Fluctuations in strongly coupled cosmologies

Silvio A. Bonometto\textsuperscript{a,b,c} and Roberto Mainini\textsuperscript{d}

\textsuperscript{a}Department of Physics, Astronomy Unit, Trieste University, Via Tiepolo 11, I 34143 Trieste, Italy
\textsuperscript{b}I.N.F.N., Sezione di Trieste, Via Valerio, 2 I 34127 Trieste, Italy
\textsuperscript{c}I.N.A.F., Astronomical Observatory of Trieste, Via Tiepolo 11, I 34143 Trieste, Italy
\textsuperscript{d}Department of Physics G. Occhialini, Milano-Bicocca University, Piazza della Scienza 3, I 20126 Milano, Italy

E-mail: bonometto@oats.inaf.it, mainini@mib.infn.it

Received December 3, 2013
Revised January 21, 2014
Accepted January 29, 2014
Published March 18, 2014

Abstract. In the early Universe, a dual component made of coupled CDM and a scalar field $\Phi$, if their coupling $\beta > \sqrt{3}/2$, owns an attractor solution, making them a stationary fraction of cosmic energy during the radiation dominated era. Along the attractor, both such components expand $\propto a^{-4}$ and have early density parameters $\Omega_d = 1/(4\beta^2)$ and $\Omega_c = 2\Omega_d$ (field and CDM, respectively). In a previous paper it was shown that, if a further component, expanding $\propto a^{-3}$, breaks such stationary expansion at $z \sim 3-5 \times 10^3$, cosmic components gradually acquire densities consistent with observations. This paper, first of all, considers the case that this component is warm. However, its main topic is the analysis of fluctuation evolution: out of horizon modes are then determined; their entry into horizon is numerically evaluated as well as the dependence of Meszaros effect on the coupling $\beta$; finally, we compute: (i) transfer function and linear spectral function; (ii) CMB $C_l$ spectra. Both are close to standard $\Lambda$CDM models; in particular, the former one can be so down to a scale smaller than Milky Way, in spite of its main DM component being made of particles of mass $< 1$ keV. The previously coupled CDM component, whose present density parameter is $\mathcal{O}(\infty/r^3)$, exhibits wider fluctuations $\delta\rho/\rho$, but approximately $\beta$-independent $\delta\rho$ values. We discuss how lower scale features of these cosmologies might ease quite a few problems that $\Lambda$CDM does not easily solve.

Keywords: dark energy theory, dark matter theory, cosmological parameters from CMBR

ArXiv ePrint: 1311.6374v1
1 Introduction

The cosmic dark components (Dark Matter: DM, and Dark Energy: DE) will be probably considered the main physical discoveries in the last decades. Understanding their nature and properties is one of the main tasks of contemporary astrophysical research. A number of options have been considered and here we shall assume DE to be a scalar field $\Phi$, interacting with DM, whose lagrangian includes a potential $V(\Phi)$, which could also be a simple mass term. The hope to detect its shape through the determination of the DE state equation $w(a)$ is remote ($a$: scale factor). The very experiment EUCLID\(^1\) \[1\] is expected to estimate the $w(a)$ derivative at $z = 0$ with an error $\sim 20\%$ \[2\]. Accordingly, although assuming DE to be a scalar field, no explicit $V(\Phi)$ expression will be taken here, rather focusing on parameters better constrained by observations.

The rational of allowing for DM-DE interactions is to insure DE a fresh energy inflow, so keeping it a significant fraction of DM density, at any redshift \[3–6\]. A fairly large DM-DE interaction scale

$$C = b/m_p = \sqrt{16\pi/3} \beta/m_p,$$

allowing DE density to steadily keep $\sim 1\%$ of CDM (Cold Dark Matter), is consistent with data: namely, if neutrinos have a mass, $\beta$ values up to $\sim 0.15–0.2$ \[7–11\] agree with observations.

In this work we shall however debate cosmologies where $C$ is still greater, allowing DE density to keep a steady fraction of radiation through radiative eras. In a previous paper (\[12\], paper I here below) this class of cosmologies was shown to be consistent with data, at background level.

Our main task, here, is to study fluctuations in these models. Out of horizon fluctuation modes are determined. The entry of fluctuations in the horizon is then numerically

\(^1\)http://www.euclid-ec.org.
discussed. In particular we compare Meszaros effect in these cosmologies with standard ΛCDM cosmologies, and, finally, work out the transfer function and the $C_l$ spectra for these cosmologies.

A fit of these cosmologies with observational data is beyond the scopes of this work. We however ascertain that discrepancies with ΛCDM, far from creating conflicts with data, could be rather exploited to ease some problems still open in ΛCDM cosmologies.

In paper I we showed that, in strongly coupled (S.C.) cosmologies, the early expansion is characterized by the presence of a dual component, made of CDM and a scalar field $\Phi$ exchanging energy, aside of ordinary radiation ($\gamma$’s & $\nu$’s: photons & neutrinos). CDM and $\Phi$ have constant early density parameters

$$
\Omega_c = \frac{1}{2\beta^2}, \quad \Omega_d = \frac{1}{4\beta^2},
$$

(1.2)

provided that $\beta$ is constant.

Quite in general, if we allow for an energy leakage from CDM to the $\Phi$ field, CDM dilution occurs faster than $\propto a^{-3}$. In turn, being almost kinetic, the field would dilute $\propto a^{-6}$, unless continuously fed fresh energy, so that its dilution becomes less frenetic. The point is that, if the energy leakage becomes so strong to yield a CDM dilution $\propto a^{-4}$, this is exactly what is needed to allow the field to dilute at the same rate, provided that CDM and $\Phi$ density parameters are those in eq. (1.2). The radiation density parameter is then $\Omega_r = 1 - 3/4\beta^2$.

Furthermore, as shown in Paper I, this is a tracker regime: starting from any initial condition, densities and dilution rates rapidly settle on the regime (1.2), provided that

$$
\beta > \sqrt{3}/2,
$$

(1.3)

and this cosmic tripartition lasts forever, unless a non-relativistic uncoupled component acquires a significant density. Let us outline that it could also be such since ever, e.g. since the end of the inflationary expansion.

As a matter of fact, in the real world baryons exist, at least, whose density dilutes $\propto a^{-3}$, and will eventually reach the radiation level. However, if only baryons are added, as non-relativistic component, the above picture fails to meet observational features. They are however met if a further DM component exist. In principle, it could be another uncoupled CDM component or warm DM (WDM), becoming non-relativistic (slightly) before a standard equality redshift.

The rise of such non-relativistic component also causes a rise of CDM and $\Phi$ densities in respect to radiation. However, until $\Phi$ keeps kinetic, its density still declines more rapidly than $a^{-3}$. If the progressive rise of the $\Phi$ field is however such to cause its transition from kinetic to potential, just a little after WDM derelativized, the background component densities easily meet the observational proportions.

Altogether, the basic parameter to be adjusted to obtain this result, is the epoch when, in the energy density of the field

$$
\rho_d = \frac{\dot{\Phi}^2}{2a^2} + V(\Phi)
$$

(1.4)

the latter term will begin to prevail on the former one. This expression assumes a metric

$$
ds^2 = a^2(\tau)(d\tau^2 - dt^2)
$$

(1.5)
with $\tau$ being the conformal time. Let us also remind that the $\Phi$ pressure then reads

$$p_d = \frac{\dot{\Phi}^2}{2a^2} - V(\Phi)$$

(1.6)

and its state parameter $w(a) = p_d/\rho_d$.

Let us notice that any coupled-DE theory (apart some peculiar low-$\beta$ cases, when the self-interaction potential is so strong to make the coupling almost negligible) requires $\Phi$ to be initially kinetic, then turning to potential at a suitably tuned redshift. In the literature, this transition has been studied by using different expressions of the potential $V(\Phi)$, e.g. Ratra-Peebles [13] or SUGRA [14–16] expressions. Although the detailed evolution of the DE state parameter $w(a)$, during the transition, exhibits some potential dependence, the epoch when the transition takes place is largely independent from the potential shape.

Let us also notice that, for coupled-DE theories, any preference granted to tracker potentials is unjustified. As a matter of fact, initial conditions however assume that the field is purely kinetic, its value being therefore independent from $V(\Phi)$. The parameters in the potential are then tuned to enable a kinetic-potential transition at a suitable epoch. This could equally be done with any potential expression, e.g. by taking $V(\Phi) = m^2\Phi^2$ or a polinomial including higher powers of $\Phi$.

Accordingly, we keep here to the approach of paper I, just assuming a parametric expression fixing the shape for the $w(a)$ transition from +1 (at early times) to -1 (close to $z = 0$). The expression depends from a single parameter ($\epsilon$), and the effects of varying $\epsilon$ mimic changes in the $V(\Phi)$ expression. The expression chosen here to follow the transition is slightly different from Paper I, for a specific reason debated in the sequel.

