Acceleration of Particles in Imbalanced Magnetohydrodynamic Turbulence

Bogdan Teaca,1,2,3,∗ Martin S. Weidl,2 Frank Jenko,2 and Reinhard Schlickeiser4

1Applied Mathematics Research Centre, Coventry University, Coventry CV1 5FB, United Kingdom
2Max-Planck/Princeton Center for Plasma Physics and Max-Planck-Institut für Plasmaphysik, D-85748 Garching, Germany
3Max-Planck für Sonnensystemforschung, Max-Planck-Str. 2, D-37191 Kaiserslautern, Germany
4Institut für Theoretische Physik, Lehrstuhl IV: Weltraum- und Astrophysik, Ruhr-Universität Bochum, D-44780 Bochum, Germany

The present work investigates the acceleration of test particles in balanced and imbalanced Alfvénic turbulence, relevant to the solar-wind problem. These turbulent states, obtained numerically by prescribing the injection rates for the ideal invariants, are evolved dynamically with the particles. While the energy spectrum for balanced and imbalanced states is known, the impact made on particle heating is a matter of debate, with different considerations giving different results. By performing direct numerical simulations, resonant and non-resonant particle accelerations are automatically considered and the correct turbulent phases are taken into account. For imbalanced turbulence, it is found that the acceleration rate of charged particles is reduced and the heating rate diminished. This behaviour is independent of the particle gyroradius, although particles that have a stronger adiabatic motion (smaller gyroradius) tend to experience a larger heating.

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Introduction. — The slow and fast streams in the solar wind represent good examples of balanced and imbalanced states of strong Alfvénic turbulence, respectively. These relatively large-scale fluctuations, compared to the proton thermal gyroradius, are captured by the magnetohydrodynamic (MHD) representation. Although a kinetic or gyrokinetic approach [1] is needed for the treatment of scales smaller than the proton gyroradius [2,3], where the interaction of kinetic Alfvén waves [4] and electron heating of the solar wind [5] become important, the self-organisation of turbulent structures remains predominantly a large-scale effect, determined by fluid-like dynamics.

In MHD turbulence, the conservation of cross-helicity (to be defined below) for the ideal systems represents a dynamical constraint of interest, as it is the quantity that leads to the acceleration of charged particles, and how does this behaviour depend on the gyroradius? In the current Letter we will numerically investigate for the first time the problem using non-relativistic test particles accelerated by the fully self-consistent electromagnetic field in time-dependent MHD fields. This study provides insight into the problem and, to a certain degree, links the fluid and kinetic approaches.

Basic equations. — The incompressible MHD equations need to be formulated in terms of two dynamical quantities. These quantities can be either the plasma velocity (u) and the self-consistent magnetic field (b, hereafter being expressed in Alfvén velocity units b → b / \sqrt{\rho m_e}, where \rho is the constant mass density) or the Elsasser variables [4], defined as \mathbf{z}^\pm = \mathbf{u} \pm \mathbf{b}. The Elsasser representation can be seen as the nonlinear scattering of counter-propagating Alfvén waves of fluctuations \mathbf{z}^\pm, travelling along the large-scale structures of the magnetic filed (not necessarily just along a mean magnetic field \mathbf{B}_0). In terms of Elsasser variables, the incompressible MHD equations can be written as

\[ \frac{\partial \mathbf{z}^\pm}{\partial t} = - (\mathbf{z}^\mp \mp \mathbf{B}_0) \cdot \nabla \mathbf{z}^\pm + \nu^+ \nabla^2 \mathbf{z}^\pm + \nu^- \nabla^2 \mathbf{z}^\mp + \mathbf{f}^\pm - \nabla \rho, \]  

where \nu^\pm = (\nu \mp \eta)/2, with \nu being the kinematic fluid viscosity and \eta being the magnetic diffusivity. The total (hydrodynamic and magnetic) pressure field (\rho) is an auxiliary variable that enforces the incompressibility condition for the velocity field. Since the magnetic field is intrinsically zero-divergent, the zero divergence conditions of the Elsasser fields read \nabla \cdot \mathbf{z}^\pm = 0. Finally, the divergence-free fields \mathbf{f}^\pm correspond to a known external forcing mechanism used to reach a stationary state of the system.

