The Pioneer Anomaly: The Measure of a Topological Phase Defect of Light in Cosmology

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Abstract

It is shown that a wave vector representing a light pulse in an adiabatically evolving expanding space should develop, after a round trip (back and forth to the emitter) a geometric phase for helicity states at a given fixed position coordinate of this expanding space. In a section of the Hopf fibration of the Poincaré sphere $S^2$ that identifies a projection to the physically allowed states, the evolution defines a parallel transported state that can be joined continuously with the initial state by means of the associated Berry-Pancharatnam connection. The connection allows to compute an anomaly in the frequency for the vector modes in terms of the scale factor of the space-time background being identical to the reported Pioneer Anomaly.

1. Introduction

Analysis of the radio-metric tracking data from the Pioneer 10/11 spacecraft at distances between 20-70 astronomical units (AU) from the Sun has consistently indicated the presence of an anomalous, small, constant Doppler frequency drift. The drift is a blue-shift, uniformly changing with rate $a_t = (2.92 \pm 0.44) \times 10^{-18} s^2$. It can also be interpreted as a constant acceleration of $a_P = (8.74 \pm 1.33) \times 10^{-8} cm/s^2$ apparently directed towards the Sun [1], [2]. This signal has become known as the Pioneer Anomaly since it does not seem to correspond to standard Newtonian dynamics (as far as this kind of anomalous acceleration has never been found perturbing the orbits of the planets in the Solar System.)

There were attempts to explain the anomaly based on the recently discovered accelerated expansion of the Universe. This association was motivated by the numerical coincidence that links the magnitude of the Pioneer Anomaly to the product of the Hubble constant and the speed of light; on the other hand, since Hubble’s flow would be for the probes vanishing small in comparison with their typical velocities in the scale of the Solar System, the effect can not be originated from Hubble’s dynamics acting on the probes $^2$. Moreover, intuitively, such a mechanism would produce an opposite sign for the effect. Yet, let us look at the

\[ \delta \omega / \omega \simeq -\hbar (R/c)(v_P/c)^2, \]
\[ \hbar \text{ denoting the local expansion rate, } R/c, \text{ standing for the Doppler delay of light signals from the spacecraft at } R \text{ and } v_P \text{ the probe velocity. See the Appendix.} \]

1 I dedicate this paper to my daughter Ana

2 $\delta \omega / \omega \simeq -\hbar (R/c)(v_P/c)^2$, $\hbar$ denoting the local expansion rate, $R/c$, standing for the Doppler delay of light signals from the spacecraft at $R$ and $v_P$ the probe velocity. See the Appendix.
problem again disregarding the motion of particles in Hubble’s flow, i.e., from a geometrical perspective, upon studying the internal states of light themselves during a measurement of the spacecraft position. Need our intuition still be valid?

In order to escape from such common intuitions, this paper analyses the true effect derived from the existence of some small, non vanishing, local space expansion rate on the adiabatic evolution of internal states of light. We will not be concerned here on the exact physical meaning of such an hypothesis, rather, we want to obtain the expected measurable consequences of it. Very surprisingly a proof will be given below in the sense that such an effect does exist and that a locally expanding space time originates a blue shift frequency anomaly of geometrical origin in the phase of light being the same for every possible polarization state. The result is in full agreement with some earlier heuristic proposals [3] [4]. Consequently theoretical and future experimental work remains to be made in order to clarify the consequences of the present proof and the measurement of the anomaly.

Let us start, then, upon considering an expanding space-time with metric given by

\[ ds^2 = c^2dt^2 - \chi(t)^2(dR^2 + R^2d\Omega^2) \]  

We are interested in the mathematical description of the measurable phase of a beam of light immersed in this space-time at a given constant coordinate position \( R \), after a round trip of total time \( T \) has taken place. This corresponds to the measurement of the approximate distance \( cT/2 \) to some remote mirror on which light bounces at \( t = T/2 \), back to the emitter as in the tracking signal experiment to the space probes. The problem is, then, different from that of obtaining the phase of a light wave emitted from a distant source (galaxies, say.) In such an expanding space it is well known that there exists a red shift in the frequency of such distant light source which is Hubble’s law. Now, instead, we want to compare the phase of a given photon emitted from \( R \) at time \( t = 0 \) and observed at this same position after a round trip of time \( t = T \). In order to cope with this problem, we should model \( \chi(t) \) as a external slow adiabatic parameter for the internal evolution of the phase states of light. This is obviously a geometric (instead of dynamic) problem for the internal state of light that can be visualized as that of determining the parallel transported state of light after such an adiabatic evolution in the external parameter space has taken place.

