Measurement of the relative motion of two mirrors in presence of an optical spring

A. DI VIRGILIO

INFN, Sez. di Pisa, Pisa, Largo B. Pontecorvo 3, 56127 Pisa, Italy
angela.divirgilio@pi.infn.it

The Low Frequency Facility (LFF) experimental set-up consists of one 1 cm long cavity hanging from a mechanical insulation system, that damps seismic noise transmission to the optical components of the VIRGO interferometer. Radiation pressure generates an opto-mechanical coupling between the two mirrors of the cavity, that we call an optical spring. The measured relative displacement power spectrum is compatible with a system at thermal equilibrium within its environment; the optical spring has a stiffness $k_{opt}$ of the order of $10^4$ N/m.

An upper limits of $10^{-15}$ m/sqrt(Hz) at 10 Hz for seismic and thermal noise contamination of the Virgo test masses suspended by a SuperAttenuator is derived from measured data.

The measurement described in this note focus on two points, very important for present and future interferometers, namely the optical spring produced by radiation pressure inside a Fabry-Perot cavity, and thermal noise spectrum – away from resonance – in the frequency region below and around 10 Hz. The radiation pressure will play a crucial role in next generation laser interferometric detectors of gravitational waves.\textsuperscript{1-5} Thermal fluctuations of mechanical systems are considered the most relevant limitation for ground based interferometers for gravitational waves detection in the low frequency region, where several gravitational wave signals are expected.\textsuperscript{6} The relative motion of the mirrors of the suspended Fabry-Perot cavity is compatible with the presence of an optical spring due to the radiation pressure at thermal equilibrium.\textsuperscript{8-10}

In the following we provide a concise description of the experimental apparatus and of data analysis; more details are available elsewhere.\textsuperscript{7,9,10,12} Comments on experimental problems and on the relevance of this kind of apparatus will be given at the end of the paper.

The last stage of the Low Frequency Facility (LFF) is sketched in fig. 1. The suspension system insulating the high finesse (ranging between 4000 and 6000) 1 cm long Fabry-Perot cavity from seismic noise is equal to the suspensions used in the VIRGO interferometer (SuperAttenuator SA).

The flat mirror of the cavity (AX, auxiliary mirror) is hung to the last mechanical
seismic filter of the chain (called Filter7), by means of an independent three-stage suspension. The other mirror, called VM (Virgo mirror), is similar to a Virgo test mass. The control of the longitudinal motion is done by acting on the VM mirror only, as done in the VIRGO interferometer. Fig. 1 shows the cavity, the input beam, the longitudinal control loop scheme and the acquired signals. The feedback control loop is based on a Digital Signal Processor (DSP).

A mechanical model of the dynamical system formed by the two mechanical branches hung to Filter7 (see fig. 1) has been developed; it includes the feedback loop circuit, and the optical spring is modeled as a mechanical spring of appropriate elastic constant \( k_{\text{opt}} \), acting between the two mirrors. The model predicts the contributions to the power spectrum coming from external noise sources (electronic and seismic noise, from the Laser etc.) and from thermal noise, using the Fluctuation Dissipation Theorem (FDT). Evidence of an optical spring effect emerged from the observation that it was possible to lock the cavity only for positive de-tuning (cavity longer than the closest resonance), corresponding to a positive value of \( k_{\text{opt}} \). A static detuning, varying from run to run, ranging between \( 10^{-11} \) and \( 10^{-12} \) m, was measured, corresponding to a stiffness constant \( k_{\text{opt}} \) ranging between 70000 and 10000 N/m. In different spectra, a resonance changing its position in accordance with the change of the static detuning has been observed.

The error signal exhibits all the statistical characteristics of the displacement power spectrum of a system at thermal equilibrium: it is Gaussian and stationary all over the acquisition time, which was of the order of 1 hour. Longer acquisition runs
are not possible, since the apparatus is not thermally stabilized. Figure 2 shows the result of one such run. Thermal noise is estimated by the model assuming an optical gain $1.56 \times 10^{10} \text{ V/m}$, $k_b = 56000 \text{ N/m}$ (this parameter is found by fitting the data with the model), and the typical working conditions.