Balanced and imbalanced turbulence. — For ideal MHD fluctuations, three quadratic invariants exist: total energy (\mathcal{E} = \frac{1}{2} \langle \mathbf{u}(x) \cdot \mathbf{u}(x) + \mathbf{b}(x) \cdot \mathbf{b}(x) \rangle_x), cross-helicity (\mathcal{H}^c = \langle \mathbf{u}(x) \cdot \mathbf{b}(x) \rangle_x), and magnetic helicity (\mathcal{H}^m = \langle \mathbf{a}(x) \cdot \mathbf{b}(x) \rangle_x), where \mathbf{b} = \nabla \times \mathbf{a}. Here, \langle \cdot \rangle_x refers to spatial averages. Ideal MHD refers to the case of absent external forces and dissipative effects (\nu = \eta = 0) in Eq. (1). For dissipative systems, ideal invariants lead to constant spectral

* bogdan.teaca@coventry.ac.uk
fluxes that redistribute invariant quantities between different scales and link the scaling of the dynamical fields, a fact evident from the Politano-Pouquet equations [15]. This link is responsible for the self-organized nature of Alfvénic turbulence. In the Elsasser representation, the cross-helicity level, defined as the energy injection rate and total energy information is contained in the definition of two ideal invariants \( \mathcal{E}^+ = \langle E^+(x) \rangle_x = \frac{1}{2} \langle z^+(x) \cdot z^+(x) \rangle_x \) and \( \mathcal{E}^- = \langle E^-(x) \rangle_x = \frac{1}{2} \langle z^-(x) \cdot z^-(x) \rangle_x \), known as pseudo-energies. The pseudo-energies \( \mathcal{E}^\pm \) represent the energy of the counter-propagating and co-propagating Alfvén waves. While their sum will obviously give the total energy in the system \( \mathcal{E}^+ + \mathcal{E}^- = \mathcal{E} \), their difference \( \mathcal{E}^- - \mathcal{E}^- = \mathcal{H}^e \) measures the preference of the system to generate one type of wave over the other. This led to the names balanced turbulence (for \( \mathcal{H}^e = 0 \)) and imbalanced turbulence (\( \mathcal{H}^e \neq 0 \)) being used in the literature. The cross-helicity level, defined as

\[
\sigma^e = \frac{\mathcal{H}^e}{\mathcal{E}^- - \mathcal{E}^-},
\]

represents a better way to quantify MHD states exactly, as \( \sigma^e \in [-1, 1] \). A value close to \( \pm 1 \) denotes strongly imbalanced turbulent states, while in a state of \( \sigma^e = \pm 1 \) no nonlinear interaction can take place (Alfvén states).

At a point-wise level, employing a definition analogous to Eq. (2) gives information regarding the alignment of the plasma velocity and the (total) magnetic field, \( \rho^e(x) = u(x) \cdot (\mathbf{b}(x) + \mathbf{B}_0) / \sqrt{2} \langle u^2(x) \rangle + \frac{1}{2} \langle \mathbf{b}(x) + \mathbf{B}_0 \rangle^2 \). While average quantities enter in statistical scaling theories of turbulence, it is the point-wise quantities that have dynamical significance and affect the acceleration of particles. As such, it is of interest to relate these two quantities. In Fig. 1 we see that both balanced and strongly imbalanced turbulence possess highly aligned and highly anti-aligned velocity and magnetic structures. However, it is the distribution of these structures [16] that gives the overall character for the turbulence. For balanced turbulence, the alignment tends to be symmetrically distributed across positive and negative values. For imbalanced turbulence, the dominance of highly aligned structures is clearly evident from the histograms. Thus, a non-zero value for the global parameter \( \sigma^e \) denotes a preference in the generation of one type of aligned structures over the others. For high level of imbalanced turbulence, on average, particles will experience the same level of alignment.

**MHD stationary states.** Numerically, we employ the TURBO code [17] to solve the MHD equations. Since the mean field value is on the order of the rms magnetic fluctuations \( B_0 \sim \delta b \), with \( \delta b = \langle \mathbf{b}^2(x) \rangle^{1/2} \) we use a cubic \((2\pi)^3\) domain, with periodic boundary conditions, discretized using 512\(^3\) grid points. For comparison, results obtained in the absence of a mean magnetic field are indicated explicitly. The pseudo-spectral method used is consistent with a direct-numerical-simulation (DNS) approach, such that \( k_{\text{max}} \ell_K \sim 1.25 \), where \( \ell_K \) is the smallest turbulent scale of the system estimated as the Kolmogorov scale \( \ell_K = (\nu^3/\varepsilon)^{1/4} \) with \( \varepsilon \) being the energy dissipation level in the system \( \varepsilon \equiv D^\text{tot} = \langle (\nu \nabla^2 u^2 + \kappa b^2) \rangle_x \) and \( \nu = 6.6 \times 10^{-4} \). For the time integration, a third-order Runge-Kutta method is used with an adaptive time step determined by a Courant-Friedrichs-Lewy condition. The aliasing effects are suppressed by a two-thirds dealiasing method [18].