\footnote{notice that the metric (1) might be the special case, for \( \chi(t) = e^{(\Lambda/3)^{1/2}t} \) and \( M/R \ll 1 \), of the Schwarzschild-DeSitter metric}

\[ ds^2 = (1 - 2m/r - \frac{\Lambda}{3} r^2) c^2 dr^2 - \frac{1}{1 - 2m/r - \frac{\Lambda}{3} r^2} dr^2 \]

for \( r = R\chi(t) \), \( \tau = t - \frac{1}{2e^{(\Lambda/3)^{1/2}t}} \ln(1 - \frac{4}{\Lambda} R^2 \chi(t)^2) \) and \( M(t) = m\chi(t) \), that is

\[ ds^2 \simeq (1 - 2M(t)/R) dt^2 - \frac{1}{1 - 2M(t)/R} \chi^2(t) dR^2 \]
2. Helicity states of light: the Hopf fibration of $S^2$

In order to enlighten the solution to this problem, we need a completely new mathematical framework. What we require now is a description of light in terms of its internal helicity complex vector state $\mathbf{h}$, and we intend to separate the dynamic (temporal) fast evolution from the geometrical (adiabatic) evolution of the internal phase state of light.

The polarization complex vector satisfy

$$\mathbf{h} = (x, y), \mathbf{h}^\dagger \cdot \mathbf{h} = 1.$$ \hspace{1cm} (2)

$x, y$ being complex numbers. It might also conveniently be described in terms of the two component spinor

$$|\Psi\rangle = \begin{bmatrix} \Psi_+ \\ \Psi_- \end{bmatrix},$$ \hspace{1cm} (3)

for, $\Psi_\pm \equiv \frac{1}{\sqrt{2}}(x \pm iy) \exp(i\beta)$. Thus $<\Psi|\Psi> = 1$, and $\beta$ an arbitrary phase. Each such $|\Psi\rangle$ is the eigenvector of some Hermitian matrix (the polarization matrix "Hamiltonian") of the form

$$r \cdot \sigma = \begin{pmatrix} z & x - iy \\ x + iy & -z \end{pmatrix} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \exp(-i\phi) \\ \sin(\theta) \exp(+i\phi) & -\cos(\theta) \end{pmatrix}.$$ \hspace{1cm} (4)

where $\sigma$ is the vector of Pauli matrices and $r = (x, y, z)$ is a unit vector with polar angles $\theta$ and $\phi$. The geometrical coordinates $(\theta, \phi)$ define the Poincaré sphere $S^2$. The relevant spinor is an eigenstate of the Hermitian matrix above times an arbitrary phase,

$$|\Psi(t)\rangle = \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \exp(i\psi) \end{bmatrix} \exp(i\beta),$$ \hspace{1cm} (5)

Which is a vector in some enlarged space that will be defined below. To this new space belongs a set of complex spinors with three degree of freedom (hereafter we follow [5])

$$|\Psi\rangle = \begin{bmatrix} x_1 + ix_2 \\ x_3 + ix_4 \end{bmatrix},$$ \hspace{1cm} (6)

such that

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = 1,$$ \hspace{1cm} (7)

the $x_i$ are rather the coordinates of a point on an $S^3$. Upon defining $\psi = \beta + \phi$, we get from (5), (6) and (7)

$$x_1 = \cos(\theta/2) \cos(\beta)$$
$$x_2 = \cos(\theta/2) \sin(\beta)$$
$$x_3 = \sin(\theta/2) \cos(\psi)$$
$$x_4 = \sin(\theta/2) \sin(\psi),$$
The metric of this space is
\[ ds^2 = \frac{1}{4}d\theta^2 + \cos^2(\theta/2)d\beta^2 + \sin^2(\theta/2)d\psi^2, \tag{8} \]
thus, we have enlarged the two polarization degree of freedom to three by means of a \( U(1) \) gauge field \( \beta \). \( S^3 \) can be regarded as a principal bundle with base space \( S^2 \) and a \( U(1) \) structure group. This procedure is called Hopf fibration of \( S^2 \).

