Characterizations of Completely $N_{nc}$ (Weakly $N_{nc}$)-irresolute Functions via $N_{nc}$-open Sets

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Abstract. The purpose of present this paper is to introduce and investigate two new classes of $N_{nc}$-irresolute functions called completely $N_{nc}$-e-irresolute functions and completely weakly $N_{nc}$-e-irresolute functions in topological spaces by using the concept of $N_{nc}$-open sets. Several new characterizations and interesting properties concerning completely $N_{nc}$-e-irresolute functions and completely weakly $N_{nc}$-e-irresolute functions are obtained. Furthermore the relationships between these functions and some other well-known types of functions are also given.

Keywords and phrases: $N_{nc}$-e-irresolute functions, completely $N_{nc}$-e-irresolute functions and completely weakly $N_{nc}$-e-irresolute functions.

1. Introduction
Smarandache’s neutrosophic framework have wide scope of constant applications for the fields of Computer Science, Information Systems, Applied Mathematics, Artificial Intelligence, Mechanics, Medicine, dynamic, Management Science and Electrical & Electronic and so forth [1, 2, 3, 4, 30, 31]. Smarandache [24] described the Neutrosophic set on three portion (T-Truth, F-Falshood, I-Indeterminacy) Neutrosophic sets. Neutrosophic topological spaces (nts’s) presented by Salama and Alblowi [21]. Lellies Thivagar et al. [6, 7] was given the mathematical presence of $N$ topology, which is a non-empty set equipped with $N$ arbitrary topologies. Al-Hamido et al. [5] investigate the chance of extending the idea of neutrosophic crisp topological spaces into $N$-neutrosophic crisp topological spaces and examine a portion of their essential properties. In 2018, Lellies Thivagar et al. [7] introduced $N_n$ continuous in $N$ neutrosophic crisp topological spaces. In 2020, Vadivel and Thangaraja [28] the concept of $N_{nc}$-continuous in $N$ topological spaces. The aim of present this paper is to introduce and investigate other new types of irresolute functions via $N_{nc}$-e-open sets called completely $N_{nc}$-e-irresolute functions and completely weakly $N_{nc}$-e-irresolute functions. Some characterizations and interesting properties of these functions are discussed.
2. Preliminaries

**Definition 2.1** [22, 23] A neutrosophic crisp set (ncs) $H$ in a nonempty set $X$ has the form $H = (H_1, H_m, H_n)$, where $H_1, H_m, & H_n$ are subsets of $X$. Defined $\phi_n = (\phi, \phi, X)$, $X_n = (X, X, \phi)$. ncs$(X)$ means the set of all ncs’s in $X$. A ncs of Type 1 (resp. 2 & 3) (in short, ncs-Type 1 (resp. 2 & 3)), if it satisfies $H_1 \cap H_m = H_m \cap H_n = H_n \cap H_l = \phi$ (resp. $H_1 \cap H_m = H_m \cap H_n = H_n \cap H_l = \phi$ and $H_1 \cup H_m \cup H_n = X & \cap H_l \cap H_m \cap H_n = \phi$ and $H_1 \cup H_m \cup H_n = X$). ncs$(1)_3$($X$) $(ncS2(X)$ and $ncS3(X)$) means set of all ncs Type 1 (resp. 2 & 3).

**Definition 2.2** Let $H = (H_1, H_m, H_n), M = (M_l, M_m, M_n) \in ncS(X)$. Then $H \subseteq M$ (resp. $H = M$), if $H_1 \subseteq M_l, H_m \subseteq M_m$ and $H_n \supseteq M_n$ (resp. $H \subseteq M$ and $M \subseteq H$); $H^c = (H_n, H_n, H_l)$; $H \cap M = (H \cap M_l, H_m \cap M_m, H_n \cap M_n); H \cup M = (H \cup M_l, H_m \cup M_m, H_n \cup M_n)$. Let $(A_j)_{j \in J} \subseteq ncS(X)$, where $H_j = (H_{j_1}, H_{j_2}, H_{j_3})$. Then $\bigcup_{j \in J} H_j$ (simply $\bigcup H_j) = (\bigcap H_{j_1}, \bigcap H_{j_2}, \bigcup H_{j_3})$.

