GLUON FIELD STRENGTH CORRELATION FUNCTIONS WITHIN A CONstrained Instanton MODEL

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We suggest a constrained instanton (CI) solution in the physical QCD vacuum which is described by large-scale vacuum field fluctuations. This solution decays exponentially at large distances. It is stable only if the interaction of the instanton with the background vacuum field is small and additional constraints are introduced. The CI solution is explicitly constructed in the ansatz form, and the two-point vacuum correlator of gluon field strengths is calculated in the framework of the effective instanton vacuum model. At small distances the results are qualitatively similar to the single instanton case, in particular, the form factor $D_1$ is small, which is in agreement with the lattice calculations.

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I. INTRODUCTION

The non-perturbative vacuum of QCD is densely populated by long-wave fluctuations of gluon and quark fields. The order parameters of this complicated state are characterized by the vacuum matrix elements of various singlet combinations of quark and gluon fields, condensates: $\langle \bar{q}q \rangle$, $\langle F_{\mu\nu}^a F_{\mu\nu}^a \rangle$, $\langle \bar{q}(i\sigma_{\mu\nu} F_{\mu\nu}^a)\bar{q} \rangle$, etc. The nonzero quark condensate $\langle \bar{q}q \rangle$ is responsible for the spontaneous breakdown of chiral symmetry, and its value was estimated a long time ago within the current algebra approach. The nonzero gluon condensate $\langle F_{\mu\nu}^a F_{\mu\nu}^a \rangle$ through trace anomaly provides the mass scale for hadrons, and its value was estimated within the QCD sum rule (SR) approach. The importance of the QCD vacuum properties for hadron phenomenology has been established by Shifman, Vainshtein and Zakharov [1]. They used the operator product expansion to relate the behavior of hadron current correlation functions at short distances to a small set of condensates. The values of low-dimensional condensates were obtained phenomenologically from the QCD SR analysis of various hadron channels.

Later on the nonlocal vacuum condensates or vacuum correlators have been introduced [2-4]. They describe the distribution of quarks and gluons in the non-perturbative vacuum. Physically, it means that vacuum quarks and gluons can flow through the vacuum with nonzero momentum. From this point of view the standard vacuum expectation values (VEVs) like $\langle \bar{q}q \rangle$, $\langle \bar{q}D^2 q \rangle$, $\langle g^2 F^2 \rangle$, etc. appear as expansion coefficients of the quark $M(x) = \langle \bar{q}(0)\hat{E}(0,x)q(x) \rangle$ and gluon $D^{\mu\nu,\rho\sigma}(x)$ correlators in a Taylor series in the variable $x^2/4$.

The correlator $D^{\mu\nu,\rho\sigma}(x)$ of gluonic strengths

$$D^{\mu\nu,\rho\sigma}(x - y) \equiv \langle [T_F F_{\mu\nu}(x)\hat{E}(x,y)F_{\rho\sigma}(y)\hat{E}(y,x)] \rangle,$$  \hspace{1cm} (1)

may be parameterized in the form consistent with general requirements of the gauge and Lorentz symmetries as [4]:

$$D^{\mu\nu,\rho\sigma}(x) = \frac{1}{24} \langle F^2 \rangle \left\{ (\delta_{\mu\nu}\delta_{\rho\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho})[D(x^2) + D_1(x^2)] + \right.$$  
$$\left. + (x_\mu x_\nu\delta_{\rho\sigma} - x_\mu x_\rho\delta_{\nu\sigma} + x_\nu x_\rho\delta_{\mu\sigma} - x_\nu x_\sigma\delta_{\mu\rho}) \frac{\partial D_1(x^2)}{\partial x^2} \right\},$$  \hspace{1cm} (2)

$^1$We follow the convention when the coupling constant is absorbed into the gauge field $A_\mu(x)$. 

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where $E(x, y) = P \exp \left(i \int_\nu^y A_\mu(z)dz^\mu \right)$ is the path-ordered Schwinger phase factor (the integration is performed along the straight line) required for gauge invariance and $A_\mu(z) = A^a_\mu(z) \frac{\lambda^a}{2}, \quad F_{\mu\nu}(x) = F_{\mu\nu}^a(x) \frac{\lambda^a}{2}, \quad F_{\mu\nu}^a(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) + f^{abc} A^b_\mu(x) A^c_\nu(x).$ The $P-$exponential ensures the parallel transport of color from one point to other. In $\langle \cdot \rangle$, $\langle F^2 \rangle = \langle F_{\mu\nu}^a(0) F_{\mu\nu}^a(0) \rangle$ is a gluon condensate, and $D(x^2)$ and $D_1(x^2)$ are form factors which characterize nonlocal properties of the condensate in different directions. They are normalized at zero by the conditions $D(0) = 1, D_1(0) = 1 - \kappa$, that depend on the dynamics considered. For example, for the self-dual fields $\kappa = 1$, while in the Abelian theory without monopoles the Bianchi identity provides $\kappa = 0$.

The gluon correlators $D_{\mu\nu\rho\sigma}(x)$ are involved in an analysis of the spectrum of bound states of heavy $Q\bar{Q}$ systems. The level shift depends on the parameter $\lambda$, where $\tau = 4/m_Qa^2$ is the typical time of the low lying levels of the system, and $\lambda$ is the correlation length of the gluon correlator $\lambda$ defined as $D(x \to \infty) \sim \langle F^2 \rangle \exp(-|x|/\lambda).$ Thus, at large distances the physically motivated asymptotics of the correlator is exponentially decreasing. The gluon correlators are the base elements of the stochastic model of vacuum and in the description of high-energy hadron scattering.

Measuring the correlation length and vacuum field correlators was the motivation to investigate these quantities on the lattice. New high-statistical LQCD measurements of the gauge - invariant bilocal correlator of the gluon field strengths have become available down to a distance of 0.1 fm. Recently, the field strength correlators have also been studied in the effective dual Abelian Higgs model in and QCD sum rule approach. In all these approaches (see also [10]), the exponential decay of the correlators at large distances has been observed. However, these investigations still omit a small and intermediate distances behavior of the nonlocal condensates.

On the other hand, in QCD there is an instanton [11], a well known nontrivial nonlocal vacuum solution of the classical Euclidean Yang - Mills equations with finite action and size $\rho$. The importance of instantons for QCD is that it is believed that an interacting instanton ensemble provides a realistic microscopic picture of the QCD vacuum in the form of instanton liquid (see, e.g., a review [14]). It has been argued on phenomenological grounds that the distribution of instantons over sizes is peaked at a typical value $\rho_c \approx 1.7$ GeV$^{-1}$ and the liquid is dilute in the sense that mean separation between instantons is much larger than the average instanton size.

In our previous work [13], we have shown that the instanton model of the QCD vacuum provides a way to construct nonlocal vacuum condensates. Within the effective single instanton (SI) approximation we have obtained the expressions for the nonlocal gluon $D^{\mu\nu\rho\sigma}(x)$ and quark $M_f(x)$ condensates and derived the average virtualities of quarks $\lambda_q^2$ and gluons $\lambda_g^2$ in the QCD vacuum. It has been found that due to specific properties of the SI approximation (self-duality of the field) the $D_1$ gluon form factor is exactly zero. The behavior of the correlation functions demonstrates that in the SI approximation the model of nonlocal condensates can well reproduce the behavior of the quark and gluon correlators at short distances. Really, the quark and gluon average virtualities, defined via the first derivatives of the nonlocal condensates $M_f(x^2)$, $D_1(x)$ at the origin,

$$\lambda_q^2 = -\frac{8}{M_f(0)} \frac{dM_f(x^2)}{dx^2} \bigg|_{x=0} = 2 \frac{1}{\rho_c^2}, \quad \lambda_g^2 = -\frac{8}{D_1(x^2)} \frac{dD_1(x^2)}{dx^2} \bigg|_{x=0} = 24 \frac{1}{5} \frac{1}{\rho_c^2}, \quad (3)$$

are connected with VEVs that parameterize the QCD SR,

$$\lambda_q^2 = \langle \langle \bar{q}D^2q \rangle \rangle, \quad \lambda_g^2 = \langle \langle \bar{q}f^3 \rangle \rangle = 2 \langle \langle \bar{q}f^2 \rangle \rangle - 2 \langle \langle \bar{g}^3J^2 \rangle \langle \rangle \rangle.
\quad (4)$$

Where $\langle \langle \cdot \rangle \rangle$ is the distribution of physically argued distributions at large distances. In asymptotics, we have found $M_f(x \to \infty) \sim \rho_c^2/x^2$ and $D_1(x \to \infty) \sim \rho_c^4/x^4$ for the quark and gluon correlators, respectively. Thus, the SI solution over mathematical vacuum provides too slow asymptotics at large distances. We should conclude that in order to have a realistic model of vacuum correlators, the important effects of instanton interaction with the long - wave vacuum configurations have to be included.

The key point in the picture of realistic instanton vacuum is the interaction of pseudoparticles in the vacuum. In [13], the interaction of a SI with an arbitrary weak external field has been examined and dipole-dipole forces in a far separated instanton-anti-instanton system derived. Later in [20], this background field has been interpreted
as a field of large-scale QCD vacuum fluctuations, and the influence of the quark and gluon condensates on the instanton density has been considered. The main assumption of the instanton liquid models [12] is the dominance of the instanton component in the vacuum and that, in particular, the gluon condensate is saturated by weakly interacting instanton liquid. In deriving instanton ensemble properties the instanton-anti-instanton interaction at intermediate separation start to play the crucial role in stabilization of the liquid [13]. However, it turns out that the final result strongly depends on the field ansatz for an instanton-anti-instanton configuration [21][13]. Further, in all instanton-anti-instanton ansatze suggested the influence of the physical vacuum on an instanton profile function has not been taken into account and the profile has only power decreasing asymptotics, which contradicts the expectations concerning the vacuum field correlators. Moreover, it is known that the instanton liquid is not responsible for large-scale effects like confinement [14]. Another point is that the instanton density $n_c$ in the instanton liquid models is normalized by the value of the gluon condensate $\langle \sum F_{\mu\nu}^a F^{a\mu\nu} \rangle = 0.012$ GeV$^4$ obtained in [1] from an analysis of charmonium spectrum. More recently in [22], a detailed analysis based on charmonium, bottomium and heavy-light mesons have led to a twice larger value of the gluon condensate $\langle \sum F_{\mu\nu}^a F^{a\mu\nu} \rangle = 0.023$ GeV$^4$. Indefiniteness in the normalization provides a window for existence of a large-scale field component in the QCD vacuum along with short-scale instantons.

