On the peripheral-tube description of the two-particle correlations in nuclear collisions

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In this work, we study the two-particle correlations regarding a peripheral tube model. From our perspective, the main characteristics of the observed two-particle correlations are attributed to the multiplicity fluctuations and the locally disturbed one-particle distribution associated with hydrodynamic response to the geometric fluctuations in the initial conditions. We investigate the properties of the initial conditions and collective flow concerning the proposed model. It is shown that the experimental data can be reproduced by hydrodynamical simulations using appropriately constructed initial conditions. Besides, instead of numerical calibration, we extract the model parameters according to their respective physical interpretations and show that the obtained numerical values are indeed qualitatively in agreement with the observed data. Possible implications of the present approach are discussed.
I. INTRODUCTION

Event-by-event fluctuations play a crucial role in the hydrodynamic description of the relativistic heavy ion collisions \[1\,\text{[6]}\]. The experimental observations of the enhancement in correlations at intermediate and low transverse momenta \[7\,\text{[12]}\], comparing with those at high transverse momenta \[13, 14\] strongly support the hydrodynamics/transport characteristic of such phenomena \[15\,\text{[21]}\]. In particular, the observed features, referred to as “ridge” and “shoulders”, were shown to be successfully reproduced by hydrodynamical simulations with event-by-event fluctuating initial conditions (IC) but not by averaged IC \[15\]. Subsequently, it leads to the current understanding through extensive studies of event-by-event based hydrodynamic/transport analysis \[22\,\text{[27]}\], that the two-particle correlations for the lower transverse momenta can be mostly interpreted in terms of flow harmonics $v_n$. Notably, the triangular flow, $v_3$, is mostly attributed to for the appearance of the “shoulder” structure on the away side of the trigger particle \[16, 22, 28\]. Moreover, it is understood that these parameters are closely associated with the corresponding $\epsilon_n$, the anisotropies of the initial energy distribution \[16, 22\]. In fact, the observed behavior of anisotropic parameters as functions of centrality and transverse momentum has been studied at a very high quantitative level \[29, 30\], as well as the correlation between $v_n$ and $\epsilon_n$ has also been established (see \[2, 4, 5, 31\] for the recent reviews on this topic).

In spite of its success of the event-by-event hydrodynamic simulations, the linearity between $v_n$ and $\epsilon_n$ become less evident for harmonics greater than $n = 2$. To be specific, it was shown that the correlations among $v_n$ and $\epsilon_n$ become weaker for larger harmonics \[32, 33\]. In this context, there seems to exist room for alternative approach from a different angle. In fact, before the explanation regarding the triangular flow was first suggested \[16\], we proposed a peripheral tube model \[34–37\] which provides a straightforward and reasonable picture for the generation of the triangular flow and the consequent two-particle correlations. It is an approach within the general event-by-event hydrodynamic scheme. The model views the fluctuations in the IC as consisting of independent high energy tubes close to the surface of the system. Thereby, each tube affects the hydrodynamical evolution of the system independently, and their contributions are summed up linearly to the resultant two-particle correlations. In this approach, one substitutes the complex bulk of the hot matter by an average distribution over many events from the same centrality class. The above picture attempts to interpret the physical content of fluctuating IC regarding the granularity represented by peripheral high energy tubes. To be specific, if a tube locates deep inside the hot matter, the effect of its hydrodynamic expansion would be quickly absorbed by its surroundings, causing relatively less inhomogeneity in the media. On the contrary, a tube staying close to the surface leads to a significant disturbance to the one-particle distribution, resulting in an azimuthal two-particle correlation structure similar in shape and magnitude to the observed data. By numerical simulations, as shown in Fig.1, one finds that the fluid is deflected to both sides of the tube, causing two peaks separated by $\sim 120$ degrees in the one-particle azimuthal distribution. Subsequently, this leads to the desired two-particle distribution where a double peak is formed on the away side, whose height is approximately half of the single peak located on the near side. It is shown that the resultant correlation structure is robust against the variation of the model parameters \[34, 36\]. Furthermore, simulations carried out with multiple peripheral tubes show very similar features in the corresponding two-particle correlations. Therefore, it is strongly indicated that the two-particle correlations can be seen as a superposition of those contributions due to individual peripheral tubes. We note that it
is meaningful to compare the present model to another approach frequently employed in the literature, where the one-particle distribution are decomposed into different flow harmonics. Subsequently, the collective flow related to the harmonic coefficients are understood to be mostly independent and associated with the corresponding eccentricity components in the IC. By and large, the hydrodynamic evolution linearly transforms the initial state geometric inhomogeneity into the final state anisotropy in momentum space. The main point is that in our present model the effect is considered as local, as a result of how the expansion is affected by a high energy tube close to the surface of the fluid. Therefore, it is irrelevant to the fluid dynamics of the rest of the system, but localized to a specific azimuthal angle $\phi_t$ associated to the peripheral tube in question. As hydrodynamics is understood as an effective theory at the long wavelength limit, the peripheral tube interprets the two-particle correlation in terms of phenomena where the characteristic length is comparable to the system size. Concerning harmonic coefficients, the event planes of the elliptic and triangular flow coefficients generated by the tube and are therefore both correlated to the location of the tube $\phi_t$. However, as one carries out the event-by-event average, $\phi_t$ is averaged out, and the resultant expression (for instance, see Eqs. (6) and (7) below) does not explicitly depend on it.

