Neutrino masses and mixing within a $SU(3)$ family symmetry model with one or two light sterile neutrinos

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We report a global fit of parameters for fermion masses and mixing, including light sterile neutrinos, within a local vector $SU(3)$ family symmetry model. In this scenario, ordinary heavy fermions, top and bottom quarks and tau lepton, become massive at tree level from Dirac See-saw mechanisms implemented by the introduction of a new set of $SU(2)_L$ weak singlet vector-like fermions, $U, D, E, N$, with $N$ a sterile neutrino. The $N_{L,R}$ sterile neutrinos allow the implementation of a $8 \times 8$ general tree level See-saw Majorana neutrino mass matrix with four massless eigenvalues. Hence, light fermions, including light neutrinos obtain masses from one loop radiative corrections mediated by the massive $SU(3)$ gauge bosons. This BSM model is able to accommodate the known spectrum of quark masses and mixing in a $4 \times 4$ non-unitary $V_{CKM}$ as well as the charged lepton masses. The explored parameter space region provide the vector-like fermion masses: $M_D \approx 914.365$ GeV, $M_U \approx 1.5$ TeV, $M_E \approx 5.97$ TeV, $SU(3)$ family gauge boson masses of $O(1 - 10)$ TeV, the neutrino masses $(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = (0, 0.0085, 0.049, 0.22, 3.21, 1749.96, 1 \times 10^8, 1 \times 10^9)$ eV, with the squared neutrino mass differences: $m^2_2 - m^2_1 \approx 7.23 \times 10^{-5}$ eV$^2$, $m^2_3 - m^2_1 \approx 2.4 \times 10^{-3}$ eV$^2$, $m^2_4 - m^2_1 \approx 0.049$ eV$^2$, $m^2_5 - m^2_1 \approx 10.3$ eV$^2$. We also show the corresponding $U_{PMNS}$ lepton mixing matrix. However, the neutrino mixing angles are extremely sensitive to parameter space region, and an improved and detailed analysis is in progress.

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I. INTRODUCTION

The standard picture of three flavor neutrinos has been successful to account for most of the neutrino oscillation data. However, several experiments have reported new experimental results, on neutrino mixing [1], on large $\theta_{13}$ mixing from Daya Bay [2], T2K [3], MINOS [4], DOUBLE CHOOZ [5], and RENO [6], implying a deviation from TBM [7] scenario. In addition, the recent experimental results from the LSND and MiniBooNe short-baseline neutrino oscillation experiments, provide indications in favor of the existence of light sterile neutrinos in the eV-scale, in order to explain the tension in the interpretation of these data [8, 9].

The strong hierarchy of quark and charged lepton masses and quark mixing have suggested to many model building theorists that light fermion masses could be generated from radiative corrections [10], while those of the top and bottom quarks and the tau lepton are generated at tree level. This may be understood as the breaking of a symmetry among families, a horizontal symmetry. This symmetry may be discrete [11], or continuous, [12]. The radiative generation of the light fermions may be mediated by scalar particles as it is proposed, for instance, in references [13, 14] and the author in [15], or also through vectorial bosons as it happens for instance in "Dynamical Symmetry Breaking" (DSB) and theories like " Extended Technicolor " [16].

In this report, we address the problem of fermion masses and quark mixing within an extension of the SM introduced by the author in [17], which includes a vector gauged $SU(3)$ flavor symmetry commuting with the SM group. In previous reports [19] we showed that this model has the properties to accommodate a realistic spectrum of charged fermion masses and quark mixing. We introduce a hierarchical mass generation mechanism in which the light fermions obtain masses through one loop radiative corrections, mediated by the massive bosons associated to the $SU(3)$ family symmetry that is spontaneously broken, while the masses for the top and bottom quarks as well as for the tau lepton, are generated at tree level from "Dirac See-saw" [20] mechanisms implemented by the introduction of a new generation of $SU(2)_L$ weak singlets vector-like fermions.

Recently, some authors have pointed out interesting features regarding the possibility of the existence of vector-like matter, both from theory and current experiments [23]. From the fact that the vector-like quarks do not couple to the $W$ boson, the mixing of $U$ and $D$ vector-like quarks with the SM quarks gives rise to an extended $4 \times 4$ non-unitary CKM quark mixing matrix. It has pointed out for some authors that these vector-like fermions are weakly constrained from Electroweak Precision Data (EWPD) because they do not break directly the custodial symmetry, then main experimental constraints on the vector-like matter come from the direct production bounds, and their implications on flavor physics. See the ref. [23] for further details on constraints for vector-like fermions. Theories and models with extra matter may also provide attractive scenarios for present cosmological problems, such as candidates for the nature of the Dark Matter [21, 22].

In this article, we report for the first time a global fit of the free parameters of the $SU(3)$ family symmetry model to accommodate quark and lepton masses and mixing, including light sterile neutrinos.

II. MODEL WITH $SU(3)$ FLAVOR SYMMETRY

A. Fermion content

We define the gauge group symmetry $G \equiv SU(3) \otimes G_{SM}$, where $SU(3)$ is a flavor symmetry among families and $G_{SM} \equiv SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is the "Standard Model" gauge group, with $g_s$, $g$ and $g'$ the corresponding coupling constants. The content of fermions assumes the ordinary quarks and leptons assigned under $G$ as:

$$\psi_q^o = (3, 3, 2, 1/3)_L \quad , \quad \psi_u^o = (3, 3, 1, 4/3)_R \quad , \quad \psi_d^o = (3, 3, 1, -2/3)_R$$

$$\psi_l^o = (3, 1, 2, -1)_L \quad , \quad \psi_e^o = (3, 1, 1, -2)_R$$
where the last entry corresponds to the hypercharge $Y$, and the electric charge is defined by $Q = T_{3L} + \frac{1}{3}Y$. The model also includes two types of extra fermions: Right handed neutrinos $\Psi^o = (3,1,1,0)_{R}$, and the $SU(2)_L$ singlet vector-like fermions

$$U^o_{L,R} = (1,3,1, \frac{4}{3}) \quad , \quad D^o_{L,R} = (1,3,1, -\frac{2}{3}),$$

$$N^o_{L,R} = (1,1,1,0) \quad , \quad E^o_{L,R} = (1,1,1, -2).$$

The transformation of these vector-like fermions allows the mass invariant mass terms

$$M_U \bar{U}_L^o U_R^o + M_D \bar{D}_L^o D_R^o + M_E \bar{E}_L^o E_R^o + h.c.,$$

and

$$m_D \bar{N}_{L,R}^o N_{R,L}^o + m_L \bar{N}_{L,R}^o (N_{R,L}^o)^c + m_R \bar{N}_{R,L}^o (N_{L,R}^o)^c + h.c$$

The above fermion content make the model anomaly free. After the definition of the gauge symmetry group and the assignment of the ordinary fermions in the usual form under the standard model group and in the fundamental 3-representation under the $SU(3)$ family symmetry, the introduction of the right-handed neutrinos is required to cancel anomalies. The $SU(2)_L$ weak singlets vector-like fermions have been introduced to give masses at tree level only to the third family of known fermions through Dirac See-saw mechanisms. These vector like fermions play a crucial role to implement a hierarchical spectrum for quarks and charged lepton masses, together with the radiative corrections.

