Analysis of the $\frac{3}{2}^+$ Heavy and Doubly Heavy Baryon States with QCD Sum Rules

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Abstract

In this article, we study the $\frac{3}{2}^+$ heavy and doubly heavy baryon states $\Xi_{cc}^*$, $\Omega_{cc}^*$, $\Xi_{bb}^*$, $\Omega_{bb}^*$, $\Sigma_c^*$, $\Xi_c^*$, $\Omega_c^*$, $\Sigma_b^*$, $\Xi_b^*$ and $\Omega_b^*$ by subtracting the contributions from the corresponding $\frac{1}{2}^+$ heavy and doubly heavy baryon states with the QCD sum rules, and make reasonable predictions for their masses.

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1 Introduction

In 2006, the Babar collaboration reported the first observation of the $\frac{3}{2}^+$ heavy baryon state $\Omega_c^*$ in the radiative decay $\Omega_c^* \rightarrow \Omega_c^* \gamma$ [1]. By now, the $\frac{1}{2}^+$ antitriplet states ($\Lambda_c^+$, $\Xi_c^+$, $\Omega_c^*$), and the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ sextet states ($\Omega_c^*$, $\Sigma_c$, $\Xi_c'$) and ($\Omega_b^*$, $\Sigma_b$, $\Xi_b'$) have been well established [2].

In 2008, the D0 collaboration reported the first observation of the doubly strange baryon state $\Omega_{b}^-$ in the decay channel $\Omega_{b}^- \rightarrow J/\psi \Omega^-$ in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV [3]. The experimental value $M_{\Omega_{b}^-} = (6.165 \pm 0.010 \pm 0.013)$ GeV is about 0.1 GeV larger than the most theoretical calculations [4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. However, the CDF collaboration did not confirm the measured mass [16], i.e. they observed the mass of the $\Omega_{b}^-$ is about $(6.0544 \pm 0.0068 \pm 0.0090)$ GeV, which is consistent with the most theoretical calculations. On the other hand, the theoretical prediction $M_{\Omega_{b}^0} \approx 2.7$ GeV [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14] is consistent with the experimental data $M_{\Omega_{b}^0} = (2.6975 \pm 0.0026)$ GeV [2]. The S-wave bottom baryon states are far from complete, only the $\Lambda_b$, $\Sigma_b$, $\Xi_b^*$, $\Omega_b$ have been observed [2].

In 2002, the SELEX collaboration reported the first observation of a signal for the doubly charm baryon state $\Xi_{cc}^*$ in the charged decay mode $\Xi_{cc}^* \rightarrow \Lambda_c^+ K^- \pi^+$ [17], and confirmed later by the same collaboration in the decay mode $\Xi_{cc}^* \rightarrow pD^+ K^-$ with measured mass $M_{\Xi} = (3518.9 \pm 0.9)$ MeV [18]. However, the Babar and Belle collaborations have not observed any evidence for the doubly charm baryon states in $e^+e^-$ annihilations [19, 20].

No experimental evidences for the $\frac{3}{2}^+$ doubly heavy baryon states are observed [2]. There have been several approaches to deal with the doubly heavy baryon masses, such as the relativistic quark model [21, 22], the non-relativistic quark model [14, 23, 24, 25], the three-body Faddeev method [5], the potential approach combined with the QCD sum rules [26], the quark model with AdS/QCD inspired potential [27], the MIT bag model [28], the full QCD sum rules [29, 30], the Feynman-Hellmann theorem and semiempirical mass formulas [31], and the effective field theories [32], etc.
The charm and bottom baryon states which contain one (two) heavy quark(s) are particularly interesting for studying dynamics of the light quarks in the presence of the heavy quark(s), and serve as an excellent ground for testing predictions of the quark models and heavy quark symmetry. On the other hand, the QCD sum rules is a powerful theoretical tool in studying the ground state heavy baryon states [33, 34, 35].

In the QCD sum rules, the operator product expansion is used to expand the time-ordered currents into a series of quark and gluon condensates which parameterize the long distance properties of the QCD vacuum. Based on the quark-hadron duality, we can obtain copious information about the hadronic parameters at the phenomenological side [33, 34, 35]. There have been several works on the masses of the heavy baryon states with the full QCD sum rules and the QCD sum rules in the heavy quark effective theory (one can consult Ref.[36] for more literatures).

In Ref.[37], Jido et al introduce a novel approach based on the QCD sum rules to separate the contributions of the negative-parity light flavor baryons from the positive-parity light flavor baryons, as the interpolating currents may have non-vanishing couplings to both the negative- and positive-parity baryons [38]. Before the work of Jido et al, Bagan et al take the infinite mass limit for the heavy quarks to separate the contributions of the positive and negative parity heavy baryon states unambiguously [39].

In Refs.[40, 41, 42], we study the heavy baryon states \(\Omega_Q^*, \Xi_Q^*, \Sigma_Q^*, \Omega_{QQ}^*, \Xi_{QQ}^*, \Sigma_{QQ}^*\) with the full QCD sum rules, and observe that the pole residues of the \(\frac{3}{2}^+\) heavy baryons from the sum rules with different tensor structures are consistent with each other, while the pole residues of the \(\frac{1}{2}^+\) heavy baryons from the sum rules with different tensor structures differ from each other greatly. In Refs.36, 43, we follow Ref.37 and study the masses and pole residues of the \(\frac{1}{2}^+\) heavy baryon states \(\Omega_Q, \Xi_Q, \Sigma_Q, \Lambda_Q\) and \(\Xi_Q\) by subtracting the contributions of the negative parity heavy baryon states to overcome the embarrassment. Those pole residues are important parameters in studying the radiative decays \(\Omega_Q^* \rightarrow \Omega_Q \gamma, \Xi_Q^* \rightarrow \Xi_Q \gamma\) and \(\Sigma_Q^* \rightarrow \Sigma_Q \gamma\) [42, 43], etc. In Ref.35, we extend our previous works to study the \(\frac{3}{2}^+\) doubly heavy baryon states \(\Xi_{QQ}\) and \(\Omega_{QQ}\) with the full QCD sum rules.

In this article, we study the \(\frac{3}{2}^+\) heavy and doubly heavy baryon states \(\Xi_{cc}^*, \Omega_{cc}^*, \Xi_{bb}^*, \Omega_{bb}^*, \Sigma_c^*, \Sigma_b^*, \Xi_b^*\) and \(\Omega_b^*\) by subtracting the contributions from the corresponding \(\frac{3}{2}^-\) heavy and doubly heavy baryon states with the QCD sum rules.

