Asymmetric Abel inversion in imaging spectroscopy for tilted TIG arc plasma

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Abstract
An asymmetric Abel inversion processing system was developed to establish a method for diagnosing arc plasma that has an elliptical cross section and is a non-axisymmetric in the direction perpendicular to the observation direction during an arc welding process. This method was applied to an arc plasma in a tilted tungsten inert gas (TIG) welding process. As a result, when this method was applied to the non-axisymmetric intensity distribution due to integration, it was confirmed that the relative error near the central axis and the edge was large regardless of the difference in the cross-sectional shape. In addition, after applying this method to the tilted TIG arc plasma, it was shown that an error of about 350 K occurred in the temperature distribution when emission intensity distribution was converted to the temperature distribution by the Fowler-Milne method. Furthermore, the temperature distribution of the arc plasma obtained in this study was slightly wider than the previous study, while the high-temperature area where the temperature was higher than 14,000 K showed a good agreement with the previous study. Therefore, it was concluded that, by applying this method to the tilted TIG arc plasma, the equivalent result to the three-dimensional emission spectroscopy could be obtained by photographing a tilted arc plasma from two directions.

Keywords
Tilted TIG arc plasma; imaging spectroscopy; asymmetric Abel inversion; elliptical cross section; Fowler-Milne method

1. Introduction
Joining technology has become indispensable in recent industrial world. Especially, arc welding which uses arc plasma as a heat source is a typical joining technology that has supported industry for many years and is widely used in the manufacture of steel bridges and ships and so on. Until now, welding has been performed by skilled technicians using their expertise, but in recent years, with the automation of production processes, the automation of welding using industrial robots has been actively pursued. In order to automate this welding process efficiently and obtain uniform welds of high quality, it is necessary to analyze welding phenomena to be controlled not only qualitatively but also quantitatively and to understand the heat source characteristics of arc plasma. However, it is well known that the heat source characteristics of arc plasma in arc welding are complicated by metal vapor contamination and molten metal droplet transfer [1]. Non-contact spectrometry, which does not affect the arc plasma, is known to be effective in diagnosing plasma conditions, and many measurements have been made in the arc welding process.

Hiraoka et al. [2] calculated the temperature distribution at 1 mm just below the cathode tip in argon arc plasma during TIG welding by spectroscopy using line spectra of argon atom and argon ion. Shigeta et al. [3] calculated the electrical conductivity and electron density distributions from the temperature and iron vapor distributions in gas metal arc welding by image spectroscopy using three sets of spectrometers and a high-speed camera, and suggested the mechanism of spray transfer mode. Although these spectroscopic techniques have advanced understanding of welding arc plasma, they assume that the arc plasma is two-dimensionally axisymmetric in cylindrical coordinates and perform an Abel inversion to convert the integral intensity of the line spectrum into an intensity at the central cross-section of the arc plasma. Therefore, the applicability of this
method has been limited to vertical TIG arc plasmas [4–9], MIG arc plasma [10–14], and a few carbon dioxide arc plasmas [15–17] where the arc plasma shape can be considered axisymmetric.

On the other hand, the arc plasma in the actual construction process can have non-axisymmetric and non-circular cross-sectional shapes due to the tilting of the electrode and groove of workpiece. Three-dimensional emission spectroscopy has been developed as a diagnostic method for such arc plasmas. Three-dimensional emission spectroscopy has succeeded in visualizing more complex welding arc phenomena such as multi-electrode welding, which could not be handled by conventional methods based on two-dimensional measurements. However, this method places multiple cameras in a fan-like configuration to measure the object, which causes spatial restrictions [18]. Therefore, if diagnostics of arc plasma in the actual welding process can be performed using image spectroscopy, which has fewer spatial constraints, it will be possible to clarify the heat source characteristics of arc plasma under a wider range of welding conditions, leading to a better understanding of welding arc phenomena. The Abel inversion processing, which takes into account non-axisymmetry, has been applied in two-dimensional spectroscopic methods [19–21]. However, these methods are assumed to be applied to torus plasmas such as tokamaks, and there are no examples of their application to welding arc plasmas, in which the horizontal cross-sectional shape may be other than a perfect circle.

