The effect of external colour field impact on the instanton liquid is studied. In the course of this study the corresponding effective Lagrangians are derived for both regimes of weak and strong external field and in long wave-length approximation. The example of Euclidean colour point-like source is analyzed in detail and the feedback of field on the instanton liquid is estimated as a function of source intensity.

The declarations of discovering new state(s) of matter in relativistic heavy ion collisions at RHIC which are actively wandering in the papers nowadays are sometimes based on the results of different nature. From one side it is the striking result of direct experimental measurements of a strong suppression (comparing to $pp$ and $pA$ collisions) of particle production at high transverse momentum well-known as a jet quenching. And although the jet reconstruction in these experiments is a nontrivial task the (and accompanying) result(s) is(are) interpreted as a degradation of hard parton (initiating a jet) energy induced by medium (new thermalized matter) produced in collision long before hadronizing in the QCD vacuum. On the other hand the convincing success of phenomenological analysis of the other measurable characteristics based on the perfect liquid hydrodynamics results in the questions about the sort of plasma (if produced) and the origin of the QCD vacuum Ref.[1].

In the present paper we are trying to understand if the instanton liquid model of QCD vacuum could be indicative in interpreting these RHIC results. The instanton liquid here is considered in the framework of the stochastic ensemble of instantons in the singular gauge and the generating functional is evaluated by the variational principle developed in the paper Ref.[2]. Comparative simplicity of the superposition ansatz and variational procedure allows us to analyze the effects almost analytically.

As a major configuration saturating the generating functional

$$Z = \int D[A] \ e^{-S(A)}$$

where $S(A)$ is a standard Yang-Mills action we take the approximate solution for the Yang-Mills equations in the form of the following superposition

$$A^a_\mu(x) = B^a_\mu(x) + \sum_{i=1}^N A^a_\mu(x; \gamma_i),$$

(1)

here $A^a_\mu$ implies the field of (anti-)instantons in the singular gauge

$$A^a_\mu(x) = \frac{2}{g} \, \omega^{ab} \bar{\eta}_{b\mu\nu} \ a_\nu(y), \quad a_\nu(y) = \frac{\rho^2}{y^2 + \rho^2} \, \frac{y_\nu}{y^2}, \quad y = x - z,$$

(2)

with the parameters $\gamma_i = (\rho_i, z_i, \omega_i)$ describing the $i$-th instanton of the $\rho$ size centered at the pseudoparticle coordinate $z$, with the matrix of colour orientation $\omega$, and $g$ denotes the coupling constant of non-abelian field. For the anti-instanton the ’t Hooft symbols should be changed according to
$\eta \to \eta$, and the external field $B_{\mu}^a(x)$ is viewed as well as the pseudo-particle field in quasiclassical approximation.

The external field interaction with an individual pseudo-particle is defined by the following strength tensor

$$G_{\mu\nu}^a = G_{\mu\nu}^a(B) + G_{\mu\nu}^a(A) + G_{\mu\nu}^a(A, B),$$

where the first two terms correspond to standard strength tensors of non-abelian field

$$G_{\mu\nu}^a(A) = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c,$$

with entirely anti-symmetric tensor $f^{abc}$. In particular,

$$G_{\mu\nu}^a(A) = -\frac{4}{g} \omega^{ak} \bar{\eta}_{\kappa \beta} M_{\mu \alpha} M_{\nu \beta} \frac{\rho^2}{(y^2 + \rho^2)^2},$$

where $M_{\mu \nu} = \delta_{\mu \nu} - 2 \bar{y}_\mu \bar{y}_\nu$, $\bar{y}_\mu = \frac{y_\mu}{|y|}$. The 'mixed' component of the instanton strength field looks like

$$G_{\mu\nu}^a(A, B) = g f^{abc} (B_{\mu}^b A_{\nu}^c - B_{\nu}^b A_{\mu}^c) = g f^{abc} \omega^{cd} \frac{2}{g} (B_{\mu}^b \bar{\eta}_{d \alpha} - B_{\nu}^b \bar{\eta}_{d \alpha}) a_\alpha(y).$$

Calculating now $G^2$ we receive the partial contributions of external field and each separate pseudo-particle as

$$G_{\mu\nu}^a G_{\mu\nu}^a = G_{\mu\nu}^a(B) G_{\mu\nu}^a(B) + G_{\mu\nu}^a(A) G_{\mu\nu}^a(A) + 2 G_{\mu\nu}^a(A, B) G_{\mu\nu}^a(A, B) +$$

$$+ 2 G_{\mu\nu}^a(B) G_{\mu\nu}^a(A) + 2 G_{\mu\nu}^a(B) G_{\mu\nu}^a(A, B) + 2 G_{\mu\nu}^a(A) G_{\mu\nu}^a(A, B),$$

