Reliability-based Structural Safety Evaluation of Reinforced Concrete Members

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Abstract

The strength of reinforced concrete members may vary from the calculated or the nominal strength due to variations in the material strength and dimensions of the element. Statistical descriptions of the variabilities in loads and strengths are required in all studies pertaining to the safety of reinforced concrete members.

Therefore, this paper recommends a number of values for the coefficient of variation of concrete, reinforcement, dimension and load to be used in the probability analysis. Also, this study analyzes reinforced concrete members using the Advanced First Order Second Moment (AFOSM) method using the statistical characterization of variables. This study shows that Bayes’ theorem is an effective tool for updating prior probabilities when the value of a random parameter is known. Finally, the results in this paper contribute to the limit state reliabilities implied in the current design of reinforced concrete elements.

Keywords: structural safety; reliability analysis; Bayesian method; AFOSM; statistical characterization

1. Introduction

Structural design focuses on planning an optimized structure that is safe and economical and that completely fulfills its intended functions. It is true, however, that previous structural design practices have tended to rely heavily on experiences in setting safety factors and on the determination of the resistance capacity of a structure, as indicated by blueprints. In other words, the specified design strength of concrete, the resistance capacity of the reinforcements and the sectional dimensions of included members have been used to calculate the resistance capacities of members in certain designs.

Measuring a reinforced concrete structure reveals deviations from the original blueprints. These differences are in the dimensions of the members or in the arrangement of the reinforcements and are due to errors that occurred during construction or had to do with characteristics of the materials due to production or curing. In addition, it is also being reported that the strength of the concrete in a concrete structure not only exhibits irregularities within the building but also is, in general, lower than the test value of the 28-day compressive strength of a standard cured concrete specimen.

Therefore, without considering the uncertainties caused by the differences in statistical features between the materials or the loads, it is difficult to assess the safety level of a structure in a practical and reliable manner.

This is why the reliability analysis method of conducting safety assessments of structures, which takes into consideration uncertainties associated with design variables using the theory of probability and statistics, has become essential.

In this study, the authors intended to analyze the suggested statistical features of each design variable and improve design characteristics using survey data. Using the limit state function including design variables that reflect enhanced statistical features, they assessed the soundness of members used in designing and constructing structures in accordance with the requirements of current concrete structure design standards by analyzing the reliabilities of reinforced concrete members.

2. Reliability Analysis Theory

2.1 The Failure Probability of Members

Reliability analysis determines the degree of reliability of a structure taking into consideration the uncertainties of the variables used in designing or analyzing a structure or the uncertainties that have a probabilistic distribution property. When the effective external force $Q$ is greater than the resistance $R$, the stability of the structure fails. The probability of
failure, which corresponds to the area in Fig. 1. marked by shaded area, can be calculated using Eq. (1).

\[ P_f = P(R - Q \leq 0) = \int_{x(0)} f(x) dx \]  

(1)

In Fig. 1., the reliability index \( \beta \) is the ratio of the standard deviation \( \sigma_G \) and the mean \( \mu_G \) from 0 to the probability variable \( G \). Because the probability of failure decreases as \( \beta \) increases, the safety level of the structure increases. The reliability index \( \beta \) is the mean ratio relative to the standard deviation (Nowak, 1997).

2.2 Reliability Analysis

As a type of simulation method, the Monte-Carlo sampling method, through which the failure probability can be calculated directly, is used to identify the probability distribution of necessary numerical values from the statistical result of a repeatable experiment. The failure probability is directly calculated by applying the sampled random numbers, which reflect the characteristics of the probability variables, to the limit state equation. Although the Monte-Carlo sampling method has merit in that the failure probability can be calculated while maintaining the limit state equation, the number of sampling times should be sufficiently large to secure a high degree of reliability for the failure probability.

However, the failure probability is often small when it comes to the stability of a structure. Accordingly, the failure occurrence rate becomes so small that the number of significant digits decreases.