Before concluding this section let us finally remind that the cosmic components, in S.C. cosmologies, are the standard ones, apart of DM which is twofold: early DM is coupled, while an uncoupled component could be, e.g., WDM.

Also the plan of the paper is essentially twofold. A former part is devoted to deepening the background picture, taking WDM as uncoupled DM component. As a matter of fact, this class of cosmologies appears far more appealing if such upgrade is made. But, as previously outlined, detailed fits with data are delayed to further work. The latter part of the paper is then devoted to the study of density fluctuation evolution. This will be done by using two numerical programs: (i) A simple program, solving a set of 11 coupled differential equations, will enable us to follow closely the physical features of these models. (ii) We shall then present results obtained by using a suitably modified version of CMBFAST.

2 CDM-DE coupling

Coupled CDM and DE were considered by several authors [3–9, 17–25]. The point is that the stress-energy tensors of CDM and DE must fulfill the pseudo-conservation equation

$$T^{\epsilon}_{c\mu\nu} + T^{\epsilon}_{d\mu\nu} = 0 ,$$

(2.1)

but there is no direct evidence that (for CDM) $T^{\epsilon}_{c\mu\nu} = 0$ and/or (for DE) $T^{\epsilon}_{d\mu\nu} = 0$, separately. Following Paper I, here we assume that

$$T^{\epsilon}_{c\mu\nu} = -CT^{\epsilon}_{c\Phi\mu} , \quad T^{\epsilon}_{d\mu\nu} = CT^{\epsilon}_{c\Phi\mu}$$

(2.2)
which is not the only possible option (see, e.g., [19]), but is the one considered first, aiming to ease the paradox of DE being significant only in the present epoch. In a FRW frame, the equations (2.2) yield

\[ \dot{\Phi} + \frac{\ddot{a}}{a} \Phi = \frac{1 + w}{2} C \rho_c a^2, \quad \ddot{\rho}_c + 3 \frac{\dot{a}}{a} \dot{\rho}_c = -C \dot{\Phi} \rho_c. \]  

(2.3)

Here

\[ \Phi = \dot{\Phi}, \quad 2 \ddot{w} = 1 + 3w - d \log(1 + w)/d \log a, \]  

(2.4)

and this formulation is equivalent, e.g., to [3–6], if \( w(a) \) (state equation of DE) is assigned, instead of the self-interaction potential of the field \( V(\Phi) \). To obtain eq. (2.3), as in Paper I, we used the expression

\[ (1 + w) a^2 V'(\Phi) = \dot{\Phi}(1 - w) - 2 \frac{\dot{a}}{a} \Phi (1 + w - \ddot{w}) \]  

(2.5)

for the potential derivative.

The latter eq. (2.3) has then the formal integral

\[ \rho_c = \rho_{i,c} \left( \frac{a_i}{a} \right)^3 \exp \left( -C \int_{\tau_i}^{\tau} d\tau \Phi \right), \]  

(2.6)

\( \tau_i \) being a reference time when CDM density is \( \rho_{i,c} \) and the scale factor is \( a_i = a(\tau_i) \). If this expression for \( \rho_c \) is then replaced in the former eq. (2.3), we obtain a transcendental differential equation, quite hard to integrate. In Paper I, we however outlined that, if we make the ansatz

\[ \Phi_1 = \alpha m_p / \tau, \]  

(2.7)

it is \( -C \int_{\tau_i}^{\tau} d\tau \Phi_1 = \ln (\tau_i / \tau)^{ab} \), so that \( \rho_c = \rho_{i,c} (a_i / a)^{3 + ab} \) and, by replacing the expression (2.7) in the former eq. (2.3), we obtain

\[ (\ddot{w} - 1) \alpha \frac{m_p}{a^2 \tau^2} = \frac{1 + w}{2} \frac{b}{m_p} \rho_{i,c} \left( \frac{a_i}{a} \right)^{3 + ab}, \]  

(2.8)

an equation surely useful if \( w \) is constant. Then, in order that the two sides scale in the same way, it must be \( \alpha b = 1 \), i.e. \( \rho_c \propto a^{-3} \); \( \rho_c \) dilutes faster than \( \propto a^{-3} \) because of the leakage of energy onto the \( \Phi \) field; that it dilutes exactly as \( a^{-4} \), instead, is a consequence of the ansatz (2.7). From eq. (2.8) and the Friedmann equation in the radiative era

\[ \frac{8\pi}{3m_p^2} \rho a^2 \tau^2 = 1, \]  

(2.9)

\( \rho : \) total density) we then derive

\[ \frac{1}{\beta^2} \frac{\ddot{w} - 1}{w + 1} = \frac{1}{2\beta^2} \frac{3w - 1}{w + 1} = \frac{8\pi}{3m_p^2} \rho_c a^2 \tau^2 \equiv \Omega_c. \]  

(10.20)

Here \( \Omega_c = \rho_c/\rho \) is the (constant) density parameter of DM (during the radiative era). In order that the ansatz (2.7) is allowed, \( \Omega_c \) ought to have the value given by this equation.

It is then easy to show that the \( \Phi \) field energy density is

\[ \rho_d = \frac{\alpha^2 m_p^2}{a^2 \tau^2} \frac{1}{1 + w}, \]  

(2.11)
so that its constant density parameter reads

\[ \Omega_d = \frac{1}{2\beta^2(1 + w)} \]  

(2.12)

showing also that

\[ \Omega_c/\Omega_d = 3w - 1 . \]  

(2.13)

Quite in general, being

\[ w = \Phi_1/2a^2 - V(\phi) \Phi_1/2a^2 + V(\phi) , \]  

(2.14)

we see that, in general, being

\[ w = 1/(2\beta^2) , \quad \Omega_d = 1/(4\beta^2) , \quad (\Omega_c + \Omega_d)\beta^2 = 3/4 . \]  

(2.15)

In the early Universe the total density parameter is unity. Requiring then \( \Omega_c + \Omega_d < 1 \) yields

\[ \beta > \sqrt{3}/2 = 0.866 . \]  

(2.16)

In Paper I we also tested that this solution of eq. (2.3) is an attractor: starting from generic initial conditions, \( \Phi \) and \( \rho_c \) rapidly evolve to approach a regime where eqs. (2.15) are satisfied.

We denominate strongly coupled cosmologies those with \( \beta > 0.866 \), allowing \( \Omega_c \) and \( \Omega_d \) to be constant in the radiative era. On the contrary, when \( \beta^2 < 3/4 \) there is no solution with constant \( \Omega_{c,d} \); the CDM and \( \Phi \) field contributions to the overall density decrease when \( a \) tends to zero.

### 3 Early expansion end

The regime described in the previous section could be present since ever, e.g. since inflation, and would last forever unless a non-relativistic component grows to overcome radiative ones.

At variance from Paper I, in this work we shall mostly assume that this component is warm. Its energy density and pressure read

\[ \rho_w = \frac{T_w^4}{\pi^2} \int_0^\infty dx \frac{x^2 \sqrt{x^2 + (m_w/T_w)^2}}{e^x + 1} , \quad p_w = \frac{T_w^4}{\pi^2} \int_0^\infty dx \frac{x^4}{\sqrt{x^2 + (m_w/T_w)^2} e^x + 1} \]  

(3.1)

and derelativization occurs when the WDM temperature \( T_w \) shifts below the mass \( m_w \) of WDM quanta. Then we gradually achieve the regime \( \rho_w \propto T_w^2 \) and WDM density overcomes the radiative components. A little later, also baryons will do so.

This is the first stage forging the present sharing of densities among cosmic components. It is also critical to establish when the \( \Phi \) field passes from the kinetic to the potential regime. In most previous work this stage was followed by using an expression of its self-interaction potential \( V(\Phi) \). The potential was often selected so to allow tracker solutions.
Figure 1. Evolution of background components in cosmologies with coupled CDM and uncoupled WDM (spinor thermal particles with 2 spin states). In both cases we took $\epsilon = 2.9$ and, at $z = 0$, $\Omega_d = 0.7$, $h = 0.68$. Values of $\beta$ and $z_d$ shown in the frames; the latter is obtained by suitably tuning the transition from kinetic to potential regime of the $\Phi$ field. Densities are given in MeV$^4$. For $\beta = 2.5$ the contribution of the coupled components to the early density is just below 1 extra massless neutrino species. For $\beta = 20$, when WDM is relativistic and DE is kinetic, $\Omega_w \simeq \Omega_d$, $\Omega_c$.

