Interaction of two high Reynolds number axisymmetric turbulent wakes

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The interaction between turbulent axisymmetric wakes plays an important role in many industrial applications, notably in the modelling of wind farms. While the non-equilibrium high Reynolds number scalings present in the wake of axisymmetric plates has been shown to modify the averaged streamwise scalings of individual wakes, little attention has been paid to their consequences in terms of wake interactions. We propose an experimental setup that tests the presence of non-equilibrium turbulence using the streamwise variation of velocity fluctuations between two bluff bodies facing a laminar flow. We have studied two different sets of plates (one with regular and another with irregular peripheries) with hot-wire anemometry in a wind tunnel. By acquiring streamwise profiles for different plate separations and identifying the wake interaction length for each separation it is possible to show that the interaction between them is consistent with non-equilibrium scalings. This work also generalises previous studies concerned with the interaction of plane wakes to include axisymmetric wakes. We find that a simple mathematical expression for the wake interaction length based on non-equilibrium turbulence scalings can be used to collapse the streamwise developments of the second, third and fourth moments of the streamwise fluctuating velocity.

Key words: Wakes, turbulence theory, experimental fluid mechanics

1. Introduction

Recently, flow regions with non-equilibrium high Reynolds number turbulence at odds with usual Richardson-Kolmogorov phenomenology have been discovered in a number of turbulent flows (Vassilicos (2015); Chongsiripinyo & Sarkar (2020)), in particular axisymmetric and self-preserving turbulent wakes of plates with and without irregular edges. These regions are characterised by streamwise evolutions of the mean flow profiles which have only recently been documented and partially understood in experiments (Nedić et al. (2013b); Obligado et al. (2016)). The presence of a different set of scalings has many consequences, such as variations in the turbulent entrainment in free-shear flows (Zhou & Vassilicos (2017); van Reeuwijk et al. (2021); Cafiero & Vassilicos (2019, 2020)) and on eddy viscosity models (Cafiero et al. (2020)), among others. Furthermore, these regions can extend as far as about 100 plate characteristic...
lengths (defined as $\sqrt{A}$, with $A$ the frontal area of the plate) in the streamwise direction. A further study by Dairay et al. (2015), critically revised the classical theory of high Reynolds axisymmetric turbulent wakes (Townsend (1976); George (1989)) to encompass these new scalings. Both direct numerical simulations and experiments were found to agree with the theory.

In this work we focus on the interaction of turbulent axisymmetric wakes generated by two bluff bodies. This is an important configuration, present for instance in arrays of wind or marine tidal turbines, and the interaction of the two wakes can be expected to involve non-equilibrium turbulence. While some experiments in wind tunnel controlled conditions have been performed recently (Neunaber et al. (2020, 2021); Scott et al. (2020)), no attention has been paid to the relation between the energy cascade of the turbulent flow and the wake interaction length (defined as the streamwise distance at which two wakes merge).

The non-equilibrium predictions and the classical predictions rely on axisymmetry of turbulence wake statistics, self-preservation of $\frac{U_\infty - U}{U_0}$ (with $U_\infty$ the freestream velocity, $U$ the streamwise mean velocity and $U_0 = U_\infty - U$ the centreline velocity deficit), turbulent kinetic energy $K$, turbulence dissipation $\varepsilon$ and the sum of production and turbulent transport, and on a scaling law for the centreline turbulence dissipation (Dairay et al. (2015); Cafiero & Vassilicos (2019)). Both sets of predictions are obtained from the Reynolds averaged streamwise momentum and turbulent kinetic energy equations leading to a closed set of equations for $u_0(x)$ and the wake half width $\delta(x)$. The equilibrium predictions for axisymmetric turbulent wakes (see Townsend 1976; George 1989) for the streamwise evolution (along $x$) of $u_0$ and $\delta$ are

$$u_0(x) = A U_\infty \left(\frac{x - x_0}{\theta}\right)^{-2/3}, \quad (1.1)$$

$$\delta(x) = B \theta \left(\frac{x - x_0}{\theta}\right)^{1/3}, \quad (1.2)$$

where $A$ and $B$ are dimensionless constants, $\theta$ the momentum thickness and $x_0$ a virtual origin. The momentum thickness $\theta$ is defined by $\theta^2 = \frac{1}{U_\infty} \int_0^\infty U_\infty (U_\infty - U) r dr$ which is constant with $x$, and the wake’s width is here characterised by the integral wake’s half width defined by $\delta^2(x) = \frac{1}{U_0} \int_0^\infty (U_\infty - U) r dr$.