The present work is organized as follows. In the next section, we briefly review the peripheral tube model and discuss its main results on the observed two-particle correlations. Subsequently, in section III, we show that the experimental data can be reasonably reproduced by appropriately constructing the IC with the peripheral tube. Furthermore, we extract the model parameters using event-by-event simulations with various devised IC reflecting the average background and event-by-event fluctuations. It is shown that those extracted values are indeed in accordance with the observed two-particle correlation data. The last section is devoted to discussions and concluding remarks.

II. THE PERIPHERAL TUBE MODEL FOR TWO-PARTICLE CORRELATIONS

In terms of the peripheral tube model, the two-particle correlations in nuclear collisions consist of two contributions. The first part is due to the average almond shape energy distribution of the IC, and the resultant collective flow. It is treated as the background flow in our approach. Owing to the event-by-event fluctuations, the contribution from the background for different events does not cancel out after the flow background subtraction, and the residual is proportional to the standard deviation of the multiplicities. The second contribution comes from that of a peripheral tube. In terms of the language of flow harmonic coefficients, it produces aligned elliptic and triangular flow. The above picture can be used to explain the event plane dependence as well as centrality dependence of the observed two-particle correlation \[38, 39\]. To be specific, we assume that the two-particle correlation is entirely determined by the one-particle distribution. But instead of writing the latter down directly in terms of Fourier expansion \[40\], we write down the one-particle distribution as a sum of two terms, namely, the distribution of the background and that of the tube.

\[
\frac{dN}{d\phi}(\phi, \phi_t) = \frac{dN_{bgd}}{d\phi}(\phi) + \frac{dN_{tube}}{d\phi}(\phi, \phi_t),
\]  

(1)
Figure 1. (Color online) The temporal evolution of the IC consisting of one peripheral tube placed on top of an elliptic smoothed background energy distribution. The parameters for the IC used in calculation are discussed in section III.
where

\[
\frac{dN_{\text{bgd}}}{d\phi}(\phi) = \frac{N_b}{2\pi} \left(1 + 2v_2^b \cos(2\phi) \right),
\]

\[
\frac{dN_{\text{tube}}}{d\phi}(\phi, \phi_t) = \frac{N_t}{2\pi} \sum_{n=2,3} 2v_n^t \cos(n[\phi - \phi_t]).
\]

In Eq. (2) we consider the most simple case for the background flow, by parametrizing it with the elliptic flow parameter \(v_2^b\) and the overall multiplicity, denoted by \(N_b\). The contributions from the tube are measured with respect to its angular position \(\phi_t\), and since its symmetry axis is correlated to the tube location \(\phi_t\), the variation of the latter is related to the event-by-event fluctuations. We also assume that the flow components from the background and the tube are written in Fourier expansion, they are intrinsically independent distributions. In particular, the triangular flow in our model is completely generated by the tube, and since its symmetry axis is correlated to the tube location \(\phi_t\), the variation of the latter is related to the event-by-event fluctuations. We also assume that the flow components from the background are much more significant than those generated by the tube, \(\Psi_2\) is mainly determined by the elliptic flow of the background \(v_2^b\).

