Thermodynamics Inducing Massive Particles’ Tunneling and Cosmic Censorship

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By calculating the change of entropy, we prove that the first law of black hole thermodynamics leads to the tunneling probability of massive particles through the horizon, including the tunneling probability of massive charged particles from the Reissner-Nordström black hole and the Kerr-Newman black hole. Novelly, we find the trajectories of massive particles are close to that of massless particles near the horizon, although the trajectories of massive charged particles may be affected by electromagnetic forces. We show that Hawking radiation as massive particles tunneling does not lead to violation of the weak cosmic-censorship conjecture.

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INTRODUCTION

In 1974, Hawking discovered [1], when considering quantum effects, that a black hole can emit thermal radiation, which was also confirmed by many other scientists [2–4]. Hawking’s result does not coexist with energy conservation, however, since the spacetime background is fixed in Hawking’s calculation [5]. Recently, Parikh and Wilczek [6] developed the method of Hawking radiation as tunneling, in which the spacetime background was not fixed and the reaction was included. This method has been discussed in different situations [7–11], and its self-consistency has been checked by using thermodynamic relations [12–14].

The laws of thermodynamics play an important role in the area of study on the black hole. Although the method of Hawking radiation as tunneling can give the radiation rate, the relation between tunneling dynamics and black hole thermodynamics still needs to be checked. Recently, Pilling [15] suggested a method of Hawking radiation as tunneling from black hole thermodynamics. In this method, the change of entropy is calculated to obtain the tunneling probability directly from the first law of black hole thermodynamics. The connection between black hole tunneling and thermodynamics has been verified further [16], when the tunneling probability through quantum horizon [8–17] had been obtained directly from the first law of black hole thermodynamics [16]. However, these discussions only involve the massless particle tunneling. Since the massless particles do not take charge with themselves, the trajectory will not be affected by the electromagnetic forces. It is noted that the tunneling probability of massive charged particles from the Reissner-Nordström black hole [7] or Kerr-Newman black hole [10] has been obtained, but whether the tunneling probability can be obtained directly from the first law of black hole thermodynamics or not has not been considered. On the other hand, since the massive particles can take charge and angular momentum with themselves, the weak cosmic-censorship conjecture [21–23] should be revisited with the tunneling of massive particles. In classical theory, the weak cosmic censorship conjecture is able to get full support, but the quantum effect may challenge it [24]. So, it is interesting to verify whether the Hawking radiation as massive particles tunneling could lead to violate the weak cosmic-censorship conjecture or not.

In this paper, we will calculate the massive particles’ tunneling probability from the first law of thermodynamics and apply this method to the tunneling from Reissner-Nordström black hole and Kerr-Newman black hole. Novelty, we find the trajectories of massive charged particles are almost identical to those of massless particles near the horizon, although the trajectories of the massive charged particles connect with the electromagnetic forces. We will also show that the massive particles tunneling of a black hole does not violate the weak cosmic-censorship conjecture, while, in some situations, the Reissner-Nordström and the Kerr-Newman black hole may evolve into extreme black holes [25–27].

The paper is organized as follows. In the second section, we calculate the change of entropy to obtain the probability of massive particle tunneling from the first law of black hole thermodynamics for a general class of static, spherically symmetric spacetime. In the third section, we investigate the massive charged particle tunneling from the Kerr-Newman black hole and then discuss the situation of Reissner-Nordström black hole. Next, we discuss the weak cosmic censorship conjecture. Finally, we give some discussion and conclusions.

In this paper we take the unit convention $k = h = c = G = 1$. 
In this section, we will extend the method by Pilling \cite{15} to the situation of the massive particle tunneling from black hole thermodynamics. Let us start with the metric

\[ ds^2 = -f(r) \, dt^2 + \frac{dr^2}{g(r)} + r^2 d\Omega^2, \]  

(1)

where the metric describes a general class of static, spherically symmetric spacetime. It is noted that there is a coordinate singularity in the metric at the horizon \( r = R \) which is given by \( f(R) = g(R) = 0 \). For the sake of convenience, we can make the Painlevé coordinate transformation to remove the singularity at the horizon,

\[ dt \rightarrow dt_s - \sqrt{\frac{1 - g(r)}{f(r) g(r)}} \, dr, \]

(2)

where the time transformation is dependent on \( r \), not \( t \). So the metric remains stationary and the time direction is still a Killing vector. After the transformation, the metric (1) becomes of the following form:

\[ ds^2 = -f(r) \, dt^2 + 2f(r) \sqrt{\frac{1 - g(r)}{f(r) g(r)}} \, dt \, dr + dr^2 + r^2 d\Omega^2. \]

(3)

The new metric is called a Painlevé metric that is regular at the horizon. The time \( t \) here is that of radially free-falling observer through the horizon.

