Anticipating physics of ‘burning’ tokamak plasmas: Energy selective confinement of alpha particles

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In order to realize the idea of a self-sustained steady-state fusion reactor using toroidal magnetic containment, it is necessary to control the content of alpha particles produced by D-T fusion reactions. On the one hand, it is necessary to keep MeV-class alpha particles well-confined in order to heat the plasma. On the other hand, partially cooled alphas known as ‘helium ash’ must be expelled before diluting the fusion fuel. Numerical simulations of a large tokamak plasma have now uncovered a synergistic effect that allows to realize this feat of selective confinement of energetic alphas and mixing of helium ash by exploiting internal relaxation events known as ‘sawtooth crashes’. The physics discussed here exemplify some of the fascinating complex processes that may be explored and utilized in future burning plasma experiments during the next decades, beginning with ITER.

Introduction

Tokamaks confine hydrogen isotope plasmas with high temperatures (∼ 10 keV) in a toroidal volume using a combination of a strong toroidal magnetic field $B_{tor}$ produced by external coils and a weaker azimuthal (poloidal) magnetic field $B_{pol}$ produced by currents carried by the plasma itself. The helicity of the magnetic field varies across the plasma radius and is measured by the so-called safety factor $q = \nabla B_{pol}/(RB_{pol})$, where $R$ is the major radius of the torus and $\tau$ the mean minor radial distance from the center of the plasma. When the plasma current distribution reaches a certain threshold so that the field helicity in the plasma core drops below unity ($q < 1$), a self-organization process sets in that prevents further steepening of the current density profile [1]. This results in a relatively robust quasi-steady state, which future fusion reactor experiments like ITER [2] will exploit.

This self-organization process may be pictured as follows. $q = 1$ means that magnetic field lines close on themselves after one poloidal and one toroidal turn as illustrated in Fig. 1(a). The portion of the plasma located within the $q = 1$ surface can thus be thought of as a ‘tilted torus within a torus’. In other words, the $q = 1$ torus is resonant with the toroidal geometry of the tokamak as a whole and can be easily displaced by the destabilizing forces associated with gradients in the current density or thermal pressure. This is known as ‘internal kink’ instability. In the example that we study here, which has the dimension of the Joint European Torus (JET), the condition $q < 1$ is satisfied within a radius of about $\tau \approx 0.2 \text{ m}$ as indicated by the dashed lines in panels (b) and (c) of Fig. 1. Together with mechanisms facilitating magnetic reconnection [3], the formation of a region with $q < 1$ results in quasi-periodic relaxation events that can be observed in the form of ‘sawtooth oscillations’ in time.

FIG. 1: (a) Toroidal geometry of a tokamak in cylinder coordinates ($R, z, \zeta$). The black curve represents a magnetic field line with field helicity $q = 1$. (b) Shape of the full plasma cross-section in a poloidal ($R, z$) plane for our working example based on the JET tokamak. Magnetic flux surfaces (gray) and pressure contours (colors) are shown. (c) Central portion of the field helicity profile $q(\tau)$. The $q = 1$ surface (dashed black line) and simulation boundary (red) are indicated.

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traces of the central electron temperature $T_e$ as in Fig. 2. These data were acquired in a JET plasma, where the 3-ion radio-frequency (RF) heating scheme was applied to a mixed D-^3^He plasma, successfully generating fusion-born alpha particles \[4, 5\].

Benign sawtooth activity is considered to have beneficial effects in fusion-oriented applications. Besides helping to keep the core plasma near a well-defined state, this mixing process was found to prevent excessive accumulation of heavy-ion impurities that would cause radiative cooling \[6\]. By the same token, it has been proposed to use sawteeth for the expulsion of helium ash (henceforth called “slow alphas”) from the core of a deuterium-tritium (D-T) fusion reactor.

However, it is not desirable to have sawteeth flatten the profiles of all particle species. In particular, energetic $^2^He^{5+}$ ions (“fast alphas”) should ideally be left unperturbed, since they provide the heating power in a self-sustained ‘burning’ fusion plasma. After decades of research — especially during the 1990s when the Tokamak Fusion Test Reactor (TFTR) and JET operated with D-T plasmas \[8, 9\] — this conundrum of needing to confine energetic alphas while expelling helium ash is still unresolved, with or without sawteeth \[10, 11\]. Many studies considered particles whose losses can be detected directly from the outside (for instance \[12, 13\]), but those results are representative only for a small fraction of the fast ion population. Less is known about the confinement of alpha particles in the core, although it has been confirmed that sawteeth (being a mixing process) have relatively little effect on fast alphas that already have a broad density peak before a sawtooth crash \[14\].