**Definition 2.3** [22] A neutrosophic crisp topology (briefly, nct) on a non-empty set $X$ is a family $\tau$ of ncs sets of $X$ satisfying

(i) $\phi_n, X_n \in \tau$.
(ii) $H_1 \cap H_m \in \tau \forall H_1 & H_m \in \tau$.
(iii) $\bigcup_{a} H_a \in \tau$, for any $\{H_a : a \in J\} \subseteq \tau$.

Then $(X, \tau)$ is a neutrosophic crisp topological space (briefly, ncts) in $X$. Elements of $\tau$ are called neutrosophic crisp open sets (briefly, ncos) in $X$. A ncs $C$ is closed set (briefly, nccs) iff its complement $C^c$ is ncos.

**Definition 2.4** [5] $nc\tau_1, nc\tau_2, \ldots, nc\tau_N$ are $N$-arbitrary crisp topologies defined on a nonempty set $X$ and the collection $N_{nc}\tau = \{A \subseteq X : A = \bigcup_{j=1}^{N} H_j \cup \bigcap_{j=1}^{N} L_j, H_j, L_j \in nc\tau_j\}$ is called $N$ neutrosophic crisp (briefly, $N_{nc}$-)topology on $X$ if the axioms are satisfied:

(i) $\phi_n, X_n \in N_{nc}\tau$.
(ii) $\bigcup_{j=1}^{n} A_j \in N_{nc}\tau \forall \{A_j\}_{j=1}^{n} \in N_{nc}\tau$.
(iii) $\bigcap_{j=1}^{n} A_j \in N_{nc}\tau \forall \{A_j\}_{j=1}^{n} \in N_{nc}\tau$.

Then $(X, N_{nc}\tau)$ is called a $N_{nc}$-topological space (briefly, $N_{nc}$ts) on $X$. Elements of $N_{nc}\tau$ are called $N_{nc}$-open sets ($N_{nc}$os) & its complement is called $N_{nc}$-closed sets ($N_{nc}$cs) on $X$. The elements of $X$ are known as $N_{nc}$-sets ($N_{nc}$s) on $X$.

**Definition 2.5** [5] Let $H$ be an $N_{nc}$s on a $N_{nc}$ts of $X$, then the $N_{nc}$ interior of $H$ (briefly, $N_{nc}$int$(H)$) and $N_{nc}$ closure of $H$ (briefly, $N_{nc}$cl$(H)$) are defined as

(i) $N_{nc}$int$(H) = \bigcup\{A : A \subseteq H & A$ is a $N_{nc}$os in $X\}$ & $N_{nc}$cl$(H) = \bigcap\{C : H \subseteq C & C$ is a $N_{nc}$cs in $X\}$.

(ii) $N_{nc}$-regular open [25] set (briefly, $N_{nc}$ros) if $H = N_{nc}$int$(N_{nc}$cl$(H)$).

The complement of an $N_{nc}$ros is called an $N_{nc}$-regular closed set (briefly, $N_{nc}$rcs) in $X$. The family of all $N_{nc}$ros of $X$ is denoted by $N_{nc}$ROS$(X)$.

**Definition 2.6** [26] A set $H$ is said to be a
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Throughout this section, let $(X, N_{nc})$ and $(Y, N_{nc})$ be any two $N_{nc}$ sets.

**Definition 3.1** A map $f : (X, N_{nc}) \to (Y, N_{nc})$ is said to be $N_{nc}$-continuous [7] (resp. $N_{nc}$-irresolute [29]) if $f^{-1}(H)$ is both $N_{nc}$-open and $N_{nc}$-closed (resp. $N_{nc}$-irresolute) in $X$ for each $N_{nc}$ set $H$ of $Y$.