On the other hand, the non-equilibrium predictions are

$$u_0(x) = A U_\infty \left(\frac{x - x_0}{\theta}\right)^{-1}, \quad (1.3)$$

$$\delta(x) = B \theta \left(\frac{x - x_0}{\theta}\right)^{1/2}. \quad (1.4)$$

The only difference between equilibrium and non-equilibrium scalings is in the scaling of the centreline value of $\varepsilon$ (Vassilicos (2015)), that will be different according to the nature of the energy cascade. It is then possible to model the interaction between wakes via the streamwise scaling of $\delta$. It is expected that within the equilibrium cascade, the wake interaction length $x^*$ defined by $2\delta(x^*) \propto S$ (see Mazellier & Vassilicos (2010); Gomes-Fernandes et al. (2012)) will evolve as $S^3$, with $S$ the separation between the centre of the plates. Accordingly, the presence of non-equilibrium energy cascade implies that $x^* \propto S^2$.

We present experimental evidence that the non-equilibrium theory in Dairay et al. (2015) properly models the interaction of two axisymmetric wakes, for plates with both regular and irregular edges. We show that, by having knowledge of the values of the wake width $\delta$ and the centreline velocity deficit $u_0$ for a single wake, it is possible to predict the $x^*$ which quantifies the position where the wakes meet. Furthermore, it is also possible to predict the intensity of the fluctuations at that particular point. For this purpose,
Figure 1. (a) Sketch of the experimental wind tunnel set-up; (b) Irregular plate used herein. They are the second iteration of a fractal plate with dimension $D_f = 1.5$ and a square initial pattern, as characterised by Nedić et al. (2013a).

we propose an experimental setup where streamwise profiles of streamwise fluctuating velocities are acquired via hot-wire anemometry in a wind tunnel. We have tested two different sets of plates, one with square regular and another with irregular edges.

This work generalizes previous studies on the interaction of plane wakes (see Gomes-Fernandes et al. (2012)) to include axisymmetric wakes. We find that the derivation of the wake interaction length proposed in this cited work can be used to collapse the streamwise development of the first three fluctuating velocity moments. Our results suggests that non-equilibrium scalings are in good agreement with the interaction of wakes for both sets of plates studied.

2. Experimental setup

The experiments were conducted in a low turbulence wind tunnel with a test-section of $3 \times 3 \text{ ft}^2$ (0.91 × 0.91 m$^2$) and 4.25 m long. The bluff plates were placed at the beginning of the test-section. Figure 1a presents a sketch of the wind tunnel set-up. Two solid iron bars (with a diameter of 16 mm) fixed to the wind tunnel sidewalls close to ceiling and bottom served as main support for the plates. The plates, each fixed to a thin iron rod (diameter 1.5 mm, length 750 mm), were connected to the main support by two pairs of T-like casings. These casings were attached movably to the main support bars to allow changes in plate separation. By means of small grub screws, the T-casings could be fixed to the main support bars.

We tested two different sets of plates: two with regular square peripheries and other two with irregular peripheries. These irregular plates, shown in figure 1b, are identical to those used in some previous works (Dairay et al. (2015); Obligado et al. (2016); Nedić et al. (2013b)). They have a reference length of $L_b = \sqrt{A} = 64$ mm (with $A$ the frontal area of the plates) and a thickness of 1.25 mm. Only interactions of the turbulent wakes of the same type of plate were studied, therefore we did measurements with two regular and two irregular plates only. The plates were located vertically in the symmetry plane of the wind tunnel, normal to the laminar freestream velocity. In spanwise direction, they were equally spaced to the streamwise symmetry plane. The total blockage of the set-up remains low, close to 4.3%, and therefore we do not consider any blockage corrections to our results.