The First-Order-Second-Moment (FOSM) method, developed by Cornell, can be used to determine an approximate solution. If the probability variables have a normal distribution and the limit state equation is linear, the stability of a structure can be accurately calculated using this method. However, if the limit state equation is non-linear, this method is able to calculate the failure probability of a structure after liberalization using the Taylor expansion. However, the FOSM method has an important limitation. The limit state equation applied to failure modes with the same mechanical features calculates different failure probabilities depending on the manner in which the features are presented.

The AFOSM method published by Hasofer and Lind in 1974 adopts a technique for determining the reliability index using the transformation parameters of the statistically independent normal distribution probability variables and presents an optimized means for calculating the reliability index, the geodesic distance from the 0 point. Among all of the points in the limit state equation, the point for calculating the reliability index has the highest frequency and is often called the MPFP (Most Probable Failure Point or Design Point). Nonetheless, this method had a problem: the accuracy decreases when the limit state function has a log-normal, gamma or extreme distribution instead of a normal distribution or when the limit state function is non-linear.

Rackwitz and Fiessler suggested a method for estimating the mean value and standard deviation of equivalent normal distribution probability variables assuming that the values of the density function and the fractional function of the non-normal distribution probability variables were the same as those of the density function and the distribution function of the equivalent normal distribution probability variables, respectively. Suppose \( \mu_X \) and \( \sigma_X \) are the mean value and the standard deviation of normal distribution probability variables, respectively. The density function \( f_X(X') \) and the distribution function \( F_X(X') \) of the probability variables are then assumed to satisfy the following equations in MPFP.

\[ F_X(X') = \Phi \left( \frac{X' - \mu_X}{\sigma_X} \right) = F_X(X') \]  

(2)

\[ f_X(X') = \frac{1}{\sigma_X} \phi \left( \frac{X' - \mu_X}{\sigma_X} \right) = f_X(X') \]  

(3)

Also, \( \mu_X \) and \( \sigma_X \) can be induced as follows:

\[ \mu_X = X' - \sigma_X \left[ \Phi^{-1}(F_X(X')) \right] \]  

(4)

\[ \sigma_X = \frac{1}{f_X(X')} \phi \left[ \Phi^{-1}(F_X(X')) \right] \]  

(5)

In the above equations, \( \phi \) refers to the standard normal probability density function, and \( \Phi \) indicates
the standard normal cumulative distribution function.

Rackwitz-Fiessler’s transformation method allows for the calculation of the reliability index by converting the non-normal distribution to the equivalent normal distribution and then to the standard normal distribution probability variables using Eqs. (4) and (5). Fig. 3 illustrates the process for calculating the reliability index based upon the method of Rackwitz and Fiessler.

2.3 Bayesian Updating

To accurately estimate the statistical characteristic values of the probability variables by constructing a limit state function, which is essential in a reliability analysis, a tremendous amount of data is required. However, given the limitation of available observational data in most cases, the statistically estimated values need to be supplemented by judgmental data.

The Bayesian method is useful for solving engineering problems which often require a subjective judgment in a data-limited situation (Ang, 1984). When the random variable \( \Theta \) is the parameter of the distribution with a prior density function in consecutive type probability variables as presented in Fig. 4, the prior probability that \( \Theta \) will be in between \( \theta_i \) and \( \theta_i + \Delta \theta \) is \( f(\theta) \Delta \theta \).

