Quantification of pore volume and degree of saturation in partially saturated sands with different bulk density

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ABSTRACT

In the present study, water retention tests on Toyoura sand with high bulk density and low bulk density are conducted. Three-phase microstructures in sand specimens are visualized using an x-ray micro tomography at the different water retention states. Distributions of the degree of saturation in each pore are quantified using a segmentation technique and a Voronoi tessellation technique. The pore-scale distribution of degree of saturation in partially saturated sands with different bulk density and its variation during drying and wetting processes are discussed.

Keywords: partially saturated sand, water retention characteristic, x-ray micro tomography, Voronoi tessellation

1 INTRODUCTION

Water retention characteristics of partially saturated soil is key to modelling the flow characteristics and the strength characteristics of the partially saturated soil. It is known that a main drying curve is located above a wetting curve during; namely, hysteresis occurs during drying and wetting processes. It is also known that bulk density influences on the water retention characteristics of partially saturated soil. Previous studies have confirmed the hysteresis (e.g., Vachaud et al., 1971) and have modeled the water retention curve considering the density dependence (e.g., Sun et al., 2006). On the other hand, the specific mechanisms of the hysteresis and the density dependence on the water retention characteristics are not sufficiently clear. It is important to investigate the water retention characteristics from the microscopic viewpoint in order to clarify causes of the hysteresis and the density dependence since partially saturated soil is granular material.

In the present study, water retention tests on Toyoura sand specimens with high bulk density and low bulk density are conducted. At the different water retention states, the soil particle phase, the pore water phase and the pore air phase in the Toyoura sand are visualized using an x-ray micro tomography with high spatial resolution. The three phases are segmented and then distributions of the degree of saturation in each pore are quantified using a Voronoi tessellation technique (Higo et al. 2018). The pore-scale distribution of degree of saturation in the sand specimens and its variation during drying and wetting processes are quantified.

2 METHODS

2.1 Water retention tests

The sample used in the present study is Toyoura sand. Physical properties of Toyoura sand are listed in Table 1. The diameter $D_{50}$ of Toyoura sand is 190 $\mu$m. In the present study, a low bulk density (loose) sand specimen and a high bulk density (dense) sand specimen were prepared using the water retention test apparatuses shown in Fig. 1(a) and Fig. 1(b), respectively. An acrylic hollow cylinder, with a

Table 1. Physical properties of Toyoura sand.

| Property                  | Value  |
|---------------------------|--------|
| Particle density (g/cm$^3$) | 2.64   |
| Maximum void ratio        | 0.975  |
| Minimum void ratio        | 0.614  |
| $D_{50}$                   | 0.185  |
| Uniformity coefficient    | 1.6    |
| Fines content (%)         | 0.1    |

Fig. 1. Water retention test apparatuses.
diameter of 18.0 mm and a height of 18.0 mm, was fixed on an acrylic pedestal with a diameter of 35.0 mm, as shown in Fig. 1(a). Another acrylic hollow cylinder was placed on an acrylic pedestal with a diameter of 20.0 mm, as shown in Fig. 1(b). A ceramic disc with an air entry value (AEV) of 50 kPa was equipped at the top of the pedestal in which a drainage path was prepared.

Sand specimens were prepared by pouring Toyoura sand from a certain height into the hollow cylinder that was initially filled with water. For the loose sand specimen, the sand was poured slowly into the hollow cylinder shown in Fig. 1(a). For the dense sand specimen, the sand was poured rapidly into the cylinder shown in Fig. 1(b); and subsequently, vibration was applied to the acrylic cylinder to densely pack the sand. The specimen conditions are listed in Table 2.

Water retention tests on both the specimens were performed using a negative water column technique (Vanapalli et al., 2008); namely, suction was applied by the water head difference between the top of the specimen and the water level in a double tube burette connected to the bottom of the specimen, as shown in Fig. 2. In the present study, the drying curve was firstly obtained by increasing suction, after which the wetting curve was obtained by decreasing suction. The amount of water drainage and that of water absorption from the specimens were measured using a differential pressure gauge installed on the burette until an equilibrium was reached at a given level of suction.

Table 2. Specimen conditions.

| Case         | Loose | Dense |
|--------------|-------|-------|
| Diameter (mm) | 18.00 | 20.00 |
| Height (mm)  | 17.74 | 20.07 |
| Initial void ratio | 0.822 | 0.637 |
| Relative density $D_r$ (%) | 42.38 | 93.63 |

2.2 X-ray micro tomography

Three-phase microstructures in the sand specimens were visualized by the x-ray micro tomography facility KYOTO-GEOμXCT (TOSCANER-32250μhdk). The tomographic area and scan conditions are shown in Fig. 3 and Table 3, respectively. The local region of interest focusing on the middle height of the specimens was observed by the x-ray micro tomography. The materials in the sand specimens are distinguished from each other based on the level of attenuation of the x-ray, that is, CT value.

2.3 Image processing techniques

The size and shape of the pores in partially saturated sand are not uniform due to the packing of soil particles whose particle diameters and shapes are different from each other, leading to the inhomogeneous water retention states in each pore. It is important, therefore, to investigate the water retention behavior with a pore-scale observation. In the present study, two kinds of image processing techniques, namely, trinarization and Voronoi tessellation, are applied to the tomographic volumes obtained in the water retention tests.

The soil particle phase, the pore water phase and the pore air phase in the tomographic volumes are separated using trinarization technique (Kido and Higo 2017). In this technique, a region growing technique is applied to the soil particle phase and the pore air phase, and then the remaining region is regarded as the pore water phase. Prior to the trinarization, a median filter with $5^3$ voxels is applied to the tomographic volumes to reduce noise. The region growing and the median filter are performed using a 3D image analysis software VGStudioMax3.1 (Volume Graphics GmbH).

