Anisotropic flow: Achievements, Difficulties, Expectations

Sergei A. Voloshin
Department of Physics and Astronomy
Wayne State University, Michigan 48201 USA
E-mail: voloshin@wayne.edu

Abstract. Anisotropic flow measurements play a crucial role in understanding the physics and bulk properties of the system created in heavy ion collisions. In this talk I briefly review the most important results obtained so far, recent developments in the analysis techniques and the interpretation of the results, and what should we expect next, both at RHIC and LHC. I also discuss event anisotropies sensitive to the strong parity violation effects.

Analysis of the event anisotropies (anisotropic flow) in multiparticle production in non-central nuclear collisions appeared to be one of the most informative paths in understanding the physics and characterizing the properties of the dense and hot strongly interacting medium. It has been observed a continuous increase in the value of in-plane elliptic flow ($v_2 > 0$) [1] from the top AGS energies to RHIC. At RHIC, strong elliptic flow [2] comparable in strength to the predictions of ideal hydrodynamics, and the hadronization via quark coalescence following from constituent quark number scaling of differential flow [3, 4], together with the jet quenching, are the key ingredients in the picture of sQGP (strongly coupled Quark Gluon Plasma).

The field is rapidly developing and evolving. From the “discovery phase” of the first years of RHIC operations it has been transforming into detailed quantitative description of the sQGP phase and subsequent hadronization. The plots of $v_2/\varepsilon$ vs particle density [3, 5, 6] that have been extensively used in the assessment of the level of thermalization reached in the relativistic nuclear collisions and applicability of ideal hydrodynamics comes under scrutiny: are the measurements of elliptic flow precise enough, are the anisotropies what we think they are and how much are they modified by fluctuation processes? When comparing to hydrodynamical calculations, are the proper initial conditions used in the calculations? Could it be that we have missed some important physics building the models? In the last couple years significant progress has been reached answering each and every of these questions. The role of viscosity, flow fluctuations, initial conditions (eccentricity and initial flow (velocity) field) are a few questions to report in this review. I also discuss preliminary measurements of azimuthal “out-of-plane” anisotropies that are related to one of the fundamental questions of the strong parity violation. Finally, I briefly overview future measurements at RHIC and
LHC. Unfortunately, the space does not allow any detailed discussion of many other very important developments, such as flow of \( \phi \)-mesons \cite{7,8} that is important to understand the relative flow development in the partonic and hadronic phases, flow of the deuterons \cite{7} and \( K^* \)-mesons as tests of coalescence, heavy flavor flow, etc. The KE scaling of elliptic flow observed by PHENIX Collaboration \cite{9} is still not fully understood/appreciated. There is no doubt that these measurements will enrich our understanding of the dense QCD medium even further.

Taking into account the significance of the \( v_2/\varepsilon \) plot in establishing the sQGP picture, I spend most of my time discussing recent developments related to this plot. There have been a number of important findings: (a) Along with several indirect indications that even in central Au+Au collisions the thermal equilibrium is not complete, it was found that even very small, compared to the conjectured low limit of \( \eta/s \) values, the viscous effects lead to a significant reduction in the predicted elliptic flow compared to the ideal hydro case. This would lead to a contradiction with experimental measurements if other effects, responsible for an increase of elliptic flow (compared to the “standard” hydrodynamical calculation) would not be identified. Several of such effects have been reported. (b) First, it was noticed that ideal hydro calculations, if tuned to describe spectra, yield larger elliptic flow than thought previously. (c) It was shown, that in some models, e.g. CGC, the initial eccentricity can take significantly larger values than in optical Glauber model that is usually used in hydro calculations. The larger eccentricities inevitably lead to larger elliptic flow. (d) Flow fluctuations, the nature of which is much better understood in the last years, lead to an increase of apparent flow. (e) Finally, it was noticed that the gradients in the initial velocity field also lead to the increase in final values of elliptic flow. Any of the above mentioned effects can be quite significant, leading each to 20-30% or even larger change in values of \( v_2 \). The final “assembly” of these effects into one reliable model is still under way.