Eleven different plate separations $S$ were tested: 230, 240, 250, 260, 270, 280, 285, 290, 295, 300 and 305 mm. A right-handed coordinate system serves as reference, with $+x$
pointing downstream, +y pointing to the bottom, and +z pointing in spanwise direction. The origin is set at the centre point of the wind tunnel at the streamwise position of the plates. Thus, x marks the streamwise distance to the plane of the plates. Freestream velocity was kept constant at $U_\infty = 10$ m/s throughout the tests and was controlled and stabilised with a PID feedback system using the static pressure difference across the 9 : 1 wind tunnel contraction and the temperature inside the test-section measured half way along it. At that velocity, the fluctuations around the mean are below 0.1% when the test-section is empty.

Hot-wire anemometry measurements were conducted downstream of the pair of plates using a Dantec Dynamics 55P01 single hot wire, driven by a Dantec StreamLine CTA system. The probe has a Pt-W wire, 5 $\mu$m in diameter, 3 mm long with a sensing length of 1.25 mm. The probe was placed with a levelling laser at the wind tunnel centre line and velocity profiles in streamwise direction were recorded with intervals of 10 mm. The probe can be located as close as 100 mm from the plates and up to 3020 mm away from them. For each probe location, the acquisition time was 60 s with a sampling frequency of 20 kHz. The traverse system is modular, allowing to automatize the acquisition of streamwise profiles over a 540 mm span. It can then be moved to cover different regions of the test section. Therefore, to measure sufficiently long streamwise distances, two or more profiles were recorded for each plate separation $S$. For the particular case, a set of streamwise profiles covering the entire test section were performed for both kinds of plate. Therefore, for $S = 285$ mm, we have access to the whole streamwise evolution of the velocity temporal signal of the streamwise velocities.

To ensure continuity between individual profiles, they overlapped for approximately 100 mm. At the beginning of each single profile, a vertical wake profile was acquired between $-250$ mm $< y < 250$ mm with $\Delta y = 20$ mm to verify $U_\infty$ (acquisition time 30 s, frequency 20 kHz), and a new calibration of the hot-wire was made to account for possible thermal drifts of the wind tunnel. Calibrations were made with a reference from a pitot tube located 50 mm below the hot-wire and for 9 equispaced velocities between 5 and 15 m/s. Temperature was monitored during the streamwise profiles, so it never changed by more than 0.2$^\circ$C.

The acquisition time of the centreline single wire measurements being 60 sec, an order of 1,000 integral time scales at each streamwise position were recorded thereby allowing good large-scale resolution. The Kolmogorov frequency was always smaller than half our sampling frequency (which is 20kHz) and the fluctuations are always below 10% (see figure 2).

3. Results - Plates with irregular edges

3.1. Streamwise profiles and wake scale $x_{12}$

For a plate separation of 285 mm, streamwise velocity measurements along the entire test-section have been conducted. To capture this length, eleven single velocity profiles have been recorded. These measurements yield directly the local streamwise mean velocity $\langle U(x) \rangle$ and the corresponding fluctuating velocity around this mean $u(t)$. Figure 2 shows the streamwise distribution of the velocity fluctuations $u/\langle U(x) \rangle$, with $u$ the standard deviation of $u(t)$.

From figure 2 we identify three different regimes in the flow (highlighted in the figure with different colours and labelled 1, 2 and 3). In region 1, both wakes have not met yet. It is important to remark that the boundaries of the wake are of statistical nature, and the flow can be affected by the generator at radial distances much higher than $\delta$. This
Figure 2. Streamwise distribution of $u/\langle U(x) \rangle$ for the irregular plates at $S = 285\,\text{mm}$.

is the reason why the fluctuations show a monotonic increase in this region. Then, the profile of $u/U_\infty(x)$ suggests that in region 2 both wakes start to significantly interact and their boundaries meet more often than not, so that the velocity fluctuations reach a peak. This change of regime is characterised by a sudden loss of linearity (present for example in the range $x \subset [500; 900] \,\text{mm}$), as better detailed below. Further downstream, in region 3, both wakes are fully merged and the fluctuations exhibit a monotonic decrease.

