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Coupling between criticality and gelation in “sticky” spheres: A structural analysis

David Richard,1 James Hallett,2 Thomas Speck,1 and C. Patrick Royall2,3,4
1Institut für Physik, Johannes Gutenberg-Universität Mainz, Staudingerweg 7-9, 55128 Mainz, Germany
2HH Wills Physics Laboratory, Tyndall Avenue, Bristol BS8 1TL, UK
3School of Chemistry, University of Bristol, Cantock’s Close, Bristol, UK
4Centre for Nanoscience and Quantum Information, Tyndall Avenue, Bristol, UK

We combine experiments and simulations to study the link between criticality and gelation in sticky spheres. We employ confocal microscopy to image colloid-polymer mixtures, and Monte Carlo simulations of the square-well (SW) potential as a reference model. To this end, we map our experimental samples onto the SW model. We find excellent structural agreement between experiments and simulations, both for locally favored structures at the single particle level and large-scale fluctuations at criticality. We follow in detail the rapid structural change of the critical fluid when approaching the gas-liquid binodal and highlight the role of critical density fluctuations for this structural crossover. Our results link the arrested spinodal decomposition to long-lived energetically favored structures, which grow even away from the binodal due to the critical scaling of the bulk correlation length and static susceptibility.

I. INTRODUCTION

Understanding how an amorphous system becomes dynamically arrested upon compression or cooling is a long-standing challenge in statistical physics. Such amorphous solids encompass states of matter such as glasses, films, plastics, and gels, among others. Despite the fact that these systems are of technological importance and have received a lot of attention in the literature [11, 2], the microscopic mechanism responsible for macroscopic arrest remains elusive. For instance, the question whether the glass transition can be explained in the context of a thermodynamic or structural phase transition is still debated [3–7]. Interestingly, gels, in contrast to glasses, allow a thermodynamic or structural phase transition to be studied [10–18]. For instance, the question whether the glass transition can be explained in the context of a thermodynamic or structural phase transition is still debated [3–7]. Interestingly, gels, in contrast to glasses, allow a thermodynamic or structural phase transition to be studied [10–18]. For instance, the question whether the glass transition can be explained in the context of a thermodynamic or structural phase transition is still debated [3–7]. Interestingly, gels, in contrast to glasses, allow a thermodynamic or structural phase transition to be studied [10–18].
in the sticky sphere limit, immediately upon quenching through the binodal, the density of the colloidal “liquid” is sufficient that the system undergoes dynamical arrest [54]. Under these circumstances, gelation may couple to critical fluctuations and this forms the subject of our study.

II. METHODS

A. Experiments

We employ confocal microscopy and particle tracking to resolve the positions of the colloidal particles. Colloid-polymer mixtures are composed of polystyrene (PS) polymer chains and sterically stabilized polymethyl methacrylate (PMMA) spheres with diameter \( \sigma_{SEM} = 2950 \text{ nm} \) and polydispersity \( \Delta = 5\% \), determined via scanning electronic microscopy (SEM). We use rhodamine as fluorescent label. Polystyrene has a molecular weight \( M_w = 1.3 \times 10^6 \) corresponding to an effective radius of gyration \( R_g = 35 \text{ nm} \) under \( \theta \) conditions. We use a solvent mixture of cis-decalin and cyclohexyl bromide which is density and refractive index-matched. We additionally screen electrostatic interactions using 4 mMol of tetrabutyl ammonium bromide salt. From previous work [17], we estimate \( R_g \approx 50 \text{ nm} \) at room temperature. This leads to a polymer-colloid size ratio \( q \approx 0.03 \) approaching the sticky sphere limit. Samples are first shear-melted. We then seal each sample into a borosilicate glass capillary with epoxy resin. We let samples equilibrate for 30 minutes before imaging. From previous work, this time is estimated to correspond to more than 200 Brownian times [34].