In the present work, assuming dominance of the weak interacting instanton liquid in the QCD vacuum, we suggest that there is also a weak residual component of the vacuum field with a large correlation length $R$ of the order of the confinement size. We are going to show, assuming only very general properties of a weak large-scale vacuum field, that it deforms an instanton at large distances leading to exponentially decreasing asymptotics of the instanton profile and the instanton induced vacuum field correlators. The vacuum model considered is a two-phase one. The large-scale phase is described by the background field and dominates at distances compared with the confinement size. The short-scale phase is dominated by instantons and is responsible for the spontaneous breaking of chiral symmetry and the solution of the $U_A(1)$ problem. The vacuum model suggested reveals a definite non-locality mechanism in the framework of QCD. We shall illustrate this by analyzing the vacuum gluon form factors $D(x)$ and $D_1(x)$. Unfortunately, the normalization of contributions from different phases to the gluon condensate is not fixed by the instanton model and remains as a free parameter, but the form of the correlation functions can be described in detail. The latter is the main motivation of the present work. The determination of the vacuum field correlators is one of the main tasks of the theory in describing the non-perturbative dynamics at large distances compatible with the typical hadron size.

The paper is organized as follows: in Section 2 the instanton field in the background of weak large-scale vacuum fluctuations is considered; solutions of the vacuum field equations for small and large distances are analyzed separately and the constrained instanton solution interpolating two asymptotics is suggested in an ansatz form. In Section 3, the space coordinate behavior of the nonlocal correlators of the gluon field strengths is found and the main asymptotics of the correlators $D(x)$, $D_1(x)$ at large distances are derived. These asymptotics are driven by the strength and correlation length of the large-scale vacuum fluctuations, rather than by their form.

II. CONSTRAINED INSTANTONS IN QCD VACUUM

The classical Yang-Mills equations in the Euclidean space

$$D_\mu F_{\mu\nu}(x) = 0,$$

where the covariant derivative is $D_\mu = \partial_\mu - iA_\mu^a \tau_a/2$, have an instanton (+) (−) for an anti-instanton as a self-dual finite-action solution with topological charge, $Q = \pm 1$:

$$A^{a,\pm}_{\text{sing},\mu}(x; x_0) = 2O^{b}_{I} \eta^{a,\pm}_{\mu\nu} (x - x_0)_\nu \varphi^{I}_{b}(x - x_0)$$

$$\varphi^{I}_{b}(x) = \frac{\rho^2}{x^2(x^2 + \rho^2)},$$

(5)

(6)

localized in size $\rho$. In $I$, $x_0$ is the position of the instanton center, $O^{ab}_{I}$ is the orthogonal matrix of instanton global orientation in the color space and $\eta^{a,\pm}_{\mu\nu} = \epsilon_{\alpha\mu\nu} \mp \frac{1}{2} \epsilon_{abc} \epsilon_{b\mu\nu}$ are 't Hooft symbols. The solution (5) is written in the singular gauge within the $SU(2)$ subgroup (with generators $\tau^a$) of the $SU_c(3)$ theory. This classical field configuration reflects the symmetries of the initial theory in terms of collective variables corresponding to translational transformations, rotational symmetry in color space and conformal transformations.

The solution (5) is given in mathematical vacuum and has an unpleasant property of a very slow decay at large distances noted in the introduction. This situation is inadequate since the physical vacuum is not empty but looks like a medium densely populated by large-scale vacuum field fluctuations. In the background of the large scale fluctuations

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there are developed non-perturbative fluctuations of a smaller size among those instantons which play a dominating role. The long-wave gluon vacuum field, which is the background for a selected instanton, may be of a more general origin and phenomenologically can be parameterized by the vacuum correlation functions of the gluon operators contributing to the corresponding nonlocal condensates.

What is important is that at random (stochastic) background vacuum field with fixed vacuum expectation values of singlet operators the scale invariance of the effective theory is spoiled already at the quasi-classical level and the instantons are no longer exact solutions of the field equations and the Dirac operator has no zero modes.

The similar situation has been observed in the standard electroweak Yang-Mills-Higgs model. There, the background of Higgs field with nonzero vacuum value \( \langle \varphi \rangle \) and coupling \( \lambda \) affects the instanton configuration \[23\]. In the presence of (even small) effects violating scale invariance of the initial theory, instanton solution does not exist at all. Nevertheless, as it was stated in \[23\] and fairly explained in \[24\]–\[26\], if the Higgs field is rather weak and some additional constraints are introduced, there may be constructed an approximate solution, so-called, constrained instanton (CI). These constraints limit the degree of freedom along the size \( \rho \) parameter. The constrained solution at small distances \( |x| << \sqrt{\lambda} \langle \varphi \rangle^{-1} \) approximately has the form of an instanton, and at large distances \( |x| >> \sqrt{\lambda} \langle \varphi \rangle^{-1} >> \rho \) has the asymptotics of massive (gauge boson) particle \( \exp(-g \langle \varphi \rangle |x|/\sqrt{\lambda}) \). In \[24\], it has also been noted that the gauge field propagator of the CIs is exponentially large at large \( |x| \) and thus does not affect the long-range behavior of the theory.

The aim of this section is to show that an analogous phenomenon takes place in QCD considering an instanton field \( A^I_\mu(x) \) in the physical vacuum. In distinction with the Yang-Mills-Higgs model, in pure QCD there is no Higgs field from the beginning and it is the long-scale vacuum gluon field, \( b_\mu(x) \), that models a source perturbing an instanton at large distances. The deep reason of this effect is in an existence of the quantum anomaly in the trace of energy-momentum tensor \[27\]. It will be shown that the presence of this background field, characterized by its vacuum expectation value \( \left\langle \left( F_{b,\mu\nu}^a \right)^2 \right\rangle_b \) and correlation length \( R \) (introduced below), sets the final scale and defines the deformation of the instanton solution in the asymptotic region \( |x| \gtrsim \left[ \left( \left\langle \left( F_{b,\mu\nu}^a \right)^2 \right\rangle_b \right)^{1/3} R \right]^{-1/3} \ll \rho \). Here and below, \( \langle ... \rangle_b = \int d\sigma [b] ... \) means the average over nonperturbative random background field weighted with some measure \( d\sigma [b] \). This solution is stable against shrinking the instanton to a point if some constraints are added. By analogy with the solutions analyzed in the Yang-Mills-Higgs model \[24\], we shall call these interpolating fields constrained (or deformed) instanton solutions.

In the following we analyze the vacuum field configuration of the single (constrained) instanton \( A^I_\mu(x) \) of fixed size and orientation in the color space in the background of the large-scale topologically neutral random vacuum field \( b_\mu(x) \)

\[
A_\mu(x) = A^I_\mu(x, x_0) + b_\mu(x),
\]

with gauge transformation property

\[
A_\mu(x) \rightarrow U^\dagger(x) A_\mu(x) U(x) + iU^\dagger(x) \partial_\mu U(x),
\]

where \( U(x) \) is a gauge transformation matrix. Considering the field ansatz \[3\], one has to take the instanton field \( A^I_\mu(x) \) in the singular gauge \[13\]

\[
A^{CI,\pm\pm}_{\text{sing},\mu}(x) = 2O^\pm_b \eta_{\mu\nu}^b (x - x_0)_{\nu} \varphi^I_g(x - x_0),
\]

\[
x^2 \varphi^I_g(x) \big|_{x \rightarrow 0} = 1, \quad \varphi^I_g(x) \big|_{x \rightarrow \infty} = 0.
\]

The last conditions mean that the constrained instanton has a finite action and a modulo unit topological charge, but, in general, ceases to be self-dual field. In the coordinate space the instantons in this specific gauge fall off rapidly enough to provide a weak interaction with the background field and the quasi-classical approach is justified. The weakness of the interaction allow us in the following to neglect the back reaction of the instanton on the background field. As to nonperturbative background field it is convenient to choose it in the Fock-Schwinger gauge \[28\]

\[
(x - x_0)_\mu b^\mu(x - x_0) = 0.
\]

In the following we put the instanton center \( x_0 \) to the origin of coordinates \( x_0 = 0 \).

\[2\] The similar model has been considered earlier in \[37\].
As an illustrative model for the background field, one can keep in mind the self-dual homogeneous vacuum gluon field $b_{\mu}(x) = \frac{2}{n^a}b_{\mu
u}x_\nu$, where $n^a$ is the orientation vector in color space and $b^{\mu\nu}$ is the constant field-strength tensor [30]. The corresponding measure $\int d\sigma\left[b\right] = \int_0^\infty db dD \left(b\right) \int d\Omega \int d\Omega_e$ averages over field amplitude and its orientations in configuration and color spaces. This field configuration with infinite correlation length $R = \infty$ and infinite topological charge quite correctly describes situation at small and intermediate distances comparable with instanton size, but at larger distances the effect of finite correlation length of physical background becomes important. Introduction of finite correlation length can be imagined as the inclusion of domain structure in the vacuum [29]. This kind of considerations are in the base of the stochastic vacuum model [3]. In the absence of a consistent theoretical approach to the large distance dynamics one is led to elaborate the problem phenomenologically.

We assume that at small distances the CI field dominates and the background field $b_\mu(x)$ is regarded as a perturbation on $A^{CI}_\mu(x)$. At distances much larger compared to the instanton size $\rho$ the background field $b_\mu(x)$ is still weak, but strong enough to deform and suppress the instanton field.

