Cautious Monotonicity in Case-Based Reasoning with Abstract Argumentation

Guilherme Paulino-Passos\textsuperscript{1}, Francesca Toni\textsuperscript{1}
\textsuperscript{1}Imperial College London, Department of Computing
\{g.passos18, f.toni\}@imperial.ac.uk

Abstract

Recently, abstract argumentation-based models of case-based reasoning (\textit{AA-CBR} in short) have been proposed, originally inspired by the legal domain, but also applicable as classifiers in different scenarios, including image classification, sentiment analysis of text, and in predicting the passage of bills in the UK Parliament. However, the formal properties of \textit{AA-CBR} as a reasoning system remain largely unexplored. In this paper, we focus on analysing the non-monotonicity properties of a \textit{regular} version of \textit{AA-CBR} (that we call \textit{AA-CBR}\textsubscript{$\geq$}). Specifically, we prove that \textit{AA-CBR}\textsubscript{$\geq$} is not cautiously monotonic, a property frequently considered desirable in the literature of non-monotonic reasoning. We then define a variation of \textit{AA-CBR}\textsubscript{$\geq$} which is cautiously monotonic, and provide an algorithm for obtaining it. Further, we prove that such variation is equivalent to using \textit{AA-CBR}\textsubscript{$\geq$} with a restricted casbase consisting of all ”surprising” cases in the original casbase.

1 Introduction

Case-based reasoning (\textit{CBR}) relies upon known solutions for problems (past cases) to infer solutions for unseen problems (new cases), based upon retrieving past cases which are “similar” to the new cases. It is widely used in legal settings (e.g. see (Prakken et al. 2015; Čyraš, Satoh, and Toni 2016a)), for classification (e.g. via the k-NN algorithm) and, more recently, within the DEAr methodology (Cocarascu et al. 2020)) and for explanation (e.g. see (Nugent and Cunningham 2005; Kenny and Keane 2019; Cocarascu et al. 2020)).

In this paper we focus on a recent approach to \textit{CBR} based upon an argumentative reading of (past and new) cases (Čyraš, Satoh, and Toni 2016a; Čyraš, Satoh, and Toni 2016b; Cocarascu, Čyraš, and Toni 2018; Čyraš et al. 2019; Cocarascu et al. 2020), and using Abstract Argumentation (\textit{AA}) (Dung 1995) as the underpinning machinery. In this paper, we will refer to all proposed incarnations of this approach in the literature generically as \textit{AA-CBR} (the acronym used in the original paper (Čyraš, Satoh, and Toni 2016a)): they all generate an AA framework from a CBR problem, with attacks from “more specific” past cases to “less specific” past cases or to a “default argument” (embedding a sort of bias), and attacks from new cases to “irrelevant” past cases; then, they all reduce CBR to membership of the “default argument” in the grounded extension (Dung 1995), and use fragments of the AA framework for explanation (e.g. dispute trees as in (Čyraš, Satoh, and Toni 2016b; Cocarascu et al. 2020) or excess features in (Čyraš et al. 2019)). Different incarnations of \textit{AA-CBR} use different mechanisms for defining “specificity”, “irrelevance” and "default argument": the original version in (Čyraš, Satoh, and Toni 2016a) defines all three notions in terms of $\geq$ (and is thus referred to in this paper as \textit{AA-CBR}\textsubscript{$\geq$}); thus, \textit{AA-CBR}\textsubscript{$\geq$} is applicable only to cases characterised by sets of features; the version used for classification in (Cocarascu et al. 2020) defines “specificity” in terms of a generic partial order $\geq$, “irrelevance” in terms of a generic relation $\not\sim$ and ”default argument” in terms of a generic characterisation $\delta_C$ (and is thus referred to in this paper as \textit{AA-CBR}\textsubscript{$\geq$, $\not\sim$, $\delta_C$}). Thus, \textit{AA-CBR}\textsubscript{$\geq$, $\not\sim$, $\delta_C$} is in principle applicable to cases characterised in any way, as sets of features or unstructured (Cocarascu et al. 2020). Here we will study a special, regular instance of \textit{AA-CBR}\textsubscript{$\geq$, $\not\sim$, $\delta_C$} (which we refer to as \textit{AA-CBR}\textsubscript{$\geq$}) in which “irrelevance” and the ”default argument” are both defined in terms of “specificity” (and in particular the “default argument” is defined in terms of the “most specific” case). \textit{AA-CBR}\textsubscript{$\geq$} admits \textit{AA-CBR}\textsubscript{$\geq$} as an instance, obtained by choosing $\geq=\geq$ and by restricting attention to “coherent” casbases (whereby there is no ”noise”, in that no two cases with different outcomes are characterised by the same set of features).

\textit{AA-CBR} was originally inspired by the legal domain in (Čyraš, Satoh, and Toni 2016a), but some incarnations of \textit{AA-CBR}, integrating dynamic features, have proven useful in predicting and explaining the passage of bills in the UK Parliament (Čyraš et al. 2019), and some instances of \textit{AA-CBR}\textsubscript{$\geq$, $\not\sim$, $\delta_C$} have also shown to be fruitfully applicable as classifiers in a number of scenarios, including classification with categorical data, with images and for sentiment analysis of text (Cocarascu et al. 2020).

In this paper we study non-monotonicity properties of \textit{AA-CBR}\textsubscript{$\geq$} understood at the same time as a reasoning system and as a classifier. These properties, typically considered for logical systems, intuitively characterise in which sense systems may stop inferring some
conclusions when more information is made available to them (Makinson 1994). These properties are thus related to modelling inference which is tentative and defeasible, as opposed to the indefeasible form of inference of classical logic. Non-monotonicity properties have already been studied in argumentation systems, such as ABA and ABA+ (Čyraš and Toni 2015; Čyraš and Toni 2016), ASPiC+ (Dung 2014; Dung 2016) and logic-based argumentation systems (Hunter 2010). In this paper, we study those properties for the application of argumentation to classification, in particular in the form of AA-CBR.

The following example illustrates AA-CBR (and AA-CBR+) in particular) as well as its non-monotonicity,
in a legal setting.

![Initial AA framework for Example 1](image1)

Figure 1: Initial AA framework for Example 1. Past cases (with their outcomes) and the new case (with no outcome, indicated by a question mark) are represented as arguments. AA-CBR predicts outcome + for the new case. (Grounded extension in colour.)

![Revised AA framework for Example 1](image2)

Figure 2: Revised AA framework for Example 1. Here, the added past case changes the AA-CBR-predicted outcome to – by limiting the applicability of the previous past case. (Again, grounded extension in colour.)