For a better comprehension of the interactions regimes, we study the properties of the flow at three representative streamwise locations: $x_1 = 500 \,\text{mm}$, $x_2 = 1500 \,\text{mm}$ and $x_3 = 2500 \,\text{mm}$. Figures 3a, b & c show the time-signal of the streamwise fluctuating velocity $u(t)$ at these points. At $x_1$, the velocity signal is almost flat, with some extreme events. The spectrum is not turbulent yet (figure 3d), while the probability density function (PDF) of $u(t)$ (figure 3e) is far from Gaussian, and positively skewed. This behaviour is reminiscent of the findings in Laizet et al. (2015) for a turbulent grid flow, very close to the grid bars. Regions 2 and 3 show a better developed turbulent spectrum and a PDF which is still non-Gaussian, but has become negatively skewed. Figures 2 and 3 therefore suggest that the point $x_{12}$ which demarcates between regions 1 and 2 might be related with the interaction of the two wakes and might be a good candidate for the wake interaction length $x^*$.

In the following, we test whether the wake width of the irregular plates scales according to the non-equilibrium dissipation law or not. To do so, we assume that the average edges of the two wakes meet where their wake extent $n\delta$ (with $n$ close to 2) is equal to $S/2$. Figure 4 visualizes this configuration. Regarding this wake interaction process, we identify the transition from regime 1 to regime 2 (defined as $x_{12}$) as the beginning of significant interactions.

Figure 5b shows an example of the streamwise evolution of the normalised velocity fluctuations for the particular case $S = 270\,\text{mm}$ (others are shown in figure 6a and present a similar shape). Regarding the wake interaction process, we interpret the transition point $x_{12}$ from region 1 to 2 along the centreline streamwise axis to be the beginning of significant interactions between the two wakes in terms of turbulence velocity fluctuations. It can be observed that region 1 is characterised by a linear relation between $u/\langle U(x) \rangle$ and $x$. We identify the streamwise transition point $x_{12}$ as the point where this linearity is lost and use this property to determine $x_{12}$ automatically for every case. To do so, the linear regime is fitted locally with increasing number of points. Each linear fit is continuously extrapolated on all points of the curve and the average quadratic deviation
Figure 3. Streamwise fluctuating velocity time signal \( u(t) \) and its spectra and PDF at different locations for irregular plates at \( S = 285 \) mm. Time signal at \( x_1 = 500 \) mm (a), \( x_2 = 1500 \) mm (b) and \( x_3 = 2500 \) mm (c). Spectra at the three different locations (d), blue lines corresponds to \( x_1 \), the red one to \( x_2 \) and the black one to \( x_3 \). PDF of \( u(t) \) (e) at the same locations and represented with the same colours as in (d). At \( x_1 \), the signal has a skewness of 3.0 and a flatness of 39.0. At \( x_2 \) they are -1.9 and 8.5, respectively. Finally, the signal at \( x_3 \) has a skewness of -1.1 and a flatness of 4.4.

Figure 4. Sketch of the interaction of both wakes when \( 0.5 \cdot S = \eta \cdot \delta \).

is calculated. Plotting the location of lowest quadratic deviation \( x_{\text{min}} \) versus the number of points used for the linear interpolation, the location \( x_{12} \) can easily be detected by the first plateau region within this plot (other plateau’s may exist for larger \( x_{\text{min}} \) due to the change of concavity that occurs in regime 2). An example of this procedure is shown in figures 5a & b.

The streamwise locations found by this automatic procedure are represented by the black squares in figure 6a. A summary of the locations \( x_{12} \) identified by this process is given in table 1. To determine the factor \( \eta \), we use the scaling law of the wake half width:

\[
\frac{\delta(x)}{\theta} = B \left( \frac{x - x_0}{\theta} \right)^\beta
\]

where \( \delta(x) \) is the wake-width at position \( x \), \( \theta \) is momentum thickness of the plate, \( x_0 \) stands for a virtual origin of the plate, and \( B \) and \( \beta \) are fitting parameters. For the
irregular plates used herein, Dairay et al. (2015) found the following values in agreement with the non-equilibrium dissipation law: $\theta = 21$ mm, $x_0/\theta = -5.35$, $B = 0.37$, and $\beta = 0.52$. Using these values and equation 3.1 we can set up equation 3.2 and solve it for $n$ for every plate separation tested. The values thus obtained for $n$ are given in table 1.