Using numerical simulations, we have now discovered a scenario where MeV-class alphas can sustain a sharply peaked density profile even inside the sawtooth mixing radius, whereas partially slowed-down alphas in the 10...100 keV range are strongly mixed. After describing the simulation setup and numerical results, we show that this finding can be explained in terms of a synergistic effect. In a nutshell: the typical time scale of a sawtooth crash turns out to be just right to act differently on slow and fast alphas in a configuration where the field helicity always remains close to unity ($q \sim 1$).

**Simulation setup**

Our simulations are performed using a so-called hybrid code \[16, 17\], which solves visco-resistive magnetohydrodynamic (MHD) equations for the bulk plasma and kinetic equations for the fast ion minority species, whose inertia is assumed to be negligible compared to that of the bulk plasma. The plasma size and field strength are based on JET. The magnetic axis has a major radius of $R_0 = 3$ m and a field strength of $B_0 = 3.7$ T. The plasma current is $I_p = 2.5$ MA. The exact plasma composition is irrelevant in our single-fluid MHD model; here, the chosen bulk ion density and effective particle mass yield a central Alfvén speed of $v_{A0} \approx 8 \times 10^6$ m/s. (For pure deuterium, this corresponds to a central density of about $5.3 \times 10^{19}$ m$^{-3}$.) Figure 1(a) shows the plasma torus schematically in right-handed cylinder coordinates $(R, z, \zeta)$. Panel (b) shows the plasma cross-section in the poloidal $(R, z)$ plane, where the toroidal angle $\zeta$, magnetic field vector $B$ and plasma current are all pointing out of the plane. Black contours represent magnetic flux surfaces and color contours represent the plasma beta, which measures the ratio of thermal to magnetic pressure as $\beta = 2\mu_0 P / B_0^2$ with $\mu_0 = 4\pi \times 10^{-7}$ H/m.

Figure 1(c) shows the assumed profile of the field helicity $q$. The $q$ profile is treated here as a free parameter. Moreover, for the purpose of this proof-of-principle study, it is not necessary to simulate the reconnection process in all its multi-scale detail, which is extremely challenging even with modern supercomputers \[18\]. Considering the type of sawtooth that is associated with an internal kink instability and proceeds in a fashion similar to that envisioned by Kadomtsev \[3\] and Wesson \[19\], it suffices here to simulate the associated changes in the magnetic topology and the global electric drift. We require that the plasma is mixed in a similar volume and on a similar time scale of a few 100 $\mu$s as in experiments (Fig. 2). By matching the crash time scale and the size of the relaxing domain, we can match the speed of the displacement.

The size of the relaxing domain can be inferred from electron temperature measurements, which were used to set the location of the $q = 1$ surface indicated by dashed lines in Fig. 1. The relaxation region is relatively small, so it suffices to simulate only the inner 25% of the plasma’s magnetic flux space \[32\]. An artificial non-slip boundary is placed along the red line in Fig. 1. Inside that region we choose the $q$ profile to be close to unity and relatively flat, which gives us a configuration that is both numerically tractable and practically relevant for steady-state scenarios in future ITER experiments.

![FIG. 2: Examples of sawtooth crashes in JET pulse #95679 as seen by electron temperature fluctuation measurements in the plasma core using (a) electron cyclotron emission (ECE) and (b-d) soft x-ray (SXR) diagnostics.](image-url)
Internal kink and magnetic reconnection

Figure 3 shows the evolution of the vertical ($z$) component of the local MHD velocity $v_{z0} = v_{z}(R_0, z_0, \zeta_0)$ evaluated at the plasma center (magnetic axis) and $\zeta_0 = 0$. (b–d) Poincaré plots at $\zeta_0$ showing the topology of the magnetic field for the snapshots labeled (A), (B) and (C), taken at times 0.1, 0.2 and 0.3 ms measured from the instant where macroscopic displacement becomes visible. Red contours have helicities $q < 1$. Green islands and blue periphery have $q > 1$. Arrows roughly indicate $E \times B$ flow directions. (e) Contour plot of the bulk plasma beta $\beta(R, z, \zeta_0)$ at 0.3 ms (C), which behaves here as an MHD fluid. The dashed circle is the initial $q = 1$ surface. See the supplementary material for further details.