An attempt of model independent analysis of \( v_2/\varepsilon \) dependence on particle density based on parametrization in terms of Knudsen number has been developed in \cite{10,11}. Using an expression \( v_2/\varepsilon = (v_2/\varepsilon)_{\text{hydro}}(1 + K/K_0)^{-1} \) (where the parameter \( K_0 \approx 0.7 \) is independently estimated from comparison to model calculations) to fit the data, see Fig. 1, the authors conclude that at RHIC we might be still up to 30% below the ideal “hydro limit” even for the most central collisions. Their estimate of the viscosity yields values of \( \eta/s = 0.11 - 0.19 \) depending on the CGC or Glauber initial conditions. Similar fits to the STAR data performed by R. Snellings \cite{12} and collaborators lead to similar conclusions.

The magnitude of the viscous effects could be judged already from the early calculations \cite{13} where the hydro dynamical evolution at some intermediate stage was joined to the transport model to simulate late (viscous) evolution of the system. Later, viscosity was attempted to be introduced directly into hydrodynamic calculations \cite{14}. Recently there have been performed a few “full” calculations \cite{15,16,17} of the hydrodynamical expansion with viscous terms explicitly included into equations. Note that the latter (namely the form of these terms) is not that everybody agrees on \cite{16},
though the difference in the results due to use of somewhat different equations are likely small. At the same time everybody agrees on the significance of the viscous effects even for the “minimal” values of viscosity ($\eta/s = 1/(4\pi)$). The results presented in Figs. 2 and 3 show that even the minimal viscosity lead up to $\sim25-30\%$ reduction in flow values in Au+Au collisions and probably more than 50\% in Cu+Cu.

Note that viscosity coefficients calculated in pQCD are usually much larger than would be allowed by the data. In this sense, noteworthy are the recent calculations [18], which emphasize the importance of taking into account $2 \leftrightarrow 3$ processes. With these effects included, the viscosity coefficient appeared to be about an order magnitude smaller compared to previous calculations and fall into the “allowable” by the data range.

One can wonder how is it possible that with viscous effects to be that strong that the ideal hydrodynamical calculations results are not much higher than the experimental measurements? Indeed, there have been identified several effects which possibly lead to a significant increase in predicted flow. Taken together with viscous effects they may restore the agreement with experiment.

Firstly, in [19] it was explicitly demonstrated that the ideal hydro calculations can not be “tuned” to describe both spectra and elliptic flow. If one tunes the model to describe spectra, the elliptic flow values appeared too large, for about 20-30\%, compared to the data, see Fig. 4. What leaves even more space for viscous effects, is the observation [20, 21] that the initial eccentricity calculated in the CGC model yields values up to 50\% larger compared to the “standard” optical Glauber model. The effect has been further studied in [22]; in [23] it was discussed how eccentricity depends on details of CGC model, which can be taken as possibility to investigate CGC model by measuring flow.

The role of flow fluctuations and non-flow effects is another long standing problem that received a lot of attention and significant progress has been made in the recent couple years. In particular, the role of fluctuations in the initial system geometry [24]...

Figure 1: Fit to $v_2/\varepsilon$ in terms of Knudsen number [11].

Figure 2: Viscous hydro calculations [15] compared to data.
Anisotropic flow: Achievements, Difficulties, Expectations

Figure 3: $v_2(p_t)$ from ideal and viscous ($\eta/s = 1/(4\pi)$) hydrodynamics [17].

Figure 4: Pion $v_2(p_t)$ in two scenarios [19]. Solid line indicates the results using parameters best fit to spectra.

defined by nuclear participants [25] (interacting nucleons or quarks) has been greatly clarified [26, 27, 28, 29, 30]. The following picture emerges: Anisotropic flow is defined as the correlations to the reaction plane spanned by the impact parameter and the beam axis. At fixed impact parameter, the geometry of the participant zone fluctuates, both, in terms of the value of the eccentricity as well as the orientation of the major axes. Then the anisotropy develops along the plane spanned by the minor axis of the participant zone and the beam direction, the so called participant plane. As the true reaction plane is not known and the event plane is estimated from the particle azimuthal distribution “defined” by the participant plane, the apparent (participant plane) flow appears to be always bigger (and always “in-plane”, $v_{2,PP} > 0$) compared to the “true” flow as projected onto the reaction plane (see Fig. 7).