The field strength can be written as

$$F^{\mu\nu\mu
u}_a[A^{CI} + b] = F^{\mu\nu\mu
u}_a[A^{CI}] + F^{\mu\nu\mu
u}_b[A^{CI}, b] + \Delta F^{\mu\nu\mu
u}_a[A^{CI}, b],$$

where $F^{\mu\nu\mu
u}_a[A^{CI}]$, $F^{\mu\nu\mu
u}_b[A^{CI}]$ and

$$\Delta F^{\mu\nu\mu
u}_a[A^{CI}, b] = f_{abc}(A^{CI}_\mu b^c_{\nu} + A^{CI}_{\nu} b^c_{\mu}),$$

and the effective Euclidean action of the instanton in the random background field becomes

$$S_E \approx \frac{1}{4g^2} \left\langle \int d^4x \left\{ F^{\mu\nu\mu
u}_a[A^{CI}] F^{\mu\nu\mu
u}_a[A^{CI}] + F^{\mu\nu\mu
u}_b[A^{CI}, b] \Delta F^{\mu\nu\mu
u}_a[A^{CI}, b] \right\} \right\rangle_b.$$

In deriving these expressions we have used the color-singlet properties of the large-scale vacuum on an average

$$\left\langle F^\mu_{b\mu
u} \right\rangle_b = 0$$

and average over relative orientation of instanton and background field in color space. We also neglected the terms higher order in interaction. Below, these terms will effectively be accumulated in the form of constraints below and, what is important in the present consideration, they do not influence the form of the solution at asymptotics.

Similar to Affleck analysis [2], we come to the conclusion that for a background field configuration $b_\mu(x)$ with a fixed nonzero condensate value no instanton solution exists. This can be seen from the rescaling $x \rightarrow ax$, $A^{CI}_\mu(x) \rightarrow a^{-1} A^{CI}_\mu(ax)$, $b_\mu(x) \rightarrow b_\mu(ax)$, preserving finite vacuum average $\left\langle F^2_{b\mu\nu} \right\rangle_b = const$, under which $S_E$ transforms to

$$S_E \rightarrow \frac{1}{4g^2} \left\langle \int d^4x \left\{ F^{\mu\nu\mu
u}_a[A^{CI}] F^{\mu\nu\mu
u}_a[A^{CI}] + a^{-2} F^{\mu\nu\mu
u}_b[A^{CI}, b] \Delta F^{\mu\nu\mu
u}_a[A^{CI}, b] \right\} \right\rangle_b.$$

If $A^{CI}_\mu(x)$ is a stationary point, then $dS_E/da = 0$ and the action is minimized by the instanton of vanishing size. Thus, given any field configuration we can always rescale it to get smaller action, except in the trivial case.

Now, let us consider the problem from the point of view of the equations of motion for the deformed instanton in the background of large-scale random vacuum fluctuations, that follows from (12).

$$D^{\mu\nu}_a[A^{CI}] F^{\mu\nu\mu
u}_a + \int f_b x^{bkl} \left( A^{CI\nu}_a k (F^b_{\mu\nu\mu}\left(\alpha x\right)) - A^{CI\mu}_a k (F^b_{\mu\nu\mu}\left(\alpha x\right)) \right)_b = 0.\quad (15)$$

In the Fock-Schwinger gauge the background field has a representation in terms of its strength

$$b_\mu^a(x) = \int_0^1 d\alpha F^{a}_{b\nu\mu}(\alpha x) x^\nu\quad (16)$$

and the bilinear field averages become

$$\left\langle b_\mu^a(x) b_\nu^b(x) \right\rangle_b = \int_0^1 d\alpha \int_0^1 d\beta \alpha \beta x_\nu x_\sigma \left\langle F^{a}_{b\nu\mu}(\alpha x) F^{b}_{b\sigma\nu}(\beta x) \right\rangle_b.\quad (17)$$

In the non-Abelian case the correlator in the integrand of (17) is not gauge-invariant, however, in the Fock-Schwinger gauge this correlator coincides with the gauge-invariant correlator in which field-strengths are connected by the
where the gauge-invariant local operator \( O \) is defined via the form factors

\[
\tilde{B} (z^2) = \tilde{D} (z^2) + \tilde{D}_1 (z^2) + z^2 \partial \tilde{D}_1 (z^2) / \partial z^2
\]

(19)
is defined via the form factors \( \tilde{D} (z^2) \) and \( \tilde{D}_1 (z^2) \) parameterizing the gauge-invariant two-point correlator \( \langle b \rangle \) of the background field strengths, with normalization \( \tilde{D}(0) = \kappa, \tilde{D}_1 (0) = 1 - \kappa \). The contribution of the background field to the gluon condensate is denoted by \( \langle F_b^2 \rangle_b \). With these definitions the equations of motion of the CI field interacting with a random large-scale vacuum fluctuation field, (13), can be cast in the form

\[
D_{\mu} [A^{CI}] F_{\mu\nu}^{CI, b} (x) - \frac{N_c \langle F_b^2 \rangle}{24(N_c^2 - 1)} x^2 \Phi (x^2) A_{\mu}^{CI, a} (x) = 0,
\]

(20)

where

\[
\Phi (x^2) = 4 \int_0^1 d\alpha \int_0^1 d\beta \alpha \beta \tilde{B} \left[ (\alpha - \beta)^2 x^2 \right], \quad \Phi (0) = 1,
\]

(21)

and \( N_c \) is the number of colors.

Let us discuss the properties of the solution of Eq. (20). In the absence of the background field \( \langle F_b^2 \rangle_b = 0 \) there exists an instanton solution [3]. For \( \langle F_b^2 \rangle_b \) small enough, such that \( \langle F_b^2 \rangle_b \ll 1/\rho^4 \), we should expect to find a solution of (20) in perturbation theory in small parameter \( \langle F_b^2 \rangle_b \rho^4 \), which reduces to (4) when \( \langle F_b^2 \rangle_b \to 0 \). However, such a perturbative solution does not exist. The reason is that for the higher order in perturbation terms appropriate finite action boundary conditions at large distances cannot be enforced [2].

The operators that act on higher order terms possess zero mode \( \partial A_{\mu}^{CI} / \partial \rho \)

\[
\nabla_{\mu} \left( \nabla_{\rho} \frac{\partial A_{\mu}^{CI}}{\partial \rho} - \nabla_{\nu} \frac{\partial A_{\mu}^{CI}}{\partial \nu} \right) + i \left[ \frac{\partial A_{\mu}^{CI}}{\partial \rho}, F_{\mu\nu} [A^{CI}] \right] = 0
\]

(22)

which determines \textit{a priori} the behavior of perturbative around instanton terms at infinity. A way of getting around this difficulty [24] is to extremize the action \( S_E \), (12), subject to a constraint. The choice of an explicit form of the constraint is quite arbitrary. In [24] it has been proposed the global constraint of the general form

\[
C_{constr}^{nl} (A) = \int d^4 x \left[ O_d (A) - O_d (A^{CI}) \right] = 0,
\]

where the gauge-invariant local operator \( O_d (A) \) has a canonical dimension \( d > 4 \). The relevant stationary configuration will be a solution of the equations of motion (20) but with the constraint term added into the right-hand side

\[
\frac{\delta S_E (A)}{\delta A_{\mu} (x)} \bigg|_{A_{\mu}^{CI}} = \sigma \frac{\delta C_{constr} (A)}{\delta A_{\mu} (x)} \bigg|_{A_{\mu}^{CI}}.
\]

(23)

The Lagrange multiplier \( \sigma \) in (23) is to be determined order by order in perturbative theory in \( \langle F_b^2 \rangle_b \rho^4 \), which provides the correct boundary conditions for the higher order terms. The constrained instanton \( A^{CI} \) is the unique solution of (23) obtained by this procedure, the \( A^{CI} \)-solution turns to (6) when \( \langle F_b^2 \rangle_b \rho^4 \to 0 \). Unfortunately, this kind of constraints is nonlinear and the higher order terms in \( \langle F_b^2 \rangle_b \rho^4 \), depending on the constraint, are difficult to evaluate in practice.

Another way has been suggested in [26], where a linear constraint of the general form

\[
C_{constr}^l (A) = \int d^4 x tr \left\{ \left( A_{\mu} (x) - A_{\mu}^{CI} (x) \right) F_{\mu}^o (x) \right\} = 0
\]

(24)
has been introduced. It has been proposed,\cite{24,25}, instead of fixing the constraint to solve the equation for $A_{\mu}^{CI}(x)$, which is almost an impossible task, to choose $A_{\mu}^{CI}(x)$ first and then find the constraint $f_{\nu}^\mu(x)$ itself by substituting $A_{\mu}^{CI}(x)$ into the left hand side of the constrained equation (24). In this way, the freedom in choosing the constraint can be used to find it by a given solution.

One can also restrict by considering only the local operators defining constraints that fall down at infinity more rapidly than the interaction term in Eq. (23). Under this condition, it is easy to obtain the behavior of the instanton in the background field at distances far from the instanton center. This large-distance asymptotics of CI, like its behavior at small distances $A_{\mu}^I(x;\rho)$, is a constraint-independent part of the solution.

We are interested in the asymptotic behavior of the function $\Phi(x^2)$, Eq. (21), where the interaction term becomes dominant over the instanton self-interaction. It is nice that the leading asymptotics of $\Phi(x^2)$, $\Phi(x^2) \sim R/|x|$, where $R$ is a correlation length, is independent of a particular form of the function $R(x)$ like its behavior at small distances.