On the other hand, in order to identify these geometrical state coordinates with relevant physical quantities defined in the physical space-time, recall that, for the metric (1), the Eikonal of a light wave at the space-time point of physical coordinates \((R, T)\) is given by\(^4\)
\[ \Xi(r, t) = -[\omega T - \frac{\omega}{c} \mathbf{l} \cdot \mathbf{R} \chi] . \tag{9} \]
Where \( \mathbf{l} \cdot \mathbf{R} = \pm R \) for circular positive and negative polarization states corresponding to the North and the South Pole of \( S^2 \). In the spinor formalism, it is equivalent to taking \( \beta = -\phi(R, T)/2, \psi = +\phi(R, T)/2 \) and \( \phi(R, T) = -2\frac{\omega}{c} \chi R \). Restricting \( \beta \) and \( \psi \) in this way is called taking a section \( \mathfrak{C} \) of the fibre bundle. Therefore, after (9)
\[ |\Psi_+> = |\epsilon_+> \exp(i\frac{\omega}{c}R\chi), \tag{10} \]
and
\[ |\Psi_-> = |\epsilon_-> \exp(-i\frac{\omega}{c}R\chi), \tag{11} \]
\((|\epsilon_+> \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix} \) and \( |\epsilon_-> \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix} \)). These are the counterpropagating (positive and negative helicities) wave modes as required for an electromagnetic field propagating in a space-time whose metric is given by equation (1).

In order to clarify the formalism let us introduce the "magnetic-like field" \(^5\)
\[ d\mathfrak{B} = \partial[\phi/2], \tag{12} \]
\(^4\)It is easily verified. Given that,
\[ \Xi = -g_{\mu\nu}k^\mu x^\nu \]
and that, for \( k_0 = \omega/c \)
\[ k_\mu k^\mu = \omega^2/c^2 - \chi^2(k^R)^2 = 0, \]
it follows that
\[ (k^R)^2 = (\omega/c\chi)^2 \]
and
\[ k_R = \pm \chi \omega/c \]
\(^5\)Hereafter we will use the notation \( \partial \)-variation with respect to the external parameter \( \chi \), it should correspond to a geometrical variation. this is different with respect to \( d \)-differentiation.
then, we rewrite equations. (10) and (11) as
\[
|\Psi(\chi)\rangle = e^{-i \int d\mathcal{S}} |\Psi\rangle ,
\]
for the "Hamiltonian"
\[
d\mathcal{S} \equiv d\mathfrak{W} \cdot \sigma_3 ,
\]
\[|\Psi(\chi)\rangle \text{ is then, the solution of a Schrödinger equation}
\]
\[
id|\Psi(\chi)\rangle = d\mathcal{S}|\Psi(\chi)\rangle .
\]

Equations (10)-(15) define the unitary evolution of the polarization state in the expanding space-time. Notice that the evolution of the helicity states of light, satisfying Maxwell equations, within an expanding space-time is formally equivalent to that of a quantum spinor. We will now explore more deeply this similarity.

3. Adiabatic evolution: the Berry Connection.

Let us consider changes in the state of polarization accomplished continuously and consider the curved space-time as a dielectric medium. If the dielectric variation \(\chi(t)\) is slow enough, the beam remains in a polarization state. In a cycle on the Poincaré sphere the displacement unit vector \(\mathfrak{h}\) will be accompanied by a phase, the Pancharatnam’s phase given by
\[
< A|A' > = \exp[-i \frac{\Omega(C)}{2}] .
\]
where, the cycle \(C\) connects the state \(|A\rangle\) with \(|A'\rangle\) and \(\Omega(C)\) is the solid angle of the circuit \(C\) on the Poincaré sphere. During adiabatic evolution, the local eigenstates are continued by means of the differential equation
\[
\mathfrak{h}^\dagger \cdot d\mathfrak{h} = 0 ,
\]
i.e., the polarization state is parallel transported through the cycle. This is consequence of the field being governed by Maxwell’s equations as was shown by Berry. Using the definitions of the spinor Schrödinger-like evolution of the previous section, we see that, (after subtraction of the trivial dynamic contribution) one equivalently obtains
\[
< \tilde{\Psi}|\tilde{d}\Psi > =0 ,
\]
for
\[
|\tilde{\Psi}(T)\rangle = \exp\{i \int_0^T \hat{\mathfrak{H}}(t')dt'\} |\Psi(T)\rangle ,
\]
and
\[
\hat{\mathfrak{H}}[t'] = \frac{\partial}{\partial t'} \text{Re} < \tilde{\Psi}(\chi,t')|\int_1^{\chi(t')} d\mathcal{S}(\chi)|\Psi(\chi,t') > .
\]
Equation (18) is the Berry connection\textsuperscript{6}. We are now interested on the adiabatic evolution of a spinor state when $\beta(\chi)$ and $\psi(\chi)$ vary continuously and slowly enough so that if the system is initially in the state $|\Psi>$, eigenstate of the Hamiltonian, $|\Psi(\chi)>$ will also instantaneously be an eigenstate of the same Hamiltonian; this is the condition of the adiabatic theorem. The formal equivalence of this system with the quantum spinor allows using Berry’s theorem\textsuperscript{7}: During adiabatic evolution, the total phase change of $|\Psi(\chi)>$ round $C(\beta,\psi)$ is given by

