A new zero-knowledge code based identification scheme with reduced communication

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Abstract—In this paper we present a new 5-pass identification scheme with asymptotic cheating probability \( \frac{1}{2} \) based on the syndrome decoding problem. Our protocol is related to the Stern identification scheme but has a reduced communication cost compared to previous code-based zero-knowledge schemes, moreover our scheme permits to obtain a very low size of public key and secret key. The contribution of this paper is twofold, first we propose a variation on the Stern authentication scheme which permits to decrease asymptotically the cheating probability to 1/2 rather than 2/3 (and very close to 1/2 in practice) but with less communication. Our solution is based on deriving new challenges from the secret key through cyclic shifts of the initial public key syndrome; a new proof of soundness for this case is given Secondly we propose a new way to deal with hashed commitments in zero-knowledge schemes based on Stern’s scheme, so that in terms of communication, on the average, only one hash value is sent rather than two or three. Overall our new scheme has the good features of having a zero-knowledge security proof based on well known hard problem of coding theory, a small size of secret and public key (a few hundred bits), a small calculation complexity, for an overall communication cost of 19kb for authentication (for a \( 2^{16} \) security) and a signature of size of 93kb (11.5kB) (for security \( 2^{80} \)), an improvement of 40\% compared to previous schemes based on coding theory.

Keywords : Zero-knowledge protocols, coding theory, Stern SD scheme.

I. INTRODUCTION

The use of coding theory for public key cryptography was initiated by McEliece more than 30 years ago, although the system has often be considered as too costly and impractical because of the size of the public key, code-based cryptography has received much more attention in recent years. Besides the fact that code-based cryptography can possibly resist to a quantum computer, code-based systems have also inherent interests: they are very fast and are usually easy to implement compared to number theory based systems. Such features make code-based systems good candidates for low-cost cryptography.

There are two main types of code-based cryptosystems: systems with hidden structure like the McEliece cryptosystems (analogous to RSA) and systems with no hidden structure (analogous to discrete log-based cryptosystems) like for instance the Stern code-based authentication scheme \([\text{Ste93}]\). This second type of system is not vulnerable to structural attacks which are the main cause of attacks on McEliece-like cryptosystems. In practice as for the Stern scheme, they have not been attacked beneath the usual improvement on the attack of the underlying hard problem.

In the case of coding theory the underlying hard problem (the Syndrome decoding problem SD) is now well studied and considered as very secure.

Code-based Zero-knowledge authentication schemes are very interesting since their security is directly related to a hard problem, moreover they can be turned into signature schemes through the Fiat-Shamir paradigm. Meanwhile there are two strong drawbacks for these schemes. The first drawback is the size of the public key which can attain several hundred thousand bits and the second drawback is the size of the communication induced by the cheating probability, more than 150kb in practice for a \( 2^{80} \) security level.

The first drawback was resolved in part by Gaborit and Girault \([\text{GC07}]\) who proposed to use structured matrices like double-circulant matrices (matrices of the form \((IA)\) for \( A \) a random circulant matrix) to reduce the size of the public key to only a few hundred bits. The second drawback, the high cost of communications, largely remains. In this paper we make a step further to obtain a small communication cost, our new algorithm, with the same type of security than previous algorithm and small size of
II. BACKGROUND ON CODE-BASED AUTHENTICATION SCHEMES

A. Previous work

There are several protocols based on the syndrome decoding problem, which we quickly survey the main advances in this area. The first efficient protocol was proposed by Stern [Ste93]: his idea was a new way to prove the knowledge of a word with small weight and fixed syndrome. The idea consists of revealing one of the three statements, the adequate weight with a masked syndrome, the adequate syndrome with a wrong weight or a way the weight and the syndrome can be masked. The 3 challenges structure implies a cheating probability equal to 2/3 instead of 1/2 for the well-known scheme of Fiat-Shamir. The Stern protocol is also uncommon by the use of hash functions. In [Ste93], Stern presents another protocol which aims at reducing the cheating probability to 1/2 by cutting the challenge step into 2 parts. Indeed, adding this challenge in the scheme prevents the prover to reveal the third statement and reduce the probability close to 1/2. The next improvement was a reduction of communication due to Véron in [Ver90], the reduction is due to a different formulation of the secret, which decreases the cost of communication but increases the size of the key. In [GG07], Gaborit-Girault proposed to use particular compact matrices (doubly circulant matrices) in order to obtain a very short public matrix. The last improvement appeared with the protocol of Cayrel-Véron-El Yousfi where the aim was to reduce the cheating probability to 1/2 as well as in the second protocol of Stern but using fields with cardinality higher than 2. Our protocol uses the Véron variation that we recall here.

