An Automatically Verified Prototype of a Landing Gear System

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Abstract. In this paper we show how \{log\} (read ‘setlog’), a Constraint Logic Programming (CLP) language based on set theory, can be used as an automated verifier for B specifications. In particular we encode in \{log\} an Event-B specification, developed by Mammar and Laleau, of the case study known as the Landing Gear System (LGS). Next we use \{log\} to discharge all the proof obligations proposed in the Event-B specification by the Rodin platform. In this way, the \{log\} program can be regarded as an automatically verified prototype of the LGS. We believe this case study provides empirical evidence on how CLP and set theory can be used in tandem as a vehicle for program verification.

1 Introduction

In the fourth edition of the ABZ Conference held in Toulouse (France) in 2014, ONERA’s Boniol and Wiels proposed a real-life, industrial-strength case study, known as the Landing Gear System (LGS) [1]. The initial objectives of the proposal were “to learn from experiences and know-how of the users of the ASM, Alloy, B, TLA, VDM and Z methods” and to disseminate the use of verification techniques based on those methods, in the aeronautic and space industries. The LGS was the core of an ABZ track of the same name. The track received a number of submissions of which eleven were published [1]. Six of the published papers approached the problem with methods and tools rooted in the B notation [2] (Event-B, ProB, Hybrid Event-B and Rodin). In this paper we consider the article by Mammar and Laleau [3] and a journal version [4] as the starting point for our work. In those articles, the authors use Event-B as the specification language and Rodin [5], ProB [6] and AnimB [3] as verification tools.

The B method was introduced by Abrial [2] after his work on the Z notation [7]. B is a formal notation based on state machines, set theory and first-order logic aimed at software specification and verification. Verification is approached by discharging proof obligations generated during specification refinement. That is, the engineer starts with a first, abstract specification and refines it into a second, less abstract specification. In order to ensure that the refinement step
is correct a number of proof obligations must be discharged. This process is continued until an executable program is obtained. Given that all refinements have been proved correct, the executable program is correct by construction.

Discharging a proof obligation entails to perform a formal proof (most often) in the form of a mechanized proof. A formal proof is a proof of a mathematical theorem; a mechanized proof is a formal proof made by either an interactive theorem prover or an automated theorem prover\(^4\). That is, mechanized proofs are controlled, guided or verified by a program. Unless there are errors in the verification software, a mechanized proof is considered to be error-free because those programs are supposed to implement only sound proof steps. The mechanization of proofs is important for at least two reasons: errors cannot be tolerated in safety-critical systems and mechanization enables the possibility of proof automation which in turn reduces verification costs.

Our work starts by considering the Event-B specification of the LGS developed by Mammar and Laleau. Event-B [8] is a further development over B aimed at modeling and reasoning about discret-event systems. The basis of B are nonetheless present in Event-B: state machines, set theory, first-order logic, refinement and formal proof. The Event-B specification developed by Mammar and Laleau has an important property for us. They used the Rodin platform to write and verify the model. This implies that proof obligations were generated by Rodin according to a precise and complete algorithm [5].

In this paper we consider the LGS case study and Mammar and Laleau’s Event-B specification as a benchmark for \{log\} (read ‘setlog’). \{log\} is a Constraint Logic Programming (CLP) language and satisfiability solver based on set theory. As such, it can be used as a model animator and as an automated theorem prover. Since \{log\} is based on set theory, it should also be a good candidate to encode Event-B (or classic B) specifications; since it implements several decision procedures for set theory, it should be a good candidate to automatically discharge proof obligations generated from refinement steps of Event-B (or classic B) specifications.

Therefore, we proceed as follows: a) Mammar and Laleau’s Event-B specification is encoded as a \{log\} program; b) all the proof obligations generated by the Rodin platform are encoded as \{log\} queries; and c) \{log\} is used to automatically discharge all these queries. We say ‘encode’ and not ‘implement’ due to the similarities between the \{log\} language and the mathematical basis of the Event-B language; however, the encoding provides an implementation in the form of a prototype.

The contributions of this paper are the following:

– We provide empirical evidence on how CLP and set theory can be used in tandem as a vehicle for program verification. More specifically, \{log\} is shown to work well in practice.

\(^4\) In this context the term ‘automated theorem prover’ includes tools such as satisfiability solvers.
Given that the \{log\} prototype of the LGS has been (mechanically and automatically) proved to verify a number of properties, it can be regarded as correct w.r.t. those properties.

The paper is structured as follows. In Sect. 2 and 3 we introduce \{log\} by means of several examples. In particular, in Sect. 3 we shown the formula-program duality enjoined by \{log\}. Section 4 presents the encoding of the Event-B specification of the LGS in \{log\}. The encoding of the proof obligations generated by the Rodin tool is introduced in Sect. 5. Finally, we discuss our approach in Sect. 6 and give our conclusions in Sect. 7.

2 Overview of \{log\}

\{log\} is a publicly available satisfiability solver and a declarative set-based, constraint-based programming language implemented in Prolog [9]. \{log\} is deeply rooted in the work on Computable Set Theory [10], combined with the ideas put forward by the set-based programming language SETL [11].

\{log\} implements various decision procedures for different theories on the domain of finite sets and integer numbers. Specifically, \{log\} implements: a decision procedure for the theory of hereditarily finite sets (SET), i.e., finitely nested sets that are finite at each level of nesting [12]; a decision procedure for a very expressive fragment of the theory of finite set relation algebras (BR) [13, 14]; a decision procedure for the theory SET extended with restricted intensional sets (RIS) [15]; a decision procedure for the theory SET extended with cardinality constraints (CARD) [16]; a decision procedure for the latter extended with integer intervals (INTV) [17]; and integrates an existing decision procedure for the theory of linear integer arithmetic (LIA). All these procedures are integrated into a single solver, implemented in Prolog, which constitutes the core part of the \{log\} tool. Several in-depth empirical evaluations provide evidence that \{log\} is able to solve non-trivial problems [13–15, 18]; in particular as an automated verifier of security properties [19, 20].

2.1 The theories

Figure 1 schematically describes the stack of the first-order theories supported by \{log\}. The fact that theory $T$ is over theory $S$ means that $T$ extends $S$. For example, CARD extends both LIA and SET.

LIA provides integer linear arithmetic constraints (e.g., $2 \cdot X + 5 \leq 3 \cdot Y - 2$) by means of Prolog’s CLP(Q) library [21]. SET [12] provides the Boolean algebra of hereditarily finite sets; that is, it provides equality (=), union (un), disjointness (disj), and membership (in). In turn, these operators enable the definition of other operators, such as intersection and relative complement, as SET formulas. In all set theories, set operators are encoded as atomic predicates, and are dealt with as constraints. For example, un($A$, $B$, $C$) is a constraint interpreted as $C = A \cup B$. CARD [16] extends SET and LIA by providing the cardinality operator
(size) which allows to link sets with integer constraints⁵ (e.g., size(A, W) & W − 1 ≤ 2 * U + 1). RIS [15] extends SET by introducing the notion of restricted intensional set (RIS) into the Boolean algebra. RIS are finite sets defined by a property. For example, \( x \in \{ y : A \mid \phi(y) \} \), where \( A \) is a set and \( \phi \) is a formula of a parameter theory \( \mathcal{X} \). BR [13, 14] extends SET by introducing ordered pairs, binary relations and Cartesian products (as sets of ordered pairs), and the operators of relation algebra lacking in SET—identity (id), converse (inv) and composition (comp). For example, \( \text{comp}(R, S, \{[X, Z]\}) \& \text{inv}(R, T) \& [X, Y] \in T \). In turn, BR allows the definition of relational operators such as domain, range and domain restriction, as BR formulas. INTV [17] extends CARD by introducing finite integer intervals thus enabling the definition of several non-trivial set operators such as the minimum of a set and the partition of a set w.r.t. a given number. ARRAY and LIST are still work in progress. They should provide theories to (automatically) reason about arrays and lists from a set theoretic perspective. For example, \( A \) is an array of length \( n \) if is a function with domain in the integer interval⁷ \( \text{int}(1, n) \). Then, in ARRAY it is possible to define the predicate \( \text{array}(A, n) \equiv \text{pfun}(A) \& \text{dom}(A, \text{int}(1, n)) \), where \( \text{pfun} \) and \( \text{dom} \) are predicates definable in BR. However, it is still necessary to study the decidability of such an extension to BR + INTV as, at the bare minimum, it requires for \( \text{dom} \) to deal with integer intervals. The following subsections provide some clarifications on the syntax and semantics of the logic languages on which these theories are based on.

