Radiative Strength Functions for Dipole Transitions in $^{90}$Zr

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Received October 19, 2011; in final form, January 16, 2012

Abstract—Partial cross sections for the $(p, \gamma)$ reaction on the $^{89}$Y nucleus that were measured previously at proton energies between 2.17 and 5.00 MeV and which were averaged over resonances were used to determine the absolute values and the energy distribution of the strength of dipole transitions from compound-nucleus states to low-lying levels of the $^{90}$Zr nucleus. The data obtained in this way were compared with the predictions of various models.

DOI: 10.1134/S1063778812120022

1. INTRODUCTION

Resonance-like structures at excitation energies well below the giant dipole resonance (GDR) were discovered in the energy distributions of the $(E1, M1)$ dipole strength that were obtained from total photabsorption cross sections recently measured for nuclei in the region around $A = 90$ [1–6]. The appearance of such structures is usually associated with the fragmentation of the resonance or with the excitation of other collective modes. Some of them are observed predominantly in nuclei featuring closed shells or nearly closed shells [1–3]. This is indicative of the possible dependence of special features in the energy distribution of the dipole strength on the nuclear structure.

From this point of view, it is of great interest to study the distribution of the dipole strength in the $^{90}$Zr nucleus. This rigid spherical nucleus featuring a closed neutron shell ($N = 50$) is characterized by a very high neutron separation energy of $S_n = 11970(3)$ keV, which is substantially higher than the proton binding energy, $S_p = 8354.8(16)$ keV [7]. The total photobosorption cross section studied in [8] for $^{90}$Zr as a function of energy in experiments with monochromatic photons in the energy range between 8.5 and 12.5 MeV showed the presence of a resonance-like structure that stands out against the behavior obtained by extrapolating, to this region, the giant dipole resonance approximated by a standard Lorentz distribution. Specifically, an obvious concentration of an additional strength was observed around the energy of 11.5 MeV. The experiments performed recently at new level of sensitivity and accuracy by using bremsstrahlung photons and reported in [2] revealed an excess of the dipole strength around the energies of 6.5 and 9 MeV.

By studying the mechanism of formation of structures in the same region of the energy distribution of the dipole strength for the nucleus under study via exciting this nucleus in different reactions, one can ensure a more reliable identification of observed structures and a more accurate quantitative description of these structures. First of all, this concerns reactions inverse to photobosorption. Within the traditional problem of a decrease in the $E1$ strength over the GDR low-energy wing (especially below the particle-emission threshold), it becomes necessary in this case to study in more detail the distribution of the strength of $E1$ transitions between neutron-capture (proton-capture) states and levels of the final-state nucleus at lower energies with resort (for the sake of comparison) to data on the photoexcitation of nuclei, which is a process that relates the ground state to dipole and quadrupole excitations.

Since a maximum of the $3p$-wave neutron strength function is observed in $A \approx 90$ nuclei, there are reasons to assume that, in the case where these nuclei are excited in $(n, \gamma)$ reactions, strong valent $E1$ transitions in them are determined by particle–hole configurations that are loosely bound to respective giant dipole resonances [9] and which may be one of the reasons for the appearance of the structures that were discovered. In the presence of nonstatistical effects, features that are observed in the energy distribution of reduced partial radiative widths averaged...
over various sets of resonances and over the sets of both correlated and uncorrelated transitions in \((n,\gamma)\) reactions indicate that, in the region below the neutron binding energy, GDR fragmentation may not be a purely stochastic process [10]. In contrast to what we have for other nuclei whose mass numbers are close to \(A = 90\), the results presented in [2, 8] for \(\text{Zr}\) cannot be supplemented with data for the respective \((n,\gamma)\) inverse-photoabsorption reaction because of the absence of a stable target nucleus. Despite the different roles that neutrons and protons must play in the formation of the energy distribution of the dipole strength in \(\text{Zr}\), it therefore becomes necessary to study it with the aid of the \((p,\gamma)\) reaction on \(\text{Y}\).

The objective of the present study is to determine the absolute values and the energy distribution of the strength of dipole transitions in the \(\text{Zr}\) nucleus in the region of a possible manifestation of resonance-like structures on the basis of already available experimental data on partial and total cross sections for the reaction of proton radiative capture \([11–16]\).

Investigation of special features of the energy distribution of the dipole strength of \(\gamma\) transitions in the \(\text{Zr}\) nucleus is of interest not only as a source of unique information about the structure of nuclei featuring a nearly closed shell or the \(N = 50\) closed shell and about the effect of this structure on the processes of photon emission and absorption by such nuclei. Isotopes in the region around \(A = 90\) are in the vicinity of one of the maxima of the fission-fragment yield, and one employs yttrium, zirconium, niobium, and molybdenum in reactor structural materials. The energy distribution of the dipole strength of \(\gamma\) transitions in nuclei as expressed in terms of the radiative strength function (RSF), which is a fundamental feature of a nucleus in what is concerned with photon absorption and emission by this nucleus, is an indispensable component of a quantitative analysis of cross sections for photonuclear reactions and radiative-nucleon-capture reactions inverse to them that is performed within statistical theory. It follows that prospects for the simulation of \(p\) processes of nucleosynthesis in nuclear astrophysics and the development of new nuclear technologies depend crucially on reliably determining the radiative strength function in the energy region below the threshold for neutron separation from nuclei of mass number close to \(A = 90\).