Figure 2. Transition from $w = +1$ to $w = -1$ of DE state equation. We assume $\Omega_d = 0.7$ at $z = 0$ and consider the case $\beta = 20$. At the r.h.s. we show the $w(z)$ dependence for 3 $\epsilon$ values. At the r.h.s. the values of $z_d$ yielding $\Omega_d = 0.7$ are plotted vs. $\epsilon$.

Determining the shape of $V(\Phi)$ from observational data is however (almost) hopeless. We find that the critical feature is rather the redshift $z_d$ (scale factor $a_d = (1 + z_d)^{-1}$) when the kinetic-potential transition takes place. Through this paper we shall assume that

$$w = \frac{1 - A}{1 + A} \quad \text{with} \quad A = \left( \frac{a}{a_d} \right)^\epsilon \quad (3.2)$$
so that \( w \to +1 \) \((-1)\) for \( a \to 0 \) \((a_0 = 1)\), and \( w(a_d) = 0 \). The parameter \( \epsilon \), rather than a peculiar shape of \( V(\Phi) \), then fixes the sharpness of the transition. In most of this work we assume \( \epsilon = 2.9 \). The above expression improves the one used in Paper I, for having continuous first and second derivatives. With this \( w \), in particular,

\[
\tilde{w} = \frac{4 + (\epsilon - 2)A}{2(1 + A)} \tag{3.3}
\]

The present density of DE is essentially fixed by \( a_d \), with a milder dependence on \( \epsilon \).

In figure 1 we then show the density evolution of all cosmic components for 2 values of \( \beta \): (i) \( \beta = 2.5 \) is slightly above the value \( \beta = 2.19 \) given by

\[
\beta^2 = \frac{3}{4} \left[ 2 + \frac{8}{7} \left( \frac{11}{4} \right)^{4/3} \right], \tag{3.4}
\]

which may be considered a phenomenological lower limit, as it yields a coupled DE-CDM component whose density approaches one extra neutrino species (the contribution to the background density due to WDM is disregarded). (ii) \( \beta = 20 \), instead, is close to the coupling strength yielding a coupled DE component with the same early density of WDM.

Let us notice that WDM derelativization occurs at a redshift \( z_{der} \propto m_w \) so that, if we assume a fixed low-z density parameter for WDM, the early \( \Omega_w \propto m_w^{-1} \), while the early \( \Omega_d \propto \beta^{-2} \). Accordingly, an approximate coincidence \( \Omega_d \sim \Omega_w \) can be maintained only if the WDM particle mass \( m_w \propto \beta^2 \).

Figures 2 then illustrate the dependence of results on the choice of \( \epsilon \), which is quite mild for \( \epsilon > 2.5 \). The range of \( \epsilon \) can be interpreted as the possible dependence of results on the shape potential \( V(\Phi) \).

4 Perturbations

Let us now consider the evolution of small perturbations to this background in a synchronous gauge. Quite in general, the metric reads

\[
ds^2 = a^2(\tau) \left[ d\tau^2 - (\delta_{ij} + h_{ij}) dx^i dx^j \right] ; \tag{4.1}\]

scalar metric perturbation can then be expanded as follows [27]:

\[
h_{ij}(\tau, \mathbf{x}) = \int d^3 k \ e^{i k \cdot x} \left[ n_i n_j h(\tau, \mathbf{k}) + (n_i n_j - \delta_{ij}/3) 6 \eta(\tau, \mathbf{k}) \right] \tag{4.2}
\]

with \( \mathbf{k} = n \mathbf{k} \). Einstein equations then yield

\[
\ddot{h} + \frac{\dot{a}}{a} \dot{h} = -\frac{8\pi}{m_p^2} a^2 (\delta \rho + 3\delta p) . \tag{4.3}
\]

The gravity sources to be included in the term \( \delta \rho + 3\delta p \) are: (i) radiation, for which \( \delta \rho + 3\delta p = 2\rho_r \delta \rho \); (ii) baryons, for which \( \delta \rho + 3\delta p = \rho_b \delta \rho \); (iii) uncoupled CDM or WDM, for which \( \delta \rho + 3\delta p = c_w \rho_w \delta \rho \) \((c_w = 1 \text{ in the CDM case}; \text{ in the WDM case } c_w = 2 \text{ until it is ultrarelativistic})\); derelativization will then be followed by sharing WDM energy spectrum in
a suitable number of components, chosen to allow Gauss-Laguerre momentum integration; (iv) coupled CDM, also yielding \( \delta \rho + 3 \delta p = \rho_c \delta_c \); and, finally, (v) the DE field \( \Phi \), for which

\[
\delta(\rho_\Phi + 3p_\Phi) = \delta \left[ 4 \frac{\Phi^2}{2a^2} - 2V(\Phi) \right] = 4 \frac{\Phi_1 \Phi_1}{a^2} - 2V'(\Phi)\phi . \tag{4.4}
\]

\( \Phi_1 \) being the background field. In principle it is then \( \Phi_1(1) \) with \( \Phi(1) \) accounting for \( \Phi \) fluctuations. However, here below, scalar field fluctuations will be mostly described by the dimensionless variable \( \varphi = (b/m_p)\phi \) and its derivative \( \dot{\varphi} = (b/m_p)\dot{\phi} \); let us remind that \( b = (16\pi/3)^{1/2} \beta \), according to eq. (1.1). Furthermore the bar in top of \( \Phi \) is omitted and the background field is simply \( \Phi(1) \). From eq. (4.3), for gravity fluctuations we then obtain

\[
\ddot{\mathcal{h}} + \frac{\dot{a}}{a} \left( \dot{h} + \frac{6}{\beta^2} D \dot{\varphi} \right) = -\frac{8\pi}{m_p^2}a^2 (2\rho_r \delta_r + \rho_b \delta_b + c_w \rho_w \delta_w + \rho_c \delta_c) + \sqrt{\frac{16\pi}{3}} a^2 V'(\Phi) m_p^3 \varphi \tag{4.5}
\]

with \( D\dot{a}/a = \Phi_1 b/m_p \) and, when in the kinetic regime, the last term at the r.h.s. can be simply omitted; otherwise, we can use the expression (2.5) for \( V'(\Phi) \).

The equations of motion of the cosmic components will then be written by neglecting massless neutrinos, unessential to understand the dynamics of the model. Let us soon outline, however, that final results obtained though a suitable modification of CMBFAST take into account 3 standard massless neutrino species. The equation of motion will be written for the case when uncoupled DM is cold, and read:

\[
\dot{\delta}_r = -\frac{2}{3} \frac{2}{3} k \varphi , \quad \dot{\nu}_r = \frac{1}{4} k \delta_r , \quad \dot{\delta}_w = -\frac{1}{2} \ddot{h} , \quad \dot{\nu}_c = -\frac{\dot{a}}{a} (1 - D) v_c - k \varphi , \tag{4.6}
\]

\[
\ddot{\varphi} + 2\frac{\dot{a}}{a} \dot{\varphi} + \frac{1}{2} \Phi_1 \ddot{h} + k^2 \varphi + a^2 V''(\Phi) \varphi = 2\beta^2 \left( \frac{\dot{a}}{a} \right)^2 \Omega_c \delta_c . \tag{4.7}
\]

In order to obtain equation (4.7) we exploited the fact that, thanks to the Friedmann equation, (\( b/m_p^2 \)) \( a^2 \rho_r = 2\beta (8\pi/3m_p^2) \) \( a^2 \rho \Omega_c = 2\beta^2 (\dot{a}/a)^2 \Omega_c \). The first two equations (4.6) refer to radiation; here they assume baryons to be tightly bound to photons, an assumption surely reliable, over most significant \( k \) scales, at least up to matter-radiation equality. From these equations we easily work out

\[
\ddot{\delta}_r + \frac{1}{3} k^2 \delta_r = -\frac{2}{3} \ddot{h} , \tag{4.8}
\]

as is expected, owing to the neutrino neglect. When gravitation is negligible, therefore, we expect harmonic oscillations in the photon-baryon fluid, with period \( P = 2\pi\sqrt{3}/k \) in respect to conformal time.

The equation of motion for \( \delta_w \) assumes an uncoupled cold component. On the contrary, WDM fluctuations cannot be described by a single function \( \delta_w, \) needing suitable expansions in respect to particle momenta and spherical harmonics. In the final quantitative analysis, we shall keep to the standard treatment (see, e.g., \([26, 27]\)). Let us just notice that, for non-vanishing components, in the initial conditions (1/2)\( \ddot{h} \) shall then be replaced by (2/3)\( \ddot{h} \).

