RESEARCH ARTICLE

THE GENERALIZED Q-BESSEL MATRIX FUNCTION OF TWO VARIABLES

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Abstract

The Bessel function is probably the best known special function, within pure and applied mathematics. In this paper, we introduce the generalized q-analogue Bessel matrix function of two variables. Some properties of this function, such as generating function, q-difference equation, and recurrence relations are obtained.

Keywords: Q-analogue Bessel matrix function of two variables, Generating function, The q-difference equation and the recurrence relations.

1. Introduction.

The theory of special functions performs an essential role in the formalism of mathematical physics. The Bessel functions [10] are one of the most important special functions and have applications in number theory, lie theory and theoretical astronomy to some problems of engineering and physics.

The Bessel’s function of first kind and order defined by [10]

\[ J_\nu(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+\nu}}{\Gamma(n+\nu+1) n!} \tag{1.1} \]

and the q-analogue Bessel function of one variable is defined by [3, 4]

\[ J_\nu(x; q) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+\nu}}{\Gamma(qn+\nu+1) \Gamma(1/q) n!} \tag{1.2} \]

where \( \lim_{q \to 1} J_\nu(x; q) = J_\nu(x) \) .

Mahmoud [9] presented the following generalized q-Bessel function:

\[ J_\nu(x, a; q) = \sum_{n=0}^{\infty} \frac{x^{2n+\nu}}{\Gamma(n+\nu+1) \Gamma(1/q) n!} (a^n q^n)_n \tag{1.3} \]

which converges absolutely for all \( x \) when \( a \in \mathbb{Z}^+ \) and for \( |x| < 2 \) if \( a = 0 \).

The Bessel matrix functions \( J_\nu(z) \) of the first kind of order \( A \) is defined as follows [11, 12]:

\[ J_\nu(z) = \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n}}{\Gamma(n+\nu+1) \Gamma(1/2) n!} (A+ (n+1)!) \tag{1.4} \]

where \( A \) is a matrix such that \( A \in \mathbb{C}^{N \times N} \) satisfying the condition \( \mu \) is not a negative integer for all \( \mu \in \sigma(A) \), where \( \sigma(A) \) is the set of all eigenvalue of \( A \).

The two variable Bessel’s functions are defined by the following series representations [1]:

\[ J_{\nu_1, \nu_2}(x, y) = \sum_{m,n=0}^{\infty} \frac{(-1)^{m+n}}{\Gamma(m+n+1) \Gamma(1/2)} \left( \frac{x}{2} \right)^{2m+n} \left( \frac{y}{2} \right)^{2m+n} \tag{1.5} \]

and

\[ J_{\nu_1, \nu_2}(x, y) = \sum_{m,n=0}^{\infty} \frac{(-1)^{m+n}}{\Gamma(m+n+1) \Gamma(1/2)} \left( \frac{x}{2} \right)^{2m+n} \left( \frac{y}{2} \right)^{2m+n} \tag{1.6} \]

respectively, where \( r \) and \( s \) both integers.

Also, Tenguria and Sharma [13] introduced and studied the advanced q-Bessel function of two variables defined by

\[ J_{\nu_1, \nu_2}(x, y; q) = \sum_{m,n=0}^{\infty} \frac{(-1)^{m+n}}{\Gamma(m+n+1) \Gamma(1/2) \Gamma(1/q) n!} \left( \frac{x}{2} \right)^{2m+n} \left( \frac{y}{2} \right)^{2m+n} \tag{1.7} \]

where \( \lim_{q \to 1} J_{\nu_1, \nu_2}(x, y; q) = J_{\nu_1, \nu_2}(x, y) \).