In relation to [17], where only the region of probable values of the strength of dipole transitions in \(\text{Zr}\) could be determined, the present investigations are more detailed. A vast and diverse set of experimental data accumulated thus far serves here as a reliable basis for determining parameters used in the statistical model.

\section{EXPERIMENTAL RESULTS AND THEIR ANALYSIS}

In the present study, the energy distribution of the strength of dipole \(\gamma\) transitions in the \(\text{Zr}\) nucleus is determined in the reaction \(\text{Y}(p,\gamma)\text{Zr}\) from a comparison of the results obtained by measuring, at incident-proton energies in the range between 2.17 and 5.0 MeV \([11–14]\), and thereupon averaging partial cross sections for this reaction with the RSF-dependent cross section calculated for this reaction on the basis of Hauser–Feshbach statistical theory. This method for determining radiative strength functions, which is described in more detail in \([18]\), for example, is model-dependent, but the statistical model as such, if justifiably applied, is one of the most precise models for a quantitative analysis of nuclear data. Its parameters either are well known or can be refined in the same experiment—for example, on the basis of an analysis of the inelastic-scattering channel, where the radiative channel plays an insignificant role.

At incident-proton energies in the range of \(E_p < 6\) MeV, the partial cross sections for the \((p,\gamma)\) reaction on \(\text{Y}\) are determined primarily by processes involving the production of a compound nucleus \([19]\) in the state \(\lambda\) and its deexcitation via direct \(\gamma\) transitions to lower lying states \(f\) of the final nucleus \(\text{Zr}\). The ground-state spin–parity of the \(\text{Y}\) target nucleus is \(J_0^\pi = 1/2^-\). In this nucleus, the last unpaired proton is in the \(2p_{1/2}\) shell. The cross section for the \((p,\gamma)\) reaction in question is dominated by the contribution of the channel where the proton orbital angular momentum is \(l_p = 0\) and \(J_\lambda^\pi = 1^-\) and which is accompanied by the \(E1\) transitions to the ground state, whose quantum numbers are \(0^+_1\); the first excited state, whose quantum numbers are \(0^+_2\) and whose energy is 1760.71 keV; and to the \(2^+\) states at 2186.27, 3308.8, and 3842.2 keV. For the \(\text{Zr}\) nucleus, all other final states such that the intensities of primary \(\gamma\) transitions to them can be measured, are characterized by a higher spin and can reached only via the capture of protons having higher values of the orbital angular momentum \(l_p\) or via a higher multipole order. The threshold for the reaction \(\text{Y}(p,n)\text{Zr}\) is \(E_p = 3.65\) MeV, but the ground state of the \(\text{Zr}\) nucleus has the spin–parity of \(J_\pi^\pi = 9/2^+\), and the transition to this state for slow neutrons is improbable. The energy of \(E_p = 4.24\) MeV at which the first excited state, whose quantum numbers are \(J_\pi^\pi = 1/2^-\), corresponds to the effective threshold. The heat of the \((p,\gamma_0)\) reaction on the \(\text{Y}\) nucleus is 8.3548(16) MeV; therefore, the compound nucleus formed is excited to the energy of 12.5 MeV at incident-proton energies not exceeding the effective threshold for the opening of the competing neutron
channel (through the reaction $^{89}$Y($p$, $n_1$)$^{89}$Zr at $E_p = 4.24$ MeV). At this excitation energy, the attained density of states even in this even–even spherical nucleus of $^{90}$Zr, which is magic in the number of neutrons, $N = 50$, exceeds (see, for example, [20]) $10^4$ MeV$^{-1}$, which must satisfy the requirements of applicability of a statistical description. On the basis of an analysis of investigations of the $(p, n)$, $(p, p)$, $(p, p')$, and $(p, \gamma)$ reactions on the $^{80}$Y nucleus and on neighboring nuclei [11, 12, 19–24], $(\gamma, \gamma')$ and $(\gamma', \gamma')$ reactions on the $^{90}$Zr nucleus [2, 25], as well as the $(n, \gamma)$ reaction on the isotopes neighboring $^{90}$Zr isotope [5, 6], one can conclude that the distances between resonance levels having identical spins and parities in the product compound nucleus $^{90}$Zr obey the Wigner distribution, while the protonic and radiative widths follow two independent Porter–Thomas distributions. At the same time, the protonic width must be substantially larger than the mean radiative width. Thus, there are reasons to assume that conditions necessary for the applicability of the statistical description hold.