**Example 1.** Consider a simplified legal system built by cases and adhering, like most modern legal systems, to the principle by which, unless proven otherwise, no person is to be considered guilty of a crime. This can be represented by a “default argument” (Φ, −), indicating that, in the absence of any information about any person, the legal system should infer a negative outcome − (that the person is not guilty). (Φ, −) can be understood as an argument, in the AA sense, given that it is merely what is called a relative presumption, since it is open to proof to the contrary, e.g. by proving that the person did indeed commit a crime. Let us consider here one possible crime: homicide\(^1\) (hm). In one case, it was established that the defendant committed homicide, and he was considered guilty, represented as (hm, +). Consider now a new case (hm, sd, ?), with an unknown outcome, of a defendant who committed homicide, but for which it was proven that it was in self-defence (sd). In order to predict the new case’s outcome by CBR, AA-CBR reduces the prediction problem to that of membership of the default argument in the grounded extension G (Dung 1995) of the AA framework in Figure 1: given that (Φ, −) \(\notin G\), the predicted outcome is positive (i.e. guilty), disregarding sd and, indeed, no matter what other feature this case may have. Thus, up to this point, having the feature hm is a sufficient condition for predicting guilty. If, however, the courts decides that for this new case the defendant should be acquitted, the case (hm, sd, −) enters in our casebase. Now, having the feature hm is no longer a sufficient condition for predicting guilty, and any case with both hm and sd will be predicted a negative outcome (i.e. that the person is innocent). This is the case for predicting the outcome of a new case with again both hm and sd, in AA-CBR using the AA framework in Figure 2. Thus, adding a new case to the casebase removed some conclusions which were inferred from the previous, smaller casebase. This illustrates non-monotonicity.

In this paper we prove that the kind of inference underpinning AA-CBR\(_-\) lacks a standard non-monotonicity property, namely cautious monotonicity. Intuitively this property means that if a conclusion is added to the set of premises (here, the casebase), then no conclusion is lost, that is, everything which was inferable still is so. In terms of a supervised classifier, satisfying cautious monotonicity culminates in being “closed” under self-supervision. That is, augmenting the dataset with conclusions inferred by the classifier itself does not change the classifier.

Then, we make a two-fold contribution: we define (formally and algorithmically) a provably cautiously monotonic variant of AA-CBR\(_-\), that we call cAA-CBR\(_-\), and prove that it is equivalent to AA-CBR\(_-\) applied to a restricted casebase consisting of all “surprising” cases in the original casebase. We also show that the property of cautious monotonicity of cAA-CBR\(_-\) leads to the desirable properties of cumulativity and rational monotonicity. All results here presented are restricted to coherent casebases, in which no case characterisation (problem) occurs with more than one outcome (solution).

**2 Background**

**2.1 Abstract argumentation**

An abstract argumentation framework (AF) (Dung 1995) is a pair \((\text{Args}, \rightarrow)\), where Args is a set (of arguments) and \(\rightarrow\) is a binary relation on Args. For \(\alpha, \beta \in \text{Args}\), if \(\alpha \rightarrow \beta\), then we say that \(\alpha\) attacks \(\beta\) and that \(\alpha\) is an attacker of \(\beta\). For a set of arguments \(E \subseteq \text{Args}\) and an argument \(\alpha \in \text{Args}\), \(E\) defends \(\alpha\) if for all \(\beta \rightarrow \alpha\) there exists \(\gamma \in E\) such that \(\gamma \rightarrow \beta\). Then, the grounded extension of \((\text{Args}, \rightarrow)\) can be constructed as \(G = \bigcup\_{i \geq 0} G_i\), where \(G_0\) is the set of all

---

\(^1\)This is merely a hypothetical example, so the terms used do not correspond to a specific jurisdiction.
unattacked arguments, and \( \forall i \geq 0, G_{i+1} \) is the set of arguments that \( G_i \) defends. For any \((\text{Args}, \prec)\), the grounded extension \( \subseteq \) always exists and is unique and, if \((\text{Args}, \prec)\) is well-founded (Dung 1995), extensions under other semantics (e.g., stable extensions (Dung 1995), where \( E \subseteq \text{Args} \) is stable if \( \notin E \alpha, \beta \in E \) such that \( \alpha \prec \beta \) and, moreover, \( \forall \alpha \in \text{Args} \setminus E, \exists \beta \in E \) such that \( \beta \prec \alpha \) are equal to \( \subseteq \). In particular for finite AFs, \((\text{Args}, \prec)\) is well-founded if it is acyclic.

Given \((\text{Args}, \prec)\), we will sometimes use \( \alpha \in (\text{Args}, \prec) \) to stand for \( \alpha \in \text{Args} \).

### 2.2 Non-monotonicity properties

We will be interested in the following properties.\(^2\) An arbitrary inference relation \( \vdash \) (for a language including, in particular, sentences \( a, b \), etc., with negations \( \neg a \) and \( \neg b \), etc., and sets of sentences \( A, B \)) is said to satisfy:

1. **non-monotonicity**, iff \( A \vdash a \) and \( A \subseteq B \) do not imply that \( B \vdash a \);
2. **cautious monotonicity**, iff \( A \vdash a \) and \( A \vdash b \) imply that \( A \cup \{a\} \vdash b \);
3. **cut**, iff \( A \vdash a \) and \( A \cup \{a\} \vdash b \) imply that \( A \vdash b \);
4. **cumulativity**, iff \( \vdash \) is both cautiously monotonic and satisfies cut;
5. **rational monotonicity**, iff \( A \vdash a \) and \( A \not\vdash \neg b \) imply that \( A \cup \{b\} \vdash a \);
6. **completeness**, iff either \( A \vdash a \) or \( A \vdash \neg a \).

### 3 Setting the ground

In this section we define \( AA-CBR_{\subseteq} \), adapting definitions from (Cocarascu et al. 2020).

All incarnations of \( AA-CBR_{\subseteq} \), including \( AA-CBR_{\subseteq} \), map a database \( D \) of examples labelled with an outcome and an unlabelled example (for which the outcome is unknown) into an AF. Here, the database may be understood as a casebase, the labelled examples as past cases and the unlabelled example as a new case: we will use these terminologies interchangeably throughout. In this paper, as in (Cocarascu et al. 2020), examples/cases have a characterisation (e.g., as in (Çyras, Satoh, and Toni 2016a)), characterisations may be sets of features, and outcomes are chosen from two available ones, one of which is selected up-front as the default outcome. Finally, in the spirit of (Cocarascu et al. 2020), we assume that the set of characterisations of (past and new) cases is equipped with a partial order \( \preceq \) (whereby \( \alpha \prec \beta \) holds if \( \alpha \preceq \beta \) and \( \alpha \neq \beta \) and is read “\( \alpha \) is less specific than \( \beta \)”)) and with a relation \( \not\sim \) (whereby \( \alpha \not\sim \beta \) is read as “\( \beta \) is irrelevant to \( \alpha \)”).

**Definition 2** (Adapted from (Cocarascu et al. 2020)). Let \( X \) be a set of characterisations, equipped with a partial order \( \prec \) and a binary relation \( \sim \). Let \( Y = \{\delta_o, \delta_e\} \) be the set of (all possible) outcomes, with \( \delta_o \) the default outcome. Then, a casebase \( D \) is a finite set such that \( D \subseteq X \times Y \) (thus a past case \( \alpha \in D \) is of the form \((\alpha_C, \alpha_o) \) for \( \alpha_C \in X \) and \( \alpha_o \in Y \) and a new case is of the form \((N_C, ?) \) for \( N_C \in X \). We also discriminate a particular element \( \delta_C \in X \) and define the default argument \((\delta_C, \delta_o) \) in \( X \times Y \).

