Biodynamic Synchronized Coupled Model for Crowd-Footbridge Interaction

Marcelo André Toso
Federal Institute of Santa Catarina, Xanxerê, SC, Brazil
Herbert Martins Gomes
Federal University of Rio Grande do Sul, Porto Alegre, RS, Brazil.

Abstract

Nowadays there are growing interests in vibration serviceability assessments of composite footbridges. The new design trends of composite footbridges make them slender civil structures that may be affected by the load action of walking pedestrians resulting in large deflections or even uncomfortable vibrations. Furthermore, the presence of people on the footbridges causes the addition of mass to the structural system and due to the human body’s ability to absorb vibrational energy, an increase in structural damping. In this paper, the interaction between pedestrian and structure is modelled using data from pedestrian characteristics and vibration data from a measured footbridge as a comparison basis. A previously developed numerical model was used, this model called Biodynamic Synchronized Coupled Model (BSCM) consists of a fully synchronized force model in the longitudinal and lateral direction of pedestrian’s movement and a biodynamic model with mass, damping and stiffness parameters. The model is coupled with the structure using the Finite Element Method at the feet’s contact points. Pedestrians are treated as individuals with intrinsic kinetic and kinematic parameters following a measured correlation matrix obtained by the use of an especially designed force platform. Finally, the adequacy of the proposed model to represent the pedestrians as BSCM for the walking effects on the structure is investigated by experimentally measured accelerations on a footbridge (freely walking). The numerical results show good agreement with the experimental results.

Keywords: footbridges, human-induced vibration, crowd-footbridge interaction, walking, natural frequencies.

1. Introduction

Several cases of excessive vibrations have been studied in the past that are related to pedestrians’ footbridges. Cases of unstable footbridges such as the Millennium Bridge in London and the Solferino Footbridge in Paris have attracted professional attention. These footbridges presented large vibrations on their opening days (Dallard et al. [1]). Many footbridges have natural frequencies that are coincident with the dominant frequencies of the pedestrian induced load and therefore they have the potential to undergo excessive vibrations. It is noteworthy that humans are quite sensitive to vibration in a low-frequency range of whole-body vibrations where natural frequencies of the human body limbs and systems can be observed. Therefore, there are recommendations related to human body vibration, for example, ISO 2631 [2] which define limitation curves for exposure times in some frequency ranges. This paper presents a study on the interaction between walking pedestrians and a flexible footbridge. The paper uses some previous investigation results (Toso et al. [3] and Toso and Gomes [4]). A Biodynamic Synchronized Coupled Model (BSCM) is used to analyze the pedestrian structure interaction. This model brings together interaction in 3D with several 1 DOF models to model human crowds walking and interacting with structures. In the longitudinal and transversal direction of movement a Fully Synchronized Force Model (FSFM) is used and in the vertical direction, it is combined with a biodynamic model (mass-spring-damper parameters) that is coupled to the structure. Besides, experimental data for a real footbridge considering a pedestrian crowd are presented which confirms the trends suggested by the numerical modeling.

