Critical Route Identify Method of Intersection Group Based on Hidden Markov Model

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Abstract. In recent years, many cities cope with the rapid growth of traffic volume by building urban expressways. On the ramp of urban expressway is often tend to traffic congestion. It will lead to queue spillback at the intersection group near the ramp. The phenomenon is more obvious in rush hours of working days, which affects the traffic efficiency of urban road network seriously. In this paper, the Hidden Markov Model is used to describe the interaction between the time sequence of the traffic volume at the cross section of each intersection entrance and the hidden state sequence of the ramp and intersection. Thus, the route correlation degree model is formed and the critical route is determined. The cases showed that the method can balance the traffic demand of expressway and ground road, and improve the traffic efficiency of the hybrid road system.

Keywords: Intelligent Transportation, Hybrid System, Route Identify, Artificial Intelligence, Hidden Markov Model

1. Introduction

New urban expressway is one of the important means to solve the rapid growth of traffic volume in cities. As a necessary link between urban road network and expressway, there are heavy traffic and frequent lane changing behavior on the ramp, which is easy to cause congestion in the downstream of main road and spread to the upstream of expressway. At the same time, the queue on the entrance ramp will spread to the adjacent intersections, which will easily cause queue spillback and then reduce the traffic efficiency of the ground intersection group.

In this paper, the Hidden Markov Model (HMM) is used to establish a route correlation identify model. The initial parameters are calibrated by the improved intersection group route correlation degree model. The model is trained by the measured traffic volume data. The critical path of the local road network can be obtained by performing time aggregation and spatial aggregation on the time series of the relevant state. Which provides the basis for balancing the traffic demand of Expressway and ground road network.

2. HMM for Critical Route of Intersection Group

2.1 Construction of HMM
2.1.1 Description of correlation degree identify model. HMM represents a double Markov random process [1,2], which cannot directly observe the model state, but some variables affected by the state are visible. In this paper, the state of the intersection entrance can be divided into three cases for the same entrance ramp section: correlated, condition correlated and uncorrelated. It is assumed that the hidden state is a first-order Markov process, and they can be transformed to each other. Since different states correspond to different traffic characteristics, the HMM can be used to identify the intersection traffic volume and its relationship with the ramp.

The correlation degree identify model is determined by the initial state probability vector \( \pi \), state transition probability matrix \( A \) and observation probability matrix \( B \). \( \pi \) and \( A \) determined the state sequence, and \( B \) determined the observation sequence. Therefore, the correlation degree identify model \( \lambda \) can be expressed by ternary set, as shown in Equation (1).

\[
\lambda = (A,B,\pi)
\]

The correlation degree identify model consists of two time sequences, one is the observable time sequence \( Q \) of intersection traffic volume, and the other is the hidden time sequence \( C \) of intersection and entrance ramp correlation, as shown in Figure 1.

**Figure 1.** The diagram of correlation degree identify model

2.1.2 The hypothesis of correlation degree identify model.

Hypothesis 1: Markov hypothesis

The model assumes that the time sequence composed of the hidden cross section traffic volume related states constitute a first-order Markov chain, i.e., the related state \( c_t \) at time \( t \). It only depends on the relevant state \( c_{t-1} \) of the previous moment and the antecedent states \( c_1,c_2, ... ,c_{t-2} \) have no relevance with it, as shown in Equation (2).

\[
P(c_t = h_i|c_{t-1}, ..., c_1) = P(c_t = h_i|c_{t-1})
\]

Hypothesis 2: Immobility hypothesis

The model assumes that the hidden cross section traffic volume related state has no relevance with the specific time, as shown in Equation (3).

\[
P(c_{i+1}|c_i) = P(c_{j+1}|c_j)
\]

Hypothesis 3: Output independence hypothesis

The intersection traffic volume \( q_t \) of each observed section only depends on its corresponding state \( c_t \), as shown in Equation (4).

\[
P(q_1, ..., q_T|c_1, ..., c_T) = \prod_{t=1}^{T} P(q_t|c_t)
\]

2.1.3 Definition of elements in correlation degree identify model

Definition 1: State transition probability matrix \( A \)

The matrix \( A \) represents the probability of mutual transition among three different related states of intersection traffic volume, as shown in Equation (5).

\[
A = \{a_{ij}\}_{N \times N}
\]

where \( a_{ij} = P(c_{t+1} = h_j|c_t = h_i), 1 \leq i,j \leq N \)

Definition 2: Observation probability matrix \( B \)
The matrix $B$ represents the probability of obtaining different section traffic volume in each relevant state, as shown in Equation (6).

$$B = \{ b_i(k) \}_{N \times M}$$  \hspace{1cm} (6)

where $b_i(k) = P(q_t = v_k | c_t = h_i), 1 \leq i \leq N, 1 \leq k \leq M$

Definition 3: Initial state distribution $\pi$

The vector $\pi$ is constructed to represent the probability of the section in each relevant state at the initial time, as shown in Equation (7).

$$\pi = \{ \pi_i \}_{1 \times N}$$  \hspace{1cm} (7)

where $\pi_i = P(c_1 = h_i), 1 \leq i \leq N$

Definition 4: $H$ is the set of relevant states of all possible detection sections, $C$ is the state time sequence, and the length is $T$.

$$H = \{ h_1, h_2, ..., h \}$$

$$C = \{ c_1, c_2, ..., \}$$

Where $N$ is the number of all possible states, and the state at time $t$ is $c_t \in \{ h_1, h_2, ..., h_N \}$.