Following the methods used by the STAR experiment \[41, 42\], the subtracted di-hadron correlation is given by

\[
\left\langle \frac{dN_{\text{pair}}}{d\Delta \phi}(\phi_s) \right\rangle = \left\langle \frac{dN_{\text{pair}}}{d\Delta \phi}(\phi_s) \right\rangle^{\text{prop}} - \left\langle \frac{dN_{\text{pair}}}{d\Delta \phi}(\phi_s) \right\rangle^{\text{mix}},
\]

where \(\phi_s\) is the trigger angle (\(\phi_s = 0\) for in-plane and \(\phi_s = \pi/2\) for out-of-plane trigger). By using Eq. (3), one finds the proper two-particle correlation

\[
\left\langle \frac{dN_{\text{pair}}}{d\Delta \phi} \right\rangle^{\text{prop}} = \int \frac{d\phi_t}{2\pi} f(\phi_t) \frac{dN_T}{d\phi}(\phi_s, \phi_t) \frac{dN_A}{d\phi}(\phi_s + \Delta\phi, \phi_t),
\]

where \(f(\phi_t)\) is the distribution function of the tube, and superscripts “\(T\)” and “\(A\)” indicate “trigger” and “associated” particles respectively (c.f. subscripts “\(T\)” are shorthands for “transverse”). For simplicity, we take \(f(\phi_t) = 1\).

The combinatorial background \(\left\langle dN_{\text{pair}}/d\Delta \phi \right\rangle^{\text{mix}}\) can be calculated by using either cumulant or ZYAM method \[43\]. Though both methods yield very similar results in our model, it is more illustrative to evaluate the cumulant. Following similar arguments presented in Ref. \[38\], it is straightforward to show that the resultant correlation reads

\[
\left\langle \frac{dN_{\text{pair}}}{d\Delta \phi}(\phi_s) \right\rangle^{\text{(cmlt)}} = \frac{<N_T^b N_A^b> - <N_T^b> <N_A^b>}{(2\pi)^2} \left(1 + 2v_2^b \cos(2\phi_s) \right) \left(1 + 2v_2^A \cos(2(\Delta\phi + \phi_s)) \right)
\]

\[
+ \frac{<N_T^t N_A^t>}{(2\pi)^2} \sum_{n=2,3} 2v_n^t \cos(n\Delta\phi) \cos(n\Delta\phi).
\]

where the event average is carried out by integration in \(\phi_t\). The above expression explicitly depends on \(\phi_s\), which can be used to study the event plane dependence of the correlation. In particular, the trigonometric dependence of the background contribution on \(\phi_s\) indicates that its contribution to the out-of-plane triggers is opposite to that for the in-plane ones. As
a result, for the out-of-plane correlation, it leads to an overall suppression in the amplitude, as well as forms a double peak structure on the away side. Indeed, experimental data [41, 42] shows that the overall correlation decreases while the away-side correlation evolves from a single peak to a double peak as $\phi_s$ increases. Since these observed features are in agreement with the analytically derived results, the peripheral model is shown to be meaningful despite its simplicity.

By further averaging out $\phi_s$, one obtains

$$\left\langle \frac{dN_{\text{pair}}}{d\Delta \phi} \right\rangle^{\text{(cmlt)}} = \frac{< N_b^T N_b^A > - < N_b^T > < N_b^A >}{(2\pi)^2} (1 + 2v_2^{b,T} v_2^{b,A} \cos(2\Delta \phi)) + \frac{< N_i^T N_i^A >}{(2\pi)^2} \sum_{n=2,3} 2v_n^{i,T} v_n^{i,A} \cos(n\Delta \phi).$$

(7)

