A Consideration on Approximation Methods of Model Matching Error for Data-Driven Controller Tuning

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Abstract: This paper proposes two kinds of data-driven controller tuning. The proposed methods are derived from the approximated model matching errors expressed by the filtered ideal model matching error. The main contribution of the paper is to find out specific filters that characterize the proposed data-driven methods. Similar filters are also presented in the existing virtual reference feedback tuning and fictitious reference iterative tuning as well. The comparison among the filters for the approximations clarifies the relation among them as well as the novelty of the proposed approach. The paper shows two numerical examples: one is a flexible transmission system and the other is a plant with an unstable zero. The numerical examples show the superiority of the proposed method to existing methods.

Key Words: data-driven control, model matching problem, controller tuning, virtual reference feedback tuning, fictitious reference iterative tuning.

1. Introduction

In industrial production facilities, many controllers are implemented and play important roles in improving the performance of production systems while the controllers are suppressing their excessive energy consumption. In order to make full use of the ability of the controllers, controller parameters, such as PID (proportional, integral, and differential) gains, need to be tuned properly. Therefore, effective control parameter tuning methods have been required.

Data-driven controller tuning methods [1]–[5] attract attention as effective means to solve the problem. These methods carry out controller parameter tuning in an offline manner using only closed-loop experimental data instead of plant models. Compared with model-based approaches which start a design procedure on the condition that a plant model has already been obtained, data-driven approaches need no plant model. The feature allows us to save time and labor for obtaining a plant model.

Typical data-driven controller tuning methods with one-shot experimental data are the virtual reference feedback tuning (VRFT) [3] and the fictitious reference iterative tuning (FRIT) [4]. These methods address the model reference problem. For a given reference model specifying desired response, they derive the control parameters that make the closed-loop output follow the reference model as close as possible. In the model-based design, the model reference problem can be solved by optimizing the evaluation of the model matching error. Meanwhile, the data-driven approaches evaluate the approximated errors which can be calculated using only the closed-loop experimental data.

Several studies have examined how the approximated errors are alternatively evaluated instead of model matching errors. For the VRFT, Campi et al. [3] compared the Taylor expansion up to the second order for the $H_2$ norm of both errors and proposed a design method for the pre-filter that mitigates the difference between them. Kaneko [5] explained that the approximated errors in the VRFT and the FRIT can be regarded as relative errors of the loop transfer function and the closed-loop transfer function, respectively. These studies have examined the existing data-driven approaches and help us to understand them. However, the existing studies require individual analysis for each data-driven approach and tend to lack in the extensibility.

Therefore, the paper focuses on the comparison between the approximated error and the model matching error. By using the approach, the paper proposes two kinds of data-driven approaches by way of finding out the specific filters that relate the approximated error with the model matching error. Similar filters are also presented in the existing VRFT and FRIT as well. The comparison among the filters for the approximations clarifies the relation among them as well as the novelty of the proposed approach.

In addition, the paper provides two numerical examples that show the superiority of the proposed method to the existing methods. A flexible transmission system and a plant with an unstable zero are used as the controlled plant models. For the initial closed-loop data, both cases where high and low gain controllers are used are applied to the two kinds of the proposed methods, the VRFT and the FRIT, in order to compare the resulting control performance between several data-driven approaches.

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2. Model Matching Error for Data-Driven Control

2.1 Ideal Model Matching Error

Consider the closed-loop system shown in Fig. 1. In Fig. 1, \(P(z)\) and \(C(z, \rho)\) are an unknown discrete time linear time-invariant (LTI) plant and a controller with its parameter vector \(\rho\), respectively; \(r\), \(u\) and \(y\) are the reference, the output of the controller, and the output of the plant, respectively.

We want to find the parameter vector \(\rho = \rho^*\) to make the transfer function from \(r\) to \(y\) in Fig. 1 as close to \(T^*(z)\) which is a desired reference model or a desired complementary sensitivity function of the closed-loop system. Therefore the error \(e(t)\) in the time domain ¹ is introduced as

\[
e(t) = T^*(z)[r(t)] - \frac{P(z)C(z, \rho)}{1 + P(z)C(z, \rho)}[r(t)],
\]

where \(r(t)\) is the reference in Fig. 1.