2. Literature Review

Zivanovic et al. [5] state that the pedestrian-structure-interaction needs to consider in footbridges design, mainly in the design of slender structures that are dynamically excited by humans. Excessive vibrations may cause discomfort to pedestrians and potential deterioration of the footbridge’s structural integrity. Sloyanoff and Hunter [6], report that the
natural frequencies of short-span footbridge are usually not susceptible to pedestrian vibrations. However, when the distance between the spans increases, the natural frequencies of the structure decrease, and the human occupation and interaction on them becomes a concern due to the resonance phenomena. The authors state that, during walking, vertical forces produce frequencies between 1.5 and 4.0 Hz, while lateral forces produce frequencies between 0.75 and 2.0 Hz. Considering lateral oscillations, Bodgi et al. [7], state that a crowd on a footbridge imposes a lateral dynamic excitation on the structure at a frequency close to 1.0 Hz. When the first vibration mode decreases, being in the same range as the human step rate, the resonance may occur. Consequently, there is an increase in the oscillation amplitude of the structure and the pedestrians are forced to change their natural gait. If the amplitude of oscillation is large enough, the phenomenon called pedestrian-structure synchronization occurs. Researchers such as, Wheeler [8] state that the crowd effect is not significant unless the pedestrian's step rate is close to 2.0 Hz. The author affirms that the effect of the pedestrian crowd on a footbridge with fundamental frequency away from the typical step rate (2.0 Hz) may be negligible because the vibratory response of the structure may be lower when compared to a single pedestrian walking at a frequency identical to the fundamental frequency of the analysed structure. Qin et al., [9], evaluated the dynamic response of a footbridge (analytical study) considering a single pedestrian and the effects of interaction between human-structure. The analysed footbridge was modelled as a simply supported Euler-Bernoulli beam with uniform cross-section. The pedestrian damping varies over time, considering the individual's walking speed. A controlled force was used to compensate the energy dissipated during walking. The effects of stiffness and damping of the human body were investigated. According to the authors, for a flexible structure, the pedestrian-structure dynamic interaction is greater when compared to a rigid structure. There is also potential elastic energy of the individual's legs, "tending to separate the pedestrian from the structure," being greater when the individual is near to footbridge mid-span. As result of the interaction between pedestrians and flexible structures, it is noted that the individual must impose more external energy and modify the walking pattern to maintain his steady gait and a relatively uniform dynamic behavior of the body's center of mass. Recently, Tubino [10] proposed a numerical model that accounts for pedestrian structure coupling in the vertical direction. The footbridge was modelled as a continuous unidimensional beam dynamic system, while pedestrians were schematized as moving single-degree-of-freedom systems with random dynamic properties. The paper results show possible variations of damping ratio and natural frequency in the coupled system based on the random pedestrians' parameters. Regarding pedestrian loads, these forces have been determined from investigations using force platforms, treadmill machines, and even prototype footbridges, in which the applied force is the amount produced by a single walking pedestrian. The combined force applied by individuals is considered for groups of pedestrians or crowds. Thus, the design load is a force model. To analyse the analytic human-induced loads, most of guidelines, for instance, SETRA [11] and ISO 10137 [12] often consider three to five harmonics of the frequency spectrum of ground reaction forces (GRF). During walking, a pedestrian produces dynamic forces with components in three directions: vertical, lateral and longitudinal to the footbridge. The vertical component is generated by the impact of supporting the body weight on each leg alternately. In the lateral direction, the forces are generated by the periodic balance of the body when changing legs. Finally, in the longitudinal direction, the force is the result of friction between the foot and the floor, as well as the acceleration and deceleration of the body in this direction. Using force platforms, some researchers (Harper et al., [13]; Galbraith and Barton, [14]; Blanchard et al., [15]; Kerr, [16]) conclude that the vertical component of the resultant force of an individual has two peaks and a valley as shown in Figure 1. The other force components are also present in the figure.
3. Methodology

This paper proposes to assess the pedestrian-structure interaction using data from an experimentally measured footbridge as a basis for comparisons. In the longitudinal and transversal direction of movement a Fully Synchronized Force Model (FSFM) is used and in the vertical direction, it is combined with a biodynamic model (mass-spring-damper parameters) that is coupled to the structure (herein called, BSCM, Biodynamic Synchronized Coupled Model). The biodynamic model considers the synchronization of the three force components applied in space (positions where they should be applied) and in time (peak and valleys of the three force components occurring synchronously at the proper time) with a spring-mass-damper model, coupled continuously with the structure’s FEM. Following the use of the BSCM, the mid-span RMS (root-mean-square) acceleration of a footbridge structure is evaluated to check its serviceability. For the applied BSCM, kinetic (forces) and kinematic (speeds, pacing rate, step length, and step width) parameters are used. Then the BSCM results are compared with the FSFM and these results are compared with experimental data.

3.1. Force Models

Design Codes use simplified force models to represent the force magnitude from successive footfalls. These models assume forces acting in a straight line along the direction of walking at a constant speed (SETRA Guideline [11] and ISO 10137 [12]). This is a very common assumption for the analysis and design of footbridges. It is obvious the major disadvantages in using this simplified model as it lacks for dynamic interaction (only time-varying forces are used) and the spatiality and synchronization of application of the three force components. SETRA Guideline [11] and ISO 10137 [12] use force models to vertical, lateral and longitudinal direction of the walking, based in Fourier series. Overall, these models produces acceleration unsafe values, since they are based on excessively simplified load models. Then, it is necessary to incorporate other parameters to simulate the pedestrian-structure interaction (for instance, considering biodynamic models). Furthermore, in a recent publication Toso et al. [3] proposed a force model called Fully Synchronized Force Model (FSFM). In that model, the changes in velocities during the walking (single and double stance phase) were considered and they were synchronized in time and space. The authors pointed that the pedestrian speed in the double stance phase is greater than at single stance phase. This speed is also greater than the average speed of the pedestrian. Another important characteristic is that human walking does not occur in a straight line in the direction of walking (as proposed by Guidelines). There are parameters like step length and step width that influence the application of the resulting force. Using a specifically designed force platform, these kinematic parameters were measured, and average values for a test campaign with 54 subjects were presented by Toso et al. [3]. In that proposed force model, peak and valley values from each force component should be placed accordingly in the right position of the contact surface and

