Stability analysis of reinforced concrete building frames damaged by corrosion under static-dynamic loading

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Abstract. The paper presents a methodic for calculation reinforced concrete frame-braced structural systems, when it was damaged by corrosion, against loss of stability in over limit state caused by removal of a load bearing element. The authors obtained analytical expressions for assessment stiffness of reinforced concrete compressed-bent elements taking in account its nonlinear deforming. Such expressions allow conducting stability analysis of structural systems under consideration.

1. Introduction
Since the 70s of the twenty century, a number of publications on the problem of progressive collapse of buildings and structures steady increases in the scientific literature [1-5]. The experience and knowledge accumulated to date in this area has allowed engineering community to introduce regulatory documents [6-8] that regulate the design requirements for the protection of buildings and structures against progressive collapse under special effects caused by a possible hazard failure of one of the supporting structures in the structural system. However, a large number of important problems of this direction remain insufficiently studied. One problem among them linked with buckling of the operated reinforced concrete building frames under special effects.

The buckling problem for such a structural composite material as reinforced concrete is associated with a change in deformation characteristics due to physical nonlinearity [9, 10], developing creep deformations [9, 11-14] and not stationary processes of environmental resistance of such material [15-17]. In this regard, this article is devoted to the study of the influence of corrosion damage and loading modes on the stability of structural elements at over limit states caused by the sudden removal of one of the load bearing elements of the building frame.

2. General equations of stability problem for structure made of an elastic plastic material
When we were solving the problem of the stability of a reinforced concrete frame structural system, we used the elastic-plastic model of the material of structures. In this case, features of deformation and crack formation can be described using the same mathematical model based on idealized diagrams of deformation. Given the specifics of the deformation of structures made of elastic-plastic material, the stability equation of an element of a system, taking in account the specifics of the deformation of elastic plastic material in accordance with [18], can be written in the form:
where $E$ is initial deformation modulus of concrete, $\nu$ is a deflection of the rod in an arbitrary section, $N$ is axial force, $J_2$ is reduced second moment of a deformed cross section calculated by the tangential modulus of concrete relatively own central axis:

$$J_2 = J_0 - A_0 a_0^2,$$  

where

$$\theta = E_i / E, \quad A_0 = \int_A \theta dA, \quad S_0 = \int_A \theta y dA, \quad a_0 = S_0 / A_0, \quad J_0 = \int_A \theta y^2 dA.$$

In the expressions (1) and (2), $E_i$ is tangential modulus, $\theta$ is relative tangential modulus, $A_0$ is reduced area of a cross section, $S_0$ is reduced static moment of a cross section, $J_0$ reduced second moment of an undeformed cross section relatively own central axis.

It should be noted that $J_2$ characterizes incremental stiffness of a deformed cross section and its resistance to buckling under incremental increasing of the external forced action on structure.

3. Physical relations for static-dynamic deforming mode

In this work, the simulation of changes in the strength and deformation characteristics of the power resistance of reinforced concrete during sequential static-dynamic loading is based on the theory of plasticity of G.A. Geniev [19]. Let us assume that the structural system is designed in such a way that it does not lose stability at the stage of normal operation. We applied for simulation of rheological properties at static-dynamic deforming a model consisting of series-connected elements: quasi-elastic - 0, which corresponds to the secant modulus of elasticity $E_{sec,0}$, calculated taking into account creep deformations, and the elastic-viscous element 1 represented by the Kelvin-Voigt model (Fig. 1, a):

$$\Delta \varepsilon = \Delta \sigma \left[ 1 - e^{-\omega t} \right] / E_{sec,0}, \quad \omega = E_{sec,0} / K,$$  

where $E_{sec,0}$ is a secant modulus at stage of normal operation (Fig. 1, c), $K$ is modulus of viscous resistance, $t$ is a time of dynamic additional loading of structural system at sudden structural transformation, $\Delta \sigma$ and $\Delta \varepsilon$ are increment of stresses and deformations in a cross section of an element under dynamic loading.

Element $A_1$ of this model corresponds to pure elastic behavior of composite material, and element $B_1$ corresponds to pure viscous behavior. However, time scale for elements 0 and 1 takes the significantly different order. Therefore, it is more convenient applying multi-level calculation schemes for structural analysis of reinforced concrete building frames under special hazard effects and to simulate long-time and dynamic deformations separately.

