Perturbations of SNe Ia Light Curves, Colors, and Spectral Features by Circumstellar Dust

Rahman Amanullah and Ariel Goobar

The Oskar Klein Center, Physics Department, Stockholm University, AlbaNova University Center, SE 106 192 Stockholm, Sweden

Received 2010 October 13; accepted 2011 April 7; published 2011 June 9

ABSTRACT

It has been suggested that multiple scattering on circumstellar dust could explain the non-standard reddening observed in the line of sight to Type Ia supernovae. In this work, we use Monte Carlo simulations to examine how the scattered light would affect the shape of optical light curves and spectral features. We find that the effects on the light curve widths, apparent time evolution of color excess, and blending of spectral features originating at different photospheric velocities should allow for tests of the circumstellar dust hypothesis on a case by case basis. Our simulations also show that for circumstellar shells with radii \( r = 10^{16} - 10^{19} \) cm, the light curve modifications are well described by the empirical \( \Delta m_{15} \) parameter and intrinsic color variations of order \( \sigma_{BV} = 0.05 - 0.1 \) arise naturally. For large shell radii an excess light curve tail is expected in \( B \)-band, as observed in, e.g., SN2006X.

Key words: dust, extinction – distance scale – supernovae: general – supernovae: individual (SN2006X)

Online-only material: color figures

1. INTRODUCTION

A dramatic breakthrough in cosmology took place more than a decade ago when the accelerated expansion of the universe was discovered through Type Ia supernovae (SNe Ia) used as standard candles (Riess et al. 1998; Perlmutter et al. 1999). SNe Ia remain among the best tools to investigate the energy content of the universe, as demonstrated by recent very successful surveys of high-z SNe (Astier et al. 2006; Wood-Vasey et al. 2007; Riess et al. 2007; Kowalski et al. 2008; Kessler et al. 2009; Amanullah et al. 2010). However, the progress for precision cosmology is hampered by systematic uncertainties, notably a potential drift in the SN Ia luminosity and intrinsic colors with cosmic time and the impact of changing dust extinction properties along the line of sight (Nordin et al. 2008). The standardization of SNe Ia generally uses two parameters: the measured color, typically rest-frame \( B - V \) and often one more optical color, and an empirically defined light curve shape parameter, such as stretch, \( s \) (Perlmutter et al. 1997; Guy et al. 2005; Conley et al. 2008), \( x_1 \) (Guy et al. 2007), \( \Delta m_{15}(B) \) (Phillips 1993; Hamuy et al. 1996) or \( \Delta \) (Riess et al. 1996; Jha et al. 2007). In this work, we examine how multiple scattering of supernova (SN) photons on the surrounding material could affect both the light curve shapes and colors.

While the reddening of SNe Ia was originally thought to be entirely due to extinction by interstellar dust, there is an increasing body of evidence showing that the measured color excesses to some SNe Ia show a steeper wavelength dependence than what is observed for reddened stars in the Milky Way. Likewise, there is no generally accepted model that explains the variations in light curve shape observed in SNe Ia and their correlations with peak SN brightness (Phillips 1993).

Following up the work by Wang (2005), Goobar (2008; hereafter G08) showed that multiple scattering on circumstellar dust could potentially help explain the low values of the total-to-selective extinction parameter, \( R_V = A_V/E(B-V) < 3 \), observed in the sight lines of nearby Type Ia SNe (Elias-Rosa et al. 2006, 2008; Krisciunas et al. 2007; Wang et al. 2008; Nobili & Goobar 2008).

A Monte Carlo ray tracing technique was used in G08 to investigate the reddening effects in the presence of circumstellar dust and an approximate reddening law for optical and near-IR wavelengths was derived. Folatelli et al. (2010) compared the parameterized model in G08 with high quality \( uBgriyJHK \) data from the Carnegie Supernova Project and found that it provided particularly good fits for the two most reddened SNe Ia in their sample, SN2005A and SN2006X. The success of these comparisons calls for further scrutiny of the model. In this work, we investigate the impact of multiple scattering on the shapes of SNe Ia optical light curves. We explore how the extra wavelength dependent “random walk” of photons in the presence of scatterers affects the width of broadband filter light curves. In particular, reddened “slow decliners” may be compatible with the scenario in G08.

