Chiral gauge theories with domain wall fermions
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We have investigated a proposal to construct chiral gauge theories on the lattice using domain wall fermions. The model contains two opposite chirality zero modes, which live on two domain walls. We couple only one of them to a gauge field, but find that mirror fermions which also couple to the gauge field always seem to exist.

1. Model

A very elegant mechanism for obtaining lattice chiral fermions was recently proposed [1]. The idea is to start from Wilson fermions in \(d+1\) dimensions, where \(d\) is the dimension of space-time, and give these fermions a mass \(m_0\) of the order of the cutoff. However, a domain wall like defect is introduced by making the mass term dependent on the extra dimension, choosing \(m_s = -m_0\) for \(s < 0\) and \(m_s = m_0\) for \(s > 0\) (\(s\) labels the coordinates of the extra dimension). It was shown that a massless mode with positive chirality exists in this model [1], which is exponentially bound to the defect at \(s = 0\), which is identified with \(d\) dimensional space-time.

In a finite volume an antidomain wall exists due to the (anti)periodic boundary conditions, and consequently, a negative chirality mode, bound to this defect, exists (other choices of boundary conditions are possible, but do not alter the conclusions [3]). One can study this system coupled to external smooth gauge fields, and one finds that any anomalous charge created on the domain wall by this gauge field is carried off through a Goldstone-Wilczek current to the antidomain wall [1,3,4].

In this work, we study the coupling to dynamical gauge fields. A detailed account can be found in ref. [5], to which we also refer the reader for notations. We introduce a waveguide \(WG = \{s : s_0 \leq s \leq s_0'\}\), with \(s_0\) on one side, and \(s_0'\) on the other side of the domain wall. Only within the waveguide, the fermions are coupled to a dynamical gauge field \(U_\mu\). This breaks gauge invariance in the fermion hopping terms across the waveguide boundary, which is restored by promoting these terms to Yukawa couplings with a scalar field \(V\), which takes values in the gauge group \(G\) [6]. For another approach with \(s\)-independent gauge fields, see ref. [7]. The model is defined by the action

\[
S_\Psi = \sum_{s \in WG} \left( \mathcal{D}_s (\Psi(U) - W(U) + m^s) \Psi^s \right) + \sum_{s \in WG} \left( \mathcal{D}_s (\bar{\Psi} - w + m^s) \Psi^s + \sum_s \mathcal{D}_s \Psi^s \right) - \sum_{s \neq s_0-1, s_0'} \left[ \mathcal{D}_s P_L \Psi^{s+1} + \mathcal{D}_s P_R \Psi^{s-1} \right] - y(\mathcal{D}_s \bar{\Psi} P_L \Psi^{s_0} + \mathcal{D}_s \bar{\Psi} P_R \Psi^{s_0-1}) - y(\mathcal{D}_s \bar{\Psi} P_L \Psi^{s_0+1} + \mathcal{D}_s \bar{\Psi} P_R \Psi^{s_0'}),
\]

where \(m^s\) is the \(s\)-dependent mass.

For \(y = 0\) the regions inside and outside the waveguide decouple, and new zeromodes appear on one of the waveguide boundaries, with negative chirality just inside, and with positive chirality just outside the waveguide [1,3]. (They appear only at one of the boundaries: which one

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depends on details.) The inside mode couples to the gauge field, rendering the theory vector-like. The key question now is whether this situation pertains for all values of the Yukawa coupling \( y \), or that a PMS phase exists for large values of \( y \), where the fermion spectrum around the waveguide boundary does not contain light fermion modes. This phenomenon takes place in a number of Higgs-Yukawa models, and if this would be the situation in the case at hand, the unwanted modes at the waveguide boundary would decouple. We emphasize that there are no simple anomaly arguments to rule out this possibility.\[5]\]

2. Effective model

We have studied this question for \( U_\mu = 1 \), as it is the fermion-scalar dynamics that would be responsible for the existence of a PMS phase. This is reasonable, as \( U_\mu \) (in the Landau gauge) should be close to one in an asymptotically free theory. At \( y = 0 \) the action can be diagonalized, and one finds two massless Dirac modes, \( \omega = \omega_R + \omega_L \) and \( \xi = \xi_R + \xi_L \), where \( \omega_R(L) \) is the right(left)-handed mode at the (anti)domain wall, and \( \xi_L(R) \) is the left(right)-handed mode just inside (outside) the waveguide boundary. These modes all have a chiral spectrum for momenta in the region \( \frac{1}{2}\hat{p}^2 = \sum \mu (1 - \cos \hat{p}_\mu) < \frac{1}{2} \hat{p}^2 \equiv |2 - m_0| \) around \( p = 0 \). Modes with momenta outside this region are delocalized in \( s \), and their wavefunctions are negligible at the domain walls and waveguide boundary. This includes all species doubler momenta \( p = (\pi, 0, \ldots) \) etc. In writing an effective action for the light modes, we may therefore ignore these large \( p \) modes. We will also ignore the Yukawa couplings of the domain wall modes \( \omega_{L,R} \), since their wavefunctions are exponentially suppressed at the waveguide boundaries. This leads us to an effective model for \( \omega \) and \( \xi \):