Supposing that \( \varepsilon \) is the result of the experiment, the posterior probability that it is within the range of the prior distribution \( f(\theta) \) can be calculated using the following Bayesian equation.

\[
f^\ast(\theta) = \frac{P(\varepsilon|\theta) f(\theta)}{\sum_{\theta_i} P(\varepsilon|\theta) f(\theta) \Delta \theta}
\]

When \( P(\varepsilon|\theta) = P(\varepsilon|\theta < \theta \leq \theta + \Delta \theta) \), the above equation can be rewritten as

\[
f^\ast(\theta) = \frac{P(\varepsilon|\theta) f(\theta)}{\int_{\theta} L(\theta) f(\theta) d\theta}
\]

Where \( \int_{\theta} L(\theta) f(\theta) d\theta \) is a normalized factor, and \( L(\theta) \) is the likelihood function of the experimental result \( \varepsilon \) and \( \theta \) is given.

It is known that the prior function and the likelihood function in Eq. (8) are related to the posterior function of \( \theta \). The expected value of \( \theta \) is used as the point estimate of the parameters, in general, and the improved estimate of parameter \( \theta \) can be calculated using Eq. (9) and the observational data \( \varepsilon \).

\[
\hat{\theta} = E(\Theta|\varepsilon) = \int_{\theta} f^\ast(\theta) d\theta
\]

Upon estimating the parameter, the uncertainty can be included in the probability calculation to which the value of random variables has been related. For instance, when \( X \) is the random variable of a probability distribution that has a parameter \( \theta \), the probability is the same as in the following equation:

\[
P(X \leq a) = \int_{-\infty}^{a} f^\ast(\theta) d\theta
\]

Eq. (10) is the mean probability of \( X \), physically weighted by the posterior probability of parameter \( \theta \).

3. Statistical Characteristics of Probability Variables for Reliability Analysis

Considering that measurement errors originally existing in the design or analysis of a structure and the flaws of an analysis model resulting from the uncertainty of the material integer values and the
external loads with probability distribution features, the reliability analysis determines the degree of reliability of a structure and is required to analyze the statistical characteristic of each variable. In this study, the posterior estimate value was obtained through the use of actual survey data on reinforced concrete members and the application of the discussed Bayesian method to previous research on the compressive strength of concrete at construction sites.

3.1 Material

(1) Concrete

The compressive strength of concrete is determined by a variety of factors. In other words, even if the same design strength and mixing ratio have been applied to a concrete structure, the compressive strength of the concrete can be different depending on the placement positions. This difference is caused by such factors as the handling environment and the setting, curing and testing methods.

In order to reflect the accurate statistical characteristic values of the variables applied to the limit state equation, this study used the statistical characteristics of the actual concrete's compressive strength that were modified using the Bayesian method (Table 1.). In the survey, the statistical characteristic values of concrete compressive strength researched by the Korea Institute of Construction Technology were used as the prior distribution. The nominal strengths of concrete manufactured by four different ready-mixed concrete factories were compared with the curing strengths of concrete surveyed in two different construction sites. The result of the comparison was used for updating. The compressive strengths of concrete after being cured at the construction sites were studied by sampling the core of an experimental member for mockup testing. These tests showed 80 to 90% compressive strengths for the tested members. The dispersion of the posterior distribution was decreased from that of the prior distribution. As indicated in Fig.5.(a), the mean compressive strength ratio to the nominal strength of the concrete's compressive strength (mean nominal ratio) was determined through existing research to be 0.83, with 0.16 coefficient of variation. These values were changed to a mean nominal ratio of 0.84 and a coefficient of variation of 0.14 after the authors' data was applied, reflecting the actual survey ratio with a reduced coefficient of variation.

(2) Reinforcing Bars

Because reinforcing bars are produced in a factory, they have less strength dispersion than concrete. The statistical features of the reinforcing bars yielding strength were based on the research performed by the Korean Institutes of Construction Technology. As presented in Table 1., this data has a log-normal distribution, and its variation coefficient is 0.05. This value is smaller than the 0.11 that was determined in previous overseas studies. This variation is believed to be due to the different sample sizes of these studies.

| Variable                  | Nominal Value (MPa) | Mean (MPa) | Standard Deviation (MPa) | Variation Coefficient | Probability Distribution |
|---------------------------|---------------------|------------|--------------------------|-----------------------|--------------------------|
| Bar's Yielding Strength   | 400                 | 436        | 21.3                     | 0.05                  | Lognormal                |
| Concrete Compressive     | 24                  | 20.1       | 3.07                     | 0.14                  | Normal                   |

3.2 Construction Error

A concrete structure tends to have a variety of errors in its section dimensions depending on the type and quality of concrete forms and the skill levels of the workers involved in its construction. These errors affect the degree and the distribution of the resistances of the concrete members.

