A Review on the Failure Modes of Rock and Soil Mass under Compression and the Exploration about Constitutive Equations of Rock and Soil Mass

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The constitutive equation of rock and soil has always been the core problem in rock and soil mechanics. Up to now, the nature of nonlinear shear strength of rock and soil has not been revealed. In many engineering practices, it is still considered that the failure mode of rock and soil is always shear failure, and Coulomb linear constitutive equation is adopted, or Mohr envelope is fitted by data. However, the constitutive equation of rock mass is nonlinear, and its failure mode is not only shear failure. A large number of single triaxial tests show that there is not only shear stress, but also tensile stress in the failure process of rock and soil. Theoretical and experimental research on failure mode of rock mass under pressure are one of the important means to improve and develop soil mechanics. It is important to understand the nonlinear nature of rock and soil constitutive equation to explore the energy variation and the distribution of tensile stress and shear stress in different failure modes of rock and soil.

1. Introduction

The strength of materials under complex stress states cannot be measured experimentally, so it is necessary to hypothesize the mechanical factors of material strength failure. Strength theories often used in engineering practice include maximum tensile stress theory, maximum tensile strain theory, maximum shear stress theory, von Mises theory and Mohr–Coulomb theory [1]. Mohr–Coulomb theory is the most widely used theory in engineering practice, especially for rock and soil.

The failure modes of rock and soil mass under pressure are various and very complex. The change of the nature of rock and soil mass and the change of external pressure conditions all produce different failure modes of rock and soil mass. Many phenomena in nature are accompanied by the destruction of rock and soil mass such as landslides and collapses of mountains and foundation soil instability. Understanding the failure modes of different rock and soil mass under different conditions and exploring the internal stress state of rock and soil mass under different failure modes play an important role in more comprehensive understanding of the mechanical properties of rock and soil mass and further revealing its constitutive relations. Rock mechanics study always attaches great importance to the compression of rock and soil’s shear failure, but no matter what the failure mode of geotechnical material is, they are used to press the shear failure model to analyze and calculate the shearing strength parameters, and in the process of investigation of geotechnical material is shear failure that generally does not pay attention to or pay attention to whether the tension failure of rock and soil body is earlier than facts or possibility of shear failure. The tensile phenomenon in the process of deformation and failure of rock and soil under pressure and the mechanical mechanism of different failure modes caused by it are worth thinking of deeply.
This paper summarizes a large number of experimental results, introduces the failure modes of different rock mass under different stress conditions, and summarizes and classifies the different failure modes of rock mass in detail. Based on the summary of previous experimental results, the internal stress state of rock and soil under different failure modes is analyzed, and a reasonable explanation for the nonlinear cause of Mohr envelope is proposed. Finally, the future development and direction of the constitutive relation of rock and soil is prospected.

2. Failure Modes of Pressurized Soil Materials

2.1. Failure Modes of Soil Materials under Different Stress States. Since Taylor first found that soil samples in triaxial test would show different failure modes [2], more and more scholars began to pay attention to the failure modes and modes of rock and soil, conducted studies on various deformation and failure modes of rock and soil under pressure from different angles, and published a large number of relevant papers. Vaid and Campanella conducted triaxial tests on saturated undisturbed Marine clay under different stress paths and drainage conditions, indicating that soil samples were damaged by shear under low confining pressure and dilated under high confining pressure [3].

Since the beginning of the 21st century, people have made more detailed studies on the deformation and failure modes of soil samples under pressure. Triaxial test of Ng and Chiu showed that shear failure occurred in granite residual soil within a certain range of stress, and swelling failure occurred in soil samples after the stress was higher than the range [4]. Luo et al., when studying the influence of initial stress on clay failure, also found that the soil samples in the triaxial shear test of remolded soil samples were all destroyed by transverse swelling, and a series of triaxial compression tests of undisturbed soil and remolded soil showed similar failure mode above [5–7]. The triaxial test conducted by Xu on the silt with high liquid limit and sand under high suction showed that when the confining pressure was 0, the soil sample was split and cracked along the longitudinal direction. With the increase of confining pressure, the soil gradually changed to shear failure [8].

The failure mode of soil will also change with the change of the first principal stress. Liu conducted conventional triaxial experiments on remolded clay by controlling different confining pressures and loading rates and found that, with the increase of confining pressures, the shear failure zone of soil samples gradually increased, and the connectivity of the shear failure zone gradually improved [9], as shown in Figure 1. With the increase of the loading rate, the cracks of the soil sample gradually increased. With the further increase of the loading rate, the compressive vertebra failure zone began to appear in the soil sample.