Finally, notice also that, in the kinetic regime, the problematic \( V'' \) term can be omitted from the \( \varphi \) field equation, on the last line. We shall return on this point in section 7.
5 Out-of-horizon solutions

Before the entry in the horizon, also the term \( k^2 \varphi, kv_c \) and \( kv_r \) can be disregarded in the field and the former CDM and radiation equations, while the latter CDM and radiation equations can be disregarded. Let us then try to solve the system by making the following ansatz:

\[
\begin{align*}
    h &= A \tau^x, \\
    \delta_r &= R \tau^r, \\
    \delta_w &= W \tau^y, \\
    \delta_c &= M \tau^c, \\
    \varphi &= F \tau^f 
\end{align*}
\]  

(5.1)

From the eqs. (4.6) we obtain

\[
\begin{align*}
    r R \tau^{-1} &= \frac{2}{3} x A \tau^{-1}, \\
    y W \tau^{-1} &= -\frac{1}{2} x A \tau^{-1}, \\
    c M \tau^{-1} &= -\frac{1}{2} x A \tau^{-1} - f F \tau^{-1}, \\
    x^2 A + 3 R &= 0, \\
    A + 2 W &= 0, \\
    A + 2 M + 2 F &= 0, \\
    x A - 4 \beta^2 \Omega_c M &= 2 x(x + 1) F = 0.
\end{align*}
\]  

(5.2)

(5.3)

(5.4)

It is easy to see that eqs. (5.2), (5.3), (5.4) require that

\[
    x = f = r = y = c 
\]  

(5.5)

eqs. (5.2), (5.3) also require that

\[
\begin{align*}
    2 A + 3 R &= 0, \\
    A + 2 W &= 0, \\
    A + 2 M + 2 F &= 0, \\
    x A - 4 \beta^2 \Omega_c M &= 2 x(x + 1) F = 0.
\end{align*}
\]  

(5.6)

while eq. (5.4) yields

\[
x^2 A + 6 \Omega_r R + 3 \Omega_w W + 3 \Omega_c M + (6 x / \beta^2) F = 0. 
\]  

(5.7)

The first two eqs. (5.6) allow us to obtain \( R \) and \( W \) in terms of \( A \). We use them in eq. (5.7), taking also into account eqs. (2.15) and assuming \( \Omega_w \) to be negligible. The system (5.6), (5.7) then yields

\[
\begin{align*}
    A(x^2 - 4 + 3 / \beta^2) + F 6 x / \beta^2 + M 3 / 2 \beta^2 &= 0 \\
    A x / 2 + F x(x + 1) - M &= 0, \\
    A + 2 F + 2 M &= 0,
\end{align*}
\]

and non-vanishing solutions exist only if we fulfill the dispersion relation

\[
\begin{vmatrix}
    x^2 - 4 + 3 / \beta^2 & 6 x / \beta^2 & 3 / 2 \beta^2 \\
    x / 2 & x(x + 1) & -1 \\
    1 & 2 & 2
\end{vmatrix} = 0
\]  

(5.8)

easily reordered to obtain

\[
(x^2 - 4)[2(x^2 + x + 1) - 3 / 2 \beta^2] = 0. 
\]  

(5.9)
There are then two categories of out-of-horizon fluctuation modes:

- modes (a): $x = \pm 2$, being $\beta$ independent;
- modes (b): $x = (1/2)[-1 \pm 3^{1/2}(1/\beta^2 - 1)^{1/2}]$.

In the case of a cold uncoupled DM component, for all modes it is

$$ R = -(2/3)A, \quad W = (-1/2)A, \quad (5.10) $$

also implying $R = (4/3)W$, as expected. If, instead, the uncoupled DM component is warm and still relativistic when I.C. are built, it shall be

$$ W = R = -(2/3)A. \quad (5.11) $$

In the case of the increasing (a) mode, we have

$$ M = -(3/14)A, \quad F = -(2/7)A, \quad (5.12) $$

while, by comparing these equations with eqs. (5.10) or (5.11) we see that

$$ M = (3/7)W \quad (5.13) $$

(in the case of relativistic WDM, the coefficient $3/7$ should be replaced by $4/7$). Coupled-CDM fluctuations, therefore, are approximately half of uncoupled-DM. In coupled DE models with $\beta < \sqrt{3}/2$, coupled-CDM fluctuations are enhanced, in respect to uncoupled components, by a $\beta$ dependent factor. Here we find an opposite, $\beta$-independent behavior.

The (a) mode with $x = -2$ is clearly decreasing. (b) modes yield a real $x$ only in the interval $\sqrt{3}/2 < \beta < 1$. In this interval, the greatest possible $x$ is 0 and is found at the limit $\beta = \sqrt{3}/2$. On the contrary, for $\beta > 1$ we find complex $x$ values, which can be combined to yield

$$ (\tau/\tau_i)^x = (\tau/\tau_i)^{-1/2} \{A \cos[Q \ln(\tau/\tau_i)] + B \sin[Q \ln(\tau/\tau_i)]\} \quad \text{with} \quad Q = [3(1 - 1/\beta^2)]^{1/2}, \quad (5.14) $$

$A, B$ being arbitrary constants and $\tau_i$ a reference time. They are however decreasing solutions, comprising an oscillatory behavior whose physical meaning is hard to realize.

Therefore, out of horizon initial conditions can be set by assuming a growth $\propto \tau^2$, with fluctuation amplitudes ruled by eqs. (5.10), (5.12), and independent from $\beta$ ($> \sqrt{3}/2$) value. This is quite alike standard models in the radiation dominated expansion stages.

6 Pre- and post-recombination evolution: a semi-qualitative approach

Initial conditions can then be applied to the system (4.5)–(4.7). It is then convenient to define the variables

$$ K(\tau) = 2\beta^2\Omega_c \quad \text{and} \quad D(\tau)\frac{\dot{a}}{a} = \Phi_1 \frac{b}{m_p} \quad (6.1) $$

which, while the initial (pseudo-)stationary regime persists, are $K = D = 1$.

By using them and excluding the terms containing the potential $V$, eqs. (4.5) and (4.7) read

$$ \ddot{h} + \frac{a}{h} \left(\dot{h} + \frac{6}{\beta^2} D\dot{\varphi}\right) + \left(\frac{\dot{a}}{a}\right)^2 \left[\left(1 - \Omega_w - \frac{K}{2\beta^2}\right) 6\delta_r + 3\Omega_w c_w \delta_w + 3\frac{K}{2\beta^2} \delta_c\right] = 0, \quad (6.2) $$
Figure 3. Fluctuation evolution when uncoupled DM is cold; the moduli $|\delta|$, $|\dot{h}|$, and $|\varphi|$ are plotted for 2 values of $\beta$ and two scales $k$. All plots show that coupled DM exhibits almost no Meszaros' effect: its fluctuation continue to grow, quite rapidly, after the entry in the horizon, also when radiation density widely exceeds its density. Such growth has indirect effects also on uncoupled DM evolution, which feel the increasing gravity of coupled DM. Indirect effects however weaken for greater $\beta$, as coupled DM density $\propto \beta^{-2}$. In all plots the present $\Omega_d = 0.7$.

\[
\ddot{\varphi} + 2\frac{\dot{a}}{a}\left(\dot{\varphi} + \frac{D}{4}\dot{h}\right) + k^2\varphi = \left(\frac{\dot{a}}{a}\right)^2 K\delta_c .
\]  

(6.3)

Here, the coefficient $c_w$ depends on the state equation of uncoupled DM. The radiation and uncoupled DM eqs. are unchanged.

These equations allow us to study fluctuation evolution at any $z$, also when $D, K \neq 1$, until the kinetic-potential transition of DE. They allow also for a component breaking the inzihal (pseudo-)stationarity, provided we assume it to be cold or that we deal with a scale $k$ where WDM free streaming is absent.

We shall now briefly discuss the physical behavior of radiation, baryons, uncoupled and coupled DM, DE and gravity, by using an 11 component system of linear differential
equations whose variables are:

for the background: $a, \rho_c, \Phi_1 \equiv \dot{\Phi}$;
for density fluctuations: $\delta_r, v_r, \delta_w, \delta_c, v_c, \varphi, \dot{\varphi}, \dot{h}$

The radiation-baryon component will be treated as a fluid with state parameter $w_R \equiv 1/3$ until $z = 1100$. Afterwards, we neglect radiation, assuming that fully decoupled baryon fluctuations $\delta_b = (3/4)\delta_r$ obey a pressureless equation. This option inhibits predictions on CMB fluctuations, but allows us a few tests on the pre- and post-recombination evolution, described in the next section.

To go beyond this approximate post-recombination treatment, as well as in the quantitative treatment, we need to reintroduce the potential terms and, namely, an expression to replace the term containing $V''(\Phi)$ when we focus on the $w(a)$ behavior (approximated by the expression (3.2)), tentatively disregarding the hardly detectable shape of $V(\Phi)$.

The final part of this section will be devoted to elaborating this expression. Let us first comment on the results of the simplified dynamical system as shown in figure 3.