The q-shifted factorials are defined by [7]

\[ (a; q)_n = \frac{1}{\prod_{i=0}^{n-1}(1 - aq^i)} \quad n = 0 \]

\[ (a_1, ..., a_k; q)_k = \prod_{i=0}^{k-1}(a_1; q)_i \quad k = 0, 1, 2, ... \]

\[ (a; q)_\infty = \prod_{i=0}^{\infty}(1 - aq^i) \]

where \( a, a^j, q, \in \mathbb{R} \) such that \( 0 < q < 1 \).
Exton [5] presented the whole family of basic q-
exponential function as:
\[
E(q; x; q) = \sum_{n=0}^{\infty} q^{mn(n-1)} \frac{x^n}{[q^n]}_n
\]  
(1.12)

where
\[
[q^n]_n = \frac{q^n}{1-q^n}.
\]  
(1.13)
The one parameter family of q-exponential functions
\[
E_q^0(x) = \sum_{n=0}^{\infty} q^{an^2/2} \frac{x^n}{[q^n]_n}
\]
with \( a \in R \) has been considered in [6]. Consequently, in
the limit when \( q \to 1 \),
we have \( \lim_{q \to 1} E_q^0(1-q)x) = e^x \).

In Exton’s formula, if we replace \( z \) by \( \frac{x}{1-q} \) and \( \mu \) by \( \frac{a}{2} \), we get
\[
E_q^0(\frac{x}{1-q}; q) = E_q(x, a),
\]
where
\[
E_q(x, a) = \sum_{n=0}^{\infty} q^{an^2/2} \frac{x^n}{[q^n]_n},
\]  
(1.14)
which satisfies the functional relation [2]
\[
E_q(x, a) - E_q(qx, a) = xE_q(x^a, a).
\]  
(1.15)
The above q-function can be rewritten by the formula
\[
D_q E_q(x, a) = \frac{1}{1-q} D_q E_q(x^a, a).
\]  
(1.16)

The Jackson q-difference operator \( D_q \) is defined by
\[
D_q f(x) = \frac{f(qx) - f(x)}{q-x},
\]  
(1.17)
In this paper, we introduce a second form of the
generalized q-Bessel matrix function of two variables
and study some of its properties.

2. The generalized q-Bessel Matrix function of
two variables
We define the generalized q-Bessel matrix function of
two variables, denoted by \( J_{r,s}(x, y, a, A, B; q) \), by the
following generating function:
\[
E_q \left[ z_{\frac{\sqrt{y}x}{2}} \frac{z^{\frac{\sqrt{y}x}{2}}}{2} \right] E_q \left[ z_{\frac{1}{\sqrt{y}}} \frac{z^{\frac{1}{\sqrt{y}}}}{2} \right] E_q \left[ z_{\frac{1}{\sqrt{y}}} \frac{z^{\frac{1}{\sqrt{y}}}}{2} \right] E_q \left[ z_{\frac{1}{\sqrt{y}}} \frac{z^{\frac{1}{\sqrt{y}}}}{2} \right] \]
(1.18)
\[
E_q \left[ z_{\frac{1}{\sqrt{y}}} \frac{z^{\frac{1}{\sqrt{y}}}}{2} \right] E_q \left[ z_{\frac{1}{\sqrt{y}}} \frac{z^{\frac{1}{\sqrt{y}}}}{2} \right] E_q \left[ z_{\frac{1}{\sqrt{y}}} \frac{z^{\frac{1}{\sqrt{y}}}}{2} \right] E_q \left[ z_{\frac{1}{\sqrt{y}}} \frac{z^{\frac{1}{\sqrt{y}}}}{2} \right] \]
where \( a \in Z^+, x, y \in N, t, w \in C, t, w \neq 0 \),
\( 0, A \in C^\times, x > 0, p(x) > 0 \), satisfying the condition
of the matrix in (1.5).

Now, by using the above generating function we will
deduce the generalized q-Bessel function of two
variables \( J_{r,s}(x, y, a, A, B; q) \) in the form of the
following theorem:

**Theorem 2.1.**

Let us assume that \( A, B \in C^\times, x > 0, p(x) > 0, 0 <
q < 1 \) then the following formula for q-Bessel function of
two variables \( J_{r,s}(x, y, a, A, B; q) \) holds true:
\[
J_{r,s}(x, y, a, A, B; q) = \left( \frac{\exp(q\pi)}{q} \right)^r \left( \frac{\exp(q\pi)}{q} \right)^s
\]
\[
\times \sum_{m,n=0}^{\infty} \frac{(1+m+n)a^{m+n}q^{m+n+1}y^{m+n+1}}{(q^{m+n+1})^m(q^{m+n+1})^n} \left( \frac{\exp(q\pi)}{q} \right)^{2m} \left( \frac{\exp(q\pi)}{q} \right)^{2n},
\]  
(2.1)
which converges absolutely for all \( x \) and \( y \) when \( a \in Z^+ \) and
for \( |x| < 2, |y| < 2 \) if \( a = 0 \).