B. Scheme of Veron

private key : \((e, m)\) with \(e\) of weight \(w\) and of length \(n\) and \(m\) a random element of \(\mathbb{F}_2^n\).

public key : \((G, x, w)\) with \(G\) a random matrix of size \(k \times n\) and \(x = e + mG\).

1) [Commitment Step] \(P\) randomly chooses \(u \in \mathbb{F}_2^n\) and a permutation \(\sigma\) of \(\{1, 2, \ldots, n\}\). Then \(P\) sends to \(V\) the commitments \(c_1, c_2\) and \(c_3\) such that:

\[
\begin{align*}
    c_1 &= h(\sigma); \\
    c_2 &= h(\sigma((u + m)G)); \\
    c_3 &= h(\sigma(uG + x));
\end{align*}
\]

2) [Challenge Step] \(V\) sends \(b \in \{0, 1, 2\}\) to \(P\).

3) [Answer Step] Three possibilities:

- if \(b = 0\) : \(P\) reveals \((u + m)\) and \(\sigma\).
- if \(b = 1\) : \(P\) reveals \(\sigma((u + m)G)\) and \(\sigma(e)\).
- if \(b = 2\) : \(P\) reveals \(u\) and \(\sigma\).

4) [Verification Step] Three possibilities:

- if \(b = 0\) : \(V\) verifies that \(c_1, c_2\) have been honestly computed.
- if \(b = 1\) : \(V\) verifies that \(c_2, c_3\) have been honestly computed, and \(wt(\sigma(e)) = w\).
- if \(b = 2\) : \(V\) verifies that \(c_1, c_3\) have been honestly computed.

Fig. 1. Protocol of Veron

III. A NEW SCHEME

We now give more details and a high level overview on our two improvements.

A. High level overview: Increasing the number of challenges

At the difference of the Fiat-Shamir scheme in which the cheating probability is 1/2, this probability is 2/3 for the Stern protocol. It comes from the fact that proving that a prover knows a codeword of small weight with a given syndrome, means proving two facts: the fact that the syndrome of the secret is valid and the fact that the secret has indeed a small weight. This situation induces that if one adds a random commitment there are always two possibilities for cheating among the three cases, notably since the attacker knows the syndrome of the secret.

The small weight of the secret is proved by using a permutation and a bitwise XOR which permit to retrieve the syndrome thanks to the linearity of both operations. In all schemes based on syndrome decoding there is a statement of the form:

\[
\sigma(e) + v
\]
Here $e$ is the secret of low weight, $\sigma$ a permutation and $v$ a mask. In the Véron scheme $v$ is equal to $\sigma((u + m)G)$ which is a good mask for $\sigma(e)$ with $u$ a random word and $v$ is a random word in the Stern scheme. The idea described in the scheme of Stern 5 pass \cite{Ste93} and \cite{CVA10} is that a variation of $e$ can prevent a dependence on $v$ and $\sigma$. So there is no need to test the construction of $v$ and $\sigma$ at the same time any more. The cheating probability is now close to 1/2, indeed there is now only two challenges possible for the second query.

The variation on $e$ can be done in different ways, Stern used $e$ as a codeword of a Reed-Muller code, Cayrel et al. used a scalar multiplication, in our case we use a rotation of the two parts of $e$. Using this rotation we can deduce the syndrome of each permuted word thanks to the propriety of double circulant codes presented here Let $H = [I|A]$, for $A$ a circulant matrix of length $k$ and let the syndrome $s = H.y^t$ for $y = (y_1, y_2)$. For $r$ a cyclic shift on $n$ positions we obtain:

$$s = H \cdot (y_1, y_2)^t \Leftrightarrow r(s) = H \cdot (r(y_1), r(y_2))^t.$$

Our construction therefore leads to $2k$ possible challenges: $k$ coming from the choice of the shift and 2 possibilities for the second query (compared to 3 in the classical case) An attacker can easily cheats for $k$ challenges among the $2k$ possible, and we show that it is not possible for an attacker to cheat for more than $k + i$ challenges (for $i$ a security parameter) without knowing the secret.