The dynamics seen here resemble a reconnecting internal kink mode as envisioned in the Kadomtsev model [3], which has been observed in well-diagnosed experiments [20], although other behavior is possible (e.g., [21]). The pair of convective cells associated with the internal kink redistributes the bulk plasma (modeled as an MHD fluid) in such a way that the pressure peak of Fig. 1(c) acquires a horse-shoe-like structure at the end of the crash as shown in Fig. 3(e). (We suspect that the associated changes in the MHD force balance are responsible for the compression of the reconnected core in Fig. 3(d).)

While the $E \times B$ flows decay, they cause further interchange and mixing as described by Wesson [19] for about 0.1 ms. As the flows weaken, their effect is eventually overcome by sources towards the end of the simulation ($\gtrsim 0.5$ ms), which tend to restore the original plasma current profile.

Alpha particle transport

The sawtooth crash in our simulation lasts less than 0.5 ms. This lies in the range of experimentally observed crash times $\tau_{\text{crash}} \sim 0.1...2$ ms in Fig. 2, so we can take this as a meaningful scenario to study the redistribution of fast alphas and helium ash during such an event in JET geometry. It is known that initially peaked profiles of the thermal bulk plasma ($\lesssim 10$ keV) may become hollow during a sawtooth crash [22]. This is routinely seen in sim-
FIG. 5: The selective confinement of fast alpha particles observed in our simulations is due to a synergy between the effects of (i) near-unity helicities and (ii) magnetic drifts, and the time scales for (iii) resonance detuning and (iv) the sawtooth crash. 

Underlying synergism

Magnetic drifts ($\nabla B$ and curvature) reduce the spatial corrugation of the density field in Fig. 6(f), but are not sufficient to explain the reduced transport. This has been verified by varying the mass-to-charge ratio of our simulation particles as described in detail in the supplementary material. Both the large magnetic drifts and the high velocity $v = 13 \times 10^6$ m/s of our 3.5 MeV alphas are essential, but there are two additional key factors.

The observations in Fig. 4 can be explained in terms of a synergy between four physical factors that are summarized in Fig. 5. The first key factor, represented by box (i) in Fig. 5, is that the magnetic helicity $q \approx \nabla B/\nabla B_{\text{pol}}$ is close to unity. This has the consequence that (ii) alpha particle resonances with respect to the internal kink are sensitive to magnetic drifts, and (iii) only fast alphas have a resonance detuning time $\tau_{2\pi} \sim 0.1$ ms that is shorter than (iv) the typical sawtooth crash time $\tau_{\text{crash}} \sim 0.3$ ms. Hence:

- There exists an optimal combination of time scales, where resonant interactions between the internal kink and fast alphas can be largely avoided; thus, facilitating good confinement of fast alphas only.

Let us discuss this synergistic effect in some more detail.

The condition for a charged particle to resonate with the internal kink can be expressed as $h \equiv \omega_{\text{tor}}/\omega_{\text{pol}} = 1$, where the orbit helicity $h \approx \tau_{\text{tor}}/(R_0v_{\text{pol}})$ is the ratio of toroidal and poloidal transit frequencies and can be thought of as the kinetic counterpart of the magnetic helicity $q \approx \nabla B_{\text{tor}}/(R_0B_{\text{pol}})$ modified by the combined effect of magnetic drifts and the mirror force [30]. Figure 6 shows a few examples of orbit helicity profiles $h(X)$ with
FIG. 6: Profiles of the orbit helicity $h(X) = \omega_{\text{tor}}/\omega_{\text{pol}}$ plotted as a function of the radial position $X = R - R_0$ where an alpha particle orbit crosses the midplane $z_0 = 0.262$. Dashed vertical lines indicate the $q = 1$ radii in the midplane.