To achieve balanced or imbalanced stationary states, we use a forcing mechanism that is local in Fourier space and acts in the same manner on all the modes within a wavenumber shell \( s_f = [2.5, 3.5] \). Selecting a large number of forced modes ensures that no anisotropy effect is induced by the forcing mechanism. The forces are defined on a helical-mode basis [19] and use the injection rates of the total energy (here \( \varepsilon_{\text{inj}} = 0.1 \)), cross-helicity level \( \sigma^e \) and magnetic helicity (taken as zero for all cases) as control parameters. The forcing method used here imposes the dissipation levels for the energy (Fig. 2-a) and cross-helicity (Fig. 2-b) in the stationary regime, without modifying the phases of the fields, see [20] for details. This ensures that no change is made in the type of turbulent structures present. We should note that \( f^+ \) and \( f^- \) force the two types of Alfvén waves. Thus, an im-

![FIG. 1.](image) (color online). Examples of the real-space density of the alignment level \( \rho^e \) for various degree of imbalanced MHD turbulence (different \( \sigma^e \) values), solved for periodic boundary conditions using parameters given in the text. The lower panel depicts histograms (256 bins) of the alignment over the entire computational domain.

![FIG. 2.](image) (color online). a) the evolution of the total (sum of kinetic and magnetic) energy dissipation \( D^\text{tot} \); b) the evolution of the cross-helicity dissipation \( D^e = \langle (\nu + \kappa) \nabla^2 u \cdot \mathbf{b} \rangle_x \), normalized to the energy injection rate \( \varepsilon_{\text{inj}} \); and c) the time evolution of the cross-helicity level. Test particles are injected at \( t/t^* = 0 \).
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on the fields and an implicit fourth-order Runge-Kutta solver
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magnetic field. Numerically, a cubic-spline interpolation
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magnetic energy distribution. The five particle species have
of imbalance, we evolve the trajectories of test-particlesin
the initial maximal gyroradii to the smallest turbulent scale.

time.

Particle tracking. — In order to investigate how the stochas-
tic acceleration of charged particles differs for various degrees of
imbalance, we evolve the trajectories of test-particles in
parallel with the MHD simulations. The test-particles are in-
jected after the MHD turbulence has attained a steady-state
and are evolved using a Newtonian representation,
\[
\frac{dr}{dt} = \mathbf{v}, \quad \frac{d\mathbf{v}}{dt} = \frac{1}{t} \mathbf{r} \times [\mathbf{B}_0 + \mathbf{b}(\mathbf{r})],
\]
where \(\mathbf{r}(t)\) and \(\mathbf{v}(t)\) are the position and the velocity of a
particle at time \(t\). The electric field is computed from Ohm’s law
for resistive MHD, \(\mathbf{e} = \eta \mathbf{j} - \mathbf{u} \times (\mathbf{B}_0 + \mathbf{b})\), with
\(\mathbf{b} = \nabla \times \mathbf{b}\). The coupling parameter \(\ell = (\eta / \sqrt{\rho \mu_0})^{-1}\) repre-
sents the particle’s charge to mass ratio \((q/m)\), takes into account the use
of Alfvénic units by the electromagnetic fields and has units
of length. Intuitively, it can be seen as the Larmor radius of a
particle that moves in a constant magnetic field with the per-
pendicular velocity equal to the local Alfvén velocity \(v_A\) of
the magnetic field. Numerically, a cubic-spline interpolation
on the fields and an implicit fourth-order Runge-Kutta solver
with adaptive step-size control are employed to advance the
particle trajectories (see Ref. [21] for details).

Throughout this work, we consider five particle species
with 50,000 test-particles per species. All particles start at
a random initial position, with a random direction for the
velocity and with an initial velocity chosen as \(v_0 = v_A\)
(here \(v_A^2 = B_0^2 + \delta b^2\)), which puts their initial kinetic en-
ergy \(E_0 = v_A^2/2\) on the same level as the peak of the
magnetic energy distribution. The five particle species have
\(\ell = \{0.1, 0.3, 1.3, 10\} \times \ell_K\). Taking the values in units of the
Kolmogorov scale length is a choice that allows us to compare
the initial maximal gyroradii to the smallest turbulent scale.

Perpendicular and parallel acceleration. — Compared to
balanced turbulence, for the strong imbalanced case the reduc-
tion in the energization of the particles can be seen in Fig. 4.
Also, independently of the cross-helicity, the energy gain is
significantly stronger for particle species with low values of \(\ell\)
due to their high charge-to-mass ratio. As the electric field is
all but constant on the small length scales of the gyroradius
of particles with \(\ell \lesssim \ell_K\), the acceleration perpendicular to
the magnetic field vanishes over one gyration period and the
particles are initially accelerated only by the Ohmic field in the
parallel direction, \(\mathbf{e}_\parallel = \eta \mathbf{j}_\parallel\).

However, turbulent fluctuations of the electromagnetic
fields lead to pitch-angle scattering and isotropization, con-
verting the parallel energy gained from Ohmic heating into
perpendicular energy. Although slow at first, the pitch-angle
scattering increases the perpendicular velocity of the particles
and thus their gyroradius, which results in enhanced scatter-
ing. This effect explains the fast growth of the squared pitch-
angle cosine \(\alpha^2 = v_\perp^2/v^2\) and its subsequent decay back to its
initial isotropic value of 0.33 (Fig. 4).