The article is arranged as follows: we derive the QCD sum rules for the masses and the pole residues of the heavy and doubly heavy baryon states \(\Xi_{cc}^*, \Omega_{cc}^*, \Xi_{bb}^*, \Omega_{bb}^*, \Sigma_c^*, \Sigma_b^*, \Xi_b^*\) and \(\Omega_b^*\) in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusions.

## 2 QCD sum rules for the baryon states \(\Omega_Q^*, \Xi_{QQ}^*, \Omega_{QQ}^*, \Xi_{QQ}^*\) and \(\Sigma_Q^*\)

The \(\frac{3}{2}^+\) heavy and doubly heavy baryon states \(\Omega_{QQ}, \Xi_{QQ}, \Omega_{QQ}, \Xi_{QQ}\) and \(\Sigma_Q\) can be interpolated by the following currents \(J_{\mu}^{\Omega_{QQ}}(x), J_{\mu}^{\Xi_{QQ}}(x), J_{\mu}^{\Omega_{QQ}}(x), J_{\mu}^{\Xi_{QQ}}(x)\) and \(J_{\mu}^{\Sigma_Q}(x)\) respec-
tively,

\[
J_\mu^{\Omega QQ}(x) = \epsilon^{ijk} Q_1^T(x) C\gamma_\mu Q_j(x)s_k(x),
\]

\[
J_\mu^{\Xi QQ}(x) = \epsilon^{ijk} Q_1^T(x) C\gamma_\mu Q_j(x)q_k(x),
\]

\[
J_\mu^{\Omega Q}(x) = \epsilon^{ijk} s_1^T(x) C\gamma_\mu s_j(x)Q_k(x),
\]

\[
J_\mu^{\Xi Q}(x) = \epsilon^{ijk} q_1^T(x) C\gamma_\mu s_j(x)Q_k(x),
\]

\[
J_\mu^{\Sigma Q}(x) = \epsilon^{ijk} u_1^T(x) C\gamma_\mu d_j(x)Q_k(x),
\]

(1)

where the \(Q\) represents the heavy quarks \(c\) and \(b\), the \(i, j\) and \(k\) are color indexes, and the \(C\) is the charge conjunction matrix. In the heavy quark limit, the heavy and doubly heavy baryon states can be described by the diquark-quark model [26].

The corresponding \(3/2^-\) heavy and doubly heavy baryon states can be interpolated by the currents \(J_\mu^+ = i\gamma_5 J_\mu^-\), because multiplying \(i\gamma_5\) to the \(J_\mu^-\) changes the parity of the \(J_\mu^+\) [37], where the \(J_\mu^+\) denotes the currents \(J_\mu^{\Omega QQ}(x), J_\mu^{\Xi QQ}(x), J_\mu^{\Omega Q}(x), J_\mu^{\Xi Q}(x)\) and \(J_\mu^{\Sigma Q}(x)\).

The correlation functions \(\Pi_{\mu\nu}^\pm(p)\) are defined by

\[
\Pi_{\mu\nu}^\pm(p) = i \int d^4xe^{ip\cdot x} \langle 0 | T \left\{ J_\mu^\pm(x) J_\nu^\pm(0) \right\} | 0 \rangle .
\]

(2)

The currents \(J_\mu^\pm(x)\) couple to both the \(3/2^+\) baryon states \(B_\pm^\pm\) and the \(1/2^+\) baryon states \(B_\pm\) [38], i.e.

\[
\langle 0 | J_\mu^\pm(0) | B_\pm^\pm(p) \rangle \langle B_\pm^\pm(p) | J_\nu^\pm(0) | 0 \rangle = -\gamma_5 \langle 0 | J_\mu^- (0) | B_\pm^\pm(p) \rangle \langle B_\pm^\pm(p) | J_\nu^- (0) | 0 \rangle \gamma_5 ,
\]

\[
\langle 0 | J_\mu^\pm(0) | B_\pm(p) \rangle \langle B_\pm(p) | J_\nu^\pm(0) | 0 \rangle = -\gamma_5 \langle 0 | J_\mu^- (0) | B_\pm(p) \rangle \langle B_\pm(p) | J_\nu^- (0) | 0 \rangle \gamma_5 ,
\]

(3)

where

\[
\langle 0 | J_\mu^\pm(0) | B_\pm^\pm(p) \rangle = \lambda_\pm U_\mu(p, s),
\]

\[
\langle 0 | J_\mu^\pm(0) | B_\pm(p) \rangle = \lambda_s \left( \gamma_\mu - 4 \frac{p_\mu}{M_s} \right) U(p, s),
\]

(4)

the \(\lambda_\pm\) and \(\lambda_s\) are the pole residues and \(M_s\) are the masses, and the spinor \(U(p, s)\) satisfies the usual Dirac equation \((p - M_s)U(p) = 0\).

The \(\Pi_{\mu\nu}^\pm(p)\) have the following relation

\[
\Pi_{\mu\nu}^- = -\gamma_5 \Pi_{\mu\nu}^+(p) \gamma_5 .
\]

(5)

We insert a complete set of intermediate baryon states with the same quantum numbers as the current operators \(J_\mu^\pm(x)\) into the correlation functions \(\Pi_{\mu\nu}^\pm(p)\) to obtain the hadronic representation [33, 34]. After isolating the pole terms of the lowest states of the heavy and doubly heavy baryons, we obtain the following result [37]:

\[
\Pi_{\mu\nu}^+(p) = -\lambda_+^2 \frac{p^2 + M_+^2}{M_+^2 - p^2} g_{\mu\nu} - \lambda_-^2 \frac{p^2 - M_-^2}{M_-^2 - p^2} g_{\mu\nu} + \cdots ,
\]

\[
= -\Pi_+(p) g_{\mu\nu} + \cdots ,
\]

(6)
where the $M_{\pm}$ are the masses of the lowest states with parity $\pm$ respectively, and the $\lambda_{\pm}$ are the corresponding pole residues (or couplings). In calculations, we have used the following equations,

$$
\sum_s U_\mu(p, s) \overline{U}_\nu(p, s) = (\not{p} + M_0^*) \left( -g_{\mu\nu} + \frac{\gamma_\mu \gamma_\nu}{3} + \frac{2p_\mu p_\nu}{3M_0^*} - \frac{p_\mu \gamma_\nu - p_\nu \gamma_\mu}{3M_0^*} \right),
$$

$$
\sum_s U(p, s) \overline{U}(p, s) = \not{p} + M_0.
$$

(7)

In this article, we choose the tensor structure $g_{\mu\nu}$ for analysis, the $\frac{1}{2}^\pm$ baryon states have no contaminations.

