Connected contribution to the kernel of the evolution equation for 3-quark Wilson loop operator

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ABSTRACT: The connected contribution to the kernel of the evolution equation for the 3-quark Wilson loop operator was derived within Balitsky high energy operator expansion. Its C-odd part was linearized and transferred to the momentum space.
1 Introduction

The Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [1]–[3] governs the energy evolution of the pomeron Green function. Pomeron is the C-even bound state of two reggeized gluons whereas its C-odd counterpart consisting of three reggeized gluons is known as odderon. The evolution equation for odderon Green function is the Bartels-Kwiecinski-Praszalowicz (BKP) equation [4]-[5]. While the next to leading order (NLO) corrections to the kernel of the BFKL equation have been known for some time [6]-[9], the nonforward NLO BKP kernel for odderon exchange has been calculated only quite recently [10]. It consists of 3 pairwise octet kernels and a connected 3 → 3 contribution.

An alternative approach to the Regge limit of high energy QCD is based on the Balitsky-Kovchegov (BK) equation [11]–[12]. Its NLO form was found in [13]-[14]. The derivation of the BK equation given by I. Balitsky in [11] through Wilson line technique does not assume reggeization. However, the linear part of the BK equation coincides with the so called Moebius form of the BFKL equation valid for scattering of colorless particles in the linear regime [15]. In NLO these kernels coincide after an equivalence transformation [16], which changes the kernel without changing the observables.

The BK Green function is a color dipole. However, in the C-odd case it is not the most general Green function since it depends only on 2 coordinates, while odderon consists of 3 reggeized gluons. The 3-quark Wilson loop (3QWL) is another colorless operator which has a baryon structure $\varepsilon^{f'h'}\varepsilon_{ijk}U_{1v}^{i}U_{2j}^{j}U_{3h'}^{h'}$. Its linear evolution equation was proved equivalent to the C-odd BKP one in [17] and its nonlinear evolution equation was derived in [18]. In the momentum representation the evolution of this operator was studied in [19], [20] and the nonlinear equation was worked out in [21].
There is a prompt question of the NLO kernel for the 3QWL operator. In this paper
the connected contribution to such a kernel has been calculated within Balitsky high energy
operator expansion [14]. The linear part of this contribution for the C-odd case was trans-
ferred to the momentum representation and found to be different from the connected 3 → 3
kernel of [10]. It indicates that there should be an equivalence transformation connecting
the kernels.

In this paper the dimension of the space-time is kept equal to 4 since the connected
part of the NLO kernel does not contain the UV divergencies and the sum of the diagrams
is IR stable because the 3QWL is a gauge invariant colorless operator.

The paper is organized as follows. The next section contains the definitions and the
derivation of the leading order (LO) evolution equation for the 3QWL operator. Section
3 deals with the connected contribution proportional to the second iteration of the LO
kernel. Section 4 presents the calculation of the connected contribution with 2 gluon
intersections of the shockwave. Section 5 comprises the calculation of the diagrams with 1
gluon intersection of the shockwave. Section 6 gives the Furier transform of the linearized
result for the C-odd case. The last section concludes the paper.

2 Definitions and necessary results

We introduce the light cone vectors $n_1$ and $n_2$

\[ n_1 = (1, 0, 0, 1), \quad n_2 = \frac{1}{2} (1, 0, 0, -1), \quad n_1^+ = n_2^- = n_1 n_2 = 1, \]

(2.1)

and for any vector $p$ we have

\[ p^+ = p_\perp = m_2 = \frac{1}{2} (p^0 + p^3), \quad p_+ = p_- = m_1 = p^0 - p^3, \]

(2.2)

\[ p = p_1 + p_2 + p_\perp, \quad p^2 = 2 p^+ p^- - p_\perp^2, \]

(2.3)

\[ p k = p^\mu k_\mu = p_+ k_+ + p^- k_- - \vec{p} \vec{k} = p_+ k_+ + p_- k_- - \vec{p} \vec{k}. \]

(2.4)

We work in the light-cone gauge $A_{\mu} n_2 = 0$ and in our convention the 3-gluon interaction
Lagrangian has the form

\[ L_i = - g f^{abc} (\partial_\mu A_\nu^a) A_\mu^b A_\nu^c. \]

(2.5)

We would like to calculate the connected part of the kernel for the evolution equation for
the 3-quark Wilson loop operator

\[ B_{123}^\eta = \epsilon^{ijh} \epsilon_{ijh} U (\vec{z}_1, \eta)_{i}^1 U (\vec{z}_2, \eta)_{j}^2 U (\vec{z}_3, \eta)_{h}^3 \]

(2.6)

contributing to the evolution of a baryon Green function. Hereafter we will use the following
shorthand notation for such convolutions

\[ \epsilon^{ijh} \epsilon_{ijh} U (\vec{z}_1, \eta)_{i}^1 U (\vec{z}_2, \eta)_{j}^2 U (\vec{z}_3, \eta)_{h}^3 = U_1 \cdot U_2 \cdot U_3, \]

(2.7)

where

\[ U (\vec{z}, \eta) = P e^{ig \int_{-\infty}^{+\infty} b_\eta (z^+, \vec{z}) dz^+} \]

(2.8)
is the Wilson line with the path along the \( z^- = 0 \) line and \( b_\eta^- \) is the external shock-wave field built from only slow gluons

\[
b_\eta^- = \int \frac{d^4p}{(2\pi)^4} e^{-ipz^-} (p) \theta(e^\eta - |p^+|).
\]  

(2.9)

The parameter \( \eta \) separates the slow gluons entering the Wilson lines from the fast ones in the impact factors. The shape of the path at \( z^+ = \pm \infty \) in (2.8) is not important because the field is concentrated at \( z^+ = 0 \). The gluon field consists of the fast component \( A \) with the rapidities greater than \( \eta \) and the slow one \( b_\eta^- \)

\[A = A + b, \quad b^\mu (z) = b^- (z^+, \vec{z}) n_2^\mu = \delta (z^+) b (\vec{z}) n_2^\mu.\]  

(2.10)

To derive the evolution equation one has to calculate the operator \( B_{123}^\eta \) in the shockwave background \( \langle \rangle \), i.e. integrate it over the gluons with \( \sigma = e^\eta > p^+ > \sigma_1 \), where \( \sigma_1 \ll \sigma \) is the lower cutoff set by the target \( \langle \rangle \).

\[
\langle B_{123}^\eta \rangle = \frac{\langle 0 | T(B_{123}^\eta e^{i \int L(z) dz}) | 0 \rangle}{\langle 0 | T(e^{i \int L(z) dz}) | 0 \rangle}.
\]  

(2.11)

To this end we need the gluon propagator in the light cone gauge. The free gluon propagator reads

\[G_{0 \mu \nu}^- (p) = - \frac{i d^{\mu \nu} (p)}{p^2 + i0},\]  

(2.12)

with

\[d^{\mu \nu} (p) = g^{\mu \nu} - \frac{p^\mu n_2^\nu + p^\nu n_2^\mu}{pn_2} = g^{\mu \nu}_\perp - \frac{p^\mu n_2^\nu + p^\nu n_2^\mu}{p^+} - 2 \frac{n_2^\mu n_2^\nu p^-}{p^+},\]  

(2.13)

Then

\[G_0^- (p^+, x^+, \vec{x}) = - \frac{i x_+^j e^{i \int (x^2 + \omega) p^+}}{4\pi (x^2)^{3/2}} \left( \theta (x^+) \theta (p^+) - \theta (-x^+) \theta (-p^+) \right),\]  

(2.14)

\[G_0^- (p^+, x^+, \vec{x}) = \int \frac{d^3p^-}{(2\pi)^3} e^{-ip^+ x^+ + i \vec{p} \cdot \vec{x}} \frac{2ip^- \theta (x^+) \theta (p^+)}{p^+ (p^2 + i0)}.\]  

(2.15)

One can take this integral explicitly. However, it is not convenient for us since it introduces \( \frac{1}{(p^+)^3} \) singularity. Therefore we use (2.15) for \( G_0^- \) and integrate with respect to \( x^+ \) first. For the calculation we need the following integral with \( G_0^- \)

\[
\int_0^\infty \! dz^+ \int \frac{d^3p^-}{(2\pi)^3} e^{-i(p^- - i\epsilon)z^+ + ip^- x^+ + i \vec{p} \cdot \vec{x}} \frac{2ip^- \theta (x^+) \theta (p^+)}{p^+ (p^2 + i0)}
\]

\[
= \int \frac{d^3p^-}{(2\pi)^3} e^{ip^- x^+ + i \vec{p} \cdot \vec{x}} 2p^- \theta (x^+) \theta (p^+) \frac{p^-}{p^+ (p^2 + i0)} = 0.
\]  

(2.16)

The propagator in the shock-wave background field has the following two convenient representations which we use in this paper

\[G^{ab}_{\mu \nu} (x, y) |_{x^+ > 0 > y^+} = - \int \frac{\theta (p^+)}{(2\pi)^3} \frac{p^+}{2 x^+ y^+} \int d\vec{z} \frac{1}{e^{ip^+ x^+ + i \vec{p} \cdot \vec{x}} + \frac{(x^2 + \omega)}{2 x^+} - \frac{(x^2 + \omega)}{2 \epsilon}}.\]
The real contribution from the interaction of $\mathcal{U}$ take the integrals with respect to $z$.

When we integrate (2.15) with respect to $p$ we have

$$G_{\mu\nu}(x, y)|_{x^+ > 0, y^+} = \frac{\theta(p^+)}{(2\pi)^2} \frac{dp^+}{2p^+} \int dz e^{-ip^+ \{x^--y^--(\xi^2+10)/2x^+\}}$$

$$\times g_{\mu\nu}(x, y)|_{x^+ > 0, y^+} = \frac{\theta(p^+)}{(2\pi)^2} \frac{dp^+}{2p^+} \int dz e^{-ip^+ \{x^--y^--(\xi^2+10)/2x^+\}}$$

(2.17)

Therefore one can introduce these imaginary parts into the integral for $2^{[1}] - \pi g_\alpha(\perp G^2)\left[\int_0^1 \sigma_1 \alpha \delta_1 \right] \cdot \hat{B} \cdot \eta_1 p^+ - n_{2\mu} k_{\perp \alpha}$. 