Furthermore, also the strengths of spectroscopic features as a function of time are likely to be affected, since a varying path length would lead to a superposition of several SN phases. Throughout the paper we point out how observations could put bounds on the size of a hypothetical circumstellar dust shell.

2. CIRCUMSTELLAR DUST IN THE SINGLE DEGENERATE SCENARIO

In the currently favored Type Ia scenario, a white dwarf accretes mass from a companion star until the Chandrasekhar limit is reached, at which point instabilities lead to an explosion. However, the model requires the accreting white dwarf to expel material accumulating on its surface through fast stellar winds, \( v_w \sim 10^2 - 10^3 \) km s\(^{-1}\), to avoid the formation of a common envelope between the white dwarf and the donor star which would prevent the explosion (Hachisu et al. 1996, 1999; Hachisu & Kato 1999). Condensation of dust grains may take place in the ejected matter, possibly also in collisions with the accreting matter. For dust grain growth in stellar winds, the density as a function of distance to the star surface, \( r \), is generically expected to scale as \( \rho_{\text{dust}} \propto \frac{M}{r^{3.5}} \), where \( v_w \) is the stellar wind velocity and \( M \) is the mass loss per unit time, typically of order \( 10^{-7} M_\odot \) yr\(^{-1}\) in SNe Ia models.
3. CIRCUMSTELLAR DUST DESTRUCTION BY SN Ia LUMINOSITY

So far, we have assumed that the SN explosion site is surrounded by dust. However, the radiation from the SN itself will sublimate dust grains as these are heated up to $T_{\text{sub}} \sim 3000 \, \text{K}$. Thus, assuming that the entire optical–UV luminosity of the SN is absorbed by surrounding dust grains, we can compute the radius $r_c$ where dust grains, if present at the explosion time, would be depleted. From the radiation balance condition:

$$r_c = \left( \frac{L_{\text{bol}}(t)}{16 \pi \sigma T_{\text{sub}} (Q_{\text{IR}})} \right)^{1/2},$$

where $Q_{\text{abs}}$ is the effective absorption efficiency and $(Q_{\text{IR}})$ is the Planck-averaged emissivity at $T = T_{\text{sub}}$. Waxman & Draine (2000) studied a similar situation, but for dust surrounding gamma-ray bursts, and found that for $2000 < T < 3000 \, \text{K}$, $(Q_{\text{IR}}) \approx 0.1$ for a mixture of astronomical silicate and graphite grains with typical sizes of $0.1 \, \mu\text{m}$. Sollerman et al. (2004) have estimated the peak bolometric luminosity of SNe Ia to be $L_{\text{bol}} = 1.43 \cdot M_{\text{SN}} \cdot 10^{43} \, \text{erg s}^{-1}$, where the $56\text{Ni}$ mass (in units of solar masses), $M_{\text{SN}}$, has been measured to be between 0.1 and 1.1 $M_{\odot}$ (Leibundgut 2000). Inserting this peak luminosity ($E_{\gamma} \sim 1–5 \, \text{eV}$) into Equation (1), and assuming that all the radiation is absorbed, we find $r_c \lesssim 10^{16} \, \text{cm}$ ($\sim 0.003 \, \text{pc}$), which we will use as a rough estimate of the region where evaporation of pre-existing dust would take place as a result of the SN heating. A similar estimate of the region that ought to be depleted of dust was done by Pearce & Mayes (1986). Thus, to create a dust shell that would survive the radiation from the SN explosion, and using the stellar wind velocities and mass-loss rates from Section 2, the wind must start about 10–100 years prior to the explosion, i.e., $\gtrsim 10^{-6} \, M_{\odot}$ of mass must be expelled for any effect on circumstellar dust to be noticeable. For SN2005gj, a well-studied example of a SN with significant interaction with the circumstellar environment, Aldering et al. (2006) find evidence for mass loss about eight years before the explosion. For significant opacity, $\tau \sim 1$, the required dust mass can be estimated from the absorption cross-sections in Weingartner & Draine (2001) and Draine (2003), $\sigma_{\lambda}/m_{\text{dust}} \sim 10^4 \, \text{cm}^2 \, \text{g}^{-1}$. Thus, for a thin dust shell at $r \sim r_c \sim 10^{16} \, \text{cm}$, this corresponds to $m_{\text{dust}} \sim 10^{-4} \, M_{\odot}$.

