Modeling and Optimal Control of a Hydrogen Storage System for Wind Farm Output Power Smoothing

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Paper key contributions

• Operate devices by means of logic commands (ON, OFF, \ldots)
• The modeling recipe is given by \( \text{MLD} + \text{MPC} + \text{multi-objective} \)
• Convert/electrify suitable amounts of energy/hydrogen
• The minimum weighted power variation over the prediction horizon is computed first (high priority)
• Then it is used as a constraint in the minimization of the load tracking error stage
• Minimise costs for long term profitability
• Ensure the safety of operations
Microgrid under investigation
Mixed logical dynamical modeling I

Automaton for the electrolyzer \((i = e)\) and the fuel cell \((i = f)\).

The devices are modeled with 5 state automata described by

- logic states \(\delta_i^\alpha\)
- states transitions \(\sigma_{\alpha_i}^\beta\)

Logic states and transitions in

- dynamic models
- constraints
- cost functions

- The 5 states of the devices ON, OFF, STB, CLD, WRM characterize the model
Mixed logical dynamical modeling II

$P_i(k)$ depends on the logic state corresponding to the operating condition of the $i$-th device that is $P_i^\alpha = P_i \delta_i^\alpha(k)$

$$
\begin{cases}
    P_i(k) = 0 & \iff \delta_i^{\text{OFF}}(k) = 1 \\
    P_i(k) = P_i^{\text{CLD}} & \iff \delta_i^{\text{CLD}}(k) = 1 \\
    P_i(k) = P_i^{\text{STB}} & \iff \delta_i^{\text{STB}}(k) = 1 \\
    P_i(k) = P_i^{\text{WRM}} & \iff \delta_i^{\text{WRM}}(k) = 1 \\
    P_i(k) \in [P_i^{\text{min}}, P_i^{\text{max}}] & \iff \delta_i^{\text{ON}}(k) = 1
\end{cases}
$$

Two (boolean) slack variables $z_i^{\geq \gamma}(k), z_i^{\leq \bar{\gamma}}(k)$ are introduced

$$
\begin{align*}
    z_i^{\geq \gamma}(k) &= \begin{cases}
        1 & P_i(k) \geq \gamma \\
        0 & P_i(k) < \gamma
    \end{cases} \\
    z_i^{\leq \bar{\gamma}}(k) &= \begin{cases}
        0 & P_i(k) > \bar{\gamma} \\
        1 & P_i(k) \leq \bar{\gamma}
    \end{cases}
\end{align*}
$$

$(\gamma, \bar{\gamma}) \in \{(0, 0), (P_i^{\text{CLD}}, P_i^{\text{CLD}}), (P_i^{\text{STB}}, P_i^{\text{STB}}), (P_i^{\text{WRM}}, P_i^{\text{WRM}}), (P_i^{\text{min}}, P_i^{\text{max}})\}$
Mixed logical dynamical modeling III

- MLD constraints of the devices logic states

\( P_i(k) \) can be linked to \( \delta^\alpha_i \) with inequalities through \( z_i^{\geq \gamma}, z_i^{\leq \bar{\gamma}} \) (Big M-reformulation)

\[
\begin{align*}
P_i(k) - \gamma &< Mz_i^{\geq \gamma}(k) & -P_i(k) + \bar{\gamma} &< Mz_i^{\leq \bar{\gamma}}(k) \\
-P_i(k) + \gamma &\leq M(1 - z_i^{\geq \gamma}(k)) & P_i(k) - \bar{\gamma} &\leq M(1 - z_i^{\leq \bar{\gamma}}(k)) \\
(1 - \delta^\alpha_i(k)) + z_i^{\geq \gamma}(k) &\geq 1 & (1 - \delta^\alpha_i(k)) + z_i^{\leq \bar{\gamma}}(k) &\geq 1
\end{align*}
\]

The following constrain must be added

\[
\sum_{\alpha} \delta^\alpha_i(k) = 1
\]

where \( \alpha \in \mathcal{A}, \mathcal{A} = \{\text{OFF, CLD, STB, WRM, ON}\} \)
Mixed logical dynamical modeling IV