The loop transfer function \(L^*(z)\) is introduced to define the complementary sensitivity function \(T^*(z)\) as

\[
T^*(z) = \frac{L^*(z)}{1 + L^*(z)}. \tag{2}
\]

The model matching error \(e(t)\) given by Eq. (1) can be rewritten with \(L^*(z)\) as

\[
e(t) = \frac{L^*(z) - P(z)C(z, \rho)}{[1 + L^*(z)][1 + P(z)C(z, \rho)]}[r(t)]. \tag{3}
\]

For the following discussion, the transfer function from \(r(t)\) to \(e(t)\) in Eq. (3) is defined as \(E(z)\), that is,

\[
E(z) = \frac{L^*(z) - P(z)C(z, \rho)}{[1 + L^*(z)][1 + P(z)C(z, \rho)]}. \tag{4}
\]

The error \(e(t)\) given by Eq. (1) is the ideal model matching error and cannot be evaluated only with the experimental closed-loop data acquired from the closed-loop system due to the unknown plant \(P(z)\).

In this paper, the error \(e(t)\) is approximated so that the controller tuning can be carried out only using the closed-loop response data sets \(u_0(t)\) and \(y_0(t)\) which are the data sets \(u(t)\) and \(y(t)\) acquired from the closed-loop system with the controller whose parameter vector is \(\rho = \rho_0\) shown in Fig. 1, and they are given by

\[
u_0(t) = \frac{C(z, \rho_0)}{1 + P(z)C(z, \rho_0)}[r(t)] \tag{5}
\]

and

\[
y_0(t) = \frac{P(z)C(z, \rho_0)}{1 + P(z)C(z, \rho_0)}[r(t)], \tag{6}
\]

respectively.

Fig. 1 Closed-loop system consists of plant \(P(z)\) and controller \(C(z, \rho)\) whose parameter vector is \(\rho\).

2.2 Approximation Method I

For data-driven control, one of the possible approximated transfer functions for \(E(z)\) is given by

\[
E_1(z) = \frac{L^*(z) - P(z)C(z, \rho)}{[1 + L^*(z)][1 + P(z)C(z, \rho)]}, \tag{7}
\]

which is an approximation of \(E(z)\), where \(1 + P(z)C(z, \rho)\) in its denominator is replaced by \(1 + P(z)C(z, \rho_0)\) [6]. Thus the relationship between \(E_1(z)\) and \(E(z)\) is given by

\[
E_1(z) = \frac{1 + P(z)C(z, \rho)}{1 + P(z)C(z, \rho_0)}E(z). \tag{8}
\]

The model matching error for \(E_1(z)\) is defined as \(e_1(t)\), and it can be written as

\[
e_1(t) = E_1(z)[r(t)] = \frac{L^*(z) - P(z)C(z, \rho)}{[1 + L^*(z)][1 + P(z)C(z, \rho)]}[r(t)] = E_1(z)[r(t)] = \frac{1 + P(z)C(z, \rho)}{1 + P(z)C(z, \rho_0)}[r(t)], \tag{9}
\]

Using Eqs. (5) and (6), \(e_1(t)\) can be rewritten as

\[
e_1(t) = T^*(z)[r(t)] - T^*(z)[y_0(t)] - \frac{C(z, \rho_0)}{C(z, \rho_0)}S^*(z)[y_0(t)] = T^*(z)[r(t)] - C(z, \rho)S^*(z)\frac{C(z, \rho_0)}{C(z, \rho_0)}[y_0(t)]. \tag{10}
\]

where \(S^*(z)\) is the sensitivity function corresponding to the complementary sensitivity function \(T^*(z)\), and it is given by

\[
S^*(z) = 1 - T^*(z) = \frac{1}{1 + L^*(z)}. \tag{11}
\]

Moreover, using the relationship between \(u_0(t)\) and \(r(t) - y_0(t)\) in Fig. 1 given by

\[
u_0(t) = C(z, \rho_0)[r(t) - y_0(t)], \tag{12}
\]

\(e_1(t)\) can finally be written by

\[
e_1(t) = \frac{T^*(z)}{C(z, \rho_0)}[u_0(t)] - C(z, \rho)\frac{S^*(z)}{C(z, \rho_0)}[y_0(t)]. \tag{13}
\]

Since the error in the time domain given by Eq. (13) can be evaluated only using the closed-loop response data acquired from the closed-loop system with \(\rho_0\), the controller tuning using \(e_1(t)\) can be carried out. And the least-squares method can be used to obtain \(\rho_1^*\) which minimizes \(\|e_1(t)\|_2^2\) if the controller is linearly parameterized by the parameter vector \(\rho\).

