Cosmological Brane Systems in Warped Spacetime

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Introduction

String theory

✓ A promising candidate for the *unified theory* of the fundamental interactions

✓ Higher-dimensional spacetime

  Compactification → A large population of universes

  Multiverse

✓ Branes

  • confine the gauge interactions

  • curve the surrounding spacetime

*Time-dependent brane solutions in the gravity theory can lead to expansion of the Universe*

After brief reviews of black holes and branes, we introduce the time-dependent brane solutions.
Charged Black Holes in 4D

\[ S = \frac{1}{2\kappa^2} \int d^4 x \sqrt{-g} \left[ R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right] \]

\[ ds^2 = -f(R)dt^2 + \frac{dR^2}{f(R)} + R^2 d\Omega^2 \]

\[ f(R) = 1 - \frac{2M}{R} + \frac{Q^2}{R^2} \]

Horizons \[ f(R_{\pm}) = 0 \]

\[ R_+ = M + \sqrt{M^2 - Q^2} \quad \text{event horizon} \]

\[ R_- = M - \sqrt{M^2 - Q^2} \quad \text{inner horizon} \]

Increasing the mass of a point particle with a fixed charge \( Q \).

\( M < Q \) no horizon

\( M = Q \) the minimal BH is formed.

**Extremal BHs** Extension to multi-centered case
Multi-centered extension

\[ ds^2 = -h^{-2}(y) dt^2 + h^2(y) \delta_{ij} dy^i dy^j \]

\[ F_2 = d(h^{-1}) \wedge dt \]

\[ \Delta_y h(y) = 0 \quad \rightarrow \quad h(y) = 1 + \sum_{\ell} \frac{M_\ell}{|\vec{y} - \vec{y}_\ell|} \]

Extremal black holes at \( \vec{y} \rightarrow \vec{y}_\ell \)

A single black hole

\[ r^2 = \delta_{ij} y^i y^j \]

\[ R := r + M \]

\[ ds^2 = -\left(1 - \frac{M}{R}\right)^2 dt^2 + \left(1 - \frac{M}{R}\right)^{-2} dR^2 + R^2 d\Omega^2 \]

= extremal BH
✓ Black branes in higher-dimensional gravity

\[ F_{\mu \nu} = \partial_{[\mu} A_{\nu]} \]

\[ F_{p+2} = dA_{p+1} \]

Higher-dimensional gravity coupled to a \((p+1)\)-form gauge field has solutions of charged black objects whose horizons are extended over \(p\)-dim space.

Black-branes

![Diagram of Black Branes](image)
Time-dependent extension

Analogy between 4D and higher-dimensional gravity.

- 4D gravity
  - Charged (extremal) BH
    - Majumdar-Papapetrou
  - Time-dependent BH
    - Kastor-Traschen

- Higher-dimensional gravity
  - p-brane solution
    - Horowitz-Strominger
  - Time-dependent p-brane
    - Gibbons-Lu-Pope
Time-dependent BH in 4D

\[ S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[ R - 2\Lambda - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \right] \]

solution \( \Lambda > 0 \)

\[ ds^2 = -h^{-2}(t, y) dt^2 + h^2(t, y) \delta_{ij} dy^i dy^j \]

\[ F_2 = d\left(h^{-1}\right) \wedge dt \]

\[ h(t, y) = \pm \sqrt{\frac{\Lambda}{3}} t + b + \sum_{\ell} \frac{M_{\ell}}{|\bar{y} - \bar{y}_\ell|} \]

\( \bar{y} \rightarrow \bar{y}_\ell \) extremal black hole horizon

\( |\bar{y}| \rightarrow \infty \) Asymptotically de Sitter spacetime

Kastor & Traschen 93
\[ ds^2 = -\left( \sqrt{\frac{\Lambda}{3}} t + b + \frac{M}{r} \right)^{-2} dt^2 + \left( \sqrt{\frac{\Lambda}{3}} t + b + \frac{M}{r} \right)^2 \left( dr^2 + r^2 d\Omega^2 \right) \]

\[ r \to \infty \quad \text{de Sitter space} \]

\[ ds^2 \approx -dt^2 + \exp\left( 2\sqrt{\frac{\Lambda}{3}} t \right) \delta_{ij} dy^i dy^j \quad \tau := \sqrt{\frac{\Lambda}{3}} \ln \left( \sqrt{\frac{\Lambda}{3}} t + b \right) \]

\[ r \to 0 \quad \text{(extremal) charged BH} \quad M=Q \]

\[ ds^2 \approx -\left( 1 - \frac{M}{R} \right)^2 dt^2 + \left( 1 - \frac{M}{R} \right)^{-2} dR^2 + R^2 d\Omega^2 \]

\[ R := br + M \]