\[
S_{\text{eff}} = \sum_{|\hat{p} | < p_c} \left( i \bar{\omega}_\mu \gamma_\mu \sin p_\mu \omega + i \bar{\xi}_\mu \gamma_\mu \sin p_\mu \xi_p \right) + y \sum_{|\hat{p},|\hat{q}| < p_c} \bar{\xi}_p \left( V_{q-p}^\dagger P_L + V_{p-q} P_R \right) \xi_p.
\]

This form of the effective action is reminiscent of a hypercubic Higgs-Yukawa model, in that in both models the Yukawa couplings are suppressed for large momenta. This leads to the suspicion that no PMS phase will exist in our model, as no such phase exist in the hypercubic model.

3. Numerical results

The obvious thing for a numerical investigation, in order to find out whether a PMS phase exists or not, is to compute fermion masses. However, in particular for large \( y \) and small scalar hopping parameter \( \kappa \) (the region of interest), the propagators become very small and noisy due to the strongly fluctuating scalar field, and we have not been able to compute reliable fermion propagators in that region (we mostly used the quenched approximation). We studied the eigenvalue spectrum of the Dirac operator for the effective action, eq. (2), for \( G = U(1) \) and \( d = 1 \), and compared it with \( d = 1 \) Higgs-Yukawa models which are known to have or lack a PMS phase. The distribution of eigenvalues in the complex plane is a very good measure of the existence of the PMS phase. Results are shown in fig. 1, indicating that indeed no PMS phase exists.

Figure 1. Eigenvalue spectra for the effective model (figs. a), and for two reference models with (figs. b) and without (figs. c) a PMS phase. The left, middle and right figures are for \( y = 0.2 \), \( 1.0 \) and \( 4.0 \) resp. The lattice size is \( L^2 = 12^2 \) and \( \kappa = 0.1 \).
We have also computed the $\xi$ mass in the broken phase (i.e. for large $\kappa$) of the full model (eq. (1)), where the signal to noise ratio is better, again for $U(1)$ and $d = 2$. (For a discussion of the finite volume $d = 2$ scalar phase structure, see ref. [5].) Results for the $\kappa$ dependence are shown in fig. 2 for strong Yukawa coupling $y = 2$, which are consistent with the weak coupling behavior $m_F \approx y v$ for $\kappa \to \kappa_c \approx 0.5$. Near a FM-PMS phase transition one would have expected a fermion mass increasing with $\kappa \to \kappa_c$ [8]. Again, this indicates that no PMS phase exists in our model.

\[
\begin{align*}
\text{Figure 2. } & \kappa \text{ dependence of the waveguide fermion mass at } y = 2 \text{ on a } 12^2 26 \text{ lattice.}
\end{align*}
\]

4. Final remarks

We conclude with some remarks, most of which are more extensively discussed in ref. [5].

- We have tried unquenched computations, which resulted in very poor statistics, and very low acceptance rate of the hybrid MC algorithm. Unquenched results were not inconsistent with our quenched computations.
- We found good agreement between fermion masses computed in the full and effective models.
- The scalar field is nothing else than the longitudinal gauge field, and it is due to the fluctuations of this field that the model produces mirror fermions, rendering the model vectorlike: both $\omega_R$ and $\xi_L$ couple to the gauge field.
- A crucial role is played by the “effective momentum cutoff” $p_c$, which is critical in removing the doublers. However, from eq. (2) it is also clear that this cutoff plays an important role in the (non)existence of a PMS phase.
- We have also considered a staggered fermion formulation of the domain wall approach, based on the notion that in that case no $p_c$ appears. However, in this case, the flavor structure causes similar problems [12].
- The model considered here is more general than the original way of coupling to dynamical gauge fields proposed in ref. [5].
- There exists a proposal to keep the volume in the extra dimension strictly infinite [4,5], based on the idea that in that way the zero mode on the antdomain wall is avoided from the start. Ref. [14] discusses a possible relationship between this approach and the one reported on here.

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