### III. $SU(3)$ FAMILY SYMMETRY BREAKING

To implement a hierarchical spectrum for charged fermion masses, and simultaneously to achieve the SSB of $SU(3)$, we introduce the flavon scalar fields: $\eta_i, i = 2,3$, transforming under the gauge group as $(3,1,1,0)$ and taking the "Vacuum Expectation Values" (VEV's):

$$\langle \eta_3 \rangle^T = (0,0,\Lambda_3), \quad \langle \eta_2 \rangle^T = (0, \Lambda_2, 0).$$

The above scalar fields and VEV’s break completely the $SU(3)$ flavor symmetry. The corresponding $SU(3)$ gauge bosons are defined in Eq.(29) through their couplings to fermions. Thus, the contribution to the horizontal gauge boson masses from Eq.(30) read

- $\eta_3: \quad \frac{g_H \Lambda^2}{2} (Y_2^+ Y_2^- + Y_3^+ Y_3^-) + g_{H_3}^2 \Lambda^2 \frac{Z_2^2}{3}$
- $\eta_2: \quad \frac{g_{H_2} \Lambda^2}{2} (Y_1^+ Y_1^- + Y_3^+ Y_3^-) + \frac{g_{H_2} \Lambda^2}{4} (Z_1^2 + \frac{Z_2^2}{3} - 2Z_1 Z_3)$

Therefore, neglecting tiny contributions from electroweak symmetry breaking, we obtain the gauge boson mass terms

$$M_1^2 Y_1^+ Y_1^- + M_2^2 \frac{Z_1^2}{2} + \left( \frac{1}{3} M_1^2 + \frac{1}{3} M_1^2 \right) \frac{Z_2^2}{2} - M_1^2 \frac{Z_1}{\sqrt{3}} Z_1 + M_2^2 Y_2^+ Y_2^- + (M_1^2 + M_2^2) Y_3^+ Y_3^-$$

$$M_1^2 = \frac{g_{H_2} \Lambda^2}{2}, \quad M_2^2 = \frac{g_{H_3} \Lambda^2}{2}, \quad M_3^2 = M_1^2 + M_2^2$$

From the diagonalization of the $Z_1 - Z_2$ squared mass matrix, we obtain the eigenvalues
\[ M^2 = \frac{2}{3} \left( M_1^2 + M_2^2 - \sqrt{(M_2^2 - M_1^2)^2 + M_1^2 M_2^2} \right) , \quad M^2_+ = \frac{2}{3} \left( M_1^2 + M_2^2 + \sqrt{(M_2^2 - M_1^2)^2 + M_1^2 M_2^2} \right) \]  

(8)

\[ M_1^2 Y_1^+ Y_1^- + M_2^2 \frac{Z_2^2}{2} + M_2^2 \frac{Z_2^2}{2} + M_2^2 Y_2^+ Y_2^- + (M_1^2 + M_2^2) Y_3^+ Y_3^- \]  

(9)

where

\[
\begin{pmatrix}
Z_1 \\
Z_2
\end{pmatrix} = \begin{pmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{pmatrix} \begin{pmatrix}
Z_- \\
Z_+
\end{pmatrix}
\]

(10)

\[
\cos \phi \sin \phi = \frac{\sqrt{3}}{4} \frac{1}{\sqrt{(M_2^2 - M_1^2)^2 + M_2^2}}
\]

with the hierarchy \( M_1, M_2 \gg M_W \).

**IV. ELECTROWEAK SYMMETRY BREAKING**

Recently ATLAS[25] and CMS[26] at the Large Hadron Collider announced the discovery of a Higgs-like particle, whose properties, couplings to fermions and gauge bosons will determine whether it is the SM Higgs or a member of an extended Higgs sector associated to a BSM theory. The electroweak symmetry breaking in the \( SU(3) \) family symmetry model involves the introduction of two triplets of \( SU(2)_L \) Higgs doublets.

To achieve the spontaneous breaking of the electroweak symmetry to \( U(1)_Q \), we introduce the scalars: \( \Phi^u = (3, 1, 2, -1) \) and \( \Phi^d = (3, 1, 2, +1) \), with the VEVs: \( \langle \Phi^u \rangle^T = (\langle \Phi^u_1 \rangle, \langle \Phi^u_2 \rangle, \langle \Phi^u_3 \rangle) \), \( \langle \Phi^d \rangle^T = (\langle \Phi^d_1 \rangle, \langle \Phi^d_2 \rangle, \langle \Phi^d_3 \rangle) \), where \( T \) means transpose, and

\[
\langle \Phi^u_i \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_i \\ 0 \end{pmatrix}, \quad \langle \Phi^d_i \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ V_i \end{pmatrix} .
\]

(11)

The contributions from \( \langle \Phi^u \rangle \) and \( \langle \Phi^d \rangle \) generate the \( W \) and \( Z \) gauge boson masses

\[
\frac{g^2}{4} (v_u^2 + v_d^2) W^+ W^- + \frac{(g^2 + g'^2)}{8} (v_u^2 + v_d^2) Z_0^2
\]

(12)

\[ v_u^2 = v_1^2 + v_2^2 + v_3^2 , \quad v_d^2 = V_1^2 + V_2^2 + V_3^2 . \] Hence, if we define as usual \( M_W = \frac{1}{2} g v \), we may write \( v = \sqrt{v_u^2 + v_d^2} \approx 246 \) GeV.

**V. TREE LEVEL NEUTRINO MASSES**

Now we describe briefly the procedure to get the masses for ordinary fermions. The analysis for quarks and charged leptons has already discussed in [19]. Here, we introduce the procedure for neutrinos.
After electroweak symmetry breaking, we obtain in the interaction basis $\Psi$ couplings $h$ and the quarks and leptons global symmetry is:

\[
SU(3)_q \otimes SU(3)_{u_R} \otimes SU(3)_{d_R} \otimes SU(3)_{l_L} \otimes SU(3)_{e_R} \otimes SU(3)_{e_R}
\]

(A) Tree level Dirac neutrino masses

With the fields of particles introduced in the model, we may write the Dirac type gauge invariant Yukawa couplings

\[
h_D \tilde{\Psi}_u \Phi^u N_R^c + h_2 \tilde{\Psi}_\nu \eta_2 N_L^c + h_3 \tilde{\Psi}_\nu \eta_3 N_L^c + M_D \tilde{N}_L^c N_R^c + h.c.
\]

$h_D$, $h_2$, and $h_3$ are Yukawa couplings, and $M_D$ a Dirac type, invariant neutrino mass for the sterile neutrinos $N_{L,R}^c$. After electroweak symmetry breaking, we obtain in the interaction basis $\Psi^{\nu T}_{\nu_{L,R}} = (\nu_e^c, \nu_\mu^c, \nu_\tau^c, N^c_{L,R})$, the mass terms

\[
h_D \left[ v_1 \nu_1^{eL} + v_2 \nu_2^{\mu L} + v_3 \nu_3^{\tau L} \right] N_R^c + \left[ h_2 \Lambda_2 \nu_2^{\mu R} + h_3 \Lambda_3 \nu_3^{\tau R} \right] N_L^c + M_D \tilde{N}_L^c N_R^c + h.c.,
\]

(B) Tree level Majorana masses:

Since $N_{L,R}^c$, Eq. [2], are completely sterile neutrinos, we may also write the left and right handed Majorana type couplings

\[
h_L \tilde{\Psi}_L^c \Phi^\nu (N_L^c)^c + m_L \tilde{N}_L^c (N_L^c)^c + h.c
\]

and

\[
h_{2R} \tilde{\Psi}_\nu \eta_2 (N_R^c)^c + h_{3R} \tilde{\Psi}_\nu \eta_3 (N_R^c)^c + m_R \tilde{N}_R^c (N_R^c)^c + h.c,
\]

respectively. After spontaneous symmetry breaking, we also get the left handed and right handed Majorana mass terms

\[
h_L \left[ v_1 \nu_1^{eL} + v_2 \nu_2^{\mu L} + v_3 \nu_3^{\tau L} \right] (N_L^c)^c + m_L \tilde{N}_L^c (N_L^c)^c + h.c.,
\]

\[
+ \left[ h_{2R} \Lambda_2 \nu_2^{\mu R} + h_{3R} \Lambda_3 \nu_3^{\tau R} \right] (N_R^c)^c + m_R \tilde{N}_R^c (N_R^c)^c + h.c.,
\]

Thus, in the basis $\Psi^{\nu T} = (\nu_e^c, \nu_\mu^c, \nu_\tau^c, \nu_{eL}^c, \nu_{\mu L}^c, \nu_{\tau L}^c, \nu_{eR}^c, \nu_{\mu R}^c, \nu_{\tau R}^c, N_{L,R}^c, (N_{L,R}^c)^c)$, the Generic $8 \times 8$ tree level Majorana mass matrix for neutrinos $M_{\nu}$, from Table IV, reads

\[
M_{\nu} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & \alpha_1 & a_1 \\
0 & 0 & 0 & 0 & 0 & \alpha_2 & a_2 \\
0 & 0 & 0 & 0 & 0 & \alpha_3 & a_3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_2 & \beta_2 \\
0 & 0 & 0 & 0 & 0 & b_3 & \beta_3 & m_L & m_D \\
\alpha_1 & \alpha_2 & \alpha_3 & 0 & b_2 & b_3 & m_L & m_D & m_R
\end{pmatrix}
\]

Diagonalization of $M_{\nu}$, Eq. [20], yields four zero eigenvalues, associated to the neutrino fields: $a p = \sqrt{a_1^2 + a_2^2}$
\[
\begin{array}{cccccccc}
\nu_{eL}^c & (\nu_{\mu L}^c)^c & (\nu_{eR}^c)^c & \nu_{\mu R}^c & \nu_{\tau R}^c & (N_L^c)^c & N_R^c \\
0 & 0 & 0 & 0 & 0 & h_{Lv_1} & h_{Dv_1} \\
0 & 0 & 0 & 0 & 0 & h_{Lv_2} & h_{Dv_2} \\
0 & 0 & 0 & 0 & 0 & h_{Lv_3} & h_{Dv_3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & h_{2A_2} & h_{2R}\Lambda_2 \\
0 & 0 & 0 & 0 & 0 & h_{3A_3} & h_{3R}\Lambda_3 \\
h_{Lv_1} & h_{Lv_2} & h_{Lv_3} & 0 & h_{2A_2} & h_{3A_3} & m_L & M_D \\
h_{Dv_1} & h_{Dv_2} & h_{Dv_3} & 0 & h_{2R}\Lambda_2 & h_{3R}\Lambda_3 & M_D & m_R \\
\end{array}
\]

**TABLE I: Tree Level Majorana masses**

\[
\frac{a_2}{a p} \nu_{eL}^c - \frac{a_1}{a p} \nu_{\mu L}^c , \quad \frac{a_1 a_3}{a p a} \nu_{eL}^c + \frac{a_2 a_3}{a p a} \nu_{\mu L}^c - \frac{a p}{a} \nu_{\tau L}^c
\]

assumed for simplicity, \( \frac{h_{2R}}{h_{3R}} = \frac{h_{2L}}{h_{3L}} \), the Characteristic Polynomial for the nonzero eigenvalues of \( M^o_\nu \) reduce to the one of the matrix \( m_4 \), Eq. (21), where

\[
m_4 = \begin{pmatrix}
0 & 0 & \alpha & a \\
0 & 0 & b & \beta \\
\alpha & b & m_L & m_D \\
a & \beta & m_D & m_R \\
\end{pmatrix}, \quad U_4 = \begin{pmatrix}
u_{11} & u_{12} & u_{13} & u_{14} \\
u_{21} & u_{22} & u_{23} & u_{24} \\
u_{31} & u_{32} & u_{33} & u_{34} \\
u_{41} & u_{42} & u_{43} & u_{44} \\
\end{pmatrix}
\]

(21)

\[
a = \sqrt{a_1^2 + a_2^2 + a_3^2}, \quad \alpha = \sqrt{\alpha_1^2 + \alpha_2^2 + \alpha_3^2},
\]

\[
b = \sqrt{b_2^2 + b_3^2}, \quad \beta = \sqrt{\beta_2^2 + \beta_3^2}
\]

\[
U_4^T m_4 U_4 = \text{Diag}(m_5^o, m_6^o, m_7^o, m_8^o) \equiv d_4 \quad m_4 = U_4 d_4 U_4^T
\]

(22)

Eq. (22) impose the constrains

\[
u_{11}^2 m_5^o + u_{12}^2 m_6^o + u_{13}^2 m_7^o + u_{14}^2 m_8^o = 0
\]

(23)

\[
u_{21}^2 m_5^o + u_{22}^2 m_6^o + u_{23}^2 m_7^o + u_{24}^2 m_8^o = 0
\]

(24)

\[
u_{11} u_{21} m_5^o + u_{12} u_{22} m_6^o + u_{13} u_{23} m_7^o + u_{14} u_{24} m_8^o = 0
\]

(25)
corresponding to the \((m_4)^{11} = (m_4)^{22} = (m_4)^{12} = 0\) zero entries, respectively.

In this form, we diagonalize \(M_\nu\) by using the orthogonal matrix

\[
U_\nu^\alpha = \begin{pmatrix}
\frac{a_1}{a_0} & \frac{a_1}{a_0} & 0 & 0 & \frac{a_1}{a_0} & \frac{a_1}{a_0} & \frac{a_1}{a_0} & \frac{a_1}{a_0} & \frac{a_1}{a_0} & \frac{a_1}{a_0}
-\frac{a_1}{a_0} & \frac{a_1}{a_0} & 0 & 0 & \frac{a_1}{a_0} & \frac{a_1}{a_0} & \frac{a_1}{a_0} & \frac{a_1}{a_0} & \frac{a_1}{a_0} & \frac{a_1}{a_0}
0 & -\frac{a_0}{a_0} & 0 & 0 & \frac{a_0}{a_0} & \frac{a_0}{a_0} & \frac{a_0}{a_0} & \frac{a_0}{a_0} & \frac{a_0}{a_0} & \frac{a_0}{a_0}
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
& 0 & 0 & \frac{b_1}{b_1} & \frac{b_1}{b_1} & \frac{b_1}{b_1} & \frac{b_1}{b_1} & \frac{b_1}{b_1} & \frac{b_1}{b_1} & \frac{b_1}{b_1} & \frac{b_1}{b_1}
0 & 0 & 0 & -\frac{b_0}{b_0} & \frac{b_0}{b_0} & \frac{b_0}{b_0} & \frac{b_0}{b_0} & \frac{b_0}{b_0} & \frac{b_0}{b_0} & \frac{b_0}{b_0}
0 & 0 & 0 & 0 & u_{31} & u_{32} & u_{33} & u_{34} & u_{41} & u_{42}
0 & 0 & 0 & 0 & u_{41} & u_{42} & u_{43} & u_{44} & u_{41} & u_{42}
\end{pmatrix}
\]

\[
(U_\nu^\alpha)^T M_\nu U_\nu^\alpha = \text{Diag}(0, 0, 0, 0, m_0^0, m_0^0, m_0^0, m_0^0)
\]

Notice that the first four columns in \(U_\nu^\alpha\) correspond to the four massless eigenvectors. Hence, the tree level mixing, \(U_\nu^\alpha\), depends on the ordering we define for these four degenerated massless eigenvectors. However, it turns out that the final mixing product \(U_\nu^\alpha U_\nu\), as well as the final mass eigenvalues are independent of the choice of this ordering.

