Moving Wigner Glasses and Smectics: Dynamics of Disordered Wigner Crystals

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We examine the dynamics of driven classical Wigner solids interacting with quenched disorder from charged impurities. For strong disorder, the initial motion is plastic—in the form of crossing winding channels. For increasing drive, the disordered Wigner glass can reorder to a moving Wigner smectic—with the electrons moving in non-crossing 1D channels. These different dynamic phases can be related to the conduction noise and $I(V)$ curves. For strong disorder, we show criticality in the voltage onset just above depinning. We also obtain the dynamic phase diagram for driven Wigner solids and prove that there is a finite threshold for transverse sliding, recently found experimentally.

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Ordered arrays of charged particles have been studied in the context of colloidal suspensions, ion rings, atomic-ion Wigner crystals, quantum computers, astrophysics, biophysics, plasmas, electrons deposited on liquid helium surfaces, semiconductor heterostructures, Wigner droplets, and arrays of metallic islands interconnected by tunnel junctions [1]. A revival of general interest in charged arrays has been fueled by the observation, in 2D heterostructures, of nonlinear $I$-$V$ curves exhibiting a threshold as a function of an externally applied electric field [2,3], indicating the presence of a Wigner solid (WS) [4] that has been pinned by the disorder in the sample. The depinning threshold can vary up to two orders of magnitude in different samples [3]. Indeed, experiments [1] based on transport and photoluminescence provide indirect evidence (without observing the location of the charges) for the existence of the WS, and demonstrate the very important role disorder plays in the dynamics of the WS [5]. It is the purpose of this work to study how disorder affects the transport properties of the driven WS.

The onset of broadband conduction noise has been interpreted as a signature of the sliding of a defected WS [2]. If the electrons were to retain their order and slide collectively, narrow band noise resembling that seen in sliding charge-density wave systems should be observed instead. Simulations by Cha and Fertig on classical Wigner crystals interacting with charged defects indicate that a disorder-induced transition, from a clean to a defected WS, can occur as a function of increasing pinning strength [6]. For strong pinning, the initial depinning is plastic and involves tearing of the electron crystal [6(b)]. However, many aspects of this transport have not been systematically characterized, including the current-voltage $I(V)$ curves, conduction noise, transverse meandering, transverse threshold for sliding, and the electron lattice structure and motion for varying applied drives.

Similar plastic depinning transitions have been observed in the related system of driven vortex lattices in disordered superconductors. The plastic depinning of the vortices is associated with flux motion through intricate river-like channels [7,12]. Defects in the vortex lattice strongly affect the depinning thresholds and the voltage noise signatures produced by the system. For increasing drive the initially defected vortex lattice can reorder to a moving lattice or a moving smectic phase [8,17]. In the fast moving lattice phase the vortex lattice regains order in both transverse and longitudinal directions with respect to the driving force, while in the smectic state only order transverse to the driving direction appears.

It is unclear a priori whether the same type of reordering transitions can occur in a system containing long-range pinning, such as in the case of the WS interacting with charged impurities. This is in contrast with the vortex system, where the pinning interacts with the flux lines only on a very short length scale. Additionally, in the strong pinning limit, critical behavior may occur near the depinning threshold, leading to velocity-force relations [3] of the form $v \sim (F - F_T)^c$. For instance, for transport in metallic dots, $\xi = 5/3$ theoretically [14], and $\xi = 2.0$ and 1.58 experimentally [15].

In this work, we use numerical simulations to study dynamical transitions in a 2D electron system forming a classical WS in the presence of charged impurities. For strong pinning, the electron crystal is highly defected and depins plastically, with certain electrons flowing in well-defined channels or interconnecting rivers, while others remain immobile. For increasing applied driving force, the initially disordered electrons can partially reorder to a moving Wigner smectic state where the electrons flow in 1D non-crossing winding channels. The reordering transition is accompanied by a saturation in the $dI/dV$ curves and by a change from a broad-band to a narrow-band voltage noise signature. For weak disorder, the depinning is elastic and a narrow-band noise signal can be observed at all drives above depinning. For strong disorder, where the depinning is plastic, we find criticality in the velocity force curves in agreement with transport in metallic dots [14,17]. We map out the dynamic phase diagram as a function of disorder strength and applied driving force. Also, we find unequivocal evidence for a transverse finite threshold for sliding conduction, in agreement with recent experiments [17]. Our results can also be tested in...
other systems listed in the first sentence of this paper (e.g., with charged colloidal systems suspended in water in the presence of a disordered substrate).