$X = R - R_0$ for co- and counter-passing alpha particles with energies $K = 35$ keV, $350$ keV and $3.5$ MeV, and velocity pitch $v_B^0/v = v \cdot B/(vB) = \sin(\pm 0.48\pi) \approx \pm 1$. One can see that, with increasing kinetic energy $K$, the orbit helicity profiles $h$ deviate increasingly from the field helicity profile $q(X) \approx h(X/35$ keV$)$. Helicities of counter-passing particles are shifted upward, and Fig. 6 shows that 3.5 MeV alphas have $h > 1$ everywhere, eliminating the $h = 1$ resonance entirely. For co-passing particles, the $h$ profiles are shifted downward.

These differences between the field and orbit helicities have the consequence that magnetic drifts reduce the amount of reconnection that occurs in the topology of fast alpha particle orbits as highlighted in box (ii) of Fig. 5. Panel (a) shows that reconnection in co-passing orbit topology is incomplete, so that a large region with $h < 1$ survives throughout the crash. For counter-passing orbits in panel (b), we have always have $h > 1$ everywhere, which means that there is no resonance and the orbit topology is merely distorted by the magnetic perturbations associated with the internal kink. More details can be found in the supplementary material.

The second implication of having $q \sim h \sim 1$ in box (i) in Fig. 5 is that field trajectories nearly close on themselves during one toroidal transit as illustrated schematically in panel (c). Taking $h \sim 1 \pm 0.01$ for illustration, this means that a particle has to perform on the order of $N(2\pi) \sim 1/[1 - h] \sim 100$ toroidal transits before it covers a toroidal orbit surface [44].

During these $N(2\pi)$ transits, the phase of the radial $E \times B$ drift velocity $U_r(t)$ measured in the frame of reference moving with a chosen particle will also complete a $2\pi$ phase shift. This is illustrated in panels (d)–(f) of Fig. 5. In our simplified example, we assume that the electric drift in the moving frame has a sinusoidal form $U_r(t) = v_{E0}(t)\sin(\Theta(t))$. The envelope $v_{E0}(t)$ is chosen to be given by the vertical MHD velocity from Fig. 3(a) as $v_{E0} = -v_{E0}^0$, which had been measured at the reference point $(R_0, z_0, \zeta_0)$. The evolution of the phase is modeled as $\Theta(t) = 2\pi t/\tau_2\pi$, where $\tau_2\pi = \tau_{\text{tor}}N(2\pi) \sim \tau_{\text{tor}}/[1 - h] \sim 100 \times \tau_{\text{tor}}$ is the time scale for performing $N(2\pi)$ transits and plays the role of a resonance detuning time, so its inverse $1/\tau_2\pi$ can be thought of as the resonance width.

As indicated in box (iii) of Fig. 5, the resonance detuning times of newly born 3.5 MeV alphas and 35 keV helium ash differ by a factor 10. Incidentally, the typical sawtooth crash time $\tau_{\text{crash}}$ of a few 100 $\mu$s happens to be shorter than the 1 ms detuning time of 35 keV alphas and longer than the 0.1 ms detuning time of 3.5 MeV alphas. As stated in box (iv) of Fig. 5, this has the consequence that only fast alphas may undergo no net $E \times B$ displacement since there is a high probability for the positive and negative peaks of $U_r(t)$ in panel (f) to cancel. There is still a fair amount of cancellation at 350 keV in panel (e), where $\tau_2\pi \sim \tau_{\text{crash}}$, whereas 35 keV alphas in panel (d) remain in more or less in phase with the kink’s electric field for the entire duration of the crash. This explains why the density field of slow 35 keV alphas in Fig. 4(b) develops the same horse-shoe-like structure like the MHD fluid in Fig. 3(e). At the intermediate energy of 350 keV in Fig. 4(d), the density field has a swirling tear-drop structure indicating that poloidal spreading competes with the displacement caused by $E \times B$ drifts. At 3.5 MeV, the toroidal speed is so high that poloidal spreading outpaces $E \times B$ drifts, so the density field in Fig. 4(f) maintains a compact peak with only a minor helical distortion.

