Specific Scalar Mass Relations in $SU(3) \times SU(2) \times U(1)$ Orbifold Model

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Abstract

We study flat directions and soft scalar masses using a $Z_3$ orbifold model with $SU(3) \times SU(2) \times U(1)$ gauge group and extra gauge symmetries including an anomalous $U(1)$ symmetry. Soft scalar masses contain $D$-term contributions and particle mixing effects after symmetry breaking and they are parametrized by a few number of parameters. Some specific relations among scalar masses are obtained.

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1 Introduction

Superstring theories (SSTs) are powerful candidates for the unification theory of all forces including gravity. There are various approaches to explore 4-dimensional (4-D) string models, for example, the compactification on Calabi-Yau manifolds \cite{1}, the construction of orbifold models \cite{2, 3} and so on. Effective supergravity theories (SUGRAs) have been derived by taking field theory limit \cite{4, 5, 6}.

Effective low-energy theories have been derived under the assumption that supersymmetry (SUSY) is broken by $F$-term condensations of the dilaton field $S$ and/or moduli fields $T$ \cite{7, 8, 9} from effective SUGRAs. Some phenomenologically interesting features are predicted from the structure of soft SUSY breaking terms which are parameterized by a few number of parameters, for example, only two parameters such as a goldstino angle $\theta$ and the gravitino mass $m_{3/2}$ in the case with the overall moduli and the vanishing vacuum energy \cite{10}. The cases with multimoduli fields are also discussed in Refs.\cite{11}. Recently study on soft scalar masses has been extended in the presence of an anomalous $U(1)$ symmetry \cite{12, 13, 14}.

Now we have thousands of effective low-energy theories corresponding to 4-D string models following the above approach. It is much important to select a realistic string model by some experiments. Soft SUSY breaking parameters can be powerful probes. For example, string models with the SUSY breaking due to dilaton $F$-term lead to the highly restricted pattern such as \cite{8, 10, 15}

$$|A| = |M_{1/2}| = \sqrt{3}|m_{3/2}|$$

(1)

where $A$ is a universal $A$-parameter, and gauginos and scalars get masses with common values $M_{1/2}$ and $m_{3/2}$, respectively.

In a recent paper \cite{16}, the formula of soft SUSY breaking scalar masses has been derived from 4-D string models with flat directions \cite{17} within a more generic framework. The effects of extra gauge symmetry breakings, that is, $D$-term and $F$-term contributions, particle mixing effects and heavy-light mass mixing effects are considered. The above prediction (1) does not hold in string models with an anomalous $U(1)$ symmetry \cite{13, 16}.

The purpose of this paper is to apply the formula to a semi-realistic string model including gauge groups and particle contents of the minimal supersymmetric standard model (MSSM) and to explore some excellent features. Using a $Z_3$ orbifold model, we study flat directions and calculate soft scalar masses incorporating $D$-term contributions and particle mixing effects. Specific relations among scalar masses are obtained.

This paper is organized as follows. In the next section, we review the formula of soft SUSY breaking scalar masses derived from 4-D string models. In section 3, we study flat directions and soft scalar masses using a $Z_3$ orbifold model. Section 4 is devoted to conclusions and discussions.
2 Formula of soft scalar masses

We assume the existence of a realistic effective SUGRA, that is, our starting theory has the following excellent feature.

The gauge group is \( G = G_{SM} \times U(1)^n \times U(1)_A \times H' \) where \( G_{SM} \) is the standard model gauge group \( G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y \), \( U(1)^n \) are anomaly-free, \( U(1)_A \) is anomalous and \( H' \) is a direct product of some non-abelian symmetries. The anomalies related to \( U(1)_A \) are canceled by the Green-Schwarz mechanism [18].

Chiral multiplets are classified into two categories. One is a set of chiral multiplets whose scalar components \( \phi^i \) have large VEVs of \( O(M) \). Here \( M \) is the gravitational scale defined as \( M \equiv M_{Pl}/\sqrt{8\pi} \) and \( M_{Pl} \) is the Planck scale. The dilaton field \( S \) and the moduli fields \( T_i \) belong to \( \{ \Phi^i \} \). We treat only the overall moduli field \( T \). It is assumed that SUSY is broken by \( F \)-term condensations of \( \Phi^i \) such that \( \langle F^i \rangle = O(m_{3/2}^2) \). The other is a set of matter multiplets denoted as \( \Phi^\iota \) which contains the MSSM matter multiplets and Higgs multiplets. We denote the above two types of multiplets as \( \Phi^I \) together.

We suppose the following situations related to extra gauge symmetry breaking.

1. The \( U(1)_A \) symmetry is broken by VEVs of \( S \) and some chiral matter multiplets.

2. Some parts of \( U(1)^n \) and \( H' \) are broken at much higher energy scales of \( O(M_I) \) than the weak scale by VEVs of some chiral matter multiplets \( \Phi^\iota \). Those VEVs are smaller than those of \( S \) and \( T \), i.e.

\[
\langle \phi^\iota \rangle \ll \langle S \rangle, \langle T \rangle = O(M).
\]

This condition is justified from the fact that a \( D \)-term condensation of \( U(1)_A \) vanishes up to \( O(m_{3/2}^2) \).

