Research on Mooring System under Static Equilibrium

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Abstract. As an important treasure house of territorial resources, the ocean has been paid more and more attention in recent years. In this paper, according to the actual working situation of the transmission node of the near shallow water observation network, the anchor chain system, the steel barrel system, the steel pipe system and the buoy system are analyzed. The model is established through the connection between the systems, and then the model is improved according to the specific situation of the three problems. The state of each system is analyzed and solved, and the good design of the mooring system is given according to the data The scheme of calculation.

Keyword: Mooring System, Static Equilibrium Law, Difference Equation, Grey Correlation Analysis

1. Restatement of the Problem

1.1. Problem Background

The transmission node of the near shallow sea observation network is composed of buoy system, mooring system and underwater acoustic communication system. The buoy system of a certain transmission node can be simplified into a cylinder with a bottom diameter of 2 meters and a height of 2 meters, with a mass of 1000kg. The mooring system consists of steel pipe, steel drum, heavy weight ball, welded anchor chain and special anti-towing anchor [1].

The known weight of the anchor is 600kg, and the anchor chain is a common chain link without gear. There are 4 steel tubes, each of which has a length of 1 meter, a diameter of 50mm and a mass of 10kg. It is required that the Angle between the tangential direction of the link between the end of the anchor chain and the anchor and the seabed should be less than 16°, otherwise the anchor will be towed and the node shift will be lost [2].

The underwater acoustic communication system is installed in a cylindrical steel drum with a length of 1 meter and an outer diameter of 30cm. The total weight of the equipment and the steel drum is 100kg. The steel barrel is connected with the fourth steel pipe and the welded anchor chain. The
underwater acoustic communication equipment works best when the steel drum is upright. If the steel barrel tilts, the working effect of the equipment will be affected. When the inclination Angle of the steel barrel, the included Angle between the steel barrel and the vertical line, exceeds 5°, the working effect of the equipment is poor. A heavy ball is hung at the link between the steel drum and the welded anchor chain to control the tilt Angle of the steel drum [3].

1.2. Question Restatement

Question 1: If a certain transmission node chooses type II anchor chain, that is, the length of each link is 105mm, the mass of 1m anchor chain is 7kg, the total chain length is 22.05m, and the mass of the selected weight ball is 1200kg. Now, the transmission node of this type is placed in the sea water at a depth of 18m, the sea bed is flat, and the sea water density is uniform at $1.025 \times 10^3$ kg/m$^3$ in the sea. Assuming that the sea water is stationary and the wind speed of 12m/s and 24m/s on the sea surface exert force on the buoy respectively, the inclination Angle of the steel barrel and each steel pipe, the shape of the anchor chain, the draft depth of the buoy and the swimming area are considered.

Question 2: Based on the hypothesis of question 1, calculate the inclination Angle of the steel barrel and each steel pipe, the shape of the anchor chain and the swimming area of the buoy if the sea wind speed is 36m/s. The mass of the heavy ball which makes the tilt Angle of the steel barrel less than 5 degrees and the Angle of the anchor chain between the anchor point and the sea floor less than 16 degrees is solved.

Question 3: Due to the influence of tide and other factors, the measured water depth of the distributed sea area is between 16m and 20m. The maximum speed of seawater at the distribution point can reach 1.5m/s and the maximum wind speed can reach 36m/s. Please give the design of the mooring system considering wind force, current force and water depth, and analyze the tilt Angle of the steel barrel and steel pipe, the shape of the anchor chain, the draft depth of the buoy and the swimming area under different conditions.

2. Symbol Description

| symbol | Symbolic meaning | symbol | Symbolic meaning |
|--------|------------------|--------|------------------|
| $M_1$  | Mass of buoy     | $\phi$ | Inclination angle of steel drum |
| $M_2$  | The mass of the heavy sphere | $\theta$ | Chain inclination angle |
| $m_1$  | Quality of steel pipe | $\theta_i$ | Chain inclination angle |
| $m_2$  | Quality of steel drums and equipment | $l_1$ | Length of steel pipe |
| $h$    | Draft depth of buoy | $l_2$ | Barrel length |
| $\alpha$ | Tangent angle of anchor chain end | $F_w$ | Wind force on buoy |
| $T$    | Tension of anchor chain | $q$ | Gravity load on anchor chain |
| $T_i$  | Tension of the i-th steel tube joint | $F_B, F_b$ | The buoyancy applied to a component of the system |
| $\phi_i$ | The inclination Angle of the I steel pipe | | |

3. The Establishment of the Model

Because the anchor chain is a combination of links, and each link is very short. Therefore, we can idealize the anchor chain as a continuous integral flexible rope. The Cartesian coordinate system is established with the coincidence point between the anchor chain connection point and the departure point as the coordinate origin and the image is made:
One section of the anchor chain is taken for force analysis, and a picture is drawn and a differential equation model is established by taking the coincidence point of anchor chain connection point and departure point as the coordinate original point. Each segment of the anchor chain is regarded as a small element, and the continuous element constitutes the whole anchor chain.