Since Eq. (8) shows that if \(C(z, \rho) = C(z, \rho_0)\) then \(E_1(z) = E(z)\), the search for the optimal \(\rho\) with \(e_1(t)\) is equivalent to the search with \(e(t)\) if the optimal \(\rho\) which minimizes \(\|e\|_2^2\) is in the vicinity of \(\rho_0\) in the \(\rho\)-space.

¹ In this paper, the output \(y(t)\) of the system \(G(z)\) to the input \(u(t)\) is expressed as \(y(t) = G(z)u(t)\) instead of the convolution sum of \(u(t)\) and \(g(t) = Z^{-1}(G(z))\) which is the impulse response of \(G(z)\) or the inverse z-transform of \(G(z)\).
2.3 Approximation Method II

For the data-driven control, another approximated model matching error in the z-domain is given by

$$E_2(z) = \frac{L' - P(z)C(z, \rho)}{[1 + L'(z)]^2},$$

(14)

which is another possible approximation of $E(z)$, where $1 + P(z)C(z, \rho)$ in its denominator is replaced by $1 + L'(z)$. Thus, the relationship between $E_2(z)$ and $E(z)$ is given by

$$E_2(z) = \frac{1 + P(z)C(z, \rho) - 1}{1 + L'(z)} E(z).$$

(15)

The model matching error for $E_2(z)$ is defined as $e_2(t)$, and it can be written as

$$e_2(t) = E_2(z)[\rho(t)] = \frac{L'(z) - P(z)C(z, \rho)}{[1 + L'(z)]^2} [\rho(t)]$$

$$= \frac{1}{1 + L'(z)} \frac{1}{1 + L'(z)} [\rho(t)]$$

$$- \frac{C(z, \rho)}{[1 + L'(z)]^2} [P(z)[\rho(t)]$$

$$= T'(z) S'[\rho(t)] - C(z, \rho) [S'(z)]^2 P(z)[\rho(t)].$$

(16)

In order to evaluate $e_2(t)$ using only the closed-loop data, $P(z)$ and $\rho(t)$ in Eq. (16) are replaced by $p(t) = Z^{-1}[P(z)]$ and $R(z) = Z[\rho(t)]$, respectively. They are the impulse response of $P(z)$ and the z-transform of $\rho(t)$, respectively. By the replacements, $e_2(t)$ can be rewritten as

$$e_2(t) = T'(z) S'[\rho(t)] - C(z, \rho) [S'(z)]^2 R(z)[p(t)].$$

(17)

In order to evaluate $\|e_2(t)\|^2$, the impulse response of $P(z)$ is necessary. Since the relationship between $y_0(t)$ and $u_0(t)$ in Fig. 1 is given by

$$y_0(t) = P(z)[u_0(t)],$$

(18)

the impulse response of $P(z)$ can be obtained by

$$p(t) = U_0^{-1}(z)[y_0(t)],$$

(19)

where the inverse filter $U_0^{-1}(z)$ converts $u_0(t)$ to the unit impulse function $\delta(t)$, that is,

$$\delta(t) = U_0^{-1}(z)[u_0(t)].$$

(20)

The inverse filter $U_0^{-1}(z)$ given by Eq. (20) was introduced and its design method was shown in [7], and the validity of $U_0^{-1}(z)$ was confirmed through numerical simulations and experiments using an actual positioning system of a servo motor in [8].

It is expected that the controller tuning results with the approximation do not depend on the experimental data used for tuning since $U_0^{-1}(z)$ converts $u_0(t)$ and $y_0(t)$ to $\delta(t)$ and $p(t)$, respectively.