We also note that since $r_c \propto \sqrt{M_{\text{SN}}}$, intrinsically brighter explosions are likely to have larger $r_c$, i.e., thinner circumstellar shells. In the following sections we will see that, depending on the size of the outer shell, multiple scattering could generate relations between brightness and light curve shapes and colors qualitatively similar to the observed ranges. We note, however, that interaction with dust in the CS material can only broaden the pristine light curve, i.e., it is not possible to produce faster decline than what is in the original SN explosion.

4. MULTIPLE SCATTERING OF PHOTONS AROUND THE SUPERNOVA

A natural effect of light scattering is that it adds flight time to the photons, thereby affecting the observed light curve (Patat 2005; Patat et al. 2006). Next, we start by generalizing the Goobar (2005) model: we consider a scenario where the exploding white dwarf ($r_{\text{WD}} \sim 10^6 \, \text{cm}$) is at the center of a homogeneous dust shell with an outer radius $r$ ($r \gg r_{\text{WD}}$) and an inner radius $r_i$, where $r_{\text{WD}} \leq r_i \leq r$. Note that this will not change any of the results in G08, where $r_i \equiv r_{\text{WD}} \approx 0$ was assumed, but it will affect the distribution of photon flight times, i.e., the subject of this paper.

The technique in G08 is extended to investigate the time domain by calculating the total traveled path length, $D$, for each photon before it crosses a plane, $P$, that is perpendicular to the line of sight between the observer and the source and is located at the distance, $r$, from the source which is illustrated in Figure A1. The details of the calculation of the photon travel times are outlined in the Appendix. The traveled photon path length, $D$, is given by Equation (A1).

The traveled photon path length is distributed in the interval $r \leq D < \infty$, with $D = r$ for photons that do not interact with the dust at all. The shape of the distribution of $D$ will depend on the radius, $r$, the thickness of the shell, the scattering and absorption properties of the dust, and the dust density. In this work we will quantify the latter in terms of the observed color excess, $E(B - V)$, relative to the pristine source color. For the scattering and absorption properties, we use models consisting of carbonaceous grains and amorphous silicate grains from Weingartner & Draine (2001) and Draine (2003).

We begin by exploring the scattering distribution for the two extreme scenarios: (1) $r_i = r_{\text{WD}} \approx 0$ (i.e., a homogeneous dust sphere as in G08) and (2) $r_i = 0.95 \cdot r$, i.e., a very thin dust shell. Clearly, the scenario (1) is already challenged by the conclusions in Section 3, nevertheless, it serves as a point of reference. The left panel of Figure 1 shows the distribution of $D/r$ from Monte Carlo simulations of photons with $\lambda = 5000 \, \text{Å}$. Most photons travel unscattered, and for the rest, there is a rough exponential fall-off of delay times. The dust density was chosen to cause an observed color excess $E(B - V) \approx 0.5$. Furthermore, we are assuming that the dust grain properties correspond to LMC-type dust, although the time distributions for MW-type dust are very similar.

A local minimum can be noted for case (2) around $D/r \sim 2$. This can be understood by realizing that a photon from the source can only double the path length, $D/r \sim 2$, by having a total scattering angle of $\sim \pi/2$ for this particular setup, which will take it through more dust than for other angles. It is then likely to scatter again, which would result in either a shorter or a longer path. The significance of the minimum will decrease with the density of the dust and the inner radius of the shell.

5. LIGHT CURVE PERTURBATION FROM MULTIPLE SCATTERING

Since the scattering cross-section, scattering angle, and albedo are wavelength dependent, the light curve perturbations are expected to vary between broadband filters.

We use the time dependent spectral energy distribution (SED) SN Ia templates from Hsiao et al. (2007; hereafter H07) to build up observed broadband light curves in the presence of circumstellar dust. Thus, we make the simplifying assumption that the empirically derived spectral template is pristine, i.e., not already affected by dust. This is not a major limitation for this analysis, since we are here primarily concerned with relative quantities, such as time delays or color excess, rather than absolute times or fluxes.

Like in G08, we only consider dust grain models from Weingartner & Draine (2001) and Draine (2003) for Milky Way dust and LMC dust. In particular, it was found in G08 that LMC dust grains in the circumstellar medium would lead to $R_V \approx 1.65$, while Milky Way dust leads to $R_V \approx 2.56$. These two grain models approximately bracket the empirically found values for $R_V$ for a large fraction of SNe Ia. In both cases, the
extinction law deviated significantly from Galactic extinction (Cardelli et al. 1989) in the UV and NIR regions.