Samples are imaged by confocal microscopy using a Leica SP8. We image different parts of the suspension at least 15 \( \mu \text{m} \) from the wall. To extract the colloidal particle positions we employ three dimensional particle tracking using the difference of Gaussian method. This tracking is performed using the package “Colloid” developed by Leocmach et al. [19]. From the tracking we estimate the sample packing fraction as \( \phi \approx \pi N \sigma^3/(6V) \), with \( N \) the number of tracked particles and \( V \) the volume of the sample. It is well known that sterically stabilized PMMA colloidal particles can exhibit swelling and unscreened electrostatic interactions [50, 51] which can lead to an intrinsic softness modeled by an effective diameter \( \sigma > \sigma_{SEM} \). To estimate \( \sigma \) under our experimental conditions, we apply a method similar to the one used in Ref. [52]. In this study, the authors matched the pair correlation function \( g(r) \) of dense hard spheres with the known Percus-Yevick expression by varying the effective diameter \( \sigma \). In our work, we extend this mapping with matching the \( g(r) \) of the colloid-polymer mixture with an attractive square-well fluid, which we adopt throughout this study as a reference system for our mixtures. In addition, our mapping allows us to determine the systematic tracking errors of the colloidal positions, responsible for the broadness of \( g(r) \) at contact [40]. More details can be found in the appendix A. We find \( \sigma = 3100 \text{ nm} \), which corresponds to an effective diameter that is \( \sim 5\% \) larger than for the SEM estimation.

B. Computer simulations

We study the behavior of the square-well (SW) model serving as a reference model for our colloid-polymer mixtures. We perform standard Monte Carlo simulations in the NVT ensemble employing local moves. The system is composed of \( N = 5000 \) particles with diameters drawn from a Gaussian distribution with polydispersity \( \Delta = 5\% \). The interaction potential between two particles \( i \) and \( j \) is

\[
V(r) = \begin{cases} 
\infty & \text{if } r \leq \sigma_{ij} \\
-V_{SW} & \text{if } \sigma_{ij} < r < \sigma_{ij} + \delta \\
0 & \text{if } r \geq \sigma_{ij} + \delta,
\end{cases}
\]

where \( \sigma_{ij} = (\sigma_i + \sigma_j)/2 \), \( \sigma_i \) and \( \sigma_j \) being the diameter of particle \( i \) and \( j \) respectively. The attraction range \( \delta \) is set by the polymers and fixed to 0.03\( \sigma \) and independ of \( \sigma_{ij} \). If not mentioned otherwise, simulations are performed in a cubic box. Additionally, we use a slab geometry to compute the coexistence packing fraction between the gas and liquid phase with box lengths: \( L_x = L_y = L_z/2 \). Initial configurations are randomly drawn and any overlap between particles is removed using the algorithm by Clarke and Wiley [53]. We determine the binodal by fitting the density profile along the gas-liquid coexistence through

\[
\phi(z) = \frac{\phi_l - \phi_g}{2} + \frac{\phi_l - \phi_g}{2} \tanh \left( \frac{z - z_0}{w} \right).
\]

Here, \( \phi_g \) and \( \phi_l \) denote the gas and liquid coexistence packing fractions, and \( z_0 \) and \( w \) the interface position and width, respectively. Since for this polydispersity the liquid is still metastable with respect to the crystal, we check that all liquid slabs remained in the liquid phase. All distances and energies are expressed in units of \( \sigma \) and \( k_BT \), \( k_B \) being the Boltzmann constant. Throughout, we denote the dimensionless attraction strength by \( V = V_{SW}/k_BT \).

