Rebuttal to Bell non-locality

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Abstract

Bell’s theorem is habitually used as an advertisement for non-local formulations of quantum mechanics and the statement that local realist interpretations are untenable. We identify four categories of interpretations that are permissible by Bell’s theorem. Many local and deterministic descriptions remain seemingly ignored. For three of those categories, we present an example of an interpretation where a local flow of quantum information is possible. We assess whether current experimental proposals and an improved philosophy of science can contrast interpretations and distinguish between them.

Keywords: Bell’s theorem, superdeterminism, Bohmian mechanics, Everettian mechanics, locality.

1 Introduction

John Bell published his celebrated inequality in 1964. Its implications remain eagerly discussed to this day [1–3]. The derivation of the inequality is simple, but its meaning has a history of misunderstandings [4]. Perhaps for the first time after the development of quantum mechanics, and the proliferation of the first interpretations, one was able to rule out a large category of quantum interpretations, namely: local hidden variable theories admitting statistical independence. Bell showed that the field of foundations of quantum mechanics is not to be restricted to philosophy only, but that physics has its part in this endeavour [2, 3, 5]. Based on Bell’s result, one then hopes to develop refined no-go theorems and contrast Bell-compatible categories of interpretations with constraints imposed by both quantum field theory and general relativity to narrow down the menu of interpretations.
Recently, a new wave of no-go theorems involving thought experiments [3, 5–7], developments in the theory of decoherence and quantum computing [8], and prizes awarded to the foundations of quantum mechanics\(^1\) reignited the interest in interpretational issues. Even orthodox philosophical stances are undergoing revisions [3]. However, half a century after Bell’s paper [1], his results are advertised in favour of non-local formulations of quantum mechanics [4, 9, 10], as if there were no other alternatives, even if they predict possible experimental tests [11, 12]. For a recent rebuttal to Bell non-locality, see Ref. [10].

Motivated by the apparent unjustified conclusion that Bell’s theorem necessarily leads to non-locality, we analyse the categories of interpretations that are Bell-compatible: superdeterministic hidden variables, non-local hidden variables, collapse-type theories, and perhaps the largest category: unitary quantum mechanics. We find that even in explicitly non-local hidden variable theories such as Bohmian mechanics, a local flow of quantum information is possible. We discuss three examples of theories in categories that admit such a local information flow, namely: the superdeterministic \(\psi\)-ensemble interpretation, Bohmian mechanics, and Everettian mechanics. We then argue that it seems unfitting to use Bell’s theorem to discard local and deterministic interpretations.

In Sec. 2 we re-examine Bell’s theorem and establish four Bell-compatible categories of quantum interpretations. In Sec. 3 we present one example interpretation from each of the three categories admitting a local information flow. In Sec. 4, we contrast the categories and, based on recent developments, evaluate the prospect of distinguishing categories of interpretations defending deterministic and local options. We conclude by using Deutsch’s philosophy of science [13] to contrast superdeterministic and Everettian type theories.

Given that different interpretations require distinct conditions for testability and philosophical distinction, some interpretations might be more adequately called interpretations, and others theories. As the subject likely lies at the border of physics and philosophy, for this paper, we use these two terms interchangeably.

## 2 Four ways to violate Bell’s inequality

### 2.1 Bell’s inequality

We briefly recollect Bell’s original result [1]. A pair of identical particles evolve into the entangled spin-singlet state \(|\psi\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}\) through local interactions. One particle moves to lab \(A\), and the other to lab \(B\). Labs \(A\) and \(B\) occupy space-like separated regions; see Fig. 2. The unit vector \(a\) sets up the reference frame (or the detector setting) of lab \(A\) to measure the spin observable \(\sigma\) of the particle. Similarly, \(b\) sets up \(B\). Therefore, the setups at \(A\) and \(B\) measure the observables \(\sigma \cdot a\) and \(\sigma \cdot b\), respectively. Bell calls the