4. Modelling of environmental resistance of reinforced concrete

In this paper, we accepted phenomenological model of environmental resistance suggested by V.M. Bondarenko [20] in order to account long-time processes of corrosion destruction of reinforced concrete elements. In accordance with this model, the depth of corrosion damage $\delta(t, t_0)$ of a reinforced concrete structure at descending in time process of corrosion can be written in the form (Fig. 2, a):

$$\delta(t, t_0) = \delta_{cr} \left\{ 1 - \left[ \alpha (m-1)(t-t_0) + \left( 1 - \frac{\delta(t, t_0)}{\delta_{cr}} \right)^{1-m} \right] \right\}^{1/m},$$  

where $\delta_{cr}$ is limit depth of corrosion damage for fading in time corrosion process; $\alpha$ and $m$ are parameters characterizing speed of corrosion process; $t$ and $t_0$ are the current time and time, when we
has begun the observation of reinforced concrete element with corrosion damages. It should be noted that inequality \( m < 0 \) corresponds to progressive damaging; \( m > 0 \) corresponds to fading (colmatation) damaging; \( m = 0 \) is boundary line corresponding to filtration process of damaging. In the common case, the parameters \( \delta_{cr} \), \( \alpha \) and \( m \) can be approximated by polynomials [20] (Fig. 2, b):

\[
\delta_{cr} = \sum_{i=0}^{3} q_{\delta} \cdot \eta^i, \quad \alpha = \sum_{i=0}^{3} q_{\alpha} \cdot \eta^i, \quad m = \sum_{i=0}^{3} q_{m} \cdot \eta^i,
\]

where \( q_{\delta}, q_{\alpha}, q_{m} \) are parameters defining from experiments for accepted conditions of aggressive media and initial physical and mechanical parameters of concrete; \( \eta = \sigma/R_b \) is a ratio between current and ultimate stresses in concrete in compression.

\[\text{Figure 1.} \] Rheological model of static-dynamic (a) and dynamic (b) power resistance of reinforced concrete; diagram of static-dynamic deformations (c) of reinforced concrete element under axial compression

In this case, an account of decreasing strength and deformation parameters through the depth of corrosion damages at colmatation can be performed multiplying modulus of concrete in this zone on function of corrosion damages \( K(z, t, \eta) \), which can be written similarly to [20] in the following form (Fig. 2, d):

\[
K(z, t, \eta) = \sum_{i=0}^{3} a_i \cdot z^i,
\]

where \( z \in [0; \delta(t)] \), \( a_i \) is parameter satisfying to boundary conditions for function of corrosion damages.
**Figure 2.** Scheme of damage kinetics during concrete compression (a); scheme for determining the parameters \( \delta_{cr}, \alpha, m \) (b); cross section of a corrosion-damaged reinforced concrete element (c); general view of the corrosion damage function \( K(z, t) \) (d): 1 – region of progressive development \( (m < 0) \); 2 – region of damped development \( (m > 0) \); 3 – boundary line \( (m = 0) \).

If we separate the preserved part of the section and part of the section damaged by corrosion processes, the flexural stiffness of the section of the compressed-bent reinforced concrete element, taking into account (4) and (5), can be determined from the following expression:

\[
B_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} E_{r,d} (y)^2 dy \int_{-\frac{b}{2}}^{\frac{b}{2}} \delta(t) dx + \int_{-\frac{h}{2}}^{\frac{h}{2}} \left[ E_{r,d} (y) \cdot K(y, t, \eta) \right]^2 dy \int_{-\frac{b}{2}}^{\frac{b}{2}} dx. \tag{6}
\]

Since finding of the depth of corrosion damaging through the cross-section for real structures operated at variable in time values of parameter \( \eta \) and at decreasing of ultimate strength \( R_b \) under compression caused by corrosion damages is a very difficult problem, then we propose using fixed value of parameter \( \eta = 0.6 \) in the first approximation. Such a value corresponds to boundary between
the first and the second sections of three-line deformation diagram of concrete in compression. In this way thermodynamic and chemical parameters are constant and time sections $t$ are fixed.