**Example 3.2** Let $X = \{d, c, b, a\}$, $N_{nc} \Gamma_1 = \{\phi, X, C, B, A\}$, $N_{nc} \Gamma_2 = \{\phi, X, N\}$. Define $f : (X, 2_{nc}) \to (X, 2_{nc})$ as an identity map, then it is $2_{nc}$-irresolute but not $sN_{nc}$-cts, the set $f^{-1}(\{a, \phi\}) = \{a, \phi\}$ is a $2_{nc}$-open but not $2_{nc}$-closed in $X$.

**Remark 3.1** Every $sN_{nc}$-cts function is $cN_{nc}$-irresolute but every $cN_{nc}$-irresolute function is $N_{nc}$-irresolute. But not conversely.

**Theorem 3.1** Let $f : (X, N_{nc}) \to (Y, N_{nc})$ be a function. Then

(i) $f$ is $cN_{nc}$-irresolute;

(ii) $f^{-1}(M)$ is $N_{nc}$-open in $X \forall N_{nc}$ set $M$ of $Y$ are equivalent.

**Proof.** (i) $\Rightarrow$ (ii): Let $M$ be any $N_{nc}$-open set of $Y$, then $Y \setminus M \in N_{nc}OS(X)$. By (i), $f^{-1}(Y \setminus M) = X \setminus f^{-1}(M) \in N_{nc}ROS(X)$. We have $f^{-1}(M) \in N_{nc}RCS(X)$.

(ii) $\Rightarrow$ (i): is similar to proof (i) $\Rightarrow$ (ii).

**Lemma 3.1** Let $H$ be an $N_{nc}$-open set of a $N_{nc}$-cts $X$. Then the following hold:

(i) If $L$ is $N_{nc}$-open in $X$, then so is $L \cap H$ in the $N_{nc}$ subspace $(H, N_{nc} \Psi_H)$. 

**Example 3.1** Let $X = \{c, b, a\}$, $N_{nc} \Gamma_1 = \{\phi, X, N\}$, $N_{nc} \Gamma_2 = \{\phi, X, n\}$. Define $f : (X, 2_{nc}) \to (X, 2_{nc})$ as an identity map, then it is $2_{nc}$-cts but not $sN_{nc}$-cts, the set $f^{-1}(\{a, \phi\}) = \{a, \phi\}$ is a $2_{nc}$-open but not $2_{nc}$-closed in $X$. 

**Example 3.2** Let $X = \{d, c, b, a\}$, $N_{nc} \Gamma_1 = \{\phi, X, C, B, A\}$, $N_{nc} \Gamma_2 = \{\phi, X, N\}$. Define $f : (X, 2_{nc}) \to (X, 2_{nc})$ as an identity map, then it is $2_{nc}$-irresolute but not $cN_{nc}$-irresolute, the set $f^{-1}(\{b, \phi, \{d, c\}\}) = \{b, \phi, \{d, c\}\}$ is a $2_{nc}$-open but not $2_{nc}$-closed in $X$. 

**Remark 3.2** From the above arguments, we obtain the following diagrams:

$$sN_{nc}$-cts $\implies cN_{nc}$-irresolute $\implies N_{nc}$-irresolute

**Theorem 3.1** Let $f : (X, N_{nc}) \to (Y, N_{nc})$ be a function. Then

(i) $f$ is $cN_{nc}$-irresolute;

(ii) $f^{-1}(M)$ is $N_{nc}$-open in $X \forall N_{nc}$ set $M$ of $Y$ are equivalent.

**Proof.** (i) $\Rightarrow$ (ii): Let $M$ be any $N_{nc}$-open set of $Y$, then $Y \setminus M \in N_{nc}OS(X)$. By (i), $f^{-1}(Y \setminus M) = X \setminus f^{-1}(M) \in N_{nc}ROS(X)$. We have $f^{-1}(M) \in N_{nc}RCS(X)$.

