Synchronization Analysis for Stochastic Inertial Memristor-Based Neural Networks with Linear Coupling

Lixia Ye, Yonghui Xia, Jin-liang Yan, and Haidong Liu

1Department of Mathematics and Computer, Wuyi University, Wuyishan, Nanping 354300, China
2Department of Mathematics, Zhejiang Normal University, Jinhua 321004, China
3School of Mathematical Sciences, Qufu Normal University, Qufu 273165, China

Correspondence should be addressed to Yonghui Xia; xiadoc@163.com

Received 28 May 2020; Accepted 29 June 2020; Published 23 July 2020

Academic Editor: Jianquan Lu

Copyright © 2020 Lixia Ye et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper concerns the synchronization problem for a class of stochastic memristive neural networks with inertial term, linear coupling, and time-varying delay. Based on the interval parametric uncertainty theory, the stochastic inertial memristor-based neural networks (IMNNs for short) with linear coupling are transformed to a stochastic interval parametric uncertain system. Furthermore, by applying the Lyapunov stability theorem, the stochastic analysis approach, and the Halanay inequality, some sufficient conditions are obtained to realize synchronization in mean square. The established criteria show that stochastic perturbation is designed to ensure that the coupled IMNNs can be synchronized better by changing the state coefficients of stochastic perturbation. Finally, an illustrative example is presented to demonstrate the efficiency of the theoretical results.

1. Introduction

The memristor [1] is a kind of a nonlinear resistor with memory and nanoscale, which is widely applied in chaotic circuits, artificial neural networks, and so on. In [2], the relevant mechanisms of neural networks, such as long-term potentiation and spike time-dependent plasticity, are presented by applying basic electric circuits, and more complex mechanisms are constructed to mimic the synaptic connections in a (human) brain. During neuron transmission, synchronous resonance is a very important biological phenomenon. In recent years, a lot of systems have been investigated to realize synchronization such as time-varying switched systems, MNNs, and BAM neural networks [3–19].

In [8], a new switching pinning controller was designed to finite-time synchronization in nonlinear coupled neural networks by regulating a parameter. Therefore, it is necessary for synchronization to design a suitable controller, such as impulsive controller [11, 12, 20–22], nonchattering controller [19], and switching controller [6–8]. Based on parametric uncertainty and state dependency in the connection weight matrices of MNNs, the connection weight matrices jump in certain intervals. Duan and Huang [23] proposed periodicity and dissipativity for memristor-based neural networks with mixed delays involving both time-varying delays and distributed delays via using Mawhin-like coincidence theorem, inclusion theory, and M-matrix properties. The authors established two different types of exponential synchronization criteria for the coupled MNNs based on the master-slave (drive response) concept and discontinuous state feedback controller, and simultaneously, an estimation of the exponential synchronization rate was estimated (see [24]). It is worth pointing out that, the authors in [25, 26] added the linear coupling and interval term into MNNs to achieve two different synchronization via applying the Halanay inequality [27] and the Lyapunov method. However, there were essential differences between the synchronization results established by these two literature studies. In [25], the differential inclusion method was applied to transform the coupled connection weight matrices; moreover, a discontinuous controller was designed to ensure that multiple IMNNs can be synchronized. Li and Zheng [26] demonstrated that the coupled connection weight matrices can be decomposed by interval analysis [28],
which by weakening the matrices satisfies the conditions. Besides, the new synchronization criteria for IMMNs with linear coupling were established.

As we know, noise plays an important role in synchronization since it can stabilize an unstable system. In recent years, many scholars are very interested in synchronization of stochastic networks with time-varying delays [2, 29–38]. In 2013, the Jensen integral inequality was improved by the so-called Wirtinger-based integral inequality [33]. Furthermore, Gao et al. [29] showed that a state feedback controller and an adaptive updated law used to guarantee stochastic memristor-based neural networks with noise disturbance can be asymptotically synchronized. In [38], the authors investigated the synchronization of a stochastic multilayer dynamic network with time-varying delays and additive couplings by designing two pinning controllers. Therefore, taking stochastic perturbation into complex neural networks is very necessary and important.

Note that the stochastic systems were mainly first-order neural networks in previous works. In this paper, based on the model of [26], considering \(x_i(t), x_i(t - \tau(t)), f_i(x_i(t)), f_j(x_i(t - \tau(t)))\) will produce errors, the new model of the stochastic coupled inertial memristor-based neural networks is constructed. Meanwhile, new results on synchronization in mean square are proposed. The main contributions of this paper are high-lighted as follows:

(i) Stochastic perturbation is taken into account in the second-order [39] coupled memristor-based neural networks with inertial term. Synchronization analysis becomes more challenging for the system with higher order and higher dimension.

(ii) The criterion for stochastic inertial memristor-based neural networks with linear coupling is proposed by applying the stochastic analysis techniques and the vector Lyapunov function method to realize synchronization in mean square.

(iii) An illustrative example is given to illustrate that system (1) can be synchronized under the coupled network with five nodes. Besides, system (1) has strong anti-interference.

2. Model Formulation and Preliminaries

In this paper, we consider the model of stochastic coupled inertial memristor-based neural networks (IMMNs for short) with \(N\) coupled identical nodes described by the following equation:

\[
\frac{dx_i(t)}{dt} = -D \frac{dx_i(t)}{dt} - Cx_i(t) + A(x_i(t))f(x_i(t)) + B(x_i(t))f(x_i(t - \tau(t)))
\]

\[
+ c \sum_{j=1}^{N} G_{ij} \left( \frac{dx_j(t)}{dt} + x_j(t) \right) + \sigma_1 f(x_i(t)) + \sigma_2 f(x_i(t - \tau(t))) + \sigma_3 f(x_j(t)) + \sigma_4 f(x_j(t - \tau(t))) \mathrm{d}w_i, \quad i \in N,
\]

where \(x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^\top \in \mathbb{R}^n, i \in N,\)

\(D = \text{diag}(\beta_1, \beta_2, \ldots, \beta_n),\)

\(C = \text{diag}(c_1, c_2, \ldots, c_n),\)