Horizon \quad R = M \quad \text{or} \quad r = 0 \]
Time-dependent p-branes

Gibbons, Lu & Pope (05), Binetruy, Uzawa & Sasaki (09)

We consider the higher-dimensional gravity theory

\[ S = \frac{1}{2\kappa^2} \int d^D x \sqrt{-g} \left[ R(X) - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2(p+2)!} e^{c\phi} F_{p+2}^2 \right] \]

graviton scalar (p+2)-form field strength

\[ c^2 = 4 - \frac{2(p+1)(D-p-3)}{D-2} \]

Coupling constant is chosen, so that in \( D = 10, 11 \) the theory becomes supergravity theories in Einstein frame.
Time-dependent p-branes

We consider the higher-dimensional gravity theory

\[
S = \frac{1}{2\kappa^2} \int d^{D}x \sqrt{-g} \left[ R(X) - \frac{1}{2}(\partial \phi)^2 - \frac{1}{2(p+2)!} e^{c_{\phi}} F_{p+2}^2 \right]
\]

graviton scalar (p+2)-form field strength

\[
c^2 = 4 - \frac{2(p+1)(D-p-3)}{D-2}
\]

Coupling constant is chosen, so that in \( D = 10, 11 \) the theory becomes supergravity theories in Einstein frame.
Time-dependent p-branes

\[ ds^2 = h \left( \frac{D-3}{D-2} \right) (x, y) \eta_{\mu \nu} dx^\mu dx^\nu + h^{D-2} (x, y) \delta_{ij} (Y) dy^i dy^j \]

(p+1)-dim worldvolume

\[ e^\phi = h^{-\frac{c}{2}} \]

\[ F_{(p+2)} = \sqrt{-q} d(h^{-1}) \wedge dx^0 \wedge \cdots \wedge dx^p \]

\[ \Delta_y h = 0 \]

\[ h(x, y) = A_\mu x^\mu + B + \sum_{\ell} \frac{M_{\ell}}{|\vec{y} - \vec{y}_\ell|^{D-p-3}} \]

\[ A_\mu, B : \text{integration constants} \]

✓ \[ \vec{y} \rightarrow \vec{y}_\ell \quad \text{p-branes} \]

✓ Time-dependence appears as a linear function

\[ A_\mu x^\mu = A_0 t + A_i x^i \]
Dynamical D3-branes \[ D = 10 \quad p = 3 \]
Gibbons, Lu and Pope (05)

\[ A_0 < 0 \quad A_i = B = 0 \]

\[ h(x, y) = -|A_0| t + \sum_{\ell} \frac{M_\ell}{|\vec{y} - \vec{y}_\ell|^4} \]

10-dim Kasner solution if all \( M_\ell = 0 \)

Singularity appears at infinity at \( t=0 \)
\[ h = 0 : \text{singularity} \]

Regular region shrinks

Separation into domains, and each contains a single D3-brane.
Dynamical intersecting branes

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Intersecting branes

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Guven (92), Bergshoeff, et.al (97)
The metric of the intersecting brane systems

\[ ds^2 = h_r^\alpha h_s^\beta \left( \left( h_r h_s \right)^{-1} q_{\mu \nu} (X) dx^\mu dx^\nu + h_s^{-1} \gamma_{ij} (Y_1) dy^i dy^j + h_r^{-1} w_{mn} (Y_2) dv^m dv^n + u_{ab} (Z) dz^a dz^b \right) \]

\[ \alpha = \frac{p_r + 1}{D - 2} \]
\[ \beta = \frac{p_s + 1}{D - 2} \]
The metric of the intersecting brane systems

\[ ds^2 = h_r^\alpha h_s^\beta \left[ \left(h_r h_s \right)^{-1} q_{\mu \nu}(X) dx^\mu dx^\nu + h_s^{-1} \gamma_{ij}(Y_1) dy^i dy^j \\
+ h_r^{-1} w_{mn}(Y_2) dv^m dv^n + u_{ab}(Z) dz^a dz^b \right] \]

\[ \alpha = \frac{p_r + 1}{D - 2} \quad \beta = \frac{p_s + 1}{D - 2} \]
The metric of the intersecting brane systems

\[ p_r - \text{brane} \]

\[ ds^2 = h_r^\alpha h_s^\beta \left( (h_r h_s)^{-1} q_{\mu \nu}(X) dx^\mu dx^\nu + h_s^{-1} \gamma_{ij}(Y_1) dy^i dy^j \right. \]

\[ + h_r^{-1} w_{mn}(Y_2) dy^n dy^m + u_{ab}(Z) dz^a dz^b \right] \]

\[ \alpha = \frac{p_r + 1}{D - 2} \]

\[ \beta = \frac{p_s + 1}{D - 2} \]
The metric of the intersecting brane systems

$$ds^2 = h_r^\alpha h_s^\beta \left[ \left( h_r h_s \right)^{-1} q_{\mu \nu}(X)dx^\mu dx^\nu + h_s^{-1} \gamma_{ij}(Y_1)dy^i dy^j + h_r^{-1} w_{mn}(Y_2)dv^m dv^n + u_{ab}(Z)dz^a dz^b \right]$$