(ii) $\Rightarrow$ (i): is similar to proof (i) $\Rightarrow$ (ii).
If $M \subseteq H$ is $N_{nc}ro$ in $(H, N_{nc}\Psi_H)$, then there exists a $N_{nc}ro$ set $L$ in $(X, N_{nc}\Psi) \ni M = L \cap H$.

**Theorem 3.2** If $f : (X, N_{nc}\Psi) \to (Y, N_{nc}\Psi^*)$ is a $cN_{nc}eIrr$ function and $H$ is any $N_{nc}o$ set of $X$, then the restriction $f|_H : H \to Y$ is $cN_{nc}eIrr$.

**Proof.** Let $M$ be an $N_{nc}e$ set of $Y$. By hypothesis $f^{-1}(M)$ is $N_{nc}ro$ in $X$. Since $H$ is $N_{nc}o$ in $X$, it follows from Lemma 3.1 that $(f|_H)^{-1}(M) = H \cap f^{-1}(M)$, which is $N_{nc}ro$ in $H$. Therefore, $f|_H$ is $cN_{nc}eIrr$.

**Theorem 3.3** For functions $h_1 : (X, N_{nc}\Psi) \to (Y, N_{nc}\Psi^*)$ and $h_2 : (Y, N_{nc}\Psi^*) \to (Z, N_{nc}\Psi^{**})$ the following hold:

(i) If $h_1$ is $cN_{nc}eIrr$ (resp. $cN_{nc}Cts, cN_{nc}eIrr$ & $sN_{nc}Cts$ ) and $h_2$ is $N_{nc}eIrr$ (resp. $cN_{nc}eIrr$, $N_{nc}eCts$ & $cN_{nc}eIrr$), then $h_2 \circ h_1 : X \to Z$ is $cN_{nc}eIrr$ (resp. $cN_{nc}eIrr$, $cN_{nc}Cts$ & $cN_{nc}eIrr$),

(ii) If $h_1$ and $h_2$ are $cN_{nc}eIrr$, then $h_2 \circ h_1$ is $cN_{nc}eIrr$.

**Proof.** The proof is obvious and easy thus omitted.

**Definition 3.2** A $N_{nc}ts (X, N_{nc}\Psi)$ is said to be $N_{nc}$ almost (resp. $N_{nc}e$) connected if $X$ cannot be written as the union of two non-empty disjoint $N_{nc}ro$ (resp. $N_{nc}e$) sets.

**Theorem 3.4** If $f : (X, N_{nc}\Psi) \to (Y, N_{nc}\Psi^*)$ is a $cN_{nc}eIrr$ surjective function and $X$ is $N_{nc}$ almost connected, then $Y$ is $N_{nc}e$-connected.

**Definition 3.3** A $N_{nc}ts (X, N_{nc}\Psi)$ is said to be:

(i) $N_{nc}$ Nearly-compact if every $N_{nc}ro$ cover of $X$ has a finite $N_{nc}$ subcover,

(ii) $N_{nc}$ Nearly-countably compact if every $N_{nc}$ countable cover of $X$ by $N_{nc}ro$ sets has a finite $N_{nc}$ subcover,

(iii) $N_{nc}$ Nearly-Lindelöf if every $N_{nc}$ cover of $X$ by $N_{nc}ro$ sets has a countable $N_{nc}$ subcover,

(iv) $N_{nc}$-compact if every $N_{nc}$ cover of $X$ by $N_{nc}eo$ sets has a finite $N_{nc}$ subcover,

(v) countably $N_{nc}$-compact if every $N_{nc}$ countable cover of $X$ by $N_{nc}eo$ sets has a finite $N_{nc}$ subcover,

(vi) $N_{nc}$-Lindelöf if every $N_{nc}$ cover of $X$ by $N_{nc}eo$ sets has a countable $N_{nc}$ subcover.

