Bounds on R-Parity Violating Parameters from Fermion EDM’s

M. Frank\textsuperscript{a} and H. Hamidian\textsuperscript{b}

\textsuperscript{a}Department of Physics, Concordia University, 1455 De Maisonneuve Blvd. W. Montreal, Quebec, Canada, H3G 1M8
\textsuperscript{b}Department of Physics, Stockholm University, Box 6730, S-113 85 Stockholm, Sweden

Abstract

We study one-loop contributions to the fermion electric dipole moments in the Minimal Supersymmetric Standard Model with explicit $R$-parity violating interactions. We obtain new individual bounds on $R$-parity violating Yukawa couplings and put more stringent limits on certain parameters than those obtained previously.

\textsuperscript{1}e-mail mfrank@vax2.concordia.ca
\textsuperscript{2}e-mail hamidian@vanosf.physto.se
Introduction

The Standard Model (SM) of electroweak interactions is constructed in such a way that it automatically conserves both the baryon number $B$ and the lepton flavor number $L$. These (accidental) global symmetries of the SM result from the particle content and the $SU(2)_L \times U(1)_Y$ gauge invariance of the theory and naturally explain the non-observation of $B$- and $L$-violating processes, as well as the stability of the nucleon. However, in supersymmetric (SUSY) extensions of the SM these features no longer follow. In fact, by promoting the SM fields to superfields, additional gauge- and Lorentz-invariant terms will be generated which violate $B$ and $L$ conservation. For example, in the SM the Higgs doublet and the lepton doublet have the same $SU(2) \times U(1)$ quantum numbers, but the spins of the particles are different; whereas in the SUSY extensions of the SM the distinction between the Higgs doublet and the lepton doublet disappears and one would naturally expect lepton number violation. In order to forbid $B$- and $L$-violating interactions in SUSY extensions of the SM a parity quantum number defined as $R = (-1)^{3B+L+2S}$, where $S$ is the spin, is assigned to each component field and invariance under $R$ transformation is imposed [1]. Although the assignment of the *ad hoc* $R$-parity to superfields in the SUSY extensions of the SM reproduces the global $B$ and $L \ U(1)$ symmetries of the SM, it is by no means the only symmetry which allows the construction of phenomenologically viable SUSY extensions of the SM [2]. In fact, from a phenomenological point of view, it is most important to ensure that there are no interaction terms in the Lagrangian which lead to rapid proton decay and, in this respect, other discrete symmetries can be used which are even more effective than $R$-parity. This is simply due the fact that $R$-parity only forbids dimension-four $B$- and $L$-violating operators in the Lagrangian, while dimension-five operators can still remain dangerous, even if suppressed with cut-off’s as large as the Planck mass (see Aulakh *et al.* in Ref.[3] for an interesting alternative to $R$-parity which forbids dimension-four and dimension-five $B$-violating interactions while allowing $L$-violating operators of the same dimensions in the Lagrangian). Since there is no fundamental theoretical basis for imposing $R$-parity conservation on the minimal supersymmetric extension of the Standard Model (MSSM), it is worthwhile to investigate the phenomenological constraints on theoretically allowed $R$ parity breaking couplings in the MSSM.

Unlike the SM in which all the elementary matter fields are fermions and, consequently, $B$
and $L$ quantum numbers are separately conserved, the MSSM has a particle content which
contains scalar leptons and quarks, thus allowing separate $B$ and $L$ violating interaction
terms in the Lagrangian.

Using only the MSSM superfields, the most general renormalizable $R$-parity violating
superpotential can be written as:

$$W = \lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k, \quad (1)$$

where $i, j, k$ are generation indices and we have rotated away a term of the form $\mu_{ij} L_i H_j$.

In (1) $L$ and $Q$ denote the lepton and quark doublet superfields respectively, and $\bar{U}_i$, $\bar{D}_j$ and
$\bar{D}_k$ are singlet $SU(2)$ superfields. The couplings $\lambda_{ijk}$ and $\lambda''_{ijk}$ are antisymmetric with respect
to the interchange of $SU(2)$ flavor indices: $\lambda_{ijk} = -\lambda_{jik}$ and $\lambda''_{ijk} = -\lambda''_{ikj}$.

To avoid rapid proton decay, it is not possible for both $\lambda$, $\lambda'$ type and $\lambda''$ type couplings
to have nonzero values. We shall assume here—as is often done in the literature—that only
the lepton number is violated (see [3] for restrictions on the $\lambda''$ type couplings).

