Characteristic Polynomial of Antiadjacency Matrix of Directed Cyclic Wheel Graph \((\overrightarrow{W}_n)\)

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Abstract. A directed cyclic wheel graph with order \(n\), where \(n \geq 4\) can be represented by an anti-adjacency matrix. The anti-adjacency matrix is a square matrix that has entries only 0 and 1. The number 0 denotes an edge that connects two vertices, whereas the number 1 denotes otherwise. The norm of every coefficient in characteristic polynomial of the anti-adjacency matrix of a directed cyclic wheel graph represents the number of Hamiltonian paths contained in the induced sub-graphs minus the number of the cyclic induced sub-graphs. In addition, the eigenvalues can be found through the anti-adjacency matrix of directed cyclic wheel graph. The result is, the anti-adjacency matrix of directed cyclic wheel graph has two real eigenvalues and some complex eigenvalues that conjugate to each other. The real eigenvalues are obtained by Horner method, while the complex eigenvalues are obtained by finding the complex roots from the factorization of the characteristic polynomial.

1. Introduction
Graph theory is a branch of mathematics that develops very rapidly. There are still many properties of a graph that have not been studied further, such as the antiadjacency matrices of directed graphs. Based on its direction, directed graphs are divided into directed acyclic graphs (a graph that does not contain a cycle graph) and directed cyclic graphs (a graph that contains a cycle graph). A directed graph can be represented in the form of an adjacency matrix or antiadjacency matrix. In a previous study, Fery had discussed about the antiadjacency matrix of some directed acyclic graphs [1]. Today’s researches have not discussed widely about the antiadjacency matrix of directed cyclic graphs. Therefore, in this paper, the antiadjacency matrix of a directed cyclic wheel graph \(\overrightarrow{W}_n\) will be discussed. From the antiadjacency matrix of a directed cyclic wheel graph \(\overrightarrow{W}_n\), the general form of its characteristic polynomial can be found. The characteristic polynomial of directed cyclic wheel graph \(\overrightarrow{W}_n\) has \(n\) coefficients, those are \(b_1, b_2, b_3, \ldots, b_n\). The coefficients represent structures that belong to the directed cyclic wheel graph \(\overrightarrow{W}_n\). Moreover, we find the eigenvalues of the antiadjacency matrix of a directed cyclic wheel graph \(\overrightarrow{W}_n\) which both real and complex eigenvalues. Eigenvalues have many applications in various fields. In graph theory, eigenvalues characterized the topological structures of a graph. In addition, we can use the eigenvalues to calculate the number of structures that belong to a graph, such as spanning trees, Hamiltonian paths, Hamiltonian circuits, induced sub-graphs and etc.

2. Basic Theory
The antiadjacency matrix of directed graph \(G\) is a matrix \(B(G) = J - A(G)\), where \(A(G)\) is an adjacency matrix of graph \(G\) and \(J\) is a \(n \times n\) matrix whose all entries are 1 [2]. Suppose \(A\) is a \(n \times n\) matrix. The principal-submatrix of \(A\) of \((n - r) \times (n - r)\) matrix is the submatrix that obtained
deleting \( r \) rows & \( r \) columns with the same index of \( A \) simultaneously. The principal-minor of \( A \) is the determinant of the principal submatrix of \( A \) [3]. A wheel graph \( W_\text{n} \) with order \( n \) is a graph that contains a cycle graph \( C_{n-1} \), and every vertex in the cycle graph \( C_{n-1} \) is connected to the middle vertex \[4\]. Thus, a directed cyclic wheel graph \( \overrightarrow{W_\text{n}} \) is a wheel graph that has a certain directed path with a cyclic direction.

![Figure 1](image)

**Figure 1.** A directed cyclic wheel graph \( \overrightarrow{W_\text{n}} \) with order \( n, (n \geq 4) \), that has a certain named vertex

Here are some theorems, lemma and corollary of previous studies that support the research in this paper.

**Theorem 2.1:** Suppose \( B \) is the antiadjacency matrix of a directed cyclic cycle graph \( \overrightarrow{C_n} \), then \( \det(B) = n - 1 \) [5].

