Orbifolded $SU(7)$ and unification of families

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Abstract

A 5D $SU(7)$ family unification model with two spinor representations of $SO(14)$ is presented. The fifth dimension is compactified on $S^1/Z_2 \times Z'_{2}$. The orbifolding is used to obtain 4D $SO(10)$ chiral fermions. The 4D grand unification group is the flipped $SU(5) \times U(1)$. The doublet-triplet splitting through the missing partner mechanism is achieved. Also, fermion mass matrices are considered.

[Key words: family unification, orbifold, SU(7) grand unification, mass matrix]
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The idea of grand unified theories (GUT’s) is probably the most influential one in particle physics in the last three decades [1]. It was so attractive that some obstacles in simple GUT models are expected to be resolved in a more complete theory. One of the problems is the proton decay problem. In the $SU(5)$ model, the proton lifetime is predicted to be of order $M_{GUT}^4$ in units of GeV. The current experimental upper bound on the partial decay rate into the $e^+\pi^0$ decay mode is $(1.6 \times 10^{33} \text{ yr})^{-1}$, which implies a huge $M_{GUT} > 10^{15}$ GeV. It is consistent with the significant separation of the coupling constants of the strong, weak, and electromagnetic interactions. This was considered as one of the successes of GUT’s. But this huge mass $M_{GUT}$ led to the so-called gauge hierarchy problem, which in turn led to the developments of technicolor, supersymmetry, and superstring in the last two decades. Another problem in this huge $M_{GUT}$ is the doublet-triplet splitting problem in the quintet($5_H$) Higgs that the standard model doublet Higgs boson is light($\sim 100$ GeV) while the accompanying color triplet boson is needed to be supermassive. In most GUT models, one needs a fine-tuning to achieve this doublet-triplet splitting.

Because of the dramatic success of GUT’s in the unification of coupling constants, the flavor problem (or the family problem), which is the most important problem in the standard model, has been expected to be resolved with the GUT idea [2]. Let us call this kind of unification the grand unification of families (GUF). There have been attempts toward flavor unification in larger GUT groups such as $SU(7)$ GUF [3], $SU(8)$ GUF [4], etc., but the predictions given in any of these models have not been confirmed. Therefore, it is fair to say that the GUF attempts along this line has not led to any convincing theory so far. On the other hand, in the heterotic superstring models the representation $248$ of $E_8$ is so large that the known three families are believed to be contained in $248$. Indeed, the superstring compactifications led to phenomenologically interesting multi generation models [5–7]. In particular, the $Z_3$ orbifold compactification has been very attractive since they give the family number as multiples of 3. Also, it has been noted that the doublet-triplet splitting problem is resolved in some orbifold compactifications [8].

The orbifold compactification is one of the efficient and simple way to break down the
huge heterotic string group $E_8 \times E'_8$ [1]. However, the ten dimensional(10D) superstring world is too far separated away from our low energy four dimensional(4D) world. Therefore, the field theoretic orbifold compactification (FTOC) [8] in five dimension(5D) has attracted a great deal of attention recently because of its simplicity, requiring only the field theoretic information. In a sense, the FTOC is a bottom-up approach. In this paper, we consider the FTOC even though a more fundamental theory is based on the string theoretic orbifold compactification (STOC) [6].

The initiation of FTOC started from the observation that the doublet-triplet splitting can be understood by making the color triplet boson superheavy, while the doublet Higgs boson can be made a Kaluza-Klein (KK) zero mode by appropriately choosing the charges of the discrete group in consideration. As noted in STOC, the orbifold is known to have the mechanisms both for the doublet-triplet splitting [7] and for the unification of flavor [6,7]. In this regard, it is not unreasonable to attempt the flavor unification also in FTOC as first tried in [9].

Along this FTOC line, we attempt to understand the flavor problem in a 5D extended GUT, compactified on the orbifold $S^1/Z_2 \times Z'_2$ [10]. The group $SU(6)$ cannot unify the flavor since $\mathbf{15}$ of $SU(6)$ contains only one $\mathbf{10}$ of $SU(5)$. The simplest GUT unifying the flavor is $SU(7)$. The $SU(7)$ model of Ref. [3] contains two standard families and two non-standard families [11] among which one lepton family becomes standard, but the others are unfamiliar ones. Alas, due to the orbifolding in 5D instead of twisting the group, all the unfamiliar families can be made familiar ones which can be removed or kept depending on the $Z'_2$ charge. We note that the $\mathbf{10} \oplus \mathbf{3} \oplus \mathbf{5}$ of $SU(5)$ [3] and $\mathbf{35} \oplus \mathbf{21} \oplus \mathbf{7}$ of $SU(7)$ [4] models are basically the $SO(10)$ and $SO(14)$ models with the spinor representations for fermions, breaking down to $SU(5)$ and $SU(7)$, respectively. Thus, the family unification hints toward the chain $SU(2n + 1)$ or $SO(4n + 2)$. In this paper, we choose the simplest generalization and construct a GUF model in 5D $SU(7)$ gauge group with the spinor representation(s) as the matter assignment. In this paper, $SO(14)$ is considered interchangeably with $SU(7)$ up to a singlet [3].
\[ 64 = \psi^{ABC} + \psi_{AB} + \psi^A + 1 \]  

