Chaotic Particle Swarm Optimization for Solving Reactive Power Optimization Problem

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1. Introduction

Reactive Power Optimization (RPO) problem is considered a complex and non-linear optimization problem. It plays a vital role in the power system operation and control. The main aims of RPO problem in this study are to decrease power loss and enhance voltage profile for the system, and these aims can be attained via a proper alteration for reactive power control parameters like generators voltage value (V_G), the value of VAR sources that injected from the
shunt capacitor \((Q_C)\) and transformer taps \((Tap)\) while dealing with some equality and inequality constraints containing load flow equations at the same time [1].

The RPO problem calculations are considered part of the Optimal Load Flow or (OPF) calculations. Carpenter has first introduced the OPF calculations [2][3]. Since then, many researchers have been working on solving the OPF problem by utilizing multiple methods, such as recursive quadratic, linear and nonlinear programming, and the interior point method [4][5][6][7]. Sun et al. have presented the Newton approach for the solution of OPF [8]. Lai et al. have presented an improved Genetic Algorithm (GA) for solving OPF [9].

In the past, several traditional optimization techniques have been presented for solving RPO problem like Interior Point Methods (IPM) [10], Linear Programming (LP) [11], non–linear programming [12], Gradient Search (GS) [13], Quadratic Programming (QP) [14] and Dynamic Programming (DP) [15]. These algorithms have several limitations, such as being unable to deal with non-continuous and complex optimization problems and dealing with problems that include a vast number of variables, huge calculations, big implementation time, and convergence to the nearby local optima. So, it becomes essential for finding and developing methods that can avoid these limitations.

Recently, several computational optimization techniques have been presented in order to prevent these limitations of the traditional optimization algorithms like Genetic Algorithm (GA) [16], hybrid GA–IPM [17], fuzzy technique [18], Moth Flame Optimization (MFO) [19] and Particle Swarm Optimization (PSO) [20]. PSO has appeared as a beneficial tool for engineering global optimization in solving this problem among all these algorithms. The benefits of the PSO algorithm are simple, fast, easy to implement; it has a flexible and balanced mechanism to improve the local and global exploration capabilities. However, it does not mean that PSO algorithm does not contain any disadvantages. In solving non-continuous and complex problems, this algorithm is declining very easily to the local minima at the premature convergence; on the other hand, its performance is also dependent on its parameters settings.

So, in this study, to avoid these disadvantages and enhance and develop the searching ability and quality of the Simple PSO algorithm, PSO and chaotic theory were merged to form a hybrid algorithm called Chaotic PSO (CPSO) algorithm. Undeniably, this merging of chaotic theory with PSO algorithm can be an efficient method to slip very easily from local optima compared to simple PSO algorithm due to remarkable behavior and great ability of the chaotic theory while helping decrease the calculation time. The CPSO algorithm is utilized as an optimization tool to obtain the best values of reactive power control parameters \((i.e. V_G, Tap, and Q_C)\), decreasing power loss \((P_{loss})\) and enhancing voltage profile. The CPSO was evaluated and examined on IEEE Node \(-14\) system for solving the RPO problem. The simulation implications confirm that the results in the CPSO algorithm are best in terms of decreasing \(P_{loss}\) and enhancing the voltage profile for the power system. Moreover, these results prove the efficiency and ability of the CPSO algorithm in solving RPO problem and any complex problem that include a vast number of variables in a power system. It also had the ability and superiority to get the best solution with the least iteration compared to those in Simple PSO and other techniques in the literature.

2. Problem Formulation

2.1. Objective Function

The great objective in this work is to decrease the Power Losses \((P_{loss})\) for the system via proper management of reactive power control parameters \((V_G, Tap, and Q_C)\) while dealing with numbers of equality and inequality constraints at the same time. The \(P_{loss}\) can be expressed as [21]
Min \( P_{loss} = \sum_{K=1}^{Nt} G_K (V_i^2 + V_j^2 - 2V_iV_j \cos(\phi_i - \phi_j)) \) \( (1) \)

where \( P_{loss} \) is the active power loss function, \( Nt \) depicts the number of branches, and \( G_K \) depicts the conductance of branch \( K \). The \( V_i, V_j \) are voltage magnitudes at nodes \( i \) and \( j \). The \( \phi_i, \phi_j \) are the difference angles voltage at node \( i \) and \( j \).