Finally, we note that Fig. 5 shows a highly simplified representation of reality. It is only meant to convey the basic idea. For instance, the transit number $N(2\pi)$ that determines the detuning time $\tau_2\pi$ varies with particle energy and radial location as one can readily infer from the helicity profiles in Fig. 6. Moreover, the actual $E \times B$ flow pattern is nonuniform in space. Thus, along particle orbits that perform large magnetic drifts, the direction of the electric drift may vary even during a single transit. This reduces the effective magnitude of the electric drifts for fast alphas. However, this well-known orbit-averaging effect alone is not sufficient to prevent strong mixing as can be verified from the parameter scans reported in the supplementary material. The selective confinement of fast alphas is realized only through the synergistic effect that arises from the four factors in Fig. 5 combined.

Discussion

Our results suggest that the majority of MeV-class alpha particles can remain well-confined in the plasma core during a benign sawtooth crash that strongly redis-
tributes less energetic ions and electrons. Our finding is of interest for DEMO fusion reactor R&D programs that are currently pursued around the world, since it demonstrates the possibility of using sawteeth for removing helium ash and other impurities without deteriorating fast alpha confinement. All else being similar, the time scales in Fig. 3 would be 2–3 times longer in ITER and DEMO due to their larger major radii $R_0 \approx 6 \text{ m}$ and $9 \text{ m}$, respectively. Since the magnetic drifts play an important role, the mechanism described here may be utilizable only when the plasma current is not too high, the advantages and disadvantages of will have to be weighed.

More detailed parameter scans are underway and will be reported elsewhere. The physical explanations we proposed imply that the results will be sensitive to the profile of the field helicity $q$. In order to utilize the described mechanism for ash removal in a reactor, it is necessary to ensure that only certain types of sawteeth occur, namely those for which the field helicity remains close to unity ($q \approx 1$). This requires both a better understanding of sawtooth physics and reliable plasma control schemes. The effect that fast alphas in realistic concentrations exert on the internal kink and various other MHD modes also requires further study.

Since a sawtooth crash is a mixing process and, thus, does not affect an already flat profile, it is clear that the mechanism we have described becomes noticeable only if the density profiles of the fusion-born alphas and resulting ash are peaked within the sawtoothing domain. The formation of such a peaked profile in the density of fusion-born alphas requires that the D-T fusion fuel has a peaked pressure profile in the first place. The sustained core-localized alpha heating can help to recover this condition after a sawtooth crash, but the interval between successive sawteeth must be sufficiently long to allow for a recovery of a sharply peaked plasma pressure. Although it is still unclear how to realize this situation in practice, the observation of impurity accumulation in tokamak plasmas with enhanced overall confinement suggests that there exists a ‘density pinch’ mechanisms that can lead to central peaking and requires further study. Techniques to deliver fusion fuel (D-T pellets) deep into the plasma core would help as well, but are yet another challenging topic of fusion research.

These practical considerations and the physics described in this paper exemplify the rich dynamic interplay and feedback between all the components of a fusion plasma and the instabilities and relaxation phenomena that such a plasma supports. Physicists are eagerly exploring these fascinating complex processes to learn more about magnetically confined burning plasmas. We dare say that a successful realization of ITER experiments has not only the potential to yield valuable insights needed for the construction of demonstration power plants but is also likely to inspire new experimental devices for burning plasma physics studies.

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SUPPLEMENTARY MATERIAL

1. Simulation method and numerical parameters

The version of the hybrid code MEGA used in this work has been documented in detail in [28]. MEGA has been validated against several experiments for different types of fast-ion-driven instabilities [29, 30].

The simulation setup is described in some detail in [26], especially with regard to the alpha particle distribution. The profiles of both the fast ion pressure and the MHD pressure are based on a reconstruction of a JET plasma using the code TRANSP-EFTP as reported in [27]. The MHD pressure in the original integrated TRANSP-EFTP simulations included the fast ion component, but we have subtracted that part as shown in Fig. S1 and recomputed the equilibrium using only the estimated pressure of the bulk plasma using the code CHEASE [31]. This reduces resistive interchange modes that would otherwise overwhelm the internal kink mode in our visco-resistive MHD simulations with Lundquist and Reynolds numbers $S = \text{Re} = 10^6$. Here, $S = \tau_\eta / \tau_A0$ is the ratio of the resistive and toroidal Alfvénic time scales, $\tau_\eta = \mu_0 R_0^2 / \eta$ and $\tau_A0 = R_0 / v_A0 \approx 0.4 \mu s$, where $\eta$ is the electrical resistivity.