The energy distribution of the strength of primary $\gamma$ transitions accompanying the deexcitation of states of the compound nucleus can be described, irrespec-
tive of the way of its formation, in terms of quantities averaged over the corresponding interval of excitation energies, such as the density $\rho = D_\lambda^{-1}$ of states whose spins and parities lie in specific intervals and radiative strength functions; that is, the mean reduced probability for an $E1$ or an $M1$ transition of mean width $\Gamma_\lambda^{XL}$ and energy $E_\gamma$ connecting the $\lambda$ and $f$ states is

$$S^{XL}(E_\gamma) = \frac{1}{\lambda_f^{XL}} \left( E_\gamma^3 A^{2/3} D_\lambda \right),$$  

(1)

where $D_\lambda$ is the mean spacing between the $\lambda$ states having a specific spin–parity.

According to statistical theory, the cross section for the process in which the capture of a proton with energy $E_p$ is followed by the emission of a photon having an energy $E_\gamma$ and corresponding to a primary transition from the state $\lambda$ of the compound nucleus that reached equilibrium to the final state $f$ can be represented in the form

$$\sigma_{\gamma\lambda} = \frac{\pi \lambda_p^2}{2(2I + 1)} \sum_J (2J + 1) \sum_{l_pj_p} \frac{T_{lpj_p} T_{\gamma\lambda f}}{T_\lambda},$$  

(2)

where $\lambda_p$ is the reduced wavelength of the incident proton, $I$ is the target spin, $J$ is the compound-nucleus spin, $T_{l_pj_p}$ is the coefficient of transmission (sticking) for protons in the entrance channel, $T_{\gamma\lambda f}$ is the coefficient of transmission for photons of energy $E_\gamma = E_\lambda - E_f$ that correspond to primary transitions from the group of $\lambda$ states to the final state $f$, and $T_\lambda$ is the sum of transmission coefficients corresponding to all open channels of deexcitation of $\lambda$ states. Summation in expression (2) is performed over all open reaction channels and over compound-nucleus states whose quantum numbers $J$ and $\pi$ are allowed by relevant selection rules. In the present study, the calculations are performed with allowance for corrections for possible fluctuations of the cross section that are due to a small number of open channels [26–28]. At low energies, this correction may prove to be significant. The transmission coefficient $T_{\gamma\lambda f}$, which is the quantity obtained by averaging, over compound-nucleus resonances, the probability for a $\gamma$ transition of multipolarity $L$, can be expressed in terms of the partial radiative strength function $S_{\lambda f}(E_\gamma)$ as

$$T_{\gamma\lambda f} = 2\pi S_{\lambda f}(E_\gamma) E_\gamma^{2L+1}. \quad (3)$$

The coefficient $T_\lambda$ then has the form

$$T_\lambda = \sum_{l_pj_p} T_{l pj_p} + \sum_{l_\lambda j_f} T_{l_\lambda j_f} \quad (4)$$

$$+ 2\pi \sum_J \int_0^{E_\lambda} \rho_J(E_{\lambda} - E_\gamma) S_{\lambda f}(E_\gamma) E_\gamma^{2L+1} dE_\gamma,$$

where $T_{l_pj_p}$ are the transmission coefficients for protons in the exit channel; $T_{l_\lambda j_f}$ are the transmission coefficients for the neutron channel; and $\rho_J(E_{\lambda} - E_\gamma)$ is the density of levels characterized by a spin $J$, a parity $\pi$, and an excitation energy $E_f$.

By using relations (2)–(4) and experimental data on partial cross sections for respective $(p, \gamma)$ reactions, one can determine absolute values of the radiative strength function versus the photon energy and properties of states between which the $\gamma$ transition being considered occurs. Because of the dominance of the dipole mode in the radiative decay of compound-nucleus states, the quantity $S_{\lambda f}(E_\gamma) = S^{E1}_{\lambda f}(E_\gamma) + S^{M1}_{\lambda f}(E_\gamma)$, which is the sum of radiative strength functions for $E1$ and $M1$ transitions, is extracted from experimental data on cross sections. The reliability of the values obtained in this way for radiative strength functions depends not only on the errors in the experimental data used but also on the justifiability and accuracy of the transmission coefficients and level densities used in Eqs. (2)–(4).

Our present calculations of the transmission coefficients for protons relied on the traditional optical model and employed as inputs the parameters of the optical potential (OP) that were obtained phenomenologically within one of the global systematics compiled recently [29]. For incident–proton energies below the Coulomb barrier, the use of OP parameters
directly from the global systematics would be poorly justified, irrespective of whether they were obtained on the basis of purely phenomenological considerations or were calculated on the basis of a realistic nucleon–nucleon interaction and specific ideas of nuclear matter [30, 31]. By investigating the absorptive properties of spherical nuclei in this region [22, 24], it was shown that, in the mass-number region around \( A \approx 90 \), the absorptive potential at subbarrier energies is substantially smaller than that which would follow from the global systematics. It would be natural to assume [24] that, in the region of closure of the \( N = 50 \) shell, the decrease in the absorptive potential, as well as the decrease in the diffuseness parameter, as will be seen below, is due to shell effects.