**Corollary 2.2:** Suppose \( G \) is an directed acyclic graph and \( B \) is the antiadjacency matrix of \( G \). If \( G \) contains a Hamiltonian path then \( \det(B) = 1 \), and otherwise \( \det(B) = 0 \) [2].

**Lemma 2.3:** Suppose \( B \) is a \( n \times n \) matrix that has entries \((0, 1)\) such that \( b_{ij} = 1 \) if \( i \geq j \). If \( b_{12} = b_{23} = b_{34} = b_{45} = \ldots = b_{(n-1)n} = 0 \) then \( \det(B) = 1 \), and otherwise \( \det(B) = 0 \) [2].

**Theorem 2.4:** If \[2\alpha^n + c_1\lambda^{n-1} + c_2\lambda^{n-2} + \ldots + c_{n-1}\lambda + c_n = 0 \] is the characteristic equation of a matrix \( A_{\text{n}x\text{n}} \), then \( c_i = (-1)^i \sum_{j=1}^{w} |A_i^{(j)}|, i = 1, 2, \ldots, n \), where \( |A_i^{(j)}| \) is a principal minor of size \( i \times i \) of \( A \) and \( j = 1, 2, 3, \ldots, w \), and \( w \) is the number of the principal minor of size \( i \times i \) of \( A \) [3].

**Theorem 2.5:** If \( A \) is a \( n \times n \) triangular matrix, then \( \det(A) = a_{11}a_{22}\ldots a_{nn} \) [6].

**Theorem 2.6:** Let \( A \) be a \( n \times n \) matrix, then (a) If \( B \) is the matrix that has results when one row or one column of \( A \) is multiplied by a scalar \( k \), then \( \det(B) = k \det(A) \); (b) If \( B \) is the matrix that has results when two rows or two columns of \( A \) are interchanged, then \( \det(B) = -\det(A) \); (c) If \( B \) is the matrix that has results when a multiple of one row or one column of \( A \) is added to another, then \( \det(B) = \det(A) \) [7].

3. Main Result

In this section we discuss about the theorems related to the main results. The following is a general form of an antiadjacency matrix of graph \( \overrightarrow{W_n} \) of \( n \times n \) matrix that denoted by \( B(\overrightarrow{W_n}) = [b_{ij}] \), where

\[
b_{ij} = \begin{cases} 
0, & i = n - 1, j = 1; \text{or } i = 1, 2, \ldots, n - 2, j = i + 1; \text{or } i = n, j = 1, 2, \ldots, n - 1 \\
1, & \text{other}
\end{cases}
\]

**Theorem 3.1**

Suppose \( B(\overrightarrow{W_n}) \) is the antiadjacency matrix of a graph \( \overrightarrow{W_n} \), then \( \det(B(\overrightarrow{W_n})) = n - 2 \).

**Proof**

By using cofactor expansion, applying row reduction process and column reduction process, it was obtained the upper triangular matrix. By applying Lemma 2.3, Theorem 2.5 and Theorem 2.6, it followed that \( \det(B(\overrightarrow{W_n})) = n - 2 \). □
Theorem 3.2
Suppose \( P(B(\overline{W}'_n)) = \lambda^n + b_1\lambda^{n-1} + \sum_{i=2}^{n-2} b_i\lambda^{n-i} + b_{n-1}\lambda + b_n \) is the characteristic polynomial of \( B(\overline{W}'_n) \). Then \( ||b_i|| \), \( i = 1, \ldots, n \) represents the number of Hamiltonian paths, each of them has length equal to \((i - 1)\), contained in the induced sub-graphs minus the number of the cyclic induced sub-graphs.

