Stock mechanics: predicting recession in S&P500, DJIA, and NASDAQ

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Abstract
An original method, assuming potential and kinetic energy for prices and conservation of their sum is developed for forecasting exchanges. Connections with power law are shown. Semiempirical applications on S&P500, DJIA, and NASDAQ predict a coming recession in them. An emerging market, Istanbul Stock Exchange index ISE-100 is found involving a potential to continue to rise.

Keywords: Potential and kinetic energy; Equations of motion; Power law; Oscillations; Crashes; Portfolio growths
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1. Introduction
As is well known, prices, market indices evolve from one mood to another in time. They may be in calm oscillatory fluctuation mood for some time, then they may increase (or decrease), then again turn into a new oscillatory period, then a crash or a crisis may come, and finally, the current era closes up. Then a new one takes stage; new conditions and new formations take place, so forth ad infinitum. The time terms between long range drastic changes are named as era in the present work, and comparatively medium or short range characteristic time periods in eras are cited as epoch. The whole past and future history of exchanges may be far from obeying analytical functions. Time series may be predicted epoch by epoch, as presently proposed, in terms of some simple analytical functions. These equations of motion for prices are derived in the next section and power law connections are displayed in the third one. In the fourth section three indices S&P500, DJIA, and NASDAQ from New York Stock Exchange (NYSE) and ISE-100 from Istanbul Stock Exchange (ISE) are predicted by the present method. Last section is devoted to conclusion.
2. Potential and kinetic energies and conservation of their sum

A potential energy (taking mass as unity, see $^1$) in terms of difference in prices ($\chi = \chi(t)$) may be defined as

$$U(\chi - \chi_{av}) = h (\chi - \chi_{av})^\alpha, \quad (1)$$

where $h$ and $\alpha$ are some price independent parameters designating the current epoch, and $\chi_{av}$ is some time average of prices, which defines the zero-potential-level. Eq. (1) describes a kind of “gravitational” potential (as on the earth surface) for $\alpha=1$, where $h$ becomes the gravitational (anti-gravitational) constant ($g$) with $0|h=g$ ($0|h=-g$). Whereas for $\alpha=2$ we have a spring-mass potential energy with a spring constant (Hooke constant) equal to $2h$. It is worth to underline that, the parameters of Eq. (1) i.e., $h$, $\alpha$, and $\chi_{av}$ may differ from one epoch to the other for any share and also from one share to another in any epoch. Note that, potential energy in Eq. (1) satisfies a power law for any $\alpha$, as discussed to some extend in the next section.

Moreover, again by taking mass as unity, one may define kinetic energy for prices as

$$K = \frac{1}{2}v^2, \quad (2)$$

where $v$ is the usual speed i.e., $v(t)=d\chi(t)/dt$. The dimensional unit of $K$ may be taken as (local currency unit/time)$^2$ for shares, e.g., ($$/c$/day)$^2$ or ($$//day)$^2$ in USA. For indices lcu may be kept in the units or it may be substituted by “value”. Potential energy of Eq. (1) will obviously have the same unit as (lcu/time)$^2$, and for $\alpha=1$ the factor $h$ will have the unit (lcu/time$^2$), where lcu stands for local currency unit. For $\alpha=2$, $h$ will have the unit of (time$^{-2}$) i.e., frequency squared.

We may assume conservation of the sum of potential and kinetic energies, as long as friction forces, damping etc. are negligible,

$$U + K = h(\chi - \chi_{av})^\alpha + \frac{1}{2}v^2 = E = \text{constant} \quad (3)$$

Differentiation of Eq. (3) with respect to time, for $\alpha=1$ yields

$$h(d\chi/dt) + v(dv/dt) = 0 \quad (4)$$

from which, after substituting $v=d\chi/dt$ and $a=dv/dt$, one may obtain the familiar equation of motion for azimuthal rises and falls as in classical mechanics