- MLD constraints of the devices state transitions

The AND operator ($\wedge$) can be written in terms of inequalities

$$
\mu_3 = \mu_1 \mu_2 \iff \begin{cases} 
\mu_3 \leq \mu_1 \\
\mu_3 \leq \mu_2 \\
\mu_3 \geq \mu_1 + \mu_2 - 1
\end{cases}
$$

where $\mu_j \in \{0, 1\}, \ j \in \{1, 2, 3\}$, $\mu_1 = \delta_i^\alpha(k - 1)$, $\mu_2 = \delta_i^\beta(k)$ and $\mu_3 = \sigma_{\alpha,i}^\beta(k)$. All the inadmissible transitions, i.e., those not shown in automata, are set to 0

$$
\sigma_{\text{OFF},i}^{\text{STB},i}(k) = \sigma_{\text{OFF},i}^{\text{WRM},i}(k) = \sigma_{\text{OFF},i}^{\text{ON},i}(k) = \sigma_{\text{OFF},i}^{\text{CLD},i}(k) \\
= \sigma_{\text{WRM},i}^{\text{ON},i}(k) = \sigma_{\text{WRM},i}^{\text{CLD},i}(k) = \sigma_{\text{STB},i}^{\text{ON},i}(k) = \sigma_{\text{STB},i}^{\text{CLD},i}(k) \\
= \sigma_{\text{OFF},i}^{\text{WRM},i}(k) = \sigma_{\text{ON},i}^{\text{WRM},i}(k) = \sigma_{\text{ON},i}^{\text{CLD},i}(k) = 0
$$
Mixed logical dynamical modeling V

- **Operating constraints**

  \[
  \delta_{i}^{\text{CLD}}(k) - \delta_{i}^{\text{CLD}}(k - 1) \leq \delta_{i}^{\text{CLD}}(\tau^{\text{CLD}}) \\
  \delta_{i}^{\text{CLD}}(k) + \cdots + \delta_{i}^{\text{CLD}}(k - T^{\text{CLD}}) \leq T^{\text{CLD}} \\
  \tau^{\text{CLD}} = k + 1, \ldots, k + T^{\text{CLD}} \\
  \delta_{i}^{\text{WRM}}(k) - \delta_{i}^{\text{WRM}}(k - 1) \leq \delta_{i}^{\text{WRM}}(\tau^{\text{WRM}}) \\
  \delta_{i}^{\text{WRM}}(k) + \cdots + \delta_{i}^{\text{WRM}}(k - T^{\text{WRM}}) \leq T^{\text{WRM}} \\
  \tau^{\text{WRM}} = k + 1, \ldots, k + T^{\text{WRM}}
  \]

- **Ramp up constraints**

  \[|((P_i(k + 1) - P_i(k))\delta_{i}^{\text{ON}}| \leq R_i\]
Mixed logical dynamical modeling VI

• Hydrogen dynamics

\[ H(k + 1) = H(k) + \eta_e(k) P_e(k) \delta_e^{ON}(k) T_s - \frac{P_f(k) \delta_f^{ON}(k) T_s}{\eta_f(k)} \]

• Operating ranges

\[ P_{i_{\text{min}}} \leq P_i(k) \leq P_{i_{\text{max}}} \]

\[ H_{\text{min}} \leq H(k) \leq H_{\text{max}} \]