Therefore, the purpose of this study is to investigate whether the asymmetric Abel inversion can consider the non-axisymmetry due to torch tilt in image spectroscopy for tilted TIG arc plasma. In addition to the non-axisymmetric shape of the plasma in a tilted TIG arc plasma, it is necessary to take into account that the horizontal cross-sectional shape is elliptical [22]. Therefore, an asymmetric Abel inversion processing system that can consider the above two factors is developed, and its validity is verified.

2. Measurement method

2.1. Abel inversion processing considering non-axisymmetry

Here details of the Abel inversion that takes into account non-axisymmetry used in this study is described, using the coordinate system shown in Figure 1(a). On the screen shown in Figure 1(a), the integrated intensity is recorded along the red arrows, and Figure 1(b) is an example of the integrated intensity recorded at the screen position shown in Figure 1(a). Yasutomo et al. showed that in a distribution where the integrated intensity was asymmetric only in the y direction, the local line spectral intensity could be separated into the local line spectral intensity calculated assuming that the arc plasma was axisymmetric and a weight function representing the asymmetry [19]. That is, if $e(r_1)$ is the local line spectral intensity at radius $r_1$ under the assumption of axisymmetry and $g(y_1)$ is the weight function representing asymmetry at $y$-coordinate $y_1$ of that radius, the local line spectral intensity $n(r_1, y_1)$ in the non-axisymmetric case can be expressed as follows.

$$n(r_1, y_1) = e(r_1)g(y_1)$$

Where the weight function $g(y_1)$ for the asymmetry is expressed.

$$g(y_1) = 1 + \frac{I''(y_1)}{I'(y_1)}$$

$$I'(y_1) = \frac{I(y_1) + I(-y_1)}{2}$$

$$I''(y_1) = \frac{I(y_1) - I(-y_1)}{2}$$

Figure 1. Schematic illustration of asymmetrical Abel inversion.
However, under the assumption that the x-direction is symmetrical, the spectral image obtained can be compared with. By applying Equation (1) to the camera-photographed integrated intensity in a cross-section through the center of the arc plasma. It is converted to an intensity, the integral intensity captured by the camera is converted to the line spectral intensity in the cross section through the center of the arc plasma. This process is repeated in the height direction z to calculate the line spectral intensity distribution in the two-dimensional yz plane.

2.2. Abel inversion processing considering elliptical cross section

The previous section describes a method for calculating local line spectral intensities from non-axisymmetric integrated intensities. Yasutomo et al. did not consider the cross-sectional shape when calculating the local line spectrum intensity w(r1) under the assumption of axisymmetry, while a circular cross-section was assumed and applied in previous studies [19–21]. On the other hand, in tilted TIG arc plasma, the horizontal cross-section of the plasma is elliptical, and this elliptical cross-section must be taken into account in the Abel inversion. In this study, the Abel inversion can take into account elliptical cross-sections by incorporating a local line spectral intensity w that can assume the cross-sectional shape. In this section, an overview of the local line spectral intensities that can assume the cross-sectional shape is explained. When the cross-sectional shape of the arc plasma is elliptical, the integrated intensity I’y) of the line spectrum at any position y is expressed by adding the product of the local spectral intensity w and the small area ΔxΔy, as shown in Figure 2.

\[
I’(y)Δy = \sum_{m=-N}^{N} e_{m}x_{m}Δy
\]  
(5)

Here, the minute area S in the first quadrant of the ellipse: \(x^2/(b' l)^2 + y^2/(a l)^2 = 1(0 < l \leq 1)\) shown in Figure 3 where the intersection with the x axis is \((±b l, 0)\) and the intersection with the y axis is \((0, ±a l)\) is given by the following equation [23].