\(A(x_i(t)) = [a_{ij}(x_i(t))]_{n \times n}\), and the connection weight matrix \(B(x_i(t)) = [b_{kj}(x_{ij}(t))]_{n \times n}\) satisfy the following conditions:

\[
a_{kj}(x_{ij}) = \begin{cases} \alpha_{kj}^+, & |x_{ij}| < T_j, \\ \alpha_{kj}^-, & |x_{ij}| > T_j, \end{cases}
\]

\[
b_{kj}(x_{ij}) = \begin{cases} \beta_{kj}^+, & |x_{ij}| < T_j, \\ \beta_{kj}^-, & |x_{ij}| > T_j, \end{cases}
\]

where \(T > 0\) is the switching jump, and \(\alpha_{kj}^+, \alpha_{kj}^-, \beta_{kj}^+, \beta_{kj}^-\) are all constants, \(k, j \in n\). The network coupling strength \(c > 0\) is a constant, and \(\Gamma = \text{diag}(\gamma_1, \gamma_2, \ldots, \gamma_n)\) is the inner coupling matrix. \(G = (G_{ij})_{N \times N} = \text{diag}(G_{ij})_{N \times N}\) is the constant coupling configuration matrix representing the topological structure of the system. \(G_{ij} > 0\) is defined as a link from node \(i\) to node \(j\), otherwise, \(G_{ij} = 0\). Besides, \(G\) satisfies

\[
G_{ii} = \sum_{j=1, j \neq i}^{N} G_{ij}, \quad i \in N.
\]

The initial condition associated with system (1) is given as \(x_i(s) = \varphi_i(s) \in \mathbb{C}^n \cap [-\tau, 0], R^n, i \in N\). And, \(f(x_i(t)) = (f_1(x_{i1}(t)), f_2(x_{i2}(t)), \ldots, f_n(x_{in}(t)))^\top\) denotes the output of the neuron unit, which satisfies the following assumption:

\((H_1)\): for any two different \(u, v \in R\), there exists a positive scalar \(\bar{l}_j > 0 (i \in n)\) such that \(|f_j(u) - f_j(v)| \leq \bar{l}_j|u - v|\).

For stochastic systems, the Itô formula plays an important role in the synchronization. Consider a general stochastic system \(dx(t) = f(x(t), t)dt + g(x(t), t)dw(t)\) on \(t > t_0\) with an initial value \(x(t_0) = x_0 \in \mathbb{R}^n\), where \(f: \mathbb{R}^n \times R^+ \rightarrow \mathbb{R}^n\) and \(g: \mathbb{R}^n \rightarrow \mathbb{R}^n\). Denote a general nonnegative function \(V(x, t)\) on \(\mathbb{R}^n \times R^+\) to be continuously twice differentiable in \(x\) and once differentiable in \(t\), an stochastic differential operator \(dV(x, t) = \mathcal{L}V(x, t)dt + \mathcal{V}_x(x, t)dw(t)\),

\[
\mathcal{L}V(x, t) = V_t(x, t) + V_x(x, t)f(x, t) + (1/2)\text{tr} [g^\top(x, t)V_{xx}(x, t)g(x, t)], \quad V_t(x, t) = \frac{\partial V(x, t)}{\partial t}, \quad V_x(x, t) = \frac{\partial V(x, t)}{\partial x}, \quad \mathcal{V}_x(x, t) = \frac{\partial V(x, t)}{\partial x}, \quad \mathcal{V}_x(x, t) = \frac{\partial V(x, t)}{\partial x}.
\]
\[ V_{xx}(x,t) = (\partial^2 V(x,t)/\partial x_i \partial x_j)_{\text{non}} \] and \( E[V(x,t)] = E[\mathcal{D}V((x,t)dt)] \).

Considering \( a_{kj}(x_{ij}) \) and \( b_{kj}(x_{ij}) \) are bounded, therefore, \( A(x(t)) = [\underline{A}, \overline{A}], B(x(t)) = [\underline{B}, \overline{B}] \), where \( \underline{A} = (a_{kj})_{\text{non}} \), \( \overline{A} = (\overline{a}_{kj})_{\text{non}} \), \( \underline{B} = (\underline{b}_{kj})_{\text{non}} \) and \( \overline{B} = (\overline{b}_{kj})_{\text{non}} \) with \( \underline{a}_{kj} = \min \{ a_{kj}, \overline{a}_{kj} \} \), \( \overline{a}_{kj} = \max \{ a_{kj}, \overline{a}_{kj} \} \), \( \underline{b}_{kj} = \min \{ b_{kj}, \overline{b}_{kj} \} \), and \( \overline{b}_{kj} = \max \{ b_{kj}, \overline{b}_{kj} \} \).

\[
\begin{align*}
dx_i(t) &= [-x_i(t) + r_i(t)]dt, \\
\bar{r}_i(t) &= \begin{pmatrix} -\Theta x_i(t) - \Lambda r_i(t) + [\underline{A}, \overline{A}] f(x_i(t)) + [\underline{B}, \overline{B}] f(x_i(t - \tau(t))) + c \sum_{j=1}^{N} G_{ij} \bar{r}_j(t) \\ + (\sigma_1 x_i(t) + \sigma_2 x_i(t - \tau(t)) + \sigma_3 f(x_i(t)) + \sigma_4 f(x_i(t - \tau(t)))) \end{pmatrix} dt, \\
i \in N,
\end{align*}
\]

where \( \Theta = I + C - D, \Lambda = D - I \).

Based on interval uncertainty theory, the intervals \([\underline{A}, \overline{A}] \) and \([\underline{B}, \overline{B}] \) can be decomposed into \([\underline{A}, \overline{A}] = A_0 + [-1,1]H_A \) and \([\underline{B}, \overline{B}] = B_0 + [-1,1]H_B \), where \( A_0 = (\underline{A} + \overline{A})/2, H_A = (1/2)(\overline{A} - \underline{A}) \) and \( B_0 = (\overline{B} + \underline{B})/2, H_B = (1/2)(\overline{B} - \underline{B}) \).

Then, system (5) can be equivalently expressed as

\[
\begin{align*}
dx_i(t) &= [-x_i(t) + r_i(t)]dt, \\
\bar{r}_i(t) &= \begin{pmatrix} -\Theta x_i(t) - \Lambda r_i(t) + A_0 f(x_i(t)) + B_0 f(x_i(t - \tau(t))) + E \Delta(t) + c \sum_{j=1}^{N} G_{ij} \bar{r}_j(t) \\ + (\sigma_1 x_i(t) + \sigma_2 x_i(t - \tau(t)) + \sigma_3 f(x_i(t)) + \sigma_4 f(x_i(t - \tau(t)))) \end{pmatrix} dt, \\
i \in N,
\end{align*}
\]

where

\[
E \Delta(t) = \begin{pmatrix} 1/2 & -1 \end{pmatrix} [\underline{A} - \overline{A}] f(x_i(t)) + \begin{pmatrix} 1/2 \end{pmatrix} [\underline{B} - \overline{B}] f(x_i(t - \tau(t)))
\]

and \( E \Delta(t) \) is satisfied:

\[
(\bar{E} \Delta(t))^\top (\bar{E} \Delta(t)) \leq (H_A f(x_i(t)))^\top (H_A f(x_i(t))) + (H_B f(x_i(t - \tau(t))))^\top (H_B f(x_i(t - \tau(t)))).
\]