The change of the second principal stress also has a great influence on the failure mode of rock mass. Sun et al.’s triaxial compression test results in the hardening and softening process of loading and unloading soil samples show that the size of the second principal stress has a great impact on the location of shear deformation [10]. Yu et al. performed the true triaxial test on the viscous Q2 loess with weak collapsible and structural properties and found that the soil samples with strong structure appeared trapezoidal failure in the case of \( \sigma_2 > \sigma_3 \), while the soil showed hexagon failure in the case of harder soil [11].

In addition to the triaxial stress state, the loading mode of soil also has an important influence on its failure mode. Dong et al. conducted triaxial compression tests on modified silt soil by unconsolidated undrained (UU), consolidated undrained (CU), and consolidated drainage (CD), as shown in Figure 2. And the results showed that the swelling failure of modified silt soil occurred to different degrees under UU and CU experiments, and the swelling degree decreased with the increase of confining pressure [12]. The consolidated drainage samples show cleavage failure at low confining pressure, and with the increase of confining pressure, single shear failure or conjugate shear zone will occur.

The deformation of soil consists of bulk strain and shear strain, and the corresponding energy is bulk strain energy and distortion energy, both of which can damage the structure of soil. In the initial shearing phase, the bulk energy of the under consolidated sample is larger than the distortion energy, and the bulk compression is the main factor; the distortion energy of the overconsolidated sample is larger than the bulk energy, and the shear energy is the main one; the two energies of the normally consolidated sample are approximately equal; the dilatancy phenomena is the phenomenon that when the volume of the specimen can no longer be compressed, the energy consumed by the volume expansion comes from the distortion energy of the specimen [13]. The influence of confining pressure on failure mode of soil is the most concerned and studied in geotechnical mechanics field. With the increase of confining pressure, the soil mass mostly develops from shear failure to dilatancy failure (lateral deformation appeared first, and then shear failure zone appeared with the increase of lateral deformation) and then to bulging failure (there is only lateral deformation and no obvious failure zone). For some soils with strong brittleness, they also exhibit splitting failure under uniaxial compression. With the increase of confining pressure, the failure mode changes from splitting failure to shear failure. And with the increase of the first principal stress, the failure zone of soil gradually increases and
becomes more obvious, and the failure mode of soil gradually changes from shear failure to punching shear failure or even splitting failure (single split zone or multiple split zone). With the increase of the second principal stress, the swelling failure of soil changes to lateral swelling failure and even shear failure. In addition, the loading mode of soil also has a certain influence on the failure mode of soil. The fracture zone and brittleness of soil under CD loading are more obvious than those under CU loading andUU loading.

2.2. Failure Modes of Different Types of Soil under Compression. Under the same stress conditions, different kinds of soil materials also show different failure modes, such as shear failure of granite residual soil under low confining pressure [4, 5] and splitting failure of silty soil containing sand and high liquid limit [8]. Even for the same soil materials, different physical properties may lead to change of failure modes. Under the same stress conditions, different kinds of soil materials will also show different failure modes, such as shear failure of granite residual soil under low confining pressure.

For the same kind of soil, the failure modes of undisturbed soil and remolded soil are very different. For example, in the ring shear test and triaxial test, the remolded homogeneous clay is destroyed in the form of swelling [14]. Li et al. (2003) through the triaxial test results show that the original state and restore the damage form of frozen clay existence very big difference, undisturbed frozen soil usually has the typical characteristics of brittle failure (fracturing, shear failure or tensile shear failure), and reshaping the permafrost usually has a typical lateral strain larger tensile-shear failure characteristics; namely, the reshape permafrost showed a certain degree of ballooning [15]. The study on the compressive failure characteristics of granite eluvial soil in South China also shows that the undisturbed soil of granite eluvial soil is shear failure, and the remolded soil is bulging failure [7, 16, 17].

Water, as a very common fluid in nature, often participates in the deformation process of stressed rock and soil. As an important index of soil physical properties, the influence of water content on soil mechanical properties not only becomes an important research topic, but also has important significance for the analysis of failure modes. Yu et al. conducted a true triaxial experiment on the viscous Q2 loess with weak collapsible and structural properties, and the results showed that the sample with water content of 20% presented lateral tension failure (bulging failure), and the sample with water content of 10% presented conical shear surface [11]. Luo (2014) also carried out uniaxial compression tests on undisturbed and remolded loess soil samples with different water content, and the results showed that, with the increase of water content, the failure mode of loess gradually changed from splitting failure to bulging failure, among which the undisturbed loess is more prone to splitting failure [18]. It can be inferred that the soil moisture content also has a great influence on the failure mode of compression soil mass. For low moisture content soil mass, its compression failure mode tends to shear failure, while the soil mass with high moisture content tends to bulge failure. Dong et al. experimental results of UU, CU, and CD of soil mass can also explain the influence of moisture content on soil failure mode to a certain extent [12]. Also, the uniaxial experiment of granite residual soil with different moisture content is carried out. With the increase of moisture content, the soil sample gradually changes from splitting failure to dilatancy failure, as shown in Figure 3.