(1)

where the multi-indices imply the antisymmetric combinations, and \( A = 1, 2, \cdots, 7 \). When we say an \( SU(7) \) spinor, it is meant Eq. (1) without the singlet.

**Orbifold compactification:** In 5D, the fifth dimension \( y = R x_5 \) is compactified on the circle \( S^1: x_5 \equiv x_5 + 2\pi \). Points on \( S_1 \) are identified under the \( Z_2(x_5 \rightarrow -x_5) \) and \( Z'_2(x_5 \rightarrow \pi - x_5) \).

Let any fermion in \( SU(7) \) tensor representation has the following parity symmetry,

\[ Z_2 : \psi^{AB\cdots}(-x_5) = \lambda \psi \gamma_5 P^A P^B \cdots \psi'^{A'B'\cdots}(x_5), \quad P \equiv \text{diag}(I_5, I_2), \]

\[ Z'_2 : \psi^{AB\cdots}(\pi - x_5) = \lambda' \psi \gamma_5 P^A P^B \cdots \psi'^{A'B'\cdots}(x_5), \quad P' \equiv \text{diag}(I_5, -I_2), \]

(2)

(3)

where \( I_n \) is the \( n \) dimensional identity matrix, and \( \lambda \) and \( \lambda' \) are either +1 or −1. Due to the non-commuting boundary conditions given by \( P' \) in the group space, the gauge group breaks down to

\[ SU(7) \longrightarrow SU(5) \times SU(2)_F \times U(1), \]

(4)

where \( SU(2)_F \) plays the role of family symmetry. Because of the \( SU(2)_F \), we expect that light two generations and the third heavy generation are discriminated.

Since we start with a group containing \( SU(5) \), there exists a possibility that \( U(1) \)-electromagnetism contains an \( SU(5) \) singlet piece which is called the flipped \( SU(5) \). The flipped \( SU(5) \) was extensively studied in fermionic construction of 4D string models. The merit of the flipped \( SU(5) \) in string models is that one does not need an adjoint representation of \( SU(5) \) for breaking \( SU(5) \) down to the standard model(SM). The \( \psi^{\alpha\beta}(10) \) has a \( Q_{em} = 0 \) element \( \psi^{6\bar{7}} = \nu^c \) which can have a GUT scale vacuum expectation value(VEV), hence breaks the unified group to the SM. At the same time, this VEV gives a large mass to the color triplet Higgs fields through the missing partner mechanism as discussed below. Note that orbifolding is not needed for the doublet-triplet splitting.

Therefore, let us choose the matter representation and the \( Z'_2 \) parity assignment \( \lambda' \) so that \( SU(5) \times U(1) \) (the flipped \( SU(5) \)) is the GUT group. Under this choice of \( Z'_2 \) eigenvalues, the
resulting zero modes automatically form an anomaly free combination of $SO(10)$ spinors. The 4D chiral anomaly depends not only on the bulk matter but also on the $Z_2'$ parity assignment \([15]\). However, our selection of $Z_2'$ parity will give no anomaly since the zero mode fermions form $SO(10)$ spinors. This property may be understood better if we consider the connection between the two symmetry breaking chains

\[
\begin{array}{ccc}
SO(14) & \overset{\text{SO}(10) \times SU(2)_F \times SU(2)'}{\longleftrightarrow} & SU(7) \times U(1)'
\end{array}
\]

\[
\begin{array}{ccc}
& \overset{SU(5) \times SU(2)_F \times U(1) \times U(1)'}{\rightarrow} & \\
\end{array}
\]

Matter content: A spinor of $SO(14)$ under the breaking chain of Eq.(3) is

\[
\Psi^{ABC} \oplus \Psi_{AB} \oplus \Psi^A \oplus \Psi = 16 \otimes 2_F \oplus \overline{16} \otimes 2',
\]

where the RHS is the decomposition into $SO(10) \times SU(2) \times SU(2)'$ and the anti-symmetrization of the indices are assumed. Since we are dealing with $SO(4n+2)$ groups, the models considered do not have the anomaly problem.