![Figure 5](image)

**Figure 5.** Example illustrating the procedure of automatic detection of $x_{12}$. Location of minimum quadratic deviation as a function of the number of points used for the linear fit (a). $x_{12}$ determined and its corresponding linear fit (b).

| $S$ [mm] | 230 240 250 260 270 280 285 290 295 300 305 |
|----------|------------------------------------------------|
| $x_{12}$ [mm] | 670 710 790 890 950 1030 1040 1110 1150 1190 1230 |
| $n$ | 2.26 2.29 2.28 2.24 2.26 2.26 2.29 2.26 2.26 2.26 2.26 |
| $\langle n \rangle$ | 2.26 |
| $\sigma_n/\langle n \rangle$ | 0.0068 |

Perhaps remarkably, all $n$-factors are found to be close to the mean value $\langle n \rangle = 2.26$. This result indicates that significant wake interactions take place where $2.26 \cdot \delta(x)$ is equal to half the plate separation. Moreover, this result is supported by previous findings of Dairay et al. (2015) who conducted vertical velocity profiles at distinct locations downstream of an irregular plate and found self-similar profiles for mean velocity as well as streamwise fluctuations: the radial location $2.26 \delta$ coincides well with relatively high levels of turbulence fluctuations and relatively high cross-stream gradients of the mean streamwise velocity.

This result supports the validity of the non-equilibrium dissipation law in the wake of the irregular plates and determines the beginning of wake interactions on the basis of the non-equilibrium scalings of the wake half width. To underscore this result, figure 6b shows the streamwise evolution of the wake half width with increasing streamwise
Figure 6. (a) Streamwise profile of the velocity fluctuations for all plate separations $S$ in table 1. The black squares represent $x_{12}$, i.e. the $x$-locations of the transition between regimes 1 and 2. (b) Comparison of the wake half width profile obtained for a single irregular plate by Dairay et al. (2015) (blue line) with the results obtained by the wake-interaction setup used here (red line & circles).

position from the plate as obtained for a single plate by Dairay et al. (2015) (blue line). To compare it with the result we just obtained with our two wake interaction set-up, we calculate $\delta_{\text{wake}}(x) = S/(2 \langle n \rangle) \left( \frac{x-x_0}{x_{12}-x_0} \right)^{0.52}$ for $x$ larger than $x_{12}$ and plot it for each plate separation against $x$ normalised by the reference length $L_b = \sqrt{A} = 64$ mm. Again, the results coincide. It is therefore possible to predict the overall level of the streamwise profile of $\delta(x)$, which is a characteristic of the mean flow cross-stream profile of a single wake, by acquiring velocity fluctuation measurements in regions 1 and 2 of a two-wake set-up.

3.2. Higher-Order Moments

In previous studies, wake interactions have been investigated in terms of higher-order moments, such as velocity skewness and flatness (Mazellier & Vassilicos (2010); Laizet et al. (2015)). Figure 7(a) and 7(b) show the skewness and flatness profiles for all plate separations tested. The behavior of both moments is quite similar to the results of Mazellier & Vassilicos (2010); Laizet et al. (2015). Consider that these two studies both investigated 2D, planar wakes of the bars of turbulence grids. In contrast, the wakes in the current study are 3D and axisymmetric. Hence, the similarity of the high-order moments is non-trivial. The detailed analysis of this finding is left for future studies.

Mazellier & Vassilicos (2010) showed that the skewness and flatness profiles of different turbulence grids could be collapsed by scaling the streamwise position with a wake-interaction length scale $x_\ast$. Subsequently, Gomes-Fernandes et al. (2012) collapsed velocity fluctuation profiles of different experiments with an improved wake-interaction length scale $x'_\ast$. In figures 8(a) and 8(b) the skewness and flatness profiles are replotted with the $x$-axis scaled by the length-scale $x_{12}$ discussed earlier. We find that all values of $S$ collapse, with the exception of the smallest separation; $S = 230$ mm. The reason for this exception may be that the wakes meet before having become self-similar or axisymmetric for this low value of $S$.

In the following sub-section, we further test the idea that the length-scale $x_{12}$ is a wake-interaction length scale and show also that it can be used to scale the streamwise profile of the turbulent velocity fluctuations.