Asymptotic behavior of the instanton solution deformed by large-scale vacuum fluctuations at large Euclidean distances $|x| \to \infty$ can be derived from the analysis of the equation

$$\partial_\mu(\partial_\mu A_{\mu}^{CI} - \partial_\nu A_{\mu}^{CI}) - \eta_0^3 |x| A_{\mu}^{CI} = 0,$$

which follows from Eq. (20). Due to a decreasing character of the field asymptotics, $A_{\mu}^{CI} \to 0$ at $|x| \to \infty$, only linear terms in the short-wave CI field $A_{\mu}^{CI}$ are kept in the kinetic part of equation (27). For the profile function $\varphi_g^{as}(x^2)$ defined by $\varphi_{g\mu} = \eta_{\nu\mu}^{\alpha} A^\alpha_{\mu}$ in the following)

$$A_{\mu}^{CI,\alpha}(x) = \varphi_{\alpha\mu} \frac{\eta_{\nu\mu}}{\nu^2} \varphi_{g}^{as}(x^2),$$

we find from the asymptotic equation (27) the large distance solution

$$\varphi_{g}^{as}(x^2) \sim K_{4/3} \left[ \frac{2}{3} (\eta_0 |x|)^{3/2} \right],$$

where $K_\nu(z)$ is the modified Bessel function with index $\nu$ having the asymptotic behavior $K_\nu(z) \to \sqrt{\frac{\pi}{2z}} e^{-z}$ as $z \to \infty$. We have to note that in the case of the homogeneous background field with infinite correlation length one get the equation similar to (27), but with the coefficient proportional to $x^2$ in the last term that results in the Gaussian form of asymptotics (c.f., (27)).

\footnote{At this point we have to note that the influence of the instanton ensemble on the instanton profile has been discussed in \cite{13}. The authors found that this interaction perturbs the self-interaction part by the term $\mu_{\nu}^2 A_{\mu}$. It turns out that numerically the coefficient $\mu_{\nu}^2$ strongly depends on the instanton-anti-instanton ansatz chosen \cite{21} and, as it is seen from (27), has subleading behavior in the limit of large $x$.}
The requirement of the instanton-background interaction being weak allows us to consider the dimensionless parameter \( \alpha_g \equiv \eta_g \rho \) a small value. It means that the region, where the instanton field dominates (small distances), and the region, where the background field dominates (large distances), are well separated and the large distance effects do not destroy the instanton. Then, the overall constant is determined by matching, at distances \( \rho \ll |x| \ll \eta_g^{-1} \), the leading terms of the expansions of \( A'^{CI}_a(x) \) at small distances, which is an instanton \( A'_a(x) \), and at large distances, which is an asymptotics \((29)\), \((30)\),

\[
A'^{CI,a}(x) = \frac{2x_{\mu}}{x^2} a_{4/3}^2 K_{4/3} \left[ \frac{2}{3} \eta_g |x| \right]^{3/2}, \quad \text{where} \quad a_{4/3} = \frac{2}{\Gamma (1/3) 3^{1/3}} \tag{31}
\]

is the normalization coefficient and \( \Gamma(z) \) is the Gamma-function.

Thus, we find that in the singular gauge the CI solution has an exponential decreasing character \((31)\) far from the instanton center. This behavior sharply differs from the power decreasing asymptotics of SI \((6)\). This modification of behavior follows from the fact that the instanton solution is considered in the physical vacuum populated by the large-scale gluon field fluctuations. The background field modifies the long-distance behavior of the instanton and instanton center. This behavior sharply differs from the power decreasing asymptotics of SI \((6)\). This modification of behavior follows from the fact that the instanton solution is considered in the physical vacuum populated by the large-scale gluon field fluctuations. The background field modifies the long-distance behavior of the instanton and leads to appearance of the “second scale” parameter \( \Lambda_g = 1/\eta_g \) (see \([15]\)) in gluon distributions. At the same time, the effect of long-wave vacuum fluctuations is not very essential for the behavior of the instanton at short distances.

Now we are looking for the constrained solution of \((20)\) in the ansatz form

\[
A'^{CI}_{\mu a}(x) = 2\eta_{\mu \nu} x_{\nu} \varphi_g(x^2) \tag{32}
\]

which has the behavior at small and large distances

\[
\varphi_g(x^2) = \frac{1}{x^2} \left\{ \frac{\rho^2}{(x^2 + \rho^2)} + (\eta_g |x|)^3 \varphi_1(1) \left( \frac{\rho}{|x|} \right)^2 \right\} \quad \text{at} \quad |x| \to 0,
\]

\[
\varphi_g(x^2) = \frac{2}{3} (\eta_g |x|)^{3/2} + \frac{\rho^2}{x^2} \varphi_1(1) (\eta_g |x|) \quad \text{at} \quad |x| \to \infty, \tag{33}
\]

where the forms of the expansions can be deduced from expanding the leading terms with respect to \( \frac{\rho}{|x|} \) and \( \eta_g |x| \), respectively. The first terms in the expansions are exact and constraint-independent ones, however, as it was shown in \([24]\), all higher order terms have dependence on the constraint chosen.

The condition of finiteness of the action, which is also constraint-independent one, should be imposed on the desirable solution. To this goal, let us rewrite the CI field strength as

\[
F'^{CI}_{\mu \nu}(x) = 4 \left[ \eta_{\mu \nu} \omega_1(x) + (x_{\mu} \eta_{\nu \rho} - x_{\nu} \eta_{\mu \rho}) x_\mu \omega_2(x) \right] \tag{34}
\]

with forms

\[
\omega_1(x) = x^2 \varphi_g^2(x^2) - \varphi_g(x^2), \quad \omega_2(x) = \varphi_g^2(x^2) + \frac{\partial \varphi_g(x^2)}{\partial x^2}, \quad x_\mu = x^2 + \rho^2. \tag{35}
\]

to be used further. With this parameterization we have the action

\[
S'^{CI}_{E} = \frac{1}{4g^2} \int d^4x \left[ F'^{\mu \nu}_{a}(x) F'^{\mu \nu}_{a}(x) \right] \tag{36}
\]

with the action density

\[
\left[ F'^{\mu \nu}_{a}(x) F'^{\mu \nu}_{a}(x) \right]^{CI} = 96 \left[ \omega_1^2(x) + \omega_2^2(x) \right], \tag{37}
\]

where

\[
\omega_3(x) = x^2 \omega_2(x) - \omega_1(x).
\]

Now, if we use the singular gauge for \( A'^{CI}_{\mu} \), then, to guarantee finiteness of the action, the condition has to be fulfilled

\[
x^2 \varphi_g(x^2) \mid_{x^2 \to 0} \to 1 + O(x^2). \tag{38}
\]

For further references we present here the well-known expressions for the SI profiles in the singular and regular gauges
\[ \varphi_{s}^{\text{sing.I}} (x^2) = \frac{\rho^2}{x^2 \rho^2}, \quad \varphi_{r}^{\text{reg.I}} (x^2) = \frac{1}{x^2}, \]  
\[ \omega_{1}^{\text{sing.I}} (x) = -\frac{\rho^2}{(x_\rho^2)^2}, \quad x^2 \omega_{2}^{\text{sing.I}} (x) = 2 \omega_{1}^{\text{sing.I}} (x), \]  
\[ \omega_{1}^{\text{reg.I}} (x) = -\frac{\rho^2}{(x_\rho^2)^2}, \quad \omega_{2}^{\text{reg.I}} (x) = 0, \]  
and gauge-independent expressions for the density action and the action itself

\[ [F_{\mu\nu}^a (x) F_{\mu\nu}^a (x)]^I = \frac{192 \rho^4}{(x_\rho^2)^4}, \quad S_E^I = \frac{8 \pi}{g^2}. \]  

By using the asymptotic properties of the CI solution \([83]\) and finite-action condition \([83]\), which are constraint-independent, we are able to construct ansatz. Certainly, this procedure is not unique and in principle one can impose further physical requirements to constrain the behavior of the solution in the intermediate region. These details, however, can be taken into account by choosing proper constraints. Thus, the freedom in choosing the constraint can be used to find it by a given solution, instead of solving complicated equations \([26]\).

Let us consider the following ansatze for the CI profile written in the singular gauge

\[ \varphi_1 (x^2) = \frac{1}{x^2} \frac{K_{4/3} [z_{\rho,x}]}{K_{4/3} [z_{\rho,0}]}, \]  
\[ \varphi_2 (x^2) = \frac{\mathcal{P}^2 (x^2)}{x^2 \rho^2}, \]  
\[ \varphi_3 (x^2) = \frac{\mathcal{P}^2 (x^2)}{x^2 (x^2 + \mathcal{P}^2 (x^2))}, \]  

where we have introduced the notation

\[ z_{\rho,x} = \frac{2}{3} \eta^{3/2} \left( x^2 + \rho^2 \right)^{3/4}, \]  
\[ \mathcal{P}^2 (x^2) = a_{4/3} \alpha_g^2 \eta^2 K_{4/3} [z_{0,x}], \]  
\[ \mathcal{P}^2 (0) = \rho^2. \]  

Note that all ansatze have easily identifiable instanton parameters. By translational invariance the center of CI can be shifted in \([83]\) - \([83]\) from the origin to an arbitrary position \(x_0\): \(x \rightarrow x - x_0\). The CI profile functions corresponding to these ansatze are shown in Figs. \([\text{1}]\) and \([\text{2}]\) along with the instanton profile \([89]\). Figs. \([\text{1}]\) and \([\text{3}]\) display the dependence of the profile function on the external field parameter \(\rho \eta\) and on the form of ansatz, respectively.

To make the difference more clear, we also take for illustration a large value of the parameter \((\rho \eta)^2 = 3\). We see that if at small distances the CI is close to the instanton form, then at large distances this solution has exponential asymptotics instead of power-like for the instanton.