(2.18)

The operator $\langle B^\alpha_{123} \rangle$ in the shockwave background has virtual the $B_v$ and the real $B_r$ contributions

$$\langle B^\alpha_{123} \rangle = B_v + B_r. \quad (2.19)$$

One can find the real contribution using (2.17) and integrating with respect to $z_1^+$ and $z_2^+$. The real contribution from the interaction of $U_1$ and $U_2$ with the shockwave reads

$$B_{r12} = -g^2(U_1 t^a) \cdot (U_2 t^a) \cdot U_3 \int_{-\infty}^{0} dz_1^+ \int_{0}^{\infty} dz_2^+ G^{-}(z_2, z_1)$$

$$-g^2(U_1 t^a) \cdot (U_2 t^a) \cdot U_3 \int_{-\infty}^{0} dz_1^+ \int_{0}^{\infty} dz_2^+ G^{-}(z_1, z_2)$$

$$= \frac{\alpha_s}{\pi^2} \int_{\sigma_1}^{\sigma} \frac{dp^+}{p^+} \int \frac{dz_1}{z_1^2} U^{t^a_1 t^a_2} U_0 \left( (U_1 t^a) \cdot (U_2 t^a) \cdot U_0 \right) . \quad (2.20)$$

Similarly, the real contribution from the interaction of $U_1$ and the shockwave reads

$$B_{r1} = -g^2(U_1 t^a) \cdot (t^a U_2) \cdot U_3 \int_{-\infty}^{0} dz_1^+ \int_{0}^{\infty} dz_2^+ G^{-}(z_1, z_1)$$

$$= \frac{\alpha_s}{\pi^2} \int_{\sigma_1}^{\sigma} \frac{dp^+}{p^+} \int \frac{dz_1}{z_1^2} U^{t^a_1 t^a_2} (U_1 t^a) \cdot U_2 . \quad (2.21)$$

The virtual contribution from the $U_1$ and $U_2$ interaction reads

$$B_{v12} = -g^2(U_1 t^a) \cdot (U_2 t^a) \cdot U_3 \int_{-\infty}^{0} dz_1^+ \int_{-\infty}^{0} dz_2^+ G_0^{-}(z_2, z_1)$$

$$-g^2(U_1 t^a) \cdot (U_2 t^a) \cdot U_3 \int_{0}^{\infty} dz_2^+ \int_{0}^{\infty} dz_1^+ G_0^{-}(z_1, z_2)$$

$$= -g^2 \left( (U_1 t^a) \cdot (U_2 t^a) \cdot U_3 \int_{-\infty}^{0} dz_1^+ \int_{-\infty}^{0} dz_2^+ + (t^a U_1) \cdot (t^a U_2) \cdot U_3 \int_{0}^{\infty} dz_2^+ \int_{0}^{\infty} dz_1^+ \right)$$

$$\times \int \frac{d^4 p}{(2\pi)^4} e^{-ip^+ z_1^+ + p\cdot z_2^+} \frac{2ip^-}{p^+ (p^2 + i0)}. \quad (2.22)$$

When we integrate (2.15) with respect to $p^-$ via residues, we see that $p^-$ has a tiny positive imaginary part $p^- \rightarrow p^- + i\varepsilon$ if $p^+ < 0$ and $x^+ < 0$ whereas $p^- \rightarrow p^- - i\varepsilon$ if $p^+ > 0$ and $x^+ > 0$. Therefore one can introduce these imaginary parts into the integral for $B_v$ and take the integrals with respect to $z_1^+$ and $z_2^+$ first. We have

$$B_{v12} = -g^2 \int \frac{d^4 p}{(2\pi)^4} \left( (U_1 t^a) \cdot (U_2 t^a) \cdot U_3 \int_{-\infty}^{0} e^{-i(p^-+i\varepsilon)z_1^+} dz_1^+ \int_{-\infty}^{0} e^{i(p^-+i\varepsilon)z_2^+} dz_2^+ \right).$$

(2.23)
Integrating its explicit form reads
\[
(t^a U_1) \cdot (t^a U_2) \cdot U_3 \int_0^\infty \int_0^\infty e^{i(p^- + i\varepsilon)z_1^+} e^{-i(p^- - i\varepsilon)z_1'^+} d\bar{z}_1^+ d\bar{z}_1'^+(2\pi)^3 \int_0^\infty \int_0^\infty e^{-i(p^- - i\varepsilon)z_2^+} dz_2^+ dz_2'^+(2\pi)^3 \int_0^\infty e^{i(p^- + i\varepsilon)z_2'^+} dz_2'^+.
\]

Then we can use the SU(3) identity
\[
\eta U \frac{\partial}{\partial \eta} \langle B_{123}^9 \rangle = \langle K \otimes B_{123}^9 \rangle.
\]

Quite similarly, the virtual contribution from the interaction of $U_1$ reads
\[
B_{v12} = -g^2 \int \frac{d^4p}{(2\pi)^3} \eta \left[ (t^a U_1) \cdot (t^a U_2) + (U_1 t^a) \cdot (U_2 t^a) \right] \cdot U_3 \int_{\sigma_1} \int_{\sigma_2} \frac{dp^+}{\sqrt{p^+}} \int_{\sigma_1} \int_{\sigma_2} \frac{dp'^+}{\sqrt{p'^+}} \cdot \frac{dz_4}{z_1' z_2' z_3' z_4'}
\]

Collecting all the contributions and differentiating with respect to $\eta = \ln \sigma$, one gets the equation
\[
\frac{\partial}{\partial \eta} \langle B_{123}^9 \rangle = \langle K \otimes B_{123}^9 \rangle.
\]

Its explicit form reads
\[
\frac{\partial}{\partial \eta} \langle B_{123}^9 \rangle = \frac{\alpha_s}{\pi^2} \int d\bar{z}_4 \left[ \left( \frac{\bar{z}_1 \bar{z}_2}{z_1' z_2'} \right) U_3 \cdot \left\{ U_4 \left( t^b U_1 \cdot (U_2 t^a) + (U_1 t^a) \cdot (t^b U_2) \right) \right. \right.
\]
\[
- \left. \left( (t^a U_1) \cdot (t^a U_2) + (U_1 t^a) \cdot (U_2 t^a) \right) \right\} \right\} + \left( 1 \leftrightarrow 3 \right) + \left( 2 \leftrightarrow 3 \right)
\]
\[
+ \left( \frac{1}{z_4^2} \right) \left( U_4 \left( t^b U_1 t^a \right) - \frac{1}{2} (t^a t^a U_1) - \frac{1}{2} (U_1 t^a t^a) \right) \cdot U_2 \cdot U_3 + \left( 1 \leftrightarrow 2 \right) + \left( 1 \leftrightarrow 3 \right)
\]

Then we can use the SU(3) identity
\[
(U_2 U_1^I U_1 + U_1 U_1^I U_2) \cdot U_4 \cdot U_3 = -B_{123}^9 + \frac{1}{2} (B_{144}^9 B_{324}^9 + B_{244}^9 B_{314}^9 - B_{344}^9 B_{214}^9),
\]
and
\[ U_4^{ij} = 2tr(t^b U_4^i t^a U_4^j), \quad t^a_{ij} t^a_{kl} = \frac{1}{2} \delta_{il} \delta_{kj} - \frac{1}{2N_c} \delta_{ij} \delta_{kl} \] (2.31)

to rearrange this expression in the following way
\[
\frac{\partial B_{123}^\eta}{\partial \eta} = \frac{\alpha_s}{4\pi^2} \int dz_4 \left[ \frac{z_4^2}{z_2^2 z_2^2} (-B_{123}^\eta + \frac{1}{6} (B_{144}^\eta B_{324}^\eta + B_{244}^\eta B_{314}^\eta - B_{344}^\eta B_{214}^\eta)) \right. \\
left. + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \right],
\] (2.32)
where we dropped the angular brackets for brevity.

3 Diagrams proportional to the LO2 kernel

![Diagram](image)

**Figure 1.** Diagrams contributing to LO2 kernel. The dashed lines represent the Wilson lines with \( z^- = 0 \) and \( \vec{z}_{1,2,3} \) along the \( z^+ \) axis from \( z^+ = -\infty \) to the left to \( z^+ = +\infty \) to the right. The grey ellipse stands for the shockwave at \( z^+ = 0 \).

The connected part of the NLO kernel comes from the diagrams where all the three Wilson lines have nontrivial evolution. The first group of such diagrams is depicted in fig 1. These diagrams and the diagrams which they come into after the reflection with respect to the shock wave and after all possible permutations of \( U_1, U_2, U_3 \) can be totally reduced to the second iteration of the LO kernel. Indeed, the first diagram reads
\[
\langle B_{123}^\eta \rangle |_1 = g^4 (U_1 t^a) \cdot (t^{b'} U_2 t^b) \cdot (U_3 t^a) \\
\times \int_{-\infty}^{0} dz_1^+ \int_{-\infty}^{0} dz_3^+ \int_{-\infty}^{0} dz_2^+ \int_{0}^{\sigma} dz^+_4 G^{--}(z_2, z_2') \cdot U^{b'} b^4 \int d\vec{z}_4 \cdot \frac{z_4^2}{z_4^2}.
\] (3.1)

Hence
\[
\frac{\partial}{\partial \eta} \langle B_{123}^\eta \rangle |_1 = -2 \ln \frac{\sigma^2}{\sigma_1^2} (U_1 t^a) \cdot (t^{b'} U_2 t^b) \cdot (U_3 t^a) \int \frac{dz^+_4}{z^+_4} U^{b'} b^4 \int d\vec{z}_4 \cdot \frac{z_4^2}{z_4^2}.
\] (3.2)
Here we used expressions (2.21) and (2.26) from the previous section. In the NLO equation (2.28) changes into

$$\frac{\partial}{\partial \eta} \langle B_{123}^{\eta} \rangle = \langle K_{LO} \otimes B_{123}^{\eta} \rangle + \langle K_{NLO} \otimes B_{123}^{\eta} \rangle.$$  

(3.3)

Therefore

$$\frac{\partial}{\partial \eta} \langle B_{123}^{\eta} \rangle - \langle K_{LO} \otimes B_{123}^{\eta} \rangle = \langle K_{NLO} \otimes B_{123}^{\eta} \rangle.$$  

(3.4)

One can obtain $\langle K_{LO} \otimes B_{123}^{\eta} \rangle$ applying the LO evolution to the Wilson lines in the r.h.s. of LO evolution equation (2.29). Among others, it contains these 2 terms

$$\frac{\alpha_s}{\pi^2} \int \frac{d\vec{z}_4}{z_4^2} \left[ U_4^{ba} U_1 \cdot (t^b U_2 t^a) \cdot U_3 - \frac{(z_{41} z_{43})}{z_{41}^2 z_{43}^2} \frac{1}{z_{02}^2} \frac{1}{z_{04}^2} \frac{1}{z_{02}^2} \frac{1}{z_{04}^2} \frac{1}{z_{02}^2} \frac{1}{z_{04}^2} \frac{1}{z_{02}^2} \frac{1}{z_{04}^2} \right].$$  

(3.5)

If we calculate these terms in the shock wave background in the LO, we will have among others the contribution where we dress the Wilson lines $U_1$ and $U_3$ from the first term and $U_2$ from the second term

$$\langle K_{LO} \otimes B_{123}^{\eta} \rangle |_{1} = \frac{\alpha_s^2}{\pi^2} \int_{1}^{\sigma} \frac{dp^+}{p^+} \int d\vec{z}_0 \int d\vec{z}_4 \times \left[ -\frac{1}{z_{42}^2} \frac{(z_{01} z_{03})}{z_{01}^2 z_{03}^2} U_4^{ba} (U_1 t^c) \cdot (t^b U_2 t^a) \cdot (U_3 t^a) - \frac{(z_{41} z_{43})}{z_{41}^2 z_{43}^2} \frac{1}{z_{02}^2} \frac{1}{z_{04}^2} \frac{1}{z_{02}^2} \frac{1}{z_{04}^2} \frac{1}{z_{02}^2} \frac{1}{z_{04}^2} \frac{1}{z_{02}^2} \frac{1}{z_{04}^2} \right]$$

$$= \frac{\partial}{\partial \eta} \langle B_{123}^{\eta} \rangle |_{1}.$$  

(3.6)

As a result, this diagram does not contribute to the NLO kernel. The same is true for all the diagrams in fig. 1.