Generally, for the static coordinates, the coordinate time can represent the global time due to Einstein’s simultaneity. For example, the Schwarzschild coordinate time in global space can be regarded as the time recorded by a standard clock that is resting at spatial infinity. The coordinates \( \mathcal{E} \) is stationary and not static (the component \( g_{0i} \) of the metric tensor does not vanish in the metric \( \mathcal{E} \)), so we must check whether the time is global. According to the general theory \cite{28} of the coordinate clock synchronization in a spacetime, made by Laudau and Lifshitz, the condition that the simultaneity of coordinate clocks can be transmitted from one place to another is either the integral

\[ \int g_{0i} \, dx_i = 0 \]

or the integral \( \int \frac{g_{0i}}{g_{00}} \, dx^i \) is independent on the path. The condition can be described as another form \cite{7} equivalently,

\[ \frac{\partial}{\partial x^j} \left( \frac{g_{0i}}{g_{00}} \right) = \frac{\partial}{\partial x^i} \left( \frac{g_{0j}}{g_{00}} \right). \]

(4)

Obviously, the coordinate \( \mathcal{E} \) satisfies the condition so that the coordinate time in Painlevé coordinates can be used as the global time and the coordinate clock synchronization can be transmitted from one place to another. This is important for tunneling process in quantum mechanics.

From the coordinate \( \mathcal{E} \), the radial null geodesics can be obtained as

\[ \dot{r} = \frac{dr}{dt} = \frac{f(r)}{g(r)} \left( \pm 1 - \sqrt{1 - g(r)} \right), \]

(5)

where the positive (negative) sign gives the outgoing (incoming) radial geodesic, under the implicit assumption that \( t \) increases towards the future.

When considering the massive test particles moving radially in the background, the radial null geodesics is improper for such particles. In analogy to Ref. \cite{7}, we treat the massive particle as de Broglie wave and obtain the expression for \( \dot{r} \),

\[ \dot{r} = v_p = \frac{1}{2} v_p = -\frac{1}{2} \frac{g_{00}}{g_{01}} = \frac{1}{2} \frac{f(r) g(r)}{1 - g(r)}, \]

(6)
where $v_p$ is the phase velocity, and $v_e$ is the group velocity. It is obvious that the geodesics of massive particles are different from radial null geodesics. The surface gravity of the black hole for the transformed metric $\text{III}$ at the horizon is given by

$$\kappa_0 = \Gamma^0_{00}(r=R) = \frac{1}{2} \left(\sqrt{1 - g(r)} \frac{df(r)}{dr} g(r) \right) |_{r=R}. \quad (7)$$

Generally, when discussing the surface gravity of a black hole, we are defining a notion that behaves analogously to the Newtonian surface gravity, while these two things are not the same because the acceleration of a test body at the event horizon of a black hole turns out to be infinite in relativity. Generally, the surface gravity of a black hole is not well defined. However, we can define the surface gravity for a black hole whose event horizon is a Killing horizon. According to the equation in terms of time-like Killing $K$ as $K_a \nabla_b K_a = \kappa_0 K_b$, we can obtain the surface gravity in terms of Christoffel components as $\text{(7)}$. Note that, in static spacetime, a more general expression for black hole surface gravity should be taken as the geometrical surface gravity $\text{[29]}$

$$\kappa = \gamma \kappa_0 \quad (8)$$

which gives an additional factor comparing with the Eq. $\text{(7)}$ and this factor is a constant at horizon.

Due to $f(R) = g(R) = 0$, we can expand $f(R)$ and $g(R)$ near the horizon in powers of $r - R$,

$$f(r) = f'(R)(r-R) + O((r-R)^2),$$
$$g(r) = g'(R)(r-R) + O((r-R)^2). \quad (9)$$

According to the laws of black hole thermodynamics, the Hawking temperature is expressed in terms of the surface gravity via

$$T_H = \frac{\kappa_0}{2\pi} = \frac{\sqrt{f'(R)g'(R)}}{4\pi}. \quad (10)$$

Note that the Eq. $\text{(10)}$ is not general local temperature expression at horizon, since it lacks the relative factor stemmed from the use of Kodama vector instead of the static Killing vector $\text{[29]}$. The general expression for the temperature can be taken as

$$T = \frac{\kappa}{2\pi} = \frac{\gamma \kappa_0}{2\pi}. \quad (11)$$

Thus we must treat the first law of black hole thermodynamics and the entropy carefully. The entropy should be given by the area, $A$, of event horizon as $S = \frac{A}{4\pi} = \frac{\pi}{7} R^2$. Thus the first law of black hole thermodynamics maintains its form $TdS = dM$.