Last two ansatze are similar to ones suggested in \([26]\), where the preference has been given to the \(\varphi_3 (x^2)\) form, since it has better convergent properties in the expansion of CI \([83]\). Moreover, with this profile the constrained solution in the regular gauge looks similar to the SI case

\[ \varphi_3^{\text{reg.CI}} (x^2) = \frac{1}{x^2 + \mathcal{P}^2 (x^2)}. \]  

In order to pass from the constraint instanton in a singular gauge to the instanton in a regular gauge one can translate the general gauge transformation \([8]\) into the form

\[ A_\mu^{CI} (x) \rightarrow \Omega^\dagger (x) A_\mu^{CI} (x) \Omega (x) + i \Omega^\dagger (x) \partial_\mu \Omega (x), \]  
\[ b_\mu (x) \rightarrow \Omega^\dagger (x) b_\mu (x) \Omega (x), \]  

with transformation matrix

\[ \Omega (x) = \frac{i \gamma^- x_\mu}{|x|}. \]
We have numerically calculated the dependence of the CI classical action, Eqs. (36) and (37), on the instanton size \( \rho \). This dependence for the three ansätze (13), (14) and (15) is shown in Figs. (3) and (4). We have to stress, that the profiles of the field \( A_{\mu}^{CI} \) and the action \( S_{E}^{CI} \) depend on the choice of constraint. However, the full effective action, with the terms coming from the Jacobian included, is constraint-independent [26]. The weak dependence of the action, \( S_{E}^{CI} \), on the profiles \( \varphi_{1,2,3} (x^2) \) (see Fig. (3)) indicates that in the region of parameters \( \rho \eta_g \lesssim 1 \), the influence of these additional terms is small and the exponential part of the action, \( S_{E}^{CI} \), can be used as a good approximation. We see in Fig. (3) that the CI action is larger than the instanton one and monotonically grows with the instanton size. It is natural because the CI-“solution” does not self-dual one and does not realize the minimum of the action. Instead, it represents the bottom of the valley parameterized by the quasi-zero mode \( \rho \).

We are not going to discuss further details in construction of the total effective CI action, that takes into account small quantum oscillations around the nonperturbative configuration (6) and the interaction of constrained instantons, and postpone them until further publication. Just point out that other effects dominating the effective action at small \( \rho \) come from the running coupling constant \( g^2 (\rho) \) and the path integral measure over the size of instanton \( d\rho / \rho^2 \). It is well known that in the model of the coupling constant, which freezes it to a constant at some large \( \rho_0 \), the corrected action

\[
S_{E}^{TOT} (\rho) = \frac{1}{4g^2 (\rho)} \int d^4x \left[ F^a_{\mu \nu} (x) F^a_{\mu \nu} (x) \right]^{CI} + 5 \ln \rho,
\]

as a function of the instanton size, has a minimum. The position of the minimum is correlated with the freezing parameter \( \rho_0 \), which can be chosen to provide the value \( \rho_{\text{min}} \approx 2 \text{ GeV}^{-1} \), and the environment of the large-scale vacuum fluctuations makes the minimum more prominent (see [31] for recent discussion of similar results). However, the typical CI action at \( \rho_{\text{min}} \) is rather large, its numerical value being around 25. This means that the configurations with small number of instantons and anti-instantons are not important statistically and suggests that the interacting instanton and anti-instanton ensemble could be a more important type of configurations. The leading interaction term of a widely separated instanton - anti-instanton pair in physical vacuum as described by Eqs. (13) - (15) falls with separation \( L \) like \( \exp (-4/3 (\rho_0 L)^{3/2}) \) and differs from power-like decreasing behavior found in [13] - [15] in unconstrained case. In the following, considering the nonlocal properties of gluon condensate, we accept that the instanton liquid is formed due to the instanton-anti-instanton interaction and the instanton density \( n_c \) and size \( \rho_c \) are fixed [13]: \( n_c \approx 1 \text{ fm}^{-4}, \rho_c \approx 1/3 \text{ fm} \).

III. SHORT-RANGE VACUUM CORRELATORS IN THE CONSTRAINED INSTANTON MODEL.

Within the model considered the full gluon correlator may be conventionally written by using the expression for the field strength (1) as

\[
\langle : F_{\mu \nu} [A^{CI} + b] (x) F_{\rho \sigma} [A^{CI} + b] (y) : \rangle = \langle F^{CI}_{\mu \nu} (x) F^{CI}_{\rho \sigma} (y) \rangle + \langle : F_{\mu \nu} (x) F_{\rho \sigma} (y) : \rangle + \langle : \Delta F_{\mu \nu} [A^{CI}, b] (x) \Delta F_{\rho \sigma} [A^{CI}, b] (y) : \rangle^{\text{interf}},
\]

where the brackets \( \langle \cdot \rangle \) mean averaging over vacuum fluctuations (6) and we do not display Schwinger phase factors explicitly. The last term represents the interference of short- and large-scale fields and will be discussed below. For the large-scale correlator (13) we have already suggested the model for the form factor \( \hat{B} (x^2) \) in (25). Now, let us calculate the short-range part of the gluon correlator.

Let us construct the correlator \( D_{\mu \nu, \rho \sigma} (x - y) \) of gluonic strengths (6) in the quasi-classical approximation by using the CI solutions given by Eqs. (12) and (13) - (15). We will use a reference frame where the instanton sits at the origin and a relative coordinate \( (x - y)^\mu \) with respect to the position of the instanton center is parallel to one of the coordinate axes, say \( \mu = 4 \), serving as a “time” direction (i.e., \( \vec{x} - \vec{y} = 0, x_4 - y_4 = |x - y| \), and reduce the path ordered exponential to an ordinary exponential

\[
\hat{E} (x, y) = P \exp \left(i \int_x^y A_{\mu}^{CI} (z) dx^\mu \right) = L (x) L (y)
\]

with

\[
L (x) = \exp \left( \mp i \tau_+ \frac{\vec{x}}{|x|} (|x|, x_4) \right) = \mp i \tau_+ \vec{x} (x),
\]

\[10\]
where
\[
\alpha \left( \left| x \right|, x_4 \right) = \left| x \right| \int_0^{x_4} dt \varphi_g \left( \left| x \right|^2 + t^2 \right),
\]
\[
\tau^\pm = (\mp i, \tau), \quad \bar{x}_0 \left( x \right) = \cos \alpha \left( x \right), \quad \bar{x}_i \left( x \right) = \left( x^i / \left| x \right| \right) \sin \alpha \left( x \right).
\]

The factor \( L \left( x \right) \) coming from the Schwinger exponent can be accumulated in the definition of the field. This representation of the field may be called the axial gauge representation \( A_\mu \left( z \right) n^\mu = 0 \), since in this gauge with the vector \( n_\mu = x_\mu - y_\mu \) the Schwinger factor \( E \left( x, y \right) = 1 \).

In the CI background the bilocal gluon correlator acquires the form
\[
D^{\mu\nu,\rho\sigma} \left( x \right) = \left< \cdot \cdot \cdot \right> = \sum_{\pm} n_c^\pm \int d^2z \int d\Omega_T \left( F^{\mu\nu}_{\left( ax \right)k}(z - \frac{x}{2}) F^{\rho\sigma}_{\left( ax \right)k}(z + \frac{x}{2}) \right),
\]
where \( n_c^\pm \) is the effective instanton / anti-instanton density, \( z \) is the collective coordinate of the instanton center and \( \Omega \) is its color space orientation.

To extract form factors \( D \left( x^2 \right) \) and \( D_1 \left( x^2 \right) \) it is easier first to average over the instanton orientations in the color space and take the trace over color matrices by using the relations
\[
\int d\Omega_T^C O^C O^C d = \frac{1}{N_c} \delta^a_0 \delta^b_0, \quad \tau_\mu^\pm \tau_\nu^\mp = \delta_{\mu\nu} + in_{\mu\nu}^a \tau^a, \quad \tau^a \tau^b = \delta^{ab} + i\epsilon^{abc} \tau^c.
\]

Then, it is convenient to define the combinations of functions \( D \left( x^2 \right) \) and \( D_1 \left( x^2 \right) \)
\[
A \left( x^2 \right) = \delta_{\mu\nu} \delta_{\rho\sigma} \frac{D^{\mu\nu,\rho\sigma} \left( x \right)}{\left< 0 \left| F^2_{\mu\nu} \right| 0 \right>}, \quad D \left( x^2 \right) = D_1 \left( x^2 \right) + \frac{1}{2} x^2 \frac{\partial D_1 \left( x^2 \right)}{\partial x^2},
\]
\[
B \left( x^2 \right) = 4 x^2 \delta_{\mu\rho} \delta_{\mu\sigma} \frac{D^{\mu\nu,\rho\sigma} \left( x \right)}{\left< 0 \left| F^2_{\mu\nu} \right| 0 \right>}, \quad D \left( x^2 \right) = D_1 \left( x^2 \right) + x^2 \frac{\partial D_1 \left( x^2 \right)}{\partial x^2},
\]

taking the boundary condition, \( D \left( 0 \right) + D_1 \left( 0 \right) = 1 \) and the asymptotic conditions \( D \left( \infty \right) = D_1 \left( \infty \right) = 0 \). After direct, but cumbersome calculations we come to the expressions for the functions \( A \) and \( B \):
\[
A \left( x^2 \right) = \frac{8}{\pi} N_D \int_0^{\infty} drr^2 \int_0^{\infty} dt \left\{ \omega_1 \left( z_+ \right) \omega_1 \left( z_- \right) + \omega_3 \left( z_+ \right) \omega_3 \left( z_- \right) \left( 3 - 4 \sin^2 \left( \alpha_2 \right) \right) \right\} \left( 1 - 2 \cos \left( \alpha_2 \right) \right) \left( 1 - 2 \cos \left( \alpha_3 \right) \right) \left( 1 - 2 \cos \left( \alpha_4 \right) \right) \left( 1 - 2 \cos \left( \alpha_5 \right) \right),
\]
\[
B \left( x^2 \right) = \frac{16}{\pi} N_D \int_0^{\infty} drr^2 \int_0^{\infty} dt \left\{ \omega_1 \left( z_+ \right) \omega_1 \left( z_- \right) \left( 3 - 4 \sin^2 \left( \alpha_2 \right) \right) \right\} \left( 1 - 2 \cos \left( \alpha_2 \right) \right) \left( 1 - 2 \cos \left( \alpha_3 \right) \right) \left( 1 - 2 \cos \left( \alpha_4 \right) \right) \left( 1 - 2 \cos \left( \alpha_5 \right) \right),
\]
where \( z_\pm = (r, t_\pm), \quad t_\pm = t \pm \frac{x}{2} \) the forms \( \omega_1 \left( z \right), \omega_2 \left( z \right), \) and \( \omega_3 \left( z \right) \) are defined in [55], \( N_D \) is the normalization factor
\[
N_D^{-1} = 6 \int_0^{\infty} dy y^3 \left( \omega_1^2 \left( y \right) + \omega_3^2 \left( y \right) \right),
\]
and the phase factor
\[
\alpha \left( x \right) = r \int \frac{d7}{2} d\tau \varphi_g \left( \tau^2 + \left( t + \tau \right)^2 \right),
\]
\[
11
\]
reflects the presence of the $\hat{E}$ exponent in the definition of the bilocal correlator. These expressions for the field strength correlators are general for any field given in the form (12). The gauge invariance of the functions $A(x^2)$ and $B(x^2)$ can explicitly be checked for Eq. (13) by transforming, for example, the field $A_\mu$ from the singular to regular gauges, Eq. (14). The expressions for $A(x^2)$ and $B(x^2)$ may be considered as generation functions to obtain condensates of higher dimensions in the instanton model approach. From a technical point of view this procedure is more convenient than their direct calculations [32, 14].