Proof
An induced sub-graph that contains Hamiltonian paths, each of them has length equal to \((i - 1)\), of graph \( \overline{W}'_n \) can be obtained by deleting \((n - i)\) connected vertices, included all the edges that connected with the \((n - i)\) vertices on graph \( \overline{W}'_n \), where \( i = 1, \ldots, n \). The proof was divided into 4 cases:

i. \( ||b_1|| = n \) was obtained from deleting \((n - 1)\) vertices.

ii. \( ||b_i|| = 2(n - 1) \), \( i = 2, \ldots, n - 2 \) was obtained from deleting \((n - i)\) vertices.

iii. \( ||b_{n-1}|| = 2n - 3 \) was obtained from deleting \( n - (n - 1) = 1 \) vertex.

iv. \( ||b_n|| = n - 2 \) was obtained from deleting \( n - n = 0 \) vertex.

In every case, except the fourth case, there were two types of deletion. In the first type, the middle vertex was deleted first. In the second type, the vertices on the cycle path were deleted first. The deletion of \((n - i)\) vertices \((i = 1, \ldots, n)\) generated some induced sub-graphs which each of them could be represented in the form of a \( i \times i \) principal-submatrix. Referring to Theorem 2.4 by expanding \( \sum_{j=1}^{w} A_1^{(j)} \), the norm of every coefficient of the characteristic polynomial, \( ||b_i|| \), \( i = 1, \ldots, n \), was obtained.

i. In the first type, the middle vertex was deleted, then \((n - 2)\) vertices on the cycle path were deleted, it remained one vertex on the cycle path. Because there were \((n - 1)\) vertices on the cycle path, then there were \((n - 1)\) ways of deleting \((n - 2)\) vertices on the cycle path. In the second type, after deleting \((n - 1)\) vertices on the cycle path, it remained the middle vertex. From these two types of deletion, it followed that there were \( n - 1 + 1 = n \) induced sub-graphs which each of them contained one vertex and a Hamiltonian path with length equal to zero. Every induced sub-graph could be represented by the principal-submatrix of \( 1 \times 1 \) matrix. Since the induced sub-graph was acyclic and contained a Hamiltonian path, then according to Corollary 2.2 the determinant of the principal-submatrix was equal to 1. As there were \( n \) induced sub-graphs, according to Theorem 2.4, it follows that \( ||b_1|| = n \). Since there was no cyclic induced sub-graph, it could be concluded that \( ||b_1|| = n \) denotes the number of Hamiltonian paths which each of them had length equal to zero and contained in the induced sub-graphs minus the number of the cyclic induced subgraphs.

ii. In the First type, the middle vertex was deleted, then \((n - i - 1)\) vertices \((i = 2, \ldots, n - 2)\) on the cycle path were deleted, it remains \( i \) vertices on the cycle path that connected to each other. Since there were \((n - 1)\) vertices on the cycle path, there were \((n - 1)\) ways of deleting \((n - i - 1)\) vertices on the cycle path. In the second type, after deleting \((n - i)\) vertices \((i = 2, \ldots, n - 2)\) on the cycle path, it remained \((i - 1)\) vertices on the cycle path where every vertex was connected to the middle vertex. By taking the middle vertex as the first vertex, it was obtained a Hamiltonian path with length equal to \((i - 1)\). As there were \((n - 1)\) vertices on the cycle path, then there were \((n - 1)\) ways of deleting \((n - i)\) vertices on the cycle path.

Figure 2. The induced sub-graphs that contained \( i \) vertices \((i = 2, \ldots, n - 2)\) and Hamiltonian paths which each of them had length equal to \((i - 1)\) as a result of deleting \((n - i)\) connected vertices. (a) The result of the first type. (b) The result of the second type.
From those two following types, there were \( n - 1 + n - 1 = 2(n - 1) \) induced sub-graphs which each of them contains \( i \) vertices and contains a Hamiltonian path with length equal to \((i - 1)\). Every induced sub-graph could be represented by the principal-submatrix of \( i \times i \) matrix. Since the induced sub-graph was acyclic and contained a Hamiltonian path, then according to corollary 2.2 the determinant of the principal-submatrix was equal to 1. Because there were \( 2(n - 1) \) induced sub-graphs, according to Theorem 2.4, it followed \( \|b_i\| = 2(n - 1), i = 2, ..., n - 2 \). As there was no cyclic induced sub-graph, it could be concluded that \( \|b_i\| = 2(n - 1), i = 2, ..., n - 2 \) denoted the number of Hamiltonian paths which each of them has length equals to \((i - 1)\) and contained in the induced sub-graphs minus the number of the cyclic induced sub-graphs.