A casebase \( D \) is coherent if there are no two cases \((\alpha_C, \alpha_o), (\beta_C, \beta_o) \in D \) such that \( \alpha_C = \beta_C \) but \( \alpha_o \neq \beta_o \).

For simplicity of notation, we sometimes extend the definition of \( \preceq \) to \( X \times Y \), by setting \((\alpha_e, \alpha_o) \preceq (\beta_e, \beta_o) \) iff \( \alpha_e \preceq \beta_e \).\(^3\)

**Definition 3** (Adapted from (Cocarascu et al. 2020)). The AF mined from a dataset \( D \) and a new case \((N_C, ?) \) is \((\text{Args}, \prec)\), in which:

- \( \text{Args} = D \cup \{(\delta_C, \delta_o)\} \cup \{(N_C, ?)\} \);
- \((\alpha_C, \alpha_o), (\beta_C, \beta_o) \in D \cup \{(\delta_C, \delta_o)\} \), it holds that \((\alpha_C, \alpha_o) \not\sim (\beta_C, \beta_o) \) iff \( \alpha_C \not\preceq \beta_C \).

The AF mined from a dataset \( D \) alone is \((\text{Args}, \not\sim)\), with \( \text{Args} = D \setminus \{(N_C, ?)\} \) and \( \not\sim \rightarrow \cap(\text{Args} \times \text{Args}) \).

Note that if \( D \) is coherent, then the “equals” case in the item 2 of the definition of attack will never apply. As a result, the AF mined from a coherent \( D \) (and any \((N_C, ?)\) ) is guaranteed to be well-founded.

**Definition 4** (Adapted from (Cocarascu et al. 2020)). Let \( \subseteq \) be the grounded extension of the AF mined from \( D \) and \((N_C, ?)\), with default argument \((\delta_C, \delta_o)\). The outcome for \( N_C \) is \( \delta_o \) if \((\delta_C, \delta_o) \) is in \( \subseteq \), and \( \delta_o \) otherwise.

In this paper we focus on a particular case of this scenario:

**Definition 5**. The AF mined from \( D \) alone and the AF mined from \( D \) and \((N_C, ?)\), with default argument \((\delta_C, \delta_o)\), are regular when the following requirements are satisfied:

1. the irrelevance relation \( \not\sim \) is defined as: \( x_1 \not\sim x_2 \) iff \( x_1 \not\preceq x_2 \) and
2. \( \delta_C \) is the least element of \( X \).

This restriction connects the treatment of a characterisation \( \alpha_C \) as a new case and as a past case. We will see below that these conditions are necessary in order to satisfy desirable properties, such as Theorem 7.

In the remainder, we will restrict attention to regular mined AFs. We will refer to the (regular) AF mined from \( D \) and \((N_C, ?)\), with default argument \((\delta_C, \delta_o)\), as

\(^3\)In (Cocarascu et al. 2020) \( \preceq \) was directly given over \( X \times Y \). Note that, in \( X \times Y \), anti-symmetry may fail for two cases with different outcomes but the same characterisation, if \( \alpha \not\preceq \beta \) then all cases \((\alpha_C, \alpha_o) \) in the casebase could be removed, as they would never change an outcome. On the other hand, assuming also the first condition in Definition 5, if \((\alpha_C, ?) \) is the new case and \( \alpha_C \not\preceq \delta_C \), then the outcome is \( \delta_o \) necessarily.
AF_<(D, N_C), and to the (regular) AF mined from D alone as AF_<(D). Also, for short, given AF_<(D, N_C), with default argument (_, δ_o), we will refer to the outcome for N_C as AA-CBR_<(D, N_C). In the remainder of the paper we assume as given arbitrary X, Y, D, (N_C, ?), (_, δ, δ_o) (satisfying the previously defined constraints), unless otherwise stated.

In the remainder of this section we will identify some properties of AA-CBR_<(, concerning its behaviour as a form of CBR.

Agreement with nearest cases. Our first property regards the predictions of AA-CBR_<( in relation to the “most similar” (or nearest) cases to the new case, when these nearest cases all agree on an outcome. This property generalises (ˇCyras, Satoh, and Toni 2016a, Proposition 2) in two ways: by considering the entire set of nearest cases, instead of requiring a unique nearest case, for AA-CBR_<(, instead of its instance AA-CBR_©. As in (ˇCyras, Satoh, and Toni 2016a), we prove this property for coherent casebases. We first define the notion of nearest case.

Definition 6. A case (α_C, α_o) ∈ D is nearest to N_C if α_C ≤ N_C and it is maximally so, that is, there is no (β_C, β_o) ∈ D such that α_C < β_C ≤ N_C.

Theorem 7. If D is coherent and every nearest case to N_C is of the form (α_C, o) for some outcome o ∈ Y (that is, all nearest cases to the new case agree on the same outcome), then AA-CBR_<(D, N_C) = o (that is, the outcome for N_C is o).

Proof. Let G be the grounded extension of AF_<(D, N_C).

An outline of the proof is as follows:

1. We will first prove that each argument in G is either (N_C, ?) or of the form (β_C, o) (that is, agreeing in outcome with all nearest cases).
2. Then we will prove that if o = δ_o (that is, o is the non-default outcome), then (δ_C, δ_o) ∈ G (and thus AA-CBR_<(D, N_C) = δ_o, as envisaged by the theorem).
3. Finally, by using the fact that AF_<(D, N_C) is well-founded (given that D is coherent), and thus G is also stable, we will prove that if o = δ_o (that is, o is the default outcome), then (δ_C, δ_o) ∈ G (and thus AA-CBR_<(D, N_C) = δ_o, as envisaged by the theorem).

We will now prove 1-3.

1. By definition G = ∪_{i≥0} G_i. We prove by induction that, for every i, each argument in G_i is either (N_C, ?) or of the form (β_C, o). Then, given that each element of G belongs to some G_i, the property holds for G.

(a) For the base case, consider G_0, (N_C, ?) and all nearest cases are unattacked, and thus in G_0 (notice how this requires the AF to be regular, otherwise nearest cases could be irrelevant). G_0 may however contain further unattacked cases. Let β = (β_C, β_o) be such a case. If N_C ∉ β_C, then (δ_C, δ_o) ∉ β and thus (N_C, ?) attacks β, contradicting that β in unattacked. So β_C ≤ N_C.

As β is not a nearest case, there is a nearest case α = (α_C, α_o) such that β_C < α_C. By contradiction, assume β_o = α_o. Let Γ = {γ ∈ Args | γ ∈ (γ_C, γ_o), β_C < γ_C ≤ α_C and γ_o = 0}. Notice that Γ is non-empty, as α_C ∈ Γ. The set is the set of “potential attackers” of β, but only ≤-minimal elements of Γ do actually attack β. Let γ be such a ≤-minimal element of Γ. By construction, γ attacks β. Thus β is attacked and not in G_0, a contradiction. Hence, β_o = α_o, as required.