Viewing the SM as a low-energy effective theory, one often searches for potential contributions
arising from the physics beyond the SM. In this manner, numerous studies on $R$-parity violating
decays have either resulted in separate bounds on the $\lambda$, $\lambda''$ couplings, or on their
products. Some of the most important studies involve limits coming from proton stability,
$n-\bar{n}$ oscillations, $\nu_e$-Majorana mass, neutrinoless double $\beta$ decays, charged current universality,
$\nu_\mu-\nu_e$ deep inelastic scattering, atomic parity violation, $e-\mu-\tau$ universality, $K^+$-decays,
$\tau$-decays, $D$-decays and precision measurements of LEP electroweak observables [4]. In ad-
dition, when the assumption of $R$-parity conservation is relaxed, the superpartner spectrum
for the MSSM is expected to be dramatically different from the one with $R$-conservation,
the most important consequence being that the lightest supersymmetric particle can decay.

The strongest bounds so far on the $\lambda$ and $\lambda'$ couplings come from cosmological con-
siderations on the survival of the cosmic $\Delta B$ [5]. They are usually obtained assuming the
$L$-violating interactions to be constantly out of equilibrium until the weak scale, so that they
cannot wash out any $(B-L)$ asymmetry previously generated. They are relaxed in the case
where the $L$-violating interactions are allowed to survive at the weak scale to give rise to
lepton-number violations. Lepton-number violating interactions in the context of $R$-parity
breaking have been studied recently and more stringent bounds than those reported earlier
have been found [5].
The most precise low-energy measurements in leptonic physics are the lepton flavor-violating decays such as $\mu \to e\gamma$, $\mu \to eee$, the anomalous magnetic moment of the muon $a_\mu$, and the EDM of the electron. These are all very sensitive probes and are often used to explore physics beyond the SM. In particular, the electric dipole moment (EDM) of the leptons (especially the electron) and the neutron are strictly bound by experiments with the currently available upper limits given by\cite{7,8,3},

\begin{align}
 d_e &= (3 \pm 8) \times 10^{-27} \text{ e cm}, \\
 d_\mu &= (3.7 \pm 3.4) \times 10^{-19} \text{ e cm}, \\
 d_\tau &< (3.7 \pm 3.4) \times 10^{-17} \text{ e cm},
\end{align}

(2)

for the leptons, and

\begin{equation}
 d_n < 1.1 \times 10^{-25} \text{ e cm}
\end{equation}

(3)

for the neutron. In this letter we investigate bounds on $R$-parity violating interactions by using the available experimental upper limits on the EDM’s of the fermions. As we shall discuss below, the individual bounds that we put on certain $R$-parity violating couplings are more stringent than those found prior to this work. We shall also briefly discuss the significance of the EDM bounds compared to the ones obtained by using the—also accurately measured—values of the lepton anomalous magnetic moments.

$R$-Parity Violating Interactions and Fermion EDM’s

We shall begin with a brief review of the fermion EDM’s and then proceed to evaluate the leading order contributions that arise by including $R$-parity violating interactions in the MSSM.

The electric dipole moment of an elementary fermion is defined through its electromagnetic form factor $F_3(q^2)$ found from the (current) matrix element

\begin{equation}
 \langle f(p')|J_\mu(0)|f(p)\rangle = \bar{u}(p')\Gamma_\mu(q)u(p),
\end{equation}

(4)

where $q = p' - p$ and

\begin{equation}
 \Gamma_\mu(q) = F_1(q^2)\gamma_\mu + F_2(q^2)i\sigma_\mu\gamma_5q'/2m + F_A(q^2)(\gamma_\mu\gamma_5q^2 - 2m\gamma_5q_\mu) + F_3(q^2)\sigma_\mu\gamma_5q'/2m,
\end{equation}

(5)
with $m$ the mass of the fermion. The EDM of the fermion field $f$ is then given by

$$d_f = -F_3(0)/2m,$$

(6)

corresponding to the effective dipole interaction

$$\mathcal{L}_I = -\frac{i}{2} d_f \bar{f} \sigma_\mu \gamma_5 f F^{\mu\nu},$$

(7)
in the static limit.

