A semiclassical resonator model for terahertz generation

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Abstract. The paper states a simple and analytically solvable model for the terahertz generation via a localized excited ensemble of identical three-level quantum particles within the resonator.

1. Introduction
Electromagnetic frequencies ranging from 0.1 to 10 THz (wavelengths of the order of 1-0.01 mm) are conventionally related to the terahertz emission. Terahertz emission interacts effectively with vibrational, rotational, tunneling, etc. quantum transitions, which is quite promising in spectroscopy problems. However, there is a difficulty in obtaining terahertz emission during laser generation of coherent emission [1,2].

The present paper proposes a simple and analytically solvable model for generating low-frequency emission by an ensemble of particles with a permanent dipole moment (PDM), which are localized in a cavity, with quantum dots serving as particles with PDM [1]. As the theoretical basis the author proposes that the model [3-5] applied earlier to study coherence transfer under conditions of optical instability be complemented with PDM.

PDM–based systems lead to the generation of terahertz emission when exposed to electromagnetic fields of optical frequencies. In contrast to the consideration of the influence of a permanent dipole moment on radiative processes in terms of the detailed quantum dot theory [6], the algebraic perturbation theory [7] was used to solve the problem of terahertz emission of a PDM–based quantum system in resonant fields of optical frequencies as in works [8,9].

As a result, the application of the algebraic perturbation theory and model [3-5] with allowance for PDM, expressions for the low-frequency emission intensity have been obtained, which occurs when a superradiant quantum level is pumped by a coherent pulse and results in further development of both superradiance and terahertz emission. In contrast to the cases of generating terahertz emission during resonance onto two-level systems, a three-level scheme is considered when a transition $E_1 \rightarrow E_3$ is resonantly excited by an optical pulse, and then superradiance occurs at an adjacent transition $E_3 \rightarrow E_2$. The generation stage of terahertz emission during the action of an exciting pulse was considered in terms of the algebraic perturbation theory in [8,9]. The generation of terahertz emission in the subsequent period of time is to be considered in this article. The quantum levels $E_1 < E_2 < E_3$ are assumed to form the so-called $\Lambda$ -configuration. Terahertz emission accompanies superradiance being developed during optical pumping of a quantum level $E_3$. Due to the given ratio between the energies of quantum levels, the energy of a superradiance quantum of the described model can be
significantly lower than that of one pump quantum, providing an opportunity to increase the energy efficiency of terahertz emission generation.

2. The model and its basic equations

Let an environment containing $N_p$ three-level particles with level energies $E_1 < E_2 < E_3$ be placed inside the cavity, with the cavity mentioned only for describing emission at the transition frequency $E_2 \rightarrow E_3$ and the cavity mirrors having a transmittance factor $T$. It is to be noted that the faces of crystal can serve as mirrors. A distinctive feature of the model of emission interaction with such a system is the approximation of an optical thin environment for the pump transition $E_1 \rightarrow E_2$ and the generated terahertz emission. At the same time, at the transition the environment $E_2 \rightarrow E_3$ appears to be effectively optically dense, and the average field approximation is used for the values characterizing the optical processes at this transition. The interaction of the field of electromagnetic pulses with these transitions is considered in the electric dipole approximation under the assumption that these transitions are adjacent and optically allowed transitions, with the remaining transition regarded as the optically forbidden one. Works [8,9] show that, according to the algebraic perturbation theory, PDM does not affect the dynamics of the system in the first order, but gives rise to a low-frequency component in the polarization of a quantum system. Therefore, regardless of the parity of the given levels and PDM, we assume the electromagnetic field of the crystal at the transition frequency $E_2 \rightarrow E_3$ to be described by the approximation of the average field, whereas at the transition $E_1 \rightarrow E_2$ the crystal is an optically thin environment. The algebraic perturbation theory [7-9] provides grounds for the basic equations of the problem to be described below.

Let the carrier frequency of the exciting field be $\omega_p$ and the carrier frequency of emission at the transition $E_2 \rightarrow E_3$ coincide with the transition frequency and equal $\omega$, as that is spontaneous emission. We shall introduce slowly varying amplitudes $\mathcal{E}^{(\pm)}_p$ and $\mathcal{E}^{(z)}$ of electromagnetic field intensities at frequencies $\omega_p$ and $\omega$,

$$E_p = \mathcal{E}_p e^{-i\omega_p t} + k.c., \quad E = \mathcal{E}^{(z)} e^{-i\omega t} + k.c., \quad \mathcal{E}_p = \mathcal{E}_p^{(+)} e^{ikz} + \mathcal{E}_p^{(-)} e^{-ikz}, \quad \mathcal{E}^{(z)} = \mathcal{E}^{(+)} e^{ikz} + \mathcal{E}^{(-)} e^{-ikz}.$$  \hspace{1cm} (1)