All plots show that coupled-DM exhibits almost no Meszaros’ effect [28]: on all mass scales below that entering the horizon at matter-radiation equality, fluctuation amplitudes are almost frozen until matter exceeds radiation density. Let us remind that Meszaros’ effect is critical in shaping the transfer functions.

As a matter of fact, radiation fluctuations, after entering the horizon, turn into sonic waves, so that their average amplitude vanishes. When the dominant cosmic component is no gravity source, other components can cause just a modest push.

The freezing period is unavoidably longer for smaller scales, spending more time below the horizon scale before matter-radiation equality, so that the transfer function decreases at increasing $k$ values. Of course, in top of this basic ingredient a number of other effects play a suitable role. Baryons, neutrinos or other specific component add specific details on the above basic structure.

The reason why coupled-DM fluctuation continue to grow, quite rapidly, after the entry in the horizon, is visible in the 4-th eq. (4.6). The gravitational push set by $\dot{h}/2$ is there increased by $\dot{\phi}$, i.e., there is an additional force acting just between CDM particles. Accordingly, coupled CDM fluctuations are a sufficient source to cause their own growth.

Such growth has indirect effects also on uncoupled DM and baryon evolution: CDM particles act on them just through ordinary gravity, but both of them feel the increasing gravity of wider coupled DM fluctuations. If we compare different plots, we however see that indirect effects however weaken for greater $\beta$, as coupled the DM density $\rho_c \propto \beta^{-2}$.

Before concluding this section, let us focus on the expression to be used in place of $V''$, when the potential is unknown, but $w(a)$ is given. Let us then consider eq. (2.3), derived in Paper I, in association with the equation used there to eliminate the $V'$ term from it. The two equations, reading

$$-a^2 V' = \ddot{\Phi} + 2\frac{\dot{a}}{a} \dot{\Phi} - C \rho_c a^2, \quad (6.4)$$
$$1 + w a^2 V' = \ddot{\Phi} - \frac{\dot{a}}{a} \dot{\Phi} - \frac{1}{1 - w^2} \frac{dw}{da} \dot{a} \dot{\Phi}, \quad (6.5)$$

can be subtracted to obtain an expression of $V'$ not including $\ddot{\Phi}$ (this will prevent the need
of considering triple derivatives of $\Phi$). In this way we obtain

$$2V' = -\left[ \frac{a}{(1+w)} \frac{dw}{da} + 3(1-w) \right] \frac{\dot{a}}{a^3} \dot{\Phi} + (1-w)C_{p_c}. \quad (6.6)$$

$V'$ can then be derived in respect to $\tau$ and divided by $\dot{\Phi}$, so obtaining $V''(\Phi)$. The general expression is however cumbersome and useless; it is rather convenient to replace soon the expression (3.2) in eq. (6.6). This yields

$$2V' = \frac{A}{1+A} \left[ \epsilon_6 \frac{\dot{a}}{a^3} \Phi + 2C_{p_c} \right] \quad (6.7)$$

with $\epsilon_6 = \epsilon - 6$. It will then be

$$2V'' = \frac{A}{1+A} \left\{ \frac{\dot{a}}{a} \frac{\epsilon}{1+A} \left[ \epsilon_6 \frac{\dot{a}}{a^3} + 2C_{p_c} \right] + \frac{\dot{a}}{a^3} \Phi + \frac{d}{d\tau} \frac{\dot{a}}{a^3} \right\} \epsilon_6 + 2C \frac{\dot{b}_c}{\Phi} \quad (6.8)$$

and this equation is soon suitable to numerical evaluations; in fact, it contains variables any numerical algorithm however needs to evaluate, apart of

$$\frac{d}{d\tau} \left( \frac{\dot{a}}{a^3} \right) = -\frac{4\pi}{3 m_p^2} (5\rho + 3p) \quad (6.9)$$

which requires an explicit expression of the pressures $p$ of all cosmic components.

We shall however further discuss this point after giving some numerical results. In figure 3 we show the expected evolution of fluctuations for all components, under the above assumptions. Here also a case with a low $\beta$ value is considered. Low $\beta$’s weaken the Meszaros effect and cause a smaller binding of the transferred spectra.

7 Quantitative results

The simplified 11-eqs. treatment is effective to understand the basic physical effects, but unsuitable to predicting CMB fluctuation spectra, treating the case when the uncoupled DM component is warm, and to allow us a detailed comparison with other models.

In order to achieve such aims, we suitably corrected the public program CMBFAST. The program allows for both massless and massive neutrinos, and the latter facility can soon be used to follow WDM fluctuations; CDM and DE equations are however to be widely modified, starting from initial conditions, both for fluctuations and background components.

In figure 4 and 5 we give two examples of transfer functions. In both of them the uncoupled DM is warm, being made of particles with mass $m_w = 218.6$ or $437.2$ eV (corresponding to $g^* = 400$ or $800$ for 2 spin states; $g^*$ is the assumed number of effective spin states at WDM hot decoupling), respectively. The coupling is also different, being $\beta = 20$ and 30, respectively. The simultaneous shift of $m_w$ and $\beta$ aims to provide similar early density parameters for WDM and the coupled components, in both cases. There is no cogent reason to follow such prescription, which just alludes to a possible correlated origin for all dark components, however characterized by different spin and different numbers of spin states.

The transfer functions of strongly coupled cosmologies are widely different for the coupled CDM and the other components (WDM and baryons). The difference starts at a scale $k \sim 10^{-3}$, the scale entering the horizon at matter-radiation equality. For any greater $k$,
coupled CDM fluctuations, growing between horizon entry and equality, gradually become greater and greater than WDM and baryons. The excess amplitude, however, is somehow moderated by the fact that WDM fluctuations do not grow significantly and, at variance from radiation, are also gravity sources. When we approach a scale, where WDM freely streams, the coupled CDM growth has a further burst.

In both plots, coupled cosmologies are compared with a standard ΛCDM cosmology and with a ΛWDM cosmology with the same parameters. The latter cosmologies differ from ΛCDM above a suitable $k$, corresponding to the scale where WDM start to undergo a free streaming process.

In strongly coupled cosmologies, the free streaming suppression is soon balanced by the effect of coupled CDM fluctuation gravity. As soon as WDM particles become non-relativistic, they re-fall in the potential wells created by CDM. This is similar to baryons falling in CDM potential wells after decoupling from radiation, in ΛCDM models. The main difference is that, in these latter cosmologies, soon after decoupling, CDM is the dominant component by far. On the contrary, in S.C. cosmologies, the coupled CDM component is just a minimal part of the cosmic substance. Therefore, WDM and baryon fluctuations, although becoming greater than in ΛCDM, never attain the CDM fluctuation level.
Figure 5. As the previous figure, for $\beta = 30$ and $m_w = 437.2\,\text{eV}$. Notice the (slight) shift on the scale where the S.C. cosmology deviates from $\Lambda$CDM.

Notice also that, up to $k \sim 10\,h\text{Mpc}^{-1}$, the discrepancy between the total transfer function and the transfer function for WDM and baryons is negligible. At greater $k$ values, however, although CDM is just a few permils of the cosmic materials, its fluctuations are so wider, to cause a discrepancy between baryons-WDM and total transfer functions.

Let us however soon outline that the large $\delta_c$ values do not correspond to a large overall density fluctuation $\delta \rho_c = \rho_c \delta_c$. In fact, although the ratio $\delta_c/\delta_w$ nearly increases $\propto \beta^2$, the density $\rho_c$ is almost proportional to the early density parameter $\Omega_c \propto \beta^{-2}$. Altogether, therefore, the density fluctuations $\delta \rho_c$ are just slightly decreasing with $\beta$.

A comparison between figures 4 and 5 shows a similar trend in the modification of the transfer function in respect to $\Lambda$CDM. At a scale fixed by $m_w$ the S.C. transfer function becomes smaller than $\Lambda$CDM, with a deficit never exceeding one order of magnitude. This deficit is however localized and, at greater $k$’s, the transfer function regains and overcomes the $\Lambda$CDM level. This behavior has been found to be typical off all S.C. cosmologies where early $\Omega_c$ and $\Omega_w$ are similar. By increasing $\beta$ (and consequently $m_w$), the feature gradually displaces towards greater $k$ values.

In figure 6 we then compare the CMB angular spectra $C_\ell$ among the same S.C. cosmologies of the previous two figures and $\Lambda$CDM. The upper figure shows $C_{TT}^\ell$, $C_{TE}^\ell$, and $C_{EE}^\ell$, from top to bottom. The lower figure, instead, just concerns $C_{TT}^\ell$. 
Figure 6. \( C_l \) angular spectra compared. Color selection in the bottom plot as in the top one. The only tenuous discrepancies between S.C. cosmologies and \( \Lambda \)CDM are visible in the lower plot, where the \( EE \) spectrum is expanded. They could be easily compensated by a slight change in \( n_s \) or the optical depth \( \tau \).