C. Mapping procedure

The attraction strength between colloidal particles is controlled by the polymer concentration \( c_p \). However, it is challenging to determine \( c_p \) precisely enough to map it directly. Instead, we map every sample individually through matching the experimentally measured total correlation function \( h_{exp}(r) = g_{exp}(r) - 1 \) and distribution \( P_{exp}(n) \) of bond number \( n \) to the SW fluid [29, 40], yielding an effective attraction strength \( V \) for every experimental sample.
Specifically, we compute $r b_{\text{sim}}(r)$ and $P_{\text{sim}}(n)$ on a grid in the $(\phi, V)$ plane around the critical point. We evaluate $P(n)$ by constructing a bond network using a Voronoi decomposition considering only direct Voronoi neighbors with the bond distance cutoff $r_c$ set to 1.5σ. Following the work of Largo et al., we estimate the location of the critical point to be at $\phi_c \approx 0.275$ and $V_c \approx 3.22$ for zero polydispersity [20]. We shall show later on that this value is very close to the critical point of our model with $\Delta = 5\%$. We use packing fractions ranging from $\phi = 0.05$ to 0.5 with an interval $\Delta \phi = 0.025$ and $V$ values ranging from 0 to 4 with $\Delta V = 0.2$ for $0 < V < 2$ and $\Delta V = 0.1$ for $2 < V < 4$. Overall, we end up with a grid of 608 state points. Beyond $V \approx 3.2$, the fluid crosses the binodal and starts to form a gel. Thus, structural observables such as $h(r)$ and $P(n)$ might evolve due to aging. We fix for every state point a MC relaxation time of $10^6$ steps before we compute any observables. Another $10^6$ steps is used to compute observables. We then pick for each sample the two numerical packing fractions $\phi_-$ and $\phi_+$, which encompass our sample density. We then compute for each $V$ value $h_{\text{sim}}$ and $P_{\text{sim}}$ as a linear combination of $\phi_-$ and $\phi_+$. We finally compute as a “goodness” parameter for our matching procedure the least squares

$$\chi_h^2 = \sum_i [r_i(h_{\text{exp}}(r_i) - h_{\text{sim}}(r_i))]^2$$  \hspace{1cm} (3)$$

and

$$\chi_P^2 = \sum_i [P_{\text{exp}}(n_i) - P_{\text{sim}}(n_i)]^2.$$  \hspace{1cm} (4)$$

In practice, $\chi_h^2$ is computed for $r < 4\sigma$, while an additional Gaussian ($\Delta = 5\%$) noise is added on numerical positions to mimic particle tracking errors [66], see appendix A. The global minima of $\chi_h^2$ and $\chi_P^2$ give us two independent evaluations of $V_{\exp}$. We then assign for each sample the mean of those two, $V_{\exp} = (V_h + V_P)/2$, whereas $|V_h - V_P|/2$ serves as an estimation for errors.

In Fig. 1(a) and (b), we present $\chi_h^2$ and $\chi_P^2$ as functions of $V$ for three samples picked along the critical isochore. From a hard sphere fluid (gray curves) to the gel (orange curves), we always observe a global minimum for $\chi^2$ and find a good agreement between $V_h$ and $V_P$. Results of the SW model are shown in Fig 1(c,d) and compared to experimental data. The correspondence between experiments and simulations is excellent, we observe only small discrepancies in the bond-number distribution for the gel sample for small bond numbers $n$, see Fig 1(d). Since the fluid is ergodic, the mapping to the SW fluid should be robust (as indeed it is). In contrast, the gel is non-ergodic and thus the correspondence would depend on the dynamics and history [70]. Hence, these deviations are not surprising since we do not include the effect of aging in our mapping method.

III. RESULTS

A. Phase diagram

We start by discussing the phase behavior of our system in Fig. 2. We choose to present our data in the $\phi-V$ plane. The gas-liquid binodal of our reference system is indicated by the black solid line. The location of the critical point is indicated by a black star and was determined via the block distribution functions method [55]. We shall come back to this point later. From our mapping procedure, we can place each sample at a given $\phi$ and $V$. We distinguish an ergodic fluid in contrast to a gel by respectively blue and orange square symbols. We also indicate samples without polymers, i.e. hard spheres, in gray. The typical gel sample shows a clear dynamical arrest with a relaxation time beyond 100τB (Supplementary Video) with τB being the Brownian time. This is consistent with a previous dynamical experimental study for a very similar system [33].