If the model is indeed realistic, one should be able to obtain the parameters in Eq.(7) according to their respective physical content, while the resulting correlations should be still quantitatively in agreement with the data. This is the principal object of the present work. In what follows, by carrying out numerical simulations, we first show that an appropriately constructed IC can reasonably reproduce the observed data. Furthermore, we attempt to calculate the model parameters according to their definitions. This is done by using various IC tailored to match the respective physical properties of IC in question. In particular, we study the multiplicity fluctuations as well as the flow harmonics of different IC associated with the background as well as the peripheral tube. The two-particle correlations are then evaluated by using the obtained values.
III. The model parameters extracted by comparing hydrodynamic simulations against the data

For the present study, we focus on mid-central 200 AGeV Au+Au collisions. We first show that one can devise the IC according to the peripheral tube model and subsequently evaluate the two-particle correlation. This is done by estimating the background energy distribution by averaging over the 343 events generated by a microscopic event generator, NeXuS [44], for the centrality window 20%-40% as shown in Fig. 2. The obtained almond shaped energy distribution is then fitted by the following parametrization

\[ \epsilon_{\text{bgd}} = (9.33 + 7r^2 + 2r^4)e^{-1.8r}, \]  

with

\[ r = \sqrt{0.41x^2 + 0.186y^2}. \]

The profile of the high energy tube is calibrated to that of a typical peripheral tube in NeXuS IC as follows

\[ \epsilon_{\text{tube}} = 12e^{-(x-x_{\text{tube}})^2-(y-y_{\text{tube}})^2}, \]  

where the tube is located at a given value of energy density close to the surface, determined by a free parameter \( r_{\text{tube}} \), so that its coordinates on the transverse plane read

\[ x_{\text{tube}} = \frac{r_{\text{tube}} \cos \theta}{\sqrt{0.41 \cos^2 \theta + 0.186 \sin^2 \theta}}, \]

\[ y_{\text{tube}} = \frac{r_{\text{tube}} \sin \theta}{\sqrt{0.41 \cos^2 \theta + 0.186 \sin^2 \theta}}. \]

Here \( r_{\text{tube}} \) is used as a free parameter whose value is determined below, and the azimuthal angle of the tube \( \theta \) is randomized among different events.

By combining the two pieces together, the IC for the present model read

\[ \epsilon = \epsilon_{\text{bgd}} + \epsilon_{\text{tube}}. \]  

Subsequently, we carry out hydrodynamical simulations by using the SPheRIO code [1, 15, 34, 36, 45–48], it is an ideal hydrodynamic code using the Smoothed Particle Dynamics algorithm [49–51]. For the present study, only the temporal evolution on the transverse plane is considered as Bjorken scaling is assumed. We generate a total of 2000 events according to IC profile discussed above. Then the hydrodynamical simulation is carried out for each event, by the end of which, a Monte-Carlo generator is evoked 200 times for hadronization in order to further increase the statistics. The resultant two-particle correlations, evaluated by cumulant method, are shown in Fig. 3 in comparison with the PHENIX data [8]. The parameter \( r_{\text{tube}} \) is chosen to be 2.3 in the calculations. As the hydrodynamic simulations are of two-dimension, the obtained correlations are multiplied by a factor related to the longitudinal scaling of the system.

Now, it is interesting to verify that the parameters given in Eq. (7) are indeed quantitatively meaningful. To achieve this, we extract the model parameters in Eq. (7) using mostly the same arguments leading to very expression. We first estimate those for the background distribution \( \frac{dN_{\text{bgd}}}{d\phi} \). The background flow coefficients \( v_2^{\text{bgd}} \) can be obtained directly by investigating the hydrodynamic evolution of IC solely determined by \( \epsilon_{\text{bgd}} \). A total of 2000 events
Figure 3. (Color online) The calculated two-particle correlations by using one-tube IC for 20%−40% centrality window in comparison with the corresponding data by PHINEX Collaboration [8], and those obtained by using the extracted parameters in Table I and Eqs.(12) and (13). The SPheRIO results from using cumulant method are shown in red solid curves, the data are shown in solid squares, and those obtained by the estimated parameters are shown by the blue solid curves. Left: the results for the momentum intervals $0.4 < p_T^{A} < 1$ Gev and $2 < p_T^{T} < 3$ Gev. Right: those for the momentum intervals $1 < p_T^{A} < 2$ Gev and $2 < p_T^{T} < 3$ Gev.