4 Diagrams with 2 gluons intersecting the shockwave

![Diagram](image-url)
Next we consider the diagrams with two gluons intersecting the shockwave depicted in fig 2. Diagram 7 reads (here $z_2' = (z_{21}^+, 0, \vec{z}_2^2)$)

$$
\langle B_{123}^\eta \rangle_7 = g^4 (U_1 t^a) \cdot (t^{b'} t^{d'} U_2) \cdot (U_3 t^b)
$$

$$
\times \int_{-\infty}^0 d\zeta_1^+ \int_{-\infty}^0 d\zeta_3^+ \int_0^{\infty} d\zeta_2^+ \int_{z_2}^{\infty} d\zeta_2'^+ \, G^{--} \langle (z_2, z_1) t^a G^{--} (z_2', z_3) t^{b'} \rangle
$$

$$
= g^4 (U_1 t^a) \cdot (t^{b'} t^{d'} U_2) \cdot (U_3 t^b) \int_{\sigma_1} \int_{\sigma_1} \frac{2 p^+ dp^+}{(2\pi)^2} \int_{\sigma_1} \frac{2 k^+ dk^+}{(2\pi)^2} \int \langle \vec{z}_{20} \vec{z}_{10} \rangle \, U_0^{a' d} d\zeta_0
$$

$$
\times \int \langle \vec{z}_{24} \vec{z}_{34} \rangle U_4^{b' \vec{d}} d\zeta_4 \int_{-\infty}^0 \frac{d\zeta_1^+}{2(z_1^+)^2} \int_{-\infty}^0 \frac{d\zeta_3^+}{2(z_3^+)^2} \int_{z_2}^{\infty} \frac{d\zeta_2'^+}{2(z_2'^+)^2} e^{-i p^+ \frac{z_0 t^a}{2 z_1^+}} \int_{-\infty}^0 \frac{d\zeta_1^+}{2(z_1^+)^2} \int_{-\infty}^0 \frac{d\zeta_3^+}{2(z_3^+)^2} \int_{z_2}^{\infty} \frac{d\zeta_2'^+}{2(z_2'^+)^2} e^{-i k^+ \frac{z_0 t^a}{2 z_3^+}}.
$$

$$
(4.1)
$$

$$
\langle B_{123}^\eta \rangle_7 = 4 g^4 (U_1 t^a) \cdot (t^{b'} t^{d'} U_2) \cdot (U_3 t^b) \int \frac{\langle \vec{z}_{20} \vec{z}_{10} \rangle}{z_0^2 z_{01}^2} U_0^{a' d} \int \frac{\langle \vec{z}_{24} \vec{z}_{34} \rangle}{z_4^2 z_{42}^2} U_4^{b' \vec{d}} d\zeta_0 d\zeta_4
$$

$$
\times \int_{\sigma_1} \frac{dp^+}{p^+} \int_{\sigma_1} \frac{dk^+}{k^+ + z_{24}^2 + z_{02}^2 p^+}.
$$

$$
(4.2)
$$

The corresponding term in $\langle K_{LO} \otimes B_{123}^\eta \rangle$ comes from the following term in LO evolution equation (2.29)

$$
\alpha_s \int \frac{d\vec{z}_1}{4 \pi^2} \frac{\langle \vec{z}_{43} \vec{z}_{32} \rangle}{z_3^2 z_{24}^2} U_4^{a' d} U_{2a'} (U_3 t^a) \cdot (t^{b'} U_2) \cdot U_1.
$$

$$
(4.3)
$$

If we dress $U_1$ and $U_2$ in this expression, we get one of the contributions which acts as a subtraction term for diagram 7

$$
\langle K_{LO} \otimes B_{123}^\eta \rangle_7 = \frac{\alpha_s^2}{4 \pi^2} \int_{\sigma_1} \frac{dp^+}{p^+} \int \frac{d\zeta_0}{\zeta_0^2} \int d\zeta_4 \frac{\langle \vec{z}_{20} \vec{z}_{10} \rangle}{z_0^2 z_{01}^2} U_0^{a' d} \int d\zeta_4 \frac{\langle \vec{z}_{24} \vec{z}_{34} \rangle}{z_4^2 z_{42}^2} U_4^{b' \vec{d}} (U_1 t^a) \cdot (t^{b'} U_2) \cdot (U_3 t^a).
$$

$$
(4.4)
$$

Then

$$
\langle K_{NLO} \otimes B_{123}^\eta \rangle_7 = \frac{\partial}{\partial \eta} \langle B_{123}^\eta \rangle_7 - \langle K_{LO} \otimes B_{123}^\eta \rangle_7
$$

$$
= \frac{\alpha_s^2}{4 \pi^2} (U_1 t^a) \cdot (t^{b'} U_2) \cdot (U_3 t^b) \int d\zeta_0 \frac{\langle \vec{z}_{20} \vec{z}_{10} \rangle}{z_0^2 z_{01}^2} U_0^{a' d} \int d\zeta_4 \frac{\langle \vec{z}_{24} \vec{z}_{34} \rangle}{z_4^2 z_{42}^2} U_4^{b' \vec{d}} \ln \frac{\zeta_4^2}{\zeta_0^2}.
$$

$$
(4.5)
$$

The contribution of diagram 7 with the interchange of 1st and 3rd Wilson lines can be obtained via the interchange 1 $\leftrightarrow$ 3 in this result. For the sum of these diagrams one gets

$$
\langle K_{NLO} \otimes B_{123}^\eta \rangle_7_{7(1\leftrightarrow3)}
$$

$$
= \frac{\alpha_s^2}{4 \pi^2} i f^{a' b' c'} (U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^c) \int d\zeta_0 \frac{\langle \vec{z}_{20} \vec{z}_{10} \rangle}{z_0^2 z_{01}^2} U_0^{a' d} \int d\zeta_4 \frac{\langle \vec{z}_{24} \vec{z}_{34} \rangle}{z_4^2 z_{42}^2} U_4^{b' \vec{d}} \ln \frac{\zeta_4^2}{\zeta_0^2}.
$$

$$
(4.6)
$$

The contribution of the diagram which is a mirror reflection of diagram 7 with respect to the shockwave reads

$$
\langle B_{123}^\eta \rangle_7 m = g^4 (t^{a'} U_1) \cdot (U_2 t^a t^b) \cdot (t^{b'} U_3)
$$

- 8 -
\[ \times \int_{-\infty}^{\infty} d\vec{z}_1^+ \int_{0}^{\infty} d\vec{z}_3^+ \int_{-\infty}^{0} d\vec{z}_2^+ \int_{0}^{\infty} d\vec{z}_2^- G^{--}(z_1, z_2)^a a G^{--}(z_1, z_2)^b b. \] (4.7)

Its integrated form differs from the contribution of diagram 7 only in the $t$ matrix order. One gets

\[ \langle K_{NLO} \otimes B_{123}^\eta \rangle_{7m} = \frac{\alpha_s^2}{\pi^4} (t^U_1) \cdot (U_2 t^{a b}) \cdot (t^{b'} U_3) \int d\vec{z}_0 (\frac{\vec{z}_2}{\vec{z}_0^2}) U_{0}^{a a} \int d\vec{z}_4 (\frac{\vec{z}_2}{\vec{z}_4^2}) U_{4}^{b b} \ln \frac{\vec{z}_2^2}{\vec{z}_0^2}. \] (4.8)

The corresponding diagram with the interchange of 1st and 3rd Wilson lines appears from this expression after the substitution $(U_2 t^{a b}) \rightarrow -(U_2 t^{b a})$. The sum of these diagrams is

\[ \langle K_{NLO} \otimes B_{123}^\eta \rangle_{7m+7m(1\leftrightarrow 3)} = \frac{\alpha_s^2}{\pi^4} i f^{abc} (t^U_1) \cdot (U_2 t^{c b}) \cdot (t^{b'} U_3) \int d\vec{z}_0 (\frac{\vec{z}_2}{\vec{z}_0^2}) U_{0}^{a a} \int d\vec{z}_4 (\frac{\vec{z}_2}{\vec{z}_4^2}) U_{4}^{b b} \ln \frac{\vec{z}_2^2}{\vec{z}_0^2}. \] (4.9)

Finally, the contribution of the four diagrams: 7, 7(1 $\leftrightarrow$ 3) and their mirror reflections with respect to the shockwave has the form

\[ \langle K_{NLO} \otimes B_{123}^\eta \rangle_{7+7(1\leftrightarrow 3)+7m+7m(1\leftrightarrow 3)m} = \frac{\alpha_s^2}{\pi^4} \int d\vec{z}_0 (\frac{\vec{z}_2}{\vec{z}_0^2}) U_{0}^{a a} \int d\vec{z}_4 (\frac{\vec{z}_2}{\vec{z}_4^2}) U_{4}^{b b} \ln \frac{\vec{z}_2^2}{\vec{z}_0^2} \times i \left\{ f^{abc} (U_1 t^c) \cdot (t^{b} U_2) \cdot (U_3 t^c) - f^{abc} (t^U_1) \cdot (U_2 t^{b}) \cdot (t^{c'} U_3) \right\}. \] (4.10)

Let us turn to diagram 8. It reads

\[ \langle B_{123}^\eta \rangle_8 = -g^4 f^{a b' c'} (U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^c) \int_{-\infty}^{\infty} d\vec{z}_1^+ \int_{-\infty}^{0} d\vec{z}_3^+ \int_{0}^{\infty} d\vec{z}_2^+ \int_{-\infty}^{\infty} \theta(x^+) d^4x \times \left\{ \frac{\partial G^{a a}(x, z_1)}{\partial x^\mu} - \frac{G^{b b'}(z_2, x)^{a^\mu} G^{c c}(x, z_3)^{j^\mu} - G^{b b'}(z_2, x)^{a^\mu} G^{c c}(x, z_3)^{j^\mu}}{\theta(x^+) d^4x} \right\}. \] (4.11)

Here we sum over $j = 1, 2$ and $\mu = -, 1, 2$. It is convenient to split this expression into two parts.

\[ \langle B_{123}^\eta \rangle_8 = \langle B_{123}^\eta \rangle_{8_1} + \langle B_{123}^\eta \rangle_{8_2}. \] (4.12)

\[ \langle B_{123}^\eta \rangle_{8_1} = -g^4 f^{a b' c'} (U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^c) \int_{-\infty}^{0} d\vec{z}_1^+ \int_{-\infty}^{0} d\vec{z}_3^+ \int_{0}^{\infty} d\vec{z}_2^+ \int_{-\infty}^{\infty} \theta(x^+) d^4x \times G^{b b'}(z_2, x)^{a^\mu} - \frac{\partial G^{a a}(x, z_1)}{\partial x^\mu} - \frac{G^{b b'}(z_2, x)^{a^\mu} - G^{a a}(x, z_1)^{a^\mu} G^{b b'}(z_2, x)^{j^\mu}}{\theta(x^+) d^4x} \right\}. \] (4.13)

\[ \langle B_{123}^\eta \rangle_{8_1} = -g^4 f^{a b' c'} (U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^c) \int_{-\infty}^{0} d\vec{z}_1^+ \int_{-\infty}^{0} d\vec{z}_3^+ \int_{0}^{\infty} d\vec{z}_2^+ \int_{-\infty}^{\infty} \theta(x^+) d^4x \]
\[\int_{\sigma}^{\bar{\sigma}} \frac{dp^+}{2\pi} e^{ix-p^+} \int \frac{dp^-}{(2\pi)^3} \frac{2ip^- e^{-i(p^- -ix)z_{12}^+ + ip^- x^+ + i\hat{p}\hat{z}_{12}^\star}}{p^+(p^2 + i0)} \{ \ldots \} = 0. \quad (4.14)\]