**Proof.** Let us denote the left hand side of (2.1) by \( W \) and
by using (1.14), we have
\[
W = \sum_{r,s=0}^{\infty} \frac{x^{r+s}}{(q^{r+s})^r} \left( \frac{\exp(q\pi)}{q} \right)^{2m} \left( \frac{\exp(q\pi)}{q} \right)^{2n} \times \sum_{m,n=0}^{\infty} \frac{(1+m+n)a^{m+n}q^{m+n+1}y^{m+n+1}}{(q^{m+n+1})^m(q^{m+n+1})^n} \left( \frac{\exp(q\pi)}{q} \right)^{2m} \left( \frac{\exp(q\pi)}{q} \right)^{2n},
\]
(2.2)
Replace \( r \) and \( s \) by \( r + m \) and \( s + n \) respectively in the
right hand side of equation (2.3), we get
\[
W = \sum_{r,s=0}^{\infty} \frac{x^{r+s+2m}}{(q^{r+s+2m})^r} \left( \frac{\exp(q\pi)}{q} \right)^{2m} \left( \frac{\exp(q\pi)}{q} \right)^{2n} \times \sum_{m,n=0}^{\infty} \frac{(1+m+n)a^{m+n}q^{m+n+1}y^{m+n+1}}{(q^{m+n+1})^m(q^{m+n+1})^n} \left( \frac{\exp(q\pi)}{q} \right)^{2m} \left( \frac{\exp(q\pi)}{q} \right)^{2n},
\]
(2.3)
By using relation (1.11) in (2.4), we obtain
\[
W = \sum_{r,s=0}^{\infty} \frac{x^{r+s+2m}}{(q^{r+s+2m})^r} \left( \frac{\exp(q\pi)}{q} \right)^{2m} \left( \frac{\exp(q\pi)}{q} \right)^{2n} \times \sum_{m,n=0}^{\infty} \frac{(1+m+n)a^{m+n}q^{m+n+1}y^{m+n+1}}{(q^{m+n+1})^m(q^{m+n+1})^n} \left( \frac{\exp(q\pi)}{q} \right)^{2m} \left( \frac{\exp(q\pi)}{q} \right)^{2n},
\]
(2.4)
By equating the coefficients of \( t^r w^s \) with the right hand
side of (2.1), we get the relation (2.2).

**Remark 2.1.**

Putting \( a = 0, A = \frac{1}{2}, B = 1 \) in equation (2.1) and in view of
equations (1.8) and (1.4), we get
\[
J_{r,s}(x, y, 0, \frac{1}{2}, 1, 1) = J_{r,s}(x, y, q),
\]  
(2.6)
and
\[
J_{r,s}(x, a, 0, \frac{1}{2}, 1, 1) = J_{r,s}(x, a, q),
\]  
(2.7)
respectively.

**Corollary 2.1.**

The function \( J_{r,s}(x, y, a, A, B; q) \) is a q-anaology of each of
the Bessel matrix function and the modified Bessel
matrix function.

**Proof.** Using (1.11) and (1.15) in (2.2) and taking the
limit as \( q \to 1 \) and setting \( x \to (1 - q)x \) and \( y \to (1 - q)y \), we get
\[
\lim_{q \to 1} J_{r,s}(x(1 - q)x, (1 - q)y, a, A, B; q) = \lim_{q \to 1} \left( \frac{\exp(q\pi)}{q} \right)^r \left( \frac{\exp(q\pi)}{q} \right)^s \times \sum_{m,n=0}^{\infty} \frac{(1+m+n)a^{m+n}q^{m+n+1}y^{m+n+1}}{(q^{m+n+1})^m(q^{m+n+1})^n} \left( \frac{\exp(q\pi)}{q} \right)^{2m} \left( \frac{\exp(q\pi)}{q} \right)^{2n},
\]
(2.8)
}\[
= \left( \frac{\exp(q\pi)}{q} \right)^r \left( \frac{\exp(q\pi)}{q} \right)^s \sum_{m,n=0}^{\infty} \frac{(1+m+n)a^{m+n}q^{m+n+1}y^{m+n+1}}{(q^{m+n+1})^m(q^{m+n+1})^n} \left( \frac{\exp(q\pi)}{q} \right)^{2m} \left( \frac{\exp(q\pi)}{q} \right)^{2n},
\]  
(2.9)
Hence, \[
\lim_{q \to 1} J_{r,s}(1 - q)x(1 - q)y, a, A, B; q) = J_{r,s}(x, y, a, A, B) \quad (2.8)
\]