As alignment of \(\mathbf{u}\) and \(\mathbf{b}\) reduces the intensity of the per-
pendicular component of the electric field, \(\mathbf{e}_\perp = \eta \mathbf{j}_\perp - \mathbf{u} \times
(\mathbf{B}_0 + \mathbf{b})\), the isotropization process takes longer in strongly
imbalanced turbulence than in balanced turbulence. On the

FIG. 3. (color online). Histograms of the kinetic energy distributions of an initially mono-energetic particle ensemble with \(E_p(0) = v_A^2/2\), for balanced turbulence (\(\sigma = 0\)) on top row and strongly imbalanced turbulence (\(\sigma = 0.9\)) on bottom row.

FIG. 4. (color online). Evolution of the pitch-angle cosine square \((\alpha^2 = v_\perp^2/v^2)\). Top to bottom \(\sigma^2 = \{0.0, 0.4, 0.6, 0.9\}\).
we measure the momentum diffusion coefficient
the effect of imbalanced turbulence on particle energization.

The magnetic field varies more slowly for non-zero
distribution is isotropized again. Since the direction of the lo-
period to be neglected. Thus

Momentum diffusion estimation. — In order to estimate
the effect of imbalanced turbulence on particle energization,
we measure the momentum diffusion coefficient

other hand, the presence of an external magnetic mean-field
increases the pitch-angle scattering rate and results in a faster
isotropization than in MHD turbulence without a magnetic
mean-field.

For particle species with \( \ell > \ell_K \), the gyroradius is too large
for the contribution of the motional electric field over one gy-
roperiod to be neglected. Thus \( \alpha \) decreases initially, reflecting
a period of perpendicular acceleration dominating over paral-
ellel acceleration, and then increases slowly as the pitch-angle
distribution is isotropized again. Since the direction of the lo-
cal magnetic field varies more slowly for non-zero \( B_0 \) than for
\( B_0 \equiv 0 \), the initial perpendicular acceleration is much more
pronounced in the runs with an external mean-field.

Momentum diffusion estimation. — In order to estimate
the effect of imbalanced turbulence on particle energization,
(Dpp = )

\[
D_{pp} \sim v_A^2 \delta \nu \frac{B^2_0}{2 \delta b^2} \left[ 1 - (\sigma^e)^2 \right].
\]

The scattering time \( \tau \) depends on the gyroradius, gyro-
frequency, and the spectral properties of the turbulence. For
isospectral slab turbulence with spectral index \( s \) and a mono-
energetic particle distribution, the model of Dung and Schlickei-
er predicts \( \tau \sim \ell^{2-s} \), which agrees surprisingly well with
our DNS simulations at larger values of \( \ell \) if we take \( s = 5/3 \),
especially considering the anisotropy of our directly simulated
MHD turbulence is at odds with Dung and Schlickeiser’s tur-
bulence model.

Discussion and conclusions. — Using numerical simula-
tions of MHD turbulence, we observed that the energization of
Alfvénic test particles \( v \approx v_A \) for balanced turbulence is
more pronounced compared to a strongly imbalanced state.
This is indicative of a systematic acceleration loss affecting
the particles in the imbalanced case, where structures of simi-
lar velocity and magnetic field alignment tend to dominate
the turbulence. This implies a weaker ion heating rate in plasmas
characterized by strong imbalance, such as the fast solar wind.
For fast particles \( v \gg v_A \), for which the electric field accel-
eration is small, the spatial diffusion is not expected to vary
with the imbalance, as reported by Ref. [22].

In our study, compared to the smallest scale of turbulence
(\( \ell_K \)), particles with various initial gyro-radii are selected. As
the particles are non-adiabatic, the gyro-radius will change
in time and the particles will resonate with different Alfvén
wavelengths, of different energies. Gyro-radii of the order of
the Kolmogorov scale and smaller give particles a stronger
adiabatic characteristic. While smaller gyro-radii particles ex-
perience a better heating, the tendency of imbalanced turbu-
lencto suppress the acceleration is shown to be present at all
scales. This result implies a need for kinetic simulations to
account for the level of imbalance of the larger plasma scales
that act as an energy source in the system.

In Ref. [13] it was conjectured that the perpendicular heat-
ing rate of ions due to low-frequency Alfvén waves with wave-
lengths on the scale of the ion gyroradius was independent
of the degree of imbalance of the turbulence, as long as the
root-mean-square fluctuations of the velocity and the mag-
netic field stay constant. The results of this letter imply that,
on the contrary, the heating rate is strongly reduced in imbal-
anced turbulence.

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