$$|\Psi(\chi)> = \exp(i\gamma(C))\exp[-i\int_1^X d\xi(\chi)]|\Psi(1)>,$$

for $\gamma(C)$, the Berry phase given by\textsuperscript{7}

$$\gamma(C) = i \int_\xi <\Psi|\partial \Psi>,$$

$$|\Psi(\chi)> = \exp(i\gamma(C))|\Psi(1)>.$$

or, after Equation (5),

$$\gamma(C) = - \int_\xi \cos^2(\theta/2) d\beta + \sin^2(\theta/2) d\psi.$$

This defines a vector potential

$$\gamma(C) = - \int_\xi A \cdot \partial r$$

where, via the $S^3$ metric in Equation (8)

$$A_\theta = 0, A_\beta = \cos(\theta/2), A_\psi = \sin(\theta/2).$$

These potentials are manifestly non-singular. For helicity states, $\theta = 0$, and $\theta = \pi$

$$\gamma(C) = - \int_\xi d\beta, \ \theta = 0$$

$$\gamma(C) = - \int_\xi d\psi, \ \theta = \pi$$

\textsuperscript{6}Clairly, $|\Psi(\tau)>$ satisfies the equation

$$i\hbar|\Psi(\tau)> = \{d\xi - \hbar(t) dt\}|\Psi(\tau)>.$$

Contracting this with $<\Psi(\tau)|$ (also using $<\Psi|\Psi> = 1$) yields the paralell transport law (18).

$$|\Psi(1)> = \exp(-i\int d\xi - \hbar(t) dt)|\Psi(1)>.$$
for the section \( \mathcal{C} \) of the fibre bundle \( \beta(\chi) = -\psi(\chi) \), equivalently \( \beta = -\phi/2 \), i.e., the physically allowed states, this obtains

\[
\gamma(\mathcal{C}) = \pm \frac{\partial \phi(\chi)}{2} \simeq \mp R \frac{\omega}{c} \chi T ,
\]

(28)

Recall \( \partial \phi = -2 R \frac{\omega}{c} \chi (T) - 1 \). This phase shift corresponds to positive, negative helicities respectively. Moreover, for a general polarization state we get

\[
\gamma(\mathcal{C}) = \cos(\theta) \frac{\partial \phi}{2} = -\cos(\theta) R \frac{\omega}{c} \chi T .
\]

(29)

4. The frequency Anomaly.

We will now obtain a remarkable consequence of equations (22) and (29). The parallel transported positive and negative helicity states are given by

\[
|\tilde{\Psi}(T) > = \exp[\mp i R \frac{\omega}{c} (\pm \chi T - 1)]|\varepsilon_\pm > .
\]

(30)

and, as a result of the Berry connection,

\[
< \tilde{\Psi} | d \tilde{\Psi} > = 0 ,
\]

(31)

one directly gets, for both cases

\[
\omega_\pm = \omega_\mp \dot{\chi}
\]

(32)

For general polarization states we get again from equations (22) and (29)

\[
|\Psi(\theta, T) > = e^{-i \cos(\theta) R \frac{\omega}{c} \chi T} \{ \cos(\theta/2) e^{i R \omega/c} |\varepsilon_+ > + \sin(\theta/2) e^{-i R \omega/c} |\varepsilon_- > \} .
\]

(33)

And the Berry connection (31) indicates that

\[
\cos^2(\theta/2) a_\chi(\theta, T) + \sin^2(\theta/2) b_\chi(\theta, T) = 0
\]

(34)

where \( a_\chi(\theta, T) = -\frac{\omega}{\omega} (1 - \dot{\chi} T \cos \theta) + \dot{\chi} \cos(\theta) \) and \( b_\chi(\theta, T) = \frac{\omega}{\omega} (1 + \dot{\chi} T \cos \theta) + \dot{\chi} \cos(\theta) \). Solving this for \( \frac{\omega}{\omega} \) we finally obtain the general expression of the frequency anomaly

\[
\dot{\omega}_\theta = \omega_\theta \dot{\chi} + O(\dot{\chi}),
\]

(35)

which is independent of \( \theta \) for every polarization state. This remarkable result coincides with the observed blue shift known as the Pioneer anomaly for \( a_t = \dot{\chi} \).
5. Conclusions.