• Power balance

\[ P_w(k) - P_e(k) \delta_e^{ON}(k) + P_f(k) \delta_f^{ON}(k) - P_{\text{avl}}(k) - P_{\text{dump}}(k) = 0 \]
ESS operating costs

\[ J_i(k + j) := \left( \frac{S_{\text{rep},i}}{NH_i} + C_{i}^{\text{OM}} \right) \delta_{i}^{\text{ON}}(k + j) + C_{\text{ON},i}^{\text{OFF}} \sigma_{\text{ON},i}(k + j) + C_{\text{CLD},i}^{\text{STB}} \sigma_{\text{CLD},i}(k + j) + C_{\text{STB},i}^{\text{OFF}} \sigma_{\text{STB},i}(k + j) + s(k + j)P_{i}^{\text{STB}} \delta_{i}^{\text{STB}}(k + j) + s(k + j)P_{i}^{\text{CLD}} \delta_{i}^{\text{CLD}}(k + j) + s(k + j)P_{i}^{\text{WRM}} \delta_{i}^{\text{WRM}}(k + j) \]

where

- \( S_{\text{rep},i} \): the \( i \)-device stack replacement cost
- \( C_{i}^{\text{OM}} \): the \( i \)-device O&M cost
- \( C_{\text{ON},i}^{\text{OFF}}, C_{\text{CLD},i}^{\text{STB}}, C_{\text{STB},i}^{\text{OFF}} \): the \( i \)-th device cycle costs
- \( P_{i}^{\text{STB}}(k) \): the power at standby
- \( s(k) \): the power spot price
Cost functions

- Power smoothing cost function

\[ J_s(k) := \sum_{j=0}^{T-1} \sum_{\tau=1}^{\tau_B} \omega^{k+j,\tau} y^{k+j,\tau}, \]

where \( y^{k+j,\tau} \) is the power increment such that

- \( y^{k+j,\tau} \geq 0 \)
- \( y^{k+j,\tau} \geq |P_{avl}(k+j) - P_{avl}(k+j-\tau)| - \bar{y}^\tau \)

with \( \bar{y}^\tau \) is a given power reference depending on \( \tau \)

- Load tracking cost function

\[ J_l(k) := \frac{1}{T} \sum_{j=0}^{T-1} \left( P_{avl}(k+j) - P_{ref}(k+j) \right)^2. \]

- Global cost function

\[ J(k) := \sum_{j=0}^{T-1} \rho_l J_l(k+j) + \rho_e J_e(k+j) + \rho_f J_f(k+j), \]
Multi-objective optimization I

$$\min_{C_k} \left\{ J_s(k), J(k) \right\}$$

Subject to
Discrete logical states constraints,
Mode transitions constraints,
Physical constraints,
Hydrogen level dynamics,
Power smoothing constraints.

$$\min_{C_k} J_s(k)$$

s.t.
All constraints.

$$\min_{C_k} J(k)$$

s.t.
All constraints,
$$J_s(k) \leq J^*_s.$$
Multi-objective optimization II

- At each time step $k$, given the initial state $H(k)$, the MPC provides the optimal control sequences

$$C_k := \{ P_{i,k}^{T-1}, P_{avl,k}^{T-1}, P_{dump,k}^{T-1}, \delta_{i,k}^{\alpha,T-1}, \sigma_{\alpha i,k}^{\beta,T-1}, z_{i,k}^{\gamma,T-1} \},$$

where

$$P_{i,k}^{T-1} = (P_i(k), \ldots, P_i(k + T - 1))^T$$
$$P_{avl,k}^{T-1} = (P_{avl}(k), \ldots, P_{avl}(k + T - 1))^T$$
$$P_{dump,k}^{T-1} = (P_{dump}(k), \ldots, P_{dump}(k + T - 1))^T$$
$$\delta_{i,k}^{\alpha,T-1} = (\delta_{i}^{\alpha}(k), \ldots, \delta_{i}^{\alpha}(k + T - 1))^T$$
$$\sigma_{\alpha i,k}^{\beta,T-1} = (\sigma_{\alpha i}^{\beta}(k), \ldots, \sigma_{\alpha i}^{\beta}(k + T - 1))^T$$
$$z_{i,k}^{\gamma,T-1} = (z_{i}^{\gamma}(k), \ldots, z_{i}^{\gamma}(k + T - 1))^T$$
Figure: Wind and operator power profiles
Figure: Smoothed available power profiles
Figure: Hydrogen levels
Hydrogen-Aeolic Energy with Optimised eLectrolysers
Upstream of Substation

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Thank you for your attention