Numerical study of random material properties within a soil specimen

M Amad¹, R P Ray¹

¹Széchenyi István University, Egyetem tér 1, Győr, 9026, Hungary

E-mail: ahmad.majd@hallgato.sze.hu

Abstract. Material properties derived from laboratory soil tests often assume that the property is uniform throughout the specimen. For some exceptional soils, this may hold true, but for many others it is obviously false. We have been performing cyclic and irregular torsional simple shear (TOSS) tests on hollow cylinder samples for decades and were intrigued by the idea of how to model inherently non-uniform specimens. As an added corollary, we wanted to understand the influence of imperfections (voids, inclusions) on the measured stress-strain behaviour in these tests. This paper examines two general classes of problems: (a) uniform specimens with inclusions of voids or imperfections, and (b) non-uniform specimens with random distributions of material properties within the specimen. Finite element modelling was performed on a TOSS specimen (ID=4cm, OD=6cm, L=14cm) using a set of over 500 different elastoplastic material properties within the specimen. Various distributions (Normal, Log-normal, Bimodal) of stiffness and strength properties were examined. The results were examined as torque vs. twist curves since those values are typically measured in the TOSS laboratory test before being converted (with assumptions of uniformity) to shear stress-shear strain hysteresis. The Bimodal distributions were used to represent soils with distinct hard and soft zones. Additionally, distributions with some degree of spatial correlation were also examined.

1. Introduction
In a paper written in 1926 [1], Masing discussed the behaviour of a brass specimen undergoing loading and unloading. His underlying premise about modelling the stress-strain response of the specimen was that it behaved as a collection of elasto-plastic elements that were loaded to their yield point at slightly different times during the test. This resulted in the load-deformation curve in figure 1a where location \( I \) is the onset of yielding. He believed that the specimen did not fail uniformly because the stresses along the specimen cross-section were not uniform and therefore, some failed while others were still undergoing elastic loading. The progress of gradual transition from fully elastic condition to (nearly) fully plastic could be seen by observing the normalized tangent modulus versus strain shown in red (figure 1b). Presently, this is a common way to represent soil behaviour under small strain conditions as a modulus reduction curve where \( G/G_{\text{max}} \) would replace the parameter \( a \) on the y-axis.
2. Concept
Masing’s idea prompted the start of a numerical study where we assigned various simple elasto-plastic material properties throughout a specimen to generate nonlinear stress-strain behaviour. The purpose was to better understand the impacts of varied simple material properties on the collective behaviour of a soil specimen. This presentation is part of a broader investigation on modelling the torsional simple shear (TOSS) and resonant column (RC) tests by finite element methods. Some results have been published while other publications are still in progress. Here we present the results of modelling our test specimen by random distributions of simple elasto-plastic material behaviour.

3. Tresca Model and Implementation in Midas GTS
The TOSS-RC test specimen is a hollow cylinder with inner and outer diameters of 4 and 6 cm. Specimen height is approximately 14 cm but may vary slightly due to sample preparation methods. Variations of this test are described in Szilvagyi [2], Szilvagyi and Ray [3], Woods and Ray [4], Ray [5]. The advantage of a hollow cylinder is to create better shear strain uniformity across the specimen cross section. This is helpful when shear strains ($\gamma$) reach levels where significant softening occurs ($\gamma \sim 10^{-3} \text{ to } 10^{-2} \text{ mm/mm}$) in the realm of small strain behaviour. At these levels, shear modulus ($G$) may reduce to 20-30% of its original, low-strain value ($G_{\text{max}}$).