\(^1\)The Nobel Prize in Physics 2022 and The 2023 Breakthrough Prize in Fundamental Physics.
respective eigenvalues $A(a) = \pm 1$ and $B(a) = \pm 1$. The quantum mechanical expectation value for the entangled state $|\psi\rangle$ of the product of the observables at different labs is

$$P(a, b) = \langle \psi | (\sigma \cdot a)(\sigma \cdot b) | \psi \rangle = -a \cdot b, \quad (1)$$

which shows that the expectation value depends on the detector settings. So far, there is no mention of hidden variables. A virtue of Eq. (1) is that $P(a, b)$ does not depend on whether one is working in the Schrödinger or Heisenberg picture.

Now, assume a set of local hidden variables $\lambda$, which compile all the necessary information to supposedly complete quantum mechanics. The variables now affect the eigenvalues $A(a, \lambda)$ and $B(b, \lambda)$. The writing of $A(a, \lambda)$ and $B(b, \lambda)$ contains an implicit assumption: the eigenvalue at lab $A$ does not depend on $b$. Similarly, $B(b, \lambda)$ does not depend on $a$. Superdeterministic interpretations relax this assumption \[14–17\]. But, if the assumption holds, the hidden variable dependent expectation value can be calculated using

$$P_H(a, b) = \int d\lambda \rho(\lambda) A(a, \lambda) B(b, \lambda), \quad (2)$$

where $\rho(\lambda)$ is a normalized hidden variable distribution. One then expects $P_H(a, b) = P(a, b)$. But within the assumptions, this is not possible. Using simple manipulations \[1\], Bell showed that $P_H(a, b)$ must satisfy the inequality

$$1 + P_H(b, c) \geq |P_H(a, b) - P_H(a, c)|, \quad (3)$$

which today bears his name. By using the quantum mechanical expectation value $P(a, b)$ instead of the hidden variable expectation value $P_H(a, b)$ in Eq. (3), it is easy to produce special cases where $P(a, b)$—usually, simply referred to as quantum mechanics—violates the inequality. Therefore, Bell’s theorem states that statistical predictions of quantum mechanics are incompatible with local hidden variable formulations that admit their statistical independence.

### 2.2 Categories of interpretations

Bell’s original results summarized in Sec. 2.1 exclude the possibility of a local hidden variable theory admitting statistical independence. We identify four remaining Bell-compatible categories:

1. Superdeterministic hidden variable interpretations (blue);
2. Non-local hidden variable interpretations (yellow);
3. Local non-hidden variable interpretations consistent with Eq. (1) (green);
4. Non-local non-hidden variable interpretations consistent with Eq. (1) (red).

Based on surveys on interpretations of quantum mechanics \[5, 18, 19\], in Fig. 1 we distribute a sample of interpretations according to the four categories. We scanned the literature for recent developments related to Bell’s
inequality with interpretive claims, either explicit or implicit; see Ref. [20] and Refs. therein. We noticed the following trends: the possibility of non-local hidden variable theories (yellow) is usually recognized and either embraced [21], or rejected based on arguments beyond Bell’s theorem; and the possibility of superdeterminism (blue) is usually ignored or rejected on philosophical grounds [4, 9]. Yet, a resurgence in superdeterminism is undergoing [11, 15–17, 22]. Rejecting hidden variables completely, current literature typically falls back to non-local non-hidden variable interpretations (red), declaring that quantum mechanics must be intrinsically non-local [23, 24]. However, while Copenhagen and collapse-type interpretations are popular ways of violating Bell’s inequality, Fig. 1 shows a large category of local non-hidden variable interpretations (green) that remain ignored. Simply rejecting hidden variables does not imply the necessity of a non-local interpretation.