Table 2. contains the posterior distribution obtained through the on-site-measured dimensions of members. The results from previous studies were used as the prior distribution in this study. The field is the result of the safety checks carried out in ten construction sites located in the metropolitan area. Columns were measured according to the width and the depth of the interior, and beams were measured according to the widths and depths of both the exterior and interior parts.
Fig. 5.(b) contains the comparison result of construction errors between the prior distribution and the posterior distribution. As shown in the figure, the ratio of the mean values of measured section dimensions against the nominal values of member dimensions presented in the drawings (the mean nominal ratio) was decreased by a small amount, along with the coefficient of variation.

Table 2. Corrected Statistical Features of Construction Errors in Beams and Columns

| Variables | Construction Error (mm) | Standard Deviation (mm) | Probability Distribution |
|-----------|------------------------|------------------------|-------------------------|
| Beam      | h 2.54                 | 3.81                   | Normal                  |
|           | h 3.04                 | 6.35                   | Normal                  |
| Column    | h 11.0                 | 12.5                   | Normal                  |

3.3 Load
The loads being applied to a structure affect the soundness of the structure according to their size. In this study, the dead load and the live load, which are the measures most universally applied to reinforced concrete structures, were used to evaluate the soundness of members. The statistical characteristics of the loads were based upon the research results of the Korean Institutes of Construction Technology (KICT, 1997) and are presented in Table 3.

Table 3. The Statistical Features of the Loads

| Variables     | Mean     | Standard Deviation | Variation Coefficient | Probability Distribution |
|---------------|----------|--------------------|-----------------------|-------------------------|
| Dead Load     | 1.05W_r  | 0.10W_r            | 50.1                  | Normal                  |
| Live Load     | 1.2W_r   | 0.3W_r             | 0.25                  | Lognormal               |

4. The Limit State Function of Reinforced Concrete Members
4.1 Flexural Member
In the case of doubly reinforced beams, the ultimate moment is classified according to whether or not the compressive reinforcement has yielded. In other words, there are several cases that involve both tension and compression to the centroid of the steel on the tension side of the member.

\[ d' = \text{the distance from the top of the member to the centroid of the layer of compression steel} \]
\[ C_c = \text{the compressive strength of the compression side reinforcement} \]
\[ Q_b = \text{the flexural moment applied to the beam} \]
\[ = M_{DL} + M_{LL} \]

4.2 Compression Member
The limit state function of the member to which only compressive strength is applied can be calculated using Eq. (12).

\[ g_c = R_c - Q_c \]

where
\[ g_c = \text{the limit state function of the column member to which the compressive strength is applied} \]
\[ R_c = \text{the resistant capacity of the column member against the compressive force} \]
\[ = 0.85 f_c' ab + A_f f_c - A_s f_s \]
\[ A_c = \text{the cross sectional area of compression side reinforcement} \]
\[ f_s = \text{the stress of compression side reinforcement} \]

