Applying GPS Time-Differenced Carrier Phase velocity estimation for Calibration of LDV/INS Integrated Navigation Systems in land vehicles

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Abstract. The laser Doppler velocimeter (LDV) is integrated with inertial navigation systems (INS) to form an integrated navigation system to acquire the land vehicle’s orientation, velocity, and position. As an integrated navigation system, the calibration items, including scale factor error of the LDV and the installation angle error between the INS and the LDV, should be well calibrated; otherwise the accuracy of navigation performance will be seriously affected. The paper represents a method about applying time-differenced carrier phase observations to deliver accurate reference velocity for calibration instead of differenced GPS (DGPS) or precise point position (PPP). The differential carrier phase between successive GPS epochs eliminates the effort of ambiguity and it is able to achieve millimetre-level relative velocity accuracy within minutes. In order to make up for the insufficient of calibration time, a iteration method is implemented for fast convergence. The navigation experiment shows that the calibrated LDV/INS navigation system can reach 34.7m/25.5km in horizon and 4.8m/25.5km in vertical.

1. Introduction
As a critical component of intelligent transportation systems (ITS), the Global Positioning System (GPS) has been widely applied in land vehicles navigation, including unmanned vehicles[1]. However, the multi-path phenomenon and outages of GPS signals occurs frequently in dense urban areas, called urban canyon effect, which requires another navigation method as substitution[2]. Inertial navigation system (INS) is able to provide complete navigation result autonomously but the position error accumulates with time rapidly[3]. Therefore, other navigation techniques are required to be integrated with INS to obtain better positioning performance in urban canyons[4].

The odometer (OD)/INS integrated navigation systems have been widely used in autonomous vehicle navigation fields[5]. However, affected by the uncertainty of the odometer’s scale factor caused by the wheel slipping and skidding as well as wheel radius changing, the navigation performance will decrease with time[6]. The visual/INS integrated system is able to obtain the passive navigation information using Simultaneous Localization and Mapping (SLAM)[7]. However, the SLAM often suffers from dynamic environment, environments with too many or very few salient features, erratic movement of camera or sensors’ occlusions that the navigation performance is seriously affected[8]. Light Detection and Ranging (LiDAR)-based SLAM can achieve high positioning accuracy and it is implemented in GPS denied environment navigation, but the heavy burden of point cloud resolution makes the real-time systems difficult[9]-[10].
Based on the principle of laser Doppler effect, the LDV has the advantages of contactless measurements and well linear response, thus it has been extensively used for precise measurement field[11]. The LDV is usually integrated with INS to construct an INS/LDV integrated system for autonomous navigation application. Before applying INS/LDV integrated system for navigation, the installation angle and scale factor should be properly calibrated, otherwise the positioning error will accumulate with time rapidly [12]-[13]. Conventionally, the DGPS or converged PPP is supposed to provide accurate absolute reference velocity or position in calibration of integrated navigation system[14]. Actually, the LDV/INS integrated navigation system’s calibration requires relative velocity rather than absolute velocity. As introduced in[15], the TDCP observation is supposed to provide relative velocity with millimetre-level accuracy and it can already meet the LDV calibration requirements.

2. LDV/INS Calibration Model

2.1. The update function of LDV/INS Integrated Calibration System

The velocity in LDV’s frame (m-frame) is expressed as

$$V_{LDV}^m = \begin{bmatrix} 0 & V & 0 \end{bmatrix}^T$$

(1)

where \(V\) is the relative velocity between LDV and the ground. With the transformation matrix given by orientation, the related velocity given by LDV can be orthogonal decomposed into local navigation frame. The velocity given by LDV is given as

$$V_{LDV}^n = C_b^c C_m^b V_{LDV}^m = C_b^n V_{LDV}^n$$

(2)

Similar to position update function of INS, the position update function in DR system is expressed as

$$\dot{L}_{LDV} = V _{LDV,E}^n / (R_n + h_{LDV})$$

$$\dot{h}_{LDV} = V_{LDV,U}^n$$

(3)

in which \(V_{LDV,E}^n, V_{LDV,N}^n, \) and \(V_{LDV,U}^n\) represent the component of vehicle’s velocity in eastern, northern, and upward. \(L_{LDV}, \hat{\lambda}_{LDV}, \) and \(h_{LDV}\) is the latitude, longitude and height of the DR system.

Since LDV gives more accuracy velocity information and it has no error accumulate effect, the INS/LDV integrated DR system is expected to have better performance in positioning.