3. Other extra gauge symmetries are broken spontaneously or radiatively by SUSY breaking effects at lower scales.

In general, the fields diagonalizing SUSY mass terms, \( \hat{\phi}^\kappa \), are given as linear combinations of original string states \( \phi^\lambda \) such as

\[
\hat{\phi}^\kappa = R^\kappa_\lambda \phi^\lambda.
\]

Coefficients \( R^\kappa_\lambda \)'s depend on VEVs of moduli fields.

Assuming the vanishing cosmological constant, we have the following mass formula for light scalar fields \( \hat{\phi}^k \) at the energy scale \( M_I \)

\[
(m^2)^I_k \big|_{M_I} = m_{3/2}^2 \delta^I_k + m_{3/2}^2 \cos^2 \theta N^I_k
+ \left( V^D_{\text{Soft Mass}} \right)^I_k + \left( V^{\text{Extra F Soft Mass}} \right)^I_k
+ \left( V^{\text{Mix Soft Mass}} \right)^I_k + \left( V^{\text{Ren Soft Mass}} \right)^I_k
\]

(4)
where $(V_{\text{Soft Mass}}^D)^l_k$, $(V_{\text{Soft Mass}}^{\text{Extra F}})^l_k$, $(V_{\text{Soft Mass}}^{\text{Mix}})^l_k$ and $(V_{\text{Soft Mass}}^{\text{Ren}})^l_k$ are $D$-term contributions, extra $F$-term contributions, the contributions due to heavy-light mass mixing and contributions related to renormalization effects from $M$ to $M_I$, respectively. Here $\theta$ is the so-called goldstino angle parametrizing the ratio of $F_S$ and $F_T$ and $\hat{N}_k^l$ is obtained as $\hat{N}_k^l = (R^{-1})^\nu_\kappa n_\nu R^\kappa_\mu$ by modular weights $n_\nu$ in the $\phi^\kappa$-basis.

$D$-term contributions play an important role for later discussions and are given as \[19, 20, 21\]

\[ (V_{\text{Soft Mass}}^D)^l_k = \sum_\alpha g_\alpha^2 \langle D^\alpha \rangle (\hat{Q}^\alpha)^l_k \]  \hspace{1cm} (5)

where $g_\alpha$'s are gauge coupling constants and

\[ (\hat{Q}^\alpha)^l_k \equiv \langle \hat{K}_{\alpha}^{\mu} \rangle \langle q^\alpha \rangle_{\mu}, \quad (\hat{q}^\alpha)_{\kappa} \equiv (R^{-1})^\nu_\kappa q^\alpha R^\kappa_\nu. \]  \hspace{1cm} (6)

Here $\hat{K}^I_\kappa$ is a Kähler metric and $q^\alpha_{\kappa}$'s are diagonal charges. The $D$-term condensations are written as

\[ g_\alpha^2 \langle D^\alpha \rangle = 2g_\alpha m^2_{3/2} \{ (M_V^{-2})^{\hat{\alpha}\hat{A}} g_A (1 - 6C^2 \sin^2 \theta) \langle \sum_\kappa q^A_{\kappa} (T + T^*)^{n_\kappa} |\phi^\kappa|^2 \rangle 
- \sum_\beta (M_V^{-2})^{\hat{\alpha}\hat{\beta}} g_\beta C^2 \cos^2 \theta \langle \sum_\kappa q^\beta_{\kappa} (T + T^*)^{n_\kappa} |\phi^\kappa|^2 \rangle \} \]  \hspace{1cm} (7)

where $(M_V^{-2})^{\alpha\beta}$ is the inverse matrix of gauge boson mass matrix $(M_V^2)^{\alpha\beta}$ given as

\[ (M^2_V)^{\alpha\beta} = 2g_\alpha g_\beta \langle (T^{\beta}(\phi)) (T^\alpha(\phi)) \rangle. \]  \hspace{1cm} (8)

Here the gauge transformation of $\phi^I$ is given as $\delta \phi^I = i g_\alpha (T^\alpha(\phi))^{I}$ up to space-time dependent infinitesimal parameters. Here indices $\hat{\alpha}$ and $\hat{\beta}$ run over broken generators. Further the gaugino mass is obtained as \[14\]

\[ M_{1/2}^2 = 3m^2_{3/2} \sin^2 \theta. \]  \hspace{1cm} (9)