In order to keep the further steps as simple and transparent as possible we limit ourselves with the standard sum of partial contributions in the superposition ansatz action and hold the highest in IL density (precisely in packing fraction parameter $n \rho^4$) one particle contributions

$$S(B, \gamma) = \int dx \frac{G_{\mu\nu}^a G_{\mu\nu}^a}{4} \simeq \sum_i \int dx \frac{G_{\mu\nu}^a(i) G_{\mu\nu}^a(i)}{4}. $$

The crossing terms of different pseudo-particles (which are proportional to the IL density squared) are neglected here because of very small packing fraction parameter characteristic to IL, i.e. $n \rho^4 \sim 0.01$. Thus, the regularized generating functional for the IL model takes the following form (for denotations see Ref.[2])

$$Y = \int D[B] \frac{1}{N!} \int \prod_{i=1}^N d\gamma_i \ e^{-S(B, \gamma)}. $$

The analysis can be easily done for the weak external field assuming the characteristic IL parameters, averaged pseudo-particle size $\bar{\rho}$ and the IL density $n$, unchanged and coinciding with their vacuum magnitudes. Those are fixed by some repulsive mechanism (see, however, the remark at the end of paper) for the particular choice of saturating configuration done above. In this case, from now on the integration over pseudo-particle sizes becomes unessential and all the pseudo-particles might be considered as having the same size $\bar{\rho}$. It allows us to average over the pseudo-particle positions and colour orientations only by calculating the generating functional (9). Making use the cluster decomposition we obtain the corresponding average of exponential as

$$\langle \exp(-S) \rangle_{\omega_z} = \exp \left( \sum_k \frac{(-1)^k}{k!} \langle (S^k) \rangle_{\omega_z} \right),$$

where $\langle S_1 \rangle = \langle (S_1) \rangle, \langle S_1 S_2 \rangle = \langle (S_1) \langle S_2 \rangle + \langle (S_1 S_2) \rangle, \ldots$. The first cumulant is simply defined by the action averaged. Taking into account the direct form of
field strength tensors \( (5) \) and \( (6) \) it is evident that the following terms will only be present in the partial contribution after averaging over colour orientation

\[
\langle G_{\mu\nu}^a G_{\mu\nu}^a \rangle_\omega = G_{\mu\nu}^a (B) G_{\mu\nu}^a (B) + \langle G_{\mu\nu}^a (A) G_{\mu\nu}^a (A) \rangle_\omega + \langle G_{\mu\nu}^a (A, B) G_{\mu\nu}^a (A, B) \rangle_\omega + 2 \langle G_{\mu\nu}^a (A) G_{\mu\nu}^a (A, B) \rangle_\omega .
\]

Performing the colour averaging we use the equality

\[
\langle \omega^{ak} \omega^{cd} \rangle = \frac{\delta^{ac} \delta^{bd}}{N_c^2 - 1} ,
\]

implying \( N_c \) as the number of colours. As a result we have that the last term

\[
2 G_{\mu\nu}^a (A) G_{\mu\nu}^a (A, B) = -\frac{16}{g} \omega^{ak} \bar{\eta}_{\alpha \beta} M_{\mu \alpha} M_{\nu \beta} \frac{\rho^2}{(y^2 + \rho^2)^2} f^{abc} \omega^{cd} (B^b_{\mu} \bar{\eta}_{d\alpha} - B^b_{\nu} \bar{\eta}_{d\alpha}) a_\alpha (y)
\]
disappears due to the antisymmetric tensor \( f^{abc} \),

\[
\langle G_{\mu\nu}^a (A) G_{\mu\nu}^a (A, B) \rangle_\omega = 0 .
\]

Analyzing now the contribution of 'mixed' (repulsive) component

\[
G_{\mu\nu}^a (A, B) G_{\mu\nu}^a (A, B) = 4 f^{abc} \omega^{cd} (B^b_{\mu} \bar{\eta}_{d\alpha} - B^b_{\nu} \bar{\eta}_{d\alpha}) a_\alpha (y) f^{akm} \omega^{mn} (B^k_{\mu} \bar{\eta}_{m\alpha} - B^k_{\nu} \bar{\eta}_{m\alpha}) a_\gamma (y) ,
\]

and averaging Eq. (11) over the colour orientation (making use another equality)

\[
f^{abc} f^{akc} = N_c \delta^{bc} ,
\]

we have

\[
\langle G_{\mu\nu}^a (A, B) G_{\mu\nu}^a (A, B) \rangle_\omega = \frac{4 N_c}{N_c^2 - 1} (B^b_{\mu} \bar{\eta}_{d\alpha} - B^b_{\nu} \bar{\eta}_{d\alpha}) a_\alpha (y) (B^b_{\mu} \bar{\eta}_{m\alpha} - B^b_{\nu} \bar{\eta}_{m\alpha}) a_\gamma (y) .
\]

