A full electromagnetic simulation study of near-field imaging using silver films

Shi-hong Jiang\textsuperscript{1} and Roy Pike\textsuperscript{2}

\textsuperscript{1} School of Electrical and Electronic Engineering, University of Nottingham, Nottingham NG7 2RD, UK
\textsuperscript{2} Department of Physics, King’s College, Strand, London WC2R 2LS, UK
E-mail: eexshj@nottingham.ac.uk and roy.pike@kcl.ac.uk

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Abstract. A full electromagnetic calculation is performed to investigate two-dimensional near-field imaging using a ‘Pendry’ lens. We demonstrate that in spite of a positive magnetic permeability satisfactory resolution is possible for a thin silver slab, or a multilayer structure, at certain optical frequencies. These analytical results are confirmed by using a novel finite-element numerical simulation, which makes use of linear superpositions of one-dimensional field components. The simulation displays clearly the excited surface plasmons. The results in the electrostatic limit are also given for comparison purposes.

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1. Introduction

Surface plasmons can be excited in an electrically conducting material when electromagnetic radiation of a specific frequency impinges upon it. The material will then behave as a plasma and its dielectric permittivity can be negative. If both the dielectric permittivity $\varepsilon_r$ and the magnetic permeability $\mu_r$ are negative, the result is that the refractive index of the medium $n = \sqrt{\varepsilon_r} \sqrt{\mu_r}$ will be negative. A parallel-sided thin slab with a refractive index of $-1$, proposed by Pendry as a perfect lens [1], would bend light to a negative angle with the surface normal and bring the transmitted light to a perfect focus. Unfortunately, it seems unlikely that negative-permeability media at optical frequencies will be available in the near future, although new techniques for creating ‘metamaterials’ are being developed [2]–[6].

One of the potential applications of this perfect-lens idea is in near-field imaging and Pendry’s calculation showed that a 40 nm thick silver slab operating at an optical frequency of $\omega = 3.48$ eV (or a wavelength of 356 nm), with $\varepsilon_r = -1 - 0.4i$, could resolve two slits spaced a few tens of nanometres apart. This assumed the electrostatic approximation, i.e., the wavelength of the incident light is assumed to be infinite, so that the P-polarization (TM mode) transmission coefficient of the slab becomes independent of its magnetic permeability. Recently, optical imaging using a planar silver lens in a near-field lithography arrangement has been reported. Sub-wavelength resolution down to 145 nm has been achieved by Melville et al through a 25/50/10 PMMA/Ag/SiO$_2$ lens stack [7]–[9]. The authors pointed out that higher resolution may be achieved by using thinner silver layers, which are difficult to deposit with low levels of surface roughness. Fang et al report on an experiment with a 35 nm silver superlens that allows 60 nm half-pitch resolution to be reached [10].

Here, we first address the question of whether 40 nm is sufficiently small, compared with an optical wavelength, to approximate this electrostatic limit. If this is the case, then what is the optimal resolution that can be achieved with tolerable lens geometry? We have already pointed out [11] that, for the TE mode, improved resolution cannot be achieved at the frequency of visible light unless the slab thickness is reduced to 20 nm or so. At this point, however, the improvement over the diffraction-limited image is not great, as may be seen later in figure 6. The situation is quite different for the TM mode. As will be demonstrated in section 5 and illustrated in figure 5, with a total object-to-image distance of 40 nm, an inserted 20 nm-thick silver slab can resolve well two slits with width of 20 nm and separation of 60 nm, while the diffraction losses are still large. Moreover, a number of such thin slabs can make a multilayer lens, which relays the image over longer distances with improved resolution over that of free-space diffraction [12]. Gain media can also be considered as the separating layers of dielectric [13]. We have investigated such systems with a new finite-element numerical simulation, which calculates the propagating electromagnetic field using linear superpositions of one-dimensional field components. We will distinguish between calculated performances on the basis of a given working distance and performances in which the total distance the image is relayed from the object may be of interest. If total distance is not of relevance then we shall see that multilayer systems, either with or without gain, do not have a great deal to offer over the performance of a single-thin slab.

We have restricted our discussions to the case of a silver slab at its nominal plasmon resonance frequency. Although calculations around this frequency would be straightforward, we have not investigated sensitivity to wavelength, since the resonance is sufficiently broad (a few per cent) for this not to be a problem in practice. Similarly, considerations of finite detector
In fact the results, as we have stated, are independent of the position of the slab between the object and the image planes. Other small effects would require routine perturbation calculations for any particular implementation and are not felt to justify additional space here.