Results of additional test simulations indicate that the MHD pressure also has a numerically stabilizing effect: the simulations terminated abnormally during the non-linear saturation stage when a zero-pressure MHD equilibrium was used. We suspect that pressure-gradient-induced dynamics help to wipe out fine structures in the reconnection layer, that may otherwise not be resolved by our grid. Our cylindrical mesh consists of $N_R \times N_\phi \times N_z = 220 \times 220 \times 96$ points; very similar results were obtained with $520 \times 520 \times 96$ and $220 \times 220 \times 192$. 
The 4th-order Runge-Kutta time step is about 1 ns for the MHD solver, which is constrained by the Courant-Friedrichs-Lewy (CFL) condition for fast magnetosonic waves. Larger time steps can be used for the particle motion, especially at low energies.

As stated in the text of the main article, we treat the \( q \) profile as a free parameter. Besides being an essential physics ingredient for our study, our choice to use a fairly flat \( q \) profile with \( q \sim 1 \) inside the \( q = 1 \) surface also has computational advantages. In combination with the suspected regularizing effect of MHD pressure gradients mentioned in the previous paragraph, a flat \( q \) profile prevents the magnetic reconnection layer from collapsing rapidly into a narrow sheet, which would be difficult to resolve numerically even with the relatively small Lundquist and Reynolds numbers \( S = Re = 10^6 \) in our simulation.

As is typical for simulations like ours, the MHD model includes a source term representing the tokamak’s loop voltage, which balances the resistive dissipation of the initial current profile. Without this term, our central \( q_0 = 0.98 \) would rise above unity on the time scale of a few \( 100 \) μs, which implies that resistive dissipation participates in the simulated dynamics on a global scale, helping to avoid the formation of a narrow current sheet and making our simulation numerically robust.

2. Fourier harmonics of the internal kink instability

With \( N_{\zeta} = 96 \) grid points along the toroidal angle \( \zeta \), our simulation can effectively capture toroidal Fourier harmonics \( \exp(in\zeta) \) with toroidal mode numbers \( -48 \leq n \leq 48 \). Our simulation starts from a small initial perturbation applied to the Fourier harmonics \( n = 1, 2, 3, 4 \), whose evolution is shown in Fig. S2 in terms of the MHD fluctuation energy \( W_n(t) \) as defined in Eq. (18) of [28]. After the initial mode structure formation process, the internal kink instability in this simulation is clearly dominated by the \( n = 1 \) harmonic throughout the simulation.

The simulation runs for a few \( 100 \) μs before macroscopic transport becomes visible, and we measure the crash time \( t - t_0 \) starting approximately from that instant. Here we have chosen \( t_0 = 0.38 \) ms, which corresponds to \( t_0 = l_0/\tau_A = 1000 \) in units of the toroidal Alfvén time \( \tau_A = R_0/v_A \). Figure S2 shows that the dominant harmonics are still growing exponentially at that time.

3. Magnetic drift effect: I. Particle transport

Magnetic drifts (\( \nabla B \) and curvature) reduce the spatial corrugation of the density field in Fig. 4(f) of the main article, but are not sufficient to prevent profile flattening. This is demonstrated in Fig. S3(a), which shows the redistribution of an artificial particle species with charge number \( Q = 2 \), mass \( M_{10} = 10 \times M_s \) and speed \( v = 1.3 \times 10^6 \) m/s. Its magnetic drift is as large as that of fast 3.5 MeV alphas, while the transit frequency equals to that of slow 35 keV alphas. Although the density field in Fig. S3(b) is blurred as in Fig. 4(f), the density profile in Fig. S3(a) is subject to much stronger flattening than in Fig. 4(e), which must be due to the particle speed being...
smaller by a factor 10.

Gyroaveraging also contributes to blurring, but was found to be negligible for transport in the present case. This was confirmed by running simulations with and without 4-point averaging over the Larmor radius as described in [28]. The results were found to be similar when comparing the guiding center density profiles.

For completeness, Fig. S3(c) shows that in the opposite limit, where light particles ($M_{0,1} = M_\alpha/10, Q = 2$) travel at high speed ($v = 13 \times 10^6 \text{ m/s}$), the density profile is also flattened. This, of course, is consistent with the experimentally known fact that electrons undergo radial mixing during sawtooth crashes. The results in Fig. S3 thus show that the effect of magnetic drifts, which (for a given $v$) is larger for particles with smaller charge-to-mass ratio $Qe/M$, is not sufficient but necessary for preventing profile flattening.