![Figure 2: Failure modes of UU, CU, and CD specimens under different confining pressures, from Dong et al. [12].](image)
In addition, pore ratio, initial stress, stress path, consolidation degree density, and other factors have different degrees of influence on the failure mode of soil [19, 20]. Yamamuro et al. (2012) concluded that the failure mode of kaolinite clay under triaxial stress was closely related to its degree of consolidation, and the greater the degree of consolidation, the more likely it was to produce shear failure [21]. And low-density samples showed hardening-shear shrinkage deformation. With the increase of soil sample density, the deformation of samples changed from hardening-shear shrinkage to softening-shear dilatation, indicating that the sample density had a significant influence on its failure mode [22].

Similarly, the influence of modified material on soil strength is reflected in the failure mode of soil. Ou et al. studied the influence of cement content on soil failure and showed that when the cement content was low, the soil was plastic failure, which was manifested as oblique shear failure. When the cement content is high, the soil is brittle failure, manifested as tensile failure, and compressive vertebral appears at the bottom of the sample [23], as shown in Figure 4.

The failure modes of undisturbed and remolded soil samples are different under low confining stress and high confining stress. The compressive failure of the original sample is inclined to shear failure with shear plane at low confining pressure, while the compressive failure at high confining pressure is usually referred to as dilatancy failure. To be more precise, the compression failure of the original soil sample under high confining pressure is tensile failure, while that of the remolded soil sample is tensile failure no matter how the confining pressure changes. The failure modes of undisturbed soil and remolded soil under different confining pressures are due to the fact that the remolded soil has destroyed the original skeleton of the soil, making the tensile strength of the remolded soil less than that of the undisturbed soil sample. Therefore, under low confining pressure, the original soil is shear failure, while the remolded soil is tensile failure. The failure mode of soil samples is also affected by many factors, such as degree of consolidation, pore ratio, loading stress state, temperature, saturation, and medium principal stress [24, 25]. With the decrease of water content, the increase of improved materials that strengthen the structures (such as cement), and the decrease of density, the soil failure will change from brittle failure to ductile failure; that is, the soil failure will change from shear failure and shear failure to shear failure and swelling failure.

3. Failure Modes of Materials in Pressurized Rock Masses

3.1. Failure Modes of Compressive Rock Mass Materials under Different Stress States. The rock failure form difference is more significant. In the 1960s, Griggs and Handin noticed that there were two possible failure modes, shear fracture and tensile fracture, in compression tests [26]. Wang et al. observed that the failure of sandstone samples was a mixture of shear failure and tensile failure [27]. Haimson et al. conducted the true triaxial compression test of granite and obtained the strength and deformation characteristics of the rock [28]. At the beginning of the 21st century, the Rock Mechanics Laboratory of China University of Petroleum studied the failure mode and deformation characteristics of rocks in detail through a large number of triaxial experiments and analyses [29–31]. Mogi carried out a triaxial rock mechanical test to analyze the relationship between rock failure modes and stress conditions and test environment [32]. Paterson and Wong made a very detailed summary of the study on brittle failure of rocks and believed that rock failure had multiple forms such as single shear plane failure, double shear plane failure, and split failure [33]. Raynaud et al. studied the failure characteristics of rock samples under different confining pressures of three axes based on CT scanning and analyzed the embrittlement transformation characteristics of rock samples [34].