By the introduction of $U_0^{-1}(z)$, $e_2(t)$ is finally given by

$$e_2(t) = T'(z) S'[\rho(t)]$$

$$- C(z, \rho) [S'(z)]^2 R(z)U_0^{-1}(z)[y_0(t)].$$

(21)

In this case, the least-squares method can be used to obtain $\rho_2^*$ which minimizes $\|e_2(t)\|^2$ if the controller is linearly parameterized by the parameter vector $\rho$.

Since Eq. (15) shows that if $P(z)C(z, \rho) = L'(z)$ then $E_2(z) = E(z)$, the search for the optimal $\rho$ with $e_2(t)$ is equivalent to the search with $e(t)$ if the optimal $\rho$ which minimizes $\|e_2(t)\|^2$ is in the vicinity where $P(z)C(z, \rho) = L'(z)$ holds in the $\rho$-space.

The signal $U_0^{-1}(z)[y_0(t)]$ represents the impulse response of the plant $P$. However, $U_0^{-1}(z)[y_0(t)]$ is not used as a plant model. Namely, the signal neither generates the prediction of the future response nor calculates the closed-loop transfer function. Hence, it follows that the proposed method conforms to the data-driven approaches. Meanwhile, the $U_0$ is the closed-loop transfer function the impulse response of which results in the initial input data $u_0$. Since $U_0$ consists of a mathematical model of the closed-loop system, the proposed method may have an aspect to use the mathematical plant model indirectly. However, the proposed method only uses extracted information regarding the plant model from data. Thus, it is entirely different from the model based design where the design procedure starts from a mathematical plant model. From the point, the proposed method conforms to the data-driven approaches.

2.4 Model Matching Errors of VRFT and FRIT

2.4.1 Model matching error of VRFT [3]

As the model matching error of VRFT without a prefilter in the time domain is defined as $e(t)$, it is given by

$$e(t) = u_0(t) - C(\rho) \left\{ \frac{1}{T(z)} [y_0(t)] - [y_0(t)] \right\}$$

$$= u_0(t) - C(\rho) S^* [y_0(t)].$$

(22)

Compared with Eqs. (13) and (22),

$$e(t) = \frac{C(z, \rho_0)}{T'(z)} e_1(t)$$

(23)

is obtained. Due to Eqs. (7), (9), and (23), as $E(z)$ is defined as the transfer function from $r(t)$ to $e(t)$, it is given by

$$E(z) = \frac{C(z, \rho_0)}{T'(z)} E_1(z)$$

$$= \frac{C(z, \rho_0)}{T'(z)} + P(z)C(z, \rho) \frac{E(z)}{T'(z)} + \frac{E(z)}{T'(z)} \frac{E(z)}{T'(z)}.$$  

(24)

The transfer function $E(z)$ shows that the model matching error of VRFT is much different from the ideal model matching error $E(z)$. The design method of the prefilter which converted $e(t)$ to $e_1(t)$ was proposed in [7].

2.4.2 Model matching error of FRIT [4]

As the model matching error of FRIT in the time domain is defined as $e(t)$, it is given by

$$e(t) = y_0(t) - T'(z) \left\{ \frac{1}{C(z, \rho)} [u_0(t)] + [y_0(t)] \right\}$$

$$= \{1 - T'(z)[y_0(t)] - \frac{1}{C(z, \rho)} T'(z)[u_0(t)]$$

$$= S^*[y_0(t)] - \frac{1}{C(z, \rho)} T'(z)[u_0(t)].$$

(25)
Compared with Eqs. (13) and (25),
\[ e_f(t) = \frac{C(z, \rho_0)}{C(z, \rho)} e_f(t) \]  \hspace{1cm} (26)

is obtained\(^1\). Due to Eqs. (7), (9), and (26), as \( E_f(z) \) is defined as the transfer function from \( r(t) \) to \( e_f(t) \), \( E_f(z) \) is given by
\[ E_f(z) = \frac{C(z, \rho_0)}{C(z, \rho)} 1 + P(z)C(z, \rho) E(z). \]  \hspace{1cm} (27)

Since Eq. (27) shows that if \( C(z, \rho) = C(z, \rho_0) \) then \( E_f(z) = -E(z) \), the search for the optimal \( \rho \) with \( e_f(t) \) is equivalent to the search with \( e(t) \) if the optimal \( \rho \) which minimizes \( \|e\|_2^2 \) is in the vicinity of \( \rho_0 \) in the \( \rho \)-space.