If we take $\not{p} = 0$, then

$$
\lim_{\epsilon \to 0} \frac{\text{Im} \Pi^+(p_0 + i\epsilon)}{\pi} = \lambda_+^2 \frac{\gamma_0 + 1}{2} \delta(p_0 - M_+) + \lambda_-^2 \frac{\gamma_0 - 1}{2} \delta(p_0 - M_-) + \cdots
$$

$$
= \gamma_0 A(p_0) + B(p_0) + \cdots,
$$

(8)

where

$$
A(p_0) = \frac{1}{2} \left[ \lambda_+^2 \delta(p_0 - M_+) + \lambda_-^2 \delta(p_0 - M_-) \right],
$$

$$
B(p_0) = \frac{1}{2} \left[ \lambda_+^2 \delta(p_0 - M_+) - \lambda_-^2 \delta(p_0 - M_-) \right],
$$

(9)

the $A(p_0) + B(p_0)$ and $A(p_0) - B(p_0)$ contain the contributions from the positive-parity states and negative-parity baryon states respectively.

We calculate the light quark parts of the correlation functions $\Pi^+(p)$ in the coordinate space and use the momentum space expression for the heavy quark propagators, i.e. we take

$$
S_{ij}(x) = \frac{i\delta_{ij} \not{x}}{2\pi^2 x^2} - \frac{\delta_{ij} m_s}{4\pi^2 x^2} - \frac{i\delta_{ij} (ss)}{12} + \frac{i\delta_{ij} m_s (ss)}{48} \not{x} - \frac{i}{32\pi^2 x^2} G^{ij}_\mu(x) \not{\sigma} \not{x} + \sigma_\mu \not{x} + \cdots,
$$

$$
S_{ij}^{ij}(x) = \frac{i}{(2\pi)^4} \int d^4 k e^{-ik \cdot x} \left\{ \frac{\delta_{ij}}{(k - m_Q)^4} - \frac{g_s G^{\alpha\beta}_{ij}}{4} \sigma_{\alpha\beta} \frac{(k + m_Q) + (k + m_Q)^4}{(k^2 - m_Q^2)^2} \right\},
$$

(10)

where $\langle \alpha G \rangle = \langle \alpha G_{\alpha\beta} G^{\alpha\beta} \rangle$, then resort to the Fourier integral to transform the light quark parts into the momentum space in $D$ dimensions, take $\not{p} = 0$, and use the dispersion relation to obtain the spectral densities $\rho^+(p_0)$ and $\rho^B(p_0)$ (which correspond to the tensor structures $\gamma_0$ and 1 respectively) at the level of quark-gluon degrees of freedom. Finally we introduce the weight functions $\exp \left[ -\frac{p_0^2}{T^2} \right]$, $p_0^2 \exp \left[ -\frac{p_0^2}{T^2} \right]$, and obtain the following sum rules,

$$
\lambda_+^2 \exp \left[ -\frac{M_0^2}{T^2} \right] = \int_0^{\sqrt{\gamma_0}} dp_0 \left[ \rho^+(p_0) + \rho^B(p_0) \right] \exp \left[ -\frac{p_0^2}{T^2} \right],
$$

(11)
\[
\lambda^2 M_m^2 \exp \left[ -\frac{M^2}{T^2} \right] = \int_{\Delta}^{\sqrt{s_0}} dp_0 \left[ \rho^A(p_0) + \rho^B(p_0) \right] p_0^2 \exp \left[ -\frac{p_0^2}{T^2} \right],
\]

where the \( s_0 \) are the threshold parameters, \( T^2 \) are the Borel parameters, and \( \Delta = 2m_Q + m_s, 2m_q, m_Q + 2m_s, m_Q + m_s \) and \( m_Q \) in the channels \( \Omega_{QQ}, \Xi_{QQ}, \Omega^\prime_{QQ}, \Xi^\prime_{QQ}, \) respectively. The spectral densities \( \rho^A(p_0) \) and \( \rho^B(p_0) \) at the level of quark-gluon degrees of freedom are given explicitly in the Appendix.

### 3 Numerical results and discussions

The input parameters are taken to be the standard values \( \langle \bar qq \rangle = -(0.24 \pm 0.01 \text{ GeV})^3, \langle \bar ss \rangle = (0.8 \pm 0.2)\langle \bar qq \rangle, \langle \bar gg, \sigma Gq \rangle = m_0^2 \langle \bar qq \rangle, \langle \bar sg, \sigma Gs \rangle = m_0^2 \langle \bar ss \rangle, \) and \( m_0 = (0.8 \pm 0.2) \text{ GeV}^2 \) \[46, 47\], \( \langle \bar q \bar q G \rangle = (0.012 \pm 0.004) \text{ GeV}^4 \) \[16, 47\], \( m_s = (0.14 \pm 0.01) \text{ GeV}, m_c = (1.35 \pm 0.10) \text{ GeV} \) and \( m_b = (4.7 \pm 0.1) \text{ GeV} \) \[2\] at the energy scale \( \mu = 1 \text{ GeV} \).

The value of the gluon condensate \( \langle \bar q \bar q G \rangle \) has been updated from time to time, and changes greatly \[35\]. At the present case, the gluon condensate makes tiny contribution, the updated value \( \langle \bar q \bar q G \rangle = (0.023 \pm 0.003) \text{ GeV}^4 \) \[35\] and the standard value \( \langle \bar q \bar q G \rangle = (0.012 \pm 0.004) \text{ GeV}^4 \) \[17\] lead to a tiny difference and can be neglected safely. The values of the quark condensates determined from the Gell-Mann-Oakes-Renner relation, the spectral functions of the \( \tau \) decay, and the QCD sum rules for baryon masses are consistent with each other within uncertainties \[46\], we usually take the value from the Gell-Mann-Oakes-Renner relation in the QCD sum rules \[47\]. For the mixed condensates, we take the value from the QCD sum rules for the baryonic resonances, which is still accepted in the literatures \[46, 47\]. Those values are not accurate, and there are much room for improvement; the update of the vacuum condensates should be combined with a more delicate procedure in dealing with the perturbative and non-perturbative contributions, and beyond the present work.

The \( Q \)-quark masses appearing in the perturbative terms are usually taken to be the pole masses in the QCD sum rules, while the choice of the \( m_Q \) in the leading-order coefficients of the higher-dimensional terms is arbitrary \[35, 48\]. The \( MS \) mass \( m_c(m_c^2) \) relates with the pole mass \( \hat m_c \) through the relation 

\[
m_c(m_c^2) = \hat m_c \left[ 1 + \frac{C_{\bar n c}(m_c^2)}{\pi} + \cdots \right]^{-1}.
\]

In this article, we take the approximation \( m_c \approx \hat m_c \) without the \( \alpha_s \) corrections for consistency. The value listed in the Particle Data Group is \( m_c(m_c^2) = 1.27^{+0.07}_{-0.11} \text{ GeV} \) \[2\], it is reasonable to take \( m_c = m_c(1 \text{ GeV}^2) = (1.35 \pm 0.10) \text{ GeV} \). The value of the \( m_b \) can be understood analogously.