$p_r$-brane

$\alpha = \frac{p_r + 1}{D - 2}$

$\beta = \frac{p_s + 1}{D - 2}$

In general, $h_r = h_r(x, y, z)$ and $h_s = h_s(x, v, z)$

The metric of the intersecting brane systems
The metric of the intersecting brane systems

\[ ds^2 = h_r^\alpha h_s^\beta \left[ (h_r h_s)^{-1} q_{\mu \nu}(X) dx^\mu dx^\nu + h_s^{-1} \gamma_{ij}(Y_1) dy^i dy^j + h_r^{-1} w_{mn}(Y_2) dv^m dv^n + u_{ab}(Z) dz^a dz^b \right] \]

\[ \alpha = \frac{p_r + 1}{D - 2} \quad \beta = \frac{p_s + 1}{D - 2} \]

In general, \( h_r = h_r(x, y, z) \) \( h_s = h_s(x, v, z) \)

Here we focus on

- **Case I** \( h_r = h_r(x, z) \) \( h_s = h_s(x, z) \)
- **Case II** \( h_r = h_r(x, y) \) \( h_s = h_s(x, z) \)
- **Case III** \( h_r = h_r(x, y) \) \( h_s = h_s(x, v) \)
Classification of the intersecting branes

Behrndt, Bergshoeff & Janssen (96)

Time-dependent generalization

Case I: Both $h_r$ and $h_s$ depend on the overall transverse coordinates

Maeda, Ohta & Uzawa (09)

\[
h_r = h_r(x, z) \quad h_s = h_s(z) \quad \text{or} \quad h_r = h_r(z) \quad h_s = h_s(x, z)
\]

\[
p + 1 - \frac{(p_r + 1)(p_s + 1)}{D - 2} + \frac{1}{2} \varepsilon_r \varepsilon_s c_r c_s = 0 \quad \text{Argurio, et. al (97), Ohta (97)}
\]

\[
c^2 = 4 - \frac{2(p_I + 1)(D - p_I - 3)}{D - 2} \quad \varepsilon_I = +1: \text{electric} \quad \varepsilon_I = -1: \text{magnetic}
\]

Case II: $h_s$ depends on the overall transverse space $z^a$

$h_r$ depends on the relative transverse space $y^i$

\[
h_r = h_r(x, y) \quad h_s = h_s(z)
\]

no \quad $h_r = h_r(y) \quad h_s = h_s(x, z)$
Case III:

\[
h_r = h_r(x, y) \quad h_s = h_s(v) \quad \text{or} \quad h_r = h_r(y) \quad h_s = h_s(x, v)
\]

Intersecting rule

\[
p + 1 - \frac{(p_r + 1)(p_s + 1)}{D - 2} + \frac{1}{2} \varepsilon_r \varepsilon_s c_r c_s = -2
\]

Common features for all cases

- Linear dependence \( h_r(x, z) = A_\mu x^\mu + g_r(z) = A_0 t + A_i x^i + g_r(z) \)
- \( g_r(z) \) is by the harmonic functions, \( \Delta_z g_r(z) = 0 \)
- Transverse spaces are Ricci-flat.

\[
R_{\mu \nu}(X) = 0 \quad R_{ij}(Y_1) = 0 \\
R_{mn}(Y_2) = 0 \quad R_{ab}(Z) = 0
\]
Partially localized branes

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Localized branes

If there is solution such as $h_s = h_s(x, v, z)$ and $h_r = h_r(x, y, z)$, each brane is localized in both the relative and overall transverse space.

Difficult to solve analytically

We could find the partially localized branes.