**Theorem 3.5** If $f : (X, N_{nc}\Psi) \to (Y, N_{nc}\Psi^*)$ is a $cN_{nc}eIrr$ surjective function. Then the following statements hold: If $X$ is

(i) $N_{nc}$ nearly-compact, then $Y$ is $N_{nc}e$-compact.

(ii) $N_{nc}$ nearly-Lindelöf, then $Y$ is $N_{nc}$-Lindelöf.

(iii) $N_{nc}$ nearly-countably compact, then $Y$ is $cN_{nc}e$-compact.

**Definition 3.4** A $N_{nc}ts (X, N_{nc}\Psi)$ is said to be:

(i) $N_{nc}$ $S$-closed (resp. $N_{nc}e$-closed compact) if every $N_{nc}rc$ (resp. $N_{nc}ec$) cover of $X$ has a finite $N_{nc}$ subcover,

(ii) countably $N_{nc}$ $S$-closed-compact (resp. countably $N_{nc}e$-closed compact) if every countable $N_{nc}$ cover of $X$ by $N_{nc}rc$ (resp. $N_{nc}ec$) sets has a finite $N_{nc}$ subcover,

(iii) $S$ $N_{nc}$-Lindelöf (resp. $N_{nc}e$-closed Lindelöf) if every $N_{nc}$ cover of $X$ by $N_{nc}rc$ (resp. $N_{nc}ec$) sets has a countable $N_{nc}$ subcover.
Theorem 3.6 If \( f : (X, N_{nc}\Psi) \to (Y, N_{nc}\Psi^*) \) is a \( cN_{nc}eIrr \) surjective function. Then the following statements hold: If \( X \) is

(i) \( N_{nc} \) \( S \)-closed, then \( Y \) is \( N_{nc}e \)-closed compact.

(ii) \( N_{nc} \) \( S \)-Lindelöf, then \( Y \) is \( N_{nc}e \)-closed Lindelöf.

(iii) countably \( N_{nc} \) \( S \)-closed-compact, then \( Y \) is countably \( N_{nc}e \)-closed compact.

Definition 3.5 A \( N_{nc} \) \( X \) is said to be almost \( N_{nc} \) regular (resp. strongly \( N_{nc}e \)-regular) if \( \forall \) \( N_{nc}r \) \( c \) (resp. \( N_{nc}ec \) \( c \)) set \( M \subseteq X \) and any point \( x \in X \setminus M \), there exists disjoint \( N_{nc}e \) \( o \) (resp. \( N_{nc}eo \)) sets \( U \) \& \( V \) \( \ni x \in U \) and \( M \subseteq V \).

Definition 3.6 A function \( f : (X, N_{nc}\Psi) \to (Y, N_{nc}\Psi^*) \) is called \( \rho \) \( N_{nc}e \)-closed if the image of each \( N_{nc}ec \) set of \( X \) is an \( N_{nc}e \) \( ec \) set in \( Y \).

Theorem 3.7 If a mapping \( f : (X, N_{nc}\Psi) \to (Y, N_{nc}\Psi^*) \) is \( N_{nc}e \)-closed, then for each subset \( B \) of \( Y \) and an \( N_{nc}e \) \( o \) \( set \( U \) \( of \) \( X \) containing \( f^{-1}(B) \), \( \exists \) a \( N_{nc}e \) \( o \) \( set \( V \) \( in \) \( Y \) containing \( B \) \( \ni f^{-1}(B) \subseteq U \).

Theorem 3.8 If \( f : (X, N_{nc}\Psi) \to (Y, N_{nc}\Psi^*) \) is \( cN_{nc}eIrr \) e-open from an almost \( N_{nc} \) regular space \( X \) onto a space \( Y \), then \( Y \) is \( sN_{nc}e \)-regular.