The proof given here indicates that the "Pioneer effect" detected in radar signals [1], [2], [3] should have nothing to do with the probe but only with the fact that the spacecraft is acting as a "mirror" for light signals, thus, being a consequence of the adiabatic evolution of the internal states of light locally monitored by the global expanding space-time metric. The Anomaly should more properly be described as corresponding to the measure of a topological phase defect of light. There is a formal identification between the polarization vector temporal evolution in an expanding space with that of a two state quantum spinor, so we explore their common algebra to discover that, upon considering the expanding space as an adiabatic dielectric, it yields to obtaining a "quantum Berry’s phase" in a section of the Hopf fibration of the Poincaré sphere $S^2$ that identifies a projection to the physically allowed states. It becomes the frequency blue shift anomaly upon using the algebra of parallel transported states (i.e., the Berry connection). Thus, the Doppler anomalous phase shift finds its explanation on the grounds of a Berry phase, a geometrical effect. This is just a non dynamic element of the evolution of helicity states of light in the expanding space background. This demonstrates entirely that the effect should not affect to the planets but only to light and that it is wrongly interpreted as a dynamic acceleration, being fully equivalent to a calibration effect similarly to the Foucault Pendulum angle defect in measuring Earth rotation (the Hannay’s angle which, indeed, is the classical analog to the Berry’s quantum phase, also a geometrical effect). In this sense, light rays play a similar rôle in the expanding space than Foucault’s Pendulum does while determining Earth’s rotation. On the other hand, given that the result has nothing to do with dynamics, it does not violate Birkhoff’s theorem. Moreover, the anomaly only depends on the "Time of Flight" of light, since the location of the spacecraft has not enter into the proof. This predicts that a geostationary system of satellites (LISA mission, for instance) or perhaps other specific more advanced mission as recently proposed would obtain the same result [3], [4]. A completely new type of optical experiments becomes, indeed, also possible, for instance, optical laser ranging with active mirrors able to accumulate this phase topological defect in n-way round trips configurations. A kind of such an experimental arrangement could be a future sophistication of the Moon Laser Ranging Device.

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APPENDIX

We want to obtain in this appendix the order of magnitude of the perturbing effect, for the motion of a probe with radial velocity $v_r$, of the existence of a local expanding space-time. In order to do this, let us scale out the expansion of the space for the metric $ds^2 = c^2 dt^2 - \chi(t)^2 dR^2$, i.e., let us consider $R_\ast \equiv \chi R$.

The physical meaning of this choice of coordinates is that, during the motion of the spacecrafts, we use the Newtonian metric as parametrically static. In terms of this scaled radial coordinate, one gets

$$ds^2 = c^2(1 - R_\ast^2/c^2 h^2)dt^2 - dR_\ast^2 + 2hR_\ast dR_\ast dt$$

and $h \equiv \dot{\chi}/\chi$. This defines the radial vector $g_\ast \equiv -g_{0R_\ast}/g_{\ast\ast} \approx -hR_\ast/c$.

The radial velocity of the probe is, using the scaled coordinates,

$$(g_{\ast\ast})^{1/2}v_\ast = \frac{dR_\ast}{d\tau}$$

where $d\tau = dt - g_\ast dR_\ast/c$ is the proper time at $R_\ast$ for this curved space-time. This obtains

$$\dot{R}_\ast \approx v_\ast(1 + hR_\ast v_\ast/c^2) + O(\dot{\chi}^2).$$
The Doppler expected effect for the probes is

$$\omega' = \omega(1 - \dot{R}_*/c) \simeq \omega(1 - v_*/c) - \omega h(R_*/c)(v_*/c)^2.$$

This corresponds to an anomalous red shift $\delta \omega/\omega \simeq -ht(v_*/c)^2$ for $t = R_*/c$. That is why the Pioneer Anomaly can not be originated from the dynamic effect of the expansion acting on the probes. The link between the figures of Hubble’s ($h$) and Anderson’s ($a_t$) constants can not be dynamic.