In the upper figure the models are barely undistinguishable. Some slight discrepancy is appreciable in the lower plot, thanks to the magnification in the ordinate scale. Such minimal discrepancy would be easily compensated by a tiny shift in the primeval spectral index \( n_s \).
or in the cosmic opacity, here assumed to be \( \tau = 0.089 \) anywhere. We may guess that such — almost unperceivably — greater \( C_\ell \) amplitude in S.C. cosmologies reflects an increase of radiation fluctuation amplitude due to the large fluctuations in coupled-CDM.

Altogether we may conclude that, while the transfer function peculiarities are significant and deserve further discussion, there is no significant change for CMB spectra in respect, e.g., to \( \Lambda \)CDM.

### 8 Discussion

Let us compare the cosmological picture described here with more standard scenarios, by distinguishing between (a) conceptual issues and (b) data fittings.

Let us start from the issue which could appear more controversial and contrived: the presence of two DM components. The presence of multiple DM components, however, has been recently advocated by several authors (see, e.g., [29–32]), for precise observational reasons on which we shall focus when coming to the (b) point. Our whole approach to the dark cosmic components, however, avoids any reference to hardly measurable entities; e.g., no specific self-interacting potential for the \( \Phi \) field is assumed, preferring to refer to its state equation \( w(a) \), surely closer to observations. In a similar way, even if the dual DE-CDM component could be a single substance (e.g., modulus and phase of a complex scalar field [33–35]), here we keep on the phenomenological side and treat it as 2 separate components, with a constant coupling \( \beta \) allowing them a suitable energy exchange.

Quite a few advantages of this approach, in respect to more standard ones, seem however clear. First of all, almost no component needed to describe today’s phenomenology is peculiar of our epoch. This is surely true for DE, but also for CDM and WDM, the only exception being baryons. Moreover, radiation, neutrinos, DE, CDM and WDM, until a fairly recent epoch, kept fixed proportions: the scale factor increase diluted them all \( \propto a^{-4} \). Photons and neutrinos are surely the dominant components in this (pseudo-)stationary expansion, but the reason is clear: their densities were enriched by the heating due to heavier particle decays. This agrees with our choice to take close values for DE, CDM and WDM early densities, just assuming a suppression factor for WDM due to its early (hot) decoupling. Accordingly, this approach allows for a simultaneous — and, therefore, possibly correlated — origin for all dark components.

The break of the early (pseudo-)stationary expansion apparently requires some tuning, to meet observational features: the kinetic-potential transition of DE, the rise of baryons, and WDM derelativization must have followed a precise order. Once again, baryons are a problem; not worse than in any other cosmological framework, however. On the contrary, when we come to transitions concerning DE and WDM, their quasi-coincidence does not appear so awkward. If

\[
V(\Phi) \simeq m^2 \Phi^2,
\]

or a similar term is part of the self-interaction potential, we meet the transition when the decrease of the kinetic energy density \( \frac{\dot{\Phi}^2}{2m^2} \propto a^{-4} \) lends relevance to the field mass. (Notice that, during the (pseudo-)stationary expansion it is \( \dot{\Phi} \propto \ln(a) \) and this moderate growth continue also after WDM — and baryons — become dominant.) The fact that this occurs slightly after the time when the dilution of the kinetic energy of WDM particles lends relevance to their mass, might not be casual.

This class of cosmologies is however peculiar for the very (pseudo-)stationary expansion process. The cosmic components keep steadily fixed proportions, as we delve into earlier and
earlier eras. One might even tentatively guess that the observed distribution was fixed at the end of inflation. If we tentatively argue that the \( \Phi \) field we are perceiving as DE coincides with the inflationary field, we face a number of problems that we plan to discuss elsewhere.

The above issues concern background features. When we come to fluctuation dynamics, the first point is that the \( C_\ell \) CMB spectra appear barely indistinguishable from \( \Lambda \)CDM. A priori this is not obvious, as coupled CDM fluctuations, on the last scattering band and — even more — later on, already significantly exceed WDM, and the low-\( \ell \) region, where fluctuations are essentially due to gravity, could be influenced.

Let us now come to data fit. The point will be discussed here from a semi-qualitative side, namely to argue whether: (i) there is any perspective that S.C. cosmologies may ease the problems met by \( \Lambda \)CDM models; (ii) the large-\( z \) behavior is substantially affected.

A first difficulty of \( \Lambda \)CDM concerns the amount of substructure in Milky Way sized haloes [36, 37]. Models involving CDM overpredict their abundance by approximately one order of magnitude. A second issue concerns the density profiles of CDM haloes in simulations, exhibiting a cuspy behavior [38–42], while the density profiles inferred from rotation curves suggest a core like structure [43–45]. A third issue concerns dwarf galaxies in large voids: recent studies [46–48] re-emphasized that they are overabundant.

We believe that this class of cosmologies might substantially ease the hardest of these problems, the only one which seemingly found no reasonable solution yet: the question of density profiles.

Figure 7. \( \Delta^2(k) \) spectral functions in S.C. cosmologies and \( \Lambda \)CDM.
As a matter of fact, it is known that replacing CDM with a “warmer” DM component, as a thermal relic of particles whose mass is \( \sim 2–3 \text{ keV} \), yields predictions better than \( \Lambda \text{CDM} \). There is a number of “thermal” candidates for such WDM; among them, a sterile neutrino and a gravitino \([49, 50]\) find a reasonable motivation in particle theory \([51, 52]\).

The free streaming of such particles, in \( \Lambda \text{WDM} \) cosmologies causes a strong suppression of the power spectrum on galactic and sub-galactic scales \([26, 53–57]\), as we also saw in figures 4 and 5 (green-yellow-black dotted curves). As a consequence, N-body simulations of these models show a shortage of galactic satellites, partially easing the observed lack of substructure in the Milky Way. However, the very figures 4 and 5 — as well as, more clearly, figure 7 — show a spectral gap (up to \( \sim 1 \) order of magnitude) followed by a power recovery at greater \( k \)'s, in the models discussed here. It is then unclear whether and how such conclusions can be extrapolated to this case.

As far as halo profiles are concerned, they are expected to be similar to CDM haloes in the outer regions, but flattening towards a constant value in the inner regions; this was predicted in \([58]\) and found in simulations \([59, 60]\). However, the core size found is 30–50 pc, while the observed cores in dwarf galaxies are around the 1000 pc scale \([61]\). A dwarf galaxy core in this scale range would be produced by higher velocity particles, as those belonging to a thermal distribution if their mass is \( \sim 0.1–0.4 \text{ keV} \). \( \Lambda \text{WDM} \) cosmologies whose warm component is made of particles with such a mass, however, yield a greater streaming length, exceeding the size of fluctuations able to generate these very dwarf galaxies, in the first place \([62]\).

In view of these difficulties, the idea that WDM is accompanied by a smaller amount of CDM has already been put forward \([29–32]\). The WDM particle velocities could then be greater, while a low-mass population is however produced by the re-infall of later derelativizing WDM particles in persisting CDM potential holes. This suggestion was put forward quite independently of any particle or cosmic model, although assuming \textit{ad hoc} a twofold dark matter component does not certainly ease coincidence problems.

It seems clear that S.C. cosmologies have no apparent difficulty to explain the observed cores, therefore. On the contrary, to obtain a fluctuation suppression on the scale of galactic subhalos one needs a suitable tuning, which might even be insufficient.

It must be however clear that a greater \( \Delta^2(k) \) yields a larger amount of galactic objects on the mass scale

\[
M = \frac{4\pi}{3} \left( \frac{2\pi}{k} \right)^3 \rho_c \Omega_m = 2.88 \times 10^{14} \left( \frac{h \text{Mpc}^{-1}}{k} \right)^3 \Omega_m M_{\odot} h^{-1}
\]

(here \( \Omega_m = \Omega_b + \Omega_w + \Omega_c \)) only in the mass variance \( \sigma^2(M) < 1 \). For greater \( k \)'s, on the contrary, it simply means an earlier formation of the related galactic systems. Accordingly, the prediction of S.C. cosmologies amounts to stating an early formation of small mass objects; this prediction could extend to the Milky Way satellite scale (\( k = 100 h \text{Mpc}^{-1} \) corresponds to a mass \( \simeq 7 \times 10^7 M_{\odot} h^{-1} \)), however keeping their total number at a level similar (or slightly inferior) to \( \Lambda \text{CDM} \) predictions. A tentative explanation of their observational scarcity could then be related to a longer lifetime, in respect to \( \Lambda \text{CDM} \) predictions. In a sense, the satellites we observe should then be the latest to form, while the older ones have become dark.