(b) For the inductive step, let us assume that the property holds for a generic G_i, and let us prove it for G_{i+1}. Let β = (β_C, β_o) ∈ G_{i+1} \ G_i (if β ∈ G_i, the property holds by the induction hypothesis). (N_C, ?) does not attack β, as otherwise β would not be defended by G_i, as G_i is conflict-free. Thus, once again, as β is not a nearest case, there is a nearest case α = (α_C, α_o) such that β_C < α_C. Again, assume that β_o = α_o. Then let Γ = {γ ∈ Args | γ ∈ (γ_C, γ_o), β_C < γ_C ≤ α_C and γ_o = 0}, with γ a ≤-minimal element of Γ. Then γ attacks β. However, as G_i defends β, there is then θ ∈ G_i such that θ attacks β. By inductive hypothesis, θ is either (N_C, ?) or θ = (θ_C, o). The first option is not possible, as θ ∈ Γ, and thus γ_C ≤ θ_C, and of course α_C ≤ N_C. Thus, γ_o ≤ N_C and is thus not attacked by (N_C, ?). This means that (θ_C, o) attacks δ_o = (θ, o). But this is absurd as well, as θ ∈ Γ and thus θ_o = o = δ_o. Therefore, our assumption that β_o = α_o was false, that is, β_o = α_o, as required.

2. If o = δ_o, the default argument (δ_C, δ_o) is not in G, since we have just proven that all arguments in G other than (N_C, ?) have outcome o.

3. If o = δ_o, then let β be an attacker of (δ_C, δ_o), and thus of the form β = (β_C, β_o) (again see how regularity is necessary, since otherwise (N_C, ?) could be the attacker). β is not in G and, since G is also a stable extension, some argument in G attacks β. This is true for any attacker β of the default argument, and thus the default argument is defended by G. As G contains every argument it defends, the default argument is in the grounded extension, confirming that the outcome for N_C is δ_o.

Addition of new cases. The next result characterises the set of past cases/arguments attacked when the dataset is extended with a new labelled case/argument. In particular, this result compares the effect of predicting the outcome of some N_2 from D alone and from D extended with (N_1, o_1), when there is no case in D with characterisation N_1 already and moreover D is coherent.

This result will be used later in the paper and is interesting in its own right as it shows that, any argument attacked by the “newly added” case (N_1, o_1) is easily identified in the sets

Note that we omit to indicate in the notations the default argument (_, δ_o), and leave it implicit instead for readability.
Lemma 8. Let $D$ be coherent, $N_1, N_2 \in X$, $o_1 \in Y$, and suppose that there is no case in $D$ with characterisation $N_1$. Consider $AF_1 = AF_2(D, N_1)$ and $AF_2 = AF_2(D \cup \{(N_1, o_1)\}, N_2)$. Finally, let $G(AF_1)$ and $G(AF_2)$ be the respective grounded extensions. Let $\beta$ be such that $(N_1, o_1) \rightarrow \beta$ in $AF_2$. Then,

1. For every $\gamma$ that attacks $\beta$ in $AF_1$, $N_1 \not\sim \gamma$ (that is, $\gamma$ is irrelevant to $N_1$ and, by regularity, $N_1 \not\sim \gamma$);
2. In $AF_1$, $(N_1, ?)$ defends $\beta$;
3. $\beta \in G(AF_1)$ and, for $G(AF_1) = \bigcup_{i \geq 0} G_i$, $\beta$ is either in $G_0$ (that it, it is unattacked), or in $G_1$;
4. For every $\theta = (\theta_C, \theta_o) \in D$ such that $(N_1, ?)$ defends $\theta$ in $AF_1$, if $\theta_o \neq o_1$, then, in $AF_2$, $(N_1, o_1) \rightarrow \theta$.

Proof. Let $\beta = (\beta_C, \beta_o)$. From the definition of attack:

(i) $N_1 \beta \beta_C$, (ii) $o_1 \neq \beta_o$, and (iii) there is no $(o_C, x_o)$ such that $x_o = o_1$ and $N_1 \alpha_C \beta_C$. Consider $\eta = (\eta_C, \eta_o)$ such that $\eta$ attacks $\beta$ in $AF_1$ (if there is no such $\eta$ then the result trivially holds).

Assume by contradiction that $\eta$ is relevant to $N_1$. Then by regularity $N_1 \not\sim \eta_C$. But since $D$ is coherent and $(N_1, o_1) \notin D$, $\eta$ and $N_1$ are distinct, and thus $N_1 \not\sim \eta_C$.

As $\eta$ attacks $\beta$, $\eta_o \neq \beta_o$, but this in turn implies that $\eta_o = o_1$, since $(N_1, o_1)$ also attacks $\beta$ in $AF_2$. But then $N_1 \not\sim \eta_C \beta_C$, with $\eta_o = o_1$. This contradicts requirement 3 in the second bullet of Definition 3 of the attack between $(N_1, o_1)$ and $\beta$. Therefore, $\eta$ is not relevant to $N_1$, as we wanted to prove.

2. Trivially, true by 1 (as, if $\eta$ is an attacker $\beta$, then $N_1 \not\sim \eta$; but then $(N_1, ?) \rightarrow \eta$).
3. Trivially, true by 2.
4. Since $(N_1, ?)$ defends $\theta$ in $AF_1$, then any attacker $\eta$ of $\theta$ is irrelevant to $N_1$, and by regularity, $N_1 \not\sim \eta$. Thus requirement 3 in the second bullet of Definition 3 is satisfied. Requirement 1 is the hypothesis and requirement 2 is satisfied since $(N_1, ?)$ defends $\theta$ in $AF_1$.

Coinciding predictions. The last result (also used later in the paper) identifies a “core” in the casebase for the purposes of outcome prediction: this amounts to all past cases that are (less or equally) specific than the new case for which the prediction is sought. In other words, irrelevant cases in the casebase do not affect the prediction in regular AFs.

Lemma 9. Let $D_1$ and $D_2$ be two datasets. Let $N_C \in X$ be a characterisation, and $D_{iN_C} = \{\alpha \in D_i \mid \alpha \not\sim N_C\}$ for $i = 1, 2$. If $D_{1N_C} = D_{2N_C}$, then $AA-CBR_{D_1} \subseteq AA-CBR_{D_2}$ (that is, $AA-CBR_{D_1}$ predicts the same outcome for $N_C$ given the two datasets).

Proof. For $i = 1, 2$, let $AF_i = AF_2(D_i, N_C)$ and the grounded extensions be $G_i = \bigcup_{j \geq 0} G_{i,j}$. We will prove that $\forall j: G_{i,j} \subseteq G_{i+1}$ and $G_{i+1} \subseteq G_{i+1}$. This allows us to prove that $G_{i+1} \subseteq G_2$, which in turn implies the outcomes are the same. Here we consider only $G_{i,j} \subseteq G_{i+1}$, as the other case is entirely symmetric. By induction on $j$.

• For the base case $j = 0$: If $G_{i,0} \subseteq G_{i+1}$, we are done, since we always have that $G_{i,j} \subseteq G_{i+1}$. If not, there is an $\alpha \in G_{i,0} \backslash G_{i+1}$. Since $\alpha \in G_{i,0}$, it is relevant to $N_C$, and thus $\alpha \not\sim N_C$, which in turn implies that $\alpha \in D_{i+1}$, since $D_{iN_C} = D_{i+1}$.