At this moment, the force balance of the anchor chain under water is at a static state, so the horizontal force force of the anchor chain is zero, and the vertical force is also zero. The mathematical formula can be constructed according to Figure 1:

\[ \sum F_x = 0, \quad [T \cos \theta + \frac{d(T \cos \theta)}{ds} \cdot ds] - T \cos \theta = 0 \]
\[ \sum F_y = 0, \quad [T \sin \theta + \frac{d(T \sin \theta)}{ds} \cdot ds] - T \sin \theta - q \cdot ds = 0 \]

It can be obtained from the first formula in (1):

\[ \frac{d(T \cos \theta)}{ds} = 0 \] (2)

From Equation (2), it can be obtained that the horizontal component of each small element in the equation of indistinguishments is equal to the horizontal component of the whole system. Then it can be deduced that:

\[ T \cos \theta = \text{Const} \] (3)

When \( x=0, \theta=\alpha \), the formula can be obtained:

\[ T \cos \alpha = T_x \] (4)

The whole segment of anchor chain is regarded as a smooth and continuous derivable curve, and the arc length of each segment element is calculated. Finally, the arc length of several segments is added together (the integral operation of the differential equation is carried out), and the result is obtained:

\[ ds = \sqrt{(dx)^2 + (dy)^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \cdot dx \] (5)

Further derivation of (2) can be obtained as follows:

\[ \frac{d(T \sin \theta)}{ds} = q \] (6)

By simultaneous analysis of (3) and (6), the following can be obtained:

\[ \begin{cases} T \cos \theta = \text{Const} \\ \frac{d(T \sin \theta)}{ds} = q \end{cases} \quad \text{And} \quad T \cos \theta \cdot d\left(\frac{T \sin \theta}{T \cos \theta}\right) = dsq \] (7)
Substituting (4) and (5) into (8), we can get:

$$T \cos \theta \frac{dy}{dx} = qdx \sqrt{1 + y^2} \tag{8}$$

From (3) and (4) to (8), we can get:

$$\frac{1}{\sqrt{1 + y^2}} dy = \frac{q}{T_x} dx \tag{9}$$

Substitute the two values obtained in (9) to obtain

$$\cosh t \frac{dt}{dx} = \frac{q}{T_x} \sqrt{1 + \sinh^2 t} \tag{10}$$

Equation (10) can be calculated from (9):

$$t = \frac{q}{T_x} x + C_1 \quad (14) \quad \rightarrow \quad y' = \sinh\left(\frac{q}{h} x + C_1\right) \tag{11}$$

When $x=0$, put this value into Equation (11) and integrate it to obtain: $y' = \tan \vartheta$

$$y = \frac{T_x}{q} \int \sinh\left(\frac{q}{T_x} x + C_1\right) d\left(\frac{q}{T_x} x + C_1\right) = \frac{T_x}{q} \cosh\left(\frac{q}{T_x} x + C_1\right) + C_0 \tag{12}$$

Thus, the integral constant can be determined as follows: $C_2 = -\frac{T_x}{q} \sec \vartheta$

Therefore, the differential equation of the anchor chain can be obtained from the integral constant and the boundary of $y$ when $x=0$: $C_1$

$$y = \frac{T_x}{q} \cosh\left[\frac{q}{T_x} x + \ln(\tan \alpha + \sec \alpha)\right] - \frac{T_x}{q} \sec \alpha \tag{13}$$

Substituting (13) into (5), the expression of anchor chain can be obtained:

$$ds = \cosh\left[\frac{q}{T_x} x + \ln(\tan \alpha + \sec \alpha)\right] dx \tag{14}$$

$$s = \int ds = \frac{T_x}{q} \sinh\left[\frac{q}{T_x} x + \ln(\tan \alpha + \sec \alpha)\right] + D \tag{15}$$

According to the force equilibrium theorem. Because the buoy is in equilibrium at the moment, the buoy receives horizontal and 1 vertical directions.

The resultant force is zero and hence the equilibrium equation [4]

$$\begin{align*}
\sum F_x &= 0, \quad F_y - T_{i,x} = 0 \\
\sum F_y &= 0, \quad F_{y1} - M_{1y} - T_{i,y} = 0
\end{align*} \tag{16}$$

The horizontal and vertical forces and torque of steel pipe are zero, the equilibrium equation of steel pipe can be obtained:

$$\begin{align*}
\sum F_x &= 0, \quad T_{i,x} - T_{i+1,x} = 0 \\
\sum F_y &= 0, \quad T_{i,y} + F_{y1} - T_{i+1,y} - m_i g = 0 \\
\sum M &= 0, \quad T_{i,x} l_1 \cos \varphi_i - T_{i,y} l_1 \sin \varphi_i + \frac{1}{2} (m_i g - F_{y1}) l_1 \sin \varphi_i = 0
\end{align*} \tag{17}$$