5. Technique of stability analysis

At the first stage, the structural system is calculated according to the accepted design scheme (primary design scheme of the first level) on the basis of the accepted physical relationships. The calculation results obtained at this stage are applied to select the potentially most dangerous zones for modelling a possible local destruction in the considered structural system. At the next stage of the calculation, a sub-structure (frame fragment) is selected from the entire frame in the area of a possible removal of one of the load bearing structures (design scheme of the second level) using decomposition for a detailed assessment of the stress-strain state of the reinforced concrete elements of the frame. The initial data for advanced computational analysis of the deformation and fracture of the elements of this fragment of the reinforced concrete frame are the calculation results obtained by the first level design scheme. Moreover, structural elements of the frame in the primary calculation scheme of the second level should be divided along the length into several rod finite elements, which are modeled taking into account creep, geometric and physical nonlinearity. In this paper, we mean geometrical nonlinearity as affecting of displacements on the efforts in the system elements [9]. The result of this calculation is diagrams and patterns of stresses and deformations in the cross sections of the elements of calculation model in which the design combinations of loads for a special limit state [6] act (Fig. 3, a).

Determination of deformations of the building frame subjected to action of special hazard effects is carried out by the quasi static method. In this case, we assign to rod elements the stiffness and coordinates of sections (nodal points) (Fig. 3, b) in accordance with the results of the physically nonlinear calculation of the primary scheme of the second level. However, it should be attached only a generalized force in a place of the discarded link instead the load acting there at the stage of normal operation, another forces should be deleted from scheme. For determining a generalized force, we take into account the forces acting in the removed element of the structural system at the stage of normal operation and attach it in the opposite direction, according to the condition of constancy of a total energy of deformation of the structural element.

The stiffness of the rod elements of a fragment of the structural system during prolonged deformation can be calculated using the secant modulus $E_{\text{sec}}$ obtained applying one of the nonlinear calculation methods: the method of variable elasticity parameters (MVEP) [21], the finite increment calculus method (FIC) [22], or applying of an integrated modulus proposed by V.M. Bondarenko [23]. Work [24] shows a comparison of the accuracy of MVEP, FIC and exact solution to the problem of beam flexure. It is about 10% for MVEP and 2% for FIC at a limited number of steps. Below, we give a brief description of the MVEP technique, since this method is relatively simple and has enough accuracy for stability problems.

At the first stage, it should be carried out a calculation of a fragment of the structural system for design combination of loads and initial value of modulus. Further, parameters of stress-strain state should be refined applying values of internal efforts obtained in calculation:

$$\varepsilon_i = \varepsilon_{\text{cr}}(t) + \varepsilon_{\text{i,j}},$$  \hspace{1cm} (7)

where $\varepsilon_{\text{cr}}(t)$ is creep deformation at the time moment $t$, which is calculated in accordance with regulatory documents [11] or on the basis of calculation relationships from scientific literature [9, 16, 17];

$\varepsilon_{\text{i,j}}$ is deformation caused by conventionally ‘instantaneous’ attaching of a static load and calculated for the respective iteration by formula:

$$\varepsilon_{\text{i,j}} = \varepsilon_N \pm \varepsilon_M = N_i/(E_i A) \pm M_i/h_y(2E_i I) .$$  \hspace{1cm} (8)
Figure 3. Levels of calculation schemes for stability analysis of structures subjected to special hazard effects: the second level calculation scheme obtained as result of physically nonlinear calculation when design combination of loads acts (a), the second level secondary calculation scheme (b), diagrams of deformation and stresses in the cross-section of compressed-bent element if there is stretched zone of the cross-section (c), the same ones if there is not a stretched zone (d)

In the formula (8), $N_i$ and $M_i$ are respectively axial force and bending moment in a rod element at $i$-th iteration of calculation; $A$ and $J$ are respectively area and second moment of the cross-section of an element; $h$ is height of the cross-section of an element.