We conduct overdamped molecular dynamics (MD) simulations using a model similar to that studied by Cha and Fertig. The energy from interactions is

\[ U = \sum_{i \neq j} \frac{e^2}{|r_i - r_j|} - \sum_{ij} \frac{e^2}{\sqrt{(|r_i - r_j|)^2 + d^2}} \]  

(1)

The first term is the electron-electron (Coulomb) repulsion and the second term is the electron-impurity interaction, where the impurities are positively charged defects out of plane. Here, \( r_i \) is the location of electron \( i \), and \( r_j^{(p)} \) is the in-plane location of a positive impurity which is located at an out-of-plane distance \( d \) (measured here in units of \( a_0 \), the average lattice constant of the WS). The number of electrons \( N_i \) equals the number of impurities \( N_p \), and the disorder strength is varied by changing \( d \). We have also considered many other cases (e.g., \( \pm \) charged impurities, \( N_i \neq N_p \), etc.) with consistent results. The long-range Coulomb interactions are evaluated with a fast converging sum technique by Jensen which is computationally more efficient than the Ewald sum technique used in previous studies.

To obtain the initial electron positions, we perform simulated annealing in which we start from a high temperature above the electron lattice melting transition and slowly cool to a low temperature. Temperature is implemented via Langevin random kicks. Once the electron configuration has been initialized, the critical depinning force is determined by applying a very slowly increasing uniform driving force \( f_d \) which would correspond to an applied electric field. After each drive increment, we wait \( 10^4 \) MD time steps before taking data, which we average over the next \( 10^4 \) time steps. For each drive increment, we measure the average electron velocity (\( \propto \) current) in the direction of drive, \( V_x = (1/N_i) \sum_{i=1}^{N_i} \hat{x} \cdot v_i \). The \( V_x \) versus \( f_d \) curve corresponds to an \( I(V) \) experimental curve and we will thus use the notation \( V_x = I \) and \( f_d = V \).

The depinning force \( f_d^* \) is defined as the drive \( f_d \) at which \( V_x \) reaches a value of 0.01 that of the ohmic response. We have studied system sizes \( N_i \) from 64 to 800 and find similar behavior at all sizes. Most of the results presented here are for systems with \( N_i = 256 \).

When driven through a sample containing strong pinning, the electron lattice undergoes a gradual reordering transition as the driving force is increased. We illustrate such a reordering transition in Fig. (a) by plotting the eastbound electron trajectories at different driving forces for a sample containing strong disorder of \( d = 0.65 \). In Fig. (a) the onset of motion occurs through the opening of a single winding channel. Electrons outside of the channel remain pinned, and the overall electron lattice is disordered. At \( f_d/f_d^* = 1.5 \), shown in Fig. (b), several channels have opened, some of which are interconnecting.

The original channel in Fig. (a) has grown in width, but regions of pinned electrons are still present. Electrons moving past a pinned electron perturb it, causing it to move (like a revolving door) in a circular orbit around the center of the potential minima in which it is trapped. Several of these electron turnstiles can be observed in Fig. (b). In Fig. (c) for \( f_d/f_d^* = 2.25 \) there appear to be regions where electrons do not flow, but none of the electrons are permanently pinned. Some electrons become pinned for a period of time before moving again. If the trajectories are drawn for a sufficiently long time, the electron flow appears everywhere in the sample, although there are still preferred paths in which more electrons flow. In Fig. (d) for \( f_d/f_d^* = 3.0 \) the electron flow occurs more uniformly across the sample. In Fig. (e) at \( f_d/f_d^* = 4.0 \) the electrons begin to flow predominantly in certain non-crossing channels, although some electrons jump from channel to channel. In Fig. (f) for \( f_d/f_d^* = 5.0 \) the electron flow occurs in well defined non-
crossing channels, which can contain different numbers of electrons. A similar channel motion exists for driven vortices in disordered superconductors\[4, 12\].

For samples containing very weak disorder, the pinned WS has six-fold ordering and depins elastically, without generating defects in the lattice. In this regime, the electron crystal flows in 1D channels with each channel containing the same number of electrons. Here, the transverse wanderings of the electrons are considerably reduced compared to the case of strong pinning.