VI. ONE LOOP NEUTRINO MASSES

After tree level contributions the fermion global symmetry is broken down to:

\[
SU(2)_{l_L} \otimes SU(2)_{u_R} \otimes SU(2)_{d_R} \otimes SU(2)_{l_L} \otimes SU(2)_{u_R} \otimes SU(2)_{d_R} \otimes SU(2)_{l_R} \otimes SU(2)_{e_R}
\]

Therefore, in this scenario light neutrinos may get extremely small masses from radiative corrections mediated by the \(SU(3)\) heavy gauge bosons.

A. One loop Dirac Neutrino masses

After the breakdown of the electroweak symmetry, neutrinos may get tiny Dirac mass terms from the generic one loop diagram in Fig. 1. The internal fermion line in this diagram represent the tree level see-saw mechanisms, Eqs. [14-19]. The vertices read from the \(SU(3)\) family symmetry interaction Lagrangian

\[
i\mathcal{L}_{\text{int}} = \frac{gH}{2} (\bar{e}^\alpha \gamma_{\mu} e^\alpha - \bar{\mu}^\alpha \gamma_{\mu} \mu^\alpha) Z^\mu_1 + \frac{gH}{2\sqrt{3}} (\bar{e}^\alpha \gamma_{\mu} e^\alpha + \bar{\mu}^\alpha \gamma_{\mu} \mu^\alpha - 2\bar{\tau}^\alpha \gamma_{\mu} \tau^\alpha) Z^\mu_2
+ \frac{gH}{\sqrt{2}} (\bar{e}^\alpha \gamma_{\mu} \mu^\alpha Y^+_1 + \bar{\mu}^\alpha \gamma_{\mu} \tau^\alpha Y^+_2 + \bar{\mu}^\alpha \gamma_{\mu} \tau^\alpha Y^+_3 + \text{h.c.})
\]

The contribution from these diagrams may be written as
8

\[ c_Y \alpha_H \frac{\alpha_H}{\pi} m_\nu(M_Y)_{ij}, \quad \alpha_H = \frac{g_H^2}{4\pi}, \]

\[ m_\nu(M_Y)_{ij} = \sum_{k=5,6,7,8} m_k^0 U_{ik}^o U_{jk}^o f(M_Y, m_k^0) \]

and \( f_{Y_k} = \frac{M_Y^2 - m_{Y_k}^2}{M_Y^2 - m_{Y_k}^2} \ln \frac{M_Y^2}{m_{Y_k}^2} \approx \ln \frac{M_Y^2}{m_{Y_k}^2} \)

\[ \nu_{jR} \quad \nu_{kR} \quad M \quad \nu_{iL} \]

\[ Y \]

\[ < \eta_k > \quad < \Phi^u > \]

FIG. 1: Generic one loop diagram contribution to the Dirac mass term \( m_{ij} \bar{\nu}_{iL} \nu_{jR} \). \( M = M_D, m_L, m_R \)

\[ \begin{array}{cccc}
\nu_{\nu_R} & \nu_{\nu_R} & \nu_{\nu_R} & N_R^o \\
\nu_{\nu_L} & D_{\nu_{14}} & D_{\nu_{15}} & D_{\nu_{16}} & 0 \\
\bar{\nu}_{\nu_L} & 0 & D_{\nu_{25}} & D_{\nu_{26}} & 0 \\
\bar{\nu}_{\nu_L} & 0 & D_{\nu_{35}} & D_{\nu_{36}} & 0 \\
N_L^o & 0 & 0 & 0 & 0
\end{array} \]

TABLE II: One loop Dirac mass terms \( m_{ij} \bar{\nu}_{iL} \nu_{jR} \)

\[ m_\nu(M_{Z_1})_{ij} = \cos \phi m_\nu(M_-)_{ij} - \sin \phi m_\nu(M_+)_{ij} \]

\[ m_\nu(M_{Z_2})_{ij} = \sin \phi m_\nu(M_-)_{ij} + \cos \phi m_\nu(M_+)_{ij} \]

\[ G_{\nu, m_{ij}} = \frac{\sqrt{\alpha_2 \alpha_3}}{\pi} \frac{1}{2\sqrt{3}} \cos \phi \sin \phi \left[ m_\nu(M_-)_{ij} - m_\nu(M_+)_{ij} \right] \]

\[ \mathcal{F}(M_Y) = m_5^o u_{11}^2 f_{Y_5} + m_6^o u_{12}^2 f_{Y_6} + m_7^o u_{13}^2 f_{Y_7} + m_8^o u_{14}^2 f_{Y_8} \]

\[ \mathcal{G}(M_Y) = m_5^o u_{21}^2 f_{Y_5} + m_6^o u_{22}^2 f_{Y_6} + m_7^o u_{23}^2 f_{Y_7} + m_8^o u_{24}^2 f_{Y_8} \]

\[ \mathcal{H}(M_Y) = m_5^o u_{11} u_{21} f_{Y_5} + m_6^o u_{12} u_{22} f_{Y_6} + m_7^o u_{13} u_{23} f_{Y_7} + m_8^o u_{14} u_{24} f_{Y_8} \]

\[ G_{\nu, m_{ij}} = \frac{\sqrt{\alpha_2 \alpha_3}}{\pi} \frac{1}{2\sqrt{3}} \cos \phi \sin \phi \left[ m_\nu(M_-)_{ij} - m_\nu(M_+)_{ij} \right] \]
\[ m_\nu(M_Y)_{15} = \frac{a_1 b_2}{a b} \mathcal{H}(M_Y) \ ; \quad m_\nu(M_Y)_{16} = \frac{a_2 b_3}{a b} \mathcal{H}(M_Y) \]

\[ m_\nu(M_Y)_{25} = \frac{a_2 b_2}{a b} \mathcal{H}(M_Y) \ ; \quad m_\nu(M_Y)_{26} = \frac{a_2 b_3}{a b} \mathcal{H}(M_Y) \]

\[ m_\nu(M_Y)_{35} = \frac{a_3 b_2}{a b} \mathcal{H}(M_Y) \ ; \quad m_\nu(M_Y)_{36} = \frac{a_3 b_3}{a b} \mathcal{H}(M_Y) \]

\[ D_\nu_{14} = \frac{1}{2} \left[ \frac{a_2 b_2}{a b} \mathcal{H}(M_1) + \frac{a_3 b_3}{a b} \mathcal{H}(M_2) \right] \],

\[ D_\nu_{15} = \frac{a_1 b_2}{a b} \left[ -\frac{1}{4} \mathcal{H}(M_{Z_1}) + \frac{1}{12} \mathcal{H}(M_{Z_2}) \right] \],