$p_r$-branes are localized on a single $p_s$-brane

$$h_s = h_s(z) \quad h_r = h_r(x, y, z)$$

~single brane

Static solutions Youm (97)

Time-dependence appears as a linear function

$$h_r(x, y, z) = A_\mu x^\mu + g_r(y, z)$$

$$h_s(z) \Delta_{y_i} g_r(y, z) + \Delta_z g_r(y, z) = 0$$
\[ S = \frac{1}{2\kappa^2} \int d^p x \sqrt{-g} \left[ R(X) - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2(p_r + 2)!} e^{c_r \phi} F_{p_r+2}^2 - \frac{1}{2(p_s + 2)!} e^{c_s \phi} F_{p_s+2}^2 \right] \]

\[ c_l^2 = N_l - \frac{2(p_l + 1)(D - p_l - 3)}{D - 2} \]

\[ \varepsilon_l = +1: \text{ electric} \]

\[ \varepsilon_l = -1: \text{ magnetic} \]

| Case | 0 | 1 | … | p | p + 1 | … | ps | ps + 1 | … | ps + pr − p | ps + pr − p + 1 | … | D − 1 |
|------|---|---|---|---|------|---|-----|--------|---|--------------|-----------------|---|-------|
| pr−ps | pr | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| ps | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| x^n | t | x^1 | … | x^p | y^1 | … | y^{ps−p−1} | y^1 | … | y^{pr−p−1} | z^1 | … | z^{D+p−pr−ps−1} |
Ansatz

\[ e^\phi = h_r^{2\varepsilon_r c_r/N_r} h_s^{2\varepsilon_s c_s/N_s} \]

\[ F_{(p_r+2)} = \frac{2}{\sqrt{N_r}} d [h_r^{-1}(x, y, z)] \wedge \Omega(X) \wedge \Omega(Y_2) \]

\[ F_{(p_s+2)} = \frac{2}{\sqrt{N_s}} d [h_s^{-1}(x, v, z)] \wedge \Omega(X) \wedge \Omega(Y_2) \]

Intersecting rule

\[ p + 1 - \frac{(p_r + 1)(p_s + 1)}{D - 2} + \frac{1}{2} \varepsilon_r \varepsilon_s c_r c_s = 0 \]

Off-diagonal Einstein equations

\[ \frac{2}{N_r} h_r^{-1} \left( \partial_\mu \partial_i h_r + \frac{4}{N_s} \partial_\mu \ln h_s \partial_i h_r \right) = 0, \]

\[ \frac{2}{N_s} h_s^{-1} \left( \partial_\mu \partial_m h_s + \frac{4}{N_r} \partial_\mu \ln h_r \partial_m h_s \right) = 0, \]

\[ \frac{2}{N_r} h_r^{-1} \partial_\mu \partial_a h_r + \frac{2}{N_s} h_s^{-1} \partial_\mu \partial_a h_s = 0, \]

\[ \partial_i \ln h_r \partial_m \ln h_s = 0 \]
Ansatz

\[ e^\phi = h_r^{2\varepsilon_r c_r/N_r} h_s^{2\varepsilon_s c_s/N_s} \]

\[ F_{(p_r+2)} = \frac{2}{\sqrt{N_r}} d\left[h_r^{-1}(x, y, z)\right] \wedge \Omega(X) \wedge \Omega(Y_2) \]

\[ F_{(p_s+2)} = \frac{2}{\sqrt{N_s}} d\left[h_s^{-1}(x, v, z)\right] \wedge \Omega(X) \wedge \Omega(Y_2) \]

Intersecting rule

\[ p + 1 - \frac{(p_r + 1)(p_s + 1)}{D - 2} + \frac{1}{2} \varepsilon_r \varepsilon_s c_r c_s = 0 \]

Off-diagonal Einstein equations

\[ \frac{2}{N_r} h_r^{-1} \left( \partial_\mu \partial_i h_r + \frac{4}{N_s} \partial_\mu \ln h_s \partial_i h_r \right) = 0, \]
\[ \frac{2}{N_s} h_s^{-1} \left( \partial_\mu \partial_m h_s + \frac{4}{N_r} \partial_\mu \ln h_r \partial_m h_s \right) = 0, \]
\[ \frac{2}{N_r} h_r^{-1} \partial_\mu \partial_a h_r + \frac{2}{N_s} h_s^{-1} \partial_\mu \partial_a h_s = 0, \]
\[ \partial_i \ln h_r \partial_m \ln h_s = 0 \]

For \( \partial_\mu h_s = 0 \), \( h_r = h_0(x) + h_1(y, z) \quad h_s = h_s(v, z) \)
Ansatz

\[ e^\phi = h_r ^{2\varepsilon_r c_r / N_r} h_s ^{2\varepsilon_s c_s / N_s} \]

\[ F_{(p_r+2)} = \frac{2}{\sqrt{N_r}} d \left[ h_r^{-1}(x, y, z) \right] \wedge \Omega(X) \wedge \Omega(Y_2) \]

\[ F_{(p_s+2)} = \frac{2}{\sqrt{N_s}} d \left[ h_s^{-1}(x, v, z) \right] \wedge \Omega(X) \wedge \Omega(Y_2) \]

Intersecting rule

\[ p + 1 - \frac{(p_r + 1)(p_s + 1)}{D - 2} + \frac{1}{2} \varepsilon_r \varepsilon_s c_r c_s = 0 \]