Definition 3.7 A \( N_{nc} \) \( (X, N_{nc}\Psi) \) is said to be:

(i) Almost \( N_{nc} \)-normal if for each \( N_{nc}c \) \( set \( L \) and each \( N_{nc}r \) \( c \) \( set \( M \) such that \( L \cap M = \phi \), there exist disjoint \( N_{nc}o \) \( sets \( U \) and \( V \) such that \( L \subseteq U \) and \( M \subseteq V \).

(ii) strongly \( N_{nc}e \)-normal if for each pair of disjoint \( N_{nc}ec \) \( sets \( L \) and \( M \) of \( X \), there exist disjoint \( N_{nc}eo \) \( sets \( U \) and \( V \) such that \( L \subseteq U \) and \( M \subseteq V \).

Theorem 3.9 If \( f : X \to Y \) is \( cN_{nc}eIrr \) e-open from an almost \( N_{nc} \)-normal space \( X \) onto a space \( Y \), then \( Y \) is strongly \( N_{nc}e \)-normal.

Definition 3.8 A \( N_{nc} \) \( (X, N_{nc}\Psi) \) is said to be \( N_{nc}e-T_1 \) (resp. \( N_{nc}r-T_1 \)) if for each pair of distinct points \( x \) and \( y \) of \( X \), there exist \( N_{nc}e \) \( o \) (resp. \( N_{nc}ro \)) \( sets \( L \) and \( M \) containing \( x \) and \( y \), respectively, such that \( x \notin M \) and \( y \notin L \).

Theorem 3.10 If \( f : (X, N_{nc}\Psi) \to (Y, N_{nc}\Psi^*) \) is a \( cN_{nc}eIrr \) injective function and \( Y \) is \( N_{nc}e-T_1 \), then \( X \) is \( N_{nc}r-T_1 \).

Definition 3.9 A \( N_{nc} \) \( (X, N_{nc}\Psi) \) is said to be \( N_{nc}e-T_2 \) (resp. \( N_{nc}r-T_2 \)) for each pair of distinct points \( x \) and \( y \) in \( X \), there exist disjoint \( N_{nc}e \) \( o \) (resp. \( N_{nc}ro \)) \( sets \( L \) and \( M \) in \( X \) such that \( x \in L \) and \( y \in M \).

Theorem 3.11 If \( f : (X, N_{nc}\Psi) \to (Y, N_{nc}\Psi^*) \) is a \( cN_{nc}eIrr \) injective function and \( Y \) is \( N_{nc}e-T_2 \), then \( X \) is \( N_{nc}r-T_2 \).

Theorem 3.12 Let \( Y \) be an \( N_{nc}e-T_2 \) space. Then the following statements hold:

(i) If \( h_1, h_2 : (X, N_{nc}\Psi) \to (Y, N_{nc}\Psi^*) \) are \( cN_{nc}eIrr \) functions, then the set \( H = \{ x \in X : h_1(x) = h_2(x) \} \) is \( N_{nc}e dc \) in \( X \).

(ii) If \( h_1 : (X, N_{nc}\Psi) \to (Y, N_{nc}\Psi^*) \) is a \( cN_{nc}eIrr \) function, then the set \( K = \{ (x, y) \in X \times X : h_1(x) = h_1(y) \} \) is \( N_{nc}e dc \) in \( X \times X \).
4. Characterizations of completely weakly $N_{nc}$-irresolute functions

**Definition 4.1** A function $f : (X, N_{nc}\Psi) \rightarrow (Y, N_{nc}\Psi')$ is said to be completely weakly $N_{nc}$-irresolute (briefly $cwN_{nc}eIrr$) if for each $x \in X$ and $\forall N_{nc}o$ set $V$ containing $f(x)$, $\exists$ an $N_{nc}o$ set $U$ containing $x \in f(U) \subseteq V$.