In the SI approximation the form factors $B'(x^2) = A'(x^2)$ reproduce the expression, Eq. (21) from [15], for the gauge-invariant correlator. As it has been shown in [17] (see also [24]), in the SI approximation the term with the second Lorentz structure, $D_1(x^2)$, parameterizing the gluon correlator (3) does not appear. This fact is due to the specific topological structure (self-duality) of the instanton solution. Both the Lorentz structures arise in the r.h.s. of (22) if one takes into account the background fields.

The form factors $D(x^2)$ and $D_1(x^2)$ are determined numerically by solving the equations (54) and plotted in Fig. 3 in coordinate space and in Fig. 4 in momentum space. The constant $\kappa$ defining the relative weight of $D$ functions $(D(0) = \kappa, D_1(0) = 1 - \kappa)$ depends on the background field strength parameter $\eta_g\rho$ and it is close to one in the region of reasonable physical parameters. It is equal to $\kappa = 1$ at $(\eta_g\rho)^2 = 0$ (SI case), $\kappa = 0.997$ at $(\eta_g\rho)^2 = 0.1$ and $\kappa = 0.926$ at $(\eta_g\rho)^2 = 3$. In all the cases the parameter $\kappa$ is close to one in accordance to the fits of lattice data [33]. It means that in the weak background field the dominant role of the $D(x^2)$ function remains as in the SI case; it is close to the SI form at small distances and exponentially rapidly decays at large distances. The $D_1(x^2)$ function is small and positive everywhere, its behavior is very sensitive not only to external field but also to the gauge phase factor effect [11]. In Appendix A we show that the reason for smallness of the $D_1(x^2)$ function is “almost” self-duality property of the CI solutions. Both the form factors possess two zeros at large distances and at very large $x$ develop positive asymptotics

$$D(x^2), D_1(x^2) \sim |x|^{-3/4}(\eta_g\rho)^4 \exp \left(-0.473(\eta_g |x|)^{3/2}\right),$$

where $\eta_g \neq 0$ and the constant in the exponent is found numerically.

In order to have contact with the QCD vacuum phenomenology and specify further the instanton-induced model of the gluon correlator, let us discuss the contributions of different terms in (17) to the gluon condensate

$$\langle : F_{\mu\nu}[A^{CI} + b](0) F_{\mu\nu}[A^{CI} + b](0) : \rangle = \langle F_{\mu\nu}^{CI}(0) F_{\mu\nu}^{CI}(0) \rangle +$$

$$+ \langle : F_{b,\mu\nu}(0) F_{b,\mu\nu}(0) : \rangle + \langle : \Delta F_{\mu\nu}[A^{CI} + b](0) \Delta F_{\mu\nu}[A^{CI} + b](0) : \rangle^{\text{interf}}.$$  

The background contribution to the gluon condensate

$$\langle : F_{b,\mu\nu}(0) F_{b,\mu\nu}(0) : \rangle = \langle F_b^2 \rangle_b$$

serves as a parameter of the model and, by assumption, is much smaller than the CI contribution given by

$$\langle F_{\mu\nu}^{CI}(0) F_{\mu\nu}^{CI}(0) \rangle = 32\pi^2 n_c N_c^{-1},$$

where $N_c^{-1} \approx 1$ (see Fig. 3). The interference term after averaging over relative color orientations and using relations (33), (34) acquires the form

$$\langle : F_{\mu\nu}[A^{CI} + b] F_{\mu\nu}[A^{CI} + b] : \rangle^{\text{interf}} =$$

$$= \frac{N_c}{16(N_c^2 - 1)} (32\pi^2 n_c) \langle F_b^2 \rangle_b \int_0^{\infty} dz z^7 \varphi_g^2(z^2) \Phi(z^2),$$

where $\Phi(z^2)$ is defined in (23) (the explicit forms of $\Phi(z^2)$ are outlined in Appendix B).

The interference term depends on two dimensionless parameters $\alpha_g = \rho_c \eta_g$ and $\beta = \rho_c/R$ and the background field condensate can be parameterized as $\langle F_b^2 \rangle_b \rho_c^4 = \frac{9(N_c^2 - 1)}{a_\Phi N_c} \alpha_g^3 \beta$. The instanton size $\rho_c$ comes in the last formula.

\footnote{In ref. [34], in the calculations of the form factors $D, D_1$ based on the instanton-anti-instanton ansatz both the influence of the physical vacuum on the instantons and the gluon correlators as an effect of the $P_-$-exponential factor on the form factors has been ignored. As a result, the negative $D_1$ has been obtained in [34]. However, both the facts are important in determining the correct norm and forms of the form factors, in particular, in obtaining a small $D_1$.}
with high power and leads to indefiniteness of the factor of order 2 in the relation of the external field condensate to the parameters $\alpha_g$ and $\beta$. To reduce this uncertainty, we can use the physical information about the vacuum properties provided by the QCD SRs and lattice QCD. Indeed, as it has been shown in \cite{13}, in the SI case there is a relation between the instanton size and the average virtuality of quarks in the vacuum, Eq. (3). The value of the average quark virtuality has been estimated in the QCD sum rule analysis, $\lambda_q^2 = 0.5 \pm 0.05$ GeV$^2$ in \cite{14}; $\lambda_q^2 = 0.4 \pm 0.1$ GeV$^2$ in \cite{17}, and from the lattice QCD calculations $\lambda_q^2 = 0.55 \pm 0.05$ GeV$^2$ in \cite{18}. The relations remain good approximation in the CI case if the external field does not strongly deform the instanton. Numerical calculations of $\lambda_q^2$, defined in \cite{3}, lead to estimates

$$\lambda_g^2 = 4.8 \frac{1}{\rho_c^2} (\alpha_g^2 = 0), \quad \lambda_g^2 = 5.7 \frac{1}{\rho_c^2} (\alpha_g^2 = 1).$$

(63)

We show below that physically motivated background field has the strength parameter $\alpha_g < 1$ and thus the value of $\lambda_g^2$ increases not more than 20%.

The relation for $\lambda_q^2$ in \cite{3} can be used to get the scale for the background field condensate

$$\langle F_{b}^2 \rangle_b = \frac{9 (N_c^2 - 1)}{4 N_c a_q} \left( \lambda_q^2 \right)^2 \alpha_g^2 \beta$$

and we accept in the following

$$\lambda_q^2 = 0.5 \text{ GeV}^2.$$  

What is the expected range for the parameters $\alpha_g$ and $\beta$? By analogy with the instanton liquid vacuum, the parameter $\beta$ can be interpreted as a ratio of the instanton size to the inter-instanton distance and it is adjusted as $\beta \approx 1/3$. Then, the estimate of the upper limit for the strength parameter $\alpha_g$ follows from the assumption that the contribution of the background field to the total gluon condensate $(\langle 0 | F^2_b | 0 \rangle)^{\text{total}} \approx 1$ GeV$^4$ \cite{22} is quite small. This assumption reduces the influence of the model dependent part of correlator and leads to the bound $\alpha_g < 1$, or for the dimensional parameter, $\eta_g^2 = \alpha_g^2 \lambda_g^2 / 4$, $\eta_g < 0.35$ GeV. In Fig. \cite{7} we present the values of the interference term as a two parametric plot and see that its contribution to the gluon condensate is small if the short-range and large-scale fluctuations are well separated: $\alpha_g < 1$ and $\beta \ll 1$. For completeness in Appendix B we present the small interference contributions to the functions $A(x^2)$ and $B(x^2)$.

Thus, we construct the model of the gluon correlators. Within this model the functions $A(x^2)$ and $B(x^2)$ are the sum of the short-range instanton induced contributions \cite{5} and \cite{6} multiplied by the weight factor $32\pi^2 n_c / (\langle 0 | F^2_b | 0 \rangle)^{\text{total}}$ and the long-range contribution \cite{19} modeled by Exp. \cite{25} with the weight factor $\langle F_{b}^2 \rangle_b / (\langle 0 | F^2_b | 0 \rangle)^{\text{total}}$. The parameters of the model are the average instanton size $\rho_c \approx 0.3$ fm, the effective instanton density $n_c \approx 1$ fm$^{-4}$, the strength $\langle F_{b}^2 \rangle_b \leq 32\pi^2 n_c$ and the correlation length $R \approx 3\rho_c$ of the background field. The first two parameters are estimated within the instanton liquid models, being in the dilute liquid limit expressed through the vacuum averages: $\rho_c^2 = 2 \lambda_q^{-2}$ and $n_c = (2\pi I/N_c) \left( (\bar{q}q)^2 / \lambda_q^2 \right)$, where the numerical constant $I \approx 0.6$ \cite{33}. The latter relation is a consequence of the gap equation \cite{13}. The form of the short-range correlator is defined by $\rho_c$, at small distances and by the long-range parameter $\eta_g$ at large distances. The form of the long-range correlator at large distances can be motivated by the results obtained in the dual effective model of QCD \cite{8}, where they have exponential decrease (modulo powers) similar to exponential ansatz in \cite{27}. Basing on the results of the dual model and lattice measurements one can expect that $A(x^2) \approx B(x^2)$ for the long-range part of the correlator.