In calculation, we also neglect the contributions from the perturbative \( \mathcal{O}(\alpha_s^n) \) corrections. Those perturbative corrections can be taken into account in the leading logarithmic approximations through the anomalous dimension factors. After the Borel transform, the effects of those corrections are to multiply each term on the operator product expansion side by the factor, 

\[
\left[ \frac{\alpha_s(T^2)}{\alpha_s(\mu^2)} \right]^{2\Gamma_J - \Gamma_{\mathcal{O}_n}},
\]

where the \( \Gamma_J \) is the anomalous dimension of the interpolating current \( J(x) \), and \( \Gamma_{\mathcal{O}_n} \) is the anomalous dimension of the local operator \( \mathcal{O}_n(0) \), which governs the evolution of the vacuum condensate \( \langle \mathcal{O}_n(0) \rangle_\mu \) with the energy scale through the re-normalization group equation.

If the perturbative \( \mathcal{O}(\alpha_s) \) corrections and the anomalous dimension factors are taken
into account consistently, the spectral densities in the QCD side should be replaced with

\[ O_0(0) \rightarrow \left[ \frac{\alpha_s(T^2)}{\alpha_s(\mu^2)} \right]^{2\Gamma_j} \left[ 1 + A(p_0^2, m_Q^2) \frac{\alpha_s(T^2)}{\pi} \right] O_0(0), \]

\[ \langle \mathcal{O}_n(0) \rangle_\mu \rightarrow \left[ \frac{\alpha_s(T^2)}{\alpha_s(\mu^2)} \right]^{2\Gamma_j - \Gamma_{O_n}} \left[ 1 + B(p_0^2, m_Q^2) \frac{\alpha_s(T^2)}{\pi} \right] \langle \mathcal{O}_n(0) \rangle_\mu, \]

where the \( A(p_0^2, m_Q^2) \) and \( B(p_0^2, m_Q^2) \) are some notations for the coefficients of the perturbative corrections, the average virtuality of the quarks in the correlation functions is characterized by the Borel parameter \( T^2 \). We cannot estimate the corrections and the uncertainties originate from the corrections with confidence without explicit calculations. In Ref. [49], Ovchinnikov et al calculate the perturbative \( \mathcal{O}(\alpha_s) \) corrections to the correlation functions of the light-flavor baryon, and observe that the corrections change the numerical values of the mass and the pole residue of the proton considerably and improve the agreement between the theoretical estimation and the experimental data. In the present case, including the \( \alpha_s \) corrections maybe improve the predictions.

In this article, we carry out the operator product expansion at the special energy scale \( \mu^2 = 1 \text{ GeV}^2 \), and set the factor \( \left[ \frac{\alpha_s(T^2)}{\alpha_s(\mu^2)} \right]^{2\Gamma_j - \Gamma_{O_n}} \approx 1 \) for consistency, as the \( \alpha_s \) corrections have not been calculated yet. Such an approximation maybe result in some scale dependence and weaken the prediction ability. In this article, we study the upper bound of the Borel parameters \( T^2 \) and double heavy baryon states in a systematic way, the predictions are still robust as we take the analogous criteria in those sum rules.

The separation of the perturbative and non-perturbative contributions to the vacuum correlation functions has some arbitrariness, and we can introduce some renormalization point \( \mu^2 (\mu^2 \sim 1 \text{ GeV}^2) \) as the boundary. The non-perturbative contributions are parameterized by the vacuum condensates, furthermore, the infrared logarithms of the form \( \log^k \left( \frac{m_Q^2}{\mu^2} \right) \) are also absorbed into the vacuum condensates in the perturbative calculations. Perturbative calculations are reliable at the special energy scale \( \mu^2 = 1 \text{ GeV}^2 \), which characterizes the chiral symmetry breaking.

In the conventional QCD sum rules [33, 34], there are two criteria (pole dominance and convergence of the operator product expansion) for choosing the Borel parameter \( T^2 \) and threshold parameter \( s_0 \). We impose the two criteria on the heavy and doubly heavy baryon states to choose the Borel parameter \( T^2 \) and threshold parameter \( s_0 \).

In Fig. 1, we plot the contributions from the pole terms with variations of the Borel parameters \( T^2 \) and the threshold parameters \( s_0 \). The pole contributions are larger than (or about) 50% at the values which are denoted by the vertical lines for central values (\( \beta \)) of the threshold parameters \( s_0 \). We can set the upper bound of the Borel parameters \( T^2_{max} \) as the values indicated by the vertical lines.

In Fig. 2, we plot the contributions from the different terms in the operator product expansion in the doubly heavy baryon channels \( \Xi_{cc}, \Omega_{cc}^+, \Xi_{bb}^+ \) and \( \Omega_{bb}^+ \). In the heavy baryon channels \( \Sigma^+_c, \Xi^+_c, \Omega^+_c, \Sigma^+_b, \Xi^+_b \) and \( \Omega^+_b \), the convergent behaviors are very good, it is no use to plot them, we only show the contributions from the perturbative terms explicitly in Table 1. From the Fig.2, we can see that the convergent behaviors in the channels \( \Omega_{QQ}^0 \) are better than the corresponding ones in the channels \( \Xi_{QQ}^0 \), this is mainly due to the fact that the values of the quark condensates, \( |\langle \bar{q}q \rangle| > |\langle \bar{s}s \rangle| \). The lower bound of the
Figure 1: The contributions of the pole terms with variations of the Borel parameters $T^2$, the $A$, $B$, $C$, $D$, $E$, $F$, $G$, $H$, $I$ and $J$ correspond to the channels $\Xi_{cc}^*$, $\Omega_{cc}^*$, $\Xi_{bb}^*$, $\Omega_{bb}^*$, $\Sigma_{c}^*$, $\Xi_{c}^*$, $\Omega_{c}^*$, $\Sigma_{b}^*$, $\Xi_{b}^*$ and $\Omega_{b}^*$ respectively, the $\beta$ corresponds to the central values of the threshold parameters, the energy gap among $\alpha$, $\beta$ and $\gamma$ is 0.1 GeV.
Borel parameters $T_{min}^2$ can be determined by the channels $\Xi_{QQ}^*$ in the regions where the contributions from the perturbative terms are larger than the corresponding ones from the quark condensates. From Figs.1-2, we can see that the Borel windows are different for the doubly charm and doubly bottom baryon states.