![Image of measured power spectrum](image.png)

**Fig. 2.** Measured power spectrum, 10 mHz frequency resolution, compared with the thermal noise estimated by the model, assuming an optical gain $1.56 \times 10^{10} \text{ V/m}$, $k_b = 55000 \text{ N/m}$, and the typical working conditions of the present set of runs. Losses are described by frequency independent parameters, associated to the AM longitudinal and rotational degree of freedom. Fitted values are respectively $5.8 \pm 1$ s and $6.5 \pm 1$.

The region of the spectrum below 3 Hz, where seismic noise dominates, has been cut off by a high pass filter. As shown in figure 2, the result of the fit and the data agree well below 90 Hz; at higher frequency the higher order modes are relevant, and the model is no longer able to reproduce the data. The measured spectrum is incompatible with external noises, as seismic, laser source and electronic noise. The result of the fit gives quite large absorption coefficients, and is compatible with a viscous damping (as opposed to structural) damping, in contradiction with the model used to predict thermal noise limits for gravitational wave antennas. Seismic noise contamination and thermal noise are very important points for the Virgo suspensions. The present measurement with the help of the model provides an upper limit$^{12}$ of $10^{-15} \text{ m/}\sqrt{\text{Hz}}$ at 10 Hz for seismic noise contamination thermal noise for the test masses of the Virgo mirrors.

To the best of our knowledge, similar experiment in the low frequency region, 3 – 10 Hz, have not been reported in the literature. This region is very important to detect gravitational wave signals, but at the moment too noisy for present-generation antennas. The development of suspensions for gravitational wave research has allowed to observe the behavior of high finesse cavity, made with suspend
mirrors, i.e. with mirrors freely moving above just a few Hz. This fact has allowed the
direct observation of the resonance due to the optical spring and a power spectrum
compatible with thermal noise in the low frequency region. Moreover the model
shows dissipation coefficients showing a viscous rather than structural behaviour
with values too large to be compatible with the standard prediction of thermal
noise in present and future gravitational waves antennas. Our results were limited
by the fact that of the LFF was not calibrated during data taking and the part of the
apparatus was not well performing from the point of view of thermal noise. In order
to extend and refine our results, it will be important in the future to develop simi-
lar facilities, including an appropriate calibration system and consider test masses
much more similar to the ones actually used in gravitational waves antennas.

References
1. A. Buonanno and Y. Chen, Class. Quantum Gravity 18, L95 (2001)
2. V. B. Braginsky, M.L. Gorodetsky and F. Ya. Khalili, Phys. Lett. A 232, 340 (1997)
3. V. B. Braginsky, and F. Ya. Khalili, Phys. Lett. A 257, 341(1999)
4. D. Vitali et al. Physics Review A, 65, 063803.
5. O.Arcizet et al., Nature, 444, 71-74, 2006.
6. K. Thorne, gr-qc/9704042 and B. Shutz, Clas. Quan. Grav., 16, 1999, A131-A156.
7. A. Di Virgilio et al., J. Physics: Conference Series, Vol. 32 (2006), 346-352.
8. B.S. Sheard et al, phys. reav. A 69, 051801 (2004);
9. A. Di Vigilio et al.,Phys. Rev. A, 74, 13813 (2006);
10. A. Di Virgilio et al.,Displacement power spectrum measurement of a macroscopic op-
tomechanical system at the thermal equilibrium, preprint gr-qc/0612130,
11. A. Di Virgilio et al.,Phys. Rev. D 69, 051801 (2007)
12. A. Di Virgilio Seismic and thermal noise upper limits at 10 Hz for the Virgo suspen-
sions, Virgo Note, VIR-NOT-PIS-1390-334
13. Callen H.B. And Welton T.A. , Phys. Rev. 83 34-40
14. R Kubo 1966 Rep. Prog. Phys. 29 255-284
15. P. Saulson, Phys. Rev. D 42, 2437 (1990).