A 5D $SO(14)$ spinor has four left-handed and four right-handed 4D $SO(10)$ spinors. Under the torus compacification, these eight $SO(10)$ spinors form four massive Dirac spinors and are removed from the low energy spectrum. But twisting can allow some zero modes. Let the $Z_2$ action in Eq.(2) makes the right-handed component of a 5D spinor heavy (breaking one supersymmetry if there was). In other words, only 4 left-handed $SO(10)$ spinors(one left-handed $SU(7)$ spinor) in 4D remain as zero modes. It is represented under $SU(5) \times SU(2) \times U(1)$ as:

\[
\begin{alignat}{2}
\Psi^{ABC} &= \psi^{\alpha\beta\gamma} \ (10,1)_6 \oplus \psi^{\alpha\beta i} \ (10,2)_{-1} \oplus \psi^{\alpha i j} \ (5,1)_{-8} \\
\Psi_{AB} &= \psi_{\alpha\beta} \ (10,1)_{-4} \oplus \psi_{\alpha i} \ (5,2)_3 \oplus \psi_{ij} \ (1,1)_{10} \\
\Psi^A &= \psi^\alpha \ (5,1)_2 \oplus \psi^i \ (1,2)_{-5}
\end{alignat}
\]

where the total number of $10$ and $\overline{10}$ is four which is the number of massless $SO(10)$ spinor zero modes. Here, the upper case Roman letters $A,B,C,\cdots$ are the $SU(7)$ indices$(1,2,\cdots,7)$, the lower case Greek letters $\alpha,\beta,\gamma,\cdots$ are the $SU(5)$ indices$(3,4,\cdots,7)$,
and the lower case Roman letters $i, j$ are the $SU(2)_F$ indices 1, 2. We can assign $\lambda' = -1$ to the $Z'_2$ parity of the whole $SU(7)$ spinor $(\Psi^{ABC}, \Psi_{AB}, \Psi^A)$, leaving the following zero modes

$$(10, 2)_{-1}, \quad (\overline{5}, 2)_3, \quad (1, 2)_{-5},$$

which is exactly the anomaly free combination of the flipped $SU(5)$ model \[12\]. Thus, this consistent choice of $Z'_2$ parity picks up one irreducible representation of $16 \otimes 2$ of $SO(10) \times SU(2)$ in 4D among the full spinor of $SO(14)$ shown in Eq. (3). The reason for this consistent selection is in that a spinor of $SO(4n + 2)$ can be decomposed into the sum of alternating totally antisymmetric tensors of $SU(2n + 1)$ as shown in Eq. (6) \[16\].

The 5D $SU(7)$ model presented above has two families, neatly unified in a doublet of $SU(2)_F$ in Eq. (8). We need to introduce the third family. A simple choice is that the third family is a singlet under $SU(2)_F$. We can put this $SU(2)_F$ singlet, $(10, 1)_{-1} \oplus (\overline{5}, 1)_3 \oplus (1, 1)_{-5}$ under $SU(5) \times U(1)$, at the asymmetric fixed point. Then we need to put Higgs fields with the gauge charges $10_{-1}, \quad \overline{10}_1, \quad 5_2, \quad \overline{5}_{-2}$ at the asymmetric fixed brain also. $10$ and $\overline{10}$ are required to break $SU(5) \times U(1) \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$. $5$ and $\overline{5}$ contain the doublet Higgs for the $SU(2)_L \times U(1)_Y$ breaking into $U(1)_{em}$.

In the remainder of this paper, however, we study a more interesting case that the third family is also a member of an $SU(2)_F$ doublet. In addition, let us extend to the supersymmetric case so that the discussion on the Higgs multiplets is neat. Put the same $SU(7)$ combination of Eq. (8) in the bulk again, from which we obtain the additional zero modes given in Eq. (8). Below the $SU(2)_F$ breaking scale, one set of the $SU(2)_F$ doublet becomes the third family fermions. The superpartners of the remaining $SU(2)_F$ doublet can be Higgs multiplets: $H(10_{-1}), \overline{h}(\overline{5}_3), \phi(1_{-5})$. However, $\overline{h}(\overline{5}_3)$ in the flipped $SU(5)$ does not have a color triplet with $Q_{em} = -1/3$; hence the $\overline{5}$ component of $H(10_{-1})$ with $Q_{em} = +1/3$ does not have a partner in $\overline{h}(\overline{5}_3)$, and the doublet-triplet problem is not solved. To solve this doublet-triplet splitting problem, we introduce $\overline{5}_2$ and $\overline{5}_{-2}$ which have color triplets with the needed electric charge. These may come from $7 \oplus \overline{7}$ of $SU(7)$, or $14$ of $SO(14)$.