Let \( x(t) = (x_1(t))^\top, x_2(t)^\top, \ldots, x_N(t)^\top \), \( r(t) = (r_1(t))^\top, r_2(t)^\top, \ldots, r_N(t)^\top \), \( f(x(t)) = (f(x_1(t))^\top, f(x_2(t))^\top, \ldots, f(x_N(t))^\top) \), \( \bar{r}(t) = (\Theta x(t) - \Lambda r(t) + A_0 f(x(t)) + B_0 f(x(t - \tau(t))) + E \Delta(t) + c \Gamma) dt \)

\[
\begin{pmatrix} \sigma_1 x(t) + \sigma_2 x(t - \tau(t)) + \sigma_3 f(x(t)) + \sigma_4 f(x(t - \tau(t))) \end{pmatrix} dt, \\
i \in N.
\]
**Definition 1.** The stochastic coupled IMMNs (5) are said to be globally synchronized in the mean square sense if \( E[\| x_i(t) - x_j(t) \|^2] \rightarrow 0 \) as \( t \rightarrow +\infty \) for any given initial conditions \( \psi_i(0) \), where \( i, j = 1, 2, \ldots, N \).

**Lemma 1** (see [1]). Let \( G \) be an \( N \times N \) matrix in the set \( T(\mathbb{R}; k) \). Then, the \((N - 1) \times (N - 1)\) matrix \( H = MGJ \) satisfies \( MG = HM \), where \( G \) and \( J \) are given, respectively, by

\[
M = \begin{bmatrix}
1 & -1 & 0 & \cdots & 0 & 0 \\
0 & 1 & -1 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 1 & -1 \\
\end{bmatrix}_{(N-1) \times N},
\]

and \( T(\mathbb{R}, K) \) is the set of matrices with entries in \( \mathbb{R} \) such that the sum of the entries in each row is equal to \( R \).

**Lemma 2.** For any positive definite symmetric constant matrix \( M \in \mathbb{R}^{m \times m} \), if there exist the scalars \( r_1 < r_2 \) and vector function \( w: [r_1, r_2] \rightarrow \mathbb{R}^n \) such that the concerned integrations are well defined, then the following inequality holds:

\[
\mathbf{\Phi} = \begin{bmatrix}
\Phi_{11} & \Phi_{12} & \Phi_{13} \\
\Phi_{22} & \Phi_{23} \\
\Phi_{33} \\
\end{bmatrix},
\]

\[
\mathbf{\Psi} = \begin{bmatrix}
-\mathbf{V}_2 + \frac{1}{r} \mathbf{V}_2 \mathbf{S}^{-1} \mathbf{V}_4 & \mathbf{V}_2 + \frac{1}{r} \mathbf{V}_2 \mathbf{S}^{-1} \mathbf{V}_5 & \mathbf{V}_2 + \frac{1}{r} \mathbf{V}_2 \mathbf{S}^{-1} \mathbf{V}_6 \\
-\mathbf{V}_3 + \frac{1}{r} \mathbf{V}_3 \mathbf{S}^{-1} \mathbf{V}_4 & \mathbf{V}_3 + \frac{1}{r} \mathbf{V}_3 \mathbf{S}^{-1} \mathbf{V}_5 & \mathbf{V}_3 + \frac{1}{r} \mathbf{V}_3 \mathbf{S}^{-1} \mathbf{V}_6 \\
\end{bmatrix},
\]

\[
\mathbf{\Pi} = \begin{bmatrix}
\Pi_{44} + y_{44} & \mathbf{V}_4 + \frac{1}{r} \mathbf{V}_4 \mathbf{S}^{-1} \mathbf{V}_5 & \mathbf{V}_4 + \frac{1}{r} \mathbf{V}_4 \mathbf{S}^{-1} \mathbf{V}_6 \\
* & 2 \mathbf{V}_5 + \frac{1}{r} \mathbf{V}_5 \mathbf{S}^{-1} \mathbf{V}_5 & \mathbf{V}_5 + \frac{1}{r} \mathbf{V}_5 \mathbf{S}^{-1} \mathbf{V}_6 \\
* & * & 2 \mathbf{V}_6 - \bar{R} + \frac{1}{r} \mathbf{V}_6 \mathbf{S}^{-1} \mathbf{V}_6 \\
\end{bmatrix},
\]

where \( \bar{R} = r_2 - r_1 \).

**Lemma 3.** Given any real matrices \( X \) and \( Y \) and \( Q > 0 \) with appropriate dimensions, then the following matrix inequality holds:

\[
X^T Y + Y^T X \leq X^T Q X + Y^T Q^{-1} Y.
\]

**Lemma 4** (see [40]). The LMI

\[
\begin{bmatrix}
\mathbf{S}_{11}(x) & \mathbf{S}_{12}(x) \\
\mathbf{S}_{12}^T(x) & \mathbf{S}_{22}(x) \\
\end{bmatrix} > 0,
\]

where \( \mathbf{S}_{11}(x) = \mathbf{S}_{11}^T(x), \mathbf{S}_{22}(x) = \mathbf{F}^T(x), \) and \( \mathbf{S}_{12}(x) \) depend on \( x \), is equivalent to each of the following conditions:

(i) \( \mathbf{S}_{11}(x) > 0, \mathbf{S}_{22}(x) - \mathbf{S}_{12}^T(x) \mathbf{S}_{11}^{-1}(x) \mathbf{S}_{12}(x) > 0 \)

(ii) \( \mathbf{S}_{22}(x) > 0, \mathbf{S}_{11}(x) - \mathbf{S}_{12}^T(x) \mathbf{S}_{12}^{-1}(x) \mathbf{S}_{12}(x) > 0 \)

### 3. Main Results

**Theorem 1.** Under the assumption \( (H_1), 0 < r(t) < r, \) and \( \tau(t) \leq \mu \mu > 0 \), the stochastic coupled IMMNs (1) are globally synchronized in mean square sense if there exist positive definite symmetric matrices \( P, Q, V_i \in \mathbb{R}^{m \times m}, i = 1, 2, 3, 4, 5, 6 \) and positive diagonal matrices \( R, S, T, \bar{R}, S_1, S_2, S_3, S_4, S_5 \in \mathbb{R}^{n \times n} \), such that the following matrix inequalities hold: \( \Phi < 0 \) and \( \Pi - \Psi \Phi^{-1} \Psi > 0 \), where

\[
\left( \int_{r_1}^{r_2} \mathbf{w}(s) ds \right)^T \mathbf{M} \int_{r_1}^{r_2} \mathbf{w}(s) ds \leq r_{12} \int_{r_1}^{r_2} \mathbf{w}^T(s) \mathbf{M} \mathbf{w}(s) ds,
\]