In this paper, we adopt the approach that interpretations should not be rejected on grounds of psychology, but by taking each one of them seriously, strive to eventually produce further no-go refinements and experimental tests that allow one to distinguish between categories of interpretations. In the following sections, we take one interpretation from each of the categories that admit a local information flow [25] (blue, yellow, and green), are universal and deterministic. Then, we assess whether these interpretations are expected to be distinguishable only in the philosophical domain.
3 Examples with local information flow

3.1 The $\psi$-ensemble interpretation

In superdeterministic theories, the eigenvalue $A(a, b, \lambda)$ depends on the detector setting $b$, and similarly for $B(a, b, \lambda)$. This implies that statistical independence lacks because Eq. (2) would have to be adapted to a generalized hidden variable distribution $\rho(a, b, \lambda) \neq \rho(\lambda)$ [14–17]. Therefore, the derivations leading to Bell’s inequality (3) are inapplicable. A superdeterministic expectation value is exempt from violating Bell’s inequality. Technology permitting, expectation values that satisfy Eq. (3) could experimentally favour superdeterminism. Contrary to the name’s suggestion, superdeterminism is no more deterministic than say classical mechanics, but because $\rho(a, b, \lambda)$ depends on the detector’s settings, superdeterministic theories possess more correlations than theories enjoying statistical independence.

Let us adopt the hidden variables collectively described by $\kappa(t)$ as presented in the $\psi$-ensemble interpretation [17]. We denote the hidden variables by $\kappa(t)$ (just as in Ref. [17]), because they are slightly less general than Bell’s hidden variables $\lambda(t)$. The difference does not concern us here. The $\kappa(t)$ determine the outcomes at the time of measurement. For the pair of particles in the Bell test, the two possible outcomes are specified in the state $|\psi\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$. Let us denote the cluster of hidden variables leading to the outcome $|\uparrow\downarrow\rangle$ as $\{\kappa\}_{\uparrow\downarrow}$, and $\{\kappa\}_{\downarrow\uparrow}$ to $|\downarrow\uparrow\rangle$. The hidden variables deterministically map to a definite measurement outcome at the time of measurement. This is how superdeterminism solves the problem of outcomes [8]. Observing the state $|\uparrow\downarrow\rangle$ reveals information (through the mapping) about the value of the hidden variable at the time of measurement. These hidden variables are causally local because the information about the settings ($a, b$) was already available at the time the entangled state was prepared through local interactions.

Because the $\psi$-ensemble interpretation has more elements than non-hidden variable quantum mechanics, predictions other than standard quantum mechanics are theoretically possible. In standard quantum mechanics, an ensemble of identically prepared states will generally admit different outcomes. In the $\psi$-ensemble interpretation, they are not identical, because they had different hidden variable values that originated different outcomes. One could then think of situations between consecutive measurements where standard quantum mechanics predicts different results between measurements, but where in the $\psi$-ensemble interpretation, the hidden variable would not have time to change its cluster. Therefore, superdeterminism could be testable in situations where the hidden variables do not change between measurements [11, 26].
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Fig. 2 Meiosis of the measurer C. Entanglement through local interactions sets up the relative states $\phi_\uparrow(x_1, t)\phi_\downarrow(x_2, t)$ and $\phi_\downarrow(x_1, t)\phi_\uparrow(x_2, t)$. The particles illustrated by blue disks move to space-like separated regions indicated by the shaded triangles. A detector C verifies the $B$ particle’s spin eigenvalue by interacting with it in the space-like intersecting region. From $t$ to $t'$, the only new foliation occurs in the measurer within the space-like intersecting region. The dashed line indicates the unfoliated detector $C$ at time $t$, and the two thinner solid lines show the two new foliations joining the already foliated entangled state. Nothing happens to $A$ while $B$ and $C$ interact. The new spatially extended preferred foliation $\phi_\uparrow(x_1, t')\phi_\downarrow(x_2, t')\chi_\downarrow(x_3, t')$ is shown as the thick grey line.

3.2 Bohmian mechanics

If one seeks a hidden variable theory without the correlations imposed by statistical dependence, then Bell’s theorem constrains the theory to have elements with action between space-like separated regions. The most prominent example is Bohmian mechanics [21, 27, 28].