Definition 5: $V$ is the set of all possible observed traffic volume of the section, and $Q$ is the time sequence of corresponding observed traffic volume.

$$V = \{ v_1, v_2, ..., v_M \}$$

$$Q = \{ q_1, q_2, ..., q_T \}$$

Where $M$ is the number of observed traffic volume, and the observed traffic volume at time $t$ is $q_t$, $q_t \in \{ v_1, v_2, ..., v_M \}$.

2.2 Training Method

The correlation between different intersections and entrance ramp is a dynamic variable process, and the corresponding observation probability matrix $B$, state transition matrix $A$ and initial vector $\pi$ in HMM are changing in different periods. In the paper, the day is divided into five periods and a union model composed of five HMM is established.

In the paper, the observed time sequence of the section traffic volume was used to find the optimal solution through the EM method $[3]$, and unsupervised learning was carried out. Among them, Baum Welch algorithm is adopted $[4]$.

3. Applicability Analysis and Initial Parameter Calibrate

3.1 Improved Intersection Group Path Correlation Model

In this paper, the coincidence degree of the key flow direction with the maximum flow direction of each section of the route, the discrete of the fleet, the degree of cooperative control and other indicators are evaluated respectively. Considering the above factors, the route correlation degree model of intersection group is constructed.

(1) Index of flow direction coincidence $I_1$

In order to consider the relationship between the route traffic volume and the total traffic volume of road section. In this paper, the route direction of the entrance in a downstream intersection of a section is defined as the critical flow direction. The coincidence index of the road section flow direction is defined as the ratio of the flow of critical flow direction to the total flow of the section$[5]$.

(2) Index of route discrete $I_2$

Different bandwidth settings are often used in the application of green wave control in intersection group. Assuming that MULTIBAND model is used to set the discreteness index to the ratio of the number of vehicles passing through the minimum green light time bandwidth at the beginning and end points of the route in a signal period$[6]$.
(3) Index of route cooperative $I_3$

The correlation between the intersections that make up the route determines whether the route is coordinated and controlled. Therefore, the cooperative index is taken as the average value of the intersection correlation degree of multiple road sections in a signal period\(^7\).

(4) Calculation of route correlation degree

The coordination index is standardized to evaluate the correlation degree of multiple routes, as shown in Equation (8).

\[
I_3^* = \frac{I_3 - I_{3\text{min}}}{I_{3\text{max}} - I_{3\text{min}}} \tag{8}
\]

Then the correlation degree $I$ in the intersection group is shown in Equation (9).

\[
I = I_1 + I_2 + I_3^* \tag{9}
\]

3.2 Initial Parameter Calibrate Method

The time sequence of the same period $T$ in $n$ days is selected as the sample, and the time sequence of the $n$th day are as follows: $Q_n = \{q_{n1}, q_{n2}, ..., q_{nT}\}, n = 1, 2, ..., N$. The state sequence of the $n$th day are as follows: $C_n = \{c_{n1}, c_{n2}, ..., c_{nT}\}, n = 1, 2, ..., N$. Then the calculation method of initial parameters $A_0, B_0, \pi_0$ are as follows:

(1) For the state transition matrix: $A_0 = \{a_{ij}\}_{N \times N}$

\[
a_{ij} = P(c_{t+1} = j | c_t = i) = \frac{\sum_{n=1}^{N} f_n(c_{nt+1} = j, c_{nt} = i)}{\sum_{n=1}^{N} f_n(c_{nt} = i)} \tag{10}
\]

where $f_n(c_{nt} = i)$ is the frequency of state $i$ in correlated state sequence on the $n$th day.

$f_n(c_{nt} = i)$ is the frequency of the transition from state $i$ to state $j$ in the correlated state sequence on the $n$th day.

$t = 1, 2, ..., T - 1; 1 \leq i, j \leq N$.

(2) For observation probability matrix: $B_0 = \{b_i(k)\}_{N \times M}$

\[
b_i(k) = P(q_t = k | c_t = i) = \frac{\sum_{n=1}^{N} f_n(q_{nt} = k, c_{nt} = i)}{\sum_{n=1}^{N} f_n(c_{nt} = i)} \tag{11}
\]

where $f_n(q_{nt} = k, c_{nt} = i)$ is the frequency of traffic volume $K$ when the correlation state is $i$ on the $n$th day.