3.3. Wake-Interaction Length Scale

As stated above, Mazellier & Vassilicos (2010) and Gomes-Fernandes et al. (2012) proposed formulae for a wake-interaction length scale characterising the interaction
of planar wakes emanating from fractal and regular grids. This length-scale is easily constructed for any free-shear flow. In our case, two axisymmetric wakes can be surely expected to interact at a streamwise distance \( x^* \) from the plates where \( \delta \) is equal to half the distance between the centres of the plates (defined as \( S \) in this work). This implies that the wake interaction length \( x^* \) scales as \( x^* \propto S^2 \) on the basis of equation 1.4 resulting in

\[
\frac{x^* - x_0}{\theta} \propto \left( \frac{S}{2\theta} \right)^2.
\]

A crucial step in the wake interaction approach of Gomes-Fernandes et al. (2012) is the neglect of the virtual origin \( x_0 \) because the “distance downstream where the wakes meet is very much larger than the virtual origin. We therefore ignore the virtual origin \( x_0 \) and effectively set it equal to zero.” However, Nedic (2013) showed that simply setting the virtual origin to zero is not valid for centreline measurements of axisymmetric wakes and can yield false results. Nevertheless, since the current measurements are conducted between 1.8 and 2.7 characteristic lengths off the centerlines of the wakes, the influence of the virtual origin \( x_0 \) is indeed negligible. To verify this, we consider the derivative of the velocity deficit with respect to the virtual origin \( \partial(u_0/U_\infty)/\partial(x_0/\theta) \). According
to Cafiero et al. (2020), the evolution of the velocity deficit in the streamwise direction \(x\) and the radial direction \(y\) for both sets of plates in the relevant streamwise range is,

\[
\frac{u_0}{U_\infty} = A \left( \frac{x - x_0}{\theta} \right)^\alpha e\left(\frac{-\alpha y^2}{\delta^2}\right). \tag{3.4}
\]

The differentiation yields

\[
\frac{\partial \frac{u_0}{U_\infty}}{\partial \frac{x_0}{\theta}} = -A\alpha \left( \frac{x - x_0}{\theta} \right)^{\alpha-1} e\left(\frac{-\alpha y^2}{\delta^2}\right) \\
+ A\alpha \left( \frac{x - x_0}{\theta} \right)^\alpha e\left(\frac{-\alpha y^2}{\delta^2}\right) \left[-2a\beta y^2(B\theta)^{-2} \left( \frac{x - x_0}{\theta} \right)^{-2\beta-1}\right]. \tag{3.5}
\]

Setting the non-equilibrium values of the scaling exponents \(\alpha = -1\) and \(\beta = 1/2\) (see Dairay et al. (2015)), the result is that

\[
\frac{\partial \frac{u_0}{U_\infty}}{\partial \frac{x_0}{\theta}} = -A \left( \frac{x - x_0}{\theta} \right)^{-2} e\left(\frac{-\alpha y^2}{\delta^2}\right) \left[1 - ay^2\right]. \tag{3.6}
\]

The function \(f(y/\delta)\) determines the radial distribution of the derivative \(\partial(u_0/U_\infty)/\partial(x_0/\theta)\). A plot of \(f(y/\delta)\) is given in figure 9 with \(a = 0.6\) (Nedić et al. (2013b)). As mentioned above, we identify the meeting of the two plate wakes at the radial location \(y = 2.26\delta\). At this radial location, the modulus of the derivative has dropped to approximately 9.6% of its centerline value. Hence, the influence of the virtual origin \(x_0/\theta\) on the velocity deficit is marginal for small variations and can be neglected.

We can then define a modified wake interaction length for two axisymmetric plates in the non-equilibrium regime by,

\[
x'_* = \phi \frac{S^2}{4\theta}. \tag{3.7}
\]

where \(\phi\) is a constant that depends on the geometry of the plates and accounts for the contributions of \(B\) and \(\langle n \rangle\).

The wake-interaction length scale \(x'_*\) based on equation 3.7 is given in table 2 for
different values of $S$ together with the length scale $x_{12}$ determined earlier. It can be seen that $x_{12}$ and $x'_* ∼ \sqrt{2/3} R_0 F(x/x_{12})$, where $R_0 = U_\infty u_0 (d/dx \delta(x))$. Thus, we attempt the scaling $u' \sim \sqrt{U_\infty u_0 (d/dx \delta(x)) F(x/x_{12})}$ where $F$ is a dimensionless function of $x/x_{12}$. In this work we did not measure the values of $u_0(x)$ and $\delta(x)$ and we therefore used their fits from Dairay et al. (2015) for the irregular plates.