The ontology of Bohmian mechanics is straightforward: there is a pilot-wave $\Psi$ that evolves according to the Schrödinger equation in coordinate space,

$$i\hbar \frac{\partial}{\partial t} \Psi(r_1, r_2, \ldots, r_N; t) = \hat{H} \Psi(r_1, r_2, \ldots, r_N; t).$$

(4)

The field $\Psi$ pilots $N$ particles, which are also part of the ontology. Each particle $i$ moves according to its guiding equation,

$$m_i \frac{d\mathbf{R}_i}{dt} = \hbar \text{Im} \nabla_i \ln \Psi.$$  

(5)

The solution of Eq. (5) determines each particle’s Bohmian trajectory $\mathbf{R}_i(t)$. Finding the set of all trajectories $\{\mathbf{R}_i(t)\}$ is a formidable task; yet unnecessary for practical purposes. Similarly to classical statistical mechanics, the collective properties can be obtained from typicality arguments, even though the microstate $\{\mathbf{R}_i(t), m_i \frac{d\mathbf{R}_i}{dt}\}$ remains unknown [21].

Let us now describe the entangled spin-singlet state $|\psi\rangle$ in the context of Bohmian mechanics. The pilot-wave evolves autonomously according to Eq. (4). For the two-particle field, we write

$$\Psi(x_1, x_2; t) = \langle x_1, x_2 | \psi(t) \rangle = \frac{1}{\sqrt{2}} \left[ \phi_\uparrow(x_1, t)\phi_\downarrow(x_2, t) - \phi_\downarrow(x_1, t)\phi_\uparrow(x_2, t) \right].$$

(6)

The underlined term will be explained shortly. Fig. 2 illustrates the position-dependent wave-packets in Eq. (6). We know that one particle has the position
$X_A(t)$ at lab $A$, and the other particle $X_B(t)$ at lab $B$. The particles positions are indicated by the blue disks in Fig. 2. The positions are determined by their respective guiding equations. For the particle at lab $A$, we have

$$m_A \frac{dX_A}{dt} = \hbar \Im \left( \frac{\partial}{\partial x_1} \ln \Psi \left( x_1, X_B(t), t \right) \right) \bigg|_{x_1=X_A(t)} . \quad (7)$$

The spin-singlet state ensures two possibilities:

1. The field $\phi^\uparrow(x_1,t)$ pilots the particle at $A$ and the field $\phi^\downarrow(x_2,t)$ pilots the particle at $B$;
2. The field $\phi^\downarrow(x_1,t)$ pilots the particle at $A$ and $\phi^\uparrow(x_2,t)$ pilots the particle at $B$.

These two options accessible to the particles determine the relative states $\phi^\uparrow(x_1,t)\phi^\downarrow(x_2,t)$ and $\phi^\downarrow(x_1,t)\phi^\uparrow(x_2,t)$ in Eq. (6). The initial conditions of the Bohmian system determine which relative state the particles populate. Without loss of generality, we assume possibility one, which corresponds to the occupation of the underlined relative state in Eq. (6). Also, from Eq. (7) we see that the particle at $A$ likely accommodates ($\frac{dX_A}{dt} \approx 0$) where the derivative of the logarithm is zero, which happens when $\phi^\uparrow(x_1,t)$ is extreme; as shown in Fig. 2.

Since the particles occupy the underlined relative state in Eq. (6), one frequently labels it as the preferred foliation of the entangled state [29, 30]. The second empty foliation could have been the preferred one if the initial conditions were different, or could still evolve to be the preferred foliation if populated in the future.

Due to the guiding Eq. (7) (the particles), Bohmian mechanics is non-local. The trajectory $X_A(t)$ depends on the global pilot-wave $\Psi$, which includes field deformations that occur in space-like separated regions. Yet, a frequently underappreciated property of pilot-waves is that fields of subsystems $\phi^{\upsilon\downarrow}(x,t)$ only interact locally. As long as the information is processed by the pilot wave, a local information flow is possible, even in a theory with non-local elements.