**corollary 2.2.**

If \( R, S \) be integer, then \( J_{r,s}(x, y, a, A, B; q) \) satisfies the following relations:

\[
J_{r,s}(x, y, a, A, B; q) = (-1)^{(r+1)}J_{r,s}(x, y, a, A, B; q) \quad (2.9)
\]

\[
J_{r,s}(x, y, a, A, B; q) = (-1)^{(s+1)}J_{r,s}(x, y, a, A, B; q) \quad (2.10)
\]

\[
J_{r,s}(x, y, a, A, B; q) = (-1)^{(r+s)}J_{r,s}(x, y, a, A, B; q) \quad (2.11)
\]

**Proof.** Using (2.2), we get

\[
J_{r,s}(x, y, a, A, B; q) = \sum_{m,n=0} \frac{(-1)^{m+n+x+y}a^{m+n+r+s+1}}{(q)_m(q)_n(q)_r(q)_s(q)_x(q)_y)} \left( \frac{\alpha(q)}{2} \right)^{2m+n} \left( \frac{\beta(q)}{2} \right)^{2n+m},
\]

replacing \( m \) by \( m + r \) in the r.h.s. of (2.12), we obtain

\[
J_{r,s}(x, y, a, A, B; q) = \sum_{m,n=0} \frac{(-1)^{m+n+x+y}a^{m+n+r+s+1}}{(q)_m(q)_n(q)_{m+r+n+r}(q)_s(q)_x(q)_y)} \left( \frac{\alpha(q)}{2} \right)^{2m+r+n} \left( \frac{\beta(q)}{2} \right)^{2n+r+m},
\]

which on using definition (2.2) gives yields the required relation (2.9).

Similarly, we can prove the relations (2.10) and (2.11).

**corollary 2.3.**

The function \( J_{r,s}(x, y, a, A, B; q) \) satisfies the relations:

\[
J_{r,s}(-x, y, a, A, B; q) = (-1)^r J_{r,s}(x, y, a, A, B; q),
\]

(2.14)

\[
J_{r,s}(-x, -y, a, A, B; q) = (-1)^s J_{r,s}(x, y, a, A, B; q),
\]

(2.15)

\[
J_{r,s}(-x, -y, a, A, B; q) = (-1)^{r+s} J_{r,s}(x, y, a, A, B; q).
\]

(2.16)

**Proof.** Since,

\[
J_{r,s}(-x, y, a, A, B; q) = \frac{(\alpha(q))^{2m+n}}{(q)_m(q)_n(q)_r(q)_s(q)_x(q)_y)} \sum_{m,n=0} \frac{(-1)^{m+n+x+y}a^{m+n+r+s+1}}{(q)_m(q)_n(q)_{m+r+n+r}(q)_s(q)_x(q)_y)} \left( \frac{\alpha(q)}{2} \right)^{2m+n} \left( \frac{\beta(q)}{2} \right)^{2n+m},
\]

\[
J_{r,s}(-x, -y, a, A, B; q) = \frac{(\alpha(q))^{2m+r+n}}{(q)_m(q)_n(q)_{m+r+n+r}(q)_s(q)_x(q)_y)} \sum_{m,n=0} \frac{(-1)^{m+n+x+y}a^{m+n+r+s+1}}{(q)_m(q)_n(q)_{m+r+n+r}(q)_s(q)_x(q)_y)} \left( \frac{\alpha(q)}{2} \right)^{2m+r+n} \left( \frac{\beta(q)}{2} \right)^{2n+r+m},
\]

which in view of (2.2) yields relation (2.14).