They are described by the usual Maxwell equations

$$\frac{\partial \mathcal{E}^{(z)}}{\partial t} + \frac{1}{c} \frac{\partial \mathcal{E}^{(z)}}{\partial z} = \pm \frac{2\pi \omega_n N_p}{c} d_{31} R_{31}^z, \quad \frac{\partial \mathcal{E}^{(+)}}{\partial t} + \frac{1}{c} \frac{\partial \mathcal{E}^{(+)}}{\partial z} = \pm \frac{2\pi \omega_n N_p}{c} d_{32} R_{32}^z.$$ \hspace{1cm} (2)

Here, the positively directed axis $z$ orients the propagation of the exciting optical pulse $E_p$, and, as with resonant processes [7], the off-diagonal matrix elements of the density matrix $\rho$ of quantum particles are represented as the slow amplitudes of the density matrix:

$$\rho_{31} = R_{31} \exp(-i\omega_p t), \quad \rho_{32} = R_{32} \exp(-i\omega t), \quad \rho_{21} = R_{21} \exp(-i(\omega_p - \omega)t).$$

$$R_{31}^{\pm} = \frac{k_p}{2\pi} \int dz R_{31} e^{-ikz}, \quad R_{32}^{\pm} = \frac{k_p}{2\pi} \int dz R_{32} e^{-ikz}.$$ 

The very density matrix obeys the usual equation for the density matrix [8].

When waves are reflected from crystal faces at the transition frequency $E_2 \rightarrow E_3$, a quasi-resonator mode is formed ( $L$ is the distance between the faces)

$$\mathcal{E}^{(z)}(t, L) = \sqrt{1-T} \mathcal{E}^{(+)}(t, L), \quad \mathcal{E}^{(+)}(t, 0) = \sqrt{1-T} \mathcal{E}^{(+)}(t, 0).$$

According to the average field approximation [10]

$$\mathcal{E} = \frac{1}{L} \int_0^L dz \mathcal{E}^{(z)} \approx \frac{1}{L} \int_0^L dz \mathcal{E}^{(z)}, \quad \bar{R}_{31} = \frac{1}{L} \int_0^L dz R_{31}^z \approx \frac{1}{L} \int_0^L dz R_{31}^z, \quad \bar{R}_{32}^z \approx \frac{1}{L} \bar{R}_{32}^z,$$ \hspace{1cm} (3)
we obtain

\[
\frac{d\vec{\sigma}}{dt} + \sigma \vec{\sigma} (t) = i\sigma \vec{R}_{32},
\]

(4)

\[
\frac{d\vec{R}_{31}}{dt} = i\hbar \frac{d_3 - d_3^*}{\hbar} + i\vec{R}_{21} \frac{d_3}{\hbar}, \quad \frac{d\vec{R}_{32}}{dt} = \vec{R}_{32} + i\hbar \frac{d_3}{\hbar} + \frac{i\vec{R}}{2} \frac{d_3}{\hbar}.
\]

(5)

where \( \vec{\sigma} = \vec{p}_1 - \vec{p}_3 \), \( \vec{\sigma}_2 = \vec{p}_2 - \vec{p}_3 \) by neglecting the spread of dipole moments, inhomogeneous broadening of the spectral line of the given transitions, detuning and relaxation;

\[
\sigma = cT / L(1 + \sqrt{1 - T}), \quad s_0 = 2\pi cN_p d_{23}.
\]

Notice that all the dipole moments are aligned along the same axis.

A low-frequency component of the environment response to its optical processes can be found in various ways. Works [1,2,8,9] used a unidirectional approximation to find the PDM-biased electric field intensity. Here, in terms of the average field approximation (3), we use the classical concepts of dipole emission of the dipole ensemble in the given model. The classical formula for the total intensity of dipole emission leads to the expression:

\[
P_{\text{LF}}^t = \frac{2}{3c^3} \left( \frac{d^2 P_{\text{LF}}^t(t)}{dt^2} \right)^2.
\]

Here, \( P_{\text{LF}}^t(t) \) is the low-frequency component of the environment’s polarization due to the fact that the environment particles have PDM. In the considered case, according to the basic order of the algebraic perturbation theory, we have

\[
P_{\text{LF}}^t(t) = N_p \sum_{n=1,2,3} d_{nm} \rho_{nm}.
\]

(7)

Below we will not draw a line over the averaged values (3).

3. The generation of terahertz emission

Let a rectangular-shaped optical pulse resonant to the transition \( E_i \rightarrow E_3 \) and the following amplitude affect the described environment

\[
\vec{\sigma} = \begin{cases} a_p \exp(i\varphi_p), & 0 \leq t \leq \tau_p; \\ 0, & \tau_p \leq t, \end{cases}
\]

where \( \varphi_p \) and \( a_p \) are the constant field phase and amplitude, respectively. The pulse duration \( \tau_p \) is much shorter than the times of irreversible relaxation of the polarization and transition populations \( E_3 \rightarrow E_1 \). The time of the signal motion inside the environment is neglected. Below we shall list the solutions to the basic equations (2), (3), (5) during the action of the pump pulse and after its action.