We now consider a third subsystem $C$ with its wave $\chi(x_3,t)$ guiding its particle (or collection of particles), that will act as the measurer of the particle at lab $B$. In Fig. 2, $\chi(x_3,t)$ is represented by the dashed line. The measurer $C$ is sufficiently close to interact with $B$, but not with $A$, which is emphasized by the intersecting light-cones between $B$ and $C$. At a certain time $t$, we must then consider the evolution of the augmented field $\Psi(x_1,x_2,t)\chi(x_3,t)$. The evolution of the pilot-wave is unitary, such that

$$\Psi(x_1,x_2,t)\chi(x_3,t) \rightarrow \frac{1}{\sqrt{2}} \left[ \phi^\uparrow(x_1,t')\phi^\downarrow(x_2,t')\chi^\downarrow(x_3,t') - \phi^\downarrow(x_1,t')\phi^\uparrow(x_2,t')\chi^\uparrow(x_3,t') \right], \quad t' > t. \quad (8)$$

The local interaction between $B$ and $C$ causes $\chi(x_3,t)$ to undergo meiosis. That is, in Fig. 2, the dashed wave splits into two thinner copies that are indicated by the solid lines at $C$. Meiosis only happens in the regions of the intersecting light-cones. The wave of the measurer produces a thinner and empty copy
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of itself $\chi_\uparrow(x_3,t')$, whose only difference to $\chi_\downarrow(x_3,t')$ is that it would have measured (if populated) $\phi_\uparrow(x_2,t')$ instead of $\phi_\downarrow(x_2,t')$. The augmented underlined relative state in Eq. (8) shows that $\chi_\downarrow(x_3,t')$ has now locally joined the evolved preferred foliation. If not for the guiding equations and the particles they describe, Bohmian mechanics would be a local theory.

In Bohmian mechanics, although some foliations are preferred over others, the importance of empty foliations cannot be overstated. It is the empty foliations that generate interference effects in the double-slit experiment. Without empty foliations, there would be no Pauli exclusion principle and no periodic table. It is precisely a second empty foliation of the form in Eq. (6) that guarantees zero resistivity in macroscopic superconductors. Quantum computations that explore the size of the Hilbert space, imply that in Bohmian mechanics, quantum information must be processed by the pilot-wave – the foliations – not by the particles.

Typical objections to Bohmian mechanics highlight the incompatibility with relativity, the autonomy of the pilot-wave, and the obsolescence of the particles. Possible responses to these objections can be found in Ref. [31]. Perhaps a less-mentioned objection is that Bohmian mechanics is necessarily formulated in the Schrödinger picture. This complicates the connection of Bohmian mechanics with the methods of quantum field theory. Perhaps one might attempt to formulate Bohmian mechanics in the Heisenberg picture by replacing the Schrödinger equation with the Heisenberg equation of motion. Then, what would a guiding equation look like? Perhaps this is a project idea for the interested reader. However, we speculate that a guiding equation in the Heisenberg picture would be an artificial introduction.

### 3.3 Everettian mechanics

Given that foliations appear to play the central role in quantum phenomena, one can examine what happens if one gets rid of the particles, and with them, the guiding equations. Then one is left with a local non-hidden variable theory called Everettian mechanics [30, 32, 33]. At most, one might use Bohmian test particles to track a particular foliation; in analogy to electrostatics, where we use test charges to track electric fields at a particular position.

The content of this section might well be adapted to other interpretations of unitary quantum mechanics, such as relational mechanics [34], modal interpretations [35], or the Montevideo interpretation [36]. One advantage of Everettian mechanics is that, since foliations are in some way part of all interpretations, it can be easily mapped to other interpretations.

The entangled state in Eq. (8) still defines two foliations, but none of them now receive a preferred status. The part of $\chi(x_3,t)$ involved in the interaction with $B$ (the region of the intersecting light-cones) foliates into the thinner $\chi_\uparrow(x_3,t')$ and $\chi_\downarrow(x_3,t')$ states. It is the characteristic length scale of the local interactions that sets the physical size of local foliations [30, 37]. Unlike frequently advertised, foliations are better thought of as local bubbles, not as
the entire universe splitting [30], which would be a non-local process. In the
absence of further interactions, the two foliations evolve autonomously.