Here as in the LO calculation, we changed
\[e^{-ip^- z_{12}^+} \to e^{-i(p^- -ix)z_{12}^+ + ip^- x^+} \quad (4.15)\]

and used (2.16) to get zero. Therefore
\[
\langle B_1^{q1}\rangle |_{8} = \langle B_1^{q1}\rangle |_{82} = g^4 f^{a'b'c'}(U_1^a) \cdot (t^bU_2) \cdot (U_3c')
\]

\[\times \int_{\sigma_1}^{\bar{\sigma}} \frac{dp^+}{p^+(2\pi)^2} \int_{\sigma_1}^{\bar{\sigma}} \frac{dk^+}{(2\pi)^3} \frac{dq^+}{(2\pi)^3} \int d\vec{x} e^{ix-p^+} e^{i\hat{p}\hat{z}_{12}^\star} \frac{1}{z_{12}^2 + k^+ z_{12}^2}\]

\[\times \left[ p^+ \left[ (\bar{z}_{10}\bar{z}_{20}) (\bar{z}_{34}\bar{z}_{42}) - (\bar{z}_{20}\bar{z}_{24}) (\bar{z}_{10}\bar{z}_{34}) \right] + k^+ \left[ - (\bar{z}_{20}\bar{z}_{34}) (\bar{z}_{10}\bar{z}_{24}) + (\bar{z}_{24}\bar{z}_{20}) (\bar{z}_{10}\bar{z}_{34}) \right] \right]. \quad (4.16)\]

Then integrating with respect to \( z_{12}^+ \) and \( x^+ \) we arrive to
\[\langle B_1^{q1}\rangle |_{8} = -4g^4 i f^{a'b'c'}(U_1^a) \cdot (t^bU_2) \cdot (U_3c') \int \frac{dp^+}{p^+(2\pi)^2} \int_{\sigma_1}^{\bar{\sigma}} \frac{dk^+}{(2\pi)^3} \frac{dq^+}{(2\pi)^3} \int d\vec{x} e^{ix-p^+} e^{i\hat{p}\hat{z}_{12}^\star} \frac{1}{z_{12}^2 + k^+ z_{12}^2}\]

\[\times \left[ p^+ \left[ (\bar{z}_{10}\bar{z}_{20}) (\bar{z}_{34}\bar{z}_{42}) - (\bar{z}_{20}\bar{z}_{24}) (\bar{z}_{10}\bar{z}_{34}) \right] + k^+ \left[ - (\bar{z}_{20}\bar{z}_{34}) (\bar{z}_{10}\bar{z}_{24}) + (\bar{z}_{24}\bar{z}_{20}) (\bar{z}_{10}\bar{z}_{34}) \right] \right]. \quad (4.17)\]

The integral with respect to \( \vec{x} \) can be calculated by the Feynman parameter technique. As a result
\[\langle B_1^{q1}\rangle |_{8} = -2g^4 i f^{a'b'c'}(U_1^a) \cdot (t^bU_2) \cdot (U_3c') \int \frac{dp^+}{p^+(2\pi)^2} \int_{\sigma_1}^{\bar{\sigma}} \frac{dk^+}{(2\pi)^3} \frac{dq^+}{(2\pi)^3} \int d\vec{x} e^{ix-p^+} e^{i\hat{p}\hat{z}_{12}^\star} \frac{1}{z_{12}^2 + k^+ z_{12}^2}\]

\[\times \left[ \frac{2}{z_{12}^2 + k^+ z_{12}^2} \left[ (\bar{z}_{10}\bar{z}_{34}) (\bar{z}_{20}^2 - \bar{z}_{24}^2) + (\bar{z}_{10}\bar{z}_{24}) (\bar{z}_{24}^2 - \bar{z}_{20}^2) \right] \right]. \quad (4.18)\]

Therefore
\[\frac{\partial}{\partial \eta} \langle B_1^{q1}\rangle |_{8} = -2g^4 i f^{a'b'c'}(U_1^a) \cdot (t^bU_2) \cdot (U_3c') \int \frac{dp^+}{p^+(2\pi)^2} \int_{\sigma_1}^{\bar{\sigma}} \frac{dk^+}{(2\pi)^3} \frac{dq^+}{(2\pi)^3} \int d\vec{x} e^{ix-p^+} e^{i\hat{p}\hat{z}_{12}^\star} \frac{1}{z_{12}^2 + k^+ z_{12}^2}\]

\[\times \left[ \frac{2}{z_{12}^2 + k^+ z_{12}^2} \left[ (\bar{z}_{10}\bar{z}_{34}) (\bar{z}_{20}^2 - \bar{z}_{24}^2) + (\bar{z}_{10}\bar{z}_{24}) (\bar{z}_{24}^2 - \bar{z}_{20}^2) \right] \right]. \quad (4.19)\]
The corresponding subtraction term comes from the following terms in the LO kernel (2.29)

\[
\frac{\alpha_s}{\pi^2} \int d\vec{z}_1 d\vec{z}_2 \left\{ \frac{1}{z_{14}^2} \frac{1}{z_{34}^2} \left[ (U_1 t^a) \cdot (t^b U_2) \cdot U_3 + \frac{1}{z_{13}^2} \frac{1}{z_{34}^2} U_1 \cdot (t^b U_2) \cdot (U_3 t^a) \right] \right\}.
\]

(4.20)

Rewriting \( U_4^{ba} \) as the trace of the Wilson lines in the fundamental representation (2.31) and dressing \( U_4, U_4^d \) and \( U_3 \) or \( U_1 \) we get in particular

\[
\langle K_{LO} \otimes B_{123}^o \rangle | s = \frac{\alpha_s^2}{\pi^4} \int d\vec{z}_1 d\vec{z}_2 \left\{ \frac{1}{z_{14}^2} \frac{1}{z_{34}^2} \left[ \langle U_1 t^a \rangle \cdot (t^b U_2) \cdot (U_3 t^c) \right] \right\}.
\]

(4.21)

One can trim it to be

\[
\langle K_{LO} \otimes B_{123}^o \rangle | s = \frac{\alpha_s^2}{\pi^4} \int d\vec{z}_1 d\vec{z}_2 \left\{ \frac{1}{z_{14}^2} \frac{1}{z_{34}^2} \left[ \langle U_1 t^a \rangle \cdot (t^b U_2) \cdot (U_3 t^c) \right] \right\}
\]

(4.22)

Then

\[
\langle K_{NLO} \otimes B_{123}^o \rangle | s = \frac{\partial}{\partial \eta} \langle B_{123}^o \rangle | s - \langle K_{LO} \otimes B_{123}^o \rangle | s
\]

\[
\times \left\{ \frac{1}{2(\sigma - k^+ \frac{1}{z_{14}^2} + k^+ \frac{1}{z_{20}^2})} \left\{ \frac{\sigma}{\sigma - k^+} \right\} \left\{ \frac{\sigma}{\sigma - k^+} \right\} \right\}.
\]

(4.23)

Thus we get the NLO contribution of diagram 8

\[
\langle K_{NLO} \otimes B_{123}^o \rangle | s = \frac{\alpha_s^2}{\pi^4} \int d\vec{z}_1 d\vec{z}_2 \left\{ \frac{1}{z_{14}^2} \frac{1}{z_{34}^2} \left[ \langle U_1 t^a \rangle \cdot (t^b U_2) \cdot (U_3 t^c) \right] \right\}
\]

\[
\times \left\{ \frac{1}{2(\sigma - k^+ \frac{1}{z_{14}^2} + k^+ \frac{1}{z_{20}^2})} \left\{ \frac{\sigma}{\sigma - k^+} \right\} \left\{ \frac{\sigma}{\sigma - k^+} \right\} \right\}.
\]

(4.24)

The contribution of the diagram which is a mirror reflection of diagram 8 with respect to the shockwave reads

\[
\langle B_{123}^o \rangle | s_m = -g^4 f^{ab'c} (t^{a'} U_1) \cdot (t^{b'} U_2) \cdot (t^{c'} U_3) \int_{z_1}^\infty d\vec{z}_1 \int_{z_2}^\infty d\vec{z}_2 \int_{-\infty}^0 d\vec{z}_3 \int_0^\infty \theta(-x^+ + d^+ x)
\]

\[
\times \left\{ \frac{\partial G^{a'\mu}(z_1, x)}{\partial x^\mu} \right\} \left[ G^{b'^\mu}(x, z_2)^\mu - G^{c'^\mu}(x, z_3)^\mu - G^{b'^\mu}(x, z_2)^\mu - G^{c'^\mu}(x, z_3)^\mu \right].
\]
\[ \frac{\partial G_{0}^{\mu}(x,z_{2})}{\partial x_{\mu}} \left[ G^{\alpha}(z_{1},x)^{-j} G^{\mu}(z_{3},x)^{-j} - G^{\alpha}(z_{1},x)^{-j} G^{\mu}(z_{3},x)^{-j} \right] + \frac{\partial G^{\mu}(z_{2},x)^{-j}}{\partial x_{\mu}} \left[ G^{\alpha}(z_{1},x)^{-j} G^{\mu}(z_{3},x)^{-j} - G^{\alpha}(z_{1},x)^{-j} G^{\mu}(z_{3},x)^{-j} \right] \right] . \]  

(4.25)

\[ \langle B_{123}^{\eta} \rangle_{8m} = g^{4} f^{abc}(t^{a} U_{1}) \cdot (U_{2} b^{b}) \cdot (t^{c} U_{3}) \int_{-\infty}^{0} \frac{d z_{2}^{+}}{(x^{+})^{2}} \int d^{4} x \theta(-z_{2}^{+}) \times \int_{\sigma_{1}} d p^{+} \int_{\sigma_{1}} d k^{+} \int_{\sigma_{1}} d q^{+} e^{-i q^{+} z_{2}^{+} + i k^{+} z_{2}^{+} - i p^{+} z_{2}^{+}} \times U_{0}^{a} d z_{0} \int \frac{c}{z_{4}^{+}} \int U_{0}^{c} d z_{4} \times \left[ k^{+} (z_{10} z_{2}^{+}) (z_{34} z_{4}^{+}) + p^{+} (z_{10} z_{2}^{+}) (z_{34} z_{4}^{+}) - q^{+} (z_{10} z_{0}^{+}) (z_{2} z_{34}^{+}) + k^{+} (z_{10} z_{34}) (z_{2} z_{34}) + q^{+} (z_{10} z_{34}) (z_{2} z_{34}) \right] . \]  

(4.26)

One can see that the structure in the brackets does not change and the integration with respect to \( z_{2}^{+} \) gives the same contribution as for diagram 8 while the integration with respect to \( x^{+} \) gives the same contribution with the opposite sign. Therefore the result for the diagram which is a mirror reflection of diagram 8 with respect to the shockwave reads

\[ \langle K_{NLO} \otimes B_{123}^{\eta} \rangle_{8m} = \frac{\alpha_{2}^{2}}{\pi^{4}} f^{abc}(t^{a} U_{1}) \cdot (U_{2} b^{b}) \cdot (t^{c} U_{3}) \int \vec{u}_{0}^{a} d z_{0} \int U_{4}^{c} d z_{4} \times \left[ \frac{1}{2 z^{04}_2} (z_{10} z_{34}_2 + (z_{10} z_{40}_2) (z_{24} z_{34}_2) + (z_{04} z_{34}_2) (z_{10} z_{20}_2) \right] \ln \frac{z_{02}^{2}}{z_{24}^{2}}. \]  

(4.27)

Adding the contribution of diagram 8 we get

\[ \langle K_{NLO} \otimes B_{123}^{\eta} \rangle_{8s+8m} = \frac{\alpha_{2}^{2}}{\pi^{4}} \int \vec{u}_{0}^{a} d z_{0} \int U_{4}^{c} d z_{4} \times \left[ \frac{1}{2 z^{04}_2} (z_{10} z_{34}_2 + (z_{10} z_{40}_2) (z_{24} z_{34}_2) + (z_{04} z_{34}_2) (z_{10} z_{20}_2) \right] \ln \frac{z_{02}^{2}}{z_{24}^{2}} \times i \left\{ f^{abc}(U_{1} t^{a} U_{2}) \cdot (U_{3} b^{c}) - f^{abc}(t^{a} U_{1}) \cdot (U_{2} b^{b}) \cdot (t^{c} U_{3}) \right\}. \]  