Let us consider the black hole thermodynamics in the region near the horizon. When the mass of the black hole changes from $M_i$ to $M_f$, the change of the entropy is given as

$$\Delta S = \int dS = \int_{M_i}^{M_f} \frac{dS}{dM} dM = \int_{M_i}^{M_f} \frac{2\pi R}{\gamma} \frac{dR}{dM} dM. \quad (12)$$

Considering the small path near $R$, we can insert the mathematical identity $\text{Im} \int_{r_i}^{r_f} \frac{1}{r-R} dr = -\pi$ in the formula $\text{(12)}$. Thus we obtain

$$\Delta S = -\frac{2}{\gamma} \text{Im} \int_{M_i}^{M_f} \int_{r_i}^{r_f} \frac{R}{r-R} \frac{dR}{dM} dM. \quad (13)$$

With $\text{(11)}$ and the expression of the temperature in thermodynamics $\frac{d}{dT} = \frac{\partial S}{\partial E}$, we can get
Using the Taylor series, we can gain the geodesic of a massive particle near the horizon,\[ \dot{r} = -\frac{1}{2} \sqrt{f'(R)g'(R)} (r - R) + O ((r - R)^2). \] (15)

It is noticed that the geodesic of massive particle is equivalent to that of massless particle in the first order. That is to say, no matter whether the particles have mass or not, their trajectories will be close to each other when they approach the horizon. In particular, their geodesics coincide precisely with each other at the horizon. One reason for this phenomenon is that the wavelength of the outgoing particle is infinitely blueshifted at the horizon. This is also the key argument that the WKB approximation can be used for Hawking radiation as tunneling. Sometimes, the infinite blueshift property of the event horizon could lead to the so-called “trans-Planckian puzzle”, which is considered as a mathematical artifact of horizon calculations nowadays. On the other hand, the tunneling process is instantaneous, so the outgoing particles, both the massless and massive particles, take the same quickest path across the horizon.

With equations (14) and (15), we can obtain the final form of the change of entropy as
\[ \Delta S = -\frac{2}{\gamma} \text{Im} \int_{M_f}^{M_i} \frac{dR}{r} dM = -\frac{2}{\gamma} \text{Im} I, \] (16)

where \( I \) is the action for an s-wave outgoing positive particle in WKB approximation. Consequently, the tunneling probability is given as
\[ \Gamma \sim e^{-\frac{\Delta S}{\gamma}} = e^{\Delta S}. \] (17)

Thus we obtain the massive particles tunneling probability from the change of entropy as a direct consequence of the first law of black hole thermodynamics, along the line presented in Ref. Note that when \( \gamma = 1 \), the entropy obeys the Bekenstein-Hawking expression. But the black hole entropy may have other expressions, for example, when \( \gamma = \frac{3}{4} \), one has \( S = \frac{A}{4} \), which can be obtained from black hole solution in modified \( f(R) \) gravity. This implies that it isn’t always true in general that the related black hole entropy is given by the Bekenstein-Hawking area law, while the laws of black hole thermodynamics play an important role in Pilling’s argument.

THERMODYNAMICS ABOUT KERR-NEWMAN BLACK HOLE

The Painlevé line element of the Kerr-Newman black hole is
\[ ds^2 = -\frac{\Lambda \Sigma}{(r^2 + a^2)^2} \frac{\Lambda \Sigma}{\Sigma} \frac{\Lambda \Sigma}{\Sigma} dt^2 + \frac{\Sigma}{r^2 + a^2} dr^2 + 2 \sqrt{(2Mr - Q^2) (r^2 + a^2)} \frac{\Sigma}{(r^2 + a^2)^2} \frac{\Sigma}{\Lambda a^2 \sin^2 \theta} dt dr + \Sigma d\theta^2, \] (18)
\[ A = \frac{Qr (r^2 + a^2)}{(r^2 + a^2)^2 - \Lambda a^2 \sin^2 \theta} dt = A_t dt, \] (19)