3 Examples

3.1 Flat direction

We study the $Z_3$ orbifold model with a shift vector $V$ and Wilson lines $a_1$ and $a_3$ such as \[22, 23\]

\[ V = \frac{1}{3}(1, 1, 1, 2, 0, 0, 0, 0, 0, 0, 0), \]
\[ a_1 = \frac{1}{3}(0, 0, 0, 0, 0, 0, 2, 0, 1, 1, 0, 0, 0), \]
exist several types of flat directions in the SUSY limit [22, 23]. Thus the fields $Y$ fields in Ref. [22]. The fields $G$ particular, the following charge generators are defined in Table 1 and one of them is anomalous. This model has matter multiplets as

$$U - \text{sec.} : 3[(3, 2)_0 + (3, 1)_0 + (1, 2)_0] + 3(16)'_9,$$

$$T - \text{sec.} : 3[4(3, 1)_4 + 5(3, 1)_4] + 3[11(1, 2)_4 + (1, 2)_{-8}]$$

$$(N_{OSC} = 0) + 114(1, 1)_4 + 30(1, 1)_{-8},$$

$$T - \text{sec.}(N_{OSC} = -1/3) : 27(1, 1)_4$$

where the number of suffix denotes the anomalous $U(1)$ charge and $N_{OSC}$ is the oscillator number. Thus there are many $G_{SM}$-singlets in this model. In particular, the following $G_{SM}$-singlets play an important role for study of flat directions leading to realistic vacua

$$S_1 : Q_a = (-6, 0, 0, 2, 0, 4, 0, -8),$$

$$S_2 : Q_a = (0, -4, 0, -2, -2, 0, 4, -8),$$

$$S_3 : Q_a = (0, -4, 0, -2, 2, -4, -4, -8),$$

$$S_6 : Q_a = (6, 4, 0, 0, -2, 0, -2, -8),$$

$$S_8 : Q_a = (6, 4, 0, 0, 2, 2, 2, -8),$$

$$S_{10} : Q_a = (-6, 0, 0, 2, -4, -4, -8),$$

$$S_{11} : Q_a = (-6, 0, 0, 2, 4, 0, 4, 4),$$

$$Y_1 : Q_a = (-6, 0, 0, 2, 0, -2, 0, 4),$$

$$Y_3 : Q_a = (0, -4, 0, -2, 2, 2, 2, 4)$$

$$A_5 : Q_a = (-3, -2, 3, 3, 1, 0, -2, -8)$$

$$\overline{A}_5 : Q_a = (-3, -2, -3, -3, 1, 0, -2, -8)$$

where $Q_a (a = 1, 2, ..., 7, A)$ are $U(1)$ charges and we follow the notation of the fields in Ref. [22]. The fields $S_i, A_i$ and $\overline{A}_i$ correspond to the non-oscillated twisted sector with $n_\kappa = -2$ and $Y_j$ corresponds to the twisted sector with a nonvanishing oscillator number. Thus the fields $Y_j$ have the modular weight $n_Y = -3$. There exist several types of flat directions in the SUSY limit [22, 23].

Let us take one example where the flat direction is given as [22]

$$\langle (T + T^*)^{-3}|Y_3|^2 \rangle = v,$$

$$\langle (T + T^*)^{-3}|Y_4|^2 \rangle = \langle (T + T^*)^{-2}|S_6|^2 \rangle = u + v,$$

$$\langle (T + T^*)^{-2}|S_1|^2 \rangle = \langle (T + T^*)^{-2}|S_2|^2 \rangle = \langle (T + T^*)^{-2}|S_3|^2 \rangle =$$

$$\langle (T + T^*)^{-2}|S_8|^2 \rangle = u$$

(10)
where $u$ and $v$ are positive constants which are determined by the $D$-flatness condition and the minimum of scalar potential after SUSY breaking. Along this flat direction, the $U(1)$ symmetries break as $U(1)^8 \rightarrow U(1)^2$. One of unbroken $U(1)^2$ corresponds to $Q_3$. The other is a linear combination of $Q_1$, $Q_2$ and $Q_4$ as $Q_1/3 - Q_2/2 + Q_4$ which is regarded as the hypercharge.

We define broken $U(1)$ charges as

$$Q'^1 \equiv \frac{1}{\sqrt{5}}(Q_1 + Q_2), \quad Q'^2 \equiv \frac{1}{\sqrt{55}}(2Q_1 - 3Q_2 - 5Q_4),$$

$$Q'^5 \equiv Q_5, \quad Q'^6 \equiv \frac{1}{\sqrt{2}}Q_6, \quad Q'^7 \equiv \frac{1}{\sqrt{2}}Q_7, \quad Q'^A \equiv \frac{1}{\sqrt{3}}Q_A.$$  \hspace{1cm} (11)

Note that the gauge boson mass matrix is not diagonalized in this definition. The modular weights and broken $U(1)$ charges of the fields with VEVs are given in Table 2.