with 200 Monte Carlo each are considered in the evaluation. To estimate the multiplicity fluctuations of the background, $< N_{b}^{T} N_{b}^{A} > - < N_{b}^{T} > < N_{b}^{A} >$, we count, on an event-by-event basis, the number of particles of corresponding momentum intervals. The events in question are those generated by NeXuS of 20% - 40% centrality window, and we made use of a total of 1000 events. By using Fourier expansion of the two particle correlation, and extracting the second and third order coefficients, one obtains the parameters related to $v_{2}^{T}$, $v_{2}^{A}$, $v_{3}^{T}$, and $v_{3}^{A}$. The obtained values are summarized in Table I and Eqs.(12) and (13).

Table I. The calculated elliptic flow coefficients for corresponding transverse momentum intervals of trigger and associated particles. The calculations are carried out by using IC of smooth elliptic energy distribution as described in the text.

| $p_T$ Interval | $v_{2}^{b}$ | $v_{2}^{A}$ | $v_{3}^{T}$ | $v_{3}^{A}$ |
|---------------|-------------|-------------|-------------|-------------|
| $0.4 < p_T < 1$ Gev | 0.11 | 0.21 | 0.36 |
| $1 < p_T < 2$ Gev | | | |
| $2 < p_T < 3$ Gev | | | |

On the other hand, however, some of the parameters can also be inferred straightforwardly from the experimental data. Therefore, we carry out these comparisons with the published data as follow. The elliptic flow coefficient of the background $v_{2}^{b}$ should be consistent with the collisions of the same centrality window. This is confirmed by comparing the value against the data of 20%-60% Au+Au collisions obtained by PHENIX [52]. The multiplicity fluctuations estimated from simulations and for the corresponding momentum intervals are
found to be

\[ < N_b^T N_b^A > - < N_b^T > < N_b^A > = 14.67, \]
\[ < N_t^T N_t^A > v_2^{t,T} v_2^{t,A} = 1.62, \]
\[ < N_t^T N_t^A > v_3^{t,T} v_3^{t,A} = 1.63, \]

for \( 0.4 < p_t^A < 1, 2 < p_T^T < 3 \), and

\[ < N_b^T N_b^A > - < N_b^T > < N_b^A > = 5.07, \]
\[ < N_t^T N_t^A > v_2^{t,T} v_2^{t,A} = 1.36, \]
\[ < N_t^T N_t^A > v_3^{t,T} v_3^{t,A} = 1.48, \]

for \( 1 < p_t^A < 2, 2 < p_T^T < 3 \) respectively. Finally, by substituting the above parameters back into Eq. (7), one obtains the two-particle correlation as also shown in Fig. 3. It is found that the two approaches are in good agreement with each other.

IV. CONCLUDING REMARKS

To summarize, in this work, we show that the data on two-particle correlations are consistent with those obtained by event-by-event hydrodynamical simulations using appropriately devised IC of the peripheral tube model. The latter is further shown to be in qualitative agreement with those by using the analytic result of a simplified model with the parameters extracted from flow coefficients and multiplicity fluctuations. This indicates that the peripheral tube model is a reasonable approach for the interpretation of the observed features of the two-particle correlations in nuclear collisions.

The hydrodynamical approach is shown to be a success for the description of many experimental data of relativistic heavy ion collisions. In particular, it provides a reasonable description for the observed two-particle correlations. One distinct feature of the observables in question, as we understand, is that they usually involve an event-average procedure, and thus may not necessarily carry all the information on genuine nonlinear nature of hydrodynamics. In other words, although the event averaged harmonic coefficients display a strong linear relation to those averaged initial state eccentricities \[32\], some essential characteristics associated with the nonlinearity may still be hidden and failed to be captured through averaged values. Subsequently, the distinction between different approaches may become less visible. It is also interesting to note that the transport models such as AMPT or PHSD have shown to have similar properties as viscous hydrodynamic calculations \[4, 20, 21\]. However, when one looks closely on an event-by-event basis, a state close to the local thermal equilibrium only corresponds to a tiny space-time domain during the entire dynamical evolution (Ref. \[4, 55\]). To clarify up to what extent the genuine event-by-event hydrodynamics is valid, it may require a new set of observables which are sensitive to the non-linear evolution of the system. Works in this direction are under progress.