2. Reflection and transmission coefficients

Consider a plane wave of frequency $\omega$ incident at an angle $\theta_0$ on a slab of thickness $h$ as illustrated in figure 1, where the $y$-axis is out of the page. The size of slab in the $x$-direction is assumed to extend to infinity. We denote by $E_{inc}^0$, $E_r^0$, $E_t^1$, $E_r^1$ and $E_t^2$ the incident field, the reflected field at the front surface, the transmitted field through the front surface, the reflected field at the back surface and the transmitted field through the back surface, respectively. They can be expressed as

$$E_{inc}^0 = a_0 \exp[-ik_0(z \cos \theta_0 + x \sin \theta_0)] \exp(i\omega t), \quad (2.1)$$

$$E_r^0 = b_0 \exp[ik_0(z \cos \theta_0 - x \sin \theta_0)] \exp(i\omega t), \quad (2.2)$$

$$E_t^1 = a_1 \exp[-ik_1(z \cos \theta_1 + x \sin \theta_1)] \exp(i\omega t), \quad (2.3)$$

$$E_r^1 = b_1 \exp[ik_1(z \cos \theta_1 - x \sin \theta_1)] \exp(i\omega t), \quad (2.4)$$

$$E_t^2 = a_2 \exp[-ik_0(z \cos \theta_0 + x \sin \theta_0)] \exp(i\omega t), \quad (2.5)$$

where $\omega$ is the light frequency, $k_0$ is the wave propagation constant in free space, $k_1 = k_0 \sqrt{\varepsilon_r \mu_r}$, $a_0$, $b_0$, $a_1$, $b_1$ and $a_2$ are unknown coefficients. For the case of the TE mode, the total electric field in the object space will be the sum of the incident field with the reflected field from the front surface

$$E_0 = E_{inc}^0 + E_r^0. \quad (2.6)$$

Similarly, the electric field inside the slab is

$$E_1 = E_t^1 + E_r^1 \quad (2.7)$$
The reflection and transmission coefficients for the TM mode incident wave are

\[ R_\parallel = \frac{b_0}{a_0} = \frac{N_0(N_1 + N_0 \tanh u_1 h) - N_1(N_0 + N_1 \tanh u_1 h)}{N_0(N_1 + N_0 \tanh u_1 h) + N_1(N_0 + N_1 \tanh u_1 h)} e^{-2i\nu h \cos \theta_0}, \quad (2.9) \]

\[ T_\parallel = \frac{a_0}{a_0} = \frac{2N_0}{N_1 + N_0} \frac{2N_1}{N_1 + N_0} \frac{\cos \theta_0}{\sin \theta_0} e^{-2i\nu h \cos \theta_0}, \quad (2.10) \]

where \( N_0 = N_2 = \cos \theta_0 / \eta_0, N_1 = \sqrt{\varepsilon_r / \mu_r} \cos \theta_1 / \eta_0, u_1 = i k_0 \sqrt{\mu_r \varepsilon_r} \cos \theta_1, \sin \theta_1 = \sin \theta_0 / \sqrt{\mu_r \varepsilon_r}. \) By definition, \( \eta_0 = \sqrt{\mu_0 / \varepsilon_0} \approx 120 \pi \Omega \) is the characteristic impedance of the medium. The reflection and transmission coefficients for the TM mode incident wave are

\[ R_\perp = \frac{K_0(K_1 + K_2 \tanh u_1 h) - K_1(K_2 + K_1 \tanh u_1 h)}{K_0(K_1 + K_2 \tanh u_1 h) + K_1(K_2 + K_1 \tanh u_1 h)} e^{-2i\nu h \cos \theta_0}, \quad (2.11) \]

\[ T_\perp = \frac{2K_0}{K_1 + K_0} \frac{2K_1}{K_1 + K_0} \frac{\cos \theta_0}{\sin \theta_0} e^{-2i\nu h \cos \theta_0}, \quad (2.12) \]

where \( K_0 = K_2 = \eta_0 \cos \theta_0 \) and \( K_1 = \sqrt{\mu_r \varepsilon_r} \eta_0 \cos \theta_1. \) In the case of normal incidence, we have \( R_\parallel = -R_\perp \) and \( T_\parallel = T_\perp. \)