Figure 1: Typical forces during walking: (a) vertical, (b) lateral and (c) longitudinal direction. (Zivanovic et al. [5]).
the model’s reference time adjusted to the correct phase. Thus, there is a spatial and temporal synchronization of the three ground reaction force components. More details about the Fully Synchronized Force Model (FSFM) can be found in Toso et al. [3].

3.2. Biodynamic Synchronized Coupled Model

In the literature, there is a consensus that shifts in the structural natural frequencies are not observed when modelling the pedestrians using force-only models. This will only be present, as indicated by experimental measurements of a crowd of pedestrians crossing footbridges, if mass, dissipative effects, and synchronism of applied loads are taken into account. Another observed feature is the structural damping increase when considering pedestrians like biodynamic models. This is attributed to the human body’s ability to absorb energy, a feature represented only when introducing the biodynamic models. In this paper, the FSFM (Fully Synchronized Force model) is merged to a biodynamic model resulting in the so-called BSCM (Biodynamic Synchronized Coupled Model) composed of mass, damping, and stiffness, with a single degree of freedom (SDOF) that represents the action of a walking pedestrian in the vertical direction. Afterward, the model’s degree of freedom is continuously coupled to the structure in the places where the feet have contact with the floor and replicated to represent a pedestrian crowd. This replication takes into account intra and inter-subject variability (mainly due to subject’s weight, height, step length, step width and pacing rate). This makes possible to analyze the structural behaviour according to distinct crowd densities in a systematic way. Toso and Gomes [4] proposed this model, where a biodynamic model with synchronization of the three force components applied in space (positions where they should be applied) and in time (peak and valleys of the three force components occurring at the proper time) was presented. This biodynamic model is coupled continuously with the structure’s FE Model. The biodynamic model has been conceived in a way that the actuator force is the main source that drives the human will for walking, generating oscillations in the vertical direction. The actuator reasoning is based on experimentally obtained vertical force (using a force platform) and acceleration data (using an accelerometer attached to the pedestrian’s body waist). This allows the fit of the biodynamic model (with actuator) in order to match measured vertical ground reaction force and acceleration along time, in a rigid platform. It is important to note that, for flexible structures, the interaction force will change as the flexible structure add a relative displacement to the biodynamic model. A biodynamic model for flexible structure, including mass, damping, stiffness and actuator that is coupled to the footbridge in the vertical direction is considered, according to Figure 2.

![Figure 2: The biodynamic single degree of freedom system composed of mass, damping, stiffness and actuator and flexible structure situation.](image)

In this figure, \( u_p \) is the pedestrian vertical displacement around the center of mass in the initial equilibrium rest configuration, considering a rigid and a flexible structure; \( c_p \) and \( k_p \) are the damping and stiffness of pedestrian; \( F_A \) is the vertical actuator force; \( \nu \) is the pedestrian speed; \( m_p \) is the pedestrian modal mass; \( u_s \) is the structural displacement and \( F_{int} \) is the interaction force.

The model proposed allows assessing interactions with both rigid and flexible structures. Toso and Gomes [4] presented a complete description of the model; here it is showed some equations to evaluate the pedestrian structure interaction. Equation (1) represents the pedestrian’s interaction force \( \{F_{int}\} \) when walking on a flexible structure, according to presented in Figure 2.
\[
(F_{\text{int}}) = \begin{bmatrix} H(e, \xi_0) \end{bmatrix}^T \left[ -c_p(\ddot{u}_p - \{H(e, \xi_0)\}|\dot{u}_s) - k_p(u_p - \{H(e, \xi_0)\}|u_s) + F_R \right]
\]

Here: \( \{H(e, \xi_0)\} = [0 \ 1 \ 0]_{1 \times 3}[N^e(\xi_0)]_{3 \times 6} \{Q\}_{6 \times n} \). Where \([N^e(\xi_0)]_{3 \times 6} \) is the shape function, assuming a plane structural beam Euler-Bernoulli finite element; \([Q]_{6 \times n} \) is used in order to evaluate only the displacements in the vertical direction at the contact point; \(u_p, \dot{u}_p\) are the displacements and velocities related to the interaction between pedestrian and flexible structure; \(u_s\) and \(\dot{u}_s\) are the displacements and velocities related to structure; \(F_R\) is the pedestrian vertical reaction force, measured in a rigid structure.