Averaging over the pseudo-particle positions results in the following integral

\[
\int \frac{dz}{V} a_\alpha (y) a_\gamma (y) = \delta_{\alpha \gamma} \frac{I}{V} ,
\]

where

\[
I = \frac{\pi^2}{4} \rho^2 ,
\]

because the basic IL parameters, as we agreed, are unchanged. Handling the 'mixed' component average we have it in the form as reads

\[
\langle G_{\mu\nu}^a (A, B) G_{\mu\nu}^a (A, B) \rangle_\omega = \frac{18 \pi^2 \rho^2}{V} \frac{N_c}{N_c^2 - 1} B^b_{\mu} B^b_{\nu} .
\]

Finally, collecting all appropriate terms we find the effective action for the external field in IL as

\[
\langle \langle S \rangle \rangle_\omega = \int dx \left( \frac{G (B) G (B)}{4} + \frac{m^2}{2} B^2 \right) + N \beta ,
\]

\[
m^2 = 9 \pi^2 \frac{N_c}{N_c^2 - 1} ,
\]

here \( N \) is the full number of particles in volume \( V \) with \( n = N/V \) and a single pseudo-particle action \( \beta = 8 \pi^2 / g^2 \). The last term of Eq. (15) introduces the contribution of purely instanton component \( \langle G(A) G(A) \rangle_\omega \). The contribution of repulsive term which fixes the pseudo-particle size in IL is omitted.
in Eq.(15) so long as it is not a principal point in this context and adding it, leads to the insignificant correction to the last condensate term in Eq.(15). An amusing point is that the mass term of Eq.(16) has been well-known for rather long time and as a matter of fact fixing the pseudo-particle size in the variational procedure of Ref.[2] is provided just by this mechanism of mass generation. With the characteristic IL parameters ($N_c = 3$ and number of flavours $N_f = 2$) $n/\Lambda_{QCD}^4 = 1.2$, $\bar{\rho}\Lambda_{QCD} = 0.27$, $\beta = 18$ the mass estimate $m \sim 0.44\text{GeV}$ looks pretty encouraging for $\bar{\rho} \sim 1\text{GeV}$ and $\Lambda_{QCD}$ in the interval of $200 - 300 \text{GeV}$. The compatibility conditions for the equations resulting from Eq.(15) is $\partial_\mu B_\mu = 0$ which is satisfied by the pseudo-particle field Eq.(2) as well.

Turning now to the next term of cluster decomposition to calculate the effective Lagrangian corrections we conclude immediately that in the second cumulant

$$\frac{1}{2} \left\langle \left\langle \int dx_1 G^{\mu\nu}(B) G^{\alpha\beta}(A) \right\rangle \int dx_2 G^{\alpha\beta}(B_2) G^{\mu\nu}(A_2) \right\rangle, \tag{17}$$

$$\frac{1}{2} \left\langle \left\langle \int dx_1 G^{\mu\nu}(A) G^{\alpha\beta}(B, A) \right\rangle \int dx_2 G^{\alpha\beta}(A_2) G^{\mu\nu}(B_2) \right\rangle, \tag{18}$$

here the index 2 underlines the fact that corresponding functions are dependent on $x_2$. The remaining terms originate from either the interference terms (and are cancelled by the contribution of the first cumulant squared) or lead to the contributions anharmonic in $B$ which are not in our interest for this paper. It was analyzed for the first time in Ref.[3] that $G(B)G(A)$ in (17) generates the dipole interaction. However, this interaction does not manifest itself in the first term of cluster decomposition if the averaging over the colour orientation is performed. It comes into focus starting on the second order of decomposing. In particular Eq.(17) can be presented in the following form

$$\frac{1}{2} \left\langle \left\langle \int dx_1 G^{\mu\nu}(B) G^{\alpha\beta}(A) \right\rangle \int dx_2 G^{\alpha\beta}(B_2) G^{\mu\nu}(A_2) \right\rangle \omega =$$

$$= \frac{1}{2} \frac{1}{N_c^2 - 1} \int dx_1 dx_2 \frac{G^{\mu\nu}(B) G^{\alpha\beta}(B_2)}{4} G^{\mu\nu}(A) G^{\alpha\beta}(A_2), \tag{19}$$

if one exploits Eq.(2) and Eq.(11) keeping in mind that $G^{\mu\nu}(A) G^{\alpha\beta}(A_2)$ is colour independent because of the identity $\omega^{ab} \omega^{ac} = \delta^{bc}$. Eq.(19) should be also averaged over the pseudo-particle positions which results in the correlation function for the instantons in singular gauge developing the following form obtained in Ref.[4]

$$\int dz \ G^{\alpha\beta}(A) G^{\alpha\beta}(A_2) = \frac{1}{g^2} \frac{16}{N_c} \left\langle \left\langle \int dx_1 dx_2 \ G^{\mu\nu}(B) G^{\alpha\beta}(A_2) \right\rangle \right\rangle$$