**Missing partner mechanism:** We introduced two $SU(2)_F$–doublet spinors of $SU(7)$. For the
Higgs fields, let us introduce $\mathbf{5}_2$ and $\overline{\mathbf{5}}_{-2}$ in the bulk, and in addition $\{10_1 \oplus \overline{5}_{-3} \oplus \mathbf{1}_5\}$ at the asymmetric fixed point, which are $SU(2)_F$–singlets. Toward a detailed discussion on the mass matrices of light fermions and the doublet-triplet splitting mechanism, let us name two $SU(2)_F$–doublets of $SO(14)$ spinor as

$$T_i(\mathbf{10}_{-1}), \overline{F}_i(\mathbf{5}_3), E_i^c(\mathbf{1}_{-5}), \quad \text{and} \quad T'_i(\mathbf{10}_{-1}), \overline{F}'_i(\overline{\mathbf{5}}_3), E_i'^c(\mathbf{1}_{-5}),$$

where the family indices $i = 1, 2$ and $SU(2)_F$–singlets as

$$H(\mathbf{10}_1), h'(\mathbf{5}_{-3}), \phi(\mathbf{1}_5), \quad \text{and} \quad h(\mathbf{5}_2), \overline{h}(\overline{\mathbf{5}}_{-2}),$$

and the components of each multiplet as

$$\mathbf{10}_{-1} : \begin{pmatrix} d^c \\ q \\ \nu^c \end{pmatrix}, \quad \mathbf{5}_3 : \begin{pmatrix} u^c \\ \ell \end{pmatrix}, \quad \overline{\mathbf{5}}_{-2} : \begin{pmatrix} \overline{D} \\ h^+ \end{pmatrix}, \quad \mathbf{5}_{+2} : \begin{pmatrix} D \\ h^- \end{pmatrix}$$

where $\overline{D}$ and $h^+$ carries the hypercharge $1/3$ and $1/2$, respectively.

In order to break the unified gauge group, we need two additional $SU(2)_F$–doublet fields $\{\chi_1^i, \chi_2^i\} = 2(1, 2)_0$ at the asymmetric fixed point. The superpotential relevant to the GUT symmetry breaking and the masses of the third generation fermions, written in the asymmetric fixed point, are given by

$$W_H = \overline{H}Hh + T'T'h + T'\overline{F}'h + \overline{F}'\overline{E}'h + h'\chi^2 + E'\phi \chi^1$$

This superpotential contains the most general cubic terms of the singlet fields in Eq.(10) and the primed doublet fields in Eq.(9) consistent with the following two discrete symmetries

$$Z_2^\chi : \chi^1 \rightarrow -\chi^1, \quad \phi \rightarrow -\phi, \quad Z_2^H : \overline{H} \rightarrow -\overline{H}$$

while the other fields are invariant under $Z_2^{\overline{\chi}}$ and $Z_2^H$. We do not allow $h\overline{H}$ term in the superpotential, which is anticipated in the superstring models. By the development of VEV along the $D$-flat (and $F$-flat) direction $(T'\overline{H} \chi^1)(\chi^1 \chi^2),$

$$\langle \nu_{T'_1}^c \rangle = \langle \overline{\mathbf{5}}_3 \rangle = \frac{1}{\sqrt{2}} \langle \chi_2^1 \rangle = \langle \chi_1^2 \rangle = M_G,$$
both $q_{T_1}$ and $q_{\overline{T}}$ are either eaten by the heavy gauge bosons or made heavy by the supersymmetric Higgs mechanism. From the superpotential terms in Eq. (12) the components $d_{T_2}^c, D_h, \overline{D}_{\overline{h}}, \overline{F}_{12}, E_{1}^c$ and $h'$ become massive after the symmetry breaking, while $h^+$ and $h^-$ remain massless and fulfill the doublet-triplet splitting. The rest massless components \{d_{T_1}^c, q_{T_2}^c, u_{F_1}^c, \ell_{F_1}^c, E_{2}^c\} form the third generation family.