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⁵ ‘&’ stands for conjunction (\( \land \)); see Sect. 2.4.
⁶ \([x, y]\) stands for the ordered pair \((x, y)\).
⁷ \(\text{int}(a, b)\) stands for the integer interval \([a, b]\); see Sect. 2.2.
2.2 Set terms

The integrated constraint language offered by \{\text{log}\} is a first-order predicate language with terms of three sorts: terms designating sets (i.e., set terms), terms designating integer numbers, and terms designating ur-elements (including ordered pairs, written as \([x,y]\)). Terms of either sort are allowed to enter in the formation of set terms (in this sense, the designated sets are hybrid), no nesting restrictions being enforced (in particular, membership chains of any finite length can be modeled).

Set terms in \{\text{log}\} can be of the following forms:

- A variable is a set term; variable names start with an uppercase letter.
- \{\}
  - is the term interpreted as the empty set.
- \{t/A\}, where \(A\) is a set term and \(t\) is any term accepted by \{\text{log}\} (basically, any Prolog uninterpreted term, integers, ordered pairs, other set terms, etc.), is called extensional set and is interpreted as \(\{t\} \cup A\). As a notational convention, set terms of the form \(\{t_1/t_1 \ldots t_n/t\}\) are abbreviated as \(\{t_1,t_2\ldots t_n/t\}\).
- \text{ris}(X\ \text{in}\ A, \phi)\), where \(\phi\) is any \{\text{log}\} formula, \(A\) is a any set term among the first three, and \(X\) is a bound variable local to the \text{ris} term, is called restricted intensional set (RIS) and is interpreted as \(\{x : x \in A \land \phi\}\). Actually, RIS have a more complex and expressive structure [15].
- \text{cp}(A,B)\), where \(A\) and \(B\) are any set term among the first three, is interpreted as \(A \times B\), i.e., the Cartesian product between \(A\) and \(B\).
- \text{int}(m, n)\), where \(m\) and \(n\) are either integer constants or variables, is interpreted as \(\{x \in \mathbb{Z} : m \leq x \leq n\}\).

Set terms can be combined in several ways: binary relations are hereditarily finite sets whose elements are ordered pairs and so set operators can take binary relations as arguments; RIS and integer intervals can be passed as arguments to \text{SET} operators and freely combined with extensional sets.

2.3 Set and relational operators

\{\text{log}\} implements a wide range of set and relational operators covering most of those used in B. Some of the basic operators are provided as primitive constraints. For instance, \text{pfun}(F)\) constrains \(F\) to be a (partial) function; \text{dom}(F, D)\) corresponds to \(\text{dom} F = D\); \text{subset}(A,B)\) corresponds to \(A \subseteq B\); \text{comp}(R, S, T)\) is interpreted as \(T = R \circ S\) (i.e., relational composition); and \text{apply}(F, X, Y)\) is equivalent to \text{pfun}(F) \& \[X, Y\] \text{in} F.\)

A number of other set, relational and integer operators (in the form of predicates) are defined as \{\text{log}\} formulas, thus making it simpler for the user to write complex formulas. Dovier et al. [12] proved that the collection of predicate symbols implementing \(\{=, \neq, \in, \notin, \cup, \cap\}\) is sufficient to define constraints for the set operators \(\cap, \subseteq\) and \(\setminus\). This result has been extended to binary relations [13] by
showing that adding to the previous collection the predicate symbols implementing \{id, :, \} is sufficient to define constraints for most of the classical relational operators, such as dom, ran, \(<, >, \leq\), etc. Similarly, \{=, \neq, \leq\} is sufficient to define \(<, >\) and \(\geq\). We call predicates defined in this way derived constraints.

\{log\} provides also so-called negated constraints. For example, \text{num}(A, B, C)\) is interpreted as \(C \neq A \cup B\) and \text{nin}\) corresponds to \(\notin\) — in general, a constraint beginning with ‘\text{n}’ identifies a negated constraint. Most of these constraints are defined as derived constraints in terms of the existing primitive constraints; thus their introduction does not really require extending the constraint language.

### 2.4 \{log\} Formulas

Formulas in \{log\} are built in the usual way by using the propositional connectives (e.g., \&, \|, or\) between atomic constraints. More precisely, \{log\} formulas are defined according to the following grammar:

\[
F ::= \text{true} | \text{false} | C | F \& F | F \| F | P | \text{neg}(F) | F \text{ implies } F | Q(F)
\]

where: \(C\) is any \{log\} atomic constraint; \(P\) is any atomic predicate which is defined by a Horn clause of the form:

\[
H ::= P | P :- F
\]

and the user-defined predicates possibly occurring in the body of the clause do not contain any direct or indirect recursive call to the predicate in \(P\); \(Q(F)\) is the quantified version of \(F\), using either restricted universal quantifiers (RUQ) or restricted existentially quantifiers (REQ). The definition and usage of RUQ and REQ in \{log\} will be discussed in Section 4.4. It is worth noting that fresh variables occurring in the body of a clause but not in its head are all implicitly existentially quantified.

\text{neg} computes the propositional negation of its argument. In particular, if \(F\) is an atomic constraint, \text{neg}(F)\) returns the corresponding negated constraint. For example, \text{neg}(x \text{ in } A \& z \text{ nin } C)\) becomes \(x \text{ nin } A\) or \(z \text{ in } C\). However, the result of \text{neg} is not always correct because, in general, the negated formula may involve existentially quantified variables, whose negation calls into play (general) universal quantification that \{log\} cannot handle properly. Existentially quantified variables may appear in the bodies of clauses or in the formula part of RIS and RUQ. Hence, there are cases where \{log\} users must manually compute the negation of some formulas.

The same may happen for some logical connectives, such as \text{implies}, whose implementation uses the predicate \text{neg}.\footnote{Indeed, \(F \text{ implies } G\) is implemented in \{log\} as \text{neg}(F) \text{ or } G.}

### 2.5 Types in \{log\}

\{log\} is an untyped formalism. This means that, for example, a set such as \{(x, y), 1, ‘red’\} is accepted by the language. However, recently, a type system
and a type checker have been defined and implemented in \{log\}. Typed as well as untyped formalisms have advantages and disadvantages [22]. For this reason, \{log\} users can activate and deactivate the typechecker according to their needs. \{log\} types are defined according to the following grammar:

\[
\tau := \text{int} \mid \text{str} \mid \text{Atom} \mid \text{etype}([\text{Atom}, \ldots, \text{Atom}]) \mid [\tau, \ldots, \tau] \mid \text{stype}(\tau)
\]

where \text{Atom} is any Prolog atom other than \text{int} and \text{str}. \text{int} corresponds to the type of integer numbers; \text{str} corresponds to the type of Prolog strings; if \text{atom} \in \text{Atom}, then it defines the type given by the set \{\text{atom}\?t \mid t \text{ is a Prolog atom}\}, where '?' is a functor of arity 2. \text{etype}(c_1, \ldots, c_n), with 2 \leq n, defines an enumerated type; \text{type}([T_1, \ldots, T_n]) with 2 \leq n defines the Cartesian product of types \text{T_1}, \ldots, \text{T_n}; and \text{stype} (T) defines the powerset type of type \text{T}. As can be seen, this type system is similar to B’s.