### 3.2.1. Linking the biodynamic model to the structural equations of motion

Equations (2), (3) and (4) represent the matrices of the coupled system (pedestrian and structure), that takes into account mass, damping and stiffness respectively.

\[
[M]_{n+1 \times n+1} = \begin{bmatrix} [M]_{n \times n} & \{0\}_{n \times 1} \\ \{0\}^T_{1 \times n} & m_p \end{bmatrix}
\]

\[
[C(t)]_{n+1 \times n+1} = \begin{bmatrix} [C]_{n \times n} + C^*(i)_{n \times n} & -[H(e, \xi_0)]^T \{c_p\} \\ -[H(e, \xi_0)]c_p & c_p \end{bmatrix}
\]

\[
[K(t)]_{n+1 \times n+1} = \begin{bmatrix} [K]_{n \times n} + K^*(i)_{n \times n} & -[H(e, \xi_0)]^T \{k_p\} \\ -[H(e, \xi_0)]k_p & k_p \end{bmatrix}
\]

One way to solve the time domain Equation of motion is by using direct numerical integration. In this paper, the Newmark integration scheme (Bathe [17]) is used in order to evaluate the structural response considering all the structural vibration modes.

Toso and Gomes [4] showed that considering a three-dimensional truss element, the human-structure interaction model can be evaluated using the Equation (5):

\[
\begin{bmatrix} [M] & \{0\} & \{\dot{u}_s\} \\ \{0\} & \text{diag}(m_p) & \{\ddot{u}_p\} \end{bmatrix} \{\dddot{u}_s\} + \begin{bmatrix} [C] + [H]^T \text{diag}(c_p) [H] \\ \text{diag}(c_p) [H] \end{bmatrix} \{\dddot{u}_p\} + \begin{bmatrix} [K] + [H]^T \text{diag}(k_p) [H] \\ \text{diag}(k_p) [H] \end{bmatrix} \{\dddot{u}_p\} = \begin{bmatrix} \{F(t)\} - \{F_R(t)\} \\ \{0\} \end{bmatrix}
\]

where \(\text{diag}(m_p), \text{diag}(c_p)\) and \(\text{diag}(k_p)\) will contain the mass, damping and stiffness parameters of each of the pedestrians.

### 4. Numerical and experimental results

The analyzed structure is located in the city of Brasília, Brazil. It is a composite footbridge (34.08 m in length, 2.4 m in width and 2.25 m in height). The footbridge roof is built with a curved reinforced concrete shell. The deck floor consists of reinforced concrete planks that are simply supported on the truss members that link the left and right side of the footbridge. The handrails consist of hollow tubular steel bars. Additional information, design details, materials properties, etc. can be found in Brasiliano et al. [18]. Figure 3 shows the analyzed footbridge.
4.1. Numerical model results

In this topic, a dynamic analysis is performed to investigate the footbridge dynamic behaviour under a pedestrian crowd (0.25 pedestrians/m²). This analysis consider: (a) a Biodynamic Synchronized Coupled Model (BSCM) that considers an actuator in the vertical direction of each pedestrian, and a fully synchronized force model as mechanisms for the willingness of walk, in a number that represents some crowd; (b) a Fully Synchronized Force model (FSFM) to vertical, longitudinal and lateral directions. The two models use the dynamic load factors proposed to SETRA Guideline [11] for force estimation. Both models assume the entry of the pedestrians on the footbridge is randomly generated, considering the effective width of the structure and keeping the specified crowd densities. Pedestrians and their characteristics are represented using random variables, following a Gaussian distribution based on average parameters, coefficients of variation and a correlation matrix of experimental data. Monte Carlo method is used to obtain the pedestrian’s kinematic parameters, which results in different pacing frequency, step length, step width etc. for each individual (Toso et al. [3]). Regarding to biodynamic parameters, Toso et al. [19] proposed a single degree of freedom (SDOF) biodynamic model to obtain the following parameters: mass (m), damping (c) and stiffness (k). In this paper, those equations were used to corresponding to the data of the analysed crowd. These biodynamic parameters are used in Equation 5.