The $D$-flatness condition for $U(1)_A$ requires

$$\langle \frac{\delta_{GS}}{S + S^*} \rangle - 36u = 0 \hspace{1cm} (12)$$

where $\delta_{GS}^A$ is a coefficient of the Green-Schwarz mechanism \[18\] to cancel the $U(1)_A$ anomaly and is given as

$$\delta_{GS}^A = \frac{1}{96\pi^2} Tr Q_A.$$  \hspace{1cm} (13)

In addition, we have

$$f(n^2_\kappa) = 29u + 22v \hspace{1cm} (14)$$

where $f(a_\kappa)$ is defined as

$$f(a_\kappa) = \langle \sum_\kappa a_\kappa(T + T^*)^{n_\kappa} |\phi^\kappa|^2 \rangle.$$  \hspace{1cm} (15)

The scalar potential includes $f(n^2_\kappa)$ \[16\]. The minimum of scalar potential is obtained at the following point:

$$u = \frac{1}{36} \langle \frac{\delta_{GS}^A}{S + S^*} \rangle, \quad v \sim O(m^2_{3/2}).$$  \hspace{1cm} (16)

Using $Tr Q_A = 1296$ and $\langle Re S \rangle \sim 2$, we estimate $u \sim M^2/105$. From Eqs.(16), the breaking scale of $U(1)_i$'s is estimated as $M_i = O(u^{1/2})$.

Along this flat direction, several fields gain mass terms. For example, we consider mass terms among $(3, 1)$ and $(\overline{3}, 1)$ fields in the twisted sector. Here we follow the notation of fields in Ref. \[22\]. The $D_1$ field appears as a massless $(3, 1)$ field in the twisted sector with the Wilson line $(m_1, m_3) = (0, 0)$, where $(m_1, m_3)$
denotes the Wilson line \( a = m_1 a_1 + m_3 a_3 \). Further \((3,1)\) fields include the \( d_1 \) and \( d_2 \) fields, which have \((m_1, m_3) = (0, 1)\) and \((-1, 1)\), respectively. Selection rules due to space group invariance allow couplings satisfying the following condition:

\[
\sum m_1 = 3\ell, \quad \sum m_3 = 3\ell'
\]  

where \( \ell \) and \( \ell' \) are integers. Further its coupling strength is obtained for each \( i \)-th plane as \( h_i \sim e^{-a(T_i)} \) where \( a = 0 \) in the case that \( m_i \) takes a same number for all species and \( a > 0 \) for other cases \([24]\). The singlet fields \( S_2 \) and \( S_3 \) have \((m_1, m_3) = (0, -1)\) and \((-1, -1)\), respectively. The space group invariance (17) as well as gauge invariance allows the following couplings:

\[
D_1 d_1 S_2, \quad D_1 d_2 S_3.
\]  

These couplings include suppression factors as \( h_2 \) and \( h_1 h_2 \), respectively. Their mass terms along the flat direction are written as

\[
\langle S_2 \rangle h_2 (d_1 + h_1 \frac{\langle S_3 \rangle}{\langle S_2 \rangle} d_2) D_1.
\]  

The light field is obtained as the linear combination \( h_1 d_1 - d_2 \), up to normalization. Thus the matrix \( R^\lambda_k \) involves the moduli-dependent function \( h_2 \). Furthermore, the matrix \( R^\lambda_k \) is, in general, dependent of ratios of VEVs. The other \( SU(3) \) triplets fields in the twisted sector become massive. Similarly we obtain the Higgs field \( H \) by string states \( G_1 \) and \( G_2 \) in the twisted sector as \( h_1 G_2 - G_3 \). Here \( H \) and \( \overline{H} \) are the Higgs doublets with hypercharge \(-1/2\) and \(1/2\), respectively. The other MSSM matter fields coincide with the string states. If we take into account nonrenormalizable couplings, up-type quarks are obtained as linear combinations of string states \([22]\). However, mass terms induced by nonrenormalizable couplings include suppression factors of \( O((u/M)^{1/2}) \sim 1/10 \). Here we neglect such effects.

This orbifold model has another flat direction as \([23]\)

\[
\langle (T + T^*)^{-2} | S_6 |^2 \rangle = 2 \langle (T + T^*)^{-2} | S_{11} |^2 \rangle = 4\lambda a, \\
\langle (T + T^*)^{-2} | A_5 |^2 \rangle = \langle (T + T^*)^{-2} | A_5 |^2 \rangle = (2 + 2\lambda) a, \\
\langle (T + T^*)^{-2} | S_8 |^2 \rangle = 2 \langle (T + T^*)^{-2} | S_{10} |^2 \rangle = (8 - 4\lambda) a, \\
\langle (T + T^*)^{-2} | S_1 |^2 \rangle = \langle (T + T^*)^{-2} | S_3 |^2 \rangle = (2 - 2\lambda) a, \\
\langle (T + T^*)^{-2} | S_2 |^2 \rangle = 4 a
\]  

where \( a \) and \( \lambda \) satisfy

\[
\left( \frac{\delta_{GS}}{S + S^*} \right) - 18 a = 0, \quad 0 \leq \lambda \leq 1.
\]  

* This moduli-dependence of the diagonalizing matrix has not been discussed in Ref. [23].

\[ \]
Along this flat direction we have $f(n_k^2) = 48a$. It is notable that $f(n_k^2)$ is independent of $\lambda$. Thus the direction corresponding to the parameter $\lambda$ is still a flat direction at this level, although this vacuum has a larger $f(n_k^2)$ than the previous one.