where $\Delta = |x_1 - x_2|$, and for the anti-instanton the substitution $\varepsilon \to -\varepsilon$ should be done. The analytical form of function $I_s$ is not our priority here, however, it is shown in Fig.1. If the numbers of instantons and anti-instantons are balanced then the term proportional to the tensor $\varepsilon$ disappears and the contribution of correlator in IL is given by

$$N \int dz \ G^{\alpha\beta}(A) G^{\alpha\beta}(A_2) = \frac{16}{g^2} n \left( \delta_{\mu\alpha} \delta_{\nu\beta} - \delta_{\mu\beta} \delta_{\nu\alpha} + \varepsilon_{\mu\alpha\beta} \right) I_s \left( \frac{\Delta}{\rho} \right),$$

where $\Delta = |x_1 - x_2|$, and for the anti-instanton the substitution $\varepsilon \to -\varepsilon$ should be done. The analytical form of function $I_s$ is not our priority here, however, it is shown in Fig.1. If the numbers of instantons and anti-instantons are balanced then the term proportional to the tensor $\varepsilon$ disappears and the contribution of correlator in IL is given by

$$N \int dz \ G^{\alpha\beta}(A) G^{\alpha\beta}(A_2) = \frac{16}{g^2} n \left( \delta_{\mu\alpha} \delta_{\nu\beta} - \delta_{\mu\beta} \delta_{\nu\alpha} + \varepsilon_{\mu\alpha\beta} \right) I_s \left( \frac{\Delta}{\rho} \right).$$

Now collecting the terms together we find the contribution of Eq.(17) in the IL approach as

$$\frac{16}{g^2} \frac{1}{N_c^2 - 1} n \int dx_1 dx_2 I_s \left( \frac{\Delta}{\rho} \right) G^{\mu\nu}(B) G^{\alpha\beta}(A_2).$$
Figure 1: Correlation function $I_s$ is given by solid line and the correlation functions $J_1$ and $J_2$ are given by the dashed lines.

Clearly, it leads to an abatement of initial action and it is more convenient for analyzing to present the non-local factor of dielectrical susceptibility type in the Fourier components Ref.\[3\]

$$
\int \frac{dk}{2\pi} \left( 1 - \frac{16}{g^2} \frac{1}{N_c^2 - 1} n \tilde{I}_s(k\rho) \right) G^a_{\mu\nu}[B(k)] G^a_{\alpha\beta}[B(-k)].
$$

Numerical estimate of $\tilde{I}_s(k\rho)$ at the zero value of argument is

$$
\tilde{I}_s(0) \sim 6 \rho^4,
$$

and at $N_c = 3$, $N_f = 2$ the correction coefficient can be estimated as $\kappa = \frac{16}{g^2} \frac{1}{N_c^2 - 1} n \tilde{I}_s(0) \sim 0.013$.

Analyzing now the term Eq.\[18\] we present it as

$$
\frac{1}{2} \left\langle \left\langle \int dx_1 \int dx_2 \omega^{ak} G^k_{\mu\nu}(A) f^{amn} \omega^{nl}(B^m_{\mu} \tilde{\eta}_{\nu\gamma} - B^m_{\nu} \tilde{\eta}_{\mu\gamma}) a_{\gamma} \times
\right. \right.
\left. \left. \times \omega^{bc} G^b_{\alpha\beta}(A_2) f^{bde} \omega^{ef}(B^d_{2\alpha} \tilde{\eta}_{f\beta\delta} - B^d_{2\beta} \tilde{\eta}_{f\alpha\delta}) a_{2\delta} \right\rangle \right\rangle_{\omega}.
$$

and imply the dependence of $G$ on the colour matrix $\omega$ might be given by the common factor (without introducing new symbol for $G$). Formally, this term looks like the next one expanding in $1/N_c$, i.e.
\( f^{\text{man}} \omega^{ak}\omega^{nl} = \varepsilon^{klq} \omega^{mq} \),

we have

\[
\langle f^{\text{man}} \omega^{ak}\omega^{nl} f^{\text{die}} \omega^{be}\omega^{cf} \rangle = \frac{\delta^{n d}}{N_c^2 - 1} \left( \delta^{k c} \delta^{j f} - \delta^{k f} \delta^{j c} \right),
\]

and then Eq.(20) receives the following form

\[
\frac{1}{2} \left( \int dx_1 \int dx_2 \frac{G^{\alpha\beta}_{\mu \nu}(A)}{4} \int dx_2 \frac{G^{\alpha\beta}_{\mu \nu}(A, B)}{4} \right) = \\
= 2 \frac{1}{N_c^2 - 1} \int dx_1 dx_2 \left[ G^{\alpha\beta}_{\mu \nu}(A) G^{\alpha\beta}_{\mu \nu}(A) \tilde{\eta}_{l \nu \gamma} \tilde{\eta}_{\beta \delta} - G^{\alpha\beta}_{\mu \nu}(A) G^{\alpha\beta}_{\mu \nu}(A) \tilde{\eta}_{k \beta \delta} \tilde{\eta}_{l \nu \gamma} \right] a_{\alpha} a_{2 \beta} B_{\mu} B_{2 \alpha}.
\]