where \( r_{12} = r_2 - r_1 \).
with

\[
\begin{align*}
\Phi_{11} &= -2\rho + (r^2 - 1)\rho + H_1^2(Q + r^2S)H_1 + \frac{1}{r}V_1S^{-1}V_1 + r^2\Theta_1^T\hat{R}\Theta_1 + \gamma_{11}, \\
\Phi_{12} &= H_1^2(Q + r^2S)H_2 + \frac{1}{r}V_2S^{-1}V_2, \\
\Phi_{13} &= R + \frac{1}{r}V_1S^{-1}V_3, \\
\Phi_{22} &= H_2^2(Q + r^2S)H_2 + \frac{1}{r}V_2S^{-1}V_2 + \gamma_{22}, \\
\Phi_{23} &= \frac{1}{r}V_2S^{-1}V_3, \\
\Pi_{44} &= r^2(cH - A_1)^\top\hat{R}(cH - A_1) + r^2R + 2Q(cH - A_1) - 2V_4 + \frac{1}{r}V_4S^{-1}V_4, \\
\gamma_{44} &= (Q + r^2(cH^\top - A_1^\top)\hat{R})(A_0^{-1} + H_A^{-1})S_1\left( A_0^{-1} + H_A^{-1} \right) + (Q + r^2(cH^\top - A_1^\top)\hat{R})(B_0^{-1} + H_B^{-1})S_1\left( B_0^{-1} + H_B^{-1} \right), \\
\gamma_{11} &= 2(H_1^2(Q + r^2S)H_3 + r^2\Theta_1^T\hat{R}(A_0^{-1} + H_A^{-1}))L + L^\top(S_1 + S_4 + T)L \\
&+ L^\top(H_3^2(Q + r^2S)H_3 + r^2(A_0^{-1} + H_A^{-1})\hat{R}(A_0^{-1} + H_A^{-1}))L \\
&+ L^\top(H_4^2(Q + r^2S)H_4 + r^2(A_0^{-1} + H_A^{-1})\hat{R}(B_0^{-1} + H_B^{-1}))L \\
&+ \hat{R}(H_3^2(Q + r^2S)H_3 + r^2(B_0^{-1} + H_B^{-1})\hat{R}(A_0^{-1} + H_A^{-1})), \\
\gamma_{22} &= L^\top(S_1 + S_4 + H_B^{-1})L + \frac{1}{r}(Q + r^2S)H_4 - (1 - \rho)T)L + 2H_3^2(Q + r^2S)H_4L \\
&+ r^2L^\top(B_0^{-1} + H_B^{-1})\hat{R}(B_0^{-1} + H_B^{-1})L + H_2^2(Q + r^2S)H_5L. \\
\end{align*}
\]

Proof. For convenience, we set

\[
\begin{align*}
g(t) &= -\Theta x - \Delta r + A_0f(x) + B_1f(x_s) + E\Delta(t) + cGr(t), \\
y(t) &= \sigma_1x + \sigma_2x_r + \sigma_3f(x) + \sigma_4f(x_s), \\
\end{align*}
\]

(15)

Consider the following Lyapunov–Krasovskii functional:

\[
V(t, x) = \sum_{i=1}^{6} V_i(t, x),
\]

(16)

where

\[
\begin{align*}
V_1(t, x) &= x(t)^\top M^T P M x(t), \\
V_2(t, x) &= r(t)^\top M^T Q M r(t), \\
V_3(t, x) &= \int_{t-\tau(t)}^{t} f(x(s))^\top M^T T M f(x(s)) ds, \\
V_4(t, x) &= \tau \int_{-\tau}^{0} \int_{t+\theta}^{t} \hat{x}(s)^\top M^T R M \hat{x}(s) ds d\theta, \\
V_5(t, x) &= \tau \int_{-\tau}^{0} \int_{t+\theta}^{t} y(s)^\top M^T S M y(s) ds d\theta, \\
V_6(t, x) &= \tau \int_{-\tau}^{0} \int_{t+\theta}^{t} g(s)^\top M^T R M g(s) ds d\theta, \\
\end{align*}
\]

(17)
for simplicity, we use \( \dot{x}(t) \) instead of \( dx(t)/dt \) in the paper. By the Itô formula, we can calculate \( \mathcal{L}V(t, x) \) along system (9), and then we have

\[
\mathcal{L}V(t, x) = \sum_{i=1}^{6} \mathcal{L}V_i(t, x),
\]

(18)

and \( \mathcal{L}V_i(t, x), i = 1, 2, 3, 4, 5, 6 \) are calculated along system (9) as follows:

\[
\mathcal{L}V_1(t) = 2x^T M^T P(-x + r) \triangleq \xi(t)^T \Omega_1 \xi(t),
\]

(19)

where

\[
\xi(t) = \left( x(t)^T M^T, x(t - \tau(t))^T M^T, r(t)^T M^T, r(t - \tau(t))^T M^T, \left( \int_{t-\tau}^{t} Mg(s)ds \right)^T \right),
\]

(20)

\[
\Omega_1 = \begin{bmatrix}
-2P & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
P & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]

\[
\mathcal{L}V_2(t) = 2r^T M^T QMg(t) + y(t)^T M^T QM y(t)
\]

\[
\leq \xi(t)^T \Omega_2 \xi(t) + 2x^T M^T H_1^T QH_1 Mf(x) + 2x^T M^T H_3^T QH_3 Mf(x_t) + 2r^T M^T Q(\Lambda^{-1}_0 + H^{-1}_A)QH_3 Mf(x_t)
\]

\[
+ 2x^T M^T H_5 QH_4 Mf(x) + 2x^T M^T H_3^T QH_3 Mf(x) + 2f(x)^T M^T H_4^T QH_4 Mf(x_t) + f(x_t)^T M^T H_4^T QH_4 Mf(x_t),
\]

(21)

where

\[
M_{\sigma_i} = H_i M, (i = 1, 2, 3, 4), M\Theta = \Theta_1 M, MA = \Lambda_1 M, MA_0 = \Lambda_0^{-1} M,
\]

\[
MB_0 = B_0^{-1} M, MH_A = H_A^{-1} M, MH_B = H_B^{-1} M,
\]

\[
\Omega_2 = \begin{bmatrix}
H_1^T QH_1 & H_1^T QH_2 & 0 & -Q\Theta_1 & 0 \\
H_1^T QH_2 & H_2^T QH_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
-Q\Theta_1 & 0 & 0 & 2Q(cH - \Lambda_1) & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]

(22)

\[
\mathcal{L}V_3(t) = f ((x))^T T Mf(x) - (1 - \tau(t))f(x_t)^T M^T T Mf(x_t)
\]

\[
\leq f(x)^T M^T T Mf(x) - (1 - \mu)f(x_t)^T M^T T Mf(x_t),
\]

(23)

\[
\mathcal{L}V_4(t) = \dot{r} x(t)^T M^T R Mx(t) - r \int_{t-\tau}^{t} \dot{x}(s)^T M^T R M \dot{x}(s) ds,
\]

(24)

where \( \dot{r}(t) \) is the derivative of \( \tau(t) \).