\[
S_{mn} = \left\{ \begin{array}{ll}
(P_{e_{m}+1,e_{m}} - P_{e_{m}-1,e_{m}}) & (n \leq m) \\
0 & (n > m)
\end{array} \right.
\]  
(6)

\[
P_{e_{m}} = \left\{ \begin{array}{l}
\frac{π}{4} a m b m - \frac{1}{2} a m b m \arctan \left( \frac{b m \sin \theta_{e_{m}}}{a m \cos \theta_{e_{m}}} \right) \quad (n \leq m) \\
0 & (n > m)
\end{array} \right.
\]  
(7)

\[
x_{0} = \begin{cases}
\frac{b m \sqrt{1 - \frac{a m^2 - 1}{a m^2}}}{a m} & (n \leq m) \\
0 & (n > m)
\end{cases}
\]  
(8)

\[
\theta_{e_{m}} = \arctan \left( \frac{a m - 1}{x_{0}} \right) & (n \leq m)
\]  
(9)

Where \(e_{k}\) be the spectral intensity at \(S_{ak}\), then Equation (5) can be rewritten as follows [23].

\[
\begin{align*}
I’_{1} &= 2(S_{11}e_{1} + S_{12}e_{2} + \cdots + S_{1k}e_{k})/d_{a} \\
I’_{2} &= 2(S_{22}e_{2} + S_{23}e_{3} + \cdots + S_{2k}e_{k})/d_{a} \\
&\vdots \\
I’_{k} &= 2(S_{kk}e_{k})/d_{a}
\end{align*}
\]  
(10)

Where \(d_{a}\) represents spatial resolution. Furthermore, rewriting Equation (10) in matrix form gives.

\[
\begin{bmatrix}
S_{11} & S_{12} & \cdots & S_{1k} \\
0 & S_{22} & \cdots & S_{2k} \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & S_{kk}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{1} \\
\varepsilon_{2} \\
\vdots \\
\varepsilon_{k}
\end{bmatrix} = \frac{d_{a}}{2}
\begin{bmatrix}
I’_{1} \\
I’_{2} \\
\vdots \\
I’_{k}
\end{bmatrix}
\]  
(11)

By calculating the inverse matrix of the matrix Sij, the Equation (11) can be represented by a simple linear matrix equation.

\[
[\varepsilon] = \frac{d_{a}}{2} [S]^{-1} [I’]
\]  
(12)
2.3. Temperature measurement method

This section explains how to convert the line spectral intensity distribution obtained in the previous section into a temperature distribution. In this study, the Fowler-Milne method (off-Axis maximum radiation coefficient method) [24] is used to measure the temperature distribution in plasma. The Fowler-Milne method finds the maximum value of radiation intensity at a distance from the arc center and normalizes all measurements to that maximum value. Since the spectral intensity is normalized by the maximum intensity value, data such as transition probabilities are not required, and temperatures can be determined relatively easily. The intensity \( I_{ji} \) of the line spectrum emitted by an excited particle’s outer nuclear electron as it transitions from the \( j \) energy level to the \( i \) lower level is expressed as Equation (13) [24].

\[
I_{ji} = \frac{A_{ji}hc}{4\pi\lambda_{ji}} \frac{N_0g_i}{Z(T)} \cdot \exp\left(-\frac{E_j}{kT}\right)
\]

\( A_{ji} \) is the probability that the extranuclear electrons of the excitation particles will transition from the \( j \) energy level to the \( i \) lower level, \( h \) is the Planck constant, \( c \) is the speed of light, and \( \lambda_{ji} \) is spectral wavelength when transitioning from the \( j \) energy level to the \( i \) lower level, \( N_0 \) is the particle number density of the atom or ion, \( g_i \) is the statistical weight, \( Z(T) \) is the partition function at temperature \( T \), \( E_j \) is the energy level of \( j \) th, and \( k \) is the Boltzmann constant. \( A_{ji} \), \( g_i \), and \( E_j \) were using NIST data [25]. Assuming that the plasma is in local thermal equilibrium (LTE), the number of particles density at each temperature can be obtained from Saha’s thermo-ionizing equilibrium equation [26]. After assigning these values to Equation (13), the calculated results are normalized by the maximum radiation intensity to obtain a relationship between the relative intensity ratio and temperature.

As an example, Figure 4 shows the relationship between relative intensity ratio and temperature in the line spectrum of argon atoms (696.5 nm) [27]. By using such a relationship, it is possible to determine the temperature distribution from the line spectral intensity distribution of a tilted TIG arc plasma.