Individuals have different characteristics, for instance height, mass, pacing rate etc. According to Toso et al. [3], the crowd characteristics are modelled based on mean value and standard deviation of the group of pedestrians being modelled. Each random variable follows a correlated Gaussian distribution based on experimentally obtained data. So, for the number of pedestrians that are to be modelled, samples are generated based on a correlated Gaussian distribution for these parameters. Besides, it is also reasonable to assume that for a specific individual, there are some trends in the kinetic and kinematic parameters such as, body mass, step length, step width, pacing rate, etc., that justifies the use of a correlation matrix (Toso et al. [3]).

Figure 4 presents the results of the mid-span accelerations in the longitudinal, vertical and lateral directions. This numerical simulation uses the Biodynamic Synchronized Coupled Model (BSCM). This model is couple to the structure in the vertical degree of freedom. In other directions remaining the fully synchronized force model. This analysis assumes that the three force components are completely synchronized (time and space), and uses the kinetic and kinematic parameters for the pedestrians. A pedestrian crowd of 0.25 pedestrians/m² is use in this simulation.
Figure 4. Acceleration response for BSCM and crowd density 0.25 ped/m²:

a) longitudinal, b) vertical and c) lateral directions.

Figure 5 shows the results of mid-span acceleration for longitudinal, vertical and lateral direction, considering just the Fully Synchronized Force Model (FSFM) in three directions. As in the previous example, there is no synchronism between pedestrians, that have their own pacing rates, and crosses the footbridge only once.
Figure 5. Acceleration response for FSFM and crowd density 0.25 ped/m²:
a) longitudinal; b) vertical; c) lateral direction.

The numerical results presented in Figures 4 and 5 shows natural frequencies to around 2.0 Hz. These frequencies are associated with pedestrians’ pacing rates. A frequency about 3.91 Hz corresponds to the natural frequency of the first vertical bending mode of the structure that was excited by the pedestrian crowd. Furthermore, the fundamental frequency of the structure (frequency about 3.0 Hz which corresponds to the lateral bending mode) was also excited by the pedestrian crowd, using both models. Considering the lateral and longitudinal directions the previous results shows frequencies around 11.0 and 13.0 Hz. These frequencies are excited due to the presence of the pedestrian thrust and deceleration phases in the human step and the zig-zag pattern in the force application. These frequencies are not excited if Design Codes (simplified force models) are used, because Design Codes assumes that the forces acting in a straight line along the direction of walking at a constant speed. Furthermore, the torsional mode (6.20 Hz) was also excited using both models, due to the spatiality of application of the pedestrian load that produces torsion in the footbridge. These frequencies, again, will not appear if simplified force models be used, since this model assumes the application of the load along a straight line.

Table 1 shows the RMS acceleration (footbridge’s mid-span response) obtained with the two models (FSFM and BSCM). The acceleration was obtained in longitudinal, vertical and lateral directions. These results show BSCM presented lower acceleration values than FSFM in all directions.

Table 1. RMS acceleration response for a density of 0.25 pedestrians/m².

| Model  | Longitudinal | Vertical | Lateral |
|--------|--------------|----------|---------|
| FSFM   | 0.0026       | 0.0526   | 0.0158  |
| BSCM   | 0.0019       | 0.0394   | 0.0070  |

* FSFM: Fully Synchronized Force model; BSCM: Biodynamic Synchronized Coupled Model.

Regarding the influence of the BSCM, considering a crowd density of 0.25 ped./m², it is noted that both models (Biodynamic Synchronized Coupled Model and Fully Synchronized Force model) presented a fundamental frequency about 3.91 Hz, i.e., there was no modification in the structural fundamental frequency. Additional simulations (Table 2) show that for the crowd density of 0.50 and 0.75 ped./m² the use of FSFM still results in structural fundamental frequencies at 3.91 Hz, as expected. However, the BSCM presents a decrease in the structural fundamental frequency, according to Table 2, proving the previous trend. It is concluded that this is due to increase in overall mass as the crowd density increases. Maybe such decrease in natural frequency was not observed in a crowd density of 0.25 ped./m² due to a low number of pedestrians.