The failure modes of rock under different confining pressures are different. For example, marble and red sandstone show cleavage failure at low confining pressure, then gradually transition to shear failure with increasing confining pressure, and present bulging failure when
confining pressure exceeds the critical confining pressure of brittle and ductile transformation [18, 35–37]. In triaxial test, the rock samples of marl show shear failure at low confining pressure and dilation failure at medium confining pressure [38]. Adelinet et al. also found that when basalt was triaxial compressed, and its failure showed several different forms, such as brittle failure, horizontal deformation zone, and pure compression zone, when the stress level was different [39]. Wang et al. studied the failure modes of rocks under triaxial stress, such as low-pressure brittle fracture, stick-slip brittle failure, and high-pressure embrittlement failure. By quartz sandstone and feldspar sandstone triaxial experiments, they found the feldspar sandstone in single fracture under low confining pressure, and the cross section and the axial angle are small; regarding the cross section throughout the upper and lower end faces of the sample, with increasing confining pressure, the sample generally produces two groups of conjugate faults, confining pressure continues to increase, and the sample on the basis of a drum shape deformation produces conjugate rupture or single fracture [40]. Li conducted a triaxial compression test on the gas shale and found that, under low confining pressure, the shale showed splitting type and double shear plane failure, with large fracture angle and more cracks, resulting in higher degree of fracture of the sample [41]. With the increase of confining pressure, the fracture of shale changes from cleavage failure to shear failure, and the fracture angle is small. The failure of sandstone is mainly split failure, or shear failure of the upper and lower end faces of the sandstone samples with shear planes. With the increase of confining pressure, compressive vertebra appeared at the bottom of sandstone. As the confining pressure continues to increase, the sandstone is gradually transformed from conjugate shear failure to single shear failure. The failure pattern of coal and rock is similar to that of sandstone [42]. The coal was mainly manifested as splitting failure dominated by tensile stress under low confining pressures. With the slight increase of confining pressure, the failure mode was accompanied by shear failure on the original basis. With the continuous increase of confining pressure, the failure mode was mainly shear failure accompanied by tensile failure. When the confining pressure rose to 3.0 MPa, the failure mode of coal and rock completely changed to shear failure [43]. The failure mode of phyllite rocks under low confining pressure was shear failure along the joint plane and local tensile failure. With the increase of axial strain, the proportion of elastic energy decreases gradually, but it is always at a high proportion until it reaches the stress peak. After the peak, the proportion of elastic energy decreases rapidly, and the proportion of dissipated energy increases rapidly. With the increase of confining pressure, the failure mode of phyllite rocks changed to shear failure along the joint plane [44]. Gong et al. carried out static load loading and triaxial dynamic impact loading on sandstone through the triaxial SHPB combined loading experimental system and found that the rock under conventional impact mainly formed splitting failure along the loading direction. With the increase of confining pressure, the degree of rock failure would be lower and lower. The change of confining pressure will also change the number and area of fractures in rock mass [45]. Yang et al. conducted triaxial experiments on tight sandstone under different confining pressures and combined with microscopic analysis and found that, under low confining pressures, sandstone samples had numerous failure cracks and complex morphology and finally formed a crack network structure with cross-distribution of main and secondary cracks. The main failure modes were split failure and shear failure. When the confining pressure is high, the number of primary cracks eventually formed decreases, secondary cracks disappeared, and the complex crack network is replaced by the failure crack of approximate straight line. The failure mode of the sample is mainly shear failure [46], as shown in Figure 5.

The first principal stress also has great influence on the failure mode and energy change of rock mass. With the increase of pressure level, both dissipated energy and releasable strain energy in rock mass increase. The hard rock mass has a relatively small internal energy dissipation value before its failure, and almost all the work done by external force is converted into the released strain energy in the rock mass. However, before the failure of soft rock mass, a large part of the work done by external force will be dissipated, and the released strain energy is relatively small [47].
More attention has been paid to the influence of the second principal stress on the failure mode of rock mass. Kun et al. performed a true triaxial compression test on the red sandstone by controlling the size of the intermediate principal stress and found that the second principal stress was small and was a shear failure. With the increase of the second principal stress, the red sandstone changed from shear failure to plate (split) fracture failure [48]. Granite is similar to red sandstone. When the second principal stress is small, it is the conjugate shear failure. With the increase of the second principal stress, the red sandstone changes from shear failure to plate fracture failure (splitting failure). With the intermediate principal stress increases, the internal fracture surfaces of sandstone are basically the same in shape. The fracture surface follows the $\sigma_2$ direction and tends to follow the $\sigma_3$ direction, and there is almost no macroscopic crack surface in the $\sigma_3$ direction [49].

According to the statistics of fracture surface area, the strength of sandstone increases first and then decreases. It is difficult to judge the influence of intermediate principal stress on the strength of rock and soil, but we can use the double shear strength criterion to explain this phenomenon. The effect of intermediate principal stress on shear strength of rock can be explained in two ways. On the one hand, with the increase of intermediate principal stress, it leads to the change of lord angle, which also leads to the change of function value $(f_1(\theta_3))$. On the other hand, the increase of intermediate principal stress also causes the change of $\sigma_m$, which eventually leads to the change of function value $(f_2(\theta_3))$. Therefore, the influence of intermediate principal stress on the shear strength of rock is the comprehensive result of the product of function $(f_1(\theta_3))$ and function $(f_2(\theta_3))$ [49].