(4.28)

Combining this result with the contribution of diagram 7 and all the diagrams obtained from it (4.10), we get the complete contribution of all diagrams with two gluon intersecting the shockwave to the connected part of the NLO kernel

\[ \langle K_{NLO}^{\text{conn}} \otimes B_{123}^{\eta} \rangle_{29} = \frac{\alpha_{2}^{2}}{\pi^{4}} \left\{ f^{abc}(U_{1} t^{a} U_{2}) \cdot (U_{3} b^{c}) - f^{abc}(t^{a} U_{1}) \cdot (U_{2} b^{b}) \cdot (t^{c} U_{3}) \right\} \times \int d z_{0} \int d z_{4} C^{a}_{0} \times \left[ \frac{1}{2 z^{04}_2} (z_{10} z_{34}_2 + (z_{10} z_{40}_2) (z_{24} z_{34}_2) + (z_{04} z_{34}_2) (z_{10} z_{20}_2) \right] \ln \frac{z_{02}^{2}}{z_{24}^{2}}. \]
Using (2.31), one can rewrite it as

\[
\langle K^{\text{LO}} \otimes B|_{123}\rangle|_{29} = \frac{\alpha_s^2}{4\pi^4} \int d\hat{z}_0 \int d\hat{z}_4 \left\{ (U_2 U_0^0 U_1) \cdot U_4 \cdot (U_0 U_4^4 U_3) + (U_3 U_0^0 U_0) \cdot U_4 \cdot (U_0 U_4^4 U_2) \right. \\
\left. - (1 \leftrightarrow 3, 0 \leftrightarrow 4) \right\}
\times \left[ \frac{1}{2} \left( \hat{z}_{10}^2 \hat{z}_{34}^2 + \hat{z}_{10}^2 \hat{z}_{34}^2 + \hat{z}_{10}^2 \hat{z}_{34}^2 + \hat{z}_{10}^2 \hat{z}_{34}^2 \right) - \hat{z}_{20}^2 \hat{z}_{10}^2 \hat{z}_{24}^2 \hat{z}_{24}^2 \right] \ln \frac{\hat{z}_{20}^2}{\hat{z}_{24}^2}
\]

\[+ (2 \leftrightarrow 1) + (2 \leftrightarrow 3). \quad (4.29)\]

5 Diagrams with 1 gluon intersecting the shockwave

Let us turn to the diagrams in fig. 3. Hereafter we will not write the LO subtraction terms explicitly. Instead we will set \( \sigma_1 = 0 \) and use the \( \left[ \frac{1}{p^+} \right]_+ \) and \( \left[ \frac{1}{\sigma - p^+} \right]_+ \) prescriptions where necessary

\[
\int_0^\sigma dp^+ f(p^+) \left[ \frac{1}{p^+} \right]_+ = \int_0^\sigma dp^+ f(p^+) \frac{f(0)}{p^+}, \quad (5.1)
\]

\[
\int_0^\sigma dp^+ f(p^+) \left[ \frac{1}{\sigma - p^+} \right]_+ = \int_0^\sigma dp^+ f(p^+) \frac{f(\sigma)}{\sigma - p^+}. \quad (5.2)
\]

Diagram 9 reads

\[
\langle B|_{123}^{\eta}|_9 = g^4 (U_1 t^a) \cdot (U_2 t^b U_2) \cdot (U_0 t^b t^c)
\times \int_{-\infty}^0 dz_1^+ \int_0^\infty dz_2^+ \int_0^\infty dz_3^+ \int_{-\infty}^\infty dz_4^+ G^{-} \cdot (z_2, z_3) G^{-} \cdot (z_3, z_1) (\sigma, \tau). \quad (5.3)
\]

Using the momentum representation of the propagators (2.15) and (2.18) and as in the LO calculation changing \( -ik^- z_3^+ \to -i(k^- + \iota\varepsilon) z_3^+ + i(k^- - \iota\varepsilon) z_3^+ \) and integrating first with respect to \( z_3^+ \) and then to \( k^- \) next, we get

\[
\int_0^\sigma dp^+ \frac{f(p^+)}{(2\pi)^2} \int \frac{dz_2}{\hat{z}_{20}^2} U_0^b \hat{z}_0 \int_{-\infty}^0 dz_3^+ \]

\text{Figure 3.} Diagrams with one gluon intersecting the shockwave.
\[
\times \int \frac{d\tilde{p}}{(2\pi)^2} e^{i\tilde{p} \cdot \tilde{z}_3^+} e^{\frac{2}{\alpha} (\tilde{p}^2 - m^2)p_{z^+}} \int \frac{dk^+}{2\pi k^+} \int \frac{dk}{(2\pi)^2} e^{i\tilde{k} \cdot \tilde{z}_{\alpha}^+} \frac{2\theta(-k^+)}{k^2} e^{-i\tilde{k} \cdot \bar{z}^+_{3\alpha}}. \tag{5.4}
\]

Then one can integrate w.r.t. \(z^+_{3\alpha}\)

\[
\langle B^n_{123} \rangle|_9 = 2ig^4(U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^b t^a) \int_0^\sigma \frac{dp^+}{p^+} \int \frac{dz_0}{(2\pi)\bar{z}_0^2} \int_0^\sigma \frac{2}{p^2 k^+ + k^2 p^+} \tag{5.5}
\]

Therefore using the \(\left[ \frac{1}{p^+} \right] \) prescription one gets

\[
\langle K_{NLO} \otimes B^n_{123} \rangle|_9 = 4ig^4(U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^b t^a) \int_0^\sigma \frac{dz_0}{(2\pi)\bar{z}_0^2} \int_0^\sigma \frac{2}{p^2 k^+ + k^2 p^+} \tag{5.6}
\]

Now we consider diagram 10. It has the form

\[
\langle B^n_{123} \rangle|_{10} = g^4(U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^a t^b) \times \int_0^0 d\tilde{z}_1^+ \int_0^0 d\tilde{z}_2^+ \int_0^0 d\tilde{z}_3^+ \int_0^0 d\tilde{z}_3^+ G^{--}(z_2, z_3) t^b G^{--}(z_3, z_1) t^a. \tag{5.7}
\]

Going through the same steps as for diagram 9 one gets

\[
\langle K_{NLO} \otimes B^n_{123} \rangle|_{10} = 4ig^4(U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^a t^b) \times \int_0^\sigma \frac{dz_0}{(2\pi)\bar{z}_0^2} \int_0^\sigma \frac{2}{p^2 k^+ + k^2 p^+} \tag{5.8}
\]

Then the contribution of both diagrams and the diagrams with the 3 \(\leftrightarrow 1\) Wilson line substitution reads

\[
\langle K_{NLO} \otimes B^n_{123} \rangle|_{9+10+(9+10)(3\leftrightarrow 1)} = 4f^{abc} g^4(U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^c) \int_0^\sigma \frac{dz_0}{(2\pi)\bar{z}_0^2} \int_0^\sigma \frac{2}{p^2 k^+ + k^2 p^+} \tag{5.9}
\]

We turn to diagram 11.

\[
\langle B^n_{123} \rangle|_{11} = -g^4 f^{ab'c'}(U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^c) \int_0^0 d\tilde{z}_1^+ \int_0^0 d\tilde{z}_2^+ \int_0^\infty d\tilde{z}_3^+ \int_0^\infty \theta(-x^+) d^4x
\]

\[
\times \left\{ \frac{\partial G^{a'c'}(x, z_1)}{\partial x^\mu} (x, z_1) \right\} \left[ G^{bb'}(z_2, x) \delta^\mu - G^{bb'}(z_2, x) \delta^\mu - G^{bb'}(z_2, x) \delta^\mu - G^{aa'}(x, z_1) \delta^\mu - G^{aa'}(x, z_1) \delta^\mu \right]
\]

\[
+ \frac{\partial G^{aa'}(z_2, x)}{\partial x^\mu} (x, z_1) \right\} \left[ G^{bb'}(z_2, x) \delta^\mu - G^{bb'}(z_2, x) \delta^\mu - G^{bb'}(z_2, x) \delta^\mu - G^{aa'}(x, z_1) \delta^\mu - G^{aa'}(x, z_1) \delta^\mu \right]
\]

\[
= -4f^{ab'c'}(U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^c) \int_0^\infty d\tilde{z}_1^+ \int_0^\infty d\tilde{z}_2^+ \int_0^\infty d\tilde{z}_3^+ \int_0^\infty \theta(-x^+) d^4x
\]

\[
\times \left\{ \frac{\partial G^{a'c'}(x, z_1)}{\partial x^\mu} (x, z_1) \right\} \left[ G^{bb'}(z_2, x) \delta^\mu - G^{bb'}(z_2, x) \delta^\mu - G^{bb'}(z_2, x) \delta^\mu - G^{aa'}(x, z_1) \delta^\mu - G^{aa'}(x, z_1) \delta^\mu \right]
\]

\[
= -4f^{ab'c'}(U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^c) \int_0^\infty d\tilde{z}_1^+ \int_0^\infty d\tilde{z}_2^+ \int_0^\infty d\tilde{z}_3^+ \int_0^\infty \theta(-x^+) d^4x
\]

\[
\times \left\{ \frac{\partial G^{a'c'}(x, z_1)}{\partial x^\mu} (x, z_1) \right\} \left[ G^{bb'}(z_2, x) \delta^\mu - G^{bb'}(z_2, x) \delta^\mu - G^{bb'}(z_2, x) \delta^\mu - G^{aa'}(x, z_1) \delta^\mu - G^{aa'}(x, z_1) \delta^\mu \right]
\]
\[
+ \frac{\partial G_0^{\prime\prime}(x, z_3)}{\partial x^\mu} \left[ G_0^{\prime\prime}(x, z_1) \mu - G^{bb'}(z_2, x)^{-j} - G_0^{\prime\prime}(x, z_1)^{-j} G^{bb'}(z_2, x)^{-\mu} \right]. \tag{5.10}
\]

Here we sum over \( j = 1, 2 \) and \( \mu = -1, 2 \). It is convenient to split this expression into two parts.

\[
\langle B_{123}^\prime \rangle_{|11} = \langle B_{123}^0 \rangle_{|11} + \langle B_{123}^\prime \rangle_{|12}. \tag{5.11}
\]

\[
\langle B_{123}^0 \rangle_{|11} = -g^4 f^{ab'}c(U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^c) \int_{-\infty}^{0} dz_1 \int_{-\infty}^{0} dz_2 \int_{-\infty}^{0} dz_3 \int_{0}^{0} d\theta \langle -x^+ \rangle d^4 x \times \left\{ \begin{array}{l}
G_0^{ab'}(x, z_2, x) \frac{\partial G_0^{ab'}(z_2, x)}{\partial x^\mu} - G_0^{ab'}(z_2, x) \frac{\partial G_0^{ab'}(x, z_1)}{\partial x^\mu} \\
+ G_0^{ab'}(x, z_2, x) \frac{\partial G_0^{ab'}(z_2, x)}{\partial x^\mu} + G_0^{ab'}(z_2, x) \frac{\partial G_0^{ab'}(x, z_1)}{\partial x^\mu} \end{array} \right\}. \tag{5.12}
\]

\[
\langle B_{123}^\prime \rangle_{|11} = -g^4 f^{ab'}c(U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^c) \int_{-\infty}^{0} dz_1 \int_{-\infty}^{0} dz_2 \int_{-\infty}^{0} dz_3 \int_{0}^{0} d\theta \langle -x^+ \rangle d^4 x \times \left\{ \begin{array}{l}
\int \frac{dq^+}{2\pi} e^{-i z \cdot q} \int \frac{d\vec{k} d\vec{q}}{(2\pi)^3} \left[ G_0^{ab'}(z_2, x) \frac{\partial G_0^{ab'}(z_2, x)}{\partial x^\mu} - G_0^{ab'}(z_2, x) \frac{\partial G_0^{ab'}(x, z_1)}{\partial x^\mu} \right] + \int \frac{dk^+}{2\pi} e^{-i z \cdot k} \int \frac{d\vec{k} d\vec{q}}{(2\pi)^3} \left[ G_0^{ab'}(z_2, x) \frac{\partial G_0^{ab'}(x, z_1)}{\partial x^\mu} - G_0^{ab'}(z_2, x) \frac{\partial G_0^{ab'}(x, z_1)}{\partial x^\mu} \right] \end{array} \right\}. \tag{5.13}
\]