$^1$Some kind of inertia as “resistance” against any impact to move the price do exist in time charts of shares.[1-4, and references therein]. Yet, not any reasonable “force equation” connecting inertia and acceleration of stock prices is proposed. In any case, whether massless potential and kinetic energies are defined, or they are defined per mass, or even mass is taken as unity in them will not be worth much in the present scheme.
\[ \chi(t_m) = \chi_0 + v_0 \, t_m + \frac{1}{2} \, h \, t_m^2, \quad (5) \]

where \( \chi_0 \), and \( v_0 \) designate initial price and speed, respectively. Time \( t_m \) runs over exchange process days and may be set to zero at the beginning of any epoch. For \( \alpha=2 \), Eq. (1) yields oscillations. In the expansion of \( \chi_{av}(t_m) = \chi_{av}(t_m=0)+v_{av} \, t_m \) sign and magnitude of \( v_{av} \) indicates the up, down or horizontal character of oscillatory trends;

\[ \chi(t_m) = \chi_{av}(t_m=0) + v_{av} \, t_m + A \sin(w t_m + \Phi), \quad (6) \]

where \( A \), \( w \), and \( \Phi \) is the usual amplitude, angular frequency (here, \( (2h)^{1/2} \)), and phase, respectively. They are observed in general to have some medium range time periods about some ten days or so, and fade away after a few (two, or three) full periods.

3. Potential energy, power law, and log-periodic equations of motion

The form of potential energy in Eq. (1) satisfies a power law, which is known in physics for a long time as effective subject, utilized especially to express critical phenomena in statistical mechanics. Some special forms of power law appears outside the physical fields as Pareto’s Law and Zipf’s Law.\[6, 7\]. It is also utilized in seismic predictions for the rupture times.\[8, 9\]

Power law states that if the argument \( x \) of any observable \( O(x) \) is scaled by some \( \lambda \), (i.e., if \( x'/x=\lambda \)) and if \( O(x)/O(x)=\zeta \), then \( O(x)=x^\alpha \) is a solution with \( \lambda^\alpha = \zeta \) and \( \alpha=\log \zeta/\log \lambda \). Note that, \( \lambda \) and \( \zeta \) are independent of \( x \) and the relative value of the observable at two different scales depend only on the ratio of the scaling parameters. This is the way scale invariance is associated to self-similarity and criticality. Note also that there is no condition on \( \alpha \) to be real. Incorporating \( \lambda^\alpha/\zeta=1=\exp(i2\pi m)\)\[10, 11\], where \( m \) is any integer, one may generalize the standard scaling \( O(x)=x^\alpha \) to a log-periodic one, \( O(x)=x^\alpha P(\log x/\log \lambda) \), where \( P \) is a function of period 1. Fourier expansion of \( P \) can be performed to obtain the most general form of the relevant function. (For detailed and complete treatment, and for various applications see \[8-31\].) For the sake of simplicity, one may take into account only the first Fourier term;

\[ O(\tau) \approx (1-\tau)^\alpha \{d_0 + d_1 \cos[2\pi \Omega \ln(1-\tau) + \Psi] \}, \quad (7) \]

where, \( \tau \) stands for \( t/T_c \) and \( t \) is the general independent time variable, and \( T_c \) is the critical time; \( \alpha=\ln \zeta/\ln \lambda \), \( \Omega=1/\ln \lambda \), and \( \Psi \) is some general phase term. What is in Eq. (7) relevant to time series of shares and indices is that, near the crash (which is considered as a failure time for the log-periodicity)
the frequency of the oscillations and the volatility increases, which can be considered as one of the hall-marks of the coming crash.

Let’s approximate the \((1–\tau)^{\alpha}\) factor and \(\ln(1-\tau)\) by \((1-\alpha\tau)\) and \((-\tau)\) for \(\tau \approx 0\), i.e. much before (and by \(t \rightarrow -t\), much after) the critical time. After some simple mathematical manipulations Eq. (7) can be written as

\[
O(\tau) \approx D_0 + D_1\tau + D_2\cos(2\pi\Omega\tau + \Psi') ,
\]

where, the constants \(D_0, D_1, D_2\) can be calculated out of \(d_0, d_1,\) and the others of Eq. (7). Close similarity between Eqs. (8) and (6) is a consequence of Eq. (1) obeying power law.

4. Applications

The three NYSE indices S&P500, DJIA, and NASDAQ[33] are extensively studied in literature especially within the formalism of power law. For similar log-periodic predictions performed on ISE-100[34] see [27]. In all of these indices (as in many other world markets) a severe crash dated about the year of 2000 is common. The present state of the same indices will be investigated utilizing the original method of stock mechanics.