In figure 10 we show the velocity fluctuations normalised according to $u' \sim \sqrt{U_\infty u_0 (d/dx \delta(x)) F(x/x_{12})}$. Most of the profiles collapse fairly well. Consistently with the discussion in sub-section 3.2, the collapse is worse for smaller values of $S$.

### 4. Results - Regular Square Plates

In addition to the two plates with irregular peripheries, two regular square plates with the same area ($\sqrt{A} = 64$ mm) were also investigated. Again eleven different separations were set and the same measurements as for the irregular plates were conducted. Figure 11(a) shows the streamwise development of the velocity fluctuations for all separations. The shapes of these profiles are similar to those for the irregular plates.

| $S$ [mm] | 230 | 240 | 250 | 260 | 270 | 280 | 285 | 290 | 295 | 300 | 305 |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $x_{12}$ [mm] | 670 | 710 | 790 | 890 | 950 | 1030 | 1040 | 1110 | 1150 | 1190 | 1230 |
| $x'_*$ [mm] | 630 | 686 | 744 | 805 | 868 | 933 | 967 | 1001 | 1036 | 1071 | 1107 |
| $x'_*/x_{12}$ | 0.94 | 0.97 | 0.94 | 0.90 | 0.91 | 0.91 | 0.93 | 0.90 | 0.90 | 0.90 | 0.90 |
Figure 11. (a) Streamwise distribution of the velocity fluctuations for all square plate’s separations given in Table 3. (b) Comparison of the wake-width distribution found for a single regular plate by Obligado et al. (2016) (blue line) and the results gained by the wake-interaction setup used herein (red line & circles). The parameters of equation 3.1 corresponding to square plates are $\theta = 19.7 \text{mm}$, $x_0/\theta = -6.20$, $B = 0.494$, and $\beta = 0.50$ (Obligado et al. (2016)).

Table 3. Overview of the length-scale $x_{12}$ and other parameters characterizing the wake’s interaction for the square plates

| $S$ [mm] | $230$ | $240$ | $250$ | $260$ | $270$ | $280$ | $285$ | $290$ | $295$ | $300$ | $305$ |
|---------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $x_{12}$ [mm] | $540$ | $570$ | $630$ | $660$ | $730$ | $800$ | $830$ | $860$ | $900$ | $980$ | $1050$ |
| $n$ | $2.11$ | $2.15$ | $2.14$ | $2.17$ | $2.15$ | $2.14$ | $2.14$ | $2.14$ | $2.09$ | $2.05$ |
| $\langle n \rangle$ | $2.13$ |
| $\sigma_n/\langle n \rangle$ | $0.016$ |
| $x_\prime$ [mm] | $671$ | $731$ | $793$ | $858$ | $925$ | $995$ | $1031$ | $1067$ | $1104$ | $1142$ | $1181$ |
| $x_\prime/x_{12}$ | $1.24$ | $1.28$ | $1.26$ | $1.30$ | $1.27$ | $1.24$ | $1.24$ | $1.24$ | $1.23$ | $1.17$ | $1.12$ |

Again, we determine the length-scale $x_{12}$ at the point of transition from regime 1 to regime 2 in the turbulent fluctuations streamwise profile (see Figure 11(a)). The results of this automated procedure are given in Table 3. The $n$-factor is once again very similar for all values of $S$, but slightly different from the irregular plates: $\langle n \rangle = 2.13$. Figure 11(b) compares the wake-width based on the identified length-scale $x_{12}$ with the results of the single-plate measurements of Obligado et al. (2016). Equation 3.7 yields the modified wake-interaction length scale $x_\prime$ for these regular square plates (see Table 3). As found for the irregular plates, our length-scale $x_{12}$ is a multiple of $x_\prime$, in this regular case with $x_{12} = 1.24 \cdot x_\prime$.

To evaluate the collapse capability of $x_{12}$, we scale the streamwise profile of the higher-order moments (skewness and flatness) of the velocity fluctuations with $x_{12}$. In Figures 12(a) and 12(b) we plot the original skewness and flatness profiles and in Figures 12(c) and 12(d) we plot the streamwise skewness and flatness profiles scaled with $x_{12}$. Some collapse, particularly for the larger values of $S$ is, once again, observed.