iii. In the first type, after deleting the middle vertex, it remained a cyclic induced sub-graph \( \overrightarrow{C_{n-1}} \). Moreover, by deleting one edge of \( \overrightarrow{C_{n-1}} \), it was obtained a Hamiltonian path with length equal to \((n - 2)\), and those deletions were related to the choice of a vertex as the first vertex of any Hamiltonian path. Since \( \overrightarrow{C_{n-1}} \) contained \((n - 1)\) vertices, there were \((n - 1)\) ways to choose a vertex as the first vertex. In the second type, after deleting one vertex on the cycle path, it remained \((n - 2)\) vertices on the cycle path where every vertex was connected to the middle vertex. By taking the middle vertex as the first vertex, it was obtained a Hamiltonian path with length equal to \((n - 2)\). Since there were \((n - 1)\) vertices on the cycle path, there were \((n - 1)\) ways of deleting one vertex on the cycle path.

![Figure 3](image-url)

**Figure 3.** The induced sub-graphs that contained \( n - 1 \) vertices and Hamiltonian paths, each of them had length equal to \((n - 2)\), as a result of deleting one vertex. (a) The result of the first type. (b) The result of the second type.

According to theorem 2.1, the determinant of the antiadjacency matrix of \( \overrightarrow{C_{n-1}} \) was \((n - 2)\). Secondly, it was obtained \((n - 1)\) induced sub-graphs which each of them contains \((n - 1)\) vertices and could be represented the principal-submatrix of \((n - 1) \times (n - 1)\) matrix. Since every induced sub-graph was acyclic and contained a Hamiltonian path, according to Corollary 2.2, the determinant of the principal submatrix was equal to 1. As there were \((n - 1)\) induced subgraphs, then the sum of the determinants of the principal submatrices were \((n - 1)\). Thus, by combining the result of the first and the second steps and also referring to Theorem 2.4, it followed that \( \|b_{n-1}\| = 2n - 3 \). For there was only one cyclic induced sub-graph, it could be concluded that \( \|b_{n-1}\| = 2n - 3 \) denoted the number of Hamiltonian paths which each of them had length equal to \((n - 2)\) and contained in the induced sub-graphs minus the number of the cyclic induced sub-graphs.

iv. After deleting the zero vertexes, it was obtained \( \overrightarrow{W_n} \) as the cyclic induced sub-graph. According to Theorem 3.1, the determinant of \( B(\overrightarrow{W_n}) \) was \((n - 2)\). Since there was only one induced sub-graph, according to Theorem 2.4, it follows that \( \|b_n\| = n - 2 \). Moreover, the graph \( \overrightarrow{W_n} \) contained Hamiltonian paths which each of them had length equal to \((n - 1)\) and had the middle vertex as the starting point. For the middle vertex should be the starting point, the end point should be one of the vertices on the cycle path. As there were \((n - 1)\) vertices on the cycle path, it followed that there were \((n - 1)\) Hamiltonian paths. Furthermore, there was only one cyclic induced sub-graph. Thus, it could be concluded that \( \|b_n\| = n - 2 \).
denoted the number of Hamiltonian paths which each of them has length equal to \((n - 1)\) and contained in the induced sub-graphs minus the number of the cyclic induced sub-graphs. ■

**Corollary 3.3**

Let \( P(B(\overline{W_n})) = \lambda^n + b_1 \lambda^{n-1} + \sum_{i=2}^{n-2} b_i \lambda^{n-i} + b_{n-1} \lambda + b_n \) is the characteristic polynomial of \( B(\overline{W_n}) \). Then \( b_1 = -n; b_i = (-1)^i 2(n - 1), i = 2, ..., n - 2; b_{n-1} = (-1)^{n-1}(2n - 3)& b_n = (-1)^n(n - 2). \)