When in typechecking mode, all variables and user-defined predicates must be declared to be of a precise type. Variables are declared by means of the \text{dec} (V, \tau) constraint, meaning that variable \text{V} is of type \tau. In this mode, every \{log\} atomic constraint has a polymorphic type much as in the B notation. For example, in \text{comp}(R, S, T) the type of the arguments must be \text{stype}([\tau_1, \tau_2]), \text{stype}([\tau_2, \tau_3]) and \text{stype}([\tau_1, \tau_3]), for some types \tau_1, \tau_2, \tau_3, respectively.

Concerning user-defined predicates, if the head of a predicate is \text{p}(X_1, \ldots, X_n), then a predicate of the form := \text{dec_p_type}(\text{p}(\tau_1, \ldots, \tau_n)), where \tau_1, \ldots, \tau_n are types, must precede \text{p}’s definition. This is interpreted by the typechecker as \text{X_i} is of type \tau_i in \text{p}, for all i \in 1 \ldots n. See an example in Sect. 4.1.

More details on types in \{log\} can be found in the user’s manual [23].

## 2.6 Constraint solving

As concerns constraint solving, the \{log\} solver repeatedly applies specialized rewriting procedures to its input formula \(\Phi\) and returns either \text{false} or a formula in a simplified form which is guaranteed to be satisfiable with respect to the intended interpretation. Each rewriting procedure applies a few non-deterministic rewrite rules which reduce the syntactic complexity of primitive constraints of one kind. At the core of these procedures is set unification [24]. The execution of the solver is iterated until a fixpoint is reached, i.e., the formula is irreducible.

The disjunction of formulas returned by the solver represents all the concrete (or ground) solutions of the input formula. Any returned formula is divided into two parts: the first part is a (possibly empty) list of equalities of the form \(X = t\), where \(X\) is a variable occurring in the input formula and \(t\) is a term; and the second part is a (possibly empty) list of primitive constraints.

## 3 Using \{log\}

In this section we show examples on how \{log\} can be used as a programming language (3.1) and as an automated theorem prover (3.2). The main goal is to provide examples of the \text{formula-program duality} enjoined by \{log\} code.
3.1 \{log\} as a programming language

\{log\} is primarily a programming language at the intersection of declarative programming, set programming \cite{11} and constraint programming. Specifically, \{log\} is an instance of the general CLP scheme. As such, \{log\} programs are structured as a finite collection of clauses, whose bodies can contain both atomic constraints and user-defined predicates. The following example shows the program side of the formula-program duality of \{log\} code along with the notion of clause.

Example 1. The following clause corresponds to the Start\_GearExtend event of the Doors machine of the Event-B model of the LGS. It takes four arguments and the body is a \{log\} formula.

```prolog
start\_GearExtend(PositionsDG, Gear\_ret\_p, Door\_open\_p, Gear\_ret\_p) :-
Poo in PositionsDG &
applyTo(Gear\_ret\_p, Po, true) &
Door\_open\_p = cp(PositionsDG, \{true\}) &
foplus(Gear\_ret\_p, Po, false, Gear\_ret\_p).
```

Note that Po is an existential variable. applyTo and foplus are predicates definable in terms of the atomic constraints provided by BR. For example, applyTo is as follows:

```prolog
applyTo(F, X, Y) :-
F = \{[X, Y]\/G\} \& [X, Y] \nin G \& \comp(\{[X, X]\}, G, \{\}).
```

Now we can call start\_GearExtend by providing inputs and waiting for outputs:

```prolog
Pos = \{front, right, left\} \&
start\_GearExtend(Pos, cp(Pos, \{true\}), cp(Pos, \{true\}), Gear\_ret\_p).
```

returns:

```prolog
Gear\_ret\_p = \{[front, false]\}/cp(\{right, left\}, \{true\})
```

As a programming language, \{log\} can be used to implement set-based specifications (e.g., B or Z specifications). As a matter of fact, an industrial-strength Z specification has been translated into \{log\} \cite{20} and students of a course on Z taught both at Rosario (Argentina) and Parma (Italy) use \{log\} as the prototyping language for their Z specifications \cite{25}. This means that \{log\} can serve as a programming language in which a prototype of a set-based specification can be easily implemented. In a sense, the \{log\} implementation of a set-based specification can be seen as an executable specification.

Remark 1. A \{log\} implementation of a set-based specification is easy to get but usually it will not meet the typical performance requirements demanded by
users. Hence, we see a \{log\} implementation of a set-based specification more as a prototype than as a final program. On the other hand, given the similarities between a specification and the corresponding \{log\} program, it is reasonable to think that the prototype is a correct implementation of the specification\(^9\).

For example, in a set-based specification a table (in a database) with a primary key is usually modeled as a partial function, \(t : X \rightarrow Y\). Furthermore, one may specify the update of row \(r\) with data \(d\) by means of the \oplus\ or override (⊕) operator: \(t' = t \oplus \{ r \mapsto d\}\). All this can be easily and naturally translated into \{log\}: \(t\) is translated as variable \(T\) constrained to verify \(\text{pfun}(T)\), and the update specification is translated as \(\text{oplus}(T, \{ [R, D] \}, T)\). However, the \text{oplus}\ constraint will perform poorly compared to the \text{update}\ command of SQL, given that \text{oplus}'s implementation comprises the possibility to operate in a purely logical manner with it (e.g., it allows to compute \(\text{oplus}(T, \{ [a, D] \}, \{ [a, 1], [b, 3] \})\) while \text{update} does not).

Then, we can use these prototypes to make an early validation of the requirements. Validating user requirements by means of prototypes entails executing the prototypes together with the users so they can agree or disagree with the behavior of the prototypes. This early validation will detect many errors, ambiguities and incompleteness present in the requirements and possible misunderstandings or misinterpretations generated by the software engineers. Without this validation many of these issues would be detected in later stages of the project thus increasing the project costs.

\[3.2\] \{log\} as an automated theorem prover

\{log\} is also a satisfiability solver. This means that \{log\} is a program that can decide if formulas of some theory are satisfiable or not. In this case the theory is the combination of the decidable (fragments of the) theories of Fig. 1.

Being a satisfiability solver, \{log\} can be used as an automated theorem prover and as a counterexample generator. To prove that formula \(\phi\) is a theorem, \{log\} has to be called to prove that \(\neg \phi\) is unsatisfiable.

**Example 2.** We can prove that set union is commutative by asking \{log\} to prove the following is unsatisfiable:

\[
\neg (\text{un}(A, B, C) \& \text{un}(B, D, A) \implies C = D).
\]

\{log\} first applies \text{neg} to the formula, returning:

\[
\text{un}(A, B, C) \& \text{un}(B, A, D) \& C \neq D
\]

As there are no sets satisfying this formula, \{log\} answers \text{no}. Note that the initial formula can also be written as: \(\neg (\text{un}(A, B, C) \implies \text{un}(B, A, C))\). In this case, the result of applying \text{neg} uses the \text{nun} constraint, \(\text{un}(A, B, C) \& \text{nun}(B, A, C)\).

\(^9\) In fact, the translation process can be automated in many cases.
When \( \{\log\} \) fails to prove that a certain formula is unsatisfiable, it generates a counterexample. This is useful at early stages of the verification phase because it helps to find mismatches between the specification and its properties.

**Example 3.** If the following is run on \( \{\log\} \):

\[
\text{neg}(\text{un}(A, BBBB, C) \& \text{un}(B, A, D) \implies C = D).
\]

the tool will provide the following as the first counterexample:

\[
C = \{\_N3/\_N1\}, \_N3\neg D, \text{un}(A, \_N2, \_N1), \ldots
\]

More counterexamples can be obtained interactively.

\[\square\]

**Remark 2.** As we have mentioned in Sect. 2.4, there are cases where \( \{\log\} \)'s \texttt{neg} predicate is not able to correctly compute the negation of its argument. For instance, if we want to compute the negation of \texttt{applyTo} as defined in Example 1 with \texttt{neg}, the result will be wrong—basically due to the presence of the existentially quantified variable \( G \). In that case, the user has to compute and write down the negation manually.