2.2. Model with calibration items

According to the principle of laser Doppler effect, the related movement between the light source and detector makes a light frequency shift and the related velocity can be obtained by beat frequency measurement. The relationship between the frequency shift and related velocity is given as

$$V = V_f / \cos \theta_{LDV} = \lambda_{LDV} f_D / (2 \cos \theta_{LDV}) = K_{LDV} f_D$$

(4)

represents the velocity given by detected frequency shift alone the direction from the detector to the measurement point and \(\lambda\) is the wavelength of laser. \(\theta_{LDV}\) is the design angle of LDV which represents the included angle between laser beam and ground. The relationship between relative velocity and detecting laser beam direction is shown in Figure 1. The \(K_{LDV}\) can be regarded as the scale coefficient from frequency shift to velocity. Affected by the error of included angle, the actual scale factor is expressed as

$$\hat{K}_{LDV} = (1 + \delta K_{LDV}) K_{LDV}$$

(5)
Another main error source is the installation angle \( \Delta \phi \) between the INS and the LDV shown in Figure 2, where the \( L_{Am} \) is the lever-arm between them. The angle \( \phi_x \), \( \phi_y \), and \( \phi_z \) represent the pitch, roll and heading installation angle error thus the installation cosine matrix \( C^m_{b} \) is given as

\[
C^m_{b} = \begin{bmatrix}
\cos \phi_x & \cos \phi_x & \sin \phi_x & -\cos \phi_y & \sin \phi_x & -\sin \phi_x & -\cos \phi_z & -\sin \phi_x & \cos \phi_x \\
\cos \phi_y & \cos \phi_y & -\sin \phi_y & \sin \phi_x & \cos \phi_y & -\sin \phi_x & -\cos \phi_z & -\sin \phi_x & -\cos \phi_y \\
\sin \phi_z & -\cos \phi_z & \sin \phi_z & -\cos \phi_x & -\sin \phi_z & \cos \phi_x & -\cos \phi_y & \sin \phi_x & \cos \phi_y
\end{bmatrix}
\]

Taking the velocity in body frame \((b-frame)\) is expressed as

\[
\tilde{V}^n_{LDV} = C^m_{b}V^n = C^m_{b}(V^n_{LDV} + V_{Am})
\]

(6)

Since the lever-arm can be well estimated by total station, (7) is simplified as

\[
V^b_{LDV} = C^m_{b}V^n_{LDV} = (C^m_{b})^T V^m_{LDV} = \begin{bmatrix}
\cos \phi_x & \sin \phi_x & \cos \phi_y & \sin \phi_y & \sin \phi_z & -\cos \phi_z
\end{bmatrix}^T V
\]

(8)

It is easy to conclude from (8) that \( \phi_y \) have no effect on the calibration result thus the installation angle error items reduce to \( \phi_x \) and \( \phi_z \).

After taking the installation angle error and the scale factor error into the DR model, the velocity is rewritten as

\[
\tilde{V}^n_{LDV} = \tilde{C}^n_{a}C^m_{b}V^n_{LDV}
\]

(9)

Since the calibration lasts for just a few minutes and the high accuracy of gyroscopes, the cosine matrix \( \tilde{C}^n_{a} \) is regard as constant as \( C^m_{b} \) and (9) is simplified to

\[
\tilde{V}^n_{LDV} = C^a_{b}C^m_{a}V^n_{LDV} + \left[ C^a_{b}C^m_{a}V^n_{LDV} \times \right]\Delta \phi + C^a_{b}C^m_{a}V^n_{LDV}\delta K
\]

(10)

According to (2), (10) is written as

\[
V^n_{LDV} = V^n_{LDV} + \left[ V^n_{LDV} \times \right]\Delta \phi + V^n_{LDV}\delta K
\]

(11)

in which \( V^n_{LDV} \) is the calculated velocity given by LDV/INS integrated system and \( \tilde{V}^n_{LDV} \) is the actual velocity given by other navigation system such as GNSS. It is obvious that velocity difference can be obtained as the measurement vector for LDV calibration estimation in Kalman filter method. Base on this idea, the state vector including installation angle and scale factor is defined as

\[
X = \begin{bmatrix}
\Delta \phi_x \\
\Delta \phi_z \\
\delta K_{LDV}
\end{bmatrix}
\]

(12)

Installation angle error and scale factor error are considered to be constant, thus the state transition functions is given as

\[
\dot{X} = 0_{3 \times 3}X + u
\]

(13)

where \( u \) is the measurement noise vector and the measurement matrix defined as

\[
H = \begin{bmatrix}
-V^n_{LDV} \\
V^n_{LDV}
\end{bmatrix}
\]

(14)
3. Time-Differenced Carrier Phase observation for velocity estimation

As introduced in [13] and [14], DGPS and PPP are common applied to provide accurate reference velocity. But there are limitations for DGPS or PPP assisted calibration:

- Since reference stations are essential for DGPS positioning mode. Hence, some other GPS receivers must be placed in some already-known positions, which will increase the calibration cost and complexity.