The main observation here is that arrested samples are lying close to the binodal for a wide range of densities (0.1 < $\phi$ < 0.4). That is to say, all samples which are gels always map onto state points of the reference system which are on or above the binodal. This means that the radial distribution function and bond distribution of a gel sample never correspond to an ergodic fluid. This is in agreement with previous numerical and experimental studies, which associate the gelation to the location of
we have shown that the gelation line is located close to the binodal and that dense domains of colloidal particles grow progressively as a function of the polymer concentration. We now demonstrate how this change of behavior can be directly linked to the fluctuations present in the context of criticality. To quantify the spatial evolution of the density we use the block distribution functions method [56], which provides the location of the critical point \( V_c \) [57] and to some extent the isothermal susceptibility \( \chi \) of the fluid [58, 59]. The procedure is as follows: We divide our system into a series of cubic subcells of dimension \( l = L/b \), with \( b \) being an integer. The global density is defined as \( \bar{\rho} = \frac{1}{n_s} \sum_i \rho_i \), where \( n_s = b^3 \) is the number of subcells and \( \rho_i \) is the local density in subcell \( i \). We then extract the second and fourth moment of the density \( \langle m_2 \rangle \) and \( \langle m_4 \rangle \) computed as \( \langle m_2 \rangle = \frac{1}{n_s} \sum_i (\rho_i - \bar{\rho})^2 \). We can finally define the Binder cumulant through

\[
U_1 = \frac{\langle m_4 \rangle}{\langle m_2 \rangle^2}.
\]

One of the main properties of \( U_1 \) is its size invariance with respect to \( l \) at the critical point [56]. This allows us to determine unambiguously and accurately the critical attraction strength \( V_c \) in computer simulations. In more detail, we compute \( U_1 \) for various attraction strengths \( V \) and subcell lengths \( l \). To this end, we sample \( U_1 \) for each \( V \) values at a fixed packing fraction \( \phi \approx 0.275 \) using an extra \( 10^8 \) MC steps. In Fig. 3 we show the evolution of \( U_1 \) as a function of \( V \). We observe a crossing for \( V > 3 \), which can be resolved more accurately as shown in the inset of Fig. 3(a). We find \( V_c = 3.19(3) \) as the final value for the critical point, which is close to the value determined by Largo et al. [20] for zero polydispersity. Additionally, in Fig. 3 we show experimental results for \( U_1 \) at a fixed subcell length, \( l = 2r \). Although the experimental data suffer from a lack of statistic in comparison with simulation data, we observe an overall good agreement. More precisely, gel samples indicated by orange squares are, within the errors, located either at the cumulant crossing or at larger attraction strengths.

FIG. 2: Phase diagram. (a) Phase diagram of the square-well with \( \delta = 3\% \) and polydispersity \( \Delta = 5\% \). Black points indicate the gas-liquid coexistence and the black star indicates the critical point. The black solid line is a guide to the eye for the two phase boundary. Gray, blue, and orange squares are, respectively, hard-sphere fluid, colloid-polymer mixtures, and arrested gel samples. Confocal images are displayed in a, b, c and d: (b) Hard sphere fluid without polymers (\( V \approx 0 \)); (c) Equilibrium fluid (\( V \approx 2.0 \)); (d) Fluid close to criticality (\( V \approx 2.9 \)) and (e) Gel phase (\( V \approx 3.2 \)). The scale bars correspond to 20 \( \mu m \).
towards the coexistence densities the two maxima of the distribution move progressively
U
Critical point.

FIG. 3: Critical point. Evolution of the Binder cumulant \( U_l \) as a function of the attraction strength \( V \) for three values of \( l \). Solid lines and open symbols indicate simulation and experimental data, respectively. The hard sphere, ergodic fluid, and arrested gel samples are distinguished by gray squares, blue circles and orange triangles respectively. Inset: zoom into the crossing region for simulation data.