The index of S&P500

A crucial feature in S&P500 (Fig. 1) is the almost symmetric behavior of time series about the critical point near 01.Sept.2000 with the close value of 1520.77. Secondly, starting with the beginning of 1995, oscillatory periods decrease as closes climb to the climax. Afterwards closes start to recede and high frequency oscillations turn back into low frequency ones with increasing amplitudes. Thirdly, with the linear price axis, the time series may be fitted as a first order approximation by partial straight lines. Then simple analytical functions may be utilized for each epoch \((j=1, 2, 3, 4)\) as

\[\chi_j(t_m) = \chi_{0j} + v_j t_m,\]

corresponding to a constant potential energy, i.e. \(\alpha=0\), and \(h\) arbitrary in Eq. (1). In this picture the 1987, 1990, 1998 crashes do seem as normal fluctuations, as well as the others after 2000.

For \(\alpha=1\) in Eq. (1), the second order expression of Eq. (5) delivers very interesting results for the two epochs; one from the beginning of 1997 to the end of 2002, and the second after 2002 till the present time (May.2005), see Fig. 1. By a simple least square fit (lsf) to daily data[33], the gravity comes about the same for both of the pronounced epochs as \(h = -0.001101\) lcu/day\(^2\). The initial (shooting) speed is found to be 1.73 lcu/day for the first epoch and 1.11 lcu/day for the next one. Imagining the close values as height of a particle shot up in the given gravitational field; the particle first rises till the climax, then falls down and hits the ground at an elevation of 663 lcu. Afterwards it bounces back with a smaller speed and rises till a lesser
height of 1219 lcu. Therefore, during the collision it looses its total energy by 59% and the collision is inelastic. The maximum height in any epoch can be calculated utilizing the relation \( \chi - \chi_0 = \frac{v_0^2}{2h} \). It is worth to forecast that March.2005 values are local maximum for S&P500, and a recessional correction may be expected till the level of 800 back, within the coming 500 days. The pronounced parameters are listed in Table 1 for S&P500 as well as for the other indices.

**The index of DJIA**

DJIA has similar features as pronounced above for S&P500, see Fig. 2. Focusing on quadratic behavior (Eq. 5), gravity again comes out as common for both of the token epochs, before and after the beginning of 2003 (Fig. 2.), where \( h = -0.00606 \) lcu/day\(^2\). Hitting speed is 10.17 lcu/day, and bouncing back speed is 8.43 lcu/day, corresponding to 31% loss in total energy. So the March.2005 height is considerably close the historical top of Jan.2000. Again, in about 500 days, DJIA is forecasted to recede back to 8000’s.

**The index of NASDAQ**

Nasdaq has the most complicated appearance of all the NYSE indices studied here. Yet, it displays very many similarities with S&P500 and DJIA, Fig. 3. Moreover a common gravity comes out for the two epochs following the beginning of 1997 and separated by the beginning of 2003 (Fig. 3.), where \( h = -0.0041 \) lcu/day\(^2\). The hitting and bouncing back speeds are 6.08 lcu/day and 3.10 lcu/day, respectively. Then, the loss in total energy is 75%. So, as expected, the March.2005 heights are quite below the historical maximum. Consecutively, NASDAQ is also forecasted to recede back to 1400’s at least, within the next one and a half year.

**The index of ISE-100**

ISE is a well known world emerging market, and comparing to the NYSE indices, ISE-100 has many more different aspects than similar ones. A log-linear era (lasting about 20 years from the beginning on) has closed by the 2000 crash. Afterwards a recession with 30% loss in a year has taken place. Between Jan.2000 and Jan.2004, recession epoch has been completed and transition to a new up trend has already taken place.

Within the pronounced epoch, ISE-100 displays a dishlike form, and as can be seen in Fig. 4. a. there exist anti-gravity with \( h = 0.001811 \) lcu/day\(^2\). The work done by this constant anti-gravity results in increasing the total energy, day by day. So, one may expect ISE-100 to continue to rise with some possible decorative up and down fluctuations. It is hard to forecast the time of departure from Eq. (5) and solid curve in Fig. 4. a.; yet, it seems
that it lies in the far future.

On the other hand, it can be observed that, at the bottom of the Jan.2000 and Jan.2004 epoch, the trend is horizontal. Meanwhile, many oscillations of type Eqs. (6) and (8) may be expected to exist in ISE-100 and in many ISE shares. In Figs. 4. b. and 4. c., two typical oscillatory epochs with different time domains are exemplified, where time axis is weekly and daily, respectively. The corresponding mechanical parameters are listed in Table 2. For better fits one may take into account many coupled smaller spring-masses.