This cyclic permutation point of view is an efficient way to reduce the cheating probability close to 1/2 in a binary scheme and without rising the communication cost like it was done in the scheme of Stern 5 pass or considering non binary alphabet like in Cayrel et al. which also leads to less interesting communications.

### B. High level overview: Commitments compression

In Stern’s scheme (or Véron’s scheme), the prover has first to send 3 commitments composed of 3 hash of different values: $c_1, c_2$ and $c_3$ in Véron’s protocol (for instance). The sending of these three hashes comes at a certain cost. Meanwhile one can remark that if the protocol works well, the Verifier retrieves 2 hash values among the 3 hash values sent. This remarks shows that in fact it possible to optimize the manipulation of these commitments. The Prover first needs to compute the three hash values as usual, but then rather than sending the three hash values, he sends a hash of the three hash values. After receiving the challenge of the Verifier the Prover knows that the Verifier is able to recover 2 of the 3 hash values, then he answers to the challenge as usual, but also adds to his answer the missing hash value.

In the verification step, if all worked correctly the Verifier is able to recover the first commitment (the hash of the concatenation of the three hash values $c_1, c_2$ and $c_3$) through the two hashed values he retrieved and the third one in the answer of the Verifier. Overall only 2 hash values are sent rather than 3.

This idea can be generalized to the case of sequenced rounds, in that case for each round the Prover sends only the missing hash value when the two others are recovered by the Verifier. In that case only a general commitment for all the rounds needs to be sent: a hash value of the sequence of all hash values of the different rounds. This point of view is very efficient in particular for signature for which the average number of hash values sent per round drops from 3 to 1.

Moreover this way of proceeding in secure in the random oracle model, since an error in the final hash value implies an error in one of the hash of the round sequence.

### C. Description of the protocol

We use the same notations and the same keys as in the scheme of Veron.

- **private key** : $(e, m)$ with $e$ of weight $w$ and of length $n$ and $m$ a random element of $\mathbb{F}_2^n$.
- **public key** : $(G, x, w)$ with $G$ a random matrix of size $k \times n$ and $x = e + mG$.

For simplicity matter we describe the protocol in figure 2 only for the first improvement since the second one is generic.

The verification protocol consists in a reconstruction of the hash value committed to the first step of the algorithm. In the first case, the first and the third hash values can be constructed and in the second case it concerns the second and the third hash values. The construction of hash value are obvious except $c_3$ in the $b = 0$ case using the two answers, the word $u$ and the permutation $\sigma$. We just have to see that $c_3 = \sigma(uG + x_r)$, with $x$ the public key shifted $r$ times.

### IV. Security

In this section we first prove the ZK security of our scheme by using the usual zero-knowledge arguments and we also consider practical security.
1) [First commitment Step] P randomly chooses \( u \in \mathbb{F}^k \) and a permutation \( \sigma \) of \( \{1, 2, \ldots, n\} \). Then \( P \) sends to \( V \) the commitments \( c_1 \) and \( c_2 \) such that:
\[
c_1 = h(\sigma); \quad c_2 = h(\sigma(uG));
\]
2) [First part of the challenge] \( V \) sends a value \( 0 \leq r \leq k - 1 \) (number of shifted positions) to \( P \).
3) [Final commitment Step] \( P \) build \( e_r = \text{Rot}_r(e) \) and sends the last part of the commitment:
\[
c_3 = h(\sigma(uG + e_r))
\]
4) [Challenge Step] \( V \) sends \( b \in \{0, 1\} \) to \( P \).
5) [Answer Step] Two possibilities:
   - if \( b = 0 \) : \( P \) reveals \( (u + m_r) \) and \( \sigma \).
   - if \( b = 1 \) : \( P \) reveals \( \sigma(uG) \) and \( \sigma(e_r) \) where \( e_r = \text{Rot}_r(e) \).
6) [Verification Step] Two possibilities:
   - if \( b = 0 \) : \( V \) verifies that \( c_1, c_3 \) have been honestly computed.
   - if \( b = 1 \) : \( V \) verifies that \( c_2, c_3 \) have been honestly computed, and that the weight of \( \sigma(e_r) \) is \( w \).

**Theorem IV.1** If a prover \( B \) is able to be accepted by a verifier with a probability upper than \( \frac{k+i}{2^k} \), \( B \) can retrieve the secret key of the protocol from the public one with a probability greater than,
\[
1 - \frac{2^{n-k-i}}{2^{n-k+i}(\binom{n}{w})^i},
\]
or find a collision for the hash function in polynomial time.