### 3.2 Scalar mass relations

Let us calculate soft scalar masses using the formula (4) and derive specific relations among them. The basic idea and strategy are the same as those in Refs. [25, 20, 26]. The SUSY spectrum at the weak scale, which is expected to be measured in the near future, is translated into the soft SUSY breaking parameters. The values of these parameters at higher energy scales are obtained by using the renormalization group equations (RGEs) [27]. In many cases, there exist some relations among these parameters. They reflect the structure of high-energy physics. Hence we can specify the high-energy physics by examining these relations.

We have the same number of observable soft masses as that of species of scalar fields and gauginos. There are several unknown parameters in the RHS of Eq. (4) such as $m^2_{3/2}$ and $\cos^2 \theta$. If the number of independent equations is more than that of unknown parameters, non-trivial relations exist among soft masses. They can be obtained by eliminating unknown parameters.

In our model, the breaking scale $M_I$ is estimated as $O(10^{-1} M)$ and so renormalization effects from $M$ to $M_I$ are neglected. We assume that Yukawa couplings among heavy and light fields are small enough and the $R$-parity is conserved. In such a case, we can neglect the effect of extra $F$-term contributions. Since there are no sizable mixing terms among heavy and light fields in the Kähler potential in $Z_3$ orbifold models as shown in appendix A, there appear no heavy-light mixing terms of $O(m_{3/2} M_I)$ if Yukawa couplings among heavy, light and moduli fields are suppressed sufficiently, i.e., $O(m_{3/2}/M)$. We assume that string state mixing occurs among the same generation after the breakdown of extra gauge symmetries, that is, there is no flavor mixing.

Under the above assumptions, our soft scalar mass formula is written in a simple form such as

$$m^2_k|_{M_I} = m^2_{3/2} + m^2_{3/2} \hat{N}_k \cos^2 \theta + \sum_\alpha g^2_\alpha (D^{\hat{\alpha}})(\hat{Q}^{\hat{\alpha}})_k$$

(22)

where $\hat{N}_k = (R^{-1})^\nu_k n_\nu R^\nu_k$ and $(\hat{Q}^{\hat{\alpha}})_k = (R^{-1})^{\nu_k} q^{\hat{\alpha}} R^\nu_k$ without summations for $k$. In Table 3, the modular weights and broken $U(1)$ charges for light scalar fields are given.

The gauge boson mass matrix is represented as $(M_V^2)_{\alpha \beta} = 2g'_{\alpha} g'_{\beta} f(q^{\alpha} q^{\beta})$. Here $g'_{\alpha}$’s are gauge coupling constants defined in the basis (11) and $q^{\alpha}$’s represent $U(1)$ charges $(\hat{Q}^{\hat{\alpha}})_k$ for scalar fields $\hat{\phi}^k$. The $D$-term condensations are written
Using Eqs. (22), (23), (27) and (28), we have obtained soft scalar masses at $M_I$ as

$$g_5^2 \langle D^\alpha \rangle = m_{3/2}^2 f^{-1}(q^\alpha q^\beta) U^\beta$$

(23)

where $f^{-1}(q^\alpha q^\beta)$ is the inverse matrix of $f(q^\alpha q^\beta)$ and $U^\beta$ is given as

$$U^\beta = (1 - 6 \sin^2 \theta) f(q^\beta) \delta^A_\beta - \cos^2 \theta f(n_k q^\beta).$$

(24)

We need the values for $f(q^\alpha)$, $f(n_k q^\alpha)$ and $f(q^\alpha q^\beta)$ to calculate $D$-term contributions. The values for $f(q^\alpha)$ and $f(n_k q^\alpha)$ are calculated as

$$f(q^\alpha) = 0 \quad (\hat{\alpha} = 1, 2, 5, 6, 7), \quad f(q^A) = -12\sqrt{3}u,$n_k q^\alpha_j) = \frac{6u + 10v}{\sqrt{5}}, \quad f(n_k q^2) = \frac{2}{5} \sqrt{55} u, \quad f(n_k q^5) = -2v,$

(25)

$$f(n_k q^6) = \sqrt{2}u, \quad f(n_k q^7) = -\sqrt{2}v, \quad f(n_k q^A) = \frac{68u - 8v}{\sqrt{3}}.$$ (26)

Using the above values, $U^\beta$ is calculated as

$$(U^\beta)^T = \left( \begin{array}{cccccc} -\frac{6}{5} \sqrt{5} \cos^2 \theta, & -\frac{2}{5} \sqrt{5} \cos^2 \theta, & 0, & -\sqrt{2} \cos^2 \theta, & 0, & \frac{1}{\sqrt{3}}(180 - 284 \cos^2 \theta)u. \end{array} \right)$$