Evaluating properties with \( \{\log\} \) helps to run correct simulations by checking that the starting state is correctly defined. It also helps to test whether or not certain properties are true of the specification or not. However, by exploiting the ability to use \( \{\log\} \) as a theorem prover, we can prove that these properties are true of the specification. In particular, \( \{\log\} \) can be used to automatically discharge verification conditions in the form of state invariants. Precisely, in order to prove that state transition \( T \) (from now on called \textit{operation}) preserves state invariant \( I \) the following proof must be discharged:

\[
I \land T \Rightarrow I'
\]

(2)
where \( I' \) corresponds to the invariant in the next state. If we want to use \( \{\log\} \) to discharge (2) we have to ask \( \{\log\} \) to check if the negation of (2) is \textit{unsatisfiable}. In fact, we need to execute the following \( \{\log\} \) \texttt{query}:

\[
\text{neg}(I \land T \implies I')
\]

(3)
As we have pointed out in Example 2, \( \{\log\} \) rewrites (3) as:

\[
I \land T \land \text{neg}(I')
\]

(4)

**Example 4.** The following is the \( \{\log\} \) encoding of the state invariant labeled \texttt{inv2} in machine Doors of the LGS:

\[
\texttt{doors}._{\text{inv2}}(\text{PositionsDG}, \text{Gear}_{\text{ext}_p}, \text{Gear}_{\text{ret}_p}, \text{Door}_{\text{open}_p}) :-
\exists (P in \text{PositionsDG},
\text{applyTo}(\text{Gear}_{\text{ext}_p}, P, false) \& \text{applyTo}(\text{Gear}_{\text{ret}_p}, P, false))
\implies \text{Door}_{\text{open}_p} = \text{cp}(\text{PositionsDG}, \{true\}).
\]
Besides, Rodin generates a proof obligation as (2) for \texttt{start\_GearExtend} and \texttt{doors\_inv2}. Then, we can discharge that proof obligation by calling \{log\} on its negation:

\[
\neg(\text{doors\_inv2}(\text{PosDG}, \text{Gear\_ext\_p}, \text{Gear\_ret\_p}, \text{Door\_open\_p}) \& \\
\text{start\_GearExtend}(\text{PosDG}, \text{Gear\_ret\_p}, \text{Door\_open\_p}, \text{Gear\_ret\_p}) \implies \text{doors\_inv2}(\text{PosDG}, \text{Gear\_ext\_p}, \text{Gear\_ret\_p}, \text{Door\_open\_p}))
\]

The consequent corresponds to the invariant evaluated in the next state due to the presence of \texttt{Gear\_ret\_p} instead of \texttt{Gear\_ret\_p} in the third argument—the other arguments are not changed by \texttt{start\_GearExtend}.

Examples 1 and 4 show that \{log\} is a programming and proof platform exploiting the program-formula duality within the theories of Fig. 1. Indeed, in Example 1 \texttt{start\_GearExtend} is treated as a program (because it is executed) while in Example 4 is treated as a formula (because it participates in a theorem).

4 Encoding the Event-B Specification of the LGS in \{log\}

We say ‘encoding’ and not ‘implementing’ the LGS specification because the resulting \{log\} code: a) is a prototype rather than a production program; and b) is a formula as the LGS specification is. In particular, \{log\} provides all the logical, set and relational operators used in the LGS specification. Furthermore, these operators are not mere imperative implementations but real mathematical definitions enjoying the formula-program duality discussed in Sect. 3.

Given that we prove that the \{log\} program verifies the properties proposed by Rodin (see Section 5), we claim the prototype is a faithful encoding of the Event-B specification.

Due to space considerations we are not going to explain in detail the Event-B development of Mannar and Laleau. Instead we will provide some examples of how we have encoded it in \{log\}. The interested reader can first take a quick read to the problem description [1], and then download the Event-B development\(^{10}\) and the \{log\} program\(^{11}\) in order to make a thorough comparison. The Event-B development of the LGS consists of eleven models organized in a refinement pipeline [4]. Each model specifies the behavior and state invariants of increasingly complex and detailed versions of the LGS. For example, the first and simplest model specifies the gears of the LGS; the second one adds the doors that allow the gears to get out of the aircraft; the third one adds the hydraulic cylinders that either open (extend) or close (retract) the doors (gears); and so on and so forth. In this development each Event-B model consists of one Event-B machine.

In the following subsections we show how the main features of the Event-B model of the LGS are encoded in \{log\}.

\(^{10}\) http://deploy-eprints.ecs.soton.ac.uk/467

\(^{11}\) http://www.clpset.unipr.it/SETLOG/APPLICATIONS/lgs.zip
4.1 Encoding Event-B machines

Fig. 2 depicts at the left the Event-B machine named Gears and at the right the corresponding \( \{ \log \} \) encoding. From here on, \( \{ \log \} \) code is written in typewriter font. We tried to align as much as possible the \( \{ \log \} \) code w.r.t. the corresponding Event-B code so the reader can compare both descriptions line by line. The \( \{ \log \} \) code corresponding to a given Event-B machine is saved in a file with the name of the machine (e.g., \texttt{gears.pl}).

As can be seen in the figure, each invariant, the initialization predicate and each event are encoded as \( \{ \log \} \) clauses. In Event-B the identifiers for each of these constructs can be any word but in \( \{ \log \} \) clause predicates must begin with a lowercase letter. For instance, event Make\_GearExtended corresponds to the \( \{ \log \} \) clause predicate named \texttt{make\_GearExtended}\footnote{Actually, the name of each clause predicate is prefixed with the name of the machine. Then, is \texttt{gears\_make\_GearExtended} rather than \texttt{make\_GearExtended}. We omit the prefix whenever it is clear from context.}.

Each clause predicate receives as many arguments as variables and constants are used by the corresponding Event-B construct. For example, \texttt{inv1} waits for two arguments: \texttt{PositionsDG}, corresponding to a set declared in the Event-B context named \texttt{PositionsDoorsGears} (not shown); and \texttt{Gear\_ext\_p}, corresponding to the state variable declared in the machine. When an event changes the state of the machine, the corresponding \( \{ \log \} \) clause predicate contains as many more arguments as variables the event modifies. For example, \texttt{Make\_GearExtended} modifies variable \texttt{gear\_ext\_p}, so \texttt{make\_GearExtended} has \texttt{Gear\_ext\_p} as the third argument. \texttt{Gear\_ext\_p} corresponds to the value of \texttt{gear\_ext\_p} in the next state; i.e., the value of \texttt{gear\_ext\_p} after considering the assignment \texttt{gear\_ext\_p(po) := TRUE}. Observe that in \( \{ \log \} \) variables must begin with an uppercase letter and constants with a lowercase letter, while such restrictions do not apply in Event-B. For example, in the Event-B model we have the Boolean constant \texttt{TRUE} which in our encoding is written as \texttt{true}. Likewise, in Event-B we have variable \texttt{gear\_ext\_p} which is encoded as \texttt{Gear\_ext\_p}.

In Fig. 2 we have omitted type declarations for brevity. We will include them only when strictly necessary. For instance, the \( \{ \log \} \) clause \texttt{inv1} is actually preceded by its type declaration:

\[
:- \text{dec\_p\_type(inv1(stype(positionsdg),stype([positionsdg,bool])))).}
\]

where \texttt{positionsdg} is a synonym for \texttt{etype([front,right,left])} and \texttt{bool} is for \texttt{etype([true,false])}.