which is obtained from the Kerr-Newman black hole in the Boyer-Lindquist coordinate system by the generalized Painlevé-type coordinate transformation,
\[ dt_k = dt - \frac{\sqrt{(2Mr - Q^2) (r^2 + a^2)}}{\Lambda} dr, \]
\[ d\phi_k = d\phi - \frac{a}{\Lambda} \sqrt{\frac{2Mr - Q^2}{r^2 + a^2}} dr, \] (20)
and the dragging coordinate transformation

\[ d\phi = \frac{a (r^2 + a^2 - \Lambda)}{(r^2 + a^2)^2 - \Lambda a^2 \sin^2 \theta} dt, \tag{21} \]

where \( \Sigma = r^2 + a^2 \cos^2 \theta \), \( \Lambda = r^2 + a^2 + Q^2 - 2Mr \), \( t_k \) and \( \phi_k \) is the coordinates before transformation. The form of the vector potential \( A \) is unchanged up to a gauge transformation. The radial time-like geodesics of massive charged particles are given by

\[ \dot{r} = v_p = \frac{1}{2} v_g = -\frac{1}{2} g_{tt} = \frac{\Lambda}{2 \sqrt{(r^2 + a^2) (r^2 + a^2 - \Lambda)}}. \tag{22} \]

Near the outer horizon \( r = R^+ = M + \sqrt{M^2 - Q^2 - a^2} \), which can be obtained by solving \( \Lambda = 0 \), and the geodesics can be expanded in powers of \( r - R^+ \) as

\[ \dot{r} = \frac{\sqrt{M^2 - Q^2 - a^2}}{(M + \sqrt{M^2 - Q^2 - a^2})^2 + a^2} (r - R^+) + O \left( (r - R^+)^2 \right). \tag{23} \]

The radiation temperature is obtained as

\[ T_H = \frac{\kappa}{2\pi} = \frac{1}{2\pi} \frac{\sqrt{M^2 - Q^2 - a^2}}{(M + \sqrt{M^2 - Q^2 - a^2})^2 + a^2}. \tag{24} \]

Let us consider the black hole thermodynamics in the region near the horizon. If the energy of the black hole changes from \( H_i \) to \( H_f \), the change of the entropy \( S = \pi (R^+)^2 + a^2 \) is

\[ \Delta S = \int_{H_i}^{H_f} \left( \frac{\partial S}{\partial M} dM + \frac{\partial S}{\partial Q} dQ + \frac{\partial S}{\partial J} dJ \right). \tag{25} \]

It can be verified that Eq. (25) is another expression of the first law of black hole thermodynamics, \( dM = \kappa \frac{\Delta S}{2\pi} + \Phi dQ + \Omega dJ \), where \( \Phi \) is the electric potential which is defined by \( \Phi = A_t = \frac{Q(r^2 + a^2)}{(r^2 + a^2)^2 - \Lambda a^2 \sin^2 \theta} \), and \( \Omega \) is the dragging angular velocity which is defined by \( \Omega = \frac{d\phi}{dt} = -\frac{\partial \phi}{\partial r} = \frac{a (r^2 + a^2 - \Lambda)}{(r^2 + a^2)^2 - \Lambda a^2 \sin^2 \theta} \). At the horizon, we have \( \Phi^+ = \Phi|_{r=R^+} = \frac{Q (M + \sqrt{M^2 - Q^2 - a^2})}{(M + \sqrt{M^2 - Q^2 - a^2})^2 + a^2} \), \( \Omega^+ = \Omega|_{r=R^+} = \frac{a}{(M + \sqrt{M^2 - Q^2 - a^2})^2 + a^2} \). Considering the conservation of energy, charge and angular momentum, we have

\[ \frac{\partial S}{\partial M} = 2\pi R^+ \frac{\partial R^+}{\partial M} = \frac{2\pi}{\sqrt{M^2 - Q^2 - a^2}} \left( M + \sqrt{M^2 - Q^2 - a^2} \right)^2 + a^2, \tag{26} \]

\[ \frac{\partial S}{\partial Q} = 2\pi R^+ \frac{\partial R^+}{\partial Q} = -\frac{2\pi Q}{\sqrt{M^2 - Q^2 - a^2}} \left( M + \sqrt{M^2 - Q^2 - a^2} \right), \tag{27} \]