**Proof**

From Theorem 3.2 it followed that \( \|b_1\| = n; \|b_i\| = 2(n - 1), i = 2, ..., n - 2; \|b_{n-1}\| = 2n - 3 \) and \( \|b_n\| = n - 2. \) By using the equation \( b_i = (-1)^i \sum_{j=1}^{n} A_i^{(j)} \) in Theorem 2.4 it follows that;

- For \( i = 1, b_1 = (-1)^i \sum_{j=1}^{n} A_i^{(j)} = (-1)^i \|b_1\| = -n. \)
- For \( i = 2, ..., n - 2, b_i = (-1)^i \sum_{j=1}^{n} A_i^{(j)} = (-1)^i \|b_i\| = (-1)^i 2(n - 1). \)
- For \( i = n - 1, b_{n-1} = (-1)^{n-1} \sum_{j=1}^{n-1} A_{n-1}^{(j)} = (-1)^{n-1} \|b_{n-1}\| = (-1)^{n-1}(2n - 3). \)
- For \( i = n, b_n = (-1)^n \sum_{j=1}^{n} A_n^{(j)} = (-1)^n \|b_n\| = (-1)^n(n - 2). \)

Therefore, this corollary was proved. ■

3.4. **Theorem 3.4**

The real eigenvalues of \( B(\overline{W_n}) \) are 1 and \( n - 2 \) for every \( n. \)

**Proof**

By substituting \( \lambda = 1 \) in \( P(B(\overline{W_n})) \), it followed that \( P(B(\overline{W_n})) = 0 \) for every \( n. \) It means that \( \lambda = 1 \) was one of the real eigenvalues. By using Horner method, it was obtained that the other real eigenvalues were \( (2k - 2) \) for \( n = 2k \) (even) \& \( (2k - 1) \) for \( n = 2k + 1 \) (odd), \( k \geq 2. \) So it can be concluded that \( B(\overline{W_{2k}}) \) has two real eigenvalues namely \( \lambda = 1 \) and \( \lambda = 2k - 2. \) Furthermore, \( B(\overline{W_{2k+1}}) \) had \( \lambda = 1 \) whose multiplicity equals 2 and \( \lambda = 2k - 1 \) as its eigenvalues. Figure 4 represented the Horner method which used to find the real roots of \( P(B(\overline{W_{2k}})) \) (the real eigenvalues of \( B(\overline{W_{2k}})) \).

![Figure 4](image)

In fact, by using Horner method, the coefficients of \( \lambda^i \) of \( P(B(\overline{W_{2k}})) \) were also found (see Figure 4).

The coefficients of \( \lambda^i, i = 1, ..., 2k - 2 \) in \( P(B(\overline{W_{2k}})) \) is \((-1)^i\) while the coefficients of \( \lambda^{2i}, i = 1, ..., k - 1 \) in \( P(B(\overline{W_{2k+1}})) \) is \((-1)^{2i}\). Hence, \( P(B(\overline{W_n})) \) could be written as the following \( P(B(\overline{W_{2k}})) = (\lambda - 1)(\lambda - 2k) \) & \( P(B(\overline{W_{2k+1}})) = (\lambda - 1)^2(\lambda - 2k - 1). \)

**Theorem 3.5**

The number of the complex eigenvalues of \( B(\overline{W_n}) \) is always even and it conjugate to each other for every \( n. \)

**Proof**

The complex eigenvalues of \( B(\overline{W_{2k}}) \) could be obtained from the equation \( 1 + \sum_{i=1}^{2k-2}(-1)^i \lambda^i = 0 \) which was similar to a finite geometric series form \( (\alpha = 1, r = (-\lambda), n = 2k - 1). \) Then, the sum of the series was

\[
\frac{1((-\lambda)^{2k-1}-1)}{(-\lambda)-1} = 0, \lambda \neq -1
\] (1)
From the equation (1), it followed that the complex eigenvalues were

$$
\lambda = e^{i\frac{\pi}{2(2k-1)}}; m = 0, \ldots, 2k - 2
$$

(2)