Secant modulus for the next iteration of analysis should be determined from expression:

$$E_i = [E_{ci}^a + E_{ai}^c E_{ai}^c + E_{ci}^c] [2(a + c)].$$

where $a$ and $c$ are respectively distance from neutral axis to the most compressed fiber and the less compressed (stretched) fiber of an element (Fig. 3, c, d); $E_{ci}^a$ is secant modulus of compressed zone for $i$-th iteration of calculation; $E_{ci}^c$ is the same one for stretched zone:

$$E_{ci}^a = \sigma_{ci} / \varepsilon_{ci}, \quad E_{ci}^c = \sigma_{ci} / \varepsilon_{ci}, \quad \sigma_{ci} = E_{ci}^a \varepsilon_{ci} - H \varepsilon_{ci}^2, \quad \sigma_{ci} = E_{ci}^c \varepsilon_{ci} - H \varepsilon_{ci}^2.$$
In the formula (10), expressions for stresses in the outer fibers \( \sigma_a \) and \( \sigma_c \) correspond to approximate relationship between stresses and deformations by second order polynomial equations; \( \varepsilon_a \) and \( \varepsilon_c \) are deformations respectively in the most and the less compressed fibers; \( H_i \) is parameter determined from the condition of equality to zero of tangential modulus when stresses reached ultimate strength at compression.

Iteration process should be continued until difference between stiffness on the serial steps of calculation satisfies to accepted accuracy.

Secant deformation modulus \( E_{sec,1} \), corresponding to stage of dynamic loading of a viscous elastic element 1, takes the form in accordance with (1):

\[
E_{sec,1} = \frac{\Delta\sigma}{\Delta\varepsilon} = E_{sec,0}/\left(1-e^{-e_x}\right).
\]  

(11)

Deformations of the structural system at accidental impacts should be calculated using secant modulus \( E_{sec,1} \) as it was described above. Further, let us write expression for stiffness of reduced cross-section (for tangential modules) in the plane of the frame that obtained in accordance with approach described in [18] for elastic-plastic rods of rectangular cross-section:

\[
B_x = \frac{b \cdot E_0}{(1-e^{-e_x})} \left[ \int_{-h/2}^{h/2} \left(1 - \frac{H_1}{E_0} \left( \frac{\varepsilon_c - \varepsilon_a}{h} \left( y + \frac{h}{2} \right) \right) \right) y^2 dy + \int_{-h/2}^{h/2} \left(1 - \frac{H_1}{E_0} \left( \frac{\varepsilon_c - \varepsilon_a}{h} \left( y + \frac{h}{2} \right) \right) \right) (a_0 + a_1 y + a_2 y^2) y^2 dy \right].
\]  

(12)

Here we accepted the following designations:

- \( E_{sec}(\varepsilon) = E_0/\left(1-e^{-e_x}\right) \) is a dynamic tangential modulus;
- \( E_{tan} = \frac{d\sigma}{d\varepsilon} = E_0 - 2H_i e \) is a tangential modulus for strain state that precedes to dynamic loading;
- \( \varepsilon = \varepsilon_a + \frac{\varepsilon_c - \varepsilon_a}{h} \left( y + \frac{h}{2} \right) \) is a fiber deformation (Fig. 3), calculated for strain state of structural element taking in account dynamic additional loading;
- \( a_0, a_1, a_2 \) are parameters of equation (5) which satisfy to boundary conditions for function of corrosion damages.

Thus, nonlinear stability analysis for reinforced concrete elements of building frame under accidental impact can be substituted by linear stability analysis of a fragment of the building frame with a piece-wise variable stiffness through the length of structural elements calculated by tangential modules. Critical force obtained in such a calculation should be compared with effort acting in the structural element calculated for dynamic loading caused by sudden failure of an element of the structural system. The structural system is stable if the following inequality is satisfied:

\[
P_{cr,dyn} > N_{dyn},
\]  

(13)

where \( P_{cr,dyn} \) is a critical force causing buckling of the structural system or its element, the stiffness of which is determined on the basis of nonlinear structural analysis of the system on accidental impact, \( N_{dyn} \) is dynamic effort in the element of structural system under consideration.

The reliability of applying the proposed approach, which is based on the use of level design schemes, is confirmed by the results of a number of experiments, for example, [2, 4], which established that the dynamic effect caused by a sudden structural transformation relatively quickly damps with the distance from the area of the local destruction.
6. Conclusions
1. In the work, an analytical expression for the stiffness of a compressed-bent reinforced concrete element of a structural system operated in the over limit state caused by sudden failure of one of the loading bearing structures of the building frame is obtained.
2. The presented technique and algorithm for stability analysis of a reinforced concrete frame-braced structural system with corrosion damages of elements can be used in the structural analysis of the protection of buildings and structures against progressive collapse caused by the sudden failure of one of the supporting structures.

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