Since a non-vanishing $d_f$ in the SM results in fermion chirality flip, it requires both $CP$ violation and $SU(2)_L$ symmetry breaking. Even if one allows for $CP$-violation in the leptonic sector of the SM, the lepton EDM’s vanish to one-loop order due to the cancellation of all the $CP$-violating phases. Two-loop calculations for the electron [8] and for quarks [9] also yield a zero EDM. In the MSSM, however, there are many more sources of $CP$ violation than in the SM. In addition to the usual Kobayashi-Maskawa phase $\delta$ from the quark mixing matrix, there are phases arising from complex parameters in the superpotential and in the soft supersymmetry breaking terms. The phases of particular interest to us are those coming from the so-called $A$-terms, $A_{u,d} = |A_{u,d}| \exp(i\phi_{A_{u,d}})$ [10]. The $CP$-violating effects arise from the squark mass matrix which has the following form:

$$\mathcal{L}_{M_u} = (\bar{\tilde{u}}_L^\dagger \bar{\tilde{u}}_R)(\begin{pmatrix} \mu_L^2 + m_u^2 & A_u^* m_u \\ A_u m_u & \mu_R^2 + m_u^2 \end{pmatrix})(\begin{pmatrix} \bar{\tilde{u}}_L \\ \bar{\tilde{u}}_R \end{pmatrix}),$$

(8)

and similarly for $\mathcal{L}_{M_d}$, where the mass parameters $|A_u|$, $\mu_L$ and $\mu_R$ are expected to be of the order of the $W$-boson mass $M_W$. The fields $\bar{\tilde{u}}_L$, $\bar{\tilde{u}}_R$ can be transformed into mass eigenstates $\tilde{u}_1$, $\tilde{u}_2$,

$$\bar{\tilde{u}}_L = \exp(-\frac{1}{2}i\phi_{A_u})(\cos \theta \ \bar{u}_1 + \sin \theta \ \bar{u}_2),$$

$$\bar{\tilde{u}}_R = \exp(\frac{1}{2}i\phi_{A_u})(\cos \theta \ \bar{u}_2 - \sin \theta \ \bar{u}_1),$$

(9)

where the mixing angle $\theta$ is given by

$$\tan 2\theta = 2|A_u|m_u/(\mu_L^2 - \mu_R^2).$$

(10)

and the physical masses, $M_{1,2}$, corresponding to the eigenvalues of the mass matrix in (8) are

$$M_{1,2}^2 = \frac{1}{2}\mu_L^2 + \mu_R^2 + 2m_u^2 \pm [(\mu_L^2 - \mu_R^2)^2 + 4m_u^2|A_u|^2]^{1/2}.$$
The lepton EDM’s at one-loop order are generated by the interactions in Fig.1 (with similar Feynman diagrams for the muon and the tau) and resemble those in the MSSM with charginos or neutralinos in the loop. The contributions to the lepton EDM’s are then given by

\[
d_{e_i} = - \left| \lambda'_{ijk} \right|^2 \frac{4e m_{d_k}}{3 m_f^2} |A_{u_j}| \sin \theta \cos \theta \sin(\phi_{A_u}) f_3(x_{d_k})
- \left| \lambda'_{ijk} \right|^2 \frac{2e m_{d_k}}{3 m_f^2} |A_{u_j}| \sin \theta \cos \theta \sin(\phi_{A_d}) f_4(x_{d_k})
- \left| \lambda'_{ijk} \right|^2 \frac{2e m_{u_j}}{3 m_f^2} |A_{d_k}| \sin \theta \cos \theta \sin(\phi_{A_d}) f_3(x_{u_j})
- \left| \lambda'_{ijk} \right|^2 \frac{4e m_{u_j}}{3 m_f^2} |A_{d_k}| \sin \theta \cos \theta \sin(\phi_{A_d}) f_4(x_{u_j}),
\]

(12)

where \( x_{u,d} = (m_{u,d}/m_f)^2 \), with \( f \) the scalar quark in the loop, and the loop integrals are expressed in a familiar form in terms of the functions

\[
f_3(x) = \frac{1}{2(1-x)^2} \left[ 1 + x + \frac{2x \ln x}{1-x} \right],
\]

\[
f_4(x) = \frac{1}{2(1-x)^2} \left[ 3 - x + \frac{2 \ln x}{1-x} \right].
\]

(13)