By applying Lemma 2, one has
where \( z_{T, h} \), Based on Lemma 2 and (15), we have

\[
-\tau \int_{t-\tau}^{t} \dot{x}(s)^{\top} R M \dot{x}(s) ds \leq -\left( \int_{t-\tau}^{t} M \dot{x}(s) ds \right)^{\top} R \left( \int_{t-\tau}^{t} M \dot{x}(s) ds \right),
\]

(25)

Recalling (9) and (15), it is easy to see that the following equalities hold:

\[
0 = 2\xi(t)^{\top} V \left( r(t) - r(t - \tau) - \int_{t-\tau}^{t} g(s) ds - \int_{t-\tau}^{t} y(s) dw_s \right),
\]

(27)

where

\[
V = \left( V_1(t)^{\top} \ V_2(t)^{\top} \ V_3(t)^{\top} \ V_4(t)^{\top} \ V_5(t)^{\top} \ V_6(t)^{\top} \right)^{\top}.
\]

By Lemma 3, we have

\[
\dot{\mathcal{V}}_{\mathcal{V}}(t) = \tau^2 y(t)^{\top} M^{\top} SM y(t) - \tau \int_{t-\tau}^{t} y(s)^{\top} M^{\top} SM y(s) ds.
\]

(26)

\[
2\xi(t)^{\top} V \int_{t-\tau}^{t} y(s) dw_s \leq \frac{1}{\tau} \xi(t)^{\top} VS^{-1} V^{\top} \xi(t) + \tau \left( \int_{t-\tau}^{t} y(s) dw_s \right)^{\top} S \left( \int_{t-\tau}^{t} y(s) dw_s \right).
\]

(28)

Then,

\[
\dot{\mathcal{V}}_{\mathcal{V}}(t) \leq \xi(t)^{\top} \Omega_4 \xi(t) + \tau \left( \int_{t-\tau}^{t} M y(s) dw_s \right)^{\top} \left( \int_{t-\tau}^{t} M y(s) dw_s \right)
\]

\[
- \tau \int_{t-\tau}^{t} y(s)^{\top} M^{\top} SM y(s) ds + 2\tau^2 x^{\top} M^{\top} H_1^{\top} SH_1 M f(x)
\]

\[
+ 2\tau^2 x^{\top} M^{\top} H_1^{\top} SH_4 M f(x) + 2\tau^2 y^{\top} M^{\top} H_1^{\top} SH_3 M f(x)
\]

\[
+ 2\tau^2 x^{\top} M^{\top} H_1^{\top} SH_4 M f(x) + 2\tau^2 y^{\top} M^{\top} H_1^{\top} SH_3 M f(x)
\]

\[
+ 2\tau^2 f(x)^{\top} M^{\top} H_1^{\top} SH_4 M f(x) + 2\tau^2 f(x)^{\top} M^{\top} H_1^{\top} SH_3 M f(x),
\]

(29)

where

\[
E \left( \left( \int_{t-\tau}^{t} M y(s) dw_s \right)^{\top} S \left( \int_{t-\tau}^{t} M y(s) dw_s \right) \right) = E \left( \int_{t-\tau}^{t} y(s)^{\top} M^{\top} SM y(s) ds \right).
\]

(31)

Based on Lemma 2 and (15), we have

\[
\Omega_4 = \begin{bmatrix}
\tau^2 H_1^{\top} SH_1 & \tau^2 H_1^{\top} SH_2 & 0 & -V_1 & V_1 & 1 \\
* & \tau^2 H_2^{\top} SH_2 & 0 & -V_2 & V_2 & 2 \\
* & * & 0 & -V_3 & V_3 & 3 \\
* & * & * & -2V_4 & V_4 & 4 \\
* & * & * & * & 2V_5 & 5 \\
* & * & * & * & * & 2V_6 \\
\end{bmatrix}
\]

(30)
\[
\mathcal{L}V_b(t) = \tau^2 g(t)^T M^T \dot{R} M g(t) - \tau \int_{t-\tau}^{t} g(s)^T M^T \dot{R} M g(s) ds
\]
\[
\leq \tau^2 g(t)^T M^T \dot{R} M g(t) - \left( \int_{t-\tau}^{t} M g(s) ds \right)^T \bar{R} \left( \int_{t-\tau}^{t} M g(s) ds \right)
\]
\[
\leq \xi(t)^T \Omega_5 \xi(t) + 2r^2 x^T M^T \bar{R} \left( A_0^{N-1} + H_A^{N-1} \right) M f(x)
\]
\[
+ 2r^2 x^T M^T \bar{R} \left( B_0^{N-1} + H_B^{N-1} \right) M f(x)
\]
\[
+ 2r^2 r^T M^T (cH^T - \Lambda_t) \bar{R} \left( A_0^{N-1} + H_A^{N-1} \right) M f(x)
\]
\[
+ 2r^2 r^T M^T (cH^T - \Lambda_t) \bar{R} \left( B_0^{N-1} + H_B^{N-1} \right) M f(x)
\]
\[
+ r^2 f(x)^T M^T \left( A_0^{N-1T} + H_A^{N-1T} \right) \bar{R} \left( A_0^{N-1} + H_A^{N-1} \right) M f(x)
\]
\[
+ r^2 f(x)^T M^T \left( B_0^{N-1T} + H_B^{N-1T} \right) \bar{R} \left( B_0^{N-1} + H_B^{N-1} \right) M f(x)
\]
(32)

where

\[
MG = HM,
\]
\[
\Omega_5 = 
\begin{bmatrix}
  r^2 \bar{R} \left( cH - \Lambda_t \right) & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 \\
  -r^2 \bar{R} \left( cH - \Lambda_t \right) & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
(33)

Under assumption \((H_1)\) and Lemma 3, we obtain

\[
2r^T M^T \left( Q + r^2 (cH^T - \Lambda_t) \bar{R} \right) \left( A_0^{N-1} + H_A^{N-1} \right) M f(x)
\]
\[
= 2 \sum_{i=1}^{N-1} (r_i - r_{i+1})^T \left( Q + r^2 (cH^T - \Lambda_t) \bar{R} \right) \left( A_0^{N-1} + H_A^{N-1} \right) M f(x_i) - f(x_{i+1})
\]
\[
\leq 2 \sum_{i=1}^{N-1} (r_i - r_{i-1})^T \left( Q + r^2 (cH^T - \Lambda_t) \bar{R} \right) \left( A_0^{N-1} + H_A^{N-1} \right) f(x_i) - f(x_{i+1})
\]
\[
\leq \sum_{i=1}^{N-1} (r_i - r_{i-1})^T \left( Q + r^2 (cH^T - \Lambda_t) \bar{R} \right) \left( A_0^{N-1} + H_A^{N-1} \right) S_1 (A_0^{N-1T} + H_A^{N-1T})
\]
\[
\times \left( Q + r^2 \bar{R} \left( cH - \Lambda_t \right) \right) (r_i - r_{i+1}) + \sum_{i=1}^{N-1} (x_i - x_{i+1})^T L S_1^T L (x_i - x_{i+1})
\]
\[
= r^T M^T \left( Q + r^2 (cH^T - \Lambda_t) \bar{R} \right) \left( A_0^{N-1} + H_A^{N-1} \right) S_1 (A_0^{N-1T} + H_A^{N-1T})
\]
\[
\times \left( Q + r^2 \bar{R} \left( cH - \Lambda_t \right) \right) M r + x^T M^T L S_1^T L M x.
\]