On the other hand, as $\alpha \notin G_{i,0}$, there is a case $\beta \in D_{i+1}$ such that $\beta \rightarrow \alpha$. However, $\alpha \notin AF_i$, otherwise $\alpha$ would be attacked in $AF_i$ and thus not in $G_{i,0}$. But then, since $D_{iN_C} = D_{i+1}$, this means that $\beta \not\sim N_C$. Finally, this means that $(N_C, ?) \rightarrow \beta$, and thus $G_{i,j}$ defends it. Therefore, $\beta \in G_{i+1}$, what we wanted to prove.

• For the induction step, from $j$ to $j + 1$: Again, if $G_{i+1,j+1} \subseteq G_{i+1,j+1}$, we are done. If not, there is an $\alpha \in G_{i+1,j+1} \backslash G_{i+1,j+1}$ again we can check that this implies that $\alpha \in D_{i+1}$. Now, since $\alpha \in G_{i+1,j+1}$, then $G_{i+1,j+1}$ defends it. But now, by inductive hypothesis, $G_{i+1,j+1} \subseteq G_{i+1,j+1}$. Therefore, $G_{i+1,j+1}$ also defends $\alpha$, which implies that $\alpha \in G_{i+1,j+1}$, as we wanted.7 This concludes the induction.

To conclude, we can now see that $G_1 = G_2$, since, once more without loss of generality, if we consider $\alpha \in G_1$, by definition of $G_1$, there is a $j$ such that $\alpha \in G_{i,j}$. But since $G_{i,j} \subseteq G_{i+1,j+1}$, $\alpha \in G_2$. This proves that $G_1 \subseteq G_2$. The converse can be proven analogously.

4 Non-monotonicity analysis of classifiers

In this section we provide a generic analysis of the non-monotonicity properties of data-driven classifiers, using $D$, $X$ and $Y$ to denote generic inputs and outputs of classifiers, admitting our casebases, characterisations and outcomes as special instances. Later in the paper, we will apply this analysis to $AA-CBR_{\infty}$ and our modification thereof. Typically, a classifier can be understood as a function from an input set $X$ to an output set $Y$. In machine learning, classifiers are obtained by training with an initial, finite $D \subseteq (X \times Y)$, called the training set. In (any form of) $AA-CBR$, $D$ can also be seen as a training set of sorts. Thus, we will characterise a classifier as a two-argument function $C$ that maps from a dataset $D \subseteq (X \times Y)$ and from a new input $x \in X$ to a prediction $y \in Y$.

Notice that this function is total, in line with the common assumptions that classifiers generalise beyond their training dataset.

Let us model directly the relationship between the dataset $D$ and the predictions it makes via the classifier as an inference system in the following way:

Definition 10. Given a classifier $C: 2^{(X \times Y)} \times X \rightarrow Y$, let $L = L^+ \cup L^-$ be a language consisting of atoms $L^+ = X \times Y$.

7In abstract argumentation it can be verified that, if $E \subseteq \text{Args}$ defends an argument $\gamma$, and $E \subseteq E'$, then $E'$ also defends $\gamma$.

8Notice that this understanding relies upon the assumption that classifiers are deterministic. Of course this is not the case for many machine learning models, e.g., artificial neural networks trained using stochastic gradient descent and randomised hyperparameter search. This understanding is however in line with recent work using decision functions as approximations of classifiers whose output needs explaining (e.g., see (Shih, Choi, and Darwiche 2019)). Moreover, it works well when analysing $AA-CBR_{\infty}$. 
Y and negative sentences $L^- = \{ -(x,y) | (x,y) \in X \times Y \}$.

Then, $\vdash_C$ is an inference relation from $2^{L^+}$ to $L$ such that

- $D \vdash_C (x,y)$, iff $C(D,x) = y$;
- $D \vdash_C \neg(x,y)$, iff there is a $y'$ such that $C(D,x) = y'$ and $y' \neq y$.

Intuitively, $C$ defines a simple language $L$ consisting of atoms (representing labelled examples) and their negations, and $\vdash_C$ applies a sort of closed world assumption around $C$.

Then, we can study non-monotonicity properties from Section 2.2 of $\vdash_C$.

**Theorem 11.**

1. $\vdash_C$ is complete, i.e., for every $(x,y) \in (X \times Y)$, either $D \vdash_C (x,y)$ or $D \vdash_C \neg(x,y)$.
2. $\vdash_C$ is consistent, i.e., for every $(x,y) \in (X \times Y)$, it does not hold that both $D \vdash_C (x,y)$ and $D \vdash_C \neg(x,y)$.
3. $\vdash_C$ is cautiously monotonic if it satisfies cut.
4. $\vdash_C$ is cautiously monotonic if it is cumulative.
5. $\vdash_C$ is cautiously monotonic if it satisfies rational monotonicity.

**Proof.**

1. By definition of $\vdash_C$, directly from the totality of $C$.
2. By definition of $\vdash_C$, since $C$ is a function.
3. Let $\vdash_C$ be cautiously monotonic, $D \vdash_C p$ and $D \cup \{ p \} \vdash_C q$, for $p, q \in L$. By completeness, either $D \vdash_C q$ or $D \vdash_C \neg q$ (here $\neg q = r$ if $q = r$, and $\neg r$ if $r = q$). In the first case we are done. Suppose the second case holds. Since $D \vdash_C p$, by cautious monotonicity $D \cup \{ p \} \vdash_C \neg q$. But then $D \vdash_C q$ and $D \vdash_C \neg q$, which is absurd since $\vdash_C$ is consistent. Therefore $D \not\vdash_C \neg q$, and then $D \vdash_C q$. The converse can be proven analogously.
4. Trivial from 3.
5. Since $\vdash_C$ is complete, $D \not\vdash_C \neg p$ implies $D \not\vdash_C p$, and thus rational monotonicity reduces to cautious monotonicity.

---

**5 Cautious monotonicity in $AA-CBR_{\infty}$**

Our first main result is about (lack of) cautious monotonicity of the inference relation drawn from the classifier $AA-CBR_{\infty}(D, N_C)$.

**Theorem 12.** $\vdash_{AA-CBR_{\infty}}$ is not cautiously monotonic.

**Proof.** We will show a counterexample, instantiating in the following way: $X = 2^{\{a, b, c\}}$, $Y = \{ -, +\}$, and $\subseteq = \emptyset$.

Define $D = \{ (\{a\},+), (\{c\},+), (\{a,b\},+), (\{c,z\},+), (\{a,b,c\},+), (\{a,b,c\},-) \}$ and $(\delta_C, \delta_o) = (\emptyset, -)$ from which $AF_{\infty}(D)$ in Figure 3 is obtained, and two new cases: $N_1 = \{ a,b,c \}$ and $N_2 = \{ a,b,c,z \}$.

Let us now consider $AA-CBR_{\infty}(D, N_1)$ and $AA-CBR_{\infty}(D, N_2)$. We can see in Figure 4 that $D \vdash_{AA-CBR_{\infty}} (N_1, +)$ and in Figure 5 that $D \vdash_{AA-CBR_{\infty}} (N_2, -)$.

---

We could equivalently have defined $D \vdash_C \neg(x,y)$ iff $C(D,x) \neq y$. We have not done so as the used definition can be generalized for a scenario in which $C$ is not necessarily a total function. This scenario is left for future work.
simultaneously proven guilty of homicide, of defamation, but shown to have committed the homicide in self-defence ((\{hm, df, sd\}, ?))?