$t = 1, 2, ..., T; 1 \leq i \leq N, 1 \leq k \leq M$.

(3) For the initial state distribution: $\pi_0 = \{\pi_i\}_{1 \times N}$

\[
\pi_i = P(c_1 = i) = \frac{\sum_{n=1}^{N} f_n(c_{n1} = i)}{N} \tag{12}
\]

For different periods of different intersections, $A_0, B_0, \pi_0$ are calculated according to Equation (10), (11) and (12).

4. Method of Critical Route Identify

4.1 Identify of Route Correlation State Sequence

After the training of the correlation degree identify model, through the observed traffic volume time sequence of the intersection, the hidden route correlation state time sequence is obtained by using Viterbi algorithm\(^8\).

4.2 Critical Route Calculation
4.2.1 *Time aggregation.* The switching between different states is random, and the state switching time interval is different. Therefore, the results cannot be directly used in the cooperative control strategy, and the related states need to be aggregated in time\(^9\).

![Figure 2. Time interval of states switching](image)

As shown in Figure 2, the identify model divides the correlation into three categories: uncorrelation, conditional correlation and correlation. The conditional correlation means that the time has a certain degree of correlation, the correlation is stable, and longer than the other two types. In order to keep the stability and continuance of the effect, the control strategy of the previous period should be followed when conditional correlation. The strategy switching effect is shown in Figure 3.

![Figure 3. Time interval of control strategy switching](image)

As shown in Figure 3, the control strategy switching time interval are different, and the update period of the control strategy needs to be unified. Assuming that the data sampling interval \(t\) and the update period of the cooperative control strategy \(T\), the paper uses the ratio of the cooperative control time and the total time to express the degree of cooperative control of the route in this period, denoted as \(S_T\), as shown in Equation (13).

\[
S_T = \frac{\sum_{t=1}^{T} I(t) t}{T}
\]

(13)

where \(S_T\) is the ratio of the cooperative control time and the total time; \(I(t)\) is the cooperative control state in the period \(T\), if it is cooperative control, then \(I(t) = 1\), otherwise, \(I(t) = 0\). When \(S_T \geq 0.5\), cooperative control is carried out in period \(T\). When \(S_T < 0.5\), there is no cooperative control. The control strategy switching effects after calculation are shown in Figure 4.

![Figure 4. Control strategy switching after aggregate](image)

According to Figure 4, the time interval of control strategy switching is a fixed value, that is, the update period of cooperative control strategy is \(T\). According to the distance of the longest route of the intersection group and the vehicle speed, a more reasonable control strategy update period can be determined to ensure the control effect.
4.2.2 Space aggregation. Any entrance of each section in the intersection group has a route to make vehicles drive to the on ramp, so the same road section in the intersection group may belong to different routes, as shown in Figure 5. Then the maximum value calculated by different routes should be taken as the relevant state of the road section, that is, the relevant state should be collected in space\textsuperscript{[10]}.

![Figure 5. Diagram of overlapping of different routes](image)

The routes in the intersection group are classified according to the different state and road grade of the road section, and the route with the largest correlation degree of traffic volume status is selected as the critical route in the intersection group.

5. Case study

5.1 Influence of Sampling Frequency of Observation Data On Results
The sampling frequency of the traffic volume data of the intersection group in the research area of this paper is 1 min and 5 min respectively, and the specific points are selected as shown in Figure 6.

![Figure 6. Diagram of intersection location](image)

The sampling frequency of Xujiahui Road - Zhizaoju Road intersection is 1 min. The results of the observed traffic volume data and the identify results of correlation degree are shown in Figure 7 a) and b).
5.1 Analysis of Observed Data and Identify Results of Correlation Degree

The observed traffic volume data and the identify results of correlation degree of Xujiahui Road - Zhizaoju Road intersection are shown in Figure 7 a) and b).

The sampling frequency of Xietu Road - Luban Road intersection is 5 min. The results of the observed traffic volume data and the identify results of correlation degree are shown in Figure 8 a) and b).

The identify results of the two different sampling frequencies showed that the correlation degree identify model can be successfully applied to both 1 min and 5 min data, and the adaptability of the data is strong. At the same time, the smaller the granularity of data, the more frequent the transformation of correlation state, that is, the more sensitive the correlation degree identify model.

5.2 Analysis of Different Section Identify Results

According to the time aggregate calculation method, the identified time sequence of relevant states can be transformed into cooperative control state sequence. Then, the two intersections in the case are transformed and calculated respectively, and their respective cooperative control state sequence are shown in Figure 9 a) and b).