(27)

The values for $f(q^\alpha q^\beta)$ are calculated as

$$f(q^\alpha q^\beta) = \left( \begin{array}{cccccc} 304 \frac{5}{5} & 8\sqrt{10} \frac{5}{5} & 0 & 12\sqrt{10} \frac{5}{5} & 0 & -24\sqrt{10} \frac{5}{5} \\ 8\sqrt{10} \frac{5}{5} & 176 \frac{5}{5} & 0 & -6\sqrt{10} \frac{5}{5} & 0 & -8\sqrt{10} \frac{5}{5} \\ 0 & 0 & 16 & -2\sqrt{10} \frac{5}{5} & -4\sqrt{10} \frac{5}{5} & 0 \\ 12\sqrt{10} \frac{5}{5} & -6\sqrt{10} \frac{5}{5} & -2\sqrt{10} \frac{5}{5} & 20 & 10 & -4\sqrt{10} \frac{5}{5} \\ 0 & 0 & -4\sqrt{10} \frac{5}{5} & 10 & 20 & 0 \\ -24\sqrt{10} \frac{5}{5} & -8\sqrt{10} \frac{5}{5} & 0 & -4\sqrt{10} \frac{5}{5} & 0 & 112 \end{array} \right)$$

(28)

Using Eqs. (22), (23), (27) and (28), we have obtained soft scalar masses at $M_I$ in the following form,

$$m_k^2|_{M_I} = m_{3/2}^2(a + b \cos^2 \theta).$$

(29)

In Table 4, we give the values of $a$ and $b$ for all species. The values $a$ and $a + b$ correspond to the extreme cases $\cos^2 \theta = 0$ and $\cos^2 \theta = 1$ for mass ratios $m_k^2/m_{3/2}^2|_{M_I}$, respectively. Note that the $h_1$ dependence disappears.

\footnote{Here we assume that the values of all gauge couplings equal at $M_I$.}
Many fields can acquire negative squared masses and they could trigger a “larger” symmetry breaking including the dangerous color and/or charge symmetry breaking. The fifth column of Table 4 shows the range of $\cos^2 \theta$ leading to $m_k^2 \geq 0$ at the tree level for each sfermion. Radiative corrections due to gaugino masses are important for squark masses. The sixth column of Table 4 shows the range of $\cos^2 \theta$ leading to $m_k^2 \geq 0$ at $M_Z$ for each sfermion including one-loop radiative corrections. Here we neglect RGE effects of Yukawa couplings.‡ All of the sfermions have $m_k^2 \geq 0$ at $M_Z$ in the range with $0.61 \leq \cos^2 \theta \leq 0.87$. The $\mu$-term as well as the soft mass terms contributes to the Higgs mass terms. Hence we omit the ranges leading to $m_{H(\overline{H})}^2 \geq 0$ for soft masses in the fifth and sixth columns of Table 4. A suitable $\mu$-term could lead to a successful symmetry breaking. Here we do not discuss the $\mu$-term explicitly since that is beyond this work.

In addition we have a strong non-universality of soft masses, i.e. $m_k^2 = O\left(10^{m_{3/2}}\right)$ for some fields while $m_k^2 = O(m_{3/2}^2)$ for others. Note that we have non-universal soft masses even in the case with $\cos \theta = 0$. That is a generic feature of models with anomalous $U(1)$ symmetry.‡ As a feature of this model, soft masses are degenerate for squarks and sleptons with same quantum numbers under $G_{SM}$ because they have same quantum numbers under gauge group $G$ and same modular weights. Hence the process of flavor changing neutral current (FCNC) is sufficiently suppressed.

Let us obtain relations among scalar masses and gaugino masses. As we have eight kinds of observables ($\tilde{m}_q, \tilde{m}_u, \tilde{m}_d, \tilde{m}_l, \tilde{m}_e, m_H, m_{\overline{H}}, M_{1/2}$) and two unknown parameters ($m_{3/2}, \cos \theta$), we can obtain at least six independent relations. In fact, we have the following relations

$$3(m_l^2 - m_e^2) = m_u^2 - m_{\tilde{q}}^2,$$

$$m_q^2 + m_H = m_{\tilde{u}}^2 + m_{\tilde{d}}^2,$$

$$m_{\tilde{q}}^2 + m_{\tilde{d}}^2 + 4(m_{\tilde{u}}^2 + m_e^2) = 0,$$

$$13(m_{\tilde{q}}^2 + m_{\tilde{d}}^2) + 12(m_l^2 + m_H) = 0,$$

$$2m_{\tilde{q}}^2 + 3m_{\tilde{d}}^2 + m_{\tilde{e}}^2 = m_{\overline{H}}^2,$$

$$m_{\tilde{q}}^2 + m_{\tilde{u}}^2 + m_{\overline{H}}^2 = M_{1/2}^2$$

where the tilde represents the scalar component. Similarly we can obtained soft scalar masses for other vacua, e.g. Eq.(20).