The change of \( U_l \) is rationalized through a density distribution \( P_\rho(\rho) \) that moves from a unimodal Gaussian shape, centered at \( \rho_c = \bar{\rho} \), to a bimodal shape, where the two maxima of the distribution move progressively towards the coexistence densities \( \rho_- \) and \( \rho_+ \). In Fig. 4 we show for both experiments and simulations the density distributions for subcells of length \( l \approx 3 \sigma \) (without reaching a truly bimodal distribution, presumably due to the vicinity to the critical point). To be consistent with the phase diagram in Fig. 2, we choose to plot \( P_\rho(\rho) \) instead of \( P(\rho) \). We start with a hard-sphere sample [Fig. 4(a)] for which we observe a narrow distribution lacking low \( (\phi < 0.1) \) and high density \( (\phi > 0.5) \) regions. The same behavior continues up to \( V = 2 \) [Fig. 4(b)]. When further increasing \( V \), we start to observe broader distributions with almost empty \( (\phi < 0.1) \) and colloidal-rich regions \( (\phi > 0.5) \). This can be clearly seen for our last ergodic sample at \( V = 2.9 \) [Fig. 4(b)]. Interestingly, dense regions can reach packing fractions larger than the freezing point of a hard-sphere fluid. This explains why dense regions arising from criticality will lower the nucleation barrier \[17, 19, 60–62\]. Continuing to quench, we observe a divergence of \( P(\rho) \) for small \( q \) through diffusion light. Since the scattering intensity \( I(q) \) is proportional to \( P(q) S(q) \), where \( P(q) \) is the form factor and \( S(q) \) the structure factor. We expect for \( q > 2 \pi / \sigma \) the scaling \( I(q) \sim q^{-4} \), which is confirmed by the dashed line in Fig. 5(a). For small \( q \), the scaling depends on the shape of the diffusing clusters through \( d_f \), its fractal dimension. Previous small-angle neutron scattering (SANS) experiments have shown that the fractal dimension at the gelation is typically around \( d_f \approx 1.7 \) \[31\], which is supported by particle-resolved experiments \[63, 64\] and simulation \[9\].

We will discuss this point in detail later on. We can additionally extract \( S(q) \) for small wave vector from

FIG. 4: Density fluctuations. Packing fraction distribution \( P(\phi) \) for various samples and matched simulations along the critical path for subcell length \( l \approx 3 \sigma \). Colored histograms and empty black circles are respectively experimental and simulation distributions. The effective attraction depth \( V \) for each sample is indicated in figures.

C. Fractal dimension, bulk correlation length, and bond distribution

We now turn to discuss the characteristic cluster shape and length scale formed by colloidal particles when quenched through criticality. We first compute the scattering intensity \( I(q) \) as a function of the wave vector \( q \) from the Fourier transform of the confocal pixel map. In Fig. 5(a) we show a typical colloidal gel close to criticality and its associated Fourier spectrum in the inset. A radial average leads to \( I(q) \) as shown in Fig. 5(b) for different samples along the critical isochore. When quenching, we observe a divergence of \( I(q) \) for small \( q \) as a consequence of larger domains forming and diffusing light. The scattering intensity \( I(q) \) is proportional to \( P(q) S(q) \), where \( P(q) \) is the form factor and \( S(q) \) the structure factor. We expect for \( q > 2 \pi / \sigma \) the scaling \( I(q) \sim q^{-4} \), which is confirmed by the dashed line in Fig. 5(a). For small \( q \), the scaling depends on the shape of the diffusing clusters through \( d_f \), its fractal dimension. Previous small-angle neutron scattering (SANS) experiments have shown that the fractal dimension at the gelation is typically around \( d_f \approx 1.7 \) \[31\], which is supported by particle-resolved experiments \[63, 64\] and simulation \[9\].

We will discuss this point in detail later on. We can additionally extract \( S(q) \) for small wave vector from
FIG. 5: Evolution of the fractal dimension, bulk correlation length, and bond distribution. (a) Confocal image of a gel close to criticality. The inset shows the associated scattering pattern. (b) Scattering intensity $I(q)$ as a function of the wave vector $q$ for various samples along the critical isochore $\phi \simeq \phi_c$. Inset: Fit of the form factor to extract $S(q)$. (c) Structure factor $S(q)$ as a function of the wave vector $q$ for the same samples. Solid lines are from simulation data at $\phi \simeq \phi_c$. (d) Evolution of the fractal dimension $d_f$ as a function of the attraction strength $V$. The horizontal dashed line indicates the fractal dimension reported for SANS experiments [31]. (e) Evolution of the bulk correlation length $\xi$ as a function of the reduced attraction strength $\epsilon$. Circles and square are experimental and simulation data, respectively. The black solid line indicates the 3D Ising scaling with $\xi \sim \epsilon^{-0.63}$. (f) Variance $\langle n^2 \rangle$ of the bond distribution as a function of the attraction strength $V$. The solid black lines indicate the simulation data for the critical isochore. Inset: ergodic fluids before gelation. All panels: arrows indicate the sample in which gelation occurred first and colors indicate attraction strength $V$.