Guards and actions are encoded as \( \{ \log \} \) predicates. The identifiers associated to guards and actions can be provided as comments. \( \{ \log \} \) does not provide language constructs to label formulas. As an alternative, each guard and action can be encoded as a clause named with the Event-B identifier. These clauses are then assembled together in clauses encoding events. More on the encoding of actions in Sect. 4.3.
MACHINE Gears
SEES PositionsDoorsGears
VARIABLES gear_ext_p
INVENTARIES
inv1: gear_ext_p ∈ PositionsDG → BOOL
EVENTS
Initialisation
begin
  ACT1: gear_ext_p := PositionsDG × {TRUE}
end
Event Make_GearExtended ≡
any po
where
  GRD1: po ∈ PositionsDG ∧ gear_ext_p(po) = FALSE
then
  ACT1: gear_ext_p(po) := TRUE
end
Event Start_GearRetract ≡
any po
where
  GRD1: po ∈ PositionsDG ∧ gear_ext_p(po) = TRUE
then
  ACT1: gear_ext_p(po) := FALSE
end
END

inv1(PositionsDG,Gear_ext_p) :-
pfun(Gear_ext_p) & dom(Gear_ext_p,PositionsDG).

init(PositionsDG,Gear_ext_p) :-
  Gear_ext_p = cp(PositionsDG,{true})).

make_GearExtended(PositionsDG,Gear_ext_p,Gear_ext_p_) :-
  Po in PositionsDG &
  applyTo(Gear_ext_p,Po,false) &
  foplus(Gear_ext_p,Po,true,Gear_ext_p_).

start_GearRetract(PositionsDG,Gear_ext_p,Gear_ext_p_) :-
  Po in PositionsDG &
  applyTo(Gear_ext_p,Po,true) &
  foplus(Gear_ext_p,Po,false,Gear_ext_p_).

Fig. 2. The Event-B Gears machine at the left and its \{log\} encoding at the right
4.2 Encoding (partial) functions

Functions play a central role in Event-B specifications. In Event-B functions are sets of ordered pairs; i.e., a function is a particular kind of binary relation. \{log\} supports functions as sets of ordered pairs and supports all the related operators. Here we show how to encode functions and their operators.

**Function definition.** In order to encode functions, we use a combination of types and constraints. In general, we try to encode as much as possible with types, as typechecking performs better than constraint solving. Hence, to encode \(f \in X \rightarrow Y\), we declare it as \(\text{dec}(F, \text{stype}([X,Y]))\) and then we assert \(\text{pfun}(F)\). If \(f\) is a total function, then we conjoin \(\text{dom}(F,D)\), where \(D\) is the set representation of the domain type.

For instance, predicate \(\text{inv1}\) in Gears asserts \(\text{gear\_ext\_p} \in \text{PositionsDG} \rightarrow \text{BOOL}\) (Fig. 2). In \(\{\log\}\) we declare \(\text{Gear\_ext\_p}\) to be of type \(\text{stype(\{positionsdg,bool\})}\) (see the \(\text{dec\_p\_type}\) predicate in Sect. 4.1). This type declaration only ensures \(\text{gear\_ext\_p} \in \text{PositionsDG} \leftrightarrow \text{BOOL}\) —i.e., a binary relation between \(\text{PositionsDG}\) and \(\text{BOOL}\). The type assertion is complemented by a constraint assertion: \(\text{pfun}(\text{Gear\_ext\_p}) \& \text{dom}(\text{Gear\_ext\_p},\text{PositionsDG})\), as explained above.

**Function application.** Function application is encoded by means of the \(\text{applyTo}\) predicate defined in (1). The encoding is a little bit more general than Event-B’s notion of function application. For instance, in \(\text{Make\_GearExtended}\) (Fig. 2):

\[
gear\_ext\_p(po) = \text{FALSE}
\]

might be undefined because Event-B’s type system can only ensure \(\text{gear\_ext\_p} \in \text{PositionsDG} \leftrightarrow \text{BOOL}\) and \(po \in \text{PositionsDG}\). Actually, the following well-definedness proof obligation is required by Event-B:

\[
po \in \text{dom gear\_ext\_p} \land \text{gear\_ext\_p} \in \text{PositionsDG} \rightarrow \text{BOOL}
\]

As said, we encode (5) as \(\text{applyTo}(\text{Gear\_ext\_p},\text{Po},\text{false})\). \(\text{applyTo}\) cannot be undefined but it can fail because: \(a)\) \(\text{Po}\) does not belong to the domain of \(\text{Gear\_ext\_p}\); \(b)\) \(\text{Gear\_ext\_p}\) contains more than one pair whose first component is \(\text{Po}\); or \(c)\) \text{false} is not the image of \(\text{Po}\) in \(\text{Gear\_ext\_p}\). Then, by discharging the proof obligation required by Event-B we are sure that \(\text{applyTo}\) will not fail due to \(a)\) and \(b)\). Actually, \(\text{gear\_ext\_p} \in \text{PositionsDG} \rightarrow \text{BOOL}\) is unnecessarily strong for function application. As \(\text{applyTo}\) suggests, \(f(x)\) is meaningful when \(f\) is *locally functional* on \(x\). For example, \(\{x \mapsto 1, y \mapsto 2, y \mapsto 3\}\)\((x)\) is well-defined in spite that the set is not a function.

**Encoding membership to dom.** In different parts of the Event-B specification we find predicates such as \(po \in \text{dom gear\_ext\_p}\). There is a natural way
of encoding this in \( \{ \text{log} \} \): \( \text{dom}(\text{Gear}_\text{ext}_p, D) \text{ & Po in } D \). However, we use an encoding based on the \text{ncomp} constraint because it turns out to be more efficient in \( \{ \text{log} \} \): \text{ncomp}([[P_0,P_0],\text{Gear}_\text{ext}_p,\emptyset]). \text{ncomp} is the negation of the \text{comp} constraint (composition of binary relations). Then, \text{ncomp}([[X,X]], F, \emptyset) states that \( ((X,X)) \notin F \neq \emptyset \) which can only hold if there is a pair in \( F \) of the form \( (X, \_). \) Then, \( X \) is in the domain of \( F \).

### 4.3 Encoding action predicates

Action predicates describe the state change performed due to an event or the value of the initial state. Next, we show the encoding of two forms of action predicates: simple assignment and functional override. In events, the next state variable is implicitly given by the variable at the left of the action predicate. In \( \{ \text{log} \} \) we must make these variables explicit.

**Encoding simple assignments.** Consider a simple assignment \( x := E \). If this is part of the initialization, we interpret it as \( x = E \); if it is part of an event, we interpret it as \( x' = E \). As we have said in Sect. 4.1, \( x' \) is encoded as \( X_\_ \). Hence, simple assignments are basically encoded as equalities to before-state variables in the case of initialization and to after-state variables in the case of events.

As an example, part of the initialization of the Gears machine is:

\[
\text{act1: } \text{gear}_\text{ext}_p := \text{PositionsDG} \times \{ \text{TRUE} \}
\]  

(6)

This is simply encoded in \( \{ \text{log} \} \) as follows:

\[
\text{Gear}_\text{ext}_p = \text{cp}(\text{PositionsDG}, \{\text{true}\})
\]

An example of a simple assignment in an event, can be the following (event ReadDoors, machine Sensors):

\[
\text{act1: } \text{door}_\text{open}_\text{ind} :=
\{\text{front} \mapsto \text{door}_\text{open}_\text{sensor}_\text{valueF},
\text{left} \mapsto \text{door}_\text{open}_\text{sensor}_\text{valueL},
\text{right} \mapsto \text{door}_\text{open}_\text{sensor}_\text{valueR}\}
\]

Then, we encode it as follows:

\[
\text{Door}_\text{open}_\text{ind} =
\{[\text{front},\text{Door}_\text{open}_\text{sensor}_\text{valueF}],
[\text{left},\text{Door}_\text{open}_\text{sensor}_\text{valueL}],
[\text{right},\text{Door}_\text{open}_\text{sensor}_\text{valueR}]\}
\]

**Functional override.** The functional override \( f(x) := E \) is interpreted as \( f' = f \oplus \{ x \mapsto E \} \), which in turn is equivalent to \( f' = f \setminus \{ x \mapsto y \mid y \in \text{ran}(f) \} \cup \{ x \mapsto E \} \). This equality is encoded by means of the \text{foplus} constraint:
foplus(F,X,Y,G) :-
    F = {[X,Z]/H} & [X,Z] nin H & comp({[X,X]},H,{}) & G = {[X,Y]/H}
or  comp({[X,X]},F,{}) & G = {[X,Y]/F}.