In the present study, the optical–potential parameters from the global systematics in [29] that were used as inputs were changed in such a way as to obtain the best fit to available data on the cross sections for the \((p, n)\) reactions on \( ^{89}\text{Y} \) and \( ^{93}\text{Nb} \) nuclei for incident-proton energies in the range from the neutron threshold to 5.8 MeV [22, 23]. For some of the OP parameters already obtained, a very small modification was needed in order to reach subsequently the best fit to the cross section measured for the \((p, p)\) and \((p, n)\) reactions on \( \text{Zr} \) and \( \text{Mo} \) isotopes at incident-proton energies in the range of \( E_p = 2 – 7 \) MeV [32], as well as to the cross sections measured for the \((p, p')\), \((p, p'')\), and \((p, \gamma)\) reactions on \( ^{90}\text{Zr} \) nuclei at the incident-proton energies in the range of \( E_p = 1.9 – 5.7 \) MeV [24]. All of the parameters ultimately obtained for the real part of the optical potential, with the exception of a somewhat reduced diffuseness parameter \((a_R = 0.62 – 0.73 \, \text{fm})\), were nearly identical to their counterparts in the global systematics from [29], but the parameters of its imaginary part differed markedly from those presented in [29]. For example, the imaginary part of the absorptive potential in the energy region under investigation complied most closely with the simple energy dependence (in MeV units) \( W_D = 2.73 + 0.70E_p \). This corresponds to the change in \( W_D \) from 4 MeV at \( E_p = 2 \) MeV to about 7 MeV at \( E_p = 6 \) MeV. For \( E_p \geq 6 \) MeV, the absorptive potential approaches fast its values from the global systematics.

In order to calculate the level density in the \( ^{90}\text{Zr} \) nucleus at excitation energies in the range being studied, we employed either the semiempirical back-shifted Fermi gas model or microscopic methods. In the first case, the calculations were performed with the level-density parameter of \( a = 8.95(41) \) MeV\(^{-1} \) and the energy shift of \( \Delta = 1.97(30) \) MeV from the global systematics recently published in [33] and obtained on the basis of a fit to experimental data on the schemes of levels in nuclei at low excitation energies and on the distances between \( s\)-wave neutron resonances at the neutron binding energy. The data on the discrete section of the spectrum of levels in \( ^{90}\text{Zr} \) were taken from the NUDAT BNL database [7], which is based on current publications concerning this nucleus. Identical level densities were assumed for positive- and negative-parity states of identical spin. This assumption was recently confirmed by good agreement of the results calculated for the level density on the basis of the back-shifted Fermi gas model with experimental data obtained from the reaction \(^{90}\text{Zr}(^{4}\text{He}, \gamma)^{90}\text{Nb} \) for the density of levels corresponding to the \( 1^+ \) states in the energy range between 5 and 10 MeV, as well as with experimental densities of \( 2^+ \) and \( 2^- \) levels in the \( ^{90}\text{Zr} \) nucleus from high-resolution measurements of the inelastic scattering of protons with energy \( E_p = 200 \) MeV at forward angles and electrons with energy \( E_0 \approx 66 \) MeV at backward angles on \( ^{90}\text{Zr} \) [34].

Calculated and experimental level densities in the \( ^{90}\text{Zr} \) nucleus are given in Fig. 1 versus the excitation energy. The closed triangles and open circles in Fig. 1a represent the level densities obtained in [35] from the reactions \(^{90}\text{Zr}(p, p')\) and \(^{90}\text{Zr}(e, e')\) for the \( 2^+ \) and \( 2^- \) states, respectively. The open triangles in Fig. 1b correspond to the density of levels for the \( 1^- \) states that was obtained from a statistical analysis of partial–width fluctuations manifesting themselves in the excitation function measured for the reaction \(^{88}\text{Y}(p, \gamma)^{90}\text{Zr} \) at incident–proton energies in the range of \( E_p = 2.6 – 3.3 \) MeV [20]. The open boxes stand for data on discrete levels of respective spin–parity. The solid curve in Fig. 1 represents the results of the calculations based on the back-shifted Fermi gas model and performed in the present study at the parameter values of \( a = 8.95 \) MeV\(^{-1} \) and \( \Delta = 1.97 \) MeV. The dash-dotted curve corresponds to the analogous calculations, but the parameters were set there to the values of \( a = 9.37 \) MeV\(^{-1} \) and \( \Delta = 2.19 \) MeV, which were obtained from an analysis of the results of the parametrization used in [36] for the level density in the region of \( A \approx 90 \) nuclei. The dashed curve stands for the results available from [37], where the calculations relied on the microscopic statistical model employing the Hartree–Fock + Bardeen–Cooper–Schrieffer (HF + BCS) approximation [38].