Substituting the propagators and using \([\frac{1}{\sigma - p^+}]_+\) prescription one gets

\[
\frac{\partial}{\partial \eta} \langle B_{123}^\prime \rangle_{|11} = -2g^4 f^{ab'}c(U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^c) \int \frac{d\vec{z}_0}{(2\pi)^3} \int \frac{d\vec{k}}{(2\pi)^3} \int_{-\infty}^{0} dp^+ \times e^{-i \vec{k} \cdot \vec{z}_0 - i \vec{q} \cdot \vec{z}_0} \left[ -\frac{1}{(\sigma - p^+)(\vec{q} + \vec{k})^2 + p^+ \vec{q}^2} + \frac{k_{\alpha \perp}}{k^2} + \frac{1}{(\sigma - p^+)(\vec{q} + \vec{k})^2 + p^+ \vec{k}^2} \frac{q_{\alpha \perp}}{\vec{q}^2} \right. \right. \left. + \left. \frac{2(\vec{q}^2 - (\vec{q} + \vec{k})^2)}{(\sigma - p^+)(\vec{q} + \vec{k})^2 + p^+ \vec{k}^2} \frac{k_{\alpha \perp}}{k^2} - \frac{2(k^2 - (\vec{q} + \vec{k})^2)}{(\sigma - p^+)(\vec{q} + \vec{k})^2 + p^+ \vec{k}^2} \frac{q_{\alpha \perp}}{k^2} \right]. \tag{5.14}
\]

Integrating with respect to \( p^+ \) one comes to

\[
\langle K_{NLO} \otimes B_{123}^\prime \rangle_{|11} = -2g^4 f^{ab'}c(U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^c) \int \frac{d\vec{z}_0}{(2\pi)^3} \int \frac{d\vec{k}}{(2\pi)^3} \int_{-\infty}^{0} dp^+ \times e^{-i \vec{k} \cdot \vec{z}_0 - i \vec{q} \cdot \vec{z}_0} \left[ \frac{q_{\alpha \perp}}{\vec{q}^2} \frac{1}{k^2 - (\vec{q} + \vec{k})^2} \ln \frac{k^2}{(\vec{q} + \vec{k})^2} - \frac{k_{\alpha \perp}}{k^2} \frac{1}{k^2 - (\vec{q} + \vec{k})^2} \ln \frac{q^2}{(\vec{q} + \vec{k})^2} \right].
\]

\[-15-\]
Adding this contribution to the contribution of diagrams 9 and 10 (5.9), one gets the regular contribution

\[ \langle K_{\text{NLO}} \otimes B_{123} \rangle_{111+9+10+(9+10)(3+1)} = -2f^{abc} g^4 (U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^c) \]

\( x \times \left[ \frac{(2\pi)^2}{2} \mathcal{Z}_{20} \int \frac{dk}{2\pi} \int dq \right] \times \left[ e^{-i\bar{k}z_{10}-i\bar{q}z_{30}} \left\{ -\frac{k_{\perp}}{q^2 - (q + \bar{k})^2} \ln \frac{q^2}{(q + \bar{k})^2} + \frac{q_{\perp}}{q^2 - (q + \bar{k})^2} \ln \frac{\bar{k}^2}{q^2} \right\} \right. \]

\( + 2 \frac{k_{\perp}}{q^2} \left\{ e^{-i\bar{k}z_{10}} \ln \frac{q^2}{(q + \bar{k})^2} + e^{-i\bar{k}z_{10}} \ln \frac{\bar{k}^2}{q^2} \right\} \)

\( + \left. 2 \frac{q_{\perp}}{q^2} \left\{ e^{-i\bar{k}z_{10}} \ln \frac{q^2}{(q + \bar{k})^2} - e^{-i\bar{k}z_{30}} \ln \frac{\bar{k}^2}{q^2} \right\} \right] \). (5.16)

The second contribution to diagram 11 reads

\[ \langle B^9_{123} \rangle_{11l} = -g^4 f^{abc} g^2 (U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^c) \int_{-\infty}^{0} dz_1^+ \int_{-\infty}^{0} dz_3^+ \int_{-\infty}^{0} dz_2^+ \int \theta(-x^+) dx^+ \]

\( x \times \left\{ G^{b\ell}(z_2, x)^{-1} - \frac{\partial G^{a\ell}(x, z_1)}{\partial x} C^c_c(x, z_3) - G^{a\ell}(x, z_1) \frac{\partial G^{c\ell}(x, z_3)}{\partial x} \right\} \)

\( + \frac{\partial G^{a\ell}(x, z_1)}{\partial x} \left[ G^{b\ell}(z_2, x)^{-1} C^c_c(x, z_3) - G^{b\ell}(z_2, x)^{-1} C^c_c(x, z_3) \right] \)

\( + \frac{\partial G^{b\ell}(z_2, x)}{\partial x} \left[ G^{a\ell}(x, z_1)^{-1} G^{b\ell}(z_2, x)^{-1} - G^{a\ell}(x, z_1)^{-1} G^{b\ell}(z_2, x)^{-1} \right] \)

\( + \frac{\partial G^{c\ell}(x, z_3)}{\partial x} \left[ C^{a\ell}(x, z_1)^{-1} G^{b\ell}(z_2, x)^{-1} - C^{a\ell}(x, z_1)^{-1} G^{b\ell}(z_2, x)^{-1} \right] \). (5.17)

Substituting the propagators and integrating with respect to \( z_2^+ \), \( x, \bar{x} \), one obtains

\[ \langle B^9_{123} \rangle_{11l} = -g^4 f^{abc} g^2 (U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^c) \int_{-\infty}^{0} dz_1^+ \int_{-\infty}^{0} dz_3^+ \]

\( \times \int \theta(-x^+) dx^+ dx^- \int \mathcal{Z}_0 \frac{(2\pi)^2}{2} \int_{-\infty}^{0} \frac{dp^+}{2\pi} e^{ip^+ x^-} \int \frac{dk^+}{2\pi} e^{-ik^+ x^-} \int \frac{dq^+}{2\pi} e^{-iq^+ x^-} \]

\( \times \left( \theta(-z_{1x}) \theta(q^+) \theta(-z^+_{1x}) \theta(k^+) - \theta(-z_{3x}) \theta(-q^+) \theta(-z^+_{3x}) \theta(k^+) - \theta(z^+_{1x}) \theta(-k^+) \theta(-z^+_{3x}) \theta(q^+) \right) \)

\( \times \left( \frac{2 \mathcal{Z}_{20}(2\pi)^2}{k_\perp} \int \frac{dq}{(2\pi)^2} e^{-i\bar{k}z_1 \bar{q}^+ \frac{2}{2(k^+)^2}} \int \frac{dq^+}{(2\pi)^2} e^{-q^+ z_{3x} \frac{2}{2(k^+)^2}} e^{i(q^+ \bar{k} 2)} \right) \)
Adding to this expression the first contribution of diagram 11 and diagrams 9 and 10 with
\[
\langle \theta \rangle (e^+ NLO_123 \eta_i q \otimes \times \{ \rangle | (2) = 2 k_k^2 + (\sigma - p^+)(\tau + k^2) + p^2 q^2
\]
\[-2 \sigma (\tilde{k} q)(k + q)_{i,\perp} \int_0^\sigma dp^+ \frac{p^2}{p^2} \left[ \frac{1}{(\sigma - p^+)(\tau + k^2) + p^2 q^2} - \frac{1}{\sigma (\tau + k^2)} \right]
\]
\[+ \int_0^\sigma dp^+ \frac{-k(k + q)_{i,\perp} (\tilde{k} q) - [k_{i,\perp} (q^2 + (\tilde{k} q)) - ((\tilde{k} q) + k^2) q_{i,\perp}] \right]
\]
\[+ 2 \sigma (k + q)_{i,\perp} (\tilde{k} q) \int_0^\sigma dp^+ \frac{p^2}{p^2} \left[ \frac{1}{(k^2 p^2 + (\sigma - p^+)(\tau + k^2) + p^2 q^2)} - \frac{1}{\sigma (\tau + k^2)} \right]
\]  
(5.19)

As a result,
\[\langle K_NLO \otimes B_{123}^0 \rangle_{112} = 2 g^4 f^{a'b'c'} (U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^c) \int d\bar{z}_0 U_0^{b'b'} (z_{20})_0^a \int d\tilde{k} \int d\tilde{q} \epsilon^{i(\tilde{z}_{20} \tilde{z}_{01})} \]
\[
\int_0^\sigma dp^+ \frac{k_{i,\perp} (q^2 + (\tilde{k} q)) - ((\tilde{k} q) + k^2) q_{i,\perp} \right]
\]
\[+ 2 \sigma (k + q)_{i,\perp} (\tilde{k} q) \int_0^\sigma dp^+ \frac{p^2}{p^2} \left[ \frac{1}{(k^2 p^2 + (\sigma - p^+)(\tau + k^2) + p^2 q^2)} - \frac{1}{\sigma (\tau + k^2)} \right]
\]  
(5.20)

Adding to this expression the first contribution of diagram 11 and diagrams 9 and 10 with the corresponding (1 \leftrightarrow 3) symmetrization (5.16) one has
\[\langle K_NLO \otimes B_{123}^0 \rangle_{11+9+10+(9+10)(3 \leftrightarrow 1)} = 2 g^4 f^{a'b'c'} (U_1 t^a) \cdot (t^b U_2) \cdot (U_3 t^c) \int d\bar{z}_0 U_0^{b'b'} (z_{20})_0^a \int d\tilde{k} \int d\tilde{q} \epsilon^{i(\tilde{z}_{20} \tilde{z}_{01})} \]
\[
\int_0^\sigma dp^+ \frac{k_{i,\perp} (q^2 + (\tilde{k} q)) - ((\tilde{k} q) + k^2) q_{i,\perp} \right]
\]
\[+ 2 \sigma (k + q)_{i,\perp} (\tilde{k} q) \int_0^\sigma dp^+ \frac{p^2}{p^2} \left[ \frac{1}{(k^2 p^2 + (\sigma - p^+)(\tau + k^2) + p^2 q^2)} - \frac{1}{\sigma (\tau + k^2)} \right]
\]  
(5.21)