3. Analytical simulation

The refractive index of a material depends also on the polarization of that material by the electric field of the transmitted radiation. For limited values of the field strength, the polarization will be linear. The radiation from the object can be regarded as a linear superposition of many individual plane waves with different amplitudes and angles of incidence; hence the field generated by the object in front of the slab (not including the reflected field) can be expressed as

\[ E_{obj}(x, z) = \int_{-\infty}^{\infty} u(\sin \theta_0) E_{inc}^{\sin \theta_0} d(\sin \theta_0), \quad 0 \leq z \leq z_1, \quad (3.1) \]

where the function \( u(\sin \theta_0) \) is the Fourier transform of the object. In near-field imaging, both propagating and evanescent components must be considered. The evanescent wave is a wave that propagates parallel to the slab and behaves exponentially along the orthogonal direction. This requires that the integrand of equation (3.1) represent either a propagation wave, when \( |\sin \theta_0| < 1 \) or an evanescent wave, when \( |\sin \theta_0| > 1. \) By letting \( \cos \theta_0 = -i \sqrt{\sin^2 \theta_0 - 1}, \) equation (3.1) becomes

\[ E_{obj}(x, z) = \int_{-\infty}^{\infty} u(s) a_0 \exp \left(-k_0 z \sqrt{s^2 - 1}\right) \exp(-ik_0 x s) ds, \quad 0 \leq z \leq z_1, \quad (3.2) \]
where $s = \sin \theta_0$. It may be seen that both propagating and evanescent components have been included in equation (3.2).

Analogously, the field behind the slab can be written as

$$E_{im}(x, z) = \int_{-\infty}^{\infty} u(s) a_0 T_\perp \exp \left(-k_0 z \sqrt{s^2 - 1}\right) \exp(-i k_0 x s) ds, \quad z \geq z_2. \quad (3.5)$$

Taking the square of the amplitude of equation (3.5) gives the intensity distribution:

$$I_{im}(x, z) = E_{im}(x, z) E_{im}(x, z)^*. \quad (3.6)$$

From equations (2.9) to (2.12), it is interesting to see that if $\mu_r = \varepsilon_r = -1$, then $N_0 = N_1 = N_2$, $K_0 = K_1 = K_2$, $\sin \theta_1 = -\sin \theta_0$. As a result,

$$R_\perp = R_\parallel = 0, \quad (3.7)$$

$$|T_\perp| = |T_\parallel| = \begin{cases} 1, & |\sin \theta_0| \leq 1, \\ \exp \left(2k_0 h \sqrt{\sin^2 \theta_0 - 1}\right), & |\sin \theta_0| > 1, \end{cases} \quad (3.8, 3.9)$$

the slab becomes transparent for propagating orders and has an amplifying effect on evanescent waves. Inserting equation (3.8) into (3.5), we get

$$E_{im}(x, z) = \int_{-\infty}^{\infty} u(s) a_0 \exp \left[k_0 (2h - z) \sqrt{s^2 - 1}\right] \exp(-i k_0 x s) ds, \quad z \geq z_2. \quad (3.10)$$

If we choose $z = 2h$ in equation (3.10), the decay of the evanescent wave will be completely cancelled by the slab. The field in the image plane $z = 2h$ will be exactly the same as that in the object plane $z = 0$. This is actually irrespective of where the slab is placed between the object and the image.

For validation purposes, we consider a 40 nm slab illuminated by a point source at a wavelength of 356 nm. Since the Fourier transform of the point source is a constant, the field behind the slab is then represented in terms of the integral:

$$E_{im}(x, z) = \int_{-\infty}^{\infty} u(s) a_0 \exp \left[k_0 (2h - z) \sqrt{s^2 - 1}\right] \exp(-i k_0 x s) ds, \quad z \geq z_2. \quad (3.11)$$

This integral can be evaluated numerically with the integration interval being truncated. The intensity distribution calculated in the region $-100 \text{ nm} \leq x \leq 100 \text{ nm}$ and $60 \text{ nm} \leq z \leq 80 \text{ nm}$ with truncated interval $[-5, +5]$, is shown in figure 2. The huge local fields at the rear surface of the slab represent the excited surface plasmons. A comparison of the intensity distribution calculated with different integration intervals is shown in figure 3. The central peak becomes
Figure 2. Intensity distribution at the back of the 40 nm slab calculated with the truncated integration interval $[-5, +5]$.

