Primary analytical model for determining the electron temperature of a CuBr laser

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Abstract. A family of copper bromide metal vapor lasers used as brightness amplifiers in recently developed optical systems is considered. One of the main problems of this area is to investigate the basic amplification characteristics of the laser medium. In this study, a simplified mathematical model is developed based on the one-dimensional stationary equation for determining the electron temperature of the discharge. The proposed model allows to calculate the radial distribution of the electron temperature of the discharge by taking into account the radial distribution of the supplied electric power. Computer simulations are run to investigate the dependence of electron energy as a function of the electrical power supplied to the gas discharge tube. The natural limits of this process are specified.

1. Introduction
It is thought that copper bromide and copper compound metal vapor lasers are well-known and studied. These have numerous unique properties – still being the most powerful sources in the visible spectrum, with 516.6 nm and 578.2 nm wavelengths and high laser beam coherence and convergence. They are sources of ultraviolet radiation (248.6 nm, 259.2 nm, 260.0 nm and 270.3 nm).

Copper bromide vapor lasers have a wide range of applications in practice and scientific studies [1-6].

- Applications in medicine and medical research
  In medicine, copper bromide vapor lasers are mainly used in dermatology and for photocoagulation.
- Industrial applications
  Copper bromide lasers and laser systems are used for the microprocessing of various types of materials: drilling, cutting, marking, etching, etc.
- In scientific research
  The laser is often used in scientific studies: for isotope isolation of various chemical elements, to study the magnetic properties of materials, etc. Copper bromide vapor lasers are used to pump titanium-sapphire lasers, dye lasers, etc.
- Other applications
A modeling of processes in electron, it is necessary to solve the compiled system of discharge. In order to examine the dependence of electron temperature only followed [9
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\( T_R = \frac{T_e}{T_0} \)]

\[ \frac{\partial}{\partial t} \left( \frac{3}{2} N_e T_e \right) = \frac{E_e^2 \sigma}{e} - 2N_e \left( T_e - T_g \right) \frac{m_e}{m_{Ne}} v_{e-Ne} + Q_r - \frac{1}{r} \frac{\partial}{\partial r} \left( r \lambda_e \frac{\partial}{\partial r} T_e \right) + \]

\[ + \frac{3}{2} \frac{1}{r} \frac{\partial}{\partial r} \left( r \left( -T_e N_e \mu_e E_r + D_e \frac{\partial}{\partial r} (T_e N_e) \right) \right) \]

where \( T_e \) is the electron temperature, \( N_e \) is the concentration of the buffer gas (here neon), \( E_e \) is the longitudinal electric field strength, \( \lambda_e \) is the heat conductivity of electrons, \( D_e \) is the electron diffusion coefficient, \( \mu_e \) is the coefficient of electron mobility, \( E_r \) is the radial strength of the electric field, which maintains plasma quasineutrality, \( Q_r \) is the contribution of reactions to electron temperature, \( v_{e-Ne} \) is the electron-neon elastic scattering rate, \( m_e \) is the electron mass, \( m_{Ne} \) is the neon atom mass, \( e \) is the electron charge, \( T_g \) is the gas temperature, which is determined by the stationary equation of the heat conductivity of the neutral gas, and \( \sigma \) is the conductivity of the plasma.

The energy of the electrons is one of the most important characteristics of the gas discharge. It defines the kinetics of operating levels of copper and neon energy states and has a defining role in laser generation. One of the crucial parameters, which determines electron temperature is the electric power supplied to the gas discharge. In order to examine the dependence of electron temperature only as a function of supplied electric power following [9], it is necessary to solve the compiled system of equations every time, which impedes such evaluations.

This paper proposes a simplified analytical model, which allows for the performance of such evaluations without the need to follow the approach developed in [9].

2. Problem rationale
To derive the model for determining the electron temperature it is necessary to solve the problem

\[ \nabla \lambda_e \nabla T_e = -\xi q_e, \]

\[ \left. \frac{\partial T_e}{\partial r} \right|_{r=0} = 0, \quad T_e(R) = T_{e,w} \]
where $\xi$ indicates the portion of the total supplied power that goes to heating the electrons, $q_v$ is the power per unit volume, $T_{e,w}$ is the known electron temperature on the wall, $R$ is the radius of the discharge tube, and $0 \leq r \leq R$.

There are two main difficulties in solving problem (2):

- unlike the heat conductivity coefficient of the neutral gas $\lambda_g$, for the coefficient of electron heat conductivity $\lambda_e$, there are practically no table of experimental values.
- it is difficult to determine in advance what part of the total supplied power $\xi$ goes to direct heating of electrons (the predominant part of the energy goes to heating the neutral gas neon).

In order to solve the first problem, $\lambda_e$ needs to be determined directly. To this end, it is most adequate to use the Wiedemann-Franz law for the electron gas:

$$ \frac{\lambda_e}{\sigma} = C_c T_e $$

where the coefficient $C_c$ assumes the value $C_c = \pi^2 (k/e)^2 / 3$. In this case, $\sigma$ is the electron conductivity of the gas, $k$ is the Boltzmann constant, $e$ is the electron discharge.

Equation (1) assumes the form

$$ \nabla C_c \sigma T_e \nabla T_e = -\xi q_v $$

In order to simplify it, the mean value of the power per unit volume $q_v$ and the mean value of conductivity $\overline{\sigma}$ are introduced as quantities averaged along the cross-section of the tube which therefore do not depend on the coordinates.

After these assumptions, equation (3) takes the following form:

$$ \nabla T_e \nabla T_e = -\frac{\xi q_v}{C_c \overline{\sigma}} $$

Equation (3) is of type

$$ \text{div}(\lambda_e \text{grad } T_e) + q_v = 0 $$

where $\lambda_e = \lambda_0 T_e^m$. Here $m$ and $\lambda_0$ are the constants to be determined.