The quantity \( S_{\chi}(E_{\gamma}) \) [see Eq. (2) and (3)] was chosen in such a way as to reproduce the absolute experimental value of the partial cross section for the \((p, \gamma)\) reaction. The radiative strength function in expression (4) for the total transmission coefficient was specified in a form that was obtained in various theoretical approaches. All of the remaining parameters obtained earlier were fixed. It was assumed that the effect of the model dependence on the values
states at 4.82 and 5.02 MeV, which were identified earlier, were excluded from the averaging procedure. In Fig. 2, the closed circles represent the values of the radiative strength function that were obtained from the averaged (over the interval of $\Delta E_p = 1.2$ MeV) cross sections that were measured in [11] for the population of individual low-lying states of $^{90}$Zr at incident-proton energies $E_p$ changing from 2.2 to 3.4 MeV with a step of 15 to 18 keV. Averaging over so wide an interval was performed because of strong fluctuations of original cross sections in this energy range. In the measurements, we used a $^{89}$Y target of equivalent thickness 13.7 keV at $E_p = 3.0$ MeV. The energy range of dipole transitions between 5.9 and 11.1 MeV for which we extracted radiative strength functions was determined by the range of energies of final states for these transitions (from 0.0 to 5.2 MeV) at the initial-state energy fixed at $E_\lambda = 11.1$ MeV, which corresponds to the mean incident-proton energy of $E_p^{\text{mean}} = 2.77$ MeV.

Employing the values obtained by measuring the partial cross sections for the ($p, \gamma$) reactions on $^{89}$Y nuclei at incident-proton energies $E_p$ in the range between 2.17 and 5.0 MeV [13, 14] and thereupon averaging the results over the range of $\Delta E_p = 0.5$ MeV, we determined by the same method the radiative strength functions for the dipole transitions to the $0^+$ (0.0), $0^+$ (1.761), $2^+$ (2.186), $3^-$ (2.748), $2^+$ (3.309) and $2^+$ (3.842) states of the $^{90}$Zr nucleus (the energies of the states in MeV units being indicated parenthetically), the characteristics of these states being established reliably. In Fig. 3, the respective RSF values obtained at the fixed energy of $E_\lambda = 12.1$ MeV ($E_p^{\text{mean}} = 3.75$ MeV) are represented by closed circles. The open circles in Figs. 2 and 3 correspond to radiative strength functions extracted the total cross sections presented in [39, 40] for the photoabsorption reaction in the GDR region, which receive a dominant contribution from the ($\gamma, n$) and ($\gamma, p$) radiations. The stars in Fig. 3 stand for the radiative-strength-function values obtained from the total photoabsorption cross section measured in a beam of bremsstrahlung gamma radiation [2]. Because of strong fluctuations, those data were averaged over the interval of $\Delta E_\gamma = 500$ keV.

3. COMPARISON OF EXPERIMENTAL RESULTS WITH PREDICTIONS OF THEORETICAL MODELS

From the assumption put forth in [8, 41] that the strength for both gamma-ray absorption and gamma-ray emission in the region below the neutron separation energy is determined by the low-energy GDR tail, it follows that, at incident–proton energies
Fig. 2. Experimental and theoretical RSF values for dipole $\gamma$ transitions in $^{90}$Zr at the initial-state energy fixed at $E_{\lambda} = 11.1$ MeV and various values of the final-state energy $E_f$. The displayed points stand for (closed circles) the RSF values obtained in our present study from the measured (in [11]) intensities of primary $\gamma$ transitions in $^{90}$Zr at $E_{p,mean} = 2.77$ MeV for the averaging interval of width 1.2 MeV and (open circles) data on the giant dipole resonance from [39]. The curves (for the respective notation, see main body of the text) correspond to (1) SLO, (2) MLO2, (3) Sirotkin’s approach, (4) GFL, (5) MLO1, (6) KMF, and (7) EGLO.

Fig. 3. As in Fig. 2, but at $E_{\lambda} = 12.1$ MeV. The displayed points stand for (closed circles) the RSF values obtained in our present study from the measured (in [13, 14]) intensities of primary $\gamma$ transitions in $^{90}$Zr at $E_{p,mean} = 3.75$ MeV, (stars) the RSF values from data on the photoabsorption cross section from [2], and (open circles) data on the giant dipole resonance from [39]. Curves 1–7 are identical to those in Fig. 2; curves 8 and 9 represent the hypothesized $M1$- and $E1$-resonance contributions, respectively, parametrized in terms of a Lorentzian distribution; and curve 10 corresponds to MLO2 supplemented with an $M1$ resonance.
in the range being considered, a virtual excitation of the giant-dipole-resonance mode must have a decisive effect on the exit $\gamma$ channel of the reaction $^{89}\text{Y}(p,\gamma)^{90}\text{Zr}$. In this case, the radiative strength function for primary $\gamma$ transitions of given multipolarity is related through the detailed-balance principle to the respective cross section for photoabsorption by a nucleus in the ground state. It is obvious that, in the region below the neutron binding energy in $^{90}\text{Zr}$, the procedure of employing a unified approximation of the radiative strength function by a Lorentzian distribution such that the energy of its maximum coincides with the respective experimental value is not justified theoretically, since this procedure does not take fully into account the observed structural effects.