Off-diagonal Einstein equations

\[ \frac{2 h_r^{-1}}{N_r} \left( \partial_\mu \partial_i h_r + \frac{4}{N_s} \partial_\mu \ln h_s \partial_i h_r \right) = 0, \]

\[ \frac{2 h_s^{-1}}{N_s} \left( \partial_\mu \partial_m h_s + \frac{4}{N_r} \partial_\mu \ln h_r \partial_m h_s \right) = 0, \]

\[ \frac{2 h_r^{-1} \partial_\mu \partial_a h_r + \frac{2}{N_s} h_s^{-1} \partial_\mu \partial_a h_s}{N_r} = 0, \]

\[ \partial_i \ln h_r \partial_m \ln h_s = 0 \]

For \( \partial_\mu h_s = 0 \), \( h_r = h_0(x) + h_1(y, z) \), \( h_s = h_s(z) \)
\[ R_{\mu\nu}(X) = 0, \quad R_{ij}(Y_1) = 0, \quad R_{mn}(Y_2) = 0, \quad R_{ab}(Z) = 0, \]
\[ h_r = h_0(x) + h_1(y, z), \quad h_s = h_s(v, z), \quad \partial_i h_r \partial_m h_s = 0, \]
\[ D_\mu D_\nu h_0 = 0, \quad \left(1 - \frac{4}{N_r}\right) \partial_\mu h_0 \partial_\nu h_0 = 0, \quad h^{4/N_s}_s \Delta_{Y_1} h_1 + \Delta_Z h_1 = 0, \]
\[ h^{4/N_r}_r \Delta_{Y_2} h_s + \Delta_Z h_s = 0. \]
\[ R_{\mu\nu}(X) = 0, \quad R_{ij}(Y_1) = 0, \quad R_{mn}(Y_2) = 0, \quad R_{ab}(Z) = 0, \]

\[ h_r = h_0(x) + h_1(y, z), \quad h_s = h_s(v, z), \quad \partial_i h_r \partial_m h_s = 0, \]

\[ D_\mu D_\nu h_0 = 0, \quad \left(1 - \frac{4}{N_r}\right) \partial_\mu h_0 \partial_\nu h_0 = 0, \quad h_s^{A/\mathbb{N}_s} \Delta_{Y_1} h_1 + \Delta_Z h_1 = 0, \]

\[ h_r^{A/\mathbb{N}_r} \Delta_{Y_2} h_s + \Delta_Z h_s = 0. \]

\[ h_s = h_s(z) \]
\[ R_{\mu\nu}(X) = 0, \quad R_{ij}(Y_1) = 0, \quad R_{mn}(Y_2) = 0, \quad R_{ab}(Z) = 0, \]
\[ h_r = h_0(x) + h_1(y, z), \quad h_s = h_s(v, z), \quad \partial_v h_r \partial_m h_s = 0, \]
\[ D_\mu D_\nu h_0 = 0, \quad \left(1 - \frac{4}{N_r}\right) \partial_\mu h_0 \partial_\nu h_0 = 0, \quad h_s^{4/N_s} \Delta_{Y_1} h_1 + \Delta_Z h_1 = 0, \]
\[ h_r^{4/N_r} \Delta_{Y_2} h_s + \Delta_Z h_s = 0. \]

linear function of \( x \)

\[ N_r = 4 \quad h_s = h_s(z) \]
linear function of $x$

\[ R_{\mu\nu}(X) = 0, \quad R_{ij}(Y_1) = 0, \quad R_{mn}(Y_2) = 0, \quad R_{ab}(Z) = 0, \]

\[ h_r = h_0(x) + h_1(y, z), \quad h_s = h_s(v, z), \quad \partial_v h_r \partial_m h_s = 0, \]

\[ D_\mu D_\nu h_0 = 0, \quad \left( 1 - \frac{4}{N_r} \right) \partial_\mu h_0 \partial_\nu h_0 = 0, \quad h^{A/N_s}_s \Delta_{Y_1} h_1 + \Delta_Z h_1 = 0, \]

\[ h^{A/N_r}_r \Delta_{Y_2} h_s + \Delta_Z h_s = 0. \]

\[ N_r = 4 \]

\[ h_s = h_s(z) \]

