Teleportation of Entangled States of a Vacuum-One Photon Qubit

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We report the experimental realization of teleporting an entangled qubit. The qubit is physically implemented by a two-dimensional subspace of states of a mode of the electromagnetic field, specifically, the space spanned by the vacuum and the one photon state. Our experiment follows along lines suggested by H. W. Lee and J. Kim, Phys. Rev. A, 63, 012305 (2000) and E. Knill, R. Laflamme and G. Milburn Nature 409: 46 (2001).

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In their pioneering paper C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres and W. Wootters introduced the concept of teleportation of a quantum state [1]. Since then teleportation has come to be recognized as one of the basic methods of quantum communication and, more generally, as one of the basic ideas of the whole field of quantum information. Following the original teleportation paper and its continuous-variables ver-
an intensive experimental effort started for the practical realization of teleportation. Quantum state teleportation (QST) has been realized in a number of experiments \[3, 4, 5, 6\]. In a beautiful example of ingenuity, although starting from a common theme, each of these experiments followed a completely different route and principle. In the present paper we report a new teleportation experiment following yet another route. In our experiment we consider a qubit which is physically realized not by a particle but by a mode of the electromagnetic (e.m.) field, and whose orthogonal basis states \(|0\rangle, |1\rangle\) are the vacuum state and the one-photon state respectively. Furthermore, we teleport entangled states of this qubit. We designed our scheme by adapting a method proposed by Knill, Laflamme and Milburn \[7\] to make it experimentally easily feasible. We later learned that our method is identical to that proposed by H.W.Lee and J.Kim \[8\] and also closely related to \[9\]. In our experiment the role of the two particles in a singlet state which constitute the non-local communication channel in the original teleportation scheme \[1\] is played by a photon in an equal superposition of being at Alice and Bob \(|\Psi\rangle = 2^{-\frac{1}{2}}(|\text{Alice}\rangle + |\text{Bob}\rangle)\) where \(|\text{Alice}\rangle\) and \(|\text{Bob}\rangle\) represent the photon located at Alice and Bob respectively. The scheme seems puzzling. Indeed entanglement is considered the basis of teleportation and here we don’t even have two particles, let alone two particles in an entangled state. The puzzle is solved however by noting that in second quantization the state of the non-local channel reads as \(|\Phi\rangle_{\text{singlet}} = 2^{-\frac{1}{2}}(|1\rangle_A |0\rangle_B - |0\rangle_A |1\rangle_B\) where the labels \(A\) and \(B\) represent two different modes of the e.m. field, with wavevectors (wv) \(k_A\) and \(k_B\) one directed towards Alice and the other towards Bob. The mode indices 0 and 1 denote the Fock state population by zero (vacuum) and one
photon respectively. In effect the role of the two entangled quantum systems which form the non-local channel are played by the e.m. fields of Alice and Bob. In other words the field’s modes rather than the photons associated with them should be properly taken as the information and entanglement carriers, i.e. *qubits*. (In the context of Bell’s inequalities, the nonlocal aspects of a single photon have been discussed in [10], [11], [12] and [13].)

Of course, in order to make use of the entanglement present in this picture we need to use the second quantization procedure of creation and annihilation of particles and/or use states which are superpositions of states with different numbers of particles. Another puzzling aspect of this second quantized picture is the need to define and measure the relative phase between states with different number of photons, such as the relative phase between the vacuum and one photon state in Eq. 1 below. That we can associate a relative phase between the *vacuum* and anything else seems most surprising, but it is less so if we recall the more familiar case of a coherent state, where the relative phase between the different photon number states in the superposition is reflected physically in the phase of the classical electric field. To be able to control these relative phases we need, by analogy with classical computers, to supply all gates and all sender/receiving stations of a quantum information network with a common *clock* signal, e.g. provided by an ancillary photon or by a multi-photon, Fourier transformed coherent e.m. pulse [14]. These concepts will be fully demonstrated by the present experiment.

The quantum system whose state we want to teleport is physically represented by another mode of the e.m. field, one with wv *k*<sub>s</sub>. Again we consider only a two dimensional Hilbert space of this mode, i.e. spanned by |0⟩<sub>S</sub>
and $|1\rangle_S$. Thus the mode $k_S$ can be considered the qubit to be teleported. Suppose now that the qubit $k_S$ is in an arbitrary pure state

$$\alpha |0\rangle_S + \beta |1\rangle_S$$  \hspace{1cm} (1)