That is, if there is more than one image of X through F, foplus fails. Then, foplus(F,X,Y,G) is equivalent to:

\[
G = F \setminus \{(X,Z)\} \cup \{(X,Y)\}, \text{for some } Z
\]

This is a slight difference w.r.t. the Event-B semantics but it is correct in a context where F is intended to be a function (i.e., pfun(F) is an invariant).

As an example, consider the action part of Make_GearExtended (Fig. 2):

\[
gear\text{ext}_p(po) := TRUE.\]

As can be seen in the same figure, the encoding is:

\[
foplus(Gear\text{ext}_p, Po, true, Gear\text{ext}_p_).\]

The following is a more complex example appearing in event Start_GearExtend of machine TimedAspects:

\[
ACT2: \text{deadline GearsRetractingExtending}(po) :=
    \{\text{front} \mapsto cT + 12, \text{left} \mapsto cT + 16, \text{right} \mapsto 16\}(po)
\]

The encoding is the following:

\[
foplus(\text{Deadline Gears Retracting Extending},
    Po, M1, \text{Deadline Gears Retracting Extending}_) \&
applyTo(\{(\text{front}, M2), (\text{left}, M3), (\text{right}, 16), \}, Po, M1) \&
\]

Given that in \{log\} function application is a constraint, we cannot put in the third argument of foplus an expression encoding \{front \mapsto cT + 12, \ldots\}(po). Instead, we have to capture the result of that function application in a new variable (M1) which is used as the third argument. Along the same lines, it would be wrong to write \(CT + 12\) in place of M2 in applyTo(\{(\text{front}, M2), \ldots\}, Po, M1) because \{log\} (as Prolog) does not interpret integer expressions unless explicitly indicated. Precisely, the is constraint forces the evaluation of the integer expression at the right-hand side.

### 4.4 Encoding quantifiers

The best way of encoding quantifiers in \{log\} is by means of the foreach and exists constraints, which implement RUQ and REQ. Unrestricted existential quantification is also supported. Then, if an Event-B universally quantified formula cannot be expressed as a RUQ formula, it cannot be expressed in \{log\}.

**Universal quantifiers.** In its simplest form, the RUQ formula \(\forall x \in A : \phi\) is written in \{log\} as foreach(X in A, \phi) [15, Sect. 5.1]\(^{13}\). However, foreach

\[\text{In turn, the foreach constraint is defined in } \{\text{log}\} \text{ by using RIS and the } \subseteq \text{ constraint, by exploiting the equivalence } \forall x \in D : \mathcal{F}(x) \iff D \subseteq \{x \in D \mid \mathcal{F}(x)\}\]
can also receive four arguments: $\text{foreach}(X \in A, [vars], \phi, \psi)$, where $\psi$ is a conjunction of functional predicates. Functional predicates play the role of LET expressions; that is, they permit to define a name for an expression [15, Sect. 6.2]. In turn, that name must be one of the variables listed in $vars$. These variables are implicitly existentially quantified inside the $\text{foreach}$. A typical functional predicate is $\text{applyTo}(F, X, Y)$ because there is only one $Y$ for given $F$ and $X$. Functional predicates can be part of $\phi$ but in that case its negation will not always correct.

As an example, consider the second invariant of machine GearsIntermediate:

$$\text{inv2}: \forall po \cdot (po \in \text{PositionsDG} \Rightarrow \neg (\text{gear\_ext\_p}(po) = \text{TRUE} \land \text{gear\_ret\_p}(po) = \text{TRUE}))$$

This is equivalent to a restricted universal quantified formula:

$$\text{inv2}: \forall po \in \text{PositionsDG} \cdot \neg (\text{gear\_ext\_p}(po) = \text{TRUE} \land \text{gear\_ret\_p}(po) = \text{TRUE}))$$

(7)

Then, we encode (7) as follows:

$$\text{inv2}(\text{PositionsDG}, \text{Gear\_ext\_p}, \text{Gear\_ret\_p}) :-
\text{foreach}(Po \in \text{PositionsDG}, [M1,M2],
\text{neg}(M1 = \text{true} \land M2 = \text{true}),
\text{applyTo}(\text{Gear\_ext\_p}, Po, M1) \land \text{applyTo}(\text{Gear\_ret\_p}, Po, M2)).$$

In some quantified formulas of Mammar and Laleau’s Event-B project the restricted quantification is not as explicit as in (7). For instance, INV3 of the Cylinders machine is:

$$\forall po \cdot \text{door\_cylinder\_locked\_p}(po) = \text{TRUE} \Rightarrow \text{door\_closed\_p}(po) = \text{TRUE}$$

However, given that $po$ must be in the domain of $\text{door\_closed\_p}$ then it must belong to $\text{PositionsDG}$.

**Existential quantifiers.** Like universal quantifiers, also existential quantifiers are encoded by first rewriting them as REQ, and then by using $\{\text{log}\}$’s $\exists$ constraint. However, $\{\text{log}\}$ supports also general (i.e., unrestricted) existential quantification: in fact, all variables occurring in the body of a clause but not in its head are implicitly existentially quantified. Thus, the Event-B construct $\text{any}$ is not explicitly encoded. For instance, in the encoding of event Make_GearExtended we just state $Po \in \text{PositionsDG}$ because this declares $Po$ as an existential variable.

4.5 Encoding types

In Event-B type information is sometimes given as membership constraints. Besides, type information sometimes becomes what can be called type invariants.
That is, state invariants that convey what normally is typing information. As we have said, we try to encode as much as possible with \{log\} types rather than with constraints. Some type invariants can be enforced by the typechecker. Nevertheless, for the purpose of using this project as a benchmark for \{log\}, we have encoded all type invariants as regular invariants so we can run all the proof obligations involving them.

**Type guards and type invariants.** For example, in event ReadHandleSwitchCircuit (machine Failures) we can find the guard:

\[
\text{Circuit\_pressurized\_sensor\_value} \in \text{BOOL}
\]

Guards like this are encoded as actual type declarations and not as constraints:

\[
\text{dec(Circuit\_pressurized\_sensor\_value, bool)}
\]

In this way, the typechecker will reject any attempt to bind this variable to something different from \text{true} and \text{false}.