For the idealized case considered in this work, a fraction of the photons scatter inside the dust shell and the delay time induced by the multiple scattering will scale with $r$, \[
\Delta t_r \sim \frac{r}{c} \sim 4 \cdot \left(\frac{r}{10^{16} \text{ cm}}\right) \text{ days.} \tag{2}
\]
Since the typical rise time of SNe Ia light curves is about three weeks, and the fall time is somewhat longer, we expect significant effects on the shapes of optical light curves for $r \gtrsim 10^{16}$ cm for increasing optical depth, $\tau(\lambda)$. On the other hand, for timescales $\Delta t_r \gg 1$ month, i.e., $r \gg 10^{17}$ cm, we expect the time perturbations to de-correlate with the light curve shape parameters currently used, since these are designed to capture smaller timescale effects.

6. LIGHT CURVE SHAPE

Existing SN Ia light curve fitters (Riess et al. 1996; Goldhaber et al. 2001; Wang et al. 2003; Guy et al. 2005, 2007; Jha et al. 2007; Conley et al. 2008) use different approaches to take variations of the optical light curve shape into account for standardizing SNe Ia. To estimate the effect on the light curve shape due to circumstellar dust, we once again consider the two extreme scenarios: (1) $r_i = r_{WD} \approx 0$ and (2) $r_i = 0.95 \cdot r$, introduced above.

Figure 2 shows the time-delay distribution in the $B$-band from Monte Carlo simulations as a function of the outer shell radius $r$ for cases (1) and (2) where the LMC-type dust density was chosen to cause an observed color excess $E(B - V) \approx 0.12$.

The far left panel of this figure is closely related to the left panel of Figure 1, except that the dimensionless property $D/r$ has been converted to days by assuming different physical shell radii. First, we note that for very large outer radii, photons that scatter on dust grains have a fairly flat delay probability with respect to the unscattered ones in the first (zero) bin. We also note how the effect that certain photon paths are less likely for a thin shell, as discussed above, propagates to a sharp feature in the delay time distribution.

Also shown in Figure 2 is how the delayed photons perturb the pristine $B$-band light curve shape. This reveals how the size of the outer radius affects (primarily) the post-max shape of the light curve. Notably, the largest deviation from the pristine shape can be seen in the tail, more than a month after light curve maximum, typically beyond the time range investigated for most reddened SNe.

6.1. The Fall Time

The most straightforward method of quantifying post-maximum light curve shape is in terms of the magnitude difference between maximum and an arbitrary number of rest-frame
Figure 2. Simulated effects on the SN Ia B-band light curve from multiple scattering by circumstellar dust for three different radii: \(r = (10^{16}, 10^{17}, 10^{18})\) cm. The upper row shows results for \(r_i = 0.0\) while the lower is for \(r_i = 0.95\cdot r\). A reddening of \(E(B-V) \approx 0.12\) has been assumed for both cases. The far left column shows the photon delay distribution in the B-band. The middle left column shows the corresponding B-band light curves after the source (black) SED has been convolved by the delay distributions. The middle right column gives the B light curves normalized by source light curve at maximum. The far right column shows the B-band light curve \(\Delta m_{15}(B)\) for different values of circumstellar shell radii.

(A color version of this figure is available in the online journal.)

days past maximum in a given rest-frame passband. This method of measuring the fall time was introduced by Phillips (1993) who chose the rest-frame B-band and 15 days past maximum, respectively, \(\Delta m_{15}(B)\). The simulation results of the evolution of the \(\Delta m_{15}(B)\) parameter for a few different CS scenarios is presented in the far right panel of Figure 2 and in more detail in Figure 3.

For \(10^{16} \lesssim r \lesssim 5 \times 10^{17}\) cm, \(\Delta m_{15}(B)\) decreases with \(r\). However, for larger radii, the late photons populate the tail of the light curve and thus the original pristine light curve fall time at earlier epochs is gradually recovered, while a plateau is built up at late times. The significance and the variation of the light curve tails in the examples shown suggest further studies of the fall time, also at epochs beyond day 15. This is illustrated by the right panel of Figure 3, where two different radii for the same dust configuration can give rise to the same value of \(\Delta m_{15}(B)\), but different values at 35 days past maximum, \(\Delta m_{35}(B)\). For very large radii, both the original values \(\Delta m_{15}(B)\) and \(\Delta m_{35}(B)\) are recovered. For these extreme radii, the imprint from the CS dust will be found for even later epochs.