In this article, we take the uniform Borel windows, $T_{max}^2 - T_{min}^2 = 1.5\, \text{GeV}^2$ and $2.0\, \text{GeV}^2$ in the doubly charm and doubly bottom channels respectively. For the heavy baryon states $\Sigma_c^*$, $\Xi_c^*$, $\Omega_c^*$, $\Sigma_b^*$, $\Xi_b^*$ and $\Omega_b^*$, we take the uniform Borel windows, $T_{max}^2 - T_{min}^2 = 1.0\, \text{GeV}^2$. The values of the threshold parameters $s_0$ and the Borel parameters $T^2$ are shown in Table 1, from the table, we can see that the two criteria of the QCD sum rules are fully satisfied. In this article, we take uniform uncertainties for the threshold parameters, $\delta_{s_{0\pm}} = \pm 0.1 \, \text{GeV}$. In calculation, we observe that the predicted masses are not sensitive to the threshold parameters, although they increase with the threshold parameters.

Taking into account all uncertainties of the relevant parameters, we can obtain the values of the masses and pole residues of the $\frac{3}{2}^+$ heavy and doubly heavy baryon states $\Xi_{cc}^*$, $\Omega_{cc}^*$, $\Xi_{bb}^*$, $\Omega_{bb}^*$, $\Sigma_{c}^*$, $\Xi_{c}^*$, $\Omega_{c}^*$, $\Sigma_{b}^*$, $\Xi_{b}^*$ and $\Omega_{b}^*$, which are shown in Figs.3-4 and Tables 2-4. In Table 2 and Table 4, we also present the predictions of other theoretical approaches and the values of the experimental data respectively.

From Table 4, we can see that the present predictions for the well established heavy baryon states are in good agreement with the experimental data, the predictions for the unestablished bottom baryon states $\Xi_b^*$ and $\Omega_b^*$ are robust as we take the analogous criteria in those sum rules. For the $\frac{3}{2}^+$ doubly heavy baryon states, there are no available experimental data, our values are comparable with other theoretical predictions, see Table 4.
In this article, we take the simple pole + continuum approximation for the phenomenological spectral densities. In fact, such a simple approximation has shortcomings. In the case of the non-relativistic harmonic-oscillator potential model, the spectrum of the bound states (the masses $E_n$ and the wave functions $\Psi_n(x)$) and the exact correlation functions (and hence its operator product expansion to any order) are known precisely. The non-relativistic harmonic-oscillator potential $\frac{1}{2}m\omega^2r^2$ is highly non-perturbative, one suppose the full Green function satisfies the Lippmann-Schwinger operator equation and may be solved perturbatively. We can introduce the Borel parameter dependent effective threshold parameter $z_{eff}(T) = \omega [z_0 + z_1 \sqrt{\frac{E}{\omega}} + z_2 \frac{E}{\omega} + \cdots]$ and fit the coefficients $z_i$ to reproduce both the ground energy $E_0$ and the pole residue $R_0 = \Psi^*_0(0)\Psi_0(0)$, the phenomenologi-

| $T^2$(GeV$^2$) | $\sqrt{s_0}$(GeV) | pole | perturbative | $(qq)$ | $(\frac{\alpha s^2}{\pi})$ |
|-----------------|-----------------|------|-------------|--------|-----------------|
| $\Xi^*_{cc}$    | 2.8 - 4.3       | 4.3  | (46 - 83)%  | (47 - 58)% | (42 - 52)% < 1% |
| $\Omega^*_{cc}$ | 3.0 - 4.5       | 4.4  | (45 - 81)%  | (67 - 74)% | (26 - 32)% < 1% |
| $\Xi^*_{bb}$    | 7.8 - 9.8       | 10.9 | (46 - 73)%  | (50 - 57)% | (43 - 50)% < 1% |
| $\Omega^*_{bb}$ | 8.1 - 10.1      | 11.0 | (46 - 71)%  | (70 - 74)% | (26 - 30)% < 1% |
| $\Omega^*_c$    | 5.3 - 6.3       | 6.9  | (45 - 68)%  | (84 - 89)% |
| $\Xi^*_b$       | 5.0 - 6.0       | 6.8  | (45 - 70)%  | (78 - 85)% |
| $\Sigma^*_b$    | 4.6 - 5.6       | 6.7  | (46 - 73)%  | (71 - 83)% |
| $\Omega^*_c$    | 2.4 - 3.4       | 3.5  | (45 - 79)%  | (79 - 87)% |
| $\Xi^*_c$       | 2.2 - 3.2       | 3.4  | (44 - 81)%  | (72 - 85)% |
| $\Sigma^*_c$    | 2.0 - 3.0       | 3.3  | (43 - 83)%  | (64 - 84)% |

Table 1: The Borel parameters $T^2$ and threshold parameters $s_0$ for the heavy and doubly heavy baryon states, the ”pole” stands for the contribution from the pole term, and the ”perturbative” stands for the contribution from the perturbative term in the operator product expansion, etc. In calculating the contributions from the pole terms, we take into account the uniform uncertainties of the threshold parameters, $\delta_{\sqrt{s_0}} = \pm 0.1 \text{GeV}$.

\vspace{1cm}

2.

In this article, we take the simple pole + continuum approximation for the phenomenological spectral densities. In fact, such a simple approximation has shortcomings. In the case of the non-relativistic harmonic-oscillator potential model, the spectrum of the bound states (the masses $E_n$ and the wave functions $\Psi_n(x)$) and the exact correlation functions (and hence its operator product expansion to any order) are known precisely. The non-relativistic harmonic-oscillator potential $\frac{1}{2}m\omega^2r^2$ is highly non-perturbative, one suppose the full Green function satisfies the Lippmann-Schwinger operator equation and may be solved perturbatively. We can introduce the Borel parameter dependent effective threshold parameter $z_{eff}(T) = \omega [z_0 + z_1 \sqrt{\frac{E}{\omega}} + z_2 \frac{E}{\omega} + \cdots]$ and fit the coefficients $z_i$ to reproduce both the ground energy $E_0$ and the pole residue $R_0 = \Psi^*_0(0)\Psi_0(0)$, the phenomenologi-

| Reference | $\Xi^*_{cc}$ | $\Omega^*_{cc}$ | $\Xi^*_{bb}$ | $\Omega^*_{bb}$ |
|-----------|--------------|----------------|--------------|----------------|
| 5         | 3.656        | 3.769          | 10.218       | 10.321         |
| 14        | 3.753        | 3.876          | 10.367       | 10.486         |
| 21        | 3.727        | 3.872          | 10.237       | 10.389         |
| 23        | 3.706        | 3.783          | 10.236       | 10.297         |
| 26        | 3.61         | 3.69           | 10.13        | 10.20          |
| 27        | 3.719        | 3.770          | 10.216       | 10.289         |
| 28        | 3.630        | 3.721          | 10.337       | 10.429         |
| 30        | 3.90         | 3.81           | 10.35        | 10.28          |
| This work | 3.61 ± 0.18  | 3.76 ± 0.17    | 10.22 ± 0.15 | 10.38 ± 0.14   |