Similarly, the relations (2.15) and (2.16), can be proved.

3. **The q-difference equation of the matrix function** \( J_{r,s}(x, y, a, A, B; q) \)

In order to derive the q-difference equation of the function \( J_{r,s}(x, y, a, A, B; q) \) applying the operator \( D_{r,q} \), on both sides of generating function (2.1) and using relations (1.15) and (1.17), we get

\[
E_x \left[ \frac{(\alpha(q))^2 \alpha q}{2} \right] E_y \left[ \frac{(-1)^{(r+1)}q^2 \alpha q}{2} \right] = \left( \frac{(-1)^{r+1}q^2 \alpha q}{2} \right)^2 \left( \frac{\beta(q)}{2} \right)^2 \left( \frac{\alpha(q)}{2} \right)^2 \left( \frac{\beta(q)}{2} \right)^2
\]

\[
= \sum_{m=0}^\infty q^m \left( \frac{q^2}{2} \right)^2 D_{r,q} J_{r,s}(x, y, a, A, B; q) t^m w^m \quad (3.1)
\]

Using (1.14) in the l.h.s. of the above equation, we obtain

\[
\sum_{m=0}^\infty \frac{(-1)^m}{m!} q^{m+1} \left( \frac{q^2}{2} \right)^m \left( \frac{\beta(q)}{2} \right)^m \left( \frac{\alpha(q)}{2} \right)^m \left( \frac{\beta(q)}{2} \right)^m \left( \frac{\alpha(q)}{2} \right)^m
\]

\[
= \sum_{m=0}^\infty q^m \left( \frac{q^2}{2} \right)^2 D_{r,q} J_{r,s}(x, y, a, A, B; q) t^m w^m \quad (3.2)
\]

Replace \( r \) and \( s \) by \( r + m \) and \( s + n \) respectively in the l.h.s. of (3.2), we get

\[
\sum_{m=0}^\infty \frac{(-1)^m}{m!} q^{m+1} \left( \frac{q^2}{2} \right)^m \left( \frac{\beta(q)}{2} \right)^m \left( \frac{\alpha(q)}{2} \right)^m \left( \frac{\beta(q)}{2} \right)^m \left( \frac{\alpha(q)}{2} \right)^m
\]

\[
= \sum_{m=0}^\infty q^m \left( \frac{q^2}{2} \right)^2 D_{r,q} J_{r,s}(x, y, a, A, B; q) t^m w^m \quad (3.3)
\]

which on using definition (2.2) in the l.h.s. gives

\[
\sum_{m=0}^\infty q^m \left( \frac{q^2}{2} \right)^2 \left( \frac{-1}{n!} q^2 t^{n+1} \right) D_{r,q} J_{r,s}(x, y, a, A, B; q) + \frac{q^{m+1} \beta(q)}{2} J_{r,s}(x, q^{a+2} y, a, A, B; q)
\]

\[
= \sum_{m=0}^\infty q^m \left( \frac{q^2}{2} \right)^2 D_{r,q} J_{r,s}(x, y, a, A, B; q) t^m w^m \quad (3.4)
\]

By equating the coefficients of \( t^m w^m \) in the above equation, we get

\[
D_{r,q} J_{r,s}(x, y, a, A, B; q) = \frac{(-1)^m q^{m+1} \alpha q}{2(1-q) \beta(q)} J_{r,s+1}(x, q^{a+2} y, a, A, B; q) + \frac{q^{m+1} \beta(q)}{2} J_{r,s}(x, q^{a+2} y, a, A, B; q)
\]

Hence the q-difference equation for \( J_{r,s}(x, y, a, A, B; q) \) is given by

\[
D_{r,q} J_{r,s}(x, y, a, A, B; q) = \frac{q^{m+1} \beta(q)}{2} \left( \frac{-1}{n!} q^2 \right) J_{r,s+1}(x, q^{a+2} y, a, A, B; q) + \frac{q^{m+1} \beta(q)}{2} J_{r,s}(x, q^{a+2} y, a, A, B; q)
\]

(3.5)