Foliations (or relative states) can be understood both in the Schrödinger
[38] and in the Heisenberg picture [30]. Although not yet explicitly formul-
ated, there are no expected difficulties in other representations, such as the
interaction picture and second quantization. One of the advantages of formul-
ating relative Heisenberg observables in the Heisenberg picture, is that all
observables are local, even entangled observables [25, 30, 39]. Nonetheless, the
locality of Everettian mechanics can also be understood in the Schrödinger
picture [37].

Most objections to Everettian mechanics have a psychological characteristic
[18, 40], or relate to an instrumentalist stance [13]. A common objection
is that there are too many foliations. However, assuming for instance a finite
size Hilbert space, a maximum number of foliations (or a measure) is set
by the dimension of the Hilbert space. A way to differentiate interpreta-
tions of quantum mechanics is to ask how a particular interpretation does
away with the many foliations. Everettian mechanics recognizes all foliations.
Bohmian mechanics prefers certain foliations. In the $\psi$-ensemble interpreta-
tion, foliations are an average description of the hidden variables. Collapse-type
interpretations obliterate all foliations upon measurement or event, except one.
As for the unitary quantum interpretations, relational quantum mechanics has
a reduced ontology as compared to Everettian mechanics.

Everettian mechanics assumes a philosophical realist stance by postulating
that all mathematical elements map to reality. This makes out of Everettian
mechanics a delicate interpretation, since the observation of collapse, hid-
den variables, non-local phenomena, or an unexpected saturation of quantum
computation times would falsify it. Possible experimental tests proposed for
Everettian mechanics include Wigner friend-type experiments [41] that could
perhaps soon be simulated in a quantum computer [7]. Other proposals sug-
gest looking at the energy balance of a measurement [12], and contrasts with
objective collapse theories [13]. An interesting mathematical direction might
be to design no-go theorems specifically targeted for Everettian mechanics.
4 Discussion

4.1 Ingredients of interpretations

Bell’s theorem does not exclude the possibility of quantum formulations admitting a local information flow. Yet, despite many alternative options (see Fig. 1), Bell’s results continue being advertised for quantum non-locality [24], which is a property of collapse/handshake interpretations. In Sec. 2, we showed how Bell’s result gives us four categories of interpretations. We have given one example from each of the three categories that allow for a local information flow. In the three interpretations, information flows locally via the hidden variables, the pilot-wave, and the foliations, respectively.

In superdeterminism, the Schrödinger equation is not all there is. The wavefunction is an emergent average description of a yet unresolved additional structure – the hidden variables. They are not required to comply with Bell’s theorem, because they break the assumption of statistical independence. This comes with correlations that lack in theories admitting statistical independence. Superdeterminism makes peculiar predictions, which might be tested soon [11, 26].

If one dismisses the additional correlations that come with superdeterminism, but maintains hidden variables, then Bell’s result enforces a theory with non-local effects, such as Bohmian mechanics. Unlike superdeterminism, where the hidden variables determine the quantum state, in Bohmian mechanics, the wave pilots the particles, which leads to non-local effects on the particles, but not on the waves.

Further, if one gets rid of the Bohmian particles, one is left with the Schrödinger equation only (or the Heisenberg equations of motion). One possible interpretation is Everettian mechanics, where the recording of measurement results on one foliation loses its preferred status with respect to other foliations. Other local and deterministic interpretations are possible, such as modal or relational mechanics. In Fig. 3 we depict how the stripping down of ingredients from the mathematical formalism takes us from correlated hidden variables to the bare foliations.

For the next section, we drop Bohmian mechanics from the discussion, because of its apparent incompatibility with the locality principles from relativity, and the difficulty of formulating guiding equations within the mathematical methods of quantum field theory. Then, for discussion purposes, this leaves us with superdeterministic and Everettian-type quantum theories.