2.2. Constrains

2.2.1. Equality Constrains

These constrains are the load flow equation and defined as follows [22]:

\[
\begin{align*}
P_{Di} - P_{Di} - V_i \sum_{j=1}^{NB} V_j (G_{ij} \cos(\phi_{ij}) + B_{ij} \sin(\phi_{ij})) &= 0 \\
Q_{Di} - Q_{Di} - V_i \sum_{j=1}^{NB} V_j (G_{ij} \sin(\phi_{ij}) - B_{ij} \cos(\phi_{ij})) &= 0
\end{align*}
\]

(2)

where \( P_{Gi}, Q_{Gi} \) are the real (MW) and reactive power (VAR) output from the generators at node \( i \). The \( P_{Di}, Q_{Di} \) are the real (MW) and reactive power (VAR) load demand at node \( i \). The \( G_{ij}, B_{ij} \) are the mutual and susceptance conductance among \( i \) node and \( j \) node. The \( \phi_{ij} \) depicts the voltage angle magnitude in node \( i \) and \( j \).

2.2.2. Inequality Constrains

These constrains contain:

1. Constraints of generator: these constrains have voltage in generator nodes \( (V_G) \) and reactive power output \( (Q_G) \) of all generators are limited by their \( min \) and \( max \) bounds as

\[
V_{Gi-min} \leq V_{Gi} \leq V_{Gi-max}, \quad i = 1, \ldots, N_G
\]

(3)

\[
Q_{Gi-min} \leq Q_{Gi} \leq Q_{Gi-max}, \quad i = 1, \ldots, N_G
\]

(4)

2. Transformer constrains: these constrains have lower and upper bounds as

\[
Tap_{i-min} \leq Tap_{i} \leq Tap_{i-max} \quad i = 1, \ldots, N_T
\]

(5)

3. Shunt VAR source \( Q_c \) constrains: switch-able VAR compensation \( (Q_c) \) are bounded as

\[
Q_{Ci-min} \leq Q_{Ci} \leq Q_{Ci-max} \quad i = 1, \ldots, N_T
\]

(6)

4. Security constrains: these constrains contain the limit of load node voltages as

\[
V_{Li-min} \leq V_{Li} \leq V_{Li-max} \quad i = 1, \ldots, N_{PQ}
\]

(7)

2.2.3. The Objective Function

In this problem, the dependent variables can be added to equation (1) by utilizing penalty factors to constrain, so equation (1) can be represented as [22]

\[
Min F = P_{loss} + \lambda_V \sum_{i=1}^{NL} (V_{li} - V_{lim})^2 + \lambda_Q \sum_{i=1}^{NG} (Q_{Gi} - Q_{Gi}^{lim})^2
\]

(8)
where \(\lambda_V, \lambda_Q\) are the penalty terms and these terms are big positive constants, \(NL\) depicts the number of loads nodes that violate the limits, \(NL\) depicts the number of reactive power output of generator nodes that outside the bounds and \(V_{lim}^{i}, Q_{G_i}^{lim}\) are described as

\[
V_{lim}^{i} = \begin{cases} 
    v_{lim}^{i}_{min} & \text{if } v_{lim}^{i} < v_{lim}^{i}_{min} \\
    v_{lim}^{i}_{max} & \text{if } v_{lim}^{i} > v_{lim}^{i}_{max} 
\end{cases}
\]

\[
Q_{G_i}^{lim} = \begin{cases} 
    Q_{G_i}^{min} & \text{if } v_{lim}^{i} < Q_{G_i}^{min} \\
    Q_{G_i}^{max} & \text{if } v_{lim}^{i} > Q_{G_i}^{max} 
\end{cases}
\]

2.2.4. Concept of Average Voltage

In this study, the new average voltage index is suggested to deal with all voltage nodes as well as satisfy most of the electrical utility limits. The equation of this concept can be written as

\[
V_{av} = \frac{\sum_{i=1}^{Nn} V_i}{N_n} 
\]

where \(V_{av}\) depicts the average voltage of all systems, \(V_i\) depicts the voltage in node \(i\) and \(N_n\) depicts the total number of nodes.