The overall state of the system and the non-local channel is then:

$$|\Phi_{total}\rangle = 2^{-\frac{1}{2}}(\alpha |0\rangle_S + \beta |1\rangle_S)(|1\rangle_A |0\rangle_B - |0\rangle_A |1\rangle_B)$$

$$= 2^{-\frac{1}{2}}\alpha |\Psi^1\rangle_{SA} |1\rangle_B + 2^{-\frac{1}{2}}\beta |\Psi^2\rangle_{SA} |0\rangle_B +$$

$$+ \frac{1}{2} |\Psi^3\rangle_{SA} (\alpha |0\rangle_B + \beta |1\rangle_B) + \frac{1}{2} |\Psi^4\rangle_{SA} (\alpha |0\rangle_B - \beta |1\rangle_B)$$  \hspace{1cm} (2)

where the states $|\Psi^j\rangle_{SA}$, $j = 1, 2, 3, 4$ are defined below in Eq. 3. The teleportation proceeds with Alice performing a partial Bell measurement. She combines the modes $k_S$ and $k_A$ on a symmetric (i.e. 50:50) beam splitter $BS_A$ whose output modes $k_1$ and $k_2$ are coupled to two detectors $D_1$ and $D_2$, respectively (see Figure 1). The action of $BS_A$ on the field operators is expressed by: $\hat{a}_S = 2^{-\frac{1}{2}}(\hat{a}_1 + \hat{a}_2)$; $\hat{a}_A = 2^{-\frac{1}{2}}(\hat{a}_1 - \hat{a}_2)$ where labels 1, 2 refer to modes $k_1$, $k_2$. As a consequence we obtain:

$$|\Psi^1\rangle_{SA} = |0\rangle_S |0\rangle_A = |0\rangle_1 |0\rangle_2$$

$$|\Psi^2\rangle_{SA} = |1\rangle_S |1\rangle_A = 2^{-\frac{1}{2}}(|2\rangle_1 |0\rangle_2 + |0\rangle_1 |2\rangle_2)$$

$$|\Psi^3\rangle_{SA} = 2^{-\frac{1}{2}}(|0\rangle_S |1\rangle_A - |1\rangle_S |0\rangle_A) = |1\rangle_1 |0\rangle_2$$

$$|\Psi^4\rangle_{SA} = 2^{-\frac{1}{2}}(|0\rangle_S |1\rangle_A + |1\rangle_S |0\rangle_A) = |0\rangle_1 |1\rangle_2$$  \hspace{1cm} (3)

The state $|\Psi^3\rangle_{SA}$ is a Bell type state [1]. From Eq. 3 we see that $|\Psi^3\rangle_{SA}$ leads to a single photon arriving at the detector $D_1$ and no photons at $D_2$. Similarly, $|\Psi^4\rangle_{SA}$ is a Bell type state and it leads to a single photon arriving at the detector $D_2$ and no photons at $D_1$. In both these cases the teleportation
is successful. Indeed, when Alice finds $|\Psi^3\rangle_{SA}$, Bob’s e.m. field ends up in the state $|\Phi\rangle = (\alpha |0\rangle_B + \beta |1\rangle_B)$ which is identical to the state to be teleported, while when Alice finds $|\Psi^4\rangle_{SA}$, Bob ends up with the state $|\Phi_\pi\rangle = (\alpha |0\rangle_B - \beta |1\rangle_B) = \sigma_z |\Phi\rangle$ which is identical to the state to be teleported up to a phase shift $\Delta = \pm \pi$. The states $|\Phi\rangle$ and $|\Phi_\pi\rangle$ are connected by a unitary transformation expressed by the Pauli spin operator $\sigma_z$. Bob can easily correct the phase shift $\Delta$ upon finding out Alice’s result. In practice this phase correction procedure, generally referred to as “active teleportation” [11] is carried out automatically by means of a fast electro-optic Pockels cell (EOP) inserted in mode $k_B$ and triggered by $D_2$. On the other hand, when Alice finds $|\Psi^1\rangle_{SA}$ or $|\Psi^2\rangle_{SA}$ the teleportation fails. From Eq 3 we see that teleportation is successful in 50% of the cases.

A major technical difficulty in the above teleportation scheme is the preparation and manipulation of the pure states to be teleported. Indeed, they are superpositions of the vacuum and one-photon states of the mode $k_S$. Manipulating such states and, in particular having control about the relative phase between the vacuum and one-photon states is quite problematic. This can be realized in principle, for example by homodyning techniques as described in [15]. Here however, we avoid the problem altogether, by teleporting appropriate entangled states instead of pure ones. The states we consider are of the form

$$|\Psi\rangle_{S\tilde{a}} = (\alpha |0\rangle_S |1\rangle_{\tilde{a}} + \beta |1\rangle_S |0\rangle_{\tilde{a}})$$

where $k_{\tilde{a}}$ is an “ancilla” mode. These states are in fact simple single-photon states and can be easily obtained by, say, letting a single photon impinge on a beam-splitter ($BS_S$ in Fig. 1) with reflectivity $r_S$ and transmissivity
$t_S$, $k_\tilde{a}$ being the reflected mode and $k_S$ the transmitted one. For the sake of simplicity and without loss of generality we assume that $\alpha$ and $\beta$ are real numbers.