Compared with soil, rock, as a kind of brittle material, is more prone to brittle failure mode. Under low confining pressure, the fracture mode of most rocks (such as marble, sandstone, coal rock, and shale) is mainly split failure, and many fracture zones of split failure are more cracks and more scattered. With the increase of confining pressure, the crack distribution converges gradually and then changes to shear failure, and the angle between shear failure zone and horizontal direction decreases gradually with the increase of confining pressure, and some even appear slight swelling. In the process of rock mass changing from splitting failure to shear failure, a considerable part of rock mass shows obvious splitting and shear coupling failure, forming compressive spine (splitting failure in the upper part and shear failure in the lower part), and high dip angle tension-shear failure through the top and bottom of the sample appears in the other part. With the increase of confining pressure, the failure mode in this process gradually changes from tensile failure to shear failure. The change of the second principal stress on the failure mode of rock mass is not as obvious as the confining pressure. The peak strength of the sample increases first and then decreases with the increase of the second principal stress. But Xie [50] gives the critical stress values of several typical three-dimensional stress states.

### 3.2. Failure Modes of Rock Masses of Different Types and Structures under Compression

Different types of rocks will have different failure modes under the same stress conditions. For example, in uniaxial test, it is found that quartzite (hard rock) is cleavage failure, and siliceous chlorite (soft rock) is shear failure [35]. You and Huadid more than forty samples by uniaxial compression test; the samples of rock under uniaxial compression failure mode can be divided throughout the complete pull-cut coupling disruption of shear and compressive vertebral and pull-shear coupling damage, and the outer side is similar to “compressive bar instability” local broken, and we have throughout the complete internal shear pull-cut coupling damage, and the three failure modes have been accompanied by a large number of tensile cracks [51], as shown in Figure 6. The triaxial experiments of different rocks show that the fracture surface of coal and rock under low confining pressure is complex, and the fracture surface under high confining pressure is a flat single section. The shear planes of sandstone and some marble samples are mostly near the diagonal of the samples. The shear planes of both limestone and marble samples have a large dip angle, and their single shear failure planes start and end at the upper and lower ends [52]. Wang, using the numerical simulation of Fish function prepared to calculate all deformation characteristics of rock samples, showed that high Poisson’s ratio would change the failure mode of rock samples from single shear failure to complex failure mode [53].

Even if the same rock is subjected to the same stress condition, the final failure mode may not be the same, because the occurrence, shape, extension scale, density, contact type, cementation, and filling condition of structural plane are also important factors affecting the strength and failure mode of rock mass. Zhao revealed the anisotropy of sandstone strength along with the angle between the plane and the principal stress axis and obtained that when the principal stress axis intersected with the weak plane at an angle of 30°, its strength was the lowest [54]. Gao et al.
studied the anisotropic mechanical properties of sand slate and obtained the influence of different dip angle fine bedding on rock deformation characteristics, strength characteristics, and parameters [55]. Nasseri et al., Tien et al., Liu et al., Zheng, Ghazvinian et al., and Wang et al. studied the relationship between the strength of stratified and stratified anisotropy rock mass and the inclination angle of structural plane by using indoor uniaxial and triaxial compression tests and splitting tests [56–61]. Their results were generally revealing that the mechanical strength of this kind of rock mass decreases first and then increases with the change of the inclination angle of the structural plane. The minimum value occurs when the principal stress and the structural plane skew, and the distribution position of the maximum value is not determined. Wang et al. studied the physicochemical and mechanical properties of hard and brittle shale formations with fractures that are prone to instability by means of mud chemistry, microstructure scanning, and rock mechanics tests [62]. Jia et al. and Wang et al., respectively, studied rock failure modes with different confining pressures, joint surfaces, and principal stress angles under the three-axis stress state of shale and phyllite rock [63, 64]. The experimental results show that when the angle between the joint plane and the principal stress is small between 0°–5°, many vertical splits along the bedding plane occur. With the increase of confining pressure, the tensile fracture effect gradually decreases, and the failure mode gradually changes to shear failure, but the angle between the shear zone and the joint plane is small. With the increase of the angle between the joint plane and the principal stress, the rock sample showed mainly fracture along the joint plane, and partial tensile fractures appeared at both ends of the shear failure. With the increase of the confining pressure, the tensile fractures at the end gradually decreased, and the failure mode transitioned to shear failure. When the joint plane is perpendicular to the principal stress direction, the failure mode is mainly single shear, and transverse shear failure occurs simultaneously.

According to the uniaxial and triaxial test results of the rock mass, the main factors influencing the change of rock mass failure mode are stress condition, angle between structural plane and principal stress, and lithology. With the increase of confining pressure, the rock mass gradually changes from splitting failure to pull-shear coupling failure, with compressive vertebrates appearing, and the confining pressure continues to increase. The pull-shear coupling failure begins to be dominated by shear failure and gradually changes from conjugate shear failure to single shear failure, and some even show slight bulging. The failure surface of the rock sample is mostly along the structural surface due to the weak structural surface, while the brittle rock is mostly split failure and shear failure, and the soft rock will show some swelling phenomenon [65].