Figure 3. Comparison of intensity distribution on the plane at $z = 80$ nm calculated with different truncated integration intervals: 1, $[-5, +5]$; 2, $[-10, +10]$; and 3, $[-20, +20]$. 
narrower with the expansion of the integration interval, implying that a Dirac function should appear on the plane at \( z = 80 \text{ nm} \) as the limit of integration approaches infinity.

4. Electrostatic approximation

By using the relations \( k_0 = \omega/c \) and \( c = 1/\sqrt{\varepsilon_0\mu_0} \), where \( c \) is the velocity of light in free space, \( N_0, N_1, N_2, K_0, K_1 \) and \( K_2 \) in section 2 can also be written as

\[
N_0 = N_2 = \frac{k_z}{\omega\mu_0}, \tag{4.1}
\]

\[
N_1 = \frac{k'_z}{\omega\mu_r\mu_0}, \tag{4.2}
\]

\[
K_0 = K_2 = \frac{k_z}{\omega\varepsilon_0}, \tag{4.3}
\]

\[
K_1 = \frac{k'_z}{\omega\varepsilon_r\varepsilon_0}, \tag{4.4}
\]

where \( k_z = k_0 \cos \theta_0, k'_z = k_1 \cos \theta_1 \). In the electrostatic limit, the wavelength \( \lambda \rightarrow \infty \) or \( \omega \rightarrow 0 \) and it follows that

\[
k_z = -i\sqrt{k_x^2 + k_y^2 - \omega^2c^2} \approx -i\sqrt{k_x^2 + k_y^2}, \tag{4.5}
\]

\[
k'_z = -i\sqrt{k_x^2 + k_y^2 - \varepsilon_r\mu_r\omega^2c^2} \approx -i\sqrt{k_x^2 + k_y^2}. \tag{4.6}
\]

Substituting (4.1) and (4.2) into (2.10), (4.3) and (4.4) into (2.12) respectively and recognizing that \( k_z = k'_z \), we get

\[
T_\perp = \frac{4\mu_r}{(1 + \mu_r)^2 - (1 - \mu_r)^2 \exp(-2i\omega_k z)}, \tag{4.7}
\]

\[
T_\parallel = \frac{4\varepsilon_r}{(1 + \varepsilon_r)^2 - (1 - \varepsilon_r)^2 \exp(-2i\omega_k z)}. \tag{4.8}
\]

The transmission coefficient for the S-polarized wave depends only on \( \mu_r \) and that of the P-polarized wave only on \( \varepsilon_r \). In the two-dimensional case, \( k_y = 0, k_z = -ik_x \), hence

\[
T_\perp(k_x) = \frac{4\mu_r}{(1 + \mu_r)^2 - (1 - \mu_r)^2 \exp(-2i\omega_k x)}, \tag{4.9}
\]

\[
T_\parallel(k_x) = \frac{4\varepsilon_r}{(1 + \varepsilon_r)^2 - (1 - \varepsilon_r)^2 \exp(-2i\omega_k x)}. \tag{4.10}
\]
Figure 4. Comparison of images for 40 nm single silver slab, TM mode, $\omega = 3.48$ eV, full electromagnetic simulation (solid line), electrostatic approximation (dashed line), FEM (crosses): 1, image at $z = 80$ nm with silver slab; and 2, diffracted image at $z = 80$ nm.

According to the principle of linear superpositions of field components, the total field for the TE mode and the total field for the TM mode behind the slab may be expressed as

$$E_{im}(x, z) = \sum_{k_x} u(k_x) T_{\perp}(k_x) \exp(-k_x z) \exp(-i k_x x), \quad z \geq z_2. \tag{4.11}$$

$$H_{im}(x, z) = \sum_{k_x} u(k_x) T_{\parallel}(k_x) \exp(-k_x z) \exp(-i k_x x), \quad z \geq z_2. \tag{4.12}$$

Note that we use summation here instead of integration, as Pendry did. If $\mu_r = 1, \varepsilon_r = -1$, then $T_{\perp} = 1, T_{\parallel} = \exp(2k_x h)$, equations (4.11) and (4.12) become

$$E_{im}(x, z) = \sum_{k_x} u(k_x) \exp(-k_x z) \exp(-i k_x x), \quad z \geq z_2. \tag{4.13}$$

$$H_{im}(x, z) = \sum_{k_x} u(k_x) \exp[k_x(2h - z)] \exp(-i k_x x), \quad z \geq z_2. \tag{4.14}$$
The decay of the evanescent wave for the TE mode will not be cancelled, but for the TM mode the decay will still be completely cancelled at \( z = 2h \), even if \( \mu \neq -1 \). This is in contrast to the full electromagnetic calculation, where only when both \( \varepsilon_r \) and \( \mu_r \) are equal to \(-1\) can the decay for both modes be cancelled.