\[ \frac{\partial S}{\partial J} = 2\pi R^+ \frac{\partial R^+}{\partial J} = \frac{-2\pi a}{\sqrt{M^2 - Q^2 - a^2}}. \tag{28} \]
where $a = J/M$. Similarly, these expressions (26), (27), and (28) are consistent with the first law of black hole thermodynamics, that is, $\frac{\partial S}{\partial M} = \frac{2\pi}{\kappa} \Phi^+$, and $\frac{\partial S}{\partial Q} = \frac{2\pi}{\kappa} Q$. Considering the small path near the horizon, we can insert the mathematical identity $\text{Im} \int_{r_i}^{f} \frac{1}{r-R^+} \text{dr} = -\pi$ in the formula (25). Thus we obtain

$$
\Delta S = -\frac{1}{\pi} \text{Im} \int_{H_i}^{H_f} \int_{r_i}^{r_f} \left( \frac{1}{r-R^+} \text{dr} \right) \left( \frac{\partial S}{\partial M} dM + \frac{\partial S}{\partial Q} dQ + \frac{\partial S}{\partial J} dJ \right) = -2\text{Im} \int_{H_i}^{H_f} \int_{r_i}^{r_f} \left( \frac{dr}{\kappa (r-R^+)} \frac{\Phi^+ dr}{\kappa (r-R^+)} dM - \frac{\Omega^+ dr}{\kappa (r-R^+)} dQ - \frac{\Omega^+ dr}{\kappa (r-R^+)} dJ \right),
$$

where we have used the expressions of the temperature in thermodynamics $\frac{1}{T} = \frac{\partial S}{\partial M}$, the electric potential $\Phi^+$ and the angular momentum $\Omega^+$. Then Eqs. (23) and (26)-(28) give the form of the change of entropy as

$$
\Delta S = -2\text{Im} \int_{H_i}^{H_f} \int_{r_i}^{r_f} \left( \frac{dr}{r} dM - \Phi^+ \frac{dr}{r} dQ - \Omega^+ \frac{dr}{r} dJ \right)
$$

Using the Hamilton’s equations, $\dot{r} = \frac{dt}{dA_r} (r, A_r, \phi, P_r) = \frac{dt}{dA_t} \mid_{(r, A_r, P_r, \phi)} = \Phi^+ \frac{dQ}{dA_t}$ and $\dot{\phi} = \frac{dt}{dP_{\phi}} \mid_{(\phi, r, A_r, P_{\phi})} = \Omega^+ \frac{dJ}{dA_t}$, we can express the change of entropy in an explicit form as

$$
\Delta S = -2\text{Im} \int_{r_i}^{r_f} \left( P_r \dot{r} - P_{A_r} \dot{A}_t - P_{\phi} \dot{\phi} \right) dt = -2\text{Im} I
$$

which is related to the emission rate of tunneling particle by $\Gamma \sim e^{-2\text{Im} I} = e^{\Delta S}$.

Thus we obtain the tunneling probability from the change of entropy of Kerr-Newman black hole as a direct consequence of the first law of black hole thermodynamics. It is noticed that when considering the conservation of energy, charge and angular momentum, the tunneling particle can take the charge and angular momentum. Note that the generalized coordinate $A_t$ is an ignorable one and the coordinate $\phi$ is a cyclic one, so the action can be written as

$$
\int_{r_i}^{r_f} \left( P_r \dot{r} - P_{A_r} \dot{A}_t - P_{\phi} \dot{\phi} \right) dt.
$$

In Ref. [10], it has been pointed out that the tunneling process is closely related to the first law of black hole thermodynamics, but the change of angular momentum was treated by the change of mass with the relation $\delta J = a \delta M$, i.e., the angular momentum of the unit mass is kept as a constant. In our calculation, although the specific angular momentum $a$ is not kept as a constant, the first law of black hole thermodynamics can still lead to the tunneling probability by calculating the change of entropy. So, our result seems to be more reasonable and more general. Another potential problem is that of how the angular momentum of black hole is carried away in the semiclassical WKB approximation, where only s-wave outgoing particles were considered. That is, the potential $l (l+1) / r^2$ related to the orbital angular momentum does not work when calculating the tunneling probability. The angular momentum taken by the outgoing particles with themselves only means rotation around the center of the spherically symmetric black hole, not including the self-spinning. That is the reason why the specific angular momentum is kept as a constant in Ref. [10], but our calculation shows that this restriction is not necessary.