$S(q) = \frac{I(q)}{(AP(q))}$, where we evaluate $AP(q)$ by fitting $I(q)$ for $q > 2\pi/\sigma$ by a polydisperse form factor and a prefactor $A$, as shown in the inset of Fig. 5(b). The resulting procedure is shown in Fig (c) and directly compared to simulation data. We find an overall very good agreement between experimental data and simulations. The structure factor can then be used to extract an estimation of the bulk correlation length $\xi$ through the Orstein-Zernike scaling,

$$S(q) = \frac{S_0}{1 + (\xi q)^2},$$

which holds close to criticality for small $q$. In Fig. 5(d), we plot the evolution of the fractal dimension $d_f$ as a function of the attraction strength $V$. We find when increasing $V$ a progressive increase of $d_f$ and saturates for gel samples at $d_f = 2.4 - 2.5$. These values agree with fractal dimensions found for gel at low volume fractions and large attraction strength [9]. At criticality, where gelation occurs, we find $d_f = 1.6 - 1.7$, which is consistent with a previous SANS experimental study [31].

As a measure for the distance to the critical point, we define the reduced attraction strength $\epsilon = (V_c - V)/V$. In Fig. 5(e), we present the behavior of the bulk correlation length $\xi$ as a function of $\epsilon$. From the simulations, we find that $\xi$ strongly increases as $\epsilon \to 0$ and that this increase is well modelled by the Ising universality class with $\xi \sim \epsilon^{-\nu}$ up to $\epsilon \simeq 0.2$, where in three dimensions $\nu \simeq 0.63$. In contrast, for the experimental data the equilibrium fluids follow closely the simulation data until gelation occurs close to the binodal, whereby the system is arrested and therefore the correlation length does not increase beyond $\xi \approx 2\sigma$. This rather small spatial correlation is not in contradiction with confocal images of gels, where structures are quite ramified in a network without large colloidal domains, cf. Fig. 5(c) and Fig. 5(a). This behavior would indicates that gels are, in a way, pictures of early critical fluids, where the cost of breaking bonds
does prevent relaxation and thus the growth of the correlation length $\xi$.

Finally, in Fig. 5(f) we discuss the behavior of the bond distribution through its variance $\langle n^2 \rangle$. It was shown recently for a similar colloid-polymer mixture model based on the Asakura-Oosawa potential that the variance of the bond distribution as a function of $V$ is peaked close to gelation. We observe the same behavior in our simulations crossing the binodal, where $\langle n^2 \rangle$ exhibits a maximum for $V \approx 3.5$. Quenching further the variance decreases, indicating a progressive aging of the network structure. The behavior of the variance will of course depend on time and other factors such as hydrodynamic interactions. In experiments, we also observe a growth of $\langle n^2 \rangle$ for equilibrium fluids ($V < 3$). We find larger variances for gel samples, but not as high as in simulations, which also confirms the picture found for $\xi$ in Fig. 5(e), where the gel structure corresponds to an arrested critical fluid. We return to the behaviour in the gel state in Sec. IV.C.

### IV. DISCUSSION

A. Entropy favors low-symmetry clusters

For isolated clusters it has been shown that the relative population of clusters with the same number of bonds is determined by entropy. Interestingly, this still holds for the fluid and even the gel as shown in Fig. 5(b) for $m = 6$. We find that $6Z$ clusters, which are polytetrahedra, are always significantly more prevalent in the system compared to $6A$ clusters, which are octahedrons (the population goes up from $\approx 5\%$ for isolated $6A$ to $\approx 10\%$ in the fluid). The same trend can be observed for other clusters, with the more symmetric clusters being less populated. This demonstrates that the minimum (free) energy clusters are determined by rotational and vibrational entropies. It has also been shown that the potential part of the free energy will promote both octahedral and tetrahedral order. The same observation was found in a more recent numerical study of the SW fluid, where gelation was associated with polytetrahedral order. Another important observation is that we also find a large population of clusters with fivefold symmetry ($8B$, $9B$, and $10B$) at any $V$. These structures are known to be local energy minima of the Morse potential. Hence, even though a gel is in an energy landscape far away from the equilibrium state, it can still locally minimize its free energy by forming isolated locally favored structures, which eventually will overlap and form the gel network. For larger clusters with 11 and 12 particles, we observe several Morse minima that correspond to $10 < \rho_0 < 25$ ($\rho_0$ is the Morse potential parameter controlling the range of attraction, in our case $\delta$). The range for $\rho_0$ found here is consistent with short range attraction. We found that $11E$ and $12D$ are the dominant structures for $m = 11$ and $m = 12$ respectively, which corresponds to $\rho_0 \approx 17$.