We have repeated Pendry’s calculation for a silver slab of thickness 40 nm using equation (4.12). The slab’s dielectric permittivity \( \varepsilon_r = 5.7 - 9.0^2/\omega^2 - 0.4i \) gives \( \varepsilon_r = -1 - 0.4i \) at \( \omega = 3.48 \text{ eV} \), the magnetic permeability \( \mu_r = 1 \). The object consists of two slits of width \( a = 20 \text{ nm} \) and separation \( d = 100 \text{ nm} \). Its Fourier components are given by

\[
\phi(k_x) = \frac{\sin(ak_x/2)}{ak_x/2} \cos(dk_x/2).
\]  

The results for the TM mode are shown in figure 4, where the images at \( z = 80 \text{ nm} \) with and without the silver slab in place are given. The frequency step was set to 0.01 with a bandwidth of \( k_x = \pm 20 \).

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Figure 6. Images for 20 nm single silver slab, TE mode, $\omega = 3.48$ eV, full electromagnetic simulation (solid line), electrostatic approximation (dashed line), FEM (crosses): 1, image at $z = 40$ nm with silver slab; and 2, diffracted image at $z = 40$ nm.

5. Finite-element numerical simulation

Obviously, the total electric field as given in equations (2.6)–(2.8) satisfies the two-dimensional scalar Helmholtz equation

$$
\left[ \frac{\partial}{\partial x} \left( \frac{1}{\mu_r} \frac{\partial}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{1}{\mu_r} \frac{\partial}{\partial z} \right) + k_0^2 \varepsilon_r \right] E = 0,
$$

(5.1)

where $\varepsilon_r$ and $\mu_r$ are functions of position. For an oblique incident plane wave, using equations (2.1)–(2.5), (5.1) reduces to [16]

$$
\frac{d}{dz} \left( \frac{1}{\mu_r} \frac{dE}{dz} \right) + k_0^2 \left( \varepsilon_r - \frac{1}{\mu_r} \sin^2 \theta_0 \right) E = 0.
$$

(5.2)

This is a one-dimensional problem and can be solved numerically over the calculation region $0 \leq z \leq 2h$ by using the finite-element method [17]. The benefit of using (5.2) is that the boundary
conditions along the $x$-axis are not needed, but it is still capable of solving two-dimensional problems. Taking the derivative of $E$ in equation (2.6) with respect to $z$, we obtain

$$\frac{dE_0}{dz} = -ik_0 \cos \theta_0 E_0^{inc} + ik_0 \cos \theta_0 E_0.$$  \hfill (5.3)

From (5.3) and (2.6) at $z = 0$, we have

$$\frac{dE_0}{dz} - ik_0 \cos \theta_0 E_0 = -2ik_0 \cos \theta_0 a_0 \exp(-ik_0 x \sin \theta_0) \exp(i \omega t).$$  \hfill (5.4)

This can be used as the boundary condition on the left side of the slab. Similarly, we obtain the boundary condition on the right side of the slab at $z = 2h$ as

$$\frac{dE_2}{dz} + ik_0 \cos \theta_0 E_2 = 0.$$  \hfill (5.5)

**Figure 7.** Intensity distribution over the calculation region using FEM. Single 20 nm silver slab, TM mode, $\omega = 3.48$ eV: (a) intensity plot and (b) mesh plot.
Figure 8. Intensity distribution over the calculation region using FEM. Two-layer, silver–air system, TM mode, \( \omega = 3.48 \) eV: (a) intensity plot and (b) mesh plot.

For each incident plane wave with given amplitude \( a_0 \) and angle of incidence \( \theta_0 \), we can evaluate the field at any node of the numerical calculation using equation (5.2) together with the boundary conditions (5.4) and (5.5).