4.3 Members with Flexure and Compressive Load
When both flexural moment and compression are applied to a member, the failure mode of the member varies depending on the eccentric distance \( e \), and the equation for calculating the axial ultimate resistant capacity \( P_u \) and the flexural moment ultimate resistant capacity \( M_u \) is changed. The resistant capacity of the members experiencing both flexural moment and compressive force can be defined by an equation reflecting the distance between the zero point and a point on the \( P - M \) relation diagram, and the limit state function is

\[ g_{\text{flex}} = R_{\text{flex}} - Q_{\text{flex}} \]

where
\[ g_{\text{flex}} = \text{the limit state function of the column member experiencing both flexure and compressive forces} \]
\[ R_{\text{flex}} = \text{the resistant capacity of the column member against flexure and compressive forces} \]
\[ P_u = \text{the resistant capacity of the column experiencing axial force and flexure} \]
\[ = \left[ P_u^2 + \left( \frac{M_u}{h} \right) \right]^{1/2} \]
\[ = 0.85 f_c' ab + A_f f_c - A_s f_s \]
\[ A_c = \text{the cross sectional area of compression side reinforcement} \]
\[ f_s = \text{the stress of compression side reinforcement} \]
\[ M_n = \text{the ultimate resistant capacity of the column against flexure} \]
\[ = 0.85 f_{cm} h \left( \frac{b-a}{2} \right) + A_f f_r \left( \frac{h}{2} - d \right) - A_f f_y \left( \frac{d-h}{2} \right) \]
\[ h = \text{the column height} \]
\[ d = \text{the location of the reinforcement on the compression side} \]

\[ Q = \text{flexural and axial loads applied to the column} \]
\[ = P_{DL} + P_{LL} \]
\[ P_{DL}, P_{LL} = \text{the axial load caused by the dead and live loads} \]
\[ M_{DL}, M_{LL} = \text{the flexural moment caused by the dead and live loads} \]

5. Reliability Analysis and Result
5.1 Reliability Analysis

A reliability analysis was carried out by applying the statistical features of each variable to the previously determined limit state function. The AFOSM method presented by Rackwitz and Fiessler was used in the reliability analysis. As previously mentioned, this is a method used to determine the most probable failure point using a repetitive approach after defining a limit state function composed of each design variable and supposing the failure point of each variable. The variables that did not follow the normal distribution were converted to the equivalent normal distribution, and then the reliability analysis was performed. The reliability index, which was converged by repeated calculations, was derived to become the safety level of the structure. Since the reliability index \( \beta \) is the inverse function of the cumulative distribution function (CDF), the induced reliability was directly converted to the failure probability.

A member of the first floor of an actual designed and constructed building with ten stories and an 8-m basic span was selected as the member to be analyzed. The flexural member and the member due to only compressive force were chosen from the internal members, while the external column was selected as the member affected by both bending and compressive forces. As shown in Fig.6, in the case of the flexural member, the analysis was conducted by modifying the flexural span to 7 to 10 m and the tensile reinforcement ratio of 0.2% to 1.0% for a member whose section area was 400x600 mm. For the compressive member with a 600x600 mm section, the reinforcement ratio of 1.0 to 2.0% and a flexural member span of 7 m to 10 m were applied in the analysis. The compressive strength of the concrete used was 24 MPa, and the nominal yielding strength of the reinforcing bars for the flexural member was 400 MPa. The main and hoop reinforcements of the compression member were D22 and D10, respectively.

Upon analysis, the cases in which the ratios of dead load to live load were 0.5, 1.0, 1.5 and 2.0 were considered. The safety level of a concrete member designed and constructed based upon the present design standard was evaluated by applying to each limit state function the load factors and strength reduction factors defined in the concrete structure design standard.

5.2 Results of Analysis

Fig.7. illustrates the reliability index of each member in accordance with the ratio between live load and dead load at an 8-m span. For the flexural member, when the ratio of live load to dead load (LL/DL) was between 0.5 and 1.5, a normal load condition, the reinforcement ratio was 0.86%, indicating a satisfactory value of \( \beta = 9.5 - 3.8 \). Therefore, the safety level of the flexural members that were designed based on the present design standards was estimated to be quite good.