The lower line here develops this form because of an asymmetric property of tensor \( G \). Averaging over the pseudo-particle positions we may extract the correlation function in the following form

\[
\frac{d z}{V} \int dx_1 dx_2 \left[ G^{\alpha\beta}_{\mu \nu}(A) G^{\alpha\beta}_{\mu \nu}(A) \tilde{\eta}_{l \nu \gamma} \tilde{\eta}_{\beta \delta} - G^{\alpha\beta}_{\mu \nu}(A) G^{\alpha\beta}_{\mu \nu}(A) \tilde{\eta}_{k \beta \delta} \tilde{\eta}_{l \nu \gamma} \right] a_{\alpha} a_{2 \beta} = \\
= 16 \frac{1}{g^2} \frac{d z}{V} \left[ J_1 \left( \frac{\Delta}{\rho} \right) \delta_{\mu \alpha} + J_2 \left( \frac{\Delta}{\rho} \right) \hat{\Delta}_{\mu} \hat{\Delta}_{\alpha} \right],
\]

where \( \hat{\Delta} = \frac{\vec{x} - \vec{p}}{|x - p|^2} \) is the unity vector.

The simple algebra allows us to calculate the functions

\[
J_1 = \int dy_1 \frac{\rho^4}{(y^2 + \rho^2)^3} \frac{\rho^4}{(z^2 + \rho^2)^3} \frac{1}{|y|} \frac{1}{|z|} \frac{1}{3} \left( 16 t^3 - 8 t + 4 p q + 6 (p^2 + q^2) t - 12 t^2 p q \right),
\]

\[
J_2 = \int dy_1 \frac{\rho^4}{(y^2 + \rho^2)^3} \frac{\rho^4}{(z^2 + \rho^2)^3} \frac{1}{|y|} \frac{1}{|z|} \frac{4}{3} \left( 4 t^3 + 5 t - 4 p q - 6 (p^2 + q^2) t + 12 t^2 p q \right),
\]

with \( z = y + \Delta, t = \frac{(y, z)}{|y| + |z|}, p = \frac{(y, \Delta)}{|y| + |\Delta|}, q = \frac{(z, \Delta)}{|z| + |\Delta|} \). Similarly to \( I_s \) we do not need their explicit forms here but one may estimate their behaviours looking at the dashed lines in Fig.1. Finally, the additional contribution to the mass term reads as

\[
\frac{1}{N_c^2 - 1} \frac{32}{g^2} \int dx_1 dx_2 \left[ J_1 \left( \frac{\Delta}{\rho} \right) \delta_{\mu \alpha} + J_2 \left( \frac{\Delta}{\rho} \right) \hat{\Delta}_{\mu} \hat{\Delta}_{\alpha} \right] B_{\mu} B_{2 \alpha},
\]

and in the Fourier components as

\[
\int dk \left[ \frac{m^2}{2} - \frac{32}{g^2} \frac{1}{N_c^2 - 1} N \left( \tilde{J}_1(k) \delta_{\mu \alpha} + \tilde{J}_2(k) \delta_{\mu \alpha} \right) B_{\mu}(k) B_{2 \alpha}(-k) \right].
\]
Making use the cluster decomposition one expects the possibility to calculate corresponding small contributions (if the sources are treated in the quasiclassical approximation) which are given by the correlation functions of the form \( \langle A^a_\mu(x; \gamma) A^b_\nu(y; \gamma) \rangle \) Ref.[3].

As the conclusion of this effective Lagrangian analysis Eq.(15) it is practical to address another approach to the problem Ref.[5]. Let us suppose the quasi-classical field \( B \) is described in the infra-red momentum region by the initial Yang-Mills action without the term breaking down gauge symmetry as before. In particular, we consider the field of point-like Euclidean source of intensity \( e \) with only one non-zero \( n \)-th component

\[
B^a_\mu(x) = (0, \delta^{an} \varphi), \quad \varphi = \frac{e}{4\pi} \frac{1}{|x|}.
\]

