_i & L_i \sin \theta_i \\
0 & 0 & 1
\end{bmatrix}
\]

(2)

Thus, in the inertial coordinate system, the speed at any point on the link \( i \) can be expressed as

\[
v_i = \frac{d}{dt}(r_i^0) = \left( \sum_{j=1}^{i} \frac{\partial A_i^0}{\partial \dot{q}_j} \dot{q}_j \right) r_i^i
\]

(3)

Notice that \( \dot{r}_i^i = 0 \), so we can obtain that

\[
\frac{\partial A_i^{-1}}{\partial \dot{q}_i} = Q_i A_i^{-1}
\]

(4)
where \( Q = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \).

So we can rewrite the equation (3)

\[ v_i = \left( \sum_{j=1}^{i} U_{ij} \dot{q}_j \right) r_i \]  

(5)

where \( U_{ij} = \begin{cases} A^0_j A^{-1}_j A^0_i A^{-1}_i & \text{for } j \leq i \\ 0 & \text{for } j > i \end{cases} \)

Now, we consider the interaction effects between joints as

\[ U_{jk} = \frac{\partial U_{ij}}{\partial q_k} = \begin{cases} A^0_j A^{-1}_j A^0_k A^{-1}_k & \text{for } j \leq k \leq i \\ A^0_j A^{-1}_j A^0_k A^{-1}_k & \text{for } k \leq j \leq i \\ 0 & \text{for } i < j \text{ or } i < k \end{cases} \]  

(6)

Then the Lagrange-Euler equations of Motion are given by

\[ L = K - P \]  

(7)

The element of the equation are calculated as

\[ K = \sum_{i=1}^{n} K_i = \frac{1}{2} \sum_{i=1}^{n} \left[ \text{Tr} \left( \sum_{p=1}^{i} \sum_{r=1}^{i} U_{ip} J U_{p} U_{jr} \dot{q}_j \dot{q}_r \right) \right] = \frac{1}{2} \sum_{i=1}^{n} \sum_{p=1}^{i} \sum_{r=1}^{i} \left[ \text{Tr} \left( U_{ip} J U_{p} \right) \dot{q}_j \dot{q}_r \right] \]  

(8)

\[ P = \sum_{i=1}^{n} P_i = \sum_{i=1}^{n} \left[ -m g \left( A^0_i r_i \right) \right] \]  

(9)

\[ \tau_i = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} \]  

\[ = \sum_{j=1}^{n} \sum_{k=1}^{j} \left[ \text{Tr} \left( U_{jk} J U_{jk}^T \right) \dot{q}_k \right] + \sum_{j=1}^{n} \sum_{k=1}^{j} \sum_{m=1}^{j} \left[ \text{Tr} \left( U_{jkm} J U_{jkm}^T \right) \dot{q}_k \dot{q}_m \right] - \sum_{j=1}^{n} m g U_{j} \bar{r}_j \]  

(10)

In a simplify form, we can get:

\[ \tau_i = \sum_{k=1}^{j} D_{ik} \ddot{q}_k + \sum_{k=1}^{n} \sum_{m=1}^{n} h_{ikm} \dot{q}_k \dot{q}_m + c_i \]  

(11)

We can rewrite the equation(11) in a matrix form as

\[ \tau = D(q(t)) \ddot{q}(t) + H(q(t), \dot{q}(t)) \dot{q}(t) + C(q(t)) \]  

(12)
where \( D_{ik} = \sum_{j=\max(i,k)}^n \text{Tr}(U_{jk} J U_{ji}^{T}) \) \( i,k = 1,2,\ldots,n \), \( H_{i} = \sum_{k=1}^n \sum_{m=1}^n h_{ikm} q_{i} q_{m} \) \( i = 1,2,\ldots,n \).

\[
h_{ikm} = \sum_{j=\max(i,j,k)}^n \text{Tr}(U_{jk} J U_{ji}^{T}) \quad i = 1,2,\ldots,n \text{, and } C = \sum_{j=i}^n m_{ij} q_{j} q_{j}^{T} \quad i = 1,2,\ldots,n.
\]

From these dynamic equations, we can obtain two important properties:

- \( D \) is a symmetric positive matrix.
- \( D - 2H \) is a skew-symmetric matrix.

These two properties are very useful, the design of control algorithm in the following part, we take full advantage of the nature of the equations, which makes the control algorithm simplify a lot \[14,15\].