**Remark 4.1** It is clear that, every $cN_{nc}eIrr$ function is $cwN_{nc}eIrr$ and every $cwN_{nc}eIrr$ function is $N_{nc}eIrr$. But the converses are not true in general as shown in the following examples.

**Example 4.1** In Example 3.2, then it is
(i) $cw2_{nc}eIrr$ but not $c2_{nc}eIrr$, the set $f^{-1}((\{b, a\}, \{\phi\}, \{d, c\}) = \{a, b\}, \{\phi\}, \{d, c\})$ is a $2_{nc}cos$ but not $2_{nc}ros$ in $X$.
(ii) $2_{nc}eIrr$ but not $cw2_{nc}eIrr$, the set $f((\{b, c\}, \{\phi\}, \{d, a\})) \subseteq (\{b\}, \{\phi\}, \{c, a, d\})$.
\[\{b\}, \{\phi\}, \{d, c, a\}\] is a $2_{nc}cos$ and $(\{c, b\}, \{\phi\}, \{d, a\})$ is a $2_{nc}ros$.

**Remark 4.2** From the Remarks of 3.1 & 4.1 we obtain the following diagrams:
\[sN_{nc}eIrr \rightarrow cN_{nc}eIrr \rightarrow cwN_{nc}eIrr \rightarrow N_{nc}eIrr\]

**Theorem 4.1** For a function $f : (X, N_{nc}\Psi) \rightarrow (Y, N_{nc}\Psi')$ the statements
(i) $f$ is $cwN_{nc}eIrr$.
(ii) For each $x \in X$ and each $N_{nc}o$ set $V$ of $Y$ containing $f(x)$, $\exists$ an $N_{nc}o$ set $U$ of $X$ containing $x \in f(U) \subseteq V$,
(iii) $f(N_{nc}cl(A)) \subseteq N_{nc}cl(f(A))$ subset $A$ of $X$,
(iv) $N_{nc}cl(f^{-1}(B)) \subseteq f^{-1}(N_{nc}cl(B))$ subset $B$ of $Y$,
(v) For each $N_{nc}ec$ set $V$ of $Y$, $f^{-1}(V)$ is $N_{nc}c$ in $X$,
(vi) $f^{-1}(N_{nc}int(B)) \subseteq N_{nc}int(f^{-1}(B))$ subset $B$ of $Y$
are equivalent.

**Theorem 4.2** Let $h_1 : (X, N_{nc}\Psi) \rightarrow (Y, N_{nc}\Psi')$ be functions. If the graph $h_2 : X \rightarrow X \times Y$ of $h_1$ is $cwN_{nc}eIrr$, then so is $h_1$.

**Theorem 4.3** For functions $h_1 : X \rightarrow Y$ and $h_2 : Y \rightarrow Z$ the following hold: If $h_1$ is
(i) $cwN_{nc}eIrr$ and $h_2$ is $N_{nc}eIrr$, then $h_2 \circ f : X \rightarrow Z$ is $cwN_{nc}eIrr$,
(ii) $cN_{nc}Cts$ and $h_2$ is $cwN_{nc}eIrr$, then $h_2 \circ h_1 : X \rightarrow Z$ is $cN_{nc}eIrr$,
(iii) $N_{nc}eIrr$ and $h_2$ is $cwN_{nc}eIrr$, then $h_2 \circ h_1 : X \rightarrow Z$ is $cN_{nc}eIrr$,
(iv) $cwN_{nc}eIrr$ and $h_2$ is $N_{nc}Cts$, then $h_2 \circ h_1 : X \rightarrow Z$ is $N_{nc}Cts$,
(v) $N_{nc}eIrr$ and $h_2$ is $cwN_{nc}eIrr$, then $h_2 \circ h_1 : X \rightarrow Z$ is $N_{nc}eIrr$,
(vi) $N_{nc}Cts$ and $h_2$ is $cwN_{nc}eIrr$, then $h_2 \circ h_1 : X \rightarrow Z$ is $cwN_{nc}eIrr$.