The field-strength correlators have been studied on the lattice in \cite{7}, \cite{10}. There, the following two combinations of form factors have been measured:

$$D_\perp (x^2) = D (x^2) + D_1 (x^2),$$

$$D_\parallel (x^2) = D (x^2) + D_1 (x^2) + x^2 \frac{\partial D_1 (x^2)}{\partial x^2},$$

where $D_\perp (x^2) = 2 A (x^2) - B (x^2)$ and $D_\parallel (x^2) = B (x^2)$ in terms of the combinations defined in \cite{24}. The lattice measurements of the field strength correlators are also obtained with the straight line path in the Schwinger exponent. The direct comparison of the model calculations with the lattice data is a delicate problem, since the used parameterization is rather conventional in separating the residual perturbative tail (divergent term $\sim x^{-4}$) from the non-perturbative part (pure exponential finite term). As a result, the fits with ad hoc chosen parameterizations are very unstable with respect to the extraction of the quantities of physical interest: the correlation lengths, the gluon...
condensate, etc. [33]. The reason is that the perturbative part is strongly divergent (as it is seen from lattice data), its contribution at small distances would be strongly dependent on the parameterization procedure. On the other hand, by construction we calculate the non-perturbative part of the correlators with perturbative contributions subtracted. In future, it would be quite desirable to make a new fit to the lattice data using the Eqs. (53), (56), as an input for a non-perturbative part of the correlators.

At the present stage, we restrict ourselves only to a few qualitative remarks. In ref. [33], the range of values of some physical quantities was discussed which can be fitted from the lattice data according to different parameterizations. Namely, the correlation length, the gluon condensate and the normalization of form factors have been analyzed. As it has been noted above the normalization of form factors $\kappa$, consistent with small $D_1 (x^2)$, is in agreement with the instanton model and that the value of the gluon condensate serves as a free model parameter. The values of the gluon condensate extracted from the fits to lattice data are very sensitive to the parameterization used, being within the interval $\langle \rho \rangle := (0.005 - 0.03) \text{ GeV}^4$. In the lattice “full-QCD” fit of an average correlation length $l_G$ of the gluon strength, defined as

$$l_G = \frac{1}{D(0)} \int_0^\infty dx D (x^2),$$

(64)

is in the range $l_G \approx 0.35 - 0.45 \text{ fm}$ with lattice quark mass $am_q = 0.01$ and $l_G \approx 0.3 - 0.4 \text{ fm}$ with $am_q = 0.02$, where $a$ is the lattice unit. One can expect, following linear extrapolation, that in the chiral limit $am_q \to 0$, $l_G \approx 0.4 - 0.5 \text{ fm}$. Now let us omit an important but unsolved problem about the difference of lattice and CI renormalization schemes and norms, to draw rather a rough comparison of the corresponding results. The instanton model predicts for the same quantity: $l_G = 0.43 $ fm at $\rho_c \eta_g = 0$ (SI), $l_G = 0.37 $ fm at $(\rho_c \eta_g)^2 = 1$, $l_G = 0.31 $ fm at $(\rho_c \eta_g)^2 = 3$. Thus, the predictions of the instanton model, under the considered condition $\alpha = \rho_c / \eta_g < 1$, are in qualitative agreement with information extracted from the lattice data.

IV. CONCLUSIONS

The instanton model provides a way of constructing the nonlocal vacuum condensates. We have obtained the expressions for the nonlocal gluon $<: T \rho F^\mu \nu(x) E(x, y) F^\rho \sigma(y) E(y, x) ::$ correlator beyond the single instanton (SI) approximation [15]. They have consistent properties at short as well as at large distances. The model constructed predict the behaviour of nonperturbative part of gluon correlation functions in the short and intermediate region assuming that it is dominated by instanton vacuum component. To this goal, we have suggested that the instanton $A^{CI}_\mu (x)$ is developed in the physical vacuum field $b_\mu (x)$ interpolating large-scale vacuum fluctuations. We have found that at small distances the instanton field dominates, and at large distances it decreases exponentially. We did not assume any particular properties of the long-wave vacuum field $b_\mu (x)$ but managed to reduce the effect to certain phenomenological quantities, namely, the correlation function $\langle F^\mu \nu_b \rangle$ determined by its strength $\langle F^\mu \nu_b \rangle_b$ and the correlation length $R$. Within this model, by averaging over random color vector orientations of the background field with respect to the fixed instanton field orientation, we have found equation (20) governing the deformation of the instanton under the influence of the weak background vacuum field. Following Affleck idea we have shown that, to stabilize the instanton, we need to put constraints on the system. Next, we have found the constraint independent asymptotics of the instanton solution at large distances, given by Eqs. (29) and (30), where it is exponentially suppressed $A^{CI}_\mu (x) \sim 2 \eta_n \rho_c \eta_g \langle 0 \mid \langle F^\mu \nu_b \rangle \rangle b \exp \left[- \frac{2}{3} (\eta_g |x|)^{3/2} \right]$ unlike the powerful decreasing SI. It is important to note that the form of this asymptotics is also independent on the model for the background field and the driven parameter $\eta_g \sim \left( \frac{N_c}{9} \frac{1}{N_2 - 1} R \langle F^\mu \nu_b \rangle b \right)^{1/4}$ only weakly depends on it. Assuming that the external field is weak, the CI profile function is close to SI profile at distances smaller than $\rho_c$, and it decreases exponentially at distances larger than $\eta^{-1}_g$ (see [11]). In particular, this result means that the leading interaction term of a widely separated instanton-anti-instanton pair in physical vacuum decays exponentially with separation and differs from dipole interaction term found previously in unconstrained model. The knowledge of the constraint-independent parts of CI allowed us to construct the solution in the ansatz form (22) with the profile functions (33) - (45). As it is seen from Figs. 4 - 48, the profile of the CI and its action are practically independent of the choice of the ansatz if the interference parameter is in the region, $\rho_c \eta_g < 1$, where our considerations are justified.

Then, for an arbitrary classical gauge field of the form $A^{CI}_\mu (x) = 2 \eta_n \rho_c \phi_g (x^2)$, we have found the expressions (33) and (65) for the combinations of form factors $D (x^2)$ and $D_1 (x^2)$, which parameterize the gauge-invariant gluon field strength correlator. These expressions generalize the previously known expressions for the SI model [15]. The
correlators have been calculated numerically. As it turns out, at a reasonable set of parameters, guaranteeing the smallness of the large-scale vacuum field fluctuations, the $D(x^2)$ structure is close to the SI induced function with the exponential asymptotics being developed at large distances. At the same time, the $D_1(x^2)$ structure is about two orders smaller than the $D(x^2)$ function at any reasonable choice of the parameter $\rho_c \eta_q$. As it is explained in the Appendix A, the reason is that in the dilute vacuum the CIs are “almost” self-dual. The relative strength of the form factors $D(x^2)$ and $D_1(x^2)$ is very sensitive to the accepted physical picture. The lattice data are in qualitative agreement with predictions of the constrained instanton model. It means, in particular, that in the interpretation of the lattice data more justified parameterization for the correlation functions can be used. It allows one to extract from data the values of physical interest and separate the perturbative tail from the nonperturbative contribution. Moreover, due to fast decay of the CI induced part of correlators, the exponential decay observed in lattice calculations can be attributed to the background component of the vacuum field, or be described by some other field theoretical approaches. From the other side, the SI model is inconsistent with large distance behavior. The nonperturbative part of the functions $A(x^2)$ and $B(x^2)$ are the sum of short-range instanton induced contributions (55) and (56), multiplied by the weight factor $n_c 32 \pi^2 / \langle 0 | F^2 | 0 \rangle_{\text{total}}$, and the long-range contribution (19), modeled by exponentially decreasing function (25) with the weight factor $\langle F^2_0 / b \rangle / \langle 0 | F^2 | 0 \rangle_{\text{total}}$

The constrained instanton model introduces two characteristic scales (correlation lengths). One is related to short distance behavior of the correlation functions and another with long range distance behavior. The first one, $\lambda_g^{-1}$, is predictable and expressed in terms of physical quantities. In SI approximation, given by Eqs. (3) and (4), it is proportional to the instanton size, and gains small negative corrections due to the background, Eq. (33). As to large scale it is out of the scope of our model, and we can only physically relate it to the confinement size or extract it from long distance asymptotics of lattice calculations. The microscopic description of the long distance background field needs other considerations not examined in the present work.

The calculations have been performed in a gauge-invariant manner by using the expressions for the instanton field in the axial gauge. The behavior of the correlation functions demonstrates that in the single constrained instanton approximation the model of nonlocal condensates can well reproduce the behavior of the functions at short and intermediate distances, while the large-scale asymptotics is dominated by the background field. It would be quite desirable to make a fit to the lattice data using as an input the instanton induced correlators. The important question concerning the interacting ensemble of the constrained instantons has also to be postponed for another specific work.

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APPENDIX A: APPENDIX

The question may arise why the short-range vacuum contributions to the $D_1(x^2)$ structure are negligible. As it was found in [15] in the SI case $D_1(x^2) = 0$ due to self-duality of instanton solution. The CIs are not self-dual and contribute to $D_1(x^2)$, but to what extent the self-duality is violated? We are going to show that for the reasonable set of parameters and ansatze assumed the CIs are “almost” self-dual and this is the reason for smallness of $D_1(x^2)$. On the contrary if lattice simulations detected very big contribution to $D_1(x^2)$, it would mean that self-duality is lost and there is no chances to save instantons as individual objects in the QCD vacuum. From the point of view of our model, any essential contributions to $D_1(x^2)$ can arise only from the large-scale vacuum fluctuations.