Set the coordinate of anchor chain and steel barrel as, then it can be obtained according to the upper and lower parts of the node respectively ($x_0, y_0$)
At this point, the modeling process is over, and the following six equations are obtained:

a) Slope equation of anchor chain (10)
b) Geometric shape equation of anchor chain (13)
c) Buoy balance equation (16)
d) Equation of steel pipe balance (17)
e) Geometric equations 18)

\[
y_0 = 18 - h - l_1 \cos \varphi_1 - l_1 \cos \varphi_2 - l_1 \cos \varphi_3 - l_1 \cos \varphi_4 - l_2 \cos \frac{T_x}{q} \cosh \left[ \frac{q}{T_x} x_0 + \ln(\tan \alpha + \sec \alpha) \right] - \frac{T_x}{q} \sec \alpha
\]

(18)

4. The Solution of the Model

Problem solving
Since the model of anchor chain should be considered, the diameter of chain link must be considered at this time. The difference equation of the anchor chain is calculated after the force analysis of the chain link.

\[
\begin{align*}
F_{i+1,x} &= F_{i,x} \\
F_{i+1,y} &= F_{i,y} + mg \\
\theta_i &= \arctan \left( \frac{2F_{i,y} + mg}{2F_{i,x}} \right)
\end{align*}
\]

(19)

The difference equation is applicable to each type of link, and by putting its mass and length into the recursive formula, the specific position and Angle of the link can be obtained.

Since we have to consider the model, length, shape of anchor chain, draft depth of buoy, tilt Angle of steel bucket, wind force and other factors, it will be very complicated if we directly list the formula. Therefore, it is advisable to assume a limit case in which the equipment can work normally. In the future, as long as the condition is better than the limit condition, the docking system can be used normally. So all you have to do is derive the worst case limit, and you get what you want to derive.

In this case, when the weight mass is large and the anchor chain length is long, the included Angle of the seabed and the tilt Angle of the steel bucket meet the requirements of the question, but the draft depth and the activity range of the buoy change greatly at this time, so the influence of other factors should be minimized as far as possible in this kind of limit condition.
According to (34) and (35), it can be deduced that the draft depth of the buoy is:

\[
\frac{31557.3h - 22143.1}{1.25(2 - h)v^2} = \sinh\left(\frac{q}{T_x} x_0 + \ln(\tan \alpha + \sec \alpha)\right)
\]

(20)

\[
\frac{31557.3h - 23655.7}{1.25(2 - h)v^2} = \tan \alpha
\]

(21)

H =1.32m was calculated, from which the number of chain links and the length of anchor chains of 5 types were deduced, as shown in Table 2:

Table 2. Five types of anchor chains require minimum weight mass and mass and corresponding length

| Chain models | 1   | 2   | 3   | 4   | 5   |
|--------------|-----|-----|-----|-----|-----|
| The number of link | 434 | 261 | 197 | 142 | 109 |
| Chain length (m) | 338.52 | 274.05 | 236.40 | 213.00 | 196.20 |
| Heavy ball mass (kg) | 2126.69 | 1291.61 | 254.97 | 0.00 | 0.00 |

In addition, the effect of tides on offshore current forces and the effect of wind on various factors are taken into account. We chose the way of controlling variables, assumed that the wind force was constant, and then carried out the influence of the offshore flow force on the experiment

\[
\frac{\partial H}{\partial t} + \frac{\partial (Hv)}{\partial x} + \frac{\partial (Hv)}{\partial y} = 0
\]

(22)

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - fu + g \frac{\partial H}{\partial x} + g \frac{\sqrt{u^2 + v^2}}{H} u - C_f \sqrt{F_{w_x}^2 + F_{w_y}^2} \frac{F_{w_x}}{H} - E \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0
\]

(23)

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + fu + g \frac{\partial H}{\partial y} + g \frac{\sqrt{u^2 + v^2}}{H} v - C_f \sqrt{F_{w_x}^2 + F_{w_y}^2} \frac{F_{w_y}}{H} - E \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0
\]

(24)

(24)

(24)

(where f is the Coriolis parameter; E represents the turbulence viscosity coefficient; C represents the Schaepy coefficient; \(F_{w_x}, F_{w_y}\) is the corresponding component of wind speed; \(C_f\) is the wind resistance coefficient.

By looking up information [5] The change curve of tide with time can be known (taking Dandong as an example)
Grey correlation analysis

In the mooring system, draft depth, swimming area and steel bucket inclination are all evaluation criteria for the results, so grey correlation analysis is conducted for them [6].

First, determine the weight of each parameter.
Assuming that they have the same weight (both are 1), the resolution coefficient $A$ is calculated by MATLAB. The larger the $A$ is, the greater the resolution is, while the smaller the $A$ is, the smaller the resolution is.

Second, the gray weighted correlation degree is calculated by using Matlab through the resolution coefficient.

Third, the parameters are sorted according to the size of grey correlation degree. The greater the correlation degree, the better the evaluation effect.

The correlation degree value was 0.5026 0.5398 0.7778.

Therefore, the evaluation result obtained from the correlation degree is good.

Reference

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