As an example of a type invariant we have the following one in machine HandleSwitchShockAbsorber:

\[
\text{INV5: Intermediate}1 \in \text{BOOL}
\]

This is an invariant that can be enforced solely by the typechecker. However, for the purpose of the empirical comparison, we encoded it as a state invariant:

\[
\text{:- dec\_p\_type(inv5(bool)).}
\]

\[
\text{inv5(Intermediate1) :- Intermediate1 in \{true, false\}.}
\]

**Encoding the set of natural numbers (\(\mathbb{N}\)).** The Event-B model includes type invariants involving the set of natural numbers. This requires to be treated with care because \{log\} can only deal with finite sets so there is no set representing \(\mathbb{N}\). As these invariants cannot be enforced solely by the typechecker we need to combine types and constraints. Then, an invariant such as:

\[
\text{INV1: currentTime} \in \mathbb{N}
\]

is encoded by typing the variable as an integer and then asserting that it is non-negative:

\[
\text{:- dec\_p\_type(inv1(int)).}
\]

\[
\text{inv1(CurrentTime) :- 0} \leq \text{CurrentTime}.
\]

There are, however, more complex invariants involving \(\mathbb{N}\) such as:

\[
\text{deadlineUnlockLockDoorsCylinders} \in \text{PositionsDG} \rightarrow \mathbb{N}
\]

In this case \{log\}'s type system can only ensure:

\[
\text{deadlineUnlockLockDoorsCylinders} \in \text{PositionsDG} \leftrightarrow \mathbb{Z}
\]
Then, we need to use constraints to state: a) \textit{deadlineUnlockLockDoorsCylinders} is a function; b) the domain is \textit{PositionsDG}; and c) the range is a subset of \( \mathbb{N} \). The first two points are explained in Sect. 4.2. The last one is encoded with the \texttt{foreach} constraint:

\begin{verbatim}
foreach([X,Y] in DeadlineUnlockLockDoorsCylinders, 0 <= Y)
\end{verbatim}

As can be seen, the control term of the \texttt{foreach} predicate can be an ordered pair [15, Sect. 6.1]. Then, the complete encoding of (8) is the following:

\begin{verbatim}
:- dec_p_type(inv5(stype(positionsdg),stype([positionsdg,int]))).
inv5(PositionsDG,DeadlineUnlockLockDoorsCylinders) :-
pfun(DeadlineUnlockLockDoorsCylinders) &
dom(DeadlineUnlockLockDoorsCylinders,PositionsDG) &
foreach([X,Y] in DeadlineUnlockLockDoorsCylinders, 0 <= Y).
\end{verbatim}

5 Encoding Proof Obligations in \{log\}

Rodin is one of the tools used by Mammar and Leleau in the project. Rodin automatically generates a set of proof obligations for each model according to the Event-B verification rules [5]. Among the many kinds of proof obligations defined in Event-B, in the case of the LGS project Rodin generates proof obligations of the following three kinds:

- **Well-definedness (wd).** A \texttt{wd} condition is a predicate describing when an expression or predicate can be safely evaluated. For instance, the \texttt{wd} condition for \( x \div y \) is \( y \neq 0 \). Then, if \( x \div y \) appears in some part of the specification, Rodin will generate a proof obligation asking for \( y \neq 0 \) to be proved in a certain context.

- **Invariant initialization (init).** Let \( I \) be an invariant depending on variable \( x \). Let \( x := V \) be the initialization of variable \( x \). Then, Rodin generates the following proof obligation\(^{14}\):

\begin{equation}
\vdash I[V/x]
\end{equation}

- **Invariant preservation (inv).** Let \( I \) be as above. Let \( E \equiv G \land A \) be an event where \( G \) are the preconditions (called \texttt{guards} in Event-B) and \( A \) are the postconditions (called \texttt{actions} in Event-B). Say \( E \) changes the value of \( x \); e.g., there is an assignment such as \( x := V \) in \( E \). Then, Rodin generates the following proof obligation:

\begin{equation}
I \land I \land G \land A \vdash I[V/x]
\end{equation}

where \( I \equiv \bigwedge_{j \in J} I_j \) is a conjunction of invariants in scope other than \( I \).

\(^{14}\) This is a simplification of the real situation which, nonetheless, captures the essence of the problem. All the technical details can be found in the Event-B literature [5].
Dealing with INV proof obligations deserves some attention. When confronted with a *mechanized* proof we can think on two general strategies concerning the hypothesis for that proof:

- The interactive strategy. When proving a theorem interactively, the more the hypothesis are available the easier the proof. In other words, users of an interactive theorem prover will be happy to have as many hypotheses as possible.

- The automated strategy. When using an automated theorem prover, some hypothesis can be harmful. Given that automated theorem provers do not have the intelligence to choose (only) the right hypothesis in each proof step, they might choose hypotheses that do not lead to the conclusion or that produce a long proof path. Therefore, a possible working strategy is to run the proof with just the necessary hypothesis.

Our approach is to encode proof obligations by following the ‘automated strategy’. For example, in the case of INV proofs, we first try (10) *without* \( I \). If the proof fails, the counterexample returned by \{log\} is analyzed, just the necessary \( I_j \) are added, and the proof is attempted once more. Furthermore, in extreme cases where a proof is taking too long, parts of \( G \) and \( A \) are dropped to speed up the prover. As we further discuss it in Sect. 6, this proof strategy considerably reduces the need for truly interactive proofs.

The combination between typechecking and constraint solving also helps in improving proof automation. As we have explained in Sect. 4, the encoding of INV1 in Fig. 2 does not include constraints to state that the range of \( gear\_ext\_p \) is a subset of \( BOOL \). This is so because this fact is enforced by the typechecker. Hence, the \{log\} encoding of INV1 is simpler than INV1 itself. As a consequence, proof obligations involving INV1 will be simpler, too. For example, the INV proof for INV1 and event Make\_GearExtended (of machine Gears) is the following:

\[
\begin{align*}
gear\_ext\_p \in PositionsDG &\rightarrow BOOL \land Make\_GearExtended \\
\Rightarrow gear\_ext\_p' \in PositionsDG &\rightarrow BOOL
\end{align*}
\]

However, the \{log\} encoding of the negation of (11) is the following\(^{15}\):

\[
\text{neg}(pfun(Gear\_ext\_p) \land \text{dom}(Gear\_ext\_p, PositionsDG) \land \\
\text{make\_GearExtended}(\text{PositionsDG}, Gear\_ext\_p, Gear\_ext\_p') \\
\Rightarrow pfun(Gear\_ext\_p') \land \text{dom}(Gear\_ext\_p', PositionsDG)).
\]

That is, \{log\} will not have to prove that:

\[
\text{ran}(Gear\_ext\_p, M) \land \text{subset}(M, \text{true, false})
\]

is an invariant because this is ensured by the typechecker.

*PositionsDG* is a set declared in the context named PositionsDoorsGears where it is bound to the set \{front, right, left\}. *PositionsDG* is used by the

\(^{15}\) Predicate *inv1* of Fig. 2 is expanded to make the point more evident.
vast majority of events and thus it participates in the vast majority of proof obligations. In the \{log\} encoding, instead of binding \texttt{PositionsDG} to that value we leave it free unless it is strictly necessary for a particular proof. For example, the proof obligation named \texttt{ReadDoors/grd/wd} (machine \texttt{Sensors}) cannot be discharged if \texttt{left} is not an element of \texttt{PositionsDG}. Hence, we encoded that proof obligation as follows:

\[
\text{ReadDoors\_grd2} ::-
  \text{neg(PositionsDG = \{left / M\} &}
  \text{doors\_inv1(PositionsDG, Door\_open\_p) &}
  \text{Door\_open\_sensor\_valueL = true}
  \text{implies}
  \text{ncomp(\{[left,left]\},Door\_open\_p,\{\}) & pfun(Door\_open\_p)).}
\]

Note that: we state the membership of only \texttt{left} to \texttt{PositionsDG}; \texttt{INV1} of machine \texttt{Doors} is needed as an hypothesis; the encoding based on \texttt{ncomp} is used to state membership to dom; and \texttt{pfun} ensures that \texttt{Door\_open\_p} is a function while the type system ensures its domain and range are correct (not shown).

Furthermore, binding \texttt{PositionsDG} to \{\texttt{front, right, left}\} is crucial in a handful of proof obligations because otherwise the encoding would fall outside the decision procedures implemented in \{log\}. One example is \texttt{passingTime/grd7/wd} (machine \texttt{TimedAspects}):

\[
\begin{align*}
\text{ran} & \quad \texttt{deadlineOpenCloseDoors} \neq \{0\} \\
\implies & \quad \text{ran(\texttt{deadlineOpenCloseDoors} \ni \{0\})} \neq \emptyset \\
& \quad \wedge (\exists b \in \text{ran(\texttt{deadlineOpenCloseDoors} \ni \{0\}))} \\
& \quad \quad \left(\forall x \quad \text{ran} \quad (\texttt{deadlineOpenCloseDoors} \ni \{0\}) \bullet b \leq x \right) \\
\end{align*}
\]

As can be seen, the quantification domain of both the REQ and RUQ is the same, the REQ is before the RUQ and the REQ is at the consequent of an implication. This kind of formulas lies outside the decision procedures implemented in \{log\} unless the quantification domain is a closed set—such as \{\texttt{front, right, left}\}.