4.2 Philosophical analysis

One of the reasons behind the proliferation of quantum interpretations is that there is no consensus on the philosophy of science. Contrasting superdeterminism and Everettian mechanics not only allows us to use a test to falsify one of them, but also contrast different philosophies of science. Although both superdeterminism and Everettian mechanics provide a causal, deterministic,
and local description of quantum mechanics, both are frequently rejected based on psychology alone. However, subjective psychology comes and goes in physics [33].

To avoid subjectivity, let us state two philosophies of science: instrumentalist views usually adopted by advocates of superdeterminism [15], and explanatory approaches adopted by Everettians [13, 32]. Instrumentalism is motivated by what we observe, and the role of experiments is to increase credence in favour of a particular theory. In this view, better theories are the ones that give better predictions. In contrast, Deutsch’s philosophy of science builds on Popper and regards fundamental science as explanatory [13]. The purpose of science is then not mapped to a particular prediction, but to conjecture explanations that specify an ontology and how it behaves, similar to Maudlin [28]. Concepts related to credence lack in this philosophy. Instead of finding a theory with high credence, scientific methodology finds flaws and deficiencies in a given explanation and seeks to replace it for a better one. There is no guarantee there is something as the truth, let alone a natural evolution towards it through this methodology. However, it does allow us to consistently compare different theories.

Before making the philosophical analysis using Deutsch’s philosophy, we summarize how, within this approach, one of two rival theories might be refuted. Paraphrasing: a bad theory is one that: (i) does not account for the objects of the explanation; or, (ii) conflicts with other good theories (theories that observe the opposites of criteria (i) and (iii)); or, (iii) is easy to vary [42]. If a theory displays at least one of the properties above, then it can be made problematic, which then motivates a scientific problem. A scientific theory can then only be refuted if it has a better rival according to the criteria. If it has no rival, it can at most be made problematic by the same criteria. A crucial scientific test could be an experiment that allows the identification of a better theory.

We now use Deutsch’s philosophy of Ref. [13] to contrast superdeterminism with Everettian mechanics, which according to the criteria above, allows for a crucial test to make one of them problematic. In the experiment proposal of Refs. [11, 26], superdeterminism predicts the same result between measurements, whereas Everettian mechanics generally predicts different results. If the experiment repeatedly observes the same result, this would be consistent with both superdeterminism and Everettian mechanics. However, superdeterminism would be a better explanation (according to criteria (i)), because it would clarify why only a single result was observed. The explanation is that the hidden variables remained in the same cluster. According to Deutsch’s criteria (i), Everettian mechanics would be refuted, because it could not account for the object of the explanation; in this case, the hidden variables. If only superdeterminism is left as a good unrivalled theory, it cannot be refuted. Superdeterminism could, at most, be made problematic by the criteria (i) - (iii). The argument can be repeated for other quantum descriptions, such as
collapse variants [13]. Similarly, if the experiment observed different results, then superdeterminism would be refuted by the criteria (ii) and (iii).

4.3 Conclusion

Under currently used philosophies of physics, there are at least two quantum descriptions that are Bell-local, casual and deterministic. Even so, Bell’s results continue being used to dismiss all local hidden variable theories and unitary interpretations in favour of non-local interpretations. Instead of ignoring local quantum descriptions, we suggest that all options should remain open, and instead of deliberate falling back to non-local interpretations, take the open options seriously and strive to produce crucial tests to hopefully refute and refine the available interpretations. And of course, the observation of true non-locality, such as the entangling of particles in space-like separated regions, would falsify all local descriptions described in this paper, including Bohmian mechanics.

Acknowledgments. The authors acknowledge the professors S. Dahmen, A. Franklin, N. Lima, S. Prado and S. Saunders for discussions concerning issues addressed in this paper. E.N.C. thanks the support of the National Council for Scientific and Technological Development (CNPq) and the support of the British Council through the Women in Science: Gender Equality 2022 Program.
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