The RSF values presented in Figs. 2 and 3 and obtained from an analysis of experimental data are contrasted against the radiative strength functions calculated within various theoretical approaches versus the photon energy decreasing to zero limit. Curve 1 corresponds to the radiative strength function related through the detailed-balance principle to the photoabsorption cross section in the form of a standard Lorentzian distribution. The following GDR parameters were used for the single-resonance Lorentzian function: the energy at the maximum was $E_r = 16.7$ MeV, the width was $\Gamma_r = 4.2$ MeV, and the cross section for photoabsorption at the maximum was $\sigma_r = 211$ mb. These parameters were chosen in such a way as to arrive at the best fit of a Lorentzian distribution to experimental data obtained in [39] for the energy dependence of the photoabsorption cross section in the region of the GDR maximum. Within the phenomenological standard Lorentzian model (SLO in the notation adopted in [37]), the absorptive width is taken to be a constant that coincides with the GDR width. In the calculations that are based on the use of a generalized Lorentzian distribution (EGLO [37, 42]) and whose results are represented by curve 7 in Fig. 2 and in Fig. 3, the absorptive width is dependent on energy and is assumed to be proportional to the collision component of the damping width of zero sound in an infinite Fermi liquid in the case where one takes into account only a collision-induced two-body relaxation. Curves 2 and 5 represent the radiative strength functions calculated on the basis of the modified Lorentzian models (MLO2 and MLO1, respectively) [37, 43], while curve 4 stands for the results of the calculations on the basis of the generalized Fermi liquid model (GFL [44]). Within these models, the absorptive width is also dependent on energy, but simultaneously, one takes into account contributions both from fragmentation and from collisions. These RSF models differ by the expression for the absorptive width and by the contribution of various dissipation mechanisms. Curve 6 corresponds to the results of the calculations for the radiative strength function within the Kadensky–Markushev–Furman model [45] (KMF in the notation adopted in [37]), which is based on taking phenomenologically into account the coupling of particle–hole configurations to more complex states. Within this model, the expression representing the spreading GDR width and corresponding to the estimates obtained by Landau and Eliashberg for damping within the theory of an infinite Fermi liquid has the following form for even–even nuclei:

$$\Gamma(E_\gamma, T_f) = \Gamma_r E_r^{-2} \left( E_\gamma + 4\pi^2 T_f^2 \right) ,$$

where $T_f$ is the effective temperature of the nucleus in the state to which the $\gamma$ transition in question occurs, $\Gamma_r$ is the width of the giant dipole resonance, and $E_r$ is the energy at its maximum. The results obtained by calculating the radiative strength function within Sirotkin’s statistical approach [46], which is based on Fermi liquid theory, are represented by curve 3. A feature peculiar to the approach described in [46] is that, in calculating the density of 2$p2h$ states, one takes into account the shell structure of the spectrum of single-particle levels and the effect of the nuclear temperature and transition energy on the occupation probabilities for these levels. Only states such that transitions between them were allowed by the Pauli exclusion principle were included in the calculation.

Figure 2 shows that the absolute RSF values determined from experimental data on partial cross sections for the $(p, \gamma)$ reaction on $^{89}\text{Y}$ at $E_p^\text{mean} = 2.77$ MeV do not deviate strongly from an extrapolation of the Lorentzian distribution used to parametrize experimental data in the GDR region. Even at $E_p^\text{mean} = 3.75$ MeV (see Fig. 3), however, the radiative strength functions obtained from the $(p, \gamma)$ reaction in the energy region below 11 MeV show a sizable excess in absolute value above the Lorentzian–like strength function and a deviation in shape from it. This agrees with the observation of resonance-like structures above the extrapolation of the Lorentzian distribution in the energy range between 6 and 11 MeV in the photoabsorption reaction [2, 8]. Within a microscopic description of cross sections for the photoabsorption of dipole gamma rays by near-magic nuclei in the energy range of 6–8 MeV, there are always remnants of intrinsic transitions that existed in the model of noncolliding particles. The authors of [2] associated the concentration of strength in the region of 6 to 7 MeV with the excitation of an electric pygmy dipole resonance (PDR), but, at higher energies, it becomes important to take into account the PDR and GDR coupling to multiphonon states; it is difficult to implement this in theoretical calculations, so that one
has to invoke various parametrizations. This excess of strength, fragmented to a considerable extent, can be exhausted, albeit not completely, by a contribution hypothesized as in [5] and parametrized in the form of a Lorentzian distribution from the M1 resonance having an energy of 9 MeV, a FWHM value of 2.5 MeV (1.2 MeV in our present study), and a cross section of 7 mb at the maximum. This contribution is represented by curve $\delta$ in Fig. 3. To a not lower degree of plausibility, this could be an E1 resonance having approximately the same parameters (curve $\gamma$ in Fig. 3) and resulting from the possible GDR fragmentation. The results of experiments in beams of highly polarized tagged photons [25] revealed that the strength that can be associated with a giant M1 resonance in $^{90}$Zr is broadly distributed over the excitation-energy range from 8.1 to 10.5 MeV, which is studied here. At the same time, a dominant contribution of the E1 strength was identified in this region.