6 Discussion and Comparison

The LGS Event-B project of Mammar and Leleau comprises 4.8 KLOC of \LaTeX{} code which amounts to 213 Kb. The \{log\} encoding is 7.8 KLOC long weighting 216 Kb. Although \{log\}’s LOC are quite more than the encoding of the specification in \LaTeX{}, we would say both encodings are similar in size—we tend to use very short lines. Beyond these numbers, it is worth noting that several key state variables of the Event-B specification are Boolean functions. We wonder why the authors used them instead of sets because this choice would have implied less proof obligations—for instance, many \texttt{wd} proofs would not be generated simply because function application would be absent.
Table 1 summarizes the verification process carried out with \{log\}^{16}. Each row shows the proof obligations generated by Rodin and discharged by \{log\} for each refinement level (machine). The meaning of the columns is as follows: PO stands for the total number of proof obligations; INIT, WD and INV is the number of each kind of proof obligations; and TIME shows the computing time (in seconds) needed to discharge those proof obligations (\(\epsilon\) means the time is less than one second). As can be seen, all the 465 proofs are discharged, roughly, in 290 s, meaning 0.6 s in average. Mammar and Leleau [4, Sect. 10.1] refer that there are 285 proof obligations, of which 72\% were automatically discharged. We believe that this number corresponds to the INV proofs—i.e., those concerning with invariant preservation. Then, Mammar and Leleau had to work out 80 proof obligations interactively in spite of Roding using external provers such as Atelier B and SMT solvers.

| MACHINE                  | PO INIT WD INV TIME |
|--------------------------|---------------------|
| Gears                    | 5 1 2 2 \(\epsilon\) |
| GearsIntermediate        | 13 2 5 6 \(\epsilon\) |
| Doors                    | 10 2 2 6 \(\epsilon\) |
| DoorsIntermediate        | 13 2 5 6 \(\epsilon\) |
| Cylinders                | 37 5 14 18 \(\epsilon\) |
| HandleSwitchShockAbsorber| 29 5 4 20 \(\epsilon\) |
| ValvesLights             | 12 2 2 8 \(\epsilon\) |
| Sensors                  | 52 12 17 23 55 s    |
| TimedAspects             | 98 14 34 50 4 s     |
| Failures                 | 38 9 9 20 3 s       |
| PropertyVerification     | 158 27 1 130 228 s  |
| **Totals**               | **465 81 95 289 290 s** |

The figures obtained with \{log\} for the LGS are aligned with previous results concerning the verification of a \{log\} prototype of the Tokeneer ID Station written from a Z specification [20] and the verification of the Bell-LaPadula security model [19].

There is, however, a proof obligation that, as far as we understand, cannot be discharged. This proof is HandleFromIntermediate2ToIntermediate1/INV2 of the TimedAspects machine which would prove that event HandleFromIntermediate2ToIntermediate1 preserves \(0 \leq \text{deadlineSwitch}\). However, the assignment \(\text{deadlineSwitch} := \text{currentTime} + (8 - (2/3) * (\text{deadlineSwitch} - \text{currentTime}))\), present in that event, implies that the invariant is preserved iff \(\text{deadlineSwitch} \leq (5/2) * \text{currentTime} + 12\). But there is no invariant implying that inequality. Both

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{16} The verification process was run on a Latitude E7470 with a 4 core Intel(R) Core™ i7-6600U CPU at 2.60GHz with 8 Gb of main memory, running Linux Ubuntu 18.04.5, SWI-Prolog 7.6.4 and \{log\} 4.9.8-10i.
\texttt{deadlineSwitch} and \texttt{currentTime} are first declared in the TimedAspects machine which states only two invariants concerning these variables: both of them must be non-negative integers.

Mammar and Leleau use ProB and AnimB besides Rodin during the verification process. They use these other tools in the first refinement steps to try to find obvious errors before attempting any serious proofs. For instance, they use ProB to check that invariants are not trivially falsified. In this sense, the tool is used to find counterexamples for invalid invariants. In general, ProB cannot prove that an invariant holds, it can only prove that it does not hold—as far as we know ProB does not implement a decision procedure for a significant fragment of the set theory underlying Event-B. \texttt{log} could potentially be used as a back-end system to run the checks carried out by Mammar and Leleau. It should produce more accurate and reliable results as it implements several decision procedures as stated in Sect. 2.

Nevertheless, the main point we would like to discuss is our approach to automated proof. That is, the automated strategy mentioned in Sect. 5 plus some details of the \texttt{log} encoding of the LGS. Our approach is based on the idea expressed as \emph{specify for automated proof}. In other words, when confronted with the choice between two or more ways of specifying a given requirement, we try to chose the one that improves the chances for automated proof. Sooner or later, automated proof hits a computational complexity wall that makes progress extremely difficult. However, there are language constructs that move that wall further away than others.

For example, the encoding of function application by means of \texttt{applyTo}, the encoding of functional override by means of \texttt{foplus}, and the encoding of membership to \texttt{dom} by means of \texttt{ncomp}, considerably simplify automated proofs because these are significantly simpler than encodings based on other constraints.

However, in our experience, the so-called automated strategy provides the greatest gains regarding automated proof. As we have said, we first try to discharge a proof such as (10) without \texttt{I}. If the proof fails we analyze the counterexample returned by \texttt{log} and add a suitable hypothesis—i.e., we pick the right \texttt{I}_j \in \texttt{I}. This process is iterated until the proof succeeds. Clearly, this proof strategy requires some degree of interactivity during the verification process. The question is, then, whether or not this approach is better than attempting to prove (10) as it is and if it does not succeed, an interactive proof assistant is called in. Is it simpler and faster our strategy than a truly interactive proof?

We still do not have strong evidence to give a conclusive answer, although we can provide some data. We have developed a prototype of an interactive proof tool based on the automated strategy [26]. Those results provide evidence for a positive answer to that question. Now, the data on the LGS project further contributes in the same direction. In this project, \texttt{log} discharges roughly 60\% of the proofs of Table 1 in the first attempt. In the vast majority of the remaining proofs only one evident \texttt{I}_j \in \texttt{I} is needed. For example, in many proofs we had to add the invariant $0 \leq \texttt{currentTime}$ as an hypothesis; and in many others an invariant stating that some variable is a function.
In part, our approach is feasible because it is clear what \{log\} can prove and what it cannot, because it implements decision procedures. As a matter of fact, the proof of (12) is a good example about the value of working with decision procedures: users can foresee the behavior of the tool and, if possible, they can take steps to avoid undesired behaviors.

7 Final Remarks

This paper provides evidence that many B specifications can be easily translated into \{log\}. This means that \{log\} can serve as a programming language in which prototypes of those specifications can be immediately implemented. Then, \{log\} itself can be used to automatically prove or disprove that the specifications verify the proof obligations generated by tools such as Rodin. In the case study presented in this paper, \{log\} was able to discharge all such proof obligations.

In turn, this provides evidence that CLP and set theory are valuable tools concerning formal specification, formal verification and prototyping. Indeed \{log\}, as a CLP instance, enjoys properties that are hard to find elsewhere. In particular, \{log\} code can be seen as a program but also as a set formula. This duality allows to use \{log\} as both, a programming language and an automated verifier for its own programs. In \{log\}, users do not need to switch back and forth between programs and specifications: programs are specifications and specifications are programs.

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