Table 2. Structural fundamental frequency (Hz).

| Model  | Crowd Density (ped./m²) |
|--------|-------------------------|
|        | 0.25       | 0.50    | 0.75    |
| FSFM   | 3.91       | 3.91    | 3.91    |
| BSCM   | 3.91       | 3.61    | 3.47    |

* FSFM: Fully Synchronized Force model; BSCM: Biodynamic Synchronized Coupled Model.
4.1. Experimental results

The results presented in Figure 6 correspond to the investigated pedestrian density (0.25 ped./m²). In a previous publication, Brito et al. [20] measured the vertical acceleration of this footbridge at the mid-span. In this case and crowd density, the subjects crossed the structure freely, maintaining their own pacing rates.

Figure 6. Mid-span vertical acceleration: experimental measurement (0.25 ped./m²).

Both models BSCM and FSFM presents a similar shape for the vertical acceleration. The results for BSCM from Figure 4(b) shows a peak acceleration of 0.15 m/s². Using the FSFM (Figure 5b) the acceleration peaks are 0.20 m/s² and the experimental values are about 0.40 m/s². One can note that these numerical results are underestimating actual peak vibration. In terms of vertical RMS acceleration, the values became closer to the experimental ones (0.068 m/s²), with FSFM resulting in 0.0523 m/s² and BSCM 0.0394 m/s². It should be emphasized that this comparison was harmed because most of the BSCM parameters were not available and the experimental campaign was performed previous to the development of the BSCM.

Figure 7 shows the spectrogram for the corresponding mid-span vertical acceleration: experimental measurement. This shows how the natural frequency varies along time according to the mass variation due to crowd flow. One can notice that the main lower frequency only happens during the crowd entrance in the footbridge (approximately from 6 s to 35 s). The second main frequency happens along all the experiment.

Figure 7. Spectrogram for the mid-span vertical acceleration: experimental measurement (0.25 ped./m²).
5. Conclusions

In this paper, it is used a biodynamic model with synchronization of the three force components in space and in time with a stiffness-mass-damper model coupled with the structure’s FEM to assess the pedestrian structure interaction. Both models used here (Biodynamic Synchronized Coupled Model and Fully Synchronized Force Model) underestimated the peak vertical acceleration values when compared to the experimental ones and this suggest other sources of uncertainty may affect the final behavior. The effect of the BSCM in other simulation for higher crowd densities (0.50 and 0.75 ped./m²) could be noticed since the simulated structure presented modifications in the dynamic behavior as a decrease in first natural frequency and increase in the overall damping as the crowd density increases. This suggests the human body effectively dissipates part of this vibration energy. The results of the dynamic analysis using FSFM rendered larger RMS accelerations, mainly in the lateral and longitudinal direction if compared with the BSCM. Additional frequencies present in both spectral acceleration in the lateral and longitudinal directions can be attributed to the use of the synchronized force model, used in both models. The observed differences in spectral accelerations in the vertical direction are attributed to the biodynamic effect that is present in BSCM and not in the FSFM.

Regarding to natural frequencies of footbridge, the vertical fundamental frequency of 3.91 Hz was identified experimentally and numerically. This is a relatively high frequency when considering structures with similar spans according to CEB [21]. Possibly, this characteristic is a consequence of the balance of stiffness and mass obtained when using a composite footbridge (steel truss and concrete floor). Such characteristics are desirable, avoiding that the structural fundamental frequency be a value close to the frequencies of excitation of the first harmonic of the pedestrian walking. Considering the lateral fundamental frequency, experimentally, the frequency of 3.12 Hz was obtained. This value is far from the frequency range considered to be critical, which are values close to 1.0 Hz. As for the vertical structural acceleration, the footbridge has vibration amplitudes below the values set by standards for comfort limits, which are of the order of 0.5 to 0.7 m / s² (OHBDC [22]; BS-5400 [23]; Eurocode 5 [24] and SETRA Guideline [11]). Numerically, it was observed that the excitation range of the second harmonic, excited by some pedestrians, are frequency values very close to the vertical structural fundamental frequency. However, the results of numerical simulations with different pedestrian’s densities, walking on the footbridge indicate that it is not subject to excessive vibrations.

