Frenet-Serret analysis of helical Bloch modes in N-fold rotationally symmetric rings of coupled spiralling optical waveguides

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Frenet-Serret (FS) analysis has previously been used to track the polarisation state evolution of a mode guided along a spiralling off-axis optical fibre core [1]. Here we present an analytical model, based in the local FS frame, for the properties of helical Bloch modes (HBM) guided in a rotationally symmetric ring of N coupled cores that spiral around the axis of an optical fibre. The model takes full account of the curvature and torsion of the spiralling waveguides.

We introduce a coordinate system consisting of a mesh of orthogonal helical curves on the surface of a cylinder of constant radius $\rho$, parametrized as (Fig. 1(a)):

$$
\begin{align*}
  x(\sigma_1, \sigma_2) &= \rho \cos \phi, \\
  y(\sigma_1, \sigma_2) &= \rho \sin \phi, \\
  z(\sigma_1, \sigma_2) &= -\frac{\sigma_1}{\sqrt{1 + \rho^2 \alpha^2}}, \\
  \phi &= (a \sigma_1 + \sigma_2) / \sqrt{1 + \rho^2 \alpha^2}
\end{align*}
$$

(1)

where $\alpha$ is the twist rate, $(\rho, \phi, z)$ are untwisted cylindrical coordinates, $\sigma_1$ = constant describes the rapidly spiralling red curves and $\sigma_2$ = constant the slowly spiralling blue curves (tracking the core positions for $N = 6$). Within the FS-frame, defined by local FS unit vectors $\hat{\mathbf{N}}$, $\hat{\mathbf{T}}$ and $\hat{\mathbf{B}}$, the vectorial modal fields can be conveniently expressed as a superposition of left and right circularly polarised fields; the Bloch phase evolves along the $\hat{\mathbf{B}}$ direction while the HBM propagates in the $\hat{\mathbf{T}}$ direction. After setting up coupled mode equations in the FS-frame and applying Bloch’s theorem, the dispersion relation of the HBM can be derived, yielding the propagation constants and polarization states of the HBM, which can be quite complex for elliptical cores. Good agreement with finite element modelling is obtained, except at high twist rates, when the modal fields in each core become distorted (Fig. 1(b)).

The total phase progression around the azimuth in the laboratory frame is $\ell_A 2\pi n$, for fields evaluated in cylindrical coordinates. For pure circularly polarised fields (i.e., perfectly circular cores), the topological charge of the $m$-th harmonic of the HBM is given by $\ell^{(m)} = \ell_A^{(m)} - s$. In this case the dispersion surfaces (axial index versus $\ell_A$) collapse on to one curve when expressed in terms of topological charge (Fig. 1(c)). This differs from previous heuristic results where torsion was not properly taken into account in the analysis [2].

In summary, vector coupled-mode theory based in the local Frenet-Serret frame provides an elegant and accurate means of analysing and understanding the properties of HBM in twisted N-fold rotationally symmetric fibres. We believe the analysis will provide a firm basis for future studies of more complex multi-core structures, both linear and nonlinear [3].

References
[1] J. N. Ross, “The rotation of the polarization in low birefringence monomode optical fibres due to geometric effects,” Optical and Quantum Electronics 16, 455-461 (1984).
[2] S. Loranger, Y. Chen, P. Roth, M. Frots, G. K. L. Wong, and P. St.J. Russell, “Bragg reflection and conversion between helical Bloch modes in chiral three-core photonic crystal fiber,” J. Lightwave Technol. 38, 4100-4107 (2020).
[3] C. R. Menyuk and P. K. A. Wai, “Elimination of nonlinear polarization rotation in twisted fibers,” J. Opt. Soc. Am. B 11, 1305–1309 (1994).