2. simultaneously proven guilty of homicide, of defamation, shown to have committed the homicide in self-defence, also shown to have committed defamation by a true allegation ((\{hm, df, sd, td\}, ?))?

We can map this to our counterexample in Theorem 12 by setting \( a = hm, b = sd, c = df, \) and \( z = td \). The first question is answered by the AF represented in Figure 4, with outcome +, that is, the defendant is considered guilty.

What we show in the proof of Theorem 12, given this interpretation of the counter-example, is that the answer to the second question in \( AA-CBR_\geq \) would depend on whether the case in the first question was already judged or not. If not, then the cases ((\{hm, sd\}, -) and (\{df, td\}, -) would be the nearest cases, and the outcome would be -, that is, not guilty. However, if the case in the first question was already judged and incorporated into the case law, it would serve as a counterargument for ((\{hm, sd\}, -), and guarantee that the outcome is +, that is, guilty. Intuitively this seems strange, and we focus on one reason for that: the case in the first question was judged as expected by the case law, and it may seem strange that the order in which it happens may affects the case in the second question.

The example above aims only to illustrate an interpretation in which the way \( AA-CBR_\geq \) operates does not seem appropriate. Whether this behaviour of \( AA-CBR_\geq \) in particular is desirable or not depends on other elements such as the interrelation between features (in general, for \( AA-CBR_\geq \), between the characterisations and the partial order).

### 6 A cumulative \( AA-CBR_\geq \)

We will now present \( cAA-CBR_\geq \), a novel, cumulative incarnation of \( AA-CBR \) which satisfies cautious monotonicity.

**Preliminaries.** Firstly, we present some general notions, defined in terms of the \( \vdash_\mathcal{C} \) inference relation from an arbitrary classifier \( \mathcal{C} \).

Intuitively, we are after a relation \( \vdash_\mathcal{C} \) such that if \( D \vdash_\mathcal{C} c \) and \( D \vdash_\mathcal{C} d \), then \( D \cup \{ c \} \vdash_\mathcal{C} d \) (in our concrete setting, \( \vdash_\mathcal{C} = \vdash_{AA-CBR_\geq} \) and \( \vdash_\mathcal{C} = \vdash_{cAA-CBR_\geq} \)). We also want the property that, whenever \( D \) is “well-behaved” (in a sense to be made precise later), \( D \vdash_\mathcal{C} s \iff D \vdash_\mathcal{C} s \). In this way, given that \( D \vdash_\mathcal{C} c \) and \( D \vdash_\mathcal{C} d \), then we would conclude \( D \cup \{ c \} \vdash_\mathcal{C} d \), making \( \vdash_\mathcal{C} \) a cautious monotonic relation.

We will define \( \vdash_\mathcal{C} \) by building a subset of the original dataset in such a way that cautious monotonicity is preserved. We start with the following notion of (unsurprising examples):

**Definition 14.** An example \( (x, y) \in X \times Y \) is unsurprising (or not surprising) w.r.t. \( D \) iff \( D \setminus \{ (x, y) \} \vdash_\mathcal{C} (x, y) \). Otherwise, \( (x, y) \) is called surprising.

We then define the notion of concise (subset of) the dataset, amounting to surprising cases only w.r.t. the dataset:

**Definition 15.** Let \( S \subseteq X \times Y \) be a dataset, \( S' \subseteq S \), and let \( \varphi(S') = \{ (x, y) \in S \mid (x, y) \text{ is surprising w.r.t. } S' \} \). Then \( S' \) is concise w.r.t. \( S \) whenever it is a fixed point of \( \varphi \), that is, \( \varphi(S') = S' \).

To illustrate this notion in the context of \( AA-CBR \), consider the dataset \( S \) from which the AF in Figure 6 is drawn. \( S \) is not concise w.r.t. itself, since \( \{ (a, b, c), + \} \) is unsurprising w.r.t. \( S \) (indeed, \( S \setminus \{ (a, b, c), + \} \vdash_{AA-CBR_\geq} \{ (a, b, c), + \} \), see Figure 4). Also, \( S' = S \setminus \{ (a, b), (a, b, c), + \} \) is not concise either (w.r.t. \( S \)), as \( \{ (a, b), - \} \) is surprising w.r.t. \( S' \) (the predicted outcome being +), but not an element of \( S' \). The only concise subset of \( S \) in this example is thus \( S'' = S \setminus \{ (a, b, c), + \} \).

Let us now consider \( D' \subseteq D \), for \( D \) the dataset underpinning our \( \vdash_\mathcal{C} \). If \( D' \) is concise w.r.t. \( D \), \( (x, y) \in (X \times Y) \setminus D \) is an example not in \( D \) already and \( D' \vdash_\mathcal{C} (x, y) \), then \( (x, y) \) is unsurprising w.r.t. \( D' \), and thus \( D' \) is still concise w.r.t. \( D \setminus \{ (x, y) \} \). Now, suppose that there is exactly one
such concise \( D' \subseteq D \) w.r.t. \( D \) (let us refer to this subset simply as \( \text{concise}(D) \)). Then, it seems attractive to define \( \vdash_c \), as: \( D \vdash_c (x, y) \) iff \( \text{concise}(D) \vdash_c (x, y) \). Such \( \vdash_c \) inference relation would then be cautiously monotonic if \( \text{concise}(D) = \text{concise}(D \cup \{(x, y)\}) \). This identity is indeed guaranteed given that a concise subset of \( D \) is still a concise subset of \( D \cup \{(x, y)\} \), and given our assumption that there is a unique concise subset of \( D \). In the remainder of this section we will prove uniqueness and (constructively) existence of \( \text{concise}(D) \) in the case of \( \text{AA-CA} \).

### Uniqueness of concise subsets in \( \text{AA-CA} \).

**Theorem 16.** Given a coherent dataset \( D \), if there exists a concise \( D' \subseteq D \) w.r.t. \( D \) then \( D' \) is unique.

**Proof.** By contradiction, let \( D'' \) be a concise subsets of \( D \) distinct from \( D' \). Let then \( (x, y) \in (D' \setminus D'') \cup (D'' \setminus D') \) such that \( (x, y) \) is \( \preceq \)-minimal in this set. Then the sets \{\( (x', y') \in D' \mid (x', y') \prec (x, y) \)\} and \{\( (x', y') \in D'' \mid (x', y') \prec (x, y) \)\} are equal, otherwise \((x, y)\) would not be minimal. But then, since \( D \) is coherent, by Lemma 9 we can conclude that \( D' - \{(x, y)\} \vdash_{\text{AA-CA}} (x, y) \) iff \( D'' - \{(x, y)\} \vdash_{\text{AA-CA}} (x, y) \). Thus, \((x, y)\) is surprising w.r.t. both \( D' \) and \( D'' \) w.r.t. neither. But since it is an element of one but not the other, one of them is either missing a surprising element or containing a non-surprising element. Such a set is not concise, contradicting our initial assumption. \( \square \)

### Existence of concise subsets in \( \text{AA-CA} \).

We have proven that \( \text{concise}(D) \) is unique, if it exists. Here we prove that existence is guaranteed too. We do so constructively, and by doing do we also prove that our approach is practical, giving as we so a (reasonable) algorithm that finds the concise subset of \( D \).