Table 2: The masses $M$(GeV) of the $\frac{3}{2}^+$ doubly heavy baryon states.
Figure 3: The masses of the heavy and doubly heavy baryon states, the $A, B, C, D, E, F, G, H, I$ and $J$ correspond to the channels $\Xi^*_c, \Omega^*_c, \Xi^*_{bb}, \Omega^*_{bb}, \Sigma^*_c, \Xi^*_c, \Sigma^*_b, \Xi^*_b$ and $\Omega^*_b$ respectively.
Figure 4: The pole residues of the heavy and doubly heavy baryon states, the A, B, C, D, E, F, G, H, I and J correspond to the channels $\Xi_{cc}$, $\Omega_{cc}$, $\Xi_{bb}$, $\Omega_{bb}$, $\Sigma_c$, $\Xi_c$, $\Omega_c$, $\Sigma_b$, $\Xi_b$ and $\Omega_b$ respectively.
Table 3: The pole residues $\lambda$ of the $\frac{3}{2}^-$ doubly heavy baryon states.

| $T^2$(GeV$^2$) | $\sqrt{s_0}$(GeV) | $M$(GeV) | $\lambda$(GeV$^3$) | $M$(GeV)$|_{\text{exp}}$
|----------------|-------------------|----------|-------------------|---------------------|
| $\Omega^*_{bc}$ | 5.3 – 6.3         | 6.9 ± 0.1| 6.17 ± 0.15       | 0.083 ± 0.018       | ?                   |
| $\Xi^*_{bc}$   | 5.0 – 6.0         | 6.8 ± 0.1| 6.02 ± 0.17       | 0.049 ± 0.012       | ?                   |
| $\Sigma^*_{bc}$| 4.6 – 5.6         | 6.7 ± 0.1| 5.85 ± 0.20       | 0.038 ± 0.011       | 5.833 [2]           |
| $\Omega^*_{cc}$| 2.4 – 3.4         | 3.5 ± 0.1| 2.79 ± 0.19       | 0.056 ± 0.012       | 2.766 [2]           |
| $\Xi^*_{cc}$   | 2.2 – 3.2         | 3.4 ± 0.1| 2.65 ± 0.20       | 0.033 ± 0.008       | 2.646 [2]           |
| $\Sigma^*_{cc}$| 2.0 – 3.0         | 3.3 ± 0.1| 2.48 ± 0.25       | 0.027 ± 0.008       | 2.518 [2]           |

Table 4: The masses $M$(GeV) and pole residues $\lambda$(GeV$^3$) of the $\frac{3}{2}^+$ heavy baryon states.

cal spectrum density can be described by the perturbative contributions well above the effective continuum threshold $z_{eff}(T)$, or reproduce the ground energy $E_0$ only and take the pole residue $R$ as a calculated parameter, there exists a solution for the effective continuum threshold $z_{eff}(T)$ which precisely reproduces the exact ground energy $E_0$ for any value of the pole residue $R$ within the range $R = (0.7 - 1.15)R_0$ in the limited fiducial Borel window, the value of the pole residue $R$ extracted from the sum rule is determined to a great extent by the contribution of the hadron continuum [50]. There maybe systemic uncertainties out of control.

In the real QCD world, the hadronic spectral densities are not known exactly. In the present case, the ground states in some channels have not been observed yet. So we have no confidence to introduce the Borel parameter dependent effective threshold parameter $s_{eff}(T) = \tilde{s}_0 + \tilde{s}_1 \frac{1}{T^2} + \tilde{s}_2 \frac{1}{T^4} + \cdots$ and approximate the phenomenological spectral densities with the perturbative contributions above the effective continuum threshold $s_{eff}(T)$ accurately. Furthermore, the pole residues (or the couplings of the interpolating currents to the ground state baryons) $\lambda_{\pm}$ are not experimentally measurable quantities, and should be calculated by some theoretical approaches, the true values are difficult to obtain, which are distinguished from the decay constants of the pseudoscalar mesons and the vector mesons, the decay constants can be measured with great precision in the leptonic decays (in some channels).

The spectrum of the bound states in the non-relativistic harmonic-oscillator potential model are of the Dirac $\delta$ function type, we can choose $z_{eff} < E_1$, while in the case of the QCD, the situation is rather complex, the effective continuum thresholds $s_{eff}(T)$ maybe overlap with the first radial excited states, which are usually broad. For example, in the pseudoscalar channels, the widths of the $\pi$, $\pi(1300)$, $\pi(1800)$, $\cdots$ are $\sim 0$GeV, (0.2 – 0.6)GeV, 0.208 ± 0.012GeV, $\cdots$ respectively, while the widths of the $K$, $K(1460)$, $K(1830)$, $\cdots$ are $\sim 0$GeV, $\sim (0.25 – 0.26)$GeV, $\sim 0.25$GeV, $\cdots$ respectively [2]. In this article, we prefer (or have to choose) the simple pole + continuum approximation, and cannot estimate the unknown systemic uncertainties of the QCD sum rules before
the spectral densities in both the QCD and phenomenological sides are known with great accuracy.

The properties of the charm and doubly charm baryon states would be studied at the BESIII and PANDA \cite{51,52}, where the charm baryon states are copiously produced at the $e^+e^-$ and $p\bar{p}$ collisions. The LHCb is a dedicated $b$ and $c$-physics precision experiment at the LHC (large hadron collider). The LHC will be the world’s most copious source of the $b$ hadrons, and a complete spectrum of the $b$ hadrons will be available through gluon fusion. In proton-proton collisions at $\sqrt{s} = 14$ TeV, the $bb$ cross section is expected to be $\sim 500\mu b$ producing $10^{12}$ $bb$ pairs in a standard year of running at the LHCb operational luminosity of $2 \times 10^{32}\text{cm}^{-2}\text{sec}^{-1}$ \cite{53}. The present predictions for the masses of the heavy and doubly heavy baryon states can be confronted with the experimental data in the future at the BESIII, PANDA and LHCb.