\[ D_\nu_{25} = \frac{a_2 b_2}{a b} \left[ \frac{1}{4} \mathcal{H}(M_{Z_1}) + \frac{1}{12} \mathcal{H}(M_{Z_2}) - \mathcal{H}(G_{\nu,m}) \right] + \frac{1}{2} \frac{a_3 b_3}{a b} \mathcal{H}(M_3) \],

\[ D_\nu_{36} = \frac{1}{2} \frac{a_2 b_2}{a b} \mathcal{H}(M_3) + \frac{1}{3} \frac{a_3 b_3}{a b} \mathcal{H}(M_{Z_2}) \],

\[ D_\nu_{16} = \frac{a_1 b_3}{a b} \left[ -\frac{1}{6} \mathcal{H}(M_{Z_2}) - \mathcal{H}(G_{\nu,m}) \right] \],

\[ D_\nu_{26} = \frac{a_2 b_3}{a b} \left[ -\frac{1}{6} \mathcal{H}(M_{Z_2}) + \mathcal{H}(G_{\nu,m}) \right] \],

\[ D_\nu_{35} = \frac{a_3 b_2}{a b} \left[ -\frac{1}{6} \mathcal{H}(M_{Z_2}) + \mathcal{H}(G_{\nu,m}) \right] \],

\[ \mathcal{H}(G_{\nu,m}) = \frac{\sqrt{a_2 a_3}}{\pi} \frac{1}{2 \sqrt{3}} \cos \phi \sin \phi \left( \mathcal{H}(M_-) - \mathcal{H}(M_+) \right) \]

**B. One loop L-handed Majorana masses**

Neutrinos also obtain one loop corrections to L-handed and R-handed Majorana masses from the diagrams of Fig. 2 and Fig. 3, respectively.

A similar procedure as for Dirac Neutrino masses, leads to the one loop Majorana mass terms

\[ m_\nu(M_Y)_{11} = \frac{a_1^2}{a^2} \mathcal{F}(M_Y) \ ; \quad m_\nu(M_Y)_{12} = \frac{a_1 a_2}{a^2} \mathcal{F}(M_Y) \]

\[ m_\nu(M_Y)_{13} = \frac{a_1 a_3}{a^2} \mathcal{F}(M_Y) \ ; \quad m_\nu(M_Y)_{22} = \frac{a_2^2}{a^2} \mathcal{F}(M_Y) \]

\[ m_\nu(M_Y)_{23} = \frac{a_2 a_3}{a^2} \mathcal{F}(M_Y) \ ; \quad m_\nu(M_Y)_{33} = \frac{a_3^2}{a^2} \mathcal{F}(M_Y) \]
\[ \begin{array}{c|cccc}
\nu_{eL}^o & \nu_{\mu L}^o & \nu_{\tau L}^o & N_L^o \\
\nu_{\mu L}^o & L_{\nu 11} & L_{\nu 12} & L_{\nu 13} & 0 \\
\nu_{\tau L}^o & L_{\nu 12} & L_{\nu 22} & L_{\nu 23} & 0 \\
N_L^o & L_{\nu 13} & L_{\nu 23} & L_{\nu 33} & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{array} \]

**TABLE III:** One loop L-handed Majorana mass terms \( m_{ij} \bar{\nu}_{eL}^o (\nu_{jL}^o)^T \)

\[
L_{\nu 11} = \frac{a_1^2}{a^2} \left[ \frac{1}{4} \mathcal{F}(M_{Z_1}) + \frac{1}{12} \mathcal{F}(M_{z_2}) + \mathcal{F}(G_{\nu,m}) \right],
\]

\[
L_{\nu 22} = \frac{a_2^2}{a^2} \left[ \frac{1}{4} \mathcal{F}(M_{Z_1}) + \frac{1}{12} \mathcal{F}(M_{z_2}) - \mathcal{F}(G_{\nu,m}) \right],
\]

\[
L_{\nu 33} = \frac{1}{3} \frac{a_3^2}{a^2} \mathcal{F}(M_{z_2}),
\]

\[
L_{\nu 12} = \frac{a_1 a_2}{a^2} \left[ -\frac{1}{4} \mathcal{F}(M_{Z_1}) + \frac{1}{2} \mathcal{F}(M_1) + \frac{1}{12} \mathcal{F}(M_{z_2}) \right],
\]

\[
L_{\nu 13} = \frac{a_1 a_3}{a^2} \left[ -\frac{1}{6} \mathcal{F}(M_{Z_2}) + \frac{1}{2} \mathcal{F}(M_2) - \mathcal{F}(G_{\nu,m}) \right],
\]

\[
L_{\nu 23} = \frac{a_2 a_3}{a^2} \left[ -\frac{1}{6} \mathcal{F}(M_{Z_2}) + \frac{1}{2} \mathcal{F}(M_3) + \mathcal{F}(G_{\nu,m}) \right]
\]

\[
\mathcal{F}(G_{\nu,m}) = \sqrt{\frac{a_2 a_3}{\pi}} \frac{1}{2 \sqrt{3}} \cos \phi \sin \phi \left[ \mathcal{F}(M_-) - \mathcal{F}(M_+) \right]
\] (35)
C. One loop R-handed Majorana masses

|       | $\nu_{iR}^{\nu}$ | $\nu_{R}^{\nu}$ | $\nu_{sR}^{\nu}$ | $N_{R}^{\nu}$ |
|-------|------------------|------------------|------------------|---------------|
| $\nu_{iR}^{\nu}$ | 0                | 0                | 0                | 0             |
| $\nu_{sR}^{\nu}$ | 0                | $R_{\nu 55}$  | $R_{\nu 56}$  | 0             |
| $\nu_{sR}^{\nu}$ | 0                | $R_{\nu 56}$  | $R_{\nu 66}$  | 0             |
| $N_{R}^{\nu}$ | 0                | 0                | 0                | 0             |

TABLE IV: One loop R-handed Majorana mass terms $m_{ij} \bar{\nu}_{iR}^{\nu} (\nu_{jR}^{\nu})^T$

\[
m_{\nu}(M_Y)_{55} = \frac{b_2^2}{b'^2} G(M_Y) ; \quad m_{\nu}(M_Y)_{66} = \frac{b_2^2}{b'^2} G(M_Y)
\]

\[
m_{\nu}(M_Y)_{56} = \frac{b_2 b_3}{b'^2} G(M_Y)
\]

\[
R_{\nu 55} = \frac{b_2^2}{b'^2} \left[ \frac{1}{4} G(M_{Z_1}) + \frac{1}{12} G(M_{Z_2}) - G(G_{\nu,m}) \right],
\]

\[
R_{\nu 66} = \frac{b_2^2}{3 b'^2} G(M_{Z_2}),
\]

\[
R_{\nu 56} = b_2 b_3 \frac{1}{b'^2} \left[ -\frac{1}{6} G(M_{Z_2}) + \frac{1}{2} G(M_3) + G(G_{\nu,m}) \right]
\]

\[
G(G_{\nu,m}) = \frac{\sqrt{\alpha_2 \alpha_3}}{\pi} \frac{1}{2 \sqrt{3}} \cos \phi \sin \phi \left[ G(M_-) - G(M_+) \right]
\]