Another interesting effect occurs for the dust shell scenario (2) as shown in Figure 2. The fall-off of the \(r = 10^{18}\) cm curve is initially steeper than the \(r = 10^{17}\) cm curve, but at day \(\sim 30\) past maximum, this changes and the \(r = 10^{18}\) cm curve flattens out and crosses the \(r = 10^{17}\) cm curve, while the latter continues to fall. This qualitative behavior can also be seen in real data: two SNe Ia normalized to the same maximum brightness show different relative fall-off for different epochs, as seen in Figure 4.

6.2. The Rise-time–Fall-time Relation

The impact of the time-delay effect from scattering on CS material is expected to have a stronger impact on the fall time than on the rise time of the light curve. The relation between the SNe Ia rise time and \(\Delta m_{15}(B)\) has been studied both for high-quality data of well-sampled nearby SNe Ia (Strovink 2007), as well as in a statistical approach for a much larger SDSS-II data set (Hayden et al. 2010). While Strovink (2007) found a double-peaked distribution for the relation between rise and fall time, this was not seen by Hayden et al. (2010). Figure 2 reveals that different CS dust scenarios could give rise to a relation between rise and fall time, even when a single pristine light source is assumed. In Figure 5 we show this relation for a few different scenarios, together with the results from Strovink (2007).

The absolute values of any light curve parameters determined from our simulations will depend on the corresponding values of the pristine source. Since the light curve parameters of the H07 template will roughly reflect the mean values of the SN Ia population, our simulations will only be able to produce light curves with longer rise times and smaller \(\Delta m_{15}(B)\) than average.

In the CS scenario, it is reasonable to assume that a “naked” SN Ia template has a shorter generic rise time and a faster fall time; in order to compensate for this we have also applied an ad-hoc shift to all the curves in Figure 5 in order to match the data points. Further, assuming that the generic B-band shape of a “naked” SN Ia is similar to the H07 template that we use here, any modification to the relative light curve properties for
different dust scenarios is likely to be of second order, and the general trend and size of the effect should still be valid.

The result from Figure 5 is that for SNe within a narrow range of observed color excess, CS dust could naturally give rise to a bi-modal distribution in the relation between rise and fall time for a small sample. However, if the accepted color range is extended, the gap between the two populations will be filled. More generally, it is intriguing that the rise–fall time scatter observed in normal SNe Ia is compatible with our simulations.

6.3. Time of Maximum for Different Wavelengths

Up to this point we have focused on the effect of circumstellar dust on the rest-frame $B$-band, the wavelength range that is traditionally used for doing SN cosmology. Since the scattering and absorption cross-section decreases with wavelength, we expect the delay distribution to affect bluer passbands more. One way of studying this is to investigate the impact on the SN rise time for different filters. However, this is related to comparing the time of maximum in different bands, which is also a property that can be measured quite accurately. Figure 6 shows the effect on the difference of the time of maximum between the $B$ and the $I$ bands, $t_{\text{B max}} - t_{I \text{ max}}$, for various CS scenarios. The general trend in the model prediction is that the time between the $I$-band and $B$-band light curve maxima increases for slow decliners. Furthermore, the effect is quite sensitive to optical depth, i.e., to the color excess.
7. INTRINSIC COLOR VARIATIONS

G08 explicitly both describes how circumstellar dust reddens the pristine source and derives this reddening law. However, as we have seen due to the time delay, mixing of photons originating from different epochs is expected. Since the delay time, $\Delta t$, depends on the shell radius, the observed color for a fixed dust amount will have an intrinsic scatter with a magnitude depending on the distribution of the shell size, $r$.

Figure 7 shows how a few different CS scenarios could affect both the light curve shape and the color at $B$-band maximum. As with previous results, we can see that the effects of both light curve shape and color increase with the amount of CS dust. There is also degeneracy between different CS scenarios, so no trivial correlation between fall time and color excess is expected in the CS scenario. The CS model thus naturally accommodates an intrinsic color scatter in moderately reddened SNe, $\sigma_{E(B-V)} \sim 0.05$–0.1 mag, in agreement with the observations (Nobili & Goobar 2008; Kessler et al. 2010).