**Sketch of proof :**

Suppose a malicious prover \( M \) is able to answer \( k+i \) challenges. By the pigeonhole principle he is able to answer \( 2i \) challenges of the form \( \{(r_j, b), 1 \leq j \leq i \text{ and } b \in \{0, 1\}\} \). Rewriting the commitment \( c_3 \) in two different ways shows that he is able to construct an \( (i+1) \)-uplet \( (c, z_1, \ldots, z_i) \) solution of the following problem:
\[
s_{r_j} = c + H \cdot z_j^i
\]
with \( wt(z_j) = w \). Since we choose \( i \) such that the shifted secret key is the unique solution with a very strong probability, therefore a malicious prover who knows how to answer in \( k+i \) cases under \( 2k \) will be able to retrieve the secret key with a shift by block with a very strong probability (in practice the probability is chosen up to \( 1 - 2^{-80} \)).

**C. Zero-Knowledge**

This part of the proof consists in proving that no information can be deduce in polynomial time from an execution of the protocol more than the knowledge of the public data. The idea is to prove that anyone can build a simulator of the protocol in polynomial time such that the result of the simulator cannot be distinguished from a real execution.

The simulator is build by anticipation by the challenges, for each round it is possible to make a valid
instance by anticipation of the challenge $b$ only. This implies a construction in twice the number of rounds of the protocol.

The case $b = 0$ can be anticipated by the choice of $\sigma'$ a random permutation, $v$ a random word, $h_1 = \text{hash}(\sigma')$ and $h_3 = \text{hash}(\sigma'(vG + x_r))$. We notice that $(v, \sigma')$ and $(u + n_r, \sigma)$ are indistinguishable. The case $b = 1$ can be anticipated by the choice of $v$ and $z$ such as $z$ is a word of weight $w$, $v = \pi(uG)$ with $\pi$ a random permutation, $u$ a random word, $h_2 = \text{hash}(v)$ and $h_3 = \text{hash}(v + z)$. We notice that $(v, z)$ and $(\sigma(uG), \sigma(v + z))$ are indistinguishable.

The construction’s cost of the simulator is negligible and does not affect the security parameters. When we use the commitment compression improvement the proof is different because of the complexity cost of anticipation, in this case the construction’s cost of the simulator is not negligible and it is more interesting to produce this improvement several times instead of one to not affect the security too much.

D. Practical security of double circulant codes

At the difference of the original Stern’s scheme, our protocol is based on decoding a random double-circulant matrix (SD-DC problem say), this problem at the difference of the SD problem, is not proven NP-hard (although a result is known on the hardness of decoding general quasi-cyclic codes). Meanwhile in our case the problem appears to be hard since: 1) it has been proven in [GZ08] that random double circulant codes rely on the GV bound, 2) it is not known, even with very structured codes, how to decode a code up to the GV bound in polynomial time and at last, 3), in practice, there is no known specialized algorithm which can do significantly better (besides a small linear factor $n$) for solving the SD-DC problem. The situation is the same than for lattices and ideal lattices compared to random lattices. In practice the best known algorithm to attack the SD-DC problem are the same than those for the SD problem [FS09].

V. Parameters for authentication and signature

According to the security constraints for zero-knowledge discuss earlier we choose as parameters $n = 698$, $k = 349$, $i = 19$, $w = 70$ for a security in $2^{81}$ and a probability of cheating in $2^{-16}$.

For a security in $2^{100}$ we choose $n = 838$, $k = 419$, $i = 20$, $w = 86$ and for a security in $2^{128}$ we have $n = 1094$, $k = 547$, $i = 14$, $w = 109$.

- For signature, and a probability of cheating in $2^{80}$ it is sufficient to multiply by 5 the previous data. Overall our double-circulant scheme permits to obtain a signature of length 93kb.

Remark: it is possible to decrease even more the communication cost by using a constant weight encoding when sending $\sigma(v + z)$, the cost is then $k$ bits rather than $2k$ bits, overall it decreases the authentication to 17kb and the signature to 79kb, but the encoding comes with a complexity price.

VI. Conclusion

In this paper we propose a new variation on Stern’s authentication scheme. Our protocol permits to obtain a gain of more than 40% compared to previous schemes and it is the first code based zero knowledge scheme to obtain a signature length of less than 100kb with strong security and small size of keys.

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