3. Optimization

3.1. Simple PSO Algorithm

PSO algorithm is a better type of stochastic optimization algorithm. The basic concept of this algorithm came from the social behavior of the animals when searching for food like fish schooling and bird flocking. This algorithm has beneficial characteristics; it is simple, fast, can be applied to solve optimization problems. It guarantees the best solution with less calculation time, and its convergence characteristic is very stable than other stochastic algorithms. Moreover, it is capable of dealing with continuous and discrete variables and does not have mutation and crossover operation like in the genetic algorithm. In PSO, every possible solution represents an individual. An individual represents the candidate solution, and every group of individuals represents a swarm. Kennedy and Eberhart were first enhanced and developed this algorithm in 1995 [23]. Each individual in the PSO algorithm has the best position discovered by the experience of the individual itself, and it is stored in a memory called the local best position \((P_{best})\). The best position discovered between all individuals \((P_{best})\) in the swarm also stored in a memory called global best position \((G_{best})\). At every iteration, the location of \(P_{best}\) and \(G_{best}\) are changing. Then, the velocity and position of every individual in the swarm are modified by calculating its current velocity and location based on \(P_{best}\) and \(G_{best}\). The velocity and position from \(P_{best}\) and \(G_{best}\) of the agents will be changed using equations (12) and (13) [24].

\[
V_i^{k+1} = K[W_{PSO}V_i^k + C_1 R_1 (P_{best(i)}^k - X_i^k) + C_2 R_2 (G_{best(i)}^k - X_i^k)] 
\]

\[
X_i^{k+1} = X_i^k + V_i^{k+1} 
\]

where \(W_{PSO}\) is the inertia coefficient of PSO technique, \(V_i\) represents the velocity of individual, \(C_1, C_2\) are the two learning factors that utilized to pull each agent to \(P_{best}\) location and \(G_{best}\) location within range \([0 \text{ to } 2.05]\) and \(R_1, R_2\) are the two random numbers within the limit \([0 \text{ to } 1]\), \(P_{best(i)}\) depicts the local best position, \(G_{best(i)}\) represents the global best position, \(X_i\) represents the position of the individual and \(K\) depicts the constriction factor. The constriction factor is used to warrant the convergence characteristic of Simple PSO to a stable point, avoiding the need for velocity fixing, which Shi first introduced. He indicated that utilizing this factor may be necessary and can be expressed as follows [25].
\[ K = \frac{2}{2 - \phi - \sqrt{\phi^2 - 4\phi}}, \quad \phi = C_1 + C_2, \quad \phi \geq 4 \]  

(14)

In this study, \((W_{PSO})\) is reduced from \((0.9\) to \(0.4)\) linearly through iterations to attain a balance between global exploration \((G_{\text{best}})\) and local exploitation \((P_{\text{best}})\) as follows:

\[ W_{PSO} = W_{\text{max}} - \left( \frac{W_{\text{max}} - W_{\text{min}}}{\text{max}_{\text{iteration}}} \right) \cdot \text{iter} \]  

(15)

where \(W_{\text{max}}\) depicts the max (upper) value of weight, \(W_{\text{min}}\) depicts the min (lower) value of weight, \(\text{iter}\) depicts the current iteration and \(\text{max}_{\text{iteration}}\) is the max (upper) iterations.