Since \( \lambda = e^{i\pi} \) was achieved when \( m = k - 1 \) in the Equation (2) and since \( \lambda \neq -1 \) in the Equation (1), it followed that there were \((2k - 2)\) complex eigenvalues of \( B(W_{2k}) \). Hence, the number of complex eigenvalues of \( B(W_{2k}) \) were even. It could be proved that the eigenvalues \( \lambda_{k-1+j} = e^{i\frac{\pi}{2k-1}(2k-2+2j)m} \) was the conjugate of the eigenvalue \( \lambda_{k-1-j} = e^{i\frac{\pi}{2k-1}(2k-2+2j)m}, j = 1, \ldots, k - 1. \)  

The complex eigenvalues of \( B(W_{2k+1}) \) could be obtained from the equation \( (1 + \sum_{i=1}^{k}(-1)^{2i} \lambda^{2i}) = 0 \) which was similar to a finite geometric series form \( (a = 1, r = (\lambda^{2}), n = k) \). Then, the sum of the series was

$$
\frac{1}{(\lambda^{2})^{k-1}} = 0, \lambda \neq 1 \text{ and } \lambda \neq -1
$$

(3)

From the Equation (3), it follows that the complex eigenvalues were

$$
\lambda = e^{i\frac{m\pi}{k}}; m = 0, \ldots, 2k - 1
$$

(4)

Since \( \lambda = e^{0} \& \lambda = e^{i\pi} \) were achieved when \( m = 0 \& m = k \) respectively in the Equation (4) and since \( \lambda \neq 1 & \lambda \neq -1 \text{in the Equation (3), it follows that there are (2k - 2) complex eigenvalues of } B(W_{2k+1}) \). Hence, the number of complex eigenvalues of \( B(W_{2k+1}) \) were even. It could be proved that the eigenvalues \( \lambda_{k+j} = e^{i\frac{(k+j)\pi}{k}} \) was the conjugate of the eigenvalue \( \lambda_{k-j} = e^{i\frac{(2k-j-1)\pi}{k}}, j = 1, \ldots, k - 1. \)

4. Conclusion

Based on the main results, \( \lambda^{n} + b_{1}\lambda^{n-1} + \cdots + b_{n-1}\lambda + b_{n} \) was considered as the characteristic polynomial of \( B(W_{n}) \). \(|b_{i}|, i = 1, \ldots, n \) represented the number of Hamiltonian paths which each of them had length equal to \((i - 1)\) and contained in the induced sub-graphs minus the number of the cyclic induced sub-graphs. The values of the coefficients of the characteristic polynomial of \( B(W_{n}) \) were given in Table 1.

| Coefficient | Coefficient value |
|-------------|-------------------|
| \(b_{i}\) | \((-1)^{i}2(n - 1), \quad i = 2, \ldots, n - 2\) |
| \(b_{n-1}\) | \((-1)^{n-1}(2n - 3)\) |
| \(b_{n}\) | \((-1)^{n-1}(n - 2)\) |

It was also obtained that \( B(W_{n}) \) had two real eigenvalues namely 1 and \((n - 2)\) for every \( n \). While thenumber of the complex eigenvalues were always even and it conjugate to each other for every \( n \). The eigenvalues of \( B(W_{n}) \) were given in Table 2.

| The number of n | Eigenvaues |
|-----------------|-----------|
| \(n = 2k; k \geq 2\) | \(\lambda = 1 & \lambda = 2k - 2\) |
| \(n = 2k + 1; k \geq 2\) | \(\lambda = 1 & \lambda = 2k - 1\) |

| The number of n | Complex |
|-----------------|---------|
| \(n = 2k; k \geq 2\) | \(\lambda = e^{i\frac{\pi}{2k-1}}; m = 0, \ldots, 2k - 2. \text{Except } \lambda = e^{i\pi}\) |
| \(n = 2k + 1; k \geq 2\) | \(\lambda = e^{i\frac{\pi}{2k-1}}; m = 0, \ldots, 2k - 1. \text{Except } \lambda = e^{0} & \lambda = e^{i\pi}\) |
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