5. Conclusion

The present analytical method can be applied to shares as well. In general there exists a wide diversity of epochs in world markets, in which the present analytical functions can safely be applied. For more elaborate epoch formations, some more complicated functional forms may be tried in Eq. (1). Or, the solutions of the present form for non-integer fractal powers of $\alpha$ may be taken into account. Yet, mismatches between the real and calculated values may always exist, due to unpredictability character of short-range fluctuations about longer-range ones. It is obvious that, such analytical approaches may be used together with the traditional approaches for better prediction of the markets.

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Table 1 Physical parameters of S&P500, DJIA, NASDAQ, and ISE-100
for $\alpha=1$ in Eq. (1). For the time domains, see text. (lcu) in some units stand for local currency unit.

S&P500 DJIA NASDAQ ISE-100

| Parameter | S&P500 | DJIA | NASDAQ | ISE-100 |
|-----------|--------|------|--------|---------|
| $h$ (day$^{-2}$) | -0.001101 | -0.00606 | -0.004119 | +0.001811 |
| $v_0$ (lcu/day) | 1.73 | 10.17 | 6.08 | -7.521 |

For detailed information about NYSE shares and indices, URL: http://biz.yahoo.com/i/
\( v_{02} \) (lcu/day) 1.11 8.43 3.10 not present
energy loss (%) 59 31 75 not present

Table 2 Physical parameters of ISE-100 for \( \alpha=2 \) in Eq. (1). (lcu) in some units stand for local currency unit.

2001-2003 2004
\( h \) (day\(^{-2}\)) 0.051911 0.093884
\( v_{av} \) (lcu/day) 14 18.23
A (lcu) 2950 880

**Figure captions**

Fig. 1. The Sept.2000 crash in S&P500 is described as a second order approximation by an azimuthal rise and fall in a gravity \( h= -0.001101 \) (lcu/day\(^2\)), where the initial (shooting) speed is \( v_{01}=1.73 \) (lcu/day). The price fall down after the maximum height and inelastically bounces back with \( v_{02}=1.11 \) (lcu/day) in the same gravity, and rises up to 1200’s in accordance with the expression \( \chi - \chi_0 = \frac{v_0^2}{2h} \). A recession, back to 800’s is predicted within the coming 500 days.

Fig. 2. The excursion of DJIA about the Apr-Sep.2000 climax is described as a second order approximation by an azimuthal rise and fall of the price in a gravity \( h= -0.00606 \) (lcu/day\(^2\)). The initial (shooting) speed at the beginning of 1995 is \( v_{01}=10.17 \) (lcu/day). The price fall down after the maximum height and inelastically bounces back with \( v_{02}=8.43 \) (lcu/day) in the same gravity, and rises up to 11000’s in accordance with the expression \( \chi - \chi_0 = \frac{v_0^2}{2h} \). A recession, back to 8000’s and below is predicted within the coming 500 days.

Fig. 3. NASDAQ’s azimuthal motion beginning with the year of 1995 is described by a gravity \( h= -0.004119 \) (lcu/day\(^2\)) and initial (shooting) speed of \( v_{01}=6.08 \) (lcu/day). The inelastic bouncing speed in the same gravity is \( v_{02}=3.10 \) (lcu/day). A recession, from the present heights back to 1200’s is predicted within the coming one and a half year.

Fig. 4. a. The epoch begun with the beginning of 2000 has an anti-gravity \( h=0.001811 \) (lcu/day\(^2\)). Rise is predicted to last till the departure of the price from the the solid curve.

Fig. 4. b. Long term oscillatory motions of ISE-100 within the dishlike epoch, corresponding to \( \alpha=2 \) in Eq. (1). The horizontal axis is weekly in time and \( h=0.051911 \) (week\(^{-2}\)). Oscillation fades away after three full periods (here, about two years and a half), as usual.

Fig. 4. d. Short term oscillatory motions of ISE-100 within the year of 2004 with \( h= 0.093884 \) (day\(^{-2}\)). Oscillation fades away after three full
periods (here, about two months or so), as usual. (Notice the relative increase in volume at dips of oscillations.)

Figures

Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4. a.
Fig. 4. b.
Fig. 4. c.
NASDAQ

\[ y = -0.004119x^2 + 6.077851t + 757.7 \]
\[ y = -0.004116x^2 + 3.099947t + 1459.7 \]