4. Failure Mode Analysis of Compressed Rock Mass

According to the above geotechnical tests and a large number of other geotechnical tests, the failure modes of geotechnical materials under uniaxial or triaxial compression can be classified into five categories: splitting failure, tensile shear failure, shear failure, dilatancy failure, and bulging failure (as shown in Figure 7). Among them, the tensile shear failure belongs to the type of shear failure containing tensile failure and the dilatancy failure belongs to the type of tensile failure containing shear failure. Generally speaking, the failure modes of geotechnical materials under axial compressive stress can be divided into shear failure, tensile failure, and tensile-shear coupling failure [65]. Therefore, under axial stress, the failure of rock and soil mass is mainly caused by shear action, tension action, or the joint action of both. It is often said that the rock and soil sample is crushed, in fact may be cut, or pull or pull-shear damaged. From the perspective of mechanics, there is no simple compression failure, but under the stress state of triaxial compression test, there is pure tensile failure (as shown in Figure 7(e)). Meanwhile, under the stress state, there is not only tensile fracture, but also shear fracture and mixed fracture [66].

Although a lot of progress has been made in the research on failure modes of rock-soil compression at home and abroad, and a large amount of test data have been accumulated, we still cannot explain the failure modes of rock-soil specimens according to common sense in the face of these historical and constantly emerging problems, and the tensile failure pattern in compression failure is difficult to be explained by either Mohr–Coulomb strength theory or Griffith strength theory. According to the Mohr–Coulomb strength criterion, the material only in negative confining pressure stress area will be tensile failure mode or tensile failure and shear failure phenomenon at the same time, and the other is that confining pressure stress zone should be shear failure, but the tension failure mode above has regularly appeared in confining pressure stress zone, and it is not consistent with Mohr–Coulomb strength theory. Although some scholars have explored this phenomenon from...
the perspective of its influencing factors [67, 68], the research may be related to the structure of rock and soil [69], and some scholars have proposed that the expansion of fracture tip in materials will cause local tensile stress [70–73], which will also lead to the appearance of tensile failure of compression rock and soil.

With the large number of experiments shown above, first, cast away the structural factors of the rock and soil, and as long as we note that the tensile strength of the rock and soil is usually relatively small, axial compressive stress converted into the inner stress is likely to meet the tensile strength. It is possible to cause the rock and soil in the shear stress less than the shear strength of the condition of tensile failure. Under unidirectional compression, any rock mass tends to expand to the free surface, and the length of soil mass decreases in the direction of maximum principal stress. At this time, the length of soil decreases in the direction of maximum principal stress but increases in the direction of other principal stress, which is “flattened” in macro state. Because the soil has the state of “flattening,” in the three-way stress state, the compression of the soil in one direction will inevitably cause the soil to bulge in the other two directions. At this point, it can be considered that there is a tensile stress caused by compressive stress in other directions in the soil in this direction, as shown in Figure 8.

But so far, there is a lack of research on the mechanism of tensile stress formation and the different failure modes in compression materials without considering the structure. Up to now, humans have not established the tensile strength theory of rock and soil under the stress state of the system, nor have they formed a mechanical criterion and strength theory that can not only judge the failure mode, but also give the failure of the tension and shear coupling, except Griffith theory (1924) of tensile strength for brittle fracture of fractured materials and his interpretation of it with the theory of tensile strength of Mohr–Coulomb cladding lines in tensile zones.

5. Study on Failure Constitutive Equation of Rock and Soil under Pressure

In 1900, Mohr (1900) extended the linear shear criterion \((\tau = c + \sigma \tan \phi)\) proposed by Coulomb in 1776 to consider the three-way stress state. Its main contribution is to make people realize that shear strength itself is a function of normal stress on the failure surface \((\tau = f(\sigma))\). It means that the Mohr strength envelope is nonlinear, and the shear strength index cohesion and internal friction angle are not constant. Mohr’s shear strength theory is still widely used in mechanics of materials, and a large number of later experimental studies have shown that the relationship between normal stress and shear strength is indeed nonlinear [74, 75]. In order to deeply reveal the essence of Mohr’s shear strength, since Mohr’s strength theory was put forward in 1900, many efforts have been made to find the function of nonlinear strength envelope. Balmer proposed Balmer type nonlinear strength envelope based on engineering experience and experimental data [76]. De Mello adopts a trifold line as the strength envelope of Mohr–Coulomb material of dam body [77]. The strength of rock and soil in the mining of Canadian clay ore bodies [78] and natural gas [79] is also in line with the double linear strength envelope. However, these nonlinear equations of strength envelope are just a kind of representational mathematical statistics, which
cannot reflect its essence. Moreover, these equations are not continuous functions in the stress space, and there are discontinuities when the stress changes from small to large, which increases the number of shear strength index parameters with unclear physical meaning. For a long time, along with the above problems, the method of obtaining the nonlinear equation of the strength envelope by solving the quadratic parabolic function, power function, or hyperbolic function through the fitting of experimental data has been almost accepted by scholars [80, 81]. Based on MC theory, many scholars add different parameters according to experience to obtain new MC equations. For example, Bejarbaneh got the new equation by introducing the orientation of the weakness and lamination planes (β) in respect of the major principal stress directions.