4. The recurrence relations of the matrix function \( J_{r,s}(x, y, a, A, B; q) \)

The following q-recurrence relations of the function \( J_{r,s}(x, y, a, A, B; q) \) holds true:

**Theorem 4.1.**

\[
J_{r,s}(x, y, a, A, B; q) = \frac{2(q^r t) \beta(q)}{ \alpha(q) } \left( \sum_{n=0}^\infty \frac{(-1)^n q^{n+1} \alpha q}{2(1-q) \beta(q)} \right) J_{r,s+1}(x, q^{a+2} y, a, A, B; q) + \frac{(-1)^m q^{m+1} \beta(q)}{2} J_{r,s}(x, q^{a+2} y, a, A, B; q)
\]
Proof. From (2.2), we know that
\[
J_{q,r}(x, y, a, B; q) = \frac{(\frac{2\pi}{q})^{\frac{1}{2}}}{(q)^{\frac{1}{2}}}(q)^{\frac{1}{2}}(q^{2}\pi x)^{2m}((q^{2}y)^{2})^{2n}\frac{\text{B}(a)}{2^{2n}}.
\]
By using relations (1.9) and (1.11), we get
\[
J_{q,r}(x, y, a, B; q) = \frac{(\frac{2\pi}{q})^{\frac{1}{2}}}{(q)^{\frac{1}{2}}}(q)^{\frac{1}{2}}(q^{2}\pi x)^{2m}((q^{2}y)^{2})^{2n}\frac{\text{B}(a)}{2^{2n}}
\]
Using (1.11), we get
\[
J_{q,r}(x, y, a, B; q) = \frac{(\frac{2\pi}{q})^{\frac{1}{2}}}{(q)^{\frac{1}{2}}}(q)^{\frac{1}{2}}(q^{2}\pi x)^{2m}((q^{2}y)^{2})^{2n}\frac{\text{B}(a)}{2^{2n}}
\]
Using, \(q; q)_{m} = (1 - q^{m})(q; q)_{m-1}\) and replace \(m\) by \(r + 1\) in r.h.s. in a second term, we obtain
\[
J_{q,r}(x, y, a, B; q) = \frac{(\frac{2\pi}{q})^{\frac{1}{2}}}{(q)^{\frac{1}{2}}}(q)^{\frac{1}{2}}(q^{2}\pi x)^{2m}((q^{2}y)^{2})^{2n}\frac{\text{B}(a)}{2^{2n}}
\]
Similarly, if we write \((1 - q^{m}) + q^{m} - q^{r+m+1}\) instead of \((1 - q^{r+1} + q^{r+1} - q^{r+m+1})\), we prove the following lemma.

**corollary 4.1.**
\[
J_{r,s}(x, y, a, B; q) = \frac{(\frac{2\pi}{q})^{\frac{1}{2}}}{(q)^{\frac{1}{2}}}(q)^{\frac{1}{2}}(q^{2}\pi x)^{2m}((q^{2}y)^{2})^{2n}\frac{\text{B}(a)}{2^{2n}}
\]

**Theorem 4.2.**
\[
J_{r,s}(x, y, a, B; q) = \frac{(\frac{2\pi}{q})^{\frac{1}{2}}}{(q)^{\frac{1}{2}}}(q)^{\frac{1}{2}}(q^{2}\pi x)^{2m}((q^{2}y)^{2})^{2n}\frac{\text{B}(a)}{2^{2n}}
\]

**Proof.** From (2.1), we know that
\[
J_{r,s}(x, y, a, B; q) = \frac{(\frac{2\pi}{q})^{\frac{1}{2}}}{(q)^{\frac{1}{2}}}(q)^{\frac{1}{2}}(q^{2}\pi x)^{2m}((q^{2}y)^{2})^{2n}\frac{\text{B}(a)}{2^{2n}}
\]
Using (1.11), we get
\[
J_{r,s}(x, y, a, B; q) = \frac{(\frac{2\pi}{q})^{\frac{1}{2}}}{(q)^{\frac{1}{2}}}(q)^{\frac{1}{2}}(q^{2}\pi x)^{2m}((q^{2}y)^{2})^{2n}\frac{\text{B}(a)}{2^{2n}}
\]

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