### Weak Cosmic Censorship Conjecture

The uniqueness theorem [21, 22] states that all stationary black hole solutions of Einstein-Maxwell equations are uniquely determined by three conserved parameters: the gravitational mass $M$, the electric charge $Q$, and the angular momentum $J$. The three parameters must satisfy the relation $M^2 \geq Q^2 + (J/M)^2$ to maintain the black hole as the condition demanded by the weak cosmic-censorship conjecture [23], which asserts that spacetime singularities coming from the completely gravitational collapse of a body must be encompassed by the horizon of a black hole. Particularly, $M^2 = Q^2 + (J/M)^2$ characterizes extreme black holes [23, 27], whereas $M^2 < Q^2 + (J/M)^2$ are concerned with naked singularities rather than black holes. In what follows we will check whether the weak cosmic-censorship conjecture will hold or not in the situations of massive particles’ tunneling.

For Kerr-Newman black holes, if the angular momentum of unit mass is considered as a constant in the tunneling process, the total angular momentum of black holes is not considered as independent variable and its change is closely related to the change of mass. Thus at some time the charge had been taken out of the black hole completely and the
mass continues to decrease until the extremal case $a^2 = M^2$ appears, so the temperature is absolutely zero and the radiation vanishes. In this case, the weak cosmic-censorship conjecture will not be violated. If the angular momentum of unit mass $a$ is not considered as a constant in the tunneling process, the angular momentum and the charge will be taken out of the black hole completely before the black hole vanishes because the particles may take angular momentum with themselves together with that of rotating around the center of the spherically symmetric black hole. For example, the electron’s $a$ and $Q$ (suitably specified in geometrized units) both exceed its mass $M$. In such a situation the relation $M^2 > Q^2 + a^2$ holds until the black hole vanishes and the extremal black hole will not appear.

Specially, when $a = 0$, the coordinate [15] decays into the Painlevé line element of the Reissner-Nordström black hole. One can easily approve that the first law of black hole thermodynamics can still give the massive particles’ tunneling rate. The consideration of massive particles’ tunneling is necessary since the massless particles’ tunneling may lead to the violation of weak cosmic-censorship conjecture. On one hand, the massless particles do not take the charges with themselves but take the energy by the self-gravity effect when tunneling out of the black hole. The extreme case $Q^2 = M^2$ can always be reached when the radiation temperature vanishes. On the other hand, there exist some massless neutral scalar particles that are spinning. When they tunnel outward, the black hole will rotate towards the inverse direction due to the conservation of angular momentum. In this case, no matter how small the angular momentum is, the relation $M^2 < Q^2 + (J/M)^2$ can always be reached, because the particles do not take the charge with themselves. Thus, the weak cosmic-censorship conjecture will be violated. It is similar to the case of overspinning a nearly extreme charged black hole via a quantum tunneling process [24]. The violation of weak cosmic censorship conjecture can be avoided by considering the massive charged particles tunneling. Generally speaking, the particle charge-mass ratio may be bigger than one; for example, for the electron $e/m \sim 10^{11} \gg 1$, for the proton $e/m \sim 10^8 \gg 1$. Thus, the charge will be carried completely out of black hole soon. The situation of violation of weak cosmic censorship conjecture will not appear.

In general, the tunneling particles may contain the massless, massive, massive charged particles in the tunneling process. If the rate of mass loss is quicker than that of charge and specific angular momentum loss, the condition $M^2 = Q^2 + a^2$ may be approached and the extreme black holes appear as the terminal of the tunneling. If the charge and the angular moment are always taken out of black holes, the extreme situation will not appear in the tunneling process. Fortunately, in any case, the weak cosmic-censorship conjecture will not be violated.

CONCLUSION

In summary, the self-consistency of Hawking radiation as massive particles’ tunneling has been verified by the laws of black hole thermodynamics in our paper. We have showed that the probability of massive particles’ tunneling can be obtained from the first law of thermodynamics by calculating the change of entropy, and this method has also been used to gain the tunneling probability of massive charged particles. We have also obtained the tunneling rate for Kerr-Newman black hole directly from the first law of black hole thermodynamics when the angular momentum is considered as an independent variable, i.e., the angular momentum of unit mass $a$ is not constant in our calculation. Moreover, we have proved that the massive particles will be along the same trajectory as that of massless particles when tunneling across the horizon. Finally, we have showed the massive particles’ tunneling does not violate the weak cosmic censorship, while, in some situations, both a Reissner-Nordström black hole and a Kerr-Newman black hole can evolve into extreme black holes.

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