It was noticed in [28] that in collisions of heavy nuclei the fluctuations in the eccentricity $(\varepsilon_x, \varepsilon_y) = (\langle (\sigma_y^2 - \sigma_x^2)/(\sigma_y^2 + \sigma_x^2) \rangle, \langle (2\sigma_{xy}/(\sigma_y^2 + \sigma_x^2) \rangle)$ can be well described by two-dimensional Gaussian. What is not trivial is that for such Gaussian fluctuations the higher cumulant flow $\langle v\{n\}, n \geq 4 \rangle$ is not only insensitive to non-flow but also to eccentricity fluctuations. All of higher cumulants are exactly equal to the “true” flow, namely as given by projection onto the reaction plane. At the same time, the apparent (participant plane) flow become unmeasurable in a sense that flow fluctuations could not be separated from non-flow contributions by means of correlation measurements.

An important conclusion from that study was that in most cases (except, probably, in mid-peripheral and peripheral Cu+Cu collisions, when the Gaussian approximation breaks [29], and $\varepsilon_{part}\{4\}$ does not agree with $\varepsilon_{std} [26]$) the measurements of higher cumulant flow values provide the elliptic flow relative to the true reaction plane. Similarly, the same value is given by several other methods, such as Lee-Yang Zeroes, Bessel Transform, and fit to $q$-distributions. This greatly simplifies the comparison, e.g. of hydrodynamical calculations to the data, as it says that in such calculation one should not worry how to take into account the fluctuations in the initial eccentricity.
Anisotropic flow: Achievements, Difficulties, Expectations

(which is a non-trivial task) but just compare to the “right” measurement, e.g. $v_2\{4\}$. This understanding also allowed to appreciate some earlier calculations with uRQMD model \[31, 32\]. There, it was shown that using higher cumulants and/or LYZ method one indeed can measure the elliptic flow very well, but it was not at all clear why there have been observed no traces of the effects of flow fluctuations, which were expected in this model.

Unfortunately this progress in understanding the nature of fluctuations does not help in resolving the problem of measuring flow fluctuations (in the participant plane) and non-flow. Strictly speaking, to make any estimates of those one is required to make assumptions. Most often to suppress non-flow contribution the azimuthal correlations between particles with large rapidity separation are used. The problem with this method is that there is no reliable estimates of how well it suppresses non flow and also how much the flow fluctuations (in this case correlations) change after imposing such a cut.

At this conference the PHOBOS \[33\] and the STAR \[34\] collaborations presented their revised (compared to QM’06) results on flow fluctuations. These results are in good qualitative and quantitative agreement, see Figs. 5, 6. In \[34\] a conservative approach is taken and only upper limits on fluctuations are reported. The PHOBOS Collaboration uses estimates of non-flow effects from correlations with large rapidity separations and report more restrictive range for fluctuations. Both agree that the current measurements exhaust the (nucleon) eccentricity values obtained in MC Glauber model and in this sense somewhat favor models which predict smaller relative fluctuations, such as the CGC model or MC Glauber taking into account constituent quark substructure.

Another important and interesting direction that is just started to be explored is the role of the non-zero initial flow velocity profile. The first obvious candidate here is to take into account the initial velocity gradient along the impact parameter, Fig. 8. As shown in \[35\] such a gradient directly contributes to the in-plane expansion rate (see Eq. 22 in \[35\]). I would draw attention to the fact that those effects naturally also lead to directed flow (see the same Eq. 22), which too briefly addressed in \[36\]. It will be very interesting to compare the calculations in such a model to a very precise data from STAR \[37\] on directed flow obtained with the reaction plane determined by neutrons in the Zero Degree Calorimeter. The relation to other models \[38, 39\] predicting non-trivial dependence of directed flow on rapidity would be also very interesting. Speculating on this subject one would also notice that viscous effects must also play an important role in such a scenario.

It was shown in \[40\] that the effect of the strong parity violation that lead to non equal number of right and left fermions (quarks) in the presence of strong magnetic fields of colliding nuclei would result in charge separation (preferential emission of same charge particles) in the direction perpendicular to the reaction plane. Such anisotropy, which very much resembles “out-of-plane directed flow” can be addressed with the help of three-particle correlations \[41\] by measuring $\langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle$, where $\phi_{\alpha,\beta}$ are azimuthal angles of two (same or opposite) charged particles, and $\Psi_{RP}$ is the reaction plane angle. The estimates \[42\] (see also talk by H. Warringa at this conference)
Figure 5: Elliptic flow fluctuations as estimated by STAR.