\[ h_s \Delta_{Y_1} h_1 + \Delta_Z h_1 = 0, \quad \Delta_Z h_s = 0 \]
\[ R_{\mu\nu}(X) = 0, \quad R_{ij}(Y_1) = 0, \quad R_{mn}(Y_2) = 0, \quad R_{ab}(Z) = 0, \]
\[ h_r = h_0(x) + h_1(y, z), \quad h_s = h_s(v, z), \quad \partial_v h_r \partial_m h_s = 0, \]
\[ D_\mu D_\nu h_0 = 0, \quad \left(1 - \frac{4}{N_r}\right) \partial_\mu h_0 \partial_\nu h_0 = 0, \quad h_s^{4/N_s} \Delta_{Y_1} h_1 + \Delta_Z h_1 = 0, \]
\[ h_r^{4/N_r} \Delta_{Y_2} h_s + \Delta_Z h_s = 0. \]

\[ N_r = 4 \quad h_s = h_s(z) \]

\[ h_s \Delta_{Y_1} h_1 + \Delta_Z h_1 = 0, \quad \Delta_Z h_s = 0 \]

We further assume

\[ q_{\mu\nu} = \eta_{\mu\nu}, \quad \gamma_{ij} = \delta_{ij}, \quad w_{mn} = \delta_{mn}, \quad u_{ab} = \delta_{ab}, \quad N_r = N_s = 4. \]
Partially localized brane solutions

\[ h_r(x, y, z) = A_\mu x^\mu + B + \sum_{\ell} \frac{M_{\ell}}{\left| \bar{y} - \bar{y}_\ell \right|^2 + \frac{4M}{(4 - d_z)^2} \left| \bar{z} - \bar{z}_0 \right|^{4-d_z}} \]

\[ h_s(z) = \frac{M}{\left| \bar{z} - \bar{z}_0 \right|^{d_z-2}} \]

For \( d_z \neq 2 \)

\[ h_r(x, y, z) = A_\mu x^\mu + B + \sum_{\ell} \frac{M_{\ell}}{\left| \bar{y} - \bar{y}_\ell \right|^2 + M \left| \bar{z} - \bar{z}_0 \right|^2} \left( p_s - p + 1 \right)^{1/2} \]

\[ h_s(z) = M \ln \left| \bar{z} - \bar{z}_0 \right| \]

Cosmological properties are the same as the delocalized cases.
Cosmology
(1) Cosmology from intersection of two branes

1. Higher-dimensional picture

$$\frac{\tau}{\tau_0} = (At)^{\frac{a_r+2}{2}} \quad \tau_0 = \frac{2}{(a_r+2)A}$$

Expansion law of each space can be obtained on a slice in the other spaces.

= Brane world picture
2. Lower-dimensional effective theory

\[ ds^2 = ds^2(M) + ds^2(N), \]

\[ (D-d)\text{-dim} \quad d\text{-dim} \]

\[ d = d_1 + d_2 + d_3 + d_4 \]

\[ \begin{align*}
X & \quad Y_1 & \quad Y_2 & \quad Z \\
\end{align*} \]

Einstein frame

\[ B = \frac{-(a_r + 1)d + d_1 + d_3}{D - d - 2}, \quad C = \frac{-(a_s + 1)d + d_2 + d_4}{D - d - 2}. \]

\[ ds^2(\bar{M}) = h_r^B h_s^C ds^2(\bar{M}), \]

\[ \begin{align*}
&= h_r^{B'} h_s^{C'} \left[ -dt^2 + \delta_{P'Q'}(\bar{X}')d\theta^{P'}d\theta^{Q'} + h_r\gamma_{k'l'}(Y_1')dy^{k'}dy^{l'} \\
&\quad + h_s\omega_{m'n'}(Y_2')dv^{m'}dv^{n'} + h_r h_s u_{a'b'}(Z')d\bar{z}^{a'}d\bar{z}^{b'} \right],
\end{align*} \]

\[ B' = -B + a_r, \quad C' = -C + a_s. \]