For the case of the TM mode, equation (5.2) should be changed to

\[
\frac{d}{dz} \left( \frac{1}{\varepsilon_r} \frac{dH}{dz} \right) + k_0^2 \left( \mu_r - \frac{1}{\varepsilon_r} \sin^2 \theta_0 \right) H = 0, \tag{5.6}
\]

with similar boundary conditions.

Here, we take the TE case to demonstrate the effectiveness of this method for simulation of negative-refraction imaging. We assume that the wavelength of the individual incident plane wave is 356 nm and the slab is 40 nm thick with both \( \varepsilon_r \) and \( \mu_r \) equal to \(-1\). The calculation region is divided into 20 layers each of 4 nm thickness and there are 21 node values along the \( z \)-direction, where the fields are to be determined. The number of sample points in the \( x \)-direction
is 50, spanning $-100\text{ nm}$ to $100\text{ nm}$. By choosing different values of the time factor contained in the boundary condition on the left side, we obtained animated pictures of the propagating wave and the evanescent wave. **Animation 1** shows a plane wave with $\sin \theta_0 = 0.1$ propagating through the slab. It is interesting to see that at the slab surfaces (red dashed lines) the angle of refraction of the light is equal to the angle of incidence, but in the opposite direction. In addition, the light wave travels backwards inside the slab, which is called ‘backward wave’ [18], as in the negatively refractive materials the wavevector $k_1$ is antiparallel to the Pointing vector [19]. **Animation 2** shows an incident wave with $\sin \theta_0 = 5$, which gives rise to an evanescent wave. Just as expected, the wave propagates parallel to the slab and penetrates through the rear surface since its amplitude is enhanced exponentially along the orthogonal direction.

As mentioned in section 3, the radiation from an object can be regarded as a linear superposition of many individual plane waves with different amplitudes and angles of incidence; the total fields defined in equations (2.6)–(2.8) at a nodal point can thus be obtained by summing all the field values of the corresponding individual plane waves at that point.

**Figure 9.** Intensity distribution over the calculation region using FEM. Four-layer, silver–air system, TM mode, $\omega = 3.48 \text{ eV}$: (a) intensity plot and (b) mesh plot.
Figure 10. Intensity distribution over the calculation region using FEM. Two-layer, silver–gain system, TM mode, $\omega = 3.48$ eV: (a) intensity plot and (b) mesh plot.

It is worth noting that the reflection and transmission coefficients can be extracted from the node values as

$$R_\perp = \frac{E(0) - a_0 \exp(-ik_0x \sin \theta_0) \exp(i\omega t)}{a_0 \exp(i k_0x \sin \theta_0) \exp(i\omega t)}, \quad (5.7)$$

$$T_\perp = \frac{E(2h)}{a_0 \exp(-ik_0(2h \cos \theta_0 + x \sin \theta_0)) \exp(i\omega t)} \quad (5.8)$$

Our calculations show that the results from formulae (5.7) and (5.8) are exactly the same as those from the formulae given in section 2.

6. Results and discussion

6.1. Single slab

We performed a full electromagnetic calculation for near-field imaging using the same 40 nm silver slab and the same two-slit object as mentioned in section 4 but with $k_z$ in (4.15) changed...
Figure 11. Intensity distribution over the calculation region using FEM. Four-layer, silver–gain system, TM mode, $\omega = 3.48$ eV: (a) intensity plot and (b) mesh plot.

to $k_0 \sin \theta_0$. The integral used is for the TM case, which has the same form as equation (3.5) with an integration interval of $\sin \theta_0 = \pm 20$. The results are shown in figure 4. Compared with the results from the electrostatic approximation, it may be seen that to resolve two slits with a separation of 100 nm, the actual thickness of the silver slab needs to be just less than 40 nm. A two-slit object with the same width but reduced separation of 60 nm was then used to investigate the performance of the silver lens. The object cannot be resolved for the TM mode either in the electrostatic limit or via the full electromagnetic simulation as shown in curve 1 of figure 12. However, it can be resolved for the TM mode, if the slab thickness is reduced to 20 nm, as shown in figure 5, at which total distance the diffraction losses are still great. This is in contrast to the TE case shown in figure 6, where the slab lens does not significantly improve the resolution.