3. The design of adaptive controller

Generally, the dynamics equation of the manipulator can be written as follows

\[
D(q) \ddot{q} + H(q, \dot{q}) \dot{q} + C(q) = F
\]  

(13)

where \( D \) is mass matrix, \( H \) is coupling matrix, and \( C \) is gravity matrix, \( F \) is torque applied on the manipulator. We can use below equations to describe the dynamics of manipulator in angle space \[16\]

\[
D_{x}(q) \ddot{X} + H_{x}(q, \dot{q}) \dot{X} + C_{x}(q) = J^{-T}(q) u + F
\]  

(14)

where \( u \) is control input, \( F \) is force vector between end-effect and environment, \( J \) is Jacobian matrix.

And \( D_{x}(q) = J^{-T}(q) D(q) J^{-1}(q) \), \( H_{x}(q) = J^{-T}(q) H(q, \dot{q}) J^{-1}(q) - D_{x}(q) J(q, \dot{q}) J^{-1}(q) \), \( C_{x}(q) = J^{-T}(q) C(q) \).

Assuming that all the parameters of the manipulator are given, so a target controller of impedance control is designed like this:

\[
B_{m}(\ddot{X} - \ddot{X}_{d}) + D_{m}(\dot{X} - \dot{X}_{d}) + K_{m}(X - X_{d}) = F
\]  

(15)

In fact, some uncertainties exist not only in manipulator but also in environment, if we neglect these uncertainties, the performance of robot will decrease seriously\[17-20\]. So after we considering these factors, a new controller should be designed. Let us define an error vector \( s = \dot{e} + \Lambda e \) where \( e = X - X_{d} \) and \( \Lambda = \text{diag}(\lambda_{1}, \lambda_{2}, \ldots, \lambda_{n}) \) with \( \lambda_{i} > 0 \) for all \( i = 1,2,\ldots,n \). So we rewrite the robot model:

\[
\tau = J^{T} \left[ F_{d} + \dot{C}_{x} \ddot{X}_{d} + \dot{D}_{x} \Lambda \dot{e} + \dot{H}_{x} \dot{X}_{d} - \ddot{H}_{x} \Lambda e - K_{d} s \right]
\]  

(16)

where, \( \dot{C}_{x}, \dot{D}_{x}, \dot{H}_{x} \) is the estimate of \( C_{x}, D_{x}, H_{x} \) and \( K_{d} \) is positive gain matrix. Substituting equation (16) into equation (15) and simplifying the equations:

\[
D_{x} \ddot{s} + H_{x} \dot{s} + K_{d} s = \dot{D}_{x} (\ddot{X}_{d} - \Lambda \dot{e}) + \dot{H}_{x} (\dot{X}_{d} - \Lambda e) + \dot{C}_{x}
\]  

(17)
where \( \tilde{D}_x = \dot{D}_x - D_x \). In order to justify the stability of this control system, choosing a Lyapunov function candidate as:

\[
V(s, \tilde{D}_x, \tilde{H}_x, \tilde{C}_x) = \frac{1}{2} Tr \left( \tilde{D}_x^T W_d \tilde{D}_x + \tilde{H}_x^T W_h \tilde{H}_x + \tilde{C}_x^T W_c \tilde{C}_x \right) + \frac{1}{2} s^T D_s s
\]

The time derivative of \( V \) can be computed as

\[
\dot{V} = s^T \left[ -H_s s - K_d s + \dot{D}_x^T \left( \dot{X}_d - \Lambda \dot{e} \right) + \dot{H}_x^T \left( \dot{X}_d - \Lambda \dot{e} \right) + \dot{C}_x^T \right]
+ Tr \left( \tilde{D}_x^T W_d \tilde{D}_x + \tilde{H}_x^T W_h \tilde{H}_x + \tilde{C}_x^T W_c \tilde{C}_x \right) + \frac{1}{2} s^T D_s s
\]

Using the fact that \( s^T \left( \tilde{D}_x - 2H_s \right) s = 0 \) and choosing the update laws to be

\[
\dot{D}_x = -W_d^{-1} \left( \dot{X}_d - \Lambda \dot{e} \right) s^T \hat{H}_x = -W_h^{-1} \left( \dot{X}_d - \Lambda \dot{e} \right) s^T \hat{C}_x = -W_c^{-1} s^T
\]

Then the equation (20) becomes \( \dot{V} = -s^T K_d s \leq 0 \) Since \( K_d \) is positive matrix and \( V \) is energy function, so we can conclude that \( \dot{V} < 0 \) and \( V > 0 \). According to Lyapunov theory, we can easily prove the stability of the adaptive impedance control system and asymptotic convergence of state vector \( s \). In other words, \( s \to 0 \) as \( t \to \infty \). By this method, we design an adaptive controller considering the uncertainties in both manipulator parameters and environment.