7.1. Time Dependent Color Excess

One of the salient features of the scenario in G08 is that blue photons scatter more than red photons. This implies that...
blue photons will, on average, be more delayed by multiple scattering since they have a longer path length before leaving the circumstellar dust environment. This implies that the color excess due to the dust component will acquire time dependence. With respect to a time averaged color excess, the measured color will be redder pre-max turning bluer after max, as shown in Figure 8. This effect should provide the most stringent bounds on the dust shell sizes allowed by the data. Yamanaka et al. (2009) show in their analysis of SN2006X that once they correct for the color excess around maximum light, the $V-R$ color is about 0.3 mag bluer than the normal unreddened SN2003du two months after light curve peak, in qualitative agreement with the CS model prediction.

8. PERTURBATIONS TO THE LIGHT-CURVE-SHAPE–BRIGHTNESS RELATION

Since light curve shape is the primary parameter to standardize SNe Ia as distance indicators, it is important to investigate how the relation between light curve shape and brightness may be affected by the presence of circumstellar dust. As noted in Section 3, an intrinsically brighter explosion will deplete more of the surrounding dust, emphasizing the fainter–redder relation. For shell radii $r \gtrsim 10^{16}$ cm, the optical light curve shape...
will broaden the light curves, potentially influencing the empirically derived brighter–broader relation. Is it possible to obtain a correlation between brightness and light curve shape from the interaction with the circumstellar environment? Although possible, it invokes a somewhat contrived scenario: for shell sizes $r \sim 10^{15}$–$5 \times 10^{17}$ cm, if the outward velocity of the circumstellar material, i.e., the white dwarf wind velocity, correlates with the amount of $^{56}$Ni powering the explosion, brighter SNe would have large $r$ and become slow decliners. We are not aware of any SN models that have predictions in this respect.

More generally, if the light curve shape is a combination of effects, involving both physics of the progenitor system and the material around the white dwarf, then one would expect a residual dependence of brightness on light curve shape, even after the main component has been calibrated out. In particular, we note that the optical light curve shape beyond 30 days after $B$-band peak will be noticeably affected for a wide range of shell sizes.

9. THE SECONDARY “BUMP” IN THE $I$-BAND LIGHT CURVE

The Type Ia $I$-band light curve exhibits a secondary maximum approximately 15–30 days after the $B$-band light curve maximum which, in turn, typically happens within 2 days from the primary $I$-band peak (Nobili et al. 2005). The time gap between the two $I$-band peaks, as well as the relative strength of the secondary peak, has been shown to correlate with the $B$-band light curve shape (Nobili et al. 2005).

The implication of the imposed time delay from scattering on circumstellar dust on the $I$-band peaks is shown in Figure 9. We note that for a constant shell size we expect the brightness difference (upper row) between the peaks, as well as the significance of the secondary peak, to decrease with an increased amount of dust. The effect comes from the fact that photons from the first peak are shifted to both fill the gap between two peaks and emphasize the second bump, and is therefore also correlated with $B$-band fall time and $E(B − V)$.

If the shell size is allowed to vary, the usual degeneracy between the shell size and the amount of dust is obtained for the observables. Despite this degeneracy, it is interesting to note that for a broad range of dust properties and shell sizes, a relation between the $I$-band light curve properties and $E(B − V)$ is expected.

10. SPECTRAL FEATURES

Folatelli et al. (2010) found that different fitted reddening corrections were obtained for their data set depending on whether the reddest SNe were included in the fit or not. For the full data set they obtained $R_V = 1.7$, while they fitted $R_V = 3.2$ if the high-reddened SNe were excluded. In contrast to this, Amanullah et al. (2010) found that redder SNe in the Union2 sample prefer a higher color correction than bluer SNe and speculate that the color correction may very well be more complex than a simple linear relation.

Further, Wang et al. (2009) found a bi-modality in the color–magnitude relation over a broad color range, with the two groups preferring different values of $R_V$ independently of the color. They also observed a correlation between the velocity of Si ii($\lambda$6355) and the fitted value of $R_V$, concluding that objects with high velocity tend to prefer a lower value of $R_V$ than SNe with low velocity.

