Three-Boson Systems in Light-Front Dynamics

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Abstract. We have applied the quasi-potential approach to project the dynamics of three bosons to the light-front (LF). The quasi-potential approach projects the Bethe-Salpeter equation at equal LF times, subtracting out the three-body reducible diagrams, avoiding double counting in the kernel. We can obtain in a systematic way both the kernel of the integral equation for the valence component of the light-front wave function, and the next-to-leading order contribution to the kernel of the integral equation for the transition matrix. We give an example for the contact interaction model.

1. Introduction
In past and ongoing works we have applied successfully the quasi-potential (QP) approach to proceed a light-front (LF) equal time projection[1] and study two-body bosonic and fermionic systems[2] along with the electromagnetic current conservation issue. The framework provided by the QP approach turns out to be a systematic tool to project the dynamics of a composite system on the LF.

Some efforts have been devoted in the past years to study 3-body systems in the LF, with special emphasis on the LF properties of the nucleon[3, 4]. However, still one has to determine the LF wave function beyond the valence component for these systems within this framework. In this way, we propose ourselves to give a systematic answer to this problem using the QP formalism, and apply it to a contact interaction model.

2. QP Approach
We write the Bethe-Salpeter equation (BSE) for the T-matrix of a 3-boson system in the ladder approximation

\[ T = V + VG_0T \quad ; \quad V = \sum V_i \quad ; \quad V_i = V^{(2)}_{jk} S^{-1}_i, \]

where \( V^{(2)}_{jk} \) stands for the 2-body interaction between the particles \( j \) and \( k \), and \( S \) is the individual propagator. We rewrite the BSE in terms of the QP:

\[ T = W + W\tilde{G}_0T; \quad W = V + W\Delta_0V; \quad \Delta_0 := G_0 - \tilde{G}_0. \]

Above, \( W \) is the QP, and \( \tilde{G}_0 \) is the auxiliary Green function. This is so far the general QP approach. The LF projection is performed through a particular choice of the auxiliary Green’s function. The LF projection is done by defining

\[ \tilde{G}_0 := G_0||g_0^{-1}||G_0; \quad g_0 := ||G_0||, \]
where, for a given three body operator $\mathcal{O}$, we have defined the double bar $\| \|$ operation as:

$$\| \mathcal{O} \| := \int dk_1 dk_2 \langle k_1, k_2 | \mathcal{O} | k_1, k_2 \rangle; \quad \mathcal{O} \| := \int dp_1 dp_2 \mathcal{O} | p_1, p_2 \rangle.$$  \hfill (4)

Hence it is straightforward to perform the LF projection and define a LF transition matrix, which yields the three-body scattering amplitude:

$$t := g_0^{-1} \| G_0 T G_0 \| g_0^{-1} = w + w g_0 t; \quad w := g_0^{-1} \| G_0 W G_0 \| g_0^{-1}.$$  \hfill (5)

Using the standard Faddeev decomposition, $t = \sum_{i=1}^{3} t_i$, we derive connected equations for the Faddeev components of the transition matrix,

$$t_i = w_i + w_i g_0 t; \quad w_i = g_0^{-1} \| G_0 W_i G_0 \| g_0^{-1},$$  \hfill (6)

where $W = \sum_{i=1}^{3} W_i$ and $i$ runs over the three possible pairs.

3. Contact Interaction

We accomplish below the study of the kernel of the LF BSE in a example: a contact interaction. For the QP expansion in first order, we get simply that, $w_1(k, k' ; p, p') = k^+ \delta^3(k - p)$, where $k \equiv (k^+, k_\perp)$. The matrix elements of the interactions $w_2$ and $w_3$ are analogous.

![Figure 1. w3 in second order of the QP expansion.](image)

The QP in second order is represented by the diagrams sketched in the fig.1. The right diagram is the only three-body irreducible term (and its permutations) which contributes for the kernel of the LF BSE in second order. This goes beyond the model used in refs. [3, 4].

4. Conclusions

We have shown the application of the QP formalism for projecting the dynamics of 3-boson systems on the LF and, in this way, one has a tool that is worth and produces a systematic expansion of the kernel of the LF Bethe-Salpeter equation. Our study is consistent with the work by Weinberg[5] in instant form, who showed that even in the ladder approximation, equal time projections generate three body forces. For the contact interaction model, the three-body interaction is due to the coupling between the valence component and a five particle state. We are able to extend easily the formalism beyond the contact interaction.

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References
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