\[ \sigma_1 - \sigma_3 = \frac{2c_u + \sigma_3 \tan \varphi}{(1 - \tan \varphi \cot \beta) \sin 2\beta} \]  

(1)

Until today, there has been no substantial breakthrough in research, and our common method is still fitting. For example, Pincus collected multiple sets of experimental data on the properties of granite, sandstone, and marble from several laboratories of the American Society of Experimental Materials and still used the fitting method to establish Mohr strength envelope in the form of linear, parabolic, and hyperbolic of three kinds of rocks [82].

\[ \tau = \frac{r_b c_a - r_a c_b}{\sqrt{(c_a - c_b)^2 - (r_a - r_b)^2}} \]  

+ \frac{(r_a - r_b)\sigma}{\sqrt{(c_a - c_b)^2 - (r_a - r_b)^2}} \]  

(2)

Rafai obtained an empirical Mohr–Coulomb nonlinear strength envelope equation based on the fitting of 195 sets of uniaxial and triaxial test data of 12 kinds of rocks [83].

\[ \sqrt{\varphi_2} = k + a\left(\frac{L_1}{3}\right) + \beta\left(\frac{L_1}{3}\right)^2. \]  

(3)

Since 2001, the research has begun to look a little deeper. Ramamurthy added two new indexes reflecting the brittle-viscosity property and the joint structure property in order to describe the nonlinearity of the strength criterion of geological materials [84].

\[ (\sigma_1' - \sigma_3') = \left[m_1 \sigma_1 \sigma_3 + s_2 \sigma_1^2 \right]^{1/2}. \]  

(4)

Singh and Singh, considering the intermediate stress and introducing the critical state concept of Patton rock, transformed the shear strength envelope into a nonlinear type [85].

\[ (\sigma_1' - \sigma_3') = \sigma_{ij} + \frac{2 \sin \varphi_{ij0}}{1 - \sin \varphi_{ij0}} \left(\frac{\sigma_3 + \sigma_2}{2}\right) \]  

- \frac{1}{\sigma_{ij}} \frac{\sin \varphi_{ij0}}{1 - \sin \varphi_{ij0}} \left(\frac{\sigma_2^2 + \sigma_3^2}{2}\right). \]  

(5)

Connor Langford and Diederichs quantified sample differences and influencing factors such as sampling and transportation process through cyclic load experiments and proposed a set of new linear and nonlinear regression methods to fit the strength envelope of rock mass [86]. Recent studies have focused on the nonlinearity of the Mohr strength envelope for different materials; for example, Espeche and León proposed an estimation method of Mohr–Coulomb strength envelope with consideration of tensile stress between new and old interfaces of reinforced concrete structures [87]. Although this method of considering tensile stress to amend Mohr strength curve is simple, it deserves our attention. Based on Mohr yield criterion, Zhao et al. derived an analytical expression of internal friction angle, which could take into account the influence of normal stress, cutting force, shear angle, and temperature [88].

\[ F_v = \frac{k \left(1 - \sqrt{\lambda \ast T / T_g}\right)t_1 \ast w \ast \cos(\beta - \gamma)}{[1 - \alpha \tan(\varphi + \beta - \gamma)] \sin \varphi \cos(\varphi + \beta - \gamma)} \]  

(6)

\[ F_p = \frac{k \left(1 - \sqrt{\lambda \ast T / T_g}\right)t_1 \ast w \ast \sin(\beta - \gamma)}{[1 - \alpha \tan(\varphi + \beta - \gamma)] \sin \varphi \cos(\varphi + \beta - \gamma)} \]  

The resulting cutting yield model could accurately predict the cutting process of metal glass.

To sum up, the existing constitutive relation of Mohr’s nonlinear strength is obtained by fitting the experimental data and correcting the empirical equation, and there is no systematic study and discussion on the nature of Mohr’s nonlinear strength envelope in any study, and no real constitutive relation has been obtained. From the failure mode of rock mass, especially uniaxial compression test, we can see that the rock mass under normal stress not only will produce shear stress, but also may produce lateral bulging or splitting lateral tensile stress. The lateral tensile stress generated by normal stress is a key detail that has not been noticed in the previous researches on the Mohr–Coulomb constitutive equation. The generation of lateral tensile stress can essentially prove the nonlinear nature of the Mohr–Coulomb strength envelope.