One possible explanation for the bi-modality could be that the color of the low-$R_V$ SNe primarily originates from CS dust reddening, while the high-$R_V$ objects are dominated by extinction from interstellar dust.

The time delay of photons from CS dust scattering is expected to blend the SEDs between different epochs, i.e., a spectral feature for a given epoch will effectively be a superposition of the feature for all previous epochs. Since time-delay distribution varies with wavelength, the amount of blending will also decrease with wavelength. Furthermore, since the minima of the SN absorption features trace the velocity of the receding photosphere during the first week after the explosion, we expect that delayed photons arriving around maximum light will originate from a region further out in the photosphere, i.e., at higher velocities. However, our simulations indicate that these effects are small (see Figure 10), typically accounting for velocity variations of a few hundred km s$^{-1}$, i.e., compatible with the velocity scatter between measurements of normal SNe Ia, but not enough to explain significant outliers like SN2006X (Yamanaka et al. 2009).
11. SUMMARY AND CONCLUSIONS

Dust layers surrounding the progenitor systems of SNe Ia could help explain the observed reddening law. In this work, we have performed Monte Carlo simulations that indicate that the scenario with circumstellar dust would also perturb the SN optical light curves in a manner that resembles empirical findings, e.g., an “intrinsic” color variation of $\sigma_{E(B-V)} \approx 0.25$ in the limit when the radius goes to zero. The inner radius is fixed to $r_i = 0.95 \cdot r$ for all cases.

(A color version of this figure is available in the online journal.)

APPENDIX

A SPHERICAL DUST SHELL

A simple model of a spherical dust shell with radius, $r$, and inner radius, $r_i = R \cdot r$, where $0 < R < 1$ is illustrated in Figure A1. Also shown in the figure is a photon leaving the sphere (solid line). The current position of the photon is defined by the vector, $\vec{r}_0 = (x, y, z)$, and the next position, $\vec{r}_{\text{next}}$, is given by $\vec{r}_{\text{next}} = \vec{r}_0 + \vec{r}_D$, where $\vec{r}_D$ is the displacement vector $\vec{r}_D = (dx, dy, dz)$.

Phonon traveling inside the shell. The impact parameter, $i$, is the minimum distance to the center of the sphere for any given photon path and can be defined as:

$$i = \begin{cases} 
    r_0 \cdot \sin \gamma & \text{if } 0 \leq \gamma \leq \frac{\pi}{2}, \\
    r_0 & \text{if } \frac{\pi}{2} < \gamma.
\end{cases}$$

Here, $\gamma$ is given by

$$\vec{r}_0 \cdot \vec{r}_D = r_0 r_D \cos(\pi - \gamma) = x \cdot dx + y \cdot dy + z \cdot dz, \quad \cos \gamma = -\cos(\pi - \gamma) = -\frac{x \cdot dx + y \cdot dy + z \cdot dz}{r_0 r_D}.$$  

If $i < r_i$, the photon will cross the inner radius of the shell and then re-enter the shell. This will in turn extend the mean free path of the photon by the amount $2q = 2\sqrt{r_i^2 + i^2}$, which is the length of the path within $r_i$.

Path length. For each distance between interactions, $r_D$ is added to the total distance traveled by the photon before leaving the sphere. For a scattered photon leaving the sphere, the last
Figure A1. Illustration of a scattered photon (thin solid line) propagating through a dust shell. The figure shows the intersecting plane of the sphere that is defined by the two vectors $\mathbf{r}_0$ and $\mathbf{r}_D$.

distance, $s$, traveled inside the sphere is given by

$$s^2 = r^2 + r_0^2 - 2rr_0 \cos(\pi - \theta - \gamma),$$

where $\cos \theta = \sin \gamma \cdot r_0 / r$. The distance traveled by a scattered photon should be compared to the distance traveled by a non-interacting photon, which means that the distance, $d$, between the surface of the sphere and the plane (thick, solid line in Figure A1) perpendicular to $\mathbf{r}_D$ at distance $r$ from the center must be added to the total distance, where $d$ is given by

$$l = 2r \cdot \sin \frac{\theta}{2},$$

$$\varphi = \pi - \frac{\pi}{2} - \frac{\theta}{2} = \frac{\pi - \theta}{2},$$

$$d = l \cdot \cos \varphi = 2r \cdot \sin \frac{\theta}{2} \cos \frac{\pi - \theta}{2} = 2r \cdot \sin^2 \frac{\theta}{2}.$$