Cosmic expansion law of each space can be read.
p=3 : higher-dimensional picture

| Branes | $x^N$ | $t$ | $x^1$ | $x^2$ | $x^3$ | $y^1$ | $y^2$ | $y^3$ | $y^4$ | $z^1$ | $z^2$ | $\tilde{M}$ | $\lambda(\tilde{M})$ | $\lambda_E(\tilde{M})$ | BW |
|--------|-------|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|---------|-----------------|----------------|-----|
| D3-D7  |       |     |       |       |       |       |       |       |       |       |       | ✓     | $X$            | $\lambda(X) = -1/3$ | $\lambda_E(X) = \frac{d_0 + d_4 - 4}{12 - 2d_1 - d_2 - d_4}$ |           |
|        |       |     |       |       |       |       |       |       |       |       |       | ✓     | $Y_1$ & $Z$    | $\lambda(Y_1) = 1/3$ | $\lambda_E(Y_1) = \frac{4 - d_4}{12 - 2d_1 - d_2 - d_4}$ |           |
|        | ✓     |     | ✓     | ✓     | ✓     | ✓     | ✓     | ✓     | ✓     | ✓     | ✓     | ✓     | ✓     | $\lambda(Z) = 1/3$ | $\lambda_E(Z) = \frac{4 - d_1}{12 - 2d_1 - d_2 - d_4}$ |           |
| D3-D7  |       |     |       |       |       |       |       |       |       |       |       | ✓     | $\tilde{X}$ & $Y_2$ | $\lambda(\tilde{X}) = 0$ | $\lambda_E(\tilde{X}) = \frac{d_4}{16 - 2d_1 - 2d_3 - d_4}$ |           |
|        |       |     |       |       |       |       |       |       |       |       |       | ✓     | $\lambda(Y_2) = 0$ | $\lambda_E(Y_2) = \frac{d_4}{16 - 2d_1 - 2d_3 - d_4}$ |           |
| D4-D6  |       |     |       |       |       |       |       |       |       |       |       | ✓     | $\tilde{X}$ & $Y_2$ | $\lambda(\tilde{X}) = \lambda(Y_2) = \frac{3}{13}$ | $\lambda_E(\tilde{X}) = \lambda_E(Y_2) = \frac{d_2 + d_4 - 1}{13 - 2d_1 - d_2 - 2d_3 - d_4}$ |           |
|        |       |     |       |       |       |       |       |       |       |       |       | ✓     | $\lambda(Y_1) = \lambda(Z) = \frac{5}{13}$ | $\lambda_E(Y_1) = \lambda_E(Z) = \frac{5 - d_1 - d_3}{13 - 2d_1 - d_2 - 2d_3 - d_4}$ |           |
| D4-D6  |       |     |       |       |       |       |       |       |       |       |       | ✓     | $\tilde{X}$ & $Y_2$ | $\lambda(\tilde{X}) = \lambda(Y_2) = \frac{1}{15}$ | $\lambda_E(\tilde{X}) = \lambda_E(Y_2) = \frac{d_2 + d_4 - 1}{15 - 2d_1 - d_2 - 2d_3 - d_4}$ |           |
|        |       |     |       |       |       |       |       |       |       |       |       | ✓     | $\lambda(Y_1) = \lambda(Z) = \frac{7}{15}$ | $\lambda_E(Y_1) = \lambda_E(Z) = \frac{7 - d_1 - d_3}{15 - 2d_1 - d_2 - 2d_3 - d_4}$ |           |
| D5-D5  |       |     |       |       |       |       |       |       |       |       |       | ✓     | $\tilde{X}$ & $Y_2$ | $\lambda(\tilde{X}) = \lambda(Y_2) = \frac{1}{7}$ | $\lambda_E(\tilde{X}) = \lambda_E(Y_2) = \frac{d_2 + d_4 - 2}{14 - 2d_1 - d_2 - 2d_3 - d_4}$ |           |
|        |       |     |       |       |       |       |       |       |       |       |       | ✓     | $\lambda(Y_1) = \lambda(Z) = \frac{3}{7}$ | $\lambda_E(Y_1) = \lambda_E(Z) = \frac{6 - d_1 - d_3}{14 - 2d_1 - d_2 - 2d_3 - d_4}$ |           |
### p=3: Lower-dimensional effective theory

| Branes | TD | dim(M) | \(\tilde{M}\)                     | \((d_1, d_2, d_3, d_4)\) | \(\lambda_E(\tilde{M})\) | Case       |
|--------|----|--------|----------------------------------|--------------------------|--------------------------|------------|
| D3-D7  | D3 | 7      | \(\tilde{X} & Y_1 & Z\)           | (0, 3, 0, 0)             | 4/9                      | I          |
| D3     | D3 | 9      | \(\tilde{X} & Y_1 & Z\)           | (1, 0, 0, 0)             | 3/10                     | II         |
| D7     | D7 | 10 \(-d\) | \(\tilde{X} & Y_2 & Z\) or \(\tilde{X} & Z\) or \(Y_2 & Z\) | \((d_1, 0, d_3, 0)\)    | 0            | I & II     |
| D4     | D4 | 8      | \(\tilde{X} & Y_1 & Y_2 & Z\)     | (0, 2, 0, 0)             | 5/11                     | I          |
| D4     | D4 | 9      | \(\tilde{X} & Y_1 & Y_2 & Z\)     | (1, 0, 0, 0)             | 4/11                     | II         |
| D4-D6  | D4 | 9      | \(\tilde{X} & Y_1 & Z\)           | (0, 0, 1, 0)             | 4/11                     | II         |
| D6     | D6 | 9      | \(\tilde{X} & Y_1 & Y_2 & Z\)     | (1, 0, 0, 0)             | 6/13                     | I & II     |
| D6     | D6 | 9      | \(\tilde{X} & Y_1 & Y_2 & Z\)     | (0, 0, 1, 0)             | 6/13                     | I & II     |
| D5-D5  | D5 | 9      | \(\tilde{X} & Y_1 & Y_2 & Z\)     | (0, 1, 0, 0)             | 6/13                     | I          |
| D5     | D5 | 9      | \(\tilde{X} & Y_1 & Y_2 & Z\)     | (1, 0, 0, 0)             | 5/12                     | II         |
| D5     | D5 | 9      | \(\tilde{X} & Y_1 & Y_2 & Z\)     | (0, 0, 1, 0)             | 5/12                     | II         |
(2) Cosmology in triple brane intersection systems