To summarize, in our experiment we have four qubits: $k_A$ and $k_B$ which constitute the non-local communication channel, $k_S$ which represents the system, i.e. the qubit to be teleported and $k_\tilde{a}$ the ancilla. The special states of these four qubits which are used in the experiment are physically implemented by exactly two photons. The state of the qubit $k_S$ is teleported to Bob into the state of the qubit $k_B$, thus the overall state $|\Psi\rangle_{S\tilde{a}}$ will now be transferred into the state of the qubits $k_B$ and $k_\tilde{a}$. To verify that the state has been teleported we transmit the qubit $k_\tilde{a}$ to Bob. The QST verification consists simply by mixing the modes $k_B$ and $k_\tilde{a}$ at a beam-splitter ($BS_B$) similar to the one which was used to produce the state to be teleported $|\Psi\rangle_{S\tilde{a}}$. We shall see that the optimum QST verification, viz implying the maximum visibility $V$ of the corresponding interferometric patterns is obtained by adopting equal optical parameters for both $BS_S$ and $BS_B$, i.e. $|r_S| = |r_B| = \alpha$ and $|t_S| = |t_B| = \beta$. This verification procedure is generally referred to as “passive teleportation”. Finally, note that the “ancillary” single photon emitted on mode $k_\tilde{a}$ indeed provides the “clock” pulse that is needed to retrieve at Bob’s side the full information content of the vacuum state $|0\rangle_B$ entangled within the nonlocal teleportation channel, i.e. with the singlet state $|\Phi\rangle_{\text{singlet}}$.

The experimental set-up is shown in Fig. 1. A nonlinear $LiO_3$ crystal slab, 1.5 mm thick with parallel anti-reflection coated faces, cut for Type I phase-matching is pumped by a single mode UV cw argon laser with wave-
length (wl) $\lambda_p = 363.8\text{nm}$ and with an average power $\simeq 100\text{mW}$. The UV laser beam was focused close to the crystal by a lens with focal length $= 2\text{m}$ in order to maximize the collection efficiency by the Alice’s detector system of the spontaneous parametric down-conversion (SPDC) fluorescence [16]. The two SPDC emitted photons have equal wl $\lambda = 727.6\text{nm}$ and are spatially selected by two pinholes with equal apertures with diameter $0.5\text{mm}$ placed at a distance of $50\text{cm}$ from the crystal. One of the photons generates on the two output modes $k_A$ and $k_B$ of a $50:50$ beam splitter ($BS$) the singlet state $|\Phi\rangle_{\text{singlet}}$ providing the nonlocal teleportation channel. The other photon generates the state $|\Psi\rangle_{\tilde{s}a}$ i.e. the quantum superposition of the state to be teleported and the one of the ancilla at the output of a variable beam-splitter $BS_S$ consisting of the combination of a $\lambda/2$ polarization rotator and of a calcite crystal. Furthermore micrometric changes of the mutual phase $\varphi$ of the $k_S$ and $k_A$ modes interfering on $BS_A$ were obtained by a piezoelectrically driven mirror $M$. All detectors were Si-avalanche EG&G-SPCM200 counting modules having nearly equal quantum efficiencies $QE \approx 0.45$. Before detection the beams were IF filtered within a bandwidth $20\text{nm}$. In Figure 1 the complete scheme for “active” teleportation is shown, including the high-voltage Pockels cell (EOP) inserted on the mode $k_B$. In the same figure is reported the interferometric scheme for “passive teleportation” which is also adopted for the verification of the correct implementation of the “active” protocol, as we shall see.

We have realized experimentally the passive teleportation protocol. By this we mean that Bob does not modify his state according to the results obtained by Alice. Instead Bob passes his state unmodified to the verification
stage. The verification stage consists in combining the mode $k_s$ (which now contains the teleported state) with the ancilla mode $k_a$ at a beam-splitter $BS_B$, as said. In order to check the overall mode alignment we first checked at Alice’s site the 2-photon Ou-Mandel interference across the beam-splitter $BS_A$ between the modes $k_S$ and $k_A$ that are coupled to detectors $D_1$ and $D_2$ respectively. We obtained a 2-photon interference pattern with a visibility $V_A \approx 0.96$. In a similar way we checked, at Bob’s site the Ou-Mandel interference across $BS_B$ between the modes $k_B$ and $k_a$ coupled to the respective detectors $D_1^*, D_2^*$ obtaining: $V_B \approx 0.92$. The QST verification experiment has been carried out first with a 50:50 beam splitter $BS_S$, i.e. with optical parameters: $|r_s|^2 = |t_s|^2 = 2^{-\frac{3}{2}}$. The maximum visibility of the verification fringe pattern is obtained by selecting the same values of the parameters for the test beam splitter $BS_B$, as said. Then we measured the coincidence counts between $D_1$, $D_2$ and $D_1^*$, $D_2^*$. By a straightforward calculation we expect