We cannot rule out the possibility that a more pronounced character of substructures observed in the energy distribution of the radiative strength function, which were estimated on the basis of experimental data on the partial cross sections for the $(p, \gamma)$ reaction on $^{89}$Y at $E_p^{\text{mean}} = 3.75$ MeV, is due not only to an averaging interval narrower than that in Fig. 2. To a still greater extent, it may be caused by special features present in original data. In Fig. 4, the experimental partial cross sections are contrasted against their counterparts calculated by formula (2). In the calculations, use was made of radiative strength functions obtained within various theoretical approaches for E1 transitions to final states of the $^{90}$Zr nucleus that were characterized by specific spin–parities $J^\pi$. The energies of states are given in MeV units.

The measured excitation functions show a pronounced fine structure not disappearing as $E_p$ increases from 2.17 to 5.0 MeV and as the level density in the compound nucleus grows accordingly, by a factor of about six. After the averaging of the excitation functions, for example, over an interval of width 270 keV, they become rather smooth in the range of $E_p < 3.3$ MeV inclusive, apart from small deviations for individual states, so that one can describe them successfully within statistical theory. However, variations belonging to the intermediate-structure type and having different shapes and magnitude for all final states, including states characterized by identical $J^\pi$, manifest themselves at higher energies.

In describing the spectrum of levels of the $^{90}$Zr nucleus on the basis of the simple shell model, one most frequently takes $^{88}$Sr as an inert core. At low excitation energies, the valent protons in $^{90}$Zr must then be active in the $2p_{1/2}$ and $1g_{9/2}$ space and, to a smaller extent, in the $1f_{5/2}$ and $2p_{3/2}$ space. The energy difference between the $2p_{1/2}$ and $1g_{9/2}$ orbitals forms the energy gap for protons, which is equal to 2670(90) keV [47], the pairing energy being 3593(8) keV. The shell-model gap for neutrons is 4445(8) keV, while the pairing energy amounts to 4093(12) keV. A strong mixing of the $\pi 2p_{1/2}$ (up to 59%) and $\pi 1g_{9/2}$ (up to 41%) configurations both in the ground state, whose spin–parity is $0^+$, and in the first excited state, whose spin–parity is $0^+$, is assumed in this model. One also assumes that the $\pi (1g_{9/2})^2$ configuration characterized by a specific degree of mixing with excited-core states corresponds to $J^\pi = 2^+$ states. Possibly, the $2^+_2$ and $2^+_3$ states feature a sizable admixture of the $2p_{3/2}$ and $1f_{5/2}$ configurations [48]. The $3^-$ state is described as an octupole phonon.

The fact that, at the proton energies in the region of $E_p > 3.3$ MeV, the energy dependence of the partial cross sections for the $\gamma$ transitions to all final states of the $^{90}$Zr nucleus in Fig. 4, with the exception of the $0^+_1$ and $0^+_2$ states, is not described within statistical theory may be indicative, first of all, of the possible contribution to the cross section for the $(p, \gamma)$ reaction from the direct process of single-particle transfer. This contribution as manifested in cross sections for $(n, \gamma)$ reactions on $A \approx 90$ nuclei is usually treated as the result of an implementation of the
Fig. 4. Experimental partial cross sections for the \((p, \gamma f)\) reaction for low-lying states of \(^{90}\text{Zr}\) (the energies are given in MeV units) and their counterparts calculated within statistical theory. The open and closed circles stand for the experimental cross sections from [13] and [14], respectively. The curves represent the cross section calculated by using radiative strength functions found within the (dashed curve) SLO, (dash-and-double-dot curve) KMF, and (solid curve) MLO2 approaches.