**Mass matrices:** In order to reproduce the realistic fermion masses and mixing angles, we need an additional global symmetry which prevents the light generation doublets $T, \overline{T}, E^c$ from acquiring the same large mass as the third generation ones $T', \overline{T}', E'^c$. Here, as a simplest option available, we just try an anomalous global $U(1)_F$ symmetry. Like the models with $U(2)_F$ family symmetry in the literature [17], if we break the $SU(2)_F \times U(1)_F$ in two steps

$$SU(2)_F \times U(1)_F \xrightarrow{\epsilon} U(1)_{\epsilon}' \xrightarrow{\epsilon'} \{\epsilon\}$$

where $\epsilon \sim 0.02$ and $\epsilon' \sim 0.004$ in units of a UV cutoff scale are the order parameters for each step, we can suppress light generation masses by small parameters $\epsilon$ and $\epsilon'$. For a model construction, let us assign $U(1)_F$ charge +1 to unprimed $SU(2)_F$–doublet fields, and 0 to the other fields. In addition, let us introduce an $SU(2)_F$ singlet $\phi(-1)$ and triplets $S_{12}^{1,2}(-2)(ij$ symmetric) with the $U(1)_F$ charges indicated inside the parenthesis. The relevant superpotential terms are given by,

$$W_Y = \sum_{a=1,2} \frac{1}{M_*} \left[ S^a T T h + \left( \frac{\phi^2}{M_*} + S^a \right) T \overline{T} h + \left( \frac{\phi^2}{M_*} + S^a \right) \overline{F} E^c h \right]$$

$$+ \frac{\phi}{M_*} \left[ T T' h + (T \overline{F}' + T' \overline{F}) h + (\overline{F} E^c + \overline{F}' E'^c) h \right]$$

where $M_*$ is the UV cutoff scale. Requiring the VEVs of the ‘flavon’ fields $\phi, S_{ij}^{1,2}$ to be

$$\langle \phi \rangle \sim \epsilon M_*, \quad \langle S_{12}^{12} \rangle \sim \epsilon M_*, \quad \langle S_{12}^{21} \rangle \sim \epsilon' M_* , \quad \langle S_{22}^{11} \rangle \sim \epsilon' M_* ,$$

the mass matrices look like

$$\frac{M_{u,d}}{M_{33}^a} \approx \begin{pmatrix} 0 & \epsilon' & 0 \\ \epsilon' & \epsilon & \epsilon \end{pmatrix}, \quad \frac{M_{e}}{M_{33}^e} \approx \begin{pmatrix} 0 & \epsilon' & \epsilon \\ \epsilon' & \epsilon & 0 \\ 0 & \epsilon & 1 \end{pmatrix} .$$
This form of mass matrices gives the qualitatively correct mass spectrum and CKM mixing matrix elements. If we let the two symmetry breaking steps in Eq.(13) occur with a single triplet $S_{ij}$ instead of two different triplets $S_{ij}^{1,2}$, the $SU(2)_F$ symmetry would enforce the unrealistic relation $m_u/m_c = m_d/m_s = m_e/m_\mu$ precisely, as long as the mixing between light two generations and the third generation remains small. In our model, however, the discrepancy between $m_u/m_c, m_d/m_s$ and $m_e/m_\mu$ as well as $m_c/m_t, m_s/m_b$ and $m_\mu/m_\tau$ can be accounted for by the numerical coefficients of tolerable size, since the up-type quark, down-type quark and lepton masses come from different superpotential terms.

In this paper, we constructed a 5D $SU(7)$(or $SO(14)$) GUF model with two spinors of $SO(14)$, with the orbifold compactification $S_1/Z_2 \times Z_2'$, which realizes the three families of fermions in the flipped $SU(5)$ and the doublet-triplet splitting of Higgs multiplet. We introduced $\bar{5}_{+2}$ and $5_{-2}$, an $SO(10)$ vector arising from the $SO(14)$ vector $14$. There may be a deep reason for the two 5D $SO(14)$ spinors. In the $E_8 \times E_8'$ heterotic string model, the adjoint or the fundamental representation of $E_8$, $248$, contains $128 \oplus 120$ of $SO(16)$, one of the maximal subgroup of $E_8$. The $SO(16)$ spinor $128$ decomposes to two $SO(14)$ spinors: $64 + \bar{64}$. For $\bar{64}(64)$, we pick up the right-handed(left-handed) components and hence assign $-(+ \text{ as before})$ for the $Z_2$ quantum number $\lambda$ so that the massless modes are the left-handed fields with the combination given in (3). This may be the reason that nature chooses two $SO(14)$ spinors. Also, the anomalous $U(1)_X$ we introduced to discriminate $\bar{64}$ from $64$ could come from $E_8/SO(16)$, which assign chiral charge to $SO(14)$ spinors. On the other hand, $120$ of $SO(16)$ breaks down to $91 \oplus$ two $14$’s $\oplus$ 1 and the needed $SO(14)$ vector $14$ can be assigned to $120$ of $SO(16)$.

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