To better illustrate the change in the amount of disorder in the electron lattice, in Fig. 2(a) we show the Delaunay triangulation for the electrons for the case of drive as in Fig. 1(f). Defects, in the form of 5-7 disclination pairs, appear with their Burgers vectors oriented perpendicular to the direction of the drive. In the structure factor there are only two prominent peaks for order in the direction transverse to the drive, consistent with a moving Wigner smectic state. In Fig. 2(b) we show the Delaunay triangulation for the moving state in a weakly pinned sample with \( d = 1.76 \) where the initial depinning is elastic. Here the moving lattice is defect free. The structure factor in this case shows four longitudinal peaks in addition to the two transverse peaks, although the transverse peaks are more prominent. Much larger systems would be necessary to determine whether the system is in a smectic state or in a moving Bragg glass in this case of weak disorder.

In order to connect the reordering sequence with a measure that is readily accessible experimentally, we plot in Fig. 3(a) the \( I(V) \) and \( dI/dV \) curves. A peak occurs in \( dI/dV \) at \( f_d/f_0^* \approx 3 \), when the electrons are undergoing very disordered plastic flow. In Fig. 3(b) we plot the fraction \( P_6 \) of six-fold coordinated electrons as a function of drive. A perfect triangular lattice would have \( P_6 = 1 \). For drives \( f_d/f_0^* < 2 \) the lattice is highly defected. For \( f_d/f_0^* > 3 \) (i.e., past the peak in \( dI/dV \)), the order in the lattice begins to increase. The value of \( P_6 \) saturates near \( f_d/f_0^* \approx 6 \) which also coincides with the saturation of the \( dI/dV \) curve. Thus the experimentally observable \( I(V) \) characteristic can be considered a good measure of the degree of order and the nature of the flow in the system. To quantify the degree of plasticity of the electron flow, we plot in Fig. 3(c) the fraction of electrons \( D_\text{tr} \) that wandered a distance of more than \( \alpha_0/2 \) in the direction transverse to the drive during an interval of 8000 MD steps. The peak in \( D_\text{tr} \) coincides with the peak in \( dI/dV \). \( D_\text{tr} \) then slowly declines until it saturates at \( f_d/f_0^* \approx 6 \), indicating the gradual formation of the noncrossing channels as seen in Fig. 3(f). The saturation in \( D_\text{tr} \) coincides with the saturations in both \( P_6 \) and \( dI/dV \).

The \( I(V) \) curve in Fig. 3(a) corresponds to a highly irregular voltage signal as a function of time when the electrons are in the plastic flow regime \( f_d/f_0^* = 1.5 \). The corresponding voltage noise spectrum in the inset of Fig. 3(c) shows that only broad-band noise is present. In contrast, at \( f_d/f_0^* = 4.0 \), in the moving smectic regime, a roughly regular signal is obtained, and narrow-band noise appears, as shown in the inset of Fig. 3(c). In a system with \( d = 1.57 \) when the depinning is elastic only, an even more pronounced narrow band noise signal is observable. In experiments, broad band noise has been observed above depinning \( \frac{V_e}{\xi} \), but narrow band noise has not been seen. Our results suggest that plasticity may be playing an important role in most experiments.
In Fig. 3(d,e) we examine the critical behavior above the depinning threshold of $V_c$ versus $f_d$ for disorder strengths $d = 0.5$ and 0.65. In Fig. 3(d), a log-log plot of $V_c \sim \left((f_d - f_d^\parallel)/f_d^\parallel\right)^\xi$, indicates that the curves are fit well by a power law over one and half decades with $\xi = 1.61 \pm 0.10$ and $1.71 \pm 0.10$, respectively. These values agree well with the predicted value of $\xi = 5/3$ [14] for electron flow through disordered arrays, and the experimentally observed values of $\xi = 1.58$ and 2.0 [15].

The theory of the moving ordered phase [14] predicts a barrier to transverse motion once channels similar to those in Fig. 1(c) form. As shown in the inset of Fig. 4, for a system in the reordered phase we observe a transverse depinning threshold that is about 1/6 the size of the longitudinal depinning threshold $f_d^\parallel$. In recent experiments, Perruchot et al. [17] also find evidence for a transverse barrier that is about 1/10 the size of the longitudinal threshold. These thresholds are much larger than those observed in vortex matter interacting with short-ranged pinning, where ratios of 1/100 are seen [16].