![Fig. 2. Schematic illustration of water retention test.](image)

![Fig. 3. Tomographic area in sand specimen.](image)

Table 3. Scan conditions.

| Case     | Loose | Dense |
|----------|-------|-------|
| Voltage (kV) | 130   | 130   |
| Current (μA) | 50    | 65    |
| Voxel size | 5.48×7.00 | 5.40×7.00 |
| Scan area (mm) | 5.61 | 5.52 |
| Scan height (mm) | 5.39 | 5.87 |
The Voronoi tessellation is a technique for subdividing a space into a finite number of cells with mutual boundaries using tessellation generators in which the number of cells is the same as the number of generators (e.g., Aste et al., 2004; Matousek et al., 2011). Figure 4 shows a schematic diagram of Voronoi cells employing tessellation generators, which means that the Voronoi cells are defined as the portions closest to a given pore center than to any other in the space. In the present study, the three-dimensional trinarized volumes are subdivided into Voronoi cells employing the pore centers as the tessellation generators, as shown in Fig. 5. The pore centers are determined by Euclidian distance mapping and thinning computed using Avizo 9.4.0 (FEI). Trinarized volumes are subdivided into the Voronoi cells based on the pore centers using Fortran programing codes coordinated by our research group. The algorithm and the validity of the Voronoi tessellation technique used in the present study is shown in Higo et al. (2018) in detail.

3 RESULTS AND DISCUSSIONS

3.1 Water retention curves

Figure 6 describes the water retention curves. The wetting paths are located below the drying paths for both specimens, i.e., hysteresis is clearly observed. The degree of saturation for the loose sand begins to decrease at a suction of 1.9 kPa in the drying process, while it begins to increase at a suction of 1.9 kPa in the wetting process. On the other hand, for the dense sand, the degree of saturation begins to decrease at a suction of 3.5 kPa in the drying process, and it begins to increase at a suction of 4.2 kPa in the wetting process. Both the drying and wetting paths for the dense sand are located above those for the loose sand. These results indicate that the sand with higher bulk density shows the higher water retention capability than that with the lower bulk density.

3.2 X-ray images and trinarized images

In Fig. 6, the symbols from “aL” to “lL”, except for “hL”, and “aD” to “mD” indicate the points at which x-ray tomography and trinarization were performed for the loose and dense sands, respectively. Point “hL” is omitted because the images were of low quality due to mechanical problems. Figure 7 shows horizontal slices of the trinarized images, which indicates that the black portion (pore air phase) increases in the drying process, whereas the blue portion (pore water phase) increases in the wetting process.

3.3 Pore-scale distribution of degree of saturation

Figures 8(a) and 8(b) describe the histograms of pore volume-degree of saturation relationships for loose and dense sands, respectively. The bin of the histogram of the pore volume for the dense sand is 200 voxels, while that for the loose sand is 400 voxels. The bin of the histogram of the degree of saturation for the dense and loose sands is 2%. In these figures, the darker blue portion indicates higher frequency and the whiter portion indicates lower frequency.

The frequency concentrates on higher degree of saturation than 0.9 at point c^L and c^D where the global degrees of saturation of each specimen are relatively high, as shown in Fig. 6. Then, the higher frequency is
observed at the lower degree of saturation than 0.4 at eL and eD where the global degrees of saturation of each specimen are relatively low; namely, the specimens are desaturated. As the opposite trend to the drying process, the higher frequency region shifts to points kL and kD where the global degrees of saturation are relatively high during the wetting process.

It is clearly seen from Fig. 8 that the frequency for the dense sand exists at smaller pore volume than 8×10³ voxels, while that for the loose sand can be observed at larger pore volume than 8×10³ voxels. Namely, there exist larger pores in the loose sand than in the dense sand. This result indicates that the difference in pore volumes of sands with different bulk density can be evaluated using the Voronoi tessellation method.

It is also seen from Fig. 8 that the higher frequency region extends in the upper left direction during the drying process (from points cL to eL and from points cD to eD); namely, the pore water tends to be desaturated from larger pores than smaller pores. The higher frequency region moves to where degrees of saturation are higher than 0.9 in smaller pore volumes, and then the frequency of lower degrees of saturation in larger pore volumes is still relatively high during the wetting process (from points lL to kL and from points lD to kD).

Therefore, the Voronoi tessellation reveals the fact that the water retention capability is lower in larger pores and vice versa.

Higo et al. (2018) confirmed through the Voronoi tessellation that some small pores showed lower degrees of saturation for the loose sand even in a globally high saturation regime (point cD) that was not confirmed by calculating local degrees of saturation for cubic subsets placed in trinarized volumes. As shown in Fig. 8, the same trend as the above-mentioned result can be also observed for the dense sand at point cD. It is therefore that the Voronoi tessellation method enables us to quantify the pore-scale distribution of degree of saturation in sands with different bulk density.

4 CONCLUSIONS

The Voronoi tessellation technique has been applied to CT images obtained during drying-wetting process for sands with different bulk density. This technique can enable us to quantify the pore-scale distribution of degree of saturation in sands with different bulk density during drying and wetting processes. It is found that smaller pores show higher water retention capability than larger pores, while some small pores show lower degrees of saturation even in a globally high saturation regime.

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Fig. 8. Histograms of pore volume-degree of saturation relationships for loose and dense sands.