The shift on the formation time is greater at lower mass scales and cosmic reionization, whose main factor are small galaxies (see, e.g., \([63]\) and references therein), could have occurred a little earlier. As a matter of fact, no spectral burst on the \( 10^9–10^8 M_{\odot} h^{-1} \) mass scale
is needed to meet observations, but an increased spectral amplitude on such mass scales does not harm current expectations.

Another point to be suitably deepened is the formation of early black holes. This question was recently discussed in [64], aiming to exclude DE state equations unable to produce enough of them. These cosmologies surely meet current lower limits, but the whole scenario of early system formation could suffer significant modifications.

A final point concerns the m.s. velocity of today’s (almost) non relativistic WDM component, reading

\[ v^2 \sim \frac{T_{\alpha, w}}{m_w}. \]  

(8.3)

Here, \( T_{\alpha, w} = T_\alpha S \) is the present WDM temperature parameter, \( S \) yielding its ratio with the CMB temperature. For instance, for the case in figure 7, yielding \( S \sim 0.15 \), it is \( v \sim 0.3 \times 10^{-3} c \), a small but non negligible value.

9 Conclusions

In this work we aim to show that cosmologies allowing for a strong energy flow from CDM to DE may be quite promising. DE is treated as a scalar field \( \Phi \), and the energy flow is fixed by a coupling constant \( \beta \gg \frac{\sqrt{3}}{2} \). This kind of coupled-DE theories is not new in cosmology, but such large couplings were excluded, up to now, as coupled CDM was supposed to be the only DM component; if so, it must be \( \beta < \sim 0.15 \).

If we lift the limit on the \( \beta \) coupling, however, we find that a dual component, made of coupled non-relativistic particles and the scalar field \( \Phi \), falls onto an attractor solution, being in equilibrium with radiative components in the radiative era. More in detail: the early density parameters converge onto fixed values

\[ \Omega_c = \frac{1}{2\beta^2} \quad \text{and} \quad \Omega_d = \frac{1}{4\beta^2}, \]  

(9.1)

for CDM and the field, respectively; such density parameters keep constant, as both dual components dilute \( \propto a^{-4} \) as the Universe expands. This solution being an attractor, if we set initial conditions violating eq. (9.1), the densities of CDM and DE fastly mutate and the condition (9.1) is restored. This attractor exists only for \( \beta > \sqrt{3}/2 \).

If we suppose that this were the actual cosmic component in the early Universe, an exit from the steady expansion can be caused either by another uncoupled CDM component, or by the derelativization of a WDM component. Diluting then \( \propto a^{-3} \), the density of such further component eventually overcomes the densities of the field and CDM. When this happens, however, also the ratio between these latter densities and radiation starts to increase, although more slowly.

In any coupled-DE theory, the DE state parameter \( w(a) \equiv +1 \), in the early Universe (there are some exceptions, when a self interaction potential \( V(\Phi) \) yields forces so intense to make the DE-CDM coupling almost negligible). In order that the \( \Phi \) field assumes DE features, \( w(a) \) must eventually turn from +1 to \( \sim -1 \) about a suitable redshift \( z_d \). Such \( z_d \) can then be easily tuned to allow all cosmic components to reach their observational ratios. The \( w(a) \) transition is expected to occur because of the progressive dilution of the field kinetic energy, while the field has a steady increase \( \propto \ln(a) \). All that is independent from any specific assumption on the form of the self-interaction potential \( V(\Phi) \).

In this paper we considered both the option that uncoupled DM is cold or warm. The latter option may however lead to a rather attractive picture, in which scalar field, coupled
CDM, and WDM have close steady density parameters in the early Universe. The main radiative components, made of photons and neutrinos, would then be significantly denser just because heated up by the progressive decays of other particles belonging to the primeval thermal soup, while WDM, coupled CDM and $\Phi$ decoupled quite early and in the same epoch.

The main topic of this paper, however, is the analysis of density fluctuation evolution in such cosmologies. This required first an analysis of initial conditions. The successive evolution was then treated both with an heuristic 11-eqs. program, able to outline the essential physical features, and by suitably modifying the public CMBFAST program.

We find that fluctuation spectra and their evolution strictly resemble $\Lambda$CDM so that, in first approximation, the present large scale picture is quite similar to $\Lambda$CDM, although microscopic quanta have a mass scale $\sim 0.1$–$0.4$ keV. Possible discrepancies therefore emerge just below the Milky Way scale, allowing for a natural explanation of flat halo profiles in dwarf galaxies and, possibly, for the observed shortage of Milky Way substructure.

Further significant differences from the $\Lambda$CDM scenario are however expected during the epoch of reionization, when first structures form. In particular, the first stars and the cosmic reionization are expected to occur earlier; also the primeval back-hole formation is expected to be more effective.

Let us finally remark that the CMB angular spectra, in S.C. cosmologies, strictly resemble $\Lambda$CDM for the same parameter choice.

Acknowledgments

Stefano Borgani and Matteo Viel are gratefully thanked for discussions. One of us (S.A.B.) acknowledges the support of CIFS.

References

[1] EUCLID collaboration, R. Laureijs et al., Euclid Definition Study Report, arXiv:1110.3193 [inSPIRE].
[2] B. Joachimi and S.L. Bridle, Simultaneous measurement of cosmology and intrinsic alignments using joint cosmic shear and galaxy number density correlations, Astron. Astrophys. 523 (2010) A1 [arXiv:0911.2454] [inSPIRE].
[3] L. Amendola, Scaling solutions in general nonminimal coupling theories, Phys. Rev. D 60 (1999) 043501 [astro-ph/9904120] [inSPIRE].
[4] L. Amendola and D. Tocchini-Valentini, Baryon bias and structure formation in an accelerating universe, Phys. Rev. D 66 (2002) 043528 [astro-ph/0111535] [inSPIRE].
[5] L. Amendola, C. Quercellini, D. Tocchini-Valentini and A. Pasqui, Constraints on the interaction and selfinteraction of dark energy from cosmic microwave background, Astrophys. J. 583 (2003) L53 [astro-ph/0205097] [inSPIRE].
[6] L. Amendola, Linear and non-linear perturbations in dark energy models, Phys. Rev. D 69 (2004) 103524 [astro-ph/0311175] [inSPIRE].
[7] J.R. Kristiansen, G. La Vacca, L.P.L. Colombo, R. Mainini and S.A. Bonometto, Coupling between cold dark matter and dark energy from neutrino mass experiments, New Astron. 15 (2010) 609 [arXiv:0902.2737] [inSPIRE].
[8] G. La Vacca, J.R. Kristiansen, L.P.L. Colombo, R. Mainini and S.A. Bonometto, Do WMAP data favor neutrino mass and a coupling between Cold Dark Matter and Dark Energy?, JCAP 04 (2009) 007 [arXiv:0902.2711] [SPIRE].

[9] G. La Vacca and S.A. Bonometto, Mildly mixed coupled models vs. WMAP7 data, Nucl. Phys. Proc. Suppl. 217 (2011) 68 [arXiv:1101.2155] [SPIRE].

[10] V. Pettorino, L. Amendola, C. Baccigalupi and C. Quercellini, Constraints on coupled dark energy using CMB data from WMAP and SPT, Phys. Rev. D 86 (2012) 103507 [arXiv:1207.3293] [SPIRE].

[11] J.-Q. Xia, Constraint on coupled dark energy models from observations, Phys. Rev. D 80 (2009) 103514 [arXiv:0911.4820] [SPIRE].

[12] S.A. Bonometto, G. Sassi and G. La Vacca, Dark energy from dark radiation in strongly coupled cosmologies with no fine tuning, JCAP 08 (2012) 015 [arXiv:1206.2281] [SPIRE].

[13] B. Ratra and P.J.E. Peebles, Cosmological Consequences of a Rolling Homogeneous Scalar Field, Phys. Rev. D 37 (1988) 3406 [SPIRE].

[14] P. Brax and J. Martin, Quintessence and supergravity, Phys. Lett. B 468 (1999) 40 [astro-ph/9905040] [SPIRE].

[15] J.R. Ellis, S. Kalara, K.A. Olive and C. Wetterich, Density Dependent Couplings and Astrophysical Bounds on Light Scalar Particles, Phys. Lett. A 103 (1984) 369.

[16] C.-P. Ma and E. Bertschinger, Cosmic microwave background temperature and polarization anisotropy in Brans-Dicke cosmology, Astrophys. J. 455 (1995) 7 [astro-ph/9506072] [SPIRE].
\[\text{References}\]

[28] R. Mainini and S. Bonometto, Limits on coupling between dark components, *JCAP* 06 (2007) 020 [astro-ph/0703303] [inSPIRE].