Similarly, we have
where

\[ E[\mathcal{Z}(t)] \leq E\left[ \xi(t)^\top \Xi(t) \right] \leq \lambda_{\max}(\Xi)E\left[ \|\xi(t)\|^2\right], \quad (43) \]
\[
\Xi = \begin{bmatrix}
\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} \\
* & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} \\
* & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & \Xi_{36} \\
* & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46} \\
* & * & * & * & \Xi_{55} & \Xi_{56} \\
* & * & * & * & * & \Xi_{66}
\end{bmatrix},
\]

with

\[
\Xi_{11} = -2P + \left(r^2 - 1\right)R + H_1^T(Q + r^2S)H_1 + \frac{1}{r}V_1S^{-1}V_1 + r^2\Theta_1^\top R\Theta_1 + y_{11},
\]

\[
\Xi_{12} = H_1^T(Q + r^2S)H_2 + \frac{1}{r}V_1S^{-1}V_2,
\]

\[
\Xi_{13} = R + \frac{1}{r}V_1S^{-1}V_3,
\]

\[
\Xi_{14} = p - Q\Theta_1 - r^2R + \frac{1}{r}V_1S^{-1}V_4 - V_1,
\]

\[
\Xi_{15} = V_1 + \frac{1}{r}V_1S^{-1}V_5,
\]

\[
\Xi_{16} = V_1 + \frac{1}{r}V_1S^{-1}V_6,
\]

\[
\Xi_{22} = H_2^T(Q + r^2S)H_2 + \frac{1}{r}V_2S^{-1}V_2 + y_{22},
\]

\[
\Xi_{23} = \frac{1}{r}V_2S^{-1}V_3,
\]

\[
\Xi_{24} = -V_2 + \frac{1}{r}V_2S^{-1}V_4,
\]

\[
\Xi_{25} = V_2 + \frac{1}{r}V_2S^{-1}V_5,
\]

\[
\Xi_{26} = V_2 + \frac{1}{r}V_2S^{-1}V_6,
\]

\[
\Xi_{33} = -R + \frac{1}{r}V_3S^{-1}V_3,
\]

\[
\Xi_{34} = -V_3 + \frac{1}{r}V_3S^{-1}V_4,
\]

\[
\Xi_{35} = V_3 + \frac{1}{r}V_3S^{-1}V_5,
\]

\[
\Xi = \begin{bmatrix}
\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} \\
* & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} & \Xi_{26} \\
* & * & \Xi_{33} & \Xi_{34} & \Xi_{35} & \Xi_{36} \\
* & * & * & \Xi_{44} & \Xi_{45} & \Xi_{46} \\
* & * & * & * & \Xi_{55} & \Xi_{56} \\
* & * & * & * & * & \Xi_{66}
\end{bmatrix},
\]

\[
\Xi_{11} = -2P + \left(r^2 - 1\right)R + H_1^T(Q + r^2S)H_1 + \frac{1}{r}V_1S^{-1}V_1 + r^2\Theta_1^\top R\Theta_1 + y_{11},
\]

\[
\Xi_{12} = H_1^T(Q + r^2S)H_2 + \frac{1}{r}V_1S^{-1}V_2,
\]

\[
\Xi_{13} = R + \frac{1}{r}V_1S^{-1}V_3,
\]

\[
\Xi_{14} = p - Q\Theta_1 - r^2R + \frac{1}{r}V_1S^{-1}V_4 - V_1,
\]

\[
\Xi_{15} = V_1 + \frac{1}{r}V_1S^{-1}V_5,
\]

\[
\Xi_{16} = V_1 + \frac{1}{r}V_1S^{-1}V_6,
\]

\[
\Xi_{22} = H_2^T(Q + r^2S)H_2 + \frac{1}{r}V_2S^{-1}V_2 + y_{22},
\]

\[
\Xi_{23} = \frac{1}{r}V_2S^{-1}V_3,
\]

\[
\Xi_{24} = -V_2 + \frac{1}{r}V_2S^{-1}V_4,
\]

\[
\Xi_{25} = V_2 + \frac{1}{r}V_2S^{-1}V_5,
\]

\[
\Xi_{26} = V_2 + \frac{1}{r}V_2S^{-1}V_6,
\]

\[
\Xi_{33} = -R + \frac{1}{r}V_3S^{-1}V_3,
\]

\[
\Xi_{34} = -V_3 + \frac{1}{r}V_3S^{-1}V_4,
\]