The results from numerical simulation using a finite-element method are also given. Unlike the analytical approach, this technique allows the whole field between object and image to be obtained by one calculation only. The frequency step chosen was 0.05 with a bandwidth of $\sin \theta_0 = \pm 100$. It may be seen that the full analytical and numerical simulations are in
Figure 12. The image at position $z = 80 \text{ nm}$, TM mode, $\omega = 3.48 \text{ eV}$: 1, single 40 nm silver slab; 2, two-layer, 20 nm silver–air system; 3, four-layer, 10 nm silver–air system; 4, two-layer, 20 nm silver–gain system; and 5, four-layer, 10 nm silver–gain system.

excellent agreement and are quite consistent with the results from the electrostatic approximation. Figure 7 shows plots of the intensity distribution for the 20 nm TM case obtained using the FEM. In this case and in later plots the plasmon fields are clearly seen. Note that the profile represents the total intensity distribution in each region. For instance, the profile in the object space represents the sum of the incident and reflected fields from the front surface of the slab.

6.2. Multiple slabs

Replacing a single slab by multiple thinner ones adding up to the same (equal) thicknesses of metal and air will considerably improve the performance [12], as the image is transferred from one point to another with fewer losses than it suffers from diffraction. Figures 8 and 9 show the intensity distribution calculated with our FEM over the calculation region for the silver–air layered structure with total object to image distance of 80 nm. If the air gaps are filled
with an amplifying positive dielectric medium, by following the requirements $\varepsilon_+ = \varepsilon'_+ + i\varepsilon''_+$, $\varepsilon_- = -\varepsilon'_- - i\varepsilon''_-$ [13], the performance will be dramatically improved. Figures 10 and 11 show the intensity distribution over the same 80 nm calculation region for the silver–gain layered structure, where we can see huge local fields near the slab surfaces, which represent the excited surface plasmons. A comparison of performance between the silver–air and silver–gain systems of figures 8–11 is shown in figures 12.

We find that the approach through linear superpositions of one-dimensional field components calculated by FEM is a simple but powerful technique, which is very suitable for the study of multi-layer structures containing different separating media, such as the planar lens stack used in optical proximity lithography.

### 6.3. Working distance

The above improvements in resolution using more layers with a given total object-to-image distance come at the expense of a decrease by half of the object to lens working distance for

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**Figure 13.** Intensity distribution for the 20–20–0 nm silver–air configuration, TM mode, (a) intensity plot and (b) mesh plot.
each doubling of the number of layers. It is not clear whether this is a useful improvement, since in many applications the working distance will be the most important factor. By comparing the result for a single 20 nm slab in figure 5 to curve 2 of figure 12 for two successive such slabs and to curve 1 of figure 15, it can be seen that a single 20 nm slab, 40 nm total distance, TM-mode system will be better than a bare detector placed at 20 nm from the object and also better than a two-layer 20 nm slab system of total distance 80 nm. It will also be much the same as a two-layer 20 nm slab system of total distance 80 nm with gain. We have stated that the resolution gains are independent of the position of the lens between the object and image so that a working distance of 20 nm may be achieved with 20 nm slab systems, either single or multiple, by placing the detector at the rear plane of the lens. In figure 13, we show plots of the intensity distribution of a 20 nm air, 20 nm silver, 0 nm air system, with the same object as above and in figure 14 the same plots for a 20 nm air, 20 nm silver, 20 nm gain medium, 20 nm silver and 0 nm air system. The images obtained with these two systems are compared in figure 15. The image for the single slab is the same as that of the centrally placed 20 nm slab shown in figure 5 due to the position invariance of the lens.

Figure 14. Intensity distribution for the 20–20–20–20–0 nm silver–gain configuration, TM mode, (a) intensity plot and (b) mesh plot.
Figure 15. Comparison of images for single-layer and two-layer gain systems: 1, bare detector placed 20 nm away from the object; 2, 20–20–0 nm single-layer, air–silver–air system; and 3, 20–20–20–20–0 nm multilayer, air–silver–gain–silver–air system.

7. Conclusions

The main conclusion drawn from our calculations is that if, in near-field imaging, a working distance between object and lens of the order of 20 nm can be tolerated and the total object to image distance is immaterial, then significant gains in resolution are possible by using a single silver slab and the TM mode, without the necessity for negative magnetic permeability, multiple layers or gain media. This might just be feasible for a scanning optical system with present-day technology.

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