[29] A.W. Brookfield, C. van de Bruck and L.M.H. Hall, New interactions in the dark sector mediated by dark energy, *Phys. Rev. D* 77 (2008) 043006 [arXiv:0709.2297] [inSPIRE].

[30] M. Baldi, Multiple Dark Matter as a self-regulating mechanism for dark sector interactions, *Annalen Phys.* 524 (2012) 602 [arXiv:1204.0514] [inSPIRE].

[31] A.V. Maccio, O. Ruchayskiy, A. Boyarsky and J.C. Muñoz-Cuartas, The inner structure of haloes in Cold+Warm dark matter models, *Mon. Not. Roy. Astron. Soc.* 428 (2013) 882 [arXiv:1202.2858] [inSPIRE].

[32] M. Blennow, E. Fernandez-Martinez, O. Mena, J. Redondo and P. Serra, Asymmetric Dark Matter and Dark Radiation, *JCAP* 07 (2012) 022 [arXiv:1203.5803] [inSPIRE].

[33] R. Mainini and S.A. Bonometto, Dark matter and dark energy from the solution of the strong-CP problem, *Phys. Rev. Lett.* 93 (2004) 121301 [astro-ph/0406114] [inSPIRE].

[34] R. Mainini, L.P.L. Colombo and S.A. Bonometto, Dark matter and dark energy from a single scalar field and CMB data, *Astrophys. J.* 632 (2005) 691 [astro-ph/0503036] [inSPIRE].

[35] R. Mainini and S. Bonometto, Dark Matter and Dark Energy from a single scalar field: CMB spectrum and transfer function, *JCAP* 09 (2007) 017 [arXiv:0709.0174] [inSPIRE].

[36] A.A. Klypin, A.V. Kravtsov, O. Valenzuela and F. Prada, Where are the missing Galactic satellites?, *Astrophys. J.* 522 (1999) 82 [astro-ph/9901240] [inSPIRE].

[37] B. Moore, S. Ghigna, F. Governato, G. Lake, T.R. Quinn et al., Dark matter substructure within galactic halos, *Astrophys. J.* 524 (1999) L19 [astro-ph/9907411] [inSPIRE].

[38] B. Moore, Evidence against dissipationless dark matter from observations of galaxy haloes, *Nature* 370 (1994) 629 [inSPIRE].

[39] R.A. Flores and J.R. Primack, Observational and theoretical constraints on singular dark matter halos, *Astrophys. J.* 427 (1994) L1 [astro-ph/9402004] [inSPIRE].

[40] J. Diemand, M. Zemp, B. Moore, J. Stadel and M. Carollo, Cusps in cold dark matter haloes, *Mon. Not. Roy. Astron. Soc.* 364 (2005) 665 [astro-ph/0504215] [inSPIRE].

[41] A.V. Maccio', A.A. Dutton, F.C. van den Bosch, B. Moore, D. Potter et al., Concentration, Spin and Shape of Dark Matter Haloes: Scatter and the Dependence on Mass and Environment, *Mon. Not. Roy. Astron. Soc.* 378 (2007) 55 [astro-ph/0608157] [inSPIRE].

[42] V. Springel, J. Wang, M. Vogelsberger, A. Ludlow, A. Jenkins et al., The Aquarius Project: the subhalos of galactic halos, *Mon. Not. Roy. Astron. Soc.* 391 (2008) 1685 [arXiv:0809.0898] [inSPIRE].

[43] W.J.G. de Blok, S.S. McGaugh, A. Bosma and V.C. Rubin, Mass density profiles of LSB galaxies, *Astrophys. J.* 552 (2001) L23 [astro-ph/0103102] [inSPIRE].

[44] R.K. de Naray, S.S. McGaugh and J.C. Milos, Constraining the NFW Potential with Observations and Modeling of LSB Galaxy Velocity Fields, *Astrophys. J.* 692 (2009) 1321 [arXiv:0810.5118] [inSPIRE].

[45] S.-H. Oh, W.J.G. de Blok, E. Brinks, F. Walter and R.C. Kennicutt Jr., Dark and luminous matter in THINGS dwarf galaxies, *Astron. J.* 141 (2011) 193 [arXiv:1011.0899] [inSPIRE].

[46] A.V. Tikhonov, S. Gottloeber, G. Yepes and Y. Hoffman, The sizes of mini-voids in the local universe: an argument in favor of a warm dark matter model?, *Mon. Not. Roy. Astron. Soc.* 399 (2009) 1611 [arXiv:0904.0175] [inSPIRE].

[47] J. Zavala, Y.P. Jing, A. Faltenbacher, G. Yepes, Y. Hoffman et al., The velocity function in the local environment from LCDM and LWDM constrained simulations, *Astrophys. J.* 700 (2009) 1779 [arXiv:0906.0585] [inSPIRE].
[48] P.J.E. Peebles and A. Nusser, "Clues from nearby galaxies to a better theory of cosmic evolution," *Nature* 465 (2010) 565 [arXiv:1101.1484] [SPIRE].

[49] K. Abazajian and S.M. Koushiappas, "Constraints on Sterile Neutrino Dark Matter," *Phys. Rev. D* 74 (2006) 023527 [astro-ph/0605271] [SPIRE].

[50] A. Boyarsky, J. Lesgourgues, O. Ruchayskiy and M. Viel, "Realistic sterile neutrino dark matter with keV mass does not contradict cosmological bounds," *Phys. Rev. Lett.* 102 (2009) 201304 [arXiv:0812.3256] [SPIRE].

[51] S. Dodelson and L.M. Widrow, "Sterile-neutrinos as dark matter," *Phys. Rev. Lett.* 72 (1994) 17 [hep-ph/9303287] [SPIRE].

[52] F. Takayama and M. Yamaguchi, "Gravitino dark matter without R-parity," *Phys. Lett.* B 485 (2000) 388 [hep-ph/0005214] [SPIRE].

[53] J.R. Bond, G. Efstathiou and J. Silk, "Massive Neutrinos and the Large Scale Structure of the Universe," *Phys. Rev. Lett.* 45 (1980) 1980 [SPIRE].

[54] S. Dodelson and L.M. Widrow, "Sterile-neutrinos as dark matter," *Phys. Rev. Lett.* 72 (1994) 17 [hep-ph/9303287] [SPIRE].

[55] C.J. Hogan and J.J. Dalcanton, "New dark matter physics: clues from halo structure," *Phys. Rev. D* 62 (2000) 063511 [astro-ph/0002330] [SPIRE].

[56] M. Viel, J. Lesgourgues, M.G. Haehnelt, S. Matarrese and A. Riotto, "Constraining warm dark matter candidates including sterile neutrinos and light gravitinos with WMAP and the Lyman-alpha forest," *Phys. Rev. D* 71 (2005) 063534 [astro-ph/0501562] [SPIRE].

[57] K. Abazajian and S.M. Koushiappas, "Constraints on Sterile Neutrino Dark Matter," *Phys. Rev. D* 74 (2006) 023527 [astro-ph/0605271] [SPIRE].

[58] F. Villaescusa-Navarro and N. Dalal, "Cores and Cusps in Warm Dark Matter Halos," *JCAP* 03 (2011) 024 [arXiv:1010.3008] [SPIRE].

[59] P. Colin, O. Valenzuela and V. Avila-Reese, "On the Structure of Dark Matter Halos at the Damping Scale of the Power Spectrum with and without Relict Velocities," *Astrophys. J.* 673 (2008) 203 [arXiv:0709.4027] [SPIRE].

[60] A.V. Macció, G. Stinson, C.B. Brook, J. Wadsley, H.M.P. Couchman et al., "Halo expansion in cosmological hydro simulations: towards a baryonic solution of the cusp/core problem in massive spirals," *Astrophys. J. Lett.* 744 (2012) L9 [arXiv:1111.5620] [SPIRE].

[61] M.G. Walker and J. Penarrubia, "A Method for Measuring (Slopes of) the Mass Profiles of Dwarf Spheroidal Galaxies," *Astrophys. J.* 742 (2011) 20 [arXiv:1108.2404] [SPIRE].

[62] A.V. Maccio and F. Fontanot, "How cold is Dark Matter? Constraints from Milky Way Satellites," *Mon. Not. Lett. Roy. Astron. Soc.* 404 (2010) L16 [arXiv:0910.2460] [SPIRE].

[63] M.A. Alvarez, K. Finlator and M. Trenti, "Constraints on the Ionizing Efficiency of the First Galaxies," *Astrophys. J.* 759 (2012) L38 [arXiv:1209.1387] [SPIRE].

[64] A. Lamastra, N. Menci, F. Fiore, C. Di Porto and L. Amendola, "Constraining Dynamical Dark Energy Models through the Abundance of High-Redshift Supermassive Black Holes," [arXiv:1111.3800] [SPIRE].