\[
\Xi_{35} = V_3 + \frac{1}{r}V_3S^{-1}V_5,
\]
\[
\Xi_{36} = V_3 + \frac{1}{\tau}V_3 S^{-1}V_6, \\
\Xi_{44} = \tau^2 (cH - A_1)^T \bar{R}_0 (cH - A_1) + \tau^2 R + 2Q (cH - A_1) - 2V_4 + \frac{1}{\tau}V_4 S^{-1}V_4 + \gamma_{44}, \\
\Xi_{45} = V_4 + \frac{1}{\tau}V_4 S^{-1}V_5, \\
\Xi_{46} = V_4 + \frac{1}{\tau}V_4 S^{-1}V_6, \\
\Xi_{55} = 2V_5 + \frac{1}{\tau}V_5 S^{-1}V_5, \\
\Xi_{56} = V_5 - \bar{R} + \frac{1}{\tau}V_5 S^{-1}V_6, \\
\Xi_{66} = 2V_6 - \bar{R} + \frac{1}{\tau}V_6 S^{-1}V_6. \\
\gamma_{11} = 2\left(\left(\tau^2 H_1^T (Q + \tau^2 S) H_3 + \tau^2 \Theta_1^T \bar{R}(A_0^{N-1} + H_A^{N-1})\right) + L^T (S_1^1 + S_4^1 + T) L \right) \\
+ \left(\left(\tau^2 H_3^T (Q + \tau^2 S) H_3 + \tau^2 (A_0^{N-1} + H_A^{N-1}) \bar{R}(B_0^{N-1} + H_B^{N-1})\right) + L^T (S_1^1 + S_4^1 + T) L \right) \\
+ \left(\left(\tau^2 H_4^T (Q + \tau^2 S) H_4 + \tau^2 (B_0^{N-1} + H_B^{N-1}) \bar{R}(A_0^{N-1} + H_A^{N-1})\right) + L^T (S_1^1 + S_4^1 + T) L \right) \\
+ \left(\left(\tau^2 H_5^T (Q + \tau^2 S) H_5 + \tau^2 (B_0^{N-1} + H_B^{N-1}) \bar{R}(A_0^{N-1} + H_A^{N-1})\right) + L^T (S_1^1 + S_4^1 + T) L \right) \\
+ \left(\left(\tau^2 H_6^T (Q + \tau^2 S) H_6 + \tau^2 (B_0^{N-1} + H_B^{N-1}) \bar{R}(A_0^{N-1} + H_A^{N-1})\right) + L^T (S_1^1 + S_4^1 + T) L \right), \\
\gamma_{22} = 2\left(\left(\tau^2 H_1^T (Q + \tau^2 S) H_3 + \tau^2 \Theta_1^T \bar{R}(A_0^{N-1} + H_A^{N-1})\right) + L^T (S_1^1 + S_4^1 + T) L \right) \\
+ \left(\left(\tau^2 H_3^T (Q + \tau^2 S) H_3 + \tau^2 (A_0^{N-1} + H_A^{N-1}) \bar{R}(B_0^{N-1} + H_B^{N-1})\right) + L^T (S_1^1 + S_4^1 + T) L \right) \\
+ \left(\left(\tau^2 H_4^T (Q + \tau^2 S) H_4 + \tau^2 (B_0^{N-1} + H_B^{N-1}) \bar{R}(A_0^{N-1} + H_A^{N-1})\right) + L^T (S_1^1 + S_4^1 + T) L \right) \\
+ \left(\left(\tau^2 H_5^T (Q + \tau^2 S) H_5 + \tau^2 (B_0^{N-1} + H_B^{N-1}) \bar{R}(A_0^{N-1} + H_A^{N-1})\right) + L^T (S_1^1 + S_4^1 + T) L \right) \\
+ \left(\left(\tau^2 H_6^T (Q + \tau^2 S) H_6 + \tau^2 (B_0^{N-1} + H_B^{N-1}) \bar{R}(A_0^{N-1} + H_A^{N-1})\right) + L^T (S_1^1 + S_4^1 + T) L \right), \\
\gamma_{44} = \left(\left(\tau^2 (cH - A_1)^T \bar{R}(A_0^{N-1} + H_A^{N-1})\right) + \left(\tau^2 (cH - A_1)^T \bar{R}(A_0^{N-1} + H_A^{N-1})\right) + L^T (S_1^1 + S_4^1 + T) L \right). \\
\]
\[
\begin{align*}
\frac{d\left( \frac{dx_i(t)}{dt} \right)}{dt} &= \left(-D \frac{dx_i(t)}{dt} - Cx_i(t) + A(x_i(t))f(x_i(t)) + B(x_i(t))f(x_i(t) - \tau(t)) \right) \\
&\quad + c \sum_{j=1}^{N} G_{ij} \left( \frac{dx_j(t)}{dt} + x_j(t) \right) dt + (\sigma_1 x_i(t) + \sigma_2 x_i(t) - \tau(t)) \\
&\quad + \sigma_3 f(x_i(t)) + \sigma_4 f(x_i(t) - \tau(t)))dw_t, \quad i = 1, 2, 3, 4, 5,
\end{align*}
\]

where the activation function \( f(x_i(t)) = 0.6 \tanh(x_i) \), the time delay \( \tau(t) = e^t/(e^t + 1) \), and the coupling strength \( c = 0.5 \). The system parameters are taken as

\[
D = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix},
\]

\[
C = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},
\]

\[
\Gamma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},
\]

\[
G = \begin{bmatrix} -2.2 & 1 & 0 & 0 & 0.2 \\ 1 & -3.2 & 1.2 & 0 & 0 \\ 0 & 2 & -3.5 & 0.5 & 0 \\ 0 & 0 & 2 & -4.4 & 1.4 \\ 0.2 & 0 & 0 & 3 & -4.2 \end{bmatrix},
\]

\[
\sigma_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},
\]

\[
\sigma_2 = \begin{bmatrix} 1.8 & 0 \\ 0 & 1.8 \end{bmatrix},
\]

\[
\sigma_3 = \begin{bmatrix} 1.6 & 0 \\ 0 & 1.6 \end{bmatrix},
\]

\[
\sigma_4 = \begin{bmatrix} 1.4 & 0 \\ 0 & 1.4 \end{bmatrix},
\]

\[
A(x_i(t)) = \begin{bmatrix} a_{11}(x_{i1}(t)) & a_{12}(x_{i2}(t)) \\ a_{21}(x_{i1}(t)) & a_{22}(x_{i2}(t)) \end{bmatrix},
\]

\[
B(x_i(t)) = \begin{bmatrix} b_{11}(x_{i1}(t)) & b_{12}(x_{i2}(t)) \\ b_{21}(x_{i1}(t)) & b_{22}(x_{i2}(t)) \end{bmatrix},
\]

with the memristor connection weights:

\[
a_{11}(x) = \begin{cases} 0.2, & |x| \leq 0.1, \\ -0.2, & |x| > 0.1, \end{cases}
\]

\[
a_{12}(x) = \begin{cases} 0.6, & |x| \leq 0.1, \\ -0.6, & |x| > 0.1, \end{cases}
\]

\[
a_{21}(x) = \begin{cases} 0.4, & |x| \leq 0.1, \\ -0.4, & |x| > 0.1, \end{cases}
\]

\[
a_{22}(x) = \begin{cases} 0.4, & |x| \leq 0.1, \\ -0.4, & |x| > 0.1, \end{cases}
\]

\[
b_{11}(x) = \begin{cases} 0.2, & |x| \leq 0.1, \\ -0.2, & |x| > 0.1, \end{cases}
\]

\[
b_{12}(x) = \begin{cases} 0.4, & |x| \leq 0.1, \\ -0.4, & |x| > 0.1, \end{cases}
\]

\[
b_{21}(x) = \begin{cases} 0.3, & |x| \leq 0.1, \\ -0.3, & |x| > 0.1, \end{cases}
\]

\[
b_{22}(x) = \begin{cases} 0.3, & |x| \leq 0.1, \\ -0.3, & |x| > 0.1. \end{cases}
\]

Obviously, by calculation, we can get the Lipschitz constants \( L = 0.6 * I_2 \), and the upper bound of the delay \( \tau = 1 \).