4 Conclusions and Discussions

We have studied flat directions and soft scalar masses using a $Z_3$ orbifold model with $SU(3) \times SU(2) \times U(1)$ gauge group and extra gauge symmetries including an anomalous $U(1)$ symmetry. Soft scalar masses contain $D$-term contributions.

‡ It is valid for the first and second families.
and particle mixing effects after extra gauge symmetry breaking and they are parametrized by a few number of parameters.

We have calculated soft scalar masses at $M_I$. It is, in general, difficult to keep the degeneracy and positivity of squared masses at the tree level. We have non-universal soft masses even for $\cos \theta = 0$ as a generic feature of string models with anomalous $U(1)$ breaking. This fact does not lead to serious problems for FCNC in our model. Because soft masses are degenerate for squarks and sleptons with same quantum numbers under $G_{SM}$. A strong non-universality of soft masses, i.e. $m^2_k = O(10m^2_{3/2})$ for some fields while $m^2_k = O(m^2_{3/2})$ for others might provide interesting implications in the phenomenological viewpoint. In fact, much work is devoted to phenomenological implications of the non-universality of soft masses [29]. The positivity of $m^2$ can be recovered by radiative corrections. It is an interesting subject to examine whether the radiative breaking scenario [27] can be realized.

We have obtained some specific relations among scalar masses. They can be powerful probes to specify a realistic model based on 4-D string models.

\section*{A Heavy-light Mixing in Kähler potential}

If the MSSM matter fields $\hat{\phi}^k_{(SM)}$ are given as linear combinations of string states with the same modular weight $n_k$, the Kähler potential of matter part is given as

$$K^{(M)} = \sum_{(SM)} (T + T^*)^{n_k}|\hat{\phi}^k_{(SM)}|^2 + \cdots$$

(36)

where the ellipses stand for terms related to fields other than the MSSM matter fields. In this case, there are no heavy-light mixing terms in $K^{(M)}$. Whether the SM matter fields $\hat{\phi}^k_{(SM)}$ are given as linear combinations of original fields with the same modular weight or not is model-dependent. We discuss this issue based on $Z_N$ orbifold models in this appendix.

The explicit model in section 3 shows the origin of particle mixing as follows. Suppose that we have the following two couplings:

$$\phi \phi_1 \prod_i \chi_i, \quad \phi \phi_2 \prod_j \chi'_j,$$

(37)

including the common field $\phi$. On the top of that, we assume this model has flat directions as $\langle \prod_i \chi_i \rangle \neq 0$ and $\langle \prod_j \chi'_j \rangle \neq 0$. Then mass eigenstates are obtained as linear combinations of $\phi_1$ and $\phi_2$. If these fields, $\phi_1$ and $\phi_2$, have the same modular weight $n_k$, the light field among their linear combinations has its Kähler potential as Eq. (36). Otherwise, its Kähler potential becomes complicated.

For example, we study $Z_3$ orbifold models. These models have two types of renormalizable couplings as

$$\phi_U \phi_U \phi_U, \quad \phi_T \phi_T \phi_T'$$

(38)
where $\phi_{U_i}$ denotes a field in one of the three untwisted sectors and $\phi_{T_1}$ corresponds to that in the twisted sector. These couplings have no common field. Thus particle mixing with different modular weights does not appear at this level. Nonrenormalizable couplings \[30\] could lead to particle mixing, but these couplings include a suppression factor \((\langle \chi \rangle/M)^n\). Thus such effects are negligible in most of cases. In the same way, the particle mixing effect is negligible in $Z_7$ orbifold models since it can appear only through nonrenormalizable couplings.

$Z_{2n}$ orbifold models are different from $Z_3$ and $Z_7$ orbifold models. Because $Z_{2n}$ orbifold models have several types of renormalizable couplings \[31\]. For example, $Z_4$ orbifold models have three types of renormalizable couplings as

$$
\phi_{T_1} \phi'_{T_1} \phi_{T_2}, \quad \phi_{T_2} \phi'_{T_2} \phi_{U_3},
$$

in addition to $\phi_{U_1} \phi_{U_2} \phi_{U_3}$. Here $\phi_{T_1}$ and $\phi_{T_2}$ correspond to fields in the $\theta$-twisted and $\theta^2$-twisted sectors, where $\theta$ denotes the $Z_4$-twist to construct $Z_4$ orbifold models. We consider the couplings \(39\) and assume the existence of flat directions as $\langle \phi_{U_3} \rangle \neq 0$ and $\langle \phi_{T_1} \rangle \neq 0$. In this case mass eigenstates are linear combinations of $\phi'_{T_1}$ and $\phi'_{T_2}$, which have modular weights $n_k = -2$ and $-1$, respectively. Therefore sizable particle mixing with different modular weights can appear in $Z_{2n}$ orbifold models.

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**Table Captions**

Table 1 $U(1)$ charge generators in terms of $E_8 \times E_8'$ lattice vectors.

