Modeling static characteristics of angular velocity measuring transducer of the “nozzle-damper” type

V Golubovsky¹, V Konovalov¹, and M Doncova¹

¹Penza State Technological University, 1A/11 proyezd Baydukov / Gagarina Street, 440039, Penza, Russia
E-mail: v.golubovsky@yandex.ru

Abstract. The article describes the design and principle of operation of the hydraulic measuring transducer of angular velocity of the "nozzle-damper" type. The aim of the study is the mathematical modeling in the MathCAD program of the static characteristics (flow and differential) of the measuring transducer, based on the Bernoulli equation. The research methodology provides a theoretical justification of the proposed expressions describing the operation of the transducer, analysis of numerical results obtained by numerical modeling in the MathCAD program, and their verification by comparison with experimental data. The developed mathematical model based on the Bernoulli fluid equation of motion, taking into account the use of the proposed correction factors, allows one to adequately describe the static characteristics of the angular velocity measuring transducer of the “nozzle-damper” type. According to the numerical values of the statistical criteria, the proposed power-law model more accurately describes the experimental data than the linear model. At a supply pressure of the measuring transducer equal to and less than 0.5 MPa, a discrepancy between the results of the model and experiments is observed.

1. Introduction
Hydraulic amplifiers are widely used in various executive bodies of machines and mechanisms [1, 2], as well as in control systems [3, 4]. This is due to a number of their known advantages: high speed; high sensitivity; ability to transfer great effort; stepless speed control of executive bodies [5].

The analysis shows that it is advisable to use hydraulic systems of automatic control for machines with hydraulic drives of working bodies, that is, to use the principle of single working medium (liquid) as there is no need for multiple transformations of information about a controlled parameter from one type of energy to another and the diagram of automatic control system itself is simplified.

Creation of hydraulic automatic control systems is impossible without a developed elemental base. The main elements of such systems are: measuring transducers, hydraulic amplifiers, actuators and other devices, which static and dynamic characteristics largely determine the characteristics of the system as a whole.

Measuring transducers created on the basis of known types of hydraulic amplifiers, in particular with elements “nozzle-damper” [6, 7], should be the most acceptable ones for these systems. In such measuring transducers, an alternating throttle “nozzle-damper” is usually used together with a constant throttle connected in series at the inlet [8].

The angular velocity measuring transducer serves as a source of information on the progress of the technological operation and to a large extent determines the quality of transients in the automatic
control system of the hydraulic drive of technological equipment during the rotational movement of working bodies. The stabilization of the controlled parameter is achieved by the development of controlling influence by the automatic controller of the system. It is aimed at the fastest transition from accelerated or slowed down movement of a working body to a steady movement – at a constant speed.

Figure 1 shows a structural diagram of an angular velocity measuring transducer based on a hydraulic amplifier “nozzle-damper” in the first cascade [9].

The transducer is mounted in rotor 14, which includes housing 1, stops 19 and 20 and covers 15 and 16 with adjusting screws 17 and 18 fixed by nuts 21 and 22. The sensor (transducer) consists of nozzles 2 and 3, constant throttles 5 and 6, inertial damper 4, springs 12 and 13.

In the initial position, when there are no inertia forces in the direction of the sensitivity axis X-X, the working fluid from line 7 enters through constant throttles 5 and 6 into the working chambers of nozzles 2 and 3, and then, passing resistance in the form of gaps between the ends of the nozzles and damper 4, pours off into the tank along line 11. The inertial damper is in equilibrium while occupying a symmetrical position in the center of housing 1 under the action of elastic forces created by centering springs 12 and 13. This leads to the creation of the same resistance to the outflow of the working fluid from nozzles 2 and 3, which ensures equal pressure in the working chambers (P1 = P2).

When rotor 14 rotates around the Y-Y axis, a centrifugal inertia force appears, which is directed along the sensitivity axis (for example, downward), under the influence of inertia, damper 4 moves down and changes the hydraulic resistance of nozzles 2 and 3. Resistance to oil outflow from nozzle 2 increases, and from nozzle 3 – decreases, which leads to the corresponding change in pressure (P1 > P2) in the working chambers of nozzles 2 and 3. The resulting pressure difference (differential) ΔP = P1 – P2 is transmitted along lines 8 and 9 and is used as a controlling signal at the inlets of actuating element 10, for example, a throttling spool valve.