In order to simplify, we will assume degenerate squark masses, \( \mu_L \approx \mu_R \approx |A_{u,d}| = \mathcal{O}(M_W) \), and expand only to the leading order in \( m_{u,d}|A_{u,d}|/M_{1,2}^2 \). Comparing the above expressions for the fermion EDM’s with the usual ones obtained in the MSSM [11], it is not difficult to see that there are two sources of enhancement: one coming from the absence of the electroweak coupling constant, \( \alpha_{ew} \), responsible for an enhancement of \( \mathcal{O}(10^2) \), and another from potentially large fermionic masses in the loop. Indeed, in this scenario it is possible to obtain a contribution to the electron EDM proportional to the mass of the top quark, in contrast to the usual one proportional to \( m_e \). (Note that even if the EDM is proportional to the mass of the up quark, there will still be an enhancement of \( \mathcal{O}(20) \).) Unfortunately estimating \( d_e, d_\mu, \) and \( d_\tau \) numerically is not completely straightforward since no model-independent experimental information is available on squark masses and mixing angles. We shall assume, without great loss of generality, that \( \cos \theta = \sin \theta = 1/\sqrt{2} \). We shall also assume, \( \phi_{A_u} = \phi_{A_d} \) and \( |A_{u_j,d_k}| \approx m_f = \mathcal{O}(M_W) \), which is in agreement with the naturalness of the MSSM.
Putting all these together, Eq. (12) becomes

\[ d_{ei} = - |\lambda'_{ijk}| \frac{1}{3} \left( \frac{m_f}{100 \text{GeV}} \right)^{-3} \left( \frac{|A_{u,d}|}{100 \text{GeV}} \right) \sin(\phi_A) \times \left\{ m_{d_k} \left[ 4f_3(x_{u_j}) + 2f_4(x_{u_j}) \right] + m_{u_j} \left[ 2f_3(x_{d_k}) + 4f_4(x_{d_k}) \right] \right\} \times 10^{-21} \text{ e cm.} \quad (14) \]

We would like to comment that in addition to these contributions, in theories with massive neutrinos one could have contributions coming from the Feynman diagrams such as those in Fig.2. These are not included here since we restrict ourselves to the particle spectrum of the MSSM. Including these contributions in other SUSY extensions of the SM could provide restrictions on the \( \lambda_{ijk} \) parameters, however they would all depend rather sensitively on the neutrino mass.

Any estimate of the lepton EDM’s must be correlated and further restricted by estimates of the neutron EDM. In the \( R \)-parity-conserving MSSM the neutron EDM severely restricts the masses of the squarks, barring accidental cancellations between the supersymmetric phases in the squark and gluino matrices. With the introduction of \( R \)-parity violating interactions, the terms contributing to the quark EDM’s are Yukawa-type only, such as those shown in Fig.3 for the up quark. Taking all the scalar quark masses to be the same, and taking \( m_u \approx m_d = 10 \text{ MeV} \), the up- and down- quark EDM’s are equal and the neutron EDM is \( d_n = \frac{4}{3} d_d - \frac{1}{3} d_u \approx d_d \). Since the experimental limit on the neutron EDM is \( |d_n| < 1.2 \times 10^{-25} \text{ e cm} \) the limits obtained on the \( \lambda_{ijk} \) parameters are weaker than those obtained from the electron EDM, in contrast with the existing situation in \( R \)-parity conserving MSSM.

The limits that we obtain on the individual \( \lambda'_{ijk} \) parameters from the electron (\( \lambda'_{1jk} \)), muon (\( \lambda'_{2jk} \)) EDM’s are given in Table 1. Unfortunately, the weak constraint on the tau EDM is insufficient to adequately restrict all the \( \lambda'_{3jk} \) coefficients. Two cases are considered in Table 1: (i) light slepton spectrum (\( m_f = 100 \text{ GeV} \)) and (ii) heavy slepton spectrum (\( m_f = 1 \text{ TeV} \)). In both cases we find strong bounds from the electron EDM and weaker ones from the \( \mu \) or \( \tau \) EDM’s. The strongest limits appear to be the ones on \( \lambda_{1j3} \) because of effects of \( \mathcal{O}(m_t) \). The difference between a light and a heavy squark spectrum is in agreement with other estimates on the \( \lambda' \) parameters. The different scenarios bridge the gap of any other possible assumptions on the superparticle spectrum. For instance, if instead of assuming a
degenerate squark mass spectrum, we assume a universal scalar mass spectrum at the GUT scale and then evolve masses at the electroweak scale as in [12], our assumptions resemble very closely the case in which $\tan \beta \leq 10$. In this case the squarks of the first two families and $\tilde{b}_R$ have very similar masses and only the stop is heavier. That would affect mostly the $\lambda_{ijk}$ coefficients; the results will be of the same order of magnitude with slightly different numerical factors.