The main idea behind the algorithm is simple: we start with the default argument, and progressively build the argumentation framework by adding cases from \( D \) following the partial order \( \preceq \). Before adding a past case, we test whether it is surprising or not w.r.t. the dataset underpinning the current AF: if it is, then it is added; otherwise, it is not added. More specifically, the algorithm works with strata over \( D \), alongside \( \preceq \). In the simplest setting where each stratum is a singleton, the algorithm works as follows: starting with \( D_0 = \{\{\delta_c, \delta_a\}\} \) and the entire dataset \( D = \{d_i\}_{i \in \{1, \ldots, |D|\}} \) unprocessed, at each step \( i + 1 \), we obtain either \( D_{i+1} = D_i \cup \{d_{i+1}\} \), if \( d_{i+1} \) is surprising w.r.t. \( D_i \), and \( D_{i+1} = D_i \), otherwise. Then \( D = D_{|D|} \subseteq D \) is the result of the algorithm. In the general case, each example of the current stratum is tested for “surprise”, and only the surprising examples are added to \( D_i \). The procedure is formally stated in Algorithm 2, using in turn Algorithm 1. We illustrate the application of the algorithms next.

**Example 17.** Once more consider the dataset \( D = \{\{a, b\}, \{c, +\}, \{a, b, +\}, \{c, +\}, \{a, b, c, +\}\} \) in Figure 6, as well as the definitions used in that example for \( X, Y, \{\delta_c, \delta_a\} \) and \( \preceq \). Let us examine the application of Algorithm 2 to it. We start with an AF consisting only of \( \{\delta_c, \delta_a\} \), that is, \( D_0 = \emptyset \), \( AF_0 = AF_\emptyset(D_0) = \emptyset \). The first stratum would consist of \( \text{stratum}_1 = \{\{a\}, +\}, \{\{c\}, +\}\). Of course, then, we have \( \text{AA-CA-CA}_1(\{\{a\}, +\}, \{\{c\}, +\}) = \emptyset \), and similarly for \( \{\{c\}, +\} \). Thus, every argument in \( \text{stratum}_1 \) is surprising, and are thus included in the next AF, resulting in \( D_1 = \{\{a\}, +\}, \{\{c\}, +\} \) and \( AF_1 = \emptyset \).

Now, the second stratum is \( \text{stratum}_2 = \{\{a, b\}, -\}, \{\{c, z\}, -\} \). We can verify that \( \text{AA-CA-CA}_2(D_1, \{\{a, b\}, -\}) + \emptyset \) and \( \text{AA-CA-CA}_2(D_1, \{\{c, z\}, -\}) + \emptyset \). Thus \( \{\{a, b\}, -\} \) and \( \{\{c, z\}, -\} \) are both surprising, and then included in next step, that is, \( D_2 = D_1 \cup \{\{a, b\}, -\}, \{\{c, z\}, -\} \), and \( AF_2 = \emptyset \).

Finally, \( \text{stratum}_3 = \{\{a, b, c\}, +\} \). Now we verify that \( \text{AA-CA-CA}_3(D_2, \{\{a, b, c\}, +\}) + \emptyset \), which means that \( \{\{a, b, c\}, +\} \) is unsurprising. Therefore it is not added in the argumentation framework, that is, \( D_3 = D_2 \) and thus \( AF_3 = AF_2(D_3) = AF_2(D_2) = AF_2 \). Now \( unprocessed = \emptyset \), and the selected subset if \( D_3 \), corresponding to \( \text{aaF} \text{one} \text{D}_3 = \text{AF}_3 \), and we are done. We can check that using \( c\text{AA-CA-CA}_3 \) the counterexample in the proof of Theorem 12 would fail, since \( \{\{a, b, c\}, +\} \) would not have been added to the AF.

Notice that we could have defined the algorithm equivalently by looking at cases one-by-one rather than grouping them in strata. However, using strata has the advantage of allowing for parallel testing of new cases.

**Theorem 18 (Convergence).** Algorithm 2 converges.

**Proof.** Obvious, since at each iteration of the while loop, the variable \( \text{stratum} \) is assigned to a non-empty set, due to the fact that \( \text{unprocessed} \) is always a finite set, and thus there is always at least one minimal element. Thus, the cardinality of \( \text{unprocessed} \) is reduced by at least 1 at each loop iteration, which guarantees that it will eventually become empty. \( \square \)

**Theorem 19 (Correctness of Algorithm 1).** Every execution of \( \text{simple_add}(\text{Args}, \text{next_case}) \) (Algorithm 1) in Algorithm 2 correctly returns \( AF_\emptyset(\text{Args} \cup \{\text{next_case}\}) \).

**Proof (sketch).** This is essentially a consequence of Lemma 8. We know that there will never be an argument in \( \text{Args} \) with the same characterisation as \( \text{next_case} \), since they will occur in the same stratum, thus the lemma applies. The lemma guarantees that Algorithm 1 adds all attacks that need to be added and only those. Finally, we need to check that it will never be necessary to remove an attack. This is true due to the requirement 3 in the second bullet of Definition 3, and since arguments are added following the partial order. Therefore the only modifications on the set of attacks are the ones in \( \text{simple_add} \).

**Theorem 20 (Correctness of Algorithm 2).** If the input dataset is coherent, then the dataset underpinning the AF resulting from Algorithm 2 is concise.

**Proof (sketch).** In order to prove that, for the returned \( \text{Args}_\text{current} \), \( \text{Args}_\text{current} \cup \{\{\delta_c, \delta_a\}\} \) is concise, we just need to prove that at the end of each loop
Lemma 9. Their prediction is not changed, that is, they keep being surprising. The same is true for every case previously not added: adding more cases afterwards does not change their prediction. For the cases added at this new iteration, as mentioned before, the order in which the cases in the same stratum are added does not affect the outcome. Thus, each case in the same stratum can be safely tested for surprise in parallel.

c_{AA-CBR_{\omega}}. All theorems in this section so far lead to the following corollary:

**Corollary 21.** Given a coherent dataset $D$, the dataset underpinning the AF resulting from Algorithm 2 is the unique concise $D' \subseteq D$, w.r.t. $D$.

To conclude, we can then define inference in $c_{AA-CBR_{\omega}}$, the classifier yielded by the strategy described until now:

**Definition 22.** Let $D$ be a coherent dataset and let $concise(D)$ be the unique concise subset of $D$, w.r.t. $D$. Let $cAF_{\omega}(D, N_C)$ be the AF mined from $concise(D)$ and $(N_C, ?)$, with default argument $(\delta_C, \delta_o)$. Then, $c_{AA-CBR_{\omega}}$ is concise w.r.t. the set of all seen examples.

As the base case, before the loop is entered, this is clearly the case, as the only seen argument is the default.