**Definition 4.2** A function $f : (X, N_{nc}\Psi) \rightarrow (Y, N_{nc}\Psi')$ is said to be almost $N_{nc}$-open (briefly, $aN_{nc}o$) if $f^{-1}(V)$ is $N_{nc}o$ in $X$ for every $N_{nc}o$ set $V$ of $Y$.

**Theorem 4.4** Let $h_1 : (X, N_{nc}\Psi) \rightarrow (Y, N_{nc}\Psi')$ be almost $N_{nc}o$ and $h_2 : (Y, N_{nc}\Psi') \rightarrow (Z, N_{nc}\Psi'')$ be any function such that $h_2 \circ h_1 : (X, N_{nc}\Psi) \rightarrow (Z, N_{nc}\Psi'')$ is $cN_{nc}eIrr$, then $h_2$ is $cwN_{nc}eIrr$.

**Proof.** Let $V$ be an $N_{nc}o$ set in $(Z, N_{nc}\Psi'')$. Since $h_2 \circ h_1$ is $cN_{nc}eIrr$, $(h_2 \circ h_1)^{-1}(V) = h_1^{-1}(h_2^{-1}(V))$ is $N_{nc}o$ in $(X, N_{nc}\Psi)$. Since $h_1$ is $aN_{nc}o$ surjection, $h_1(h_1^{-1}(h_2^{-1}(V))) = h_2^{-1}(V)$ is $N_{nc}o$ in $Y$. Therefore, $h_2$ is $cwN_{nc}eIrr$.  

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Theorem 4.5 Let \( h_1 : (X, N_{nc}Ψ) \rightarrow (Y, N_{nc}Ψ^*) \) be \( N_{nc}o \) surjection and \( h_2 : (Y, N_{nc}Ψ^*) \rightarrow (Z, N_{nc}Ψ^{**}) \) be any function such that \( h_2 \circ h_1 : (X, N_{nc}Ψ) \rightarrow (Z, N_{nc}Ψ^{**}) \) is \( cwN_{nc}eIrr \), then \( h_2 \) is \( cwN_{nc}eIrr \).

Definition 4.3 (i) A filterbase \( L \) is said to be \( L \)-convergent to a point \( x \) in \( X \) if for each \( U \in eOS(X) \), \( \exists B \in L \ni B \subseteq U \).

(ii) A filterbase \( L \) is said to be convergent to a point \( x \) in \( X \) if for each \( N_{nc}o \) set \( U \) of \( X \) containing \( x \), \( \exists B \in L \ni B \subseteq U \).

Theorem 4.6 Let \( h_1 : (X, N_{nc}Ψ) \rightarrow (Y, N_{nc}Ψ^*) \) be \( cwN_{nc}eIrr \), then for each point \( x \in X \) and each filterbase \( N_{nc}e \) in \( X \) converging to \( x \), the filterbase \( h_1(L) \) is \( N_{nc}e \)-convergent to \( h_1(x) \).

5. Conclusion

Several generalized forms of continuity and irresoluteness have been introduced during the last few years. Recently, continuity and irresoluteness of functions in topological spaces have been researched by many mathematicians and physicists (see [8, 9, 10, 11, 12, 13, 17, 18, 19]), El Naschie, [10] derived quantum gravity from set theory. Also El-Naschie in [14] has indicated that there was a contribution towards the resolution of some fundamental questions linking space-time geometry and topology. Since the mathematical theory of fuzzy sets is highly developed and used extensively in many practical and engineering problems and Furthermore, since El-Naschie has shown that the notion of fuzzy topology have very important applications in quantum particle physics especially in related to both string theory and \( \infty \) theory [9, 15, 16]. Thus, it should be mentioned that the present work in this paper may have become relevant to the works of El-Naschie [9, 10]. Also the neutrosophic topological version of the concepts and results introduced in this paper are very important.

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