If we assume, as has been sometimes done in the literature, that the $\lambda'_{ijk}$ are flavor blind, i.e. $\lambda'_{ijk} = \lambda'$, we will be able to severely restrict all the $R$-parity violating couplings, as shown in Figures 4 and 5 respectively for the light and heavy squark mass scenarios. In both cases the restrictions on the $\lambda'_{ijk}$ couplings are more stringent than previously found limits. The EDM’s do not, unfortunately, provide any limits on the $\lambda_{ijk}$ or $\lambda''_{ijk}$ couplings, so one would have to rely on limits obtained by considering other phenomena, such as those mentioned in the introduction.

We shall now briefly comment on possible restrictions coming from the anomalous magnetic moment of the muon, $a_\mu$. Its experimental value is $a_{\mu}^{exp} = 1165922(9) \times 10^{-9}$ and its measured deviation from the SM prediction lies within a range of $-2 \times 10^{-8} \leq \Delta a_{\mu}^{exp} \leq 2.6 \times 10^{-8}$. The one-loop contributions to the muon magnetic moment also come from Feynman diagrams similar to those in Fig.1 and are given by

$$a_\mu = \frac{F_2(0)}{e},$$

where $F_2(0)$ is the static limit of the electromagnetic form factor defined in (15). Unfortunately, the restrictions that $a_\mu$ would place on the $R$-parity violating couplings $\lambda$ and $\lambda'$ are extremely weak compared to those coming from the EDM’s. The EDM of the electron is expected to be known to about five orders of magnitude more accurately than its anomalous magnetic moment within a few years. Indeed the precise measurements of the electron $(g-2)$ factor yield $\Delta[(g-2)/2] = 1 \times 10^{-11}$, which corresponds to $\Delta(F_2(0)/2m_e) = 2 \times 10^{-22} e \text{ cm}$. The same is true for the anomalous magnetic moment of the muon [13]. In addition, for the calculation of the form factor $F_2(q^2)$ for the electron (muon) the helicity flip occurs on the external fermion line, giving rise to an amplitude proportional to the electron (muon) mass. These two factors combined, give weaker bounds on the $R$-parity violating Yukawa couplings than previously found, despite the fact that the anomalous magnetic moment does not require $CP$ violation.
To conclude, we have studied one-loop contributions to the lepton and neutron EDM’s in the MSSM with explicit $R$-parity violating interactions. We have used the—very accurately measured—experimental values of the lepton EDM’s to severely restrict the magnitude of the $R$-parity violating Yukawa couplings. This analysis has allowed us to obtain new bounds on individual $\lambda'_{ijk}$ couplings, rather than their combinations with other parameters.

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Figure Captions:

**Figure 1:** Graphs contributing to the electron EDM that put limits on the value of the $\lambda'_{1jk}$ couplings. Similar graphs contribute to the $\mu$- and $\tau$-EDM's and put limits on $\lambda'_{2jk}$ and $\lambda'_{3jk}$ respectively.

**Figure 2:** Graphs that could contribute to the electron EDM in theories with massive neutrinos. Similar graphs contribute to the $\mu$- and $\tau$-EDM's. These graphs could restrict the values of the $\lambda_{ijk}$ couplings.

**Figure 3:** Graphs contributing to up quark EDM’s and therefore to the neutron EDM. For the down quark the same graphs contribute, with $d \leftrightarrow u$ and $e \leftrightarrow e^c$.

**Figure 4:** The electron EDM as a function of a universal coupling $\lambda'_{ijk} = \lambda'$ (horizontal axis) for the light squark scenario, $m_{\tilde{f}} = 100 \, GeV$. We take the following values for the quark masses: $m_u = m_d = 10 \, MeV$, $m_s = 300 \, MeV$, $m_c = 1.5 \, GeV$, $m_b = 4.5 \, GeV$ and $m_t = 175 \, GeV$ [7].

**Figure 5:** The electron EDM as a function of a universal coupling $\lambda'_{ijk} = \lambda'$ (horizontal axis) for the heavy squark scenario, $m_{\tilde{f}} = 1 \, TeV$. We take the following values for the quark masses: $m_u = m_d = 10 \, MeV$, $m_s = 300 \, MeV$, $m_c = 1.5 \, GeV$, $m_b = 4.5 \, GeV$ and $m_t = 175 \, GeV$ [7].