6. Discussion

The failure modes of rock and soil mass are varied, and predecessors have done detailed experimental research and phenomenon observation. Its influence factor is also numerous; it both has the external stress condition influence, such as the size of confining pressure, the size of the second principal stress, the loading rate of the principal stress, and the loading mode and has many internal factors influence, such as density, water content, structural difference between remolded soil and undisturbed soil, occurrence and location of structural plane inside rock, cementation inside rock mass, and filling condition. Although many scholars have classified the failure modes of rock and soil according to their own observation results, the failure modes of each...
classification show that the failure of rock and soil under compression is related to the tensile stress and shear stress. However, the Mohr–Coulomb strength theory, which has been widely used in the mechanical analysis of the failure of rock mass under compression, attributes all the failures to pure shear failure, and only when the pressure value is negative will there be tensile failure or pull shear failure. Although pure shear failure is the most common failure mode of rock mass under pressure, we often see other failure modes (splitting failure, bulging failure, and Tensoshear coupling failure). It is worth noting that, in the process of uniaxial compression, some rock and soil will appear split failure (e.g., red sandstone) or swelling failure (e.g., highmoisture remolded soil). This means that the rock and soil mass produces transverse displacement only under the action of vertical compressive stress, so we can find that it produces the transverse stress that drives the rock and soil mass to produce transverse displacement inside the rock and soil mass. From the above deduction, we can know that the vertical compressive stress in the rock and soil internal parts will produce horizontal tensile stress perpendicular to its direction. In Mohr–Coulomb strength theory, the strength envelope of rock and soil mass under ideal condition should be Coulomb line, but practice shows that its strength envelope is a downward bending Mohr envelope. This may be caused by the transformation of compressive stress into tensile stress perpendicular to it in rock and soil. The compressive stress is transformed inside the rock and soil mass, which makes the actual compressive stress received by the rock and soil mass less than the external applied value, then causes the Mohr circle on the Coulomb line to shift to the right (Mohr circle size will also change accordingly), and finally forms the observed nonlinear Mohr envelope. Therefore, when studying the constitutive equation of rock-soil mass under pressure, considering the stress transformation, the nonlinear of Mohr envelope may be explained essentially, and the equation of Mohr envelope can be derived from this.

7. Existing Problems

(1) At present, there is no unified standard for compression failure mode of rock and soil mass. Although different scholars have classified the failure modes of rock and soil mass differently according to their own experiments, these classification methods obtained only by observing the morphology of soil samples after the experiment have failed to form a unified standard and have been widely recognized. Therefore, it is necessary to establish a quantifiable classification and discrimination method based on mechanics and energy.

(2) The failure process of rock mass is regarded as the result of the joint action of tensile stress and shear stress, and the dominant stress may be brittle tensile stress, shear stress, or tensile stress in swelling. Obviously, the energy changes of rock and soil mass are different in the three force-led failures (splitting failure, shear failure, and swelling failure), and the distribution of tensile stress is also different. However, in the current strength analysis process of rock and soil mass, both splitting and swelling are put into shear constitutive equation to calculate the strength, which makes the strength obtained by these failure modes have a large error with the original strength. Therefore, it is necessary to establish constitutive models with different failure modes to analyze the strength of rock mass under different failure modes.

(3) For the rock mass under uniaxial action, the splitting or swelling failure under a single stress indicates that it is subjected to tensile stress perpendicular to the direction of applied stress. It is unknown how the internal tensile stress is generated and what the quantitative equation between the applied load is.

(4) At present, there is still no unified form of the Mohr envelope nonlinear equation in the academic circle. Most Mohr envelope equation is still obtained from the fitting of experimental data, rather than from the analysis of the internal force of rock and soil mass, and the relationship between Mohr envelope equation and Coulomb line $\tau = \sigma \tan \phi + c$ has not been established.

8. Prospects

(1) Based on the measurement and calculation of internal tensile stress, establishing the quantitative relationship between tensile stress and applied stress is an important step to further improve the constitutive equation of rock mass in essence.

(2) Based on the understanding of rock mass failure modes, explore the mechanical behavior and energy changes of rock mass under different failure modes, and establish quantifiable failure mode discrimination of rock mass, which is the basis for establishing the constitutive equation of rock mass under different failure modes.

(3) Explore the energy changes on the mechanical behavior under different failure modes, the effect of combination with the transformation on tensile stress, and the quantitative relationship between the load application, and based on the criterion of soil with different failure mode, establish the Coulomb straight line and the relationship between the Mohr envelopes, for further improvement, and the development of the constitutive equation of rock and soil mass has the vital role.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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