Then \( B^2 \) integrated over the 4-dimensional space gives

\[
\int dx \left( \frac{e}{4\pi |x|} \right)^2 = \frac{e^2}{4\pi} X_4 L,
\]

where \( X_4, L \) are some formal upper limits of corresponding integrals. In this approach the contribution of the first cumulant Eq.(15) could be written down as

\[
\langle \langle S \rangle \rangle_{\omega z} = E X_4, \quad E = \frac{e^2}{4\pi} \frac{1}{r_0} + \sigma L + \beta n L^3,
\]

with \( \sigma = \frac{9\pi}{8} \frac{N_c}{N_c^2 - 1} e^2 \frac{1}{r_0^2} \). The first term in defining \( E \) comes from the Coulomb energy of point-like source and \( r_0 \) represents a formal particle radius. The last term is originated by the gluon condensate and the previous term looks like negligibly small correction to the condensate term. However, this contribution linearly increasing with \( L \) is proportional to \( e^2 \) and has different physical meaning as a term additional to the self-energy of source. In other words, it demonstrates an impossibility for the source with an open colour to be available in IL because the amplitude of such a state is very strongly suppressed \( (e^{-S}) \) comparing to the condensate contribution if the screening effects are not taken into account. For the dipole in 'isosinglet' \((s)\) and 'isotriplet' \((t)\) states (i.e. \( N_c = 2 \)) we obtain

\[
B^a_\mu(x) = (0, \delta^{3a} \varphi), \quad \varphi = \frac{e}{4\pi} \left( \frac{1}{|x - z_1|} \pm \frac{1}{|x - z_2|} \right),
\]

where \( z_1, z_2 \) are the dipole coordinates what leads to

\[
\int dx B^2_s = \frac{e^2}{4\pi} X_4 l, \quad \int dx B^2_t = \frac{e^2}{4\pi} X_4 (4L - l),
\]

with \( l = |z_1 - z_2| \) to be the distance separating sources. We have another confirmation of suppression effect for the states with open colour in IL, i.e. the energy of 'isosinglet' dipole state increases with \( l \) enlarging and the corresponding coefficient is \( \sigma \sim 0.6 \text{ GeV/fm} \) if we take \( e \sim g \).

Thus, we are quite allowed to conclude the regime of weak external field in IL is described by effective Lagrangian Eq.(15) and basic IL parameters are within a well adapted interval. Moreover all the corrections originated by the second cumulant should be certainly neglected.

Obviously, this conclusion will be considerably strengthened if a criterion of external field weakness is well defined and the point of how crucial is an assumption of the IL parameters unchanged is clarified. We are going to modify slightly the variational procedure of Ref.[2] to implement possibility of the changing IL parameters. We retain here the same designations to demonstrate precisely where the changes are introduced and imply \( S(B, \gamma) \) in Eq.(9) in the following form

\[
S(B, \gamma) = - \sum \ln d(p_i) + \beta U_{int} + \sum U_{ext}(\gamma_i, B) + S(B),
\]

(24)
The first term here describes one-instanton contributions with the following distribution function over the (anti-)instanton sizes
\[ d(\rho) = C_{N_c} A^{b} QCD \rho^{-2N_c}, \]  
(25)
where \( b = \frac{11}{3} N_c - \frac{2}{3} N_f \), \( \beta = -b \ln(\Lambda_{QCD} \rho) \),

\[ C_{N_c} \approx \frac{4.66 \exp(-1.68 N_c)}{\pi^2 (N_c - 1)! (N_c - 2)!}. \]

The second term of Eq.(24) is responsible for providing pseudo-particles with repulsive interaction which fixes their sizes. The characteristic single instanton action is defined on the scale of average pseudo-particle size \( \beta = \beta(\rho) \) where \( \beta(\rho) = - \ln C_{N_c} - b \ln(\Lambda_{QCD} \rho) \). The partial pseudo-particle contributions grouped in the third term and we take only
\[ U_{ext}(\gamma_i, B) = \int d\gamma \frac{G^a_{\mu\nu}(A_i, B) G^{a\nu}(A_i, B)}{4}, \]
because the other contributions at the standard IL parameters are small as we have seen. At last, the fourth term represents simply the Yang-Mills action of the \( B \) field
\[ S(B) = \int dx \frac{G^a_{\mu\nu}(B) G^{a\nu}(B)}{4}. \]

The well-known property of exponential makes it possible to estimate the generating functional of Eq.(24) with the approximating functional as
\[ Y \geq Y_1 \exp(-\langle S - S_1 \rangle), \]  
(26)
where
\[ Y_1 = \int D[B] \frac{1}{N!} \prod_{i=1}^{N} d\gamma_i \ e^{-S_1(B, \gamma)-S(B)}, \]

\[ S_1(B, \gamma) = - \sum \ln \mu(\rho_i), \]
and \( \mu(\rho) \) is an effective one-particle distribution function which may be derived with the variational procedure. In our particular situation a mean value of corresponding difference is given by
\[ \langle S - S_1 \rangle = \frac{1}{Y_1} \frac{1}{N!} \prod_{i=1}^{N} d\gamma_i \ [\beta U_{int} + U_{ext}(\gamma, B) - \sum \ln d(\rho_i) + \sum \ln \mu(\rho_i)] e^{-\sum \ln \mu(\rho_i)} = \]