- PPP is able to provide accuracy velocity in stand-alone receiver. However the PPP positioning usually needs a long convergence time (approximately 30 minutes) to converge to accurate performance; otherwise the position error is meter-level[16].

In addition, both the DGPS and the PPP have to face to the ambiguity fixing problem which has not been appropriately solved within short time. Corresponding to the disadvantages of DGPS and PPP, the TDCP method do not require ambiguity fixing and it is able to provide relative velocity with millimetre-level accuracy in horizon and centimetre-level accuracy in vertical[15].

3.1. Principle

The carrier phase observation from \(i\)-th GPS satellite to antenna \(n\) can expressed in meters as[16]

\[
\phi_i = r_i + c\delta t_i - c\delta t_{i,j} - I_i + T_i + \epsilon_{\phi_i} + \lambda N
\]

(15)

Where \(r_i\) is the distance between the antenna and GPS satellite. \(N\) represents the carrier phase ambiguity which remains an unknown constant if there is no signal blockage or cycle slip. \(c\delta t_i\) is the receiver clock offset in meters. \(c\delta t_{i,j}\), \(I_i\), and \(T_i\) represents satellite clock offset, ionospheric delay, and tropospheric delay correspondingly and they can be considered deterministic by stand-alone GPS in meters. The satellite clock offset and satellite position is corrected via the precise ephemeris given by IGS station. The ionospheric delay is revised by the dual-frequency measurement and the tropospheric delay is estimated by Saastamoinen model. After depriving the deterministic items from (15), the carrier phase is written as

\[
\phi_i = r_i + c\delta t_i + \epsilon_{\phi_i} + \lambda N
\]

(16)

where \(\epsilon_{\phi_i}\) represents the residual errors including the GPS receiver’s noise and the multi-path effect which is within several centimeters. Since \(N\) keep fixed if there is no signal blockage or cycle slip, the TDCP method establish a differential carrier phase observation between different GPS time epochs to avoid the problem of ambiguity fixing. The TDCP observation is expressed as

\[
\Delta \phi_i = \phi_i(t_2) - \phi_i(t_1) = \Delta r_i + c\Delta \delta t_i + \Delta \epsilon_{\phi_i}
\]

(17)
\( \Delta e_{\phi,j} \) represents the unmolded errors and it is at centimeter level for tens of minutes. The observation between \( t_1 \) and \( t_2 \) is shown in Figure 3, in which \( \Delta b \) is the position change vector. As introduced in [17], the different satellite distance \( \Delta r_i \) is expressed as

\[
\Delta r_i = \left( \{ \mathbf{R}_i(t_1), \mathbf{e}_i(t_1) \} - \{ \mathbf{R}_i(t_2), \mathbf{e}_i(t_2) \} \right) - \left( \{ \mathbf{b}(t_1), \mathbf{e}_i(t_1) \} - \{ \mathbf{b}(t_2), \mathbf{e}_i(t_2) \} \right) - \Delta \mathbf{b}, \mathbf{e}_i(t_2) \tag{18}
\]

where \( \mathbf{R}_i(t) \) is the \( i \)-th satellite position vector and \( \mathbf{b}(t) \) is the receiver position vector at epoch \( t \). The relationship between the \( i \)-th satellite and the receiver during time epoch \( t_1 \) and \( t_2 \) is shown in Figure 3. \( \mathbf{e}_i(t) \) is the line-of-sight unit vector from receiver to satellite and is expressed as

\[
\mathbf{e}_i(t) = \frac{\mathbf{R}_i(t) - \mathbf{b}(t)}{\| \mathbf{R}_i(t) - \mathbf{b}(t) \|} \tag{19}
\]

The first two items represents the Doppler effect caused by the satellite motion and the next two items represents the geometry changing between the satellite and the earth, the TDCP observation in (17) is rewritten as

\[
\Delta \phi_i = \text{Dop} - \text{Geo} - \{ \Delta \mathbf{b}, \mathbf{e}_i(t) \} + c \Delta \delta t_i + \Delta \epsilon_{\phi,j} \tag{20}
\]