We note that another factor that may affect our artificial light particle species in Fig. S3(c,d) is the parallel electric field $E_| = E \cdot B/B$ (here about 1% of $|E_|$) that is generated in the relaxing domain of our resistive MHD simulation. The larger charge-to-mass ratio $Qe/M$ of the light particles makes them more responsive to $E_|$ than alphas and heavier ions. Consequences remain to be examined.

4. Magnetic drift effect: II. Orbit topology

The topology of the charged particle orbits in Fig. S3(b,c) of the main article was visualized using conventional Poincaré plots, where we follow the guiding center motion of passive test particles. Each point in the plot represents the position where a test particle crosses the Poincaré section, which in our case is the poloidal $(R, z)$ plane at $\zeta_0 = 0$. Each test particles is launched from a different radial position in the midplane (height $z_0 \approx 0.26 \text{ m}$).

All Poincaré plots presented here were obtained by tracing test particles in the self-consistent perturbed magnetic field $B$ of the hybrid simulation. Although present in the simulations, electric drifts were not included in the Poincaré analysis and the magnetic field was not evolved while the particles traced out their orbits. This means that our Poincaré contours reflect only the instantaneous topology of the orbits, not the motion of particles in the original hybrid simulation of the sawtooth crash, where the magnetic configuration changes while a particle is tracing out a toroidal orbit surface.

The Poincaré plots for the magnetic field shown in Fig. S3(b-d) of the main article were computed by following test particles without magnetic drifts. These plots are shown again in the top row of Fig. S4 with the addition of a fourth snapshot (D) taken near the end of the simulation at 0.5 ms.

The rest of Fig. S4 shows the Poincaré contours of co-

Fig. S4: Topology of the magnetic field (top row) and alpha particle orbits at the times of snapshots (A)–(D) indicated in Fig. S2 (and Fig. 3(a) of the main article). Red and orange Poincaré contours have helicities $q, h < 1$. Reconnected green islands and unreconnected blue periphery have $q, h > 1$. The initial $q = 1$ surface is drawn as a dashed circle and the dotted circles represent the initial $h = 1$ surfaces in cases where they exist. Arrows roughly indicate the direction of electric drifts. Note that the density of the field contours and orbit contours in these plots is arbitrary, so it does not does not by any means reflect the density of magnetic flux or orbit surfaces.
and counter-passing alpha particles with energies 35 keV, 350 keV and 3.5 MeV. Panels (w) and (#) already appeared in box (ii) of Fig. 5 of the main article to illustrate that magnetic drifts reduce the amount of reconnection that occurs in orbit topology. The complete set of snapshots in Fig. [S3] confirms this statement and illustrates how the effect increases with increasing particle energy.

The orange contours appearing in snapshot (D) in the right column of Fig. [S3] are meant to represent regions where $q < 1$ has been restored due to the effect of the loop voltage (current source), which balances the global dissipation of the plasma current by the relatively large resistivity (small Lundquist number $S = 10^6$) in our simulations. The source term gradually restores the initial $q$ profile and would later lead to the growth of a new instability. However, the effect of the current source on the Poincaré plots is difficult to quantify, so the boundary between red and orange domains was chosen somewhat arbitrarily based on intuition gained from inspecting many such plots.

Finally, we note that all our Poincaré plots consist of closed orbit contours, which means that they represent conservative Hamiltonian dynamics. No chaotic domains were seen in the present case, which means that there are no significant resonance overlaps.

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[32] The reduced domain size facilitates parameter scans and numerical convergence tests. Tentative simulations of the full domain have reproduced the key results. Details will be reported elsewhere.
[33] Magnetic drift is proportional to the Larmor radius $\rho = MV_{\parallel}/(Qe)$ of a particle and the plasma current. It shifts co-(counter-)passing orbits out-(in-)ward in major radius $R$, where the attributes co and counter refer to the direction of the plasma current. The mirror force causes particles to spend more time in regions of weaker field (larger $R$).
[34] When viewed from above, the motion illustrated schematically in Fig. [S4]c is somewhat similar to the apsidal precession of a planetary orbit around the Sun.