\[ = \frac{N}{\mu_0} \int d\rho \mu(\rho) \ln \frac{\mu(\rho)}{d(\rho)} + \frac{\beta N^2}{2} \frac{1}{\mu_0^2} \int d\gamma_1 d\gamma_2 U_{int}(\gamma_1, \gamma_2) \mu(\rho_1) \mu(\rho_2) + \]

\[ + \int dx \frac{N}{\mu_0} \int d\rho \mu(\rho) \rho^2 \zeta B^2 = \]

\[ = \int dx n \left( \frac{1}{\mu_0} \int d\rho \mu(\rho) \ln \frac{\mu(\rho)}{d(\rho)} + \frac{\beta \xi^2}{2} n \left( \frac{\rho}{\mu} \right)^2 + \zeta \rho^2 B^2 \right), \]  
(27)
with \( \zeta = \frac{9}{2} \frac{N_f}{N_c - 1} \), \( \xi^2 = \frac{27}{4} \frac{N_f}{N_c - 1} \pi^2 \), \( \mu_0 = \int d\rho \mu(\rho) \). Here we estimate the functional in the adiabatic (long wave-length) approximation. It means we consider the IL elements of some characteristic size (of the same order of magnitude as the mean distance between pseudo-particles) being equilibrated by the presence of some fixed field \( B \). Then calculating the optimal configurations of pseudo-particles we found out the effective action in the mean field. Eq.(27) is given just in the form underlining that an integration is performed over liquid elements and the proper parameters describing their states could be dependent on the external field, i.e. could be the functions of coordinate \( x \). Physical meaning of such a functional is quite transparent, it implies that each separate element of IL possesses a characteristic aptitude of screening external field assessed by \( U_{ext} \).
Calculating the variation of \( \langle S - S_1 \rangle \) in \( \mu(\rho) \) we have

\[
\mu(\rho) = C \cdot d(\rho) \cdot e^{-(n \xi^2 \rho^2 + \zeta B^2)\rho^2},
\]

where \( C \) is an arbitrary constant and we fix it demanding the coincidence of its value when the external field is absent with its vacuum average. Then

\[
\mu(\rho) = C N \beta N_c \Lambda^{b-5} \cdot e^{-(n \xi^2 \rho^2 + \zeta B^2)\rho^2}.
\]

(28)

and making use the definition of an average as

\[
\overline{\rho^2} = \frac{\int d\rho \rho^2 \mu(\rho)}{\mu_0},
\]

we obtain the practical relation between mean pseudo-particle size and the IL density

\[
(n \beta \xi^2 \overline{\rho^2} + \zeta B^2) \overline{\rho^2} \simeq \nu,
\]

(29)

where \( \nu = \frac{b - 4}{2} \). Apparently, it results in a well-known form of pseudo-particle size distribution

\[
\mu(\rho) = C N \beta N_c \Lambda^{b-5} \cdot e^{-\nu \frac{\rho^2}{\overline{\rho^2}}}.
\]

(30)

Now Eq. (29) allows us to formulate the criterion we are interested in. It looks like \( \zeta B^2 \ll n \beta \xi^2 \overline{\rho^2} \) and for the IL parameters mentioned above it is \( B \ll 400 \text{ MeV} \).
Dealing with Eq.(27) and Eq.(30) the generating functional estimate Eq.(26) may be presented as

\[ Y \geq \int D[B] e^{-S(B)} e^{-F}, \tag{31} \]

where

\[ F = \int dx \ n \left\{ \ln \frac{n}{\Lambda_{QCD}^4} - 1 - \frac{\nu}{2} + \frac{\zeta \rho^2 B^2}{2} - \ln[\Gamma(\nu)C_{N_c} \beta^{2N_c}] - \nu \ln \frac{\rho^2}{\nu} \right\}. \]

Making use of the relation Eq.(29) it is not difficult to find the maximum of functional Eq.(31) in the IL parameters at the fixed $B$ value as a solution of transcendental equation ($\frac{dF}{dp} = 0$). As an information we give the simple expression of its derivative in $n$

\[ F'_n = \ln \frac{n}{\Lambda_{QCD}^4} + \frac{1}{4} \frac{n^2 \xi^4 \beta b (\rho^2)^3}{2n \beta \xi^2 \rho^2 \zeta B^2 - n \xi^2 b^2 \rho^2} - \ln[\Gamma(\nu)C_{N_c} \beta^{2N_c}] - 2N_c \frac{\beta'_n}{\beta} - \nu \ln \frac{\rho^2}{\nu}. \]