(5.22)
Then we Fourier transform this expression introducing \( \vec{x} \) integration via

\[
\langle K_{NLO} \otimes B_{123} \rangle_{11+9+10+(9+10)(3+1)} = 2g^4 f^{a'b'c'} (U_1 t^a) \cdot (t^{b'} U_2) \cdot (U_3 t^c) \int d\vec{z}_0 t_0^{b'} \frac{(z_{20})_1^a}{(2\pi)^b \vec{z}_{20}^2}
\]

\[
\times \int d\vec{k} \int d\vec{q} \int \frac{d\vec{q} d\vec{p} \, p_{1a} (\vec{k} - \vec{p}) \vec{z}_{1x}}{(2\pi)^b \vec{p}^2} e^{i(\vec{k} - \vec{p}) \vec{z}_{1x}} \left( \frac{2}{q^2} e^{-i\vec{k} \vec{z}_{10} - i\vec{q} \vec{z}_{30}} \ln \frac{q^2}{(q + \vec{k})^2} + e^{-i\vec{k} \vec{z}_{10} - i\vec{q} \vec{z}_{31}} \ln \frac{k^2}{q^2} \right)
\]

\[
\times e^{i(\vec{q} \vec{z}_{03} + \vec{k} \vec{z}_{01})} \left\{ -\frac{1}{k^2} \ln \frac{q^2}{(q + k)^2} e^{i\vec{z}_{3} (\vec{p} - \vec{q})} - \left( \frac{q\vec{k}}{q^2 k^2} \right) \ln \frac{k^2}{q^2} e^{i\vec{z}_{3x} (\vec{p} - \vec{q}) - \vec{k}} - \left( \vec{k} \leftrightarrow \vec{q} \right) \right\} (5.23).
\]

All the integrals necessary to take Fourier Transform one can find in Appendices A and B of [22]. As a result

\[
\langle K_{NLO} \otimes B_{123} \rangle_{11+9+10+(9+10)(3+1)} = if^{abc} g^4 (U_1 t^a) \cdot (t^{b'} U_2) \cdot (U_3 t^c) \int \frac{U_{0b'} d\vec{z}_0}{(2\pi)^b \vec{z}_{20}^2}
\]

\[
\times \left[ 2 \int d\vec{z} \left\{ \left( \frac{z_{20}}{z_{21}^2} \right) - \frac{2}{z_{20}^2} \left( \frac{z_{20}}{z_{03}^2} \right) \ln \frac{z_{23}^2}{z_{23}^2} \ln \frac{z_{23}^2}{z_{23}^2} \right\}
\]

\[
\times + 2\pi \frac{z_{10} z_{20}}{z_{01}^2} \ln \frac{z_{10}^2}{z_{13}^2} - (1 \leftrightarrow 3) \right].
\]

This integral can be calculated changing the variables to \( \rho = |\vec{z}_{x3}|, t = e^{i\rho \vec{z}_{x3}} \) and integrating with respect to \( t \) via residues. After that one has dilogarithmic integrals which can be combined to

\[
\langle K_{NLO} \otimes B_{123} \rangle_{11+9+10+(9+10)(3+1)} = if^{abc} g^4 (U_1 t^a) \cdot (t^{b'} U_2) \cdot (U_3 t^c) \int \frac{U_{0b'} d\vec{z}_0}{(2\pi)^b \vec{z}_{20}^2}
\]

\[
\times \left[ \left( \frac{z_{10} z_{20}}{z_{10} z_{20}} \right) - \frac{z_{10} z_{20}}{z_{10} z_{20}} \ln \frac{z_{10}^2}{z_{10}^2} \ln \frac{z_{10}^2}{z_{10}^2} \right]. (5.24)
\]

Therefore the sum of these diagrams and the diagrams which are the mirror reflection of them with respect to the shockwave reads

\[
\langle K_{NLO} \otimes B_{123} \rangle_{1g} = ig^4 \left\{ f^{abc} (U_1 t^a) \cdot (t^{b'} U_2) \cdot (U_3 t^c) - f^{abc} (U_1 t^a) \cdot (t^{b'} U_2) \cdot (t^{c'} U_3) \right\}
\]

\[
\times \int \frac{U_{0b'} d\vec{z}_0}{(2\pi)^b} \left[ \left( \frac{z_{10} z_{20}}{z_{10} z_{20}} \right) - \frac{z_{10} z_{20}}{z_{10} z_{20}} \ln \frac{z_{10}^2}{z_{10}^2} \ln \frac{z_{10}^2}{z_{10}^2} \right]. (5.25)
\]

Performing the convolution, one gets

\[
\langle K_{NLO} \otimes B_{123} \rangle_{1g} = \frac{2a_s}{8\pi^3} \int d\vec{z} \left[ \left( \frac{z_{10} z_{20}}{z_{10} z_{20}} \right) - \frac{z_{10} z_{20}}{z_{10} z_{20}} \ln \frac{z_{10}^2}{z_{10}^2} \ln \frac{z_{10}^2}{z_{10}^2} \right] \ln \frac{z_{10}^2}{z_{10}^2} \ln \frac{z_{10}^2}{z_{10}^2} (B_{100} B_{320} - B_{300} B_{210})
\]

\[
+ (2 \leftrightarrow 1) + (2 \leftrightarrow 3). (5.26)
\]

From (5.25) one can find the contribution of the disconnected diagrams, which differ from the ones in fig. 3 in the attachment of the gluon in the right hand side of the diagrams,
and the ones which they go into after the mirror reflection with respect to the shockwave. We get the sum of (5.26) and all such diagrams

\[
\langle K_{NLO} \otimes B_{123}^\eta \rangle |_{1g} = \frac{g^4}{4} \int \frac{d\vec{z}_0}{(2\pi)^5} \left[ \frac{(\vec{z}_{10} \cdot \vec{z}_{20})}{z_{10}^2 z_{20}^2} - \frac{(\vec{z}_{30} \cdot \vec{z}_{20})}{z_{30}^2 z_{20}^2} \right] \ln \frac{\vec{z}_{20}^2}{z_{21}^2} \ln \frac{\vec{z}_{10}^2}{z_{31}^2} (B_{100}B_{320} - B_{300}B_{210}) + \frac{g^4}{4} \int \frac{d\vec{z}_0}{(2\pi)^5} \left[ \frac{1}{\vec{z}_{10}^2} - \frac{(\vec{z}_{30} \cdot \vec{z}_{10})}{z_{30}^2 z_{10}^2} \right] \ln \frac{\vec{z}_{20}^2}{z_{21}^2} \ln \frac{\vec{z}_{10}^2}{z_{31}^2} \left( B_{123} - \frac{1}{2} [3B_{100}B_{320} + B_{300}B_{120} - B_{200}B_{130}] \right) + \frac{g^4}{4} \int \frac{d\vec{z}_0}{(2\pi)^5} \left[ \frac{(\vec{z}_{10} \cdot \vec{z}_{30})}{z_{10}^2 z_{30}^2} - \frac{1}{\vec{z}_{30}^2} \right] \ln \frac{\vec{z}_{20}^2}{z_{21}^2} \ln \frac{\vec{z}_{10}^2}{z_{31}^2} \left( \frac{1}{2} [3B_{300}B_{120} + B_{100}B_{320} - B_{200}B_{130}] - B_{123} \right) + (2 \leftrightarrow 1) + (2 \leftrightarrow 3).
\]

(5.27)

If we put here \( \vec{z}_3 = \vec{z}_2 \), we get

\[
\langle \tilde{K}_{NLO} \otimes \text{tr}(U_1U_2^\dagger) \rangle |_{1g} = -\frac{\alpha_s^2}{4\pi^3} \int d\vec{z}_0 \ln \frac{\vec{z}_{20}^2}{z_{21}^2} \ln \frac{\vec{z}_{10}^2}{z_{31}^2} \left( 3\text{tr}(U_1U_0^\dagger)\text{tr}(U_0U_2^\dagger) - \text{tr}(U_1U_2^\dagger) \right),
\]

(5.28)

which coincides with the corresponding contribution to the color dipole kernel (see expression (99) in [14]).

6 Linearized C-odd connected contribution in the momentum space

In this section we linearize and Fourier transform the connected part of the kernel (4.30) and (5.26) for the C-odd case. All the integrals necessary to take Fourier transform one can find in Appendices A and B of [22]. We introduce the C-odd and C-even Green functions

\[
B_{123}^- = B_{123}^\eta - B_{123}^\eta, \quad B_{123}^+ = B_{123}^\eta + B_{123}^\eta - 12,
\]

(6.1)

where

\[
B_{123}^\eta = U_1^\dagger \cdot U_2 \cdot U_3^\dagger.
\]

(6.2)

We start from (5.26). For the C-odd case in the linear regime we have

\[
\langle \tilde{K}_{NLO}^{\text{conn}} \otimes B_{123}^\eta \rangle |_{1g} = \frac{3\alpha_s^2}{4\pi^3} \int d\vec{z}_0 \left[ \frac{(\vec{z}_{10} \cdot \vec{z}_{20})}{z_{10}^2 z_{20}^2} - \frac{(\vec{z}_{30} \cdot \vec{z}_{20})}{z_{30}^2 z_{20}^2} \right] \ln \frac{\vec{z}_{20}^2}{z_{21}^2} \ln \frac{\vec{z}_{10}^2}{z_{31}^2} \times (B_{023} + B_{100} - B_{120} - B_{030}) + (2 \leftrightarrow 1) + (2 \leftrightarrow 3).
\]

(6.3)

One can rewrite it as

\[
\langle \tilde{K}_{NLO}^{\text{conn}} \otimes B_{123}^- \rangle |_{1g} = \int d\vec{z}_{1'}d\vec{z}_{2'}d\vec{z}_{3'} K_{NLO}^{\text{conn}} (\vec{z}_{1}, \vec{z}_{2}, \vec{z}_{3}; \vec{z}_{1'}, \vec{z}_{2'}, \vec{z}_{3'}) |_{1g} B_{123}^{\dagger \gamma_{23}},
\]

(6.4)

where

\[
K_{NLO}^{\text{conn}} (\vec{z}_{1}, \vec{z}_{2}, \vec{z}_{3}; \vec{z}_{1'}, \vec{z}_{2'}, \vec{z}_{3'}) |_{1g} = \frac{3\alpha_s^2}{4\pi^3} \int d\vec{z}_0 \left[ \frac{(\vec{z}_{10} \cdot \vec{z}_{20})}{z_{10}^2 z_{20}^2} - \frac{(\vec{z}_{30} \cdot \vec{z}_{20})}{z_{30}^2 z_{20}^2} \right] \ln \frac{\vec{z}_{20}^2}{z_{21}^2} \ln \frac{\vec{z}_{10}^2}{z_{31}^2} \times (\delta (\vec{z}_{22'}) + \delta (\vec{z}_{02'})) (\delta (\vec{z}_{30'}) \delta (\vec{z}_{33'}) - \delta (\vec{z}_{11'}) \delta (\vec{z}_{03'}))
\]
The kernel in the momentum representation reads
\[
K(q_1, \vec{q}_2, \vec{q}_3; \vec{q}_1', \vec{q}_2', \vec{q}_3') = \int \frac{d\vec{z}_1 d\vec{z}_2 d\vec{z}_3 d\vec{z}_4 d\vec{z}_5 d\vec{z}_6}{2\pi^2 2\pi^2 2\pi^2 2\pi^2} e^{-i[\vec{q}_1 \cdot \vec{z}_1 + \vec{q}_2 \cdot \vec{z}_2 + \vec{q}_3 \cdot \vec{z}_3 - \vec{q}_1' \cdot \vec{z}_1' - \vec{q}_2' \cdot \vec{z}_2' - \vec{q}_3' \cdot \vec{z}_3']}
\times K(z_1, z_2, z_3; z_1', z_2', z_3').
\] (6.5)