The dual field strength $\tilde{F}_{\mu \nu}^{CI,a}(x) = \frac{1}{2} \epsilon_{\mu \nu \rho \sigma} F_{\mu \nu}^{CI,a}(x)$, where $F_{\mu \nu}^{CI,a}(x)$ is defined in (34), can be expressed in the form

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[5] The similar behavior is expected for the quark correlator in the physical vacuum [36].
\[ \tilde{F}_{\mu\nu}^{CI,a}(x) = 4 \left[ \tau_{\mu\nu} \tilde{\omega}_1(x) + (x_\mu \tau_{\nu\rho} - x_\nu \tau_{\mu\rho}) x_\rho \tilde{\omega}_2(x) \right], \]  

with forms \( \tilde{\omega}(x) \)

\[ \tilde{\omega}_1(x) = \varphi_g(x^2) + x^2 \frac{\partial \varphi_g(x^2)}{\partial x^2}, \quad \tilde{\omega}_2(x) = \varphi_g^2(x^2) + \frac{\partial \varphi_g(x^2)}{\partial x^2}. \]  

Let us consider the difference of the field strengths given in the regular at zero gauge

\[ F_{\mu\nu}^{CI,a}(x) - \tilde{F}_{\mu\nu}^{CI,a}(x) = 4 \left[ \tau_{\mu\nu} + 2 \frac{(x_\mu \tau_{\nu\rho} - x_\nu \tau_{\mu\rho}) x_\rho}{x^2} \right] \omega_2^{reg}(x) x^2. \]  

The self-duality condition \( F_{\mu\nu}^{CI,a}(x) - \tilde{F}_{\mu\nu}^{CI,a}(x) = 0 \) is satisfied for the SI case, where \( x^2 \omega_2^{reg,I}(x) = 0 \) (see Eq. (31)). Comparing Eqs. (34) and (A1) with Eq. (A3) we can consider the condition

\[ |\omega_2^{reg,CI}(x)| x^2 < |\omega_1^{reg,CI}(x)| \sim |\omega_1^{reg,I}(x)| \]

as a criterion indicating that the field is “almost” self-dual. It can be checked numerically, that it is really the case at reasonable choice of background field strength parameter \( \alpha_g < 1 \) and all forms of ansatze for CI. At the same time at larger values of parameter \( \alpha_g \approx 4 \div 6 \) the inequality is not fulfilled. Thus we show that assuming diluteness of the vacuum the CIs are “almost” self-dual solutions and, as a consequence, contribute very small to the \( D_1(x^2) \) structure of the vacuum gluon field strength correlators.

**APPENDIX B: APPENDIX**

The function \( \Phi(z^2) \) for the different forms of the form factor \( \tilde{B}(x^2) \):

\[ \Phi_G(z^2) = \frac{2}{3y^2} \left[ 2\sqrt{\pi} y^3 erf(y) - 3y^2 + 1 - (1 - 2y^2) \exp(-y^2) \right], \quad \text{Gaussian}, \]  

\[ \Phi_M(z^2) = \frac{2}{3y^2} \left[ 4y^3 \arctan(y) + y^2 - (1 + 3y^2) \ln(1 + y^2) \right], \quad \text{monopole}, \]  

and

\[ \Phi_E(z^2) = \frac{4}{3y^2} \left[ 2y^3 - 3y^2 + 6 - 6(1 + y) \exp(-y) \right], \quad \text{exponential}, \]  

where \( y = z/R, \) \( R \) being the correlation length of the large-scale vacuum field.

For completeness we present here the small interference contribution to the functions \( A(x^2) \) and \( B(x^2) \)

\[ A_{\text{interf}}(x^2) = N_D \left\langle 0 \left| F_0^2 \right| 0 \right\rangle \frac{N_c}{6(N_c^2 - 1)} \int_0^\infty dr r^2 \int_0^\infty dt \Phi(z_+, z_-) \varphi_g(z_+^2) \varphi_g(z_-^2) \cdot \]  

\[ \cdot \left( z_+ \cdot z_- \right) \left\{ (z_+ \cdot z_-) (3 - 4\sin^2(\alpha_z)) - x \eta \sin(2\alpha_z) \right\}, \]  

\[ B_{\text{interf}}(x^2) = N_D \left\langle 0 \left| F_0^2 \right| 0 \right\rangle \frac{N_c}{6(N_c^2 - 1)} \int_0^\infty dr r^2 \int_0^\infty dt \Phi(z_+, z_-) \varphi_g(z_+^2) \varphi_g(z_-^2) \cdot \]  

\[ \cdot r^2 \left\{ (z_+ \cdot z_-) (2 - 3\sin^2(\alpha_z)) - x \eta \sin(2\alpha_z) \right\}, \]  

where

\[ \Phi(z_+, z_-) = 4 \int_0^1 d\alpha \int_0^1 d\beta \alpha \beta \tilde{B} \left[ (\alpha z_+ - \beta z_-)^2 \right], \]

\( z_\pm = (r, t \pm \tilde{\eta}), \ z = (r, t), \) and \( \varphi_g(z^2) \) is defined in (33). In deriving the above expressions we used the Schwinger - Fock gauge for background field, Eqs. (10) - (13), and neglected the derivative \( \tilde{D}'_i(x^2) \) in comparison with the
function $\tilde{B}(x^2)$ itself. In fact, it is consistent with $D_1(x^2) = 0$ that follows from the lattice data \cite{15} and instanton calculations \cite{15}. We find that the interference contributions to the correlators are very small in absolute value, have shorter correlation length comparing with the CI contributions (\cite{55}, \cite{56}) and do not lead to valuable appearance of the $D_1(x^2)$ structure.

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**FIGURE CAPTIONS:**

Fig. 1: The constrained instanton profile functions $x^2 \phi_g(|x|/\rho)$ \((32)\), corresponding to the ansatz \((45)\), at different values of the parameter $(\rho \eta_g)^2$: $(\rho \eta_g)^2 = 0$ solid line (instanton case), $(\rho \eta_g)^2 = 0.5$ short dashed line, $(\rho \eta_g)^2 = 3$ - long dashed line.

Fig. 2: The instanton (solid line) and constrained instanton profile functions $x^2 \phi_g(|x|/\rho)$ \((32)\) corresponding to different ansatze: \((43)\) dotted line, \((44)\) - short dashed line, \((45)\) - long dashed line, at the large value of the parameter $(\rho \eta_g)^2 = 3$.

Fig. 3: The density action of the instanton (solid line) and constrained instanton $G^2(x^2) \equiv \rho^4/192 F_{\mu \nu}^A(x) F_{\mu \nu}^A(x)$, Eq. \((37)\), as a function of $|x|/\rho$ corresponding to different ansatze: \((43)\) dotted line, \((44)\) - short dashed line, \((45)\) - long dashed line, at the value of the parameter $(\rho \eta_g)^2 = 1$.

Fig. 4: The classical action of the instanton (solid line) and the constrained instanton $S_{CI}$, Eq. \((36)\), as a function of $\rho \eta_g$ corresponding to different ansatze: \((43)\) dotted line, \((44)\) - short dashed line, \((45)\) - long dashed line. The action is given in units of $8\pi^2/g^2$.

Fig. 5: The form factors $D$ (top lines) and $D_1$ (bottom lines) (all normalized by $D(0)$) versus physical distance $x$, for the instanton size $\rho = 0.3$ fm and parameters $(\rho \eta_g)^2 = 0$ (solid lines) and $(\rho \eta_g)^2 = 1$ (dashed lines).

Fig. 6: The form factor $\tilde{D}(p)$ as a function of $\rho |p|$ corresponding to the ansatz \((45)\), at different values of the parameter $(\rho \eta_g)^2$: $(\rho \eta_g)^2 = 0$ solid line (instanton case), $(\rho \eta_g)^2 = 0.5$ - short dashed line, $(\rho \eta_g)^2 = 3$ - long dashed line.

Fig. 7: The interference term contribution to the gluon condensate normalized by the instanton contribution $F_{\mu \nu}[A^I]\Phi_{\rho \sigma}[A^I] = n_c 32\pi^2$ as function of the large-scale vacuum fluctuation correlation length $1/\beta = R/\rho_c$ and its strength parameter $\alpha_g = \eta_g \rho_c$. 
FIG. 1. The constrained instanton profile functions $x^2 \phi_x(|x|/\rho)$ (32), corresponding to the ansatz (45), at different values of the parameter $(\rho \eta)^2$ : $(\rho \eta)^2 = 0$ solid line (instanton case), $(\rho \eta)^2 = 0.5$ - short dashed line, $(\rho \eta)^2 = 3$ - long dashed line.
FIG. 2. The instanton (solid line) and constrained instanton profile functions $x^2 \phi(x/\rho)$ corresponding to different ansätze: (32) dotted line, (33) short dashed line, (34) long dashed line, at the large value of the parameter $(\rho \eta)^2 = 3$. 
Fig. 3. The density action of the instanton (solid line) and constrained instanton $G^2(x^2) \equiv \rho^4/192F^{Ia}_{\mu\nu}(x)F^{Ia\mu\nu}_\rho(x)$, Eq. (37), as a function of $|x|/\rho$ corresponding to different ansatze: (43) dotted line, (44) - short dashed line, (45) - long dashed line, at the value of the parameter $(\rho\eta_0)^2 = 1$. 
FIG. 4. The classical action of the instanton (solid line) and constrained instanton $S_{CI}$, Eq. (36), as a function of $\rho \eta_0$ corresponding to different ansatze: (43) dotted line, (44) - short dashed line, (45) - long dashed line. The action is given in units of $8\pi^2/g^2$. 
FIG. 5. The form factors $D$ (top lines) and $D_1$ (bottom lines) (all normalized by $D(0)$) versus physical distance $x$, for the instanton size $\rho = 0.3$ fm and parameters $(\rho \eta_0)^2 = 0$ (solid lines) and $(\rho \eta_0)^2 = 1$ (dashed lines).
FIG. 6. The form factor $\tilde{D}(p)$ as a function of $\rho|p|$ corresponding to the ansatz (45), at different values of the parameter $(\rho\eta)^2$: $(\rho\eta)^2 = 0$ solid line (instanton case), $(\rho\eta)^2 = 0.5$ - short dashed line, $(\rho\eta)^2 = 3$ - long dashed line.
Fig. 7. The interference term contribution to the gluon condensate normalized by the instanton contribution $F_{\mu\nu}[A^I]F_{\rho\sigma}[A^I] = n_c 32\pi^2$ as function of the large-scale vacuum fluctuation correlation length $1/\beta = R/\rho_c$ and its strength parameter $\alpha_g = \eta_g \rho_c$. 

FIG. 7.