4. Experimental study

For the purpose of verifying the proposed control algorithm, we consider a rigid planar manipulator with two links and two revolute joints. The experimental platform is setup, as shown in figure 2, the schematic diagram of signal acquisition and control of the manipulator is shown in figure 3. According to the equations derived before, we can obtain the complete dynamic equations like this

\[
\begin{bmatrix}
\tau_1 \\
\tau_2
\end{bmatrix} =
\begin{bmatrix}
m_2 l_2^3 / 3 + \pi m_2 C_2 l_2 / 2 + (m_1 / 3 + m_2)l_1^3 & m_2 l_2^3 / 3 + \pi m_2 C_2 l_2 / 2 \\
m_2 l_2^3 / 3 + \pi m_2 C_2 l_2 / 2 & m_2 l_2^3 / 3
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta}_1 \\
\ddot{\theta}_2
\end{bmatrix}
+ \begin{bmatrix}
-m_2 S_2 l_2 / 2 (2 \dot{\theta}_1 \dot{\theta}_2 - \dot{\theta}_2^2) / 2 \\
m_2 S_2 l_1 \ddot{\theta}_1^2 / 2
\end{bmatrix} + \begin{bmatrix}
(g l_1 (m_1 + 2m_2) C_1 + m_2 g l_2 C_1) / 2 \\
m_2 g l_2 C_1 / 2
\end{bmatrix}
\]

The quantities \( m_i, l_i, I_i \) represent mass, length and inertia of link \( i \), respectively. In the experiment and simulation, we select the value of the parameters are list in Table 1 below:
Figure 2: The experimental platform

Figure 3: The Schematic diagram of signal acquisition and control of the manipulator

Table 1. Three Scheme comparing.

| Parameters                              | Values                  |
|-----------------------------------------|-------------------------|
| The length of link 1 and link 2/m       | 0.5, 0.5                |
| The mass of link 1 and link 2/kg        | 1, 2                    |
| The section moment of link 1 and link 2/m^4 | 0.03575, 0.08255       |
| The Young’s modulus of link 1 and link 2/GPa | 206, 206              |
| The torque constant of motor 1 and motor 2 | 64, 32                |
| The ratio of gear 1 and gear 2          | 0.0575, 0.0542         |
In this experiment, we would like the end-point of each arm to track a sine curve in 3 seconds. Assuming that the mass is unknown, so we apply the proposed adaptive control algorithm to the manipulator. Base on the controller designed above, the control parameters are picked as

\[ \Lambda = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}, K_d = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix} \]

The parameters of target impedance are selected as:

\[ B = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, D = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}, K_m = \begin{bmatrix} 1000 & 0 \\ 0 & 1000 \end{bmatrix} \]

The adaptive update laws gain matrices can be selected as diagonal matrices.

\[ W^{-1} = W_h^{-1} = \text{diag}(1,1,1), \quad \text{and } W^{-1}_c = \text{diag}(10,10,10,10). \]

The expected force between end-effect and environment is assumed to be constant, for instance, 100N. The experimental and simulation results are shown as follows.

![Figure 4](image1.png) **Figure 4** Position and velocity tracking of arm 1

![Figure 5](image2.png) **Figure 5** Position and velocity tracking of arm 2

Fig.4 shows the actual position and velocity tracking of the desired value of link1. We can see the initial value between actual and desired are quite different both in position and velocity tracking in this figure. With time going on, the actual position and velocity are approximately equal to the desired value gradually. This is because of the adaptive update laws, the estimate parameters in the controller are convergence to the actual parameters. That is to say, according to the update laws, although the mass is unknown at the beginning, the adaptive control algorithm tune the parameters automatically until the error between estimated and actual is tolerant. We can find the same results in Fig. 5.

The estimate of mass is tuning continuously till the estimated error is acceptable according to Fig. 5. From the variation of the estimate, we can obtain the information that the control system is stable and the estimated parameter is convergence to the actual value under the adaptive update laws. The simulation results above can also verify the proposed control algorithm.
Figure 6: Estimated of the mass of manipulator

Figure 7: The torque of joint 1

Figure 8: The torque of joint 2

The figure 7 and figure 8 are torques applied on the joint 1 and joint 2, respectively. The estimation and the measure value are very close, although there is difference at some places. This reason may be the errors between dynamic modeling and the actual model, and the errors is in reasonable range. The algorithm designed is also verified to be correct.

5. Conclusion
This paper provides an impedance control of the manipulator interaction with the human, the adaptive control algorithm is applied considering the uncertainty of the robot physical parameters. A proper dynamic model is established and the corresponding controller is designed. From the simulation results, the proposed algorithm proves to be correct and easy to implement. In future work, in order to improve the performance of the manipulator, some details should be taken into accounts, such as the external disturb and flexibility of the arms or joints. To accomplish more complex tasks, the dynamic model and the controller will be even more complex, these issues will attract a great deal of attention to studying in future.

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