However, since \( C(z, \rho) \) is in the denominator of the second term on the right-hand side of Eq. (25), the least-squares method cannot be applied to search \( \rho^* \) even if the controller is linearly parameterized by the parameter vector \( \rho \).

In the FRIT, the error function \( C(z, \rho)e_f(t) \) becomes
\[ C(z, \rho)e_f(t) = C(z, \rho)S^*(z)[y_0(t)] - T'(z)[y_0(t)]. \]

It is linearly parameterized in terms of the parameter \( \rho \). Hence, the minimizer of the \( L_2 \) norm of the error function can be obtained by way of the least-squares method. However, the cost function is different from one in the FRIT. Indeed, the minimizer of the norm of \( C(z, \rho)e_f(t) \) is different from the control parameter using FRIT.

\[2.5\] Discussions

The proposed method calculates the approximation Eq. (8) and Eq. (15) by applying a respective linear filter to the model matching error \( e(z) \). In addition, the paper shows that the approximated error signal in the existing VRFT and FRIT can be described as Eq. (24) and Eq. (27). The main contribution of the paper is to find out a specific filter expressed by Eq. (8) and Eq. (15) in the proposed method. Similar filters are also presented in the existing VRFT and FRIT, as shown in Eq. (24) and Eq. (27). For the following reasons, these expressions are effective from the theoretical points of view.

1. In the ideal case where the norm of \( E(z) \) is zero at the optimized parameter \( \rho^* \), the norms of \( E_1(z) \), \( E_2(z) \), \( E_i(z) \), and \( E_f(z) \) turn out to be zero at the optimized parameter \( \rho^* \).

2. When the norm of \( E(z) \) is not zero at the optimized parameter \( \rho^* \) due to the constraint of controller structures, the optimized values of \( E_1(z) \), \( E_2(z) \), and \( E_i(z) \) result in different values from each other. Such a difference characterizes the control performances in each data-driven approach.

3. The error signals \( E_1(z), E_2(z), \) and \( E_i(z) \) can be expressed using initial input and output data. In other words, the norms of \( E_1(z) \), \( E_2(z) \), and \( E_i(z) \) can be optimized without using the plant model.

4. Comparing the approximated errors among several data-driven approaches, we can easily characterize each data-driven approach. The relations that \( E_1(z) = E(z) \) when \( \rho = \rho_0 \) in Eq. (8) and \( E_1(z) = E(z) \) when \( \rho = \rho^* \) in Eq. (15) follow the above main results. Hence, it is easy to understand the meaning of the filters.

3. Numerical Examples

The approximated model matching errors for data-driven controller tuning are evaluated through numerical simulations with two kinds of plants. One is a flexible transmission system proposed in [9] as a benchmark for digital control design, and the other is a plant having an unstable zero.

3.1 Controller Tuning for Flexible Transmission System

The transfer function of the flexible transmission system [9] which has two resonances is given by
\[ P(z) = z^{-3} \frac{N_P(z)}{D_P(z)} \]  \hspace{1cm} (28)

where
\[ D_P(z) = 1 - 1.41833z^{-1} + 1.58939z^{-2} - 1.31608z^{-3} + 0.8864z^{-4}, \]  \hspace{1cm} (29)
\[ N_P(z) = 0.28261 + 0.50666z^{-1}, \]  \hspace{1cm} (30)

and the sampling period \( T_s \) is 0.05 s. The controller is given by
\[ C(z, \rho) = B^T(z)\rho, \]  \hspace{1cm} (31)

where
\[ B^T(z) = \frac{1}{1 - z^{-1}} \begin{bmatrix} 1 & z^{-1} & z^{-2} & \cdots & z^{-5} \end{bmatrix} \]  \hspace{1cm} (32)

and \( \rho = \begin{bmatrix} \rho_1 & \rho_2 & \rho_3 & \rho_4 & \rho_5 & \rho_6 \end{bmatrix}^T \).