Fig.2 and Fig.3 demonstrate the solutions for $\bar{\rho}$ and $n$ at $N_c = 3$ and $N_f = 2$ as the functions of field $B$. Fig.4 shows the plot of free energy density $f/\Lambda_{QCD}^4$ where $F = \int dx \ f$ and convinces IL is steady as to an impact of external field. At strong external field the IL parameters are given by the following asymptotic formulae

\[ \bar{\rho}^2 \approx \frac{\nu}{\zeta B^2} \left( 1 - \frac{n \nu \beta \xi^2}{\zeta^2 B^4} \right), \]

\[ n \approx \frac{\Gamma(\nu)C_{N_c} \beta^{2N_c}}{(\zeta B^2)^{\nu}} \left( 1 + \frac{\Gamma(\nu)C_{N_c} \beta^{2N_c}}{(\zeta B^2)^{\nu}} \frac{N_c b \nu \beta \xi^2}{\zeta^2 B^4} \right). \]
This regime starts somewhere around $B \Lambda_{QCD}^{-1} \sim 10$ at all the plots given.

Thus, the effective action for the $B$ field is given by the following nonlinear functional

$$S_{\text{eff}} = \int dx \left( \frac{G^{\alpha}_{\mu \nu}(B) G^{\alpha}_{\mu \nu}(B)}{4} + f[B] \right). \quad (32)$$

This functional makes possible to calculate the external field as a function of $x$ and IL parameters $\bar{\rho}[B]$ and $n[B]$.

To get any estimate of the IL feedback on the presence of external field could be very practical for instanton liquid model. If so let us try to extract such an estimate from very simple example. Now we will search the minimum of effective action resolving the following boundary value problem

$$\Delta_r B = \frac{d f[B]}{dB}, \quad (33)$$

$$B|_{r=r_0} = p(e) , \quad \nabla_r B|_{r=r_0} = - \frac{e}{4\pi} \frac{1}{r_0^2}. $$

The source intensity here is controlled by $e$, and parameter $r_0$ sets a radius of colour ball which we take as $\sim 0.1\bar{\rho}$ (albeit it is unessential) in order to avoid the difficulties in resolving the singular boundary value problem of Eq. (33). The solution could be accomplished numerically probing such values of potential $p(e)$ which provide with the solution going to zero magnitude at large values of $r$.

The IL density as a function of $r$ is plotted in Fig.5 for ten various quantities of intensity. The extreme right hand side line corresponds to $e/4\pi = 1$ and the extreme left hand side corresponds to
e/4\pi = 0.1$. The same quantity of spacing corresponds to the lines running to the right with intensity increasing. As it was expected the solution has the Yukawa like behaviour which is well seen in Fig.6 where $\ln(Br)$ is plotted as a function of $r$ for four various values of intensity with the pace of 0.1 and $e/4\pi = 1$ for the upper line. Fitting it with the linear function gives the estimate of screening radius which looks as follows

$$R_d \sim (1.24 \Lambda_{QCD})^{-1},$$

Amazingly, this results remains practically unchanged for the whole interval of the intensities from $e/4\pi = 0.1$ to $e/4\pi = 1$ and implies that such a parameter characterizes (at least in this interval of values) the screening properties of IL itself. In a context of the model it looks like rather soft scale for the screening radius and might be taken as another confirmation of adiabatic approximation relevance for the Coulomb external field. Besides it hints the instanton vacuum could provide the significant energy loss of a parton at the later time of collision process contributing (and may be essentially) to jet quenching.

Another point which is not studied here but worth of mentioning concerns the manifestation of instanton ensemble screening properties. It turns out rather unexpected because saturating instanton configuration is randomly oriented in colour space both with external field and with it switched off. Actually, anisotropy in colour space is rooted (and is playing a role analogous to distribution function of colour charge) in the gluon field action of corresponding exponent, namely in the ‘mixed component of gluon field. It is intuitively clear other instanton-like solutions (sensitive to the presence...
of external colour field) could be even more adequate configurations. Such solutions were investigated in Ref.[6] and was demonstrated that suitable scale and proper configurations affected by external field (crumpled instantons) could appear, indeed.

Figure 6: $\ln(Br)$ as a function of $r$ for four various solutions. The upper line corresponds to $e/4\pi = 1$. Going down the lines correspond to decreasing $e/4\pi$ with spacing 0.1.

Eventually let us comment on how it is essential that we are dealing with singular (anti-)instanton ensemble as a saturating configuration. Apparently, the screening properties of effective Lagrangian for external field $B$ could be provided by any stochastic configuration of small characteristic size. The assumption of superposition ansatz validity occurs crucial to have all the leading contributions coming from the 'mixed' (repulsive) component of $G(A,B)$ again. Another solution of the problem may appear, of course, in the quantum approach but this discussion is out of this paper scope. Studying the pseudo-particle behaviour while inside (anti-)instanton medium ($n \neq 0$) one could explore the interrelation of two mechanisms (the repulsive interaction and freezing the coupling constant out Ref.[7]) of fixing instanton size.

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