2.4. Experimental conditions

Table 1 shows experimental conditions. In this study, the static TIG arc on a water-cooled copper plate was used as the measurement target. Pure argon was used as the shielding gas, and a tungsten electrode with thoria (electrode diameter 3.2 mm, tip angle 60°) was used as the cathode, and the welding current was 150 A and the arc length was 5 mm. Figure 5(a) shows the experimental setup used in this study, and
Figure 5(b) shows a schematic diagram of the TIG torch and arc plasma viewed from the high-speed camera with spectrometer. The black dashed line in Figure 5(a) shows the light path. In this experiment, the radiation from the arc plasma on the base metal was focused by an objective lens and formed an image by a relay lens. The imaged radiation was split into two light paths by a half-mirror. Each of the bifurcated imaged radiations was spectrally split into the wavelengths of interest inside the spectrometer. A high-resolution image was obtained by capturing the light at the spectral wavelengths of the line spectrum with a 12-bit (4096-levels) high-speed camera.

The instrument also has two sets of spectrographs and a high-speed camera. These combinations made it possible to measure not only element-specific line spectra but also continuous spectra simultaneously. Since the measurement of argon arc plasma had the problem of simultaneously capturing the continuous spectrum in addition to the line spectrum of argon atoms [9], the continuous spectrum intensity was subtracted from the line spectrum intensity to obtain results with higher accuracy. Furthermore, as shown in Figure 5(a), a high-speed camera equipped with a bandpass filter was used to capture images of the arc plasma from a 90-transverse direction to the imaging spectrometer system. The cross-sectional shape of the arc plasma was estimated by obtaining the value of $a$ from the image taken by the high-speed camera on the spectrograph side and the value of $b$ from the image taken by the high-speed camera with bandpass filter.

3. Experimental results and discussion

3.1. Verifying the validity of the method

In order to verify the validity of this method, the following equation was given as a local value as test data. Figure 6(a) is the distribution of local values. Also, at this time the integral value was calculated assuming the ellipse...
(a, b) = (1.0, 2.0). Figure 6(b) shows the integral of the local values shown in Figure 6(a).

\[ n(l, y) = \frac{1}{b} \left[ 1 + 0.5\sin(\pi y) \right] \left( 1 + 10l^2 - 23l^4 + 12l^6 \right) \]  \hspace{1cm} (14)

\[ \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad (0 < l \leq 1) \]  \hspace{1cm} (15)

\[ I(y) = \frac{16}{105} \left[ 1 + 0.5\sin(\pi y) \right] \left( 19 + 34y^2 - 125y^4 + 72y^6 \right)(1 - y^2)^{0.5} \]  \hspace{1cm} (16)

Integrated over the assumed elliptical section, denoted by Figure 6(c) is the result obtained by applying this method to Equation (16), which is accurate compared with Figure 6(a). From this, it is considered that the distribution reconstructed by this method has its validity. Also, as shown in Figure 7(a), the error increases for all cross-sectional shapes are consistent, and the error tends to be larger near the central axis and the boundary. However, the error near the boundary of the intensity distribution is acceptable because the difference in intensity values is small, as shown in Figure 7(b), even though the relative error is large, because the local values are quite small near the boundary. In addition, although the relative error is slightly larger at the center, it is less than 5% for all locations except for those near the boundary. Based on the above, it is considered that this method is applicable to non-axisymmetric arc plasma with an elliptical shape.

### 3.2. Study on application to inclined TIG-arc plasma

Figure 9(a) shows the line spectral intensity distribution of TIG-arc plasma with the electrode tilted 30°, 1 mm below the electrode tip. The method was verified using the intensity distribution shown in Figure 9(b), which was a polynomial approximation of the intensity distribution of the line spectrum of this tilted TIG arc plasma, as a test curve. The polynomial was as follows.

\[ I(y) = \frac{164}{27} \left[ 1 + 0.5\sin(\pi y) \right] \left[ 1 + \cos[0.5\pi(y' - 1)] \right] \]  
\[ \left( 19 + 34y'^2 - 125y'^4 + 72y'^6 \right)(1 - y'^2)^{0.5} \]  \hspace{1cm} \text{while } y' = \frac{y}{\sqrt{3}} - 1 \]  \hspace{1cm} (17)