B. Hard sphere fluid

Having obtained a set of minimum energy clusters, we use these to follow the overall structural change in the
FIG. 6: Identification of locally favored structures. (a) Populations of local clusters for three different samples along the critical density, $\phi_c \simeq 0.275$. From top to bottom: a hard-spheres sample, a critical fluid, and an arrested gel. (b) Experimental probability to observe octahedron and poly-tetrahedron in clusters composed of 6 particles. Colors distinguish the three different samples along the critical isochore. (c) Snapshots of selected locally favored structures.

fluid. As a reference, we first consider the compression of the hard sphere fluid ($V = 0$) towards the glass shown in Fig. 7(a). At low density, $\phi \simeq 0.05$, there is an absence of clusters as one can expect since particles are mostly isolated without any neighbors. At this low density, the system exhibits as a larger cluster only a few percents of $4A$ (tetrahedron). The typical spatial arrangement of clusters is illustrated with an experimental snapshot in Fig. 8(a1). We then observe a rapid change from $\phi = 0.05$ to 0.25, where medium sized clusters such as $5A$, $6Z$, $7C$, and $8B$ start to appear, cf. Fig. 8(a1-a3). This continues until a crossover at $\phi > 0.25$, where isolated and small clusters ($3A$ and $4A$) are converted with a combination of medium clusters ($5A$, $6Z$, $7C$, and $8B$) to larger structures: $9B$, $10B$, $11E$, and $12D$, see Fig. 8(a4). This kind of conversion will continue at higher packing fractions with the extinction of $5A$, $6Z$, and $7C$ to promote clusters sharing the same sub-structures. We also notice that the agreement between experiments and simulation is excellent. This gives another confirmation for the robustness of the mapping procedure employed.

C. En route to the gel

In Fig. 7(b), we follow the same idea but now approaching the gel. We fix the density to the critical packing fraction $\phi_c = 0.275$ and progressively increase the attraction strength $V$ towards the binodal. We find a plateau with little change in structure until $V \simeq 2$. We show the spatial arrangement of clusters at $V = 1.1$ in Fig. 8(b1), where we only display clusters with $m > 4$. For higher attraction strengths, $V > 2$, we observe a structural crossover (cf. the compression of hard spheres). Clusters with $m > 4$ increase quickly until reaching the binodal. Interestingly, this change starts where we have located the beginning of the critical scaling for the bulk correlation length $\xi$ and the static susceptibility $\chi$. The main contribution to this structural evolution comes from the creation of a large amount of $6Z$, $7C$, and $8B$ clusters, see Fig. 8(b1-b3). These clusters are not necessarily localized in space, but we do observe denser regions with even the presence of $9B$ at $V = 2.9$, see Fig. 8(b3). This is consistent with our previous finding in Fig. 4(c), where the density distribution exhibits a broader tail toward large packing fractions $\phi > 0.4$. Therefore, we can expect a link between the local structures of dense hard spheres and the colloidal-rich regions arising from critical density fluctuations. Finally at $V > V_c$, which corresponds to our first identification of a gel, clusters percolate the whole system, see Fig. 8(b4). The inner parts of the percolating structure are rich in large clusters such as $8B$, $9B$, and $10B$, which are defective icosahedra sharing a fivefold symmetry. They are known to play an important role in the slowing down of the dynamics for dense hard-spheres approaching the glass transition [18, 68, 69]. This may help to explain the origin of the rigidity of the network leading to gelation [29, 34].