The total distance traveled, $D$, can then be summarized as

$$D = \begin{cases} r_{\text{Der}} \quad \text{for non-interacting photons,} \\ r_{\text{Der}} + s + d \quad \text{for scattered photons} \end{cases} \quad (A1)$$

REFERENCES

Aldering, G., et al. 2000, ApJ, 560, 510
Amanullah, R., et al. 2010, ApJ, 716, 712
Astier, P., et al. 2006, A&A, 447, 31
Cardelli, J. A., Clayton, G. C., & Mathis, J. S. 1989, ApJ, 345, 245
Conley, A., et al. 2008, ApJ, 681, 482
Draine, B. T. 2003, ApJ, 598, 1017
Dwek, E. 1983, ApJ, 274, 175
Elias-Rosa, N., et al. 2006, MNRAS, 369, 1880
Elias-Rosa, N., et al. 2008, MNRAS, 384, 107
Folatelli, G., et al. 2010, AJ, 139, 120
Goldhaber, G., et al. 2001, ApJ, 558, 359
Goobar, A. 2008, ApJ, 686, L103
Guy, J., et al. 2005, A&A, 443, 781
Guy, J., et al. 2007, A&A, 466, 11
Hachisu, I., & Kato, M. 1999, ApJ, 517, L47
Hachisu, I., Kato, M., & Nomoto, K. 1996, ApJ, 470, L97
Hachisu, I., Kato, M., Nomoto, K., & Umeda, H. 1999, ApJ, 519, 314
Hamuy, M., Phillips, M. M., Suntzeff, N. B., Schommer, R. A., Maza, J., Smith, R. C., Lira, P., & Aviles, R. 1996, AJ, 112, 2438
Hayden, B. T., et al. 2010, ApJ, 712, 350
Hsiao, E. Y., Conley, A., Howell, D. A., Sullivan, M., Pritchet, C. J., Carlberg, R. G., Nugent, P. E., & Phillips, M. M. 2007, ApJ, 663, 1187
Jha, S., Riess, A. G., & Kirshner, R. P. 2007, ApJ, 659, 122
Kessler, R., et al. 2009, ApJS, 185, 32
Kessler, R., et al. 2010, ApJ, 717, 40
Kowalski, M., et al. 2008, ApJ, 686, 749
Krivonos, K., et al. 2007, AJ, 133, 58
Leibundgut, B. 2000, A&AR, 10, 179
Nobili, S., & Goobar, A. 2008, A&A, 487, 19
Nobili, S., et al. 2005, A&A, 437, 789
Nordin, J., Goobar, A., & Jönsson, J. 2008, J. Cosmol. Astropart. Phys., JCAP02(2008)008
Patat, F. 2005, MNRAS, 357, 1161
Patat, F., Benetti, S., Cappellaro, E., & Turatto, M. 2006, MNRAS, 369, 1949
Pearce, G., & Hayes, A. J. 1986, A&A, 155, 291
Perlmutter, S., et al. 1997, BAAS, 29, 1351
Perlmutter, S., et al. 1999, ApJ, 517, 565
Phillips, M. M. 1993, ApJ, 413, L105
Riess, A. G., et al. 1998, AJ, 116, 1009
Riess, A. G., Press, W. H., & Kirshner, R. P. 1996, ApJ, 473, 88
Riess, A. G., et al. 2007, ApJ, 659, 98
Sollerman, J., et al. 2004, A&A, 428, 555
Strovink, M. 2007, ApJ, 671, 1084
Wang, L. 2005, ApJ, 635, L33
Wang, L., Goldhaber, G., Aldering, G., & Perlmutter, S. 2003, ApJ, 590, 944
Wang, X., et al. 2009, ApJ, 699, L139
Wang, X., et al. 2006, ApJ, 675, 626
Waxman, E., & Draine, B. T. 2000, ApJ, 537, 796
Weingartner, J. C., & Draine, B. T. 2001, ApJ, 548, 296
Wood-Vasey, W. M., et al. 2007, ApJ, 666, 694
Yamanaka, M., et al. 2009, PASJ, 61, 713