We have
\[
K^{\text{NNLO}}(q_1, \vec{q}_2, \vec{q}_3; \vec{q}_1', \vec{q}_2', \vec{q}_3') |_{1g} = \frac{3\alpha_s^2}{4\pi^3 (2\pi)} \delta(q_{11'} + q_{22'} + q_{33'}) \left[ \frac{\vec{q}_2}{\vec{q}_2'} - \frac{\vec{q}_2}{\vec{q}_2'} \right]
\times \left\{ - \left[ \frac{1}{q_{35}^2} \frac{\partial}{\partial q_{11}} - \frac{1}{q_{11}^2} \frac{\partial}{\partial q_{35}} \right] \ln \left( \frac{q^2_{33'}}{q_1 + q_{33'}} \right)^2 \ln \left( \frac{q^2_{11}}{q_1 + q_{33'}} \right)^2 
+ \left[ \frac{1}{q_{33}^2} \frac{\partial}{\partial q_{11}} - \frac{1}{q_{11}^2} \frac{\partial}{\partial q_{33}} \right] \ln \left( \frac{q^2_{33'}}{q_{11'} + q_{33'}} \right)^2 \ln \left( \frac{q^2_{11}}{q_{11'} + q_{33'}} \right)^2 \right\}
+ (2 \leftrightarrow 1) + (2 \leftrightarrow 3).
\] (6.6)

To deal with (4.30) one has to linearize the color structure first. It reads
\[
M = (U_2 U_0^\dagger U_1) \cdot U_4 \cdot (U_0 U_1 U_3) + (U_1 U_0^\dagger U_2) \cdot U_4 \cdot (U_3 U_1 U_0)
= U_1 \cdot U_4 \cdot (U_3 U_1^\dagger U_0 + U_0 U_1^\dagger U_3) + U_1 U_0^\dagger U_2 + U_2 U_0^\dagger U_1 \cdot U_4 \cdot U_0
+ (U_2 - U_0) \cdot U_4 \cdot ((U_0 - U_4) U_1^\dagger U_3 - U_4) + (U_3 - U_4) U_1^\dagger (U_0 - U_4))
+ ((U_1 - U_0) U_0^\dagger U_3 - U_4) \cdot U_4 \cdot ((U_3 - U_4) U_1^\dagger U_0)
+ ((U_2 - U_0) U_0^\dagger (U_1 - U_0)) \cdot U_4 \cdot (U_0 U_1^\dagger (U_3 - U_4))
\[+ 2(U_2 - U_0) \cdot U_4 \cdot (U_3 - U_4) - 2U_4 \cdot U_4 \cdot U_0.
\] (6.7)

Then we can use SU(3) identity (2.30) and take into account that in the 3-gluon approximation
\[
(U_2 - U_0) \cdot U_4 \cdot ((U_0 - U_4) U_1^\dagger U_3 - U_4) + (U_3 - U_4) U_1^\dagger (U_0 - U_4))
= (U_2 - U_0) \cdot E \cdot ((U_0 - U_4)(U_3 - U_4) + (U_3 - U_4)(U_0 - U_4))
= -(U_2 - U_0) \cdot (U_0 - U_4) \cdot (U_3 - U_4),
\] (6.8)

and
\[
((U_1 - U_0) U_0^\dagger (U_2 - U_0)) \cdot U_4 \cdot ((U_3 - U_4) U_1^\dagger U_0)
+ ((U_2 - U_0) U_0^\dagger (U_1 - U_0)) \cdot U_4 \cdot (U_0 U_1^\dagger (U_3 - U_4))
= ((U_1 - U_0)(U_2 - U_0) + (U_2 - U_0)(U_1 - U_0)) \cdot E \cdot (U_3 - U_4)
= -(U_2 - U_0) \cdot (U_1 - U_0) \cdot (U_3 - U_4).
\] (6.9)

Then in the 3-gluon approximation
\[
M - M^\dagger - (1, 0 \leftrightarrow 3, 4) = 3(6B_{044} + 3B_{014} - 3B_{043})
\]
\[
+ 2B_{200} - 2B_{244} + 2B_{243} - 2B_{210} + B_{314} - B_{310} - B_{144} + B_{300}, \quad \text{(6.11)}
\]

and (4.30) reads

\[
\langle K_{NLO}^\text{conn} \otimes B_{223}^n \rangle |_{2g} = \frac{3\alpha_s^2}{4\pi^4} \int d\vec{z}_0 \int d\vec{z}_1 \\left\{ 3B_{044} - 3B_{004} + 3B_{104} - 3B_{014} \right\} \\
+ 2B_{243} - 2B_{210} + 2B_{200} - 2B_{242} + B_{143} - B_{103} + B_{003} - B_{144} \}
\times \left[ \frac{1}{2z_{04}^2 z_{10}^2 z_{24}^2} \left( \frac{z_{04}^2 z_{10}^2 z_{24}^2}{z_{04}^2 z_{10}^2 z_{24}^2} \right) + \frac{z_{04}^2 z_{10}^2 z_{24}^2}{z_{04}^2 z_{10}^2 z_{24}^2} - \frac{z_{04}^2 z_{10}^2 z_{24}^2}{z_{04}^2 z_{10}^2 z_{24}^2} \right] \ln \frac{z_{02}^2 z_{04}^2}{z_{24}^2} \\
+ (2 \leftrightarrow 1) + (2 \leftrightarrow 3). \quad \text{(6.12)}
\]

Therefore

\[
K_{NLO}^\text{conn} (\hat{z}_1, \hat{z}_2, \hat{z}_3; \hat{z}_1', \hat{z}_2', \hat{z}_3') |_{2g} = \frac{3\alpha_s^2}{4\pi^4} \int d\vec{z}_0 \int d\vec{z}_1 \ln \frac{z_{02}^2 z_{04}^2}{z_{24}^2} \\
\times \left[ \frac{1}{2z_{04}^2 z_{10}^2 z_{24}^2} \left( \frac{z_{04}^2 z_{10}^2 z_{24}^2}{z_{04}^2 z_{10}^2 z_{24}^2} \right) + \frac{z_{04}^2 z_{10}^2 z_{24}^2}{z_{04}^2 z_{10}^2 z_{24}^2} - \frac{z_{04}^2 z_{10}^2 z_{24}^2}{z_{04}^2 z_{10}^2 z_{24}^2} \right] \ln \frac{z_{02}^2 z_{04}^2}{z_{24}^2} \\
+ (2 \leftrightarrow 1) + (2 \leftrightarrow 3). \quad \text{(6.13)}
\]

Hence this contribution to the kernel in the momentum representation (6.6) reads

\[
K_{NLO}^\text{conn} (\vec{q}_1, \vec{q}_2, \vec{q}_3; \hat{q}_{11'}, \hat{q}_{22'}, \hat{q}_{33'}) |_{2g} = \left( 2\pi \right)^2 \delta (\hat{q}_{11'} + \hat{q}_{22'} + \hat{q}_{33'}) \frac{3\alpha_s^2}{4\pi^4} \int \frac{d\vec{z}_{10}}{(2\pi)^2} \frac{d\vec{z}_{20}}{(2\pi)^2} \frac{d\vec{z}_{34}}{(2\pi)^2} \\
\times \int d\vec{z}_{40} \ln \frac{z_{20}^2}{(z_{20} - z_{40})^2} \cdot \frac{z_{10}^2 z_{24}^2}{z_{10}^2 z_{24}^2} \left[ \frac{\delta_{ij}}{2z_{40}^2} + \frac{z_{40}^2 - z_{20}^2}{z_{40}^2 - z_{20}^2} \right] \ln \frac{z_{02}^2 z_{04}^2}{z_{24}^2} \\
\times \left( e^{-i\vec{q}_3 \cdot \vec{z}_{40}} + (1 \leftrightarrow 3, 0 \leftrightarrow 4, 1' \leftrightarrow 3') \right) \right\} + (2 \leftrightarrow 1) + (2 \leftrightarrow 3). \quad \text{(6.14)}
\]

We can rewrite it as

\[
K_{NLO}^\text{conn} (\vec{q}_1, \vec{q}_2, \vec{q}_3; \hat{q}_{11'}, \hat{q}_{22'}, \hat{q}_{33'}) |_{2g} = \left( 2\pi \right)^2 \delta (\hat{q}_{11'} + \hat{q}_{22'} + \hat{q}_{33'}) \frac{3\alpha_s^2}{4\pi^4} \int \frac{d\vec{z}_{20}}{(2\pi)^2} \frac{d\vec{z}_{34}}{(2\pi)^2} \\
\times \ln \frac{z_{20}^2}{(z_{20} - z_{40})^2} \left( \frac{\vec{q}_{11'} \cdot \vec{q}_{34} - \vec{q}_{11'} \cdot \vec{q}_{34}^2}{\vec{q}_{11'} \cdot \vec{q}_{34}^2} \right) \left[ \frac{\delta_{ij}}{2z_{40}^2} + \frac{z_{40}^2 - z_{20}^2}{z_{40}^2 - z_{20}^2} \right] \ln \frac{z_{02}^2 z_{04}^2}{z_{24}^2} \\
\times \left( e^{-i\vec{q}_3 \cdot \vec{z}_{40}} + (1 \leftrightarrow 3, 0 \leftrightarrow 4, 1' \leftrightarrow 3') \right) \right\} + (2 \leftrightarrow 1) + (2 \leftrightarrow 3). \quad \text{(6.15)}
\]
Finally

\[
K_{NLO}^{conn} (\vec{q}_1, \vec{q}_2, \vec{q}_3; \vec{q}_1', \vec{q}_2', \vec{q}_3') \big|_{2g} = \delta (\vec{q}_{11} + \vec{q}_{22} + \vec{q}_{33}) \frac{3\alpha_s^2}{4\pi^4}
\]

\[
\times \left\{ \left( \frac{q_{11}'}{q_{11}} - \frac{q_1}{q_{11}} \right) \left( \frac{q_{33}'}{q_{33}} - \frac{q_3}{q_{33}} \right) A^{ij} (q_2, q_{33'}) + 2 \frac{q_{33}'}{q_3} (A^{ij} (q_{22'}, q_3) - A^{ij} (q_2, q_{33'})) \right\}
\]

\[
+ \left( \frac{q_{33}'}{q_{33}} - \frac{q_3}{q_{33}} \right) \left( \frac{q_{11}'}{q_{11}} - \frac{q_1}{q_{11}} \right) A^{ij} (q_2, q_{11'}) + 2 \frac{q_{11}'}{q_1} (A^{ij} (q_{22'}, q_1) - A^{ij} (q_2, q_{11'})) \right\}
\]

\[
+ (2, 2' \leftrightarrow 1, 1') + (2, 2' \leftrightarrow 3, 3'),
\]

(6.16)

where

\[
A^{ij} (q_2, q_{33'}) = \ln \frac{q_{33}^2}{(q_{33'} + q_2)^2} \left\{ \frac{q_{33}^2}{q_{33'} + q_2} + \frac{q_{2}^2 q_{33'}^2}{q_2^2 q_{33}^2} - \frac{(q_{33'} + q_2)^2}{q_{33'} + q_2} \right\}.
\]

(6.17)

7 Conclusion

The connected part of the NLO kernel for 3QWL operator has been calculated here within Balitsky high energy operator expansion formalism [14]. The result consists of two parts (5.26) and (4.30), which represent the contribution of diagrams with 1 and 2 gluon states crossing the shockwave. The momentum representation of these contributions in the linear limit for C-odd case is given in (6.7) and (6.16). Comparing these expressions with the connected 3 \(\rightarrow\) 3 contribution to odderon kernel, obtained in [10] in the momentum representation, one can see that they do not coincide. This fact indicates that there should be an equivalence transformation connecting the whole kernels obtained in the high energy operator expansion formalism and in the formalism based on reggeization. Moreover, the construction of a matrix element of a gauge invariant operator in the momentum representation from its Mobius form in the coordinate space consists of two steps [23]. First one does the Fourier transform and then adds to the result such terms that restore its gauge invariance but vanish after the convolution with the colorless impact factors. This procedure is to be applied to the whole kernel after its calculation.

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