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[1] Y. Hama, T. Kodama, and O. Socolowski Jr., Braz.J.Phys. 35, 24 (2005), arXiv:hep-ph/0407264.
[2] U. W. Heinz and R. Snellings, Annu. Rev. Nucl. Part. Sci. 63, 123 (2013), arXiv:1301.2826.
[3] C. Gale, S. Jeon, and B. Schenke, Int.J.Mod.Phys. A28, 1340011 (2013), arXiv:1301.5893.
[4] R. Derradi de Souza, T. Koide, and T. Kodama, Prog. Part. Nucl. Phys. 86, 35 (2016), arXiv:1506.03863.
[5] T. Kodama, H. Stocker, and N. Xu, Journal of Physics G: Nuclear and Particle Physics 41, 120301 (2014).
[6] W. Florkowski, M. P. Heller, and M. Spalinski, (2017), arXiv:1707.02282.
[7] STAR Collaboration, B. Abelev et al., Phys.Rev.Lett. 102, 052302 (2009), arXiv:0805.0622.
[8] PHENIX Collaboration, A. Adare et al., Phys.Rev. C78, 014901 (2008), arXiv:0801.4545.
[9] PHOBOS Collaboration, B. Alver et al., Phys.Rev. C81, 024904 (2010), arXiv:0812.1172.
[10] ALICE Collaboration, K. Aamodt et al., Phys.Rev.Lett. 107, 032301 (2011), arXiv:1105.3865.
[11] CMS, V. Khachatryan et al., JHEP 09, 091 (2010), arXiv:1009.4122.
[12] ATLAS Collaboration, G. Aad et al., Phys.Lett. B707, 330 (2012), arXiv:1108.6018.
[13] STAR Collaboration, C. Adler et al., Phys.Rev.Lett. 90, 082302 (2003), arXiv:nucl-ex/0210033.
[14] STAR Collaboration, J. Adams et al., Phys.Rev.Lett. 97, 162301 (2006), arXiv:nucl-ex/0604018.
[15] J. Takahashi et al., Phys.Rev.Lett. 103, 242301 (2009), arXiv:0902.4870.
[16] B. Alver and G. Roland, Phys.Rev. C81, 054905 (2010), arXiv:1003.0194.
[17] P. Sorensen, J.Phys.G G37, 094011 (2010), arXiv:1002.4878.
[18] M. Luzum, Phys.Lett. B696, 499 (2011), arXiv:1011.5773.
[19] L. Pang, Q. Wang, and X.-N. Wang, Phys.Rev. C86, 024911 (2012), arXiv:1205.5019.
[20] G.-L. Ma and X.-N. Wang, Phys.Rev.Lett. 106, 162301 (2011), arXiv:1011.5249.
[21] J. Xu and C. M. Ko, Phys.Rev. C84, 014903 (2011), arXiv:1103.5187.
[22] D. Teaney and L. Yan, Phys.Rev. C83, 064904 (2011), arXiv:1010.1876.
[23] G.-Y. Qin, H. Petersen, S. A. Bass, and B. Muller, Phys.Rev. C82, 064903 (2010), arXiv:1009.1847.
[24] B. Schenke, S. Jeon, and C. Gale, Phys.Rev.Lett. 106, 042301 (2011), arXiv:1009.3244.
[25] J. Xu and C. M. Ko, Phys.Rev. C83, 021903 (2011), arXiv:1011.3750.
[26] G.-L. Ma and X.-N. Wang, Phys.Rev.Lett. 106, 162301 (2011), arXiv:1011.5249.
[27] H. Petersen, G.-Y. Qin, S. A. Bass, and B. Muller, Phys. Rev. C82, 041901 (2010), arXiv:1008.0625.
[28] B. H. Alver, C. Gombeaud, M. Luzum, and J.-Y. Ollitrault, Phys.Rev. C82, 034913 (2010), arXiv:1007.5469.