The stress strain model that we have relied on has been the Ramberg-Osgood model. While it was originally developed for non-linear behaviour of aluminium alloys [6], the model was adapted for soils and used extensively by Richart and Woods [7]. Another, similar model is the hyperbolic model adopted and modified by Hardin and Drnevich [8] and later by Stokoe and Darendeli [9],[10] and others. The Ramberg-Osgood model is used here only as a reference for comparison. The simple elasto-plastic model used extensively in this study is the one developed by Tresca. It has an elastic modulus, Poisson’s ratio and a yield value (1D), yield line (2D), or more generally, yield surface (3D). The yield surface is shaped like a tube with a hexagonal cross section centred along the hydrostatic axis. The behaviour in torsional shear would follow a path shown in Figure 2a. The conceptual load and stress conditions for an element in the model is shown in figure 2b.
The finite element software used for this study was MidasGTS-NX. We have had a great deal of experience with the software, and it allows for 3-D modelling using cylindrical coordinates which is very helpful for torsional problems. The mesh consisted of 4033 quadratic hexahedral elements and 20,689 nodes (figure 3a). To create random properties, we specified 512 different elasto-plastic materials in the original model. The Midas program could export the mesh and materials to a text file (.fpm) that was identical to a (.csv) format file and easily imported into Excel. The material properties could then be randomized and inserted using a VBA program within Excel. Once the material was randomized, the file could be imported back into Midas. A work-flow chart and example of randomized elements from Excel is shown in figure 3b. One should note that step (4) could generate, say, a normal distribution of yield stresses and moduli based on prescribed mean and standard deviations for that material. There could also be up to ten different distributions of Tresca materials, each with their own mean and standard deviation values for yield and modulus. The only requirement was that the total number of possible property combinations (yield stress and modulus) had to be 512. At this stage, log-normal, and random (flat histogram) distributions could be created as well. For step (5) the 512 properties created from step (4) were distributed throughout the specimen with even randomness, thereby creating a specimen with a normal distribution of properties. The property map shown (step 5) is the inner “cylinder” of the finite element mesh laid out flat so that we could visually inspect the distribution of properties. Each cell has a property number from 1 to 512 and a corresponding colour. The middle and outer cylinders were also mapped in a similar fashion.
Once the randomized mesh was imported into Midas, the remaining analysis parameters were input. Boundary conditions were added at the base as pinned nodes that represented the fixed base of the experimental apparatus. The top of the specimen was connected to a series of rigid links (figure 3a light blue lines). These links crossed the top of the mesh along every diameter, connecting nodes on opposite sides; a total of 529 nodes. The links were all connected to a single node at the centre where a single moment \((M_z)\) or prescribed rotation \((\phi_z)\) was applied. This arrangement avoided any difficulty with having a uniform rotation along the top of the specimen throughout the test. Generally, a rotation was prescribed since it was more likely to remain stable as the specimen approached full yield. Since Tresca material strength is not influenced by confining stress, there was no need to simulate confinement during the analysis. The prescribed rotation was entered as a static displacement; however, the final load condition was a time-varying static load. The time function was often a sine wave that had a duration of 1.25 cycles, simulating initial (one-way) loading then one full cycle to produce a hysteresis loop. The time function was set to a slow speed to avoid any inertial effects, however the analysis type was set to non-linear time history, since it was the most convenient way to model the loading process. Analysis options were set to seek convergence via normalized load, deflection, and work parameters to be below 1.0e-03, 1.0e-03, and 1.0e-06, respectively. The solution method used a full Newton Raphson stiffness update scheme, allowing for line search. A maximum number of iterations for a single loading step was set to 50, and the loading history itself was set to 1200 steps for the 1.25 cycle excursion.

4. Analysis Cases and Results
The initial analyses were performed to verify that the Tresca material model was performing in a way we understood. As a verification, three realizations with different Tresca \(\sigma_y\) and \(E\) values were analysed and centrally located element stress-strain data was extracted. The results (figure 4) showed what we expected: in pure rotational shear: a) the stiffness was equal to the shear modulus \(G\), where
and b) The yield stress in pure shear was equal to one-half the Tresca yield. Since $\nu = 0.3$ for all tests, $G$ and $E$ were related by a factor of 2.6. This is shown in the linear portions of the results in figure 4a.