The observed traffic volume data of correlation degree

Figure 7. The observed data and identify results of correlation degree of Xujiahui Road - Zhizaoju Road intersection

Figure 8. The observed data and identify results of correlation degree of Xietu Road - Luban Road intersection

Figure 9. The cooperative control sequence of Xujiahui Road – Zhizaoju Road and Xietu Road - Luban Road intersection
The analysis showed that although the routes have large traffic in the morning and evening rush hours and need to be coordinated, but the time of cooperative control presents the opposite characteristics.

5.3 Analysis of Identify Results in Different Periods
The correlation degree is different for different periods of the same route. The data of Xujiahui Road - Zhizaoju Road intersection for one week were selected for correlation identify. The results are shown in Figure 10.

![Figure 10](image)

Figure 10. The average correlation degree in different periods

It can be seen that in one day, the correlation degree of the same route changes with time, and the correlation are different in rush hours and flat periods. Among them, the correlation in the morning rush hours is the largest and the correlation time is concentrated; after 16:00, the correlation degree changes slightly and the correlation state lasts longer. The same route has different association characteristics on weekdays and weekends.

5.4 Analysis of Main Road Identify Results
The route composed of main road intersections in the group adjacent to the ramp was selected for research, and the location of each intersection is shown in Figure 11.

![Figure 11](image)

Figure 11. The diagram of critical route intersection location

In Figure 11, Intersection 4# is the boundary of the group, and Intersection 1# is connecting with the on ramp. The data of the entrance ramp volume of each intersection is selected for case analysis.

When the traffic volume data of each intersection is known, the correlation degree is identified according to the correlation degree identify method described in this paper, and then the related state of the route is transformed into the cooperative control state, as shown in Figure 12.

![Figure 12](image)

Figure 12. The cooperative control state of each intersection
It can be seen that the change law of cooperative control state of intersections on the same route is basically the same. According to the update period of the cooperative control strategy, the time aggregation of the identified results is calculated. If the update period of the cooperative control strategy is 1 hour, Equation (13) is applied to calculate the cooperative control time proportion $S_T$ of each intersection. The results are shown in Table 1.

Table 1. Proportion of cooperative control at each intersection

| Time | 1st | 2nd | 3rd | 4th | 1st | 2nd | 3rd | 4th |
|------|-----|-----|-----|-----|-----|-----|-----|-----|
| 1    | 0.60| 0.32| 0.10| 0.18| 13  | 0.35| 0.12| 0.12| 0.12|
| 2    | 1.00| 0.55| 0.42| 0.55| 14  | 1.00| 0.83| 0.65| 0.65|
| 3    | 0.55| 0.00| 0.00| 0.00| 15  | 0.77| 0.67| 0.67| 0.33|
| 4    | 1.00| 0.57| 0.57| 0.57| 16  | 1.00| 0.82| 0.65| 0.45|
| 5    | 1.00| 0.73| 0.30| 0.00| 17  | 0.85| 0.85| 0.85| 0.77|
| 6    | 0.50| 0.60| 0.55| 0.40| 18  | 1.00| 0.83| 0.50| 0.83|
| 7    | 0.38| 0.32| 0.73| 0.33| 19  | 0.92| 0.73| 0.75| 0.73|
| 8    | 0.77| 0.78| 0.87| 0.77| 20  | 0.92| 0.80| 0.90| 0.90|
| 9    | 1.00| 1.00| 0.92| 0.92| 21  | 0.92| 0.42| 0.42| 0.28|
| 10   | 0.72| 0.50| 0.80| 0.50| 22  | 1.00| 0.80| 0.75| 0.77|
| 11   | 0.92| 0.32| 0.45| 0.43| 23  | 0.28| 0.28| 0.28| 0.28|
| 12   | 0.70| 0.55| 0.62| 0.40| 24  | 0.62| 0.28| 0.28| 0.17|

It can be seen from the table that the time period for cooperative control of the same route is relatively stable, and the proportion of cooperative control in rush hours is longer. In addition, the cooperative control time of the intersection close to the entrance ramp on the selected route, and the range of cooperative control on the same route is variable.

According to the update period of the control strategy, when $S_T \geq 0.5$, the cooperative control is carried out in period $T$; when $S_T < 0.5$, the cooperative control is not carried out. The switching effect of each intersection control strategy after calculation is shown in Figure 13.

Figure 13. The switching effect of each intersection control strategy

From the analysis of the case intersection, it can be seen that the result of the correlation degree identify model is consistent with the actual traffic state, which proves that the model is effective, and the cooperative control strategy is also applicable.

6. Conclusion
This paper explored the application of HMM for critical route identify of intersection group. The route correlation degree identify model is suitable for traffic data of different sampling frequencies, and the process result can accurately describe the characteristics of intersections in different directions of intersection group. It can be concluded from the experiment case that the traffic operation between the intersection group and the urban expressway entrance ramp is closely related to each other.

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