Assuming that the dead time of the plant is known, the reference model is given by
\[ T'(z) = z^{-3} \frac{(1 - \alpha)^2}{(1 - \alpha z^{-1})^2}, \]  \hspace{1cm} (33)

where
\[ \alpha = e^{-10T_s}. \]  \hspace{1cm} (34)

The step reference responses of \( u_0(t) \) and \( y_0(t) \) whose data length is 51 are acquired from the closed-loop systems with the controller parameter vectors \( \rho_{01} \) and \( \rho_{02} \) shown in Tables 1 and 2, respectively. The response data with \( \rho_{01} \) and \( \rho_{02} \) are named as data #1 and #2, respectively, and they are shown in Fig. 2. Figure 2 shows that \( \rho_{01} \) and \( \rho_{02} \) make the closed-loop system quite conservative and unstable, respectively.

First, in order to confirm whether the controller given by Eq. (31) could make the closed-loop system similar to the reference model \( T'(z) \), \( \rho^* \) which minimizes the squared 2-norm of the ideal model matching error \( e(t) \) given by Eq. (1) with not the closed-loop data but \( P(z) \) was obtained. In this case, since the least-squares method could not be used to obtain \( \rho^* \), the MATLAB nonlinear optimization function “fminsearch” was employed to obtain \( \rho^* \). The tuned controller parameter vectors are shown in Tables 1 and 2.

The step reference responses of the closed-loop system with \( \rho^* \) are shown in Fig. 3. The upper part of Fig. 3 shows the response of \( y(t) \) of the reference model \( T'(z) \) given by

\[ y(t) = \begin{bmatrix} 1 & z^{-1} & z^{-2} & \cdots & z^{-5} \end{bmatrix} e(t), \]  \hspace{1cm} (35)

where \( e(t) = r(t) - y(t) \), and the lower part of Fig. 3 shows the response of \( y(t) \) for the closed-loop system with \( \rho^* \) given by

\[ y(t) = \begin{bmatrix} 1 & z^{-1} & z^{-2} & \cdots & z^{-5} \end{bmatrix} \begin{bmatrix} \rho_{01} & \rho_{02} & \rho_{03} & \rho_{04} & \rho_{05} \end{bmatrix} e(t). \]  \hspace{1cm} (36)

The difference between the upper and lower parts of Fig. 3 shows the effect of the controller parameter vectors on the closed-loop system.
y^*(t) = T^*(z)[r(t)]
\tag{35}

and y(t) acquired from the closed-loop system with \( \rho^* \), and the lower part shows the response of the model matching error \( y^*(t) - y(t) \). Figure 3 shows that the response of the closed-loop system with \( \rho^* \) is very similar to that of the reference model \( T^*(z) \).

The controller parameter vectors \( \rho_{1s}^*, \rho_{2s}^*, \rho_{3s}^*, \) and \( \rho_{4s}^* \) corresponding to the model matching errors \( e_1(t), e_2(t), e_3(t), \) and \( e_4(t) \) were obtained for each of the data #1 and #2 as shown in Tables 1 and 2, respectively. Only \( \rho_{1s}^* \)s were obtained with the MATLAB nonlinear optimization function “fminsearch,” and the others were obtained with the least-squares method. The step responses of \( y(t) \) and the model matching errors \( y^*(t) - y(t) \) of the closed-loop systems with \( \rho_{1s}^*, \rho_{2s}^*, \rho_{3s}^*, \) and \( \rho_{4s}^* \) obtained with the data #1 and #2 are shown in Figs. 4–7, respectively.

The squared 2-norms of \( e_1(t), e_2(t), e_3(t), \) and \( e_4(t) \) are shown in Table 3.
Fig. 7 Step reference responses of closed-loop systems with \( \rho \)'s tuned by \( e_s(t) \) given by Eq. (25) with data #1 and #2, respectively.

Table 3 Squared 2-norms of \( e(t) = y^*(t) - y(t) \) for data #1 and #2.

| Data # | \( ||e(t)||^2 \) |
|--------|--------------------|
| #1     | 0.5                |
| #2     | 0.7                |

Fig. 8 Step reference response data sets #3 and #4 acquired from the closed-loop systems with \( \rho_{03} \) and \( \rho_{04} \), respectively.