When the direction of acceleration of the rotor is reversed, damper 4 moves up. Resistance to oil outflow from nozzle 3 increases, and from nozzle 2 – decreases, which leads to the occurrence of a corresponding pressure drop ΔP = P2 – P1 in the working chambers of nozzles 2 and 3.

In the absence of acceleration, inertial damper 4 returns to its initial position under the action of the elastic forces of springs 12 and 13, which leads to equalization of pressures in the working chambers of nozzles 2 and 3. The transducer returns to its original position.

Figure 2 shows a schematic diagram of an angular velocity measuring transducer based on a hydraulic amplifier “nozzle-damper” in the first cascade.
Figure 2. Schematic diagram of a device: 1 – supply pipe; 2, 11 – branches of pipeline; 3, 8 – constant throttles; 4, 9 – working chambers; 5, 10 – nozzles; 12 – actuator; $P_{str}$ – supply pressure of measuring transducer, MPa; $P_0$ – outlet pressure, MPa; $P_1, P_2$ – pressure in working chambers, MPa; $P_{AM}$ – actuator supply pressure, MPa; $A, B$ – industrial equipment drive control lines; $T$ – line for pouring off working fluid; $Q_{str}$ – fluid flow through measuring transducer, $m^3/s$; $Q_1, Q_2$ – fluid flow through nozzles 5 and 10, $m^3/s$; $d_m$ – diameter of the opening of the constant throttle, m; $s_d$ – diameter of the nozzle opening, m; $h_{us}$ – average gap between nozzle and damper, m; $\Delta h$ – displacement of damper from the middle position, m.

Equal pressures $P_1$ and $P_2$ mean the absence of an output signal and are provided by presetting (bridge balancing) the transducer.

Pressure difference $\Delta P = P_1 - P_2 = f(\Delta h)$ in working chambers that occurs when the damper moves from the neutral position is used to activate the actuator (AM) of the automatic regulator of the control system included in the bridge diagonal.

A throttling direction control valve is used as the second cascade of amplification, which controls fluid flow at the inlet and outlet (in lines A and B) of the hydraulic motor of the machine drive, while lines $T$ are connected to the drain. It shifts under the influence of pressure difference $\Delta P = P_1 - P_2$ in the lines under the ends of the valve changing the flow sections of corresponding slots and, thereby, forming a signal that is used to automatically control the drive of industrial equipment.

In [10], a device was considered according to a structural scheme similar to the proposed device. And in theoretical studies, the authors proceeded from the fact that, the supplied pressure at the control unit dropped to the atmospheric one in the whole structure without analyzing the drop in its individual parts.

The aim of the research is to ascertain expressions describing the operation of the proposed design of a “nozzle-damper” transducer taking into account the existing features of its construction.

2. Research methods

The research methodology provided for the theoretical justification of the proposed expressions describing the operation of the transducer, the analysis of the numerical results obtained by numerical modelling in the MathCAD program and their verification by comparison with observed data. Due to
the practical application of the device’s operating characteristics reduced to a linear dependence, linear models and a mathematical model were compared based on the Bernoulli law of fluid motion.

The development of methods for calculating the static characteristics of measuring transducers is caused by the need to obtain analytical dependencies that allow determining the parameters and characteristics of transducers and their elements at the design stage.

Calculations allow determining the operating ranges for changing parameters and characteristics or ascertaining their optimal values when using transducers in automatic control systems. These characteristics include: characteristics of transducer elements (flow characteristic, pressure loss on alternating and constant throttles) and the characteristics of transducers themselves (flow, differential, power ones).

3. Research result

At the inlet of the controlling mechanism, PMT pressure (MPa) is applied, and at the outlet of the device, the spent liquid is poured off into a container with pressure P0. As a result, a pressure drop ΔP is observed, which will decrease at several sections: at permanent throttle ΔP1, at nozzle ΔP2 and in the gap between the nozzle and damper ΔP3. The pressure drop, MPa, will be determined as:

$$\Delta P = P - P_0 = \Delta P_1 + \Delta P_2 + \Delta P_3.$$  \hspace{1cm} (1)

Given the equal geodesic height of the device elements arrangement and the height of the liquid column (pressure), the short pipeline lengths, we take into account only the effect of the liquid velocity head from the Bernoulli law of fluid motion. The pressure drop at the sections of the transducer, MPa [11]:

$$\Delta P_i = \frac{\xi_i \rho_i \theta_i^2}{2},$$ \hspace{1cm} (2)

where $\theta_i$ is fluid velocity at the $i^{th}$ transducer element, m/s; $\rho_i$ is fluid density at the $i^{th}$ transducer element, kg/m$^3$. Density $\rho$ is constant for incompressible fluid.