Figure 6: Elliptic flow fluctuations as estimated by PHOBOS Collaboration.

Figure 7: Flow vector distribution at fixed \((\varepsilon_v, \varepsilon_y)\) [28].

Figure 8: Initial velocity profile in non-central nuclear collisions [35].

Figure 9: Azimuthal anisotropy correlator sensitive to strong \(P\)–violation effects.

Figure 10: Extrapolation of \(v_2\) values to LHC energies [48].
Anisotropic flow: Achievements, Difficulties, Expectations

Figure 11: “Collapse” of directed flow as discussed in [45].

indicate that the effect is strong enough to be observed in heavy ion collisions. At this conference the STAR Collaboration reported the preliminary results [43], see Fig. 9 that qualitatively agree with estimates presented in [40, 42]. Note, that the correlator \( \langle \cos(\phi_\alpha + \phi_\beta - 2\Psi_{RP}) \rangle \) is \( P \)-even and contain contributions from other effects not related to parity violation. A careful analysis of such a contribution is obviously needed before any strong conclusion can be drawn from these measurements.

Coming years promise many new interesting data from low energy RHIC run and, of course, from LHC. The main interest in the low energy RHIC scan, anisotropic flow is no exception, is the search for the QCD critical point. The scan would cover the energy region from top AGS energies, over the CERN SPS, and higher. In terms of anisotropic flow two major observables to watch would be a possible “wiggle” in \( v_2/\varepsilon \) dependence on particle density [44] and “collapse” of directed flow [45]. RHIC also has plans to extend its reach in terms of energy density using uranium beams. From the first estimates and ideas of using uranium beam we now have real detailed simulations [46] of such collisions with developed methods for a selection of desired geometry of the collision.

The predictions for the LHC are rather uncertain, though most agree that the elliptic flow will continue to increase [47], partially due relatively smaller contribution of viscous effects. Simple extrapolations [48, 49] of the collision energy dependence of \( v_2 \) look rather “reliable”. Note that there exist calculations predicting decrease of the elliptic flow [50]. Another important observation is an increase in mass dependence (splitting) of \( v_2(p_t) \) due to a strong increase of radial flow.

In summary, we have had very exciting years of anisotropic flow study, which greatly enriched our understanding of ultra-relativistic nuclear collisions and multiparticle production in general. We are looking forward for new physics with LHC and RHIC.

I thank A. Poskanzer, R. Snellings, and A. Tang for numerous fruitful discussions.

[1] J. Barrette et al. [E877 Collaboration], Phys. Rev. C 55, 1420 (1997) arXiv:nucl-ex/9610006.
[2] K. H. Ackermann et al. [STAR Collaboration], Phys. Rev. Lett. 86, 402 (2001)
[3] S. A. Voloshin, Nucl. Phys. A 715, 379 (2003) arXiv:nucl-ex/0210014.
Anisotropic flow: Achievements, Difficulties, Expectations