3.2. CPSO Algorithm

The Simple PSO algorithm mainly relies on its parameters, making it difficult and sometimes unable to reach the precise solution criteria in some cases, especially when the number of parameters of the optimization problem is enormous. A chaos theory merged with a PSO algorithm to form a hybrid algorithm called CPSO, helped the CPSO algorithm to slip very easily from the local optima due to the remarkable behavior and high ability of the chaos [26]. In this study, the logistic map equation adopted for establishing the hybrid CPSO algorithm is described by the following equation [27].

\[ \beta^{k+1} = \mu \cdot \beta^k \cdot (1 - \beta^k), \quad 0 \leq \beta^k \leq 1 \]  

(16)

where the control parameter \(\mu\) is set within a range \([0.0 - 4.0]\), \(k\) is the number of the iterations (steps). The magnitude of \(\mu\) decides whether \(\beta\) stabilizes at a constant area, oscillates within restricted limits, or behaves chaotically in an unpredictable form. Equation (17) is deterministic; it shows chaotic dynamics when \(\mu = 4.0\) and \(\beta^1 \in \{0, 0.25, 0.5, 0.75, 1\}\). It displays high sensitivity depending on its initial conditions, which are the basic features of chaos. The new inertia weight factor \((W_{CPSO})\) is calculated by multiplying the \((W_{PSO})\) in equation (15) and logistic map in equation (16) to form equation (18) as follows.

\[ W_{CPSO} = W_{PSO} \cdot \beta^{k+1} \]  

(17)

To enhance the behavior of the Simple PSO, this study presented a new velocity update by blending a logistic map equation \((\beta)\) with inertia weight factor \((W_{PSO})\). Finally, by blending equation (17) with equation (12), the following velocity changed the equation for the proposed CPSO algorithm, which can be expressed as follows:

\[ V_{i}^{k+1} = W_{CPSO} \cdot V_{i}^{k} + C_1 R_1 (P_{\text{best}(i)}^{k} - X_{i}^{k}) + C_2 R_2 (G_{\text{best}(i)}^{k} - X_{i}^{k}) \]  

(18)

In the CPSO algorithm, \(W_{CPSO}\) decreases and oscillates simultaneously from \((0.9\) to \(0.4)\) for total iteration but decreases linearly in Simple PSO. Fig. 1 shows a flowchart of the CPSO.

4. Results and Discussion

A standard IEEE Node—14 was utilized as a test system to evaluate the efficiency and consistency of the CPSO algorithm and discover the optimal solution for the RPO problem. The CPSO algorithm will be tested on this system to demonstrate the quality and flexibility. The Simple PSO and CPSO algorithms were developed and simulated in MATLAB, with the number of maximum iterations in this study being 200.

4.1. IEEE 14-Node System

This system involves 20 branches, 5 generators, 1 reactive power VAR source compensation (capacitor banks), and 3 transformers; bus, line, generator data, the bounds of reactive power
$(Q_G)$ for generators and other operating data were tabulated in reference [28]. Table 1 shows constraints of independent variables while constraints of reactive power $(Q_G)$ in MVAR for generators are illustrated in Table 2 [28].

![Flowchart of the CPSO algorithm based on RPO problem.](image)

**Fig. 1.** Flowchart of the CPSO algorithm based on RPO problem.

**Table 1.** Independent variables constraints for IEEE Node-14 system.

| Power System Type | Independent Variables          | MinMax  |
|-------------------|--------------------------------|---------|
| 14 Bus            | Generator Voltage $(V_G)$      | 0.95 1.1|
|                   | Transformer Tap $(T)$          | 0.9 1.1 |
|                   | VAR Source Compensation $(Q_C)$| 0 0.20  |

**Table 2.** Constraints of reactive power generation for IEEE Node-14 system [28].

| Power System Type | Generator Nodes | $Q_{Min}$ | $Q_{Max}$ |
|-------------------|-----------------|-----------|-----------|
| 14 Bus            | 1               | 0         | 10        |
|                   | 2               | -40       | 50        |
|                   | 3               | 0         | 40        |
|                   | 6               | -6        | 24        |
|                   | 8               | -6        | 24        |