The flow coefficient at the $i^{th}$ element of the transducer [11]:

$$\xi_i = \frac{\lambda_i l_i}{d_i},$$ \hspace{1cm} (3)

where $l_i$ is the length of the $i^{th}$ section; $\lambda_i$ is the coefficient of friction at the section; $d_i$ is the diameter of the opening of the $i^{th}$ transducer element ($d_m$ – throttle, $d_s$ – nozzle), m.

The fluid flow rate (m$^3$/s) at all the $i^{th}$ sections of the transducer is constant for incompressible fluid by virtue of the law of matter conservation and is ideally determined as, m$^3$/s:

$$Q = \frac{\pi d_i^2}{4} \cdot \theta_i = \pi \cdot d_i \cdot h_o \cdot \theta_i,$$ \hspace{1cm} (4)

where $d_i$ is the diameter of the $i^{th}$ opening, m; $h_o$ is a gap between the nozzle and the damper, m:

$$h_o = h_{os} + \Delta h,$$

where $h_{os}$ is an average clearance between the nozzle and the damper, m; $\Delta h$ is a displacement of the valve from the middle position, m.

The fluid velocity will be determined as, m/s:

$$\theta_i = \frac{4 \cdot Q}{\pi d_i^2} = \frac{Q}{\pi d_s \cdot h_o}.$$ \hspace{1cm} (5)

Accordingly, the total pressure drop in transducer $\Delta P$ will be determined taking into account the drop at permanent throttle $\Delta P_1$, nozzle $\Delta P_2$ and in the gap between the nozzle and damper $\Delta P_3$:

$$\Delta P = \frac{\rho \cdot \xi_m \cdot \theta_m^2}{2} + \frac{\rho \cdot \xi_s \cdot \theta_s^2}{2} + \frac{\rho \cdot \xi_o \cdot \theta_o^2}{2} = \frac{\rho}{2} \left( (\xi_m \cdot \theta_m^2 + \xi_s \cdot \theta_s^2 + \xi_o \cdot \theta_o^2) \right).$$
where $d_{st}$ is a diameter of the nozzle end, m.

Pressure in front of the nozzle will be determined as:

$$P_1 = P - \Delta P_1 = P - \frac{\rho Q^2}{2\pi^2} \left[ \frac{16\lambda_m l_m^2}{d_m^2} + \frac{16\lambda_s l_s^2}{d_s^2} + \frac{\lambda_0 d_{st}^2}{2d_m d_s^2 h_0^2} \right].$$

The flow rate of liquid along the transducer will be determined as, m$^3$/s:

$$Q = \mu \cdot \pi \sqrt{\frac{2 \Delta P}{\rho}} \cdot \frac{1}{\sqrt{\frac{16\lambda_m l_m^2}{d_m^2} + \frac{16\lambda_s l_s^2}{d_s^2} + \frac{\lambda_0 d_{st}^2}{2d_m d_s^2 h_0^2}}},$$

where $\mu$ is a flow coefficient. Numerical values determined empirically are presented in Figure 3. The numerical values of the correspondence between predicted and observed data agree with the values of the criteria: $F$-test = 0.990609, $R$-corr = 0.992961.

Based on the results of observed studies of the transducer, numerical values of flow and pressure are obtained. They allow clarifying the numerical value of the coefficient of friction at a section of the throttle length:

$$\lambda_m = \frac{(P - P_1)2\pi^2}{\rho Q^2} \cdot \frac{d^2_s}{d_m^2 l_m^2}.$$

Given the similarity of the material, design and manufacturing method of the constant throttle and the nozzle, it is logical to assume the equality of the numerical values of the friction coefficient at the sections of the throttle and the nozzle: $\lambda_m = \lambda_s$. In this case, the coefficient of friction in the gap between the nozzle and the damper will be determined as:

$$\lambda_0 = \frac{2\pi^2 \Delta P}{\rho Q^2} - \frac{16\lambda_m l_m^2}{d_m^2} - \frac{16\lambda_s l_s^2}{d_s^2} - \frac{\lambda_0 d_{st}^2}{2d_m d_s^2 h_0^2}.$$

We can also determine it on the basis of observed data assuming $\lambda_0 = \lambda_m \cdot k_1$, where $k_1$ is an empirical coefficient of proportionality.