4 Conclusion

In this article, we study the $\frac{3}{2}^+$ heavy and doubly heavy baryon states $\Xi_{c}^*$, $\Omega_{c}^*$, $\Xi_{bb}^*$, $\Omega_{bb}^*$, $\Sigma_c^*$, $\Xi_c^*$, $\Omega_c^*$, $\Sigma_b^*$, $\Xi_b^*$ and $\Omega_b^*$ by subtracting the contributions from the corresponding $\frac{1}{2}^+$ heavy and doubly heavy baryon states with the QCD sum rules, and make reasonable predictions for their masses. The present predictions can be confronted with the experimental data in the future at the BESIII, PANDA and LHCb, especially the LHCb. Once reasonable values of the pole residues $\lambda_\pm$ are obtained, we can take them as basic input parameters and study the relevant hadronic processes with the QCD sum rules.

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Appendix

The spectral densities of the heavy and doubly heavy baryon states $\Omega_{QQ}^*$, $\Xi_{QQ}^*$, $\Omega_Q^*$, $\Xi_Q^*$ and $\Sigma_Q^*$ at the level of quark-gluon degrees of freedom,
\[
\rho_{\mathbb{A}}^{\alpha_{Q\bar{Q}}}(p_0) = \frac{3p_0}{16\pi^4} \int_{\alpha_i}^{\alpha_f} \frac{d\alpha}{\alpha_i} \int_{1-\alpha}^{1-\alpha} d\beta \alpha \beta (1 - \alpha - \beta) (p_0^2 - \bar{m}^2_Q)(2p_0^2 - \bar{m}^2_Q) \\
+ \frac{3m^2_{Q\bar{Q}} p_0}{16\pi^4} \int_{\alpha_i}^{\alpha_f} \frac{d\alpha}{\alpha_i} \int_{1-\alpha}^{1-\alpha} d\beta (1 - \alpha - \beta) (p_0^2 - \bar{m}^2_Q) \\
- \frac{m^2_{Q\bar{Q}}}{192\pi^2} \int_{\alpha_i}^{\alpha_f} \frac{d\alpha}{\alpha_i} \int_{1-\alpha}^{1-\alpha} d\beta (1 - \alpha - \beta) \left[ \frac{1}{\beta^2} + \frac{1}{\alpha^2} \right] \frac{1}{2T} \delta(p_0 - \bar{m}_Q) \\
- \frac{m^4_{Q\bar{Q}}}{384\pi^2 p_0 T} \int_{\alpha_i}^{\alpha_f} \frac{d\alpha}{\alpha_i} \int_{1-\alpha}^{1-\alpha} d\beta (1 - \alpha - \beta) \left[ \frac{1}{\beta^3} + \frac{1}{\alpha^3} \right] \delta(p_0 - \bar{m}_Q) \\
+ \frac{m^2_{Q\bar{Q}}}{64\pi^2} \int_{\alpha_i}^{\alpha_f} \frac{d\alpha}{\alpha_i} \int_{1-\alpha}^{1-\alpha} d\beta (1 - \alpha - \beta) \left[ \frac{1}{\alpha^2} + \frac{1}{\beta^2} \right] \delta(p_0 - \bar{m}_Q) \\
+ \frac{m_\alpha m^2_{Q\bar{Q}}}{4\pi^2} \int_{\alpha_i}^{\alpha_f} \frac{d\alpha}{\alpha_i} \int_{1-\alpha}^{1-\alpha} d\beta (1 - \alpha - \beta) \left[ \frac{1}{\alpha} + \frac{p_0}{4} \right] \delta(p_0 - \bar{m}_Q) \\
\]

\[
\rho_{\mathbb{B}}^{\alpha_{Q\bar{Q}}}(p_0) = \frac{3m_\alpha m^2_{Q\bar{Q}}}{32\pi^4} \int_{\alpha_i}^{\alpha_f} \frac{d\alpha}{\alpha_i} \int_{1-\alpha}^{1-\alpha} d\beta \alpha \beta (p_0^2 - \bar{m}^2_Q)(3p_0^2 - 2\bar{m}^2_Q) \\
+ \frac{3m_\alpha m^2_{Q\bar{Q}}}{16\pi^4} \int_{\alpha_i}^{\alpha_f} \frac{d\alpha}{\alpha_i} \int_{1-\alpha}^{1-\alpha} d\beta (p_0^2 - \bar{m}^2_Q) \\
- \frac{m^2_{Q\bar{Q}}}{384\pi^2 T} \int_{\alpha_i}^{\alpha_f} \frac{d\alpha}{\alpha_i} \int_{1-\alpha}^{1-\alpha} d\beta \left[ \frac{1}{\beta^2} + \frac{1}{\alpha^2} \right] \delta(p_0 - \bar{m}_Q) \\
- \frac{m^4_{Q\bar{Q}}}{384\pi^2 p_0 T} \int_{\alpha_i}^{\alpha_f} \frac{d\alpha}{\alpha_i} \int_{1-\alpha}^{1-\alpha} d\beta \left[ \frac{1}{\beta^3} + \frac{1}{\alpha^3} \right] \delta(p_0 - \bar{m}_Q) \\
+ \frac{m_\alpha m^2_{Q\bar{Q}}}{64\pi^2 p_0} \int_{\alpha_i}^{\alpha_f} \frac{d\alpha}{\alpha_i} \int_{1-\alpha}^{1-\alpha} d\beta \left[ \frac{1}{\alpha^2} + \frac{1}{\beta^2} \right] \delta(p_0 - \bar{m}_Q) \\
- \frac{m_\alpha m^2_{Q\bar{Q}}}{4\pi^2} \int_{\alpha_i}^{\alpha_f} \frac{d\alpha}{\alpha_i} \int_{1-\alpha}^{1-\alpha} d\beta (1 + \frac{p_0}{6}) \delta(p_0 - \bar{m}_Q) \\
\]