In Fig. 4 we present the WS dynamic phase diagram as a function of disorder strength and driving force. For $d < 1.4$ there are a considerable number of defects in the WS, and the initial depinning is plastic. We label this region the pinned Wigner glass. For increasing disorder, the pinned region grows while for strong enough drive the electrons reorder to a moving Wigner smectic state. For $d > 1.4$ there are few or no defects in the pinned state and the initial depinning is elastic. The presence of a crossover from elastic to plastic depinning with decreasing disorder strength is in agreement with [16].

In summary, we have investigated the pinning and dynamics of an electron solid interacting with charged disorder. We find that for strong disorder the depinning transition is plastic with electrons flowing in a network of winding channels. For increasing drives, the electrons partially reorder and flow in non-crossing channels forming a moving Wigner smectic. We show that the onset of these different phases can be inferred from the transport characteristics. In the plastic flow regime, the noise has broad-band characteristics, while in the moving smectic or elastic flow phase, a narrow band noise signal is observable. We also show that the onset of the plastic flow phase shows critical behavior with critical exponents in agreement with predictions for transport in arrays of metallic dots [14,15]. We map out the dynamic phase diagram as a function of disorder and applied driving force. We obtain a finite threshold for transverse sliding, in agreement with recent experiments [17].

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[1] See, e.g., 2D Electron Systems on Helium and Other Substrates, E.Y. Andrei ed. (Kluwer, New York, 1997), and references therein.
[2] See, e.g., V.J. Goldman et al., Phys. Rev. Lett. 65, 2189 (1990); Y.P. Li et al., Phys. Rev. Lett. 67, 1630 (1991).
[3] See, e.g., F.I.B. Williams et al., Phys. Rev. Lett. 66, 3285 (1991); H.-W. Jiang et al., ibid. 44, 8107 (1991).
[4] See, e.g., A.H. MacDonald and S.M. Girvin, Physics World, Dec. 1990; Phys. Rev. B 38, 6295 (1988); R. Cote and A.H. MacDonald, ibid. 44, 8759 (1991); Phys. Rev. Lett. 65, 2662 (1990); M.I. Dykman and Y.G. Rubo, ibid. 78, 4813 (1997); I.V. Schweigert et al., ibid. 82, 5293 (1999); B.G.A. Normand, P.B. Littlewood, and A.J. Millis, Phys. Rev. B 46, 3920 (1992); H.A. Fertig, ibid. 59, 2120 (1999).
[5] See, e.g., D. Shahar et al, Phys. Rev. Lett. 74, 4511 (1995); S.T. Chui and B. Tanatar, ibid. 74, 458 (1995).
[6] (a) M.-C. Cha and H.A. Fertig, Phys. Rev. Lett. 73, 870 (1994); (b) Phys. Rev. B 50, 14 369 (1994).
[7] see, e.g., F. Nori, Science, 271, 1373 (1996); H.J. Jensen et al., Phys. Rev. Lett. 60, 1676 (1988); N. Grønbech-Jensen et al., ibid. 76, 2985 (1996).
[8] A.E. Koshelev and V.M. Vinokur, Phys. Rev. Lett. 73, 3580 (1994).
[9] T. Giamarchi and P. Le Doussal, Phys. Rev. Lett. 76, 3408 (1996); 78, 752 (1997); P. Le Doussal and T. Giamarchi, Phys. Rev. B 57, 11 356 (1998).
[10] L. Balents et al., Phys. Rev. Lett. 78, 751 (1997); Phys. Rev. B 57, 7705 (1998).
[11] K. Moon, R.T. Scalettar and G.T. Zimányi, Phys. Rev. Lett. 77, 2778 (1996); S. Ryu et al., ibid. 77, 5114 (1996); S. Spencer and H.J. Jensen, Phys. Rev. B 55, 8473 (1997); C.J. Olson and C. Reichhardt, ibid 61, R3811 (2000).
[12] C.J. Olson, C. Reichhardt and F. Nori, Phys. Rev. Lett. 81, 3757 (1998); A.B. Kolton, D. Dominguez and N. Grønbech-Jensen, Phys. Rev. Lett. 83, 3061 (1999).
[13] D.S. Fisher, Phys. Rev. Lett. 50, 1486 (1983).
[14] A.A. Middleton and N.S. Wingreen, Phys. Rev. Lett. 71, 3198 (1993).
[15] C. Kurdak et al., Phys. Rev. B 57, R6842 (1998).
[16] N. Grønbech-Jensen et al., Molec. Phys. 92, 941 (1997).
[17] F. Perruchot et al., Physica B 256-258, 587 (1998); 284-288, 1984 (2000).