Table 4 Tuned controller parameters using the closed-loop data #1.

| Parameter | \( \rho \) |
|-----------|------------|
| \( \rho_0 \) | 0.5        |
| \( \rho_1 \) | 0          |
| \( \rho_2 \) | 0          |
| \( \rho_3 \) | 0          |

Table 5 Tuned controller parameters using the closed-loop data #2.

| Parameter | \( \rho \) |
|-----------|------------|
| \( \rho_{04} \) | 0.9        |
| \( \rho_2 \) | 0          |
| \( \rho_3 \) | 0          |

where

\[
B^T(z) = \begin{bmatrix} 1 & T_s & 1 - z^{-1} \\ 1 - z^{-1} \\ T_s \end{bmatrix}
\]  

(38)

and \( \rho = [\rho_1 \quad \rho_2 \quad \rho_3]^T \).

Assuming that the dead time of the plant is known, the reference model is given by

\[
T^*(z) = z^{-1} \frac{(1 - \alpha^2)}{(1 - \alpha z^{-1})^2}
\]  

(39)

where

\[
\alpha = e^{-0.2T_i}
\]  

(40)

In this case, any controller parameter vector \( \rho \) cannot make the closed-loop system the same as the reference model \( T^*(z) \) given by Eq. (39) because the controller \( C(z, \rho) \) given by Eq. (37) cannot change the closed-loop zeros and the reference model \( T^*(z) \) does not have the unstable zero of \( P(z) \).

The step reference responses of \( u_0(t) \) and \( y_0(t) \) whose data length is 101 were acquired from the closed-loop system with the controller parameter vectors \( \rho_{03} \) and \( \rho_{04} \) shown in Tables 4 and 5, respectively. The response data with \( \rho_{03} \) and \( \rho_{04} \) are named as data #3 and #4, respectively, and they are shown in Fig. 8. Figure 8 shows that \( \rho_{03} \) and \( \rho_{04} \) make the closed-loop system conservative and unstable, respectively.

First, in order to confirm whether the controller given by Eq. (37) could make the closed-loop system similar to the reference model \( T^*(z) \), \( \rho^* \) which minimized the squared 2-norm of the model matching error \( e(t) \) given by Eq. (1) with not the closed-loop data but \( P(z) \) was obtained. In this case, since the
Fig. 9 Step reference response with controller tuned by the ideal model matching error $e$ given by Eq. (1) with true $P(z)$ instead of the approximated model matching errors.

Fig. 10 Step reference responses with controllers tuned by $e_1$ given by Eq. (13) using data #3 and #4, respectively.

Fig. 11 Step reference responses with controllers tuned by $e_2$ given by Eq. (21) using data #1 and #2, respectively.

The step reference responses of the closed-loop system with $\rho^*$ is similar to that of the reference model $T^*(z)$ after the undershoot is converged. The undershoot cannot be suppressed by the controller since no feedback controller can change zeros of plant generally.

The controller parameter vectors $\rho_1^*$, $\rho_2^*$, $\rho_3^*$, and $\rho_4^*$ corresponding to the model matching errors $e_1(t)$, $e_2(t)$, $e_3(t)$, and $e_4(t)$ were obtained for each of the data #3 and #4 as shown in Tables 3 and 5.

The step response of $y(t)$ and the model matching errors $y^*(t) - y(t)$ of the closed-loop systems with $\rho_1^*$, $\rho_2^*$, $\rho_3^*$, and $\rho_4^*$ obtained with the data #3 and #4 are shown in Figs. 10–13, respectively.

The squared 2-norms of $e(t) = y^*(t) - y(t)$ for data #3 and #4 are also shown in Table 6.

Table 6 Squared 2-norms of $e(t) = y^*(t) - y(t)$ for data #3 and #4.

|     | $\|y^*(t) - y(t)\|^2_2$ | $\rho_1^*$ | $\rho_2^*$ | $\rho_3^*$ | $\rho_4^*$ |
|-----|---------------------------|-----------|-----------|-----------|-----------|
| data #3 | 1.1807                    | 2.1217    | 1.2157    | 4.4377    | 6.8248    |
| data #4 | 5.9859                    | 5.9859    | 1.2157    | 100.39    | 3.4686    |

Fig. 12 Step reference responses with controllers tuned by $e_v$ given by Eq. (22) using data #1 and #2, respectively.