Further the case of a radial distribution of power per unit volume $q_v$ is considered. For this reason, some qualitative theoretical dependencies are used. From $q_v = jE$ and $j = \sigma E$ the result is $q_v = jE^2$. Here $j$ is the density of electric current and $E$ is the electric field intensity. According to [10], the intensity distribution of the electric field in the cross-section of the tube follows the law $E(r) = E_0 J_0(2.4r/R)$, where $E_0$ is a constant and $J_0$ is the zero order Bessel function of first kind.

In this way, the result is

$$ q_v(r) = c \left[ J_0 \left( \frac{2.4r}{R} \right) \right]^2 $$

where $c$ is an unknown constant subject to defining. The function $J_0(2.4r/R)$ is sufficiently well known and tabulated, for example in [11]. Since in the general case, it is not convenient for engineering calculations, the volume power density is approximated by a second order polynomial of type [12]

$$ q_v(r) = K q_0 \left( a + b r^2 \right) $$

where $K, a$ and $b$ are constants subject to defining, and $q_0$ is the mean volume power density (supposing that $Q$ is the supplied electric power in the laser medium and $V$ is the volume of the discharge), i.e. the power, obtained under the assumption that it is distributed equally along the tube's cross-section. Using the least squares method, the following values of the unknown constants are obtained: $K = 1.4383; a = 1.0183471; b = -0.001077$. 


The solution to equation of type (5) under assumption (7) has the form [12]

\[ T_e(r) = \left[ T_{e,w}^{m+1} - \frac{(m+1)K_0}{C_0} \left( \frac{r^2 - R^2}{4a + br^2 + bR^2} \right) \right]^{1/(m+1)} \]

(8)

In the considered case we take \( \lambda_0 = 1, m = 1, T_{e,w} = 0 \).

The unknown quantity in equation (8) is \( \xi \). In order to find it, the results from [9] are used. Since \( T_e \) in [9] is averaged along the tube’s cross-section, the following integral needs to be solved:

\[ \bar{T}_{e, ex} = \frac{1}{R} \int_0^R \frac{2K_0}{C_0} \left( \frac{r^2 - R^2}{4a + br^2 + bR^2} \right) \, dr \]

(9)

This integral does not have an exact solution. For this reason, it is necessary to use approximate numerical methods for its solution. If in (9) the respective values are replaced by those from [9]: \( R = 3.5\, \text{mm}, \bar{T}_{e, ex} = 1.5\, \text{eV}, \sigma = 4.1 \times 10^{-5} \, \text{\Omega}^{-1} \), for \( \xi \) the result is \( \xi = 4.1 \times 10^{-5} \).

3. Subject of investigation

Further, the derived model is applied for a real operational CuBr laser, described in [9]. The principle scheme and the geometric characteristics are given in figure 1 and Table 1.

![Figure 1](image)

**Figure 1.** Gas discharge tube (GDT): (1) working channel, (2) electrodes, (3) CuBr containers, (4) traps, (5) exit windows, (6) HBr generator, (7) CuBr container heaters; \( L \) is the active zone length [9].

**Table 1.** GDT parameters.

| Parameter                      | Value   |
|--------------------------------|---------|
| GDT diameter \( (d = 2R) \), cm | 0.7     |
| Active zone length \( (L) \), cm | 14      |
| Maximum pump PRF, MHz         | 1.1     |
| Buffer gas (Ne) pressure, Torr | 25      |

4. Application of the model and discussion

The solution and computer simulations are carried out by using *Wolfram Mathematica* software system. The distribution of the electron temperature in the tube’s cross-section for three values of the supplied electric power \( Q_1 = 800\, \text{W}, Q_2 = 970\, \text{W}, Q_3 = 1080\, \text{W} \) is shown in figure 2.

The next figure 3 shows the changes in the electron temperature at the center of the tube as a function of the electric power supplied in the GDT. The dependence is given in relative units, taking as a basis the point \( Q = 800\, \text{W}, T_e = 1.91\, \text{eV} \).
Figure 2. Calculated distribution of the electron temperature $T_e(r)$ depending on the radius for three values of the supplied electric power $Q$.

Figure 3. Dependence of the electron temperature $T_e(0)$ in tube center with respect to supplied electric power $Q$ in relative units.

This figure shows that an increase in the energy of the electrons is significantly slower than the supplied electric power. For example, when $Q$ is increased by 21%, the electron temperature $T_e$ goes up just 9%, and when $Q$ is changed by 35%, $T_e$ increases by just 15%.

Figure 4 shows a comparison of the simulation results for $T_e$ adopted from [8] with the calculated by our model at $Q = 800W$. Our model has the advantage of calculating the distribution of $T_e(r)$ by taking into account the radial distribution of the supplied electric power.

Figure 4. Comparison of the calculated distribution (curve 1) of the electron temperature $T_e(r)$ with the results from [8] (curve 2) for the case of supplied electric power $Q = 800W$. 
The obtained results have important operational significance when studying laser generation. The supplied electric power is mainly significant for the increase of laser generation. This process has its natural limits. Together with the increase in electron temperature, the temperature of the neutral gas $T_g$ also goes up. When critical values are reached for $T_g$, the gas may overheat and laser generation may cease. Other possibilities also need to be sought to increase laser generation: structural, energetic, thermodynamic and optical.

5. Conclusion
Based on experimental data and an actual existing CuBr laser, a primary mathematical model for defining the energy of the electrons $T_e$ has been developed. The degree of influence of the supplied electrical power $Q$ to the gas discharge tube is investigated. It has been found that $Q$ has a limited influence on $T_e$ from the positions on the stable operation of the laser.

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