Finally, in Figure 13, $x_{12}$ is used to collapse the streamwise profile of velocity fluctuations. Streamwise velocity fluctuations are scaled according to $u' \sim$
Figure 12. (a, b) Unscaled skewness and flatness profiles for all square plate separations tested. (c, d) Skewness and flatness profiles scaled with $x_{12}$.

Figure 13. Profiles of $u'$ obtained for plates with square peripheries normalised according to equation $u' \sim \sqrt{2/3} R_0 F(x/x_{12})$ where $R_0 = U_\infty u_0 (d/dx \delta(x))$. A fairly good collapse of the profiles is obtained again except for the smaller values of $S$. 

$\sqrt{U_\infty u_0 (d/dx \delta(x))} F(x/x_{12})$ as for the irregular plates.
5. Results - Comparison irregular and regular plates

The major results from the irregular and the regular square plates are now compared. We focus on $S \geq 260$mm, where the profile collapse is best in terms of $x_{12}$. In figure 14(a) and 14(b) we plot the streamwise profiles of the skewness and the flatness of the turbulent velocity fluctuations scaled by the interaction length scale $x_{12}$ for both the irregular and the regular set of square plates. A reasonable collapse is achieved. In figure 15, the streamwise profiles of the scaled velocity fluctuations are displayed. It can be seen that the maxima of $u'/\sqrt{(2/3R_0)}$ (where $R_0 = U_\infty u_0 (d/dx \delta(x))$) are higher for the regular plates than for the irregular ones. The scaling $u' \sim \sqrt{U_\infty u_0 (d/dx \delta(x))} F(x/x_{12})$ seems to hold for both sets of plates, but the proportionality constant is different depending on plate geometry.

6. Conclusions

In this work we have studied experimentally via hot-wire anemometry the interaction between two turbulent axisymmetric wakes in the wind tunnel. The wake generators
were pairs of plates, one pair with plates of regular peripheries and one pair with plates of irregular peripheries. We first pointed out that the wake interaction length defined from $2\delta(x^*) \propto S$ using the wake half width $\delta$ and the separation $S$ between the two plates depends on the nature of the turbulent kinetic energy cascade. For the case of the standard Richardson-Kolmogorov energy cascade, $x^*$ evolves as $S^3$ while a non-equilibrium cascade implies $x^* \propto S^2$.

By acquiring streamwise profiles for different plate separations and identifying the wake interaction length for each separation it is possible to show that the interaction between the wakes is consistent with non-equilibrium scalings. The profile of the streamwise distribution of the normalised velocity fluctuations $u'/\langle U(x) \rangle$ is used to characterise the interaction of two wakes, as it shows three clearly defined regions: a first region where the wakes have not significantly met yet and the flow is highly intermittent and a second region where the wakes start to merge and where $u'/\langle U(x) \rangle$ reaches a maximum value. Finally, in the third region both wakes are fully merged and the fluctuations exhibit a monotonic decrease with streamwise distance.

We have therefore proposed to identify the wake interaction length with the streamwise point $x_{12}$ where the flow goes from region 1 to region 2. We found that the values of $x_{12}$ are indeed consistent with the non-equilibrium cascade, as $x_{12} \propto S^2$. We also defined a wake interaction length-scale independently from $x_{12}$ and have shown that it is proportional to $x_{12}$. We then used $x_{12}$ to successfully collapse the streamwise profiles of the second, third and fourth moments of the streamwise fluctuating velocity.

Following previous theoretical developments (Townsend [1976]; George [1989]; Dairay et al. [2015]) which demonstrated that the turbulent kinetic energy and the Reynolds shear stress both scale with streamwise distance as $U_\infty u_0 (d/dx \delta(x))$ we have proposed the scaling $u' \sim \sqrt{U_\infty u_0 (d/dx \delta(x))} F(x/x_{12})$ where $F$ is a dimensionless function of $x/x_{12}$. This scaling holds for both sets of plates, but the proportionality constant is larger for the regular than the irregular plates.

Our results and analysis could contribute to the study of interactions between wakes of neighboring wind turbines and the design of wind farms.

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