Figure 10 shows the distributions normalized by the maximum intensity after applying this method. The solid black line in the figure is a distribution in which the local value obtained by calculating backwards so that the integral value is Figure 9(b) is corrected at maximum intensity,
and is given by the following equation. Note that the ratio of the major axis to the minor axis of the elliptical cross section was taken from an image taken that \( a : b \) was 0.125:0.0832.

\[
n(l, y) = \frac{40}{23} \left[ 1 + 0.5\sin(\pi y') \right] \left[ 1 + \cos(0.5\pi(y' - 1)) \right] \\
(1 + 10l^2 - 23l^4 + 12l^6) \text{ while } y' = \frac{y}{7.5} - 1
\]

(18)

The dashed red line is the distribution of calculated values obtained by applying this method to Equation (17). The intensity distribution after asymmetric Abel inversion obtained by applying this method showed results that were relatively close to the distribution of Equation (18). In addition, the error was up to about 0.1, and an error of about 350 K was generated by converting it to temperature by the Fowler-Milne method. Hiraoka et al. [2] indicated that TIG arc plasma was high-temperature in excess of 15,000 K, so an error of 350 K seemed to be acceptable.

### 3.3. Results of application to tilted TIG arc plasma

Figure 11(a) shows the results of applying this method to the spectral image obtained. The gray areas indicate the electrode and base metal. Figure 11(b) shows the \( y \)-axis distribution of the local luminescence intensity obtained by this method, just below 1 mm from the electrode tip indicated by the dashed line in Figure 11(a). A dash line shown in Figure 11(b) indicates a central axis of arc plasma. From Figure 11(b), the emission intensity decreased near the central axis of the plasma, and a peak of emission intensity could be seen at a position slightly away from the center. Figure 12(a) shows the temperature distribution of a tilted TIG arc plasma calculated by applying the Fowler-Milne method [2,9,24] to Figure 11(a). The reconstructed luminescence intensity distribution was normalized for each height because a clear line of maximum intensity could not be found in the reconstructed luminescence intensity distribution in this study. If the maximum intensity was found at a position far from the central axis, the intensity was normalized to the lower peak intensity (peak intensity on the right side of the dashed line in Figure 11(b)). The gray areas indicate the electrode and base metal. From measurement result, maximum temperature of arc plasma was about 16,500 K. Figure 12(b) shows a comparison of the \( y \)-axis temperature distribution just 1 mm below the tip of the electrode from this experiment and the results of Nomura et al. [28]. Although the temperature distribution of the arc plasma in this experiment was slightly wider in the \( y \) direction than that of Nomura et al, the temperature profiles were in good agreement, especially in the high-temperature region above 14,000 K, indicating that the present method was reasonable. This also suggested that only two-directional imaging was required to obtain the same level of results as three-dimensional emission spectroscopy.
4. Conclusions

In this study, an asymmetric Abel inversion process was developed to diagnose non-axisymmetric arc plasma in practical arc welding. The applicability of the method was examined using a tilted arc plasma in an actual TIG welding condition. The conclusions obtained in this study are shown below.

1. The developed asymmetric Abel inversion process was applied to the intensity distribution of non-axisymmetric integrals. As a result, the relative error tended to be larger near the central axis and at the outer edges, regardless of the cross-sectional geometry. However, since the local value was small at the outer edge, it was judged that the error of the outer edge was acceptable. The error near the central axis was less than 5%. In consequence, this method is applicable to nonaxially symmetric arc plasma with elliptical cross sections.

2. After applying this method to the tilted TIG arc plasma, an error of about 350 K was observed in the temperature fraction obtained by the Fowler-Milne method. However, this error magnitude was sufficiently small compared with the temperature of the TIG arc plasma. It was an acceptable range of the error.

3. The temperature distribution of the arc plasma in this experiment was slightly wider than that of the previous study. On the other hand, the maximum temperatures were equivalent for both of the experiments. Especially in the high temperature region exceeding 14,000 K, the temperature distributions showed a good agreement. This suggested that the developed method could be applied to tilted TIG arc plasma to obtain results comparable to three-dimensional emission spectroscopy only by photographing from two directions.

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