The crucial difference between a gel and a dense hard sphere liquid is that not only the defective icosahedra
FIG. 7: Evolution of LFS through compression and cooling. (a) Evolution of the fraction of locally favored structures as a function of the packing fraction $\phi$ for hard spheres ($V = 0$). (b) Evolution of the fraction of locally favored structures as a function of the reduced attraction strength $\epsilon$. The yellow area indicates the two-phase region where gelation occurs. Solid lines and open squares indicate results for simulations and experiments, respectively.

(8B, 9B, and 10B) are very stable due to entropy, but each bond breaking will also result in an energetic penalty of more than $3k_B T$. Therefore only colloids in the outer part of dense regions will be able to break bonds and diffuse (see Supplementary Video). To conclude, we find that the structural crossover starts $1k_B T$ before the location of the binodal when increasing $V$. This is a direct consequence of criticality inducing larger density fluctuations. For a monodisperse sample, these fluctuations might promote crystal precursors and lower the nucleation time [19]. For a polydisperse sample, however, they induce low-symmetry polytetrahedral backbones for the bond network. These clusters are incompatible with respect to the crystal symmetry. Hence, the system will fall into an amorphous solid state, the gel.

V. CONCLUSIONS

We have investigated the role of criticality in the gelation of sticky spheres. Combining experiment and simulations, we provide further evidence that the dynamical arrest is initiated by the onset of critical fluctuations in agreement with previous work [8, 16, 29, 34]. We have demonstrated that carefully mapping two-point structure to the square-well fluid faithfully reproduces the experimental data including higher-order local structures as identified by the topological cluster classification method [55]. In particular, we find that gel samples are (i) located at the cumulant crossing, (ii) identified by a broad distribution of densities, and (iii) have correlation lengths of $\xi \approx 2\sigma$. We find a sharp but continuous increase of locally favored structures when increasing the attraction strength. This increase occurs in concert with the increase of both the bulk correlation length and the static susceptibility, which can be extracted together with the structure factor of the fluid. More precisely, the start of the critical scaling of these two quantities coincides with the appearance of larger locally favored structures. The picture of a gel is thus that of an early critical fluid, which is arrested due to the large cost of breaking bonds. Before arrest, clusters of several particles appear, which have a low symmetry favored by entropy. The den-

FIG. 8: Experimental snapshots of the cluster evolution. Evolution of the clusters when increasing the packing fraction $\phi$ (left) and attraction strength $V$ (right). Only structures with $m > 4$ are shown for the cooling path.
sification of these clusters driven by the incipient critical fluctuations then leads to the gel.

Conflicts of interest

There are no conflicts of interest to declare.

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Appendix A: Effective colloid diameter

There are two challenges for the determination of the effective packing fraction and the effective temperature of a colloidal sample from real space imaging. First, it is difficult to know the change of the colloidal diameter in solution due to swelling and unscreened electrostatic. This leads to a poor estimation of the sample packing fraction and, e.g., a mismatch of structural oscillations in pair correlations. Second, errors due to imaging and tracking will lead to an error on the true position of a particle, resulting in a broadening of the peaks of a pair correlation. In the context of matching a sample’s g(r) onto simulations, these errors will induce a systematic overestimation of the temperature of a mixture at a given polymer concentration c_p. To handle these issues, we pick a sample close to criticality and compute its pair correlation r exp [g(r) − 1] for various effective diameters. To mimic tracking errors, we pick a simulation state point at criticality and apply different Gaussian noises of variance ∆ error to the particle positions. We then find the optimal set of parameters that minimize χ = \sum_i [r_i(h_{exp}(r_i) - h_{sim}(r_i))]^2. The evolution of χ^2 as a function of the parameters is shown in Fig. 9(a). We find a unique minimum for χ^2 leading to an effective diameter σ = 3100 nm (5% larger than for dry colloids) and a tracking error ∆ error = 5%, which is consistent with previous works. The resulting matched pair correlation function is shown in Fig. 9(b). We observe a very good agreement between experiment and simulation which validate our approach.

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FIG. 9: Effective diameter and tracking errors. (a) Colormap of ln(χ^2) against the effective diameter σ and the tracking error ∆error. (b) Resulting matching of the pair correlation between an experimental sample and simulation at criticality (φ = 0.275, V = 3.2)