$$(D_1 - D_1^*) = (D_2 - D_2^*) = \frac{1}{2} \sin^2 \frac{\varphi}{2}, (D_1 - D_2^*) = (D_2 - D_1^*) = \frac{1}{2} \cos^2 \frac{\varphi}{2}$$

(5)

where $(D_i - D_i^*)$ expresses the probability of a coincidence detected by the pair $D_i$, $D_i^*$ in correspondence with the realization of either one of the states: $|\Psi^3\rangle_{SA}$, $|\Psi^4\rangle_{SA}$. The experimental plots shown in Figure 2, obtained by varying the position $X = (2)^{-3/2} \lambda \varphi / \pi$ of the mirror M, are in agreement with the theory, Eq. 5. This agreement is further substantiated by the data reported in Fig. 3 corresponding to a similar QST verification experiment carried out with a different set of optical parameters for $BS_B$: $|r_B|^2 = 0.20$, $|t_B|^2 = 0.80$. Precisely, each experimental point of Fig. 3 corresponds to an experiment equal to the one referred to by Fig. 2. The visibility of
each sinusoidal fringe pattern is then reported. Note that the maximum visibility $V$ is attained for values of $\alpha^2 = 1 - \beta^2$ that are equal to $|r_B|^2$ or to $|t_B|^2$ depending on which pair of detectors are excited. The two different peaks collapse into only one maximum with theoretical $V = 1$ in the fully symmetric case: $|r_B|^2 = |t_B|^2 = \alpha^2 = \beta^2 = \frac{1}{2}$. In Fig. 3 is also reported a single experimental value $V \simeq 0.91$ related to the symmetric case.

Note that by assuming perfect detectors, i.e. with $QE = 1$, the above QST verification procedure involving the ancilla mode $k_\tilde{a}$ enables a fully noise-free teleportation procedure. Indeed, if no photons are detected at Alice’s site, i.e. by $D_1$ and/or $D_2$, while photons are detected at Bob’s site by $D_1^*$ and/or $D_2^*$ we can safely conclude that the “idle” Bell state $|\Psi^1\rangle_{SA}$ has been created. If, on the other hand, no photons are detected at Bob’s site while photons are detected at Alice’s site, we must conclude that the other “idle” Bell state $|\Psi^2\rangle_{SA}$ has been realized. The data collected in correspondence with these “idle” events can automatically be discarded by the electronic coincidence circuit. In addition to that, note that the effect of the above verification procedure involving the ancilla mode $k_\tilde{a}$ keeps holding within the active teleportation scheme. Indeed, if the $D_2$-driven EO phase-modulator works correctly within the active scheme, the detector $D_2^*$ should be found to be always inactive.

Our present effort is directed towards the completion of the teleportation picture by the realization of the “active” scheme. The main technical problem resides in the relatively large time needed to activate a high-voltage EO device by a single photon detection. The best result we have attained so far for the 1KVS switching time across an EO modulator is about 10 nsec.
This figure would enable us to achieve the goal in the near future by the adoption of small $\lambda/2$ – voltage EO devices possibly in conjunction with the use of optical fibers. We are greatly indebted with the FET European Network on Quantum Information and Communication (Contract IST-2000-29681:ATESIT) and with M.U.R.S.T. for funding.

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Figure Captions

1. Experimental apparatus realizing the “active” and “passive” quantum state teleportation (QST).

2. Interferometric fringe patterns obtained by coincidence experiments involving different pairs of detectors within a passive QST verification procedure.

3. Visibility $V$ of the coincidence fringe patterns $V$ vs the superposition parameter $\alpha^2 = 1 - \beta^2$ obtained by two different pairs of detectors within a passive QST experiment and for an unbalanced beam splitter $BS_B$ with optical parameters $|r_B|^2 = 0.20$, $|t_B|^2 = 0.80$. The continuous lines represent the corresponding theoretical expectations. A single experimental value of $V$ for the fully symmetric case $|r_B|^2 = |t_B|^2 = \alpha^2 = 0.50$ is also reported.