Valent-capture mechanism and the effect of doorway states [49]. In this case, there are grounds to assume that strong valent \(E1\) transitions may be determined by particle–hole configurations weakly coupled to the corresponding giant dipole resonance [7]. At the same time, the proton strength function for \(^{89}\text{Y}\) in the energy range under study is rather smooth [23, 24], since the single-particle \(3s\) and \(3p\) shape resonances are, respectively, below 2 MeV and above 5 MeV. Moreover, data on \((d, n)\) and \((^{3}\text{He}, d)\) reactions of single-particle transfer (see [7] and references therein) are insufficient for determining, to a rather high degree of precision, the spectroscopic factors for states of the \(^{90}\text{Zr}\) nucleus that are of interest, but these spectroscopic factors are necessary for a correlation analysis. In this situation, it is difficult to explain in terms of a mechanism acceptable for the \((n, \gamma)\) reaction how that part of the radiative strength which is associated with the simple component of the state wave function may undergo fragmentation over resonances (with allowance for their protonic width). Nevertheless, special features of the structure of the \(^{90}\text{Zr}\) nucleus do not rule out the possibility for explaining the observed (in Fig. 4) nonstatistical contribution to the partial cross sections for the \((p, \gamma)\) reaction on \(^{89}\text{Y}\) in terms of the formation of doorway states. We emphasize that, in the case of a dominant valent contribution, the use of the intensities of primary \(\gamma\) transitions from the reaction of radiative neutron (proton) capture in order to obtain data on radiative strength functions by the method applied in the present study becomes illegitimate.
of the choice of RSF model is ultimately tested by addressing the question of whether it can faithfully reproduce experimental data over the entire range of γ-transition energies. The results obtained by calculating the total cross sections for the \((p, \gamma)\) reaction on \(^{89}\)Y nuclei on the basis of the Hauser–Feshbach theory with radiative strength functions found within various theoretical approaches are given in Fig. 5 along with experimental data from [15, 16]. Curves 1–7 represent the results of these calculations with the radiative strength functions obtained on the basis of, respectively, the SLO model, the microscopic model from [50], the GFL model [44], Sirotkin’s approach [46], the MLO2 model [43], the KFM model [45], and the EGLO model [42]. From Fig. 5, one can see that curve 5, which was calculated with the radiative strength function based on the MLO2 model, and curve 4, which is nearly coincident with it and which was calculated with the radiative strength function based on Sirotkin’s approach, exhibit the best agreement with experimental data. At the same time, the calculations with the radiative strength function obtained on the basis of the standard Lorentzian model yield total cross sections for the \((p, \gamma)\) reaction on \(^{89}\)Y that are markedly overestimated in relation to experimental data.

4. CONCLUSIONS

By using resonance-averaged data that were obtained earlier from the \((p, \gamma)\) reaction on \(^{89}\)Y [11–14], we have determined the absolute values and energy distribution of the strength of dipole transitions in the \(^{90}\)Zr nucleus in the region of a possible manifestation of resonance-like structures observed in the photoexcitation of this nucleus [2, 8]. This determination relies, first of all, on the dependence of the cross section written within statistical theory for the \((p, \gamma)\) reaction on the spectroscopic properties of the final-state nucleus and on its statistical properties related to the radiative strength function. The data obtained in this way are compared with the results obtained by calculating radiative strength functions within various theoretical approaches.

The absolute values determined at the mean proton energy of \(E_p^\text{mean} = 2.77\) MeV for the strength of dipole transitions in the range of their energies between 5.9 and 11.1 MeV do not deviate strongly from the extrapolation of the Lorentzian distribution that is used to parametrize experimental data in the GDR region. However, the radiative strength function corresponding to \(E_p^\text{mean} = 3.75\) MeV features an obvious excess of the strength in agreement with the resonance-like structure observed earlier in experiments that studied photon scattering. We cannot rule

![Fig. 5. Total cross section for the \((p, \gamma)\) reaction on \(^{89}\)Y. The open circles and closed boxes stand for the experimental cross sections from [16] and [15], respectively. The curves represent the cross sections calculated on the basis of Hauser–Feshbach theory by using the radiative width found within various theoretical models: (1) SLO, (2) microscopic model from [50], (3) GFL, (4) Sirotkin’s approach, (5) MLO2, (6) KFM, and (7) EGLO.](image)
out the possibility that a more pronounced character of the substructures observed in this case is due to special features present in original experimental data on partial cross sections.

Additional measurements that would refine the level of the nonstatistical contribution to the partial cross sections for the \((p, \gamma)\) reaction on \(^{89}\)Y nuclei and reliable experimental data on spectroscopic factors from one-particle-transfer reactions populating the same states of the \(^{90}\)Zr nucleus as the respective \((p, \gamma)\) reaction are required for drawing definitive conclusions on the mechanism of formation of the energy distribution of the strength of dipole transitions in \(^{90}\)Zr.

The results of the calculations on the basis of Hauser–Feshbach theory that employ the total radiative width and which rely on the strength functions obtained within the modified Lorentzian model, Sirotkin’s approach, the generalized Fermi liquid model, and the microscopic model are in the best agreement with experimental data on the total cross sections for the \((p, \gamma)\) reaction on \(^{89}\)Y nuclei. It is these models, but with allowance for an excess strength parametrized in terms of a Lorentzian distribution, that provide the best description of experimental data on partial strength functions as well. The use of a standard Lorentzian distribution leads to total cross sections substantially underestimated both in relation to experimental data and in relation to the calculations based on the different theoretical models.

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Translated by A. Isaakyan