(13) (14)
\[
\rho^A_{\Xi QQ}(p_0) = \frac{3p_0}{16\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta (1 - \alpha - \beta) (p_0^2 - \bar{m}_Q^2) (2p_0^2 - \bar{m}_Q^2) \\
+ \frac{3m_0^2 p_0}{16\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (1 - \alpha - \beta) (p_0^2 - \bar{m}_Q^2) \\
- \frac{m_0^2}{192\pi^2} \left\langle \alpha_s GG \right\rangle_{\pi} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (1 - \alpha - \beta) \left[ \frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2} \right] \left[ 1 + \frac{p_0}{2T} \right] \delta(p_0 - \bar{m}_Q) \\
- \frac{m_0^4}{384\pi^2 p_0 T} \left\langle \alpha_s GG \right\rangle_{\pi} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (1 - \alpha - \beta) \left[ \frac{1}{\alpha^3} + \frac{1}{\beta^3} \right] \delta(p_0 - \bar{m}_Q) \\
+ \frac{m_0^2}{64\pi^2} \left\langle \alpha_s GG \right\rangle_{\pi} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (1 - \alpha - \beta) \left[ \frac{1}{\alpha^2} + \frac{1}{\beta^2} \right] \delta(p_0 - \bar{m}_Q) \\
- \frac{1}{48\pi^2} \left\langle \alpha_s GG \right\rangle_{\pi} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (1 - \alpha - \beta) \left[ p_0 + \frac{p_0^2}{8} \delta(p_0 - \bar{m}_Q) \right], \tag{15}
\]

\[
\rho^B_{\Xi QQ}(p_0) = -\frac{\left\langle \bar{q} q \right\rangle}{4\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \alpha (1 - \alpha) \left[ 2p_0^2 - \bar{m}_Q^2 \right] - \frac{m_0^2 \left\langle \bar{q} q \right\rangle}{4\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha, \tag{16}
\]

\[
\rho^A_{\Omega Q}(p_0) = \frac{p_0}{64\pi^4} \int_{t_i}^{t_f} dt (2 + t) (1 - t)^2 (p_0^2 - \bar{m}_Q^2)^2 - \frac{p_0 m_s \left\langle \bar{s} s \right\rangle}{4\pi^2} \int_{t_i}^{t_f} dt t (2 - t) \\
+ \frac{m_s \left\langle \bar{s} g_s \sigma Gs \right\rangle}{48\pi^2} \int_{t_i}^{t_f} dt t \delta(p_0 - \bar{m}_Q) + \frac{m_s \left\langle \bar{s} g_s \sigma Gs \right\rangle}{24\pi^2} \delta(p_0 - m_Q) + \frac{\left\langle \bar{s} s \right\rangle^2}{6} \delta(p_0 - m_Q) \\
- \frac{p_0}{192\pi^2} \left\langle \alpha_s GG \right\rangle_{\pi} \int_{t_i}^{t_f} dt (2 - t) + \frac{m_0^2}{1152\pi^2} \left\langle \alpha_s GG \right\rangle_{\pi} \int_{t_i}^{t_f} dt \left(1 - t \right)^3 \frac{1}{t^2} \delta(p_0 - \bar{m}_Q) \\
- \frac{m_0^2}{384\pi^2} \left\langle \alpha_s GG \right\rangle_{\pi} \int_{t_i}^{t_f} dt \left(1 - t \right)^2 \delta(p_0 - \bar{m}_Q), \tag{17}
\]

\[
\rho^B_{\Omega Q}(p_0) = \frac{m_0}{64\pi^4} \int_{t_i}^{t_f} dt (2 + t) (1 - t)^2 (p_0^2 - \bar{m}_Q^2)^2 - \frac{m_0 m_Q \left\langle \bar{s} s \right\rangle}{4\pi^2} \int_{t_i}^{t_f} dt (2 - t) \\
+ \frac{m_s m_Q \left\langle \bar{s} g_s \sigma Gs \right\rangle}{48\pi^2 p_0} \int_{t_i}^{t_f} dt \delta(p_0 - \bar{m}_Q) + \frac{m_s m_Q \left\langle \bar{s} g_s \sigma Gs \right\rangle}{24\pi^2} \delta(p_0 - m_Q) + \frac{\left\langle \bar{s} s \right\rangle^2}{6} \delta(p_0 - m_Q) \\
- \frac{m_0}{192\pi^2} \left\langle \alpha_s GG \right\rangle_{\pi} \int_{t_i}^{t_f} dt (2 - t) + \frac{m_0}{576\pi^2} \left\langle \alpha_s GG \right\rangle_{\pi} \int_{t_i}^{t_f} dt \left(1 - t \right)^3 \frac{1}{t^2} \delta(p_0 - \bar{m}_Q) \\
- \frac{m_0}{1152\pi^2} \left\langle \alpha_s GG \right\rangle_{\pi} \int_{t_i}^{t_f} dt \left(1 - t \right)^3 \frac{1}{t} \bar{m}_Q \delta(p_0 - \bar{m}_Q), \tag{18}
\]
\[
\rho^A_{\Xi_Q}(p_0) = \frac{p_0}{128\pi^4} \int_{t_i}^1 dt \int dt' (2 + t)(1 - t)^2 (p_0^2 - \bar{m}_Q^2)^2 + \frac{p_0 m_\alpha \langle \bar{s}\bar{s} \rangle}{16\pi^2} \int_{t_i}^1 dt \int dt' (p_0 - \bar{m}_Q)^2 - \frac{m_\alpha \langle \bar{q}\bar{q} \rangle}{8\pi^2} \int_{t_i}^1 dt \int dt' (p_0 - \bar{m}_Q) + \frac{m_\alpha \langle \bar{q}\bar{q} \rangle}{8\pi^2} \int_{t_i}^1 dt \int dt' \delta(p_0 - \bar{m}_Q),
\]

\[
\rho^B_{\Xi_Q}(p_0) = \frac{m_Q}{128\pi^4} \int_{t_i}^1 dt (2 + t)(1 - t)^2 (p_0^2 - \bar{m}_Q^2)^2 + \frac{m_\alpha m_Q \langle \bar{s}\bar{s} \rangle}{16\pi^2} \int_{t_i}^1 dt - \frac{m_\alpha m_Q \langle \bar{q}\bar{q} \rangle}{8\pi^2} \int_{t_i}^1 dt + \frac{m_\alpha \langle \bar{q}\bar{q} \rangle}{8\pi^2} \int_{t_i}^1 dt \delta(p_0 - \bar{m}_Q) + \frac{m_\alpha m_Q \langle \bar{q}\bar{q} \rangle}{8\pi^2} \int_{t_i}^1 dt \delta(p_0 - \bar{m}_Q).
\]

where \(\alpha_f = \frac{1 + \sqrt{1 - 4m_Q^2/p_0^2}}{2}, \alpha_i = \frac{1 - \sqrt{1 - 4m_Q^2/p_0^2}}{2}, \beta_i = \frac{\alpha m_Q^2}{\alpha p_0 - m_Q}, \bar{m}_Q = \frac{(\alpha + \beta)m_Q^2}{\alpha \beta}, \bar{m}_Q = \frac{m_Q^2}{\alpha (1 - \alpha)}\)

in the channels \(\Omega^*_{QQ}\) and \(\Xi^*_{QQ}\); and \(\bar{m}_Q^2 = \frac{m_Q^2}{\alpha}, t_i = \frac{m_Q^2}{p_0}\) in the channels \(\Omega^*_{Q}, \Xi^*_{Q}\) and \(\Sigma^*_{Q}\).
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