Fig. 13 Step reference responses with controllers tuned by $e_f$ (FRIT) given by Eq. (25) using data #3 and #4, respectively.

Figures 10–13 and Table 6 show the following: 1) Only $\rho_2^*$ tuned by the approximated model matching error $e_2(t)$ given by Eq. (21) with the data #3 and #4 make the closed-loop systems with $\rho^*$ similar to that of the reference model $T^*(z)$ after the undershoot is converged. The undershoot cannot be suppressed by the controller since no feedback controller can change zeros of plant generally.

The controller parameter vectors $\rho_1^*$, $\rho_2^*$, $\rho_3^*$, and $\rho_4^*$ were obtained with the MATLAB nonlinear optimization function “fminsearch,” and the others were obtained with the least-squares method. The step responses of $y(t)$ and the model matching errors $y^*(t) - y(t)$ of the closed-loop systems with $\rho_1^*$, $\rho_2^*$, $\rho_3^*$, and $\rho_4^*$ were obtained with the data #3 and #4 as shown in Tables 3 and 4, respectively. Only $\rho_2^*$s were obtained with the MATLAB nonlinear optimization function “fminsearch,” and the others were obtained with the least-squares method. The step responses of $y(t)$ and the model matching errors $y^*(t) - y(t)$ of the closed-loop systems with $\rho_1^*$, $\rho_2^*$, $\rho_3^*$, and $\rho_4^*$ were obtained with the data #3 and #4 as shown in Figs. 10–13, respectively.

The squared 2-norms of $e(t) = y^*(t) - y(t)$ acquired from the closed-loop systems with $\rho_1^*$, $\rho_2^*$, $\rho_3^*$, and $\rho_4^*$ tuned with the approximated model matching errors $e_1(t)$, $e_2(t)$, $e_3(t)$, and $e_4(t)$ are also shown in Table 6.

Figures 10–13 and Table 6 show the following: 1) Only $\rho_2^*$ tuned by the approximated model matching error $e_2(t)$ given by Eq. (21) with the data #3 and #4 make the closed-loop systems
very similar to the closed-loop system with $\rho^*$ tuned by the ideal model matching error $e(t)$ given by Eq. (1) regardless of the data used for the tuning. 2) $\rho_1$ is tuned by the approximated model matching error $e_1(t)$ with the data #3 and #4 cannot suppress the overshoot of $y(t)$, and $\rho_1$s are affected by the data used for the tuning. 3) $\rho_1$ tuned by the approximated model matching error $e_1(t)$ with the data #3 generates the oscillation of $y(t)$. 4) $\rho_*$ tuned by the approximated model matching error $e_*(t)$ with the data #4 makes the closed-loop system unstable.

3.4 Discussions

From the simulation result, the approximation method II also showed the best control performance. The reason is the same as the case of the flexible transmission system. In the case of the low gain controller, the FRIT showed the worst result. The reason is that the initial data does not include enough information due to the relatively smooth response. On the other hand, in the case of the high gain controller, the VRFT showed the worst result. That is the same reason as the case of the flexible transmission system.

4. Conclusion

In this paper, two kinds of approximated model matching errors for data-driven controller tuning derived from the ideal model matching error have been evaluated through numerical simulations for a flexible transmission system and a plant with an unstable zero.

The tuning results using the approximated errors obtained with the inverse filter of the plant input were not affected by the data used in the controller tuning for both plants as well as the results with the ideal model matching error.

The tuning results using the approximated error obtained without the filter were less affected by the data used in the controller tuning for both plants than those with the model matching error of FRIT, although the controllers tuned by the errors without the filter could not suppress the overshoot of the plant with an unstable zero.

The present work does not treat the case where the initial input and output data are contaminated by noise signals. Formentine et al. [10] introduced $L_2$ regularization to reduce the influence of noise signals to the VRFT design. Future work is to introduce some devices to mitigate the influence of the noise signal in the proposed method.

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