1. Higher-dimensional picture

| Branes   | TD | \(\tilde{M}\)   | \(\lambda(\tilde{M})\) | \(\lambda_E(\tilde{M})\) |
|----------|----|-----------------|-------------------------|--------------------------|
| D3-D5-NS5 |    | \(\tilde{X} & W\) | \(-1/3\) | \(-\frac{dy+dz-4}{12-2d_X-d_Y-2d_W-d_Z}\) |
| D5       |    | \(Y & Z\)       | \(-1/3\) | \(-\frac{dy+dz-4}{12-2d_X-d_Y-2d_W-d_Z}\) |
| NS5      |    | \(\tilde{X} & W\) | \(-1/3\) | \(-\frac{dy+dz-4}{12-2d_X-d_Y-2d_W-d_Z}\) |
| x^N      | t  | \(X^1\)         | \(1/3\) | \(-\frac{dy+dz-4}{12-2d_X-d_Y-2d_W-d_Z}\) |
|          | y  | \(X^2\)         | \(1/3\) | \(-\frac{dy+dz-4}{12-2d_X-d_Y-2d_W-d_Z}\) |
|          |    | \(X^3\)         | \(1/3\) | \(-\frac{dy+dz-4}{12-2d_X-d_Y-2d_W-d_Z}\) |
|          | w  | \(z^1\)         | \(1/3\) | \(-\frac{dy+dz-4}{12-2d_X-d_Y-2d_W-d_Z}\) |
|          | z  | \(z^2\)         | \(1/3\) | \(-\frac{dy+dz-4}{12-2d_X-d_Y-2d_W-d_Z}\) |
|          |    | \(z^3\)         | \(1/3\) | \(-\frac{dy+dz-4}{12-2d_X-d_Y-2d_W-d_Z}\) |

2. Lower-dimensional effective theory

| Branes   | TD | \(\text{dim}(M)\) | \(\tilde{M}\) | \(d\) | \(\lambda_E(\tilde{M})\) |
|----------|----|-------------------|----------------|------|--------------------------|
| D3-D5-NS5 | D3 | 7                 | \(\tilde{X} & Y & W & Z\) | \((d_{\tilde{X}}, d_Y, d_W, d_Z) = (0, 1, 0, 2)\) | 4/9 |
| D5       |    | 7                 | \(\tilde{X} & Y & W & Z\) | \((d_{\tilde{X}}, d_Y, d_W, d_Z) = (0, 2, 0, 1)\) | 4/9 |
| NS5      |    | 9                 | \(\tilde{X} & Y & W & Z\) | \((d_{\tilde{X}}, d_Y, d_W, d_Z) = (0, 1, 0, 0)\) | 6/13 |
| x^N      | t  | 6                 | \(\tilde{X} & Y & U & V & Z\) | \((d_{\tilde{X}}, d_Y, d_U, d_V, d_Z) = (0, 1, 2, 0, 2)\) | 3/7 |
|          | y  | 6                 | \(\tilde{X} & Y & V & U\)   | \((d_{\tilde{X}}, d_Y, d_V, d_Z) = (0, 2, 0, 2)\) | 3/7 |
Summary of cosmic expansion

✓ All spaces X, Y, Z can provide expanding homogeneous and isotropic 3-space, after compactification.

✓ In both pictures, expansion cannot be faster than that in the radiation-dominated universe.
Summary of cosmic expansion

✓ All spaces X,Y,Z can provide expanding homogeneous and isotropic 3-space, after compactification.

✓ In both pictures, expansion cannot be faster than that in the radiation-dominated universe.

We need a potential term to get accelerated expansion.
Summary
✓ Charged BH solutions can be extended to p-branes in higher-dimensional theory coupled to (p+2)-form anti-symmetric form field strength

✓ Time-dependence can be introduced by adding a linear function of time into the harmonic function.

✓ For intersecting brane systems, only the particular can be time-dependent.

✓ Time-dependent warp factor gives a cosmic expansion, which cannot be faster than the radiation universe.
Thank you.