In order to show the effectiveness of Theorem 1, we display the synchronization of each node \( x_{ij}(t) (i = 1, 2, 3, 4, 5; j = 1, 2) \) in Figure 1. Moreover, Figures 2 and 3 depict the synchronization error trajectories of \( x_{11}(t) - x_{12}(t) \) and \( x_{22}(t) - x_{12}(t), i = 1, 2, 3, 4 \).

**Data Availability**

No data were used to support this study.
Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Acknowledgments

This work was jointly supported by the Natural Science Foundation of Zhejiang Province under Grant (no. LY20A010016), National Natural Science Foundation of China under Grant (no. 11931016), the Scientific Research Foundation for the Introduced Senior Talents, Wuyi University (Grant no. YJ201802), Department of Education Foundation of Jiangsu Province (no. 201804), and the Natural Science Foundation of Shandong Province (China) (no. ZR2018MA018).

References

[1] Chua, “Memristor-the missing circuit element,” Trans. Circuit Theory, vol. 18, pp. 507–519, 1978.
[2] A. Thomas, “Memristor-based neural networks,” Applied Physics, vol. 46, pp. 1–12, 2013.
[3] L. Chen, C. Huang, H. Liu, and Y. Xia, “Anti-synchronization of a class of chaotic systems with application to Lorenz system: a unified analysis of the integer order and fractional order,” Mathematics, vol. 7, no. 6, p. 559, 2019.
[4] C. Huang, J. Q. Lu, G. S. Zhai, J. D. Cao, G. P. Lu, and M. Perc, “Stability and stabilization in probability of probabilistic boolean networks,” IEEE Transactions on Neural Networks and Learning Systems, 2020.
[5] C. Huang, X. Zhang, H. K. Lam, and S.-H. Tsa, “Synchronization analysis for nonlinear complex networks with reaction-diffusion terms using fuzzy-model-based approach,” IEEE Transactions on Fuzzy Systems, 2020.
[6] Y. Liu, J. D. Cao, L. Q. Wang, and Z. G. Wu, “On pinning reachability of probabilistic Boolean control networks,” Science China Information Sciences, vol. 63, no. 6, pp. 169–201, 2020.
[7] Y. Liu, L. Sun, J. Lu, and J. Liang, “Feedback controller design for the synchronization of Boolean control networks,” IEEE Transactions on Neural Networks and Learning Systems, vol. 27, no. 9, pp. 1991–1996, 2016.
[8] X. Liu, H. Su, and M. Z. Q. Chen, “A switching approach to designing finite-time synchronization controllers of coupled neural networks,” IEEE Transactions on Neural Networks and Learning Systems, vol. 27, no. 2, pp. 471–482, 2016.
[9] X. Liu, J. Lam, W. Yu, and G. Chen, “Finite-time consensus of multiagent systems with a switching protocol,” IEEE Transactions on Neural Networks and Learning Systems, vol. 27, no. 4, pp. 853–862, 2016.
[10] X. Liu, D. W. C. Ho, J. Cao, and W. Xu, “Discontinuous observers design for finite-time consensus of multiagent systems with external disturbances,” IEEE Transactions on Neural Networks and Learning Systems, vol. 28, no. 11, pp. 2826–2830, 2017.
[11] X. Li, D. W. C. Ho, and J. Cao, “Finite-time stability and settling-time estimation of nonlinear impulsive systems,” Automatica, vol. 99, pp. 361–368, 2019.
X. Li and S. Song, “Stabilization of delay systems: delay-dependent control,” *IEEE Transactions on Automatic Control*, vol. 62, no. 1, pp. 406–411, 2017.

X. Li and J. Wu, “Sufficient stability conditions of nonlinear differential systems under impulsive control with state-dependent delay,” *IEEE Transactions on Automatic Control*, vol. 63, no. 1, pp. 306–311, 2018.

J. Q. Lu, Y. Q. Wang, X. C. Shi, and J. D. Cao, “Finite-time bipartite consensus for multi-agent systems under detail-balanced antagonistic interactions,” *IEEE Transactions on Systems, Man and Cybernetics: Systems (Regular Paper)*, 2019, In press.

H. Li and X. Ding, “A control Lyapunov function approach to feedback stabilization of logical control networks,” *SIAM Journal on Control and Optimization*, vol. 57, no. 2, pp. 810–831, 2019.

Y. Li, H. Li, and X. Ding, “Set stability of switched delayed logical networks with application to finite-field consensus,” *Automatica*, vol. 113, p. 108768, 2020.

D. Wang, L. Huang, and L. Tang, “Synchronization criteria for discontinuous neural networks with mixed delays via functional differential inclusions,” *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 5, pp. 1809–1821, 2018.

D. S. Wang, L. H. Huang, and L. K. Tang, “Dissipativity and synchronization for generalized BAM neural networks with multivariate discontinuous activations,” *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 8, pp. 3815–3827, 2018.

X. Yang, J. Lam, D. W. C. Ho, and Z. Feng, “Fixed-time synchronization of complex networks with impulsive effects via nonchattering control,” *IEEE Transactions on Automatic Control*, vol. 62, no. 11, pp. 5511–5521, 2017.

J. Lu, C. Ding, J. Lou, and J. Cao, “Outer synchronization of partially coupled dynamical networks via pinning impulsive controllers,” *Journal of the Franklin Institute*, vol. 352, no. 7, pp. 2107–2115, 2013.

B. Zhang, Y. Xia, L. Zhu, H. Liu, and L. Gu, “Global stability of fractional order coupled systems with impulses via a graphic approach,” *Mathematics*, vol. 7, no. 8, p. 744, 2019.

Y. Zhang, J. Zhuang, Y. Xia, Y. Bai, J. Cao, and L. Gu, “Fixed-time synchronization of the impulsive memristor-based neural networks,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 77, pp. 40–53, 2019.

L. Duan and L. Huang, “Periodicity and dissipativity for memristor-based mixed time-varying delayed neural networks via differential inclusions,” *Neural Networks*, vol. 57, pp. 12–22, 2014.

Z. Cai, L. Huang, and L. Zhang, “New conditions on synchronization of memristor-based neural networks via differential inclusions,” *Neurocomputing*, vol. 186, pp. 235–250, 2016.

J. Lu, X. Guo, T. Huang, and Z. Wang, “Consensus of signed networked multi-agent systems with nonlinear coupling and communication delays,” *Applied Mathematics and Computation*, vol. 350, pp. 153–162, 2019.

N. Li and W. X. Zheng, “Synchronization criteria for inertial memristor-based neural networks with linear coupling,” *Neural Networks*, vol. 106, pp. 260–270, 2018.

A. Halanay, *Differential Equations: Stability, Oscillations, Time Lags*, Academic Press, New York, NY, USA, 1966.

R. E. Moore, *Interval Analysis*, Prentice-Hall, Englewood, NJ, USA, 1996.