Table 2 The modular weights and broken $U(1)$ charges of the scalar fields with VEVs. We follow the notation of fields in Ref. [22].

Table 3 The modular weights and broken $U(1)$ charges for light scalar fields. Here $h_1$ denotes $h_1 \sim e^{-a\langle T_1 \rangle}$.

Table 4 The particle contents and the ratios of $m_k^2/m_{3/2}^2$. We omit the ranges leading to $m_{H(H)}^2 \geq 0$ for soft masses.
Table 1

\[
Q_1 = 6(1, 1, 0, 0, 0, 0)(0, 0, 0, 0, 0, 0, 0, 0, 0, 0)'
\]

\[
Q_2 = 6(0, 0, 0, 0, 1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)'
\]

\[
Q_3 = 6(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)'
\]

\[
Q_4 = 6(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)'
\]

\[
Q_5 = 6(0, 0, 0, 0, 0, 0, 1)(0, 0, 0, 0, 0, 0, 0, 0)'
\]

\[
Q_6 = 6(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)'
\]

\[
Q_7 = 6(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)'
\]

\[
Q_A = 6(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)'
\]

Table 2

| String state | \( n_k \) | \( \sqrt{5Q^Q} \) | \( \sqrt{55Q^Q} \) | \( Q^B \) | \( \sqrt{2Q^B} \) | \( \sqrt{2Q^B} \) | \( \sqrt{3Q^B} \) |
|-------------|-------|----------------|----------------|------|--------|--------|--------|
| \( S_1 \)   | -2    | -6             | -22            | 0    | 4      | 0      | -8     |
| \( S_2 \)   | -2    | -4             | 22             | -2   | 0      | 4      | -8     |
| \( S_3 \)   | -2    | -4             | 22             | -4   | -4     | -4     | -8     |
| \( S_6 \)   | -2    | 10             | 0              | -2   | 0      | -2     | -8     |
| \( S_8 \)   | -2    | 10             | 0              | 2    | 2      | 2      | -8     |
| \( Y_1 \)   | -3    | -6             | -22            | 0    | -2     | 0      | 4      |
| \( Y_3 \)   | -3    | -4             | 22             | 2    | 2      | 2      | 4      |
### Table 3

| SM field | String state | $n_k$ | $\sqrt{5}Q^1$ | $\sqrt{55}Q^2$ | $Q^5$ | $\sqrt{2}Q^6$ | $\sqrt{2}Q^7$ | $\sqrt{3}Q^8$ |
|----------|--------------|-------|---------------|---------------|-------|----------------|----------------|----------------|
| $\tilde{q}$ | $Q_L$ | $-1$ | $-12$ | $6$ | $0$ | $0$ | $0$ | $0$ |
| $\tilde{u}$ | $u_L$ | $-1$ | $6$ | $42$ | $0$ | $0$ | $0$ | $0$ |
| $\tilde{d}$ | $h_d - d_2$ | $-2$ | $0$ | $-10$ | $\frac{2(h_d^2 - 1)}{1 + h_d^2}$ | $\frac{-4h_d^2}{1 + h_d^2}$ | $\frac{4(1 - h_d^2)}{1 + h_d^2}$ | $4$ |
| $\tilde{l}$ | $G_5$ | $-2$ | $2$ | $4$ | $0$ | $-2$ | $0$ | $-8$ |
| $\tilde{e}$ | $l_5$ | $-2$ | $-4$ | $-8$ | $0$ | $-2$ | $0$ | $-8$ |
| $H$ | $h_1G_2 - G_3$ | $-2$ | $2$ | $4$ | $\frac{-4h_d^2}{1 + h_d^2}$ | $\frac{2(h_d^2 - 1)}{1 + h_d^2}$ | $\frac{2(h_d^2 - 3)}{1 + h_d^2}$ | $4$ |
| $\overline{H}$ | $\overline{G}_1$ | $-1$ | $6$ | $-48$ | $0$ | $0$ | $0$ | $0$ |

### Table 4

| Rep. | 11a | 11b | $a + b$ | $\cos^2 \theta$ | $\cos^2 \theta$ (rad. corr.) |
|------|-----|-----|---------|-----------------|-------------------------------|
| $\tilde{q}$ | 26 | $-37$ | $-1$ | $[0, 0.70]$ | $[0, 0.95]$ |
| $\tilde{u}$ | 116 | $-193$ | $-7$ | $[0, 0.60]$ | $[0, 0.87]$ |
| $\tilde{d}$ | $-14$ | 25 | $1$ | $[0.56, 1]$ | $[0, 1]$ |
| $\tilde{l}$ | $-89$ | 144 | $5$ | $[0.62, 1]$ | $[0.57, 1]$ |
| $\tilde{e}$ | $-119$ | 196 | $7$ | $[0.61, 1]$ | $[0.61, 1]$ |
| $H$ | 76 | $-131$ | $-5$ | — | — |
| $\overline{H}$ | $-109$ | 197 | $8$ | — | — |