In the case of the column compression member in which the actual design and construction reinforcement ratio was 1.7%, a satisfactory result of \( \beta = 7.3 - 4.7 \) was obtained when the ratio between the live load and the dead load (LL/DL) was 0.5-1.5 when only compressive force was applied. The column to which both flexural and compressive force were applied showed a satisfactory safety level of \( \beta = 7.8 - 3.8 \) under the same conditions. According to the result of the reliability of the compression member, when the ratio of live load to dead load (LL/DL) was 0.5, the reliability was smaller than that of the flexural member. However, when the load ratio was between 1.0 and 2.0, the reliability index was higher than that of the flexural member, in general. This seems to have reflected the design purpose, inducing yielding in beams while avoiding the failure of columns.

Fig.8. indicates the change in the reliability indices according to the changes in the spans, illustrating the failure probability of the flexural member as the span increased. In columns where both flexural and compressive loads were applied, the reliability index increased while its failure probability decreased as the span increased. This is due to the increase of the strength reduction factor reflected in the capacity of members in the limited state function, in the transition area rather than the compression-controlled section as the moment load increased, despite the load increase due to the growth of the span.

Fig.6. The Section of the Concrete Member Used for Analysis
Fig. 9. and Table 4. compare the reliability analysis results prior to performance of the improvement work with the results obtained after improving the statistical feature using the surveyed data for compressive strength and construction errors. In the case of the flexural member with an 8-m span, analyzing the member whose design and construction reinforcement ratio was 0.86%, as shown in the figure, produced a reliability index increase and a failure probability decrease when the ratio of live load to dead load (LL/DL) was between 0.5 and 2.0. The mean compressive force ratio to the nominal strength of the compressive strength (mean nominal ratio), estimated in previous research prior to implementing improvement, was 0.83 with a coefficient variation of 0.16. However, the mean nominal ratio of the concrete's strength increased to 0.84 and the coefficient of variation decreased after improvements reflecting the actual survey data were conducted. This is why the resistant capability and the reliability indices increased with a decrease in the uncertainty, while the failure probability decreased. As for the construction errors, although the mean nominal ratio decreased slightly compared to the result of a previous study, the coefficient of variation decreased.

Very different values for the reliability indices and for the failure probability have been measured after the improvement of statistical characteristic values through the application of Bayesian updating with actual data. Accordingly, continuing research into the accurate statistical features of each variable used for the resistant capability of members is necessary.

6. Summary and Conclusions
In this research, the probability variables related to the compressive strength of concrete, the reinforcement yielding strength and the construction errors of members affecting the strength of reinforced concrete members have been improved by conducting
Bayesian updating on previously suggested statistical characteristics. The safety levels of reinforced concrete members that were designed and constructed according to the present design standard were evaluated through reliability analyses on reinforced concrete flexural members and compression members and members to which both flexural and compressive loads are applied using the AFOSM method for the improved statistical characteristic values. Several conclusions have been reached as a result of this study.

1. The assessment of safety levels, in which actual data is used in the reliability analysis, was conducted by improving the statistical features of probability variables through Bayesian updating. When the compressive strength of concrete and the member construction errors were analyzed, the reliability index increased from 0.4 to 0.8 because actual data on concrete strength, of which the nominal mean values were greater and the dispersion of error range was smaller than that of previous research, was used.

2. According to the results of analyzing the reinforced members that were designed and constructed based upon the present design standard, when the load ratio (LL/DL) adhered to the general load condition of 0.5 to 1.5, the reliability indices of flexural members, compression members and members experiencing both flexural and compressive forces were 9.5 to 3.8, 7.3 to 4.7, and 7.8 to 3.8, respectively, and were considered satisfactory failure probabilities. It is estimated that the reliabilities of the members were properly calculated.

3. After the statistical features of the design variables were modified according to actual data, the values of the reliability index were calculated differently from the prior values due to the growth of member resistant capacities and the decrease in uncertainty. Therefore, to carry out more accurate safety assessments of structures, it is necessary to conduct further research on the statistical features of reliable design variables.

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