[4] D. Molnar and S. A. Voloshin, Phys. Rev. Lett. 91, 092301 (2003) [arXiv:nucl-th/0302014].
[5] C. Alt et al. [NA49 Collaboration], Phys. Rev. C 68, 034903 (2003) [arXiv:nucl-ex/0303001].
[6] S. A. Voloshin [STAR Collaboration], J. Phys. G 34, S883 (2007) [arXiv:nucl-ex/0701038].
[7] S. Afanasiev et al. [PHENIX Collaboration], Phys. Rev. Lett. 99, 052301 (2007)
[8] B. I. Abelev et al. [STAR Collaboration], Phys. Rev. Lett. 99, 112301 (2007)
[9] A. Adare et al. [PHENIX Collaboration], Phys. Rev. Lett. 99, 112301 (2007)
[10] R. S. Bhalerao, J. P. Blaizot, N. Borghini and J. Y. Ollitrault, Phys. Lett. B 627, 49 (2005)
[11] H. J. J. Drescher, A. Dumitru, C. Gombeaud and J. Y. Ollitrault, Phys. Rev. C 76, 024905 (2007)
[12] R. Snellings, private communication.
[13] D. Teaney, J. Lauret and E. V. Shuryak, arXiv:nucl-th/0110037.
[14] D. Teaney, Phys. Rev. C 68, 034913 (2003) [arXiv:nucl-th/0301099].
[15] P. Romatschke, arXiv:0706.1522 [nucl-th].
[16] H. Song and U. W. Heinz, Phys. Lett. B 658, 279 (2008) [arXiv:0709.0742 [nucl-th]].
[17] H. Song and U. W. Heinz, arXiv:0712.3715 [nucl-th].
[18] Z. Xu, C. Greiner and H. Stocker, arXiv:0711.0961 [nucl-th].
[19] P. Huovinen, arXiv:0710.4379 [nucl-th].
[20] T. Hirano, U. W. Heinz, D. Kharzeev, R. Lacey and Y. Nara, Phys. Lett. B 636, 299 (2006)
[21] H. J. Drescher, A. Dumitru, A. Hayashigaki and Y. Nara, Phys. Rev. C 74, 044905 (2006)
[22] H. J. Drescher and Y. Nara, Phys. Rev. C 76, 041903 (2007) [arXiv:0707.0249 [nucl-th]].
[23] T. Lappi and R. Venugopalan, Phys. Rev. C 74, 054905 (2006) [arXiv:nucl-th/0609021].
[24] M. Miller and R. Snellings, arXiv:nucl-ex/0312008.
[25] S. Manly et al. [PHOBOS Collaboration], Nucl. Phys. A 774, 523 (2006) [arXiv:nucl-ex/0510031].
[26] S. A. Voloshin, arXiv:nucl-th/0606022.
[27] S. A. Voloshin, arXiv:nucl-th/0607009.
[28] S. A. Voloshin, A. M. Poskanzer, A. Tang and G. Wang, Phys. Lett. B 659, 337 (2008)
[29] B. Alver et al., Phys. Rev. C 77, 014906 (2008) [arXiv:0711.3724 [nucl-ex]].
[30] W. Broniowski, P. Bozek and M. Rybczynski, Phys. Rev. C 76, 054905 (2007)
[31] X. I. Zhu, M. Bleicher and H. Stoecker, Phys. Rev. C 72, 064911 (2005) [arXiv:nucl-th/0509081].
[32] X. I. Zhu, M. Bleicher and H. Stoecker, J. Phys. G 32, 2181 (2006) [arXiv:nucl-th/0601049].
[33] B. Wosiek for the PHOBOS Collaboration, this proceedings.
[34] P. Sorensen for the STAR Collaboration, this proceedings.
[35] F. Becattini, F. Piccinini and J. Rizzo, Phys. Rev. C 77, 024906 (2008) [arXiv:0711.1253 [nucl-th]].
[36] S. M. Troshin and N. E. Tyurin, arXiv:0709.4090 [hep-ph].
[37] G. Wang, J. Phys. G 34, S1093 (2007) [arXiv:nucl-ex/0701045].
[38] L. P. Csernai, J. I. Kapusta and L. D. McLerran, J. Phys. G 32, S115 (2006).
[39] R. Snellings, H. Sorge, S. Voloshin, F. Wang and N. Xu, Phys. Rev. Lett. 84, 2803 (2000)
[40] D. Kharzeev, Phys. Lett. B 633, 260 (2006) [arXiv:hep-ph/0406125].
[41] D. Kharzeev, L. D. McLerran and H. J. Warringa, arXiv:0711.0950 [hep-ph].
[42] S. Voloshin for the STAR Collaboration, poster presentation at this conference.
[43] S. A. Voloshin and A. M. Poskanzer, Phys. Lett. B 474, 27 (2000) [arXiv:nucl-th/9906075].
[44] H. Stoecker, arXiv:0710.5089 [hep-ph].
[45] C. Nepali, G. I. Fai and D. Keane, Phys. Rev. C 76, 051902 (2007) [Erratum-ibid. C 76, 069903 (2007) [arXiv:0709.1497 [hep-ph]].
[46] N. Armesto et al., J. Phys. G 35, 054001 (2008) [arXiv:0711.0974 [hep-ph]].
[47] W. Busza, J. Phys. G 35, 044040 (2008) [arXiv:0710.2293 [nucl-ex]].
[48] N. Borghini and U. A. Wiedemann, J. Phys. G 35, 023001 (2008) [arXiv:0707.0564 [hep-ph]].
[49] D. Krieg and M. Bleicher, arXiv:0708.3015 [nucl-th].