Figure 4. a) Stress-strain curves for $E=20000$ kPa $\sigma_y=120$, b) Moment rotation curves for three specimens with uniform properties

An introductory set of analyses were performed to investigate the behaviour of the test specimen when random distributions of properties were input. The series of configurations is presented in table 1. The analysis number indicates normal distribution (N), log-normal distributions (LN), bimodal normal even split (BE), and bimodal normal offset (BO). All distributions started with population of 512. Normal distributions were described by mean and standard deviation values for both $E$ and $\sigma_y$. The log-normal distributions had much higher values for standard deviation. The reader should note the drastic differences in standard deviation between the two sets. This is expected since, the whole purpose for using log-normal distributions is to account for the extensive degree of variability in geotechnical properties one may find in nature. The even bi-modal normal distributions were simply two distributions of 256 values with two mean values and two standard deviation values. Both distributions were randomly spread over the element mesh as there were no soil layers or zones separating them. The offset bi-modal normal distributions were two distributions with different populations that added to 512. Additionally, the overall mean remained the same ($\sigma_y = 120$ kPa, $E=20000$ kPa). The placement of all elements in this set was also entirely random.

Table 1. Material conditions, random distributions, loading conditions.

| Analysis Number | Number of Materials | Yield Stress, $\sigma_y$ (kPa) | Tresca Modulus $E$ (kPa) |
|-----------------|---------------------|-------------------------------|--------------------------|
|                 |                     | Mean  | Std Dev | Mean  | Std Dev |
| N-01            | 512                 | 120   | 30      | 20000 | 1000   |
| N-02            | 512                 | 120   | 20      | 20000 | 1000   |
| N-03            | 512                 | 120   | 20      | 20000 | 5000   |
| LN-01           | 512                 | 120   | 60      | 20000 | 10000  |
| LN-02           | 512                 | 120   | 120     | 20000 | 20000  |
| LN-03           | 512                 | 120   | 240     | 20000 | 40000  |
Reaction moment vs rotation plots are shown for each group in figures 5a,b and 6a,b. Figure 5 shows the reaction moment vs. rotation for specimens with normal and log normal distributions of $E$ and $\sigma_y$. Test numbers correspond to the conditions listed in table 1. The first characteristic is the “wobbly” path of the initial loading curve. This is not a numerical oscillation, but rather demonstrates the effect of different elements reaching yield, and perhaps unloading/reloading over short intervals as the main body of the specimen continues to rotate. The unloading/reloading bursts were verified by viewing the plastic states of all elements during the entire loading history. We recorded animations of each test and they proved to be quite remarkable. Since every test was a prescribed rotation between 0.1 and -0.1 radians, the plots all match along the horizontal axis.

Figure 5 presents a set of: (a) normally distributed properties, and (b) log-normally distributed properties. The effect is obvious in that the paths are more smoothly curved and mimic more closely the typical behaviour of soils in our torsional shear tests.

A similar effect occurs in figure 6a where the pairs of mean values are separated greater and greater distances. Analysis BE-03 has the widest differences and shows the smoothest curvature. In figure 6b the bimodal concept was changed so that the stiffer, stronger component had a reduced population. However, the overall average $E$ and $\sigma_y$ remained constant. The progression is similar to the previous groups but less pronounced.

Figure 5. Reaction moment vs rotation for a) Normally distributed, b) Log-Normally distributed values of $E$ and $\sigma_y$.
5. Conclusions and Further Research

The observations presented by Masing were investigated by applying a simple elasto-plastic model to a carefully analysed test specimen. His observation that such a simple material model can produce complex behaviour is also demonstrated here. Our main conclusion is that material non-linearity, as a continuous process, becomes more apparent as the material properties vary over a wider range, become more variable. This supports the intuitive notion of particulate mechanics where the particle-particle contact behaviour may be simple, but the aggregate behaviour becomes quite complex.

We anticipate our research will examine a broader range of material models and more sophisticated approaches to representing their variability in a laboratory specimen.

6. References

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