This system has 9 dimensions search space that need to be optimized, including 5 generator voltages $(V_G)$, three transformer taps setting $(T)$ and 1 reactive power injected from capacitor bank $(Q_C)$ as listed in Table 3. Simulation results of standard IEEE-14 node system were tested.
through a series of comparisons among Simple PSO and CPSO with other optimization methods such as (EP and SARGA) [29], which are tabulated in Table 2. This table proves that the CPSO algorithm had achieved the best results in minimizing loss among all competitors. Fig. 2 compares the percentage reduction power loss for various algorithms. From Fig. 2, it is clear that the reduction in $P_{\text{loss}}$ are 9.5% at CPSO, 8.8% at PSO, 1.5% at EP, and 2.5% at SARGA algorithms. Fig. 3 and Fig. 4 show the convergence in terms of power loss versus 200 iterations for the IEEE Node—14 system; these figures indicate that the convergence characteristic of CPSO was the best and most effective in terms of minimizing loss and reaching an optimal solution in fewer iterations than Simple PSO. The chaotic theory helps the system avoid the premature convergence problem and demonstrates that the system was not trapped into the local optima. Fig. 5 presents the voltage profile for this system before and after Simple PSO and CPSO algorithms. From Fig. 5, it is clear that the average voltage at initial is about 1.048, at Simple PSO is about 1.059 and at CPSO is about 1.082.

**Table 3.** Simulation results of IEEE Node—14 system.

| Control Variables | Base Case | CPSO | PSO | EP [29] | SARGA [29] |
|-------------------|-----------|------|-----|---------|------------|
| $V_{G-1}$         | 1.060     | 1.100| 1.100| NR*     | NR*        |
| $V_{G-2}$         | 1.045     | 1.008| 1.006| 1.029   | 1.060      |
| $V_{G-3}$         | 1.010     | 1.058| 1.057| 1.016   | 1.036      |
| $V_{G-6}$         | 1.070     | 1.096| 1.067| 1.097   | 1.099      |
| $V_{G-8}$         | 1.090     | 1.100| 1.061| 1.053   | 1.078      |
| $\text{Tap}_{4-7}$| 0.978     | 0.976| 1.019| 1.04    | 0.95       |
| $\text{Tap}_{4-9}$| 0.969     | 0.975| 0.989| 0.94    | 0.95       |
| $\text{Tap}_{5-6}$| 0.932     | 1.019| 1.008| 1.03    | 0.96       |
| $Q_{C-9}$         | 0.19      | 0.186| 0.184| 0.18    | 0.06       |
| Reduction in $P_L$ (%) | 0   | 9.5 | 8.8 | 1.5 | 2.5 |
| Total $P_L$ (Mw)  | 13.550    | 12.253| 12.355| 13.346 | 13.21643  |

NR* means not reported.

**Fig. 2.** Comparison of the percentage reduction power loss for various algorithms.
5. Conclusion

In this study, to enhance the performance and quality while avoiding the premature convergence of the Simple PSO algorithm, CPSO algorithm was utilized as an optimization tool to solve RPO problems. The main goal of the objective function is to decrease power loss ($P_{loss}$).
enhancing the voltage profile of the power system through a proper alteration of the reactive power devices (i.e., control variables). The CPSO algorithm was tested on the IEEE Node-14 system. The simulation implications prove that the CPSO algorithm is the best in terms of convergence characteristics, obtaining optimal values of reactive power devices (i.e., control variables) while decreasing the $P_{loss}$ as well as improving the voltage profile of the system. Furthermore, the reduction in $P_{loss}$ achieved by CPSO algorithm is the biggest than the reduction in $P_{loss}$ obtained by the Simple PSO and other techniques in the literature. In addition, the simulation implications demonstrate the efficiency and potential of the CPSO algorithm for solving the RPO problem. The CPSO algorithm had the best results with faster calculation time than the Simple PSO for solving RPO and other complex problems in power systems. Furthermore, the minimum power loss obtained by the CPSO algorithm can attain considerable economic benefits and secure power system operations. So, the CPSO algorithm has been recorded as one of the candidate algorithms with great interest from the authors due to versatility, capability, and achievement in solving complex and multi-variable problems such as RPO.

**Author Contribution:** All authors contributed equally to the main contributor to this paper. All authors read and approved the final paper.

**Funding:** This research received no external funding.

**Conflicts of Interest:** The authors declare no conflict of interest.

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