In the region \( 0 \leq t \leq \tau_p \) of the resonant exciting pulse, we have the following solutions

\[
n_1 = \cos \Omega_p t, \quad n_2 = -\sin^2 \frac{\Omega_p t}{2}, \quad \Omega_3 = i\frac{\exp(i\varphi_p)}{2} \sin \Omega_p t, \quad R_{31} = 0, \quad R_{32} = 0, \quad \Omega_p = 2a_p |d_{13}| / \hbar.
\]

In this case, as was studied in [6], terahertz emission is generated, whose distinguishing feature is its carrier frequency — the Rabi frequency \( \Omega_p \).
In the region $\tau_p \leq t$ the low-frequency emission at the Rabi frequency $\Omega_p$, ceases after the exciting pulse, and there is an inversion at the transition frequency $E_3 \rightarrow E_2$

$$n_1(t) = \cos \Omega_p \tau_p, \quad n_2(t) = -\sin^2 \frac{\Omega_p \tau_p}{2}, \quad R_{34}(t) = i \frac{\exp(i \Omega_p \tau_p)}{2} \sin \Omega_p \tau_p.$$  

As this situation is unstable, there occurs superradiance at the transition $E_3 \rightarrow E_2$. Various modes of superradiance for quantum levels characterized by a certain parity are described in [11].

Further, we shall consider the simplest case of monopulse superradiance, which takes place when neglecting relaxation and fulfillment of the following condition

$$\sigma^2 \gg |s_0 R_{32} d_{32}| / \hbar.$$  

(8)

A superradiance pulse arises in a time of the order of the time of instability development $\tau_s \sim \eta^{-1}$ and has a duration of the same order $\tau \sim \eta^{-1}$. The envelope of the superradiance pulse is standard secant:

$$\mathcal{E} = -\frac{\hbar}{d_{32}} \frac{\eta}{2 \text{ch}(\eta')} ,$$  

(9)

$$\eta = \omega_c \cos^{-1} \sin^2 \Omega_p \tau_p / 2, \quad t' = t - \tau_p - \tau_s, \quad \omega_c = 2 \pi N |d_{32}|^2 \hbar^{-1},$$

where $\omega_c$ is the cooperative frequency. In estimating the delay time $\tau_s$ of the superradiance pulse, it was taken into account that condition (8) is equivalent to $\sigma \gg \eta$. The rest parameters of a three-level system are determined by the equations

$$R_{32} = -i \sigma s_0^{-1} \mathcal{E}, \quad n_2 = \sin^2 (\Omega_p \tau_p / 2) \text{th}(\eta'),$$

$$R_{31} = i \frac{\sin \Omega_p \tau_p}{2 \sqrt{2 \text{ch} \eta'}} (\text{ch} \eta' / 2 - \text{sh} \eta' / 2) e^{i \eta'}, \quad R_{21} = \frac{\sin \Omega_p \tau_p}{2 \sqrt{2 \text{ch} \eta'}} (\text{sh} \eta' / 2 + \text{ch} \eta' / 2) e^{i \eta'} .$$  

(10)

As a result, according to formulas (6) and (7), simultaneously with the generation of a superradiance pulse, there is a change in time of the populations of quantum levels $E_2$ and $E_3$ at a constant population of the ground level $E_1$. The time-variable component $P^{LF}(t)$ of low-frequency polarization (7) takes the form

$$P^{LF}(t) = \frac{1}{3} N_p n_2 (2d_{22} - d_{33}), \quad P^{LF}(t) = P^{LF}(t) + \text{const} .$$

This polarization causes the terahertz emission intensity

$$I^{LF} = \frac{2}{27c^3} N_p (2d_{22} - d_{33})^2 \left( \frac{d^2 n_2}{dt^2} \right)^2 .$$

In contrast to terahertz emission accompanying superradiance in an ensemble of two-level particles [12], in the considered case, the dependence on the constant dipole moments of quantum levels is different, and the time profile coincides with the time profile obtained by quantum calculation of collective superradiance [12]:

$$I^{LF} = \frac{8 \omega_c^4 \omega^4 N_p^2}{27c^3 \sigma} (2d_{22} - d_{33})^2 \sin^2 \frac{[\Omega_p \tau_p / 2]}{\text{ch}^6 (\eta')} .$$

The intensity of the terahertz emission is very sensitive to the area of pump pulse $\Omega_p \tau_p$, which is an important factor in experiments.

4. Conclusion
The model does not describe the efficiency of transforming pump energy into terahertz emission energy. A similar consideration would take into account the inverse effect of terahertz emission on the dynamics of the superradiative transition. This could be taken into account by adding a term
\[
\frac{d\bar{n}_2}{dt} = -\gamma_{a_1a_2} \bar{n}_2 + \ldots \text{ to the final equation } \frac{d\bar{n}_2}{dt} \text{ for the set of equations (5). In that case equations (4), (5) could not be solved analytically.}
\]

In fact, in the present work, the cavity was used for the transition \(E_3 \rightarrow E_2\), and terahertz emission accompanied the pulse of optical superradiance. Work [13] used the cavity at the frequency of terahertz emission. The very terahertz emission resulted from the resonant generation, with inhomogeneous broadening of levels. The proposed model makes it possible to take into account the inhomogeneous broadening of levels, which, however, does not affect the generation of terahertz radiation as significantly as in [13].

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