Thus, in the $\Psi_\nu^c$ basis, we may write the one loop contribution for neutrinos as
Finally, we obtain the Majorana mass matrix for neutrinos up to one loop

\[
\mathcal{M}_\nu = (U^\nu)^T \mathcal{M}^\nu_1 U^\nu + \text{Diag}(0, 0, 0, 0, m_5^o, m_6^o, m_7^o, m_8^o), \tag{37}
\]

where explicitly

\[
\mathcal{M}_\nu = \begin{pmatrix}
N_{11} & N_{12} & N_{13} & N_{14} & N_{15} & N_{16} & N_{17} & N_{18} \\
N_{12} & N_{22} & N_{23} & N_{24} & N_{25} & N_{26} & N_{27} & N_{28} \\
N_{13} & N_{23} & 0 & 0 & N_{35} & N_{36} & N_{37} & N_{38} \\
N_{14} & N_{24} & 0 & N_{44} & N_{45} & N_{46} & N_{47} & N_{48} \\
N_{15} & N_{25} & N_{35} & N_{45} & N_{55} + m_5^o & N_{56} & N_{57} & N_{58} \\
N_{16} & N_{26} & N_{36} & N_{46} & N_{56} + m_6^o & N_{67} & N_{68} & \\
N_{17} & N_{27} & N_{37} & N_{47} & N_{57} + m_7^o & N_{67} & N_{78} & \\
N_{18} & N_{28} & N_{38} & N_{48} & N_{58} & N_{68} & N_{78} & N_{88} + m_8^o
\end{pmatrix} \tag{38}
\]

**Majorana L-handed:**

\[
N_{11} = \frac{a_3^2 a_2^2}{a_1^2 a_2^2} (F_{Z_1} - F_1) \tag{39}
\]

\[
N_{12} = -\frac{a_1 a_2 a_3}{2a_1^2} \frac{a_1^2 - a_2^2}{a_0^2} (F_{Z_1} - F_1) + F_2 - F_3 - 6F_m \tag{40}
\]
\[ N_{22} = \frac{a_3^2}{a^2} \left[ \frac{1}{4} \left( \frac{a_2^2 - a_1^2}{a_p^2 a^2} \right)^2 (F_{Z_1} - F_1) + \frac{a_2^2}{a^2} (F_2 - F_3) + \frac{a_p^2}{4 a^2} (F_1 + 3 F_{Z_2} - 4 F_2) - 3 \frac{a_2^2 - a_1^2}{a^2} F_m \right] \] (41)

**Dirac:**

\[ N_{13} = \frac{a_2}{2 a_p} \left( \frac{a_2 b_2}{a b} H_1 + \frac{a_3 b_3}{a b} H_2 \right) = q_{11} \] (42)

\[ N_{14} = -\frac{a_1 b_3}{2 a_p b} \left( \frac{a_3 b_2}{a b} H_{Z_1} + \frac{a_3 b_3}{a b} H_3 - 6 \frac{a_2 b_2}{a b} H_m \right) = q_{12} \] (43)

\[ N_{23} = \frac{a_1 a_3}{2 a_p a} \left( \frac{a_2 b_2}{a b} H_1 + \frac{a_3 b_3}{a b} H_2 \right) = q_{21} \] (44)

\[ N_{24} = \frac{a_2 \left( a_p^2 b_2^2 + a_3^2 b_3^2 \right)}{2 a_p a^2 b^2} H_3 + \left( \frac{a_2^2 - a_2^2}{4 a_p a^2 b^2} \right) H_{Z_1} + \frac{3 a_p a_3 b_2 b_3}{4 a^2 b^2} H_{Z_2} - \frac{3 a_2^2 a_3 b_2 b_3}{a_p a^2 b^2} H_m = q_{22} \] (45)

**Majorana R-handed:**

\[ N_{44} = \frac{b_2^2 b_3^2}{4 a^2} \left( G_{Z_1} + 3 G_{Z_2} - 4 G_3 - 12 G_m \right) \] (46)

**Majorana L-handed and Dirac:**

\[ N_{15} = -F_{15} u_{11} + q_{13} u_{21} \ ; \quad N_{16} = -F_{15} u_{12} + q_{13} u_{22} \] (47)

\[ N_{17} = -F_{15} u_{13} + q_{13} u_{23} \ ; \quad N_{18} = -F_{15} u_{14} + q_{13} u_{24} \] (48)

\[ F_{15} = \frac{a_1 a_2}{2 a_p a} \left[ \frac{a_2^2 - a_2^2}{a^2} (F_{Z_1} - F_1) + \frac{a_3^2}{a^2} (F_3 - F_2) + 2 \frac{2 a_2^2 - a_2^2}{a^2} F_m \right] \]

\[ q_{13} = -\frac{a_1 b_2}{2 a_p b} \left[ \frac{a_2 b_2}{a b} H_{Z_1} + \frac{a_3 b_3}{a b} H_3 - 2 \frac{a_2 b_2 - 2 b_2}{a b} - \frac{2 b_2}{b_2} H_m \right] \]

\[ N_{25} = F_{25} u_{11} + q_{23} u_{21} \ ; \quad N_{26} = F_{25} u_{12} + q_{23} u_{22} \] (49)

\[ N_{27} = F_{25} u_{13} + q_{23} u_{23} \ ; \quad N_{28} = F_{25} u_{14} + q_{23} u_{24} \] (50)

\[ F_{25} = \frac{a_3}{4 a_p a^2} \left[ \left( a_3^2 - a_1^2 \right)^2 (F_{Z_1} - F_1) + 2 a_2^2(a_3^2 - a_p^2)(F_3 - F_2) - a_4^4 (F_{Z_2} - F_1) \right. \]

\[ \left. -2 a_p^2(a_3^2 - a_p^2)(F_{Z_2} - F_2) + 4 (a_3^2 - a_1^2)(a_3^2 - 2 a_p^2) F_m \right] \]
\[ q_{23} = \frac{a_2 (a_3^2 - a_1^2)}{2 a_p a^2 b^2} b_3 b_3 + \frac{(a_2^2 - a_1^2) a_2 b_2^2}{4 a_p a^2 b^2} H_3 + \frac{a_p a_3 (b_2^2 - 2 b_3^2)}{4 a^2 b^2} H_{Z_1} + \frac{a_3 [a_2^2 b^2 + a_3^2 (b_2^2 - 2 b_3^2)]}{a_p a^2 b^2} H_m \]

Dirac:

\[ N_{35} = q_{31} u_{11} , \quad N_{36} = q_{31} u_{12} , \quad N_{37} = q_{31} u_{13} , \quad N_{38} = q_{31} u_{14} \]  

(51)

\[ q_{31} = \frac{a_1}{2a} (\frac{a_2 b_2}{a} H_1 + \frac{a_3 b_3}{a} H_2) \]

Dirac and Majorana R-handed:

\[ N_{45} = q_{32} u_{11} + G_{45} u_{21} , \quad N_{46} = q_{32} u_{12} + G_{45} u_{22} \]  

(52)

\[ N_{47} = q_{32} u_{13} + G_{45} u_{23} , \quad N_{48} = q_{32} u_{14} + G_{45} u_{24} \]  

(53)

\[ q_{32} = -\frac{a_2 a_3 (b_2^2 - b_3^2)}{2 a_p a^2 b^2} b_3 b_3 + \frac{(a_2^2 - a_1^2) a_2 b_2^2}{4 a_p a^2 b^2} H_3 \quad \frac{(2a_3^2 - a_p^2) b_2 b_3}{4 a^2 b^2} b_3 H_{Z_1} \quad \frac{(a_3^2 + a_1^2 - 2a_2^2) b_2 b_3}{a^2 b^2} H_m \]

\[ G_{45} = \frac{b_2 b_3}{4 b^2} \left[ \frac{b_3^2 - 2b_2^2}{b^2} (G_{Z_2} - G_3) + \frac{b_2^2}{b^2} (G_{Z_1} - G_3) - 4 \frac{b_3^2 - b_2^2}{b^2} G_m \right] \]