Receiver position changing is estimated by implementing the least-mean square (LMS) solution, the observability matrix \( \mathbf{H} \) and observed value \( \mathbf{Y} \) are expressed as

\[
\mathbf{Y} = \begin{bmatrix}
\Delta \phi_1^{\text{comp}} \\
\vdots \\
\Delta \phi_n^{\text{comp}}
\end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix}
\mathbf{e}_1(t)^T & c \Delta \delta t_{1,j} \\
\vdots & \vdots \\
\mathbf{e}_n(t)^T & c \Delta \delta t_{n,j}
\end{bmatrix}
\]

Receiver position changing \( \Delta \mathbf{b} \) is calculated as

\[
\Delta \mathbf{b} = \left( \mathbf{H}^T \mathbf{H} \right)^{-1} \mathbf{H}^T \cdot \mathbf{Y} \tag{22}
\]

The average velocity in east-north, and up in (ENU) frame from GPS epoch \( t_1 \) to \( t_2 \) is calculated as

\[
\mathbf{v}_{\text{ref}}^e = \mathbf{C}_e^w \Delta \mathbf{b} / (t_2 - t_1) \tag{23}
\]

Where \( \mathbf{C}_e^w \) is the transform matrix from ECEF frame to ENU frame which is given as

Figure 3. the TDCP observation during epoch \( t_1 \) and \( t_2 \)
\[
\mathbf{C}^* = \begin{bmatrix}
-\sin \lambda & -\sin \phi \cos \lambda & \cos \phi \cos \lambda \\
\cos \lambda & -\sin \phi \sin \lambda & \cos \phi \sin \lambda \\
0 & \cos \phi & \sin \phi
\end{bmatrix}
\] (24)

where \( \lambda \) is the receiver’s latitude and \( \phi \) is the receiver’s longitude.

4. Experiment

The test equipment of the integrated system in Fig composed of a high-precision INS and a LDV whose main parameters are listed in Table 1 and Table 2.

| Specification of the INS. |
|---------------------------|
| **Random walk** | **Bias instability** |
| Gyroscope | 0.003deg/√h | 0.0004deg/h |
| Accelerometer | 40mGal/s | 40mGal |

| Specification of the LDV. |
|---------------------------|
| **Noise** | **Bias** |
| 0.1% | 0.08% |

The GPS/SINS lever arm and LDV/SINS lever arm have been measured by total station. The GPS sampling rate is 1 Hz, whereas the INS and the LDV sampling rate is 100 Hz. The calibration takes 1 minute, and the journey is 665.6m. The calibration result is shown in Figure 4 and Figure 5.

![Figure 4. The installation angle after 10 times of iteration.](image1)

![Figure 5. The scale factor after 10 times of iteration.](image2)

After 10 times of iteration, the installation angle and the scale factor have converged to stable value shown in Table 3.

| Calibration result after 10 times of iteration. |
|---------------------------|
| **Pitch** | **Heading** | **Scale factor** |
| 2.435° | -9.979° | 5.847 × 10^{-2} |

In order to test the accuracy of calibration results, the installation angle and scale factor is implemented into the LDV/INS integrated navigation system. The integrated navigation experiment, including turning, straight driving and standing, takes 0.8 hour and the distance is 25.5km. The true position is given by the converged PPP and the trajectory of the experiment is shown in Figure 6, in which the red line represents trajectory of the LDV/INS integrated navigation system and the blue line represents the true position given by converged PPP. The position error in horizon and vertical is shown in Figure 7 and the maximum position error in horizon and vertical are shown in
Table 4.

| Horizon | Vertical |
|---------|----------|
| 34.7m   | 4.80m    |

Figure 6. The trajectory of the LDV/INS navigation experiment.

Figure 7. The position error in horizon and vertical.

5. Conclusion

In this paper, a calibration method using TDCP velocity estimation for LDV/SINS integrated navigation system is proposed. Without fixing carrier phase ambiguities, the TDCP observation is able to deliver high accuracy relative velocity by differenced carrier phase during two successive GPS epochs in stand-alone GPS mode. For more convenient usage and lower cost, the velocity given by TDCP is implemented in the Kalman filter as reference velocity instead of DGPS or PPP. Since the time is not enough for the calibration items to fully converge, iteration is employed in calibration method. The experimental results show that this calibration method is convenient to achieve, and has high accuracy of the scale factor error and installation errors of LDV/SINS integrated system within short time period. Therefore, the TDCP-aided calibration method in this paper is efficient to improve the accuracy of installation angle and scale factor error measurement, and makes it possible to apply it into the ITS for high accuracy positioning.

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