As the induction step, we know that every case previously added is still surprising, since the new cases added are not smaller than them according to the partial order and, thus by Lemma 9 their prediction is not changed, that is, they keep being surprising. The same is true for every case previously not added: adding more cases afterwards does not change their prediction. For the cases added at this new iteration, by definition the surprising ones are added and the unsurprising ones are not. Regarding the order in which cases of the same stratum are added, each of the surprising cases will be included and the unsurprising ones will not be. It can be seen that the order is irrelevant as, since they are all $\leq$-minimal and the dataset is coherent, they are incomparable, so each case in the list is irrelevant with respect to the other. Thus, for every case seen until this point, it is in the AF if it is surprising. As this is true for every iteration, it is true for the final, returned AF.

A full complexity analysis of the algorithm is outside the scope of this paper. However, notice here that the algorithm refrains from building the AF from scratch each time a new case is considered, as seen in Theorem 19. Still regarding Algorithm 1, notice that it is easy to compute the set DEF while checking whether the next case is surprising or not, thus we could optimise its implementation with the use of caching. Besides, the subset of minimal cases (that is, the stratum) can be extracted efficiently by representing the partial order as a directed acyclic graph and traversing this graph. Finally, as mentioned before, the order in which the cases in the same stratum are added does not affect the outcome. Thus, each case in the same stratum can be safely tested for surprise in parallel.
cAA-CBR_{\geq} (D, N_C) stand for the outcome for N_C, given cAF_{\geq} (D, \bar{N}_C).

Thus, we directly obtain the inference relation ⊢_{AA-CBR_{\geq}}.

Then, cAA-CBR_{\geq} amounts to the form of AA-CBR using this inference relation. It is easy to see, in line with the discussion before Theorem 16, and using the results in Section 11, that cAA-CBR_{\geq} satisfies several non-monotonicity properties, as follows:

Theorem 23. ⊢_{cAA-CBR_{\geq}} is cautiously monotonic and also satisfies cut, cumulativity, and rational monotonicity.

7 Conclusion

In this paper we study regular AA-CBR\_{\geq} frameworks, and propose a new form of AA-CBR, denoted cAA-CBR\_{\geq}, which is cautiously monotonic, as well as, as a by-product, cumulative and rationally monotonic. Given that AA-CBR\_{\geq} admits the original AA-CBR\_{\geq} (Čyras, Satoh, and Toni 2016a) as an instance, we have (implicitly) also defined a cautiously monotonic version thereof.

(Some incarnations of) AA-CBR have been shown successful empirically in a number of settings (see (Cocarascu et al. 2020). The formal properties we have considered in this paper do not necessarily imply better empirical results at the tasks in which AA-CBR has been applied. We thus leave for future work an empirical comparison between AA-CBR\_{\geq} and cAA-CBR\_{\geq}. Other issues open for future work are comparisons w.r.t. learnability (such as model performance in the presence of noise), as well as a full complexity analysis of the new model. Also, we conjecture that the reduced size of the AF our method generates could possibly have advantages in terms of time and space complexity: we leave investigation of this issue to future work.

8 Acknowledgements

We are very grateful to Kristijonas Čyras for very valuable discussions, as well as to Alexandre Augusto Abreu Almeida, Victor Luis Barroso Nascimento and Matheus de Elias Muller for reviewing initial drafts of this paper. The first author was supported by Capes (Brazil, Ph.D. Scholarship 88881.174481/2018-01).

References

Cocarascu, O.; Stylianou, A.; Čyras, K.; and Toni, F. 2020. Data-empowered argumentation for dialectically explainable predictions. In ECAI 2020 - 24th European Conference on Artificial Intelligence, Santiago de Compostela, Spain, 10-12 June 2020.

Cocarascu, O.; Čyras, K.; and Toni, F. 2018. Explanatory predictions with artificial neural networks and argumentation. In 2nd Workshop on XAI at the 27th IJCAI and the 23rd ECAI.

Dung, P. M. 1995. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. Artificial Intelligence 77(2):321 – 357.

Dung, P. M. 2014. An axiomatic analysis of structured argumentation for prioritized default reasoning. In Schaub, T.; Friedrich, G.; and O’Sullivan, B., eds., ECAI 2014 - 21st European Conference on Artificial Intelligence, 18-22 August 2014, Prague, Czech Republic - Including Prestigious Applications of Intelligent Systems (PAIS 2014), volume 263 of Frontiers in Artificial Intelligence and Applications, 267–272. IOS Press.

Dung, P. M. 2016. An axiomatic analysis of structured argumentation with priorities. Artificial Intelligence 231:107–150.

Hunter, A. 2010. Base logics in argumentation. In Baroni, P.; Cerutti, F.; Giacomin, M.; and Simari, G. R., eds., Computational Models of Argument: Proceedings of COMMA 2010, Desenzano del Garda, Italy, September 8-10, 2010, volume 216 of Frontiers in Artificial Intelligence and Applications, 275–286. IOS Press.

Kenny, E. M., and Keane, M. T. 2019. Twin-systems to explain artificial neural networks using case-based reasoning: Comparative tests of feature-weighting methods in ANN-CBR twins for XAI. In Kraus, S., ed., Proceedings of the Twenty-Eighth International Joint Conference on Artificial Intelligence, IJCAI 2019, Macao, China, August 10-16, 2019, 2708–2715. ijcai.org.

Makinson, D. 1994. General patterns in nonmonotonic reasoning. 35–110. Oxford University Press.

Nugent, C., and Cunningham, P. 2005. A case-based explanation system for black-box systems. Artif. Intell. Rev. 24(2):163–178.

Prakken, H.; Wyner, A. Z.; Bench-Capon, T. J. M.; and Atkinson, K. 2015. A formalization of argumentation schemes for legal case-based reasoning in ASPIC+. J. Log. Comput. 25(5):1141–1166.

Shih, A.; Choi, A.; and Darwiche, A. 2019. Compiling bayesian network classifiers into decision graphs. In The Thirty-Third AAAI Conference on Artificial Intelligence, AAAI 2019, The Thirty-First Innovative Applications of Artificial Intelligence Conference, IAAI 2019, The Ninth AAAI Symposium on Educational Advances in Artificial Intelligence, EAAI 2019, Honolulu, Hawaii, USA, January 27 - February 1, 2019, 7966–7974.

Čyras, K., and Toni, F. 2015. Non-monotonic inference properties for assumption-based argumentation. In Black, E.; Modgil, S.; and Oren, N., eds., Theory and Applications of Formal Argumentation - Third International Workshop, TAFA 2015, Buenos Aires, Argentina, July 25-26, 2015, Revised Selected Papers, volume 9524 of Lecture Notes in Computer Science, 92–111. Springer.

Čyras, K., and Toni, F. 2016. Properties of ABA+ for non-monotonic reasoning. CoRR abs/1603.08714.

Čyras, K.; Birch, D.; Guo, Y.; Toni, F.; Dulay, R.; Turvey, S.; Greenberg, D.; and Hapuarachchi, T. 2019. Explanations by arbitrated argumentative dispute. Expert Syst. Appl. 127:141–156.

Čyras, K.; Satoh, K.; and Toni, F. 2016a. Abstract argumentation for case-based reasoning. In KR 2016, 549–552.
Čyrs, K.; Satoh, K.; and Toni, F. 2016b. Explanation for case-based reasoning via abstract argumentation. In Proceedings of COMMA 2016, 243–254.