References

[1] Dallard P, Fitzpatrick AJ, Flint A, Le Bourva S, Low A, Ridsdill-Smith RM, et al. The London millennium footbridge. J Struct Eng 2001; 79 (22), 17-33.
[2] ISO 2631-2. International Organization for Standardization. Evaluation of human exposure to whole-body vibration, Part 2: human exposure to continuous and shockinduced vibrations in buildings (1 to 80 Hz), 1989.
[3] Toso MA, Gomes HM, de Brito JLV. Crowd-structure interaction: investigating the spatiality and synchronization of a pedestrian force model. Int J Constr Steel Res 2017;133: 510-21.
[4] Toso MA, Gomes HM. A coupled biodynamic model for crowd-footbridge interaction. Engineering Structures 2018; 177: 47-60.
[5] Zivanovic S, Pavic A, Reynolds P. Vibration serviceability of footbridges under human-induced excitation: a literature review. J Sound Vib 2005; 279: 1-74.
[6] Stoyanoff S, Hunter M. Footbridges: Pedestrian induced vibrations. Rowan Williams Davies and Irwin Inc, Ontario, Canada, 2003.
[7] Bodgi J, Eriecher S, Argoul P. Lateral vibration of footbridges under crowd-loading: Continuous crowd modeling approach. Key Engineering Materials 2009; 347, 685-690.
[8] Wheeler JE. Prediction and Control of Pedestrian Induced Vibration in Footbridges. J Structural Division 1982; 108, n° ST9, 2045-2065.
[9] Qin JW, Law SS, Yang QS, Yang N. Pedestrian-bridge dynamic interaction, including human participation. J. Sound Vib 2013; 332, 1107-1124.
[10] Tubino F. Probabilistic assessment of the dynamic interaction between multiple pedestrians and vertical vibrations of footbridges. J Sound Vib 2018; 417:80–96.

[11] SETRA-Service d’Études techniques des routes et autoroutes. Footbridges. Assessment of vibrational behaviour of footbridges under pedestrian loading, 2006.

[12] ISO 10137, International Organization for Standardization. Bases for Design of Structures-Serviceability of Buildings and Walkways against Vibrations, 2012.

[13] Harper FC. The mechanics of walking. Research Applied in Industry 1962; 15 (1), 23-28.

[14] Galbraith FW, Barton MV. Ground loading from footsteps. Journal of the Acoustic Society of America 1970; 48,1288-1292.

[15] Blanchard J, Davies BL, Smith JW. Design criteria and analysis for dynamic loading of footbridges. In: Proceedings of the DOE and DOT TRRL Symposium on Dynamic Behaviour of Bridges, Crowthorne, England, 1977, 90-106.

[16] Kerr SC. Human Induced Loading on Staircases. PhD Thesis (Mechanical Engineering Department) University College London, London, 1998.

[17] Bathe KJ. Finite element procedures. 2nd ed. New Jersey: Prentice-Hall; 1996.

[18] Brasiliano A, Doz G, Brito JLV, Pimentel RL. Role of non-metallic components on the dynamic behaviour of composite footbridges. In: Proceedings of the 3rd international conference footbridge; 2008.

[19] Toso MA, Gomes HM, da Silva FT, Pimentel RL. Experimentally fitted biodynamic models for pedestrian-structure interaction in walking situations. Int J Mech Syst Signal Process 2016; 72-73: 590-606.

[20] Brito JLV, Doz GN, Ávila SM, Pimentel RL, Brasiliano A, Roitman N, et al. Evaluation of the dynamic behaviour of a composite footbridge located in the city of Brasilia-Brazil. In: XXXIV Jornadas Sul Americanas de Engenharia Estrutural; 2010 [in Portuguese].

[21] CEB, Comite Euro-International Du Beton. Bulletin D’Information N° 209. Vibrations Problems in Structures. Practical Guidelines, Lausanne, Switzerland, August, 1991.

[22] OHBDC, Ontario Highway Bridge Design Code, Highway Engineering Division, Ministry of Transportation and Communication, Ontario, Canada, 1991.

[23] BS 5400, British Standards Association. Steel, Concrete and Composite Bridges - Part 2: Specification for Loads; Appendix C: Vibration Serviceability Requirements for Foot and Cycle Track Bridges, London, 1978.

[24] EUROCODE 5-2. Design of Timber Structures – Part 2: Bridges, ENV 1995-2, European Committee for Standardization, Brussels, Belgium, 1997.