[29] P. Kolb, P. Huovinen, U. W. Heinz, and H. Heiselberg, Phys.Lett. B500, 232 (2001), arXiv:hep-ph/0012137.
[30] S. A. Voloshin and A. M. Poskanzer, Phys. Lett. B474, 27 (2000), arXiv:nucl-th/9906075.
[31] T. Hirano, P. Huovinen, K. Murase, and Y. Nara, Prog. Part. Nucl. Phys. 70, 108 (2013), arXiv:1204.5814.
[32] H. Niemi, G. Denicol, H. Holopainen, and P. Huovinen, Phys.Rev. C87, 054901 (2012), arXiv:1212.1008.
[33] W.-L. Qian et al., J.Phys.G G41, 015103 (2014), arXiv:1305.4673.
[34] Y. Hama, R. P. G. Andrade, F. Grassi, and W.-L. Qian, Nonlin.Phenom.Complex Syst. 12, 466 (2009), arXiv:0911.0811.
[35] R. Andrade, F. Grassi, Y. Hama, and W.-L. Qian, J.Phys.G G37, 094043 (2010), arXiv:0912.0703.
[36] R. P. G. Andrade, F. Grassi, Y. Hama, and W.-L. Qian, Phys.Lett. B712, 226 (2012), arXiv:1008.4612.
[37] Y. Hama, R. P. Andrade, F. Grassi, J. Noronha, and W.-L. Qian, Acta Phys.Polon.Supp. 6, 513 (2013), arXiv:1212.6554.
[38] W.-L. Qian, R. Andrade, F. Gardim, F. Grassi, and Y. Hama, Phys.Rev. C87, 014904 (2013), arXiv:1207.6415.
[39] W. M. Castilho, W.-L. Qian, F. G. Gardim, Y. Hama, and T. Kodama, Phys.Rev. C95, 064908 (2017), arXiv:1610.04108.
[40] N. Borghini, P. M. Dinh, and J.-Y. Ollitrault, Phys. Rev. C64, 054901 (2001), nucl-th/0105040.
[41] STAR Collaboration, A. Feng, J.Phys.G G35, 104082 (2008), arXiv:0807.4606.
[42] STAR Collaboration, H. Agakishiev et al., (2010), arXiv:1010.0690.
[43] N. Ajitanand et al., Phys.Rev. C72, 011902 (2005), arXiv:nucl-ex/0501025.
[44] H. Drescher, M. Hladik, S. Ostapchenko, T. Pierog, and K. Werner, Phys.Rept. 350, 93 (2001), arXiv:hep-ph/0007198.
[45] W.-L. Qian et al., Braz.J.Phys. 37, 767 (2007), arXiv:nucl-th/0612061.
[46] W.-L. Qian et al., Int.J.Mod.Phys. E16, 1877 (2007), arXiv:nucl-th/0703078.
[47] D. M. Dudek et al., Int. J. Mod. Phys. E27, 1850058 (2018), arXiv:1409.0278.
[48] W. M. Castilho, W.-L. Qian, Y. Hama, and T. Kodama, Phys. Lett. B777, 369 (2018), arXiv:1707.09878.
[49] J. Monaghan, Annu. Rev. Astron. and Astroph. 30, 543 (1992).
[50] C. Aguiar, T. Kodama, T. Osada, and Y. Hama, J.Phys.G G27, 75 (2001), arXiv:hep-ph/0006239.
[51] P. Mota, W. Chen, and W.-L. Qian, (2017), 1704.06165.
[52] PHENIX, S. Afanasiev et al., Phys. Rev. C80, 024909 (2009), arXiv:0905.1070.
[53] PHENIX, A. Adare et al., Phys. Rev. Lett. 105, 062301 (2010), arXiv:1003.5586.
[54] PHENIX, A. Adare et al., Phys.Rev.Lett. 107, 252301 (2011), arXiv:1105.3928.
[55] Y. Xu et al., Phys. Rev. C96, 024902 (2017), arXiv:1703.09178.