The numerical values of the correspondence between predicted and observed data $k_1$ agree with the values of the criteria: $F$-test = 0.998616, $R$-corr = 0.999047.

MathCAD will perform a numerical modelling of the effect of the ratio of the nozzle and the permanent throttle diameters and the ratio of the gap between the nozzle and the damper and the...
nozzle diameter on the flow rate and pressure in front of the nozzle. The diameter of the permanent throttle opening is constant and equal to 1 mm, the nozzle – 1.1 mm, its end – 1.34 mm. Absolute pressure at the inlet – 1.1 MPa, at the outlet – 0.1 MPa. Openings length: permanent throttle – 10 mm; nozzle – 5 mm. The results of numerical calculations are in good agreement with the observed values (Figures 5, 6, 7).

**Figure 4.** The effect of applied pressure $P$ (Pa) on the empirical coefficient of proportionality $k_1$: (a) is a graphical dependence; (b) is a compliance of predicted and observed values.

![Graph showing the effect of applied pressure on the empirical coefficient of proportionality](image1)

The convergence of the predicted values of the fluid flow rate according to the proposed mathematical model for calibration (spillage) of the permanent throttle flow rate ($m^3/s$) under overpressure $P$ (MPa) is shown in Figure 5. The numerical values of the criteria ($\chi^2$-test = 0.999676; F-test = 0.99664017; Pearson correlation coefficient R-corr = 0.99983486) indicate the adequacy of the flow model.

**Figure 5.** The results of modelling the throttle flow rate calibration ($m^3/s$) under excess pressure $P$ (MPa): $Q_1$ – observed flow rates; $Q_{1t}$ – predicted flow rates.

![Graph showing the results of modeling the throttle flow rate calibration](image2)

**Figure 6.** Change in fluid flow ($m^3/s$) through the nozzle when changing the gap between the nozzle and the damper ($h_o$, m) under the pressure at the inlet to the measuring transducer: observed values $Q_{31,j}$ – for 2.0 MPa; $Q_{32,j}$ – for 1.5 MPa; $Q_{31,j}$ – for 1.0 MPa; predicted values according to the model $Q_{31,j}$ – for 2.0 MPa; $Q_{32,j}$ – for 1.5 MPa; $Q_{31,j}$ – for 1.0 MPa.

![Graph showing the change in fluid flow through the nozzle](image3)
With an increase in the gap \( h_0 \) between the nozzle and the damper (Figure 6), the flow rate increases. The increase rate gradually decreases due to the decrease in the pressure drop at the inlet and outlet of the specified gap. With increasing the inlet pressure, the flow rate increases proportionally. The predicted flow rates are in good agreement with the observed values.

![Figure 7](image-url)  
**Figure 7.** Change in fluid pressure (MPa) in the working chambers of both (right and left) branches when changing the gap between the nozzle and the damper \( (h_0, m) \) under the pressure at the inlet to the measuring transducer: predicted values \( P_{13,j} \) – for 2.0 MPa; \( P_{12,j} \) – for 1.5 MPa; \( P_{11,j} \) – for 1.0 MPa; \( P_{10,j} \) – for 0.5 MPa; observed values \( P_{03,j} \) – for 2.0 MPa; \( P_{02,j} \) – for 1.5 MPa; \( P_{01,j} \) – for 1.0 MPa; \( P_{00,j} \) – for 0.5 MPa.

Comparing the predicted pressure values (Figure 7) in the working chamber with the observed results, one can see their good agreement. The results are somewhat worse at the inlet pressure of 0.5 MPa at the initial stage of an increase in the gap (blue line and blue dots to the value \( h_0 = 0.3 \times 10^{-4} \)).