Table Captions:

**Table 1:** Bounds on the $R$-parity violating parameters, $\lambda'_{ijk}$, from the electron ($\lambda'_{1jk}$) and muon ($\lambda'_{2jk}$) EDM’s, compared with previous bounds obtained from: (a) $K^+$-decay [14]; (b) Atomic parity violation and $eD$ asymmetry [13]; (c) $t$-decay [14]; (d) $\nu_e$-Majorana mass [16]; (e) $\nu_\mu$ deep-inelastic scattering [13].
### Table 1

| $|\lambda'_{ijk}|^2 \leq m_{\tilde{f}} = 100 \, GeV$ | $m_{\tilde{f}} = 1 \, TeV$ | Previous Limits for $m_{\tilde{f}} = 100 \, GeV$ |
|---|---|---|
| $|\lambda'_{111}|^2$ | $3 \times 10^{-9}$ | $2.4 \times 10^{-7}$ | $1.44 \times 10^{-4}$ (a) |
| $|\lambda'_{112}|^2$ | $5 \times 10^{-10}$ | $3 \times 10^{-8}$ | $1.44 \times 10^{-4}$ |
| $|\lambda'_{113}|^2$ | $9 \times 10^{-11}$ | $4 \times 10^{-9}$ | $1.44 \times 10^{-4}$ (a) |
| $|\lambda'_{121}|^2$ | $7.5 \times 10^{-11}$ | $4.3 \times 10^{-9}$ | $1.44 \times 10^{-4}$ (a) |
| $|\lambda'_{122}|^2$ | $6.6 \times 10^{-11}$ | $4 \times 10^{-9}$ | $1.44 \times 10^{-4}$ (a) |
| $|\lambda'_{123}|^2$ | $4 \times 10^{-11}$ | $2 \times 10^{-9}$ | $1.44 \times 10^{-4}$ (a) |
| $|\lambda'_{131}|^2$ | $2.6 \times 10^{-12}$ | $2.4 \times 10^{-11}$ | $1.44 \times 10^{-4}$ (a), $0.0676$ (b) |
| $|\lambda'_{132}|^2$ | $2.6 \times 10^{-12}$ | $2.4 \times 10^{-11}$ | $1.44 \times 10^{-4}$ (a), $0.16$ (c) |
| $|\lambda'_{133}|^2$ | $2.5 \times 10^{-12}$ | $2.3 \times 10^{-11}$ | $1.44 \times 10^{-4}$ (a), $10^{-6}$ (d) |
| $|\lambda'_{211}|^2$ | $3 \times 10^{-1}$ | $24$ | $1.44 \times 10^{-4}$ (a) |
| $|\lambda'_{212}|^2$ | $5 \times 10^{-2}$ | $3$ | $1.44 \times 10^{-4}$ (a) |
| $|\lambda'_{213}|^2$ | $9 \times 10^{-3}$ | $4 \times 10^{-1}$ | $1.44 \times 10^{-4}$ (a) |
| $|\lambda'_{221}|^2$ | $7.5 \times 10^{-3}$ | $4.3 \times 10^{-1}$ | $1.44 \times 10^{-4}$ (a) |
| $|\lambda'_{222}|^2$ | $6.6 \times 10^{-3}$ | $4 \times 10^{-1}$ | $1.44 \times 10^{-4}$ (a) |
| $|\lambda'_{223}|^2$ | $4 \times 10^{-3}$ | $2 \times 10^{-1}$ | $1.44 \times 10^{-4}$ (a) |
| $|\lambda'_{231}|^2$ | $2.6 \times 10^{-4}$ | $2.4 \times 10^{-3}$ | $1.44 \times 10^{-4}$ (a), $0.0484$ (e) |
| $|\lambda'_{232}|^2$ | $2.6 \times 10^{-4}$ | $2.4 \times 10^{-3}$ | $1.44 \times 10^{-4}$ (a), $0.16$ (c) |
| $|\lambda'_{233}|^2$ | $2.5 \times 10^{-4}$ | $2.3 \times 10^{-3}$ | $1.44 \times 10^{-4}$ (a), $0.16$ (c) |
Figure 1
Figure 2
Figure 3
Figure 4
Figure 5