Majorana L-handed, Dirac and Majorana R-handed:

\[ N_{55} = F_{55} u_{11}^2 + 2 q_{33} u_{11} u_{21} + G_{55} u_{21}^2 \]  

(54)

\[ N_{56} = F_{55} u_{11} u_{12} + q_{33} (u_{11} u_{22} + u_{12} u_{21}) + G_{55} u_{21} u_{22} \]  

(55)

\[ N_{57} = F_{55} u_{11} u_{13} + q_{33} (u_{11} u_{23} + u_{13} u_{21}) + G_{55} u_{21} u_{23} \]  

(56)

\[ N_{58} = F_{55} u_{11} u_{14} + q_{33} (u_{11} u_{24} + u_{14} u_{21}) + G_{55} u_{21} u_{24} \]  

(57)

\[ N_{66} = F_{55} u_{12}^2 + 2 q_{33} u_{12} u_{22} + G_{55} u_{22}^2 \]  

(58)

\[ N_{67} = F_{55} u_{12} u_{13} + q_{33} (u_{13} u_{22} + u_{12} u_{23}) + G_{55} u_{22} u_{23} \]  

(59)

\[ N_{68} = F_{55} u_{12} u_{14} + q_{33} (u_{14} u_{22} + u_{12} u_{24}) + G_{55} u_{22} u_{24} \]  

(60)

\[ N_{77} = F_{55} u_{13}^2 + 2 q_{33} u_{13} u_{23} + G_{55} u_{23}^2 \]  

(61)

\[ N_{78} = F_{55} u_{13} u_{14} + q_{33} (u_{14} u_{23} + u_{13} u_{24}) + G_{55} u_{23} u_{24} \]  

(62)

\[ N_{88} = F_{55} u_{14}^2 + 2 q_{33} u_{14} u_{24} + G_{55} u_{24}^2 \]  

(63)
mixing, coming out from a global fit of the parameter space.

For the quarks and charged leptons is

$$u_L^a = \frac{a_2 a_3}{a^2} F_1 + \frac{a_2^2 a_3^2}{a^4} F_2 + \frac{a_2^3 a_3}{a^4} F_3 + \frac{(a_2^2 - a_1^2)^2}{4 a^4} F_{Z_1} + \frac{(2a_2^2 - a_1^2)^2}{12 a^4} F_{Z_2} + \frac{(a_2^2 - a_1^2)(2a_2^2 - a_1^2)}{a^4} F_m$$

$$q_{33} = \frac{a_2 a_3 b_2 b_2}{a^2 b^2} H_3 + \frac{(a_2^2 - a_1^2)^2}{4 a^2 b^2} H_{Z_1} + \frac{(2a_2^2 - a_1^2)(b_2^2 - 2b_3^2)}{12 a^2 b^2} H_{Z_2} + \frac{a_3^2 b_2^2 - a_2^2 b_3^2 - a_2^2 (b_2^2 - 2b_3^2)}{a^2 b^2} H_m$$

$$G_{55} = \frac{b_2^2 b_3^2}{b^4} G_3 + \frac{b_2^2}{4 b^4} G_{Z_1} + \frac{(b_2^2 - 2b_3^2)^2}{12 b^4} G_{Z_2} - \frac{b_2^2 (b_2^2 - 2b_3^2)}{b^4} G_m$$

E. $(V_{CKM})_{4 \times 4}$ and $(V_{PMNS})_{4 \times 8}$ mixing matrices

Within this $SU(3)$ family symmetry model, the transformation from massless to physical mass fermion eigenfields for quarks and charged leptons is

$$\psi_L^0 = V_L^0 V_L^{(1)} \Psi_L \quad \text{and} \quad \psi_R^0 = V_R^0 V_R^{(1)} \Psi_R,$$

and for neutrinos $\Psi_L^0 = U_\nu^0 U_\nu \Psi_\nu$. Recall now that vector like quarks, Eq. (I), are $SU(2)_L$ weak singlets, and hence, they do not couple to $W$ boson in the interaction basis. In this way, the interaction of $L$-handed up and down quarks; $f^o_{uL} = (u^o, e^o, \nu^o)_L$ and $f^o_{dL} = (d^o, s^o, b^o)_L$, to the $W$ charged gauge boson is

$$\frac{g}{\sqrt{2}} f^o_{uL} \gamma_\mu f^o_{dL} W^{+\mu} = \frac{g}{\sqrt{2}} \bar{\psi}_{uL} [(V^0_{uL} V_{uL}^{(1)})_{3 \times 4}]^T (V^0_{dL} V_{dL}^{(1)})_{3 \times 4} \gamma_\mu \Psi_{dL} W^{+\mu},$$

(g is the $SU(2)_L$ gauge coupling. Hence, the non-unitary $V_{CKM}$ of dimension $4 \times 4$ is identified as

$$(V_{CKM})_{4 \times 4} = [(V^0_{uL} V_{uL}^{(1)})_{3 \times 4}]^T (V^0_{dL} V_{dL}^{(1)})_{3 \times 4}$$

(65)

Similar analysis of the couplings of active $L$-handed neutrinos and $L$-handed charged leptons to $W$ boson, leads to the lepton mixing matrix

$$(U_{PMNS})_{4 \times 8} = [(V^0_{eL} V_{eL}^{(1)})_{3 \times 4}]^T (U^0_{\nu} U_{\nu})_{3 \times 8}$$

(66)

VII. NUMERICAL RESULTS

To illustrate the spectrum of masses and mixing, let us consider the following fit of space parameters at the $M_Z$ scale [27].

Using the strong hierarchy for quarks and charged leptons masses[15], here we report the fermion masses and mixing, coming out from a global fit of the parameter space.

In the approach $\alpha_2 \approx \alpha_3 = \alpha_H$, we take the input values

$$M_1 = 10 \text{ TeV} \quad M_2 = 1 \text{ TeV} \quad \frac{\alpha_H}{\pi} = 0.05$$

for the $M_1, M_2$ horizontal boson masses, Eq. (7), and the $SU(3)$ coupling constant, respectively, and the ratio of electroweak VEV’s: $V_i$ from $\Phi^i$, and $v_i$ from $\Phi^*$

$$\frac{V_1}{V_2} = 0.09981 \quad \frac{\sqrt{V_1^2 + V_2^2}}{V_3} = 0.54326$$

$$\frac{v_1}{v_2} = 0.1 \quad \frac{\sqrt{v_1^2 + v_2^2}}{v_3} = 0.5$$
A. Quark masses and mixing

u-quarks:

Tree level see-saw mass matrix:

\[
\mathcal{M}^o_u = \begin{pmatrix}
0 & 0 & 0 & 7933.76 \\
0 & 0 & 0 & 7933.76 \\
0 & 0 & 0 & 159467. \\
0 & 1.18613 \times 10^6 & -841128. & 374542.
\end{pmatrix} \text{ MeV}, \tag{67}
\]

the mass matrix up to one loop corrections:

\[
\mathcal{M}_u = \begin{pmatrix}
-1.40509 & 187.442 & -66.8139 & -255.74 \\
-0.125675 & -609.844 & 408.793 & 1564.71 \\
-0.062809 & -1197.67 & -172100. & 1825.79 \\
-0.001885 & -35.9461 & 14.3165 & 1.502 \times 10^6
\end{pmatrix} \text{ MeV} \tag{68}
\]

and the u-quark masses

\[(m_u, m_c, m_t, M_U) = (1.3802, 640.801, 172105, 1.502 \times 10^6) \text{ MeV} \tag{69}\]

d-quarks:

\[
\mathcal{M}^o_d = \begin{pmatrix}
0 & 0 & 0 & 1740.94 \\
0 & 0 & 0 & 17442.3 \\
0 & 0 & 0 & 32265.8 \\
0 & 7019.9 & -41383.4 & 910004
\end{pmatrix} \text{ MeV} \tag{70}
\]

\[
\mathcal{M}_d = \begin{pmatrix}
3.09609 & 28.1593 & -47.4565 & -4.23475 \\
0.271539 & -40.5966 & 215.617 & 19.2404 \\
0.147401 & -176.235 & -2846.26 & 37.484 \\
0.005900 & -7.05504 & 16.8159 & 914365.
\end{pmatrix} \text{ MeV} \tag{71}
\]

\[(m_d, m_s, m_b, M_D) = (2.82, 61.9998, 2860, 914365) \text{ MeV} \tag{72}\]

and the quark mixing

\[
V_{CKM} = \begin{pmatrix}
0.974352 & 0.225001 & 0.003647 & 0.000410 \\
-0.224958 & 0.973502 & 0.041031 & -0.001417 \\
-0.005632 & 0.040662 & -0.997868 & -0.039994 \\
0.000576 & -0.002325 & 0.031130 & 0.001251
\end{pmatrix} \tag{73}
\]
B. Charged leptons:

\[
\mathcal{M}_e = \begin{pmatrix}
0 & 0 & 0 & 28340.3 \\
0 & 0 & 0 & 283940. \\
0 & 0 & 0 & 525249. \\
0 & 17105.4 & -11570.9 & 5.94752 \times 10^6
\end{pmatrix} \text{ MeV}
\] (74)

\[
\mathcal{M}_e = \begin{pmatrix}
-0.499137 & 29.7086 & -43.9181 & -0.15097 \\
-0.043776 & -72.8148 & 238.953 & 0.821414 \\
-0.023663 & -183.913 & -1720.65 & 1.18425 \\
-0.002378 & -18.4839 & 34.6241 & 5.977 \times 10^6
\end{pmatrix} \text{ MeV}
\] (75)

fit the charged lepton masses:

\[(m_e, m_\mu, m_\tau, M_E) = (0.486, 102.7, 1746.17, 5.977 \times 10^6) \text{ MeV}\]

and the mixing

\[
V_{eL} V_{eL}^{(1)} = \begin{pmatrix}
0.968866 & 0.24054 & -0.0584594 & 0.00474112 \\
0.205175 & -0.912554 & -0.350561 & 0.0475013 \\
-0.138557 & 0.330471 & -0.929446 & 0.0878703 \\
-0.00217545 & 0.0132348 & 0.0990967 & 0.994987
\end{pmatrix}
\] (76)

C. Neutrino masses and mixing:

\[
\mathcal{M}_\nu = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 53594.6 & 44137.2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 535946. & 441372. \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.07 \times 10^6 & 887147. \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.80 \times 10^6 & 1.49 \times 10^6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -886604. & -730152. \\
53594.6 & 535946. & 1.07 \times 10^6 & 0 & 1.8097 \times 10^6 & -886604. & 1.97 \times 10^6 & 4.88 \times 10^8 & 4.88 \times 10^8 & 4.41 \times 10^7 & 7.02 \times 10^8
\end{pmatrix}
\] (77)

\[
\mathcal{M}_\nu = \begin{pmatrix}
-0.0119 & 0.0527 & 0.0227 & -0.0878 & -0.0693 & 0.1674 & -0.0016 & 0.0004 \\
0.0527 & -0.036 & 0.002 & 0.068 & 0.043 & -0.748 & 0.007 & -0.002 \\
0.0227 & 0.002 & 0. & 0. & 0.0008 & 0.0005 & -5.2 \times 10^{-6} & 1.5 \times 10^{-6} \\
-0.0878 & 0.068 & 0. & -0.125 & -0.1218 & 1.282 & -0.012 & 0.003 \\
-0.0693 & 0.043 & 0.0008 & -0.121 & 3.206 & -0.7430 & 0.0074 & -0.0021 \\
0.1674 & -0.748 & 0.0005 & 1.282 & -0.7430 & 1749.96 & 0.0003 & -0.0001 \\
-0.0016 & 0.007 & -5.2 \times 10^{-6} & -0.012 & 0.0074 & 0.0003 & -1. \times 10^8 & 1.1 \times 10^{-6} \\
0.0004 & -0.002 & 1.5 \times 10^{-6} & 0.003 & -0.0021 & -0.0001 & 1.1 \times 10^{-6} & 1. \times 10^9
\end{pmatrix}
\] (78)
generates the neutrino mass eigenvalues
\[
(m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8) = \text{eV} \begin{pmatrix} 0, -0.0085, 0.049, -0.22, 3.21, 1749.96, -1 \times 10^8, 1 \times 10^9 \end{pmatrix},
\]
the squared mass differences
\[
m_2^2 - m_1^2 \approx 0.0000723 \text{eV}^2, \quad m_3^2 - m_1^2 \approx 0.0024 \text{eV}^2
\]
\[
m_4^2 - m_1^2 \approx 0.0492 \text{eV}^2, \quad m_5^2 - m_4^2 \approx 10.3182 \text{eV}^2
\]
and the lepton mixing matrix
\[
U_{PMNS} = \begin{pmatrix}
0.2104 & 0.3520 & 0.8658 & -0.2861 & 0.0060 & 0.0053 & 0.00005 & 0.00001 \\
-0.8282 & 0.0030 & 0.0186 & -0.5478 & 0.1038 & -0.0507 & -0.0005 & -0.0001 \\
0.0807 & 0.0041 & 0.0052 & 0.0881 & 0.8475 & -0.5074 & -0.0050 & -0.0014 \\
0.0034 & 0.0003 & 0.0011 & -0.0021 & -0.0857 & 0.0512 & 0.0005 & 0.0001
\end{pmatrix}
\]

VIII. CONCLUSIONS

We have reported a low energy parameter space, within a local $SU(3)$ Family symmetry model, which combines tree level "Dirac See-saw" mechanisms and radiative corrections to implement a successful hierarchical spectrum, for charged fermion masses and quark mixing. In section VII we illustrated the predicted values for quark and charged lepton masses at the the LHC and neutrino oscillation experiments.

It is worth to comment here that the symmetries and the transformation of the fermion and scalar fields, all together, forbid tree level Yukawa couplings between ordinary standard model fermions. Consequently, the flavon scalar fields introduced to break the symmetries: $\Phi^u, \Phi^d, \eta_2$ and $\eta_3$, couple only ordinary fermions to their corresponding vector like fermion at tree level. Thus, FCNC scalar couplings to ordinary fermions are suppressed by light-heavy mixing angles, which as is shown in $(V_{CKM})_{4 \times 4}$, Eq. (76), may be small enough to suppress properly the FCNC mediated by the scalar fields within this scenario.

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