Analysis of the correspondence of pressure drops (Figure 8.a) between the branches (based on pressure difference shown in Figure 7) showed good agreement between the predicted values of pressure drops according to the proposed model and observed values. The model adequacy criteria (Table 1) are high (\( \chi^2 \)-test, F-test – confidence probability for the distribution of \( \chi^2 \) and Fisher; R-corr – Pearson correlation coefficient) and allow using the proposed model. Given the practical application, the use of approximated linear models, the first-order regression equations were obtained (Figure 8.b). Analysis of the graphs (Figure 8.a, b) shows a greater discrepancy of linear models compared to the proposed mathematical model. Adequacy criteria for linear models (Table 2) have lower values. Accordingly, preference should be given to the proposed mathematical model based on the Bernoulli equation of fluid motion taking into account the use of the proposed expressions of correction factors. The pressure at the inlet to the measuring transducer equal to 0.5 MPa is rejected for use.
Figure 8. Change in the difference in fluid pressure (MPa) in working chambers when changing the gap between the nozzle and the damper \((h_0, m)\) under the pressure at the inlet to the measuring transducer: (a) – predicted values of pressure difference of the mathematical model \((P1\Delta_3 – 2\text{ MPa}; P1\Delta_2 – 1.5\text{ MPa}; P1\Delta_1 – 1\text{ MPa}; P1\Delta_0 – 0.5\text{ MPa})\) and observed data \((P\Delta_3 – 2\text{ MPa}; P\Delta_2 – 1.5\text{ MPa}; P\Delta_1 – 1\text{ MPa}; P\Delta_0 – 0.5\text{ MPa})\); (b) – predicted values of pressure difference approximated by linear relationship \((Y_2 – 2\text{ MPa}; Y_{15} – 1.5\text{ MPa}; Y_1 – 1\text{ MPa}; Y_{05} – 0.5\text{ MPa})\) and observed data \((Y_2^* – 2\text{ MPa}; Y_{15}^* – 1.5\text{ MPa}; Y_1^* – 1\text{ MPa}; Y_{05}^* – 0.5\text{ MPa})\).

Table 1. Numerical values of adequacy criteria of a mathematical model.

| Pressure drop at the device | 0.5 MPa | 1.0 MPa | 1.5 MPa | 2.0 MPa |
|---------------------------|---------|---------|---------|---------|
| \(\chi^2\)-test          | 1       | 0.958999 | 1       | 0.9548888 |
| F-test                    | 0.7024332 | 0.9490535 | 0.97249454 | 0.9333652 |
| R-corr                    | 0.9707062 | 0.9990427 | 0.99994571 | 0.9999049 |

Table 2. Numerical values of adequacy criteria of linear models.

| Pressure drop at the device | 0.5 MPa | 1.0 MPa | 1.5 MPa | 2.0 MPa |
|---------------------------|---------|---------|---------|---------|
| F-test                    | 0.68964 | 0.9482965 | 0.9698593 | 0.9285141 |
| R-corr                    | 0.9926408 | 0.9977009 | 0.99862377 | 0.9964289 |

4. Conclusion
The developed mathematical model based on the Bernoulli equation of fluid motion taking into account the use of the proposed expressions of correction coefficients adequately describes the static characteristics of the angular velocity measuring transducer of the “nozzle-damper” type.

The static indicators of the adequacy of the proposed model more accurately describe the processes than similar values of linear models based on observed data.

The proposed model is not recommended for use at a measuring transducer supply pressure of less than 0.5 MPa.
References

[1] Sokolov V, Krol O, Stepanova O 2017 J. Phys. Conf. Ser. 1278(1) 012003 DOI:10.1088/1742-6596/1278/1/012003

[2] Li Y 2019 Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering 233(2) 657 DOI:10.1177/0954410017740386

[3] Bartnicki A, Klimek A 2019 IEEE Access 7 20172 DOI:10.1109/ACCESS.2019.2897148

[4] Sokolov V, Krol O 2019 Notes in Mechanical Engineering 7 364 DOI:10.1007/978-3-319-93587-4_38

[5] Du Y, Wang B 2018 Proceedings - 2018 IEEE 18th International Conference on Power Electronics and Motion 8521958 499 DOI:10.1109/EPEPEMC.2018.8521958

[6] Medvedev Y, Kuznetsoy V 2011 Russian Engineering Research 31(9) 828 DOI:10.3103/S1068798X11090206

[7] Medvedev Y, Kuznetsoy V 2011 Russian Engineering Research 31(6) 527 DOI:10.3103/S1068798X11060165

[8] Bazhenov A, Levichev E 1988 Vestnik mashinostvoeniy 7 17 https://www.scopus.com/inward/record.uri?eid=2-s2.0-0023701727&partnerID=40&md5=16b65b368baa13889ee0022be772c290

[9] Simanin N, Golubovsky V 2018 RU 88919 U1G05D 13/10 (2006.01) FIPS information Bulletin 13 http://www1.fips.ru/registers-doc-view/fips_servlet

[10] Simanin N, Golubowski V, Prohorov A 2013 XXI century: resumes of the past and challenges of the present plus 6(10) 171 https://elibrary.ru/item.asp?id=20394710

[11] South Russian state technical University (Novocherkassk Polytechnic Institute) https://studfiles.net/preview/2955391